Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/1.2.1.2-d+e-x-^m-a+b-x+c-x^2-^p

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [2590]. This is test number [33].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (2590)	% 0. (0)
Mathematica	% 99.77 (2584)	% 0.23 (6)
Maple	% 89.58 (2320)	% 10.42 (270)
Maxima	% 43.05 (1115)	% 56.95 (1475)
Fricas	% 79.73 (2065)	% 20.27 (525)
Sympy	% 40.27 (1043)	% 59.73 (1547)
Giac	% 61.47 (1592)	% 38.53 (998)

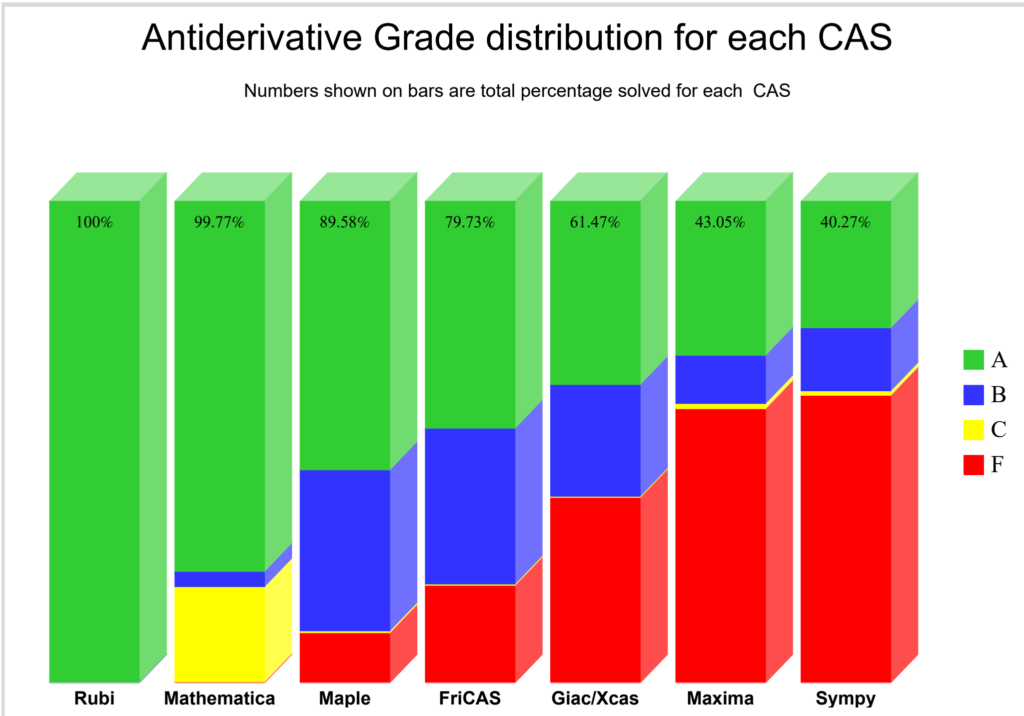
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

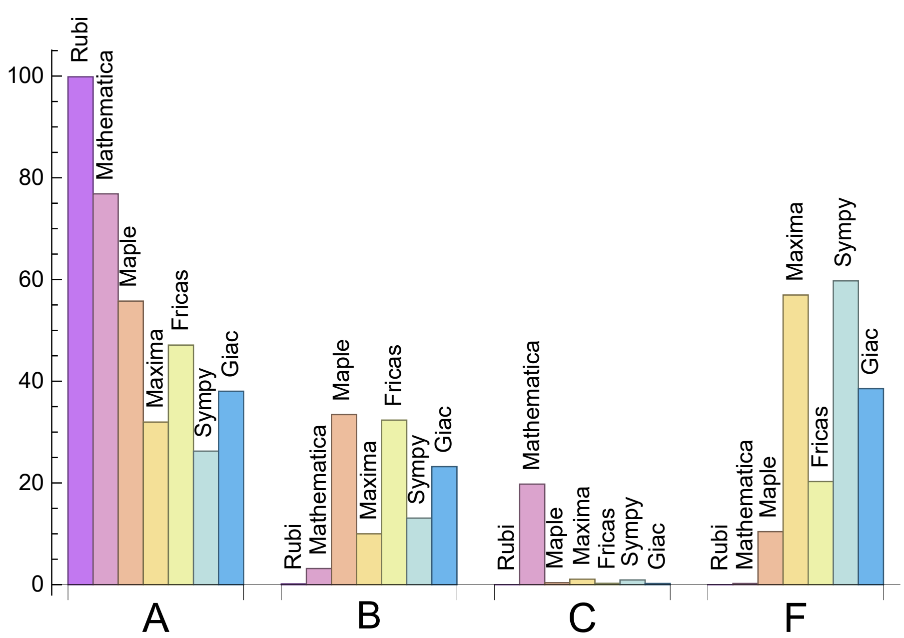
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.85	0.15	0.	0.
Mathematica	76.83	3.17	19.77	0.23
Maple	55.75	33.44	0.39	10.42
Maxima	31.97	10.	1.08	56.95
Fricas	47.1	32.36	0.27	20.27
Sympy	26.25	13.09	0.93	59.73
Giac	38.03	23.2	0.23	38.53

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.22	169.74	1.01	118.5	1.
Mathematica	0.53	203.62	1.03	90.	0.91
Maple	0.11	711.64	2.57	138.	1.29
Maxima	1.2	203.9	2.09	117.	1.61
Fricas	4.1	1043.27	5.92	375.	4.08
Sympy	10.52	381.95	3.24	146.	1.71
Giac	1.48	426.52	2.97	183.	1.95

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {70, 309, 310, 311, 313, 314, 329, 446, 447, 448, 449, 450, 699, 703, 704, 705, 709, 710, 711, 712, 713, 714, 715, 716, 717, 720, 728, 729, 730, 731, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 951, 952, 953, 973, 2447, 2448, 2449, 2452, 2453, 2455, 2456, 2457, 2458, 2459, 2460, 2476, 2480, 2481, 2493, 2494, 2495, 2496, 2500, 2501, 2502, 2506, 2507, 2508, 2509, 2520, 2521, 2532, 2533, 2534, 2546, 2547, 2557, 2558, 2559, 2560, 2561, 2562, 2563, 2564, 2565, 2567, 2568, 2569, 2570, 2571, 2572, 2573, 2574, 2575, 2582, 2584, 2585, 2587, 2588, 2590}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima abs_integrate was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

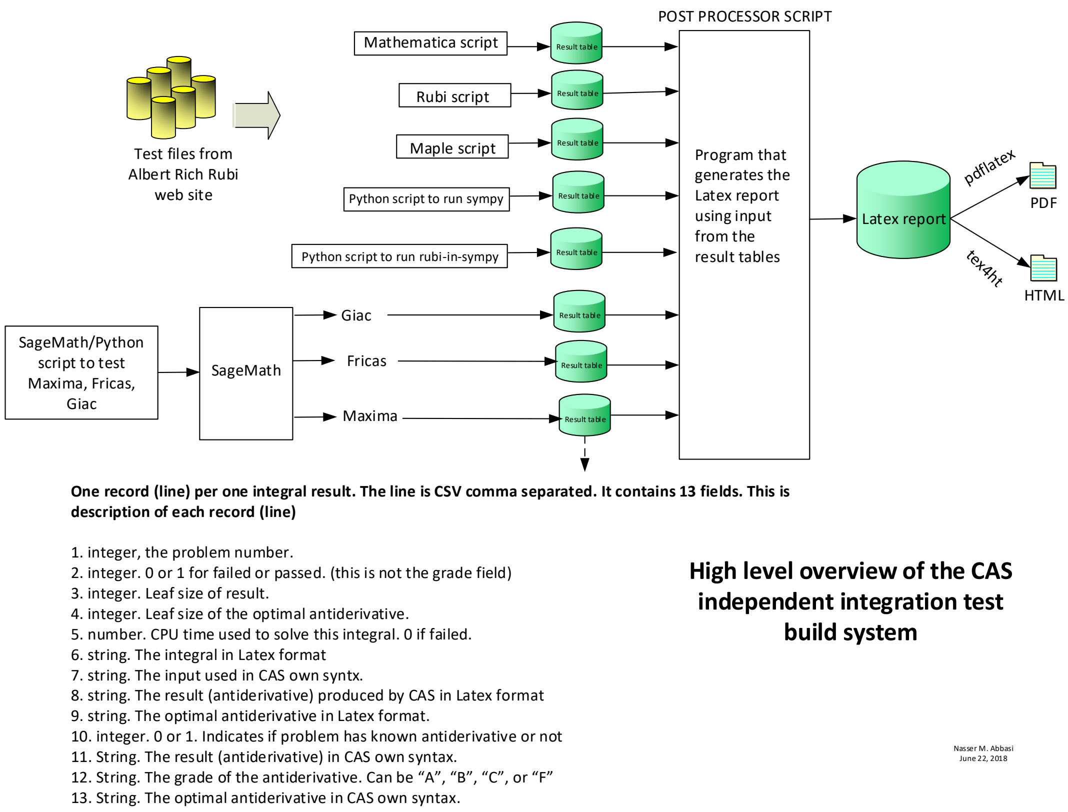
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820,

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B grade: { 2419, 2420, 2421, 2422 }

C grade: { }

F grade: { }

2.1.2 Mathematica

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B grade: { 46, 175, 306, 312, 318, 428, 430, 743, 744, 745, 821, 823, 949, 950, 960, 967, 968, 1109, 1121, 1122, 1125, 1137, 1138, 1139, 1141, 1150, 1451, 1463, 1464, 1465, 1466, 1467, 1468, 1475, 1476, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1497, 1498, 1499, 1500, 1526, 1529, 1537, 1564, 1581, 1582, 1607, 1759, 1769, 1770, 1783, 1784, 1785, 1794, 1828, 1829, 1839, 1851, 1861, 1918, 1919, 1931, 1932, 1933, 1934, 1935, 2202, 2212, 2214, 2221, 2302, 2304, 2482, 2549, 2589 }

C grade: { 17, 18, 30, 31, 32, 52, 53, 60, 61, 79, 80, 81, 89, 91, 92, 93, 100, 101, 108, 109, 110, 111, 329, 366, 367, 368, 375, 376, 377, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 431, 432, 433, 616, 617, 622, 623, 629, 630, 636, 637, 648, 649, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 720, 900, 907, 909, 910, 917, 923, 924, 925, 930, 931, 932, 937, 938, 939, 940, 951, 952, 953, 973, 1199, 1201, 1208, 1209, 1210, 1213, 1215, 1223, 1224, 1225, 1226, 1227, 1230, 1232, 1246, 1248, 1256, 1258, 1292, 1293, 1294, 1296, 1298, 1300, 1302, 1304, 1305, 1306, 1307, 1309, 1311, 1313, 1315, 1317, 1318, 1319, 1321, 1323, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1416, 1417, 1418, 1419, 1645, 1646, 1647, 1648, 1651, 1652, 1653, 1654, 1655, 1656, 1658, 1659, 1660, 1661, 1662, 1663, 1664, 1665, 1666, 1668, 1669, 1670, 1671, 1672, 1673, 1674, 1675, 1708, 1709, 1710, 1711, 1712, 1713, 1715, 1716, 1717, 1718, 1719, 1720, 1721, 1722, 1724, 1725, 1726, 1727, 1728, 1729, 1730, 1925, 1939, 1940, 1954, 1955, 1956, 1964, 1965, 1966, 1974, 1975, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2015, 2016, 2017, 2018, 2019, 2020, 2022, 2023, 2024, 2025, 2026, 2034, 2035, 2042, 2044, 2045, 2052, 2053, 2055, 2056, 2063, 2064, 2068, 2069, 2070, 2071, 2072, 2075, 2076, 2077, 2078, 2079, 2080, 2083, 2309, 2310, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2328, 2329, 2440, 2441, 2442, 2443, 2444, 2445, 2446, 2447, 2448, 2449, 2450, 2451, 2452, 2453, 2454, 2455, 2456, 2457, 2458, 2459, 2460, 2461, 2462, 2463, 2464, 2465, 2466, 2467, 2468, 2469, 2470, 2471, 2472, 2473, 2474, 2475, 2476, 2477, 2478, 2479, 2480, 2481, 2483, 2484, 2485, 2489, 2490, 2491, 2492, 2496, 2497, 2498, 2499, 2500, 2501, 2502, 2503, 2504, 2505, 2506, 2507, 2508, 2509, 2516, 2517, 2518, 2519, 2520, 2521, 2528, 2529, 2530, 2531, 2532, 2533, 2534, 2542, 2543, 2544, 2545, 2546, 2547, 2563, 2564, 2565, 2582, 2583, 2584, 2585, 2586, 2587, 2588, 2590 }

F grade: { 727, 2486, 2487, 2488, 2555, 2556}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 113, 114, 137, 138, 139, 140, 141, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 165, 166, 167, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 268, 269, 270, 271, 272, 273, 274, 277, 279, 280, 281, 282, 283, 284, 287, 288, 298, 305, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 322, 323, 324, 325, 330, 331, 332, 333, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 364, 365, 366, 367, 368, 373, 374, 375, 376, 377, 381, 383, 384, 385, 389, 409, 410, 411, 417, 432, 434, 435, 436, 437, 442, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 519, 520, 521, 522, 525, 526, 527, 528, 534, 535, 536, 537, 545, 546, 547, 548, 558, 559, 560, 561, 562, 563, 564, 569, 570, 571, 572, 577, 578, 579, 580, 581, 585, 586, 587, 588, 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1882, 1883, 1884, 1888, 1889, 1890, 1891, 1892, 1893, 1894, 1895, 1899, 1901, 1902, 1903, 1904, 1905, 1906, 1911, 1912, 1914, 1915, 1916, 1917, 1927, 1928, 1929, 1930, 1942, 1943, 1944, 1945, 1947, 1948, 1949, 1950, 1951, 1952, 1953, 1958, 1959, 1960, 1961, 1962, 1963, 1967, 1968, 1969, 1970, 1971, 1972, 1973, 1976, 1977, 1978, 1979, 1980, 1981, 1982, 1983, 1984, 1985, 1986, 1987, 1988, 1989, 1990, 1991, 1992, 1993, 1994, 1995, 1996, 1997, 1998, 1999, 2003, 2004, 2005, 2006, 2007, 2008, 2012, 2013, 2014, 2015, 2016, 2017, 2021, 2022, 2023, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038, 2039, 2040, 2041, 2043, 2046, 2047, 2048, 2049, 2050, 2057, 2058, 2059, 2060, 2061, 2062, 2063, 2064, 2065, 2066, 2067, 2068, 2069, 2070, 2071, 2073, 2074, 2075, 2076, 2081, 2082, 2086, 2101, 2102, 2103, 2105, 2106, 2108, 2109, 2110, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2118, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2127, 2128, 2129, 2130, 2132, 2133, 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B grade: { 17, 18, 29, 30, 31, 32, 62, 112, 158, 168, 175, 249, 267, 275, 276, 278, 285, 286, 289, 290, 291, 292, 293, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 306, 307, 308, 319, 320, 321, 326, 327, 328, 329, 334, 335, 361, 362, 363, 369, 370, 371, 372, 378, 379, 380, 382, 386, 387, 388, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 412, 413, 414, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 433, 440, 441, 517, 518, 523, 524, 529, 530, 531, 532, 533, 538, 539, 540, 541, 542, 543, 544, 549, 550, 551, 552, 553, 554, 555, 556, 557, 565, 566, 567, 568, 573, 574, 575, 576, 582, 583, 584, 612, 613, 614, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 648, 649, 652, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 721, 722, 771, 782, 792, 793, 794, 804, 805, 806, 807, 808, 809, 810, 848, 849, 899, 915, 916, 927, 929, 945, 978, 979, 987, 988, 989, 990, 1029, 1039, 1040, 1041, 1050, 1051, 1052, 1053, 1054, 1107, 1109, 1110, 1112, 1121, 1122, 1123, 1124, 1125, 1132, 1137, 1138, 1139, 1140, 1141, 1142, 1144, 1148, 1150, 1154, 1155, 1156, 1157, 1158, 1166, 1167, 1168, 1177, 1178, 1179, 1180, 1181, 1182, 1185, 1187, 1188, 1189, 1191, 1193, 1195, 1196, 1197, 1199, 1201, 1203, 1205, 1207, 1208, 1209, 1210, 1211, 1213, 1215, 1218, 1220, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1230, 1232, 1234, 1238, 1240, 1242, 1246, 1250, 1252, 1256, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1372, 1374, 1375, 1379, 1380, 1383, 1384, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1423, 1424, 1451, 1452, 1459, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1515, 1516, 1517, 1518, 1519, 1525, 1526, 1527, 1528, 1529, 1530, 1537, 1538, 1554, 1564, 1569, 1570, 1571, 1572, 1573, 1574, 1576, 1577, 1578, 1579, 1581, 1582, 1583, 1596, 1602, 1603, 1604, 1605, 1607, 1612, 1613, 1614, 1637, 1638, 1639, 1640, 1641, 1642, 1643, 1644, 1645, 1646, 1647, 1648, 1654, 1655, 1656, 1661, 1664, 1665, 1666, 1667, 1668, 1673, 1674, 1675, 1703, 1704, 1711, 1712, 1713, 1718, 1719, 1720, 1721, 1722, 1723, 1724, 1725, 1726, 1727, 1728, 1729, 1730, 1731, 1732, 1733, 1737, 1738, 1744, 1759, 1760, 1769, 1770, 1771, 1778, 1783, 1784, 1785, 1786, 1787, 1788, 1791, 1792, 1793, 1794, 1795, 1799, 1800, 1810, 1811, 1818, 1819, 1820, 1828, 1829, 1830, 1839, 1840, 1847, 1851, 1852, 1853, 1854, 1855, 1861, 1862, 1874, 1875, 1885, 1886, 1887, 1896, 1897, 1898, 1900, 1907, 1908, 1909, 1910, 1913, 1918, 1919, 1920, 1921, 1922, 1923, 1924, 1925, 1926, 1931, 1932, 1933, 1934, 1935, 1936, 1937, 1938, 1939, 1940, 1941, 1946, 1954, 1955, 1956, 1957, 1964, 1965, 1966, 2000, 2001, 2002, 2009, 2010, 2011, 2018, 2019, 2020, 2042, 2044, 2045, 2051, 2052, 2053, 2054, 2055, 2056, 2072, 2077, 2078, 2079, 2080, 2084, 2085, 2104, 2107, 2126, 2131, 2136, 2137, 2138, 2139, 2140, 2141, 2142, 2146, 2151, 2152, 2153, 2154, 2155, 2156, 2157, 2158, 2159, 2160, 2161, 2162, 2182, 2183, 2184, 2188, 2189, 2190, 2191, 2192, 2193, 2196, 2197, 2198, 2199, 2200, 2201, 2202, 2203, 2204, 2207, 2208, 2209, 2210, 2211, 2212, 2213, 2214, 2215, 2216, 2217, 2219, 2220, 2221, 2222, 2223, 2224, 2225, 2227, 2228, 2235, 2289, 2290, 2291, 2293, 2294, 2295, 2296, 2297, 2298, 2299, 2300, 2301, 2302, 2303, 2304, 2305, 2306, 2309, 2310, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2328, 2329, 2330, 2331, 2333, 2334, 2335, 2337, 2338, 2339, 2340, 2341, 2342, 2343, 2344, 2345, 2346, 2347, 2348, 2349, 2350, 2351, 2352, 2353, 2354, 2355, 2356, 2357, 2358, 2359, 2360, 2361, 2362, 2363, 2373, 2374, 2378, 2379, 2380, 2381, 2382, 2383, 2384, 2387, 2388, 2389, 2390, 2391, 2392, 2393, 2394, 2397, 2398, 2399, 2408, 2440, 2441, 2442, 2443, 2444, 2445, 2446, 2447, 2448, 2449, 2450, 2451, 2452, 2453, 2454, 2455, 2456, 2457, 2458, 2459, 2460, 2461, 2462, 2463, 2464, 2466, 2467, 2468, 2469, 2470, 2471, 2472, 2473, 2474, 2475, 2476, 2477, 2478, 2479, 2480, 2481, 2482, 2549, 2550, 2551 }

C grade: { 142, 143, 700, 701, 702, 706, 707, 708, 1548, 1549 }

F grade: { 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 438, 439, 443, 444, 445, 446, 447, 448, 449, 450, 634, 635, 646, 647, 699, 703, 704, 705, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 930, 931, 932, 937, 938, 939, 940, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 1412, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1437, 1439, 1441, 1442, 1443, 1444, 1445, 1446, 1734, 1735, 1736, 1740, 1741, 1742, 1743, 1748, 1749, 1750, 1751, 1752, 1753, 1754, 1755, 1756, 1974, 1975, 2083, 2087, 2088, 2089, 2090, 2091, 2092, 2093, 2094, 2095, 2096, 2097, 2098, 2099, 2100, 2483, 2484, 2485, 2486, 2487, 2488, 2489, 2490, 2491, 2492, 2493, 2494, 2495, 2496, 2497, 2498, 2499, 2500, 2501, 2502, 2503, 2504, 2505, 2506, 2507, 2508, 2509, 2510, 2511, 2512, 2513, 2514, 2515, 2516, 2517, 2518, 2519, 2520, 2521, 2522, 2523, 2524, 2525, 2526, 2527, 2528, 2529, 2530, 2531, 2532, 2533, 2534, 2535, 2536, 2537, 2538, 2539, 2540, 2541, 2542, 2543, 2544, 2545, 2546, 2547, 2548, 2553, 2554, 2555, 2556, 2557, 2558, 2559, 2560, 2561, 2562, 2563, 2564, 2565, 2566, 2567, 2568, 2569, 2570, 2571, 2572, 2573, 2574, 2575, 2576, 2577, 2578, 2579, 2580, 2581, 2582, 2583, 2584, 2585, 2586, 2587, 2588, 2589, 2590 }

2.1.4 Maxima

A grade: { 39, 40, 41, 55, 56, 64, 65, 66, 69, 70, 71, 72, 73, 74, 75, 76, 82, 83, 84, 85, 86, 94, 95, 96, 97, 112, 113, 114, 181, 182, 183, 184, 185, 190, 191, 192, 193, 194, 198, 199, 200, 202, 203, 204, 208, 209, 210, 211, 212, 213, 214, 215, 216, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 267, 268, 269, 270, 271, 272, 273, 276, 277, 278, 279, 280, 281, 282, 309, 310, 311, 313, 314, 324, 325, 333, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 441, 442, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 558, 559, 560, 572, 579, 580, 581, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 650, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 788, 789, 790, 800, 801, 802, 819, 820, 821, 822, 823, 824, 826, 827, 828, 829, 830, 836, 837, 840, 841, 852, 853, 854, 860, 861, 862, 863, 864, 865, 869, 870, 871, 872, 873, 879, 880, 881, 882, 886, 887, 888, 889, 926, 928, 977, 980, 981, 982, 983, 984, 985, 991, 992, 993, 994, 995, 996, 999, 1000, 1001, 1003, 1004, 1008, 1009, 1010, 1011, 1012, 1018, 1019, 1020, 1021, 1022, 1031, 1041, 1052, 1065, 1066, 1067, 1068, 1069, 1070, 1074, 1075, 1076, 1077, 1084, 1085, 1091, 1094, 1095, 1096, 1097, 1098, 1099, 1103, 1104, 1105, 1106, 1107, 1108, 1111, 1112, 1113, 1114, 1115, 1116, 1124, 1125, 1126, 1127, 1128, 1129, 1141, 1142, 1143, 1144, 1145, 1146, 1157, 1159, 1161, 1162, 1164, 1167, 1169, 1171, 1173, 1174, 1180, 1182, 1186, 1194, 1206, 1221, 1237, 1245, 1255, 1260, 1261, 1262, 1263, 1265, 1266, 1267, 1268, 1269, 1270, 1272, 1273, 1274, 1275, 1276, 1277, 1279, 1280, 1281, 1282, 1283, 1284, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1438, 1447, 1448, 1449, 1450, 1453, 1454, 1455, 1456, 1457, 1458, 1460, 1461, 1470, 1471, 1472, 1473, 1506, 1507, 1508, 1509, 1510, 1511, 1517, 1518, 1520, 1531, 1538, 1539, 1540, 1541, 1542, 1589, 1590, 1591, 1598, 1599, 1600, 1608, 1609, 1610, 1614, 1615, 1616, 1617, 1618, 1619, 1620, 1621, 1622, 1623, 1624, 1625, 1626, 1627, 1628, 1629, 1630, 1631, 1632, 1634, 1635, 1636, 1676, 1677, 1678, 1679, 1680, 1681, 1682, 1683, 1684, 1685, 1686, 1687, 1688, 1689, 1690, 1691, 1693, 1694, 1695, 1696, 1697, 1698, 1699, 1700, 1701, 1702, 1738, 1739, 1744, 1745, 1746, 1747, 1758, 1761, 1762, 1763, 1764, 1765, 1766, 1767, 1768, 1772, 1773, 1774, 1775, 1776, 1777, 1779, 1780, 1786, 1790, 1791, 1792, 1793, 1800, 1801, 1802, 1803, 1804, 1805, 1806, 1810, 1811, 1812, 1813, 1814, 1815, 1819, 1820, 1821, 1822, 1823, 1831, 1832, 1833, 1834, 1835, 1836, 1837, 1838, 1840, 1841, 1842, 1843, 1844, 1845, 1846, 1848, 1849, 1850, 1853, 1854, 1855, 1857, 1858, 1859, 1860, 1863, 1864, 1865, 1866, 1867, 1868, 1869, 1870, 1871, 1874, 1875, 1876, 1877, 1878, 1879, 1880, 1881, 1885, 1886, 1887, 1888, 1889, 1890, 1891, 1896, 1897, 1898, 1899, 1901, 1976, 1977, 1979, 1980, 1981, 1982, 1983, 1984, 1986, 1987, 1988, 1989, 1990, 1991, 1992, 1994, 1995, 1996, 1997, 1998, 1999, 2027, 2028, 2029, 2030, 2031, 2036, 2037, 2038, 2039, 2040, 2046, 2047, 2048, 2049, 2050, 2057, 2058, 2059, 2060, 2065, 2066, 2067, 2073, 2074, 2105, 2106, 2108, 2109, 2110, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2118, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2130, 2131, 2132, 2133, 2134, 2135, 2136, 2137, 2138, 2139, 2140, 2141, 2142, 2143, 2144, 2145, 2146, 2147, 2148, 2149, 2150, 2151, 2152, 2153, 2163, 2164, 2165, 2166, 2167, 2168, 2169, 2170, 2171, 2172, 2173, 2174, 2175, 2176, 2177, 2178, 2179, 2180, 2181, 2229, 2230, 2231, 2232, 2233, 2234, 2235, 2236, 2237, 2238, 2239, 2240, 2241, 2242, 2243, 2244, 2245, 2246, 2247, 2248, 2249, 2250, 2251, 2252, 2253, 2254, 2255, 2256, 2257, 2258, 2259, 2260, 2261, 2262, 2263, 2264, 2265, 2266, 2267, 2268, 2269, 2270, 2271, 2272, 2273, 2274, 2275, 2276, 2277, 2278, 2279, 2280, 2281, 2282, 2283, 2284, 2286, 2287, 2288, 2332, 2364, 2365, 2366, 2367, 2368, 2369, 2370, 2400, 2401, 2402, 2403, 2404, 2405, 2406, 2410, 2411, 2412, 2413, 2414, 2415, 2417, 2418, 2419, 2420, 2427, 2428, 2429, 2430, 2431, 2432, 2433, 2434, 2436, 2437, 2438

B grade: { 63, 87, 201, 266, 274, 275, 283, 284, 312, 330, 331, 332, 434, 435, 436, 440, 825, 838, 839, 846, 847, 848, 849, 850, 851, 978, 979, 986, 987, 988, 989, 990, 997, 998, 1002, 1005, 1006, 1007, 1013, 1014, 1015, 1016, 1017, 1023, 1024, 1025, 1026, 1027, 1028, 1062, 1063, 1064, 1071, 1072, 1073, 1078, 1079, 1080, 1081, 1082, 1083, 1086, 1087, 1088, 1092, 1093, 1100, 1101, 1109, 1110, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1155, 1176, 1178, 1184, 1187, 1189, 1264, 1271, 1278, 1415, 1436, 1451, 1452, 1459, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1512, 1513, 1514, 1515, 1516, 1519, 1521, 1522, 1523, 1524, 1525, 1526, 1527, 1528, 1529, 1530, 1532, 1533, 1534, 1535, 1536, 1537, 1587, 1588, 1596, 1597, 1604, 1605, 1606, 1607, 1633, 1637, 1638, 1639, 1640, 1641, 1642, 1643, 1644, 1692, 1737, 1759, 1760, 1769, 1770, 1771, 1778, 1781, 1782, 1783, 1784, 1785, 1787, 1788, 1789, 1794, 1795, 1796, 1797, 1798, 1799, 1807, 1808, 1809, 1816, 1817, 1818, 1824, 1825, 1826, 1827, 1828, 1829, 1830, 1839, 1847, 1851, 1852, 1856, 1861, 1862, 1872, 1873, 1882, 1883, 1884, 1892, 1893, 1894, 1895, 1900, 1902, 1903, 1904, 1905, 1906, 1978, 1985, 1993, 2107, 2154, 2155, 2156, 2157, 2158, 2159, 2160, 2161, 2162, 2285 }

C grade: { 217, 218, 791, 803, 804, 805, 894, 895, 896, 897, 901, 902, 903, 904, 905, 911, 912, 913, 914, 918, 919, 920, 921, 922, 2416, 2421, 2422, 2435 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 62, 67, 68, 77, 78, 79, 80, 81, 88, 89, 90, 91, 92, 93, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 186, 187, 188, 189, 195, 196, 197, 205, 206, 207, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 315, 316, 317, 318, 319, 320, 321, 322, 323, 326, 327, 328, 329, 334, 335, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 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B grade: { 33, 175, 201, 238, 239, 250, 251, 252, 253, 254, 265, 266, 267, 268, 269, 270, 273, 274, 275, 276, 277, 278, 279, 280, 281, 291, 292, 293, 294, 301, 308, 311, 312, 313, 314, 321, 327, 328, 334, 335, 337, 338, 345, 353, 366, 367, 368, 374, 375, 376, 377, 378, 379, 380, 382, 383, 384, 385, 434, 435, 440, 441, 442, 472, 480, 481, 482, 488, 504, 511, 513, 514, 517, 518, 519, 523, 530, 531, 532, 533, 540, 541, 542, 543, 544, 552, 553, 565, 566, 567, 568, 573, 574, 575, 576, 582, 583, 584, 590, 591, 598, 605, 612, 613, 614, 615, 616, 617, 618, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 645, 646, 647, 649, 653, 656, 703, 712, 713, 715, 721, 722, 723, 755, 756, 764, 766, 774, 775, 776, 783, 795, 796, 811, 812, 813, 814, 815, 821, 823, 825, 839, 841, 842, 843, 850, 854, 855, 856, 860, 861, 897, 898, 899, 900, 905, 906, 907, 908, 915, 916, 917, 922, 923, 924, 927, 929, 930, 931, 932, 937, 938, 939, 940, 945, 946, 978, 979, 986, 987, 988, 989, 990, 997, 998, 1005, 1006, 1007, 1013, 1014, 1015, 1016, 1017, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1069, 1070, 1076, 1077, 1078, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1092, 1093, 1097, 1109, 1110, 1112, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1172, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1197, 1198, 1199, 1200, 1201, 1202, 1204, 1206, 1211, 1212, 1213, 1214, 1217, 1218, 1219, 1221, 1225, 1227, 1228, 1229, 1240, 1241, 1247, 1248, 1249, 1250, 1252, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1268, 1269, 1276, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1326, 1413, 1414, 1415, 1423, 1424, 1436, 1438, 1451, 1452, 1459, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1521, 1522, 1523, 1524, 1525, 1526, 1527, 1528, 1529, 1530, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1564, 1565, 1569, 1570, 1571, 1572, 1581, 1582, 1583, 1584, 1595, 1601, 1602, 1603, 1604, 1605, 1607, 1611, 1612, 1613, 1621, 1622, 1623, 1629, 1630, 1631, 1632, 1637, 1638, 1639, 1640, 1641, 1642, 1643, 1644, 1645, 1646, 1650, 1651, 1652, 1653, 1654, 1655, 1656, 1657, 1658, 1659, 1660, 1661, 1662, 1663, 1664, 1665, 1666, 1667, 1668, 1669, 1670, 1671, 1672, 1673, 1674, 1675, 1692, 1710, 1716, 1717, 1718, 1719, 1720, 1721, 1723, 1724, 1725, 1726, 1727, 1728, 1729, 1730, 1731, 1732, 1733, 1737, 1738, 1744, 1759, 1760, 1769, 1770, 1771, 1778, 1781, 1782, 1783, 1784, 1785, 1786, 1787, 1788, 1789, 1791, 1792, 1793, 1794, 1795, 1796, 1797, 1798, 1799, 1807, 1808, 1809, 1810, 1811, 1816, 1817, 1818, 1819, 1820, 1824, 1825, 1826, 1827, 1828, 1829, 1830, 1839, 1840, 1847, 1851, 1852, 1853, 1856, 1858, 1859, 1861, 1862, 1872, 1873, 1874, 1875, 1876, 1882, 1883, 1884, 1885, 1886, 1887, 1888, 1892, 1893, 1894, 1895, 1896, 1897, 1898, 1900, 1902, 1903, 1904, 1905, 1906, 1916, 1917, 1927, 1928, 1929, 1931, 1932, 1933, 1942, 1943, 1953, 1959, 1960, 1961, 1962, 1967, 1969, 1970, 1971, 1976, 1977, 1983, 1984, 1985, 1992, 1993, 1994, 2006, 2007, 2008, 2009, 2015, 2016, 2017, 2018, 2019, 2021, 2022, 2023, 2024, 2025, 2026, 2033, 2034, 2035, 2044, 2045, 2050, 2054, 2055, 2056, 2062, 2063, 2064, 2069, 2070, 2071, 2072, 2074, 2076, 2077, 2078, 2079, 2080, 2084, 2085, 2086, 2103, 2104, 2107, 2118, 2124, 2125, 2126, 2131, 2137, 2138, 2139, 2140, 2141, 2142, 2146, 2152, 2153, 2154, 2155, 2156, 2157, 2158, 2159, 2160, 2161, 2162, 2177, 2188, 2190, 2191, 2192, 2193, 2194, 2195, 2196, 2199, 2200, 2201, 2202, 2203, 2204, 2205, 2206, 2208, 2209, 2210, 2211, 2212, 2213, 2214, 2215, 2216, 2217, 2218, 2220, 2221, 2222, 2223, 2224, 2225, 2226, 2268, 2275, 2276, 2282, 2283, 2289, 2290, 2291, 2292, 2293, 2295, 2296, 2297, 2298, 2301, 2302, 2303, 2304, 2308, 2309, 2310, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2328, 2329, 2330, 2331, 2338, 2339, 2340, 2343, 2349, 2355, 2356, 2357, 2378, 2379, 2380, 2381, 2382, 2383, 2386, 2387, 2388, 2389, 2391, 2392, 2393, 2394, 2395, 2396, 2397, 2398, 2400, 2401, 2414, 2418, 2435, 2500, 2506, 2549, 2550, 2551, 2552 }

C grade: { 217, 218, 2416, 2421, 2422, 2425, 2426 }

F grade: { 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 283, 284, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 438, 439, 443, 444, 445, 446, 447, 448, 449, 450, 524, 549, 554, 555, 556, 557, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 704, 705, 706, 707, 708, 709, 710, 711, 716, 717, 718, 719, 720, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 1215, 1216, 1230, 1231, 1232, 1233, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390,

1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1437, 1439, 1441, 1442, 1443, 1444, 1445, 1446, 1734, 1735, 1736, 1740, 1741, 1742, 1743, 1748, 1749, 1750, 1751, 1752, 1753, 1754, 1755, 1756, 1930, 1944, 1945, 1963, 1972, 1973, 1974, 1975, 2083, 2087, 2088, 2089, 2090, 2091, 2092, 2093, 2094, 2095, 2096, 2097, 2098, 2099, 2100, 2189, 2197, 2198, 2207, 2219, 2227, 2228, 2294, 2299, 2300, 2305, 2306, 2341, 2342, 2347, 2350, 2351, 2352, 2353, 2354, 2359, 2360, 2361, 2362, 2363, 2390, 2399, 2440, 2441, 2442, 2443, 2444, 2445, 2446, 2447, 2448, 2449, 2450, 2451, 2452, 2453, 2454, 2455, 2456, 2457, 2458, 2459, 2460, 2461, 2462, 2463, 2464, 2465, 2466, 2467, 2468, 2469, 2470, 2471, 2472, 2473, 2474, 2475, 2476, 2477, 2478, 2479, 2480, 2481, 2482, 2483, 2484, 2485, 2486, 2487, 2488, 2489, 2490, 2491, 2492, 2493, 2494, 2495, 2496, 2497, 2498, 2499, 2501, 2502, 2503, 2504, 2505, 2507, 2508, 2509, 2510, 2511, 2512, 2513, 2514, 2515, 2516, 2517, 2518, 2519, 2520, 2521, 2522, 2523, 2524, 2525, 2526, 2527, 2528, 2529, 2530, 2531, 2532, 2533, 2534, 2535, 2536, 2537, 2538, 2539, 2540, 2541, 2542, 2543, 2544, 2545, 2546, 2547, 2548, 2553, 2554, 2555, 2556, 2557, 2558, 2559, 2560, 2561, 2562, 2563, 2564, 2565, 2566, 2567, 2568, 2569, 2570, 2571, 2572, 2573, 2574, 2575, 2576, 2577, 2578, 2579, 2580, 2581, 2583, 2584, 2586, 2587, 2588, 2589, 2590
}

2.1.6 Sympy

A grade: { 112, 113, 114, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 181, 182, 183, 184, 185, 186, 187, 188, 189, 215, 216, 219, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 259, 262, 263, 272, 282, 337, 338, 340, 341, 342, 343, 344, 345, 348, 349, 350, 351, 352, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 434, 435, 436, 437, 441, 442, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 489, 490, 491, 492, 493, 494, 495, 496, 497, 509, 515, 516, 521, 522, 525, 526, 527, 528, 534, 535, 536, 537, 545, 546, 547, 548, 560, 561, 562, 563, 572, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 614, 618, 619, 620, 648, 649, 650, 651, 652, 653, 654, 700, 701, 702, 706, 707, 708, 721, 722, 723, 733, 734, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 773, 774, 775, 776, 819, 820, 821, 822, 823, 826, 827, 828, 829, 830, 860, 861, 926, 928, 945, 946, 947, 962, 968, 969, 977, 980, 981, 982, 983, 984, 992, 993, 994, 995, 996, 1000, 1001, 1002, 1003, 1009, 1010, 1011, 1012, 1019, 1020, 1021, 1022, 1029, 1031, 1033, 1039, 1041, 1043, 1045, 1050, 1052, 1054, 1058, 1063, 1065, 1067, 1072, 1074, 1076, 1080, 1082, 1084, 1086, 1089, 1090, 1091, 1092, 1093, 1100, 1102, 1104, 1106, 1113, 1114, 1115, 1116, 1126, 1127, 1128, 1129, 1142, 1143, 1144, 1145, 1146, 1157, 1159, 1161, 1162, 1164, 1167, 1169, 1171, 1173, 1174, 1235, 1237, 1243, 1245, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1282, 1283, 1284, 1289, 1290, 1320, 1321, 1322, 1323, 1345, 1358, 1373, 1376, 1377, 1423, 1424, 1425, 1447, 1448, 1453, 1454, 1455, 1456, 1457, 1458, 1460, 1470, 1471, 1472, 1505, 1506, 1507, 1508, 1509, 1510, 1518, 1520, 1539, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1547, 1548, 1549, 1550, 1551, 1552, 1553, 1587, 1588, 1589, 1590, 1591, 1621, 1622, 1623, 1624, 1625, 1626, 1627, 1628, 1629, 1630, 1631, 1632, 1633, 1634, 1635, 1636, 1637, 1638, 1639, 1641, 1642, 1643, 1644, 1678, 1706, 1732, 1733, 1761, 1762, 1763, 1764, 1765, 1766, 1767, 1772, 1773, 1775, 1776, 1777, 1779, 1790, 1791, 1792, 1793, 1799, 1800, 1801, 1802, 1803, 1804, 1810, 1811, 1812, 1813, 1814, 1819, 1820, 1821, 1822, 1831, 1832, 1833, 1834, 1835, 1836, 1837, 1841, 1842, 1844, 1845, 1846, 1848, 1849, 1854, 1857, 1858, 1859, 1860, 1863, 1865, 1866, 1867, 1868, 1869, 1874, 1875, 1876, 1877, 1878, 1879, 1880, 1885, 1886, 1887, 1888, 1889, 1896, 1897, 1898, 1899, 1901, 1976, 1977, 1978, 1979, 1980, 1981, 1982, 1983, 1984, 1985, 1986, 1987, 1988, 1989, 1990, 1991, 1992, 1994, 1995, 1996, 1997, 1998, 1999, 2003, 2004, 2005, 2006, 2007, 2084, 2085, 2086, 2108, 2109, 2110, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2132, 2133, 2134, 2135, 2136, 2137, 2138, 2147, 2148, 2149, 2150, 2151, 2152, 2163, 2164, 2165, 2166, 2167, 2168, 2169, 2170, 2171, 2172, 2173, 2174, 2175, 2176, 2177, 2178, 2179, 2180, 2181, 2229, 2230, 2231, 2232, 2233, 2234, 2235, 2236, 2237, 2238, 2239, 2240, 2241, 2242, 2243, 2244, 2245, 2246, 2247, 2248, 2249, 2250, 2251, 2252, 2253, 2254, 2255, 2256, 2257, 2258, 2259, 2260, 2261, 2262, 2263, 2264, 2265, 2266, 2267, 2268, 2269, 2270, 2271, 2272, 2273, 2274, 2275, 2276, 2277, 2278, 2279, 2280, 2281, 2282, 2283, 2284, 2285, 2286, 2287, 2291, 2309, 2310, 2311, 2312, 2317, 2318, 2319, 2326, 2327, 2332, 2419, 2420, 2423, 2424, 2551, 2552 }

B grade: { 221, 260, 261, 264, 265, 267, 268, 269, 270, 271, 275, 276, 277, 278, 279, 280, 281, 339, 346, 347, 353, 354, 355, 373, 472, 488, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 510, 512, 513, 514, 519, 520, 564, 581, 612, 613, 771, 772, 960, 967, 978, 979, 985, 986, 987, 988, 989, 990, 991, 997, 998, 999, 1004, 1005, 1006, 1007, 1008, 1013, 1014, 1015, 1016, 1017, 1018, 1023, 1024, 1025, 1026, 1027, 1028, 1109, 1110, 1111, 1112, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1158, 1160, 1163, 1165, 1166, 1168, 1170, 1172, 1175, 1176, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1192, 1194, 1202, 1204, 1206, 1217, 1219, 1221, 1251, 1253, 1255, 1302, 1440, 1451, 1452, 1459, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1473, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1519, 1521, 1522, 1523, 1524, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1592, 1593, 1594, 1595, 1640, 1648, 1649, 1659, 1759, 1760, 1768, 1769, 1770, 1771, 1774, 1778, 1780, 1781, 1782, 1783, 1784, 1785, 1786, 1787, 1788, 1789, 1794, 1795, 1796, 1797, 1798, 1805, 1806, 1807, 1808, 1809, 1815, 1816, 1817, 1818, 1823, 1824, 1825, 1826, 1827, 1828, 1829, 1830, 1838, 1839, 1840, 1843, 1847, 1850, 1851, 1852, 1853, 1855, 1856, 1861, 1862, 1864, 1870, 1871, 1872, 1873, 1881, 1882, 1883, 1884, 1890, 1891, 1892, 1893, 1894, 1895, 1900, 1902, 1903, 1904, 1905, 1906, 2107, 2118, 2131, 2139, 2140, 2146, 2153, 2154, 2182, 2183, 2184, 2185, 2186, 2190, 2191, 2192, 2193, 2194, 2195, 2199, 2200, 2201, 2202, 2203, 2204, 2205, 2206, 2208, 2209, 2210, 2211, 2212, 2213, 2214, 2215, 2216, 2217, 2218, 2220, 2223, 2224, 2225, 2226, 2290, 2366 }

C grade: { 732, 735, 777, 778, 779, 780, 788, 789, 790, 791, 800, 801, 802, 803, 804, 805, 817, 818, 841, 854, 961, 963, 964, 970 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 217, 218, 256, 257, 258, 266, 273, 274, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 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2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 10, 11, 12, 13, 14, 15, 16, 17, 24, 25, 26, 27, 28, 29, 30, 31, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 55, 56, 64, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 114, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 165, 166, 167, 169, 170, 171, 172, 173, 174, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 209, 210, 211, 215, 216, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 266, 267, 268, 269, 270, 271, 272, 273, 275, 276, 277, 278, 279, 280, 281, 282, 285, 286, 287, 288, 295, 296, 297, 298, 302, 303, 304, 305, 309, 310, 311, 312, 313, 314, 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B grade: { 7, 8, 9, 18, 19, 20, 21, 22, 23, 32, 33, 34, 35, 36, 37, 38, 63, 86, 87, 112, 113, 153, 158, 168, 175, 176, 208, 250, 264, 265, 274, 283, 284, 291, 292, 293, 294, 301, 308, 320, 321, 328, 337, 338, 339, 345, 346, 347, 353, 354, 355, 377, 378, 379, 380, 382, 383, 384, 385, 434, 435, 436, 440, 441, 442, 488, 511, 524, 531, 532, 533, 541, 542, 543, 544, 551, 552, 554, 555, 556, 557, 559, 566, 567, 568, 575, 576, 582, 584, 591, 592, 598, 599, 605, 606, 721, 722, 723, 783, 784, 786, 787, 796, 797, 798, 799, 825, 850, 926, 927, 945, 946, 947, 978, 979, 984, 986, 987, 988, 989, 990, 995, 996, 997, 1039, 1040, 1041, 1050, 1051, 1052, 1053, 1073, 1079, 1080, 1081, 1082, 1084, 1089, 1090, 1094, 1100, 1101, 1102, 1109, 1110, 1112, 1114, 1121, 1122, 1123, 1124, 1125, 1127, 1130, 1132, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1146, 1150, 1154, 1155, 1156, 1163, 1166, 1167, 1168, 1175, 1177, 1178, 1179, 1180, 1188, 1192, 1196, 1198, 1199, 1200, 1202, 1203, 1204, 1206, 1211, 1212, 1214, 1216, 1217, 1218, 1219, 1220, 1221, 1225, 1226, 1229, 1231, 1233, 1239, 1242, 1243, 1244, 1245, 1246, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1260, 1261, 1262, 1263, 1264, 1269, 1270, 1271, 1277, 1278, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1423, 1424, 1425, 1436, 1438, 1440, 1450, 1451, 1452, 1457, 1459, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1471, 1474, 1475, 1476, 1477, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1513, 1514, 1515, 1519, 1522, 1523, 1524, 1525, 1526, 1527, 1529, 1530, 1534, 1535, 1536, 1537, 1538, 1543, 1554, 1555, 1556, 1558, 1559, 1564, 1565, 1569, 1570, 1571, 1572, 1573, 1574, 1575, 1581, 1582, 1583, 1584, 1614, 1615, 1621, 1622, 1623, 1629, 1630, 1631, 1634, 1635, 1636, 1637, 1638, 1639, 1640, 1641, 1642, 1643, 1644, 1645, 1646, 1647, 1652, 1653, 1654, 1655, 1656, 1661, 1662, 1663, 1664, 1665, 1666, 1667, 1668, 1669, 1670, 1671, 1672, 1673, 1674, 1675, 1676, 1677, 1683, 1684, 1692, 1693, 1696, 1711, 1716, 1717, 1718, 1719, 1720, 1721, 1722, 1725, 1726, 1727, 1728, 1729, 1730, 1731, 1732, 1733, 1737, 1738, 1739, 1744, 1745, 1746, 1758, 1759, 1760, 1764, 1769, 1770, 1771, 1774, 1778, 1783, 1784, 1785, 1786, 1787, 1788, 1789, 1791, 1794, 1795, 1799, 1808, 1809, 1817, 1818, 1824, 1825, 1827, 1828, 1829, 1830, 1834, 1839, 1843, 1845, 1847, 1851, 1852, 1855, 1856, 1859, 1860, 1861, 1862, 1863, 1864, 1865, 1866, 1867, 1868, 1869, 1874, 1875, 1876, 1877, 1878, 1879, 1880, 1881, 1885, 1886, 1887, 1888, 1889, 1890, 1891, 1892, 1893, 1900, 1901, 1902, 1903, 1904, 1905, 1906, 1931, 1932, 1954, 1955, 1956, 1957, 1958, 1964, 1965, 1966, 1967, 1968, 1969, 1970, 1976, 1977, 1978, 1984, 1985, 1992, 1993, 2084, 2085, 2086, 2107, 2118, 2127, 2131, 2137, 2140, 2146, 2151, 2152, 2153, 2154, 2155, 2156, 2157, 2158, 2160, 2161, 2162, 2189, 2196, 2197, 2198, 2201, 2202, 2203, 2207, 2212, 2213, 2214, 2215, 2216, 2217, 2219, 2221, 2222, 2223, 2224, 2225, 2226, 2227, 2228, 2268, 2269, 2275, 2276, 2282, 2283, 2284, 2285, 2286, 2287, 2288, 2339, 2340, 2341, 2342, 2343, 2344, 2351, 2352, 2353, 2355, 2356, 2357, 2361, 2362, 2365, 2379, 2380, 2381, 2387, 2389, 2390, 2391, 2393, 2394, 2395, 2396, 2397, 2399, 2430, 2431, 2432, 2433, 2439, 2549, 2550, 2551, 2552 }

C grade: { 785, 795, 893, 982, 2416, 2435 }

F grade: { 52, 53, 54, 57, 58, 59, 60, 61, 62, 65, 67, 68, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 212, 213, 214, 217, 218, 289, 290, 299, 300, 306, 307, 327, 335, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 438, 439, 443, 444, 445, 446, 447, 448, 449, 450, 530, 539, 540, 550, 553, 574, 583, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 782, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 824, 831, 832, 833, 834, 835, 842, 843, 844, 845, 855, 856, 857, 858, 859, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 930, 931, 932, 937, 938, 939, 940, 941, 942, 943, 944, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 998, 999, 1000, 1001, 1002, 1003, 1004, 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1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1437, 1439, 1441, 1442, 1443, 1444, 1445, 1446, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603, 1604, 1605, 1606, 1607, 1608, 1609, 1610, 1611, 1612, 1613, 1618, 1619, 1620, 1734, 1735, 1736, 1740, 1741, 1742, 1743, 1748, 1749, 1750, 1751, 1752, 1753, 1754, 1755, 1756, 1757, 1805, 1816, 1826, 1871, 1872, 1873, 1883, 1884, 1895, 1896, 1897, 1898, 1899, 1912, 1913, 1914, 1915, 1916, 1917, 1923, 1924, 1925, 1926, 1927, 1928, 1929, 1930, 1936, 1937, 1938, 1939, 1940, 1941, 1942, 1943, 1944, 1945, 1950, 1951, 1952, 1953, 1960, 1961, 1962, 1963, 1971, 1972, 1973, 1974, 1975, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020, 2021, 2022, 2023, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038, 2039, 2040, 2041, 2042, 2043, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2057, 2058, 2059, 2060, 2061, 2062, 2063, 2064, 2065, 2066, 2067, 2068, 2069, 2070, 2071, 2072, 2073, 2074, 2075, 2076, 2077, 2078, 2079, 2080, 2081, 2082, 2083, 2087, 2088, 2089, 2090, 2091, 2092, 2093, 2094, 2095, 2096, 2097, 2098, 2099, 2100, 2101, 2102, 2103, 2104, 2289, 2290, 2291, 2292, 2293, 2294, 2295, 2296, 2297, 2298, 2299, 2300, 2301, 2302, 2303, 2304, 2305, 2306, 2307, 2308, 2309, 2310, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2328, 2329, 2330, 2331, 2337, 2338, 2347, 2348, 2349, 2350, 2354, 2359, 2360, 2363, 2388, 2398, 2407, 2409, 2421, 2422, 2425, 2426, 2440, 2441, 2442, 2443, 2444, 2445, 2446, 2447, 2448, 2449, 2450, 2451, 2452, 2453, 2454, 2455, 2456, 2457, 2458, 2459, 2460, 2461, 2462, 2463, 2464, 2465, 2466, 2467, 2468, 2469, 2470, 2471, 2472, 2473, 2474, 2475, 2476, 2477, 2478, 2479, 2480, 2481, 2482, 2483, 2484, 2485, 2486, 2487, 2488, 2489, 2490, 2491, 2492, 2493, 2494, 2495, 2496, 2497, 2498, 2499, 2500, 2501, 2502, 2503, 2504, 2505, 2506, 2507, 2508, 2509, 2510, 2511, 2512, 2513, 2514, 2515, 2516, 2517, 2518, 2519, 2520, 2521, 2522, 2523, 2524, 2525, 2526, 2527, 2528, 2529, 2530, 2531, 2532, 2533, 2534, 2535, 2536, 2537, 2538, 2539, 2540, 2541, 2542, 2543, 2544, 2545, 2546, 2547, 2548, 2553, 2554, 2555, 2556, 2557, 2558, 2559, 2560, 2561, 2562, 2563, 2564, 2565, 2566, 2567, 2568, 2569, 2570, 2571, 2572, 2573, 2574, 2575, 2576, 2577, 2578, 2579, 2580, 2581, 2582, 2583, 2584, 2585, 2586, 2587, 2588, 2589, 2590 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	109	129	0	459	0	131
normalized size	1	1.	0.83	0.98	0.	3.5	0.	1.
time (sec)	N/A	0.06	0.154	0.047	0.	2.073	0.	1.303

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	98	107	0	398	0	115
normalized size	1	1.	0.93	1.02	0.	3.79	0.	1.1
time (sec)	N/A	0.037	0.132	0.044	0.	2.066	0.	1.357

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	87	87	0	343	0	99
normalized size	1	1.	1.07	1.07	0.	4.23	0.	1.22
time (sec)	N/A	0.024	0.113	0.053	0.	2.074	0.	1.32

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	74	56	0	285	0	82
normalized size	1	1.	1.23	0.93	0.	4.75	0.	1.37
time (sec)	N/A	0.014	0.1	0.047	0.	2.048	0.	1.4

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	66	43	0	232	0	65
normalized size	1	1.	1.57	1.02	0.	5.52	0.	1.55
time (sec)	N/A	0.019	0.099	0.045	0.	1.896	0.	1.466

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	63	66	0	224	0	81
normalized size	1	1.	1.34	1.4	0.	4.77	0.	1.72
time (sec)	N/A	0.018	0.084	0.046	0.	2.027	0.	1.376

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	21	25	0	57	0	103
normalized size	1	1.	0.91	1.09	0.	2.48	0.	4.48
time (sec)	N/A	0.006	0.009	0.045	0.	2.002	0.	1.402

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	29	33	0	84	0	144
normalized size	1	1.	0.6	0.69	0.	1.75	0.	3.
time (sec)	N/A	0.017	0.011	0.052	0.	2.127	0.	1.265

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	51	44	0	112	0	184
normalized size	1	1.	0.69	0.59	0.	1.51	0.	2.49
time (sec)	N/A	0.028	0.011	0.049	0.	1.994	0.	1.34

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	62	55	0	134	0	223
normalized size	1	1.	0.62	0.55	0.	1.34	0.	2.23
time (sec)	N/A	0.041	0.013	0.047	0.	2.085	0.	1.356

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	73	66	0	166	0	262
normalized size	1	1.	0.58	0.52	0.	1.32	0.	2.08
time (sec)	N/A	0.053	0.014	0.054	0.	1.945	0.	1.331

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	120	146	0	514	0	146
normalized size	1	1.	0.9	1.09	0.	3.84	0.	1.09
time (sec)	N/A	0.055	0.167	0.048	0.	1.947	0.	1.35

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	109	126	0	451	0	128
normalized size	1	1.	0.99	1.15	0.	4.1	0.	1.16
time (sec)	N/A	0.035	0.152	0.045	0.	1.95	0.	1.334

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	98	95	0	393	0	112
normalized size	1	1.	1.1	1.07	0.	4.42	0.	1.26
time (sec)	N/A	0.023	0.131	0.048	0.	2.022	0.	1.397

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	87	81	0	344	0	97
normalized size	1	1.	1.12	1.04	0.	4.41	0.	1.24
time (sec)	N/A	0.027	0.118	0.047	0.	2.014	0.	1.454

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	69	99	0	292	0	81
normalized size	1	1.	0.96	1.38	0.	4.06	0.	1.12
time (sec)	N/A	0.028	0.107	0.046	0.	1.82	0.	1.382

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	46	124	0	270	0	103
normalized size	1	1.	0.72	1.94	0.	4.22	0.	1.61
time (sec)	N/A	0.027	0.012	0.045	0.	1.974	0.	1.456

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	48	149	0	284	0	155
normalized size	1	1.	0.71	2.19	0.	4.18	0.	2.28
time (sec)	N/A	0.028	0.014	0.046	0.	2.036	0.	1.343

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	21	25	0	78	0	181
normalized size	1	1.	0.91	1.09	0.	3.39	0.	7.87
time (sec)	N/A	0.008	0.012	0.044	0.	1.988	0.	1.253

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	29	33	0	105	0	223
normalized size	1	1.	0.6	0.69	0.	2.19	0.	4.65
time (sec)	N/A	0.016	0.012	0.051	0.	2.068	0.	1.252

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	40	44	0	135	0	262
normalized size	1	1.	0.54	0.59	0.	1.82	0.	3.54
time (sec)	N/A	0.027	0.016	0.046	0.	1.93	0.	1.234

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	51	55	0	161	0	301
normalized size	1	1.	0.51	0.55	0.	1.61	0.	3.01
time (sec)	N/A	0.041	0.017	0.046	0.	1.963	0.	1.28

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	62	66	0	194	0	340
normalized size	1	1.	0.49	0.52	0.	1.54	0.	2.7
time (sec)	N/A	0.057	0.019	0.046	0.	1.93	0.	1.213

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	142	185	0	633	0	177
normalized size	1	1.	0.87	1.13	0.	3.88	0.	1.09
time (sec)	N/A	0.074	0.207	0.053	0.	2.101	0.	1.225

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	131	165	0	567	0	162
normalized size	1	1.	0.94	1.19	0.	4.08	0.	1.17
time (sec)	N/A	0.05	0.177	0.049	0.	2.015	0.	1.292

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	120	134	0	501	0	144
normalized size	1	1.	1.02	1.14	0.	4.25	0.	1.22
time (sec)	N/A	0.036	0.183	0.047	0.	1.939	0.	1.275

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	109	120	0	458	0	130
normalized size	1	1.	1.02	1.12	0.	4.28	0.	1.21
time (sec)	N/A	0.039	0.146	0.047	0.	2.035	0.	1.307

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	98	135	0	406	0	113
normalized size	1	1.	0.97	1.34	0.	4.02	0.	1.12
time (sec)	N/A	0.041	0.131	0.046	0.	2.119	0.	1.183

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	80	158	0	346	0	97
normalized size	1	1.	0.85	1.68	0.	3.68	0.	1.03
time (sec)	N/A	0.039	0.125	0.05	0.	1.923	0.	1.25

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	48	185	0	332	0	120
normalized size	1	1.	0.52	2.01	0.	3.61	0.	1.3
time (sec)	N/A	0.041	0.012	0.048	0.	2.083	0.	1.265

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	50	209	0	343	0	180
normalized size	1	1.	0.56	2.35	0.	3.85	0.	2.02
time (sec)	N/A	0.041	0.013	0.05	0.	1.933	0.	1.213

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	50	232	0	346	0	236
normalized size	1	1.	0.55	2.55	0.	3.8	0.	2.59
time (sec)	N/A	0.041	0.013	0.048	0.	2.038	0.	1.296

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	21	25	0	100	0	259
normalized size	1	1.	0.91	1.09	0.	4.35	0.	11.26
time (sec)	N/A	0.007	0.015	0.044	0.	1.982	0.	1.224

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	36	33	0	130	0	301
normalized size	1	1.	0.75	0.69	0.	2.71	0.	6.27
time (sec)	N/A	0.017	0.011	0.046	0.	1.99	0.	1.216

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	47	44	0	161	0	340
normalized size	1	1.	0.64	0.59	0.	2.18	0.	4.59
time (sec)	N/A	0.028	0.013	0.045	0.	2.042	0.	1.369

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	58	55	0	185	0	379
normalized size	1	1.	0.58	0.55	0.	1.85	0.	3.79
time (sec)	N/A	0.042	0.014	0.043	0.	1.944	0.	1.282

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	69	66	0	220	0	419
normalized size	1	1.	0.55	0.52	0.	1.75	0.	3.33
time (sec)	N/A	0.057	0.017	0.047	0.	1.98	0.	1.193

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	80	77	0	247	0	458
normalized size	1	1.	0.53	0.51	0.	1.62	0.	3.01
time (sec)	N/A	0.073	0.019	0.053	0.	1.891	0.	1.252

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	39	39	66	90	0	39
normalized size	1	1.	0.78	0.78	1.32	1.8	0.	0.78
time (sec)	N/A	0.011	0.048	0.05	1.783	1.805	0.	1.214

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	63	41	66	122	0	43
normalized size	1	1.	1.21	0.79	1.27	2.35	0.	0.83
time (sec)	N/A	0.015	0.043	0.044	1.818	1.966	0.	1.211

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	43	38	62	104	0	51
normalized size	1	1.	0.9	0.79	1.29	2.17	0.	1.06
time (sec)	N/A	0.01	0.031	0.045	1.149	1.941	0.	1.156

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	98	112	0	408	0	120
normalized size	1	1.	0.77	0.88	0.	3.19	0.	0.94
time (sec)	N/A	0.058	0.214	0.048	0.	1.976	0.	1.22

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	87	90	0	350	0	104
normalized size	1	1.	0.85	0.88	0.	3.43	0.	1.02
time (sec)	N/A	0.043	0.173	0.046	0.	1.969	0.	1.264

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	88	68	0	297	0	88
normalized size	1	1.	1.16	0.89	0.	3.91	0.	1.16
time (sec)	N/A	0.026	0.054	0.045	0.	2.013	0.	1.266

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	71	47	0	236	0	70
normalized size	1	1.	1.51	1.	0.	5.02	0.	1.49
time (sec)	N/A	0.015	0.041	0.046	0.	2.032	0.	1.315

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	57	29	0	154	0	47
normalized size	1	1.	2.04	1.04	0.	5.5	0.	1.68
time (sec)	N/A	0.008	0.016	0.049	0.	1.913	0.	1.324

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	0	38	0	31
normalized size	1	1.	1.	1.05	0.	1.81	0.	1.48
time (sec)	N/A	0.007	0.007	0.047	0.	1.745	0.	1.197

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	29	31	0	61	0	66
normalized size	1	1.	0.6	0.65	0.	1.27	0.	1.38
time (sec)	N/A	0.016	0.013	0.046	0.	2.012	0.	1.228

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	40	44	0	88	0	105
normalized size	1	1.	0.54	0.59	0.	1.19	0.	1.42
time (sec)	N/A	0.026	0.012	0.046	0.	1.885	0.	1.205

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	51	55	0	109	0	144
normalized size	1	1.	0.51	0.55	0.	1.09	0.	1.44
time (sec)	N/A	0.039	0.014	0.046	0.	1.883	0.	1.338

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	62	66	0	140	0	184
normalized size	1	1.	0.49	0.52	0.	1.11	0.	1.46
time (sec)	N/A	0.055	0.017	0.051	0.	1.946	0.	1.188

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	50	93	0	410	0	0
normalized size	1	1.	0.52	0.96	0.	4.23	0.	0.
time (sec)	N/A	0.04	0.012	0.063	0.	2.085	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	50	68	0	344	0	0
normalized size	1	1.	0.72	0.99	0.	4.99	0.	0.
time (sec)	N/A	0.027	0.013	0.05	0.	1.944	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	67	47	0	289	0	0
normalized size	1	1.	1.4	0.98	0.	6.02	0.	0.
time (sec)	N/A	0.018	0.081	0.048	0.	1.905	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	17	25	23	47	0	43
normalized size	1	1.	0.89	1.32	1.21	2.47	0.	2.26
time (sec)	N/A	0.005	0.006	0.047	1.093	1.965	0.	1.21

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	29	47	73	0	32
normalized size	1	1.	0.92	1.21	1.96	3.04	0.	1.33
time (sec)	N/A	0.003	0.007	0.045	1.069	1.904	0.	1.304

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	40	39	0	99	0	0
normalized size	1	1.	0.78	0.76	0.	1.94	0.	0.
time (sec)	N/A	0.013	0.011	0.045	0.	1.902	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	49	53	0	123	0	0
normalized size	1	1.	0.64	0.69	0.	1.6	0.	0.
time (sec)	N/A	0.024	0.012	0.045	0.	1.867	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	62	66	0	151	0	0
normalized size	1	1.	0.6	0.64	0.	1.47	0.	0.
time (sec)	N/A	0.039	0.014	0.045	0.	1.936	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	50	176	0	556	0	0
normalized size	1	1.	0.41	1.44	0.	4.56	0.	0.
time (sec)	N/A	0.058	0.015	0.049	0.	1.989	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	50	149	0	493	0	0
normalized size	1	1.	0.53	1.59	0.	5.24	0.	0.
time (sec)	N/A	0.041	0.013	0.047	0.	1.922	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	84	123	0	432	0	0
normalized size	1	1.	1.18	1.73	0.	6.08	0.	0.
time (sec)	N/A	0.03	0.16	0.044	0.	2.028	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	21	25	100	74	0	120
normalized size	1	1.	0.91	1.09	4.35	3.22	0.	5.22
time (sec)	N/A	0.008	0.01	0.043	1.137	1.842	0.	1.16

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	29	33	73	93	0	82
normalized size	1	1.	0.57	0.65	1.43	1.82	0.	1.61
time (sec)	N/A	0.013	0.011	0.045	1.158	2.072	0.	1.214

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	38	44	70	123	0	0
normalized size	1	1.	0.79	0.92	1.46	2.56	0.	0.
time (sec)	N/A	0.011	0.013	0.049	1.042	1.87	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	48	51	97	144	0	68
normalized size	1	1.	0.89	0.94	1.8	2.67	0.	1.26
time (sec)	N/A	0.009	0.014	0.045	1.184	2.01	0.	1.179

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	62	63	0	176	0	0
normalized size	1	1.	0.78	0.79	0.	2.2	0.	0.
time (sec)	N/A	0.022	0.018	0.044	0.	1.957	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	73	77	0	197	0	0
normalized size	1	1.	0.69	0.73	0.	1.86	0.	0.
time (sec)	N/A	0.036	0.02	0.047	0.	2.039	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	27	23	30	68	0	30
normalized size	1	1.	1.04	0.88	1.15	2.62	0.	1.15
time (sec)	N/A	0.011	0.03	0.044	1.695	1.975	0.	1.159

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	28	28	40	26	39	69	0	38
normalized size	1	1.	1.43	0.93	1.39	2.46	0.	1.36
time (sec)	N/A	0.007	0.013	0.046	1.116	2.038	0.	1.166

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	47	35	49	84	0	31
normalized size	1	1.	1.02	0.76	1.07	1.83	0.	0.67
time (sec)	N/A	0.015	0.044	0.044	1.738	1.948	0.	1.227

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	64	66	86	170	0	95
normalized size	1	1.	0.47	0.49	0.63	1.25	0.	0.7
time (sec)	N/A	0.055	0.035	0.046	1.16	1.885	0.	1.162

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	53	55	72	139	0	78
normalized size	1	1.	0.49	0.51	0.67	1.29	0.	0.72
time (sec)	N/A	0.04	0.029	0.046	1.15	2.205	0.	1.206

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	42	44	57	116	0	62
normalized size	1	1.	0.52	0.55	0.71	1.45	0.	0.78
time (sec)	N/A	0.027	0.022	0.049	1.159	2.289	0.	1.168

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	31	33	41	89	0	46
normalized size	1	1.	0.6	0.63	0.79	1.71	0.	0.88
time (sec)	N/A	0.015	0.017	0.046	1.028	2.186	0.	1.215

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	25	16	61	0	28
normalized size	1	1.	0.92	1.	0.64	2.44	0.	1.12
time (sec)	N/A	0.006	0.011	0.042	1.166	2.218	0.	1.197

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	60	48	0	273	0	82
normalized size	1	1.	1.13	0.91	0.	5.15	0.	1.55
time (sec)	N/A	0.021	0.025	0.207	0.	2.056	0.	1.125

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	51	53	0	308	0	51
normalized size	1	1.	0.94	0.98	0.	5.7	0.	0.94
time (sec)	N/A	0.021	0.041	0.183	0.	2.028	0.	1.21

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	42	71	0	363	0	76
normalized size	1	1.	0.49	0.83	0.	4.22	0.	0.88
time (sec)	N/A	0.033	0.014	0.182	0.	2.133	0.	1.183

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	42	90	0	419	0	97
normalized size	1	1.	0.37	0.79	0.	3.68	0.	0.85
time (sec)	N/A	0.052	0.014	0.188	0.	2.099	0.	1.315

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	42	108	0	477	0	113
normalized size	1	1.	0.3	0.76	0.	3.36	0.	0.8
time (sec)	N/A	0.065	0.015	0.18	0.	2.176	0.	1.208

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	75	77	227	223	0	246
normalized size	1	1.	0.46	0.47	1.38	1.36	0.	1.5
time (sec)	N/A	0.075	0.046	0.054	1.14	2.044	0.	1.326

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	64	66	198	198	0	213
normalized size	1	1.	0.47	0.49	1.46	1.46	0.	1.57
time (sec)	N/A	0.056	0.038	0.047	1.062	2.012	0.	1.22

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	53	55	167	166	0	181
normalized size	1	1.	0.49	0.51	1.55	1.54	0.	1.68
time (sec)	N/A	0.043	0.032	0.047	1.156	1.973	0.	1.194

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	42	44	138	139	0	149
normalized size	1	1.	0.52	0.55	1.72	1.74	0.	1.86
time (sec)	N/A	0.027	0.026	0.046	1.151	1.903	0.	1.296

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	31	33	104	111	0	116
normalized size	1	1.	0.6	0.63	2.	2.13	0.	2.23
time (sec)	N/A	0.016	0.02	0.045	1.128	1.839	0.	1.279

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	25	66	82	0	81
normalized size	1	1.	0.92	1.	2.64	3.28	0.	3.24
time (sec)	N/A	0.007	0.014	0.044	1.135	1.973	0.	1.323

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	70	61	0	321	0	104
normalized size	1	1.	0.92	0.8	0.	4.22	0.	1.37
time (sec)	N/A	0.034	0.044	0.193	0.	2.088	0.	1.261

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	40	68	0	329	0	68
normalized size	1	1.	0.53	0.91	0.	4.39	0.	0.91
time (sec)	N/A	0.032	0.016	0.195	0.	2.098	0.	1.344

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	72	72	0	367	0	74
normalized size	1	1.	0.87	0.87	0.	4.42	0.	0.89
time (sec)	N/A	0.032	0.049	0.192	0.	2.193	0.	1.304

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	42	90	0	423	0	97
normalized size	1	1.	0.38	0.81	0.	3.81	0.	0.87
time (sec)	N/A	0.046	0.019	0.189	0.	2.154	0.	1.339

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	42	108	0	468	0	113
normalized size	1	1.	0.3	0.78	0.	3.37	0.	0.81
time (sec)	N/A	0.065	0.018	0.186	0.	2.097	0.	1.346

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	42	126	0	529	0	130
normalized size	1	1.	0.25	0.75	0.	3.17	0.	0.78
time (sec)	N/A	0.083	0.018	0.218	0.	2.118	0.	1.356

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	53	55	72	115	0	78
normalized size	1	1.	0.49	0.51	0.67	1.06	0.	0.72
time (sec)	N/A	0.04	0.032	0.047	1.146	2.007	0.	1.192

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	42	44	57	92	0	62
normalized size	1	1.	0.52	0.55	0.71	1.15	0.	0.78
time (sec)	N/A	0.027	0.023	0.049	1.153	2.087	0.	1.281

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	30	33	41	66	0	43
normalized size	1	1.	0.58	0.63	0.79	1.27	0.	0.83
time (sec)	N/A	0.016	0.017	0.055	1.125	1.909	0.	1.252

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	21	25	16	45	0	28
normalized size	1	1.	0.91	1.09	0.7	1.96	0.	1.22
time (sec)	N/A	0.007	0.009	0.045	1.189	1.994	0.	1.263

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	48	37	0	181	0	53
normalized size	1	1.	1.5	1.16	0.	5.66	0.	1.66
time (sec)	N/A	0.012	0.012	0.181	0.	1.968	0.	1.243

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	63	52	0	315	0	59
normalized size	1	1.	1.12	0.93	0.	5.62	0.	1.05
time (sec)	N/A	0.022	0.058	0.196	0.	1.93	0.	1.25

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	40	72	0	373	0	81
normalized size	1	1.	0.45	0.81	0.	4.19	0.	0.91
time (sec)	N/A	0.034	0.012	0.225	0.	2.067	0.	1.31

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	40	90	0	428	0	97
normalized size	1	1.	0.34	0.77	0.	3.66	0.	0.83
time (sec)	N/A	0.049	0.012	0.189	0.	2.18	0.	1.325

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	75	77	0	185	0	111
normalized size	1	1.	0.46	0.47	0.	1.13	0.	0.68
time (sec)	N/A	0.075	0.036	0.051	0.	1.909	0.	1.16

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	64	66	0	159	0	95
normalized size	1	1.	0.47	0.49	0.	1.17	0.	0.7
time (sec)	N/A	0.057	0.029	0.046	0.	1.981	0.	1.177

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	52	54	0	130	0	76
normalized size	1	1.	0.48	0.5	0.	1.2	0.	0.7
time (sec)	N/A	0.041	0.024	0.048	0.	2.002	0.	1.228

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	41	44	0	107	0	59
normalized size	1	1.	0.51	0.55	0.	1.34	0.	0.74
time (sec)	N/A	0.028	0.018	0.047	0.	1.999	0.	1.289

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	28	32	0	82	0	42
normalized size	1	1.	0.58	0.67	0.	1.71	0.	0.88
time (sec)	N/A	0.018	0.013	0.045	0.	2.025	0.	1.24

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	21	25	0	65	0	28
normalized size	1	1.	0.91	1.09	0.	2.83	0.	1.22
time (sec)	N/A	0.007	0.007	0.049	0.	2.085	0.	1.215

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	37	51	0	359	0	90
normalized size	1	1.	0.66	0.91	0.	6.41	0.	1.61
time (sec)	N/A	0.022	0.009	0.181	0.	2.073	0.	1.239

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	38	60	0	428	0	78
normalized size	1	1.	0.47	0.74	0.	5.28	0.	0.96
time (sec)	N/A	0.033	0.01	0.218	0.	2.244	0.	1.36

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	40	76	0	491	0	97
normalized size	1	1.	0.34	0.65	0.	4.2	0.	0.83
time (sec)	N/A	0.048	0.01	0.19	0.	2.143	0.	1.315

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	40	87	0	549	0	113
normalized size	1	1.	0.28	0.6	0.	3.79	0.	0.78
time (sec)	N/A	0.066	0.01	0.191	0.	2.065	0.	1.298

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	57	173	104	363	738	356
normalized size	1	1.	0.7	2.14	1.28	4.48	9.11	4.4
time (sec)	N/A	0.067	0.036	0.058	1.172	2.003	3.84	1.279

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	41	90	74	193	345	190
normalized size	1	1.	0.71	1.55	1.28	3.33	5.95	3.28
time (sec)	N/A	0.041	0.031	0.05	1.163	2.132	2.609	1.291

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	25	35	45	82	112	76
normalized size	1	1.	0.71	1.	1.29	2.34	3.2	2.17
time (sec)	N/A	0.014	0.015	0.046	1.155	2.031	0.793	1.338

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.006	0.374	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	30	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.009	0.39	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	32	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	0.009	0.409	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	70	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	0.123	0.429	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	60	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.117	0.403	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	60	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.068	0.427	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	58	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.058	0.4	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	58	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.093	0.438	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	60	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.109	0.425	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	53	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.013	0.479	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	47	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.008	0.366	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	47	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.009	0.371	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	83	47	0	0	0	0	0
normalized size	1	1.69	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.008	0.413	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	40	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	0.006	0.332	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	45	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.008	0.374	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	47	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.008	0.369	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	58	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.014	0.362	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	58	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.013	0.342	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	58	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.01	0.355	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	58	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.012	0.361	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	58	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.012	0.342	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	58	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.015	0.335	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	33	30	0	31	12	53
normalized size	1	1.	0.46	0.42	0.	0.44	0.17	0.75
time (sec)	N/A	0.026	0.013	0.168	0.	1.969	0.207	1.242

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	33	30	0	31	12	53
normalized size	1	1.	0.46	0.42	0.	0.44	0.17	0.75
time (sec)	N/A	0.024	0.009	0.049	0.	1.865	0.209	1.254

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	33	30	0	31	12	53
normalized size	1	1.	0.46	0.42	0.	0.44	0.17	0.75
time (sec)	N/A	0.022	0.009	0.046	0.	1.968	0.148	1.239

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	33	30	0	31	12	53
normalized size	1	1.	0.54	0.49	0.	0.51	0.2	0.87
time (sec)	N/A	0.015	0.008	0.042	0.	1.845	0.149	1.241

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	30	27	0	23	8	45
normalized size	1	1.	0.94	0.84	0.	0.72	0.25	1.41
time (sec)	N/A	0.006	0.008	0.042	0.	1.857	0.246	1.179

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	27	19	0	22	7	28
normalized size	1	1.	0.44	0.31	0.	0.35	0.11	0.45
time (sec)	N/A	0.017	0.01	0.328	0.	1.864	0.264	1.214

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	31	22	0	27	7	32
normalized size	1	1.	0.48	0.34	0.	0.42	0.11	0.49
time (sec)	N/A	0.018	0.01	0.235	0.	1.834	0.31	1.262

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	31	28	0	30	12	53
normalized size	1	1.	0.89	0.8	0.	0.86	0.34	1.51
time (sec)	N/A	0.013	0.007	0.045	0.	2.022	0.582	1.231

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	33	30	0	32	14	54
normalized size	1	1.	0.46	0.42	0.	0.45	0.2	0.76
time (sec)	N/A	0.019	0.007	0.044	0.	1.89	0.889	1.241

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	33	30	0	34	14	54
normalized size	1	1.	0.46	0.42	0.	0.48	0.2	0.76
time (sec)	N/A	0.019	0.008	0.044	0.	1.882	0.665	1.243

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	33	30	0	34	14	54
normalized size	1	1.	0.46	0.42	0.	0.48	0.2	0.76
time (sec)	N/A	0.023	0.007	0.047	0.	2.08	0.986	1.158

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	55	52	0	80	0	99
normalized size	1	1.	0.36	0.34	0.	0.53	0.	0.66
time (sec)	N/A	0.046	0.019	0.186	0.	1.745	0.	1.179

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	55	52	0	80	0	99
normalized size	1	1.	0.36	0.34	0.	0.53	0.	0.66
time (sec)	N/A	0.042	0.014	0.182	0.	1.732	0.	1.195

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	55	52	0	80	0	99
normalized size	1	1.	0.36	0.34	0.	0.53	0.	0.66
time (sec)	N/A	0.041	0.014	0.176	0.	2.029	0.	1.252

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	107	55	52	0	80	0	99
normalized size	1	1.11	0.57	0.54	0.	0.83	0.	1.03
time (sec)	N/A	0.031	0.014	0.184	0.	1.965	0.	1.3

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	55	52	0	74	0	97
normalized size	1	1.	0.9	0.85	0.	1.21	0.	1.59
time (sec)	N/A	0.015	0.014	0.171	0.	1.968	0.	1.197

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	23	49	0	66	0	93
normalized size	1	1.	0.72	1.53	0.	2.06	0.	2.91
time (sec)	N/A	0.005	0.01	0.048	0.	1.927	0.	1.202

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	52	51	0	73	0	76
normalized size	1	1.	0.36	0.36	0.	0.51	0.	0.53
time (sec)	N/A	0.034	0.016	0.219	0.	1.968	0.	1.134

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	56	53	0	78	0	77
normalized size	1	1.	0.39	0.37	0.	0.55	0.	0.54
time (sec)	N/A	0.034	0.017	0.222	0.	1.883	0.	1.288

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	55	54	0	81	0	76
normalized size	1	1.	0.39	0.38	0.	0.57	0.	0.54
time (sec)	N/A	0.036	0.016	0.209	0.	1.993	0.	1.247

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	57	54	0	85	0	80
normalized size	1	1.	0.39	0.37	0.	0.59	0.	0.55
time (sec)	N/A	0.034	0.019	0.223	0.	2.019	0.	1.264

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	53	50	0	73	0	99
normalized size	1	1.	1.43	1.35	0.	1.97	0.	2.68
time (sec)	N/A	0.014	0.012	0.166	0.	1.958	0.	1.376

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	55	52	0	81	0	100
normalized size	1	1.	0.72	0.68	0.	1.07	0.	1.32
time (sec)	N/A	0.023	0.015	0.184	0.	2.012	0.	1.229

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	55	52	0	82	0	100
normalized size	1	1.	0.36	0.34	0.	0.54	0.	0.66
time (sec)	N/A	0.035	0.014	0.176	0.	2.011	0.	1.491

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	55	52	0	84	0	100
normalized size	1	1.	0.36	0.34	0.	0.56	0.	0.66
time (sec)	N/A	0.037	0.012	0.184	0.	1.738	0.	1.253

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	55	52	0	86	0	100
normalized size	1	1.	0.36	0.34	0.	0.57	0.	0.66
time (sec)	N/A	0.035	0.017	0.181	0.	1.574	0.	1.395

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	77	74	0	134	0	144
normalized size	1	1.	0.33	0.32	0.	0.58	0.	0.62
time (sec)	N/A	0.062	0.023	0.176	0.	1.555	0.	1.227

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	77	74	0	132	0	144
normalized size	1	1.	0.43	0.41	0.	0.73	0.	0.8
time (sec)	N/A	0.051	0.019	0.174	0.	1.652	0.	1.371

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	77	74	0	124	0	143
normalized size	1	1.	0.53	0.51	0.	0.86	0.	0.99
time (sec)	N/A	0.052	0.019	0.188	0.	1.778	0.	1.343

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	77	74	0	126	0	144
normalized size	1	1.	0.72	0.69	0.	1.18	0.	1.35
time (sec)	N/A	0.041	0.02	0.176	0.	1.587	0.	1.339

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	77	74	0	126	0	144
normalized size	1	1.	1.26	1.21	0.	2.07	0.	2.36
time (sec)	N/A	0.015	0.019	0.168	0.	1.658	0.	1.358

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	23	71	0	116	0	139
normalized size	1	1.	0.72	2.22	0.	3.62	0.	4.34
time (sec)	N/A	0.005	0.01	0.048	0.	1.6	0.	1.393

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	74	73	0	120	0	122
normalized size	1	1.	0.33	0.33	0.	0.54	0.	0.55
time (sec)	N/A	0.049	0.021	0.227	0.	1.659	0.	1.388

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	79	76	0	134	0	123
normalized size	1	1.	0.36	0.35	0.	0.61	0.	0.56
time (sec)	N/A	0.05	0.025	0.224	0.	1.66	0.	1.181

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	79	76	0	132	0	123
normalized size	1	1.	0.36	0.34	0.	0.59	0.	0.55
time (sec)	N/A	0.051	0.022	0.235	0.	1.708	0.	1.368

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	79	76	0	132	0	124
normalized size	1	1.	0.36	0.34	0.	0.59	0.	0.56
time (sec)	N/A	0.053	0.022	0.225	0.	1.697	0.	1.325

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	79	76	0	136	0	123
normalized size	1	1.	0.36	0.35	0.	0.62	0.	0.56
time (sec)	N/A	0.053	0.02	0.225	0.	1.74	0.	1.473

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	79	76	0	140	0	126
normalized size	1	1.	0.35	0.34	0.	0.63	0.	0.57
time (sec)	N/A	0.053	0.025	0.231	0.	1.776	0.	1.329

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	75	72	0	120	0	144
normalized size	1	1.	2.03	1.95	0.	3.24	0.	3.89
time (sec)	N/A	0.014	0.015	0.175	0.	1.731	0.	1.258

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	77	74	0	128	0	146
normalized size	1	1.	1.01	0.97	0.	1.68	0.	1.92
time (sec)	N/A	0.023	0.016	0.203	0.	1.665	0.	1.389

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	77	74	0	135	0	146
normalized size	1	1.	0.66	0.64	0.	1.16	0.	1.26
time (sec)	N/A	0.033	0.016	0.184	0.	1.618	0.	1.417

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	77	74	0	136	0	146
normalized size	1	1.	0.34	0.32	0.	0.59	0.	0.64
time (sec)	N/A	0.053	0.019	0.176	0.	1.667	0.	1.325

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	77	74	0	144	0	146
normalized size	1	1.	0.33	0.32	0.	0.62	0.	0.63
time (sec)	N/A	0.053	0.016	0.183	0.	1.648	0.	1.302

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	77	74	0	146	0	146
normalized size	1	1.	0.33	0.32	0.	0.63	0.	0.63
time (sec)	N/A	0.056	0.016	0.237	0.	1.747	0.	1.262

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	68	67	201	117	49	112
normalized size	1	1.	0.37	0.37	1.1	0.64	0.27	0.62
time (sec)	N/A	0.062	0.025	0.246	1.256	1.724	1.068	1.354

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	57	56	159	92	37	90
normalized size	1	1.	0.4	0.39	1.1	0.64	0.26	0.62
time (sec)	N/A	0.047	0.018	0.225	1.225	1.633	0.626	1.292

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	45	44	55	68	26	65
normalized size	1	1.	0.42	0.42	0.52	0.64	0.25	0.61
time (sec)	N/A	0.037	0.015	0.222	1.255	1.651	1.087	1.266

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	33	33	57	38	14	42
normalized size	1	1.	0.53	0.53	0.92	0.61	0.23	0.68
time (sec)	N/A	0.017	0.01	0.22	1.241	1.632	0.483	1.342

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	26	25	19	22	7	23
normalized size	1	1.	0.74	0.71	0.54	0.63	0.2	0.66
time (sec)	N/A	0.008	0.007	0.171	1.213	1.724	0.169	1.265

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	31	30	0	38	10	38
normalized size	1	1.	0.46	0.44	0.	0.56	0.15	0.56
time (sec)	N/A	0.022	0.01	0.194	0.	1.657	0.532	1.278

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	41	40	0	61	19	50
normalized size	1	1.	0.4	0.39	0.	0.59	0.18	0.49
time (sec)	N/A	0.035	0.013	0.177	0.	1.669	1.083	1.325

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	59	58	0	103	31	73
normalized size	1	1.	0.42	0.41	0.	0.73	0.22	0.51
time (sec)	N/A	0.045	0.018	0.178	0.	1.762	0.911	1.398

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	72	69	0	126	44	88
normalized size	1	1.	0.4	0.38	0.	0.7	0.24	0.49
time (sec)	N/A	0.052	0.022	0.181	0.	1.733	0.726	1.361

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	83	101	223	200	0	0
normalized size	1	1.	0.48	0.59	1.3	1.16	0.	0.
time (sec)	N/A	0.073	0.027	0.229	1.621	1.589	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	71	89	180	176	0	0
normalized size	1	1.	0.53	0.67	1.35	1.32	0.	0.
time (sec)	N/A	0.058	0.023	0.224	1.189	1.679	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	51	67	76	132	0	0
normalized size	1	1.	0.52	0.68	0.77	1.33	0.	0.
time (sec)	N/A	0.045	0.018	0.222	1.197	1.505	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	33	26	59	68	0	0
normalized size	1	1.	0.54	0.43	0.97	1.11	0.	0.
time (sec)	N/A	0.015	0.01	0.176	1.831	1.581	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	23	20	22	49	0	0
normalized size	1	1.	0.68	0.59	0.65	1.44	0.	0.
time (sec)	N/A	0.004	0.009	0.044	1.47	1.616	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	62	91	0	182	0	0
normalized size	1	1.	0.49	0.72	0.	1.44	0.	0.
time (sec)	N/A	0.058	0.023	0.226	0.	1.696	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	81	117	0	232	0	0
normalized size	1	1.	0.49	0.71	0.	1.41	0.	0.
time (sec)	N/A	0.068	0.029	0.242	0.	1.725	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	99	136	0	269	0	0
normalized size	1	1.	0.47	0.65	0.	1.29	0.	0.
time (sec)	N/A	0.08	0.036	0.239	0.	1.699	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	106	158	170	344	0	0
normalized size	1	1.	0.43	0.65	0.7	1.41	0.	0.
time (sec)	N/A	0.117	0.034	0.264	1.295	1.563	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	93	145	153	320	0	0
normalized size	1	1.	0.45	0.71	0.75	1.56	0.	0.
time (sec)	N/A	0.093	0.029	0.221	1.251	1.717	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	73	123	124	271	0	0
normalized size	1	1.	0.43	0.72	0.73	1.58	0.	0.
time (sec)	N/A	0.078	0.026	0.223	1.184	1.619	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	55	48	181	154	0	0
normalized size	1	1.	1.49	1.3	4.89	4.16	0.	0.
time (sec)	N/A	0.016	0.015	0.175	1.123	1.64	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	44	37	77	134	0	0
normalized size	1	1.	0.41	0.35	0.72	1.25	0.	0.
time (sec)	N/A	0.048	0.014	0.177	1.182	1.599	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	33	26	59	112	0	0
normalized size	1	1.	0.52	0.41	0.94	1.78	0.	0.
time (sec)	N/A	0.015	0.01	0.174	1.109	1.519	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	23	20	22	92	0	0
normalized size	1	1.	0.68	0.59	0.65	2.71	0.	0.
time (sec)	N/A	0.004	0.01	0.044	1.138	1.699	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	84	173	0	365	0	0
normalized size	1	1.	0.43	0.89	0.	1.88	0.	0.
time (sec)	N/A	0.086	0.035	0.238	0.	1.791	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	103	199	0	421	0	0
normalized size	1	1.	0.44	0.85	0.	1.79	0.	0.
time (sec)	N/A	0.102	0.04	0.225	0.	1.787	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	121	218	0	455	0	0
normalized size	1	1.	0.44	0.78	0.	1.64	0.	0.
time (sec)	N/A	0.114	0.043	0.231	0.	1.741	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	47	47	59	82	0	101
normalized size	1	1.	1.12	1.12	1.4	1.95	0.	2.4
time (sec)	N/A	0.009	0.017	0.064	1.706	1.614	0.	1.306

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	37	37	59	50	0	72
normalized size	1	1.	0.88	0.88	1.4	1.19	0.	1.71
time (sec)	N/A	0.009	0.012	0.067	1.72	1.651	0.	1.265

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	30	27	59	26	0	42
normalized size	1	1.	0.71	0.64	1.4	0.62	0.	1.
time (sec)	N/A	0.008	0.006	0.062	1.707	1.539	0.	1.313

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	33	29	28	35	0	49
normalized size	1	1.	0.69	0.6	0.58	0.73	0.	1.02
time (sec)	N/A	0.011	0.009	0.158	1.683	1.738	0.	1.311

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	27	22	32	47	0	0
normalized size	1	1.	0.61	0.5	0.73	1.07	0.	0.
time (sec)	N/A	0.009	0.008	0.08	1.71	1.631	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	27	22	32	78	0	0
normalized size	1	1.	0.61	0.5	0.73	1.77	0.	0.
time (sec)	N/A	0.009	0.008	0.067	1.59	1.685	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	27	22	32	113	0	0
normalized size	1	1.	0.61	0.5	0.73	2.57	0.	0.
time (sec)	N/A	0.009	0.009	0.066	1.729	1.59	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	33	29	28	35	12	34
normalized size	1	1.	0.69	0.6	0.58	0.73	0.25	0.71
time (sec)	N/A	0.011	0.011	0.12	1.674	1.685	0.174	1.408

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	33	29	28	35	12	34
normalized size	1	1.	0.69	0.6	0.58	0.73	0.25	0.71
time (sec)	N/A	0.011	0.016	0.095	1.757	1.581	0.251	1.233

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	35	31	28	42	0	0
normalized size	1	1.	0.73	0.65	0.58	0.88	0.	0.
time (sec)	N/A	0.011	0.011	0.096	1.708	1.596	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	35	31	28	42	0	0
normalized size	1	1.	0.73	0.65	0.58	0.88	0.	0.
time (sec)	N/A	0.012	0.008	0.116	1.714	1.616	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	15	9	14	27	8	18
normalized size	1	1.	1.25	0.75	1.17	2.25	0.67	1.5
time (sec)	N/A	0.003	0.003	0.042	1.117	1.512	0.163	1.249

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	9	9	14	24	8	15
normalized size	1	1.	0.9	0.9	1.4	2.4	0.8	1.5
time (sec)	N/A	0.004	0.004	0.043	1.113	1.553	0.982	1.335

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	99	100	134	231	107	130
normalized size	1	1.	1.6	1.61	2.16	3.73	1.73	2.1
time (sec)	N/A	0.067	0.015	0.042	1.121	1.411	0.278	1.265

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	67	76	99	177	80	100
normalized size	1	1.	1.08	1.23	1.6	2.85	1.29	1.61
time (sec)	N/A	0.049	0.016	0.045	1.136	1.43	0.288	1.296

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	49	52	69	128	54	72
normalized size	1	1.	0.89	0.95	1.25	2.33	0.98	1.31
time (sec)	N/A	0.038	0.012	0.041	1.105	1.479	0.119	1.339

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	29	28	36	74	29	42
normalized size	1	1.	0.88	0.85	1.09	2.24	0.88	1.27
time (sec)	N/A	0.022	0.005	0.043	1.117	1.435	0.283	1.332

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	31	12	18
normalized size	1	1.	1.	0.82	1.06	1.82	0.71	1.06
time (sec)	N/A	0.003	0.	0.043	1.159	1.357	0.153	1.287

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	41	52	61	103	37	63
normalized size	1	1.	0.91	1.16	1.36	2.29	0.82	1.4
time (sec)	N/A	0.034	0.015	0.044	1.111	1.527	1.245	1.382

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	41	61	72	149	44	126
normalized size	1	1.	0.85	1.27	1.5	3.1	0.92	2.62
time (sec)	N/A	0.037	0.026	0.049	1.117	1.647	1.393	1.413

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	52	70	88	173	63	74
normalized size	1	1.	0.95	1.27	1.6	3.15	1.15	1.35
time (sec)	N/A	0.037	0.018	0.049	1.16	1.631	2.279	1.319

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	44	56	96	149	75	61
normalized size	1	1.	0.73	0.93	1.6	2.48	1.25	1.02
time (sec)	N/A	0.036	0.017	0.048	1.128	1.453	2.537	1.354

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	43	56	108	166	85	101
normalized size	1	1.	0.69	0.9	1.74	2.68	1.37	1.63
time (sec)	N/A	0.036	0.016	0.048	1.093	1.662	1.327	1.251

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	159	166	217	387	178	228
normalized size	1	1.	1.16	1.21	1.58	2.82	1.3	1.66
time (sec)	N/A	0.141	0.026	0.044	1.161	1.389	0.375	1.253

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	127	128	171	301	138	177
normalized size	1	1.	1.	1.01	1.35	2.37	1.09	1.39
time (sec)	N/A	0.093	0.017	0.042	1.156	1.553	0.235	1.262

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	90	115	217	94	127
normalized size	1	1.	1.	1.03	1.32	2.49	1.08	1.46
time (sec)	N/A	0.076	0.014	0.045	1.119	1.401	0.266	1.169

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	50	52	69	128	54	76
normalized size	1	1.	0.91	0.95	1.25	2.33	0.98	1.38
time (sec)	N/A	0.04	0.01	0.043	1.155	1.452	0.278	1.469

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	55	24	32
normalized size	1	1.	1.	0.83	1.07	1.83	0.8	1.07
time (sec)	N/A	0.009	0.002	0.04	1.155	1.227	0.253	1.313

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	106	152	177	279	112	181
normalized size	1	1.	1.14	1.63	1.9	3.	1.2	1.95
time (sec)	N/A	0.082	0.042	0.045	1.157	1.533	0.959	1.282

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	114	164	186	416	122	248
normalized size	1	1.	1.07	1.53	1.74	3.89	1.14	2.32
time (sec)	N/A	0.102	0.097	0.052	1.105	1.656	1.738	1.352

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	116	178	198	486	153	181
normalized size	1	1.	0.97	1.5	1.66	4.08	1.29	1.52
time (sec)	N/A	0.106	0.069	0.05	1.137	1.6	2.536	1.293

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	134	189	215	494	163	177
normalized size	1	1.	1.12	1.58	1.79	4.12	1.36	1.48
time (sec)	N/A	0.096	0.056	0.05	1.055	1.612	4.223	1.324

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	126	197	239	456	180	289
normalized size	1	1.	0.96	1.5	1.82	3.48	1.37	2.21
time (sec)	N/A	0.092	0.045	0.05	1.172	1.579	5.067	1.347

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	116	143	244	374	192	178
normalized size	1	1.	0.88	1.08	1.85	2.83	1.45	1.35
time (sec)	N/A	0.09	0.049	0.048	1.032	1.678	10.562	1.297

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	116	143	258	396	204	178
normalized size	1	1.	0.85	1.04	1.88	2.89	1.49	1.3
time (sec)	N/A	0.092	0.041	0.052	1.158	1.631	13.827	1.285

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	117	143	279	435	218	180
normalized size	1	1.	0.85	1.04	2.04	3.18	1.59	1.31
time (sec)	N/A	0.088	0.048	0.056	1.166	1.569	22.319	1.254

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	225	232	309	556	257	327
normalized size	1	1.	1.	1.03	1.37	2.47	1.14	1.45
time (sec)	N/A	0.215	0.035	0.043	1.185	1.463	0.219	1.332

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	169	180	231	433	199	255
normalized size	1	1.	1.04	1.11	1.43	2.67	1.23	1.57
time (sec)	N/A	0.156	0.026	0.042	1.142	1.376	0.336	1.265

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	127	128	171	301	138	181
normalized size	1	1.	1.	1.01	1.35	2.37	1.09	1.43
time (sec)	N/A	0.11	0.024	0.043	1.131	1.421	0.238	1.313

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	76	99	182	80	109
normalized size	1	1.	1.	1.01	1.32	2.43	1.07	1.45
time (sec)	N/A	0.063	0.01	0.044	1.113	1.353	0.222	1.228

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	47	80	37	47
normalized size	1	1.	1.	0.84	1.09	1.86	0.86	1.09
time (sec)	N/A	0.015	0.002	0.041	1.125	1.336	0.154	1.28

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	144	302	356	537	231	365
normalized size	1	1.	0.95	2.	2.36	3.56	1.53	2.42
time (sec)	N/A	0.151	0.083	0.046	1.126	1.669	1.733	1.304

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	160	318	369	751	248	448
normalized size	1	1.	0.96	1.92	2.22	4.52	1.49	2.7
time (sec)	N/A	0.186	0.062	0.056	1.037	1.449	3.467	1.439

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	207	335	378	868	277	356
normalized size	1	1.	1.03	1.68	1.89	4.34	1.38	1.78
time (sec)	N/A	0.219	0.131	0.051	1.156	1.66	5.792	1.361

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	210	353	397	996	298	352
normalized size	1	1.	0.99	1.66	1.86	4.68	1.4	1.65
time (sec)	N/A	0.215	0.108	0.054	1.166	1.615	12.896	1.316

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	210	370	409	976	314	522
normalized size	1	1.	0.99	1.74	1.92	4.58	1.47	2.45
time (sec)	N/A	0.197	0.114	0.054	1.203	1.584	14.425	1.278

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	242	379	420	953	326	339
normalized size	1	1.	1.11	1.74	1.93	4.37	1.5	1.56
time (sec)	N/A	0.184	0.108	0.053	1.186	1.741	38.99	1.266

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	231	387	447	840	343	351
normalized size	1	1.	1.01	1.7	1.96	3.68	1.5	1.54
time (sec)	N/A	0.175	0.081	0.05	1.204	1.663	103.83	1.262

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	221	274	451	701	0	360
normalized size	1	1.	0.96	1.19	1.96	3.05	0.	1.57
time (sec)	N/A	0.159	0.071	0.048	1.171	1.758	0.	1.239

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	221	274	464	710	0	360
normalized size	1	1.	0.96	1.19	2.01	3.07	0.	1.56
time (sec)	N/A	0.158	0.081	0.049	1.079	1.838	0.	1.318

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	222	274	487	776	0	362
normalized size	1	1.	0.95	1.17	2.08	3.32	0.	1.55
time (sec)	N/A	0.157	0.075	0.051	1.234	1.606	0.	1.257

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	90	162	192	312	165	184
normalized size	1	1.	0.91	1.64	1.94	3.15	1.67	1.86
time (sec)	N/A	0.091	0.045	0.053	1.141	1.704	5.847	1.245

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	59	103	123	204	112	117
normalized size	1	1.	0.92	1.61	1.92	3.19	1.75	1.83
time (sec)	N/A	0.057	0.028	0.048	1.096	1.705	5.588	1.305

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	61	72	115	73	73
normalized size	1	1.	1.	1.45	1.71	2.74	1.74	1.74
time (sec)	N/A	0.034	0.019	0.052	1.103	1.689	2.639	1.282

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	29	32	41	63	41	45
normalized size	1	1.	0.97	1.07	1.37	2.1	1.37	1.5
time (sec)	N/A	0.021	0.01	0.052	1.135	1.68	2.027	1.32

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	24	38	10	27
normalized size	1	1.	1.	1.06	1.33	2.11	0.56	1.5
time (sec)	N/A	0.003	0.003	0.045	1.11	1.658	0.59	1.265

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	48	54	72	109	583	169
normalized size	1	1.	0.91	1.02	1.36	2.06	11.	3.19
time (sec)	N/A	0.046	0.023	0.052	1.105	2.231	78.829	1.369

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	83	105	173	420	1238	389
normalized size	1	1.	0.95	1.21	1.99	4.83	14.23	4.47
time (sec)	N/A	0.078	0.108	0.085	1.172	6.314	161.156	1.363

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	116	184	359	1008	0	306
normalized size	1	1.	0.87	1.37	2.68	7.52	0.	2.28
time (sec)	N/A	0.12	0.258	0.056	1.158	45.312	0.	1.27

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	116	251	292	707	379	282
normalized size	1	1.	0.98	2.13	2.47	5.99	3.21	2.39
time (sec)	N/A	0.138	0.057	0.061	0.998	1.754	8.457	1.303

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	95	188	220	504	306	216
normalized size	1	1.	1.01	2.	2.34	5.36	3.26	2.3
time (sec)	N/A	0.1	0.094	0.055	1.124	1.746	7.363	1.36

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	79	141	178	392	250	174
normalized size	1	1.	0.91	1.62	2.05	4.51	2.87	2.
time (sec)	N/A	0.082	0.075	0.057	1.147	1.722	3.594	1.378

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	67	106	126	297	173	136
normalized size	1	1.	0.92	1.45	1.73	4.07	2.37	1.86
time (sec)	N/A	0.058	0.077	0.057	1.145	1.743	1.601	1.238

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	56	78	93	227	128	104
normalized size	1	1.	0.86	1.2	1.43	3.49	1.97	1.6
time (sec)	N/A	0.05	0.04	0.054	1.131	1.689	1.453	1.25

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	35	43	61	138	36	61
normalized size	1	1.	0.81	1.	1.42	3.21	0.84	1.42
time (sec)	N/A	0.01	0.038	0.056	1.137	1.719	1.224	1.303

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	111	132	239	574	0	273
normalized size	1	1.	1.01	1.2	2.17	5.22	0.	2.48
time (sec)	N/A	0.118	0.096	0.105	1.177	24.745	0.	1.298

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	145	185	504	1262	0	749
normalized size	1	1.	1.01	1.28	3.5	8.76	0.	5.2
time (sec)	N/A	0.167	0.179	0.065	1.283	85.786	0.	1.84

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	202	481	551	1384	685	517
normalized size	1	1.	1.	2.37	2.71	6.82	3.37	2.55
time (sec)	N/A	0.269	0.123	0.066	1.172	2.127	61.728	2.011

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	179	396	460	1154	597	427
normalized size	1	1.	1.	2.21	2.57	6.45	3.34	2.39
time (sec)	N/A	0.222	0.087	0.062	1.004	1.985	30.123	3.197

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	165	329	400	991	524	371
normalized size	1	1.	0.96	1.92	2.34	5.8	3.06	2.17
time (sec)	N/A	0.178	0.108	0.06	1.136	2.018	12.101	1.327

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	130	278	338	834	389	343
normalized size	1	1.	0.96	2.04	2.49	6.13	2.86	2.52
time (sec)	N/A	0.152	0.084	0.062	1.024	1.86	8.518	1.276

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	138	238	293	770	371	296
normalized size	1	1.	1.01	1.74	2.14	5.62	2.71	2.16
time (sec)	N/A	0.147	0.155	0.058	1.083	1.935	4.002	1.268

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	144	207	243	664	345	246
normalized size	1	1.	1.	1.44	1.69	4.61	2.4	1.71
time (sec)	N/A	0.133	0.094	0.056	1.112	1.8	2.589	1.265

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	102	138	184	467	219	178
normalized size	1	1.	0.93	1.25	1.67	4.25	1.99	1.62
time (sec)	N/A	0.103	0.081	0.056	1.123	1.878	1.745	1.258

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	68	73	116	269	78	99
normalized size	1	1.	0.94	1.01	1.61	3.74	1.08	1.38
time (sec)	N/A	0.019	0.046	0.051	1.125	1.69	1.506	1.296

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	192	254	593	0	0	559
normalized size	1	1.	0.99	1.32	3.07	0.	0.	2.9
time (sec)	N/A	0.227	0.196	0.061	1.238	0.	0.	1.334

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	230	306	1015	0	0	1133
normalized size	1	1.	1.	1.33	4.41	0.	0.	4.93
time (sec)	N/A	0.339	0.29	0.069	1.291	0.	0.	1.356

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	230	444	0	1112	0	338
normalized size	1	1.	1.1	2.11	0.	5.3	0.	1.61
time (sec)	N/A	0.269	0.537	0.053	0.	2.763	0.	1.319

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	164	287	0	757	0	232
normalized size	1	1.	1.01	1.77	0.	4.67	0.	1.43
time (sec)	N/A	0.126	0.298	0.052	0.	2.666	0.	1.32

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	108	157	0	467	0	146
normalized size	1	1.	1.09	1.59	0.	4.72	0.	1.47
time (sec)	N/A	0.036	0.19	0.05	0.	2.489	0.	1.313

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	74	56	0	285	0	82
normalized size	1	1.	1.23	0.93	0.	4.75	0.	1.37
time (sec)	N/A	0.015	0.099	0.045	0.	2.229	0.	1.157

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	137	490	0	1102	0	0
normalized size	1	1.	1.06	3.8	0.	8.54	0.	0.
time (sec)	N/A	0.141	0.524	0.249	0.	2.598	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	147	885	0	1775	0	0
normalized size	1	1.	1.05	6.32	0.	12.68	0.	0.
time (sec)	N/A	0.111	0.601	0.213	0.	2.595	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	121	1963	0	977	0	552
normalized size	1	1.	0.95	15.46	0.	7.69	0.	4.35
time (sec)	N/A	0.088	0.216	0.23	0.	2.279	0.	1.403

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	191	2891	0	1925	0	1114
normalized size	1	1.	1.04	15.8	0.	10.52	0.	6.09
time (sec)	N/A	0.138	0.293	0.214	0.	2.417	0.	1.488

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	243	4819	0	3313	0	1661
normalized size	1	1.	0.94	18.68	0.	12.84	0.	6.44
time (sec)	N/A	0.319	0.761	0.222	0.	2.551	0.	1.928

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	309	6533	0	5138	0	2836
normalized size	1	1.	0.92	19.39	0.	15.25	0.	8.42
time (sec)	N/A	0.456	1.277	0.241	0.	2.789	0.	1.668

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	312	629	0	1586	0	493
normalized size	1	1.	1.15	2.32	0.	5.85	0.	1.82
time (sec)	N/A	0.35	0.702	0.056	0.	2.409	0.	1.342

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	197	420	0	1108	0	354
normalized size	1	1.	0.92	1.96	0.	5.18	0.	1.65
time (sec)	N/A	0.179	0.465	0.055	0.	2.33	0.	1.389

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	146	239	0	689	0	231
normalized size	1	1.	1.07	1.74	0.	5.03	0.	1.69
time (sec)	N/A	0.053	0.265	0.049	0.	2.353	0.	1.278

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	98	95	0	393	0	112
normalized size	1	1.	1.1	1.07	0.	4.42	0.	1.26
time (sec)	N/A	0.025	0.088	0.051	0.	2.368	0.	1.286

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	225	1090	0	1979	0	0
normalized size	1	1.	1.04	5.05	0.	9.16	0.	0.
time (sec)	N/A	0.242	0.826	0.209	0.	6.067	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	218	1569	0	2187	0	0
normalized size	1	1.	1.1	7.92	0.	11.05	0.	0.
time (sec)	N/A	0.234	1.045	0.216	0.	2.968	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	205	3466	0	3626	0	675
normalized size	1	1.	1.	16.91	0.	17.69	0.	3.29
time (sec)	N/A	0.198	1.266	0.235	0.	2.894	0.	2.326

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	332	395	813	0	2109	0	648
normalized size	1	1.	1.19	2.45	0.	6.35	0.	1.95
time (sec)	N/A	0.454	0.893	0.056	0.	2.222	0.	1.735

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	219	553	0	1481	0	473
normalized size	1	1.	0.82	2.08	0.	5.57	0.	1.78
time (sec)	N/A	0.22	0.591	0.052	0.	2.164	0.	1.9

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	171	321	0	913	0	315
normalized size	1	1.	0.98	1.83	0.	5.22	0.	1.8
time (sec)	N/A	0.074	0.317	0.049	0.	2.073	0.	1.556

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	120	134	0	501	0	144
normalized size	1	1.	1.02	1.14	0.	4.25	0.	1.22
time (sec)	N/A	0.037	0.122	0.049	0.	2.022	0.	1.358

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	727	1932	0	3449	0	0
normalized size	1	1.	2.04	5.43	0.	9.69	0.	0.
time (sec)	N/A	0.434	3.783	0.233	0.	31.844	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	334	2534	0	4049	0	0
normalized size	1	1.	1.06	8.07	0.	12.89	0.	0.
time (sec)	N/A	0.393	1.827	0.272	0.	10.587	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	322	5534	0	4356	0	988
normalized size	1	1.	1.14	19.62	0.	15.45	0.	3.5
time (sec)	N/A	0.326	2.124	0.23	0.	4.169	0.	1.669

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	26	26	38	21	23	73	0	36
normalized size	1	1.	1.46	0.81	0.88	2.81	0.	1.38
time (sec)	N/A	0.014	0.029	0.06	1.727	2.092	0.	1.321

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	53	53	48	42	78	150	0	63
normalized size	1	1.	0.91	0.79	1.47	2.83	0.	1.19
time (sec)	N/A	0.024	0.06	0.052	1.678	2.13	0.	1.315

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	36	36	46	29	61	130	0	54
normalized size	1	1.	1.28	0.81	1.69	3.61	0.	1.5
time (sec)	N/A	0.017	0.039	0.047	1.692	2.21	0.	1.378

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	38	15	42	97	0	35
normalized size	1	1.	2.11	0.83	2.33	5.39	0.	1.94
time (sec)	N/A	0.01	0.011	0.046	1.169	1.998	0.	1.418

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	36	36	42	29	61	169	0	66
normalized size	1	1.	1.17	0.81	1.69	4.69	0.	1.83
time (sec)	N/A	0.017	0.02	0.048	1.121	2.015	0.	1.359

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	53	53	50	42	78	239	0	77
normalized size	1	1.	0.94	0.79	1.47	4.51	0.	1.45
time (sec)	N/A	0.024	0.043	0.049	1.072	1.89	0.	1.406

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	152	265	0	666	0	198
normalized size	1	1.	1.02	1.78	0.	4.47	0.	1.33
time (sec)	N/A	0.149	0.465	0.057	0.	2.031	0.	1.324

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	111	158	0	432	0	131
normalized size	1	1.	1.01	1.44	0.	3.93	0.	1.19
time (sec)	N/A	0.077	0.105	0.053	0.	2.117	0.	1.449

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	80	78	0	274	0	85
normalized size	1	1.	1.45	1.42	0.	4.98	0.	1.55
time (sec)	N/A	0.022	0.062	0.05	0.	1.899	0.	1.364

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	57	29	0	154	0	47
normalized size	1	1.	2.04	1.04	0.	5.5	0.	1.68
time (sec)	N/A	0.009	0.017	0.05	0.	1.889	0.	1.226

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	77	132	0	284	0	82
normalized size	1	1.	1.13	1.94	0.	4.18	0.	1.21
time (sec)	N/A	0.031	0.026	0.216	0.	2.059	0.	1.293

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	122	355	0	716	0	540
normalized size	1	1.	1.04	3.03	0.	6.12	0.	4.62
time (sec)	N/A	0.075	0.144	0.257	0.	2.046	0.	2.004

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	183	798	0	1501	0	657
normalized size	1	1.	1.02	4.43	0.	8.34	0.	3.65
time (sec)	N/A	0.173	0.257	0.265	0.	2.033	0.	1.316

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	129	177	0	730	0	169
normalized size	1	1.	0.93	1.27	0.	5.25	0.	1.22
time (sec)	N/A	0.093	0.112	0.054	0.	2.112	0.	1.322

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	100	97	0	505	0	120
normalized size	1	1.	0.99	0.96	0.	5.	0.	1.19
time (sec)	N/A	0.062	0.096	0.061	0.	2.055	0.	1.265

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	30	37	74	89	0	46
normalized size	1	1.	0.91	1.12	2.24	2.7	0.	1.39
time (sec)	N/A	0.01	0.012	0.047	1.053	1.965	0.	1.33

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	29	47	73	0	32
normalized size	1	1.	0.92	1.21	1.96	3.04	0.	1.33
time (sec)	N/A	0.004	0.006	0.044	1.129	1.819	0.	1.441

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	129	403	0	909	0	225
normalized size	1	1.	1.02	3.2	0.	7.21	0.	1.79
time (sec)	N/A	0.096	0.101	0.258	0.	1.987	0.	1.4

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	206	893	0	1791	0	0
normalized size	1	1.	1.	4.31	0.	8.65	0.	0.
time (sec)	N/A	0.235	0.189	0.228	0.	2.135	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	247	1612	0	3347	0	975
normalized size	1	1.	0.83	5.45	0.	11.31	0.	3.29
time (sec)	N/A	0.344	0.892	0.244	0.	2.538	0.	1.972

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	208	208	389	447	0	1050	0	282
normalized size	1	1.	1.87	2.15	0.	5.05	0.	1.36
time (sec)	N/A	0.198	2.011	0.055	0.	2.058	0.	1.937

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	105	136	401	298	0	188
normalized size	1	1.	1.21	1.56	4.61	3.43	0.	2.16
time (sec)	N/A	0.036	0.048	0.047	1.173	1.945	0.	1.285

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	95	117	274	259	0	166
normalized size	1	1.	1.22	1.5	3.51	3.32	0.	2.13
time (sec)	N/A	0.033	0.04	0.047	1.077	2.009	0.	1.33

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	67	83	176	209	0	136
normalized size	1	1.	0.94	1.17	2.48	2.94	0.	1.92
time (sec)	N/A	0.018	0.024	0.046	1.149	1.969	0.	1.395

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	48	51	97	144	0	68
normalized size	1	1.	0.89	0.94	1.8	2.67	0.	1.26
time (sec)	N/A	0.01	0.012	0.044	1.105	1.937	0.	1.386

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	238	950	0	1993	0	562
normalized size	1	1.	1.03	4.13	0.	8.67	0.	2.44
time (sec)	N/A	0.179	0.298	0.219	0.	2.115	0.	1.411

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	316	1857	0	3553	0	0
normalized size	1	1.	0.91	5.34	0.	10.21	0.	0.
time (sec)	N/A	0.349	0.989	0.219	0.	2.746	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	11	12	20	47	0	24
normalized size	1	1.	0.65	0.71	1.18	2.76	0.	1.41
time (sec)	N/A	0.006	0.005	0.046	1.112	1.867	0.	1.617

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	50	47	73	325	292	598
normalized size	1	1.	0.74	0.69	1.07	4.78	4.29	8.79
time (sec)	N/A	0.029	0.044	0.046	1.057	1.883	15.1	1.466

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	50	47	73	266	245	393
normalized size	1	1.	0.74	0.69	1.07	3.91	3.6	5.78
time (sec)	N/A	0.028	0.037	0.046	1.103	1.997	4.957	1.533

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	50	47	73	212	178	224
normalized size	1	1.	0.74	0.69	1.07	3.12	2.62	3.29
time (sec)	N/A	0.028	0.033	0.043	1.08	1.864	9.544	2.414

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	50	47	73	161	66	96
normalized size	1	1.	0.74	0.69	1.07	2.37	0.97	1.41
time (sec)	N/A	0.026	0.031	0.045	1.057	1.876	3.144	2.294

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	49	47	90	112	182	93
normalized size	1	1.	0.74	0.71	1.36	1.7	2.76	1.41
time (sec)	N/A	0.027	0.031	0.048	1.062	1.822	11.91	1.317

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	48	46	82	123	60	93
normalized size	1	1.	0.75	0.72	1.28	1.92	0.94	1.45
time (sec)	N/A	0.027	0.031	0.049	1.085	1.889	10.588	1.286

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	50	47	78	147	211	80
normalized size	1	1.	0.78	0.73	1.22	2.3	3.3	1.25
time (sec)	N/A	0.028	0.032	0.046	0.998	1.85	1.598	1.347

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	49	47	68	173	314	77
normalized size	1	1.	0.74	0.71	1.03	2.62	4.76	1.17
time (sec)	N/A	0.027	0.031	0.046	1.	1.85	4.171	1.234

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	124	141	188	662	590	1237
normalized size	1	1.	0.84	0.96	1.28	4.5	4.01	8.41
time (sec)	N/A	0.072	0.096	0.048	1.099	1.959	21.854	1.356

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	124	141	188	564	695	844
normalized size	1	1.	0.84	0.96	1.28	3.84	4.73	5.74
time (sec)	N/A	0.065	0.078	0.049	1.136	1.902	30.128	1.359

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	125	141	188	493	413	506
normalized size	1	1.	0.85	0.96	1.28	3.35	2.81	3.44
time (sec)	N/A	0.061	0.078	0.05	1.218	1.894	18.719	1.425

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	124	141	188	392	173	227
normalized size	1	1.	0.84	0.96	1.28	2.67	1.18	1.54
time (sec)	N/A	0.059	0.072	0.05	1.118	1.918	4.651	1.282

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	124	141	216	312	418	227
normalized size	1	1.	0.86	0.97	1.49	2.15	2.88	1.57
time (sec)	N/A	0.059	0.076	0.05	1.175	1.917	62.131	1.277

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	123	141	198	324	150	254
normalized size	1	1.	0.86	0.99	1.38	2.27	1.05	1.78
time (sec)	N/A	0.058	0.071	0.049	1.194	1.811	24.154	1.285

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	123	141	196	343	139	246
normalized size	1	1.	0.86	0.99	1.37	2.4	0.97	1.72
time (sec)	N/A	0.058	0.066	0.049	1.039	1.946	41.188	1.386

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	123	141	198	359	787	242
normalized size	1	1.	0.86	0.99	1.38	2.51	5.5	1.69
time (sec)	N/A	0.058	0.066	0.048	1.161	2.003	4.844	1.347

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	206	286	366	1141	1741	2086
normalized size	1	1.	0.83	1.15	1.48	4.6	7.02	8.41
time (sec)	N/A	0.139	0.197	0.049	1.063	2.07	83.63	1.514

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	206	286	366	1002	1207	1454
normalized size	1	1.	0.83	1.15	1.48	4.04	4.87	5.86
time (sec)	N/A	0.108	0.161	0.048	1.132	1.949	56.681	1.469

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	206	286	366	841	738	892
normalized size	1	1.	0.83	1.15	1.48	3.39	2.98	3.6
time (sec)	N/A	0.106	0.161	0.049	1.149	2.032	34.445	1.354

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	206	286	366	720	326	410
normalized size	1	1.	0.83	1.15	1.48	2.9	1.31	1.65
time (sec)	N/A	0.106	0.15	0.048	1.135	2.232	6.264	1.336

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	206	286	389	630	745	412
normalized size	1	1.	0.84	1.17	1.59	2.58	3.05	1.69
time (sec)	N/A	0.102	0.135	0.053	1.148	2.197	115.984	1.316

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	206	286	377	629	284	501
normalized size	1	1.	0.85	1.18	1.56	2.6	1.17	2.07
time (sec)	N/A	0.104	0.143	0.047	1.157	2.304	52.028	1.369

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	206	286	374	645	260	487
normalized size	1	1.	0.84	1.17	1.53	2.64	1.07	2.
time (sec)	N/A	0.101	0.14	0.048	1.158	2.413	56.73	1.39

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	206	286	374	645	248	485
normalized size	1	1.	0.86	1.19	1.56	2.69	1.03	2.02
time (sec)	N/A	0.106	0.139	0.049	1.129	2.431	82.596	1.308

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	138	336	0	1820	162	309
normalized size	1	1.	0.88	2.14	0.	11.59	1.03	1.97
time (sec)	N/A	0.387	0.206	0.246	0.	9.517	92.468	1.252

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	107	237	0	1350	119	217
normalized size	1	1.	0.91	2.01	0.	11.44	1.01	1.84
time (sec)	N/A	0.228	0.123	0.223	0.	3.867	56.124	1.344

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	97	159	0	1007	92	151
normalized size	1	1.	1.05	1.73	0.	10.95	1.	1.64
time (sec)	N/A	0.196	0.061	0.223	0.	2.609	33.784	1.375

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	75	100	0	788	78	108
normalized size	1	1.	0.97	1.3	0.	10.23	1.01	1.4
time (sec)	N/A	0.072	0.036	0.217	0.	2.385	4.907	1.18

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	75	62	0	880	80	96
normalized size	1	1.	0.97	0.81	0.	11.43	1.04	1.25
time (sec)	N/A	0.058	0.084	0.25	0.	2.332	17.203	1.18

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	81	97	0	1553	94	153
normalized size	1	1.	0.79	0.95	0.	15.23	0.92	1.5
time (sec)	N/A	0.318	0.027	0.231	0.	2.733	17.973	1.258

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	83	147	0	3040	133	235
normalized size	1	1.	0.6	1.07	0.	22.03	0.96	1.7
time (sec)	N/A	0.261	0.027	0.224	0.	3.591	21.77	1.321

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	83	228	0	5191	182	389
normalized size	1	1.	0.44	1.22	0.	27.76	0.97	2.08
time (sec)	N/A	0.377	0.03	0.235	0.	8.155	48.79	1.304

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	202	515	0	3251	0	589
normalized size	1	1.	0.8	2.05	0.	12.95	0.	2.35
time (sec)	N/A	0.534	0.396	0.28	0.	22.86	0.	1.44

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	167	403	0	2628	0	464
normalized size	1	1.	0.84	2.02	0.	13.14	0.	2.32
time (sec)	N/A	0.395	0.311	0.229	0.	6.248	0.	1.384

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	159	313	0	2122	0	369
normalized size	1	1.	1.	1.97	0.	13.35	0.	2.32
time (sec)	N/A	0.302	0.276	0.242	0.	3.038	0.	1.388

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	127	237	0	1679	0	285
normalized size	1	1.	0.95	1.77	0.	12.53	0.	2.13
time (sec)	N/A	0.183	0.214	0.264	0.	2.218	0.	1.383

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	169	167	0	1750	790	244
normalized size	1	1.	1.35	1.34	0.	14.	6.32	1.95
time (sec)	N/A	0.234	0.217	0.221	0.	2.451	49.294	1.324

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	148	202	0	2396	0	342
normalized size	1	1.	0.96	1.31	0.	15.56	0.	2.22
time (sec)	N/A	0.261	0.499	0.227	0.	7.389	0.	1.321

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	166	229	0	4633	0	477
normalized size	1	1.	0.81	1.11	0.	22.49	0.	2.32
time (sec)	N/A	0.353	0.107	0.243	0.	6.296	0.	1.361

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	170	280	0	7775	0	649
normalized size	1	1.	0.64	1.05	0.	29.12	0.	2.43
time (sec)	N/A	0.515	0.115	0.254	0.	18.61	0.	1.446

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	171	364	0	11756	0	868
normalized size	1	1.	0.49	1.04	0.	33.68	0.	2.49
time (sec)	N/A	0.739	0.118	0.237	0.	45.955	0.	1.405

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	275	703	0	4871	0	852
normalized size	1	1.	0.92	2.34	0.	16.24	0.	2.84
time (sec)	N/A	0.52	0.642	0.27	0.	12.682	0.	1.367

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	263	627	0	4234	0	745
normalized size	1	1.	1.06	2.53	0.	17.07	0.	3.
time (sec)	N/A	0.386	0.502	0.226	0.	5.157	0.	1.525

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	222	521	0	3510	0	605
normalized size	1	1.	0.96	2.25	0.	15.13	0.	2.61
time (sec)	N/A	0.307	0.427	0.244	0.	2.912	0.	1.442

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	289	414	0	3532	0	529
normalized size	1	1.	1.17	1.68	0.	14.36	0.	2.15
time (sec)	N/A	0.465	0.533	0.232	0.	3.016	0.	1.371

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	361	436	0	4645	0	686
normalized size	1	1.	1.47	1.78	0.	18.96	0.	2.8
time (sec)	N/A	0.378	0.976	0.234	0.	4.484	0.	1.275

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	299	526	0	5762	0	836
normalized size	1	1.	1.	1.76	0.	19.27	0.	2.8
time (sec)	N/A	0.43	1.222	0.358	0.	11.389	0.	1.286

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	299	530	0	9646	0	1062
normalized size	1	1.	0.81	1.43	0.	26.07	0.	2.87
time (sec)	N/A	0.6	0.289	0.399	0.	39.569	0.	1.44

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	470	470	301	582	0	14445	0	1272
normalized size	1	1.	0.64	1.24	0.	30.73	0.	2.71
time (sec)	N/A	0.879	0.294	0.253	0.	105.111	0.	1.844

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	362	362	372	920	0	0	0	0
normalized size	1	1.	1.03	2.54	0.	0.	0.	0.
time (sec)	N/A	0.394	1.765	0.343	0.	0.	0.	0.

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	294	681	0	0	0	0
normalized size	1	1.	0.95	2.21	0.	0.	0.	0.
time (sec)	N/A	0.341	1.126	0.289	0.	0.	0.	0.

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	226	467	0	0	0	0
normalized size	1	1.	0.92	1.9	0.	0.	0.	0.
time (sec)	N/A	0.19	1.398	0.273	0.	0.	0.	0.

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	195	353	0	0	0	0
normalized size	1	1.	0.84	1.53	0.	0.	0.	0.
time (sec)	N/A	0.156	0.561	0.338	0.	0.	0.	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	265	887	0	0	0	0
normalized size	1	1.	0.88	2.95	0.	0.	0.	0.
time (sec)	N/A	0.28	1.143	0.293	0.	0.	0.	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	398	398	362	1897	0	0	0	0
normalized size	1	1.	0.91	4.77	0.	0.	0.	0.
time (sec)	N/A	0.491	1.316	0.303	0.	0.	0.	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	521	521	559	1359	0	0	0	0
normalized size	1	1.	1.07	2.61	0.	0.	0.	0.
time (sec)	N/A	0.729	2.826	0.288	0.	0.	0.	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	457	457	463	1170	0	0	0	0
normalized size	1	1.	1.01	2.56	0.	0.	0.	0.
time (sec)	N/A	0.578	2.36	0.29	0.	0.	0.	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	380	918	0	0	0	0
normalized size	1	1.	1.06	2.55	0.	0.	0.	0.
time (sec)	N/A	0.353	2.02	0.271	0.	0.	0.	0.

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	340	685	0	0	0	0
normalized size	1	1.	1.1	2.22	0.	0.	0.	0.
time (sec)	N/A	0.338	1.688	0.283	0.	0.	0.	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	279	1051	0	0	0	0
normalized size	1	1.	0.94	3.53	0.	0.	0.	0.
time (sec)	N/A	0.3	1.207	0.285	0.	0.	0.	0.

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	354	369	1885	0	0	0	0
normalized size	1	1.	1.04	5.32	0.	0.	0.	0.
time (sec)	N/A	0.374	1.198	0.302	0.	0.	0.	0.

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	476	476	479	3267	0	0	0	0
normalized size	1	1.	1.01	6.86	0.	0.	0.	0.
time (sec)	N/A	0.588	2.278	0.324	0.	0.	0.	0.

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	666	666	663	1728	0	0	0	0
normalized size	1	1.	1.	2.59	0.	0.	0.	0.
time (sec)	N/A	0.833	3.283	0.287	0.	0.	0.	0.

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	537	537	557	1441	0	0	0	0
normalized size	1	1.	1.04	2.68	0.	0.	0.	0.
time (sec)	N/A	0.604	2.876	0.279	0.	0.	0.	0.

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	457	457	498	1170	0	0	0	0
normalized size	1	1.	1.09	2.56	0.	0.	0.	0.
time (sec)	N/A	0.588	2.414	0.29	0.	0.	0.	0.

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	401	401	442	1692	0	0	0	0
normalized size	1	1.	1.1	4.22	0.	0.	0.	0.
time (sec)	N/A	0.477	2.205	0.3	0.	0.	0.	0.

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	392	392	401	2170	0	0	0	0
normalized size	1	1.	1.02	5.54	0.	0.	0.	0.
time (sec)	N/A	0.429	1.629	0.301	0.	0.	0.	0.

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	474	474	500	3284	0	0	0	0
normalized size	1	1.	1.05	6.93	0.	0.	0.	0.
time (sec)	N/A	0.562	2.529	0.307	0.	0.	0.	0.

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	570	570	610	5005	0	0	0	0
normalized size	1	1.	1.07	8.78	0.	0.	0.	0.
time (sec)	N/A	0.68	3.554	0.337	0.	0.	0.	0.

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	379	379	388	918	0	0	0	0
normalized size	1	1.	1.02	2.42	0.	0.	0.	0.
time (sec)	N/A	0.503	2.229	0.294	0.	0.	0.	0.

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	314	682	0	0	0	0
normalized size	1	1.	1.04	2.25	0.	0.	0.	0.
time (sec)	N/A	0.33	1.285	0.272	0.	0.	0.	0.

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	246	460	0	0	0	0
normalized size	1	1.	1.02	1.91	0.	0.	0.	0.
time (sec)	N/A	0.192	1.02	0.278	0.	0.	0.	0.

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	121	121	0	0	0	0
normalized size	1	1.	1.29	1.29	0.	0.	0.	0.
time (sec)	N/A	0.037	0.564	0.262	0.	0.	0.	0.

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	94	113	0	0	0	0
normalized size	1	1.	1.	1.2	0.	0.	0.	0.
time (sec)	N/A	0.04	0.122	0.268	0.	0.	0.	0.

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	127	216	0	0	0	0
normalized size	1	1.	0.87	1.48	0.	0.	0.	0.
time (sec)	N/A	0.081	0.161	0.291	0.	0.	0.	0.

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	290	897	0	0	0	0
normalized size	1	1.	0.91	2.83	0.	0.	0.	0.
time (sec)	N/A	0.288	1.183	0.319	0.	0.	0.	0.

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	403	381	1912	0	0	0	0
normalized size	1	1.	0.95	4.74	0.	0.	0.	0.
time (sec)	N/A	0.465	1.283	0.317	0.	0.	0.	0.

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	395	360	863	0	0	0	0
normalized size	1	1.	0.91	2.18	0.	0.	0.	0.
time (sec)	N/A	0.514	1.245	0.361	0.	0.	0.	0.

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	262	652	0	0	0	0
normalized size	1	1.	0.85	2.1	0.	0.	0.	0.
time (sec)	N/A	0.339	1.165	0.31	0.	0.	0.	0.

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	210	451	0	0	0	0
normalized size	1	1.	0.84	1.81	0.	0.	0.	0.
time (sec)	N/A	0.191	1.105	0.298	0.	0.	0.	0.

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	186	352	0	0	0	0
normalized size	1	1.	0.81	1.52	0.	0.	0.	0.
time (sec)	N/A	0.154	0.416	0.286	0.	0.	0.	0.

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	220	480	0	0	0	0
normalized size	1	1.	0.8	1.75	0.	0.	0.	0.
time (sec)	N/A	0.219	0.592	0.317	0.	0.	0.	0.

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	266	698	0	0	0	0
normalized size	1	1.	0.72	1.89	0.	0.	0.	0.
time (sec)	N/A	0.387	0.994	0.296	0.	0.	0.	0.

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	478	478	420	1708	0	0	0	0
normalized size	1	1.	0.88	3.57	0.	0.	0.	0.
time (sec)	N/A	0.588	1.631	0.313	0.	0.	0.	0.

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	470	470	451	2086	0	0	0	0
normalized size	1	1.	0.96	4.44	0.	0.	0.	0.
time (sec)	N/A	0.59	2.714	0.349	0.	0.	0.	0.

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	383	383	405	1687	0	0	0	0
normalized size	1	1.	1.06	4.4	0.	0.	0.	0.
time (sec)	N/A	0.392	1.993	0.318	0.	0.	0.	0.

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	353	1318	0	0	0	0
normalized size	1	1.	1.03	3.84	0.	0.	0.	0.
time (sec)	N/A	0.356	1.102	0.336	0.	0.	0.	0.

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	290	1099	0	0	0	0
normalized size	1	1.	0.84	3.19	0.	0.	0.	0.
time (sec)	N/A	0.416	1.15	0.313	0.	0.	0.	0.

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	359	375	1362	0	0	0	0
normalized size	1	1.	1.04	3.79	0.	0.	0.	0.
time (sec)	N/A	0.398	1.071	0.297	0.	0.	0.	0.

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	451	451	429	1763	0	0	0	0
normalized size	1	1.	0.95	3.91	0.	0.	0.	0.
time (sec)	N/A	0.433	1.436	0.323	0.	0.	0.	0.

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	567	567	504	2189	0	0	0	0
normalized size	1	1.	0.89	3.86	0.	0.	0.	0.
time (sec)	N/A	0.645	2.955	0.315	0.	0.	0.	0.

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	117	215	0	0	0	0
normalized size	1	1.	2.29	4.22	0.	0.	0.	0.
time (sec)	N/A	0.026	0.314	0.2	0.	0.	0.	0.

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	76	115	0	0	0	0
normalized size	1	1.	1.49	2.25	0.	0.	0.	0.
time (sec)	N/A	0.025	0.121	0.207	0.	0.	0.	0.

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	117	215	0	0	0	0
normalized size	1	1.	2.21	4.06	0.	0.	0.	0.
time (sec)	N/A	0.026	0.211	0.245	0.	0.	0.	0.

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	82	115	0	0	0	0
normalized size	1	1.	1.55	2.17	0.	0.	0.	0.
time (sec)	N/A	0.026	0.109	0.207	0.	0.	0.	0.

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	66	17	0	0	0	0
normalized size	1	1.	5.5	1.42	0.	0.	0.	0.
time (sec)	N/A	0.004	0.026	0.097	0.	0.	0.	0.

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	66	38	0	0	0	0
normalized size	1	1.	5.5	3.17	0.	0.	0.	0.
time (sec)	N/A	0.01	0.012	0.105	0.	0.	0.	0.

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	70	129	910	699	2270	713
normalized size	1	1.	0.74	1.36	9.58	7.36	23.89	7.51
time (sec)	N/A	0.074	0.059	0.046	1.375	2.066	8.114	1.253

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	60	76	436	394	1047	394
normalized size	1	1.	0.87	1.1	6.32	5.71	15.17	5.71
time (sec)	N/A	0.044	0.031	0.047	1.304	2.091	3.873	1.224

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	34	37	154	163	299	159
normalized size	1	1.	0.83	0.9	3.76	3.98	7.29	3.88
time (sec)	N/A	0.022	0.02	0.044	1.234	2.089	1.503	1.277

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	17	19	0	45	20	24
normalized size	1	1.	0.94	1.06	0.	2.5	1.11	1.33
time (sec)	N/A	0.003	0.009	0.043	0.	1.988	0.062	1.233

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.008	0.547	0.	0.	0.	0.

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	36	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.013	0.563	0.	0.	0.	0.

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	236	1528	903	3106	0	3426
normalized size	1	1.	0.88	5.72	3.38	11.63	0.	12.83
time (sec)	N/A	0.165	0.206	0.055	1.326	2.183	0.	1.387

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	138	547	429	1220	6205	1353
normalized size	1	1.	0.87	3.44	2.7	7.67	39.03	8.51
time (sec)	N/A	0.083	0.095	0.057	1.253	1.974	7.871	1.344

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	116	153	323	1095	355
normalized size	1	1.	1.	1.55	2.04	4.31	14.6	4.73
time (sec)	N/A	0.036	0.053	0.053	1.211	2.02	2.318	1.34

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	86	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.029	0.591	0.	0.	0.	0.

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	174	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.181	0.148	0.61	0.	0.	0.	0.

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	350	350	337	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.428	0.531	0.71	0.	0.	0.	0.

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	111	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	0.087	0.655	0.	0.	0.	0.

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	76	0	0	0	0	0
normalized size	1	1.	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.035	0.614	0.	0.	0.	0.

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	74	0	0	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.04	0.613	0.	0.	0.	0.

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	138	0	0	0	0	0
normalized size	1	1.	1.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.134	0.613	0.	0.	0.	0.

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	76	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.102	0.664	0.	0.	0.	0.

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	101	97	126	212	100	124
normalized size	1	1.	1.77	1.7	2.21	3.72	1.75	2.18
time (sec)	N/A	0.063	0.016	0.042	1.167	1.64	0.127	1.178

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	74	73	97	169	80	96
normalized size	1	1.	1.3	1.28	1.7	2.96	1.4	1.68
time (sec)	N/A	0.046	0.014	0.04	1.154	1.601	0.125	1.311

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	53	48	63	115	51	66
normalized size	1	1.	0.93	0.84	1.11	2.02	0.89	1.16
time (sec)	N/A	0.032	0.008	0.042	1.29	1.712	0.109	1.26

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	32	27	35	66	29	38
normalized size	1	1.	1.03	0.87	1.13	2.13	0.94	1.23
time (sec)	N/A	0.007	0.001	0.042	1.12	1.675	0.069	1.267

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	38	44	53	89	36	53
normalized size	1	1.	0.93	1.07	1.29	2.17	0.88	1.29
time (sec)	N/A	0.03	0.011	0.044	1.143	2.122	0.355	1.245

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	39	50	62	122	41	88
normalized size	1	1.	0.91	1.16	1.44	2.84	0.95	2.05
time (sec)	N/A	0.028	0.021	0.054	1.172	1.806	0.517	1.384

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	48	56	77	157	56	62
normalized size	1	1.	0.94	1.1	1.51	3.08	1.1	1.22
time (sec)	N/A	0.032	0.015	0.052	1.165	1.784	0.614	1.42

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	39	51	85	130	66	50
normalized size	1	1.	0.75	0.98	1.63	2.5	1.27	0.96
time (sec)	N/A	0.03	0.013	0.048	1.189	1.913	0.774	1.281

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	40	52	101	155	80	80
normalized size	1	1.	0.7	0.91	1.77	2.72	1.4	1.4
time (sec)	N/A	0.03	0.014	0.046	1.252	1.846	0.914	1.346

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	167	169	220	375	182	224
normalized size	1	1.	1.43	1.44	1.88	3.21	1.56	1.91
time (sec)	N/A	0.131	0.019	0.044	1.053	1.607	0.138	1.337

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	131	176	293	141	173
normalized size	1	1.	1.	1.12	1.5	2.5	1.21	1.48
time (sec)	N/A	0.102	0.017	0.043	1.184	1.738	0.12	1.373

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	91	88	117	198	95	120
normalized size	1	1.	1.14	1.1	1.46	2.48	1.19	1.5
time (sec)	N/A	0.046	0.012	0.048	1.16	1.661	0.123	1.353

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	60	51	68	120	58	72
normalized size	1	1.	1.33	1.13	1.51	2.67	1.29	1.6
time (sec)	N/A	0.015	0.002	0.041	1.139	1.574	0.123	1.241

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	79	114	142	223	90	135
normalized size	1	1.	0.84	1.21	1.51	2.37	0.96	1.44
time (sec)	N/A	0.075	0.03	0.045	1.185	1.818	0.58	1.277

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	91	126	151	309	105	201
normalized size	1	1.	0.97	1.34	1.61	3.29	1.12	2.14
time (sec)	N/A	0.076	0.056	0.049	1.174	1.819	0.721	1.28

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	111	136	162	360	122	143
normalized size	1	1.	1.11	1.36	1.62	3.6	1.22	1.43
time (sec)	N/A	0.081	0.037	0.049	1.328	1.93	1.489	1.304

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	110	140	176	373	134	136
normalized size	1	1.	1.09	1.39	1.74	3.69	1.33	1.35
time (sec)	N/A	0.075	0.054	0.048	1.291	1.947	1.437	1.218

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	100	146	197	397	150	220
normalized size	1	1.	0.92	1.34	1.81	3.64	1.38	2.02
time (sec)	N/A	0.075	0.036	0.049	1.171	1.848	1.85	1.214

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	90	119	204	312	160	132
normalized size	1	1.	0.82	1.08	1.85	2.84	1.45	1.2
time (sec)	N/A	0.069	0.037	0.048	1.339	1.801	2.319	1.296

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	89	120	215	328	170	131
normalized size	1	1.	0.76	1.03	1.84	2.8	1.45	1.12
time (sec)	N/A	0.069	0.03	0.046	1.134	1.808	3.057	1.291

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	90	119	234	363	184	132
normalized size	1	1.	0.79	1.04	2.05	3.18	1.61	1.16
time (sec)	N/A	0.07	0.036	0.046	1.608	1.945	3.761	1.346

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	338	345	454	779	371	467
normalized size	1	1.	1.78	1.82	2.39	4.1	1.95	2.46
time (sec)	N/A	0.329	0.048	0.043	1.18	1.777	0.114	1.346

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	252	293	394	667	321	393
normalized size	1	1.	1.33	1.54	2.07	3.51	1.69	2.07
time (sec)	N/A	0.261	0.055	0.043	1.385	1.628	0.125	1.336

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	206	241	313	529	255	321
normalized size	1	1.	1.1	1.28	1.66	2.81	1.36	1.71
time (sec)	N/A	0.233	0.049	0.041	1.875	1.71	0.111	1.32

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	155	189	244	414	202	248
normalized size	1	1.	0.96	1.17	1.52	2.57	1.25	1.54
time (sec)	N/A	0.14	0.048	0.039	1.165	1.706	0.123	1.313

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	116	129	170	282	139	174
normalized size	1	1.	1.12	1.24	1.63	2.71	1.34	1.67
time (sec)	N/A	0.063	0.023	0.043	1.144	1.608	0.099	1.288

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	85	74	99	169	85	104
normalized size	1	1.	1.52	1.32	1.77	3.02	1.52	1.86
time (sec)	N/A	0.021	0.002	0.042	1.17	1.811	0.087	1.313

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	142	220	267	410	173	259
normalized size	1	1.	0.82	1.27	1.54	2.37	1.	1.5
time (sec)	N/A	0.145	0.054	0.046	1.146	2.064	0.72	1.329

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	193	233	278	558	189	351
normalized size	1	1.	1.22	1.47	1.76	3.53	1.2	2.22
time (sec)	N/A	0.152	0.058	0.05	1.156	2.183	1.041	1.295

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	198	249	289	640	216	259
normalized size	1	1.	1.21	1.53	1.77	3.93	1.33	1.59
time (sec)	N/A	0.168	0.069	0.052	1.498	2.173	1.924	1.226

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	197	258	305	679	235	259
normalized size	1	1.	1.19	1.56	1.85	4.12	1.42	1.57
time (sec)	N/A	0.165	0.066	0.056	1.102	2.107	2.886	1.304

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	185	268	323	716	243	389
normalized size	1	1.	1.08	1.57	1.89	4.19	1.42	2.27
time (sec)	N/A	0.157	0.065	0.053	1.208	2.085	4.598	1.322

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	182	272	332	672	257	254
normalized size	1	1.	1.06	1.58	1.93	3.91	1.49	1.48
time (sec)	N/A	0.149	0.092	0.053	1.176	2.146	7.589	1.306

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	172	278	355	710	272	265
normalized size	1	1.	0.93	1.51	1.93	3.86	1.48	1.44
time (sec)	N/A	0.135	0.062	0.049	1.252	1.982	11.546	1.291

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	161	216	355	540	280	255
normalized size	1	1.	0.9	1.21	1.99	3.03	1.57	1.43
time (sec)	N/A	0.122	0.053	0.052	1.216	2.034	15.254	1.222

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	163	218	381	590	296	258
normalized size	1	1.	0.87	1.16	2.03	3.14	1.57	1.37
time (sec)	N/A	0.125	0.062	0.05	1.215	2.062	26.274	1.203

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	163	218	396	625	308	258
normalized size	1	1.	0.86	1.15	2.08	3.29	1.62	1.36
time (sec)	N/A	0.121	0.054	0.049	1.254	2.209	44.454	1.332

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	423	511	689	1218	571	705
normalized size	1	1.	1.52	1.84	2.48	4.38	2.05	2.54
time (sec)	N/A	0.516	0.083	0.049	1.147	1.865	0.138	1.334

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	361	445	595	1023	486	610
normalized size	1	1.	1.31	1.61	2.16	3.71	1.76	2.21
time (sec)	N/A	0.458	0.123	0.043	1.192	1.875	0.126	1.287

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	307	379	505	879	418	516
normalized size	1	1.	1.1	1.36	1.82	3.16	1.5	1.86
time (sec)	N/A	0.407	0.096	0.05	1.208	1.758	0.125	1.27

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	300	313	413	699	340	421
normalized size	1	1.	1.11	1.16	1.53	2.59	1.26	1.56
time (sec)	N/A	0.252	0.045	0.043	1.174	1.672	0.121	1.337

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	197	247	329	548	270	327
normalized size	1	1.	0.94	1.18	1.57	2.62	1.29	1.56
time (sec)	N/A	0.192	0.064	0.041	1.134	1.532	0.113	1.359

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	148	170	228	375	187	231
normalized size	1	1.	1.12	1.29	1.73	2.84	1.42	1.75
time (sec)	N/A	0.095	0.03	0.045	1.162	1.598	0.107	1.356

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	110	97	130	220	112	136
normalized size	1	1.	1.51	1.33	1.78	3.01	1.53	1.86
time (sec)	N/A	0.029	0.003	0.042	1.203	1.711	0.087	1.329

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	227	358	431	662	287	427
normalized size	1	1.	0.86	1.36	1.63	2.51	1.09	1.62
time (sec)	N/A	0.281	0.105	0.046	1.181	1.89	0.734	1.336

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	289	378	446	872	308	535
normalized size	1	1.	1.13	1.48	1.75	3.42	1.21	2.1
time (sec)	N/A	0.255	0.094	0.052	1.148	1.86	1.396	1.291

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	110	97	130	220	112	130
normalized size	1	1.	1.51	1.33	1.78	3.01	1.53	1.78
time (sec)	N/A	0.031	0.004	0.042	1.069	1.627	0.082	1.271

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	111	150	0	595	401	153
normalized size	1	1.	0.9	1.22	0.	4.84	3.26	1.24
time (sec)	N/A	0.097	0.087	0.048	0.	2.017	1.242	1.255

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	80	99	0	414	308	105
normalized size	1	1.	0.89	1.1	0.	4.6	3.42	1.17
time (sec)	N/A	0.075	0.057	0.045	0.	1.772	0.97	1.344

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	56	65	0	309	185	70
normalized size	1	1.	0.95	1.1	0.	5.24	3.14	1.19
time (sec)	N/A	0.051	0.04	0.046	0.	1.912	0.701	1.425

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	32	0	225	124	43
normalized size	1	1.	1.	0.76	0.	5.36	2.95	1.02
time (sec)	N/A	0.015	0.013	0.047	0.	1.915	0.279	1.245

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	63	77	0	298	1134	107
normalized size	1	1.	0.73	0.9	0.	3.47	13.19	1.24
time (sec)	N/A	0.038	0.035	0.049	0.	2.148	3.802	1.184

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	113	143	0	733	2508	252
normalized size	1	1.	0.92	1.16	0.	5.96	20.39	2.05
time (sec)	N/A	0.103	0.088	0.056	0.	3.037	22.057	1.299

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	140	233	0	1716	4996	363
normalized size	1	1.	0.8	1.32	0.	9.75	28.39	2.06
time (sec)	N/A	0.166	0.289	0.062	0.	8.889	116.536	1.357

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	164	248	0	1142	515	236
normalized size	1	1.	0.86	1.31	0.	6.01	2.71	1.24
time (sec)	N/A	0.177	0.125	0.056	0.	1.975	3.547	1.317

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	137	192	0	879	403	177
normalized size	1	1.	0.92	1.29	0.	5.9	2.7	1.19
time (sec)	N/A	0.12	0.083	0.051	0.	1.901	2.714	1.302

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	107	115	0	639	298	140
normalized size	1	1.	0.98	1.06	0.	5.86	2.73	1.28
time (sec)	N/A	0.081	0.079	0.049	0.	1.926	1.402	1.304

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	77	85	0	452	129	93
normalized size	1	1.	1.07	1.18	0.	6.28	1.79	1.29
time (sec)	N/A	0.021	0.058	0.05	0.	1.884	0.723	1.314

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	49	0	301	90	65
normalized size	1	1.	1.	0.86	0.	5.28	1.58	1.14
time (sec)	N/A	0.014	0.026	0.046	0.	1.717	0.793	1.233

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	138	244	0	900	3225	259
normalized size	1	1.	0.97	1.72	0.	6.34	22.71	1.82
time (sec)	N/A	0.126	0.1	0.087	0.	5.881	120.809	1.325

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	162	314	0	2225	0	521
normalized size	1	1.	0.79	1.53	0.	10.85	0.	2.54
time (sec)	N/A	0.189	0.214	0.064	0.	16.346	0.	1.256

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	199	233	0	1443	520	279
normalized size	1	1.	1.01	1.18	0.	7.29	2.63	1.41
time (sec)	N/A	0.185	0.152	0.059	0.	2.098	4.432	1.261

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	148	189	0	1104	328	217
normalized size	1	1.	1.23	1.58	0.	9.2	2.73	1.81
time (sec)	N/A	0.048	0.102	0.051	0.	1.908	2.63	1.334

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	127	133	0	822	272	167
normalized size	1	1.	1.3	1.36	0.	8.39	2.78	1.7
time (sec)	N/A	0.038	0.104	0.051	0.	2.052	1.72	1.34

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	101	108	0	716	172	128
normalized size	1	1.	0.89	0.96	0.	6.34	1.52	1.13
time (sec)	N/A	0.049	0.07	0.047	0.	1.986	1.118	1.254

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	71	65	0	455	124	82
normalized size	1	1.	0.95	0.87	0.	6.07	1.65	1.09
time (sec)	N/A	0.02	0.049	0.044	0.	1.943	0.658	1.319

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	180	532	0	2045	0	462
normalized size	1	1.	0.85	2.51	0.	9.65	0.	2.18
time (sec)	N/A	0.221	0.146	0.059	0.	24.15	0.	1.372

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	241	680	0	4594	0	768
normalized size	1	1.	0.8	2.27	0.	15.31	0.	2.56
time (sec)	N/A	0.357	0.389	0.06	0.	67.57	0.	1.363

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	197	225	0	1480	413	277
normalized size	1	1.	1.27	1.45	0.	9.55	2.66	1.79
time (sec)	N/A	0.085	0.147	0.053	0.	2.308	3.749	1.282

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	155	158	0	1148	320	208
normalized size	1	1.	0.99	1.01	0.	7.36	2.05	1.33
time (sec)	N/A	0.118	0.153	0.05	0.	2.202	2.359	1.188

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	127	129	0	983	214	166
normalized size	1	1.	0.88	0.89	0.	6.78	1.48	1.14
time (sec)	N/A	0.064	0.091	0.049	0.	2.12	1.406	1.317

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	83	81	0	608	150	99
normalized size	1	1.	0.89	0.87	0.	6.54	1.61	1.06
time (sec)	N/A	0.028	0.046	0.044	0.	2.097	0.839	1.304

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	265	941	0	3580	0	716
normalized size	1	1.	0.9	3.19	0.	12.14	0.	2.43
time (sec)	N/A	0.382	0.225	0.062	0.	97.594	0.	1.371

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	430	430	336	1126	0	0	0	1156
normalized size	1	1.	0.78	2.62	0.	0.	0.	2.69
time (sec)	N/A	0.534	0.432	0.063	0.	0.	0.	1.355

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	177	260	0	887	411	266
normalized size	1	1.	0.86	1.26	0.	4.29	1.99	1.29
time (sec)	N/A	0.209	0.133	0.051	0.	2.714	11.85	1.368

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	132	164	0	662	265	194
normalized size	1	1.	0.92	1.14	0.	4.6	1.84	1.35
time (sec)	N/A	0.115	0.084	0.051	0.	2.289	6.527	1.409

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	99	122	0	491	184	130
normalized size	1	1.	0.83	1.03	0.	4.13	1.55	1.09
time (sec)	N/A	0.054	0.064	0.048	0.	2.519	6.089	1.301

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	53	0	316	70	77
normalized size	1	1.	1.	0.79	0.	4.72	1.04	1.15
time (sec)	N/A	0.018	0.035	0.043	0.	2.329	2.872	1.498

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	99	381	0	1274	0	147
normalized size	1	1.	0.96	3.7	0.	12.37	0.	1.43
time (sec)	N/A	0.081	0.048	0.257	0.	2.982	0.	1.389

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	134	649	0	1891	0	0
normalized size	1	1.	1.22	5.9	0.	17.19	0.	0.
time (sec)	N/A	0.068	0.137	0.203	0.	3.25	0.	0.

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	127	1174	0	1064	0	421
normalized size	1	1.	1.23	11.4	0.	10.33	0.	4.09
time (sec)	N/A	0.044	0.104	0.213	0.	3.5	0.	1.229

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	173	1262	0	1719	0	699
normalized size	1	1.	1.2	8.76	0.	11.94	0.	4.85
time (sec)	N/A	0.086	0.17	0.207	0.	7.256	0.	1.412

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	248	2073	0	2958	0	1297
normalized size	1	1.	1.2	10.06	0.	14.36	0.	6.3
time (sec)	N/A	0.126	0.236	0.205	0.	24.853	0.	4.215

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	231	322	0	1219	734	374
normalized size	1	1.	0.91	1.26	0.	4.78	2.88	1.47
time (sec)	N/A	0.25	0.173	0.056	0.	2.283	32.254	1.311

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	174	205	0	902	551	286
normalized size	1	1.	0.97	1.14	0.	5.01	3.06	1.59
time (sec)	N/A	0.145	0.121	0.052	0.	2.278	17.095	1.298

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	132	161	0	675	372	192
normalized size	1	1.	0.86	1.05	0.	4.38	2.42	1.25
time (sec)	N/A	0.069	0.093	0.052	0.	2.13	15.533	1.37

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	88	69	0	425	219	107
normalized size	1	1.	1.01	0.79	0.	4.89	2.52	1.23
time (sec)	N/A	0.025	0.052	0.046	0.	2.155	6.858	1.241

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	195	745	0	1716	0	238
normalized size	1	1.	1.23	4.69	0.	10.79	0.	1.5
time (sec)	N/A	0.179	0.463	0.249	0.	16.478	0.	1.336

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	179	1154	0	1933	0	0
normalized size	1	1.	1.17	7.54	0.	12.63	0.	0.
time (sec)	N/A	0.136	0.192	0.223	0.	5.682	0.	0.

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	189	2117	0	3198	0	0
normalized size	1	1.	1.17	13.15	0.	19.86	0.	0.
time (sec)	N/A	0.124	0.219	0.208	0.	7.396	0.	0.

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	242	2490	0	5117	0	795
normalized size	1	1.	1.21	12.45	0.	25.58	0.	3.98
time (sec)	N/A	0.177	0.283	0.216	0.	54.17	0.	1.576

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	198	3528	0	2221	0	1278
normalized size	1	1.	1.29	23.06	0.	14.52	0.	8.35
time (sec)	N/A	0.063	0.278	0.24	0.	20.229	0.	4.537

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	272	3622	0	3316	0	1682
normalized size	1	1.	1.39	18.57	0.	17.01	0.	8.63
time (sec)	N/A	0.095	0.375	0.217	0.	74.406	0.	1.458

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	358	5087	0	5085	0	2454
normalized size	1	1.	1.33	18.91	0.	18.9	0.	9.12
time (sec)	N/A	0.184	0.65	0.26	0.	157.252	0.	1.662

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	283	386	0	1577	1062	486
normalized size	1	1.	0.92	1.26	0.	5.14	3.46	1.58
time (sec)	N/A	0.293	0.252	0.054	0.	1.978	75.149	1.385

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	216	245	0	1185	843	378
normalized size	1	1.	1.	1.13	0.	5.49	3.9	1.75
time (sec)	N/A	0.164	0.165	0.053	0.	1.992	42.598	1.322

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	162	200	0	879	539	257
normalized size	1	1.	0.86	1.06	0.	4.65	2.85	1.36
time (sec)	N/A	0.088	0.122	0.056	0.	1.838	39.103	1.324

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	108	85	0	549	348	142
normalized size	1	1.	1.01	0.79	0.	5.13	3.25	1.33
time (sec)	N/A	0.034	0.07	0.044	0.	1.676	17.069	1.306

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	332	1225	0	0	0	381
normalized size	1	1.	1.47	5.42	0.	0.	0.	1.69
time (sec)	N/A	0.259	0.967	0.191	0.	0.	0.	1.358

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	239	1796	0	2967	0	0
normalized size	1	1.	1.09	8.2	0.	13.55	0.	0.
time (sec)	N/A	0.244	0.247	0.194	0.	89.519	0.	0.

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	281	3342	0	3295	0	703
normalized size	1	1.	1.32	15.69	0.	15.47	0.	3.3
time (sec)	N/A	0.239	0.303	0.195	0.	15.445	0.	2.054

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	260	3789	0	5146	0	776
normalized size	1	1.	1.17	17.07	0.	23.18	0.	3.5
time (sec)	N/A	0.218	0.305	0.196	0.	26.525	0.	2.181

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	299	5406	0	7640	0	0
normalized size	1	1.	1.04	18.84	0.	26.62	0.	0.
time (sec)	N/A	0.303	0.545	0.201	0.	103.694	0.	0.

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	359	5921	0	0	0	1875
normalized size	1	1.	1.14	18.86	0.	0.	0.	5.97
time (sec)	N/A	0.343	0.559	0.207	0.	0.	0.	2.182

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	305	7616	0	0	0	2558
normalized size	1	1.	1.5	37.52	0.	0.	0.	12.6
time (sec)	N/A	0.105	0.537	0.214	0.	0.	0.	2.346

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	403	7718	0	0	0	3178
normalized size	1	1.	1.64	31.37	0.	0.	0.	12.92
time (sec)	N/A	0.124	0.563	0.24	0.	0.	0.	2.294

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	332	489	9978	0	0	0	4207
normalized size	1	1.	1.47	30.05	0.	0.	0.	12.67
time (sec)	N/A	0.258	1.058	0.239	0.	0.	0.	3.04

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	57	72	190	0	96
normalized size	1	1.	1.	1.02	1.29	3.39	0.	1.71
time (sec)	N/A	0.033	0.028	0.043	1.535	2.512	0.	1.48

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	57	56	73	212	0	142
normalized size	1	1.	0.85	0.84	1.09	3.16	0.	2.12
time (sec)	N/A	0.044	0.045	0.042	1.941	2.48	0.	1.404

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	50	45	63	136	56	58
normalized size	1	1.	0.91	0.82	1.15	2.47	1.02	1.05
time (sec)	N/A	0.015	0.04	0.041	1.891	2.416	0.416	1.332

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	126	198	0	603	330	180
normalized size	1	1.	0.78	1.23	0.	3.75	2.05	1.12
time (sec)	N/A	0.161	0.107	0.049	0.	1.946	8.75	1.322

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	92	126	0	428	216	122
normalized size	1	1.	0.84	1.15	0.	3.89	1.96	1.11
time (sec)	N/A	0.075	0.062	0.048	0.	1.907	5.991	1.36

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	71	84	0	327	158	85
normalized size	1	1.	0.83	0.98	0.	3.8	1.84	0.99
time (sec)	N/A	0.04	0.042	0.047	0.	1.924	4.904	1.373

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	46	37	0	223	102	54
normalized size	1	1.	1.07	0.86	0.	5.19	2.37	1.26
time (sec)	N/A	0.013	0.021	0.042	0.	1.845	1.126	1.521

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	127	0	435	0	80
normalized size	1	1.	1.	2.35	0.	8.06	0.	1.48
time (sec)	N/A	0.017	0.012	0.189	0.	1.986	0.	1.373

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	115	210	0	783	0	439
normalized size	1	1.	1.26	2.31	0.	8.6	0.	4.82
time (sec)	N/A	0.034	0.101	0.193	0.	2.491	0.	5.032

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	161	426	0	1432	0	466
normalized size	1	1.	1.11	2.94	0.	9.88	0.	3.21
time (sec)	N/A	0.071	0.14	0.193	0.	4.109	0.	1.43

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	209	573	0	2290	0	780
normalized size	1	1.	1.06	2.89	0.	11.57	0.	3.94
time (sec)	N/A	0.164	0.179	0.196	0.	8.502	0.	1.38

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	127	189	0	768	0	186
normalized size	1	1.	0.78	1.17	0.	4.74	0.	1.15
time (sec)	N/A	0.148	0.142	0.049	0.	1.964	0.	1.344

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	91	118	0	531	0	135
normalized size	1	1.	0.86	1.11	0.	5.01	0.	1.27
time (sec)	N/A	0.057	0.105	0.049	0.	1.969	0.	1.554

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	69	76	0	428	0	93
normalized size	1	1.	0.83	0.92	0.	5.16	0.	1.12
time (sec)	N/A	0.035	0.07	0.046	0.	1.995	0.	1.393

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	27	42	69	46	32
normalized size	1	1.	0.96	0.96	1.5	2.46	1.64	1.14
time (sec)	N/A	0.007	0.014	0.042	1.124	1.825	3.989	1.342

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	94	260	0	913	0	232
normalized size	1	1.	1.	2.77	0.	9.71	0.	2.47
time (sec)	N/A	0.058	0.058	0.19	0.	2.75	0.	1.409

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	139	400	0	1777	0	0
normalized size	1	1.	0.92	2.65	0.	11.77	0.	0.
time (sec)	N/A	0.084	0.125	0.194	0.	3.419	0.	0.

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	240	681	0	3078	0	876
normalized size	1	1.	1.08	3.05	0.	13.8	0.	3.93
time (sec)	N/A	0.198	0.399	0.193	0.	7.516	0.	1.695

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	279	898	0	4508	0	1384
normalized size	1	1.	0.95	3.06	0.	15.39	0.	4.72
time (sec)	N/A	0.374	0.652	0.201	0.	18.529	0.	1.97

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	167	270	0	1008	0	269
normalized size	1	1.	0.87	1.41	0.	5.28	0.	1.41
time (sec)	N/A	0.163	0.246	0.053	0.	2.471	0.	1.329

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	130	202	0	825	0	203
normalized size	1	1.	0.81	1.25	0.	5.12	0.	1.26
time (sec)	N/A	0.118	0.212	0.053	0.	2.325	0.	1.326

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	78	83	180	213	0	119
normalized size	1	1.	0.99	1.05	2.28	2.7	0.	1.51
time (sec)	N/A	0.027	0.105	0.045	1.707	2.254	0.	1.272

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	57	55	124	153	0	74
normalized size	1	1.	0.98	0.95	2.14	2.64	0.	1.28
time (sec)	N/A	0.02	0.08	0.043	1.172	2.164	0.	1.35

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	43	39	65	126	146	51
normalized size	1	1.	0.84	0.76	1.27	2.47	2.86	1.
time (sec)	N/A	0.011	0.018	0.043	1.172	2.188	13.391	1.671

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	153	454	0	1690	0	1262
normalized size	1	1.	0.99	2.95	0.	10.97	0.	8.19
time (sec)	N/A	0.126	0.15	0.192	0.	4.05	0.	1.352

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	237	667	0	3310	0	0
normalized size	1	1.	0.97	2.73	0.	13.57	0.	0.
time (sec)	N/A	0.233	0.299	0.2	0.	6.185	0.	0.

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	327	296	1088	0	5269	0	2759
normalized size	1	1.	0.91	3.33	0.	16.11	0.	8.44
time (sec)	N/A	0.372	0.901	0.199	0.	13.752	0.	2.577

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	22	70	12	22
normalized size	1	1.	1.	0.94	1.22	3.89	0.67	1.22
time (sec)	N/A	0.005	0.011	0.043	1.748	1.729	0.136	1.381

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	70	12	22
normalized size	1	1.	1.	0.85	1.1	3.5	0.6	1.1
time (sec)	N/A	0.005	0.012	0.045	1.727	1.829	0.145	1.68

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	58	14	30
normalized size	1	1.	1.	0.83	1.06	3.22	0.78	1.67
time (sec)	N/A	0.004	0.01	0.042	1.985	1.734	0.143	2.184

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	38	42	57	113	42	47
normalized size	1	1.	0.7	0.78	1.06	2.09	0.78	0.87
time (sec)	N/A	0.023	0.032	0.046	1.756	1.858	0.244	1.764

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	36	38	51	108	42	61
normalized size	1	1.	0.69	0.73	0.98	2.08	0.81	1.17
time (sec)	N/A	0.02	0.026	0.045	1.69	1.812	0.251	1.509

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	16	13	27	66	20	16
normalized size	1	1.	0.89	0.72	1.5	3.67	1.11	0.89
time (sec)	N/A	0.004	0.009	0.039	1.201	1.803	2.664	2.208

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	44	41	63	244	218	331
normalized size	1	1.	0.7	0.65	1.	3.87	3.46	5.25
time (sec)	N/A	0.024	0.049	0.043	1.155	1.917	3.192	1.405

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	44	41	63	194	155	182
normalized size	1	1.	0.7	0.65	1.	3.08	2.46	2.89
time (sec)	N/A	0.021	0.037	0.043	1.192	1.713	5.609	1.419

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	44	41	63	143	61	74
normalized size	1	1.	0.7	0.65	1.	2.27	0.97	1.17
time (sec)	N/A	0.021	0.031	0.042	1.127	1.662	1.735	1.325

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	44	41	72	96	150	74
normalized size	1	1.	0.72	0.67	1.18	1.57	2.46	1.21
time (sec)	N/A	0.021	0.033	0.04	1.147	1.788	5.989	1.384

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	43	41	73	107	58	73
normalized size	1	1.	0.73	0.69	1.24	1.81	0.98	1.24
time (sec)	N/A	0.021	0.034	0.041	1.127	1.781	5.838	1.721

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	44	40	70	130	168	65
normalized size	1	1.	0.75	0.68	1.19	2.2	2.85	1.1
time (sec)	N/A	0.022	0.037	0.041	1.113	1.848	1.196	1.311

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	44	41	59	158	252	61
normalized size	1	1.	0.72	0.67	0.97	2.59	4.13	1.
time (sec)	N/A	0.023	0.039	0.042	1.096	1.827	2.841	1.355

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	96	106	153	504	566	676
normalized size	1	1.	0.76	0.83	1.2	3.97	4.46	5.32
time (sec)	N/A	0.057	0.112	0.044	1.124	1.759	17.529	1.712

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	97	106	153	423	328	394
normalized size	1	1.	0.76	0.83	1.2	3.33	2.58	3.1
time (sec)	N/A	0.048	0.087	0.044	1.148	1.813	10.626	1.417

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	96	106	153	325	148	170
normalized size	1	1.	0.76	0.83	1.2	2.56	1.17	1.34
time (sec)	N/A	0.046	0.061	0.046	1.16	1.783	2.566	1.653

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	96	106	162	242	330	170
normalized size	1	1.	0.77	0.85	1.3	1.94	2.64	1.36
time (sec)	N/A	0.045	0.061	0.045	1.109	1.799	27.291	1.239

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	97	106	163	261	126	185
normalized size	1	1.	0.79	0.86	1.33	2.12	1.02	1.5
time (sec)	N/A	0.047	0.056	0.043	1.104	1.7	12.279	1.303

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	96	106	161	270	121	180
normalized size	1	1.	0.78	0.86	1.31	2.2	0.98	1.46
time (sec)	N/A	0.047	0.057	0.044	1.183	1.772	19.936	1.322

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	96	106	163	289	592	176
normalized size	1	1.	0.78	0.86	1.33	2.35	4.81	1.43
time (sec)	N/A	0.046	0.064	0.044	1.173	1.87	3.145	2.102

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	188	205	282	863	945	1129
normalized size	1	1.	0.92	1.	1.38	4.23	4.63	5.53
time (sec)	N/A	0.103	0.254	0.044	1.187	1.83	27.755	1.432

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	188	205	282	707	564	675
normalized size	1	1.	0.92	1.	1.38	3.47	2.76	3.31
time (sec)	N/A	0.085	0.196	0.045	1.174	1.83	16.661	1.354

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	170	205	282	581	265	301
normalized size	1	1.	0.83	1.	1.38	2.85	1.3	1.48
time (sec)	N/A	0.083	0.144	0.046	1.145	1.809	4.064	1.368

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	171	205	286	478	563	302
normalized size	1	1.	0.86	1.02	1.43	2.39	2.82	1.51
time (sec)	N/A	0.082	0.118	0.046	1.479	1.861	48.8	1.34

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	171	205	293	479	224	352
normalized size	1	1.	0.86	1.04	1.48	2.42	1.13	1.78
time (sec)	N/A	0.083	0.116	0.045	1.537	1.872	23.68	1.484

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	171	205	290	495	206	344
normalized size	1	1.	0.86	1.02	1.45	2.48	1.03	1.72
time (sec)	N/A	0.083	0.109	0.046	1.603	1.847	31.14	1.4

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	170	205	290	498	197	339
normalized size	1	1.	0.87	1.05	1.48	2.54	1.01	1.73
time (sec)	N/A	0.081	0.115	0.044	1.613	1.766	48.507	1.348

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	158	460	0	3356	418	0
normalized size	1	1.	0.95	2.75	0.	20.1	2.5	0.
time (sec)	N/A	0.398	0.259	0.267	0.	2.587	70.188	0.

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	147	335	0	1970	316	0
normalized size	1	1.	0.99	2.25	0.	13.22	2.12	0.
time (sec)	N/A	0.282	0.115	0.209	0.	2.037	37.876	0.

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	125	203	0	725	76	0
normalized size	1	1.	0.93	1.51	0.	5.41	0.57	0.
time (sec)	N/A	0.109	0.061	0.207	0.	1.983	5.732	0.

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	125	110	0	1791	0	0
normalized size	1	1.	0.93	0.82	0.	13.37	0.	0.
time (sec)	N/A	0.114	0.087	0.205	0.	1.941	0.	0.

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	136	291	0	5617	0	0
normalized size	1	1.	0.85	1.82	0.	35.11	0.	0.
time (sec)	N/A	0.208	0.086	0.209	0.	2.523	0.	0.

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	130	472	0	10656	0	0
normalized size	1	1.	0.68	2.48	0.	56.08	0.	0.
time (sec)	N/A	0.38	0.059	0.211	0.	3.039	0.	0.

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	781	781	226	3931	0	3383	418	0
normalized size	1	1.	0.29	5.03	0.	4.33	0.54	0.
time (sec)	N/A	3.043	0.348	0.313	0.	2.526	66.978	0.

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	689	689	159	2763	0	1997	316	0
normalized size	1	1.	0.23	4.01	0.	2.9	0.46	0.
time (sec)	N/A	1.477	0.155	0.248	0.	2.097	34.984	0.

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	478	478	135	1176	0	743	75	0
normalized size	1	1.	0.28	2.46	0.	1.55	0.16	0.
time (sec)	N/A	0.405	0.092	0.245	0.	1.915	5.044	0.

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	538	538	135	1548	0	1808	0	0
normalized size	1	1.	0.25	2.88	0.	3.36	0.	0.
time (sec)	N/A	0.447	0.137	0.24	0.	2.049	0.	0.

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	663	663	132	5659	0	5651	0	0
normalized size	1	1.	0.2	8.54	0.	8.52	0.	0.
time (sec)	N/A	0.969	0.131	0.232	0.	2.482	0.	0.

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	736	736	135	7264	0	10683	0	0
normalized size	1	1.	0.18	9.87	0.	14.51	0.	0.
time (sec)	N/A	1.732	0.065	0.254	0.	3.111	0.	0.

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	251	717	0	4626	0	0
normalized size	1	1.	0.95	2.73	0.	17.59	0.	0.
time (sec)	N/A	0.534	0.512	0.226	0.	3.51	0.	0.

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	247	575	0	2897	0	0
normalized size	1	1.	1.07	2.49	0.	12.54	0.	0.
time (sec)	N/A	0.375	0.349	0.221	0.	2.346	0.	0.

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	218	432	0	1423	0	0
normalized size	1	1.	1.04	2.07	0.	6.81	0.	0.
time (sec)	N/A	0.27	0.246	0.214	0.	2.054	0.	0.

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	267	287	0	2573	0	0
normalized size	1	1.	1.38	1.48	0.	13.26	0.	0.
time (sec)	N/A	0.186	0.309	0.268	0.	2.062	0.	0.

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	248	375	0	6498	0	0
normalized size	1	1.	1.12	1.69	0.	29.27	0.	0.
time (sec)	N/A	0.339	0.492	0.227	0.	3.069	0.	0.

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	331	783	0	12070	0	0
normalized size	1	1.	1.25	2.95	0.	45.55	0.	0.
time (sec)	N/A	0.479	0.451	0.249	0.	6.315	0.	0.

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	327	1021	0	18263	0	0
normalized size	1	1.	1.05	3.28	0.	58.72	0.	0.
time (sec)	N/A	0.723	0.303	0.24	0.	16.41	0.	0.

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	887	887	311	5653	0	4660	0	0
normalized size	1	1.	0.35	6.37	0.	5.25	0.	0.
time (sec)	N/A	6.449	0.885	0.247	0.	4.169	0.	0.

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	811	811	248	4065	0	2917	0	0
normalized size	1	1.	0.31	5.01	0.	3.6	0.	0.
time (sec)	N/A	2.839	0.612	0.238	0.	2.838	0.	0.

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	726	726	208	2851	0	1432	0	0
normalized size	1	1.	0.29	3.93	0.	1.97	0.	0.
time (sec)	N/A	1.298	0.584	0.23	0.	2.39	0.	0.

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	675	675	265	0	0	2593	0	0
normalized size	1	1.	0.39	0.	0.	3.84	0.	0.
time (sec)	N/A	1.018	0.588	180.	0.	2.468	0.	0.

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	739	739	255	0	0	6525	0	0
normalized size	1	1.	0.35	0.	0.	8.83	0.	0.
time (sec)	N/A	1.498	0.601	180.	0.	3.672	0.	0.

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	845	845	346	8744	0	12104	0	0
normalized size	1	1.	0.41	10.35	0.	14.32	0.	0.
time (sec)	N/A	3.344	0.559	0.281	0.	7.464	0.	0.

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	930	930	321	10403	0	18290	0	0
normalized size	1	1.	0.35	11.19	0.	19.67	0.	0.
time (sec)	N/A	6.489	0.538	0.268	0.	18.974	0.	0.

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	319	986	0	3876	0	0
normalized size	1	1.	1.09	3.35	0.	13.18	0.	0.
time (sec)	N/A	0.525	0.711	0.234	0.	3.286	0.	0.

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	260	792	0	2178	0	0
normalized size	1	1.	0.93	2.84	0.	7.81	0.	0.
time (sec)	N/A	0.459	0.52	0.226	0.	2.563	0.	0.

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	428	608	0	3357	0	0
normalized size	1	1.	1.6	2.27	0.	12.53	0.	0.
time (sec)	N/A	0.506	0.994	0.218	0.	2.714	0.	0.

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	368	803	0	7752	0	0
normalized size	1	1.	1.31	2.86	0.	27.59	0.	0.
time (sec)	N/A	0.49	0.735	0.276	0.	4.572	0.	0.

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	335	956	0	12446	0	0
normalized size	1	1.	1.06	3.03	0.	39.51	0.	0.
time (sec)	N/A	0.634	0.7	0.267	0.	13.216	0.	0.

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	905	905	343	5915	0	3903	0	0
normalized size	1	1.	0.38	6.54	0.	4.31	0.	0.
time (sec)	N/A	5.635	1.1	0.247	0.	3.298	0.	0.

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	846	846	277	4264	0	2194	0	0
normalized size	1	1.	0.33	5.04	0.	2.59	0.	0.
time (sec)	N/A	3.396	0.725	0.245	0.	2.522	0.	0.

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	769	769	430	7538	0	3384	0	0
normalized size	1	1.	0.56	9.8	0.	4.4	0.	0.
time (sec)	N/A	1.936	1.227	0.254	0.	2.776	0.	0.

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	849	849	412	0	0	7772	0	0
normalized size	1	1.	0.49	0.	0.	9.15	0.	0.
time (sec)	N/A	2.883	0.871	180.	0.	4.637	0.	0.

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	920	920	464	0	0	12473	0	0
normalized size	1	1.	0.5	0.	0.	13.56	0.	0.
time (sec)	N/A	5.851	0.763	180.	0.	13.305	0.	0.

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	59	360	0	990	32	0
normalized size	1	1.	0.28	1.68	0.	4.63	0.15	0.
time (sec)	N/A	0.23	0.03	0.173	0.	2.623	3.088	0.

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	75	570	0	2176	53	0
normalized size	1	1.	0.24	1.8	0.	6.89	0.17	0.
time (sec)	N/A	0.347	0.042	0.209	0.	2.987	3.88	0.

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	29	61	116	70	65
normalized size	1	1.	1.	0.83	1.74	3.31	2.	1.86
time (sec)	N/A	0.02	0.015	0.047	1.655	2.329	3.097	1.322

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	47	0	741	61	84
normalized size	1	1.	1.	0.81	0.	12.78	1.05	1.45
time (sec)	N/A	0.058	0.035	0.131	0.	2.407	3.657	1.367

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	427	427	133	944	0	721	56	0
normalized size	1	1.	0.31	2.21	0.	1.69	0.13	0.
time (sec)	N/A	0.531	0.139	0.227	0.	2.231	4.252	0.

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	123	182	0	703	58	0
normalized size	1	1.	0.93	1.38	0.	5.33	0.44	0.
time (sec)	N/A	0.132	0.093	0.158	0.	2.239	4.566	0.

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	55	336	0	860	31	0
normalized size	1	1.	0.27	1.64	0.	4.2	0.15	0.
time (sec)	N/A	0.223	0.026	0.107	0.	2.315	2.774	0.

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	55	420	0	868	0	0
normalized size	1	1.	0.28	2.12	0.	4.38	0.	0.
time (sec)	N/A	0.132	0.023	0.058	0.	2.291	0.	0.

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	90	639	0	1778	0	0
normalized size	1	1.	0.33	2.35	0.	6.54	0.	0.
time (sec)	N/A	0.364	0.094	0.365	0.	2.425	0.	0.

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	398	398	582	1386	0	0	0	0
normalized size	1	1.	1.46	3.48	0.	0.	0.	0.
time (sec)	N/A	0.441	3.413	0.394	0.	0.	0.	0.

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	362	362	536	1162	0	0	0	0
normalized size	1	1.	1.48	3.21	0.	0.	0.	0.
time (sec)	N/A	0.299	2.888	0.244	0.	0.	0.	0.

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	456	688	0	0	0	0
normalized size	1	1.	1.42	2.14	0.	0.	0.	0.
time (sec)	N/A	0.204	1.985	0.26	0.	0.	0.	0.

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	419	646	0	0	0	0
normalized size	1	1.	1.37	2.12	0.	0.	0.	0.
time (sec)	N/A	0.183	1.309	0.323	0.	0.	0.	0.

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	366	366	504	1309	0	0	0	0
normalized size	1	1.	1.38	3.58	0.	0.	0.	0.
time (sec)	N/A	0.273	1.175	0.278	0.	0.	0.	0.

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	444	444	602	3409	0	0	0	0
normalized size	1	1.	1.36	7.68	0.	0.	0.	0.
time (sec)	N/A	0.382	2.132	0.286	0.	0.	0.	0.

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	497	497	695	1970	0	0	0	0
normalized size	1	1.	1.4	3.96	0.	0.	0.	0.
time (sec)	N/A	0.559	3.982	0.274	0.	0.	0.	0.

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	448	448	646	1731	0	0	0	0
normalized size	1	1.	1.44	3.86	0.	0.	0.	0.
time (sec)	N/A	0.467	4.167	0.25	0.	0.	0.	0.

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	393	575	1385	0	0	0	0
normalized size	1	1.	1.46	3.52	0.	0.	0.	0.
time (sec)	N/A	0.357	3.244	0.25	0.	0.	0.	0.

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	565	1168	0	0	0	0
normalized size	1	1.	1.53	3.17	0.	0.	0.	0.
time (sec)	N/A	0.307	3.01	0.257	0.	0.	0.	0.

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	358	358	494	1597	0	0	0	0
normalized size	1	1.	1.38	4.46	0.	0.	0.	0.
time (sec)	N/A	0.255	2.37	0.261	0.	0.	0.	0.

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	410	602	3411	0	0	0	0
normalized size	1	1.	1.47	8.32	0.	0.	0.	0.
time (sec)	N/A	0.314	2.863	0.273	0.	0.	0.	0.

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	491	491	659	5277	0	0	0	0
normalized size	1	1.	1.34	10.75	0.	0.	0.	0.
time (sec)	N/A	0.476	3.812	0.306	0.	0.	0.	0.

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	566	566	790	2332	0	0	0	0
normalized size	1	1.	1.4	4.12	0.	0.	0.	0.
time (sec)	N/A	0.666	4.926	0.268	0.	0.	0.	0.

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	494	494	634	1970	0	0	0	0
normalized size	1	1.	1.28	3.99	0.	0.	0.	0.
time (sec)	N/A	0.482	3.679	0.258	0.	0.	0.	0.

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	457	457	684	1736	0	0	0	0
normalized size	1	1.	1.5	3.8	0.	0.	0.	0.
time (sec)	N/A	0.455	3.939	0.287	0.	0.	0.	0.

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	430	430	637	2646	0	0	0	0
normalized size	1	1.	1.48	6.15	0.	0.	0.	0.
time (sec)	N/A	0.438	3.579	0.273	0.	0.	0.	0.

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	420	420	613	3421	0	0	0	0
normalized size	1	1.	1.46	8.15	0.	0.	0.	0.
time (sec)	N/A	0.362	3.415	0.278	0.	0.	0.	0.

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	498	498	677	5303	0	0	0	0
normalized size	1	1.	1.36	10.65	0.	0.	0.	0.
time (sec)	N/A	0.455	3.997	0.286	0.	0.	0.	0.

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	553	553	762	8244	0	0	0	0
normalized size	1	1.	1.38	14.91	0.	0.	0.	0.
time (sec)	N/A	0.492	5.139	0.323	0.	0.	0.	0.

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	413	413	548	1534	0	0	0	0
normalized size	1	1.	1.33	3.71	0.	0.	0.	0.
time (sec)	N/A	0.435	3.117	0.27	0.	0.	0.	0.

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	359	557	1312	0	0	0	0
normalized size	1	1.	1.55	3.65	0.	0.	0.	0.
time (sec)	N/A	0.325	3.057	0.287	0.	0.	0.	0.

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	445	978	0	0	0	0
normalized size	1	1.	1.4	3.09	0.	0.	0.	0.
time (sec)	N/A	0.212	2.339	0.26	0.	0.	0.	0.

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	294	396	0	0	0	0
normalized size	1	1.	2.16	2.91	0.	0.	0.	0.
time (sec)	N/A	0.053	0.472	0.263	0.	0.	0.	0.

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	186	200	0	0	0	0
normalized size	1	1.	1.37	1.47	0.	0.	0.	0.
time (sec)	N/A	0.057	0.243	0.264	0.	0.	0.	0.

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	331	655	0	0	0	0
normalized size	1	1.	1.78	3.52	0.	0.	0.	0.
time (sec)	N/A	0.09	0.388	0.273	0.	0.	0.	0.

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	382	494	1904	0	0	0	0
normalized size	1	1.	1.29	4.98	0.	0.	0.	0.
time (sec)	N/A	0.266	1.661	0.284	0.	0.	0.	0.

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	447	447	618	3863	0	0	0	0
normalized size	1	1.	1.38	8.64	0.	0.	0.	0.
time (sec)	N/A	0.408	3.259	0.307	0.	0.	0.	0.

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	426	426	586	1362	0	0	0	0
normalized size	1	1.	1.38	3.2	0.	0.	0.	0.
time (sec)	N/A	0.469	3.472	0.35	0.	0.	0.	0.

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	363	495	1150	0	0	0	0
normalized size	1	1.	1.36	3.17	0.	0.	0.	0.
time (sec)	N/A	0.314	3.047	0.27	0.	0.	0.	0.

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	414	685	0	0	0	0
normalized size	1	1.	1.29	2.13	0.	0.	0.	0.
time (sec)	N/A	0.213	5.028	0.266	0.	0.	0.	0.

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	408	649	0	0	0	0
normalized size	1	1.	1.37	2.18	0.	0.	0.	0.
time (sec)	N/A	0.209	2.056	0.268	0.	0.	0.	0.

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	430	696	0	0	0	0
normalized size	1	1.	1.3	2.1	0.	0.	0.	0.
time (sec)	N/A	0.22	0.974	0.279	0.	0.	0.	0.

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	406	406	583	1167	0	0	0	0
normalized size	1	1.	1.44	2.87	0.	0.	0.	0.
time (sec)	N/A	0.304	1.379	0.295	0.	0.	0.	0.

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	485	485	634	2623	0	0	0	0
normalized size	1	1.	1.31	5.41	0.	0.	0.	0.
time (sec)	N/A	0.495	3.63	0.305	0.	0.	0.	0.

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	475	475	700	3304	0	0	0	0
normalized size	1	1.	1.47	6.96	0.	0.	0.	0.
time (sec)	N/A	0.505	4.594	0.329	0.	0.	0.	0.

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	418	627	2660	0	0	0	0
normalized size	1	1.	1.5	6.36	0.	0.	0.	0.
time (sec)	N/A	0.38	3.677	0.3	0.	0.	0.	0.

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	392	392	597	2278	0	0	0	0
normalized size	1	1.	1.52	5.81	0.	0.	0.	0.
time (sec)	N/A	0.325	3.501	0.287	0.	0.	0.	0.

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	368	504	1633	0	0	0	0
normalized size	1	1.	1.37	4.44	0.	0.	0.	0.
time (sec)	N/A	0.351	3.495	0.278	0.	0.	0.	0.

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	392	392	619	2292	0	0	0	0
normalized size	1	1.	1.58	5.85	0.	0.	0.	0.
time (sec)	N/A	0.322	2.117	0.277	0.	0.	0.	0.

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	450	450	570	2673	0	0	0	0
normalized size	1	1.	1.27	5.94	0.	0.	0.	0.
time (sec)	N/A	0.385	2.038	0.299	0.	0.	0.	0.

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	532	532	669	3322	0	0	0	0
normalized size	1	1.	1.26	6.24	0.	0.	0.	0.
time (sec)	N/A	0.531	4.009	0.324	0.	0.	0.	0.

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	151	151	176	0	0	0	0	0
normalized size	1	1.	1.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.134	0.542	0.	0.	0.	0.

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	558	558	53	40	0	0	85	0
normalized size	1	1.	0.09	0.07	0.	0.	0.15	0.
time (sec)	N/A	0.41	0.033	0.244	0.	0.	4.74	0.

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	551	551	47	35	0	0	68	0
normalized size	1	1.	0.09	0.06	0.	0.	0.12	0.
time (sec)	N/A	0.313	0.025	0.218	0.	0.	3.433	0.

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	529	529	40	29	0	0	36	0
normalized size	1	1.	0.08	0.05	0.	0.	0.07	0.
time (sec)	N/A	0.266	0.016	0.215	0.	0.	2.359	0.

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	121	0	0	571	0	0
normalized size	1	1.	1.25	0.	0.	5.89	0.	0.
time (sec)	N/A	0.014	0.078	0.362	0.	28.365	0.	0.

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	634	634	211	0	0	0	0	0
normalized size	1	1.	0.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.364	0.243	0.378	0.	0.	0.	0.

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	656	656	223	0	0	0	0	0
normalized size	1	1.	0.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.412	0.271	0.375	0.	0.	0.	0.

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	564	564	60	49	0	0	150	0
normalized size	1	1.	0.11	0.09	0.	0.	0.27	0.
time (sec)	N/A	0.363	0.035	0.267	0.	0.	4.992	0.

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	557	557	51	43	0	0	73	0
normalized size	1	1.	0.09	0.08	0.	0.	0.13	0.
time (sec)	N/A	0.315	0.031	0.273	0.	0.	3.471	0.

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	531	531	42	37	0	0	39	0
normalized size	1	1.	0.08	0.07	0.	0.	0.07	0.
time (sec)	N/A	0.261	0.019	0.247	0.	0.	1.958	0.

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	125	0	0	0	0	0
normalized size	1	1.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.083	0.411	0.	0.	0.	0.

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	650	650	132	0	0	0	0	0
normalized size	1	1.	0.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.35	0.108	0.436	0.	0.	0.	0.

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	676	676	134	0	0	0	0	0
normalized size	1	1.	0.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.413	0.101	0.463	0.	0.	0.	0.

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	104	104	102	0	0	799	0	0
normalized size	1	1.	0.98	0.	0.	7.68	0.	0.
time (sec)	N/A	0.013	0.08	0.377	0.	43.65	0.	0.

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	101	101	110	0	0	1006	0	0
normalized size	1	1.	1.09	0.	0.	9.96	0.	0.
time (sec)	N/A	0.014	0.083	0.413	0.	40.936	0.	0.

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	68	0	0	350	0	0
normalized size	1	1.	0.87	0.	0.	4.49	0.	0.
time (sec)	N/A	0.016	0.053	0.418	0.	5.596	0.	0.

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	76	76	68	0	0	350	0	0
normalized size	1	1.	0.89	0.	0.	4.61	0.	0.
time (sec)	N/A	0.014	0.04	0.392	0.	5.343	0.	0.

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	206	206	155	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	0.105	0.521	0.	0.	0.	0.

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	278	278	126	0	0	0	0	0
normalized size	1	1.	0.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.283	0.103	0.567	0.	0.	0.	0.

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	237	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.27	0.202	0.615	0.	0.	0.	0.

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	108	0	0	0	0	0
normalized size	1	1.	0.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	0.517	0.668	0.	0.	0.	0.

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	203	203	72	0	0	0	0	0
normalized size	1	1.	0.35	0.	0.	0.	0.	0.
time (sec)	N/A	0.368	0.039	0.366	0.	0.	0.	0.

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	379	1140	0	2692	15781	2808
normalized size	1	1.	1.7	5.11	0.	12.07	70.77	12.59
time (sec)	N/A	0.133	0.699	0.049	0.	2.283	18.135	1.181

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	176	420	0	1092	5049	1145
normalized size	1	1.	1.26	3.	0.	7.8	36.06	8.18
time (sec)	N/A	0.069	0.188	0.047	0.	2.224	5.753	1.151

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	59	100	0	308	952	319
normalized size	1	1.	0.84	1.43	0.	4.4	13.6	4.56
time (sec)	N/A	0.03	0.04	0.043	0.	2.074	1.426	1.126

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	145	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.208	0.158	0.587	0.	0.	0.	0.

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	253	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.381	0.361	0.545	0.	0.	0.	0.

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	472	472	396	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.812	0.924	0.604	0.	0.	0.	0.

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	154	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.135	0.081	0.558	0.	0.	0.	0.

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	154	154	159	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	0.11	0.555	0.	0.	0.	0.

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	154	154	159	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	0.113	180.	0.	0.	0.	0.

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	154	154	159	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	0.194	0.531	0.	0.	0.	0.

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	152	152	157	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.137	0.599	0.	0.	0.	0.

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	169	223	0	0	0	468	0
normalized size	1	0.95	1.25	0.	0.	0.	2.63	0.
time (sec)	N/A	0.152	0.229	0.452	0.	0.	17.266	0.

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	125	133	0	0	0	97	0
normalized size	1	0.94	1.	0.	0.	0.	0.73	0.
time (sec)	N/A	0.073	0.1	0.45	0.	0.	12.74	0.

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	70	98	0	0	0	61	0
normalized size	1	1.15	1.61	0.	0.	0.	1.	0.
time (sec)	N/A	0.02	0.05	0.309	0.	0.	6.736	0.

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	44	44	0	0	0	22	0
normalized size	1	1.26	1.26	0.	0.	0.	0.63	0.
time (sec)	N/A	0.009	0.003	0.312	0.	0.	2.598	0.

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	125	125	131	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.118	0.089	0.564	0.	0.	0.	0.

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	191	191	141	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.179	0.084	0.555	0.	0.	0.	0.

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	322	322	142	0	0	0	0	0
normalized size	1	1.	0.44	0.	0.	0.	0.	0.
time (sec)	N/A	0.316	0.121	0.58	0.	0.	0.	0.

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	160	160	166	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.085	0.126	0.583	0.	0.	0.	0.

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	155	155	160	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	0.1	0.593	0.	0.	0.	0.

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	209	209	200	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.084	0.134	0.583	0.	0.	0.	0.

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	270	270	368	0	0	0	0	0
normalized size	1	1.	1.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	49.557	0.614	0.	0.	0.	0.

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	347	347	1439	0	0	0	0	0
normalized size	1	1.	4.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.183	25.074	0.594	0.	0.	0.	0.

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	436	436	2500	0	0	0	0	0
normalized size	1	1.	5.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.454	98.197	0.597	0.	0.	0.	0.

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	559	559	5685	0	0	0	0	0
normalized size	1	1.	10.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.731	53.793	0.599	0.	0.	0.	0.

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	43	43	48	0	0	0	0	0
normalized size	1	1.	1.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.032	0.415	0.	0.	0.	0.

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	65	61	81	147	65	97
normalized size	1	1.	0.78	0.73	0.98	1.77	0.78	1.17
time (sec)	N/A	0.032	0.007	0.041	1.206	1.948	0.351	1.172

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	54	50	66	122	53	82
normalized size	1	1.	0.82	0.76	1.	1.85	0.8	1.24
time (sec)	N/A	0.025	0.006	0.041	1.206	1.821	0.328	1.179

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	39	39	51	90	37	66
normalized size	1	1.	0.8	0.8	1.04	1.84	0.76	1.35
time (sec)	N/A	0.021	0.005	0.041	1.23	1.712	0.315	1.222

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	28	36	66	26	51
normalized size	1	1.	1.	1.	1.29	2.36	0.93	1.82
time (sec)	N/A	0.015	0.005	0.041	1.234	1.623	0.291	1.232

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	19	24	39	14	26
normalized size	1	1.	1.	1.12	1.41	2.29	0.82	1.53
time (sec)	N/A	0.014	0.003	0.039	1.038	1.834	0.27	1.183

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	14	18	23	8	19
normalized size	1	1.	1.	1.17	1.5	1.92	0.67	1.58
time (sec)	N/A	0.004	0.001	0.038	1.41	1.737	0.068	1.203

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	47	47	63	109	39	65
normalized size	1	1.	1.34	1.34	1.8	3.11	1.11	1.86
time (sec)	N/A	0.031	0.011	0.047	1.348	1.731	0.396	1.207

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	58	62	90	192	58	69
normalized size	1	1.	1.12	1.19	1.73	3.69	1.12	1.33
time (sec)	N/A	0.037	0.017	0.047	1.147	1.858	0.493	1.209

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	71	77	122	288	83	95
normalized size	1	1.	1.03	1.12	1.77	4.17	1.2	1.38
time (sec)	N/A	0.046	0.022	0.049	1.027	1.785	0.609	1.243

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	82	92	151	375	107	109
normalized size	1	1.	0.95	1.07	1.76	4.36	1.24	1.27
time (sec)	N/A	0.053	0.024	0.049	1.033	1.723	0.708	1.223

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	71	67	89	192	65	105
normalized size	1	1.	1.01	0.96	1.27	2.74	0.93	1.5
time (sec)	N/A	0.051	0.027	0.044	1.095	1.701	0.443	1.229

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	56	56	74	157	49	89
normalized size	1	1.	1.02	1.02	1.35	2.85	0.89	1.62
time (sec)	N/A	0.041	0.028	0.046	1.118	1.663	0.398	1.219

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	45	45	59	134	37	74
normalized size	1	1.	1.02	1.02	1.34	3.05	0.84	1.68
time (sec)	N/A	0.031	0.02	0.046	1.048	1.735	0.384	1.229

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	32	34	45	97	26	46
normalized size	1	1.	1.03	1.1	1.45	3.13	0.84	1.48
time (sec)	N/A	0.021	0.02	0.043	1.021	1.759	0.341	1.243

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	23	29	38	62	19	39
normalized size	1	1.	0.88	1.12	1.46	2.38	0.73	1.5
time (sec)	N/A	0.017	0.008	0.043	1.08	1.75	0.321	1.257

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	15	19	24	10	19
normalized size	1	1.	1.	1.25	1.58	2.	0.83	1.58
time (sec)	N/A	0.006	0.002	0.037	1.324	1.706	0.296	1.2

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	50	49	65	109	37	68
normalized size	1	1.	1.39	1.36	1.81	3.03	1.03	1.89
time (sec)	N/A	0.026	0.01	0.047	1.066	1.843	0.392	1.234

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	87	79	122	269	85	107
normalized size	1	1.	1.24	1.13	1.74	3.84	1.21	1.53
time (sec)	N/A	0.046	0.02	0.051	1.042	1.746	0.579	1.243

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	75	94	136	308	92	134
normalized size	1	1.	0.86	1.08	1.56	3.54	1.06	1.54
time (sec)	N/A	0.053	0.04	0.052	1.018	1.795	0.669	1.199

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	112	109	182	471	133	136
normalized size	1	1.	1.08	1.05	1.75	4.53	1.28	1.31
time (sec)	N/A	0.066	0.028	0.051	1.067	1.766	0.865	1.182

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	73	73	101	231	70	103
normalized size	1	1.	1.03	1.03	1.42	3.25	0.99	1.45
time (sec)	N/A	0.051	0.033	0.046	1.088	1.706	0.513	1.235

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	62	62	86	203	58	88
normalized size	1	1.	1.03	1.03	1.43	3.38	0.97	1.47
time (sec)	N/A	0.041	0.028	0.047	1.085	1.626	0.495	1.225

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	41	53	74	167	46	62
normalized size	1	1.	0.84	1.08	1.51	3.41	0.94	1.27
time (sec)	N/A	0.03	0.044	0.045	1.038	1.741	0.426	1.175

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	30	47	66	122	39	54
normalized size	1	1.	0.7	1.09	1.53	2.84	0.91	1.26
time (sec)	N/A	0.025	0.02	0.044	1.003	1.772	0.42	1.159

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	29	27	39	17	15
normalized size	1	1.	1.	2.9	2.7	3.9	1.7	1.5
time (sec)	N/A	0.007	0.004	0.044	1.008	1.659	0.358	1.19

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	15	32	47	24	19
normalized size	1	1.	1.	1.	2.13	3.13	1.6	1.27
time (sec)	N/A	0.007	0.002	0.039	1.025	1.654	0.348	1.198

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	62	66	90	192	58	81
normalized size	1	1.	1.15	1.22	1.67	3.56	1.07	1.5
time (sec)	N/A	0.035	0.015	0.046	1.07	1.752	0.496	1.17

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	65	81	122	269	87	107
normalized size	1	1.	0.92	1.14	1.72	3.79	1.23	1.51
time (sec)	N/A	0.042	0.032	0.048	1.117	1.802	0.584	1.24

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	87	111	178	454	134	136
normalized size	1	1.	0.83	1.06	1.7	4.32	1.28	1.3
time (sec)	N/A	0.066	0.043	0.051	1.293	1.817	0.859	1.17

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	98	126	211	547	158	169
normalized size	1	1.	0.8	1.03	1.73	4.48	1.3	1.39
time (sec)	N/A	0.079	0.053	0.049	1.313	1.706	1.042	1.208

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	123	139	177	234	706	123
normalized size	1	1.	0.71	0.8	1.02	1.35	4.08	0.71
time (sec)	N/A	0.076	0.21	0.058	1.609	1.838	12.388	1.17

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	112	114	143	208	442	109
normalized size	1	1.	0.8	0.81	1.02	1.49	3.16	0.78
time (sec)	N/A	0.05	0.218	0.05	1.555	1.819	6.8	1.199

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	101	91	112	177	354	93
normalized size	1	1.	0.94	0.85	1.05	1.65	3.31	0.87
time (sec)	N/A	0.034	0.18	0.048	1.674	1.822	6.285	1.244

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	90	71	85	150	146	76
normalized size	1	1.	1.18	0.93	1.12	1.97	1.92	1.
time (sec)	N/A	0.02	0.077	0.048	1.738	1.81	3.343	1.201

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	77	0	101	0	49
normalized size	1	1.	0.93	1.67	0.	2.2	0.	1.07
time (sec)	N/A	0.016	0.036	0.044	0.	1.778	0.	1.288

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	51	126	0	142	0	0
normalized size	1	1.	0.94	2.33	0.	2.63	0.	0.
time (sec)	N/A	0.014	0.055	0.049	0.	1.934	0.	0.

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	39	36	0	134	0	100
normalized size	1	1.	1.18	1.09	0.	4.06	0.	3.03
time (sec)	N/A	0.009	0.033	0.043	0.	1.776	0.	1.282

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	51	43	0	213	0	223
normalized size	1	1.	0.76	0.64	0.	3.18	0.	3.33
time (sec)	N/A	0.021	0.037	0.044	0.	1.876	0.	1.223

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	63	55	0	290	0	120
normalized size	1	1.	0.63	0.55	0.	2.9	0.	1.2
time (sec)	N/A	0.036	0.041	0.043	0.	1.911	0.	1.212

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	74	66	0	366	0	390
normalized size	1	1.	0.56	0.5	0.	2.75	0.	2.93
time (sec)	N/A	0.053	0.045	0.043	0.	2.072	0.	1.205

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	85	77	0	456	0	474
normalized size	1	1.	0.51	0.46	0.	2.75	0.	2.86
time (sec)	N/A	0.071	0.045	0.043	0.	2.588	0.	1.242

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	134	134	170	259	821	140
normalized size	1	1.	0.82	0.82	1.04	1.58	5.01	0.85
time (sec)	N/A	0.063	0.225	0.055	1.568	2.05	13.69	1.266

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	123	111	139	231	498	124
normalized size	1	1.	0.94	0.85	1.06	1.76	3.8	0.95
time (sec)	N/A	0.044	0.183	0.051	1.606	2.082	9.461	1.245

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	112	91	112	200	439	109
normalized size	1	1.	1.12	0.91	1.12	2.	4.39	1.09
time (sec)	N/A	0.026	0.146	0.047	1.682	2.111	7.835	1.237

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	90	113	119	150	146	76
normalized size	1	1.	1.18	1.49	1.57	1.97	1.92	1.
time (sec)	N/A	0.022	0.078	0.047	1.735	2.012	4.005	1.234

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	60	158	0	126	0	163
normalized size	1	1.	0.71	1.86	0.	1.48	0.	1.92
time (sec)	N/A	0.029	0.056	0.05	0.	2.082	0.	1.262

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	60	206	0	174	0	104
normalized size	1	1.	0.79	2.71	0.	2.29	0.	1.37
time (sec)	N/A	0.028	0.085	0.047	0.	2.156	0.	1.219

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	61	248	0	235	0	116
normalized size	1	1.	0.73	2.99	0.	2.83	0.	1.4
time (sec)	N/A	0.023	0.07	0.053	0.	2.079	0.	1.258

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	41	36	0	198	0	139
normalized size	1	1.	1.24	1.09	0.	6.	0.	4.21
time (sec)	N/A	0.009	0.046	0.043	0.	2.132	0.	1.298

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	48	43	0	281	0	306
normalized size	1	1.	0.72	0.64	0.	4.19	0.	4.57
time (sec)	N/A	0.021	0.051	0.045	0.	2.233	0.	1.289

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	60	55	0	366	0	390
normalized size	1	1.	0.6	0.55	0.	3.66	0.	3.9
time (sec)	N/A	0.037	0.052	0.045	0.	2.522	0.	1.249

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	71	66	0	448	0	474
normalized size	1	1.	0.53	0.5	0.	3.37	0.	3.56
time (sec)	N/A	0.055	0.058	0.045	0.	3.148	0.	1.248

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	82	77	0	544	0	558
normalized size	1	1.	0.49	0.46	0.	3.28	0.	3.36
time (sec)	N/A	0.075	0.059	0.046	0.	3.918	0.	1.258

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	178	174	224	405	1503	186
normalized size	1	1.	0.84	0.82	1.06	1.91	7.09	0.88
time (sec)	N/A	0.09	0.315	0.079	1.6	2.289	34.62	1.262

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	118	151	193	355	1420	173
normalized size	1	1.	0.66	0.84	1.08	1.98	7.93	0.97
time (sec)	N/A	0.067	0.355	0.05	1.72	2.159	31.363	1.266

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	114	131	166	311	1290	161
normalized size	1	1.	0.77	0.89	1.12	2.1	8.72	1.09
time (sec)	N/A	0.047	0.263	0.046	1.525	2.185	24.189	1.224

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	113	181	174	257	818	0
normalized size	1	1.	0.91	1.46	1.4	2.07	6.6	0.
time (sec)	N/A	0.039	0.097	0.047	1.597	2.155	14.159	0.

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	C	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	102	228	188	231	498	0
normalized size	1	1.	0.77	1.73	1.42	1.75	3.77	0.
time (sec)	N/A	0.051	0.078	0.05	1.585	2.18	15.211	0.

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	91	274	216	208	442	0
normalized size	1	1.	0.64	1.93	1.52	1.46	3.11	0.
time (sec)	N/A	0.059	0.075	0.051	1.582	2.29	21.563	0.

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	80	317	0	181	0	0
normalized size	1	1.	0.59	2.33	0.	1.33	0.	0.
time (sec)	N/A	0.057	0.078	0.05	0.	2.199	0.	0.

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	85	364	0	244	0	0
normalized size	1	1.	0.64	2.76	0.	1.85	0.	0.
time (sec)	N/A	0.051	0.099	0.051	0.	2.236	0.	0.

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	87	407	0	312	0	0
normalized size	1	1.	0.6	2.81	0.	2.15	0.	0.
time (sec)	N/A	0.059	0.097	0.051	0.	2.304	0.	0.

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	87	454	0	379	0	0
normalized size	1	1.	0.63	3.29	0.	2.75	0.	0.
time (sec)	N/A	0.059	0.087	0.051	0.	2.567	0.	0.

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	85	496	0	432	0	0
normalized size	1	1.	0.59	3.47	0.	3.02	0.	0.
time (sec)	N/A	0.041	0.095	0.051	0.	2.559	0.	0.

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	41	36	0	333	0	0
normalized size	1	1.	1.24	1.09	0.	10.09	0.	0.
time (sec)	N/A	0.009	0.063	0.042	0.	2.371	0.	0.

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	48	43	0	428	0	0
normalized size	1	1.	0.72	0.64	0.	6.39	0.	0.
time (sec)	N/A	0.021	0.067	0.043	0.	2.805	0.	0.

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	60	55	0	522	0	0
normalized size	1	1.	0.6	0.55	0.	5.22	0.	0.
time (sec)	N/A	0.036	0.072	0.044	0.	3.666	0.	0.

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	71	66	0	612	0	0
normalized size	1	1.	0.53	0.5	0.	4.6	0.	0.
time (sec)	N/A	0.056	0.074	0.045	0.	5.071	0.	0.

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	82	77	0	720	0	0
normalized size	1	1.	0.49	0.46	0.	4.34	0.	0.
time (sec)	N/A	0.076	0.082	0.045	0.	7.099	0.	0.

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	82	0	101	0	50
normalized size	1	1.	1.	1.74	0.	2.15	0.	1.06
time (sec)	N/A	0.015	0.045	0.044	0.	2.06	0.	1.415

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	103	113	0	506	411	136
normalized size	1	1.	0.79	0.87	0.	3.89	3.16	1.05
time (sec)	N/A	0.053	0.15	0.264	0.	2.221	6.3	1.307

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	114	169	0	568	495	153
normalized size	1	1.	0.68	1.01	0.	3.4	2.96	0.92
time (sec)	N/A	0.078	0.177	0.154	0.	2.319	6.716	1.311

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	49	33	49	90	39	46
normalized size	1	1.	1.11	0.75	1.11	2.05	0.89	1.05
time (sec)	N/A	0.007	0.031	0.04	1.022	2.134	0.202	1.333

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	31	29	38	96	39	34
normalized size	1	1.	0.82	0.76	1.	2.53	1.03	0.89
time (sec)	N/A	0.006	0.009	0.04	1.513	2.393	0.194	1.194

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	30	18	16	69	15	16
normalized size	1	1.	2.14	1.29	1.14	4.93	1.07	1.14
time (sec)	N/A	0.006	0.049	0.04	1.513	2.316	1.683	1.206

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	32	29	38	97	39	34
normalized size	1	1.	0.84	0.76	1.	2.55	1.03	0.89
time (sec)	N/A	0.006	0.015	0.043	1.56	2.326	0.195	1.205

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	44	20	19	70	17	19
normalized size	1	1.	2.75	1.25	1.19	4.38	1.06	1.19
time (sec)	N/A	0.006	0.05	0.04	1.708	2.214	1.824	1.212

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	50	40	28	104	0	0
normalized size	1	1.	2.	1.6	1.12	4.16	0.	0.
time (sec)	N/A	0.007	0.055	0.045	1.53	2.143	0.	0.

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	23	20	51	84	0	55
normalized size	1	1.	1.05	0.91	2.32	3.82	0.	2.5
time (sec)	N/A	0.006	0.017	0.041	0.995	2.116	0.	1.255

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	92	144	184	207	644	99
normalized size	1	1.	0.51	0.79	1.01	1.14	3.54	0.54
time (sec)	N/A	0.079	0.121	0.061	1.69	2.149	9.075	1.276

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	81	119	150	181	549	85
normalized size	1	1.	0.54	0.8	1.01	1.21	3.68	0.57
time (sec)	N/A	0.062	0.084	0.056	1.691	2.192	8.198	1.3

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	70	94	116	153	338	70
normalized size	1	1.	0.6	0.81	1.	1.32	2.91	0.6
time (sec)	N/A	0.043	0.06	0.05	1.796	2.162	4.37	1.37

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	83	58	71	85	126	270	54
normalized size	1	1.12	0.78	0.96	1.15	1.7	3.65	0.73
time (sec)	N/A	0.027	0.037	0.049	1.735	2.158	3.729	1.339

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	50	57	101	42	43
normalized size	1	1.	1.	1.06	1.21	2.15	0.89	0.91
time (sec)	N/A	0.012	0.015	0.044	1.967	2.138	1.529	1.264

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	32	29	0	72	0	0
normalized size	1	1.	1.03	0.94	0.	2.32	0.	0.
time (sec)	N/A	0.009	0.006	0.045	0.	2.016	0.	0.

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	40	43	0	144	0	0
normalized size	1	1.	0.6	0.64	0.	2.15	0.	0.
time (sec)	N/A	0.022	0.042	0.044	0.	2.126	0.	0.

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	52	55	0	216	0	0
normalized size	1	1.	0.52	0.55	0.	2.16	0.	0.
time (sec)	N/A	0.038	0.039	0.043	0.	2.115	0.	0.

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	63	66	0	288	0	0
normalized size	1	1.	0.47	0.5	0.	2.17	0.	0.
time (sec)	N/A	0.055	0.042	0.043	0.	2.283	0.	0.

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	74	77	0	369	0	0
normalized size	1	1.	0.45	0.46	0.	2.22	0.	0.
time (sec)	N/A	0.072	0.048	0.045	0.	2.4	0.	0.

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	121	189	284	313	0	131
normalized size	1	1.	0.85	1.32	1.99	2.19	0.	0.92
time (sec)	N/A	0.058	0.214	0.08	1.657	2.224	0.	1.264

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	109	160	244	277	0	116
normalized size	1	1.	1.01	1.48	2.26	2.56	0.	1.07
time (sec)	N/A	0.04	0.157	0.062	1.875	2.147	0.	1.344

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	100	132	207	236	0	89
normalized size	1	1.	1.23	1.63	2.56	2.91	0.	1.1
time (sec)	N/A	0.023	0.12	0.059	1.686	2.031	0.	1.416

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	41	36	108	132	0	76
normalized size	1	1.	1.24	1.09	3.27	4.	0.	2.3
time (sec)	N/A	0.01	0.045	0.042	1.199	2.061	0.	1.305

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	47	44	78	143	0	65
normalized size	1	1.	0.89	0.83	1.47	2.7	0.	1.23
time (sec)	N/A	0.013	0.028	0.044	1.148	2.039	0.	1.455

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	51	53	81	192	298	70
normalized size	1	1.	0.91	0.95	1.45	3.43	5.32	1.25
time (sec)	N/A	0.012	0.024	0.044	1.202	2.083	4.954	1.428

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	70	0	342	0	0
normalized size	1	1.	1.	0.85	0.	4.17	0.	0.
time (sec)	N/A	0.021	0.065	0.045	0.	2.174	0.	0.

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	93	88	0	413	0	0
normalized size	1	1.	0.81	0.77	0.	3.59	0.	0.
time (sec)	N/A	0.035	0.061	0.046	0.	2.35	0.	0.

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	104	99	0	493	0	0
normalized size	1	1.	0.7	0.67	0.	3.33	0.	0.
time (sec)	N/A	0.054	0.068	0.045	0.	2.99	0.	0.

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	115	110	0	582	0	0
normalized size	1	1.	0.64	0.61	0.	3.22	0.	0.
time (sec)	N/A	0.075	0.068	0.047	0.	3.695	0.	0.

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	144	309	518	447	0	174
normalized size	1	1.	0.7	1.5	2.51	2.17	0.	0.84
time (sec)	N/A	0.101	0.343	0.209	1.878	2.544	0.	1.323

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	131	284	485	409	0	159
normalized size	1	1.	0.76	1.64	2.8	2.36	0.	0.92
time (sec)	N/A	0.077	0.283	0.148	1.818	2.316	0.	1.341

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	119	253	443	378	0	144
normalized size	1	1.	0.86	1.83	3.21	2.74	0.	1.04
time (sec)	N/A	0.057	0.24	0.106	2.284	2.288	0.	1.34

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	113	225	405	333	0	128
normalized size	1	1.	1.01	2.01	3.62	2.97	0.	1.14
time (sec)	N/A	0.033	0.193	0.084	1.984	2.204	0.	1.377

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	41	36	200	197	0	103
normalized size	1	1.	1.24	1.09	6.06	5.97	0.	3.12
time (sec)	N/A	0.009	0.094	0.043	1.179	1.965	0.	1.305

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	49	44	166	212	0	95
normalized size	1	1.	0.73	0.66	2.48	3.16	0.	1.42
time (sec)	N/A	0.023	0.069	0.043	1.173	2.086	0.	1.346

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	58	55	136	215	0	95
normalized size	1	1.	0.56	0.53	1.32	2.09	0.	0.92
time (sec)	N/A	0.044	0.063	0.043	1.302	2.136	0.	1.279

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	63	65	105	230	0	82
normalized size	1	1.	0.82	0.84	1.36	2.99	0.	1.06
time (sec)	N/A	0.018	0.044	0.045	1.19	2.112	0.	1.426

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	82	77	108	340	605	88
normalized size	1	1.	1.02	0.96	1.35	4.25	7.56	1.1
time (sec)	N/A	0.019	0.032	0.045	1.276	2.283	10.657	1.331

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	104	92	0	487	0	0
normalized size	1	1.	0.98	0.87	0.	4.59	0.	0.
time (sec)	N/A	0.028	0.08	0.045	0.	3.053	0.	0.

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	115	110	0	524	0	0
normalized size	1	1.	0.83	0.79	0.	3.77	0.	0.
time (sec)	N/A	0.047	0.076	0.048	0.	3.848	0.	0.

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	126	121	0	575	0	0
normalized size	1	1.	0.73	0.7	0.	3.34	0.	0.
time (sec)	N/A	0.064	0.083	0.046	0.	5.721	0.	0.

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	137	132	0	709	0	0
normalized size	1	1.	0.67	0.64	0.	3.46	0.	0.
time (sec)	N/A	0.089	0.089	0.047	0.	7.736	0.	0.

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	148	143	0	863	0	0
normalized size	1	1.	0.62	0.6	0.	3.63	0.	0.
time (sec)	N/A	0.113	0.097	0.048	0.	11.67	0.	0.

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	70	10	19
normalized size	1	1.	1.	0.94	1.19	4.38	0.62	1.19
time (sec)	N/A	0.004	0.006	0.042	1.642	2.031	0.138	1.271

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	69	10	16
normalized size	1	1.	1.	0.93	1.14	4.93	0.71	1.14
time (sec)	N/A	0.004	0.006	0.041	1.525	2.045	0.131	1.198

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	75	66	111	174	0	0
normalized size	1	1.	0.47	0.41	0.69	1.09	0.	0.
time (sec)	N/A	0.073	0.063	0.044	1.109	2.086	0.	0.

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	64	55	92	149	0	0
normalized size	1	1.	0.54	0.46	0.77	1.25	0.	0.
time (sec)	N/A	0.048	0.053	0.043	1.181	2.059	0.	0.

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	53	44	73	120	0	0
normalized size	1	1.	0.68	0.56	0.94	1.54	0.	0.
time (sec)	N/A	0.028	0.043	0.045	1.059	2.08	0.	0.

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	40	36	35	92	0	0
normalized size	1	1.	1.05	0.95	0.92	2.42	0.	0.
time (sec)	N/A	0.013	0.037	0.04	1.061	2.016	0.	0.

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	98	97	0	533	0	0
normalized size	1	1.	0.99	0.98	0.	5.38	0.	0.
time (sec)	N/A	0.055	0.093	0.243	0.	2.301	0.	0.

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	101	127	0	640	0	0
normalized size	1	1.	1.03	1.3	0.	6.53	0.	0.
time (sec)	N/A	0.048	0.112	0.171	0.	2.375	0.	0.

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	111	190	0	776	0	0
normalized size	1	1.	0.79	1.35	0.	5.5	0.	0.
time (sec)	N/A	0.078	0.145	0.169	0.	2.387	0.	0.

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	84	77	149	259	0	0
normalized size	1	1.	0.42	0.38	0.74	1.29	0.	0.
time (sec)	N/A	0.1	0.067	0.043	1.277	2.341	0.	0.

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	73	66	130	220	0	0
normalized size	1	1.	0.46	0.41	0.81	1.38	0.	0.
time (sec)	N/A	0.07	0.057	0.049	1.285	2.465	0.	0.

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	62	55	111	189	0	0
normalized size	1	1.	0.52	0.46	0.93	1.59	0.	0.
time (sec)	N/A	0.05	0.048	0.046	1.217	2.258	0.	0.

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	51	44	74	153	0	0
normalized size	1	1.	0.65	0.56	0.95	1.96	0.	0.
time (sec)	N/A	0.031	0.049	0.041	1.084	1.999	0.	0.

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	43	36	53	123	0	0
normalized size	1	1.	1.13	0.95	1.39	3.24	0.	0.
time (sec)	N/A	0.013	0.05	0.042	1.056	2.077	0.	0.

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	110	122	0	609	0	0
normalized size	1	1.	0.81	0.9	0.	4.48	0.	0.
time (sec)	N/A	0.072	0.211	0.167	0.	2.221	0.	0.

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	107	154	0	689	0	0
normalized size	1	1.	0.8	1.16	0.	5.18	0.	0.
time (sec)	N/A	0.079	0.204	0.171	0.	2.213	0.	0.

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	109	190	0	803	0	0
normalized size	1	1.	0.78	1.37	0.	5.78	0.	0.
time (sec)	N/A	0.071	0.173	0.175	0.	2.163	0.	0.

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	134	259	0	965	0	0
normalized size	1	1.	0.75	1.46	0.	5.42	0.	0.
time (sec)	N/A	0.097	0.247	0.174	0.	2.255	0.	0.

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	145	325	0	1130	0	0
normalized size	1	1.	0.67	1.5	0.	5.21	0.	0.
time (sec)	N/A	0.134	0.272	0.176	0.	2.288	0.	0.

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	70	66	77	154	0	0
normalized size	1	1.	0.44	0.41	0.48	0.96	0.	0.
time (sec)	N/A	0.08	0.075	0.043	1.068	2.113	0.	0.

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	59	55	78	130	0	0
normalized size	1	1.	0.5	0.46	0.66	1.09	0.	0.
time (sec)	N/A	0.049	0.058	0.042	1.089	2.049	0.	0.

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	47	43	46	101	0	0
normalized size	1	1.	0.6	0.55	0.59	1.29	0.	0.
time (sec)	N/A	0.03	0.048	0.042	1.046	2.138	0.	0.

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	35	36	39	82	0	0
normalized size	1	1.	0.97	1.	1.08	2.28	0.	0.
time (sec)	N/A	0.013	0.044	0.042	1.13	2.116	0.	0.

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	86	74	0	371	0	0
normalized size	1	1.	1.32	1.14	0.	5.71	0.	0.
time (sec)	N/A	0.029	0.048	0.162	0.	2.111	0.	0.

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	122	133	0	668	0	0
normalized size	1	1.	1.12	1.22	0.	6.13	0.	0.
time (sec)	N/A	0.051	0.081	0.169	0.	2.297	0.	0.

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	140	195	0	811	0	0
normalized size	1	1.	0.93	1.3	0.	5.41	0.	0.
time (sec)	N/A	0.072	0.11	0.165	0.	2.1	0.	0.

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	66	66	61	157	0	0
normalized size	1	1.	0.41	0.41	0.38	0.98	0.	0.
time (sec)	N/A	0.073	0.083	0.043	1.117	2.183	0.	0.

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	55	55	58	135	0	0
normalized size	1	1.	0.46	0.46	0.49	1.13	0.	0.
time (sec)	N/A	0.049	0.059	0.042	1.138	2.116	0.	0.

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	43	44	31	108	0	0
normalized size	1	1.	0.58	0.59	0.42	1.46	0.	0.
time (sec)	N/A	0.029	0.056	0.039	1.142	2.044	0.	0.

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	35	36	22	93	0	0
normalized size	1	1.	0.97	1.	0.61	2.58	0.	0.
time (sec)	N/A	0.013	0.045	0.04	1.166	2.222	0.	0.

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	110	98	0	609	0	0
normalized size	1	1.	1.06	0.94	0.	5.86	0.	0.
time (sec)	N/A	0.051	0.054	0.169	0.	2.235	0.	0.

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	128	152	0	805	0	0
normalized size	1	1.	0.85	1.01	0.	5.37	0.	0.
time (sec)	N/A	0.073	0.087	0.194	0.	2.244	0.	0.

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	143	217	0	886	0	0
normalized size	1	1.	0.75	1.14	0.	4.64	0.	0.
time (sec)	N/A	0.107	0.117	0.175	0.	2.265	0.	0.

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	46	39	0	84	0	49
normalized size	1	1.	1.48	1.26	0.	2.71	0.	1.58
time (sec)	N/A	0.01	0.028	0.085	0.	2.131	0.	1.246

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	58	52	96	154	0	0
normalized size	1	1.	0.67	0.6	1.1	1.77	0.	0.
time (sec)	N/A	0.027	0.075	0.043	1.715	2.117	0.	0.

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	50	44	81	132	0	0
normalized size	1	1.	0.77	0.68	1.25	2.03	0.	0.
time (sec)	N/A	0.02	0.055	0.042	1.712	2.095	0.	0.

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	42	36	66	109	0	0
normalized size	1	1.	0.98	0.84	1.53	2.53	0.	0.
time (sec)	N/A	0.015	0.044	0.039	2.162	2.12	0.	0.

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	34	30	34	88	0	0
normalized size	1	1.	1.7	1.5	1.7	4.4	0.	0.
time (sec)	N/A	0.009	0.038	0.04	1.877	2.045	0.	0.

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	63	66	0	239	0	0
normalized size	1	1.	1.37	1.43	0.	5.2	0.	0.
time (sec)	N/A	0.02	0.06	0.143	0.	2.136	0.	0.

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	74	88	0	277	0	0
normalized size	1	1.	1.35	1.6	0.	5.04	0.	0.
time (sec)	N/A	0.02	0.089	0.149	0.	1.883	0.	0.

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	54	125	0	327	0	0
normalized size	1	1.	0.63	1.45	0.	3.8	0.	0.
time (sec)	N/A	0.029	0.068	0.15	0.	1.875	0.	0.

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	67	60	126	215	0	0
normalized size	1	1.	0.61	0.55	1.16	1.97	0.	0.
time (sec)	N/A	0.027	0.068	0.043	1.775	1.845	0.	0.

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	59	52	111	181	0	0
normalized size	1	1.	0.68	0.6	1.28	2.08	0.	0.
time (sec)	N/A	0.024	0.06	0.043	1.959	1.845	0.	0.

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	52	44	96	155	0	0
normalized size	1	1.	0.8	0.68	1.48	2.38	0.	0.
time (sec)	N/A	0.02	0.045	0.043	1.995	1.868	0.	0.

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	43	36	63	128	0	0
normalized size	1	1.	0.96	0.8	1.4	2.84	0.	0.
time (sec)	N/A	0.016	0.055	0.04	1.789	1.776	0.	0.

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	37	30	49	105	0	0
normalized size	1	1.	1.68	1.36	2.23	4.77	0.	0.
time (sec)	N/A	0.009	0.045	0.042	1.781	1.792	0.	0.

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	69	77	0	258	0	0
normalized size	1	1.	1.05	1.17	0.	3.91	0.	0.
time (sec)	N/A	0.029	0.069	0.124	0.	1.893	0.	0.

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	55	101	0	290	0	0
normalized size	1	1.	0.75	1.38	0.	3.97	0.	0.
time (sec)	N/A	0.028	0.063	0.131	0.	1.816	0.	0.

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	88	126	0	332	0	0
normalized size	1	1.	1.02	1.47	0.	3.86	0.	0.
time (sec)	N/A	0.029	0.132	0.132	0.	1.879	0.	0.

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	55	167	0	390	0	0
normalized size	1	1.	0.49	1.48	0.	3.45	0.	0.
time (sec)	N/A	0.039	0.085	0.154	0.	1.859	0.	0.

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	55	208	0	450	0	0
normalized size	1	1.	0.38	1.44	0.	3.12	0.	0.
time (sec)	N/A	0.052	0.085	0.149	0.	1.78	0.	0.

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	57	52	61	135	0	0
normalized size	1	1.	0.67	0.61	0.72	1.59	0.	0.
time (sec)	N/A	0.023	0.083	0.043	1.901	1.819	0.	0.

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	49	44	63	113	0	0
normalized size	1	1.	0.75	0.68	0.97	1.74	0.	0.
time (sec)	N/A	0.02	0.059	0.043	1.881	1.82	0.	0.

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	40	35	38	90	0	0
normalized size	1	1.	0.93	0.81	0.88	2.09	0.	0.
time (sec)	N/A	0.017	0.051	0.04	1.962	1.728	0.	0.

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	33	30	34	76	0	0
normalized size	1	1.	1.65	1.5	1.7	3.8	0.	0.
time (sec)	N/A	0.009	0.035	0.039	1.732	1.751	0.	0.

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	50	50	0	158	0	0
normalized size	1	1.	2.	2.	0.	6.32	0.	0.
time (sec)	N/A	0.015	0.042	0.125	0.	1.899	0.	0.

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	54	88	0	278	0	0
normalized size	1	1.	0.95	1.54	0.	4.88	0.	0.
time (sec)	N/A	0.021	0.061	0.127	0.	1.79	0.	0.

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	53	126	0	335	0	0
normalized size	1	1.	0.62	1.47	0.	3.9	0.	0.
time (sec)	N/A	0.028	0.061	0.128	0.	1.901	0.	0.

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	60	60	78	158	0	0
normalized size	1	1.	0.54	0.54	0.7	1.42	0.	0.
time (sec)	N/A	0.029	0.102	0.043	1.671	1.849	0.	0.

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	51	51	49	131	0	0
normalized size	1	1.	0.59	0.59	0.56	1.51	0.	0.
time (sec)	N/A	0.025	0.081	0.044	1.579	1.751	0.	0.

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	43	43	49	111	0	0
normalized size	1	1.	0.64	0.64	0.73	1.66	0.	0.
time (sec)	N/A	0.021	0.064	0.042	1.688	1.857	0.	0.

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	35	35	27	90	0	0
normalized size	1	1.	0.78	0.78	0.6	2.	0.	0.
time (sec)	N/A	0.017	0.057	0.04	1.555	1.828	0.	0.

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	30	30	20	78	0	0
normalized size	1	1.	1.36	1.36	0.91	3.55	0.	0.
time (sec)	N/A	0.01	0.046	0.04	1.719	1.678	0.	0.

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	48	60	0	254	0	0
normalized size	1	1.	0.96	1.2	0.	5.08	0.	0.
time (sec)	N/A	0.021	0.042	0.054	0.	1.845	0.	0.

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	86	48	93	0	331	0	0
normalized size	1	1.09	0.61	1.18	0.	4.19	0.	0.
time (sec)	N/A	0.03	0.05	0.058	0.	1.853	0.	0.

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	115	48	135	0	359	0	0
normalized size	1	1.06	0.44	1.25	0.	3.32	0.	0.
time (sec)	N/A	0.041	0.05	0.059	0.	1.84	0.	0.

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	46	80	44	51
normalized size	1	1.	1.	0.83	2.	3.48	1.91	2.22
time (sec)	N/A	0.006	0.004	0.041	1.745	1.858	1.517	1.206

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	45	40	0	122	0	50
normalized size	1	1.	1.96	1.74	0.	5.3	0.	2.17
time (sec)	N/A	0.01	0.026	0.084	0.	1.743	0.	1.255

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	23	53	90	65	57
normalized size	1	1.	1.	0.85	1.96	3.33	2.41	2.11
time (sec)	N/A	0.008	0.007	0.043	1.673	1.787	5.717	1.148

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	53	50	0	149	0	58
normalized size	1	1.	1.96	1.85	0.	5.52	0.	2.15
time (sec)	N/A	0.014	0.047	0.122	0.	1.773	0.	1.242

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	60	0	0	1532	0	0
normalized size	1	1.	0.19	0.	0.	4.96	0.	0.
time (sec)	N/A	0.335	0.049	0.171	0.	2.064	0.	0.

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	60	0	0	1511	0	0
normalized size	1	1.	0.22	0.	0.	5.62	0.	0.
time (sec)	N/A	0.261	0.048	0.066	0.	2.183	0.	0.

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	60	0	0	1593	0	0
normalized size	1	1.	0.22	0.	0.	5.9	0.	0.
time (sec)	N/A	0.263	0.056	0.485	0.	2.056	0.	0.

Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	30	0	107	0	62
normalized size	1	1.	1.	0.86	0.	3.06	0.	1.77
time (sec)	N/A	0.01	0.053	0.049	0.	1.773	0.	1.26

Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	45	35	0	143	0	124
normalized size	1	1.	0.63	0.49	0.	2.01	0.	1.75
time (sec)	N/A	0.026	0.064	0.042	0.	1.753	0.	1.261

Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	57	44	0	186	0	197
normalized size	1	1.	0.54	0.42	0.	1.75	0.	1.86
time (sec)	N/A	0.041	0.076	0.042	0.	1.842	0.	1.321

Problem 936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	65	52	0	225	0	284
normalized size	1	1.	0.46	0.37	0.	1.6	0.	2.01
time (sec)	N/A	0.058	0.077	0.041	0.	1.823	0.	1.366

Problem 937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	60	0	0	1804	0	0
normalized size	1	1.	0.18	0.	0.	5.31	0.	0.
time (sec)	N/A	0.298	0.07	0.159	0.	2.121	0.	0.

Problem 938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	60	0	0	1828	0	0
normalized size	1	1.	0.19	0.	0.	5.92	0.	0.
time (sec)	N/A	0.269	0.055	0.067	0.	2.224	0.	0.

Problem 939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	60	0	0	1810	0	0
normalized size	1	1.	0.22	0.	0.	6.7	0.	0.
time (sec)	N/A	0.239	0.046	0.075	0.	2.077	0.	0.

Problem 940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	60	0	0	1654	0	0
normalized size	1	1.	0.25	0.	0.	6.86	0.	0.
time (sec)	N/A	0.245	0.05	0.465	0.	2.158	0.	0.

Problem 941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	30	0	95	0	0
normalized size	1	1.	1.	0.86	0.	2.71	0.	0.
time (sec)	N/A	0.014	0.046	0.043	0.	1.855	0.	0.

Problem 942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	40	35	0	127	0	0
normalized size	1	1.	0.56	0.49	0.	1.79	0.	0.
time (sec)	N/A	0.026	0.057	0.043	0.	1.815	0.	0.

Problem 943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	49	44	0	169	0	0
normalized size	1	1.	0.46	0.42	0.	1.59	0.	0.
time (sec)	N/A	0.051	0.069	0.042	0.	1.862	0.	0.

Problem 944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	57	52	0	208	0	0
normalized size	1	1.	0.4	0.37	0.	1.48	0.	0.
time (sec)	N/A	0.073	0.07	0.04	0.	1.93	0.	0.

Problem 945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	68	178	0	682	2096	740
normalized size	1	1.	0.81	2.12	0.	8.12	24.95	8.81
time (sec)	N/A	0.042	0.065	0.046	0.	1.999	5.326	1.157

Problem 946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	50	94	0	363	945	389
normalized size	1	1.	0.82	1.54	0.	5.95	15.49	6.38
time (sec)	N/A	0.028	0.053	0.046	0.	1.961	2.483	1.283

Problem 947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	36	40	0	151	267	159
normalized size	1	1.	0.9	1.	0.	3.78	6.68	3.98
time (sec)	N/A	0.017	0.028	0.04	0.	1.812	1.057	1.24

Problem 948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	59	0	0	0	0	0
normalized size	1	1.	1.55	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.042	0.522	0.	0.	0.	0.

Problem 949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	102	0	0	0	0	0
normalized size	1	1.	2.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.151	0.505	0.	0.	0.	0.

Problem 950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	153	0	0	0	0	0
normalized size	1	1.	3.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.185	0.536	0.	0.	0.	0.

Problem 951	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	59	83	347	0	0	0	0	0
normalized size	1	1.41	5.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.516	0.506	0.	0.	0.	0.

Problem 952	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	227	0	0	0	0	0
normalized size	1	1.	2.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.399	0.484	0.	0.	0.	0.

Problem 953	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	59	83	191	0	0	0	0	0
normalized size	1	1.41	3.24	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.238	0.479	0.	0.	0.	0.

Problem 954	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	83	86	0	0	0	0	0
normalized size	1	1.24	1.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.064	0.481	0.	0.	0.	0.

Problem 955	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	81	84	0	0	0	0	0
normalized size	1	1.21	1.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.082	0.484	0.	0.	0.	0.

Problem 956	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.07	0.483	0.	0.	0.	0.

Problem 957	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	83	92	0	0	0	0	0
normalized size	1	1.38	1.53	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.112	0.484	0.	0.	0.	0.

Problem 958	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	83	91	0	0	0	0	0
normalized size	1	1.22	1.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.103	0.479	0.	0.	0.	0.

Problem 959	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	85	85	0	0	0	0	0
normalized size	1	1.35	1.35	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.09	0.625	0.	0.	0.	0.

Problem 960	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	116	104	0	0	479	0
normalized size	1	1.	2.04	1.82	0.	0.	8.4	0.
time (sec)	N/A	0.039	0.112	0.616	0.	0.	6.267	0.

Problem 961	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	86	75	0	0	116	0
normalized size	1	1.	1.51	1.32	0.	0.	2.04	0.
time (sec)	N/A	0.035	0.066	0.495	0.	0.	3.892	0.

Problem 962	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	56	56	47	0	0	78	0
normalized size	1	0.98	0.98	0.82	0.	0.	1.37	0.
time (sec)	N/A	0.016	0.028	0.354	0.	0.	3.253	0.

Problem 963	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	0	0	24	0
normalized size	1	1.	1.	0.95	0.	0.	1.09	0.
time (sec)	N/A	0.005	0.002	0.343	0.	0.	1.056	0.

Problem 964	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	54	76	0	0	0	323	0
normalized size	1	1.32	1.85	0.	0.	0.	7.88	0.
time (sec)	N/A	0.036	0.086	0.529	0.	0.	5.181	0.

Problem 965	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	76	0	0	0	0	0
normalized size	1	1.	1.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.078	0.546	0.	0.	0.	0.

Problem 966	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	76	0	0	0	0	0
normalized size	1	1.	1.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.077	0.61	0.	0.	0.	0.

Problem 967	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	73	155	0	0	0	476	0
normalized size	1	1.22	2.58	0.	0.	0.	7.93	0.
time (sec)	N/A	0.031	0.179	0.51	0.	0.	5.848	0.

Problem 968	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	71	134	0	0	0	124	0
normalized size	1	1.18	2.23	0.	0.	0.	2.07	0.
time (sec)	N/A	0.029	0.061	0.488	0.	0.	3.719	0.

Problem 969	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	0	0	0	82	0
normalized size	1	1.	1.	0.	0.	0.	0.99	0.
time (sec)	N/A	0.022	0.046	0.381	0.	0.	2.945	0.

Problem 970	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	73	75	0	0	0	323	0
normalized size	1	1.33	1.36	0.	0.	0.	5.87	0.
time (sec)	N/A	0.03	0.052	0.532	0.	0.	5.052	0.

Problem 971	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	73	75	0	0	0	0	0
normalized size	1	1.26	1.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.053	0.541	0.	0.	0.	0.

Problem 972	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	73	75	0	0	0	0	0
normalized size	1	1.18	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.056	0.564	0.	0.	0.	0.

Problem 973	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	189	0	0	0	0	0
normalized size	1	1.	2.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.256	0.514	0.	0.	0.	0.

Problem 974	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	89	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	0.094	0.516	0.	0.	0.	0.

Problem 975	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	89	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	0.095	0.52	0.	0.	0.	0.

Problem 976	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	92	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	0.111	0.516	0.	0.	0.	0.

Problem 977	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	36	57	68	49	35
normalized size	1	1.	1.	1.29	2.04	2.43	1.75	1.25
time (sec)	N/A	0.041	0.005	0.039	1.393	1.971	3.822	1.265

Problem 978	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	48	63	99	51	61
normalized size	1	1.	1.	3.2	4.2	6.6	3.4	4.07
time (sec)	N/A	0.004	0.003	0.04	1.1	1.617	0.072	1.16

Problem 979	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	36	41	77	39	46
normalized size	1	1.	1.	2.4	2.73	5.13	2.6	3.07
time (sec)	N/A	0.004	0.002	0.039	1.101	1.562	0.075	1.184

Problem 980	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	31	50	24	31
normalized size	1	1.	1.	0.96	1.24	2.	0.96	1.24
time (sec)	N/A	0.006	0.	0.038	1.012	1.527	0.069	1.139

Problem 981	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	28	12	26
normalized size	1	1.	1.	0.93	1.14	2.	0.86	1.86
time (sec)	N/A	0.008	0.001	0.038	1.117	1.936	0.086	1.132

Problem 982	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	7	2	149
normalized size	1	1.	1.	1.33	1.33	2.33	0.67	49.67
time (sec)	N/A	0.004	0.	0.038	1.125	1.991	0.081	1.16

Problem 983	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	15	24	8	16
normalized size	1	1.	1.	1.09	1.36	2.18	0.73	1.45
time (sec)	N/A	0.006	0.001	0.038	1.16	2.052	0.083	1.204

Problem 984	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	19	24	10	46
normalized size	1	1.	1.	1.08	1.46	1.85	0.77	3.54
time (sec)	N/A	0.006	0.003	0.038	1.161	1.985	0.314	1.223

Problem 985	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	34	51	26	18
normalized size	1	1.	1.	0.93	2.27	3.4	1.73	1.2
time (sec)	N/A	0.006	0.003	0.037	1.186	1.932	0.414	1.162

Problem 986	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	49	73	37	46
normalized size	1	1.	1.	0.93	3.27	4.87	2.47	3.07
time (sec)	N/A	0.006	0.004	0.04	1.146	1.978	0.489	1.211

Problem 987	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	86	115	166	90	109
normalized size	1	1.	1.	5.06	6.76	9.76	5.29	6.41
time (sec)	N/A	0.005	0.003	0.039	1.147	1.782	0.089	1.217

Problem 988	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	72	41	149	80	92
normalized size	1	1.	1.	4.24	2.41	8.76	4.71	5.41
time (sec)	N/A	0.004	0.002	0.038	1.224	1.728	0.084	1.168

Problem 989	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	58	92	112	60	74
normalized size	1	1.	1.	3.41	5.41	6.59	3.53	4.35
time (sec)	N/A	0.004	0.001	0.039	1.224	1.86	0.074	1.192

Problem 990	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	36	58	88	46	66
normalized size	1	1.	1.	2.12	3.41	5.18	2.71	3.88
time (sec)	N/A	0.005	0.001	0.039	1.203	1.978	0.104	1.173

Problem 991	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	39	58	29	20
normalized size	1	1.	1.	0.94	2.29	3.41	1.71	1.18
time (sec)	N/A	0.005	0.001	0.038	1.226	1.952	0.116	1.168

Problem 992	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	16	15	22	34	15	31
normalized size	1	1.	0.94	0.88	1.29	2.	0.88	1.82
time (sec)	N/A	0.004	0.001	0.038	1.162	2.059	0.117	1.15

Problem 993	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	7	9	3	7
normalized size	1	1.	1.	1.2	1.4	1.8	0.6	1.4
time (sec)	N/A	0.002	0.	0.04	1.148	2.01	0.096	1.163

Problem 994	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	18	27	10	35
normalized size	1	1.	1.	1.08	1.38	2.08	0.77	2.69
time (sec)	N/A	0.004	0.001	0.039	1.173	1.982	0.138	1.198

Problem 995	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	22	27	12	89
normalized size	1	1.	1.	1.07	1.47	1.8	0.8	5.93
time (sec)	N/A	0.005	0.002	0.039	1.175	2.035	0.314	1.209

Problem 996	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	36	54	27	89
normalized size	1	1.	1.	0.94	2.12	3.18	1.59	5.24
time (sec)	N/A	0.005	0.002	0.04	1.219	1.961	0.691	1.221

Problem 997	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	51	76	39	89
normalized size	1	1.	1.	0.94	3.	4.47	2.29	5.24
time (sec)	N/A	0.005	0.002	0.039	1.197	1.931	0.663	1.262

Problem 998	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	50	74	39	0
normalized size	1	1.	1.	0.94	2.94	4.35	2.29	0.
time (sec)	N/A	0.005	0.001	0.039	1.241	1.968	0.115	0.

Problem 999	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	35	53	24	0
normalized size	1	1.	1.	0.94	2.06	3.12	1.41	0.
time (sec)	N/A	0.005	0.001	0.039	1.1	1.925	0.117	0.

Problem 1000	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	16	15	20	31	12	0
normalized size	1	1.	0.94	0.88	1.18	1.82	0.71	0.
time (sec)	N/A	0.003	0.001	0.04	1.201	1.939	0.167	0.

Problem 1001	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	7	7	2	0
normalized size	1	1.	1.	1.2	1.4	1.4	0.4	0.
time (sec)	N/A	0.002	0.	0.037	1.25	2.005	0.097	0.

Problem 1002	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	16	14	39	27	12	0
normalized size	1	1.	1.23	1.08	3.	2.08	0.92	0.
time (sec)	N/A	0.004	0.002	0.039	1.178	1.912	0.138	0.

Problem 1003	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	20	30	14	0
normalized size	1	1.	1.	1.07	1.33	2.	0.93	0.
time (sec)	N/A	0.004	0.002	0.042	1.195	1.974	0.328	0.

Problem 1004	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	36	57	31	0
normalized size	1	1.	1.	0.94	2.12	3.35	1.82	0.
time (sec)	N/A	0.005	0.002	0.041	1.083	2.091	0.507	0.

Problem 1005	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	53	81	44	0
normalized size	1	1.	1.	0.94	3.12	4.76	2.59	0.
time (sec)	N/A	0.005	0.002	0.043	1.277	2.019	0.486	0.

Problem 1006	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	69	105	58	0
normalized size	1	1.	1.	0.94	4.06	6.18	3.41	0.
time (sec)	N/A	0.005	0.005	0.043	1.005	1.851	0.653	0.

Problem 1007	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	50	77	46	0
normalized size	1	1.	1.	0.94	2.94	4.53	2.71	0.
time (sec)	N/A	0.005	0.001	0.038	1.024	2.066	0.163	0.

Problem 1008	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	35	55	29	0
normalized size	1	1.	1.	0.94	2.06	3.24	1.71	0.
time (sec)	N/A	0.005	0.001	0.039	1.004	1.879	0.131	0.

Problem 1009	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	16	15	20	34	15	0
normalized size	1	1.	0.94	0.88	1.18	2.	0.88	0.
time (sec)	N/A	0.004	0.001	0.04	1.118	2.013	0.138	0.

Problem 1010	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	7	9	3	0
normalized size	1	1.	1.	1.2	1.4	1.8	0.6	0.
time (sec)	N/A	0.003	0.	0.04	1.09	2.044	0.128	0.

Problem 1011	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	18	30	17	0
normalized size	1	1.	1.	1.08	1.38	2.31	1.31	0.
time (sec)	N/A	0.005	0.001	0.041	1.161	1.828	0.163	0.

Problem 1012	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	26	35	17	0
normalized size	1	1.	1.	1.07	1.73	2.33	1.13	0.
time (sec)	N/A	0.005	0.002	0.039	1.087	1.926	0.527	0.

Problem 1013	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	41	65	36	0
normalized size	1	1.	1.	0.94	2.41	3.82	2.12	0.
time (sec)	N/A	0.004	0.002	0.041	1.088	2.031	0.622	0.

Problem 1014	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	63	92	51	0
normalized size	1	1.	1.	0.94	3.71	5.41	3.	0.
time (sec)	N/A	0.004	0.002	0.041	1.006	1.953	0.692	0.

Problem 1015	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	82	119	66	0
normalized size	1	1.	1.	0.94	4.82	7.	3.88	0.
time (sec)	N/A	0.005	0.002	0.042	1.033	1.978	0.602	0.

Problem 1016	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	101	149	82	0
normalized size	1	1.	1.	0.94	5.94	8.76	4.82	0.
time (sec)	N/A	0.005	0.005	0.043	1.026	2.103	0.822	0.

Problem 1017	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	50	77	46	0
normalized size	1	1.	1.	0.94	2.94	4.53	2.71	0.
time (sec)	N/A	0.005	0.001	0.04	1.031	1.946	0.163	0.

Problem 1018	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	35	55	29	0
normalized size	1	1.	1.	0.94	2.06	3.24	1.71	0.
time (sec)	N/A	0.005	0.001	0.04	1.079	1.984	0.146	0.

Problem 1019	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	16	15	20	34	15	0
normalized size	1	1.	0.94	0.88	1.18	2.	0.88	0.
time (sec)	N/A	0.004	0.001	0.039	1.066	1.971	0.148	0.

Problem 1020	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	7	9	3	0
normalized size	1	1.	1.	1.2	1.4	1.8	0.6	0.
time (sec)	N/A	0.002	0.	0.038	0.987	2.014	0.134	0.

Problem 1021	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	18	30	17	0
normalized size	1	1.	1.	1.08	1.38	2.31	1.31	0.
time (sec)	N/A	0.005	0.001	0.039	0.976	2.069	0.163	0.

Problem 1022	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	26	35	17	0
normalized size	1	1.	1.	1.07	1.73	2.33	1.13	0.
time (sec)	N/A	0.005	0.002	0.038	0.992	2.002	0.402	0.

Problem 1023	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	45	65	36	0
normalized size	1	1.	1.	0.94	2.65	3.82	2.12	0.
time (sec)	N/A	0.005	0.002	0.059	0.976	1.92	0.475	0.

Problem 1024	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	63	92	51	0
normalized size	1	1.	1.	0.94	3.71	5.41	3.	0.
time (sec)	N/A	0.006	0.003	0.042	0.991	1.906	0.591	0.

Problem 1025	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	41	119	66	0
normalized size	1	1.	1.	0.94	2.41	7.	3.88	0.
time (sec)	N/A	0.004	0.003	0.042	0.979	2.025	0.772	0.

Problem 1026	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	101	149	82	0
normalized size	1	1.	1.	0.94	5.94	8.76	4.82	0.
time (sec)	N/A	0.004	0.003	0.04	0.978	1.986	0.746	0.

Problem 1027	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	120	177	97	0
normalized size	1	1.	1.	0.94	7.06	10.41	5.71	0.
time (sec)	N/A	0.005	0.005	0.041	0.992	1.993	0.745	0.

Problem 1028	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	139	205	112	0
normalized size	1	1.	1.	0.94	8.18	12.06	6.59	0.
time (sec)	N/A	0.005	0.005	0.042	1.022	2.016	1.03	0.

Problem 1029	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	27	73	0	158	187	82
normalized size	1	1.	0.79	2.15	0.	4.65	5.5	2.41
time (sec)	N/A	0.024	0.011	0.042	0.	2.153	0.823	1.204

Problem 1030	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	28	62	0	134	0	69
normalized size	1	1.	0.72	1.59	0.	3.44	0.	1.77
time (sec)	N/A	0.022	0.014	0.042	0.	2.054	0.	1.206

Problem 1031	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	23	51	41	112	107	55
normalized size	1	1.	0.68	1.5	1.21	3.29	3.15	1.62
time (sec)	N/A	0.009	0.009	0.04	0.986	2.042	0.292	1.164

Problem 1032	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	31	40	0	90	0	41
normalized size	1	1.	0.86	1.11	0.	2.5	0.	1.14
time (sec)	N/A	0.006	0.013	0.04	0.	2.029	0.	1.264

Problem 1033	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	21	32	0	66	37	0
normalized size	1	1.	0.75	1.14	0.	2.36	1.32	0.
time (sec)	N/A	0.024	0.006	0.041	0.	2.218	1.659	0.

Problem 1034	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	29	40	0	86	0	11
normalized size	1	1.	0.72	1.	0.	2.15	0.	0.28
time (sec)	N/A	0.025	0.008	0.042	0.	2.391	0.	1.353

Problem 1035	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	19	35	0	92	0	0
normalized size	1	1.	0.63	1.17	0.	3.07	0.	0.
time (sec)	N/A	0.023	0.007	0.04	0.	2.402	0.	0.

Problem 1036	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	27	35	0	119	0	0
normalized size	1	1.	0.69	0.9	0.	3.05	0.	0.
time (sec)	N/A	0.019	0.006	0.04	0.	2.373	0.	0.

Problem 1037	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	27	35	0	140	0	0
normalized size	1	1.	0.79	1.03	0.	4.12	0.	0.
time (sec)	N/A	0.023	0.007	0.041	0.	2.316	0.	0.

Problem 1038	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	27	35	0	165	0	0
normalized size	1	1.	0.66	0.85	0.	4.02	0.	0.
time (sec)	N/A	0.019	0.007	0.041	0.	2.344	0.	0.

Problem 1039	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	27	95	0	223	277	119
normalized size	1	1.	0.79	2.79	0.	6.56	8.15	3.5
time (sec)	N/A	0.024	0.017	0.042	0.	2.219	4.62	1.3

Problem 1040	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	28	84	0	197	0	104
normalized size	1	1.	0.72	2.15	0.	5.05	0.	2.67
time (sec)	N/A	0.021	0.024	0.04	0.	2.279	0.	1.174

Problem 1041	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	23	73	41	171	194	89
normalized size	1	1.	0.68	2.15	1.21	5.03	5.71	2.62
time (sec)	N/A	0.009	0.013	0.042	1.019	2.306	2.241	1.207

Problem 1042	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	25	62	0	144	0	74
normalized size	1	1.	0.69	1.72	0.	4.	0.	2.06
time (sec)	N/A	0.006	0.002	0.04	0.	2.182	0.	1.297

Problem 1043	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	20	51	0	120	39	0
normalized size	1	1.	0.65	1.65	0.	3.87	1.26	0.
time (sec)	N/A	0.023	0.003	0.041	0.	2.142	3.007	0.

Problem 1044	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	33	40	0	96	0	0
normalized size	1	1.	0.89	1.08	0.	2.59	0.	0.
time (sec)	N/A	0.021	0.002	0.039	0.	2.278	0.	0.

Problem 1045	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	22	32	0	69	39	0
normalized size	1	1.	0.76	1.1	0.	2.38	1.34	0.
time (sec)	N/A	0.022	0.009	0.04	0.	2.349	3.657	0.

Problem 1046	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	31	40	0	89	0	0
normalized size	1	1.	0.74	0.95	0.	2.12	0.	0.
time (sec)	N/A	0.025	0.004	0.043	0.	2.292	0.	0.

Problem 1047	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	25	35	0	95	0	0
normalized size	1	1.	0.78	1.09	0.	2.97	0.	0.
time (sec)	N/A	0.023	0.011	0.04	0.	2.525	0.	0.

Problem 1048	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	27	35	0	122	0	0
normalized size	1	1.	0.66	0.85	0.	2.98	0.	0.
time (sec)	N/A	0.02	0.009	0.041	0.	2.419	0.	0.

Problem 1049	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	27	35	0	143	0	0
normalized size	1	1.	0.79	1.03	0.	4.21	0.	0.
time (sec)	N/A	0.024	0.009	0.04	0.	2.445	0.	0.

Problem 1050	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	27	117	0	301	374	173
normalized size	1	1.	0.79	3.44	0.	8.85	11.	5.09
time (sec)	N/A	0.024	0.026	0.04	0.	2.376	19.236	1.207

Problem 1051	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	28	106	0	270	0	155
normalized size	1	1.	0.72	2.72	0.	6.92	0.	3.97
time (sec)	N/A	0.021	0.02	0.04	0.	2.505	0.	1.208

Problem 1052	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	23	95	41	242	287	138
normalized size	1	1.	0.68	2.79	1.21	7.12	8.44	4.06
time (sec)	N/A	0.009	0.019	0.04	1.012	2.341	8.196	1.22

Problem 1053	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	25	84	0	213	0	120
normalized size	1	1.	0.69	2.33	0.	5.92	0.	3.33
time (sec)	N/A	0.007	0.002	0.041	0.	2.356	0.	1.31

Problem 1054	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	20	73	0	185	39	0
normalized size	1	1.	0.65	2.35	0.	5.97	1.26	0.
time (sec)	N/A	0.024	0.005	0.04	0.	2.371	5.578	0.

Problem 1055	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	26	62	0	155	0	0
normalized size	1	1.	0.7	1.68	0.	4.19	0.	0.
time (sec)	N/A	0.021	0.005	0.042	0.	2.255	0.	0.

Problem 1056	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	21	51	0	128	0	0
normalized size	1	1.	0.66	1.59	0.	4.	0.	0.
time (sec)	N/A	0.023	0.006	0.04	0.	2.334	0.	0.

Problem 1057	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	33	40	0	101	0	0
normalized size	1	1.	0.85	1.03	0.	2.59	0.	0.
time (sec)	N/A	0.021	0.002	0.039	0.	2.217	0.	0.

Problem 1058	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	23	32	0	72	41	0
normalized size	1	1.	0.74	1.03	0.	2.32	1.32	0.
time (sec)	N/A	0.023	0.006	0.041	0.	2.264	8.023	0.

Problem 1059	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	31	40	0	92	0	0
normalized size	1	1.	0.74	0.95	0.	2.19	0.	0.
time (sec)	N/A	0.025	0.004	0.041	0.	2.299	0.	0.

Problem 1060	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	25	35	0	97	0	0
normalized size	1	1.	0.78	1.09	0.	3.03	0.	0.
time (sec)	N/A	0.024	0.013	0.039	0.	2.253	0.	0.

Problem 1061	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	27	35	0	124	0	0
normalized size	1	1.	0.66	0.85	0.	3.02	0.	0.
time (sec)	N/A	0.022	0.011	0.04	0.	2.392	0.	0.

Problem 1062	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	27	60	246	139	0	85
normalized size	1	1.	0.69	1.54	6.31	3.56	0.	2.18
time (sec)	N/A	0.022	0.009	0.042	1.175	2.444	0.	1.382

Problem 1063	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	27	49	197	117	114	68
normalized size	1	1.	0.79	1.44	5.79	3.44	3.35	2.
time (sec)	N/A	0.023	0.006	0.041	1.213	2.335	1.079	1.357

Problem 1064	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	30	38	147	96	0	50
normalized size	1	1.	0.77	0.97	3.77	2.46	0.	1.28
time (sec)	N/A	0.022	0.003	0.04	1.16	2.385	0.	1.394

Problem 1065	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	20	30	39	72	39	38
normalized size	1	1.	0.65	0.97	1.26	2.32	1.26	1.23
time (sec)	N/A	0.009	0.003	0.04	1.204	2.272	0.827	1.345

Problem 1066	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	28	38	24	92	0	73
normalized size	1	1.	0.72	0.97	0.62	2.36	0.	1.87
time (sec)	N/A	0.009	0.007	0.04	1.141	2.332	0.	1.357

Problem 1067	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	18	28	26	100	41	0
normalized size	1	1.	0.62	0.97	0.9	3.45	1.41	0.
time (sec)	N/A	0.024	0.011	0.039	1.12	2.412	1.903	0.

Problem 1068	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	26	35	45	130	0	0
normalized size	1	1.	0.68	0.92	1.18	3.42	0.	0.
time (sec)	N/A	0.019	0.01	0.041	1.202	2.387	0.	0.

Problem 1069	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	21	35	63	154	0	0
normalized size	1	1.	0.66	1.09	1.97	4.81	0.	0.
time (sec)	N/A	0.024	0.015	0.041	1.06	2.38	0.	0.

Problem 1070	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	27	35	82	181	0	0
normalized size	1	1.	0.69	0.9	2.1	4.64	0.	0.
time (sec)	N/A	0.019	0.011	0.043	1.152	2.271	0.	0.

Problem 1071	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	33	40	134	101	0	88
normalized size	1	1.	0.85	1.03	3.44	2.59	0.	2.26
time (sec)	N/A	0.022	0.005	0.039	1.322	2.238	0.	1.335

Problem 1072	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	22	32	86	77	42	72
normalized size	1	1.	0.71	1.03	2.77	2.48	1.35	2.32
time (sec)	N/A	0.024	0.007	0.04	1.288	2.428	0.917	1.331

Problem 1073	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	31	40	166	97	0	119
normalized size	1	1.	0.74	0.95	3.95	2.31	0.	2.83
time (sec)	N/A	0.025	0.003	0.042	1.144	2.354	0.	1.412

Problem 1074	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	21	35	41	108	42	55
normalized size	1	1.	0.66	1.09	1.28	3.38	1.31	1.72
time (sec)	N/A	0.009	0.006	0.04	1.196	2.44	1.194	1.369

Problem 1075	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	25	33	24	140	0	0
normalized size	1	1.	0.61	0.8	0.59	3.41	0.	0.
time (sec)	N/A	0.005	0.003	0.042	1.252	2.305	0.	0.

Problem 1076	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	20	28	63	167	42	0
normalized size	1	1.	0.65	0.9	2.03	5.39	1.35	0.
time (sec)	N/A	0.025	0.02	0.046	1.138	2.408	2.664	0.

Problem 1077	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	26	35	82	197	0	0
normalized size	1	1.	0.68	0.92	2.16	5.18	0.	0.
time (sec)	N/A	0.02	0.018	0.041	1.203	2.426	0.	0.

Problem 1078	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	21	35	101	225	0	0
normalized size	1	1.	0.66	1.09	3.16	7.03	0.	0.
time (sec)	N/A	0.024	0.026	0.041	1.201	2.36	0.	0.

Problem 1079	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	33	40	313	101	0	146
normalized size	1	1.	0.85	1.03	8.03	2.59	0.	3.74
time (sec)	N/A	0.023	0.003	0.04	1.276	2.309	0.	1.488

Problem 1080	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	23	32	266	77	42	126
normalized size	1	1.	0.74	1.03	8.58	2.48	1.35	4.06
time (sec)	N/A	0.024	0.005	0.039	1.254	2.384	1.707	1.46

Problem 1081	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	31	40	606	97	0	149
normalized size	1	1.	0.74	0.95	14.43	2.31	0.	3.55
time (sec)	N/A	0.026	0.007	0.042	1.579	2.323	0.	1.559

Problem 1082	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	21	35	215	108	70	107
normalized size	1	1.	0.66	1.09	6.72	3.38	2.19	3.34
time (sec)	N/A	0.025	0.013	0.04	1.573	2.407	1.528	1.402

Problem 1083	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	28	35	167	140	0	95
normalized size	1	1.	0.68	0.85	4.07	3.41	0.	2.32
time (sec)	N/A	0.022	0.01	0.041	1.273	2.38	0.	1.54

Problem 1084	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	35	41	167	124	86
normalized size	1	1.	0.88	1.03	1.21	4.91	3.65	2.53
time (sec)	N/A	0.009	0.011	0.04	1.578	2.186	1.647	1.332

Problem 1085	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	25	33	24	197	0	0
normalized size	1	1.	0.61	0.8	0.59	4.8	0.	0.
time (sec)	N/A	0.005	0.003	0.039	1.145	2.283	0.	0.

Problem 1086	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	20	28	101	225	42	0
normalized size	1	1.	0.65	0.9	3.26	7.26	1.35	0.
time (sec)	N/A	0.024	0.027	0.042	1.213	2.322	3.663	0.

Problem 1087	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	26	35	120	254	0	0
normalized size	1	1.	0.68	0.92	3.16	6.68	0.	0.
time (sec)	N/A	0.021	0.018	0.042	1.162	2.27	0.	0.

Problem 1088	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	21	35	139	282	0	0
normalized size	1	1.	0.66	1.09	4.34	8.81	0.	0.
time (sec)	N/A	0.023	0.027	0.042	1.144	2.365	0.	0.

Problem 1089	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	22	40	0	169	185	169
normalized size	1	1.	1.05	1.9	0.	8.05	8.81	8.05
time (sec)	N/A	0.009	0.017	0.04	0.	2.446	1.457	1.335

Problem 1090	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	20	36	0	101	116	101
normalized size	1	1.	1.05	1.89	0.	5.32	6.11	5.32
time (sec)	N/A	0.008	0.014	0.043	0.	2.485	0.707	1.272

Problem 1091	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	21	22	36	72	63	0
normalized size	1	1.	0.88	0.92	1.5	3.	2.62	0.
time (sec)	N/A	0.012	0.014	0.041	1.182	2.339	1.183	0.

Problem 1092	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	21	40	88	196	136	0
normalized size	1	1.	0.88	1.67	3.67	8.17	5.67	0.
time (sec)	N/A	0.01	0.015	0.04	1.374	2.477	2.147	0.

Problem 1093	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	21	40	134	306	201	0
normalized size	1	1.	0.88	1.67	5.58	12.75	8.38	0.
time (sec)	N/A	0.01	0.014	0.042	1.208	2.493	3.703	0.

Problem 1094	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	31	41	95	150	0	140
normalized size	1	1.	0.74	0.98	2.26	3.57	0.	3.33
time (sec)	N/A	0.019	0.03	0.038	1.182	2.559	0.	1.275

Problem 1095	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	31	41	57	96	0	84
normalized size	1	1.	0.74	0.98	1.36	2.29	0.	2.
time (sec)	N/A	0.018	0.018	0.044	1.21	2.421	0.	1.192

Problem 1096	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	29	39	23	96	0	0
normalized size	1	1.	0.72	0.98	0.57	2.4	0.	0.
time (sec)	N/A	0.018	0.012	0.04	1.226	2.366	0.	0.

Problem 1097	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	31	41	69	244	0	0
normalized size	1	1.	0.69	0.91	1.53	5.42	0.	0.
time (sec)	N/A	0.02	0.02	0.039	1.08	2.472	0.	0.

Problem 1098	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	32	44	58	99	0	93
normalized size	1	1.	0.74	1.02	1.35	2.3	0.	2.16
time (sec)	N/A	0.018	0.017	0.041	1.1	2.432	0.	1.243

Problem 1099	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	31	44	39	54	0	93
normalized size	1	1.	0.7	1.	0.89	1.23	0.	2.11
time (sec)	N/A	0.017	0.016	0.04	1.156	2.448	0.	1.237

Problem 1100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	30	40	423	147	233	216
normalized size	1	1.	0.77	1.03	10.85	3.77	5.97	5.54
time (sec)	N/A	0.026	0.017	0.04	1.97	2.29	1.069	1.287

Problem 1101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	32	41	246	123	0	173
normalized size	1	1.	0.74	0.95	5.72	2.86	0.	4.02
time (sec)	N/A	0.026	0.019	0.039	1.293	2.619	0.	1.211

Problem 1102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	28	40	0	101	139	127
normalized size	1	1.	0.72	1.03	0.	2.59	3.56	3.26
time (sec)	N/A	0.01	0.014	0.042	0.	2.622	0.473	1.197

Problem 1103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	25	39	43	77	0	84
normalized size	1	1.	0.66	1.03	1.13	2.03	0.	2.21
time (sec)	N/A	0.01	0.01	0.039	1.24	2.525	0.	1.322

Problem 1104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	21	31	27	61	48	0
normalized size	1	1.	0.66	0.97	0.84	1.91	1.5	0.
time (sec)	N/A	0.021	0.006	0.039	1.155	2.518	0.348	0.

Problem 1105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	30	41	46	96	0	0
normalized size	1	1.	0.71	0.98	1.1	2.29	0.	0.
time (sec)	N/A	0.026	0.017	0.039	1.317	2.443	0.	0.

Problem 1106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	26	40	61	136	100	0
normalized size	1	1.	0.67	1.03	1.56	3.49	2.56	0.
time (sec)	N/A	0.026	0.012	0.039	1.274	2.487	1.168	0.

Problem 1107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	30	95	18	27	0	0
normalized size	1	1.	0.73	2.32	0.44	0.66	0.	0.
time (sec)	N/A	0.015	0.007	0.079	1.349	2.405	0.	0.

Problem 1108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	32	74	20	30	0	0
normalized size	1	1.	0.74	1.72	0.47	0.7	0.	0.
time (sec)	N/A	0.015	0.006	0.077	1.164	2.367	0.	0.

Problem 1109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	102	137	162	286	143	185
normalized size	1	1.	2.27	3.04	3.6	6.36	3.18	4.11
time (sec)	N/A	0.075	0.017	0.04	1.228	2.023	0.123	1.216

Problem 1110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	66	108	131	225	114	146
normalized size	1	1.	1.47	2.4	2.91	5.	2.53	3.24
time (sec)	N/A	0.054	0.015	0.039	1.232	2.023	0.12	1.222

Problem 1111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	64	79	96	169	85	107
normalized size	1	1.	1.42	1.76	2.13	3.76	1.89	2.38
time (sec)	N/A	0.04	0.01	0.039	1.268	1.968	0.084	1.194

Problem 1112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	22	39	20	88	39	51
normalized size	1	1.	1.29	2.29	1.18	5.18	2.29	3.
time (sec)	N/A	0.004	0.005	0.038	1.171	1.942	0.097	1.243

Problem 1113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	37	54	55	89	37	63
normalized size	1	1.	0.77	1.12	1.15	1.85	0.77	1.31
time (sec)	N/A	0.035	0.013	0.041	1.173	2.221	0.434	1.176

Problem 1114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	41	35	54	90	36	230
normalized size	1	1.	1.08	0.92	1.42	2.37	0.95	6.05
time (sec)	N/A	0.028	0.011	0.043	1.348	2.205	0.47	1.143

Problem 1115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	37	53	81	153	60	55
normalized size	1	1.	0.84	1.2	1.84	3.48	1.36	1.25
time (sec)	N/A	0.033	0.015	0.044	1.187	2.334	0.622	1.222

Problem 1116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	38	42	96	149	75	50
normalized size	1	1.	0.84	0.93	2.13	3.31	1.67	1.11
time (sec)	N/A	0.032	0.016	0.044	1.163	2.356	0.652	1.255

Problem 1117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	38	42	115	178	90	81
normalized size	1	1.	1.03	1.14	3.11	4.81	2.43	2.19
time (sec)	N/A	0.01	0.015	0.046	1.225	2.499	0.869	1.184

Problem 1118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	43	42	134	211	105	50
normalized size	1	1.	0.96	0.93	2.98	4.69	2.33	1.11
time (sec)	N/A	0.033	0.017	0.044	1.456	2.477	1.144	1.195

Problem 1119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	43	42	153	243	121	50
normalized size	1	1.	0.96	0.93	3.4	5.4	2.69	1.11
time (sec)	N/A	0.033	0.014	0.045	1.142	2.36	1.489	1.207

Problem 1120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	39	42	171	277	136	50
normalized size	1	1.	0.87	0.93	3.8	6.16	3.02	1.11
time (sec)	N/A	0.032	0.015	0.043	1.153	2.092	1.405	1.208

Problem 1121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	168	362	320	605	291	386
normalized size	1	1.	2.3	4.96	4.38	8.29	3.99	5.29
time (sec)	N/A	0.17	0.06	0.059	1.203	2.085	0.137	1.185

Problem 1122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	179	300	271	510	248	324
normalized size	1	1.	2.45	4.11	3.71	6.99	3.4	4.44
time (sec)	N/A	0.124	0.026	0.039	1.203	1.933	0.105	1.272

Problem 1123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	97	237	217	393	194	261
normalized size	1	1.	1.76	4.31	3.95	7.15	3.53	4.75
time (sec)	N/A	0.024	0.02	0.041	1.205	1.725	0.1	1.217

Problem 1124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	111	176	171	315	156	200
normalized size	1	1.	1.52	2.41	2.34	4.32	2.14	2.74
time (sec)	N/A	0.084	0.018	0.039	1.143	1.674	0.099	1.182

Problem 1125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	37	95	20	174	80	108
normalized size	1	1.	2.18	5.59	1.18	10.24	4.71	6.35
time (sec)	N/A	0.006	0.01	0.039	1.17	1.848	0.085	1.236

Problem 1126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	61	121	120	192	76	157
normalized size	1	1.	0.85	1.68	1.67	2.67	1.06	2.18
time (sec)	N/A	0.072	0.027	0.043	1.221	1.936	0.538	1.222

Problem 1127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	59	70	104	182	78	181
normalized size	1	1.	0.82	0.97	1.44	2.53	1.08	2.51
time (sec)	N/A	0.06	0.052	0.042	1.928	1.875	0.643	1.209

Problem 1128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	61	115	130	312	102	119
normalized size	1	1.	0.77	1.46	1.65	3.95	1.29	1.51
time (sec)	N/A	0.064	0.041	0.046	1.137	2.02	1.08	1.13

Problem 1129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	53	67	157	263	117	111
normalized size	1	1.	0.8	1.02	2.38	3.98	1.77	1.68
time (sec)	N/A	0.054	0.057	0.044	1.239	2.025	1.353	1.137

Problem 1130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	59	111	184	360	139	197
normalized size	1	1.	0.82	1.54	2.56	5.	1.93	2.74
time (sec)	N/A	0.058	0.035	0.045	1.201	2.054	2.079	1.175

Problem 1131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	59	74	201	313	156	117
normalized size	1	1.	0.81	1.01	2.75	4.29	2.14	1.6
time (sec)	N/A	0.057	0.034	0.045	1.282	1.984	1.894	1.19

Problem 1132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	65	74	221	351	172	117
normalized size	1	1.	1.76	2.	5.97	9.49	4.65	3.16
time (sec)	N/A	0.014	0.029	0.044	1.184	1.881	3.892	1.247

Problem 1133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	59	74	238	379	187	117
normalized size	1	1.	0.81	1.01	3.26	5.19	2.56	1.6
time (sec)	N/A	0.057	0.032	0.044	1.134	2.006	2.756	1.217

Problem 1134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	59	74	259	423	202	117
normalized size	1	1.	0.81	1.01	3.55	5.79	2.77	1.6
time (sec)	N/A	0.055	0.026	0.043	1.063	1.971	5.118	1.15

Problem 1135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	59	74	278	474	218	117
normalized size	1	1.	0.81	1.01	3.81	6.49	2.99	1.6
time (sec)	N/A	0.058	0.04	0.046	1.489	2.054	4.739	1.137

Problem 1136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	59	74	297	516	233	117
normalized size	1	1.	0.81	1.01	4.07	7.07	3.19	1.6
time (sec)	N/A	0.054	0.032	0.046	1.213	1.979	9.01	1.176

Problem 1137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	224	810	462	907	428	572
normalized size	1	1.	2.22	8.02	4.57	8.98	4.24	5.66
time (sec)	N/A	0.245	0.084	0.041	1.138	1.724	0.128	1.236

Problem 1138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	259	672	392	776	371	487
normalized size	1	1.	2.56	6.65	3.88	7.68	3.67	4.82
time (sec)	N/A	0.192	0.042	0.038	1.168	1.703	0.126	1.283

Problem 1139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	132	534	328	614	299	402
normalized size	1	1.	2.4	9.71	5.96	11.16	5.44	7.31
time (sec)	N/A	0.025	0.033	0.041	1.1	1.817	0.185	1.159

Problem 1140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	179	396	267	500	246	317
normalized size	1	1.	1.77	3.92	2.64	4.95	2.44	3.14
time (sec)	N/A	0.129	0.03	0.04	1.141	1.701	0.109	1.171

Problem 1141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	52	231	20	306	144	189
normalized size	1	1.	3.06	13.59	1.18	18.	8.47	11.12
time (sec)	N/A	0.005	0.018	0.039	1.237	1.654	0.128	1.224

Problem 1142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	111	222	220	356	141	288
normalized size	1	1.	1.11	2.22	2.2	3.56	1.41	2.88
time (sec)	N/A	0.118	0.045	0.041	1.148	1.973	1.09	1.163

Problem 1143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	101	135	186	305	143	298
normalized size	1	1.	1.07	1.44	1.98	3.24	1.52	3.17
time (sec)	N/A	0.095	0.079	0.044	1.167	1.91	0.973	1.228

Problem 1144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	90	192	198	539	150	200
normalized size	1	1.	0.9	1.92	1.98	5.39	1.5	2.
time (sec)	N/A	0.098	0.095	0.047	1.162	2.	2.451	1.168

Problem 1145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	81	116	236	417	185	221
normalized size	1	1.	0.79	1.13	2.29	4.05	1.8	2.15
time (sec)	N/A	0.094	0.064	0.046	1.168	1.922	2.405	1.201

Problem 1146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	80	195	262	624	209	354
normalized size	1	1.	0.75	1.82	2.45	5.83	1.95	3.31
time (sec)	N/A	0.098	0.043	0.047	1.134	2.08	5.384	1.211

Problem 1147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	72	114	294	493	223	216
normalized size	1	1.	0.77	1.21	3.13	5.24	2.37	2.3
time (sec)	N/A	0.085	0.063	0.047	1.234	1.934	5.235	1.186

Problem 1148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	78	191	321	643	245	220
normalized size	1	1.	0.78	1.91	3.21	6.43	2.45	2.2
time (sec)	N/A	0.083	0.042	0.069	1.144	2.04	14.764	1.187

Problem 1149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	79	121	335	529	262	223
normalized size	1	1.	0.78	1.2	3.32	5.24	2.59	2.21
time (sec)	N/A	0.081	0.049	0.046	1.236	1.952	9.303	1.188

Problem 1150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	96	121	359	585	277	223
normalized size	1	1.	2.59	3.27	9.7	15.81	7.49	6.03
time (sec)	N/A	0.013	0.052	0.046	1.432	1.989	22.523	1.179

Problem 1151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	79	121	378	636	292	223
normalized size	1	1.	0.78	1.2	3.74	6.3	2.89	2.21
time (sec)	N/A	0.082	0.058	0.048	1.896	1.938	12.05	1.178

Problem 1152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	79	121	397	678	308	223
normalized size	1	1.	0.78	1.2	3.93	6.71	3.05	2.21
time (sec)	N/A	0.081	0.057	0.046	2.357	2.057	42.611	1.18

Problem 1153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	79	121	413	717	323	223
normalized size	1	1.	0.78	1.2	4.09	7.1	3.2	2.21
time (sec)	N/A	0.085	0.063	0.047	1.36	2.071	20.548	1.171

Problem 1154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	188	432	0	1166	502	423
normalized size	1	1.	1.54	3.54	0.	9.56	4.11	3.47
time (sec)	N/A	0.15	0.113	0.151	0.	1.947	1.387	1.161

Problem 1155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	110	243	244	393	185	296
normalized size	1	1.	1.28	2.83	2.84	4.57	2.15	3.44
time (sec)	N/A	0.07	0.055	0.042	2.382	1.634	1.277	1.181

Problem 1156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	120	284	0	780	337	266
normalized size	1	1.	1.24	2.93	0.	8.04	3.47	2.74
time (sec)	N/A	0.08	0.076	0.149	0.	1.758	0.949	1.248

Problem 1157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	54	133	134	207	99	159
normalized size	1	1.	0.89	2.18	2.2	3.39	1.62	2.61
time (sec)	N/A	0.043	0.023	0.041	1.583	1.692	0.874	1.201

Problem 1158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	72	170	0	464	204	155
normalized size	1	1.	1.	2.36	0.	6.44	2.83	2.15
time (sec)	N/A	0.07	0.042	0.147	0.	1.722	0.745	1.116

Problem 1159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	33	57	58	95	44	72
normalized size	1	1.	0.92	1.58	1.61	2.64	1.22	2.
time (sec)	N/A	0.023	0.012	0.042	1.142	1.672	0.625	1.121

Problem 1160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	47	88	0	300	99	77
normalized size	1	1.	0.96	1.8	0.	6.12	2.02	1.57
time (sec)	N/A	0.031	0.024	0.148	0.	1.719	1.098	1.139

Problem 1161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	12	14	18	32	12	18
normalized size	1	1.	0.92	1.08	1.38	2.46	0.92	1.38
time (sec)	N/A	0.005	0.002	0.04	1.299	1.729	0.333	1.228

Problem 1162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	34	54	65	82	42	77
normalized size	1	1.	0.71	1.12	1.35	1.71	0.88	1.6
time (sec)	N/A	0.023	0.02	0.044	1.203	1.645	0.924	1.193

Problem 1163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	63	64	0	564	240	158
normalized size	1	1.	1.03	1.05	0.	9.25	3.93	2.59
time (sec)	N/A	0.041	0.051	0.152	0.	2.116	1.217	1.18

Problem 1164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	65	78	174	338	119	150
normalized size	1	1.	0.93	1.11	2.49	4.83	1.7	2.14
time (sec)	N/A	0.04	0.039	0.046	1.201	1.994	2.345	1.161

Problem 1165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	83	89	0	1350	442	173
normalized size	1	1.	0.97	1.03	0.	15.7	5.14	2.01
time (sec)	N/A	0.067	0.052	0.153	0.	2.028	2.573	1.215

Problem 1166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	155	479	0	1634	476	416
normalized size	1	1.	1.23	3.8	0.	12.97	3.78	3.3
time (sec)	N/A	0.105	0.082	0.155	0.	2.081	3.362	1.154

Problem 1167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	103	230	209	587	160	244
normalized size	1	1.	1.16	2.58	2.35	6.6	1.8	2.74
time (sec)	N/A	0.062	0.088	0.048	1.118	1.963	4.284	1.153

Problem 1168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	108	312	0	1085	313	266
normalized size	1	1.	1.08	3.12	0.	10.85	3.13	2.66
time (sec)	N/A	0.074	0.065	0.157	0.	2.03	2.21	1.188

Problem 1169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	64	128	116	342	90	135
normalized size	1	1.	0.98	1.97	1.78	5.26	1.38	2.08
time (sec)	N/A	0.039	0.034	0.047	1.162	1.97	2.302	1.165

Problem 1170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	77	133	0	647	173	151
normalized size	1	1.	1.01	1.75	0.	8.51	2.28	1.99
time (sec)	N/A	0.051	0.052	0.157	0.	2.084	1.658	1.161

Problem 1171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	42	58	58	136	42	63
normalized size	1	1.	0.98	1.35	1.35	3.16	0.98	1.47
time (sec)	N/A	0.02	0.021	0.045	1.011	2.015	1.213	1.15

Problem 1172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	65	77	0	667	209	89
normalized size	1	1.	1.05	1.24	0.	10.76	3.37	1.44
time (sec)	N/A	0.036	0.036	0.152	0.	2.075	1.076	1.17

Problem 1173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	14	16	20	30	12	20
normalized size	1	1.	0.93	1.07	1.33	2.	0.8	1.33
time (sec)	N/A	0.005	0.004	0.037	1.056	1.94	0.594	1.19

Problem 1174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	59	119	163	309	102	146
normalized size	1	1.	0.76	1.53	2.09	3.96	1.31	1.87
time (sec)	N/A	0.038	0.056	0.075	1.172	1.942	2.267	1.157

Problem 1175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	84	127	0	1384	457	297
normalized size	1	1.	0.86	1.3	0.	14.12	4.66	3.03
time (sec)	N/A	0.058	0.122	0.158	0.	2.155	2.676	1.237

Problem 1176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	79	144	400	864	303	274
normalized size	1	1.	0.72	1.31	3.64	7.85	2.75	2.49
time (sec)	N/A	0.062	0.09	0.056	1.13	2.051	6.448	1.179

Problem 1177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	192	751	0	2643	0	621
normalized size	1	1.	1.2	4.69	0.	16.52	0.	3.88
time (sec)	N/A	0.137	0.115	0.163	0.	2.167	0.	1.199

Problem 1178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	131	465	375	1035	320	404
normalized size	1	1.	1.07	3.78	3.05	8.41	2.6	3.28
time (sec)	N/A	0.085	0.061	0.052	1.17	1.982	31.332	1.272

Problem 1179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	142	526	0	1879	469	425
normalized size	1	1.	1.06	3.93	0.	14.02	3.5	3.17
time (sec)	N/A	0.101	0.086	0.162	0.	2.15	12.384	1.193

Problem 1180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	92	307	255	724	219	258
normalized size	1	1.	0.95	3.16	2.63	7.46	2.26	2.66
time (sec)	N/A	0.06	0.038	0.051	1.123	2.116	13.414	1.179

Problem 1181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	113	289	0	1235	299	265
normalized size	1	1.	1.05	2.68	0.	11.44	2.77	2.45
time (sec)	N/A	0.073	0.065	0.157	0.	2.322	7.118	1.179

Problem 1182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	65	181	167	385	141	149
normalized size	1	1.	0.89	2.48	2.29	5.27	1.93	2.04
time (sec)	N/A	0.038	0.042	0.048	1.227	2.352	6.814	1.168

Problem 1183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	89	173	0	1312	303	154
normalized size	1	1.	0.97	1.88	0.	14.26	3.29	1.67
time (sec)	N/A	0.057	0.065	0.157	0.	2.27	3.963	1.264

Problem 1184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	38	40	96	154	80	65
normalized size	1	1.	1.03	1.08	2.59	4.16	2.16	1.76
time (sec)	N/A	0.015	0.024	0.046	1.133	2.036	2.435	1.182

Problem 1185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	98	245	0	1485	430	181
normalized size	1	1.	0.98	2.45	0.	14.85	4.3	1.81
time (sec)	N/A	0.053	0.091	0.155	0.	2.16	2.771	1.25

Problem 1186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	16	16	20	89	44	20
normalized size	1	1.	0.94	0.94	1.18	5.24	2.59	1.18
time (sec)	N/A	0.005	0.006	0.038	1.335	2.021	1.819	1.221

Problem 1187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	90	304	359	825	246	254
normalized size	1	1.	0.8	2.71	3.21	7.37	2.2	2.27
time (sec)	N/A	0.06	0.075	0.057	1.262	2.164	6.376	1.188

Problem 1188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	119	273	0	2534	801	408
normalized size	1	1.	0.85	1.95	0.	18.1	5.72	2.91
time (sec)	N/A	0.099	0.135	0.162	0.	2.368	15.276	1.183

Problem 1189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	111	332	747	1747	597	408
normalized size	1	1.	0.72	2.16	4.85	11.34	3.88	2.65
time (sec)	N/A	0.088	0.145	0.057	1.26	2.339	27.542	1.214

Problem 1190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	140	301	0	4655	1238	420
normalized size	1	1.	0.83	1.79	0.	27.71	7.37	2.5
time (sec)	N/A	0.146	0.339	0.164	0.	2.461	97.511	1.251

Problem 1191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	203	413	0	1071	0	350
normalized size	1	1.	1.23	2.5	0.	6.49	0.	2.12
time (sec)	N/A	0.098	0.889	0.055	0.	2.279	0.	1.289

Problem 1192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	44	41	0	198	216	163
normalized size	1	1.	0.75	0.69	0.	3.36	3.66	2.76
time (sec)	N/A	0.026	0.038	0.045	0.	2.25	0.497	1.191

Problem 1193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	140	230	0	702	0	209
normalized size	1	1.	1.14	1.87	0.	5.71	0.	1.7
time (sec)	N/A	0.056	0.372	0.048	0.	2.294	0.	1.189

Problem 1194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	18	16	20	69	65	20
normalized size	1	1.	0.95	0.84	1.05	3.63	3.42	1.05
time (sec)	N/A	0.006	0.009	0.043	1.18	2.189	0.255	1.126

Problem 1195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	80	244	0	410	0	131
normalized size	1	1.	0.96	2.94	0.	4.94	0.	1.58
time (sec)	N/A	0.063	0.054	0.212	0.	2.616	0.	1.169

Problem 1196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	114	291	0	450	0	261
normalized size	1	1.	1.52	3.88	0.	6.	0.	3.48
time (sec)	N/A	0.029	0.299	0.195	0.	2.712	0.	1.2

Problem 1197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	103	340	0	806	0	0
normalized size	1	1.	1.13	3.74	0.	8.86	0.	0.
time (sec)	N/A	0.052	0.201	0.196	0.	3.076	0.	0.

Problem 1198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	38	38	0	205	0	277
normalized size	1	1.	0.97	0.97	0.	5.26	0.	7.1
time (sec)	N/A	0.014	0.018	0.042	0.	5.278	0.	1.287

Problem 1199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	62	400	0	1532	0	859
normalized size	1	1.	0.47	3.01	0.	11.52	0.	6.46
time (sec)	N/A	0.089	0.029	0.197	0.	10.478	0.	1.69

Problem 1200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	62	70	0	575	0	571
normalized size	1	1.	0.78	0.89	0.	7.28	0.	7.23
time (sec)	N/A	0.035	0.029	0.043	0.	21.128	0.	1.488

Problem 1201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	62	460	0	2624	0	0
normalized size	1	1.	0.35	2.63	0.	14.99	0.	0.
time (sec)	N/A	0.125	0.025	0.202	0.	43.556	0.	0.

Problem 1202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	92	91	0	564	656	441
normalized size	1	1.	0.94	0.93	0.	5.76	6.69	4.5
time (sec)	N/A	0.051	0.077	0.045	0.	2.938	4.432	1.195

Problem 1203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	249	641	0	1540	0	528
normalized size	1	1.	1.2	3.1	0.	7.44	0.	2.55
time (sec)	N/A	0.129	2.109	0.056	0.	2.807	0.	1.195

Problem 1204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	44	41	0	320	371	269
normalized size	1	1.	0.75	0.69	0.	5.42	6.29	4.56
time (sec)	N/A	0.027	0.047	0.045	0.	2.67	1.835	1.139

Problem 1205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	211	406	0	1071	0	350
normalized size	1	1.	1.28	2.46	0.	6.49	0.	2.12
time (sec)	N/A	0.089	0.457	0.049	0.	2.865	0.	1.197

Problem 1206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	18	16	20	128	146	88
normalized size	1	1.	0.95	0.84	1.05	6.74	7.68	4.63
time (sec)	N/A	0.007	0.014	0.042	1.123	2.636	0.94	1.222

Problem 1207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	103	430	0	570	0	201
normalized size	1	1.	0.9	3.74	0.	4.96	0.	1.75
time (sec)	N/A	0.083	0.105	0.19	0.	3.697	0.	1.159

Problem 1208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	95	570	0	647	0	0
normalized size	1	1.	0.84	5.04	0.	5.73	0.	0.
time (sec)	N/A	0.046	0.045	0.193	0.	3.729	0.	0.

Problem 1209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	62	562	0	705	0	0
normalized size	1	1.	0.54	4.89	0.	6.13	0.	0.
time (sec)	N/A	0.071	0.032	0.191	0.	4.638	0.	0.

Problem 1210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	95	629	0	776	0	0
normalized size	1	1.	0.89	5.88	0.	7.25	0.	0.
time (sec)	N/A	0.05	0.047	0.192	0.	7.302	0.	0.

Problem 1211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	162	622	0	1320	0	911
normalized size	1	1.	1.32	5.06	0.	10.73	0.	7.41
time (sec)	N/A	0.073	0.246	0.194	0.	12.449	0.	1.796

Problem 1212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	38	38	0	383	0	802
normalized size	1	1.	0.97	0.97	0.	9.82	0.	20.56
time (sec)	N/A	0.016	0.025	0.043	0.	24.228	0.	1.843

Problem 1213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	62	682	0	2276	0	0
normalized size	1	1.	0.38	4.13	0.	13.79	0.	0.
time (sec)	N/A	0.114	0.032	0.23	0.	48.142	0.	0.

Problem 1214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	62	70	0	841	0	1354
normalized size	1	1.	0.78	0.89	0.	10.65	0.	17.14
time (sec)	N/A	0.035	0.037	0.044	0.	119.981	0.	2.004

Problem 1215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	62	742	0	0	0	0
normalized size	1	1.	0.3	3.58	0.	0.	0.	0.
time (sec)	N/A	0.146	0.03	0.238	0.	0.	0.	0.

Problem 1216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	110	133	0	0	0	1881
normalized size	1	1.	0.93	1.13	0.	0.	0.	15.94
time (sec)	N/A	0.057	0.063	0.046	0.	0.	0.	3.097

Problem 1217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	92	91	0	778	913	599
normalized size	1	1.	0.94	0.93	0.	7.94	9.32	6.11
time (sec)	N/A	0.051	0.094	0.046	0.	4.035	15.488	2.113

Problem 1218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	265	920	0	2192	0	738
normalized size	1	1.	1.06	3.69	0.	8.8	0.	2.96
time (sec)	N/A	0.171	3.74	0.056	0.	3.829	0.	1.21

Problem 1219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	44	41	0	467	559	389
normalized size	1	1.	0.75	0.69	0.	7.92	9.47	6.59
time (sec)	N/A	0.026	0.061	0.044	0.	3.111	7.865	1.175

Problem 1220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	225	634	0	1569	0	525
normalized size	1	1.	1.09	3.06	0.	7.58	0.	2.54
time (sec)	N/A	0.124	0.989	0.049	0.	3.123	0.	1.215

Problem 1221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	18	16	20	207	260	161
normalized size	1	1.	0.95	0.84	1.05	10.89	13.68	8.47
time (sec)	N/A	0.007	0.013	0.041	1.094	2.841	4.39	1.193

Problem 1222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	150	660	0	852	0	320
normalized size	1	1.	1.01	4.43	0.	5.72	0.	2.15
time (sec)	N/A	0.121	0.175	0.19	0.	4.468	0.	1.184

Problem 1223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	97	961	0	977	0	0
normalized size	1	1.	0.63	6.28	0.	6.39	0.	0.
time (sec)	N/A	0.064	0.052	0.192	0.	5.965	0.	0.

Problem 1224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	62	840	0	1056	0	0
normalized size	1	1.	0.42	5.71	0.	7.18	0.	0.
time (sec)	N/A	0.112	0.04	0.196	0.	7.401	0.	0.

Problem 1225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	97	1022	0	1170	0	841
normalized size	1	1.	0.67	7.05	0.	8.07	0.	5.8
time (sec)	N/A	0.07	0.047	0.196	0.	14.809	0.	1.926

Problem 1226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	62	900	0	1165	0	1014
normalized size	1	1.	0.42	6.12	0.	7.93	0.	6.9
time (sec)	N/A	0.102	0.037	0.198	0.	18.135	0.	2.156

Problem 1227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	97	1080	0	1237	0	0
normalized size	1	1.	0.7	7.77	0.	8.9	0.	0.
time (sec)	N/A	0.075	0.051	0.204	0.	30.033	0.	0.

Problem 1228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	233	960	0	1985	0	0
normalized size	1	1.	1.5	6.19	0.	12.81	0.	0.
time (sec)	N/A	0.099	0.385	0.2	0.	46.172	0.	0.

Problem 1229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	38	38	0	563	0	1683
normalized size	1	1.	0.97	0.97	0.	14.44	0.	43.15
time (sec)	N/A	0.016	0.027	0.042	0.	112.052	0.	2.513

Problem 1230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	62	1020	0	0	0	0
normalized size	1	1.	0.31	5.18	0.	0.	0.	0.
time (sec)	N/A	0.15	0.04	0.225	0.	0.	0.	0.

Problem 1231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	62	70	0	0	0	2496
normalized size	1	1.	0.78	0.89	0.	0.	0.	31.59
time (sec)	N/A	0.034	0.055	0.042	0.	0.	0.	4.578

Problem 1232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	62	1080	0	0	0	0
normalized size	1	1.	0.26	4.52	0.	0.	0.	0.
time (sec)	N/A	0.185	0.037	0.262	0.	0.	0.	0.

Problem 1233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	110	133	0	0	0	3201
normalized size	1	1.	0.93	1.13	0.	0.	0.	27.13
time (sec)	N/A	0.056	0.082	0.048	0.	0.	0.	11.911

Problem 1234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	100	242	0	710	0	215
normalized size	1	1.	0.85	2.07	0.	6.07	0.	1.84
time (sec)	N/A	0.055	0.107	0.053	0.	3.367	0.	1.176

Problem 1235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	43	41	0	107	97	78
normalized size	1	1.	0.73	0.69	0.	1.81	1.64	1.32
time (sec)	N/A	0.026	0.036	0.046	0.	2.848	0.387	1.189

Problem 1236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	71	108	0	462	0	105
normalized size	1	1.	0.95	1.44	0.	6.16	0.	1.4
time (sec)	N/A	0.03	0.055	0.047	0.	3.029	0.	1.159

Problem 1237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	16	16	20	36	15	20
normalized size	1	1.	0.94	0.94	1.18	2.12	0.88	1.18
time (sec)	N/A	0.006	0.006	0.041	2.277	2.633	0.194	1.181

Problem 1238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	54	101	0	381	0	88
normalized size	1	1.	0.98	1.84	0.	6.93	0.	1.6
time (sec)	N/A	0.033	0.034	0.192	0.	2.852	0.	1.151

Problem 1239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	36	38	0	101	0	194
normalized size	1	1.	0.97	1.03	0.	2.73	0.	5.24
time (sec)	N/A	0.015	0.015	0.045	0.	3.049	0.	1.175

Problem 1240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	107	174	0	936	0	0
normalized size	1	1.	1.13	1.83	0.	9.85	0.	0.
time (sec)	N/A	0.056	0.284	0.192	0.	4.188	0.	0.

Problem 1241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	60	70	0	346	0	0
normalized size	1	1.	0.76	0.89	0.	4.38	0.	0.
time (sec)	N/A	0.032	0.028	0.044	0.	7.412	0.	0.

Problem 1242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	139	340	0	788	0	336
normalized size	1	1.	1.36	3.33	0.	7.73	0.	3.29
time (sec)	N/A	0.053	0.411	0.052	0.	4.597	0.	1.212

Problem 1243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	40	41	0	101	92	188
normalized size	1	1.	0.83	0.85	0.	2.1	1.92	3.92
time (sec)	N/A	0.023	0.029	0.044	0.	3.557	1.368	1.174

Problem 1244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	118	72	0	517	0	153
normalized size	1	1.	1.79	1.09	0.	7.83	0.	2.32
time (sec)	N/A	0.027	0.195	0.048	0.	3.477	0.	1.236

Problem 1245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	16	16	20	38	17	47
normalized size	1	1.	0.94	0.94	1.18	2.24	1.	2.76
time (sec)	N/A	0.006	0.007	0.042	1.137	3.195	1.493	1.163

Problem 1246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	60	158	0	695	0	213
normalized size	1	1.	0.7	1.84	0.	8.08	0.	2.48
time (sec)	N/A	0.058	0.028	0.196	0.	4.432	0.	1.157

Problem 1247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	56	68	0	327	0	0
normalized size	1	1.	0.74	0.89	0.	4.3	0.	0.
time (sec)	N/A	0.034	0.028	0.045	0.	5.75	0.	0.

Problem 1248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	60	218	0	1496	0	0
normalized size	1	1.	0.45	1.65	0.	11.33	0.	0.
time (sec)	N/A	0.085	0.029	0.196	0.	13.623	0.	0.

Problem 1249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	108	133	0	824	0	0
normalized size	1	1.	0.92	1.13	0.	6.98	0.	0.
time (sec)	N/A	0.055	0.05	0.048	0.	27.77	0.	0.

Problem 1250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	204	997	0	1482	0	713
normalized size	1	1.	1.5	7.33	0.	10.9	0.	5.24
time (sec)	N/A	0.077	1.506	0.063	0.	11.541	0.	1.2

Problem 1251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	91	91	0	302	615	521
normalized size	1	1.	1.08	1.08	0.	3.6	7.32	6.2
time (sec)	N/A	0.044	0.053	0.046	0.	8.911	2.626	1.156

Problem 1252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	142	531	0	979	0	458
normalized size	1	1.	1.48	5.53	0.	10.2	0.	4.77
time (sec)	N/A	0.05	0.348	0.053	0.	8.8	0.	1.17

Problem 1253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	42	39	0	186	264	275
normalized size	1	1.	0.81	0.75	0.	3.58	5.08	5.29
time (sec)	N/A	0.023	0.029	0.044	0.	5.683	1.996	1.143

Problem 1254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	38	38	0	304	0	296
normalized size	1	1.	0.97	0.97	0.	7.79	0.	7.59
time (sec)	N/A	0.014	0.021	0.047	0.	5.554	0.	1.195

Problem 1255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	18	16	20	119	60	86
normalized size	1	1.	0.95	0.84	1.05	6.26	3.16	4.53
time (sec)	N/A	0.006	0.007	0.043	1.144	4.774	2.451	1.152

Problem 1256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	62	207	0	1341	0	753
normalized size	1	1.	0.53	1.75	0.	11.36	0.	6.38
time (sec)	N/A	0.077	0.036	0.192	0.	7.548	0.	1.216

Problem 1257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	108	133	0	794	0	0
normalized size	1	1.	0.89	1.09	0.	6.51	0.	0.
time (sec)	N/A	0.056	0.052	0.048	0.	22.365	0.	0.

Problem 1258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	62	267	0	2759	0	0
normalized size	1	1.	0.35	1.52	0.	15.68	0.	0.
time (sec)	N/A	0.106	0.036	0.2	0.	40.87	0.	0.

Problem 1259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	178	218	0	1494	0	0
normalized size	1	1.	1.1	1.35	0.	9.22	0.	0.
time (sec)	N/A	0.079	0.084	0.052	0.	74.785	0.	0.

Problem 1260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	19	24	19	157	0	120
normalized size	1	1.	0.7	0.89	0.7	5.81	0.	4.44
time (sec)	N/A	0.021	0.008	0.045	2.573	2.051	0.	1.236

Problem 1261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	45	46	62	265	289	508
normalized size	1	1.	0.82	0.84	1.13	4.82	5.25	9.24
time (sec)	N/A	0.024	0.044	0.041	1.098	1.983	5.245	1.144

Problem 1262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	45	46	62	194	274	308
normalized size	1	1.	0.82	0.84	1.13	3.53	4.98	5.6
time (sec)	N/A	0.023	0.029	0.043	1.119	2.061	10.278	1.176

Problem 1263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	45	46	62	128	48	157
normalized size	1	1.	0.82	0.84	1.13	2.33	0.87	2.85
time (sec)	N/A	0.022	0.024	0.042	1.021	1.901	3.236	1.143

Problem 1264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	43	44	157	88	258	157
normalized size	1	1.	0.78	0.8	2.85	1.6	4.69	2.85
time (sec)	N/A	0.022	0.025	0.041	1.166	1.96	12.085	1.142

Problem 1265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	41	46	69	112	49	63
normalized size	1	1.	0.75	0.84	1.25	2.04	0.89	1.15
time (sec)	N/A	0.023	0.025	0.041	1.014	1.959	12.549	1.114

Problem 1266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	43	45	69	142	235	63
normalized size	1	1.	0.78	0.82	1.25	2.58	4.27	1.15
time (sec)	N/A	0.022	0.027	0.042	1.169	2.032	1.718	1.159

Problem 1267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	44	43	61	171	298	63
normalized size	1	1.	0.8	0.78	1.11	3.11	5.42	1.15
time (sec)	N/A	0.022	0.029	0.046	1.026	2.011	4.466	1.148

Problem 1268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	51	44	62	205	360	65
normalized size	1	1.	0.93	0.8	1.13	3.73	6.55	1.18
time (sec)	N/A	0.023	0.037	0.042	1.014	2.006	10.618	1.153

Problem 1269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	63	96	109	377	695	783
normalized size	1	1.	0.72	1.09	1.24	4.28	7.9	8.9
time (sec)	N/A	0.055	0.061	0.043	1.054	2.021	21.662	1.165

Problem 1270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	92	96	109	271	94	471
normalized size	1	1.	1.05	1.09	1.24	3.08	1.07	5.35
time (sec)	N/A	0.039	0.044	0.044	1.193	1.997	3.299	1.147

Problem 1271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	92	96	474	201	668	471
normalized size	1	1.	1.05	1.09	5.39	2.28	7.59	5.35
time (sec)	N/A	0.038	0.043	0.044	1.022	2.026	63.115	1.139

Problem 1272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	91	96	120	223	82	147
normalized size	1	1.	1.03	1.09	1.36	2.53	0.93	1.67
time (sec)	N/A	0.038	0.038	0.044	1.253	2.011	27.482	1.198

Problem 1273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	91	96	122	251	82	147
normalized size	1	1.	1.03	1.09	1.39	2.85	0.93	1.67
time (sec)	N/A	0.04	0.04	0.044	1.104	2.02	49.934	1.189

Problem 1274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	92	96	126	281	688	134
normalized size	1	1.	1.05	1.09	1.43	3.19	7.82	1.52
time (sec)	N/A	0.038	0.043	0.046	1.001	2.014	5.933	1.185

Problem 1275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	63	96	124	311	826	135
normalized size	1	1.	0.72	1.09	1.41	3.53	9.39	1.53
time (sec)	N/A	0.037	0.048	0.049	0.993	2.019	9.505	1.164

Problem 1276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	63	96	109	342	966	134
normalized size	1	1.	0.72	1.09	1.24	3.89	10.98	1.52
time (sec)	N/A	0.037	0.051	0.044	1.074	1.971	20.56	1.161

Problem 1277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	83	174	171	462	151	1049
normalized size	1	1.	0.69	1.44	1.41	3.82	1.25	8.67
time (sec)	N/A	0.051	0.084	0.046	1.043	2.039	4.474	1.187

Problem 1278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	83	174	1050	363	1363	1050
normalized size	1	1.	0.69	1.44	8.68	3.	11.26	8.68
time (sec)	N/A	0.05	0.076	0.044	1.12	2.025	161.051	1.176

Problem 1279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	83	173	184	375	128	252
normalized size	1	1.	0.69	1.43	1.52	3.1	1.06	2.08
time (sec)	N/A	0.049	0.068	0.044	1.08	2.07	68.842	1.166

Problem 1280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	83	173	184	405	128	252
normalized size	1	1.	0.69	1.43	1.52	3.35	1.06	2.08
time (sec)	N/A	0.05	0.069	0.043	1.049	2.103	95.096	1.186

Problem 1281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	83	174	192	435	0	251
normalized size	1	1.	0.69	1.44	1.59	3.6	0.	2.07
time (sec)	N/A	0.049	0.076	0.047	1.016	1.956	0.	1.173

Problem 1282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	83	163	192	440	1394	251
normalized size	1	1.	0.69	1.35	1.59	3.64	11.52	2.07
time (sec)	N/A	0.05	0.07	0.045	1.146	2.076	15.291	1.207

Problem 1283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	83	174	186	498	1731	238
normalized size	1	1.	0.69	1.44	1.54	4.12	14.31	1.97
time (sec)	N/A	0.05	0.074	0.044	1.131	2.067	24.107	1.218

Problem 1284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	83	174	186	528	1975	238
normalized size	1	1.	0.69	1.44	1.54	4.36	16.32	1.97
time (sec)	N/A	0.053	0.076	0.045	1.251	2.169	40.559	1.21

Problem 1285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	157	1287	0	3090	0	779
normalized size	1	1.	0.9	7.35	0.	17.66	0.	4.45
time (sec)	N/A	0.215	0.196	0.213	0.	2.51	0.	1.267

Problem 1286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	131	922	0	3245	0	612
normalized size	1	1.	0.89	6.27	0.	22.07	0.	4.16
time (sec)	N/A	0.138	0.145	0.194	0.	2.197	0.	1.259

Problem 1287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	141	922	0	1671	0	612
normalized size	1	1.	0.97	6.36	0.	11.52	0.	4.22
time (sec)	N/A	0.121	0.135	0.196	0.	2.101	0.	1.192

Problem 1288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	104	582	0	1369	0	478
normalized size	1	1.	0.87	4.89	0.	11.5	0.	4.02
time (sec)	N/A	0.096	0.136	0.196	0.	2.049	0.	1.239

Problem 1289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	113	582	0	509	212	478
normalized size	1	1.	0.97	4.97	0.	4.35	1.81	4.09
time (sec)	N/A	0.096	0.048	0.19	0.	1.893	37.303	1.238

Problem 1290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	87	271	0	518	65	531
normalized size	1	1.	0.86	2.68	0.	5.13	0.64	5.26
time (sec)	N/A	0.072	0.039	0.193	0.	2.01	4.546	1.147

Problem 1291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	85	271	0	1033	0	531
normalized size	1	1.	0.84	2.68	0.	10.23	0.	5.26
time (sec)	N/A	0.077	0.05	0.19	0.	2.269	0.	1.133

Problem 1292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	54	341	0	1912	0	671
normalized size	1	1.	0.42	2.64	0.	14.82	0.	5.2
time (sec)	N/A	0.108	0.045	0.195	0.	2.386	0.	1.151

Problem 1293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	56	341	0	2742	0	671
normalized size	1	1.	0.43	2.6	0.	20.93	0.	5.12
time (sec)	N/A	0.107	0.053	0.192	0.	2.343	0.	1.198

Problem 1294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	56	369	0	4031	0	817
normalized size	1	1.	0.35	2.32	0.	25.35	0.	5.14
time (sec)	N/A	0.132	0.066	0.197	0.	2.311	0.	1.206

Problem 1295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	189	1512	0	3564	0	972
normalized size	1	1.	0.9	7.2	0.	16.97	0.	4.63
time (sec)	N/A	0.18	0.602	0.201	0.	2.285	0.	1.268

Problem 1296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	171	1090	0	3710	0	767
normalized size	1	1.	0.94	6.02	0.	20.5	0.	4.24
time (sec)	N/A	0.16	0.183	0.202	0.	2.309	0.	1.311

Problem 1297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	167	1090	0	2026	0	767
normalized size	1	1.	0.93	6.09	0.	11.32	0.	4.28
time (sec)	N/A	0.15	0.365	0.201	0.	2.178	0.	1.261

Problem 1298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	92	693	0	1721	0	595
normalized size	1	1.	0.61	4.56	0.	11.32	0.	3.91
time (sec)	N/A	0.121	0.091	0.2	0.	2.066	0.	1.242

Problem 1299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	168	693	0	803	0	595
normalized size	1	1.	1.12	4.62	0.	5.35	0.	3.97
time (sec)	N/A	0.12	0.194	0.198	0.	1.862	0.	1.19

Problem 1300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	83	327	0	805	0	593
normalized size	1	1.	0.63	2.5	0.	6.15	0.	4.53
time (sec)	N/A	0.099	0.085	0.222	0.	1.973	0.	1.265

Problem 1301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	126	326	0	1197	0	591
normalized size	1	1.	0.96	2.49	0.	9.14	0.	4.51
time (sec)	N/A	0.099	0.231	0.198	0.	1.978	0.	1.171

Problem 1302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	57	344	0	2037	279	680
normalized size	1	1.	0.4	2.41	0.	14.24	1.95	4.76
time (sec)	N/A	0.097	0.038	0.194	0.	2.194	98.01	1.236

Problem 1303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	167	345	0	2684	0	684
normalized size	1	1.	1.17	2.41	0.	18.77	0.	4.78
time (sec)	N/A	0.102	0.164	0.191	0.	2.227	0.	1.165

Problem 1304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	55	404	0	4028	0	878
normalized size	1	1.	0.32	2.35	0.	23.42	0.	5.1
time (sec)	N/A	0.123	0.051	0.227	0.	2.333	0.	1.209

Problem 1305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	57	404	0	5273	0	872
normalized size	1	1.	0.33	2.32	0.	30.3	0.	5.01
time (sec)	N/A	0.142	0.066	0.203	0.	2.362	0.	1.207

Problem 1306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	57	433	0	7275	0	1052
normalized size	1	1.	0.28	2.13	0.	35.84	0.	5.18
time (sec)	N/A	0.172	0.081	0.202	0.	3.108	0.	1.215

Problem 1307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	138	1310	0	4231	0	903
normalized size	1	1.	0.62	5.9	0.	19.06	0.	4.07
time (sec)	N/A	0.191	0.324	0.219	0.	2.711	0.	1.401

Problem 1308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	225	1310	0	2502	0	903
normalized size	1	1.	1.01	5.9	0.	11.27	0.	4.07
time (sec)	N/A	0.182	0.756	0.211	0.	1.85	0.	1.298

Problem 1309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	145	857	0	2148	0	703
normalized size	1	1.	0.75	4.44	0.	11.13	0.	3.64
time (sec)	N/A	0.148	0.16	0.203	0.	1.901	0.	1.27

Problem 1310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	226	857	0	1170	0	703
normalized size	1	1.	1.18	4.49	0.	6.13	0.	3.68
time (sec)	N/A	0.148	0.349	0.199	0.	1.788	0.	1.259

Problem 1311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	119	435	0	1138	0	689
normalized size	1	1.	0.7	2.56	0.	6.69	0.	4.05
time (sec)	N/A	0.123	0.155	0.199	0.	1.803	0.	1.269

Problem 1312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	196	435	0	1530	0	689
normalized size	1	1.	1.15	2.56	0.	9.	0.	4.05
time (sec)	N/A	0.124	0.281	0.224	0.	1.758	0.	1.304

Problem 1313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	77	431	0	2589	0	765
normalized size	1	1.	0.43	2.42	0.	14.54	0.	4.3
time (sec)	N/A	0.128	0.117	0.198	0.	2.027	0.	1.251

Problem 1314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	171	431	0	3089	0	765
normalized size	1	1.	0.96	2.42	0.	17.35	0.	4.3
time (sec)	N/A	0.129	0.378	0.2	0.	2.071	0.	1.231

Problem 1315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	59	419	0	4201	0	871
normalized size	1	1.	0.31	2.18	0.	21.88	0.	4.54
time (sec)	N/A	0.13	0.045	0.194	0.	2.111	0.	1.278

Problem 1316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	179	419	0	4983	0	871
normalized size	1	1.	0.93	2.18	0.	25.95	0.	4.54
time (sec)	N/A	0.135	0.38	0.197	0.	2.025	0.	1.193

Problem 1317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	57	534	0	7175	0	1098
normalized size	1	1.	0.26	2.39	0.	32.17	0.	4.92
time (sec)	N/A	0.168	0.069	0.209	0.	2.47	0.	1.238

Problem 1318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	59	534	0	8675	0	1098
normalized size	1	1.	0.26	2.37	0.	38.56	0.	4.88
time (sec)	N/A	0.191	0.084	0.206	0.	2.454	0.	1.277

Problem 1319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	59	569	0	11348	0	1293
normalized size	1	1.	0.23	2.22	0.	44.33	0.	5.05
time (sec)	N/A	0.228	0.09	0.21	0.	2.708	0.	1.32

Problem 1320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	182	129	203	651	180	186
normalized size	1	1.	0.99	0.7	1.11	3.56	0.98	1.02
time (sec)	N/A	0.171	0.093	0.046	1.8	1.71	54.725	1.182

Problem 1321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	34	120	190	682	163	174
normalized size	1	1.	0.2	0.71	1.12	4.01	0.96	1.02
time (sec)	N/A	0.137	0.01	0.043	1.912	1.678	32.994	1.195

Problem 1322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	148	120	190	622	162	174
normalized size	1	1.	0.88	0.71	1.13	3.7	0.96	1.04
time (sec)	N/A	0.135	0.04	0.041	1.821	1.603	19.481	1.175

Problem 1323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	32	111	178	621	155	162
normalized size	1	1.	0.2	0.71	1.13	3.96	0.99	1.03
time (sec)	N/A	0.121	0.006	0.041	2.181	1.597	2.766	1.163

Problem 1324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	108	111	178	676	0	162
normalized size	1	1.	0.69	0.71	1.13	4.31	0.	1.03
time (sec)	N/A	0.119	0.026	0.043	1.888	1.63	0.	1.22

Problem 1325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	32	120	190	706	0	174
normalized size	1	1.	0.18	0.67	1.06	3.92	0.	0.97
time (sec)	N/A	0.139	0.008	0.047	1.878	1.611	0.	1.106

Problem 1326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	32	120	190	811	0	174
normalized size	1	1.	0.18	0.67	1.06	4.51	0.	0.97
time (sec)	N/A	0.133	0.008	0.046	2.013	1.688	0.	1.171

Problem 1327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	155	798	0	0	0	0
normalized size	1	1.	0.68	3.52	0.	0.	0.	0.
time (sec)	N/A	0.207	0.407	0.381	0.	0.	0.	0.

Problem 1328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	110	564	0	0	0	0
normalized size	1	1.	0.61	3.13	0.	0.	0.	0.
time (sec)	N/A	0.149	0.137	0.207	0.	0.	0.	0.

Problem 1329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	91	364	0	0	0	0
normalized size	1	1.	0.66	2.66	0.	0.	0.	0.
time (sec)	N/A	0.12	0.058	0.232	0.	0.	0.	0.

Problem 1330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	91	319	0	0	0	0
normalized size	1	1.	0.66	2.33	0.	0.	0.	0.
time (sec)	N/A	0.117	0.054	0.312	0.	0.	0.	0.

Problem 1331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	99	659	0	0	0	0
normalized size	1	1.	0.54	3.58	0.	0.	0.	0.
time (sec)	N/A	0.155	0.073	0.268	0.	0.	0.	0.

Problem 1332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	99	1016	0	0	0	0
normalized size	1	1.	0.43	4.4	0.	0.	0.	0.
time (sec)	N/A	0.187	0.078	0.274	0.	0.	0.	0.

Problem 1333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	110	703	0	0	0	0
normalized size	1	1.	0.39	2.52	0.	0.	0.	0.
time (sec)	N/A	0.268	0.22	0.211	0.	0.	0.	0.

Problem 1334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	91	492	0	0	0	0
normalized size	1	1.	0.39	2.08	0.	0.	0.	0.
time (sec)	N/A	0.215	0.05	0.207	0.	0.	0.	0.

Problem 1335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	91	325	0	0	0	0
normalized size	1	1.	0.4	1.42	0.	0.	0.	0.
time (sec)	N/A	0.213	0.046	0.314	0.	0.	0.	0.

Problem 1336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	91	874	0	0	0	0
normalized size	1	1.	0.32	3.09	0.	0.	0.	0.
time (sec)	N/A	0.252	0.059	0.277	0.	0.	0.	0.

Problem 1337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	161	1057	0	0	0	0
normalized size	1	1.	0.59	3.86	0.	0.	0.	0.
time (sec)	N/A	0.224	0.411	0.271	0.	0.	0.	0.

Problem 1338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	117	796	0	0	0	0
normalized size	1	1.	0.52	3.51	0.	0.	0.	0.
time (sec)	N/A	0.183	0.089	0.214	0.	0.	0.	0.

Problem 1339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	99	566	0	0	0	0
normalized size	1	1.	0.54	3.11	0.	0.	0.	0.
time (sec)	N/A	0.15	0.071	0.207	0.	0.	0.	0.

Problem 1340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	99	626	0	0	0	0
normalized size	1	1.	0.57	3.6	0.	0.	0.	0.
time (sec)	N/A	0.144	0.058	0.216	0.	0.	0.	0.

Problem 1341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	107	678	0	0	0	0
normalized size	1	1.	0.61	3.9	0.	0.	0.	0.
time (sec)	N/A	0.142	0.082	0.22	0.	0.	0.	0.

Problem 1342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	107	1046	0	0	0	0
normalized size	1	1.	0.48	4.73	0.	0.	0.	0.
time (sec)	N/A	0.18	0.087	0.218	0.	0.	0.	0.

Problem 1343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	107	1431	0	0	0	0
normalized size	1	1.	0.4	5.34	0.	0.	0.	0.
time (sec)	N/A	0.219	0.094	0.284	0.	0.	0.	0.

Problem 1344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	117	938	0	0	0	0
normalized size	1	1.	0.36	2.88	0.	0.	0.	0.
time (sec)	N/A	0.296	0.155	0.214	0.	0.	0.	0.

Problem 1345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	99	700	0	0	264	0
normalized size	1	1.	0.35	2.49	0.	0.	0.94	0.
time (sec)	N/A	0.249	0.059	0.209	0.	0.	9.779	0.

Problem 1346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	99	504	0	0	0	0
normalized size	1	1.	0.37	1.86	0.	0.	0.	0.
time (sec)	N/A	0.248	0.056	0.22	0.	0.	0.	0.

Problem 1347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	99	893	0	0	0	0
normalized size	1	1.	0.36	3.27	0.	0.	0.	0.
time (sec)	N/A	0.244	0.06	0.223	0.	0.	0.	0.

Problem 1348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	107	1501	0	0	0	0
normalized size	1	1.	0.33	4.69	0.	0.	0.	0.
time (sec)	N/A	0.291	0.086	0.29	0.	0.	0.	0.

Problem 1349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	161	1344	0	0	0	0
normalized size	1	1.	0.5	4.19	0.	0.	0.	0.
time (sec)	N/A	0.296	0.41	0.26	0.	0.	0.	0.

Problem 1350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	117	1055	0	0	0	0
normalized size	1	1.	0.43	3.85	0.	0.	0.	0.
time (sec)	N/A	0.221	0.119	0.222	0.	0.	0.	0.

Problem 1351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	101	798	0	0	0	0
normalized size	1	1.	0.44	3.48	0.	0.	0.	0.
time (sec)	N/A	0.187	0.08	0.243	0.	0.	0.	0.

Problem 1352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	101	1000	0	0	0	0
normalized size	1	1.	0.46	4.57	0.	0.	0.	0.
time (sec)	N/A	0.183	0.063	0.245	0.	0.	0.	0.

Problem 1353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	109	1310	0	0	0	0
normalized size	1	1.	0.52	6.21	0.	0.	0.	0.
time (sec)	N/A	0.179	0.087	0.22	0.	0.	0.	0.

Problem 1354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	109	1035	0	0	0	0
normalized size	1	1.	0.52	4.91	0.	0.	0.	0.
time (sec)	N/A	0.179	0.091	0.222	0.	0.	0.	0.

Problem 1355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	109	1431	0	0	0	0
normalized size	1	1.	0.42	5.55	0.	0.	0.	0.
time (sec)	N/A	0.214	0.095	0.234	0.	0.	0.	0.

Problem 1356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	109	1843	0	0	0	0
normalized size	1	1.	0.36	6.04	0.	0.	0.	0.
time (sec)	N/A	0.263	0.085	0.287	0.	0.	0.	0.

Problem 1357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	373	373	117	1190	0	0	0	0
normalized size	1	1.	0.31	3.19	0.	0.	0.	0.
time (sec)	N/A	0.373	0.186	0.227	0.	0.	0.	0.

Problem 1358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	328	101	924	0	0	539	0
normalized size	1	1.	0.31	2.82	0.	0.	1.64	0.
time (sec)	N/A	0.299	0.067	0.219	0.	0.	47.042	0.

Problem 1359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	101	700	0	0	0	0
normalized size	1	1.	0.32	2.22	0.	0.	0.	0.
time (sec)	N/A	0.292	0.054	0.227	0.	0.	0.	0.

Problem 1360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	101	1362	0	0	0	0
normalized size	1	1.	0.33	4.39	0.	0.	0.	0.
time (sec)	N/A	0.286	0.065	0.228	0.	0.	0.	0.

Problem 1361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	109	1489	0	0	0	0
normalized size	1	1.	0.35	4.8	0.	0.	0.	0.
time (sec)	N/A	0.287	0.087	0.24	0.	0.	0.	0.

Problem 1362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	109	2125	0	0	0	0
normalized size	1	1.	0.31	5.95	0.	0.	0.	0.
time (sec)	N/A	0.337	0.088	0.33	0.	0.	0.	0.

Problem 1363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	166	567	0	0	0	0
normalized size	1	1.	0.95	3.26	0.	0.	0.	0.
time (sec)	N/A	0.144	0.181	0.232	0.	0.	0.	0.

Problem 1364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	111	362	0	0	0	0
normalized size	1	1.	0.84	2.74	0.	0.	0.	0.
time (sec)	N/A	0.116	0.109	0.218	0.	0.	0.	0.

Problem 1365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	89	189	0	0	0	0
normalized size	1	1.	0.92	1.95	0.	0.	0.	0.
time (sec)	N/A	0.092	0.059	0.212	0.	0.	0.	0.

Problem 1366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	91	362	0	0	0	0
normalized size	1	1.	0.63	2.51	0.	0.	0.	0.
time (sec)	N/A	0.117	0.051	0.217	0.	0.	0.	0.

Problem 1367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	99	691	0	0	0	0
normalized size	1	1.	0.53	3.68	0.	0.	0.	0.
time (sec)	N/A	0.147	0.075	0.224	0.	0.	0.	0.

Problem 1368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	167	703	0	0	0	0
normalized size	1	1.	0.61	2.58	0.	0.	0.	0.
time (sec)	N/A	0.252	0.202	0.23	0.	0.	0.	0.

Problem 1369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	111	496	0	0	0	0
normalized size	1	1.	0.48	2.15	0.	0.	0.	0.
time (sec)	N/A	0.215	0.127	0.208	0.	0.	0.	0.

Problem 1370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	91	186	0	0	0	0
normalized size	1	1.	0.47	0.95	0.	0.	0.	0.
time (sec)	N/A	0.179	0.047	0.193	0.	0.	0.	0.

Problem 1371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	89	336	0	0	0	0
normalized size	1	1.	0.38	1.42	0.	0.	0.	0.
time (sec)	N/A	0.209	0.044	0.251	0.	0.	0.	0.

Problem 1372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	91	874	0	0	0	0
normalized size	1	1.	0.32	3.05	0.	0.	0.	0.
time (sec)	N/A	0.248	0.061	0.258	0.	0.	0.	0.

Problem 1373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	76	118	0	0	41	0
normalized size	1	1.	0.96	1.49	0.	0.	0.52	0.
time (sec)	N/A	0.039	0.028	0.167	0.	0.	10.545	0.

Problem 1374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	63	102	0	0	0	0
normalized size	1	1.	1.24	2.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.014	0.162	0.	0.	0.	0.

Problem 1375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	65	172	0	0	0	0
normalized size	1	1.	0.82	2.18	0.	0.	0.	0.
time (sec)	N/A	0.034	0.015	0.206	0.	0.	0.	0.

Problem 1376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	76	127	0	0	41	0
normalized size	1	1.	0.59	0.99	0.	0.	0.32	0.
time (sec)	N/A	0.071	0.023	0.252	0.	0.	25.758	0.

Problem 1377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	65	105	0	0	41	0
normalized size	1	1.	0.63	1.02	0.	0.	0.4	0.
time (sec)	N/A	0.052	0.013	0.1	0.	0.	3.585	0.

Problem 1378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	63	116	0	0	0	0
normalized size	1	1.	0.49	0.91	0.	0.	0.	0.
time (sec)	N/A	0.068	0.013	0.174	0.	0.	0.	0.

Problem 1379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	172	569	0	0	0	0
normalized size	1	1.	0.84	2.78	0.	0.	0.	0.
time (sec)	N/A	0.176	0.198	0.343	0.	0.	0.	0.

Problem 1380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	122	366	0	0	0	0
normalized size	1	1.	0.75	2.26	0.	0.	0.	0.
time (sec)	N/A	0.142	0.127	0.302	0.	0.	0.	0.

Problem 1381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	88	194	0	0	0	0
normalized size	1	1.	0.7	1.55	0.	0.	0.	0.
time (sec)	N/A	0.113	0.065	0.282	0.	0.	0.	0.

Problem 1382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	100	206	0	0	0	0
normalized size	1	1.	0.73	1.5	0.	0.	0.	0.
time (sec)	N/A	0.113	0.07	0.226	0.	0.	0.	0.

Problem 1383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	98	365	0	0	0	0
normalized size	1	1.	0.53	1.98	0.	0.	0.	0.
time (sec)	N/A	0.144	0.062	0.23	0.	0.	0.	0.

Problem 1384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	122	498	0	0	0	0
normalized size	1	1.	0.47	1.93	0.	0.	0.	0.
time (sec)	N/A	0.246	0.146	0.277	0.	0.	0.	0.

Problem 1385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	88	323	0	0	0	0
normalized size	1	1.	0.4	1.47	0.	0.	0.	0.
time (sec)	N/A	0.205	0.073	0.219	0.	0.	0.	0.

Problem 1386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	98	332	0	0	0	0
normalized size	1	1.	0.42	1.44	0.	0.	0.	0.
time (sec)	N/A	0.206	0.062	0.212	0.	0.	0.	0.

Problem 1387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	96	339	0	0	0	0
normalized size	1	1.	0.35	1.24	0.	0.	0.	0.
time (sec)	N/A	0.245	0.056	0.254	0.	0.	0.	0.

Problem 1388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	98	876	0	0	0	0
normalized size	1	1.	0.3	2.7	0.	0.	0.	0.
time (sec)	N/A	0.294	0.07	0.237	0.	0.	0.	0.

Problem 1389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	252	1473	0	0	0	0
normalized size	1	1.	1.02	5.96	0.	0.	0.	0.
time (sec)	N/A	0.203	0.324	0.313	0.	0.	0.	0.

Problem 1390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	201	958	0	0	0	0
normalized size	1	1.	1.03	4.89	0.	0.	0.	0.
time (sec)	N/A	0.166	0.228	0.314	0.	0.	0.	0.

Problem 1391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	122	479	0	0	0	0
normalized size	1	1.	0.74	2.9	0.	0.	0.	0.
time (sec)	N/A	0.136	0.133	0.286	0.	0.	0.	0.

Problem 1392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	129	487	0	0	0	0
normalized size	1	1.	0.75	2.82	0.	0.	0.	0.
time (sec)	N/A	0.138	0.163	0.279	0.	0.	0.	0.

Problem 1393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	132	491	0	0	0	0
normalized size	1	1.	0.71	2.63	0.	0.	0.	0.
time (sec)	N/A	0.14	0.167	0.235	0.	0.	0.	0.

Problem 1394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	99	797	0	0	0	0
normalized size	1	1.	0.43	3.5	0.	0.	0.	0.
time (sec)	N/A	0.177	0.072	0.24	0.	0.	0.	0.

Problem 1395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	202	1328	0	0	0	0
normalized size	1	1.	0.68	4.44	0.	0.	0.	0.
time (sec)	N/A	0.286	0.229	0.292	0.	0.	0.	0.

Problem 1396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	122	859	0	0	0	0
normalized size	1	1.	0.47	3.33	0.	0.	0.	0.
time (sec)	N/A	0.238	0.132	0.229	0.	0.	0.	0.

Problem 1397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	116	871	0	0	0	0
normalized size	1	1.	0.44	3.3	0.	0.	0.	0.
time (sec)	N/A	0.237	0.116	0.227	0.	0.	0.	0.

Problem 1398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	99	866	0	0	0	0
normalized size	1	1.	0.36	3.12	0.	0.	0.	0.
time (sec)	N/A	0.247	0.059	0.217	0.	0.	0.	0.

Problem 1399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	97	877	0	0	0	0
normalized size	1	1.	0.3	2.7	0.	0.	0.	0.
time (sec)	N/A	0.284	0.058	0.239	0.	0.	0.	0.

Problem 1400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	125	641	0	0	0	0
normalized size	1	1.	0.74	3.77	0.	0.	0.	0.
time (sec)	N/A	0.146	0.102	0.384	0.	0.	0.	0.

Problem 1401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	86	463	0	0	0	0
normalized size	1	1.	0.69	3.73	0.	0.	0.	0.
time (sec)	N/A	0.095	0.063	0.179	0.	0.	0.	0.

Problem 1402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	54	308	0	0	0	0
normalized size	1	1.	0.69	3.95	0.	0.	0.	0.
time (sec)	N/A	0.059	0.02	0.172	0.	0.	0.	0.

Problem 1403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	38	209	0	0	0	0
normalized size	1	1.	1.23	6.74	0.	0.	0.	0.
time (sec)	N/A	0.029	0.016	0.23	0.	0.	0.	0.

Problem 1404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	40	494	0	0	0	0
normalized size	1	1.	0.5	6.18	0.	0.	0.	0.
time (sec)	N/A	0.058	0.02	0.286	0.	0.	0.	0.

Problem 1405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	40	1002	0	0	0	0
normalized size	1	1.	0.32	7.95	0.	0.	0.	0.
time (sec)	N/A	0.086	0.023	0.254	0.	0.	0.	0.

Problem 1406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	40	1532	0	0	0	0
normalized size	1	1.	0.23	8.91	0.	0.	0.	0.
time (sec)	N/A	0.118	0.026	0.298	0.	0.	0.	0.

Problem 1407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	86	500	0	0	0	0
normalized size	1	1.	0.55	3.18	0.	0.	0.	0.
time (sec)	N/A	0.134	0.08	0.205	0.	0.	0.	0.

Problem 1408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	54	333	0	0	0	0
normalized size	1	1.	0.49	3.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.029	0.176	0.	0.	0.	0.

Problem 1409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	40	204	0	0	0	0
normalized size	1	1.	0.63	3.24	0.	0.	0.	0.
time (sec)	N/A	0.055	0.017	0.17	0.	0.	0.	0.

Problem 1410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	38	292	0	0	0	0
normalized size	1	1.	0.36	2.73	0.	0.	0.	0.
time (sec)	N/A	0.085	0.016	0.288	0.	0.	0.	0.

Problem 1411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	40	768	0	0	0	0
normalized size	1	1.	0.25	4.83	0.	0.	0.	0.
time (sec)	N/A	0.118	0.023	0.258	0.	0.	0.	0.

Problem 1412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	468	104	0	0	0	0	0
normalized size	1	1.46	0.32	0.	0.	0.	0.	0.
time (sec)	N/A	1.282	0.095	1.25	0.	0.	0.	0.

Problem 1413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	50	44	0	416	0	0
normalized size	1	1.	1.14	1.	0.	9.45	0.	0.
time (sec)	N/A	0.018	0.05	0.046	0.	2.953	0.	0.

Problem 1414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	74	76	0	868	0	0
normalized size	1	1.	0.83	0.85	0.	9.75	0.	0.
time (sec)	N/A	0.04	0.078	0.044	0.	3.581	0.	0.

Problem 1415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	122	139	1057	1536	0	0
normalized size	1	1.	0.92	1.05	7.95	11.55	0.	0.
time (sec)	N/A	0.066	0.104	0.048	2.558	4.362	0.	0.

Problem 1416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	597	597	104	0	0	0	0	0
normalized size	1	1.	0.17	0.	0.	0.	0.	0.
time (sec)	N/A	2.52	0.077	0.091	0.	0.	0.	0.

Problem 1417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	581	581	104	0	0	0	0	0
normalized size	1	1.	0.18	0.	0.	0.	0.	0.
time (sec)	N/A	1.95	0.065	0.776	0.	0.	0.	0.

Problem 1418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	591	591	112	0	0	0	0	0
normalized size	1	1.	0.19	0.	0.	0.	0.	0.
time (sec)	N/A	2.005	0.091	1.164	0.	0.	0.	0.

Problem 1419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	637	637	112	0	0	0	0	0
normalized size	1	1.	0.18	0.	0.	0.	0.	0.
time (sec)	N/A	2.14	0.108	1.153	0.	0.	0.	0.

Problem 1420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	104	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	0.067	0.413	0.	0.	0.	0.

Problem 1421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	104	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.113	0.068	1.148	0.	0.	0.	0.

Problem 1422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	112	0	0	0	0	0
normalized size	1	1.	1.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	0.09	1.184	0.	0.	0.	0.

Problem 1423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	103	653	0	1474	10271	2033
normalized size	1	1.	0.73	4.63	0.	10.45	72.84	14.42
time (sec)	N/A	0.079	0.091	0.048	0.	2.198	28.514	1.2

Problem 1424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	77	255	0	641	3434	879
normalized size	1	1.	0.75	2.48	0.	6.22	33.34	8.53
time (sec)	N/A	0.051	0.047	0.046	0.	2.213	6.704	1.207

Problem 1425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	64	76	0	223	707	282
normalized size	1	1.	0.98	1.17	0.	3.43	10.88	4.34
time (sec)	N/A	0.028	0.033	0.041	0.	2.229	3.538	1.162

Problem 1426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	68	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.038	1.204	0.	0.	0.	0.

Problem 1427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	69	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.037	1.211	0.	0.	0.	0.

Problem 1428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	71	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.044	1.237	0.	0.	0.	0.

Problem 1429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	114	115	0	0	0	0	0
normalized size	1	1.39	1.4	0.	0.	0.	0.	0.
time (sec)	N/A	0.119	0.084	1.167	0.	0.	0.	0.

Problem 1430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	112	113	0	0	0	0	0
normalized size	1	1.14	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.12	0.064	1.196	0.	0.	0.	0.

Problem 1431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	104	105	0	0	0	0	0
normalized size	1	1.06	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.108	0.055	1.16	0.	0.	0.	0.

Problem 1432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	104	102	0	0	0	0	0
normalized size	1	1.08	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	0.046	1.159	0.	0.	0.	0.

Problem 1433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	109	110	0	0	0	0	0
normalized size	1	1.16	1.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	0.053	1.164	0.	0.	0.	0.

Problem 1434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	110	111	0	0	0	0	0
normalized size	1	1.15	1.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	0.049	1.163	0.	0.	0.	0.

Problem 1435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	102	107	0	0	0	0	0
normalized size	1	0.95	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	0.061	1.272	0.	0.	0.	0.

Problem 1436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	145	233	398	828	0	1184
normalized size	1	1.	1.2	1.93	3.29	6.84	0.	9.79
time (sec)	N/A	0.073	0.101	0.051	1.289	2.224	0.	1.217

Problem 1437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	85	92	0	0	0	0	0
normalized size	1	0.94	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	0.04	1.141	0.	0.	0.	0.

Problem 1438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	58	74	166	309	0	417
normalized size	1	1.	0.85	1.09	2.44	4.54	0.	6.13
time (sec)	N/A	0.027	0.05	0.047	1.213	2.185	0.	1.228

Problem 1439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	85	92	0	0	0	0	0
normalized size	1	0.94	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	0.034	1.095	0.	0.	0.	0.

Problem 1440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	20	22	0	72	112	76
normalized size	1	1.	0.95	1.05	0.	3.43	5.33	3.62
time (sec)	N/A	0.006	0.007	0.042	0.	2.435	90.246	1.232

Problem 1441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	64	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	0.027	1.157	0.	0.	0.	0.

Problem 1442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	85	90	0	0	0	0	0
normalized size	1	0.97	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	0.039	1.168	0.	0.	0.	0.

Problem 1443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	64	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	0.038	1.158	0.	0.	0.	0.

Problem 1444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	85	92	0	0	0	0	0
normalized size	1	0.94	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.044	1.174	0.	0.	0.	0.

Problem 1445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	64	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	0.043	1.23	0.	0.	0.	0.

Problem 1446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	85	92	0	0	0	0	0
normalized size	1	0.94	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.048	1.259	0.	0.	0.	0.

Problem 1447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	19	16	36	14	16
normalized size	1	1.	1.	1.19	1.	2.25	0.88	1.
time (sec)	N/A	0.004	0.004	0.04	1.176	2.021	0.158	1.214

Problem 1448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	42	17	20
normalized size	1	1.	1.	0.84	1.05	2.21	0.89	1.05
time (sec)	N/A	0.006	0.011	0.044	1.285	2.005	0.371	1.139

Problem 1449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	23	27	36	63	0	50
normalized size	1	1.	0.88	1.04	1.38	2.42	0.	1.92
time (sec)	N/A	0.011	0.011	0.04	1.31	2.182	0.	1.239

Problem 1450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	54	58	107	150	0	140
normalized size	1	1.	1.08	1.16	2.14	3.	0.	2.8
time (sec)	N/A	0.021	0.027	0.044	1.665	2.192	0.	1.187

Problem 1451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	148	163	211	363	168	221
normalized size	1	1.	2.28	2.51	3.25	5.58	2.58	3.4
time (sec)	N/A	0.098	0.026	0.04	1.141	1.701	0.1	1.19

Problem 1452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	122	125	167	285	133	171
normalized size	1	1.	1.88	1.92	2.57	4.38	2.05	2.63
time (sec)	N/A	0.068	0.018	0.039	1.135	1.761	0.081	1.159

Problem 1453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	79	87	109	198	87	120
normalized size	1	1.	1.22	1.34	1.68	3.05	1.34	1.85
time (sec)	N/A	0.075	0.014	0.039	1.159	1.779	0.08	1.166

Problem 1454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	46	49	65	115	49	70
normalized size	1	1.	1.21	1.29	1.71	3.03	1.29	1.84
time (sec)	N/A	0.03	0.01	0.039	1.186	1.698	0.075	1.165

Problem 1455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	27	42	19	27
normalized size	1	1.	1.	0.95	1.23	1.91	0.86	1.23
time (sec)	N/A	0.005	0.	0.038	1.18	1.642	0.065	1.115

Problem 1456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	43	74	81	135	44	82
normalized size	1	1.	0.86	1.48	1.62	2.7	0.88	1.64
time (sec)	N/A	0.021	0.018	0.04	1.191	1.642	0.401	1.145

Problem 1457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	47	86	90	184	60	150
normalized size	1	1.	0.92	1.69	1.76	3.61	1.18	2.94
time (sec)	N/A	0.048	0.037	0.046	1.182	1.749	0.553	1.145

Problem 1458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	48	92	108	205	80	93
normalized size	1	1.	0.81	1.56	1.83	3.47	1.36	1.58
time (sec)	N/A	0.038	0.024	0.046	1.144	1.603	0.795	1.2

Problem 1459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	53	71	113	170	88	78
normalized size	1	1.	1.89	2.54	4.04	6.07	3.14	2.79
time (sec)	N/A	0.005	0.025	0.045	1.109	1.688	0.868	1.17

Problem 1460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	55	71	132	201	104	130
normalized size	1	1.	0.85	1.09	2.03	3.09	1.6	2.
time (sec)	N/A	0.037	0.02	0.045	1.176	1.689	1.102	1.14

Problem 1461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	55	71	147	227	116	81
normalized size	1	1.	0.85	1.09	2.26	3.49	1.78	1.25
time (sec)	N/A	0.036	0.027	0.045	1.248	1.765	1.283	1.141

Problem 1462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	55	71	162	251	128	81
normalized size	1	1.	0.85	1.09	2.49	3.86	1.97	1.25
time (sec)	N/A	0.037	0.021	0.045	1.2	1.777	1.578	1.152

Problem 1463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	398	427	564	992	462	608
normalized size	1	1.	3.34	3.59	4.74	8.34	3.88	5.11
time (sec)	N/A	0.267	0.061	0.045	1.17	1.477	0.131	1.1

Problem 1464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	350	361	486	853	401	514
normalized size	1	1.	2.94	3.03	4.08	7.17	3.37	4.32
time (sec)	N/A	0.208	0.049	0.041	1.141	1.487	0.12	1.104

Problem 1465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	273	295	385	680	318	420
normalized size	1	1.	2.29	2.48	3.24	5.71	2.67	3.53
time (sec)	N/A	0.165	0.039	0.04	1.097	1.512	0.115	1.132

Problem 1466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	217	229	304	520	243	324
normalized size	1	1.	2.36	2.49	3.3	5.65	2.64	3.52
time (sec)	N/A	0.13	0.031	0.041	1.187	1.511	0.11	1.166

Problem 1467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	148	163	211	363	168	230
normalized size	1	1.	2.28	2.51	3.25	5.58	2.58	3.54
time (sec)	N/A	0.086	0.026	0.04	1.186	1.504	0.097	1.126

Problem 1468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	84	97	130	217	100	138
normalized size	1	1.	2.21	2.55	3.42	5.71	2.63	3.63
time (sec)	N/A	0.016	0.016	0.039	1.17	1.529	0.077	1.155

Problem 1469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	43	72	85	42	57
normalized size	1	1.	1.	3.07	5.14	6.07	3.	4.07
time (sec)	N/A	0.002	0.001	0.039	1.166	1.395	0.066	1.144

Problem 1470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	115	209	239	369	134	238
normalized size	1	1.	1.17	2.13	2.44	3.77	1.37	2.43
time (sec)	N/A	0.042	0.045	0.042	1.152	1.789	0.707	1.151

Problem 1471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	165	230	247	540	151	323
normalized size	1	1.	1.59	2.21	2.38	5.19	1.45	3.11
time (sec)	N/A	0.1	0.061	0.049	1.163	1.949	1.103	1.183

Problem 1472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	167	245	258	586	184	236
normalized size	1	1.	1.62	2.38	2.5	5.69	1.79	2.29
time (sec)	N/A	0.085	0.056	0.049	1.114	2.036	1.704	1.139

Problem 1473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	163	255	271	581	209	230
normalized size	1	1.	1.58	2.48	2.63	5.64	2.03	2.23
time (sec)	N/A	0.08	0.075	0.05	1.047	2.038	2.673	1.174

Problem 1474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	119	260	297	544	230	377
normalized size	1	1.	1.07	2.34	2.68	4.9	2.07	3.4
time (sec)	N/A	0.077	0.065	0.047	1.226	1.999	3.707	1.176

Problem 1475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	140	186	290	428	233	230
normalized size	1	1.	5.	6.64	10.36	15.29	8.32	8.21
time (sec)	N/A	0.005	0.054	0.046	1.17	1.924	6.007	1.171

Problem 1476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	144	186	319	482	252	235
normalized size	1	1.	2.48	3.21	5.5	8.31	4.34	4.05
time (sec)	N/A	0.014	0.045	0.045	1.182	1.634	11.019	1.214

Problem 1477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	144	186	333	512	264	235
normalized size	1	1.	1.62	2.09	3.74	5.75	2.97	2.64
time (sec)	N/A	0.021	0.05	0.046	1.183	1.769	16.6	1.11

Problem 1478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	144	186	348	540	275	235
normalized size	1	1.	1.23	1.59	2.97	4.62	2.35	2.01
time (sec)	N/A	0.071	0.047	0.046	1.155	1.653	23.146	1.092

Problem 1479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	144	186	363	568	287	235
normalized size	1	1.	1.21	1.56	3.05	4.77	2.41	1.97
time (sec)	N/A	0.068	0.051	0.046	1.14	1.69	44.319	1.199

Problem 1480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	144	186	378	608	299	235
normalized size	1	1.	1.21	1.56	3.18	5.11	2.51	1.97
time (sec)	N/A	0.068	0.046	0.045	1.068	1.709	69.031	1.13

Problem 1481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	771	803	1076	1967	884	1166
normalized size	1	1.	4.46	4.64	6.22	11.37	5.11	6.74
time (sec)	N/A	0.517	0.111	0.042	1.087	1.505	0.216	1.118

Problem 1482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	684	709	953	1736	796	1030
normalized size	1	1.	3.95	4.1	5.51	10.03	4.6	5.95
time (sec)	N/A	0.433	0.096	0.04	1.147	1.554	0.204	1.183

Problem 1483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	573	615	809	1473	677	892
normalized size	1	1.	3.35	3.6	4.73	8.61	3.96	5.22
time (sec)	N/A	0.358	0.082	0.04	1.042	1.515	0.159	1.108

Problem 1484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	501	521	698	1251	580	753
normalized size	1	1.	3.5	3.64	4.88	8.75	4.06	5.27
time (sec)	N/A	0.306	0.071	0.039	1.199	1.529	0.215	1.137

Problem 1485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	398	427	564	992	462	616
normalized size	1	1.	3.34	3.59	4.74	8.34	3.88	5.18
time (sec)	N/A	0.252	0.061	0.04	1.173	1.503	0.15	1.123

Problem 1486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	276	333	441	776	364	479
normalized size	1	1.	3.	3.62	4.79	8.43	3.96	5.21
time (sec)	N/A	0.192	0.081	0.04	1.072	1.622	0.125	1.181

Problem 1487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	199	239	316	539	252	342
normalized size	1	1.	3.06	3.68	4.86	8.29	3.88	5.26
time (sec)	N/A	0.136	0.067	0.041	1.094	1.551	0.121	1.176

Problem 1488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	122	145	192	316	148	205
normalized size	1	1.	3.21	3.82	5.05	8.32	3.89	5.39
time (sec)	N/A	0.016	0.036	0.04	1.141	1.49	0.096	1.144

Problem 1489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	65	131	128	66	86
normalized size	1	1.	1.	4.64	9.36	9.14	4.71	6.14
time (sec)	N/A	0.002	0.001	0.04	1.155	1.574	0.086	1.142

Problem 1490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	230	412	471	726	286	478
normalized size	1	1.	1.58	2.82	3.23	4.97	1.96	3.27
time (sec)	N/A	0.064	0.087	0.043	1.148	1.825	1.605	1.182

Problem 1491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	302	440	482	1018	303	579
normalized size	1	1.	1.94	2.82	3.09	6.53	1.94	3.71
time (sec)	N/A	0.213	0.103	0.049	1.204	1.811	1.831	1.188

Problem 1492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	303	464	491	1110	335	460
normalized size	1	1.	1.92	2.94	3.11	7.03	2.12	2.91
time (sec)	N/A	0.179	0.109	0.052	1.162	1.75	5.166	1.147

Problem 1493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	302	483	505	1161	364	452
normalized size	1	1.	1.94	3.1	3.24	7.44	2.33	2.9
time (sec)	N/A	0.161	0.115	0.051	1.053	1.767	6.696	1.114

Problem 1494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	301	498	522	1158	393	694
normalized size	1	1.	1.94	3.21	3.37	7.47	2.54	4.48
time (sec)	N/A	0.146	0.121	0.052	1.085	1.831	13.4	1.149

Problem 1495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	297	508	536	1095	420	447
normalized size	1	1.	1.92	3.28	3.46	7.06	2.71	2.88
time (sec)	N/A	0.14	0.124	0.052	1.095	1.887	36.014	1.237

Problem 1496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	233	513	562	1022	439	458
normalized size	1	1.	1.4	3.07	3.37	6.12	2.63	2.74
time (sec)	N/A	0.132	0.126	0.048	1.271	1.845	96.22	1.151

Problem 1497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	271	357	537	787	0	467
normalized size	1	1.	9.68	12.75	19.18	28.11	0.	16.68
time (sec)	N/A	0.005	0.099	0.048	1.245	1.746	0.	1.136

Problem 1498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	277	357	581	871	0	475
normalized size	1	1.	4.78	6.16	10.02	15.02	0.	8.19
time (sec)	N/A	0.012	0.093	0.047	1.271	1.575	0.	1.198

Problem 1499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	277	357	595	915	0	475
normalized size	1	1.	3.11	4.01	6.69	10.28	0.	5.34
time (sec)	N/A	0.021	0.095	0.049	1.217	1.784	0.	1.117

Problem 1500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	277	357	610	960	0	475
normalized size	1	1.	2.31	2.98	5.08	8.	0.	3.96
time (sec)	N/A	0.035	0.095	0.048	1.301	1.742	0.	1.141

Problem 1501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	277	357	625	994	0	475
normalized size	1	1.	1.63	2.1	3.68	5.85	0.	2.79
time (sec)	N/A	0.129	0.095	0.054	1.103	1.742	0.	1.167

Problem 1502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	277	357	640	1026	0	475
normalized size	1	1.	1.6	2.06	3.7	5.93	0.	2.75
time (sec)	N/A	0.118	0.092	0.047	1.273	1.764	0.	1.172

Problem 1503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	277	357	655	1064	0	475
normalized size	1	1.	1.62	2.09	3.83	6.22	0.	2.78
time (sec)	N/A	0.131	0.098	0.048	1.144	1.758	0.	1.135

Problem 1504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	277	357	670	1104	0	475
normalized size	1	1.	1.6	2.06	3.87	6.38	0.	2.75
time (sec)	N/A	0.115	0.095	0.048	1.245	1.731	0.	1.155

Problem 1505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	230	326	358	767	224	347
normalized size	1	1.	1.76	2.49	2.73	5.85	1.71	2.65
time (sec)	N/A	0.146	0.078	0.048	1.161	1.813	1.201	1.15

Problem 1506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	166	230	248	540	151	240
normalized size	1	1.	1.6	2.21	2.38	5.19	1.45	2.31
time (sec)	N/A	0.102	0.064	0.048	1.132	1.758	0.978	1.194

Problem 1507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	72	149	159	354	100	154
normalized size	1	1.	0.96	1.99	2.12	4.72	1.33	2.05
time (sec)	N/A	0.064	0.052	0.048	1.114	1.728	0.773	1.114

Problem 1508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	47	86	90	184	60	86
normalized size	1	1.	0.92	1.69	1.76	3.61	1.18	1.69
time (sec)	N/A	0.039	0.037	0.047	1.136	1.677	0.592	1.243

Problem 1509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	31	39	47	80	27	47
normalized size	1	1.	0.97	1.22	1.47	2.5	0.84	1.47
time (sec)	N/A	0.022	0.011	0.043	1.155	1.697	0.391	1.17

Problem 1510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	18	24	10	16
normalized size	1	1.	1.	1.08	1.5	2.	0.83	1.33
time (sec)	N/A	0.002	0.003	0.041	1.14	1.705	0.305	1.165

Problem 1511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	53	57	124	200	233	128
normalized size	1	1.	0.93	1.	2.18	3.51	4.09	2.25
time (sec)	N/A	0.035	0.025	0.057	1.119	1.761	0.83	1.146

Problem 1512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	66	82	281	486	405	211
normalized size	1	1.	0.81	1.01	3.47	6.	5.	2.6
time (sec)	N/A	0.053	0.068	0.052	1.021	1.801	1.328	1.218

Problem 1513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	97	108	521	991	632	335
normalized size	1	1.	0.88	0.98	4.74	9.01	5.75	3.05
time (sec)	N/A	0.079	0.1	0.055	1.166	1.78	2.133	1.168

Problem 1514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	120	131	809	1501	881	455
normalized size	1	1.	0.9	0.98	6.08	11.29	6.62	3.42
time (sec)	N/A	0.108	0.125	0.055	1.331	1.858	3.767	1.122

Problem 1515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	301	483	505	1161	364	450
normalized size	1	1.	1.93	3.1	3.24	7.44	2.33	2.88
time (sec)	N/A	0.18	0.119	0.051	1.259	1.795	5.618	1.152

Problem 1516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	228	361	379	867	282	332
normalized size	1	1.	1.77	2.8	2.94	6.72	2.19	2.57
time (sec)	N/A	0.125	0.082	0.049	1.148	1.668	3.744	1.124

Problem 1517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	166	255	271	581	209	225
normalized size	1	1.	1.61	2.48	2.63	5.64	2.03	2.18
time (sec)	N/A	0.088	0.061	0.049	1.124	1.7	2.466	1.14

Problem 1518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	80	166	192	360	148	153
normalized size	1	1.	0.93	1.93	2.23	4.19	1.72	1.78
time (sec)	N/A	0.06	0.041	0.046	1.208	1.747	1.529	1.162

Problem 1519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	53	70	113	170	88	78
normalized size	1	1.	1.89	2.5	4.04	6.07	3.14	2.79
time (sec)	N/A	0.005	0.024	0.046	1.13	1.742	0.959	1.219

Problem 1520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	27	35	68	105	53	36
normalized size	1	1.	0.71	0.92	1.79	2.76	1.39	0.95
time (sec)	N/A	0.022	0.008	0.043	2.026	1.695	0.518	1.177

Problem 1521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	47	70	37	16
normalized size	1	1.	1.	0.93	3.36	5.	2.64	1.14
time (sec)	N/A	0.002	0.003	0.043	1.382	1.701	0.395	1.201

Problem 1522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	107	103	487	856	570	316
normalized size	1	1.	1.	0.96	4.55	8.	5.33	2.95
time (sec)	N/A	0.068	0.045	0.05	1.213	1.762	2.204	1.173

Problem 1523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	121	132	807	1501	881	377
normalized size	1	1.	0.92	1.	6.11	11.37	6.67	2.86
time (sec)	N/A	0.101	0.125	0.056	1.11	1.819	3.598	1.239

Problem 1524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	154	165	1200	2313	1217	587
normalized size	1	1.	0.91	0.97	7.06	13.61	7.16	3.45
time (sec)	N/A	0.148	0.214	0.055	1.36	2.032	5.096	1.126

Problem 1525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	195	820	845	1952	0	737
normalized size	1	1.	0.94	3.94	4.06	9.38	0.	3.54
time (sec)	N/A	0.311	0.17	0.057	1.286	1.866	0.	1.177

Problem 1526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	389	656	680	1517	522	583
normalized size	1	1.	2.15	3.62	3.76	8.38	2.88	3.22
time (sec)	N/A	0.235	0.149	0.056	1.295	1.723	111.638	1.148

Problem 1527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	300	508	536	1095	420	443
normalized size	1	1.	1.94	3.28	3.46	7.06	2.71	2.86
time (sec)	N/A	0.169	0.163	0.052	1.253	1.921	37.57	1.157

Problem 1528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	171	377	419	775	326	335
normalized size	1	1.	1.24	2.73	3.04	5.62	2.36	2.43
time (sec)	N/A	0.117	0.089	0.052	1.186	2.091	14.382	1.177

Problem 1529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	140	185	290	428	233	230
normalized size	1	1.	5.	6.61	10.36	15.29	8.32	8.21
time (sec)	N/A	0.005	0.048	0.046	1.188	2.029	5.995	1.198

Problem 1530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	97	121	216	328	170	147
normalized size	1	1.	1.67	2.09	3.72	5.66	2.93	2.53
time (sec)	N/A	0.011	0.036	0.046	1.169	1.808	2.689	1.13

Problem 1531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	57	71	147	227	116	81
normalized size	1	1.	0.88	1.09	2.26	3.49	1.78	1.25
time (sec)	N/A	0.04	0.024	0.045	1.081	1.574	1.307	1.181

Problem 1532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	27	35	97	153	76	36
normalized size	1	1.	0.71	0.92	2.55	4.03	2.	0.95
time (sec)	N/A	0.024	0.009	0.045	1.144	1.501	0.773	1.135

Problem 1533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	77	116	61	16
normalized size	1	1.	1.	0.93	5.5	8.29	4.36	1.14
time (sec)	N/A	0.002	0.003	0.041	1.115	1.496	0.558	1.121

Problem 1534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	152	147	1087	1894	1081	568
normalized size	1	1.	0.98	0.95	7.01	12.22	6.97	3.66
time (sec)	N/A	0.114	0.076	0.055	1.317	1.709	4.856	1.122

Problem 1535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	167	178	1558	2889	1516	609
normalized size	1	1.	0.92	0.98	8.61	15.96	8.38	3.36
time (sec)	N/A	0.184	0.133	0.059	1.391	1.888	9.336	1.153

Problem 1536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	204	215	2103	4157	1974	907
normalized size	1	1.	0.93	0.98	9.56	18.9	8.97	4.12
time (sec)	N/A	0.252	0.167	0.059	1.889	2.142	16.31	1.223

Problem 1537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	81	84	104	231	76	122
normalized size	1	1.	2.61	2.71	3.35	7.45	2.45	3.94
time (sec)	N/A	0.012	0.015	0.04	1.09	1.344	0.089	1.137

Problem 1538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	59	60	72	155	58	86
normalized size	1	1.	1.9	1.94	2.32	5.	1.87	2.77
time (sec)	N/A	0.011	0.009	0.039	1.163	1.34	0.078	1.186

Problem 1539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	36	35	43	80	32	50
normalized size	1	1.	1.16	1.13	1.39	2.58	1.03	1.61
time (sec)	N/A	0.021	0.006	0.038	1.114	1.363	0.078	1.119

Problem 1540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	31	35	76	20	39
normalized size	1	1.	1.	1.03	1.17	2.53	0.67	1.3
time (sec)	N/A	0.017	0.007	0.044	1.107	1.378	0.335	1.133

Problem 1541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	22	28	41	76	27	30
normalized size	1	1.	0.71	0.9	1.32	2.45	0.87	0.97
time (sec)	N/A	0.016	0.008	0.044	1.156	1.398	0.374	1.195

Problem 1542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	22	28	54	111	37	30
normalized size	1	1.	0.71	0.9	1.74	3.58	1.19	0.97
time (sec)	N/A	0.017	0.007	0.045	1.015	1.672	0.496	1.13

Problem 1543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	111	114	0	212	100	207
normalized size	1	1.	1.21	1.24	0.	2.3	1.09	2.25
time (sec)	N/A	0.04	0.038	0.042	0.	1.765	0.119	1.243

Problem 1544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	89	90	0	150	73	159
normalized size	1	1.	0.97	0.98	0.	1.63	0.79	1.73
time (sec)	N/A	0.036	0.028	0.042	0.	1.858	0.131	1.164

Problem 1545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	67	66	0	109	49	115
normalized size	1	1.	0.73	0.72	0.	1.18	0.53	1.25
time (sec)	N/A	0.049	0.022	0.041	0.	1.64	0.112	1.167

Problem 1546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	45	42	0	58	26	70
normalized size	1	1.	0.65	0.61	0.	0.84	0.38	1.01
time (sec)	N/A	0.021	0.017	0.041	0.	1.51	0.116	1.127

Problem 1547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	30	27	0	23	8	45
normalized size	1	1.	0.94	0.84	0.	0.72	0.25	1.41
time (sec)	N/A	0.005	0.009	0.041	0.	1.455	0.112	1.199

Problem 1548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	42	44	0	54	20	61
normalized size	1	1.	0.52	0.55	0.	0.68	0.25	0.76
time (sec)	N/A	0.037	0.016	0.201	0.	1.547	0.586	1.202

Problem 1549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	50	51	0	78	27	69
normalized size	1	1.	0.59	0.6	0.	0.92	0.32	0.81
time (sec)	N/A	0.041	0.019	0.227	0.	1.58	0.379	1.222

Problem 1550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	44	41	0	81	39	59
normalized size	1	1.	0.96	0.89	0.	1.76	0.85	1.28
time (sec)	N/A	0.02	0.016	0.041	0.	1.553	0.452	1.137

Problem 1551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	45	42	0	105	53	61
normalized size	1	1.	0.49	0.46	0.	1.14	0.58	0.66
time (sec)	N/A	0.042	0.018	0.042	0.	1.545	0.536	1.226

Problem 1552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	45	42	0	128	65	61
normalized size	1	1.	0.49	0.46	0.	1.39	0.71	0.66
time (sec)	N/A	0.041	0.018	0.044	0.	1.495	0.653	1.166

Problem 1553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	45	42	0	153	76	61
normalized size	1	1.	0.49	0.46	0.	1.66	0.83	0.66
time (sec)	N/A	0.042	0.019	0.043	0.	1.591	0.754	1.263

Problem 1554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	259	322	0	585	0	587
normalized size	1	1.	1.3	1.61	0.	2.92	0.	2.94
time (sec)	N/A	0.198	0.093	0.156	0.	1.518	0.	1.198

Problem 1555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	215	264	0	471	0	482
normalized size	1	1.	1.08	1.32	0.	2.36	0.	2.41
time (sec)	N/A	0.16	0.076	0.154	0.	1.571	0.	1.171

Problem 1556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	171	206	0	344	0	378
normalized size	1	1.	0.99	1.2	0.	2.	0.	2.2
time (sec)	N/A	0.132	0.062	0.155	0.	1.525	0.	1.155

Problem 1557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	125	127	148	0	266	0	273
normalized size	1	1.1	1.11	1.3	0.	2.33	0.	2.39
time (sec)	N/A	0.05	0.045	0.154	0.	1.447	0.	1.173

Problem 1558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	83	90	0	150	0	167
normalized size	1	1.	1.2	1.3	0.	2.17	0.	2.42
time (sec)	N/A	0.021	0.033	0.178	0.	1.529	0.	1.135

Problem 1559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	23	49	0	66	0	93
normalized size	1	1.	0.72	1.53	0.	2.06	0.	2.91
time (sec)	N/A	0.005	0.009	0.039	0.	1.424	0.	1.115

Problem 1560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	92	149	0	238	0	234
normalized size	1	1.	0.55	0.9	0.	1.43	0.	1.41
time (sec)	N/A	0.068	0.046	0.199	0.	1.61	0.	1.219

Problem 1561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	132	216	0	354	0	236
normalized size	1	1.	0.72	1.18	0.	1.93	0.	1.29
time (sec)	N/A	0.092	0.09	0.202	0.	1.49	0.	1.171

Problem 1562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	131	219	0	375	0	230
normalized size	1	1.	0.7	1.18	0.	2.02	0.	1.24
time (sec)	N/A	0.086	0.068	0.202	0.	1.507	0.	1.213

Problem 1563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	104	186	0	359	0	239
normalized size	1	1.	0.54	0.96	0.	1.85	0.	1.23
time (sec)	N/A	0.088	0.058	0.199	0.	1.503	0.	1.132

Problem 1564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	109	128	0	284	0	224
normalized size	1	1.	2.27	2.67	0.	5.92	0.	4.67
time (sec)	N/A	0.019	0.042	0.157	0.	1.537	0.	1.181

Problem 1565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	112	131	0	328	0	228
normalized size	1	1.	1.14	1.34	0.	3.35	0.	2.33
time (sec)	N/A	0.038	0.047	0.155	0.	1.596	0.	1.189

Problem 1566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	200	112	131	0	354	0	228
normalized size	1	1.4	0.78	0.92	0.	2.48	0.	1.59
time (sec)	N/A	0.086	0.044	0.159	0.	1.846	0.	1.14

Problem 1567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	112	131	0	382	0	228
normalized size	1	1.	0.56	0.66	0.	1.91	0.	1.14
time (sec)	N/A	0.089	0.041	0.155	0.	1.854	0.	1.185

Problem 1568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	112	131	0	406	0	228
normalized size	1	1.	0.56	0.66	0.	2.03	0.	1.14
time (sec)	N/A	0.087	0.047	0.155	0.	1.842	0.	1.179

Problem 1569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	385	506	0	883	0	926
normalized size	1	1.	1.45	1.9	0.	3.32	0.	3.48
time (sec)	N/A	0.303	0.132	0.157	0.	1.86	0.	1.303

Problem 1570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	319	414	0	760	0	761
normalized size	1	1.	1.46	1.89	0.	3.47	0.	3.47
time (sec)	N/A	0.09	0.108	0.157	0.	1.608	0.	1.244

Problem 1571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	253	322	0	585	0	595
normalized size	1	1.	1.47	1.87	0.	3.4	0.	3.46
time (sec)	N/A	0.18	0.087	0.156	0.	1.492	0.	1.191

Problem 1572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	187	230	0	417	0	432
normalized size	1	1.	1.5	1.84	0.	3.34	0.	3.46
time (sec)	N/A	0.14	0.068	0.153	0.	1.594	0.	1.143

Problem 1573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	121	138	0	247	0	269
normalized size	1	1.	1.75	2.	0.	3.58	0.	3.9
time (sec)	N/A	0.024	0.045	0.16	0.	1.51	0.	1.241

Problem 1574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	23	71	0	116	0	139
normalized size	1	1.	0.72	2.22	0.	3.62	0.	4.34
time (sec)	N/A	0.006	0.013	0.042	0.	1.535	0.	1.177

Problem 1575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	185	318	0	537	0	520
normalized size	1	1.	0.73	1.25	0.	2.11	0.	2.05
time (sec)	N/A	0.125	0.096	0.197	0.	1.693	0.	1.247

Problem 1576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	246	456	0	767	0	516
normalized size	1	1.	0.84	1.56	0.	2.63	0.	1.77
time (sec)	N/A	0.211	0.161	0.203	0.	1.502	0.	1.215

Problem 1577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	248	502	0	840	0	508
normalized size	1	1.	0.84	1.7	0.	2.85	0.	1.72
time (sec)	N/A	0.19	0.114	0.203	0.	1.626	0.	1.141

Problem 1578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	247	502	0	867	0	506
normalized size	1	1.	0.85	1.72	0.	2.97	0.	1.73
time (sec)	N/A	0.157	0.118	0.2	0.	1.61	0.	1.259

Problem 1579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	243	458	0	838	0	500
normalized size	1	1.	0.83	1.57	0.	2.87	0.	1.71
time (sec)	N/A	0.148	0.129	0.232	0.	1.623	0.	1.216

Problem 1580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	196	383	0	774	0	510
normalized size	1	1.	0.65	1.28	0.	2.58	0.	1.7
time (sec)	N/A	0.144	0.13	0.2	0.	1.56	0.	1.16

Problem 1581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	218	283	0	595	0	508
normalized size	1	1.	4.54	5.9	0.	12.4	0.	10.58
time (sec)	N/A	0.019	0.078	0.155	0.	1.646	0.	1.211

Problem 1582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	223	288	0	663	0	514
normalized size	1	1.	2.28	2.94	0.	6.77	0.	5.24
time (sec)	N/A	0.036	0.08	0.157	0.	1.581	0.	1.202

Problem 1583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	223	288	0	697	0	514
normalized size	1	1.	1.5	1.93	0.	4.68	0.	3.45
time (sec)	N/A	0.051	0.08	0.158	0.	1.621	0.	1.172

Problem 1584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	223	288	0	732	0	514
normalized size	1	1.	1.12	1.44	0.	3.66	0.	2.57
time (sec)	N/A	0.068	0.097	0.155	0.	1.628	0.	1.203

Problem 1585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	223	288	0	771	0	514
normalized size	1	1.	0.72	0.94	0.	2.5	0.	1.67
time (sec)	N/A	0.14	0.08	0.159	0.	1.607	0.	1.164

Problem 1586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	223	288	0	802	0	514
normalized size	1	1.	0.72	0.94	0.	2.6	0.	1.67
time (sec)	N/A	0.138	0.079	0.156	0.	1.617	0.	1.172

Problem 1587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	130	223	590	369	134	356
normalized size	1	1.	0.59	1.	2.66	1.66	0.6	1.6
time (sec)	N/A	0.08	0.081	0.158	1.154	1.584	0.647	1.22

Problem 1588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	90	147	346	238	82	230
normalized size	1	1.	0.52	0.85	2.	1.38	0.47	1.33
time (sec)	N/A	0.062	0.053	0.154	1.146	1.558	0.549	1.159

Problem 1589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	59	87	153	135	44	128
normalized size	1	1.	0.48	0.7	1.23	1.09	0.35	1.03
time (sec)	N/A	0.046	0.034	0.154	1.062	1.563	0.453	1.183

Problem 1590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	40	45	80	54	20	62
normalized size	1	1.	0.58	0.65	1.16	0.78	0.29	0.9
time (sec)	N/A	0.023	0.017	0.164	1.02	1.488	0.369	1.133

Problem 1591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	26	25	19	22	7	23
normalized size	1	1.	0.74	0.71	0.54	0.63	0.2	0.66
time (sec)	N/A	0.008	0.007	0.043	1.059	1.65	0.088	1.157

Problem 1592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	42	41	0	58	128	101
normalized size	1	1.	0.49	0.48	0.	0.67	1.49	1.17
time (sec)	N/A	0.031	0.019	0.157	0.	1.596	0.402	1.241

Problem 1593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	69	82	0	198	233	139
normalized size	1	1.	0.53	0.63	0.	1.51	1.78	1.06
time (sec)	N/A	0.062	0.044	0.162	0.	1.596	0.953	1.19

Problem 1594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	97	163	0	490	381	235
normalized size	1	1.	0.53	0.9	0.	2.69	2.09	1.29
time (sec)	N/A	0.081	0.065	0.165	0.	1.614	1.473	1.165

Problem 1595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	124	256	0	855	570	332
normalized size	1	1.	0.54	1.11	0.	3.7	2.47	1.44
time (sec)	N/A	0.102	0.075	0.165	0.	1.624	2.157	1.168

Problem 1596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	174	341	657	586	0	0
normalized size	1	1.	0.83	1.62	3.13	2.79	0.	0.
time (sec)	N/A	0.135	0.084	0.204	1.084	1.58	0.	0.

Problem 1597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	125	209	390	375	0	0
normalized size	1	1.	0.78	1.3	2.42	2.33	0.	0.
time (sec)	N/A	0.094	0.057	0.2	1.052	1.611	0.	0.

Problem 1598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	67	104	176	207	0	0
normalized size	1	1.	0.57	0.89	1.5	1.77	0.	0.
time (sec)	N/A	0.069	0.032	0.201	1.089	1.536	0.	0.

Problem 1599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	39	32	85	81	0	0
normalized size	1	1.	0.57	0.46	1.23	1.17	0.	0.
time (sec)	N/A	0.022	0.015	0.154	1.013	1.532	0.	0.

Problem 1600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	23	20	22	49	0	0
normalized size	1	1.	0.68	0.59	0.65	1.44	0.	0.
time (sec)	N/A	0.005	0.009	0.042	1.071	1.535	0.	0.

Problem 1601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	92	155	0	491	0	0
normalized size	1	1.	0.56	0.94	0.	2.98	0.	0.
time (sec)	N/A	0.097	0.055	0.203	0.	1.493	0.	0.

Problem 1602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	141	329	0	991	0	0
normalized size	1	1.	0.65	1.52	0.	4.57	0.	0.
time (sec)	N/A	0.126	0.099	0.204	0.	1.703	0.	0.

Problem 1603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	163	508	0	1488	0	0
normalized size	1	1.	0.59	1.84	0.	5.39	0.	0.
time (sec)	N/A	0.152	0.098	0.212	0.	1.755	0.	0.

Problem 1604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	302	313	661	844	1158	0	0
normalized size	1	1.	1.04	2.19	2.79	3.83	0.	0.
time (sec)	N/A	0.239	0.148	0.205	1.406	1.65	0.	0.

Problem 1605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	242	449	664	838	0	0
normalized size	1	1.	0.96	1.77	2.62	3.31	0.	0.
time (sec)	N/A	0.175	0.129	0.204	1.19	1.638	0.	0.

Problem 1606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	138	267	502	545	0	0
normalized size	1	1.	0.66	1.28	2.4	2.61	0.	0.
time (sec)	N/A	0.127	0.076	0.198	1.144	1.517	0.	0.

Problem 1607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	106	119	392	284	0	0
normalized size	1	1.	2.21	2.48	8.17	5.92	0.	0.
time (sec)	N/A	0.022	0.043	0.155	1.14	1.621	0.	0.

Problem 1608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	69	69	178	201	0	0
normalized size	1	1.	0.55	0.55	1.42	1.61	0.	0.
time (sec)	N/A	0.07	0.028	0.154	1.149	1.52	0.	0.

Problem 1609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	39	33	85	128	0	0
normalized size	1	1.	0.55	0.46	1.2	1.8	0.	0.
time (sec)	N/A	0.021	0.016	0.15	1.007	1.598	0.	0.

Problem 1610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	23	20	22	92	0	0
normalized size	1	1.	0.68	0.59	0.65	2.71	0.	0.
time (sec)	N/A	0.005	0.011	0.041	1.036	1.556	0.	0.

Problem 1611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	163	359	0	1320	0	0
normalized size	1	1.	0.64	1.42	0.	5.22	0.	0.
time (sec)	N/A	0.157	0.124	0.204	0.	1.696	0.	0.

Problem 1612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	167	651	0	2196	0	0
normalized size	1	1.	0.54	2.12	0.	7.15	0.	0.
time (sec)	N/A	0.21	0.119	0.211	0.	1.912	0.	0.

Problem 1613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	365	209	982	0	3136	0	0
normalized size	1	1.	0.57	2.69	0.	8.59	0.	0.
time (sec)	N/A	0.263	0.158	0.235	0.	2.133	0.	0.

Problem 1614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	81	86	105	176	0	223
normalized size	1	1.	1.62	1.72	2.1	3.52	0.	4.46
time (sec)	N/A	0.013	0.031	0.081	1.602	1.476	0.	1.153

Problem 1615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	57	62	105	107	0	155
normalized size	1	1.	1.14	1.24	2.1	2.14	0.	3.1
time (sec)	N/A	0.012	0.022	0.08	1.556	1.58	0.	1.193

Problem 1616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	38	38	105	55	0	86
normalized size	1	1.	0.76	0.76	2.1	1.1	0.	1.72
time (sec)	N/A	0.012	0.012	0.079	1.592	1.555	0.	1.188

Problem 1617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	42	40	41	54	0	62
normalized size	1	1.	0.75	0.71	0.73	0.96	0.	1.11
time (sec)	N/A	0.016	0.015	0.12	1.558	1.601	0.	1.158

Problem 1618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	34	28	49	61	0	0
normalized size	1	1.	0.65	0.54	0.94	1.17	0.	0.
time (sec)	N/A	0.014	0.011	0.079	1.56	1.544	0.	0.

Problem 1619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	34	28	49	92	0	0
normalized size	1	1.	0.65	0.54	0.94	1.77	0.	0.
time (sec)	N/A	0.013	0.014	0.078	1.602	1.511	0.	0.

Problem 1620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	34	28	49	131	0	0
normalized size	1	1.	0.65	0.54	0.94	2.52	0.	0.
time (sec)	N/A	0.014	0.015	0.08	1.641	1.489	0.	0.

Problem 1621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	61	63	92	474	432	803
normalized size	1	1.	0.86	0.89	1.3	6.68	6.08	11.31
time (sec)	N/A	0.028	0.053	0.046	1.218	1.52	9.584	1.162

Problem 1622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	61	63	92	382	355	518
normalized size	1	1.	0.86	0.89	1.3	5.38	5.	7.3
time (sec)	N/A	0.023	0.041	0.046	1.062	1.533	3.879	1.164

Problem 1623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	61	63	92	300	240	288
normalized size	1	1.	0.86	0.89	1.3	4.23	3.38	4.06
time (sec)	N/A	0.023	0.037	0.046	1.035	1.538	9.398	1.218

Problem 1624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	61	63	92	220	85	117
normalized size	1	1.	0.86	0.89	1.3	3.1	1.2	1.65
time (sec)	N/A	0.023	0.034	0.046	1.083	1.568	2.834	1.183

Problem 1625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	60	63	111	146	236	115
normalized size	1	1.	0.87	0.91	1.61	2.12	3.42	1.67
time (sec)	N/A	0.022	0.033	0.047	1.056	1.53	9.331	1.159

Problem 1626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	59	63	101	157	65	112
normalized size	1	1.	0.88	0.94	1.51	2.34	0.97	1.67
time (sec)	N/A	0.022	0.031	0.045	1.104	1.57	10.298	1.212

Problem 1627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	62	62	97	174	265	101
normalized size	1	1.	0.93	0.93	1.45	2.6	3.96	1.51
time (sec)	N/A	0.022	0.036	0.046	1.036	1.441	1.455	1.241

Problem 1628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	61	63	88	204	389	97
normalized size	1	1.	0.88	0.91	1.28	2.96	5.64	1.41
time (sec)	N/A	0.026	0.033	0.044	1.03	1.472	3.136	1.192

Problem 1629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	101	186	244	1022	903	1729
normalized size	1	1.	0.78	1.44	1.89	7.92	7.	13.4
time (sec)	N/A	0.071	0.118	0.048	1.086	1.525	19.738	1.355

Problem 1630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	101	186	244	855	960	1157
normalized size	1	1.	0.78	1.44	1.89	6.63	7.44	8.97
time (sec)	N/A	0.042	0.094	0.046	1.045	1.541	33.506	1.254

Problem 1631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	101	186	244	702	559	675
normalized size	1	1.	0.78	1.44	1.89	5.44	4.33	5.23
time (sec)	N/A	0.043	0.085	0.046	1.104	1.49	18.913	1.26

Problem 1632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	101	186	244	547	223	292
normalized size	1	1.	0.78	1.44	1.89	4.24	1.73	2.26
time (sec)	N/A	0.054	0.085	0.045	1.17	1.501	4.666	1.196

Problem 1633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	101	186	333	405	561	289
normalized size	1	1.	0.8	1.46	2.62	3.19	4.42	2.28
time (sec)	N/A	0.041	0.071	0.044	1.177	1.493	61.624	1.209

Problem 1634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	101	186	255	412	168	320
normalized size	1	1.	0.82	1.51	2.07	3.35	1.37	2.6
time (sec)	N/A	0.041	0.07	0.046	1.048	1.504	40.107	1.182

Problem 1635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	101	186	252	431	136	309
normalized size	1	1.	0.81	1.49	2.02	3.45	1.09	2.47
time (sec)	N/A	0.042	0.072	0.049	1.122	1.619	40.379	1.155

Problem 1636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	101	186	255	447	1008	305
normalized size	1	1.	0.81	1.49	2.04	3.58	8.06	2.44
time (sec)	N/A	0.043	0.072	0.046	1.216	1.496	4.301	1.241

Problem 1637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	145	377	473	1746	2450	2936
normalized size	1	1.	0.78	2.02	2.53	9.34	13.1	15.7
time (sec)	N/A	0.083	0.159	0.049	1.07	1.604	78.906	1.363

Problem 1638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	145	377	473	1497	1671	2007
normalized size	1	1.	0.78	2.04	2.56	8.09	9.03	10.85
time (sec)	N/A	0.061	0.113	0.046	1.063	1.651	51.213	1.324

Problem 1639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	145	377	473	1256	1000	1202
normalized size	1	1.	0.78	2.02	2.53	6.72	5.35	6.43
time (sec)	N/A	0.061	0.104	0.047	1.071	1.618	33.121	1.24

Problem 1640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	145	377	473	1027	422	536
normalized size	1	1.	0.78	2.02	2.53	5.49	2.26	2.87
time (sec)	N/A	0.059	0.101	0.047	1.075	1.6	6.471	1.222

Problem 1641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	145	377	729	815	1003	533
normalized size	1	1.	0.8	2.08	4.03	4.5	5.54	2.94
time (sec)	N/A	0.06	0.081	0.048	1.039	1.686	98.016	1.184

Problem 1642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	145	377	483	814	333	640
normalized size	1	1.	0.81	2.11	2.7	4.55	1.86	3.58
time (sec)	N/A	0.059	0.08	0.047	1.017	1.863	55.166	1.303

Problem 1643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	145	377	481	822	270	624
normalized size	1	1.	0.8	2.08	2.66	4.54	1.49	3.45
time (sec)	N/A	0.059	0.083	0.048	1.048	1.816	66.336	1.221

Problem 1644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	145	377	481	834	221	618
normalized size	1	1.	0.81	2.11	2.69	4.66	1.23	3.45
time (sec)	N/A	0.059	0.081	0.048	1.176	1.85	123.569	1.239

Problem 1645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	50	539	0	1450	0	522
normalized size	1	1.	0.31	3.33	0.	8.95	0.	3.22
time (sec)	N/A	0.183	0.025	0.213	0.	2.03	0.	1.264

Problem 1646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	50	387	0	1049	0	379
normalized size	1	1.	0.36	2.82	0.	7.66	0.	2.77
time (sec)	N/A	0.076	0.017	0.202	0.	1.92	0.	1.28

Problem 1647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	50	258	0	707	0	258
normalized size	1	1.	0.45	2.35	0.	6.43	0.	2.35
time (sec)	N/A	0.058	0.017	0.202	0.	1.848	0.	1.189

Problem 1648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	50	148	0	455	923	165
normalized size	1	1.	0.59	1.74	0.	5.35	10.86	1.94
time (sec)	N/A	0.04	0.014	0.2	0.	1.892	108.324	1.164

Problem 1649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	69	64	0	498	573	108
normalized size	1	1.	0.99	0.91	0.	7.11	8.19	1.54
time (sec)	N/A	0.031	0.075	0.199	0.	1.979	19.193	1.227

Problem 1650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	77	0	603	0	131
normalized size	1	1.	1.	1.01	0.	7.93	0.	1.72
time (sec)	N/A	0.044	0.056	0.196	0.	1.993	0.	1.171

Problem 1651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	48	101	0	883	0	207
normalized size	1	1.	0.48	1.02	0.	8.92	0.	2.09
time (sec)	N/A	0.054	0.013	0.207	0.	2.018	0.	1.187

Problem 1652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	50	125	0	1592	0	302
normalized size	1	1.	0.4	1.01	0.	12.84	0.	2.44
time (sec)	N/A	0.064	0.015	0.204	0.	2.02	0.	1.24

Problem 1653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	50	149	0	2471	0	410
normalized size	1	1.	0.33	0.99	0.	16.36	0.	2.72
time (sec)	N/A	0.1	0.017	0.207	0.	2.135	0.	1.203

Problem 1654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	52	719	0	2140	0	663
normalized size	1	1.	0.26	3.58	0.	10.65	0.	3.3
time (sec)	N/A	0.131	0.025	0.209	0.	2.035	0.	1.223

Problem 1655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	52	525	0	1528	0	486
normalized size	1	1.	0.3	3.05	0.	8.88	0.	2.83
time (sec)	N/A	0.097	0.02	0.205	0.	1.901	0.	1.242

Problem 1656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	52	352	0	1065	0	335
normalized size	1	1.	0.36	2.43	0.	7.34	0.	2.31
time (sec)	N/A	0.069	0.019	0.208	0.	1.943	0.	1.24

Problem 1657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	119	204	0	1162	0	223
normalized size	1	1.	0.94	1.62	0.	9.22	0.	1.77
time (sec)	N/A	0.059	0.144	0.2	0.	2.028	0.	1.255

Problem 1658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	52	163	0	1368	0	258
normalized size	1	1.	0.38	1.2	0.	10.06	0.	1.9
time (sec)	N/A	0.072	0.016	0.207	0.	1.858	0.	1.192

Problem 1659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	52	170	0	1581	4592	285
normalized size	1	1.	0.36	1.16	0.	10.83	31.45	1.95
time (sec)	N/A	0.069	0.013	0.204	0.	2.105	108.858	1.257

Problem 1660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	50	147	0	1805	0	315
normalized size	1	1.	0.34	1.	0.	12.28	0.	2.14
time (sec)	N/A	0.072	0.011	0.196	0.	2.112	0.	1.138

Problem 1661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	50	292	0	2441	0	437
normalized size	1	1.	0.29	1.69	0.	14.11	0.	2.53
time (sec)	N/A	0.084	0.015	0.21	0.	2.469	0.	1.22

Problem 1662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	52	319	0	3717	0	576
normalized size	1	1.	0.26	1.6	0.	18.58	0.	2.88
time (sec)	N/A	0.14	0.017	0.212	0.	2.563	0.	1.223

Problem 1663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	52	344	0	5176	0	635
normalized size	1	1.	0.23	1.5	0.	22.6	0.	2.77
time (sec)	N/A	0.185	0.022	0.212	0.	2.778	0.	1.208

Problem 1664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	52	1164	0	3553	0	1060
normalized size	1	1.	0.21	4.6	0.	14.04	0.	4.19
time (sec)	N/A	0.183	0.027	0.244	0.	2.282	0.	1.381

Problem 1665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	52	908	0	2743	0	828
normalized size	1	1.	0.23	4.05	0.	12.25	0.	3.7
time (sec)	N/A	0.139	0.022	0.216	0.	2.418	0.	1.323

Problem 1666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	52	673	0	1976	0	620
normalized size	1	1.	0.26	3.42	0.	10.03	0.	3.15
time (sec)	N/A	0.103	0.023	0.212	0.	2.229	0.	1.344

Problem 1667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	178	463	0	2122	0	451
normalized size	1	1.	1.	2.6	0.	11.92	0.	2.53
time (sec)	N/A	0.09	0.315	0.209	0.	2.141	0.	1.267

Problem 1668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	52	360	0	2458	0	486
normalized size	1	1.	0.28	1.91	0.	13.07	0.	2.59
time (sec)	N/A	0.106	0.02	0.258	0.	1.898	0.	1.265

Problem 1669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	52	305	0	2743	0	518
normalized size	1	1.	0.26	1.54	0.	13.85	0.	2.62
time (sec)	N/A	0.103	0.018	0.205	0.	1.919	0.	1.248

Problem 1670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	52	300	0	3065	0	556
normalized size	1	1.	0.25	1.44	0.	14.74	0.	2.67
time (sec)	N/A	0.123	0.018	0.232	0.	1.868	0.	1.248

Problem 1671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	52	337	0	3452	0	583
normalized size	1	1.	0.24	1.55	0.	15.83	0.	2.67
time (sec)	N/A	0.15	0.014	0.204	0.	1.853	0.	1.182

Problem 1672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	50	211	0	3794	0	613
normalized size	1	1.	0.23	0.99	0.	17.81	0.	2.88
time (sec)	N/A	0.116	0.013	0.198	0.	1.919	0.	1.196

Problem 1673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	50	641	0	4783	0	771
normalized size	1	1.	0.21	2.68	0.	20.01	0.	3.23
time (sec)	N/A	0.163	0.016	0.219	0.	2.337	0.	1.206

Problem 1674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	52	668	0	6792	0	860
normalized size	1	1.	0.2	2.51	0.	25.53	0.	3.23
time (sec)	N/A	0.243	0.021	0.217	0.	2.5	0.	1.321

Problem 1675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	52	693	0	9030	0	1193
normalized size	1	1.	0.18	2.35	0.	30.61	0.	4.04
time (sec)	N/A	0.27	0.026	0.218	0.	2.749	0.	1.212

Problem 1676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	48	43	126	205	0	327
normalized size	1	1.	0.5	0.45	1.31	2.14	0.	3.41
time (sec)	N/A	0.04	0.035	0.04	1.132	1.556	0.	1.182

Problem 1677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	48	43	93	155	0	180
normalized size	1	1.	0.5	0.45	0.97	1.61	0.	1.88
time (sec)	N/A	0.038	0.026	0.041	1.152	1.495	0.	1.186

Problem 1678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	48	43	62	108	49	73
normalized size	1	1.	0.5	0.45	0.65	1.12	0.51	0.76
time (sec)	N/A	0.037	0.023	0.04	1.108	1.437	13.521	1.12

Problem 1679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	47	42	62	63	0	70
normalized size	1	1.	0.5	0.45	0.66	0.67	0.	0.74
time (sec)	N/A	0.036	0.028	0.043	1.071	1.555	0.	1.149

Problem 1680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	45	42	34	74	0	72
normalized size	1	1.	0.49	0.46	0.37	0.8	0.	0.78
time (sec)	N/A	0.036	0.024	0.041	1.091	1.425	0.	1.155

Problem 1681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	47	42	47	103	0	65
normalized size	1	1.	0.5	0.45	0.5	1.1	0.	0.69
time (sec)	N/A	0.037	0.023	0.043	1.191	1.557	0.	1.136

Problem 1682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	48	43	63	128	0	66
normalized size	1	1.	0.5	0.45	0.66	1.33	0.	0.69
time (sec)	N/A	0.035	0.024	0.04	1.073	1.539	0.	1.13

Problem 1683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	120	132	362	595	0	910
normalized size	1	1.	0.58	0.63	1.74	2.86	0.	4.38
time (sec)	N/A	0.073	0.092	0.154	1.094	1.663	0.	1.226

Problem 1684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	120	132	292	474	0	528
normalized size	1	1.	0.58	0.63	1.4	2.28	0.	2.54
time (sec)	N/A	0.065	0.073	0.153	1.087	1.546	0.	1.158

Problem 1685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	120	132	221	359	0	228
normalized size	1	1.	0.58	0.63	1.06	1.73	0.	1.1
time (sec)	N/A	0.066	0.069	0.184	1.117	1.555	0.	1.205

Problem 1686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	119	132	221	251	0	225
normalized size	1	1.	0.58	0.65	1.08	1.23	0.	1.1
time (sec)	N/A	0.064	0.065	0.152	1.076	1.518	0.	1.226

Problem 1687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	119	132	154	259	0	284
normalized size	1	1.	0.59	0.65	0.76	1.28	0.	1.41
time (sec)	N/A	0.065	0.06	0.154	1.168	1.591	0.	1.196

Problem 1688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	119	131	169	281	0	273
normalized size	1	1.	0.58	0.64	0.83	1.38	0.	1.34
time (sec)	N/A	0.068	0.063	0.153	1.108	1.52	0.	1.198

Problem 1689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	118	131	185	298	0	265
normalized size	1	1.	0.58	0.65	0.92	1.48	0.	1.31
time (sec)	N/A	0.066	0.065	0.153	1.182	1.645	0.	1.216

Problem 1690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	119	132	198	329	0	262
normalized size	1	1.	0.58	0.65	0.97	1.61	0.	1.28
time (sec)	N/A	0.063	0.063	0.184	1.269	1.541	0.	1.226

Problem 1691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	120	132	215	366	0	262
normalized size	1	1.	0.58	0.63	1.03	1.76	0.	1.26
time (sec)	N/A	0.066	0.065	0.153	1.254	1.561	0.	1.178

Problem 1692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	141	289	671	1148	0	1702
normalized size	1	1.	0.44	0.9	2.1	3.59	0.	5.32
time (sec)	N/A	0.116	0.183	0.155	1.14	1.636	0.	1.28

Problem 1693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	141	289	564	953	0	1017
normalized size	1	1.	0.44	0.9	1.76	2.98	0.	3.18
time (sec)	N/A	0.096	0.149	0.17	1.101	1.593	0.	1.22

Problem 1694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	235	289	456	761	0	454
normalized size	1	1.	0.74	0.91	1.43	2.39	0.	1.43
time (sec)	N/A	0.094	0.124	0.155	1.123	1.633	0.	1.192

Problem 1695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	234	289	456	586	0	451
normalized size	1	1.	0.74	0.91	1.44	1.85	0.	1.43
time (sec)	N/A	0.097	0.124	0.155	1.095	1.608	0.	1.183

Problem 1696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	232	289	352	590	0	637
normalized size	1	1.	0.74	0.92	1.12	1.88	0.	2.03
time (sec)	N/A	0.095	0.134	0.157	1.066	1.637	0.	1.247

Problem 1697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	235	289	367	603	0	621
normalized size	1	1.	0.75	0.92	1.17	1.92	0.	1.98
time (sec)	N/A	0.097	0.116	0.153	1.139	1.51	0.	1.218

Problem 1698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	236	289	382	621	0	620
normalized size	1	1.	0.75	0.91	1.21	1.97	0.	1.96
time (sec)	N/A	0.097	0.116	0.154	1.095	1.575	0.	1.216

Problem 1699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	235	289	396	635	0	616
normalized size	1	1.	0.75	0.92	1.26	2.02	0.	1.96
time (sec)	N/A	0.094	0.115	0.157	1.163	1.621	0.	1.241

Problem 1700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	235	289	412	657	0	608
normalized size	1	1.	0.75	0.92	1.31	2.09	0.	1.94
time (sec)	N/A	0.096	0.12	0.154	1.138	1.66	0.	1.187

Problem 1701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	234	289	425	698	0	603
normalized size	1	1.	0.74	0.91	1.34	2.21	0.	1.91
time (sec)	N/A	0.096	0.117	0.155	1.131	1.573	0.	1.216

Problem 1702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	235	289	441	745	0	603
normalized size	1	1.	0.74	0.91	1.39	2.34	0.	1.9
time (sec)	N/A	0.098	0.121	0.155	1.206	1.633	0.	1.187

Problem 1703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	150	462	0	940	0	478
normalized size	1	1.	0.57	1.76	0.	3.57	0.	1.82
time (sec)	N/A	0.169	0.255	0.232	0.	1.71	0.	1.178

Problem 1704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	127	309	0	644	0	324
normalized size	1	1.	0.6	1.46	0.	3.04	0.	1.53
time (sec)	N/A	0.109	0.106	0.232	0.	1.717	0.	1.191

Problem 1705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	96	188	0	424	0	200
normalized size	1	1.	0.6	1.17	0.	2.63	0.	1.24
time (sec)	N/A	0.078	0.055	0.233	0.	1.688	0.	1.199

Problem 1706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	81	104	0	306	95	99
normalized size	1	1.	0.72	0.93	0.	2.73	0.85	0.88
time (sec)	N/A	0.053	0.021	0.23	0.	1.652	9.136	1.144

Problem 1707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	63	51	0	266	0	63
normalized size	1	1.	0.88	0.71	0.	3.69	0.	0.88
time (sec)	N/A	0.039	0.019	0.23	0.	1.703	0.	1.144

Problem 1708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	62	90	0	456	0	109
normalized size	1	1.	0.52	0.76	0.	3.83	0.	0.92
time (sec)	N/A	0.067	0.017	0.233	0.	1.656	0.	1.174

Problem 1709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	64	130	0	841	0	170
normalized size	1	1.	0.38	0.77	0.	5.01	0.	1.01
time (sec)	N/A	0.071	0.017	0.249	0.	1.599	0.	1.228

Problem 1710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	64	202	0	1434	0	265
normalized size	1	1.	0.29	0.92	0.	6.55	0.	1.21
time (sec)	N/A	0.091	0.02	0.266	0.	1.773	0.	1.145

Problem 1711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	67	1115	0	1535	0	602
normalized size	1	1.	0.22	3.62	0.	4.98	0.	1.95
time (sec)	N/A	0.174	0.046	0.282	0.	1.695	0.	1.288

Problem 1712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	67	714	0	1094	0	455
normalized size	1	1.	0.26	2.81	0.	4.31	0.	1.79
time (sec)	N/A	0.129	0.036	0.279	0.	1.75	0.	1.228

Problem 1713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	67	413	0	728	0	332
normalized size	1	1.	0.33	2.04	0.	3.6	0.	1.64
time (sec)	N/A	0.094	0.039	0.275	0.	1.649	0.	1.225

Problem 1714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	110	194	0	795	0	216
normalized size	1	1.	0.7	1.23	0.	5.03	0.	1.37
time (sec)	N/A	0.075	0.108	0.271	0.	1.669	0.	1.191

Problem 1715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	67	200	0	949	0	297
normalized size	1	1.	0.4	1.19	0.	5.65	0.	1.77
time (sec)	N/A	0.088	0.025	0.271	0.	1.706	0.	1.211

Problem 1716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	65	203	0	1119	0	383
normalized size	1	1.	0.38	1.18	0.	6.51	0.	2.23
time (sec)	N/A	0.087	0.02	0.27	0.	1.767	0.	1.177

Problem 1717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	65	285	0	1582	0	674
normalized size	1	1.	0.29	1.28	0.	7.09	0.	3.02
time (sec)	N/A	0.107	0.022	0.283	0.	1.636	0.	1.257

Problem 1718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	67	388	0	2472	0	844
normalized size	1	1.	0.24	1.41	0.	8.99	0.	3.07
time (sec)	N/A	0.125	0.024	0.279	0.	1.81	0.	1.279

Problem 1719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	67	518	0	3753	0	1046
normalized size	1	1.	0.2	1.57	0.	11.41	0.	3.18
time (sec)	N/A	0.161	0.027	0.307	0.	1.913	0.	1.292

Problem 1720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	400	400	67	2192	0	2846	0	945
normalized size	1	1.	0.17	5.48	0.	7.12	0.	2.36
time (sec)	N/A	0.238	0.055	0.284	0.	1.818	0.	1.382

Problem 1721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	67	1471	0	2079	0	740
normalized size	1	1.	0.19	4.25	0.	6.01	0.	2.14
time (sec)	N/A	0.188	0.049	0.288	0.	1.7	0.	1.337

Problem 1722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	67	892	0	1453	0	560
normalized size	1	1.	0.23	3.03	0.	4.94	0.	1.9
time (sec)	N/A	0.141	0.045	0.282	0.	1.675	0.	1.318

Problem 1723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	182	467	0	1602	0	387
normalized size	1	1.	0.73	1.87	0.	6.41	0.	1.55
time (sec)	N/A	0.118	0.223	0.277	0.	1.71	0.	1.201

Problem 1724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	67	477	0	1859	0	477
normalized size	1	1.	0.26	1.83	0.	7.15	0.	1.83
time (sec)	N/A	0.139	0.04	0.275	0.	1.738	0.	1.235

Problem 1725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	67	477	0	2101	0	568
normalized size	1	1.	0.25	1.77	0.	7.78	0.	2.1
time (sec)	N/A	0.139	0.034	0.279	0.	1.854	0.	1.242

Problem 1726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	67	500	0	2404	0	660
normalized size	1	1.	0.24	1.79	0.	8.59	0.	2.36
time (sec)	N/A	0.151	0.028	0.276	0.	1.831	0.	1.289

Problem 1727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	65	497	0	2709	0	744
normalized size	1	1.	0.23	1.79	0.	9.74	0.	2.68
time (sec)	N/A	0.143	0.024	0.268	0.	1.735	0.	1.263

Problem 1728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	65	602	0	3549	0	1129
normalized size	1	1.	0.2	1.83	0.	10.79	0.	3.43
time (sec)	N/A	0.182	0.025	0.285	0.	2.01	0.	1.341

Problem 1729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	67	763	0	5126	0	1299
normalized size	1	1.	0.18	2.	0.	13.45	0.	3.41
time (sec)	N/A	0.237	0.028	0.286	0.	2.146	0.	1.325

Problem 1730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	435	435	67	951	0	7048	0	1504
normalized size	1	1.	0.15	2.19	0.	16.2	0.	3.46
time (sec)	N/A	0.256	0.035	0.291	0.	2.327	0.	1.423

Problem 1731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	175	2157	0	4810	0	5234
normalized size	1	1.	0.85	10.47	0.	23.35	0.	25.41
time (sec)	N/A	0.11	0.179	0.053	0.	1.897	0.	1.237

Problem 1732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	121	768	0	1890	8531	2064
normalized size	1	1.	0.85	5.41	0.	13.31	60.08	14.54
time (sec)	N/A	0.067	0.104	0.051	0.	1.768	9.751	1.18

Problem 1733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	67	159	0	478	1506	524
normalized size	1	1.	0.86	2.04	0.	6.13	19.31	6.72
time (sec)	N/A	0.033	0.071	0.047	0.	1.759	2.079	1.207

Problem 1734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	52	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.014	1.005	0.	0.	0.	0.

Problem 1735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	54	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.015	1.092	0.	0.	0.	0.

Problem 1736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	54	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.016	1.235	0.	0.	0.	0.

Problem 1737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	167	1361	1069	3060	0	4316
normalized size	1	1.	0.5	4.04	3.17	9.08	0.	12.81
time (sec)	N/A	0.142	0.264	0.16	1.243	1.843	0.	1.387

Problem 1738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	113	402	412	1011	0	1451
normalized size	1	1.	0.52	1.84	1.88	4.62	0.	6.63
time (sec)	N/A	0.088	0.118	0.155	1.181	1.672	0.	1.245

Problem 1739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	59	62	84	171	0	248
normalized size	1	1.	0.58	0.61	0.83	1.69	0.	2.46
time (sec)	N/A	0.045	0.044	0.152	1.078	1.639	0.	1.162

Problem 1740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	67	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.024	1.032	0.	0.	0.	0.

Problem 1741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	72	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.031	1.041	0.	0.	0.	0.

Problem 1742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	72	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	0.034	1.085	0.	0.	0.	0.

Problem 1743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	75	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.035	1.316	0.	0.	0.	0.

Problem 1744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	107	405	373	1053	0	1710
normalized size	1	1.	0.59	2.24	2.06	5.82	0.	9.45
time (sec)	N/A	0.092	0.086	0.049	1.122	1.726	0.	1.237

Problem 1745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	75	175	212	509	0	821
normalized size	1	1.	0.59	1.38	1.67	4.01	0.	6.46
time (sec)	N/A	0.061	0.078	0.046	1.127	1.653	0.	1.146

Problem 1746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	54	65	105	204	0	308
normalized size	1	1.	0.71	0.86	1.38	2.68	0.	4.05
time (sec)	N/A	0.024	0.034	0.041	1.08	1.682	0.	1.156

Problem 1747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	23	35	34	69	0	69
normalized size	1	1.	0.68	1.03	1.	2.03	0.	2.03
time (sec)	N/A	0.006	0.011	0.041	1.111	1.606	0.	1.163

Problem 1748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	62	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	0.017	1.215	0.	0.	0.	0.

Problem 1749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	63	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	0.016	1.188	0.	0.	0.	0.

Problem 1750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	65	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	0.015	1.209	0.	0.	0.	0.

Problem 1751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	73	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	0.03	1.185	0.	0.	0.	0.

Problem 1752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	73	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.022	1.162	0.	0.	0.	0.

Problem 1753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	71	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.017	1.143	0.	0.	0.	0.

Problem 1754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	71	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.018	1.161	0.	0.	0.	0.

Problem 1755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	73	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.021	1.18	0.	0.	0.	0.

Problem 1756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	87	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	0.058	1.21	0.	0.	0.	0.

Problem 1757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	72	139	0	451	0	0
normalized size	1	1.	0.63	1.21	0.	3.92	0.	0.
time (sec)	N/A	0.048	0.04	0.044	0.	1.925	0.	0.

Problem 1758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	48	52	88	154	0	205
normalized size	1	1.	0.8	0.87	1.47	2.57	0.	3.42
time (sec)	N/A	0.015	0.025	0.042	1.07	1.825	0.	1.162

Problem 1759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	84	133	130	217	100	131
normalized size	1	1.	2.21	3.5	3.42	5.71	2.63	3.45
time (sec)	N/A	0.018	0.018	0.039	1.099	1.537	0.14	1.163

Problem 1760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	67	94	93	163	73	97
normalized size	1	1.	1.76	2.47	2.45	4.29	1.92	2.55
time (sec)	N/A	0.016	0.01	0.041	1.16	1.662	0.085	1.172

Problem 1761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	46	55	65	115	49	66
normalized size	1	1.	1.21	1.45	1.71	3.03	1.29	1.74
time (sec)	N/A	0.028	0.007	0.04	1.063	1.588	0.072	1.201

Problem 1762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	32	25	32	66	26	32
normalized size	1	1.	1.14	0.89	1.14	2.36	0.93	1.14
time (sec)	N/A	0.008	0.	0.039	1.043	1.602	0.123	1.246

Problem 1763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	23	8	14
normalized size	1	1.	1.	0.92	1.17	1.92	0.67	1.17
time (sec)	N/A	0.007	0.001	0.038	1.151	1.774	0.111	1.217

Problem 1764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	32	34	54	20	158
normalized size	1	1.	1.	1.28	1.36	2.16	0.8	6.32
time (sec)	N/A	0.021	0.007	0.041	1.075	1.717	0.578	1.168

Problem 1765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	31	39	47	80	27	45
normalized size	1	1.	0.97	1.22	1.47	2.5	0.84	1.41
time (sec)	N/A	0.025	0.01	0.045	1.024	1.824	0.511	1.24

Problem 1766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	35	51	81	39	32
normalized size	1	1.	0.93	1.25	1.82	2.89	1.39	1.14
time (sec)	N/A	0.011	0.009	0.045	1.062	1.671	0.525	1.175

Problem 1767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	27	35	68	105	53	55
normalized size	1	1.	0.71	0.92	1.79	2.76	1.39	1.45
time (sec)	N/A	0.024	0.009	0.044	1.089	1.788	0.574	1.149

Problem 1768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	27	35	82	128	65	34
normalized size	1	1.	0.71	0.92	2.16	3.37	1.71	0.89
time (sec)	N/A	0.026	0.009	0.044	1.046	1.779	0.738	1.175

Problem 1769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	189	315	266	460	218	286
normalized size	1	1.	2.91	4.85	4.09	7.08	3.35	4.4
time (sec)	N/A	0.13	0.031	0.039	1.053	1.57	0.219	1.157

Problem 1770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	148	231	211	363	168	230
normalized size	1	1.	2.28	3.55	3.25	5.58	2.58	3.54
time (sec)	N/A	0.09	0.02	0.039	1.045	1.688	0.127	1.191

Problem 1771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	122	147	167	285	133	176
normalized size	1	1.	1.88	2.26	2.57	4.38	2.05	2.71
time (sec)	N/A	0.066	0.014	0.038	1.105	1.582	0.103	1.206

Problem 1772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	79	69	97	198	87	120
normalized size	1	1.	1.22	1.06	1.49	3.05	1.34	1.85
time (sec)	N/A	0.069	0.01	0.039	1.045	1.466	0.086	1.215

Problem 1773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	47	55	65	109	49	66
normalized size	1	1.	1.24	1.45	1.71	2.87	1.29	1.74
time (sec)	N/A	0.035	0.009	0.038	1.055	1.795	0.128	1.209

Problem 1774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	27	42	19	113
normalized size	1	1.	1.	0.93	1.93	3.	1.36	8.07
time (sec)	N/A	0.01	0.001	0.038	1.042	1.667	0.19	1.222

Problem 1775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	43	74	82	135	44	81
normalized size	1	1.	0.88	1.51	1.67	2.76	0.9	1.65
time (sec)	N/A	0.028	0.015	0.04	1.094	1.472	0.832	1.202

Problem 1776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	47	86	90	184	60	88
normalized size	1	1.	0.92	1.69	1.76	3.61	1.18	1.73
time (sec)	N/A	0.044	0.036	0.045	1.037	1.525	0.766	1.221

Problem 1777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	49	92	107	207	80	149
normalized size	1	1.	0.83	1.56	1.81	3.51	1.36	2.53
time (sec)	N/A	0.043	0.024	0.045	0.995	1.608	0.853	1.194

Problem 1778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	53	70	113	170	88	80
normalized size	1	1.	1.89	2.5	4.04	6.07	3.14	2.86
time (sec)	N/A	0.011	0.022	0.043	1.184	1.58	1.265	1.18

Problem 1779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	56	71	132	201	104	82
normalized size	1	1.	0.86	1.09	2.03	3.09	1.6	1.26
time (sec)	N/A	0.044	0.019	0.044	1.174	1.623	1.376	1.248

Problem 1780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	57	71	147	227	116	82
normalized size	1	1.	0.88	1.09	2.26	3.49	1.78	1.26
time (sec)	N/A	0.044	0.024	0.044	1.029	1.429	1.632	1.209

Problem 1781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	58	71	162	251	128	82
normalized size	1	1.	0.89	1.09	2.49	3.86	1.97	1.26
time (sec)	N/A	0.041	0.02	0.044	1.035	1.532	1.943	1.237

Problem 1782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	57	71	177	277	139	82
normalized size	1	1.	0.88	1.09	2.72	4.26	2.14	1.26
time (sec)	N/A	0.041	0.023	0.049	1.054	1.491	2.812	1.201

Problem 1783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	276	811	441	776	364	489
normalized size	1	1.	3.	8.82	4.79	8.43	3.96	5.32
time (sec)	N/A	0.22	0.082	0.04	1.048	1.286	0.246	1.219

Problem 1784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	235	601	374	651	308	409
normalized size	1	1.	2.55	6.53	4.07	7.08	3.35	4.45
time (sec)	N/A	0.157	0.059	0.04	1.036	1.389	0.191	1.23

Problem 1785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	217	391	304	520	243	331
normalized size	1	1.	2.36	4.25	3.3	5.65	2.64	3.6
time (sec)	N/A	0.121	0.025	0.04	1.046	1.393	0.28	1.186

Problem 1786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	161	194	189	409	190	254
normalized size	1	1.	1.75	2.11	2.05	4.45	2.07	2.76
time (sec)	N/A	0.117	0.018	0.041	1.082	1.393	0.22	1.196

Problem 1787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	122	147	167	266	133	176
normalized size	1	1.	1.88	2.26	2.57	4.09	2.05	2.71
time (sec)	N/A	0.072	0.013	0.04	1.028	1.56	0.256	1.215

Problem 1788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	67	94	93	150	73	209
normalized size	1	1.	1.76	2.47	2.45	3.95	1.92	5.5
time (sec)	N/A	0.023	0.008	0.041	1.011	1.587	0.241	1.161

Problem 1789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	42	66	32	42
normalized size	1	1.	1.	0.93	3.	4.71	2.29	3.
time (sec)	N/A	0.01	0.002	0.04	1.035	1.475	0.145	1.224

Problem 1790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	74	133	154	238	82	155
normalized size	1	1.	1.01	1.82	2.11	3.26	1.12	2.12
time (sec)	N/A	0.037	0.029	0.042	1.033	1.528	0.799	1.268

Problem 1791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	72	149	159	354	100	225
normalized size	1	1.	0.96	1.99	2.12	4.72	1.33	3.
time (sec)	N/A	0.067	0.049	0.046	1.032	1.518	1.149	1.235

Problem 1792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	114	160	169	375	128	151
normalized size	1	1.	1.46	2.05	2.17	4.81	1.64	1.94
time (sec)	N/A	0.061	0.04	0.048	1.047	1.643	1.578	1.201

Problem 1793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	80	166	192	360	148	159
normalized size	1	1.	0.93	1.93	2.23	4.19	1.72	1.85
time (sec)	N/A	0.06	0.041	0.045	1.141	1.556	2.222	1.208

Problem 1794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	91	122	193	284	153	150
normalized size	1	1.	3.25	4.36	6.89	10.14	5.46	5.36
time (sec)	N/A	0.012	0.029	0.045	1.235	1.461	2.903	1.163

Problem 1795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	97	121	216	328	170	154
normalized size	1	1.	1.67	2.09	3.72	5.66	2.93	2.66
time (sec)	N/A	0.019	0.034	0.044	1.009	1.555	5.887	1.238

Problem 1796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	97	122	231	354	182	154
normalized size	1	1.	1.05	1.33	2.51	3.85	1.98	1.67
time (sec)	N/A	0.061	0.03	0.044	1.13	1.525	8.094	1.221

Problem 1797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	97	122	246	382	194	154
normalized size	1	1.	1.05	1.33	2.67	4.15	2.11	1.67
time (sec)	N/A	0.058	0.031	0.045	1.11	1.527	13.703	1.263

Problem 1798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	97	122	261	406	206	154
normalized size	1	1.	1.05	1.33	2.84	4.41	2.24	1.67
time (sec)	N/A	0.055	0.035	0.044	1.09	1.627	25.574	1.294

Problem 1799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	167	302	348	537	202	369
normalized size	1	1.	1.37	2.48	2.85	4.4	1.66	3.02
time (sec)	N/A	0.063	0.064	0.043	1.049	1.619	1.051	1.254

Problem 1800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	115	209	239	369	134	248
normalized size	1	1.	1.17	2.13	2.44	3.77	1.37	2.53
time (sec)	N/A	0.046	0.04	0.041	1.09	1.622	0.892	1.265

Problem 1801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	133	154	238	82	157
normalized size	1	1.	1.	1.8	2.08	3.22	1.11	2.12
time (sec)	N/A	0.036	0.026	0.04	1.063	1.575	0.536	1.16

Problem 1802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	43	74	81	135	44	81
normalized size	1	1.	0.86	1.48	1.62	2.7	0.88	1.62
time (sec)	N/A	0.025	0.016	0.041	1.042	1.566	0.603	1.19

Problem 1803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	25	32	35	54	20	36
normalized size	1	1.	0.96	1.23	1.35	2.08	0.77	1.38
time (sec)	N/A	0.022	0.007	0.04	1.066	1.501	0.507	1.191

Problem 1804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	22	7	15
normalized size	1	1.	1.	1.1	1.4	2.2	0.7	1.5
time (sec)	N/A	0.005	0.001	0.039	1.044	1.409	0.14	1.187

Problem 1805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	26	37	49	58	128	0
normalized size	1	1.	0.72	1.03	1.36	1.61	3.56	0.
time (sec)	N/A	0.012	0.011	0.043	1.007	1.567	0.411	0.

Problem 1806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	53	57	124	200	233	127
normalized size	1	1.	0.93	1.	2.18	3.51	4.09	2.23
time (sec)	N/A	0.036	0.024	0.05	1.046	1.68	1.173	1.214

Problem 1807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	67	81	273	491	381	196
normalized size	1	1.	0.82	0.99	3.33	5.99	4.65	2.39
time (sec)	N/A	0.056	0.063	0.051	1.044	1.659	1.921	1.2

Problem 1808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	107	103	487	856	570	328
normalized size	1	1.	1.	0.96	4.55	8.	5.33	3.07
time (sec)	N/A	0.071	0.042	0.051	1.248	1.601	2.873	1.215

Problem 1809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	130	125	753	1320	802	456
normalized size	1	1.	1.	0.96	5.79	10.15	6.17	3.51
time (sec)	N/A	0.092	0.051	0.052	1.279	1.736	4.337	1.196

Problem 1810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	165	230	247	540	151	254
normalized size	1	1.	1.59	2.21	2.38	5.19	1.45	2.44
time (sec)	N/A	0.105	0.06	0.048	1.028	1.531	1.276	1.207

Problem 1811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	114	149	158	354	100	159
normalized size	1	1.	1.52	1.99	2.11	4.72	1.33	2.12
time (sec)	N/A	0.066	0.034	0.046	1.053	1.523	1.133	1.175

Problem 1812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	47	86	90	184	60	88
normalized size	1	1.	0.92	1.69	1.76	3.61	1.18	1.73
time (sec)	N/A	0.043	0.035	0.044	1.128	1.471	0.744	1.261

Problem 1813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	39	46	78	27	43
normalized size	1	1.	1.	1.26	1.48	2.52	0.87	1.39
time (sec)	N/A	0.025	0.01	0.044	1.049	1.585	0.448	1.198

Problem 1814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	18	24	10	16
normalized size	1	1.	1.	1.08	1.5	2.	0.83	1.33
time (sec)	N/A	0.008	0.003	0.037	1.021	1.511	0.326	1.175

Problem 1815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	53	58	122	198	233	126
normalized size	1	1.	0.95	1.04	2.18	3.54	4.16	2.25
time (sec)	N/A	0.032	0.024	0.055	1.14	1.656	1.199	1.183

Problem 1816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	66	82	281	486	405	0
normalized size	1	1.	0.77	0.95	3.27	5.65	4.71	0.
time (sec)	N/A	0.026	0.066	0.051	1.105	1.619	1.932	0.

Problem 1817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	98	109	521	991	632	342
normalized size	1	1.	0.9	1.	4.78	9.09	5.8	3.14
time (sec)	N/A	0.078	0.072	0.052	1.104	1.613	3.873	1.218

Problem 1818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	230	346	366	840	253	356
normalized size	1	1.	1.73	2.6	2.75	6.32	1.9	2.68
time (sec)	N/A	0.136	0.08	0.051	1.111	1.596	3.393	1.259

Problem 1819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	167	245	258	586	184	247
normalized size	1	1.	1.62	2.38	2.5	5.69	1.79	2.4
time (sec)	N/A	0.088	0.054	0.048	1.086	1.499	1.802	1.264

Problem 1820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	114	160	169	375	128	151
normalized size	1	1.	1.46	2.05	2.17	4.81	1.64	1.94
time (sec)	N/A	0.061	0.039	0.046	1.041	1.628	1.588	1.132

Problem 1821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	48	92	108	205	80	93
normalized size	1	1.	0.81	1.56	1.83	3.47	1.36	1.58
time (sec)	N/A	0.043	0.024	0.046	1.157	1.601	1.156	1.236

Problem 1822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	35	51	81	39	32
normalized size	1	1.	0.93	1.25	1.82	2.89	1.39	1.14
time (sec)	N/A	0.009	0.009	0.043	1.084	1.62	0.567	1.173

Problem 1823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	32	49	26	16
normalized size	1	1.	1.	0.93	2.29	3.5	1.86	1.14
time (sec)	N/A	0.008	0.003	0.039	1.112	1.592	0.45	1.276

Problem 1824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	67	81	273	490	381	223
normalized size	1	1.	0.82	0.99	3.33	5.98	4.65	2.72
time (sec)	N/A	0.051	0.047	0.05	1.173	1.626	2.38	1.172

Problem 1825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	97	108	521	991	632	343
normalized size	1	1.	0.88	0.98	4.74	9.01	5.75	3.12
time (sec)	N/A	0.076	0.095	0.053	1.305	1.653	3.534	1.228

Problem 1826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	128	140	802	1488	881	0
normalized size	1	1.	0.9	0.98	5.61	10.41	6.16	0.
time (sec)	N/A	0.049	0.109	0.053	1.153	1.755	3.792	0.

Problem 1827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	154	165	1200	2313	1217	618
normalized size	1	1.	0.91	0.97	7.06	13.61	7.16	3.64
time (sec)	N/A	0.16	0.212	0.054	1.352	1.728	7.185	1.208

Problem 1828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	117	198	163	286	136	162
normalized size	1	1.	3.	5.08	4.18	7.33	3.49	4.15
time (sec)	N/A	0.023	0.031	0.041	1.085	1.345	0.107	1.268

Problem 1829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	95	155	138	228	107	132
normalized size	1	1.	2.44	3.97	3.54	5.85	2.74	3.38
time (sec)	N/A	0.018	0.019	0.04	1.017	1.275	0.093	1.234

Problem 1830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	73	112	101	174	80	101
normalized size	1	1.	1.87	2.87	2.59	4.46	2.05	2.59
time (sec)	N/A	0.017	0.02	0.04	1.161	1.327	0.115	1.25

Problem 1831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	51	69	73	126	56	73
normalized size	1	1.	1.31	1.77	1.87	3.23	1.44	1.87
time (sec)	N/A	0.034	0.012	0.039	1.025	1.38	0.164	1.164

Problem 1832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	38	31	41	77	32	42
normalized size	1	1.	1.12	0.91	1.21	2.26	0.94	1.24
time (sec)	N/A	0.009	0.	0.039	1.098	1.285	0.189	1.153

Problem 1833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	28	12	26
normalized size	1	1.	1.	0.93	1.14	2.	0.86	1.86
time (sec)	N/A	0.008	0.001	0.038	1.032	1.596	0.128	1.151

Problem 1834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	30	32	42	62	26	158
normalized size	1	1.	1.15	1.23	1.62	2.38	1.	6.08
time (sec)	N/A	0.028	0.009	0.04	1.019	1.56	0.393	1.242

Problem 1835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	36	39	53	89	32	70
normalized size	1	1.	1.09	1.18	1.61	2.7	0.97	2.12
time (sec)	N/A	0.028	0.013	0.046	1.125	1.526	0.538	1.208

Problem 1836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	29	40	58	89	44	62
normalized size	1	1.	0.83	1.14	1.66	2.54	1.26	1.77
time (sec)	N/A	0.014	0.011	0.043	1.056	1.591	0.831	1.172

Problem 1837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	30	40	74	113	58	57
normalized size	1	1.	0.77	1.03	1.9	2.9	1.49	1.46
time (sec)	N/A	0.028	0.012	0.044	1.125	1.561	0.856	1.274

Problem 1838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	30	40	89	136	70	65
normalized size	1	1.	0.77	1.03	2.28	3.49	1.79	1.67
time (sec)	N/A	0.028	0.012	0.043	1.174	1.449	0.842	1.225

Problem 1839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	160	295	232	390	185	238
normalized size	1	1.	2.08	3.83	3.01	5.06	2.4	3.09
time (sec)	N/A	0.132	0.032	0.041	1.134	1.386	0.136	1.2

Problem 1840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	120	195	189	312	150	188
normalized size	1	1.	1.56	2.53	2.45	4.05	1.95	2.44
time (sec)	N/A	0.079	0.032	0.039	1.637	1.424	0.107	1.159

Problem 1841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	87	93	126	225	104	136
normalized size	1	1.	1.13	1.21	1.64	2.92	1.35	1.77
time (sec)	N/A	0.088	0.024	0.044	1.083	1.377	0.12	1.184

Problem 1842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	77	86	136	66	97
normalized size	1	1.	1.	1.43	1.59	2.52	1.22	1.8
time (sec)	N/A	0.049	0.013	0.038	1.106	1.529	0.168	1.164

Problem 1843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	38	58	29	116
normalized size	1	1.	1.	0.95	1.9	2.9	1.45	5.8
time (sec)	N/A	0.013	0.002	0.039	0.989	1.512	0.149	1.194

Problem 1844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	52	77	97	151	56	96
normalized size	1	1.	0.84	1.24	1.56	2.44	0.9	1.55
time (sec)	N/A	0.039	0.022	0.042	1.114	1.582	0.539	1.252

Problem 1845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	59	92	107	208	71	181
normalized size	1	1.	0.94	1.46	1.7	3.3	1.13	2.87
time (sec)	N/A	0.052	0.041	0.046	1.013	1.507	0.647	1.232

Problem 1846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	59	98	122	224	90	159
normalized size	1	1.	0.83	1.38	1.72	3.15	1.27	2.24
time (sec)	N/A	0.052	0.031	0.046	1.073	1.658	0.967	1.226

Problem 1847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	59	83	127	186	99	185
normalized size	1	1.	1.69	2.37	3.63	5.31	2.83	5.29
time (sec)	N/A	0.014	0.028	0.043	1.072	1.667	1.54	1.225

Problem 1848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	61	83	146	217	114	189
normalized size	1	1.	0.79	1.08	1.9	2.82	1.48	2.45
time (sec)	N/A	0.05	0.023	0.043	1.046	1.511	1.472	1.239

Problem 1849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	61	83	161	243	126	189
normalized size	1	1.	0.79	1.08	2.09	3.16	1.64	2.45
time (sec)	N/A	0.049	0.03	0.044	1.058	1.63	2.086	1.224

Problem 1850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	61	83	176	267	138	189
normalized size	1	1.	0.79	1.08	2.29	3.47	1.79	2.45
time (sec)	N/A	0.049	0.025	0.045	1.056	1.589	3.187	1.205

Problem 1851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	255	801	409	694	335	419
normalized size	1	1.	2.3	7.22	3.68	6.25	3.02	3.77
time (sec)	N/A	0.233	0.073	0.04	0.999	1.372	0.197	1.147

Problem 1852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	211	531	339	563	270	346
normalized size	1	1.	1.9	4.78	3.05	5.07	2.43	3.12
time (sec)	N/A	0.16	0.063	0.04	1.017	1.301	0.18	1.214

Problem 1853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	167	266	247	452	218	274
normalized size	1	1.	1.5	2.4	2.23	4.07	1.96	2.47
time (sec)	N/A	0.149	0.049	0.038	1.102	1.38	0.145	1.163

Problem 1854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	123	205	203	309	160	213
normalized size	1	1.	1.35	2.25	2.23	3.4	1.76	2.34
time (sec)	N/A	0.099	0.039	0.038	1.079	1.527	0.153	1.243

Problem 1855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	79	136	128	193	100	197
normalized size	1	1.	1.46	2.52	2.37	3.57	1.85	3.65
time (sec)	N/A	0.031	0.026	0.04	1.085	1.378	0.144	1.2

Problem 1856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	61	93	49	69
normalized size	1	1.	1.	0.95	3.05	4.65	2.45	3.45
time (sec)	N/A	0.013	0.003	0.04	1.085	1.522	0.138	1.239

Problem 1857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	85	138	177	262	100	173
normalized size	1	1.	0.96	1.55	1.99	2.94	1.12	1.94
time (sec)	N/A	0.049	0.036	0.043	1.126	1.674	0.651	1.345

Problem 1858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	129	156	184	386	119	240
normalized size	1	1.	1.37	1.66	1.96	4.11	1.27	2.55
time (sec)	N/A	0.081	0.041	0.046	1.067	1.628	1.098	1.244

Problem 1859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	129	167	192	408	144	352
normalized size	1	1.	1.33	1.72	1.98	4.21	1.48	3.63
time (sec)	N/A	0.074	0.045	0.048	1.092	1.529	1.928	1.207

Problem 1860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	92	173	213	386	163	365
normalized size	1	1.	0.88	1.65	2.03	3.68	1.55	3.48
time (sec)	N/A	0.075	0.047	0.044	1.078	1.607	3.261	1.225

Problem 1861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	100	141	213	308	168	373
normalized size	1	1.	2.86	4.03	6.09	8.8	4.8	10.66
time (sec)	N/A	0.015	0.04	0.045	1.139	1.522	4.725	1.19

Problem 1862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	103	141	236	352	185	378
normalized size	1	1.	1.41	1.93	3.23	4.82	2.53	5.18
time (sec)	N/A	0.026	0.041	0.046	1.13	1.506	18.493	1.215

Problem 1863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	103	141	251	378	197	378
normalized size	1	1.	0.93	1.27	2.26	3.41	1.77	3.41
time (sec)	N/A	0.071	0.035	0.045	1.154	1.539	24.911	1.201

Problem 1864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	103	141	266	406	209	378
normalized size	1	1.	0.93	1.27	2.4	3.66	1.88	3.41
time (sec)	N/A	0.068	0.034	0.047	1.069	1.522	100.358	1.207

Problem 1865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	134	239	277	414	162	486
normalized size	1	1.	1.02	1.82	2.11	3.16	1.24	3.71
time (sec)	N/A	0.067	0.053	0.043	1.03	1.624	0.672	1.279

Problem 1866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	91	157	182	275	104	374
normalized size	1	1.	0.91	1.57	1.82	2.75	1.04	3.74
time (sec)	N/A	0.045	0.034	0.042	1.042	1.498	0.607	1.213

Problem 1867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	58	93	104	165	61	282
normalized size	1	1.	0.84	1.35	1.51	2.39	0.88	4.09
time (sec)	N/A	0.033	0.021	0.041	1.053	1.58	0.439	1.272

Problem 1868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	35	45	51	76	32	215
normalized size	1	1.	0.92	1.18	1.34	2.	0.84	5.66
time (sec)	N/A	0.032	0.009	0.041	1.006	1.558	0.376	1.26

Problem 1869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	22	32	12	170
normalized size	1	1.	1.	1.06	1.38	2.	0.75	10.62
time (sec)	N/A	0.006	0.002	0.039	1.024	1.63	0.084	1.223

Problem 1870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	33	48	63	69	172	101
normalized size	1	1.	0.7	1.02	1.34	1.47	3.66	2.15
time (sec)	N/A	0.015	0.014	0.045	1.065	1.547	0.361	1.282

Problem 1871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	66	75	144	225	301	0
normalized size	1	1.	0.9	1.03	1.97	3.08	4.12	0.
time (sec)	N/A	0.051	0.029	0.076	0.999	1.917	1.004	0.

Problem 1872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	102	107	308	528	471	0
normalized size	1	1.	0.94	0.99	2.85	4.89	4.36	0.
time (sec)	N/A	0.072	0.048	0.05	1.143	1.947	1.526	0.

Problem 1873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	135	137	531	906	672	0
normalized size	1	1.	0.97	0.99	3.82	6.52	4.83	0.
time (sec)	N/A	0.099	0.068	0.053	1.155	1.869	2.402	0.

Problem 1874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	339	502	537	1112	348	1075
normalized size	1	1.	1.55	2.29	2.45	5.08	1.59	4.91
time (sec)	N/A	0.275	0.124	0.052	1.022	1.801	2.33	1.548

Problem 1875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	263	378	405	841	265	909
normalized size	1	1.	1.43	2.05	2.2	4.57	1.44	4.94
time (sec)	N/A	0.197	0.086	0.049	1.105	1.966	1.586	1.462

Problem 1876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	196	275	289	602	187	757
normalized size	1	1.	1.35	1.9	1.99	4.15	1.29	5.22
time (sec)	N/A	0.152	0.076	0.048	1.032	1.941	1.311	1.47

Problem 1877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	142	184	193	408	131	629
normalized size	1	1.	1.35	1.75	1.84	3.89	1.25	5.99
time (sec)	N/A	0.093	0.048	0.047	1.033	1.821	1.086	1.372

Problem 1878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	65	114	120	227	85	525
normalized size	1	1.	0.88	1.54	1.62	3.07	1.15	7.09
time (sec)	N/A	0.059	0.046	0.053	1.02	1.92	0.689	1.299

Problem 1879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	47	55	70	109	46	435
normalized size	1	1.	0.98	1.15	1.46	2.27	0.96	9.06
time (sec)	N/A	0.036	0.013	0.045	1.021	1.889	0.445	1.209

Problem 1880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	24	35	17	147
normalized size	1	1.	1.	1.06	1.33	1.94	0.94	8.17
time (sec)	N/A	0.01	0.004	0.039	1.016	1.738	0.372	1.193

Problem 1881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	74	75	153	238	287	262
normalized size	1	1.	0.99	1.	2.04	3.17	3.83	3.49
time (sec)	N/A	0.045	0.03	0.074	1.025	1.904	1.056	1.179

Problem 1882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	86	107	319	545	484	238
normalized size	1	1.	0.75	0.94	2.8	4.78	4.25	2.09
time (sec)	N/A	0.033	0.095	0.052	1.065	1.928	1.532	1.316

Problem 1883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	130	144	571	1077	734	0
normalized size	1	1.	0.89	0.99	3.91	7.38	5.03	0.
time (sec)	N/A	0.11	0.095	0.054	1.121	2.009	2.544	0.

Problem 1884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	160	174	865	1615	994	0
normalized size	1	1.	0.91	0.99	4.91	9.18	5.65	0.
time (sec)	N/A	0.153	0.141	0.056	1.142	2.061	3.979	0.

Problem 1885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	337	544	551	1220	386	1442
normalized size	1	1.	1.52	2.46	2.49	5.52	1.75	6.52
time (sec)	N/A	0.261	0.129	0.051	1.075	1.849	16.166	18.497

Problem 1886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	262	412	419	930	299	1260
normalized size	1	1.	1.42	2.23	2.26	5.03	1.62	6.81
time (sec)	N/A	0.184	0.09	0.051	1.064	1.899	4.577	15.072

Problem 1887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	191	302	302	664	224	1084
normalized size	1	1.	1.35	2.13	2.13	4.68	1.58	7.63
time (sec)	N/A	0.135	0.07	0.05	1.071	1.89	2.652	14.795

Problem 1888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	139	201	211	443	163	944
normalized size	1	1.	1.25	1.81	1.9	3.99	1.47	8.5
time (sec)	N/A	0.092	0.053	0.048	1.097	1.857	1.493	1.346

Problem 1889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	65	123	142	255	109	811
normalized size	1	1.	0.76	1.45	1.67	3.	1.28	9.54
time (sec)	N/A	0.061	0.034	0.046	1.07	1.989	0.853	1.332

Problem 1890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	51	76	113	60	509
normalized size	1	1.	1.	1.46	2.17	3.23	1.71	14.54
time (sec)	N/A	0.013	0.013	0.042	1.081	1.837	0.618	3.101

Problem 1891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	47	70	39	350
normalized size	1	1.	1.	0.95	2.35	3.5	1.95	17.5
time (sec)	N/A	0.01	0.004	0.039	1.068	1.948	0.451	1.371

Problem 1892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	83	106	321	556	457	535
normalized size	1	1.	0.78	0.99	3.	5.2	4.27	5.
time (sec)	N/A	0.071	0.077	0.05	1.098	2.	1.611	1.249

Problem 1893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	127	142	579	1099	734	483
normalized size	1	1.	0.89	1.	4.08	7.74	5.17	3.4
time (sec)	N/A	0.113	0.103	0.055	1.227	2.037	2.629	1.256

Problem 1894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	168	186	867	1628	1001	440
normalized size	1	1.	0.88	0.97	4.54	8.52	5.24	2.3
time (sec)	N/A	0.065	0.151	0.055	1.163	2.091	3.741	1.228

Problem 1895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	201	218	1278	2485	1353	0
normalized size	1	1.	0.9	0.98	5.73	11.14	6.07	0.
time (sec)	N/A	0.224	0.195	0.053	1.274	2.418	6.329	0.

Problem 1896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	335	578	572	1292	422	0
normalized size	1	1.	1.54	2.66	2.64	5.95	1.94	0.
time (sec)	N/A	0.271	0.142	0.054	1.09	1.952	134.566	0.

Problem 1897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	259	436	440	976	335	0
normalized size	1	1.	1.45	2.44	2.46	5.45	1.87	0.
time (sec)	N/A	0.174	0.094	0.049	1.095	1.939	42.751	0.

Problem 1898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	194	318	328	675	257	0
normalized size	1	1.	1.33	2.18	2.25	4.62	1.76	0.
time (sec)	N/A	0.133	0.074	0.048	1.144	1.853	11.77	0.

Problem 1899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	99	210	242	433	189	0
normalized size	1	1.	0.81	1.72	1.98	3.55	1.55	0.
time (sec)	N/A	0.088	0.052	0.045	1.175	1.809	3.172	0.

Problem 1900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	65	96	153	221	121	1110
normalized size	1	1.	1.86	2.74	4.37	6.31	3.46	31.71
time (sec)	N/A	0.013	0.032	0.044	1.095	1.815	1.315	90.613

Problem 1901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	37	51	100	149	80	865
normalized size	1	1.	0.69	0.94	1.85	2.76	1.48	16.02
time (sec)	N/A	0.039	0.017	0.043	1.074	1.983	0.851	1.525

Problem 1902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	70	103	58	639
normalized size	1	1.	1.	0.95	3.5	5.15	2.9	31.95
time (sec)	N/A	0.011	0.004	0.038	1.118	1.787	0.672	3.415

Problem 1903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	117	135	556	961	668	973
normalized size	1	1.	0.84	0.97	4.	6.91	4.81	7.
time (sec)	N/A	0.098	0.101	0.052	1.078	1.998	2.689	1.345

Problem 1904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	157	173	886	1669	1005	907
normalized size	1	1.	0.91	1.	5.12	9.65	5.81	5.24
time (sec)	N/A	0.153	0.149	0.056	1.24	2.246	4.495	1.282

Problem 1905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	206	221	1291	2518	1363	791
normalized size	1	1.	0.91	0.98	5.71	11.14	6.03	3.5
time (sec)	N/A	0.208	0.194	0.056	1.289	2.374	7.053	1.235

Problem 1906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	234	253	1725	3263	1742	718
normalized size	1	1.	0.89	0.97	6.58	12.45	6.65	2.74
time (sec)	N/A	0.104	0.261	0.055	1.232	2.273	11.443	1.218

Problem 1907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	388	388	320	1327	0	2280	0	655
normalized size	1	1.	0.82	3.42	0.	5.88	0.	1.69
time (sec)	N/A	0.441	2.461	0.102	0.	2.107	0.	1.265

Problem 1908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	328	291	968	0	1813	0	521
normalized size	1	1.	0.89	2.95	0.	5.53	0.	1.59
time (sec)	N/A	0.276	1.916	0.053	0.	2.091	0.	1.265

Problem 1909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	316	730	0	1424	0	402
normalized size	1	1.	1.18	2.72	0.	5.31	0.	1.5
time (sec)	N/A	0.189	0.95	0.05	0.	1.845	0.	1.241

Problem 1910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	262	465	0	1123	0	304
normalized size	1	1.	1.25	2.21	0.	5.35	0.	1.45
time (sec)	N/A	0.104	0.701	0.05	0.	1.729	0.	1.263

Problem 1911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	214	265	0	888	0	219
normalized size	1	1.	1.35	1.67	0.	5.58	0.	1.38
time (sec)	N/A	0.05	0.488	0.044	0.	1.794	0.	1.352

Problem 1912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	155	205	0	725	0	0
normalized size	1	1.	1.18	1.56	0.	5.53	0.	0.
time (sec)	N/A	0.067	0.787	0.045	0.	1.703	0.	0.

Problem 1913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	164	317	0	697	0	0
normalized size	1	1.	1.32	2.56	0.	5.62	0.	0.
time (sec)	N/A	0.054	0.692	0.048	0.	2.187	0.	0.

Problem 1914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	43	58	0	182	0	0
normalized size	1	1.	0.8	1.07	0.	3.37	0.	0.
time (sec)	N/A	0.02	0.021	0.044	0.	2.57	0.	0.

Problem 1915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	61	90	0	413	0	0
normalized size	1	1.	0.55	0.81	0.	3.72	0.	0.
time (sec)	N/A	0.047	0.031	0.043	0.	5.942	0.	0.

Problem 1916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	124	146	0	740	0	0
normalized size	1	1.	0.73	0.85	0.	4.33	0.	0.
time (sec)	N/A	0.081	0.044	0.045	0.	17.096	0.	0.

Problem 1917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	180	217	0	1161	0	0
normalized size	1	1.	0.78	0.94	0.	5.03	0.	0.
time (sec)	N/A	0.115	0.064	0.047	0.	68.509	0.	0.

Problem 1918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	461	461	1276	2065	0	3494	0	992
normalized size	1	1.	2.77	4.48	0.	7.58	0.	2.15
time (sec)	N/A	0.549	6.409	0.061	0.	2.699	0.	1.372

Problem 1919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	401	401	1196	1586	0	2804	0	824
normalized size	1	1.	2.98	3.96	0.	6.99	0.	2.05
time (sec)	N/A	0.373	6.254	0.057	0.	2.454	0.	1.312

Problem 1920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	328	1302	0	2271	0	671
normalized size	1	1.	0.96	3.82	0.	6.66	0.	1.97
time (sec)	N/A	0.264	3.224	0.053	0.	2.182	0.	1.26

Problem 1921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	299	917	0	1790	0	536
normalized size	1	1.	1.06	3.24	0.	6.33	0.	1.89
time (sec)	N/A	0.162	2.231	0.051	0.	2.023	0.	1.316

Problem 1922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	317	671	0	1399	0	414
normalized size	1	1.	1.37	2.89	0.	6.03	0.	1.78
time (sec)	N/A	0.089	0.955	0.043	0.	1.909	0.	1.336

Problem 1923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	264	566	0	1125	0	0
normalized size	1	1.	1.31	2.82	0.	5.6	0.	0.
time (sec)	N/A	0.111	0.681	0.047	0.	1.767	0.	0.

Problem 1924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	209	757	0	895	0	0
normalized size	1	1.	1.12	4.05	0.	4.79	0.	0.
time (sec)	N/A	0.126	0.38	0.049	0.	1.704	0.	0.

Problem 1925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	108	837	0	873	0	0
normalized size	1	1.	0.62	4.78	0.	4.99	0.	0.
time (sec)	N/A	0.104	0.076	0.052	0.	2.535	0.	0.

Problem 1926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	184	914	0	907	0	0
normalized size	1	1.	1.09	5.41	0.	5.37	0.	0.
time (sec)	N/A	0.092	0.707	0.049	0.	4.39	0.	0.

Problem 1927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	43	58	0	258	0	0
normalized size	1	1.	0.8	1.07	0.	4.78	0.	0.
time (sec)	N/A	0.021	0.032	0.043	0.	7.514	0.	0.

Problem 1928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	61	90	0	548	0	0
normalized size	1	1.	0.55	0.81	0.	4.94	0.	0.
time (sec)	N/A	0.048	0.048	0.045	0.	22.5	0.	0.

Problem 1929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	94	146	0	937	0	0
normalized size	1	1.	0.55	0.85	0.	5.48	0.	0.
time (sec)	N/A	0.078	0.07	0.047	0.	89.46	0.	0.

Problem 1930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	138	217	0	0	0	0
normalized size	1	1.	0.6	0.94	0.	0.	0.	0.
time (sec)	N/A	0.116	0.091	0.049	0.	0.	0.	0.

Problem 1931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	534	534	1439	2973	0	5203	0	1397
normalized size	1	1.	2.69	5.57	0.	9.74	0.	2.62
time (sec)	N/A	0.676	6.812	0.065	0.	4.029	0.	2.013

Problem 1932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	474	474	1359	2368	0	4213	0	1193
normalized size	1	1.	2.87	5.	0.	8.89	0.	2.52
time (sec)	N/A	0.493	6.51	0.063	0.	3.309	0.	1.745

Problem 1933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	414	414	1279	2044	0	3398	0	1006
normalized size	1	1.	3.09	4.94	0.	8.21	0.	2.43
time (sec)	N/A	0.354	6.318	0.054	0.	3.043	0.	1.849

Problem 1934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	1199	1533	0	2782	0	836
normalized size	1	1.	3.37	4.31	0.	7.81	0.	2.35
time (sec)	N/A	0.239	6.2	0.05	0.	2.695	0.	1.755

Problem 1935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	1119	1247	0	2199	0	678
normalized size	1	1.	3.67	4.09	0.	7.21	0.	2.22
time (sec)	N/A	0.148	6.118	0.045	0.	2.191	0.	1.292

Problem 1936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	384	1123	0	1789	0	0
normalized size	1	1.	1.4	4.1	0.	6.53	0.	0.
time (sec)	N/A	0.179	1.246	0.049	0.	1.914	0.	0.

Problem 1937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	316	1455	0	1424	0	0
normalized size	1	1.	1.21	5.57	0.	5.46	0.	0.
time (sec)	N/A	0.22	0.899	0.049	0.	1.913	0.	0.

Problem 1938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	252	1531	0	1130	0	0
normalized size	1	1.	1.03	6.27	0.	4.63	0.	0.
time (sec)	N/A	0.202	0.525	0.051	0.	1.824	0.	0.

Problem 1939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	108	1617	0	1160	0	0
normalized size	1	1.	0.46	6.88	0.	4.94	0.	0.
time (sec)	N/A	0.212	0.078	0.05	0.	3.839	0.	0.

Problem 1940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	112	1695	0	1231	0	0
normalized size	1	1.	0.5	7.5	0.	5.45	0.	0.
time (sec)	N/A	0.148	0.074	0.051	0.	5.646	0.	0.

Problem 1941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	217	1764	0	1210	0	0
normalized size	1	1.	1.	8.09	0.	5.55	0.	0.
time (sec)	N/A	0.133	1.118	0.052	0.	13.557	0.	0.

Problem 1942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	43	58	0	333	0	0
normalized size	1	1.	0.8	1.07	0.	6.17	0.	0.
time (sec)	N/A	0.021	0.046	0.046	0.	26.535	0.	0.

Problem 1943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	71	90	0	686	0	0
normalized size	1	1.	0.64	0.81	0.	6.18	0.	0.
time (sec)	N/A	0.049	0.041	0.045	0.	96.091	0.	0.

Problem 1944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	104	146	0	0	0	0
normalized size	1	1.	0.61	0.85	0.	0.	0.	0.
time (sec)	N/A	0.08	0.061	0.046	0.	0.	0.	0.

Problem 1945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	148	217	0	0	0	0
normalized size	1	1.	0.64	0.94	0.	0.	0.	0.
time (sec)	N/A	0.121	0.08	0.05	0.	0.	0.	0.

Problem 1946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	214	513	0	1137	0	316
normalized size	1	1.	0.84	2.01	0.	4.46	0.	1.24
time (sec)	N/A	0.197	0.586	0.056	0.	2.542	0.	1.397

Problem 1947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	182	318	0	900	0	238
normalized size	1	1.	0.93	1.63	0.	4.62	0.	1.22
time (sec)	N/A	0.115	0.547	0.051	0.	2.143	0.	1.371

Problem 1948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	203	171	0	730	0	181
normalized size	1	1.	1.51	1.28	0.	5.45	0.	1.35
time (sec)	N/A	0.056	0.204	0.049	0.	1.966	0.	1.309

Problem 1949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	148	62	0	521	0	116
normalized size	1	1.	1.8	0.76	0.	6.35	0.	1.41
time (sec)	N/A	0.025	0.074	0.046	0.	1.911	0.	1.32

Problem 1950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	42	51	0	117	0	0
normalized size	1	1.	0.81	0.98	0.	2.25	0.	0.
time (sec)	N/A	0.02	0.017	0.045	0.	2.199	0.	0.

Problem 1951	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	61	89	0	285	0	0
normalized size	1	1.	0.55	0.8	0.	2.57	0.	0.
time (sec)	N/A	0.047	0.041	0.044	0.	3.739	0.	0.

Problem 1952	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	94	146	0	551	0	0
normalized size	1	1.	0.55	0.85	0.	3.22	0.	0.
time (sec)	N/A	0.078	0.048	0.046	0.	9.183	0.	0.

Problem 1953	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	138	217	0	910	0	0
normalized size	1	1.	0.6	0.94	0.	3.94	0.	0.
time (sec)	N/A	0.115	0.069	0.049	0.	34.008	0.	0.

Problem 1954	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	302	100	1850	0	1574	0	840
normalized size	1	1.	0.33	6.13	0.	5.21	0.	2.78
time (sec)	N/A	0.244	0.071	0.065	0.	10.076	0.	1.343

Problem 1955	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	100	1428	0	1216	0	648
normalized size	1	1.	0.41	5.93	0.	5.05	0.	2.69
time (sec)	N/A	0.162	0.058	0.059	0.	5.277	0.	1.314

Problem 1956	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	93	1047	0	932	0	478
normalized size	1	1.	0.52	5.82	0.	5.18	0.	2.66
time (sec)	N/A	0.099	0.051	0.056	0.	2.823	0.	1.287

Problem 1957	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	164	717	0	737	0	309
normalized size	1	1.	1.31	5.74	0.	5.9	0.	2.47
time (sec)	N/A	0.058	0.357	0.051	0.	2.691	0.	1.296

Problem 1958	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	39	58	0	130	0	147
normalized size	1	1.	0.78	1.16	0.	2.6	0.	2.94
time (sec)	N/A	0.025	0.015	0.044	0.	3.14	0.	1.225

Problem 1959	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	49	86	0	305	0	134
normalized size	1	1.	0.79	1.39	0.	4.92	0.	2.16
time (sec)	N/A	0.009	0.021	0.043	0.	6.557	0.	1.249

Problem 1960	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	95	138	0	599	0	0
normalized size	1	1.	0.79	1.14	0.	4.95	0.	0.
time (sec)	N/A	0.039	0.053	0.046	0.	14.695	0.	0.

Problem 1961	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	136	216	0	991	0	0
normalized size	1	1.	0.75	1.19	0.	5.48	0.	0.
time (sec)	N/A	0.071	0.063	0.048	0.	41.281	0.	0.

Problem 1962	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	193	307	0	1509	0	0
normalized size	1	1.	0.8	1.27	0.	6.26	0.	0.
time (sec)	N/A	0.111	0.089	0.051	0.	115.071	0.	0.

Problem 1963	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	258	412	0	0	0	0
normalized size	1	1.	0.86	1.37	0.	0.	0.	0.
time (sec)	N/A	0.159	0.121	0.051	0.	0.	0.	0.

Problem 1964	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	112	4008	0	1717	0	1391
normalized size	1	1.	0.38	13.63	0.	5.84	0.	4.73
time (sec)	N/A	0.212	0.069	0.076	0.	27.412	0.	1.591

Problem 1965	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	112	3215	0	1311	0	1116
normalized size	1	1.	0.48	13.92	0.	5.68	0.	4.83
time (sec)	N/A	0.14	0.065	0.08	0.	12.285	0.	1.438

Problem 1966	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	98	2538	0	980	0	837
normalized size	1	1.	0.57	14.76	0.	5.7	0.	4.87
time (sec)	N/A	0.089	0.072	0.061	0.	6.767	0.	1.499

Problem 1967	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	46	58	0	205	0	552
normalized size	1	1.	0.85	1.07	0.	3.8	0.	10.22
time (sec)	N/A	0.021	0.02	0.045	0.	4.693	0.	1.315

Problem 1968	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	59	90	0	312	0	521
normalized size	1	1.	0.51	0.78	0.	2.69	0.	4.49
time (sec)	N/A	0.043	0.033	0.045	0.	6.32	0.	1.297

Problem 1969	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	90	146	0	614	0	497
normalized size	1	1.	0.76	1.24	0.	5.2	0.	4.21
time (sec)	N/A	0.05	0.057	0.045	0.	16.324	0.	1.256

Problem 1970	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	132	213	0	972	0	466
normalized size	1	1.	1.	1.61	0.	7.36	0.	3.53
time (sec)	N/A	0.024	0.066	0.045	0.	58.02	0.	1.306

Problem 1971	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	193	300	0	1566	0	0
normalized size	1	1.	1.01	1.56	0.	8.16	0.	0.
time (sec)	N/A	0.062	0.096	0.057	0.	172.752	0.	0.

Problem 1972	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	259	412	0	0	0	0
normalized size	1	1.	1.03	1.63	0.	0.	0.	0.
time (sec)	N/A	0.102	0.125	0.051	0.	0.	0.	0.

Problem 1973	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	336	536	0	0	0	0
normalized size	1	1.	1.08	1.72	0.	0.	0.	0.
time (sec)	N/A	0.152	0.154	0.055	0.	0.	0.	0.

Problem 1974	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1485	1485	88	0	0	0	0	0
normalized size	1	1.	0.06	0.	0.	0.	0.	0.
time (sec)	N/A	2.236	0.059	0.992	0.	0.	0.	0.

Problem 1975	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1432	1432	95	0	0	0	0	0
normalized size	1	1.	0.07	0.	0.	0.	0.	0.
time (sec)	N/A	1.309	0.043	2.056	0.	0.	0.	0.

Problem 1976	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	34	32	51	213	235	290
normalized size	1	1.	0.79	0.74	1.19	4.95	5.47	6.74
time (sec)	N/A	0.02	0.034	0.042	0.985	2.339	12.753	1.139

Problem 1977	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	34	32	51	163	41	157
normalized size	1	1.	0.79	0.74	1.19	3.79	0.95	3.65
time (sec)	N/A	0.019	0.025	0.041	0.977	2.278	2.841	1.138

Problem 1978	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	34	32	122	116	221	151
normalized size	1	1.	0.79	0.74	2.84	2.7	5.14	3.51
time (sec)	N/A	0.019	0.022	0.041	0.998	2.141	22.596	1.12

Problem 1979	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	33	31	50	72	124	63
normalized size	1	1.	0.8	0.76	1.22	1.76	3.02	1.54
time (sec)	N/A	0.025	0.021	0.042	0.989	2.138	9.618	1.162

Problem 1980	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	31	31	57	82	58	68
normalized size	1	1.	0.79	0.79	1.46	2.1	1.49	1.74
time (sec)	N/A	0.023	0.019	0.041	1.009	2.082	1.568	1.165

Problem 1981	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	33	31	45	111	126	63
normalized size	1	1.	0.8	0.76	1.1	2.71	3.07	1.54
time (sec)	N/A	0.022	0.021	0.041	1.033	2.143	3.624	1.15

Problem 1982	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	34	32	46	136	187	65
normalized size	1	1.	0.79	0.74	1.07	3.16	4.35	1.51
time (sec)	N/A	0.023	0.022	0.042	1.014	1.971	8.882	1.137

Problem 1983	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	34	32	46	158	248	65
normalized size	1	1.	0.79	0.74	1.07	3.67	5.77	1.51
time (sec)	N/A	0.022	0.023	0.042	0.997	1.846	17.58	1.155

Problem 1984	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	67	73	108	398	97	529
normalized size	1	1.	0.81	0.88	1.3	4.8	1.17	6.37
time (sec)	N/A	0.058	0.056	0.046	0.984	1.923	3.83	1.166

Problem 1985	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	67	73	378	316	631	524
normalized size	1	1.	0.81	0.88	4.55	3.81	7.6	6.31
time (sec)	N/A	0.039	0.043	0.046	1.009	1.866	85.495	1.157

Problem 1986	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	67	73	108	236	411	143
normalized size	1	1.	0.81	0.88	1.3	2.84	4.95	1.72
time (sec)	N/A	0.039	0.04	0.043	1.034	1.923	50.845	1.203

Problem 1987	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	66	73	108	162	236	143
normalized size	1	1.	0.81	0.9	1.33	2.	2.91	1.77
time (sec)	N/A	0.039	0.039	0.045	0.988	1.858	40.099	1.18

Problem 1988	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	65	73	117	173	133	155
normalized size	1	1.	0.82	0.92	1.48	2.19	1.68	1.96
time (sec)	N/A	0.041	0.035	0.044	1.028	1.833	5.879	1.13

Problem 1989	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	68	72	113	190	264	150
normalized size	1	1.	0.86	0.91	1.43	2.41	3.34	1.9
time (sec)	N/A	0.039	0.039	0.044	1.026	1.823	11.515	1.163

Problem 1990	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	67	73	104	220	388	146
normalized size	1	1.	0.83	0.9	1.28	2.72	4.79	1.8
time (sec)	N/A	0.04	0.037	0.044	1.025	1.724	27.477	1.166

Problem 1991	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	67	73	104	248	510	146
normalized size	1	1.	0.81	0.88	1.25	2.99	6.14	1.76
time (sec)	N/A	0.039	0.038	0.045	1.022	1.896	65.397	1.244

Problem 1992	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	111	131	185	743	165	1258
normalized size	1	1.	0.93	1.1	1.55	6.24	1.39	10.57
time (sec)	N/A	0.082	0.092	0.046	1.037	1.917	7.856	1.27

Problem 1993	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	98	131	825	620	0	1254
normalized size	1	1.	0.82	1.1	6.93	5.21	0.	10.54
time (sec)	N/A	0.057	0.078	0.045	1.017	1.917	0.	1.236

Problem 1994	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	98	131	185	498	971	250
normalized size	1	1.	0.82	1.1	1.55	4.18	8.16	2.1
time (sec)	N/A	0.058	0.071	0.043	1.046	1.83	90.104	1.184

Problem 1995	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	98	131	185	383	644	250
normalized size	1	1.	0.82	1.1	1.55	3.22	5.41	2.1
time (sec)	N/A	0.052	0.067	0.048	1.074	1.875	102.341	1.204

Problem 1996	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	110	131	185	275	376	250
normalized size	1	1.	0.96	1.14	1.61	2.39	3.27	2.17
time (sec)	N/A	0.055	0.07	0.043	1.01	1.95	138.656	1.254

Problem 1997	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	109	131	194	284	230	263
normalized size	1	1.	0.96	1.16	1.72	2.51	2.04	2.33
time (sec)	N/A	0.052	0.057	0.046	1.051	1.867	16.461	1.215

Problem 1998	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	110	130	190	305	450	261
normalized size	1	1.	0.96	1.13	1.65	2.65	3.91	2.27
time (sec)	N/A	0.052	0.06	0.044	1.017	1.875	31.487	1.192

Problem 1999	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	109	130	189	323	654	252
normalized size	1	1.	0.96	1.15	1.67	2.86	5.79	2.23
time (sec)	N/A	0.053	0.056	0.044	1.049	1.933	47.232	1.236

Problem 2000	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	175	455	0	1081	0	0
normalized size	1	1.	0.97	2.53	0.	6.01	0.	0.
time (sec)	N/A	0.237	0.29	0.213	0.	1.962	0.	0.

Problem 2001	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	135	324	0	771	0	0
normalized size	1	1.	0.92	2.2	0.	5.24	0.	0.
time (sec)	N/A	0.092	0.112	0.194	0.	1.955	0.	0.

Problem 2002	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	102	211	0	533	0	0
normalized size	1	1.	0.89	1.85	0.	4.68	0.	0.
time (sec)	N/A	0.071	0.078	0.192	0.	1.939	0.	0.

Problem 2003	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	122	0	385	80	0
normalized size	1	1.	1.	1.47	0.	4.64	0.96	0.
time (sec)	N/A	0.055	0.034	0.192	0.	1.97	12.677	0.

Problem 2004	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	48	0	325	48	0
normalized size	1	1.	1.	0.74	0.	5.	0.74	0.
time (sec)	N/A	0.045	0.016	0.194	0.	1.835	2.712	0.

Problem 2005	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	55	88	0	529	82	0
normalized size	1	1.	0.6	0.97	0.	5.81	0.9	0.
time (sec)	N/A	0.083	0.015	0.195	0.	1.856	7.513	0.

Problem 2006	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	57	117	0	941	107	0
normalized size	1	1.	0.48	0.98	0.	7.84	0.89	0.
time (sec)	N/A	0.095	0.012	0.197	0.	2.02	9.909	0.

Problem 2007	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	57	146	0	1555	141	0
normalized size	1	1.	0.37	0.95	0.	10.16	0.92	0.
time (sec)	N/A	0.13	0.013	0.198	0.	2.054	40.852	0.

Problem 2008	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	57	175	0	2345	0	0
normalized size	1	1.	0.31	0.94	0.	12.61	0.	0.
time (sec)	N/A	0.164	0.014	0.199	0.	2.032	0.	0.

Problem 2009	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	59	628	0	1629	0	0
normalized size	1	1.	0.28	2.99	0.	7.76	0.	0.
time (sec)	N/A	0.176	0.025	0.203	0.	2.035	0.	0.

Problem 2010	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	59	457	0	1214	0	0
normalized size	1	1.	0.33	2.57	0.	6.82	0.	0.
time (sec)	N/A	0.135	0.019	0.205	0.	1.812	0.	0.

Problem 2011	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	59	314	0	855	0	0
normalized size	1	1.	0.41	2.18	0.	5.94	0.	0.
time (sec)	N/A	0.095	0.018	0.204	0.	1.971	0.	0.

Problem 2012	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	59	183	0	576	0	0
normalized size	1	1.	0.53	1.63	0.	5.14	0.	0.
time (sec)	N/A	0.07	0.015	0.203	0.	1.966	0.	0.

Problem 2013	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	93	84	0	625	0	0
normalized size	1	1.	0.99	0.89	0.	6.65	0.	0.
time (sec)	N/A	0.057	0.11	0.201	0.	1.997	0.	0.

Problem 2014	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	101	99	0	725	0	0
normalized size	1	1.	1.	0.98	0.	7.18	0.	0.
time (sec)	N/A	0.057	0.077	0.193	0.	1.843	0.	0.

Problem 2015	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	57	129	0	1010	0	0
normalized size	1	1.	0.45	1.01	0.	7.89	0.	0.
time (sec)	N/A	0.064	0.015	0.236	0.	2.064	0.	0.

Problem 2016	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	59	162	0	1751	0	0
normalized size	1	1.	0.37	1.03	0.	11.08	0.	0.
time (sec)	N/A	0.093	0.017	0.204	0.	2.115	0.	0.

Problem 2017	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	59	193	0	2678	0	0
normalized size	1	1.	0.31	1.01	0.	13.95	0.	0.
time (sec)	N/A	0.129	0.018	0.204	0.	2.133	0.	0.

Problem 2018	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	61	635	0	1748	0	0
normalized size	1	1.	0.27	2.86	0.	7.87	0.	0.
time (sec)	N/A	0.186	0.026	0.239	0.	2.03	0.	0.

Problem 2019	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	61	449	0	1289	0	0
normalized size	1	1.	0.33	2.41	0.	6.93	0.	0.
time (sec)	N/A	0.138	0.02	0.204	0.	2.031	0.	0.

Problem 2020	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	61	288	0	892	0	0
normalized size	1	1.	0.4	1.89	0.	5.87	0.	0.
time (sec)	N/A	0.1	0.018	0.202	0.	2.008	0.	0.

Problem 2021	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	118	149	0	976	0	0
normalized size	1	1.	0.91	1.15	0.	7.51	0.	0.
time (sec)	N/A	0.081	0.138	0.201	0.	2.069	0.	0.

Problem 2022	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	61	142	0	1129	0	0
normalized size	1	1.	0.42	0.99	0.	7.84	0.	0.
time (sec)	N/A	0.088	0.016	0.201	0.	1.976	0.	0.

Problem 2023	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	59	144	0	1295	0	0
normalized size	1	1.	0.4	0.99	0.	8.87	0.	0.
time (sec)	N/A	0.086	0.013	0.199	0.	1.97	0.	0.

Problem 2024	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	59	226	0	1767	0	0
normalized size	1	1.	0.34	1.28	0.	10.04	0.	0.
time (sec)	N/A	0.119	0.015	0.207	0.	2.136	0.	0.

Problem 2025	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	61	262	0	2707	0	0
normalized size	1	1.	0.29	1.26	0.	13.01	0.	0.
time (sec)	N/A	0.138	0.016	0.206	0.	2.15	0.	0.

Problem 2026	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	61	294	0	4073	0	0
normalized size	1	1.	0.25	1.2	0.	16.69	0.	0.
time (sec)	N/A	0.223	0.021	0.234	0.	2.386	0.	0.

Problem 2027	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	187	243	400	679	0	0
normalized size	1	1.	0.63	0.82	1.36	2.3	0.	0.
time (sec)	N/A	0.263	0.157	0.044	1.091	1.912	0.	0.

Problem 2028	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	131	168	285	483	0	0
normalized size	1	1.	0.56	0.72	1.22	2.07	0.	0.
time (sec)	N/A	0.189	0.106	0.044	1.121	1.863	0.	0.

Problem 2029	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	88	110	189	336	0	0
normalized size	1	1.	0.51	0.64	1.11	1.96	0.	0.
time (sec)	N/A	0.113	0.074	0.044	1.088	1.848	0.	0.

Problem 2030	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	55	69	112	216	0	0
normalized size	1	1.	0.5	0.63	1.03	1.98	0.	0.
time (sec)	N/A	0.06	0.048	0.042	1.081	1.803	0.	0.

Problem 2031	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	37	50	24	128	0	0
normalized size	1	1.	0.77	1.04	0.5	2.67	0.	0.
time (sec)	N/A	0.021	0.025	0.04	1.029	1.761	0.	0.

Problem 2032	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	105	163	0	659	0	0
normalized size	1	1.	0.82	1.27	0.	5.15	0.	0.
time (sec)	N/A	0.098	0.119	0.26	0.	1.949	0.	0.

Problem 2033	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	112	163	0	1004	0	0
normalized size	1	1.	0.87	1.26	0.	7.78	0.	0.
time (sec)	N/A	0.073	0.159	0.244	0.	1.873	0.	0.

Problem 2034	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	83	292	0	1528	0	0
normalized size	1	1.	0.42	1.47	0.	7.68	0.	0.
time (sec)	N/A	0.131	0.036	0.261	0.	2.135	0.	0.

Problem 2035	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	83	457	0	2246	0	0
normalized size	1	1.	0.31	1.73	0.	8.51	0.	0.
time (sec)	N/A	0.177	0.037	0.251	0.	2.042	0.	0.

Problem 2036	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	187	243	504	853	0	0
normalized size	1	1.	0.63	0.82	1.71	2.89	0.	0.
time (sec)	N/A	0.296	0.181	0.046	1.115	1.9	0.	0.

Problem 2037	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	132	168	369	614	0	0
normalized size	1	1.	0.57	0.72	1.58	2.64	0.	0.
time (sec)	N/A	0.208	0.125	0.046	1.086	1.951	0.	0.

Problem 2038	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	88	110	255	432	0	0
normalized size	1	1.	0.51	0.64	1.49	2.53	0.	0.
time (sec)	N/A	0.122	0.092	0.046	1.09	1.883	0.	0.

Problem 2039	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	55	69	132	279	0	0
normalized size	1	1.	0.5	0.63	1.21	2.56	0.	0.
time (sec)	N/A	0.061	0.063	0.043	1.068	1.824	0.	0.

Problem 2040	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	37	50	58	161	0	0
normalized size	1	1.	0.77	1.04	1.21	3.35	0.	0.
time (sec)	N/A	0.025	0.033	0.042	1.04	1.849	0.	0.

Problem 2041	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	137	275	0	841	0	0
normalized size	1	1.	0.76	1.52	0.	4.65	0.	0.
time (sec)	N/A	0.163	0.24	0.245	0.	1.82	0.	0.

Problem 2042	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	79	314	0	867	0	0
normalized size	1	1.	0.45	1.79	0.	4.95	0.	0.
time (sec)	N/A	0.119	0.051	0.247	0.	2.015	0.	0.

Problem 2043	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	176	281	0	1323	0	0
normalized size	1	1.	0.95	1.52	0.	7.15	0.	0.
time (sec)	N/A	0.118	0.178	0.247	0.	2.076	0.	0.

Problem 2044	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	83	457	0	1968	0	0
normalized size	1	1.	0.33	1.83	0.	7.87	0.	0.
time (sec)	N/A	0.184	0.053	0.26	0.	2.096	0.	0.

Problem 2045	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	83	662	0	2807	0	0
normalized size	1	1.	0.26	2.1	0.	8.91	0.	0.
time (sec)	N/A	0.223	0.053	0.257	0.	2.188	0.	0.

Problem 2046	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	197	243	605	1026	0	0
normalized size	1	1.	0.67	0.82	2.05	3.48	0.	0.
time (sec)	N/A	0.265	0.154	0.045	1.208	1.924	0.	0.

Problem 2047	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	142	168	452	752	0	0
normalized size	1	1.	0.61	0.72	1.94	3.23	0.	0.
time (sec)	N/A	0.179	0.122	0.047	1.139	1.887	0.	0.

Problem 2048	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	98	110	293	531	0	0
normalized size	1	1.	0.57	0.64	1.71	3.11	0.	0.
time (sec)	N/A	0.114	0.084	0.046	1.047	1.881	0.	0.

Problem 2049	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	65	69	178	346	0	0
normalized size	1	1.	0.6	0.63	1.63	3.17	0.	0.
time (sec)	N/A	0.057	0.056	0.053	1.098	1.823	0.	0.

Problem 2050	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	37	50	81	193	0	0
normalized size	1	1.	0.77	1.04	1.69	4.02	0.	0.
time (sec)	N/A	0.021	0.042	0.043	1.077	1.877	0.	0.

Problem 2051	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	152	437	0	1131	0	0
normalized size	1	1.	0.63	1.82	0.	4.71	0.	0.
time (sec)	N/A	0.214	0.329	0.245	0.	2.014	0.	0.

Problem 2052	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	79	521	0	1195	0	0
normalized size	1	1.	0.34	2.24	0.	5.13	0.	0.
time (sec)	N/A	0.182	0.07	0.281	0.	2.071	0.	0.

Problem 2053	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	83	527	0	1166	0	0
normalized size	1	1.	0.35	2.23	0.	4.94	0.	0.
time (sec)	N/A	0.154	0.073	0.286	0.	2.497	0.	0.

Problem 2054	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	220	443	0	1709	0	0
normalized size	1	1.	0.93	1.88	0.	7.24	0.	0.
time (sec)	N/A	0.152	0.259	0.247	0.	1.998	0.	0.

Problem 2055	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	83	662	0	2495	0	0
normalized size	1	1.	0.28	2.2	0.	8.29	0.	0.
time (sec)	N/A	0.234	0.073	0.292	0.	2.062	0.	0.

Problem 2056	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	366	366	83	910	0	3484	0	0
normalized size	1	1.	0.23	2.49	0.	9.52	0.	0.
time (sec)	N/A	0.28	0.075	0.278	0.	2.303	0.	0.

Problem 2057	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	131	168	259	362	0	0
normalized size	1	1.	0.56	0.72	1.11	1.55	0.	0.
time (sec)	N/A	0.178	0.106	0.045	1.033	1.817	0.	0.

Problem 2058	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	87	110	165	248	0	0
normalized size	1	1.	0.51	0.64	0.96	1.45	0.	0.
time (sec)	N/A	0.114	0.073	0.043	1.051	1.745	0.	0.

Problem 2059	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	54	69	88	158	0	0
normalized size	1	1.	0.5	0.63	0.81	1.45	0.	0.
time (sec)	N/A	0.056	0.041	0.041	1.096	1.819	0.	0.

Problem 2060	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	35	50	24	107	0	0
normalized size	1	1.	0.76	1.09	0.52	2.33	0.	0.
time (sec)	N/A	0.02	0.017	0.042	1.07	1.961	0.	0.

Problem 2061	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	97	91	0	500	0	0
normalized size	1	1.	1.15	1.08	0.	5.95	0.	0.
time (sec)	N/A	0.041	0.035	0.247	0.	2.292	0.	0.

Problem 2062	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	135	172	0	1160	0	0
normalized size	1	1.	0.97	1.24	0.	8.35	0.	0.
time (sec)	N/A	0.077	0.083	0.242	0.	2.32	0.	0.

Problem 2063	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	81	292	0	1758	0	0
normalized size	1	1.	0.39	1.41	0.	8.49	0.	0.
time (sec)	N/A	0.113	0.032	0.243	0.	2.28	0.	0.

Problem 2064	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	81	454	0	2535	0	0
normalized size	1	1.	0.3	1.69	0.	9.42	0.	0.
time (sec)	N/A	0.166	0.032	0.243	0.	2.449	0.	0.

Problem 2065	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	87	110	107	292	0	0
normalized size	1	1.	0.51	0.64	0.63	1.71	0.	0.
time (sec)	N/A	0.116	0.06	0.044	1.059	2.184	0.	0.

Problem 2066	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	51	68	49	201	0	0
normalized size	1	1.	0.49	0.65	0.47	1.91	0.	0.
time (sec)	N/A	0.058	0.034	0.041	1.124	2.046	0.	0.

Problem 2067	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	35	50	24	157	0	0
normalized size	1	1.	0.76	1.09	0.52	3.41	0.	0.
time (sec)	N/A	0.021	0.013	0.042	1.095	2.113	0.	0.

Problem 2068	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	75	136	0	1019	0	0
normalized size	1	1.	0.54	0.98	0.	7.33	0.	0.
time (sec)	N/A	0.094	0.021	0.207	0.	2.159	0.	0.

Problem 2069	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	77	235	0	1555	0	0
normalized size	1	1.	0.39	1.2	0.	7.93	0.	0.
time (sec)	N/A	0.132	0.024	0.214	0.	2.258	0.	0.

Problem 2070	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	81	384	0	2273	0	0
normalized size	1	1.	0.3	1.43	0.	8.45	0.	0.
time (sec)	N/A	0.177	0.027	0.248	0.	2.472	0.	0.

Problem 2071	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	81	559	0	3243	0	0
normalized size	1	1.	0.24	1.69	0.	9.8	0.	0.
time (sec)	N/A	0.246	0.026	0.251	0.	2.469	0.	0.

Problem 2072	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	393	81	767	0	4335	0	0
normalized size	1	1.	0.21	1.95	0.	11.03	0.	0.
time (sec)	N/A	0.32	0.03	0.224	0.	2.671	0.	0.

Problem 2073	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	53	68	68	270	0	0
normalized size	1	1.	0.5	0.64	0.64	2.52	0.	0.
time (sec)	N/A	0.065	0.046	0.043	1.111	2.078	0.	0.

Problem 2074	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	37	50	38	221	0	0
normalized size	1	1.	0.77	1.04	0.79	4.6	0.	0.
time (sec)	N/A	0.022	0.029	0.041	1.063	2.1	0.	0.

Problem 2075	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	77	234	0	1592	0	0
normalized size	1	1.	0.31	0.93	0.	6.34	0.	0.
time (sec)	N/A	0.174	0.025	0.208	0.	2.295	0.	0.

Problem 2076	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	79	433	0	2452	0	0
normalized size	1	1.	0.31	1.68	0.	9.54	0.	0.
time (sec)	N/A	0.209	0.031	0.248	0.	2.155	0.	0.

Problem 2077	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	83	668	0	3603	0	0
normalized size	1	1.	0.25	2.03	0.	10.95	0.	0.
time (sec)	N/A	0.281	0.032	0.22	0.	2.079	0.	0.

Problem 2078	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	395	83	930	0	4867	0	0
normalized size	1	1.	0.21	2.35	0.	12.32	0.	0.
time (sec)	N/A	0.349	0.035	0.22	0.	2.225	0.	0.

Problem 2079	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	457	457	83	1225	0	6433	0	0
normalized size	1	1.	0.18	2.68	0.	14.08	0.	0.
time (sec)	N/A	0.501	0.041	0.227	0.	2.453	0.	0.

Problem 2080	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	519	519	83	1553	0	8296	0	0
normalized size	1	1.	0.16	2.99	0.	15.98	0.	0.
time (sec)	N/A	0.598	0.038	0.259	0.	2.805	0.	0.

Problem 2081	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	58	0	333	0	0
normalized size	1	1.	1.	1.12	0.	6.4	0.	0.
time (sec)	N/A	0.024	0.052	0.164	0.	1.879	0.	0.

Problem 2082	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	63	0	332	0	0
normalized size	1	1.	1.	1.19	0.	6.26	0.	0.
time (sec)	N/A	0.022	0.063	0.166	0.	1.892	0.	0.

Problem 2083	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	566	566	95	0	0	0	0	0
normalized size	1	1.	0.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.765	0.061	0.326	0.	0.	0.	0.

Problem 2084	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	114	436	0	2418	7844	2696
normalized size	1	1.	0.88	3.35	0.	18.6	60.34	20.74
time (sec)	N/A	0.077	0.091	0.047	0.	2.035	13.793	1.25

Problem 2085	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	79	183	0	979	2839	1085
normalized size	1	1.	0.88	2.03	0.	10.88	31.54	12.06
time (sec)	N/A	0.05	0.083	0.046	0.	1.868	4.902	1.191

Problem 2086	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	45	55	0	277	556	296
normalized size	1	1.	0.87	1.06	0.	5.33	10.69	5.69
time (sec)	N/A	0.025	0.036	0.045	0.	1.934	1.355	1.127

Problem 2087	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.013	1.179	0.	0.	0.	0.

Problem 2088	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.023	1.269	0.	0.	0.	0.

Problem 2089	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	63	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.023	1.609	0.	0.	0.	0.

Problem 2090	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	63	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.028	1.294	0.	0.	0.	0.

Problem 2091	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	123	107	0	0	0	0	0
normalized size	1	1.23	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	0.054	1.319	0.	0.	0.	0.

Problem 2092	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	124	112	0	0	0	0	0
normalized size	1	1.31	1.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	0.053	1.162	0.	0.	0.	0.

Problem 2093	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	124	112	0	0	0	0	0
normalized size	1	1.31	1.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.043	1.089	0.	0.	0.	0.

Problem 2094	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	152	94	0	0	0	0	0
normalized size	1	1.6	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.052	0.972	0.	0.	0.	0.

Problem 2095	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	96	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.025	0.953	0.	0.	0.	0.

Problem 2096	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	81	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.027	1.174	0.	0.	0.	0.

Problem 2097	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	120	108	0	0	0	0	0
normalized size	1	1.3	1.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	0.038	1.165	0.	0.	0.	0.

Problem 2098	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	124	112	0	0	0	0	0
normalized size	1	1.29	1.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.042	1.201	0.	0.	0.	0.

Problem 2099	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	97	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	0.044	1.261	0.	0.	0.	0.

Problem 2100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	95	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	0.022	1.219	0.	0.	0.	0.

Problem 2101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	49	75	0	189	0	0
normalized size	1	1.	0.82	1.25	0.	3.15	0.	0.
time (sec)	N/A	0.016	0.029	0.043	0.	2.22	0.	0.

Problem 2102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	76	170	0	505	0	0
normalized size	1	1.	0.59	1.33	0.	3.95	0.	0.
time (sec)	N/A	0.052	0.055	0.043	0.	2.308	0.	0.

Problem 2103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	131	381	0	1145	0	0
normalized size	1	1.	0.64	1.85	0.	5.56	0.	0.
time (sec)	N/A	0.093	0.091	0.044	0.	2.404	0.	0.

Problem 2104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	217	745	0	2107	0	0
normalized size	1	1.	0.75	2.59	0.	7.32	0.	0.
time (sec)	N/A	0.164	0.149	0.048	0.	2.481	0.	0.

Problem 2105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	42	57	45	116	0	117
normalized size	1	1.	0.78	1.06	0.83	2.15	0.	2.17
time (sec)	N/A	0.018	0.026	0.04	1.01	2.156	0.	1.141

Problem 2106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	41	56	41	115	0	115
normalized size	1	1.	0.79	1.08	0.79	2.21	0.	2.21
time (sec)	N/A	0.016	0.021	0.043	1.029	2.132	0.	1.149

Problem 2107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	135	136	182	331	146	189
normalized size	1	1.	1.96	1.97	2.64	4.8	2.12	2.74
time (sec)	N/A	0.098	0.039	0.039	0.978	1.777	0.091	1.108

Problem 2108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	104	103	138	255	110	144
normalized size	1	1.	1.51	1.49	2.	3.7	1.59	2.09
time (sec)	N/A	0.069	0.025	0.04	0.975	1.757	0.079	1.125

Problem 2109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	73	70	93	180	73	101
normalized size	1	1.	1.06	1.01	1.35	2.61	1.06	1.46
time (sec)	N/A	0.054	0.02	0.041	0.999	1.775	0.073	1.101

Problem 2110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	49	104	39	58
normalized size	1	1.	1.	0.88	1.17	2.48	0.93	1.38
time (sec)	N/A	0.029	0.013	0.04	0.967	1.817	0.063	1.083

Problem 2111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	39	15	22
normalized size	1	1.	1.	0.85	1.1	1.95	0.75	1.1
time (sec)	N/A	0.004	0.	0.039	1.013	1.717	0.055	1.13

Problem 2112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	48	63	68	113	44	69
normalized size	1	1.	0.92	1.21	1.31	2.17	0.85	1.33
time (sec)	N/A	0.043	0.016	0.042	1.013	2.022	0.364	1.09

Problem 2113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	49	74	78	159	49	143
normalized size	1	1.	0.89	1.35	1.42	2.89	0.89	2.6
time (sec)	N/A	0.045	0.027	0.049	0.995	1.967	0.509	1.103

Problem 2114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	57	83	96	184	68	81
normalized size	1	1.	0.92	1.34	1.55	2.97	1.1	1.31
time (sec)	N/A	0.045	0.021	0.045	0.983	1.986	0.695	1.159

Problem 2115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	50	63	104	162	82	68
normalized size	1	1.	0.75	0.94	1.55	2.42	1.22	1.01
time (sec)	N/A	0.044	0.019	0.045	0.992	2.029	1.029	1.135

Problem 2116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	49	63	116	180	92	116
normalized size	1	1.	0.71	0.91	1.68	2.61	1.33	1.68
time (sec)	N/A	0.046	0.018	0.045	1.003	1.899	1.5	1.079

Problem 2117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	51	63	136	217	107	69
normalized size	1	1.	0.74	0.91	1.97	3.14	1.55	1.
time (sec)	N/A	0.044	0.02	0.044	1.002	2.016	2.106	1.115

Problem 2118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	283	283	374	756	337	443
normalized size	1	1.	1.81	1.81	2.4	4.85	2.16	2.84
time (sec)	N/A	0.25	0.08	0.038	0.975	1.728	0.11	1.105

Problem 2119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	223	219	294	587	260	342
normalized size	1	1.	1.43	1.4	1.88	3.76	1.67	2.19
time (sec)	N/A	0.174	0.065	0.039	0.977	1.802	0.099	1.099

Problem 2120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	153	155	197	412	173	240
normalized size	1	1.	0.98	0.99	1.26	2.64	1.11	1.54
time (sec)	N/A	0.14	0.043	0.039	0.956	1.727	0.089	1.087

Problem 2121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	96	91	122	244	100	142
normalized size	1	1.	1.	0.95	1.27	2.54	1.04	1.48
time (sec)	N/A	0.075	0.02	0.04	0.968	1.701	0.078	1.112

Problem 2122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	41	61	99	42	57
normalized size	1	1.	1.	0.89	1.33	2.15	0.91	1.24
time (sec)	N/A	0.023	0.006	0.039	0.979	1.702	0.066	1.096

Problem 2123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	128	221	227	369	144	243
normalized size	1	1.	0.99	1.71	1.76	2.86	1.12	1.88
time (sec)	N/A	0.155	0.063	0.042	1.004	2.011	0.61	1.114

Problem 2124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	127	246	236	548	167	336
normalized size	1	1.	0.97	1.88	1.8	4.18	1.27	2.56
time (sec)	N/A	0.141	0.111	0.047	1.019	2.109	1.185	1.117

Problem 2125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	176	266	250	602	209	238
normalized size	1	1.	1.28	1.93	1.81	4.36	1.51	1.72
time (sec)	N/A	0.136	0.074	0.048	1.021	2.108	2.884	1.135

Problem 2126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	176	279	262	578	218	230
normalized size	1	1.	1.27	2.01	1.88	4.16	1.57	1.65
time (sec)	N/A	0.123	0.083	0.046	0.993	1.943	7.455	1.105

Problem 2127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	170	287	290	548	238	410
normalized size	1	1.	1.13	1.91	1.93	3.65	1.59	2.73
time (sec)	N/A	0.122	0.073	0.048	1.	2.027	17.116	1.126

Problem 2128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	160	195	296	466	250	242
normalized size	1	1.	1.06	1.29	1.96	3.09	1.66	1.6
time (sec)	N/A	0.111	0.072	0.044	0.988	2.039	38.715	1.118

Problem 2129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	159	195	309	489	262	242
normalized size	1	1.	1.02	1.25	1.98	3.13	1.68	1.55
time (sec)	N/A	0.109	0.064	0.046	1.008	2.021	85.418	1.092

Problem 2130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	161	195	331	531	0	243
normalized size	1	1.	1.03	1.25	2.12	3.4	0.	1.56
time (sec)	N/A	0.108	0.068	0.046	1.064	2.013	0.	1.091

Problem 2131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	497	631	653	1377	620	815
normalized size	1	1.	1.83	2.32	2.4	5.06	2.28	3.
time (sec)	N/A	0.509	0.155	0.04	1.002	1.729	0.15	1.122

Problem 2132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	372	495	495	1075	484	633
normalized size	1	1.	1.37	1.82	1.82	3.95	1.78	2.33
time (sec)	N/A	0.368	0.136	0.039	1.009	1.748	0.141	1.133

Problem 2133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	282	359	371	743	332	446
normalized size	1	1.	1.04	1.32	1.36	2.73	1.22	1.64
time (sec)	N/A	0.271	0.095	0.041	0.978	1.808	0.117	1.139

Problem 2134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	161	223	220	444	190	266
normalized size	1	1.	1.	1.39	1.37	2.76	1.18	1.65
time (sec)	N/A	0.157	0.037	0.04	1.005	1.784	0.098	1.078

Problem 2135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	81	108	115	188	85	111
normalized size	1	1.	1.	1.33	1.42	2.32	1.05	1.37
time (sec)	N/A	0.056	0.013	0.039	1.019	1.697	0.078	1.075

Problem 2136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	308	546	541	829	376	621
normalized size	1	1.	1.18	2.1	2.08	3.19	1.45	2.39
time (sec)	N/A	0.323	0.16	0.044	0.968	2.023	1.12	1.121

Problem 2137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	255	585	554	1197	403	730
normalized size	1	1.	1.	2.29	2.16	4.68	1.57	2.85
time (sec)	N/A	0.321	0.104	0.05	0.989	2.111	2.643	1.127

Problem 2138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	265	624	563	1318	457	583
normalized size	1	1.	1.04	2.45	2.21	5.17	1.79	2.29
time (sec)	N/A	0.343	0.13	0.053	1.059	2.062	9.02	1.101

Problem 2139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	260	653	581	1393	498	572
normalized size	1	1.	1.04	2.6	2.31	5.55	1.98	2.28
time (sec)	N/A	0.296	0.115	0.054	1.082	2.041	34.234	1.113

Problem 2140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	402	678	595	1308	518	927
normalized size	1	1.	1.6	2.7	2.37	5.21	2.06	3.69
time (sec)	N/A	0.274	0.176	0.051	1.041	1.982	141.101	1.166

Problem 2141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	396	688	606	1247	0	560
normalized size	1	1.	1.55	2.69	2.37	4.87	0.	2.19
time (sec)	N/A	0.245	0.255	0.051	1.07	2.093	0.	1.132

Problem 2142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	385	695	633	1135	0	574
normalized size	1	1.	1.45	2.61	2.38	4.27	0.	2.16
time (sec)	N/A	0.226	0.166	0.051	1.156	1.951	0.	1.135

Problem 2143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	377	461	637	999	0	618
normalized size	1	1.	1.41	1.72	2.38	3.73	0.	2.31
time (sec)	N/A	0.207	0.143	0.049	1.188	2.143	0.	1.099

Problem 2144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	375	461	651	1008	0	618
normalized size	1	1.	1.39	1.71	2.42	3.75	0.	2.3
time (sec)	N/A	0.224	0.154	0.05	1.109	2.109	0.	1.102

Problem 2145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	378	461	674	1083	0	620
normalized size	1	1.	1.39	1.69	2.48	3.98	0.	2.28
time (sec)	N/A	0.209	0.152	0.052	1.093	1.98	0.	1.123

Problem 2146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	443	443	766	949	1029	2202	998	1311
normalized size	1	1.	1.73	2.14	2.32	4.97	2.25	2.96
time (sec)	N/A	0.871	0.26	0.041	1.116	1.73	0.181	1.158

Problem 2147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	443	443	611	747	829	1704	777	1018
normalized size	1	1.	1.38	1.69	1.87	3.85	1.75	2.3
time (sec)	N/A	0.674	0.204	0.041	0.999	1.738	0.152	1.103

Problem 2148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	441	441	428	545	589	1195	537	725
normalized size	1	1.	0.97	1.24	1.34	2.71	1.22	1.64
time (sec)	N/A	0.523	0.135	0.041	0.968	1.742	0.133	1.092

Problem 2149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	268	343	373	713	313	435
normalized size	1	1.	1.	1.28	1.39	2.66	1.17	1.62
time (sec)	N/A	0.291	0.069	0.038	1.013	1.751	0.114	1.092

Problem 2150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	133	168	184	308	141	186
normalized size	1	1.	1.	1.26	1.38	2.32	1.06	1.4
time (sec)	N/A	0.117	0.019	0.041	0.999	1.517	0.083	1.084

Problem 2151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	428	428	616	1096	1076	1662	796	1276
normalized size	1	1.	1.44	2.56	2.51	3.88	1.86	2.98
time (sec)	N/A	0.708	0.404	0.048	1.05	1.775	1.832	1.167

Problem 2152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	426	426	780	1159	1089	2288	824	1368
normalized size	1	1.	1.83	2.72	2.56	5.37	1.93	3.21
time (sec)	N/A	0.722	0.339	0.055	1.044	1.85	5.001	1.098

Problem 2153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	430	430	440	1216	1106	2573	892	1202
normalized size	1	1.	1.02	2.83	2.57	5.98	2.07	2.8
time (sec)	N/A	0.667	0.205	0.058	1.122	1.968	22.844	1.126

Problem 2154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	417	417	425	1265	1116	2708	933	1168
normalized size	1	1.	1.02	3.03	2.68	6.49	2.24	2.8
time (sec)	N/A	0.652	0.226	0.059	1.206	1.914	111.556	1.102

Problem 2155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	426	426	430	1306	1138	2774	0	1732
normalized size	1	1.	1.01	3.07	2.67	6.51	0.	4.07
time (sec)	N/A	0.635	0.206	0.059	1.154	1.852	0.	1.166

Problem 2156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	414	414	419	1341	1148	2645	0	1135
normalized size	1	1.	1.01	3.24	2.77	6.39	0.	2.74
time (sec)	N/A	0.557	0.234	0.06	1.138	1.779	0.	1.122

Problem 2157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	426	426	764	1364	1169	2498	0	1137
normalized size	1	1.	1.79	3.2	2.74	5.86	0.	2.67
time (sec)	N/A	0.507	0.35	0.056	1.145	1.763	0.	1.123

Problem 2158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	424	424	748	1374	1177	2275	0	1125
normalized size	1	1.	1.76	3.24	2.78	5.37	0.	2.65
time (sec)	N/A	0.482	0.549	0.054	1.123	1.807	0.	1.154

Problem 2159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	435	435	740	1382	1207	2138	0	1138
normalized size	1	1.	1.7	3.18	2.77	4.91	0.	2.62
time (sec)	N/A	0.462	0.352	0.053	1.139	1.789	0.	1.11

Problem 2160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	436	436	730	914	1206	1879	0	1274
normalized size	1	1.	1.67	2.1	2.77	4.31	0.	2.92
time (sec)	N/A	0.41	0.331	0.05	1.21	1.736	0.	1.126

Problem 2161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	443	443	731	914	1223	1926	0	1274
normalized size	1	1.	1.65	2.06	2.76	4.35	0.	2.88
time (sec)	N/A	0.401	0.296	0.048	1.189	1.839	0.	1.122

Problem 2162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	440	440	731	914	1247	1995	0	1276
normalized size	1	1.	1.66	2.08	2.83	4.53	0.	2.9
time (sec)	N/A	0.397	0.324	0.05	1.197	1.777	0.	1.112

Problem 2163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	35	59	26	35
normalized size	1	1.	1.	0.84	1.09	1.84	0.81	1.09
time (sec)	N/A	0.014	0.001	0.038	0.971	1.46	0.062	1.105

Problem 2164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	27	35	69	31	35
normalized size	1	1.	1.	0.75	0.97	1.92	0.86	0.97
time (sec)	N/A	0.013	0.001	0.039	0.993	1.488	0.059	1.123

Problem 2165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	35	62	27	35
normalized size	1	1.	1.	0.84	1.09	1.94	0.84	1.09
time (sec)	N/A	0.013	0.001	0.04	1.023	1.445	0.06	1.104

Problem 2166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	27	35	65	29	35
normalized size	1	1.	1.	0.79	1.03	1.91	0.85	1.03
time (sec)	N/A	0.011	0.001	0.04	1.031	1.607	0.059	1.105

Problem 2167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	32	58	24	32
normalized size	1	1.	1.	0.89	1.14	2.07	0.86	1.14
time (sec)	N/A	0.011	0.001	0.039	0.979	1.425	0.058	1.129

Problem 2168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	31	62	24	32
normalized size	1	1.	1.	0.89	1.15	2.3	0.89	1.19
time (sec)	N/A	0.009	0.001	0.038	0.961	1.631	0.082	1.14

Problem 2169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	31	68	20	32
normalized size	1	1.	1.	0.96	1.24	2.72	0.8	1.28
time (sec)	N/A	0.012	0.001	0.044	0.955	1.686	0.088	1.104

Problem 2170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	31	69	22	32
normalized size	1	1.	1.	0.89	1.15	2.56	0.81	1.19
time (sec)	N/A	0.011	0.001	0.043	1.063	1.724	0.096	1.141

Problem 2171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	28	62	19	30
normalized size	1	1.	1.	1.05	1.33	2.95	0.9	1.43
time (sec)	N/A	0.011	0.001	0.045	0.985	1.689	0.101	1.132

Problem 2172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	31	72	22	32
normalized size	1	1.	1.	0.96	1.24	2.88	0.88	1.28
time (sec)	N/A	0.011	0.001	0.043	0.995	1.852	0.106	1.136

Problem 2173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	34	69	26	34
normalized size	1	1.	1.	0.9	1.13	2.3	0.87	1.13
time (sec)	N/A	0.012	0.001	0.043	1.006	1.885	0.106	1.119

Problem 2174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	27	34	70	26	34
normalized size	1	1.	1.	0.75	0.94	1.94	0.72	0.94
time (sec)	N/A	0.012	0.001	0.041	1.009	1.909	0.117	1.105

Problem 2175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	15	13	16	32	10	18
normalized size	1	1.	1.07	0.93	1.14	2.29	0.71	1.29
time (sec)	N/A	0.008	0.003	0.039	0.984	2.011	0.078	1.114

Problem 2176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	18	16	20	42	14	22
normalized size	1	1.	0.95	0.84	1.05	2.21	0.74	1.16
time (sec)	N/A	0.01	0.003	0.04	0.96	1.928	0.077	1.13

Problem 2177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	23	76	15	18
normalized size	1	1.	1.	0.93	1.64	5.43	1.07	1.29
time (sec)	N/A	0.007	0.005	0.045	0.991	1.816	0.101	1.091

Problem 2178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	26	70	15	20
normalized size	1	1.	1.	1.05	1.37	3.68	0.79	1.05
time (sec)	N/A	0.011	0.008	0.044	1.016	1.658	0.092	1.079

Problem 2179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	12	30	8	14
normalized size	1	1.	1.	0.91	1.09	2.73	0.73	1.27
time (sec)	N/A	0.003	0.001	0.039	0.998	1.693	0.073	1.141

Problem 2180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	15	35	8	16
normalized size	1	1.	1.	1.09	1.36	3.18	0.73	1.45
time (sec)	N/A	0.005	0.001	0.044	0.976	1.748	0.091	1.131

Problem 2181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	20	38	15	20
normalized size	1	1.	1.	0.94	1.11	2.11	0.83	1.11
time (sec)	N/A	0.005	0.001	0.042	1.008	1.678	0.088	1.131

Problem 2182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	240	595	0	1693	1554	358
normalized size	1	1.	0.99	2.45	0.	6.97	6.4	1.47
time (sec)	N/A	0.439	0.193	0.154	0.	2.206	6.107	1.121

Problem 2183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	148	366	0	1107	892	217
normalized size	1	1.	0.98	2.42	0.	7.33	5.91	1.44
time (sec)	N/A	0.172	0.131	0.153	0.	1.913	3.449	1.139

Problem 2184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	101	207	0	713	588	135
normalized size	1	1.	1.	2.05	0.	7.06	5.82	1.34
time (sec)	N/A	0.123	0.067	0.153	0.	2.154	1.758	1.103

Problem 2185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	66	93	0	464	280	88
normalized size	1	1.	1.	1.41	0.	7.03	4.24	1.33
time (sec)	N/A	0.035	0.058	0.15	0.	2.029	0.662	1.104

Problem 2186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	38	35	0	277	124	46
normalized size	1	1.	1.12	1.03	0.	8.15	3.65	1.35
time (sec)	N/A	0.018	0.007	0.148	0.	1.899	0.196	1.114

Problem 2187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	105	168	0	697	0	170
normalized size	1	1.	0.86	1.38	0.	5.71	0.	1.39
time (sec)	N/A	0.094	0.081	0.155	0.	3.855	0.	1.113

Problem 2188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	151	386	0	2295	0	447
normalized size	1	1.	0.81	2.08	0.	12.34	0.	2.4
time (sec)	N/A	0.285	0.224	0.171	0.	19.638	0.	1.113

Problem 2189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	272	719	0	0	0	801
normalized size	1	1.	1.	2.64	0.	0.	0.	2.94
time (sec)	N/A	0.425	0.346	0.162	0.	0.	0.	1.118

Problem 2190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	374	374	422	1542	0	5738	2669	690
normalized size	1	1.	1.13	4.12	0.	15.34	7.14	1.84
time (sec)	N/A	0.725	0.708	0.165	0.	3.332	26.313	1.128

Problem 2191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	298	1037	0	3791	1924	479
normalized size	1	1.	1.15	3.99	0.	14.58	7.4	1.84
time (sec)	N/A	0.567	0.495	0.161	0.	2.734	13.023	1.134

Problem 2192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	201	416	0	2435	1238	319
normalized size	1	1.	1.16	2.4	0.	14.08	7.16	1.84
time (sec)	N/A	0.285	0.34	0.157	0.	1.942	5.315	1.115

Problem 2193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	128	212	0	1393	517	188
normalized size	1	1.	1.29	2.14	0.	14.07	5.22	1.9
time (sec)	N/A	0.051	0.134	0.155	0.	1.834	1.733	1.118

Problem 2194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	88	118	0	988	359	134
normalized size	1	1.	1.01	1.36	0.	11.36	4.13	1.54
time (sec)	N/A	0.038	0.082	0.154	0.	1.904	1.1	1.149

Problem 2195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	70	68	0	745	265	103
normalized size	1	1.	1.06	1.03	0.	11.29	4.02	1.56
time (sec)	N/A	0.028	0.078	0.151	0.	1.873	0.71	1.102

Problem 2196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	223	1037	0	4417	0	635
normalized size	1	1.	1.	4.63	0.	19.72	0.	2.83
time (sec)	N/A	0.381	0.357	0.193	0.	159.644	0.	1.109

Problem 2197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	339	1617	0	0	0	1220
normalized size	1	1.	0.99	4.7	0.	0.	0.	3.55
time (sec)	N/A	0.652	0.744	0.174	0.	0.	0.	1.181

Problem 2198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	485	485	489	2554	0	0	0	2176
normalized size	1	1.	1.01	5.27	0.	0.	0.	4.49
time (sec)	N/A	1.057	1.289	0.177	0.	0.	0.	1.177

Problem 2199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	299	1187	0	4757	1875	448
normalized size	1	1.	1.02	4.04	0.	16.18	6.38	1.52
time (sec)	N/A	0.425	0.444	0.166	0.	3.022	5.695	1.141

Problem 2200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	260	1040	0	4120	1714	381
normalized size	1	1.	1.09	4.37	0.	17.31	7.2	1.6
time (sec)	N/A	0.287	0.382	0.181	0.	2.725	4.383	1.127

Problem 2201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	388	388	628	1444	0	8014	3403	1087
normalized size	1	1.	1.62	3.72	0.	20.65	8.77	2.8
time (sec)	N/A	1.11	1.2	0.168	0.	3.366	138.706	1.136

Problem 2202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	413	932	0	5349	1355	845
normalized size	1	1.	2.44	5.51	0.	31.65	8.02	5.
time (sec)	N/A	0.123	0.599	0.162	0.	2.344	38.264	1.131

Problem 2203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	308	695	0	4259	1180	602
normalized size	1	1.	1.95	4.4	0.	26.96	7.47	3.81
time (sec)	N/A	0.088	0.44	0.16	0.	2.228	10.749	1.195

Problem 2204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	203	508	0	3270	1052	414
normalized size	1	1.	1.03	2.57	0.	16.52	5.31	2.09
time (sec)	N/A	0.218	0.31	0.159	0.	2.155	3.863	1.107

Problem 2205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	128	242	0	2340	651	278
normalized size	1	1.	0.98	1.85	0.	17.86	4.97	2.12
time (sec)	N/A	0.05	0.138	0.152	0.	2.153	2.266	1.122

Problem 2206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	97	129	0	1685	474	184
normalized size	1	1.	0.96	1.28	0.	16.68	4.69	1.82
time (sec)	N/A	0.036	0.103	0.154	0.	2.194	1.364	1.092

Problem 2207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	429	429	429	4701	0	0	0	1871
normalized size	1	1.	1.	10.96	0.	0.	0.	4.36
time (sec)	N/A	1.016	1.301	0.178	0.	0.	0.	1.161

Problem 2208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	221	954	0	4849	5722	417
normalized size	1	1.	0.92	3.99	0.	20.29	23.94	1.74
time (sec)	N/A	0.314	0.462	0.172	0.	8.997	50.92	1.128

Problem 2209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	269	1110	0	5724	7465	554
normalized size	1	1.	0.88	3.63	0.	18.71	24.4	1.81
time (sec)	N/A	0.46	0.526	0.171	0.	16.81	52.243	1.129

Problem 2210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	435	2336	0	7313	2769	630
normalized size	1	1.	1.25	6.69	0.	20.95	7.93	1.81
time (sec)	N/A	0.64	0.719	0.173	0.	2.876	11.631	1.128

Problem 2211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	386	1013	0	6337	2565	564
normalized size	1	1.	1.33	3.48	0.	21.78	8.81	1.94
time (sec)	N/A	0.505	0.585	0.168	0.	2.663	7.286	1.122

Problem 2212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	314	531	0	3584	938	521
normalized size	1	1.	2.15	3.64	0.	24.55	6.42	3.57
time (sec)	N/A	0.083	0.209	0.165	0.	2.285	4.363	1.144

Problem 2213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	266	486	0	3379	898	440
normalized size	1	1.	1.83	3.35	0.	23.3	6.19	3.03
time (sec)	N/A	0.064	0.278	0.163	0.	2.216	3.49	1.128

Problem 2214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	572	1666	0	9528	2547	1524
normalized size	1	1.	2.21	6.43	0.	36.79	9.83	5.88
time (sec)	N/A	0.452	1.075	0.162	0.	2.724	121.526	1.14

Problem 2215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	401	1213	0	7605	2057	1118
normalized size	1	1.	1.31	3.96	0.	24.85	6.72	3.65
time (sec)	N/A	0.429	0.823	0.165	0.	2.689	27.188	1.139

Problem 2216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	258	850	0	5802	1635	814
normalized size	1	1.	0.99	3.27	0.	22.32	6.29	3.13
time (sec)	N/A	0.262	0.563	0.161	0.	2.444	7.969	1.127

Problem 2217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	168	369	0	4157	1062	510
normalized size	1	1.	0.97	2.13	0.	24.03	6.14	2.95
time (sec)	N/A	0.074	0.216	0.153	0.	2.348	4.365	1.185

Problem 2218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	134	189	0	2898	777	297
normalized size	1	1.	0.99	1.39	0.	21.31	5.71	2.18
time (sec)	N/A	0.051	0.156	0.153	0.	2.246	2.577	1.099

Problem 2219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	771	771	769	14396	0	0	0	4361
normalized size	1	1.	1.	18.67	0.	0.	0.	5.66
time (sec)	N/A	7.3	3.682	0.222	0.	0.	0.	1.276

Problem 2220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	352	329	2162	0	8609	9418	670
normalized size	1	1.	0.93	6.14	0.	24.46	26.76	1.9
time (sec)	N/A	0.52	0.907	0.175	0.	17.903	114.677	1.133

Problem 2221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	388	388	985	3092	0	18823	0	3162
normalized size	1	1.	2.54	7.97	0.	48.51	0.	8.15
time (sec)	N/A	0.561	2.691	0.171	0.	2.915	0.	1.158

Problem 2222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	545	545	713	2430	0	15663	0	2483
normalized size	1	1.	1.31	4.46	0.	28.74	0.	4.56
time (sec)	N/A	0.88	1.886	0.169	0.	3.118	0.	1.179

Problem 2223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	378	378	467	1742	0	12459	2994	1889
normalized size	1	1.	1.24	4.61	0.	32.96	7.92	5.
time (sec)	N/A	0.502	1.301	0.173	0.	2.44	58.215	1.126

Problem 2224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	326	1243	0	9765	2407	1359
normalized size	1	1.	0.99	3.77	0.	29.59	7.29	4.12
time (sec)	N/A	0.387	0.908	0.168	0.	2.389	16.992	1.167

Problem 2225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	209	496	0	6942	1564	826
normalized size	1	1.	0.95	2.26	0.	31.7	7.14	3.77
time (sec)	N/A	0.096	0.307	0.157	0.	2.237	8.586	1.146

Problem 2226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	167	249	0	4934	1153	454
normalized size	1	1.	0.98	1.46	0.	28.85	6.74	2.65
time (sec)	N/A	0.07	0.173	0.162	0.	2.148	4.93	1.1

Problem 2227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1324	1324	1601	32834	0	0	0	8597
normalized size	1	1.	1.21	24.8	0.	0.	0.	6.49
time (sec)	N/A	13.901	6.59	0.213	0.	0.	0.	1.398

Problem 2228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1761	1761	2147	39599	0	0	0	10855
normalized size	1	1.	1.22	22.49	0.	0.	0.	6.16
time (sec)	N/A	15.196	7.13	0.227	0.	0.	0.	4.222

Problem 2229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	78	68	104	432	90	92
normalized size	1	1.	0.88	0.76	1.17	4.85	1.01	1.03
time (sec)	N/A	0.083	0.096	0.049	1.504	2.809	0.241	1.108

Problem 2230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	98	77	117	517	100	146
normalized size	1	1.	0.86	0.68	1.03	4.54	0.88	1.28
time (sec)	N/A	0.088	0.067	0.05	1.515	2.974	0.25	1.105

Problem 2231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	88	78	131	633	110	105
normalized size	1	1.	0.8	0.71	1.19	5.75	1.	0.95
time (sec)	N/A	0.099	0.112	0.051	1.517	2.38	0.275	1.077

Problem 2232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	119	87	144	726	121	170
normalized size	1	1.	0.84	0.61	1.01	5.11	0.85	1.2
time (sec)	N/A	0.109	0.09	0.05	1.564	2.434	0.3	1.092

Problem 2233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	29	47	151	44	59
normalized size	1	1.	1.	0.62	1.	3.21	0.94	1.26
time (sec)	N/A	0.027	0.038	0.041	1.509	2.277	0.111	1.124

Problem 2234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	33	43	113	41	45
normalized size	1	1.	1.	0.8	1.05	2.76	1.	1.1
time (sec)	N/A	0.027	0.008	0.043	1.455	2.267	0.13	1.101

Problem 2235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	111	311	282	302	177	273
normalized size	1	1.	0.97	2.73	2.47	2.65	1.55	2.39
time (sec)	N/A	0.262	0.072	0.274	1.5	2.576	1.076	1.199

Problem 2236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	31	41	108	34	43
normalized size	1	1.	1.	0.78	1.02	2.7	0.85	1.08
time (sec)	N/A	0.017	0.005	0.044	0.972	2.278	0.105	1.094

Problem 2237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	34	85	27	36
normalized size	1	1.	1.	0.79	1.03	2.58	0.82	1.09
time (sec)	N/A	0.016	0.004	0.044	0.977	2.161	0.108	1.111

Problem 2238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	27	66	20	30
normalized size	1	1.	1.	0.81	1.04	2.54	0.77	1.15
time (sec)	N/A	0.012	0.003	0.044	0.985	2.235	0.103	1.107

Problem 2239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	54	17	26
normalized size	1	1.	1.	0.86	1.1	2.57	0.81	1.24
time (sec)	N/A	0.006	0.003	0.043	0.96	2.068	0.098	1.129

Problem 2240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	50	15	26
normalized size	1	1.	1.	0.86	1.1	2.38	0.71	1.24
time (sec)	N/A	0.006	0.002	0.043	0.989	2.357	0.093	1.148

Problem 2241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	22	28	70	24	32
normalized size	1	1.	1.	0.81	1.04	2.59	0.89	1.19
time (sec)	N/A	0.012	0.004	0.045	0.97	2.329	0.127	1.104

Problem 2242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	27	35	90	31	39
normalized size	1	1.	1.	0.79	1.03	2.65	0.91	1.15
time (sec)	N/A	0.026	0.004	0.045	0.955	2.199	0.147	1.086

Problem 2243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	32	42	117	36	46
normalized size	1	1.	1.	0.78	1.02	2.85	0.88	1.12
time (sec)	N/A	0.028	0.005	0.046	0.962	2.229	0.154	1.089

Problem 2244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	37	49	138	41	53
normalized size	1	1.	1.	0.77	1.02	2.88	0.85	1.1
time (sec)	N/A	0.037	0.004	0.047	0.983	2.277	0.163	1.101

Problem 2245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	31	41	108	34	43
normalized size	1	1.	1.	0.78	1.02	2.7	0.85	1.08
time (sec)	N/A	0.025	0.004	0.043	0.988	2.299	0.111	1.1

Problem 2246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	34	85	27	36
normalized size	1	1.	1.	0.79	1.03	2.58	0.82	1.09
time (sec)	N/A	0.023	0.004	0.043	1.057	2.185	0.11	1.091

Problem 2247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	27	66	20	30
normalized size	1	1.	1.	0.81	1.04	2.54	0.77	1.15
time (sec)	N/A	0.019	0.004	0.043	1.019	2.005	0.11	1.099

Problem 2248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	54	17	26
normalized size	1	1.	1.	0.86	1.1	2.57	0.81	1.24
time (sec)	N/A	0.013	0.003	0.043	1.011	2.305	0.102	1.085

Problem 2249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	50	15	26
normalized size	1	1.	1.	0.86	1.1	2.38	0.71	1.24
time (sec)	N/A	0.011	0.003	0.044	0.981	2.375	0.102	1.118

Problem 2250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	22	28	70	24	32
normalized size	1	1.	1.	0.81	1.04	2.59	0.89	1.19
time (sec)	N/A	0.014	0.004	0.046	0.985	2.492	0.129	1.128

Problem 2251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	27	35	90	31	39
normalized size	1	1.	1.	0.79	1.03	2.65	0.91	1.15
time (sec)	N/A	0.028	0.004	0.048	1.003	2.468	0.144	1.134

Problem 2252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	32	42	117	36	46
normalized size	1	1.	1.	0.78	1.02	2.85	0.88	1.12
time (sec)	N/A	0.031	0.004	0.045	1.002	2.24	0.158	1.12

Problem 2253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	37	49	138	41	53
normalized size	1	1.	1.	0.77	1.02	2.88	0.85	1.1
time (sec)	N/A	0.034	0.004	0.047	0.974	2.438	0.166	1.093

Problem 2254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	16	46	8	18
normalized size	1	1.	1.	1.08	1.33	3.83	0.67	1.5
time (sec)	N/A	0.006	0.003	0.042	0.956	2.348	0.081	1.11

Problem 2255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	27	69	20	27
normalized size	1	1.	1.	0.81	1.04	2.65	0.77	1.04
time (sec)	N/A	0.013	0.004	0.04	1.506	2.445	0.103	1.091

Problem 2256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	41	12	20
normalized size	1	1.	1.	0.82	1.06	2.41	0.71	1.18
time (sec)	N/A	0.005	0.003	0.046	0.949	2.694	0.096	1.097

Problem 2257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	28	25	30	84	20	30
normalized size	1	1.	1.08	0.96	1.15	3.23	0.77	1.15
time (sec)	N/A	0.007	0.012	0.04	1.45	2.582	0.112	1.132

Problem 2258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	49	48	73	189	63	57
normalized size	1	1.	0.91	0.89	1.35	3.5	1.17	1.06
time (sec)	N/A	0.021	0.025	0.041	1.441	2.873	0.135	1.118

Problem 2259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	28	36	96	36	36
normalized size	1	1.	1.	0.88	1.12	3.	1.12	1.12
time (sec)	N/A	0.019	0.008	0.04	1.502	2.239	0.105	1.08

Problem 2260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	42	12	22
normalized size	1	1.	1.	0.83	1.06	2.33	0.67	1.22
time (sec)	N/A	0.008	0.003	0.043	0.956	2.252	0.094	1.092

Problem 2261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	19	50	17	22
normalized size	1	1.	1.	0.75	0.95	2.5	0.85	1.1
time (sec)	N/A	0.008	0.003	0.044	0.981	2.26	0.097	1.096

Problem 2262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	23	15	16	20	72	14	20
normalized size	1	1.53	1.	1.07	1.33	4.8	0.93	1.33
time (sec)	N/A	0.008	0.008	0.042	1.536	2.129	0.109	1.089

Problem 2263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	22	28	58	19	31
normalized size	1	1.	1.	0.81	1.04	2.15	0.7	1.15
time (sec)	N/A	0.012	0.004	0.043	0.974	2.263	0.096	1.094

Problem 2264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	26	21	27	81	19	28
normalized size	1	1.	1.18	0.95	1.23	3.68	0.86	1.27
time (sec)	N/A	0.01	0.009	0.043	0.957	2.248	0.079	1.124

Problem 2265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	25	23	30	81	19	31
normalized size	1	1.	0.96	0.88	1.15	3.12	0.73	1.19
time (sec)	N/A	0.01	0.007	0.046	1.022	2.121	0.079	1.091

Problem 2266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	30	28	36	97	24	38
normalized size	1	1.	1.03	0.97	1.24	3.34	0.83	1.31
time (sec)	N/A	0.015	0.009	0.044	0.985	2.23	0.086	1.116

Problem 2267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	29	38	103	37	39
normalized size	1	1.	1.	0.88	1.15	3.12	1.12	1.18
time (sec)	N/A	0.019	0.007	0.043	1.462	2.267	0.126	1.122

Problem 2268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	55	53	80	340	326	498
normalized size	1	1.	0.73	0.71	1.07	4.53	4.35	6.64
time (sec)	N/A	0.038	0.062	0.042	1.044	2.19	3.352	1.15

Problem 2269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	55	53	80	266	230	274
normalized size	1	1.	0.73	0.71	1.07	3.55	3.07	3.65
time (sec)	N/A	0.029	0.047	0.041	0.97	2.243	8.006	1.118

Problem 2270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	55	53	80	193	71	111
normalized size	1	1.	0.73	0.71	1.07	2.57	0.95	1.48
time (sec)	N/A	0.03	0.048	0.041	0.971	2.245	2.278	1.097

Problem 2271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	54	53	104	127	223	108
normalized size	1	1.	0.74	0.73	1.42	1.74	3.05	1.48
time (sec)	N/A	0.03	0.051	0.042	0.978	2.272	8.247	1.1

Problem 2272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	54	53	89	136	70	99
normalized size	1	1.	0.76	0.75	1.25	1.92	0.99	1.39
time (sec)	N/A	0.031	0.05	0.042	0.975	2.186	9.1	1.11

Problem 2273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	55	52	85	158	252	86
normalized size	1	1.	0.77	0.73	1.2	2.23	3.55	1.21
time (sec)	N/A	0.032	0.047	0.042	1.038	2.317	1.298	1.102

Problem 2274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	54	53	76	186	376	84
normalized size	1	1.	0.74	0.73	1.04	2.55	5.15	1.15
time (sec)	N/A	0.03	0.049	0.042	0.994	2.303	3.043	1.135

Problem 2275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	173	194	238	860	1129	1353
normalized size	1	1.	1.04	1.17	1.43	5.18	6.8	8.15
time (sec)	N/A	0.093	0.164	0.048	0.983	2.355	34.434	1.158

Problem 2276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	174	194	238	724	654	786
normalized size	1	1.	1.05	1.17	1.43	4.36	3.94	4.73
time (sec)	N/A	0.073	0.15	0.046	0.978	2.286	21.058	1.169

Problem 2277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	172	194	238	554	230	338
normalized size	1	1.	1.04	1.17	1.43	3.34	1.39	2.04
time (sec)	N/A	0.072	0.151	0.043	0.972	2.313	4.051	1.162

Problem 2278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	172	194	320	412	644	335
normalized size	1	1.	1.05	1.18	1.95	2.51	3.93	2.04
time (sec)	N/A	0.073	0.16	0.044	1.013	2.256	53.352	1.109

Problem 2279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	171	194	248	425	182	342
normalized size	1	1.	1.06	1.2	1.53	2.62	1.12	2.11
time (sec)	N/A	0.07	0.134	0.045	0.972	2.322	29.953	1.118

Problem 2280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	170	194	246	436	160	329
normalized size	1	1.	1.05	1.2	1.52	2.69	0.99	2.03
time (sec)	N/A	0.075	0.128	0.046	0.975	2.001	43.371	1.143

Problem 2281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	172	194	250	451	1180	325
normalized size	1	1.	1.06	1.2	1.54	2.78	7.28	2.01
time (sec)	N/A	0.071	0.137	0.044	0.965	1.994	3.812	1.138

Problem 2282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	321	495	549	1733	2363	2830
normalized size	1	1.	1.12	1.73	1.92	6.06	8.26	9.9
time (sec)	N/A	0.181	0.909	0.046	1.019	2.031	65.28	1.237

Problem 2283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	320	495	549	1426	1411	1690
normalized size	1	1.	1.12	1.73	1.92	4.99	4.93	5.91
time (sec)	N/A	0.129	0.882	0.045	1.033	2.042	39.504	1.187

Problem 2284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	396	495	549	1170	539	752
normalized size	1	1.	1.38	1.73	1.92	4.09	1.88	2.63
time (sec)	N/A	0.13	0.387	0.047	1.	2.003	7.269	1.122

Problem 2285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	317	495	709	956	1406	751
normalized size	1	1.	1.12	1.76	2.51	3.39	4.99	2.66
time (sec)	N/A	0.128	0.559	0.044	1.029	2.019	120.098	1.126

Problem 2286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	394	495	560	950	428	852
normalized size	1	1.	1.41	1.77	2.	3.39	1.53	3.04
time (sec)	N/A	0.128	0.412	0.045	1.046	2.017	99.205	1.138

Problem 2287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	395	495	558	964	348	826
normalized size	1	1.	1.4	1.76	1.98	3.42	1.23	2.93
time (sec)	N/A	0.132	0.382	0.044	1.086	2.092	117.06	1.14

Problem 2288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	391	495	558	950	0	822
normalized size	1	1.	1.41	1.78	2.01	3.42	0.	2.96
time (sec)	N/A	0.134	0.365	0.047	1.031	1.994	0.	1.132

Problem 2289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	459	459	455	1929	0	13779	0	0
normalized size	1	1.	0.99	4.2	0.	30.02	0.	0.
time (sec)	N/A	4.432	1.082	0.335	0.	17.164	0.	0.

Problem 2290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	317	1138	0	5581	1443	0
normalized size	1	1.	0.98	3.53	0.	17.33	4.48	0.
time (sec)	N/A	1.216	0.773	0.264	0.	3.777	162.641	0.

Problem 2291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	175	545	0	1466	155	0
normalized size	1	1.	0.88	2.75	0.	7.4	0.78	0.
time (sec)	N/A	0.289	0.467	0.253	0.	2.416	10.4	0.

Problem 2292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	176	194	0	5416	0	0
normalized size	1	1.	0.88	0.97	0.	27.22	0.	0.
time (sec)	N/A	0.296	0.534	0.242	0.	2.834	0.	0.

Problem 2293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	273	689	0	23024	0	0
normalized size	1	1.	0.88	2.22	0.	74.27	0.	0.
time (sec)	N/A	0.729	0.412	0.258	0.	6.499	0.	0.

Problem 2294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	414	414	377	1444	0	0	0	0
normalized size	1	1.	0.91	3.49	0.	0.	0.	0.
time (sec)	N/A	1.546	1.336	0.256	0.	0.	0.	0.

Problem 2295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	691	691	1116	3838	0	22507	0	0
normalized size	1	1.	1.62	5.55	0.	32.57	0.	0.
time (sec)	N/A	15.962	6.492	0.294	0.	64.651	0.	0.

Problem 2296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	504	504	549	2557	0	11088	0	0
normalized size	1	1.	1.09	5.07	0.	22.	0.	0.
time (sec)	N/A	4.618	5.796	0.278	0.	5.061	0.	0.

Problem 2297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	363	418	1503	0	5069	0	0
normalized size	1	1.	1.15	4.14	0.	13.96	0.	0.
time (sec)	N/A	1.293	3.186	0.293	0.	2.456	0.	0.

Problem 2298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	283	869	0	13504	0	0
normalized size	1	1.	0.99	3.03	0.	47.05	0.	0.
time (sec)	N/A	0.671	0.651	0.329	0.	3.28	0.	0.

Problem 2299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	428	428	366	1120	0	0	0	0
normalized size	1	1.	0.86	2.62	0.	0.	0.	0.
time (sec)	N/A	1.472	1.449	0.283	0.	0.	0.	0.

Problem 2300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	604	604	552	3212	0	0	0	0
normalized size	1	1.	0.91	5.32	0.	0.	0.	0.
time (sec)	N/A	4.943	2.933	0.3	0.	0.	0.	0.

Problem 2301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	337	929	0	7992	0	0
normalized size	1	1.	0.94	2.58	0.	22.2	0.	0.
time (sec)	N/A	3.693	0.913	0.226	0.	8.589	0.	0.

Problem 2302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	751	751	17950	5849	0	19941	0	0
normalized size	1	1.	23.9	7.79	0.	26.55	0.	0.
time (sec)	N/A	13.512	6.695	0.31	0.	15.97	0.	0.

Problem 2303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	577	577	506	3925	0	11310	0	0
normalized size	1	1.	0.88	6.8	0.	19.6	0.	0.
time (sec)	N/A	5.057	5.055	0.287	0.	4.538	0.	0.

Problem 2304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	441	441	4707	2582	0	24935	0	0
normalized size	1	1.	10.67	5.85	0.	56.54	0.	0.
time (sec)	N/A	1.942	6.267	0.277	0.	6.198	0.	0.

Problem 2305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	634	634	580	3360	0	0	0	0
normalized size	1	1.	0.91	5.3	0.	0.	0.	0.
time (sec)	N/A	4.974	4.298	0.352	0.	0.	0.	0.

Problem 2306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	835	834	1056	4488	0	0	0	0
normalized size	1	1.	1.26	5.37	0.	0.	0.	0.
time (sec)	N/A	13.919	6.209	0.341	0.	0.	0.	0.

Problem 2307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	629	629	197	609	0	1515	0	0
normalized size	1	1.	0.31	0.97	0.	2.41	0.	0.
time (sec)	N/A	1.227	0.498	0.379	0.	2.796	0.	0.

Problem 2308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	705	705	198	673	0	5848	0	0
normalized size	1	1.	0.28	0.95	0.	8.3	0.	0.
time (sec)	N/A	0.791	0.627	0.38	0.	3.264	0.	0.

Problem 2309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	134	634	0	2519	109	0
normalized size	1	1.	0.48	2.27	0.	9.03	0.39	0.
time (sec)	N/A	0.608	0.315	0.303	0.	2.674	61.425	0.

Problem 2310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	133	625	0	2144	97	0
normalized size	1	1.	0.5	2.35	0.	8.06	0.36	0.
time (sec)	N/A	0.436	0.276	0.078	0.	2.714	33.903	0.

Problem 2311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	128	616	0	2006	85	0
normalized size	1	1.	0.51	2.43	0.	7.93	0.34	0.
time (sec)	N/A	0.358	0.266	0.073	0.	2.551	17.613	0.

Problem 2312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	112	486	0	1442	32	0
normalized size	1	1.	0.5	2.19	0.	6.5	0.14	0.
time (sec)	N/A	0.247	0.203	0.087	0.	2.656	2.513	0.

Problem 2313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	112	607	0	1427	0	0
normalized size	1	1.	0.51	2.78	0.	6.55	0.	0.
time (sec)	N/A	0.272	0.246	0.071	0.	2.611	0.	0.

Problem 2314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	122	616	0	2148	0	0
normalized size	1	1.	0.48	2.43	0.	8.49	0.	0.
time (sec)	N/A	0.337	0.45	0.077	0.	2.772	0.	0.

Problem 2315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	133	625	0	2337	0	0
normalized size	1	1.	0.5	2.35	0.	8.79	0.	0.
time (sec)	N/A	0.382	0.527	0.075	0.	2.76	0.	0.

Problem 2316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	199	651	0	2880	0	0
normalized size	1	1.	0.67	2.2	0.	9.73	0.	0.
time (sec)	N/A	0.446	0.434	0.078	0.	3.002	0.	0.

Problem 2317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	141	642	0	2557	246	0
normalized size	1	1.	0.5	2.27	0.	9.04	0.87	0.
time (sec)	N/A	0.41	0.673	0.076	0.	2.891	127.154	0.

Problem 2318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	141	642	0	2195	211	0
normalized size	1	1.	0.52	2.38	0.	8.13	0.78	0.
time (sec)	N/A	0.364	0.718	0.082	0.	3.194	58.901	0.

Problem 2319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	145	972	0	2176	83	0
normalized size	1	1.	0.54	3.6	0.	8.06	0.31	0.
time (sec)	N/A	0.34	0.369	0.243	0.	3.094	5.229	0.

Problem 2320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	166	968	0	2591	0	0
normalized size	1	1.	0.61	3.59	0.	9.6	0.	0.
time (sec)	N/A	0.324	0.309	0.223	0.	3.066	0.	0.

Problem 2321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	158	651	0	3001	0	0
normalized size	1	1.	0.56	2.3	0.	10.6	0.	0.
time (sec)	N/A	0.367	0.393	0.081	0.	2.929	0.	0.

Problem 2322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	176	660	0	3170	0	0
normalized size	1	1.	0.59	2.23	0.	10.71	0.	0.
time (sec)	N/A	0.443	0.518	0.081	0.	2.624	0.	0.

Problem 2323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	236	662	0	3374	0	0
normalized size	1	1.	0.75	2.12	0.	10.78	0.	0.
time (sec)	N/A	0.522	0.698	0.077	0.	2.841	0.	0.

Problem 2324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	223	662	0	2979	0	0
normalized size	1	1.	0.74	2.21	0.	9.93	0.	0.
time (sec)	N/A	0.438	0.626	0.076	0.	2.712	0.	0.

Problem 2325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	209	662	0	2738	0	0
normalized size	1	1.	0.7	2.21	0.	9.13	0.	0.
time (sec)	N/A	0.435	0.565	0.083	0.	2.737	0.	0.

Problem 2326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	198	662	0	2778	490	0
normalized size	1	1.	0.66	2.21	0.	9.26	1.63	0.
time (sec)	N/A	0.422	0.485	0.078	0.	2.846	92.255	0.

Problem 2327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	151	1108	0	3016	199	0
normalized size	1	1.	0.5	3.69	0.	10.05	0.66	0.
time (sec)	N/A	0.385	0.661	0.428	0.	2.599	8.838	0.

Problem 2328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	151	1100	0	3406	0	0
normalized size	1	1.	0.48	3.5	0.	10.85	0.	0.
time (sec)	N/A	0.386	0.652	0.444	0.	2.84	0.	0.

Problem 2329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	156	671	0	3787	0	0
normalized size	1	1.	0.5	2.14	0.	12.1	0.	0.
time (sec)	N/A	0.424	0.871	0.082	0.	2.958	0.	0.

Problem 2330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	389	518	1166	0	9743	0	0
normalized size	1	1.	1.33	3.	0.	25.05	0.	0.
time (sec)	N/A	1.763	1.591	0.203	0.	7.069	0.	0.

Problem 2331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	458	458	440	1571	0	11857	0	0
normalized size	1	1.	0.96	3.43	0.	25.89	0.	0.
time (sec)	N/A	9.722	1.262	0.201	0.	17.034	0.	0.

Problem 2332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	19	16	22	45	24	22
normalized size	1	1.	0.68	0.57	0.79	1.61	0.86	0.79
time (sec)	N/A	0.004	0.005	0.038	1.	2.182	1.967	1.891

Problem 2333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	206	795	0	1762	0	513
normalized size	1	1.	0.83	3.21	0.	7.1	0.	2.07
time (sec)	N/A	0.255	0.349	0.052	0.	3.118	0.	1.955

Problem 2334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	162	484	0	1137	0	317
normalized size	1	1.	0.85	2.53	0.	5.95	0.	1.66
time (sec)	N/A	0.234	0.179	0.05	0.	2.615	0.	1.684

Problem 2335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	114	229	0	683	0	174
normalized size	1	1.	0.99	1.99	0.	5.94	0.	1.51
time (sec)	N/A	0.043	0.106	0.046	0.	2.421	0.	1.174

Problem 2336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	71	89	0	435	0	92
normalized size	1	1.	0.95	1.19	0.	5.8	0.	1.23
time (sec)	N/A	0.02	0.064	0.042	0.	2.439	0.	1.176

Problem 2337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	145	715	0	2244	0	0
normalized size	1	1.	0.95	4.7	0.	14.76	0.	0.
time (sec)	N/A	0.167	0.212	0.331	0.	14.026	0.	0.

Problem 2338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	152	1519	0	3155	0	0
normalized size	1	1.	0.95	9.49	0.	19.72	0.	0.
time (sec)	N/A	0.127	0.187	0.229	0.	20.73	0.	0.

Problem 2339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	149	3269	0	1972	0	926
normalized size	1	1.	0.97	21.37	0.	12.89	0.	6.05
time (sec)	N/A	0.098	0.171	0.229	0.	11.502	0.	1.265

Problem 2340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	206	4844	0	4016	0	2631
normalized size	1	1.	0.96	22.53	0.	18.68	0.	12.24
time (sec)	N/A	0.148	0.433	0.233	0.	49.915	0.	1.582

Problem 2341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	276	7991	0	0	0	4086
normalized size	1	1.	0.9	25.94	0.	0.	0.	13.27
time (sec)	N/A	0.415	0.694	0.238	0.	0.	0.	21.673

Problem 2342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	402	402	367	10791	0	0	0	11410
normalized size	1	1.	0.91	26.84	0.	0.	0.	28.38
time (sec)	N/A	0.525	2.892	0.242	0.	0.	0.	2.941

Problem 2343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	231	1437	0	3075	0	965
normalized size	1	1.	0.72	4.48	0.	9.58	0.	3.01
time (sec)	N/A	0.347	0.528	0.052	0.	4.968	0.	1.152

Problem 2344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	188	922	0	2044	0	625
normalized size	1	1.	0.73	3.59	0.	7.95	0.	2.43
time (sec)	N/A	0.333	0.338	0.05	0.	4.051	0.	1.134

Problem 2345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	144	469	0	1215	0	356
normalized size	1	1.	0.89	2.91	0.	7.55	0.	2.21
time (sec)	N/A	0.062	0.298	0.045	0.	3.061	0.	1.123

Problem 2346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	114	201	0	657	0	166
normalized size	1	1.	1.02	1.79	0.	5.87	0.	1.48
time (sec)	N/A	0.033	0.355	0.044	0.	2.732	0.	1.139

Problem 2347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	236	1946	0	0	0	0
normalized size	1	1.	0.94	7.72	0.	0.	0.	0.
time (sec)	N/A	0.343	0.454	0.225	0.	0.	0.	0.

Problem 2348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	214	3450	0	3494	0	0
normalized size	1	1.	0.94	15.2	0.	15.39	0.	0.
time (sec)	N/A	0.275	0.441	0.231	0.	142.45	0.	0.

Problem 2349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	311	7299	0	5808	0	0
normalized size	1	1.	1.32	30.93	0.	24.61	0.	0.
time (sec)	N/A	0.252	0.99	0.236	0.	179.376	0.	0.

Problem 2350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	421	10401	0	0	0	0
normalized size	1	1.	1.37	33.88	0.	0.	0.	0.
time (sec)	N/A	0.355	1.711	0.246	0.	0.	0.	0.

Problem 2351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	222	15932	0	0	0	3803
normalized size	1	1.	0.99	70.81	0.	0.	0.	16.9
time (sec)	N/A	0.15	0.769	0.24	0.	0.	0.	18.765

Problem 2352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	275	20477	0	0	0	11048
normalized size	1	1.	0.93	69.18	0.	0.	0.	37.32
time (sec)	N/A	0.235	1.254	0.245	0.	0.	0.	4.634

Problem 2353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	409	409	349	28629	0	0	0	19012
normalized size	1	1.	0.85	70.	0.	0.	0.	46.48
time (sec)	N/A	0.596	1.866	0.269	0.	0.	0.	89.385

Problem 2354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	510	510	687	35234	0	0	0	0
normalized size	1	1.	1.35	69.09	0.	0.	0.	0.
time (sec)	N/A	0.766	6.069	0.275	0.	0.	0.	0.

Problem 2355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	400	400	280	2294	0	4959	0	1566
normalized size	1	1.	0.7	5.74	0.	12.4	0.	3.92
time (sec)	N/A	0.44	0.821	0.056	0.	6.855	0.	1.178

Problem 2356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	238	1517	0	3314	0	1035
normalized size	1	1.	0.74	4.7	0.	10.26	0.	3.2
time (sec)	N/A	0.44	0.452	0.05	0.	5.137	0.	1.167

Problem 2357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	180	807	0	1974	0	601
normalized size	1	1.	0.87	3.9	0.	9.54	0.	2.9
time (sec)	N/A	0.089	0.278	0.047	0.	3.281	0.	1.158

Problem 2358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	162	360	0	1000	0	281
normalized size	1	1.	1.09	2.42	0.	6.71	0.	1.89
time (sec)	N/A	0.05	0.58	0.044	0.	2.731	0.	1.144

Problem 2359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	459	459	440	4126	0	0	0	0
normalized size	1	1.	0.96	8.99	0.	0.	0.	0.
time (sec)	N/A	0.75	1.284	0.23	0.	0.	0.	0.

Problem 2360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	388	388	370	6711	0	0	0	0
normalized size	1	1.	0.95	17.3	0.	0.	0.	0.
time (sec)	N/A	0.663	1.171	0.233	0.	0.	0.	0.

Problem 2361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	445	14002	0	0	0	1968
normalized size	1	1.	1.34	42.3	0.	0.	0.	5.95
time (sec)	N/A	0.548	1.971	0.237	0.	0.	0.	2.126

Problem 2362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	559	18718	0	0	0	2862
normalized size	1	1.	1.66	55.54	0.	0.	0.	8.49
time (sec)	N/A	0.429	3.403	0.238	0.	0.	0.	27.197

Problem 2363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	492	492	593	28635	0	0	0	0
normalized size	1	1.	1.21	58.2	0.	0.	0.	0.
time (sec)	N/A	0.735	3.928	0.252	0.	0.	0.	0.

Problem 2364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	84	71	81	232	0	104
normalized size	1	1.	0.95	0.81	0.92	2.64	0.	1.18
time (sec)	N/A	0.049	0.036	0.042	1.515	2.967	0.	1.102

Problem 2365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	88	80	232	0	227
normalized size	1	1.	1.	1.29	1.18	3.41	0.	3.34
time (sec)	N/A	0.041	0.034	0.043	1.536	2.622	0.	1.106

Problem 2366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	26	23	55	85	85	38
normalized size	1	1.	0.68	0.61	1.45	2.24	2.24	1.
time (sec)	N/A	0.012	0.009	0.04	1.512	2.339	0.26	1.089

Problem 2367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	40	43	70	123	0	61
normalized size	1	1.	0.74	0.8	1.3	2.28	0.	1.13
time (sec)	N/A	0.018	0.02	0.052	1.494	2.265	0.	1.094

Problem 2368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	43	43	70	135	0	43
normalized size	1	1.	0.77	0.77	1.25	2.41	0.	0.77
time (sec)	N/A	0.017	0.026	0.046	1.492	2.396	0.	1.088

Problem 2369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	63	54	84	169	0	90
normalized size	1	1.	1.03	0.89	1.38	2.77	0.	1.48
time (sec)	N/A	0.043	0.017	0.042	0.993	2.376	0.	1.087

Problem 2370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	46	78	136	0	70
normalized size	1	1.	1.	0.71	1.2	2.09	0.	1.08
time (sec)	N/A	0.043	0.014	0.045	1.514	2.485	0.	1.104

Problem 2371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	263	394	0	1075	0	281
normalized size	1	1.	1.01	1.51	0.	4.12	0.	1.08
time (sec)	N/A	0.382	0.323	0.049	0.	2.907	0.	1.167

Problem 2372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	213	290	0	848	0	217
normalized size	1	1.	1.05	1.44	0.	4.2	0.	1.07
time (sec)	N/A	0.217	0.208	0.048	0.	2.749	0.	1.101

Problem 2373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	352	627	0	1368	0	373
normalized size	1	1.	1.19	2.12	0.	4.62	0.	1.26
time (sec)	N/A	0.377	0.477	0.055	0.	3.246	0.	1.114

Problem 2374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	210	366	0	890	0	230
normalized size	1	1.	1.21	2.1	0.	5.11	0.	1.32
time (sec)	N/A	0.151	0.224	0.05	0.	2.609	0.	1.13

Problem 2375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	103	198	0	582	0	142
normalized size	1	1.	0.81	1.56	0.	4.58	0.	1.12
time (sec)	N/A	0.116	0.121	0.049	0.	2.528	0.	1.13

Problem 2376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	66	81	0	400	0	88
normalized size	1	1.	0.97	1.19	0.	5.88	0.	1.29
time (sec)	N/A	0.024	0.079	0.045	0.	2.326	0.	1.101

Problem 2377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	34	30	0	263	0	49
normalized size	1	1.	0.94	0.83	0.	7.31	0.	1.36
time (sec)	N/A	0.011	0.022	0.043	0.	2.49	0.	1.108

Problem 2378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	78	157	0	743	0	97
normalized size	1	1.	0.99	1.99	0.	9.41	0.	1.23
time (sec)	N/A	0.039	0.037	0.228	0.	3.074	0.	1.104

Problem 2379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	131	432	0	1397	0	859
normalized size	1	1.	0.98	3.22	0.	10.43	0.	6.41
time (sec)	N/A	0.078	0.108	0.228	0.	6.002	0.	18.762

Problem 2380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	205	959	0	2838	0	1060
normalized size	1	1.	0.98	4.59	0.	13.58	0.	5.07
time (sec)	N/A	0.219	0.304	0.237	0.	19.766	0.	1.201

Problem 2381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	288	1665	0	5632	0	2807
normalized size	1	1.	0.98	5.68	0.	19.22	0.	9.58
time (sec)	N/A	0.359	0.673	0.236	0.	97.89	0.	1.339

Problem 2382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	316	913	0	2488	0	509
normalized size	1	1.	1.1	3.19	0.	8.7	0.	1.78
time (sec)	N/A	0.318	0.451	0.054	0.	6.107	0.	1.176

Problem 2383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	196	541	0	1574	0	315
normalized size	1	1.	1.11	3.06	0.	8.89	0.	1.78
time (sec)	N/A	0.178	0.419	0.05	0.	4.845	0.	1.145

Problem 2384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	127	264	0	995	0	178
normalized size	1	1.	0.98	2.05	0.	7.71	0.	1.38
time (sec)	N/A	0.08	0.25	0.047	0.	4.529	0.	1.154

Problem 2385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	43	45	0	162	0	77
normalized size	1	1.	0.96	1.	0.	3.6	0.	1.71
time (sec)	N/A	0.011	0.199	0.042	0.	3.449	0.	1.117

Problem 2386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	31	33	0	135	0	55
normalized size	1	1.	0.97	1.03	0.	4.22	0.	1.72
time (sec)	N/A	0.004	0.017	0.043	0.	2.767	0.	1.137

Problem 2387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	162	603	0	2768	0	603
normalized size	1	1.	1.05	3.89	0.	17.86	0.	3.89
time (sec)	N/A	0.1	0.304	0.229	0.	9.398	0.	1.12

Problem 2388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	250	1313	0	6063	0	0
normalized size	1	1.	0.98	5.15	0.	23.78	0.	0.
time (sec)	N/A	0.275	0.407	0.267	0.	21.794	0.	0.

Problem 2389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	371	353	2380	0	11780	0	3475
normalized size	1	1.	0.95	6.42	0.	31.75	0.	9.37
time (sec)	N/A	0.476	1.419	0.236	0.	92.894	0.	1.459

Problem 2390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	519	519	486	3823	0	0	0	6974
normalized size	1	1.	0.94	7.37	0.	0.	0.	13.44
time (sec)	N/A	0.858	2.564	0.244	0.	0.	0.	2.72

Problem 2391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	389	566	2395	0	4651	0	1064
normalized size	1	1.	1.46	6.16	0.	11.96	0.	2.74
time (sec)	N/A	0.465	1.891	0.058	0.	15.039	0.	1.181

Problem 2392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	397	1550	0	3106	0	741
normalized size	1	1.	1.35	5.25	0.	10.53	0.	2.51
time (sec)	N/A	0.345	0.863	0.053	0.	11.092	0.	1.159

Problem 2393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	190	296	0	783	0	474
normalized size	1	1.	1.61	2.51	0.	6.64	0.	4.02
time (sec)	N/A	0.042	0.816	0.047	0.	8.75	0.	1.123

Problem 2394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	167	215	0	644	0	387
normalized size	1	1.	1.7	2.19	0.	6.57	0.	3.95
time (sec)	N/A	0.037	0.731	0.047	0.	8.821	0.	1.114

Problem 2395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	110	131	0	521	0	306
normalized size	1	1.	1.21	1.44	0.	5.73	0.	3.36
time (sec)	N/A	0.021	0.122	0.045	0.	8.441	0.	1.129

Problem 2396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	57	78	0	406	0	194
normalized size	1	1.	0.81	1.11	0.	5.8	0.	2.77
time (sec)	N/A	0.012	0.029	0.043	0.	2.886	0.	1.134

Problem 2397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	309	1408	0	9372	0	4895
normalized size	1	1.	1.	4.54	0.	30.23	0.	15.79
time (sec)	N/A	0.273	0.503	0.23	0.	27.255	0.	1.744

Problem 2398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	473	473	482	2765	0	18303	0	0
normalized size	1	1.	1.02	5.85	0.	38.7	0.	0.
time (sec)	N/A	0.614	1.938	0.239	0.	102.847	0.	0.

Problem 2399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	621	621	626	4942	0	0	0	16020
normalized size	1	1.	1.01	7.96	0.	0.	0.	25.8
time (sec)	N/A	1.041	3.534	0.241	0.	0.	0.	17.856

Problem 2400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	25	22	31	105	0	28
normalized size	1	1.	0.86	0.76	1.07	3.62	0.	0.97
time (sec)	N/A	0.013	0.008	0.044	1.486	2.342	0.	1.516

Problem 2401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	31	120	0	31
normalized size	1	1.	1.	0.96	1.24	4.8	0.	1.24
time (sec)	N/A	0.011	0.009	0.045	1.444	2.393	0.	1.583

Problem 2402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	25	20	26	80	0	42
normalized size	1	1.	1.09	0.87	1.13	3.48	0.	1.83
time (sec)	N/A	0.011	0.006	0.044	1.475	2.505	0.	1.469

Problem 2403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	26	39	77	0	41
normalized size	1	1.	0.88	0.76	1.15	2.26	0.	1.21
time (sec)	N/A	0.009	0.019	0.043	0.995	2.298	0.	1.382

Problem 2404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	22	23	36	55	0	24
normalized size	1	1.	1.16	1.21	1.89	2.89	0.	1.26
time (sec)	N/A	0.012	0.006	0.044	1.492	2.036	0.	1.385

Problem 2405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	12	13	12	54	0	24
normalized size	1	1.	0.92	1.	0.92	4.15	0.	1.85
time (sec)	N/A	0.008	0.004	0.044	1.533	2.058	0.	1.484

Problem 2406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	19	21	35	55	0	26
normalized size	1	1.	1.12	1.24	2.06	3.24	0.	1.53
time (sec)	N/A	0.005	0.04	0.041	0.962	2.015	0.	1.279

Problem 2407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	51	58	0	626	0	0
normalized size	1	1.	0.53	0.6	0.	6.52	0.	0.
time (sec)	N/A	0.035	0.032	0.187	0.	2.187	0.	0.

Problem 2408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	55	98	0	385	0	88
normalized size	1	1.	0.98	1.75	0.	6.88	0.	1.57
time (sec)	N/A	0.053	0.04	0.193	0.	2.293	0.	1.145

Problem 2409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	45	59	0	126	0	0
normalized size	1	1.	0.94	1.23	0.	2.62	0.	0.
time (sec)	N/A	0.027	0.057	0.19	0.	2.795	0.	0.

Problem 2410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	21	29	45	103	0	59
normalized size	1	1.	0.78	1.07	1.67	3.81	0.	2.19
time (sec)	N/A	0.028	0.013	0.153	0.984	1.969	0.	1.099

Problem 2411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	30	42	142	0	65
normalized size	1	1.	1.	0.75	1.05	3.55	0.	1.62
time (sec)	N/A	0.018	0.012	0.043	1.476	2.022	0.	1.18

Problem 2412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	30	42	154	0	42
normalized size	1	1.	1.	0.75	1.05	3.85	0.	1.05
time (sec)	N/A	0.023	0.016	0.043	1.514	2.035	0.	1.155

Problem 2413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	52	45	58	151	0	66
normalized size	1	1.	0.91	0.79	1.02	2.65	0.	1.16
time (sec)	N/A	0.015	0.019	0.041	1.576	2.083	0.	1.161

Problem 2414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	27	35	155	0	35
normalized size	1	1.	1.	0.71	0.92	4.08	0.	0.92
time (sec)	N/A	0.013	0.009	0.04	1.441	2.493	0.	1.152

Problem 2415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	49	45	58	143	0	66
normalized size	1	1.	0.91	0.83	1.07	2.65	0.	1.22
time (sec)	N/A	0.015	0.017	0.041	1.483	2.459	0.	1.177

Problem 2416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	49	41	42	230	0	42
normalized size	1	1.	0.91	0.76	0.78	4.26	0.	0.78
time (sec)	N/A	0.015	0.019	0.043	1.447	2.054	0.	1.119

Problem 2417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	52	45	58	150	0	66
normalized size	1	1.	0.91	0.79	1.02	2.63	0.	1.16
time (sec)	N/A	0.015	0.018	0.043	1.451	2.016	0.	1.116

Problem 2418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	27	35	155	0	35
normalized size	1	1.	1.	0.79	1.03	4.56	0.	1.03
time (sec)	N/A	0.009	0.009	0.044	1.412	2.096	0.	1.12

Problem 2419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	55	31	28	32	43	12	28
normalized size	1	2.04	1.15	1.04	1.19	1.59	0.44	1.04
time (sec)	N/A	0.012	0.012	0.106	1.469	2.032	0.127	1.105

Problem 2420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	55	31	28	32	42	12	28
normalized size	1	2.04	1.15	1.04	1.19	1.56	0.44	1.04
time (sec)	N/A	0.016	0.016	0.106	1.42	2.088	0.119	1.127

Problem 2421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	A	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	55	33	30	32	49	0	0
normalized size	1	2.04	1.22	1.11	1.19	1.81	0.	0.
time (sec)	N/A	0.013	0.01	0.103	1.468	2.091	0.	0.

Problem 2422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	A	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	55	33	30	32	47	0	0
normalized size	1	2.04	1.22	1.11	1.19	1.74	0.	0.
time (sec)	N/A	0.014	0.007	0.103	1.514	2.003	0.	0.

Problem 2423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	31	30	0	38	10	38
normalized size	1	1.	0.46	0.44	0.	0.56	0.15	0.56
time (sec)	N/A	0.022	0.016	0.16	0.	2.029	0.16	1.097

Problem 2424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	34	37	0	36	10	42
normalized size	1	1.	0.48	0.52	0.	0.51	0.14	0.59
time (sec)	N/A	0.024	0.019	0.238	0.	2.085	0.175	1.088

Problem 2425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	36	39	0	161	0	0
normalized size	1	1.	0.47	0.51	0.	2.09	0.	0.
time (sec)	N/A	0.024	0.013	0.194	0.	2.093	0.	0.

Problem 2426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	33	32	0	161	0	0
normalized size	1	1.	0.45	0.43	0.	2.18	0.	0.
time (sec)	N/A	0.026	0.013	0.192	0.	2.076	0.	0.

Problem 2427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	37	43	70	134	0	43
normalized size	1	1.	0.71	0.83	1.35	2.58	0.	0.83
time (sec)	N/A	0.014	0.017	0.042	1.497	2.08	0.	1.091

Problem 2428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	42	43	70	138	0	43
normalized size	1	1.	0.75	0.77	1.25	2.46	0.	0.77
time (sec)	N/A	0.014	0.02	0.041	1.478	2.039	0.	1.095

Problem 2429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	38	42	66	109	0	54
normalized size	1	1.	0.76	0.84	1.32	2.18	0.	1.08
time (sec)	N/A	0.015	0.019	0.042	1.592	1.969	0.	1.106

Problem 2430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	28	29	32	111	0	81
normalized size	1	1.	0.9	0.94	1.03	3.58	0.	2.61
time (sec)	N/A	0.012	0.008	0.042	1.568	2.006	0.	1.127

Problem 2431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	28	29	47	111	0	132
normalized size	1	1.	0.9	0.94	1.52	3.58	0.	4.26
time (sec)	N/A	0.011	0.008	0.042	1.494	2.064	0.	1.269

Problem 2432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	31	29	47	119	0	82
normalized size	1	1.	0.86	0.81	1.31	3.31	0.	2.28
time (sec)	N/A	0.011	0.01	0.041	1.527	2.169	0.	1.134

Problem 2433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	31	29	47	116	0	113
normalized size	1	1.	0.86	0.81	1.31	3.22	0.	3.14
time (sec)	N/A	0.013	0.01	0.04	1.504	2.112	0.	1.201

Problem 2434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	27	29	35	80	0	41
normalized size	1	1.	0.82	0.88	1.06	2.42	0.	1.24
time (sec)	N/A	0.011	0.009	0.042	1.544	2.069	0.	1.136

Problem 2435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	27	29	34	178	0	158
normalized size	1	1.	0.82	0.88	1.03	5.39	0.	4.79
time (sec)	N/A	0.011	0.008	0.043	1.466	2.001	0.	1.244

Problem 2436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	30	29	27	88	0	41
normalized size	1	1.	0.83	0.81	0.75	2.44	0.	1.14
time (sec)	N/A	0.011	0.009	0.048	1.511	2.056	0.	1.117

Problem 2437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	30	29	27	115	0	59
normalized size	1	1.	0.83	0.81	0.75	3.19	0.	1.64
time (sec)	N/A	0.011	0.01	0.042	1.5	2.035	0.	1.134

Problem 2438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	43	44	68	169	0	113
normalized size	1	1.	0.75	0.77	1.19	2.96	0.	1.98
time (sec)	N/A	0.024	0.018	0.042	1.488	2.047	0.	1.109

Problem 2439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	27	27	24	0	73	0	68
normalized size	1	1.17	1.17	1.04	0.	3.17	0.	2.96
time (sec)	N/A	0.015	0.055	0.041	0.	2.075	0.	1.149

Problem 2440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	504	504	594	2062	0	0	0	0
normalized size	1	1.	1.18	4.09	0.	0.	0.	0.
time (sec)	N/A	0.645	3.534	0.26	0.	0.	0.	0.

Problem 2441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	581	581	5328	6516	0	0	0	0
normalized size	1	1.	9.17	11.22	0.	0.	0.	0.
time (sec)	N/A	0.868	13.067	0.398	0.	0.	0.	0.

Problem 2442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	513	513	697	4356	0	0	0	0
normalized size	1	1.	1.36	8.49	0.	0.	0.	0.
time (sec)	N/A	0.492	10.782	0.349	0.	0.	0.	0.

Problem 2443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	444	444	936	1854	0	0	0	0
normalized size	1	1.	2.11	4.18	0.	0.	0.	0.
time (sec)	N/A	0.308	8.489	0.481	0.	0.	0.	0.

Problem 2444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	419	419	865	1595	0	0	0	0
normalized size	1	1.	2.06	3.81	0.	0.	0.	0.
time (sec)	N/A	0.266	4.699	0.427	0.	0.	0.	0.

Problem 2445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	497	497	1088	3645	0	0	0	0
normalized size	1	1.	2.19	7.33	0.	0.	0.	0.
time (sec)	N/A	0.394	10.244	0.366	0.	0.	0.	0.

Problem 2446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	617	617	3493	12980	0	0	0	0
normalized size	1	1.	5.66	21.04	0.	0.	0.	0.
time (sec)	N/A	0.625	12.757	0.424	0.	0.	0.	0.

Problem 2447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	816	816	10848	11933	0	0	0	0
normalized size	1	1.	13.29	14.62	0.	0.	0.	0.
time (sec)	N/A	2.836	13.961	0.407	0.	0.	0.	0.

Problem 2448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	712	712	7541	9177	0	0	0	0
normalized size	1	1.	10.59	12.89	0.	0.	0.	0.
time (sec)	N/A	1.273	13.684	0.332	0.	0.	0.	0.

Problem 2449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	579	579	5338	6516	0	0	0	0
normalized size	1	1.	9.22	11.25	0.	0.	0.	0.
time (sec)	N/A	0.672	13.14	0.314	0.	0.	0.	0.

Problem 2450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	515	515	732	4364	0	0	0	0
normalized size	1	1.	1.42	8.47	0.	0.	0.	0.
time (sec)	N/A	0.521	10.826	0.331	0.	0.	0.	0.

Problem 2451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	499	499	978	5874	0	0	0	0
normalized size	1	1.	1.96	11.77	0.	0.	0.	0.
time (sec)	N/A	0.452	8.973	0.395	0.	0.	0.	0.

Problem 2452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	578	578	3506	12946	0	0	0	0
normalized size	1	1.	6.07	22.4	0.	0.	0.	0.
time (sec)	N/A	0.519	12.793	0.376	0.	0.	0.	0.

Problem 2453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	721	721	5469	25722	0	0	0	0
normalized size	1	1.	7.59	35.68	0.	0.	0.	0.
time (sec)	N/A	0.881	13.4	0.441	0.	0.	0.	0.

Problem 2454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	616	616	708	2810	0	0	0	0
normalized size	1	1.	1.15	4.56	0.	0.	0.	0.
time (sec)	N/A	0.959	4.29	0.286	0.	0.	0.	0.

Problem 2455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	847	847	10879	12152	0	0	0	0
normalized size	1	1.	12.84	14.35	0.	0.	0.	0.
time (sec)	N/A	2.701	13.976	0.401	0.	0.	0.	0.

Problem 2456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	716	716	7946	9187	0	0	0	0
normalized size	1	1.	11.1	12.83	0.	0.	0.	0.
time (sec)	N/A	1.141	13.732	0.351	0.	0.	0.	0.

Problem 2457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	622	622	5407	12847	0	0	0	0
normalized size	1	1.	8.69	20.65	0.	0.	0.	0.
time (sec)	N/A	0.805	13.518	0.398	0.	0.	0.	0.

Problem 2458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	603	603	8961	14453	0	0	0	0
normalized size	1	1.	14.86	23.97	0.	0.	0.	0.
time (sec)	N/A	0.689	13.268	0.378	0.	0.	0.	0.

Problem 2459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	731	731	5482	25728	0	0	0	0
normalized size	1	1.	7.5	35.2	0.	0.	0.	0.
time (sec)	N/A	0.887	13.342	0.456	0.	0.	0.	0.

Problem 2460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	923	923	8108	44994	0	0	0	0
normalized size	1	1.	8.78	48.75	0.	0.	0.	0.
time (sec)	N/A	1.188	14.008	0.576	0.	0.	0.	0.

Problem 2461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	600	600	5340	6947	0	0	0	0
normalized size	1	1.	8.9	11.58	0.	0.	0.	0.
time (sec)	N/A	0.849	13.164	0.341	0.	0.	0.	0.

Problem 2462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	509	509	868	4786	0	0	0	0
normalized size	1	1.	1.71	9.4	0.	0.	0.	0.
time (sec)	N/A	0.541	9.089	0.33	0.	0.	0.	0.

Problem 2463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	439	439	609	2926	0	0	0	0
normalized size	1	1.	1.39	6.67	0.	0.	0.	0.
time (sec)	N/A	0.321	5.659	0.31	0.	0.	0.	0.

Problem 2464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	365	747	0	0	0	0
normalized size	1	1.	1.94	3.97	0.	0.	0.	0.
time (sec)	N/A	0.069	0.787	0.306	0.	0.	0.	0.

Problem 2465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	308	287	0	0	0	0
normalized size	1	1.	1.63	1.52	0.	0.	0.	0.
time (sec)	N/A	0.071	0.694	0.306	0.	0.	0.	0.

Problem 2466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	408	1365	0	0	0	0
normalized size	1	1.	1.65	5.5	0.	0.	0.	0.
time (sec)	N/A	0.129	0.793	0.334	0.	0.	0.	0.

Problem 2467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	523	523	643	5824	0	0	0	0
normalized size	1	1.	1.23	11.14	0.	0.	0.	0.
time (sec)	N/A	0.412	7.742	0.39	0.	0.	0.	0.

Problem 2468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	629	629	983	14311	0	0	0	0
normalized size	1	1.	1.56	22.75	0.	0.	0.	0.
time (sec)	N/A	0.687	9.092	0.412	0.	0.	0.	0.

Problem 2469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	641	641	5433	6486	0	0	0	0
normalized size	1	1.	8.48	10.12	0.	0.	0.	0.
time (sec)	N/A	0.949	12.98	0.404	0.	0.	0.	0.

Problem 2470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	533	533	732	4352	0	0	0	0
normalized size	1	1.	1.37	8.17	0.	0.	0.	0.
time (sec)	N/A	0.512	11.823	0.368	0.	0.	0.	0.

Problem 2471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	457	457	964	1863	0	0	0	0
normalized size	1	1.	2.11	4.08	0.	0.	0.	0.
time (sec)	N/A	0.329	8.41	0.345	0.	0.	0.	0.

Problem 2472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	426	426	996	1612	0	0	0	0
normalized size	1	1.	2.34	3.78	0.	0.	0.	0.
time (sec)	N/A	0.249	8.543	0.36	0.	0.	0.	0.

Problem 2473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	480	480	976	1894	0	0	0	0
normalized size	1	1.	2.03	3.95	0.	0.	0.	0.
time (sec)	N/A	0.32	8.929	0.369	0.	0.	0.	0.

Problem 2474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	607	607	784	4415	0	0	0	0
normalized size	1	1.	1.29	7.27	0.	0.	0.	0.
time (sec)	N/A	0.529	10.963	0.386	0.	0.	0.	0.

Problem 2475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	744	744	5565	12895	0	0	0	0
normalized size	1	1.	7.48	17.33	0.	0.	0.	0.
time (sec)	N/A	0.911	13.326	0.436	0.	0.	0.	0.

Problem 2476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	659	659	5598	19258	0	0	0	0
normalized size	1	1.	8.49	29.22	0.	0.	0.	0.
time (sec)	N/A	0.811	14.343	0.474	0.	0.	0.	0.

Problem 2477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	590	590	3577	12990	0	0	0	0
normalized size	1	1.	6.06	22.02	0.	0.	0.	0.
time (sec)	N/A	0.588	13.083	0.437	0.	0.	0.	0.

Problem 2478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	542	542	1141	8889	0	0	0	0
normalized size	1	1.	2.11	16.4	0.	0.	0.	0.
time (sec)	N/A	0.564	10.307	0.383	0.	0.	0.	0.

Problem 2479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	605	605	3560	13071	0	0	0	0
normalized size	1	1.	5.88	21.6	0.	0.	0.	0.
time (sec)	N/A	0.576	12.629	0.407	0.	0.	0.	0.

Problem 2480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	725	725	5566	19400	0	0	0	0
normalized size	1	1.	7.68	26.76	0.	0.	0.	0.
time (sec)	N/A	0.736	13.077	0.464	0.	0.	0.	0.

Problem 2481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	918	918	7870	27157	0	0	0	0
normalized size	1	1.	8.57	29.58	0.	0.	0.	0.
time (sec)	N/A	1.463	14.042	0.517	0.	0.	0.	0.

Problem 2482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	86	77	0	0	0	0
normalized size	1	1.	2.87	2.57	0.	0.	0.	0.
time (sec)	N/A	0.021	0.077	0.119	0.	0.	0.	0.

Problem 2483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	638	638	164	0	0	0	0	0
normalized size	1	1.	0.26	0.	0.	0.	0.	0.
time (sec)	N/A	1.072	0.384	1.145	0.	0.	0.	0.

Problem 2484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	539	539	113	0	0	0	0	0
normalized size	1	1.	0.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.514	0.222	1.012	0.	0.	0.	0.

Problem 2485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	490	490	95	0	0	0	0	0
normalized size	1	1.	0.19	0.	0.	0.	0.	0.
time (sec)	N/A	0.391	0.059	2.244	0.	0.	0.	0.

Problem 2486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	180	180	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.198	1.308	1.302	0.	0.	0.	0.

Problem 2487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	189	189	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.9	1.816	0.	0.	0.	0.

Problem 2488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	187	187	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	0.876	1.074	0.	0.	0.	0.

Problem 2489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1224	1224	446	0	0	0	0	0
normalized size	1	1.	0.36	0.	0.	0.	0.	0.
time (sec)	N/A	1.556	1.257	1.211	0.	0.	0.	0.

Problem 2490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1153	1153	342	0	0	0	0	0
normalized size	1	1.	0.3	0.	0.	0.	0.	0.
time (sec)	N/A	1.368	0.849	1.213	0.	0.	0.	0.

Problem 2491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1043	1043	200	0	0	0	0	0
normalized size	1	1.	0.19	0.	0.	0.	0.	0.
time (sec)	N/A	0.989	0.53	1.058	0.	0.	0.	0.

Problem 2492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	993	993	138	0	0	0	0	0
normalized size	1	1.	0.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.954	0.203	2.279	0.	0.	0.	0.

Problem 2493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	182	182	180	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	0.667	1.304	0.	0.	0.	0.

Problem 2494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	189	189	190	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	1.095	1.274	0.	0.	0.	0.

Problem 2495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	189	189	190	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.09	2.166	1.21	0.	0.	0.	0.

Problem 2496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	242	242	317	0	0	0	0	0
normalized size	1	1.	1.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.139	0.491	1.477	0.	0.	0.	0.

Problem 2497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	635	635	59	0	0	0	0	0
normalized size	1	1.	0.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.733	0.035	3.566	0.	0.	0.	0.

Problem 2498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	628	628	54	0	0	0	0	0
normalized size	1	1.	0.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.541	0.024	1.437	0.	0.	0.	0.

Problem 2499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	603	603	47	0	0	0	0	0
normalized size	1	1.	0.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.485	0.011	1.403	0.	0.	0.	0.

Problem 2500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	108	108	126	0	0	663	0	0
normalized size	1	1.	1.17	0.	0.	6.14	0.	0.
time (sec)	N/A	0.019	0.077	1.619	0.	22.499	0.	0.

Problem 2501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	719	719	243	0	0	0	0	0
normalized size	1	1.	0.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.605	0.285	1.412	0.	0.	0.	0.

Problem 2502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	744	744	233	0	0	0	0	0
normalized size	1	1.	0.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.675	0.297	1.249	0.	0.	0.	0.

Problem 2503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	589	589	53	0	0	0	0	0
normalized size	1	1.	0.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.563	0.031	3.254	0.	0.	0.	0.

Problem 2504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	585	585	47	0	0	0	0	0
normalized size	1	1.	0.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.486	0.018	1.506	0.	0.	0.	0.

Problem 2505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	560	560	41	0	0	0	0	0
normalized size	1	1.	0.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.378	0.007	1.478	0.	0.	0.	0.

Problem 2506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	127	0	0	626	0	0
normalized size	1	1.	1.23	0.	0.	6.08	0.	0.
time (sec)	N/A	0.019	0.077	1.678	0.	25.442	0.	0.

Problem 2507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	671	671	240	0	0	0	0	0
normalized size	1	1.	0.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.595	0.252	1.715	0.	0.	0.	0.

Problem 2508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	696	696	233	0	0	0	0	0
normalized size	1	1.	0.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.596	0.235	1.77	0.	0.	0.	0.

Problem 2509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	564	564	290	0	0	0	0	0
normalized size	1	1.	0.51	0.	0.	0.	0.	0.
time (sec)	N/A	0.405	0.422	1.313	0.	0.	0.	0.

Problem 2510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	374	374	235	0	0	0	0	0
normalized size	1	1.	0.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.518	0.925	1.073	0.	0.	0.	0.

Problem 2511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	191	0	0	0	0	0
normalized size	1	1.	0.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.423	0.46	0.978	0.	0.	0.	0.

Problem 2512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	140	0	0	0	0	0
normalized size	1	1.	0.58	0.	0.	0.	0.	0.
time (sec)	N/A	0.184	0.258	0.99	0.	0.	0.	0.

Problem 2513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	100	0	0	0	0	0
normalized size	1	1.	0.5	0.	0.	0.	0.	0.
time (sec)	N/A	0.131	0.243	2.113	0.	0.	0.	0.

Problem 2514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	881	881	617	0	0	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	2.638	2.273	1.296	0.	0.	0.	0.

Problem 2515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	944	944	646	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	2.026	2.484	1.272	0.	0.	0.	0.

Problem 2516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	703	703	234	0	0	0	0	0
normalized size	1	1.	0.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.867	0.637	1.04	0.	0.	0.	0.

Problem 2517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	630	630	196	0	0	0	0	0
normalized size	1	1.	0.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.848	0.328	1.011	0.	0.	0.	0.

Problem 2518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	510	510	141	0	0	0	0	0
normalized size	1	1.	0.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.439	0.243	1.049	0.	0.	0.	0.

Problem 2519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	452	452	119	0	0	0	0	0
normalized size	1	1.	0.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.362	0.095	2.293	0.	0.	0.	0.

Problem 2520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1209	1209	180	0	0	0	0	0
normalized size	1	1.	0.15	0.	0.	0.	0.	0.
time (sec)	N/A	2.513	0.424	1.278	0.	0.	0.	0.

Problem 2521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1220	1220	185	0	0	0	0	0
normalized size	1	1.	0.15	0.	0.	0.	0.	0.
time (sec)	N/A	2.179	0.339	1.217	0.	0.	0.	0.

Problem 2522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	448	448	286	0	0	0	0	0
normalized size	1	1.	0.64	0.	0.	0.	0.	0.
time (sec)	N/A	0.559	1.18	1.085	0.	0.	0.	0.

Problem 2523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	384	384	233	0	0	0	0	0
normalized size	1	1.	0.61	0.	0.	0.	0.	0.
time (sec)	N/A	0.541	0.605	1.01	0.	0.	0.	0.

Problem 2524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	175	0	0	0	0	0
normalized size	1	1.	0.61	0.	0.	0.	0.	0.
time (sec)	N/A	0.212	0.348	1.121	0.	0.	0.	0.

Problem 2525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	125	0	0	0	0	0
normalized size	1	1.	0.53	0.	0.	0.	0.	0.
time (sec)	N/A	0.157	0.378	2.196	0.	0.	0.	0.

Problem 2526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1014	1014	705	0	0	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	2.694	4.872	1.305	0.	0.	0.	0.

Problem 2527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	975	975	663	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	2.328	4.716	1.625	0.	0.	0.	0.

Problem 2528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	637	637	211	0	0	0	0	0
normalized size	1	1.	0.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.728	0.307	0.994	0.	0.	0.	0.

Problem 2529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	573	573	165	0	0	0	0	0
normalized size	1	1.	0.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.725	0.207	0.954	0.	0.	0.	0.

Problem 2530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	469	469	112	0	0	0	0	0
normalized size	1	1.	0.24	0.	0.	0.	0.	0.
time (sec)	N/A	0.381	0.123	1.037	0.	0.	0.	0.

Problem 2531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	418	84	0	0	0	0	0
normalized size	1	1.	0.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.305	0.033	2.052	0.	0.	0.	0.

Problem 2532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	733	733	178	0	0	0	0	0
normalized size	1	1.	0.24	0.	0.	0.	0.	0.
time (sec)	N/A	1.56	0.327	1.245	0.	0.	0.	0.

Problem 2533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1280	1280	187	0	0	0	0	0
normalized size	1	1.	0.15	0.	0.	0.	0.	0.
time (sec)	N/A	2.394	0.463	1.189	0.	0.	0.	0.

Problem 2534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1465	1465	187	0	0	0	0	0
normalized size	1	1.	0.13	0.	0.	0.	0.	0.
time (sec)	N/A	3.218	0.559	1.189	0.	0.	0.	0.

Problem 2535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	244	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.33	0.424	0.95	0.	0.	0.	0.

Problem 2536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	150	0	0	0	0	0
normalized size	1	1.	0.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.255	0.265	0.944	0.	0.	0.	0.

Problem 2537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	111	0	0	0	0	0
normalized size	1	1.	0.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.142	0.172	0.999	0.	0.	0.	0.

Problem 2538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	88	0	0	0	0	0
normalized size	1	1.	0.52	0.	0.	0.	0.	0.
time (sec)	N/A	0.112	0.046	2.028	0.	0.	0.	0.

Problem 2539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	709	709	533	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	1.574	1.74	1.242	0.	0.	0.	0.

Problem 2540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	970	970	657	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	2.014	3.038	1.252	0.	0.	0.	0.

Problem 2541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1134	1134	1696	0	0	0	0	0
normalized size	1	1.	1.5	0.	0.	0.	0.	0.
time (sec)	N/A	2.549	6.24	1.217	0.	0.	0.	0.

Problem 2542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	662	662	247	0	0	0	0	0
normalized size	1	1.	0.37	0.	0.	0.	0.	0.
time (sec)	N/A	0.724	0.571	2.526	0.	0.	0.	0.

Problem 2543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	594	594	177	0	0	0	0	0
normalized size	1	1.	0.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.737	0.337	1.151	0.	0.	0.	0.

Problem 2544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	490	490	167	0	0	0	0	0
normalized size	1	1.	0.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.397	0.23	0.957	0.	0.	0.	0.

Problem 2545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	451	451	100	0	0	0	0	0
normalized size	1	1.	0.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.361	0.071	2.167	0.	0.	0.	0.

Problem 2546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1299	1299	180	0	0	0	0	0
normalized size	1	1.	0.14	0.	0.	0.	0.	0.
time (sec)	N/A	2.518	0.551	1.271	0.	0.	0.	0.

Problem 2547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1485	1485	187	0	0	0	0	0
normalized size	1	1.	0.13	0.	0.	0.	0.	0.
time (sec)	N/A	3.17	0.937	1.247	0.	0.	0.	0.

Problem 2548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	235	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.154	0.265	1.284	0.	0.	0.	0.

Problem 2549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	485	485	1309	7696	0	13875	0	17743
normalized size	1	1.	2.7	15.87	0.	28.61	0.	36.58
time (sec)	N/A	0.386	4.382	0.083	0.	3.32	0.	2.197

Problem 2550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	476	2927	0	5573	0	7272
normalized size	1	1.	1.56	9.6	0.	18.27	0.	23.84
time (sec)	N/A	0.212	1.704	0.059	0.	1.98	0.	1.512

Problem 2551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	178	822	0	1897	9853	2318
normalized size	1	1.	1.	4.62	0.	10.66	55.35	13.02
time (sec)	N/A	0.114	0.4	0.052	0.	1.753	11.132	1.255

Problem 2552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	83	135	0	416	1416	477
normalized size	1	1.	1.01	1.65	0.	5.07	17.27	5.82
time (sec)	N/A	0.045	0.082	0.043	0.	1.592	2.038	1.166

Problem 2553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	163	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.384	0.452	1.28	0.	0.	0.	0.

Problem 2554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	425	425	339	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	1.069	1.289	1.241	0.	0.	0.	0.

Problem 2555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	189	189	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.252	0.122	1.225	0.	0.	0.	0.

Problem 2556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	189	189	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	0.068	1.331	0.	0.	0.	0.

Problem 2557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	189	189	207	0	0	0	0	0
normalized size	1	1.	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	0.214	1.208	0.	0.	0.	0.

Problem 2558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	189	189	207	0	0	0	0	0
normalized size	1	1.	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	0.193	1.22	0.	0.	0.	0.

Problem 2559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	189	189	267	0	0	0	0	0
normalized size	1	1.	1.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	0.375	1.202	0.	0.	0.	0.

Problem 2560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	189	189	225	0	0	0	0	0
normalized size	1	1.	1.19	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	0.355	1.259	0.	0.	0.	0.

Problem 2561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	137	137	160	0	0	0	0	0
normalized size	1	1.	1.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.278	0.272	1.112	0.	0.	0.	0.

Problem 2562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	187	187	205	0	0	0	0	0
normalized size	1	1.	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	0.401	1.395	0.	0.	0.	0.

Problem 2563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	327	327	558	0	0	0	0	0
normalized size	1	1.	1.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.423	1.521	1.213	0.	0.	0.	0.

Problem 2564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	248	248	414	0	0	0	0	0
normalized size	1	1.	1.67	0.	0.	0.	0.	0.
time (sec)	N/A	0.209	0.812	1.245	0.	0.	0.	0.

Problem 2565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	160	160	268	0	0	0	0	0
normalized size	1	1.	1.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.385	1.055	0.	0.	0.	0.

Problem 2566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	126	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.061	0.042	0.	0.	0.	0.

Problem 2567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	184	184	182	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.184	0.262	1.313	0.	0.	0.	0.

Problem 2568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	196	196	191	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	0.265	1.339	0.	0.	0.	0.

Problem 2569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	200	200	193	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	0.386	1.32	0.	0.	0.	0.

Problem 2570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	185	185	239	0	0	0	0	0
normalized size	1	1.	1.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.15	1.447	1.269	0.	0.	0.	0.

Problem 2571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	185	185	211	0	0	0	0	0
normalized size	1	1.	1.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	0.483	1.284	0.	0.	0.	0.

Problem 2572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	183	183	208	0	0	0	0	0
normalized size	1	1.	1.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	0.371	1.238	0.	0.	0.	0.

Problem 2573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	183	183	209	0	0	0	0	0
normalized size	1	1.	1.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	0.406	1.276	0.	0.	0.	0.

Problem 2574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	195	195	214	0	0	0	0	0
normalized size	1	1.	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	0.331	1.26	0.	0.	0.	0.

Problem 2575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	190	190	208	0	0	0	0	0
normalized size	1	1.	1.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	0.373	1.25	0.	0.	0.	0.

Problem 2576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	242	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.122	0.375	1.238	0.	0.	0.	0.

Problem 2577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	332	294	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.136	0.332	1.329	0.	0.	0.	0.

Problem 2578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	442	442	399	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.353	1.482	1.299	0.	0.	0.	0.

Problem 2579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	577	577	731	0	0	0	0	0
normalized size	1	1.	1.27	0.	0.	0.	0.	0.
time (sec)	N/A	0.868	5.974	1.311	0.	0.	0.	0.

Problem 2580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	809	809	1577	0	0	0	0	0
normalized size	1	1.	1.95	0.	0.	0.	0.	0.
time (sec)	N/A	1.72	6.274	1.268	0.	0.	0.	0.

Problem 2581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	440	440	379	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.385	3.783	1.258	0.	0.	0.	0.

Problem 2582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	132	0	0	333	0	0
normalized size	1	1.	1.29	0.	0.	3.26	0.	0.
time (sec)	N/A	0.017	0.07	1.582	0.	3.309	0.	0.

Problem 2583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	143	0	0	0	0	0
normalized size	1	1.	3.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.074	1.603	0.	0.	0.	0.

Problem 2584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	41	41	132	0	0	0	0	0
normalized size	1	1.	3.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.112	1.566	0.	0.	0.	0.

Problem 2585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	132	0	0	339	0	0
normalized size	1	1.	1.21	0.	0.	3.11	0.	0.
time (sec)	N/A	0.019	0.065	1.691	0.	3.569	0.	0.

Problem 2586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	145	0	0	0	0	0
normalized size	1	1.	3.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.086	1.712	0.	0.	0.	0.

Problem 2587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	41	41	133	0	0	0	0	0
normalized size	1	1.	3.24	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.121	1.783	0.	0.	0.	0.

Problem 2588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	196	196	241	0	0	0	0	0
normalized size	1	1.	1.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	0.191	3.069	0.	0.	0.	0.

Problem 2589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	258	0	0	0	0	0
normalized size	1	1.	3.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.217	2.985	0.	0.	0.	0.

Problem 2590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	67	67	243	0	0	0	0	0
normalized size	1	1.	3.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.307	3.016	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [2534] had the largest ratio of [0.8636]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	5	1.	17	0.294
2	A	5	5	1.	17	0.294
3	A	4	4	1.	15	0.267
4	A	3	3	1.	13	0.231
5	A	3	3	1.	17	0.176
6	A	3	3	1.	17	0.176
7	A	1	1	1.	17	0.059
8	A	2	2	1.	17	0.118
9	A	3	2	1.	17	0.118
10	A	4	2	1.	17	0.118
11	A	5	2	1.	17	0.118
12	A	6	5	1.	17	0.294
13	A	5	4	1.	15	0.267
14	A	4	3	1.	13	0.231
15	A	4	4	1.	17	0.235
16	A	4	3	1.	17	0.176
17	A	4	4	1.	17	0.235
18	A	4	3	1.	17	0.176
19	A	1	1	1.	17	0.059
20	A	2	2	1.	17	0.118
21	A	3	2	1.	17	0.118
22	A	4	2	1.	17	0.118
23	A	5	2	1.	17	0.118
24	A	7	5	1.	17	0.294
25	A	6	4	1.	15	0.267
26	A	5	3	1.	13	0.231
27	A	5	4	1.	17	0.235
28	A	5	4	1.	17	0.235
29	A	5	5	1.	17	0.294
30	A	5	4	1.	17	0.235
31	A	5	4	1.	17	0.235
32	A	5	3	1.	17	0.176
33	A	1	1	1.	17	0.059
34	A	2	2	1.	17	0.118
35	A	3	2	1.	17	0.118
36	A	4	2	1.	17	0.118
37	A	5	2	1.	17	0.118
38	A	6	2	1.	17	0.118
39	A	4	4	1.	15	0.267
40	A	4	4	1.	15	0.267
41	A	4	4	1.	11	0.364
42	A	6	4	1.	17	0.235
43	A	5	4	1.	17	0.235

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
44	A	4	4	1.	17	0.235
45	A	3	3	1.	15	0.2
46	A	2	2	1.	13	0.154
47	A	1	1	1.	17	0.059
48	A	2	2	1.	17	0.118
49	A	3	2	1.	17	0.118
50	A	4	2	1.	17	0.118
51	A	5	2	1.	17	0.118
52	A	5	5	1.	17	0.294
53	A	4	4	1.	17	0.235
54	A	3	3	1.	17	0.176
55	A	1	1	1.	15	0.067
56	A	1	1	1.	13	0.077
57	A	2	2	1.	17	0.118
58	A	3	2	1.	17	0.118
59	A	4	2	1.	17	0.118
60	A	6	5	1.	17	0.294
61	A	5	4	1.	17	0.235
62	A	4	4	1.	17	0.235
63	A	1	1	1.	17	0.059
64	A	2	2	1.	17	0.118
65	A	2	2	1.	15	0.133
66	A	2	2	1.	13	0.154
67	A	3	3	1.	17	0.176
68	A	4	3	1.	17	0.176
69	A	3	3	1.	15	0.2
70	A	3	3	1.	13	0.231
71	A	4	4	1.	17	0.235
72	A	5	2	1.	19	0.105
73	A	4	2	1.	19	0.105
74	A	3	2	1.	19	0.105
75	A	2	2	1.	19	0.105
76	A	1	1	1.	19	0.053
77	A	3	3	1.	19	0.158
78	A	3	3	1.	19	0.158
79	A	4	4	1.	19	0.21
80	A	5	4	1.	19	0.21
81	A	6	4	1.	19	0.21
82	A	6	2	1.	19	0.105
83	A	5	2	1.	19	0.105
84	A	4	2	1.	19	0.105
85	A	3	2	1.	19	0.105
86	A	2	2	1.	19	0.105
87	A	1	1	1.	19	0.053

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	4	3	1.	19	0.158
89	A	4	4	1.	19	0.21
90	A	4	3	1.	19	0.158
91	A	5	4	1.	19	0.21
92	A	6	4	1.	19	0.21
93	A	7	4	1.	19	0.21
94	A	4	2	1.	19	0.105
95	A	3	2	1.	19	0.105
96	A	2	2	1.	19	0.105
97	A	1	1	1.	19	0.053
98	A	2	2	1.	19	0.105
99	A	3	3	1.	19	0.158
100	A	4	3	1.	19	0.158
101	A	5	3	1.	19	0.158
102	A	6	2	1.	19	0.105
103	A	5	2	1.	19	0.105
104	A	4	2	1.	19	0.105
105	A	3	2	1.	19	0.105
106	A	2	2	1.	19	0.105
107	A	1	1	1.	19	0.053
108	A	3	3	1.	19	0.158
109	A	4	4	1.	19	0.21
110	A	5	4	1.	19	0.21
111	A	6	4	1.	19	0.21
112	A	3	2	1.	17	0.118
113	A	3	2	1.	17	0.118
114	A	2	1	1.	15	0.067
115	A	2	2	1.	17	0.118
116	A	2	2	1.	17	0.118
117	A	2	2	1.	17	0.118
118	A	3	3	1.	19	0.158
119	A	3	3	1.	19	0.158
120	A	3	3	1.	19	0.158
121	A	3	3	1.	19	0.158
122	A	3	3	1.	19	0.158
123	A	3	3	1.	19	0.158
124	A	3	3	1.	17	0.176
125	A	3	3	1.	15	0.2
126	A	3	3	1.	15	0.2
127	A	2	2	1.69	13	0.154
128	A	3	3	1.	15	0.2
129	A	3	3	1.	15	0.2
130	A	3	3	1.	15	0.2
131	A	3	3	1.	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
132	A	3	3	1.	19	0.158
133	A	3	3	1.	19	0.158
134	A	3	3	1.	19	0.158
135	A	3	3	1.	19	0.158
136	A	3	3	1.	19	0.158
137	A	3	2	1.	24	0.083
138	A	3	2	1.	24	0.083
139	A	3	2	1.	24	0.083
140	A	2	2	1.	22	0.091
141	A	1	1	1.	20	0.05
142	A	3	2	1.	24	0.083
143	A	3	2	1.	24	0.083
144	A	2	2	1.	24	0.083
145	A	3	2	1.	24	0.083
146	A	3	2	1.	24	0.083
147	A	3	2	1.	24	0.083
148	A	3	2	1.	24	0.083
149	A	3	2	1.	24	0.083
150	A	3	2	1.	24	0.083
151	A	2	1	1.11	24	0.042
152	A	2	2	1.	22	0.091
153	A	1	1	1.	20	0.05
154	A	3	2	1.	24	0.083
155	A	3	2	1.	24	0.083
156	A	3	2	1.	24	0.083
157	A	3	2	1.	24	0.083
158	A	2	2	1.	24	0.083
159	A	3	3	1.	24	0.125
160	A	3	2	1.	24	0.083
161	A	3	2	1.	24	0.083
162	A	3	2	1.	24	0.083
163	A	3	2	1.	24	0.083
164	A	2	1	1.	24	0.042
165	A	3	2	1.	24	0.083
166	A	3	2	1.	24	0.083
167	A	2	2	1.	22	0.091
168	A	1	1	1.	20	0.05
169	A	3	2	1.	24	0.083
170	A	3	2	1.	24	0.083
171	A	3	2	1.	24	0.083
172	A	3	2	1.	24	0.083
173	A	3	2	1.	24	0.083
174	A	3	2	1.	24	0.083
175	A	2	2	1.	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
176	A	3	3	1.	24	0.125
177	A	4	3	1.	24	0.125
178	A	3	2	1.	24	0.083
179	A	3	2	1.	24	0.083
180	A	3	2	1.	24	0.083
181	A	3	2	1.	24	0.083
182	A	3	2	1.	24	0.083
183	A	3	2	1.	24	0.083
184	A	3	3	1.	22	0.136
185	A	2	2	1.	20	0.1
186	A	4	4	1.	24	0.167
187	A	3	2	1.	24	0.083
188	A	3	2	1.	24	0.083
189	A	3	2	1.	24	0.083
190	A	3	2	1.	24	0.083
191	A	3	2	1.	24	0.083
192	A	3	2	1.	24	0.083
193	A	2	2	1.	22	0.091
194	A	1	1	1.	20	0.05
195	A	3	2	1.	24	0.083
196	A	3	2	1.	24	0.083
197	A	3	2	1.	24	0.083
198	A	3	2	1.	24	0.083
199	A	3	2	1.	24	0.083
200	A	3	2	1.	24	0.083
201	A	2	2	1.	24	0.083
202	A	3	2	1.	24	0.083
203	A	2	2	1.	22	0.091
204	A	1	1	1.	20	0.05
205	A	3	2	1.	24	0.083
206	A	3	2	1.	24	0.083
207	A	3	2	1.	24	0.083
208	A	2	2	1.	16	0.125
209	A	2	2	1.	16	0.125
210	A	2	2	1.	16	0.125
211	A	3	3	1.	16	0.188
212	A	2	2	1.	16	0.125
213	A	2	2	1.	16	0.125
214	A	2	2	1.	16	0.125
215	A	3	3	1.	16	0.188
216	A	3	3	1.	16	0.188
217	A	3	3	1.	16	0.188
218	A	3	3	1.	16	0.188
219	A	1	1	1.	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
220	A	1	1	1.	18	0.056
221	A	2	1	1.	17	0.059
222	A	2	1	1.	17	0.059
223	A	2	1	1.	17	0.059
224	A	2	1	1.	15	0.067
225	A	1	0	1.	9	0.
226	A	2	1	1.	17	0.059
227	A	2	1	1.	17	0.059
228	A	2	1	1.	17	0.059
229	A	2	1	1.	17	0.059
230	A	2	1	1.	17	0.059
231	A	2	1	1.	19	0.053
232	A	2	1	1.	19	0.053
233	A	2	1	1.	19	0.053
234	A	2	1	1.	17	0.059
235	A	2	1	1.	11	0.091
236	A	2	1	1.	19	0.053
237	A	2	1	1.	19	0.053
238	A	2	1	1.	19	0.053
239	A	2	1	1.	19	0.053
240	A	2	1	1.	19	0.053
241	A	2	1	1.	19	0.053
242	A	2	1	1.	19	0.053
243	A	2	1	1.	19	0.053
244	A	2	1	1.	19	0.053
245	A	2	1	1.	19	0.053
246	A	2	1	1.	19	0.053
247	A	2	1	1.	17	0.059
248	A	2	1	1.	11	0.091
249	A	2	1	1.	19	0.053
250	A	2	1	1.	19	0.053
251	A	2	1	1.	19	0.053
252	A	2	1	1.	19	0.053
253	A	2	1	1.	19	0.053
254	A	2	1	1.	19	0.053
255	A	2	1	1.	19	0.053
256	A	2	1	1.	19	0.053
257	A	2	1	1.	19	0.053
258	A	2	1	1.	19	0.053
259	A	2	1	1.	19	0.053
260	A	2	1	1.	19	0.053
261	A	2	1	1.	19	0.053
262	A	2	1	1.	17	0.059
263	A	1	1	1.	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
264	A	2	1	1.	19	0.053
265	A	2	1	1.	19	0.053
266	A	2	1	1.	19	0.053
267	A	2	1	1.	19	0.053
268	A	2	1	1.	19	0.053
269	A	2	1	1.	19	0.053
270	A	2	1	1.	19	0.053
271	A	2	1	1.	17	0.059
272	A	2	2	1.	11	0.182
273	A	2	1	1.	19	0.053
274	A	2	1	1.	19	0.053
275	A	2	1	1.	19	0.053
276	A	2	1	1.	19	0.053
277	A	2	1	1.	19	0.053
278	A	2	1	1.	19	0.053
279	A	2	1	1.	19	0.053
280	A	2	1	1.	19	0.053
281	A	2	1	1.	17	0.059
282	A	3	2	1.	11	0.182
283	A	2	1	1.	19	0.053
284	A	2	1	1.	19	0.053
285	A	5	5	1.	21	0.238
286	A	5	5	1.	21	0.238
287	A	4	4	1.	19	0.21
288	A	3	3	1.	13	0.231
289	A	6	5	1.	21	0.238
290	A	6	5	1.	21	0.238
291	A	3	3	1.	21	0.143
292	A	4	4	1.	21	0.19
293	A	5	5	1.	21	0.238
294	A	6	6	1.	21	0.286
295	A	6	5	1.	21	0.238
296	A	6	5	1.	21	0.238
297	A	5	4	1.	19	0.21
298	A	4	3	1.	13	0.231
299	A	7	6	1.	21	0.286
300	A	7	6	1.	21	0.286
301	A	7	6	1.	21	0.286
302	A	7	5	1.	21	0.238
303	A	7	5	1.	21	0.238
304	A	6	4	1.	19	0.21
305	A	5	3	1.	13	0.231
306	A	8	6	1.	21	0.286
307	A	8	6	1.	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
308	A	8	7	1.	21	0.333
309	A	3	3	1.	17	0.176
310	A	4	3	1.	21	0.143
311	A	3	3	1.	21	0.143
312	A	2	2	1.	21	0.095
313	A	3	3	1.	21	0.143
314	A	4	3	1.	21	0.143
315	A	4	4	1.	21	0.19
316	A	4	4	1.	21	0.19
317	A	3	3	1.	19	0.158
318	A	2	2	1.	13	0.154
319	A	2	2	1.	21	0.095
320	A	3	3	1.	21	0.143
321	A	4	4	1.	21	0.19
322	A	4	4	1.	21	0.19
323	A	4	4	1.	21	0.19
324	A	1	1	1.	19	0.053
325	A	1	1	1.	13	0.077
326	A	4	4	1.	21	0.19
327	A	4	4	1.	21	0.19
328	A	5	5	1.	21	0.238
329	A	5	5	1.	21	0.238
330	A	2	2	1.	21	0.095
331	A	2	2	1.	21	0.095
332	A	2	2	1.	19	0.105
333	A	2	2	1.	13	0.154
334	A	5	5	1.	21	0.238
335	A	5	5	1.	21	0.238
336	A	1	1	1.	17	0.059
337	A	2	1	1.	19	0.053
338	A	2	1	1.	19	0.053
339	A	2	1	1.	19	0.053
340	A	2	1	1.	19	0.053
341	A	2	1	1.	19	0.053
342	A	2	1	1.	19	0.053
343	A	2	1	1.	19	0.053
344	A	2	1	1.	19	0.053
345	A	2	1	1.	21	0.048
346	A	2	1	1.	21	0.048
347	A	2	1	1.	21	0.048
348	A	2	1	1.	21	0.048
349	A	2	1	1.	21	0.048
350	A	2	1	1.	21	0.048
351	A	2	1	1.	21	0.048

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
352	A	2	1	1.	21	0.048
353	A	2	1	1.	21	0.048
354	A	2	1	1.	21	0.048
355	A	2	1	1.	21	0.048
356	A	2	1	1.	21	0.048
357	A	2	1	1.	21	0.048
358	A	2	1	1.	21	0.048
359	A	2	1	1.	21	0.048
360	A	2	1	1.	21	0.048
361	A	7	5	1.	21	0.238
362	A	6	5	1.	21	0.238
363	A	5	4	1.	21	0.19
364	A	4	3	1.	21	0.143
365	A	4	3	1.	21	0.143
366	A	5	4	1.	21	0.19
367	A	6	5	1.	21	0.238
368	A	7	5	1.	21	0.238
369	A	8	5	1.	21	0.238
370	A	7	5	1.	21	0.238
371	A	6	5	1.	21	0.238
372	A	5	4	1.	21	0.19
373	A	5	4	1.	21	0.19
374	A	5	4	1.	21	0.19
375	A	6	5	1.	21	0.238
376	A	7	5	1.	21	0.238
377	A	8	5	1.	21	0.238
378	A	7	6	1.	21	0.286
379	A	6	5	1.	21	0.238
380	A	6	5	1.	21	0.238
381	A	6	5	1.	21	0.238
382	A	6	5	1.	21	0.238
383	A	6	5	1.	21	0.238
384	A	7	6	1.	21	0.286
385	A	8	6	1.	21	0.286
386	A	9	8	1.	23	0.348
387	A	9	8	1.	23	0.348
388	A	8	7	1.	23	0.304
389	A	8	7	1.	23	0.304
390	A	9	8	1.	23	0.348
391	A	10	8	1.	23	0.348
392	A	10	8	1.	23	0.348
393	A	10	9	1.	23	0.391
394	A	9	8	1.	23	0.348
395	A	9	8	1.	23	0.348

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
396	A	9	8	1.	23	0.348
397	A	9	8	1.	23	0.348
398	A	10	9	1.	23	0.391
399	A	11	9	1.	23	0.391
400	A	10	8	1.	23	0.348
401	A	10	8	1.	23	0.348
402	A	10	9	1.	23	0.391
403	A	10	8	1.	23	0.348
404	A	10	9	1.	23	0.391
405	A	10	8	1.	23	0.348
406	A	10	8	1.	23	0.348
407	A	9	8	1.	23	0.348
408	A	8	7	1.	23	0.304
409	A	3	3	1.	23	0.13
410	A	3	3	1.	23	0.13
411	A	5	5	1.	23	0.217
412	A	9	8	1.	23	0.348
413	A	10	8	1.	23	0.348
414	A	10	8	1.	23	0.348
415	A	9	8	1.	23	0.348
416	A	8	7	1.	23	0.304
417	A	8	7	1.	23	0.304
418	A	8	7	1.	23	0.304
419	A	9	8	1.	23	0.348
420	A	10	8	1.	23	0.348
421	A	10	9	1.	23	0.391
422	A	9	8	1.	23	0.348
423	A	9	8	1.	23	0.348
424	A	9	8	1.	23	0.348
425	A	9	8	1.	23	0.348
426	A	9	8	1.	23	0.348
427	A	10	9	1.	23	0.391
428	A	4	4	1.	23	0.174
429	A	4	4	1.	23	0.174
430	A	4	4	1.	23	0.174
431	A	4	4	1.	23	0.174
432	A	1	1	1.	24	0.042
433	A	2	2	1.	23	0.087
434	A	4	3	1.	21	0.143
435	A	4	3	1.	21	0.143
436	A	4	3	1.	19	0.158
437	A	1	1	1.	7	0.143
438	A	3	3	1.	21	0.143
439	A	3	3	1.	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
440	A	2	1	1.	19	0.053
441	A	2	1	1.	19	0.053
442	A	2	1	1.	17	0.059
443	A	4	3	1.	19	0.158
444	A	5	4	1.	19	0.21
445	A	6	5	1.	19	0.263
446	A	2	2	1.	21	0.095
447	A	2	2	1.	21	0.095
448	A	2	2	1.	21	0.095
449	A	2	2	1.	21	0.095
450	A	2	2	1.	19	0.105
451	A	2	1	1.	15	0.067
452	A	2	1	1.	15	0.067
453	A	2	1	1.	15	0.067
454	A	2	1	1.	13	0.077
455	A	2	1	1.	15	0.067
456	A	2	1	1.	15	0.067
457	A	2	1	1.	15	0.067
458	A	2	1	1.	15	0.067
459	A	2	1	1.	15	0.067
460	A	2	1	1.	17	0.059
461	A	2	1	1.	17	0.059
462	A	3	2	1.	17	0.118
463	A	3	2	1.	15	0.133
464	A	2	1	1.	17	0.059
465	A	2	1	1.	17	0.059
466	A	2	1	1.	17	0.059
467	A	2	1	1.	17	0.059
468	A	2	1	1.	17	0.059
469	A	2	1	1.	17	0.059
470	A	2	1	1.	17	0.059
471	A	2	1	1.	17	0.059
472	A	2	1	1.	17	0.059
473	A	2	1	1.	17	0.059
474	A	2	1	1.	17	0.059
475	A	3	2	1.	17	0.118
476	A	3	2	1.	17	0.118
477	A	3	2	1.	15	0.133
478	A	2	1	1.	17	0.059
479	A	2	1	1.	17	0.059
480	A	2	1	1.	17	0.059
481	A	2	1	1.	17	0.059
482	A	2	1	1.	17	0.059
483	A	2	1	1.	17	0.059

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
484	A	2	1	1.	17	0.059
485	A	2	1	1.	17	0.059
486	A	2	1	1.	17	0.059
487	A	2	1	1.	17	0.059
488	A	2	1	1.	17	0.059
489	A	2	1	1.	17	0.059
490	A	2	1	1.	17	0.059
491	A	3	2	1.	17	0.118
492	A	3	2	1.	17	0.118
493	A	3	2	1.	17	0.118
494	A	3	2	1.	15	0.133
495	A	2	1	1.	17	0.059
496	A	2	1	1.	17	0.059
497	A	3	2	1.	15	0.133
498	A	5	4	1.	17	0.235
499	A	5	4	1.	17	0.235
500	A	5	4	1.	17	0.235
501	A	3	3	1.	15	0.2
502	A	5	5	1.	17	0.294
503	A	6	5	1.	17	0.294
504	A	6	5	1.	17	0.294
505	A	6	5	1.	17	0.294
506	A	6	5	1.	17	0.294
507	A	5	5	1.	17	0.294
508	A	2	2	1.	17	0.118
509	A	2	2	1.	15	0.133
510	A	6	5	1.	17	0.294
511	A	6	5	1.	17	0.294
512	A	6	6	1.	17	0.353
513	A	3	2	1.	17	0.118
514	A	3	3	1.	17	0.176
515	A	3	3	1.	17	0.176
516	A	3	3	1.	15	0.2
517	A	7	6	1.	17	0.353
518	A	7	6	1.	17	0.353
519	A	4	4	1.	17	0.235
520	A	4	4	1.	17	0.235
521	A	4	4	1.	17	0.235
522	A	4	3	1.	15	0.2
523	A	8	6	1.	17	0.353
524	A	8	6	1.	17	0.353
525	A	6	6	1.	19	0.316
526	A	5	5	1.	19	0.263
527	A	5	5	1.	19	0.263

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
528	A	4	4	1.	17	0.235
529	A	6	5	1.	19	0.263
530	A	6	5	1.	19	0.263
531	A	3	3	1.	19	0.158
532	A	4	4	1.	19	0.21
533	A	5	5	1.	19	0.263
534	A	7	6	1.	19	0.316
535	A	6	5	1.	19	0.263
536	A	6	5	1.	19	0.263
537	A	5	4	1.	17	0.235
538	A	7	6	1.	19	0.316
539	A	7	6	1.	19	0.316
540	A	7	6	1.	19	0.316
541	A	7	6	1.	19	0.316
542	A	4	3	1.	19	0.158
543	A	5	4	1.	19	0.21
544	A	6	5	1.	19	0.263
545	A	8	6	1.	19	0.316
546	A	7	5	1.	19	0.263
547	A	7	5	1.	19	0.263
548	A	6	4	1.	17	0.235
549	A	8	6	1.	19	0.316
550	A	8	6	1.	19	0.316
551	A	8	7	1.	19	0.368
552	A	8	6	1.	19	0.316
553	A	8	7	1.	19	0.368
554	A	8	6	1.	19	0.316
555	A	5	3	1.	19	0.158
556	A	6	4	1.	19	0.21
557	A	7	5	1.	19	0.263
558	A	5	5	1.	17	0.294
559	A	5	5	1.	19	0.263
560	A	4	4	1.	17	0.235
561	A	5	5	1.	19	0.263
562	A	4	4	1.	19	0.21
563	A	4	4	1.	19	0.21
564	A	3	3	1.	17	0.176
565	A	2	2	1.	19	0.105
566	A	3	3	1.	19	0.158
567	A	4	4	1.	19	0.21
568	A	5	5	1.	19	0.263
569	A	5	5	1.	19	0.263
570	A	4	4	1.	19	0.21
571	A	4	4	1.	19	0.21

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
572	A	1	1	1.	17	0.059
573	A	4	4	1.	19	0.21
574	A	4	4	1.	19	0.21
575	A	5	5	1.	19	0.263
576	A	6	5	1.	19	0.263
577	A	5	5	1.	19	0.263
578	A	5	5	1.	19	0.263
579	A	2	2	1.	19	0.105
580	A	2	2	1.	19	0.105
581	A	2	2	1.	17	0.118
582	A	5	5	1.	19	0.263
583	A	5	5	1.	19	0.263
584	A	6	6	1.	19	0.316
585	A	2	2	1.	15	0.133
586	A	2	2	1.	15	0.133
587	A	2	2	1.	13	0.154
588	A	3	3	1.	19	0.158
589	A	3	3	1.	17	0.176
590	A	1	1	1.	15	0.067
591	A	2	1	1.	17	0.059
592	A	2	1	1.	17	0.059
593	A	2	1	1.	17	0.059
594	A	2	1	1.	17	0.059
595	A	2	1	1.	17	0.059
596	A	2	1	1.	17	0.059
597	A	2	1	1.	17	0.059
598	A	2	1	1.	19	0.053
599	A	2	1	1.	19	0.053
600	A	2	1	1.	19	0.053
601	A	2	1	1.	19	0.053
602	A	2	1	1.	19	0.053
603	A	2	1	1.	19	0.053
604	A	2	1	1.	19	0.053
605	A	2	1	1.	19	0.053
606	A	2	1	1.	19	0.053
607	A	2	1	1.	19	0.053
608	A	2	1	1.	19	0.053
609	A	2	1	1.	19	0.053
610	A	2	1	1.	19	0.053
611	A	2	1	1.	19	0.053
612	A	6	5	1.	20	0.25
613	A	5	4	1.	20	0.2
614	A	4	3	1.	20	0.15
615	A	4	3	1.	20	0.15

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
616	A	5	4	1.	20	0.2
617	A	6	5	1.	20	0.25
618	A	12	8	1.	19	0.421
619	A	11	7	1.	19	0.368
620	A	10	6	1.	19	0.316
621	A	10	6	1.	19	0.316
622	A	11	7	1.	19	0.368
623	A	12	8	1.	19	0.421
624	A	7	5	1.	20	0.25
625	A	6	5	1.	20	0.25
626	A	5	4	1.	20	0.2
627	A	5	4	1.	20	0.2
628	A	5	4	1.	20	0.2
629	A	6	5	1.	20	0.25
630	A	7	5	1.	20	0.25
631	A	13	8	1.	19	0.421
632	A	12	8	1.	19	0.421
633	A	11	7	1.	19	0.368
634	A	11	7	1.	19	0.368
635	A	11	7	1.	19	0.368
636	A	12	8	1.	19	0.421
637	A	13	8	1.	19	0.421
638	A	6	5	1.	20	0.25
639	A	6	5	1.	20	0.25
640	A	6	5	1.	20	0.25
641	A	6	5	1.	20	0.25
642	A	6	5	1.	20	0.25
643	A	12	8	1.	19	0.421
644	A	12	8	1.	19	0.421
645	A	12	8	1.	19	0.421
646	A	12	8	1.	19	0.421
647	A	12	8	1.	19	0.421
648	A	10	7	1.	17	0.412
649	A	10	6	1.	17	0.353
650	A	4	4	1.	19	0.21
651	A	4	3	1.	19	0.158
652	A	10	6	1.	19	0.316
653	A	4	4	1.	20	0.2
654	A	10	7	1.	15	0.467
655	A	10	6	1.	15	0.4
656	A	12	8	1.	15	0.533
657	A	7	6	1.	21	0.286
658	A	7	6	1.	21	0.286
659	A	6	5	1.	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
660	A	6	5	1.	21	0.238
661	A	7	6	1.	21	0.286
662	A	8	6	1.	21	0.286
663	A	8	6	1.	21	0.286
664	A	8	7	1.	21	0.333
665	A	7	6	1.	21	0.286
666	A	7	6	1.	21	0.286
667	A	7	6	1.	21	0.286
668	A	7	6	1.	21	0.286
669	A	8	7	1.	21	0.333
670	A	9	7	1.	21	0.333
671	A	8	6	1.	21	0.286
672	A	8	6	1.	21	0.286
673	A	8	7	1.	21	0.333
674	A	8	6	1.	21	0.286
675	A	8	7	1.	21	0.333
676	A	8	6	1.	21	0.286
677	A	8	6	1.	21	0.286
678	A	7	6	1.	21	0.286
679	A	6	5	1.	21	0.238
680	A	2	2	1.	21	0.095
681	A	2	2	1.	21	0.095
682	A	4	4	1.	21	0.19
683	A	7	6	1.	21	0.286
684	A	8	6	1.	21	0.286
685	A	8	6	1.	21	0.286
686	A	7	6	1.	21	0.286
687	A	6	5	1.	21	0.238
688	A	7	6	1.	21	0.286
689	A	6	5	1.	21	0.238
690	A	7	6	1.	21	0.286
691	A	8	6	1.	21	0.286
692	A	8	7	1.	21	0.333
693	A	7	6	1.	21	0.286
694	A	7	6	1.	21	0.286
695	A	7	6	1.	21	0.286
696	A	7	6	1.	21	0.286
697	A	7	6	1.	21	0.286
698	A	8	7	1.	21	0.333
699	A	1	1	1.	24	0.042
700	A	6	6	1.	19	0.316
701	A	6	6	1.	19	0.316
702	A	5	5	1.	17	0.294
703	A	1	1	1.	19	0.053

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
704	A	7	7	1.	19	0.368
705	A	8	8	1.	19	0.421
706	A	6	6	1.	21	0.286
707	A	6	6	1.	21	0.286
708	A	5	5	1.	19	0.263
709	A	1	1	1.	21	0.048
710	A	7	7	1.	21	0.333
711	A	8	8	1.	21	0.381
712	A	1	1	1.	19	0.053
713	A	1	1	1.	21	0.048
714	A	2	2	1.	19	0.105
715	A	2	2	1.	17	0.118
716	A	3	3	1.	24	0.125
717	A	10	9	1.	19	0.474
718	A	11	10	1.	19	0.526
719	A	1	1	1.	21	0.048
720	A	15	10	1.	15	0.667
721	A	2	1	1.	17	0.059
722	A	2	1	1.	17	0.059
723	A	2	1	1.	15	0.067
724	A	4	2	1.	17	0.118
725	A	5	3	1.	17	0.176
726	A	6	4	1.	17	0.235
727	A	2	2	1.	19	0.105
728	A	2	2	1.	19	0.105
729	A	2	2	1.	19	0.105
730	A	2	2	1.	19	0.105
731	A	2	2	1.	17	0.118
732	A	4	4	0.95	17	0.235
733	A	4	4	0.94	17	0.235
734	A	3	3	1.15	15	0.2
735	A	2	2	1.26	9	0.222
736	A	6	5	1.	17	0.294
737	A	8	7	1.	17	0.412
738	A	11	9	1.	17	0.529
739	A	2	2	1.	19	0.105
740	A	2	2	1.	21	0.095
741	A	1	1	1.	21	0.048
742	A	2	2	1.	21	0.095
743	A	3	3	1.	21	0.143
744	A	4	4	1.	21	0.19
745	A	5	4	1.	21	0.19
746	A	2	2	1.	19	0.105
747	A	3	2	1.	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
748	A	3	2	1.	22	0.091
749	A	3	2	1.	22	0.091
750	A	3	2	1.	22	0.091
751	A	3	2	1.	22	0.091
752	A	2	2	1.	20	0.1
753	A	4	3	1.	22	0.136
754	A	4	3	1.	22	0.136
755	A	4	3	1.	22	0.136
756	A	4	3	1.	22	0.136
757	A	3	2	1.	22	0.091
758	A	3	2	1.	22	0.091
759	A	3	2	1.	22	0.091
760	A	3	2	1.	22	0.091
761	A	3	2	1.	22	0.091
762	A	2	2	1.	22	0.091
763	A	4	3	1.	20	0.15
764	A	4	3	1.	22	0.136
765	A	4	3	1.	22	0.136
766	A	4	3	1.	22	0.136
767	A	3	2	1.	22	0.091
768	A	3	2	1.	22	0.091
769	A	3	2	1.	22	0.091
770	A	3	2	1.	22	0.091
771	A	2	2	1.	22	0.091
772	A	2	2	1.	22	0.091
773	A	4	3	1.	22	0.136
774	A	4	3	1.	20	0.15
775	A	4	3	1.	22	0.136
776	A	4	3	1.	22	0.136
777	A	7	5	1.	24	0.208
778	A	6	5	1.	24	0.208
779	A	5	5	1.	24	0.208
780	A	4	4	1.	22	0.182
781	A	3	3	1.	24	0.125
782	A	3	3	1.	24	0.125
783	A	1	1	1.	24	0.042
784	A	2	2	1.	24	0.083
785	A	3	2	1.	24	0.083
786	A	4	2	1.	24	0.083
787	A	5	2	1.	24	0.083
788	A	7	5	1.	24	0.208
789	A	6	5	1.	24	0.208
790	A	5	4	1.	22	0.182
791	A	4	4	1.	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
792	A	4	3	1.	24	0.125
793	A	4	4	1.	24	0.167
794	A	4	3	1.	24	0.125
795	A	1	1	1.	24	0.042
796	A	2	2	1.	24	0.083
797	A	3	2	1.	24	0.083
798	A	4	2	1.	24	0.083
799	A	5	2	1.	24	0.083
800	A	9	5	1.	24	0.208
801	A	8	5	1.	24	0.208
802	A	7	4	1.	22	0.182
803	A	6	4	1.	24	0.167
804	A	7	6	1.	24	0.25
805	A	7	6	1.	24	0.25
806	A	7	7	1.	24	0.292
807	A	6	5	1.	24	0.208
808	A	6	4	1.	24	0.167
809	A	6	4	1.	24	0.167
810	A	6	3	1.	24	0.125
811	A	1	1	1.	24	0.042
812	A	2	2	1.	24	0.083
813	A	3	2	1.	24	0.083
814	A	4	2	1.	24	0.083
815	A	5	2	1.	24	0.083
816	A	3	3	1.	25	0.12
817	A	5	5	1.	27	0.185
818	A	6	5	1.	27	0.185
819	A	4	4	1.	13	0.308
820	A	3	3	1.	15	0.2
821	A	2	2	1.	17	0.118
822	A	3	3	1.	17	0.176
823	A	2	2	1.	19	0.105
824	A	2	2	1.	19	0.105
825	A	1	1	1.	19	0.053
826	A	7	4	1.	24	0.167
827	A	6	4	1.	24	0.167
828	A	5	4	1.	24	0.167
829	A	4	4	1.12	24	0.167
830	A	3	3	1.	22	0.136
831	A	1	1	1.	24	0.042
832	A	2	2	1.	24	0.083
833	A	3	2	1.	24	0.083
834	A	4	2	1.	24	0.083
835	A	5	2	1.	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
836	A	6	5	1.	24	0.208
837	A	5	4	1.	24	0.167
838	A	4	4	1.	24	0.167
839	A	1	1	1.	24	0.042
840	A	2	2	1.	24	0.083
841	A	2	2	1.	22	0.091
842	A	3	3	1.	24	0.125
843	A	4	3	1.	24	0.125
844	A	5	3	1.	24	0.125
845	A	6	3	1.	24	0.125
846	A	8	5	1.	24	0.208
847	A	7	5	1.	24	0.208
848	A	6	4	1.	24	0.167
849	A	5	4	1.	24	0.167
850	A	1	1	1.	24	0.042
851	A	2	2	1.	24	0.083
852	A	4	3	1.	24	0.125
853	A	3	3	1.	24	0.125
854	A	3	3	1.	22	0.136
855	A	4	3	1.	24	0.125
856	A	5	3	1.	24	0.125
857	A	6	3	1.	24	0.125
858	A	7	3	1.	24	0.125
859	A	8	3	1.	24	0.125
860	A	2	2	1.	15	0.133
861	A	2	2	1.	17	0.118
862	A	4	2	1.	29	0.069
863	A	3	2	1.	29	0.069
864	A	2	2	1.	29	0.069
865	A	1	1	1.	29	0.034
866	A	3	3	1.	29	0.103
867	A	3	3	1.	29	0.103
868	A	4	4	1.	29	0.138
869	A	5	2	1.	29	0.069
870	A	4	2	1.	29	0.069
871	A	3	2	1.	29	0.069
872	A	2	2	1.	29	0.069
873	A	1	1	1.	29	0.034
874	A	4	3	1.	29	0.103
875	A	4	4	1.	29	0.138
876	A	4	3	1.	29	0.103
877	A	5	4	1.	29	0.138
878	A	6	4	1.	29	0.138
879	A	4	2	1.	29	0.069

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
880	A	3	2	1.	29	0.069
881	A	2	2	1.	29	0.069
882	A	1	1	1.	29	0.034
883	A	2	2	1.	29	0.069
884	A	3	3	1.	29	0.103
885	A	4	3	1.	29	0.103
886	A	4	2	1.	29	0.069
887	A	3	2	1.	29	0.069
888	A	2	2	1.	29	0.069
889	A	1	1	1.	29	0.034
890	A	3	3	1.	29	0.103
891	A	4	4	1.	29	0.138
892	A	5	4	1.	29	0.138
893	A	2	2	1.	19	0.105
894	A	3	2	1.	24	0.083
895	A	3	2	1.	24	0.083
896	A	3	2	1.	24	0.083
897	A	2	2	1.	24	0.083
898	A	4	4	1.	24	0.167
899	A	4	4	1.	24	0.167
900	A	5	5	1.	24	0.208
901	A	3	2	1.	24	0.083
902	A	3	2	1.	24	0.083
903	A	3	2	1.	24	0.083
904	A	3	2	1.	24	0.083
905	A	2	2	1.	24	0.083
906	A	5	4	1.	24	0.167
907	A	5	5	1.	24	0.208
908	A	5	4	1.	24	0.167
909	A	6	5	1.	24	0.208
910	A	7	5	1.	24	0.208
911	A	3	2	1.	24	0.083
912	A	3	2	1.	24	0.083
913	A	3	2	1.	24	0.083
914	A	2	2	1.	24	0.083
915	A	3	3	1.	24	0.125
916	A	4	4	1.	24	0.167
917	A	5	4	1.	24	0.167
918	A	3	2	1.	24	0.083
919	A	3	2	1.	24	0.083
920	A	3	2	1.	24	0.083
921	A	3	2	1.	24	0.083
922	A	2	2	1.	24	0.083
923	A	4	4	1.	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
924	A	5	4	1.09	24	0.167
925	A	6	4	1.06	24	0.167
926	A	2	2	1.	15	0.133
927	A	3	3	1.	19	0.158
928	A	2	2	1.	18	0.111
929	A	3	3	1.	24	0.125
930	A	14	10	1.	24	0.417
931	A	13	10	1.	24	0.417
932	A	13	10	1.	24	0.417
933	A	1	1	1.	24	0.042
934	A	2	2	1.	24	0.083
935	A	3	2	1.	24	0.083
936	A	4	2	1.	24	0.083
937	A	15	10	1.	24	0.417
938	A	14	10	1.	24	0.417
939	A	13	10	1.	24	0.417
940	A	12	9	1.	24	0.375
941	A	1	1	1.	24	0.042
942	A	2	2	1.	24	0.083
943	A	3	2	1.	24	0.083
944	A	4	2	1.	24	0.083
945	A	3	2	1.	22	0.091
946	A	3	2	1.	22	0.091
947	A	3	2	1.	20	0.1
948	A	2	2	1.	22	0.091
949	A	2	2	1.	22	0.091
950	A	2	2	1.	22	0.091
951	A	3	3	1.41	24	0.125
952	A	3	3	1.	24	0.125
953	A	3	3	1.41	24	0.125
954	A	3	3	1.24	24	0.125
955	A	3	3	1.21	24	0.125
956	A	3	3	1.	24	0.125
957	A	3	3	1.38	24	0.125
958	A	3	3	1.22	24	0.125
959	A	3	3	1.35	22	0.136
960	A	2	2	1.	23	0.087
961	A	2	2	1.	23	0.087
962	A	2	2	0.98	21	0.095
963	A	1	1	1.	15	0.067
964	A	2	2	1.32	23	0.087
965	A	2	2	1.	23	0.087
966	A	2	2	1.	23	0.087
967	A	2	2	1.22	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
968	A	2	2	1.18	22	0.091
969	A	3	3	1.	20	0.15
970	A	2	2	1.33	22	0.091
971	A	2	2	1.26	22	0.091
972	A	2	2	1.18	22	0.091
973	A	3	3	1.	24	0.125
974	A	3	3	1.	24	0.125
975	A	3	3	1.	24	0.125
976	A	3	3	1.	24	0.125
977	A	6	3	1.	38	0.079
978	A	3	3	1.	28	0.107
979	A	3	3	1.	26	0.115
980	A	1	0	1.	20	0.
981	A	2	1	1.	28	0.036
982	A	3	3	1.	28	0.107
983	A	3	3	1.	28	0.107
984	A	3	3	1.	28	0.107
985	A	3	3	1.	28	0.107
986	A	3	3	1.	28	0.107
987	A	3	3	1.	30	0.1
988	A	3	3	1.	28	0.107
989	A	3	3	1.	22	0.136
990	A	3	3	1.	30	0.1
991	A	3	3	1.	30	0.1
992	A	2	2	1.	30	0.067
993	A	2	2	1.	30	0.067
994	A	3	3	1.	30	0.1
995	A	3	3	1.	30	0.1
996	A	3	3	1.	30	0.1
997	A	3	3	1.	30	0.1
998	A	3	3	1.	30	0.1
999	A	3	3	1.	30	0.1
1000	A	2	2	1.	30	0.067
1001	A	2	2	1.	30	0.067
1002	A	3	3	1.	28	0.107
1003	A	3	3	1.	22	0.136
1004	A	3	3	1.	30	0.1
1005	A	3	3	1.	30	0.1
1006	A	3	3	1.	30	0.1
1007	A	3	3	1.	30	0.1
1008	A	3	3	1.	30	0.1
1009	A	2	2	1.	30	0.067
1010	A	2	2	1.	30	0.067
1011	A	3	3	1.	30	0.1

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1012	A	3	3	1.	30	0.1
1013	A	3	3	1.	28	0.107
1014	A	3	3	1.	22	0.136
1015	A	3	3	1.	30	0.1
1016	A	3	3	1.	30	0.1
1017	A	3	3	1.	30	0.1
1018	A	3	3	1.	30	0.1
1019	A	2	2	1.	30	0.067
1020	A	2	2	1.	30	0.067
1021	A	3	3	1.	30	0.1
1022	A	3	3	1.	30	0.1
1023	A	3	3	1.	30	0.1
1024	A	3	3	1.	30	0.1
1025	A	3	3	1.	28	0.107
1026	A	3	3	1.	22	0.136
1027	A	3	3	1.	30	0.1
1028	A	3	3	1.	30	0.1
1029	A	2	2	1.	32	0.062
1030	A	2	2	1.	32	0.062
1031	A	1	1	1.	30	0.033
1032	A	1	1	1.	24	0.042
1033	A	2	2	1.	32	0.062
1034	A	3	3	1.	32	0.094
1035	A	2	2	1.	32	0.062
1036	A	2	2	1.	32	0.062
1037	A	2	2	1.	32	0.062
1038	A	2	2	1.	32	0.062
1039	A	2	2	1.	32	0.062
1040	A	2	2	1.	32	0.062
1041	A	1	1	1.	30	0.033
1042	A	1	1	1.	24	0.042
1043	A	2	2	1.	32	0.062
1044	A	2	2	1.	32	0.062
1045	A	2	2	1.	32	0.062
1046	A	3	3	1.	32	0.094
1047	A	2	2	1.	32	0.062
1048	A	2	2	1.	32	0.062
1049	A	2	2	1.	32	0.062
1050	A	2	2	1.	32	0.062
1051	A	2	2	1.	32	0.062
1052	A	1	1	1.	30	0.033
1053	A	1	1	1.	24	0.042
1054	A	2	2	1.	32	0.062
1055	A	2	2	1.	32	0.062

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1056	A	2	2	1.	32	0.062
1057	A	2	2	1.	32	0.062
1058	A	2	2	1.	32	0.062
1059	A	3	3	1.	32	0.094
1060	A	2	2	1.	32	0.062
1061	A	2	2	1.	32	0.062
1062	A	2	2	1.	32	0.062
1063	A	2	2	1.	32	0.062
1064	A	2	2	1.	32	0.062
1065	A	1	1	1.	30	0.033
1066	A	2	2	1.	24	0.083
1067	A	2	2	1.	32	0.062
1068	A	2	2	1.	32	0.062
1069	A	2	2	1.	32	0.062
1070	A	2	2	1.	32	0.062
1071	A	2	2	1.	32	0.062
1072	A	2	2	1.	32	0.062
1073	A	3	3	1.	32	0.094
1074	A	1	1	1.	30	0.033
1075	A	1	1	1.	24	0.042
1076	A	2	2	1.	32	0.062
1077	A	2	2	1.	32	0.062
1078	A	2	2	1.	32	0.062
1079	A	2	2	1.	32	0.062
1080	A	2	2	1.	32	0.062
1081	A	3	3	1.	32	0.094
1082	A	2	2	1.	32	0.062
1083	A	2	2	1.	32	0.062
1084	A	1	1	1.	30	0.033
1085	A	1	1	1.	24	0.042
1086	A	2	2	1.	32	0.062
1087	A	2	2	1.	32	0.062
1088	A	2	2	1.	32	0.062
1089	A	3	3	1.	30	0.1
1090	A	3	3	1.	28	0.107
1091	A	3	3	1.	30	0.1
1092	A	3	3	1.	30	0.1
1093	A	3	3	1.	30	0.1
1094	A	2	2	1.	32	0.062
1095	A	2	2	1.	32	0.062
1096	A	2	2	1.	32	0.062
1097	A	2	2	1.	32	0.062
1098	A	2	2	1.	30	0.067
1099	A	2	2	1.	32	0.062

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1100	A	2	2	1.	30	0.067
1101	A	2	2	1.	30	0.067
1102	A	1	1	1.	28	0.036
1103	A	1	1	1.	22	0.045
1104	A	2	2	1.	30	0.067
1105	A	2	2	1.	30	0.067
1106	A	2	2	1.	30	0.067
1107	A	2	2	1.	34	0.059
1108	A	2	2	1.	36	0.056
1109	A	2	1	1.	22	0.045
1110	A	2	1	1.	22	0.045
1111	A	2	1	1.	22	0.045
1112	A	1	1	1.	20	0.05
1113	A	2	1	1.	22	0.045
1114	A	2	1	1.	22	0.045
1115	A	2	1	1.	22	0.045
1116	A	2	1	1.	22	0.045
1117	A	1	1	1.	22	0.045
1118	A	2	1	1.	22	0.045
1119	A	2	1	1.	22	0.045
1120	A	2	1	1.	22	0.045
1121	A	2	1	1.	24	0.042
1122	A	2	1	1.	24	0.042
1123	A	2	2	1.	24	0.083
1124	A	2	1	1.	24	0.042
1125	A	1	1	1.	22	0.045
1126	A	2	1	1.	24	0.042
1127	A	2	1	1.	24	0.042
1128	A	2	1	1.	24	0.042
1129	A	2	1	1.	24	0.042
1130	A	2	1	1.	24	0.042
1131	A	2	1	1.	24	0.042
1132	A	1	1	1.	24	0.042
1133	A	2	1	1.	24	0.042
1134	A	2	1	1.	24	0.042
1135	A	2	1	1.	24	0.042
1136	A	2	1	1.	24	0.042
1137	A	2	1	1.	24	0.042
1138	A	2	1	1.	24	0.042
1139	A	2	2	1.	24	0.083
1140	A	2	1	1.	24	0.042
1141	A	1	1	1.	22	0.045
1142	A	2	1	1.	24	0.042
1143	A	2	1	1.	24	0.042

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1144	A	2	1	1.	24	0.042
1145	A	2	1	1.	24	0.042
1146	A	2	1	1.	24	0.042
1147	A	2	1	1.	24	0.042
1148	A	2	1	1.	24	0.042
1149	A	2	1	1.	24	0.042
1150	A	1	1	1.	24	0.042
1151	A	2	1	1.	24	0.042
1152	A	2	1	1.	24	0.042
1153	A	2	1	1.	24	0.042
1154	A	6	3	1.	24	0.125
1155	A	4	2	1.	24	0.083
1156	A	5	3	1.	24	0.125
1157	A	3	2	1.	24	0.083
1158	A	4	3	1.	24	0.125
1159	A	2	2	1.	24	0.083
1160	A	3	3	1.	24	0.125
1161	A	1	1	1.	22	0.045
1162	A	3	3	1.	24	0.125
1163	A	3	3	1.	24	0.125
1164	A	4	4	1.	24	0.167
1165	A	4	3	1.	24	0.125
1166	A	6	4	1.	24	0.167
1167	A	4	3	1.	24	0.125
1168	A	5	4	1.	24	0.167
1169	A	3	3	1.	24	0.125
1170	A	4	4	1.	24	0.167
1171	A	2	2	1.	24	0.083
1172	A	3	3	1.	24	0.125
1173	A	1	1	1.	22	0.045
1174	A	4	4	1.	24	0.167
1175	A	4	4	1.	24	0.167
1176	A	5	5	1.	24	0.208
1177	A	7	4	1.	24	0.167
1178	A	5	3	1.	24	0.125
1179	A	6	4	1.	24	0.167
1180	A	4	3	1.	24	0.125
1181	A	5	4	1.	24	0.167
1182	A	3	2	1.	24	0.083
1183	A	4	3	1.	24	0.125
1184	A	1	1	1.	24	0.042
1185	A	4	4	1.	24	0.167
1186	A	1	1	1.	22	0.045
1187	A	5	4	1.	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1188	A	5	4	1.	24	0.167
1189	A	6	5	1.	24	0.208
1190	A	6	4	1.	24	0.167
1191	A	5	4	1.	26	0.154
1192	A	2	2	1.	26	0.077
1193	A	4	4	1.	26	0.154
1194	A	1	1	1.	24	0.042
1195	A	3	3	1.	26	0.115
1196	A	3	3	1.	26	0.115
1197	A	3	3	1.	26	0.115
1198	A	1	1	1.	26	0.038
1199	A	4	4	1.	26	0.154
1200	A	2	2	1.	26	0.077
1201	A	5	4	1.	26	0.154
1202	A	3	2	1.	26	0.077
1203	A	6	4	1.	26	0.154
1204	A	2	2	1.	26	0.077
1205	A	5	4	1.	26	0.154
1206	A	1	1	1.	24	0.042
1207	A	4	3	1.	26	0.115
1208	A	4	4	1.	26	0.154
1209	A	4	4	1.	26	0.154
1210	A	4	3	1.	26	0.115
1211	A	4	3	1.	26	0.115
1212	A	1	1	1.	26	0.038
1213	A	5	4	1.	26	0.154
1214	A	2	2	1.	26	0.077
1215	A	6	4	1.	26	0.154
1216	A	3	2	1.	26	0.077
1217	A	3	2	1.	26	0.077
1218	A	7	4	1.	26	0.154
1219	A	2	2	1.	26	0.077
1220	A	6	4	1.	26	0.154
1221	A	1	1	1.	24	0.042
1222	A	5	3	1.	26	0.115
1223	A	5	4	1.	26	0.154
1224	A	5	4	1.	26	0.154
1225	A	5	4	1.	26	0.154
1226	A	5	4	1.	26	0.154
1227	A	5	3	1.	26	0.115
1228	A	5	3	1.	26	0.115
1229	A	1	1	1.	26	0.038
1230	A	6	4	1.	26	0.154
1231	A	2	2	1.	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1232	A	7	4	1.	26	0.154
1233	A	3	2	1.	26	0.077
1234	A	4	3	1.	26	0.115
1235	A	2	2	1.	26	0.077
1236	A	3	3	1.	26	0.115
1237	A	1	1	1.	24	0.042
1238	A	2	2	1.	26	0.077
1239	A	1	1	1.	26	0.038
1240	A	3	3	1.	26	0.115
1241	A	2	2	1.	26	0.077
1242	A	4	4	1.	26	0.154
1243	A	2	2	1.	26	0.077
1244	A	3	3	1.	26	0.115
1245	A	1	1	1.	24	0.042
1246	A	3	3	1.	26	0.115
1247	A	2	2	1.	26	0.077
1248	A	4	4	1.	26	0.154
1249	A	3	3	1.	26	0.115
1250	A	5	4	1.	26	0.154
1251	A	3	2	1.	26	0.077
1252	A	4	3	1.	26	0.115
1253	A	2	2	1.	26	0.077
1254	A	1	1	1.	26	0.038
1255	A	1	1	1.	24	0.042
1256	A	4	3	1.	26	0.115
1257	A	3	2	1.	26	0.077
1258	A	5	4	1.	26	0.154
1259	A	4	3	1.	26	0.115
1260	A	2	2	1.	29	0.069
1261	A	2	1	1.	24	0.042
1262	A	2	1	1.	24	0.042
1263	A	2	1	1.	24	0.042
1264	A	2	1	1.	24	0.042
1265	A	2	1	1.	24	0.042
1266	A	2	1	1.	24	0.042
1267	A	2	1	1.	24	0.042
1268	A	2	1	1.	24	0.042
1269	A	2	1	1.	26	0.038
1270	A	2	1	1.	26	0.038
1271	A	2	1	1.	26	0.038
1272	A	2	1	1.	26	0.038
1273	A	2	1	1.	26	0.038
1274	A	2	1	1.	26	0.038
1275	A	2	1	1.	26	0.038

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1276	A	2	1	1.	26	0.038
1277	A	2	1	1.	26	0.038
1278	A	2	1	1.	26	0.038
1279	A	2	1	1.	26	0.038
1280	A	2	1	1.	26	0.038
1281	A	2	1	1.	26	0.038
1282	A	2	1	1.	26	0.038
1283	A	2	1	1.	26	0.038
1284	A	2	1	1.	26	0.038
1285	A	8	6	1.	26	0.231
1286	A	7	6	1.	26	0.231
1287	A	7	6	1.	26	0.231
1288	A	6	6	1.	26	0.231
1289	A	6	6	1.	26	0.231
1290	A	5	5	1.	26	0.192
1291	A	5	5	1.	26	0.192
1292	A	6	6	1.	26	0.231
1293	A	6	6	1.	26	0.231
1294	A	7	6	1.	26	0.231
1295	A	9	7	1.	26	0.269
1296	A	8	7	1.	26	0.269
1297	A	8	7	1.	26	0.269
1298	A	7	7	1.	26	0.269
1299	A	7	7	1.	26	0.269
1300	A	6	6	1.	26	0.231
1301	A	6	6	1.	26	0.231
1302	A	6	6	1.	26	0.231
1303	A	6	6	1.	26	0.231
1304	A	7	7	1.	26	0.269
1305	A	7	7	1.	26	0.269
1306	A	8	7	1.	26	0.269
1307	A	9	7	1.	26	0.269
1308	A	9	7	1.	26	0.269
1309	A	8	7	1.	26	0.269
1310	A	8	7	1.	26	0.269
1311	A	7	6	1.	26	0.231
1312	A	7	6	1.	26	0.231
1313	A	7	7	1.	26	0.269
1314	A	7	7	1.	26	0.269
1315	A	7	6	1.	26	0.231
1316	A	7	6	1.	26	0.231
1317	A	8	7	1.	26	0.269
1318	A	8	7	1.	26	0.269
1319	A	9	7	1.	26	0.269

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1320	A	13	9	1.	18	0.5
1321	A	12	9	1.	18	0.5
1322	A	12	9	1.	18	0.5
1323	A	11	8	1.	18	0.444
1324	A	11	8	1.	18	0.444
1325	A	12	9	1.	18	0.5
1326	A	12	9	1.	18	0.5
1327	A	6	5	1.	28	0.179
1328	A	5	5	1.	28	0.179
1329	A	4	4	1.	28	0.143
1330	A	4	4	1.	28	0.143
1331	A	5	5	1.	28	0.179
1332	A	6	5	1.	28	0.179
1333	A	8	8	1.	28	0.286
1334	A	7	7	1.	28	0.25
1335	A	7	7	1.	28	0.25
1336	A	8	8	1.	28	0.286
1337	A	7	5	1.	28	0.179
1338	A	6	5	1.	28	0.179
1339	A	5	4	1.	28	0.143
1340	A	5	5	1.	28	0.179
1341	A	5	4	1.	28	0.143
1342	A	6	5	1.	28	0.179
1343	A	7	5	1.	28	0.179
1344	A	9	8	1.	28	0.286
1345	A	8	7	1.	28	0.25
1346	A	8	8	1.	28	0.286
1347	A	8	7	1.	28	0.25
1348	A	9	8	1.	28	0.286
1349	A	8	5	1.	28	0.179
1350	A	7	5	1.	28	0.179
1351	A	6	4	1.	28	0.143
1352	A	6	5	1.	28	0.179
1353	A	6	5	1.	28	0.179
1354	A	6	4	1.	28	0.143
1355	A	7	5	1.	28	0.179
1356	A	8	5	1.	28	0.179
1357	A	10	8	1.	28	0.286
1358	A	9	7	1.	28	0.25
1359	A	9	8	1.	28	0.286
1360	A	9	8	1.	28	0.286
1361	A	9	7	1.	28	0.25
1362	A	10	8	1.	28	0.286
1363	A	5	4	1.	28	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1364	A	4	4	1.	28	0.143
1365	A	3	3	1.	28	0.107
1366	A	4	4	1.	28	0.143
1367	A	5	4	1.	28	0.143
1368	A	8	7	1.	28	0.25
1369	A	7	7	1.	28	0.25
1370	A	6	6	1.	28	0.214
1371	A	7	7	1.	28	0.25
1372	A	8	7	1.	28	0.25
1373	A	4	4	1.	22	0.182
1374	A	3	3	1.	22	0.136
1375	A	4	4	1.	22	0.182
1376	A	8	8	1.	22	0.364
1377	A	7	7	1.	22	0.318
1378	A	8	8	1.	22	0.364
1379	A	6	5	1.	28	0.179
1380	A	5	5	1.	28	0.179
1381	A	4	4	1.	28	0.143
1382	A	4	4	1.	28	0.143
1383	A	5	5	1.	28	0.179
1384	A	8	8	1.	28	0.286
1385	A	7	7	1.	28	0.25
1386	A	7	7	1.	28	0.25
1387	A	8	8	1.	28	0.286
1388	A	9	8	1.	28	0.286
1389	A	7	5	1.	28	0.179
1390	A	6	5	1.	28	0.179
1391	A	5	4	1.	28	0.143
1392	A	5	5	1.	28	0.179
1393	A	5	4	1.	28	0.143
1394	A	6	5	1.	28	0.179
1395	A	9	8	1.	28	0.286
1396	A	8	7	1.	28	0.25
1397	A	8	8	1.	28	0.286
1398	A	8	7	1.	28	0.25
1399	A	9	8	1.	28	0.286
1400	A	5	3	1.	37	0.081
1401	A	4	3	1.	37	0.081
1402	A	3	3	1.	37	0.081
1403	A	2	2	1.	37	0.054
1404	A	3	3	1.	37	0.081
1405	A	4	3	1.	37	0.081
1406	A	5	3	1.	37	0.081
1407	A	7	6	1.	37	0.162

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1408	A	6	6	1.	37	0.162
1409	A	5	5	1.	37	0.135
1410	A	6	6	1.	37	0.162
1411	A	7	6	1.	37	0.162
1412	A	14	12	1.46	28	0.429
1413	A	1	1	1.	28	0.036
1414	A	2	2	1.	28	0.071
1415	A	3	2	1.	28	0.071
1416	A	6	5	1.	28	0.179
1417	A	6	6	1.	28	0.214
1418	A	6	5	1.	28	0.179
1419	A	7	6	1.	28	0.214
1420	A	3	3	1.	28	0.107
1421	A	3	3	1.	28	0.107
1422	A	3	3	1.	28	0.107
1423	A	2	1	1.	24	0.042
1424	A	2	1	1.	24	0.042
1425	A	2	1	1.	22	0.045
1426	A	2	2	1.	24	0.083
1427	A	2	2	1.	24	0.083
1428	A	2	2	1.	24	0.083
1429	A	3	3	1.39	26	0.115
1430	A	3	3	1.14	26	0.115
1431	A	3	3	1.06	26	0.115
1432	A	3	3	1.08	26	0.115
1433	A	3	3	1.16	26	0.115
1434	A	3	3	1.15	26	0.115
1435	A	3	3	0.95	24	0.125
1436	A	3	2	1.	24	0.083
1437	A	3	3	0.94	24	0.125
1438	A	2	2	1.	24	0.083
1439	A	3	3	0.94	24	0.125
1440	A	1	1	1.	22	0.045
1441	A	3	3	1.	24	0.125
1442	A	3	3	0.97	24	0.125
1443	A	3	3	1.	24	0.125
1444	A	3	3	0.94	24	0.125
1445	A	3	3	1.	24	0.125
1446	A	3	3	0.94	24	0.125
1447	A	1	1	1.	16	0.062
1448	A	1	1	1.	21	0.048
1449	A	2	2	1.	16	0.125
1450	A	2	2	1.	37	0.054
1451	A	3	2	1.	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1452	A	3	2	1.	24	0.083
1453	A	3	2	1.	24	0.083
1454	A	3	2	1.	22	0.091
1455	A	1	0	1.	16	0.
1456	A	3	2	1.	24	0.083
1457	A	3	2	1.	24	0.083
1458	A	3	2	1.	24	0.083
1459	A	2	2	1.	24	0.083
1460	A	3	2	1.	24	0.083
1461	A	3	2	1.	24	0.083
1462	A	3	2	1.	24	0.083
1463	A	3	2	1.	26	0.077
1464	A	3	2	1.	26	0.077
1465	A	3	2	1.	26	0.077
1466	A	3	2	1.	26	0.077
1467	A	3	2	1.	26	0.077
1468	A	3	2	1.	24	0.083
1469	A	2	2	1.	18	0.111
1470	A	3	2	1.	26	0.077
1471	A	3	2	1.	26	0.077
1472	A	3	2	1.	26	0.077
1473	A	3	2	1.	26	0.077
1474	A	3	2	1.	26	0.077
1475	A	2	2	1.	26	0.077
1476	A	3	3	1.	26	0.115
1477	A	4	3	1.	26	0.115
1478	A	3	2	1.	26	0.077
1479	A	3	2	1.	26	0.077
1480	A	3	2	1.	26	0.077
1481	A	3	2	1.	26	0.077
1482	A	3	2	1.	26	0.077
1483	A	3	2	1.	26	0.077
1484	A	3	2	1.	26	0.077
1485	A	3	2	1.	26	0.077
1486	A	3	2	1.	26	0.077
1487	A	3	2	1.	26	0.077
1488	A	3	2	1.	24	0.083
1489	A	2	2	1.	18	0.111
1490	A	3	2	1.	26	0.077
1491	A	3	2	1.	26	0.077
1492	A	3	2	1.	26	0.077
1493	A	3	2	1.	26	0.077
1494	A	3	2	1.	26	0.077
1495	A	3	2	1.	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1496	A	3	2	1.	26	0.077
1497	A	2	2	1.	26	0.077
1498	A	3	3	1.	26	0.115
1499	A	4	3	1.	26	0.115
1500	A	5	3	1.	26	0.115
1501	A	3	2	1.	26	0.077
1502	A	3	2	1.	26	0.077
1503	A	3	2	1.	26	0.077
1504	A	3	2	1.	26	0.077
1505	A	3	2	1.	26	0.077
1506	A	3	2	1.	26	0.077
1507	A	3	2	1.	26	0.077
1508	A	3	2	1.	26	0.077
1509	A	3	2	1.	24	0.083
1510	A	2	2	1.	18	0.111
1511	A	3	2	1.	26	0.077
1512	A	3	2	1.	26	0.077
1513	A	3	2	1.	26	0.077
1514	A	3	2	1.	26	0.077
1515	A	3	2	1.	26	0.077
1516	A	3	2	1.	26	0.077
1517	A	3	2	1.	26	0.077
1518	A	3	2	1.	26	0.077
1519	A	2	2	1.	26	0.077
1520	A	3	2	1.	24	0.083
1521	A	2	2	1.	18	0.111
1522	A	3	2	1.	26	0.077
1523	A	3	2	1.	26	0.077
1524	A	3	2	1.	26	0.077
1525	A	3	2	1.	26	0.077
1526	A	3	2	1.	26	0.077
1527	A	3	2	1.	26	0.077
1528	A	3	2	1.	26	0.077
1529	A	2	2	1.	26	0.077
1530	A	3	3	1.	26	0.115
1531	A	3	2	1.	26	0.077
1532	A	3	2	1.	24	0.083
1533	A	2	2	1.	18	0.111
1534	A	3	2	1.	26	0.077
1535	A	3	2	1.	26	0.077
1536	A	3	2	1.	26	0.077
1537	A	3	2	1.	18	0.111
1538	A	3	2	1.	18	0.111
1539	A	3	2	1.	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1540	A	3	2	1.	18	0.111
1541	A	3	2	1.	18	0.111
1542	A	3	2	1.	18	0.111
1543	A	3	2	1.	28	0.071
1544	A	3	2	1.	28	0.071
1545	A	3	2	1.	28	0.071
1546	A	2	2	1.	26	0.077
1547	A	1	1	1.	20	0.05
1548	A	3	2	1.	28	0.071
1549	A	3	2	1.	28	0.071
1550	A	2	2	1.	28	0.071
1551	A	3	2	1.	28	0.071
1552	A	3	2	1.	28	0.071
1553	A	3	2	1.	28	0.071
1554	A	3	2	1.	28	0.071
1555	A	3	2	1.	28	0.071
1556	A	3	2	1.	28	0.071
1557	A	2	1	1.1	28	0.036
1558	A	2	2	1.	26	0.077
1559	A	1	1	1.	20	0.05
1560	A	3	2	1.	28	0.071
1561	A	3	2	1.	28	0.071
1562	A	3	2	1.	28	0.071
1563	A	3	2	1.	28	0.071
1564	A	2	2	1.	28	0.071
1565	A	3	3	1.	28	0.107
1566	A	3	2	1.4	28	0.071
1567	A	3	2	1.	28	0.071
1568	A	3	2	1.	28	0.071
1569	A	3	2	1.	28	0.071
1570	A	2	1	1.	28	0.036
1571	A	3	2	1.	28	0.071
1572	A	3	2	1.	28	0.071
1573	A	2	2	1.	26	0.077
1574	A	1	1	1.	20	0.05
1575	A	3	2	1.	28	0.071
1576	A	3	2	1.	28	0.071
1577	A	3	2	1.	28	0.071
1578	A	3	2	1.	28	0.071
1579	A	3	2	1.	28	0.071
1580	A	3	2	1.	28	0.071
1581	A	2	2	1.	28	0.071
1582	A	3	3	1.	28	0.107
1583	A	4	3	1.	28	0.107

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1584	A	5	3	1.	28	0.107
1585	A	3	2	1.	28	0.071
1586	A	3	2	1.	28	0.071
1587	A	3	2	1.	28	0.071
1588	A	3	2	1.	28	0.071
1589	A	3	2	1.	28	0.071
1590	A	3	3	1.	26	0.115
1591	A	2	2	1.	20	0.1
1592	A	4	3	1.	28	0.107
1593	A	3	2	1.	28	0.071
1594	A	3	2	1.	28	0.071
1595	A	3	2	1.	28	0.071
1596	A	3	2	1.	28	0.071
1597	A	3	2	1.	28	0.071
1598	A	3	2	1.	28	0.071
1599	A	2	2	1.	26	0.077
1600	A	1	1	1.	20	0.05
1601	A	3	2	1.	28	0.071
1602	A	3	2	1.	28	0.071
1603	A	3	2	1.	28	0.071
1604	A	3	2	1.	28	0.071
1605	A	3	2	1.	28	0.071
1606	A	3	2	1.	28	0.071
1607	A	2	2	1.	28	0.071
1608	A	3	2	1.	28	0.071
1609	A	2	2	1.	26	0.077
1610	A	1	1	1.	20	0.05
1611	A	3	2	1.	28	0.071
1612	A	3	2	1.	28	0.071
1613	A	3	2	1.	28	0.071
1614	A	2	2	1.	20	0.1
1615	A	2	2	1.	20	0.1
1616	A	2	2	1.	20	0.1
1617	A	3	3	1.	20	0.15
1618	A	2	2	1.	20	0.1
1619	A	2	2	1.	20	0.1
1620	A	2	2	1.	20	0.1
1621	A	3	2	1.	26	0.077
1622	A	3	2	1.	26	0.077
1623	A	3	2	1.	26	0.077
1624	A	3	2	1.	26	0.077
1625	A	3	2	1.	26	0.077
1626	A	3	2	1.	26	0.077
1627	A	3	2	1.	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1628	A	3	2	1.	26	0.077
1629	A	3	2	1.	28	0.071
1630	A	3	2	1.	28	0.071
1631	A	3	2	1.	28	0.071
1632	A	3	2	1.	28	0.071
1633	A	3	2	1.	28	0.071
1634	A	3	2	1.	28	0.071
1635	A	3	2	1.	28	0.071
1636	A	3	2	1.	28	0.071
1637	A	3	2	1.	28	0.071
1638	A	3	2	1.	28	0.071
1639	A	3	2	1.	28	0.071
1640	A	3	2	1.	28	0.071
1641	A	3	2	1.	28	0.071
1642	A	3	2	1.	28	0.071
1643	A	3	2	1.	28	0.071
1644	A	3	2	1.	28	0.071
1645	A	8	5	1.	28	0.179
1646	A	7	5	1.	28	0.179
1647	A	6	5	1.	28	0.179
1648	A	5	5	1.	28	0.179
1649	A	4	4	1.	28	0.143
1650	A	4	4	1.	28	0.143
1651	A	5	4	1.	28	0.143
1652	A	6	4	1.	28	0.143
1653	A	7	4	1.	28	0.143
1654	A	9	5	1.	28	0.179
1655	A	8	5	1.	28	0.179
1656	A	7	5	1.	28	0.179
1657	A	6	4	1.	28	0.143
1658	A	6	5	1.	28	0.179
1659	A	6	5	1.	28	0.179
1660	A	6	4	1.	28	0.143
1661	A	7	4	1.	28	0.143
1662	A	8	4	1.	28	0.143
1663	A	9	4	1.	28	0.143
1664	A	11	5	1.	28	0.179
1665	A	10	5	1.	28	0.179
1666	A	9	5	1.	28	0.179
1667	A	8	4	1.	28	0.143
1668	A	8	5	1.	28	0.179
1669	A	8	5	1.	28	0.179
1670	A	8	5	1.	28	0.179
1671	A	8	5	1.	28	0.179

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1672	A	8	4	1.	28	0.143
1673	A	9	4	1.	28	0.143
1674	A	10	4	1.	28	0.143
1675	A	11	4	1.	28	0.143
1676	A	3	2	1.	30	0.067
1677	A	3	2	1.	30	0.067
1678	A	3	2	1.	30	0.067
1679	A	3	2	1.	30	0.067
1680	A	3	2	1.	30	0.067
1681	A	3	2	1.	30	0.067
1682	A	3	2	1.	30	0.067
1683	A	3	2	1.	30	0.067
1684	A	3	2	1.	30	0.067
1685	A	3	2	1.	30	0.067
1686	A	3	2	1.	30	0.067
1687	A	3	2	1.	30	0.067
1688	A	3	2	1.	30	0.067
1689	A	3	2	1.	30	0.067
1690	A	3	2	1.	30	0.067
1691	A	3	2	1.	30	0.067
1692	A	3	2	1.	30	0.067
1693	A	3	2	1.	30	0.067
1694	A	3	2	1.	30	0.067
1695	A	3	2	1.	30	0.067
1696	A	3	2	1.	30	0.067
1697	A	3	2	1.	30	0.067
1698	A	3	2	1.	30	0.067
1699	A	3	2	1.	30	0.067
1700	A	3	2	1.	30	0.067
1701	A	3	2	1.	30	0.067
1702	A	3	2	1.	30	0.067
1703	A	7	4	1.	30	0.133
1704	A	6	4	1.	30	0.133
1705	A	5	4	1.	30	0.133
1706	A	4	4	1.	30	0.133
1707	A	3	3	1.	30	0.1
1708	A	4	4	1.	30	0.133
1709	A	5	4	1.	30	0.133
1710	A	6	4	1.	30	0.133
1711	A	8	5	1.	30	0.167
1712	A	7	5	1.	30	0.167
1713	A	6	5	1.	30	0.167
1714	A	5	4	1.	30	0.133
1715	A	5	5	1.	30	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1716	A	5	4	1.	30	0.133
1717	A	6	4	1.	30	0.133
1718	A	7	4	1.	30	0.133
1719	A	8	4	1.	30	0.133
1720	A	10	5	1.	30	0.167
1721	A	9	5	1.	30	0.167
1722	A	8	5	1.	30	0.167
1723	A	7	4	1.	30	0.133
1724	A	7	5	1.	30	0.167
1725	A	7	5	1.	30	0.167
1726	A	7	5	1.	30	0.167
1727	A	7	4	1.	30	0.133
1728	A	8	4	1.	30	0.133
1729	A	9	4	1.	30	0.133
1730	A	10	4	1.	30	0.133
1731	A	3	2	1.	26	0.077
1732	A	3	2	1.	26	0.077
1733	A	3	2	1.	24	0.083
1734	A	2	2	1.	26	0.077
1735	A	2	2	1.	26	0.077
1736	A	2	2	1.	26	0.077
1737	A	3	2	1.	28	0.071
1738	A	3	2	1.	28	0.071
1739	A	3	2	1.	28	0.071
1740	A	2	2	1.	28	0.071
1741	A	2	2	1.	28	0.071
1742	A	2	2	1.	28	0.071
1743	A	3	3	1.	26	0.115
1744	A	3	2	1.	26	0.077
1745	A	3	2	1.	26	0.077
1746	A	2	2	1.	24	0.083
1747	A	1	1	1.	18	0.056
1748	A	2	2	1.	26	0.077
1749	A	2	2	1.	26	0.077
1750	A	2	2	1.	26	0.077
1751	A	3	3	1.	28	0.107
1752	A	3	3	1.	28	0.107
1753	A	3	3	1.	28	0.107
1754	A	3	3	1.	28	0.107
1755	A	3	3	1.	28	0.107
1756	A	3	3	1.	28	0.107
1757	A	3	3	1.	30	0.1
1758	A	2	2	1.	18	0.111
1759	A	3	2	1.	27	0.074

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1760	A	3	2	1.	27	0.074
1761	A	3	2	1.	25	0.08
1762	A	1	0	1.	19	0.
1763	A	2	1	1.	27	0.037
1764	A	3	2	1.	27	0.074
1765	A	3	2	1.	27	0.074
1766	A	2	2	1.	27	0.074
1767	A	3	2	1.	27	0.074
1768	A	3	2	1.	27	0.074
1769	A	3	2	1.	29	0.069
1770	A	3	2	1.	29	0.069
1771	A	3	2	1.	27	0.074
1772	A	3	2	1.	21	0.095
1773	A	3	2	1.	29	0.069
1774	A	2	2	1.	29	0.069
1775	A	3	2	1.	29	0.069
1776	A	3	2	1.	29	0.069
1777	A	3	2	1.	29	0.069
1778	A	2	2	1.	29	0.069
1779	A	3	2	1.	29	0.069
1780	A	3	2	1.	29	0.069
1781	A	3	2	1.	29	0.069
1782	A	3	2	1.	29	0.069
1783	A	3	2	1.	29	0.069
1784	A	3	2	1.	29	0.069
1785	A	3	2	1.	27	0.074
1786	A	3	2	1.	21	0.095
1787	A	3	2	1.	29	0.069
1788	A	3	2	1.	29	0.069
1789	A	2	2	1.	29	0.069
1790	A	3	2	1.	29	0.069
1791	A	3	2	1.	29	0.069
1792	A	3	2	1.	29	0.069
1793	A	3	2	1.	29	0.069
1794	A	2	2	1.	29	0.069
1795	A	3	3	1.	29	0.103
1796	A	3	2	1.	29	0.069
1797	A	3	2	1.	29	0.069
1798	A	3	2	1.	29	0.069
1799	A	3	2	1.	29	0.069
1800	A	3	2	1.	29	0.069
1801	A	3	2	1.	29	0.069
1802	A	3	2	1.	29	0.069
1803	A	3	2	1.	29	0.069

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1804	A	2	2	1.	27	0.074
1805	A	3	2	1.	21	0.095
1806	A	3	2	1.	29	0.069
1807	A	3	2	1.	29	0.069
1808	A	3	2	1.	29	0.069
1809	A	3	2	1.	29	0.069
1810	A	3	2	1.	29	0.069
1811	A	3	2	1.	29	0.069
1812	A	3	2	1.	29	0.069
1813	A	3	2	1.	29	0.069
1814	A	2	2	1.	29	0.069
1815	A	3	2	1.	27	0.074
1816	A	4	3	1.	21	0.143
1817	A	3	2	1.	29	0.069
1818	A	3	2	1.	29	0.069
1819	A	3	2	1.	29	0.069
1820	A	3	2	1.	29	0.069
1821	A	3	2	1.	29	0.069
1822	A	2	2	1.	29	0.069
1823	A	2	2	1.	29	0.069
1824	A	3	2	1.	29	0.069
1825	A	3	2	1.	27	0.074
1826	A	5	3	1.	21	0.143
1827	A	3	2	1.	29	0.069
1828	A	3	2	1.	33	0.061
1829	A	3	2	1.	33	0.061
1830	A	3	2	1.	33	0.061
1831	A	3	2	1.	31	0.065
1832	A	1	0	1.	25	0.
1833	A	2	1	1.	33	0.03
1834	A	3	2	1.	33	0.061
1835	A	3	2	1.	33	0.061
1836	A	2	2	1.	33	0.061
1837	A	3	2	1.	33	0.061
1838	A	3	2	1.	33	0.061
1839	A	3	2	1.	35	0.057
1840	A	3	2	1.	33	0.061
1841	A	3	2	1.	27	0.074
1842	A	3	2	1.	35	0.057
1843	A	2	2	1.	35	0.057
1844	A	3	2	1.	35	0.057
1845	A	3	2	1.	35	0.057
1846	A	3	2	1.	35	0.057
1847	A	2	2	1.	35	0.057

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1848	A	3	2	1.	35	0.057
1849	A	3	2	1.	35	0.057
1850	A	3	2	1.	35	0.057
1851	A	3	2	1.	35	0.057
1852	A	3	2	1.	33	0.061
1853	A	3	2	1.	27	0.074
1854	A	3	2	1.	35	0.057
1855	A	3	2	1.	35	0.057
1856	A	2	2	1.	35	0.057
1857	A	3	2	1.	35	0.057
1858	A	3	2	1.	35	0.057
1859	A	3	2	1.	35	0.057
1860	A	3	2	1.	35	0.057
1861	A	2	2	1.	35	0.057
1862	A	3	3	1.	35	0.086
1863	A	3	2	1.	35	0.057
1864	A	3	2	1.	35	0.057
1865	A	3	2	1.	35	0.057
1866	A	3	2	1.	35	0.057
1867	A	3	2	1.	35	0.057
1868	A	3	2	1.	35	0.057
1869	A	2	2	1.	33	0.061
1870	A	3	2	1.	27	0.074
1871	A	3	2	1.	35	0.057
1872	A	3	2	1.	35	0.057
1873	A	3	2	1.	35	0.057
1874	A	3	2	1.	35	0.057
1875	A	3	2	1.	35	0.057
1876	A	3	2	1.	35	0.057
1877	A	3	2	1.	35	0.057
1878	A	3	2	1.	35	0.057
1879	A	3	2	1.	35	0.057
1880	A	2	2	1.	35	0.057
1881	A	3	2	1.	33	0.061
1882	A	4	3	1.	27	0.111
1883	A	3	2	1.	35	0.057
1884	A	3	2	1.	35	0.057
1885	A	3	2	1.	35	0.057
1886	A	3	2	1.	35	0.057
1887	A	3	2	1.	35	0.057
1888	A	3	2	1.	35	0.057
1889	A	3	2	1.	35	0.057
1890	A	2	2	1.	35	0.057
1891	A	2	2	1.	35	0.057

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1892	A	3	2	1.	35	0.057
1893	A	3	2	1.	33	0.061
1894	A	5	3	1.	27	0.111
1895	A	3	2	1.	35	0.057
1896	A	3	2	1.	35	0.057
1897	A	3	2	1.	35	0.057
1898	A	3	2	1.	35	0.057
1899	A	3	2	1.	35	0.057
1900	A	2	2	1.	35	0.057
1901	A	3	2	1.	35	0.057
1902	A	2	2	1.	35	0.057
1903	A	3	2	1.	35	0.057
1904	A	3	2	1.	35	0.057
1905	A	3	2	1.	33	0.061
1906	A	6	3	1.	27	0.111
1907	A	7	5	1.	37	0.135
1908	A	6	5	1.	37	0.135
1909	A	5	5	1.	37	0.135
1910	A	4	4	1.	35	0.114
1911	A	3	3	1.	29	0.103
1912	A	3	3	1.	37	0.081
1913	A	3	3	1.	37	0.081
1914	A	1	1	1.	37	0.027
1915	A	2	2	1.	37	0.054
1916	A	3	2	1.	37	0.054
1917	A	4	2	1.	37	0.054
1918	A	8	5	1.	37	0.135
1919	A	7	5	1.	37	0.135
1920	A	6	5	1.	37	0.135
1921	A	5	4	1.	35	0.114
1922	A	4	3	1.	29	0.103
1923	A	4	4	1.	37	0.108
1924	A	4	3	1.	37	0.081
1925	A	4	4	1.	37	0.108
1926	A	4	3	1.	37	0.081
1927	A	1	1	1.	37	0.027
1928	A	2	2	1.	37	0.054
1929	A	3	2	1.	37	0.054
1930	A	4	2	1.	37	0.054
1931	A	9	5	1.	37	0.135
1932	A	8	5	1.	37	0.135
1933	A	7	5	1.	37	0.135
1934	A	6	4	1.	35	0.114
1935	A	5	3	1.	29	0.103

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1936	A	5	4	1.	37	0.108
1937	A	6	6	1.	37	0.162
1938	A	5	5	1.	37	0.135
1939	A	5	4	1.	37	0.108
1940	A	5	4	1.	37	0.108
1941	A	5	3	1.	37	0.081
1942	A	1	1	1.	37	0.027
1943	A	2	2	1.	37	0.054
1944	A	3	2	1.	37	0.054
1945	A	4	2	1.	37	0.054
1946	A	5	4	1.	37	0.108
1947	A	4	4	1.	37	0.108
1948	A	3	3	1.	35	0.086
1949	A	2	2	1.	29	0.069
1950	A	1	1	1.	37	0.027
1951	A	2	2	1.	37	0.054
1952	A	3	2	1.	37	0.054
1953	A	4	2	1.	37	0.054
1954	A	6	5	1.	37	0.135
1955	A	5	5	1.	37	0.135
1956	A	4	4	1.	37	0.108
1957	A	3	3	1.	37	0.081
1958	A	1	1	1.	35	0.029
1959	A	1	1	1.	29	0.034
1960	A	2	2	1.	37	0.054
1961	A	3	2	1.	37	0.054
1962	A	4	2	1.	37	0.054
1963	A	5	2	1.	37	0.054
1964	A	6	5	1.	37	0.135
1965	A	5	4	1.	37	0.108
1966	A	4	4	1.	37	0.108
1967	A	1	1	1.	37	0.027
1968	A	2	2	1.	37	0.054
1969	A	2	2	1.	35	0.057
1970	A	2	2	1.	29	0.069
1971	A	3	3	1.	37	0.081
1972	A	4	3	1.	37	0.081
1973	A	5	3	1.	37	0.081
1974	A	5	5	1.	35	0.143
1975	A	4	4	1.	29	0.138
1976	A	3	2	1.	35	0.057
1977	A	3	2	1.	35	0.057
1978	A	3	2	1.	35	0.057
1979	A	3	2	1.	35	0.057

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1980	A	3	2	1.	35	0.057
1981	A	3	2	1.	35	0.057
1982	A	3	2	1.	35	0.057
1983	A	3	2	1.	35	0.057
1984	A	3	2	1.	37	0.054
1985	A	3	2	1.	37	0.054
1986	A	3	2	1.	37	0.054
1987	A	3	2	1.	37	0.054
1988	A	3	2	1.	37	0.054
1989	A	3	2	1.	37	0.054
1990	A	3	2	1.	37	0.054
1991	A	3	2	1.	37	0.054
1992	A	3	2	1.	37	0.054
1993	A	3	2	1.	37	0.054
1994	A	3	2	1.	37	0.054
1995	A	3	2	1.	37	0.054
1996	A	3	2	1.	37	0.054
1997	A	3	2	1.	37	0.054
1998	A	3	2	1.	37	0.054
1999	A	3	2	1.	37	0.054
2000	A	7	4	1.	37	0.108
2001	A	6	4	1.	37	0.108
2002	A	5	4	1.	37	0.108
2003	A	4	4	1.	37	0.108
2004	A	3	3	1.	37	0.081
2005	A	4	4	1.	37	0.108
2006	A	5	4	1.	37	0.108
2007	A	6	4	1.	37	0.108
2008	A	7	4	1.	37	0.108
2009	A	8	5	1.	37	0.135
2010	A	7	5	1.	37	0.135
2011	A	6	5	1.	37	0.135
2012	A	5	5	1.	37	0.135
2013	A	4	4	1.	37	0.108
2014	A	4	4	1.	37	0.108
2015	A	5	4	1.	37	0.108
2016	A	6	4	1.	37	0.108
2017	A	7	4	1.	37	0.108
2018	A	8	5	1.	37	0.135
2019	A	7	5	1.	37	0.135
2020	A	6	5	1.	37	0.135
2021	A	5	4	1.	37	0.108
2022	A	5	5	1.	37	0.135
2023	A	5	4	1.	37	0.108

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2024	A	6	4	1.	37	0.108
2025	A	7	4	1.	37	0.108
2026	A	8	4	1.	37	0.108
2027	A	5	2	1.	39	0.051
2028	A	4	2	1.	39	0.051
2029	A	3	2	1.	39	0.051
2030	A	2	2	1.	39	0.051
2031	A	1	1	1.	39	0.026
2032	A	3	3	1.	39	0.077
2033	A	3	3	1.	39	0.077
2034	A	4	4	1.	39	0.103
2035	A	5	4	1.	39	0.103
2036	A	5	2	1.	39	0.051
2037	A	4	2	1.	39	0.051
2038	A	3	2	1.	39	0.051
2039	A	2	2	1.	39	0.051
2040	A	1	1	1.	39	0.026
2041	A	4	3	1.	39	0.077
2042	A	4	4	1.	39	0.103
2043	A	4	3	1.	39	0.077
2044	A	5	4	1.	39	0.103
2045	A	6	4	1.	39	0.103
2046	A	5	2	1.	39	0.051
2047	A	4	2	1.	39	0.051
2048	A	3	2	1.	39	0.051
2049	A	2	2	1.	39	0.051
2050	A	1	1	1.	39	0.026
2051	A	5	3	1.	39	0.077
2052	A	5	4	1.	39	0.103
2053	A	5	4	1.	39	0.103
2054	A	5	3	1.	39	0.077
2055	A	6	4	1.	39	0.103
2056	A	7	4	1.	39	0.103
2057	A	4	2	1.	39	0.051
2058	A	3	2	1.	39	0.051
2059	A	2	2	1.	39	0.051
2060	A	1	1	1.	39	0.026
2061	A	2	2	1.	39	0.051
2062	A	3	3	1.	39	0.077
2063	A	4	3	1.	39	0.077
2064	A	5	3	1.	39	0.077
2065	A	3	2	1.	39	0.051
2066	A	2	2	1.	39	0.051
2067	A	1	1	1.	39	0.026

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
2068	A	3	3	1.	39	0.077
2069	A	4	4	1.	39	0.103
2070	A	5	4	1.	39	0.103
2071	A	6	4	1.	39	0.103
2072	A	7	4	1.	39	0.103
2073	A	2	2	1.	39	0.051
2074	A	1	1	1.	39	0.026
2075	A	5	5	1.	39	0.128
2076	A	5	4	1.	39	0.103
2077	A	6	4	1.	39	0.103
2078	A	7	4	1.	39	0.103
2079	A	8	4	1.	39	0.103
2080	A	9	4	1.	39	0.103
2081	A	2	2	1.	26	0.077
2082	A	2	2	1.	28	0.071
2083	A	5	5	1.	39	0.128
2084	A	3	2	1.	35	0.057
2085	A	3	2	1.	35	0.057
2086	A	3	2	1.	33	0.061
2087	A	2	2	1.	35	0.057
2088	A	2	2	1.	35	0.057
2089	A	2	2	1.	35	0.057
2090	A	2	2	1.	35	0.057
2091	A	4	4	1.23	35	0.114
2092	A	3	3	1.31	35	0.086
2093	A	3	3	1.31	35	0.086
2094	A	2	2	1.6	33	0.061
2095	A	1	1	1.	27	0.037
2096	A	3	3	1.	35	0.086
2097	A	3	3	1.3	35	0.086
2098	A	3	3	1.29	35	0.086
2099	A	4	4	1.	37	0.108
2100	A	4	4	1.	39	0.103
2101	A	1	1	1.	39	0.026
2102	A	2	2	1.	39	0.051
2103	A	3	2	1.	39	0.051
2104	A	4	2	1.	39	0.051
2105	A	1	1	1.	37	0.027
2106	A	1	1	1.	37	0.027
2107	A	2	1	1.	18	0.056
2108	A	2	1	1.	18	0.056
2109	A	2	1	1.	18	0.056
2110	A	2	1	1.	16	0.062
2111	A	1	0	1.	10	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2112	A	2	1	1.	18	0.056
2113	A	2	1	1.	18	0.056
2114	A	2	1	1.	18	0.056
2115	A	2	1	1.	18	0.056
2116	A	2	1	1.	18	0.056
2117	A	2	1	1.	18	0.056
2118	A	2	1	1.	20	0.05
2119	A	2	1	1.	20	0.05
2120	A	2	1	1.	20	0.05
2121	A	2	1	1.	18	0.056
2122	A	2	1	1.	12	0.083
2123	A	2	1	1.	20	0.05
2124	A	2	1	1.	20	0.05
2125	A	2	1	1.	20	0.05
2126	A	2	1	1.	20	0.05
2127	A	2	1	1.	20	0.05
2128	A	2	1	1.	20	0.05
2129	A	2	1	1.	20	0.05
2130	A	2	1	1.	20	0.05
2131	A	2	1	1.	20	0.05
2132	A	2	1	1.	20	0.05
2133	A	2	1	1.	20	0.05
2134	A	2	1	1.	18	0.056
2135	A	2	1	1.	12	0.083
2136	A	2	1	1.	20	0.05
2137	A	2	1	1.	20	0.05
2138	A	2	1	1.	20	0.05
2139	A	2	1	1.	20	0.05
2140	A	2	1	1.	20	0.05
2141	A	2	1	1.	20	0.05
2142	A	2	1	1.	20	0.05
2143	A	2	1	1.	20	0.05
2144	A	2	1	1.	20	0.05
2145	A	2	1	1.	20	0.05
2146	A	2	1	1.	20	0.05
2147	A	2	1	1.	20	0.05
2148	A	2	1	1.	20	0.05
2149	A	2	1	1.	18	0.056
2150	A	2	1	1.	12	0.083
2151	A	2	1	1.	20	0.05
2152	A	2	1	1.	20	0.05
2153	A	2	1	1.	20	0.05
2154	A	2	1	1.	20	0.05
2155	A	2	1	1.	20	0.05

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
2156	A	2	1	1.	20	0.05
2157	A	2	1	1.	20	0.05
2158	A	2	1	1.	20	0.05
2159	A	2	1	1.	20	0.05
2160	A	2	1	1.	20	0.05
2161	A	2	1	1.	20	0.05
2162	A	2	1	1.	20	0.05
2163	A	2	1	1.	14	0.071
2164	A	2	1	1.	14	0.071
2165	A	2	1	1.	14	0.071
2166	A	2	1	1.	12	0.083
2167	A	3	2	1.	10	0.2
2168	A	2	1	1.	14	0.071
2169	A	2	1	1.	14	0.071
2170	A	2	1	1.	14	0.071
2171	A	2	1	1.	14	0.071
2172	A	2	1	1.	14	0.071
2173	A	2	1	1.	14	0.071
2174	A	2	1	1.	14	0.071
2175	A	2	1	1.	14	0.071
2176	A	2	1	1.	14	0.071
2177	A	2	1	1.	14	0.071
2178	A	2	1	1.	16	0.062
2179	A	2	1	1.	10	0.1
2180	A	2	1	1.	12	0.083
2181	A	2	1	1.	12	0.083
2182	A	6	5	1.	20	0.25
2183	A	6	5	1.	20	0.25
2184	A	6	5	1.	20	0.25
2185	A	4	4	1.	18	0.222
2186	A	2	2	1.	12	0.167
2187	A	6	6	1.	20	0.3
2188	A	7	6	1.	20	0.3
2189	A	7	6	1.	20	0.3
2190	A	7	6	1.	20	0.3
2191	A	7	6	1.	20	0.3
2192	A	6	6	1.	20	0.3
2193	A	3	3	1.	20	0.15
2194	A	3	3	1.	18	0.167
2195	A	3	3	1.	12	0.25
2196	A	7	6	1.	20	0.3
2197	A	7	6	1.	20	0.3
2198	A	7	6	1.	20	0.3
2199	A	8	7	1.	16	0.438

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
2200	A	8	7	1.	16	0.438
2201	A	7	7	1.	20	0.35
2202	A	4	3	1.	20	0.15
2203	A	4	4	1.	20	0.2
2204	A	4	4	1.	20	0.2
2205	A	4	4	1.	18	0.222
2206	A	4	3	1.	12	0.25
2207	A	8	7	1.	20	0.35
2208	A	8	7	1.	16	0.438
2209	A	8	7	1.	16	0.438
2210	A	9	7	1.	16	0.438
2211	A	8	7	1.	16	0.438
2212	A	5	3	1.	16	0.188
2213	A	5	4	1.	16	0.25
2214	A	5	5	1.	20	0.25
2215	A	5	5	1.	20	0.25
2216	A	5	5	1.	20	0.25
2217	A	5	4	1.	18	0.222
2218	A	5	3	1.	12	0.25
2219	A	9	7	1.	20	0.35
2220	A	9	7	1.	16	0.438
2221	A	6	6	1.	20	0.3
2222	A	6	5	1.	20	0.25
2223	A	6	6	1.	20	0.3
2224	A	6	5	1.	20	0.25
2225	A	6	4	1.	18	0.222
2226	A	6	3	1.	12	0.25
2227	A	10	7	1.	20	0.35
2228	A	10	7	1.	20	0.35
2229	A	8	7	1.	20	0.35
2230	A	8	7	1.	20	0.35
2231	A	9	7	1.	20	0.35
2232	A	9	7	1.	20	0.35
2233	A	3	2	1.	16	0.125
2234	A	6	6	1.	14	0.429
2235	A	5	5	1.	66	0.076
2236	A	5	3	1.	16	0.188
2237	A	5	3	1.	16	0.188
2238	A	4	3	1.	16	0.188
2239	A	3	2	1.	14	0.143
2240	A	3	2	1.	12	0.167
2241	A	5	4	1.	16	0.25
2242	A	3	2	1.	16	0.125
2243	A	3	2	1.	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
2244	A	3	2	1.	16	0.125
2245	A	6	4	1.	20	0.2
2246	A	6	4	1.	20	0.2
2247	A	5	4	1.	20	0.2
2248	A	4	3	1.	20	0.15
2249	A	4	3	1.	18	0.167
2250	A	6	5	1.	16	0.312
2251	A	4	3	1.	20	0.15
2252	A	4	3	1.	20	0.15
2253	A	4	3	1.	20	0.15
2254	A	3	2	1.	12	0.167
2255	A	4	4	1.	12	0.333
2256	A	3	2	1.	12	0.167
2257	A	3	3	1.	12	0.25
2258	A	4	4	1.	10	0.4
2259	A	5	5	1.	12	0.417
2260	A	4	3	1.	14	0.214
2261	A	4	3	1.	12	0.25
2262	A	3	3	1.53	14	0.214
2263	A	5	3	1.	14	0.214
2264	A	3	2	1.	14	0.143
2265	A	3	2	1.	14	0.143
2266	A	3	2	1.	14	0.143
2267	A	6	6	1.	12	0.5
2268	A	2	1	1.	20	0.05
2269	A	2	1	1.	20	0.05
2270	A	2	1	1.	20	0.05
2271	A	2	1	1.	20	0.05
2272	A	2	1	1.	20	0.05
2273	A	2	1	1.	20	0.05
2274	A	2	1	1.	20	0.05
2275	A	2	1	1.	22	0.045
2276	A	2	1	1.	22	0.045
2277	A	2	1	1.	22	0.045
2278	A	2	1	1.	22	0.045
2279	A	2	1	1.	22	0.045
2280	A	2	1	1.	22	0.045
2281	A	2	1	1.	22	0.045
2282	A	2	1	1.	22	0.045
2283	A	2	1	1.	22	0.045
2284	A	2	1	1.	22	0.045
2285	A	2	1	1.	22	0.045
2286	A	2	1	1.	22	0.045
2287	A	2	1	1.	22	0.045

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2288	A	2	1	1.	22	0.045
2289	A	6	5	1.	22	0.227
2290	A	5	4	1.	22	0.182
2291	A	4	3	1.	22	0.136
2292	A	4	3	1.	22	0.136
2293	A	5	4	1.	22	0.182
2294	A	6	5	1.	22	0.227
2295	A	7	5	1.	22	0.227
2296	A	6	5	1.	22	0.227
2297	A	5	4	1.	22	0.182
2298	A	5	4	1.	22	0.182
2299	A	5	4	1.	22	0.182
2300	A	6	5	1.	22	0.227
2301	A	7	5	1.	18	0.278
2302	A	6	5	1.	22	0.227
2303	A	6	5	1.	22	0.227
2304	A	6	5	1.	22	0.227
2305	A	6	5	1.	22	0.227
2306	A	6	5	1.	22	0.227
2307	A	10	6	1.	25	0.24
2308	A	10	6	1.	25	0.24
2309	A	13	8	1.	22	0.364
2310	A	12	8	1.	22	0.364
2311	A	11	7	1.	22	0.318
2312	A	10	7	1.	22	0.318
2313	A	10	6	1.	22	0.273
2314	A	11	7	1.	22	0.318
2315	A	12	8	1.	22	0.364
2316	A	13	8	1.	22	0.364
2317	A	12	8	1.	22	0.364
2318	A	11	7	1.	22	0.318
2319	A	11	7	1.	22	0.318
2320	A	11	7	1.	22	0.318
2321	A	12	8	1.	22	0.364
2322	A	13	8	1.	22	0.364
2323	A	13	9	1.	22	0.409
2324	A	12	8	1.	22	0.364
2325	A	12	8	1.	22	0.364
2326	A	12	8	1.	22	0.364
2327	A	12	8	1.	22	0.364
2328	A	12	8	1.	22	0.364
2329	A	13	9	1.	22	0.409
2330	A	7	6	1.	18	0.333
2331	A	7	6	1.	18	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
2332	A	2	1	1.	14	0.071
2333	A	5	5	1.	22	0.227
2334	A	5	5	1.	22	0.227
2335	A	4	4	1.	20	0.2
2336	A	3	3	1.	14	0.214
2337	A	6	5	1.	22	0.227
2338	A	6	5	1.	22	0.227
2339	A	3	3	1.	22	0.136
2340	A	4	4	1.	22	0.182
2341	A	5	5	1.	22	0.227
2342	A	6	6	1.	22	0.273
2343	A	6	5	1.	22	0.227
2344	A	6	5	1.	22	0.227
2345	A	5	4	1.	20	0.2
2346	A	4	3	1.	14	0.214
2347	A	7	6	1.	22	0.273
2348	A	7	6	1.	22	0.273
2349	A	7	6	1.	22	0.273
2350	A	7	6	1.	22	0.273
2351	A	4	3	1.	22	0.136
2352	A	5	4	1.	22	0.182
2353	A	6	5	1.	22	0.227
2354	A	7	6	1.	22	0.273
2355	A	7	5	1.	22	0.227
2356	A	7	5	1.	22	0.227
2357	A	6	4	1.	20	0.2
2358	A	5	3	1.	14	0.214
2359	A	8	6	1.	22	0.273
2360	A	8	6	1.	22	0.273
2361	A	8	7	1.	22	0.318
2362	A	8	6	1.	22	0.273
2363	A	8	7	1.	22	0.318
2364	A	6	6	1.	18	0.333
2365	A	6	6	1.	18	0.333
2366	A	2	2	1.	18	0.111
2367	A	4	4	1.	20	0.2
2368	A	4	4	1.	20	0.2
2369	A	6	5	1.	18	0.278
2370	A	6	6	1.	20	0.3
2371	A	7	5	1.	18	0.278
2372	A	6	5	1.	18	0.278
2373	A	5	5	1.	22	0.227
2374	A	4	4	1.	22	0.182
2375	A	4	4	1.	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
2376	A	3	3	1.	20	0.15
2377	A	2	2	1.	14	0.143
2378	A	2	2	1.	22	0.091
2379	A	3	3	1.	22	0.136
2380	A	4	4	1.	22	0.182
2381	A	5	5	1.	22	0.227
2382	A	5	5	1.	22	0.227
2383	A	4	4	1.	22	0.182
2384	A	4	4	1.	22	0.182
2385	A	1	1	1.	20	0.05
2386	A	1	1	1.	14	0.071
2387	A	4	4	1.	22	0.182
2388	A	4	4	1.	22	0.182
2389	A	5	5	1.	22	0.227
2390	A	6	5	1.	22	0.227
2391	A	5	5	1.	22	0.227
2392	A	5	5	1.	22	0.227
2393	A	2	2	1.	22	0.091
2394	A	2	2	1.	22	0.091
2395	A	2	2	1.	20	0.1
2396	A	2	2	1.	14	0.143
2397	A	5	5	1.	22	0.227
2398	A	5	5	1.	22	0.227
2399	A	6	6	1.	22	0.273
2400	A	3	3	1.	18	0.167
2401	A	3	3	1.	20	0.15
2402	A	3	3	1.	18	0.167
2403	A	3	3	1.	16	0.188
2404	A	2	2	1.	20	0.1
2405	A	2	2	1.	18	0.111
2406	A	1	1	1.	16	0.062
2407	A	4	3	1.	31	0.097
2408	A	2	2	1.	30	0.067
2409	A	1	1	1.	36	0.028
2410	A	2	2	1.	39	0.051
2411	A	3	3	1.	16	0.188
2412	A	3	3	1.	16	0.188
2413	A	3	3	1.	16	0.188
2414	A	3	3	1.	16	0.188
2415	A	3	3	1.	16	0.188
2416	A	3	3	1.	16	0.188
2417	A	3	3	1.	16	0.188
2418	A	3	3	1.	16	0.188
2419	B	4	4	2.04	18	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
2420	B	4	4	2.04	18	0.222
2421	B	4	4	2.04	18	0.222
2422	B	4	4	2.04	18	0.222
2423	A	4	4	1.	24	0.167
2424	A	4	4	1.	24	0.167
2425	A	4	4	1.	27	0.148
2426	A	4	4	1.	27	0.148
2427	A	4	4	1.	16	0.25
2428	A	4	4	1.	16	0.25
2429	A	4	4	1.	14	0.286
2430	A	2	2	1.	18	0.111
2431	A	2	2	1.	18	0.111
2432	A	2	2	1.	18	0.111
2433	A	2	2	1.	18	0.111
2434	A	2	2	1.	18	0.111
2435	A	2	2	1.	18	0.111
2436	A	2	2	1.	18	0.111
2437	A	2	2	1.	18	0.111
2438	A	4	4	1.	14	0.286
2439	A	3	2	1.17	23	0.087
2440	A	8	8	1.	22	0.364
2441	A	7	6	1.	24	0.25
2442	A	7	6	1.	24	0.25
2443	A	6	5	1.	24	0.208
2444	A	6	5	1.	24	0.208
2445	A	7	6	1.	24	0.25
2446	A	8	6	1.	24	0.25
2447	A	8	6	1.	24	0.25
2448	A	8	7	1.	24	0.292
2449	A	7	6	1.	24	0.25
2450	A	7	6	1.	24	0.25
2451	A	7	6	1.	24	0.25
2452	A	7	6	1.	24	0.25
2453	A	8	7	1.	24	0.292
2454	A	9	8	1.	22	0.364
2455	A	8	6	1.	24	0.25
2456	A	8	6	1.	24	0.25
2457	A	8	7	1.	24	0.292
2458	A	8	6	1.	24	0.25
2459	A	8	7	1.	24	0.292
2460	A	8	6	1.	24	0.25
2461	A	8	6	1.	24	0.25
2462	A	7	6	1.	24	0.25
2463	A	6	5	1.	24	0.208

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2464	A	2	2	1.	24	0.083
2465	A	2	2	1.	24	0.083
2466	A	4	4	1.	24	0.167
2467	A	7	6	1.	24	0.25
2468	A	8	6	1.	24	0.25
2469	A	8	6	1.	24	0.25
2470	A	7	6	1.	24	0.25
2471	A	6	5	1.	24	0.208
2472	A	6	5	1.	24	0.208
2473	A	6	5	1.	24	0.208
2474	A	7	6	1.	24	0.25
2475	A	8	6	1.	24	0.25
2476	A	7	6	1.	24	0.25
2477	A	7	6	1.	24	0.25
2478	A	7	6	1.	24	0.25
2479	A	7	6	1.	24	0.25
2480	A	7	6	1.	24	0.25
2481	A	8	7	1.	24	0.292
2482	A	2	2	1.	24	0.083
2483	A	6	5	1.	22	0.227
2484	A	5	4	1.	20	0.2
2485	A	4	3	1.	14	0.214
2486	A	2	2	1.	22	0.091
2487	A	2	2	1.	22	0.091
2488	A	2	2	1.	22	0.091
2489	A	6	6	1.	22	0.273
2490	A	6	6	1.	22	0.273
2491	A	6	6	1.	20	0.3
2492	A	6	5	1.	14	0.357
2493	A	2	2	1.	22	0.091
2494	A	2	2	1.	22	0.091
2495	A	2	2	1.	22	0.091
2496	A	1	1	1.	52	0.019
2497	A	7	7	1.	22	0.318
2498	A	8	8	1.	22	0.364
2499	A	6	6	1.	20	0.3
2500	A	1	1	1.	22	0.045
2501	A	8	8	1.	22	0.364
2502	A	9	9	1.	22	0.409
2503	A	7	7	1.	22	0.318
2504	A	7	7	1.	22	0.318
2505	A	6	6	1.	20	0.3
2506	A	1	1	1.	22	0.045
2507	A	9	9	1.	22	0.409

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
2508	A	9	9	1.	22	0.409
2509	A	2	2	1.	53	0.038
2510	A	5	5	1.	22	0.227
2511	A	5	5	1.	22	0.227
2512	A	4	4	1.	20	0.2
2513	A	3	3	1.	14	0.214
2514	A	19	17	1.	22	0.773
2515	A	19	17	1.	22	0.773
2516	A	7	7	1.	22	0.318
2517	A	7	7	1.	22	0.318
2518	A	6	6	1.	20	0.3
2519	A	5	5	1.	14	0.357
2520	A	20	18	1.	22	0.818
2521	A	20	18	1.	22	0.818
2522	A	6	5	1.	22	0.227
2523	A	6	5	1.	22	0.227
2524	A	5	4	1.	20	0.2
2525	A	4	3	1.	14	0.214
2526	A	20	18	1.	22	0.818
2527	A	20	18	1.	22	0.818
2528	A	6	6	1.	22	0.273
2529	A	6	6	1.	22	0.273
2530	A	5	5	1.	20	0.25
2531	A	4	4	1.	14	0.286
2532	A	14	12	1.	22	0.546
2533	A	20	18	1.	22	0.818
2534	A	21	19	1.	22	0.864
2535	A	4	4	1.	22	0.182
2536	A	4	4	1.	22	0.182
2537	A	3	3	1.	20	0.15
2538	A	2	2	1.	14	0.143
2539	A	15	13	1.	22	0.591
2540	A	19	17	1.	22	0.773
2541	A	20	18	1.	22	0.818
2542	A	6	6	1.	22	0.273
2543	A	6	6	1.	22	0.273
2544	A	5	5	1.	20	0.25
2545	A	5	5	1.	14	0.357
2546	A	20	18	1.	22	0.818
2547	A	21	19	1.	22	0.864
2548	A	1	1	1.	24	0.042
2549	A	2	1	1.	20	0.05
2550	A	2	1	1.	20	0.05
2551	A	2	1	1.	20	0.05

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2552	A	2	1	1.	18	0.056
2553	A	4	2	1.	20	0.1
2554	A	5	3	1.	20	0.15
2555	A	2	2	1.	22	0.091
2556	A	2	2	1.	22	0.091
2557	A	2	2	1.	22	0.091
2558	A	2	2	1.	22	0.091
2559	A	2	2	1.	22	0.091
2560	A	2	2	1.	22	0.091
2561	A	2	2	1.	18	0.111
2562	A	2	2	1.	20	0.1
2563	A	3	3	1.	20	0.15
2564	A	3	3	1.	20	0.15
2565	A	2	2	1.	18	0.111
2566	A	1	1	1.	12	0.083
2567	A	2	2	1.	20	0.1
2568	A	2	2	1.	20	0.1
2569	A	2	2	1.	20	0.1
2570	A	2	2	1.	22	0.091
2571	A	2	2	1.	22	0.091
2572	A	2	2	1.	22	0.091
2573	A	2	2	1.	22	0.091
2574	A	2	2	1.	22	0.091
2575	A	2	2	1.	24	0.083
2576	A	1	1	1.	24	0.042
2577	A	2	2	1.	24	0.083
2578	A	3	3	1.	24	0.125
2579	A	4	4	1.	24	0.167
2580	A	5	4	1.	24	0.167
2581	A	3	3	1.	26	0.115
2582	A	2	2	1.	20	0.1
2583	A	2	2	1.	20	0.1
2584	A	2	2	1.	16	0.125
2585	A	2	2	1.	20	0.1
2586	A	2	2	1.	20	0.1
2587	A	2	2	1.	16	0.125
2588	A	2	2	1.	33	0.061
2589	A	3	3	1.	33	0.091
2590	A	3	3	1.	29	0.103

Chapter 3

Listing of integrals

3.1 $\int x^3\sqrt{bx+cx^2}\,dx$

Optimal. Leaf size=131

$$-\frac{7b^3(b+2cx)\sqrt{bx+cx^2}}{128c^4}+\frac{7b^2\left(bx+cx^2\right)^{3/2}}{48c^3}+\frac{7b^5\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{128c^{9/2}}-\frac{7bx\left(bx+cx^2\right)^{3/2}}{40c^2}+\frac{x^2\left(bx+cx^2\right)^{3/2}}{5c}$$

```
[Out] (-7*b^3*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(128*c^4) + (7*b^2*(b*x + c*x^2)^(3/
2))/(48*c^3) - (7*b*x*(b*x + c*x^2)^(3/2))/(40*c^2) + (x^2*(b*x + c*x^2)^(3
/2))/(5*c) + (7*b^5*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(128*c^(9/2))
```

Rubi [A] time = 0.0597608, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {670, 640, 612, 620, 206}

$$-\frac{7b^3(b+2cx)\sqrt{bx+cx^2}}{128c^4}+\frac{7b^2\left(bx+cx^2\right)^{3/2}}{48c^3}+\frac{7b^5\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{128c^{9/2}}-\frac{7bx\left(bx+cx^2\right)^{3/2}}{40c^2}+\frac{x^2\left(bx+cx^2\right)^{3/2}}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[b*x + c*x^2],x]
```

```
[Out] (-7*b^3*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(128*c^4) + (7*b^2*(b*x + c*x^2)^(3/
2))/(48*c^3) - (7*b*x*(b*x + c*x^2)^(3/2))/(40*c^2) + (x^2*(b*x + c*x^2)^(3
/2))/(5*c) + (7*b^5*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(128*c^(9/2))
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3\sqrt{bx+cx^2}\,dx &= \frac{x^2\left(bx+cx^2\right)^{3/2}}{5c}-\frac{(7b)\int x^2\sqrt{bx+cx^2}\,dx}{10c}\\
&= -\frac{7bx\left(bx+cx^2\right)^{3/2}}{40c^2}+\frac{x^2\left(bx+cx^2\right)^{3/2}}{5c}+\frac{\left(7b^2\right)\int x\sqrt{bx+cx^2}\,dx}{16c^2}\\
&= \frac{7b^2\left(bx+cx^2\right)^{3/2}}{48c^3}-\frac{7bx\left(bx+cx^2\right)^{3/2}}{40c^2}+\frac{x^2\left(bx+cx^2\right)^{3/2}}{5c}-\frac{\left(7b^3\right)\int\sqrt{bx+cx^2}\,dx}{32c^3}\\
&= -\frac{7b^3(b+2cx)\sqrt{bx+cx^2}}{128c^4}+\frac{7b^2\left(bx+cx^2\right)^{3/2}}{48c^3}-\frac{7bx\left(bx+cx^2\right)^{3/2}}{40c^2}+\frac{x^2\left(bx+cx^2\right)^{3/2}}{5c}+\frac{\left(7b^5\right)\int\frac{1}{\sqrt{bx+}}}{256c^4}\\
&= -\frac{7b^3(b+2cx)\sqrt{bx+cx^2}}{128c^4}+\frac{7b^2\left(bx+cx^2\right)^{3/2}}{48c^3}-\frac{7bx\left(bx+cx^2\right)^{3/2}}{40c^2}+\frac{x^2\left(bx+cx^2\right)^{3/2}}{5c}+\frac{\left(7b^5\right)\text{Subst}}{}\\
&= -\frac{7b^3(b+2cx)\sqrt{bx+cx^2}}{128c^4}+\frac{7b^2\left(bx+cx^2\right)^{3/2}}{48c^3}-\frac{7bx\left(bx+cx^2\right)^{3/2}}{40c^2}+\frac{x^2\left(bx+cx^2\right)^{3/2}}{5c}+\frac{7b^5\tanh^{-1}(}{128c^9}
\end{aligned}$$

Mathematica [A] time = 0.154078, size = 109, normalized size = 0.83

$$\frac{\sqrt{x(b+cx)}\left(\sqrt{c}\left(-56b^2c^2x^2+70b^3cx-105b^4+48bc^3x^3+384c^4x^4\right)+\frac{105b^{9/2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}\right)}{1920c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[b*x + c*x^2],x]
```

```
[Out] (Sqrt[x*(b + c*x)]*(Sqrt[c]*(-105*b^4 + 70*b^3*c*x - 56*b^2*c^2*x^2 + 48*b*
c^3*x^3 + 384*c^4*x^4) + (105*b^(9/2)*ArcSinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(
Sqrt[x]*Sqrt[1 + (c*x)/b])))/(1920*c^(9/2))
```

Maple [A] time = 0.047, size = 129, normalized size = 1.

$$\frac{x^2}{5c}\left(cx^2+bx\right)^{\frac{3}{2}}-\frac{7bx}{40c^2}\left(cx^2+bx\right)^{\frac{3}{2}}+\frac{7b^2}{48c^3}\left(cx^2+bx\right)^{\frac{3}{2}}-\frac{7b^3x}{64c^3}\sqrt{cx^2+bx}-\frac{7b^4}{128c^4}\sqrt{cx^2+bx}+\frac{7b^5}{256}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2+b*x)^(1/2),x)
```

```
[Out] 1/5*x^2*(c*x^2+b*x)^(3/2)/c-7/40*b*x*(c*x^2+b*x)^(3/2)/c^2+7/48*b^2*(c*x^2+
b*x)^(3/2)/c^3-7/64*b^3/c^3*(c*x^2+b*x)^(1/2)*x-7/128*b^4/c^4*(c*x^2+b*x)^(
1/2)+7/256*b^5/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.07286, size = 459, normalized size = 3.5

$$\left[\frac{105\,b^5\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\left(384\,c^5x^4+48\,bc^4x^3-56\,b^2c^3x^2+70\,b^3c^2x-105\,b^4c\right)\sqrt{cx^2+bx}}{3840\,c^5},-\frac{105}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3840*(105*b^5*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*(
384*c^5*x^4 + 48*b*c^4*x^3 - 56*b^2*c^3*x^2 + 70*b^3*c^2*x - 105*b^4*c)*sqr
t(c*x^2 + b*x))/c^5, -1/1920*(105*b^5*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqr
t(-c)/(c*x)) - (384*c^5*x^4 + 48*b*c^4*x^3 - 56*b^2*c^3*x^2 + 70*b^3*c^2*x
- 105*b^4*c)*sqrt(c*x^2 + b*x))/c^5]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{x\left(b+cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(x*(b + c*x)), x)
```

Giac [A] time = 1.30265, size = 131, normalized size = 1.

$$\frac{1}{1920}\sqrt{cx^2+bx}\left(2\left(4\left(6\left(8\,x+\frac{b}{c}\right)x-\frac{7\,b^2}{c^2}\right)x+\frac{35\,b^3}{c^3}\right)x-\frac{105\,b^4}{c^4}\right)-\frac{7\,b^5\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}-b\right|\right)}{256\,c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(c*x^2 + b*x)*(2*(4*(6*(8*x + b/c)*x - 7*b^2/c^2)*x + 35*b^3/c^3
)*x - 105*b^4/c^4) - 7/256*b^5*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*s
qrt(c) - b))/c^(9/2)
```

3.2 $\int x^2\sqrt{bx+cx^2}\,dx$

Optimal. Leaf size=105

$$\frac{5b^2(b+2cx)\sqrt{bx+cx^2}}{64c^3}-\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{64c^{7/2}}-\frac{5b\left(bx+cx^2\right)^{3/2}}{24c^2}+\frac{x\left(bx+cx^2\right)^{3/2}}{4c}$$

```
[Out] (5*b^2*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(64*c^3) - (5*b*(b*x + c*x^2)^(3/2))/
(24*c^2) + (x*(b*x + c*x^2)^(3/2))/(4*c) - (5*b^4*ArcTanh[(Sqrt[c]*x)/Sqrt[
b*x + c*x^2]])/(64*c^(7/2))
```

Rubi [A] time = 0.0374605, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {670, 640, 612, 620, 206}

$$\frac{5b^2(b+2cx)\sqrt{bx+cx^2}}{64c^3}-\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{64c^{7/2}}-\frac{5b\left(bx+cx^2\right)^{3/2}}{24c^2}+\frac{x\left(bx+cx^2\right)^{3/2}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[b*x + c*x^2],x]
```

```
[Out] (5*b^2*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(64*c^3) - (5*b*(b*x + c*x^2)^(3/2))/
(24*c^2) + (x*(b*x + c*x^2)^(3/2))/(4*c) - (5*b^4*ArcTanh[(Sqrt[c]*x)/Sqrt[
b*x + c*x^2]])/(64*c^(7/2))
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{bx+cx^2}\,dx &= \frac{x\left(bx+cx^2\right)^{3/2}}{4c}-\frac{(5b)\int x\sqrt{bx+cx^2}\,dx}{8c}\\ &= -\frac{5b\left(bx+cx^2\right)^{3/2}}{24c^2}+\frac{x\left(bx+cx^2\right)^{3/2}}{4c}+\frac{\left(5b^2\right)\int\sqrt{bx+cx^2}\,dx}{16c^2}\\ &= \frac{5b^2(b+2cx)\sqrt{bx+cx^2}}{64c^3}-\frac{5b\left(bx+cx^2\right)^{3/2}}{24c^2}+\frac{x\left(bx+cx^2\right)^{3/2}}{4c}-\frac{\left(5b^4\right)\int\frac{1}{\sqrt{bx+cx^2}}\,dx}{128c^3}\\ &= \frac{5b^2(b+2cx)\sqrt{bx+cx^2}}{64c^3}-\frac{5b\left(bx+cx^2\right)^{3/2}}{24c^2}+\frac{x\left(bx+cx^2\right)^{3/2}}{4c}-\frac{\left(5b^4\right)\text{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{bx+cx^2}}\right)}{64c^3}\\ &= \frac{5b^2(b+2cx)\sqrt{bx+cx^2}}{64c^3}-\frac{5b\left(bx+cx^2\right)^{3/2}}{24c^2}+\frac{x\left(bx+cx^2\right)^{3/2}}{4c}-\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{64c^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.13163, size = 98, normalized size = 0.93

$$\frac{\sqrt{x(b+cx)}\left(\sqrt{c}\left(-10b^2cx+15b^3+8bc^2x^2+48c^3x^3\right)-\frac{15b^{7/2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}\right)}{192c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[b*x + c*x^2],x]
```

```
[Out] (Sqrt[x*(b + c*x)]*(Sqrt[c]*(15*b^3 - 10*b^2*c*x + 8*b*c^2*x^2 + 48*c^3*x^3
) - (15*b^(7/2)*ArcSinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(Sqrt[x]*Sqrt[1 + (c*x)
/b])))/(192*c^(7/2))
```

Maple [A] time = 0.044, size = 107, normalized size = 1.

$$\frac{x}{4c}\left(cx^2+bx\right)^{\frac{3}{2}}-\frac{5b}{24c^2}\left(cx^2+bx\right)^{\frac{3}{2}}+\frac{5b^2x}{32c^2}\sqrt{cx^2+bx}+\frac{5b^3}{64c^3}\sqrt{cx^2+bx}-\frac{5b^4}{128}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx}\right)c^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2+b*x)^(1/2),x)
```

```
[Out] 1/4*x*(c*x^2+b*x)^(3/2)/c-5/24*b*(c*x^2+b*x)^(3/2)/c^2+5/32*b^2/c^2*(c*x^2+
b*x)^(1/2)*x+5/64*b^3/c^3*(c*x^2+b*x)^(1/2)-5/128*b^4/c^(7/2)*ln((1/2*b+c*x
)/c^(1/2)+(c*x^2+b*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.0657, size = 398, normalized size = 3.79

$$\left[\frac{15\,b^4\sqrt{c}\log\left(2\,cx+b-2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\left(48\,c^4x^3+8\,bc^3x^2-10\,b^2c^2x+15\,b^3c\right)\sqrt{cx^2+bx}}{384\,c^4}, \frac{15\,b^4\sqrt{-c}\arctan\left(\frac{\sqrt{cx}}{\ldots}\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/384*(15*b^4*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*(48
*c^4*x^3 + 8*b*c^3*x^2 - 10*b^2*c^2*x + 15*b^3*c)*sqrt(c*x^2 + b*x))/c^4, 1
/192*(15*b^4*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)) + (48*c^4*x^
3 + 8*b*c^3*x^2 - 10*b^2*c^2*x + 15*b^3*c)*sqrt(c*x^2 + b*x))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{x\,(b+cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(x*(b + c*x)), x)
```

Giac [A] time = 1.35702, size = 115, normalized size = 1.1

$$\frac{1}{192}\sqrt{cx^2+bx}\left(2\left(4\left(6\,x+\frac{b}{c}\right)x-\frac{5\,b^2}{c^2}\right)x+\frac{15\,b^3}{c^3}\right)+\frac{5\,b^4\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}-b\right|\right)}{128\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x)*(2*(4*(6*x + b/c)*x - 5*b^2/c^2)*x + 15*b^3/c^3) +
5/128*b^4*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b))/c^(7/2)
```

3.3 $\int x\sqrt{bx+cx^2}\,dx$

Optimal. Leaf size=81

$$\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{8c^{5/2}}-\frac{b(b+2cx)\sqrt{bx+cx^2}}{8c^2}+\frac{\left(bx+cx^2\right)^{3/2}}{3c}$$

```
[Out] -(b*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(8*c^2) + (b*x + c*x^2)^(3/2)/(3*c) + (b
^3*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(8*c^(5/2))
```

Rubi [A] time = 0.0238282, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {640, 612, 620, 206}

$$\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{8c^{5/2}}-\frac{b(b+2cx)\sqrt{bx+cx^2}}{8c^2}+\frac{\left(bx+cx^2\right)^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[b*x + c*x^2],x]
```

```
[Out] -(b*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(8*c^2) + (b*x + c*x^2)^(3/2)/(3*c) + (b
^3*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(8*c^(5/2))
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x\sqrt{bx+cx^2}\,dx = \frac{\left(bx+cx^2\right)^{3/2}}{3c} - \frac{b\int\sqrt{bx+cx^2}\,dx}{2c}$$
$$= -\frac{b(b+2cx)\sqrt{bx+cx^2}}{8c^2} + \frac{\left(bx+cx^2\right)^{3/2}}{3c} + \frac{b^3\int\frac{1}{\sqrt{bx+cx^2}}\,dx}{16c^2}$$
$$= -\frac{b(b+2cx)\sqrt{bx+cx^2}}{8c^2} + \frac{\left(bx+cx^2\right)^{3/2}}{3c} + \frac{b^3\,\mathrm{Subst}\left(\int\frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{bx+cx^2}}\right)}{8c^2}$$
$$= -\frac{b(b+2cx)\sqrt{bx+cx^2}}{8c^2} + \frac{\left(bx+cx^2\right)^{3/2}}{3c} + \frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{8c^{5/2}}$$

Mathematica [A] time = 0.112991, size = 87, normalized size = 1.07

$$\frac{\sqrt{x(b+cx)}\left(\sqrt{c}\left(-3b^2+2bcx+8c^2x^2\right)+\frac{3b^{5/2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}\right)}{24c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[b*x + c*x^2],x]
```

```
[Out] (Sqrt[x*(b + c*x)]*(Sqrt[c]*(-3*b^2 + 2*b*c*x + 8*c^2*x^2) + (3*b^(5/2)*Arc
Sinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(Sqrt[x]*Sqrt[1 + (c*x)/b])))/(24*c^(5/2))
```

Maple [A] time = 0.053, size = 87, normalized size = 1.1

$$\frac{1}{3c}\left(cx^2+bx\right)^{\frac{3}{2}} - \frac{bx}{4c}\sqrt{cx^2+bx} - \frac{b^2}{8c^2}\sqrt{cx^2+bx} + \frac{b^3}{16}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx}\right)c^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x)^(1/2),x)
```

```
[Out] 1/3*(c*x^2+b*x)^(3/2)/c-1/4*b/c*x*(c*x^2+b*x)^(1/2)-1/8*b^2/c^2*(c*x^2+b*x)
^(1/2)+1/16*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.07372, size = 343, normalized size = 4.23

$$\left[\frac{3\,b^{3}\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^{2}+bx}\sqrt{c}\right)+2\left(8\,c^{3}x^{2}+2\,bc^{2}x-3\,b^{2}c\right)\sqrt{cx^{2}+bx}}{48\,c^{3}},\,-\frac{3\,b^{3}\sqrt{-c}\arctan\left(\frac{\sqrt{cx^{2}+bx}\sqrt{-c}}{cx}\right)-\left(8\,c^{3}x^{2}\right.}{24\,c^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*b^3*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*(8*c^
3*x^2 + 2*b*c^2*x - 3*b^2*c)*sqrt(c*x^2 + b*x))/c^3, -1/24*(3*b^3*sqrt(-c)*
arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)) - (8*c^3*x^2 + 2*b*c^2*x - 3*b^2*c
)*sqrt(c*x^2 + b*x))/c^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{x\,(b+cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x*(b + c*x)), x)
```

Giac [A] time = 1.31978, size = 99, normalized size = 1.22

$$\frac{1}{24}\,\sqrt{cx^{2}+bx}\left(2\left(4\,x+\frac{b}{c}\right)x-\frac{3\,b^{2}}{c^{2}}\right)-\frac{b^{3}\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^{2}+bx}\right)\sqrt{c}-b\right|\right)}{16\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + b*x)*(2*(4*x + b/c)*x - 3*b^2/c^2) - 1/16*b^3*log(abs(-2*
(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b))/c^(5/2)
```

3.4 $\int \sqrt{bx+cx^2}\,dx$

Optimal. Leaf size=60

$$\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}$$

```
[Out] ((b + 2*c*x)*Sqrt[b*x + c*x^2])/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x +
 c*x^2]])/(4*c^(3/2))
```

Rubi [A] time = 0.0136197, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {612, 620, 206}

$$\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x + c*x^2],x]
```

```
[Out] ((b + 2*c*x)*Sqrt[b*x + c*x^2])/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x +
 c*x^2]])/(4*c^(3/2))
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{bx+cx^2}\,dx &= \frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int \frac{1}{\sqrt{bx+cx^2}}\,dx}{8c}\\ &= \frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{bx+cx^2}}\right)}{4c}\\ &= \frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0999567, size = 74, normalized size = 1.23

$$\frac{\sqrt{x(b+cx)}\left(\sqrt{c}(b+2cx)-\frac{b^{3/2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}\right)}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x + c*x^2],x]
```

```
[Out] (Sqrt[x*(b + c*x)]*(Sqrt[c]*(b + 2*c*x) - (b^(3/2)*ArcSinh[(Sqrt[c]*Sqrt[x]
)/Sqrt[b]])/(Sqrt[x]*Sqrt[1 + (c*x)/b])))/(4*c^(3/2))
```

Maple [A] time = 0.047, size = 56, normalized size = 0.9

$$\frac{2\,cx+b}{4\,c}\sqrt{cx^2+bx}-\frac{b^2}{8}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx}\right)c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(1/2),x)
```

```
[Out] 1/4*(2*c*x+b)*(c*x^2+b*x)^(1/2)/c-1/8*b^2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c
*x^2+b*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.048, size = 285, normalized size = 4.75

$$\left[\frac{b^2\sqrt{c}\log\left(2\,cx+b-2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\left(2\,c^2x+bc\right)\sqrt{cx^2+bx}}{8\,c^2},\frac{b^2\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx}\right)+\left(2\,c^2x+bc\right)\sqrt{cx^2+bx}}{4\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(b^2*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*(2*c^2*x
 + b*c)*sqrt(c*x^2 + b*x))/c^2, 1/4*(b^2*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*
sqrt(-c)/(c*x)) + (2*c^2*x + b*c)*sqrt(c*x^2 + b*x))/c^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx + cx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(sqrt(b*x + c*x**2), x)
```

Giac [A] time = 1.40028, size = 82, normalized size = 1.37

$$\frac{1}{4}\sqrt{cx^2 + bx}\left(2x + \frac{b}{c}\right) + \frac{b^2 \log\left(\left|-2\left(\sqrt{c}x - \sqrt{cx^2 + bx}\right)\sqrt{c} - b\right|\right)}{8c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^2 + b*x)*(2*x + b/c) + 1/8*b^2*log(abs(-2*(sqrt(c)*x - sqrt(c*
x^2 + b*x))*sqrt(c) - b))/c^(3/2)
```

3.5 $\int \frac{\sqrt{bx+cx^2}}{x} \, dx$

Optimal. Leaf size=42

$$\sqrt{bx+cx^2}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{\sqrt{c}}$$

[Out] Sqrt[b*x + c*x^2] + (b*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/Sqrt[c]

Rubi [A] time = 0.019189, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {664, 620, 206}

$$\sqrt{bx+cx^2}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*x + c*x^2]/x,x]

[Out] Sqrt[b*x + c*x^2] + (b*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/Sqrt[c]

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{bx+cx^2}}{x} \, dx &= \sqrt{bx+cx^2}+\frac{1}{2}b\int \frac{1}{\sqrt{bx+cx^2}} \, dx\\
&= \sqrt{bx+cx^2}+b\,\text{Subst}\left(\int \frac{1}{1-cx^2} \, dx,x,\frac{x}{\sqrt{bx+cx^2}}\right)\\
&= \sqrt{bx+cx^2}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.0988608, size = 66, normalized size = 1.57

$$\sqrt{x(b+cx)}\left(\frac{b^{3/2}\sqrt{\frac{cx}{b}+1}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{c}\sqrt{x}(b+cx)}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x + c*x^2]/x,x]
```

```
[Out] Sqrt[x*(b + c*x)]*(1 + (b^(3/2)*Sqrt[1 + (c*x)/b]*ArcSinh[(Sqrt[c]*Sqrt[x])
/Sqrt[b]])/(Sqrt[c]*Sqrt[x]*(b + c*x)))
```

Maple [A] time = 0.045, size = 43, normalized size = 1.

$$\sqrt{cx^2+bx}+\frac{b}{2}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(1/2)/x,x)
```

```
[Out] (c*x^2+b*x)^(1/2)+1/2*b*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89567, size = 232, normalized size = 5.52

$$\left[\frac{b\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\,\sqrt{cx^2+bx}c}{2\,c}, -\frac{b\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx}\right)-\sqrt{cx^2+bx}c}{c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*(b*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*sqrt(c*x^2
 + b*x)*c)/c, -(b*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)) - sqrt(
c*x^2 + b*x)*c)/c]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x\,(b+cx)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x*(b + c*x))/x, x)
```

Giac [A] time = 1.46597, size = 65, normalized size = 1.55

$$-\frac{b\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}-b\right|\right)}{2\sqrt{c}}+\sqrt{cx^2+bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -1/2*b*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b))/sqrt(c) + s
qrt(c*x^2 + b*x)
```

3.6 $\int \frac{\sqrt{bx+cx^2}}{x^2}\,dx$

Optimal. Leaf size=47

$$2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)-\frac{2\sqrt{bx+cx^2}}{x}$$

```
[Out] (-2*Sqrt[b*x + c*x^2])/x + 2*Sqrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]]
```

Rubi [A] time = 0.0177092, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {662, 620, 206}

$$2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)-\frac{2\sqrt{bx+cx^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x + c*x^2]/x^2,x]
```

```
[Out] (-2*Sqrt[b*x + c*x^2])/x + 2*Sqrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]]
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{bx+cx^2}}{x^2}\,dx &= -\frac{2\sqrt{bx+cx^2}}{x}+c\int \frac{1}{\sqrt{bx+cx^2}}\,dx\\
&= -\frac{2\sqrt{bx+cx^2}}{x}+(2c)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{bx+cx^2}}\right)\\
&= -\frac{2\sqrt{bx+cx^2}}{x}+2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0839122, size = 63, normalized size = 1.34

$$\frac{2\sqrt{x(b+cx)}\left(\frac{\sqrt{c}\sqrt{x}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{\frac{cx}{b}+1}}-1\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x + c*x^2]/x^2,x]
```

```
[Out] (2*Sqrt[x*(b + c*x)]*(-1 + (Sqrt[c]*Sqrt[x]*ArcSinh[(Sqrt[c]*Sqrt[x])/Sqrt[
b]])/(Sqrt[b]*Sqrt[1 + (c*x)/b])))/x
```

Maple [A] time = 0.046, size = 66, normalized size = 1.4

$$-2\frac{\left(cx^2+bx\right)^{3/2}}{bx^2}+2\frac{c\sqrt{cx^2+bx}}{b}+\sqrt{c}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(1/2)/x^2,x)
```

```
[Out] -2/b/x^2*(c*x^2+b*x)^(3/2)+2*c/b*(c*x^2+b*x)^(1/2)+c^(1/2)*ln((1/2*b+c*x)/c
^(1/2)+(c*x^2+b*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.02655, size = 224, normalized size = 4.77

$$\left[\frac{\sqrt{c}x\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\,\sqrt{cx^2+bx}}{x},-\frac{2\left(\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx}\right)+\sqrt{cx^2+bx}\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [(sqrt(c)*x*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 2*sqrt(c*x^2 + b
*x))/x, -2*(sqrt(-c)*x*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)) + sqrt(c*x^
2 + b*x))/x]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x\,(b+cx)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(x*(b + c*x))/x**2, x)
```

Giac [A] time = 1.37555, size = 81, normalized size = 1.72

$$-\sqrt{c}\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}-b\right|\right)+\frac{2\,b}{\sqrt{c}x-\sqrt{cx^2+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -sqrt(c)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b)) + 2*b/(sq
rt(c)*x - sqrt(c*x^2 + b*x))
```

3.7 $\int \frac{\sqrt{bx+cx^2}}{x^3} dx$

Optimal. Leaf size=23

$$-\frac{2\left(bx+cx^2\right)^{3/2}}{3bx^3}$$

[Out] (-2*(b*x + c*x^2)^(3/2))/(3*b*x^3)

Rubi [A] time = 0.0064643, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {650}

$$-\frac{2\left(bx+cx^2\right)^{3/2}}{3bx^3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*x + c*x^2]/x^3,x]

[Out] (-2*(b*x + c*x^2)^(3/2))/(3*b*x^3)

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\sqrt{bx+cx^2}}{x^3} dx = -\frac{2\left(bx+cx^2\right)^{3/2}}{3bx^3}$$

Mathematica [A] time = 0.0094617, size = 21, normalized size = 0.91

$$-\frac{2(x(b+cx))^{3/2}}{3bx^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*x + c*x^2]/x^3,x]

[Out] (-2*(x*(b + c*x))^(3/2))/(3*b*x^3)

Maple [A] time = 0.045, size = 25, normalized size = 1.1

$$-\frac{2\,cx+2\,b}{3\,bx^2}\sqrt{cx^2+bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(1/2)/x^3,x)
```

```
[Out] -2/3/x^2*(c*x+b)/b*(c*x^2+b*x)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.00175, size = 57, normalized size = 2.48

$$-\frac{2\sqrt{cx^2+bx}(cx+b)}{3\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(c*x^2 + b*x)*(c*x + b)/(b*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x(b+cx)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(x*(b + c*x))/x**3, x)
```

Giac [B] time = 1.40153, size = 103, normalized size = 4.48

$$\frac{2\left(3\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^2c+3\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)b\sqrt{c}+b^2\right)}{3\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 2/3*(3*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*c + 3*(sqrt(c)*x - sqrt(c*x^2 + b*
x))*b*sqrt(c) + b^2)/(sqrt(c)*x - sqrt(c*x^2 + b*x))^3
```

3.8 $\int \frac{\sqrt{bx+cx^2}}{x^4}\,dx$

Optimal. Leaf size=48

$$\frac{4c\left(bx+cx^2\right)^{3/2}}{15b^2x^3}-\frac{2\left(bx+cx^2\right)^{3/2}}{5bx^4}$$

```
[Out] (-2*(b*x + c*x^2)^(3/2))/(5*b*x^4) + (4*c*(b*x + c*x^2)^(3/2))/(15*b^2*x^3)
```

Rubi [A] time = 0.0165422, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {658, 650}

$$\frac{4c\left(bx+cx^2\right)^{3/2}}{15b^2x^3}-\frac{2\left(bx+cx^2\right)^{3/2}}{5bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x + c*x^2]/x^4,x]
```

```
[Out] (-2*(b*x + c*x^2)^(3/2))/(5*b*x^4) + (4*c*(b*x + c*x^2)^(3/2))/(15*b^2*x^3)
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx+cx^2}}{x^4}\,dx &= -\frac{2\left(bx+cx^2\right)^{3/2}}{5bx^4}-\frac{(2c)\int \frac{\sqrt{bx+cx^2}}{x^3}\,dx}{5b}\\ &= -\frac{2\left(bx+cx^2\right)^{3/2}}{5bx^4}+\frac{4c\left(bx+cx^2\right)^{3/2}}{15b^2x^3}\end{aligned}$$

Mathematica [A] time = 0.0110612, size = 29, normalized size = 0.6

$$\frac{2(x(b+cx))^{3/2}(2cx-3b)}{15b^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x + c*x^2]/x^4,x]
```

```
[Out] (2*(x*(b + c*x))^(3/2)*(-3*b + 2*c*x))/(15*b^2*x^4)
```

Maple [A] time = 0.052, size = 33, normalized size = 0.7

$$-\frac{(2\,cx+2\,b)\,(-2\,cx+3\,b)}{15\,b^2x^3}\sqrt{cx^2+bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(1/2)/x^4,x)
```

```
[Out] -2/15*(c*x+b)*(-2*c*x+3*b)*(c*x^2+b*x)^(1/2)/b^2/x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.12743, size = 84, normalized size = 1.75

$$\frac{2\left(2\,c^2x^2-bcx-3\,b^2\right)\sqrt{cx^2+bx}}{15\,b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 2/15*(2*c^2*x^2 - b*c*x - 3*b^2)*sqrt(c*x^2 + b*x)/(b^2*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x\,(b+cx)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(x*(b + c*x))/x**4, x)
```

Giac [B] time = 1.26466, size = 144, normalized size = 3.

$$\frac{2\left(15\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3c^{\frac{3}{2}}+25\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^2bc+15\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)b^2\sqrt{c}+3\,b^3\right)}{15\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 2/15*(15*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*c^(3/2) + 25*(sqrt(c)*x - sqrt(c
*x^2 + b*x))^2*b*c + 15*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b^2*sqrt(c) + 3*b^3
)/(sqrt(c)*x - sqrt(c*x^2 + b*x))^5
```

3.9 $\int \frac{\sqrt{bx+cx^2}}{x^5} \, dx$

Optimal. Leaf size=74

$$-\frac{16c^2\left(bx+cx^2\right)^{3/2}}{105b^3x^3}+\frac{8c\left(bx+cx^2\right)^{3/2}}{35b^2x^4}-\frac{2\left(bx+cx^2\right)^{3/2}}{7bx^5}$$

```
[Out] (-2*(b*x + c*x^2)^(3/2))/(7*b*x^5) + (8*c*(b*x + c*x^2)^(3/2))/(35*b^2*x^4)
 - (16*c^2*(b*x + c*x^2)^(3/2))/(105*b^3*x^3)
```

Rubi [A] time = 0.0282151, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {658, 650}

$$-\frac{16c^2\left(bx+cx^2\right)^{3/2}}{105b^3x^3}+\frac{8c\left(bx+cx^2\right)^{3/2}}{35b^2x^4}-\frac{2\left(bx+cx^2\right)^{3/2}}{7bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x + c*x^2]/x^5,x]
```

```
[Out] (-2*(b*x + c*x^2)^(3/2))/(7*b*x^5) + (8*c*(b*x + c*x^2)^(3/2))/(35*b^2*x^4)
 - (16*c^2*(b*x + c*x^2)^(3/2))/(105*b^3*x^3)
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{bx+cx^2}}{x^5} \, dx &= -\frac{2\left(bx+cx^2\right)^{3/2}}{7bx^5}-\frac{(4c)\int \frac{\sqrt{bx+cx^2}}{x^4} \, dx}{7b}\\
&= -\frac{2\left(bx+cx^2\right)^{3/2}}{7bx^5}+\frac{8c\left(bx+cx^2\right)^{3/2}}{35b^2x^4}+\frac{\left(8c^2\right)\int \frac{\sqrt{bx+cx^2}}{x^3} \, dx}{35b^2}\\
&= -\frac{2\left(bx+cx^2\right)^{3/2}}{7bx^5}+\frac{8c\left(bx+cx^2\right)^{3/2}}{35b^2x^4}-\frac{16c^2\left(bx+cx^2\right)^{3/2}}{105b^3x^3}
\end{aligned}$$

Mathematica [A] time = 0.0108996, size = 51, normalized size = 0.69

$$-\frac{2\sqrt{x(b+cx)}\left(3b^2cx+15b^3-4bc^2x^2+8c^3x^3\right)}{105b^3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x + c*x^2]/x^5,x]
```

```
[Out] (-2*Sqrt[x*(b + c*x)]*(15*b^3 + 3*b^2*c*x - 4*b*c^2*x^2 + 8*c^3*x^3))/(105*
b^3*x^4)
```

Maple [A] time = 0.049, size = 44, normalized size = 0.6

$$-\frac{(2\,cx+2\,b)\left(8\,c^2x^2-12\,bcx+15\,b^2\right)}{105\,b^3x^4}\sqrt{cx^2+bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(1/2)/x^5,x)
```

```
[Out] -2/105*(c*x+b)*(8*c^2*x^2-12*b*c*x+15*b^2)*(c*x^2+b*x)^(1/2)/b^3/x^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.99353, size = 112, normalized size = 1.51

$$-\frac{2\left(8\,c^3x^3-4\,bc^2x^2+3\,b^2cx+15\,b^3\right)\sqrt{cx^2+bx}}{105\,b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -2/105*(8*c^3*x^3 - 4*b*c^2*x^2 + 3*b^2*c*x + 15*b^3)*sqrt(c*x^2 + b*x)/(b^
3*x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x\,(b+cx)}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(x*(b + c*x))/x**5, x)
```

Giac [B] time = 1.34038, size = 184, normalized size = 2.49

$$\frac{2\left(140\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^4c^2+315\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3bc^{\frac{3}{2}}+273\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^2b^2c+105\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)b}{105\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 2/105*(140*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*c^2 + 315*(sqrt(c)*x - sqrt(c*
x^2 + b*x))^3*b*c^(3/2) + 273*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*b^2*c + 105
*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b^3*sqrt(c) + 15*b^4)/(sqrt(c)*x - sqrt(c*
x^2 + b*x))^7
```

3.10 $\int \frac{\sqrt{bx+cx^2}}{x^6}\,dx$

Optimal. Leaf size=100

$$\frac{32c^3\left(bx+cx^2\right)^{3/2}}{315b^4x^3}-\frac{16c^2\left(bx+cx^2\right)^{3/2}}{105b^3x^4}+\frac{4c\left(bx+cx^2\right)^{3/2}}{21b^2x^5}-\frac{2\left(bx+cx^2\right)^{3/2}}{9bx^6}$$

```
[Out] (-2*(b*x + c*x^2)^(3/2))/(9*b*x^6) + (4*c*(b*x + c*x^2)^(3/2))/(21*b^2*x^5)
 - (16*c^2*(b*x + c*x^2)^(3/2))/(105*b^3*x^4) + (32*c^3*(b*x + c*x^2)^(3/2)
)/(315*b^4*x^3)
```

Rubi [A] time = 0.0407514, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {658, 650}

$$\frac{32c^3\left(bx+cx^2\right)^{3/2}}{315b^4x^3}-\frac{16c^2\left(bx+cx^2\right)^{3/2}}{105b^3x^4}+\frac{4c\left(bx+cx^2\right)^{3/2}}{21b^2x^5}-\frac{2\left(bx+cx^2\right)^{3/2}}{9bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x + c*x^2]/x^6,x]
```

```
[Out] (-2*(b*x + c*x^2)^(3/2))/(9*b*x^6) + (4*c*(b*x + c*x^2)^(3/2))/(21*b^2*x^5)
 - (16*c^2*(b*x + c*x^2)^(3/2))/(105*b^3*x^4) + (32*c^3*(b*x + c*x^2)^(3/2)
)/(315*b^4*x^3)
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{bx+cx^2}}{x^6}\,dx &= -\frac{2\left(bx+cx^2\right)^{3/2}}{9bx^6}-\frac{(2c)\int \frac{\sqrt{bx+cx^2}}{x^5}\,dx}{3b}\\
&= -\frac{2\left(bx+cx^2\right)^{3/2}}{9bx^6}+\frac{4c\left(bx+cx^2\right)^{3/2}}{21b^2x^5}+\frac{\left(8c^2\right)\int \frac{\sqrt{bx+cx^2}}{x^4}\,dx}{21b^2}\\
&= -\frac{2\left(bx+cx^2\right)^{3/2}}{9bx^6}+\frac{4c\left(bx+cx^2\right)^{3/2}}{21b^2x^5}-\frac{16c^2\left(bx+cx^2\right)^{3/2}}{105b^3x^4}-\frac{\left(16c^3\right)\int \frac{\sqrt{bx+cx^2}}{x^3}\,dx}{105b^3}\\
&= -\frac{2\left(bx+cx^2\right)^{3/2}}{9bx^6}+\frac{4c\left(bx+cx^2\right)^{3/2}}{21b^2x^5}-\frac{16c^2\left(bx+cx^2\right)^{3/2}}{105b^3x^4}+\frac{32c^3\left(bx+cx^2\right)^{3/2}}{315b^4x^3}
\end{aligned}$$

Mathematica [A] time = 0.0125828, size = 62, normalized size = 0.62

$$\frac{2\sqrt{x(b+cx)}\left(6b^2c^2x^2-5b^3cx-35b^4-8bc^3x^3+16c^4x^4\right)}{315b^4x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x + c*x^2]/x^6,x]
```

```
[Out] (2*Sqrt[x*(b + c*x)]*(-35*b^4 - 5*b^3*c*x + 6*b^2*c^2*x^2 - 8*b*c^3*x^3 + 1
6*c^4*x^4))/(315*b^4*x^5)
```

Maple [A] time = 0.047, size = 55, normalized size = 0.6

$$-\frac{(2\,cx+2\,b)\left(-16\,x^3c^3+24\,bx^2c^2-30\,b^2xc+35\,b^3\right)}{315\,b^4x^5}\sqrt{cx^2+bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(1/2)/x^6,x)
```

```
[Out] -2/315*(c*x+b)*(-16*c^3*x^3+24*b*c^2*x^2-30*b^2*c*x+35*b^3)*(c*x^2+b*x)^(1/
2)/b^4/x^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.08463, size = 134, normalized size = 1.34

$$\frac{2\left(16\,c^4x^4-8\,bc^3x^3+6\,b^2c^2x^2-5\,b^3cx-35\,b^4\right)\sqrt{cx^2+bx}}{315\,b^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] 2/315*(16*c^4*x^4 - 8*b*c^3*x^3 + 6*b^2*c^2*x^2 - 5*b^3*c*x - 35*b^4)*sqrt(
c*x^2 + b*x)/(b^4*x^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x\,(b+cx)}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(1/2)/x**6,x)
```

```
[Out] Integral(sqrt(x*(b + c*x))/x**6, x)
```

Giac [A] time = 1.35586, size = 223, normalized size = 2.23

$$\frac{2\left(630\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^5c^{\frac{5}{2}}+1764\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^4bc^2+1995\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3b^2c^{\frac{3}{2}}+1125\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\right.}{315\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] 2/315*(630*(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*c^(5/2) + 1764*(sqrt(c)*x - sq
rt(c*x^2 + b*x))^4*b*c^2 + 1995*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b^2*c^(3/
2) + 1125*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*b^3*c + 315*(sqrt(c)*x - sqrt(c
*x^2 + b*x))*b^4*sqrt(c) + 35*b^5)/(sqrt(c)*x - sqrt(c*x^2 + b*x))^9
```

3.11 $\int \frac{\sqrt{bx+cx^2}}{x^7}\,dx$

Optimal. Leaf size=126

$$-\frac{256c^4\left(bx+cx^2\right)^{3/2}}{3465b^5x^3}+\frac{128c^3\left(bx+cx^2\right)^{3/2}}{1155b^4x^4}-\frac{32c^2\left(bx+cx^2\right)^{3/2}}{231b^3x^5}+\frac{16c\left(bx+cx^2\right)^{3/2}}{99b^2x^6}-\frac{2\left(bx+cx^2\right)^{3/2}}{11bx^7}$$

```
[Out] (-2*(b*x + c*x^2)^(3/2))/(11*b*x^7) + (16*c*(b*x + c*x^2)^(3/2))/(99*b^2*x^
6) - (32*c^2*(b*x + c*x^2)^(3/2))/(231*b^3*x^5) + (128*c^3*(b*x + c*x^2)^(3
/2))/(1155*b^4*x^4) - (256*c^4*(b*x + c*x^2)^(3/2))/(3465*b^5*x^3)
```

Rubi [A] time = 0.0527275, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {658, 650}

$$-\frac{256c^4\left(bx+cx^2\right)^{3/2}}{3465b^5x^3}+\frac{128c^3\left(bx+cx^2\right)^{3/2}}{1155b^4x^4}-\frac{32c^2\left(bx+cx^2\right)^{3/2}}{231b^3x^5}+\frac{16c\left(bx+cx^2\right)^{3/2}}{99b^2x^6}-\frac{2\left(bx+cx^2\right)^{3/2}}{11bx^7}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x + c*x^2]/x^7,x]
```

```
[Out] (-2*(b*x + c*x^2)^(3/2))/(11*b*x^7) + (16*c*(b*x + c*x^2)^(3/2))/(99*b^2*x^
6) - (32*c^2*(b*x + c*x^2)^(3/2))/(231*b^3*x^5) + (128*c^3*(b*x + c*x^2)^(3
/2))/(1155*b^4*x^4) - (256*c^4*(b*x + c*x^2)^(3/2))/(3465*b^5*x^3)
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\sqrt{bx+cx^2}}{x^7}\,dx = -\frac{2\left(bx+cx^2\right)^{3/2}}{11bx^7} - \frac{(8c)\int \frac{\sqrt{bx+cx^2}}{x^6}\,dx}{11b}$$
$$= -\frac{2\left(bx+cx^2\right)^{3/2}}{11bx^7} + \frac{16c\left(bx+cx^2\right)^{3/2}}{99b^2x^6} + \frac{\left(16c^2\right)\int \frac{\sqrt{bx+cx^2}}{x^5}\,dx}{33b^2}$$
$$= -\frac{2\left(bx+cx^2\right)^{3/2}}{11bx^7} + \frac{16c\left(bx+cx^2\right)^{3/2}}{99b^2x^6} - \frac{32c^2\left(bx+cx^2\right)^{3/2}}{231b^3x^5} - \frac{\left(64c^3\right)\int \frac{\sqrt{bx+cx^2}}{x^4}\,dx}{231b^3}$$
$$= -\frac{2\left(bx+cx^2\right)^{3/2}}{11bx^7} + \frac{16c\left(bx+cx^2\right)^{3/2}}{99b^2x^6} - \frac{32c^2\left(bx+cx^2\right)^{3/2}}{231b^3x^5} + \frac{128c^3\left(bx+cx^2\right)^{3/2}}{1155b^4x^4} + \frac{\left(128c^4\right)\int \frac{\sqrt{bx+cx^2}}{x^3}\,d}{1155b^4}$$
$$= -\frac{2\left(bx+cx^2\right)^{3/2}}{11bx^7} + \frac{16c\left(bx+cx^2\right)^{3/2}}{99b^2x^6} - \frac{32c^2\left(bx+cx^2\right)^{3/2}}{231b^3x^5} + \frac{128c^3\left(bx+cx^2\right)^{3/2}}{1155b^4x^4} - \frac{256c^4\left(bx+cx^2\right)^{3/2}}{3465b^5x^3}$$

Mathematica [A] time = 0.0141014, size = 73, normalized size = 0.58

$$-\frac{2\sqrt{x(b+cx)}\left(-40b^3c^2x^2+48b^2c^3x^3+35b^4cx+315b^5-64bc^4x^4+128c^5x^5\right)}{3465b^5x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x + c*x^2]/x^7,x]
```

```
[Out] (-2*Sqrt[x*(b + c*x)]*(315*b^5 + 35*b^4*c*x - 40*b^3*c^2*x^2 + 48*b^2*c^3*x
^3 - 64*b*c^4*x^4 + 128*c^5*x^5))/(3465*b^5*x^6)
```

Maple [A] time = 0.054, size = 66, normalized size = 0.5

$$-\frac{(2\,cx+2\,b)\left(128\,c^4x^4-192\,x^3c^3b+240\,c^2x^2b^2-280\,cxb^3+315\,b^4\right)}{3465\,x^6b^5}\sqrt{cx^2+bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(1/2)/x^7,x)
```

```
[Out] -2/3465*(c*x+b)*(128*c^4*x^4-192*b*c^3*x^3+240*b^2*c^2*x^2-280*b^3*c*x+315*
b^4)*(c*x^2+b*x)^(1/2)/x^6/b^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94495, size = 166, normalized size = 1.32

$$-\frac{2\left(128\,c^5x^5-64\,bc^4x^4+48\,b^2c^3x^3-40\,b^3c^2x^2+35\,b^4cx+315\,b^5\right)\sqrt{cx^2+bx}}{3465\,b^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] -2/3465*(128*c^5*x^5 - 64*b*c^4*x^4 + 48*b^2*c^3*x^3 - 40*b^3*c^2*x^2 + 35*
b^4*c*x + 315*b^5)*sqrt(c*x^2 + b*x)/(b^5*x^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x\,(b+cx)}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(1/2)/x**7,x)
```

```
[Out] Integral(sqrt(x*(b + c*x))/x**7, x)
```

Giac [A] time = 1.33142, size = 262, normalized size = 2.08

$$\frac{2\left(11088\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^6c^3+36960\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^5bc^{\frac{5}{2}}+51480\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^4b^2c^2+38115\left(\sqrt{c}x-\sqrt{\right.\right.}{3465\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 2/3465*(11088*(sqrt(c)*x - sqrt(c*x^2 + b*x))^6*c^3 + 36960*(sqrt(c)*x - sq
rt(c*x^2 + b*x))^5*b*c^(5/2) + 51480*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*b^2*
c^2 + 38115*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b^3*c^(3/2) + 15785*(sqrt(c)*
x - sqrt(c*x^2 + b*x))^2*b^4*c + 3465*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b^5*s
qrt(c) + 315*b^6)/(sqrt(c)*x - sqrt(c*x^2 + b*x))^11
```

3.12 $\int x^2 \left(bx+cx^2\right)^{3/2} dx$

Optimal. Leaf size=134

$$-\frac{7b^4(b+2cx)\sqrt{bx+cx^2}}{512c^4}+\frac{7b^2(b+2cx)\left(bx+cx^2\right)^{3/2}}{192c^3}+\frac{7b^6\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{512c^{9/2}}-\frac{7b\left(bx+cx^2\right)^{5/2}}{60c^2}+\frac{x\left(bx+cx^2\right)^{5/2}}{6c}$$

```
[Out] (-7*b^4*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(512*c^4) + (7*b^2*(b + 2*c*x)*(b*x
+ c*x^2)^(3/2))/(192*c^3) - (7*b*(b*x + c*x^2)^(5/2))/(60*c^2) + (x*(b*x +
c*x^2)^(5/2))/(6*c) + (7*b^6*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(512*c
^(9/2))
```

Rubi [A] time = 0.0551097, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {670, 640, 612, 620, 206}

$$-\frac{7b^4(b+2cx)\sqrt{bx+cx^2}}{512c^4}+\frac{7b^2(b+2cx)\left(bx+cx^2\right)^{3/2}}{192c^3}+\frac{7b^6\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{512c^{9/2}}-\frac{7b\left(bx+cx^2\right)^{5/2}}{60c^2}+\frac{x\left(bx+cx^2\right)^{5/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b*x + c*x^2)^(3/2),x]
```

```
[Out] (-7*b^4*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(512*c^4) + (7*b^2*(b + 2*c*x)*(b*x
+ c*x^2)^(3/2))/(192*c^3) - (7*b*(b*x + c*x^2)^(5/2))/(60*c^2) + (x*(b*x +
c*x^2)^(5/2))/(6*c) + (7*b^6*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(512*c
^(9/2))
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2\left(bx+cx^2\right)^{3/2}dx &= \frac{x\left(bx+cx^2\right)^{5/2}}{6c}-\frac{(7b)\int x\left(bx+cx^2\right)^{3/2}dx}{12c}\\
&= -\frac{7b\left(bx+cx^2\right)^{5/2}}{60c^2}+\frac{x\left(bx+cx^2\right)^{5/2}}{6c}+\frac{\left(7b^2\right)\int\left(bx+cx^2\right)^{3/2}dx}{24c^2}\\
&= \frac{7b^2(b+2cx)\left(bx+cx^2\right)^{3/2}}{192c^3}-\frac{7b\left(bx+cx^2\right)^{5/2}}{60c^2}+\frac{x\left(bx+cx^2\right)^{5/2}}{6c}-\frac{\left(7b^4\right)\int\sqrt{bx+cx^2}\,dx}{128c^3}\\
&= -\frac{7b^4(b+2cx)\sqrt{bx+cx^2}}{512c^4}+\frac{7b^2(b+2cx)\left(bx+cx^2\right)^{3/2}}{192c^3}-\frac{7b\left(bx+cx^2\right)^{5/2}}{60c^2}+\frac{x\left(bx+cx^2\right)^{5/2}}{6c}+\\
&= -\frac{7b^4(b+2cx)\sqrt{bx+cx^2}}{512c^4}+\frac{7b^2(b+2cx)\left(bx+cx^2\right)^{3/2}}{192c^3}-\frac{7b\left(bx+cx^2\right)^{5/2}}{60c^2}+\frac{x\left(bx+cx^2\right)^{5/2}}{6c}+\\
&= -\frac{7b^4(b+2cx)\sqrt{bx+cx^2}}{512c^4}+\frac{7b^2(b+2cx)\left(bx+cx^2\right)^{3/2}}{192c^3}-\frac{7b\left(bx+cx^2\right)^{5/2}}{60c^2}+\frac{x\left(bx+cx^2\right)^{5/2}}{6c}+
\end{aligned}$$

Mathematica [A] time = 0.167331, size = 120, normalized size = 0.9

$$\frac{\sqrt{x(b+cx)}\left(\sqrt{c}\left(-56b^3c^2x^2+48b^2c^3x^3+70b^4cx-105b^5+1664bc^4x^4+1280c^5x^5\right)+\frac{105b^{11/2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}\right)}{7680c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b*x + c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[x*(b + c*x)]*(Sqrt[c]*(-105*b^5 + 70*b^4*c*x - 56*b^3*c^2*x^2 + 48*b^
2*c^3*x^3 + 1664*b*c^4*x^4 + 1280*c^5*x^5) + (105*b^(11/2)*ArcSinh[(Sqrt[c]
*Sqrt[x])/Sqrt[b]])/(Sqrt[x]*Sqrt[1 + (c*x)/b])))/(7680*c^(9/2))
```

Maple [A] time = 0.048, size = 146, normalized size = 1.1

$$\frac{x}{6c}\left(cx^2+bx\right)^{\frac{5}{2}}-\frac{7b}{60c^2}\left(cx^2+bx\right)^{\frac{5}{2}}+\frac{7b^2x}{96c^2}\left(cx^2+bx\right)^{\frac{3}{2}}+\frac{7b^3}{192c^3}\left(cx^2+bx\right)^{\frac{3}{2}}-\frac{7b^4x}{256c^3}\sqrt{cx^2+bx}-\frac{7b^5}{512c^4}\sqrt{cx^2+bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2+b*x)^(3/2),x)
```

```
[Out] 1/6*x*(c*x^2+b*x)^(5/2)/c-7/60*b*(c*x^2+b*x)^(5/2)/c^2+7/96*b^2/c^2*(c*x^2+
b*x)^(3/2)*x+7/192*b^3/c^3*(c*x^2+b*x)^(3/2)-7/256*b^4/c^3*(c*x^2+b*x)^(1/2
)*x-7/512*b^5/c^4*(c*x^2+b*x)^(1/2)+7/1024*b^6/c^(9/2)*ln((1/2*b+c*x)/c^(1/
2)+(c*x^2+b*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94666, size = 514, normalized size = 3.84

$$\left[\frac{105\,b^6\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\left(1280\,c^6x^5+1664\,bc^5x^4+48\,b^2c^4x^3-56\,b^3c^3x^2+70\,b^4c^2x-105\,b^5c\right)\sqrt{cx}}{15360\,c^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/15360*(105*b^6*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*
(1280*c^6*x^5 + 1664*b*c^5*x^4 + 48*b^2*c^4*x^3 - 56*b^3*c^3*x^2 + 70*b^4*c
^2*x - 105*b^5*c)*sqrt(c*x^2 + b*x))/c^5, -1/7680*(105*b^6*sqrt(-c)*arctan(
sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)) - (1280*c^6*x^5 + 1664*b*c^5*x^4 + 48*b^2
*c^4*x^3 - 56*b^3*c^3*x^2 + 70*b^4*c^2*x - 105*b^5*c)*sqrt(c*x^2 + b*x))/c^
5]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\,(x\,(b+cx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral(x**2*(x*(b + c*x))**(3/2), x)
```

Giac [A] time = 1.34954, size = 146, normalized size = 1.09

$$-\frac{7\,b^6\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}-b\right|\right)}{1024\,c^{\frac{9}{2}}}+\frac{1}{7680}\,\sqrt{cx^2+bx}\left(2\left(4\left(2\left(8\,(10\,cx+13\,b)x+\frac{3\,b^2}{c}\right)x-\frac{7\,b^3}{c^2}\right)x+\frac{35\,b^4}{c^3}\right)x-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] -7/1024*b^6*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b))/c^(9/2
) + 1/7680*sqrt(c*x^2 + b*x)*(2*(4*(2*(8*(10*c*x + 13*b)*x + 3*b^2/c)*x - 7
*b^3/c^2)*x + 35*b^4/c^3)*x - 105*b^5/c^4)
```

3.13 $\int x\left(bx+cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=110

$$\frac{3b^3(b+2cx)\sqrt{bx+cx^2}}{128c^3}-\frac{3b^5\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{128c^{7/2}}-\frac{b(b+2cx)\left(bx+cx^2\right)^{3/2}}{16c^2}+\frac{\left(bx+cx^2\right)^{5/2}}{5c}$$

```
[Out] (3*b^3*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(128*c^3) - (b*(b + 2*c*x)*(b*x + c*x
^2)^(3/2))/(16*c^2) + (b*x + c*x^2)^(5/2)/(5*c) - (3*b^5*ArcTanh[(Sqrt[c]*x
)/Sqrt[b*x + c*x^2]])/(128*c^(7/2))
```

Rubi [A] time = 0.0350046, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {640, 612, 620, 206}

$$\frac{3b^3(b+2cx)\sqrt{bx+cx^2}}{128c^3}-\frac{3b^5\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{128c^{7/2}}-\frac{b(b+2cx)\left(bx+cx^2\right)^{3/2}}{16c^2}+\frac{\left(bx+cx^2\right)^{5/2}}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b*x + c*x^2)^(3/2),x]
```

```
[Out] (3*b^3*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(128*c^3) - (b*(b + 2*c*x)*(b*x + c*x
^2)^(3/2))/(16*c^2) + (b*x + c*x^2)^(5/2)/(5*c) - (3*b^5*ArcTanh[(Sqrt[c]*x
)/Sqrt[b*x + c*x^2]])/(128*c^(7/2))
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x\left(bx+cx^2\right)^{3/2}dx=\frac{\left(bx+cx^2\right)^{5/2}}{5c}-\frac{b\int\left(bx+cx^2\right)^{3/2}dx}{2c}$$
$$=-\frac{b(b+2cx)\left(bx+cx^2\right)^{3/2}}{16c^2}+\frac{\left(bx+cx^2\right)^{5/2}}{5c}+\frac{\left(3b^3\right)\int\sqrt{bx+cx^2}dx}{32c^2}$$
$$=\frac{3b^3(b+2cx)\sqrt{bx+cx^2}}{128c^3}-\frac{b(b+2cx)\left(bx+cx^2\right)^{3/2}}{16c^2}+\frac{\left(bx+cx^2\right)^{5/2}}{5c}-\frac{\left(3b^5\right)\int\frac{1}{\sqrt{bx+cx^2}}dx}{256c^3}$$
$$=\frac{3b^3(b+2cx)\sqrt{bx+cx^2}}{128c^3}-\frac{b(b+2cx)\left(bx+cx^2\right)^{3/2}}{16c^2}+\frac{\left(bx+cx^2\right)^{5/2}}{5c}-\frac{\left(3b^5\right)\text{Subst}\left(\int\frac{1}{1-cx^2}dx,x,\right.}{128c^3}$$
$$=\frac{3b^3(b+2cx)\sqrt{bx+cx^2}}{128c^3}-\frac{b(b+2cx)\left(bx+cx^2\right)^{3/2}}{16c^2}+\frac{\left(bx+cx^2\right)^{5/2}}{5c}-\frac{3b^5\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{128c^{7/2}}$$

Mathematica [A] time = 0.152296, size = 109, normalized size = 0.99

$$\frac{\sqrt{x(b+cx)}\left(\sqrt{c}\left(8b^2c^2x^2-10b^3cx+15b^4+176bc^3x^3+128c^4x^4\right)-\frac{15b^{9/2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}\right)}{640c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b*x + c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[x*(b + c*x)]*(Sqrt[c]*(15*b^4 - 10*b^3*c*x + 8*b^2*c^2*x^2 + 176*b*c^
3*x^3 + 128*c^4*x^4) - (15*b^(9/2)*ArcSinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(Sqr
t[x]*Sqrt[1 + (c*x)/b])))/(640*c^(7/2))
```

Maple [A] time = 0.045, size = 126, normalized size = 1.2

$$\frac{1}{5c}\left(cx^2+bx\right)^{\frac{5}{2}}-\frac{bx}{8c}\left(cx^2+bx\right)^{\frac{3}{2}}-\frac{b^2}{16c^2}\left(cx^2+bx\right)^{\frac{3}{2}}+\frac{3b^3x}{64c^2}\sqrt{cx^2+bx}+\frac{3b^4}{128c^3}\sqrt{cx^2+bx}-\frac{3b^5}{256}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x)^(3/2),x)
```

```
[Out] 1/5*(c*x^2+b*x)^(5/2)/c-1/8*b/c*x*(c*x^2+b*x)^(3/2)-1/16*b^2/c^2*(c*x^2+b*x
)^(3/2)+3/64*b^3/c^2*(c*x^2+b*x)^(1/2)*x+3/128*b^4/c^3*(c*x^2+b*x)^(1/2)-3/
256*b^5/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.95037, size = 451, normalized size = 4.1

$$\left[\frac{15\,b^5\sqrt{c}\log\left(2\,cx+b-2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\left(128\,c^5x^4+176\,bc^4x^3+8\,b^2c^3x^2-10\,b^3c^2x+15\,b^4c\right)\sqrt{cx^2+bx}}{1280\,c^4},\ \frac{15\,b^5\sqrt{-}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/1280*(15*b^5*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*(1
28*c^5*x^4 + 176*b*c^4*x^3 + 8*b^2*c^3*x^2 - 10*b^3*c^2*x + 15*b^4*c)*sqrt(
c*x^2 + b*x))/c^4, 1/640*(15*b^5*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)
/(c*x)) + (128*c^5*x^4 + 176*b*c^4*x^3 + 8*b^2*c^3*x^2 - 10*b^3*c^2*x + 15*
b^4*c)*sqrt(c*x^2 + b*x))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(x\left(b+cx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral(x*(x*(b + c*x))**(3/2), x)
```

Giac [A] time = 1.33394, size = 128, normalized size = 1.16

$$\frac{3\,b^5\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}-b\right|\right)}{256\,c^{\frac{7}{2}}}+\frac{1}{640}\,\sqrt{cx^2+bx}\left(2\left(4\left(2\,(8\,cx+11\,b)x+\frac{b^2}{c}\right)x-\frac{5\,b^3}{c^2}\right)x+\frac{15\,b^4}{c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] 3/256*b^5*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b))/c^(7/2)
+ 1/640*sqrt(c*x^2 + b*x)*(2*(4*(2*(8*c*x + 11*b)*x + b^2/c)*x - 5*b^3/c^2)
*x + 15*b^4/c^3)
```

3.14 $\int \left(bx+cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=89

$$-\frac{3b^2(b+2cx)\sqrt{bx+cx^2}}{64c^2}+\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{64c^{5/2}}+\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}$$

```
[Out] (-3*b^2*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(64*c^2) + ((b + 2*c*x)*(b*x + c*x^2
)^(3/2))/(8*c) + (3*b^4*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(64*c^(5/2)
)
```

Rubi [A] time = 0.0232197, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {612, 620, 206}

$$-\frac{3b^2(b+2cx)\sqrt{bx+cx^2}}{64c^2}+\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{64c^{5/2}}+\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(3/2),x]
```

```
[Out] (-3*b^2*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(64*c^2) + ((b + 2*c*x)*(b*x + c*x^2
)^(3/2))/(8*c) + (3*b^4*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(64*c^(5/2)
)
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(bx+cx^2\right)^{3/2} dx = \frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c} - \frac{\left(3b^2\right)\int \sqrt{bx+cx^2}\,dx}{16c}$$
$$= -\frac{3b^2(b+2cx)\sqrt{bx+cx^2}}{64c^2} + \frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c} + \frac{\left(3b^4\right)\int \frac{1}{\sqrt{bx+cx^2}}\,dx}{128c^2}$$
$$= -\frac{3b^2(b+2cx)\sqrt{bx+cx^2}}{64c^2} + \frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c} + \frac{\left(3b^4\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{bx+cx^2}}\right)}{64c^2}$$
$$= -\frac{3b^2(b+2cx)\sqrt{bx+cx^2}}{64c^2} + \frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c} + \frac{3b^4\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{64c^{5/2}}$$

Mathematica [A] time = 0.130844, size = 98, normalized size = 1.1

$$\frac{\sqrt{x(b+cx)}\left(\sqrt{c}\left(2b^2cx-3b^3+24bc^2x^2+16c^3x^3\right)+\frac{3b^{7/2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}\right)}{64c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[x*(b + c*x)]*(Sqrt[c]*(-3*b^3 + 2*b^2*c*x + 24*b*c^2*x^2 + 16*c^3*x^3
) + (3*b^(7/2)*ArcSinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(Sqrt[x]*Sqrt[1 + (c*x)/
b])))/(64*c^(5/2))
```

Maple [A] time = 0.048, size = 95, normalized size = 1.1

$$\frac{2\,cx+b}{8\,c}\left(cx^2+bx\right)^{\frac{3}{2}} - \frac{3\,b^2x}{32\,c}\sqrt{cx^2+bx} - \frac{3\,b^3}{64\,c^2}\sqrt{cx^2+bx} + \frac{3\,b^4}{128}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx}\right)c^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(3/2),x)
```

```
[Out] 1/8*(2*c*x+b)*(c*x^2+b*x)^(3/2)/c-3/32*b^2/c*(c*x^2+b*x)^(1/2)*x-3/64*b^3/c
^2*(c*x^2+b*x)^(1/2)+3/128*b^4/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(
1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.02213, size = 393, normalized size = 4.42

$$\left[\frac{3\,b^4\sqrt{c}\log\left(2\,cx + b + 2\,\sqrt{cx^2+bx}\sqrt{c}\right) + 2\left(16\,c^4x^3 + 24\,bc^3x^2 + 2\,b^2c^2x - 3\,b^3c\right)\sqrt{cx^2+bx}}{128\,c^3}, -\frac{3\,b^4\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{}}{cx}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/128*(3*b^4*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*(16*
c^4*x^3 + 24*b*c^3*x^2 + 2*b^2*c^2*x - 3*b^3*c)*sqrt(c*x^2 + b*x))/c^3, -1/
64*(3*b^4*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)) - (16*c^4*x^3 +
 24*b*c^3*x^2 + 2*b^2*c^2*x - 3*b^3*c)*sqrt(c*x^2 + b*x))/c^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx + cx^2\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral((b*x + c*x**2)**(3/2), x)
```

Giac [A] time = 1.39659, size = 112, normalized size = 1.26

$$-\frac{3\,b^4\log\left(\left|-2\left(\sqrt{c}x - \sqrt{cx^2+bx}\right)\sqrt{c} - b\right|\right)}{128\,c^{\frac{5}{2}}} + \frac{1}{64}\,\sqrt{cx^2+bx}\left(2\left(4\,(2\,cx+3\,b)x + \frac{b^2}{c}\right)x - \frac{3\,b^3}{c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] -3/128*b^4*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b))/c^(5/2)
 + 1/64*sqrt(c*x^2 + b*x)*(2*(4*(2*c*x + 3*b)*x + b^2/c)*x - 3*b^3/c^2)
```

3.15 $\int \frac{\left(bx+cx^2\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=78

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{8c^{3/2}}+\frac{b(b+2cx)\sqrt{bx+cx^2}}{8c}+\frac{1}{3}\left(bx+cx^2\right)^{3/2}$$

```
[Out] (b*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(8*c) + (b*x + c*x^2)^(3/2)/3 - (b^3*ArcT
anh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(8*c^(3/2))
```

Rubi [A] time = 0.0265645, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {664, 612, 620, 206}

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{8c^{3/2}}+\frac{b(b+2cx)\sqrt{bx+cx^2}}{8c}+\frac{1}{3}\left(bx+cx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(3/2)/x,x]
```

```
[Out] (b*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(8*c) + (b*x + c*x^2)^(3/2)/3 - (b^3*ArcT
anh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(8*c^(3/2))
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(bx+cx^2\right)^{3/2}}{x}\,dx = \frac{1}{3}\left(bx+cx^2\right)^{3/2} + \frac{1}{2}b\int \sqrt{bx+cx^2}\,dx$$
$$= \frac{b(b+2cx)\sqrt{bx+cx^2}}{8c} + \frac{1}{3}\left(bx+cx^2\right)^{3/2} - \frac{b^3\int \frac{1}{\sqrt{bx+cx^2}}\,dx}{16c}$$
$$= \frac{b(b+2cx)\sqrt{bx+cx^2}}{8c} + \frac{1}{3}\left(bx+cx^2\right)^{3/2} - \frac{b^3\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{bx+cx^2}}\right)}{8c}$$
$$= \frac{b(b+2cx)\sqrt{bx+cx^2}}{8c} + \frac{1}{3}\left(bx+cx^2\right)^{3/2} - \frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{8c^{3/2}}$$

Mathematica [A] time = 0.117933, size = 87, normalized size = 1.12

$$\frac{\sqrt{x(b+cx)}\left(\sqrt{c}\left(3b^2+14bcx+8c^2x^2\right) - \frac{3b^{5/2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}\right)}{24c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(3/2)/x,x]
```

```
[Out] (Sqrt[x*(b + c*x)]*(Sqrt[c]*(3*b^2 + 14*b*c*x + 8*c^2*x^2) - (3*b^(5/2)*Arc
Sinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(Sqrt[x]*Sqrt[1 + (c*x)/b])))/(24*c^(3/2))
```

Maple [A] time = 0.047, size = 81, normalized size = 1.

$$\frac{1}{3}\left(cx^2+bx\right)^{\frac{3}{2}} + \frac{bx}{4}\sqrt{cx^2+bx} + \frac{b^2}{8c}\sqrt{cx^2+bx} - \frac{b^3}{16}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}} + \sqrt{cx^2+bx}\right)c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(3/2)/x,x)
```

```
[Out] 1/3*(c*x^2+b*x)^(3/2)+1/4*b*(c*x^2+b*x)^(1/2)*x+1/8/c*(c*x^2+b*x)^(1/2)*b^2
-1/16*b^3/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.01422, size = 344, normalized size = 4.41

$$\left[\frac{3\,b^{3}\sqrt{c}\log\left(2\,cx+b-2\,\sqrt{cx^{2}+bx}\sqrt{c}\right)+2\,\left(8\,c^{3}x^{2}+14\,bc^{2}x+3\,b^{2}c\right)\sqrt{cx^{2}+bx}}{48\,c^{2}},\ \frac{3\,b^{3}\sqrt{-c}\arctan\left(\frac{\sqrt{cx^{2}+bx}\sqrt{-c}}{cx}\right)+\left(8\,c^{3}\right.}{24\,c^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/48*(3*b^3*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*(8*c^
3*x^2 + 14*b*c^2*x + 3*b^2*c)*sqrt(c*x^2 + b*x))/c^2, 1/24*(3*b^3*sqrt(-c)*
arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)) + (8*c^3*x^2 + 14*b*c^2*x + 3*b^2*
c)*sqrt(c*x^2 + b*x))/c^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x\,(b+cx))^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(3/2)/x,x)
```

```
[Out] Integral((x*(b + c*x))**(3/2)/x, x)
```

Giac [A] time = 1.45398, size = 97, normalized size = 1.24

$$\frac{b^{3}\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^{2}+bx}\right)\sqrt{c}-b\right|\right)}{16\,c^{\frac{3}{2}}}+\frac{1}{24}\,\sqrt{cx^{2}+bx}\left(2\,(4\,cx+7\,b)x+\frac{3\,b^{2}}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/16*b^3*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b))/c^(3/2) +
 1/24*sqrt(c*x^2 + b*x)*(2*(4*c*x + 7*b)*x + 3*b^2/c)
```

3.16 $\int \frac{(bx+cx^2)^{3/2}}{x^2} dx$

Optimal. Leaf size=72

$$\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4\sqrt{c}}+\frac{3}{4}b\sqrt{bx+cx^2}+\frac{(bx+cx^2)^{3/2}}{2x}$$

```
[Out] (3*b*Sqrt[b*x + c*x^2])/4 + (b*x + c*x^2)^(3/2)/(2*x) + (3*b^2*ArcTanh[(Sqr
t[c]*x)/Sqrt[b*x + c*x^2]])/(4*Sqrt[c])
```

Rubi [A] time = 0.0279387, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {664, 620, 206}

$$\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4\sqrt{c}}+\frac{3}{4}b\sqrt{bx+cx^2}+\frac{(bx+cx^2)^{3/2}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(3/2)/x^2,x]
```

```
[Out] (3*b*Sqrt[b*x + c*x^2])/4 + (b*x + c*x^2)^(3/2)/(2*x) + (3*b^2*ArcTanh[(Sqr
t[c]*x)/Sqrt[b*x + c*x^2]])/(4*Sqrt[c])
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(bx+cx^2\right)^{3/2}}{x^2}\,dx &= \frac{\left(bx+cx^2\right)^{3/2}}{2x}+\frac{1}{4}(3b)\int \frac{\sqrt{bx+cx^2}}{x}\,dx \\
&= \frac{3}{4}b\sqrt{bx+cx^2}+\frac{\left(bx+cx^2\right)^{3/2}}{2x}+\frac{1}{8}\left(3b^2\right)\int \frac{1}{\sqrt{bx+cx^2}}\,dx \\
&= \frac{3}{4}b\sqrt{bx+cx^2}+\frac{\left(bx+cx^2\right)^{3/2}}{2x}+\frac{1}{4}\left(3b^2\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{bx+cx^2}}\right) \\
&= \frac{3}{4}b\sqrt{bx+cx^2}+\frac{\left(bx+cx^2\right)^{3/2}}{2x}+\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4\sqrt{c}}
\end{aligned}
$$

Mathematica [A] time = 0.106998, size = 69, normalized size = 0.96

$$
\frac{1}{4}\sqrt{x(b+cx)}\left(\frac{3b^{3/2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}}+5b+2cx\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(3/2)/x^2,x]
```

```
[Out] (Sqrt[x*(b + c*x)]*(5*b + 2*c*x + (3*b^(3/2)*ArcSinh[(Sqrt[c]*Sqrt[x])/Sqrt
[b]])/(Sqrt[c]*Sqrt[x]*Sqrt[1 + (c*x)/b])))/4
```

Maple [A] time = 0.046, size = 99, normalized size = 1.4

$$
2\frac{\left(cx^2+bx\right)^{5/2}}{bx^2}-2\frac{c\left(cx^2+bx\right)^{3/2}}{b}-\frac{3\,cx}{2}\sqrt{cx^2+bx}-\frac{3\,b}{4}\sqrt{cx^2+bx}+\frac{3\,b^2}{8}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx}\right)\frac{1}{\sqrt{c}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(3/2)/x^2,x)
```

```
[Out] 2/b/x^2*(c*x^2+b*x)^(5/2)-2*c/b*(c*x^2+b*x)^(3/2)-3/2*c*(c*x^2+b*x)^(1/2)*x
-3/4*b*(c*x^2+b*x)^(1/2)+3/8/c^(1/2)*b^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)
^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8205, size = 292, normalized size = 4.06

$$\left[\frac{3\,b^2\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\left(2\,c^2x+5\,bc\right)\sqrt{cx^2+bx}}{8\,c}, -\frac{3\,b^2\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx}\right)-\left(2\,c^2x+5\,bc\right)\sqrt{cx^2+bx}}{4\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/8*(3*b^2*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*(2*c^2
*x + 5*b*c)*sqrt(c*x^2 + b*x))/c, -1/4*(3*b^2*sqrt(-c)*arctan(sqrt(c*x^2 +
b*x)*sqrt(-c)/(c*x)) - (2*c^2*x + 5*b*c)*sqrt(c*x^2 + b*x))/c]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(x\left(b+cx\right)\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(3/2)/x**2,x)
```

```
[Out] Integral((x*(b + c*x))**(3/2)/x**2, x)
```

Giac [A] time = 1.38224, size = 81, normalized size = 1.12

$$-\frac{3\,b^2\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}-b\right|\right)}{8\,\sqrt{c}}+\frac{1}{4}\,\sqrt{cx^2+bx}(2\,cx+5\,b)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] -3/8*b^2*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b))/sqrt(c) +
 1/4*sqrt(c*x^2 + b*x)*(2*c*x + 5*b)
```

3.17 $\int \frac{(bx+cx^2)^{3/2}}{x^3} \, dx$

Optimal. Leaf size=64

$$-\frac{2\left(bx+cx^2\right)^{3/2}}{x^2}+3c\sqrt{bx+cx^2}+3b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)$$

```
[Out] 3*c*Sqrt[b*x + c*x^2] - (2*(b*x + c*x^2)^(3/2))/x^2 + 3*b*Sqrt[c]*ArcTanh[(
Sqrt[c]*x)/Sqrt[b*x + c*x^2]]
```

Rubi [A] time = 0.027478, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {662, 664, 620, 206}

$$-\frac{2\left(bx+cx^2\right)^{3/2}}{x^2}+3c\sqrt{bx+cx^2}+3b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(3/2)/x^3,x]
```

```
[Out] 3*c*Sqrt[b*x + c*x^2] - (2*(b*x + c*x^2)^(3/2))/x^2 + 3*b*Sqrt[c]*ArcTanh[(
Sqrt[c]*x)/Sqrt[b*x + c*x^2]]
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx+cx^2\right)^{3/2}}{x^3}\,dx &= -\frac{2\left(bx+cx^2\right)^{3/2}}{x^2}+(3c)\int \frac{\sqrt{bx+cx^2}}{x}\,dx\\ &= 3c\sqrt{bx+cx^2}-\frac{2\left(bx+cx^2\right)^{3/2}}{x^2}+\frac{1}{2}(3bc)\int \frac{1}{\sqrt{bx+cx^2}}\,dx\\ &= 3c\sqrt{bx+cx^2}-\frac{2\left(bx+cx^2\right)^{3/2}}{x^2}+(3bc)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{bx+cx^2}}\right)\\ &= 3c\sqrt{bx+cx^2}-\frac{2\left(bx+cx^2\right)^{3/2}}{x^2}+3b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)\end{aligned}$$

Mathematica [C] time = 0.0120287, size = 46, normalized size = 0.72

$$-\frac{2b\sqrt{x(b+cx)}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{cx}{b}\right)}{x\sqrt{\frac{cx}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(3/2)/x^3,x]
```

```
[Out] (-2*b*Sqrt[x*(b + c*x)]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((c*x)/b)])/(x*
Sqrt[1 + (c*x)/b])
```

Maple [B] time = 0.045, size = 124, normalized size = 1.9

$$-2\,\frac{\left(cx^2+bx\right)^{5/2}}{bx^3}+8\,\frac{c\left(cx^2+bx\right)^{5/2}}{b^2x^2}-8\,\frac{c^2\left(cx^2+bx\right)^{3/2}}{b^2}-6\,\frac{c^2\sqrt{cx^2+bx}x}{b}-3\,c\sqrt{cx^2+bx}+\frac{3\,b}{2}\sqrt{c}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(3/2)/x^3,x)
```

```
[Out] -2/b/x^3*(c*x^2+b*x)^(5/2)+8*c/b^2/x^2*(c*x^2+b*x)^(5/2)-8*c^2/b^2*(c*x^2+b
*x)^(3/2)-6*c^2/b*(c*x^2+b*x)^(1/2)*x-3*c*(c*x^2+b*x)^(1/2)+3/2*c^(1/2)*b*l
n((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.97435, size = 270, normalized size = 4.22

$$\left[\frac{3\,b\sqrt{c}x\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\,\sqrt{cx^2+bx}(cx-2\,b)}{2\,x},-\frac{3\,b\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx}\right)-\sqrt{cx^2+bx}(cx-2\,b}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/2*(3*b*sqrt(c)*x*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*sqrt(c
*x^2 + b*x)*(c*x - 2*b))/x, -(3*b*sqrt(-c)*x*arctan(sqrt(c*x^2 + b*x)*sqrt(
-c)/(c*x)) - sqrt(c*x^2 + b*x)*(c*x - 2*b))/x]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x\,(b+cx))^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(3/2)/x**3,x)
```

```
[Out] Integral((x*(b + c*x))**(3/2)/x**3, x)
```

Giac [A] time = 1.45613, size = 103, normalized size = 1.61

$$-\frac{3}{2}\,b\sqrt{c}\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}-b\right|\right)+\sqrt{cx^2+bx}c+\frac{2\,b^2}{\sqrt{c}x-\sqrt{cx^2+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -3/2*b*sqrt(c)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b)) + s
qrt(c*x^2 + b*x)*c + 2*b^2/(sqrt(c)*x - sqrt(c*x^2 + b*x))
```

3.18 $\int \frac{(bx+cx^2)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=68

$$2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)-\frac{2c\sqrt{bx+cx^2}}{x}-\frac{2\left(bx+cx^2\right)^{3/2}}{3x^3}$$

```
[Out] (-2*c*Sqrt[b*x + c*x^2])/x - (2*(b*x + c*x^2)^(3/2))/(3*x^3) + 2*c^(3/2)*Ar
cTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]]
```

Rubi [A] time = 0.0278444, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {662, 620, 206}

$$2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)-\frac{2c\sqrt{bx+cx^2}}{x}-\frac{2\left(bx+cx^2\right)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(3/2)/x^4,x]
```

```
[Out] (-2*c*Sqrt[b*x + c*x^2])/x - (2*(b*x + c*x^2)^(3/2))/(3*x^3) + 2*c^(3/2)*Ar
cTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]]
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(bx+cx^2)^{3/2}}{x^4}\,dx &= -\frac{2(bx+cx^2)^{3/2}}{3x^3} + c\int \frac{\sqrt{bx+cx^2}}{x^2}\,dx \\
&= -\frac{2c\sqrt{bx+cx^2}}{x} - \frac{2(bx+cx^2)^{3/2}}{3x^3} + c^2\int \frac{1}{\sqrt{bx+cx^2}}\,dx \\
&= -\frac{2c\sqrt{bx+cx^2}}{x} - \frac{2(bx+cx^2)^{3/2}}{3x^3} + (2c^2)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{bx+cx^2}}\right) \\
&= -\frac{2c\sqrt{bx+cx^2}}{x} - \frac{2(bx+cx^2)^{3/2}}{3x^3} + 2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.0135384, size = 48, normalized size = 0.71

$$
-\frac{2b\sqrt{x(b+cx)}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{cx}{b}\right)}{3x^2\sqrt{\frac{cx}{b}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(3/2)/x^4,x]
```

```
[Out] (-2*b*Sqrt[x*(b + c*x)]*Hypergeometric2F1[-3/2, -3/2, -1/2, -((c*x)/b)])/(3
*x^2*Sqrt[1 + (c*x)/b])
```

Maple [B] time = 0.046, size = 149, normalized size = 2.2

$$
-\frac{2}{3\,bx^4}\left(cx^2+bx\right)^{\frac{5}{2}} - \frac{4\,c}{3\,b^2x^3}\left(cx^2+bx\right)^{\frac{5}{2}} + \frac{16\,c^2}{3\,b^3x^2}\left(cx^2+bx\right)^{\frac{5}{2}} - \frac{16\,c^3}{3\,b^3}\left(cx^2+bx\right)^{\frac{3}{2}} - 4\,\frac{c^3\sqrt{cx^2+bx}x}{b^2} - 2\,\frac{c^2\sqrt{cx^2+bx}}{b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(3/2)/x^4,x)
```

```
[Out] -2/3/b/x^4*(c*x^2+b*x)^(5/2)-4/3*c/b^2/x^3*(c*x^2+b*x)^(5/2)+16/3*c^2/b^3/x
^2*(c*x^2+b*x)^(5/2)-16/3*c^3/b^3*(c*x^2+b*x)^(3/2)-4*c^3/b^2*(c*x^2+b*x)^(
1/2)*x-2*c^2/b*(c*x^2+b*x)^(1/2)+c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)
^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.0361, size = 284, normalized size = 4.18

$$\left[\frac{3\,c^{\frac{3}{2}}x^2\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\,\sqrt{cx^2+bx}(4\,cx+b)}{3\,x^2},\,-\frac{2\left(3\,\sqrt{-c}cx^2\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx}\right)+\sqrt{cx^2+bx}(4\,cx+b\right.}{3\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/3*(3*c^(3/2)*x^2*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 2*sqrt(c
*x^2 + b*x)*(4*c*x + b))/x^2, -2/3*(3*sqrt(-c)*c*x^2*arctan(sqrt(c*x^2 + b*
x)*sqrt(-c)/(c*x)) + sqrt(c*x^2 + b*x)*(4*c*x + b))/x^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x\,(b+cx))^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(3/2)/x**4,x)
```

```
[Out] Integral((x*(b + c*x))**(3/2)/x**4, x)
```

Giac [B] time = 1.34296, size = 155, normalized size = 2.28

$$-c^{\frac{3}{2}}\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}-b\right|\right)+\frac{2\left(6\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^2bc+3\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)b^2\sqrt{c}+b^3\right)}{3\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] -c^(3/2)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b)) + 2/3*(6*
(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*b*c + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b
^2*sqrt(c) + b^3)/(sqrt(c)*x - sqrt(c*x^2 + b*x))^3
```

3.19 $\int \frac{(bx+cx^2)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=23

$$-\frac{2\left(bx+cx^2\right)^{5/2}}{5bx^5}$$

```
[Out] (-2*(b*x + c*x^2)^(5/2))/(5*b*x^5)
```

Rubi [A] time = 0.0075107, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {650}

$$-\frac{2\left(bx+cx^2\right)^{5/2}}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(3/2)/x^5,x]
```

```
[Out] (-2*(b*x + c*x^2)^(5/2))/(5*b*x^5)
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\left(bx+cx^2\right)^{3/2}}{x^5}\,dx=-\frac{2\left(bx+cx^2\right)^{5/2}}{5bx^5}$$

Mathematica [A] time = 0.0120201, size = 21, normalized size = 0.91

$$-\frac{2(x(b+cx))^{5/2}}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(3/2)/x^5,x]
```

```
[Out] (-2*(x*(b + c*x))^(5/2))/(5*b*x^5)
```

Maple [A] time = 0.044, size = 25, normalized size = 1.1

$$-\frac{2\,cx+2\,b}{5\,bx^4}\left(cx^2+bx\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(3/2)/x^5,x)
```

```
[Out] -2/5/x^4*(c*x+b)/b*(c*x^2+b*x)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.98778, size = 78, normalized size = 3.39

$$-\frac{2\left(c^2x^2+2\,bcx+b^2\right)\sqrt{cx^2+bx}}{5\,bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] -2/5*(c^2*x^2 + 2*b*c*x + b^2)*sqrt(c*x^2 + b*x)/(b*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(x\,(b+cx))^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(3/2)/x**5,x)
```

```
[Out] Integral((x*(b + c*x))**(3/2)/x**5, x)
```

Giac [B] time = 1.25282, size = 181, normalized size = 7.87

$$\frac{2\left(5\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^4c^2+10\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3bc^{\frac{3}{2}}+10\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^2b^2c+5\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)b^3\sqrt{c}+b^4\right)}{5\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 2/5*(5*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*c^2 + 10*(sqrt(c)*x - sqrt(c*x^2 +
 b*x))^3*b*c^(3/2) + 10*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*b^2*c + 5*(sqrt(c
)*x - sqrt(c*x^2 + b*x))*b^3*sqrt(c) + b^4)/(sqrt(c)*x - sqrt(c*x^2 + b*x))
^5
```

3.20 $\int \frac{(bx+cx^2)^{3/2}}{x^6} \, dx$

Optimal. Leaf size=48

$$\frac{4c(bx+cx^2)^{5/2}}{35b^2x^5} - \frac{2(bx+cx^2)^{5/2}}{7bx^6}$$

```
[Out] (-2*(b*x + c*x^2)^(5/2))/(7*b*x^6) + (4*c*(b*x + c*x^2)^(5/2))/(35*b^2*x^5)
```

Rubi [A] time = 0.0164416, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {658, 650}

$$\frac{4c(bx+cx^2)^{5/2}}{35b^2x^5} - \frac{2(bx+cx^2)^{5/2}}{7bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(3/2)/x^6,x]
```

```
[Out] (-2*(b*x + c*x^2)^(5/2))/(7*b*x^6) + (4*c*(b*x + c*x^2)^(5/2))/(35*b^2*x^5)
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(bx+cx^2)^{3/2}}{x^6} \, dx &= -\frac{2(bx+cx^2)^{5/2}}{7bx^6} - \frac{(2c)\int \frac{(bx+cx^2)^{3/2}}{x^5} \, dx}{7b} \\ &= -\frac{2(bx+cx^2)^{5/2}}{7bx^6} + \frac{4c(bx+cx^2)^{5/2}}{35b^2x^5} \end{aligned}$$

Mathematica [A] time = 0.0120312, size = 29, normalized size = 0.6

$$\frac{2(x(b+cx))^{5/2}(2cx-5b)}{35b^2x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(3/2)/x^6,x]
```

```
[Out] (2*(x*(b + c*x))^(5/2)*(-5*b + 2*c*x))/(35*b^2*x^6)
```

Maple [A] time = 0.051, size = 33, normalized size = 0.7

$$-{\frac{ \left(2\,cx+2\,b \right) \left(-2\,cx+5\,b \right) }{35\,{x}^{5}{b}^{2}} \left(c{x}^{2}+bx \right) ^{{\frac{3}{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(3/2)/x^6,x)
```

```
[Out] -2/35*(c*x+b)*(-2*c*x+5*b)*(c*x^2+b*x)^(3/2)/x^5/b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.06845, size = 105, normalized size = 2.19

$$\frac{2 \, {\left(2 \, c^{3} x^{3} - b c^{2} x^{2} - 8 \, b^{2} c x - 5 \, b^{3}\right)} \sqrt{c x^{2} + b x}}{35 \, b^{2} x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] 2/35*(2*c^3*x^3 - b*c^2*x^2 - 8*b^2*c*x - 5*b^3)*sqrt(c*x^2 + b*x)/(b^2*x^4
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x \left(b + c x\right)\right)^{\frac{3}{2}}}{x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(3/2)/x**6,x)
```

```
[Out] Integral((x*(b + c*x))**(3/2)/x**6, x)
```

Giac [B] time = 1.25156, size = 223, normalized size = 4.65

$$\frac{2\left(35\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^5c^{\frac{5}{2}}+105\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^4bc^2+140\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3b^2c^{\frac{3}{2}}+98\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^2b^3c-\right.}{35\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 2/35*(35*(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*c^(5/2) + 105*(sqrt(c)*x - sqrt(
c*x^2 + b*x))^4*b*c^2 + 140*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b^2*c^(3/2) +
 98*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*b^3*c + 35*(sqrt(c)*x - sqrt(c*x^2 +
b*x))*b^4*sqrt(c) + 5*b^5)/(sqrt(c)*x - sqrt(c*x^2 + b*x))^7
```

3.21 $\int \frac{(bx+cx^2)^{3/2}}{x^7} dx$

Optimal. Leaf size=74

$$-\frac{16c^2(bx+cx^2)^{5/2}}{315b^3x^5}+\frac{8c(bx+cx^2)^{5/2}}{63b^2x^6}-\frac{2(bx+cx^2)^{5/2}}{9bx^7}$$

```
[Out] (-2*(b*x + c*x^2)^(5/2))/(9*b*x^7) + (8*c*(b*x + c*x^2)^(5/2))/(63*b^2*x^6)
 - (16*c^2*(b*x + c*x^2)^(5/2))/(315*b^3*x^5)
```

Rubi [A] time = 0.0270773, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {658, 650}

$$-\frac{16c^2(bx+cx^2)^{5/2}}{315b^3x^5}+\frac{8c(bx+cx^2)^{5/2}}{63b^2x^6}-\frac{2(bx+cx^2)^{5/2}}{9bx^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(3/2)/x^7,x]
```

```
[Out] (-2*(b*x + c*x^2)^(5/2))/(9*b*x^7) + (8*c*(b*x + c*x^2)^(5/2))/(63*b^2*x^6)
 - (16*c^2*(b*x + c*x^2)^(5/2))/(315*b^3*x^5)
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx+cx^2)^{3/2}}{x^7} dx &= -\frac{2(bx+cx^2)^{5/2}}{9bx^7}-\frac{(4c)\int \frac{(bx+cx^2)^{3/2}}{x^6} dx}{9b}\\
&= -\frac{2(bx+cx^2)^{5/2}}{9bx^7}+\frac{8c(bx+cx^2)^{5/2}}{63b^2x^6}+\frac{(8c^2)\int \frac{(bx+cx^2)^{3/2}}{x^5} dx}{63b^2}\\
&= -\frac{2(bx+cx^2)^{5/2}}{9bx^7}+\frac{8c(bx+cx^2)^{5/2}}{63b^2x^6}-\frac{16c^2(bx+cx^2)^{5/2}}{315b^3x^5}
\end{aligned}$$

Mathematica [A] time = 0.0157896, size = 40, normalized size = 0.54

$$-\frac{2(x(b+cx))^{5/2}\left(35b^2-20bcx+8c^2x^2\right)}{315b^3x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(3/2)/x^7,x]
```

```
[Out] (-2*(x*(b + c*x))^(5/2)*(35*b^2 - 20*b*c*x + 8*c^2*x^2))/(315*b^3*x^7)
```

Maple [A] time = 0.046, size = 44, normalized size = 0.6

$$-\frac{(2\,cx+2\,b)\left(8\,c^2x^2-20\,bcx+35\,b^2\right)}{315\,x^6b^3}\left(cx^2+bx\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(3/2)/x^7,x)
```

```
[Out] -2/315*(c*x+b)*(8*c^2*x^2-20*b*c*x+35*b^2)*(c*x^2+b*x)^(3/2)/x^6/b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.93022, size = 135, normalized size = 1.82

$$-\frac{2\left(8\,c^4x^4-4\,bc^3x^3+3\,b^2c^2x^2+50\,b^3cx+35\,b^4\right)\sqrt{cx^2+bx}}{315\,b^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] -2/315*(8*c^4*x^4 - 4*b*c^3*x^3 + 3*b^2*c^2*x^2 + 50*b^3*c*x + 35*b^4)*sqrt
(c*x^2 + b*x)/(b^3*x^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(x\,(b+cx))^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(3/2)/x**7,x)
```

```
[Out] Integral((x*(b + c*x))**(3/2)/x**7, x)
```

Giac [B] time = 1.23412, size = 262, normalized size = 3.54

$$\frac{2\left(420\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^6c^3+1575\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^5bc^{\frac{5}{2}}+2583\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^4b^2c^2+2310\left(\sqrt{c}x-\sqrt{cx^2+}\right.\right.}{315\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 2/315*(420*(sqrt(c)*x - sqrt(c*x^2 + b*x))^6*c^3 + 1575*(sqrt(c)*x - sqrt(c
*x^2 + b*x))^5*b*c^(5/2) + 2583*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*b^2*c^2 +
 2310*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b^3*c^(3/2) + 1170*(sqrt(c)*x - sqr
t(c*x^2 + b*x))^2*b^4*c + 315*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b^5*sqrt(c) +
 35*b^6)/(sqrt(c)*x - sqrt(c*x^2 + b*x))^9
```

3.22 $\int \frac{\left(bx+cx^2\right)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=100

$$\frac{32c^3\left(bx+cx^2\right)^{5/2}}{1155b^4x^5}-\frac{16c^2\left(bx+cx^2\right)^{5/2}}{231b^3x^6}+\frac{4c\left(bx+cx^2\right)^{5/2}}{33b^2x^7}-\frac{2\left(bx+cx^2\right)^{5/2}}{11bx^8}$$

```
[Out] (-2*(b*x + c*x^2)^(5/2))/(11*b*x^8) + (4*c*(b*x + c*x^2)^(5/2))/(33*b^2*x^7
) - (16*c^2*(b*x + c*x^2)^(5/2))/(231*b^3*x^6) + (32*c^3*(b*x + c*x^2)^(5/2
))/(1155*b^4*x^5)
```

Rubi [A] time = 0.0411665, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {658, 650}

$$\frac{32c^3\left(bx+cx^2\right)^{5/2}}{1155b^4x^5}-\frac{16c^2\left(bx+cx^2\right)^{5/2}}{231b^3x^6}+\frac{4c\left(bx+cx^2\right)^{5/2}}{33b^2x^7}-\frac{2\left(bx+cx^2\right)^{5/2}}{11bx^8}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(3/2)/x^8,x]
```

```
[Out] (-2*(b*x + c*x^2)^(5/2))/(11*b*x^8) + (4*c*(b*x + c*x^2)^(5/2))/(33*b^2*x^7
) - (16*c^2*(b*x + c*x^2)^(5/2))/(231*b^3*x^6) + (32*c^3*(b*x + c*x^2)^(5/2
))/(1155*b^4*x^5)
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{(bx+cx^2)^{3/2}}{x^8}dx = -\frac{2(bx+cx^2)^{5/2}}{11bx^8} - \frac{(6c)\int \frac{(bx+cx^2)^{3/2}}{x^7}dx}{11b}$$
$$= -\frac{2(bx+cx^2)^{5/2}}{11bx^8} + \frac{4c(bx+cx^2)^{5/2}}{33b^2x^7} + \frac{(8c^2)\int \frac{(bx+cx^2)^{3/2}}{x^6}dx}{33b^2}$$
$$= -\frac{2(bx+cx^2)^{5/2}}{11bx^8} + \frac{4c(bx+cx^2)^{5/2}}{33b^2x^7} - \frac{16c^2(bx+cx^2)^{5/2}}{231b^3x^6} - \frac{(16c^3)\int \frac{(bx+cx^2)^{3/2}}{x^5}dx}{231b^3}$$
$$= -\frac{2(bx+cx^2)^{5/2}}{11bx^8} + \frac{4c(bx+cx^2)^{5/2}}{33b^2x^7} - \frac{16c^2(bx+cx^2)^{5/2}}{231b^3x^6} + \frac{32c^3(bx+cx^2)^{5/2}}{1155b^4x^5}$$

Mathematica [A] time = 0.0174287, size = 51, normalized size = 0.51

$$\frac{2(x(b+cx))^{5/2}(70b^2cx-105b^3-40bc^2x^2+16c^3x^3)}{1155b^4x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(3/2)/x^8,x]

[Out] (2*(x*(b + c*x))^(5/2)*(-105*b^3 + 70*b^2*c*x - 40*b*c^2*x^2 + 16*c^3*x^3))
/(1155*b^4*x^8)
```

Maple [A] time = 0.046, size = 55, normalized size = 0.6

$$-\frac{(2cx+2b)(-16x^3c^3+40bx^2c^2-70b^2xc+105b^3)}{1155x^7b^4}(cx^2+bx)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(3/2)/x^8,x)

[Out] -2/1155*(c*x+b)*(-16*c^3*x^3+40*b*c^2*x^2-70*b^2*c*x+105*b^3)*(c*x^2+b*x)^(
3/2)/x^7/b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^8,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 1.96342, size = 161, normalized size = 1.61

$$\frac{2(16c^5x^5-8bc^4x^4+6b^2c^3x^3-5b^3c^2x^2-140b^4cx-105b^5)\sqrt{cx^2+bx}}{1155b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] 2/1155*(16*c^5*x^5 - 8*b*c^4*x^4 + 6*b^2*c^3*x^3 - 5*b^3*c^2*x^2 - 140*b^4*
c*x - 105*b^5)*sqrt(c*x^2 + b*x)/(b^4*x^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x(b+cx))^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(3/2)/x**8,x)
```

```
[Out] Integral((x*(b + c*x))**(3/2)/x**8, x)
```

Giac [B] time = 1.27978, size = 301, normalized size = 3.01

$$\frac{2\left(2310\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^7c^{\frac{7}{2}}+10164\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^6bc^3+19635\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^5b^2c^{\frac{5}{2}}+21285\left(\sqrt{c}x-\sqrt{cx^2\cdots}\right.\right.}{1155\left(\sqrt{c}x-\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] 2/1155*(2310*(sqrt(c)*x - sqrt(c*x^2 + b*x))^7*c^(7/2) + 10164*(sqrt(c)*x -
 sqrt(c*x^2 + b*x))^6*b*c^3 + 19635*(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*b^2*c
^(5/2) + 21285*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*b^3*c^2 + 13860*(sqrt(c)*x
 - sqrt(c*x^2 + b*x))^3*b^4*c^(3/2) + 5390*(sqrt(c)*x - sqrt(c*x^2 + b*x))^
2*b^5*c + 1155*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b^6*sqrt(c) + 105*b^7)/(sqrt
(c)*x - sqrt(c*x^2 + b*x))^11
```

3.23 $\int \frac{(bx+cx^2)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=126

$$-\frac{256c^4(bx+cx^2)^{5/2}}{15015b^5x^5}+\frac{128c^3(bx+cx^2)^{5/2}}{3003b^4x^6}-\frac{32c^2(bx+cx^2)^{5/2}}{429b^3x^7}+\frac{16c(bx+cx^2)^{5/2}}{143b^2x^8}-\frac{2(bx+cx^2)^{5/2}}{13bx^9}$$

```
[Out] (-2*(b*x + c*x^2)^(5/2))/(13*b*x^9) + (16*c*(b*x + c*x^2)^(5/2))/(143*b^2*x
^8) - (32*c^2*(b*x + c*x^2)^(5/2))/(429*b^3*x^7) + (128*c^3*(b*x + c*x^2)^(
5/2))/(3003*b^4*x^6) - (256*c^4*(b*x + c*x^2)^(5/2))/(15015*b^5*x^5)
```

Rubi [A] time = 0.0574311, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {658, 650}

$$-\frac{256c^4(bx+cx^2)^{5/2}}{15015b^5x^5}+\frac{128c^3(bx+cx^2)^{5/2}}{3003b^4x^6}-\frac{32c^2(bx+cx^2)^{5/2}}{429b^3x^7}+\frac{16c(bx+cx^2)^{5/2}}{143b^2x^8}-\frac{2(bx+cx^2)^{5/2}}{13bx^9}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(3/2)/x^9,x]
```

```
[Out] (-2*(b*x + c*x^2)^(5/2))/(13*b*x^9) + (16*c*(b*x + c*x^2)^(5/2))/(143*b^2*x
^8) - (32*c^2*(b*x + c*x^2)^(5/2))/(429*b^3*x^7) + (128*c^3*(b*x + c*x^2)^(
5/2))/(3003*b^4*x^6) - (256*c^4*(b*x + c*x^2)^(5/2))/(15015*b^5*x^5)
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{(bx+cx^2)^{3/2}}{x^9}\,dx = -\frac{2(bx+cx^2)^{5/2}}{13bx^9} - \frac{(8c)\int \frac{(bx+cx^2)^{3/2}}{x^8}\,dx}{13b}$$
$$= -\frac{2(bx+cx^2)^{5/2}}{13bx^9} + \frac{16c(bx+cx^2)^{5/2}}{143b^2x^8} + \frac{(48c^2)\int \frac{(bx+cx^2)^{3/2}}{x^7}\,dx}{143b^2}$$
$$= -\frac{2(bx+cx^2)^{5/2}}{13bx^9} + \frac{16c(bx+cx^2)^{5/2}}{143b^2x^8} - \frac{32c^2(bx+cx^2)^{5/2}}{429b^3x^7} - \frac{(64c^3)\int \frac{(bx+cx^2)^{3/2}}{x^6}\,dx}{429b^3}$$
$$= -\frac{2(bx+cx^2)^{5/2}}{13bx^9} + \frac{16c(bx+cx^2)^{5/2}}{143b^2x^8} - \frac{32c^2(bx+cx^2)^{5/2}}{429b^3x^7} + \frac{128c^3(bx+cx^2)^{5/2}}{3003b^4x^6} + \frac{(128c^4)\int \frac{(bx+cx}{x^5}}{3003b^4}$$
$$= -\frac{2(bx+cx^2)^{5/2}}{13bx^9} + \frac{16c(bx+cx^2)^{5/2}}{143b^2x^8} - \frac{32c^2(bx+cx^2)^{5/2}}{429b^3x^7} + \frac{128c^3(bx+cx^2)^{5/2}}{3003b^4x^6} - \frac{256c^4(bx+cx^2}{15015b^5x^5}$$

Mathematica [A] time = 0.0187573, size = 62, normalized size = 0.49

$$-\frac{2(x(b+cx))^{5/2}\left(560b^2c^2x^2-840b^3cx+1155b^4-320bc^3x^3+128c^4x^4\right)}{15015b^5x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(3/2)/x^9,x]
```

```
[Out] (-2*(x*(b + c*x))^(5/2)*(1155*b^4 - 840*b^3*c*x + 560*b^2*c^2*x^2 - 320*b*c
^3*x^3 + 128*c^4*x^4))/(15015*b^5*x^9)
```

Maple [A] time = 0.046, size = 66, normalized size = 0.5

$$-\frac{(2cx+2b)\left(128c^4x^4-320x^3c^3b+560c^2x^2b^2-840cxb^3+1155b^4\right)}{15015x^8b^5}\left(cx^2+bx\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(3/2)/x^9,x)
```

```
[Out] -2/15015*(c*x+b)*(128*c^4*x^4-320*b*c^3*x^3+560*b^2*c^2*x^2-840*b^3*c*x+115
5*b^4)*(c*x^2+b*x)^(3/2)/x^8/b^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.92996, size = 194, normalized size = 1.54

$$-\frac{2\left(128\,c^{6}x^{6}-64\,bc^{5}x^{5}+48\,b^{2}c^{4}x^{4}-40\,b^{3}c^{3}x^{3}+35\,b^{4}c^{2}x^{2}+1470\,b^{5}cx+1155\,b^{6}\right)\sqrt{cx^{2}+bx}}{15015\,b^{5}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] -2/15015*(128*c^6*x^6 - 64*b*c^5*x^5 + 48*b^2*c^4*x^4 - 40*b^3*c^3*x^3 + 35
*b^4*c^2*x^2 + 1470*b^5*c*x + 1155*b^6)*sqrt(c*x^2 + b*x)/(b^5*x^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(x\,(b+cx))^{\frac{3}{2}}}{x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(3/2)/x**9,x)
```

```
[Out] Integral((x*(b + c*x))**(3/2)/x**9, x)
```

Giac [B] time = 1.21335, size = 340, normalized size = 2.7

$$2\left(48048\left(\sqrt{c}x-\sqrt{cx^{2}+bx}\right)^{8}c^{4}+240240\left(\sqrt{c}x-\sqrt{cx^{2}+bx}\right)^{7}bc^{\frac{7}{2}}+531960\left(\sqrt{c}x-\sqrt{cx^{2}+bx}\right)^{6}b^{2}c^{3}+675675\left(\sqrt{c}x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] 2/15015*(48048*(sqrt(c)*x - sqrt(c*x^2 + b*x))^8*c^4 + 240240*(sqrt(c)*x -
sqrt(c*x^2 + b*x))^7*b*c^(7/2) + 531960*(sqrt(c)*x - sqrt(c*x^2 + b*x))^6*b
^2*c^3 + 675675*(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*b^3*c^(5/2) + 535535*(sqr
t(c)*x - sqrt(c*x^2 + b*x))^4*b^4*c^2 + 270270*(sqrt(c)*x - sqrt(c*x^2 + b*
x))^3*b^5*c^(3/2) + 84630*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*b^6*c + 15015*(
sqrt(c)*x - sqrt(c*x^2 + b*x))*b^7*sqrt(c) + 1155*b^8)/(sqrt(c)*x - sqrt(c*
x^2 + b*x))^13
```

3.24 $\int x^2 \left(ax+bx^2\right)^{5/2} dx$

Optimal. Leaf size=163

$$\frac{45a^6(a+2bx)\sqrt{ax+bx^2}}{16384b^5}-\frac{15a^4(a+2bx)\left(ax+bx^2\right)^{3/2}}{2048b^4}+\frac{3a^2(a+2bx)\left(ax+bx^2\right)^{5/2}}{128b^3}-\frac{45a^8\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{16384b^{11/2}}-\frac{9a\left(ax\right.}{11}$$

```
[Out] (45*a^6*(a + 2*b*x)*Sqrt[a*x + b*x^2])/(16384*b^5) - (15*a^4*(a + 2*b*x)*(a
*x + b*x^2)^(3/2))/(2048*b^4) + (3*a^2*(a + 2*b*x)*(a*x + b*x^2)^(5/2))/(12
8*b^3) - (9*a*(a*x + b*x^2)^(7/2))/(112*b^2) + (x*(a*x + b*x^2)^(7/2))/(8*b
) - (45*a^8*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(16384*b^(11/2))
```

Rubi [A] time = 0.0736416, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {670, 640, 612, 620, 206}

$$\frac{45a^6(a+2bx)\sqrt{ax+bx^2}}{16384b^5}-\frac{15a^4(a+2bx)\left(ax+bx^2\right)^{3/2}}{2048b^4}+\frac{3a^2(a+2bx)\left(ax+bx^2\right)^{5/2}}{128b^3}-\frac{45a^8\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{16384b^{11/2}}-\frac{9a\left(ax\right.}{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x + b*x^2)^(5/2),x]
```

```
[Out] (45*a^6*(a + 2*b*x)*Sqrt[a*x + b*x^2])/(16384*b^5) - (15*a^4*(a + 2*b*x)*(a
*x + b*x^2)^(3/2))/(2048*b^4) + (3*a^2*(a + 2*b*x)*(a*x + b*x^2)^(5/2))/(12
8*b^3) - (9*a*(a*x + b*x^2)^(7/2))/(112*b^2) + (x*(a*x + b*x^2)^(7/2))/(8*b
) - (45*a^8*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(16384*b^(11/2))
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^2\left(ax+bx^2\right)^{5/2}dx = \frac{x\left(ax+bx^2\right)^{7/2}}{8b} - \frac{(9a)\int x\left(ax+bx^2\right)^{5/2}dx}{16b}$$
$$= -\frac{9a\left(ax+bx^2\right)^{7/2}}{112b^2} + \frac{x\left(ax+bx^2\right)^{7/2}}{8b} + \frac{\left(9a^2\right)\int\left(ax+bx^2\right)^{5/2}dx}{32b^2}$$
$$= \frac{3a^2(a+2bx)\left(ax+bx^2\right)^{5/2}}{128b^3} - \frac{9a\left(ax+bx^2\right)^{7/2}}{112b^2} + \frac{x\left(ax+bx^2\right)^{7/2}}{8b} - \frac{\left(15a^4\right)\int\left(ax+bx^2\right)^{3/2}dx}{256b^3}$$
$$= -\frac{15a^4(a+2bx)\left(ax+bx^2\right)^{3/2}}{2048b^4} + \frac{3a^2(a+2bx)\left(ax+bx^2\right)^{5/2}}{128b^3} - \frac{9a\left(ax+bx^2\right)^{7/2}}{112b^2} + \frac{x\left(ax+bx^2\right.}{8b}$$
$$= \frac{45a^6(a+2bx)\sqrt{ax+bx^2}}{16384b^5} - \frac{15a^4(a+2bx)\left(ax+bx^2\right)^{3/2}}{2048b^4} + \frac{3a^2(a+2bx)\left(ax+bx^2\right)^{5/2}}{128b^3} - \frac{9a\,(a}{}$$
$$= \frac{45a^6(a+2bx)\sqrt{ax+bx^2}}{16384b^5} - \frac{15a^4(a+2bx)\left(ax+bx^2\right)^{3/2}}{2048b^4} + \frac{3a^2(a+2bx)\left(ax+bx^2\right)^{5/2}}{128b^3} - \frac{9a\,(a}{}$$
$$= \frac{45a^6(a+2bx)\sqrt{ax+bx^2}}{16384b^5} - \frac{15a^4(a+2bx)\left(ax+bx^2\right)^{3/2}}{2048b^4} + \frac{3a^2(a+2bx)\left(ax+bx^2\right)^{5/2}}{128b^3} - \frac{9a\,(a}{}$$

Mathematica [A] time = 0.206821, size = 142, normalized size = 0.87

$$\frac{\sqrt{x(a+bx)}\left(\sqrt{b}\left(168a^5b^2x^2-144a^4b^3x^3+128a^3b^4x^4+20736a^2b^5x^5-210a^6bx+315a^7+33792ab^6x^6+14336b^7x^7\right)\right.}{114688b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[x*(a + b*x)]*(Sqrt[b]*(315*a^7 - 210*a^6*b*x + 168*a^5*b^2*x^2 - 144*
a^4*b^3*x^3 + 128*a^3*b^4*x^4 + 20736*a^2*b^5*x^5 + 33792*a*b^6*x^6 + 14336
*b^7*x^7) - (315*a^(15/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[x]*Sqrt
[1 + (b*x)/a])))/(114688*b^(11/2))
```

Maple [A] time = 0.053, size = 185, normalized size = 1.1

$$\frac{x}{8\,b}\left(bx^2+ax\right)^{\frac{7}{2}} - \frac{9\,a}{112\,b^2}\left(bx^2+ax\right)^{\frac{7}{2}} + \frac{3\,a^2x}{64\,b^2}\left(bx^2+ax\right)^{\frac{5}{2}} + \frac{3\,a^3}{128\,b^3}\left(bx^2+ax\right)^{\frac{5}{2}} - \frac{15\,a^4x}{1024\,b^3}\left(bx^2+ax\right)^{\frac{3}{2}} - \frac{15\,a^5}{2048\,b^4}\left(bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a*x)^(5/2),x)
```

```
[Out] 1/8*x*(b*x^2+a*x)^(7/2)/b-9/112*a*(b*x^2+a*x)^(7/2)/b^2+3/64/b^2*a^2*(b*x^2
+a*x)^(5/2)*x+3/128/b^3*a^3*(b*x^2+a*x)^(5/2)-15/1024/b^3*a^4*(b*x^2+a*x)^(
```

```
3/2)*x-15/2048/b^4*a^5*(b*x^2+a*x)^(3/2)+45/8192/b^4*a^6*(b*x^2+a*x)^(1/2)*
x+45/16384/b^5*a^7*(b*x^2+a*x)^(1/2)-45/32768/b^(11/2)*a^8*ln((1/2*a+b*x)/b
^(1/2)+(b*x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.10095, size = 633, normalized size = 3.88

$$\left[\frac{315\,a^8\sqrt{b}\log\left(2\,bx + a - 2\,\sqrt{bx^2+ax}\sqrt{b}\right) + 2\left(14336\,b^8x^7 + 33792\,ab^7x^6 + 20736\,a^2b^6x^5 + 128\,a^3b^5x^4 - 144\,a^4b^4x^3 + 1\right.}{229376\,b^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/229376*(315*a^8*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2
*(14336*b^8*x^7 + 33792*a*b^7*x^6 + 20736*a^2*b^6*x^5 + 128*a^3*b^5*x^4 - 1
44*a^4*b^4*x^3 + 168*a^5*b^3*x^2 - 210*a^6*b^2*x + 315*a^7*b)*sqrt(b*x^2 +
a*x))/b^6, 1/114688*(315*a^8*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*
x)) + (14336*b^8*x^7 + 33792*a*b^7*x^6 + 20736*a^2*b^6*x^5 + 128*a^3*b^5*x^
4 - 144*a^4*b^4*x^3 + 168*a^5*b^3*x^2 - 210*a^6*b^2*x + 315*a^7*b)*sqrt(b*x
^2 + a*x))/b^6]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\,(x\,(a+bx))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a*x)**(5/2),x)
```

```
[Out] Integral(x**2*(x*(a + b*x))**(5/2), x)
```

Giac [A] time = 1.22452, size = 177, normalized size = 1.09

$$\frac{45\,a^8\log\left(\left|-2\left(\sqrt{b}x - \sqrt{bx^2+ax}\right)\sqrt{b} - a\right|\right)}{32768\,b^{\frac{11}{2}}} + \frac{1}{114688}\sqrt{bx^2+ax}\left(\frac{315\,a^7}{b^5} - 2\left(\frac{105\,a^6}{b^4} - 4\left(\frac{21\,a^5}{b^3} - 2\left(\frac{9\,a^4}{b^2} - 8\left(\frac{a^3}{b} + 2\,(81\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
[Out] 45/32768*a^8*log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/b^(11
/2) + 1/114688*sqrt(b*x^2 + a*x)*(315*a^7/b^5 - 2*(105*a^6/b^4 - 4*(21*a^5/
b^3 - 2*(9*a^4/b^2 - 8*(a^3/b + 2*(81*a^2 + 4*(14*b^2*x + 33*a*b)*x)*x)*x)*
x)*x)*x)
```

3.25 $\int x \left(ax + bx^2\right)^{5/2} dx$

Optimal. Leaf size=139

$$-\frac{5a^5(a+2bx)\sqrt{ax+bx^2}}{1024b^4}+\frac{5a^3(a+2bx)\left(ax+bx^2\right)^{3/2}}{384b^3}+\frac{5a^7\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{1024b^{9/2}}-\frac{a(a+2bx)\left(ax+bx^2\right)^{5/2}}{24b^2}+\frac{\left(ax+bx^2\right)^{7/2}}{7b}$$

```
[Out] (-5*a^5*(a + 2*b*x)*Sqrt[a*x + b*x^2])/(1024*b^4) + (5*a^3*(a + 2*b*x)*(a*x
 + b*x^2)^(3/2))/(384*b^3) - (a*(a + 2*b*x)*(a*x + b*x^2)^(5/2))/(24*b^2) +
 (a*x + b*x^2)^(7/2)/(7*b) + (5*a^7*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])
/(1024*b^(9/2))
```

Rubi [A] time = 0.0504348, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {640, 612, 620, 206}

$$-\frac{5a^5(a+2bx)\sqrt{ax+bx^2}}{1024b^4}+\frac{5a^3(a+2bx)\left(ax+bx^2\right)^{3/2}}{384b^3}+\frac{5a^7\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{1024b^{9/2}}-\frac{a(a+2bx)\left(ax+bx^2\right)^{5/2}}{24b^2}+\frac{\left(ax+bx^2\right)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x + b*x^2)^(5/2),x]
```

```
[Out] (-5*a^5*(a + 2*b*x)*Sqrt[a*x + b*x^2])/(1024*b^4) + (5*a^3*(a + 2*b*x)*(a*x
 + b*x^2)^(3/2))/(384*b^3) - (a*(a + 2*b*x)*(a*x + b*x^2)^(5/2))/(24*b^2) +
 (a*x + b*x^2)^(7/2)/(7*b) + (5*a^7*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])
/(1024*b^(9/2))
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x\left(ax+bx^2\right)^{5/2}dx = \frac{\left(ax+bx^2\right)^{7/2}}{7b} - \frac{a\int\left(ax+bx^2\right)^{5/2}dx}{2b}$$
$$= -\frac{a(a+2bx)\left(ax+bx^2\right)^{5/2}}{24b^2} + \frac{\left(ax+bx^2\right)^{7/2}}{7b} + \frac{\left(5a^3\right)\int\left(ax+bx^2\right)^{3/2}dx}{48b^2}$$
$$= \frac{5a^3(a+2bx)\left(ax+bx^2\right)^{3/2}}{384b^3} - \frac{a(a+2bx)\left(ax+bx^2\right)^{5/2}}{24b^2} + \frac{\left(ax+bx^2\right)^{7/2}}{7b} - \frac{\left(5a^5\right)\int\sqrt{ax+bx^2}\,dx}{256b^3}$$
$$= -\frac{5a^5(a+2bx)\sqrt{ax+bx^2}}{1024b^4} + \frac{5a^3(a+2bx)\left(ax+bx^2\right)^{3/2}}{384b^3} - \frac{a(a+2bx)\left(ax+bx^2\right)^{5/2}}{24b^2} + \frac{\left(ax+bx^2\right.}{7b}$$
$$= -\frac{5a^5(a+2bx)\sqrt{ax+bx^2}}{1024b^4} + \frac{5a^3(a+2bx)\left(ax+bx^2\right)^{3/2}}{384b^3} - \frac{a(a+2bx)\left(ax+bx^2\right)^{5/2}}{24b^2} + \frac{\left(ax+bx\right.}{7b}$$
$$= -\frac{5a^5(a+2bx)\sqrt{ax+bx^2}}{1024b^4} + \frac{5a^3(a+2bx)\left(ax+bx^2\right)^{3/2}}{384b^3} - \frac{a(a+2bx)\left(ax+bx^2\right)^{5/2}}{24b^2} + \frac{\left(ax+bx\right.}{7b}$$

Mathematica [A] time = 0.177274, size = 131, normalized size = 0.94

$$\frac{\sqrt{x(a+bx)}\left(\sqrt{b}\left(-56a^4b^2x^2+48a^3b^3x^3+4736a^2b^4x^4+70a^5bx-105a^6+7424ab^5x^5+3072b^6x^6\right)+\frac{105a^{13/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{}}{\sqrt{a}}\right.}{\sqrt{x}\sqrt{\frac{bx}{a}+1}}\right.}{21504b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[x*(a + b*x)]*(Sqrt[b]*(-105*a^6 + 70*a^5*b*x - 56*a^4*b^2*x^2 + 48*a^
3*b^3*x^3 + 4736*a^2*b^4*x^4 + 7424*a*b^5*x^5 + 3072*b^6*x^6) + (105*a^(13/
2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[x]*Sqrt[1 + (b*x)/a])))/(21504
*b^(9/2))
```

Maple [A] time = 0.049, size = 165, normalized size = 1.2

$$\frac{1}{7b}\left(bx^2+ax\right)^{\frac{7}{2}} - \frac{ax}{12b}\left(bx^2+ax\right)^{\frac{5}{2}} - \frac{a^2}{24b^2}\left(bx^2+ax\right)^{\frac{5}{2}} + \frac{5xa^3}{192b^2}\left(bx^2+ax\right)^{\frac{3}{2}} + \frac{5a^4}{384b^3}\left(bx^2+ax\right)^{\frac{3}{2}} - \frac{5a^5x}{512b^3}\sqrt{bx^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a*x)^(5/2),x)
```

```
[Out] 1/7*(b*x^2+a*x)^(7/2)/b-1/12/b*a*x*(b*x^2+a*x)^(5/2)-1/24/b^2*a^2*(b*x^2+a*
x)^(5/2)+5/192/b^2*a^3*(b*x^2+a*x)^(3/2)*x+5/384/b^3*a^4*(b*x^2+a*x)^(3/2)-
5/512/b^3*a^5*(b*x^2+a*x)^(1/2)*x-5/1024/b^4*a^6*(b*x^2+a*x)^(1/2)+5/2048/b
^(9/2)*a^7*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.01484, size = 567, normalized size = 4.08

$$\left[\frac{105\,a^7\sqrt{b}\log\left(2\,bx + a + 2\,\sqrt{bx^2 + ax}\sqrt{b}\right) + 2\left(3072\,b^7x^6 + 7424\,ab^6x^5 + 4736\,a^2b^5x^4 + 48\,a^3b^4x^3 - 56\,a^4b^3x^2 + 70\,a^5b^2\right.}{43008\,b^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/43008*(105*a^7*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*
(3072*b^7*x^6 + 7424*a*b^6*x^5 + 4736*a^2*b^5*x^4 + 48*a^3*b^4*x^3 - 56*a^4
*b^3*x^2 + 70*a^5*b^2*x - 105*a^6*b)*sqrt(b*x^2 + a*x))/b^5, -1/21504*(105*
a^7*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x)) - (3072*b^7*x^6 + 742
4*a*b^6*x^5 + 4736*a^2*b^5*x^4 + 48*a^3*b^4*x^3 - 56*a^4*b^3*x^2 + 70*a^5*b
^2*x - 105*a^6*b)*sqrt(b*x^2 + a*x))/b^5]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\,(x\,(a + bx))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a*x)**(5/2),x)
```

```
[Out] Integral(x*(x*(a + b*x))**(5/2), x)
```

Giac [A] time = 1.29223, size = 162, normalized size = 1.17

$$-\frac{5\,a^7\log\left(\left|-2\left(\sqrt{b}x - \sqrt{bx^2 + ax}\right)\sqrt{b} - a\right|\right)}{2048\,b^{\frac{9}{2}}} - \frac{1}{21504}\sqrt{bx^2 + ax}\left(\frac{105\,a^6}{b^4} - 2\left(\frac{35\,a^5}{b^3} - 4\left(\frac{7\,a^4}{b^2} - 2\left(\frac{3\,a^3}{b} + 8\left(37\,a^2 + 2\left(12\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
[Out] -5/2048*a^7*log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/b^(9/2
) - 1/21504*sqrt(b*x^2 + a*x)*(105*a^6/b^4 - 2*(35*a^5/b^3 - 4*(7*a^4/b^2 -
 2*(3*a^3/b + 8*(37*a^2 + 2*(12*b^2*x + 29*a*b)*x)*x)*x)*x)*x)
```

3.26 $\int \left(ax+bx^2\right)^{5/2} dx$

Optimal. Leaf size=118

$$\frac{5a^4(a+2bx)\sqrt{ax+bx^2}}{512b^3}-\frac{5a^2(a+2bx)\left(ax+bx^2\right)^{3/2}}{192b^2}-\frac{5a^6\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{512b^{7/2}}+\frac{(a+2bx)\left(ax+bx^2\right)^{5/2}}{12b}$$

```
[Out] (5*a^4*(a + 2*b*x)*Sqrt[a*x + b*x^2])/(512*b^3) - (5*a^2*(a + 2*b*x)*(a*x +
 b*x^2)^(3/2))/(192*b^2) + ((a + 2*b*x)*(a*x + b*x^2)^(5/2))/(12*b) - (5*a^
6*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(512*b^(7/2))
```

Rubi [A] time = 0.0359948, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {612, 620, 206}

$$\frac{5a^4(a+2bx)\sqrt{ax+bx^2}}{512b^3}-\frac{5a^2(a+2bx)\left(ax+bx^2\right)^{3/2}}{192b^2}-\frac{5a^6\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{512b^{7/2}}+\frac{(a+2bx)\left(ax+bx^2\right)^{5/2}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^2)^(5/2),x]
```

```
[Out] (5*a^4*(a + 2*b*x)*Sqrt[a*x + b*x^2])/(512*b^3) - (5*a^2*(a + 2*b*x)*(a*x +
 b*x^2)^(3/2))/(192*b^2) + ((a + 2*b*x)*(a*x + b*x^2)^(5/2))/(12*b) - (5*a^
6*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(512*b^(7/2))
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(ax+bx^2\right)^{5/2} dx = \frac{(a+2bx)\left(ax+bx^2\right)^{5/2}}{12b} - \frac{\left(5a^2\right)\int \left(ax+bx^2\right)^{3/2} dx}{24b}$$

$$= -\frac{5a^2(a+2bx)\left(ax+bx^2\right)^{3/2}}{192b^2} + \frac{(a+2bx)\left(ax+bx^2\right)^{5/2}}{12b} + \frac{\left(5a^4\right)\int \sqrt{ax+bx^2}\,dx}{128b^2}$$

$$= \frac{5a^4(a+2bx)\sqrt{ax+bx^2}}{512b^3} - \frac{5a^2(a+2bx)\left(ax+bx^2\right)^{3/2}}{192b^2} + \frac{(a+2bx)\left(ax+bx^2\right)^{5/2}}{12b} - \frac{\left(5a^6\right)\int \frac{1}{\sqrt{ax+bx^2}}}{1024b^3}$$

$$= \frac{5a^4(a+2bx)\sqrt{ax+bx^2}}{512b^3} - \frac{5a^2(a+2bx)\left(ax+bx^2\right)^{3/2}}{192b^2} + \frac{(a+2bx)\left(ax+bx^2\right)^{5/2}}{12b} - \frac{\left(5a^6\right)\text{Subst}\left(\int\right.}{5}$$

$$= \frac{5a^4(a+2bx)\sqrt{ax+bx^2}}{512b^3} - \frac{5a^2(a+2bx)\left(ax+bx^2\right)^{3/2}}{192b^2} + \frac{(a+2bx)\left(ax+bx^2\right)^{5/2}}{12b} - \frac{5a^6\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{ax}}\right.}{512b^{7/2}}$$

Mathematica [A] time = 0.183326, size = 120, normalized size = 1.02

$$\frac{\sqrt{x(a+bx)}\left(\sqrt{b}\left(8a^3b^2x^2+432a^2b^3x^3-10a^4bx+15a^5+640ab^4x^4+256b^5x^5\right)-\frac{15a^{11/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{x}\sqrt{\frac{bx}{a}+1}}\right)}{1536b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[x*(a + b*x)]*(Sqrt[b]*(15*a^5 - 10*a^4*b*x + 8*a^3*b^2*x^2 + 432*a^2*
b^3*x^3 + 640*a*b^4*x^4 + 256*b^5*x^5) - (15*a^(11/2)*ArcSinh[(Sqrt[b]*Sqrt
[x])/Sqrt[a]])/(Sqrt[x]*Sqrt[1 + (b*x)/a])))/(1536*b^(7/2))
```

Maple [A] time = 0.047, size = 134, normalized size = 1.1

$$\frac{2bx+a}{12b}\left(bx^2+ax\right)^{\frac{5}{2}} - \frac{5a^2x}{96b}\left(bx^2+ax\right)^{\frac{3}{2}} - \frac{5a^3}{192b^2}\left(bx^2+ax\right)^{\frac{3}{2}} + \frac{5a^4x}{256b^2}\sqrt{bx^2+ax} + \frac{5a^5}{512b^3}\sqrt{bx^2+ax} - \frac{5a^6}{1024}\ln\left(\left(\frac{a}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a*x)^(5/2),x)
```

```
[Out] 1/12*(2*b*x+a)*(b*x^2+a*x)^(5/2)/b-5/96/b*a^2*(b*x^2+a*x)^(3/2)*x-5/192/b^2
*a^3*(b*x^2+a*x)^(3/2)+5/256/b^2*a^4*(b*x^2+a*x)^(1/2)*x+5/512/b^3*a^5*(b*x
^2+a*x)^(1/2)-5/1024/b^(7/2)*a^6*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.93928, size = 501, normalized size = 4.25

$$\left[\frac{15\,a^{6}\sqrt{b}\log\left(2\,bx+a-2\,\sqrt{bx^{2}+ax}\sqrt{b}\right)+2\left(256\,b^{6}x^{5}+640\,ab^{5}x^{4}+432\,a^{2}b^{4}x^{3}+8\,a^{3}b^{3}x^{2}-10\,a^{4}b^{2}x+15\,a^{5}b\right)\sqrt{bx}}{3072\,b^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3072*(15*a^6*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(2
56*b^6*x^5 + 640*a*b^5*x^4 + 432*a^2*b^4*x^3 + 8*a^3*b^3*x^2 - 10*a^4*b^2*x
 + 15*a^5*b)*sqrt(b*x^2 + a*x))/b^4, 1/1536*(15*a^6*sqrt(-b)*arctan(sqrt(b*
x^2 + a*x)*sqrt(-b)/(b*x)) + (256*b^6*x^5 + 640*a*b^5*x^4 + 432*a^2*b^4*x^3
 + 8*a^3*b^3*x^2 - 10*a^4*b^2*x + 15*a^5*b)*sqrt(b*x^2 + a*x))/b^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(ax+bx^{2}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a*x)**(5/2),x)
```

```
[Out] Integral((a*x + b*x**2)**(5/2), x)
```

Giac [A] time = 1.27501, size = 144, normalized size = 1.22

$$\frac{5\,a^{6}\log\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^{2}+ax}\right)\sqrt{b}-a\right|\right)}{1024\,b^{\frac{7}{2}}}+\frac{1}{1536}\sqrt{bx^{2}+ax}\left(\frac{15\,a^{5}}{b^{3}}-2\left(\frac{5\,a^{4}}{b^{2}}-4\left(\frac{a^{3}}{b}+2\left(27\,a^{2}+8\left(2\,b^{2}x+5\,ab\right)x\right)x\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
[Out] 5/1024*a^6*log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/b^(7/2)
 + 1/1536*sqrt(b*x^2 + a*x)*(15*a^5/b^3 - 2*(5*a^4/b^2 - 4*(a^3/b + 2*(27*a
^2 + 8*(2*b^2*x + 5*a*b)*x)*x)*x)*x)
```

3.27 $\int \frac{(ax+bx^2)^{5/2}}{x} dx$

Optimal. Leaf size=107

$$-\frac{3a^3(a+2bx)\sqrt{ax+bx^2}}{128b^2}+\frac{3a^5\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{128b^{5/2}}+\frac{a(a+2bx)\left(ax+bx^2\right)^{3/2}}{16b}+\frac{1}{5}\left(ax+bx^2\right)^{5/2}$$

```
[Out] (-3*a^3*(a + 2*b*x)*Sqrt[a*x + b*x^2])/(128*b^2) + (a*(a + 2*b*x)*(a*x + b*
x^2)^(3/2))/(16*b) + (a*x + b*x^2)^(5/2)/5 + (3*a^5*ArcTanh[(Sqrt[b]*x)/Sqr
t[a*x + b*x^2]])/(128*b^(5/2))
```

Rubi [A] time = 0.0389775, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {664, 612, 620, 206}

$$-\frac{3a^3(a+2bx)\sqrt{ax+bx^2}}{128b^2}+\frac{3a^5\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{128b^{5/2}}+\frac{a(a+2bx)\left(ax+bx^2\right)^{3/2}}{16b}+\frac{1}{5}\left(ax+bx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^2)^(5/2)/x,x]
```

```
[Out] (-3*a^3*(a + 2*b*x)*Sqrt[a*x + b*x^2])/(128*b^2) + (a*(a + 2*b*x)*(a*x + b*
x^2)^(3/2))/(16*b) + (a*x + b*x^2)^(5/2)/5 + (3*a^5*ArcTanh[(Sqrt[b]*x)/Sqr
t[a*x + b*x^2]])/(128*b^(5/2))
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ax+bx^2\right)^{5/2}}{x}\,dx = \frac{1}{5}\left(ax+bx^2\right)^{5/2} + \frac{1}{2}a\int \left(ax+bx^2\right)^{3/2}\,dx$$

$$= \frac{a(a+2bx)\left(ax+bx^2\right)^{3/2}}{16b} + \frac{1}{5}\left(ax+bx^2\right)^{5/2} - \frac{\left(3a^3\right)\int \sqrt{ax+bx^2}\,dx}{32b}$$

$$= -\frac{3a^3(a+2bx)\sqrt{ax+bx^2}}{128b^2} + \frac{a(a+2bx)\left(ax+bx^2\right)^{3/2}}{16b} + \frac{1}{5}\left(ax+bx^2\right)^{5/2} + \frac{\left(3a^5\right)\int \frac{1}{\sqrt{ax+bx^2}}\,dx}{256b^2}$$

$$= -\frac{3a^3(a+2bx)\sqrt{ax+bx^2}}{128b^2} + \frac{a(a+2bx)\left(ax+bx^2\right)^{3/2}}{16b} + \frac{1}{5}\left(ax+bx^2\right)^{5/2} + \frac{\left(3a^5\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\right.}{128b^2}$$

$$= -\frac{3a^3(a+2bx)\sqrt{ax+bx^2}}{128b^2} + \frac{a(a+2bx)\left(ax+bx^2\right)^{3/2}}{16b} + \frac{1}{5}\left(ax+bx^2\right)^{5/2} + \frac{3a^5\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{128b^{5/2}}$$

Mathematica [A] time = 0.14581, size = 109, normalized size = 1.02

$$\frac{\sqrt{x(a+bx)}\left(\sqrt{b}\left(248a^2b^2x^2+10a^3bx-15a^4+336ab^3x^3+128b^4x^4\right)+\frac{15a^{9/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{x}\sqrt{\frac{bx}{a}+1}}\right)}{640b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^2)^(5/2)/x,x]
```

```
[Out] (Sqrt[x*(a + b*x)]*(Sqrt[b]*(-15*a^4 + 10*a^3*b*x + 248*a^2*b^2*x^2 + 336*a
*b^3*x^3 + 128*b^4*x^4) + (15*a^(9/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(
Sqrt[x]*Sqrt[1 + (b*x)/a])))/(640*b^(5/2))
```

Maple [A] time = 0.047, size = 120, normalized size = 1.1

$$\frac{1}{5}\left(bx^2+ax\right)^{\frac{5}{2}} + \frac{ax}{8}\left(bx^2+ax\right)^{\frac{3}{2}} + \frac{a^2}{16\,b}\left(bx^2+ax\right)^{\frac{3}{2}} - \frac{3\,xa^3}{64\,b}\sqrt{bx^2+ax} - \frac{3\,a^4}{128\,b^2}\sqrt{bx^2+ax} + \frac{3\,a^5}{256}\ln\left(\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a*x)^(5/2)/x,x)
```

```
[Out] 1/5*(b*x^2+a*x)^(5/2)+1/8*a*(b*x^2+a*x)^(3/2)*x+1/16/b*(b*x^2+a*x)^(3/2)*a^
2-3/64/b*a^3*(b*x^2+a*x)^(1/2)*x-3/128/b^2*a^4*(b*x^2+a*x)^(1/2)+3/256/b^(5
/2)*a^5*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.0351, size = 458, normalized size = 4.28

$$\left[\frac{15\,a^5\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\left(128\,b^5x^4+336\,ab^4x^3+248\,a^2b^3x^2+10\,a^3b^2x-15\,a^4b\right)\sqrt{bx^2+ax}}{1280\,b^3},-\frac{15\,a^5}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/1280*(15*a^5*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(1
28*b^5*x^4 + 336*a*b^4*x^3 + 248*a^2*b^3*x^2 + 10*a^3*b^2*x - 15*a^4*b)*sqr
t(b*x^2 + a*x))/b^3, -1/640*(15*a^5*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(
-b)/(b*x)) - (128*b^5*x^4 + 336*a*b^4*x^3 + 248*a^2*b^3*x^2 + 10*a^3*b^2*x
- 15*a^4*b)*sqrt(b*x^2 + a*x))/b^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x\left(a+bx\right)\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a*x)**(5/2)/x,x)
```

```
[Out] Integral((x*(a + b*x))**(5/2)/x, x)
```

Giac [A] time = 1.30692, size = 130, normalized size = 1.21

$$-\frac{3\,a^5\log\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{256\,b^{\frac{5}{2}}}-\frac{1}{640}\sqrt{bx^2+ax}\left(\frac{15\,a^4}{b^2}-2\left(\frac{5\,a^3}{b}+4\left(31\,a^2+2\left(8\,b^2x+21\,ab\right)x\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x,x, algorithm="giac")
```

```
[Out] -3/256*a^5*log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/b^(5/2)
 - 1/640*sqrt(b*x^2 + a*x)*(15*a^4/b^2 - 2*(5*a^3/b + 4*(31*a^2 + 2*(8*b^2*
x + 21*a*b)*x)*x)*x)
```

3.28 $\int \frac{\left(ax+bx^2\right)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=101

$$-\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{64b^{3/2}}+\frac{5a^2(a+2bx)\sqrt{ax+bx^2}}{64b}+\frac{5}{24}a\left(ax+bx^2\right)^{3/2}+\frac{\left(ax+bx^2\right)^{5/2}}{4x}$$

```
[Out] (5*a^2*(a + 2*b*x)*Sqrt[a*x + b*x^2])/(64*b) + (5*a*(a*x + b*x^2)^(3/2))/24
 + (a*x + b*x^2)^(5/2)/(4*x) - (5*a^4*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]
])/(64*b^(3/2))
```

Rubi [A] time = 0.041171, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {664, 612, 620, 206}

$$-\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{64b^{3/2}}+\frac{5a^2(a+2bx)\sqrt{ax+bx^2}}{64b}+\frac{5}{24}a\left(ax+bx^2\right)^{3/2}+\frac{\left(ax+bx^2\right)^{5/2}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^2)^(5/2)/x^2,x]
```

```
[Out] (5*a^2*(a + 2*b*x)*Sqrt[a*x + b*x^2])/(64*b) + (5*a*(a*x + b*x^2)^(3/2))/24
 + (a*x + b*x^2)^(5/2)/(4*x) - (5*a^4*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]
])/(64*b^(3/2))
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(ax+bx^2)^{5/2}}{x^2}\,dx = \frac{(ax+bx^2)^{5/2}}{4x} + \frac{1}{8}(5a)\int \frac{(ax+bx^2)^{3/2}}{x}\,dx$$
$$= \frac{5}{24}a\left(ax+bx^2\right)^{3/2} + \frac{(ax+bx^2)^{5/2}}{4x} + \frac{1}{16}\left(5a^2\right)\int \sqrt{ax+bx^2}\,dx$$
$$= \frac{5a^2(a+2bx)\sqrt{ax+bx^2}}{64b} + \frac{5}{24}a\left(ax+bx^2\right)^{3/2} + \frac{(ax+bx^2)^{5/2}}{4x} - \frac{\left(5a^4\right)\int \frac{1}{\sqrt{ax+bx^2}}\,dx}{128b}$$
$$= \frac{5a^2(a+2bx)\sqrt{ax+bx^2}}{64b} + \frac{5}{24}a\left(ax+bx^2\right)^{3/2} + \frac{(ax+bx^2)^{5/2}}{4x} - \frac{\left(5a^4\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{ax+bx^2}}\right)}{64b}$$
$$= \frac{5a^2(a+2bx)\sqrt{ax+bx^2}}{64b} + \frac{5}{24}a\left(ax+bx^2\right)^{3/2} + \frac{(ax+bx^2)^{5/2}}{4x} - \frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{64b^{3/2}}$$

Mathematica [A] time = 0.131444, size = 98, normalized size = 0.97

$$\frac{\sqrt{x(a+bx)}\left(\sqrt{b}\left(118a^2bx+15a^3+136ab^2x^2+48b^3x^3\right)-\frac{15a^{7/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{x}\sqrt{\frac{bx}{a}+1}}\right)}{192b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^2)^(5/2)/x^2,x]
```

```
[Out] (Sqrt[x*(a + b*x)]*(Sqrt[b]*(15*a^3 + 118*a^2*b*x + 136*a*b^2*x^2 + 48*b^3*
x^3) - (15*a^(7/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[x]*Sqrt[1 + (b
*x)/a])))/(192*b^(3/2))
```

Maple [A] time = 0.046, size = 135, normalized size = 1.3

$$\frac{2}{3\,ax^2}\left(bx^2+ax\right)^{\frac{7}{2}} - \frac{2\,b}{3\,a}\left(bx^2+ax\right)^{\frac{5}{2}} - \frac{5\,bx}{12}\left(bx^2+ax\right)^{\frac{3}{2}} - \frac{5\,a}{24}\left(bx^2+ax\right)^{\frac{3}{2}} + \frac{5\,a^2x}{32}\sqrt{bx^2+ax} + \frac{5\,a^3}{64\,b}\sqrt{bx^2+ax} - \frac{5\,a^4}{128}\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a*x)^(5/2)/x^2,x)
```

```
[Out] 2/3/a/x^2*(b*x^2+a*x)^(7/2)-2/3*b/a*(b*x^2+a*x)^(5/2)-5/12*b*(b*x^2+a*x)^(3
/2)*x-5/24*a*(b*x^2+a*x)^(3/2)+5/32*a^2*(b*x^2+a*x)^(1/2)*x+5/64/b*a^3*(b*x
^2+a*x)^(1/2)-5/128/b^(3/2)*a^4*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.11876, size = 406, normalized size = 4.02

$$\left[\frac{15\,a^4\sqrt{b}\log\left(2\,bx+a-2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\left(48\,b^4x^3+136\,ab^3x^2+118\,a^2b^2x+15\,a^3b\right)\sqrt{bx^2+ax}}{384\,b^2},\ \frac{15\,a^4\sqrt{-b}\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/384*(15*a^4*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(48
*b^4*x^3 + 136*a*b^3*x^2 + 118*a^2*b^2*x + 15*a^3*b)*sqrt(b*x^2 + a*x))/b^2
, 1/192*(15*a^4*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x)) + (48*b^4
*x^3 + 136*a*b^3*x^2 + 118*a^2*b^2*x + 15*a^3*b)*sqrt(b*x^2 + a*x))/b^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x\,(a+bx))^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a*x)**(5/2)/x**2,x)
```

```
[Out] Integral((x*(a + b*x))**(5/2)/x**2, x)
```

Giac [A] time = 1.18341, size = 113, normalized size = 1.12

$$\frac{5\,a^4\log\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{128\,b^{\frac{3}{2}}}+\frac{1}{192}\,\sqrt{bx^2+ax}\left(\frac{15\,a^3}{b}+2\left(59\,a^2+4\left(6\,b^2x+17\,ab\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 5/128*a^4*log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/b^(3/2)
+ 1/192*sqrt(b*x^2 + a*x)*(15*a^3/b + 2*(59*a^2 + 4*(6*b^2*x + 17*a*b)*x)*x
)
```

3.29 $\int \frac{\left(ax+bx^2\right)^{5/2}}{x^3}\,dx$

Optimal. Leaf size=94

$$\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{8\sqrt{b}}-\frac{5}{8}a(a+2bx)\sqrt{ax+bx^2}+\frac{2\left(ax+bx^2\right)^{5/2}}{x^2}-\frac{5}{3}b\left(ax+bx^2\right)^{3/2}$$

```
[Out] (-5*a*(a + 2*b*x)*Sqrt[a*x + b*x^2])/8 - (5*b*(a*x + b*x^2)^(3/2))/3 + (2*(
a*x + b*x^2)^(5/2))/x^2 + (5*a^3*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(8
*Sqrt[b])
```

Rubi [A] time = 0.0391494, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {662, 664, 612, 620, 206}

$$\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{8\sqrt{b}}-\frac{5}{8}a(a+2bx)\sqrt{ax+bx^2}+\frac{2\left(ax+bx^2\right)^{5/2}}{x^2}-\frac{5}{3}b\left(ax+bx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^2)^(5/2)/x^3,x]
```

```
[Out] (-5*a*(a + 2*b*x)*Sqrt[a*x + b*x^2])/8 - (5*b*(a*x + b*x^2)^(3/2))/3 + (2*(
a*x + b*x^2)^(5/2))/x^2 + (5*a^3*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(8
*Sqrt[b])
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ax+bx^2)^{5/2}}{x^3}\,dx &= \frac{2(ax+bx^2)^{5/2}}{x^2}-(5b)\int \frac{(ax+bx^2)^{3/2}}{x}\,dx\\ &= -\frac{5}{3}b(ax+bx^2)^{3/2}+\frac{2(ax+bx^2)^{5/2}}{x^2}-\frac{1}{2}(5ab)\int \sqrt{ax+bx^2}\,dx\\ &= -\frac{5}{8}a(a+2bx)\sqrt{ax+bx^2}-\frac{5}{3}b(ax+bx^2)^{3/2}+\frac{2(ax+bx^2)^{5/2}}{x^2}+\frac{1}{16}(5a^3)\int \frac{1}{\sqrt{ax+bx^2}}\,dx\\ &= -\frac{5}{8}a(a+2bx)\sqrt{ax+bx^2}-\frac{5}{3}b(ax+bx^2)^{3/2}+\frac{2(ax+bx^2)^{5/2}}{x^2}+\frac{1}{8}(5a^3)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x\right.\\ &= -\frac{5}{8}a(a+2bx)\sqrt{ax+bx^2}-\frac{5}{3}b(ax+bx^2)^{3/2}+\frac{2(ax+bx^2)^{5/2}}{x^2}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{8\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.124836, size = 80, normalized size = 0.85

$$\frac{1}{24}\sqrt{x(a+bx)}\left(\frac{15a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{x}\sqrt{\frac{bx}{a}+1}}+33a^2+26abx+8b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^2)^(5/2)/x^3,x]
```

```
[Out] (Sqrt[x*(a + b*x)]*(33*a^2 + 26*a*b*x + 8*b^2*x^2 + (15*a^(5/2)*ArcSinh[(Sq
rt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[b]*Sqrt[x]*Sqrt[1 + (b*x)/a])))/24
```

Maple [B] time = 0.05, size = 158, normalized size = 1.7

$$2\,\frac{(bx^2+ax)^{7/2}}{ax^3}-\frac{16\,b}{3\,a^2x^2}(bx^2+ax)^{\frac{7}{2}}+\frac{16\,b^2}{3\,a^2}(bx^2+ax)^{\frac{5}{2}}+\frac{10\,b^2x}{3\,a}(bx^2+ax)^{\frac{3}{2}}+\frac{5\,b}{3}(bx^2+ax)^{\frac{3}{2}}-\frac{5\,abx}{4}\sqrt{bx^2+ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a*x)^(5/2)/x^3,x)
```

```
[Out] 2/a/x^3*(b*x^2+a*x)^(7/2)-16/3*b/a^2/x^2*(b*x^2+a*x)^(7/2)+16/3*b^2/a^2*(b*
x^2+a*x)^(5/2)+10/3*b^2/a*(b*x^2+a*x)^(3/2)*x+5/3*b*(b*x^2+a*x)^(3/2)-5/4*b
*a*(b*x^2+a*x)^(1/2)*x-5/8*a^2*(b*x^2+a*x)^(1/2)+5/16/b^(1/2)*a^3*ln((1/2*a
+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.92348, size = 346, normalized size = 3.68

$$\left[\frac{15\,a^3\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\left(8\,b^3x^2+26\,ab^2x+33\,a^2b\right)\sqrt{bx^2+ax}}{48\,b}, -\frac{15\,a^3\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx}\right)-(\right.}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/48*(15*a^3*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(8*b
^3*x^2 + 26*a*b^2*x + 33*a^2*b)*sqrt(b*x^2 + a*x))/b, -1/24*(15*a^3*sqrt(-b
)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x)) - (8*b^3*x^2 + 26*a*b^2*x + 33*a
^2*b)*sqrt(b*x^2 + a*x))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x\,(a+bx))^{\frac{5}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a*x)**(5/2)/x**3,x)
```

```
[Out] Integral((x*(a + b*x))**(5/2)/x**3, x)
```

Giac [A] time = 1.24978, size = 97, normalized size = 1.03

$$-\frac{5\,a^3\log\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{16\,\sqrt{b}}+\frac{1}{24}\,\sqrt{bx^2+ax}\left(33\,a^2+2\left(4\,b^2x+13\,ab\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] -5/16*a^3*log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/sqrt(b)
+ 1/24*sqrt(b*x^2 + a*x)*(33*a^2 + 2*(4*b^2*x + 13*a*b)*x)
```

3.30 $\int \frac{(ax+bx^2)^{5/2}}{x^4}\,dx$

Optimal. Leaf size=92

$$\frac{15}{4}a^2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)-\frac{2\left(ax+bx^2\right)^{5/2}}{x^3}+\frac{5b\left(ax+bx^2\right)^{3/2}}{2x}+\frac{15}{4}ab\sqrt{ax+bx^2}$$

```
[Out] (15*a*b*Sqrt[a*x + b*x^2])/4 + (5*b*(a*x + b*x^2)^(3/2))/(2*x) - (2*(a*x +
b*x^2)^(5/2))/x^3 + (15*a^2*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])
/4
```

Rubi [A] time = 0.04064, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {662, 664, 620, 206}

$$\frac{15}{4}a^2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)-\frac{2\left(ax+bx^2\right)^{5/2}}{x^3}+\frac{5b\left(ax+bx^2\right)^{3/2}}{2x}+\frac{15}{4}ab\sqrt{ax+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^2)^(5/2)/x^4,x]
```

```
[Out] (15*a*b*Sqrt[a*x + b*x^2])/4 + (5*b*(a*x + b*x^2)^(3/2))/(2*x) - (2*(a*x +
b*x^2)^(5/2))/x^3 + (15*a^2*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])
/4
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(ax+bx^2)^{5/2}}{x^4}\,dx &= -\frac{2(ax+bx^2)^{5/2}}{x^3}+(5b)\int \frac{(ax+bx^2)^{3/2}}{x^2}\,dx\\
&= \frac{5b(ax+bx^2)^{3/2}}{2x}-\frac{2(ax+bx^2)^{5/2}}{x^3}+\frac{1}{4}(15ab)\int \frac{\sqrt{ax+bx^2}}{x}\,dx\\
&= \frac{15}{4}ab\sqrt{ax+bx^2}+\frac{5b(ax+bx^2)^{3/2}}{2x}-\frac{2(ax+bx^2)^{5/2}}{x^3}+\frac{1}{8}(15a^2b)\int \frac{1}{\sqrt{ax+bx^2}}\,dx\\
&= \frac{15}{4}ab\sqrt{ax+bx^2}+\frac{5b(ax+bx^2)^{3/2}}{2x}-\frac{2(ax+bx^2)^{5/2}}{x^3}+\frac{1}{4}(15a^2b)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{ax+b}}\right.\\
&= \frac{15}{4}ab\sqrt{ax+bx^2}+\frac{5b(ax+bx^2)^{3/2}}{2x}-\frac{2(ax+bx^2)^{5/2}}{x^3}+\frac{15}{4}a^2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.0124159, size = 48, normalized size = 0.52

$$
-\frac{2a^2\sqrt{x(a+bx)}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx}{a}\right)}{x\sqrt{\frac{bx}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^2)^(5/2)/x^4,x]
```

```
[Out] (-2*a^2*Sqrt[x*(a + b*x)]*Hypergeometric2F1[-5/2, -1/2, 1/2, -((b*x)/a)])/(
x*Sqrt[1 + (b*x)/a])
```

Maple [B] time = 0.048, size = 185, normalized size = 2.

$$
-2\frac{(bx^2+ax)^{7/2}}{ax^4}+12\frac{b(bx^2+ax)^{7/2}}{a^2x^3}-32\frac{b^2(bx^2+ax)^{7/2}}{x^2a^3}+32\frac{b^3(bx^2+ax)^{5/2}}{a^3}+20\frac{b^3(bx^2+ax)^{3/2}x}{a^2}+10\frac{b^2(bx^2+a}{a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a*x)^(5/2)/x^4,x)
```

```
[Out] -2/a/x^4*(b*x^2+a*x)^(7/2)+12*b/a^2/x^3*(b*x^2+a*x)^(7/2)-32*b^2/a^3/x^2*(b
*x^2+a*x)^(7/2)+32*b^3/a^3*(b*x^2+a*x)^(5/2)+20*b^3/a^2*(b*x^2+a*x)^(3/2)*x
+10*b^2/a*(b*x^2+a*x)^(3/2)-15/2*b^2*(b*x^2+a*x)^(1/2)*x-15/4*a*b*(b*x^2+a*
x)^(1/2)+15/8*b^(1/2)*a^2*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.08301, size = 332, normalized size = 3.61

$$\left[\frac{15\,a^2\sqrt{bx}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\left(2\,b^2x^2+9\,abx-8\,a^2\right)\sqrt{bx^2+ax}}{8\,x},-\frac{15\,a^2\sqrt{-bx}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx}\right)-}{4.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/8*(15*a^2*sqrt(b)*x*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(2*
b^2*x^2 + 9*a*b*x - 8*a^2)*sqrt(b*x^2 + a*x))/x, -1/4*(15*a^2*sqrt(-b)*x*ar
ctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x)) - (2*b^2*x^2 + 9*a*b*x - 8*a^2)*sqrt
(b*x^2 + a*x))/x]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(x\,(a+bx))^{\frac{5}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a*x)**(5/2)/x**4,x)
```

```
[Out] Integral((x*(a + b*x))**(5/2)/x**4, x)
```

Giac [A] time = 1.26531, size = 120, normalized size = 1.3

$$-\frac{15}{8}\,a^2\sqrt{b}\log\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)+\frac{2\,a^3}{\sqrt{b}x-\sqrt{bx^2+ax}}+\frac{1}{4}\left(2\,b^2x+9\,ab\right)\sqrt{bx^2+ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] -15/8*a^2*sqrt(b)*log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))
+ 2*a^3/(sqrt(b)*x - sqrt(b*x^2 + a*x)) + 1/4*(2*b^2*x + 9*a*b)*sqrt(b*x^2
+ a*x)
```

3.31 $\int \frac{(ax+bx^2)^{5/2}}{x^5} dx$

Optimal. Leaf size=89

$$5b^2\sqrt{ax+bx^2}+5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)-\frac{2\left(ax+bx^2\right)^{5/2}}{3x^4}-\frac{10b\left(ax+bx^2\right)^{3/2}}{3x^2}$$

```
[Out] 5*b^2*Sqrt[a*x + b*x^2] - (10*b*(a*x + b*x^2)^(3/2))/(3*x^2) - (2*(a*x + b*
x^2)^(5/2))/(3*x^4) + 5*a*b^(3/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]]
```

Rubi [A] time = 0.0405703, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {662, 664, 620, 206}

$$5b^2\sqrt{ax+bx^2}+5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)-\frac{2\left(ax+bx^2\right)^{5/2}}{3x^4}-\frac{10b\left(ax+bx^2\right)^{3/2}}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^2)^(5/2)/x^5,x]
```

```
[Out] 5*b^2*Sqrt[a*x + b*x^2] - (10*b*(a*x + b*x^2)^(3/2))/(3*x^2) - (2*(a*x + b*
x^2)^(5/2))/(3*x^4) + 5*a*b^(3/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]]
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ax+bx^2\right)^{5/2}}{x^5}\,dx &= -\frac{2\left(ax+bx^2\right)^{5/2}}{3x^4}+\frac{1}{3}(5b)\int \frac{\left(ax+bx^2\right)^{3/2}}{x^3}\,dx\\ &= -\frac{10b\left(ax+bx^2\right)^{3/2}}{3x^2}-\frac{2\left(ax+bx^2\right)^{5/2}}{3x^4}+\left(5b^2\right)\int \frac{\sqrt{ax+bx^2}}{x}\,dx\\ &= 5b^2\sqrt{ax+bx^2}-\frac{10b\left(ax+bx^2\right)^{3/2}}{3x^2}-\frac{2\left(ax+bx^2\right)^{5/2}}{3x^4}+\frac{1}{2}\left(5ab^2\right)\int \frac{1}{\sqrt{ax+bx^2}}\,dx\\ &= 5b^2\sqrt{ax+bx^2}-\frac{10b\left(ax+bx^2\right)^{3/2}}{3x^2}-\frac{2\left(ax+bx^2\right)^{5/2}}{3x^4}+\left(5ab^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{ax+b}}\right.\\ &= 5b^2\sqrt{ax+bx^2}-\frac{10b\left(ax+bx^2\right)^{3/2}}{3x^2}-\frac{2\left(ax+bx^2\right)^{5/2}}{3x^4}+5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)\end{aligned}$$

Mathematica [C] time = 0.0128478, size = 50, normalized size = 0.56

$$-\frac{2a^2\sqrt{x(a+bx)}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};-\frac{bx}{a}\right)}{3x^2\sqrt{\frac{bx}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^2)^(5/2)/x^5,x]
```

```
[Out] (-2*a^2*Sqrt[x*(a + b*x)]*Hypergeometric2F1[-5/2, -3/2, -1/2, -((b*x)/a)])/
(3*x^2*Sqrt[1 + (b*x)/a])
```

Maple [B] time = 0.05, size = 209, normalized size = 2.4

$$-\frac{2}{3\,a x^5}\left(bx^2+ax\right)^{\frac{7}{2}}-\frac{8\,b}{3\,a^2x^4}\left(bx^2+ax\right)^{\frac{7}{2}}+16\,\frac{b^2\left(bx^2+ax\right)^{7/2}}{a^3x^3}-\frac{128\,b^3}{3\,a^4x^2}\left(bx^2+ax\right)^{\frac{7}{2}}+\frac{128\,b^4}{3\,a^4}\left(bx^2+ax\right)^{\frac{5}{2}}+\frac{80\,b^4x}{3\,a^3}($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a*x)^(5/2)/x^5,x)
```

```
[Out] -2/3/a/x^5*(b*x^2+a*x)^(7/2)-8/3*b/a^2/x^4*(b*x^2+a*x)^(7/2)+16*b^2/a^3/x^3
*(b*x^2+a*x)^(7/2)-128/3*b^3/a^4/x^2*(b*x^2+a*x)^(7/2)+128/3*b^4/a^4*(b*x^2
+a*x)^(5/2)+80/3*b^4/a^3*(b*x^2+a*x)^(3/2)*x+40/3*b^3/a^2*(b*x^2+a*x)^(3/2)
-10*b^3/a*(b*x^2+a*x)^(1/2)*x-5*b^2*(b*x^2+a*x)^(1/2)+5/2*b^(3/2)*a*ln((1/2
*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.9333, size = 343, normalized size = 3.85

$$\left[\frac{15\,ab^{\frac{3}{2}}x^{2}\log\left(2\,bx+a+2\,\sqrt{bx^{2}+ax}\sqrt{b}\right)+2\left(3\,b^{2}x^{2}-14\,abx-2\,a^{2}\right)\sqrt{bx^{2}+ax}}{6\,x^{2}},-\frac{15\,a\sqrt{-b}bx^{2}\arctan\left(\frac{\sqrt{bx^{2}+ax}\sqrt{-b}}{bx}\right)-(\cdots}{3\,x^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/6*(15*a*b^(3/2)*x^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(3*
b^2*x^2 - 14*a*b*x - 2*a^2)*sqrt(b*x^2 + a*x))/x^2, -1/3*(15*a*sqrt(-b)*b*x
^2*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x)) - (3*b^2*x^2 - 14*a*b*x - 2*a^2
)*sqrt(b*x^2 + a*x))/x^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(x\,(a+bx))^{\frac{5}{2}}}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a*x)**(5/2)/x**5,x)
```

```
[Out] Integral((x*(a + b*x))**(5/2)/x**5, x)
```

Giac [A] time = 1.21262, size = 180, normalized size = 2.02

$$-\frac{5}{2}\,ab^{\frac{3}{2}}\log\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^{2}+ax}\right)\sqrt{b}-a\right|\right)+\sqrt{bx^{2}+ax}b^{2}+\frac{2\left(9\left(\sqrt{b}x-\sqrt{bx^{2}+ax}\right)^{2}a^{2}b+3\left(\sqrt{b}x-\sqrt{bx^{2}+ax}\right)a^{3}\sqrt{b}\cdots}{3\left(\sqrt{b}x-\sqrt{bx^{2}+ax}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] -5/2*a*b^(3/2)*log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a)) + s
qrt(b*x^2 + a*x)*b^2 + 2/3*(9*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^2*b + 3*(
sqrt(b)*x - sqrt(b*x^2 + a*x))*a^3*sqrt(b) + a^4)/(sqrt(b)*x - sqrt(b*x^2 +
 a*x))^3
```

3.32 $\int \frac{(ax+bx^2)^{5/2}}{x^6} \, dx$

Optimal. Leaf size=91

$$-\frac{2b^2\sqrt{ax+bx^2}}{x}+2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)-\frac{2b\left(ax+bx^2\right)^{3/2}}{3x^3}-\frac{2\left(ax+bx^2\right)^{5/2}}{5x^5}$$

```
[Out] (-2*b^2*Sqrt[a*x + b*x^2])/x - (2*b*(a*x + b*x^2)^(3/2))/(3*x^3) - (2*(a*x
+ b*x^2)^(5/2))/(5*x^5) + 2*b^(5/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]]
```

Rubi [A] time = 0.0406987, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {662, 620, 206}

$$-\frac{2b^2\sqrt{ax+bx^2}}{x}+2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)-\frac{2b\left(ax+bx^2\right)^{3/2}}{3x^3}-\frac{2\left(ax+bx^2\right)^{5/2}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^2)^(5/2)/x^6,x]
```

```
[Out] (-2*b^2*Sqrt[a*x + b*x^2])/x - (2*b*(a*x + b*x^2)^(3/2))/(3*x^3) - (2*(a*x
+ b*x^2)^(5/2))/(5*x^5) + 2*b^(5/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]]
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ax+bx^2)^{5/2}}{x^6}\,dx &= -\frac{2(ax+bx^2)^{5/2}}{5x^5} + b\int \frac{(ax+bx^2)^{3/2}}{x^4}\,dx\\ &= -\frac{2b(ax+bx^2)^{3/2}}{3x^3} - \frac{2(ax+bx^2)^{5/2}}{5x^5} + b^2\int \frac{\sqrt{ax+bx^2}}{x^2}\,dx\\ &= -\frac{2b^2\sqrt{ax+bx^2}}{x} - \frac{2b(ax+bx^2)^{3/2}}{3x^3} - \frac{2(ax+bx^2)^{5/2}}{5x^5} + b^3\int \frac{1}{\sqrt{ax+bx^2}}\,dx\\ &= -\frac{2b^2\sqrt{ax+bx^2}}{x} - \frac{2b(ax+bx^2)^{3/2}}{3x^3} - \frac{2(ax+bx^2)^{5/2}}{5x^5} + (2b^3)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{ax+bx^2}}\right)\\ &= -\frac{2b^2\sqrt{ax+bx^2}}{x} - \frac{2b(ax+bx^2)^{3/2}}{3x^3} - \frac{2(ax+bx^2)^{5/2}}{5x^5} + 2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)\end{aligned}$$

Mathematica [C] time = 0.013242, size = 50, normalized size = 0.55

$$-\frac{2a^2\sqrt{x(a+bx)}\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};-\frac{bx}{a}\right)}{5x^3\sqrt{\frac{bx}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^2)^(5/2)/x^6,x]
```

```
[Out] (-2*a^2*Sqrt[x*(a + b*x)]*Hypergeometric2F1[-5/2, -5/2, -3/2, -((b*x)/a)])/
(5*x^3*Sqrt[1 + (b*x)/a])
```

Maple [B] time = 0.048, size = 232, normalized size = 2.6

$$-\frac{2}{5\,ax^6}\left(bx^2+ax\right)^{\frac{7}{2}}-\frac{4\,b}{15\,a^2x^5}\left(bx^2+ax\right)^{\frac{7}{2}}-\frac{16\,b^2}{15\,a^3x^4}\left(bx^2+ax\right)^{\frac{7}{2}}+\frac{32\,b^3}{5\,a^4x^3}\left(bx^2+ax\right)^{\frac{7}{2}}-\frac{256\,b^4}{15\,a^5x^2}\left(bx^2+ax\right)^{\frac{7}{2}}+\frac{256\,b^5}{15\,a^5}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a*x)^(5/2)/x^6,x)
```

```
[Out] -2/5/a/x^6*(b*x^2+a*x)^(7/2)-4/15*b/a^2/x^5*(b*x^2+a*x)^(7/2)-16/15*b^2/a^3
/x^4*(b*x^2+a*x)^(7/2)+32/5*b^3/a^4/x^3*(b*x^2+a*x)^(7/2)-256/15*b^4/a^5/x^
2*(b*x^2+a*x)^(7/2)+256/15*b^5/a^5*(b*x^2+a*x)^(5/2)+32/3*b^5/a^4*(b*x^2+a*
x)^(3/2)*x+16/3*b^4/a^3*(b*x^2+a*x)^(3/2)-4*b^4/a^2*(b*x^2+a*x)^(1/2)*x-2*b
^3/a*(b*x^2+a*x)^(1/2)+b^(5/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.03784, size = 346, normalized size = 3.8

$$\left[\frac{15\,b^{\frac{5}{2}}x^3\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\left(23\,b^2x^2+11\,abx+3\,a^2\right)\sqrt{bx^2+ax}}{15\,x^3},-\frac{2\left(15\,\sqrt{-b}b^2x^3\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx}\right)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/15*(15*b^(5/2)*x^3*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2*(23*
b^2*x^2 + 11*a*b*x + 3*a^2)*sqrt(b*x^2 + a*x))/x^3, -2/15*(15*sqrt(-b)*b^2*
x^3*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x)) + (23*b^2*x^2 + 11*a*b*x + 3*a
^2)*sqrt(b*x^2 + a*x))/x^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x\left(a+bx\right)\right)^{\frac{5}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a*x)**(5/2)/x**6,x)
```

```
[Out] Integral((x*(a + b*x))**(5/2)/x**6, x)
```

Giac [B] time = 1.29556, size = 236, normalized size = 2.59

$$-b^{\frac{5}{2}}\log\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)+\frac{2\left(45\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4ab^2+45\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3a^2b^{\frac{3}{2}}+35\left(\sqrt{b}x-\right.\right.}{15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] -b^(5/2)*log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a)) + 2/15*(4
5*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a*b^2 + 45*(sqrt(b)*x - sqrt(b*x^2 + a*
x))^3*a^2*b^(3/2) + 35*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^3*b + 15*(sqrt(b
)*x - sqrt(b*x^2 + a*x))*a^4*sqrt(b) + 3*a^5)/(sqrt(b)*x - sqrt(b*x^2 + a*x
))^5
```

3.33 $\int \frac{(ax+bx^2)^{5/2}}{x^7} dx$

Optimal. Leaf size=23

$$-\frac{2(ax+bx^2)^{7/2}}{7ax^7}$$

```
[Out] (-2*(a*x + b*x^2)^(7/2))/(7*a*x^7)
```

Rubi [A] time = 0.0074363, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {650}

$$-\frac{2(ax+bx^2)^{7/2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^2)^(5/2)/x^7,x]
```

```
[Out] (-2*(a*x + b*x^2)^(7/2))/(7*a*x^7)
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{(ax+bx^2)^{5/2}}{x^7} dx = -\frac{2(ax+bx^2)^{7/2}}{7ax^7}$$

Mathematica [A] time = 0.0154628, size = 21, normalized size = 0.91

$$-\frac{2(x(a+bx))^{7/2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^2)^(5/2)/x^7,x]
```

```
[Out] (-2*(x*(a + b*x))^(7/2))/(7*a*x^7)
```

Maple [A] time = 0.044, size = 25, normalized size = 1.1

$$-\frac{2bx+2a}{7x^6a}(bx^2+ax)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a*x)^(5/2)/x^7,x)
```

```
[Out] -2/7/x^6*(b*x+a)/a*(b*x^2+a*x)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.9822, size = 100, normalized size = 4.35

$$-\frac{2\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)\sqrt{bx^2+ax}}{7\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^7,x, algorithm="fricas")
```

```
[Out] -2/7*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b*x^2 + a*x)/(a*x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x(a+bx))^{\frac{5}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a*x)**(5/2)/x**7,x)
```

```
[Out] Integral((x*(a + b*x))**(5/2)/x**7, x)
```

Giac [B] time = 1.22441, size = 259, normalized size = 11.26

$$\frac{2\left(7\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^6b^3+21\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^5ab^{\frac{5}{2}}+35\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4a^2b^2+35\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3a^3b}{7\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^7,x, algorithm="giac")
```

```
[Out] 2/7*(7*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*b^3 + 21*(sqrt(b)*x - sqrt(b*x^2 +
 a*x))^5*a*b^(5/2) + 35*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^2*b^2 + 35*(sqr
t(b)*x - sqrt(b*x^2 + a*x))^3*a^3*b^(3/2) + 21*(sqrt(b)*x - sqrt(b*x^2 + a*
x))^2*a^4*b + 7*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^5*sqrt(b) + a^6)/(sqrt(b)
*x - sqrt(b*x^2 + a*x))^7
```

3.34 $\int \frac{\left(ax+bx^2\right)^{5/2}}{x^8}\,dx$

Optimal. Leaf size=48

$$\frac{4b\left(ax+bx^2\right)^{7/2}}{63a^2x^7}-\frac{2\left(ax+bx^2\right)^{7/2}}{9ax^8}$$

```
[Out] (-2*(a*x + b*x^2)^(7/2))/(9*a*x^8) + (4*b*(a*x + b*x^2)^(7/2))/(63*a^2*x^7)
```

Rubi [A] time = 0.0173728, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {658, 650}

$$\frac{4b\left(ax+bx^2\right)^{7/2}}{63a^2x^7}-\frac{2\left(ax+bx^2\right)^{7/2}}{9ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^2)^(5/2)/x^8,x]
```

```
[Out] (-2*(a*x + b*x^2)^(7/2))/(9*a*x^8) + (4*b*(a*x + b*x^2)^(7/2))/(63*a^2*x^7)
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ax+bx^2\right)^{5/2}}{x^8}\,dx &= -\frac{2\left(ax+bx^2\right)^{7/2}}{9ax^8}-\frac{(2b)\int \frac{\left(ax+bx^2\right)^{5/2}}{x^7}\,dx}{9a}\\ &= -\frac{2\left(ax+bx^2\right)^{7/2}}{9ax^8}+\frac{4b\left(ax+bx^2\right)^{7/2}}{63a^2x^7}\end{aligned}$$

Mathematica [A] time = 0.0107597, size = 36, normalized size = 0.75

$$\frac{2(a+bx)^3\sqrt{x(a+bx)}(2bx-7a)}{63a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^2)^(5/2)/x^8,x]
```

```
[Out] (2*(a + b*x)^3*Sqrt[x*(a + b*x)]*(-7*a + 2*b*x))/(63*a^2*x^5)
```

Maple [A] time = 0.046, size = 33, normalized size = 0.7

$$-\frac{(2\,bx+2\,a)\,(-2\,bx+7\,a)}{63\,a^2x^7}\left(bx^2+ax\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a*x)^(5/2)/x^8,x)
```

```
[Out] -2/63*(b*x+a)*(-2*b*x+7*a)*(b*x^2+a*x)^(5/2)/a^2/x^7
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.99, size = 130, normalized size = 2.71

$$\frac{2\left(2\,b^4x^4-ab^3x^3-15\,a^2b^2x^2-19\,a^3bx-7\,a^4\right)\sqrt{bx^2+ax}}{63\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^8,x, algorithm="fricas")
```

```
[Out] 2/63*(2*b^4*x^4 - a*b^3*x^3 - 15*a^2*b^2*x^2 - 19*a^3*b*x - 7*a^4)*sqrt(b*x
^2 + a*x)/(a^2*x^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(x\,(a+bx))^{\frac{5}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a*x)**(5/2)/x**8,x)
```

```
[Out] Integral((x*(a + b*x))**(5/2)/x**8, x)
```

Giac [B] time = 1.21591, size = 301, normalized size = 6.27

$$\frac{2\left(63\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^7b^{\frac{7}{2}}+273\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^6ab^3+567\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^5a^2b^{\frac{5}{2}}+693\left(\sqrt{b}x-\sqrt{bx^2+ax}\right.\right.}{63\left(\sqrt{b}x-\sqrt{bx^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^8,x, algorithm="giac")
```

```
[Out] 2/63*(63*(sqrt(b)*x - sqrt(b*x^2 + a*x))^7*b^(7/2) + 273*(sqrt(b)*x - sqrt(
b*x^2 + a*x))^6*a*b^3 + 567*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a^2*b^(5/2) +
 693*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^3*b^2 + 525*(sqrt(b)*x - sqrt(b*x^
2 + a*x))^3*a^4*b^(3/2) + 243*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^5*b + 63*
(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^6*sqrt(b) + 7*a^7)/(sqrt(b)*x - sqrt(b*x^
2 + a*x))^9
```

3.35 $\int \frac{(ax+bx^2)^{5/2}}{x^9}\,dx$

Optimal. Leaf size=74

$$-\frac{16b^2\left(ax+bx^2\right)^{7/2}}{693a^3x^7}+\frac{8b\left(ax+bx^2\right)^{7/2}}{99a^2x^8}-\frac{2\left(ax+bx^2\right)^{7/2}}{11ax^9}$$

```
[Out] (-2*(a*x + b*x^2)^(7/2))/(11*a*x^9) + (8*b*(a*x + b*x^2)^(7/2))/(99*a^2*x^8
) - (16*b^2*(a*x + b*x^2)^(7/2))/(693*a^3*x^7)
```

Rubi [A] time = 0.0281318, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {658, 650}

$$-\frac{16b^2\left(ax+bx^2\right)^{7/2}}{693a^3x^7}+\frac{8b\left(ax+bx^2\right)^{7/2}}{99a^2x^8}-\frac{2\left(ax+bx^2\right)^{7/2}}{11ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^2)^(5/2)/x^9,x]
```

```
[Out] (-2*(a*x + b*x^2)^(7/2))/(11*a*x^9) + (8*b*(a*x + b*x^2)^(7/2))/(99*a^2*x^8
) - (16*b^2*(a*x + b*x^2)^(7/2))/(693*a^3*x^7)
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ax+bx^2\right)^{5/2}}{x^9}\,dx &= -\frac{2\left(ax+bx^2\right)^{7/2}}{11ax^9}-\frac{(4b)\int \frac{\left(ax+bx^2\right)^{5/2}}{x^8}\,dx}{11a}\\
&= -\frac{2\left(ax+bx^2\right)^{7/2}}{11ax^9}+\frac{8b\left(ax+bx^2\right)^{7/2}}{99a^2x^8}+\frac{\left(8b^2\right)\int \frac{\left(ax+bx^2\right)^{5/2}}{x^7}\,dx}{99a^2}\\
&= -\frac{2\left(ax+bx^2\right)^{7/2}}{11ax^9}+\frac{8b\left(ax+bx^2\right)^{7/2}}{99a^2x^8}-\frac{16b^2\left(ax+bx^2\right)^{7/2}}{693a^3x^7}
\end{aligned}$$

Mathematica [A] time = 0.0126369, size = 47, normalized size = 0.64

$$-\frac{2(a+bx)^3\sqrt{x(a+bx)}\left(63a^2-28abx+8b^2x^2\right)}{693a^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^2)^(5/2)/x^9,x]
```

```
[Out] (-2*(a + b*x)^3*Sqrt[x*(a + b*x)]*(63*a^2 - 28*a*b*x + 8*b^2*x^2))/(693*a^3
*x^6)
```

Maple [A] time = 0.045, size = 44, normalized size = 0.6

$$-\frac{(2\,bx+2\,a)\left(8\,b^2x^2-28\,abx+63\,a^2\right)}{693\,x^8a^3}\left(bx^2+ax\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a*x)^(5/2)/x^9,x)
```

```
[Out] -2/693*(b*x+a)*(8*b^2*x^2-28*a*b*x+63*a^2)*(b*x^2+a*x)^(5/2)/x^8/a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.04175, size = 161, normalized size = 2.18

$$-\frac{2\left(8\,b^5x^5-4\,ab^4x^4+3\,a^2b^3x^3+113\,a^3b^2x^2+161\,a^4bx+63\,a^5\right)\sqrt{bx^2+ax}}{693\,a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^9,x, algorithm="fricas")
```

```
[Out] -2/693*(8*b^5*x^5 - 4*a*b^4*x^4 + 3*a^2*b^3*x^3 + 113*a^3*b^2*x^2 + 161*a^4
*b*x + 63*a^5)*sqrt(b*x^2 + a*x)/(a^3*x^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(x\,(a+bx))^{\frac{5}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a*x)**(5/2)/x**9,x)
```

```
[Out] Integral((x*(a + b*x))**(5/2)/x**9, x)
```

Giac [B] time = 1.36916, size = 340, normalized size = 4.59

$$2\left(924\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^8 b^4+4851\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^7 ab^{\frac{7}{2}}+11781\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^6 a^2b^3+16863\left(\sqrt{b}x-\sqrt{bx^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^9,x, algorithm="giac")
```

```
[Out] 2/693*(924*(sqrt(b)*x - sqrt(b*x^2 + a*x))^8*b^4 + 4851*(sqrt(b)*x - sqrt(b
*x^2 + a*x))^7*a*b^(7/2) + 11781*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a^2*b^3
+ 16863*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a^3*b^(5/2) + 15345*(sqrt(b)*x -
sqrt(b*x^2 + a*x))^4*a^4*b^2 + 9009*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^5*b
^(3/2) + 3311*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^6*b + 693*(sqrt(b)*x - sq
rt(b*x^2 + a*x))*a^7*sqrt(b) + 63*a^8)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^11
```

3.36 $\int \frac{(ax+bx^2)^{5/2}}{x^{10}} dx$

Optimal. Leaf size=100

$$\frac{32b^3(ax+bx^2)^{7/2}}{3003a^4x^7}-\frac{16b^2(ax+bx^2)^{7/2}}{429a^3x^8}+\frac{12b(ax+bx^2)^{7/2}}{143a^2x^9}-\frac{2(ax+bx^2)^{7/2}}{13ax^{10}}$$

```
[Out] (-2*(a*x + b*x^2)^(7/2))/(13*a*x^10) + (12*b*(a*x + b*x^2)^(7/2))/(143*a^2*
x^9) - (16*b^2*(a*x + b*x^2)^(7/2))/(429*a^3*x^8) + (32*b^3*(a*x + b*x^2)^(
7/2))/(3003*a^4*x^7)
```

Rubi [A] time = 0.0415272, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {658, 650}

$$\frac{32b^3(ax+bx^2)^{7/2}}{3003a^4x^7}-\frac{16b^2(ax+bx^2)^{7/2}}{429a^3x^8}+\frac{12b(ax+bx^2)^{7/2}}{143a^2x^9}-\frac{2(ax+bx^2)^{7/2}}{13ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^2)^(5/2)/x^10,x]
```

```
[Out] (-2*(a*x + b*x^2)^(7/2))/(13*a*x^10) + (12*b*(a*x + b*x^2)^(7/2))/(143*a^2*
x^9) - (16*b^2*(a*x + b*x^2)^(7/2))/(429*a^3*x^8) + (32*b^3*(a*x + b*x^2)^(
7/2))/(3003*a^4*x^7)
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(ax+bx^2\right)^{5/2}}{x^{10}}\,dx &= -\frac{2\left(ax+bx^2\right)^{7/2}}{13ax^{10}} - \frac{(6b)\int \frac{\left(ax+bx^2\right)^{5/2}}{x^9}\,dx}{13a} \\
&= -\frac{2\left(ax+bx^2\right)^{7/2}}{13ax^{10}} + \frac{12b\left(ax+bx^2\right)^{7/2}}{143a^2x^9} + \frac{\left(24b^2\right)\int \frac{\left(ax+bx^2\right)^{5/2}}{x^8}\,dx}{143a^2} \\
&= -\frac{2\left(ax+bx^2\right)^{7/2}}{13ax^{10}} + \frac{12b\left(ax+bx^2\right)^{7/2}}{143a^2x^9} - \frac{16b^2\left(ax+bx^2\right)^{7/2}}{429a^3x^8} - \frac{\left(16b^3\right)\int \frac{\left(ax+bx^2\right)^{5/2}}{x^7}\,dx}{429a^3} \\
&= -\frac{2\left(ax+bx^2\right)^{7/2}}{13ax^{10}} + \frac{12b\left(ax+bx^2\right)^{7/2}}{143a^2x^9} - \frac{16b^2\left(ax+bx^2\right)^{7/2}}{429a^3x^8} + \frac{32b^3\left(ax+bx^2\right)^{7/2}}{3003a^4x^7}
\end{aligned}
$$

Mathematica [A] time = 0.0143286, size = 58, normalized size = 0.58

$$
\frac{2(a+bx)^3\sqrt{x(a+bx)}\left(126a^2bx-231a^3-56ab^2x^2+16b^3x^3\right)}{3003a^4x^7}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^2)^(5/2)/x^10,x]
```

```
[Out] (2*(a + b*x)^3*Sqrt[x*(a + b*x)]*(-231*a^3 + 126*a^2*b*x - 56*a*b^2*x^2 + 1
6*b^3*x^3))/(3003*a^4*x^7)
```

Maple [A] time = 0.043, size = 55, normalized size = 0.6

$$
-\frac{(2\,bx+2\,a)\left(-16\,b^3x^3+56\,ab^2x^2-126\,bxa^2+231\,a^3\right)}{3003\,x^9a^4}\left(bx^2+ax\right)^{\frac{5}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a*x)^(5/2)/x^10,x)
```

```
[Out] -2/3003*(b*x+a)*(-16*b^3*x^3+56*a*b^2*x^2-126*a^2*b*x+231*a^3)*(b*x^2+a*x)^
(5/2)/x^9/a^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.944, size = 185, normalized size = 1.85

$$
\frac{2\left(16\,b^6x^6-8\,ab^5x^5+6\,a^2b^4x^4-5\,a^3b^3x^3-371\,a^4b^2x^2-567\,a^5bx-231\,a^6\right)\sqrt{bx^2+ax}}{3003\,a^4x^7}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^10,x, algorithm="fricas")
```

```
[Out] 2/3003*(16*b^6*x^6 - 8*a*b^5*x^5 + 6*a^2*b^4*x^4 - 5*a^3*b^3*x^3 - 371*a^4*
b^2*x^2 - 567*a^5*b*x - 231*a^6)*sqrt(b*x^2 + a*x)/(a^4*x^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x(a+bx))^{\frac{5}{2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a*x)**(5/2)/x**10,x)
```

```
[Out] Integral((x*(a + b*x))**(5/2)/x**10, x)
```

Giac [B] time = 1.28221, size = 379, normalized size = 3.79

$$\frac{2\left(6006\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^9 b^{\frac{9}{2}}+36036\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^8 ab^4+99099\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^7 a^2 b^{\frac{7}{2}}+161733\left(\sqrt{b}x-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^10,x, algorithm="giac")
```

```
[Out] 2/3003*(6006*(sqrt(b)*x - sqrt(b*x^2 + a*x))^9*b^(9/2) + 36036*(sqrt(b)*x -
 sqrt(b*x^2 + a*x))^8*a*b^4 + 99099*(sqrt(b)*x - sqrt(b*x^2 + a*x))^7*a^2*b
^(7/2) + 161733*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a^3*b^3 + 171171*(sqrt(b)
*x - sqrt(b*x^2 + a*x))^5*a^4*b^(5/2) + 121121*(sqrt(b)*x - sqrt(b*x^2 + a*
x))^4*a^5*b^2 + 57057*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^6*b^(3/2) + 17199
*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^7*b + 3003*(sqrt(b)*x - sqrt(b*x^2 + a
*x))*a^8*sqrt(b) + 231*a^9)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^13
```

3.37 $\int \frac{(ax+bx^2)^{5/2}}{x^{11}} dx$

Optimal. Leaf size=126

$$-\frac{256b^4(ax+bx^2)^{7/2}}{45045a^5x^7}+\frac{128b^3(ax+bx^2)^{7/2}}{6435a^4x^8}-\frac{32b^2(ax+bx^2)^{7/2}}{715a^3x^9}+\frac{16b(ax+bx^2)^{7/2}}{195a^2x^{10}}-\frac{2(ax+bx^2)^{7/2}}{15ax^{11}}$$

```
[Out] (-2*(a*x + b*x^2)^(7/2))/(15*a*x^11) + (16*b*(a*x + b*x^2)^(7/2))/(195*a^2*
x^10) - (32*b^2*(a*x + b*x^2)^(7/2))/(715*a^3*x^9) + (128*b^3*(a*x + b*x^2)
^(7/2))/(6435*a^4*x^8) - (256*b^4*(a*x + b*x^2)^(7/2))/(45045*a^5*x^7)
```

Rubi [A] time = 0.0568929, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {658, 650}

$$-\frac{256b^4(ax+bx^2)^{7/2}}{45045a^5x^7}+\frac{128b^3(ax+bx^2)^{7/2}}{6435a^4x^8}-\frac{32b^2(ax+bx^2)^{7/2}}{715a^3x^9}+\frac{16b(ax+bx^2)^{7/2}}{195a^2x^{10}}-\frac{2(ax+bx^2)^{7/2}}{15ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^2)^(5/2)/x^11,x]
```


```
[Out] (-2*(a*x + b*x^2)^(7/2))/(15*a*x^11) + (16*b*(a*x + b*x^2)^(7/2))/(195*a^2*
x^10) - (32*b^2*(a*x + b*x^2)^(7/2))/(715*a^3*x^9) + (128*b^3*(a*x + b*x^2)
^(7/2))/(6435*a^4*x^8) - (256*b^4*(a*x + b*x^2)^(7/2))/(45045*a^5*x^7)
```


Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{(ax+bx^2)^{5/2}}{x^{11}}\,dx = -\frac{2(ax+bx^2)^{7/2}}{15ax^{11}} - \frac{(8b)\int \frac{(ax+bx^2)^{5/2}}{x^{10}}dx}{15a}$$
$$= -\frac{2(ax+bx^2)^{7/2}}{15ax^{11}} + \frac{16b(ax+bx^2)^{7/2}}{195a^2x^{10}} + \frac{(16b^2)\int \frac{(ax+bx^2)^{5/2}}{x^9}dx}{65a^2}$$
$$= -\frac{2(ax+bx^2)^{7/2}}{15ax^{11}} + \frac{16b(ax+bx^2)^{7/2}}{195a^2x^{10}} - \frac{32b^2(ax+bx^2)^{7/2}}{715a^3x^9} - \frac{(64b^3)\int \frac{(ax+bx^2)^{5/2}}{x^8}dx}{715a^3}$$
$$= -\frac{2(ax+bx^2)^{7/2}}{15ax^{11}} + \frac{16b(ax+bx^2)^{7/2}}{195a^2x^{10}} - \frac{32b^2(ax+bx^2)^{7/2}}{715a^3x^9} + \frac{128b^3(ax+bx^2)^{7/2}}{6435a^4x^8} + \frac{(128b^4)\int \ldots}{643\ldots}$$
$$= -\frac{2(ax+bx^2)^{7/2}}{15ax^{11}} + \frac{16b(ax+bx^2)^{7/2}}{195a^2x^{10}} - \frac{32b^2(ax+bx^2)^{7/2}}{715a^3x^9} + \frac{128b^3(ax+bx^2)^{7/2}}{6435a^4x^8} - \frac{256b^4(ax+\ldots}{45045a\ldots}$$

Mathematica [A] time = 0.0166837, size = 69, normalized size = 0.55

$$-\frac{2(a+bx)^3\sqrt{x(a+bx)}\left(1008a^2b^2x^2-1848a^3bx+3003a^4-448ab^3x^3+128b^4x^4\right)}{45045a^5x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^2)^(5/2)/x^11,x]
```

```
[Out] (-2*(a + b*x)^3*Sqrt[x*(a + b*x)]*(3003*a^4 - 1848*a^3*b*x + 1008*a^2*b^2*x
^2 - 448*a*b^3*x^3 + 128*b^4*x^4))/(45045*a^5*x^8)
```

Maple [A] time = 0.047, size = 66, normalized size = 0.5

$$-\frac{(2\,bx+2\,a)\left(128\,b^4x^4-448\,ab^3x^3+1008\,b^2x^2a^2-1848\,xa^3b+3003\,a^4\right)}{45045\,x^{10}a^5}\left(bx^2+ax\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a*x)^(5/2)/x^11,x)
```

```
[Out] -2/45045*(b*x+a)*(128*b^4*x^4-448*a*b^3*x^3+1008*a^2*b^2*x^2-1848*a^3*b*x+3
003*a^4)*(b*x^2+a*x)^(5/2)/x^10/a^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.98007, size = 220, normalized size = 1.75

$$-\frac{2\left(128\,b^{7}x^{7}-64\,ab^{6}x^{6}+48\,a^{2}b^{5}x^{5}-40\,a^{3}b^{4}x^{4}+35\,a^{4}b^{3}x^{3}+4473\,a^{5}b^{2}x^{2}+7161\,a^{6}bx+3003\,a^{7}\right)\sqrt{bx^{2}+ax}}{45045\,a^{5}x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^11,x, algorithm="fricas")
```

```
[Out] -2/45045*(128*b^7*x^7 - 64*a*b^6*x^6 + 48*a^2*b^5*x^5 - 40*a^3*b^4*x^4 + 35
*a^4*b^3*x^3 + 4473*a^5*b^2*x^2 + 7161*a^6*b*x + 3003*a^7)*sqrt(b*x^2 + a*x
)/(a^5*x^8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x(a+bx))^{\frac{5}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a*x)**(5/2)/x**11,x)
```

```
[Out] Integral((x*(a + b*x))**(5/2)/x**11, x)
```

Giac [B] time = 1.1926, size = 419, normalized size = 3.33

$$2\left(144144\left(\sqrt{b}x-\sqrt{bx^{2}+ax}\right)^{10}b^{5}+960960\left(\sqrt{b}x-\sqrt{bx^{2}+ax}\right)^{9}ab^{\frac{9}{2}}+2934360\left(\sqrt{b}x-\sqrt{bx^{2}+ax}\right)^{8}a^{2}b^{4}+5360355\left(\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^11,x, algorithm="giac")
```

```
[Out] 2/45045*(144144*(sqrt(b)*x - sqrt(b*x^2 + a*x))^10*b^5 + 960960*(sqrt(b)*x
- sqrt(b*x^2 + a*x))^9*a*b^(9/2) + 2934360*(sqrt(b)*x - sqrt(b*x^2 + a*x))^
8*a^2*b^4 + 5360355*(sqrt(b)*x - sqrt(b*x^2 + a*x))^7*a^3*b^(7/2) + 6451445
*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a^4*b^3 + 5324319*(sqrt(b)*x - sqrt(b*x^
2 + a*x))^5*a^5*b^(5/2) + 3042585*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^6*b^2
 + 1186185*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^7*b^(3/2) + 301455*(sqrt(b)*
x - sqrt(b*x^2 + a*x))^2*a^8*b + 45045*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^9*
sqrt(b) + 3003*a^10)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^15
```

3.38 $\int \frac{(ax+bx^2)^{5/2}}{x^{12}}\,dx$

Optimal. Leaf size=152

$$\frac{512b^5(ax+bx^2)^{7/2}}{153153a^6x^7}-\frac{256b^4(ax+bx^2)^{7/2}}{21879a^5x^8}+\frac{64b^3(ax+bx^2)^{7/2}}{2431a^4x^9}-\frac{32b^2(ax+bx^2)^{7/2}}{663a^3x^{10}}+\frac{4b(ax+bx^2)^{7/2}}{51a^2x^{11}}-\frac{2(ax+bx^2)^{7/2}}{17ax^{12}}$$

```
[Out] (-2*(a*x + b*x^2)^(7/2))/(17*a*x^12) + (4*b*(a*x + b*x^2)^(7/2))/(51*a^2*x^
11) - (32*b^2*(a*x + b*x^2)^(7/2))/(663*a^3*x^10) + (64*b^3*(a*x + b*x^2)^(
7/2))/(2431*a^4*x^9) - (256*b^4*(a*x + b*x^2)^(7/2))/(21879*a^5*x^8) + (512
*b^5*(a*x + b*x^2)^(7/2))/(153153*a^6*x^7)
```

Rubi [A] time = 0.0730637, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {658, 650}

$$\frac{512b^5(ax+bx^2)^{7/2}}{153153a^6x^7}-\frac{256b^4(ax+bx^2)^{7/2}}{21879a^5x^8}+\frac{64b^3(ax+bx^2)^{7/2}}{2431a^4x^9}-\frac{32b^2(ax+bx^2)^{7/2}}{663a^3x^{10}}+\frac{4b(ax+bx^2)^{7/2}}{51a^2x^{11}}-\frac{2(ax+bx^2)^{7/2}}{17ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^2)^(5/2)/x^12,x]
```

```
[Out] (-2*(a*x + b*x^2)^(7/2))/(17*a*x^12) + (4*b*(a*x + b*x^2)^(7/2))/(51*a^2*x^
11) - (32*b^2*(a*x + b*x^2)^(7/2))/(663*a^3*x^10) + (64*b^3*(a*x + b*x^2)^(
7/2))/(2431*a^4*x^9) - (256*b^4*(a*x + b*x^2)^(7/2))/(21879*a^5*x^8) + (512
*b^5*(a*x + b*x^2)^(7/2))/(153153*a^6*x^7)
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{(ax+bx^2)^{5/2}}{x^{12}}\,dx = -\frac{2(ax+bx^2)^{7/2}}{17ax^{12}} - \frac{(10b)\int \frac{(ax+bx^2)^{5/2}}{x^{11}}\,dx}{17a}$$

$$= -\frac{2(ax+bx^2)^{7/2}}{17ax^{12}} + \frac{4b(ax+bx^2)^{7/2}}{51a^2x^{11}} + \frac{(16b^2)\int \frac{(ax+bx^2)^{5/2}}{x^{10}}\,dx}{51a^2}$$

$$= -\frac{2(ax+bx^2)^{7/2}}{17ax^{12}} + \frac{4b(ax+bx^2)^{7/2}}{51a^2x^{11}} - \frac{32b^2(ax+bx^2)^{7/2}}{663a^3x^{10}} - \frac{(32b^3)\int \frac{(ax+bx^2)^{5/2}}{x^9}\,dx}{221a^3}$$

$$= -\frac{2(ax+bx^2)^{7/2}}{17ax^{12}} + \frac{4b(ax+bx^2)^{7/2}}{51a^2x^{11}} - \frac{32b^2(ax+bx^2)^{7/2}}{663a^3x^{10}} + \frac{64b^3(ax+bx^2)^{7/2}}{2431a^4x^9} + \frac{(128b^4)\int \frac{(ax+bx^2}{x^8}}{2431a^4}$$

$$= -\frac{2(ax+bx^2)^{7/2}}{17ax^{12}} + \frac{4b(ax+bx^2)^{7/2}}{51a^2x^{11}} - \frac{32b^2(ax+bx^2)^{7/2}}{663a^3x^{10}} + \frac{64b^3(ax+bx^2)^{7/2}}{2431a^4x^9} - \frac{256b^4(ax+bx^2)}{21879a^5x^8}$$

$$= -\frac{2(ax+bx^2)^{7/2}}{17ax^{12}} + \frac{4b(ax+bx^2)^{7/2}}{51a^2x^{11}} - \frac{32b^2(ax+bx^2)^{7/2}}{663a^3x^{10}} + \frac{64b^3(ax+bx^2)^{7/2}}{2431a^4x^9} - \frac{256b^4(ax+bx^2)}{21879a^5x^8}$$

Mathematica [A] time = 0.0188533, size = 80, normalized size = 0.53

$$\frac{2(a+bx)^3\sqrt{x(a+bx)}\left(-3696a^3b^2x^2+2016a^2b^3x^3+6006a^4bx-9009a^5-896ab^4x^4+256b^5x^5\right)}{153153a^6x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^2)^(5/2)/x^12,x]
```

```
[Out] (2*(a + b*x)^3*Sqrt[x*(a + b*x)]*(-9009*a^5 + 6006*a^4*b*x - 3696*a^3*b^2*x
^2 + 2016*a^2*b^3*x^3 - 896*a*b^4*x^4 + 256*b^5*x^5))/(153153*a^6*x^9)
```

Maple [A] time = 0.053, size = 77, normalized size = 0.5

$$-\frac{(2\,bx+2\,a)\left(-256\,b^5x^5+896\,b^4x^4a-2016\,b^3x^3a^2+3696\,b^2x^2a^3-6006\,bxa^4+9009\,a^5\right)}{153153\,x^{11}a^6}\left(bx^2+ax\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a*x)^(5/2)/x^12,x)
```

```
[Out] -2/153153*(b*x+a)*(-256*b^5*x^5+896*a*b^4*x^4-2016*a^2*b^3*x^3+3696*a^3*b^2
*x^2-6006*a^4*b*x+9009*a^5)*(b*x^2+a*x)^(5/2)/x^11/a^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^12,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89062, size = 247, normalized size = 1.62

$$\frac{2\left(256\,b^8x^8-128\,ab^7x^7+96\,a^2b^6x^6-80\,a^3b^5x^5+70\,a^4b^4x^4-63\,a^5b^3x^3-12705\,a^6b^2x^2-21021\,a^7bx-9009\,a^8\right)\sqrt{bx^2+ax}}{153153\,a^6x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^12,x, algorithm="fricas")
```

```
[Out] 2/153153*(256*b^8*x^8 - 128*a*b^7*x^7 + 96*a^2*b^6*x^6 - 80*a^3*b^5*x^5 + 7
0*a^4*b^4*x^4 - 63*a^5*b^3*x^3 - 12705*a^6*b^2*x^2 - 21021*a^7*b*x - 9009*a
^8)*sqrt(b*x^2 + a*x)/(a^6*x^9)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x(a+bx))^{\frac{5}{2}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a*x)**(5/2)/x**12,x)
```

```
[Out] Integral((x*(a + b*x))**(5/2)/x**12, x)
```

Giac [B] time = 1.25247, size = 458, normalized size = 3.01

$$2\left(816816\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^{11}b^{\frac{11}{2}}+5951088\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^{10}ab^5+19909890\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^{9}a^2b^{\frac{9}{2}}+4016\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a*x)^(5/2)/x^12,x, algorithm="giac")
```

```
[Out] 2/153153*(816816*(sqrt(b)*x - sqrt(b*x^2 + a*x))^11*b^(11/2) + 5951088*(sqr
t(b)*x - sqrt(b*x^2 + a*x))^10*a*b^5 + 19909890*(sqrt(b)*x - sqrt(b*x^2 + a
*x))^9*a^2*b^(9/2) + 40160120*(sqrt(b)*x - sqrt(b*x^2 + a*x))^8*a^3*b^4 + 5
4063009*(sqrt(b)*x - sqrt(b*x^2 + a*x))^7*a^4*b^(7/2) + 50860719*(sqrt(b)*x
 - sqrt(b*x^2 + a*x))^6*a^5*b^3 + 34051017*(sqrt(b)*x - sqrt(b*x^2 + a*x))^
5*a^6*b^(5/2) + 16198875*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^7*b^2 + 536035
5*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^8*b^(3/2) + 1174173*(sqrt(b)*x - sqrt
(b*x^2 + a*x))^2*a^9*b + 153153*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^10*sqrt(b
) + 9009*a^11)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^17
```

3.39 $\int x\sqrt{2x-x^2}\,dx$

Optimal. Leaf size=50

$$-\frac{1}{3}\left(2x-x^2\right)^{3/2}-\frac{1}{2}(1-x)\sqrt{2x-x^2}-\frac{1}{2}\sin^{-1}(1-x)$$

```
[Out] -((1 - x)*Sqrt[2*x - x^2])/2 - (2*x - x^2)^(3/2)/3 - ArcSin[1 - x]/2
```

Rubi [A] time = 0.0109595, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {640, 612, 619, 216}

$$-\frac{1}{3}\left(2x-x^2\right)^{3/2}-\frac{1}{2}(1-x)\sqrt{2x-x^2}-\frac{1}{2}\sin^{-1}(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[2*x - x^2],x]
```

```
[Out] -((1 - x)*Sqrt[2*x - x^2])/2 - (2*x - x^2)^(3/2)/3 - ArcSin[1 - x]/2
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int x\sqrt{2x-x^2}\,dx = -\frac{1}{3}\left(2x-x^2\right)^{3/2} + \int \sqrt{2x-x^2}\,dx$$
$$= -\frac{1}{2}(1-x)\sqrt{2x-x^2} - \frac{1}{3}\left(2x-x^2\right)^{3/2} + \frac{1}{2}\int \frac{1}{\sqrt{2x-x^2}}\,dx$$
$$= -\frac{1}{2}(1-x)\sqrt{2x-x^2} - \frac{1}{3}\left(2x-x^2\right)^{3/2} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, 2-2x\right)$$
$$= -\frac{1}{2}(1-x)\sqrt{2x-x^2} - \frac{1}{3}\left(2x-x^2\right)^{3/2} - \frac{1}{2}\sin^{-1}(1-x)$$

Mathematica [A] time = 0.0484477, size = 39, normalized size = 0.78

$$\frac{1}{6}\sqrt{-(x-2)x}\left(2x^2-x-3\right) - \sin^{-1}\left(\sqrt{1-\frac{x}{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[2*x - x^2],x]
```

```
[Out] (Sqrt[-((-2 + x)*x)]*(-3 - x + 2*x^2))/6 - ArcSin[Sqrt[1 - x/2]]
```

Maple [A] time = 0.05, size = 39, normalized size = 0.8

$$-\frac{1}{3}\left(-x^2+2x\right)^{\frac{3}{2}} - \frac{2-2x}{4}\sqrt{-x^2+2x} + \frac{\arcsin(-1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^2+2*x)^(1/2),x)
```

```
[Out] -1/3*(-x^2+2*x)^(3/2)-1/4*(2-2*x)*(-x^2+2*x)^(1/2)+1/2*arcsin(-1+x)
```

Maxima [A] time = 1.78255, size = 66, normalized size = 1.32

$$-\frac{1}{3}\left(-x^2+2x\right)^{\frac{3}{2}} + \frac{1}{2}\sqrt{-x^2+2x}\,x - \frac{1}{2}\sqrt{-x^2+2x} - \frac{1}{2}\arcsin(-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-x^2 + 2*x)^(3/2) + 1/2*sqrt(-x^2 + 2*x)*x - 1/2*sqrt(-x^2 + 2*x) - 1
/2*arcsin(-x + 1)
```

Fricas [A] time = 1.80531, size = 90, normalized size = 1.8

$$\frac{1}{6}\left(2x^2-x-3\right)\sqrt{-x^2+2x} - \arctan\left(\frac{\sqrt{-x^2+2x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^2 - x - 3)*sqrt(-x^2 + 2*x) - arctan(sqrt(-x^2 + 2*x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{-x\left(x-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**2+2*x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-x*(x - 2)), x)
```

Giac [A] time = 1.21428, size = 39, normalized size = 0.78

$$\frac{1}{6}\left((2\,x-1)x-3\right)\sqrt{-x^2+2\,x}+\frac{1}{2}\,\arcsin\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*((2*x - 1)*x - 3)*sqrt(-x^2 + 2*x) + 1/2*arcsin(x - 1)
```

3.40 $\int x\sqrt{3x-4x^2}\,dx$

Optimal. Leaf size=52

$$-\frac{1}{12}\left(3x-4x^2\right)^{3/2}-\frac{3}{128}(3-8x)\sqrt{3x-4x^2}-\frac{27}{512}\sin^{-1}\left(1-\frac{8x}{3}\right)$$

```
[Out] (-3*(3 - 8*x)*Sqrt[3*x - 4*x^2])/128 - (3*x - 4*x^2)^(3/2)/12 - (27*ArcSin[
1 - (8*x)/3])/512
```

Rubi [A] time = 0.0145858, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {640, 612, 619, 216}

$$-\frac{1}{12}\left(3x-4x^2\right)^{3/2}-\frac{3}{128}(3-8x)\sqrt{3x-4x^2}-\frac{27}{512}\sin^{-1}\left(1-\frac{8x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[3*x - 4*x^2],x]
```

```
[Out] (-3*(3 - 8*x)*Sqrt[3*x - 4*x^2])/128 - (3*x - 4*x^2)^(3/2)/12 - (27*ArcSin[
1 - (8*x)/3])/512
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x\sqrt{3x-4x^2}\,dx &= -\frac{1}{12}\left(3x-4x^2\right)^{3/2}+\frac{3}{8}\int \sqrt{3x-4x^2}\,dx\\
&= -\frac{3}{128}(3-8x)\sqrt{3x-4x^2}-\frac{1}{12}\left(3x-4x^2\right)^{3/2}+\frac{27}{256}\int \frac{1}{\sqrt{3x-4x^2}}\,dx\\
&= -\frac{3}{128}(3-8x)\sqrt{3x-4x^2}-\frac{1}{12}\left(3x-4x^2\right)^{3/2}-\frac{9}{512}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx,x,3-8x\right)\\
&= -\frac{3}{128}(3-8x)\sqrt{3x-4x^2}-\frac{1}{12}\left(3x-4x^2\right)^{3/2}-\frac{27}{512}\sin^{-1}\left(1-\frac{8x}{3}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0434005, size = 63, normalized size = 1.21

$$
\frac{2x\left(-512x^3+480x^2+36x-81\right)-81\sqrt{3-4x}\sqrt{x}\sin^{-1}\left(\sqrt{1-\frac{4x}{3}}\right)}{768\sqrt{-x(4x-3)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[3*x - 4*x^2],x]
```

```
[Out] (2*x*(-81 + 36*x + 480*x^2 - 512*x^3) - 81*Sqrt[3 - 4*x]*Sqrt[x]*ArcSin[Sqr
t[1 - (4*x)/3]])/(768*Sqrt[-(x*(-3 + 4*x))])
```

Maple [A] time = 0.044, size = 41, normalized size = 0.8

$$
-\frac{1}{12}\left(-4x^2+3x\right)^{\frac{3}{2}}+\frac{27}{512}\arcsin\left(-1+\frac{8x}{3}\right)-\frac{9-24x}{128}\sqrt{-4x^2+3x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-4*x^2+3*x)^(1/2),x)
```

```
[Out] -1/12*(-4*x^2+3*x)^(3/2)+27/512*arcsin(-1+8/3*x)-3/128*(3-8*x)*(-4*x^2+3*x)
^(1/2)
```

Maxima [A] time = 1.81818, size = 66, normalized size = 1.27

$$
-\frac{1}{12}\left(-4x^2+3x\right)^{\frac{3}{2}}+\frac{3}{16}\sqrt{-4x^2+3x}x-\frac{9}{128}\sqrt{-4x^2+3x}-\frac{27}{512}\arcsin\left(-\frac{8}{3}x+1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x^2+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*(-4*x^2 + 3*x)^(3/2) + 3/16*sqrt(-4*x^2 + 3*x)*x - 9/128*sqrt(-4*x^2
+ 3*x) - 27/512*arcsin(-8/3*x + 1)
```

Fricas [A] time = 1.96582, size = 122, normalized size = 2.35

$$\frac{1}{384}\left(128\,x^2-24\,x-27\right)\sqrt{-4\,x^2+3\,x}-\frac{27}{256}\,\arctan\left(\frac{\sqrt{-4\,x^2+3\,x}}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x^2+3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/384*(128*x^2 - 24*x - 27)*sqrt(-4*x^2 + 3*x) - 27/256*arctan(1/2*sqrt(-4*
x^2 + 3*x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{-x\,(4x-3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x**2+3*x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-x*(4*x - 3)), x)
```

Giac [A] time = 1.21139, size = 43, normalized size = 0.83

$$\frac{1}{384}\,(8\,(16\,x-3)x-27)\sqrt{-4\,x^2+3\,x}+\frac{27}{512}\,\arcsin\left(\frac{8}{3}\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x^2+3*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(8*(16*x - 3)*x - 27)*sqrt(-4*x^2 + 3*x) + 27/512*arcsin(8/3*x - 1)
```

3.41 $\int x\sqrt{x+x^2}\,dx$

Optimal. Leaf size=48

$$\frac{1}{3}\left(x^2+x\right)^{3/2}-\frac{1}{8}(2x+1)\sqrt{x^2+x}+\frac{1}{8}\tanh^{-1}\left(\frac{x}{\sqrt{x^2+x}}\right)$$

```
[Out] -((1 + 2*x)*Sqrt[x + x^2])/8 + (x + x^2)^(3/2)/3 + ArcTanh[x/Sqrt[x + x^2]]
/8
```

Rubi [A] time = 0.0099011, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {640, 612, 620, 206}

$$\frac{1}{3}\left(x^2+x\right)^{3/2}-\frac{1}{8}(2x+1)\sqrt{x^2+x}+\frac{1}{8}\tanh^{-1}\left(\frac{x}{\sqrt{x^2+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[x + x^2],x]
```

```
[Out] -((1 + 2*x)*Sqrt[x + x^2])/8 + (x + x^2)^(3/2)/3 + ArcTanh[x/Sqrt[x + x^2]]
/8
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x\sqrt{x+x^2}\,dx = \frac{1}{3}\left(x+x^2\right)^{3/2} - \frac{1}{2}\int \sqrt{x+x^2}\,dx$$
$$= -\frac{1}{8}(1+2x)\sqrt{x+x^2} + \frac{1}{3}\left(x+x^2\right)^{3/2} + \frac{1}{16}\int \frac{1}{\sqrt{x+x^2}}\,dx$$
$$= -\frac{1}{8}(1+2x)\sqrt{x+x^2} + \frac{1}{3}\left(x+x^2\right)^{3/2} + \frac{1}{8}\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{x+x^2}}\right)$$
$$= -\frac{1}{8}(1+2x)\sqrt{x+x^2} + \frac{1}{3}\left(x+x^2\right)^{3/2} + \frac{1}{8}\tanh^{-1}\left(\frac{x}{\sqrt{x+x^2}}\right)$$

Mathematica [A] time = 0.0306674, size = 43, normalized size = 0.9

$$\frac{1}{24}\sqrt{x(x+1)}\left(8x^2+2x+\frac{3\sinh^{-1}\left(\sqrt{x}\right)}{\sqrt{x+1}\sqrt{x}}-3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[x + x^2],x]
```

```
[Out] (Sqrt[x*(1 + x)]*(-3 + 2*x + 8*x^2 + (3*ArcSinh[Sqrt[x]])/(Sqrt[x]*Sqrt[1 +
 x])))/24
```

Maple [A] time = 0.045, size = 38, normalized size = 0.8

$$\frac{1}{3}\left(x^2+x\right)^{\frac{3}{2}} - \frac{1+2x}{8}\sqrt{x^2+x} + \frac{1}{16}\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+x)^(1/2),x)
```

```
[Out] 1/3*(x^2+x)^(3/2)-1/8*(1+2*x)*(x^2+x)^(1/2)+1/16*ln(x+1/2+(x^2+x)^(1/2))
```

Maxima [A] time = 1.14893, size = 62, normalized size = 1.29

$$\frac{1}{3}\left(x^2+x\right)^{\frac{3}{2}} - \frac{1}{4}\sqrt{x^2+x}x - \frac{1}{8}\sqrt{x^2+x} + \frac{1}{16}\log\left(2x+2\sqrt{x^2+x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(x^2 + x)^(3/2) - 1/4*sqrt(x^2 + x)*x - 1/8*sqrt(x^2 + x) + 1/16*log(2*
x + 2*sqrt(x^2 + x) + 1)
```

Fricas [A] time = 1.94107, size = 104, normalized size = 2.17

$$\frac{1}{24}\left(8x^2+2x-3\right)\sqrt{x^2+x} - \frac{1}{16}\log\left(-2x+2\sqrt{x^2+x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(8*x^2 + 2*x - 3)*sqrt(x^2 + x) - 1/16*log(-2*x + 2*sqrt(x^2 + x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{x(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x*(x + 1)), x)
```

Giac [A] time = 1.1555, size = 51, normalized size = 1.06

$$\frac{1}{24}(2(4x+1)x-3)\sqrt{x^2+x}-\frac{1}{16}\log\left(\left|-2x+2\sqrt{x^2+x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(2*(4*x + 1)*x - 3)*sqrt(x^2 + x) - 1/16*log(abs(-2*x + 2*sqrt(x^2 + x
) - 1))
```

3.42 $\int \frac{x^4}{\sqrt{bx+cx^2}}\,dx$

Optimal. Leaf size=128

$$-\frac{35b^3\sqrt{bx+cx^2}}{64c^4}+\frac{35b^2x\sqrt{bx+cx^2}}{96c^3}+\frac{35b^4\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{64c^{9/2}}-\frac{7bx^2\sqrt{bx+cx^2}}{24c^2}+\frac{x^3\sqrt{bx+cx^2}}{4c}$$

```
[Out] (-35*b^3*Sqrt[b*x + c*x^2])/(64*c^4) + (35*b^2*x*Sqrt[b*x + c*x^2])/(96*c^3
) - (7*b*x^2*Sqrt[b*x + c*x^2])/(24*c^2) + (x^3*Sqrt[b*x + c*x^2])/(4*c) +
(35*b^4*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(64*c^(9/2))
```

Rubi [A] time = 0.0576066, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {670, 640, 620, 206}

$$-\frac{35b^3\sqrt{bx+cx^2}}{64c^4}+\frac{35b^2x\sqrt{bx+cx^2}}{96c^3}+\frac{35b^4\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{64c^{9/2}}-\frac{7bx^2\sqrt{bx+cx^2}}{24c^2}+\frac{x^3\sqrt{bx+cx^2}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[b*x + c*x^2],x]
```

```
[Out] (-35*b^3*Sqrt[b*x + c*x^2])/(64*c^4) + (35*b^2*x*Sqrt[b*x + c*x^2])/(96*c^3
) - (7*b*x^2*Sqrt[b*x + c*x^2])/(24*c^2) + (x^3*Sqrt[b*x + c*x^2])/(4*c) +
(35*b^4*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(64*c^(9/2))
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4}{\sqrt{bx+cx^2}}\,dx = \frac{x^3\sqrt{bx+cx^2}}{4c} - \frac{(7b)\int \frac{x^3}{\sqrt{bx+cx^2}}\,dx}{8c}$$

$$= -\frac{7bx^2\sqrt{bx+cx^2}}{24c^2} + \frac{x^3\sqrt{bx+cx^2}}{4c} + \frac{\left(35b^2\right)\int \frac{x^2}{\sqrt{bx+cx^2}}\,dx}{48c^2}$$

$$= \frac{35b^2x\sqrt{bx+cx^2}}{96c^3} - \frac{7bx^2\sqrt{bx+cx^2}}{24c^2} + \frac{x^3\sqrt{bx+cx^2}}{4c} - \frac{\left(35b^3\right)\int \frac{x}{\sqrt{bx+cx^2}}\,dx}{64c^3}$$

$$= -\frac{35b^3\sqrt{bx+cx^2}}{64c^4} + \frac{35b^2x\sqrt{bx+cx^2}}{96c^3} - \frac{7bx^2\sqrt{bx+cx^2}}{24c^2} + \frac{x^3\sqrt{bx+cx^2}}{4c} + \frac{\left(35b^4\right)\int \frac{1}{\sqrt{bx+cx^2}}\,dx}{128c^4}$$

$$= -\frac{35b^3\sqrt{bx+cx^2}}{64c^4} + \frac{35b^2x\sqrt{bx+cx^2}}{96c^3} - \frac{7bx^2\sqrt{bx+cx^2}}{24c^2} + \frac{x^3\sqrt{bx+cx^2}}{4c} + \frac{\left(35b^4\right)\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x\right.}{64c^4}$$

$$= -\frac{35b^3\sqrt{bx+cx^2}}{64c^4} + \frac{35b^2x\sqrt{bx+cx^2}}{96c^3} - \frac{7bx^2\sqrt{bx+cx^2}}{24c^2} + \frac{x^3\sqrt{bx+cx^2}}{4c} + \frac{35b^4\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{64c^{9/2}}$$

Mathematica [A] time = 0.213593, size = 98, normalized size = 0.77

$$\frac{\sqrt{x(b+cx)}\left(\sqrt{c}\left(70b^2cx-105b^3-56bc^2x^2+48c^3x^3\right)+\frac{105b^{7/2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}\right)}{192c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[b*x + c*x^2],x]
```

```
[Out] (Sqrt[x*(b + c*x)]*(Sqrt[c]*(-105*b^3 + 70*b^2*c*x - 56*b*c^2*x^2 + 48*c^3*
x^3) + (105*b^(7/2)*ArcSinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(Sqrt[x]*Sqrt[1 + (
c*x)/b])))/(192*c^(9/2))
```

Maple [A] time = 0.048, size = 112, normalized size = 0.9

$$\frac{x^3}{4\,c}\sqrt{cx^2+bx} - \frac{7\,bx^2}{24\,c^2}\sqrt{cx^2+bx} + \frac{35\,b^2x}{96\,c^3}\sqrt{cx^2+bx} - \frac{35\,b^3}{64\,c^4}\sqrt{cx^2+bx} + \frac{35\,b^4}{128}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}} + \sqrt{cx^2+bx}\right)c^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^2+b*x)^(1/2),x)
```

```
[Out] 1/4*x^3*(c*x^2+b*x)^(1/2)/c-7/24*b*x^2*(c*x^2+b*x)^(1/2)/c^2+35/96*b^2*x*(c
*x^2+b*x)^(1/2)/c^3-35/64*b^3*(c*x^2+b*x)^(1/2)/c^4+35/128*b^4/c^(9/2)*ln((
1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.97581, size = 408, normalized size = 3.19

$$\left[\frac{105\,b^4\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\left(48\,c^4x^3-56\,bc^3x^2+70\,b^2c^2x-105\,b^3c\right)\sqrt{cx^2+bx}}{384\,c^5}, -\frac{105\,b^4\sqrt{-c}\,\mathrm{arcta}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/384*(105*b^4*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*(4
8*c^4*x^3 - 56*b*c^3*x^2 + 70*b^2*c^2*x - 105*b^3*c)*sqrt(c*x^2 + b*x))/c^5
, -1/192*(105*b^4*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)) - (48*c
^4*x^3 - 56*b*c^3*x^2 + 70*b^2*c^2*x - 105*b^3*c)*sqrt(c*x^2 + b*x))/c^5]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{x\,(b+cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(x*(b + c*x)), x)
```

Giac [A] time = 1.22022, size = 120, normalized size = 0.94

$$\frac{1}{192}\,\sqrt{cx^2+bx}\left(2\left(4\,x\left(\frac{6\,x}{c}-\frac{7\,b}{c^2}\right)+\frac{35\,b^2}{c^3}\right)x-\frac{105\,b^3}{c^4}\right)-\frac{35\,b^4\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}-b\right|\right)}{128\,c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x)*(2*(4*x*(6*x/c - 7*b/c^2) + 35*b^2/c^3)*x - 105*b^3
/c^4) - 35/128*b^4*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b))
/c^(9/2)
```

3.43 $\int \frac{x^3}{\sqrt{bx+cx^2}}\,dx$

Optimal. Leaf size=102

$$\frac{5b^2\sqrt{bx+cx^2}}{8c^3}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{8c^{7/2}}-\frac{5bx\sqrt{bx+cx^2}}{12c^2}+\frac{x^2\sqrt{bx+cx^2}}{3c}$$

```
[Out] (5*b^2*Sqrt[b*x + c*x^2])/(8*c^3) - (5*b*x*Sqrt[b*x + c*x^2])/(12*c^2) + (x
^2*Sqrt[b*x + c*x^2])/(3*c) - (5*b^3*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]]
)/(8*c^(7/2))
```

Rubi [A] time = 0.0425747, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {670, 640, 620, 206}

$$\frac{5b^2\sqrt{bx+cx^2}}{8c^3}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{8c^{7/2}}-\frac{5bx\sqrt{bx+cx^2}}{12c^2}+\frac{x^2\sqrt{bx+cx^2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[b*x + c*x^2],x]
```

```
[Out] (5*b^2*Sqrt[b*x + c*x^2])/(8*c^3) - (5*b*x*Sqrt[b*x + c*x^2])/(12*c^2) + (x
^2*Sqrt[b*x + c*x^2])/(3*c) - (5*b^3*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]]
)/(8*c^(7/2))
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3}{\sqrt{bx+cx^2}}\,dx = \frac{x^2\sqrt{bx+cx^2}}{3c} - \frac{(5b)\int \frac{x^2}{\sqrt{bx+cx^2}}\,dx}{6c}$$
$$= -\frac{5bx\sqrt{bx+cx^2}}{12c^2} + \frac{x^2\sqrt{bx+cx^2}}{3c} + \frac{\left(5b^2\right)\int \frac{x}{\sqrt{bx+cx^2}}\,dx}{8c^2}$$
$$= \frac{5b^2\sqrt{bx+cx^2}}{8c^3} - \frac{5bx\sqrt{bx+cx^2}}{12c^2} + \frac{x^2\sqrt{bx+cx^2}}{3c} - \frac{\left(5b^3\right)\int \frac{1}{\sqrt{bx+cx^2}}\,dx}{16c^3}$$
$$= \frac{5b^2\sqrt{bx+cx^2}}{8c^3} - \frac{5bx\sqrt{bx+cx^2}}{12c^2} + \frac{x^2\sqrt{bx+cx^2}}{3c} - \frac{\left(5b^3\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{bx+cx^2}}\right)}{8c^3}$$
$$= \frac{5b^2\sqrt{bx+cx^2}}{8c^3} - \frac{5bx\sqrt{bx+cx^2}}{12c^2} + \frac{x^2\sqrt{bx+cx^2}}{3c} - \frac{5b^3\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{8c^{7/2}}$$

Mathematica [A] time = 0.173059, size = 87, normalized size = 0.85

$$\frac{\sqrt{x(b+cx)}\left(\sqrt{c}\left(15b^2-10bcx+8c^2x^2\right)-\frac{15b^{5/2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}\right)}{24c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[b*x + c*x^2],x]
```

```
[Out] (Sqrt[x*(b + c*x)]*(Sqrt[c]*(15*b^2 - 10*b*c*x + 8*c^2*x^2) - (15*b^(5/2)*A
rcSinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(Sqrt[x]*Sqrt[1 + (c*x)/b])))/(24*c^(7/2
))
```

Maple [A] time = 0.046, size = 90, normalized size = 0.9

$$\frac{x^2}{3c}\sqrt{cx^2+bx} - \frac{5bx}{12c^2}\sqrt{cx^2+bx} + \frac{5b^2}{8c^3}\sqrt{cx^2+bx} - \frac{5b^3}{16}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}} + \sqrt{cx^2+bx}\right)c^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^2+b*x)^(1/2),x)
```

```
[Out] 1/3*x^2*(c*x^2+b*x)^(1/2)/c-5/12*b*x*(c*x^2+b*x)^(1/2)/c^2+5/8*b^2*(c*x^2+b
*x)^(1/2)/c^3-5/16*b^3/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.96853, size = 350, normalized size = 3.43

$$\left[\frac{15\,b^3\sqrt{c}\log\left(2\,cx+b-2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\left(8\,c^3x^2-10\,bc^2x+15\,b^2c\right)\sqrt{cx^2+bx}}{48\,c^4},\ \frac{15\,b^3\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx}\right)+\left(8\,c^3\right.}{24\,c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*b^3*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*(8*c
^3*x^2 - 10*b*c^2*x + 15*b^2*c)*sqrt(c*x^2 + b*x))/c^4, 1/24*(15*b^3*sqrt(-
c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)) + (8*c^3*x^2 - 10*b*c^2*x + 15*
b^2*c)*sqrt(c*x^2 + b*x))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{x\,(b+cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x*(b + c*x)), x)
```

Giac [A] time = 1.26409, size = 104, normalized size = 1.02

$$\frac{1}{24}\sqrt{cx^2+bx}\left(2\,x\left(\frac{4\,x}{c}-\frac{5\,b}{c^2}\right)+\frac{15\,b^2}{c^3}\right)+\frac{5\,b^3\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}-b\right|\right)}{16\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + b*x)*(2*x*(4*x/c - 5*b/c^2) + 15*b^2/c^3) + 5/16*b^3*log(
abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b))/c^(7/2)
```

3.44 $\int \frac{x^2}{\sqrt{bx+cx^2}} dx$

Optimal. Leaf size=76

$$\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{5/2}} - \frac{3b\sqrt{bx+cx^2}}{4c^2} + \frac{x\sqrt{bx+cx^2}}{2c}$$

```
[Out] (-3*b*Sqrt[b*x + c*x^2])/(4*c^2) + (x*Sqrt[b*x + c*x^2])/(2*c) + (3*b^2*Arc
Tanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(4*c^(5/2))
```

Rubi [A] time = 0.0264775, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {670, 640, 620, 206}

$$\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{5/2}} - \frac{3b\sqrt{bx+cx^2}}{4c^2} + \frac{x\sqrt{bx+cx^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[b*x + c*x^2],x]
```

```
[Out] (-3*b*Sqrt[b*x + c*x^2])/(4*c^2) + (x*Sqrt[b*x + c*x^2])/(2*c) + (3*b^2*Arc
Tanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(4*c^(5/2))
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\sqrt{bx+cx^2}}\,dx &= \frac{x\sqrt{bx+cx^2}}{2c} - \frac{(3b)\int \frac{x}{\sqrt{bx+cx^2}}\,dx}{4c} \\
&= -\frac{3b\sqrt{bx+cx^2}}{4c^2} + \frac{x\sqrt{bx+cx^2}}{2c} + \frac{\left(3b^2\right)\int \frac{1}{\sqrt{bx+cx^2}}\,dx}{8c^2} \\
&= -\frac{3b\sqrt{bx+cx^2}}{4c^2} + \frac{x\sqrt{bx+cx^2}}{2c} + \frac{\left(3b^2\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{bx+cx^2}}\right)}{4c^2} \\
&= -\frac{3b\sqrt{bx+cx^2}}{4c^2} + \frac{x\sqrt{bx+cx^2}}{2c} + \frac{3b^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0536501, size = 88, normalized size = 1.16

$$
\frac{\sqrt{c}x\left(-3b^2-bcx+2c^2x^2\right)+3b^{5/2}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{4c^{5/2}\sqrt{x(b+cx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[b*x + c*x^2],x]
```

```
[Out] (Sqrt[c]*x*(-3*b^2 - b*c*x + 2*c^2*x^2) + 3*b^(5/2)*Sqrt[x]*Sqrt[1 + (c*x)/
b]*ArcSinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(4*c^(5/2)*Sqrt[x*(b + c*x)])
```

Maple [A] time = 0.045, size = 68, normalized size = 0.9

$$
\frac{x}{2\,c}\sqrt{cx^2+bx} - \frac{3\,b}{4\,c^2}\sqrt{cx^2+bx} + \frac{3\,b^2}{8}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}} + \sqrt{cx^2+bx}\right)c^{-\frac{5}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^2+b*x)^(1/2),x)
```

```
[Out] 1/2*x*(c*x^2+b*x)^(1/2)/c-3/4*b*(c*x^2+b*x)^(1/2)/c^2+3/8*b^2/c^(5/2)*ln((1
/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.01287, size = 297, normalized size = 3.91

$$\left[\frac{3\,b^{2}\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^{2}+bx}\sqrt{c}\right)+2\,\left(2\,c^{2}x-3\,bc\right)\sqrt{cx^{2}+bx}}{8\,c^{3}}, -\frac{3\,b^{2}\sqrt{-c}\arctan\left(\frac{\sqrt{cx^{2}+bx}\sqrt{-c}}{cx}\right)-\left(2\,c^{2}x-3\,bc\right)\sqrt{}}{4\,c^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*b^2*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*(2*c^2
*x - 3*b*c)*sqrt(c*x^2 + b*x))/c^3, -1/4*(3*b^2*sqrt(-c)*arctan(sqrt(c*x^2
+ b*x)*sqrt(-c)/(c*x)) - (2*c^2*x - 3*b*c)*sqrt(c*x^2 + b*x))/c^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\sqrt{x\,(b+cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(x*(b + c*x)), x)
```

Giac [A] time = 1.26556, size = 88, normalized size = 1.16

$$\frac{1}{4}\,\sqrt{cx^{2}+bx}\left(\frac{2\,x}{c}-\frac{3\,b}{c^{2}}\right)-\frac{3\,b^{2}\log\left(\left|-2\,\left(\sqrt{c}x-\sqrt{cx^{2}+bx}\right)\sqrt{c}-b\right|\right)}{8\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^2 + b*x)*(2*x/c - 3*b/c^2) - 3/8*b^2*log(abs(-2*(sqrt(c)*x - s
qrt(c*x^2 + b*x))*sqrt(c) - b))/c^(5/2)
```

3.45 $\int \frac{x}{\sqrt{bx+cx^2}}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{bx+cx^2}}{c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{3/2}}$$

```
[Out] Sqrt[b*x + c*x^2]/c - (b*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/c^(3/2)
```

Rubi [A] time = 0.0151187, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {640, 620, 206}

$$\frac{\sqrt{bx+cx^2}}{c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[b*x + c*x^2],x]
```

```
[Out] Sqrt[b*x + c*x^2]/c - (b*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/c^(3/2)
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{bx+cx^2}}\,dx &= \frac{\sqrt{bx+cx^2}}{c}-\frac{b\int \frac{1}{\sqrt{bx+cx^2}}\,dx}{2c}\\ &= \frac{\sqrt{bx+cx^2}}{c}-\frac{b\,\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{bx+cx^2}}\right)}{c}\\ &= \frac{\sqrt{bx+cx^2}}{c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0414106, size = 71, normalized size = 1.51

$$\frac{\sqrt{c}x(b+cx)-b^{3/2}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{c^{3/2}\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[b*x + c*x^2],x]
```

```
[Out] (Sqrt[c]*x*(b + c*x) - b^(3/2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*ArcSinh[(Sqrt[c]*S
qrt[x])/Sqrt[b]])/(c^(3/2)*Sqrt[x*(b + c*x)])
```

Maple [A] time = 0.046, size = 47, normalized size = 1.

$$\frac{1}{c}\sqrt{cx^2+bx}-\frac{b}{2}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx}\right)c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^2+b*x)^(1/2),x)
```

```
[Out] (c*x^2+b*x)^(1/2)/c-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.03164, size = 236, normalized size = 5.02

$$\left[\frac{b\sqrt{c}\log\left(2\,cx+b-2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\,\sqrt{cx^2+bx}c}{2\,c^2}, \frac{b\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx}\right)+\sqrt{cx^2+bx}c}{c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(b*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*sqrt(c*x^2
 + b*x)*c)/c^2, (b*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)) + sqrt
(c*x^2 + b*x)*c)/c^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x\,(b+cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x*(b + c*x)), x)
```

Giac [A] time = 1.31484, size = 70, normalized size = 1.49

$$\frac{b\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}-b\right|\right)}{2\,c^{\frac{3}{2}}}+\frac{\sqrt{cx^2+bx}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*b*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b))/c^(3/2) + sq
rt(c*x^2 + b*x)/c
```

3.46 $\int \frac{1}{\sqrt{bx+cx^2}}\,dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{\sqrt{c}}$$

```
[Out] (2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/Sqrt[c]
```

Rubi [A] time = 0.0081746, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*x + c*x^2],x]
```

```
[Out] (2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/Sqrt[c]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{bx+cx^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{bx+cx^2}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{\sqrt{c}}\end{aligned}$$

Mathematica [B] time = 0.0159234, size = 57, normalized size = 2.04

$$\frac{2\sqrt{b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{c}\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*x + c*x^2],x]
```

[Out] (2*Sqrt[b]*Sqrt[x]*Sqrt[1 + (c*x)/b]*ArcSinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(S
qrt[c]*Sqrt[x*(b + c*x)])

Maple [A] time = 0.049, size = 29, normalized size = 1.

$$\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+b*x)^(1/2),x)

[Out] ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))/c^(1/2)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.91292, size = 154, normalized size = 5.5

$$\left[\frac{\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{\sqrt{c}}, -\frac{2\,\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx}\right)}{c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(1/2),x, algorithm="fricas")

[Out] [log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/sqrt(c), -2*sqrt(-c)*arctan(s
qrt(c*x^2 + b*x)*sqrt(-c)/(c*x))/c]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**2+b*x)**(1/2),x)

[Out] Integral(1/sqrt(b*x + c*x**2), x)

Giac [A] time = 1.32406, size = 47, normalized size = 1.68

$$-\frac{\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}-b\right|\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b))/sqrt(c)
```

3.47 $\int \frac{1}{x\sqrt{bx+cx^2}}\,dx$

Optimal. Leaf size=21

$$-\frac{2\sqrt{bx+cx^2}}{bx}$$

```
[Out] (-2*Sqrt[b*x + c*x^2])/(b*x)
```

Rubi [A] time = 0.0073153, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {650}

$$-\frac{2\sqrt{bx+cx^2}}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[b*x + c*x^2]),x]

[Out] (-2*Sqrt[b*x + c*x^2])/(b*x)
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt{bx+cx^2}}\,dx = -\frac{2\sqrt{bx+cx^2}}{bx}$$

Mathematica [A] time = 0.006572, size = 21, normalized size = 1.

$$-\frac{2(b+cx)}{b\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[b*x + c*x^2]),x]

[Out] (-2*(b + c*x))/(b*Sqrt[x*(b + c*x)])
```

Maple [A] time = 0.047, size = 22, normalized size = 1.1

$$-2\,\frac{cx+b}{b\sqrt{cx^2+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^2+b*x)^(1/2),x)
```

```
[Out] -2*(c*x+b)/b/(c*x^2+b*x)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7453, size = 38, normalized size = 1.81

$$-\frac{2\sqrt{cx^2+bx}}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(c*x^2 + b*x)/(b*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{x(b+cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x*(b + c*x))), x)
```

Giac [A] time = 1.19682, size = 31, normalized size = 1.48

$$\frac{2}{\sqrt{c}x-\sqrt{cx^2+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/(sqrt(c)*x - sqrt(c*x^2 + b*x))
```

3.48 $\int \frac{1}{x^2\sqrt{bx+cx^2}}\,dx$

Optimal. Leaf size=48

$$\frac{4c\sqrt{bx+cx^2}}{3b^2x}-\frac{2\sqrt{bx+cx^2}}{3bx^2}$$

```
[Out] (-2*Sqrt[b*x + c*x^2])/(3*b*x^2) + (4*c*Sqrt[b*x + c*x^2])/(3*b^2*x)
```

Rubi [A] time = 0.0163138, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {658, 650}

$$\frac{4c\sqrt{bx+cx^2}}{3b^2x}-\frac{2\sqrt{bx+cx^2}}{3bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[b*x + c*x^2]),x]
```

```
[Out] (-2*Sqrt[b*x + c*x^2])/(3*b*x^2) + (4*c*Sqrt[b*x + c*x^2])/(3*b^2*x)
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{bx+cx^2}}\,dx &= -\frac{2\sqrt{bx+cx^2}}{3bx^2}-\frac{(2c)\int \frac{1}{x\sqrt{bx+cx^2}}\,dx}{3b}\\ &= -\frac{2\sqrt{bx+cx^2}}{3bx^2}+\frac{4c\sqrt{bx+cx^2}}{3b^2x}\end{aligned}$$

Mathematica [A] time = 0.0130494, size = 29, normalized size = 0.6

$$\frac{2\sqrt{x(b+cx)}(2cx-b)}{3b^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[b*x + c*x^2]),x]
```

```
[Out] (2*Sqrt[x*(b + c*x)]*(-b + 2*c*x))/(3*b^2*x^2)
```

Maple [A] time = 0.046, size = 31, normalized size = 0.7

$$-\frac{(2\,cx+2\,b)\,(-2\,cx+b)}{3\,b^2x}\frac{1}{\sqrt{cx^2+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^2+b*x)^(1/2),x)
```

```
[Out] -2/3*(c*x+b)*(-2*c*x+b)/x/b^2/(c*x^2+b*x)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.01151, size = 61, normalized size = 1.27

$$\frac{2\,\sqrt{cx^2+bx}(2\,cx-b)}{3\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(c*x^2 + b*x)*(2*c*x - b)/(b^2*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^2\sqrt{x\,(b+cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x*(b + c*x))), x)
```

Giac [A] time = 1.22839, size = 66, normalized size = 1.38

$$\frac{2\left(3\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right)}{3\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b)/(sqrt(c)*x - sqrt(c*x^2
 + b*x))^3
```

3.49 $\int \frac{1}{x^3\sqrt{bx+cx^2}}\,dx$

Optimal. Leaf size=74

$$-\frac{16c^2\sqrt{bx+cx^2}}{15b^3x}+\frac{8c\sqrt{bx+cx^2}}{15b^2x^2}-\frac{2\sqrt{bx+cx^2}}{5bx^3}$$

```
[Out] (-2*Sqrt[b*x + c*x^2])/(5*b*x^3) + (8*c*Sqrt[b*x + c*x^2])/(15*b^2*x^2) - (
16*c^2*Sqrt[b*x + c*x^2])/(15*b^3*x)
```

Rubi [A] time = 0.0263216, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {658, 650}

$$-\frac{16c^2\sqrt{bx+cx^2}}{15b^3x}+\frac{8c\sqrt{bx+cx^2}}{15b^2x^2}-\frac{2\sqrt{bx+cx^2}}{5bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[b*x + c*x^2]),x]
```

```
[Out] (-2*Sqrt[b*x + c*x^2])/(5*b*x^3) + (8*c*Sqrt[b*x + c*x^2])/(15*b^2*x^2) - (
16*c^2*Sqrt[b*x + c*x^2])/(15*b^3*x)
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{bx+cx^2}}\,dx &= -\frac{2\sqrt{bx+cx^2}}{5bx^3}-\frac{(4c)\int \frac{1}{x^2\sqrt{bx+cx^2}}\,dx}{5b}\\ &= -\frac{2\sqrt{bx+cx^2}}{5bx^3}+\frac{8c\sqrt{bx+cx^2}}{15b^2x^2}+\frac{\left(8c^2\right)\int \frac{1}{x\sqrt{bx+cx^2}}\,dx}{15b^2}\\ &= -\frac{2\sqrt{bx+cx^2}}{5bx^3}+\frac{8c\sqrt{bx+cx^2}}{15b^2x^2}-\frac{16c^2\sqrt{bx+cx^2}}{15b^3x}\end{aligned}$$

Mathematica [A] time = 0.011888, size = 40, normalized size = 0.54

$$-\frac{2\sqrt{x(b+cx)}\left(3b^2-4bcx+8c^2x^2\right)}{15b^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[b*x + c*x^2]),x]
```

```
[Out] (-2*Sqrt[x*(b + c*x)]*(3*b^2 - 4*b*c*x + 8*c^2*x^2))/(15*b^3*x^3)
```

Maple [A] time = 0.046, size = 44, normalized size = 0.6

$$-\frac{(2\,cx+2\,b)\left(8\,c^{2}x^{2}-4\,bcx+3\,b^{2}\right)}{15\,x^{2}b^{3}}\frac{1}{\sqrt{cx^{2}+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^2+b*x)^(1/2),x)
```

```
[Out] -2/15*(c*x+b)*(8*c^2*x^2-4*b*c*x+3*b^2)/x^2/b^3/(c*x^2+b*x)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88494, size = 88, normalized size = 1.19

$$-\frac{2\left(8\,c^{2}x^{2}-4\,bcx+3\,b^{2}\right)\sqrt{cx^{2}+bx}}{15\,b^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(8*c^2*x^2 - 4*b*c*x + 3*b^2)*sqrt(c*x^2 + b*x)/(b^3*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^{3}\sqrt{x\,(b+cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x*(b + c*x))), x)
```

Giac [A] time = 1.20529, size = 105, normalized size = 1.42

$$\frac{2\left(20\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^2c+15\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)b\sqrt{c}+3\,b^2\right)}{15\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(20*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*c + 15*(sqrt(c)*x - sqrt(c*x^2 +
 b*x))*b*sqrt(c) + 3*b^2)/(sqrt(c)*x - sqrt(c*x^2 + b*x))^5
```

3.50 $\int \frac{1}{x^4\sqrt{bx+cx^2}}\,dx$

Optimal. Leaf size=100

$$\frac{32c^3\sqrt{bx+cx^2}}{35b^4x}-\frac{16c^2\sqrt{bx+cx^2}}{35b^3x^2}+\frac{12c\sqrt{bx+cx^2}}{35b^2x^3}-\frac{2\sqrt{bx+cx^2}}{7bx^4}$$

```
[Out] (-2*Sqrt[b*x + c*x^2])/(7*b*x^4) + (12*c*Sqrt[b*x + c*x^2])/(35*b^2*x^3) -
(16*c^2*Sqrt[b*x + c*x^2])/(35*b^3*x^2) + (32*c^3*Sqrt[b*x + c*x^2])/(35*b^
4*x)
```

Rubi [A] time = 0.0388309, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {658, 650}

$$\frac{32c^3\sqrt{bx+cx^2}}{35b^4x}-\frac{16c^2\sqrt{bx+cx^2}}{35b^3x^2}+\frac{12c\sqrt{bx+cx^2}}{35b^2x^3}-\frac{2\sqrt{bx+cx^2}}{7bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[b*x + c*x^2]),x]
```

```
[Out] (-2*Sqrt[b*x + c*x^2])/(7*b*x^4) + (12*c*Sqrt[b*x + c*x^2])/(35*b^2*x^3) -
(16*c^2*Sqrt[b*x + c*x^2])/(35*b^3*x^2) + (32*c^3*Sqrt[b*x + c*x^2])/(35*b^
4*x)
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\sqrt{bx+cx^2}}\,dx &= -\frac{2\sqrt{bx+cx^2}}{7bx^4}-\frac{(6c)\int \frac{1}{x^3\sqrt{bx+cx^2}}\,dx}{7b}\\
&= -\frac{2\sqrt{bx+cx^2}}{7bx^4}+\frac{12c\sqrt{bx+cx^2}}{35b^2x^3}+\frac{\left(24c^2\right)\int \frac{1}{x^2\sqrt{bx+cx^2}}\,dx}{35b^2}\\
&= -\frac{2\sqrt{bx+cx^2}}{7bx^4}+\frac{12c\sqrt{bx+cx^2}}{35b^2x^3}-\frac{16c^2\sqrt{bx+cx^2}}{35b^3x^2}-\frac{\left(16c^3\right)\int \frac{1}{x\sqrt{bx+cx^2}}\,dx}{35b^3}\\
&= -\frac{2\sqrt{bx+cx^2}}{7bx^4}+\frac{12c\sqrt{bx+cx^2}}{35b^2x^3}-\frac{16c^2\sqrt{bx+cx^2}}{35b^3x^2}+\frac{32c^3\sqrt{bx+cx^2}}{35b^4x}
\end{aligned}$$

Mathematica [A] time = 0.0136753, size = 51, normalized size = 0.51

$$\frac{2\sqrt{x(b+cx)}\left(6b^2cx-5b^3-8bc^2x^2+16c^3x^3\right)}{35b^4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[b*x + c*x^2]),x]
```

```
[Out] (2*Sqrt[x*(b + c*x)]*(-5*b^3 + 6*b^2*c*x - 8*b*c^2*x^2 + 16*c^3*x^3))/(35*b
^4*x^4)
```

Maple [A] time = 0.046, size = 55, normalized size = 0.6

$$-\frac{(2cx+2b)\left(-16x^3c^3+8bx^2c^2-6b^2xc+5b^3\right)}{35x^3b^4}\frac{1}{\sqrt{cx^2+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^2+b*x)^(1/2),x)
```

```
[Out] -2/35*(c*x+b)*(-16*c^3*x^3+8*b*c^2*x^2-6*b^2*c*x+5*b^3)/x^3/b^4/(c*x^2+b*x)
^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88261, size = 109, normalized size = 1.09

$$\frac{2\left(16c^3x^3-8bc^2x^2+6b^2cx-5b^3\right)\sqrt{cx^2+bx}}{35b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*c^3*x^3 - 8*b*c^2*x^2 + 6*b^2*c*x - 5*b^3)*sqrt(c*x^2 + b*x)/(b^4*
x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\sqrt{x(b+cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(x*(b + c*x))), x)
```

Giac [A] time = 1.33767, size = 144, normalized size = 1.44

$$\frac{2\left(70\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3c^{\frac{3}{2}}+84\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^2bc+35\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)b^2\sqrt{c}+5\,b^3\right)}{35\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/35*(70*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*c^(3/2) + 84*(sqrt(c)*x - sqrt(c
*x^2 + b*x))^2*b*c + 35*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b^2*sqrt(c) + 5*b^3
)/(sqrt(c)*x - sqrt(c*x^2 + b*x))^7
```

3.51 $\int \frac{1}{x^5\sqrt{bx+cx^2}}\,dx$

Optimal. Leaf size=126

$$-\frac{256c^4\sqrt{bx+cx^2}}{315b^5x}+\frac{128c^3\sqrt{bx+cx^2}}{315b^4x^2}-\frac{32c^2\sqrt{bx+cx^2}}{105b^3x^3}+\frac{16c\sqrt{bx+cx^2}}{63b^2x^4}-\frac{2\sqrt{bx+cx^2}}{9bx^5}$$

```
[Out] (-2*Sqrt[b*x + c*x^2])/(9*b*x^5) + (16*c*Sqrt[b*x + c*x^2])/(63*b^2*x^4) -
(32*c^2*Sqrt[b*x + c*x^2])/(105*b^3*x^3) + (128*c^3*Sqrt[b*x + c*x^2])/(315
*b^4*x^2) - (256*c^4*Sqrt[b*x + c*x^2])/(315*b^5*x)
```

Rubi [A] time = 0.0547222, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {658, 650}

$$-\frac{256c^4\sqrt{bx+cx^2}}{315b^5x}+\frac{128c^3\sqrt{bx+cx^2}}{315b^4x^2}-\frac{32c^2\sqrt{bx+cx^2}}{105b^3x^3}+\frac{16c\sqrt{bx+cx^2}}{63b^2x^4}-\frac{2\sqrt{bx+cx^2}}{9bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[b*x + c*x^2]),x]
```

```
[Out] (-2*Sqrt[b*x + c*x^2])/(9*b*x^5) + (16*c*Sqrt[b*x + c*x^2])/(63*b^2*x^4) -
(32*c^2*Sqrt[b*x + c*x^2])/(105*b^3*x^3) + (128*c^3*Sqrt[b*x + c*x^2])/(315
*b^4*x^2) - (256*c^4*Sqrt[b*x + c*x^2])/(315*b^5*x)
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^5\sqrt{bx+cx^2}}\,dx &= -\frac{2\sqrt{bx+cx^2}}{9bx^5} - \frac{(8c)\int \frac{1}{x^4\sqrt{bx+cx^2}}\,dx}{9b} \\
&= -\frac{2\sqrt{bx+cx^2}}{9bx^5} + \frac{16c\sqrt{bx+cx^2}}{63b^2x^4} + \frac{\left(16c^2\right)\int \frac{1}{x^3\sqrt{bx+cx^2}}\,dx}{21b^2} \\
&= -\frac{2\sqrt{bx+cx^2}}{9bx^5} + \frac{16c\sqrt{bx+cx^2}}{63b^2x^4} - \frac{32c^2\sqrt{bx+cx^2}}{105b^3x^3} - \frac{\left(64c^3\right)\int \frac{1}{x^2\sqrt{bx+cx^2}}\,dx}{105b^3} \\
&= -\frac{2\sqrt{bx+cx^2}}{9bx^5} + \frac{16c\sqrt{bx+cx^2}}{63b^2x^4} - \frac{32c^2\sqrt{bx+cx^2}}{105b^3x^3} + \frac{128c^3\sqrt{bx+cx^2}}{315b^4x^2} + \frac{\left(128c^4\right)\int \frac{1}{x\sqrt{bx+cx^2}}\,dx}{315b^4} \\
&= -\frac{2\sqrt{bx+cx^2}}{9bx^5} + \frac{16c\sqrt{bx+cx^2}}{63b^2x^4} - \frac{32c^2\sqrt{bx+cx^2}}{105b^3x^3} + \frac{128c^3\sqrt{bx+cx^2}}{315b^4x^2} - \frac{256c^4\sqrt{bx+cx^2}}{315b^5x}
\end{aligned}
$$

Mathematica [A] time = 0.01652, size = 62, normalized size = 0.49

$$
-\frac{2\sqrt{x(b+cx)}\left(48b^2c^2x^2-40b^3cx+35b^4-64bc^3x^3+128c^4x^4\right)}{315b^5x^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[b*x + c*x^2]),x]
```

```
[Out] (-2*Sqrt[x*(b + c*x)]*(35*b^4 - 40*b^3*c*x + 48*b^2*c^2*x^2 - 64*b*c^3*x^3
+ 128*c^4*x^4))/(315*b^5*x^5)
```

Maple [A] time = 0.051, size = 66, normalized size = 0.5

$$
-\frac{(2\,cx+2\,b)\left(128\,c^4x^4-64\,x^3c^3b+48\,c^2x^2b^2-40\,cxb^3+35\,b^4\right)}{315\,x^4b^5}\frac{1}{\sqrt{cx^2+bx}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^2+b*x)^(1/2),x)
```

```
[Out] -2/315*(c*x+b)*(128*c^4*x^4-64*b*c^3*x^3+48*b^2*c^2*x^2-40*b^3*c*x+35*b^4)/
x^4/b^5/(c*x^2+b*x)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.946, size = 140, normalized size = 1.11

$$-\frac{2\left(128\,c^4x^4-64\,bc^3x^3+48\,b^2c^2x^2-40\,b^3cx+35\,b^4\right)\sqrt{cx^2+bx}}{315\,b^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*(128*c^4*x^4 - 64*b*c^3*x^3 + 48*b^2*c^2*x^2 - 40*b^3*c*x + 35*b^4)*
sqrt(c*x^2 + b*x)/(b^5*x^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^5\sqrt{x\,(b+cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(1/(x**5*sqrt(x*(b + c*x))), x)
```

Giac [A] time = 1.18812, size = 184, normalized size = 1.46

$$\frac{2\left(1008\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^4c^2+1680\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3bc^{\frac{3}{2}}+1080\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^2b^2c+315\left(\sqrt{c}x-\sqrt{cx^2+b}\right.}{315\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(1008*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*c^2 + 1680*(sqrt(c)*x - sqrt(
c*x^2 + b*x))^3*b*c^(3/2) + 1080*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*b^2*c +
315*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b^3*sqrt(c) + 35*b^4)/(sqrt(c)*x - sqrt
(c*x^2 + b*x))^9
```

3.52 $\int \frac{x^4}{\left(bx+cx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=97

$$\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{7/2}}+\frac{5x\sqrt{bx+cx^2}}{2c^2}-\frac{15b\sqrt{bx+cx^2}}{4c^3}-\frac{2x^3}{c\sqrt{bx+cx^2}}$$

```
[Out] (-2*x^3)/(c*Sqrt[b*x + c*x^2]) - (15*b*Sqrt[b*x + c*x^2])/(4*c^3) + (5*x*Sq
rt[b*x + c*x^2])/(2*c^2) + (15*b^2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/
(4*c^(7/2))
```

Rubi [A] time = 0.0398188, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {668, 670, 640, 620, 206}

$$\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{7/2}}+\frac{5x\sqrt{bx+cx^2}}{2c^2}-\frac{15b\sqrt{bx+cx^2}}{4c^3}-\frac{2x^3}{c\sqrt{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*x^3)/(c*Sqrt[b*x + c*x^2]) - (15*b*Sqrt[b*x + c*x^2])/(4*c^3) + (5*x*Sq
rt[b*x + c*x^2])/(2*c^2) + (15*b^2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/
(4*c^(7/2))
```

Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\left(bx+cx^2\right)^{3/2}}\,dx &= -\frac{2x^3}{c\sqrt{bx+cx^2}}+\frac{5\int \frac{x^2}{\sqrt{bx+cx^2}}\,dx}{c}\\
&= -\frac{2x^3}{c\sqrt{bx+cx^2}}+\frac{5x\sqrt{bx+cx^2}}{2c^2}-\frac{(15b)\int \frac{x}{\sqrt{bx+cx^2}}\,dx}{4c^2}\\
&= -\frac{2x^3}{c\sqrt{bx+cx^2}}-\frac{15b\sqrt{bx+cx^2}}{4c^3}+\frac{5x\sqrt{bx+cx^2}}{2c^2}+\frac{\left(15b^2\right)\int \frac{1}{\sqrt{bx+cx^2}}\,dx}{8c^3}\\
&= -\frac{2x^3}{c\sqrt{bx+cx^2}}-\frac{15b\sqrt{bx+cx^2}}{4c^3}+\frac{5x\sqrt{bx+cx^2}}{2c^2}+\frac{\left(15b^2\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{bx+cx^2}}\right)}{4c^3}\\
&= -\frac{2x^3}{c\sqrt{bx+cx^2}}-\frac{15b\sqrt{bx+cx^2}}{4c^3}+\frac{5x\sqrt{bx+cx^2}}{2c^2}+\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.0123995, size = 50, normalized size = 0.52

$$\frac{2x^4\sqrt{\frac{cx}{b}+1}\,{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};-\frac{cx}{b}\right)}{7b\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*x^4*Sqrt[1 + (c*x)/b]*Hypergeometric2F1[3/2, 7/2, 9/2, -((c*x)/b)])/(7*b
*Sqrt[x*(b + c*x)])
```

Maple [A] time = 0.063, size = 93, normalized size = 1.

$$\frac{x^3}{2c}\frac{1}{\sqrt{cx^2+bx}}-\frac{5bx^2}{4c^2}\frac{1}{\sqrt{cx^2+bx}}-\frac{15b^2x}{4c^3}\frac{1}{\sqrt{cx^2+bx}}+\frac{15b^2}{8}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx}\right)c^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^2+b*x)^(3/2),x)
```

```
[Out] 1/2*x^3/c/(c*x^2+b*x)^(1/2)-5/4*b/c^2*x^2/(c*x^2+b*x)^(1/2)-15/4*b^2/c^3/(c
*x^2+b*x)^(1/2)*x+15/8*b^2/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2)
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.08516, size = 410, normalized size = 4.23

$$\left[\frac{15\left(b^2cx+b^3\right)\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\left(2\,c^3x^2-5\,bc^2x-15\,b^2c\right)\sqrt{cx^2+bx}}{8\left(c^5x+bc^4\right)},-\frac{15\left(b^2cx+b^3\right)\sqrt{-c}\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*c*x + b^3)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c
)) + 2*(2*c^3*x^2 - 5*b*c^2*x - 15*b^2*c)*sqrt(c*x^2 + b*x))/(c^5*x + b*c^4
), -1/4*(15*(b^2*c*x + b^3)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x
)) - (2*c^3*x^2 - 5*b*c^2*x - 15*b^2*c)*sqrt(c*x^2 + b*x))/(c^5*x + b*c^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(x\,(b+cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral(x**4/(x*(b + c*x))**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.53 $\int \frac{x^3}{(bx+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=69

$$\frac{3\sqrt{bx+cx^2}}{c^2}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{5/2}}-\frac{2x^2}{c\sqrt{bx+cx^2}}$$

```
[Out] (-2*x^2)/(c*Sqrt[b*x + c*x^2]) + (3*Sqrt[b*x + c*x^2])/c^2 - (3*b*ArcTanh[(
Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/c^(5/2)
```

Rubi [A] time = 0.0266901, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {668, 640, 620, 206}

$$\frac{3\sqrt{bx+cx^2}}{c^2}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{5/2}}-\frac{2x^2}{c\sqrt{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*x^2)/(c*Sqrt[b*x + c*x^2]) + (3*Sqrt[b*x + c*x^2])/c^2 - (3*b*ArcTanh[(
Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/c^(5/2)
```

Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(bx+cx^2\right)^{3/2}}\,dx = -\frac{2x^2}{c\sqrt{bx+cx^2}} + \frac{3\int \frac{x}{\sqrt{bx+cx^2}}\,dx}{c}$$

$$= -\frac{2x^2}{c\sqrt{bx+cx^2}} + \frac{3\sqrt{bx+cx^2}}{c^2} - \frac{(3b)\int \frac{1}{\sqrt{bx+cx^2}}\,dx}{2c^2}$$

$$= -\frac{2x^2}{c\sqrt{bx+cx^2}} + \frac{3\sqrt{bx+cx^2}}{c^2} - \frac{(3b)\,\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{bx+cx^2}}\right)}{c^2}$$

$$= -\frac{2x^2}{c\sqrt{bx+cx^2}} + \frac{3\sqrt{bx+cx^2}}{c^2} - \frac{3b\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{5/2}}$$

Mathematica [C] time = 0.0126394, size = 50, normalized size = 0.72

$$\frac{2x^3\sqrt{\frac{cx}{b}+1}\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};-\frac{cx}{b}\right)}{5b\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*x^3*Sqrt[1 + (c*x)/b]*Hypergeometric2F1[3/2, 5/2, 7/2, -((c*x)/b)])/(5*b
*Sqrt[x*(b + c*x)])
```

Maple [A] time = 0.05, size = 68, normalized size = 1.

$$\frac{x^2}{c}\frac{1}{\sqrt{cx^2+bx}} + 3\frac{bx}{c^2\sqrt{cx^2+bx}} - \frac{3b}{2}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}} + \sqrt{cx^2+bx}\right)c^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^2+b*x)^(3/2),x)
```

```
[Out] x^2/c/(c*x^2+b*x)^(1/2)+3*b/c^2/(c*x^2+b*x)^(1/2)*x-3/2*b/c^(5/2)*ln((1/2*b
+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94351, size = 344, normalized size = 4.99

$$\left[\frac{3\left(bcx+b^2\right)\sqrt{c}\log\left(2\,cx+b-2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\left(c^2x+3\,bc\right)\sqrt{cx^2+bx}}{2\left(c^4x+bc^3\right)}, \frac{3\left(bcx+b^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx}\right)+\left(c\right.}{c^4x+bc^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*c*x + b^2)*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 + b*x)*sqrt(c))
+ 2*(c^2*x + 3*b*c)*sqrt(c*x^2 + b*x))/(c^4*x + b*c^3), (3*(b*c*x + b^2)*sq
rt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)) + (c^2*x + 3*b*c)*sqrt(c*x^
2 + b*x))/(c^4*x + b*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(x\,(b+cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral(x**3/(x*(b + c*x))**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.54 $\int \frac{x^2}{(bx+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=48

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{3/2}}-\frac{2x}{c\sqrt{bx+cx^2}}$$

```
[Out] (-2*x)/(c*Sqrt[b*x + c*x^2]) + (2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/c
^(3/2)
```

Rubi [A] time = 0.0175548, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {652, 620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{3/2}}-\frac{2x}{c\sqrt{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*x)/(c*Sqrt[b*x + c*x^2]) + (2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/c
^(3/2)
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))^2*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[(e*(d + e*x)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[
(e^2*(p + 2))/(c*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(bx+cx^2\right)^{3/2}}\,dx = -\frac{2x}{c\sqrt{bx+cx^2}}+\frac{\int \frac{1}{\sqrt{bx+cx^2}}\,dx}{c}$$
$$= -\frac{2x}{c\sqrt{bx+cx^2}}+\frac{2\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{bx+cx^2}}\right)}{c}$$
$$= -\frac{2x}{c\sqrt{bx+cx^2}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{3/2}}$$

Mathematica [A] time = 0.0812652, size = 67, normalized size = 1.4

$$\frac{2\sqrt{b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)-2\sqrt{c}x}{c^{3/2}\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[c]*x + 2*Sqrt[b]*Sqrt[x]*Sqrt[1 + (c*x)/b]*ArcSinh[(Sqrt[c]*Sqrt[x
])/Sqrt[b]])/(c^(3/2)*Sqrt[x*(b + c*x)])
```

Maple [A] time = 0.048, size = 47, normalized size = 1.

$$-2\,\frac{x}{c\sqrt{cx^2+bx}}+\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx}\right)c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^2+b*x)^(3/2),x)
```

```
[Out] -2*x/c/(c*x^2+b*x)^(1/2)+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2)
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.90494, size = 289, normalized size = 6.02

$$\left[\frac{(cx+b)\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\,\sqrt{cx^2+bx}c}{c^3x+bc^2},-\frac{2\left((cx+b)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx}\right)+\sqrt{cx^2+bx}c\right)}{c^3x+bc^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [((c*x + b)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 2*sqrt(c
*x^2 + b*x)*c)/(c^3*x + b*c^2), -2*((c*x + b)*sqrt(-c)*arctan(sqrt(c*x^2 +
b*x)*sqrt(-c)/(c*x)) + sqrt(c*x^2 + b*x)*c)/(c^3*x + b*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(x(b+cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral(x**2/(x*(b + c*x))**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.55 $\int \frac{x}{(bx+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=19

$$\frac{2x}{b\sqrt{bx+cx^2}}$$

```
[Out] (2*x)/(b*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.0053672, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {636}

$$\frac{2x}{b\sqrt{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*x)/(b*Sqrt[b*x + c*x^2])
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x}{(bx+cx^2)^{3/2}}\,dx = \frac{2x}{b\sqrt{bx+cx^2}}$$

Mathematica [A] time = 0.006217, size = 17, normalized size = 0.89

$$\frac{2x}{b\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*x)/(b*Sqrt[x*(b + c*x)])
```

Maple [A] time = 0.047, size = 25, normalized size = 1.3

$$2\,\frac{x^2\,(cx+b)}{b\left(cx^2+bx\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^2+b*x)^(3/2),x)
```

```
[Out] 2*x^2*(c*x+b)/b/(c*x^2+b*x)^(3/2)
```

Maxima [A] time = 1.09251, size = 23, normalized size = 1.21

$$\frac{2\,x}{\sqrt{cx^2+bx}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*x/(sqrt(c*x^2 + b*x)*b)
```

Fricas [A] time = 1.96491, size = 47, normalized size = 2.47

$$\frac{2\sqrt{cx^2+bx}}{bcx+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(c*x^2 + b*x)/(b*c*x + b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x(b+cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral(x/(x*(b + c*x))**(3/2), x)
```

Giac [A] time = 1.20963, size = 43, normalized size = 2.26

$$\frac{2}{\left(\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right)\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] 2/(((sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b)*sqrt(c))
```

3.56 $\int \frac{1}{(bx+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=24

$$-\frac{2(b+2cx)}{b^2\sqrt{bx+cx^2}}$$

```
[Out] (-2*(b + 2*c*x))/(b^2*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.0033446, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {613}

$$-\frac{2(b+2cx)}{b^2\sqrt{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(-3/2),x]
```

```
[Out] (-2*(b + 2*c*x))/(b^2*Sqrt[b*x + c*x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{(bx+cx^2)^{3/2}}\,dx = -\frac{2(b+2cx)}{b^2\sqrt{bx+cx^2}}$$

Mathematica [A] time = 0.0072823, size = 22, normalized size = 0.92

$$-\frac{2(b+2cx)}{b^2\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(-3/2),x]
```

```
[Out] (-2*(b + 2*c*x))/(b^2*Sqrt[x*(b + c*x)])
```

Maple [A] time = 0.045, size = 29, normalized size = 1.2

$$-2\,\frac{x\,(cx+b)\,(2\,cx+b)}{b^2\,(cx^2+bx)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^(3/2),x)
```

```
[Out] -2*x*(c*x+b)*(2*c*x+b)/b^2/(c*x^2+b*x)^(3/2)
```

Maxima [A] time = 1.06922, size = 47, normalized size = 1.96

$$-\frac{4\,cx}{\sqrt{cx^2+bx}b^2}-\frac{2}{\sqrt{cx^2+bx}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] -4*c*x/(sqrt(c*x^2 + b*x)*b^2) - 2/(sqrt(c*x^2 + b*x)*b)
```

Fricas [A] time = 1.90418, size = 73, normalized size = 3.04

$$-\frac{2\,\sqrt{cx^2+bx}(2\,cx+b)}{b^2cx^2+b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(c*x^2 + b*x)*(2*c*x + b)/(b^2*c*x^2 + b^3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral((b*x + c*x**2)**(-3/2), x)
```

Giac [A] time = 1.30407, size = 32, normalized size = 1.33

$$-\frac{2\left(\frac{2\,cx}{b^2}+\frac{1}{b}\right)}{\sqrt{cx^2+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(2*c*x/b^2 + 1/b)/sqrt(c*x^2 + b*x)
```

3.57 $\int \frac{1}{x\left(bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=51

$$\frac{8c(b+2cx)}{3b^3\sqrt{bx+cx^2}}-\frac{2}{3bx\sqrt{bx+cx^2}}$$

```
[Out] -2/(3*b*x*Sqrt[b*x + c*x^2]) + (8*c*(b + 2*c*x))/(3*b^3*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.0129913, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {658, 613}

$$\frac{8c(b+2cx)}{3b^3\sqrt{bx+cx^2}}-\frac{2}{3bx\sqrt{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b*x + c*x^2)^(3/2)),x]
```

```
[Out] -2/(3*b*x*Sqrt[b*x + c*x^2]) + (8*c*(b + 2*c*x))/(3*b^3*Sqrt[b*x + c*x^2])
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(bx+cx^2\right)^{3/2}}\,dx &= -\frac{2}{3bx\sqrt{bx+cx^2}}-\frac{(4c)\int \frac{1}{\left(bx+cx^2\right)^{3/2}}\,dx}{3b}\\ &= -\frac{2}{3bx\sqrt{bx+cx^2}}+\frac{8c(b+2cx)}{3b^3\sqrt{bx+cx^2}}\end{aligned}$$

Mathematica [A] time = 0.011478, size = 40, normalized size = 0.78

$$\frac{2\left(-b^2+4bcx+8c^2x^2\right)}{3b^3x\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*x + c*x^2)^(3/2)),x]

[Out] (2*(-b^2 + 4*b*c*x + 8*c^2*x^2))/(3*b^3*x*Sqrt[x*(b + c*x)])
```

Maple [A] time = 0.045, size = 39, normalized size = 0.8

$$-\frac{(2\,cx+2\,b)\left(-8\,c^2x^2-4\,bcx+b^2\right)}{3\,b^3}\left(cx^2+bx\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^2+b*x)^(3/2),x)

[Out] -2/3*(c*x+b)*(-8*c^2*x^2-4*b*c*x+b^2)/b^3/(c*x^2+b*x)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 1.90221, size = 99, normalized size = 1.94

$$\frac{2\left(8\,c^2x^2+4\,bcx-b^2\right)\sqrt{cx^2+bx}}{3\left(b^3cx^3+b^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x)^(3/2),x, algorithm="fricas")

[Out] 2/3*(8*c^2*x^2 + 4*b*c*x - b^2)*sqrt(c*x^2 + b*x)/(b^3*c*x^3 + b^4*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(x\left(b+cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**2+b*x)**(3/2),x)

[Out] Integral(1/(x*(x*(b + c*x))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2 + bx\right)^{\frac{3}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^2 + b*x)^(3/2)*x), x)
```

3.58 $\int \frac{1}{x^2(bx+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{16c^2(b+2cx)}{5b^4\sqrt{bx+cx^2}}+\frac{4c}{5b^2x\sqrt{bx+cx^2}}-\frac{2}{5bx^2\sqrt{bx+cx^2}}$$

```
[Out] -2/(5*b*x^2*Sqrt[b*x + c*x^2]) + (4*c)/(5*b^2*x*Sqrt[b*x + c*x^2]) - (16*c^
2*(b + 2*c*x))/(5*b^4*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.0244401, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {658, 613}

$$-\frac{16c^2(b+2cx)}{5b^4\sqrt{bx+cx^2}}+\frac{4c}{5b^2x\sqrt{bx+cx^2}}-\frac{2}{5bx^2\sqrt{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(b*x + c*x^2)^(3/2)),x]
```

```
[Out] -2/(5*b*x^2*Sqrt[b*x + c*x^2]) + (4*c)/(5*b^2*x*Sqrt[b*x + c*x^2]) - (16*c^
2*(b + 2*c*x))/(5*b^4*Sqrt[b*x + c*x^2])
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(bx+cx^2\right)^{3/2}}\,dx &= -\frac{2}{5bx^2\sqrt{bx+cx^2}}-\frac{(6c)\int \frac{1}{x\left(bx+cx^2\right)^{3/2}}\,dx}{5b}\\
&= -\frac{2}{5bx^2\sqrt{bx+cx^2}}+\frac{4c}{5b^2x\sqrt{bx+cx^2}}+\frac{\left(8c^2\right)\int \frac{1}{\left(bx+cx^2\right)^{3/2}}\,dx}{5b^2}\\
&= -\frac{2}{5bx^2\sqrt{bx+cx^2}}+\frac{4c}{5b^2x\sqrt{bx+cx^2}}-\frac{16c^2(b+2cx)}{5b^4\sqrt{bx+cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0122747, size = 49, normalized size = 0.64

$$-\frac{2\left(-2b^2cx+b^3+8bc^2x^2+16c^3x^3\right)}{5b^4x^2\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(b^3 - 2*b^2*c*x + 8*b*c^2*x^2 + 16*c^3*x^3))/(5*b^4*x^2*Sqrt[x*(b + c*
x)])
```

Maple [A] time = 0.045, size = 53, normalized size = 0.7

$$-\frac{(2cx+2b)\left(16x^3c^3+8bx^2c^2-2b^2xc+b^3\right)}{5xb^4}\left(cx^2+bx\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^2+b*x)^(3/2),x)
```

```
[Out] -2/5*(c*x+b)*(16*c^3*x^3+8*b*c^2*x^2-2*b^2*c*x+b^3)/x/b^4/(c*x^2+b*x)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86682, size = 123, normalized size = 1.6

$$-\frac{2\left(16c^3x^3+8bc^2x^2-2b^2cx+b^3\right)\sqrt{cx^2+bx}}{5\left(b^4cx^4+b^5x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(16*c^3*x^3 + 8*b*c^2*x^2 - 2*b^2*c*x + b^3)*sqrt(c*x^2 + b*x)/(b^4*c*
x^4 + b^5*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^2\left(x\left(b+cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(x*(b + c*x))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^2 + b*x)^(3/2)*x^2), x)
```

3.59 $\int \frac{1}{x^3(bx+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=103

$$\frac{128c^3(b+2cx)}{35b^5\sqrt{bx+cx^2}}-\frac{32c^2}{35b^3x\sqrt{bx+cx^2}}+\frac{16c}{35b^2x^2\sqrt{bx+cx^2}}-\frac{2}{7bx^3\sqrt{bx+cx^2}}$$

```
[Out] -2/(7*b*x^3*Sqrt[b*x + c*x^2]) + (16*c)/(35*b^2*x^2*Sqrt[b*x + c*x^2]) - (3
2*c^2)/(35*b^3*x*Sqrt[b*x + c*x^2]) + (128*c^3*(b + 2*c*x))/(35*b^5*Sqrt[b*
x + c*x^2])
```

Rubi [A] time = 0.0394125, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {658, 613}

$$\frac{128c^3(b+2cx)}{35b^5\sqrt{bx+cx^2}}-\frac{32c^2}{35b^3x\sqrt{bx+cx^2}}+\frac{16c}{35b^2x^2\sqrt{bx+cx^2}}-\frac{2}{7bx^3\sqrt{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(b*x + c*x^2)^(3/2)),x]
```

```
[Out] -2/(7*b*x^3*Sqrt[b*x + c*x^2]) + (16*c)/(35*b^2*x^2*Sqrt[b*x + c*x^2]) - (3
2*c^2)/(35*b^3*x*Sqrt[b*x + c*x^2]) + (128*c^3*(b + 2*c*x))/(35*b^5*Sqrt[b*
x + c*x^2])
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^3 \left(bx+cx^2\right)^{3/2}}\,dx = -\frac{2}{7bx^3\sqrt{bx+cx^2}} - \frac{(8c)\int \frac{1}{x^2\left(bx+cx^2\right)^{3/2}}\,dx}{7b}$$

$$= -\frac{2}{7bx^3\sqrt{bx+cx^2}} + \frac{16c}{35b^2x^2\sqrt{bx+cx^2}} + \frac{\left(48c^2\right)\int \frac{1}{x\left(bx+cx^2\right)^{3/2}}\,dx}{35b^2}$$

$$= -\frac{2}{7bx^3\sqrt{bx+cx^2}} + \frac{16c}{35b^2x^2\sqrt{bx+cx^2}} - \frac{32c^2}{35b^3x\sqrt{bx+cx^2}} - \frac{\left(64c^3\right)\int \frac{1}{\left(bx+cx^2\right)^{3/2}}\,dx}{35b^3}$$

$$= -\frac{2}{7bx^3\sqrt{bx+cx^2}} + \frac{16c}{35b^2x^2\sqrt{bx+cx^2}} - \frac{32c^2}{35b^3x\sqrt{bx+cx^2}} + \frac{128c^3(b+2cx)}{35b^5\sqrt{bx+cx^2}}$$

Mathematica [A] time = 0.0144757, size = 62, normalized size = 0.6

$$\frac{2\left(-16b^2c^2x^2+8b^3cx-5b^4+64bc^3x^3+128c^4x^4\right)}{35b^5x^3\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(-5*b^4 + 8*b^3*c*x - 16*b^2*c^2*x^2 + 64*b*c^3*x^3 + 128*c^4*x^4))/(35*
b^5*x^3*Sqrt[x*(b + c*x)])
```

Maple [A] time = 0.045, size = 66, normalized size = 0.6

$$-\frac{(2\,cx+2\,b)\left(-128\,c^4x^4-64\,x^3c^3b+16\,c^2x^2b^2-8\,cxb^3+5\,b^4\right)}{35\,x^2b^5}\left(cx^2+bx\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^2+b*x)^(3/2),x)
```

```
[Out] -2/35*(c*x+b)*(-128*c^4*x^4-64*b*c^3*x^3+16*b^2*c^2*x^2-8*b^3*c*x+5*b^4)/x^
2/b^5/(c*x^2+b*x)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.93629, size = 151, normalized size = 1.47

$$\frac{2\left(128\,c^4x^4+64\,bc^3x^3-16\,b^2c^2x^2+8\,b^3cx-5\,b^4\right)\sqrt{cx^2+bx}}{35\left(b^5cx^5+b^6x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(128*c^4*x^4 + 64*b*c^3*x^3 - 16*b^2*c^2*x^2 + 8*b^3*c*x - 5*b^4)*sqrt
(c*x^2 + b*x)/(b^5*c*x^5 + b^6*x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(x\left(b+cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(x*(b + c*x))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^2 + b*x)^(3/2)*x^3), x)
```

3.60 $\int \frac{x^6}{\left(ax+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=122

$$\frac{35a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{9/2}}-\frac{14x^3}{3b^2\sqrt{ax+bx^2}}+\frac{35x\sqrt{ax+bx^2}}{6b^3}-\frac{35a\sqrt{ax+bx^2}}{4b^4}-\frac{2x^5}{3b\left(ax+bx^2\right)^{3/2}}$$

```
[Out] (-2*x^5)/(3*b*(a*x + b*x^2)^(3/2)) - (14*x^3)/(3*b^2*Sqrt[a*x + b*x^2]) - (
35*a*Sqrt[a*x + b*x^2])/(4*b^4) + (35*x*Sqrt[a*x + b*x^2])/(6*b^3) + (35*a^
2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(4*b^(9/2))
```

Rubi [A] time = 0.0582204, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {668, 670, 640, 620, 206}

$$\frac{35a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{9/2}}-\frac{14x^3}{3b^2\sqrt{ax+bx^2}}+\frac{35x\sqrt{ax+bx^2}}{6b^3}-\frac{35a\sqrt{ax+bx^2}}{4b^4}-\frac{2x^5}{3b\left(ax+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a*x + b*x^2)^(5/2),x]
```


```
[Out] (-2*x^5)/(3*b*(a*x + b*x^2)^(3/2)) - (14*x^3)/(3*b^2*Sqrt[a*x + b*x^2]) - (
35*a*Sqrt[a*x + b*x^2])/(4*b^4) + (35*x*Sqrt[a*x + b*x^2])/(6*b^3) + (35*a^
2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(4*b^(9/2))
```


Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{\left(ax+bx^2\right)^{5/2}}\,dx &= -\frac{2x^5}{3b\left(ax+bx^2\right)^{3/2}}+\frac{7\int \frac{x^4}{\left(ax+bx^2\right)^{3/2}}\,dx}{3b}\\ &= -\frac{2x^5}{3b\left(ax+bx^2\right)^{3/2}}-\frac{14x^3}{3b^2\sqrt{ax+bx^2}}+\frac{35\int \frac{x^2}{\sqrt{ax+bx^2}}\,dx}{3b^2}\\ &= -\frac{2x^5}{3b\left(ax+bx^2\right)^{3/2}}-\frac{14x^3}{3b^2\sqrt{ax+bx^2}}+\frac{35x\sqrt{ax+bx^2}}{6b^3}-\frac{(35a)\int \frac{x}{\sqrt{ax+bx^2}}\,dx}{4b^3}\\ &= -\frac{2x^5}{3b\left(ax+bx^2\right)^{3/2}}-\frac{14x^3}{3b^2\sqrt{ax+bx^2}}-\frac{35a\sqrt{ax+bx^2}}{4b^4}+\frac{35x\sqrt{ax+bx^2}}{6b^3}+\frac{\left(35a^2\right)\int \frac{1}{\sqrt{ax+bx^2}}\,dx}{8b^4}\\ &= -\frac{2x^5}{3b\left(ax+bx^2\right)^{3/2}}-\frac{14x^3}{3b^2\sqrt{ax+bx^2}}-\frac{35a\sqrt{ax+bx^2}}{4b^4}+\frac{35x\sqrt{ax+bx^2}}{6b^3}+\frac{\left(35a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\right.}{4b^4}\\ &= -\frac{2x^5}{3b\left(ax+bx^2\right)^{3/2}}-\frac{14x^3}{3b^2\sqrt{ax+bx^2}}-\frac{35a\sqrt{ax+bx^2}}{4b^4}+\frac{35x\sqrt{ax+bx^2}}{6b^3}+\frac{35a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{9/2}}\end{aligned}$$

Mathematica [C] time = 0.0145022, size = 50, normalized size = 0.41

$$\frac{2x^5\sqrt{\frac{bx}{a}+1}\,{}_2F_1\left(\frac{5}{2},\frac{9}{2};\frac{11}{2};-\frac{bx}{a}\right)}{9a^2\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a*x + b*x^2)^(5/2),x]
```

```
[Out] (2*x^5*Sqrt[1 + (b*x)/a]*Hypergeometric2F1[5/2, 9/2, 11/2, -((b*x)/a)])/(9*
a^2*Sqrt[x*(a + b*x)])
```

Maple [A] time = 0.049, size = 176, normalized size = 1.4

$$\frac{x^5}{2b}\left(bx^2+ax\right)^{-\frac{3}{2}}-\frac{7ax^4}{4b^2}\left(bx^2+ax\right)^{-\frac{3}{2}}-\frac{35a^2x^3}{24b^3}\left(bx^2+ax\right)^{-\frac{3}{2}}+\frac{35x^2a^3}{16b^4}\left(bx^2+ax\right)^{-\frac{3}{2}}+\frac{35a^4x}{48b^5}\left(bx^2+ax\right)^{-\frac{3}{2}}-\frac{245}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a*x)^(5/2),x)
```

```
[Out] 1/2*x^5/b/(b*x^2+a*x)^(3/2)-7/4/b^2*a*x^4/(b*x^2+a*x)^(3/2)-35/24/b^3*a^2*x
^3/(b*x^2+a*x)^(3/2)+35/16/b^4*a^3*x^2/(b*x^2+a*x)^(3/2)+35/48/b^5*a^4/(b*x
^2+a*x)^(3/2)*x-245/24/b^4*a^2/(b*x^2+a*x)^(1/2)*x-35/48/b^5*a^3/(b*x^2+a*x
)^(1/2)+35/8/b^(9/2)*a^2*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.98898, size = 556, normalized size = 4.56

$$\left[\frac{105\left(a^2b^2x^2+2\,a^3bx+a^4\right)\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\left(6\,b^4x^3-21\,ab^3x^2-140\,a^2b^2x-105\,a^3b\right)\sqrt{bx^2+ax}}{24\left(b^7x^2+2\,ab^6x+a^2b^5\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(105*(a^2*b^2*x^2 + 2*a^3*b*x + a^4)*sqrt(b)*log(2*b*x + a + 2*sqrt(b
*x^2 + a*x)*sqrt(b)) + 2*(6*b^4*x^3 - 21*a*b^3*x^2 - 140*a^2*b^2*x - 105*a^
3*b)*sqrt(b*x^2 + a*x))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5), -1/12*(105*(a^2*b^
2*x^2 + 2*a^3*b*x + a^4)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x))
- (6*b^4*x^3 - 21*a*b^3*x^2 - 140*a^2*b^2*x - 105*a^3*b)*sqrt(b*x^2 + a*x))
/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(x\,(a+bx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a*x)**(5/2),x)
```

```
[Out] Integral(x**6/(x*(a + b*x))**(5/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.61 $\int \frac{x^5}{(ax+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{10x^2}{3b^2\sqrt{ax+bx^2}}+\frac{5\sqrt{ax+bx^2}}{b^3}-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{7/2}}-\frac{2x^4}{3b\left(ax+bx^2\right)^{3/2}}$$

```
[Out] (-2*x^4)/(3*b*(a*x + b*x^2)^(3/2)) - (10*x^2)/(3*b^2*Sqrt[a*x + b*x^2]) + (
5*Sqrt[a*x + b*x^2])/b^3 - (5*a*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/b^(
7/2)
```

Rubi [A] time = 0.0414738, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {668, 640, 620, 206}

$$-\frac{10x^2}{3b^2\sqrt{ax+bx^2}}+\frac{5\sqrt{ax+bx^2}}{b^3}-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{7/2}}-\frac{2x^4}{3b\left(ax+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*x + b*x^2)^(5/2),x]
```

```
[Out] (-2*x^4)/(3*b*(a*x + b*x^2)^(3/2)) - (10*x^2)/(3*b^2*Sqrt[a*x + b*x^2]) + (
5*Sqrt[a*x + b*x^2])/b^3 - (5*a*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/b^(
7/2)
```

Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5}{\left(ax+bx^2\right)^{5/2}}\,dx = -\frac{2x^4}{3b\left(ax+bx^2\right)^{3/2}}+\frac{5\int \frac{x^3}{\left(ax+bx^2\right)^{3/2}}\,dx}{3b}$$

$$= -\frac{2x^4}{3b\left(ax+bx^2\right)^{3/2}}-\frac{10x^2}{3b^2\sqrt{ax+bx^2}}+\frac{5\int \frac{x}{\sqrt{ax+bx^2}}\,dx}{b^2}$$

$$= -\frac{2x^4}{3b\left(ax+bx^2\right)^{3/2}}-\frac{10x^2}{3b^2\sqrt{ax+bx^2}}+\frac{5\sqrt{ax+bx^2}}{b^3}-\frac{(5a)\int \frac{1}{\sqrt{ax+bx^2}}\,dx}{2b^3}$$

$$= -\frac{2x^4}{3b\left(ax+bx^2\right)^{3/2}}-\frac{10x^2}{3b^2\sqrt{ax+bx^2}}+\frac{5\sqrt{ax+bx^2}}{b^3}-\frac{(5a)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{ax+bx^2}}\right)}{b^3}$$

$$= -\frac{2x^4}{3b\left(ax+bx^2\right)^{3/2}}-\frac{10x^2}{3b^2\sqrt{ax+bx^2}}+\frac{5\sqrt{ax+bx^2}}{b^3}-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{7/2}}$$

Mathematica [C] time = 0.0131385, size = 50, normalized size = 0.53

$$\frac{2x^4\sqrt{\frac{bx}{a}+1}\,{}_2F_1\left(\frac{5}{2},\frac{7}{2};\frac{9}{2};-\frac{bx}{a}\right)}{7a^2\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*x + b*x^2)^(5/2),x]
```

```
[Out] (2*x^4*Sqrt[1 + (b*x)/a]*Hypergeometric2F1[5/2, 7/2, 9/2, -((b*x)/a)])/(7*a
^2*Sqrt[x*(a + b*x)])
```

Maple [A] time = 0.047, size = 149, normalized size = 1.6

$$\frac{x^4}{b}\left(bx^2+ax\right)^{-\frac{3}{2}}+\frac{5\,ax^3}{6\,b^2}\left(bx^2+ax\right)^{-\frac{3}{2}}-\frac{5\,a^2x^2}{4\,b^3}\left(bx^2+ax\right)^{-\frac{3}{2}}-\frac{5\,xa^3}{12\,b^4}\left(bx^2+ax\right)^{-\frac{3}{2}}+\frac{35\,ax}{6\,b^3}\frac{1}{\sqrt{bx^2+ax}}+\frac{5\,a^2}{12\,b^4}\frac{1}{\sqrt{bx^2 -}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a*x)^(5/2),x)
```

```
[Out] x^4/b/(b*x^2+a*x)^(3/2)+5/6/b^2*a*x^3/(b*x^2+a*x)^(3/2)-5/4/b^3*a^2*x^2/(b*
x^2+a*x)^(3/2)-5/12/b^4*a^3/(b*x^2+a*x)^(3/2)*x+35/6/b^3*a/(b*x^2+a*x)^(1/2
)*x+5/12/b^4*a^2/(b*x^2+a*x)^(1/2)-5/2/b^(7/2)*a*ln((1/2*a+b*x)/b^(1/2)+(b*
x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.92178, size = 493, normalized size = 5.24

$$\left[\frac{15\left(ab^{2}x^{2}+2\,a^{2}bx+a^{3}\right)\sqrt{b}\log\left(2\,bx+a-2\,\sqrt{bx^{2}+ax}\sqrt{b}\right)+2\left(3\,b^{3}x^{2}+20\,ab^{2}x+15\,a^{2}b\right)\sqrt{bx^{2}+ax}}{6\left(b^{6}x^{2}+2\,ab^{5}x+a^{2}b^{4}\right)}, \frac{15\left(ab^{2}x^{2}+2\,a^{2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a*b^2*x^2 + 2*a^2*b*x + a^3)*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2
 + a*x)*sqrt(b)) + 2*(3*b^3*x^2 + 20*a*b^2*x + 15*a^2*b)*sqrt(b*x^2 + a*x))
/(b^6*x^2 + 2*a*b^5*x + a^2*b^4), 1/3*(15*(a*b^2*x^2 + 2*a^2*b*x + a^3)*sqr
t(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x)) + (3*b^3*x^2 + 20*a*b^2*x +
15*a^2*b)*sqrt(b*x^2 + a*x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{5}}{\left(x\left(a+bx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a*x)**(5/2),x)
```

```
[Out] Integral(x**5/(x*(a + b*x))**(5/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.62 $\int \frac{x^4}{(ax+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=71

$$-\frac{2x}{b^2\sqrt{ax+bx^2}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{5/2}}-\frac{2x^3}{3b\left(ax+bx^2\right)^{3/2}}$$

```
[Out] (-2*x^3)/(3*b*(a*x + b*x^2)^(3/2)) - (2*x)/(b^2*Sqrt[a*x + b*x^2]) + (2*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/b^(5/2)
```

Rubi [A] time = 0.0300053, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {668, 652, 620, 206}

$$-\frac{2x}{b^2\sqrt{ax+bx^2}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{5/2}}-\frac{2x^3}{3b\left(ax+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x + b*x^2)^(5/2),x]
```

```
[Out] (-2*x^3)/(3*b*(a*x + b*x^2)^(3/2)) - (2*x)/(b^2*Sqrt[a*x + b*x^2]) + (2*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/b^(5/2)
```

Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))^2*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[(e*(d + e*x)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[
(e^2*(p + 2))/(c*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4}{(ax+bx^2)^{5/2}}\,dx = -\frac{2x^3}{3b(ax+bx^2)^{3/2}} + \frac{\int \frac{x^2}{(ax+bx^2)^{3/2}}\,dx}{b}$$
$$= -\frac{2x^3}{3b(ax+bx^2)^{3/2}} - \frac{2x}{b^2\sqrt{ax+bx^2}} + \frac{\int \frac{1}{\sqrt{ax+bx^2}}\,dx}{b^2}$$
$$= -\frac{2x^3}{3b(ax+bx^2)^{3/2}} - \frac{2x}{b^2\sqrt{ax+bx^2}} + \frac{2\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{ax+bx^2}}\right)}{b^2}$$
$$= -\frac{2x^3}{3b(ax+bx^2)^{3/2}} - \frac{2x}{b^2\sqrt{ax+bx^2}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{5/2}}$$

Mathematica [A] time = 0.160041, size = 84, normalized size = 1.18

$$\frac{x\left(6\sqrt{a}\sqrt{x}(a+bx)\sqrt{\frac{bx}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)-2\sqrt{b}x(3a+4bx)\right)}{3b^{5/2}(x(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x + b*x^2)^(5/2),x]
```

```
[Out] (x*(-2*Sqrt[b]*x*(3*a + 4*b*x) + 6*Sqrt[a]*Sqrt[x]*(a + b*x)*Sqrt[1 + (b*x)
/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]))/(3*b^(5/2)*(x*(a + b*x))^(3/2))
```

Maple [B] time = 0.044, size = 123, normalized size = 1.7

$$-\frac{x^3}{3b}\left(bx^2+ax\right)^{-\frac{3}{2}}+\frac{ax^2}{2b^2}\left(bx^2+ax\right)^{-\frac{3}{2}}+\frac{a^2x}{6b^3}\left(bx^2+ax\right)^{-\frac{3}{2}}-\frac{7x}{3b^2}\frac{1}{\sqrt{bx^2+ax}}-\frac{a}{6b^3}\frac{1}{\sqrt{bx^2+ax}}+\ln\left(\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}}+\sqrt{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a*x)^(5/2),x)
```

```
[Out] -1/3*x^3/b/(b*x^2+a*x)^(3/2)+1/2/b^2*a*x^2/(b*x^2+a*x)^(3/2)+1/6/b^3*a^2/(b
*x^2+a*x)^(3/2)*x-7/3*x/b^2/(b*x^2+a*x)^(1/2)-1/6/b^3*a/(b*x^2+a*x)^(1/2)+1
/b^(5/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.02773, size = 432, normalized size = 6.08

$$\left[\frac{3\left(b^2x^2+2\,abx+a^2\right)\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\left(4\,b^2x+3\,ab\right)\sqrt{bx^2+ax}}{3\left(b^5x^2+2\,ab^4x+a^2b^3\right)},-\frac{2\left(3\left(b^2x^2+2\,abx+a^2\right)\sqrt{-b}\right.}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*
x)*sqrt(b)) - 2*(4*b^2*x + 3*a*b)*sqrt(b*x^2 + a*x))/(b^5*x^2 + 2*a*b^4*x +
 a^2*b^3), -2/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-b)*arctan(sqrt(b*x^2 + a
*x)*sqrt(-b)/(b*x)) + (4*b^2*x + 3*a*b)*sqrt(b*x^2 + a*x))/(b^5*x^2 + 2*a*b
^4*x + a^2*b^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(x\left(a+bx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a*x)**(5/2),x)
```

```
[Out] Integral(x**4/(x*(a + b*x))**(5/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.63 $\int \frac{x^3}{(ax+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=23

$$\frac{2x^3}{3a\left(ax+bx^2\right)^{3/2}}$$

```
[Out] (2*x^3)/(3*a*(a*x + b*x^2)^(3/2))
```

Rubi [A] time = 0.0076737, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {650}

$$\frac{2x^3}{3a\left(ax+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x + b*x^2)^(5/2),x]
```

```
[Out] (2*x^3)/(3*a*(a*x + b*x^2)^(3/2))
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(ax+bx^2\right)^{5/2}}\,dx = \frac{2x^3}{3a\left(ax+bx^2\right)^{3/2}}$$

Mathematica [A] time = 0.0098519, size = 21, normalized size = 0.91

$$\frac{2x^3}{3a(x(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x + b*x^2)^(5/2),x]
```

```
[Out] (2*x^3)/(3*a*(x*(a + b*x))^(3/2))
```

Maple [A] time = 0.043, size = 25, normalized size = 1.1

$$\frac{2\,x^4\,(bx+a)}{3\,a}\left(bx^2+ax\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a*x)^(5/2),x)
```

```
[Out] 2/3*x^4*(b*x+a)/a/(b*x^2+a*x)^(5/2)
```

Maxima [B] time = 1.13721, size = 100, normalized size = 4.35

$$-\frac{x^2}{\left(bx^2+ax\right)^{\frac{3}{2}}b}-\frac{ax}{3\left(bx^2+ax\right)^{\frac{3}{2}}b^2}+\frac{2x}{3\sqrt{bx^2+ax}ab}+\frac{1}{3\sqrt{bx^2+ax}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -x^2/((b*x^2 + a*x)^(3/2)*b) - 1/3*a*x/((b*x^2 + a*x)^(3/2)*b^2) + 2/3*x/(s
qrt(b*x^2 + a*x)*a*b) + 1/3/(sqrt(b*x^2 + a*x)*b^2)
```

Fricas [A] time = 1.84234, size = 74, normalized size = 3.22

$$\frac{2\sqrt{bx^2+ax}x}{3\left(ab^2x^2+2a^2bx+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x^2 + a*x)*x/(a*b^2*x^2 + 2*a^2*b*x + a^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(x\left(a+bx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a*x)**(5/2),x)
```

```
[Out] Integral(x**3/(x*(a + b*x))**(5/2), x)
```

Giac [B] time = 1.15981, size = 120, normalized size = 5.22

$$\frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2b^{\frac{3}{2}}+3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)ab+a^2\sqrt{b}\right)}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right)^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*b^(3/2) + 3*(sqrt(b)*x - sqrt(b*x^
2 + a*x))*a*b + a^2*sqrt(b))/(((sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a)
^3*b^2)
```

3.64 $\int \frac{x^2}{\left(ax+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=51

$$\frac{2(a+2bx)}{3a^2b\sqrt{ax+bx^2}}-\frac{2x}{3b\left(ax+bx^2\right)^{3/2}}$$

```
[Out] (-2*x)/(3*b*(a*x + b*x^2)^(3/2)) + (2*(a + 2*b*x))/(3*a^2*b*Sqrt[a*x + b*x^
2])
```

Rubi [A] time = 0.0134114, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {652, 613}

$$\frac{2(a+2bx)}{3a^2b\sqrt{ax+bx^2}}-\frac{2x}{3b\left(ax+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x + b*x^2)^(5/2),x]
```

```
[Out] (-2*x)/(3*b*(a*x + b*x^2)^(3/2)) + (2*(a + 2*b*x))/(3*a^2*b*Sqrt[a*x + b*x^
2])
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))^2*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[(e*(d + e*x)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[
(e^2*(p + 2))/(c*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(ax+bx^2\right)^{5/2}}\,dx &= -\frac{2x}{3b\left(ax+bx^2\right)^{3/2}}-\frac{\int \frac{1}{\left(ax+bx^2\right)^{3/2}}\,dx}{3b}\\ &= -\frac{2x}{3b\left(ax+bx^2\right)^{3/2}}+\frac{2(a+2bx)}{3a^2b\sqrt{ax+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.0110211, size = 29, normalized size = 0.57

$$\frac{2x^2(3a+2bx)}{3a^2(x(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x + b*x^2)^(5/2),x]
```

```
[Out] (2*x^2*(3*a + 2*b*x))/(3*a^2*(x*(a + b*x))^(3/2))
```

Maple [A] time = 0.045, size = 33, normalized size = 0.7

$$\frac{2\,x^{3}\,(bx+a)\,(2\,bx+3\,a)}{3\,a^{2}}\left(bx^{2}+ax\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a*x)^(5/2),x)
```

```
[Out] 2/3*x^3*(b*x+a)*(2*b*x+3*a)/a^2/(b*x^2+a*x)^(5/2)
```

Maxima [A] time = 1.1577, size = 73, normalized size = 1.43

$$\frac{4\,x}{3\,\sqrt{bx^{2}+ax}a^{2}}-\frac{2\,x}{3\left(bx^{2}+ax\right)^{\frac{3}{2}}b}+\frac{2}{3\,\sqrt{bx^{2}+ax}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 4/3*x/(sqrt(b*x^2 + a*x)*a^2) - 2/3*x/((b*x^2 + a*x)^(3/2)*b) + 2/3/(sqrt(b
*x^2 + a*x)*a*b)
```

Fricas [A] time = 2.07175, size = 93, normalized size = 1.82

$$\frac{2\,\sqrt{bx^{2}+ax}(2\,bx+3\,a)}{3\left(a^{2}b^{2}x^{2}+2\,a^{3}bx+a^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x^2 + a*x)*(2*b*x + 3*a)/(a^2*b^2*x^2 + 2*a^3*b*x + a^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{2}}{(x\,(a+bx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a*x)**(5/2),x)
```

```
[Out] Integral(x**2/(x*(a + b*x))**(5/2), x)
```

Giac [A] time = 1.21426, size = 82, normalized size = 1.61

$$\frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)b+2\,a\sqrt{b}\right)}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a*x))*b + 2*a*sqrt(b))/(((sqrt(b)*x - sqrt
(b*x^2 + a*x))*sqrt(b) + a)^3*b)
```

3.65 $\int \frac{x}{\left(ax+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=48

$$\frac{2x}{3a\left(ax+bx^2\right)^{3/2}}-\frac{8(a+2bx)}{3a^3\sqrt{ax+bx^2}}$$

```
[Out] (2*x)/(3*a*(a*x + b*x^2)^(3/2)) - (8*(a + 2*b*x))/(3*a^3*Sqrt[a*x + b*x^2])
```

Rubi [A] time = 0.0111226, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {638, 613}

$$\frac{2x}{3a\left(ax+bx^2\right)^{3/2}}-\frac{8(a+2bx)}{3a^3\sqrt{ax+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x + b*x^2)^(5/2),x]
```

```
[Out] (2*x)/(3*a*(a*x + b*x^2)^(3/2)) - (8*(a + 2*b*x))/(3*a^3*Sqrt[a*x + b*x^2])
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(ax+bx^2\right)^{5/2}}\,dx &= \frac{2x}{3a\left(ax+bx^2\right)^{3/2}}+\frac{4\int \frac{1}{\left(ax+bx^2\right)^{3/2}}\,dx}{3a}\\ &= \frac{2x}{3a\left(ax+bx^2\right)^{3/2}}-\frac{8(a+2bx)}{3a^3\sqrt{ax+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.0128015, size = 38, normalized size = 0.79

$$-\frac{2x\left(3a^2+12abx+8b^2x^2\right)}{3a^3(x(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x + b*x^2)^(5/2),x]
```

```
[Out] (-2*x*(3*a^2 + 12*a*b*x + 8*b^2*x^2))/(3*a^3*(x*(a + b*x))^(3/2))
```

Maple [A] time = 0.049, size = 44, normalized size = 0.9

$$-\frac{2\,x^2\,(bx+a)\left(8\,b^2x^2+12\,bxa+3\,a^2\right)}{3\,a^3}\left(bx^2+ax\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a*x)^(5/2),x)
```

```
[Out] -2/3*x^2*(b*x+a)*(8*b^2*x^2+12*a*b*x+3*a^2)/a^3/(b*x^2+a*x)^(5/2)
```

Maxima [A] time = 1.04203, size = 70, normalized size = 1.46

$$\frac{2\,x}{3\left(bx^2+ax\right)^{\frac{3}{2}}a}-\frac{16\,bx}{3\,\sqrt{bx^2+ax}a^3}-\frac{8}{3\,\sqrt{bx^2+ax}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x/((b*x^2 + a*x)^(3/2)*a) - 16/3*b*x/(sqrt(b*x^2 + a*x)*a^3) - 8/3/(sqr
t(b*x^2 + a*x)*a^2)
```

Fricas [A] time = 1.86979, size = 123, normalized size = 2.56

$$-\frac{2\left(8\,b^2x^2+12\,abx+3\,a^2\right)\sqrt{bx^2+ax}}{3\left(a^3b^2x^3+2\,a^4bx^2+a^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(8*b^2*x^2 + 12*a*b*x + 3*a^2)*sqrt(b*x^2 + a*x)/(a^3*b^2*x^3 + 2*a^4*
b*x^2 + a^5*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x\,(a+bx)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a*x)**(5/2),x)
```

```
[Out] Integral(x/(x*(a + b*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(bx^2 + ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*x^2 + a*x)^(5/2), x)
```

3.66 $\int \frac{1}{\left(ax+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=54

$$\frac{16b(a+2bx)}{3a^4\sqrt{ax+bx^2}}-\frac{2(a+2bx)}{3a^2\left(ax+bx^2\right)^{3/2}}$$

```
[Out] (-2*(a + 2*b*x))/(3*a^2*(a*x + b*x^2)^(3/2)) + (16*b*(a + 2*b*x))/(3*a^4*Sq
rt[a*x + b*x^2])
```

Rubi [A] time = 0.0090646, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {614, 613}

$$\frac{16b(a+2bx)}{3a^4\sqrt{ax+bx^2}}-\frac{2(a+2bx)}{3a^2\left(ax+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^2)^(-5/2),x]
```

```
[Out] (-2*(a + 2*b*x))/(3*a^2*(a*x + b*x^2)^(3/2)) + (16*b*(a + 2*b*x))/(3*a^4*Sq
rt[a*x + b*x^2])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(ax+bx^2\right)^{5/2}}\,dx &= -\frac{2(a+2bx)}{3a^2\left(ax+bx^2\right)^{3/2}}-\frac{(8b)\int \frac{1}{\left(ax+bx^2\right)^{3/2}}\,dx}{3a^2}\\ &= -\frac{2(a+2bx)}{3a^2\left(ax+bx^2\right)^{3/2}}+\frac{16b(a+2bx)}{3a^4\sqrt{ax+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.0139228, size = 48, normalized size = 0.89

$$\frac{12a^2bx-2a^3+48ab^2x^2+32b^3x^3}{3a^4(x(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^2)^(-5/2),x]
```

```
[Out] (-2*a^3 + 12*a^2*b*x + 48*a*b^2*x^2 + 32*b^3*x^3)/(3*a^4*(x*(a + b*x))^(3/2
))
```

Maple [A] time = 0.045, size = 51, normalized size = 0.9

$$-\frac{2\,x\,(bx+a)\left(-16\,b^3x^3-24\,ab^2x^2-6\,bxa^2+a^3\right)}{3\,a^4}\left(bx^2+ax\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a*x)^(5/2),x)
```

```
[Out] -2/3*x*(b*x+a)*(-16*b^3*x^3-24*a*b^2*x^2-6*a^2*b*x+a^3)/a^4/(b*x^2+a*x)^(5/
2)
```

Maxima [A] time = 1.18368, size = 97, normalized size = 1.8

$$-\frac{4\,bx}{3\left(bx^2+ax\right)^{\frac{3}{2}}a^2}+\frac{32\,b^2x}{3\,\sqrt{bx^2+ax}a^4}-\frac{2}{3\left(bx^2+ax\right)^{\frac{3}{2}}a}+\frac{16\,b}{3\,\sqrt{bx^2+ax}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -4/3*b*x/((b*x^2 + a*x)^(3/2)*a^2) + 32/3*b^2*x/(sqrt(b*x^2 + a*x)*a^4) - 2
/3/((b*x^2 + a*x)^(3/2)*a) + 16/3*b/(sqrt(b*x^2 + a*x)*a^3)
```

Fricas [A] time = 2.01023, size = 144, normalized size = 2.67

$$\frac{2\left(16\,b^3x^3+24\,ab^2x^2+6\,a^2bx-a^3\right)\sqrt{bx^2+ax}}{3\left(a^4b^2x^4+2\,a^5bx^3+a^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(16*b^3*x^3 + 24*a*b^2*x^2 + 6*a^2*b*x - a^3)*sqrt(b*x^2 + a*x)/(a^4*b^
2*x^4 + 2*a^5*b*x^3 + a^6*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(ax+bx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a*x)**(5/2),x)
```

```
[Out] Integral((a*x + b*x**2)**(-5/2), x)
```

Giac [A] time = 1.17887, size = 68, normalized size = 1.26

$$\frac{2\left(2\left(4x\left(\frac{2b^3x}{a^4}+\frac{3b^2}{a^3}\right)+\frac{3b}{a^2}\right)x-\frac{1}{a}\right)}{3\left(bx^2+ax\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(2*(4*x*(2*b^3*x/a^4 + 3*b^2/a^3) + 3*b/a^2)*x - 1/a)/(b*x^2 + a*x)^(3/
2)
```

3.67 $\int \frac{1}{x\left(ax+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=80

$$-\frac{128b^2(a+2bx)}{15a^5\sqrt{ax+bx^2}}+\frac{16b(a+2bx)}{15a^3\left(ax+bx^2\right)^{3/2}}-\frac{2}{5ax\left(ax+bx^2\right)^{3/2}}$$

```
[Out] -2/(5*a*x*(a*x + b*x^2)^(3/2)) + (16*b*(a + 2*b*x))/(15*a^3*(a*x + b*x^2)^(
3/2)) - (128*b^2*(a + 2*b*x))/(15*a^5*Sqrt[a*x + b*x^2])
```

Rubi [A] time = 0.0220632, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {658, 614, 613}

$$-\frac{128b^2(a+2bx)}{15a^5\sqrt{ax+bx^2}}+\frac{16b(a+2bx)}{15a^3\left(ax+bx^2\right)^{3/2}}-\frac{2}{5ax\left(ax+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x + b*x^2)^(5/2)),x]
```

```
[Out] -2/(5*a*x*(a*x + b*x^2)^(3/2)) + (16*b*(a + 2*b*x))/(15*a^3*(a*x + b*x^2)^(
3/2)) - (128*b^2*(a + 2*b*x))/(15*a^5*Sqrt[a*x + b*x^2])
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\left(ax+bx^2\right)^{5/2}}\,dx = -\frac{2}{5ax\left(ax+bx^2\right)^{3/2}} - \frac{(8b)\int \frac{1}{\left(ax+bx^2\right)^{5/2}}\,dx}{5a}$$
$$= -\frac{2}{5ax\left(ax+bx^2\right)^{3/2}} + \frac{16b(a+2bx)}{15a^3\left(ax+bx^2\right)^{3/2}} + \frac{\left(64b^2\right)\int \frac{1}{\left(ax+bx^2\right)^{3/2}}\,dx}{15a^3}$$
$$= -\frac{2}{5ax\left(ax+bx^2\right)^{3/2}} + \frac{16b(a+2bx)}{15a^3\left(ax+bx^2\right)^{3/2}} - \frac{128b^2(a+2bx)}{15a^5\sqrt{ax+bx^2}}$$

Mathematica [A] time = 0.0183507, size = 62, normalized size = 0.78

$$-\frac{2\left(48a^2b^2x^2-8a^3bx+3a^4+192ab^3x^3+128b^4x^4\right)}{15a^5x(x(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x + b*x^2)^(5/2)),x]
```

```
[Out] (-2*(3*a^4 - 8*a^3*b*x + 48*a^2*b^2*x^2 + 192*a*b^3*x^3 + 128*b^4*x^4))/(15
*a^5*x*(x*(a + b*x))^(3/2))
```

Maple [A] time = 0.044, size = 63, normalized size = 0.8

$$-\frac{(2\,bx+2\,a)\left(128\,b^4x^4+192\,ab^3x^3+48\,b^2x^2a^2-8\,xa^3b+3\,a^4\right)}{15\,a^5}\left(bx^2+ax\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a*x)^(5/2),x)
```

```
[Out] -2/15*(b*x+a)*(128*b^4*x^4+192*a*b^3*x^3+48*a^2*b^2*x^2-8*a^3*b*x+3*a^4)/a^
5/(b*x^2+a*x)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.95703, size = 176, normalized size = 2.2

$$-\frac{2\left(128\,b^4x^4+192\,ab^3x^3+48\,a^2b^2x^2-8\,a^3bx+3\,a^4\right)\sqrt{bx^2+ax}}{15\left(a^5b^2x^5+2\,a^6bx^4+a^7x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(128*b^4*x^4 + 192*a*b^3*x^3 + 48*a^2*b^2*x^2 - 8*a^3*b*x + 3*a^4)*sq
rt(b*x^2 + a*x)/(a^5*b^2*x^5 + 2*a^6*b*x^4 + a^7*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(x\left(a+bx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a*x)**(5/2),x)
```

```
[Out] Integral(1/(x*(x*(a + b*x))**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+ax\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a*x)^(5/2)*x), x)
```

3.68 $\int \frac{1}{x^2(ax+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=106

$$\frac{256b^3(a+2bx)}{21a^6\sqrt{ax+bx^2}}-\frac{32b^2(a+2bx)}{21a^4\left(ax+bx^2\right)^{3/2}}+\frac{4b}{7a^2x\left(ax+bx^2\right)^{3/2}}-\frac{2}{7ax^2\left(ax+bx^2\right)^{3/2}}$$

```
[Out] -2/(7*a*x^2*(a*x + b*x^2)^(3/2)) + (4*b)/(7*a^2*x*(a*x + b*x^2)^(3/2)) - (3
2*b^2*(a + 2*b*x))/(21*a^4*(a*x + b*x^2)^(3/2)) + (256*b^3*(a + 2*b*x))/(21
*a^6*Sqrt[a*x + b*x^2])
```

Rubi [A] time = 0.0362233, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {658, 614, 613}

$$\frac{256b^3(a+2bx)}{21a^6\sqrt{ax+bx^2}}-\frac{32b^2(a+2bx)}{21a^4\left(ax+bx^2\right)^{3/2}}+\frac{4b}{7a^2x\left(ax+bx^2\right)^{3/2}}-\frac{2}{7ax^2\left(ax+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x + b*x^2)^(5/2)),x]
```

```
[Out] -2/(7*a*x^2*(a*x + b*x^2)^(3/2)) + (4*b)/(7*a^2*x*(a*x + b*x^2)^(3/2)) - (3
2*b^2*(a + 2*b*x))/(21*a^4*(a*x + b*x^2)^(3/2)) + (256*b^3*(a + 2*b*x))/(21
*a^6*Sqrt[a*x + b*x^2])
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(ax+bx^2\right)^{5/2}}\,dx = -\frac{2}{7ax^2\left(ax+bx^2\right)^{3/2}} - \frac{(10b)\int \frac{1}{x\left(ax+bx^2\right)^{5/2}}\,dx}{7a}$$

$$= -\frac{2}{7ax^2\left(ax+bx^2\right)^{3/2}} + \frac{4b}{7a^2x\left(ax+bx^2\right)^{3/2}} + \frac{\left(16b^2\right)\int \frac{1}{\left(ax+bx^2\right)^{5/2}}\,dx}{7a^2}$$

$$= -\frac{2}{7ax^2\left(ax+bx^2\right)^{3/2}} + \frac{4b}{7a^2x\left(ax+bx^2\right)^{3/2}} - \frac{32b^2(a+2bx)}{21a^4\left(ax+bx^2\right)^{3/2}} - \frac{\left(128b^3\right)\int \frac{1}{\left(ax+bx^2\right)^{3/2}}\,dx}{21a^4}$$

$$= -\frac{2}{7ax^2\left(ax+bx^2\right)^{3/2}} + \frac{4b}{7a^2x\left(ax+bx^2\right)^{3/2}} - \frac{32b^2(a+2bx)}{21a^4\left(ax+bx^2\right)^{3/2}} + \frac{256b^3(a+2bx)}{21a^6\sqrt{ax+bx^2}}$$

Mathematica [A] time = 0.0199907, size = 73, normalized size = 0.69

$$\frac{2\left(-16a^3b^2x^2+96a^2b^3x^3+6a^4bx-3a^5+384ab^4x^4+256b^5x^5\right)}{21a^6x^2(x(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x + b*x^2)^(5/2)),x]
```

```
[Out] (2*(-3*a^5 + 6*a^4*b*x - 16*a^3*b^2*x^2 + 96*a^2*b^3*x^3 + 384*a*b^4*x^4 +
256*b^5*x^5))/(21*a^6*x^2*(x*(a + b*x))^(3/2))
```

Maple [A] time = 0.047, size = 77, normalized size = 0.7

$$-\frac{(2\,bx+2\,a)\left(-256\,b^5x^5-384\,b^4x^4a-96\,b^3x^3a^2+16\,b^2x^2a^3-6\,bxa^4+3\,a^5\right)}{21\,xa^6}\left(bx^2+ax\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a*x)^(5/2),x)
```

```
[Out] -2/21*(b*x+a)*(-256*b^5*x^5-384*a*b^4*x^4-96*a^2*b^3*x^3+16*a^3*b^2*x^2-6*a
^4*b*x+3*a^5)/x/a^6/(b*x^2+a*x)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.03942, size = 197, normalized size = 1.86

$$\frac{2\left(256\,b^{5}x^{5}+384\,ab^{4}x^{4}+96\,a^{2}b^{3}x^{3}-16\,a^{3}b^{2}x^{2}+6\,a^{4}bx-3\,a^{5}\right)\sqrt{bx^{2}+ax}}{21\left(a^{6}b^{2}x^{6}+2\,a^{7}bx^{5}+a^{8}x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/21*(256*b^5*x^5 + 384*a*b^4*x^4 + 96*a^2*b^3*x^3 - 16*a^3*b^2*x^2 + 6*a^4
*b*x - 3*a^5)*sqrt(b*x^2 + a*x)/(a^6*b^2*x^6 + 2*a^7*b*x^5 + a^8*x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{2}\left(x\left(a+bx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a*x)**(5/2),x)
```

```
[Out] Integral(1/(x**2*(x*(a + b*x))**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^{2}+ax\right)^{\frac{5}{2}}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a*x)^(5/2)*x^2), x)
```

3.69 $\int \frac{x}{\sqrt{4x-x^2}}\,dx$

Optimal. Leaf size=26

$$-\sqrt{4x-x^2}-2\sin^{-1}\left(1-\frac{x}{2}\right)$$

```
[Out] -Sqrt[4*x - x^2] - 2*ArcSin[1 - x/2]
```

Rubi [A] time = 0.010801, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {640, 619, 216}

$$-\sqrt{4x-x^2}-2\sin^{-1}\left(1-\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[4*x - x^2],x]
```

```
[Out] -Sqrt[4*x - x^2] - 2*ArcSin[1 - x/2]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{4x-x^2}}\,dx &= -\sqrt{4x-x^2}+2\int \frac{1}{\sqrt{4x-x^2}}\,dx\\
&= -\sqrt{4x-x^2}-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{16}}}\,dx, x, 4-2x\right)\\
&= -\sqrt{4x-x^2}-2\sin^{-1}\left(1-\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0298684, size = 27, normalized size = 1.04

$$-\sqrt{-(x-4)x}-4\sin^{-1}\left(\sqrt{1-\frac{x}{4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[4*x - x^2],x]
```

```
[Out] -Sqrt[-((-4 + x)*x)] - 4*ArcSin[Sqrt[1 - x/4]]
```

Maple [A] time = 0.044, size = 23, normalized size = 0.9

$$2 \arcsin\left(-1 + x/2\right) - \sqrt{-x^2 + 4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2+4*x)^(1/2),x)
```

```
[Out] 2*arcsin(-1+1/2*x)-(-x^2+4*x)^(1/2)
```

Maxima [A] time = 1.69455, size = 30, normalized size = 1.15

$$-\sqrt{-x^2 + 4x} - 2 \arcsin\left(-\frac{1}{2}x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+4*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 4*x) - 2*arcsin(-1/2*x + 1)
```

Fricas [A] time = 1.9748, size = 68, normalized size = 2.62

$$-\sqrt{-x^2 + 4x} - 4 \arctan\left(\frac{\sqrt{-x^2 + 4x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+4*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 + 4*x) - 4*arctan(sqrt(-x^2 + 4*x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-x(x-4)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+4*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(-x*(x - 4)), x)
```

Giac [A] time = 1.15947, size = 30, normalized size = 1.15

$$-\sqrt{-x^2+4x}+2\arcsin\left(\frac{1}{2}x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+4*x)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 4*x) + 2*arcsin(1/2*x - 1)
```

3.70 $\int \frac{x}{\sqrt{-4x+x^2}}\,dx$

Optimal. Leaf size=28

$$\sqrt{x^2-4x}+4\tanh^{-1}\left(\frac{x}{\sqrt{x^2-4x}}\right)$$

```
[Out] Sqrt[-4*x + x^2] + 4*ArcTanh[x/Sqrt[-4*x + x^2]]
```

Rubi [A] time = 0.0074961, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {640, 620, 206}

$$\sqrt{x^2-4x}+4\tanh^{-1}\left(\frac{x}{\sqrt{x^2-4x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[-4*x + x^2],x]
```

```
[Out] Sqrt[-4*x + x^2] + 4*ArcTanh[x/Sqrt[-4*x + x^2]]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{-4x+x^2}}\,dx &= \sqrt{-4x+x^2}+2\int \frac{1}{\sqrt{-4x+x^2}}\,dx\\ &= \sqrt{-4x+x^2}+4\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{-4x+x^2}}\right)\\ &= \sqrt{-4x+x^2}+4\tanh^{-1}\left(\frac{x}{\sqrt{-4x+x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0130569, size = 40, normalized size = 1.43

$$\frac{(x-4)x-4\sqrt{-(x-4)x}\sin^{-1}\left(\sqrt{1-\frac{x}{4}}\right)}{\sqrt{(x-4)x}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Sqrt[-4*x + x^2],x]
```

```
[Out] ((-4 + x)*x - 4*Sqrt[-((-4 + x)*x)]*ArcSin[Sqrt[1 - x/4]])/Sqrt[(-4 + x)*x]
```

Maple [A] time = 0.046, size = 26, normalized size = 0.9

$$\sqrt{x^2-4x}+2\ln\left(x-2+\sqrt{x^2-4x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2-4*x)^(1/2),x)
```

```
[Out] (x^2-4*x)^(1/2)+2*ln(x-2+(x^2-4*x)^(1/2))
```

Maxima [A] time = 1.11566, size = 39, normalized size = 1.39

$$\sqrt{x^2-4x}+2\log\left(2x+2\sqrt{x^2-4x}-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-4*x)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 - 4*x) + 2*log(2*x + 2*sqrt(x^2 - 4*x) - 4)
```

Fricas [A] time = 2.0381, size = 69, normalized size = 2.46

$$\sqrt{x^2-4x}-2\log\left(-x+\sqrt{x^2-4x}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-4*x)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x^2 - 4*x) - 2*log(-x + sqrt(x^2 - 4*x) + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{x(x-4)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2-4*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x*(x - 4)), x)
```

Giac [A] time = 1.16571, size = 38, normalized size = 1.36

$$\sqrt{x^2 - 4\,x} - 2\,\log\left(\left|-x + \sqrt{x^2 - 4\,x} + 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-4*x)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x^2 - 4*x) - 2*log(abs(-x + sqrt(x^2 - 4*x) + 2))
```

3.71 $\int \frac{x^2}{\sqrt{2x-x^2}}\,dx$

Optimal. Leaf size=46

$$-\frac{1}{2}\sqrt{2x-x^2}x-\frac{3}{2}\sqrt{2x-x^2}-\frac{3}{2}\sin^{-1}(1-x)$$

```
[Out] (-3*Sqrt[2*x - x^2])/2 - (x*Sqrt[2*x - x^2])/2 - (3*ArcSin[1 - x])/2
```

Rubi [A] time = 0.0153809, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {670, 640, 619, 216}

$$-\frac{1}{2}\sqrt{2x-x^2}x-\frac{3}{2}\sqrt{2x-x^2}-\frac{3}{2}\sin^{-1}(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[2*x - x^2],x]
```

```
[Out] (-3*Sqrt[2*x - x^2])/2 - (x*Sqrt[2*x - x^2])/2 - (3*ArcSin[1 - x])/2
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\sqrt{2x-x^2}}\,dx &= -\frac{1}{2}x\sqrt{2x-x^2}+\frac{3}{2}\int \frac{x}{\sqrt{2x-x^2}}\,dx \\
&= -\frac{3}{2}\sqrt{2x-x^2}-\frac{1}{2}x\sqrt{2x-x^2}+\frac{3}{2}\int \frac{1}{\sqrt{2x-x^2}}\,dx \\
&= -\frac{3}{2}\sqrt{2x-x^2}-\frac{1}{2}x\sqrt{2x-x^2}-\frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx,x,2-2x\right) \\
&= -\frac{3}{2}\sqrt{2x-x^2}-\frac{1}{2}x\sqrt{2x-x^2}-\frac{3}{2}\sin^{-1}(1-x)
\end{aligned}
$$

Mathematica [A] time = 0.0435854, size = 47, normalized size = 1.02

$$
\frac{1}{2}\left(-\sqrt{2-x}x^{3/2}-3\sqrt{-(x-2)x}-6\sin^{-1}\left(\sqrt{1-\frac{x}{2}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[2*x - x^2],x]
```

```
[Out] (-(Sqrt[2 - x]*x^(3/2)) - 3*Sqrt[-((-2 + x)*x)] - 6*ArcSin[Sqrt[1 - x/2]])/
2
```

Maple [A] time = 0.044, size = 35, normalized size = 0.8

$$
\frac{3\arcsin(-1+x)}{2}-\frac{3}{2}\sqrt{-x^2+2x}-\frac{x}{2}\sqrt{-x^2+2x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+2*x)^(1/2),x)
```

```
[Out] 3/2*arcsin(-1+x)-3/2*(-x^2+2*x)^(1/2)-1/2*x*(-x^2+2*x)^(1/2)
```

Maxima [A] time = 1.73757, size = 49, normalized size = 1.07

$$
-\frac{1}{2}\sqrt{-x^2+2x}x-\frac{3}{2}\sqrt{-x^2+2x}-\frac{3}{2}\arcsin(-x+1)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-x^2 + 2*x)*x - 3/2*sqrt(-x^2 + 2*x) - 3/2*arcsin(-x + 1)
```

Fricas [A] time = 1.94772, size = 84, normalized size = 1.83

$$
-\frac{1}{2}\sqrt{-x^2+2x}(x+3)-3\arctan\left(\frac{\sqrt{-x^2+2x}}{x}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-x^2 + 2*x)*(x + 3) - 3*arctan(sqrt(-x^2 + 2*x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-x\,(x-2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+2*x)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(-x*(x - 2)), x)
```

Giac [A] time = 1.22651, size = 31, normalized size = 0.67

$$-\frac{1}{2}\sqrt{-x^2+2\,x}(x+3)+\frac{3}{2}\,\arcsin\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-x^2 + 2*x)*(x + 3) + 3/2*arcsin(x - 1)
```

3.72 $\int x^{7/2}\sqrt{bx+cx^2}\,dx$

Optimal. Leaf size=136

$$\frac{256b^4\left(bx+cx^2\right)^{3/2}}{3465c^5x^{3/2}}-\frac{128b^3\left(bx+cx^2\right)^{3/2}}{1155c^4\sqrt{x}}+\frac{32b^2\sqrt{x}\left(bx+cx^2\right)^{3/2}}{231c^3}-\frac{16bx^{3/2}\left(bx+cx^2\right)^{3/2}}{99c^2}+\frac{2x^{5/2}\left(bx+cx^2\right)^{3/2}}{11c}$$

```
[Out] (256*b^4*(b*x + c*x^2)^(3/2))/(3465*c^5*x^(3/2)) - (128*b^3*(b*x + c*x^2)^(
3/2))/(1155*c^4*Sqrt[x]) + (32*b^2*Sqrt[x]*(b*x + c*x^2)^(3/2))/(231*c^3) -
 (16*b*x^(3/2)*(b*x + c*x^2)^(3/2))/(99*c^2) + (2*x^(5/2)*(b*x + c*x^2)^(3/
2))/(11*c)
```

Rubi [A] time = 0.0551495, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {656, 648}

$$\frac{256b^4\left(bx+cx^2\right)^{3/2}}{3465c^5x^{3/2}}-\frac{128b^3\left(bx+cx^2\right)^{3/2}}{1155c^4\sqrt{x}}+\frac{32b^2\sqrt{x}\left(bx+cx^2\right)^{3/2}}{231c^3}-\frac{16bx^{3/2}\left(bx+cx^2\right)^{3/2}}{99c^2}+\frac{2x^{5/2}\left(bx+cx^2\right)^{3/2}}{11c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*Sqrt[b*x + c*x^2],x]
```

```
[Out] (256*b^4*(b*x + c*x^2)^(3/2))/(3465*c^5*x^(3/2)) - (128*b^3*(b*x + c*x^2)^(
3/2))/(1155*c^4*Sqrt[x]) + (32*b^2*Sqrt[x]*(b*x + c*x^2)^(3/2))/(231*c^3) -
 (16*b*x^(3/2)*(b*x + c*x^2)^(3/2))/(99*c^2) + (2*x^(5/2)*(b*x + c*x^2)^(3/
2))/(11*c)
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int x^{7/2}\sqrt{bx+cx^2}\,dx = \frac{2x^{5/2}\left(bx+cx^2\right)^{3/2}}{11c} - \frac{(8b)\int x^{5/2}\sqrt{bx+cx^2}\,dx}{11c}$$
$$= -\frac{16bx^{3/2}\left(bx+cx^2\right)^{3/2}}{99c^2} + \frac{2x^{5/2}\left(bx+cx^2\right)^{3/2}}{11c} + \frac{\left(16b^2\right)\int x^{3/2}\sqrt{bx+cx^2}\,dx}{33c^2}$$
$$= \frac{32b^2\sqrt{x}\left(bx+cx^2\right)^{3/2}}{231c^3} - \frac{16bx^{3/2}\left(bx+cx^2\right)^{3/2}}{99c^2} + \frac{2x^{5/2}\left(bx+cx^2\right)^{3/2}}{11c} - \frac{\left(64b^3\right)\int \sqrt{x}\sqrt{bx+cx^2}\,dx}{231c^3}$$
$$= -\frac{128b^3\left(bx+cx^2\right)^{3/2}}{1155c^4\sqrt{x}} + \frac{32b^2\sqrt{x}\left(bx+cx^2\right)^{3/2}}{231c^3} - \frac{16bx^{3/2}\left(bx+cx^2\right)^{3/2}}{99c^2} + \frac{2x^{5/2}\left(bx+cx^2\right)^{3/2}}{11c} + \frac{(128}{}$$
$$= \frac{256b^4\left(bx+cx^2\right)^{3/2}}{3465c^5x^{3/2}} - \frac{128b^3\left(bx+cx^2\right)^{3/2}}{1155c^4\sqrt{x}} + \frac{32b^2\sqrt{x}\left(bx+cx^2\right)^{3/2}}{231c^3} - \frac{16bx^{3/2}\left(bx+cx^2\right)^{3/2}}{99c^2} + \frac{2x^{5/2}}{}$$

Mathematica [A] time = 0.0346805, size = 64, normalized size = 0.47

$$\frac{2(x(b+cx))^{3/2}\left(240b^2c^2x^2-192b^3cx+128b^4-280bc^3x^3+315c^4x^4\right)}{3465c^5x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*Sqrt[b*x + c*x^2],x]
```

```
[Out] (2*(x*(b + c*x))^(3/2)*(128*b^4 - 192*b^3*c*x + 240*b^2*c^2*x^2 - 280*b*c^3
*x^3 + 315*c^4*x^4))/(3465*c^5*x^(3/2))
```

Maple [A] time = 0.046, size = 66, normalized size = 0.5

$$\frac{(2\,cx+2\,b)\left(315\,x^4c^4-280\,bx^3c^3+240\,b^2x^2c^2-192\,b^3xc+128\,b^4\right)}{3465\,c^5}\sqrt{cx^2+bx}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(c*x^2+b*x)^(1/2),x)
```

```
[Out] 2/3465*(c*x+b)*(315*c^4*x^4-280*b*c^3*x^3+240*b^2*c^2*x^2-192*b^3*c*x+128*b
^4)*(c*x^2+b*x)^(1/2)/c^5/x^(1/2)
```

Maxima [A] time = 1.16032, size = 86, normalized size = 0.63

$$\frac{2\left(315\,c^5x^5+35\,bc^4x^4-40\,b^2c^3x^3+48\,b^3c^2x^2-64\,b^4cx+128\,b^5\right)\sqrt{cx+b}}{3465\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3465*(315*c^5*x^5 + 35*b*c^4*x^4 - 40*b^2*c^3*x^3 + 48*b^3*c^2*x^2 - 64*b
^4*c*x + 128*b^5)*sqrt(c*x + b)/c^5
```

Fricas [A] time = 1.88514, size = 170, normalized size = 1.25

$$\frac{2\left(315\,c^{5}x^{5}+35\,bc^{4}x^{4}-40\,b^{2}c^{3}x^{3}+48\,b^{3}c^{2}x^{2}-64\,b^{4}cx+128\,b^{5}\right)\sqrt{cx^{2}+bx}}{3465\,c^{5}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*c^5*x^5 + 35*b*c^4*x^4 - 40*b^2*c^3*x^3 + 48*b^3*c^2*x^2 - 64*b
^4*c*x + 128*b^5)*sqrt(c*x^2 + b*x)/(c^5*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(c*x**2+b*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16155, size = 95, normalized size = 0.7

$$-\frac{256\,b^{\frac{11}{2}}}{3465\,c^{5}}+\frac{2\left(315\,(cx+b)^{\frac{11}{2}}-1540\,(cx+b)^{\frac{9}{2}}b+2970\,(cx+b)^{\frac{7}{2}}b^{2}-2772\,(cx+b)^{\frac{5}{2}}b^{3}+1155\,(cx+b)^{\frac{3}{2}}b^{4}\right)}{3465\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] -256/3465*b^(11/2)/c^5 + 2/3465*(315*(c*x + b)^(11/2) - 1540*(c*x + b)^(9/2
)*b + 2970*(c*x + b)^(7/2)*b^2 - 2772*(c*x + b)^(5/2)*b^3 + 1155*(c*x + b)^
(3/2)*b^4)/c^5
```

3.73 $\int x^{5/2}\sqrt{bx+cx^2}\,dx$

Optimal. Leaf size=108

$$-\frac{32b^3\left(bx+cx^2\right)^{3/2}}{315c^4x^{3/2}}+\frac{16b^2\left(bx+cx^2\right)^{3/2}}{105c^3\sqrt{x}}-\frac{4b\sqrt{x}\left(bx+cx^2\right)^{3/2}}{21c^2}+\frac{2x^{3/2}\left(bx+cx^2\right)^{3/2}}{9c}$$

```
[Out] (-32*b^3*(b*x + c*x^2)^(3/2))/(315*c^4*x^(3/2)) + (16*b^2*(b*x + c*x^2)^(3/
2))/(105*c^3*Sqrt[x]) - (4*b*Sqrt[x]*(b*x + c*x^2)^(3/2))/(21*c^2) + (2*x^(
3/2)*(b*x + c*x^2)^(3/2))/(9*c)
```

Rubi [A] time = 0.0402275, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {656, 648}

$$-\frac{32b^3\left(bx+cx^2\right)^{3/2}}{315c^4x^{3/2}}+\frac{16b^2\left(bx+cx^2\right)^{3/2}}{105c^3\sqrt{x}}-\frac{4b\sqrt{x}\left(bx+cx^2\right)^{3/2}}{21c^2}+\frac{2x^{3/2}\left(bx+cx^2\right)^{3/2}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*Sqrt[b*x + c*x^2],x]
```

```
[Out] (-32*b^3*(b*x + c*x^2)^(3/2))/(315*c^4*x^(3/2)) + (16*b^2*(b*x + c*x^2)^(3/
2))/(105*c^3*Sqrt[x]) - (4*b*Sqrt[x]*(b*x + c*x^2)^(3/2))/(21*c^2) + (2*x^(
3/2)*(b*x + c*x^2)^(3/2))/(9*c)
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^{5/2}\sqrt{bx+cx^2}\,dx &= \frac{2x^{3/2}\left(bx+cx^2\right)^{3/2}}{9c}-\frac{(2b)\int x^{3/2}\sqrt{bx+cx^2}\,dx}{3c}\\
&= -\frac{4b\sqrt{x}\left(bx+cx^2\right)^{3/2}}{21c^2}+\frac{2x^{3/2}\left(bx+cx^2\right)^{3/2}}{9c}+\frac{\left(8b^2\right)\int\sqrt{x}\sqrt{bx+cx^2}\,dx}{21c^2}\\
&= \frac{16b^2\left(bx+cx^2\right)^{3/2}}{105c^3\sqrt{x}}-\frac{4b\sqrt{x}\left(bx+cx^2\right)^{3/2}}{21c^2}+\frac{2x^{3/2}\left(bx+cx^2\right)^{3/2}}{9c}-\frac{\left(16b^3\right)\int\frac{\sqrt{bx+cx^2}}{\sqrt{x}}\,dx}{105c^3}\\
&= -\frac{32b^3\left(bx+cx^2\right)^{3/2}}{315c^4x^{3/2}}+\frac{16b^2\left(bx+cx^2\right)^{3/2}}{105c^3\sqrt{x}}-\frac{4b\sqrt{x}\left(bx+cx^2\right)^{3/2}}{21c^2}+\frac{2x^{3/2}\left(bx+cx^2\right)^{3/2}}{9c}
\end{aligned}$$

Mathematica [A] time = 0.028814, size = 53, normalized size = 0.49

$$\frac{2(x(b+cx))^{3/2}\left(24b^2cx-16b^3-30bc^2x^2+35c^3x^3\right)}{315c^4x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*Sqrt[b*x + c*x^2],x]
```

```
[Out] (2*(x*(b + c*x))^(3/2)*(-16*b^3 + 24*b^2*c*x - 30*b*c^2*x^2 + 35*c^3*x^3))/
(315*c^4*x^(3/2))
```

Maple [A] time = 0.046, size = 55, normalized size = 0.5

$$-\frac{(2\,cx+2\,b)\left(-35\,x^3c^3+30\,bx^2c^2-24\,b^2xc+16\,b^3\right)}{315\,c^4}\sqrt{cx^2+bx}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^2+b*x)^(1/2),x)
```

```
[Out] -2/315*(c*x+b)*(-35*c^3*x^3+30*b*c^2*x^2-24*b^2*c*x+16*b^3)*(c*x^2+b*x)^(1/
2)/c^4/x^(1/2)
```

Maxima [A] time = 1.14971, size = 72, normalized size = 0.67

$$\frac{2\left(35\,c^4x^4+5\,bc^3x^3-6\,b^2c^2x^2+8\,b^3cx-16\,b^4\right)\sqrt{cx+b}}{315\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*c^4*x^4 + 5*b*c^3*x^3 - 6*b^2*c^2*x^2 + 8*b^3*c*x - 16*b^4)*sqrt(
c*x + b)/c^4
```

Fricas [A] time = 2.20523, size = 139, normalized size = 1.29

$$\frac{2\left(35\,c^4x^4+5\,bc^3x^3-6\,b^2c^2x^2+8\,b^3cx-16\,b^4\right)\sqrt{cx^2+bx}}{315\,c^4\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*c^4*x^4 + 5*b*c^3*x^3 - 6*b^2*c^2*x^2 + 8*b^3*c*x - 16*b^4)*sqrt(
c*x^2 + b*x)/(c^4*sqrt(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{\frac{5}{2}}\sqrt{x\,(b+cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(x**(5/2)*sqrt(x*(b + c*x)), x)
```

Giac [A] time = 1.20601, size = 78, normalized size = 0.72

$$\frac{32\,b^{\frac{9}{2}}}{315\,c^4}+\frac{2\left(35\,(cx+b)^{\frac{9}{2}}-135\,(cx+b)^{\frac{7}{2}}b+189\,(cx+b)^{\frac{5}{2}}b^2-105\,(cx+b)^{\frac{3}{2}}b^3\right)}{315\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] 32/315*b^(9/2)/c^4 + 2/315*(35*(c*x + b)^(9/2) - 135*(c*x + b)^(7/2)*b + 18
9*(c*x + b)^(5/2)*b^2 - 105*(c*x + b)^(3/2)*b^3)/c^4
```

3.74 $\int x^{3/2}\sqrt{bx+cx^2}\,dx$

Optimal. Leaf size=80

$$\frac{16b^2\left(bx+cx^2\right)^{3/2}}{105c^3x^{3/2}}-\frac{8b\left(bx+cx^2\right)^{3/2}}{35c^2\sqrt{x}}+\frac{2\sqrt{x}\left(bx+cx^2\right)^{3/2}}{7c}$$

```
[Out] (16*b^2*(b*x + c*x^2)^(3/2))/(105*c^3*x^(3/2)) - (8*b*(b*x + c*x^2)^(3/2))/
(35*c^2*Sqrt[x]) + (2*Sqrt[x]*(b*x + c*x^2)^(3/2))/(7*c)
```

Rubi [A] time = 0.0267062, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {656, 648}

$$\frac{16b^2\left(bx+cx^2\right)^{3/2}}{105c^3x^{3/2}}-\frac{8b\left(bx+cx^2\right)^{3/2}}{35c^2\sqrt{x}}+\frac{2\sqrt{x}\left(bx+cx^2\right)^{3/2}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Sqrt[b*x + c*x^2],x]
```

```
[Out] (16*b^2*(b*x + c*x^2)^(3/2))/(105*c^3*x^(3/2)) - (8*b*(b*x + c*x^2)^(3/2))/
(35*c^2*Sqrt[x]) + (2*Sqrt[x]*(b*x + c*x^2)^(3/2))/(7*c)
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\sqrt{bx+cx^2}\,dx &= \frac{2\sqrt{x}\left(bx+cx^2\right)^{3/2}}{7c}-\frac{(4b)\int\sqrt{x}\sqrt{bx+cx^2}\,dx}{7c}\\ &= -\frac{8b\left(bx+cx^2\right)^{3/2}}{35c^2\sqrt{x}}+\frac{2\sqrt{x}\left(bx+cx^2\right)^{3/2}}{7c}+\frac{\left(8b^2\right)\int\frac{\sqrt{bx+cx^2}}{\sqrt{x}}\,dx}{35c^2}\\ &= \frac{16b^2\left(bx+cx^2\right)^{3/2}}{105c^3x^{3/2}}-\frac{8b\left(bx+cx^2\right)^{3/2}}{35c^2\sqrt{x}}+\frac{2\sqrt{x}\left(bx+cx^2\right)^{3/2}}{7c}\end{aligned}$$

Mathematica [A] time = 0.0217325, size = 42, normalized size = 0.52

$$\frac{2(x(b+cx))^{3/2}\left(8b^2-12bcx+15c^2x^2\right)}{105c^3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Sqrt[b*x + c*x^2],x]
```

```
[Out] (2*(x*(b + c*x))^(3/2)*(8*b^2 - 12*b*c*x + 15*c^2*x^2))/(105*c^3*x^(3/2))
```

Maple [A] time = 0.049, size = 44, normalized size = 0.6

$$\frac{(2\,cx+2\,b)\left(15\,c^2x^2-12\,bcx+8\,b^2\right)}{105\,c^3}\sqrt{cx^2+bx}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^2+b*x)^(1/2),x)
```

```
[Out] 2/105*(c*x+b)*(15*c^2*x^2-12*b*c*x+8*b^2)*(c*x^2+b*x)^(1/2)/c^3/x^(1/2)
```

Maxima [A] time = 1.15948, size = 57, normalized size = 0.71

$$\frac{2\left(15\,c^3x^3+3\,bc^2x^2-4\,b^2cx+8\,b^3\right)\sqrt{cx+b}}{105\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*c^3*x^3 + 3*b*c^2*x^2 - 4*b^2*c*x + 8*b^3)*sqrt(c*x + b)/c^3
```

Fricas [A] time = 2.28884, size = 116, normalized size = 1.45

$$\frac{2\left(15\,c^3x^3+3\,bc^2x^2-4\,b^2cx+8\,b^3\right)\sqrt{cx^2+bx}}{105\,c^3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*c^3*x^3 + 3*b*c^2*x^2 - 4*b^2*c*x + 8*b^3)*sqrt(c*x^2 + b*x)/(c^3
*sqrt(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{\frac{3}{2}}\sqrt{x\,(b+cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(x**(3/2)*sqrt(x*(b + c*x)), x)
```

Giac [A] time = 1.16809, size = 62, normalized size = 0.78

$$-\frac{16\, b^{\frac{7}{2}}}{105\, c^3} + \frac{2\left(15\,(cx+b)^{\frac{7}{2}} - 42\,(cx+b)^{\frac{5}{2}} b + 35\,(cx+b)^{\frac{3}{2}} b^2\right)}{105\, c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] -16/105*b^(7/2)/c^3 + 2/105*(15*(c*x + b)^(7/2) - 42*(c*x + b)^(5/2)*b + 35
*(c*x + b)^(3/2)*b^2)/c^3
```

3.75 $\int \sqrt{x}\sqrt{bx+cx^2}\,dx$

Optimal. Leaf size=52

$$\frac{2\left(bx+cx^2\right)^{3/2}}{5c\sqrt{x}}-\frac{4b\left(bx+cx^2\right)^{3/2}}{15c^2x^{3/2}}$$

```
[Out] (-4*b*(b*x + c*x^2)^(3/2))/(15*c^2*x^(3/2)) + (2*(b*x + c*x^2)^(3/2))/(5*c*
Sqrt[x])
```

Rubi [A] time = 0.0152971, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {656, 648}

$$\frac{2\left(bx+cx^2\right)^{3/2}}{5c\sqrt{x}}-\frac{4b\left(bx+cx^2\right)^{3/2}}{15c^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[b*x + c*x^2],x]
```

```
[Out] (-4*b*(b*x + c*x^2)^(3/2))/(15*c^2*x^(3/2)) + (2*(b*x + c*x^2)^(3/2))/(5*c*
Sqrt[x])
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\sqrt{bx+cx^2}\,dx &= \frac{2\left(bx+cx^2\right)^{3/2}}{5c\sqrt{x}}-\frac{(2b)\int \frac{\sqrt{bx+cx^2}}{\sqrt{x}}\,dx}{5c}\\ &= -\frac{4b\left(bx+cx^2\right)^{3/2}}{15c^2x^{3/2}}+\frac{2\left(bx+cx^2\right)^{3/2}}{5c\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.017424, size = 31, normalized size = 0.6

$$\frac{2(x(b+cx))^{3/2}(3cx-2b)}{15c^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[b*x + c*x^2],x]
```

```
[Out] (2*(x*(b + c*x))^(3/2)*(-2*b + 3*c*x))/(15*c^2*x^(3/2))
```

Maple [A] time = 0.046, size = 33, normalized size = 0.6

$$-\frac{(2\,cx+2\,b)\,(-3\,cx+2\,b)}{15\,c^2}\sqrt{cx^2+bx}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^2+b*x)^(1/2),x)
```

```
[Out] -2/15*(c*x+b)*(-3*c*x+2*b)*(c*x^2+b*x)^(1/2)/c^2/x^(1/2)
```

Maxima [A] time = 1.02837, size = 41, normalized size = 0.79

$$\frac{2\left(3\,c^2x^2+bcx-2\,b^2\right)\sqrt{cx+b}}{15\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*c^2*x^2 + b*c*x - 2*b^2)*sqrt(c*x + b)/c^2
```

Fricas [A] time = 2.18565, size = 89, normalized size = 1.71

$$\frac{2\left(3\,c^2x^2+bcx-2\,b^2\right)\sqrt{cx^2+bx}}{15\,c^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*c^2*x^2 + b*c*x - 2*b^2)*sqrt(c*x^2 + b*x)/(c^2*sqrt(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x}\sqrt{x\,(b+cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x)*sqrt(x*(b + c*x)), x)
```

Giac [A] time = 1.21536, size = 46, normalized size = 0.88

$$\frac{4 b^{\frac{5}{2}}}{15 c^2} + \frac{2 \left(3 (cx + b)^{\frac{5}{2}} - 5 (cx + b)^{\frac{3}{2}} b\right)}{15 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*b^(5/2)/c^2 + 2/15*(3*(c*x + b)^(5/2) - 5*(c*x + b)^(3/2)*b)/c^2
```

3.76 $\int \frac{\sqrt{bx+cx^2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=25

$$\frac{2\left(bx+cx^2\right)^{3/2}}{3cx^{3/2}}$$

```
[Out] (2*(b*x + c*x^2)^(3/2))/(3*c*x^(3/2))
```

Rubi [A] time = 0.0062485, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {648}

$$\frac{2\left(bx+cx^2\right)^{3/2}}{3cx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x + c*x^2]/Sqrt[x],x]
```

```
[Out] (2*(b*x + c*x^2)^(3/2))/(3*c*x^(3/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{\sqrt{bx+cx^2}}{\sqrt{x}}\,dx=\frac{2\left(bx+cx^2\right)^{3/2}}{3cx^{3/2}}$$

Mathematica [A] time = 0.0108567, size = 23, normalized size = 0.92

$$\frac{2(x(b+cx))^{3/2}}{3cx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x + c*x^2]/Sqrt[x],x]
```

```
[Out] (2*(x*(b + c*x))^(3/2))/(3*c*x^(3/2))
```

Maple [A] time = 0.042, size = 25, normalized size = 1.

$$\frac{2\,cx+2\,b}{3\,c}\sqrt{cx^2+bx}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(1/2)/x^(1/2),x)
```

```
[Out] 2/3*(c*x+b)*(c*x^2+b*x)^(1/2)/c/x^(1/2)
```

Maxima [A] time = 1.16604, size = 16, normalized size = 0.64

$$\frac{2\,(cx+b)^{\frac{3}{2}}}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(c*x + b)^(3/2)/c
```

Fricas [A] time = 2.21826, size = 61, normalized size = 2.44

$$\frac{2\,\sqrt{cx^2+bx}(cx+b)}{3\,c\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(c*x^2 + b*x)*(c*x + b)/(c*sqrt(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x\,(b+cx)}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(1/2)/x**(1/2),x)
```

```
[Out] Integral(sqrt(x*(b + c*x))/sqrt(x), x)
```

Giac [A] time = 1.19745, size = 28, normalized size = 1.12

$$\frac{2\,(cx+b)^{\frac{3}{2}}}{3\,c}-\frac{2\,b^{\frac{3}{2}}}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(c*x + b)^(3/2)/c - 2/3*b^(3/2)/c
```

3.77 $\int \frac{\sqrt{bx+cx^2}}{x^{3/2}}\,dx$

Optimal. Leaf size=53

$$\frac{2\sqrt{bx+cx^2}}{\sqrt{x}}-2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)$$

```
[Out] (2*Sqrt[b*x + c*x^2])/Sqrt[x] - 2*Sqrt[b]*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b
]*Sqrt[x])]
```

Rubi [A] time = 0.0205161, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {664, 660, 207}

$$\frac{2\sqrt{bx+cx^2}}{\sqrt{x}}-2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x + c*x^2]/x^(3/2),x]
```

```
[Out] (2*Sqrt[b*x + c*x^2])/Sqrt[x] - 2*Sqrt[b]*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b
]*Sqrt[x])]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{bx+cx^2}}{x^{3/2}}\,dx = \frac{2\sqrt{bx+cx^2}}{\sqrt{x}} + b\int \frac{1}{\sqrt{x}\sqrt{bx+cx^2}}\,dx$$
$$= \frac{2\sqrt{bx+cx^2}}{\sqrt{x}} + (2b)\,\text{Subst}\left(\int \frac{1}{-b+x^2}\,dx, x, \frac{\sqrt{bx+cx^2}}{\sqrt{x}}\right)$$
$$= \frac{2\sqrt{bx+cx^2}}{\sqrt{x}} - 2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)$$

Mathematica [A] time = 0.0251797, size = 60, normalized size = 1.13

$$\frac{2\sqrt{x}\sqrt{b+cx}\left(\sqrt{b+cx} - \sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b+cx}}{\sqrt{b}}\right)\right)}{\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x + c*x^2]/x^(3/2),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[b + c*x]*(Sqrt[b + c*x] - Sqrt[b]*ArcTanh[Sqrt[b + c*x]/Sqr
t[b]]))/Sqrt[x*(b + c*x)]
```

Maple [A] time = 0.207, size = 48, normalized size = 0.9

$$-2\,\frac{\sqrt{x\,(cx+b)}}{\sqrt{x}\sqrt{cx+b}}\left(\sqrt{b}\text{Artanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right) - \sqrt{cx+b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(1/2)/x^(3/2),x)
```

```
[Out] -2*(x*(c*x+b))^(1/2)/x^(1/2)*(b^(1/2)*arctanh((c*x+b)^(1/2)/b^(1/2))-(c*x+b
)^(1/2))/(c*x+b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+bx}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x)/x^(3/2), x)
```

Fricas [A] time = 2.05607, size = 273, normalized size = 5.15

$$\left[\frac{\sqrt{b}x\log\left(-\frac{cx^2+2\,bx-2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right) + 2\,\sqrt{cx^2+bx}\sqrt{x}}{x}, \frac{2\left(\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{cx^2+bx}}\right) + \sqrt{cx^2+bx}\sqrt{x}\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(b)*x*log(-(c*x^2 + 2*b*x - 2*sqrt(c*x^2 + b*x)*sqrt(b)*sqrt(x))/x^2)
 + 2*sqrt(c*x^2 + b*x)*sqrt(x))/x, 2*(sqrt(-b)*x*arctan(sqrt(-b)*sqrt(x)/sq
rt(c*x^2 + b*x)) + sqrt(c*x^2 + b*x)*sqrt(x))/x]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x\left(b+cx\right)}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(1/2)/x**(3/2),x)
```

```
[Out] Integral(sqrt(x*(b + c*x))/x**(3/2), x)
```

Giac [A] time = 1.12536, size = 82, normalized size = 1.55

$$\frac{2\,b\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}+2\,\sqrt{cx+b}-\frac{2\left(b\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+\sqrt{-b}\sqrt{b}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2*b*arctan(sqrt(c*x + b)/sqrt(-b))/sqrt(-b) + 2*sqrt(c*x + b) - 2*(b*arctan
(sqrt(b)/sqrt(-b)) + sqrt(-b)*sqrt(b))/sqrt(-b)
```

3.78 $\int \frac{\sqrt{bx+cx^2}}{x^{5/2}}\,dx$

Optimal. Leaf size=54

$$-\frac{\sqrt{bx+cx^2}}{x^{3/2}}-\frac{c\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{b}}$$

```
[Out] -(Sqrt[b*x + c*x^2]/x^(3/2)) - (c*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x
])])/Sqrt[b]
```

Rubi [A] time = 0.0207981, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {662, 660, 207}

$$-\frac{\sqrt{bx+cx^2}}{x^{3/2}}-\frac{c\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x + c*x^2]/x^(5/2),x]
```

```
[Out] -(Sqrt[b*x + c*x^2]/x^(3/2)) - (c*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x
])])/Sqrt[b]
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{bx+cx^2}}{x^{5/2}}\,dx &= -\frac{\sqrt{bx+cx^2}}{x^{3/2}}+\frac{1}{2}c\int \frac{1}{\sqrt{x}\sqrt{bx+cx^2}}\,dx \\
&= -\frac{\sqrt{bx+cx^2}}{x^{3/2}}+c\,\text{Subst}\left(\int \frac{1}{-b+x^2}\,dx,x,\frac{\sqrt{bx+cx^2}}{\sqrt{x}}\right) \\
&= -\frac{\sqrt{bx+cx^2}}{x^{3/2}}-\frac{c\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{b}}
\end{aligned}
$$

Mathematica [A] time = 0.0408061, size = 51, normalized size = 0.94

$$
-\frac{cx\sqrt{\frac{cx}{b}+1}\tanh^{-1}\left(\sqrt{\frac{cx}{b}+1}\right)+b+cx}{\sqrt{x}\sqrt{x(b+cx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x + c*x^2]/x^(5/2),x]
```

```
[Out] -((b + c*x + c*x*Sqrt[1 + (c*x)/b]*ArcTanh[Sqrt[1 + (c*x)/b]])/(Sqrt[x]*Sqr
t[x*(b + c*x)]))
```

Maple [A] time = 0.183, size = 53, normalized size = 1.

$$
\left(-\text{Artanh}\left(\sqrt{cx+b}\frac{1}{\sqrt{b}}\right)xc-\sqrt{cx+b}\sqrt{b}\right)\sqrt{x\,(cx+b)}x^{-\frac{3}{2}}\frac{1}{\sqrt{cx+b}}\frac{1}{\sqrt{b}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(1/2)/x^(5/2),x)
```

```
[Out] (-arctanh((c*x+b)^(1/2)/b^(1/2))*x*c-(c*x+b)^(1/2)*b^(1/2))*(x*(c*x+b))^(1/
2)/x^(3/2)/(c*x+b)^(1/2)/b^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\sqrt{cx^2+bx}}{x^{\frac{5}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x)/x^(5/2), x)
```

Fricas [A] time = 2.02776, size = 308, normalized size = 5.7

$$
\left[\frac{\sqrt{b}cx^2\log\left(-\frac{cx^2+2\,bx-2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)-2\,\sqrt{cx^2+bx}b\sqrt{x}}{2\,bx^2},\frac{\sqrt{-b}cx^2\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{cx^2+bx}}\right)-\sqrt{cx^2+bx}b\sqrt{x}}{bx^2}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b)*c*x^2*log(-(c*x^2 + 2*b*x - 2*sqrt(c*x^2 + b*x)*sqrt(b)*sqrt(
x))/x^2) - 2*sqrt(c*x^2 + b*x)*b*sqrt(x))/(b*x^2), (sqrt(-b)*c*x^2*arctan(s
qrt(-b)*sqrt(x)/sqrt(c*x^2 + b*x)) - sqrt(c*x^2 + b*x)*b*sqrt(x))/(b*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x\,(b+cx)}}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(1/2)/x**(5/2),x)
```

```
[Out] Integral(sqrt(x*(b + c*x))/x**(5/2), x)
```

Giac [A] time = 1.21015, size = 51, normalized size = 0.94

$$c\left(\frac{\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{\sqrt{cx+b}}{cx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] c*(arctan(sqrt(c*x + b)/sqrt(-b))/sqrt(-b) - sqrt(c*x + b)/(c*x))
```

3.79 $\int \frac{\sqrt{bx+cx^2}}{x^{7/2}}\,dx$

Optimal. Leaf size=86

$$\frac{c^2\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{4b^{3/2}}-\frac{c\sqrt{bx+cx^2}}{4bx^{3/2}}-\frac{\sqrt{bx+cx^2}}{2x^{5/2}}$$

```
[Out] -Sqrt[b*x + c*x^2]/(2*x^(5/2)) - (c*Sqrt[b*x + c*x^2])/(4*b*x^(3/2)) + (c^2
*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/(4*b^(3/2))
```

Rubi [A] time = 0.0331965, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {662, 672, 660, 207}

$$\frac{c^2\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{4b^{3/2}}-\frac{c\sqrt{bx+cx^2}}{4bx^{3/2}}-\frac{\sqrt{bx+cx^2}}{2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x + c*x^2]/x^(7/2),x]
```

```
[Out] -Sqrt[b*x + c*x^2]/(2*x^(5/2)) - (c*Sqrt[b*x + c*x^2])/(4*b*x^(3/2)) + (c^2
*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/(4*b^(3/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*(m + 2*p + 2))/((m + p + 1)*(2*c*d - b*e)), Int[(d
 + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && Ne
Q[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{bx+cx^2}}{x^{7/2}}\,dx &= -\frac{\sqrt{bx+cx^2}}{2x^{5/2}}+\frac{1}{4}c\int \frac{1}{x^{3/2}\sqrt{bx+cx^2}}\,dx\\
&= -\frac{\sqrt{bx+cx^2}}{2x^{5/2}}-\frac{c\sqrt{bx+cx^2}}{4bx^{3/2}}-\frac{c^2\int \frac{1}{\sqrt{x}\sqrt{bx+cx^2}}\,dx}{8b}\\
&= -\frac{\sqrt{bx+cx^2}}{2x^{5/2}}-\frac{c\sqrt{bx+cx^2}}{4bx^{3/2}}-\frac{c^2\,\mathrm{Subst}\left(\int \frac{1}{-b+x^2}\,dx,x,\frac{\sqrt{bx+cx^2}}{\sqrt{x}}\right)}{4b}\\
&= -\frac{\sqrt{bx+cx^2}}{2x^{5/2}}-\frac{c\sqrt{bx+cx^2}}{4bx^{3/2}}+\frac{c^2\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{4b^{3/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0139201, size = 42, normalized size = 0.49

$$
-\frac{2c^2(x(b+cx))^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{cx}{b}+1\right)}{3b^3x^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x + c*x^2]/x^(7/2),x]
```

```
[Out] (-2*c^2*(x*(b + c*x))^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (c*x)/b])/(3
*b^3*x^(3/2))
```

Maple [A] time = 0.182, size = 71, normalized size = 0.8

$$
\frac{1}{4}\sqrt{x(cx+b)}\left(\mathrm{Artanh}\left(\sqrt{cx+b}\frac{1}{\sqrt{b}}\right)x^2c^2-xc\sqrt{cx+b}\sqrt{b}-2\,b^{3/2}\sqrt{cx+b}\right)b^{-\frac{3}{2}}x^{-\frac{5}{2}}\frac{1}{\sqrt{cx+b}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(1/2)/x^(7/2),x)
```

```
[Out] 1/4*(x*(c*x+b))^(1/2)/b^(3/2)*(arctanh((c*x+b)^(1/2)/b^(1/2))*x^2*c^2-x*c*(
c*x+b)^(1/2)*b^(1/2)-2*b^(3/2)*(c*x+b)^(1/2))/x^(5/2)/(c*x+b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\sqrt{cx^2+bx}}{x^{\frac{7}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x)/x^(7/2), x)
```

Fricas [A] time = 2.13271, size = 363, normalized size = 4.22

$$\left[\frac{\sqrt{b}c^2x^3\log\left(-\frac{cx^2+2\,bx+2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)-2\left(bcx+2\,b^2\right)\sqrt{cx^2+bx}\sqrt{x}}{8\,b^2x^3}, -\frac{\sqrt{-b}c^2x^3\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{cx^2+bx}}\right)+\left(bcx+2\,b^2\right)\sqrt{cx^2\cdot}}{4\,b^2x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(b)*c^2*x^3*log(-(c*x^2 + 2*b*x + 2*sqrt(c*x^2 + b*x)*sqrt(b)*sqr
t(x))/x^2) - 2*(b*c*x + 2*b^2)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^2*x^3), -1/4*(
sqrt(-b)*c^2*x^3*arctan(sqrt(-b)*sqrt(x)/sqrt(c*x^2 + b*x)) + (b*c*x + 2*b^
2)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^2*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x\,(b+cx)}}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(1/2)/x**(7/2),x)
```

```
[Out] Integral(sqrt(x*(b + c*x))/x**(7/2), x)
```

Giac [A] time = 1.18345, size = 76, normalized size = 0.88

$$-\frac{1}{4}\,c^2\left(\frac{\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b}+\frac{(cx+b)^{\frac{3}{2}}+\sqrt{cx+b}b}{bc^2x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] -1/4*c^2*(arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b) + ((c*x + b)^(3/2) +
sqrt(c*x + b)*b)/(b*c^2*x^2))
```

3.80 $\int \frac{\sqrt{bx+cx^2}}{x^{9/2}} dx$

Optimal. Leaf size=114

$$\frac{c^2\sqrt{bx+cx^2}}{8b^2x^{3/2}} - \frac{c^3\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{8b^{5/2}} - \frac{c\sqrt{bx+cx^2}}{12bx^{5/2}} - \frac{\sqrt{bx+cx^2}}{3x^{7/2}}$$

```
[Out] -Sqrt[b*x + c*x^2]/(3*x^(7/2)) - (c*Sqrt[b*x + c*x^2])/(12*b*x^(5/2)) + (c^
2*Sqrt[b*x + c*x^2])/(8*b^2*x^(3/2)) - (c^3*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt
[b]*Sqrt[x])])/(8*b^(5/2))
```

Rubi [A] time = 0.0516426, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {662, 672, 660, 207}

$$\frac{c^2\sqrt{bx+cx^2}}{8b^2x^{3/2}} - \frac{c^3\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{8b^{5/2}} - \frac{c\sqrt{bx+cx^2}}{12bx^{5/2}} - \frac{\sqrt{bx+cx^2}}{3x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x + c*x^2]/x^(9/2),x]
```

```
[Out] -Sqrt[b*x + c*x^2]/(3*x^(7/2)) - (c*Sqrt[b*x + c*x^2])/(12*b*x^(5/2)) + (c^
2*Sqrt[b*x + c*x^2])/(8*b^2*x^(3/2)) - (c^3*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt
[b]*Sqrt[x])])/(8*b^(5/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*(m + 2*p + 2))/((m + p + 1)*(2*c*d - b*e)), Int[(d
 + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && Ne
Q[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{bx+cx^2}}{x^{9/2}}\,dx &= -\frac{\sqrt{bx+cx^2}}{3x^{7/2}}+\frac{1}{6}c\int \frac{1}{x^{5/2}\sqrt{bx+cx^2}}\,dx\\
&= -\frac{\sqrt{bx+cx^2}}{3x^{7/2}}-\frac{c\sqrt{bx+cx^2}}{12bx^{5/2}}-\frac{c^2\int \frac{1}{x^{3/2}\sqrt{bx+cx^2}}\,dx}{8b}\\
&= -\frac{\sqrt{bx+cx^2}}{3x^{7/2}}-\frac{c\sqrt{bx+cx^2}}{12bx^{5/2}}+\frac{c^2\sqrt{bx+cx^2}}{8b^2x^{3/2}}+\frac{c^3\int \frac{1}{\sqrt{x}\sqrt{bx+cx^2}}\,dx}{16b^2}\\
&= -\frac{\sqrt{bx+cx^2}}{3x^{7/2}}-\frac{c\sqrt{bx+cx^2}}{12bx^{5/2}}+\frac{c^2\sqrt{bx+cx^2}}{8b^2x^{3/2}}+\frac{c^3\,\mathrm{Subst}\left(\int \frac{1}{-b+x^2}\,dx,x,\frac{\sqrt{bx+cx^2}}{\sqrt{x}}\right)}{8b^2}\\
&= -\frac{\sqrt{bx+cx^2}}{3x^{7/2}}-\frac{c\sqrt{bx+cx^2}}{12bx^{5/2}}+\frac{c^2\sqrt{bx+cx^2}}{8b^2x^{3/2}}-\frac{c^3\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{8b^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0141477, size = 42, normalized size = 0.37

$$\frac{2c^3(x(b+cx))^{3/2}\,{}_2F_1\left(\frac{3}{2},4;\frac{5}{2};\frac{cx}{b}+1\right)}{3b^4x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x + c*x^2]/x^(9/2),x]

[Out] (2*c^3*(x*(b + c*x))^(3/2)*Hypergeometric2F1[3/2, 4, 5/2, 1 + (c*x)/b])/(3*
b^4*x^(3/2))
```

Maple [A] time = 0.188, size = 90, normalized size = 0.8

$$-\frac{1}{24}\sqrt{x(cx+b)}\left(3\,\mathrm{Artanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)x^3c^3-3\,x^2c^2\sqrt{b}\sqrt{cx+b}+2\,xb^{3/2}c\sqrt{cx+b}+8\,b^{5/2}\sqrt{cx+b}\right)b^{-\frac{5}{2}}x^{-\frac{7}{2}}\frac{1}{\sqrt{cx+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(1/2)/x^(9/2),x)

[Out] -1/24*(x*(c*x+b))^(1/2)/b^(5/2)*(3*arctanh((c*x+b)^(1/2)/b^(1/2))*x^3*c^3-3
*x^2*c^2*b^(1/2)*(c*x+b)^(1/2)+2*x*b^(3/2)*c*(c*x+b)^(1/2)+8*b^(5/2)*(c*x+b
)^(1/2))/x^(7/2)/(c*x+b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+bx}}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x)/x^(9/2), x)
```

Fricas [A] time = 2.09861, size = 419, normalized size = 3.68

$$\left[\frac{3\sqrt{b}c^3x^4\log\left(-\frac{cx^2+2bx-2\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)+2\left(3bc^2x^2-2b^2cx-8b^3\right)\sqrt{cx^2+bx}\sqrt{x}}{48b^3x^4}, \frac{3\sqrt{-b}c^3x^4\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{cx^2+bx}}\right)+\left(3bc^2x^2\right.}{24b^3x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(b)*c^3*x^4*log(-(c*x^2 + 2*b*x - 2*sqrt(c*x^2 + b*x)*sqrt(b)*
sqrt(x))/x^2) + 2*(3*b*c^2*x^2 - 2*b^2*c*x - 8*b^3)*sqrt(c*x^2 + b*x)*sqrt(
x))/(b^3*x^4), 1/24*(3*sqrt(-b)*c^3*x^4*arctan(sqrt(-b)*sqrt(x)/sqrt(c*x^2
+ b*x)) + (3*b*c^2*x^2 - 2*b^2*c*x - 8*b^3)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^3
*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(1/2)/x**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31509, size = 97, normalized size = 0.85

$$\frac{1}{24}c^3\left(\frac{3\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^2}+\frac{3(cx+b)^{\frac{5}{2}}-8(cx+b)^{\frac{3}{2}}b-3\sqrt{cx+b}b^2}{b^2c^3x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] 1/24*c^3*(3*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b^2) + (3*(c*x + b)^(5
/2) - 8*(c*x + b)^(3/2)*b - 3*sqrt(c*x + b)*b^2)/(b^2*c^3*x^3))
```

3.81 $\int \frac{\sqrt{bx+cx^2}}{x^{11/2}} \, dx$

Optimal. Leaf size=142

$$-\frac{5c^3\sqrt{bx+cx^2}}{64b^3x^{3/2}}+\frac{5c^2\sqrt{bx+cx^2}}{96b^2x^{5/2}}+\frac{5c^4\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{64b^{7/2}}-\frac{c\sqrt{bx+cx^2}}{24bx^{7/2}}-\frac{\sqrt{bx+cx^2}}{4x^{9/2}}$$

```
[Out] -Sqrt[b*x + c*x^2]/(4*x^(9/2)) - (c*Sqrt[b*x + c*x^2])/(24*b*x^(7/2)) + (5*
c^2*Sqrt[b*x + c*x^2])/(96*b^2*x^(5/2)) - (5*c^3*Sqrt[b*x + c*x^2])/(64*b^3
*x^(3/2)) + (5*c^4*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/(64*b^(7/2
))
```

Rubi [A] time = 0.0648671, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {662, 672, 660, 207}

$$-\frac{5c^3\sqrt{bx+cx^2}}{64b^3x^{3/2}}+\frac{5c^2\sqrt{bx+cx^2}}{96b^2x^{5/2}}+\frac{5c^4\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{64b^{7/2}}-\frac{c\sqrt{bx+cx^2}}{24bx^{7/2}}-\frac{\sqrt{bx+cx^2}}{4x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x + c*x^2]/x^(11/2),x]
```

```
[Out] -Sqrt[b*x + c*x^2]/(4*x^(9/2)) - (c*Sqrt[b*x + c*x^2])/(24*b*x^(7/2)) + (5*
c^2*Sqrt[b*x + c*x^2])/(96*b^2*x^(5/2)) - (5*c^3*Sqrt[b*x + c*x^2])/(64*b^3
*x^(3/2)) + (5*c^4*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/(64*b^(7/2
))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*(m + 2*p + 2))/((m + p + 1)*(2*c*d - b*e)), Int[(d
 + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && Ne
Q[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx+cx^2}}{x^{11/2}}\,dx &= -\frac{\sqrt{bx+cx^2}}{4x^{9/2}}+\frac{1}{8}c\int \frac{1}{x^{7/2}\sqrt{bx+cx^2}}\,dx\\ &= -\frac{\sqrt{bx+cx^2}}{4x^{9/2}}-\frac{c\sqrt{bx+cx^2}}{24bx^{7/2}}-\frac{(5c^2)\int \frac{1}{x^{5/2}\sqrt{bx+cx^2}}\,dx}{48b}\\ &= -\frac{\sqrt{bx+cx^2}}{4x^{9/2}}-\frac{c\sqrt{bx+cx^2}}{24bx^{7/2}}+\frac{5c^2\sqrt{bx+cx^2}}{96b^2x^{5/2}}+\frac{(5c^3)\int \frac{1}{x^{3/2}\sqrt{bx+cx^2}}\,dx}{64b^2}\\ &= -\frac{\sqrt{bx+cx^2}}{4x^{9/2}}-\frac{c\sqrt{bx+cx^2}}{24bx^{7/2}}+\frac{5c^2\sqrt{bx+cx^2}}{96b^2x^{5/2}}-\frac{5c^3\sqrt{bx+cx^2}}{64b^3x^{3/2}}-\frac{(5c^4)\int \frac{1}{\sqrt{x}\sqrt{bx+cx^2}}\,dx}{128b^3}\\ &= -\frac{\sqrt{bx+cx^2}}{4x^{9/2}}-\frac{c\sqrt{bx+cx^2}}{24bx^{7/2}}+\frac{5c^2\sqrt{bx+cx^2}}{96b^2x^{5/2}}-\frac{5c^3\sqrt{bx+cx^2}}{64b^3x^{3/2}}-\frac{(5c^4)\,\mathrm{Subst}\left(\int \frac{1}{-b+x^2}\,dx,x,\frac{\sqrt{bx+cx^2}}{\sqrt{x}}\right)}{64b^3}\\ &= -\frac{\sqrt{bx+cx^2}}{4x^{9/2}}-\frac{c\sqrt{bx+cx^2}}{24bx^{7/2}}+\frac{5c^2\sqrt{bx+cx^2}}{96b^2x^{5/2}}-\frac{5c^3\sqrt{bx+cx^2}}{64b^3x^{3/2}}+\frac{5c^4\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{64b^{7/2}}\end{aligned}$$

Mathematica [C] time = 0.0147189, size = 42, normalized size = 0.3

$$-\frac{2c^4(x(b+cx))^{3/2}\,{}_2F_1\left(\frac{3}{2},5;\frac{5}{2};\frac{cx}{b}+1\right)}{3b^5x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x + c*x^2]/x^(11/2),x]
```

```
[Out] (-2*c^4*(x*(b + c*x))^(3/2)*Hypergeometric2F1[3/2, 5, 5/2, 1 + (c*x)/b])/(3
*b^5*x^(3/2))
```

Maple [A] time = 0.18, size = 108, normalized size = 0.8

$$\frac{1}{192}\sqrt{x(cx+b)}\left(15\,\mathrm{Artanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)x^4c^4-15\,x^3c^3\sqrt{cx+b}\sqrt{b}+10\,x^2b^{3/2}c^2\sqrt{cx+b}-8\,xb^{5/2}c\sqrt{cx+b}-48\,b^{7/2}\sqrt{cx+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(1/2)/x^(11/2),x)
```

```
[Out] 1/192*(x*(c*x+b))^(1/2)/b^(7/2)*(15*arctanh((c*x+b)^(1/2)/b^(1/2))*x^4*c^4-
15*x^3*c^3*(c*x+b)^(1/2)*b^(1/2)+10*x^2*b^(3/2)*c^2*(c*x+b)^(1/2)-8*x*b^(5/
2)*c*(c*x+b)^(1/2)-48*b^(7/2)*(c*x+b)^(1/2))/x^(9/2)/(c*x+b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+bx}}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x)/x^(11/2), x)
```

Fricas [A] time = 2.17614, size = 477, normalized size = 3.36

$$\left[\frac{15\sqrt{b}c^4x^5\log\left(-\frac{cx^2+2\,bx+2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)-2\left(15\,bc^3x^3-10\,b^2c^2x^2+8\,b^3cx+48\,b^4\right)\sqrt{cx^2+bx}\sqrt{x}}{384\,b^4x^5}, -\frac{15\sqrt{-b}c^4x^5\,\text{arcta}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] [1/384*(15*sqrt(b)*c^4*x^5*log(-(c*x^2 + 2*b*x + 2*sqrt(c*x^2 + b*x)*sqrt(b
)*sqrt(x))/x^2) - 2*(15*b*c^3*x^3 - 10*b^2*c^2*x^2 + 8*b^3*c*x + 48*b^4)*sq
rt(c*x^2 + b*x)*sqrt(x))/(b^4*x^5), -1/192*(15*sqrt(-b)*c^4*x^5*arctan(sqrt
(-b)*sqrt(x)/sqrt(c*x^2 + b*x)) + (15*b*c^3*x^3 - 10*b^2*c^2*x^2 + 8*b^3*c*
x + 48*b^4)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^4*x^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(1/2)/x**(11/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20823, size = 113, normalized size = 0.8

$$-\frac{1}{192}c^4\left(\frac{15\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^3}+\frac{15\,(cx+b)^{\frac{7}{2}}-55\,(cx+b)^{\frac{5}{2}}b+73\,(cx+b)^{\frac{3}{2}}b^2+15\,\sqrt{cx+b}b^3}{b^3c^4x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/x^(11/2),x, algorithm="giac")
```

```
[Out] -1/192*c^4*(15*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b^3) + (15*(c*x + b
)^(7/2) - 55*(c*x + b)^(5/2)*b + 73*(c*x + b)^(3/2)*b^2 + 15*sqrt(c*x + b)*
b^3)/(b^3*c^4*x^4))
```

3.82 $\int x^{7/2} \left(bx + cx^2\right)^{3/2} dx$

Optimal. Leaf size=164

$$-\frac{512b^5\left(bx+cx^2\right)^{5/2}}{45045c^6x^{5/2}}+\frac{256b^4\left(bx+cx^2\right)^{5/2}}{9009c^5x^{3/2}}-\frac{64b^3\left(bx+cx^2\right)^{5/2}}{1287c^4\sqrt{x}}+\frac{32b^2\sqrt{x}\left(bx+cx^2\right)^{5/2}}{429c^3}-\frac{4bx^{3/2}\left(bx+cx^2\right)^{5/2}}{39c^2}+\frac{2x^{5/2}\left(\right.}{}$$

```
[Out] (-512*b^5*(b*x + c*x^2)^(5/2))/(45045*c^6*x^(5/2)) + (256*b^4*(b*x + c*x^2)
^(5/2))/(9009*c^5*x^(3/2)) - (64*b^3*(b*x + c*x^2)^(5/2))/(1287*c^4*Sqrt[x]
) + (32*b^2*Sqrt[x]*(b*x + c*x^2)^(5/2))/(429*c^3) - (4*b*x^(3/2)*(b*x + c*
x^2)^(5/2))/(39*c^2) + (2*x^(5/2)*(b*x + c*x^2)^(5/2))/(15*c)
```

Rubi [A] time = 0.075216, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {656, 648}

$$-\frac{512b^5\left(bx+cx^2\right)^{5/2}}{45045c^6x^{5/2}}+\frac{256b^4\left(bx+cx^2\right)^{5/2}}{9009c^5x^{3/2}}-\frac{64b^3\left(bx+cx^2\right)^{5/2}}{1287c^4\sqrt{x}}+\frac{32b^2\sqrt{x}\left(bx+cx^2\right)^{5/2}}{429c^3}-\frac{4bx^{3/2}\left(bx+cx^2\right)^{5/2}}{39c^2}+\frac{2x^{5/2}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(b*x + c*x^2)^(3/2),x]
```


```
[Out] (-512*b^5*(b*x + c*x^2)^(5/2))/(45045*c^6*x^(5/2)) + (256*b^4*(b*x + c*x^2)
^(5/2))/(9009*c^5*x^(3/2)) - (64*b^3*(b*x + c*x^2)^(5/2))/(1287*c^4*Sqrt[x]
) + (32*b^2*Sqrt[x]*(b*x + c*x^2)^(5/2))/(429*c^3) - (4*b*x^(3/2)*(b*x + c*
x^2)^(5/2))/(39*c^2) + (2*x^(5/2)*(b*x + c*x^2)^(5/2))/(15*c)
```


Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int x^{7/2}\left(bx+cx^2\right)^{3/2}dx = \frac{2x^{5/2}\left(bx+cx^2\right)^{5/2}}{15c} - \frac{(2b)\int x^{5/2}\left(bx+cx^2\right)^{3/2}dx}{3c}$$
$$= -\frac{4bx^{3/2}\left(bx+cx^2\right)^{5/2}}{39c^2} + \frac{2x^{5/2}\left(bx+cx^2\right)^{5/2}}{15c} + \frac{\left(16b^2\right)\int x^{3/2}\left(bx+cx^2\right)^{3/2}dx}{39c^2}$$
$$= \frac{32b^2\sqrt{x}\left(bx+cx^2\right)^{5/2}}{429c^3} - \frac{4bx^{3/2}\left(bx+cx^2\right)^{5/2}}{39c^2} + \frac{2x^{5/2}\left(bx+cx^2\right)^{5/2}}{15c} - \frac{\left(32b^3\right)\int\sqrt{x}\left(bx+cx^2\right)}{143c^3}$$
$$= -\frac{64b^3\left(bx+cx^2\right)^{5/2}}{1287c^4\sqrt{x}} + \frac{32b^2\sqrt{x}\left(bx+cx^2\right)^{5/2}}{429c^3} - \frac{4bx^{3/2}\left(bx+cx^2\right)^{5/2}}{39c^2} + \frac{2x^{5/2}\left(bx+cx^2\right)^{5/2}}{15c} +$$
$$= \frac{256b^4\left(bx+cx^2\right)^{5/2}}{9009c^5x^{3/2}} - \frac{64b^3\left(bx+cx^2\right)^{5/2}}{1287c^4\sqrt{x}} + \frac{32b^2\sqrt{x}\left(bx+cx^2\right)^{5/2}}{429c^3} - \frac{4bx^{3/2}\left(bx+cx^2\right)^{5/2}}{39c^2} +$$
$$= -\frac{512b^5\left(bx+cx^2\right)^{5/2}}{45045c^6x^{5/2}} + \frac{256b^4\left(bx+cx^2\right)^{5/2}}{9009c^5x^{3/2}} - \frac{64b^3\left(bx+cx^2\right)^{5/2}}{1287c^4\sqrt{x}} + \frac{32b^2\sqrt{x}\left(bx+cx^2\right)^{5/2}}{429c^3} -$$

Mathematica [A] time = 0.0462373, size = 75, normalized size = 0.46

$$\frac{2(x(b+cx))^{5/2}\left(-1120b^3c^2x^2+1680b^2c^3x^3+640b^4cx-256b^5-2310bc^4x^4+3003c^5x^5\right)}{45045c^6x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(x*(b + c*x))^(5/2)*(-256*b^5 + 640*b^4*c*x - 1120*b^3*c^2*x^2 + 1680*b^
2*c^3*x^3 - 2310*b*c^4*x^4 + 3003*c^5*x^5))/(45045*c^6*x^(5/2))
```

Maple [A] time = 0.054, size = 77, normalized size = 0.5

$$-\frac{(2\,cx+2\,b)\left(-3003\,x^5c^5+2310\,bx^4c^4-1680\,b^2x^3c^3+1120\,b^3x^2c^2-640\,b^4xc+256\,b^5\right)}{45045\,c^6}\left(cx^2+bx\right)^{\frac{3}{2}}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(c*x^2+b*x)^(3/2),x)
```

```
[Out] -2/45045*(c*x+b)*(-3003*c^5*x^5+2310*b*c^4*x^4-1680*b^2*c^3*x^3+1120*b^3*c^
2*x^2-640*b^4*c*x+256*b^5)*(c*x^2+b*x)^(3/2)/c^6/x^(3/2)
```

Maxima [A] time = 1.1401, size = 227, normalized size = 1.38

$$\frac{2\left(\left(3003\,c^7x^7+231\,bc^6x^6-252\,b^2c^5x^5+280\,b^3c^4x^4-320\,b^4c^3x^3+384\,b^5c^2x^2-512\,b^6cx+1024\,b^7\right)x^6+5\left(693\,bc^6x\right.}{45045\,c^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

[Out] 2/45045*((3003*c^7*x^7 + 231*b*c^6*x^6 - 252*b^2*c^5*x^5 + 280*b^3*c^4*x^4 - 320*b^4*c^3*x^3 + 384*b^5*c^2*x^2 - 512*b^6*c*x + 1024*b^7)*x^6 + 5*(693*b*c^6*x^7 + 63*b^2*c^5*x^6 - 70*b^3*c^4*x^5 + 80*b^4*c^3*x^4 - 96*b^5*c^2*x^3 + 128*b^6*c*x^2 - 256*b^7*x)*x^5)*sqrt(c*x + b)/(c^6*x^6)

Fricas [A] time = 2.04401, size = 223, normalized size = 1.36

$$\frac{2\left(3003\,c^7x^7+3696\,bc^6x^6+63\,b^2c^5x^5-70\,b^3c^4x^4+80\,b^4c^3x^3-96\,b^5c^2x^2+128\,b^6cx-256\,b^7\right)\sqrt{cx^2+bx}}{45045\,c^6\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)*(c*x^2+b*x)^(3/2),x, algorithm="fricas")

[Out] 2/45045*(3003*c^7*x^7 + 3696*b*c^6*x^6 + 63*b^2*c^5*x^5 - 70*b^3*c^4*x^4 + 80*b^4*c^3*x^3 - 96*b^5*c^2*x^2 + 128*b^6*c*x - 256*b^7)*sqrt(c*x^2 + b*x)/(c^6*sqrt(x))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(7/2)*(c*x**2+b*x)**(3/2),x)

[Out] Timed out

Giac [A] time = 1.32596, size = 246, normalized size = 1.5

$$-\frac{2}{45045}c\left(\frac{1024\,b^{\frac{15}{2}}}{c^7}-\frac{3003\,(cx+b)^{\frac{15}{2}}-20790\,(cx+b)^{\frac{13}{2}}b+61425\,(cx+b)^{\frac{11}{2}}b^2-100100\,(cx+b)^{\frac{9}{2}}b^3+96525\,(cx+b)^{\frac{7}{2}}b}{c^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)*(c*x^2+b*x)^(3/2),x, algorithm="giac")

[Out] -2/45045*c*(1024*b^(15/2)/c^7 - (3003*(c*x + b)^(15/2) - 20790*(c*x + b)^(13/2)*b + 61425*(c*x + b)^(11/2)*b^2 - 100100*(c*x + b)^(9/2)*b^3 + 96525*(c*x + b)^(7/2)*b^4 - 54054*(c*x + b)^(5/2)*b^5 + 15015*(c*x + b)^(3/2)*b^6)/c^7) + 2/9009*b*(256*b^(13/2)/c^6 + (693*(c*x + b)^(13/2) - 4095*(c*x + b)^(11/2)*b + 10010*(c*x + b)^(9/2)*b^2 - 12870*(c*x + b)^(7/2)*b^3 + 9009*(c*x + b)^(5/2)*b^4 - 3003*(c*x + b)^(3/2)*b^5)/c^6)

3.83 $\int x^{5/2} \left(bx + cx^2\right)^{3/2} dx$

Optimal. Leaf size=136

$$\frac{256b^4\left(bx+cx^2\right)^{5/2}}{15015c^5x^{5/2}}-\frac{128b^3\left(bx+cx^2\right)^{5/2}}{3003c^4x^{3/2}}+\frac{32b^2\left(bx+cx^2\right)^{5/2}}{429c^3\sqrt{x}}-\frac{16b\sqrt{x}\left(bx+cx^2\right)^{5/2}}{143c^2}+\frac{2x^{3/2}\left(bx+cx^2\right)^{5/2}}{13c}$$

```
[Out] (256*b^4*(b*x + c*x^2)^(5/2))/(15015*c^5*x^(5/2)) - (128*b^3*(b*x + c*x^2)^
(5/2))/(3003*c^4*x^(3/2)) + (32*b^2*(b*x + c*x^2)^(5/2))/(429*c^3*Sqrt[x])
- (16*b*Sqrt[x]*(b*x + c*x^2)^(5/2))/(143*c^2) + (2*x^(3/2)*(b*x + c*x^2)^(
5/2))/(13*c)
```

Rubi [A] time = 0.0560172, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {656, 648}

$$\frac{256b^4\left(bx+cx^2\right)^{5/2}}{15015c^5x^{5/2}}-\frac{128b^3\left(bx+cx^2\right)^{5/2}}{3003c^4x^{3/2}}+\frac{32b^2\left(bx+cx^2\right)^{5/2}}{429c^3\sqrt{x}}-\frac{16b\sqrt{x}\left(bx+cx^2\right)^{5/2}}{143c^2}+\frac{2x^{3/2}\left(bx+cx^2\right)^{5/2}}{13c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(b*x + c*x^2)^(3/2),x]
```

```
[Out] (256*b^4*(b*x + c*x^2)^(5/2))/(15015*c^5*x^(5/2)) - (128*b^3*(b*x + c*x^2)^
(5/2))/(3003*c^4*x^(3/2)) + (32*b^2*(b*x + c*x^2)^(5/2))/(429*c^3*Sqrt[x])
- (16*b*Sqrt[x]*(b*x + c*x^2)^(5/2))/(143*c^2) + (2*x^(3/2)*(b*x + c*x^2)^(
5/2))/(13*c)
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int x^{5/2}\left(bx+cx^2\right)^{3/2}dx=\frac{2x^{3/2}\left(bx+cx^2\right)^{5/2}}{13c}-\frac{(8b)\int x^{3/2}\left(bx+cx^2\right)^{3/2}dx}{13c}$$
$$=-\frac{16b\sqrt{x}\left(bx+cx^2\right)^{5/2}}{143c^2}+\frac{2x^{3/2}\left(bx+cx^2\right)^{5/2}}{13c}+\frac{\left(48b^2\right)\int\sqrt{x}\left(bx+cx^2\right)^{3/2}dx}{143c^2}$$
$$=\frac{32b^2\left(bx+cx^2\right)^{5/2}}{429c^3\sqrt{x}}-\frac{16b\sqrt{x}\left(bx+cx^2\right)^{5/2}}{143c^2}+\frac{2x^{3/2}\left(bx+cx^2\right)^{5/2}}{13c}-\frac{\left(64b^3\right)\int\frac{\left(bx+cx^2\right)^{3/2}}{\sqrt{x}}dx}{429c^3}$$
$$=-\frac{128b^3\left(bx+cx^2\right)^{5/2}}{3003c^4x^{3/2}}+\frac{32b^2\left(bx+cx^2\right)^{5/2}}{429c^3\sqrt{x}}-\frac{16b\sqrt{x}\left(bx+cx^2\right)^{5/2}}{143c^2}+\frac{2x^{3/2}\left(bx+cx^2\right)^{5/2}}{13c}+(128$$
$$=\frac{256b^4\left(bx+cx^2\right)^{5/2}}{15015c^5x^{5/2}}-\frac{128b^3\left(bx+cx^2\right)^{5/2}}{3003c^4x^{3/2}}+\frac{32b^2\left(bx+cx^2\right)^{5/2}}{429c^3\sqrt{x}}-\frac{16b\sqrt{x}\left(bx+cx^2\right)^{5/2}}{143c^2}+2x^{3/2}$$

Mathematica [A] time = 0.0381884, size = 64, normalized size = 0.47

$$\frac{2(x(b+cx))^{5/2}\left(560b^2c^2x^2-320b^3cx+128b^4-840bc^3x^3+1155c^4x^4\right)}{15015c^5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(x*(b + c*x))^(5/2)*(128*b^4 - 320*b^3*c*x + 560*b^2*c^2*x^2 - 840*b*c^3
*x^3 + 1155*c^4*x^4))/(15015*c^5*x^(5/2))
```

Maple [A] time = 0.047, size = 66, normalized size = 0.5

$$\frac{(2\,cx+2\,b)\left(1155\,x^4c^4-840\,bx^3c^3+560\,b^2x^2c^2-320\,b^3xc+128\,b^4\right)}{15015\,c^5}\left(cx^2+bx\right)^{\frac{3}{2}}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^2+b*x)^(3/2),x)
```

```
[Out] 2/15015*(c*x+b)*(1155*c^4*x^4-840*b*c^3*x^3+560*b^2*c^2*x^2-320*b^3*c*x+128
*b^4)*(c*x^2+b*x)^(3/2)/c^5/x^(3/2)
```

Maxima [A] time = 1.06191, size = 198, normalized size = 1.46

$$\frac{2\left(5\left(693\,c^6x^6+63\,bc^5x^5-70\,b^2c^4x^4+80\,b^3c^3x^3-96\,b^4c^2x^2+128\,b^5cx-256\,b^6\right)x^5+13\left(315\,bc^5x^6+35\,b^2c^4x^5-40\,b^3c\right.}{45045\,c^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(5*(693*c^6*x^6 + 63*b*c^5*x^5 - 70*b^2*c^4*x^4 + 80*b^3*c^3*x^3 -
96*b^4*c^2*x^2 + 128*b^5*c*x - 256*b^6)*x^5 + 13*(315*b*c^5*x^6 + 35*b^2*c^
4*x^5 - 40*b^3*c^3*x^4 + 48*b^4*c^2*x^3 - 64*b^5*c*x^2 + 128*b^6*x)*x^4)*sq
```

```
rt(c*x + b)/(c^5*x^5)
```

Fricas [A] time = 2.01196, size = 198, normalized size = 1.46

$$\frac{2\left(1155\,c^6x^6+1470\,bc^5x^5+35\,b^2c^4x^4-40\,b^3c^3x^3+48\,b^4c^2x^2-64\,b^5cx+128\,b^6\right)\sqrt{cx^2+bx}}{15015\,c^5\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15015*(1155*c^6*x^6 + 1470*b*c^5*x^5 + 35*b^2*c^4*x^4 - 40*b^3*c^3*x^3 +
48*b^4*c^2*x^2 - 64*b^5*c*x + 128*b^6)*sqrt(c*x^2 + b*x)/(c^5*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**2+b*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22036, size = 213, normalized size = 1.57

$$\frac{2}{9009}c\left(\frac{256\,b^{\frac{13}{2}}}{c^6}+\frac{693\,(cx+b)^{\frac{13}{2}}-4095\,(cx+b)^{\frac{11}{2}}b+10010\,(cx+b)^{\frac{9}{2}}b^2-12870\,(cx+b)^{\frac{7}{2}}b^3+9009\,(cx+b)^{\frac{5}{2}}b^4-30}{c^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] 2/9009*c*(256*b^(13/2)/c^6 + (693*(c*x + b)^(13/2) - 4095*(c*x + b)^(11/2)*
b + 10010*(c*x + b)^(9/2)*b^2 - 12870*(c*x + b)^(7/2)*b^3 + 9009*(c*x + b)^
(5/2)*b^4 - 3003*(c*x + b)^(3/2)*b^5)/c^6) - 2/3465*b*(128*b^(11/2)/c^5 - (
315*(c*x + b)^(11/2) - 1540*(c*x + b)^(9/2)*b + 2970*(c*x + b)^(7/2)*b^2 -
2772*(c*x + b)^(5/2)*b^3 + 1155*(c*x + b)^(3/2)*b^4)/c^5)
```

3.84 $\int x^{3/2} \left(bx + cx^2\right)^{3/2} dx$

Optimal. Leaf size=108

$$-\frac{32b^3 \left(bx + cx^2\right)^{5/2}}{1155c^4x^{5/2}} + \frac{16b^2 \left(bx + cx^2\right)^{5/2}}{231c^3x^{3/2}} - \frac{4b \left(bx + cx^2\right)^{5/2}}{33c^2\sqrt{x}} + \frac{2\sqrt{x} \left(bx + cx^2\right)^{5/2}}{11c}$$

```
[Out] (-32*b^3*(b*x + c*x^2)^(5/2))/(1155*c^4*x^(5/2)) + (16*b^2*(b*x + c*x^2)^(5
/2))/(231*c^3*x^(3/2)) - (4*b*(b*x + c*x^2)^(5/2))/(33*c^2*Sqrt[x]) + (2*Sq
rt[x]*(b*x + c*x^2)^(5/2))/(11*c)
```

Rubi [A] time = 0.0429493, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {656, 648}

$$-\frac{32b^3 \left(bx + cx^2\right)^{5/2}}{1155c^4x^{5/2}} + \frac{16b^2 \left(bx + cx^2\right)^{5/2}}{231c^3x^{3/2}} - \frac{4b \left(bx + cx^2\right)^{5/2}}{33c^2\sqrt{x}} + \frac{2\sqrt{x} \left(bx + cx^2\right)^{5/2}}{11c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(b*x + c*x^2)^(3/2),x]
```

```
[Out] (-32*b^3*(b*x + c*x^2)^(5/2))/(1155*c^4*x^(5/2)) + (16*b^2*(b*x + c*x^2)^(5
/2))/(231*c^3*x^(3/2)) - (4*b*(b*x + c*x^2)^(5/2))/(33*c^2*Sqrt[x]) + (2*Sq
rt[x]*(b*x + c*x^2)^(5/2))/(11*c)
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^{3/2}\left(bx+cx^2\right)^{3/2}dx &= \frac{2\sqrt{x}\left(bx+cx^2\right)^{5/2}}{11c}-\frac{(6b)\int \sqrt{x}\left(bx+cx^2\right)^{3/2}dx}{11c}\\
&= -\frac{4b\left(bx+cx^2\right)^{5/2}}{33c^2\sqrt{x}}+\frac{2\sqrt{x}\left(bx+cx^2\right)^{5/2}}{11c}+\frac{\left(8b^2\right)\int \frac{\left(bx+cx^2\right)^{3/2}}{\sqrt{x}}dx}{33c^2}\\
&= \frac{16b^2\left(bx+cx^2\right)^{5/2}}{231c^3x^{3/2}}-\frac{4b\left(bx+cx^2\right)^{5/2}}{33c^2\sqrt{x}}+\frac{2\sqrt{x}\left(bx+cx^2\right)^{5/2}}{11c}-\frac{\left(16b^3\right)\int \frac{\left(bx+cx^2\right)^{3/2}}{x^{3/2}}dx}{231c^3}\\
&= -\frac{32b^3\left(bx+cx^2\right)^{5/2}}{1155c^4x^{5/2}}+\frac{16b^2\left(bx+cx^2\right)^{5/2}}{231c^3x^{3/2}}-\frac{4b\left(bx+cx^2\right)^{5/2}}{33c^2\sqrt{x}}+\frac{2\sqrt{x}\left(bx+cx^2\right)^{5/2}}{11c}
\end{aligned}
$$

Mathematica [A] time = 0.0324024, size = 53, normalized size = 0.49

$$
\frac{2(x(b+cx))^{5/2}\left(40b^2cx-16b^3-70bc^2x^2+105c^3x^3\right)}{1155c^4x^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(x*(b + c*x))^(5/2)*(-16*b^3 + 40*b^2*c*x - 70*b*c^2*x^2 + 105*c^3*x^3))
/(1155*c^4*x^(5/2))
```

Maple [A] time = 0.047, size = 55, normalized size = 0.5

$$
-\frac{(2\,cx+2\,b)\left(-105\,x^3c^3+70\,bx^2c^2-40\,b^2xc+16\,b^3\right)}{1155\,c^4}\left(cx^2+bx\right)^{\frac{3}{2}}x^{-\frac{3}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^2+b*x)^(3/2),x)
```

```
[Out] -2/1155*(c*x+b)*(-105*c^3*x^3+70*b*c^2*x^2-40*b^2*c*x+16*b^3)*(c*x^2+b*x)^(
3/2)/c^4/x^(3/2)
```

Maxima [A] time = 1.15566, size = 167, normalized size = 1.55

$$
\frac{2\left(\left(315\,c^5x^5+35\,bc^4x^4-40\,b^2c^3x^3+48\,b^3c^2x^2-64\,b^4cx+128\,b^5\right)x^4+11\left(35\,bc^4x^5+5\,b^2c^3x^4-6\,b^3c^2x^3+8\,b^4cx^2-\right.\right.}{3465\,c^4x^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3465*((315*c^5*x^5 + 35*b*c^4*x^4 - 40*b^2*c^3*x^3 + 48*b^3*c^2*x^2 - 64*
b^4*c*x + 128*b^5)*x^4 + 11*(35*b*c^4*x^5 + 5*b^2*c^3*x^4 - 6*b^3*c^2*x^3 +
 8*b^4*c*x^2 - 16*b^5*x)*x^3)*sqrt(c*x + b)/(c^4*x^4)
```

Fricas [A] time = 1.97309, size = 166, normalized size = 1.54

$$\frac{2\left(105\,c^5x^5+140\,bc^4x^4+5\,b^2c^3x^3-6\,b^3c^2x^2+8\,b^4cx-16\,b^5\right)\sqrt{cx^2+bx}}{1155\,c^4\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*c^5*x^5 + 140*b*c^4*x^4 + 5*b^2*c^3*x^3 - 6*b^3*c^2*x^2 + 8*b^4
*c*x - 16*b^5)*sqrt(c*x^2 + b*x)/(c^4*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**2+b*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19372, size = 181, normalized size = 1.68

$$-\frac{2}{3465}\,c\left(\frac{128\,b^{\frac{11}{2}}}{c^5}-\frac{315\,(cx+b)^{\frac{11}{2}}-1540\,(cx+b)^{\frac{9}{2}}b+2970\,(cx+b)^{\frac{7}{2}}b^2-2772\,(cx+b)^{\frac{5}{2}}b^3+1155\,(cx+b)^{\frac{3}{2}}b^4}{c^5}\right)+\frac{2}{315}\,b\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3465*c*(128*b^(11/2)/c^5 - (315*(c*x + b)^(11/2) - 1540*(c*x + b)^(9/2)*
b + 2970*(c*x + b)^(7/2)*b^2 - 2772*(c*x + b)^(5/2)*b^3 + 1155*(c*x + b)^(3
/2)*b^4)/c^5) + 2/315*b*(16*b^(9/2)/c^4 + (35*(c*x + b)^(9/2) - 135*(c*x +
b)^(7/2)*b + 189*(c*x + b)^(5/2)*b^2 - 105*(c*x + b)^(3/2)*b^3)/c^4)
```

3.85 $\int \sqrt{x}\left(bx+cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=80

$$\frac{16b^2\left(bx+cx^2\right)^{5/2}}{315c^3x^{5/2}}-\frac{8b\left(bx+cx^2\right)^{5/2}}{63c^2x^{3/2}}+\frac{2\left(bx+cx^2\right)^{5/2}}{9c\sqrt{x}}$$

```
[Out] (16*b^2*(b*x + c*x^2)^(5/2))/(315*c^3*x^(5/2)) - (8*b*(b*x + c*x^2)^(5/2))/
(63*c^2*x^(3/2)) + (2*(b*x + c*x^2)^(5/2))/(9*c*Sqrt[x])
```

Rubi [A] time = 0.0269342, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {656, 648}

$$\frac{16b^2\left(bx+cx^2\right)^{5/2}}{315c^3x^{5/2}}-\frac{8b\left(bx+cx^2\right)^{5/2}}{63c^2x^{3/2}}+\frac{2\left(bx+cx^2\right)^{5/2}}{9c\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(b*x + c*x^2)^(3/2),x]
```

```
[Out] (16*b^2*(b*x + c*x^2)^(5/2))/(315*c^3*x^(5/2)) - (8*b*(b*x + c*x^2)^(5/2))/
(63*c^2*x^(3/2)) + (2*(b*x + c*x^2)^(5/2))/(9*c*Sqrt[x])
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}\left(bx+cx^2\right)^{3/2}\,dx &= \frac{2\left(bx+cx^2\right)^{5/2}}{9c\sqrt{x}}-\frac{(4b)\int \frac{\left(bx+cx^2\right)^{3/2}}{\sqrt{x}}\,dx}{9c}\\
&= -\frac{8b\left(bx+cx^2\right)^{5/2}}{63c^2x^{3/2}}+\frac{2\left(bx+cx^2\right)^{5/2}}{9c\sqrt{x}}+\frac{\left(8b^2\right)\int \frac{\left(bx+cx^2\right)^{3/2}}{x^{3/2}}\,dx}{63c^2}\\
&= \frac{16b^2\left(bx+cx^2\right)^{5/2}}{315c^3x^{5/2}}-\frac{8b\left(bx+cx^2\right)^{5/2}}{63c^2x^{3/2}}+\frac{2\left(bx+cx^2\right)^{5/2}}{9c\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.0255045, size = 42, normalized size = 0.52

$$\frac{2(x(b+cx))^{5/2}\left(8b^2-20bcx+35c^2x^2\right)}{315c^3x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(x*(b + c*x))^(5/2)*(8*b^2 - 20*b*c*x + 35*c^2*x^2))/(315*c^3*x^(5/2))
```

Maple [A] time = 0.046, size = 44, normalized size = 0.6

$$\frac{(2\,cx+2\,b)\left(35\,c^2x^2-20\,bcx+8\,b^2\right)}{315\,c^3}\left(cx^2+bx\right)^{\frac{3}{2}}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^2+b*x)^(3/2),x)
```

```
[Out] 2/315*(c*x+b)*(35*c^2*x^2-20*b*c*x+8*b^2)*(c*x^2+b*x)^(3/2)/c^3/x^(3/2)
```

Maxima [A] time = 1.15089, size = 138, normalized size = 1.72

$$\frac{2\left(\left(35\,c^4x^4+5\,bc^3x^3-6\,b^2c^2x^2+8\,b^3cx-16\,b^4\right)x^3+3\left(15\,bc^3x^4+3\,b^2c^2x^3-4\,b^3cx^2+8\,b^4x\right)x^2\right)\sqrt{cx+b}}{315\,c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/315*((35*c^4*x^4 + 5*b*c^3*x^3 - 6*b^2*c^2*x^2 + 8*b^3*c*x - 16*b^4)*x^3
+ 3*(15*b*c^3*x^4 + 3*b^2*c^2*x^3 - 4*b^3*c*x^2 + 8*b^4*x)*x^2)*sqrt(c*x +
b)/(c^3*x^3)
```

Fricas [A] time = 1.90306, size = 139, normalized size = 1.74

$$\frac{2\left(35\,c^4x^4+50\,bc^3x^3+3\,b^2c^2x^2-4\,b^3cx+8\,b^4\right)\sqrt{cx^2+bx}}{315\,c^3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*c^4*x^4 + 50*b*c^3*x^3 + 3*b^2*c^2*x^2 - 4*b^3*c*x + 8*b^4)*sqrt(
c*x^2 + b*x)/(c^3*sqrt(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x}\,(x\,(b+cx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral(sqrt(x)*(x*(b + c*x))**(3/2), x)
```

Giac [A] time = 1.29596, size = 149, normalized size = 1.86

$$\frac{2}{315}\,c\left(\frac{16\,b^{\frac{9}{2}}}{c^4}+\frac{35\,(cx+b)^{\frac{9}{2}}-135\,(cx+b)^{\frac{7}{2}}b+189\,(cx+b)^{\frac{5}{2}}b^2-105\,(cx+b)^{\frac{3}{2}}b^3}{c^4}\right)-\frac{2}{105}\,b\left(\frac{8\,b^{\frac{7}{2}}}{c^3}-\frac{15\,(cx+b)^{\frac{7}{2}}-42\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] 2/315*c*(16*b^(9/2)/c^4 + (35*(c*x + b)^(9/2) - 135*(c*x + b)^(7/2)*b + 189
*(c*x + b)^(5/2)*b^2 - 105*(c*x + b)^(3/2)*b^3)/c^4) - 2/105*b*(8*b^(7/2)/c
^3 - (15*(c*x + b)^(7/2) - 42*(c*x + b)^(5/2)*b + 35*(c*x + b)^(3/2)*b^2)/c
^3)
```

3.86 $\int \frac{(bx+cx^2)^{3/2}}{\sqrt{x}} \, dx$

Optimal. Leaf size=52

$$\frac{2(bx+cx^2)^{5/2}}{7cx^{3/2}} - \frac{4b(bx+cx^2)^{5/2}}{35c^2x^{5/2}}$$

```
[Out] (-4*b*(b*x + c*x^2)^(5/2))/(35*c^2*x^(5/2)) + (2*(b*x + c*x^2)^(5/2))/(7*c*
x^(3/2))
```

Rubi [A] time = 0.0163527, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {656, 648}

$$\frac{2(bx+cx^2)^{5/2}}{7cx^{3/2}} - \frac{4b(bx+cx^2)^{5/2}}{35c^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(3/2)/Sqrt[x],x]
```

```
[Out] (-4*b*(b*x + c*x^2)^(5/2))/(35*c^2*x^(5/2)) + (2*(b*x + c*x^2)^(5/2))/(7*c*
x^(3/2))
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(bx+cx^2)^{3/2}}{\sqrt{x}} \, dx &= \frac{2(bx+cx^2)^{5/2}}{7cx^{3/2}} - \frac{(2b)\int \frac{(bx+cx^2)^{3/2}}{x^{3/2}} \, dx}{7c} \\ &= -\frac{4b(bx+cx^2)^{5/2}}{35c^2x^{5/2}} + \frac{2(bx+cx^2)^{5/2}}{7cx^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.0198447, size = 31, normalized size = 0.6

$$\frac{2(x(b+cx))^{5/2}(5cx-2b)}{35c^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(3/2)/Sqrt[x],x]
```

```
[Out] (2*(x*(b + c*x))^(5/2)*(-2*b + 5*c*x))/(35*c^2*x^(5/2))
```

Maple [A] time = 0.045, size = 33, normalized size = 0.6

$$-\frac{(2\,cx+2\,b)\,(-5\,cx+2\,b)}{35\,c^2}\left(cx^2+bx\right)^{\frac{3}{2}}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(3/2)/x^(1/2),x)
```

```
[Out] -2/35*(c*x+b)*(-5*c*x+2*b)*(c*x^2+b*x)^(3/2)/c^2/x^(3/2)
```

Maxima [A] time = 1.12821, size = 104, normalized size = 2.

$$\frac{2\left(\left(15\,c^3x^3+3\,bc^2x^2-4\,b^2cx+8\,b^3\right)x^2+7\left(3\,bc^2x^3+b^2cx^2-2\,b^3x\right)x\right)\sqrt{cx+b}}{105\,c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*((15*c^3*x^3 + 3*b*c^2*x^2 - 4*b^2*c*x + 8*b^3)*x^2 + 7*(3*b*c^2*x^3
+ b^2*c*x^2 - 2*b^3*x)*x)*sqrt(c*x + b)/(c^2*x^2)
```

Fricas [A] time = 1.83903, size = 111, normalized size = 2.13

$$\frac{2\left(5\,c^3x^3+8\,bc^2x^2+b^2cx-2\,b^3\right)\sqrt{cx^2+bx}}{35\,c^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*c^3*x^3 + 8*b*c^2*x^2 + b^2*c*x - 2*b^3)*sqrt(c*x^2 + b*x)/(c^2*sqr
t(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(x\,(b+cx))^{\frac{3}{2}}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(3/2)/x**(1/2),x)
```

```
[Out] Integral((x*(b + c*x))**(3/2)/sqrt(x), x)
```

Giac [B] time = 1.27858, size = 116, normalized size = 2.23

$$-\frac{2}{105}c\left(\frac{8b^{\frac{7}{2}}}{c^3}-\frac{15(cx+b)^{\frac{7}{2}}-42(cx+b)^{\frac{5}{2}}b+35(cx+b)^{\frac{3}{2}}b^2}{c^3}\right)+\frac{2}{15}b\left(\frac{2b^{\frac{5}{2}}}{c^2}+\frac{3(cx+b)^{\frac{5}{2}}-5(cx+b)^{\frac{3}{2}}b}{c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] -2/105*c*(8*b^(7/2)/c^3 - (15*(c*x + b)^(7/2) - 42*(c*x + b)^(5/2)*b + 35*(
c*x + b)^(3/2)*b^2)/c^3) + 2/15*b*(2*b^(5/2)/c^2 + (3*(c*x + b)^(5/2) - 5*(
c*x + b)^(3/2)*b)/c^2)
```

3.87 $\int \frac{(bx+cx^2)^{3/2}}{x^{3/2}}\,dx$

Optimal. Leaf size=25

$$\frac{2(bx+cx^2)^{5/2}}{5cx^{5/2}}$$

```
[Out] (2*(b*x + c*x^2)^(5/2))/(5*c*x^(5/2))
```

Rubi [A] time = 0.0069109, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {648}

$$\frac{2(bx+cx^2)^{5/2}}{5cx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(3/2)/x^(3/2),x]
```

```
[Out] (2*(b*x + c*x^2)^(5/2))/(5*c*x^(5/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(bx+cx^2)^{3/2}}{x^{3/2}}\,dx = \frac{2(bx+cx^2)^{5/2}}{5cx^{5/2}}$$

Mathematica [A] time = 0.0138718, size = 23, normalized size = 0.92

$$\frac{2(x(b+cx))^{5/2}}{5cx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(3/2)/x^(3/2),x]
```

```
[Out] (2*(x*(b + c*x))^(5/2))/(5*c*x^(5/2))
```

Maple [A] time = 0.044, size = 25, normalized size = 1.

$$\frac{2\,cx+2\,b}{5\,c}\left(cx^2+bx\right)^{\frac{3}{2}}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(3/2)/x^(3/2),x)
```

```
[Out] 2/5*(c*x+b)*(c*x^2+b*x)^(3/2)/c/x^(3/2)
```

Maxima [B] time = 1.13455, size = 66, normalized size = 2.64

$$\frac{2\left(5\,bcx^2+5\,b^2x+\left(3\,c^2x^2+bcx-2\,b^2\right)x\right)\sqrt{cx+b}}{15\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*(5*b*c*x^2 + 5*b^2*x + (3*c^2*x^2 + b*c*x - 2*b^2)*x)*sqrt(c*x + b)/(c
*x)
```

Fricas [A] time = 1.97336, size = 82, normalized size = 3.28

$$\frac{2\left(c^2x^2+2\,bcx+b^2\right)\sqrt{cx^2+bx}}{5\,c\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(c^2*x^2 + 2*b*c*x + b^2)*sqrt(c*x^2 + b*x)/(c*sqrt(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x\,(b+cx))^{\frac{3}{2}}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(3/2)/x**(3/2),x)
```

```
[Out] Integral((x*(b + c*x))**(3/2)/x**(3/2), x)
```

Giac [B] time = 1.32269, size = 81, normalized size = 3.24

$$\frac{2}{15}\,c\left(\frac{2\,b^{\frac{5}{2}}}{c^2}+\frac{3\,(cx+b)^{\frac{5}{2}}-5\,(cx+b)^{\frac{3}{2}}b}{c^2}\right)+\frac{2}{3}\,b\left(\frac{(cx+b)^{\frac{3}{2}}}{c}-\frac{b^{\frac{3}{2}}}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/15*c*(2*b^(5/2)/c^2 + (3*(c*x + b)^(5/2) - 5*(c*x + b)^(3/2)*b)/c^2) + 2/
3*b*((c*x + b)^(3/2)/c - b^(3/2)/c)
```

3.88 $\int \frac{(bx+cx^2)^{3/2}}{x^{5/2}} \, dx$

Optimal. Leaf size=76

$$-2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)+\frac{2b\sqrt{bx+cx^2}}{\sqrt{x}}+\frac{2\left(bx+cx^2\right)^{3/2}}{3x^{3/2}}$$

```
[Out] (2*b*Sqrt[b*x + c*x^2])/Sqrt[x] + (2*(b*x + c*x^2)^(3/2))/(3*x^(3/2)) - 2*b
^(3/2)*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])]
```

Rubi [A] time = 0.0335446, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {664, 660, 207}

$$-2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)+\frac{2b\sqrt{bx+cx^2}}{\sqrt{x}}+\frac{2\left(bx+cx^2\right)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(3/2)/x^(5/2),x]
```

```
[Out] (2*b*Sqrt[b*x + c*x^2])/Sqrt[x] + (2*(b*x + c*x^2)^(3/2))/(3*x^(3/2)) - 2*b
^(3/2)*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx+cx^2\right)^{3/2}}{x^{5/2}}\,dx &= \frac{2\left(bx+cx^2\right)^{3/2}}{3x^{3/2}}+b\int \frac{\sqrt{bx+cx^2}}{x^{3/2}}\,dx\\ &= \frac{2b\sqrt{bx+cx^2}}{\sqrt{x}}+\frac{2\left(bx+cx^2\right)^{3/2}}{3x^{3/2}}+b^2\int \frac{1}{\sqrt{x}\sqrt{bx+cx^2}}\,dx\\ &= \frac{2b\sqrt{bx+cx^2}}{\sqrt{x}}+\frac{2\left(bx+cx^2\right)^{3/2}}{3x^{3/2}}+\left(2b^2\right)\text{Subst}\left(\int \frac{1}{-b+x^2}\,dx,x,\frac{\sqrt{bx+cx^2}}{\sqrt{x}}\right)\\ &= \frac{2b\sqrt{bx+cx^2}}{\sqrt{x}}+\frac{2\left(bx+cx^2\right)^{3/2}}{3x^{3/2}}-2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)\end{aligned}$$

Mathematica [A] time = 0.0444276, size = 70, normalized size = 0.92

$$\frac{2\sqrt{x}\sqrt{b+cx}\left(\sqrt{b+cx}(4b+cx)-3b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b+cx}}{\sqrt{b}}\right)\right)}{3\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(3/2)/x^(5/2),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[b + c*x]*(Sqrt[b + c*x]*(4*b + c*x) - 3*b^(3/2)*ArcTanh[Sqr
t[b + c*x]/Sqrt[b]]))/(3*Sqrt[x*(b + c*x)])
```

Maple [A] time = 0.193, size = 61, normalized size = 0.8

$$-\frac{2}{3}\sqrt{x\,(cx+b)}\left(3\,b^{3/2}\text{Artanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)-xc\sqrt{cx+b}-4\,b\sqrt{cx+b}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{cx+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(3/2)/x^(5/2),x)
```

```
[Out] -2/3*(x*(c*x+b))^(1/2)*(3*b^(3/2)*arctanh((c*x+b)^(1/2)/b^(1/2))-x*c*(c*x+b
)^(1/2)-4*b*(c*x+b)^(1/2))/x^(1/2)/(c*x+b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b\int \frac{\sqrt{cx+b}}{x}\,dx+\frac{2}{3}\,(cx+b)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] b*integrate(sqrt(c*x + b)/x, x) + 2/3*(c*x + b)^(3/2)
```

Fricas [A] time = 2.08767, size = 321, normalized size = 4.22

$$\left[\frac{3\,b^{\frac{3}{2}}x\log\left(-\frac{cx^2+2\,bx-2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)+2\,\sqrt{cx^2+bx}(cx+4\,b)\sqrt{x}}{3\,x},\ \frac{2\left(3\,\sqrt{-b}bx\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{cx^2+bx}}\right)+\sqrt{cx^2+bx}(cx+4\,b)\sqrt{x}\right)}{3\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*b^(3/2)*x*log(-(c*x^2 + 2*b*x - 2*sqrt(c*x^2 + b*x)*sqrt(b)*sqrt(x)
)/x^2) + 2*sqrt(c*x^2 + b*x)*(c*x + 4*b)*sqrt(x))/x, 2/3*(3*sqrt(-b)*b*x*ar
ctan(sqrt(-b)*sqrt(x)/sqrt(c*x^2 + b*x)) + sqrt(c*x^2 + b*x)*(c*x + 4*b)*sq
rt(x))/x]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(x\,(b+cx))^{\frac{3}{2}}}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(3/2)/x**(5/2),x)
```

```
[Out] Integral((x*(b + c*x))**(3/2)/x**(5/2), x)
```

Giac [A] time = 1.26144, size = 104, normalized size = 1.37

$$\frac{2\,b^2\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}+\frac{2}{3}\,(cx+b)^{\frac{3}{2}}+2\,\sqrt{cx+b}b-\frac{2\left(3\,b^2\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+4\,\sqrt{-b}b^{\frac{3}{2}}\right)}{3\,\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2*b^2*arctan(sqrt(c*x + b)/sqrt(-b))/sqrt(-b) + 2/3*(c*x + b)^(3/2) + 2*sqr
t(c*x + b)*b - 2/3*(3*b^2*arctan(sqrt(b)/sqrt(-b)) + 4*sqrt(-b)*b^(3/2))/sq
rt(-b)
```

3.89 $\int \frac{(bx+cx^2)^{3/2}}{x^{7/2}} \, dx$

Optimal. Leaf size=75

$$-\frac{(bx+cx^2)^{3/2}}{x^{5/2}}+\frac{3c\sqrt{bx+cx^2}}{\sqrt{x}}-3\sqrt{b}c\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)$$

```
[Out] (3*c*Sqrt[b*x + c*x^2])/Sqrt[x] - (b*x + c*x^2)^(3/2)/x^(5/2) - 3*Sqrt[b]*c
*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])]
```

Rubi [A] time = 0.0315328, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {662, 664, 660, 207}

$$-\frac{(bx+cx^2)^{3/2}}{x^{5/2}}+\frac{3c\sqrt{bx+cx^2}}{\sqrt{x}}-3\sqrt{b}c\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(3/2)/x^(7/2),x]
```

```
[Out] (3*c*Sqrt[b*x + c*x^2])/Sqrt[x] - (b*x + c*x^2)^(3/2)/x^(5/2) - 3*Sqrt[b]*c
*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])]
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(bx+cx^2\right)^{3/2}}{x^{7/2}}\,dx &= -\frac{\left(bx+cx^2\right)^{3/2}}{x^{5/2}}+\frac{1}{2}(3c)\int \frac{\sqrt{bx+cx^2}}{x^{3/2}}\,dx\\
&= \frac{3c\sqrt{bx+cx^2}}{\sqrt{x}}-\frac{\left(bx+cx^2\right)^{3/2}}{x^{5/2}}+\frac{1}{2}(3bc)\int \frac{1}{\sqrt{x}\sqrt{bx+cx^2}}\,dx\\
&= \frac{3c\sqrt{bx+cx^2}}{\sqrt{x}}-\frac{\left(bx+cx^2\right)^{3/2}}{x^{5/2}}+(3bc)\,\text{Subst}\left(\int \frac{1}{-b+x^2}\,dx,x,\frac{\sqrt{bx+cx^2}}{\sqrt{x}}\right)\\
&= \frac{3c\sqrt{bx+cx^2}}{\sqrt{x}}-\frac{\left(bx+cx^2\right)^{3/2}}{x^{5/2}}-3\sqrt{b}c\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.0162617, size = 40, normalized size = 0.53

$$
\frac{2c(x(b+cx))^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{cx}{b}+1\right)}{5b^2x^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(3/2)/x^(7/2),x]
```

```
[Out] (2*c*(x*(b + c*x))^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + (c*x)/b])/(5*b^
2*x^(5/2))
```

Maple [A] time = 0.195, size = 68, normalized size = 0.9

$$
\left(2\,xc\sqrt{cx+b}\sqrt{b}-3\,\text{Artanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)xbc-b^{\frac{3}{2}}\sqrt{cx+b}\right)\sqrt{x\,(cx+b)}x^{-\frac{3}{2}}\frac{1}{\sqrt{cx+b}}\frac{1}{\sqrt{b}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(3/2)/x^(7/2),x)
```

```
[Out] (2*x*c*(c*x+b)^(1/2)*b^(1/2)-3*arctanh((c*x+b)^(1/2)/b^(1/2))*x*b*c-b^(3/2)
*(c*x+b)^(1/2))*(x*(c*x+b))^(1/2)/x^(3/2)/(c*x+b)^(1/2)/b^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\left(cx^2+bx\right)^{\frac{3}{2}}}{x^{\frac{7}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(3/2)/x^(7/2), x)
```

Fricas [A] time = 2.09823, size = 329, normalized size = 4.39

$$\left[\frac{3\sqrt{b}cx^2\log\left(-\frac{cx^2+2\,bx-2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)+2\,\sqrt{cx^2+bx}(2\,cx-b)\sqrt{x}}{2\,x^2}, \frac{3\,\sqrt{-b}cx^2\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{cx^2+bx}}\right)+\sqrt{cx^2+bx}(2\,cx-b)\sqrt{x}}{x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*sqrt(b)*c*x^2*log(-(c*x^2 + 2*b*x - 2*sqrt(c*x^2 + b*x)*sqrt(b)*sqr
t(x))/x^2) + 2*sqrt(c*x^2 + b*x)*(2*c*x - b)*sqrt(x))/x^2, (3*sqrt(-b)*c*x^
2*arctan(sqrt(-b)*sqrt(x)/sqrt(c*x^2 + b*x)) + sqrt(c*x^2 + b*x)*(2*c*x - b
)*sqrt(x))/x^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x(b+cx))^{\frac{3}{2}}}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(3/2)/x**(7/2),x)
```

```
[Out] Integral((x*(b + c*x))**(3/2)/x**(7/2), x)
```

Giac [A] time = 1.3444, size = 68, normalized size = 0.91

$$\left(\frac{3\,b\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}+2\,\sqrt{cx+b}-\frac{\sqrt{cx+b}b}{cx}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] (3*b*arctan(sqrt(c*x + b)/sqrt(-b))/sqrt(-b) + 2*sqrt(c*x + b) - sqrt(c*x +
 b)*b/(c*x))*c
```

3.90 $\int \frac{(bx+cx^2)^{3/2}}{x^{9/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{4\sqrt{b}}-\frac{3c\sqrt{bx+cx^2}}{4x^{3/2}}-\frac{(bx+cx^2)^{3/2}}{2x^{7/2}}$$

```
[Out] (-3*c*Sqrt[b*x + c*x^2])/(4*x^(3/2)) - (b*x + c*x^2)^(3/2)/(2*x^(7/2)) - (3
*c^2*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/(4*Sqrt[b])
```

Rubi [A] time = 0.0323445, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {662, 660, 207}

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{4\sqrt{b}}-\frac{3c\sqrt{bx+cx^2}}{4x^{3/2}}-\frac{(bx+cx^2)^{3/2}}{2x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(3/2)/x^(9/2),x]
```

```
[Out] (-3*c*Sqrt[b*x + c*x^2])/(4*x^(3/2)) - (b*x + c*x^2)^(3/2)/(2*x^(7/2)) - (3
*c^2*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/(4*Sqrt[b])
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(bx+cx^2\right)^{3/2}}{x^{9/2}}\,dx &= -\frac{\left(bx+cx^2\right)^{3/2}}{2x^{7/2}}+\frac{1}{4}(3c)\int \frac{\sqrt{bx+cx^2}}{x^{5/2}}\,dx \\
&= -\frac{3c\sqrt{bx+cx^2}}{4x^{3/2}}-\frac{\left(bx+cx^2\right)^{3/2}}{2x^{7/2}}+\frac{1}{8}\left(3c^2\right)\int \frac{1}{\sqrt{x}\sqrt{bx+cx^2}}\,dx \\
&= -\frac{3c\sqrt{bx+cx^2}}{4x^{3/2}}-\frac{\left(bx+cx^2\right)^{3/2}}{2x^{7/2}}+\frac{1}{4}\left(3c^2\right)\text{Subst}\left(\int \frac{1}{-b+x^2}\,dx,x,\frac{\sqrt{bx+cx^2}}{\sqrt{x}}\right) \\
&= -\frac{3c\sqrt{bx+cx^2}}{4x^{3/2}}-\frac{\left(bx+cx^2\right)^{3/2}}{2x^{7/2}}-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{4\sqrt{b}}
\end{aligned}
$$

Mathematica [A] time = 0.0491958, size = 72, normalized size = 0.87

$$
-\frac{2b^2+3c^2x^2\sqrt{\frac{cx}{b}+1}\tanh^{-1}\left(\sqrt{\frac{cx}{b}+1}\right)+7bcx+5c^2x^2}{4x^{3/2}\sqrt{x(b+cx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(3/2)/x^(9/2),x]
```

```
[Out] -(2*b^2 + 7*b*c*x + 5*c^2*x^2 + 3*c^2*x^2*Sqrt[1 + (c*x)/b]*ArcTanh[Sqrt[1
+ (c*x)/b]])/(4*x^(3/2)*Sqrt[x*(b + c*x)])
```

Maple [A] time = 0.192, size = 72, normalized size = 0.9

$$
-\frac{1}{4}\sqrt{x\,(cx+b)}\left(3\,\text{Artanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)x^2c^2+5\,xc\sqrt{cx+b}\sqrt{b}+2\,b^{3/2}\sqrt{cx+b}\right)x^{-\frac{5}{2}}\frac{1}{\sqrt{cx+b}}\frac{1}{\sqrt{b}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(3/2)/x^(9/2),x)
```

```
[Out] -1/4*(x*(c*x+b))^(1/2)*(3*arctanh((c*x+b)^(1/2)/b^(1/2))*x^2*c^2+5*x*c*(c*x
+b)^(1/2)*b^(1/2)+2*b^(3/2)*(c*x+b)^(1/2))/x^(5/2)/(c*x+b)^(1/2)/b^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\left(cx^2+bx\right)^{\frac{3}{2}}}{x^{\frac{9}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(3/2)/x^(9/2), x)
```

Fricas [A] time = 2.19315, size = 367, normalized size = 4.42

$$\left[\frac{3\sqrt{b}c^2x^3\log\left(-\frac{cx^2+2bx-2\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)-2\left(5bcx+2b^2\right)\sqrt{cx^2+bx}\sqrt{x}}{8bx^3}, \frac{3\sqrt{-b}c^2x^3\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{cx^2+bx}}\right)-\left(5bcx+2b^2\right)\sqrt{cx^2+bx}\sqrt{x}}{4bx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(b)*c^2*x^3*log(-(c*x^2 + 2*b*x - 2*sqrt(c*x^2 + b*x)*sqrt(b)*s
qrt(x))/x^2) - 2*(5*b*c*x + 2*b^2)*sqrt(c*x^2 + b*x)*sqrt(x))/(b*x^3), 1/4*
(3*sqrt(-b)*c^2*x^3*arctan(sqrt(-b)*sqrt(x)/sqrt(c*x^2 + b*x)) - (5*b*c*x +
 2*b^2)*sqrt(c*x^2 + b*x)*sqrt(x))/(b*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(3/2)/x**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30394, size = 74, normalized size = 0.89

$$\frac{1}{4}c^2\left(\frac{3\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}-\frac{5(cx+b)^{\frac{3}{2}}-3\sqrt{cx+b}b}{c^2x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] 1/4*c^2*(3*arctan(sqrt(c*x + b)/sqrt(-b))/sqrt(-b) - (5*(c*x + b)^(3/2) - 3
*sqrt(c*x + b)*b)/(c^2*x^2))
```

3.91 $\int \frac{(bx+cx^2)^{3/2}}{x^{11/2}}\,dx$

Optimal. Leaf size=111

$$\frac{c^3\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{8b^{3/2}}-\frac{c^2\sqrt{bx+cx^2}}{8bx^{3/2}}-\frac{c\sqrt{bx+cx^2}}{4x^{5/2}}-\frac{(bx+cx^2)^{3/2}}{3x^{9/2}}$$

```
[Out] -(c*Sqrt[b*x + c*x^2])/(4*x^(5/2)) - (c^2*Sqrt[b*x + c*x^2])/(8*b*x^(3/2))
- (b*x + c*x^2)^(3/2)/(3*x^(9/2)) + (c^3*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]
*Sqrt[x])])/(8*b^(3/2))
```

Rubi [A] time = 0.0460828, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {662, 672, 660, 207}

$$\frac{c^3\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{8b^{3/2}}-\frac{c^2\sqrt{bx+cx^2}}{8bx^{3/2}}-\frac{c\sqrt{bx+cx^2}}{4x^{5/2}}-\frac{(bx+cx^2)^{3/2}}{3x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(3/2)/x^(11/2),x]
```

```
[Out] -(c*Sqrt[b*x + c*x^2])/(4*x^(5/2)) - (c^2*Sqrt[b*x + c*x^2])/(8*b*x^(3/2))
- (b*x + c*x^2)^(3/2)/(3*x^(9/2)) + (c^3*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]
*Sqrt[x])])/(8*b^(3/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*(m + 2*p + 2))/((m + p + 1)*(2*c*d - b*e)), Int[(d
 + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && Ne
Q[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(bx+cx^2)^{3/2}}{x^{11/2}}\,dx &= -\frac{(bx+cx^2)^{3/2}}{3x^{9/2}}+\frac{1}{2}c\int \frac{\sqrt{bx+cx^2}}{x^{7/2}}\,dx\\ &= -\frac{c\sqrt{bx+cx^2}}{4x^{5/2}}-\frac{(bx+cx^2)^{3/2}}{3x^{9/2}}+\frac{1}{8}c^2\int \frac{1}{x^{3/2}\sqrt{bx+cx^2}}\,dx\\ &= -\frac{c\sqrt{bx+cx^2}}{4x^{5/2}}-\frac{c^2\sqrt{bx+cx^2}}{8bx^{3/2}}-\frac{(bx+cx^2)^{3/2}}{3x^{9/2}}-\frac{c^3\int \frac{1}{\sqrt{x}\sqrt{bx+cx^2}}\,dx}{16b}\\ &= -\frac{c\sqrt{bx+cx^2}}{4x^{5/2}}-\frac{c^2\sqrt{bx+cx^2}}{8bx^{3/2}}-\frac{(bx+cx^2)^{3/2}}{3x^{9/2}}-\frac{c^3\,\mathrm{Subst}\left(\int \frac{1}{-b+x^2}\,dx,x,\frac{\sqrt{bx+cx^2}}{\sqrt{x}}\right)}{8b}\\ &= -\frac{c\sqrt{bx+cx^2}}{4x^{5/2}}-\frac{c^2\sqrt{bx+cx^2}}{8bx^{3/2}}-\frac{(bx+cx^2)^{3/2}}{3x^{9/2}}+\frac{c^3\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{8b^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.0185051, size = 42, normalized size = 0.38

$$\frac{2c^3(x(b+cx))^{5/2}\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{cx}{b}+1\right)}{5b^4x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(3/2)/x^(11/2),x]
```

```
[Out] (2*c^3*(x*(b + c*x))^(5/2)*Hypergeometric2F1[5/2, 4, 7/2, 1 + (c*x)/b])/(5*
b^4*x^(5/2))
```

Maple [A] time = 0.189, size = 90, normalized size = 0.8

$$\frac{1}{24}\sqrt{x(cx+b)}\left(3\,\mathrm{Artanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)x^3c^3-3\,x^2c^2\sqrt{b}\sqrt{cx+b}-14\,xb^{3/2}c\sqrt{cx+b}-8\,b^{5/2}\sqrt{cx+b}\right)b^{-\frac{3}{2}}x^{-\frac{7}{2}}\frac{1}{\sqrt{cx+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(3/2)/x^(11/2),x)
```

```
[Out] 1/24*(x*(c*x+b))^(1/2)/b^(3/2)*(3*arctanh((c*x+b)^(1/2)/b^(1/2))*x^3*c^3-3*
x^2*c^2*b^(1/2)*(c*x+b)^(1/2)-14*x*b^(3/2)*c*(c*x+b)^(1/2)-8*b^(5/2)*(c*x+b
)^(1/2))/x^(7/2)/(c*x+b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2+bx)^{\frac{3}{2}}}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(3/2)/x^(11/2), x)
```

Fricas [A] time = 2.15364, size = 423, normalized size = 3.81

$$\left[\frac{3\sqrt{b}c^3x^4\log\left(-\frac{cx^2+2\,bx+2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)-2\left(3\,bc^2x^2+14\,b^2cx+8\,b^3\right)\sqrt{cx^2+bx}\sqrt{x}}{48\,b^2x^4}, -\frac{3\sqrt{-b}c^3x^4\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{cx^2+bx}}\right)+(\ldots}{2\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(b)*c^3*x^4*log(-(c*x^2 + 2*b*x + 2*sqrt(c*x^2 + b*x)*sqrt(b)*
sqrt(x))/x^2) - 2*(3*b*c^2*x^2 + 14*b^2*c*x + 8*b^3)*sqrt(c*x^2 + b*x)*sqrt
(x))/(b^2*x^4), -1/24*(3*sqrt(-b)*c^3*x^4*arctan(sqrt(-b)*sqrt(x)/sqrt(c*x^
2 + b*x)) + (3*b*c^2*x^2 + 14*b^2*c*x + 8*b^3)*sqrt(c*x^2 + b*x)*sqrt(x))/(
b^2*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(3/2)/x**(11/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33937, size = 97, normalized size = 0.87

$$-\frac{1}{24}c^3\left(\frac{3\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b}+\frac{3\,(cx+b)^{\frac{5}{2}}+8\,(cx+b)^{\frac{3}{2}}b-3\,\sqrt{cx+b}b^2}{bc^3x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^(11/2),x, algorithm="giac")
```

```
[Out] -1/24*c^3*(3*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b) + (3*(c*x + b)^(5/
2) + 8*(c*x + b)^(3/2)*b - 3*sqrt(c*x + b)*b^2)/(b*c^3*x^3))
```

3.92 $\int \frac{(bx+cx^2)^{3/2}}{x^{13/2}} dx$

Optimal. Leaf size=139

$$\frac{3c^3\sqrt{bx+cx^2}}{64b^2x^{3/2}} - \frac{3c^4\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{64b^{5/2}} - \frac{c^2\sqrt{bx+cx^2}}{32bx^{5/2}} - \frac{c\sqrt{bx+cx^2}}{8x^{7/2}} - \frac{(bx+cx^2)^{3/2}}{4x^{11/2}}$$

```
[Out] -(c*Sqrt[b*x + c*x^2])/(8*x^(7/2)) - (c^2*Sqrt[b*x + c*x^2])/(32*b*x^(5/2))
 + (3*c^3*Sqrt[b*x + c*x^2])/(64*b^2*x^(3/2)) - (b*x + c*x^2)^(3/2)/(4*x^(1
1/2)) - (3*c^4*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/(64*b^(5/2))
```

Rubi [A] time = 0.0646788, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {662, 672, 660, 207}

$$\frac{3c^3\sqrt{bx+cx^2}}{64b^2x^{3/2}} - \frac{3c^4\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{64b^{5/2}} - \frac{c^2\sqrt{bx+cx^2}}{32bx^{5/2}} - \frac{c\sqrt{bx+cx^2}}{8x^{7/2}} - \frac{(bx+cx^2)^{3/2}}{4x^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(3/2)/x^(13/2),x]
```

```
[Out] -(c*Sqrt[b*x + c*x^2])/(8*x^(7/2)) - (c^2*Sqrt[b*x + c*x^2])/(32*b*x^(5/2))
 + (3*c^3*Sqrt[b*x + c*x^2])/(64*b^2*x^(3/2)) - (b*x + c*x^2)^(3/2)/(4*x^(1
1/2)) - (3*c^4*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/(64*b^(5/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*(m + 2*p + 2))/((m + p + 1)*(2*c*d - b*e)), Int[(d
 + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && Ne
Q[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx+cx^2)^{3/2}}{x^{13/2}}\,dx &= -\frac{(bx+cx^2)^{3/2}}{4x^{11/2}} + \frac{1}{8}(3c)\int \frac{\sqrt{bx+cx^2}}{x^{9/2}}\,dx\\
&= -\frac{c\sqrt{bx+cx^2}}{8x^{7/2}} - \frac{(bx+cx^2)^{3/2}}{4x^{11/2}} + \frac{1}{16}c^2\int \frac{1}{x^{5/2}\sqrt{bx+cx^2}}\,dx\\
&= -\frac{c\sqrt{bx+cx^2}}{8x^{7/2}} - \frac{c^2\sqrt{bx+cx^2}}{32bx^{5/2}} - \frac{(bx+cx^2)^{3/2}}{4x^{11/2}} - \frac{(3c^3)\int \frac{1}{x^{3/2}\sqrt{bx+cx^2}}\,dx}{64b}\\
&= -\frac{c\sqrt{bx+cx^2}}{8x^{7/2}} - \frac{c^2\sqrt{bx+cx^2}}{32bx^{5/2}} + \frac{3c^3\sqrt{bx+cx^2}}{64b^2x^{3/2}} - \frac{(bx+cx^2)^{3/2}}{4x^{11/2}} + \frac{(3c^4)\int \frac{1}{\sqrt{x}\sqrt{bx+cx^2}}\,dx}{128b^2}\\
&= -\frac{c\sqrt{bx+cx^2}}{8x^{7/2}} - \frac{c^2\sqrt{bx+cx^2}}{32bx^{5/2}} + \frac{3c^3\sqrt{bx+cx^2}}{64b^2x^{3/2}} - \frac{(bx+cx^2)^{3/2}}{4x^{11/2}} + \frac{(3c^4)\,\mathrm{Subst}\left(\int \frac{1}{-b+x^2}\,dx, x, \frac{\sqrt{bx}}{}\right.}{64b^2}\\
&= -\frac{c\sqrt{bx+cx^2}}{8x^{7/2}} - \frac{c^2\sqrt{bx+cx^2}}{32bx^{5/2}} + \frac{3c^3\sqrt{bx+cx^2}}{64b^2x^{3/2}} - \frac{(bx+cx^2)^{3/2}}{4x^{11/2}} - \frac{3c^4\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{64b^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.0176101, size = 42, normalized size = 0.3

$$-\frac{2c^4(x(b+cx))^{5/2}\,{}_2F_1\left(\frac{5}{2},5;\frac{7}{2};\frac{cx}{b}+1\right)}{5b^5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(3/2)/x^(13/2),x]
```

```
[Out] (-2*c^4*(x*(b + c*x))^(5/2)*Hypergeometric2F1[5/2, 5, 7/2, 1 + (c*x)/b])/(5
*b^5*x^(5/2))
```

Maple [A] time = 0.186, size = 108, normalized size = 0.8

$$-\frac{1}{64}\sqrt{x(cx+b)}\left(3\,\mathrm{Artanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)x^4c^4 - 3\,x^3c^3\sqrt{cx+b}\sqrt{b} + 2\,x^2b^{3/2}c^2\sqrt{cx+b} + 24\,xb^{5/2}c\sqrt{cx+b} + 16\,b^{7/2}\sqrt{cx+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(3/2)/x^(13/2),x)
```

```
[Out] -1/64*(x*(c*x+b))^(1/2)/b^(5/2)*(3*arctanh((c*x+b)^(1/2)/b^(1/2))*x^4*c^4-3
*x^3*c^3*(c*x+b)^(1/2)*b^(1/2)+2*x^2*b^(3/2)*c^2*(c*x+b)^(1/2)+24*x*b^(5/2)
*c*(c*x+b)^(1/2)+16*b^(7/2)*(c*x+b)^(1/2))/x^(9/2)/(c*x+b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2+bx)^{\frac{3}{2}}}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^(13/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(3/2)/x^(13/2), x)
```

Fricas [A] time = 2.09697, size = 468, normalized size = 3.37

$$\left[\frac{3\sqrt{b}c^4x^5\log\left(-\frac{cx^2+2bx-2\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)+2\left(3bc^3x^3-2b^2c^2x^2-24b^3cx-16b^4\right)\sqrt{cx^2+bx}\sqrt{x}}{128b^3x^5}, \frac{3\sqrt{-b}c^4x^5\arctan\left(\frac{\sqrt{-b}\sqrt{}}{\sqrt{cx^2+}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^(13/2),x, algorithm="fricas")
```

```
[Out] [1/128*(3*sqrt(b)*c^4*x^5*log(-(c*x^2 + 2*b*x - 2*sqrt(c*x^2 + b*x)*sqrt(b)
*sqrt(x))/x^2) + 2*(3*b*c^3*x^3 - 2*b^2*c^2*x^2 - 24*b^3*c*x - 16*b^4)*sqrt
(c*x^2 + b*x)*sqrt(x))/(b^3*x^5), 1/64*(3*sqrt(-b)*c^4*x^5*arctan(sqrt(-b)*
sqrt(x)/sqrt(c*x^2 + b*x)) + (3*b*c^3*x^3 - 2*b^2*c^2*x^2 - 24*b^3*c*x - 16
*b^4)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^3*x^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(3/2)/x**(13/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3455, size = 113, normalized size = 0.81

$$\frac{1}{64}c^4\left(\frac{3\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^2}+\frac{3(cx+b)^{\frac{7}{2}}-11(cx+b)^{\frac{5}{2}}b-11(cx+b)^{\frac{3}{2}}b^2+3\sqrt{cx+b}b^3}{b^2c^4x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^(13/2),x, algorithm="giac")
```

```
[Out] 1/64*c^4*(3*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b^2) + (3*(c*x + b)^(7
/2) - 11*(c*x + b)^(5/2)*b - 11*(c*x + b)^(3/2)*b^2 + 3*sqrt(c*x + b)*b^3)/
(b^2*c^4*x^4))
```

3.93 $\int \frac{(bx+cx^2)^{3/2}}{x^{15/2}} dx$

Optimal. Leaf size=167

$$-\frac{3c^4\sqrt{bx+cx^2}}{128b^3x^{3/2}}+\frac{c^3\sqrt{bx+cx^2}}{64b^2x^{5/2}}+\frac{3c^5\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{128b^{7/2}}-\frac{c^2\sqrt{bx+cx^2}}{80bx^{7/2}}-\frac{3c\sqrt{bx+cx^2}}{40x^{9/2}}-\frac{(bx+cx^2)^{3/2}}{5x^{13/2}}$$

```
[Out] (-3*c*Sqrt[b*x + c*x^2])/(40*x^(9/2)) - (c^2*Sqrt[b*x + c*x^2])/(80*b*x^(7/
2)) + (c^3*Sqrt[b*x + c*x^2])/(64*b^2*x^(5/2)) - (3*c^4*Sqrt[b*x + c*x^2])/
(128*b^3*x^(3/2)) - (b*x + c*x^2)^(3/2)/(5*x^(13/2)) + (3*c^5*ArcTanh[Sqrt[
b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/(128*b^(7/2))
```

Rubi [A] time = 0.0833269, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {662, 672, 660, 207}

$$-\frac{3c^4\sqrt{bx+cx^2}}{128b^3x^{3/2}}+\frac{c^3\sqrt{bx+cx^2}}{64b^2x^{5/2}}+\frac{3c^5\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{128b^{7/2}}-\frac{c^2\sqrt{bx+cx^2}}{80bx^{7/2}}-\frac{3c\sqrt{bx+cx^2}}{40x^{9/2}}-\frac{(bx+cx^2)^{3/2}}{5x^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(3/2)/x^(15/2),x]
```

```
[Out] (-3*c*Sqrt[b*x + c*x^2])/(40*x^(9/2)) - (c^2*Sqrt[b*x + c*x^2])/(80*b*x^(7/
2)) + (c^3*Sqrt[b*x + c*x^2])/(64*b^2*x^(5/2)) - (3*c^4*Sqrt[b*x + c*x^2])/
(128*b^3*x^(3/2)) - (b*x + c*x^2)^(3/2)/(5*x^(13/2)) + (3*c^5*ArcTanh[Sqrt[
b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/(128*b^(7/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*(m + 2*p + 2))/((m + p + 1)*(2*c*d - b*e)), Int[(d
 + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && Ne
Q[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(bx+cx^2\right)^{3/2}}{x^{15/2}}\,dx = -\frac{\left(bx+cx^2\right)^{3/2}}{5x^{13/2}}+\frac{1}{10}(3c)\int \frac{\sqrt{bx+cx^2}}{x^{11/2}}\,dx$$
$$= -\frac{3c\sqrt{bx+cx^2}}{40x^{9/2}}-\frac{\left(bx+cx^2\right)^{3/2}}{5x^{13/2}}+\frac{1}{80}\left(3c^2\right)\int \frac{1}{x^{7/2}\sqrt{bx+cx^2}}\,dx$$
$$= -\frac{3c\sqrt{bx+cx^2}}{40x^{9/2}}-\frac{c^2\sqrt{bx+cx^2}}{80bx^{7/2}}-\frac{\left(bx+cx^2\right)^{3/2}}{5x^{13/2}}-\frac{c^3\int \frac{1}{x^{5/2}\sqrt{bx+cx^2}}\,dx}{32b}$$
$$= -\frac{3c\sqrt{bx+cx^2}}{40x^{9/2}}-\frac{c^2\sqrt{bx+cx^2}}{80bx^{7/2}}+\frac{c^3\sqrt{bx+cx^2}}{64b^2x^{5/2}}-\frac{\left(bx+cx^2\right)^{3/2}}{5x^{13/2}}+\frac{\left(3c^4\right)\int \frac{1}{x^{3/2}\sqrt{bx+cx^2}}\,dx}{128b^2}$$
$$= -\frac{3c\sqrt{bx+cx^2}}{40x^{9/2}}-\frac{c^2\sqrt{bx+cx^2}}{80bx^{7/2}}+\frac{c^3\sqrt{bx+cx^2}}{64b^2x^{5/2}}-\frac{3c^4\sqrt{bx+cx^2}}{128b^3x^{3/2}}-\frac{\left(bx+cx^2\right)^{3/2}}{5x^{13/2}}-\frac{\left(3c^5\right)\int \frac{1}{\sqrt{x}\sqrt{bx+cx^2}}}{256b^3}$$
$$= -\frac{3c\sqrt{bx+cx^2}}{40x^{9/2}}-\frac{c^2\sqrt{bx+cx^2}}{80bx^{7/2}}+\frac{c^3\sqrt{bx+cx^2}}{64b^2x^{5/2}}-\frac{3c^4\sqrt{bx+cx^2}}{128b^3x^{3/2}}-\frac{\left(bx+cx^2\right)^{3/2}}{5x^{13/2}}-\frac{\left(3c^5\right)\text{Subst}\left(\int -\right.}{12}$$
$$= -\frac{3c\sqrt{bx+cx^2}}{40x^{9/2}}-\frac{c^2\sqrt{bx+cx^2}}{80bx^{7/2}}+\frac{c^3\sqrt{bx+cx^2}}{64b^2x^{5/2}}-\frac{3c^4\sqrt{bx+cx^2}}{128b^3x^{3/2}}-\frac{\left(bx+cx^2\right)^{3/2}}{5x^{13/2}}+\frac{3c^5\tanh^{-1}\left(\frac{\sqrt{bx+}}{\sqrt{b}}\right.}{128b^{7/2}}$$

Mathematica [C] time = 0.0180711, size = 42, normalized size = 0.25

$$\frac{2c^5(x(b+cx))^{5/2}\,{}_2F_1\left(\frac{5}{2},6;\frac{7}{2};\frac{cx}{b}+1\right)}{5b^6x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(3/2)/x^(15/2),x]
```

```
[Out] (2*c^5*(x*(b + c*x))^(5/2)*Hypergeometric2F1[5/2, 6, 7/2, 1 + (c*x)/b])/(5*
b^6*x^(5/2))
```

Maple [A] time = 0.218, size = 126, normalized size = 0.8

$$\frac{1}{640}\sqrt{x(cx+b)}\left(15\,\text{Artanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)x^5c^5-15\,x^4c^4\sqrt{b}\sqrt{cx+b}+10\,x^3b^{3/2}c^3\sqrt{cx+b}-8\,x^2b^{5/2}c^2\sqrt{cx+b}-176\,xb^{7/2}c\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(3/2)/x^(15/2),x)
```

```
[Out] 1/640*(x*(c*x+b))^(1/2)/b^(7/2)*(15*arctanh((c*x+b)^(1/2)/b^(1/2))*x^5*c^5-
15*x^4*c^4*b^(1/2)*(c*x+b)^(1/2)+10*x^3*b^(3/2)*c^3*(c*x+b)^(1/2)-8*x^2*b^(
5/2)*c^2*(c*x+b)^(1/2)-176*x*b^(7/2)*c*(c*x+b)^(1/2)-128*b^(9/2)*(c*x+b)^(1
/2))/x^(11/2)/(c*x+b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2+bx)^{\frac{3}{2}}}{x^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^(15/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(3/2)/x^(15/2), x)
```

Fricas [A] time = 2.1178, size = 529, normalized size = 3.17

$$\left[\frac{15\,\sqrt{b}c^5x^6\log\left(-\frac{cx^2+2\,bx+2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)-2\left(15\,bc^4x^4-10\,b^2c^3x^3+8\,b^3c^2x^2+176\,b^4cx+128\,b^5\right)\sqrt{cx^2+bx}\sqrt{x}}{1280\,b^4x^6},-\frac{15}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^(15/2),x, algorithm="fricas")
```

```
[Out] [1/1280*(15*sqrt(b)*c^5*x^6*log(-(c*x^2 + 2*b*x + 2*sqrt(c*x^2 + b*x)*sqrt(
b)*sqrt(x))/x^2) - 2*(15*b*c^4*x^4 - 10*b^2*c^3*x^3 + 8*b^3*c^2*x^2 + 176*b
^4*c*x + 128*b^5)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^4*x^6), -1/640*(15*sqrt(-b)
*c^5*x^6*arctan(sqrt(-b)*sqrt(x)/sqrt(c*x^2 + b*x)) + (15*b*c^4*x^4 - 10*b^
2*c^3*x^3 + 8*b^3*c^2*x^2 + 176*b^4*c*x + 128*b^5)*sqrt(c*x^2 + b*x)*sqrt(x
))/(b^4*x^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(3/2)/x**(15/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35627, size = 130, normalized size = 0.78

$$-\frac{1}{640}\,c^5\left(\frac{15\,\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^3}+\frac{15\,(cx+b)^{\frac{9}{2}}-70\,(cx+b)^{\frac{7}{2}}b+128\,(cx+b)^{\frac{5}{2}}b^2+70\,(cx+b)^{\frac{3}{2}}b^3-15\,\sqrt{cx+b}b^4}{b^3c^5x^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/x^(15/2),x, algorithm="giac")
```

```
[Out] -1/640*c^5*(15*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b^3) + (15*(c*x + b
)^(9/2) - 70*(c*x + b)^(7/2)*b + 128*(c*x + b)^(5/2)*b^2 + 70*(c*x + b)^(3/
2)*b^3 - 15*sqrt(c*x + b)*b^4)/(b^3*c^5*x^5))
```

3.94 $\int \frac{x^{7/2}}{\sqrt{bx+cx^2}}\,dx$

Optimal. Leaf size=108

$$-\frac{32b^3\sqrt{bx+cx^2}}{35c^4\sqrt{x}}+\frac{16b^2\sqrt{x}\sqrt{bx+cx^2}}{35c^3}-\frac{12bx^{3/2}\sqrt{bx+cx^2}}{35c^2}+\frac{2x^{5/2}\sqrt{bx+cx^2}}{7c}$$

```
[Out] (-32*b^3*Sqrt[b*x + c*x^2])/(35*c^4*Sqrt[x]) + (16*b^2*Sqrt[x]*Sqrt[b*x + c
*x^2])/(35*c^3) - (12*b*x^(3/2)*Sqrt[b*x + c*x^2])/(35*c^2) + (2*x^(5/2)*Sq
rt[b*x + c*x^2])/(7*c)
```

Rubi [A] time = 0.040222, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {656, 648}

$$-\frac{32b^3\sqrt{bx+cx^2}}{35c^4\sqrt{x}}+\frac{16b^2\sqrt{x}\sqrt{bx+cx^2}}{35c^3}-\frac{12bx^{3/2}\sqrt{bx+cx^2}}{35c^2}+\frac{2x^{5/2}\sqrt{bx+cx^2}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/Sqrt[b*x + c*x^2],x]
```

```
[Out] (-32*b^3*Sqrt[b*x + c*x^2])/(35*c^4*Sqrt[x]) + (16*b^2*Sqrt[x]*Sqrt[b*x + c
*x^2])/(35*c^3) - (12*b*x^(3/2)*Sqrt[b*x + c*x^2])/(35*c^2) + (2*x^(5/2)*Sq
rt[b*x + c*x^2])/(7*c)
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{7/2}}{\sqrt{bx+cx^2}}\,dx &= \frac{2x^{5/2}\sqrt{bx+cx^2}}{7c} - \frac{(6b)\int \frac{x^{5/2}}{\sqrt{bx+cx^2}}\,dx}{7c} \\ &= -\frac{12bx^{3/2}\sqrt{bx+cx^2}}{35c^2} + \frac{2x^{5/2}\sqrt{bx+cx^2}}{7c} + \frac{\left(24b^2\right)\int \frac{x^{3/2}}{\sqrt{bx+cx^2}}\,dx}{35c^2} \\ &= \frac{16b^2\sqrt{x}\sqrt{bx+cx^2}}{35c^3} - \frac{12bx^{3/2}\sqrt{bx+cx^2}}{35c^2} + \frac{2x^{5/2}\sqrt{bx+cx^2}}{7c} - \frac{\left(16b^3\right)\int \frac{\sqrt{x}}{\sqrt{bx+cx^2}}\,dx}{35c^3} \\ &= -\frac{32b^3\sqrt{bx+cx^2}}{35c^4\sqrt{x}} + \frac{16b^2\sqrt{x}\sqrt{bx+cx^2}}{35c^3} - \frac{12bx^{3/2}\sqrt{bx+cx^2}}{35c^2} + \frac{2x^{5/2}\sqrt{bx+cx^2}}{7c}\end{aligned}$$

Mathematica [A] time = 0.0323661, size = 53, normalized size = 0.49

$$\frac{2\sqrt{x(b+cx)}\left(8b^2cx-16b^3-6bc^2x^2+5c^3x^3\right)}{35c^4\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/Sqrt[b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[x*(b + c*x)]*(-16*b^3 + 8*b^2*c*x - 6*b*c^2*x^2 + 5*c^3*x^3))/(35*c
^4*Sqrt[x])
```

Maple [A] time = 0.047, size = 55, normalized size = 0.5

$$-\frac{(2\,cx+2\,b)\left(-5\,x^3c^3+6\,bx^2c^2-8\,b^2xc+16\,b^3\right)}{35\,c^4}\sqrt{x}\frac{1}{\sqrt{cx^2+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^2+b*x)^(1/2),x)
```

```
[Out] -2/35*(c*x+b)*(-5*c^3*x^3+6*b*c^2*x^2-8*b^2*c*x+16*b^3)*x^(1/2)/c^4/(c*x^2+
b*x)^(1/2)
```

Maxima [A] time = 1.14649, size = 72, normalized size = 0.67

$$\frac{2\left(5\,c^4x^4-bc^3x^3+2\,b^2c^2x^2-8\,b^3cx-16\,b^4\right)}{35\,\sqrt{cx+b}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*c^4*x^4 - b*c^3*x^3 + 2*b^2*c^2*x^2 - 8*b^3*c*x - 16*b^4)/(sqrt(c*x
 + b)*c^4)
```

Fricas [A] time = 2.0068, size = 115, normalized size = 1.06

$$\frac{2\left(5\,c^3x^3 - 6\,bc^2x^2 + 8\,b^2cx - 16\,b^3\right)\sqrt{cx^2 + bx}}{35\,c^4\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*c^3*x^3 - 6*b*c^2*x^2 + 8*b^2*c*x - 16*b^3)*sqrt(c*x^2 + b*x)/(c^4*
sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19238, size = 78, normalized size = 0.72

$$\frac{32\,b^{\frac{7}{2}}}{35\,c^4} + \frac{2\left(5\,(cx + b)^{\frac{7}{2}} - 21\,(cx + b)^{\frac{5}{2}}b + 35\,(cx + b)^{\frac{3}{2}}b^2 - 35\,\sqrt{cx + b}b^3\right)}{35\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] 32/35*b^(7/2)/c^4 + 2/35*(5*(c*x + b)^(7/2) - 21*(c*x + b)^(5/2)*b + 35*(c*
x + b)^(3/2)*b^2 - 35*sqrt(c*x + b)*b^3)/c^4
```

3.95 $\int \frac{x^{5/2}}{\sqrt{bx+cx^2}} dx$

Optimal. Leaf size=80

$$\frac{16b^2\sqrt{bx+cx^2}}{15c^3\sqrt{x}} - \frac{8b\sqrt{x}\sqrt{bx+cx^2}}{15c^2} + \frac{2x^{3/2}\sqrt{bx+cx^2}}{5c}$$

```
[Out] (16*b^2*Sqrt[b*x + c*x^2])/(15*c^3*Sqrt[x]) - (8*b*Sqrt[x]*Sqrt[b*x + c*x^2
])/(15*c^2) + (2*x^(3/2)*Sqrt[b*x + c*x^2])/(5*c)
```

Rubi [A] time = 0.0265036, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {656, 648}

$$\frac{16b^2\sqrt{bx+cx^2}}{15c^3\sqrt{x}} - \frac{8b\sqrt{x}\sqrt{bx+cx^2}}{15c^2} + \frac{2x^{3/2}\sqrt{bx+cx^2}}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[b*x + c*x^2],x]
```

```
[Out] (16*b^2*Sqrt[b*x + c*x^2])/(15*c^3*Sqrt[x]) - (8*b*Sqrt[x]*Sqrt[b*x + c*x^2
])/(15*c^2) + (2*x^(3/2)*Sqrt[b*x + c*x^2])/(5*c)
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{\sqrt{bx+cx^2}} dx &= \frac{2x^{3/2}\sqrt{bx+cx^2}}{5c} - \frac{(4b)\int \frac{x^{3/2}}{\sqrt{bx+cx^2}} dx}{5c} \\
&= -\frac{8b\sqrt{x}\sqrt{bx+cx^2}}{15c^2} + \frac{2x^{3/2}\sqrt{bx+cx^2}}{5c} + \frac{\left(8b^2\right)\int \frac{\sqrt{x}}{\sqrt{bx+cx^2}} dx}{15c^2} \\
&= \frac{16b^2\sqrt{bx+cx^2}}{15c^3\sqrt{x}} - \frac{8b\sqrt{x}\sqrt{bx+cx^2}}{15c^2} + \frac{2x^{3/2}\sqrt{bx+cx^2}}{5c}
\end{aligned}$$

Mathematica [A] time = 0.0227216, size = 42, normalized size = 0.52

$$\frac{2\sqrt{x(b+cx)}\left(8b^2-4bcx+3c^2x^2\right)}{15c^3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[x*(b + c*x)]*(8*b^2 - 4*b*c*x + 3*c^2*x^2))/(15*c^3*Sqrt[x])
```

Maple [A] time = 0.049, size = 44, normalized size = 0.6

$$\frac{(2\,cx+2\,b)\left(3\,c^2x^2-4\,bcx+8\,b^2\right)}{15\,c^3}\sqrt{x}\frac{1}{\sqrt{cx^2+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^2+b*x)^(1/2),x)
```

```
[Out] 2/15*(c*x+b)*(3*c^2*x^2-4*b*c*x+8*b^2)*x^(1/2)/c^3/(c*x^2+b*x)^(1/2)
```

Maxima [A] time = 1.15306, size = 57, normalized size = 0.71

$$\frac{2\left(3\,c^3x^3-bc^2x^2+4\,b^2cx+8\,b^3\right)}{15\,\sqrt{cx+b}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*c^3*x^3 - b*c^2*x^2 + 4*b^2*c*x + 8*b^3)/(sqrt(c*x + b)*c^3)
```

Fricas [A] time = 2.0872, size = 92, normalized size = 1.15

$$\frac{2\left(3\,c^2x^2-4\,bcx+8\,b^2\right)\sqrt{cx^2+bx}}{15\,c^3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*c^2*x^2 - 4*b*c*x + 8*b^2)*sqrt(c*x^2 + b*x)/(c^3*sqrt(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{\frac{5}{2}}}{\sqrt{x\,(b+cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(x**(5/2)/sqrt(x*(b + c*x)), x)
```

Giac [A] time = 1.28111, size = 62, normalized size = 0.78

$$-\frac{16\,b^{\frac{5}{2}}}{15\,c^3}+\frac{2\left(3\,(cx+b)^{\frac{5}{2}}-10\,(cx+b)^{\frac{3}{2}}b+15\,\sqrt{cx+b}b^2\right)}{15\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] -16/15*b^(5/2)/c^3 + 2/15*(3*(c*x + b)^(5/2) - 10*(c*x + b)^(3/2)*b + 15*sq
rt(c*x + b)*b^2)/c^3
```

3.96 $\int \frac{x^{3/2}}{\sqrt{bx+cx^2}}\,dx$

Optimal. Leaf size=52

$$\frac{2\sqrt{x}\sqrt{bx+cx^2}}{3c}-\frac{4b\sqrt{bx+cx^2}}{3c^2\sqrt{x}}$$

```
[Out] (-4*b*Sqrt[b*x + c*x^2])/(3*c^2*Sqrt[x]) + (2*Sqrt[x]*Sqrt[b*x + c*x^2])/(3
*c)
```

Rubi [A] time = 0.0157122, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {656, 648}

$$\frac{2\sqrt{x}\sqrt{bx+cx^2}}{3c}-\frac{4b\sqrt{bx+cx^2}}{3c^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[b*x + c*x^2],x]
```

```
[Out] (-4*b*Sqrt[b*x + c*x^2])/(3*c^2*Sqrt[x]) + (2*Sqrt[x]*Sqrt[b*x + c*x^2])/(3
*c)
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{\sqrt{bx+cx^2}}\,dx &= \frac{2\sqrt{x}\sqrt{bx+cx^2}}{3c}-\frac{(2b)\int \frac{\sqrt{x}}{\sqrt{bx+cx^2}}\,dx}{3c}\\ &= -\frac{4b\sqrt{bx+cx^2}}{3c^2\sqrt{x}}+\frac{2\sqrt{x}\sqrt{bx+cx^2}}{3c}\end{aligned}$$

Mathematica [A] time = 0.0166868, size = 30, normalized size = 0.58

$$\frac{2(cx-2b)\sqrt{x(b+cx)}}{3c^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[b*x + c*x^2],x]
```

```
[Out] (2*(-2*b + c*x)*Sqrt[x*(b + c*x)])/(3*c^2*Sqrt[x])
```

Maple [A] time = 0.055, size = 33, normalized size = 0.6

$$-\frac{(2\,cx+2\,b)\,(-cx+2\,b)}{3\,c^2}\sqrt{x}\frac{1}{\sqrt{cx^2+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^2+b*x)^(1/2),x)
```

```
[Out] -2/3*(c*x+b)*(-c*x+2*b)*x^(1/2)/c^2/(c*x^2+b*x)^(1/2)
```

Maxima [A] time = 1.12455, size = 41, normalized size = 0.79

$$\frac{2\left(c^2x^2-bcx-2\,b^2\right)}{3\,\sqrt{cx+b}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(c^2*x^2 - b*c*x - 2*b^2)/(sqrt(c*x + b)*c^2)
```

Fricas [A] time = 1.909, size = 66, normalized size = 1.27

$$\frac{2\,\sqrt{cx^2+bx}(cx-2\,b)}{3\,c^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(c*x^2 + b*x)*(c*x - 2*b)/(c^2*sqrt(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{\frac{3}{2}}}{\sqrt{x\,(b+cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(x**(3/2)/sqrt(x*(b + c*x)), x)
```

Giac [A] time = 1.25214, size = 43, normalized size = 0.83

$$\frac{4\,b^{\frac{3}{2}}}{3\,c^2} + \frac{2\left((cx+b)^{\frac{3}{2}} - 3\,\sqrt{cx+b}b\right)}{3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/3*b^(3/2)/c^2 + 2/3*((c*x + b)^(3/2) - 3*sqrt(c*x + b)*b)/c^2
```

3.97 $\int \frac{\sqrt{x}}{\sqrt{bx+cx^2}}\,dx$

Optimal. Leaf size=23

$$\frac{2\sqrt{bx+cx^2}}{c\sqrt{x}}$$

```
[Out] (2*Sqrt[b*x + c*x^2])/(c*Sqrt[x])
```

Rubi [A] time = 0.0068375, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {648}

$$\frac{2\sqrt{bx+cx^2}}{c\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[b*x + c*x^2],x]

[Out] (2*Sqrt[b*x + c*x^2])/(c*Sqrt[x])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\sqrt{bx+cx^2}}\,dx = \frac{2\sqrt{bx+cx^2}}{c\sqrt{x}}$$

Mathematica [A] time = 0.008983, size = 21, normalized size = 0.91

$$\frac{2\sqrt{x(b+cx)}}{c\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[b*x + c*x^2],x]

[Out] (2*Sqrt[x*(b + c*x)])/(c*Sqrt[x])
```

Maple [A] time = 0.045, size = 25, normalized size = 1.1

$$2\,\frac{(cx+b)\sqrt{x}}{c\sqrt{cx^2+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^2+b*x)^(1/2),x)
```

```
[Out] 2*(c*x+b)*x^(1/2)/c/(c*x^2+b*x)^(1/2)
```

Maxima [A] time = 1.18947, size = 16, normalized size = 0.7

$$\frac{2\sqrt{cx+b}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(c*x + b)/c
```

Fricas [A] time = 1.99411, size = 45, normalized size = 1.96

$$\frac{2\sqrt{cx^2+bx}}{c\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(c*x^2 + b*x)/(c*sqrt(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{x(b+cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x)/sqrt(x*(b + c*x)), x)
```

Giac [A] time = 1.26256, size = 28, normalized size = 1.22

$$\frac{2\sqrt{cx+b}}{c}-\frac{2\sqrt{b}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(c*x + b)/c - 2*sqrt(b)/c
```

3.98 $\int \frac{1}{\sqrt{x}\sqrt{bx+cx^2}}\,dx$

Optimal. Leaf size=32

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{b}}$$

```
[Out] (-2*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/Sqrt[b]
```

Rubi [A] time = 0.0124113, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {660, 207}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[b*x + c*x^2]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/Sqrt[b]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{bx+cx^2}}\,dx &= 2\,\mathrm{Subst}\left(\int \frac{1}{-b+x^2}\,dx, x, \frac{\sqrt{bx+cx^2}}{\sqrt{x}}\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0115571, size = 48, normalized size = 1.5

$$-\frac{2\sqrt{x}\sqrt{b+cx}\tanh^{-1}\left(\frac{\sqrt{b+cx}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[b*x + c*x^2]),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[b + c*x]*ArcTanh[Sqrt[b + c*x]/Sqrt[b]])/(Sqrt[b]*Sqrt[x*(
b + c*x)])
```

Maple [A] time = 0.181, size = 37, normalized size = 1.2

$$-2\frac{\sqrt{x(cx+b)}}{\sqrt{x}\sqrt{cx+b}\sqrt{b}}\mathrm{Artanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(c*x^2+b*x)^(1/2),x)
```

```
[Out] -2/x^(1/2)*(x*(c*x+b))^(1/2)/(c*x+b)^(1/2)/b^(1/2)*arctanh((c*x+b)^(1/2)/b^
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+bx}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x)*sqrt(x)), x)
```

Fricas [A] time = 1.96808, size = 181, normalized size = 5.66

$$\left[\frac{\log\left(-\frac{cx^2+2\,bx-2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)}{\sqrt{b}}, \frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{cx^2+bx}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(-(c*x^2 + 2*b*x - 2*sqrt(c*x^2 + b*x)*sqrt(b)*sqrt(x))/x^2)/sqrt(b), 2
*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(c*x^2 + b*x))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x}\sqrt{x(b+cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*sqrt(x*(b + c*x))), x)
```

Giac [A] time = 1.24326, size = 53, normalized size = 1.66

$$\frac{2 \arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{2 \arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(c*x + b)/sqrt(-b))/sqrt(-b) - 2*arctan(sqrt(b)/sqrt(-b))/sqrt
(-b)
```

3.99 $\int \frac{1}{x^{3/2}\sqrt{bx+cx^2}}\,dx$

Optimal. Leaf size=56

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{3/2}}-\frac{\sqrt{bx+cx^2}}{bx^{3/2}}$$

```
[Out] -(Sqrt[b*x + c*x^2]/(b*x^(3/2))) + (c*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sq
rt[x])])/b^(3/2)
```

Rubi [A] time = 0.0220336, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {672, 660, 207}

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{3/2}}-\frac{\sqrt{bx+cx^2}}{bx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[b*x + c*x^2]),x]
```

```
[Out] -(Sqrt[b*x + c*x^2]/(b*x^(3/2))) + (c*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sq
rt[x])])/b^(3/2)
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*(m + 2*p + 2))/((m + p + 1)*(2*c*d - b*e)), Int[(d
 + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && Ne
Q[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{bx+cx^2}}\,dx = -\frac{\sqrt{bx+cx^2}}{bx^{3/2}} - \frac{c\int \frac{1}{\sqrt{x}\sqrt{bx+cx^2}}\,dx}{2b}$$
$$= -\frac{\sqrt{bx+cx^2}}{bx^{3/2}} - \frac{c\,\text{Subst}\left(\int \frac{1}{-b+x^2}\,dx, x, \frac{\sqrt{bx+cx^2}}{\sqrt{x}}\right)}{b}$$
$$= -\frac{\sqrt{bx+cx^2}}{bx^{3/2}} + \frac{c\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{3/2}}$$

Mathematica [A] time = 0.0583343, size = 63, normalized size = 1.12

$$\frac{2c\sqrt{x(b+cx)}\left(\frac{\tanh^{-1}\left(\sqrt{\frac{cx}{b}+1}\right)}{2\sqrt{\frac{cx}{b}+1}} - \frac{b}{2cx}\right)}{b^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[b*x + c*x^2]),x]
```

```
[Out] (2*c*Sqrt[x*(b + c*x)]*(-b/(2*c*x) + ArcTanh[Sqrt[1 + (c*x)/b]]/(2*Sqrt[1 +
 (c*x)/b])))/(b^2*Sqrt[x])
```

Maple [A] time = 0.196, size = 52, normalized size = 0.9

$$\sqrt{x(cx+b)}\left(\text{Artanh}\left(\sqrt{cx+b}\frac{1}{\sqrt{b}}\right)xc - \sqrt{cx+b}\sqrt{b}\right)b^{-\frac{3}{2}}x^{-\frac{3}{2}}\frac{1}{\sqrt{cx+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^2+b*x)^(1/2),x)
```

```
[Out] (x*(c*x+b))^(1/2)/b^(3/2)*(arctanh((c*x+b)^(1/2)/b^(1/2))*x*c-(c*x+b)^(1/2)
*b^(1/2))/x^(3/2)/(c*x+b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+bx}\,x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x)*x^(3/2)), x)
```

Fricas [A] time = 1.93033, size = 315, normalized size = 5.62

$$\left[\frac{\sqrt{b}cx^2\log\left(-\frac{cx^2+2\,bx+2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)-2\,\sqrt{cx^2+bx}b\sqrt{x}}{2\,b^2x^2}, -\frac{\sqrt{-b}cx^2\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{cx^2+bx}}\right)+\sqrt{cx^2+bx}b\sqrt{x}}{b^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b)*c*x^2*log(-(c*x^2 + 2*b*x + 2*sqrt(c*x^2 + b*x)*sqrt(b)*sqrt(
x))/x^2) - 2*sqrt(c*x^2 + b*x)*b*sqrt(x))/(b^2*x^2), -(sqrt(-b)*c*x^2*arcta
n(sqrt(-b)*sqrt(x)/sqrt(c*x^2 + b*x)) + sqrt(c*x^2 + b*x)*b*sqrt(x))/(b^2*x
^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{3}{2}}\sqrt{x\,(b+cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(1/(x**(3/2)*sqrt(x*(b + c*x))), x)
```

Giac [A] time = 1.25034, size = 59, normalized size = 1.05

$$-c\left(\frac{\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b}+\frac{\sqrt{cx+b}}{bcx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] -c*(arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b) + sqrt(c*x + b)/(b*c*x))
```

3.100 $\int \frac{1}{x^{5/2}\sqrt{bx+cx^2}}\,dx$

Optimal. Leaf size=89

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{4b^{5/2}}+\frac{3c\sqrt{bx+cx^2}}{4b^2x^{3/2}}-\frac{\sqrt{bx+cx^2}}{2bx^{5/2}}$$

```
[Out] -Sqrt[b*x + c*x^2]/(2*b*x^(5/2)) + (3*c*Sqrt[b*x + c*x^2])/(4*b^2*x^(3/2))
 - (3*c^2*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/(4*b^(5/2))
```

Rubi [A] time = 0.0343466, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {672, 660, 207}

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{4b^{5/2}}+\frac{3c\sqrt{bx+cx^2}}{4b^2x^{3/2}}-\frac{\sqrt{bx+cx^2}}{2bx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[b*x + c*x^2]),x]
```

```
[Out] -Sqrt[b*x + c*x^2]/(2*b*x^(5/2)) + (3*c*Sqrt[b*x + c*x^2])/(4*b^2*x^(3/2))
 - (3*c^2*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/(4*b^(5/2))
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*(m + 2*p + 2))/((m + p + 1)*(2*c*d - b*e)), Int[(d
 + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && Ne
Q[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^{5/2}\sqrt{bx+cx^2}}\,dx = -\frac{\sqrt{bx+cx^2}}{2bx^{5/2}} - \frac{(3c)\int \frac{1}{x^{3/2}\sqrt{bx+cx^2}}\,dx}{4b}$$
$$= -\frac{\sqrt{bx+cx^2}}{2bx^{5/2}} + \frac{3c\sqrt{bx+cx^2}}{4b^2x^{3/2}} + \frac{(3c^2)\int \frac{1}{\sqrt{x}\sqrt{bx+cx^2}}\,dx}{8b^2}$$
$$= -\frac{\sqrt{bx+cx^2}}{2bx^{5/2}} + \frac{3c\sqrt{bx+cx^2}}{4b^2x^{3/2}} + \frac{(3c^2)\,\text{Subst}\left(\int \frac{1}{-b+x^2}\,dx, x, \frac{\sqrt{bx+cx^2}}{\sqrt{x}}\right)}{4b^2}$$
$$= -\frac{\sqrt{bx+cx^2}}{2bx^{5/2}} + \frac{3c\sqrt{bx+cx^2}}{4b^2x^{3/2}} - \frac{3c^2\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{4b^{5/2}}$$

Mathematica [C] time = 0.0123243, size = 40, normalized size = 0.45

$$-\frac{2c^2\sqrt{x(b+cx)}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{cx}{b}+1\right)}{b^3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[b*x + c*x^2]),x]
```

```
[Out] (-2*c^2*Sqrt[x*(b + c*x)]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (c*x)/b])/(b^3
*Sqrt[x])
```

Maple [A] time = 0.225, size = 72, normalized size = 0.8

$$-\frac{1}{4}\sqrt{x(cx+b)}\left(3\,\text{Artanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)x^2c^2 - 3\,xc\sqrt{cx+b}\sqrt{b} + 2\,b^{3/2}\sqrt{cx+b}\right)b^{-\frac{5}{2}}x^{-\frac{5}{2}}\frac{1}{\sqrt{cx+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(c*x^2+b*x)^(1/2),x)
```

```
[Out] -1/4*(x*(c*x+b))^(1/2)/b^(5/2)*(3*arctanh((c*x+b)^(1/2)/b^(1/2))*x^2*c^2-3*
x*c*(c*x+b)^(1/2)*b^(1/2)+2*b^(3/2)*(c*x+b)^(1/2))/x^(5/2)/(c*x+b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+bx}\,x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x)*x^(5/2)), x)
```

Fricas [A] time = 2.06695, size = 373, normalized size = 4.19

$$\left[\frac{3\sqrt{b}c^2x^3\log\left(-\frac{cx^2+2bx-2\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)+2\left(3bcx-2b^2\right)\sqrt{cx^2+bx}\sqrt{x}}{8b^3x^3}, \frac{3\sqrt{-b}c^2x^3\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{cx^2+bx}}\right)+\left(3bcx-2b^2\right)\sqrt{cx^2}}{4b^3x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(b)*c^2*x^3*log(-(c*x^2 + 2*b*x - 2*sqrt(c*x^2 + b*x)*sqrt(b)*s
qrt(x))/x^2) + 2*(3*b*c*x - 2*b^2)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^3*x^3), 1/
4*(3*sqrt(-b)*c^2*x^3*arctan(sqrt(-b)*sqrt(x)/sqrt(c*x^2 + b*x)) + (3*b*c*x
 - 2*b^2)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^3*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{5}{2}}\sqrt{x\,(b+cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(1/(x**(5/2)*sqrt(x*(b + c*x))), x)
```

Giac [A] time = 1.30997, size = 81, normalized size = 0.91

$$\frac{1}{4}c^2\left(\frac{3\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^2}+\frac{3\,(cx+b)^{\frac{3}{2}}-5\sqrt{cx+b}\,b}{b^2c^2x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*c^2*(3*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b^2) + (3*(c*x + b)^(3/
2) - 5*sqrt(c*x + b)*b)/(b^2*c^2*x^2))
```

3.101 $\int \frac{1}{x^{7/2}\sqrt{bx+cx^2}}\,dx$

Optimal. Leaf size=117

$$-\frac{5c^2\sqrt{bx+cx^2}}{8b^3x^{3/2}}+\frac{5c^3\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{8b^{7/2}}+\frac{5c\sqrt{bx+cx^2}}{12b^2x^{5/2}}-\frac{\sqrt{bx+cx^2}}{3bx^{7/2}}$$

```
[Out] -Sqrt[b*x + c*x^2]/(3*b*x^(7/2)) + (5*c*Sqrt[b*x + c*x^2])/(12*b^2*x^(5/2))
 - (5*c^2*Sqrt[b*x + c*x^2])/(8*b^3*x^(3/2)) + (5*c^3*ArcTanh[Sqrt[b*x + c*
x^2]/(Sqrt[b]*Sqrt[x])])/(8*b^(7/2))
```

Rubi [A] time = 0.0492188, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {672, 660, 207}

$$-\frac{5c^2\sqrt{bx+cx^2}}{8b^3x^{3/2}}+\frac{5c^3\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{8b^{7/2}}+\frac{5c\sqrt{bx+cx^2}}{12b^2x^{5/2}}-\frac{\sqrt{bx+cx^2}}{3bx^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*Sqrt[b*x + c*x^2]),x]
```

```
[Out] -Sqrt[b*x + c*x^2]/(3*b*x^(7/2)) + (5*c*Sqrt[b*x + c*x^2])/(12*b^2*x^(5/2))
 - (5*c^2*Sqrt[b*x + c*x^2])/(8*b^3*x^(3/2)) + (5*c^3*ArcTanh[Sqrt[b*x + c*
x^2]/(Sqrt[b]*Sqrt[x])])/(8*b^(7/2))
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*(m + 2*p + 2))/((m + p + 1)*(2*c*d - b*e)), Int[(d
 + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && Ne
Q[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^{7/2}\sqrt{bx+cx^2}}\,dx = -\frac{\sqrt{bx+cx^2}}{3bx^{7/2}} - \frac{(5c)\int \frac{1}{x^{5/2}\sqrt{bx+cx^2}}\,dx}{6b}$$
$$= -\frac{\sqrt{bx+cx^2}}{3bx^{7/2}} + \frac{5c\sqrt{bx+cx^2}}{12b^2x^{5/2}} + \frac{\left(5c^2\right)\int \frac{1}{x^{3/2}\sqrt{bx+cx^2}}\,dx}{8b^2}$$
$$= -\frac{\sqrt{bx+cx^2}}{3bx^{7/2}} + \frac{5c\sqrt{bx+cx^2}}{12b^2x^{5/2}} - \frac{5c^2\sqrt{bx+cx^2}}{8b^3x^{3/2}} - \frac{\left(5c^3\right)\int \frac{1}{\sqrt{x}\sqrt{bx+cx^2}}\,dx}{16b^3}$$
$$= -\frac{\sqrt{bx+cx^2}}{3bx^{7/2}} + \frac{5c\sqrt{bx+cx^2}}{12b^2x^{5/2}} - \frac{5c^2\sqrt{bx+cx^2}}{8b^3x^{3/2}} - \frac{\left(5c^3\right)\operatorname{Subst}\left(\int \frac{1}{-b+x^2}\,dx, x, \frac{\sqrt{bx+cx^2}}{\sqrt{x}}\right)}{8b^3}$$
$$= -\frac{\sqrt{bx+cx^2}}{3bx^{7/2}} + \frac{5c\sqrt{bx+cx^2}}{12b^2x^{5/2}} - \frac{5c^2\sqrt{bx+cx^2}}{8b^3x^{3/2}} + \frac{5c^3\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{8b^{7/2}}$$

Mathematica [C] time = 0.0121609, size = 40, normalized size = 0.34

$$\frac{2c^3\sqrt{x(b+cx)}\,{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};\frac{cx}{b}+1\right)}{b^4\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*Sqrt[b*x + c*x^2]),x]
```

```
[Out] (2*c^3*Sqrt[x*(b + c*x)]*Hypergeometric2F1[1/2, 4, 3/2, 1 + (c*x)/b])/(b^4*
Sqrt[x])
```

Maple [A] time = 0.189, size = 90, normalized size = 0.8

$$\frac{1}{24}\sqrt{x\,(cx+b)}\left(15\,\mathrm{Artanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)x^3c^3 - 15\,x^2c^2\sqrt{b}\sqrt{cx+b} + 10\,xb^{3/2}c\sqrt{cx+b} - 8\,b^{5/2}\sqrt{cx+b}\right)b^{-\frac{7}{2}}x^{-\frac{7}{2}}\frac{1}{\sqrt{cx+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(c*x^2+b*x)^(1/2),x)
```

```
[Out] 1/24*(x*(c*x+b))^(1/2)/b^(7/2)*(15*arctanh((c*x+b)^(1/2)/b^(1/2))*x^3*c^3-1
5*x^2*c^2*b^(1/2)*(c*x+b)^(1/2)+10*x*b^(3/2)*c*(c*x+b)^(1/2)-8*b^(5/2)*(c*x
+b)^(1/2))/x^(7/2)/(c*x+b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+bx}\,x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x)*x^(7/2)), x)
```

Fricas [A] time = 2.18035, size = 428, normalized size = 3.66

$$\left[\frac{15\sqrt{b}c^3x^4\log\left(-\frac{cx^2+2\,bx+2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)-2\left(15\,bc^2x^2-10\,b^2cx+8\,b^3\right)\sqrt{cx^2+bx}\sqrt{x}}{48\,b^4x^4},-\frac{15\sqrt{-b}c^3x^4\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{cx^2+bx}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(b)*c^3*x^4*log(-(c*x^2 + 2*b*x + 2*sqrt(c*x^2 + b*x)*sqrt(b)
*sqrt(x))/x^2) - 2*(15*b*c^2*x^2 - 10*b^2*c*x + 8*b^3)*sqrt(c*x^2 + b*x)*sq
rt(x))/(b^4*x^4), -1/24*(15*sqrt(-b)*c^3*x^4*arctan(sqrt(-b)*sqrt(x)/sqrt(c
*x^2 + b*x)) + (15*b*c^2*x^2 - 10*b^2*c*x + 8*b^3)*sqrt(c*x^2 + b*x)*sqrt(x
))/(b^4*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{7}{2}}\sqrt{x\,(b+cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(1/(x**(7/2)*sqrt(x*(b + c*x))), x)
```

Giac [A] time = 1.32539, size = 97, normalized size = 0.83

$$-\frac{1}{24}\,c^3\left(\frac{15\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^3}+\frac{15\,(cx+b)^{\frac{5}{2}}-40\,(cx+b)^{\frac{3}{2}}b+33\,\sqrt{cx+b}b^2}{b^3c^3x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*c^3*(15*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b^3) + (15*(c*x + b)
^(5/2) - 40*(c*x + b)^(3/2)*b + 33*sqrt(c*x + b)*b^2)/(b^3*c^3*x^3))
```

3.102 $\int \frac{x^{13/2}}{(bx+cx^2)^{3/2}} dx$

Optimal. Leaf size=164

$$\frac{32b^2x^{7/2}}{63c^3\sqrt{bx+cx^2}} - \frac{64b^3x^{5/2}}{63c^4\sqrt{bx+cx^2}} + \frac{256b^4x^{3/2}}{63c^5\sqrt{bx+cx^2}} + \frac{512b^5\sqrt{x}}{63c^6\sqrt{bx+cx^2}} - \frac{20bx^{9/2}}{63c^2\sqrt{bx+cx^2}} + \frac{2x^{11/2}}{9c\sqrt{bx+cx^2}}$$

```
[Out] (512*b^5*Sqrt[x])/(63*c^6*Sqrt[b*x + c*x^2]) + (256*b^4*x^(3/2))/(63*c^5*Sq
rt[b*x + c*x^2]) - (64*b^3*x^(5/2))/(63*c^4*Sqrt[b*x + c*x^2]) + (32*b^2*x^
(7/2))/(63*c^3*Sqrt[b*x + c*x^2]) - (20*b*x^(9/2))/(63*c^2*Sqrt[b*x + c*x^2
]) + (2*x^(11/2))/(9*c*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.0748355, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {656, 648}

$$\frac{32b^2x^{7/2}}{63c^3\sqrt{bx+cx^2}} - \frac{64b^3x^{5/2}}{63c^4\sqrt{bx+cx^2}} + \frac{256b^4x^{3/2}}{63c^5\sqrt{bx+cx^2}} + \frac{512b^5\sqrt{x}}{63c^6\sqrt{bx+cx^2}} - \frac{20bx^{9/2}}{63c^2\sqrt{bx+cx^2}} + \frac{2x^{11/2}}{9c\sqrt{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/(b*x + c*x^2)^(3/2),x]
```


```
[Out] (512*b^5*Sqrt[x])/(63*c^6*Sqrt[b*x + c*x^2]) + (256*b^4*x^(3/2))/(63*c^5*Sq
rt[b*x + c*x^2]) - (64*b^3*x^(5/2))/(63*c^4*Sqrt[b*x + c*x^2]) + (32*b^2*x^
(7/2))/(63*c^3*Sqrt[b*x + c*x^2]) - (20*b*x^(9/2))/(63*c^2*Sqrt[b*x + c*x^2
]) + (2*x^(11/2))/(9*c*Sqrt[b*x + c*x^2])
```


Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{x^{13/2}}{\left(bx+cx^2\right)^{3/2}}\,dx = \frac{2x^{11/2}}{9c\sqrt{bx+cx^2}} - \frac{(10b)\int \frac{x^{11/2}}{\left(bx+cx^2\right)^{3/2}}\,dx}{9c}$$

$$= -\frac{20bx^{9/2}}{63c^2\sqrt{bx+cx^2}} + \frac{2x^{11/2}}{9c\sqrt{bx+cx^2}} + \frac{\left(80b^2\right)\int \frac{x^{9/2}}{\left(bx+cx^2\right)^{3/2}}\,dx}{63c^2}$$

$$= \frac{32b^2x^{7/2}}{63c^3\sqrt{bx+cx^2}} - \frac{20bx^{9/2}}{63c^2\sqrt{bx+cx^2}} + \frac{2x^{11/2}}{9c\sqrt{bx+cx^2}} - \frac{\left(32b^3\right)\int \frac{x^{7/2}}{\left(bx+cx^2\right)^{3/2}}\,dx}{21c^3}$$

$$= -\frac{64b^3x^{5/2}}{63c^4\sqrt{bx+cx^2}} + \frac{32b^2x^{7/2}}{63c^3\sqrt{bx+cx^2}} - \frac{20bx^{9/2}}{63c^2\sqrt{bx+cx^2}} + \frac{2x^{11/2}}{9c\sqrt{bx+cx^2}} + \frac{\left(128b^4\right)\int \frac{x^{5/2}}{\left(bx+cx^2\right)^{3/2}}\,dx}{63c^4}$$

$$= \frac{256b^4x^{3/2}}{63c^5\sqrt{bx+cx^2}} - \frac{64b^3x^{5/2}}{63c^4\sqrt{bx+cx^2}} + \frac{32b^2x^{7/2}}{63c^3\sqrt{bx+cx^2}} - \frac{20bx^{9/2}}{63c^2\sqrt{bx+cx^2}} + \frac{2x^{11/2}}{9c\sqrt{bx+cx^2}} - \frac{\left(256b^5\right.}{}$$

$$= \frac{512b^5\sqrt{x}}{63c^6\sqrt{bx+cx^2}} + \frac{256b^4x^{3/2}}{63c^5\sqrt{bx+cx^2}} - \frac{64b^3x^{5/2}}{63c^4\sqrt{bx+cx^2}} + \frac{32b^2x^{7/2}}{63c^3\sqrt{bx+cx^2}} - \frac{20bx^{9/2}}{63c^2\sqrt{bx+cx^2}} + \frac{2}{9c\sqrt{}}$$

Mathematica [A] time = 0.0357231, size = 75, normalized size = 0.46

$$\frac{2\sqrt{x}\left(-32b^3c^2x^2+16b^2c^3x^3+128b^4cx+256b^5-10bc^4x^4+7c^5x^5\right)}{63c^6\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/(b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[x]*(256*b^5 + 128*b^4*c*x - 32*b^3*c^2*x^2 + 16*b^2*c^3*x^3 - 10*b*
c^4*x^4 + 7*c^5*x^5))/(63*c^6*Sqrt[x*(b + c*x)])
```

Maple [A] time = 0.051, size = 77, normalized size = 0.5

$$\frac{(2\,cx+2\,b)\left(7\,x^5c^5-10\,bx^4c^4+16\,b^2x^3c^3-32\,b^3x^2c^2+128\,b^4xc+256\,b^5\right)}{63\,c^6}x^{\frac{3}{2}}\left(cx^2+bx\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(c*x^2+b*x)^(3/2),x)
```

```
[Out] 2/63*(c*x+b)*(7*c^5*x^5-10*b*c^4*x^4+16*b^2*c^3*x^3-32*b^3*c^2*x^2+128*b^4*
c*x+256*b^5)*x^(3/2)/c^6/(c*x^2+b*x)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(\left(35\,c^6x^5-5\,bc^5x^4+8\,b^2c^4x^3-16\,b^3c^3x^2+64\,b^4c^2x+128\,b^5c\right)x^5-2\left(5\,bc^5x^5-2\,b^2c^4x^4+5\,b^3c^3x^3-28\,b^4c^2x^2-1\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/315*((35*c^6*x^5 - 5*b*c^5*x^4 + 8*b^2*c^4*x^3 - 16*b^3*c^3*x^2 + 64*b^4*
c^2*x + 128*b^5*c)*x^5 - 2*(5*b*c^5*x^5 - 2*b^2*c^4*x^4 + 5*b^3*c^3*x^3 - 2
8*b^4*c^2*x^2 - 104*b^5*c*x - 64*b^6)*x^4 + 6*(3*b^2*c^4*x^5 - 2*b^3*c^3*x^
4 + 11*b^4*c^2*x^3 + 40*b^5*c*x^2 + 24*b^6*x)*x^3 - 42*(b^3*c^3*x^5 - 2*b^4
*c^2*x^4 - 7*b^5*c*x^3 - 4*b^6*x^2)*x^2 + 210*(b^4*c^2*x^5 + 2*b^5*c*x^4 +
b^6*x^3)*x)/((c^7*x^5 + b*c^6*x^4)*sqrt(c*x + b)) - integrate(2*(b^5*c*x +
b^6)*x/((c^7*x^3 + 2*b*c^6*x^2 + b^2*c^5*x)*sqrt(c*x + b)), x)
```

Fricas [A] time = 1.90933, size = 185, normalized size = 1.13

$$\frac{2\left(7\,c^5x^5 - 10\,bc^4x^4 + 16\,b^2c^3x^3 - 32\,b^3c^2x^2 + 128\,b^4cx + 256\,b^5\right)\sqrt{cx^2+bx}\sqrt{x}}{63\left(c^7x^2 + bc^6x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/63*(7*c^5*x^5 - 10*b*c^4*x^4 + 16*b^2*c^3*x^3 - 32*b^3*c^2*x^2 + 128*b^4*
c*x + 256*b^5)*sqrt(c*x^2 + b*x)*sqrt(x)/(c^7*x^2 + b*c^6*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15987, size = 111, normalized size = 0.68

$$-\frac{512\,b^{\frac{9}{2}}}{63\,c^6} + \frac{2\left(7\,(cx+b)^{\frac{9}{2}} - 45\,(cx+b)^{\frac{7}{2}}b + 126\,(cx+b)^{\frac{5}{2}}b^2 - 210\,(cx+b)^{\frac{3}{2}}b^3 + 315\,\sqrt{cx+b}\,b^4 + \frac{63\,b^5}{\sqrt{cx+b}}\right)}{63\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] -512/63*b^(9/2)/c^6 + 2/63*(7*(c*x + b)^(9/2) - 45*(c*x + b)^(7/2)*b + 126*
(c*x + b)^(5/2)*b^2 - 210*(c*x + b)^(3/2)*b^3 + 315*sqrt(c*x + b)*b^4 + 63*
b^5/sqrt(c*x + b))/c^6
```

3.103 $\int \frac{x^{11/2}}{\left(bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=136

$$\frac{32b^2x^{5/2}}{35c^3\sqrt{bx+cx^2}}-\frac{128b^3x^{3/2}}{35c^4\sqrt{bx+cx^2}}-\frac{256b^4\sqrt{x}}{35c^5\sqrt{bx+cx^2}}-\frac{16bx^{7/2}}{35c^2\sqrt{bx+cx^2}}+\frac{2x^{9/2}}{7c\sqrt{bx+cx^2}}$$

```
[Out] (-256*b^4*Sqrt[x])/(35*c^5*Sqrt[b*x + c*x^2]) - (128*b^3*x^(3/2))/(35*c^4*S
qrt[b*x + c*x^2]) + (32*b^2*x^(5/2))/(35*c^3*Sqrt[b*x + c*x^2]) - (16*b*x^(
7/2))/(35*c^2*Sqrt[b*x + c*x^2]) + (2*x^(9/2))/(7*c*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.0567674, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {656, 648}

$$\frac{32b^2x^{5/2}}{35c^3\sqrt{bx+cx^2}}-\frac{128b^3x^{3/2}}{35c^4\sqrt{bx+cx^2}}-\frac{256b^4\sqrt{x}}{35c^5\sqrt{bx+cx^2}}-\frac{16bx^{7/2}}{35c^2\sqrt{bx+cx^2}}+\frac{2x^{9/2}}{7c\sqrt{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/(b*x + c*x^2)^(3/2),x]
```


```
[Out] (-256*b^4*Sqrt[x])/(35*c^5*Sqrt[b*x + c*x^2]) - (128*b^3*x^(3/2))/(35*c^4*S
qrt[b*x + c*x^2]) + (32*b^2*x^(5/2))/(35*c^3*Sqrt[b*x + c*x^2]) - (16*b*x^(
7/2))/(35*c^2*Sqrt[b*x + c*x^2]) + (2*x^(9/2))/(7*c*Sqrt[b*x + c*x^2])
```


Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{x^{11/2}}{\left(bx+cx^2\right)^{3/2}}\,dx = \frac{2x^{9/2}}{7c\sqrt{bx+cx^2}} - \frac{(8b)\int \frac{x^{9/2}}{\left(bx+cx^2\right)^{3/2}}\,dx}{7c}$$
$$= -\frac{16bx^{7/2}}{35c^2\sqrt{bx+cx^2}} + \frac{2x^{9/2}}{7c\sqrt{bx+cx^2}} + \frac{\left(48b^2\right)\int \frac{x^{7/2}}{\left(bx+cx^2\right)^{3/2}}\,dx}{35c^2}$$
$$= \frac{32b^2x^{5/2}}{35c^3\sqrt{bx+cx^2}} - \frac{16bx^{7/2}}{35c^2\sqrt{bx+cx^2}} + \frac{2x^{9/2}}{7c\sqrt{bx+cx^2}} - \frac{\left(64b^3\right)\int \frac{x^{5/2}}{\left(bx+cx^2\right)^{3/2}}\,dx}{35c^3}$$
$$= -\frac{128b^3x^{3/2}}{35c^4\sqrt{bx+cx^2}} + \frac{32b^2x^{5/2}}{35c^3\sqrt{bx+cx^2}} - \frac{16bx^{7/2}}{35c^2\sqrt{bx+cx^2}} + \frac{2x^{9/2}}{7c\sqrt{bx+cx^2}} + \frac{\left(128b^4\right)\int \frac{x^{3/2}}{\left(bx+cx^2\right)^{3/2}}\,dx}{35c^4}$$
$$= -\frac{256b^4\sqrt{x}}{35c^5\sqrt{bx+cx^2}} - \frac{128b^3x^{3/2}}{35c^4\sqrt{bx+cx^2}} + \frac{32b^2x^{5/2}}{35c^3\sqrt{bx+cx^2}} - \frac{16bx^{7/2}}{35c^2\sqrt{bx+cx^2}} + \frac{2x^{9/2}}{7c\sqrt{bx+cx^2}}$$

Mathematica [A] time = 0.0291625, size = 64, normalized size = 0.47

$$\frac{2\sqrt{x}\left(16b^2c^2x^2-64b^3cx-128b^4-8bc^3x^3+5c^4x^4\right)}{35c^5\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/(b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[x]*(-128*b^4 - 64*b^3*c*x + 16*b^2*c^2*x^2 - 8*b*c^3*x^3 + 5*c^4*x^
4))/(35*c^5*Sqrt[x*(b + c*x)])
```

Maple [A] time = 0.046, size = 66, normalized size = 0.5

$$-\frac{(2\,cx+2\,b)\left(-5\,x^4c^4+8\,bx^3c^3-16\,b^2x^2c^2+64\,b^3xc+128\,b^4\right)}{35\,c^5}x^{\frac{3}{2}}\left(cx^2+bx\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(c*x^2+b*x)^(3/2),x)
```

```
[Out] -2/35*(c*x+b)*(-5*c^4*x^4+8*b*c^3*x^3-16*b^2*c^2*x^2+64*b^3*c*x+128*b^4)*x^
(3/2)/c^5/(c*x^2+b*x)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(3\left(5\,c^5x^4-bc^4x^3+2\,b^2c^3x^2-8\,b^3c^2x-16\,b^4c\right)x^4-2\left(3\,bc^4x^4-2\,b^2c^3x^3+11\,b^3c^2x^2+40\,b^4cx+24\,b^5\right)x^3+14\left(b^2c^3x^4\right.\right.}{105\left(c^6x^4+bc^5x^3\right)\sqrt{cx+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/105*(3*(5*c^5*x^4 - b*c^4*x^3 + 2*b^2*c^3*x^2 - 8*b^3*c^2*x - 16*b^4*c)*x
^4 - 2*(3*b*c^4*x^4 - 2*b^2*c^3*x^3 + 11*b^3*c^2*x^2 + 40*b^4*c*x + 24*b^5)
*x^3 + 14*(b^2*c^3*x^4 - 2*b^3*c^2*x^3 - 7*b^4*c*x^2 - 4*b^5*x)*x^2 - 70*(b
^3*c^2*x^4 + 2*b^4*c*x^3 + b^5*x^2)*x)/((c^6*x^4 + b*c^5*x^3)*sqrt(c*x + b)
) + integrate(2*(b^4*c*x + b^5)*x/((c^6*x^3 + 2*b*c^5*x^2 + b^2*c^4*x)*sqrt
(c*x + b)), x)
```

Fricas [A] time = 1.98118, size = 159, normalized size = 1.17

$$\frac{2\left(5\,c^4x^4 - 8\,bc^3x^3 + 16\,b^2c^2x^2 - 64\,b^3cx - 128\,b^4\right)\sqrt{cx^2 + bx}\sqrt{x}}{35\left(c^6x^2 + bc^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*c^4*x^4 - 8*b*c^3*x^3 + 16*b^2*c^2*x^2 - 64*b^3*c*x - 128*b^4)*sqrt
(c*x^2 + b*x)*sqrt(x)/(c^6*x^2 + b*c^5*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17735, size = 95, normalized size = 0.7

$$\frac{256\,b^{\frac{7}{2}}}{35\,c^5} + \frac{2\left(5\,(cx+b)^{\frac{7}{2}} - 28\,(cx+b)^{\frac{5}{2}}b + 70\,(cx+b)^{\frac{3}{2}}b^2 - 140\,\sqrt{cx+b}\,b^3 - \frac{35\,b^4}{\sqrt{cx+b}}\right)}{35\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] 256/35*b^(7/2)/c^5 + 2/35*(5*(c*x + b)^(7/2) - 28*(c*x + b)^(5/2)*b + 70*(c
*x + b)^(3/2)*b^2 - 140*sqrt(c*x + b)*b^3 - 35*b^4/sqrt(c*x + b))/c^5
```

3.104 $\int \frac{x^{9/2}}{(bx+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=108

$$\frac{16b^2x^{3/2}}{5c^3\sqrt{bx+cx^2}}+\frac{32b^3\sqrt{x}}{5c^4\sqrt{bx+cx^2}}-\frac{4bx^{5/2}}{5c^2\sqrt{bx+cx^2}}+\frac{2x^{7/2}}{5c\sqrt{bx+cx^2}}$$

```
[Out] (32*b^3*Sqrt[x])/(5*c^4*Sqrt[b*x + c*x^2]) + (16*b^2*x^(3/2))/(5*c^3*Sqrt[b
*x + c*x^2]) - (4*b*x^(5/2))/(5*c^2*Sqrt[b*x + c*x^2]) + (2*x^(7/2))/(5*c*S
qrt[b*x + c*x^2])
```

Rubi [A] time = 0.0413416, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {656, 648}

$$\frac{16b^2x^{3/2}}{5c^3\sqrt{bx+cx^2}}+\frac{32b^3\sqrt{x}}{5c^4\sqrt{bx+cx^2}}-\frac{4bx^{5/2}}{5c^2\sqrt{bx+cx^2}}+\frac{2x^{7/2}}{5c\sqrt{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(b*x + c*x^2)^(3/2),x]
```

```
[Out] (32*b^3*Sqrt[x])/(5*c^4*Sqrt[b*x + c*x^2]) + (16*b^2*x^(3/2))/(5*c^3*Sqrt[b
*x + c*x^2]) - (4*b*x^(5/2))/(5*c^2*Sqrt[b*x + c*x^2]) + (2*x^(7/2))/(5*c*S
qrt[b*x + c*x^2])
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{x^{9/2}}{(bx+cx^2)^{3/2}}\,dx = \frac{2x^{7/2}}{5c\sqrt{bx+cx^2}} - \frac{(6b)\int \frac{x^{7/2}}{(bx+cx^2)^{3/2}}\,dx}{5c}$$

$$= -\frac{4bx^{5/2}}{5c^2\sqrt{bx+cx^2}} + \frac{2x^{7/2}}{5c\sqrt{bx+cx^2}} + \frac{(8b^2)\int \frac{x^{5/2}}{(bx+cx^2)^{3/2}}\,dx}{5c^2}$$

$$= \frac{16b^2x^{3/2}}{5c^3\sqrt{bx+cx^2}} - \frac{4bx^{5/2}}{5c^2\sqrt{bx+cx^2}} + \frac{2x^{7/2}}{5c\sqrt{bx+cx^2}} - \frac{(16b^3)\int \frac{x^{3/2}}{(bx+cx^2)^{3/2}}\,dx}{5c^3}$$

$$= \frac{32b^3\sqrt{x}}{5c^4\sqrt{bx+cx^2}} + \frac{16b^2x^{3/2}}{5c^3\sqrt{bx+cx^2}} - \frac{4bx^{5/2}}{5c^2\sqrt{bx+cx^2}} + \frac{2x^{7/2}}{5c\sqrt{bx+cx^2}}$$

Mathematica [A] time = 0.0237896, size = 52, normalized size = 0.48

$$\frac{2\sqrt{x}\left(8b^2cx+16b^3-2bc^2x^2+c^3x^3\right)}{5c^4\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[x]*(16*b^3 + 8*b^2*c*x - 2*b*c^2*x^2 + c^3*x^3))/(5*c^4*Sqrt[x*(b +
 c*x)])
```

Maple [A] time = 0.048, size = 54, normalized size = 0.5

$$\frac{(2\,cx+2\,b)\left(x^3c^3-2\,bx^2c^2+8\,b^2xc+16\,b^3\right)}{5\,c^4}x^{\frac{3}{2}}\left(cx^2+bx\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^2+b*x)^(3/2),x)
```

```
[Out] 2/5*(c*x+b)*(c^3*x^3-2*b*c^2*x^2+8*b^2*c*x+16*b^3)*x^(3/2)/c^4/(c*x^2+b*x)^
(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(\left(3\,c^4x^3-bc^3x^2+4\,b^2c^2x+8\,b^3c\right)x^3-2\left(bc^3x^3-2\,b^2c^2x^2-7\,b^3cx-4\,b^4\right)x^2+10\left(b^2c^2x^3+2\,b^3cx^2+b^4x\right)x\right)}{15\left(c^5x^3+bc^4x^2\right)\sqrt{cx+b}} - \int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*((3*c^4*x^3 - b*c^3*x^2 + 4*b^2*c^2*x + 8*b^3*c)*x^3 - 2*(b*c^3*x^3 -
2*b^2*c^2*x^2 - 7*b^3*c*x - 4*b^4)*x^2 + 10*(b^2*c^2*x^3 + 2*b^3*c*x^2 + b^
4*x)*x)/((c^5*x^3 + b*c^4*x^2)*sqrt(c*x + b)) - integrate(2*(b^3*c*x + b^4)
```

```
*x/((c^5*x^3 + 2*b*c^4*x^2 + b^2*c^3*x)*sqrt(c*x + b)), x)
```

Fricas [A] time = 2.00206, size = 130, normalized size = 1.2

$$\frac{2\left(c^3x^3 - 2\,bc^2x^2 + 8\,b^2cx + 16\,b^3\right)\sqrt{cx^2 + bx}\sqrt{x}}{5\left(c^5x^2 + bc^4x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(c^3*x^3 - 2*b*c^2*x^2 + 8*b^2*c*x + 16*b^3)*sqrt(c*x^2 + b*x)*sqrt(x)/
(c^5*x^2 + b*c^4*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22841, size = 76, normalized size = 0.7

$$-\frac{32\,b^{\frac{5}{2}}}{5\,c^4} + \frac{2\left((cx + b)^{\frac{5}{2}} - 5\,(cx + b)^{\frac{3}{2}}b + 15\,\sqrt{cx + b}b^2 + \frac{5\,b^3}{\sqrt{cx+b}}\right)}{5\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] -32/5*b^(5/2)/c^4 + 2/5*((c*x + b)^(5/2) - 5*(c*x + b)^(3/2)*b + 15*sqrt(c*
x + b)*b^2 + 5*b^3/sqrt(c*x + b))/c^4
```

3.105 $\int \frac{x^{7/2}}{\left(bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=80

$$-\frac{16b^2\sqrt{x}}{3c^3\sqrt{bx+cx^2}}-\frac{8bx^{3/2}}{3c^2\sqrt{bx+cx^2}}+\frac{2x^{5/2}}{3c\sqrt{bx+cx^2}}$$

```
[Out] (-16*b^2*Sqrt[x])/(3*c^3*Sqrt[b*x + c*x^2]) - (8*b*x^(3/2))/(3*c^2*Sqrt[b*x
 + c*x^2]) + (2*x^(5/2))/(3*c*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.0275102, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {656, 648}

$$-\frac{16b^2\sqrt{x}}{3c^3\sqrt{bx+cx^2}}-\frac{8bx^{3/2}}{3c^2\sqrt{bx+cx^2}}+\frac{2x^{5/2}}{3c\sqrt{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(b*x + c*x^2)^(3/2),x]
```

```
[Out] (-16*b^2*Sqrt[x])/(3*c^3*Sqrt[b*x + c*x^2]) - (8*b*x^(3/2))/(3*c^2*Sqrt[b*x
 + c*x^2]) + (2*x^(5/2))/(3*c*Sqrt[b*x + c*x^2])
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{7/2}}{\left(bx+cx^2\right)^{3/2}}\,dx &= \frac{2x^{5/2}}{3c\sqrt{bx+cx^2}}-\frac{(4b)\int \frac{x^{5/2}}{\left(bx+cx^2\right)^{3/2}}\,dx}{3c}\\
&= -\frac{8bx^{3/2}}{3c^2\sqrt{bx+cx^2}}+\frac{2x^{5/2}}{3c\sqrt{bx+cx^2}}+\frac{\left(8b^2\right)\int \frac{x^{3/2}}{\left(bx+cx^2\right)^{3/2}}\,dx}{3c^2}\\
&= -\frac{16b^2\sqrt{x}}{3c^3\sqrt{bx+cx^2}}-\frac{8bx^{3/2}}{3c^2\sqrt{bx+cx^2}}+\frac{2x^{5/2}}{3c\sqrt{bx+cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0184767, size = 41, normalized size = 0.51

$$\frac{2\sqrt{x}\left(-8b^2-4bcx+c^2x^2\right)}{3c^3\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[x]*(-8*b^2 - 4*b*c*x + c^2*x^2))/(3*c^3*Sqrt[x*(b + c*x)])
```

Maple [A] time = 0.047, size = 44, normalized size = 0.6

$$-\frac{(2\,cx+2\,b)\left(-c^2x^2+4\,bcx+8\,b^2\right)}{3\,c^3}x^{\frac{3}{2}}\left(cx^2+bx\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^2+b*x)^(3/2),x)
```

```
[Out] -2/3*(c*x+b)*(-c^2*x^2+4*b*c*x+8*b^2)*x^(3/2)/c^3/(c*x^2+b*x)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(\left(c^3x^2-bc^2x-2\,b^2c\right)x^2-2\left(bc^2x^2+2\,b^2cx+b^3\right)x\right)}{3\left(c^4x^2+bc^3x\right)\sqrt{cx+b}}+\int\frac{2\left(b^2cx+b^3\right)x}{\left(c^4x^3+2\,bc^3x^2+b^2c^2x\right)\sqrt{cx+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*((c^3*x^2 - b*c^2*x - 2*b^2*c)*x^2 - 2*(b*c^2*x^2 + 2*b^2*c*x + b^3)*x)
/((c^4*x^2 + b*c^3*x)*sqrt(c*x + b)) + integrate(2*(b^2*c*x + b^3)*x/((c^4*
x^3 + 2*b*c^3*x^2 + b^2*c^2*x)*sqrt(c*x + b)), x)
```

Fricas [A] time = 1.99862, size = 107, normalized size = 1.34

$$\frac{2\left(c^2x^2-4\,bcx-8\,b^2\right)\sqrt{cx^2+bx}\sqrt{x}}{3\left(c^4x^2+bc^3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(c^2*x^2 - 4*b*c*x - 8*b^2)*sqrt(c*x^2 + b*x)*sqrt(x)/(c^4*x^2 + b*c^3*
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28931, size = 59, normalized size = 0.74

$$\frac{16\,b^{\frac{3}{2}}}{3\,c^3}+\frac{2\left((cx+b)^{\frac{3}{2}}-6\,\sqrt{cx+b}b-\frac{3\,b^2}{\sqrt{cx+b}}\right)}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] 16/3*b^(3/2)/c^3 + 2/3*((c*x + b)^(3/2) - 6*sqrt(c*x + b)*b - 3*b^2/sqrt(c*
x + b))/c^3
```

3.106 $\int \frac{x^{5/2}}{\left(bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=48

$$\frac{4b\sqrt{x}}{c^2\sqrt{bx+cx^2}}+\frac{2x^{3/2}}{c\sqrt{bx+cx^2}}$$

```
[Out] (4*b*Sqrt[x])/(c^2*Sqrt[b*x + c*x^2]) + (2*x^(3/2))/(c*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.0181612, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {656, 648}

$$\frac{4b\sqrt{x}}{c^2\sqrt{bx+cx^2}}+\frac{2x^{3/2}}{c\sqrt{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(b*x + c*x^2)^(3/2),x]

[Out] (4*b*Sqrt[x])/(c^2*Sqrt[b*x + c*x^2]) + (2*x^(3/2))/(c*Sqrt[b*x + c*x^2])
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] && !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{5/2}}{\left(bx+cx^2\right)^{3/2}}\,dx &= \frac{2x^{3/2}}{c\sqrt{bx+cx^2}}-\frac{(2b)\int \frac{x^{3/2}}{\left(bx+cx^2\right)^{3/2}}\,dx}{c}\\ &= \frac{4b\sqrt{x}}{c^2\sqrt{bx+cx^2}}+\frac{2x^{3/2}}{c\sqrt{bx+cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0129798, size = 28, normalized size = 0.58

$$\frac{2\sqrt{x}(2b+cx)}{c^2\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[x]*(2*b + c*x))/(c^2*Sqrt[x*(b + c*x)])
```

Maple [A] time = 0.045, size = 32, normalized size = 0.7

$$2\,{\frac{ \left(cx+b \right) \left(cx+2\,b \right) {x}^{3/2}}{{c}^{2}} \left(c{x}^{2}+bx \right) ^{-3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^2+b*x)^(3/2),x)
```

```
[Out] 2*(c*x+b)*(c*x+2*b)*x^(3/2)/c^2/(c*x^2+b*x)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,\sqrt{cx+b}x}{c^{2}x+bc} - \int \frac{2\,{\left(bcx + b^{2}\right)}x}{{\left(c^{3}x^{3} + 2\,bc^{2}x^{2} + b^{2}cx\right)}\sqrt{cx+b}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(c*x + b)*x/(c^2*x + b*c) - integrate(2*(b*c*x + b^2)*x/((c^3*x^3 + 2
*b*c^2*x^2 + b^2*c*x)*sqrt(c*x + b)), x)
```

Fricas [A] time = 2.02542, size = 82, normalized size = 1.71

$$\frac{2\,\sqrt{cx^{2} + bx}{\left(cx + 2\,b\right)}\sqrt{x}}{c^{3}x^{2} + bc^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(c*x^2 + b*x)*(c*x + 2*b)*sqrt(x)/(c^3*x^2 + b*c^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\left(x \left(b + cx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral(x**(5/2)/(x*(b + c*x))**(3/2), x)
```

Giac [A] time = 1.2398, size = 42, normalized size = 0.88

$$\frac{2\left(\sqrt{cx+b}+\frac{b}{\sqrt{cx+b}}\right)}{c^2}-\frac{4\sqrt{b}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] 2*(sqrt(c*x + b) + b/sqrt(c*x + b))/c^2 - 4*sqrt(b)/c^2
```

3.107 $\int \frac{x^{3/2}}{\left(bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=23

$$-\frac{2\sqrt{x}}{c\sqrt{bx+cx^2}}$$

```
[Out] (-2*Sqrt[x])/(c*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.0070982, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {648}

$$-\frac{2\sqrt{x}}{c\sqrt{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[x])/(c*Sqrt[b*x + c*x^2])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(bx+cx^2\right)^{3/2}}\,dx=-\frac{2\sqrt{x}}{c\sqrt{bx+cx^2}}$$

Mathematica [A] time = 0.0067679, size = 21, normalized size = 0.91

$$-\frac{2\sqrt{x}}{c\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[x])/(c*Sqrt[x*(b + c*x)])
```

Maple [A] time = 0.049, size = 25, normalized size = 1.1

$$-2\,\frac{(cx+b)\,x^{3/2}}{c\left(cx^2+bx\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^2+b*x)^(3/2),x)
```

```
[Out] -2*(c*x+b)*x^(3/2)/c/(c*x^2+b*x)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\left(cx^2 + bx\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/(c*x^2 + b*x)^(3/2), x)
```

Fricas [A] time = 2.08504, size = 65, normalized size = 2.83

$$-\frac{2\sqrt{cx^2 + bx}\sqrt{x}}{c^2x^2 + bcx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(c*x^2 + b*x)*sqrt(x)/(c^2*x^2 + b*c*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\left(x\left(b + cx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral(x**(3/2)/(x*(b + c*x))**(3/2), x)
```

Giac [A] time = 1.21502, size = 28, normalized size = 1.22

$$-\frac{2}{\sqrt{cx + b}c} + \frac{2}{\sqrt{b}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/(sqrt(c*x + b)*c) + 2/(sqrt(b)*c)
```

3.108 $\int \frac{\sqrt{x}}{\left(bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=56

$$\frac{2\sqrt{x}}{b\sqrt{bx+cx^2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{3/2}}$$

```
[Out] (2*Sqrt[x])/(b*Sqrt[b*x + c*x^2]) - (2*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*S
qrt[x])])/b^(3/2)
```

Rubi [A] time = 0.0216995, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {666, 660, 207}

$$\frac{2\sqrt{x}}{b\sqrt{bx+cx^2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[x])/(b*Sqrt[b*x + c*x^2]) - (2*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*S
qrt[x])])/b^(3/2)
```

Rule 666

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((2*c*d - b*e)*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/(e*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[((2*c*d - b*e)*(m + 2*p + 2))/((p + 1)*(b^2
- 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0
] && LtQ[p, -1] && LtQ[0, m, 1] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(bx+cx^2\right)^{3/2}}\,dx = \frac{2\sqrt{x}}{b\sqrt{bx+cx^2}} + \frac{\int \frac{1}{\sqrt{x}\sqrt{bx+cx^2}}\,dx}{b}$$
$$= \frac{2\sqrt{x}}{b\sqrt{bx+cx^2}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-b+x^2}\,dx, x, \frac{\sqrt{bx+cx^2}}{\sqrt{x}}\right)}{b}$$
$$= \frac{2\sqrt{x}}{b\sqrt{bx+cx^2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{3/2}}$$

Mathematica [C] time = 0.0088132, size = 37, normalized size = 0.66

$$\frac{2\sqrt{x}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{cx}{b}+1\right)}{b\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[x]*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (c*x)/b])/(b*Sqrt[x*(b + c*x
)])
```

Maple [A] time = 0.181, size = 51, normalized size = 0.9

$$-2\,\frac{\sqrt{x\,(cx+b)}}{b^{3/2}\sqrt{x}\,(cx+b)}\left(\text{Artanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)\sqrt{cx+b}-\sqrt{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^2+b*x)^(3/2),x)
```

```
[Out] -2*(x*(c*x+b))^(1/2)/b^(3/2)*(arctanh((c*x+b)^(1/2)/b^(1/2))*(c*x+b)^(1/2)-
b^(1/2))/x^(1/2)/(c*x+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\left(cx^2+bx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/(c*x^2 + b*x)^(3/2), x)
```

Fricas [A] time = 2.0726, size = 359, normalized size = 6.41

$$\left[\frac{\left(cx^2+bx\right)\sqrt{b}\log\left(-\frac{cx^2+2\,bx-2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)+2\,\sqrt{cx^2+bx}b\sqrt{x}}{b^2cx^2+b^3x},\ \frac{2\left(\left(cx^2+bx\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{cx^2+bx}}\right)+\sqrt{cx^2+bx}b\sqrt{x}\right)}{b^2cx^2+b^3x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [((c*x^2 + b*x)*sqrt(b)*log(-(c*x^2 + 2*b*x - 2*sqrt(c*x^2 + b*x)*sqrt(b)*s
qrt(x))/x^2) + 2*sqrt(c*x^2 + b*x)*b*sqrt(x))/(b^2*c*x^2 + b^3*x), 2*((c*x^
2 + b*x)*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(c*x^2 + b*x)) + sqrt(c*x^2 +
 b*x)*b*sqrt(x))/(b^2*c*x^2 + b^3*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\left(x\left(b+cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral(sqrt(x)/(x*(b + c*x))**(3/2), x)
```

Giac [A] time = 1.23874, size = 90, normalized size = 1.61

$$\frac{2\,\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b}-\frac{2\left(\sqrt{b}\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+\sqrt{-b}\right)}{\sqrt{-b}b^{\frac{3}{2}}}+\frac{2}{\sqrt{cx+b}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b) - 2*(sqrt(b)*arctan(sqrt(b)/s
qrt(-b)) + sqrt(-b))/(sqrt(-b)*b^(3/2)) + 2/(sqrt(c*x + b)*b)
```

3.109 $\int \frac{1}{\sqrt{x}(bx+cx^2)^{3/2}} dx$

Optimal. Leaf size=81

$$-\frac{3c\sqrt{x}}{b^2\sqrt{bx+cx^2}}+\frac{3c\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{5/2}}-\frac{1}{b\sqrt{x}\sqrt{bx+cx^2}}$$

```
[Out] -(1/(b*Sqrt[x]*Sqrt[b*x + c*x^2])) - (3*c*Sqrt[x])/(b^2*Sqrt[b*x + c*x^2])
+ (3*c*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/b^(5/2)
```

Rubi [A] time = 0.0334128, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {672, 666, 660, 207}

$$-\frac{3c\sqrt{x}}{b^2\sqrt{bx+cx^2}}+\frac{3c\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{5/2}}-\frac{1}{b\sqrt{x}\sqrt{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(b*x + c*x^2)^(3/2)),x]
```

```
[Out] -(1/(b*Sqrt[x]*Sqrt[b*x + c*x^2])) - (3*c*Sqrt[x])/(b^2*Sqrt[b*x + c*x^2])
+ (3*c*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/b^(5/2)
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*(m + 2*p + 2))/((m + p + 1)*(2*c*d - b*e)), Int[(d
 + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && Ne
Q[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 666

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((2*c*d - b*e)*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/(e*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[((2*c*d - b*e)*(m + 2*p + 2))/((p + 1)*(b^2
- 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0
] && LtQ[p, -1] && LtQ[0, m, 1] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(bx+cx^2\right)^{3/2}}\,dx = -\frac{1}{b\sqrt{x}\sqrt{bx+cx^2}} - \frac{(3c)\int \frac{\sqrt{x}}{\left(bx+cx^2\right)^{3/2}}\,dx}{2b}$$
$$= -\frac{1}{b\sqrt{x}\sqrt{bx+cx^2}} - \frac{3c\sqrt{x}}{b^2\sqrt{bx+cx^2}} - \frac{(3c)\int \frac{1}{\sqrt{x}\sqrt{bx+cx^2}}\,dx}{2b^2}$$
$$= -\frac{1}{b\sqrt{x}\sqrt{bx+cx^2}} - \frac{3c\sqrt{x}}{b^2\sqrt{bx+cx^2}} - \frac{(3c)\,\text{Subst}\left(\int \frac{1}{-b+x^2}\,dx, x, \frac{\sqrt{bx+cx^2}}{\sqrt{x}}\right)}{b^2}$$
$$= -\frac{1}{b\sqrt{x}\sqrt{bx+cx^2}} - \frac{3c\sqrt{x}}{b^2\sqrt{bx+cx^2}} + \frac{3c\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{5/2}}$$

Mathematica [C] time = 0.0096923, size = 38, normalized size = 0.47

$$-\frac{2c\sqrt{x}\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{cx}{b}+1\right)}{b^2\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*c*Sqrt[x]*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (c*x)/b])/(b^2*Sqrt[x*(b
+ c*x)])
```

Maple [A] time = 0.218, size = 60, normalized size = 0.7

$$\frac{1}{cx+b}\sqrt{x\left(cx+b\right)}\left(3\,\text{Artanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)\sqrt{cx+b}xc-3\,xc\sqrt{b}-b^{\frac{3}{2}}\right)x^{-\frac{3}{2}}b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(c*x^2+b*x)^(3/2),x)
```

```
[Out] (x*(c*x+b))^(1/2)*(3*arctanh((c*x+b)^(1/2)/b^(1/2))*(c*x+b)^(1/2)*x*c-3*x*c
*b^(1/2)-b^(3/2))/x^(3/2)/(c*x+b)/b^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx\right)^{\frac{3}{2}}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x)^(3/2)*sqrt(x)), x)
```

Fricas [A] time = 2.24426, size = 428, normalized size = 5.28

$$\left[\frac{3\left(c^2x^3 + bcx^2\right)\sqrt{b}\log\left(-\frac{cx^2+2\,bx+2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right) - 2\left(3\,bcx + b^2\right)\sqrt{cx^2 + bx}\sqrt{x}}{2\left(b^3cx^3 + b^4x^2\right)}, -\frac{3\left(c^2x^3 + bcx^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{cx^2+bx}}\right) +}{b^3cx^3 + b^4x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(c^2*x^3 + b*c*x^2)*sqrt(b)*log(-(c*x^2 + 2*b*x + 2*sqrt(c*x^2 + b*
x)*sqrt(b)*sqrt(x))/x^2) - 2*(3*b*c*x + b^2)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^
3*c*x^3 + b^4*x^2), -(3*(c^2*x^3 + b*c*x^2)*sqrt(-b)*arctan(sqrt(-b)*sqrt(x
)/sqrt(c*x^2 + b*x)) + (3*b*c*x + b^2)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^3*c*x^
3 + b^4*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x}\left(x\left(b + cx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral(1/(sqrt(x)*(x*(b + c*x))**(3/2)), x)
```

Giac [A] time = 1.36039, size = 78, normalized size = 0.96

$$-c\left(\frac{3\,\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^2} + \frac{3\,cx + b}{\left((cx + b)^{\frac{3}{2}} - \sqrt{cx + b}b\right)b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] -c*(3*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b^2) + (3*c*x + b)/(((c*x +
b)^(3/2) - sqrt(c*x + b)*b)*b^2))
```

3.110 $\int \frac{1}{x^{3/2}(bx+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=117

$$\frac{15c^2\sqrt{x}}{4b^3\sqrt{bx+cx^2}}-\frac{15c^2\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{4b^{7/2}}+\frac{5c}{4b^2\sqrt{x}\sqrt{bx+cx^2}}-\frac{1}{2bx^{3/2}\sqrt{bx+cx^2}}$$

```
[Out] -1/(2*b*x^(3/2)*Sqrt[b*x + c*x^2]) + (5*c)/(4*b^2*Sqrt[x]*Sqrt[b*x + c*x^2]
) + (15*c^2*Sqrt[x])/(4*b^3*Sqrt[b*x + c*x^2]) - (15*c^2*ArcTanh[Sqrt[b*x +
 c*x^2]/(Sqrt[b]*Sqrt[x])])/(4*b^(7/2))
```

Rubi [A] time = 0.048457, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {672, 666, 660, 207}

$$\frac{15c^2\sqrt{x}}{4b^3\sqrt{bx+cx^2}}-\frac{15c^2\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{4b^{7/2}}+\frac{5c}{4b^2\sqrt{x}\sqrt{bx+cx^2}}-\frac{1}{2bx^{3/2}\sqrt{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(b*x + c*x^2)^(3/2)),x]
```

```
[Out] -1/(2*b*x^(3/2)*Sqrt[b*x + c*x^2]) + (5*c)/(4*b^2*Sqrt[x]*Sqrt[b*x + c*x^2]
) + (15*c^2*Sqrt[x])/(4*b^3*Sqrt[b*x + c*x^2]) - (15*c^2*ArcTanh[Sqrt[b*x +
 c*x^2]/(Sqrt[b]*Sqrt[x])])/(4*b^(7/2))
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*(m + 2*p + 2))/((m + p + 1)*(2*c*d - b*e)), Int[(d
 + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && Ne
Q[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 666

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((2*c*d - b*e)*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/(e*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[((2*c*d - b*e)*(m + 2*p + 2))/((p + 1)*(b^2
- 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0
] && LtQ[p, -1] && LtQ[0, m, 1] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{3/2}\left(bx+cx^2\right)^{3/2}}\,dx &= -\frac{1}{2bx^{3/2}\sqrt{bx+cx^2}}-\frac{(5c)\int \frac{1}{\sqrt{x}\left(bx+cx^2\right)^{3/2}}\,dx}{4b}\\
&= -\frac{1}{2bx^{3/2}\sqrt{bx+cx^2}}+\frac{5c}{4b^2\sqrt{x}\sqrt{bx+cx^2}}+\frac{\left(15c^2\right)\int \frac{\sqrt{x}}{\left(bx+cx^2\right)^{3/2}}\,dx}{8b^2}\\
&= -\frac{1}{2bx^{3/2}\sqrt{bx+cx^2}}+\frac{5c}{4b^2\sqrt{x}\sqrt{bx+cx^2}}+\frac{15c^2\sqrt{x}}{4b^3\sqrt{bx+cx^2}}+\frac{\left(15c^2\right)\int \frac{1}{\sqrt{x}\sqrt{bx+cx^2}}\,dx}{8b^3}\\
&= -\frac{1}{2bx^{3/2}\sqrt{bx+cx^2}}+\frac{5c}{4b^2\sqrt{x}\sqrt{bx+cx^2}}+\frac{15c^2\sqrt{x}}{4b^3\sqrt{bx+cx^2}}+\frac{\left(15c^2\right)\text{Subst}\left(\int \frac{1}{-b+x^2}\,dx,x,\frac{\sqrt{bx+cx^2}}{\sqrt{x}}\right)}{4b^3}\\
&= -\frac{1}{2bx^{3/2}\sqrt{bx+cx^2}}+\frac{5c}{4b^2\sqrt{x}\sqrt{bx+cx^2}}+\frac{15c^2\sqrt{x}}{4b^3\sqrt{bx+cx^2}}-\frac{15c^2\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{4b^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.0100784, size = 40, normalized size = 0.34

$$\frac{2c^2\sqrt{x}\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\frac{cx}{b}+1\right)}{b^3\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*c^2*Sqrt[x]*Hypergeometric2F1[-1/2, 3, 1/2, 1 + (c*x)/b])/(b^3*Sqrt[x*(b
 + c*x)])
```

Maple [A] time = 0.19, size = 76, normalized size = 0.7

$$-\frac{1}{4\,cx+4\,b}\sqrt{x\,(cx+b)}\left(15\,\text{Artanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)\sqrt{cx+b}x^2c^2-5\,b^{3/2}xc-15\,x^2c^2\sqrt{b}+2\,b^{5/2}\right)x^{-\frac{5}{2}}b^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^2+b*x)^(3/2),x)
```

```
[Out] -1/4/x^(5/2)*(x*(c*x+b))^(1/2)*(15*arctanh((c*x+b)^(1/2)/b^(1/2))*(c*x+b)^(
1/2)*x^2*c^2-5*b^(3/2)*x*c-15*x^2*c^2*b^(1/2)+2*b^(5/2))/(c*x+b)/b^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx\right)^{\frac{3}{2}}x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x)^(3/2)*x^(3/2)), x)
```

Fricas [A] time = 2.14278, size = 491, normalized size = 4.2

$$\left[\frac{15\left(c^3x^4+bc^2x^3\right)\sqrt{b}\log\left(-\frac{cx^2+2\,bx-2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)+2\left(15\,bc^2x^2+5\,b^2cx-2\,b^3\right)\sqrt{cx^2+bx}\sqrt{x}}{8\left(b^4cx^4+b^5x^3\right)},\ \frac{15\left(c^3x^4+bc^2x^3\right)\sqrt{-b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*(c^3*x^4 + b*c^2*x^3)*sqrt(b)*log(-(c*x^2 + 2*b*x - 2*sqrt(c*x^2 +
 b*x)*sqrt(b)*sqrt(x))/x^2) + 2*(15*b*c^2*x^2 + 5*b^2*c*x - 2*b^3)*sqrt(c*x
^2 + b*x)*sqrt(x))/(b^4*c*x^4 + b^5*x^3), 1/4*(15*(c^3*x^4 + b*c^2*x^3)*sqr
t(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(c*x^2 + b*x)) + (15*b*c^2*x^2 + 5*b^2*c*
x - 2*b^3)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^4*c*x^4 + b^5*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{3}{2}}\left(x\left(b+cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral(1/(x**(3/2)*(x*(b + c*x))**(3/2)), x)
```

Giac [A] time = 1.31458, size = 97, normalized size = 0.83

$$\frac{1}{4}\,c^2\left(\frac{15\,\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^3}+\frac{8}{\sqrt{cx+b}b^3}+\frac{7\left(cx+b\right)^{\frac{3}{2}}-9\,\sqrt{cx+b}b}{b^3c^2x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*c^2*(15*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b^3) + 8/(sqrt(c*x + b
)*b^3) + (7*(c*x + b)^(3/2) - 9*sqrt(c*x + b)*b)/(b^3*c^2*x^2))
```

3.111 $\int \frac{1}{x^{5/2}\left(bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{35c^3\sqrt{x}}{8b^4\sqrt{bx+cx^2}}-\frac{35c^2}{24b^3\sqrt{x}\sqrt{bx+cx^2}}+\frac{35c^3\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{8b^{9/2}}+\frac{7c}{12b^2x^{3/2}\sqrt{bx+cx^2}}-\frac{1}{3bx^{5/2}\sqrt{bx+cx^2}}$$

```
[Out] -1/(3*b*x^(5/2)*Sqrt[b*x + c*x^2]) + (7*c)/(12*b^2*x^(3/2)*Sqrt[b*x + c*x^2
]) - (35*c^2)/(24*b^3*Sqrt[x]*Sqrt[b*x + c*x^2]) - (35*c^3*Sqrt[x])/(8*b^4*
Sqrt[b*x + c*x^2]) + (35*c^3*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/
(8*b^(9/2))
```

Rubi [A] time = 0.0660191, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {672, 666, 660, 207}

$$-\frac{35c^3\sqrt{x}}{8b^4\sqrt{bx+cx^2}}-\frac{35c^2}{24b^3\sqrt{x}\sqrt{bx+cx^2}}+\frac{35c^3\tanh^{-1}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{8b^{9/2}}+\frac{7c}{12b^2x^{3/2}\sqrt{bx+cx^2}}-\frac{1}{3bx^{5/2}\sqrt{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(b*x + c*x^2)^(3/2)),x]
```

```
[Out] -1/(3*b*x^(5/2)*Sqrt[b*x + c*x^2]) + (7*c)/(12*b^2*x^(3/2)*Sqrt[b*x + c*x^2
]) - (35*c^2)/(24*b^3*Sqrt[x]*Sqrt[b*x + c*x^2]) - (35*c^3*Sqrt[x])/(8*b^4*
Sqrt[b*x + c*x^2]) + (35*c^3*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/
(8*b^(9/2))
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*(m + 2*p + 2))/((m + p + 1)*(2*c*d - b*e)), Int[(d
 + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && Ne
Q[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 666

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((2*c*d - b*e)*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/(e*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[((2*c*d - b*e)*(m + 2*p + 2))/((p + 1)*(b^2
- 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0
] && LtQ[p, -1] && LtQ[0, m, 1] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(bx+cx^2\right)^{3/2}}\,dx = -\frac{1}{3bx^{5/2}\sqrt{bx+cx^2}} - \frac{(7c)\int \frac{1}{x^{3/2}\left(bx+cx^2\right)^{3/2}}\,dx}{6b}$$

$$= -\frac{1}{3bx^{5/2}\sqrt{bx+cx^2}} + \frac{7c}{12b^2x^{3/2}\sqrt{bx+cx^2}} + \frac{\left(35c^2\right)\int \frac{1}{\sqrt{x}\left(bx+cx^2\right)^{3/2}}\,dx}{24b^2}$$

$$= -\frac{1}{3bx^{5/2}\sqrt{bx+cx^2}} + \frac{7c}{12b^2x^{3/2}\sqrt{bx+cx^2}} - \frac{35c^2}{24b^3\sqrt{x}\sqrt{bx+cx^2}} - \frac{\left(35c^3\right)\int \frac{\sqrt{x}}{\left(bx+cx^2\right)^{3/2}}\,dx}{16b^3}$$

$$= -\frac{1}{3bx^{5/2}\sqrt{bx+cx^2}} + \frac{7c}{12b^2x^{3/2}\sqrt{bx+cx^2}} - \frac{35c^2}{24b^3\sqrt{x}\sqrt{bx+cx^2}} - \frac{35c^3\sqrt{x}}{8b^4\sqrt{bx+cx^2}} - \frac{\left(35c^3\right)\int \text{}}{1}$$

$$= -\frac{1}{3bx^{5/2}\sqrt{bx+cx^2}} + \frac{7c}{12b^2x^{3/2}\sqrt{bx+cx^2}} - \frac{35c^2}{24b^3\sqrt{x}\sqrt{bx+cx^2}} - \frac{35c^3\sqrt{x}}{8b^4\sqrt{bx+cx^2}} - \frac{\left(35c^3\right)\text{Su}}{}$$

$$= -\frac{1}{3bx^{5/2}\sqrt{bx+cx^2}} + \frac{7c}{12b^2x^{3/2}\sqrt{bx+cx^2}} - \frac{35c^2}{24b^3\sqrt{x}\sqrt{bx+cx^2}} - \frac{35c^3\sqrt{x}}{8b^4\sqrt{bx+cx^2}} + \frac{35c^3\tanh}{8}$$

Mathematica [C] time = 0.0101033, size = 40, normalized size = 0.28

$$-\frac{2c^3\sqrt{x}\,{}_2F_1\left(-\frac{1}{2},4;\frac{1}{2};\frac{cx}{b}+1\right)}{b^4\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*c^3*Sqrt[x]*Hypergeometric2F1[-1/2, 4, 1/2, 1 + (c*x)/b])/(b^4*Sqrt[x*(
b + c*x)])
```

Maple [A] time = 0.191, size = 87, normalized size = 0.6

$$\frac{1}{24\,cx+24\,b}\sqrt{x\,(cx+b)}\left(105\,\mathrm{Artanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)\sqrt{cx+b}x^3c^3-105\,x^3c^3\sqrt{b}-35\,b^{3/2}x^2c^2+14\,b^{5/2}xc-8\,b^{7/2}\right)x^{-\frac{7}{2}}b^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(c*x^2+b*x)^(3/2),x)
```

```
[Out] 1/24*(x*(c*x+b))^(1/2)*(105*arctanh((c*x+b)^(1/2)/b^(1/2))*(c*x+b)^(1/2)*x^
3*c^3-105*x^3*c^3*b^(1/2)-35*b^(3/2)*x^2*c^2+14*b^(5/2)*x*c-8*b^(7/2))/x^(7
```

```
/2)/(c*x+b)/b^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx\right)^{\frac{3}{2}}x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x)^(3/2)*x^(5/2)), x)
```

Fricas [A] time = 2.06533, size = 549, normalized size = 3.79

$$\left[\frac{105\left(c^4x^5+bc^3x^4\right)\sqrt{b}\log\left(-\frac{cx^2+2\,bx+2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)-2\left(105\,bc^3x^3+35\,b^2c^2x^2-14\,b^3cx+8\,b^4\right)\sqrt{cx^2+bx}\sqrt{x}}{48\left(b^5cx^5+b^6x^4\right)},-\frac{105\left(c^4x\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(105*(c^4*x^5 + b*c^3*x^4)*sqrt(b)*log(-(c*x^2 + 2*b*x + 2*sqrt(c*x^2
 + b*x)*sqrt(b)*sqrt(x))/x^2) - 2*(105*b*c^3*x^3 + 35*b^2*c^2*x^2 - 14*b^3*
c*x + 8*b^4)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^5*c*x^5 + b^6*x^4), -1/24*(105*(
c^4*x^5 + b*c^3*x^4)*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(c*x^2 + b*x)) +
(105*b*c^3*x^3 + 35*b^2*c^2*x^2 - 14*b^3*c*x + 8*b^4)*sqrt(c*x^2 + b*x)*sqr
t(x))/(b^5*c*x^5 + b^6*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{5}{2}}\left(x\left(b+cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral(1/(x**(5/2)*(x*(b + c*x))**(3/2)), x)
```

Giac [A] time = 1.29756, size = 113, normalized size = 0.78

$$-\frac{1}{24}c^3\left(\frac{105\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^4}+\frac{48}{\sqrt{cx+b}b^4}+\frac{57\left(cx+b\right)^{\frac{5}{2}}-136\left(cx+b\right)^{\frac{3}{2}}b+87\sqrt{cx+b}b^2}{b^4c^3x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*c^3*(105*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b^4) + 48/(sqrt(c*x
 + b)*b^4) + (57*(c*x + b)^(5/2) - 136*(c*x + b)^(3/2)*b + 87*sqrt(c*x + b)
*b^2)/(b^4*c^3*x^3))
```

3.112 $\int (dx)^m \left(bx+cx^2\right)^3 \, dx$

Optimal. Leaf size=81

$$\frac{3b^2c(dx)^{m+5}}{d^5(m+5)}+\frac{b^3(dx)^{m+4}}{d^4(m+4)}+\frac{3bc^2(dx)^{m+6}}{d^6(m+6)}+\frac{c^3(dx)^{m+7}}{d^7(m+7)}$$

```
[Out] (b^3*(d*x)^(4 + m))/(d^4*(4 + m)) + (3*b^2*c*(d*x)^(5 + m))/(d^5*(5 + m)) +
 (3*b*c^2*(d*x)^(6 + m))/(d^6*(6 + m)) + (c^3*(d*x)^(7 + m))/(d^7*(7 + m))
```

Rubi [A] time = 0.0666968, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {647, 43}

$$\frac{3b^2c(dx)^{m+5}}{d^5(m+5)}+\frac{b^3(dx)^{m+4}}{d^4(m+4)}+\frac{3bc^2(dx)^{m+6}}{d^6(m+6)}+\frac{c^3(dx)^{m+7}}{d^7(m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(b*x + c*x^2)^3,x]
```

```
[Out] (b^3*(d*x)^(4 + m))/(d^4*(4 + m)) + (3*b^2*c*(d*x)^(5 + m))/(d^5*(5 + m)) +
 (3*b*c^2*(d*x)^(6 + m))/(d^6*(6 + m)) + (c^3*(d*x)^(7 + m))/(d^7*(7 + m))
```

Rule 647

```
Int[((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist
[1/e^p, Int[(e*x)^(m + p)*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, m}, x] &&
IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(bx+cx^2\right)^3 \, dx &= \frac{\int (dx)^{3+m}(b+cx)^3 \, dx}{d^3}\\ &= \frac{\int \left(b^3(dx)^{3+m}+\frac{3b^2c(dx)^{4+m}}{d}+\frac{3bc^2(dx)^{5+m}}{d^2}+\frac{c^3(dx)^{6+m}}{d^3}\right) dx}{d^3}\\ &= \frac{b^3(dx)^{4+m}}{d^4(4+m)}+\frac{3b^2c(dx)^{5+m}}{d^5(5+m)}+\frac{3bc^2(dx)^{6+m}}{d^6(6+m)}+\frac{c^3(dx)^{7+m}}{d^7(7+m)}\end{aligned}$$

Mathematica [A] time = 0.0360907, size = 57, normalized size = 0.7

$$x^4(dx)^m\left(\frac{3b^2cx}{m+5}+\frac{b^3}{m+4}+\frac{3bc^2x^2}{m+6}+\frac{c^3x^3}{m+7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(b*x + c*x^2)^3,x]
```

```
[Out] x^4*(d*x)^m*(b^3/(4 + m) + (3*b^2*c*x)/(5 + m) + (3*b*c^2*x^2)/(6 + m) + (c
^3*x^3)/(7 + m))
```

Maple [B] time = 0.058, size = 173, normalized size = 2.1

$$\frac{(dx)^m \left(c^3m^3x^3 + 3\,bc^2m^3x^2 + 15\,c^3m^2x^3 + 3\,b^2cm^3x + 48\,bc^2m^2x^2 + 74\,c^3mx^3 + b^3m^3 + 51\,b^2cm^2x + 249\,bc^2mx^2 + 12\right.}{(7+m)(6+m)(5+m)(4+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2+b*x)^3,x)
```

```
[Out] (d*x)^m*(c^3*m^3*x^3+3*b*c^2*m^3*x^2+15*c^3*m^2*x^3+3*b^2*c*m^3*x+48*b*c^2*
m^2*x^2+74*c^3*m*x^3+b^3*m^3+51*b^2*c*m^2*x+249*b*c^2*m*x^2+120*c^3*x^3+18*
b^3*m^2+282*b^2*c*m*x+420*b*c^2*x^2+107*b^3*m+504*b^2*c*x+210*b^3)*x^4/(7+m
)/(6+m)/(5+m)/(4+m)
```

Maxima [A] time = 1.17176, size = 104, normalized size = 1.28

$$\frac{c^3d^mx^7x^m}{m+7} + \frac{3\,bc^2d^mx^6x^m}{m+6} + \frac{3\,b^2cd^mx^5x^m}{m+5} + \frac{b^3d^mx^4x^m}{m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2+b*x)^3,x, algorithm="maxima")
```

```
[Out] c^3*d^m*x^7*x^m/(m + 7) + 3*b*c^2*d^m*x^6*x^m/(m + 6) + 3*b^2*c*d^m*x^5*x^m
/(m + 5) + b^3*d^m*x^4*x^m/(m + 4)
```

Fricas [A] time = 2.00345, size = 363, normalized size = 4.48

$$\frac{\left(\left(c^3m^3 + 15\,c^3m^2 + 74\,c^3m + 120\,c^3\right)x^7 + 3\left(bc^2m^3 + 16\,bc^2m^2 + 83\,bc^2m + 140\,bc^2\right)x^6 + 3\left(b^2cm^3 + 17\,b^2cm^2 + 94\,b^2\right.\right.}{m^4 + 22\,m^3 + 179\,m^2 + 638\,m + 840}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2+b*x)^3,x, algorithm="fricas")
```

```
[Out] ((c^3*m^3 + 15*c^3*m^2 + 74*c^3*m + 120*c^3)*x^7 + 3*(b*c^2*m^3 + 16*b*c^2*
m^2 + 83*b*c^2*m + 140*b*c^2)*x^6 + 3*(b^2*c*m^3 + 17*b^2*c*m^2 + 94*b^2*c*
m + 168*b^2*c)*x^5 + (b^3*m^3 + 18*b^3*m^2 + 107*b^3*m + 210*b^3)*x^4)*(d*x
)^m/(m^4 + 22*m^3 + 179*m^2 + 638*m + 840)
```

Sympy [A] time = 3.83979, size = 738, normalized size = 9.11

$$\begin{cases} \frac{-\frac{b^3}{3x^3}-\frac{3b^2c}{2x^2}-\frac{3bc^2}{x}+c^3\log(x)}{d^7} \\ \frac{-\frac{b^3}{2x^2}-\frac{3b^2c}{x}+3bc^2\log(x)+c^3x}{d^6} \\ \frac{-\frac{b^3}{x}+3b^2c\log(x)+3bc^2x+\frac{c^3x^2}{2}}{d^5} \\ \frac{b^3\log(x)+3b^2cx+\frac{3bc^2x^2}{2}+\frac{c^3x^3}{3}}{d^4} \\ \frac{b^3d^mm^3x^4x^m}{m^4+22m^3+179m^2+638m+840}+\frac{18b^3d^mm^2x^4x^m}{m^4+22m^3+179m^2+638m+840}+\frac{107b^3d^mmx^4x^m}{m^4+22m^3+179m^2+638m+840}+\frac{210b^3d^mx^4x^m}{m^4+22m^3+179m^2+638m+840}+\frac{3b^2cd^mm^3x}{m^4+22m^3+179m^2}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2+b*x)**3,x)
```

```
[Out] Piecewise(((-b**3/(3*x**3) - 3*b**2*c/(2*x**2) - 3*b*c**2/x + c**3*log(x))/
d**7, Eq(m, -7)), ((-b**3/(2*x**2) - 3*b**2*c/x + 3*b*c**2*log(x) + c**3*x)
/d**6, Eq(m, -6)), ((-b**3/x + 3*b**2*c*log(x) + 3*b*c**2*x + c**3*x**2/2)/
d**5, Eq(m, -5)), ((b**3*log(x) + 3*b**2*c*x + 3*b*c**2*x**2/2 + c**3*x**3/
3)/d**4, Eq(m, -4)), (b**3*d**m*m**3*x**4*x**m/(m**4 + 22*m**3 + 179*m**2 +
 638*m + 840) + 18*b**3*d**m*m**2*x**4*x**m/(m**4 + 22*m**3 + 179*m**2 + 63
8*m + 840) + 107*b**3*d**m*m*x**4*x**m/(m**4 + 22*m**3 + 179*m**2 + 638*m +
 840) + 210*b**3*d**m*x**4*x**m/(m**4 + 22*m**3 + 179*m**2 + 638*m + 840) +
 3*b**2*c*d**m*m**3*x**5*x**m/(m**4 + 22*m**3 + 179*m**2 + 638*m + 840) + 5
1*b**2*c*d**m*m**2*x**5*x**m/(m**4 + 22*m**3 + 179*m**2 + 638*m + 840) + 28
2*b**2*c*d**m*m*x**5*x**m/(m**4 + 22*m**3 + 179*m**2 + 638*m + 840) + 504*b
**2*c*d**m*x**5*x**m/(m**4 + 22*m**3 + 179*m**2 + 638*m + 840) + 3*b*c**2*d
**m*m**3*x**6*x**m/(m**4 + 22*m**3 + 179*m**2 + 638*m + 840) + 48*b*c**2*d*
*m*m**2*x**6*x**m/(m**4 + 22*m**3 + 179*m**2 + 638*m + 840) + 249*b*c**2*d*
*m*m*x**6*x**m/(m**4 + 22*m**3 + 179*m**2 + 638*m + 840) + 420*b*c**2*d**m*
x**6*x**m/(m**4 + 22*m**3 + 179*m**2 + 638*m + 840) + c**3*d**m*m**3*x**7*x
**m/(m**4 + 22*m**3 + 179*m**2 + 638*m + 840) + 15*c**3*d**m*m**2*x**7*x**m
/(m**4 + 22*m**3 + 179*m**2 + 638*m + 840) + 74*c**3*d**m*m*x**7*x**m/(m**4
 + 22*m**3 + 179*m**2 + 638*m + 840) + 120*c**3*d**m*x**7*x**m/(m**4 + 22*m
**3 + 179*m**2 + 638*m + 840), True))
```

Giac [B] time = 1.27941, size = 356, normalized size = 4.4

$$\frac{(dx)^m c^3m^3x^7 + 3\,(dx)^m bc^2m^3x^6 + 15\,(dx)^m c^3m^2x^7 + 3\,(dx)^m b^2cm^3x^5 + 48\,(dx)^m bc^2m^2x^6 + 74\,(dx)^m c^3mx^7 + (dx)^m b^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2+b*x)^3,x, algorithm="giac")
```

```
[Out] ((d*x)^m*c^3*m^3*x^7 + 3*(d*x)^m*b*c^2*m^3*x^6 + 15*(d*x)^m*c^3*m^2*x^7 + 3
*(d*x)^m*b^2*c*m^3*x^5 + 48*(d*x)^m*b*c^2*m^2*x^6 + 74*(d*x)^m*c^3*m*x^7 +
(d*x)^m*b^3*m^3*x^4 + 51*(d*x)^m*b^2*c*m^2*x^5 + 249*(d*x)^m*b*c^2*m*x^6 +
120*(d*x)^m*c^3*x^7 + 18*(d*x)^m*b^3*m^2*x^4 + 282*(d*x)^m*b^2*c*m*x^5 + 42
0*(d*x)^m*b*c^2*x^6 + 107*(d*x)^m*b^3*m*x^4 + 504*(d*x)^m*b^2*c*x^5 + 210*(
d*x)^m*b^3*x^4)/(m^4 + 22*m^3 + 179*m^2 + 638*m + 840)
```

3.113 $\int (dx)^m \left(bx+cx^2\right)^2 \, dx$

Optimal. Leaf size=58

$$\frac{b^2(dx)^{m+3}}{d^3(m+3)}+\frac{2bc(dx)^{m+4}}{d^4(m+4)}+\frac{c^2(dx)^{m+5}}{d^5(m+5)}$$

```
[Out] (b^2*(d*x)^(3 + m))/(d^3*(3 + m)) + (2*b*c*(d*x)^(4 + m))/(d^4*(4 + m)) + (
c^2*(d*x)^(5 + m))/(d^5*(5 + m))
```

Rubi [A] time = 0.0408282, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {647, 43}

$$\frac{b^2(dx)^{m+3}}{d^3(m+3)}+\frac{2bc(dx)^{m+4}}{d^4(m+4)}+\frac{c^2(dx)^{m+5}}{d^5(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(b*x + c*x^2)^2,x]
```

```
[Out] (b^2*(d*x)^(3 + m))/(d^3*(3 + m)) + (2*b*c*(d*x)^(4 + m))/(d^4*(4 + m)) + (
c^2*(d*x)^(5 + m))/(d^5*(5 + m))
```

Rule 647

```
Int[((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist
[1/e^p, Int[(e*x)^(m + p)*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, m}, x] &&
IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (dx)^m \left(bx+cx^2\right)^2 \, dx &= \frac{\int (dx)^{2+m}(b+cx)^2 \, dx}{d^2}\\
&= \frac{\int \left(b^2(dx)^{2+m}+\frac{2bc(dx)^{3+m}}{d}+\frac{c^2(dx)^{4+m}}{d^2}\right) dx}{d^2}\\
&= \frac{b^2(dx)^{3+m}}{d^3(3+m)}+\frac{2bc(dx)^{4+m}}{d^4(4+m)}+\frac{c^2(dx)^{5+m}}{d^5(5+m)}
\end{aligned}$$

Mathematica [A] time = 0.0306284, size = 41, normalized size = 0.71

$$x^3(dx)^m\left(\frac{b^2}{m+3}+\frac{2bcx}{m+4}+\frac{c^2x^2}{m+5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(b*x + c*x^2)^2,x]
```

```
[Out] x^3*(d*x)^m*(b^2/(3 + m) + (2*b*c*x)/(4 + m) + (c^2*x^2)/(5 + m))
```

Maple [A] time = 0.05, size = 90, normalized size = 1.6

$$\frac{(dx)^m \left(c^2m^2x^2 + 2\,bcm^2x + 7\,c^2mx^2 + b^2m^2 + 16\,bcmx + 12\,c^2x^2 + 9\,b^2m + 30\,bcx + 20\,b^2\right) x^3}{(5+m)\,(4+m)\,(3+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2+b*x)^2,x)
```

```
[Out] (d*x)^m*(c^2*m^2*x^2+2*b*c*m^2*x+7*c^2*m*x^2+b^2*m^2+16*b*c*m*x+12*c^2*x^2+
9*b^2*m+30*b*c*x+20*b^2)*x^3/(5+m)/(4+m)/(3+m)
```

Maxima [A] time = 1.16304, size = 74, normalized size = 1.28

$$\frac{c^2 d^m x^5 x^m}{m+5} + \frac{2\,bcd^m x^4 x^m}{m+4} + \frac{b^2 d^m x^3 x^m}{m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2+b*x)^2,x, algorithm="maxima")
```

```
[Out] c^2*d^m*x^5*x^m/(m + 5) + 2*b*c*d^m*x^4*x^m/(m + 4) + b^2*d^m*x^3*x^m/(m +
3)
```

Fricas [A] time = 2.1322, size = 193, normalized size = 3.33

$$\frac{\left(\left(c^2m^2 + 7\,c^2m + 12\,c^2\right)x^5 + 2\left(bcm^2 + 8\,bcm + 15\,bc\right)x^4 + \left(b^2m^2 + 9\,b^2m + 20\,b^2\right)x^3\right)(dx)^m}{m^3 + 12\,m^2 + 47\,m + 60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2+b*x)^2,x, algorithm="fricas")
```

```
[Out] ((c^2*m^2 + 7*c^2*m + 12*c^2)*x^5 + 2*(b*c*m^2 + 8*b*c*m + 15*b*c)*x^4 + (b
^2*m^2 + 9*b^2*m + 20*b^2)*x^3)*(d*x)^m/(m^3 + 12*m^2 + 47*m + 60)
```

Sympy [A] time = 2.60889, size = 345, normalized size = 5.95

$$\begin{cases} \frac{-\frac{b^2}{2x^2} - \frac{2bc}{x} + c^2\log(x)}{d^5} \\ \frac{-\frac{b^2}{x} + 2bc\log(x) + c^2x}{d^4} \\ \frac{b^2\log(x) + 2bcx + \frac{c^2x^2}{2}}{d^3} \\ \frac{b^2d^m m^2x^3x^m}{m^3+12m^2+47m+60} + \frac{9b^2d^m mx^3x^m}{m^3+12m^2+47m+60} + \frac{20b^2d^m x^3x^m}{m^3+12m^2+47m+60} + \frac{2bcd^m m^2x^4x^m}{m^3+12m^2+47m+60} + \frac{16bcd^m mx^4x^m}{m^3+12m^2+47m+60} + \frac{30bcd^m x^4x^m}{m^3+12m^2+47m+60} + \frac{c^2d^m m^2x}{m^3+12m^2+4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2+b*x)**2,x)
```

```
[Out] Piecewise(((-b**2/(2*x**2) - 2*b*c/x + c**2*log(x))/d**5, Eq(m, -5)), ((-b*
*2/x + 2*b*c*log(x) + c**2*x)/d**4, Eq(m, -4)), ((b**2*log(x) + 2*b*c*x + c
**2*x**2/2)/d**3, Eq(m, -3)), (b**2*d**m*m**2*x**3*x**m/(m**3 + 12*m**2 + 4
7*m + 60) + 9*b**2*d**m*m*x**3*x**m/(m**3 + 12*m**2 + 47*m + 60) + 20*b**2*
d**m*x**3*x**m/(m**3 + 12*m**2 + 47*m + 60) + 2*b*c*d**m*m**2*x**4*x**m/(m*
*3 + 12*m**2 + 47*m + 60) + 16*b*c*d**m*m*x**4*x**m/(m**3 + 12*m**2 + 47*m
+ 60) + 30*b*c*d**m*x**4*x**m/(m**3 + 12*m**2 + 47*m + 60) + c**2*d**m*m**2
*x**5*x**m/(m**3 + 12*m**2 + 47*m + 60) + 7*c**2*d**m*m*x**5*x**m/(m**3 + 1
2*m**2 + 47*m + 60) + 12*c**2*d**m*x**5*x**m/(m**3 + 12*m**2 + 47*m + 60),
True))
```

Giac [B] time = 1.29118, size = 190, normalized size = 3.28

$$\frac{(dx)^m c^2m^2x^5 + 2\,(dx)^m bcm^2x^4 + 7\,(dx)^m c^2mx^5 + (dx)^m b^2m^2x^3 + 16\,(dx)^m bcmx^4 + 12\,(dx)^m c^2x^5 + 9\,(dx)^m b^2mx^3}{m^3 + 12\,m^2 + 47\,m + 60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2+b*x)^2,x, algorithm="giac")
```

```
[Out] ((d*x)^m*c^2*m^2*x^5 + 2*(d*x)^m*b*c*m^2*x^4 + 7*(d*x)^m*c^2*m*x^5 + (d*x)^
m*b^2*m^2*x^3 + 16*(d*x)^m*b*c*m*x^4 + 12*(d*x)^m*c^2*x^5 + 9*(d*x)^m*b^2*m
*x^3 + 30*(d*x)^m*b*c*x^4 + 20*(d*x)^m*b^2*x^3)/(m^3 + 12*m^2 + 47*m + 60)
```

3.114 $\int (dx)^m \left(bx+cx^2\right) dx$

Optimal. Leaf size=35

$$\frac{b(dx)^{m+2}}{d^2(m+2)}+\frac{c(dx)^{m+3}}{d^3(m+3)}$$

```
[Out] (b*(d*x)^(2 + m))/(d^2*(2 + m)) + (c*(d*x)^(3 + m))/(d^3*(3 + m))
```

Rubi [A] time = 0.0139718, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$\frac{b(dx)^{m+2}}{d^2(m+2)}+\frac{c(dx)^{m+3}}{d^3(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(b*x + c*x^2),x]
```

```
[Out] (b*(d*x)^(2 + m))/(d^2*(2 + m)) + (c*(d*x)^(3 + m))/(d^3*(3 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(bx+cx^2\right) dx &= \int \left(\frac{b(dx)^{1+m}}{d}+\frac{c(dx)^{2+m}}{d^2}\right) dx\\ &= \frac{b(dx)^{2+m}}{d^2(2+m)}+\frac{c(dx)^{3+m}}{d^3(3+m)}\end{aligned}$$

Mathematica [A] time = 0.0147581, size = 25, normalized size = 0.71

$$x^2(dx)^m\left(\frac{b}{m+2}+\frac{cx}{m+3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(b*x + c*x^2),x]
```

```
[Out] x^2*(d*x)^m*(b/(2 + m) + (c*x)/(3 + m))
```

Maple [A] time = 0.046, size = 35, normalized size = 1.

$$\frac{(dx)^m\,(cmx+bm+2\,cx+3\,b)\,x^2}{(3+m)\,(2+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2+b*x),x)
```

```
[Out] (d*x)^m*(c*m*x+b*m+2*c*x+3*b)*x^2/(3+m)/(2+m)
```

Maxima [A] time = 1.15542, size = 45, normalized size = 1.29

$$\frac{cd^m x^3 x^m}{m+3} + \frac{bd^m x^2 x^m}{m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] c*d^m*x^3*x^m/(m + 3) + b*d^m*x^2*x^m/(m + 2)
```

Fricas [A] time = 2.03065, size = 82, normalized size = 2.34

$$\frac{\left((cm+2\,c)x^3 + (bm+3\,b)x^2\right)(dx)^m}{m^2+5\,m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] ((c*m + 2*c)*x^3 + (b*m + 3*b)*x^2)*(d*x)^m/(m^2 + 5*m + 6)
```

Sympy [A] time = 0.79318, size = 112, normalized size = 3.2

$$\begin{cases} \frac{-\frac{b}{x}+c\log(x)}{d^3} & \text{for } m = -3 \\ \frac{b\log(x)+cx}{d^2} & \text{for } m = -2 \\ \frac{bd^m m x^2 x^m}{m^2+5m+6} + \frac{3bd^m x^2 x^m}{m^2+5m+6} + \frac{cd^m m x^3 x^m}{m^2+5m+6} + \frac{2cd^m x^3 x^m}{m^2+5m+6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2+b*x),x)
```

```
[Out] Piecewise(((-b/x + c*log(x))/d**3, Eq(m, -3)), ((b*log(x) + c*x)/d**2, Eq(m
, -2)), (b*d**m*m*x**2*x**m/(m**2 + 5*m + 6) + 3*b*d**m*x**2*x**m/(m**2 + 5
*m + 6) + c*d**m*m*x**3*x**m/(m**2 + 5*m + 6) + 2*c*d**m*x**3*x**m/(m**2 +
5*m + 6), True))
```

Giac [A] time = 1.33834, size = 76, normalized size = 2.17

$$\frac{(dx)^m cmx^3 + (dx)^m bmx^2 + 2\,(dx)^m cx^3 + 3\,(dx)^m bx^2}{m^2+5\,m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] ((d*x)^m*c*m*x^3 + (d*x)^m*b*m*x^2 + 2*(d*x)^m*c*x^3 + 3*(d*x)^m*b*x^2)/(m^
2 + 5*m + 6)
```

3.115 $\int \frac{(dx)^m}{bx+cx^2} \, dx$

Optimal. Leaf size=25

$$\frac{(dx)^m {}_2F_1\left(1,m;m+1;-\frac{cx}{b}\right)}{bm}$$

```
[Out] ((d*x)^m*Hypergeometric2F1[1, m, 1 + m, -((c*x)/b)])/(b*m)
```

Rubi [A] time = 0.0130205, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {647, 64}

$$\frac{(dx)^m {}_2F_1\left(1,m;m+1;-\frac{cx}{b}\right)}{bm}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(b*x + c*x^2),x]
```

```
[Out] ((d*x)^m*Hypergeometric2F1[1, m, 1 + m, -((c*x)/b)])/(b*m)
```

Rule 647

```
Int[((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist
[1/e^p, Int[(e*x)^(m + p)*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, m}, x] &&
IntegerQ[p]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int \frac{(dx)^m}{bx+cx^2} \, dx &= d \int \frac{(dx)^{-1+m}}{b+cx} \, dx\\ &= \frac{(dx)^m {}_2F_1\left(1,m;1+m;-\frac{cx}{b}\right)}{bm}\end{aligned}$$

Mathematica [A] time = 0.006013, size = 25, normalized size = 1.

$$\frac{(dx)^m {}_2F_1\left(1,m;m+1;-\frac{cx}{b}\right)}{bm}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(b*x + c*x^2),x]
```

```
[Out] ((d*x)^m*Hypergeometric2F1[1, m, 1 + m, -((c*x)/b)])/(b*m)
```

Maple [F] time = 0.374, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{cx^2 + bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(c*x^2+b*x),x)
```

```
[Out] int((d*x)^m/(c*x^2+b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{cx^2 + bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(c*x^2 + b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{cx^2 + bx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(c*x^2 + b*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{x\,(b + cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(c*x**2+b*x),x)
```

```
[Out] Integral((d*x)**m/(x*(b + c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{cx^2 + bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^2 + b*x), x)
```

3.116 $\int \frac{(dx)^m}{\left(bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=33

$$-\frac{d(dx)^{m-1}\,{}_2F_1\left(2,m-1;m;-\frac{cx}{b}\right)}{b^2(1-m)}$$

```
[Out] -((d*(d*x)^(-1 + m)*Hypergeometric2F1[2, -1 + m, m, -((c*x)/b)])/(b^2*(1 -
m)))
```

Rubi [A] time = 0.0172126, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {647, 64}

$$-\frac{d(dx)^{m-1}\,{}_2F_1\left(2,m-1;m;-\frac{cx}{b}\right)}{b^2(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(b*x + c*x^2)^2,x]
```

```
[Out] -((d*(d*x)^(-1 + m)*Hypergeometric2F1[2, -1 + m, m, -((c*x)/b)])/(b^2*(1 -
m)))
```

Rule 647

```
Int[((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist
[1/e^p, Int[(e*x)^(m + p)*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, m}, x] &&
IntegerQ[p]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int \frac{(dx)^m}{\left(bx+cx^2\right)^2}\,dx &= d^2\int \frac{(dx)^{-2+m}}{(b+cx)^2}\,dx\\ &= -\frac{d(dx)^{-1+m}\,{}_2F_1\left(2,-1+m;m;-\frac{cx}{b}\right)}{b^2(1-m)}\end{aligned}$$

Mathematica [A] time = 0.0088843, size = 30, normalized size = 0.91

$$\frac{(dx)^m\,{}_2F_1\left(2,m-1;m;-\frac{cx}{b}\right)}{b^2(m-1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(b*x + c*x^2)^2,x]
```

```
[Out] ((d*x)^m*Hypergeometric2F1[2, -1 + m, m, -((c*x)/b)])/(b^2*(-1 + m)*x)
```

Maple [F] time = 0.39, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(cx^2 + bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(c*x^2+b*x)^2,x)
```

```
[Out] int((d*x)^m/(c*x^2+b*x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(cx^2 + bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^2+b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(c*x^2 + b*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{c^2x^4 + 2\,bcx^3 + b^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^2+b*x)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(c^2*x^4 + 2*b*c*x^3 + b^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{x^2\,(b + cx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(c*x**2+b*x)**2,x)
```

```
[Out] Integral((d*x)**m/(x**2*(b + c*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(cx^2 + bx\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^2+b*x)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^2 + b*x)^2, x)
```

3.117 $\int \frac{(dx)^m}{(bx+cx^2)^3}\,dx$

Optimal. Leaf size=37

$$-\frac{d^2(dx)^{m-2}\,{}_2F_1\left(3,m-2;m-1;-\frac{cx}{b}\right)}{b^3(2-m)}$$

```
[Out] -((d^2*(d*x)^(-2 + m)*Hypergeometric2F1[3, -2 + m, -1 + m, -((c*x)/b)])/(b^
3*(2 - m)))
```

Rubi [A] time = 0.0189416, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {647, 64}

$$-\frac{d^2(dx)^{m-2}\,{}_2F_1\left(3,m-2;m-1;-\frac{cx}{b}\right)}{b^3(2-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(b*x + c*x^2)^3,x]
```

```
[Out] -((d^2*(d*x)^(-2 + m)*Hypergeometric2F1[3, -2 + m, -1 + m, -((c*x)/b)])/(b^
3*(2 - m)))
```

Rule 647

```
Int[((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist
[1/e^p, Int[(e*x)^(m + p)*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, m}, x] &&
IntegerQ[p]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int \frac{(dx)^m}{\left(bx+cx^2\right)^3}\,dx &= d^3\int \frac{(dx)^{-3+m}}{(b+cx)^3}\,dx\\ &= -\frac{d^2(dx)^{-2+m}\,{}_2F_1\left(3,-2+m;-1+m;-\frac{cx}{b}\right)}{b^3(2-m)}\end{aligned}$$

Mathematica [A] time = 0.0091514, size = 32, normalized size = 0.86

$$\frac{(dx)^m\,{}_2F_1\left(3,m-2;m-1;-\frac{cx}{b}\right)}{b^3(m-2)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(b*x + c*x^2)^3,x]
```

```
[Out] ((d*x)^m*Hypergeometric2F1[3, -2 + m, -1 + m, -((c*x)/b)])/(b^3*(-2 + m)*x^
2)
```

Maple [F] time = 0.409, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(cx^2 + bx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(c*x^2+b*x)^3,x)
```

```
[Out] int((d*x)^m/(c*x^2+b*x)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(cx^2 + bx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^2+b*x)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(c*x^2 + b*x)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{c^3x^6 + 3\,bc^2x^5 + 3\,b^2cx^4 + b^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^2+b*x)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(c^3*x^6 + 3*b*c^2*x^5 + 3*b^2*c*x^4 + b^3*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{x^3\,(b + cx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(c*x**2+b*x)**3,x)
```

```
[Out] Integral((d*x)**m/(x**3*(b + c*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(cx^2 + bx)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^2+b*x)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^2 + b*x)^3, x)
```

3.118 $\int (dx)^m \left(bx+cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=73

$$\frac{2b^2(b+cx)\left(bx+cx^2\right)^{5/2}(dx)^m\left(-\frac{cx}{b}\right)^{-m-\frac{1}{2}}\,{}_2F_1\left(\frac{7}{2},-m-\frac{5}{2};\frac{9}{2};\frac{cx}{b}+1\right)}{7c^3x^2}$$

```
[Out] (2*b^2*(-((c*x)/b))^(-1/2 - m)*(d*x)^m*(b + c*x)*(b*x + c*x^2)^(5/2)*Hyperg
eometric2F1[7/2, -5/2 - m, 9/2, 1 + (c*x)/b])/(7*c^3*x^2)
```

Rubi [A] time = 0.030488, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {674, 67, 65}

$$\frac{2b^2(b+cx)\left(bx+cx^2\right)^{5/2}(dx)^m\left(-\frac{cx}{b}\right)^{-m-\frac{1}{2}}\,{}_2F_1\left(\frac{7}{2},-m-\frac{5}{2};\frac{9}{2};\frac{cx}{b}+1\right)}{7c^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*b^2*(-((c*x)/b))^(-1/2 - m)*(d*x)^m*(b + c*x)*(b*x + c*x^2)^(5/2)*Hyperg
eometric2F1[7/2, -5/2 - m, 9/2, 1 + (c*x)/b])/(7*c^3*x^2)
```

Rule 674

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(
(e*x)^m*(b*x + c*x^2)^p)/(x^(m + p)*(b + c*x)^p), Int[x^(m + p)*(b + c*x)^p
, x], x] /; FreeQ[{b, c, e, m}, x] &&  !IntegerQ[p]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (dx)^m \left(bx+cx^2\right)^{5/2} dx = \frac{\left(x^{-\frac{5}{2}-m}(dx)^m \left(bx+cx^2\right)^{5/2}\right) \int x^{\frac{5}{2}+m}(b+cx)^{5/2} dx}{(b+cx)^{5/2}}$$
$$= \frac{\left(b^2 \left(-\frac{cx}{b}\right)^{-\frac{1}{2}-m} (dx)^m \left(bx+cx^2\right)^{5/2}\right) \int \left(-\frac{cx}{b}\right)^{\frac{5}{2}+m} (b+cx)^{5/2} dx}{c^2x^2(b+cx)^{5/2}}$$
$$= \frac{2b^2 \left(-\frac{cx}{b}\right)^{-\frac{1}{2}-m} (dx)^m (b+cx) \left(bx+cx^2\right)^{5/2} {}_2F_1\left(\frac{7}{2}, -\frac{5}{2}-m; \frac{9}{2}; 1+\frac{cx}{b}\right)}{7c^3x^2}$$

Mathematica [A] time = 0.122992, size = 70, normalized size = 0.96

$$\frac{2b^2(b+cx)^3\sqrt{x(b+cx)}(dx)^m \left(-\frac{cx}{b}\right)^{-m-\frac{1}{2}} {}_2F_1\left(\frac{7}{2}, -m-\frac{5}{2}; \frac{9}{2}; \frac{cx}{b}+1\right)}{7c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*b^2*(-((c*x)/b))^(-1/2 - m)*(d*x)^m*(b + c*x)^3*Sqrt[x*(b + c*x)]*Hyperg
eometric2F1[7/2, -5/2 - m, 9/2, 1 + (c*x)/b])/(7*c^3)
```

Maple [F] time = 0.429, size = 0, normalized size = 0.

$$\int (dx)^m \left(cx^2+bx\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2+b*x)^(5/2),x)
```

```
[Out] int((d*x)^m*(c*x^2+b*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{\frac{5}{2}} (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(5/2)*(d*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(c^2x^4+2\,bcx^3+b^2x^2\right)\sqrt{cx^2+bx}\,(dx)^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + b^2*x^2)*sqrt(c*x^2 + b*x)*(d*x)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m (x(b+cx))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2+b*x)**(5/2),x)
```

```
[Out] Integral((d*x)**m*(x*(b + c*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{\frac{5}{2}} (dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(5/2)*(d*x)^m, x)
```

3.119 $\int (dx)^m \left(bx+cx^2\right)^{3/2} dx$

Optimal. Leaf size=71

$$-\frac{2b(b+cx)\left(bx+cx^2\right)^{3/2}(dx)^m\left(-\frac{cx}{b}\right)^{-m-\frac{1}{2}}\,{}_2F_1\left(\frac{5}{2},-m-\frac{3}{2};\frac{7}{2};\frac{cx}{b}+1\right)}{5c^2x}$$

```
[Out] (-2*b*(-((c*x)/b))^(-1/2 - m)*(d*x)^m*(b + c*x)*(b*x + c*x^2)^(3/2)*Hyperge
ometric2F1[5/2, -3/2 - m, 7/2, 1 + (c*x)/b])/(5*c^2*x)
```

Rubi [A] time = 0.0294293, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {674, 67, 65}

$$-\frac{2b(b+cx)\left(bx+cx^2\right)^{3/2}(dx)^m\left(-\frac{cx}{b}\right)^{-m-\frac{1}{2}}\,{}_2F_1\left(\frac{5}{2},-m-\frac{3}{2};\frac{7}{2};\frac{cx}{b}+1\right)}{5c^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*b*(-((c*x)/b))^(-1/2 - m)*(d*x)^m*(b + c*x)*(b*x + c*x^2)^(3/2)*Hyperge
ometric2F1[5/2, -3/2 - m, 7/2, 1 + (c*x)/b])/(5*c^2*x)
```

Rule 674

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(
(e*x)^m*(b*x + c*x^2)^p)/(x^(m + p)*(b + c*x)^p), Int[x^(m + p)*(b + c*x)^p
, x], x] /; FreeQ[{b, c, e, m}, x] && !IntegerQ[p]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (dx)^m \left(bx+cx^2\right)^{3/2} dx = \frac{\left(x^{-\frac{3}{2}-m}(dx)^m \left(bx+cx^2\right)^{3/2}\right) \int x^{\frac{3}{2}+m}(b+cx)^{3/2} dx}{(b+cx)^{3/2}}$$
$$= -\frac{\left(b\left(-\frac{cx}{b}\right)^{-\frac{1}{2}-m} (dx)^m \left(bx+cx^2\right)^{3/2}\right) \int \left(-\frac{cx}{b}\right)^{\frac{3}{2}+m} (b+cx)^{3/2} dx}{cx(b+cx)^{3/2}}$$
$$= -\frac{2b\left(-\frac{cx}{b}\right)^{-\frac{1}{2}-m} (dx)^m (b+cx) \left(bx+cx^2\right)^{3/2} {}_2F_1\left(\frac{5}{2}, -\frac{3}{2}-m; \frac{7}{2}; 1+\frac{cx}{b}\right)}{5c^2x}$$

Mathematica [A] time = 0.116832, size = 60, normalized size = 0.85

$$-\frac{2(x(b+cx))^{5/2}(dx)^m \left(-\frac{cx}{b}\right)^{-m-\frac{5}{2}} {}_2F_1\left(\frac{5}{2}, -m-\frac{3}{2}; \frac{7}{2}; \frac{cx}{b}+1\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*(-((c*x)/b))^(-5/2 - m)*(d*x)^m*(x*(b + c*x))^(5/2)*Hypergeometric2F1[5
/2, -3/2 - m, 7/2, 1 + (c*x)/b])/(5*b)
```

Maple [F] time = 0.403, size = 0, normalized size = 0.

$$\int (dx)^m \left(cx^2+bx\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2+b*x)^(3/2),x)
```

```
[Out] int((d*x)^m*(c*x^2+b*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{\frac{3}{2}} (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(3/2)*(d*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2+bx\right)^{\frac{3}{2}} (dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(3/2)*(d*x)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m (x(b+cx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral((d*x)**m*(x*(b + c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{\frac{3}{2}} (dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(3/2)*(d*x)^m, x)
```

3.120 $\int (dx)^m \sqrt{bx+cx^2}\,dx$

Optimal. Leaf size=67

$$\frac{2(b+cx)\sqrt{bx+cx^2}(dx)^m\left(-\frac{cx}{b}\right)^{-m-\frac{1}{2}}\,{}_2F_1\left(\frac{3}{2},-m-\frac{1}{2};\frac{5}{2};\frac{cx}{b}+1\right)}{3c}$$

```
[Out] (2*(-((c*x)/b))^(-1/2 - m)*(d*x)^m*(b + c*x)*Sqrt[b*x + c*x^2]*Hypergeometr
ic2F1[3/2, -1/2 - m, 5/2, 1 + (c*x)/b])/(3*c)
```

Rubi [A] time = 0.0251237, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {674, 67, 65}

$$\frac{2(b+cx)\sqrt{bx+cx^2}(dx)^m\left(-\frac{cx}{b}\right)^{-m-\frac{1}{2}}\,{}_2F_1\left(\frac{3}{2},-m-\frac{1}{2};\frac{5}{2};\frac{cx}{b}+1\right)}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[b*x + c*x^2],x]
```

```
[Out] (2*(-((c*x)/b))^(-1/2 - m)*(d*x)^m*(b + c*x)*Sqrt[b*x + c*x^2]*Hypergeometr
ic2F1[3/2, -1/2 - m, 5/2, 1 + (c*x)/b])/(3*c)
```

Rule 674

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(
(e*x)^m*(b*x + c*x^2)^p)/(x^(m + p)*(b + c*x)^p), Int[x^(m + p)*(b + c*x)^p
, x], x] /; FreeQ[{b, c, e, m}, x] && !IntegerQ[p]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (dx)^m\sqrt{bx+cx^2}\,dx = \frac{\left(x^{-\frac{1}{2}-m}(dx)^m\sqrt{bx+cx^2}\right)\int x^{\frac{1}{2}+m}\sqrt{b+cx}\,dx}{\sqrt{b+cx}}$$
$$= \frac{\left(\left(-\frac{cx}{b}\right)^{-\frac{1}{2}-m}(dx)^m\sqrt{bx+cx^2}\right)\int \left(-\frac{cx}{b}\right)^{\frac{1}{2}+m}\sqrt{b+cx}\,dx}{\sqrt{b+cx}}$$
$$= \frac{2\left(-\frac{cx}{b}\right)^{-\frac{1}{2}-m}(dx)^m(b+cx)\sqrt{bx+cx^2}\,{}_2F_1\left(\frac{3}{2},-\frac{1}{2}-m;\frac{5}{2};1+\frac{cx}{b}\right)}{3c}$$

Mathematica [A] time = 0.0681754, size = 60, normalized size = 0.9

$$-\frac{2(x(b+cx))^{3/2}(dx)^m\left(-\frac{cx}{b}\right)^{-m-\frac{3}{2}}\,{}_2F_1\left(\frac{3}{2},-m-\frac{1}{2};\frac{5}{2};\frac{cx}{b}+1\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[b*x + c*x^2],x]
```

```
[Out] (-2*(-((c*x)/b))^(-3/2 - m)*(d*x)^m*(x*(b + c*x))^(3/2)*Hypergeometric2F1[3
/2, -1/2 - m, 5/2, 1 + (c*x)/b])/(3*b)
```

Maple [F] time = 0.427, size = 0, normalized size = 0.

$$\int (dx)^m\sqrt{cx^2+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2+b*x)^(1/2),x)
```

```
[Out] int((d*x)^m*(c*x^2+b*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2+bx}\,(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x)*(d*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{cx^2+bx}\,(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x)*(d*x)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \sqrt{x (b + cx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral((d*x)**m*sqrt(x*(b + c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + bx}\, (dx)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x)*(d*x)^m, x)
```

3.121 $\int \frac{(dx)^m}{\sqrt{bx+cx^2}} dx$

Optimal. Leaf size=65

$$\frac{2(b+cx)(dx)^m \left(-\frac{cx}{b}\right)^{\frac{1}{2}-m} {}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{cx}{b}+1\right)}{c\sqrt{bx+cx^2}}$$

```
[Out] (2*(-((c*x)/b))^(1/2 - m)*(d*x)^m*(b + c*x)*Hypergeometric2F1[1/2, 1/2 - m,
 3/2, 1 + (c*x)/b])/(c*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.0253687, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {674, 67, 65}

$$\frac{2(b+cx)(dx)^m \left(-\frac{cx}{b}\right)^{\frac{1}{2}-m} {}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{cx}{b}+1\right)}{c\sqrt{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/Sqrt[b*x + c*x^2],x]
```

```
[Out] (2*(-((c*x)/b))^(1/2 - m)*(d*x)^m*(b + c*x)*Hypergeometric2F1[1/2, 1/2 - m,
 3/2, 1 + (c*x)/b])/(c*Sqrt[b*x + c*x^2])
```

Rule 674

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(
(e*x)^m*(b*x + c*x^2)^p)/(x^(m + p)*(b + c*x)^p), Int[x^(m + p)*(b + c*x)^p
, x], x] /; FreeQ[{b, c, e, m}, x] && !IntegerQ[p]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{(dx)^m}{\sqrt{bx+cx^2}}\,dx = \frac{\left(x^{\frac{1}{2}-m}(dx)^m\sqrt{b+cx}\right)\int \frac{x^{-\frac{1}{2}+m}}{\sqrt{b+cx}}\,dx}{\sqrt{bx+cx^2}}$$
$$= \frac{\left(\left(-\frac{cx}{b}\right)^{\frac{1}{2}-m}(dx)^m\sqrt{b+cx}\right)\int \frac{\left(-\frac{cx}{b}\right)^{-\frac{1}{2}+m}}{\sqrt{b+cx}}\,dx}{\sqrt{bx+cx^2}}$$
$$= \frac{2\left(-\frac{cx}{b}\right)^{\frac{1}{2}-m}(dx)^m(b+cx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};1+\frac{cx}{b}\right)}{c\sqrt{bx+cx^2}}$$

Mathematica [A] time = 0.0577265, size = 58, normalized size = 0.89

$$-\frac{2\sqrt{x(b+cx)}(dx)^m\left(-\frac{cx}{b}\right)^{-m-\frac{1}{2}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{cx}{b}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/Sqrt[b*x + c*x^2],x]

[Out] (-2*(-((c*x)/b))^(-1/2 - m)*(d*x)^m*Sqrt[x*(b + c*x)]*Hypergeometric2F1[1/2
, 1/2 - m, 3/2, 1 + (c*x)/b])/b
```

Maple [F] time = 0.4, size = 0, normalized size = 0.

$$\int (dx)^m \frac{1}{\sqrt{cx^2+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(c*x^2+b*x)^(1/2),x)

[Out] int((d*x)^m/(c*x^2+b*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{cx^2+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^2+b*x)^(1/2),x, algorithm="maxima")

[Out] integrate((d*x)^m/sqrt(c*x^2 + b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(dx)^m}{\sqrt{cx^2+bx}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/sqrt(c*x^2 + b*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{x(b+cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral((d*x)**m/sqrt(x*(b + c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{cx^2+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/sqrt(c*x^2 + b*x), x)
```

3.122 $\int \frac{(dx)^m}{\left(bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$\frac{2x(b+cx)(dx)^m\left(-\frac{cx}{b}\right)^{\frac{1}{2}-m}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{1}{2};\frac{cx}{b}+1\right)}{b\left(bx+cx^2\right)^{3/2}}$$

```
[Out] (2*x*(-((c*x)/b))^(1/2 - m)*(d*x)^m*(b + c*x)*Hypergeometric2F1[-1/2, 3/2 -
 m, 1/2, 1 + (c*x)/b])/(b*(b*x + c*x^2)^(3/2))
```

Rubi [A] time = 0.0265839, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {674, 67, 65}

$$\frac{2x(b+cx)(dx)^m\left(-\frac{cx}{b}\right)^{\frac{1}{2}-m}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{1}{2};\frac{cx}{b}+1\right)}{b\left(bx+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*x*(-((c*x)/b))^(1/2 - m)*(d*x)^m*(b + c*x)*Hypergeometric2F1[-1/2, 3/2 -
 m, 1/2, 1 + (c*x)/b])/(b*(b*x + c*x^2)^(3/2))
```

Rule 674

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(
(e*x)^m*(b*x + c*x^2)^p)/(x^(m + p)*(b + c*x)^p), Int[x^(m + p)*(b + c*x)^p
, x], x] /; FreeQ[{b, c, e, m}, x] && !IntegerQ[p]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{(dx)^m}{\left(bx+cx^2\right)^{3/2}}\,dx = \frac{\left(x^{\frac{3}{2}-m}(dx)^m(b+cx)^{3/2}\right)\int \frac{x^{-\frac{3}{2}+m}}{(b+cx)^{3/2}}\,dx}{\left(bx+cx^2\right)^{3/2}}$$

$$= -\frac{\left(cx\left(-\frac{cx}{b}\right)^{\frac{1}{2}-m}(dx)^m(b+cx)^{3/2}\right)\int \frac{\left(-\frac{cx}{b}\right)^{-\frac{3}{2}+m}}{(b+cx)^{3/2}}\,dx}{b\left(bx+cx^2\right)^{3/2}}$$

$$= \frac{2x\left(-\frac{cx}{b}\right)^{\frac{1}{2}-m}(dx)^m(b+cx)\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{1}{2};1+\frac{cx}{b}\right)}{b\left(bx+cx^2\right)^{3/2}}$$

Mathematica [A] time = 0.0931142, size = 58, normalized size = 0.88

$$\frac{2(dx)^m\left(-\frac{cx}{b}\right)^{\frac{1}{2}-m}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{1}{2};\frac{cx}{b}+1\right)}{b\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(-((c*x)/b))^(1/2 - m)*(d*x)^m*Hypergeometric2F1[-1/2, 3/2 - m, 1/2, 1 +
 (c*x)/b])/(b*Sqrt[x*(b + c*x)])
```

Maple [F] time = 0.438, size = 0, normalized size = 0.

$$\int (dx)^m\left(cx^2+bx\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(c*x^2+b*x)^(3/2),x)
```

```
[Out] int((d*x)^m/(c*x^2+b*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(cx^2+bx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(c*x^2 + b*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+bx}\,(dx)^m}{c^2x^4+2\,bcx^3+b^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x)*(d*x)^m/(c^2*x^4 + 2*b*c*x^3 + b^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(x\,(b+cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral((d*x)**m/(x*(b + c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(cx^2+bx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^2 + b*x)^(3/2), x)
```

3.123 $\int \frac{(dx)^m}{\left(bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=71

$$-\frac{2cx^2(b+cx)(dx)^m\left(-\frac{cx}{b}\right)^{\frac{1}{2}-m}\,{}_2F_1\left(-\frac{3}{2},\frac{5}{2}-m;-\frac{1}{2};\frac{cx}{b}+1\right)}{3b^2\left(bx+cx^2\right)^{5/2}}$$

```
[Out] (-2*c*x^2*(-((c*x)/b))^(1/2 - m)*(d*x)^m*(b + c*x)*Hypergeometric2F1[-3/2,
5/2 - m, -1/2, 1 + (c*x)/b])/(3*b^2*(b*x + c*x^2)^(5/2))
```

Rubi [A] time = 0.0281655, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {674, 67, 65}

$$-\frac{2cx^2(b+cx)(dx)^m\left(-\frac{cx}{b}\right)^{\frac{1}{2}-m}\,{}_2F_1\left(-\frac{3}{2},\frac{5}{2}-m;-\frac{1}{2};\frac{cx}{b}+1\right)}{3b^2\left(bx+cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*c*x^2*(-((c*x)/b))^(1/2 - m)*(d*x)^m*(b + c*x)*Hypergeometric2F1[-3/2,
5/2 - m, -1/2, 1 + (c*x)/b])/(3*b^2*(b*x + c*x^2)^(5/2))
```

Rule 674

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(
(e*x)^m*(b*x + c*x^2)^p)/(x^(m + p)*(b + c*x)^p), Int[x^(m + p)*(b + c*x)^p
, x], x] /; FreeQ[{b, c, e, m}, x] && !IntegerQ[p]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{(dx)^m}{\left(bx+cx^2\right)^{5/2}}\,dx = \frac{\left(x^{\frac{5}{2}-m}(dx)^m(b+cx)^{5/2}\right)\int \frac{x^{-\frac{5}{2}+m}}{(b+cx)^{5/2}}\,dx}{\left(bx+cx^2\right)^{5/2}}$$
$$= \frac{\left(c^2x^2\left(-\frac{cx}{b}\right)^{\frac{1}{2}-m}(dx)^m(b+cx)^{5/2}\right)\int \frac{\left(-\frac{cx}{b}\right)^{-\frac{5}{2}+m}}{(b+cx)^{5/2}}\,dx}{b^2\left(bx+cx^2\right)^{5/2}}$$
$$= -\frac{2cx^2\left(-\frac{cx}{b}\right)^{\frac{1}{2}-m}(dx)^m(b+cx)\,{}_2F_1\left(-\frac{3}{2},\frac{5}{2}-m;-\frac{1}{2};1+\frac{cx}{b}\right)}{3b^2\left(bx+cx^2\right)^{5/2}}$$

Mathematica [A] time = 0.108664, size = 60, normalized size = 0.85

$$\frac{2(dx)^m\left(-\frac{cx}{b}\right)^{\frac{3}{2}-m}\,{}_2F_1\left(-\frac{3}{2},\frac{5}{2}-m;-\frac{1}{2};\frac{cx}{b}+1\right)}{3b(x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(-((c*x)/b))^(3/2 - m)*(d*x)^m*Hypergeometric2F1[-3/2, 5/2 - m, -1/2, 1
+ (c*x)/b])/(3*b*(x*(b + c*x))^(3/2))
```

Maple [F] time = 0.425, size = 0, normalized size = 0.

$$\int (dx)^m\left(cx^2+bx\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(c*x^2+b*x)^(5/2),x)
```

```
[Out] int((d*x)^m/(c*x^2+b*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(cx^2+bx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(c*x^2 + b*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+bx}\,(dx)^m}{c^3x^6+3\,bc^2x^5+3\,b^2cx^4+b^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x)*(d*x)^m/(c^3*x^6 + 3*b*c^2*x^5 + 3*b^2*c*x^4 + b
^3*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(x\,(b+cx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(c*x**2+b*x)**(5/2),x)
```

```
[Out] Integral((d*x)**m/(x*(b + c*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(cx^2+bx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^2 + b*x)^(5/2), x)
```

3.124 $\int (dx)^m \left(bx+cx^2\right)^p \,dx$

Optimal. Leaf size=55

$$\frac{x(dx)^m \left(\frac{cx}{b}+1\right)^{-p} \left(bx+cx^2\right)^p {}_2F_1\left(-p,m+p+1;m+p+2;-\frac{cx}{b}\right)}{m+p+1}$$

```
[Out] (x*(d*x)^m*(b*x + c*x^2)^p*Hypergeometric2F1[-p, 1 + m + p, 2 + m + p, -((c
*x)/b)])/((1 + m + p)*(1 + (c*x)/b)^p)
```

Rubi [A] time = 0.0232367, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {674, 66, 64}

$$\frac{x(dx)^m \left(\frac{cx}{b}+1\right)^{-p} \left(bx+cx^2\right)^p {}_2F_1\left(-p,m+p+1;m+p+2;-\frac{cx}{b}\right)}{m+p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(b*x + c*x^2)^p,x]
```

```
[Out] (x*(d*x)^m*(b*x + c*x^2)^p*Hypergeometric2F1[-p, 1 + m + p, 2 + m + p, -((c
*x)/b)])/((1 + m + p)*(1 + (c*x)/b)^p)
```

Rule 674

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(
(e*x)^m*(b*x + c*x^2)^p)/(x^(m + p)*(b + c*x)^p), Int[x^(m + p)*(b + c*x)^p
, x], x] /; FreeQ[{b, c, e, m}, x] && !IntegerQ[p]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}
\int (dx)^m \left(bx+cx^2\right)^p \,dx &= \left(x^{-m-p}(dx)^m(b+cx)^{-p} \left(bx+cx^2\right)^p\right) \int x^{m+p}(b+cx)^p \,dx \\
&= \left(x^{-m-p}(dx)^m \left(1+\frac{cx}{b}\right)^{-p} \left(bx+cx^2\right)^p\right) \int x^{m+p} \left(1+\frac{cx}{b}\right)^p \,dx \\
&= \frac{x(dx)^m \left(1+\frac{cx}{b}\right)^{-p} \left(bx+cx^2\right)^p {}_2F_1\left(-p,1+m+p;2+m+p;-\frac{cx}{b}\right)}{1+m+p}
\end{aligned}$$

Mathematica [A] time = 0.0133406, size = 53, normalized size = 0.96

$$\frac{x(dx)^m(x(b+cx))^p\left(\frac{cx}{b}+1\right)^{-p}{}_2F_1\left(-p,m+p+1;m+p+2;-\frac{cx}{b}\right)}{m+p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(b*x + c*x^2)^p,x]
```

```
[Out] (x*(d*x)^m*(x*(b + c*x))^p*Hypergeometric2F1[-p, 1 + m + p, 2 + m + p, -((c
*x)/b)])/((1 + m + p)*(1 + (c*x)/b)^p)
```

Maple [F] time = 0.479, size = 0, normalized size = 0.

$$\int (dx)^m \left(cx^2+bx\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2+b*x)^p,x)
```

```
[Out] int((d*x)^m*(c*x^2+b*x)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^p (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2+b*x)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^p*(d*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2+bx\right)^p (dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2+b*x)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^p*(d*x)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m (x(b+cx))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2+b*x)**p,x)
```

```
[Out] Integral((d*x)**m*(x*(b + c*x))**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx\right)^p (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2+b*x)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^p*(d*x)^m, x)
```

3.125 $\int x^3 \left(bx+cx^2\right)^p \, dx$

Optimal. Leaf size=49

$$\frac{x^4 \left(\frac{cx}{b}+1\right)^{-p} \left(bx+cx^2\right)^p {}_2F_1\left(-p,p+4;p+5;-\frac{cx}{b}\right)}{p+4}$$

```
[Out] (x^4*(b*x + c*x^2)^p*Hypergeometric2F1[-p, 4 + p, 5 + p, -((c*x)/b)])/((4 +
 p)*(1 + (c*x)/b)^p)
```

Rubi [A] time = 0.0204716, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {674, 66, 64}

$$\frac{x^4 \left(\frac{cx}{b}+1\right)^{-p} \left(bx+cx^2\right)^p {}_2F_1\left(-p,p+4;p+5;-\frac{cx}{b}\right)}{p+4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(b*x + c*x^2)^p,x]
```

```
[Out] (x^4*(b*x + c*x^2)^p*Hypergeometric2F1[-p, 4 + p, 5 + p, -((c*x)/b)])/((4 +
 p)*(1 + (c*x)/b)^p)
```

Rule 674

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(
(e*x)^m*(b*x + c*x^2)^p)/(x^(m + p)*(b + c*x)^p), Int[x^(m + p)*(b + c*x)^p
, x], x] /; FreeQ[{b, c, e, m}, x] && !IntegerQ[p]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(bx+cx^2\right)^p \, dx &= \left(x^{-p}(b+cx)^{-p} \left(bx+cx^2\right)^p\right) \int x^{3+p}(b+cx)^p \, dx \\
&= \left(x^{-p} \left(1+\frac{cx}{b}\right)^{-p} \left(bx+cx^2\right)^p\right) \int x^{3+p} \left(1+\frac{cx}{b}\right)^p \, dx \\
&= \frac{x^4 \left(1+\frac{cx}{b}\right)^{-p} \left(bx+cx^2\right)^p {}_2F_1\left(-p,4+p;5+p;-\frac{cx}{b}\right)}{4+p}
\end{aligned}$$

Mathematica [A] time = 0.0076904, size = 47, normalized size = 0.96

$$\frac{x^4(x(b+cx))^p\left(\frac{cx}{b}+1\right)^{-p}{}_2F_1\left(-p,p+4;p+5;-\frac{cx}{b}\right)}{p+4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(b*x + c*x^2)^p,x]
```

```
[Out] (x^4*(x*(b + c*x))^p*Hypergeometric2F1[-p, 4 + p, 5 + p, -((c*x)/b)])/((4 +
 p)*(1 + (c*x)/b)^p)
```

Maple [F] time = 0.366, size = 0, normalized size = 0.

$$\int x^3\left(cx^2+bx\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2+b*x)^p,x)
```

```
[Out] int(x^3*(c*x^2+b*x)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^p x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+b*x)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^p*x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(cx^2+bx\right)^p x^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+b*x)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^p*x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\left(x\left(b+cx\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2+b*x)**p,x)
```

```
[Out] Integral(x**3*(x*(b + c*x))**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx\right)^p x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+b*x)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^p*x^3, x)
```

3.126 $\int x^2 \left(bx+cx^2\right)^p \, dx$

Optimal. Leaf size=49

$$\frac{x^3\left(\frac{cx}{b}+1\right)^{-p}\left(bx+cx^2\right)^p {}_2F_1\left(-p,p+3;p+4;-\frac{cx}{b}\right)}{p+3}$$

```
[Out] (x^3*(b*x + c*x^2)^p*Hypergeometric2F1[-p, 3 + p, 4 + p, -((c*x)/b)])/((3 +
 p)*(1 + (c*x)/b)^p)
```

Rubi [A] time = 0.0203876, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {674, 66, 64}

$$\frac{x^3\left(\frac{cx}{b}+1\right)^{-p}\left(bx+cx^2\right)^p {}_2F_1\left(-p,p+3;p+4;-\frac{cx}{b}\right)}{p+3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b*x + c*x^2)^p,x]
```

```
[Out] (x^3*(b*x + c*x^2)^p*Hypergeometric2F1[-p, 3 + p, 4 + p, -((c*x)/b)])/((3 +
 p)*(1 + (c*x)/b)^p)
```

Rule 674

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(
(e*x)^m*(b*x + c*x^2)^p)/(x^(m + p)*(b + c*x)^p), Int[x^(m + p)*(b + c*x)^p
, x], x] /; FreeQ[{b, c, e, m}, x] && !IntegerQ[p]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}
\int x^2\left(bx+cx^2\right)^p dx &= \left(x^{-p}(b+cx)^{-p}\left(bx+cx^2\right)^p\right)\int x^{2+p}(b+cx)^p\,dx\\
&= \left(x^{-p}\left(1+\frac{cx}{b}\right)^{-p}\left(bx+cx^2\right)^p\right)\int x^{2+p}\left(1+\frac{cx}{b}\right)^p dx\\
&= \frac{x^3\left(1+\frac{cx}{b}\right)^{-p}\left(bx+cx^2\right)^p {}_2F_1\left(-p,3+p;4+p;-\frac{cx}{b}\right)}{3+p}
\end{aligned}$$

Mathematica [A] time = 0.0089366, size = 47, normalized size = 0.96

$$\frac{x^3(x(b+cx))^p\left(\frac{cx}{b}+1\right)^{-p}{}_2F_1\left(-p,p+3;p+4;-\frac{cx}{b}\right)}{p+3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b*x + c*x^2)^p,x]
```

```
[Out] (x^3*(x*(b + c*x))^p*Hypergeometric2F1[-p, 3 + p, 4 + p, -((c*x)/b)])/((3 +
 p)*(1 + (c*x)/b)^p)
```

Maple [F] time = 0.371, size = 0, normalized size = 0.

$$\int x^2\left(cx^2+bx\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2+b*x)^p,x)
```

```
[Out] int(x^2*(c*x^2+b*x)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^2+bx\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^p*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(cx^2+bx\right)^p x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^p*x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(x\left(b+cx\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2+b*x)**p,x)
```

```
[Out] Integral(x**2*(x*(b + c*x))**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx\right)^p x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^p*x^2, x)
```

3.127 $\int x\left(bx+cx^2\right)^p\,dx$

Optimal. Leaf size=49

$$\frac{x^2\left(\frac{cx}{b}+1\right)^{-p}\left(bx+cx^2\right)^p\,{}_2F_1\left(-p,p+2;p+3;-\frac{cx}{b}\right)}{p+2}$$

```
[Out] (x^2*(b*x + c*x^2)^p*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x)/b)])/((2 +
 p)*(1 + (c*x)/b)^p)
```

Rubi [A] time = 0.02233, antiderivative size = 83, normalized size of antiderivative = 1.69, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {640, 624}

$$\frac{\left(bx+cx^2\right)^{p+1}\left(-\frac{cx}{b}\right)^{-p-1}\,{}_2F_1\left(-p,p+1;p+2;\frac{b+cx}{b}\right)}{2c(p+1)}+\frac{\left(bx+cx^2\right)^{p+1}}{2c(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b*x + c*x^2)^p,x]
```

```
[Out] (b*x + c*x^2)^(1 + p)/(2*c*(1 + p)) + ((-((c*x)/b))^(-1 - p)*(b*x + c*x^2)^
(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + c*x)/b])/(2*c*(1 + p))
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rubi steps

$$\begin{aligned}\int x\left(bx+cx^2\right)^p\,dx&=\frac{\left(bx+cx^2\right)^{1+p}}{2c(1+p)}-\frac{b\int\left(bx+cx^2\right)^p\,dx}{2c}\\&=\frac{\left(bx+cx^2\right)^{1+p}}{2c(1+p)}+\frac{\left(-\frac{cx}{b}\right)^{-1-p}\left(bx+cx^2\right)^{1+p}\,{}_2F_1\left(-p,1+p;2+p;\frac{b+cx}{b}\right)}{2c(1+p)}\end{aligned}$$

Mathematica [A] time = 0.007747, size = 47, normalized size = 0.96

$$\frac{x^2(x(b+cx))^p\left(\frac{cx}{b}+1\right)^{-p}\,{}_2F_1\left(-p,p+2;p+3;-\frac{cx}{b}\right)}{p+2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b*x + c*x^2)^p,x]
```

```
[Out] (x^2*(x*(b + c*x))^p*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x)/b)])/((2 +
 p)*(1 + (c*x)/b)^p)
```

Maple [F] time = 0.413, size = 0, normalized size = 0.

$$\int x\left(cx^2+bx\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x)^p,x)
```

```
[Out] int(x*(c*x^2+b*x)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^p*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(cx^2+bx\right)^p x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^p*x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(x\left(b+cx\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x)**p,x)
```

```
[Out] Integral(x*(x*(b + c*x))**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^p*x, x)
```

3.128 $\int \frac{(bx+cx^2)^p}{x}\,dx$

Optimal. Leaf size=42

$$\frac{\left(\frac{cx}{b}+1\right)^{-p}\left(bx+cx^2\right)^p {}_2F_1\left(-p,p;p+1;-\frac{cx}{b}\right)}{p}$$

```
[Out] ((b*x + c*x^2)^p*Hypergeometric2F1[-p, p, 1 + p, -((c*x)/b)])/(p*(1 + (c*x)
/b)^p)
```

Rubi [A] time = 0.0188719, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {674, 66, 64}

$$\frac{\left(\frac{cx}{b}+1\right)^{-p}\left(bx+cx^2\right)^p {}_2F_1\left(-p,p;p+1;-\frac{cx}{b}\right)}{p}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^p/x,x]
```

```
[Out] ((b*x + c*x^2)^p*Hypergeometric2F1[-p, p, 1 + p, -((c*x)/b)])/(p*(1 + (c*x)
/b)^p)
```

Rule 674

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(
(e*x)^m*(b*x + c*x^2)^p)/(x^(m + p)*(b + c*x)^p), Int[x^(m + p)*(b + c*x)^p
, x], x] /; FreeQ[{b, c, e, m}, x] && !IntegerQ[p]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(bx+cx^2\right)^p}{x}\,dx &= \left(x^{-p}(b+cx)^{-p}\left(bx+cx^2\right)^p\right)\int x^{-1+p}(b+cx)^p\,dx\\
&= \left(x^{-p}\left(1+\frac{cx}{b}\right)^{-p}\left(bx+cx^2\right)^p\right)\int x^{-1+p}\left(1+\frac{cx}{b}\right)^p\,dx\\
&= \frac{\left(1+\frac{cx}{b}\right)^{-p}\left(bx+cx^2\right)^p {}_2F_1\left(-p,p;1+p;-\frac{cx}{b}\right)}{p}
\end{aligned}$$

Mathematica [A] time = 0.0059112, size = 40, normalized size = 0.95

$$\frac{(x(b+cx))^p \left(\frac{cx}{b}+1\right)^{-p} {}_2F_1\left(-p,p;p+1;-\frac{cx}{b}\right)}{p}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^p/x,x]
```

```
[Out] ((x*(b + c*x))^p*Hypergeometric2F1[-p, p, 1 + p, -((c*x)/b)])/(p*(1 + (c*x)
/b)^p)
```

Maple [F] time = 0.332, size = 0, normalized size = 0.

$$\int \frac{(cx^2+bx)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^p/x,x)
```

```
[Out] int((c*x^2+b*x)^p/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2+bx)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^p/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(cx^2+bx)^p}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^p/x,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^p/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x(b+cx))^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**p/x,x)
```

```
[Out] Integral((x*(b + c*x))**p/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^p/x,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^p/x, x)
```

3.129 $\int \frac{(bx+cx^2)^p}{x^2} \, dx$

Optimal. Leaf size=50

$$-\frac{\left(\frac{cx}{b}+1\right)^{-p}\left(bx+cx^2\right)^p {}_2F_1\left(p-1,-p;p;-\frac{cx}{b}\right)}{(1-p)x}$$

```
[Out] -(((b*x + c*x^2)^p*Hypergeometric2F1[-1 + p, -p, p, -((c*x)/b)])/((1 - p)*x
*(1 + (c*x)/b)^p))
```

Rubi [A] time = 0.0218345, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {674, 66, 64}

$$-\frac{\left(\frac{cx}{b}+1\right)^{-p}\left(bx+cx^2\right)^p {}_2F_1\left(p-1,-p;p;-\frac{cx}{b}\right)}{(1-p)x}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^p/x^2,x]
```

```
[Out] -(((b*x + c*x^2)^p*Hypergeometric2F1[-1 + p, -p, p, -((c*x)/b)])/((1 - p)*x
*(1 + (c*x)/b)^p))
```

Rule 674

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(
(e*x)^m*(b*x + c*x^2)^p)/(x^(m + p)*(b + c*x)^p), Int[x^(m + p)*(b + c*x)^p
, x], x] /; FreeQ[{b, c, e, m}, x] && !IntegerQ[p]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(bx+cx^2\right)^p}{x^2} \, dx &= \left(x^{-p}(b+cx)^{-p}\left(bx+cx^2\right)^p\right) \int x^{-2+p}(b+cx)^p \, dx \\
&= \left(x^{-p}\left(1+\frac{cx}{b}\right)^{-p}\left(bx+cx^2\right)^p\right) \int x^{-2+p}\left(1+\frac{cx}{b}\right)^p \, dx \\
&= -\frac{\left(1+\frac{cx}{b}\right)^{-p}\left(bx+cx^2\right)^p {}_2F_1\left(-1+p,-p;p;-\frac{cx}{b}\right)}{(1-p)x}
\end{aligned}$$

Mathematica [A] time = 0.0077625, size = 45, normalized size = 0.9

$$\frac{(x(b+cx))^p\left(\frac{cx}{b}+1\right)^{-p}{}_2F_1\left(p-1,-p;p;-\frac{cx}{b}\right)}{(p-1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^p/x^2,x]
```

```
[Out] ((x*(b + c*x))^p*Hypergeometric2F1[-1 + p, -p, p, -((c*x)/b)])/((-1 + p)*x*
(1 + (c*x)/b)^p)
```

Maple [F] time = 0.374, size = 0, normalized size = 0.

$$\int \frac{(cx^2+bx)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^p/x^2,x)
```

```
[Out] int((c*x^2+b*x)^p/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2+bx)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^p/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(cx^2+bx)^p}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^p/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x(b+cx))^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**p/x**2,x)
```

```
[Out] Integral((x*(b + c*x))**p/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx)^p}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^p/x^2, x)
```

3.130 $\int \frac{(bx+cx^2)^p}{x^3}\,dx$

Optimal. Leaf size=52

$$-\frac{\left(\frac{cx}{b}+1\right)^{-p}\left(bx+cx^2\right)^p\,{}_2F_1\left(p-2,-p;p-1;-\frac{cx}{b}\right)}{(2-p)x^2}$$

```
[Out] -(((b*x + c*x^2)^p*Hypergeometric2F1[-2 + p, -p, -1 + p, -((c*x)/b)])/((2 -
 p)*x^2*(1 + (c*x)/b)^p))
```

Rubi [A] time = 0.0223019, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {674, 66, 64}

$$-\frac{\left(\frac{cx}{b}+1\right)^{-p}\left(bx+cx^2\right)^p\,{}_2F_1\left(p-2,-p;p-1;-\frac{cx}{b}\right)}{(2-p)x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^p/x^3,x]
```

```
[Out] -(((b*x + c*x^2)^p*Hypergeometric2F1[-2 + p, -p, -1 + p, -((c*x)/b)])/((2 -
 p)*x^2*(1 + (c*x)/b)^p))
```

Rule 674

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(
(e*x)^m*(b*x + c*x^2)^p)/(x^(m + p)*(b + c*x)^p), Int[x^(m + p)*(b + c*x)^p
, x], x] /; FreeQ[{b, c, e, m}, x] &&  !IntegerQ[p]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ
[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] &&  !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(bx+cx^2\right)^p}{x^3}\,dx &= \left(x^{-p}(b+cx)^{-p}\left(bx+cx^2\right)^p\right)\int x^{-3+p}(b+cx)^p\,dx\\
&= \left(x^{-p}\left(1+\frac{cx}{b}\right)^{-p}\left(bx+cx^2\right)^p\right)\int x^{-3+p}\left(1+\frac{cx}{b}\right)^p\,dx\\
&= -\frac{\left(1+\frac{cx}{b}\right)^{-p}\left(bx+cx^2\right)^p\,{}_2F_1\left(-2+p,-p;-1+p;-\frac{cx}{b}\right)}{(2-p)x^2}
\end{aligned}$$

Mathematica [A] time = 0.0081432, size = 47, normalized size = 0.9

$$\frac{(x(b+cx))^p\left(\frac{cx}{b}+1\right)^{-p}{}_2F_1\left(p-2,-p;p-1;-\frac{cx}{b}\right)}{(p-2)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^p/x^3,x]
```

```
[Out] ((x*(b + c*x))^p*Hypergeometric2F1[-2 + p, -p, -1 + p, -((c*x)/b)])/((-2 +
p)*x^2*(1 + (c*x)/b)^p)
```

Maple [F] time = 0.369, size = 0, normalized size = 0.

$$\int\frac{\left(cx^2+bx\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^p/x^3,x)
```

```
[Out] int((c*x^2+b*x)^p/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^2+bx\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^p/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+bx\right)^p}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^p/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(x(b+cx))^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**p/x**3,x)
```

```
[Out] Integral((x*(b + c*x))**p/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx)^p}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^p/x^3, x)
```

3.131 $\int (dx)^{5/2} \left(bx+cx^2\right)^p \, dx$

Optimal. Leaf size=61

$$\frac{2x(dx)^{5/2}\left(\frac{cx}{b}+1\right)^{-p}\left(bx+cx^2\right)^p {}_2F_1\left(-p,p+\frac{7}{2};p+\frac{9}{2};-\frac{cx}{b}\right)}{2p+7}$$

```
[Out] (2*x*(d*x)^(5/2)*(b*x + c*x^2)^p*Hypergeometric2F1[-p, 7/2 + p, 9/2 + p, -(
(c*x)/b)])/((7 + 2*p)*(1 + (c*x)/b)^p)
```

Rubi [A] time = 0.0246532, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {674, 66, 64}

$$\frac{2x(dx)^{5/2}\left(\frac{cx}{b}+1\right)^{-p}\left(bx+cx^2\right)^p {}_2F_1\left(-p,p+\frac{7}{2};p+\frac{9}{2};-\frac{cx}{b}\right)}{2p+7}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*(b*x + c*x^2)^p,x]
```

```
[Out] (2*x*(d*x)^(5/2)*(b*x + c*x^2)^p*Hypergeometric2F1[-p, 7/2 + p, 9/2 + p, -(
(c*x)/b)])/((7 + 2*p)*(1 + (c*x)/b)^p)
```

Rule 674

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(
(e*x)^m*(b*x + c*x^2)^p)/(x^(m + p)*(b + c*x)^p), Int[x^(m + p)*(b + c*x)^p
, x], x] /; FreeQ[{b, c, e, m}, x] && !IntegerQ[p]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}
\int (dx)^{5/2}\left(bx+cx^2\right)^p \, dx &= \left(x^{-\frac{5}{2}-p}(dx)^{5/2}(b+cx)^{-p}\left(bx+cx^2\right)^p\right)\int x^{\frac{5}{2}+p}(b+cx)^p \, dx\\
&= \left(x^{-\frac{5}{2}-p}(dx)^{5/2}\left(1+\frac{cx}{b}\right)^{-p}\left(bx+cx^2\right)^p\right)\int x^{\frac{5}{2}+p}\left(1+\frac{cx}{b}\right)^p \, dx\\
&= \frac{2x(dx)^{5/2}\left(1+\frac{cx}{b}\right)^{-p}\left(bx+cx^2\right)^p {}_2F_1\left(-p,\frac{7}{2}+p;\frac{9}{2}+p;-\frac{cx}{b}\right)}{7+2p}
\end{aligned}$$

Mathematica [A] time = 0.0135288, size = 58, normalized size = 0.95

$$\frac{x(dx)^{5/2}(x(b+cx))^p\left(\frac{cx}{b}+1\right)^{-p}\,{}_2F_1\left(-p,p+\frac{7}{2};p+\frac{9}{2};-\frac{cx}{b}\right)}{p+\frac{7}{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*(b*x + c*x^2)^p,x]
```

```
[Out] (x*(d*x)^(5/2)*(x*(b + c*x))^p*Hypergeometric2F1[-p, 7/2 + p, 9/2 + p, -((c
*x)/b)])/((7/2 + p)*(1 + (c*x)/b)^p)
```

Maple [F] time = 0.362, size = 0, normalized size = 0.

$$\int (dx)^{\frac{5}{2}}\left(cx^2+bx\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(c*x^2+b*x)^p,x)
```

```
[Out] int((d*x)^(5/2)*(c*x^2+b*x)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{5}{2}}\left(cx^2+bx\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(c*x^2+b*x)^p,x, algorithm="maxima")
```

```
[Out] integrate((d*x)^(5/2)*(c*x^2 + b*x)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{dx}\left(cx^2+bx\right)^p d^2x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(c*x^2+b*x)^p,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*(c*x^2 + b*x)^p*d^2*x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(c*x**2+b*x)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{5}{2}} \left(cx^2 + bx\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(c*x^2+b*x)^p,x, algorithm="giac")
```

```
[Out] integrate((d*x)^(5/2)*(c*x^2 + b*x)^p, x)
```

3.132 $\int (dx)^{3/2} \left(bx+cx^2\right)^p \, dx$

Optimal. Leaf size=61

$$\frac{2x(dx)^{3/2} \left(\frac{cx}{b}+1\right)^{-p} \left(bx+cx^2\right)^p \, _2F_1\left(-p,p+\frac{5}{2};p+\frac{7}{2};-\frac{cx}{b}\right)}{2p+5}$$

```
[Out] (2*x*(d*x)^(3/2)*(b*x + c*x^2)^p*Hypergeometric2F1[-p, 5/2 + p, 7/2 + p, -(
(c*x)/b)])/((5 + 2*p)*(1 + (c*x)/b)^p)
```

Rubi [A] time = 0.0243567, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {674, 66, 64}

$$\frac{2x(dx)^{3/2} \left(\frac{cx}{b}+1\right)^{-p} \left(bx+cx^2\right)^p \, _2F_1\left(-p,p+\frac{5}{2};p+\frac{7}{2};-\frac{cx}{b}\right)}{2p+5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(b*x + c*x^2)^p,x]
```

```
[Out] (2*x*(d*x)^(3/2)*(b*x + c*x^2)^p*Hypergeometric2F1[-p, 5/2 + p, 7/2 + p, -(
(c*x)/b)])/((5 + 2*p)*(1 + (c*x)/b)^p)
```

Rule 674

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(
(e*x)^m*(b*x + c*x^2)^p)/(x^(m + p)*(b + c*x)^p), Int[x^(m + p)*(b + c*x)^p
, x], x] /; FreeQ[{b, c, e, m}, x] &&  !IntegerQ[p]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ
[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] &&  !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}
\int (dx)^{3/2} \left(bx+cx^2\right)^p \, dx &= \left(x^{-\frac{3}{2}-p}(dx)^{3/2}(b+cx)^{-p} \left(bx+cx^2\right)^p\right) \int x^{\frac{3}{2}+p}(b+cx)^p \, dx \\
&= \left(x^{-\frac{3}{2}-p}(dx)^{3/2} \left(1+\frac{cx}{b}\right)^{-p} \left(bx+cx^2\right)^p\right) \int x^{\frac{3}{2}+p} \left(1+\frac{cx}{b}\right)^p \, dx \\
&= \frac{2x(dx)^{3/2} \left(1+\frac{cx}{b}\right)^{-p} \left(bx+cx^2\right)^p \, _2F_1\left(-p,\frac{5}{2}+p;\frac{7}{2}+p;-\frac{cx}{b}\right)}{5+2p}
\end{aligned}$$

Mathematica [A] time = 0.0125277, size = 58, normalized size = 0.95

$$\frac{x(dx)^{3/2}(x(b+cx))^p\left(\frac{cx}{b}+1\right)^{-p}\,{}_2F_1\left(-p,p+\frac{5}{2};p+\frac{7}{2};-\frac{cx}{b}\right)}{p+\frac{5}{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(b*x + c*x^2)^p,x]
```

```
[Out] (x*(d*x)^(3/2)*(x*(b + c*x))^p*Hypergeometric2F1[-p, 5/2 + p, 7/2 + p, -((c
*x)/b)])/((5/2 + p)*(1 + (c*x)/b)^p)
```

Maple [F] time = 0.342, size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}}\left(cx^2+bx\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(c*x^2+b*x)^p,x)
```

```
[Out] int((d*x)^(3/2)*(c*x^2+b*x)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}}\left(cx^2+bx\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(c*x^2+b*x)^p,x, algorithm="maxima")
```

```
[Out] integrate((d*x)^(3/2)*(c*x^2 + b*x)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{dx}\left(cx^2+bx\right)^p dx, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(c*x^2+b*x)^p,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*(c*x^2 + b*x)^p*d*x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}}\left(x\left(b+cx\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(c*x**2+b*x)**p,x)
```

```
[Out] Integral((d*x)**(3/2)*(x*(b + c*x))**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}} \left(cx^2 + bx\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(c*x^2+b*x)^p,x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)*(c*x^2 + b*x)^p, x)
```

3.133 $\int \sqrt{dx}\left(bx+cx^2\right)^p\,dx$

Optimal. Leaf size=61

$$\frac{2x\sqrt{dx}\left(\frac{cx}{b}+1\right)^{-p}\left(bx+cx^2\right)^p\,{}_2F_1\left(-p,p+\frac{3}{2};p+\frac{5}{2};-\frac{cx}{b}\right)}{2p+3}$$

```
[Out] (2*x*Sqrt[d*x]*(b*x + c*x^2)^p*Hypergeometric2F1[-p, 3/2 + p, 5/2 + p, -((c
*x)/b)])/((3 + 2*p)*(1 + (c*x)/b)^p)
```

Rubi [A] time = 0.0236347, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {674, 66, 64}

$$\frac{2x\sqrt{dx}\left(\frac{cx}{b}+1\right)^{-p}\left(bx+cx^2\right)^p\,{}_2F_1\left(-p,p+\frac{3}{2};p+\frac{5}{2};-\frac{cx}{b}\right)}{2p+3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(b*x + c*x^2)^p,x]
```

```
[Out] (2*x*Sqrt[d*x]*(b*x + c*x^2)^p*Hypergeometric2F1[-p, 3/2 + p, 5/2 + p, -((c
*x)/b)])/((3 + 2*p)*(1 + (c*x)/b)^p)
```

Rule 674

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(
(e*x)^m*(b*x + c*x^2)^p)/(x^(m + p)*(b + c*x)^p), Int[x^(m + p)*(b + c*x)^p
, x], x] /; FreeQ[{b, c, e, m}, x] &&  !IntegerQ[p]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ
[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] &&  !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}
\int \sqrt{dx}\left(bx+cx^2\right)^p\,dx &= \left(x^{-\frac{1}{2}-p}\sqrt{dx}(b+cx)^{-p}\left(bx+cx^2\right)^p\right)\int x^{\frac{1}{2}+p}(b+cx)^p\,dx\\
&= \left(x^{-\frac{1}{2}-p}\sqrt{dx}\left(1+\frac{cx}{b}\right)^{-p}\left(bx+cx^2\right)^p\right)\int x^{\frac{1}{2}+p}\left(1+\frac{cx}{b}\right)^p\,dx\\
&= \frac{2x\sqrt{dx}\left(1+\frac{cx}{b}\right)^{-p}\left(bx+cx^2\right)^p\,{}_2F_1\left(-p,\frac{3}{2}+p;\frac{5}{2}+p;-\frac{cx}{b}\right)}{3+2p}
\end{aligned}$$

Mathematica [A] time = 0.0104214, size = 58, normalized size = 0.95

$$\frac{x\sqrt{dx}(x(b+cx))^p\left(\frac{cx}{b}+1\right)^{-p}{}_2F_1\left(-p,p+\frac{3}{2};p+\frac{5}{2};-\frac{cx}{b}\right)}{p+\frac{3}{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(b*x + c*x^2)^p,x]
```

```
[Out] (x*Sqrt[d*x]*(x*(b + c*x))^p*Hypergeometric2F1[-p, 3/2 + p, 5/2 + p, -((c*x
)/b)])/((3/2 + p)*(1 + (c*x)/b)^p)
```

Maple [F] time = 0.355, size = 0, normalized size = 0.

$$\int \sqrt{dx}\left(cx^2+bx\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(c*x^2+b*x)^p,x)
```

```
[Out] int((d*x)^(1/2)*(c*x^2+b*x)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx}\left(cx^2+bx\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(c*x^2+b*x)^p,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x)*(c*x^2 + b*x)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{dx}\left(cx^2+bx\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(c*x^2+b*x)^p,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*(c*x^2 + b*x)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx}\,(x(b+cx))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*(c*x**2+b*x)**p,x)
```

```
[Out] Integral(sqrt(d*x)*(x*(b + c*x))**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx}\left(cx^2 + bx\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(c*x^2+b*x)^p,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)*(c*x^2 + b*x)^p, x)
```

3.134 $\int \frac{(bx+cx^2)^p}{\sqrt{dx}}\,dx$

Optimal. Leaf size=61

$$\frac{2x\left(\frac{cx}{b}+1\right)^{-p}\left(bx+cx^2\right)^p\,{}_2F_1\left(-p,p+\frac{1}{2};p+\frac{3}{2};-\frac{cx}{b}\right)}{(2p+1)\sqrt{dx}}$$

```
[Out] (2*x*(b*x + c*x^2)^p*Hypergeometric2F1[-p, 1/2 + p, 3/2 + p, -((c*x)/b)])/(
(1 + 2*p)*Sqrt[d*x]*(1 + (c*x)/b)^p)
```

Rubi [A] time = 0.0234973, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {674, 66, 64}

$$\frac{2x\left(\frac{cx}{b}+1\right)^{-p}\left(bx+cx^2\right)^p\,{}_2F_1\left(-p,p+\frac{1}{2};p+\frac{3}{2};-\frac{cx}{b}\right)}{(2p+1)\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^p/Sqrt[d*x],x]
```

```
[Out] (2*x*(b*x + c*x^2)^p*Hypergeometric2F1[-p, 1/2 + p, 3/2 + p, -((c*x)/b)])/(
(1 + 2*p)*Sqrt[d*x]*(1 + (c*x)/b)^p)
```

Rule 674

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(
(e*x)^m*(b*x + c*x^2)^p)/(x^(m + p)*(b + c*x)^p), Int[x^(m + p)*(b + c*x)^p
, x], x] /; FreeQ[{b, c, e, m}, x] && !IntegerQ[p]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int \frac{(bx+cx^2)^p}{\sqrt{dx}}\,dx = \frac{\left(x^{\frac{1}{2}-p}(b+cx)^{-p}(bx+cx^2)^p\right)\int x^{-\frac{1}{2}+p}(b+cx)^p\,dx}{\sqrt{dx}}$$
$$= \frac{\left(x^{\frac{1}{2}-p}\left(1+\frac{cx}{b}\right)^{-p}(bx+cx^2)^p\right)\int x^{-\frac{1}{2}+p}\left(1+\frac{cx}{b}\right)^p\,dx}{\sqrt{dx}}$$
$$= \frac{2x\left(1+\frac{cx}{b}\right)^{-p}(bx+cx^2)^p\,{}_2F_1\left(-p,\frac{1}{2}+p;\frac{3}{2}+p;-\frac{cx}{b}\right)}{(1+2p)\sqrt{dx}}$$

Mathematica [A] time = 0.0120283, size = 58, normalized size = 0.95

$$\frac{x(x(b+cx))^p\left(\frac{cx}{b}+1\right)^{-p}\,{}_2F_1\left(-p,p+\frac{1}{2};p+\frac{3}{2};-\frac{cx}{b}\right)}{\left(p+\frac{1}{2}\right)\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^p/Sqrt[d*x],x]
```

```
[Out] (x*(x*(b + c*x))^p*Hypergeometric2F1[-p, 1/2 + p, 3/2 + p, -((c*x)/b)])/((1
/2 + p)*Sqrt[d*x]*(1 + (c*x)/b)^p)
```

Maple [F] time = 0.361, size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^p \frac{1}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^p/(d*x)^(1/2),x)
```

```
[Out] int((c*x^2+b*x)^p/(d*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx\right)^p}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^p/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^p/sqrt(d*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}\left(cx^2+bx\right)^p}{dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^p/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*(c*x^2 + b*x)^p/(d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x(b+cx))^p}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**p/(d*x)**(1/2),x)
```

```
[Out] Integral((x*(b + c*x))**p/sqrt(d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx\right)^p}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^p/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^p/sqrt(d*x), x)
```

3.135 $\int \frac{\left(bx+cx^2\right)^p}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2x\left(\frac{cx}{b}+1\right)^{-p}\left(bx+cx^2\right)^p\,{}_2F_1\left(p-\frac{1}{2},-p;p+\frac{1}{2};-\frac{cx}{b}\right)}{(1-2p)(dx)^{3/2}}$$

```
[Out] (-2*x*(b*x + c*x^2)^p*Hypergeometric2F1[-1/2 + p, -p, 1/2 + p, -((c*x)/b)])
/((1 - 2*p)*(d*x)^(3/2)*(1 + (c*x)/b)^p)
```

Rubi [A] time = 0.0265773, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {674, 66, 64}

$$-\frac{2x\left(\frac{cx}{b}+1\right)^{-p}\left(bx+cx^2\right)^p\,{}_2F_1\left(p-\frac{1}{2},-p;p+\frac{1}{2};-\frac{cx}{b}\right)}{(1-2p)(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^p/(d*x)^(3/2),x]
```

```
[Out] (-2*x*(b*x + c*x^2)^p*Hypergeometric2F1[-1/2 + p, -p, 1/2 + p, -((c*x)/b)])
/((1 - 2*p)*(d*x)^(3/2)*(1 + (c*x)/b)^p)
```

Rule 674

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(
(e*x)^m*(b*x + c*x^2)^p)/(x^(m + p)*(b + c*x)^p), Int[x^(m + p)*(b + c*x)^p
, x], x] /; FreeQ[{b, c, e, m}, x] && !IntegerQ[p]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int \frac{\left(bx+cx^2\right)^p}{(dx)^{3/2}}\,dx = \frac{\left(x^{\frac{3}{2}-p}(b+cx)^{-p}\left(bx+cx^2\right)^p\right)\int x^{-\frac{3}{2}+p}(b+cx)^p\,dx}{(dx)^{3/2}}$$
$$= \frac{\left(x^{\frac{3}{2}-p}\left(1+\frac{cx}{b}\right)^{-p}\left(bx+cx^2\right)^p\right)\int x^{-\frac{3}{2}+p}\left(1+\frac{cx}{b}\right)^p\,dx}{(dx)^{3/2}}$$
$$= -\frac{2x\left(1+\frac{cx}{b}\right)^{-p}\left(bx+cx^2\right)^p\,{}_2F_1\left(-\frac{1}{2}+p,-p;\frac{1}{2}+p;-\frac{cx}{b}\right)}{(1-2p)(dx)^{3/2}}$$

Mathematica [A] time = 0.0121797, size = 58, normalized size = 0.95

$$\frac{x(x(b+cx))^p\left(\frac{cx}{b}+1\right)^{-p}\,{}_2F_1\left(p-\frac{1}{2},-p;p+\frac{1}{2};-\frac{cx}{b}\right)}{\left(p-\frac{1}{2}\right)(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^p/(d*x)^(3/2),x]
```

```
[Out] (x*(x*(b + c*x))^p*Hypergeometric2F1[-1/2 + p, -p, 1/2 + p, -((c*x)/b)])/((
-1/2 + p)*(d*x)^(3/2)*(1 + (c*x)/b)^p)
```

Maple [F] time = 0.342, size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^p (dx)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^p/(d*x)^(3/2),x)
```

```
[Out] int((c*x^2+b*x)^p/(d*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx\right)^p}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^p/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^p/(d*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}\left(cx^2+bx\right)^p}{d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^p/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*(c*x^2 + b*x)^p/(d^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x(b+cx))^p}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**p/(d*x)**(3/2),x)
```

```
[Out] Integral((x*(b + c*x))**p/(d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx\right)^p}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^p/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^p/(d*x)^(3/2), x)
```

3.136 $\int \frac{(bx+cx^2)^p}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2x\left(\frac{cx}{b}+1\right)^{-p}\left(bx+cx^2\right)^p\,{}_2F_1\left(p-\frac{3}{2},-p;p-\frac{1}{2};-\frac{cx}{b}\right)}{(3-2p)(dx)^{5/2}}$$

```
[Out] (-2*x*(b*x + c*x^2)^p*Hypergeometric2F1[-3/2 + p, -p, -1/2 + p, -((c*x)/b)]
)/((3 - 2*p)*(d*x)^(5/2)*(1 + (c*x)/b)^p)
```

Rubi [A] time = 0.0258183, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {674, 66, 64}

$$-\frac{2x\left(\frac{cx}{b}+1\right)^{-p}\left(bx+cx^2\right)^p\,{}_2F_1\left(p-\frac{3}{2},-p;p-\frac{1}{2};-\frac{cx}{b}\right)}{(3-2p)(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^p/(d*x)^(5/2),x]
```

```
[Out] (-2*x*(b*x + c*x^2)^p*Hypergeometric2F1[-3/2 + p, -p, -1/2 + p, -((c*x)/b)]
)/((3 - 2*p)*(d*x)^(5/2)*(1 + (c*x)/b)^p)
```

Rule 674

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(
(e*x)^m*(b*x + c*x^2)^p)/(x^(m + p)*(b + c*x)^p), Int[x^(m + p)*(b + c*x)^p
, x], x] /; FreeQ[{b, c, e, m}, x] && !IntegerQ[p]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int \frac{(bx+cx^2)^p}{(dx)^{5/2}}\,dx = \frac{\left(x^{\frac{5}{2}-p}(b+cx)^{-p}\left(bx+cx^2\right)^p\right)\int x^{-\frac{5}{2}+p}(b+cx)^p\,dx}{(dx)^{5/2}}$$
$$= \frac{\left(x^{\frac{5}{2}-p}\left(1+\frac{cx}{b}\right)^{-p}\left(bx+cx^2\right)^p\right)\int x^{-\frac{5}{2}+p}\left(1+\frac{cx}{b}\right)^p\,dx}{(dx)^{5/2}}$$
$$= -\frac{2x\left(1+\frac{cx}{b}\right)^{-p}\left(bx+cx^2\right)^p\,{}_2F_1\left(-\frac{3}{2}+p,-p;-\frac{1}{2}+p;-\frac{cx}{b}\right)}{(3-2p)(dx)^{5/2}}$$

Mathematica [A] time = 0.0145333, size = 58, normalized size = 0.95

$$\frac{x(x(b+cx))^p\left(\frac{cx}{b}+1\right)^{-p}\,{}_2F_1\left(p-\frac{3}{2},-p;p-\frac{1}{2};-\frac{cx}{b}\right)}{\left(p-\frac{3}{2}\right)(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^p/(d*x)^(5/2),x]
```

```
[Out] (x*(x*(b + c*x))^p*Hypergeometric2F1[-3/2 + p, -p, -1/2 + p, -((c*x)/b)])/(
(-3/2 + p)*(d*x)^(5/2)*(1 + (c*x)/b)^p)
```

Maple [F] time = 0.335, size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^p (dx)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^p/(d*x)^(5/2),x)
```

```
[Out] int((c*x^2+b*x)^p/(d*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx\right)^p}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^p/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^p/(d*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}\left(cx^2+bx\right)^p}{d^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^p/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*(c*x^2 + b*x)^p/(d^3*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x(b+cx))^p}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**p/(d*x)**(5/2),x)
```

```
[Out] Integral((x*(b + c*x))**p/(d*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx\right)^p}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^p/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^p/(d*x)^(5/2), x)
```

3.137 $\int x^4\sqrt{a^2+2abx+b^2x^2}\,dx$

Optimal. Leaf size=71

$$\frac{bx^6\sqrt{a^2+2abx+b^2x^2}}{6(a+bx)}+\frac{ax^5\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}$$

```
[Out] (a*x^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*(a + b*x)) + (b*x^6*Sqrt[a^2 + 2*a
*b*x + b^2*x^2])/(6*(a + b*x))
```

Rubi [A] time = 0.0256565, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$\frac{bx^6\sqrt{a^2+2abx+b^2x^2}}{6(a+bx)}+\frac{ax^5\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (a*x^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*(a + b*x)) + (b*x^6*Sqrt[a^2 + 2*a
*b*x + b^2*x^2])/(6*(a + b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4\sqrt{a^2+2abx+b^2x^2}\,dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int x^4\left(ab+b^2x\right)\,dx}{ab+b^2x}\\ &= \frac{\sqrt{a^2+2abx+b^2x^2}\int\left(abx^4+b^2x^5\right)\,dx}{ab+b^2x}\\ &= \frac{ax^5\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}+\frac{bx^6\sqrt{a^2+2abx+b^2x^2}}{6(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.0130453, size = 33, normalized size = 0.46

$$\frac{x^5\sqrt{(a+bx)^2}(6a+5bx)}{30(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (x^5*Sqrt[(a + b*x)^2]*(6*a + 5*b*x))/(30*(a + b*x))
```

Maple [A] time = 0.168, size = 30, normalized size = 0.4

$$\frac{x^5(5bx+6a)}{30bx+30a}\sqrt{(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((b*x+a)^2)^(1/2),x)
```

```
[Out] 1/30*x^5*(5*b*x+6*a)*((b*x+a)^2)^(1/2)/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.96933, size = 31, normalized size = 0.44

$$\frac{1}{6}bx^6 + \frac{1}{5}ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*b*x^6 + 1/5*a*x^5
```

Sympy [A] time = 0.206823, size = 12, normalized size = 0.17

$$\frac{ax^5}{5} + \frac{bx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*((b*x+a)**2)**(1/2),x)
```

```
[Out] a*x**5/5 + b*x**6/6
```

Giac [A] time = 1.24171, size = 53, normalized size = 0.75

$$\frac{1}{6}\,bx^6\mathrm{sgn}\,(bx+a)+\frac{1}{5}\,ax^5\mathrm{sgn}\,(bx+a)+\frac{a^6\mathrm{sgn}\,(bx+a)}{30\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*b*x^6*sgn(b*x + a) + 1/5*a*x^5*sgn(b*x + a) + 1/30*a^6*sgn(b*x + a)/b^5
```

3.138 $\int x^3\sqrt{a^2+2abx+b^2x^2}\,dx$

Optimal. Leaf size=71

$$\frac{bx^5\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}+\frac{ax^4\sqrt{a^2+2abx+b^2x^2}}{4(a+bx)}$$

```
[Out] (a*x^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*(a + b*x)) + (b*x^5*Sqrt[a^2 + 2*a
*b*x + b^2*x^2])/(5*(a + b*x))
```

Rubi [A] time = 0.0235502, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$\frac{bx^5\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}+\frac{ax^4\sqrt{a^2+2abx+b^2x^2}}{4(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (a*x^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*(a + b*x)) + (b*x^5*Sqrt[a^2 + 2*a
*b*x + b^2*x^2])/(5*(a + b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{a^2+2abx+b^2x^2}\,dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int x^3\left(ab+b^2x\right)dx}{ab+b^2x}\\ &= \frac{\sqrt{a^2+2abx+b^2x^2}\int\left(abx^3+b^2x^4\right)dx}{ab+b^2x}\\ &= \frac{ax^4\sqrt{a^2+2abx+b^2x^2}}{4(a+bx)}+\frac{bx^5\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.0087176, size = 33, normalized size = 0.46

$$\frac{x^4\sqrt{(a+bx)^2}(5a+4bx)}{20(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (x^4*Sqrt[(a + b*x)^2]*(5*a + 4*b*x))/(20*(a + b*x))
```

Maple [A] time = 0.049, size = 30, normalized size = 0.4

$$\frac{x^4 (4\,bx + 5\,a)}{20\,bx + 20\,a}\sqrt{(bx + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((b*x+a)^2)^(1/2),x)
```

```
[Out] 1/20*x^4*(4*b*x+5*a)*((b*x+a)^2)^(1/2)/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86497, size = 31, normalized size = 0.44

$$\frac{1}{5}\,bx^5 + \frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*b*x^5 + 1/4*a*x^4
```

Sympy [A] time = 0.208986, size = 12, normalized size = 0.17

$$\frac{ax^4}{4} + \frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((b*x+a)**2)**(1/2),x)
```

```
[Out] a*x**4/4 + b*x**5/5
```

Giac [A] time = 1.25429, size = 53, normalized size = 0.75

$$\frac{1}{5}\,bx^5\mathrm{sgn}\left(bx+a\right)+\frac{1}{4}\,ax^4\mathrm{sgn}\left(bx+a\right)-\frac{a^5\mathrm{sgn}\left(bx+a\right)}{20\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*b*x^5*sgn(b*x + a) + 1/4*a*x^4*sgn(b*x + a) - 1/20*a^5*sgn(b*x + a)/b^4
```

3.139 $\int x^2\sqrt{a^2+2abx+b^2x^2}\,dx$

Optimal. Leaf size=71

$$\frac{bx^4\sqrt{a^2+2abx+b^2x^2}}{4(a+bx)}+\frac{ax^3\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)}$$

```
[Out] (a*x^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*(a + b*x)) + (b*x^4*Sqrt[a^2 + 2*a
*b*x + b^2*x^2])/(4*(a + b*x))
```

Rubi [A] time = 0.0215479, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$\frac{bx^4\sqrt{a^2+2abx+b^2x^2}}{4(a+bx)}+\frac{ax^3\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (a*x^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*(a + b*x)) + (b*x^4*Sqrt[a^2 + 2*a
*b*x + b^2*x^2])/(4*(a + b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a^2+2abx+b^2x^2}\,dx&=\frac{\sqrt{a^2+2abx+b^2x^2}\int x^2\left(ab+b^2x\right)\,dx}{ab+b^2x}\\&=\frac{\sqrt{a^2+2abx+b^2x^2}\int\left(abx^2+b^2x^3\right)\,dx}{ab+b^2x}\\&=\frac{ax^3\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)}+\frac{bx^4\sqrt{a^2+2abx+b^2x^2}}{4(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.0088511, size = 33, normalized size = 0.46

$$\frac{x^3\sqrt{(a+bx)^2}(4a+3bx)}{12(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (x^3*Sqrt[(a + b*x)^2]*(4*a + 3*b*x))/(12*(a + b*x))
```

Maple [A] time = 0.046, size = 30, normalized size = 0.4

$$\frac{x^3 (3 bx + 4 a)}{12 bx + 12 a} \sqrt{(bx + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((b*x+a)^2)^(1/2),x)
```

```
[Out] 1/12*x^3*(3*b*x+4*a)*((b*x+a)^2)^(1/2)/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.96834, size = 31, normalized size = 0.44

$$\frac{1}{4} bx^4 + \frac{1}{3} ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

Sympy [A] time = 0.148424, size = 12, normalized size = 0.17

$$\frac{ax^3}{3} + \frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((b*x+a)**2)**(1/2),x)
```

```
[Out] a*x**3/3 + b*x**4/4
```

Giac [A] time = 1.23872, size = 53, normalized size = 0.75

$$\frac{1}{4}bx^4\mathrm{sgn}(bx+a)+\frac{1}{3}ax^3\mathrm{sgn}(bx+a)+\frac{a^4\mathrm{sgn}(bx+a)}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*b*x^4*sgn(b*x + a) + 1/3*a*x^3*sgn(b*x + a) + 1/12*a^4*sgn(b*x + a)/b^3
```

3.140 $\int x\sqrt{a^2+2abx+b^2x^2}\,dx$

Optimal. Leaf size=61

$$\frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{3b^2}-\frac{a(a+bx)\sqrt{a^2+2abx+b^2x^2}}{2b^2}$$

```
[Out] -(a*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*b^2) + (a^2 + 2*a*b*x + b^2
*x^2)^(3/2)/(3*b^2)
```

Rubi [A] time = 0.0152343, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {640, 609}

$$\frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{3b^2}-\frac{a(a+bx)\sqrt{a^2+2abx+b^2x^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] -(a*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*b^2) + (a^2 + 2*a*b*x + b^2
*x^2)^(3/2)/(3*b^2)
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a^2+2abx+b^2x^2}\,dx &= \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{3b^2}-\frac{a\int\sqrt{a^2+2abx+b^2x^2}\,dx}{b}\\ &= -\frac{a(a+bx)\sqrt{a^2+2abx+b^2x^2}}{2b^2}+\frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{3b^2}\end{aligned}$$

Mathematica [A] time = 0.0083166, size = 33, normalized size = 0.54

$$\frac{x^2\sqrt{(a+bx)^2}(3a+2bx)}{6(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (x^2*Sqrt[(a + b*x)^2]*(3*a + 2*b*x))/(6*(a + b*x))
```

Maple [A] time = 0.042, size = 30, normalized size = 0.5

$$\frac{x^2 (2 bx + 3 a)}{6 bx + 6 a}\sqrt{(bx + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((b*x+a)^2)^(1/2),x)
```

```
[Out] 1/6*x^2*(2*b*x+3*a)*((b*x+a)^2)^(1/2)/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.84533, size = 31, normalized size = 0.51

$$\frac{1}{3} bx^3 + \frac{1}{2} ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

Sympy [A] time = 0.148866, size = 12, normalized size = 0.2

$$\frac{ax^2}{2} + \frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)**2)**(1/2),x)
```

```
[Out] a*x**2/2 + b*x**3/3
```

Giac [A] time = 1.24059, size = 53, normalized size = 0.87

$$\frac{1}{3} bx^3 \mathrm{sgn}(bx + a) + \frac{1}{2} ax^2 \mathrm{sgn}(bx + a) - \frac{a^3 \mathrm{sgn}(bx + a)}{6 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*b*x^3*sgn(b*x + a) + 1/2*a*x^2*sgn(b*x + a) - 1/6*a^3*sgn(b*x + a)/b^2
```

3.141 $\int \sqrt{a^2+2abx+b^2x^2}\,dx$

Optimal. Leaf size=32

$$\frac{(a+bx)\sqrt{a^2+2abx+b^2x^2}}{2b}$$

```
[Out] ((a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*b)
```

Rubi [A] time = 0.0058244, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {609}

$$\frac{(a+bx)\sqrt{a^2+2abx+b^2x^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ((a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*b)
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \sqrt{a^2+2abx+b^2x^2}\,dx=\frac{(a+bx)\sqrt{a^2+2abx+b^2x^2}}{2b}$$

Mathematica [A] time = 0.0079605, size = 30, normalized size = 0.94

$$\frac{x\sqrt{(a+bx)^2}(2a+bx)}{2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (x*Sqrt[(a + b*x)^2]*(2*a + b*x))/(2*(a + b*x))
```

Maple [A] time = 0.042, size = 27, normalized size = 0.8

$$\frac{x\,(bx+2\,a)}{2\,bx+2\,a}\sqrt{(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2),x)
```

```
[Out] 1/2*x*(b*x+2*a)*((b*x+a)^2)^(1/2)/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.85692, size = 23, normalized size = 0.72

$$\frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*x
```

Sympy [A] time = 0.245859, size = 8, normalized size = 0.25

$$ax + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2),x)
```

```
[Out] a*x + b*x**2/2
```

Giac [A] time = 1.17913, size = 45, normalized size = 1.41

$$\frac{1}{2}\left(bx^2 + 2\,ax\right)\mathrm{sgn}\left(bx + a\right) + \frac{a^2\mathrm{sgn}\left(bx + a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*sgn(b*x + a) + 1/2*a^2*sgn(b*x + a)/b
```

3.142 $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x}\,dx$

Optimal. Leaf size=62

$$\frac{bx\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{a\log(x)\sqrt{a^2+2abx+b^2x^2}}{a+bx}$$

```
[Out] (b*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(a + b*x) + (a*Sqrt[a^2 + 2*a*b*x + b^2
*x^2]*Log[x])/(a + b*x)
```

Rubi [A] time = 0.0166675, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$\frac{bx\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{a\log(x)\sqrt{a^2+2abx+b^2x^2}}{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/x,x]
```

```
[Out] (b*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(a + b*x) + (a*Sqrt[a^2 + 2*a*b*x + b^2
*x^2]*Log[x])/(a + b*x)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x}\,dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{ab+b^2x}{x}\,dx}{ab+b^2x}\\
&= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(b^2+\frac{ab}{x}\right)dx}{ab+b^2x}\\
&= \frac{bx\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{a\sqrt{a^2+2abx+b^2x^2}\log(x)}{a+bx}
\end{aligned}$$

Mathematica [A] time = 0.0095251, size = 27, normalized size = 0.44

$$\frac{\sqrt{(a+bx)^2}(a\log(x)+bx)}{a+bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/x,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(b*x + a*Log[x]))/(a + b*x)
```

Maple [C] time = 0.328, size = 19, normalized size = 0.3

$$\mathrm{csgn}\left(bx+a\right)\left(bx+a+a\ln\left(bx\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)/x,x)
```

```
[Out] csgn(b*x+a)*(b*x+a+a*ln(b*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86431, size = 22, normalized size = 0.35

$$bx+a\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] b*x + a*log(x)
```

Sympy [A] time = 0.264238, size = 7, normalized size = 0.11

$$a\log\left(x\right)+bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)/x,x)
```

```
[Out] a*log(x) + b*x
```

Giac [A] time = 1.21377, size = 28, normalized size = 0.45

$$bx\operatorname{sgn}(bx+a)+a\log(|x|)\operatorname{sgn}(bx+a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] b*x*sgn(b*x + a) + a*log(abs(x))*sgn(b*x + a)
```

3.143 $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^2} \, dx$

Optimal. Leaf size=65

$$\frac{b\log(x)\sqrt{a^2+2abx+b^2x^2}}{a+bx}-\frac{a\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)}$$

```
[Out] -((a*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(x*(a + b*x))) + (b*Sqrt[a^2 + 2*a*b*x
+ b^2*x^2]*Log[x])/(a + b*x)
```

Rubi [A] time = 0.018244, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$\frac{b\log(x)\sqrt{a^2+2abx+b^2x^2}}{a+bx}-\frac{a\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/x^2,x]
```

```
[Out] -((a*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(x*(a + b*x))) + (b*Sqrt[a^2 + 2*a*b*x
+ b^2*x^2]*Log[x])/(a + b*x)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^2} \, dx &= \frac{\sqrt{a^2+2abx+b^2x^2} \int \frac{ab+b^2x}{x^2} \, dx}{ab+b^2x} \\
&= \frac{\sqrt{a^2+2abx+b^2x^2} \int \left(\frac{ab}{x^2}+\frac{b^2}{x}\right) \, dx}{ab+b^2x} \\
&= -\frac{a\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)}+\frac{b\sqrt{a^2+2abx+b^2x^2}\log(x)}{a+bx}
\end{aligned}$$

Mathematica [A] time = 0.0096034, size = 31, normalized size = 0.48

$$\frac{\sqrt{(a+bx)^2}(bx\log(x)-a)}{x(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/x^2,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(-a + b*x*Log[x]))/(x*(a + b*x))
```

Maple [C] time = 0.235, size = 22, normalized size = 0.3

$$\frac{\operatorname{csgn}(bx+a)(\ln(bx)bx-a)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)/x^2,x)
```

```
[Out] csgn(b*x+a)*(ln(b*x)*b*x-a)/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83424, size = 27, normalized size = 0.42

$$\frac{bx\log(x)-a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (b*x*log(x) - a)/x
```

Sympy [A] time = 0.310047, size = 7, normalized size = 0.11

$$-\frac{a}{x}+b\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)/x**2,x)
```

```
[Out] -a/x + b*log(x)
```

Giac [A] time = 1.26161, size = 32, normalized size = 0.49

$$b\log(|x|)\,\mathrm{sgn}(bx+a)-\frac{a\mathrm{sgn}(bx+a)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] b*log(abs(x))*sgn(b*x + a) - a*sgn(b*x + a)/x
```

3.144 $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^3}\,dx$

Optimal. Leaf size=35

$$-\frac{(a+bx)\sqrt{a^2+2abx+b^2x^2}}{2ax^2}$$

```
[Out] -((a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*a*x^2)
```

Rubi [A] time = 0.0130166, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 37}

$$-\frac{(a+bx)\sqrt{a^2+2abx+b^2x^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/x^3,x]
```

```
[Out] -((a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*a*x^2)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^3}\,dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{ab+b^2x}{x^3}\,dx}{ab+b^2x}\\ &= -\frac{(a+bx)\sqrt{a^2+2abx+b^2x^2}}{2ax^2}\end{aligned}$$

Mathematica [A] time = 0.0069257, size = 31, normalized size = 0.89

$$-\frac{\sqrt{(a+bx)^2}(a+2bx)}{2x^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/x^3,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(a + 2*b*x))/(2*x^2*(a + b*x))
```

Maple [A] time = 0.045, size = 28, normalized size = 0.8

$$-\frac{2\,bx+a}{2\,x^2\,(bx+a)}\sqrt{(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)/x^3,x)
```

```
[Out] -1/2*(2*b*x+a)*((b*x+a)^2)^(1/2)/x^2/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.02235, size = 30, normalized size = 0.86

$$-\frac{2\,bx+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x + a)/x^2
```

Sympy [A] time = 0.582048, size = 12, normalized size = 0.34

$$-\frac{a+2bx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)/x**3,x)
```

```
[Out] -(a + 2*b*x)/(2*x**2)
```

Giac [A] time = 1.23143, size = 53, normalized size = 1.51

$$-\frac{b^2\mathrm{sgn}\,(bx+a)}{2\,a}-\frac{2\,bx\mathrm{sgn}\,(bx+a)+a\mathrm{sgn}\,(bx+a)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*b^2*sgn(b*x + a)/a - 1/2*(2*b*x*sgn(b*x + a) + a*sgn(b*x + a))/x^2
```

3.145 $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^4} dx$

Optimal. Leaf size=71

$$-\frac{a\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)}-\frac{b\sqrt{a^2+2abx+b^2x^2}}{2x^2(a+bx)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*x^3*(a + b*x)) - (b*Sqrt[a^2 + 2*a*b*
x + b^2*x^2])/(2*x^2*(a + b*x))
```

Rubi [A] time = 0.0186558, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$-\frac{a\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)}-\frac{b\sqrt{a^2+2abx+b^2x^2}}{2x^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/x^4,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*x^3*(a + b*x)) - (b*Sqrt[a^2 + 2*a*b*
x + b^2*x^2])/(2*x^2*(a + b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^4} dx &= \frac{\sqrt{a^2+2abx+b^2x^2} \int \frac{ab+b^2x}{x^4} dx}{ab+b^2x} \\
&= \frac{\sqrt{a^2+2abx+b^2x^2} \int \left(\frac{ab}{x^4}+\frac{b^2}{x^3}\right) dx}{ab+b^2x} \\
&= -\frac{a\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)}-\frac{b\sqrt{a^2+2abx+b^2x^2}}{2x^2(a+bx)}
\end{aligned}$$

Mathematica [A] time = 0.0069367, size = 33, normalized size = 0.46

$$-\frac{\sqrt{(a+bx)^2}(2a+3bx)}{6x^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/x^4,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(2*a + 3*b*x))/(6*x^3*(a + b*x))
```

Maple [A] time = 0.044, size = 30, normalized size = 0.4

$$-\frac{3\,bx+2\,a}{6\,x^{3}\,(bx+a)}\sqrt{(bx+a)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)/x^4,x)
```

```
[Out] -1/6*(3*b*x+2*a)*((b*x+a)^2)^(1/2)/x^3/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89047, size = 32, normalized size = 0.45

$$-\frac{3\,bx+2\,a}{6\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b*x + 2*a)/x^3
```

Sympy [A] time = 0.888519, size = 14, normalized size = 0.2

$$-\frac{2a+3bx}{6x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)/x**4,x)
```

```
[Out] -(2*a + 3*b*x)/(6*x**3)
```

Giac [A] time = 1.24129, size = 54, normalized size = 0.76

$$\frac{b^3 \mathrm{sgn}\,(bx+a)}{6\,a^2} - \frac{3\,bx\mathrm{sgn}\,(bx+a) + 2\,a\mathrm{sgn}\,(bx+a)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/6*b^3*sgn(b*x + a)/a^2 - 1/6*(3*b*x*sgn(b*x + a) + 2*a*sgn(b*x + a))/x^3
```

3.146 $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^5} dx$

Optimal. Leaf size=71

$$-\frac{a\sqrt{a^2+2abx+b^2x^2}}{4x^4(a+bx)}-\frac{b\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*x^4*(a + b*x)) - (b*Sqrt[a^2 + 2*a*b*
x + b^2*x^2])/(3*x^3*(a + b*x))
```

Rubi [A] time = 0.0185848, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$-\frac{a\sqrt{a^2+2abx+b^2x^2}}{4x^4(a+bx)}-\frac{b\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/x^5,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*x^4*(a + b*x)) - (b*Sqrt[a^2 + 2*a*b*
x + b^2*x^2])/(3*x^3*(a + b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^5} dx &= \frac{\sqrt{a^2+2abx+b^2x^2} \int \frac{ab+b^2x}{x^5} dx}{ab+b^2x} \\
&= \frac{\sqrt{a^2+2abx+b^2x^2} \int \left(\frac{ab}{x^5}+\frac{b^2}{x^4}\right) dx}{ab+b^2x} \\
&= -\frac{a\sqrt{a^2+2abx+b^2x^2}}{4x^4(a+bx)}-\frac{b\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)}
\end{aligned}$$

Mathematica [A] time = 0.0079465, size = 33, normalized size = 0.46

$$-\frac{\sqrt{(a+bx)^2}(3a+4bx)}{12x^4(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/x^5,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(3*a + 4*b*x))/(12*x^4*(a + b*x))
```

Maple [A] time = 0.044, size = 30, normalized size = 0.4

$$-\frac{4\,bx+3\,a}{12\,x^{4}\,(bx+a)}\sqrt{(bx+a)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)/x^5,x)
```

```
[Out] -1/12*(4*b*x+3*a)*((b*x+a)^2)^(1/2)/x^4/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88232, size = 34, normalized size = 0.48

$$-\frac{4\,bx+3\,a}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(4*b*x + 3*a)/x^4
```

Sympy [A] time = 0.665221, size = 14, normalized size = 0.2

$$-\frac{3a+4bx}{12x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)/x**5,x)
```

```
[Out] -(3*a + 4*b*x)/(12*x**4)
```

Giac [A] time = 1.24318, size = 54, normalized size = 0.76

$$-\frac{b^4\mathrm{sgn}\,(bx+a)}{12\,a^3}-\frac{4\,bx\mathrm{sgn}\,(bx+a)+3\,a\mathrm{sgn}\,(bx+a)}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/12*b^4*sgn(b*x + a)/a^3 - 1/12*(4*b*x*sgn(b*x + a) + 3*a*sgn(b*x + a))/x
^4
```

3.147 $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^6}\,dx$

Optimal. Leaf size=71

$$-\frac{a\sqrt{a^2+2abx+b^2x^2}}{5x^5(a+bx)}-\frac{b\sqrt{a^2+2abx+b^2x^2}}{4x^4(a+bx)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*x^5*(a + b*x)) - (b*Sqrt[a^2 + 2*a*b*
x + b^2*x^2])/(4*x^4*(a + b*x))
```

Rubi [A] time = 0.0233714, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$-\frac{a\sqrt{a^2+2abx+b^2x^2}}{5x^5(a+bx)}-\frac{b\sqrt{a^2+2abx+b^2x^2}}{4x^4(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/x^6,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*x^5*(a + b*x)) - (b*Sqrt[a^2 + 2*a*b*
x + b^2*x^2])/(4*x^4*(a + b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^6}\,dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{ab+b^2x}{x^6}\,dx}{ab+b^2x}\\
&= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{ab}{x^6}+\frac{b^2}{x^5}\right)dx}{ab+b^2x}\\
&= -\frac{a\sqrt{a^2+2abx+b^2x^2}}{5x^5(a+bx)}-\frac{b\sqrt{a^2+2abx+b^2x^2}}{4x^4(a+bx)}
\end{aligned}$$

Mathematica [A] time = 0.0070738, size = 33, normalized size = 0.46

$$-\frac{\sqrt{(a+bx)^2}(4a+5bx)}{20x^5(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/x^6,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(4*a + 5*b*x))/(20*x^5*(a + b*x))
```

Maple [A] time = 0.047, size = 30, normalized size = 0.4

$$-\frac{5\,bx+4\,a}{20\,x^5\,(bx+a)}\sqrt{(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)/x^6,x)
```

```
[Out] -1/20*(5*b*x+4*a)*((b*x+a)^2)^(1/2)/x^5/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.08012, size = 34, normalized size = 0.48

$$-\frac{5\,bx+4\,a}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/20*(5*b*x + 4*a)/x^5
```

Sympy [A] time = 0.986136, size = 14, normalized size = 0.2

$$-\frac{4a+5bx}{20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)/x**6,x)
```

```
[Out] -(4*a + 5*b*x)/(20*x**5)
```

Giac [A] time = 1.15781, size = 54, normalized size = 0.76

$$\frac{b^5 \operatorname{sgn}(bx + a)}{20\, a^4} - \frac{5\, bx \operatorname{sgn}(bx + a) + 4\, a \operatorname{sgn}(bx + a)}{20\, x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/20*b^5*sgn(b*x + a)/a^4 - 1/20*(5*b*x*sgn(b*x + a) + 4*a*sgn(b*x + a))/x^
5
```

3.148 $\int x^5 \left(a^2+2abx+b^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=151

$$\frac{b^3x^9\sqrt{a^2+2abx+b^2x^2}}{9(a+bx)}+\frac{3ab^2x^8\sqrt{a^2+2abx+b^2x^2}}{8(a+bx)}+\frac{3a^2bx^7\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}+\frac{a^3x^6\sqrt{a^2+2abx+b^2x^2}}{6(a+bx)}$$

```
[Out] (a^3*x^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*(a + b*x)) + (3*a^2*b*x^7*Sqrt[a
^2 + 2*a*b*x + b^2*x^2])/(7*(a + b*x)) + (3*a*b^2*x^8*Sqrt[a^2 + 2*a*b*x +
b^2*x^2])/(8*(a + b*x)) + (b^3*x^9*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(9*(a + b
*x))
```

Rubi [A] time = 0.0457595, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$\frac{b^3x^9\sqrt{a^2+2abx+b^2x^2}}{9(a+bx)}+\frac{3ab^2x^8\sqrt{a^2+2abx+b^2x^2}}{8(a+bx)}+\frac{3a^2bx^7\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}+\frac{a^3x^6\sqrt{a^2+2abx+b^2x^2}}{6(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (a^3*x^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*(a + b*x)) + (3*a^2*b*x^7*Sqrt[a
^2 + 2*a*b*x + b^2*x^2])/(7*(a + b*x)) + (3*a*b^2*x^8*Sqrt[a^2 + 2*a*b*x +
b^2*x^2])/(8*(a + b*x)) + (b^3*x^9*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(9*(a + b
*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5\left(a^2+2abx+b^2x^2\right)^{3/2}dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int x^5\left(ab+b^2x\right)^3dx}{b^2\left(ab+b^2x\right)}\\ &= \frac{\sqrt{a^2+2abx+b^2x^2}\int\left(a^3b^3x^5+3a^2b^4x^6+3ab^5x^7+b^6x^8\right)dx}{b^2\left(ab+b^2x\right)}\\ &= \frac{a^3x^6\sqrt{a^2+2abx+b^2x^2}}{6(a+bx)}+\frac{3a^2bx^7\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}+\frac{3ab^2x^8\sqrt{a^2+2abx+b^2x^2}}{8(a+bx)}+\end{aligned}$$

Mathematica [A] time = 0.0188173, size = 55, normalized size = 0.36

$$\frac{x^6\sqrt{(a+bx)^2}\left(216a^2bx+84a^3+189ab^2x^2+56b^3x^3\right)}{504(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (x^6*Sqrt[(a + b*x)^2]*(84*a^3 + 216*a^2*b*x + 189*a*b^2*x^2 + 56*b^3*x^3))
/(504*(a + b*x))
```

Maple [A] time = 0.186, size = 52, normalized size = 0.3

$$\frac{x^6\left(56\,b^3x^3+189\,ab^2x^2+216\,a^2bx+84\,a^3\right)}{504\,(bx+a)^3}\left((bx+a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] 1/504*x^6*(56*b^3*x^3+189*a*b^2*x^2+216*a^2*b*x+84*a^3)*((b*x+a)^2)^(3/2)/(
b*x+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7448, size = 80, normalized size = 0.53

$$\frac{1}{9}\,b^3x^9+\frac{3}{8}\,ab^2x^8+\frac{3}{7}\,a^2bx^7+\frac{1}{6}\,a^3x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/9*b^3*x^9 + 3/8*a*b^2*x^8 + 3/7*a^2*b*x^7 + 1/6*a^3*x^6
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\left((a+bx)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral(x**5*((a + b*x)**2)**(3/2), x)
```

Giac [A] time = 1.17896, size = 99, normalized size = 0.66

$$\frac{1}{9}b^3x^9\text{sgn}\left(bx+a\right)+\frac{3}{8}ab^2x^8\text{sgn}\left(bx+a\right)+\frac{3}{7}a^2bx^7\text{sgn}\left(bx+a\right)+\frac{1}{6}a^3x^6\text{sgn}\left(bx+a\right)-\frac{a^9\text{sgn}\left(bx+a\right)}{504\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/9*b^3*x^9*sgn(b*x + a) + 3/8*a*b^2*x^8*sgn(b*x + a) + 3/7*a^2*b*x^7*sgn(b
*x + a) + 1/6*a^3*x^6*sgn(b*x + a) - 1/504*a^9*sgn(b*x + a)/b^6
```

3.149 $\int x^4 \left(a^2+2abx+b^2x^2\right)^{3/2} dx$

Optimal. Leaf size=151

$$\frac{b^3x^8\sqrt{a^2+2abx+b^2x^2}}{8(a+bx)}+\frac{3ab^2x^7\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}+\frac{a^2bx^6\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)}+\frac{a^3x^5\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}$$

```
[Out] (a^3*x^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*(a + b*x)) + (a^2*b*x^6*Sqrt[a^2
 + 2*a*b*x + b^2*x^2])/(2*(a + b*x)) + (3*a*b^2*x^7*Sqrt[a^2 + 2*a*b*x + b^
2*x^2])/(7*(a + b*x)) + (b^3*x^8*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(8*(a + b*x
))
```

Rubi [A] time = 0.0415061, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$\frac{b^3x^8\sqrt{a^2+2abx+b^2x^2}}{8(a+bx)}+\frac{3ab^2x^7\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}+\frac{a^2bx^6\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)}+\frac{a^3x^5\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (a^3*x^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*(a + b*x)) + (a^2*b*x^6*Sqrt[a^2
 + 2*a*b*x + b^2*x^2])/(2*(a + b*x)) + (3*a*b^2*x^7*Sqrt[a^2 + 2*a*b*x + b^
2*x^2])/(7*(a + b*x)) + (b^3*x^8*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(8*(a + b*x
))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4\left(a^2+2abx+b^2x^2\right)^{3/2}dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int x^4\left(ab+b^2x\right)^3 dx}{b^2\left(ab+b^2x\right)}\\ &= \frac{\sqrt{a^2+2abx+b^2x^2}\int\left(a^3b^3x^4+3a^2b^4x^5+3ab^5x^6+b^6x^7\right)dx}{b^2\left(ab+b^2x\right)}\\ &= \frac{a^3x^5\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}+\frac{a^2bx^6\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)}+\frac{3ab^2x^7\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}+\frac{b^3x^8}{}\end{aligned}$$

Mathematica [A] time = 0.0141467, size = 55, normalized size = 0.36

$$\frac{x^5\sqrt{(a+bx)^2}\left(140a^2bx+56a^3+120ab^2x^2+35b^3x^3\right)}{280(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (x^5*Sqrt[(a + b*x)^2]*(56*a^3 + 140*a^2*b*x + 120*a*b^2*x^2 + 35*b^3*x^3))
/(280*(a + b*x))
```

Maple [A] time = 0.182, size = 52, normalized size = 0.3

$$\frac{x^5\left(35\,b^3x^3+120\,ab^2x^2+140\,a^2bx+56\,a^3\right)}{280\,(bx+a)^3}\left((bx+a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] 1/280*x^5*(35*b^3*x^3+120*a*b^2*x^2+140*a^2*b*x+56*a^3)*((b*x+a)^2)^(3/2)/(
b*x+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73161, size = 80, normalized size = 0.53

$$\frac{1}{8}\,b^3x^8+\frac{3}{7}\,ab^2x^7+\frac{1}{2}\,a^2bx^6+\frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*b^3*x^8 + 3/7*a*b^2*x^7 + 1/2*a^2*b*x^6 + 1/5*a^3*x^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\left((a+bx)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral(x**4*((a + b*x)**2)**(3/2), x)
```

Giac [A] time = 1.19534, size = 99, normalized size = 0.66

$$\frac{1}{8}b^3x^8\text{sgn}(bx+a)+\frac{3}{7}ab^2x^7\text{sgn}(bx+a)+\frac{1}{2}a^2bx^6\text{sgn}(bx+a)+\frac{1}{5}a^3x^5\text{sgn}(bx+a)+\frac{a^8\text{sgn}(bx+a)}{280\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*b^3*x^8*sgn(b*x + a) + 3/7*a*b^2*x^7*sgn(b*x + a) + 1/2*a^2*b*x^6*sgn(b
*x + a) + 1/5*a^3*x^5*sgn(b*x + a) + 1/280*a^8*sgn(b*x + a)/b^5
```

3.150 $\int x^3 \left(a^2+2abx+b^2x^2\right)^{3/2} dx$

Optimal. Leaf size=151

$$\frac{b^3x^7\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}+\frac{ab^2x^6\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)}+\frac{3a^2bx^5\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}+\frac{a^3x^4\sqrt{a^2+2abx+b^2x^2}}{4(a+bx)}$$

```
[Out] (a^3*x^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*(a + b*x)) + (3*a^2*b*x^5*Sqrt[a
^2 + 2*a*b*x + b^2*x^2])/(5*(a + b*x)) + (a*b^2*x^6*Sqrt[a^2 + 2*a*b*x + b^
2*x^2])/(2*(a + b*x)) + (b^3*x^7*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*(a + b*x
))
```

Rubi [A] time = 0.0413578, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$\frac{b^3x^7\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}+\frac{ab^2x^6\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)}+\frac{3a^2bx^5\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}+\frac{a^3x^4\sqrt{a^2+2abx+b^2x^2}}{4(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```


```
[Out] (a^3*x^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*(a + b*x)) + (3*a^2*b*x^5*Sqrt[a
^2 + 2*a*b*x + b^2*x^2])/(5*(a + b*x)) + (a*b^2*x^6*Sqrt[a^2 + 2*a*b*x + b^
2*x^2])/(2*(a + b*x)) + (b^3*x^7*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*(a + b*x
))
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(a^2+2abx+b^2x^2\right)^{3/2} dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int x^3\left(ab+b^2x\right)^3 dx}{b^2\left(ab+b^2x\right)}\\
&= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(a^3b^3x^3+3a^2b^4x^4+3ab^5x^5+b^6x^6\right) dx}{b^2\left(ab+b^2x\right)}\\
&= \frac{a^3x^4\sqrt{a^2+2abx+b^2x^2}}{4(a+bx)}+\frac{3a^2bx^5\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}+\frac{ab^2x^6\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)}+
\end{aligned}$$

Mathematica [A] time = 0.013957, size = 55, normalized size = 0.36

$$\frac{x^4\sqrt{(a+bx)^2}\left(84a^2bx+35a^3+70ab^2x^2+20b^3x^3\right)}{140(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (x^4*Sqrt[(a + b*x)^2]*(35*a^3 + 84*a^2*b*x + 70*a*b^2*x^2 + 20*b^3*x^3))/(
140*(a + b*x))
```

Maple [A] time = 0.176, size = 52, normalized size = 0.3

$$\frac{x^4\left(20\,b^3x^3+70\,ab^2x^2+84\,a^2bx+35\,a^3\right)}{140\,(bx+a)^3}\left((bx+a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] 1/140*x^4*(20*b^3*x^3+70*a*b^2*x^2+84*a^2*b*x+35*a^3)*((b*x+a)^2)^(3/2)/(b*
x+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.02851, size = 80, normalized size = 0.53

$$\frac{1}{7}\,b^3x^7+\frac{1}{2}\,ab^2x^6+\frac{3}{5}\,a^2bx^5+\frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/7*b^3*x^7 + 1/2*a*b^2*x^6 + 3/5*a^2*b*x^5 + 1/4*a^3*x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\left((a+bx)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral(x**3*((a + b*x)**2)**(3/2), x)
```

Giac [A] time = 1.25165, size = 99, normalized size = 0.66

$$\frac{1}{7}\,b^3x^7\text{sgn}\left(bx+a\right)+\frac{1}{2}\,ab^2x^6\text{sgn}\left(bx+a\right)+\frac{3}{5}\,a^2bx^5\text{sgn}\left(bx+a\right)+\frac{1}{4}\,a^3x^4\text{sgn}\left(bx+a\right)-\frac{a^7\text{sgn}\left(bx+a\right)}{140\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/7*b^3*x^7*sgn(b*x + a) + 1/2*a*b^2*x^6*sgn(b*x + a) + 3/5*a^2*b*x^5*sgn(b
*x + a) + 1/4*a^3*x^4*sgn(b*x + a) - 1/140*a^7*sgn(b*x + a)/b^4
```

3.151 $\int x^2 \left(a^2+2abx+b^2x^2\right)^{3/2} dx$

Optimal. Leaf size=96

$$\frac{(a+bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{6b^3}-\frac{2a\left(a^2+2abx+b^2x^2\right)^{5/2}}{5b^3}+\frac{a^2(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{4b^3}$$

```
[Out] (a^2*(a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/(4*b^3) - (2*a*(a^2 + 2*a*b
*x + b^2*x^2)^(5/2))/(5*b^3) + ((a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/
(6*b^3)
```

Rubi [A] time = 0.0310885, antiderivative size = 107, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {645}

$$\frac{\sqrt{a^2+2abx+b^2x^2}(a+bx)^5}{6b^3}-\frac{2a\sqrt{a^2+2abx+b^2x^2}(a+bx)^4}{5b^3}+\frac{a^2\sqrt{a^2+2abx+b^2x^2}(a+bx)^3}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (a^2*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*b^3) - (2*a*(a + b*x)^4*
Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*b^3) + ((a + b*x)^5*Sqrt[a^2 + 2*a*b*x +
b^2*x^2])/(6*b^3)
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[ExpandLinearProduct[(b/2 + c*x)^(2*p), (d + e*x)^m, b/2, c
, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&
!IntegerQ[p] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 0] && EqQ[m - 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a^2+2abx+b^2x^2\right)^{3/2} dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int\left(\frac{a^2\left(ab+b^2x\right)^3}{b^2}-\frac{2a\left(ab+b^2x\right)^4}{b^3}+\frac{\left(ab+b^2x\right)^5}{b^4}\right)dx}{b^2\left(ab+b^2x\right)}\\ &= \frac{a^2(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{4b^3}-\frac{2a(a+bx)^4\sqrt{a^2+2abx+b^2x^2}}{5b^3}+\frac{(a+bx)^5\sqrt{a^2+2abx}}{6b^3}\end{aligned}$$

Mathematica [A] time = 0.0137966, size = 55, normalized size = 0.57

$$\frac{x^3\sqrt{(a+bx)^2}\left(45a^2bx+20a^3+36ab^2x^2+10b^3x^3\right)}{60(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (x^3*Sqrt[(a + b*x)^2]*(20*a^3 + 45*a^2*b*x + 36*a*b^2*x^2 + 10*b^3*x^3))/(
60*(a + b*x))
```

Maple [A] time = 0.184, size = 52, normalized size = 0.5

$$\frac{x^3 \left(10\, b^3 x^3 + 36\, a b^2 x^2 + 45\, a^2 b x + 20\, a^3\right)}{60\, (bx + a)^3} \left((bx + a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] 1/60*x^3*(10*b^3*x^3+36*a*b^2*x^2+45*a^2*b*x+20*a^3)*((b*x+a)^2)^(3/2)/(b*x
+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.965, size = 80, normalized size = 0.83

$$\frac{1}{6}\, b^3 x^6 + \frac{3}{5}\, a b^2 x^5 + \frac{3}{4}\, a^2 b x^4 + \frac{1}{3}\, a^3 x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*b^3*x^6 + 3/5*a*b^2*x^5 + 3/4*a^2*b*x^4 + 1/3*a^3*x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left((a + bx)^2\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral(x**2*((a + b*x)**2)**(3/2), x)
```

<u>Giac</u> [A] time = 1.30003, size = 99, normalized size = 1.03

$$\frac{1}{6}\,b^3x^6\mathrm{sgn}\left(bx+a\right)+\frac{3}{5}\,ab^2x^5\mathrm{sgn}\left(bx+a\right)+\frac{3}{4}\,a^2bx^4\mathrm{sgn}\left(bx+a\right)+\frac{1}{3}\,a^3x^3\mathrm{sgn}\left(bx+a\right)+\frac{a^6\mathrm{sgn}\left(bx+a\right)}{60\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/6*b^3*x^6*sgn(b*x + a) + 3/5*a*b^2*x^5*sgn(b*x + a) + 3/4*a^2*b*x^4*sgn(b
*x + a) + 1/3*a^3*x^3*sgn(b*x + a) + 1/60*a^6*sgn(b*x + a)/b^3
```

3.152 $\int x\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=61

$$\frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{5b^2}-\frac{a(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{4b^2}$$

```
[Out] -(a*(a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/(4*b^2) + (a^2 + 2*a*b*x + b
^2*x^2)^(5/2)/(5*b^2)
```

Rubi [A] time = 0.014963, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {640, 609}

$$\frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{5b^2}-\frac{a(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] -(a*(a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/(4*b^2) + (a^2 + 2*a*b*x + b
^2*x^2)^(5/2)/(5*b^2)
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int x\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx&=\frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{5b^2}-\frac{a\int\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx}{b}\\&=-\frac{a(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{4b^2}+\frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{5b^2}\end{aligned}$$

Mathematica [A] time = 0.0140692, size = 55, normalized size = 0.9

$$\frac{x^2\sqrt{(a+bx)^2}\left(20a^2bx+10a^3+15ab^2x^2+4b^3x^3\right)}{20(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (x^2*Sqrt[(a + b*x)^2]*(10*a^3 + 20*a^2*b*x + 15*a*b^2*x^2 + 4*b^3*x^3))/(2
0*(a + b*x))
```

Maple [A] time = 0.171, size = 52, normalized size = 0.9

$$\frac{x^2\left(4x^3b^3+15ab^2x^2+20a^2bx+10a^3\right)}{20(bx+a)^3}\left((bx+a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] 1/20*x^2*(4*b^3*x^3+15*a*b^2*x^2+20*a^2*b*x+10*a^3)*((b*x+a)^2)^(3/2)/(b*x+
a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.96812, size = 74, normalized size = 1.21

$$\frac{1}{5}b^3x^5+\frac{3}{4}ab^2x^4+a^2bx^3+\frac{1}{2}a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*b^3*x^5 + 3/4*a*b^2*x^4 + a^2*b*x^3 + 1/2*a^3*x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left((a+bx)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral(x*((a + b*x)**2)**(3/2), x)
```

Giac [A] time = 1.19716, size = 97, normalized size = 1.59

$$\frac{1}{5}\,b^{3}x^{5}\mathrm{sgn}\left(bx+a\right)+\frac{3}{4}\,ab^{2}x^{4}\mathrm{sgn}\left(bx+a\right)+a^{2}bx^{3}\mathrm{sgn}\left(bx+a\right)+\frac{1}{2}\,a^{3}x^{2}\mathrm{sgn}\left(bx+a\right)-\frac{a^{5}\mathrm{sgn}\left(bx+a\right)}{20\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/5*b^3*x^5*sgn(b*x + a) + 3/4*a*b^2*x^4*sgn(b*x + a) + a^2*b*x^3*sgn(b*x +
 a) + 1/2*a^3*x^2*sgn(b*x + a) - 1/20*a^5*sgn(b*x + a)/b^2
```

3.153 $\int \left(a^2+2abx+b^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=32

$$\frac{(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{4b}$$

```
[Out] ((a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/(4*b)
```

Rubi [A] time = 0.0048205, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {609}

$$\frac{(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] ((a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/(4*b)
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{4b}$$

Mathematica [A] time = 0.0101818, size = 23, normalized size = 0.72

$$\frac{(a+bx)\left((a+bx)^2\right)^{3/2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] ((a + b*x)*((a + b*x)^2)^(3/2))/(4*b)
```

Maple [A] time = 0.048, size = 49, normalized size = 1.5

$$\frac{x\left(b^3x^3+4\,ab^2x^2+6\,a^2bx+4\,a^3\right)}{4\,(bx+a)^3}\left((bx+a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] 1/4*x*(b^3*x^3+4*a*b^2*x^2+6*a^2*b*x+4*a^3)*((b*x+a)^2)^(3/2)/(b*x+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.92746, size = 66, normalized size = 2.06

$$\frac{1}{4}\,b^3x^4 + ab^2x^3 + \frac{3}{2}\,a^2bx^2 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2 + 2abx + b^2x^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x + b**2*x**2)**(3/2), x)
```

Giac [B] time = 1.20249, size = 93, normalized size = 2.91

$$\frac{1}{4}\,b^3x^4\mathrm{sgn}\left(bx+a\right) + ab^2x^3\mathrm{sgn}\left(bx+a\right) + \frac{3}{2}\,a^2bx^2\mathrm{sgn}\left(bx+a\right) + a^3x\mathrm{sgn}\left(bx+a\right) + \frac{a^4\mathrm{sgn}\left(bx+a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*b^3*x^4*sgn(b*x + a) + a*b^2*x^3*sgn(b*x + a) + 3/2*a^2*b*x^2*sgn(b*x +
 a) + a^3*x*sgn(b*x + a) + 1/4*a^4*sgn(b*x + a)/b
```

3.154 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=143

$$\frac{3a^2bx\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{3ab^2x^2\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)}+\frac{b^3x^3\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)}+\frac{a^3\log(x)\sqrt{a^2+2abx+b^2x^2}}{a+bx}$$

```
[Out] (3*a^2*b*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(a + b*x) + (3*a*b^2*x^2*Sqrt[a^2
 + 2*a*b*x + b^2*x^2])/(2*(a + b*x)) + (b^3*x^3*Sqrt[a^2 + 2*a*b*x + b^2*x^
2])/(3*(a + b*x)) + (a^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Log[x])/(a + b*x)
```

Rubi [A] time = 0.0341437, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$\frac{3a^2bx\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{3ab^2x^2\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)}+\frac{b^3x^3\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)}+\frac{a^3\log(x)\sqrt{a^2+2abx+b^2x^2}}{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x,x]
```

```
[Out] (3*a^2*b*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(a + b*x) + (3*a*b^2*x^2*Sqrt[a^2
 + 2*a*b*x + b^2*x^2])/(2*(a + b*x)) + (b^3*x^3*Sqrt[a^2 + 2*a*b*x + b^2*x^
2])/(3*(a + b*x)) + (a^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Log[x])/(a + b*x)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x}\,dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^3}{x}\,dx}{b^2\left(ab+b^2x\right)}\\ &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(3a^2b^4+\frac{a^3b^3}{x}+3ab^5x+b^6x^2\right)dx}{b^2\left(ab+b^2x\right)}\\ &= \frac{3a^2bx\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{3ab^2x^2\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)}+\frac{b^3x^3\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)}+\frac{a^3\sqrt{a^2}}{}\end{aligned}$$

Mathematica [A] time = 0.0159683, size = 52, normalized size = 0.36

$$\frac{\sqrt{(a+bx)^2}\left(bx\left(18a^2+9abx+2b^2x^2\right)+6a^3\log(x)\right)}{6(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(b*x*(18*a^2 + 9*a*b*x + 2*b^2*x^2) + 6*a^3*Log[x]))/(6*
(a + b*x))
```

Maple [A] time = 0.219, size = 51, normalized size = 0.4

$$\frac{2\,b^3x^3+9\,ab^2x^2+6\,a^3\ln(x)+18\,ba^2x}{6\,(bx+a)^3}\left((bx+a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(3/2)/x,x)
```

```
[Out] 1/6*((b*x+a)^2)^(3/2)*(2*b^3*x^3+9*a*b^2*x^2+6*a^3*ln(x)+18*b*a^2*x)/(b*x+a
)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.96813, size = 73, normalized size = 0.51

$$\frac{1}{3}\,b^3x^3+\frac{3}{2}\,ab^2x^2+3\,a^2bx+a^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left((a+bx)^2\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/x,x)
```

```
[Out] Integral(((a + b*x)**2)**(3/2)/x, x)
```

Giac [A] time = 1.13416, size = 76, normalized size = 0.53

$$\frac{1}{3}\,b^3x^3\mathrm{sgn}\left(bx+a\right)+\frac{3}{2}\,ab^2x^2\mathrm{sgn}\left(bx+a\right)+3\,a^2bx\mathrm{sgn}\left(bx+a\right)+a^3\log\left(|x|\right)\mathrm{sgn}\left(bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/3*b^3*x^3*sgn(b*x + a) + 3/2*a*b^2*x^2*sgn(b*x + a) + 3*a^2*b*x*sgn(b*x +
 a) + a^3*log(abs(x))*sgn(b*x + a)
```

3.155 $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^2} dx$

Optimal. Leaf size=142

$$-\frac{a^3\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)}+\frac{3ab^2x\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{b^3x^2\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)}+\frac{3a^2b\log(x)\sqrt{a^2+2abx+b^2x^2}}{a+bx}$$

```
[Out] -((a^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(x*(a + b*x))) + (3*a*b^2*x*Sqrt[a^2
+ 2*a*b*x + b^2*x^2])/(a + b*x) + (b^3*x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(
2*(a + b*x)) + (3*a^2*b*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Log[x])/(a + b*x)
```

Rubi [A] time = 0.0344887, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$-\frac{a^3\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)}+\frac{3ab^2x\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{b^3x^2\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)}+\frac{3a^2b\log(x)\sqrt{a^2+2abx+b^2x^2}}{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^2,x]
```

```
[Out] -((a^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(x*(a + b*x))) + (3*a*b^2*x*Sqrt[a^2
+ 2*a*b*x + b^2*x^2])/(a + b*x) + (b^3*x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(
2*(a + b*x)) + (3*a^2*b*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Log[x])/(a + b*x)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^2}\,dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^3}{x^2}\,dx}{b^2\left(ab+b^2x\right)}\\ &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(3ab^5+\frac{a^3b^3}{x^2}+\frac{3a^2b^4}{x}+b^6x\right)dx}{b^2\left(ab+b^2x\right)}\\ &= -\frac{a^3\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)}+\frac{3ab^2x\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{b^3x^2\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)}+\frac{3a^2b\sqrt{}}{}\end{aligned}$$

Mathematica [A] time = 0.0171186, size = 56, normalized size = 0.39

$$\frac{\sqrt{(a+bx)^2}\left(6a^2bx\log(x)-2a^3+6ab^2x^2+b^3x^3\right)}{2x(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(-2*a^3 + 6*a*b^2*x^2 + b^3*x^3 + 6*a^2*b*x*Log[x]))/(2*
x*(a + b*x))
```

Maple [A] time = 0.222, size = 53, normalized size = 0.4

$$\frac{b^3x^3+6\,ba^2\ln(x)\,x+6\,ab^2x^2-2\,a^3}{2\,(bx+a)^3\,x}\left((bx+a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^2,x)
```

```
[Out] 1/2*((b*x+a)^2)^(3/2)*(b^3*x^3+6*b*a^2*ln(x)*x+6*a*b^2*x^2-2*a^3)/(b*x+a)^3
/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88336, size = 78, normalized size = 0.55

$$\frac{b^3x^3+6\,ab^2x^2+6\,a^2bx\log(x)-2\,a^3}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 + 6*a*b^2*x^2 + 6*a^2*b*x*log(x) - 2*a^3)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left((a+bx)^2\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/x**2,x)
```

```
[Out] Integral(((a + b*x)**2)**(3/2)/x**2, x)
```

Giac [A] time = 1.28843, size = 77, normalized size = 0.54

$$\frac{1}{2}b^3x^2\mathrm{sgn}\left(bx+a\right)+3\,ab^2x\mathrm{sgn}\left(bx+a\right)+3\,a^2b\log\left(|x|\right)\mathrm{sgn}\left(bx+a\right)-\frac{a^3\mathrm{sgn}\left(bx+a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^2*sgn(b*x + a) + 3*a*b^2*x*sgn(b*x + a) + 3*a^2*b*log(abs(x))*sgn
(b*x + a) - a^3*sgn(b*x + a)/x
```

3.156 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=141

$$-\frac{a^3\sqrt{a^2+2abx+b^2x^2}}{2x^2(a+bx)}-\frac{3a^2b\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)}+\frac{b^3x\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{3ab^2\log(x)\sqrt{a^2+2abx+b^2x^2}}{a+bx}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*x^2*(a + b*x)) - (3*a^2*b*Sqrt[a^2
+ 2*a*b*x + b^2*x^2])/(x*(a + b*x)) + (b^3*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
/(a + b*x) + (3*a*b^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Log[x])/(a + b*x)
```

Rubi [A] time = 0.0355548, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$-\frac{a^3\sqrt{a^2+2abx+b^2x^2}}{2x^2(a+bx)}-\frac{3a^2b\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)}+\frac{b^3x\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{3ab^2\log(x)\sqrt{a^2+2abx+b^2x^2}}{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^3,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*x^2*(a + b*x)) - (3*a^2*b*Sqrt[a^2
+ 2*a*b*x + b^2*x^2])/(x*(a + b*x)) + (b^3*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
/(a + b*x) + (3*a*b^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Log[x])/(a + b*x)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^3}\,dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^3}{x^3}\,dx}{b^2\left(ab+b^2x\right)}\\ &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(b^6+\frac{a^3b^3}{x^3}+\frac{3a^2b^4}{x^2}+\frac{3ab^5}{x}\right)dx}{b^2\left(ab+b^2x\right)}\\ &= -\frac{a^3\sqrt{a^2+2abx+b^2x^2}}{2x^2(a+bx)}-\frac{3a^2b\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)}+\frac{b^3x\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{3ab^2\sqrt{a^2+}}{}\end{aligned}$$

Mathematica [A] time = 0.0155769, size = 55, normalized size = 0.39

$$-\frac{\sqrt{(a+bx)^2}\left(6a^2bx+a^3-6ab^2x^2\log(x)-2b^3x^3\right)}{2x^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^3,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(a^3 + 6*a^2*b*x - 2*b^3*x^3 - 6*a*b^2*x^2*Log[x]))/(2*
x^2*(a + b*x))
```

Maple [A] time = 0.209, size = 54, normalized size = 0.4

$$\frac{6\,b^2a\ln(x)\,x^2+2\,b^3x^3-6\,ba^2x-a^3}{2\,(bx+a)^3\,x^2}\left((bx+a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^3,x)
```

```
[Out] 1/2*((b*x+a)^2)^(3/2)*(6*b^2*a*ln(x)*x^2+2*b^3*x^3-6*b*a^2*x-a^3)/(b*x+a)^3
/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.99319, size = 81, normalized size = 0.57

$$\frac{2\,b^3x^3+6\,ab^2x^2\log(x)-6\,a^2bx-a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*x^3 + 6*a*b^2*x^2*log(x) - 6*a^2*b*x - a^3)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left((a+bx)^2\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/x**3,x)
```

```
[Out] Integral(((a + b*x)**2)**(3/2)/x**3, x)
```

Giac [A] time = 1.24739, size = 76, normalized size = 0.54

$$b^3x\mathrm{sgn}\,(bx+a)+3\,ab^2\log\left(|x|\right)\mathrm{sgn}\,(bx+a)-\frac{6\,a^2bx\mathrm{sgn}\,(bx+a)+a^3\mathrm{sgn}\,(bx+a)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] b^3*x*sgn(b*x + a) + 3*a*b^2*log(abs(x))*sgn(b*x + a) - 1/2*(6*a^2*b*x*sgn(
b*x + a) + a^3*sgn(b*x + a))/x^2
```

3.157 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^4} \, dx$

Optimal. Leaf size=145

$$-\frac{a^3\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)}-\frac{3a^2b\sqrt{a^2+2abx+b^2x^2}}{2x^2(a+bx)}-\frac{3ab^2\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)}+\frac{b^3\log(x)\sqrt{a^2+2abx+b^2x^2}}{a+bx}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*x^3*(a + b*x)) - (3*a^2*b*Sqrt[a^2
+ 2*a*b*x + b^2*x^2])/(2*x^2*(a + b*x)) - (3*a*b^2*Sqrt[a^2 + 2*a*b*x + b^2
*x^2])/(x*(a + b*x)) + (b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Log[x])/(a + b*x)
```

Rubi [A] time = 0.0344629, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$-\frac{a^3\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)}-\frac{3a^2b\sqrt{a^2+2abx+b^2x^2}}{2x^2(a+bx)}-\frac{3ab^2\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)}+\frac{b^3\log(x)\sqrt{a^2+2abx+b^2x^2}}{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^4,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*x^3*(a + b*x)) - (3*a^2*b*Sqrt[a^2
+ 2*a*b*x + b^2*x^2])/(2*x^2*(a + b*x)) - (3*a*b^2*Sqrt[a^2 + 2*a*b*x + b^2
*x^2])/(x*(a + b*x)) + (b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Log[x])/(a + b*x)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^4}\,dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^3}{x^4}\,dx}{b^2\left(ab+b^2x\right)}\\
&= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{a^3b^3}{x^4}+\frac{3a^2b^4}{x^3}+\frac{3ab^5}{x^2}+\frac{b^6}{x}\right)dx}{b^2\left(ab+b^2x\right)}\\
&= -\frac{a^3\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)}-\frac{3a^2b\sqrt{a^2+2abx+b^2x^2}}{2x^2(a+bx)}-\frac{3ab^2\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)}+\frac{b^3\sqrt{a^2}}{}
\end{aligned}$$

Mathematica [A] time = 0.0188272, size = 57, normalized size = 0.39

$$-\frac{\sqrt{(a+bx)^2}\left(a\left(2a^2+9abx+18b^2x^2\right)-6b^3x^3\log(x)\right)}{6x^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^4,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(a*(2*a^2 + 9*a*b*x + 18*b^2*x^2) - 6*b^3*x^3*Log[x]))/
(6*x^3*(a + b*x))
```

Maple [A] time = 0.223, size = 54, normalized size = 0.4

$$\frac{6\,b^3\ln(x)\,x^3-18\,ab^2x^2-9\,ba^2x-2\,a^3}{6\,(bx+a)^3\,x^3}\left((bx+a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^4,x)
```

```
[Out] 1/6*((b*x+a)^2)^(3/2)*(6*b^3*ln(x)*x^3-18*a*b^2*x^2-9*b*a^2*x-2*a^3)/(b*x+a
)^3/x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.01924, size = 85, normalized size = 0.59

$$\frac{6\,b^3x^3\log(x)-18\,ab^2x^2-9\,a^2bx-2\,a^3}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*x^3*log(x) - 18*a*b^2*x^2 - 9*a^2*b*x - 2*a^3)/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left((a+bx)^2\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/x**4,x)
```

```
[Out] Integral(((a + b*x)**2)**(3/2)/x**4, x)
```

Giac [A] time = 1.26428, size = 80, normalized size = 0.55

$$b^3 \log(|x|) \operatorname{sgn}(bx + a) - \frac{18\, ab^2x^2 \operatorname{sgn}(bx + a) + 9\, a^2bx \operatorname{sgn}(bx + a) + 2\, a^3 \operatorname{sgn}(bx + a)}{6\, x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] b^3*log(abs(x))*sgn(b*x + a) - 1/6*(18*a*b^2*x^2*sgn(b*x + a) + 9*a^2*b*x*s
gn(b*x + a) + 2*a^3*sgn(b*x + a))/x^3
```

3.158 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=37

$$-\frac{(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{4ax^4}$$

```
[Out] -((a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*a*x^4)
```

Rubi [A] time = 0.0136598, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 37}

$$-\frac{(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^5,x]
```

```
[Out] -((a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*a*x^4)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^5}\,dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^3}{x^5}\,dx}{b^2\left(ab+b^2x\right)}\\ &= -\frac{(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{4ax^4}\end{aligned}$$

Mathematica [A] time = 0.0119043, size = 53, normalized size = 1.43

$$-\frac{\sqrt{(a+bx)^2}\left(4a^2bx+a^3+6ab^2x^2+4b^3x^3\right)}{4x^4(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^5,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(a^3 + 4*a^2*b*x + 6*a*b^2*x^2 + 4*b^3*x^3))/(4*x^4*(a
+ b*x))
```

Maple [B] time = 0.166, size = 50, normalized size = 1.4

$$-\frac{4\,b^{3}x^{3}+6\,ab^{2}x^{2}+4\,ba^{2}x+a^{3}}{4\,x^{4}\left(bx+a\right)^{3}}\left(\left(bx+a\right)^{2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^5,x)
```

```
[Out] -1/4*(4*b^3*x^3+6*a*b^2*x^2+4*a^2*b*x+a^3)*((b*x+a)^2)^(3/2)/x^4/(b*x+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.95822, size = 73, normalized size = 1.97

$$-\frac{4\,b^{3}x^{3}+6\,ab^{2}x^{2}+4\,a^{2}bx+a^{3}}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx\right)^{2}\right)^{\frac{3}{2}}}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/x**5,x)
```

```
[Out] Integral(((a + b*x)**2)**(3/2)/x**5, x)
```

Giac [B] time = 1.37638, size = 99, normalized size = 2.68

$$-\frac{b^4\mathrm{sgn}\,(bx+a)}{4\,a}-\frac{4\,b^3x^3\mathrm{sgn}\,(bx+a)+6\,ab^2x^2\mathrm{sgn}\,(bx+a)+4\,a^2bx\mathrm{sgn}\,(bx+a)+a^3\mathrm{sgn}\,(bx+a)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/4*b^4*sgn(b*x + a)/a - 1/4*(4*b^3*x^3*sgn(b*x + a) + 6*a*b^2*x^2*sgn(b*x
 + a) + 4*a^2*b*x*sgn(b*x + a) + a^3*sgn(b*x + a))/x^4
```

3.159 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=76

$$\frac{b(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{20a^2x^4}-\frac{(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{5ax^5}$$

```
[Out] -((a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*a*x^5) + (b*(a + b*x)^3*Sqr
t[a^2 + 2*a*b*x + b^2*x^2])/(20*a^2*x^4)
```

Rubi [A] time = 0.0234828, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {646, 45, 37}

$$\frac{b(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{20a^2x^4}-\frac{(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^6,x]
```

```
[Out] -((a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*a*x^5) + (b*(a + b*x)^3*Sqr
t[a^2 + 2*a*b*x + b^2*x^2])/(20*a^2*x^4)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^6}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^3}{x^6}\,dx}{b^2\left(ab+b^2x\right)}$$
$$= -\frac{(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{5ax^5} - \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^3}{x^5}\,dx}{5ab\left(ab+b^2x\right)}$$
$$= -\frac{(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{5ax^5} + \frac{b(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{20a^2x^4}$$

Mathematica [A] time = 0.0154006, size = 55, normalized size = 0.72

$$-\frac{\sqrt{(a+bx)^2}\left(15a^2bx+4a^3+20ab^2x^2+10b^3x^3\right)}{20x^5(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^6,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(4*a^3 + 15*a^2*b*x + 20*a*b^2*x^2 + 10*b^3*x^3))/(20*x
^5*(a + b*x))
```

Maple [A] time = 0.184, size = 52, normalized size = 0.7

$$-\frac{10\,b^3x^3+20\,ab^2x^2+15\,ba^2x+4\,a^3}{20\,x^5\,(bx+a)^3}\left((bx+a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^6,x)
```

```
[Out] -1/20*(10*b^3*x^3+20*a*b^2*x^2+15*a^2*b*x+4*a^3)*((b*x+a)^2)^(3/2)/x^5/(b*x
+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.01163, size = 81, normalized size = 1.07

$$-\frac{10\,b^3x^3+20\,ab^2x^2+15\,a^2bx+4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/20*(10*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 4*a^3)/x^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((a+bx)^2\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/x**6,x)
```

```
[Out] Integral(((a + b*x)**2)**(3/2)/x**6, x)
```

Giac [A] time = 1.22869, size = 100, normalized size = 1.32

$$\frac{b^5\operatorname{sgn}(bx+a)}{20\,a^2}-\frac{10\,b^3x^3\operatorname{sgn}(bx+a)+20\,ab^2x^2\operatorname{sgn}(bx+a)+15\,a^2bx\operatorname{sgn}(bx+a)+4\,a^3\operatorname{sgn}(bx+a)}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/20*b^5*sgn(b*x + a)/a^2 - 1/20*(10*b^3*x^3*sgn(b*x + a) + 20*a*b^2*x^2*sg
n(b*x + a) + 15*a^2*b*x*sgn(b*x + a) + 4*a^3*sgn(b*x + a))/x^5
```

3.160 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=151

$$-\frac{a^3\sqrt{a^2+2abx+b^2x^2}}{6x^6(a+bx)}-\frac{3a^2b\sqrt{a^2+2abx+b^2x^2}}{5x^5(a+bx)}-\frac{3ab^2\sqrt{a^2+2abx+b^2x^2}}{4x^4(a+bx)}-\frac{b^3\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*x^6*(a + b*x)) - (3*a^2*b*Sqrt[a^2
+ 2*a*b*x + b^2*x^2])/(5*x^5*(a + b*x)) - (3*a*b^2*Sqrt[a^2 + 2*a*b*x + b^2
*x^2])/(4*x^4*(a + b*x)) - (b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*x^3*(a +
b*x))
```

Rubi [A] time = 0.0347421, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$-\frac{a^3\sqrt{a^2+2abx+b^2x^2}}{6x^6(a+bx)}-\frac{3a^2b\sqrt{a^2+2abx+b^2x^2}}{5x^5(a+bx)}-\frac{3ab^2\sqrt{a^2+2abx+b^2x^2}}{4x^4(a+bx)}-\frac{b^3\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^7,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*x^6*(a + b*x)) - (3*a^2*b*Sqrt[a^2
+ 2*a*b*x + b^2*x^2])/(5*x^5*(a + b*x)) - (3*a*b^2*Sqrt[a^2 + 2*a*b*x + b^2
*x^2])/(4*x^4*(a + b*x)) - (b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*x^3*(a +
b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^7}\,dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^3}{x^7}\,dx}{b^2\left(ab+b^2x\right)}\\ &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{a^3b^3}{x^7}+\frac{3a^2b^4}{x^6}+\frac{3ab^5}{x^5}+\frac{b^6}{x^4}\right)dx}{b^2\left(ab+b^2x\right)}\\ &= -\frac{a^3\sqrt{a^2+2abx+b^2x^2}}{6x^6(a+bx)}-\frac{3a^2b\sqrt{a^2+2abx+b^2x^2}}{5x^5(a+bx)}-\frac{3ab^2\sqrt{a^2+2abx+b^2x^2}}{4x^4(a+bx)}-\frac{b^3\sqrt{a^2+2a}}{3x^3(a}\end{aligned}$$

Mathematica [A] time = 0.0138948, size = 55, normalized size = 0.36

$$-\frac{\sqrt{(a+bx)^2}\left(36a^2bx+10a^3+45ab^2x^2+20b^3x^3\right)}{60x^6(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^7,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(10*a^3 + 36*a^2*b*x + 45*a*b^2*x^2 + 20*b^3*x^3))/(60*
x^6*(a + b*x))
```

Maple [A] time = 0.176, size = 52, normalized size = 0.3

$$-\frac{20\,b^3x^3+45\,ab^2x^2+36\,ba^2x+10\,a^3}{60\,x^6\,(bx+a)^3}\left((bx+a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^7,x)
```

```
[Out] -1/60*(20*b^3*x^3+45*a*b^2*x^2+36*a^2*b*x+10*a^3)*((b*x+a)^2)^(3/2)/x^6/(b*
x+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.01139, size = 82, normalized size = 0.54

$$-\frac{20\,b^3x^3+45\,ab^2x^2+36\,a^2bx+10\,a^3}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/60*(20*b^3*x^3 + 45*a*b^2*x^2 + 36*a^2*b*x + 10*a^3)/x^6
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left((a+bx)^2\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/x**7,x)
```

```
[Out] Integral(((a + b*x)**2)**(3/2)/x**7, x)
```

Giac [A] time = 1.49118, size = 100, normalized size = 0.66

$$-\frac{b^6 \operatorname{sgn}(bx+a)}{60\, a^3} - \frac{20\, b^3 x^3 \operatorname{sgn}(bx+a) + 45\, ab^2 x^2 \operatorname{sgn}(bx+a) + 36\, a^2 bx \operatorname{sgn}(bx+a) + 10\, a^3 \operatorname{sgn}(bx+a)}{60\, x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/60*b^6*sgn(b*x + a)/a^3 - 1/60*(20*b^3*x^3*sgn(b*x + a) + 45*a*b^2*x^2*s
gn(b*x + a) + 36*a^2*b*x*sgn(b*x + a) + 10*a^3*sgn(b*x + a))/x^6
```

3.161 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=151

$$-\frac{a^3\sqrt{a^2+2abx+b^2x^2}}{7x^7(a+bx)}-\frac{a^2b\sqrt{a^2+2abx+b^2x^2}}{2x^6(a+bx)}-\frac{3ab^2\sqrt{a^2+2abx+b^2x^2}}{5x^5(a+bx)}-\frac{b^3\sqrt{a^2+2abx+b^2x^2}}{4x^4(a+bx)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*x^7*(a + b*x)) - (a^2*b*Sqrt[a^2 +
2*a*b*x + b^2*x^2])/(2*x^6*(a + b*x)) - (3*a*b^2*Sqrt[a^2 + 2*a*b*x + b^2*x
^2])/(5*x^5*(a + b*x)) - (b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*x^4*(a + b*
x))
```

Rubi [A] time = 0.0372512, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$-\frac{a^3\sqrt{a^2+2abx+b^2x^2}}{7x^7(a+bx)}-\frac{a^2b\sqrt{a^2+2abx+b^2x^2}}{2x^6(a+bx)}-\frac{3ab^2\sqrt{a^2+2abx+b^2x^2}}{5x^5(a+bx)}-\frac{b^3\sqrt{a^2+2abx+b^2x^2}}{4x^4(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^8,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*x^7*(a + b*x)) - (a^2*b*Sqrt[a^2 +
2*a*b*x + b^2*x^2])/(2*x^6*(a + b*x)) - (3*a*b^2*Sqrt[a^2 + 2*a*b*x + b^2*x
^2])/(5*x^5*(a + b*x)) - (b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*x^4*(a + b*
x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^8}\,dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^3}{x^8}\,dx}{b^2\left(ab+b^2x\right)}\\ &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{a^3b^3}{x^8}+\frac{3a^2b^4}{x^7}+\frac{3ab^5}{x^6}+\frac{b^6}{x^5}\right)dx}{b^2\left(ab+b^2x\right)}\\ &= -\frac{a^3\sqrt{a^2+2abx+b^2x^2}}{7x^7(a+bx)}-\frac{a^2b\sqrt{a^2+2abx+b^2x^2}}{2x^6(a+bx)}-\frac{3ab^2\sqrt{a^2+2abx+b^2x^2}}{5x^5(a+bx)}-\frac{b^3\sqrt{a^2+4x^4(}}{}\end{aligned}$$

Mathematica [A] time = 0.0118509, size = 55, normalized size = 0.36

$$-\frac{\sqrt{(a+bx)^2}\left(70a^2bx+20a^3+84ab^2x^2+35b^3x^3\right)}{140x^7(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^8,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(20*a^3 + 70*a^2*b*x + 84*a*b^2*x^2 + 35*b^3*x^3))/(140
*x^7*(a + b*x))
```

Maple [A] time = 0.184, size = 52, normalized size = 0.3

$$-\frac{35\,b^3x^3+84\,ab^2x^2+70\,ba^2x+20\,a^3}{140\,x^7\,(bx+a)^3}\left((bx+a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^8,x)
```

```
[Out] -1/140*(35*b^3*x^3+84*a*b^2*x^2+70*a^2*b*x+20*a^3)*((b*x+a)^2)^(3/2)/x^7/(b
*x+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73816, size = 84, normalized size = 0.56

$$-\frac{35\,b^3x^3+84\,ab^2x^2+70\,a^2bx+20\,a^3}{140\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/140*(35*b^3*x^3 + 84*a*b^2*x^2 + 70*a^2*b*x + 20*a^3)/x^7
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left((a+bx)^2\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/x**8,x)
```

```
[Out] Integral(((a + b*x)**2)**(3/2)/x**8, x)
```

Giac [A] time = 1.25297, size = 100, normalized size = 0.66

$$\frac{b^7\operatorname{sgn}(bx+a)}{140\,a^4}-\frac{35\,b^3x^3\operatorname{sgn}(bx+a)+84\,ab^2x^2\operatorname{sgn}(bx+a)+70\,a^2bx\operatorname{sgn}(bx+a)+20\,a^3\operatorname{sgn}(bx+a)}{140\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/140*b^7*sgn(b*x + a)/a^4 - 1/140*(35*b^3*x^3*sgn(b*x + a) + 84*a*b^2*x^2*
sgn(b*x + a) + 70*a^2*b*x*sgn(b*x + a) + 20*a^3*sgn(b*x + a))/x^7
```

3.162 $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=151

$$-\frac{a^3\sqrt{a^2+2abx+b^2x^2}}{8x^8(a+bx)}-\frac{3a^2b\sqrt{a^2+2abx+b^2x^2}}{7x^7(a+bx)}-\frac{ab^2\sqrt{a^2+2abx+b^2x^2}}{2x^6(a+bx)}-\frac{b^3\sqrt{a^2+2abx+b^2x^2}}{5x^5(a+bx)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(8*x^8*(a + b*x)) - (3*a^2*b*Sqrt[a^2
+ 2*a*b*x + b^2*x^2])/(7*x^7*(a + b*x)) - (a*b^2*Sqrt[a^2 + 2*a*b*x + b^2*x
^2])/(2*x^6*(a + b*x)) - (b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*x^5*(a + b*
x))
```

Rubi [A] time = 0.0348462, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$-\frac{a^3\sqrt{a^2+2abx+b^2x^2}}{8x^8(a+bx)}-\frac{3a^2b\sqrt{a^2+2abx+b^2x^2}}{7x^7(a+bx)}-\frac{ab^2\sqrt{a^2+2abx+b^2x^2}}{2x^6(a+bx)}-\frac{b^3\sqrt{a^2+2abx+b^2x^2}}{5x^5(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^9,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(8*x^8*(a + b*x)) - (3*a^2*b*Sqrt[a^2
+ 2*a*b*x + b^2*x^2])/(7*x^7*(a + b*x)) - (a*b^2*Sqrt[a^2 + 2*a*b*x + b^2*x
^2])/(2*x^6*(a + b*x)) - (b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*x^5*(a + b*
x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^9}\,dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(ab+b^2x)^3}{x^9}\,dx}{b^2(ab+b^2x)}\\ &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{a^3b^3}{x^9}+\frac{3a^2b^4}{x^8}+\frac{3ab^5}{x^7}+\frac{b^6}{x^6}\right)dx}{b^2(ab+b^2x)}\\ &= -\frac{a^3\sqrt{a^2+2abx+b^2x^2}}{8x^8(a+bx)}-\frac{3a^2b\sqrt{a^2+2abx+b^2x^2}}{7x^7(a+bx)}-\frac{ab^2\sqrt{a^2+2abx+b^2x^2}}{2x^6(a+bx)}-\frac{b^3\sqrt{a^2+2ab}}{5x^5(a+}\end{aligned}$$

Mathematica [A] time = 0.0169757, size = 55, normalized size = 0.36

$$-\frac{\sqrt{(a+bx)^2}\left(120a^2bx+35a^3+140ab^2x^2+56b^3x^3\right)}{280x^8(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/x^9,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(35*a^3 + 120*a^2*b*x + 140*a*b^2*x^2 + 56*b^3*x^3))/(2
80*x^8*(a + b*x))
```

Maple [A] time = 0.181, size = 52, normalized size = 0.3

$$-\frac{56\,b^3x^3+140\,ab^2x^2+120\,ba^2x+35\,a^3}{280\,x^8\,(bx+a)^3}\left((bx+a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^9,x)
```

```
[Out] -1/280*(56*b^3*x^3+140*a*b^2*x^2+120*a^2*b*x+35*a^3)*((b*x+a)^2)^(3/2)/x^8/
(b*x+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57423, size = 86, normalized size = 0.57

$$-\frac{56\,b^3x^3+140\,ab^2x^2+120\,a^2bx+35\,a^3}{280\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/280*(56*b^3*x^3 + 140*a*b^2*x^2 + 120*a^2*b*x + 35*a^3)/x^8
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left((a+bx)^2\right)^{\frac{3}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/x**9,x)
```

```
[Out] Integral(((a + b*x)**2)**(3/2)/x**9, x)
```

Giac [A] time = 1.39493, size = 100, normalized size = 0.66

$$-\frac{b^8 \mathrm{sgn}\,(bx+a)}{280\,a^5} - \frac{56\,b^3x^3\mathrm{sgn}\,(bx+a) + 140\,ab^2x^2\mathrm{sgn}\,(bx+a) + 120\,a^2bx\mathrm{sgn}\,(bx+a) + 35\,a^3\mathrm{sgn}\,(bx+a)}{280\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] -1/280*b^8*sgn(b*x + a)/a^5 - 1/280*(56*b^3*x^3*sgn(b*x + a) + 140*a*b^2*x^
2*sgn(b*x + a) + 120*a^2*b*x*sgn(b*x + a) + 35*a^3*sgn(b*x + a))/x^8
```

3.163 $\int x^5 \left(a^2+2abx+b^2x^2\right)^{5/2} dx$

Optimal. Leaf size=231

$$\frac{b^5x^{11}\sqrt{a^2+2abx+b^2x^2}}{11(a+bx)}+\frac{ab^4x^{10}\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)}+\frac{10a^2b^3x^9\sqrt{a^2+2abx+b^2x^2}}{9(a+bx)}+\frac{5a^3b^2x^8\sqrt{a^2+2abx+b^2x^2}}{4(a+bx)}+$$

```
[Out] (a^5*x^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*(a + b*x)) + (5*a^4*b*x^7*Sqrt[a
^2 + 2*a*b*x + b^2*x^2])/(7*(a + b*x)) + (5*a^3*b^2*x^8*Sqrt[a^2 + 2*a*b*x
+ b^2*x^2])/(4*(a + b*x)) + (10*a^2*b^3*x^9*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/
(9*(a + b*x)) + (a*b^4*x^10*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*(a + b*x)) +
(b^5*x^11*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(11*(a + b*x))
```

Rubi [A] time = 0.061848, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$\frac{b^5x^{11}\sqrt{a^2+2abx+b^2x^2}}{11(a+bx)}+\frac{ab^4x^{10}\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)}+\frac{10a^2b^3x^9\sqrt{a^2+2abx+b^2x^2}}{9(a+bx)}+\frac{5a^3b^2x^8\sqrt{a^2+2abx+b^2x^2}}{4(a+bx)}+$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (a^5*x^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*(a + b*x)) + (5*a^4*b*x^7*Sqrt[a
^2 + 2*a*b*x + b^2*x^2])/(7*(a + b*x)) + (5*a^3*b^2*x^8*Sqrt[a^2 + 2*a*b*x
+ b^2*x^2])/(4*(a + b*x)) + (10*a^2*b^3*x^9*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/
(9*(a + b*x)) + (a*b^4*x^10*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*(a + b*x)) +
(b^5*x^11*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(11*(a + b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(a^2+2abx+b^2x^2\right)^{5/2} dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int x^5 \left(ab+b^2x\right)^5 dx}{b^4\left(ab+b^2x\right)} \\
&= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(a^5b^5x^5+5a^4b^6x^6+10a^3b^7x^7+10a^2b^8x^8+5ab^9x^9+b^{10}x^{10}\right)}{b^4\left(ab+b^2x\right)} \\
&= \frac{a^5x^6\sqrt{a^2+2abx+b^2x^2}}{6(a+bx)}+\frac{5a^4bx^7\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}+\frac{5a^3b^2x^8\sqrt{a^2+2abx+b^2x^2}}{4(a+bx)}
\end{aligned}$$

Mathematica [A] time = 0.0229274, size = 77, normalized size = 0.33

$$\frac{x^6\sqrt{(a+bx)^2}\left(3465a^3b^2x^2+3080a^2b^3x^3+1980a^4bx+462a^5+1386ab^4x^4+252b^5x^5\right)}{2772(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (x^6*Sqrt[(a + b*x)^2]*(462*a^5 + 1980*a^4*b*x + 3465*a^3*b^2*x^2 + 3080*a^
2*b^3*x^3 + 1386*a*b^4*x^4 + 252*b^5*x^5))/(2772*(a + b*x))
```

Maple [A] time = 0.176, size = 74, normalized size = 0.3

$$\frac{x^6\left(252\,b^5x^5+1386\,ab^4x^4+3080\,a^2b^3x^3+3465\,a^3b^2x^2+1980\,a^4bx+462\,a^5\right)}{2772\,(bx+a)^5}\left((bx+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] 1/2772*x^6*(252*b^5*x^5+1386*a*b^4*x^4+3080*a^2*b^3*x^3+3465*a^3*b^2*x^2+19
80*a^4*b*x+462*a^5)*((b*x+a)^2)^(5/2)/(b*x+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55523, size = 134, normalized size = 0.58

$$\frac{1}{11}\,b^5x^{11}+\frac{1}{2}\,ab^4x^{10}+\frac{10}{9}\,a^2b^3x^9+\frac{5}{4}\,a^3b^2x^8+\frac{5}{7}\,a^4bx^7+\frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/11*b^5*x^11 + 1/2*a*b^4*x^10 + 10/9*a^2*b^3*x^9 + 5/4*a^3*b^2*x^8 + 5/7*a
^4*b*x^7 + 1/6*a^5*x^6
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\left((a+bx)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Integral(x**5*((a + b*x)**2)**(5/2), x)
```

Giac [A] time = 1.22666, size = 144, normalized size = 0.62

$$\frac{1}{11}\,b^5x^{11}\mathrm{sgn}\,(bx+a)+\frac{1}{2}\,ab^4x^{10}\mathrm{sgn}\,(bx+a)+\frac{10}{9}\,a^2b^3x^9\mathrm{sgn}\,(bx+a)+\frac{5}{4}\,a^3b^2x^8\mathrm{sgn}\,(bx+a)+\frac{5}{7}\,a^4bx^7\mathrm{sgn}\,(bx+a)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/11*b^5*x^11*sgn(b*x + a) + 1/2*a*b^4*x^10*sgn(b*x + a) + 10/9*a^2*b^3*x^9
*sgn(b*x + a) + 5/4*a^3*b^2*x^8*sgn(b*x + a) + 5/7*a^4*b*x^7*sgn(b*x + a) +
 1/6*a^5*x^6*sgn(b*x + a) - 1/2772*a^11*sgn(b*x + a)/b^6
```

3.164 $\int x^4 \left(a^2+2abx+b^2x^2\right)^{5/2} dx$

Optimal. Leaf size=181

$$\frac{\sqrt{a^2+2abx+b^2x^2}(a+bx)^9}{10b^5}-\frac{4a\sqrt{a^2+2abx+b^2x^2}(a+bx)^8}{9b^5}+\frac{3a^2\sqrt{a^2+2abx+b^2x^2}(a+bx)^7}{4b^5}-\frac{4a^3\sqrt{a^2+2abx+b^2x}}{7b^5}$$

```
[Out] (a^4*(a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*b^5) - (4*a^3*(a + b*x)^
6*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*b^5) + (3*a^2*(a + b*x)^7*Sqrt[a^2 + 2*
a*b*x + b^2*x^2])/(4*b^5) - (4*a*(a + b*x)^8*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
/(9*b^5) + ((a + b*x)^9*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(10*b^5)
```

Rubi [A] time = 0.051446, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {645}

$$\frac{\sqrt{a^2+2abx+b^2x^2}(a+bx)^9}{10b^5}-\frac{4a\sqrt{a^2+2abx+b^2x^2}(a+bx)^8}{9b^5}+\frac{3a^2\sqrt{a^2+2abx+b^2x^2}(a+bx)^7}{4b^5}-\frac{4a^3\sqrt{a^2+2abx+b^2x}}{7b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (a^4*(a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*b^5) - (4*a^3*(a + b*x)^
6*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*b^5) + (3*a^2*(a + b*x)^7*Sqrt[a^2 + 2*
a*b*x + b^2*x^2])/(4*b^5) - (4*a*(a + b*x)^8*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
/(9*b^5) + ((a + b*x)^9*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(10*b^5)
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[ExpandLinearProduct[(b/2 + c*x)^(2*p), (d + e*x)^m, b/2, c
, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&
!IntegerQ[p] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 0] && EqQ[m - 2*p + 1, 0]
```

Rubi steps

$$\int x^4\left(a^2+2abx+b^2x^2\right)^{5/2}dx=\frac{\sqrt{a^2+2abx+b^2x^2}\int\left(\frac{a^4\left(ab+b^2x\right)^5}{b^4}-\frac{4a^3\left(ab+b^2x\right)^6}{b^5}+\frac{6a^2\left(ab+b^2x\right)^7}{b^6}-\frac{4a\left(ab+b^2x\right)^8}{b^7}+\frac{\left(ab+b^2x\right)^9}{b^8}\right)dx}{b^4\left(ab+b^2x\right)}$$
$$=\frac{a^4(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{6b^5}-\frac{4a^3(a+bx)^6\sqrt{a^2+2abx+b^2x^2}}{7b^5}+\frac{3a^2(a+bx)^7\sqrt{a^2+}}{4b^5}$$

Mathematica [A] time = 0.0186447, size = 77, normalized size = 0.43

$$\frac{x^5\sqrt{(a+bx)^2}\left(1800a^3b^2x^2+1575a^2b^3x^3+1050a^4bx+252a^5+700ab^4x^4+126b^5x^5\right)}{1260(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (x^5*Sqrt[(a + b*x)^2]*(252*a^5 + 1050*a^4*b*x + 1800*a^3*b^2*x^2 + 1575*a^
2*b^3*x^3 + 700*a*b^4*x^4 + 126*b^5*x^5))/(1260*(a + b*x))
```

Maple [A] time = 0.174, size = 74, normalized size = 0.4

$$\frac{x^5\left(126\,b^5x^5+700\,ab^4x^4+1575\,a^2b^3x^3+1800\,a^3b^2x^2+1050\,a^4bx+252\,a^5\right)}{1260\,(bx+a)^5}\left((bx+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] 1/1260*x^5*(126*b^5*x^5+700*a*b^4*x^4+1575*a^2*b^3*x^3+1800*a^3*b^2*x^2+105
0*a^4*b*x+252*a^5)*((b*x+a)^2)^(5/2)/(b*x+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65244, size = 132, normalized size = 0.73

$$\frac{1}{10}\,b^5x^{10}+\frac{5}{9}\,ab^4x^9+\frac{5}{4}\,a^2b^3x^8+\frac{10}{7}\,a^3b^2x^7+\frac{5}{6}\,a^4bx^6+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/10*b^5*x^10 + 5/9*a*b^4*x^9 + 5/4*a^2*b^3*x^8 + 10/7*a^3*b^2*x^7 + 5/6*a^
4*b*x^6 + 1/5*a^5*x^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\left((a+bx)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Integral(x**4*((a + b*x)**2)**(5/2), x)
```

Giac [A] time = 1.3708, size = 144, normalized size = 0.8

$$\frac{1}{10}b^5x^{10}\text{sgn}(bx+a)+\frac{5}{9}ab^4x^9\text{sgn}(bx+a)+\frac{5}{4}a^2b^3x^8\text{sgn}(bx+a)+\frac{10}{7}a^3b^2x^7\text{sgn}(bx+a)+\frac{5}{6}a^4bx^6\text{sgn}(bx+a)+\frac{1}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/10*b^5*x^10*sgn(b*x + a) + 5/9*a*b^4*x^9*sgn(b*x + a) + 5/4*a^2*b^3*x^8*s
gn(b*x + a) + 10/7*a^3*b^2*x^7*sgn(b*x + a) + 5/6*a^4*b*x^6*sgn(b*x + a) +
1/5*a^5*x^5*sgn(b*x + a) + 1/1260*a^10*sgn(b*x + a)/b^5
```

3.165 $\int x^3 \left(a^2+2abx+b^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=144

$$\frac{\sqrt{a^2+2abx+b^2x^2}(a+bx)^8}{9b^4}-\frac{3a\sqrt{a^2+2abx+b^2x^2}(a+bx)^7}{8b^4}+\frac{3a^2\sqrt{a^2+2abx+b^2x^2}(a+bx)^6}{7b^4}-\frac{a^3\sqrt{a^2+2abx+b^2}}{6b^4}$$

```
[Out] -(a^3*(a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*b^4) + (3*a^2*(a + b*x)
^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*b^4) - (3*a*(a + b*x)^7*Sqrt[a^2 + 2*a
*b*x + b^2*x^2])/(8*b^4) + ((a + b*x)^8*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(9*b
^4)
```

Rubi [A] time = 0.0519569, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$\frac{\sqrt{a^2+2abx+b^2x^2}(a+bx)^8}{9b^4}-\frac{3a\sqrt{a^2+2abx+b^2x^2}(a+bx)^7}{8b^4}+\frac{3a^2\sqrt{a^2+2abx+b^2x^2}(a+bx)^6}{7b^4}-\frac{a^3\sqrt{a^2+2abx+b^2}}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] -(a^3*(a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*b^4) + (3*a^2*(a + b*x)
^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*b^4) - (3*a*(a + b*x)^7*Sqrt[a^2 + 2*a
*b*x + b^2*x^2])/(8*b^4) + ((a + b*x)^8*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(9*b
^4)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a^2+2abx+b^2x^2\right)^{5/2}dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int x^3\left(ab+b^2x\right)^5 dx}{b^4\left(ab+b^2x\right)}\\
&= \frac{\sqrt{a^2+2abx+b^2x^2}\int\left(-\frac{a^3\left(ab+b^2x\right)^5}{b^3}+\frac{3a^2\left(ab+b^2x\right)^6}{b^4}-\frac{3a\left(ab+b^2x\right)^7}{b^5}+\frac{\left(ab+b^2x\right)^8}{b^6}\right)dx}{b^4\left(ab+b^2x\right)}\\
&= -\frac{a^3(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{6b^4}+\frac{3a^2(a+bx)^6\sqrt{a^2+2abx+b^2x^2}}{7b^4}-\frac{3a(a+bx)^7\sqrt{a}}{8}
\end{aligned}$$

Mathematica [A] time = 0.0190502, size = 77, normalized size = 0.53

$$\frac{x^4\sqrt{(a+bx)^2}\left(840a^3b^2x^2+720a^2b^3x^3+504a^4bx+126a^5+315ab^4x^4+56b^5x^5\right)}{504(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (x^4*Sqrt[(a + b*x)^2]*(126*a^5 + 504*a^4*b*x + 840*a^3*b^2*x^2 + 720*a^2*b
^3*x^3 + 315*a*b^4*x^4 + 56*b^5*x^5))/(504*(a + b*x))
```

Maple [A] time = 0.188, size = 74, normalized size = 0.5

$$\frac{x^4\left(56\,b^5x^5+315\,ab^4x^4+720\,a^2b^3x^3+840\,a^3b^2x^2+504\,a^4bx+126\,a^5\right)}{504\,(bx+a)^5}\left((bx+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] 1/504*x^4*(56*b^5*x^5+315*a*b^4*x^4+720*a^2*b^3*x^3+840*a^3*b^2*x^2+504*a^4
*b*x+126*a^5)*((b*x+a)^2)^(5/2)/(b*x+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7779, size = 124, normalized size = 0.86

$$\frac{1}{9}\,b^5x^9+\frac{5}{8}\,ab^4x^8+\frac{10}{7}\,a^2b^3x^7+\frac{5}{3}\,a^3b^2x^6+a^4bx^5+\frac{1}{4}\,a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/9*b^5*x^9 + 5/8*a*b^4*x^8 + 10/7*a^2*b^3*x^7 + 5/3*a^3*b^2*x^6 + a^4*b*x^
5 + 1/4*a^5*x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\left((a+bx)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Integral(x**3*((a + b*x)**2)**(5/2), x)
```

Giac [A] time = 1.34331, size = 143, normalized size = 0.99

$$\frac{1}{9}b^5x^9\operatorname{sgn}(bx+a)+\frac{5}{8}ab^4x^8\operatorname{sgn}(bx+a)+\frac{10}{7}a^2b^3x^7\operatorname{sgn}(bx+a)+\frac{5}{3}a^3b^2x^6\operatorname{sgn}(bx+a)+a^4bx^5\operatorname{sgn}(bx+a)+\frac{1}{4}a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/9*b^5*x^9*sgn(b*x + a) + 5/8*a*b^4*x^8*sgn(b*x + a) + 10/7*a^2*b^3*x^7*sg
n(b*x + a) + 5/3*a^3*b^2*x^6*sgn(b*x + a) + a^4*b*x^5*sgn(b*x + a) + 1/4*a^
5*x^4*sgn(b*x + a) - 1/504*a^9*sgn(b*x + a)/b^4
```

3.166 $\int x^2 \left(a^2+2abx+b^2x^2\right)^{5/2} \, dx$

Optimal. Leaf size=107

$$\frac{\sqrt{a^2+2abx+b^2x^2}(a+bx)^7}{8b^3}-\frac{2a\sqrt{a^2+2abx+b^2x^2}(a+bx)^6}{7b^3}+\frac{a^2\sqrt{a^2+2abx+b^2x^2}(a+bx)^5}{6b^3}$$

```
[Out] (a^2*(a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*b^3) - (2*a*(a + b*x)^6*
Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*b^3) + ((a + b*x)^7*Sqrt[a^2 + 2*a*b*x +
b^2*x^2])/(8*b^3)
```

Rubi [A] time = 0.0409676, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$\frac{\sqrt{a^2+2abx+b^2x^2}(a+bx)^7}{8b^3}-\frac{2a\sqrt{a^2+2abx+b^2x^2}(a+bx)^6}{7b^3}+\frac{a^2\sqrt{a^2+2abx+b^2x^2}(a+bx)^5}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (a^2*(a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*b^3) - (2*a*(a + b*x)^6*
Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*b^3) + ((a + b*x)^7*Sqrt[a^2 + 2*a*b*x +
b^2*x^2])/(8*b^3)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a^2+2abx+b^2x^2\right)^{5/2} \, dx &= \frac{\sqrt{a^2+2abx+b^2x^2} \int x^2 \left(ab+b^2x\right)^5 \, dx}{b^4 \left(ab+b^2x\right)}\\ &= \frac{\sqrt{a^2+2abx+b^2x^2} \int \left(\frac{a^2\left(ab+b^2x\right)^5}{b^2}-\frac{2a\left(ab+b^2x\right)^6}{b^3}+\frac{\left(ab+b^2x\right)^7}{b^4}\right) dx}{b^4 \left(ab+b^2x\right)}\\ &= \frac{a^2(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{6b^3}-\frac{2a(a+bx)^6\sqrt{a^2+2abx+b^2x^2}}{7b^3}+\frac{(a+bx)^7\sqrt{a^2+2abx+b^2x^2}}{8b^3}\end{aligned}$$

Mathematica [A] time = 0.0195258, size = 77, normalized size = 0.72

$$\frac{x^3\sqrt{(a+bx)^2}\left(336a^3b^2x^2+280a^2b^3x^3+210a^4bx+56a^5+120ab^4x^4+21b^5x^5\right)}{168(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (x^3*Sqrt[(a + b*x)^2]*(56*a^5 + 210*a^4*b*x + 336*a^3*b^2*x^2 + 280*a^2*b^
3*x^3 + 120*a*b^4*x^4 + 21*b^5*x^5))/(168*(a + b*x))
```

Maple [A] time = 0.176, size = 74, normalized size = 0.7

$$\frac{x^3\left(21\,b^5x^5+120\,ab^4x^4+280\,a^2b^3x^3+336\,a^3b^2x^2+210\,a^4bx+56\,a^5\right)}{168\,(bx+a)^5}\left((bx+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] 1/168*x^3*(21*b^5*x^5+120*a*b^4*x^4+280*a^2*b^3*x^3+336*a^3*b^2*x^2+210*a^4
*b*x+56*a^5)*((b*x+a)^2)^(5/2)/(b*x+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58719, size = 126, normalized size = 1.18

$$\frac{1}{8}\,b^5x^8+\frac{5}{7}\,ab^4x^7+\frac{5}{3}\,a^2b^3x^6+2\,a^3b^2x^5+\frac{5}{4}\,a^4bx^4+\frac{1}{3}\,a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/8*b^5*x^8 + 5/7*a*b^4*x^7 + 5/3*a^2*b^3*x^6 + 2*a^3*b^2*x^5 + 5/4*a^4*b*x
^4 + 1/3*a^5*x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left((a+bx)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Integral(x**2*((a + b*x)**2)**(5/2), x)
```

Giac [A] time = 1.3392, size = 144, normalized size = 1.35

$$\frac{1}{8}\,b^5x^8\mathrm{sgn}\left(bx+a\right)+\frac{5}{7}\,ab^4x^7\mathrm{sgn}\left(bx+a\right)+\frac{5}{3}\,a^2b^3x^6\mathrm{sgn}\left(bx+a\right)+2\,a^3b^2x^5\mathrm{sgn}\left(bx+a\right)+\frac{5}{4}\,a^4bx^4\mathrm{sgn}\left(bx+a\right)+\frac{1}{3}\,a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/8*b^5*x^8*sgn(b*x + a) + 5/7*a*b^4*x^7*sgn(b*x + a) + 5/3*a^2*b^3*x^6*sgn
(b*x + a) + 2*a^3*b^2*x^5*sgn(b*x + a) + 5/4*a^4*b*x^4*sgn(b*x + a) + 1/3*a
^5*x^3*sgn(b*x + a) + 1/168*a^8*sgn(b*x + a)/b^3
```

3.167 $\int x\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=61

$$\frac{\left(a^2+2abx+b^2x^2\right)^{7/2}}{7b^2}-\frac{a(a+bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{6b^2}$$

```
[Out] -(a*(a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/(6*b^2) + (a^2 + 2*a*b*x + b
^2*x^2)^(7/2)/(7*b^2)
```

Rubi [A] time = 0.0145801, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {640, 609}

$$\frac{\left(a^2+2abx+b^2x^2\right)^{7/2}}{7b^2}-\frac{a(a+bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] -(a*(a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/(6*b^2) + (a^2 + 2*a*b*x + b
^2*x^2)^(7/2)/(7*b^2)
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int x\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx &= \frac{\left(a^2+2abx+b^2x^2\right)^{7/2}}{7b^2}-\frac{a\int\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx}{b}\\ &= -\frac{a(a+bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{6b^2}+\frac{\left(a^2+2abx+b^2x^2\right)^{7/2}}{7b^2}\end{aligned}$$

Mathematica [A] time = 0.0194926, size = 77, normalized size = 1.26

$$\frac{x^2\sqrt{(a+bx)^2}\left(105a^3b^2x^2+84a^2b^3x^3+70a^4bx+21a^5+35ab^4x^4+6b^5x^5\right)}{42(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (x^2*Sqrt[(a + b*x)^2]*(21*a^5 + 70*a^4*b*x + 105*a^3*b^2*x^2 + 84*a^2*b^3*
x^3 + 35*a*b^4*x^4 + 6*b^5*x^5))/(42*(a + b*x))
```

Maple [A] time = 0.168, size = 74, normalized size = 1.2

$$\frac{x^2\left(6\,b^5x^5+35\,ab^4x^4+84\,a^2b^3x^3+105\,a^3b^2x^2+70\,a^4bx+21\,a^5\right)}{42\,\left(bx+a\right)^5}\left(\left(bx+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] 1/42*x^2*(6*b^5*x^5+35*a*b^4*x^4+84*a^2*b^3*x^3+105*a^3*b^2*x^2+70*a^4*b*x+
21*a^5)*((b*x+a)^2)^(5/2)/(b*x+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65767, size = 126, normalized size = 2.07

$$\frac{1}{7}\,b^5x^7+\frac{5}{6}\,ab^4x^6+2\,a^2b^3x^5+\frac{5}{2}\,a^3b^2x^4+\frac{5}{3}\,a^4bx^3+\frac{1}{2}\,a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/7*b^5*x^7 + 5/6*a*b^4*x^6 + 2*a^2*b^3*x^5 + 5/2*a^3*b^2*x^4 + 5/3*a^4*b*x
^3 + 1/2*a^5*x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\left(a+bx\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Integral(x*((a + b*x)**2)**(5/2), x)
```

Giac [A] time = 1.35764, size = 144, normalized size = 2.36

$$\frac{1}{7}\,b^5x^7\mathrm{sgn}\left(bx+a\right)+\frac{5}{6}\,ab^4x^6\mathrm{sgn}\left(bx+a\right)+2\,a^2b^3x^5\mathrm{sgn}\left(bx+a\right)+\frac{5}{2}\,a^3b^2x^4\mathrm{sgn}\left(bx+a\right)+\frac{5}{3}\,a^4bx^3\mathrm{sgn}\left(bx+a\right)+\frac{1}{2}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/7*b^5*x^7*sgn(b*x + a) + 5/6*a*b^4*x^6*sgn(b*x + a) + 2*a^2*b^3*x^5*sgn(b
*x + a) + 5/2*a^3*b^2*x^4*sgn(b*x + a) + 5/3*a^4*b*x^3*sgn(b*x + a) + 1/2*a
^5*x^2*sgn(b*x + a) - 1/42*a^7*sgn(b*x + a)/b^2
```

3.168 $\int \left(a^2+2abx+b^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=32

$$\frac{(a+bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{6b}$$

```
[Out] ((a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/(6*b)
```

Rubi [A] time = 0.0053773, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {609}

$$\frac{(a+bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] ((a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/(6*b)
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{(a+bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{6b}$$

Mathematica [A] time = 0.0104349, size = 23, normalized size = 0.72

$$\frac{(a+bx)\left((a+bx)^2\right)^{5/2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] ((a + b*x)*((a + b*x)^2)^(5/2))/(6*b)
```

Maple [B] time = 0.048, size = 71, normalized size = 2.2

$$\frac{x\left(b^5x^5+6\,ab^4x^4+15\,a^2b^3x^3+20\,a^3b^2x^2+15\,a^4bx+6\,a^5\right)}{6\,(bx+a)^5}\left((bx+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] 1/6*x*(b^5*x^5+6*a*b^4*x^4+15*a^2*b^3*x^3+20*a^3*b^2*x^2+15*a^4*b*x+6*a^5)*
((b*x+a)^2)^(5/2)/(b*x+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59964, size = 116, normalized size = 3.62

$$\frac{1}{6}\,b^{5}x^{6} + ab^{4}x^{5} + \frac{5}{2}\,a^{2}b^{3}x^{4} + \frac{10}{3}\,a^{3}b^{2}x^{3} + \frac{5}{2}\,a^{4}bx^{2} + a^{5}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/2*a^4*b*x^
2 + a^5*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^{2} + 2abx + b^{2}x^{2}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x + b**2*x**2)**(5/2), x)
```

Giac [B] time = 1.39333, size = 139, normalized size = 4.34

$$\frac{1}{6}\,b^{5}x^{6}\mathrm{sgn}\left(bx + a\right) + ab^{4}x^{5}\mathrm{sgn}\left(bx + a\right) + \frac{5}{2}\,a^{2}b^{3}x^{4}\mathrm{sgn}\left(bx + a\right) + \frac{10}{3}\,a^{3}b^{2}x^{3}\mathrm{sgn}\left(bx + a\right) + \frac{5}{2}\,a^{4}bx^{2}\mathrm{sgn}\left(bx + a\right) + a^{5}x\mathrm{s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*b^5*x^6*sgn(b*x + a) + a*b^4*x^5*sgn(b*x + a) + 5/2*a^2*b^3*x^4*sgn(b*x
 + a) + 10/3*a^3*b^2*x^3*sgn(b*x + a) + 5/2*a^4*b*x^2*sgn(b*x + a) + a^5*x*
sgn(b*x + a) + 1/6*a^6*sgn(b*x + a)/b
```

3.169 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x} dx$

Optimal. Leaf size=221

$$\frac{5a^4bx\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{5a^3b^2x^2\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{10a^2b^3x^3\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)}+\frac{5ab^4x^4\sqrt{a^2+2abx+b^2x^2}}{4(a+bx)}+\frac{b^5}{}$$

```
[Out] (5*a^4*b*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(a + b*x) + (5*a^3*b^2*x^2*Sqrt[a
^2 + 2*a*b*x + b^2*x^2])/(a + b*x) + (10*a^2*b^3*x^3*Sqrt[a^2 + 2*a*b*x + b
^2*x^2])/(3*(a + b*x)) + (5*a*b^4*x^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*(a
+ b*x)) + (b^5*x^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*(a + b*x)) + (a^5*Sqrt
[a^2 + 2*a*b*x + b^2*x^2]*Log[x])/(a + b*x)
```

Rubi [A] time = 0.0493987, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$\frac{5a^4bx\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{5a^3b^2x^2\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{10a^2b^3x^3\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)}+\frac{5ab^4x^4\sqrt{a^2+2abx+b^2x^2}}{4(a+bx)}+\frac{b^5}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x,x]
```


```
[Out] (5*a^4*b*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(a + b*x) + (5*a^3*b^2*x^2*Sqrt[a
^2 + 2*a*b*x + b^2*x^2])/(a + b*x) + (10*a^2*b^3*x^3*Sqrt[a^2 + 2*a*b*x + b
^2*x^2])/(3*(a + b*x)) + (5*a*b^4*x^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*(a
+ b*x)) + (b^5*x^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*(a + b*x)) + (a^5*Sqrt
[a^2 + 2*a*b*x + b^2*x^2]*Log[x])/(a + b*x)
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^5}{x}\,dx}{b^4\left(ab+b^2x\right)}$$
$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(5a^4b^6+\frac{a^5b^5}{x}+10a^3b^7x+10a^2b^8x^2+5ab^9x^3+b^{10}x^4\right)dx}{b^4\left(ab+b^2x\right)}$$
$$= \frac{5a^4bx\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{5a^3b^2x^2\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{10a^2b^3x^3\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)}$$

Mathematica [A] time = 0.0212908, size = 74, normalized size = 0.33

$$\frac{\sqrt{(a+bx)^2}\left(bx\left(200a^2b^2x^2+300a^3bx+300a^4+75ab^3x^3+12b^4x^4\right)+60a^5\log(x)\right)}{60(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(b*x*(300*a^4 + 300*a^3*b*x + 200*a^2*b^2*x^2 + 75*a*b^3
*x^3 + 12*b^4*x^4) + 60*a^5*Log[x]))/(60*(a + b*x))
```

Maple [A] time = 0.227, size = 73, normalized size = 0.3

$$\frac{12\,b^5x^5+75\,ab^4x^4+200\,a^2b^3x^3+300\,a^3b^2x^2+60\,a^5\ln(x)+300\,a^4bx}{60\,(bx+a)^5}\left((bx+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x,x)
```

```
[Out] 1/60*((b*x+a)^2)^(5/2)*(12*b^5*x^5+75*a*b^4*x^4+200*a^2*b^3*x^3+300*a^3*b^2
*x^2+60*a^5*ln(x)+300*a^4*b*x)/(b*x+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65914, size = 120, normalized size = 0.54

$$\frac{1}{5}\,b^5x^5+\frac{5}{4}\,ab^4x^4+\frac{10}{3}\,a^2b^3x^3+5\,a^3b^2x^2+5\,a^4bx+a^5\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] 1/5*b^5*x^5 + 5/4*a*b^4*x^4 + 10/3*a^2*b^3*x^3 + 5*a^3*b^2*x^2 + 5*a^4*b*x
+ a^5*log(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((a+bx)^2\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/x,x)
```

```
[Out] Integral(((a + b*x)**2)**(5/2)/x, x)
```

Giac [A] time = 1.38778, size = 122, normalized size = 0.55

$$\frac{1}{5}\,b^5x^5\mathrm{sgn}\left(bx+a\right)+\frac{5}{4}\,ab^4x^4\mathrm{sgn}\left(bx+a\right)+\frac{10}{3}\,a^2b^3x^3\mathrm{sgn}\left(bx+a\right)+5\,a^3b^2x^2\mathrm{sgn}\left(bx+a\right)+5\,a^4bx\mathrm{sgn}\left(bx+a\right)+a^5\log($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x,x, algorithm="giac")
```

```
[Out] 1/5*b^5*x^5*sgn(b*x + a) + 5/4*a*b^4*x^4*sgn(b*x + a) + 10/3*a^2*b^3*x^3*sg
n(b*x + a) + 5*a^3*b^2*x^2*sgn(b*x + a) + 5*a^4*b*x*sgn(b*x + a) + a^5*log(
abs(x))*sgn(b*x + a)
```

3.170 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^2} dx$

Optimal. Leaf size=220

$$-\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)}+\frac{10a^3b^2x\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{5a^2b^3x^2\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{5ab^4x^3\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)}+\frac{b^5}{}$$

```
[Out] -((a^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(x*(a + b*x))) + (10*a^3*b^2*x*Sqrt[a
^2 + 2*a*b*x + b^2*x^2])/(a + b*x) + (5*a^2*b^3*x^2*Sqrt[a^2 + 2*a*b*x + b^
2*x^2])/(a + b*x) + (5*a*b^4*x^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*(a + b*x
)) + (b^5*x^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*(a + b*x)) + (5*a^4*b*Sqrt[
a^2 + 2*a*b*x + b^2*x^2]*Log[x])/(a + b*x)
```

Rubi [A] time = 0.0504543, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$-\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)}+\frac{10a^3b^2x\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{5a^2b^3x^2\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{5ab^4x^3\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)}+\frac{b^5}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^2,x]
```


```
[Out] -((a^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(x*(a + b*x))) + (10*a^3*b^2*x*Sqrt[a
^2 + 2*a*b*x + b^2*x^2])/(a + b*x) + (5*a^2*b^3*x^2*Sqrt[a^2 + 2*a*b*x + b^
2*x^2])/(a + b*x) + (5*a*b^4*x^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*(a + b*x
)) + (b^5*x^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*(a + b*x)) + (5*a^4*b*Sqrt[
a^2 + 2*a*b*x + b^2*x^2]*Log[x])/(a + b*x)
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^2}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^5}{x^2}\,dx}{b^4\left(ab+b^2x\right)}$$
$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(10a^3b^7+\frac{a^5b^5}{x^2}+\frac{5a^4b^6}{x}+10a^2b^8x+5ab^9x^2+b^{10}x^3\right)dx}{b^4\left(ab+b^2x\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)}+\frac{10a^3b^2x\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{5a^2b^3x^2\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{5ab}{}$$

Mathematica [A] time = 0.0254742, size = 79, normalized size = 0.36

$$\frac{\sqrt{(a+bx)^2}\left(120a^3b^2x^2+60a^2b^3x^3+60a^4bx\log(x)-12a^5+20ab^4x^4+3b^5x^5\right)}{12x(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(-12*a^5 + 120*a^3*b^2*x^2 + 60*a^2*b^3*x^3 + 20*a*b^4*x
^4 + 3*b^5*x^5 + 60*a^4*b*x*Log[x]))/(12*x*(a + b*x))
```

Maple [A] time = 0.224, size = 76, normalized size = 0.4

$$\frac{3\,b^5x^5+20\,ab^4x^4+60\,a^2b^3x^3+60\,a^4b\ln(x)\,x+120\,a^3b^2x^2-12\,a^5}{12\,(bx+a)^5\,x}\left((bx+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^2,x)
```

```
[Out] 1/12*((b*x+a)^2)^(5/2)*(3*b^5*x^5+20*a*b^4*x^4+60*a^2*b^3*x^3+60*a^4*b*ln(x
)*x+120*a^3*b^2*x^2-12*a^5)/(b*x+a)^5/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66037, size = 134, normalized size = 0.61

$$\frac{3\,b^5x^5+20\,ab^4x^4+60\,a^2b^3x^3+120\,a^3b^2x^2+60\,a^4bx\log(x)-12\,a^5}{12\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^5*x^5 + 20*a*b^4*x^4 + 60*a^2*b^3*x^3 + 120*a^3*b^2*x^2 + 60*a^4*
b*x*log(x) - 12*a^5)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((a+bx)^2\right)^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/x**2,x)
```

```
[Out] Integral(((a + b*x)**2)**(5/2)/x**2, x)
```

Giac [A] time = 1.18102, size = 123, normalized size = 0.56

$$\frac{1}{4}\,b^5x^4\mathrm{sgn}\left(bx+a\right)+\frac{5}{3}\,ab^4x^3\mathrm{sgn}\left(bx+a\right)+5\,a^2b^3x^2\mathrm{sgn}\left(bx+a\right)+10\,a^3b^2x\mathrm{sgn}\left(bx+a\right)+5\,a^4b\log\left(|x|\right)\mathrm{sgn}\left(bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*b^5*x^4*sgn(b*x + a) + 5/3*a*b^4*x^3*sgn(b*x + a) + 5*a^2*b^3*x^2*sgn(b
*x + a) + 10*a^3*b^2*x*sgn(b*x + a) + 5*a^4*b*log(abs(x))*sgn(b*x + a) - a^
5*sgn(b*x + a)/x
```

3.171 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^3}\,dx$

Optimal. Leaf size=222

$$-\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{2x^2(a+bx)}-\frac{5a^4b\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)}+\frac{10a^2b^3x\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{5ab^4x^2\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)}+\frac{b^5x^3\sqrt{a^2}}{3(}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*x^2*(a + b*x)) - (5*a^4*b*Sqrt[a^2
+ 2*a*b*x + b^2*x^2])/(x*(a + b*x)) + (10*a^2*b^3*x*Sqrt[a^2 + 2*a*b*x + b^
2*x^2])/(a + b*x) + (5*a*b^4*x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*(a + b*x
)) + (b^5*x^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*(a + b*x)) + (10*a^3*b^2*Sq
rt[a^2 + 2*a*b*x + b^2*x^2]*Log[x])/(a + b*x)
```

Rubi [A] time = 0.0509013, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$-\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{2x^2(a+bx)}-\frac{5a^4b\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)}+\frac{10a^2b^3x\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{5ab^4x^2\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)}+\frac{b^5x^3\sqrt{a^2}}{3(}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^3,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*x^2*(a + b*x)) - (5*a^4*b*Sqrt[a^2
+ 2*a*b*x + b^2*x^2])/(x*(a + b*x)) + (10*a^2*b^3*x*Sqrt[a^2 + 2*a*b*x + b^
2*x^2])/(a + b*x) + (5*a*b^4*x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*(a + b*x
)) + (b^5*x^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*(a + b*x)) + (10*a^3*b^2*Sq
rt[a^2 + 2*a*b*x + b^2*x^2]*Log[x])/(a + b*x)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^3}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^5}{x^3}\,dx}{b^4\left(ab+b^2x\right)}$$

$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(10a^2b^8+\frac{a^5b^5}{x^3}+\frac{5a^4b^6}{x^2}+\frac{10a^3b^7}{x}+5ab^9x+b^{10}x^2\right)dx}{b^4\left(ab+b^2x\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{2x^2(a+bx)}-\frac{5a^4b\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)}+\frac{10a^2b^3x\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{5ab}{}$$

Mathematica [A] time = 0.0220932, size = 79, normalized size = 0.36

$$\frac{\sqrt{(a+bx)^2}\left(60a^2b^3x^3+60a^3b^2x^2\log(x)-30a^4bx-3a^5+15ab^4x^4+2b^5x^5\right)}{6x^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^3,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(-3*a^5 - 30*a^4*b*x + 60*a^2*b^3*x^3 + 15*a*b^4*x^4 + 2
*b^5*x^5 + 60*a^3*b^2*x^2*Log[x]))/(6*x^2*(a + b*x))
```

Maple [A] time = 0.235, size = 76, normalized size = 0.3

$$\frac{2b^5x^5+15ab^4x^4+60a^3b^2\ln(x)x^2+60a^2b^3x^3-30a^4bx-3a^5}{6(bx+a)^5x^2}\left((bx+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^3,x)
```

```
[Out] 1/6*((b*x+a)^2)^(5/2)*(2*b^5*x^5+15*a*b^4*x^4+60*a^3*b^2*ln(x)*x^2+60*a^2*b
^3*x^3-30*a^4*b*x-3*a^5)/(b*x+a)^5/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7083, size = 132, normalized size = 0.59

$$\frac{2b^5x^5+15ab^4x^4+60a^2b^3x^3+60a^3b^2x^2\log(x)-30a^4bx-3a^5}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^5*x^5 + 15*a*b^4*x^4 + 60*a^2*b^3*x^3 + 60*a^3*b^2*x^2*log(x) - 30
*a^4*b*x - 3*a^5)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((a+bx)^2\right)^{\frac{5}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/x**3,x)
```

```
[Out] Integral(((a + b*x)**2)**(5/2)/x**3, x)
```

Giac [A] time = 1.36792, size = 123, normalized size = 0.55

$$\frac{1}{3}\,b^5x^3\mathrm{sgn}\left(bx+a\right)+\frac{5}{2}\,ab^4x^2\mathrm{sgn}\left(bx+a\right)+10\,a^2b^3x\mathrm{sgn}\left(bx+a\right)+10\,a^3b^2\log\left(|x|\right)\mathrm{sgn}\left(bx+a\right)-\frac{10\,a^4bx\mathrm{sgn}\left(bx+a\right)+}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/3*b^5*x^3*sgn(b*x + a) + 5/2*a*b^4*x^2*sgn(b*x + a) + 10*a^2*b^3*x*sgn(b*
x + a) + 10*a^3*b^2*log(abs(x))*sgn(b*x + a) - 1/2*(10*a^4*b*x*sgn(b*x + a)
 + a^5*sgn(b*x + a))/x^2
```

3.172 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^4}\,dx$

Optimal. Leaf size=222

$$-\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)}-\frac{5a^4b\sqrt{a^2+2abx+b^2x^2}}{2x^2(a+bx)}-\frac{10a^3b^2\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)}+\frac{5ab^4x\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{b^5x^2\sqrt{a^2}}{2}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*x^3*(a + b*x)) - (5*a^4*b*Sqrt[a^2
+ 2*a*b*x + b^2*x^2])/(2*x^2*(a + b*x)) - (10*a^3*b^2*Sqrt[a^2 + 2*a*b*x +
b^2*x^2])/(x*(a + b*x)) + (5*a*b^4*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(a + b*
x) + (b^5*x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*(a + b*x)) + (10*a^2*b^3*Sq
rt[a^2 + 2*a*b*x + b^2*x^2]*Log[x])/(a + b*x)
```

Rubi [A] time = 0.0532879, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$-\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)}-\frac{5a^4b\sqrt{a^2+2abx+b^2x^2}}{2x^2(a+bx)}-\frac{10a^3b^2\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)}+\frac{5ab^4x\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{b^5x^2\sqrt{a^2}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^4,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*x^3*(a + b*x)) - (5*a^4*b*Sqrt[a^2
+ 2*a*b*x + b^2*x^2])/(2*x^2*(a + b*x)) - (10*a^3*b^2*Sqrt[a^2 + 2*a*b*x +
b^2*x^2])/(x*(a + b*x)) + (5*a*b^4*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(a + b*
x) + (b^5*x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*(a + b*x)) + (10*a^2*b^3*Sq
rt[a^2 + 2*a*b*x + b^2*x^2]*Log[x])/(a + b*x)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^4}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^5}{x^4}\,dx}{b^4\left(ab+b^2x\right)}$$
$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(5ab^9+\frac{a^5b^5}{x^4}+\frac{5a^4b^6}{x^3}+\frac{10a^3b^7}{x^2}+\frac{10a^2b^8}{x}+b^{10}x\right)dx}{b^4\left(ab+b^2x\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)}-\frac{5a^4b\sqrt{a^2+2abx+b^2x^2}}{2x^2(a+bx)}-\frac{10a^3b^2\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)}+\frac{5ab^4x\sqrt{a}}{}$$

Mathematica [A] time = 0.0224764, size = 79, normalized size = 0.36

$$\frac{\sqrt{(a+bx)^2}\left(-60a^3b^2x^2+60a^2b^3x^3\log(x)-15a^4bx-2a^5+30ab^4x^4+3b^5x^5\right)}{6x^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^4,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(-2*a^5 - 15*a^4*b*x - 60*a^3*b^2*x^2 + 30*a*b^4*x^4 + 3
*b^5*x^5 + 60*a^2*b^3*x^3*Log[x]))/(6*x^3*(a + b*x))
```

Maple [A] time = 0.225, size = 76, normalized size = 0.3

$$\frac{3\,b^5x^5+60\,a^2b^3\ln(x)\,x^3+30\,ab^4x^4-60\,a^3b^2x^2-15\,a^4bx-2\,a^5}{6\,(bx+a)^5\,x^3}\left((bx+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^4,x)
```

```
[Out] 1/6*((b*x+a)^2)^(5/2)*(3*b^5*x^5+60*a^2*b^3*ln(x)*x^3+30*a*b^4*x^4-60*a^3*b
^2*x^2-15*a^4*b*x-2*a^5)/(b*x+a)^5/x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69692, size = 132, normalized size = 0.59

$$\frac{3\,b^5x^5+30\,ab^4x^4+60\,a^2b^3x^3\log(x)-60\,a^3b^2x^2-15\,a^4bx-2\,a^5}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^5*x^5 + 30*a*b^4*x^4 + 60*a^2*b^3*x^3*log(x) - 60*a^3*b^2*x^2 - 15
*a^4*b*x - 2*a^5)/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((a+bx)^2\right)^{\frac{5}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/x**4,x)
```

```
[Out] Integral(((a + b*x)**2)**(5/2)/x**4, x)
```

Giac [A] time = 1.3254, size = 124, normalized size = 0.56

$$\frac{1}{2}\,b^5x^2\operatorname{sgn}(bx+a)+5\,ab^4x\operatorname{sgn}(bx+a)+10\,a^2b^3\log(|x|)\operatorname{sgn}(bx+a)-\frac{60\,a^3b^2x^2\operatorname{sgn}(bx+a)+15\,a^4bx\operatorname{sgn}(bx+a)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*b^5*x^2*sgn(b*x + a) + 5*a*b^4*x*sgn(b*x + a) + 10*a^2*b^3*log(abs(x))*
sgn(b*x + a) - 1/6*(60*a^3*b^2*x^2*sgn(b*x + a) + 15*a^4*b*x*sgn(b*x + a) +
 2*a^5*sgn(b*x + a))/x^3
```

3.173 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^5}\,dx$

Optimal. Leaf size=219

$$-\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{4x^4(a+bx)}-\frac{5a^4b\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)}-\frac{5a^3b^2\sqrt{a^2+2abx+b^2x^2}}{x^2(a+bx)}-\frac{10a^2b^3\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)}+\frac{b^5x\sqrt{a^2+2}}{a+}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*x^4*(a + b*x)) - (5*a^4*b*Sqrt[a^2
+ 2*a*b*x + b^2*x^2])/(3*x^3*(a + b*x)) - (5*a^3*b^2*Sqrt[a^2 + 2*a*b*x + b
^2*x^2])/(x^2*(a + b*x)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(x*(a
 + b*x)) + (b^5*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(a + b*x) + (5*a*b^4*Sqrt[
a^2 + 2*a*b*x + b^2*x^2]*Log[x])/(a + b*x)
```

Rubi [A] time = 0.0528697, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$-\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{4x^4(a+bx)}-\frac{5a^4b\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)}-\frac{5a^3b^2\sqrt{a^2+2abx+b^2x^2}}{x^2(a+bx)}-\frac{10a^2b^3\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)}+\frac{b^5x\sqrt{a^2+2}}{a+}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^5,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*x^4*(a + b*x)) - (5*a^4*b*Sqrt[a^2
+ 2*a*b*x + b^2*x^2])/(3*x^3*(a + b*x)) - (5*a^3*b^2*Sqrt[a^2 + 2*a*b*x + b
^2*x^2])/(x^2*(a + b*x)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(x*(a
 + b*x)) + (b^5*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(a + b*x) + (5*a*b^4*Sqrt[
a^2 + 2*a*b*x + b^2*x^2]*Log[x])/(a + b*x)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^5}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^5}{x^5}\,dx}{b^4\left(ab+b^2x\right)}$$
$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(b^{10}+\frac{a^5b^5}{x^5}+\frac{5a^4b^6}{x^4}+\frac{10a^3b^7}{x^3}+\frac{10a^2b^8}{x^2}+\frac{5ab^9}{x}\right)dx}{b^4\left(ab+b^2x\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{4x^4(a+bx)} - \frac{5a^4b\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)} - \frac{5a^3b^2\sqrt{a^2+2abx+b^2x^2}}{x^2(a+bx)} - \frac{10a^2b^3}{}$$

Mathematica [A] time = 0.019687, size = 79, normalized size = 0.36

$$-\frac{\sqrt{(a+bx)^2}\left(60a^3b^2x^2+120a^2b^3x^3+20a^4bx+3a^5-60ab^4x^4\log(x)-12b^5x^5\right)}{12x^4(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^5,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(3*a^5 + 20*a^4*b*x + 60*a^3*b^2*x^2 + 120*a^2*b^3*x^3
- 12*b^5*x^5 - 60*a*b^4*x^4*Log[x]))/(12*x^4*(a + b*x))
```

Maple [A] time = 0.225, size = 76, normalized size = 0.4

$$\frac{60\,ab^4\ln(x)\,x^4+12\,b^5x^5-120\,a^2b^3x^3-60\,a^3b^2x^2-20\,a^4bx-3\,a^5}{12\,(bx+a)^5\,x^4}\left((bx+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^5,x)
```

```
[Out] 1/12*((b*x+a)^2)^(5/2)*(60*a*b^4*ln(x)*x^4+12*b^5*x^5-120*a^2*b^3*x^3-60*a^
3*b^2*x^2-20*a^4*b*x-3*a^5)/(b*x+a)^5/x^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74022, size = 136, normalized size = 0.62

$$\frac{12\,b^5x^5+60\,ab^4x^4\log(x)-120\,a^2b^3x^3-60\,a^3b^2x^2-20\,a^4bx-3\,a^5}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(12*b^5*x^5 + 60*a*b^4*x^4*log(x) - 120*a^2*b^3*x^3 - 60*a^3*b^2*x^2 -
 20*a^4*b*x - 3*a^5)/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((a+bx)^2\right)^{\frac{5}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/x**5,x)
```

```
[Out] Integral(((a + b*x)**2)**(5/2)/x**5, x)
```

Giac [A] time = 1.47272, size = 123, normalized size = 0.56

$$b^5 x \operatorname{sgn}(bx+a) + 5\,ab^4 \log(|x|) \operatorname{sgn}(bx+a) - \frac{120\,a^2b^3x^3 \operatorname{sgn}(bx+a) + 60\,a^3b^2x^2 \operatorname{sgn}(bx+a) + 20\,a^4bx \operatorname{sgn}(bx+a) + 3}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] b^5*x*sgn(b*x + a) + 5*a*b^4*log(abs(x))*sgn(b*x + a) - 1/12*(120*a^2*b^3*x
^3*sgn(b*x + a) + 60*a^3*b^2*x^2*sgn(b*x + a) + 20*a^4*b*x*sgn(b*x + a) + 3
*a^5*sgn(b*x + a))/x^4
```

3.174 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^6} dx$

Optimal. Leaf size=223

$$-\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{5x^5(a+bx)}-\frac{5a^4b\sqrt{a^2+2abx+b^2x^2}}{4x^4(a+bx)}-\frac{10a^3b^2\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)}-\frac{5a^2b^3\sqrt{a^2+2abx+b^2x^2}}{x^2(a+bx)}-\frac{5ab^4\sqrt{a^2}}{x}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*x^5*(a + b*x)) - (5*a^4*b*Sqrt[a^2
+ 2*a*b*x + b^2*x^2])/(4*x^4*(a + b*x)) - (10*a^3*b^2*Sqrt[a^2 + 2*a*b*x +
b^2*x^2])/(3*x^3*(a + b*x)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(x^
2*(a + b*x)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(x*(a + b*x)) + (b^5
*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Log[x])/(a + b*x)
```

Rubi [A] time = 0.0532962, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$-\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{5x^5(a+bx)}-\frac{5a^4b\sqrt{a^2+2abx+b^2x^2}}{4x^4(a+bx)}-\frac{10a^3b^2\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)}-\frac{5a^2b^3\sqrt{a^2+2abx+b^2x^2}}{x^2(a+bx)}-\frac{5ab^4\sqrt{a^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^6,x]
```


```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*x^5*(a + b*x)) - (5*a^4*b*Sqrt[a^2
+ 2*a*b*x + b^2*x^2])/(4*x^4*(a + b*x)) - (10*a^3*b^2*Sqrt[a^2 + 2*a*b*x +
b^2*x^2])/(3*x^3*(a + b*x)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(x^
2*(a + b*x)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(x*(a + b*x)) + (b^5
*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Log[x])/(a + b*x)
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^6}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^5}{x^6}\,dx}{b^4\left(ab+b^2x\right)}$$
$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{a^5b^5}{x^6}+\frac{5a^4b^6}{x^5}+\frac{10a^3b^7}{x^4}+\frac{10a^2b^8}{x^3}+\frac{5ab^9}{x^2}+\frac{b^{10}}{x}\right)dx}{b^4\left(ab+b^2x\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{5x^5(a+bx)}-\frac{5a^4b\sqrt{a^2+2abx+b^2x^2}}{4x^4(a+bx)}-\frac{10a^3b^2\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)}-\frac{5a^2b^3\sqrt{a}}{x}$$

Mathematica [A] time = 0.0252238, size = 79, normalized size = 0.35

$$-\frac{\sqrt{(a+bx)^2}\left(a\left(200a^2b^2x^2+75a^3bx+12a^4+300ab^3x^3+300b^4x^4\right)-60b^5x^5\log(x)\right)}{60x^5(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^6,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(a*(12*a^4 + 75*a^3*b*x + 200*a^2*b^2*x^2 + 300*a*b^3*x
^3 + 300*b^4*x^4) - 60*b^5*x^5*Log[x]))/(60*x^5*(a + b*x))
```

Maple [A] time = 0.231, size = 76, normalized size = 0.3

$$\frac{60\,b^5\ln(x)\,x^5-300\,ab^4x^4-300\,a^2b^3x^3-200\,a^3b^2x^2-75\,a^4bx-12\,a^5}{60\,(bx+a)^5\,x^5}\left((bx+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^6,x)
```

```
[Out] 1/60*((b*x+a)^2)^(5/2)*(60*b^5*ln(x)*x^5-300*a*b^4*x^4-300*a^2*b^3*x^3-200*
a^3*b^2*x^2-75*a^4*b*x-12*a^5)/(b*x+a)^5/x^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77558, size = 140, normalized size = 0.63

$$\frac{60\,b^5x^5\log(x)-300\,ab^4x^4-300\,a^2b^3x^3-200\,a^3b^2x^2-75\,a^4bx-12\,a^5}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/60*(60*b^5*x^5*log(x) - 300*a*b^4*x^4 - 300*a^2*b^3*x^3 - 200*a^3*b^2*x^2
 - 75*a^4*b*x - 12*a^5)/x^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((a+bx)^2\right)^{\frac{5}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/x**6,x)
```

```
[Out] Integral(((a + b*x)**2)**(5/2)/x**6, x)
```

Giac [A] time = 1.32917, size = 126, normalized size = 0.57

$$b^5\log(|x|)\operatorname{sgn}(bx+a) - \frac{300\,ab^4x^4\operatorname{sgn}(bx+a) + 300\,a^2b^3x^3\operatorname{sgn}(bx+a) + 200\,a^3b^2x^2\operatorname{sgn}(bx+a) + 75\,a^4bx\operatorname{sgn}(bx}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] b^5*log(abs(x))*sgn(b*x + a) - 1/60*(300*a*b^4*x^4*sgn(b*x + a) + 300*a^2*b
^3*x^3*sgn(b*x + a) + 200*a^3*b^2*x^2*sgn(b*x + a) + 75*a^4*b*x*sgn(b*x + a
) + 12*a^5*sgn(b*x + a))/x^5
```

3.175 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^7} \, dx$

Optimal. Leaf size=37

$$-\frac{(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{6ax^6}$$

```
[Out] -((a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*a*x^6)
```

Rubi [A] time = 0.0137379, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 37}

$$-\frac{(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^7,x]
```

```
[Out] -((a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*a*x^6)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^7} \, dx &= \frac{\sqrt{a^2+2abx+b^2x^2} \int \frac{\left(ab+b^2x\right)^5}{x^7} \, dx}{b^4\left(ab+b^2x\right)}\\ &= -\frac{(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{6ax^6}\end{aligned}$$

Mathematica [B] time = 0.0150135, size = 75, normalized size = 2.03

$$-\frac{\sqrt{(a+bx)^2}\left(15a^3b^2x^2+20a^2b^3x^3+6a^4bx+a^5+15ab^4x^4+6b^5x^5\right)}{6x^6(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^7,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(a^5 + 6*a^4*b*x + 15*a^3*b^2*x^2 + 20*a^2*b^3*x^3 + 15
*a*b^4*x^4 + 6*b^5*x^5))/(6*x^6*(a + b*x))
```

Maple [B] time = 0.175, size = 72, normalized size = 2.

$$-\frac{6\,b^{5}x^{5}+15\,ab^{4}x^{4}+20\,a^{2}b^{3}x^{3}+15\,a^{3}b^{2}x^{2}+6\,a^{4}bx+a^{5}}{6\,x^{6}\left(bx+a\right)^{5}}\left(\left(bx+a\right)^{2}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^7,x)
```

```
[Out] -1/6*(6*b^5*x^5+15*a*b^4*x^4+20*a^2*b^3*x^3+15*a^3*b^2*x^2+6*a^4*b*x+a^5)*(
(b*x+a)^2)^(5/2)/x^6/(b*x+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.73067, size = 120, normalized size = 3.24

$$-\frac{6\,b^{5}x^{5}+15\,ab^{4}x^{4}+20\,a^{2}b^{3}x^{3}+15\,a^{3}b^{2}x^{2}+6\,a^{4}bx+a^{5}}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b*
x + a^5)/x^6
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx\right)^{2}\right)^{\frac{5}{2}}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/x**7,x)
```

```
[Out] Integral(((a + b*x)**2)**(5/2)/x**7, x)
```

Giac [B] time = 1.25764, size = 144, normalized size = 3.89

$$-\frac{b^6 \operatorname{sgn}(bx+a)}{6a} - \frac{6 b^5 x^5 \operatorname{sgn}(bx+a) + 15 a b^4 x^4 \operatorname{sgn}(bx+a) + 20 a^2 b^3 x^3 \operatorname{sgn}(bx+a) + 15 a^3 b^2 x^2 \operatorname{sgn}(bx+a) + 6 a^4 b x \operatorname{sg}}{6 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/6*b^6*sgn(b*x + a)/a - 1/6*(6*b^5*x^5*sgn(b*x + a) + 15*a*b^4*x^4*sgn(b*
x + a) + 20*a^2*b^3*x^3*sgn(b*x + a) + 15*a^3*b^2*x^2*sgn(b*x + a) + 6*a^4*
b*x*sgn(b*x + a) + a^5*sgn(b*x + a))/x^6
```

3.176 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^8} dx$

Optimal. Leaf size=76

$$\frac{b(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{42a^2x^6}-\frac{(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{7ax^7}$$

```
[Out] -((a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*a*x^7) + (b*(a + b*x)^5*Sqr
t[a^2 + 2*a*b*x + b^2*x^2])/(42*a^2*x^6)
```

Rubi [A] time = 0.0230251, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {646, 45, 37}

$$\frac{b(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{42a^2x^6}-\frac{(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^8,x]
```

```
[Out] -((a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*a*x^7) + (b*(a + b*x)^5*Sqr
t[a^2 + 2*a*b*x + b^2*x^2])/(42*a^2*x^6)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^8}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^5}{x^8}\,dx}{b^4\left(ab+b^2x\right)}$$
$$= -\frac{(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{7ax^7} - \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^5}{x^7}\,dx}{7ab^3\left(ab+b^2x\right)}$$
$$= -\frac{(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{7ax^7} + \frac{b(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{42a^2x^6}$$

Mathematica [A] time = 0.0157314, size = 77, normalized size = 1.01

$$-\frac{\sqrt{(a+bx)^2}\left(84a^3b^2x^2+105a^2b^3x^3+35a^4bx+6a^5+70ab^4x^4+21b^5x^5\right)}{42x^7(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^8,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(6*a^5 + 35*a^4*b*x + 84*a^3*b^2*x^2 + 105*a^2*b^3*x^3
+ 70*a*b^4*x^4 + 21*b^5*x^5))/(42*x^7*(a + b*x))
```

Maple [A] time = 0.203, size = 74, normalized size = 1.

$$-\frac{21\,b^5x^5+70\,ab^4x^4+105\,a^2b^3x^3+84\,a^3b^2x^2+35\,a^4bx+6\,a^5}{42\,x^7\,(bx+a)^5}\left((bx+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^8,x)
```

```
[Out] -1/42*(21*b^5*x^5+70*a*b^4*x^4+105*a^2*b^3*x^3+84*a^3*b^2*x^2+35*a^4*b*x+6*
a^5)*((b*x+a)^2)^(5/2)/x^7/(b*x+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66511, size = 128, normalized size = 1.68

$$-\frac{21\,b^5x^5+70\,ab^4x^4+105\,a^2b^3x^3+84\,a^3b^2x^2+35\,a^4bx+6\,a^5}{42\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/42*(21*b^5*x^5 + 70*a*b^4*x^4 + 105*a^2*b^3*x^3 + 84*a^3*b^2*x^2 + 35*a^
4*b*x + 6*a^5)/x^7
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((a+bx)^2\right)^{\frac{5}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/x**8,x)
```

```
[Out] Integral(((a + b*x)**2)**(5/2)/x**8, x)
```

Giac [B] time = 1.38857, size = 146, normalized size = 1.92

$$\frac{b^7 \operatorname{sgn}(bx+a)}{42\,a^2} - \frac{21\,b^5x^5 \operatorname{sgn}(bx+a) + 70\,ab^4x^4 \operatorname{sgn}(bx+a) + 105\,a^2b^3x^3 \operatorname{sgn}(bx+a) + 84\,a^3b^2x^2 \operatorname{sgn}(bx+a) + 35\,a}{42\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/42*b^7*sgn(b*x + a)/a^2 - 1/42*(21*b^5*x^5*sgn(b*x + a) + 70*a*b^4*x^4*sg
n(b*x + a) + 105*a^2*b^3*x^3*sgn(b*x + a) + 84*a^3*b^2*x^2*sgn(b*x + a) + 3
5*a^4*b*x*sgn(b*x + a) + 6*a^5*sgn(b*x + a))/x^7
```

3.177 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^9}\,dx$

Optimal. Leaf size=116

$$-\frac{b^2\sqrt{a^2+2abx+b^2x^2}(a+bx)^5}{168a^3x^6}+\frac{b\sqrt{a^2+2abx+b^2x^2}(a+bx)^5}{28a^2x^7}-\frac{\sqrt{a^2+2abx+b^2x^2}(a+bx)^5}{8ax^8}$$

```
[Out] -((a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(8*a*x^8) + (b*(a + b*x)^5*Sqr
t[a^2 + 2*a*b*x + b^2*x^2])/(28*a^2*x^7) - (b^2*(a + b*x)^5*Sqrt[a^2 + 2*a*
b*x + b^2*x^2])/(168*a^3*x^6)
```

Rubi [A] time = 0.0329352, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {646, 45, 37}

$$-\frac{b^2\sqrt{a^2+2abx+b^2x^2}(a+bx)^5}{168a^3x^6}+\frac{b\sqrt{a^2+2abx+b^2x^2}(a+bx)^5}{28a^2x^7}-\frac{\sqrt{a^2+2abx+b^2x^2}(a+bx)^5}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^9,x]
```

```
[Out] -((a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(8*a*x^8) + (b*(a + b*x)^5*Sqr
t[a^2 + 2*a*b*x + b^2*x^2])/(28*a^2*x^7) - (b^2*(a + b*x)^5*Sqrt[a^2 + 2*a*
b*x + b^2*x^2])/(168*a^3*x^6)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^9}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(ab+b^2x)^5}{x^9}\,dx}{b^4(ab+b^2x)}$$
$$= -\frac{(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{8ax^8} - \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(ab+b^2x)^5}{x^8}\,dx}{4ab^3(ab+b^2x)}$$
$$= -\frac{(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{8ax^8} + \frac{b(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{28a^2x^7} + \frac{\sqrt{a^2+2abx+b^2x^2}\int}{28a^2b^2(ab+b}$$
$$= -\frac{(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{8ax^8} + \frac{b(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{28a^2x^7} - \frac{b^2(a+bx)^5\sqrt{a^2+2ab}}{168a^3x^6}$$

Mathematica [A] time = 0.015702, size = 77, normalized size = 0.66

$$-\frac{\sqrt{(a+bx)^2}\left(280a^3b^2x^2+336a^2b^3x^3+120a^4bx+21a^5+210ab^4x^4+56b^5x^5\right)}{168x^8(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^9,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(21*a^5 + 120*a^4*b*x + 280*a^3*b^2*x^2 + 336*a^2*b^3*x
^3 + 210*a*b^4*x^4 + 56*b^5*x^5))/(168*x^8*(a + b*x))
```

Maple [A] time = 0.184, size = 74, normalized size = 0.6

$$-\frac{56\,b^5x^5+210\,ab^4x^4+336\,a^2b^3x^3+280\,a^3b^2x^2+120\,a^4bx+21\,a^5}{168\,x^8\,(bx+a)^5}\left((bx+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^9,x)
```

```
[Out] -1/168*(56*b^5*x^5+210*a*b^4*x^4+336*a^2*b^3*x^3+280*a^3*b^2*x^2+120*a^4*b*
x+21*a^5)*((b*x+a)^2)^(5/2)/x^8/(b*x+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.61849, size = 135, normalized size = 1.16

$$-\frac{56\,b^5x^5 + 210\,ab^4x^4 + 336\,a^2b^3x^3 + 280\,a^3b^2x^2 + 120\,a^4bx + 21\,a^5}{168\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/168*(56*b^5*x^5 + 210*a*b^4*x^4 + 336*a^2*b^3*x^3 + 280*a^3*b^2*x^2 + 12
0*a^4*b*x + 21*a^5)/x^8
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((a+bx)^2\right)^{\frac{5}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/x**9,x)
```

```
[Out] Integral(((a + b*x)**2)**(5/2)/x**9, x)
```

Giac [A] time = 1.41711, size = 146, normalized size = 1.26

$$-\frac{b^8\operatorname{sgn}(bx+a)}{168\,a^3} - \frac{56\,b^5x^5\operatorname{sgn}(bx+a) + 210\,ab^4x^4\operatorname{sgn}(bx+a) + 336\,a^2b^3x^3\operatorname{sgn}(bx+a) + 280\,a^3b^2x^2\operatorname{sgn}(bx+a) + 120}{168\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^9,x, algorithm="giac")
```

```
[Out] -1/168*b^8*sgn(b*x + a)/a^3 - 1/168*(56*b^5*x^5*sgn(b*x + a) + 210*a*b^4*x^
4*sgn(b*x + a) + 336*a^2*b^3*x^3*sgn(b*x + a) + 280*a^3*b^2*x^2*sgn(b*x + a
) + 120*a^4*b*x*sgn(b*x + a) + 21*a^5*sgn(b*x + a))/x^8
```

3.178 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^{10}} dx$

Optimal. Leaf size=229

$$-\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{9x^9(a+bx)}-\frac{5a^4b\sqrt{a^2+2abx+b^2x^2}}{8x^8(a+bx)}-\frac{10a^3b^2\sqrt{a^2+2abx+b^2x^2}}{7x^7(a+bx)}-\frac{5a^2b^3\sqrt{a^2+2abx+b^2x^2}}{3x^6(a+bx)}-\frac{ab^4\sqrt{a^2}}{x^5}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(9*x^9*(a + b*x)) - (5*a^4*b*Sqrt[a^2
+ 2*a*b*x + b^2*x^2])/(8*x^8*(a + b*x)) - (10*a^3*b^2*Sqrt[a^2 + 2*a*b*x +
b^2*x^2])/(7*x^7*(a + b*x)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*
x^6*(a + b*x)) - (a*b^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(x^5*(a + b*x)) - (b
^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*x^4*(a + b*x))
```

Rubi [A] time = 0.0529566, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$-\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{9x^9(a+bx)}-\frac{5a^4b\sqrt{a^2+2abx+b^2x^2}}{8x^8(a+bx)}-\frac{10a^3b^2\sqrt{a^2+2abx+b^2x^2}}{7x^7(a+bx)}-\frac{5a^2b^3\sqrt{a^2+2abx+b^2x^2}}{3x^6(a+bx)}-\frac{ab^4\sqrt{a^2}}{x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^10,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(9*x^9*(a + b*x)) - (5*a^4*b*Sqrt[a^2
+ 2*a*b*x + b^2*x^2])/(8*x^8*(a + b*x)) - (10*a^3*b^2*Sqrt[a^2 + 2*a*b*x +
b^2*x^2])/(7*x^7*(a + b*x)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*
x^6*(a + b*x)) - (a*b^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(x^5*(a + b*x)) - (b
^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*x^4*(a + b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{10}}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^5}{x^{10}}\,dx}{b^4\left(ab+b^2x\right)}$$
$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{a^5b^5}{x^{10}}+\frac{5a^4b^6}{x^9}+\frac{10a^3b^7}{x^8}+\frac{10a^2b^8}{x^7}+\frac{5ab^9}{x^6}+\frac{b^{10}}{x^5}\right)dx}{b^4\left(ab+b^2x\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{9x^9(a+bx)} - \frac{5a^4b\sqrt{a^2+2abx+b^2x^2}}{8x^8(a+bx)} - \frac{10a^3b^2\sqrt{a^2+2abx+b^2x^2}}{7x^7(a+bx)} - \frac{5a^2b^3\sqrt{a}}{3x}$$

Mathematica [A] time = 0.018933, size = 77, normalized size = 0.34

$$-\frac{\sqrt{(a+bx)^2}\left(720a^3b^2x^2+840a^2b^3x^3+315a^4bx+56a^5+504ab^4x^4+126b^5x^5\right)}{504x^9(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^10,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(56*a^5 + 315*a^4*b*x + 720*a^3*b^2*x^2 + 840*a^2*b^3*x
^3 + 504*a*b^4*x^4 + 126*b^5*x^5))/(504*x^9*(a + b*x))
```

Maple [A] time = 0.176, size = 74, normalized size = 0.3

$$-\frac{126\,b^5x^5+504\,ab^4x^4+840\,a^2b^3x^3+720\,a^3b^2x^2+315\,a^4bx+56\,a^5}{504\,x^9\,(bx+a)^5}\left((bx+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^10,x)
```

```
[Out] -1/504*(126*b^5*x^5+504*a*b^4*x^4+840*a^2*b^3*x^3+720*a^3*b^2*x^2+315*a^4*b
*x+56*a^5)*((b*x+a)^2)^(5/2)/x^9/(b*x+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66672, size = 136, normalized size = 0.59

$$-\frac{126\,b^5x^5+504\,ab^4x^4+840\,a^2b^3x^3+720\,a^3b^2x^2+315\,a^4bx+56\,a^5}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^10,x, algorithm="fricas")
```

```
[Out] -1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 + 3
15*a^4*b*x + 56*a^5)/x^9
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((a+bx)^2\right)^{\frac{5}{2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/x**10,x)
```

```
[Out] Integral(((a + b*x)**2)**(5/2)/x**10, x)
```

Giac [A] time = 1.32478, size = 146, normalized size = 0.64

$$\frac{b^9\operatorname{sgn}(bx+a)}{504\,a^4} - \frac{126\,b^5x^5\operatorname{sgn}(bx+a) + 504\,ab^4x^4\operatorname{sgn}(bx+a) + 840\,a^2b^3x^3\operatorname{sgn}(bx+a) + 720\,a^3b^2x^2\operatorname{sgn}(bx+a) +}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^10,x, algorithm="giac")
```

```
[Out] 1/504*b^9*sgn(b*x + a)/a^4 - 1/504*(126*b^5*x^5*sgn(b*x + a) + 504*a*b^4*x^
4*sgn(b*x + a) + 840*a^2*b^3*x^3*sgn(b*x + a) + 720*a^3*b^2*x^2*sgn(b*x + a
) + 315*a^4*b*x*sgn(b*x + a) + 56*a^5*sgn(b*x + a))/x^9
```

3.179 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{11}}\,dx$

Optimal. Leaf size=231

$$-\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{10x^{10}(a+bx)}-\frac{5a^4b\sqrt{a^2+2abx+b^2x^2}}{9x^9(a+bx)}-\frac{5a^3b^2\sqrt{a^2+2abx+b^2x^2}}{4x^8(a+bx)}-\frac{10a^2b^3\sqrt{a^2+2abx+b^2x^2}}{7x^7(a+bx)}-\frac{5ab^4\sqrt{a^2+}}{6x^6(a}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(10*x^10*(a + b*x)) - (5*a^4*b*Sqrt[a^
2 + 2*a*b*x + b^2*x^2])/(9*x^9*(a + b*x)) - (5*a^3*b^2*Sqrt[a^2 + 2*a*b*x +
 b^2*x^2])/(4*x^8*(a + b*x)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(
7*x^7*(a + b*x)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*x^6*(a + b*x)
) - (b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*x^5*(a + b*x))
```

Rubi [A] time = 0.0528161, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$-\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{10x^{10}(a+bx)}-\frac{5a^4b\sqrt{a^2+2abx+b^2x^2}}{9x^9(a+bx)}-\frac{5a^3b^2\sqrt{a^2+2abx+b^2x^2}}{4x^8(a+bx)}-\frac{10a^2b^3\sqrt{a^2+2abx+b^2x^2}}{7x^7(a+bx)}-\frac{5ab^4\sqrt{a^2+}}{6x^6(a}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^11,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(10*x^10*(a + b*x)) - (5*a^4*b*Sqrt[a^
2 + 2*a*b*x + b^2*x^2])/(9*x^9*(a + b*x)) - (5*a^3*b^2*Sqrt[a^2 + 2*a*b*x +
 b^2*x^2])/(4*x^8*(a + b*x)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(
7*x^7*(a + b*x)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*x^6*(a + b*x)
) - (b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*x^5*(a + b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^{11}}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(ab+b^2x)^5}{x^{11}}\,dx}{b^4(ab+b^2x)}$$
$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{a^5b^5}{x^{11}}+\frac{5a^4b^6}{x^{10}}+\frac{10a^3b^7}{x^9}+\frac{10a^2b^8}{x^8}+\frac{5ab^9}{x^7}+\frac{b^{10}}{x^6}\right)dx}{b^4(ab+b^2x)}$$
$$= -\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{10x^{10}(a+bx)}-\frac{5a^4b\sqrt{a^2+2abx+b^2x^2}}{9x^9(a+bx)}-\frac{5a^3b^2\sqrt{a^2+2abx+b^2x^2}}{4x^8(a+bx)}-\frac{10a^2b^3}{}$$

Mathematica [A] time = 0.0159684, size = 77, normalized size = 0.33

$$-\frac{\sqrt{(a+bx)^2}\,(1575a^3b^2x^2+1800a^2b^3x^3+700a^4bx+126a^5+1050ab^4x^4+252b^5x^5)}{1260x^{10}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^11,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(126*a^5 + 700*a^4*b*x + 1575*a^3*b^2*x^2 + 1800*a^2*b^
3*x^3 + 1050*a*b^4*x^4 + 252*b^5*x^5))/(1260*x^10*(a + b*x))
```

Maple [A] time = 0.183, size = 74, normalized size = 0.3

$$-\frac{252\,b^5x^5+1050\,ab^4x^4+1800\,a^2b^3x^3+1575\,a^3b^2x^2+700\,a^4bx+126\,a^5}{1260\,x^{10}\,(bx+a)^5}\left((bx+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^11,x)
```

```
[Out] -1/1260*(252*b^5*x^5+1050*a*b^4*x^4+1800*a^2*b^3*x^3+1575*a^3*b^2*x^2+700*a
^4*b*x+126*a^5)*((b*x+a)^2)^(5/2)/x^10/(b*x+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64799, size = 144, normalized size = 0.62

$$-\frac{252\,b^5x^5+1050\,ab^4x^4+1800\,a^2b^3x^3+1575\,a^3b^2x^2+700\,a^4bx+126\,a^5}{1260\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^11,x, algorithm="fricas")
```

```
[Out] -1/1260*(252*b^5*x^5 + 1050*a*b^4*x^4 + 1800*a^2*b^3*x^3 + 1575*a^3*b^2*x^2
 + 700*a^4*b*x + 126*a^5)/x^10
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((a+bx)^2\right)^{\frac{5}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/x**11,x)
```

```
[Out] Integral(((a + b*x)**2)**(5/2)/x**11, x)
```

Giac [A] time = 1.30161, size = 146, normalized size = 0.63

$$-\frac{b^{10}\operatorname{sgn}(bx+a)}{1260\,a^5}-\frac{252\,b^5x^5\operatorname{sgn}(bx+a)+1050\,ab^4x^4\operatorname{sgn}(bx+a)+1800\,a^2b^3x^3\operatorname{sgn}(bx+a)+1575\,a^3b^2x^2\operatorname{sgn}(bx+a)}{1260\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^11,x, algorithm="giac")
```

```
[Out] -1/1260*b^10*sgn(b*x + a)/a^5 - 1/1260*(252*b^5*x^5*sgn(b*x + a) + 1050*a*b
^4*x^4*sgn(b*x + a) + 1800*a^2*b^3*x^3*sgn(b*x + a) + 1575*a^3*b^2*x^2*sgn(
b*x + a) + 700*a^4*b*x*sgn(b*x + a) + 126*a^5*sgn(b*x + a))/x^10
```

3.180 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^{12}}\,dx$

Optimal. Leaf size=231

$$-\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{11x^{11}(a+bx)}-\frac{a^4b\sqrt{a^2+2abx+b^2x^2}}{2x^{10}(a+bx)}-\frac{10a^3b^2\sqrt{a^2+2abx+b^2x^2}}{9x^9(a+bx)}-\frac{5a^2b^3\sqrt{a^2+2abx+b^2x^2}}{4x^8(a+bx)}-\frac{5ab^4\sqrt{a^2}}{7x^7}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(11*x^11*(a + b*x)) - (a^4*b*Sqrt[a^2
+ 2*a*b*x + b^2*x^2])/(2*x^10*(a + b*x)) - (10*a^3*b^2*Sqrt[a^2 + 2*a*b*x +
 b^2*x^2])/(9*x^9*(a + b*x)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4
*x^8*(a + b*x)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*x^7*(a + b*x))
 - (b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*x^6*(a + b*x))
```

Rubi [A] time = 0.0560486, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$-\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{11x^{11}(a+bx)}-\frac{a^4b\sqrt{a^2+2abx+b^2x^2}}{2x^{10}(a+bx)}-\frac{10a^3b^2\sqrt{a^2+2abx+b^2x^2}}{9x^9(a+bx)}-\frac{5a^2b^3\sqrt{a^2+2abx+b^2x^2}}{4x^8(a+bx)}-\frac{5ab^4\sqrt{a^2}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^12,x]
```


```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(11*x^11*(a + b*x)) - (a^4*b*Sqrt[a^2
+ 2*a*b*x + b^2*x^2])/(2*x^10*(a + b*x)) - (10*a^3*b^2*Sqrt[a^2 + 2*a*b*x +
 b^2*x^2])/(9*x^9*(a + b*x)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4
*x^8*(a + b*x)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*x^7*(a + b*x))
 - (b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*x^6*(a + b*x))
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{12}}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^5}{x^{12}}\,dx}{b^4\left(ab+b^2x\right)}$$
$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{a^5b^5}{x^{12}}+\frac{5a^4b^6}{x^{11}}+\frac{10a^3b^7}{x^{10}}+\frac{10a^2b^8}{x^9}+\frac{5ab^9}{x^8}+\frac{b^{10}}{x^7}\right)dx}{b^4\left(ab+b^2x\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx+b^2x^2}}{11x^{11}(a+bx)} - \frac{a^4b\sqrt{a^2+2abx+b^2x^2}}{2x^{10}(a+bx)} - \frac{10a^3b^2\sqrt{a^2+2abx+b^2x^2}}{9x^9(a+bx)} - \frac{5a^2b^3\sqrt{a^2}}{4x^8}$$

Mathematica [A] time = 0.0163567, size = 77, normalized size = 0.33

$$-\frac{\sqrt{(a+bx)^2}\left(3080a^3b^2x^2+3465a^2b^3x^3+1386a^4bx+252a^5+1980ab^4x^4+462b^5x^5\right)}{2772x^{11}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/x^12,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(252*a^5 + 1386*a^4*b*x + 3080*a^3*b^2*x^2 + 3465*a^2*b
^3*x^3 + 1980*a*b^4*x^4 + 462*b^5*x^5))/(2772*x^11*(a + b*x))
```

Maple [A] time = 0.237, size = 74, normalized size = 0.3

$$-\frac{462\,b^5x^5+1980\,ab^4x^4+3465\,a^2b^3x^3+3080\,a^3b^2x^2+1386\,a^4bx+252\,a^5}{2772\,x^{11}\,(bx+a)^5}\left((bx+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^12,x)
```

```
[Out] -1/2772*(462*b^5*x^5+1980*a*b^4*x^4+3465*a^2*b^3*x^3+3080*a^3*b^2*x^2+1386*
a^4*b*x+252*a^5)*((b*x+a)^2)^(5/2)/x^11/(b*x+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^12,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74668, size = 146, normalized size = 0.63

$$-\frac{462\,b^5x^5+1980\,ab^4x^4+3465\,a^2b^3x^3+3080\,a^3b^2x^2+1386\,a^4bx+252\,a^5}{2772\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^12,x, algorithm="fricas")
```

```
[Out] -1/2772*(462*b^5*x^5 + 1980*a*b^4*x^4 + 3465*a^2*b^3*x^3 + 3080*a^3*b^2*x^2
 + 1386*a^4*b*x + 252*a^5)/x^11
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((a+bx)^2\right)^{\frac{5}{2}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/x**12,x)
```

```
[Out] Integral(((a + b*x)**2)**(5/2)/x**12, x)
```

Giac [A] time = 1.26166, size = 146, normalized size = 0.63

$$\frac{b^{11}\mathrm{sgn}\left(bx+a\right)}{2772\,a^{6}} - \frac{462\,b^{5}x^{5}\mathrm{sgn}\left(bx+a\right) + 1980\,ab^{4}x^{4}\mathrm{sgn}\left(bx+a\right) + 3465\,a^{2}b^{3}x^{3}\mathrm{sgn}\left(bx+a\right) + 3080\,a^{3}b^{2}x^{2}\mathrm{sgn}\left(bx+\right.}{2772\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/x^12,x, algorithm="giac")
```

```
[Out] 1/2772*b^11*sgn(b*x + a)/a^6 - 1/2772*(462*b^5*x^5*sgn(b*x + a) + 1980*a*b^
4*x^4*sgn(b*x + a) + 3465*a^2*b^3*x^3*sgn(b*x + a) + 3080*a^3*b^2*x^2*sgn(b
*x + a) + 1386*a^4*b*x*sgn(b*x + a) + 252*a^5*sgn(b*x + a))/x^11
```

3.181 $\int \frac{x^4}{\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=182

$$-\frac{a^3x(a+bx)}{b^4\sqrt{a^2+2abx+b^2x^2}}+\frac{a^2x^2(a+bx)}{2b^3\sqrt{a^2+2abx+b^2x^2}}-\frac{ax^3(a+bx)}{3b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{x^4(a+bx)}{4b\sqrt{a^2+2abx+b^2x^2}}+\frac{a^4(a+bx)\log(a+}{b^5\sqrt{a^2+2abx+b^2}}$$

```
[Out] -((a^3*x*(a + b*x))/(b^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])) + (a^2*x^2*(a + b*
x))/(2*b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (a*x^3*(a + b*x))/(3*b^2*Sqrt[a
^2 + 2*a*b*x + b^2*x^2]) + (x^4*(a + b*x))/(4*b*Sqrt[a^2 + 2*a*b*x + b^2*x^
2]) + (a^4*(a + b*x)*Log[a + b*x])/(b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0616439, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$-\frac{a^3x(a+bx)}{b^4\sqrt{a^2+2abx+b^2x^2}}+\frac{a^2x^2(a+bx)}{2b^3\sqrt{a^2+2abx+b^2x^2}}-\frac{ax^3(a+bx)}{3b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{x^4(a+bx)}{4b\sqrt{a^2+2abx+b^2x^2}}+\frac{a^4(a+bx)\log(a+}{b^5\sqrt{a^2+2abx+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] -((a^3*x*(a + b*x))/(b^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])) + (a^2*x^2*(a + b*
x))/(2*b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (a*x^3*(a + b*x))/(3*b^2*Sqrt[a
^2 + 2*a*b*x + b^2*x^2]) + (x^4*(a + b*x))/(4*b*Sqrt[a^2 + 2*a*b*x + b^2*x^
2]) + (a^4*(a + b*x)*Log[a + b*x])/(b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{a^2+2abx+b^2x^2}}\,dx &= \frac{\left(ab+b^2x\right)\int \frac{x^4}{ab+b^2x}\,dx}{\sqrt{a^2+2abx+b^2x^2}}\\
&= \frac{\left(ab+b^2x\right)\int \left(-\frac{a^3}{b^5}+\frac{a^2x}{b^4}-\frac{ax^2}{b^3}+\frac{x^3}{b^2}+\frac{a^4}{b^5(a+bx)}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}\\
&= -\frac{a^3x(a+bx)}{b^4\sqrt{a^2+2abx+b^2x^2}}+\frac{a^2x^2(a+bx)}{2b^3\sqrt{a^2+2abx+b^2x^2}}-\frac{ax^3(a+bx)}{3b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{x^4(a+bx)}{4b\sqrt{a^2+2abx-}}
\end{aligned}$$

Mathematica [A] time = 0.0252535, size = 68, normalized size = 0.37

$$\frac{(a+bx)\left(bx\left(6a^2bx-12a^3-4ab^2x^2+3b^3x^3\right)+12a^4\log(a+bx)\right)}{12b^5\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ((a + b*x)*(b*x*(-12*a^3 + 6*a^2*b*x - 4*a*b^2*x^2 + 3*b^3*x^3) + 12*a^4*Lo
g[a + b*x]))/(12*b^5*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.246, size = 67, normalized size = 0.4

$$\frac{(bx+a)\left(3\,b^4x^4-4\,ab^3x^3+6\,x^2a^2b^2+12\,a^4\ln(bx+a)-12\,xa^3b\right)}{12\,b^5}\frac{1}{\sqrt{(bx+a)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((b*x+a)^2)^(1/2),x)
```

```
[Out] 1/12*(b*x+a)*(3*b^4*x^4-4*a*b^3*x^3+6*x^2*a^2*b^2+12*a^4*ln(b*x+a)-12*x*a^3
*b)/((b*x+a)^2)^(1/2)/b^5
```

Maxima [A] time = 1.25644, size = 201, normalized size = 1.1

$$\frac{13\,a^4\log\left(x+\frac{a}{b}\right)}{6\left(b^2\right)^{\frac{5}{2}}}-\frac{13\,a^3x}{6\left(b^2\right)^{\frac{3}{2}}b}+\frac{13\,a^2x^2}{12\sqrt{b^2}b^2}+\frac{\sqrt{b^2x^2+2\,abx+a^2}x^3}{4\,b^2}-\frac{7\,a^4\sqrt{\frac{1}{b^2}}\log\left(x+\frac{a}{b}\right)}{6\,b^4}-\frac{7\sqrt{b^2x^2+2\,abx+a^2}ax^2}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 13/6*a^4*log(x + a/b)/(b^2)^(5/2) - 13/6*a^3*x/((b^2)^(3/2)*b) + 13/12*a^2*
x^2/(sqrt(b^2)*b^2) + 1/4*sqrt(b^2*x^2 + 2*a*b*x + a^2)*x^3/b^2 - 7/6*a^4*s
qrt(b^(-2))*log(x + a/b)/b^4 - 7/12*sqrt(b^2*x^2 + 2*a*b*x + a^2)*a*x^2/b^3
 + 7/6*sqrt(b^2*x^2 + 2*a*b*x + a^2)*a^3/b^5
```

Fricas [A] time = 1.72371, size = 117, normalized size = 0.64

$$\frac{3\,b^4x^4-4\,ab^3x^3+6\,a^2b^2x^2-12\,a^3bx+12\,a^4\log(bx+a)}{12\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*x^4 - 4*a*b^3*x^3 + 6*a^2*b^2*x^2 - 12*a^3*b*x + 12*a^4*log(b*x
 + a))/b^5
```

Sympy [A] time = 1.06767, size = 49, normalized size = 0.27

$$\frac{a^4 \log(a+bx)}{b^5} - \frac{a^3 x}{b^4} + \frac{a^2 x^2}{2b^3} - \frac{a x^3}{3b^2} + \frac{x^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/((b*x+a)**2)**(1/2),x)
```

```
[Out] a**4*log(a + b*x)/b**5 - a**3*x/b**4 + a**2*x**2/(2*b**3) - a*x**3/(3*b**2)
 + x**4/(4*b)
```

Giac [A] time = 1.35437, size = 112, normalized size = 0.62

$$\frac{a^4 \log(|bx+a|)\,\mathrm{sgn}(bx+a)}{b^5} + \frac{3\,b^3 x^4 \mathrm{sgn}(bx+a) - 4\,ab^2 x^3 \mathrm{sgn}(bx+a) + 6\,a^2 b x^2 \mathrm{sgn}(bx+a) - 12\,a^3 x \mathrm{sgn}(bx+a)}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] a^4*log(abs(b*x + a))*sgn(b*x + a)/b^5 + 1/12*(3*b^3*x^4*sgn(b*x + a) - 4*a
*b^2*x^3*sgn(b*x + a) + 6*a^2*b*x^2*sgn(b*x + a) - 12*a^3*x*sgn(b*x + a))/b
^4
```

3.182 $\int \frac{x^3}{\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=144

$$\frac{a^2x(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}}-\frac{ax^2(a+bx)}{2b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{x^3(a+bx)}{3b\sqrt{a^2+2abx+b^2x^2}}-\frac{a^3(a+bx)\log(a+bx)}{b^4\sqrt{a^2+2abx+b^2x^2}}$$

```
[Out] (a^2*x*(a + b*x))/(b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (a*x^2*(a + b*x))/(
2*b^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (x^3*(a + b*x))/(3*b*Sqrt[a^2 + 2*a*
b*x + b^2*x^2]) - (a^3*(a + b*x)*Log[a + b*x])/(b^4*Sqrt[a^2 + 2*a*b*x + b^
2*x^2])
```

Rubi [A] time = 0.0466961, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$\frac{a^2x(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}}-\frac{ax^2(a+bx)}{2b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{x^3(a+bx)}{3b\sqrt{a^2+2abx+b^2x^2}}-\frac{a^3(a+bx)\log(a+bx)}{b^4\sqrt{a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (a^2*x*(a + b*x))/(b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (a*x^2*(a + b*x))/(
2*b^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (x^3*(a + b*x))/(3*b*Sqrt[a^2 + 2*a*
b*x + b^2*x^2]) - (a^3*(a + b*x)*Log[a + b*x])/(b^4*Sqrt[a^2 + 2*a*b*x + b^
2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{a^2+2abx+b^2x^2}}\,dx &= \frac{\left(ab+b^2x\right)\int \frac{x^3}{ab+b^2x}\,dx}{\sqrt{a^2+2abx+b^2x^2}}\\ &= \frac{\left(ab+b^2x\right)\int \left(\frac{a^2}{b^4}-\frac{ax}{b^3}+\frac{x^2}{b^2}-\frac{a^3}{b^4(a+bx)}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}\\ &= \frac{a^2x(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}}-\frac{ax^2(a+bx)}{2b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{x^3(a+bx)}{3b\sqrt{a^2+2abx+b^2x^2}}-\frac{a^3(a+bx)\log(}{b^4\sqrt{a^2+2abx}}\end{aligned}$$

Mathematica [A] time = 0.0183377, size = 57, normalized size = 0.4

$$\frac{(a+bx)\left(bx\left(6a^2-3abx+2b^2x^2\right)-6a^3\log(a+bx)\right)}{6b^4\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ((a + b*x)*(b*x*(6*a^2 - 3*a*b*x + 2*b^2*x^2) - 6*a^3*Log[a + b*x]))/(6*b^4
*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.225, size = 56, normalized size = 0.4

$$-\frac{(bx+a)\left(-2\,b^3x^3+3\,ab^2x^2+6\,a^3\ln(bx+a)-6\,ba^2x\right)}{6\,b^4}\frac{1}{\sqrt{(bx+a)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((b*x+a)^2)^(1/2),x)
```

```
[Out] -1/6*(b*x+a)*(-2*b^3*x^3+3*a*b^2*x^2+6*a^3*ln(b*x+a)-6*b*a^2*x)/((b*x+a)^2)
^(1/2)/b^4
```

Maxima [A] time = 1.22456, size = 159, normalized size = 1.1

$$-\frac{5\,a^3b\log\left(x+\frac{a}{b}\right)}{3\left(b^2\right)^{\frac{5}{2}}}+\frac{5\,a^2x}{3\left(b^2\right)^{\frac{3}{2}}}-\frac{5\,ax^2}{6\,\sqrt{b^2}b}+\frac{2\,a^3\sqrt{\frac{1}{b^2}}\log\left(x+\frac{a}{b}\right)}{3\,b^3}+\frac{\sqrt{b^2x^2+2\,abx+a^2}x^2}{3\,b^2}-\frac{2\,\sqrt{b^2x^2+2\,abx+a^2}a^2}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -5/3*a^3*b*log(x + a/b)/(b^2)^(5/2) + 5/3*a^2*x/(b^2)^(3/2) - 5/6*a*x^2/(sq
rt(b^2)*b) + 2/3*a^3*sqrt(b^(-2))*log(x + a/b)/b^3 + 1/3*sqrt(b^2*x^2 + 2*a
*b*x + a^2)*x^2/b^2 - 2/3*sqrt(b^2*x^2 + 2*a*b*x + a^2)*a^2/b^4
```

Fricas [A] time = 1.6327, size = 92, normalized size = 0.64

$$\frac{2\,b^3x^3-3\,ab^2x^2+6\,a^2bx-6\,a^3\log(bx+a)}{6\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))/b^4
```

Sympy [A] time = 0.626093, size = 37, normalized size = 0.26

$$-\frac{a^3\log(a+bx)}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((b*x+a)**2)**(1/2),x)
```

```
[Out] -a**3*log(a + b*x)/b**4 + a**2*x/b**3 - a*x**2/(2*b**2) + x**3/(3*b)
```

Giac [A] time = 1.29153, size = 90, normalized size = 0.62

$$-\frac{a^3\log(|bx+a|)\,\mathrm{sgn}(bx+a)}{b^4}+\frac{2\,b^2x^3\mathrm{sgn}(bx+a)-3\,abx^2\mathrm{sgn}(bx+a)+6\,a^2x\mathrm{sgn}(bx+a)}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -a^3*log(abs(b*x + a))*sgn(b*x + a)/b^4 + 1/6*(2*b^2*x^3*sgn(b*x + a) - 3*a
*b*x^2*sgn(b*x + a) + 6*a^2*x*sgn(b*x + a))/b^3
```

3.183 $\int \frac{x^2}{\sqrt{a^2+2abx+b^2x^2}} \, dx$

Optimal. Leaf size=106

$$-\frac{ax(a+bx)}{b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{x^2(a+bx)}{2b\sqrt{a^2+2abx+b^2x^2}}+\frac{a^2(a+bx)\log(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}}$$

```
[Out] -((a*x*(a + b*x))/(b^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])) + (x^2*(a + b*x))/(2
*b*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (a^2*(a + b*x)*Log[a + b*x])/(b^3*Sqrt[
a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0371791, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$-\frac{ax(a+bx)}{b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{x^2(a+bx)}{2b\sqrt{a^2+2abx+b^2x^2}}+\frac{a^2(a+bx)\log(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] -((a*x*(a + b*x))/(b^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])) + (x^2*(a + b*x))/(2
*b*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (a^2*(a + b*x)*Log[a + b*x])/(b^3*Sqrt[
a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{a^2+2abx+b^2x^2}} \, dx &= \frac{\left(ab+b^2x\right)\int \frac{x^2}{ab+b^2x} \, dx}{\sqrt{a^2+2abx+b^2x^2}}\\
&= \frac{\left(ab+b^2x\right)\int \left(-\frac{a}{b^3}+\frac{x}{b^2}+\frac{a^2}{b^3(a+bx)}\right) dx}{\sqrt{a^2+2abx+b^2x^2}}\\
&= -\frac{ax(a+bx)}{b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{x^2(a+bx)}{2b\sqrt{a^2+2abx+b^2x^2}}+\frac{a^2(a+bx)\log(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.0149632, size = 45, normalized size = 0.42

$$\frac{(a+bx)\left(2a^2\log(a+bx)+bx(bx-2a)\right)}{2b^3\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ((a + b*x)*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x]))/(2*b^3*Sqrt[(a + b*x)^2
])
```

Maple [A] time = 0.222, size = 44, normalized size = 0.4

$$\frac{(bx+a)\left(b^2x^2+2\,a^2\ln(bx+a)-2\,abx\right)}{2\,b^3}\frac{1}{\sqrt{(bx+a)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x+a)^2)^(1/2),x)
```

```
[Out] 1/2*(b*x+a)*(b^2*x^2+2*a^2*ln(b*x+a)-2*a*b*x)/((b*x+a)^2)^(1/2)/b^3
```

Maxima [A] time = 1.25525, size = 55, normalized size = 0.52

$$\frac{a^2b^2\log\left(x+\frac{a}{b}\right)}{\left(b^2\right)^{\frac{5}{2}}}-\frac{abx}{\left(b^2\right)^{\frac{3}{2}}}+\frac{x^2}{2\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] a^2*b^2*log(x + a/b)/(b^2)^(5/2) - a*b*x/(b^2)^(3/2) + 1/2*x^2/sqrt(b^2)
```

Fricas [A] time = 1.65108, size = 68, normalized size = 0.64

$$\frac{b^2x^2-2\,abx+2\,a^2\log(bx+a)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))/b^3
```

Sympy [A] time = 1.08682, size = 26, normalized size = 0.25

$$\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x+a)**2)**(1/2),x)
```

```
[Out] a**2*log(a + b*x)/b**3 - a*x/b**2 + x**2/(2*b)
```

Giac [A] time = 1.26573, size = 65, normalized size = 0.61

$$\frac{a^2\log\left(|bx+a|\right)\mathrm{sgn}\left(bx+a\right)}{b^3}+\frac{bx^2\mathrm{sgn}\left(bx+a\right)-2\,ax\mathrm{sgn}\left(bx+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] a^2*log(abs(b*x + a))*sgn(b*x + a)/b^3 + 1/2*(b*x^2*sgn(b*x + a) - 2*a*x*sg
n(b*x + a))/b^2
```

3.184 $\int \frac{x}{\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=62

$$\frac{\sqrt{a^2+2abx+b^2x^2}}{b^2}-\frac{a(a+bx)\log(a+bx)}{b^2\sqrt{a^2+2abx+b^2x^2}}$$

```
[Out] Sqrt[a^2 + 2*a*b*x + b^2*x^2]/b^2 - (a*(a + b*x)*Log[a + b*x])/(b^2*Sqrt[a^
2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0172623, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {640, 608, 31}

$$\frac{\sqrt{a^2+2abx+b^2x^2}}{b^2}-\frac{a(a+bx)\log(a+bx)}{b^2\sqrt{a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] Sqrt[a^2 + 2*a*b*x + b^2*x^2]/b^2 - (a*(a + b*x)*Log[a + b*x])/(b^2*Sqrt[a^
2 + 2*a*b*x + b^2*x^2])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{a^2+2abx+b^2x^2}}\,dx &= \frac{\sqrt{a^2+2abx+b^2x^2}}{b^2}-\frac{a\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx}{b}\\
&= \frac{\sqrt{a^2+2abx+b^2x^2}}{b^2}-\frac{\left(a\left(ab+b^2x\right)\right)\int \frac{1}{ab+b^2x}\,dx}{b\sqrt{a^2+2abx+b^2x^2}}\\
&= \frac{\sqrt{a^2+2abx+b^2x^2}}{b^2}-\frac{a(a+bx)\log(a+bx)}{b^2\sqrt{a^2+2abx+b^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.0103491, size = 33, normalized size = 0.53

$$\frac{(a+bx)(bx-a\log(a+bx))}{b^2\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ((a + b*x)*(b*x - a*Log[a + b*x]))/(b^2*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.22, size = 33, normalized size = 0.5

$$-\frac{(bx+a)(a\ln(bx+a)-bx)}{b^2}\frac{1}{\sqrt{(bx+a)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x+a)^2)^(1/2),x)
```

```
[Out] -(b*x+a)*(a*ln(b*x+a)-b*x)/((b*x+a)^2)^(1/2)/b^2
```

Maxima [A] time = 1.24071, size = 57, normalized size = 0.92

$$-\frac{a\sqrt{\frac{1}{b^2}}\log\left(x+\frac{a}{b}\right)}{b}+\frac{\sqrt{b^2x^2+2\,abx+a^2}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -a*sqrt(b^(-2))*log(x + a/b)/b + sqrt(b^2*x^2 + 2*a*b*x + a^2)/b^2
```

Fricas [A] time = 1.63242, size = 38, normalized size = 0.61

$$\frac{bx-a\log(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (b*x - a*log(b*x + a))/b^2
```

Sympy [A] time = 0.482535, size = 14, normalized size = 0.23

$$-\frac{a\log(a+bx)}{b^2}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)**2)**(1/2),x)
```

```
[Out] -a*log(a + b*x)/b**2 + x/b
```

Giac [A] time = 1.34215, size = 42, normalized size = 0.68

$$\frac{x\text{sgn}\left(bx+a\right)}{b}-\frac{a\log\left(|bx+a|\right)\text{sgn}\left(bx+a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] x*sgn(b*x + a)/b - a*log(abs(b*x + a))*sgn(b*x + a)/b^2
```

3.185 $\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=35

$$\frac{(a+bx)\log(a+bx)}{b\sqrt{a^2+2abx+b^2x^2}}$$

```
[Out] ((a + b*x)*Log[a + b*x])/(b*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0075376, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {608, 31}

$$\frac{(a+bx)\log(a+bx)}{b\sqrt{a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ((a + b*x)*Log[a + b*x])/(b*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx &= \frac{\left(ab+b^2x\right)\int \frac{1}{ab+b^2x}\,dx}{\sqrt{a^2+2abx+b^2x^2}}\\ &= \frac{(a+bx)\log(a+bx)}{b\sqrt{a^2+2abx+b^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0067045, size = 26, normalized size = 0.74

$$\frac{(a+bx)\log(a+bx)}{b\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ((a + b*x)*Log[a + b*x])/(b*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.171, size = 25, normalized size = 0.7

$$\frac{(bx+a)\ln(bx+a)}{b}\frac{1}{\sqrt{(bx+a)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^2)^(1/2),x)
```

```
[Out] (b*x+a)*ln(b*x+a)/b/((b*x+a)^2)^(1/2)
```

Maxima [A] time = 1.21254, size = 19, normalized size = 0.54

$$\sqrt{\frac{1}{b^2}}\log\left(x+\frac{a}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(b^(-2))*log(x + a/b)
```

Fricas [A] time = 1.72418, size = 22, normalized size = 0.63

$$\frac{\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] log(b*x + a)/b
```

Sympy [A] time = 0.168525, size = 7, normalized size = 0.2

$$\frac{\log(a+bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)**2)**(1/2),x)
```

```
[Out] log(a + b*x)/b
```

Giac [A] time = 1.26493, size = 23, normalized size = 0.66

$$\frac{\log(|bx+a|)\,\mathrm{sgn}(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))*sgn(b*x + a)/b
```

3.186 $\int \frac{1}{x\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=68

$$\frac{\log(x)(a+bx)}{a\sqrt{a^2+2abx+b^2x^2}}-\frac{(a+bx)\log(a+bx)}{a\sqrt{a^2+2abx+b^2x^2}}$$

```
[Out] ((a + b*x)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - ((a + b*x)*Log[a + b
*x])/(a*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0215129, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {646, 36, 29, 31}

$$\frac{\log(x)(a+bx)}{a\sqrt{a^2+2abx+b^2x^2}}-\frac{(a+bx)\log(a+bx)}{a\sqrt{a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

```
[Out] ((a + b*x)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - ((a + b*x)*Log[a + b
*x])/(a*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{\left(ab+b^2x\right)\int \frac{1}{x\left(ab+b^2x\right)}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$
$$= \frac{\left(ab+b^2x\right)\int \frac{1}{x}\,dx}{ab\sqrt{a^2+2abx+b^2x^2}} - \frac{\left(b\left(ab+b^2x\right)\right)\int \frac{1}{ab+b^2x}\,dx}{a\sqrt{a^2+2abx+b^2x^2}}$$
$$= \frac{(a+bx)\log(x)}{a\sqrt{a^2+2abx+b^2x^2}} - \frac{(a+bx)\log(a+bx)}{a\sqrt{a^2+2abx+b^2x^2}}$$

Mathematica [A] time = 0.0097142, size = 31, normalized size = 0.46

$$\frac{(a+bx)(\log(x)-\log(a+bx))}{a\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

```
[Out] ((a + b*x)*(Log[x] - Log[a + b*x]))/(a*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.194, size = 30, normalized size = 0.4

$$\frac{(bx+a)(\ln(x)-\ln(bx+a))}{a}\frac{1}{\sqrt{(bx+a)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x+a)^2)^(1/2),x)
```

```
[Out] (b*x+a)*(ln(x)-ln(b*x+a))/((b*x+a)^2)^(1/2)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65685, size = 38, normalized size = 0.56

$$-\frac{\log(bx+a)-\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(log(b*x + a) - log(x))/a
```

Sympy [A] time = 0.532106, size = 10, normalized size = 0.15

$$\frac{\log(x) - \log\left(\frac{a}{b} + x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)**2)**(1/2),x)
```

```
[Out] (log(x) - log(a/b + x))/a
```

Giac [A] time = 1.27777, size = 38, normalized size = 0.56

$$-\left(\frac{\log(|bx + a|)}{a} - \frac{\log(|x|)}{a}\right)\operatorname{sgn}(bx + a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(log(abs(b*x + a))/a - log(abs(x))/a)*sgn(b*x + a)
```

3.187 $\int \frac{1}{x^2\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=103

$$-\frac{a+bx}{ax\sqrt{a^2+2abx+b^2x^2}}-\frac{b\log(x)(a+bx)}{a^2\sqrt{a^2+2abx+b^2x^2}}+\frac{b(a+bx)\log(a+bx)}{a^2\sqrt{a^2+2abx+b^2x^2}}$$

```
[Out] -((a + b*x)/(a*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])) - (b*(a + b*x)*Log[x])/(a^
2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (b*(a + b*x)*Log[a + b*x])/(a^2*Sqrt[a^2
 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0350412, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 44}

$$-\frac{a+bx}{ax\sqrt{a^2+2abx+b^2x^2}}-\frac{b\log(x)(a+bx)}{a^2\sqrt{a^2+2abx+b^2x^2}}+\frac{b(a+bx)\log(a+bx)}{a^2\sqrt{a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

```
[Out] -((a + b*x)/(a*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])) - (b*(a + b*x)*Log[x])/(a^
2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (b*(a + b*x)*Log[a + b*x])/(a^2*Sqrt[a^2
 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\sqrt{a^2+2abx+b^2x^2}}\,dx &= \frac{\left(ab+b^2x\right)\int \frac{1}{x^2\left(ab+b^2x\right)}\,dx}{\sqrt{a^2+2abx+b^2x^2}}\\
&= \frac{\left(ab+b^2x\right)\int \left(\frac{1}{abx^2}-\frac{1}{a^2x}+\frac{b}{a^2(a+bx)}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}\\
&= -\frac{a+bx}{ax\sqrt{a^2+2abx+b^2x^2}}-\frac{b(a+bx)\log(x)}{a^2\sqrt{a^2+2abx+b^2x^2}}+\frac{b(a+bx)\log(a+bx)}{a^2\sqrt{a^2+2abx+b^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.0131405, size = 41, normalized size = 0.4

$$-\frac{(a+bx)(-bx\log(a+bx)+a+bx\log(x))}{a^2x\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

```
[Out] -(((a + b*x)*(a + b*x*Log[x] - b*x*Log[a + b*x]))/(a^2*x*Sqrt[(a + b*x)^2])
)
```

Maple [A] time = 0.177, size = 40, normalized size = 0.4

$$-\frac{(bx+a)\left(b\ln(x)x-b\ln(bx+a)x+a\right)}{a^2x}\frac{1}{\sqrt{(bx+a)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x+a)^2)^(1/2),x)
```

```
[Out] -(b*x+a)*(b*ln(x)*x-b*ln(b*x+a)*x+a)/((b*x+a)^2)^(1/2)/a^2/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66927, size = 61, normalized size = 0.59

$$\frac{bx\log(bx+a)-bx\log(x)-a}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

Sympy [A] time = 1.08252, size = 19, normalized size = 0.18

$$-\frac{1}{ax}+\frac{b\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x+a)**2)**(1/2),x)
```

```
[Out] -1/(a*x) + b*(-log(x) + log(a/b + x))/a**2
```

Giac [A] time = 1.3248, size = 50, normalized size = 0.49

$$\left(\frac{b\log\left(|bx+a|\right)}{a^2}-\frac{b\log\left(|x|\right)}{a^2}-\frac{1}{ax}\right)\operatorname{sgn}\left(bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] (b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x))*sgn(b*x + a)
```

3.188 $\int \frac{1}{x^3\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=142

$$\frac{b(a+bx)}{a^2x\sqrt{a^2+2abx+b^2x^2}}-\frac{a+bx}{2ax^2\sqrt{a^2+2abx+b^2x^2}}+\frac{b^2\log(x)(a+bx)}{a^3\sqrt{a^2+2abx+b^2x^2}}-\frac{b^2(a+bx)\log(a+bx)}{a^3\sqrt{a^2+2abx+b^2x^2}}$$

```
[Out] -(a + b*x)/(2*a*x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (b*(a + b*x))/(a^2*x*S
qrt[a^2 + 2*a*b*x + b^2*x^2]) + (b^2*(a + b*x)*Log[x])/(a^3*Sqrt[a^2 + 2*a*
b*x + b^2*x^2]) - (b^2*(a + b*x)*Log[a + b*x])/(a^3*Sqrt[a^2 + 2*a*b*x + b^
2*x^2])
```

Rubi [A] time = 0.0447885, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 44}

$$\frac{b(a+bx)}{a^2x\sqrt{a^2+2abx+b^2x^2}}-\frac{a+bx}{2ax^2\sqrt{a^2+2abx+b^2x^2}}+\frac{b^2\log(x)(a+bx)}{a^3\sqrt{a^2+2abx+b^2x^2}}-\frac{b^2(a+bx)\log(a+bx)}{a^3\sqrt{a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

```
[Out] -(a + b*x)/(2*a*x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (b*(a + b*x))/(a^2*x*S
qrt[a^2 + 2*a*b*x + b^2*x^2]) + (b^2*(a + b*x)*Log[x])/(a^3*Sqrt[a^2 + 2*a*
b*x + b^2*x^2]) - (b^2*(a + b*x)*Log[a + b*x])/(a^3*Sqrt[a^2 + 2*a*b*x + b^
2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{a^2+2abx+b^2x^2}}\,dx &= \frac{\left(ab+b^2x\right)\int \frac{1}{x^3\left(ab+b^2x\right)}\,dx}{\sqrt{a^2+2abx+b^2x^2}}\\ &= \frac{\left(ab+b^2x\right)\int \left(\frac{1}{abx^3}-\frac{1}{a^2x^2}+\frac{b}{a^3x}-\frac{b^2}{a^3(a+bx)}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}\\ &= -\frac{a+bx}{2ax^2\sqrt{a^2+2abx+b^2x^2}}+\frac{b(a+bx)}{a^2x\sqrt{a^2+2abx+b^2x^2}}+\frac{b^2(a+bx)\log(x)}{a^3\sqrt{a^2+2abx+b^2x^2}}-\frac{b^2(a+b}{a^3\sqrt{a^2+}}\end{aligned}$$

Mathematica [A] time = 0.0179001, size = 59, normalized size = 0.42

$$-\frac{(a+bx)\left(2b^2x^2\log(a+bx)+a(a-2bx)-2b^2x^2\log(x)\right)}{2a^3x^2\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

```
[Out] -((a + b*x)*(a*(a - 2*b*x) - 2*b^2*x^2*Log[x] + 2*b^2*x^2*Log[a + b*x]))/(2
*a^3*x^2*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.178, size = 58, normalized size = 0.4

$$\frac{(bx+a)\left(2\,b^2\ln(x)\,x^2-2\,b^2\ln(bx+a)\,x^2+2\,abx-a^2\right)}{2\,x^2a^3}\frac{1}{\sqrt{(bx+a)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/((b*x+a)^2)^(1/2),x)
```

```
[Out] 1/2*(b*x+a)*(2*b^2*ln(x)*x^2-2*b^2*ln(b*x+a)*x^2+2*a*b*x-a^2)/((b*x+a)^2)^(
1/2)/a^3/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.76156, size = 103, normalized size = 0.73

$$-\frac{2\,b^2x^2\log(bx+a)-2\,b^2x^2\log(x)-2\,abx+a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*x^2*log(b*x + a) - 2*b^2*x^2*log(x) - 2*a*b*x + a^2)/(a^3*x^2)
```

Sympy [A] time = 0.911211, size = 31, normalized size = 0.22

$$\frac{-a+2bx}{2a^2x^2}+\frac{b^2\left(\log(x)-\log\left(\frac{a}{b}+x\right)\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/((b*x+a)**2)**(1/2),x)
```

```
[Out] (-a + 2*b*x)/(2*a**2*x**2) + b**2*(log(x) - log(a/b + x))/a**3
```

Giac [A] time = 1.3977, size = 73, normalized size = 0.51

$$-\frac{1}{2}\left(\frac{2\,b^2\log\left(|bx+a|\right)}{a^3}-\frac{2\,b^2\log\left(|x|\right)}{a^3}-\frac{2\,abx-a^2}{a^3x^2}\right)\operatorname{sgn}\left(bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^2*log(abs(b*x + a))/a^3 - 2*b^2*log(abs(x))/a^3 - (2*a*b*x - a^2)
/(a^3*x^2))*sgn(b*x + a)
```

3.189 $\int \frac{1}{x^4\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=181

$$-\frac{b^2(a+bx)}{a^3x\sqrt{a^2+2abx+b^2x^2}}+\frac{b(a+bx)}{2a^2x^2\sqrt{a^2+2abx+b^2x^2}}-\frac{a+bx}{3ax^3\sqrt{a^2+2abx+b^2x^2}}-\frac{b^3\log(x)(a+bx)}{a^4\sqrt{a^2+2abx+b^2x^2}}+\frac{b^3(a+bx)\log(}{a^4\sqrt{a^2+2abx}}$$

```
[Out] -(a + b*x)/(3*a*x^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (b*(a + b*x))/(2*a^2*x
^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (b^2*(a + b*x))/(a^3*x*Sqrt[a^2 + 2*a*b
*x + b^2*x^2]) - (b^3*(a + b*x)*Log[x])/(a^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
 + (b^3*(a + b*x)*Log[a + b*x])/(a^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0522904, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 44}

$$-\frac{b^2(a+bx)}{a^3x\sqrt{a^2+2abx+b^2x^2}}+\frac{b(a+bx)}{2a^2x^2\sqrt{a^2+2abx+b^2x^2}}-\frac{a+bx}{3ax^3\sqrt{a^2+2abx+b^2x^2}}-\frac{b^3\log(x)(a+bx)}{a^4\sqrt{a^2+2abx+b^2x^2}}+\frac{b^3(a+bx)\log(}{a^4\sqrt{a^2+2abx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```


```
[Out] -(a + b*x)/(3*a*x^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (b*(a + b*x))/(2*a^2*x
^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (b^2*(a + b*x))/(a^3*x*Sqrt[a^2 + 2*a*b
*x + b^2*x^2]) - (b^3*(a + b*x)*Log[x])/(a^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
 + (b^3*(a + b*x)*Log[a + b*x])/(a^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{a^2+2abx+b^2x^2}}\,dx &= \frac{\left(ab+b^2x\right)\int \frac{1}{x^4\left(ab+b^2x\right)}\,dx}{\sqrt{a^2+2abx+b^2x^2}}\\ &= \frac{\left(ab+b^2x\right)\int \left(\frac{1}{abx^4}-\frac{1}{a^2x^3}+\frac{b}{a^3x^2}-\frac{b^2}{a^4x}+\frac{b^3}{a^4(a+bx)}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}\\ &= -\frac{a+bx}{3ax^3\sqrt{a^2+2abx+b^2x^2}}+\frac{b(a+bx)}{2a^2x^2\sqrt{a^2+2abx+b^2x^2}}-\frac{b^2(a+bx)}{a^3x\sqrt{a^2+2abx+b^2x^2}}-\frac{b^3(a+}{a^4\sqrt{a^2+}}\end{aligned}$$

Mathematica [A] time = 0.0220217, size = 72, normalized size = 0.4

$$-\frac{(a+bx)\left(a\left(2a^2-3abx+6b^2x^2\right)-6b^3x^3\log(a+bx)+6b^3x^3\log(x)\right)}{6a^4x^3\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

```
[Out] -((a + b*x)*(a*(2*a^2 - 3*a*b*x + 6*b^2*x^2) + 6*b^3*x^3*Log[x] - 6*b^3*x^3
*Log[a + b*x]))/(6*a^4*x^3*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.181, size = 69, normalized size = 0.4

$$-\frac{(bx+a)\left(6\,b^3\ln(x)\,x^3-6\,b^3\ln(bx+a)\,x^3+6\,ab^2x^2-3\,ba^2x+2\,a^3\right)}{6\,a^4x^3}\frac{1}{\sqrt{(bx+a)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/((b*x+a)^2)^(1/2),x)
```

```
[Out] -1/6*(b*x+a)*(6*b^3*ln(x)*x^3-6*b^3*ln(b*x+a)*x^3+6*a*b^2*x^2-3*b*a^2*x+2*a
^3)/((b*x+a)^2)^(1/2)/a^4/x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73287, size = 126, normalized size = 0.7

$$\frac{6\,b^3x^3\log(bx+a)-6\,b^3x^3\log(x)-6\,ab^2x^2+3\,a^2bx-2\,a^3}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*x^3*log(b*x + a) - 6*b^3*x^3*log(x) - 6*a*b^2*x^2 + 3*a^2*b*x -
2*a^3)/(a^4*x^3)
```

Sympy [A] time = 0.725751, size = 44, normalized size = 0.24

$$-\frac{2a^2-3abx+6b^2x^2}{6a^3x^3}+\frac{b^3\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/((b*x+a)**2)**(1/2),x)
```

```
[Out] -(2*a**2 - 3*a*b*x + 6*b**2*x**2)/(6*a**3*x**3) + b**3*(-log(x) + log(a/b +
 x))/a**4
```

Giac [A] time = 1.36114, size = 88, normalized size = 0.49

$$\frac{1}{6}\left(\frac{6\,b^{3}\log\left(|bx+a|\right)}{a^{4}}-\frac{6\,b^{3}\log\left(|x|\right)}{a^{4}}-\frac{6\,ab^{2}x^{2}-3\,a^{2}bx+2\,a^{3}}{a^{4}x^{3}}\right)\operatorname{sgn}\left(bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(6*b^3*log(abs(b*x + a))/a^4 - 6*b^3*log(abs(x))/a^4 - (6*a*b^2*x^2 - 3
*a^2*b*x + 2*a^3)/(a^4*x^3))*sgn(b*x + a)
```

3.190 $\int \frac{x^4}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\, dx$

Optimal. Leaf size=172

$$-\frac{a^4}{2b^5(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{4a^3}{b^5\sqrt{a^2+2abx+b^2x^2}}-\frac{3ax(a+bx)}{b^4\sqrt{a^2+2abx+b^2x^2}}+\frac{x^2(a+bx)}{2b^3\sqrt{a^2+2abx+b^2x^2}}+\frac{6a^2(a+}{b^5\sqrt{a^2}}$$

```
[Out] (4*a^3)/(b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - a^4/(2*b^5*(a + b*x)*Sqrt[a^2
 + 2*a*b*x + b^2*x^2]) - (3*a*x*(a + b*x))/(b^4*Sqrt[a^2 + 2*a*b*x + b^2*x^
2]) + (x^2*(a + b*x))/(2*b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (6*a^2*(a + b
*x)*Log[a + b*x])/(b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0727534, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$-\frac{a^4}{2b^5(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{4a^3}{b^5\sqrt{a^2+2abx+b^2x^2}}-\frac{3ax(a+bx)}{b^4\sqrt{a^2+2abx+b^2x^2}}+\frac{x^2(a+bx)}{2b^3\sqrt{a^2+2abx+b^2x^2}}+\frac{6a^2(a+}{b^5\sqrt{a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (4*a^3)/(b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - a^4/(2*b^5*(a + b*x)*Sqrt[a^2
 + 2*a*b*x + b^2*x^2]) - (3*a*x*(a + b*x))/(b^4*Sqrt[a^2 + 2*a*b*x + b^2*x^
2]) + (x^2*(a + b*x))/(2*b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (6*a^2*(a + b
*x)*Log[a + b*x])/(b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\, dx &= \frac{\left(b^2\left(ab+b^2x\right)\right)\int \frac{x^4}{\left(ab+b^2x\right)^3}\, dx}{\sqrt{a^2+2abx+b^2x^2}}\\
&= \frac{\left(b^2\left(ab+b^2x\right)\right)\int \left(-\frac{3a}{b^7}+\frac{x}{b^6}+\frac{a^4}{b^7(a+bx)^3}-\frac{4a^3}{b^7(a+bx)^2}+\frac{6a^2}{b^7(a+bx)}\right) dx}{\sqrt{a^2+2abx+b^2x^2}}\\
&= \frac{4a^3}{b^5\sqrt{a^2+2abx+b^2x^2}}-\frac{a^4}{2b^5(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{3ax(a+bx)}{b^4\sqrt{a^2+2abx+b^2x^2}}+\frac{}{2b^3}
\end{aligned}$$

Mathematica [A] time = 0.0266041, size = 83, normalized size = 0.48

$$\frac{-11a^2b^2x^2+2a^3bx+12a^2(a+bx)^2\log(a+bx)+7a^4-4ab^3x^3+b^4x^4}{2b^5(a+bx)\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (7*a^4 + 2*a^3*b*x - 11*a^2*b^2*x^2 - 4*a*b^3*x^3 + b^4*x^4 + 12*a^2*(a + b
*x)^2*Log[a + b*x])/(2*b^5*(a + b*x)*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.229, size = 101, normalized size = 0.6

$$\frac{\left(b^4x^4+12\,\ln(bx+a)\,x^2a^2b^2-4\,ab^3x^3+24\,\ln(bx+a)\,xa^3b-11\,x^2a^2b^2+12\,a^4\ln(bx+a)+2\,xa^3b+7\,a^4\right)(bx+a)}{2\,b^5}\left((b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] 1/2*(b^4*x^4+12*ln(b*x+a)*x^2*a^2*b^2-4*a*b^3*x^3+24*ln(b*x+a)*x*a^3*b-11*x
^2*a^2*b^2+12*a^4*ln(b*x+a)+2*x*a^3*b+7*a^4)*(b*x+a)/b^5/((b*x+a)^2)^(3/2)
```

Maxima [A] time = 1.62069, size = 223, normalized size = 1.3

$$\frac{x^3}{2\sqrt{b^2x^2+2\,abx+a^2}b^2}-\frac{5\,ax^2}{2\sqrt{b^2x^2+2\,abx+a^2}b^3}+\frac{6\,a^2\log\left(x+\frac{a}{b}\right)}{\left(b^2\right)^{\frac{3}{2}}b^2}+\frac{9\,a^4}{\left(b^2\right)^{\frac{7}{2}}\left(x+\frac{a}{b}\right)^2}+\frac{12\,a^3x}{\left(b^2\right)^{\frac{5}{2}}b\left(x+\frac{a}{b}\right)^2}-\frac{5\,a^3}{\sqrt{b^2x^2+2\,ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^3/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^2) - 5/2*a*x^2/(sqrt(b^2*x^2 + 2*a
*b*x + a^2)*b^3) + 6*a^2*log(x + a/b)/((b^2)^(3/2)*b^2) + 9*a^4/((b^2)^(7/2
)*(x + a/b)^2) + 12*a^3*x/((b^2)^(5/2)*b*(x + a/b)^2) - 5*a^3/(sqrt(b^2*x^2
 + 2*a*b*x + a^2)*b^5) + 5/2*a^4/((b^2)^(3/2)*b^4*(x + a/b)^2)
```

Fricas [A] time = 1.58875, size = 200, normalized size = 1.16

$$\frac{b^4x^4-4\,ab^3x^3-11\,a^2b^2x^2+2\,a^3bx+7\,a^4+12\left(a^2b^2x^2+2\,a^3bx+a^4\right)\log(bx+a)}{2\left(b^7x^2+2\,ab^6x+a^2b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*x^4 - 4*a*b^3*x^3 - 11*a^2*b^2*x^2 + 2*a^3*b*x + 7*a^4 + 12*(a^2*b
^2*x^2 + 2*a^3*b*x + a^4)*log(b*x + a))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left((a+bx)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral(x**4/((a + b*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.191 $\int \frac{x^3}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=133

$$\frac{a^3}{2b^4(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{3a^2}{b^4\sqrt{a^2+2abx+b^2x^2}}+\frac{x(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}}-\frac{3a(a+bx)\log(a+bx)}{b^4\sqrt{a^2+2abx+b^2x^2}}$$

```
[Out] (-3*a^2)/(b^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + a^3/(2*b^4*(a + b*x)*Sqrt[a^
2 + 2*a*b*x + b^2*x^2]) + (x*(a + b*x))/(b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
 - (3*a*(a + b*x)*Log[a + b*x])/(b^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0580191, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$\frac{a^3}{2b^4(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{3a^2}{b^4\sqrt{a^2+2abx+b^2x^2}}+\frac{x(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}}-\frac{3a(a+bx)\log(a+bx)}{b^4\sqrt{a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (-3*a^2)/(b^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + a^3/(2*b^4*(a + b*x)*Sqrt[a^
2 + 2*a*b*x + b^2*x^2]) + (x*(a + b*x))/(b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
 - (3*a*(a + b*x)*Log[a + b*x])/(b^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx &= \frac{\left(b^2\left(ab+b^2x\right)\right)\int \frac{x^3}{\left(ab+b^2x\right)^3}\,dx}{\sqrt{a^2+2abx+b^2x^2}}\\
&= \frac{\left(b^2\left(ab+b^2x\right)\right)\int \left(\frac{1}{b^6}-\frac{a^3}{b^6(a+bx)^3}+\frac{3a^2}{b^6(a+bx)^2}-\frac{3a}{b^6(a+bx)}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}\\
&= -\frac{3a^2}{b^4\sqrt{a^2+2abx+b^2x^2}}+\frac{a^3}{2b^4(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{x(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}}-\frac{3a(a+bx)}{b^4\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.02345, size = 71, normalized size = 0.53

$$\frac{-4a^2bx - 5a^3 + 4ab^2x^2 - 6a(a+bx)^2\log(a+bx) + 2b^3x^3}{2b^4(a+bx)\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (-5*a^3 - 4*a^2*b*x + 4*a*b^2*x^2 + 2*b^3*x^3 - 6*a*(a + b*x)^2*Log[a + b*x
])/(2*b^4*(a + b*x)*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.224, size = 89, normalized size = 0.7

$$-\frac{\left(6\ln(bx+a)x^2ab^2 - 2b^3x^3 + 12\ln(bx+a)xa^2b - 4ab^2x^2 + 6a^3\ln(bx+a) + 4ba^2x + 5a^3\right)(bx+a)}{2b^4}\left((bx+a)^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] -1/2*(6*ln(b*x+a)*x^2*a*b^2-2*b^3*x^3+12*ln(b*x+a)*x*a^2*b-4*a*b^2*x^2+6*a^
3*ln(b*x+a)+4*b*a^2*x+5*a^3)*(b*x+a)/b^4/((b*x+a)^2)^(3/2)
```

Maxima [A] time = 1.18906, size = 180, normalized size = 1.35

$$\frac{x^2}{\sqrt{b^2x^2+2abx+a^2}b^2} - \frac{3a\log\left(x+\frac{a}{b}\right)}{\left(b^2\right)^{\frac{3}{2}}b} - \frac{9a^3b}{2\left(b^2\right)^{\frac{7}{2}}\left(x+\frac{a}{b}\right)^2} - \frac{6a^2x}{\left(b^2\right)^{\frac{5}{2}}\left(x+\frac{a}{b}\right)^2} + \frac{2a^2}{\sqrt{b^2x^2+2abx+a^2}b^4} - \frac{a^3}{\left(b^2\right)^{\frac{3}{2}}b^3\left(x+\frac{a}{b}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^2) - 3*a*log(x + a/b)/((b^2)^(3/2)*b)
- 9/2*a^3*b/((b^2)^(7/2)*(x + a/b)^2) - 6*a^2*x/((b^2)^(5/2)*(x + a/b)^2) +
 2*a^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^4) - a^3/((b^2)^(3/2)*b^3*(x + a/b)
^2)
```

Fricas [A] time = 1.67851, size = 176, normalized size = 1.32

$$\frac{2b^3x^3 + 4ab^2x^2 - 4a^2bx - 5a^3 - 6\left(ab^2x^2 + 2a^2bx + a^3\right)\log(bx+a)}{2\left(b^6x^2 + 2ab^5x + a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*x^3 + 4*a*b^2*x^2 - 4*a^2*b*x - 5*a^3 - 6*(a*b^2*x^2 + 2*a^2*b*x
 + a^3)*log(b*x + a))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left((a+bx)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral(x**3/((a + b*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.192 $\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=99

$$-\frac{a^2}{2b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{2a}{b^3\sqrt{a^2+2abx+b^2x^2}}+\frac{(a+bx)\log(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}}$$

```
[Out] (2*a)/(b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - a^2/(2*b^3*(a + b*x)*Sqrt[a^2 +
 2*a*b*x + b^2*x^2]) + ((a + b*x)*Log[a + b*x])/(b^3*Sqrt[a^2 + 2*a*b*x + b
^2*x^2])
```

Rubi [A] time = 0.0447602, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$-\frac{a^2}{2b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{2a}{b^3\sqrt{a^2+2abx+b^2x^2}}+\frac{(a+bx)\log(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (2*a)/(b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - a^2/(2*b^3*(a + b*x)*Sqrt[a^2 +
 2*a*b*x + b^2*x^2]) + ((a + b*x)*Log[a + b*x])/(b^3*Sqrt[a^2 + 2*a*b*x + b
^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx &= \frac{\left(b^2\left(ab+b^2x\right)\right)\int \frac{x^2}{\left(ab+b^2x\right)^3}\,dx}{\sqrt{a^2+2abx+b^2x^2}}\\
&= \frac{\left(b^2\left(ab+b^2x\right)\right)\int \left(\frac{a^2}{b^5(a+bx)^3}-\frac{2a}{b^5(a+bx)^2}+\frac{1}{b^5(a+bx)}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}\\
&= \frac{2a}{b^3\sqrt{a^2+2abx+b^2x^2}}-\frac{a^2}{2b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{(a+bx)\log(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.0178568, size = 51, normalized size = 0.52

$$\frac{a(3a+4bx)+2(a+bx)^2\log(a+bx)}{2b^3(a+bx)\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (a*(3*a + 4*b*x) + 2*(a + b*x)^2*Log[a + b*x])/(2*b^3*(a + b*x)*Sqrt[(a + b
*x)^2])
```

Maple [A] time = 0.222, size = 67, normalized size = 0.7

$$\frac{\left(2\,b^2\ln\left(bx+a\right)x^2+4\,\ln\left(bx+a\right)xab+2\,a^2\ln\left(bx+a\right)+4\,abx+3\,a^2\right)\left(bx+a\right)}{2\,b^3}\left(\left(bx+a\right)^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] 1/2*(2*b^2*ln(b*x+a)*x^2+4*ln(b*x+a)*x*a*b+2*a^2*ln(b*x+a)+4*a*b*x+3*a^2)*(
b*x+a)/b^3/((b*x+a)^2)^(3/2)
```

Maxima [A] time = 1.19661, size = 76, normalized size = 0.77

$$\frac{\log\left(x+\frac{a}{b}\right)}{\left(b^2\right)^{\frac{3}{2}}}+\frac{3\,a^2b^2}{2\left(b^2\right)^{\frac{7}{2}}\left(x+\frac{a}{b}\right)^2}+\frac{2\,abx}{\left(b^2\right)^{\frac{5}{2}}\left(x+\frac{a}{b}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] log(x + a/b)/(b^2)^(3/2) + 3/2*a^2*b^2/((b^2)^(7/2)*(x + a/b)^2) + 2*a*b*x/
((b^2)^(5/2)*(x + a/b)^2)
```

Fricas [A] time = 1.50534, size = 132, normalized size = 1.33

$$\frac{4\,abx+3\,a^2+2\left(b^2x^2+2\,abx+a^2\right)\log\left(bx+a\right)}{2\left(b^5x^2+2\,ab^4x+a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*b*x + 3*a^2 + 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a))/(b^5*x^2 +
 2*a*b^4*x + a^2*b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left((a+bx)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral(x**2/((a + b*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.193 $\int \frac{x}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=61

$$\frac{a}{2b^2(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{1}{b^2\sqrt{a^2+2abx+b^2x^2}}$$

```
[Out] -(1/(b^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])) + a/(2*b^2*(a + b*x)*Sqrt[a^2 + 2*
a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0146494, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {640, 607}

$$\frac{a}{2b^2(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{1}{b^2\sqrt{a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] -(1/(b^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])) + a/(2*b^2*(a + b*x)*Sqrt[a^2 + 2*
a*b*x + b^2*x^2])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx &= -\frac{1}{b^2\sqrt{a^2+2abx+b^2x^2}}-\frac{a\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx}{b}\\ &= -\frac{1}{b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{a}{2b^2(a+bx)\sqrt{a^2+2abx+b^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0096303, size = 33, normalized size = 0.54

$$\frac{-a-2bx}{2b^2(a+bx)\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]

[Out] (-a - 2*b*x)/(2*b^2*(a + b*x)*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.176, size = 26, normalized size = 0.4

$$-\frac{(bx+a)(2bx+a)}{2b^2}\left((bx+a)^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)

[Out] -1/2*(b*x+a)*(2*b*x+a)/b^2/((b*x+a)^2)^(3/2)
```

Maxima [A] time = 1.83118, size = 59, normalized size = 0.97

$$-\frac{1}{\sqrt{b^2x^2+2abx+a^2}b^2}+\frac{a}{2\left(b^2\right)^{\frac{3}{2}}b\left(x+\frac{a}{b}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")

[Out] -1/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^2) + 1/2*a/((b^2)^(3/2)*b*(x + a/b)^2)
```

Fricas [A] time = 1.58122, size = 68, normalized size = 1.11

$$-\frac{2bx+a}{2\left(b^4x^2+2ab^3x+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")

[Out] -1/2*(2*b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\left((a+bx)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)

[Out] Integral(x/((a + b*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.194 $\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{3/2}} dx$

Optimal. Leaf size=34

$$-\frac{1}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}}$$

```
[Out] -1/(2*b*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0043055, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {607}

$$-\frac{1}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(-3/2),x]
```

```
[Out] -1/(2*b*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{3/2}} dx = -\frac{1}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}}$$

Mathematica [A] time = 0.0085033, size = 23, normalized size = 0.68

$$-\frac{a+bx}{2b\left((a+bx)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(-3/2),x]
```

```
[Out] -(a + b*x)/(2*b*((a + b*x)^2)^(3/2))
```

Maple [A] time = 0.044, size = 20, normalized size = 0.6

$$-\frac{bx+a}{2b}\left((bx+a)^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] -1/2*(b*x+a)/b/((b*x+a)^2)^(3/2)
```

Maxima [A] time = 1.46967, size = 22, normalized size = 0.65

$$-\frac{1}{2\left(b^{2}\right)^{\frac{3}{2}}\left(x+\frac{a}{b}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2/((b^2)^(3/2)*(x + a/b)^2)
```

Fricas [A] time = 1.61633, size = 49, normalized size = 1.44

$$-\frac{1}{2\left(b^{3}x^{2}+2\,ab^{2}x+a^{2}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^{2}+2abx+b^{2}x^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x + b**2*x**2)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.195 $\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=126

$$\frac{1}{2a(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{1}{a^2\sqrt{a^2+2abx+b^2x^2}}+\frac{\log(x)(a+bx)}{a^3\sqrt{a^2+2abx+b^2x^2}}-\frac{(a+bx)\log(a+bx)}{a^3\sqrt{a^2+2abx+b^2x^2}}$$

```
[Out] 1/(a^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + 1/(2*a*(a + b*x)*Sqrt[a^2 + 2*a*b*x
 + b^2*x^2]) + ((a + b*x)*Log[x])/(a^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - ((a
 + b*x)*Log[a + b*x])/(a^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0579414, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 44}

$$\frac{1}{2a(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{1}{a^2\sqrt{a^2+2abx+b^2x^2}}+\frac{\log(x)(a+bx)}{a^3\sqrt{a^2+2abx+b^2x^2}}-\frac{(a+bx)\log(a+bx)}{a^3\sqrt{a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```


```
[Out] 1/(a^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + 1/(2*a*(a + b*x)*Sqrt[a^2 + 2*a*b*x
 + b^2*x^2]) + ((a + b*x)*Log[x])/(a^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - ((a
 + b*x)*Log[a + b*x])/(a^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx &= \frac{\left(b^2\left(ab+b^2x\right)\right)\int \frac{1}{x\left(ab+b^2x\right)^3}\,dx}{\sqrt{a^2+2abx+b^2x^2}}\\ &= \frac{\left(b^2\left(ab+b^2x\right)\right)\int \left(\frac{1}{a^3b^3x}-\frac{1}{ab^2(a+bx)^3}-\frac{1}{a^2b^2(a+bx)^2}-\frac{1}{a^3b^2(a+bx)}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}\\ &= \frac{1}{a^2\sqrt{a^2+2abx+b^2x^2}}+\frac{1}{2a(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{(a+bx)\log(x)}{a^3\sqrt{a^2+2abx+b^2x^2}}-\frac{(a}{a^3\sqrt{}}\end{aligned}$$

Mathematica [A] time = 0.022777, size = 62, normalized size = 0.49

$$\frac{a(3a+2bx)+2\log(x)(a+bx)^2-2(a+bx)^2\log(a+bx)}{2a^3(a+bx)\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

```
[Out] (a*(3*a + 2*b*x) + 2*(a + b*x)^2*Log[x] - 2*(a + b*x)^2*Log[a + b*x])/(2*a^
3*(a + b*x)*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.226, size = 91, normalized size = 0.7

$$\frac{\left(2\,b^2\ln(x)\,x^2-2\,b^2\ln(bx+a)\,x^2+4\,\ln(x)\,xab-4\,\ln(bx+a)\,xab+2\,\ln(x)\,a^2-2\,a^2\ln(bx+a)+2\,abx+3\,a^2\right)(bx+a}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] 1/2*(2*b^2*ln(x)*x^2-2*b^2*ln(b*x+a)*x^2+4*ln(x)*x*a*b-4*ln(b*x+a)*x*a*b+2*
ln(x)*a^2-2*a^2*ln(b*x+a)+2*a*b*x+3*a^2)*(b*x+a)/a^3/((b*x+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69607, size = 182, normalized size = 1.44

$$\frac{2\,abx+3\,a^2-2\left(b^2x^2+2\,abx+a^2\right)\log(bx+a)+2\left(b^2x^2+2\,abx+a^2\right)\log(x)}{2\left(a^3b^2x^2+2\,a^4bx+a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*x + 3*a^2 - 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a) + 2*(b^2*x^
2 + 2*a*b*x + a^2)*log(x))/(a^3*b^2*x^2 + 2*a^4*b*x + a^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\left((a+bx)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x*((a + b*x)**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.196 $\int \frac{1}{x^2\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=165

$$-\frac{b}{2a^2(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{2b}{a^3\sqrt{a^2+2abx+b^2x^2}}-\frac{a+bx}{a^3x\sqrt{a^2+2abx+b^2x^2}}-\frac{3b\log(x)(a+bx)}{a^4\sqrt{a^2+2abx+b^2x^2}}+\frac{3b(a+bx)\,\mathrm{l}}{a^4\sqrt{a^2+2a}}$$

```
[Out] (-2*b)/(a^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - b/(2*a^2*(a + b*x)*Sqrt[a^2 +
2*a*b*x + b^2*x^2]) - (a + b*x)/(a^3*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (3*
b*(a + b*x)*Log[x])/(a^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (3*b*(a + b*x)*Lo
g[a + b*x])/(a^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0675738, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 44}

$$-\frac{b}{2a^2(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{2b}{a^3\sqrt{a^2+2abx+b^2x^2}}-\frac{a+bx}{a^3x\sqrt{a^2+2abx+b^2x^2}}-\frac{3b\log(x)(a+bx)}{a^4\sqrt{a^2+2abx+b^2x^2}}+\frac{3b(a+bx)\,\mathrm{l}}{a^4\sqrt{a^2+2a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

```
[Out] (-2*b)/(a^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - b/(2*a^2*(a + b*x)*Sqrt[a^2 +
2*a*b*x + b^2*x^2]) - (a + b*x)/(a^3*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (3*
b*(a + b*x)*Log[x])/(a^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (3*b*(a + b*x)*Lo
g[a + b*x])/(a^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx &= \frac{\left(b^2\left(ab+b^2x\right)\right)\int \frac{1}{x^2\left(ab+b^2x\right)^3}\,dx}{\sqrt{a^2+2abx+b^2x^2}}\\ &= \frac{\left(b^2\left(ab+b^2x\right)\right)\int \left(\frac{1}{a^3b^3x^2}-\frac{3}{a^4b^2x}+\frac{1}{a^2b(a+bx)^3}+\frac{2}{a^3b(a+bx)^2}+\frac{3}{a^4b(a+bx)}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}\\ &= -\frac{2b}{a^3\sqrt{a^2+2abx+b^2x^2}}-\frac{b}{2a^2(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{a+bx}{a^3x\sqrt{a^2+2abx+b^2x^2}}-\frac{3}{a^4}\end{aligned}$$

Mathematica [A] time = 0.0287591, size = 81, normalized size = 0.49

$$\frac{-a\left(2a^2+9abx+6b^2x^2\right)-6bx\log(x)(a+bx)^2+6bx(a+bx)^2\log(a+bx)}{2a^4x(a+bx)\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

```
[Out] (-(a*(2*a^2 + 9*a*b*x + 6*b^2*x^2)) - 6*b*x*(a + b*x)^2*Log[x] + 6*b*x*(a +
 b*x)^2*Log[a + b*x])/(2*a^4*x*(a + b*x)*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.242, size = 117, normalized size = 0.7

$$-\frac{\left(6\,b^3\ln(x)\,x^3-6\,b^3\ln(bx+a)\,x^3+12\,b^2a\ln(x)\,x^2-12\,\ln(bx+a)\,x^2ab^2+6\,ba^2\ln(x)\,x-6\,\ln(bx+a)\,xa^2b+6\,ab}{2\,a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] -1/2*(6*b^3*ln(x)*x^3-6*b^3*ln(b*x+a)*x^3+12*b^2*a*ln(x)*x^2-12*ln(b*x+a)*x
^2*a*b^2+6*b*a^2*ln(x)*x-6*ln(b*x+a)*x*a^2*b+6*a*b^2*x^2+9*b*a^2*x+2*a^3)*(
b*x+a)/a^4/x/((b*x+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72541, size = 232, normalized size = 1.41

$$-\frac{6\,ab^2x^2+9\,a^2bx+2\,a^3-6\left(b^3x^3+2\,ab^2x^2+a^2bx\right)\log(bx+a)+6\left(b^3x^3+2\,ab^2x^2+a^2bx\right)\log(x)}{2\left(a^4b^2x^3+2\,a^5bx^2+a^6x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(6*a*b^2*x^2 + 9*a^2*b*x + 2*a^3 - 6*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)
*log(b*x + a) + 6*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)*log(x))/(a^4*b^2*x^3 +
2*a^5*b*x^2 + a^6*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left((a+bx)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x**2*((a + b*x)**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.197 $\int \frac{1}{x^3(a^2+2abx+b^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=209

$$\frac{b^2}{2a^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{3b^2}{a^4\sqrt{a^2+2abx+b^2x^2}}+\frac{3b(a+bx)}{a^4x\sqrt{a^2+2abx+b^2x^2}}-\frac{a+bx}{2a^3x^2\sqrt{a^2+2abx+b^2x^2}}+\frac{6b^2 \text{]}}{a^5\sqrt{a^2}}$$

```
[Out] (3*b^2)/(a^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + b^2/(2*a^3*(a + b*x)*Sqrt[a^2
 + 2*a*b*x + b^2*x^2]) - (a + b*x)/(2*a^3*x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]
) + (3*b*(a + b*x))/(a^4*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (6*b^2*(a + b*x
)*Log[x])/(a^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (6*b^2*(a + b*x)*Log[a + b*
x])/(a^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0795661, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 44}

$$\frac{b^2}{2a^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{3b^2}{a^4\sqrt{a^2+2abx+b^2x^2}}+\frac{3b(a+bx)}{a^4x\sqrt{a^2+2abx+b^2x^2}}-\frac{a+bx}{2a^3x^2\sqrt{a^2+2abx+b^2x^2}}+\frac{6b^2 \text{]}}{a^5\sqrt{a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```


```
[Out] (3*b^2)/(a^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + b^2/(2*a^3*(a + b*x)*Sqrt[a^2
 + 2*a*b*x + b^2*x^2]) - (a + b*x)/(2*a^3*x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]
) + (3*b*(a + b*x))/(a^4*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (6*b^2*(a + b*x
)*Log[x])/(a^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (6*b^2*(a + b*x)*Log[a + b*
x])/(a^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x\right)\right)\int \frac{1}{x^3\left(ab+b^2x\right)^3}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$
$$= \frac{\left(b^2\left(ab+b^2x\right)\right)\int \left(\frac{1}{a^3b^3x^3}-\frac{3}{a^4b^2x^2}+\frac{6}{a^5bx}-\frac{1}{a^3(a+bx)^3}-\frac{3}{a^4(a+bx)^2}-\frac{6}{a^5(a+bx)}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$
$$= \frac{3b^2}{a^4\sqrt{a^2+2abx+b^2x^2}}+\frac{b^2}{2a^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{a+bx}{2a^3x^2\sqrt{a^2+2abx+b^2x^2}}+\frac{\ }{a}$$

Mathematica [A] time = 0.0362402, size = 99, normalized size = 0.47

$$\frac{a\left(4a^2bx-a^3+18ab^2x^2+12b^3x^3\right)+12b^2x^2\log(x)(a+bx)^2-12b^2x^2(a+bx)^2\log(a+bx)}{2a^5x^2(a+bx)\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

```
[Out] (a*(-a^3 + 4*a^2*b*x + 18*a*b^2*x^2 + 12*b^3*x^3) + 12*b^2*x^2*(a + b*x)^2*
Log[x] - 12*b^2*x^2*(a + b*x)^2*Log[a + b*x])/(2*a^5*x^2*(a + b*x)*Sqrt[(a
+ b*x)^2])
```

Maple [A] time = 0.239, size = 136, normalized size = 0.7

$$\frac{\left(12\ln(x)x^4b^4-12\ln(bx+a)x^4b^4+24\ln(x)x^3ab^3-24\ln(bx+a)x^3ab^3+12\ln(x)x^2a^2b^2-12\ln(bx+a)x^2a^2b^2+\right.}{2x^2a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] 1/2*(12*ln(x)*x^4*b^4-12*ln(b*x+a)*x^4*b^4+24*ln(x)*x^3*a*b^3-24*ln(b*x+a)*
x^3*a*b^3+12*ln(x)*x^2*a^2*b^2-12*ln(b*x+a)*x^2*a^2*b^2+12*a*b^3*x^3+18*x^2
*a^2*b^2+4*x*a^3*b-a^4)*(b*x+a)/x^2/a^5/((b*x+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69949, size = 269, normalized size = 1.29

$$\frac{12\,ab^3x^3+18\,a^2b^2x^2+4\,a^3bx-a^4-12\left(b^4x^4+2\,ab^3x^3+a^2b^2x^2\right)\log(bx+a)+12\left(b^4x^4+2\,ab^3x^3+a^2b^2x^2\right)\log(x)}{2\left(a^5b^2x^4+2\,a^6bx^3+a^7x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(12*a*b^3*x^3 + 18*a^2*b^2*x^2 + 4*a^3*b*x - a^4 - 12*(b^4*x^4 + 2*a*b^
3*x^3 + a^2*b^2*x^2)*log(b*x + a) + 12*(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x^2
)*log(x))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{3}\left(\left(a + bx\right)^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x**3*((a + b*x)**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_{0}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.198 $\int \frac{x^6}{(a^2+2abx+b^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=244

$$-\frac{a^6}{4b^7(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}+\frac{2a^5}{b^7(a+bx)^2\sqrt{a^2+2abx+b^2x^2}}-\frac{15a^4}{2b^7(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{20a^3}{b^7\sqrt{a^2+2abx+b}}$$

```
[Out] (20*a^3)/(b^7*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - a^6/(4*b^7*(a + b*x)^3*Sqrt[
a^2 + 2*a*b*x + b^2*x^2]) + (2*a^5)/(b^7*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b
^2*x^2]) - (15*a^4)/(2*b^7*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (5*a*
x*(a + b*x))/(b^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (x^2*(a + b*x))/(2*b^5*S
qrt[a^2 + 2*a*b*x + b^2*x^2]) + (15*a^2*(a + b*x)*Log[a + b*x])/(b^7*Sqrt[a
^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.117451, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$-\frac{a^6}{4b^7(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}+\frac{2a^5}{b^7(a+bx)^2\sqrt{a^2+2abx+b^2x^2}}-\frac{15a^4}{2b^7(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{20a^3}{b^7\sqrt{a^2+2abx+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (20*a^3)/(b^7*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - a^6/(4*b^7*(a + b*x)^3*Sqrt[
a^2 + 2*a*b*x + b^2*x^2]) + (2*a^5)/(b^7*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b
^2*x^2]) - (15*a^4)/(2*b^7*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (5*a*
x*(a + b*x))/(b^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (x^2*(a + b*x))/(2*b^5*S
qrt[a^2 + 2*a*b*x + b^2*x^2]) + (15*a^2*(a + b*x)*Log[a + b*x])/(b^7*Sqrt[a
^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^6}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x\right)\right)\int \frac{x^6}{\left(ab+b^2x\right)^5}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= \frac{\left(b^4\left(ab+b^2x\right)\right)\int \left(-\frac{5a}{b^{11}}+\frac{x}{b^{10}}+\frac{a^6}{b^{11}(a+bx)^5}-\frac{6a^5}{b^{11}(a+bx)^4}+\frac{15a^4}{b^{11}(a+bx)^3}-\frac{20a^3}{b^{11}(a+bx)^2}+\frac{15a^2}{b^{11}(a+bx)}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= \frac{20a^3}{b^7\sqrt{a^2+2abx+b^2x^2}} - \frac{a^6}{4b^7(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{2a^5}{b^7(a+bx)^2\sqrt{a^2+2abx+b^2}}$$

Mathematica [A] time = 0.0342297, size = 106, normalized size = 0.43

$$\frac{132a^4b^2x^2-32a^3b^3x^3-68a^2b^4x^4+168a^5bx+60a^2(a+bx)^4\log(a+bx)+57a^6-12ab^5x^5+2b^6x^6}{4b^7(a+bx)^3\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (57*a^6 + 168*a^5*b*x + 132*a^4*b^2*x^2 - 32*a^3*b^3*x^3 - 68*a^2*b^4*x^4 -
 12*a*b^5*x^5 + 2*b^6*x^6 + 60*a^2*(a + b*x)^4*Log[a + b*x])/(4*b^7*(a + b*
x)^3*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.264, size = 158, normalized size = 0.7

$$\frac{\left(2\,b^6x^6+60\,\ln(bx+a)\,x^4a^2b^4-12\,x^5ab^5+240\,\ln(bx+a)\,x^3a^3b^3-68\,a^2x^4b^4+360\,\ln(bx+a)\,x^2a^4b^2-32\,a^3x^3b^3\right.}{4\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] 1/4*(2*b^6*x^6+60*ln(b*x+a)*x^4*a^2*b^4-12*x^5*a*b^5+240*ln(b*x+a)*x^3*a^3*
b^3-68*a^2*x^4*b^4+360*ln(b*x+a)*x^2*a^4*b^2-32*a^3*x^3*b^3+240*ln(b*x+a)*x
*a^5*b+132*a^4*x^2*b^2+60*ln(b*x+a)*a^6+168*a^5*x*b+57*a^6)*(b*x+a)/b^7/((b
*x+a)^2)^(5/2)
```

Maxima [A] time = 1.29544, size = 170, normalized size = 0.7

$$\frac{2\,b^6x^6-12\,ab^5x^5-68\,a^2b^4x^4-32\,a^3b^3x^3+132\,a^4b^2x^2+168\,a^5bx+57\,a^6}{4\left(b^{11}x^4+4\,ab^{10}x^3+6\,a^2b^9x^2+4\,a^3b^8x+a^4b^7\right)}+\frac{15\,a^2\log(bx+a)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*b^6*x^6 - 12*a*b^5*x^5 - 68*a^2*b^4*x^4 - 32*a^3*b^3*x^3 + 132*a^4*b
^2*x^2 + 168*a^5*b*x + 57*a^6)/(b^11*x^4 + 4*a*b^10*x^3 + 6*a^2*b^9*x^2 + 4
*a^3*b^8*x + a^4*b^7) + 15*a^2*log(b*x + a)/b^7
```

Fricas [A] time = 1.56303, size = 344, normalized size = 1.41

$$\frac{2\,b^{6}x^{6}-12\,ab^{5}x^{5}-68\,a^{2}b^{4}x^{4}-32\,a^{3}b^{3}x^{3}+132\,a^{4}b^{2}x^{2}+168\,a^{5}bx+57\,a^{6}+60\left(a^{2}b^{4}x^{4}+4\,a^{3}b^{3}x^{3}+6\,a^{4}b^{2}x^{2}+4\,a^{5}bx+\right.}{4\left(b^{11}x^{4}+4\,ab^{10}x^{3}+6\,a^{2}b^{9}x^{2}+4\,a^{3}b^{8}x+a^{4}b^{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^6*x^6 - 12*a*b^5*x^5 - 68*a^2*b^4*x^4 - 32*a^3*b^3*x^3 + 132*a^4*b
^2*x^2 + 168*a^5*b*x + 57*a^6 + 60*(a^2*b^4*x^4 + 4*a^3*b^3*x^3 + 6*a^4*b^2
*x^2 + 4*a^5*b*x + a^6)*log(b*x + a))/(b^11*x^4 + 4*a*b^10*x^3 + 6*a^2*b^9*
x^2 + 4*a^3*b^8*x + a^4*b^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{6}}{\left(\left(a+bx\right)^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Integral(x**6/((a + b*x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_{0}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.199 $\int \frac{x^5}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=205

$$\frac{a^5}{4b^6(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}-\frac{5a^4}{3b^6(a+bx)^2\sqrt{a^2+2abx+b^2x^2}}+\frac{5a^3}{b^6(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{10a^2}{b^6\sqrt{a^2+2abx+}}$$

```
[Out] (-10*a^2)/(b^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + a^5/(4*b^6*(a + b*x)^3*Sqrt
[a^2 + 2*a*b*x + b^2*x^2]) - (5*a^4)/(3*b^6*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x
+ b^2*x^2]) + (5*a^3)/(b^6*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (x*(a
 + b*x))/(b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (5*a*(a + b*x)*Log[a + b*x])
/(b^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0930691, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$\frac{a^5}{4b^6(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}-\frac{5a^4}{3b^6(a+bx)^2\sqrt{a^2+2abx+b^2x^2}}+\frac{5a^3}{b^6(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{10a^2}{b^6\sqrt{a^2+2abx+}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (-10*a^2)/(b^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + a^5/(4*b^6*(a + b*x)^3*Sqrt
[a^2 + 2*a*b*x + b^2*x^2]) - (5*a^4)/(3*b^6*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x
+ b^2*x^2]) + (5*a^3)/(b^6*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (x*(a
 + b*x))/(b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (5*a*(a + b*x)*Log[a + b*x])
/(b^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^5}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x\right)\right)\int \frac{x^5}{\left(ab+b^2x\right)^5}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= \frac{\left(b^4\left(ab+b^2x\right)\right)\int \left(\frac{1}{b^{10}}-\frac{a^5}{b^{10}(a+bx)^5}+\frac{5a^4}{b^{10}(a+bx)^4}-\frac{10a^3}{b^{10}(a+bx)^3}+\frac{10a^2}{b^{10}(a+bx)^2}-\frac{5a}{b^{10}(a+bx)}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{10a^2}{b^6\sqrt{a^2+2abx+b^2x^2}}+\frac{a^5}{4b^6(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}-\frac{5a^4}{3b^6(a+bx)^2\sqrt{a^2+2abx+b^2x^2}}$$

Mathematica [A] time = 0.0294583, size = 93, normalized size = 0.45

$$\frac{-252a^3b^2x^2-48a^2b^3x^3-248a^4bx-77a^5+48ab^4x^4-60a(a+bx)^4\log(a+bx)+12b^5x^5}{12b^6(a+bx)^3\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (-77*a^5 - 248*a^4*b*x - 252*a^3*b^2*x^2 - 48*a^2*b^3*x^3 + 48*a*b^4*x^4 +
12*b^5*x^5 - 60*a*(a + b*x)^4*Log[a + b*x])/(12*b^6*(a + b*x)^3*Sqrt[(a + b
*x)^2])
```

Maple [A] time = 0.221, size = 145, normalized size = 0.7

$$-\frac{\left(60\ln(bx+a)x^4ab^4-12b^5x^5+240\ln(bx+a)x^3a^2b^3-48ab^4x^4+360\ln(bx+a)x^2a^3b^2+48a^2b^3x^3+240\ln(bx+\right.}{12b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] -1/12*(60*ln(b*x+a)*x^4*a*b^4-12*b^5*x^5+240*ln(b*x+a)*x^3*a^2*b^3-48*a*b^4
*x^4+360*ln(b*x+a)*x^2*a^3*b^2+48*a^2*b^3*x^3+240*ln(b*x+a)*x*a^4*b+252*a^3
*b^2*x^2+60*ln(b*x+a)*a^5+248*a^4*b*x+77*a^5)*(b*x+a)/b^6/((b*x+a)^2)^(5/2)
```

Maxima [A] time = 1.25088, size = 153, normalized size = 0.75

$$\frac{12b^5x^5+48ab^4x^4-48a^2b^3x^3-252a^3b^2x^2-248a^4bx-77a^5}{12\left(b^{10}x^4+4ab^9x^3+6a^2b^8x^2+4a^3b^7x+a^4b^6\right)}-\frac{5a\log(bx+a)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(12*b^5*x^5 + 48*a*b^4*x^4 - 48*a^2*b^3*x^3 - 252*a^3*b^2*x^2 - 248*a^
4*b*x - 77*a^5)/(b^10*x^4 + 4*a*b^9*x^3 + 6*a^2*b^8*x^2 + 4*a^3*b^7*x + a^4
*b^6) - 5*a*log(b*x + a)/b^6
```

Fricas [A] time = 1.71691, size = 320, normalized size = 1.56

$$\frac{12\,b^5x^5 + 48\,ab^4x^4 - 48\,a^2b^3x^3 - 252\,a^3b^2x^2 - 248\,a^4bx - 77\,a^5 - 60\left(ab^4x^4 + 4\,a^2b^3x^3 + 6\,a^3b^2x^2 + 4\,a^4bx + a^5\right)\log(}{12\left(b^{10}x^4 + 4\,ab^9x^3 + 6\,a^2b^8x^2 + 4\,a^3b^7x + a^4b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(12*b^5*x^5 + 48*a*b^4*x^4 - 48*a^2*b^3*x^3 - 252*a^3*b^2*x^2 - 248*a^
4*b*x - 77*a^5 - 60*(a*b^4*x^4 + 4*a^2*b^3*x^3 + 6*a^3*b^2*x^2 + 4*a^4*b*x
+ a^5)*log(b*x + a))/(b^10*x^4 + 4*a*b^9*x^3 + 6*a^2*b^8*x^2 + 4*a^3*b^7*x
+ a^4*b^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left((a+bx)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Integral(x**5/((a + b*x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.200 $\int \frac{x^4}{(a^2+2abx+b^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=171

$$-\frac{a^4}{4b^5(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}+\frac{4a^3}{3b^5(a+bx)^2\sqrt{a^2+2abx+b^2x^2}}-\frac{3a^2}{b^5(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{4a}{b^5\sqrt{a^2+2abx+b}}$$

```
[Out] (4*a)/(b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - a^4/(4*b^5*(a + b*x)^3*Sqrt[a^2
 + 2*a*b*x + b^2*x^2]) + (4*a^3)/(3*b^5*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^
2*x^2]) - (3*a^2)/(b^5*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + ((a + b*x
)*Log[a + b*x])/(b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0781741, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$-\frac{a^4}{4b^5(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}+\frac{4a^3}{3b^5(a+bx)^2\sqrt{a^2+2abx+b^2x^2}}-\frac{3a^2}{b^5(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{4a}{b^5\sqrt{a^2+2abx+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (4*a)/(b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - a^4/(4*b^5*(a + b*x)^3*Sqrt[a^2
 + 2*a*b*x + b^2*x^2]) + (4*a^3)/(3*b^5*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^
2*x^2]) - (3*a^2)/(b^5*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + ((a + b*x
)*Log[a + b*x])/(b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx &= \frac{\left(b^4\left(ab+b^2x\right)\right)\int \frac{x^4}{\left(ab+b^2x\right)^5}\,dx}{\sqrt{a^2+2abx+b^2x^2}}\\ &= \frac{\left(b^4\left(ab+b^2x\right)\right)\int \left(\frac{a^4}{b^9(a+bx)^5}-\frac{4a^3}{b^9(a+bx)^4}+\frac{6a^2}{b^9(a+bx)^3}-\frac{4a}{b^9(a+bx)^2}+\frac{1}{b^9(a+bx)}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}\\ &= \frac{4a}{b^5\sqrt{a^2+2abx+b^2x^2}}-\frac{a^4}{4b^5(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}+\frac{4a^3}{3b^5(a+bx)^2\sqrt{a^2+2abx+b^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0256027, size = 73, normalized size = 0.43

$$\frac{a\left(88a^2bx+25a^3+108ab^2x^2+48b^3x^3\right)+12(a+bx)^4\log(a+bx)}{12b^5(a+bx)^3\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (a*(25*a^3 + 88*a^2*b*x + 108*a*b^2*x^2 + 48*b^3*x^3) + 12*(a + b*x)^4*Log[
a + b*x])/(12*b^5*(a + b*x)^3*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.223, size = 123, normalized size = 0.7

$$\frac{\left(12\ \ln(bx+a)x^4b^4+48\ \ln(bx+a)x^3ab^3+72\ \ln(bx+a)x^2a^2b^2+48\,ab^3x^3+48\ \ln(bx+a)xa^3b+108\,x^2a^2b^2+12}{12\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] 1/12*(12*ln(b*x+a)*x^4*b^4+48*ln(b*x+a)*x^3*a*b^3+72*ln(b*x+a)*x^2*a^2*b^2+
48*a*b^3*x^3+48*ln(b*x+a)*x*a^3*b+108*x^2*a^2*b^2+12*a^4*ln(b*x+a)+88*x*a^3
*b+25*a^4)*(b*x+a)/b^5/((b*x+a)^2)^(5/2)
```

Maxima [A] time = 1.18393, size = 124, normalized size = 0.73

$$\frac{48\,ab^3x^3+108\,a^2b^2x^2+88\,a^3bx+25\,a^4}{12\left(b^9x^4+4\,ab^8x^3+6\,a^2b^7x^2+4\,a^3b^6x+a^4b^5\right)}+\frac{\log(bx+a)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(48*a*b^3*x^3 + 108*a^2*b^2*x^2 + 88*a^3*b*x + 25*a^4)/(b^9*x^4 + 4*a*
b^8*x^3 + 6*a^2*b^7*x^2 + 4*a^3*b^6*x + a^4*b^5) + log(b*x + a)/b^5
```

Fricas [A] time = 1.61907, size = 271, normalized size = 1.58

$$\frac{48\,ab^3x^3+108\,a^2b^2x^2+88\,a^3bx+25\,a^4+12\left(b^4x^4+4\,ab^3x^3+6\,a^2b^2x^2+4\,a^3bx+a^4\right)\log(bx+a)}{12\left(b^9x^4+4\,ab^8x^3+6\,a^2b^7x^2+4\,a^3b^6x+a^4b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(48*a*b^3*x^3 + 108*a^2*b^2*x^2 + 88*a^3*b*x + 25*a^4 + 12*(b^4*x^4 +
4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*log(b*x + a))/(b^9*x^4 + 4*a
*b^8*x^3 + 6*a^2*b^7*x^2 + 4*a^3*b^6*x + a^4*b^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left((a+bx)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Integral(x**4/((a + b*x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.201 $\int \frac{x^3}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=37

$$\frac{x^4}{4a(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}$$

```
[Out] x^4/(4*a*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0161654, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 37}

$$\frac{x^4}{4a(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] x^4/(4*a*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx &= \frac{\left(b^4\left(ab+b^2x\right)\right)\int \frac{x^3}{\left(ab+b^2x\right)^5}\,dx}{\sqrt{a^2+2abx+b^2x^2}}\\
&= \frac{x^4}{4a(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.0147132, size = 55, normalized size = 1.49

$$\frac{-4a^2bx-a^3-6ab^2x^2-4b^3x^3}{4b^4(a+bx)^3\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (-a^3 - 4*a^2*b*x - 6*a*b^2*x^2 - 4*b^3*x^3)/(4*b^4*(a + b*x)^3*Sqrt[(a + b
*x)^2])
```

Maple [A] time = 0.175, size = 48, normalized size = 1.3

$$-\frac{(bx+a)\left(4\,b^3x^3+6\,ab^2x^2+4\,ba^2x+a^3\right)}{4\,b^4}\left((bx+a)^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] -1/4*(b*x+a)*(4*b^3*x^3+6*a*b^2*x^2+4*a^2*b*x+a^3)/b^4/((b*x+a)^2)^(5/2)
```

Maxima [B] time = 1.12336, size = 181, normalized size = 4.89

$$-\frac{x^2}{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}b^2}-\frac{2\,a^2}{3\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}b^4}-\frac{a^3b}{4\left(b^2\right)^{\frac{9}{2}}\left(x+\frac{a}{b}\right)^4}+\frac{2\,a^2}{3\left(b^2\right)^{\frac{7}{2}}\left(x+\frac{a}{b}\right)^3}-\frac{a}{2\left(b^2\right)^{\frac{5}{2}}b\left(x+\frac{a}{b}\right)^2}+\frac{}{2\left(b^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -x^2/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^2) - 2/3*a^2/((b^2*x^2 + 2*a*b*x +
a^2)^(3/2)*b^4) - 1/4*a^3*b/((b^2)^(9/2)*(x + a/b)^4) + 2/3*a^2/((b^2)^(7/2
)*(x + a/b)^3) - 1/2*a/((b^2)^(5/2)*b*(x + a/b)^2) + 1/2*a^3/((b^2)^(5/2)*b
^3*(x + a/b)^4)
```

Fricas [B] time = 1.64007, size = 154, normalized size = 4.16

$$-\frac{4\,b^3x^3+6\,ab^2x^2+4\,a^2bx+a^3}{4\left(b^8x^4+4\,ab^7x^3+6\,a^2b^6x^2+4\,a^3b^5x+a^4b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/(b^8*x^4 + 4*a*b^7*x^3 + 6
*a^2*b^6*x^2 + 4*a^3*b^5*x + a^4*b^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left((a+bx)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Integral(x**3/((a + b*x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.202 $\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=107

$$-\frac{a^2}{4b^3(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}+\frac{2a}{3b^3(a+bx)^2\sqrt{a^2+2abx+b^2x^2}}-\frac{1}{2b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}$$

```
[Out] -a^2/(4*b^3*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (2*a)/(3*b^3*(a +
b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - 1/(2*b^3*(a + b*x)*Sqrt[a^2 + 2*a*b
*x + b^2*x^2])
```

Rubi [A] time = 0.0478855, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 43}

$$-\frac{a^2}{4b^3(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}+\frac{2a}{3b^3(a+bx)^2\sqrt{a^2+2abx+b^2x^2}}-\frac{1}{2b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```


```
[Out] -a^2/(4*b^3*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (2*a)/(3*b^3*(a +
b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - 1/(2*b^3*(a + b*x)*Sqrt[a^2 + 2*a*b
*x + b^2*x^2])
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx &= \frac{\left(b^4\left(ab+b^2x\right)\right)\int \frac{x^2}{\left(ab+b^2x\right)^5}\,dx}{\sqrt{a^2+2abx+b^2x^2}}\\
&= \frac{\left(b^4\left(ab+b^2x\right)\right)\int \left(\frac{a^2}{b^7(a+bx)^5}-\frac{2a}{b^7(a+bx)^4}+\frac{1}{b^7(a+bx)^3}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}\\
&= -\frac{a^2}{4b^3(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}+\frac{2a}{3b^3(a+bx)^2\sqrt{a^2+2abx+b^2x^2}}-\frac{1}{2b^3(a+bx)\sqrt{a^2+2}}
\end{aligned}$$

Mathematica [A] time = 0.0135012, size = 44, normalized size = 0.41

$$\frac{-a^2-4abx-6b^2x^2}{12b^3(a+bx)^3\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (-a^2 - 4*a*b*x - 6*b^2*x^2)/(12*b^3*(a + b*x)^3*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.177, size = 37, normalized size = 0.4

$$-\frac{(bx+a)\left(6\,b^2x^2+4\,abx+a^2\right)}{12\,b^3}\left((bx+a)^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] -1/12*(b*x+a)*(6*b^2*x^2+4*a*b*x+a^2)/b^3/((b*x+a)^2)^(5/2)
```

Maxima [A] time = 1.18181, size = 77, normalized size = 0.72

$$-\frac{a^2b^2}{4\left(b^2\right)^{\frac{9}{2}}\left(x+\frac{a}{b}\right)^4}+\frac{2\,ab}{3\left(b^2\right)^{\frac{7}{2}}\left(x+\frac{a}{b}\right)^3}-\frac{1}{2\left(b^2\right)^{\frac{5}{2}}\left(x+\frac{a}{b}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*a^2*b^2/((b^2)^(9/2)*(x + a/b)^4) + 2/3*a*b/((b^2)^(7/2)*(x + a/b)^3)
- 1/2/((b^2)^(5/2)*(x + a/b)^2)
```

Fricas [A] time = 1.59905, size = 134, normalized size = 1.25

$$-\frac{6\,b^2x^2+4\,abx+a^2}{12\left(b^7x^4+4\,ab^6x^3+6\,a^2b^5x^2+4\,a^3b^4x+a^4b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^2*x^2 + 4*a*b*x + a^2)/(b^7*x^4 + 4*a*b^6*x^3 + 6*a^2*b^5*x^2 +
4*a^3*b^4*x + a^4*b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left((a+bx)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Integral(x**2/((a + b*x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.203 $\int \frac{x}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=63

$$\frac{a}{4b^2(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}-\frac{1}{3b^2\left(a^2+2abx+b^2x^2\right)^{3/2}}$$

```
[Out] -1/(3*b^2*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)) + a/(4*b^2*(a + b*x)*(a^2 + 2*a*
b*x + b^2*x^2)^(3/2))
```

Rubi [A] time = 0.0145728, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {640, 607}

$$\frac{a}{4b^2(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}-\frac{1}{3b^2\left(a^2+2abx+b^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] -1/(3*b^2*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)) + a/(4*b^2*(a + b*x)*(a^2 + 2*a*
b*x + b^2*x^2)^(3/2))
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx &= -\frac{1}{3b^2\left(a^2+2abx+b^2x^2\right)^{3/2}}-\frac{a\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx}{b}\\ &= -\frac{1}{3b^2\left(a^2+2abx+b^2x^2\right)^{3/2}}+\frac{a}{4b^2(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0100249, size = 33, normalized size = 0.52

$$\frac{-a-4bx}{12b^2(a+bx)^3\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (-a - 4*b*x)/(12*b^2*(a + b*x)^3*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.174, size = 26, normalized size = 0.4

$$-\frac{(bx+a)(4bx+a)}{12b^2}\left((bx+a)^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] -1/12*(b*x+a)*(4*b*x+a)/b^2/((b*x+a)^2)^(5/2)
```

Maxima [A] time = 1.10861, size = 59, normalized size = 0.94

$$-\frac{1}{3\left(b^2x^2+2abx+a^2\right)^{\frac{3}{2}}b^2}+\frac{a}{4\left(b^2\right)^{\frac{5}{2}}b\left(x+\frac{a}{b}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^2) + 1/4*a/((b^2)^(5/2)*b*(x + a/b)
^4)
```

Fricas [A] time = 1.51872, size = 112, normalized size = 1.78

$$-\frac{4bx+a}{12\left(b^6x^4+4ab^5x^3+6a^2b^4x^2+4a^3b^3x+a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(4*b*x + a)/(b^6*x^4 + 4*a*b^5*x^3 + 6*a^2*b^4*x^2 + 4*a^3*b^3*x + a^
4*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\left((a+bx)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Integral(x/((a + b*x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.204 $\int \frac{1}{(a^2+2abx+b^2x^2)^{5/2}} dx$

Optimal. Leaf size=34

$$-\frac{1}{4b(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}$$

```
[Out] -1/(4*b*(a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))
```

Rubi [A] time = 0.0043447, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {607}

$$-\frac{1}{4b(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(-5/2),x]
```

```
[Out] -1/(4*b*(a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = -\frac{1}{4b(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}$$

Mathematica [A] time = 0.0095918, size = 23, normalized size = 0.68

$$-\frac{a+bx}{4b\left((a+bx)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(-5/2),x]
```

```
[Out] -(a + b*x)/(4*b*((a + b*x)^2)^(5/2))
```

Maple [A] time = 0.044, size = 20, normalized size = 0.6

$$-\frac{bx+a}{4\,b}\left((bx+a)^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] -1/4*(b*x+a)/b/((b*x+a)^2)^(5/2)
```

Maxima [A] time = 1.13813, size = 22, normalized size = 0.65

$$-\frac{1}{4\left(b^2\right)^{\frac{5}{2}}\left(x+\frac{a}{b}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4/((b^2)^(5/2)*(x + a/b)^4)
```

Fricas [A] time = 1.69939, size = 92, normalized size = 2.71

$$-\frac{1}{4\left(b^5x^4+4\,ab^4x^3+6\,a^2b^3x^2+4\,a^3b^2x+a^4b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/4/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(a^2+2abx+b^2x^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x + b**2*x**2)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.205 $\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=194

$$\frac{1}{2a^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{1}{3a^2(a+bx)^2\sqrt{a^2+2abx+b^2x^2}}+\frac{1}{4a(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}+\frac{1}{a^4\sqrt{a^2+2abx+b^2}}$$

```
[Out] 1/(a^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + 1/(4*a*(a + b*x)^3*Sqrt[a^2 + 2*a*b
*x + b^2*x^2]) + 1/(3*a^2*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + 1/(2
*a^3*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + ((a + b*x)*Log[x])/(a^5*Sqr
t[a^2 + 2*a*b*x + b^2*x^2]) - ((a + b*x)*Log[a + b*x])/(a^5*Sqrt[a^2 + 2*a*
b*x + b^2*x^2])
```

Rubi [A] time = 0.0863049, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 44}

$$\frac{1}{2a^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{1}{3a^2(a+bx)^2\sqrt{a^2+2abx+b^2x^2}}+\frac{1}{4a(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}+\frac{1}{a^4\sqrt{a^2+2abx+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

```
[Out] 1/(a^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + 1/(4*a*(a + b*x)^3*Sqrt[a^2 + 2*a*b
*x + b^2*x^2]) + 1/(3*a^2*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + 1/(2
*a^3*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + ((a + b*x)*Log[x])/(a^5*Sqr
t[a^2 + 2*a*b*x + b^2*x^2]) - ((a + b*x)*Log[a + b*x])/(a^5*Sqrt[a^2 + 2*a*
b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x\right)\right)\int \frac{1}{x\left(ab+b^2x\right)^5}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$
$$= \frac{\left(b^4\left(ab+b^2x\right)\right)\int \left(\frac{1}{a^5b^5x}-\frac{1}{ab^4(a+bx)^5}-\frac{1}{a^2b^4(a+bx)^4}-\frac{1}{a^3b^4(a+bx)^3}-\frac{1}{a^4b^4(a+bx)^2}-\frac{1}{a^5b^4(a+bx)}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$
$$= \frac{1}{a^4\sqrt{a^2+2abx+b^2x^2}}+\frac{1}{4a(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}+\frac{1}{3a^2(a+bx)^2\sqrt{a^2+2abx+}}$$

Mathematica [A] time = 0.0346088, size = 84, normalized size = 0.43

$$\frac{a\left(52a^2bx+25a^3+42ab^2x^2+12b^3x^3\right)+12\log(x)(a+bx)^4-12(a+bx)^4\log(a+bx)}{12a^5(a+bx)^3\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

```
[Out] (a*(25*a^3 + 52*a^2*b*x + 42*a*b^2*x^2 + 12*b^3*x^3) + 12*(a + b*x)^4*Log[x
] - 12*(a + b*x)^4*Log[a + b*x])/(12*a^5*(a + b*x)^3*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.238, size = 173, normalized size = 0.9

$$\frac{\left(12\ln(x)x^4b^4-12\ln(bx+a)x^4b^4+48\ln(x)x^3ab^3-48\ln(bx+a)x^3ab^3+72\ln(x)x^2a^2b^2-72\ln(bx+a)x^2a^2b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] 1/12*(12*ln(x)*x^4*b^4-12*ln(b*x+a)*x^4*b^4+48*ln(x)*x^3*a*b^3-48*ln(b*x+a)
*x^3*a*b^3+72*ln(x)*x^2*a^2*b^2-72*ln(b*x+a)*x^2*a^2*b^2+12*a*b^3*x^3+48*ln
(x)*x*a^3*b-48*ln(b*x+a)*x*a^3*b+42*x^2*a^2*b^2+12*ln(x)*a^4-12*a^4*ln(b*x+
a)+52*x*a^3*b+25*a^4)*(b*x+a)/a^5/((b*x+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7908, size = 365, normalized size = 1.88

$$\frac{12ab^3x^3+42a^2b^2x^2+52a^3bx+25a^4-12\left(b^4x^4+4ab^3x^3+6a^2b^2x^2+4a^3bx+a^4\right)\log(bx+a)+12\left(b^4x^4+4ab^3\right.}{12\left(a^5b^4x^4+4a^6b^3x^3+6a^7b^2x^2+4a^8bx+a^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(12*a*b^3*x^3 + 42*a^2*b^2*x^2 + 52*a^3*b*x + 25*a^4 - 12*(b^4*x^4 + 4
*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*log(b*x + a) + 12*(b^4*x^4 +
4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*log(x))/(a^5*b^4*x^4 + 4*a^6
*b^3*x^3 + 6*a^7*b^2*x^2 + 4*a^8*b*x + a^9)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left((a+bx)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x*((a + b*x)**2)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.206 $\int \frac{1}{x^2\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=235

$$-\frac{3b}{2a^4(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{2b}{3a^3(a+bx)^2\sqrt{a^2+2abx+b^2x^2}}-\frac{b}{4a^2(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}-\frac{4b}{a^5\sqrt{a^2+2ab}}$$

```
[Out] (-4*b)/(a^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - b/(4*a^2*(a + b*x)^3*Sqrt[a^2
+ 2*a*b*x + b^2*x^2]) - (2*b)/(3*a^3*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x
^2]) - (3*b)/(2*a^4*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (a + b*x)/(a
^5*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (5*b*(a + b*x)*Log[x])/(a^6*Sqrt[a^2
+ 2*a*b*x + b^2*x^2]) + (5*b*(a + b*x)*Log[a + b*x])/(a^6*Sqrt[a^2 + 2*a*b*
x + b^2*x^2])
```

Rubi [A] time = 0.102017, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 44}

$$-\frac{3b}{2a^4(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{2b}{3a^3(a+bx)^2\sqrt{a^2+2abx+b^2x^2}}-\frac{b}{4a^2(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}-\frac{4b}{a^5\sqrt{a^2+2ab}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

```
[Out] (-4*b)/(a^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - b/(4*a^2*(a + b*x)^3*Sqrt[a^2
+ 2*a*b*x + b^2*x^2]) - (2*b)/(3*a^3*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x
^2]) - (3*b)/(2*a^4*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (a + b*x)/(a
^5*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (5*b*(a + b*x)*Log[x])/(a^6*Sqrt[a^2
+ 2*a*b*x + b^2*x^2]) + (5*b*(a + b*x)*Log[a + b*x])/(a^6*Sqrt[a^2 + 2*a*b*
x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^2 \left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x\right)\right)\int \frac{1}{x^2\left(ab+b^2x\right)^5}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= \frac{\left(b^4\left(ab+b^2x\right)\right)\int \left(\frac{1}{a^5b^5x^2}-\frac{5}{a^6b^4x}+\frac{1}{a^2b^3(a+bx)^5}+\frac{2}{a^3b^3(a+bx)^4}+\frac{3}{a^4b^3(a+bx)^3}+\frac{4}{a^5b^3(a+bx)^2}+\frac{}{a^6b}\right.}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{4b}{a^5\sqrt{a^2+2abx+b^2x^2}}-\frac{b}{4a^2(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}-\frac{2b}{3a^3(a+bx)^2\sqrt{a^2+2abx+}}$$

Mathematica [A] time = 0.0402118, size = 103, normalized size = 0.44

$$\frac{-a\left(260a^2b^2x^2+125a^3bx+12a^4+210ab^3x^3+60b^4x^4\right)-60bx\log(x)(a+bx)^4+60bx(a+bx)^4\log(a+bx)}{12a^6x(a+bx)^3\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

```
[Out] (-(a*(12*a^4 + 125*a^3*b*x + 260*a^2*b^2*x^2 + 210*a*b^3*x^3 + 60*b^4*x^4))
 - 60*b*x*(a + b*x)^4*Log[x] + 60*b*x*(a + b*x)^4*Log[a + b*x])/(12*a^6*x*(
a + b*x)^3*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.225, size = 199, normalized size = 0.9

$$-\frac{\left(60\,b^5\ln(x)\,x^5-60\,\ln(bx+a)\,x^5b^5+240\,ab^4\ln(x)\,x^4-240\,\ln(bx+a)\,x^4ab^4+360\,a^2b^3\ln(x)\,x^3-360\,\ln(bx+a)\,x^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] -1/12*(60*b^5*ln(x)*x^5-60*ln(b*x+a)*x^5*b^5+240*a*b^4*ln(x)*x^4-240*ln(b*x
+a)*x^4*a*b^4+360*a^2*b^3*ln(x)*x^3-360*ln(b*x+a)*x^3*a^2*b^3+60*a*b^4*x^4+
240*a^3*b^2*ln(x)*x^2-240*ln(b*x+a)*x^2*a^3*b^2+210*a^2*b^3*x^3+60*a^4*b*ln
(x)*x-60*ln(b*x+a)*x*a^4*b+260*a^3*b^2*x^2+125*a^4*b*x+12*a^5)*(b*x+a)/x/a^
6/((b*x+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7869, size = 421, normalized size = 1.79

$$-\frac{60\,ab^4x^4 + 210\,a^2b^3x^3 + 260\,a^3b^2x^2 + 125\,a^4bx + 12\,a^5 - 60\left(b^5x^5 + 4\,ab^4x^4 + 6\,a^2b^3x^3 + 4\,a^3b^2x^2 + a^4bx\right)\log\left(bx + \right.}{12\left(a^6b^4x^5 + 4\,a^7b^3x^4 + 6\,a^8b^2x^3 + 4\,a^9bx^2 + a^{10}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(60*a*b^4*x^4 + 210*a^2*b^3*x^3 + 260*a^3*b^2*x^2 + 125*a^4*b*x + 12*
a^5 - 60*(b^5*x^5 + 4*a*b^4*x^4 + 6*a^2*b^3*x^3 + 4*a^3*b^2*x^2 + a^4*b*x)*
log(b*x + a) + 60*(b^5*x^5 + 4*a*b^4*x^4 + 6*a^2*b^3*x^3 + 4*a^3*b^2*x^2 +
a^4*b*x)*log(x))/(a^6*b^4*x^5 + 4*a^7*b^3*x^4 + 6*a^8*b^2*x^3 + 4*a^9*b*x^2
 + a^10*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\left(a + bx\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**2*((a + b*x)**2)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.207 $\int \frac{1}{x^3\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=278

$$\frac{3b^2}{a^5(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{b^2}{a^4(a+bx)^2\sqrt{a^2+2abx+b^2x^2}}+\frac{b^2}{4a^3(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}+\frac{10b^2}{a^6\sqrt{a^2+2abx+b^2x}}$$

```
[Out] (10*b^2)/(a^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + b^2/(4*a^3*(a + b*x)^3*Sqrt[
a^2 + 2*a*b*x + b^2*x^2]) + b^2/(a^4*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x
^2]) + (3*b^2)/(a^5*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (a + b*x)/(2
*a^5*x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (5*b*(a + b*x))/(a^6*x*Sqrt[a^2 +
 2*a*b*x + b^2*x^2]) + (15*b^2*(a + b*x)*Log[x])/(a^7*Sqrt[a^2 + 2*a*b*x +
b^2*x^2]) - (15*b^2*(a + b*x)*Log[a + b*x])/(a^7*Sqrt[a^2 + 2*a*b*x + b^2*x
^2])
```

Rubi [A] time = 0.114067, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {646, 44}

$$\frac{3b^2}{a^5(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{b^2}{a^4(a+bx)^2\sqrt{a^2+2abx+b^2x^2}}+\frac{b^2}{4a^3(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}+\frac{10b^2}{a^6\sqrt{a^2+2abx+b^2x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

```
[Out] (10*b^2)/(a^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + b^2/(4*a^3*(a + b*x)^3*Sqrt[
a^2 + 2*a*b*x + b^2*x^2]) + b^2/(a^4*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x
^2]) + (3*b^2)/(a^5*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (a + b*x)/(2
*a^5*x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (5*b*(a + b*x))/(a^6*x*Sqrt[a^2 +
 2*a*b*x + b^2*x^2]) + (15*b^2*(a + b*x)*Log[x])/(a^7*Sqrt[a^2 + 2*a*b*x +
b^2*x^2]) - (15*b^2*(a + b*x)*Log[a + b*x])/(a^7*Sqrt[a^2 + 2*a*b*x + b^2*x
^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x\right)\right)\int \frac{1}{x^3\left(ab+b^2x\right)^5}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= \frac{\left(b^4\left(ab+b^2x\right)\right)\int \left(\frac{1}{a^5b^5x^3}-\frac{5}{a^6b^4x^2}+\frac{15}{a^7b^3x}-\frac{1}{a^3b^2(a+bx)^5}-\frac{3}{a^4b^2(a+bx)^4}-\frac{6}{a^5b^2(a+bx)^3}-\frac{}{a^6b^2(}\right.}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= \frac{10b^2}{a^6\sqrt{a^2+2abx+b^2x^2}}+\frac{b^2}{4a^3(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}+\frac{b^2}{a^4(a+bx)^2\sqrt{a^2+2abx+}}$$

Mathematica [A] time = 0.0433473, size = 121, normalized size = 0.44

$$\frac{a\left(125a^3b^2x^2+260a^2b^3x^3+12a^4bx-2a^5+210ab^4x^4+60b^5x^5\right)+60b^2x^2\log(x)(a+bx)^4-60b^2x^2(a+bx)^4\log(a+}{4a^7x^2(a+bx)^3\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

```
[Out] (a*(-2*a^5 + 12*a^4*b*x + 125*a^3*b^2*x^2 + 260*a^2*b^3*x^3 + 210*a*b^4*x^4
 + 60*b^5*x^5) + 60*b^2*x^2*(a + b*x)^4*Log[x] - 60*b^2*x^2*(a + b*x)^4*Log
[a + b*x])/(4*a^7*x^2*(a + b*x)^3*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.231, size = 218, normalized size = 0.8

$$\frac{\left(60\ln(x)x^6b^6-60\ln(bx+a)x^6b^6+240\ln(x)x^5ab^5-240\ln(bx+a)x^5ab^5+360\ln(x)x^4a^2b^4-360\ln(bx+a)x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] 1/4*(60*ln(x)*x^6*b^6-60*ln(b*x+a)*x^6*b^6+240*ln(x)*x^5*a*b^5-240*ln(b*x+a
)*x^5*a*b^5+360*ln(x)*x^4*a^2*b^4-360*ln(b*x+a)*x^4*a^2*b^4+60*x^5*a*b^5+24
0*ln(x)*x^3*a^3*b^3-240*ln(b*x+a)*x^3*a^3*b^3+210*a^2*x^4*b^4+60*ln(x)*x^2*
a^4*b^2-60*ln(b*x+a)*x^2*a^4*b^2+260*a^3*x^3*b^3+125*a^4*x^2*b^2+12*a^5*x*b
-2*a^6)*(b*x+a)/x^2/a^7/((b*x+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74119, size = 455, normalized size = 1.64

$$\frac{60\,ab^5x^5 + 210\,a^2b^4x^4 + 260\,a^3b^3x^3 + 125\,a^4b^2x^2 + 12\,a^5bx - 2\,a^6 - 60\left(b^6x^6 + 4\,ab^5x^5 + 6\,a^2b^4x^4 + 4\,a^3b^3x^3 + a^4b^2x^2\right)\mathrm{lo}}{4\left(a^7b^4x^6 + 4\,a^8b^3x^5 + 6\,a^9b^2x^4 + 4\,a^{10}bx^3 + a^{11}x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(60*a*b^5*x^5 + 210*a^2*b^4*x^4 + 260*a^3*b^3*x^3 + 125*a^4*b^2*x^2 + 1
2*a^5*b*x - 2*a^6 - 60*(b^6*x^6 + 4*a*b^5*x^5 + 6*a^2*b^4*x^4 + 4*a^3*b^3*x
^3 + a^4*b^2*x^2)*log(b*x + a) + 60*(b^6*x^6 + 4*a*b^5*x^5 + 6*a^2*b^4*x^4
+ 4*a^3*b^3*x^3 + a^4*b^2*x^2)*log(x))/(a^7*b^4*x^6 + 4*a^8*b^3*x^5 + 6*a^9
*b^2*x^4 + 4*a^10*b*x^3 + a^11*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left((a+bx)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**3*((a + b*x)**2)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.208 $\int x\left(9+12x+4x^2\right)^{5/2}\,dx$

Optimal. Leaf size=42

$$\frac{1}{28}\left(4x^2+12x+9\right)^{7/2}-\frac{1}{8}(2x+3)\left(4x^2+12x+9\right)^{5/2}$$

```
[Out] -((3 + 2*x)*(9 + 12*x + 4*x^2)^(5/2))/8 + (9 + 12*x + 4*x^2)^(7/2)/28
```

Rubi [A] time = 0.0088291, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {640, 609}

$$\frac{1}{28}\left(4x^2+12x+9\right)^{7/2}-\frac{1}{8}(2x+3)\left(4x^2+12x+9\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(9 + 12*x + 4*x^2)^(5/2),x]
```

```
[Out] -((3 + 2*x)*(9 + 12*x + 4*x^2)^(5/2))/8 + (9 + 12*x + 4*x^2)^(7/2)/28
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int x\left(9+12x+4x^2\right)^{5/2}\,dx &= \frac{1}{28}\left(9+12x+4x^2\right)^{7/2}-\frac{3}{2}\int\left(9+12x+4x^2\right)^{5/2}\,dx\\ &= -\frac{1}{8}(3+2x)\left(9+12x+4x^2\right)^{5/2}+\frac{1}{28}\left(9+12x+4x^2\right)^{7/2}\end{aligned}$$

Mathematica [A] time = 0.0174827, size = 47, normalized size = 1.12

$$\frac{x^2\sqrt{(2x+3)^2}\left(64x^5+560x^4+2016x^3+3780x^2+3780x+1701\right)}{28x+42}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(9 + 12*x + 4*x^2)^(5/2),x]
```

```
[Out] (x^2*Sqrt[(3 + 2*x)^2]*(1701 + 3780*x + 3780*x^2 + 2016*x^3 + 560*x^4 + 64*
x^5))/(42 + 28*x)
```

Maple [A] time = 0.064, size = 47, normalized size = 1.1

$$\frac{x^2\left(64x^5+560x^4+2016x^3+3780x^2+3780x+1701\right)}{14\left(3+2x\right)^5}\left(\left(3+2x\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(4*x^2+12*x+9)^(5/2),x)
```

```
[Out] 1/14*x^2*(64*x^5+560*x^4+2016*x^3+3780*x^2+3780*x+1701)*((3+2*x)^2)^(5/2)/(
3+2*x)^5
```

Maxima [A] time = 1.70609, size = 59, normalized size = 1.4

$$\frac{1}{28}\left(4x^2+12x+9\right)^{\frac{7}{2}}-\frac{1}{4}\left(4x^2+12x+9\right)^{\frac{5}{2}}x-\frac{3}{8}\left(4x^2+12x+9\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2+12*x+9)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/28*(4*x^2 + 12*x + 9)^(7/2) - 1/4*(4*x^2 + 12*x + 9)^(5/2)*x - 3/8*(4*x^2
 + 12*x + 9)^(5/2)
```

Fricas [A] time = 1.61433, size = 82, normalized size = 1.95

$$\frac{32}{7}x^7+40x^6+144x^5+270x^4+270x^3+\frac{243}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2+12*x+9)^(5/2),x, algorithm="fricas")
```

```
[Out] 32/7*x^7 + 40*x^6 + 144*x^5 + 270*x^4 + 270*x^3 + 243/2*x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\left(2x+3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x**2+12*x+9)**(5/2),x)
```

```
[Out] Integral(x*((2*x + 3)**2)**(5/2), x)
```

Giac [B] time = 1.30584, size = 101, normalized size = 2.4

$$\frac{32}{7}x^7\mathrm{sgn}\left(2x+3\right)+40x^6\mathrm{sgn}\left(2x+3\right)+144x^5\mathrm{sgn}\left(2x+3\right)+270x^4\mathrm{sgn}\left(2x+3\right)+270x^3\mathrm{sgn}\left(2x+3\right)+\frac{243}{2}x^2\mathrm{sgn}\left(2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2+12*x+9)^(5/2),x, algorithm="giac")
```

```
[Out] 32/7*x^7*sgn(2*x + 3) + 40*x^6*sgn(2*x + 3) + 144*x^5*sgn(2*x + 3) + 270*x^
4*sgn(2*x + 3) + 270*x^3*sgn(2*x + 3) + 243/2*x^2*sgn(2*x + 3) - 729/56*sgn
(2*x + 3)
```

3.209 $\int x\left(9+12x+4x^2\right)^{3/2}\,dx$

Optimal. Leaf size=42

$$\frac{1}{20}\left(4x^2+12x+9\right)^{5/2}-\frac{3}{16}(2x+3)\left(4x^2+12x+9\right)^{3/2}$$

```
[Out] (-3*(3 + 2*x)*(9 + 12*x + 4*x^2)^(3/2))/16 + (9 + 12*x + 4*x^2)^(5/2)/20
```

Rubi [A] time = 0.008506, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {640, 609}

$$\frac{1}{20}\left(4x^2+12x+9\right)^{5/2}-\frac{3}{16}(2x+3)\left(4x^2+12x+9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(9 + 12*x + 4*x^2)^(3/2),x]
```

```
[Out] (-3*(3 + 2*x)*(9 + 12*x + 4*x^2)^(3/2))/16 + (9 + 12*x + 4*x^2)^(5/2)/20
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int x\left(9+12x+4x^2\right)^{3/2}\,dx&=\frac{1}{20}\left(9+12x+4x^2\right)^{5/2}-\frac{3}{2}\int\left(9+12x+4x^2\right)^{3/2}\,dx\\&=-\frac{3}{16}(3+2x)\left(9+12x+4x^2\right)^{3/2}+\frac{1}{20}\left(9+12x+4x^2\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.0116702, size = 37, normalized size = 0.88

$$\frac{x^2\sqrt{(2x+3)^2}\left(16x^3+90x^2+180x+135\right)}{20x+30}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(9 + 12*x + 4*x^2)^(3/2),x]
```

```
[Out] (x^2*Sqrt[(3 + 2*x)^2]*(135 + 180*x + 90*x^2 + 16*x^3))/(30 + 20*x)
```

Maple [A] time = 0.067, size = 37, normalized size = 0.9

$$\frac{x^{2}\left(16x^{3}+90x^{2}+180x+135\right)\left(\left(3+2x\right)^{2}\right)^{\frac{3}{2}}}{10\left(3+2x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(4*x^2+12*x+9)^(3/2),x)
```

```
[Out] 1/10*x^2*(16*x^3+90*x^2+180*x+135)*((3+2*x)^2)^(3/2)/(3+2*x)^3
```

Maxima [A] time = 1.71987, size = 59, normalized size = 1.4

$$\frac{1}{20}\left(4x^{2}+12x+9\right)^{\frac{5}{2}}-\frac{3}{8}\left(4x^{2}+12x+9\right)^{\frac{3}{2}}x-\frac{9}{16}\left(4x^{2}+12x+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2+12*x+9)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/20*(4*x^2 + 12*x + 9)^(5/2) - 3/8*(4*x^2 + 12*x + 9)^(3/2)*x - 9/16*(4*x^
2 + 12*x + 9)^(3/2)
```

Fricas [A] time = 1.65142, size = 50, normalized size = 1.19

$$\frac{8}{5}x^{5}+9x^{4}+18x^{3}+\frac{27}{2}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2+12*x+9)^(3/2),x, algorithm="fricas")
```

```
[Out] 8/5*x^5 + 9*x^4 + 18*x^3 + 27/2*x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\left(2x+3\right)^{2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x**2+12*x+9)**(3/2),x)
```

```
[Out] Integral(x*((2*x + 3)**2)**(3/2), x)
```

Giac [A] time = 1.26485, size = 72, normalized size = 1.71

$$\frac{8}{5}x^{5}\mathrm{sgn}\left(2x+3\right)+9x^{4}\mathrm{sgn}\left(2x+3\right)+18x^{3}\mathrm{sgn}\left(2x+3\right)+\frac{27}{2}x^{2}\mathrm{sgn}\left(2x+3\right)-\frac{243}{80}\mathrm{sgn}\left(2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2+12*x+9)^(3/2),x, algorithm="giac")
```

```
[Out] 8/5*x^5*sgn(2*x + 3) + 9*x^4*sgn(2*x + 3) + 18*x^3*sgn(2*x + 3) + 27/2*x^2*
sgn(2*x + 3) - 243/80*sgn(2*x + 3)
```

3.210 $\int x\sqrt{9+12x+4x^2}\,dx$

Optimal. Leaf size=42

$$\frac{1}{12}\left(4x^2+12x+9\right)^{3/2}-\frac{3}{8}(2x+3)\sqrt{4x^2+12x+9}$$

```
[Out] (-3*(3 + 2*x)*Sqrt[9 + 12*x + 4*x^2])/8 + (9 + 12*x + 4*x^2)^(3/2)/12
```

Rubi [A] time = 0.0079994, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {640, 609}

$$\frac{1}{12}\left(4x^2+12x+9\right)^{3/2}-\frac{3}{8}(2x+3)\sqrt{4x^2+12x+9}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[9 + 12*x + 4*x^2],x]
```

```
[Out] (-3*(3 + 2*x)*Sqrt[9 + 12*x + 4*x^2])/8 + (9 + 12*x + 4*x^2)^(3/2)/12
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{9+12x+4x^2}\,dx &= \frac{1}{12}\left(9+12x+4x^2\right)^{3/2}-\frac{3}{2}\int\sqrt{9+12x+4x^2}\,dx\\ &= -\frac{3}{8}(3+2x)\sqrt{9+12x+4x^2}+\frac{1}{12}\left(9+12x+4x^2\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.0064007, size = 30, normalized size = 0.71

$$\frac{x^2\sqrt{(2x+3)^2}(4x+9)}{6(2x+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[9 + 12*x + 4*x^2],x]
```

```
[Out] (x^2*Sqrt[(3 + 2*x)^2]*(9 + 4*x))/(6*(3 + 2*x))
```

Maple [A] time = 0.062, size = 27, normalized size = 0.6

$$\frac{x^2(4x+9)}{18+12x}\sqrt{(3+2x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(4*x^2+12*x+9)^(1/2),x)
```

```
[Out] 1/6*x^2*(4*x+9)*((3+2*x)^2)^(1/2)/(3+2*x)
```

Maxima [A] time = 1.70717, size = 59, normalized size = 1.4

$$\frac{1}{12}\left(4x^2+12x+9\right)^{\frac{3}{2}}-\frac{3}{4}\sqrt{4x^2+12x+9}x-\frac{9}{8}\sqrt{4x^2+12x+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2+12*x+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(4*x^2 + 12*x + 9)^(3/2) - 3/4*sqrt(4*x^2 + 12*x + 9)*x - 9/8*sqrt(4*x
^2 + 12*x + 9)
```

Fricas [A] time = 1.53914, size = 26, normalized size = 0.62

$$\frac{2}{3}x^3+\frac{3}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2+12*x+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*x^3 + 3/2*x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{(2x+3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x**2+12*x+9)**(1/2),x)
```

```
[Out] Integral(x*sqrt((2*x + 3)**2), x)
```

Giac [A] time = 1.31284, size = 42, normalized size = 1.

$$\frac{2}{3}x^3\mathrm{sgn}(2x+3)+\frac{3}{2}x^2\mathrm{sgn}(2x+3)-\frac{9}{8}\mathrm{sgn}(2x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2+12*x+9)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*x^3*sgn(2*x + 3) + 3/2*x^2*sgn(2*x + 3) - 9/8*sgn(2*x + 3)
```

3.211 $\int \frac{x}{\sqrt{9+12x+4x^2}}\,dx$

Optimal. Leaf size=48

$$\frac{1}{4}\sqrt{4x^2+12x+9}-\frac{3(2x+3)\log(2x+3)}{4\sqrt{4x^2+12x+9}}$$

```
[Out] Sqrt[9 + 12*x + 4*x^2]/4 - (3*(3 + 2*x)*Log[3 + 2*x])/(4*Sqrt[9 + 12*x + 4*
x^2])
```

Rubi [A] time = 0.0108877, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {640, 608, 31}

$$\frac{1}{4}\sqrt{4x^2+12x+9}-\frac{3(2x+3)\log(2x+3)}{4\sqrt{4x^2+12x+9}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[9 + 12*x + 4*x^2],x]
```

```
[Out] Sqrt[9 + 12*x + 4*x^2]/4 - (3*(3 + 2*x)*Log[3 + 2*x])/(4*Sqrt[9 + 12*x + 4*
x^2])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{9+12x+4x^2}}\,dx &= \frac{1}{4}\sqrt{9+12x+4x^2}-\frac{3}{2}\int \frac{1}{\sqrt{9+12x+4x^2}}\,dx\\ &= \frac{1}{4}\sqrt{9+12x+4x^2}-\frac{(3(6+4x))\int \frac{1}{6+4x}\,dx}{2\sqrt{9+12x+4x^2}}\\ &= \frac{1}{4}\sqrt{9+12x+4x^2}-\frac{3(3+2x)\log(3+2x)}{4\sqrt{9+12x+4x^2}}\end{aligned}$$

Mathematica [A] time = 0.0094597, size = 33, normalized size = 0.69

$$\frac{(2x+3)(2x-3\log(2x+3)+3)}{4\sqrt{(2x+3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[9 + 12*x + 4*x^2],x]
```

```
[Out] ((3 + 2*x)*(3 + 2*x - 3*Log[3 + 2*x]))/(4*Sqrt[(3 + 2*x)^2])
```

Maple [A] time = 0.158, size = 29, normalized size = 0.6

$$-\frac{(3+2x)(-2x+3\ln(3+2x))}{4}\frac{1}{\sqrt{(3+2x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(4*x^2+12*x+9)^(1/2),x)
```

```
[Out] -1/4*(3+2*x)*(-2*x+3*ln(3+2*x))/((3+2*x)^2)^(1/2)
```

Maxima [A] time = 1.68325, size = 28, normalized size = 0.58

$$\frac{1}{4}\sqrt{4x^2+12x+9}-\frac{3}{4}\log\left(x+\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2+12*x+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(4*x^2 + 12*x + 9) - 3/4*log(x + 3/2)
```

Fricas [A] time = 1.73834, size = 35, normalized size = 0.73

$$\frac{1}{2}x-\frac{3}{4}\log(2x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2+12*x+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*x - 3/4*log(2*x + 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{(2x+3)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x**2+12*x+9)**(1/2),x)
```

```
[Out] Integral(x/sqrt((2*x + 3)**2), x)
```

Giac [A] time = 1.31121, size = 49, normalized size = 1.02

$$\frac{1}{4}\sqrt{4x^2+12x+9}+\frac{3}{4}\log\left(\left|-2x+\sqrt{4x^2+12x+9}-3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2+12*x+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(4*x^2 + 12*x + 9) + 3/4*log(abs(-2*x + sqrt(4*x^2 + 12*x + 9) - 3)
)
```

3.212 $\int \frac{x}{(9+12x+4x^2)^{3/2}}\,dx$

Optimal. Leaf size=44

$$\frac{3}{8(2x+3)\sqrt{4x^2+12x+9}}-\frac{1}{4\sqrt{4x^2+12x+9}}$$

```
[Out] -1/(4*Sqrt[9 + 12*x + 4*x^2]) + 3/(8*(3 + 2*x)*Sqrt[9 + 12*x + 4*x^2])
```

Rubi [A] time = 0.0086371, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {640, 607}

$$\frac{3}{8(2x+3)\sqrt{4x^2+12x+9}}-\frac{1}{4\sqrt{4x^2+12x+9}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(9 + 12*x + 4*x^2)^(3/2),x]
```

```
[Out] -1/(4*Sqrt[9 + 12*x + 4*x^2]) + 3/(8*(3 + 2*x)*Sqrt[9 + 12*x + 4*x^2])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(9+12x+4x^2)^{3/2}}\,dx &= -\frac{1}{4\sqrt{9+12x+4x^2}}-\frac{3}{2}\int \frac{1}{(9+12x+4x^2)^{3/2}}\,dx\\ &= -\frac{1}{4\sqrt{9+12x+4x^2}}+\frac{3}{8(3+2x)\sqrt{9+12x+4x^2}}\end{aligned}$$

Mathematica [A] time = 0.0081203, size = 27, normalized size = 0.61

$$\frac{-4x-3}{8(2x+3)\sqrt{(2x+3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(9 + 12*x + 4*x^2)^(3/2),x]
```

```
[Out] (-3 - 4*x)/(8*(3 + 2*x)*Sqrt[(3 + 2*x)^2])
```

Maple [A] time = 0.08, size = 22, normalized size = 0.5

$$-\frac{(3+2x)(4x+3)}{8}\left((3+2x)^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(4*x^2+12*x+9)^(3/2),x)
```

```
[Out] -1/8*(3+2*x)*(4*x+3)/((3+2*x)^2)^(3/2)
```

Maxima [A] time = 1.71029, size = 32, normalized size = 0.73

$$-\frac{1}{4\sqrt{4x^2+12x+9}}+\frac{3}{8(2x+3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2+12*x+9)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4/sqrt(4*x^2 + 12*x + 9) + 3/8/(2*x + 3)^2
```

Fricas [A] time = 1.63125, size = 47, normalized size = 1.07

$$-\frac{4x+3}{8\left(4x^2+12x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2+12*x+9)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(4*x + 3)/(4*x^2 + 12*x + 9)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\left((2x+3)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x**2+12*x+9)**(3/2),x)
```

```
[Out] Integral(x/((2*x + 3)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2+12*x+9)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.213 $\int \frac{x}{(9+12x+4x^2)^{5/2}}\,dx$

Optimal. Leaf size=44

$$\frac{3}{16(2x+3)\left(4x^2+12x+9\right)^{3/2}}-\frac{1}{12\left(4x^2+12x+9\right)^{3/2}}$$

```
[Out] -1/(12*(9 + 12*x + 4*x^2)^(3/2)) + 3/(16*(3 + 2*x)*(9 + 12*x + 4*x^2)^(3/2)
)
```

Rubi [A] time = 0.0090006, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {640, 607}

$$\frac{3}{16(2x+3)\left(4x^2+12x+9\right)^{3/2}}-\frac{1}{12\left(4x^2+12x+9\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(9 + 12*x + 4*x^2)^(5/2),x]
```

```
[Out] -1/(12*(9 + 12*x + 4*x^2)^(3/2)) + 3/(16*(3 + 2*x)*(9 + 12*x + 4*x^2)^(3/2)
)
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(9+12x+4x^2\right)^{5/2}}\,dx &= -\frac{1}{12\left(9+12x+4x^2\right)^{3/2}}-\frac{3}{2}\int \frac{1}{\left(9+12x+4x^2\right)^{5/2}}\,dx\\ &= -\frac{1}{12\left(9+12x+4x^2\right)^{3/2}}+\frac{3}{16(3+2x)\left(9+12x+4x^2\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0084795, size = 27, normalized size = 0.61

$$\frac{-8x-3}{48(2x+3)^3\sqrt{(2x+3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(9 + 12*x + 4*x^2)^(5/2),x]
```

```
[Out] (-3 - 8*x)/(48*(3 + 2*x)^3*Sqrt[(3 + 2*x)^2])
```

Maple [A] time = 0.067, size = 22, normalized size = 0.5

$$-\frac{(3+2x)(8x+3)}{48}\left((3+2x)^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(4*x^2+12*x+9)^(5/2),x)
```

```
[Out] -1/48*(3+2*x)*(8*x+3)/((3+2*x)^2)^(5/2)
```

Maxima [A] time = 1.58961, size = 32, normalized size = 0.73

$$-\frac{1}{12\left(4x^2+12x+9\right)^{\frac{3}{2}}}+\frac{3}{16(2x+3)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2+12*x+9)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12/(4*x^2 + 12*x + 9)^(3/2) + 3/16/(2*x + 3)^4
```

Fricas [A] time = 1.68457, size = 78, normalized size = 1.77

$$-\frac{8x+3}{48\left(16x^4+96x^3+216x^2+216x+81\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2+12*x+9)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/48*(8*x + 3)/(16*x^4 + 96*x^3 + 216*x^2 + 216*x + 81)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\left((2x+3)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x**2+12*x+9)**(5/2),x)
```

```
[Out] Integral(x/((2*x + 3)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2+12*x+9)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.214 $\int \frac{x}{(9+12x+4x^2)^{7/2}}\,dx$

Optimal. Leaf size=44

$$\frac{1}{8(2x+3)\left(4x^2+12x+9\right)^{5/2}}-\frac{1}{20\left(4x^2+12x+9\right)^{5/2}}$$

```
[Out] -1/(20*(9 + 12*x + 4*x^2)^(5/2)) + 1/(8*(3 + 2*x)*(9 + 12*x + 4*x^2)^(5/2))
```

Rubi [A] time = 0.0091762, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {640, 607}

$$\frac{1}{8(2x+3)\left(4x^2+12x+9\right)^{5/2}}-\frac{1}{20\left(4x^2+12x+9\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(9 + 12*x + 4*x^2)^(7/2),x]
```

```
[Out] -1/(20*(9 + 12*x + 4*x^2)^(5/2)) + 1/(8*(3 + 2*x)*(9 + 12*x + 4*x^2)^(5/2))
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(9+12x+4x^2\right)^{7/2}}\,dx &= -\frac{1}{20\left(9+12x+4x^2\right)^{5/2}}-\frac{3}{2}\int \frac{1}{\left(9+12x+4x^2\right)^{7/2}}\,dx\\ &= -\frac{1}{20\left(9+12x+4x^2\right)^{5/2}}+\frac{1}{8(3+2x)\left(9+12x+4x^2\right)^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.0086804, size = 27, normalized size = 0.61

$$\frac{-4x-1}{40(2x+3)^5\sqrt{(2x+3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(9 + 12*x + 4*x^2)^(7/2),x]
```

```
[Out] (-1 - 4*x)/(40*(3 + 2*x)^5*Sqrt[(3 + 2*x)^2])
```

Maple [A] time = 0.066, size = 22, normalized size = 0.5

$$-\frac{(3+2x)(4x+1)}{40}\left((3+2x)^2\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(4*x^2+12*x+9)^(7/2),x)
```

```
[Out] -1/40*(3+2*x)*(4*x+1)/((3+2*x)^2)^(7/2)
```

Maxima [A] time = 1.72885, size = 32, normalized size = 0.73

$$-\frac{1}{20\left(4x^2+12x+9\right)^{\frac{5}{2}}}+\frac{1}{8(2x+3)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2+12*x+9)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/20/(4*x^2 + 12*x + 9)^(5/2) + 1/8/(2*x + 3)^6
```

Fricas [A] time = 1.59023, size = 113, normalized size = 2.57

$$-\frac{4x+1}{40\left(64x^6+576x^5+2160x^4+4320x^3+4860x^2+2916x+729\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2+12*x+9)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/40*(4*x + 1)/(64*x^6 + 576*x^5 + 2160*x^4 + 4320*x^3 + 4860*x^2 + 2916*x
 + 729)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\left((2x+3)^2\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x**2+12*x+9)**(7/2),x)
```

```
[Out] Integral(x/((2*x + 3)**2)**(7/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2+12*x+9)^(7/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.215 $\int \frac{x}{\sqrt{4+12x+9x^2}}\,dx$

Optimal. Leaf size=48

$$\frac{1}{9}\sqrt{9x^2+12x+4}-\frac{2(3x+2)\log(3x+2)}{9\sqrt{9x^2+12x+4}}$$

```
[Out] Sqrt[4 + 12*x + 9*x^2]/9 - (2*(2 + 3*x)*Log[2 + 3*x])/(9*Sqrt[4 + 12*x + 9*
x^2])
```

Rubi [A] time = 0.0113589, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {640, 608, 31}

$$\frac{1}{9}\sqrt{9x^2+12x+4}-\frac{2(3x+2)\log(3x+2)}{9\sqrt{9x^2+12x+4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[4 + 12*x + 9*x^2],x]
```

```
[Out] Sqrt[4 + 12*x + 9*x^2]/9 - (2*(2 + 3*x)*Log[2 + 3*x])/(9*Sqrt[4 + 12*x + 9*
x^2])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{4+12x+9x^2}}\,dx &= \frac{1}{9}\sqrt{4+12x+9x^2}-\frac{2}{3}\int \frac{1}{\sqrt{4+12x+9x^2}}\,dx\\
&= \frac{1}{9}\sqrt{4+12x+9x^2}-\frac{(2(6+9x))\int \frac{1}{6+9x}\,dx}{3\sqrt{4+12x+9x^2}}\\
&= \frac{1}{9}\sqrt{4+12x+9x^2}-\frac{2(2+3x)\log(2+3x)}{9\sqrt{4+12x+9x^2}}
\end{aligned}$$

Mathematica [A] time = 0.0110616, size = 33, normalized size = 0.69

$$\frac{(3x+2)(3x-2\log(3x+2)+2)}{9\sqrt{(3x+2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[4 + 12*x + 9*x^2],x]
```

```
[Out] ((2 + 3*x)*(2 + 3*x - 2*Log[2 + 3*x]))/(9*Sqrt[(2 + 3*x)^2])
```

Maple [A] time = 0.12, size = 29, normalized size = 0.6

$$-\frac{(2+3x)(-3x+2\ln(2+3x))}{9}\frac{1}{\sqrt{(2+3x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((2+3*x)^2)^(1/2),x)
```

```
[Out] -1/9*(2+3*x)*(-3*x+2*ln(2+3*x))/((2+3*x)^2)^(1/2)
```

Maxima [A] time = 1.67392, size = 28, normalized size = 0.58

$$\frac{1}{9}\sqrt{9x^2+12x+4}-\frac{2}{9}\log\left(x+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((2+3*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(9*x^2 + 12*x + 4) - 2/9*log(x + 2/3)
```

Fricas [A] time = 1.68502, size = 35, normalized size = 0.73

$$\frac{1}{3}x-\frac{2}{9}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((2+3*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*x - 2/9*log(3*x + 2)
```

Sympy [A] time = 0.173751, size = 12, normalized size = 0.25

$$\frac{x}{3}-\frac{2\log(3x+2)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((2+3*x)**2)**(1/2),x)
```

```
[Out] x/3 - 2*log(3*x + 2)/9
```

Giac [A] time = 1.40761, size = 34, normalized size = 0.71

$$\frac{1}{3}\,x\mathrm{sgn}\left(3\,x+2\right)-\frac{2}{9}\,\log\left(|3\,x+2|\right)\mathrm{sgn}\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((2+3*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*x*sgn(3*x + 2) - 2/9*log(abs(3*x + 2))*sgn(3*x + 2)
```

3.216 $\int \frac{x}{\sqrt{4-12x+9x^2}}\,dx$

Optimal. Leaf size=48

$$\frac{1}{9}\sqrt{9x^2-12x+4}-\frac{2(2-3x)\log(2-3x)}{9\sqrt{9x^2-12x+4}}$$

```
[Out] Sqrt[4 - 12*x + 9*x^2]/9 - (2*(2 - 3*x)*Log[2 - 3*x])/(9*Sqrt[4 - 12*x + 9*
x^2])
```

Rubi [A] time = 0.0109042, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {640, 608, 31}

$$\frac{1}{9}\sqrt{9x^2-12x+4}-\frac{2(2-3x)\log(2-3x)}{9\sqrt{9x^2-12x+4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[4 - 12*x + 9*x^2],x]
```

```
[Out] Sqrt[4 - 12*x + 9*x^2]/9 - (2*(2 - 3*x)*Log[2 - 3*x])/(9*Sqrt[4 - 12*x + 9*
x^2])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{4-12x+9x^2}}\,dx &= \frac{1}{9}\sqrt{4-12x+9x^2}+\frac{2}{3}\int \frac{1}{\sqrt{4-12x+9x^2}}\,dx\\ &= \frac{1}{9}\sqrt{4-12x+9x^2}+\frac{(2(-6+9x))\int \frac{1}{-6+9x}\,dx}{3\sqrt{4-12x+9x^2}}\\ &= \frac{1}{9}\sqrt{4-12x+9x^2}-\frac{2(2-3x)\log(2-3x)}{9\sqrt{4-12x+9x^2}}\end{aligned}$$

Mathematica [A] time = 0.0162329, size = 33, normalized size = 0.69

$$\frac{(3x-2)(3x+2\log(3x-2)-2)}{9\sqrt{(2-3x)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[4 - 12*x + 9*x^2],x]
```

```
[Out] ((-2 + 3*x)*(-2 + 3*x + 2*Log[-2 + 3*x]))/(9*Sqrt[(2 - 3*x)^2])
```

Maple [A] time = 0.095, size = 29, normalized size = 0.6

$$\frac{(-2+3x)(3x+2\ln(-2+3x))}{9}\frac{1}{\sqrt{(-2+3x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((-2+3*x)^2)^(1/2),x)
```

```
[Out] 1/9*(-2+3*x)*(3*x+2*ln(-2+3*x))/((-2+3*x)^2)^(1/2)
```

Maxima [A] time = 1.75691, size = 28, normalized size = 0.58

$$\frac{1}{9}\sqrt{9x^2-12x+4}+\frac{2}{9}\log\left(x-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((-2+3*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(9*x^2 - 12*x + 4) + 2/9*log(x - 2/3)
```

Fricas [A] time = 1.58107, size = 35, normalized size = 0.73

$$\frac{1}{3}x+\frac{2}{9}\log(3x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((-2+3*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*x + 2/9*log(3*x - 2)
```

Sympy [A] time = 0.250986, size = 12, normalized size = 0.25

$$\frac{x}{3}+\frac{2\log(3x-2)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((-2+3*x)**2)**(1/2),x)
```

```
[Out] x/3 + 2*log(3*x - 2)/9
```

Giac [A] time = 1.23288, size = 34, normalized size = 0.71

$$\frac{1}{3}\,x\mathrm{sgn}\left(3\,x-2\right)+\frac{2}{9}\,\log\left(\left|3\,x-2\right|\right)\mathrm{sgn}\left(3\,x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((-2+3*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*x*sgn(3*x - 2) + 2/9*log(abs(3*x - 2))*sgn(3*x - 2)
```

3.217 $\int \frac{x}{\sqrt{-4+12x-9x^2}}\,dx$

Optimal. Leaf size=48

$$-\frac{1}{9}\sqrt{-9x^2+12x-4}-\frac{2(2-3x)\log(2-3x)}{9\sqrt{-9x^2+12x-4}}$$

```
[Out] -Sqrt[-4 + 12*x - 9*x^2]/9 - (2*(2 - 3*x)*Log[2 - 3*x])/(9*Sqrt[-4 + 12*x -
 9*x^2])
```

Rubi [A] time = 0.0105578, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {640, 608, 31}

$$-\frac{1}{9}\sqrt{-9x^2+12x-4}-\frac{2(2-3x)\log(2-3x)}{9\sqrt{-9x^2+12x-4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[-4 + 12*x - 9*x^2],x]
```

```
[Out] -Sqrt[-4 + 12*x - 9*x^2]/9 - (2*(2 - 3*x)*Log[2 - 3*x])/(9*Sqrt[-4 + 12*x -
 9*x^2])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{-4+12x-9x^2}}\,dx &= -\frac{1}{9}\sqrt{-4+12x-9x^2}+\frac{2}{3}\int \frac{1}{\sqrt{-4+12x-9x^2}}\,dx\\ &= -\frac{1}{9}\sqrt{-4+12x-9x^2}+\frac{(2(6-9x))\int \frac{1}{6-9x}\,dx}{3\sqrt{-4+12x-9x^2}}\\ &= -\frac{1}{9}\sqrt{-4+12x-9x^2}-\frac{2(2-3x)\log(2-3x)}{9\sqrt{-4+12x-9x^2}}\end{aligned}$$

Mathematica [A] time = 0.0111983, size = 35, normalized size = 0.73

$$\frac{(3x-2)(3x+2\log(2-3x)-2)}{9\sqrt{-(2-3x)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[-4 + 12*x - 9*x^2],x]
```

```
[Out] ((-2 + 3*x)*(-2 + 3*x + 2*Log[2 - 3*x]))/(9*Sqrt[-(2 - 3*x)^2])
```

Maple [A] time = 0.096, size = 31, normalized size = 0.7

$$\frac{(-2+3x)(3x+2\ln(-2+3x))}{9}\frac{1}{\sqrt{-(-2+3x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-(-2+3*x)^2)^(1/2),x)
```

```
[Out] 1/9*(-2+3*x)*(3*x+2*ln(-2+3*x))/(-(-2+3*x)^2)^(1/2)
```

Maxima [C] time = 1.70781, size = 28, normalized size = 0.58

$$-\frac{1}{9}\sqrt{-9x^2+12x-4}+\frac{2}{9}i\log\left(x-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-(-2+3*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(-9*x^2 + 12*x - 4) + 2/9*I*log(x - 2/3)
```

Fricas [C] time = 1.59583, size = 42, normalized size = 0.88

$$-\frac{1}{3}ix-\frac{2}{9}i\log\left(x-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-(-2+3*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*I*x - 2/9*I*log(x - 2/3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{-(3x-2)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-(-2+3*x)**2)**(1/2),x)
```

```
[Out] Integral(x/sqrt(-(3*x - 2)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-(-2+3*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] undef
```

3.218 $\int \frac{x}{\sqrt{-4-12x-9x^2}}\,dx$

Optimal. Leaf size=48

$$-\frac{1}{9}\sqrt{-9x^2-12x-4}-\frac{2(3x+2)\log(3x+2)}{9\sqrt{-9x^2-12x-4}}$$

```
[Out] -Sqrt[-4 - 12*x - 9*x^2]/9 - (2*(2 + 3*x)*Log[2 + 3*x])/(9*Sqrt[-4 - 12*x -
 9*x^2])
```

Rubi [A] time = 0.0119671, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {640, 608, 31}

$$-\frac{1}{9}\sqrt{-9x^2-12x-4}-\frac{2(3x+2)\log(3x+2)}{9\sqrt{-9x^2-12x-4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[-4 - 12*x - 9*x^2],x]
```

```
[Out] -Sqrt[-4 - 12*x - 9*x^2]/9 - (2*(2 + 3*x)*Log[2 + 3*x])/(9*Sqrt[-4 - 12*x -
 9*x^2])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{-4-12x-9x^2}}\,dx &= -\frac{1}{9}\sqrt{-4-12x-9x^2}-\frac{2}{3}\int \frac{1}{\sqrt{-4-12x-9x^2}}\,dx\\ &= -\frac{1}{9}\sqrt{-4-12x-9x^2}+-\frac{(2(-6-9x))\int \frac{1}{-6-9x}\,dx}{3\sqrt{-4-12x-9x^2}}\\ &= -\frac{1}{9}\sqrt{-4-12x-9x^2}-\frac{2(2+3x)\log(2+3x)}{9\sqrt{-4-12x-9x^2}}\end{aligned}$$

Mathematica [A] time = 0.0075156, size = 35, normalized size = 0.73

$$\frac{(3x+2)(3x-2\log(3x+2)+2)}{9\sqrt{-(3x+2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[-4 - 12*x - 9*x^2],x]
```

```
[Out] ((2 + 3*x)*(2 + 3*x - 2*Log[2 + 3*x]))/(9*Sqrt[-(2 + 3*x)^2])
```

Maple [A] time = 0.116, size = 31, normalized size = 0.7

$$-\frac{(2+3x)(-3x+2\ln(2+3x))}{9}\frac{1}{\sqrt{-(2+3x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-(2+3*x)^2)^(1/2),x)
```

```
[Out] -1/9*(2+3*x)*(-3*x+2*ln(2+3*x))/(-(2+3*x)^2)^(1/2)
```

Maxima [C] time = 1.71448, size = 28, normalized size = 0.58

$$-\frac{1}{9}\sqrt{-9x^2-12x-4}-\frac{2}{9}i\log\left(x+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-(2+3*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(-9*x^2 - 12*x - 4) - 2/9*I*log(x + 2/3)
```

Fricas [C] time = 1.61575, size = 42, normalized size = 0.88

$$-\frac{1}{3}ix+\frac{2}{9}i\log\left(x+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-(2+3*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*I*x + 2/9*I*log(x + 2/3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{-(3x+2)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-(2+3*x)**2)**(1/2),x)
```

```
[Out] Integral(x/sqrt(-(3*x + 2)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-(2+3*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] undef
```

3.219 $\int \frac{1+x}{2x+x^2} \, dx$

Optimal. Leaf size=12

$$\frac{1}{2}\log\left(x^2+2x\right)$$

```
[Out] Log[2*x + x^2]/2
```

Rubi [A] time = 0.0032206, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {628}

$$\frac{1}{2}\log\left(x^2+2x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(2*x + x^2),x]

[Out] Log[2*x + x^2]/2
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1+x}{2x+x^2} \, dx = \frac{1}{2}\log\left(2x+x^2\right)$$

Mathematica [A] time = 0.0025538, size = 15, normalized size = 1.25

$$\frac{\log(x)}{2}+\frac{1}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(2*x + x^2),x]

[Out] Log[x]/2 + Log[2 + x]/2
```

Maple [A] time = 0.042, size = 9, normalized size = 0.8

$$\frac{\ln\left(x\left(2+x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(x^2+2*x),x)
```

```
[Out] 1/2*ln(x*(2+x))
```

Maxima [A] time = 1.11713, size = 14, normalized size = 1.17

$$\frac{1}{2}\log\left(x^2+2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+2*x),x, algorithm="maxima")
```

```
[Out] 1/2*log(x^2 + 2*x)
```

Fricas [A] time = 1.51227, size = 27, normalized size = 2.25

$$\frac{1}{2}\log\left(x^2+2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+2*x),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^2 + 2*x)
```

Sympy [A] time = 0.163188, size = 8, normalized size = 0.67

$$\frac{\log\left(x^2+2x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**2+2*x),x)
```

```
[Out] log(x**2 + 2*x)/2
```

Giac [A] time = 1.24889, size = 18, normalized size = 1.5

$$\frac{1}{2}\log\left(|x+2|\right)+\frac{1}{2}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+2*x),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(x + 2)) + 1/2*log(abs(x))
```

3.220 $\int \frac{a+2bx}{ax+bx^2} dx$

Optimal. Leaf size=10

$$\log\left(ax+bx^2\right)$$

```
[Out] Log[a*x + b*x^2]
```

Rubi [A] time = 0.0039895, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {628}

$$\log\left(ax+bx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + 2*b*x)/(a*x + b*x^2),x]
```

```
[Out] Log[a*x + b*x^2]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{a+2bx}{ax+bx^2} dx = \log\left(ax+bx^2\right)$$

Mathematica [A] time = 0.0039173, size = 9, normalized size = 0.9

$$\log(a+bx)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + 2*b*x)/(a*x + b*x^2),x]
```

```
[Out] Log[x] + Log[a + b*x]
```

Maple [A] time = 0.043, size = 9, normalized size = 0.9

$$\ln(x(bx+a))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*b*x+a)/(b*x^2+a*x),x)
```

```
[Out] ln(x*(b*x+a))
```

Maxima [A] time = 1.11328, size = 14, normalized size = 1.4

$$\log\left(bx^2 + ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*b*x+a)/(b*x^2+a*x),x, algorithm="maxima")
```

```
[Out] log(b*x^2 + a*x)
```

Fricas [A] time = 1.55282, size = 24, normalized size = 2.4

$$\log\left(bx^2 + ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*b*x+a)/(b*x^2+a*x),x, algorithm="fricas")
```

```
[Out] log(b*x^2 + a*x)
```

Sympy [A] time = 0.981668, size = 8, normalized size = 0.8

$$\log\left(ax + bx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*b*x+a)/(b*x**2+a*x),x)
```

```
[Out] log(a*x + b*x**2)
```

Giac [A] time = 1.33478, size = 15, normalized size = 1.5

$$\log\left(|bx + a|\right) + \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*b*x+a)/(b*x^2+a*x),x, algorithm="giac")
```

```
[Out] log(abs(b*x + a)) + log(abs(x))
```

3.221 $\int (d+ex)^4 \left(bx+cx^2\right) dx$

Optimal. Leaf size=62

$$-\frac{(d+ex)^6(2cd-be)}{6e^3}+\frac{d(d+ex)^5(cd-be)}{5e^3}+\frac{c(d+ex)^7}{7e^3}$$

```
[Out] (d*(c*d - b*e)*(d + e*x)^5)/(5*e^3) - ((2*c*d - b*e)*(d + e*x)^6)/(6*e^3) +
 (c*(d + e*x)^7)/(7*e^3)
```

Rubi [A] time = 0.0667449, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {698}

$$-\frac{(d+ex)^6(2cd-be)}{6e^3}+\frac{d(d+ex)^5(cd-be)}{5e^3}+\frac{c(d+ex)^7}{7e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4*(b*x + c*x^2),x]
```

```
[Out] (d*(c*d - b*e)*(d + e*x)^5)/(5*e^3) - ((2*c*d - b*e)*(d + e*x)^6)/(6*e^3) +
 (c*(d + e*x)^7)/(7*e^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int (d+ex)^4 \left(bx+cx^2\right) dx &= \int \left(\frac{d(cd-be)(d+ex)^4}{e^2}+\frac{(-2cd+be)(d+ex)^5}{e^2}+\frac{c(d+ex)^6}{e^2}\right) dx\\ &= \frac{d(cd-be)(d+ex)^5}{5e^3}-\frac{(2cd-be)(d+ex)^6}{6e^3}+\frac{c(d+ex)^7}{7e^3}\end{aligned}$$

Mathematica [A] time = 0.0153295, size = 99, normalized size = 1.6

$$\frac{1}{2}d^2ex^4(3be+2cd)+\frac{1}{3}d^3x^3(4be+cd)+\frac{1}{6}e^3x^6(be+4cd)+\frac{2}{5}de^2x^5(2be+3cd)+\frac{1}{2}bd^4x^2+\frac{1}{7}ce^4x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4*(b*x + c*x^2),x]
```

```
[Out] (b*d^4*x^2)/2 + (d^3*(c*d + 4*b*e)*x^3)/3 + (d^2*e*(2*c*d + 3*b*e)*x^4)/2 +
 (2*d*e^2*(3*c*d + 2*b*e)*x^5)/5 + (e^3*(4*c*d + b*e)*x^6)/6 + (c*e^4*x^7)/
7
```

Maple [A] time = 0.042, size = 100, normalized size = 1.6

$$\frac{e^4cx^7}{7}+\frac{\left(e^4b+4\,de^3c\right)x^6}{6}+\frac{\left(4\,de^3b+6\,d^2e^2c\right)x^5}{5}+\frac{\left(6\,d^2e^2b+4\,d^3ec\right)x^4}{4}+\frac{\left(4\,d^3eb+d^4c\right)x^3}{3}+\frac{bd^4x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(c*x^2+b*x),x)
```

```
[Out] 1/7*e^4*c*x^7+1/6*(b*e^4+4*c*d*e^3)*x^6+1/5*(4*b*d*e^3+6*c*d^2*e^2)*x^5+1/4
*(6*b*d^2*e^2+4*c*d^3*e)*x^4+1/3*(4*b*d^3*e+c*d^4)*x^3+1/2*b*d^4*x^2
```

Maxima [A] time = 1.12101, size = 134, normalized size = 2.16

$$\frac{1}{7}\,ce^4x^7+\frac{1}{2}\,bd^4x^2+\frac{1}{6}\left(4\,cde^3+be^4\right)x^6+\frac{2}{5}\left(3\,cd^2e^2+2\,bde^3\right)x^5+\frac{1}{2}\left(2\,cd^3e+3\,bd^2e^2\right)x^4+\frac{1}{3}\left(cd^4+4\,bd^3e\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] 1/7*c*e^4*x^7 + 1/2*b*d^4*x^2 + 1/6*(4*c*d*e^3 + b*e^4)*x^6 + 2/5*(3*c*d^2*
e^2 + 2*b*d*e^3)*x^5 + 1/2*(2*c*d^3*e + 3*b*d^2*e^2)*x^4 + 1/3*(c*d^4 + 4*b
*d^3*e)*x^3
```

Fricas [A] time = 1.41085, size = 231, normalized size = 3.73

$$\frac{1}{7}x^7e^4c+\frac{2}{3}x^6e^3dc+\frac{1}{6}x^6e^4b+\frac{6}{5}x^5e^2d^2c+\frac{4}{5}x^5e^3db+x^4ed^3c+\frac{3}{2}x^4e^2d^2b+\frac{1}{3}x^3d^4c+\frac{4}{3}x^3ed^3b+\frac{1}{2}x^2d^4b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*e^4*c + 2/3*x^6*e^3*d*c + 1/6*x^6*e^4*b + 6/5*x^5*e^2*d^2*c + 4/5*x
^5*e^3*d*b + x^4*e*d^3*c + 3/2*x^4*e^2*d^2*b + 1/3*x^3*d^4*c + 4/3*x^3*e*d^
3*b + 1/2*x^2*d^4*b
```

Sympy [B] time = 0.278048, size = 107, normalized size = 1.73

$$\frac{bd^4x^2}{2}+\frac{ce^4x^7}{7}+x^6\left(\frac{be^4}{6}+\frac{2cde^3}{3}\right)+x^5\left(\frac{4bde^3}{5}+\frac{6cd^2e^2}{5}\right)+x^4\left(\frac{3bd^2e^2}{2}+cd^3e\right)+x^3\left(\frac{4bd^3e}{3}+\frac{cd^4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(c*x**2+b*x),x)
```

```
[Out] b*d**4*x**2/2 + c*e**4*x**7/7 + x**6*(b*e**4/6 + 2*c*d*e**3/3) + x**5*(4*b*
d*e**3/5 + 6*c*d**2*e**2/5) + x**4*(3*b*d**2*e**2/2 + c*d**3*e) + x**3*(4*b
*d**3*e/3 + c*d**4/3)
```

Giac [A] time = 1.26549, size = 130, normalized size = 2.1

$$\frac{1}{7}cx^7e^4 + \frac{2}{3}cdx^6e^3 + \frac{6}{5}cd^2x^5e^2 + cd^3x^4e + \frac{1}{3}cd^4x^3 + \frac{1}{6}bx^6e^4 + \frac{4}{5}bdx^5e^3 + \frac{3}{2}bd^2x^4e^2 + \frac{4}{3}bd^3x^3e + \frac{1}{2}bd^4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] 1/7*c*x^7*e^4 + 2/3*c*d*x^6*e^3 + 6/5*c*d^2*x^5*e^2 + c*d^3*x^4*e + 1/3*c*d
^4*x^3 + 1/6*b*x^6*e^4 + 4/5*b*d*x^5*e^3 + 3/2*b*d^2*x^4*e^2 + 4/3*b*d^3*x^
3*e + 1/2*b*d^4*x^2
```

3.222 $\int (d+ex)^3 \left(bx+cx^2\right) dx$

Optimal. Leaf size=62

$$-\frac{(d+ex)^5(2cd-be)}{5e^3}+\frac{d(d+ex)^4(cd-be)}{4e^3}+\frac{c(d+ex)^6}{6e^3}$$

```
[Out] (d*(c*d - b*e)*(d + e*x)^4)/(4*e^3) - ((2*c*d - b*e)*(d + e*x)^5)/(5*e^3) +
 (c*(d + e*x)^6)/(6*e^3)
```

Rubi [A] time = 0.0488904, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {698}

$$-\frac{(d+ex)^5(2cd-be)}{5e^3}+\frac{d(d+ex)^4(cd-be)}{4e^3}+\frac{c(d+ex)^6}{6e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(b*x + c*x^2),x]
```

```
[Out] (d*(c*d - b*e)*(d + e*x)^4)/(4*e^3) - ((2*c*d - b*e)*(d + e*x)^5)/(5*e^3) +
 (c*(d + e*x)^6)/(6*e^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int (d+ex)^3 \left(bx+cx^2\right) dx &= \int \left(\frac{d(cd-be)(d+ex)^3}{e^2}+\frac{(-2cd+be)(d+ex)^4}{e^2}+\frac{c(d+ex)^5}{e^2}\right) dx\\ &= \frac{d(cd-be)(d+ex)^4}{4e^3}-\frac{(2cd-be)(d+ex)^5}{5e^3}+\frac{c(d+ex)^6}{6e^3}\end{aligned}$$

Mathematica [A] time = 0.0162647, size = 67, normalized size = 1.08

$$\frac{1}{60}x^2\left(20d^2x(3be+cd)+12e^2x^3(be+3cd)+45dex^2(be+cd)+30bd^3+10ce^3x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(b*x + c*x^2),x]
```

```
[Out] (x^2*(30*b*d^3 + 20*d^2*(c*d + 3*b*e)*x + 45*d*e*(c*d + b*e)*x^2 + 12*e^2*(
3*c*d + b*e)*x^3 + 10*c*e^3*x^4))/60
```

Maple [A] time = 0.045, size = 76, normalized size = 1.2

$$\frac{e^3cx^6}{6}+\frac{(e^3b+3\,de^2c)\,x^5}{5}+\frac{(3\,de^2b+3\,d^2ec)\,x^4}{4}+\frac{(3\,d^2eb+d^3c)\,x^3}{3}+\frac{d^3bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x),x)
```

```
[Out] 1/6*e^3*c*x^6+1/5*(b*e^3+3*c*d*e^2)*x^5+1/4*(3*b*d*e^2+3*c*d^2*e)*x^4+1/3*(
3*b*d^2*e+c*d^3)*x^3+1/2*d^3*b*x^2
```

Maxima [A] time = 1.1356, size = 99, normalized size = 1.6

$$\frac{1}{6}\,ce^3x^6+\frac{1}{2}\,bd^3x^2+\frac{1}{5}\left(3\,cde^2+be^3\right)x^5+\frac{3}{4}\left(cd^2e+bde^2\right)x^4+\frac{1}{3}\left(cd^3+3\,bd^2e\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] 1/6*c*e^3*x^6 + 1/2*b*d^3*x^2 + 1/5*(3*c*d*e^2 + b*e^3)*x^5 + 3/4*(c*d^2*e
+ b*d*e^2)*x^4 + 1/3*(c*d^3 + 3*b*d^2*e)*x^3
```

Fricas [A] time = 1.42969, size = 177, normalized size = 2.85

$$\frac{1}{6}x^6e^3c+\frac{3}{5}x^5e^2dc+\frac{1}{5}x^5e^3b+\frac{3}{4}x^4ed^2c+\frac{3}{4}x^4e^2db+\frac{1}{3}x^3d^3c+x^3ed^2b+\frac{1}{2}x^2d^3b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*e^3*c + 3/5*x^5*e^2*d*c + 1/5*x^5*e^3*b + 3/4*x^4*e*d^2*c + 3/4*x^4
*e^2*d*b + 1/3*x^3*d^3*c + x^3*e*d^2*b + 1/2*x^2*d^3*b
```

Sympy [A] time = 0.288298, size = 80, normalized size = 1.29

$$\frac{bd^3x^2}{2}+\frac{ce^3x^6}{6}+x^5\left(\frac{be^3}{5}+\frac{3cde^2}{5}\right)+x^4\left(\frac{3bde^2}{4}+\frac{3cd^2e}{4}\right)+x^3\left(bd^2e+\frac{cd^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x),x)
```

```
[Out] b*d**3*x**2/2 + c*e**3*x**6/6 + x**5*(b*e**3/5 + 3*c*d*e**2/5) + x**4*(3*b*
d*e**2/4 + 3*c*d**2*e/4) + x**3*(b*d**2*e + c*d**3/3)
```

Giac [A] time = 1.29579, size = 100, normalized size = 1.61

$$\frac{1}{6}\,cx^6e^3+\frac{3}{5}\,cdx^5e^2+\frac{3}{4}\,cd^2x^4e+\frac{1}{3}\,cd^3x^3+\frac{1}{5}\,bx^5e^3+\frac{3}{4}\,bdx^4e^2+bd^2x^3e+\frac{1}{2}\,bd^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] 1/6*c*x^6*e^3 + 3/5*c*d*x^5*e^2 + 3/4*c*d^2*x^4*e + 1/3*c*d^3*x^3 + 1/5*b*x
^5*e^3 + 3/4*b*d*x^4*e^2 + b*d^2*x^3*e + 1/2*b*d^3*x^2
```

3.223 $\int (d+ex)^2 \left(bx+cx^2\right) \, dx$

Optimal. Leaf size=55

$$\frac{1}{4}ex^4(be+2cd)+\frac{1}{3}dx^3(2be+cd)+\frac{1}{2}bd^2x^2+\frac{1}{5}ce^2x^5$$

```
[Out] (b*d^2*x^2)/2 + (d*(c*d + 2*b*e)*x^3)/3 + (e*(2*c*d + b*e)*x^4)/4 + (c*e^2*
x^5)/5
```

Rubi [A] time = 0.0376067, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {698}

$$\frac{1}{4}ex^4(be+2cd)+\frac{1}{3}dx^3(2be+cd)+\frac{1}{2}bd^2x^2+\frac{1}{5}ce^2x^5$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(b*x + c*x^2),x]
```

```
[Out] (b*d^2*x^2)/2 + (d*(c*d + 2*b*e)*x^3)/3 + (e*(2*c*d + b*e)*x^4)/4 + (c*e^2*
x^5)/5
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int (d+ex)^2 \left(bx+cx^2\right) \, dx &= \int \left(bd^2x+d(cd+2be)x^2+e(2cd+be)x^3+ce^2x^4\right) \, dx\\ &= \frac{1}{2}bd^2x^2+\frac{1}{3}d(cd+2be)x^3+\frac{1}{4}e(2cd+be)x^4+\frac{1}{5}ce^2x^5\end{aligned}$$

Mathematica [A] time = 0.0119075, size = 49, normalized size = 0.89

$$\frac{1}{60}x^2\left(15ex^2(be+2cd)+20dx(2be+cd)+30bd^2+12ce^2x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(b*x + c*x^2),x]
```

```
[Out] (x^2*(30*b*d^2 + 20*d*(c*d + 2*b*e)*x + 15*e*(2*c*d + b*e)*x^2 + 12*c*e^2*x
^3))/60
```

Maple [A] time = 0.041, size = 52, normalized size = 1.

$$\frac{ce^2x^5}{5}+\frac{\left(e^2b+2\,dec\right)x^4}{4}+\frac{\left(2\,bde+cd^2\right)x^3}{3}+\frac{bd^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+b*x),x)
```

```
[Out] 1/5*c*e^2*x^5+1/4*(b*e^2+2*c*d*e)*x^4+1/3*(2*b*d*e+c*d^2)*x^3+1/2*b*d^2*x^2
```

Maxima [A] time = 1.10459, size = 69, normalized size = 1.25

$$\frac{1}{5}\,ce^2x^5+\frac{1}{2}\,bd^2x^2+\frac{1}{4}\left(2\,cde+be^2\right)x^4+\frac{1}{3}\left(cd^2+2\,bde\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] 1/5*c*e^2*x^5 + 1/2*b*d^2*x^2 + 1/4*(2*c*d*e + b*e^2)*x^4 + 1/3*(c*d^2 + 2*
b*d*e)*x^3
```

Fricas [A] time = 1.47921, size = 128, normalized size = 2.33

$$\frac{1}{5}x^5e^2c+\frac{1}{2}x^4edc+\frac{1}{4}x^4e^2b+\frac{1}{3}x^3d^2c+\frac{2}{3}x^3edb+\frac{1}{2}x^2d^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] 1/5*x^5*e^2*c + 1/2*x^4*e*d*c + 1/4*x^4*e^2*b + 1/3*x^3*d^2*c + 2/3*x^3*e*d
*b + 1/2*x^2*d^2*b
```

Sympy [A] time = 0.118933, size = 54, normalized size = 0.98

$$\frac{bd^2x^2}{2}+\frac{ce^2x^5}{5}+x^4\left(\frac{be^2}{4}+\frac{cde}{2}\right)+x^3\left(\frac{2bde}{3}+\frac{cd^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+b*x),x)
```

```
[Out] b*d**2*x**2/2 + c*e**2*x**5/5 + x**4*(b*e**2/4 + c*d*e/2) + x**3*(2*b*d*e/3
 + c*d**2/3)
```

Giac [A] time = 1.3394, size = 72, normalized size = 1.31

$$\frac{1}{5}\,cx^5e^2+\frac{1}{2}\,cdx^4e+\frac{1}{3}\,cd^2x^3+\frac{1}{4}\,bx^4e^2+\frac{2}{3}\,bdx^3e+\frac{1}{2}\,bd^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] 1/5*c*x^5*e^2 + 1/2*c*d*x^4*e + 1/3*c*d^2*x^3 + 1/4*b*x^4*e^2 + 2/3*b*d*x^3
*e + 1/2*b*d^2*x^2
```

3.224 $\int (d+ex)\left(bx+cx^2\right)\,dx$

Optimal. Leaf size=33

$$\frac{1}{3}x^3(be+cd)+\frac{1}{2}bdx^2+\frac{1}{4}cex^4$$

```
[Out] (b*d*x^2)/2 + ((c*d + b*e)*x^3)/3 + (c*e*x^4)/4
```

Rubi [A] time = 0.0219701, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {631}

$$\frac{1}{3}x^3(be+cd)+\frac{1}{2}bdx^2+\frac{1}{4}cex^4$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(b*x + c*x^2),x]
```

```
[Out] (b*d*x^2)/2 + ((c*d + b*e)*x^3)/3 + (c*e*x^4)/4
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(bx+cx^2\right)\,dx &= \int \left(bdx+(cd+be)x^2+cex^3\right)\,dx\\ &= \frac{1}{2}bdx^2+\frac{1}{3}(cd+be)x^3+\frac{1}{4}cex^4\end{aligned}$$

Mathematica [A] time = 0.0050811, size = 29, normalized size = 0.88

$$\frac{1}{12}x^2(b(6d+4ex)+cx(4d+3ex))$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(b*x + c*x^2),x]
```

```
[Out] (x^2*(c*x*(4*d + 3*e*x) + b*(6*d + 4*e*x)))/12
```

Maple [A] time = 0.043, size = 28, normalized size = 0.9

$$\frac{bdx^2}{2}+\frac{(be+cd)\,x^3}{3}+\frac{cex^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+b*x),x)
```

```
[Out] 1/2*b*d*x^2+1/3*(b*e+c*d)*x^3+1/4*c*e*x^4
```

Maxima [A] time = 1.11657, size = 36, normalized size = 1.09

$$\frac{1}{4}cex^4 + \frac{1}{2}bdx^2 + \frac{1}{3}(cd + be)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] 1/4*c*e*x^4 + 1/2*b*d*x^2 + 1/3*(c*d + b*e)*x^3
```

Fricas [A] time = 1.43521, size = 74, normalized size = 2.24

$$\frac{1}{4}x^4ec + \frac{1}{3}x^3dc + \frac{1}{3}x^3eb + \frac{1}{2}x^2db$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*e*c + 1/3*x^3*d*c + 1/3*x^3*e*b + 1/2*x^2*d*b
```

Sympy [A] time = 0.282943, size = 29, normalized size = 0.88

$$\frac{bdx^2}{2} + \frac{cex^4}{4} + x^3\left(\frac{be}{3} + \frac{cd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+b*x),x)
```

```
[Out] b*d*x**2/2 + c*e*x**4/4 + x**3*(b*e/3 + c*d/3)
```

Giac [A] time = 1.3321, size = 42, normalized size = 1.27

$$\frac{1}{4}cx^4e + \frac{1}{3}cdx^3 + \frac{1}{3}bx^3e + \frac{1}{2}bdx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] 1/4*c*x^4*e + 1/3*c*d*x^3 + 1/3*b*x^3*e + 1/2*b*d*x^2
```

3.225 $\int (bx + cx^2) \, dx$

Optimal. Leaf size=17

$$\frac{bx^2}{2} + \frac{cx^3}{3}$$

```
[Out] (b*x^2)/2 + (c*x^3)/3
```

Rubi [A] time = 0.00323, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{bx^2}{2} + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[b*x + c*x^2,x]
```

```
[Out] (b*x^2)/2 + (c*x^3)/3
```

Rubi steps

$$\int (bx + cx^2) \, dx = \frac{bx^2}{2} + \frac{cx^3}{3}$$

Mathematica [A] time = 0.0000396, size = 17, normalized size = 1.

$$\frac{bx^2}{2} + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[b*x + c*x^2,x]
```

```
[Out] (b*x^2)/2 + (c*x^3)/3
```

Maple [A] time = 0.043, size = 14, normalized size = 0.8

$$\frac{bx^2}{2} + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*x^2+b*x,x)
```

```
[Out] 1/2*b*x^2+1/3*c*x^3
```

Maxima [A] time = 1.15944, size = 18, normalized size = 1.06

$$\frac{1}{3}cx^3 + \frac{1}{2}bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2+b*x,x, algorithm="maxima")
```

```
[Out] 1/3*c*x^3 + 1/2*b*x^2
```

Fricas [A] time = 1.3568, size = 31, normalized size = 1.82

$$\frac{1}{3}x^3c + \frac{1}{2}x^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2+b*x,x, algorithm="fricas")
```

```
[Out] 1/3*x^3*c + 1/2*x^2*b
```

Sympy [A] time = 0.15272, size = 12, normalized size = 0.71

$$\frac{bx^2}{2} + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x**2+b*x,x)
```

```
[Out] b*x**2/2 + c*x**3/3
```

Giac [A] time = 1.28656, size = 18, normalized size = 1.06

$$\frac{1}{3}cx^3 + \frac{1}{2}bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2+b*x,x, algorithm="giac")
```

```
[Out] 1/3*c*x^3 + 1/2*b*x^2
```

3.226 $\int \frac{bx+cx^2}{d+ex}\,dx$

Optimal. Leaf size=45

$$-\frac{x(cd-be)}{e^2}+\frac{d(cd-be)\log(d+ex)}{e^3}+\frac{cx^2}{2e}$$

```
[Out] -(((c*d - b*e)*x)/e^2) + (c*x^2)/(2*e) + (d*(c*d - b*e)*Log[d + e*x])/e^3
```

Rubi [A] time = 0.0340222, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {698}

$$-\frac{x(cd-be)}{e^2}+\frac{d(cd-be)\log(d+ex)}{e^3}+\frac{cx^2}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)/(d + e*x),x]
```

```
[Out] -(((c*d - b*e)*x)/e^2) + (c*x^2)/(2*e) + (d*(c*d - b*e)*Log[d + e*x])/e^3
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{bx+cx^2}{d+ex}\,dx &= \int \left(\frac{-cd+be}{e^2}+\frac{cx}{e}+\frac{d(cd-be)}{e^2(d+ex)}\right)dx\\ &= -\frac{(cd-be)x}{e^2}+\frac{cx^2}{2e}+\frac{d(cd-be)\log(d+ex)}{e^3}\end{aligned}$$

Mathematica [A] time = 0.0146041, size = 41, normalized size = 0.91

$$\frac{ex(2be-2cd+cex)+2d(cd-be)\log(d+ex)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)/(d + e*x),x]
```

```
[Out] (e*x*(-2*c*d + 2*b*e + c*e*x) + 2*d*(c*d - b*e)*Log[d + e*x])/(2*e^3)
```

Maple [A] time = 0.044, size = 52, normalized size = 1.2

$$\frac{cx^2}{2e}+\frac{bx}{e}-\frac{cdx}{e^2}-\frac{d\ln(ex+d)\,b}{e^2}+\frac{d^2\ln(ex+d)\,c}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)/(e*x+d),x)
```

```
[Out] 1/2*c*x^2/e+1/e*x*b-c*d*x/e^2-d/e^2*ln(e*x+d)*b+d^2/e^3*ln(e*x+d)*c
```

Maxima [A] time = 1.11148, size = 61, normalized size = 1.36

$$\frac{cex^2 - 2\,(cd - be)x}{2\,e^2} + \frac{\left(cd^2 - bde\right)\log\left(ex + d\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*(c*e*x^2 - 2*(c*d - b*e)*x)/e^2 + (c*d^2 - b*d*e)*log(e*x + d)/e^3
```

Fricas [A] time = 1.52674, size = 103, normalized size = 2.29

$$\frac{ce^2x^2 - 2\left(cde - be^2\right)x + 2\left(cd^2 - bde\right)\log\left(ex + d\right)}{2\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/2*(c*e^2*x^2 - 2*(c*d*e - b*e^2)*x + 2*(c*d^2 - b*d*e)*log(e*x + d))/e^3
```

Sympy [A] time = 1.24481, size = 37, normalized size = 0.82

$$\frac{cx^2}{2e} - \frac{d\,(be - cd)\log\left(d + ex\right)}{e^3} + \frac{x\,(be - cd)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)/(e*x+d),x)
```

```
[Out] c*x**2/(2*e) - d*(b*e - c*d)*log(d + e*x)/e**3 + x*(b*e - c*d)/e**2
```

Giac [A] time = 1.38182, size = 63, normalized size = 1.4

$$\left(cd^2 - bde\right)e^{(-3)}\log\left(|xe + d|\right) + \frac{1}{2}\left(cx^2e - 2\,cdx + 2\,bxe\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x+d),x, algorithm="giac")
```

```
[Out] (c*d^2 - b*d*e)*e^(-3)*log(abs(x*e + d)) + 1/2*(c*x^2*e - 2*c*d*x + 2*b*x*e
)*e^(-2)
```

3.227 $\int \frac{bx+cx^2}{(d+ex)^2} dx$

Optimal. Leaf size=48

$$-\frac{d(cd-be)}{e^3(d+ex)}-\frac{(2cd-be)\log(d+ex)}{e^3}+\frac{cx}{e^2}$$

```
[Out] (c*x)/e^2 - (d*(c*d - b*e))/(e^3*(d + e*x)) - ((2*c*d - b*e)*Log[d + e*x])/
e^3
```

Rubi [A] time = 0.0367024, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {698}

$$-\frac{d(cd-be)}{e^3(d+ex)}-\frac{(2cd-be)\log(d+ex)}{e^3}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)/(d + e*x)^2,x]
```

```
[Out] (c*x)/e^2 - (d*(c*d - b*e))/(e^3*(d + e*x)) - ((2*c*d - b*e)*Log[d + e*x])/
e^3
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{bx+cx^2}{(d+ex)^2}\,dx &= \int \left(\frac{c}{e^2}+\frac{d(cd-be)}{e^2(d+ex)^2}+\frac{-2cd+be}{e^2(d+ex)}\right)dx\\ &= \frac{cx}{e^2}-\frac{d(cd-be)}{e^3(d+ex)}-\frac{(2cd-be)\log(d+ex)}{e^3}\end{aligned}$$

Mathematica [A] time = 0.0261806, size = 41, normalized size = 0.85

$$\frac{\frac{d(be-cd)}{d+ex}+(be-2cd)\log(d+ex)+cex}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)/(d + e*x)^2,x]
```

```
[Out] (c*e*x + (d*(-(c*d) + b*e))/(d + e*x) + (-2*c*d + b*e)*Log[d + e*x])/e^3
```

Maple [A] time = 0.049, size = 61, normalized size = 1.3

$$\frac{cx}{e^2}+\frac{\ln(ex+d)\,b}{e^2}-2\frac{cd\ln(ex+d)}{e^3}+\frac{bd}{e^2(ex+d)}-\frac{cd^2}{e^3(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)/(e*x+d)^2,x)
```

```
[Out] c*x/e^2+1/e^2*ln(e*x+d)*b-2*c*d*ln(e*x+d)/e^3+d/e^2/(e*x+d)*b-d^2/e^3/(e*x+
d)*c
```

Maxima [A] time = 1.11739, size = 72, normalized size = 1.5

$$-\frac{cd^2-bde}{e^4x+de^3}+\frac{cx}{e^2}-\frac{(2\,cd-be)\log(ex+d)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(c*d^2 - b*d*e)/(e^4*x + d*e^3) + c*x/e^2 - (2*c*d - b*e)*log(e*x + d)/e^3
```

Fricas [A] time = 1.64656, size = 149, normalized size = 3.1

$$\frac{ce^2x^2+cdex-cd^2+bde-\left(2\,cd^2-bde+\left(2\,cde-be^2\right)x\right)\log(ex+d)}{e^4x+de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] (c*e^2*x^2 + c*d*e*x - c*d^2 + b*d*e - (2*c*d^2 - b*d*e + (2*c*d*e - b*e^2)
*x)*log(e*x + d))/(e^4*x + d*e^3)
```

Sympy [A] time = 1.39345, size = 44, normalized size = 0.92

$$\frac{cx}{e^2}+\frac{bde-cd^2}{de^3+e^4x}+\frac{(be-2cd)\log(d+ex)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)/(e*x+d)**2,x)
```

```
[Out] c*x/e**2 + (b*d*e - c*d**2)/(d*e**3 + e**4*x) + (b*e - 2*c*d)*log(d + e*x)/
e**3
```

Giac [A] time = 1.41269, size = 126, normalized size = 2.62

$$-\left(e^{(-1)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^2}\right)-\frac{de^{(-1)}}{xe+d}\right)be^{(-1)}+\left(2\,de^{(-3)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^2}\right)+(xe+d)e^{(-3)}-\frac{d^2e^{(-3)}}{xe+d}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -(e^(-1)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2) - d*e^(-1)/(x*e + d))*b*e^(-1
) + (2*d*e^(-3)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2) + (x*e + d)*e^(-3) - d
^2*e^(-3)/(x*e + d))*c
```

3.228 $\int \frac{bx+cx^2}{(d+ex)^3} dx$

Optimal. Leaf size=55

$$-\frac{d(cd-be)}{2e^3(d+ex)^2}+\frac{2cd-be}{e^3(d+ex)}+\frac{c\log(d+ex)}{e^3}$$

```
[Out] -(d*(c*d - b*e))/(2*e^3*(d + e*x)^2) + (2*c*d - b*e)/(e^3*(d + e*x)) + (c*L
og[d + e*x])/e^3
```

Rubi [A] time = 0.0373487, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {698}

$$-\frac{d(cd-be)}{2e^3(d+ex)^2}+\frac{2cd-be}{e^3(d+ex)}+\frac{c\log(d+ex)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)/(d + e*x)^3,x]
```

```
[Out] -(d*(c*d - b*e))/(2*e^3*(d + e*x)^2) + (2*c*d - b*e)/(e^3*(d + e*x)) + (c*L
og[d + e*x])/e^3
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{bx+cx^2}{(d+ex)^3}\,dx &= \int \left(\frac{d(cd-be)}{e^2(d+ex)^3}+\frac{-2cd+be}{e^2(d+ex)^2}+\frac{c}{e^2(d+ex)}\right)dx\\ &= -\frac{d(cd-be)}{2e^3(d+ex)^2}+\frac{2cd-be}{e^3(d+ex)}+\frac{c\log(d+ex)}{e^3}\end{aligned}$$

Mathematica [A] time = 0.017849, size = 52, normalized size = 0.95

$$\frac{-be(d+2ex)+cd(3d+4ex)+2c(d+ex)^2\log(d+ex)}{2e^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)/(d + e*x)^3,x]
```

```
[Out] (-(b*e*(d + 2*e*x)) + c*d*(3*d + 4*e*x) + 2*c*(d + e*x)^2*Log[d + e*x])/(2*
e^3*(d + e*x)^2)
```

Maple [A] time = 0.049, size = 70, normalized size = 1.3

$$\frac{bd}{2\,e^{2}\,(ex+d)^{2}}-\frac{cd^{2}}{2\,e^{3}\,(ex+d)^{2}}+\frac{c\ln(ex+d)}{e^{3}}-\frac{b}{e^{2}\,(ex+d)}+2\,\frac{cd}{e^{3}\,(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)/(e*x+d)^3,x)
```

```
[Out] 1/2*d/e^2/(e*x+d)^2*b-1/2*d^2/e^3/(e*x+d)^2*c+c*ln(e*x+d)/e^3-1/e^2/(e*x+d)
*b+2*c*d/e^3/(e*x+d)
```

Maxima [A] time = 1.15968, size = 88, normalized size = 1.6

$$\frac{3\,cd^{2}-bde+2\left(2\,cde-be^{2}\right)x}{2\left(e^{5}x^{2}+2\,de^{4}x+d^{2}e^{3}\right)}+\frac{c\log(ex+d)}{e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(3*c*d^2 - b*d*e + 2*(2*c*d*e - b*e^2)*x)/(e^5*x^2 + 2*d*e^4*x + d^2*e^
3) + c*log(e*x + d)/e^3
```

Fricas [A] time = 1.63066, size = 173, normalized size = 3.15

$$\frac{3\,cd^{2}-bde+2\left(2\,cde-be^{2}\right)x+2\left(ce^{2}x^{2}+2\,cdex+cd^{2}\right)\log(ex+d)}{2\left(e^{5}x^{2}+2\,de^{4}x+d^{2}e^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*c*d^2 - b*d*e + 2*(2*c*d*e - b*e^2)*x + 2*(c*e^2*x^2 + 2*c*d*e*x + c
*d^2)*log(e*x + d))/(e^5*x^2 + 2*d*e^4*x + d^2*e^3)
```

Sympy [A] time = 2.27856, size = 63, normalized size = 1.15

$$\frac{c\log(d+ex)}{e^{3}}-\frac{bde-3cd^{2}+x\left(2be^{2}-4cde\right)}{2d^{2}e^{3}+4de^{4}x+2e^{5}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)/(e*x+d)**3,x)
```

```
[Out] c*log(d + e*x)/e**3 - (b*d*e - 3*c*d**2 + x*(2*b*e**2 - 4*c*d*e))/(2*d**2*e
**3 + 4*d*e**4*x + 2*e**5*x**2)
```

Giac [A] time = 1.3188, size = 74, normalized size = 1.35

$$ce^{(-3)}\log(|xe+d|)+\frac{\left(2\left(2\,cd-be\right)x+\left(3\,cd^2-bde\right)e^{(-1)}\right)e^{(-2)}}{2\left(xe+d\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] c*e^(-3)*log(abs(x*e + d)) + 1/2*(2*(2*c*d - b*e)*x + (3*c*d^2 - b*d*e)*e^(
-1))*e^(-2)/(x*e + d)^2
```

3.229 $\int \frac{bx+cx^2}{(d+ex)^4} \, dx$

Optimal. Leaf size=60

$$\frac{2cd-be}{2e^3(d+ex)^2}-\frac{d(cd-be)}{3e^3(d+ex)^3}-\frac{c}{e^3(d+ex)}$$

```
[Out] -(d*(c*d - b*e))/(3*e^3*(d + e*x)^3) + (2*c*d - b*e)/(2*e^3*(d + e*x)^2) -
c/(e^3*(d + e*x))
```

Rubi [A] time = 0.0356667, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {698}

$$\frac{2cd-be}{2e^3(d+ex)^2}-\frac{d(cd-be)}{3e^3(d+ex)^3}-\frac{c}{e^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)/(d + e*x)^4,x]
```

```
[Out] -(d*(c*d - b*e))/(3*e^3*(d + e*x)^3) + (2*c*d - b*e)/(2*e^3*(d + e*x)^2) -
c/(e^3*(d + e*x))
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{bx+cx^2}{(d+ex)^4}\,dx &= \int \left(\frac{d(cd-be)}{e^2(d+ex)^4}+\frac{-2cd+be}{e^2(d+ex)^3}+\frac{c}{e^2(d+ex)^2}\right)dx\\ &= -\frac{d(cd-be)}{3e^3(d+ex)^3}+\frac{2cd-be}{2e^3(d+ex)^2}-\frac{c}{e^3(d+ex)}\end{aligned}$$

Mathematica [A] time = 0.0165066, size = 44, normalized size = 0.73

$$-\frac{be(d+3ex)+2c\left(d^2+3dex+3e^2x^2\right)}{6e^3(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)/(d + e*x)^4,x]
```

```
[Out] -(b*e*(d + 3*e*x) + 2*c*(d^2 + 3*d*e*x + 3*e^2*x^2))/(6*e^3*(d + e*x)^3)
```

Maple [A] time = 0.048, size = 56, normalized size = 0.9

$$-\frac{be-2\,cd}{2\,e^3\,(ex+d)^2}+\frac{d\,(be-cd)}{3\,e^3\,(ex+d)^3}-\frac{c}{e^3\,(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)/(e*x+d)^4,x)
```

```
[Out] -1/2*(b*e-2*c*d)/e^3/(e*x+d)^2+1/3*d*(b*e-c*d)/e^3/(e*x+d)^3-c/e^3/(e*x+d)
```

Maxima [A] time = 1.12797, size = 96, normalized size = 1.6

$$-\frac{6\,ce^2x^2+2\,cd^2+bde+3\,(2\,cde+be^2)x}{6\,(e^6x^3+3\,de^5x^2+3\,d^2e^4x+d^3e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(6*c*e^2*x^2 + 2*c*d^2 + b*d*e + 3*(2*c*d*e + b*e^2)*x)/(e^6*x^3 + 3*d
*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3)
```

Fricas [A] time = 1.45312, size = 149, normalized size = 2.48

$$-\frac{6\,ce^2x^2+2\,cd^2+bde+3\,(2\,cde+be^2)x}{6\,(e^6x^3+3\,de^5x^2+3\,d^2e^4x+d^3e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(6*c*e^2*x^2 + 2*c*d^2 + b*d*e + 3*(2*c*d*e + b*e^2)*x)/(e^6*x^3 + 3*d
*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3)
```

Sympy [A] time = 2.53654, size = 75, normalized size = 1.25

$$-\frac{bde+2cd^2+6ce^2x^2+x\,(3be^2+6cde)}{6d^3e^3+18d^2e^4x+18de^5x^2+6e^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)/(e*x+d)**4,x)
```

```
[Out] -(b*d*e + 2*c*d**2 + 6*c*e**2*x**2 + x*(3*b*e**2 + 6*c*d*e))/(6*d**3*e**3 +
 18*d**2*e**4*x + 18*d*e**5*x**2 + 6*e**6*x**3)
```

Giac [A] time = 1.35359, size = 61, normalized size = 1.02

$$-\frac{\left(6\,cx^2e^2+6\,cdxe+2\,cd^2+3\,bxe^2+bde\right)e^{(-3)}}{6\,(xe+d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -1/6*(6*c*x^2*e^2 + 6*c*d*x*e + 2*c*d^2 + 3*b*x*e^2 + b*d*e)*e^(-3)/(x*e +
d)^3
```

3.230 $\int \frac{bx+cx^2}{(d+ex)^5}\,dx$

Optimal. Leaf size=62

$$\frac{2cd-be}{3e^3(d+ex)^3}-\frac{d(cd-be)}{4e^3(d+ex)^4}-\frac{c}{2e^3(d+ex)^2}$$

```
[Out] -(d*(c*d - b*e))/(4*e^3*(d + e*x)^4) + (2*c*d - b*e)/(3*e^3*(d + e*x)^3) -
c/(2*e^3*(d + e*x)^2)
```

Rubi [A] time = 0.0362365, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {698}

$$\frac{2cd-be}{3e^3(d+ex)^3}-\frac{d(cd-be)}{4e^3(d+ex)^4}-\frac{c}{2e^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)/(d + e*x)^5,x]
```

```
[Out] -(d*(c*d - b*e))/(4*e^3*(d + e*x)^4) + (2*c*d - b*e)/(3*e^3*(d + e*x)^3) -
c/(2*e^3*(d + e*x)^2)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{bx+cx^2}{(d+ex)^5}\,dx &= \int \left(\frac{d(cd-be)}{e^2(d+ex)^5}+\frac{-2cd+be}{e^2(d+ex)^4}+\frac{c}{e^2(d+ex)^3}\right)dx\\ &= -\frac{d(cd-be)}{4e^3(d+ex)^4}+\frac{2cd-be}{3e^3(d+ex)^3}-\frac{c}{2e^3(d+ex)^2}\end{aligned}$$

Mathematica [A] time = 0.0157057, size = 43, normalized size = 0.69

$$-\frac{be(d+4ex)+c\left(d^2+4dex+6e^2x^2\right)}{12e^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)/(d + e*x)^5,x]
```

```
[Out] -(b*e*(d + 4*e*x) + c*(d^2 + 4*d*e*x + 6*e^2*x^2))/(12*e^3*(d + e*x)^4)
```

Maple [A] time = 0.048, size = 56, normalized size = 0.9

$$-\frac{c}{2\,e^{3}\left(ex+d\right)^{2}}-\frac{be-2\,cd}{3\,e^{3}\left(ex+d\right)^{3}}+\frac{d\left(be-cd\right)}{4\,e^{3}\left(ex+d\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)/(e*x+d)^5,x)
```

```
[Out] -1/2*c/e^3/(e*x+d)^2-1/3*(b*e-2*c*d)/e^3/(e*x+d)^3+1/4*d*(b*e-c*d)/e^3/(e*x
+d)^4
```

Maxima [A] time = 1.09339, size = 108, normalized size = 1.74

$$-\frac{6\,ce^{2}x^{2}+cd^{2}+bde+4\left(cde+be^{2}\right)x}{12\left(e^{7}x^{4}+4\,de^{6}x^{3}+6\,d^{2}e^{5}x^{2}+4\,d^{3}e^{4}x+d^{4}e^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x+d)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(6*c*e^2*x^2 + c*d^2 + b*d*e + 4*(c*d*e + b*e^2)*x)/(e^7*x^4 + 4*d*e^
6*x^3 + 6*d^2*e^5*x^2 + 4*d^3*e^4*x + d^4*e^3)
```

Fricas [A] time = 1.66231, size = 166, normalized size = 2.68

$$-\frac{6\,ce^{2}x^{2}+cd^{2}+bde+4\left(cde+be^{2}\right)x}{12\left(e^{7}x^{4}+4\,de^{6}x^{3}+6\,d^{2}e^{5}x^{2}+4\,d^{3}e^{4}x+d^{4}e^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x+d)^5,x, algorithm="fricas")
```

```
[Out] -1/12*(6*c*e^2*x^2 + c*d^2 + b*d*e + 4*(c*d*e + b*e^2)*x)/(e^7*x^4 + 4*d*e^
6*x^3 + 6*d^2*e^5*x^2 + 4*d^3*e^4*x + d^4*e^3)
```

Sympy [A] time = 1.32717, size = 85, normalized size = 1.37

$$-\frac{bde+cd^{2}+6ce^{2}x^{2}+x\left(4be^{2}+4cde\right)}{12d^{4}e^{3}+48d^{3}e^{4}x+72d^{2}e^{5}x^{2}+48de^{6}x^{3}+12e^{7}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)/(e*x+d)**5,x)
```

```
[Out] -(b*d*e + c*d**2 + 6*c*e**2*x**2 + x*(4*b*e**2 + 4*c*d*e))/(12*d**4*e**3 +
48*d**3*e**4*x + 72*d**2*e**5*x**2 + 48*d*e**6*x**3 + 12*e**7*x**4)
```

Giac [A] time = 1.25089, size = 101, normalized size = 1.63

$$-\frac{1}{12}\left(\frac{6\,ce^{(-2)}}{(xe+d)^2}-\frac{8\,cde^{(-2)}}{(xe+d)^3}+\frac{3\,cd^2e^{(-2)}}{(xe+d)^4}+\frac{4\,be^{(-1)}}{(xe+d)^3}-\frac{3\,bde^{(-1)}}{(xe+d)^4}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] -1/12*(6*c*e^(-2)/(x*e + d)^2 - 8*c*d*e^(-2)/(x*e + d)^3 + 3*c*d^2*e^(-2)/(
x*e + d)^4 + 4*b*e^(-1)/(x*e + d)^3 - 3*b*d*e^(-1)/(x*e + d)^4)*e^(-1)
```

3.231 $\int (d+ex)^4 \left(bx+cx^2\right)^2 \, dx$

Optimal. Leaf size=137

$$\frac{(d+ex)^7 \left(b^2e^2-6bcde+6c^2d^2\right)}{7e^5}+\frac{d^2(d+ex)^5(cd-be)^2}{5e^5}-\frac{c(d+ex)^8(2cd-be)}{4e^5}-\frac{d(d+ex)^6(cd-be)(2cd-be)}{3e^5}+\frac{c^2(}{}$$

```
[Out] (d^2*(c*d - b*e)^2*(d + e*x)^5)/(5*e^5) - (d*(c*d - b*e)*(2*c*d - b*e)*(d +
 e*x)^6)/(3*e^5) + ((6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(d + e*x)^7)/(7*e^5)
- (c*(2*c*d - b*e)*(d + e*x)^8)/(4*e^5) + (c^2*(d + e*x)^9)/(9*e^5)
```

Rubi [A] time = 0.140859, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$\frac{(d+ex)^7 \left(b^2e^2-6bcde+6c^2d^2\right)}{7e^5}+\frac{d^2(d+ex)^5(cd-be)^2}{5e^5}-\frac{c(d+ex)^8(2cd-be)}{4e^5}-\frac{d(d+ex)^6(cd-be)(2cd-be)}{3e^5}+\frac{c^2(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4*(b*x + c*x^2)^2,x]
```

```
[Out] (d^2*(c*d - b*e)^2*(d + e*x)^5)/(5*e^5) - (d*(c*d - b*e)*(2*c*d - b*e)*(d +
 e*x)^6)/(3*e^5) + ((6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(d + e*x)^7)/(7*e^5)
- (c*(2*c*d - b*e)*(d + e*x)^8)/(4*e^5) + (c^2*(d + e*x)^9)/(9*e^5)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^4 \left(bx+cx^2\right)^2 \, dx = \int \left(\frac{d^2(cd-be)^2(d+ex)^4}{e^4}+\frac{2d(cd-be)(-2cd+be)(d+ex)^5}{e^4}+\frac{\left(6c^2d^2-6bcde+b^2e^2\right)(}{e^4}\right.$$

$$=\frac{d^2(cd-be)^2(d+ex)^5}{5e^5}-\frac{d(cd-be)(2cd-be)(d+ex)^6}{3e^5}+\frac{\left(6c^2d^2-6bcde+b^2e^2\right)(d+ex)^7}{7e^5}$$

Mathematica [A] time = 0.0255949, size = 159, normalized size = 1.16

$$\frac{1}{7}e^2x^7\left(b^2e^2+8bcde+6c^2d^2\right)+\frac{2}{3}dex^6\left(b^2e^2+3bcde+c^2d^2\right)+\frac{1}{5}d^2x^5\left(6b^2e^2+8bcde+c^2d^2\right)+\frac{1}{3}b^2d^4x^3+\frac{1}{2}bd^3x^4(2b$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4*(b*x + c*x^2)^2,x]
```

```
[Out] (b^2*d^4*x^3)/3 + (b*d^3*(c*d + 2*b*e)*x^4)/2 + (d^2*(c^2*d^2 + 8*b*c*d*e +
 6*b^2*e^2)*x^5)/5 + (2*d*e*(c^2*d^2 + 3*b*c*d*e + b^2*e^2)*x^6)/3 + (e^2*(
```

```
6*c^2*d^2 + 8*b*c*d*e + b^2*e^2)*x^7)/7 + (c*e^3*(2*c*d + b*e)*x^8)/4 + (c^
2*e^4*x^9)/9
```

Maple [A] time = 0.044, size = 166, normalized size = 1.2

$$\frac{e^4c^2x^9}{9}+\frac{\left(2\,e^4bc+4\,de^3c^2\right)x^8}{8}+\frac{\left(e^4b^2+8\,de^3bc+6\,d^2e^2c^2\right)x^7}{7}+\frac{\left(4\,de^3b^2+12\,d^2e^2bc+4\,d^3ec^2\right)x^6}{6}+\frac{\left(6\,d^2e^2b^2+8\,d^3eb\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(c*x^2+b*x)^2,x)
```

```
[Out] 1/9*e^4*c^2*x^9+1/8*(2*b*c*e^4+4*c^2*d*e^3)*x^8+1/7*(b^2*e^4+8*b*c*d*e^3+6*
c^2*d^2*e^2)*x^7+1/6*(4*b^2*d*e^3+12*b*c*d^2*e^2+4*c^2*d^3*e)*x^6+1/5*(6*b^
2*d^2*e^2+8*b*c*d^3*e+c^2*d^4)*x^5+1/4*(4*b^2*d^3*e+2*b*c*d^4)*x^4+1/3*d^4*
b^2*x^3
```

Maxima [A] time = 1.16143, size = 217, normalized size = 1.58

$$\frac{1}{9}\,c^2e^4x^9+\frac{1}{3}\,b^2d^4x^3+\frac{1}{4}\left(2\,c^2de^3+bce^4\right)x^8+\frac{1}{7}\left(6\,c^2d^2e^2+8\,bcde^3+b^2e^4\right)x^7+\frac{2}{3}\left(c^2d^3e+3\,bcd^2e^2+b^2de^3\right)x^6+\frac{1}{5}\left(c^2d^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+b*x)^2,x, algorithm="maxima")
```

```
[Out] 1/9*c^2*e^4*x^9 + 1/3*b^2*d^4*x^3 + 1/4*(2*c^2*d*e^3 + b*c*e^4)*x^8 + 1/7*(
6*c^2*d^2*e^2 + 8*b*c*d*e^3 + b^2*e^4)*x^7 + 2/3*(c^2*d^3*e + 3*b*c*d^2*e^2
 + b^2*d*e^3)*x^6 + 1/5*(c^2*d^4 + 8*b*c*d^3*e + 6*b^2*d^2*e^2)*x^5 + 1/2*(
b*c*d^4 + 2*b^2*d^3*e)*x^4
```

Fricas [A] time = 1.38888, size = 387, normalized size = 2.82

$$\frac{1}{9}x^9e^4c^2+\frac{1}{2}x^8e^3dc^2+\frac{1}{4}x^8e^4cb+\frac{6}{7}x^7e^2d^2c^2+\frac{8}{7}x^7e^3dcb+\frac{1}{7}x^7e^4b^2+\frac{2}{3}x^6ed^3c^2+2x^6e^2d^2cb+\frac{2}{3}x^6e^3db^2+\frac{1}{5}x^5d^4c^2+\frac{8}{5}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*e^4*c^2 + 1/2*x^8*e^3*d*c^2 + 1/4*x^8*e^4*c*b + 6/7*x^7*e^2*d^2*c^2
 + 8/7*x^7*e^3*d*c*b + 1/7*x^7*e^4*b^2 + 2/3*x^6*e*d^3*c^2 + 2*x^6*e^2*d^2*
c*b + 2/3*x^6*e^3*d*b^2 + 1/5*x^5*d^4*c^2 + 8/5*x^5*e*d^3*c*b + 6/5*x^5*e^2
*d^2*b^2 + 1/2*x^4*d^4*c*b + x^4*e*d^3*b^2 + 1/3*x^3*d^4*b^2
```

Sympy [A] time = 0.374781, size = 178, normalized size = 1.3

$$\frac{b^2d^4x^3}{3}+\frac{c^2e^4x^9}{9}+x^8\left(\frac{bce^4}{4}+\frac{c^2de^3}{2}\right)+x^7\left(\frac{b^2e^4}{7}+\frac{8bcde^3}{7}+\frac{6c^2d^2e^2}{7}\right)+x^6\left(\frac{2b^2de^3}{3}+2bcd^2e^2+\frac{2c^2d^3e}{3}\right)+x^5\left(\frac{6b^2d^2e^2}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(c*x**2+b*x)**2,x)
```

```
[Out] b**2*d**4*x**3/3 + c**2*e**4*x**9/9 + x**8*(b*c*e**4/4 + c**2*d*e**3/2) + x
**7*(b**2*e**4/7 + 8*b*c*d*e**3/7 + 6*c**2*d**2*e**2/7) + x**6*(2*b**2*d*e*
*3/3 + 2*b*c*d**2*e**2 + 2*c**2*d**3*e/3) + x**5*(6*b**2*d**2*e**2/5 + 8*b*
c*d**3*e/5 + c**2*d**4/5) + x**4*(b**2*d**3*e + b*c*d**4/2)
```

Giac [A] time = 1.25349, size = 228, normalized size = 1.66

$$\frac{1}{9}\,c^2x^9e^4+\frac{1}{2}\,c^2dx^8e^3+\frac{6}{7}\,c^2d^2x^7e^2+\frac{2}{3}\,c^2d^3x^6e+\frac{1}{5}\,c^2d^4x^5+\frac{1}{4}\,bcx^8e^4+\frac{8}{7}\,bcdx^7e^3+2\,bcd^2x^6e^2+\frac{8}{5}\,bcd^3x^5e+\frac{1}{2}\,bcd^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+b*x)^2,x, algorithm="giac")
```

```
[Out] 1/9*c^2*x^9*e^4 + 1/2*c^2*d*x^8*e^3 + 6/7*c^2*d^2*x^7*e^2 + 2/3*c^2*d^3*x^6
*e + 1/5*c^2*d^4*x^5 + 1/4*b*c*x^8*e^4 + 8/7*b*c*d*x^7*e^3 + 2*b*c*d^2*x^6*
e^2 + 8/5*b*c*d^3*x^5*e + 1/2*b*c*d^4*x^4 + 1/7*b^2*x^7*e^4 + 2/3*b^2*d*x^6
*e^3 + 6/5*b^2*d^2*x^5*e^2 + b^2*d^3*x^4*e + 1/3*b^2*d^4*x^3
```

3.232 $\int (d+ex)^3 \left(bx+cx^2\right)^2 \, dx$

Optimal. Leaf size=127

$$\frac{1}{6}ex^6\left(b^2e^2+6bcde+3c^2d^2\right)+\frac{1}{5}dx^5\left(3b^2e^2+6bcde+c^2d^2\right)+\frac{1}{3}b^2d^3x^3+\frac{1}{4}bd^2x^4(3be+2cd)+\frac{1}{7}ce^2x^7(2be+3cd)+\frac{1}{8}c^2e$$

```
[Out] (b^2*d^3*x^3)/3 + (b*d^2*(2*c*d + 3*b*e)*x^4)/4 + (d*(c^2*d^2 + 6*b*c*d*e +
 3*b^2*e^2)*x^5)/5 + (e*(3*c^2*d^2 + 6*b*c*d*e + b^2*e^2)*x^6)/6 + (c*e^2*(
3*c*d + 2*b*e)*x^7)/7 + (c^2*e^3*x^8)/8
```

Rubi [A] time = 0.0928371, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$\frac{1}{6}ex^6\left(b^2e^2+6bcde+3c^2d^2\right)+\frac{1}{5}dx^5\left(3b^2e^2+6bcde+c^2d^2\right)+\frac{1}{3}b^2d^3x^3+\frac{1}{4}bd^2x^4(3be+2cd)+\frac{1}{7}ce^2x^7(2be+3cd)+\frac{1}{8}c^2e$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(b*x + c*x^2)^2,x]
```

```
[Out] (b^2*d^3*x^3)/3 + (b*d^2*(2*c*d + 3*b*e)*x^4)/4 + (d*(c^2*d^2 + 6*b*c*d*e +
 3*b^2*e^2)*x^5)/5 + (e*(3*c^2*d^2 + 6*b*c*d*e + b^2*e^2)*x^6)/6 + (c*e^2*(
3*c*d + 2*b*e)*x^7)/7 + (c^2*e^3*x^8)/8
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int (d+ex)^3\left(bx+cx^2\right)^2\,dx &= \int \left(b^2d^3x^2+bd^2(2cd+3be)x^3+d\left(c^2d^2+6bcde+3b^2e^2\right)x^4+e\left(3c^2d^2+6bcde+b^2e^2\right)x^5\right.\\ &= \frac{1}{3}b^2d^3x^3+\frac{1}{4}bd^2(2cd+3be)x^4+\frac{1}{5}d\left(c^2d^2+6bcde+3b^2e^2\right)x^5+\frac{1}{6}e\left(3c^2d^2+6bcde+b^2e^2\right)\end{aligned}$$

Mathematica [A] time = 0.0174962, size = 127, normalized size = 1.

$$\frac{1}{6}ex^6\left(b^2e^2+6bcde+3c^2d^2\right)+\frac{1}{5}dx^5\left(3b^2e^2+6bcde+c^2d^2\right)+\frac{1}{3}b^2d^3x^3+\frac{1}{4}bd^2x^4(3be+2cd)+\frac{1}{7}ce^2x^7(2be+3cd)+\frac{1}{8}c^2e$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(b*x + c*x^2)^2,x]
```

```
[Out] (b^2*d^3*x^3)/3 + (b*d^2*(2*c*d + 3*b*e)*x^4)/4 + (d*(c^2*d^2 + 6*b*c*d*e +
 3*b^2*e^2)*x^5)/5 + (e*(3*c^2*d^2 + 6*b*c*d*e + b^2*e^2)*x^6)/6 + (c*e^2*(
3*c*d + 2*b*e)*x^7)/7 + (c^2*e^3*x^8)/8
```

Maple [A] time = 0.042, size = 128, normalized size = 1.

$$\frac{c^2e^3x^8}{8}+\frac{\left(2\,e^3bc+3\,de^2c^2\right)x^7}{7}+\frac{\left(e^3b^2+6\,de^2bc+3\,d^2ec^2\right)x^6}{6}+\frac{\left(3\,de^2b^2+6\,d^2ebc+c^2d^3\right)x^5}{5}+\frac{\left(3\,d^2eb^2+2\,d^3bc\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x)^2,x)
```

```
[Out] 1/8*c^2*e^3*x^8+1/7*(2*b*c*e^3+3*c^2*d*e^2)*x^7+1/6*(b^2*e^3+6*b*c*d*e^2+3*
c^2*d^2*e)*x^6+1/5*(3*b^2*d*e^2+6*b*c*d^2*e+c^2*d^3)*x^5+1/4*(3*b^2*d^2*e+2
*b*c*d^3)*x^4+1/3*b^2*d^3*x^3
```

Maxima [A] time = 1.15568, size = 171, normalized size = 1.35

$$\frac{1}{8}\,c^2e^3x^8+\frac{1}{3}\,b^2d^3x^3+\frac{1}{7}\left(3\,c^2de^2+2\,bce^3\right)x^7+\frac{1}{6}\left(3\,c^2d^2e+6\,bcde^2+b^2e^3\right)x^6+\frac{1}{5}\left(c^2d^3+6\,bcd^2e+3\,b^2de^2\right)x^5+\frac{1}{4}\,(2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x)^2,x, algorithm="maxima")
```

```
[Out] 1/8*c^2*e^3*x^8 + 1/3*b^2*d^3*x^3 + 1/7*(3*c^2*d*e^2 + 2*b*c*e^3)*x^7 + 1/6
*(3*c^2*d^2*e + 6*b*c*d*e^2 + b^2*e^3)*x^6 + 1/5*(c^2*d^3 + 6*b*c*d^2*e + 3
*b^2*d*e^2)*x^5 + 1/4*(2*b*c*d^3 + 3*b^2*d^2*e)*x^4
```

Fricas [A] time = 1.55294, size = 301, normalized size = 2.37

$$\frac{1}{8}x^8e^3c^2+\frac{3}{7}x^7e^2dc^2+\frac{2}{7}x^7e^3cb+\frac{1}{2}x^6ed^2c^2+x^6e^2dcb+\frac{1}{6}x^6e^3b^2+\frac{1}{5}x^5d^3c^2+\frac{6}{5}x^5ed^2cb+\frac{3}{5}x^5e^2db^2+\frac{1}{2}x^4d^3cb+\frac{3}{4}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*e^3*c^2 + 3/7*x^7*e^2*d*c^2 + 2/7*x^7*e^3*c*b + 1/2*x^6*e*d^2*c^2 +
 x^6*e^2*d*c*b + 1/6*x^6*e^3*b^2 + 1/5*x^5*d^3*c^2 + 6/5*x^5*e*d^2*c*b + 3/
5*x^5*e^2*d*b^2 + 1/2*x^4*d^3*c*b + 3/4*x^4*e*d^2*b^2 + 1/3*x^3*d^3*b^2
```

Sympy [A] time = 0.234868, size = 138, normalized size = 1.09

$$\frac{b^2d^3x^3}{3}+\frac{c^2e^3x^8}{8}+x^7\left(\frac{2bce^3}{7}+\frac{3c^2de^2}{7}\right)+x^6\left(\frac{b^2e^3}{6}+bcde^2+\frac{c^2d^2e}{2}\right)+x^5\left(\frac{3b^2de^2}{5}+\frac{6bcd^2e}{5}+\frac{c^2d^3}{5}\right)+x^4\left(\frac{3b^2d^2e}{4}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x)**2,x)
```

```
[Out] b**2*d**3*x**3/3 + c**2*e**3*x**8/8 + x**7*(2*b*c*e**3/7 + 3*c**2*d*e**2/7)
 + x**6*(b**2*e**3/6 + b*c*d*e**2 + c**2*d**2*e/2) + x**5*(3*b**2*d*e**2/5
```

```
+ 6*b*c*d**2*e/5 + c**2*d**3/5) + x**4*(3*b**2*d**2*e/4 + b*c*d**3/2)
```

Giac [A] time = 1.26225, size = 177, normalized size = 1.39

$$\frac{1}{8}c^2x^8e^3 + \frac{3}{7}c^2dx^7e^2 + \frac{1}{2}c^2d^2x^6e + \frac{1}{5}c^2d^3x^5 + \frac{2}{7}bcx^7e^3 + bcdx^6e^2 + \frac{6}{5}bcd^2x^5e + \frac{1}{2}bcd^3x^4 + \frac{1}{6}b^2x^6e^3 + \frac{3}{5}b^2dx^5e^2 + \frac{3}{4}b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x)^2,x, algorithm="giac")
```

```
[Out] 1/8*c^2*x^8*e^3 + 3/7*c^2*d*x^7*e^2 + 1/2*c^2*d^2*x^6*e + 1/5*c^2*d^3*x^5 +
 2/7*b*c*x^7*e^3 + b*c*d*x^6*e^2 + 6/5*b*c*d^2*x^5*e + 1/2*b*c*d^3*x^4 + 1/
6*b^2*x^6*e^3 + 3/5*b^2*d*x^5*e^2 + 3/4*b^2*d^2*x^4*e + 1/3*b^2*d^3*x^3
```

3.233 $\int (d+ex)^2 \left(bx+cx^2\right)^2 \, dx$

Optimal. Leaf size=87

$$\frac{1}{5}x^5\left(b^2e^2+4bcde+c^2d^2\right)+\frac{1}{3}b^2d^2x^3+\frac{1}{3}cex^6(be+cd)+\frac{1}{2}bdx^4(be+cd)+\frac{1}{7}c^2e^2x^7$$

```
[Out] (b^2*d^2*x^3)/3 + (b*d*(c*d + b*e)*x^4)/2 + ((c^2*d^2 + 4*b*c*d*e + b^2*e^2
)*x^5)/5 + (c*e*(c*d + b*e)*x^6)/3 + (c^2*e^2*x^7)/7
```

Rubi [A] time = 0.0764796, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$\frac{1}{5}x^5\left(b^2e^2+4bcde+c^2d^2\right)+\frac{1}{3}b^2d^2x^3+\frac{1}{3}cex^6(be+cd)+\frac{1}{2}bdx^4(be+cd)+\frac{1}{7}c^2e^2x^7$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(b*x + c*x^2)^2,x]
```

```
[Out] (b^2*d^2*x^3)/3 + (b*d*(c*d + b*e)*x^4)/2 + ((c^2*d^2 + 4*b*c*d*e + b^2*e^2
)*x^5)/5 + (c*e*(c*d + b*e)*x^6)/3 + (c^2*e^2*x^7)/7
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int (d+ex)^2\left(bx+cx^2\right)^2\,dx &= \int \left(b^2d^2x^2+2bd(cd+be)x^3+\left(c^2d^2+4bcde+b^2e^2\right)x^4+2ce(cd+be)x^5+c^2e^2x^6\right)dx\\ &= \frac{1}{3}b^2d^2x^3+\frac{1}{2}bd(cd+be)x^4+\frac{1}{5}\left(c^2d^2+4bcde+b^2e^2\right)x^5+\frac{1}{3}ce(cd+be)x^6+\frac{1}{7}c^2e^2x^7\end{aligned}$$

Mathematica [A] time = 0.0141169, size = 87, normalized size = 1.

$$\frac{1}{5}x^5\left(b^2e^2+4bcde+c^2d^2\right)+\frac{1}{3}b^2d^2x^3+\frac{1}{3}cex^6(be+cd)+\frac{1}{2}bdx^4(be+cd)+\frac{1}{7}c^2e^2x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(b*x + c*x^2)^2,x]
```

```
[Out] (b^2*d^2*x^3)/3 + (b*d*(c*d + b*e)*x^4)/2 + ((c^2*d^2 + 4*b*c*d*e + b^2*e^2
)*x^5)/5 + (c*e*(c*d + b*e)*x^6)/3 + (c^2*e^2*x^7)/7
```

Maple [A] time = 0.045, size = 90, normalized size = 1.

$$\frac{c^2e^2x^7}{7}+\frac{(2\,e^2bc+2\,dec^2)\,x^6}{6}+\frac{(b^2e^2+4\,bcde+c^2d^2)\,x^5}{5}+\frac{(2\,b^2de+2\,d^2bc)\,x^4}{4}+\frac{b^2d^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+b*x)^2,x)
```

```
[Out] 1/7*c^2*e^2*x^7+1/6*(2*b*c*e^2+2*c^2*d*e)*x^6+1/5*(b^2*e^2+4*b*c*d*e+c^2*d^
2)*x^5+1/4*(2*b^2*d*e+2*b*c*d^2)*x^4+1/3*b^2*d^2*x^3
```

Maxima [A] time = 1.11869, size = 115, normalized size = 1.32

$$\frac{1}{7}\,c^2e^2x^7+\frac{1}{3}\,b^2d^2x^3+\frac{1}{3}\,(c^2de+bce^2)x^6+\frac{1}{5}\,(c^2d^2+4\,bcde+b^2e^2)x^5+\frac{1}{2}\,(bcd^2+b^2de)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x)^2,x, algorithm="maxima")
```

```
[Out] 1/7*c^2*e^2*x^7 + 1/3*b^2*d^2*x^3 + 1/3*(c^2*d*e + b*c*e^2)*x^6 + 1/5*(c^2*
d^2 + 4*b*c*d*e + b^2*e^2)*x^5 + 1/2*(b*c*d^2 + b^2*d*e)*x^4
```

Fricas [A] time = 1.40058, size = 217, normalized size = 2.49

$$\frac{1}{7}x^7e^2c^2+\frac{1}{3}x^6edc^2+\frac{1}{3}x^6e^2cb+\frac{1}{5}x^5d^2c^2+\frac{4}{5}x^5edcb+\frac{1}{5}x^5e^2b^2+\frac{1}{2}x^4d^2cb+\frac{1}{2}x^4edb^2+\frac{1}{3}x^3d^2b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*e^2*c^2 + 1/3*x^6*e*d*c^2 + 1/3*x^6*e^2*c*b + 1/5*x^5*d^2*c^2 + 4/5
*x^5*e*d*c*b + 1/5*x^5*e^2*b^2 + 1/2*x^4*d^2*c*b + 1/2*x^4*e*d*b^2 + 1/3*x^
3*d^2*b^2
```

Sympy [A] time = 0.266324, size = 94, normalized size = 1.08

$$\frac{b^2d^2x^3}{3}+\frac{c^2e^2x^7}{7}+x^6\left(\frac{bce^2}{3}+\frac{c^2de}{3}\right)+x^5\left(\frac{b^2e^2}{5}+\frac{4bcde}{5}+\frac{c^2d^2}{5}\right)+x^4\left(\frac{b^2de}{2}+\frac{bcd^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+b*x)**2,x)
```

```
[Out] b**2*d**2*x**3/3 + c**2*e**2*x**7/7 + x**6*(b*c*e**2/3 + c**2*d*e/3) + x**5
*(b**2*e**2/5 + 4*b*c*d*e/5 + c**2*d**2/5) + x**4*(b**2*d*e/2 + b*c*d**2/2)
```

Giac [A] time = 1.16886, size = 127, normalized size = 1.46

$$\frac{1}{7}c^2x^7e^2+\frac{1}{3}c^2dx^6e+\frac{1}{5}c^2d^2x^5+\frac{1}{3}bcx^6e^2+\frac{4}{5}bcdx^5e+\frac{1}{2}bcd^2x^4+\frac{1}{5}b^2x^5e^2+\frac{1}{2}b^2dx^4e+\frac{1}{3}b^2d^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x)^2,x, algorithm="giac")
```

```
[Out] 1/7*c^2*x^7*e^2 + 1/3*c^2*d*x^6*e + 1/5*c^2*d^2*x^5 + 1/3*b*c*x^6*e^2 + 4/5
*b*c*d*x^5*e + 1/2*b*c*d^2*x^4 + 1/5*b^2*x^5*e^2 + 1/2*b^2*d*x^4*e + 1/3*b^
2*d^2*x^3
```

3.234 $\int (d+ex)\left(bx+cx^2\right)^2\,dx$

Optimal. Leaf size=55

$$\frac{1}{3}b^2dx^3+\frac{1}{5}cx^5(2be+cd)+\frac{1}{4}bx^4(be+2cd)+\frac{1}{6}c^2ex^6$$

```
[Out] (b^2*d*x^3)/3 + (b*(2*c*d + b*e)*x^4)/4 + (c*(c*d + 2*b*e)*x^5)/5 + (c^2*e*
x^6)/6
```

Rubi [A] time = 0.0400556, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {631}

$$\frac{1}{3}b^2dx^3+\frac{1}{5}cx^5(2be+cd)+\frac{1}{4}bx^4(be+2cd)+\frac{1}{6}c^2ex^6$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(b*x + c*x^2)^2,x]
```

```
[Out] (b^2*d*x^3)/3 + (b*(2*c*d + b*e)*x^4)/4 + (c*(c*d + 2*b*e)*x^5)/5 + (c^2*e*
x^6)/6
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(bx+cx^2\right)^2\,dx &= \int \left(b^2dx^2+b(2cd+be)x^3+c(cd+2be)x^4+c^2ex^5\right)dx\\ &= \frac{1}{3}b^2dx^3+\frac{1}{4}b(2cd+be)x^4+\frac{1}{5}c(cd+2be)x^5+\frac{1}{6}c^2ex^6\end{aligned}$$

Mathematica [A] time = 0.0101526, size = 50, normalized size = 0.91

$$\frac{1}{60}x^3\left(5b^2(4d+3ex)+6bcx(5d+4ex)+2c^2x^2(6d+5ex)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(b*x + c*x^2)^2,x]
```

```
[Out] (x^3*(5*b^2*(4*d + 3*e*x) + 6*b*c*x*(5*d + 4*e*x) + 2*c^2*x^2*(6*d + 5*e*x)
))/60
```

Maple [A] time = 0.043, size = 52, normalized size = 1.

$$\frac{c^2ex^6}{6}+\frac{\left(2\,bce+dc^2\right)x^5}{5}+\frac{\left(b^2e+2\,bcd\right)x^4}{4}+\frac{b^2dx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+b*x)^2,x)
```

```
[Out] 1/6*c^2*e*x^6+1/5*(2*b*c*e+c^2*d)*x^5+1/4*(b^2*e+2*b*c*d)*x^4+1/3*b^2*d*x^3
```

Maxima [A] time = 1.15484, size = 69, normalized size = 1.25

$$\frac{1}{6}\,c^2ex^6+\frac{1}{3}\,b^2dx^3+\frac{1}{5}\,\left(c^2d+2\,bce\right)x^5+\frac{1}{4}\,\left(2\,bcd+b^2e\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*c^2*e*x^6 + 1/3*b^2*d*x^3 + 1/5*(c^2*d + 2*b*c*e)*x^5 + 1/4*(2*b*c*d +
b^2*e)*x^4
```

Fricas [A] time = 1.45162, size = 128, normalized size = 2.33

$$\frac{1}{6}x^6ec^2+\frac{1}{5}x^5dc^2+\frac{2}{5}x^5ecb+\frac{1}{2}x^4dcb+\frac{1}{4}x^4eb^2+\frac{1}{3}x^3db^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/6*x^6*e*c^2 + 1/5*x^5*d*c^2 + 2/5*x^5*e*c*b + 1/2*x^4*d*c*b + 1/4*x^4*e*b
^2 + 1/3*x^3*d*b^2
```

Sympy [A] time = 0.278477, size = 54, normalized size = 0.98

$$\frac{b^2dx^3}{3}+\frac{c^2ex^6}{6}+x^5\left(\frac{2bce}{5}+\frac{c^2d}{5}\right)+x^4\left(\frac{b^2e}{4}+\frac{bcd}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+b*x)**2,x)
```

```
[Out] b**2*d*x**3/3 + c**2*e*x**6/6 + x**5*(2*b*c*e/5 + c**2*d/5) + x**4*(b**2*e/
4 + b*c*d/2)
```

Giac [A] time = 1.4691, size = 76, normalized size = 1.38

$$\frac{1}{6}\,c^2x^6e+\frac{1}{5}\,c^2dx^5+\frac{2}{5}\,bcx^5e+\frac{1}{2}\,bcdx^4+\frac{1}{4}\,b^2x^4e+\frac{1}{3}\,b^2dx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x)^2,x, algorithm="giac")
```

```
[Out] 1/6*c^2*x^6*e + 1/5*c^2*d*x^5 + 2/5*b*c*x^5*e + 1/2*b*c*d*x^4 + 1/4*b^2*x^4
*e + 1/3*b^2*d*x^3
```

3.235 $\int \left(bx+cx^2\right)^2 \, dx$

Optimal. Leaf size=30

$$\frac{b^2x^3}{3}+\frac{1}{2}bcx^4+\frac{c^2x^5}{5}$$

```
[Out] (b^2*x^3)/3 + (b*c*x^4)/2 + (c^2*x^5)/5
```

Rubi [A] time = 0.0093515, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {611}

$$\frac{b^2x^3}{3}+\frac{1}{2}bcx^4+\frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^2,x]

[Out] (b^2*x^3)/3 + (b*c*x^4)/2 + (c^2*x^5)/5
```

Rule 611

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && IGtQ[p, 0] && (EqQ[a, 0] ||  !PerfectSquareQ[b^2 - 4*a*c])
```

Rubi steps

$$\begin{aligned}\int \left(bx+cx^2\right)^2 \, dx &= \int \left(b^2x^2+2bcx^3+c^2x^4\right) \, dx\\ &= \frac{b^2x^3}{3}+\frac{1}{2}bcx^4+\frac{c^2x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.0016168, size = 30, normalized size = 1.

$$\frac{b^2x^3}{3}+\frac{1}{2}bcx^4+\frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^2,x]

[Out] (b^2*x^3)/3 + (b*c*x^4)/2 + (c^2*x^5)/5
```

Maple [A] time = 0.04, size = 25, normalized size = 0.8

$$\frac{b^2x^3}{3}+\frac{bcx^4}{2}+\frac{c^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^2,x)
```

```
[Out] 1/3*b^2*x^3+1/2*b*c*x^4+1/5*c^2*x^5
```

Maxima [A] time = 1.15469, size = 32, normalized size = 1.07

$$\frac{1}{5}\,c^2x^5 + \frac{1}{2}\,bcx^4 + \frac{1}{3}\,b^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2,x, algorithm="maxima")
```

```
[Out] 1/5*c^2*x^5 + 1/2*b*c*x^4 + 1/3*b^2*x^3
```

Fricas [A] time = 1.22747, size = 55, normalized size = 1.83

$$\frac{1}{5}x^5c^2 + \frac{1}{2}x^4cb + \frac{1}{3}x^3b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*c^2 + 1/2*x^4*c*b + 1/3*x^3*b^2
```

Sympy [A] time = 0.252658, size = 24, normalized size = 0.8

$$\frac{b^2x^3}{3} + \frac{bcx^4}{2} + \frac{c^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**2,x)
```

```
[Out] b**2*x**3/3 + b*c*x**4/2 + c**2*x**5/5
```

Giac [A] time = 1.31254, size = 32, normalized size = 1.07

$$\frac{1}{5}\,c^2x^5 + \frac{1}{2}\,bcx^4 + \frac{1}{3}\,b^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2,x, algorithm="giac")
```

```
[Out] 1/5*c^2*x^5 + 1/2*b*c*x^4 + 1/3*b^2*x^3
```

3.236 $\int \frac{(bx+cx^2)^2}{d+ex}\,dx$

Optimal. Leaf size=93

$$\frac{d^2(cd-be)^2\log(d+ex)}{e^5}-\frac{cx^3(cd-2be)}{3e^2}+\frac{x^2(cd-be)^2}{2e^3}-\frac{dx(cd-be)^2}{e^4}+\frac{c^2x^4}{4e}$$

```
[Out] -((d*(c*d - b*e)^2*x)/e^4) + ((c*d - b*e)^2*x^2)/(2*e^3) - (c*(c*d - 2*b*e)
*x^3)/(3*e^2) + (c^2*x^4)/(4*e) + (d^2*(c*d - b*e)^2*Log[d + e*x])/e^5
```

Rubi [A] time = 0.0815552, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$\frac{d^2(cd-be)^2\log(d+ex)}{e^5}-\frac{cx^3(cd-2be)}{3e^2}+\frac{x^2(cd-be)^2}{2e^3}-\frac{dx(cd-be)^2}{e^4}+\frac{c^2x^4}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^2/(d + e*x),x]
```

```
[Out] -((d*(c*d - b*e)^2*x)/e^4) + ((c*d - b*e)^2*x^2)/(2*e^3) - (c*(c*d - 2*b*e)
*x^3)/(3*e^2) + (c^2*x^4)/(4*e) + (d^2*(c*d - b*e)^2*Log[d + e*x])/e^5
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{(bx+cx^2)^2}{d+ex}\,dx &= \int \left(-\frac{d(cd-be)^2}{e^4}+\frac{(-cd+be)^2x}{e^3}-\frac{c(cd-2be)x^2}{e^2}+\frac{c^2x^3}{e}+\frac{d^2(cd-be)^2}{e^4(d+ex)}\right)dx\\ &= -\frac{d(cd-be)^2x}{e^4}+\frac{(cd-be)^2x^2}{2e^3}-\frac{c(cd-2be)x^3}{3e^2}+\frac{c^2x^4}{4e}+\frac{d^2(cd-be)^2\log(d+ex)}{e^5}\end{aligned}$$

Mathematica [A] time = 0.0420754, size = 106, normalized size = 1.14

$$\frac{\left(b^2d^2e^2-2bcd^3e+c^2d^4\right)\log(d+ex)}{e^5}-\frac{cx^3(cd-2be)}{3e^2}+\frac{x^2(be-cd)^2}{2e^3}-\frac{dx(cd-be)^2}{e^4}+\frac{c^2x^4}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^2/(d + e*x),x]
```

```
[Out] -((d*(c*d - b*e)^2*x)/e^4) + ((-(c*d) + b*e)^2*x^2)/(2*e^3) - (c*(c*d - 2*b
*e)*x^3)/(3*e^2) + (c^2*x^4)/(4*e) + ((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2)
*Log[d + e*x])/e^5
```

Maple [A] time = 0.045, size = 152, normalized size = 1.6

$$\frac{c^2x^4}{4\,e}+\frac{2\,bcx^3}{3\,e}-\frac{c^2dx^3}{3\,e^2}+\frac{b^2x^2}{2\,e}-\frac{bcx^2d}{e^2}+\frac{c^2x^2d^2}{2\,e^3}-\frac{b^2dx}{e^2}+2\,\frac{bcd^2x}{e^3}-\frac{c^2d^3x}{e^4}+\frac{d^2\ln(ex+d)\,b^2}{e^3}-2\,\frac{d^3\ln(ex+d)\,bc}{e^4}+\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^2/(e*x+d),x)
```

```
[Out] 1/4*c^2*x^4/e+2/3/e*x^3*b*c-1/3*c^2*d*x^3/e^2+1/2/e*x^2*b^2-1/e^2*x^2*b*c*d
+1/2/e^3*x^2*c^2*d^2-1/e^2*b^2*d*x+2/e^3*b*c*d^2*x-1/e^4*c^2*d^3*x+d^2/e^3*
ln(e*x+d)*b^2-2*d^3/e^4*ln(e*x+d)*b*c+d^4/e^5*ln(e*x+d)*c^2
```

Maxima [A] time = 1.15742, size = 177, normalized size = 1.9

$$\frac{3\,c^2e^3x^4-4\left(c^2de^2-2\,bce^3\right)x^3+6\left(c^2d^2e-2\,bcde^2+b^2e^3\right)x^2-12\left(c^2d^3-2\,bcd^2e+b^2de^2\right)x}{12\,e^4}+\frac{\left(c^2d^4-2\,bcd^3e+b^2d^2e^2\right)\ldots}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/12*(3*c^2*e^3*x^4 - 4*(c^2*d*e^2 - 2*b*c*e^3)*x^3 + 6*(c^2*d^2*e - 2*b*c*
d*e^2 + b^2*e^3)*x^2 - 12*(c^2*d^3 - 2*b*c*d^2*e + b^2*d*e^2)*x)/e^4 + (c^2
*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2)*log(e*x + d)/e^5
```

Fricas [A] time = 1.53304, size = 279, normalized size = 3.

$$\frac{3\,c^2e^4x^4-4\left(c^2de^3-2\,bce^4\right)x^3+6\left(c^2d^2e^2-2\,bcde^3+b^2e^4\right)x^2-12\left(c^2d^3e-2\,bcd^2e^2+b^2de^3\right)x+12\left(c^2d^4-2\,bcd^3e+b^2\ldots\right.}{12\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/12*(3*c^2*e^4*x^4 - 4*(c^2*d*e^3 - 2*b*c*e^4)*x^3 + 6*(c^2*d^2*e^2 - 2*b*
c*d*e^3 + b^2*e^4)*x^2 - 12*(c^2*d^3*e - 2*b*c*d^2*e^2 + b^2*d*e^3)*x + 12*
(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2)*log(e*x + d))/e^5
```

Sympy [A] time = 0.958715, size = 112, normalized size = 1.2

$$\frac{c^2x^4}{4e}+\frac{d^2\,(be-cd)^2\log(d+ex)}{e^5}+\frac{x^3\left(2bce-c^2d\right)}{3e^2}+\frac{x^2\left(b^2e^2-2bcde+c^2d^2\right)}{2e^3}-\frac{x\left(b^2de^2-2bcd^2e+c^2d^3\right)}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**2/(e*x+d),x)
```

```
[Out] c**2*x**4/(4*e) + d**2*(b*e - c*d)**2*log(d + e*x)/e**5 + x**3*(2*b*c*e - c
**2*d)/(3*e**2) + x**2*(b**2*e**2 - 2*b*c*d*e + c**2*d**2)/(2*e**3) - x*(b*
```

```
*2*d*e**2 - 2*b*c*d**2*e + c**2*d**3)/e**4
```

Giac [A] time = 1.28223, size = 181, normalized size = 1.95

$$\left(c^2d^4 - 2\,bcd^3e + b^2d^2e^2\right)e^{(-5)}\log\left(|xe + d|\right) + \frac{1}{12}\left(3\,c^2x^4e^3 - 4\,c^2dx^3e^2 + 6\,c^2d^2x^2e - 12\,c^2d^3x + 8\,bcx^3e^3 - 12\,bcdx^2e^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d),x, algorithm="giac")
```

```
[Out] (c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2)*e^(-5)*log(abs(x*e + d)) + 1/12*(3*c^
2*x^4*e^3 - 4*c^2*d*x^3*e^2 + 6*c^2*d^2*x^2*e - 12*c^2*d^3*x + 8*b*c*x^3*e^
3 - 12*b*c*d*x^2*e^2 + 24*b*c*d^2*x*e + 6*b^2*x^2*e^3 - 12*b^2*d*x*e^2)*e^(
-4)
```

3.237 $\int \frac{(bx+cx^2)^2}{(d+ex)^2}\,dx$

Optimal. Leaf size=107

$$-\frac{d^2(cd-be)^2}{e^5(d+ex)}-\frac{cx^2(cd-be)}{e^3}+\frac{x(cd-be)(3cd-be)}{e^4}-\frac{2d(cd-be)(2cd-be)\log(d+ex)}{e^5}+\frac{c^2x^3}{3e^2}$$

```
[Out] ((c*d - b*e)*(3*c*d - b*e)*x)/e^4 - (c*(c*d - b*e)*x^2)/e^3 + (c^2*x^3)/(3*
e^2) - (d^2*(c*d - b*e)^2)/(e^5*(d + e*x)) - (2*d*(c*d - b*e)*(2*c*d - b*e)
*Log[d + e*x])/e^5
```

Rubi [A] time = 0.102462, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$-\frac{d^2(cd-be)^2}{e^5(d+ex)}-\frac{cx^2(cd-be)}{e^3}+\frac{x(cd-be)(3cd-be)}{e^4}-\frac{2d(cd-be)(2cd-be)\log(d+ex)}{e^5}+\frac{c^2x^3}{3e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^2/(d + e*x)^2,x]
```

```
[Out] ((c*d - b*e)*(3*c*d - b*e)*x)/e^4 - (c*(c*d - b*e)*x^2)/e^3 + (c^2*x^3)/(3*
e^2) - (d^2*(c*d - b*e)^2)/(e^5*(d + e*x)) - (2*d*(c*d - b*e)*(2*c*d - b*e)
*Log[d + e*x])/e^5
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{(bx+cx^2)^2}{(d+ex)^2}\,dx &= \int \left(\frac{(cd-be)(3cd-be)}{e^4}-\frac{2c(cd-be)x}{e^3}+\frac{c^2x^2}{e^2}+\frac{d^2(cd-be)^2}{e^4(d+ex)^2}+\frac{2d(cd-be)(-2cd+be)}{e^4(d+ex)}\right)dx\\ &= \frac{(cd-be)(3cd-be)x}{e^4}-\frac{c(cd-be)x^2}{e^3}+\frac{c^2x^3}{3e^2}-\frac{d^2(cd-be)^2}{e^5(d+ex)}-\frac{2d(cd-be)(2cd-be)\log(d+ex)}{e^5}\end{aligned}$$

Mathematica [A] time = 0.0968287, size = 114, normalized size = 1.07

$$\frac{3ex\left(b^2e^2-4bcde+3c^2d^2\right)-6d\left(b^2e^2-3bcde+2c^2d^2\right)\log(d+ex)-\frac{3d^2(cd-be)^2}{d+ex}-3ce^2x^2(cd-be)+c^2e^3x^3}{3e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^2/(d + e*x)^2,x]
```

```
[Out] (3*e*(3*c^2*d^2 - 4*b*c*d*e + b^2*e^2)*x - 3*c*e^2*(c*d - b*e)*x^2 + c^2*e^
3*x^3 - (3*d^2*(c*d - b*e)^2)/(d + e*x) - 6*d*(2*c^2*d^2 - 3*b*c*d*e + b^2*
```

```
e^2)*Log[d + e*x])/(3*e^5)
```

Maple [A] time = 0.052, size = 164, normalized size = 1.5

$$\frac{c^2x^3}{3\,e^2}+\frac{bcx^2}{e^2}-\frac{c^2dx^2}{e^3}+\frac{b^2x}{e^2}-4\,\frac{bcdx}{e^3}+3\,\frac{c^2d^2x}{e^4}-2\,\frac{d\ln(ex+d)\,b^2}{e^3}+6\,\frac{d^2\ln(ex+d)\,bc}{e^4}-4\,\frac{d^3\ln(ex+d)\,c^2}{e^5}-\frac{b^2d}{e^3\,(ex}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^2/(e*x+d)^2,x)
```

```
[Out] 1/3*c^2*x^3/e^2+1/e^2*x^2*b*c-c^2*d*x^2/e^3+b^2*x/e^2-4/e^3*b*c*d*x+3/e^4*c
^2*d^2*x-2*d/e^3*ln(e*x+d)*b^2+6*d^2/e^4*ln(e*x+d)*b*c-4*d^3/e^5*ln(e*x+d)*
c^2-d^2/e^3/(e*x+d)*b^2+2*d^3/e^4/(e*x+d)*b*c-d^4/e^5/(e*x+d)*c^2
```

Maxima [A] time = 1.10502, size = 186, normalized size = 1.74

$$-\frac{c^2d^4-2\,bcd^3e+b^2d^2e^2}{e^6x+de^5}+\frac{c^2e^2x^3-3\,(c^2de-bce^2)x^2+3\,(3\,c^2d^2-4\,bcde+b^2e^2)x}{3\,e^4}-\frac{2\,(2\,c^2d^3-3\,bcd^2e+b^2de^2)\log}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2)/(e^6*x + d*e^5) + 1/3*(c^2*e^2*x^3 -
 3*(c^2*d*e - b*c*e^2)*x^2 + 3*(3*c^2*d^2 - 4*b*c*d*e + b^2*e^2)*x)/e^4 - 2
*(2*c^2*d^3 - 3*b*c*d^2*e + b^2*d*e^2)*log(e*x + d)/e^5
```

Fricas [A] time = 1.65597, size = 416, normalized size = 3.89

$$\frac{c^2e^4x^4-3\,c^2d^4+6\,bcd^3e-3\,b^2d^2e^2-(2\,c^2de^3-3\,bce^4)x^3+3\,(2\,c^2d^2e^2-3\,bcde^3+b^2e^4)x^2+3\,(3\,c^2d^3e-4\,bcd^2e^2+b}{3\,(e^6x+de^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(c^2*e^4*x^4 - 3*c^2*d^4 + 6*b*c*d^3*e - 3*b^2*d^2*e^2 - (2*c^2*d*e^3 -
 3*b*c*e^4)*x^3 + 3*(2*c^2*d^2*e^2 - 3*b*c*d*e^3 + b^2*e^4)*x^2 + 3*(3*c^2*
d^3*e - 4*b*c*d^2*e^2 + b^2*d*e^3)*x - 6*(2*c^2*d^4 - 3*b*c*d^3*e + b^2*d^2
*e^2 + (2*c^2*d^3*e - 3*b*c*d^2*e^2 + b^2*d*e^3)*x)*log(e*x + d))/(e^6*x +
d*e^5)
```

Sympy [A] time = 1.7384, size = 122, normalized size = 1.14

$$\frac{c^2x^3}{3e^2}-\frac{2d\,(be-2cd)\,(be-cd)\log(d+ex)}{e^5}-\frac{b^2d^2e^2-2bcd^3e+c^2d^4}{de^5+e^6x}+\frac{x^2\left(bce-c^2d\right)}{e^3}+\frac{x\left(b^2e^2-4bcde+3c^2d^2\right)}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**2/(e*x+d)**2,x)
```

```
[Out] c**2*x**3/(3*e**2) - 2*d*(b*e - 2*c*d)*(b*e - c*d)*log(d + e*x)/e**5 - (b**
2*d**2*e**2 - 2*b*c*d**3*e + c**2*d**4)/(d*e**5 + e**6*x) + x**2*(b*c*e - c
**2*d)/e**3 + x*(b**2*e**2 - 4*b*c*d*e + 3*c**2*d**2)/e**4
```

Giac [A] time = 1.35191, size = 248, normalized size = 2.32

$$\frac{1}{3}\left(c^2 - \frac{3\left(2\,c^2de - bce^2\right)e^{(-1)}}{xe+d} + \frac{3\left(6\,c^2d^2e^2 - 6\,bcde^3 + b^2e^4\right)e^{(-2)}}{(xe+d)^2}\right)(xe+d)^3e^{(-5)} + 2\left(2\,c^2d^3 - 3\,bcd^2e + b^2de^2\right)e^{(-5)}\log\left(\frac{|x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/3*(c^2 - 3*(2*c^2*d*e - b*c*e^2)*e^(-1)/(x*e + d) + 3*(6*c^2*d^2*e^2 - 6*
b*c*d*e^3 + b^2*e^4)*e^(-2)/(x*e + d)^2)*(x*e + d)^3*e^(-5) + 2*(2*c^2*d^3
- 3*b*c*d^2*e + b^2*d*e^2)*e^(-5)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2) - (c
^2*d^4*e^3/(x*e + d) - 2*b*c*d^3*e^4/(x*e + d) + b^2*d^2*e^5/(x*e + d))*e^(
-8)
```

3.238 $\int \frac{(bx+cx^2)^2}{(d+ex)^3}\,dx$

Optimal. Leaf size=119

$$\frac{(b^2e^2-6bcde+6c^2d^2)\log(d+ex)}{e^5}-\frac{d^2(cd-be)^2}{2e^5(d+ex)^2}+\frac{2d(2cd-be)(cd-be)}{e^5(d+ex)}-\frac{cx(3cd-2be)}{e^4}+\frac{c^2x^2}{2e^3}$$

```
[Out] -((c*(3*c*d - 2*b*e)*x)/e^4) + (c^2*x^2)/(2*e^3) - (d^2*(c*d - b*e)^2)/(2*e
^5*(d + e*x)^2) + (2*d*(c*d - b*e)*(2*c*d - b*e))/(e^5*(d + e*x)) + ((6*c^2
*d^2 - 6*b*c*d*e + b^2*e^2)*Log[d + e*x])/e^5
```

Rubi [A] time = 0.106206, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$\frac{(b^2e^2-6bcde+6c^2d^2)\log(d+ex)}{e^5}-\frac{d^2(cd-be)^2}{2e^5(d+ex)^2}+\frac{2d(2cd-be)(cd-be)}{e^5(d+ex)}-\frac{cx(3cd-2be)}{e^4}+\frac{c^2x^2}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^2/(d + e*x)^3,x]
```

```
[Out] -((c*(3*c*d - 2*b*e)*x)/e^4) + (c^2*x^2)/(2*e^3) - (d^2*(c*d - b*e)^2)/(2*e
^5*(d + e*x)^2) + (2*d*(c*d - b*e)*(2*c*d - b*e))/(e^5*(d + e*x)) + ((6*c^2
*d^2 - 6*b*c*d*e + b^2*e^2)*Log[d + e*x])/e^5
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(bx+cx^2)^2}{(d+ex)^3}\,dx=\int \left(-\frac{c(3cd-2be)}{e^4}+\frac{c^2x}{e^3}+\frac{d^2(cd-be)^2}{e^4(d+ex)^3}+\frac{2d(cd-be)(-2cd+be)}{e^4(d+ex)^2}+\frac{6c^2d^2-6bcde+b^2e^2}{e^4(d+ex)}\right)dx$$
$$=-\frac{c(3cd-2be)x}{e^4}+\frac{c^2x^2}{2e^3}-\frac{d^2(cd-be)^2}{2e^5(d+ex)^2}+\frac{2d(cd-be)(2cd-be)}{e^5(d+ex)}+\frac{(6c^2d^2-6bcde+b^2e^2)\log(d+}{e^5}$$

Mathematica [A] time = 0.0690577, size = 116, normalized size = 0.97

$$\frac{\frac{4d(b^2e^2-3bcde+2c^2d^2)}{d+ex}+2(b^2e^2-6bcde+6c^2d^2)\log(d+ex)-\frac{d^2(cd-be)^2}{(d+ex)^2}-2cex(3cd-2be)+c^2e^2x^2}{2e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^2/(d + e*x)^3,x]
```

```
[Out] (-2*c*e*(3*c*d - 2*b*e)*x + c^2*e^2*x^2 - (d^2*(c*d - b*e)^2)/(d + e*x)^2 +
 (4*d*(2*c^2*d^2 - 3*b*c*d*e + b^2*e^2))/(d + e*x) + 2*(6*c^2*d^2 - 6*b*c*d
*e + b^2*e^2)*Log[d + e*x])/(2*e^5)
```

Maple [A] time = 0.05, size = 178, normalized size = 1.5

$$\frac{c^2x^2}{2\,e^3}+2\,\frac{bcx}{e^3}-3\,\frac{c^2dx}{e^4}-\frac{b^2d^2}{2\,e^3\,(ex+d)^2}+\frac{d^3bc}{e^4\,(ex+d)^2}-\frac{c^2d^4}{2\,e^5\,(ex+d)^2}+\frac{b^2\ln(ex+d)}{e^3}-6\,\frac{\ln(ex+d)\,bcd}{e^4}+6\,\frac{\ln(ex+d)\,c}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^2/(e*x+d)^3,x)
```

```
[Out] 1/2*c^2*x^2/e^3+2*c/e^3*x*b-3*c^2*d*x/e^4-1/2*d^2/e^3/(e*x+d)^2*b^2+d^3/e^4
/(e*x+d)^2*b*c-1/2*d^4/e^5/(e*x+d)^2*c^2+b^2*ln(e*x+d)/e^3-6/e^4*ln(e*x+d)*
b*c*d+6/e^5*ln(e*x+d)*c^2*d^2+2*d/e^3/(e*x+d)*b^2-6*d^2/e^4/(e*x+d)*b*c+4*d
^3/e^5/(e*x+d)*c^2
```

Maxima [A] time = 1.13679, size = 198, normalized size = 1.66

$$\frac{7\,c^2d^4-10\,bcd^3e+3\,b^2d^2e^2+4\left(2\,c^2d^3e-3\,bcd^2e^2+b^2de^3\right)x}{2\left(e^7x^2+2\,de^6x+d^2e^5\right)}+\frac{c^2ex^2-2\left(3\,c^2d-2\,bce\right)x}{2\,e^4}+\frac{\left(6\,c^2d^2-6\,bcde+b^2e^2\right)\log}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(7*c^2*d^4 - 10*b*c*d^3*e + 3*b^2*d^2*e^2 + 4*(2*c^2*d^3*e - 3*b*c*d^2*
e^2 + b^2*d*e^3)*x)/(e^7*x^2 + 2*d*e^6*x + d^2*e^5) + 1/2*(c^2*e*x^2 - 2*(3
*c^2*d - 2*b*c*e)*x)/e^4 + (6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*log(e*x + d)/e
^5
```

Fricas [B] time = 1.59996, size = 486, normalized size = 4.08

$$\frac{c^2e^4x^4+7\,c^2d^4-10\,bcd^3e+3\,b^2d^2e^2-4\left(c^2de^3-bce^4\right)x^3-\left(11\,c^2d^2e^2-8\,bcde^3\right)x^2+2\left(c^2d^3e-4\,bcd^2e^2+2\,b^2de^3\right)x+2}{2\left(e^7x^2+2\,de^6x+d^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(c^2*e^4*x^4 + 7*c^2*d^4 - 10*b*c*d^3*e + 3*b^2*d^2*e^2 - 4*(c^2*d*e^3
- b*c*e^4)*x^3 - (11*c^2*d^2*e^2 - 8*b*c*d*e^3)*x^2 + 2*(c^2*d^3*e - 4*b*c*
d^2*e^2 + 2*b^2*d*e^3)*x + 2*(6*c^2*d^4 - 6*b*c*d^3*e + b^2*d^2*e^2 + (6*c^
2*d^2*e^2 - 6*b*c*d*e^3 + b^2*e^4)*x^2 + 2*(6*c^2*d^3*e - 6*b*c*d^2*e^2 + b
^2*d*e^3)*x)*log(e*x + d))/(e^7*x^2 + 2*d*e^6*x + d^2*e^5)
```

Sympy [A] time = 2.5355, size = 153, normalized size = 1.29

$$\frac{c^2x^2}{2e^3}+\frac{3b^2d^2e^2-10bcd^3e+7c^2d^4+x\left(4b^2de^3-12bcd^2e^2+8c^2d^3e\right)}{2d^2e^5+4de^6x+2e^7x^2}+\frac{x\left(2bce-3c^2d\right)}{e^4}+\frac{\left(b^2e^2-6bcde+6c^2d^2\right)\log\left(d+ex\right)}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**2/(e*x+d)**3,x)
```

```
[Out] c**2*x**2/(2*e**3) + (3*b**2*d**2*e**2 - 10*b*c*d**3*e + 7*c**2*d**4 + x*(4
*b**2*d*e**3 - 12*b*c*d**2*e**2 + 8*c**2*d**3*e))/(2*d**2*e**5 + 4*d*e**6*x
 + 2*e**7*x**2) + x*(2*b*c*e - 3*c**2*d)/e**4 + (b**2*e**2 - 6*b*c*d*e + 6*
c**2*d**2)*log(d + e*x)/e**5
```

Giac [A] time = 1.29311, size = 181, normalized size = 1.52

$$\left(6\,c^2d^2-6\,bcde+b^2e^2\right)e^{(-5)}\log\left(|xe+d|\right)+\frac{1}{2}\left(c^2x^2e^3-6\,c^2dxe^2+4\,bcxe^3\right)e^{(-6)}+\frac{\left(7\,c^2d^4-10\,bcd^3e+3\,b^2d^2e^2+4\left(2\,c^2d^3e-3\,bcd^2e^2+b^2de^3\right)x\right)e^{(-5)}}{2\left(xe+d\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] (6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*e^(-5)*log(abs(x*e + d)) + 1/2*(c^2*x^2*e
^3 - 6*c^2*d*x*e^2 + 4*b*c*x*e^3)*e^(-6) + 1/2*(7*c^2*d^4 - 10*b*c*d^3*e +
3*b^2*d^2*e^2 + 4*(2*c^2*d^3*e - 3*b*c*d^2*e^2 + b^2*d*e^3)*x)*e^(-5)/(x*e
+ d)^2
```

3.239 $\int \frac{(bx+cx^2)^2}{(d+ex)^4}\,dx$

Optimal. Leaf size=120

$$-\frac{b^2e^2-6bcde+6c^2d^2}{e^5(d+ex)}-\frac{d^2(cd-be)^2}{3e^5(d+ex)^3}+\frac{d(cd-be)(2cd-be)}{e^5(d+ex)^2}-\frac{2c(2cd-be)\log(d+ex)}{e^5}+\frac{c^2x}{e^4}$$

```
[Out] (c^2*x)/e^4 - (d^2*(c*d - b*e)^2)/(3*e^5*(d + e*x)^3) + (d*(c*d - b*e)*(2*c
*d - b*e))/(e^5*(d + e*x)^2) - (6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)/(e^5*(d +
e*x)) - (2*c*(2*c*d - b*e)*Log[d + e*x])/e^5
```

Rubi [A] time = 0.0962984, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$-\frac{b^2e^2-6bcde+6c^2d^2}{e^5(d+ex)}-\frac{d^2(cd-be)^2}{3e^5(d+ex)^3}+\frac{d(cd-be)(2cd-be)}{e^5(d+ex)^2}-\frac{2c(2cd-be)\log(d+ex)}{e^5}+\frac{c^2x}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^2/(d + e*x)^4,x]
```

```
[Out] (c^2*x)/e^4 - (d^2*(c*d - b*e)^2)/(3*e^5*(d + e*x)^3) + (d*(c*d - b*e)*(2*c
*d - b*e))/(e^5*(d + e*x)^2) - (6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)/(e^5*(d +
e*x)) - (2*c*(2*c*d - b*e)*Log[d + e*x])/e^5
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{(bx+cx^2)^2}{(d+ex)^4}\,dx &= \int \left(\frac{c^2}{e^4}+\frac{d^2(cd-be)^2}{e^4(d+ex)^4}+\frac{2d(cd-be)(-2cd+be)}{e^4(d+ex)^3}+\frac{6c^2d^2-6bcde+b^2e^2}{e^4(d+ex)^2}-\frac{2c(2cd-be)}{e^4(d+ex)}\right)dx\\ &= \frac{c^2x}{e^4}-\frac{d^2(cd-be)^2}{3e^5(d+ex)^3}+\frac{d(cd-be)(2cd-be)}{e^5(d+ex)^2}-\frac{6c^2d^2-6bcde+b^2e^2}{e^5(d+ex)}-\frac{2c(2cd-be)\log(d+ex)}{e^5}\end{aligned}$$

Mathematica [A] time = 0.0558831, size = 134, normalized size = 1.12

$$\frac{-b^2e^2\left(d^2+3dex+3e^2x^2\right)+bcde\left(11d^2+27dex+18e^2x^2\right)-6c(d+ex)^3(2cd-be)\log(d+ex)+c^2\left(-9d^2e^2x^2-27d^3ex-\right.}{3e^5(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^2/(d + e*x)^4,x]
```

```
[Out] (-(b^2*e^2*(d^2 + 3*d*e*x + 3*e^2*x^2)) + b*c*d*e*(11*d^2 + 27*d*e*x + 18*e
^2*x^2) + c^2*(-13*d^4 - 27*d^3*e*x - 9*d^2*e^2*x^2 + 9*d*e^3*x^3 + 3*e^4*x
```

```
^4) - 6*c*(2*c*d - b*e)*(d + e*x)^3*Log[d + e*x])/(3*e^5*(d + e*x)^3)
```

Maple [A] time = 0.05, size = 189, normalized size = 1.6

$$\frac{c^2x}{e^4}+\frac{b^2d}{e^3\,(ex+d)^2}-3\,\frac{d^2bc}{e^4\,(ex+d)^2}+2\,\frac{c^2d^3}{e^5\,(ex+d)^2}+2\,\frac{c\ln(ex+d)\,b}{e^4}-4\,\frac{c^2d\ln(ex+d)}{e^5}-\frac{b^2d^2}{3\,e^3\,(ex+d)^3}+\frac{2\,d^3bc}{3\,e^4\,(ex+ \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^2/(e*x+d)^4,x)
```

```
[Out] c^2*x/e^4+d/e^3/(e*x+d)^2*b^2-3*d^2/e^4/(e*x+d)^2*b*c+2*d^3/e^5/(e*x+d)^2*c
^2+2*c/e^4*ln(e*x+d)*b-4*c^2*d*ln(e*x+d)/e^5-1/3*d^2/e^3/(e*x+d)^3*b^2+2/3*
d^3/e^4/(e*x+d)^3*b*c-1/3*d^4/e^5/(e*x+d)^3*c^2-1/e^3/(e*x+d)*b^2+6/e^4/(e*
x+d)*b*c*d-6/e^5/(e*x+d)*c^2*d^2
```

Maxima [A] time = 1.05529, size = 215, normalized size = 1.79

$$-\frac{13\,c^2d^4-11\,bcd^3e+b^2d^2e^2+3\left(6\,c^2d^2e^2-6\,bcde^3+b^2e^4\right)x^2+3\left(10\,c^2d^3e-9\,bcd^2e^2+b^2de^3\right)x}{3\left(e^8x^3+3\,de^7x^2+3\,d^2e^6x+d^3e^5\right)}+\frac{c^2x}{e^4}-\frac{2\left(2\,c^2d-b \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(13*c^2*d^4 - 11*b*c*d^3*e + b^2*d^2*e^2 + 3*(6*c^2*d^2*e^2 - 6*b*c*d*
e^3 + b^2*e^4)*x^2 + 3*(10*c^2*d^3*e - 9*b*c*d^2*e^2 + b^2*d*e^3)*x)/(e^8*x
^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3*e^5) + c^2*x/e^4 - 2*(2*c^2*d - b*c*e)
*log(e*x + d)/e^5
```

Fricas [B] time = 1.61198, size = 494, normalized size = 4.12

$$\frac{3\,c^2e^4x^4+9\,c^2de^3x^3-13\,c^2d^4+11\,bcd^3e-b^2d^2e^2-3\left(3\,c^2d^2e^2-6\,bcde^3+b^2e^4\right)x^2-3\left(9\,c^2d^3e-9\,bcd^2e^2+b^2de^3\right)x- \cdots}{3\left(e^8x^3+3\,de^7x^2+3\,d^2e^6x \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*c^2*e^4*x^4 + 9*c^2*d*e^3*x^3 - 13*c^2*d^4 + 11*b*c*d^3*e - b^2*d^2*
e^2 - 3*(3*c^2*d^2*e^2 - 6*b*c*d*e^3 + b^2*e^4)*x^2 - 3*(9*c^2*d^3*e - 9*b*
c*d^2*e^2 + b^2*d*e^3)*x - 6*(2*c^2*d^4 - b*c*d^3*e + (2*c^2*d*e^3 - b*c*e^
4)*x^3 + 3*(2*c^2*d^2*e^2 - b*c*d*e^3)*x^2 + 3*(2*c^2*d^3*e - b*c*d^2*e^2)*
x)*log(e*x + d))/(e^8*x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3*e^5)
```

Sympy [A] time = 4.22294, size = 163, normalized size = 1.36

$$\frac{c^2x}{e^4}+\frac{2c\,(be-2cd)\log(d+ex)}{e^5}-\frac{b^2d^2e^2-11bcd^3e+13c^2d^4+x^2\left(3b^2e^4-18bcde^3+18c^2d^2e^2\right)+x\left(3b^2de^3-27bcd^2e \cdots\right.}{3d^3e^5+9d^2e^6x+9de^7x^2+3e^8x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**2/(e*x+d)**4,x)
```

```
[Out] c**2*x/e**4 + 2*c*(b*e - 2*c*d)*log(d + e*x)/e**5 - (b**2*d**2*e**2 - 11*b*
c*d**3*e + 13*c**2*d**4 + x**2*(3*b**2*e**4 - 18*b*c*d*e**3 + 18*c**2*d**2*
e**2) + x*(3*b**2*d*e**3 - 27*b*c*d**2*e**2 + 30*c**2*d**3*e))/(3*d**3*e**5
 + 9*d**2*e**6*x + 9*d*e**7*x**2 + 3*e**8*x**3)
```

Giac [A] time = 1.3237, size = 177, normalized size = 1.48

$$c^2xe^{(-4)} - 2\left(2\,c^2d - bce\right)e^{(-5)}\log\left(|xe + d|\right) - \frac{\left(13\,c^2d^4 - 11\,bcd^3e + b^2d^2e^2 + 3\left(6\,c^2d^2e^2 - 6\,bcde^3 + b^2e^4\right)x^2 + 3\left(10\,c^2d^3e - \right.\right.}{3\left(xe + d\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] c^2*x*e^(-4) - 2*(2*c^2*d - b*c*e)*e^(-5)*log(abs(x*e + d)) - 1/3*(13*c^2*d
^4 - 11*b*c*d^3*e + b^2*d^2*e^2 + 3*(6*c^2*d^2*e^2 - 6*b*c*d*e^3 + b^2*e^4)
*x^2 + 3*(10*c^2*d^3*e - 9*b*c*d^2*e^2 + b^2*d*e^3)*x)*e^(-5)/(x*e + d)^3
```

3.240 $\int \frac{(bx+cx^2)^2}{(d+ex)^5} dx$

Optimal. Leaf size=131

$$-\frac{b^2e^2-6bcde+6c^2d^2}{2e^5(d+ex)^2}-\frac{d^2(cd-be)^2}{4e^5(d+ex)^4}+\frac{2c(2cd-be)}{e^5(d+ex)}+\frac{2d(cd-be)(2cd-be)}{3e^5(d+ex)^3}+\frac{c^2\log(d+ex)}{e^5}$$

```
[Out] -(d^2*(c*d - b*e)^2)/(4*e^5*(d + e*x)^4) + (2*d*(c*d - b*e)*(2*c*d - b*e))/
(3*e^5*(d + e*x)^3) - (6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)/(2*e^5*(d + e*x)^2)
 + (2*c*(2*c*d - b*e))/(e^5*(d + e*x)) + (c^2*Log[d + e*x])/e^5
```

Rubi [A] time = 0.091824, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$-\frac{b^2e^2-6bcde+6c^2d^2}{2e^5(d+ex)^2}-\frac{d^2(cd-be)^2}{4e^5(d+ex)^4}+\frac{2c(2cd-be)}{e^5(d+ex)}+\frac{2d(cd-be)(2cd-be)}{3e^5(d+ex)^3}+\frac{c^2\log(d+ex)}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^2/(d + e*x)^5,x]
```

```
[Out] -(d^2*(c*d - b*e)^2)/(4*e^5*(d + e*x)^4) + (2*d*(c*d - b*e)*(2*c*d - b*e))/
(3*e^5*(d + e*x)^3) - (6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)/(2*e^5*(d + e*x)^2)
 + (2*c*(2*c*d - b*e))/(e^5*(d + e*x)) + (c^2*Log[d + e*x])/e^5
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{(bx+cx^2)^2}{(d+ex)^5}dx &= \int \left(\frac{d^2(cd-be)^2}{e^4(d+ex)^5}+\frac{2d(cd-be)(-2cd+be)}{e^4(d+ex)^4}+\frac{6c^2d^2-6bcde+b^2e^2}{e^4(d+ex)^3}-\frac{2c(2cd-be)}{e^4(d+ex)^2}+\frac{c^2}{e^4(d+ex)}\right)\\ &= -\frac{d^2(cd-be)^2}{4e^5(d+ex)^4}+\frac{2d(cd-be)(2cd-be)}{3e^5(d+ex)^3}-\frac{6c^2d^2-6bcde+b^2e^2}{2e^5(d+ex)^2}+\frac{2c(2cd-be)}{e^5(d+ex)}+\frac{c^2\log(d+ex)}{e^5}\end{aligned}$$

Mathematica [A] time = 0.0447406, size = 126, normalized size = 0.96

$$\frac{-b^2e^2\left(d^2+4dex+6e^2x^2\right)-6bce\left(4d^2ex+d^3+6de^2x^2+4e^3x^3\right)+c^2d\left(88d^2ex+25d^3+108de^2x^2+48e^3x^3\right)+12c^2(d}{12e^5(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^2/(d + e*x)^5,x]
```

```
[Out] (-(b^2*e^2*(d^2 + 4*d*e*x + 6*e^2*x^2)) - 6*b*c*e*(d^3 + 4*d^2*e*x + 6*d*e^
2*x^2 + 4*e^3*x^3) + c^2*d*(25*d^3 + 88*d^2*e*x + 108*d*e^2*x^2 + 48*e^3*x^
```

```
3) + 12*c^2*(d + e*x)^4*Log[d + e*x])/(12*e^5*(d + e*x)^4)
```

Maple [A] time = 0.05, size = 197, normalized size = 1.5

$$-\frac{b^2}{2e^3(ex+d)^2}+3\frac{bcd}{e^4(ex+d)^2}-3\frac{c^2d^2}{e^5(ex+d)^2}+\frac{c^2\ln(ex+d)}{e^5}+\frac{2b^2d}{3e^3(ex+d)^3}-2\frac{bcd^2}{e^4(ex+d)^3}+\frac{4c^2d^3}{3e^5(ex+d)^3}-2\frac{}{e^4(e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^2/(e*x+d)^5,x)
```

```
[Out] -1/2*b^2/e^3/(e*x+d)^2+3/e^4/(e*x+d)^2*b*c*d-3/e^5/(e*x+d)^2*c^2*d^2+c^2*ln
(e*x+d)/e^5+2/3*d/e^3/(e*x+d)^3*b^2-2*d^2/e^4/(e*x+d)^3*b*c+4/3*d^3/e^5/(e*
x+d)^3*c^2-2*c/e^4/(e*x+d)*b+4*c^2*d/e^5/(e*x+d)-1/4*d^2/e^3/(e*x+d)^4*b^2+
1/2*d^3/e^4/(e*x+d)^4*b*c-1/4*d^4/e^5/(e*x+d)^4*c^2
```

Maxima [A] time = 1.17247, size = 239, normalized size = 1.82

$$\frac{25c^2d^4-6bcd^3e-b^2d^2e^2+24\left(2c^2de^3-bce^4\right)x^3+6\left(18c^2d^2e^2-6bcde^3-b^2e^4\right)x^2+4\left(22c^2d^3e-6bcd^2e^2-b^2de^3\right)x}{12\left(e^9x^4+4de^8x^3+6d^2e^7x^2+4d^3e^6x+d^4e^5\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^5,x, algorithm="maxima")
```

```
[Out] 1/12*(25*c^2*d^4 - 6*b*c*d^3*e - b^2*d^2*e^2 + 24*(2*c^2*d*e^3 - b*c*e^4)*x
^3 + 6*(18*c^2*d^2*e^2 - 6*b*c*d*e^3 - b^2*e^4)*x^2 + 4*(22*c^2*d^3*e - 6*b
*c*d^2*e^2 - b^2*d*e^3)*x)/(e^9*x^4 + 4*d*e^8*x^3 + 6*d^2*e^7*x^2 + 4*d^3*e
^6*x + d^4*e^5) + c^2*log(e*x + d)/e^5
```

Fricas [A] time = 1.57932, size = 456, normalized size = 3.48

$$\frac{25c^2d^4-6bcd^3e-b^2d^2e^2+24\left(2c^2de^3-bce^4\right)x^3+6\left(18c^2d^2e^2-6bcde^3-b^2e^4\right)x^2+4\left(22c^2d^3e-6bcd^2e^2-b^2de^3\right)x+}{12\left(e^9x^4+4de^8x^3+6d^2e^7x^2+4d^3e^6x+d^4e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(25*c^2*d^4 - 6*b*c*d^3*e - b^2*d^2*e^2 + 24*(2*c^2*d*e^3 - b*c*e^4)*x
^3 + 6*(18*c^2*d^2*e^2 - 6*b*c*d*e^3 - b^2*e^4)*x^2 + 4*(22*c^2*d^3*e - 6*b
*c*d^2*e^2 - b^2*d*e^3)*x + 12*(c^2*e^4*x^4 + 4*c^2*d*e^3*x^3 + 6*c^2*d^2*e
^2*x^2 + 4*c^2*d^3*e*x + c^2*d^4)*log(e*x + d))/(e^9*x^4 + 4*d*e^8*x^3 + 6*
d^2*e^7*x^2 + 4*d^3*e^6*x + d^4*e^5)
```

Sympy [A] time = 5.06712, size = 180, normalized size = 1.37

$$\frac{c^2\log(d+ex)}{e^5}-\frac{b^2d^2e^2+6bcd^3e-25c^2d^4+x^3\left(24bce^4-48c^2de^3\right)+x^2\left(6b^2e^4+36bcde^3-108c^2d^2e^2\right)+x\left(4b^2de^3+24\right.}{12d^4e^5+48d^3e^6x+72d^2e^7x^2+48de^8x^3+12e^9x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**2/(e*x+d)**5,x)
```

```
[Out] c**2*log(d + e*x)/e**5 - (b**2*d**2*e**2 + 6*b*c*d**3*e - 25*c**2*d**4 + x*
*3*(24*b*c*e**4 - 48*c**2*d*e**3) + x**2*(6*b**2*e**4 + 36*b*c*d*e**3 - 108
*c**2*d**2*e**2) + x*(4*b**2*d*e**3 + 24*b*c*d**2*e**2 - 88*c**2*d**3*e))/(
12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**
9*x**4)
```

Giac [A] time = 1.34748, size = 289, normalized size = 2.21

$$-c^2e^{(-5)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^2}\right)+\frac{1}{12}\left(\frac{48\,c^2de^{15}}{xe+d}-\frac{36\,c^2d^2e^{15}}{(xe+d)^2}+\frac{16\,c^2d^3e^{15}}{(xe+d)^3}-\frac{3\,c^2d^4e^{15}}{(xe+d)^4}-\frac{24\,bce^{16}}{xe+d}+\frac{36\,bcde^{16}}{(xe+d)^2}-\frac{24\,bcd^2e^{16}}{(xe+d)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] -c^2*e^(-5)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2) + 1/12*(48*c^2*d*e^15/(x*e
 + d) - 36*c^2*d^2*e^15/(x*e + d)^2 + 16*c^2*d^3*e^15/(x*e + d)^3 - 3*c^2*d
^4*e^15/(x*e + d)^4 - 24*b*c*e^16/(x*e + d) + 36*b*c*d*e^16/(x*e + d)^2 - 2
4*b*c*d^2*e^16/(x*e + d)^3 + 6*b*c*d^3*e^16/(x*e + d)^4 - 6*b^2*e^17/(x*e +
 d)^2 + 8*b^2*d*e^17/(x*e + d)^3 - 3*b^2*d^2*e^17/(x*e + d)^4)*e^(-20)
```

3.241 $\int \frac{(bx+cx^2)^2}{(d+ex)^6}\,dx$

Optimal. Leaf size=132

$$-\frac{b^2e^2-6bcde+6c^2d^2}{3e^5(d+ex)^3}-\frac{d^2(cd-be)^2}{5e^5(d+ex)^5}+\frac{c(2cd-be)}{e^5(d+ex)^2}+\frac{d(cd-be)(2cd-be)}{2e^5(d+ex)^4}-\frac{c^2}{e^5(d+ex)}$$

```
[Out] -(d^2*(c*d - b*e)^2)/(5*e^5*(d + e*x)^5) + (d*(c*d - b*e)*(2*c*d - b*e))/(2
*e^5*(d + e*x)^4) - (6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)/(3*e^5*(d + e*x)^3) +
 (c*(2*c*d - b*e))/(e^5*(d + e*x)^2) - c^2/(e^5*(d + e*x))
```

Rubi [A] time = 0.0898765, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$-\frac{b^2e^2-6bcde+6c^2d^2}{3e^5(d+ex)^3}-\frac{d^2(cd-be)^2}{5e^5(d+ex)^5}+\frac{c(2cd-be)}{e^5(d+ex)^2}+\frac{d(cd-be)(2cd-be)}{2e^5(d+ex)^4}-\frac{c^2}{e^5(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^2/(d + e*x)^6,x]
```

```
[Out] -(d^2*(c*d - b*e)^2)/(5*e^5*(d + e*x)^5) + (d*(c*d - b*e)*(2*c*d - b*e))/(2
*e^5*(d + e*x)^4) - (6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)/(3*e^5*(d + e*x)^3) +
 (c*(2*c*d - b*e))/(e^5*(d + e*x)^2) - c^2/(e^5*(d + e*x))
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{(bx+cx^2)^2}{(d+ex)^6}\,dx &= \int \left(\frac{d^2(cd-be)^2}{e^4(d+ex)^6}+\frac{2d(cd-be)(-2cd+be)}{e^4(d+ex)^5}+\frac{6c^2d^2-6bcde+b^2e^2}{e^4(d+ex)^4}-\frac{2c(2cd-be)}{e^4(d+ex)^3}+\frac{c^2}{e^4(d+ex)^2}\right)dx\\ &= -\frac{d^2(cd-be)^2}{5e^5(d+ex)^5}+\frac{d(cd-be)(2cd-be)}{2e^5(d+ex)^4}-\frac{6c^2d^2-6bcde+b^2e^2}{3e^5(d+ex)^3}+\frac{c(2cd-be)}{e^5(d+ex)^2}-\frac{c^2}{e^5(d+ex)}\end{aligned}$$

Mathematica [A] time = 0.0494653, size = 116, normalized size = 0.88

$$-\frac{b^2e^2\left(d^2+5dex+10e^2x^2\right)+3bce\left(5d^2ex+d^3+10de^2x^2+10e^3x^3\right)+6c^2\left(10d^2e^2x^2+5d^3ex+d^4+10de^3x^3+5e^4x^4\right)}{30e^5(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^2/(d + e*x)^6,x]
```

```
[Out] -(b^2*e^2*(d^2 + 5*d*e*x + 10*e^2*x^2) + 3*b*c*e*(d^3 + 5*d^2*e*x + 10*d*e^
2*x^2 + 10*e^3*x^3) + 6*c^2*(d^4 + 5*d^3*e*x + 10*d^2*e^2*x^2 + 10*d*e^3*x^
```

```
3 + 5*e^4*x^4))/(30*e^5*(d + e*x)^5)
```

Maple [A] time = 0.048, size = 143, normalized size = 1.1

$$-\frac{c(be-2cd)}{e^5(ex+d)^2}-\frac{d^2(b^2e^2-2bcde+c^2d^2)}{5e^5(ex+d)^5}-\frac{b^2e^2-6bcde+6c^2d^2}{3e^5(ex+d)^3}-\frac{c^2}{e^5(ex+d)}+\frac{d(b^2e^2-3bcde+2c^2d^2)}{2e^5(ex+d)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^2/(e*x+d)^6,x)
```

```
[Out] -c*(b*e-2*c*d)/e^5/(e*x+d)^2-1/5*d^2*(b^2*e^2-2*b*c*d*e+c^2*d^2)/e^5/(e*x+d
)^5-1/3*(b^2*e^2-6*b*c*d*e+6*c^2*d^2)/e^5/(e*x+d)^3-c^2/e^5/(e*x+d)+1/2*d*(
b^2*e^2-3*b*c*d*e+2*c^2*d^2)/e^5/(e*x+d)^4
```

Maxima [A] time = 1.0318, size = 244, normalized size = 1.85

$$-\frac{30c^2e^4x^4+6c^2d^4+3bcd^3e+b^2d^2e^2+30(2c^2de^3+bce^4)x^3+10(6c^2d^2e^2+3bcde^3+b^2e^4)x^2+5(6c^2d^3e+3bcd^2\cdots}{30(e^{10}x^5+5de^9x^4+10d^2e^8x^3+10d^3e^7x^2+5d^4e^6x+d^5e^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^6,x, algorithm="maxima")
```

```
[Out] -1/30*(30*c^2*e^4*x^4 + 6*c^2*d^4 + 3*b*c*d^3*e + b^2*d^2*e^2 + 30*(2*c^2*d
*e^3 + b*c*e^4)*x^3 + 10*(6*c^2*d^2*e^2 + 3*b*c*d*e^3 + b^2*e^4)*x^2 + 5*(6
*c^2*d^3*e + 3*b*c*d^2*e^2 + b^2*d*e^3)*x)/(e^10*x^5 + 5*d*e^9*x^4 + 10*d^2
*e^8*x^3 + 10*d^3*e^7*x^2 + 5*d^4*e^6*x + d^5*e^5)
```

Fricas [A] time = 1.67753, size = 374, normalized size = 2.83

$$-\frac{30c^2e^4x^4+6c^2d^4+3bcd^3e+b^2d^2e^2+30(2c^2de^3+bce^4)x^3+10(6c^2d^2e^2+3bcde^3+b^2e^4)x^2+5(6c^2d^3e+3bcd^2\cdots}{30(e^{10}x^5+5de^9x^4+10d^2e^8x^3+10d^3e^7x^2+5d^4e^6x+d^5e^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^6,x, algorithm="fricas")
```

```
[Out] -1/30*(30*c^2*e^4*x^4 + 6*c^2*d^4 + 3*b*c*d^3*e + b^2*d^2*e^2 + 30*(2*c^2*d
*e^3 + b*c*e^4)*x^3 + 10*(6*c^2*d^2*e^2 + 3*b*c*d*e^3 + b^2*e^4)*x^2 + 5*(6
*c^2*d^3*e + 3*b*c*d^2*e^2 + b^2*d*e^3)*x)/(e^10*x^5 + 5*d*e^9*x^4 + 10*d^2
*e^8*x^3 + 10*d^3*e^7*x^2 + 5*d^4*e^6*x + d^5*e^5)
```

Sympy [A] time = 10.562, size = 192, normalized size = 1.45

$$-\frac{b^2d^2e^2+3bcd^3e+6c^2d^4+30c^2e^4x^4+x^3(30bce^4+60c^2de^3)+x^2(10b^2e^4+30bcde^3+60c^2d^2e^2)+x(5b^2de^3+15bc\cdots}{30d^5e^5+150d^4e^6x+300d^3e^7x^2+300d^2e^8x^3+150de^9x^4+30e^{10}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**2/(e*x+d)**6,x)
```

```
[Out] -(b**2*d**2*e**2 + 3*b*c*d**3*e + 6*c**2*d**4 + 30*c**2*e**4*x**4 + x**3*(3
0*b*c*e**4 + 60*c**2*d*e**3) + x**2*(10*b**2*e**4 + 30*b*c*d*e**3 + 60*c**2
*d**2*e**2) + x*(5*b**2*d*e**3 + 15*b*c*d**2*e**2 + 30*c**2*d**3*e))/(30*d*
*5*e**5 + 150*d**4*e**6*x + 300*d**3*e**7*x**2 + 300*d**2*e**8*x**3 + 150*d
*e**9*x**4 + 30*e**10*x**5)
```

Giac [A] time = 1.29727, size = 178, normalized size = 1.35

$$-\frac{\left(30\,c^2x^4e^4+60\,c^2dx^3e^3+60\,c^2d^2x^2e^2+30\,c^2d^3xe+6\,c^2d^4+30\,bcx^3e^4+30\,bcdx^2e^3+15\,bcd^2xe^2+3\,bcd^3e+10\,b^2x^2e^4}{30\,(xe+d)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^6,x, algorithm="giac")
```

```
[Out] -1/30*(30*c^2*x^4*e^4 + 60*c^2*d*x^3*e^3 + 60*c^2*d^2*x^2*e^2 + 30*c^2*d^3*
x*e + 6*c^2*d^4 + 30*b*c*x^3*e^4 + 30*b*c*d*x^2*e^3 + 15*b*c*d^2*x*e^2 + 3*
b*c*d^3*e + 10*b^2*x^2*e^4 + 5*b^2*d*x*e^3 + b^2*d^2*e^2)*e^(-5)/(x*e + d)^
5
```

3.242 $\int \frac{(bx+cx^2)^2}{(d+ex)^7}\,dx$

Optimal. Leaf size=137

$$-\frac{b^2e^2-6bcde+6c^2d^2}{4e^5(d+ex)^4}-\frac{d^2(cd-be)^2}{6e^5(d+ex)^6}+\frac{2c(2cd-be)}{3e^5(d+ex)^3}+\frac{2d(cd-be)(2cd-be)}{5e^5(d+ex)^5}-\frac{c^2}{2e^5(d+ex)^2}$$

```
[Out] -(d^2*(c*d - b*e)^2)/(6*e^5*(d + e*x)^6) + (2*d*(c*d - b*e)*(2*c*d - b*e))/
(5*e^5*(d + e*x)^5) - (6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)/(4*e^5*(d + e*x)^4)
 + (2*c*(2*c*d - b*e))/(3*e^5*(d + e*x)^3) - c^2/(2*e^5*(d + e*x)^2)
```

Rubi [A] time = 0.0921652, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$-\frac{b^2e^2-6bcde+6c^2d^2}{4e^5(d+ex)^4}-\frac{d^2(cd-be)^2}{6e^5(d+ex)^6}+\frac{2c(2cd-be)}{3e^5(d+ex)^3}+\frac{2d(cd-be)(2cd-be)}{5e^5(d+ex)^5}-\frac{c^2}{2e^5(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^2/(d + e*x)^7,x]
```

```
[Out] -(d^2*(c*d - b*e)^2)/(6*e^5*(d + e*x)^6) + (2*d*(c*d - b*e)*(2*c*d - b*e))/
(5*e^5*(d + e*x)^5) - (6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)/(4*e^5*(d + e*x)^4)
 + (2*c*(2*c*d - b*e))/(3*e^5*(d + e*x)^3) - c^2/(2*e^5*(d + e*x)^2)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{(bx+cx^2)^2}{(d+ex)^7}\,dx &= \int \left(\frac{d^2(cd-be)^2}{e^4(d+ex)^7}+\frac{2d(cd-be)(-2cd+be)}{e^4(d+ex)^6}+\frac{6c^2d^2-6bcde+b^2e^2}{e^4(d+ex)^5}-\frac{2c(2cd-be)}{e^4(d+ex)^4}+\frac{c^2}{e^4(d+ex)^3}\right)\\ &= -\frac{d^2(cd-be)^2}{6e^5(d+ex)^6}+\frac{2d(cd-be)(2cd-be)}{5e^5(d+ex)^5}-\frac{6c^2d^2-6bcde+b^2e^2}{4e^5(d+ex)^4}+\frac{2c(2cd-be)}{3e^5(d+ex)^3}-\frac{c^2}{2e^5(d+ex)^2}\end{aligned}$$

Mathematica [A] time = 0.040776, size = 116, normalized size = 0.85

$$-\frac{b^2e^2\left(d^2+6dex+15e^2x^2\right)+2bce\left(6d^2ex+d^3+15de^2x^2+20e^3x^3\right)+2c^2\left(15d^2e^2x^2+6d^3ex+d^4+20de^3x^3+15e^4x^4\right.}{60e^5(d+ex)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^2/(d + e*x)^7,x]
```

```
[Out] -(b^2*e^2*(d^2 + 6*d*e*x + 15*e^2*x^2) + 2*b*c*e*(d^3 + 6*d^2*e*x + 15*d*e^
2*x^2 + 20*e^3*x^3) + 2*c^2*(d^4 + 6*d^3*e*x + 15*d^2*e^2*x^2 + 20*d*e^3*x^
```

```
3 + 15*e^4*x^4))/(60*e^5*(d + e*x)^6)
```

Maple [A] time = 0.052, size = 143, normalized size = 1.

$$-\frac{c^2}{2e^5(ex+d)^2}+\frac{2d\left(b^2e^2-3bcde+2c^2d^2\right)}{5e^5(ex+d)^5}-\frac{2c(be-2cd)}{3e^5(ex+d)^3}-\frac{d^2\left(b^2e^2-2bcde+c^2d^2\right)}{6e^5(ex+d)^6}-\frac{b^2e^2-6bcde+6c^2d^2}{4e^5(ex+d)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^2/(e*x+d)^7,x)
```

```
[Out] -1/2*c^2/e^5/(e*x+d)^2+2/5*d*(b^2*e^2-3*b*c*d*e+2*c^2*d^2)/e^5/(e*x+d)^5-2/
3*c*(b*e-2*c*d)/e^5/(e*x+d)^3-1/6*d^2*(b^2*e^2-2*b*c*d*e+c^2*d^2)/e^5/(e*x+
d)^6-1/4*(b^2*e^2-6*b*c*d*e+6*c^2*d^2)/e^5/(e*x+d)^4
```

Maxima [A] time = 1.15776, size = 258, normalized size = 1.88

$$-\frac{30c^2e^4x^4+2c^2d^4+2bcd^3e+b^2d^2e^2+40\left(c^2de^3+bce^4\right)x^3+15\left(2c^2d^2e^2+2bcde^3+b^2e^4\right)x^2+6\left(2c^2d^3e+2bcd^2e^2+\right.}{60\left(e^{11}x^6+6de^{10}x^5+15d^2e^9x^4+20d^3e^8x^3+15d^4e^7x^2+6d^5e^6x+d^6e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(30*c^2*e^4*x^4 + 2*c^2*d^4 + 2*b*c*d^3*e + b^2*d^2*e^2 + 40*(c^2*d*e
^3 + b*c*e^4)*x^3 + 15*(2*c^2*d^2*e^2 + 2*b*c*d*e^3 + b^2*e^4)*x^2 + 6*(2*c
^2*d^3*e + 2*b*c*d^2*e^2 + b^2*d*e^3)*x)/(e^11*x^6 + 6*d*e^10*x^5 + 15*d^2*
e^9*x^4 + 20*d^3*e^8*x^3 + 15*d^4*e^7*x^2 + 6*d^5*e^6*x + d^6*e^5)
```

Fricas [A] time = 1.63062, size = 396, normalized size = 2.89

$$-\frac{30c^2e^4x^4+2c^2d^4+2bcd^3e+b^2d^2e^2+40\left(c^2de^3+bce^4\right)x^3+15\left(2c^2d^2e^2+2bcde^3+b^2e^4\right)x^2+6\left(2c^2d^3e+2bcd^2e^2+\right.}{60\left(e^{11}x^6+6de^{10}x^5+15d^2e^9x^4+20d^3e^8x^3+15d^4e^7x^2+6d^5e^6x+d^6e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^7,x, algorithm="fricas")
```

```
[Out] -1/60*(30*c^2*e^4*x^4 + 2*c^2*d^4 + 2*b*c*d^3*e + b^2*d^2*e^2 + 40*(c^2*d*e
^3 + b*c*e^4)*x^3 + 15*(2*c^2*d^2*e^2 + 2*b*c*d*e^3 + b^2*e^4)*x^2 + 6*(2*c
^2*d^3*e + 2*b*c*d^2*e^2 + b^2*d*e^3)*x)/(e^11*x^6 + 6*d*e^10*x^5 + 15*d^2*
e^9*x^4 + 20*d^3*e^8*x^3 + 15*d^4*e^7*x^2 + 6*d^5*e^6*x + d^6*e^5)
```

Sympy [A] time = 13.8266, size = 204, normalized size = 1.49

$$-\frac{b^2d^2e^2+2bcd^3e+2c^2d^4+30c^2e^4x^4+x^3\left(40bce^4+40c^2de^3\right)+x^2\left(15b^2e^4+30bcde^3+30c^2d^2e^2\right)+x\left(6b^2de^3+12bcd^2e\right.}{60d^6e^5+360d^5e^6x+900d^4e^7x^2+1200d^3e^8x^3+900d^2e^9x^4+360de^{10}x^5+60e^{11}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**2/(e*x+d)**7,x)
```

```
[Out] -(b**2*d**2*e**2 + 2*b*c*d**3*e + 2*c**2*d**4 + 30*c**2*e**4*x**4 + x**3*(4
0*b*c*e**4 + 40*c**2*d*e**3) + x**2*(15*b**2*e**4 + 30*b*c*d*e**3 + 30*c**2
*d**2*e**2) + x*(6*b**2*d*e**3 + 12*b*c*d**2*e**2 + 12*c**2*d**3*e))/(60*d*
*6*e**5 + 360*d**5*e**6*x + 900*d**4*e**7*x**2 + 1200*d**3*e**8*x**3 + 900*
d**2*e**9*x**4 + 360*d*e**10*x**5 + 60*e**11*x**6)
```

Giac [A] time = 1.28522, size = 178, normalized size = 1.3

$$-\frac{(30\,c^2x^4e^4 + 40\,c^2dx^3e^3 + 30\,c^2d^2x^2e^2 + 12\,c^2d^3xe + 2\,c^2d^4 + 40\,bcx^3e^4 + 30\,bcdx^2e^3 + 12\,bcd^2xe^2 + 2\,bcd^3e + 15\,b^2x}{60\,(xe+d)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^7,x, algorithm="giac")
```

```
[Out] -1/60*(30*c^2*x^4*e^4 + 40*c^2*d*x^3*e^3 + 30*c^2*d^2*x^2*e^2 + 12*c^2*d^3*
x*e + 2*c^2*d^4 + 40*b*c*x^3*e^4 + 30*b*c*d*x^2*e^3 + 12*b*c*d^2*x*e^2 + 2*
b*c*d^3*e + 15*b^2*x^2*e^4 + 6*b^2*d*x*e^3 + b^2*d^2*e^2)*e^(-5)/(x*e + d)^
6
```

3.243 $\int \frac{(bx+cx^2)^2}{(d+ex)^8} dx$

Optimal. Leaf size=137

$$-\frac{b^2e^2-6bcde+6c^2d^2}{5e^5(d+ex)^5}-\frac{d^2(cd-be)^2}{7e^5(d+ex)^7}+\frac{c(2cd-be)}{2e^5(d+ex)^4}+\frac{d(cd-be)(2cd-be)}{3e^5(d+ex)^6}-\frac{c^2}{3e^5(d+ex)^3}$$

```
[Out] -(d^2*(c*d - b*e)^2)/(7*e^5*(d + e*x)^7) + (d*(c*d - b*e)*(2*c*d - b*e))/(3
*e^5*(d + e*x)^6) - (6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)/(5*e^5*(d + e*x)^5) +
 (c*(2*c*d - b*e))/(2*e^5*(d + e*x)^4) - c^2/(3*e^5*(d + e*x)^3)
```

Rubi [A] time = 0.0880271, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$-\frac{b^2e^2-6bcde+6c^2d^2}{5e^5(d+ex)^5}-\frac{d^2(cd-be)^2}{7e^5(d+ex)^7}+\frac{c(2cd-be)}{2e^5(d+ex)^4}+\frac{d(cd-be)(2cd-be)}{3e^5(d+ex)^6}-\frac{c^2}{3e^5(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^2/(d + e*x)^8,x]
```

```
[Out] -(d^2*(c*d - b*e)^2)/(7*e^5*(d + e*x)^7) + (d*(c*d - b*e)*(2*c*d - b*e))/(3
*e^5*(d + e*x)^6) - (6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)/(5*e^5*(d + e*x)^5) +
 (c*(2*c*d - b*e))/(2*e^5*(d + e*x)^4) - c^2/(3*e^5*(d + e*x)^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{(bx+cx^2)^2}{(d+ex)^8}dx &= \int \left(\frac{d^2(cd-be)^2}{e^4(d+ex)^8}+\frac{2d(cd-be)(-2cd+be)}{e^4(d+ex)^7}+\frac{6c^2d^2-6bcde+b^2e^2}{e^4(d+ex)^6}-\frac{2c(2cd-be)}{e^4(d+ex)^5}+\frac{c^2}{e^4(d+ex)^4}\right)dx\\ &= -\frac{d^2(cd-be)^2}{7e^5(d+ex)^7}+\frac{d(cd-be)(2cd-be)}{3e^5(d+ex)^6}-\frac{6c^2d^2-6bcde+b^2e^2}{5e^5(d+ex)^5}+\frac{c(2cd-be)}{2e^5(d+ex)^4}-\frac{c^2}{3e^5(d+ex)^3}\end{aligned}$$

Mathematica [A] time = 0.0477294, size = 117, normalized size = 0.85

$$-\frac{2b^2e^2\left(d^2+7dex+21e^2x^2\right)+3bce\left(7d^2ex+d^3+21de^2x^2+35e^3x^3\right)+2c^2\left(21d^2e^2x^2+7d^3ex+d^4+35de^3x^3+35e^4x^4\right)}{210e^5(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^2/(d + e*x)^8,x]
```

```
[Out] -(2*b^2*e^2*(d^2 + 7*d*e*x + 21*e^2*x^2) + 3*b*c*e*(d^3 + 7*d^2*e*x + 21*d*
e^2*x^2 + 35*e^3*x^3) + 2*c^2*(d^4 + 7*d^3*e*x + 21*d^2*e^2*x^2 + 35*d*e^3*
```

```
x^3 + 35*e^4*x^4))/(210*e^5*(d + e*x)^7)
```

Maple [A] time = 0.056, size = 143, normalized size = 1.

$$-\frac{d^2\left(b^2e^2-2\,bcde+c^2d^2\right)}{7\,e^5\left(ex+d\right)^7}-\frac{b^2e^2-6\,bcde+6\,c^2d^2}{5\,e^5\left(ex+d\right)^5}-\frac{c^2}{3\,e^5\left(ex+d\right)^3}+\frac{d\left(b^2e^2-3\,bcde+2\,c^2d^2\right)}{3\,e^5\left(ex+d\right)^6}-\frac{c\left(be-2\,cd\right)}{2\,e^5\left(ex+d\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^2/(e*x+d)^8,x)
```

```
[Out] -1/7*d^2*(b^2*e^2-2*b*c*d*e+c^2*d^2)/e^5/(e*x+d)^7-1/5*(b^2*e^2-6*b*c*d*e+6
*c^2*d^2)/e^5/(e*x+d)^5-1/3*c^2/e^5/(e*x+d)^3+1/3*d*(b^2*e^2-3*b*c*d*e+2*c^
2*d^2)/e^5/(e*x+d)^6-1/2*c*(b*e-2*c*d)/e^5/(e*x+d)^4
```

Maxima [A] time = 1.16599, size = 279, normalized size = 2.04

$$-\frac{70\,c^2e^4x^4+2\,c^2d^4+3\,bcd^3e+2\,b^2d^2e^2+35\left(2\,c^2de^3+3\,bce^4\right)x^3+21\left(2\,c^2d^2e^2+3\,bcde^3+2\,b^2e^4\right)x^2+7\left(2\,c^2d^3e+3\right.}{210\left(e^{12}x^7+7\,de^{11}x^6+21\,d^2e^{10}x^5+35\,d^3e^9x^4+35\,d^4e^8x^3+21\,d^5e^7x^2+7\,d^6e^6x+d^7e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^8,x, algorithm="maxima")
```

```
[Out] -1/210*(70*c^2*e^4*x^4 + 2*c^2*d^4 + 3*b*c*d^3*e + 2*b^2*d^2*e^2 + 35*(2*c^
2*d*e^3 + 3*b*c*e^4)*x^3 + 21*(2*c^2*d^2*e^2 + 3*b*c*d*e^3 + 2*b^2*e^4)*x^2
 + 7*(2*c^2*d^3*e + 3*b*c*d^2*e^2 + 2*b^2*d*e^3)*x)/(e^12*x^7 + 7*d*e^11*x^
6 + 21*d^2*e^10*x^5 + 35*d^3*e^9*x^4 + 35*d^4*e^8*x^3 + 21*d^5*e^7*x^2 + 7*
d^6*e^6*x + d^7*e^5)
```

Fricas [A] time = 1.56862, size = 435, normalized size = 3.18

$$-\frac{70\,c^2e^4x^4+2\,c^2d^4+3\,bcd^3e+2\,b^2d^2e^2+35\left(2\,c^2de^3+3\,bce^4\right)x^3+21\left(2\,c^2d^2e^2+3\,bcde^3+2\,b^2e^4\right)x^2+7\left(2\,c^2d^3e+3\right.}{210\left(e^{12}x^7+7\,de^{11}x^6+21\,d^2e^{10}x^5+35\,d^3e^9x^4+35\,d^4e^8x^3+21\,d^5e^7x^2+7\,d^6e^6x+d^7e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^8,x, algorithm="fricas")
```

```
[Out] -1/210*(70*c^2*e^4*x^4 + 2*c^2*d^4 + 3*b*c*d^3*e + 2*b^2*d^2*e^2 + 35*(2*c^
2*d*e^3 + 3*b*c*e^4)*x^3 + 21*(2*c^2*d^2*e^2 + 3*b*c*d*e^3 + 2*b^2*e^4)*x^2
 + 7*(2*c^2*d^3*e + 3*b*c*d^2*e^2 + 2*b^2*d*e^3)*x)/(e^12*x^7 + 7*d*e^11*x^
6 + 21*d^2*e^10*x^5 + 35*d^3*e^9*x^4 + 35*d^4*e^8*x^3 + 21*d^5*e^7*x^2 + 7*
d^6*e^6*x + d^7*e^5)
```

Sympy [A] time = 22.3192, size = 218, normalized size = 1.59

$$-\frac{2b^2d^2e^2+3bcd^3e+2c^2d^4+70c^2e^4x^4+x^3\left(105bce^4+70c^2de^3\right)+x^2\left(42b^2e^4+63bcde^3+42c^2d^2e^2\right)+x\left(14b^2de^3+2\right.}{210d^7e^5+1470d^6e^6x+4410d^5e^7x^2+7350d^4e^8x^3+7350d^3e^9x^4+4410d^2e^{10}x^5+1470de^{11}x^6+210e^{1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**2/(e*x+d)**8,x)
```

```
[Out] -(2*b**2*d**2*e**2 + 3*b*c*d**3*e + 2*c**2*d**4 + 70*c**2*e**4*x**4 + x**3*
(105*b*c*e**4 + 70*c**2*d*e**3) + x**2*(42*b**2*e**4 + 63*b*c*d*e**3 + 42*c
**2*d**2*e**2) + x*(14*b**2*d*e**3 + 21*b*c*d**2*e**2 + 14*c**2*d**3*e))/(2
10*d**7*e**5 + 1470*d**6*e**6*x + 4410*d**5*e**7*x**2 + 7350*d**4*e**8*x**3
 + 7350*d**3*e**9*x**4 + 4410*d**2*e**10*x**5 + 1470*d*e**11*x**6 + 210*e**
12*x**7)
```

Giac [A] time = 1.25388, size = 180, normalized size = 1.31

$$-\frac{(70\,c^2x^4e^4 + 70\,c^2dx^3e^3 + 42\,c^2d^2x^2e^2 + 14\,c^2d^3xe + 2\,c^2d^4 + 105\,bcx^3e^4 + 63\,bcdx^2e^3 + 21\,bcd^2xe^2 + 3\,bcd^3e + 42\,b^2x^2e}{210\,(xe+d)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^8,x, algorithm="giac")
```

```
[Out] -1/210*(70*c^2*x^4*e^4 + 70*c^2*d*x^3*e^3 + 42*c^2*d^2*x^2*e^2 + 14*c^2*d^3
*x*e + 2*c^2*d^4 + 105*b*c*x^3*e^4 + 63*b*c*d*x^2*e^3 + 21*b*c*d^2*x*e^2 +
3*b*c*d^3*e + 42*b^2*x^2*e^4 + 14*b^2*d*x*e^3 + 2*b^2*d^2*e^2)*e^(-5)/(x*e
+ d)^7
```

3.244 $\int (d+ex)^4 \left(bx+cx^2\right)^3 dx$

Optimal. Leaf size=225

$$\frac{1}{3}ce^2x^9\left(b^2e^2+4bcde+2c^2d^2\right)+\frac{1}{8}ex^8\left(12b^2cde^2+b^3e^3+18bc^2d^2e+4c^3d^3\right)+\frac{1}{7}dx^7\left(18b^2cde^2+4b^3e^3+12bc^2d^2e+c\right.$$

```
[Out] (b^3*d^4*x^4)/4 + (b^2*d^3*(3*c*d + 4*b*e)*x^5)/5 + (b*d^2*(c^2*d^2 + 4*b*c
*d*e + 2*b^2*e^2)*x^6)/2 + (d*(c^3*d^3 + 12*b*c^2*d^2*e + 18*b^2*c*d*e^2 +
4*b^3*e^3)*x^7)/7 + (e*(4*c^3*d^3 + 18*b*c^2*d^2*e + 12*b^2*c*d*e^2 + b^3*e
^3)*x^8)/8 + (c*e^2*(2*c^2*d^2 + 4*b*c*d*e + b^2*e^2)*x^9)/3 + (c^2*e^3*(4*
c*d + 3*b*e)*x^10)/10 + (c^3*e^4*x^11)/11
```

Rubi [A] time = 0.215499, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$\frac{1}{3}ce^2x^9\left(b^2e^2+4bcde+2c^2d^2\right)+\frac{1}{8}ex^8\left(12b^2cde^2+b^3e^3+18bc^2d^2e+4c^3d^3\right)+\frac{1}{7}dx^7\left(18b^2cde^2+4b^3e^3+12bc^2d^2e+c\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4*(b*x + c*x^2)^3,x]
```


```
[Out] (b^3*d^4*x^4)/4 + (b^2*d^3*(3*c*d + 4*b*e)*x^5)/5 + (b*d^2*(c^2*d^2 + 4*b*c
*d*e + 2*b^2*e^2)*x^6)/2 + (d*(c^3*d^3 + 12*b*c^2*d^2*e + 18*b^2*c*d*e^2 +
4*b^3*e^3)*x^7)/7 + (e*(4*c^3*d^3 + 18*b*c^2*d^2*e + 12*b^2*c*d*e^2 + b^3*e
^3)*x^8)/8 + (c*e^2*(2*c^2*d^2 + 4*b*c*d*e + b^2*e^2)*x^9)/3 + (c^2*e^3*(4*
c*d + 3*b*e)*x^10)/10 + (c^3*e^4*x^11)/11
```


Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^4\left(bx+cx^2\right)^3 dx=\int \left(b^3d^4x^3+b^2d^3(3cd+4be)x^4+3bd^2\left(c^2d^2+4bcde+2b^2e^2\right)x^5+d\left(c^3d^3+12bc^2d^2e+\right.\right.$$

$$=\frac{1}{4}b^3d^4x^4+\frac{1}{5}b^2d^3(3cd+4be)x^5+\frac{1}{2}bd^2\left(c^2d^2+4bcde+2b^2e^2\right)x^6+\frac{1}{7}d\left(c^3d^3+12bc^2d^2e\right.$$

Mathematica [A] time = 0.0347559, size = 225, normalized size = 1.

$$\frac{1}{3}ce^2x^9\left(b^2e^2+4bcde+2c^2d^2\right)+\frac{1}{8}ex^8\left(12b^2cde^2+b^3e^3+18bc^2d^2e+4c^3d^3\right)+\frac{1}{7}dx^7\left(18b^2cde^2+4b^3e^3+12bc^2d^2e+c\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4*(b*x + c*x^2)^3,x]
```

```
[Out] (b^3*d^4*x^4)/4 + (b^2*d^3*(3*c*d + 4*b*e)*x^5)/5 + (b*d^2*(c^2*d^2 + 4*b*c
*d*e + 2*b^2*e^2)*x^6)/2 + (d*(c^3*d^3 + 12*b*c^2*d^2*e + 18*b^2*c*d*e^2 +
4*b^3*e^3)*x^7)/7 + (e*(4*c^3*d^3 + 18*b*c^2*d^2*e + 12*b^2*c*d*e^2 + b^3*e
^3)*x^8)/8 + (c*e^2*(2*c^2*d^2 + 4*b*c*d*e + b^2*e^2)*x^9)/3 + (c^2*e^3*(4*
c*d + 3*b*e)*x^10)/10 + (c^3*e^4*x^11)/11
```

Maple [A] time = 0.043, size = 232, normalized size = 1.

$$\frac{c^3e^4x^{11}}{11}+\frac{\left(3\,e^4bc^2+4\,de^3c^3\right)x^{10}}{10}+\frac{\left(3\,e^4b^2c+12\,de^3bc^2+6\,d^2e^2c^3\right)x^9}{9}+\frac{\left(e^4b^3+12\,de^3b^2c+18\,d^2e^2bc^2+4\,d^3ec^3\right)x^8}{8}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(c*x^2+b*x)^3,x)
```

```
[Out] 1/11*c^3*e^4*x^11+1/10*(3*b*c^2*e^4+4*c^3*d*e^3)*x^10+1/9*(3*b^2*c*e^4+12*b
*c^2*d*e^3+6*c^3*d^2*e^2)*x^9+1/8*(b^3*e^4+12*b^2*c*d*e^3+18*b*c^2*d^2*e^2+
4*c^3*d^3*e)*x^8+1/7*(4*b^3*d*e^3+18*b^2*c*d^2*e^2+12*b*c^2*d^3*e+c^3*d^4)*
x^7+1/6*(6*b^3*d^2*e^2+12*b^2*c*d^3*e+3*b*c^2*d^4)*x^6+1/5*(4*b^3*d^3*e+3*b
^2*c*d^4)*x^5+1/4*b^3*d^4*x^4
```

Maxima [A] time = 1.18534, size = 309, normalized size = 1.37

$$\frac{1}{11}\,c^3e^4x^{11}+\frac{1}{4}\,b^3d^4x^4+\frac{1}{10}\left(4\,c^3de^3+3\,bc^2e^4\right)x^{10}+\frac{1}{3}\left(2\,c^3d^2e^2+4\,bc^2de^3+b^2ce^4\right)x^9+\frac{1}{8}\left(4\,c^3d^3e+18\,bc^2d^2e^2+12\,b^2cd\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+b*x)^3,x, algorithm="maxima")
```

```
[Out] 1/11*c^3*e^4*x^11 + 1/4*b^3*d^4*x^4 + 1/10*(4*c^3*d*e^3 + 3*b*c^2*e^4)*x^10
 + 1/3*(2*c^3*d^2*e^2 + 4*b*c^2*d*e^3 + b^2*c*e^4)*x^9 + 1/8*(4*c^3*d^3*e +
 18*b*c^2*d^2*e^2 + 12*b^2*c*d*e^3 + b^3*e^4)*x^8 + 1/7*(c^3*d^4 + 12*b*c^2
*d^3*e + 18*b^2*c*d^2*e^2 + 4*b^3*d*e^3)*x^7 + 1/2*(b*c^2*d^4 + 4*b^2*c*d^3
*e + 2*b^3*d^2*e^2)*x^6 + 1/5*(3*b^2*c*d^4 + 4*b^3*d^3*e)*x^5
```

Fricas [A] time = 1.46311, size = 556, normalized size = 2.47

$$\frac{1}{11}x^{11}e^4c^3+\frac{2}{5}x^{10}e^3dc^3+\frac{3}{10}x^{10}e^4c^2b+\frac{2}{3}x^9e^2d^2c^3+\frac{4}{3}x^9e^3dc^2b+\frac{1}{3}x^9e^4cb^2+\frac{1}{2}x^8ed^3c^3+\frac{9}{4}x^8e^2d^2c^2b+\frac{3}{2}x^8e^3dcb^2+\frac{1}{8}x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+b*x)^3,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*e^4*c^3 + 2/5*x^10*e^3*d*c^3 + 3/10*x^10*e^4*c^2*b + 2/3*x^9*e^2*
d^2*c^3 + 4/3*x^9*e^3*d*c^2*b + 1/3*x^9*e^4*c*b^2 + 1/2*x^8*e*d^3*c^3 + 9/4
*x^8*e^2*d^2*c^2*b + 3/2*x^8*e^3*d*c*b^2 + 1/8*x^8*e^4*b^3 + 1/7*x^7*d^4*c^
3 + 12/7*x^7*e*d^3*c^2*b + 18/7*x^7*e^2*d^2*c*b^2 + 4/7*x^7*e^3*d*b^3 + 1/2
*x^6*d^4*c^2*b + 2*x^6*e*d^3*c*b^2 + x^6*e^2*d^2*b^3 + 3/5*x^5*d^4*c*b^2 +
4/5*x^5*e*d^3*b^3 + 1/4*x^4*d^4*b^3
```

Sympy [A] time = 0.218664, size = 257, normalized size = 1.14

$$\frac{b^3d^4x^4}{4}+\frac{c^3e^4x^{11}}{11}+x^{10}\left(\frac{3bc^2e^4}{10}+\frac{2c^3de^3}{5}\right)+x^9\left(\frac{b^2ce^4}{3}+\frac{4bc^2de^3}{3}+\frac{2c^3d^2e^2}{3}\right)+x^8\left(\frac{b^3e^4}{8}+\frac{3b^2cde^3}{2}+\frac{9bc^2d^2e^2}{4}+\frac{c^3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(c*x**2+b*x)**3,x)
```

```
[Out] b**3*d**4*x**4/4 + c**3*e**4*x**11/11 + x**10*(3*b*c**2*e**4/10 + 2*c**3*d*
e**3/5) + x**9*(b**2*c*e**4/3 + 4*b*c**2*d*e**3/3 + 2*c**3*d**2*e**2/3) + x
**8*(b**3*e**4/8 + 3*b**2*c*d*e**3/2 + 9*b*c**2*d**2*e**2/4 + c**3*d**3*e/2
) + x**7*(4*b**3*d*e**3/7 + 18*b**2*c*d**2*e**2/7 + 12*b*c**2*d**3*e/7 + c*
*3*d**4/7) + x**6*(b**3*d**2*e**2 + 2*b**2*c*d**3*e + b*c**2*d**4/2) + x**5
*(4*b**3*d**3*e/5 + 3*b**2*c*d**4/5)
```

Giac [A] time = 1.33182, size = 327, normalized size = 1.45

$$\frac{1}{11}c^3x^{11}e^4+\frac{2}{5}c^3dx^{10}e^3+\frac{2}{3}c^3d^2x^9e^2+\frac{1}{2}c^3d^3x^8e+\frac{1}{7}c^3d^4x^7+\frac{3}{10}bc^2x^{10}e^4+\frac{4}{3}bc^2dx^9e^3+\frac{9}{4}bc^2d^2x^8e^2+\frac{12}{7}bc^2d^3x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+b*x)^3,x, algorithm="giac")
```

```
[Out] 1/11*c^3*x^11*e^4 + 2/5*c^3*d*x^10*e^3 + 2/3*c^3*d^2*x^9*e^2 + 1/2*c^3*d^3*
x^8*e + 1/7*c^3*d^4*x^7 + 3/10*b*c^2*x^10*e^4 + 4/3*b*c^2*d*x^9*e^3 + 9/4*b
*c^2*d^2*x^8*e^2 + 12/7*b*c^2*d^3*x^7*e + 1/2*b*c^2*d^4*x^6 + 1/3*b^2*c*x^9
*e^4 + 3/2*b^2*c*d*x^8*e^3 + 18/7*b^2*c*d^2*x^7*e^2 + 2*b^2*c*d^3*x^6*e + 3
/5*b^2*c*d^4*x^5 + 1/8*b^3*x^8*e^4 + 4/7*b^3*d*x^7*e^3 + b^3*d^2*x^6*e^2 +
4/5*b^3*d^3*x^5*e + 1/4*b^3*d^4*x^4
```

3.245 $\int (d+ex)^3 \left(bx+cx^2\right)^3 \, dx$

Optimal. Leaf size=162

$$\frac{3}{8}cex^8 \left(b^2e^2+3bcde+c^2d^2\right)+\frac{1}{7}x^7(be+cd)\left(b^2e^2+8bcde+c^2d^2\right)+\frac{1}{2}bdx^6\left(b^2e^2+3bcde+c^2d^2\right)+\frac{3}{5}b^2d^2x^5(be+cd)+\frac{1}{4}$$

```
[Out] (b^3*d^3*x^4)/4 + (3*b^2*d^2*(c*d + b*e)*x^5)/5 + (b*d*(c^2*d^2 + 3*b*c*d*e
 + b^2*e^2)*x^6)/2 + ((c*d + b*e)*(c^2*d^2 + 8*b*c*d*e + b^2*e^2)*x^7)/7 +
(3*c*e*(c^2*d^2 + 3*b*c*d*e + b^2*e^2)*x^8)/8 + (c^2*e^2*(c*d + b*e)*x^9)/3
 + (c^3*e^3*x^10)/10
```

Rubi [A] time = 0.15603, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$\frac{3}{8}cex^8 \left(b^2e^2+3bcde+c^2d^2\right)+\frac{1}{7}x^7(be+cd)\left(b^2e^2+8bcde+c^2d^2\right)+\frac{1}{2}bdx^6\left(b^2e^2+3bcde+c^2d^2\right)+\frac{3}{5}b^2d^2x^5(be+cd)+\frac{1}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(b*x + c*x^2)^3,x]
```

```
[Out] (b^3*d^3*x^4)/4 + (3*b^2*d^2*(c*d + b*e)*x^5)/5 + (b*d*(c^2*d^2 + 3*b*c*d*e
 + b^2*e^2)*x^6)/2 + ((c*d + b*e)*(c^2*d^2 + 8*b*c*d*e + b^2*e^2)*x^7)/7 +
(3*c*e*(c^2*d^2 + 3*b*c*d*e + b^2*e^2)*x^8)/8 + (c^2*e^2*(c*d + b*e)*x^9)/3
 + (c^3*e^3*x^10)/10
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int (d+ex)^3\left(bx+cx^2\right)^3\,dx &= \int \left(b^3d^3x^3+3b^2d^2(cd+be)x^4+3bd\left(c^2d^2+3bcde+b^2e^2\right)x^5+(cd+be)\left(c^2d^2+8bcde+b^2\right.\right.\\ &= \frac{1}{4}b^3d^3x^4+\frac{3}{5}b^2d^2(cd+be)x^5+\frac{1}{2}bd\left(c^2d^2+3bcde+b^2e^2\right)x^6+\frac{1}{7}(cd+be)\left(c^2d^2+8bcde+b\right.\end{aligned}$$

Mathematica [A] time = 0.0257193, size = 169, normalized size = 1.04

$$\frac{3}{8}cex^8\left(b^2e^2+3bcde+c^2d^2\right)+\frac{1}{7}x^7\left(9b^2cde^2+b^3e^3+9bc^2d^2e+c^3d^3\right)+\frac{1}{2}bdx^6\left(b^2e^2+3bcde+c^2d^2\right)+\frac{3}{5}b^2d^2x^5(be+cd)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(b*x + c*x^2)^3,x]
```

```
[Out] (b^3*d^3*x^4)/4 + (3*b^2*d^2*(c*d + b*e)*x^5)/5 + (b*d*(c^2*d^2 + 3*b*c*d*e
 + b^2*e^2)*x^6)/2 + ((c^3*d^3 + 9*b*c^2*d^2*e + 9*b^2*c*d*e^2 + b^3*e^3)*x
```

```
^7)/7 + (3*c*e*(c^2*d^2 + 3*b*c*d*e + b^2*e^2)*x^8)/8 + (c^2*e^2*(c*d + b*e
)*x^9)/3 + (c^3*e^3*x^10)/10
```

Maple [A] time = 0.042, size = 180, normalized size = 1.1

$$\frac{c^3e^3x^{10}}{10}+\frac{\left(3\,e^3bc^2+3\,de^2c^3\right)x^9}{9}+\frac{\left(3\,e^3b^2c+9\,de^2bc^2+3\,d^2ec^3\right)x^8}{8}+\frac{\left(b^3e^3+9\,b^2cde^2+9\,bc^2d^2e+c^3d^3\right)x^7}{7}+\frac{(3\,b^3d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x)^3,x)
```

```
[Out] 1/10*c^3*e^3*x^10+1/9*(3*b*c^2*e^3+3*c^3*d*e^2)*x^9+1/8*(3*b^2*c*e^3+9*b*c^
2*d*e^2+3*c^3*d^2*e)*x^8+1/7*(b^3*e^3+9*b^2*c*d*e^2+9*b*c^2*d^2*e+c^3*d^3)*
x^7+1/6*(3*b^3*d*e^2+9*b^2*c*d^2*e+3*b*c^2*d^3)*x^6+1/5*(3*b^3*d^2*e+3*b^2*
c*d^3)*x^5+1/4*b^3*d^3*x^4
```

Maxima [A] time = 1.14228, size = 231, normalized size = 1.43

$$\frac{1}{10}\,c^3e^3x^{10}+\frac{1}{4}\,b^3d^3x^4+\frac{1}{3}\,\left(c^3de^2+bc^2e^3\right)x^9+\frac{3}{8}\,\left(c^3d^2e+3\,bc^2de^2+b^2ce^3\right)x^8+\frac{1}{7}\,\left(c^3d^3+9\,bc^2d^2e+9\,b^2cde^2+b^3e^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x)^3,x, algorithm="maxima")
```

```
[Out] 1/10*c^3*e^3*x^10 + 1/4*b^3*d^3*x^4 + 1/3*(c^3*d*e^2 + b*c^2*e^3)*x^9 + 3/8
*(c^3*d^2*e + 3*b*c^2*d*e^2 + b^2*c*e^3)*x^8 + 1/7*(c^3*d^3 + 9*b*c^2*d^2*e
 + 9*b^2*c*d*e^2 + b^3*e^3)*x^7 + 1/2*(b*c^2*d^3 + 3*b^2*c*d^2*e + b^3*d*e^
2)*x^6 + 3/5*(b^2*c*d^3 + b^3*d^2*e)*x^5
```

Fricas [A] time = 1.37637, size = 433, normalized size = 2.67

$$\frac{1}{10}x^{10}e^3c^3+\frac{1}{3}x^9e^2dc^3+\frac{1}{3}x^9e^3c^2b+\frac{3}{8}x^8ed^2c^3+\frac{9}{8}x^8e^2dc^2b+\frac{3}{8}x^8e^3cb^2+\frac{1}{7}x^7d^3c^3+\frac{9}{7}x^7ed^2c^2b+\frac{9}{7}x^7e^2dcb^2+\frac{1}{7}x^7e^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x)^3,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*e^3*c^3 + 1/3*x^9*e^2*d*c^3 + 1/3*x^9*e^3*c^2*b + 3/8*x^8*e*d^2*c
^3 + 9/8*x^8*e^2*d*c^2*b + 3/8*x^8*e^3*c*b^2 + 1/7*x^7*d^3*c^3 + 9/7*x^7*e*
d^2*c^2*b + 9/7*x^7*e^2*d*c*b^2 + 1/7*x^7*e^3*b^3 + 1/2*x^6*d^3*c^2*b + 3/2
*x^6*e*d^2*c*b^2 + 1/2*x^6*e^2*d*b^3 + 3/5*x^5*d^3*c*b^2 + 3/5*x^5*e*d^2*b^
3 + 1/4*x^4*d^3*b^3
```

Sympy [A] time = 0.335741, size = 199, normalized size = 1.23

$$\frac{b^3d^3x^4}{4}+\frac{c^3e^3x^{10}}{10}+x^9\left(\frac{bc^2e^3}{3}+\frac{c^3de^2}{3}\right)+x^8\left(\frac{3b^2ce^3}{8}+\frac{9bc^2de^2}{8}+\frac{3c^3d^2e}{8}\right)+x^7\left(\frac{b^3e^3}{7}+\frac{9b^2cde^2}{7}+\frac{9bc^2d^2e}{7}+\frac{c^3d^3}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x)**3,x)
```

```
[Out] b**3*d**3*x**4/4 + c**3*e**3*x**10/10 + x**9*(b*c**2*e**3/3 + c**3*d*e**2/3
) + x**8*(3*b**2*c*e**3/8 + 9*b*c**2*d*e**2/8 + 3*c**3*d**2*e/8) + x**7*(b*
*3*e**3/7 + 9*b**2*c*d*e**2/7 + 9*b*c**2*d**2*e/7 + c**3*d**3/7) + x**6*(b*
*3*d*e**2/2 + 3*b**2*c*d**2*e/2 + b*c**2*d**3/2) + x**5*(3*b**3*d**2*e/5 +
3*b**2*c*d**3/5)
```

Giac [A] time = 1.2647, size = 255, normalized size = 1.57

$$\frac{1}{10}c^3x^{10}e^3+\frac{1}{3}c^3dx^9e^2+\frac{3}{8}c^3d^2x^8e+\frac{1}{7}c^3d^3x^7+\frac{1}{3}bc^2x^9e^3+\frac{9}{8}bc^2dx^8e^2+\frac{9}{7}bc^2d^2x^7e+\frac{1}{2}bc^2d^3x^6+\frac{3}{8}b^2cx^8e^3+\frac{9}{7}b^2cd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x)^3,x, algorithm="giac")
```

```
[Out] 1/10*c^3*x^10*e^3 + 1/3*c^3*d*x^9*e^2 + 3/8*c^3*d^2*x^8*e + 1/7*c^3*d^3*x^7
 + 1/3*b*c^2*x^9*e^3 + 9/8*b*c^2*d*x^8*e^2 + 9/7*b*c^2*d^2*x^7*e + 1/2*b*c^
2*d^3*x^6 + 3/8*b^2*c*x^8*e^3 + 9/7*b^2*c*d*x^7*e^2 + 3/2*b^2*c*d^2*x^6*e +
 3/5*b^2*c*d^3*x^5 + 1/7*b^3*x^7*e^3 + 1/2*b^3*d*x^6*e^2 + 3/5*b^3*d^2*x^5*
e + 1/4*b^3*d^3*x^4
```

3.246 $\int (d+ex)^2 \left(bx+cx^2\right)^3 \, dx$

Optimal. Leaf size=127

$$\frac{1}{7}cx^7\left(3b^2e^2+6bcde+c^2d^2\right)+\frac{1}{6}bx^6\left(b^2e^2+6bcde+3c^2d^2\right)+\frac{1}{5}b^2dx^5(2be+3cd)+\frac{1}{4}b^3d^2x^4+\frac{1}{8}c^2ex^8(3be+2cd)+\frac{1}{9}$$

```
[Out] (b^3*d^2*x^4)/4 + (b^2*d*(3*c*d + 2*b*e)*x^5)/5 + (b*(3*c^2*d^2 + 6*b*c*d*e
 + b^2*e^2)*x^6)/6 + (c*(c^2*d^2 + 6*b*c*d*e + 3*b^2*e^2)*x^7)/7 + (c^2*e*(
2*c*d + 3*b*e)*x^8)/8 + (c^3*e^2*x^9)/9
```

Rubi [A] time = 0.109911, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$\frac{1}{7}cx^7\left(3b^2e^2+6bcde+c^2d^2\right)+\frac{1}{6}bx^6\left(b^2e^2+6bcde+3c^2d^2\right)+\frac{1}{5}b^2dx^5(2be+3cd)+\frac{1}{4}b^3d^2x^4+\frac{1}{8}c^2ex^8(3be+2cd)+\frac{1}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(b*x + c*x^2)^3,x]
```

```
[Out] (b^3*d^2*x^4)/4 + (b^2*d*(3*c*d + 2*b*e)*x^5)/5 + (b*(3*c^2*d^2 + 6*b*c*d*e
 + b^2*e^2)*x^6)/6 + (c*(c^2*d^2 + 6*b*c*d*e + 3*b^2*e^2)*x^7)/7 + (c^2*e*(
2*c*d + 3*b*e)*x^8)/8 + (c^3*e^2*x^9)/9
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^2\left(bx+cx^2\right)^3\,dx=\int \left(b^3d^2x^3+b^2d(3cd+2be)x^4+b\left(3c^2d^2+6bcde+b^2e^2\right)x^5+c\left(c^2d^2+6bcde+3b^2e^2\right)$$
$$=\frac{1}{4}b^3d^2x^4+\frac{1}{5}b^2d(3cd+2be)x^5+\frac{1}{6}b\left(3c^2d^2+6bcde+b^2e^2\right)x^6+\frac{1}{7}c\left(c^2d^2+6bcde+3b^2e\right.$$

Mathematica [A] time = 0.0239765, size = 127, normalized size = 1.

$$\frac{1}{7}cx^7\left(3b^2e^2+6bcde+c^2d^2\right)+\frac{1}{6}bx^6\left(b^2e^2+6bcde+3c^2d^2\right)+\frac{1}{5}b^2dx^5(2be+3cd)+\frac{1}{4}b^3d^2x^4+\frac{1}{8}c^2ex^8(3be+2cd)+\frac{1}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(b*x + c*x^2)^3,x]
```

```
[Out] (b^3*d^2*x^4)/4 + (b^2*d*(3*c*d + 2*b*e)*x^5)/5 + (b*(3*c^2*d^2 + 6*b*c*d*e
 + b^2*e^2)*x^6)/6 + (c*(c^2*d^2 + 6*b*c*d*e + 3*b^2*e^2)*x^7)/7 + (c^2*e*(
2*c*d + 3*b*e)*x^8)/8 + (c^3*e^2*x^9)/9
```

Maple [A] time = 0.043, size = 128, normalized size = 1.

$$\frac{c^3e^2x^9}{9}+\frac{\left(3\,e^2bc^2+2\,dec^3\right)x^8}{8}+\frac{\left(3\,e^2b^2c+6\,debc^2+d^2c^3\right)x^7}{7}+\frac{\left(b^3e^2+6\,b^2cde+3\,d^2bc^2\right)x^6}{6}+\frac{\left(2\,deb^3+3\,d^2b^2c\right)x^5}{5}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+b*x)^3,x)
```

```
[Out] 1/9*c^3*e^2*x^9+1/8*(3*b*c^2*e^2+2*c^3*d*e)*x^8+1/7*(3*b^2*c*e^2+6*b*c^2*d*
e+c^3*d^2)*x^7+1/6*(b^3*e^2+6*b^2*c*d*e+3*b*c^2*d^2)*x^6+1/5*(2*b^3*d*e+3*b
^2*c*d^2)*x^5+1/4*b^3*d^2*x^4
```

Maxima [A] time = 1.13131, size = 171, normalized size = 1.35

$$\frac{1}{9}\,c^3e^2x^9+\frac{1}{4}\,b^3d^2x^4+\frac{1}{8}\left(2\,c^3de+3\,bc^2e^2\right)x^8+\frac{1}{7}\left(c^3d^2+6\,bc^2de+3\,b^2ce^2\right)x^7+\frac{1}{6}\left(3\,bc^2d^2+6\,b^2cde+b^3e^2\right)x^6+\frac{1}{5}\left(3\,b^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x)^3,x, algorithm="maxima")
```

```
[Out] 1/9*c^3*e^2*x^9 + 1/4*b^3*d^2*x^4 + 1/8*(2*c^3*d*e + 3*b*c^2*e^2)*x^8 + 1/7
*(c^3*d^2 + 6*b*c^2*d*e + 3*b^2*c*e^2)*x^7 + 1/6*(3*b*c^2*d^2 + 6*b^2*c*d*e
 + b^3*e^2)*x^6 + 1/5*(3*b^2*c*d^2 + 2*b^3*d*e)*x^5
```

Fricas [A] time = 1.42058, size = 301, normalized size = 2.37

$$\frac{1}{9}x^9e^2c^3+\frac{1}{4}x^8edc^3+\frac{3}{8}x^8e^2c^2b+\frac{1}{7}x^7d^2c^3+\frac{6}{7}x^7edc^2b+\frac{3}{7}x^7e^2cb^2+\frac{1}{2}x^6d^2c^2b+x^6edcb^2+\frac{1}{6}x^6e^2b^3+\frac{3}{5}x^5d^2cb^2+\frac{2}{5}x^5ed$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x)^3,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*e^2*c^3 + 1/4*x^8*e*d*c^3 + 3/8*x^8*e^2*c^2*b + 1/7*x^7*d^2*c^3 + 6
/7*x^7*e*d*c^2*b + 3/7*x^7*e^2*c*b^2 + 1/2*x^6*d^2*c^2*b + x^6*e*d*c*b^2 +
1/6*x^6*e^2*b^3 + 3/5*x^5*d^2*c*b^2 + 2/5*x^5*e*d*b^3 + 1/4*x^4*d^2*b^3
```

Sympy [A] time = 0.237614, size = 138, normalized size = 1.09

$$\frac{b^3d^2x^4}{4}+\frac{c^3e^2x^9}{9}+x^8\left(\frac{3bc^2e^2}{8}+\frac{c^3de}{4}\right)+x^7\left(\frac{3b^2ce^2}{7}+\frac{6bc^2de}{7}+\frac{c^3d^2}{7}\right)+x^6\left(\frac{b^3e^2}{6}+b^2cde+\frac{bc^2d^2}{2}\right)+x^5\left(\frac{2b^3de}{5}+\frac{3b^2c}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+b*x)**3,x)
```

```
[Out] b**3*d**2*x**4/4 + c**3*e**2*x**9/9 + x**8*(3*b*c**2*e**2/8 + c**3*d*e/4) +
 x**7*(3*b**2*c*e**2/7 + 6*b*c**2*d*e/7 + c**3*d**2/7) + x**6*(b**3*e**2/6
```

```
+ b**2*c*d*e + b*c**2*d**2/2) + x**5*(2*b**3*d*e/5 + 3*b**2*c*d**2/5)
```

Giac [A] time = 1.31292, size = 181, normalized size = 1.43

$$\frac{1}{9}c^3x^9e^2+\frac{1}{4}c^3dx^8e+\frac{1}{7}c^3d^2x^7+\frac{3}{8}bc^2x^8e^2+\frac{6}{7}bc^2dx^7e+\frac{1}{2}bc^2d^2x^6+\frac{3}{7}b^2cx^7e^2+b^2cdx^6e+\frac{3}{5}b^2cd^2x^5+\frac{1}{6}b^3x^6e^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x)^3,x, algorithm="giac")
```

```
[Out] 1/9*c^3*x^9*e^2 + 1/4*c^3*d*x^8*e + 1/7*c^3*d^2*x^7 + 3/8*b*c^2*x^8*e^2 + 6
/7*b*c^2*d*x^7*e + 1/2*b*c^2*d^2*x^6 + 3/7*b^2*c*x^7*e^2 + b^2*c*d*x^6*e +
3/5*b^2*c*d^2*x^5 + 1/6*b^3*x^6*e^2 + 2/5*b^3*d*x^5*e + 1/4*b^3*d^2*x^4
```

3.247 $\int (d+ex)\left(bx+cx^2\right)^3\,dx$

Optimal. Leaf size=75

$$\frac{1}{5}b^2x^5(be+3cd)+\frac{1}{4}b^3dx^4+\frac{1}{7}c^2x^7(3be+cd)+\frac{1}{2}bcx^6(be+cd)+\frac{1}{8}c^3ex^8$$

```
[Out] (b^3*d*x^4)/4 + (b^2*(3*c*d + b*e)*x^5)/5 + (b*c*(c*d + b*e)*x^6)/2 + (c^2*
(c*d + 3*b*e)*x^7)/7 + (c^3*e*x^8)/8
```

Rubi [A] time = 0.062641, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {631}

$$\frac{1}{5}b^2x^5(be+3cd)+\frac{1}{4}b^3dx^4+\frac{1}{7}c^2x^7(3be+cd)+\frac{1}{2}bcx^6(be+cd)+\frac{1}{8}c^3ex^8$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(b*x + c*x^2)^3,x]
```

```
[Out] (b^3*d*x^4)/4 + (b^2*(3*c*d + b*e)*x^5)/5 + (b*c*(c*d + b*e)*x^6)/2 + (c^2*
(c*d + 3*b*e)*x^7)/7 + (c^3*e*x^8)/8
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(bx+cx^2\right)^3\,dx &= \int \left(b^3dx^3+b^2(3cd+be)x^4+3bc(cd+be)x^5+c^2(cd+3be)x^6+c^3ex^7\right)dx\\ &= \frac{1}{4}b^3dx^4+\frac{1}{5}b^2(3cd+be)x^5+\frac{1}{2}bc(cd+be)x^6+\frac{1}{7}c^2(cd+3be)x^7+\frac{1}{8}c^3ex^8\end{aligned}$$

Mathematica [A] time = 0.0102053, size = 75, normalized size = 1.

$$\frac{1}{5}b^2x^5(be+3cd)+\frac{1}{4}b^3dx^4+\frac{1}{7}c^2x^7(3be+cd)+\frac{1}{2}bcx^6(be+cd)+\frac{1}{8}c^3ex^8$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(b*x + c*x^2)^3,x]
```

```
[Out] (b^3*d*x^4)/4 + (b^2*(3*c*d + b*e)*x^5)/5 + (b*c*(c*d + b*e)*x^6)/2 + (c^2*
(c*d + 3*b*e)*x^7)/7 + (c^3*e*x^8)/8
```

Maple [A] time = 0.044, size = 76, normalized size = 1.

$$\frac{c^3ex^8}{8}+\frac{\left(3\,ebc^2+dc^3\right)x^7}{7}+\frac{\left(3\,eb^2c+3\,dbc^2\right)x^6}{6}+\frac{\left(eb^3+3\,db^2c\right)x^5}{5}+\frac{b^3dx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+b*x)^3,x)
```

```
[Out] 1/8*c^3*e*x^8+1/7*(3*b*c^2*e+c^3*d)*x^7+1/6*(3*b^2*c*e+3*b*c^2*d)*x^6+1/5*(
b^3*e+3*b^2*c*d)*x^5+1/4*b^3*d*x^4
```

Maxima [A] time = 1.11304, size = 99, normalized size = 1.32

$$\frac{1}{8}c^3ex^8 + \frac{1}{4}b^3dx^4 + \frac{1}{7}\left(c^3d + 3\,bc^2e\right)x^7 + \frac{1}{2}\left(bc^2d + b^2ce\right)x^6 + \frac{1}{5}\left(3\,b^2cd + b^3e\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x)^3,x, algorithm="maxima")
```

```
[Out] 1/8*c^3*e*x^8 + 1/4*b^3*d*x^4 + 1/7*(c^3*d + 3*b*c^2*e)*x^7 + 1/2*(b*c^2*d
+ b^2*c*e)*x^6 + 1/5*(3*b^2*c*d + b^3*e)*x^5
```

Fricas [A] time = 1.35286, size = 182, normalized size = 2.43

$$\frac{1}{8}x^8ec^3 + \frac{1}{7}x^7dc^3 + \frac{3}{7}x^7ec^2b + \frac{1}{2}x^6dc^2b + \frac{1}{2}x^6ecb^2 + \frac{3}{5}x^5dcb^2 + \frac{1}{5}x^5eb^3 + \frac{1}{4}x^4db^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x)^3,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*e*c^3 + 1/7*x^7*d*c^3 + 3/7*x^7*e*c^2*b + 1/2*x^6*d*c^2*b + 1/2*x^6
*e*c*b^2 + 3/5*x^5*d*c*b^2 + 1/5*x^5*e*b^3 + 1/4*x^4*d*b^3
```

Sympy [A] time = 0.22159, size = 80, normalized size = 1.07

$$\frac{b^3dx^4}{4} + \frac{c^3ex^8}{8} + x^7\left(\frac{3bc^2e}{7} + \frac{c^3d}{7}\right) + x^6\left(\frac{b^2ce}{2} + \frac{bc^2d}{2}\right) + x^5\left(\frac{b^3e}{5} + \frac{3b^2cd}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+b*x)**3,x)
```

```
[Out] b**3*d*x**4/4 + c**3*e*x**8/8 + x**7*(3*b*c**2*e/7 + c**3*d/7) + x**6*(b**2
*c*e/2 + b*c**2*d/2) + x**5*(b**3*e/5 + 3*b**2*c*d/5)
```

Giac [A] time = 1.22847, size = 109, normalized size = 1.45

$$\frac{1}{8}c^3x^8e + \frac{1}{7}c^3dx^7 + \frac{3}{7}bc^2x^7e + \frac{1}{2}bc^2dx^6 + \frac{1}{2}b^2cx^6e + \frac{3}{5}b^2cdx^5 + \frac{1}{5}b^3x^5e + \frac{1}{4}b^3dx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x)^3,x, algorithm="giac")
```

```
[Out] 1/8*c^3*x^8*e + 1/7*c^3*d*x^7 + 3/7*b*c^2*x^7*e + 1/2*b*c^2*d*x^6 + 1/2*b^2
*c*x^6*e + 3/5*b^2*c*d*x^5 + 1/5*b^3*x^5*e + 1/4*b^3*d*x^4
```

3.248 $\int \left(bx+cx^2\right)^3 \, dx$

Optimal. Leaf size=43

$$\frac{3}{5}b^2cx^5+\frac{b^3x^4}{4}+\frac{1}{2}bc^2x^6+\frac{c^3x^7}{7}$$

```
[Out] (b^3*x^4)/4 + (3*b^2*c*x^5)/5 + (b*c^2*x^6)/2 + (c^3*x^7)/7
```

Rubi [A] time = 0.0146033, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {611}

$$\frac{3}{5}b^2cx^5+\frac{b^3x^4}{4}+\frac{1}{2}bc^2x^6+\frac{c^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^3,x]
```

```
[Out] (b^3*x^4)/4 + (3*b^2*c*x^5)/5 + (b*c^2*x^6)/2 + (c^3*x^7)/7
```

Rule 611

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && IGtQ[p, 0] && (EqQ[a, 0] ||  !PerfectSquareQ[b^2 - 4*a*c])
```

Rubi steps

$$\begin{aligned}\int \left(bx+cx^2\right)^3 \, dx &= \int \left(b^3x^3+3b^2cx^4+3bc^2x^5+c^3x^6\right) dx\\ &= \frac{b^3x^4}{4}+\frac{3}{5}b^2cx^5+\frac{1}{2}bc^2x^6+\frac{c^3x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.0020258, size = 43, normalized size = 1.

$$\frac{3}{5}b^2cx^5+\frac{b^3x^4}{4}+\frac{1}{2}bc^2x^6+\frac{c^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^3,x]
```

```
[Out] (b^3*x^4)/4 + (3*b^2*c*x^5)/5 + (b*c^2*x^6)/2 + (c^3*x^7)/7
```

Maple [A] time = 0.041, size = 36, normalized size = 0.8

$$\frac{b^3x^4}{4}+\frac{3\,b^2cx^5}{5}+\frac{bc^2x^6}{2}+\frac{c^3x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^3,x)
```

```
[Out] 1/4*b^3*x^4+3/5*b^2*c*x^5+1/2*b*c^2*x^6+1/7*c^3*x^7
```

Maxima [A] time = 1.12471, size = 47, normalized size = 1.09

$$\frac{1}{7}c^3x^7 + \frac{1}{2}bc^2x^6 + \frac{3}{5}b^2cx^5 + \frac{1}{4}b^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3,x, algorithm="maxima")
```

```
[Out] 1/7*c^3*x^7 + 1/2*b*c^2*x^6 + 3/5*b^2*c*x^5 + 1/4*b^3*x^4
```

Fricas [A] time = 1.33609, size = 80, normalized size = 1.86

$$\frac{1}{7}x^7c^3 + \frac{1}{2}x^6c^2b + \frac{3}{5}x^5cb^2 + \frac{1}{4}x^4b^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*c^3 + 1/2*x^6*c^2*b + 3/5*x^5*c*b^2 + 1/4*x^4*b^3
```

Sympy [A] time = 0.153519, size = 37, normalized size = 0.86

$$\frac{b^3x^4}{4} + \frac{3b^2cx^5}{5} + \frac{bc^2x^6}{2} + \frac{c^3x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**3,x)
```

```
[Out] b**3*x**4/4 + 3*b**2*c*x**5/5 + b*c**2*x**6/2 + c**3*x**7/7
```

Giac [A] time = 1.28007, size = 47, normalized size = 1.09

$$\frac{1}{7}c^3x^7 + \frac{1}{2}bc^2x^6 + \frac{3}{5}b^2cx^5 + \frac{1}{4}b^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3,x, algorithm="giac")
```

```
[Out] 1/7*c^3*x^7 + 1/2*b*c^2*x^6 + 3/5*b^2*c*x^5 + 1/4*b^3*x^4
```

3.249 $\int \frac{(bx+cx^2)^3}{d+ex}\,dx$

Optimal. Leaf size=151

$$\frac{cx^4(3b^2e^2-3bcde+c^2d^2)}{4e^3}-\frac{c^2x^5(cd-3be)}{5e^2}-\frac{d^2x(cd-be)^3}{e^6}+\frac{d^3(cd-be)^3\log(d+ex)}{e^7}-\frac{x^3(cd-be)^3}{3e^4}+\frac{dx^2(cd-be)}{2e^5}$$

```
[Out] -((d^2*(c*d - b*e)^3*x)/e^6) + (d*(c*d - b*e)^3*x^2)/(2*e^5) - ((c*d - b*e)
^3*x^3)/(3*e^4) + (c*(c^2*d^2 - 3*b*c*d*e + 3*b^2*e^2)*x^4)/(4*e^3) - (c^2*
(c*d - 3*b*e)*x^5)/(5*e^2) + (c^3*x^6)/(6*e) + (d^3*(c*d - b*e)^3*Log[d + e
*x])/e^7
```

Rubi [A] time = 0.151383, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$\frac{cx^4(3b^2e^2-3bcde+c^2d^2)}{4e^3}-\frac{c^2x^5(cd-3be)}{5e^2}-\frac{d^2x(cd-be)^3}{e^6}+\frac{d^3(cd-be)^3\log(d+ex)}{e^7}-\frac{x^3(cd-be)^3}{3e^4}+\frac{dx^2(cd-be)}{2e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^3/(d + e*x),x]
```


```
[Out] -((d^2*(c*d - b*e)^3*x)/e^6) + (d*(c*d - b*e)^3*x^2)/(2*e^5) - ((c*d - b*e)
^3*x^3)/(3*e^4) + (c*(c^2*d^2 - 3*b*c*d*e + 3*b^2*e^2)*x^4)/(4*e^3) - (c^2*
(c*d - 3*b*e)*x^5)/(5*e^2) + (c^3*x^6)/(6*e) + (d^3*(c*d - b*e)^3*Log[d + e
*x])/e^7
```


Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(bx+cx^2)^3}{d+ex}\,dx=\int \left(-\frac{d^2(cd-be)^3}{e^6}+\frac{d(cd-be)^3x}{e^5}+\frac{(-cd+be)^3x^2}{e^4}+\frac{c(c^2d^2-3bcde+3b^2e^2)x^3}{e^3}-\frac{c^2(cd-3be)x^4}{e^2}\right.$$

$$=-\frac{d^2(cd-be)^3x}{e^6}+\frac{d(cd-be)^3x^2}{2e^5}-\frac{(cd-be)^3x^3}{3e^4}+\frac{c(c^2d^2-3bcde+3b^2e^2)x^4}{4e^3}-\frac{c^2(cd-3be)x^5}{5e^2}+$$

Mathematica [A] time = 0.0830686, size = 144, normalized size = 0.95

$$\frac{15ce^4x^4(3b^2e^2-3bcde+c^2d^2)-12c^2e^5x^5(cd-3be)-60d^2ex(cd-be)^3+60d^3(cd-be)^3\log(d+ex)+20e^3x^3(be-cd}{60e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^3/(d + e*x),x]
```

```
[Out] (-60*d^2*e*(c*d - b*e)^3*x + 30*d*e^2*(c*d - b*e)^3*x^2 + 20*e^3*(-(c*d) +
b*e)^3*x^3 + 15*c*e^4*(c^2*d^2 - 3*b*c*d*e + 3*b^2*e^2)*x^4 - 12*c^2*e^5*(c
*d - 3*b*e)*x^5 + 10*c^3*e^6*x^6 + 60*d^3*(c*d - b*e)^3*Log[d + e*x])/(60*e
^7)
```

Maple [B] time = 0.046, size = 302, normalized size = 2.

$$\frac{c^3x^6}{6e}+\frac{3bx^5c^2}{5e}-\frac{c^3dx^5}{5e^2}+\frac{3x^4b^2c}{4e}-\frac{3bx^4c^2d}{4e^2}+\frac{x^4c^3d^2}{4e^3}+\frac{x^3b^3}{3e}-\frac{b^2cx^3d}{e^2}+\frac{bx^3c^2d^2}{e^3}-\frac{x^3c^3d^3}{3e^4}-\frac{x^2b^3d}{2e^2}+\frac{3b^2x^2cd^2}{2e^3}-3b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^3/(e*x+d),x)
```

```
[Out] 1/6*c^3*x^6/e+3/5/e*x^5*b*c^2-1/5*c^3*d*x^5/e^2+3/4/e*x^4*b^2*c-3/4/e^2*x^4
*b*c^2*d+1/4/e^3*x^4*c^3*d^2+1/3/e*x^3*b^3-1/e^2*x^3*b^2*c*d+1/e^3*x^3*b*c^
2*d^2-1/3/e^4*x^3*c^3*d^3-1/2/e^2*x^2*b^3*d+3/2/e^3*x^2*b^2*c*d^2-3/2/e^4*x
^2*b*c^2*d^3+1/2/e^5*x^2*c^3*d^4+1/e^3*b^3*d^2*x-3/e^4*b^2*c*d^3*x+3/e^5*b*
c^2*d^4*x-1/e^6*c^3*d^5*x-d^3/e^4*ln(e*x+d)*b^3+3*d^4/e^5*ln(e*x+d)*b^2*c-3
*d^5/e^6*ln(e*x+d)*b*c^2+d^6/e^7*ln(e*x+d)*c^3
```

Maxima [A] time = 1.12637, size = 356, normalized size = 2.36

$$\frac{10\,c^3e^5x^6-12\left(c^3de^4-3\,bc^2e^5\right)x^5+15\left(c^3d^2e^3-3\,bc^2de^4+3\,b^2ce^5\right)x^4-20\left(c^3d^3e^2-3\,bc^2d^2e^3+3\,b^2cde^4-b^3e^5\right)x^3+30}{60\,e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/60*(10*c^3*e^5*x^6 - 12*(c^3*d*e^4 - 3*b*c^2*e^5)*x^5 + 15*(c^3*d^2*e^3 -
 3*b*c^2*d*e^4 + 3*b^2*c*e^5)*x^4 - 20*(c^3*d^3*e^2 - 3*b*c^2*d^2*e^3 + 3*b
^2*c*d*e^4 - b^3*e^5)*x^3 + 30*(c^3*d^4*e - 3*b*c^2*d^3*e^2 + 3*b^2*c*d^2*e
^3 - b^3*d*e^4)*x^2 - 60*(c^3*d^5 - 3*b*c^2*d^4*e + 3*b^2*c*d^3*e^2 - b^3*d
^2*e^3)*x)/e^6 + (c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 - b^3*d^3*e^3)*
log(e*x + d)/e^7
```

Fricas [A] time = 1.66888, size = 537, normalized size = 3.56

$$10\,c^3e^6x^6-12\left(c^3de^5-3\,bc^2e^6\right)x^5+15\left(c^3d^2e^4-3\,bc^2de^5+3\,b^2ce^6\right)x^4-20\left(c^3d^3e^3-3\,bc^2d^2e^4+3\,b^2cde^5-b^3e^6\right)x^3+30$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/60*(10*c^3*e^6*x^6 - 12*(c^3*d*e^5 - 3*b*c^2*e^6)*x^5 + 15*(c^3*d^2*e^4 -
 3*b*c^2*d*e^5 + 3*b^2*c*e^6)*x^4 - 20*(c^3*d^3*e^3 - 3*b*c^2*d^2*e^4 + 3*b
^2*c*d*e^5 - b^3*e^6)*x^3 + 30*(c^3*d^4*e^2 - 3*b*c^2*d^3*e^3 + 3*b^2*c*d^2
*e^4 - b^3*d*e^5)*x^2 - 60*(c^3*d^5*e - 3*b*c^2*d^4*e^2 + 3*b^2*c*d^3*e^3 -
 b^3*d^2*e^4)*x + 60*(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 - b^3*d^3*e
^3)*log(e*x + d))/e^7
```

Sympy [A] time = 1.73305, size = 231, normalized size = 1.53

$$\frac{c^3x^6}{6e}-\frac{d^3\,(be-cd)^3\log{(d+ex)}}{e^7}+\frac{x^5\left(3bc^2e-c^3d\right)}{5e^2}+\frac{x^4\left(3b^2ce^2-3bc^2de+c^3d^2\right)}{4e^3}+\frac{x^3\left(b^3e^3-3b^2cde^2+3bc^2d^2e-c\right.}{3e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**3/(e*x+d),x)
```

```
[Out] c**3*x**6/(6*e) - d**3*(b*e - c*d)**3*log(d + e*x)/e**7 + x**5*(3*b*c**2*e
- c**3*d)/(5*e**2) + x**4*(3*b**2*c*e**2 - 3*b*c**2*d*e + c**3*d**2)/(4*e**
3) + x**3*(b**3*e**3 - 3*b**2*c*d*e**2 + 3*b*c**2*d**2*e - c**3*d**3)/(3*e*
*4) - x**2*(b**3*d*e**3 - 3*b**2*c*d**2*e**2 + 3*b*c**2*d**3*e - c**3*d**4)
/(2*e**5) + x*(b**3*d**2*e**3 - 3*b**2*c*d**3*e**2 + 3*b*c**2*d**4*e - c**3
*d**5)/e**6
```

Giac [A] time = 1.30434, size = 365, normalized size = 2.42

$$\left(c^3d^6-3\,bc^2d^5e+3\,b^2cd^4e^2-b^3d^3e^3\right)e^{(-7)}\log\left(|xe+d|\right)+\frac{1}{60}\left(10\,c^3x^6e^5-12\,c^3dx^5e^4+15\,c^3d^2x^4e^3-20\,c^3d^3x^3e^2+30\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d),x, algorithm="giac")
```

```
[Out] (c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 - b^3*d^3*e^3)*e^(-7)*log(abs(x*
e + d)) + 1/60*(10*c^3*x^6*e^5 - 12*c^3*d*x^5*e^4 + 15*c^3*d^2*x^4*e^3 - 20
*c^3*d^3*x^3*e^2 + 30*c^3*d^4*x^2*e - 60*c^3*d^5*x + 36*b*c^2*x^5*e^5 - 45*
b*c^2*d*x^4*e^4 + 60*b*c^2*d^2*x^3*e^3 - 90*b*c^2*d^3*x^2*e^2 + 180*b*c^2*d
^4*x*e + 45*b^2*c*x^4*e^5 - 60*b^2*c*d*x^3*e^4 + 90*b^2*c*d^2*x^2*e^3 - 180
*b^2*c*d^3*x*e^2 + 20*b^3*x^3*e^5 - 30*b^3*d*x^2*e^4 + 60*b^3*d^2*x*e^3)*e^
(-6)
```

3.250 $\int \frac{(bx+cx^2)^3}{(d+ex)^2}\,dx$

Optimal. Leaf size=166

$$-\frac{c^2x^4(2cd-3be)}{4e^3}-\frac{d^3(cd-be)^3}{e^7(d+ex)}-\frac{3d^2(cd-be)^2(2cd-be)\log(d+ex)}{e^7}+\frac{cx^3(cd-be)^2}{e^4}-\frac{x^2(cd-be)^2(4cd-be)}{2e^5}+\frac{dx(5cd}{}$$

```
[Out] (d*(5*c*d - 2*b*e)*(c*d - b*e)^2*x)/e^6 - ((c*d - b*e)^2*(4*c*d - b*e)*x^2)
/(2*e^5) + (c*(c*d - b*e)^2*x^3)/e^4 - (c^2*(2*c*d - 3*b*e)*x^4)/(4*e^3) +
(c^3*x^5)/(5*e^2) - (d^3*(c*d - b*e)^3)/(e^7*(d + e*x)) - (3*d^2*(c*d - b*e
)^2*(2*c*d - b*e)*Log[d + e*x])/e^7
```

Rubi [A] time = 0.186282, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$-\frac{c^2x^4(2cd-3be)}{4e^3}-\frac{d^3(cd-be)^3}{e^7(d+ex)}-\frac{3d^2(cd-be)^2(2cd-be)\log(d+ex)}{e^7}+\frac{cx^3(cd-be)^2}{e^4}-\frac{x^2(cd-be)^2(4cd-be)}{2e^5}+\frac{dx(5cd}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^3/(d + e*x)^2,x]
```

```
[Out] (d*(5*c*d - 2*b*e)*(c*d - b*e)^2*x)/e^6 - ((c*d - b*e)^2*(4*c*d - b*e)*x^2)
/(2*e^5) + (c*(c*d - b*e)^2*x^3)/e^4 - (c^2*(2*c*d - 3*b*e)*x^4)/(4*e^3) +
(c^3*x^5)/(5*e^2) - (d^3*(c*d - b*e)^3)/(e^7*(d + e*x)) - (3*d^2*(c*d - b*e
)^2*(2*c*d - b*e)*Log[d + e*x])/e^7
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(bx+cx^2)^3}{(d+ex)^2}\,dx=\int \left(\frac{d(5cd-2be)(cd-be)^2}{e^6}+\frac{(-4cd+be)(-cd+be)^2x}{e^5}+\frac{3c(cd-be)^2x^2}{e^4}-\frac{c^2(2cd-3be)x^3}{e^3}+\frac{c^3x^4}{e^2}+\right.$$
$$=\frac{d(5cd-2be)(cd-be)^2x}{e^6}-\frac{(cd-be)^2(4cd-be)x^2}{2e^5}+\frac{c(cd-be)^2x^3}{e^4}-\frac{c^2(2cd-3be)x^4}{4e^3}+\frac{c^3x^5}{5e^2}-\frac{d^3(cd}{e^7(d}$$

Mathematica [A] time = 0.0618319, size = 160, normalized size = 0.96

$$\frac{-5c^2e^4x^4(2cd-3be)-\frac{20d^3(cd-be)^3}{d+ex}-60d^2(cd-be)^2(2cd-be)\log(d+ex)+20ce^3x^3(cd-be)^2+10e^2x^2(cd-be)^2(be-4cd)}{20e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^3/(d + e*x)^2,x]
```

```
[Out] (20*d*e*(5*c*d - 2*b*e)*(c*d - b*e)^2*x + 10*e^2*(c*d - b*e)^2*(-4*c*d + b*
e)*x^2 + 20*c*e^3*(c*d - b*e)^2*x^3 - 5*c^2*e^4*(2*c*d - 3*b*e)*x^4 + 4*c^3
*e^5*x^5 - (20*d^3*(c*d - b*e)^3)/(d + e*x) - 60*d^2*(c*d - b*e)^2*(2*c*d -
 b*e)*Log[d + e*x])/(20*e^7)
```

Maple [A] time = 0.056, size = 318, normalized size = 1.9

$$\frac{c^3x^5}{5\,e^2}+\frac{3\,bx^4c^2}{4\,e^2}-\frac{c^3dx^4}{2\,e^3}+\frac{b^2cx^3}{e^2}-2\,\frac{bx^3c^2d}{e^3}+\frac{x^3c^3d^2}{e^4}+\frac{x^2b^3}{2\,e^2}-3\,\frac{b^2x^2cd}{e^3}+\frac{9\,bx^2c^2d^2}{2\,e^4}-2\,\frac{x^2c^3d^3}{e^5}-2\,\frac{b^3dx}{e^3}+9\,\frac{b^2cd^2x}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^3/(e*x+d)^2,x)
```

```
[Out] 1/5*c^3*x^5/e^2+3/4/e^2*x^4*b*c^2-1/2*c^3*d*x^4/e^3+1/e^2*x^3*b^2*c-2/e^3*x
^3*b*c^2*d+1/e^4*x^3*c^3*d^2+1/2/e^2*x^2*b^3-3/e^3*x^2*b^2*c*d+9/2/e^4*x^2*
b*c^2*d^2-2/e^5*x^2*c^3*d^3-2/e^3*d*b^3*x+9/e^4*b^2*c*d^2*x-12/e^5*b*c^2*d^
3*x+5/e^6*c^3*d^4*x+3*d^2/e^4*ln(e*x+d)*b^3-12*d^3/e^5*ln(e*x+d)*b^2*c+15*d
^4/e^6*ln(e*x+d)*b*c^2-6*d^5/e^7*ln(e*x+d)*c^3+d^3/e^4/(e*x+d)*b^3-3*d^4/e^
5/(e*x+d)*b^2*c+3*d^5/e^6/(e*x+d)*b*c^2-d^6/e^7/(e*x+d)*c^3
```

Maxima [A] time = 1.0369, size = 369, normalized size = 2.22

$$-\frac{c^3d^6-3\,bc^2d^5e+3\,b^2cd^4e^2-b^3d^3e^3}{e^8x+de^7}+\frac{4\,c^3e^4x^5-5\left(2\,c^3de^3-3\,bc^2e^4\right)x^4+20\left(c^3d^2e^2-2\,bc^2de^3+b^2ce^4\right)x^3-10\left(4\,c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 - b^3*d^3*e^3)/(e^8*x + d*e^7)
+ 1/20*(4*c^3*e^4*x^5 - 5*(2*c^3*d*e^3 - 3*b*c^2*e^4)*x^4 + 20*(c^3*d^2*e^2
 - 2*b*c^2*d*e^3 + b^2*c*e^4)*x^3 - 10*(4*c^3*d^3*e - 9*b*c^2*d^2*e^2 + 6*b
^2*c*d*e^3 - b^3*e^4)*x^2 + 20*(5*c^3*d^4 - 12*b*c^2*d^3*e + 9*b^2*c*d^2*e^
2 - 2*b^3*d*e^3)*x)/e^6 - 3*(2*c^3*d^5 - 5*b*c^2*d^4*e + 4*b^2*c*d^3*e^2 -
b^3*d^2*e^3)*log(e*x + d)/e^7
```

Fricas [B] time = 1.44909, size = 751, normalized size = 4.52

$$\frac{4\,c^3e^6x^6-20\,c^3d^6+60\,bc^2d^5e-60\,b^2cd^4e^2+20\,b^3d^3e^3-3\left(2\,c^3de^5-5\,bc^2e^6\right)x^5+5\left(2\,c^3d^2e^4-5\,bc^2de^5+4\,b^2ce^6\right)x^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/20*(4*c^3*e^6*x^6 - 20*c^3*d^6 + 60*b*c^2*d^5*e - 60*b^2*c*d^4*e^2 + 20*b
^3*d^3*e^3 - 3*(2*c^3*d*e^5 - 5*b*c^2*e^6)*x^5 + 5*(2*c^3*d^2*e^4 - 5*b*c^2
*d*e^5 + 4*b^2*c*e^6)*x^4 - 10*(2*c^3*d^3*e^3 - 5*b*c^2*d^2*e^4 + 4*b^2*c*d
*e^5 - b^3*e^6)*x^3 + 30*(2*c^3*d^4*e^2 - 5*b*c^2*d^3*e^3 + 4*b^2*c*d^2*e^4
 - b^3*d*e^5)*x^2 + 20*(5*c^3*d^5*e - 12*b*c^2*d^4*e^2 + 9*b^2*c*d^3*e^3 -
```

```
2*b^3*d^2*e^4)*x - 60*(2*c^3*d^6 - 5*b*c^2*d^5*e + 4*b^2*c*d^4*e^2 - b^3*d^
3*e^3 + (2*c^3*d^5*e - 5*b*c^2*d^4*e^2 + 4*b^2*c*d^3*e^3 - b^3*d^2*e^4)*x)*
log(e*x + d))/(e^8*x + d*e^7)
```

Sympy [A] time = 3.46736, size = 248, normalized size = 1.49

$$\frac{c^3x^5}{5e^2} + \frac{3d^2\left(be-2cd\right)\left(be-cd\right)^2\log\left(d+ex\right)}{e^7} + \frac{b^3d^3e^3-3b^2cd^4e^2+3bc^2d^5e-c^3d^6}{de^7+e^8x} + \frac{x^4\left(3bc^2e-2c^3d\right)}{4e^3} + \frac{x^3\left(b^2ce^2-2bc\right.}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**3/(e*x+d)**2,x)
```

```
[Out] c**3*x**5/(5*e**2) + 3*d**2*(b*e - 2*c*d)*(b*e - c*d)**2*log(d + e*x)/e**7
+ (b**3*d**3*e**3 - 3*b**2*c*d**4*e**2 + 3*b*c**2*d**5*e - c**3*d**6)/(d*e*
*7 + e**8*x) + x**4*(3*b*c**2*e - 2*c**3*d)/(4*e**3) + x**3*(b**2*c*e**2 -
2*b*c**2*d*e + c**3*d**2)/e**4 + x**2*(b**3*e**3 - 6*b**2*c*d*e**2 + 9*b*c*
*2*d**2*e - 4*c**3*d**3)/(2*e**5) - x*(2*b**3*d*e**3 - 9*b**2*c*d**2*e**2 +
 12*b*c**2*d**3*e - 5*c**3*d**4)/e**6
```

Giac [B] time = 1.43872, size = 448, normalized size = 2.7

$$\frac{1}{20}\left(4\,c^3 - \frac{15\left(2\,c^3de-bc^2e^2\right)e^{(-1)}}{xe+d} + \frac{20\left(5\,c^3d^2e^2-5\,bc^2de^3+b^2ce^4\right)e^{(-2)}}{(xe+d)^2} - \frac{10\left(20\,c^3d^3e^3-30\,bc^2d^2e^4+12\,b^2cde^5-b^3e^6\right.}{(xe+d)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/20*(4*c^3 - 15*(2*c^3*d*e - b*c^2*e^2)*e^(-1)/(x*e + d) + 20*(5*c^3*d^2*e
^2 - 5*b*c^2*d*e^3 + b^2*c*e^4)*e^(-2)/(x*e + d)^2 - 10*(20*c^3*d^3*e^3 - 3
0*b*c^2*d^2*e^4 + 12*b^2*c*d*e^5 - b^3*e^6)*e^(-3)/(x*e + d)^3 + 60*(5*c^3*
d^4*e^4 - 10*b*c^2*d^3*e^5 + 6*b^2*c*d^2*e^6 - b^3*d*e^7)*e^(-4)/(x*e + d)^
4)*(x*e + d)^5*e^(-7) + 3*(2*c^3*d^5 - 5*b*c^2*d^4*e + 4*b^2*c*d^3*e^2 - b^
3*d^2*e^3)*e^(-7)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2) - (c^3*d^6*e^5/(x*e
+ d) - 3*b*c^2*d^5*e^6/(x*e + d) + 3*b^2*c*d^4*e^7/(x*e + d) - b^3*d^3*e^8/
(x*e + d))*e^(-12)
```

3.251 $\int \frac{(bx+cx^2)^3}{(d+ex)^3}\,dx$

Optimal. Leaf size=200

$$-\frac{x(cd-be)\left(b^2e^2-8bcde+10c^2d^2\right)}{e^6}+\frac{3d(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)\log(d+ex)}{e^7}-\frac{c^2x^3(cd-be)}{e^4}+\frac{3d^2(cd-be)^2}{e^7(d+$$

```
[Out] -(((c*d - b*e)*(10*c^2*d^2 - 8*b*c*d*e + b^2*e^2)*x)/e^6) + (3*c*(c*d - b*e
)*(2*c*d - b*e)*x^2)/(2*e^5) - (c^2*(c*d - b*e)*x^3)/e^4 + (c^3*x^4)/(4*e^3
) - (d^3*(c*d - b*e)^3)/(2*e^7*(d + e*x)^2) + (3*d^2*(c*d - b*e)^2*(2*c*d -
 b*e))/(e^7*(d + e*x)) + (3*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)
*Log[d + e*x])/e^7
```

Rubi [A] time = 0.21873, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$-\frac{x(cd-be)\left(b^2e^2-8bcde+10c^2d^2\right)}{e^6}+\frac{3d(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)\log(d+ex)}{e^7}-\frac{c^2x^3(cd-be)}{e^4}+\frac{3d^2(cd-be)^2}{e^7(d+$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^3/(d + e*x)^3,x]
```

```
[Out] -(((c*d - b*e)*(10*c^2*d^2 - 8*b*c*d*e + b^2*e^2)*x)/e^6) + (3*c*(c*d - b*e
)*(2*c*d - b*e)*x^2)/(2*e^5) - (c^2*(c*d - b*e)*x^3)/e^4 + (c^3*x^4)/(4*e^3
) - (d^3*(c*d - b*e)^3)/(2*e^7*(d + e*x)^2) + (3*d^2*(c*d - b*e)^2*(2*c*d -
 b*e))/(e^7*(d + e*x)) + (3*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)
*Log[d + e*x])/e^7
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(bx+cx^2)^3}{(d+ex)^3}\,dx=\int \left(\frac{(cd-be)\left(-10c^2d^2+8bcde-b^2e^2\right)}{e^6}+\frac{3c(cd-be)(2cd-be)x}{e^5}-\frac{3c^2(cd-be)x^2}{e^4}+\frac{c^3x^3}{e^3}+\frac{d^3(c}{e^6(d}\right.$$

$$=-\frac{(cd-be)\left(10c^2d^2-8bcde+b^2e^2\right)x}{e^6}+\frac{3c(cd-be)(2cd-be)x^2}{2e^5}-\frac{c^2(cd-be)x^3}{e^4}+\frac{c^3x^4}{4e^3}-\frac{d^3(cd-}{2e^7(d+}$$

Mathematica [A] time = 0.131485, size = 207, normalized size = 1.03

$$\frac{6ce^2x^2\left(b^2e^2-3bcde+2c^2d^2\right)+4ex\left(-9b^2cde^2+b^3e^3+18bc^2d^2e-10c^3d^3\right)+12d\left(6b^2cde^2-b^3e^3-10bc^2d^2e+5c^3d^3\right)}{4e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^3/(d + e*x)^3,x]
```

```
[Out] (4*e*(-10*c^3*d^3 + 18*b*c^2*d^2*e - 9*b^2*c*d*e^2 + b^3*e^3)*x + 6*c*e^2*(
2*c^2*d^2 - 3*b*c*d*e + b^2*e^2)*x^2 - 4*c^2*e^3*(c*d - b*e)*x^3 + c^3*e^4*
x^4 - (2*d^3*(c*d - b*e)^3)/(d + e*x)^2 + (12*d^2*(c*d - b*e)^2*(2*c*d - b*
e))/(d + e*x) + 12*d*(5*c^3*d^3 - 10*b*c^2*d^2*e + 6*b^2*c*d*e^2 - b^3*e^3)
*Log[d + e*x])/(4*e^7)
```

Maple [A] time = 0.051, size = 335, normalized size = 1.7

$$\frac{c^3x^4}{4\,e^3}+\frac{bx^3c^2}{e^3}-\frac{c^3dx^3}{e^4}+\frac{3\,b^2x^2c}{2\,e^3}-\frac{9\,bx^2c^2d}{2\,e^4}+3\,\frac{x^2c^3d^2}{e^5}+\frac{b^3x}{e^3}-9\,\frac{b^2cdx}{e^4}+18\,\frac{bc^2d^2x}{e^5}-10\,\frac{c^3d^3x}{e^6}+\frac{d^3b^3}{2\,e^4\,(ex+d)^2}-\frac{3}{2\,e^5\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^3/(e*x+d)^3,x)
```

```
[Out] 1/4*c^3*x^4/e^3+1/e^3*x^3*b*c^2-c^3*d*x^3/e^4+3/2/e^3*x^2*b^2*c-9/2/e^4*x^2
*b*c^2*d+3/e^5*x^2*c^3*d^2+1/e^3*b^3*x-9/e^4*b^2*c*d*x+18/e^5*b*c^2*d^2*x-1
0/e^6*c^3*d^3*x+1/2*d^3/e^4/(e*x+d)^2*b^3-3/2*d^4/e^5/(e*x+d)^2*b^2*c+3/2*d
^5/e^6/(e*x+d)^2*b*c^2-1/2*d^6/e^7/(e*x+d)^2*c^3-3*d/e^4*ln(e*x+d)*b^3+18*d
^2/e^5*ln(e*x+d)*b^2*c-30*d^3/e^6*ln(e*x+d)*b*c^2+15*d^4/e^7*ln(e*x+d)*c^3-
3*d^2/e^4/(e*x+d)*b^3+12*d^3/e^5/(e*x+d)*b^2*c-15*d^4/e^6/(e*x+d)*b*c^2+6*d
^5/e^7/(e*x+d)*c^3
```

Maxima [A] time = 1.156, size = 378, normalized size = 1.89

$$\frac{11\,c^3d^6-27\,bc^2d^5e+21\,b^2cd^4e^2-5\,b^3d^3e^3+6\left(2\,c^3d^5e-5\,bc^2d^4e^2+4\,b^2cd^3e^3-b^3d^2e^4\right)x}{2\left(e^9x^2+2\,de^8x+d^2e^7\right)}+\frac{c^3e^3x^4-4\left(c^3de^2-bc^2e^3\right)x^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(11*c^3*d^6 - 27*b*c^2*d^5*e + 21*b^2*c*d^4*e^2 - 5*b^3*d^3*e^3 + 6*(2*
c^3*d^5*e - 5*b*c^2*d^4*e^2 + 4*b^2*c*d^3*e^3 - b^3*d^2*e^4)*x)/(e^9*x^2 +
2*d*e^8*x + d^2*e^7) + 1/4*(c^3*e^3*x^4 - 4*(c^3*d*e^2 - b*c^2*e^3)*x^3 + 6
*(2*c^3*d^2*e - 3*b*c^2*d*e^2 + b^2*c*e^3)*x^2 - 4*(10*c^3*d^3 - 18*b*c^2*d
^2*e + 9*b^2*c*d*e^2 - b^3*e^3)*x)/e^6 + 3*(5*c^3*d^4 - 10*b*c^2*d^3*e + 6*
b^2*c*d^2*e^2 - b^3*d*e^3)*log(e*x + d)/e^7
```

Fricas [B] time = 1.66007, size = 868, normalized size = 4.34

$$\frac{c^3e^6x^6+22\,c^3d^6-54\,bc^2d^5e+42\,b^2cd^4e^2-10\,b^3d^3e^3-2\left(c^3de^5-2\,bc^2e^6\right)x^5+\left(5\,c^3d^2e^4-10\,bc^2de^5+6\,b^2ce^6\right)x^4-4\left(5\,c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(c^3*e^6*x^6 + 22*c^3*d^6 - 54*b*c^2*d^5*e + 42*b^2*c*d^4*e^2 - 10*b^3*
d^3*e^3 - 2*(c^3*d*e^5 - 2*b*c^2*e^6)*x^5 + (5*c^3*d^2*e^4 - 10*b*c^2*d*e^5
 + 6*b^2*c*e^6)*x^4 - 4*(5*c^3*d^3*e^3 - 10*b*c^2*d^2*e^4 + 6*b^2*c*d*e^5 -
 b^3*e^6)*x^3 - 2*(34*c^3*d^4*e^2 - 63*b*c^2*d^3*e^3 + 33*b^2*c*d^2*e^4 - 4
*b^3*d*e^5)*x^2 - 4*(4*c^3*d^5*e - 3*b*c^2*d^4*e^2 - 3*b^2*c*d^3*e^3 + 2*b^
3*d^2*e^4)*x + 12*(5*c^3*d^6 - 10*b*c^2*d^5*e + 6*b^2*c*d^4*e^2 - b^3*d^3*e
^3 + (5*c^3*d^4*e^2 - 10*b*c^2*d^3*e^3 + 6*b^2*c*d^2*e^4 - b^3*d*e^5)*x^2 +
 2*(5*c^3*d^5*e - 10*b*c^2*d^4*e^2 + 6*b^2*c*d^3*e^3 - b^3*d^2*e^4)*x)*log(
e*x + d))/(e^9*x^2 + 2*d*e^8*x + d^2*e^7)
```

Sympy [A] time = 5.79209, size = 277, normalized size = 1.38

$$\frac{c^3x^4}{4e^3} - \frac{3d\,(be-cd)\left(b^2e^2-5bcde+5c^2d^2\right)\log\left(d+ex\right)}{e^7} - \frac{5b^3d^3e^3-21b^2cd^4e^2+27bc^2d^5e-11c^3d^6+x\left(6b^3d^2e^4-24b\right.}{2d^2e^7+4de^8x+2e^9x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**3/(e*x+d)**3,x)
```

```
[Out] c**3*x**4/(4*e**3) - 3*d*(b*e - c*d)*(b**2*e**2 - 5*b*c*d*e + 5*c**2*d**2)*
log(d + e*x)/e**7 - (5*b**3*d**3*e**3 - 21*b**2*c*d**4*e**2 + 27*b*c**2*d**
5*e - 11*c**3*d**6 + x*(6*b**3*d**2*e**4 - 24*b**2*c*d**3*e**3 + 30*b*c**2*
d**4*e**2 - 12*c**3*d**5*e))/(2*d**2*e**7 + 4*d*e**8*x + 2*e**9*x**2) + x**
3*(b*c**2*e - c**3*d)/e**4 + x**2*(3*b**2*c*e**2 - 9*b*c**2*d*e + 6*c**3*d*
*2)/(2*e**5) + x*(b**3*e**3 - 9*b**2*c*d*e**2 + 18*b*c**2*d**2*e - 10*c**3*
d**3)/e**6
```

Giac [A] time = 1.3611, size = 356, normalized size = 1.78

$$3\left(5\,c^3d^4-10\,bc^2d^3e+6\,b^2cd^2e^2-b^3de^3\right)e^{(-7)}\log\left(|xe+d|\right)+\frac{1}{4}\left(c^3x^4e^9-4\,c^3dx^3e^8+12\,c^3d^2x^2e^7-40\,c^3d^3xe^6+4\,bc^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 3*(5*c^3*d^4 - 10*b*c^2*d^3*e + 6*b^2*c*d^2*e^2 - b^3*d*e^3)*e^(-7)*log(abs
(x*e + d)) + 1/4*(c^3*x^4*e^9 - 4*c^3*d*x^3*e^8 + 12*c^3*d^2*x^2*e^7 - 40*c
^3*d^3*x*e^6 + 4*b*c^2*x^3*e^9 - 18*b*c^2*d*x^2*e^8 + 72*b*c^2*d^2*x*e^7 +
6*b^2*c*x^2*e^9 - 36*b^2*c*d*x*e^8 + 4*b^3*x*e^9)*e^(-12) + 1/2*(11*c^3*d^6
 - 27*b*c^2*d^5*e + 21*b^2*c*d^4*e^2 - 5*b^3*d^3*e^3 + 6*(2*c^3*d^5*e - 5*b
*c^2*d^4*e^2 + 4*b^2*c*d^3*e^3 - b^3*d^2*e^4)*x)*e^(-7)/(x*e + d)^2
```

3.252 $\int \frac{(bx+cx^2)^3}{(d+ex)^4} \, dx$

Optimal. Leaf size=213

$$-\frac{3d\left(b^2e^2-5bcde+5c^2d^2\right)(cd-be)}{e^7(d+ex)}+\frac{cx\left(3b^2e^2-12bcde+10c^2d^2\right)}{e^6}-\frac{(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)\log(d+ex)}{e^7}-$$

```
[Out] (c*(10*c^2*d^2 - 12*b*c*d*e + 3*b^2*e^2)*x)/e^6 - (c^2*(4*c*d - 3*b*e)*x^2)
/(2*e^5) + (c^3*x^3)/(3*e^4) - (d^3*(c*d - b*e)^3)/(3*e^7*(d + e*x)^3) + (3
*d^2*(c*d - b*e)^2*(2*c*d - b*e))/(2*e^7*(d + e*x)^2) - (3*d*(c*d - b*e)*(5
*c^2*d^2 - 5*b*c*d*e + b^2*e^2))/(e^7*(d + e*x)) - ((2*c*d - b*e)*(10*c^2*d
^2 - 10*b*c*d*e + b^2*e^2)*Log[d + e*x])/e^7
```

Rubi [A] time = 0.214941, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$-\frac{3d\left(b^2e^2-5bcde+5c^2d^2\right)(cd-be)}{e^7(d+ex)}+\frac{cx\left(3b^2e^2-12bcde+10c^2d^2\right)}{e^6}-\frac{(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)\log(d+ex)}{e^7}-$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^3/(d + e*x)^4,x]
```

```
[Out] (c*(10*c^2*d^2 - 12*b*c*d*e + 3*b^2*e^2)*x)/e^6 - (c^2*(4*c*d - 3*b*e)*x^2)
/(2*e^5) + (c^3*x^3)/(3*e^4) - (d^3*(c*d - b*e)^3)/(3*e^7*(d + e*x)^3) + (3
*d^2*(c*d - b*e)^2*(2*c*d - b*e))/(2*e^7*(d + e*x)^2) - (3*d*(c*d - b*e)*(5
*c^2*d^2 - 5*b*c*d*e + b^2*e^2))/(e^7*(d + e*x)) - ((2*c*d - b*e)*(10*c^2*d
^2 - 10*b*c*d*e + b^2*e^2)*Log[d + e*x])/e^7
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(bx+cx^2\right)^3}{(d+ex)^4}\,dx=\int \left(\frac{c\left(10c^2d^2-12bcde+3b^2e^2\right)}{e^6}-\frac{c^2(4cd-3be)x}{e^5}+\frac{c^3x^2}{e^4}+\frac{d^3(cd-be)^3}{e^6(d+ex)^4}-\frac{3d^2(cd-be)^2(2cd-be)}{e^6(d+ex)^3}-\right.$$

$$=\frac{c\left(10c^2d^2-12bcde+3b^2e^2\right)x}{e^6}-\frac{c^2(4cd-3be)x^2}{2e^5}+\frac{c^3x^3}{3e^4}-\frac{d^3(cd-be)^3}{3e^7(d+ex)^3}+\frac{3d^2(cd-be)^2(2cd-be)}{2e^7(d+ex)^2}-$$

Mathematica [A] time = 0.107645, size = 210, normalized size = 0.99

$$\frac{6cex\left(3b^2e^2-12bcde+10c^2d^2\right)+\frac{18d\left(-6b^2cde^2+b^3e^3+10bc^2d^2e-5c^3d^3\right)}{d+ex}+6\left(-12b^2cde^2+b^3e^3+30bc^2d^2e-20c^3d^3\right)\log(d+ex)-}{6e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^3/(d + e*x)^4,x]
```

```
[Out] (6*c*e*(10*c^2*d^2 - 12*b*c*d*e + 3*b^2*e^2)*x - 3*c^2*e^2*(4*c*d - 3*b*e)*
x^2 + 2*c^3*e^3*x^3 - (2*d^3*(c*d - b*e)^3)/(d + e*x)^3 + (9*d^2*(c*d - b*e
)^2*(2*c*d - b*e))/(d + e*x)^2 + (18*d*(-5*c^3*d^3 + 10*b*c^2*d^2*e - 6*b^2
*c*d*e^2 + b^3*e^3))/(d + e*x) + 6*(-20*c^3*d^3 + 30*b*c^2*d^2*e - 12*b^2*c
*d*e^2 + b^3*e^3)*Log[d + e*x])/(6*e^7)
```

Maple [A] time = 0.054, size = 353, normalized size = 1.7

$$\frac{x^3c^3}{3\,e^4}+\frac{3\,bx^2c^2}{2\,e^4}-2\,\frac{c^3dx^2}{e^5}+3\,\frac{b^2xc}{e^4}-12\,\frac{bc^2dx}{e^5}+10\,\frac{c^3d^2x}{e^6}-\frac{3\,d^2b^3}{2\,e^4\,(ex+d)^2}+6\,\frac{b^2cd^3}{e^5\,(ex+d)^2}-\frac{15\,bc^2d^4}{2\,e^6\,(ex+d)^2}+3\,\frac{c^3 d}{e^7\,(ex}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^3/(e*x+d)^4,x)
```

```
[Out] 1/3*c^3*x^3/e^4+3/2*c^2/e^4*x^2*b-2*c^3*d*x^2/e^5+3*c/e^4*b^2*x-12*c^2/e^5*
b*d*x+10*c^3/e^6*d^2*x-3/2*d^2/e^4/(e*x+d)^2*b^3+6*d^3/e^5/(e*x+d)^2*b^2*c-
15/2*d^4/e^6/(e*x+d)^2*b*c^2+3*d^5/e^7/(e*x+d)^2*c^3+1/e^4*ln(e*x+d)*b^3-12
/e^5*ln(e*x+d)*b^2*c*d+30/e^6*ln(e*x+d)*b*c^2*d^2-20/e^7*ln(e*x+d)*c^3*d^3+
1/3*d^3/e^4/(e*x+d)^3*b^3-d^4/e^5/(e*x+d)^3*b^2*c+d^5/e^6/(e*x+d)^3*b*c^2-1
/3*d^6/e^7/(e*x+d)^3*c^3+3*d/e^4/(e*x+d)*b^3-18*d^2/e^5/(e*x+d)*b^2*c+30*d^
3/e^6/(e*x+d)*b*c^2-15*d^4/e^7/(e*x+d)*c^3
```

Maxima [A] time = 1.16581, size = 397, normalized size = 1.86

$$-\frac{74\,c^3d^6-141\,bc^2d^5e+78\,b^2cd^4e^2-11\,b^3d^3e^3+18\left(5\,c^3d^4e^2-10\,bc^2d^3e^3+6\,b^2cd^2e^4-b^3de^5\right)x^2+9\left(18\,c^3d^5e-35\,bc\right.}{6\left(e^{10}x^3+3\,de^9x^2+3\,d^2e^8x+d^3e^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(74*c^3*d^6 - 141*b*c^2*d^5*e + 78*b^2*c*d^4*e^2 - 11*b^3*d^3*e^3 + 18
*(5*c^3*d^4*e^2 - 10*b*c^2*d^3*e^3 + 6*b^2*c*d^2*e^4 - b^3*d*e^5)*x^2 + 9*(
18*c^3*d^5*e - 35*b*c^2*d^4*e^2 + 20*b^2*c*d^3*e^3 - 3*b^3*d^2*e^4)*x)/(e^1
0*x^3 + 3*d*e^9*x^2 + 3*d^2*e^8*x + d^3*e^7) + 1/6*(2*c^3*e^2*x^3 - 3*(4*c^
3*d*e - 3*b*c^2*e^2)*x^2 + 6*(10*c^3*d^2 - 12*b*c^2*d*e + 3*b^2*c*e^2)*x)/e
^6 - (20*c^3*d^3 - 30*b*c^2*d^2*e + 12*b^2*c*d*e^2 - b^3*e^3)*log(e*x + d)/
e^7
```

Fricas [B] time = 1.61528, size = 996, normalized size = 4.68

$$\frac{2\,c^3e^6x^6-74\,c^3d^6+141\,bc^2d^5e-78\,b^2cd^4e^2+11\,b^3d^3e^3-3\left(2\,c^3de^5-3\,bc^2e^6\right)x^5+3\left(10\,c^3d^2e^4-15\,bc^2de^5+6\,b^2ce^6\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*c^3*e^6*x^6 - 74*c^3*d^6 + 141*b*c^2*d^5*e - 78*b^2*c*d^4*e^2 + 11*b
^3*d^3*e^3 - 3*(2*c^3*d*e^5 - 3*b*c^2*e^6)*x^5 + 3*(10*c^3*d^2*e^4 - 15*b*c
^2*d*e^5 + 6*b^2*c*e^6)*x^4 + (146*c^3*d^3*e^3 - 189*b*c^2*d^2*e^4 + 54*b^2
*c*d*e^5)*x^3 + 3*(26*c^3*d^4*e^2 - 9*b*c^2*d^3*e^3 - 18*b^2*c*d^2*e^4 + 6*
b^3*d*e^5)*x^2 - 3*(34*c^3*d^5*e - 81*b*c^2*d^4*e^2 + 54*b^2*c*d^3*e^3 - 9*
b^3*d^2*e^4)*x - 6*(20*c^3*d^6 - 30*b*c^2*d^5*e + 12*b^2*c*d^4*e^2 - b^3*d^
3*e^3 + (20*c^3*d^3*e^3 - 30*b*c^2*d^2*e^4 + 12*b^2*c*d*e^5 - b^3*e^6)*x^3
+ 3*(20*c^3*d^4*e^2 - 30*b*c^2*d^3*e^3 + 12*b^2*c*d^2*e^4 - b^3*d*e^5)*x^2
+ 3*(20*c^3*d^5*e - 30*b*c^2*d^4*e^2 + 12*b^2*c*d^3*e^3 - b^3*d^2*e^4)*x)*l
og(e*x + d))/(e^10*x^3 + 3*d*e^9*x^2 + 3*d^2*e^8*x + d^3*e^7)
```

Sympy [A] time = 12.896, size = 298, normalized size = 1.4

$$\frac{c^3x^3}{3e^4} + \frac{11b^3d^3e^3 - 78b^2cd^4e^2 + 141bc^2d^5e - 74c^3d^6 + x^2\left(18b^3de^5 - 108b^2cd^2e^4 + 180bc^2d^3e^3 - 90c^3d^4e^2\right) + x\left(27b^3d^2e^4 - \right.}{6d^3e^7 + 18d^2e^8x + 18de^9x^2 + 6e^{10}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**3/(e*x+d)**4,x)
```

```
[Out] c**3*x**3/(3*e**4) + (11*b**3*d**3*e**3 - 78*b**2*c*d**4*e**2 + 141*b*c**2*
d**5*e - 74*c**3*d**6 + x**2*(18*b**3*d*e**5 - 108*b**2*c*d**2*e**4 + 180*b
*c**2*d**3*e**3 - 90*c**3*d**4*e**2) + x*(27*b**3*d**2*e**4 - 180*b**2*c*d*
*3*e**3 + 315*b*c**2*d**4*e**2 - 162*c**3*d**5*e))/(6*d**3*e**7 + 18*d**2*e
**8*x + 18*d*e**9*x**2 + 6*e**10*x**3) + x**2*(3*b*c**2*e - 4*c**3*d)/(2*e*
*5) + x*(3*b**2*c*e**2 - 12*b*c**2*d*e + 10*c**3*d**2)/e**6 + (b*e - 2*c*d)
*(b**2*e**2 - 10*b*c*d*e + 10*c**2*d**2)*log(d + e*x)/e**7
```

Giac [A] time = 1.31645, size = 352, normalized size = 1.65

$$-\left(20\,c^3d^3 - 30\,bc^2d^2e + 12\,b^2cde^2 - b^3e^3\right)e^{(-7)}\log\left(|xe + d|\right) + \frac{1}{6}\left(2\,c^3x^3e^8 - 12\,c^3dx^2e^7 + 60\,c^3d^2xe^6 + 9\,bc^2x^2e^8 - 72\,bc^2d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -(20*c^3*d^3 - 30*b*c^2*d^2*e + 12*b^2*c*d*e^2 - b^3*e^3)*e^(-7)*log(abs(x*
e + d)) + 1/6*(2*c^3*x^3*e^8 - 12*c^3*d*x^2*e^7 + 60*c^3*d^2*x*e^6 + 9*b*c^
2*x^2*e^8 - 72*b*c^2*d*x*e^7 + 18*b^2*c*x*e^8)*e^(-12) - 1/6*(74*c^3*d^6 -
141*b*c^2*d^5*e + 78*b^2*c*d^4*e^2 - 11*b^3*d^3*e^3 + 18*(5*c^3*d^4*e^2 - 1
0*b*c^2*d^3*e^3 + 6*b^2*c*d^2*e^4 - b^3*d*e^5)*x^2 + 9*(18*c^3*d^5*e - 35*b
*c^2*d^4*e^2 + 20*b^2*c*d^3*e^3 - 3*b^3*d^2*e^4)*x)*e^(-7)/(x*e + d)^3
```

3.253 $\int \frac{(bx+cx^2)^3}{(d+ex)^5}\,dx$

Optimal. Leaf size=213

$$\frac{(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{e^7(d+ex)}-\frac{3d(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)}{2e^7(d+ex)^2}+\frac{3c\left(b^2e^2-5bcde+5c^2d^2\right)\log(d+ex)}{e^7}-\frac{c^{}}{}$$

```
[Out] -((c^2*(5*c*d - 3*b*e)*x)/e^6) + (c^3*x^2)/(2*e^5) - (d^3*(c*d - b*e)^3)/(4
*e^7*(d + e*x)^4) + (d^2*(c*d - b*e)^2*(2*c*d - b*e))/(e^7*(d + e*x)^3) - (
3*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2))/(2*e^7*(d + e*x)^2) + ((
2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e + b^2*e^2))/(e^7*(d + e*x)) + (3*c*(5
*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*Log[d + e*x])/e^7
```

Rubi [A] time = 0.197327, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$\frac{(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{e^7(d+ex)}-\frac{3d(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)}{2e^7(d+ex)^2}+\frac{3c\left(b^2e^2-5bcde+5c^2d^2\right)\log(d+ex)}{e^7}-\frac{c^{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^3/(d + e*x)^5,x]
```



```
[Out] -((c^2*(5*c*d - 3*b*e)*x)/e^6) + (c^3*x^2)/(2*e^5) - (d^3*(c*d - b*e)^3)/(4
*e^7*(d + e*x)^4) + (d^2*(c*d - b*e)^2*(2*c*d - b*e))/(e^7*(d + e*x)^3) - (
3*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2))/(2*e^7*(d + e*x)^2) + ((
2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e + b^2*e^2))/(e^7*(d + e*x)) + (3*c*(5
*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*Log[d + e*x])/e^7
```


Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(bx+cx^2\right)^3}{(d+ex)^5}\,dx=\int \left(-\frac{c^2(5cd-3be)}{e^6}+\frac{c^3x}{e^5}+\frac{d^3(cd-be)^3}{e^6(d+ex)^5}-\frac{3d^2(cd-be)^2(2cd-be)}{e^6(d+ex)^4}+\frac{3d(cd-be)\left(5c^2d^2-5bcde\right.}{e^6(d+ex)^3}\right.$$

$$=-\frac{c^2(5cd-3be)x}{e^6}+\frac{c^3x^2}{2e^5}-\frac{d^3(cd-be)^3}{4e^7(d+ex)^4}+\frac{d^2(cd-be)^2(2cd-be)}{e^7(d+ex)^3}-\frac{3d(cd-be)\left(5c^2d^2-5bcde+\right.}{2e^7(d+ex)^2}$$

Mathematica [A] time = 0.114337, size = 210, normalized size = 0.99

$$\frac{\frac{48b^2cde^2-4b^3e^3-120bc^2d^2e+80c^3d^3}{d+ex}+\frac{6d\left(-6b^2cde^2+b^3e^3+10bc^2d^2e-5c^3d^3\right)}{(d+ex)^2}+12c\left(b^2e^2-5bcde+5c^2d^2\right)\log(d+ex)-4c^2ex(5cd-3be}{4e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^3/(d + e*x)^5,x]
```

```
[Out] (-4*c^2*e*(5*c*d - 3*b*e)*x + 2*c^3*e^2*x^2 - (d^3*(c*d - b*e)^3)/(d + e*x)
^4 + (4*d^2*(c*d - b*e)^2*(2*c*d - b*e))/(d + e*x)^3 + (6*d*(-5*c^3*d^3 + 1
0*b*c^2*d^2*e - 6*b^2*c*d*e^2 + b^3*e^3))/(d + e*x)^2 + (80*c^3*d^3 - 120*b
*c^2*d^2*e + 48*b^2*c*d*e^2 - 4*b^3*e^3)/(d + e*x) + 12*c*(5*c^2*d^2 - 5*b*
c*d*e + b^2*e^2)*Log[d + e*x])/(4*e^7)
```

Maple [A] time = 0.054, size = 370, normalized size = 1.7

$$\frac{c^3x^2}{2e^5} + 3\frac{c^2xb}{e^5} - 5\frac{c^3dx}{e^6} + \frac{3db^3}{2e^4(ex+d)^2} - 9\frac{b^2d^2c}{e^5(ex+d)^2} + 15\frac{bc^2d^3}{e^6(ex+d)^2} - \frac{15c^3d^4}{2e^7(ex+d)^2} + 3\frac{c\ln(ex+d)b^2}{e^5} - 15\frac{c^2\ln(\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^3/(e*x+d)^5,x)
```

```
[Out] 1/2*c^3*x^2/e^5+3*c^2/e^5*x*b-5*c^3*d*x/e^6+3/2*d/e^4/(e*x+d)^2*b^3-9*d^2/e
^5/(e*x+d)^2*b^2*c+15*d^3/e^6/(e*x+d)^2*b*c^2-15/2*d^4/e^7/(e*x+d)^2*c^3+3*
c/e^5*ln(e*x+d)*b^2-15*c^2/e^6*ln(e*x+d)*b*d+15*c^3/e^7*ln(e*x+d)*d^2-d^2/e
^4/(e*x+d)^3*b^3+4*d^3/e^5/(e*x+d)^3*b^2*c-5*d^4/e^6/(e*x+d)^3*b*c^2+2*d^5/
e^7/(e*x+d)^3*c^3-1/e^4/(e*x+d)*b^3+12/e^5/(e*x+d)*b^2*c*d-30/e^6/(e*x+d)*b
*c^2*d^2+20/e^7/(e*x+d)*c^3*d^3+1/4*d^3/e^4/(e*x+d)^4*b^3-3/4*d^4/e^5/(e*x+
d)^4*b^2*c+3/4*d^5/e^6/(e*x+d)^4*b*c^2-1/4*d^6/e^7/(e*x+d)^4*c^3
```

Maxima [A] time = 1.2026, size = 409, normalized size = 1.92

$$\frac{57c^3d^6 - 77bc^2d^5e + 25b^2cd^4e^2 - b^3d^3e^3 + 4\left(20c^3d^3e^3 - 30bc^2d^2e^4 + 12b^2cde^5 - b^3e^6\right)x^3 + 6\left(35c^3d^4e^2 - 50bc^2d^3e^3 + \cdots\right.}{4\left(e^{11}x^4 + 4de^{10}x^3 + 6d^2e^9x^2 + 4d^3e^8x + d^4e^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^5,x, algorithm="maxima")
```

```
[Out] 1/4*(57*c^3*d^6 - 77*b*c^2*d^5*e + 25*b^2*c*d^4*e^2 - b^3*d^3*e^3 + 4*(20*c
^3*d^3*e^3 - 30*b*c^2*d^2*e^4 + 12*b^2*c*d*e^5 - b^3*e^6)*x^3 + 6*(35*c^3*d
^4*e^2 - 50*b*c^2*d^3*e^3 + 18*b^2*c*d^2*e^4 - b^3*d*e^5)*x^2 + 4*(47*c^3*d
^5*e - 65*b*c^2*d^4*e^2 + 22*b^2*c*d^3*e^3 - b^3*d^2*e^4)*x)/(e^11*x^4 + 4*
d*e^10*x^3 + 6*d^2*e^9*x^2 + 4*d^3*e^8*x + d^4*e^7) + 1/2*(c^3*e*x^2 - 2*(5
*c^3*d - 3*b*c^2*e)*x)/e^6 + 3*(5*c^3*d^2 - 5*b*c^2*d*e + b^2*c*e^2)*log(e*
x + d)/e^7
```

Fricas [B] time = 1.58442, size = 976, normalized size = 4.58

$$\frac{2c^3e^6x^6 + 57c^3d^6 - 77bc^2d^5e + 25b^2cd^4e^2 - b^3d^3e^3 - 12\left(c^3de^5 - bc^2e^6\right)x^5 - 4\left(17c^3d^2e^4 - 12bc^2de^5\right)x^4 - 4\left(8c^3d^3e^3 + 1\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^5,x, algorithm="fricas")
```

```
[Out] 1/4*(2*c^3*e^6*x^6 + 57*c^3*d^6 - 77*b*c^2*d^5*e + 25*b^2*c*d^4*e^2 - b^3*d
^3*e^3 - 12*(c^3*d*e^5 - b*c^2*e^6)*x^5 - 4*(17*c^3*d^2*e^4 - 12*b*c^2*d*e^
5)*x^4 - 4*(8*c^3*d^3*e^3 + 12*b*c^2*d^2*e^4 - 12*b^2*c*d*e^5 + b^3*e^6)*x^
3 + 6*(22*c^3*d^4*e^2 - 42*b*c^2*d^3*e^3 + 18*b^2*c*d^2*e^4 - b^3*d*e^5)*x^
2 + 4*(42*c^3*d^5*e - 62*b*c^2*d^4*e^2 + 22*b^2*c*d^3*e^3 - b^3*d^2*e^4)*x
+ 12*(5*c^3*d^6 - 5*b*c^2*d^5*e + b^2*c*d^4*e^2 + (5*c^3*d^2*e^4 - 5*b*c^2*
d*e^5 + b^2*c*e^6)*x^4 + 4*(5*c^3*d^3*e^3 - 5*b*c^2*d^2*e^4 + b^2*c*d*e^5)*
x^3 + 6*(5*c^3*d^4*e^2 - 5*b*c^2*d^3*e^3 + b^2*c*d^2*e^4)*x^2 + 4*(5*c^3*d^
5*e - 5*b*c^2*d^4*e^2 + b^2*c*d^3*e^3)*x)*log(e*x + d))/(e^11*x^4 + 4*d*e^1
0*x^3 + 6*d^2*e^9*x^2 + 4*d^3*e^8*x + d^4*e^7)
```

Sympy [A] time = 14.4251, size = 314, normalized size = 1.47

$$\frac{c^3x^2}{2e^5} + \frac{3c\left(b^2e^2 - 5bcde + 5c^2d^2\right)\log(d+ex)}{e^7} - \frac{b^3d^3e^3 - 25b^2cd^4e^2 + 77bc^2d^5e - 57c^3d^6 + x^3\left(4b^3e^6 - 48b^2cde^5 + 12\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**3/(e*x+d)**5,x)
```

```
[Out] c**3*x**2/(2*e**5) + 3*c*(b**2*e**2 - 5*b*c*d*e + 5*c**2*d**2)*log(d + e*x)
/e**7 - (b**3*d**3*e**3 - 25*b**2*c*d**4*e**2 + 77*b*c**2*d**5*e - 57*c**3*
d**6 + x**3*(4*b**3*e**6 - 48*b**2*c*d*e**5 + 120*b*c**2*d**2*e**4 - 80*c**
3*d**3*e**3) + x**2*(6*b**3*d*e**5 - 108*b**2*c*d**2*e**4 + 300*b*c**2*d**3
*e**3 - 210*c**3*d**4*e**2) + x*(4*b**3*d**2*e**4 - 88*b**2*c*d**3*e**3 + 2
60*b*c**2*d**4*e**2 - 188*c**3*d**5*e))/(4*d**4*e**7 + 16*d**3*e**8*x + 24*
d**2*e**9*x**2 + 16*d*e**10*x**3 + 4*e**11*x**4) + x*(3*b*c**2*e - 5*c**3*d
)/e**6
```

Giac [A] time = 1.27752, size = 522, normalized size = 2.45

$$\frac{1}{2}\left(c^3 - \frac{6\left(2c^3de - bc^2e^2\right)e^{(-1)}}{xe+d}\right)(xe+d)^2e^{(-7)} - 3\left(5c^3d^2 - 5bc^2de + b^2ce^2\right)e^{(-7)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^2}\right) + \frac{1}{4}\left(\frac{80c^3d^3e^{29}}{xe+d} - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] 1/2*(c^3 - 6*(2*c^3*d*e - b*c^2*e^2)*e^(-1)/(x*e + d))*(x*e + d)^2*e^(-7) -
 3*(5*c^3*d^2 - 5*b*c^2*d*e + b^2*c*e^2)*e^(-7)*log(abs(x*e + d)*e^(-1)/(x*
e + d)^2) + 1/4*(80*c^3*d^3*e^29/(x*e + d) - 30*c^3*d^4*e^29/(x*e + d)^2 +
8*c^3*d^5*e^29/(x*e + d)^3 - c^3*d^6*e^29/(x*e + d)^4 - 120*b*c^2*d^2*e^30/
(x*e + d) + 60*b*c^2*d^3*e^30/(x*e + d)^2 - 20*b*c^2*d^4*e^30/(x*e + d)^3 +
 3*b*c^2*d^5*e^30/(x*e + d)^4 + 48*b^2*c*d*e^31/(x*e + d) - 36*b^2*c*d^2*e^
31/(x*e + d)^2 + 16*b^2*c*d^3*e^31/(x*e + d)^3 - 3*b^2*c*d^4*e^31/(x*e + d)
^4 - 4*b^3*e^32/(x*e + d) + 6*b^3*d*e^32/(x*e + d)^2 - 4*b^3*d^2*e^32/(x*e
+ d)^3 + b^3*d^3*e^32/(x*e + d)^4)*e^(-36)
```

3.254 $\int \frac{(bx+cx^2)^3}{(d+ex)^6} dx$

Optimal. Leaf size=218

$$-\frac{3c\left(b^2e^2-5bcde+5c^2d^2\right)}{e^7(d+ex)}+\frac{(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{2e^7(d+ex)^2}-\frac{d(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)}{e^7(d+ex)^3}-\frac{3c^2(2cd-be)\log}{e^7}$$

```
[Out] (c^3*x)/e^6 - (d^3*(c*d - b*e)^3)/(5*e^7*(d + e*x)^5) + (3*d^2*(c*d - b*e)^
2*(2*c*d - b*e))/(4*e^7*(d + e*x)^4) - (d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*
e + b^2*e^2))/(e^7*(d + e*x)^3) + ((2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e +
 b^2*e^2))/(2*e^7*(d + e*x)^2) - (3*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2))/(e
^7*(d + e*x)) - (3*c^2*(2*c*d - b*e)*Log[d + e*x])/e^7
```

Rubi [A] time = 0.183574, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$-\frac{3c\left(b^2e^2-5bcde+5c^2d^2\right)}{e^7(d+ex)}+\frac{(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{2e^7(d+ex)^2}-\frac{d(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)}{e^7(d+ex)^3}-\frac{3c^2(2cd-be)\log}{e^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^3/(d + e*x)^6,x]
```

```
[Out] (c^3*x)/e^6 - (d^3*(c*d - b*e)^3)/(5*e^7*(d + e*x)^5) + (3*d^2*(c*d - b*e)^
2*(2*c*d - b*e))/(4*e^7*(d + e*x)^4) - (d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*
e + b^2*e^2))/(e^7*(d + e*x)^3) + ((2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e +
 b^2*e^2))/(2*e^7*(d + e*x)^2) - (3*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2))/(e
^7*(d + e*x)) - (3*c^2*(2*c*d - b*e)*Log[d + e*x])/e^7
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(bx+cx^2\right)^3}{(d+ex)^6} dx = \int \left(\frac{c^3}{e^6}+\frac{d^3(cd-be)^3}{e^6(d+ex)^6}-\frac{3d^2(cd-be)^2(2cd-be)}{e^6(d+ex)^5}+\frac{3d(cd-be)\left(5c^2d^2-5bcde+b^2e^2\right)}{e^6(d+ex)^4}+\frac{(2cd-be)\left(\right.}{}\right.$$

$$=\frac{c^3x}{e^6}-\frac{d^3(cd-be)^3}{5e^7(d+ex)^5}+\frac{3d^2(cd-be)^2(2cd-be)}{4e^7(d+ex)^4}-\frac{d(cd-be)\left(5c^2d^2-5bcde+b^2e^2\right)}{e^7(d+ex)^3}+\frac{(2cd-be)\left(10c^2\right.}{2e^7(}$$

Mathematica [A] time = 0.108065, size = 242, normalized size = 1.11

$$-\frac{12b^2ce^2\left(10d^2e^2x^2+5d^3ex+d^4+10de^3x^3+5e^4x^4\right)+b^3e^3\left(5d^2ex+d^3+10de^2x^2+10e^3x^3\right)-bc^2de\left(1100d^2e^2x^2+625\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^3/(d + e*x)^6,x]
```

```
[Out] -(b^3*e^3*(d^3 + 5*d^2*e*x + 10*d*e^2*x^2 + 10*e^3*x^3) + 12*b^2*c*e^2*(d^4
 + 5*d^3*e*x + 10*d^2*e^2*x^2 + 10*d*e^3*x^3 + 5*e^4*x^4) - b*c^2*d*e*(137*
d^4 + 625*d^3*e*x + 1100*d^2*e^2*x^2 + 900*d*e^3*x^3 + 300*e^4*x^4) + 2*c^3
*(87*d^6 + 375*d^5*e*x + 600*d^4*e^2*x^2 + 400*d^3*e^3*x^3 + 50*d^2*e^4*x^4
 - 50*d*e^5*x^5 - 10*e^6*x^6) + 60*c^2*(2*c*d - b*e)*(d + e*x)^5*Log[d + e*
x])/(20*e^7*(d + e*x)^5)
```

Maple [A] time = 0.053, size = 379, normalized size = 1.7

$$\frac{c^3x}{e^6} - \frac{b^3}{2\,e^4\,(ex+d)^2} + 6\,\frac{b^2cd}{e^5\,(ex+d)^2} - 15\,\frac{bc^2d^2}{e^6\,(ex+d)^2} + 10\,\frac{c^3d^3}{e^7\,(ex+d)^2} + \frac{d^3b^3}{5\,e^4\,(ex+d)^5} - \frac{3\,d^4b^2c}{5\,e^5\,(ex+d)^5} + \frac{3\,d^5bc^2}{5\,e^6\,(ex+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^3/(e*x+d)^6,x)
```

```
[Out] c^3*x/e^6-1/2/e^4/(e*x+d)^2*b^3+6/e^5/(e*x+d)^2*b^2*c*d-15/e^6/(e*x+d)^2*b*
c^2*d^2+10/e^7/(e*x+d)^2*c^3*d^3+1/5*d^3/e^4/(e*x+d)^5*b^3-3/5*d^4/e^5/(e*x
+d)^5*b^2*c+3/5*d^5/e^6/(e*x+d)^5*b*c^2-1/5*d^6/e^7/(e*x+d)^5*c^3+3*c^2/e^6
*ln(e*x+d)*b-6*c^3*d*ln(e*x+d)/e^7+d/e^4/(e*x+d)^3*b^3-6*d^2/e^5/(e*x+d)^3*
b^2*c+10*d^3/e^6/(e*x+d)^3*b*c^2-5*d^4/e^7/(e*x+d)^3*c^3-3*c/e^5/(e*x+d)*b^
2+15*c^2/e^6/(e*x+d)*b*d-15*c^3/e^7/(e*x+d)*d^2-3/4*d^2/e^4/(e*x+d)^4*b^3+3
*d^3/e^5/(e*x+d)^4*b^2*c-15/4*d^4/e^6/(e*x+d)^4*b*c^2+3/2*d^5/e^7/(e*x+d)^4
*c^3
```

Maxima [A] time = 1.18628, size = 420, normalized size = 1.93

$$-\frac{174\,c^3d^6 - 137\,bc^2d^5e + 12\,b^2cd^4e^2 + b^3d^3e^3 + 60\left(5\,c^3d^2e^4 - 5\,bc^2de^5 + b^2ce^6\right)x^4 + 10\left(100\,c^3d^3e^3 - 90\,bc^2d^2e^4 + 12\,b\right.}{20\left(e^{12}x^5 + 5\,de^{11}x^4 + 10\,d^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^6,x, algorithm="maxima")
```

```
[Out] -1/20*(174*c^3*d^6 - 137*b*c^2*d^5*e + 12*b^2*c*d^4*e^2 + b^3*d^3*e^3 + 60*
(5*c^3*d^2*e^4 - 5*b*c^2*d*e^5 + b^2*c*e^6)*x^4 + 10*(100*c^3*d^3*e^3 - 90*
b*c^2*d^2*e^4 + 12*b^2*c*d*e^5 + b^3*e^6)*x^3 + 10*(130*c^3*d^4*e^2 - 110*b
*c^2*d^3*e^3 + 12*b^2*c*d^2*e^4 + b^3*d*e^5)*x^2 + 5*(154*c^3*d^5*e - 125*b
*c^2*d^4*e^2 + 12*b^2*c*d^3*e^3 + b^3*d^2*e^4)*x)/(e^12*x^5 + 5*d*e^11*x^4
+ 10*d^2*e^10*x^3 + 10*d^3*e^9*x^2 + 5*d^4*e^8*x + d^5*e^7) + c^3*x/e^6 - 3
*(2*c^3*d - b*c^2*e)*log(e*x + d)/e^7
```

Fricas [B] time = 1.7414, size = 953, normalized size = 4.37

$$\frac{20\,c^3e^6x^6 + 100\,c^3de^5x^5 - 174\,c^3d^6 + 137\,bc^2d^5e - 12\,b^2cd^4e^2 - b^3d^3e^3 - 20\left(5\,c^3d^2e^4 - 15\,bc^2de^5 + 3\,b^2ce^6\right)x^4 - 10\left(80\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^6,x, algorithm="fricas")
```

```
[Out] 1/20*(20*c^3*e^6*x^6 + 100*c^3*d*e^5*x^5 - 174*c^3*d^6 + 137*b*c^2*d^5*e -
12*b^2*c*d^4*e^2 - b^3*d^3*e^3 - 20*(5*c^3*d^2*e^4 - 15*b*c^2*d*e^5 + 3*b^2
*c*e^6)*x^4 - 10*(80*c^3*d^3*e^3 - 90*b*c^2*d^2*e^4 + 12*b^2*c*d*e^5 + b^3*
e^6)*x^3 - 10*(120*c^3*d^4*e^2 - 110*b*c^2*d^3*e^3 + 12*b^2*c*d^2*e^4 + b^3
*d*e^5)*x^2 - 5*(150*c^3*d^5*e - 125*b*c^2*d^4*e^2 + 12*b^2*c*d^3*e^3 + b^3
*d^2*e^4)*x - 60*(2*c^3*d^6 - b*c^2*d^5*e + (2*c^3*d*e^5 - b*c^2*e^6)*x^5 +
 5*(2*c^3*d^2*e^4 - b*c^2*d*e^5)*x^4 + 10*(2*c^3*d^3*e^3 - b*c^2*d^2*e^4)*x
^3 + 10*(2*c^3*d^4*e^2 - b*c^2*d^3*e^3)*x^2 + 5*(2*c^3*d^5*e - b*c^2*d^4*e^
2)*x)*log(e*x + d))/(e^12*x^5 + 5*d*e^11*x^4 + 10*d^2*e^10*x^3 + 10*d^3*e^9
*x^2 + 5*d^4*e^8*x + d^5*e^7)
```

Sympy [A] time = 38.9897, size = 326, normalized size = 1.5

$$\frac{c^3x}{e^6} + \frac{3c^2\left(be - 2cd\right)\log\left(d + ex\right)}{e^7} - \frac{b^3d^3e^3 + 12b^2cd^4e^2 - 137bc^2d^5e + 174c^3d^6 + x^4\left(60b^2ce^6 - 300bc^2de^5 + 300c^3d^2e^4\right) + \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**3/(e*x+d)**6,x)
```

```
[Out] c**3*x/e**6 + 3*c**2*(b*e - 2*c*d)*log(d + e*x)/e**7 - (b**3*d**3*e**3 + 12
*b**2*c*d**4*e**2 - 137*b*c**2*d**5*e + 174*c**3*d**6 + x**4*(60*b**2*c*e**
6 - 300*b*c**2*d*e**5 + 300*c**3*d**2*e**4) + x**3*(10*b**3*e**6 + 120*b**2
*c*d*e**5 - 900*b*c**2*d**2*e**4 + 1000*c**3*d**3*e**3) + x**2*(10*b**3*d*e
**5 + 120*b**2*c*d**2*e**4 - 1100*b*c**2*d**3*e**3 + 1300*c**3*d**4*e**2) +
 x*(5*b**3*d**2*e**4 + 60*b**2*c*d**3*e**3 - 625*b*c**2*d**4*e**2 + 770*c**
3*d**5*e))/(20*d**5*e**7 + 100*d**4*e**8*x + 200*d**3*e**9*x**2 + 200*d**2*
e**10*x**3 + 100*d*e**11*x**4 + 20*e**12*x**5)
```

Giac [A] time = 1.26617, size = 339, normalized size = 1.56

$$c^3xe^{(-6)} - 3\left(2c^3d - bc^2e\right)e^{(-7)}\log\left(|xe + d|\right) - \frac{\left(174c^3d^6 - 137bc^2d^5e + 12b^2cd^4e^2 + b^3d^3e^3 + 60\left(5c^3d^2e^4 - 5bc^2de^5 + b^2\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^6,x, algorithm="giac")
```

```
[Out] c^3*x*e^(-6) - 3*(2*c^3*d - b*c^2*e)*e^(-7)*log(abs(x*e + d)) - 1/20*(174*c
^3*d^6 - 137*b*c^2*d^5*e + 12*b^2*c*d^4*e^2 + b^3*d^3*e^3 + 60*(5*c^3*d^2*e
^4 - 5*b*c^2*d*e^5 + b^2*c*e^6)*x^4 + 10*(100*c^3*d^3*e^3 - 90*b*c^2*d^2*e^
4 + 12*b^2*c*d*e^5 + b^3*e^6)*x^3 + 10*(130*c^3*d^4*e^2 - 110*b*c^2*d^3*e^3
 + 12*b^2*c*d^2*e^4 + b^3*d*e^5)*x^2 + 5*(154*c^3*d^5*e - 125*b*c^2*d^4*e^2
 + 12*b^2*c*d^3*e^3 + b^3*d^2*e^4)*x)*e^(-7)/(x*e + d)^5
```

3.255 $\int \frac{(bx+cx^2)^3}{(d+ex)^7} dx$

Optimal. Leaf size=228

$$-\frac{3c\left(b^2e^2-5bcde+5c^2d^2\right)}{2e^7(d+ex)^2}+\frac{(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{3e^7(d+ex)^3}-\frac{3d(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)}{4e^7(d+ex)^4}+\frac{3c^2(2cd-b}{e^7(d+ex)}$$

```
[Out] -(d^3*(c*d - b*e)^3)/(6*e^7*(d + e*x)^6) + (3*d^2*(c*d - b*e)^2*(2*c*d - b*
e))/(5*e^7*(d + e*x)^5) - (3*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2
))/(4*e^7*(d + e*x)^4) + ((2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e + b^2*e^2)
)/(3*e^7*(d + e*x)^3) - (3*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2))/(2*e^7*(d +
 e*x)^2) + (3*c^2*(2*c*d - b*e))/(e^7*(d + e*x)) + (c^3*Log[d + e*x])/e^7
```

Rubi [A] time = 0.174597, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$-\frac{3c\left(b^2e^2-5bcde+5c^2d^2\right)}{2e^7(d+ex)^2}+\frac{(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{3e^7(d+ex)^3}-\frac{3d(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)}{4e^7(d+ex)^4}+\frac{3c^2(2cd-b}{e^7(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^3/(d + e*x)^7,x]
```


```
[Out] -(d^3*(c*d - b*e)^3)/(6*e^7*(d + e*x)^6) + (3*d^2*(c*d - b*e)^2*(2*c*d - b*
e))/(5*e^7*(d + e*x)^5) - (3*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2
))/(4*e^7*(d + e*x)^4) + ((2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e + b^2*e^2)
)/(3*e^7*(d + e*x)^3) - (3*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2))/(2*e^7*(d +
 e*x)^2) + (3*c^2*(2*c*d - b*e))/(e^7*(d + e*x)) + (c^3*Log[d + e*x])/e^7
```


Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(bx+cx^2)^3}{(d+ex)^7}dx = \int \left(\frac{d^3(cd-be)^3}{e^6(d+ex)^7}-\frac{3d^2(cd-be)^2(2cd-be)}{e^6(d+ex)^6}+\frac{3d(cd-be)\left(5c^2d^2-5bcde+b^2e^2\right)}{e^6(d+ex)^5}+\frac{(2cd-be)\left(-1\right.}{e}\right.$$

$$= -\frac{d^3(cd-be)^3}{6e^7(d+ex)^6}+\frac{3d^2(cd-be)^2(2cd-be)}{5e^7(d+ex)^5}-\frac{3d(cd-be)\left(5c^2d^2-5bcde+b^2e^2\right)}{4e^7(d+ex)^4}+\frac{(2cd-be)\left(10c^2\right.}{3e^7(}$$

Mathematica [A] time = 0.0813611, size = 231, normalized size = 1.01

$$-6b^2ce^2\left(15d^2e^2x^2+6d^3ex+d^4+20de^3x^3+15e^4x^4\right)-b^3e^3\left(6d^2ex+d^3+15de^2x^2+20e^3x^3\right)-30bc^2e\left(15d^3e^2x^2+20\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^3/(d + e*x)^7,x]
```

```
[Out] (-(b^3*e^3*(d^3 + 6*d^2*e*x + 15*d*e^2*x^2 + 20*e^3*x^3)) - 6*b^2*c*e^2*(d^
4 + 6*d^3*e*x + 15*d^2*e^2*x^2 + 20*d*e^3*x^3 + 15*e^4*x^4) - 30*b*c^2*e*(d
^5 + 6*d^4*e*x + 15*d^3*e^2*x^2 + 20*d^2*e^3*x^3 + 15*d*e^4*x^4 + 6*e^5*x^5
) + c^3*d*(147*d^5 + 822*d^4*e*x + 1875*d^3*e^2*x^2 + 2200*d^2*e^3*x^3 + 13
50*d*e^4*x^4 + 360*e^5*x^5) + 60*c^3*(d + e*x)^6*Log[d + e*x])/(60*e^7*(d +
 e*x)^6)
```

Maple [A] time = 0.05, size = 387, normalized size = 1.7

$$-\frac{3\,b^2c}{2\,e^5\,(ex+d)^2}+\frac{15\,bc^2d}{2\,e^6\,(ex+d)^2}-\frac{15\,c^3d^2}{2\,e^7\,(ex+d)^2}-\frac{3\,b^3d^2}{5\,e^4\,(ex+d)^5}+\frac{12\,b^2cd^3}{5\,e^5\,(ex+d)^5}-3\,\frac{bc^2d^4}{e^6\,(ex+d)^5}+\frac{6\,c^3d^5}{5\,e^7\,(ex+d)^5}+\frac{c^3\ln(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^3/(e*x+d)^7,x)
```

```
[Out] -3/2*c/e^5/(e*x+d)^2*b^2+15/2*c^2/e^6/(e*x+d)^2*b*d-15/2*c^3/e^7/(e*x+d)^2*
d^2-3/5*d^2/e^4/(e*x+d)^5*b^3+12/5*d^3/e^5/(e*x+d)^5*b^2*c-3*d^4/e^6/(e*x+d
)^5*b*c^2+6/5*d^5/e^7/(e*x+d)^5*c^3+c^3*ln(e*x+d)/e^7-1/3/e^4/(e*x+d)^3*b^3
+4/e^5/(e*x+d)^3*b^2*c*d-10/e^6/(e*x+d)^3*b*c^2*d^2+20/3/e^7/(e*x+d)^3*c^3*
d^3+1/6*d^3/e^4/(e*x+d)^6*b^3-1/2*d^4/e^5/(e*x+d)^6*b^2*c+1/2*d^5/e^6/(e*x+
d)^6*b*c^2-1/6*d^6/e^7/(e*x+d)^6*c^3-3*c^2/e^6/(e*x+d)*b+6*c^3*d/e^7/(e*x+d
)+3/4*d/e^4/(e*x+d)^4*b^3-9/2*d^2/e^5/(e*x+d)^4*b^2*c+15/2*d^3/e^6/(e*x+d)^
4*b*c^2-15/4*d^4/e^7/(e*x+d)^4*c^3
```

Maxima [A] time = 1.20406, size = 447, normalized size = 1.96

$$\frac{147\,c^3d^6-30\,bc^2d^5e-6\,b^2cd^4e^2-b^3d^3e^3+180\left(2\,c^3de^5-bc^2e^6\right)x^5+90\left(15\,c^3d^2e^4-5\,bc^2de^5-b^2ce^6\right)x^4+20\left(110\,c^3d^3e^3\right.}{60\left(e^{13}x^6+6\,de^{12}x^5+15\,d^2e^{1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^7,x, algorithm="maxima")
```

```
[Out] 1/60*(147*c^3*d^6 - 30*b*c^2*d^5*e - 6*b^2*c*d^4*e^2 - b^3*d^3*e^3 + 180*(2
*c^3*d*e^5 - b*c^2*e^6)*x^5 + 90*(15*c^3*d^2*e^4 - 5*b*c^2*d*e^5 - b^2*c*e^
6)*x^4 + 20*(110*c^3*d^3*e^3 - 30*b*c^2*d^2*e^4 - 6*b^2*c*d*e^5 - b^3*e^6)*
x^3 + 15*(125*c^3*d^4*e^2 - 30*b*c^2*d^3*e^3 - 6*b^2*c*d^2*e^4 - b^3*d*e^5)
*x^2 + 6*(137*c^3*d^5*e - 30*b*c^2*d^4*e^2 - 6*b^2*c*d^3*e^3 - b^3*d^2*e^4)
*x)/(e^13*x^6 + 6*d*e^12*x^5 + 15*d^2*e^11*x^4 + 20*d^3*e^10*x^3 + 15*d^4*e
^9*x^2 + 6*d^5*e^8*x + d^6*e^7) + c^3*log(e*x + d)/e^7
```

Fricas [A] time = 1.66275, size = 840, normalized size = 3.68

$$\frac{147\,c^3d^6-30\,bc^2d^5e-6\,b^2cd^4e^2-b^3d^3e^3+180\left(2\,c^3de^5-bc^2e^6\right)x^5+90\left(15\,c^3d^2e^4-5\,bc^2de^5-b^2ce^6\right)x^4+20\left(110\,c^3d^3e^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^7,x, algorithm="fricas")
```

```
[Out] 1/60*(147*c^3*d^6 - 30*b*c^2*d^5*e - 6*b^2*c*d^4*e^2 - b^3*d^3*e^3 + 180*(2
*c^3*d*e^5 - b*c^2*e^6)*x^5 + 90*(15*c^3*d^2*e^4 - 5*b*c^2*d*e^5 - b^2*c*e^
6)*x^4 + 20*(110*c^3*d^3*e^3 - 30*b*c^2*d^2*e^4 - 6*b^2*c*d*e^5 - b^3*e^6)*
x^3 + 15*(125*c^3*d^4*e^2 - 30*b*c^2*d^3*e^3 - 6*b^2*c*d^2*e^4 - b^3*d*e^5)
*x^2 + 6*(137*c^3*d^5*e - 30*b*c^2*d^4*e^2 - 6*b^2*c*d^3*e^3 - b^3*d^2*e^4)
*x + 60*(c^3*e^6*x^6 + 6*c^3*d*e^5*x^5 + 15*c^3*d^2*e^4*x^4 + 20*c^3*d^3*e^
3*x^3 + 15*c^3*d^4*e^2*x^2 + 6*c^3*d^5*e*x + c^3*d^6)*log(e*x + d))/(e^13*x
^6 + 6*d*e^12*x^5 + 15*d^2*e^11*x^4 + 20*d^3*e^10*x^3 + 15*d^4*e^9*x^2 + 6*
d^5*e^8*x + d^6*e^7)
```

Sympy [A] time = 103.83, size = 343, normalized size = 1.5

$$\frac{c^3\log(d+ex)}{e^7} - \frac{b^3d^3e^3 + 6b^2cd^4e^2 + 30bc^2d^5e - 147c^3d^6 + x^5\left(180bc^2e^6 - 360c^3de^5\right) + x^4\left(90b^2ce^6 + 450bc^2de^5 - 135\right.}{60d^6e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**3/(e*x+d)**7,x)
```

```
[Out] c**3*log(d + e*x)/e**7 - (b**3*d**3*e**3 + 6*b**2*c*d**4*e**2 + 30*b*c**2*d
**5*e - 147*c**3*d**6 + x**5*(180*b*c**2*e**6 - 360*c**3*d*e**5) + x**4*(90
*b**2*c*e**6 + 450*b*c**2*d*e**5 - 1350*c**3*d**2*e**4) + x**3*(20*b**3*e**
6 + 120*b**2*c*d*e**5 + 600*b*c**2*d**2*e**4 - 2200*c**3*d**3*e**3) + x**2*
(15*b**3*d*e**5 + 90*b**2*c*d**2*e**4 + 450*b*c**2*d**3*e**3 - 1875*c**3*d*
*4*e**2) + x*(6*b**3*d**2*e**4 + 36*b**2*c*d**3*e**3 + 180*b*c**2*d**4*e**2
 - 822*c**3*d**5*e))/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 +
 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x
**6)
```

Giac [A] time = 1.26178, size = 351, normalized size = 1.54

$$c^3e^{(-7)}\log(|xe+d|) + \frac{\left(180\left(2c^3de^4 - bc^2e^5\right)x^5 + 90\left(15c^3d^2e^3 - 5bc^2de^4 - b^2ce^5\right)x^4 + 20\left(110c^3d^3e^2 - 30bc^2d^2e^3 - 6\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^7,x, algorithm="giac")
```

```
[Out] c^3*e^(-7)*log(abs(x*e + d)) + 1/60*(180*(2*c^3*d*e^4 - b*c^2*e^5)*x^5 + 90
*(15*c^3*d^2*e^3 - 5*b*c^2*d*e^4 - b^2*c*e^5)*x^4 + 20*(110*c^3*d^3*e^2 - 3
0*b*c^2*d^2*e^3 - 6*b^2*c*d*e^4 - b^3*e^5)*x^3 + 15*(125*c^3*d^4*e - 30*b*c
^2*d^3*e^2 - 6*b^2*c*d^2*e^3 - b^3*d*e^4)*x^2 + 6*(137*c^3*d^5 - 30*b*c^2*d
^4*e - 6*b^2*c*d^3*e^2 - b^3*d^2*e^3)*x + (147*c^3*d^6 - 30*b*c^2*d^5*e - 6
*b^2*c*d^4*e^2 - b^3*d^3*e^3)*e^(-1))*e^(-6)/(x*e + d)^6
```

3.256 $\int \frac{(bx+cx^2)^3}{(d+ex)^8} dx$

Optimal. Leaf size=230

$$-\frac{c(b^2e^2-5bcde+5c^2d^2)}{e^7(d+ex)^3}+\frac{(2cd-be)(b^2e^2-10bcde+10c^2d^2)}{4e^7(d+ex)^4}-\frac{3d(cd-be)(b^2e^2-5bcde+5c^2d^2)}{5e^7(d+ex)^5}+\frac{3c^2(2cd-be)}{2e^7(d+ex)^2}+$$

```
[Out] -(d^3*(c*d - b*e)^3)/(7*e^7*(d + e*x)^7) + (d^2*(c*d - b*e)^2*(2*c*d - b*e)
)/(2*e^7*(d + e*x)^6) - (3*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2))
/(5*e^7*(d + e*x)^5) + ((2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e + b^2*e^2))/
(4*e^7*(d + e*x)^4) - (c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2))/(e^7*(d + e*x)^
3) + (3*c^2*(2*c*d - b*e))/(2*e^7*(d + e*x)^2) - c^3/(e^7*(d + e*x))
```

Rubi [A] time = 0.159425, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$-\frac{c(b^2e^2-5bcde+5c^2d^2)}{e^7(d+ex)^3}+\frac{(2cd-be)(b^2e^2-10bcde+10c^2d^2)}{4e^7(d+ex)^4}-\frac{3d(cd-be)(b^2e^2-5bcde+5c^2d^2)}{5e^7(d+ex)^5}+\frac{3c^2(2cd-be)}{2e^7(d+ex)^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^3/(d + e*x)^8,x]
```

```
[Out] -(d^3*(c*d - b*e)^3)/(7*e^7*(d + e*x)^7) + (d^2*(c*d - b*e)^2*(2*c*d - b*e)
)/(2*e^7*(d + e*x)^6) - (3*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2))
/(5*e^7*(d + e*x)^5) + ((2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e + b^2*e^2))/
(4*e^7*(d + e*x)^4) - (c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2))/(e^7*(d + e*x)^
3) + (3*c^2*(2*c*d - b*e))/(2*e^7*(d + e*x)^2) - c^3/(e^7*(d + e*x))
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(bx+cx^2)^3}{(d+ex)^8} dx = \int \left(\frac{d^3(cd-be)^3}{e^6(d+ex)^8}-\frac{3d^2(cd-be)^2(2cd-be)}{e^6(d+ex)^7}+\frac{3d(cd-be)(5c^2d^2-5bcde+b^2e^2)}{e^6(d+ex)^6}+\frac{(2cd-be)(-10c^2}{e^6(d}\right.$$

$$=-\frac{d^3(cd-be)^3}{7e^7(d+ex)^7}+\frac{d^2(cd-be)^2(2cd-be)}{2e^7(d+ex)^6}-\frac{3d(cd-be)(5c^2d^2-5bcde+b^2e^2)}{5e^7(d+ex)^5}+\frac{(2cd-be)(10c^2d^2-}{4e^7(d+e}$$

Mathematica [A] time = 0.0711211, size = 221, normalized size = 0.96

$$-\frac{4b^2ce^2(21d^2e^2x^2+7d^3ex+d^4+35de^3x^3+35e^4x^4)+b^3e^3(7d^2ex+d^3+21de^2x^2+35e^3x^3)+10bc^2e(21d^3e^2x^2+35d^2}{140e^7(d+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^3/(d + e*x)^8,x]
```

```
[Out] -(b^3*e^3*(d^3 + 7*d^2*e*x + 21*d*e^2*x^2 + 35*e^3*x^3) + 4*b^2*c*e^2*(d^4
+ 7*d^3*e*x + 21*d^2*e^2*x^2 + 35*d*e^3*x^3 + 35*e^4*x^4) + 10*b*c^2*e*(d^5
 + 7*d^4*e*x + 21*d^3*e^2*x^2 + 35*d^2*e^3*x^3 + 35*d*e^4*x^4 + 21*e^5*x^5)
 + 20*c^3*(d^6 + 7*d^5*e*x + 21*d^4*e^2*x^2 + 35*d^3*e^3*x^3 + 35*d^2*e^4*x
^4 + 21*d*e^5*x^5 + 7*e^6*x^6))/(140*e^7*(d + e*x)^7)
```

Maple [A] time = 0.048, size = 274, normalized size = 1.2

$$-\frac{3\,c^2\,(be-2\,cd)}{2\,e^7\,(ex+d)^2}+\frac{d^3\left(b^3e^3-3\,b^2cde^2+3\,bc^2d^2e-c^3d^3\right)}{7\,e^7\,(ex+d)^7}+\frac{3\,d\left(b^3e^3-6\,b^2cde^2+10\,bc^2d^2e-5\,c^3d^3\right)}{5\,e^7\,(ex+d)^5}-\frac{c\left(b^2e^2-5\,bcd\right.}{e^7\,(ex+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^3/(e*x+d)^8,x)
```

```
[Out] -3/2*c^2*(b*e-2*c*d)/e^7/(e*x+d)^2+1/7*d^3*(b^3*e^3-3*b^2*c*d*e^2+3*b*c^2*d
^2*e-c^3*d^3)/e^7/(e*x+d)^7+3/5*d*(b^3*e^3-6*b^2*c*d*e^2+10*b*c^2*d^2*e-5*c
^3*d^3)/e^7/(e*x+d)^5-c*(b^2*e^2-5*b*c*d*e+5*c^2*d^2)/e^7/(e*x+d)^3-1/2*d^2
*(b^3*e^3-4*b^2*c*d*e^2+5*b*c^2*d^2*e-2*c^3*d^3)/e^7/(e*x+d)^6-c^3/e^7/(e*x
+d)-1/4*(b^3*e^3-12*b^2*c*d*e^2+30*b*c^2*d^2*e-20*c^3*d^3)/e^7/(e*x+d)^4
```

Maxima [A] time = 1.17091, size = 451, normalized size = 1.96

$$-\frac{140\,c^3e^6x^6+20\,c^3d^6+10\,bc^2d^5e+4\,b^2cd^4e^2+b^3d^3e^3+210\left(2\,c^3de^5+bc^2e^6\right)x^5+70\left(10\,c^3d^2e^4+5\,bc^2de^5+2\,b^2ce^6\right)}{140\left(e^{14}x^7+7\,de^{13}x^6+21\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^8,x, algorithm="maxima")
```

```
[Out] -1/140*(140*c^3*e^6*x^6 + 20*c^3*d^6 + 10*b*c^2*d^5*e + 4*b^2*c*d^4*e^2 + b
^3*d^3*e^3 + 210*(2*c^3*d*e^5 + b*c^2*e^6)*x^5 + 70*(10*c^3*d^2*e^4 + 5*b*c
^2*d*e^5 + 2*b^2*c*e^6)*x^4 + 35*(20*c^3*d^3*e^3 + 10*b*c^2*d^2*e^4 + 4*b^2
*c*d*e^5 + b^3*e^6)*x^3 + 21*(20*c^3*d^4*e^2 + 10*b*c^2*d^3*e^3 + 4*b^2*c*d
^2*e^4 + b^3*d*e^5)*x^2 + 7*(20*c^3*d^5*e + 10*b*c^2*d^4*e^2 + 4*b^2*c*d^3*
e^3 + b^3*d^2*e^4)*x)/(e^14*x^7 + 7*d*e^13*x^6 + 21*d^2*e^12*x^5 + 35*d^3*e
^11*x^4 + 35*d^4*e^10*x^3 + 21*d^5*e^9*x^2 + 7*d^6*e^8*x + d^7*e^7)
```

Fricas [A] time = 1.75796, size = 701, normalized size = 3.05

$$-\frac{140\,c^3e^6x^6+20\,c^3d^6+10\,bc^2d^5e+4\,b^2cd^4e^2+b^3d^3e^3+210\left(2\,c^3de^5+bc^2e^6\right)x^5+70\left(10\,c^3d^2e^4+5\,bc^2de^5+2\,b^2ce^6\right)}{140\left(e^{14}x^7+7\,de^{13}x^6+21\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^8,x, algorithm="fricas")
```

```
[Out] -1/140*(140*c^3*e^6*x^6 + 20*c^3*d^6 + 10*b*c^2*d^5*e + 4*b^2*c*d^4*e^2 + b
^3*d^3*e^3 + 210*(2*c^3*d*e^5 + b*c^2*e^6)*x^5 + 70*(10*c^3*d^2*e^4 + 5*b*c
```

```
^2*d*e^5 + 2*b^2*c*e^6)*x^4 + 35*(20*c^3*d^3*e^3 + 10*b*c^2*d^2*e^4 + 4*b^2
*c*d*e^5 + b^3*e^6)*x^3 + 21*(20*c^3*d^4*e^2 + 10*b*c^2*d^3*e^3 + 4*b^2*c*d
^2*e^4 + b^3*d*e^5)*x^2 + 7*(20*c^3*d^5*e + 10*b*c^2*d^4*e^2 + 4*b^2*c*d^3*
e^3 + b^3*d^2*e^4)*x)/(e^14*x^7 + 7*d*e^13*x^6 + 21*d^2*e^12*x^5 + 35*d^3*e
^11*x^4 + 35*d^4*e^10*x^3 + 21*d^5*e^9*x^2 + 7*d^6*e^8*x + d^7*e^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**3/(e*x+d)**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23886, size = 360, normalized size = 1.57

$$-\frac{\left(140\, c^3 x^6 e^6 + 420\, c^3 d x^5 e^5 + 700\, c^3 d^2 x^4 e^4 + 700\, c^3 d^3 x^3 e^3 + 420\, c^3 d^4 x^2 e^2 + 140\, c^3 d^5 x e + 20\, c^3 d^6 + 210\, b c^2 x^5 e^6 + 350\, b c^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^8,x, algorithm="giac")
```

```
[Out] -1/140*(140*c^3*x^6*e^6 + 420*c^3*d*x^5*e^5 + 700*c^3*d^2*x^4*e^4 + 700*c^3
*d^3*x^3*e^3 + 420*c^3*d^4*x^2*e^2 + 140*c^3*d^5*x*e + 20*c^3*d^6 + 210*b*c
^2*x^5*e^6 + 350*b*c^2*d*x^4*e^5 + 350*b*c^2*d^2*x^3*e^4 + 210*b*c^2*d^3*x^
2*e^3 + 70*b*c^2*d^4*x*e^2 + 10*b*c^2*d^5*e + 140*b^2*c*x^4*e^6 + 140*b^2*c
*d*x^3*e^5 + 84*b^2*c*d^2*x^2*e^4 + 28*b^2*c*d^3*x*e^3 + 4*b^2*c*d^4*e^2 +
35*b^3*x^3*e^6 + 21*b^3*d*x^2*e^5 + 7*b^3*d^2*x*e^4 + b^3*d^3*e^3)*e^(-7)/(
x*e + d)^7
```

3.257 $\int \frac{(bx+cx^2)^3}{(d+ex)^9}\,dx$

Optimal. Leaf size=231

$$-\frac{3c\left(b^2e^2-5bcde+5c^2d^2\right)}{4e^7(d+ex)^4}+\frac{(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{5e^7(d+ex)^5}-\frac{d(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)}{2e^7(d+ex)^6}+\frac{c^2(2cd-be)}{e^7(d+ex)^3}$$

```
[Out] -(d^3*(c*d - b*e)^3)/(8*e^7*(d + e*x)^8) + (3*d^2*(c*d - b*e)^2*(2*c*d - b*
e))/(7*e^7*(d + e*x)^7) - (d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2))
/(2*e^7*(d + e*x)^6) + ((2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e + b^2*e^2))/
(5*e^7*(d + e*x)^5) - (3*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2))/(4*e^7*(d + e
*x)^4) + (c^2*(2*c*d - b*e))/(e^7*(d + e*x)^3) - c^3/(2*e^7*(d + e*x)^2)
```

Rubi [A] time = 0.157936, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$-\frac{3c\left(b^2e^2-5bcde+5c^2d^2\right)}{4e^7(d+ex)^4}+\frac{(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{5e^7(d+ex)^5}-\frac{d(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)}{2e^7(d+ex)^6}+\frac{c^2(2cd-be)}{e^7(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^3/(d + e*x)^9,x]
```


```
[Out] -(d^3*(c*d - b*e)^3)/(8*e^7*(d + e*x)^8) + (3*d^2*(c*d - b*e)^2*(2*c*d - b*
e))/(7*e^7*(d + e*x)^7) - (d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2))
/(2*e^7*(d + e*x)^6) + ((2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e + b^2*e^2))/
(5*e^7*(d + e*x)^5) - (3*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2))/(4*e^7*(d + e
*x)^4) + (c^2*(2*c*d - b*e))/(e^7*(d + e*x)^3) - c^3/(2*e^7*(d + e*x)^2)
```


Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(bx+cx^2\right)^3}{(d+ex)^9}\,dx = \int \left(\frac{d^3(cd-be)^3}{e^6(d+ex)^9}-\frac{3d^2(cd-be)^2(2cd-be)}{e^6(d+ex)^8}+\frac{3d(cd-be)\left(5c^2d^2-5bcde+b^2e^2\right)}{e^6(d+ex)^7}+\frac{(2cd-be)\left(-1\right.}{e}\right.$$

$$= -\frac{d^3(cd-be)^3}{8e^7(d+ex)^8}+\frac{3d^2(cd-be)^2(2cd-be)}{7e^7(d+ex)^7}-\frac{d(cd-be)\left(5c^2d^2-5bcde+b^2e^2\right)}{2e^7(d+ex)^6}+\frac{(2cd-be)\left(10c^2d\right.}{5e^7(d}$$

Mathematica [A] time = 0.0805159, size = 221, normalized size = 0.96

$$-\frac{3b^2ce^2\left(28d^2e^2x^2+8d^3ex+d^4+56de^3x^3+70e^4x^4\right)+b^3e^3\left(8d^2ex+d^3+28de^2x^2+56e^3x^3\right)+5bc^2e\left(28d^3e^2x^2+56\right.}{280e^7(d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^3/(d + e*x)^9,x]
```

```
[Out] -(b^3*e^3*(d^3 + 8*d^2*e*x + 28*d*e^2*x^2 + 56*e^3*x^3) + 3*b^2*c*e^2*(d^4
+ 8*d^3*e*x + 28*d^2*e^2*x^2 + 56*d*e^3*x^3 + 70*e^4*x^4) + 5*b*c^2*e*(d^5
+ 8*d^4*e*x + 28*d^3*e^2*x^2 + 56*d^2*e^3*x^3 + 70*d*e^4*x^4 + 56*e^5*x^5)
+ 5*c^3*(d^6 + 8*d^5*e*x + 28*d^4*e^2*x^2 + 56*d^3*e^3*x^3 + 70*d^2*e^4*x^4
 + 56*d*e^5*x^5 + 28*e^6*x^6))/(280*e^7*(d + e*x)^8)
```

Maple [A] time = 0.049, size = 274, normalized size = 1.2

$$-\frac{c^3}{2\,e^7\,(ex+d)^2}-\frac{3\,d^2\left(b^3e^3-4\,b^2cde^2+5\,bc^2d^2e-2\,c^3d^3\right)}{7\,e^7\,(ex+d)^7}-\frac{b^3e^3-12\,b^2cde^2+30\,bc^2d^2e-20\,c^3d^3}{5\,e^7\,(ex+d)^5}+\frac{d^3\left(b^3e^3-3\,b^2cde^2\right.}{8\,e^7\,(e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^3/(e*x+d)^9,x)
```

```
[Out] -1/2*c^3/e^7/(e*x+d)^2-3/7*d^2*(b^3*e^3-4*b^2*c*d*e^2+5*b*c^2*d^2*e-2*c^3*d
^3)/e^7/(e*x+d)^7-1/5*(b^3*e^3-12*b^2*c*d*e^2+30*b*c^2*d^2*e-20*c^3*d^3)/e^
7/(e*x+d)^5+1/8*d^3*(b^3*e^3-3*b^2*c*d*e^2+3*b*c^2*d^2*e-c^3*d^3)/e^7/(e*x+
d)^8-c^2*(b*e-2*c*d)/e^7/(e*x+d)^3+1/2*d*(b^3*e^3-6*b^2*c*d*e^2+10*b*c^2*d^
2*e-5*c^3*d^3)/e^7/(e*x+d)^6-3/4*c*(b^2*e^2-5*b*c*d*e+5*c^2*d^2)/e^7/(e*x+d
)^4
```

Maxima [A] time = 1.07856, size = 464, normalized size = 2.01

$$-\frac{140\,c^3e^6x^6+5\,c^3d^6+5\,bc^2d^5e+3\,b^2cd^4e^2+b^3d^3e^3+280\left(c^3de^5+bc^2e^6\right)x^5+70\left(5\,c^3d^2e^4+5\,bc^2de^5+3\,b^2ce^6\right)x^4+56}{280\left(e^{15}x^8+8\,de^{14}x^7+28\,d^2e^{13}x^6+56\,d^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^9,x, algorithm="maxima")
```

```
[Out] -1/280*(140*c^3*e^6*x^6 + 5*c^3*d^6 + 5*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 + b^3
*d^3*e^3 + 280*(c^3*d*e^5 + b*c^2*e^6)*x^5 + 70*(5*c^3*d^2*e^4 + 5*b*c^2*d*
e^5 + 3*b^2*c*e^6)*x^4 + 56*(5*c^3*d^3*e^3 + 5*b*c^2*d^2*e^4 + 3*b^2*c*d*e^
5 + b^3*e^6)*x^3 + 28*(5*c^3*d^4*e^2 + 5*b*c^2*d^3*e^3 + 3*b^2*c*d^2*e^4 +
b^3*d*e^5)*x^2 + 8*(5*c^3*d^5*e + 5*b*c^2*d^4*e^2 + 3*b^2*c*d^3*e^3 + b^3*d
^2*e^4)*x)/(e^15*x^8 + 8*d*e^14*x^7 + 28*d^2*e^13*x^6 + 56*d^3*e^12*x^5 + 7
0*d^4*e^11*x^4 + 56*d^5*e^10*x^3 + 28*d^6*e^9*x^2 + 8*d^7*e^8*x + d^8*e^7)
```

Fricas [A] time = 1.83764, size = 710, normalized size = 3.07

$$-\frac{140\,c^3e^6x^6+5\,c^3d^6+5\,bc^2d^5e+3\,b^2cd^4e^2+b^3d^3e^3+280\left(c^3de^5+bc^2e^6\right)x^5+70\left(5\,c^3d^2e^4+5\,bc^2de^5+3\,b^2ce^6\right)x^4+56}{280\left(e^{15}x^8+8\,de^{14}x^7+28\,d^2e^{13}x^6+56\,d^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^9,x, algorithm="fricas")
```

```
[Out] -1/280*(140*c^3*e^6*x^6 + 5*c^3*d^6 + 5*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 + b^3
*d^3*e^3 + 280*(c^3*d*e^5 + b*c^2*e^6)*x^5 + 70*(5*c^3*d^2*e^4 + 5*b*c^2*d*
e^5 + 3*b^2*c*e^6)*x^4 + 56*(5*c^3*d^3*e^3 + 5*b*c^2*d^2*e^4 + 3*b^2*c*d*e^
5 + b^3*e^6)*x^3 + 28*(5*c^3*d^4*e^2 + 5*b*c^2*d^3*e^3 + 3*b^2*c*d^2*e^4 +
b^3*d*e^5)*x^2 + 8*(5*c^3*d^5*e + 5*b*c^2*d^4*e^2 + 3*b^2*c*d^3*e^3 + b^3*d
^2*e^4)*x)/(e^15*x^8 + 8*d*e^14*x^7 + 28*d^2*e^13*x^6 + 56*d^3*e^12*x^5 + 7
0*d^4*e^11*x^4 + 56*d^5*e^10*x^3 + 28*d^6*e^9*x^2 + 8*d^7*e^8*x + d^8*e^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**3/(e*x+d)**9,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3183, size = 360, normalized size = 1.56

$$-\frac{(140\,c^3x^6e^6 + 280\,c^3dx^5e^5 + 350\,c^3d^2x^4e^4 + 280\,c^3d^3x^3e^3 + 140\,c^3d^4x^2e^2 + 40\,c^3d^5xe + 5\,c^3d^6 + 280\,bc^2x^5e^6 + 350\,bc}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^9,x, algorithm="giac")
```

```
[Out] -1/280*(140*c^3*x^6*e^6 + 280*c^3*d*x^5*e^5 + 350*c^3*d^2*x^4*e^4 + 280*c^3
*d^3*x^3*e^3 + 140*c^3*d^4*x^2*e^2 + 40*c^3*d^5*x*e + 5*c^3*d^6 + 280*b*c^2
*x^5*e^6 + 350*b*c^2*d*x^4*e^5 + 280*b*c^2*d^2*x^3*e^4 + 140*b*c^2*d^3*x^2*
e^3 + 40*b*c^2*d^4*x*e^2 + 5*b*c^2*d^5*e + 210*b^2*c*x^4*e^6 + 168*b^2*c*d*
x^3*e^5 + 84*b^2*c*d^2*x^2*e^4 + 24*b^2*c*d^3*x*e^3 + 3*b^2*c*d^4*e^2 + 56*
b^3*x^3*e^6 + 28*b^3*d*x^2*e^5 + 8*b^3*d^2*x*e^4 + b^3*d^3*e^3)*e^(-7)/(x*e
 + d)^8
```

3.258 $\int \frac{(bx+cx^2)^3}{(d+ex)^{10}}\, dx$

Optimal. Leaf size=234

$$-\frac{3c\left(b^2e^2-5bcde+5c^2d^2\right)}{5e^7(d+ex)^5}+\frac{(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{6e^7(d+ex)^6}-\frac{3d(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)}{7e^7(d+ex)^7}+\frac{3c^2(2cd-be)}{4e^7(d+ex)^4}$$

```
[Out] -(d^3*(c*d - b*e)^3)/(9*e^7*(d + e*x)^9) + (3*d^2*(c*d - b*e)^2*(2*c*d - b*
e))/(8*e^7*(d + e*x)^8) - (3*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2
))/(7*e^7*(d + e*x)^7) + ((2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e + b^2*e^2)
)/(6*e^7*(d + e*x)^6) - (3*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2))/(5*e^7*(d +
 e*x)^5) + (3*c^2*(2*c*d - b*e))/(4*e^7*(d + e*x)^4) - c^3/(3*e^7*(d + e*x)
^3)
```

Rubi [A] time = 0.156602, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$-\frac{3c\left(b^2e^2-5bcde+5c^2d^2\right)}{5e^7(d+ex)^5}+\frac{(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{6e^7(d+ex)^6}-\frac{3d(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)}{7e^7(d+ex)^7}+\frac{3c^2(2cd-be)}{4e^7(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^3/(d + e*x)^10,x]
```


```
[Out] -(d^3*(c*d - b*e)^3)/(9*e^7*(d + e*x)^9) + (3*d^2*(c*d - b*e)^2*(2*c*d - b*
e))/(8*e^7*(d + e*x)^8) - (3*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2
))/(7*e^7*(d + e*x)^7) + ((2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e + b^2*e^2)
)/(6*e^7*(d + e*x)^6) - (3*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2))/(5*e^7*(d +
 e*x)^5) + (3*c^2*(2*c*d - b*e))/(4*e^7*(d + e*x)^4) - c^3/(3*e^7*(d + e*x)
^3)
```


Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(bx+cx^2\right)^3}{(d+ex)^{10}}\, dx = \int \left(\frac{d^3(cd-be)^3}{e^6(d+ex)^{10}}-\frac{3d^2(cd-be)^2(2cd-be)}{e^6(d+ex)^9}+\frac{3d(cd-be)\left(5c^2d^2-5bcde+b^2e^2\right)}{e^6(d+ex)^8}+\frac{(2cd-be)\left(-10c^2\right.}{e^6(d}\right.$$

$$= -\frac{d^3(cd-be)^3}{9e^7(d+ex)^9}+\frac{3d^2(cd-be)^2(2cd-be)}{8e^7(d+ex)^8}-\frac{3d(cd-be)\left(5c^2d^2-5bcde+b^2e^2\right)}{7e^7(d+ex)^7}+\frac{(2cd-be)\left(10c^2d^2\right.}{6e^7(d+}$$

Mathematica [A] time = 0.0746255, size = 222, normalized size = 0.95

$$-\frac{12b^2ce^2\left(36d^2e^2x^2+9d^3ex+d^4+84de^3x^3+126e^4x^4\right)+5b^3e^3\left(9d^2ex+d^3+36de^2x^2+84e^3x^3\right)+15bc^2e\left(36d^3e^2x^2+8\right.}{2520e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^3/(d + e*x)^10,x]
```

```
[Out] -(5*b^3*e^3*(d^3 + 9*d^2*e*x + 36*d*e^2*x^2 + 84*e^3*x^3) + 12*b^2*c*e^2*(d
^4 + 9*d^3*e*x + 36*d^2*e^2*x^2 + 84*d*e^3*x^3 + 126*e^4*x^4) + 15*b*c^2*e*
(d^5 + 9*d^4*e*x + 36*d^3*e^2*x^2 + 84*d^2*e^3*x^3 + 126*d*e^4*x^4 + 126*e^
5*x^5) + 10*c^3*(d^6 + 9*d^5*e*x + 36*d^4*e^2*x^2 + 84*d^3*e^3*x^3 + 126*d^
2*e^4*x^4 + 126*d*e^5*x^5 + 84*e^6*x^6))/(2520*e^7*(d + e*x)^9)
```

Maple [A] time = 0.051, size = 274, normalized size = 1.2

$$\frac{d^3\left(b^3e^3-3b^2cde^2+3bc^2d^2e-c^3d^3\right)}{9e^7(ex+d)^9}+\frac{3d\left(b^3e^3-6b^2cde^2+10bc^2d^2e-5c^3d^3\right)}{7e^7(ex+d)^7}-\frac{3c\left(b^2e^2-5bcde+5c^2d^2\right)}{5e^7(ex+d)^5}-\frac{b^3e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^3/(e*x+d)^10,x)
```

```
[Out] 1/9*d^3*(b^3*e^3-3*b^2*c*d*e^2+3*b*c^2*d^2*e-c^3*d^3)/e^7/(e*x+d)^9+3/7*d*(
b^3*e^3-6*b^2*c*d*e^2+10*b*c^2*d^2*e-5*c^3*d^3)/e^7/(e*x+d)^7-3/5*c*(b^2*e^
2-5*b*c*d*e+5*c^2*d^2)/e^7/(e*x+d)^5-1/6*(b^3*e^3-12*b^2*c*d*e^2+30*b*c^2*d
^2*e-20*c^3*d^3)/e^7/(e*x+d)^6-3/8*d^2*(b^3*e^3-4*b^2*c*d*e^2+5*b*c^2*d^2*e
-2*c^3*d^3)/e^7/(e*x+d)^8-1/3*c^3/e^7/(e*x+d)^3-3/4*c^2*(b*e-2*c*d)/e^7/(e*
x+d)^4
```

Maxima [A] time = 1.23416, size = 487, normalized size = 2.08

$$-\frac{840c^3e^6x^6+10c^3d^6+15bc^2d^5e+12b^2cd^4e^2+5b^3d^3e^3+630\left(2c^3de^5+3bc^2e^6\right)x^5+126\left(10c^3d^2e^4+15bc^2de^5+1\right.}{2520\left(e^{16}x^9+9de^{15}x^8+36d^2e^{14}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^10,x, algorithm="maxima")
```

```
[Out] -1/2520*(840*c^3*e^6*x^6 + 10*c^3*d^6 + 15*b*c^2*d^5*e + 12*b^2*c*d^4*e^2 +
 5*b^3*d^3*e^3 + 630*(2*c^3*d*e^5 + 3*b*c^2*e^6)*x^5 + 126*(10*c^3*d^2*e^4
+ 15*b*c^2*d*e^5 + 12*b^2*c*e^6)*x^4 + 84*(10*c^3*d^3*e^3 + 15*b*c^2*d^2*e^
4 + 12*b^2*c*d*e^5 + 5*b^3*e^6)*x^3 + 36*(10*c^3*d^4*e^2 + 15*b*c^2*d^3*e^3
 + 12*b^2*c*d^2*e^4 + 5*b^3*d*e^5)*x^2 + 9*(10*c^3*d^5*e + 15*b*c^2*d^4*e^2
 + 12*b^2*c*d^3*e^3 + 5*b^3*d^2*e^4)*x)/(e^16*x^9 + 9*d*e^15*x^8 + 36*d^2*e
^14*x^7 + 84*d^3*e^13*x^6 + 126*d^4*e^12*x^5 + 126*d^5*e^11*x^4 + 84*d^6*e^
10*x^3 + 36*d^7*e^9*x^2 + 9*d^8*e^8*x + d^9*e^7)
```

Fricas [A] time = 1.60613, size = 776, normalized size = 3.32

$$-\frac{840c^3e^6x^6+10c^3d^6+15bc^2d^5e+12b^2cd^4e^2+5b^3d^3e^3+630\left(2c^3de^5+3bc^2e^6\right)x^5+126\left(10c^3d^2e^4+15bc^2de^5+1\right.}{2520\left(e^{16}x^9+9de^{15}x^8+36d^2e^{14}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^10,x, algorithm="fricas")
```

```
[Out] -1/2520*(840*c^3*e^6*x^6 + 10*c^3*d^6 + 15*b*c^2*d^5*e + 12*b^2*c*d^4*e^2 +
 5*b^3*d^3*e^3 + 630*(2*c^3*d*e^5 + 3*b*c^2*e^6)*x^5 + 126*(10*c^3*d^2*e^4
+ 15*b*c^2*d*e^5 + 12*b^2*c*e^6)*x^4 + 84*(10*c^3*d^3*e^3 + 15*b*c^2*d^2*e^
4 + 12*b^2*c*d*e^5 + 5*b^3*e^6)*x^3 + 36*(10*c^3*d^4*e^2 + 15*b*c^2*d^3*e^3
 + 12*b^2*c*d^2*e^4 + 5*b^3*d*e^5)*x^2 + 9*(10*c^3*d^5*e + 15*b*c^2*d^4*e^2
 + 12*b^2*c*d^3*e^3 + 5*b^3*d^2*e^4)*x)/(e^16*x^9 + 9*d*e^15*x^8 + 36*d^2*e
^14*x^7 + 84*d^3*e^13*x^6 + 126*d^4*e^12*x^5 + 126*d^5*e^11*x^4 + 84*d^6*e^
10*x^3 + 36*d^7*e^9*x^2 + 9*d^8*e^8*x + d^9*e^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**3/(e*x+d)**10,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25734, size = 362, normalized size = 1.55

$$-\frac{(840\,c^3x^6e^6 + 1260\,c^3dx^5e^5 + 1260\,c^3d^2x^4e^4 + 840\,c^3d^3x^3e^3 + 360\,c^3d^4x^2e^2 + 90\,c^3d^5xe + 10\,c^3d^6 + 1890\,bc^2x^5e^6 + 1890}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^10,x, algorithm="giac")
```

```
[Out] -1/2520*(840*c^3*x^6*e^6 + 1260*c^3*d*x^5*e^5 + 1260*c^3*d^2*x^4*e^4 + 840*
c^3*d^3*x^3*e^3 + 360*c^3*d^4*x^2*e^2 + 90*c^3*d^5*x*e + 10*c^3*d^6 + 1890*
b*c^2*x^5*e^6 + 1890*b*c^2*d*x^4*e^5 + 1260*b*c^2*d^2*x^3*e^4 + 540*b*c^2*d
^3*x^2*e^3 + 135*b*c^2*d^4*x*e^2 + 15*b*c^2*d^5*e + 1512*b^2*c*x^4*e^6 + 10
08*b^2*c*d*x^3*e^5 + 432*b^2*c*d^2*x^2*e^4 + 108*b^2*c*d^3*x*e^3 + 12*b^2*c
*d^4*e^2 + 420*b^3*x^3*e^6 + 180*b^3*d*x^2*e^5 + 45*b^3*d^2*x*e^4 + 5*b^3*d
^3*e^3)*e^(-7)/(x*e + d)^9
```

3.259 $\int \frac{(d+ex)^4}{bx+cx^2} \, dx$

Optimal. Leaf size=99

$$\frac{e^2x\left(b^2e^2-4bcde+6c^2d^2\right)}{c^3}+\frac{e^3x^2(4cd-be)}{2c^2}-\frac{(cd-be)^4\log(b+cx)}{bc^4}+\frac{d^4\log(x)}{b}+\frac{e^4x^3}{3c}$$

```
[Out] (e^2*(6*c^2*d^2 - 4*b*c*d*e + b^2*e^2)*x)/c^3 + (e^3*(4*c*d - b*e)*x^2)/(2*
c^2) + (e^4*x^3)/(3*c) + (d^4*Log[x])/b - ((c*d - b*e)^4*Log[b + c*x])/(b*c
^4)
```

Rubi [A] time = 0.0908252, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$\frac{e^2x\left(b^2e^2-4bcde+6c^2d^2\right)}{c^3}+\frac{e^3x^2(4cd-be)}{2c^2}-\frac{(cd-be)^4\log(b+cx)}{bc^4}+\frac{d^4\log(x)}{b}+\frac{e^4x^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(b*x + c*x^2),x]
```

```
[Out] (e^2*(6*c^2*d^2 - 4*b*c*d*e + b^2*e^2)*x)/c^3 + (e^3*(4*c*d - b*e)*x^2)/(2*
c^2) + (e^4*x^3)/(3*c) + (d^4*Log[x])/b - ((c*d - b*e)^4*Log[b + c*x])/(b*c
^4)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^4}{bx+cx^2}\,dx &= \int \left(\frac{e^2\left(6c^2d^2-4bcde+b^2e^2\right)}{c^3}+\frac{d^4}{bx}+\frac{e^3(4cd-be)x}{c^2}+\frac{e^4x^2}{c}-\frac{(-cd+be)^4}{bc^3(b+cx)}\right)dx\\ &= \frac{e^2\left(6c^2d^2-4bcde+b^2e^2\right)x}{c^3}+\frac{e^3(4cd-be)x^2}{2c^2}+\frac{e^4x^3}{3c}+\frac{d^4\log(x)}{b}-\frac{(cd-be)^4\log(b+cx)}{bc^4}\end{aligned}$$

Mathematica [A] time = 0.0454209, size = 90, normalized size = 0.91

$$\frac{bce^2x\left(6b^2e^2-3bce(8d+ex)+2c^2\left(18d^2+6dex+e^2x^2\right)\right)-6(cd-be)^4\log(b+cx)+6c^4d^4\log(x)}{6bc^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(b*x + c*x^2),x]
```

```
[Out] (b*c*e^2*x*(6*b^2*e^2 - 3*b*c*e*(8*d + e*x) + 2*c^2*(18*d^2 + 6*d*e*x + e^2
*x^2)) + 6*c^4*d^4*Log[x] - 6*(c*d - b*e)^4*Log[b + c*x])/(6*b*c^4)
```

Maple [A] time = 0.053, size = 162, normalized size = 1.6

$$\frac{e^4x^3}{3c} - \frac{e^4x^2b}{2c^2} + 2\frac{de^3x^2}{c} + \frac{e^4b^2x}{c^3} - 4\frac{e^3bdx}{c^2} + 6\frac{d^2e^2x}{c} + \frac{d^4\ln(x)}{b} - \frac{b^3\ln(cx+b)e^4}{c^4} + 4\frac{b^2\ln(cx+b)de^3}{c^3} - 6\frac{b\ln(cx+b}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(c*x^2+b*x),x)
```

```
[Out] 1/3*e^4*x^3/c-1/2*e^4/c^2*x^2*b+2*d*e^3*x^2/c+e^4/c^3*b^2*x-4*e^3/c^2*b*d*x
+6*e^2/c*d^2*x+d^4*ln(x)/b-b^3/c^4*ln(c*x+b)*e^4+4*b^2/c^3*ln(c*x+b)*d*e^3-
6*b/c^2*ln(c*x+b)*d^2*e^2+4/c*ln(c*x+b)*d^3*e-1/b*ln(c*x+b)*d^4
```

Maxima [A] time = 1.14139, size = 192, normalized size = 1.94

$$\frac{d^4\log(x)}{b} + \frac{2c^2e^4x^3 + 3\left(4c^2de^3 - bce^4\right)x^2 + 6\left(6c^2d^2e^2 - 4bcde^3 + b^2e^4\right)x}{6c^3} - \frac{\left(c^4d^4 - 4bc^3d^3e + 6b^2c^2d^2e^2 - 4b^3cde^3 +\right.}{bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] d^4*log(x)/b + 1/6*(2*c^2*e^4*x^3 + 3*(4*c^2*d*e^3 - b*c*e^4)*x^2 + 6*(6*c^
2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*x)/c^3 - (c^4*d^4 - 4*b*c^3*d^3*e + 6*b^
2*c^2*d^2*e^2 - 4*b^3*c*d*e^3 + b^4*e^4)*log(c*x + b)/(b*c^4)
```

Fricas [A] time = 1.70411, size = 312, normalized size = 3.15

$$\frac{2bc^3e^4x^3 + 6c^4d^4\log(x) + 3\left(4bc^3de^3 - b^2c^2e^4\right)x^2 + 6\left(6bc^3d^2e^2 - 4b^2c^2de^3 + b^3ce^4\right)x - 6\left(c^4d^4 - 4bc^3d^3e + 6b^2c^2d^2e^2\right.}{6bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*c^3*e^4*x^3 + 6*c^4*d^4*log(x) + 3*(4*b*c^3*d*e^3 - b^2*c^2*e^4)*x
^2 + 6*(6*b*c^3*d^2*e^2 - 4*b^2*c^2*d*e^3 + b^3*c*e^4)*x - 6*(c^4*d^4 - 4*b
*c^3*d^3*e + 6*b^2*c^2*d^2*e^2 - 4*b^3*c*d*e^3 + b^4*e^4)*log(c*x + b))/(b*
c^4)
```

Sympy [A] time = 5.8469, size = 165, normalized size = 1.67

$$\frac{e^4x^3}{3c} - \frac{x^2\left(be^4 - 4cde^3\right)}{2c^2} + \frac{x\left(b^2e^4 - 4bcde^3 + 6c^2d^2e^2\right)}{c^3} + \frac{d^4\log(x)}{b} - \frac{(be-cd)^4\log\left(x + \frac{bc^3d^4 + \frac{b(be-cd)^4}{c}}{b^4e^4 - 4b^3cde^3 + 6b^2c^2d^2e^2 - 4bc^3d^3e + 2c^4d^4}\right)}{bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(c*x**2+b*x),x)
```

```
[Out] e**4*x**3/(3*c) - x**2*(b*e**4 - 4*c*d*e**3)/(2*c**2) + x*(b**2*e**4 - 4*b*
c*d*e**3 + 6*c**2*d**2*e**2)/c**3 + d**4*log(x)/b - (b*e - c*d)**4*log(x +
(b*c**3*d**4 + b*(b*e - c*d)**4/c)/(b**4*e**4 - 4*b**3*c*d*e**3 + 6*b**2*c*
*2*d**2*e**2 - 4*b*c**3*d**3*e + 2*c**4*d**4))/(b*c**4)
```

Giac [A] time = 1.24519, size = 184, normalized size = 1.86

$$\frac{d^4 \log(|x|)}{b} + \frac{2\,c^2x^3e^4 + 12\,c^2dx^2e^3 + 36\,c^2d^2xe^2 - 3\,bcx^2e^4 - 24\,bcdxe^3 + 6\,b^2xe^4}{6\,c^3} - \frac{\left(c^4d^4 - 4\,bc^3d^3e + 6\,b^2c^2d^2e^2 - 4\,l\right.}{bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] d^4*log(abs(x))/b + 1/6*(2*c^2*x^3*e^4 + 12*c^2*d*x^2*e^3 + 36*c^2*d^2*x*e^
2 - 3*b*c*x^2*e^4 - 24*b*c*d*x*e^3 + 6*b^2*x*e^4)/c^3 - (c^4*d^4 - 4*b*c^3*
d^3*e + 6*b^2*c^2*d^2*e^2 - 4*b^3*c*d*e^3 + b^4*e^4)*log(abs(c*x + b))/(b*c
^4)
```

3.260 $\int \frac{(d+ex)^3}{bx+cx^2}\,dx$

Optimal. Leaf size=64

$$\frac{e^2x(3cd-be)}{c^2}-\frac{(cd-be)^3\log(b+cx)}{bc^3}+\frac{d^3\log(x)}{b}+\frac{e^3x^2}{2c}$$

```
[Out] (e^2*(3*c*d - b*e)*x)/c^2 + (e^3*x^2)/(2*c) + (d^3*Log[x])/b - ((c*d - b*e)
^3*Log[b + c*x])/(b*c^3)
```

Rubi [A] time = 0.0574003, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$\frac{e^2x(3cd-be)}{c^2}-\frac{(cd-be)^3\log(b+cx)}{bc^3}+\frac{d^3\log(x)}{b}+\frac{e^3x^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(b*x + c*x^2),x]
```

```
[Out] (e^2*(3*c*d - b*e)*x)/c^2 + (e^3*x^2)/(2*c) + (d^3*Log[x])/b - ((c*d - b*e)
^3*Log[b + c*x])/(b*c^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^3}{bx+cx^2}\,dx &= \int \left(\frac{e^2(3cd-be)}{c^2}+\frac{d^3}{bx}+\frac{e^3x}{c}+\frac{(-cd+be)^3}{bc^2(b+cx)}\right)dx\\ &= \frac{e^2(3cd-be)x}{c^2}+\frac{e^3x^2}{2c}+\frac{d^3\log(x)}{b}-\frac{(cd-be)^3\log(b+cx)}{bc^3}\end{aligned}$$

Mathematica [A] time = 0.0275231, size = 59, normalized size = 0.92

$$\frac{bce^2x(-2be+6cd+cex)-2(cd-be)^3\log(b+cx)+2c^3d^3\log(x)}{2bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(b*x + c*x^2),x]
```

```
[Out] (b*c*e^2*x*(6*c*d - 2*b*e + c*e*x) + 2*c^3*d^3*Log[x] - 2*(c*d - b*e)^3*Log
[b + c*x])/(2*b*c^3)
```

Maple [A] time = 0.048, size = 103, normalized size = 1.6

$$\frac{e^3x^2}{2c}-\frac{e^3xb}{c^2}+3\frac{de^2x}{c}+\frac{d^3\ln(x)}{b}+\frac{b^2\ln(cx+b)e^3}{c^3}-3\frac{b\ln(cx+b)de^2}{c^2}+3\frac{\ln(cx+b)d^2e}{c}-\frac{\ln(cx+b)d^3}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^2+b*x),x)
```

```
[Out] 1/2*e^3*x^2/c-e^3/c^2*x*b+3*d*e^2*x/c+d^3*ln(x)/b+1/c^3*b^2*ln(c*x+b)*e^3-3
/c^2*b*ln(c*x+b)*d*e^2+3/c*ln(c*x+b)*d^2*e-1/b*ln(c*x+b)*d^3
```

Maxima [A] time = 1.09648, size = 123, normalized size = 1.92

$$\frac{d^3\log(x)}{b}+\frac{ce^3x^2+2\left(3\,cde^2-be^3\right)x}{2\,c^2}-\frac{\left(c^3d^3-3\,bc^2d^2e+3\,b^2cde^2-b^3e^3\right)\log(cx+b)}{bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] d^3*log(x)/b + 1/2*(c*e^3*x^2 + 2*(3*c*d*e^2 - b*e^3)*x)/c^2 - (c^3*d^3 - 3
*b*c^2*d^2*e + 3*b^2*c*d*e^2 - b^3*e^3)*log(c*x + b)/(b*c^3)
```

Fricas [A] time = 1.70464, size = 204, normalized size = 3.19

$$\frac{bc^2e^3x^2+2\,c^3d^3\log(x)+2\left(3\,bc^2de^2-b^2ce^3\right)x-2\left(c^3d^3-3\,bc^2d^2e+3\,b^2cde^2-b^3e^3\right)\log(cx+b)}{2\,bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] 1/2*(b*c^2*e^3*x^2 + 2*c^3*d^3*log(x) + 2*(3*b*c^2*d*e^2 - b^2*c*e^3)*x - 2
*(c^3*d^3 - 3*b*c^2*d^2*e + 3*b^2*c*d*e^2 - b^3*e^3)*log(c*x + b))/(b*c^3)
```

Sympy [B] time = 5.58811, size = 112, normalized size = 1.75

$$\frac{e^3x^2}{2c}-\frac{x\left(be^3-3cde^2\right)}{c^2}+\frac{d^3\log(x)}{b}+\frac{(be-cd)^3\log\left(x+\frac{-bc^2d^3+\frac{b(be-cd)^3}{c}}{b^3e^3-3b^2cde^2+3bc^2d^2e-2c^3d^3}\right)}{bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**2+b*x),x)
```

```
[Out] e**3*x**2/(2*c) - x*(b*e**3 - 3*c*d*e**2)/c**2 + d**3*log(x)/b + (b*e - c*d
)**3*log(x + (-b*c**2*d**3 + b*(b*e - c*d)**3/c)/(b**3*e**3 - 3*b**2*c*d*e*
*2 + 3*b*c**2*d**2*e - 2*c**3*d**3))/(b*c**3)
```

Giac [A] time = 1.30506, size = 117, normalized size = 1.83

$$\frac{d^3\log(|x|)}{b} + \frac{cx^2e^3 + 6\,cdxe^2 - 2\,bxe^3}{2\,c^2} - \frac{\left(c^3d^3 - 3\,bc^2d^2e + 3\,b^2cde^2 - b^3e^3\right)\log(|cx + b|)}{bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] d^3*log(abs(x))/b + 1/2*(c*x^2*e^3 + 6*c*d*x*e^2 - 2*b*x*e^3)/c^2 - (c^3*d^
3 - 3*b*c^2*d^2*e + 3*b^2*c*d*e^2 - b^3*e^3)*log(abs(c*x + b))/(b*c^3)
```

3.261 $\int \frac{(d+ex)^2}{bx+cx^2} dx$

Optimal. Leaf size=42

$$-\frac{(cd-be)^2\log(b+cx)}{bc^2}+\frac{d^2\log(x)}{b}+\frac{e^2x}{c}$$

```
[Out] (e^2*x)/c + (d^2*Log[x])/b - ((c*d - b*e)^2*Log[b + c*x])/(b*c^2)
```

Rubi [A] time = 0.0335049, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$-\frac{(cd-be)^2\log(b+cx)}{bc^2}+\frac{d^2\log(x)}{b}+\frac{e^2x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(b*x + c*x^2),x]
```

```
[Out] (e^2*x)/c + (d^2*Log[x])/b - ((c*d - b*e)^2*Log[b + c*x])/(b*c^2)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2}{bx+cx^2}\,dx &= \int \left(\frac{e^2}{c}+\frac{d^2}{bx}-\frac{(-cd+be)^2}{bc(b+cx)}\right)dx\\ &= \frac{e^2x}{c}+\frac{d^2\log(x)}{b}-\frac{(cd-be)^2\log(b+cx)}{bc^2}\end{aligned}$$

Mathematica [A] time = 0.0185061, size = 42, normalized size = 1.

$$\frac{-(cd-be)^2\log(b+cx)+bce^2x+c^2d^2\log(x)}{bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(b*x + c*x^2),x]
```

```
[Out] (b*c*e^2*x + c^2*d^2*Log[x] - (c*d - b*e)^2*Log[b + c*x])/(b*c^2)
```

Maple [A] time = 0.052, size = 61, normalized size = 1.5

$$\frac{e^2x}{c}+\frac{d^2\ln(x)}{b}-\frac{b\ln(cx+b)e^2}{c^2}+2\,\frac{\ln(cx+b)\,de}{c}-\frac{\ln(cx+b)\,d^2}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^2+b*x),x)
```

```
[Out] e^2*x/c+d^2*ln(x)/b-b/c^2*ln(c*x+b)*e^2+2/c*ln(c*x+b)*d*e-1/b*ln(c*x+b)*d^2
```

Maxima [A] time = 1.1029, size = 72, normalized size = 1.71

$$\frac{e^2x}{c} + \frac{d^2\log(x)}{b} - \frac{(c^2d^2 - 2\,bcde + b^2e^2)\log(cx+b)}{bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] e^2*x/c + d^2*log(x)/b - (c^2*d^2 - 2*b*c*d*e + b^2*e^2)*log(c*x + b)/(b*c^
2)
```

Fricas [A] time = 1.68877, size = 115, normalized size = 2.74

$$\frac{bce^2x + c^2d^2\log(x) - (c^2d^2 - 2\,bcde + b^2e^2)\log(cx+b)}{bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] (b*c*e^2*x + c^2*d^2*log(x) - (c^2*d^2 - 2*b*c*d*e + b^2*e^2)*log(c*x + b))
/(b*c^2)
```

Sympy [B] time = 2.63886, size = 73, normalized size = 1.74

$$\frac{e^2x}{c} + \frac{d^2\log(x)}{b} - \frac{(be-cd)^2\log\left(x + \frac{bcd^2 + \frac{b(be-cd)^2}{c}}{b^2e^2 - 2bcde + 2c^2d^2}\right)}{bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**2+b*x),x)
```

```
[Out] e**2*x/c + d**2*log(x)/b - (b*e - c*d)**2*log(x + (b*c*d**2 + b*(b*e - c*d)
**2/c)/(b**2*e**2 - 2*b*c*d*e + 2*c**2*d**2))/(b*c**2)
```

Giac [A] time = 1.28209, size = 73, normalized size = 1.74

$$\frac{d^2\log(|x|)}{b} + \frac{xe^2}{c} - \frac{(c^2d^2 - 2\,bcde + b^2e^2)\log(|cx+b|)}{bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] d^2*log(abs(x))/b + x*e^2/c - (c^2*d^2 - 2*b*c*d*e + b^2*e^2)*log(abs(c*x +
 b))/(b*c^2)
```

3.262 $\int \frac{d+ex}{bx+cx^2}\,dx$

Optimal. Leaf size=30

$$\frac{d\log(x)}{b}-\frac{(cd-be)\log(b+cx)}{bc}$$

```
[Out] (d*Log[x])/b - ((c*d - b*e)*Log[b + c*x])/(b*c)
```

Rubi [A] time = 0.0207153, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {631}

$$\frac{d\log(x)}{b}-\frac{(cd-be)\log(b+cx)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(b*x + c*x^2),x]

[Out] (d*Log[x])/b - ((c*d - b*e)*Log[b + c*x])/(b*c)
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex}{bx+cx^2}\,dx&=\int \left(\frac{d}{bx}+\frac{-cd+be}{b(b+cx)}\right)dx\\&=\frac{d\log(x)}{b}-\frac{(cd-be)\log(b+cx)}{bc}\end{aligned}$$

Mathematica [A] time = 0.0097819, size = 29, normalized size = 0.97

$$\frac{(be-cd)\log(b+cx)}{bc}+\frac{d\log(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(b*x + c*x^2),x]

[Out] (d*Log[x])/b + ((-(c*d) + b*e)*Log[b + c*x])/(b*c)
```

Maple [A] time = 0.052, size = 32, normalized size = 1.1

$$\frac{d\ln(x)}{b}+\frac{\ln(cx+b)e}{c}-\frac{\ln(cx+b)d}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^2+b*x),x)
```

```
[Out] 1/b*d*ln(x)+1/c*ln(c*x+b)*e-1/b*ln(c*x+b)*d
```

Maxima [A] time = 1.13507, size = 41, normalized size = 1.37

$$\frac{d\log(x)}{b} - \frac{(cd - be)\log(cx + b)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] d*log(x)/b - (c*d - b*e)*log(c*x + b)/(b*c)
```

Fricas [A] time = 1.67953, size = 63, normalized size = 2.1

$$\frac{cd\log(x) - (cd - be)\log(cx + b)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] (c*d*log(x) - (c*d - b*e)*log(c*x + b))/(b*c)
```

Sympy [A] time = 2.02744, size = 41, normalized size = 1.37

$$\frac{d\log(x)}{b} + \frac{(be - cd)\log\left(x + \frac{-bd + \frac{b(be-cd)}{c}}{be-2cd}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**2+b*x),x)
```

```
[Out] d*log(x)/b + (b*e - c*d)*log(x + (-b*d + b*(b*e - c*d)/c)/(b*e - 2*c*d))/(b
*c)
```

Giac [A] time = 1.32039, size = 45, normalized size = 1.5

$$\frac{d\log(|x|)}{b} - \frac{(cd - be)\log(|cx + b|)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] d*log(abs(x))/b - (c*d - b*e)*log(abs(c*x + b))/(b*c)
```

3.263 $\int \frac{1}{bx+cx^2} dx$

Optimal. Leaf size=18

$$\frac{\log(x)}{b} - \frac{\log(b+cx)}{b}$$

[Out] Log[x]/b - Log[b + c*x]/b

Rubi [A] time = 0.0030771, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {615}

$$\frac{\log(x)}{b} - \frac{\log(b+cx)}{b}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2)^(-1),x]

[Out] Log[x]/b - Log[b + c*x]/b

Rule 615

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[Log[x]/b, x] - Simp
[Log[RemoveContent[b + c*x, x]]/b, x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\int \frac{1}{bx+cx^2} dx = \frac{\log(x)}{b} - \frac{\log(b+cx)}{b}$$

Mathematica [A] time = 0.0032633, size = 18, normalized size = 1.

$$\frac{\log(x)}{b} - \frac{\log(b+cx)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2)^(-1),x]

[Out] Log[x]/b - Log[b + c*x]/b

Maple [A] time = 0.045, size = 19, normalized size = 1.1

$$\frac{\ln(x)}{b} - \frac{\ln(cx+b)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+b*x),x)

```
[Out] ln(x)/b-ln(c*x+b)/b
```

Maxima [A] time = 1.11035, size = 24, normalized size = 1.33

$$-\frac{\log(cx+b)}{b}+\frac{\log(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] -log(c*x + b)/b + log(x)/b
```

Fricas [A] time = 1.65764, size = 38, normalized size = 2.11

$$-\frac{\log(cx+b)-\log(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] -(log(c*x + b) - log(x))/b
```

Sympy [A] time = 0.589893, size = 10, normalized size = 0.56

$$\frac{\log(x)-\log\left(\frac{b}{c}+x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x),x)
```

```
[Out] (log(x) - log(b/c + x))/b
```

Giac [A] time = 1.26521, size = 27, normalized size = 1.5

$$-\frac{\log(|cx+b|)}{b}+\frac{\log(|x|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] -log(abs(c*x + b))/b + log(abs(x))/b
```

3.264 $\int \frac{1}{(d+ex)(bx+cx^2)} dx$

Optimal. Leaf size=53

$$-\frac{c\log(b+cx)}{b(cd-be)}+\frac{e\log(d+ex)}{d(cd-be)}+\frac{\log(x)}{bd}$$

```
[Out] Log[x]/(b*d) - (c*Log[b + c*x])/(b*(c*d - b*e)) + (e*Log[d + e*x])/(d*(c*d
- b*e))
```

Rubi [A] time = 0.0462377, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$-\frac{c\log(b+cx)}{b(cd-be)}+\frac{e\log(d+ex)}{d(cd-be)}+\frac{\log(x)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(b*x + c*x^2)),x]
```

```
[Out] Log[x]/(b*d) - (c*Log[b + c*x])/(b*(c*d - b*e)) + (e*Log[d + e*x])/(d*(c*d
- b*e))
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)\left(bx+cx^2\right)} dx &= \int \left(\frac{1}{bdx}+\frac{c^2}{b(-cd+be)(b+cx)}+\frac{e^2}{d(cd-be)(d+ex)}\right) dx\\ &= \frac{\log(x)}{bd}-\frac{c\log(b+cx)}{b(cd-be)}+\frac{e\log(d+ex)}{d(cd-be)}\end{aligned}$$

Mathematica [A] time = 0.0233908, size = 48, normalized size = 0.91

$$\frac{-cd\log(b+cx)+be\log(d+ex)-be\log(x)+cd\log(x)}{bcd^2-b^2de}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(b*x + c*x^2)),x]
```

```
[Out] (c*d*Log[x] - b*e*Log[x] - c*d*Log[b + c*x] + b*e*Log[d + e*x])/(b*c*d^2 -
b^2*d*e)
```

Maple [A] time = 0.052, size = 54, normalized size = 1.

$$\frac{\ln(x)}{bd}+\frac{c\ln(cx+b)}{b(be-cd)}-\frac{e\ln(ex+d)}{d(be-cd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+b*x),x)
```

```
[Out] ln(x)/b/d+c/b/(b*e-c*d)*ln(c*x+b)-e/d/(b*e-c*d)*ln(e*x+d)
```

Maxima [A] time = 1.10489, size = 72, normalized size = 1.36

$$-\frac{c\log(cx+b)}{bcd-b^2e}+\frac{e\log(ex+d)}{cd^2-bde}+\frac{\log(x)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] -c*log(c*x + b)/(b*c*d - b^2*e) + e*log(e*x + d)/(c*d^2 - b*d*e) + log(x)/(
b*d)
```

Fricas [A] time = 2.23065, size = 109, normalized size = 2.06

$$-\frac{cd\log(cx+b)-be\log(ex+d)-(cd-be)\log(x)}{bcd^2-b^2de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] -(c*d*log(c*x + b) - b*e*log(e*x + d) - (c*d - b*e)*log(x))/(b*c*d^2 - b^2*
d*e)
```

Sympy [B] time = 78.8291, size = 583, normalized size = 11.

$$-\frac{e\log\left(x+\frac{-\frac{2b^6e^6}{(be-cd)^2}+\frac{6b^5cde^5}{(be-cd)^2}-\frac{8b^4c^2d^2e^4}{(be-cd)^2}+\frac{3b^4cde^4}{be-cd}+2b^4e^4+\frac{6b^3c^3d^3e^3}{(be-cd)^2}-\frac{6b^3c^2d^2e^3}{be-cd}-3b^3cde^3-\frac{2b^2c^4d^4e^2}{(be-cd)^2}+\frac{3b^2c^3d^3e^2}{be-cd}+2b^2c^2d^2e^2-3bc^3d^3e+2c^4d^4}{2b^3ce^4-3b^2c^2de^3-3bc^3d^2e^2+2c^4d^3e}\right)}{d(be-cd)}+\frac{c\log\Bigg(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+b*x),x)
```

```
[Out] -e*log(x + (-2*b**6*e**6/(b*e - c*d)**2 + 6*b**5*c*d*e**5/(b*e - c*d)**2 -
8*b**4*c**2*d**2*e**4/(b*e - c*d)**2 + 3*b**4*c*d*e**4/(b*e - c*d) + 2*b**4
*e**4 + 6*b**3*c**3*d**3*e**3/(b*e - c*d)**2 - 6*b**3*c**2*d**2*e**3/(b*e -
 c*d) - 3*b**3*c*d*e**3 - 2*b**2*c**4*d**4*e**2/(b*e - c*d)**2 + 3*b**2*c**
3*d**3*e**2/(b*e - c*d) + 2*b**2*c**2*d**2*e**2 - 3*b*c**3*d**3*e + 2*c**4*
d**4)/(2*b**3*c*e**4 - 3*b**2*c**2*d*e**3 - 3*b*c**3*d**2*e**2 + 2*c**4*d**
```

```
3*e))/(d*(b*e - c*d)) + c*log(x + (-2*b**4*c**2*d**2*e**4/(b*e - c*d)**2 +
2*b**4*e**4 + 6*b**3*c**3*d**3*e**3/(b*e - c*d)**2 - 3*b**3*c**2*d**2*e**3/
(b*e - c*d) - 3*b**3*c*d*e**3 - 8*b**2*c**4*d**4*e**2/(b*e - c*d)**2 + 6*b*
*2*c**3*d**3*e**2/(b*e - c*d) + 2*b**2*c**2*d**2*e**2 + 6*b*c**5*d**5*e/(b*
e - c*d)**2 - 3*b*c**4*d**4*e/(b*e - c*d) - 3*b*c**3*d**3*e - 2*c**6*d**6/(
b*e - c*d)**2 + 2*c**4*d**4)/(2*b**3*c*e**4 - 3*b**2*c**2*d*e**3 - 3*b*c**3
*d**2*e**2 + 2*c**4*d**3*e))/(b*(b*e - c*d)) + log(x)/(b*d)
```

Giac [B] time = 1.36874, size = 169, normalized size = 3.19

$$-\frac{(cd+be)\log\left(\frac{|2\,cxe+cd+be-|cd-be||}{|2\,cxe+cd+be+|cd-be||}\right)}{2\,bd|cd-be|}-\frac{\log\left(\left|cx^2e+cdx+bxe+bd\right|\right)}{2\,bd}+\frac{\log\left(|x|\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] -1/2*(c*d + b*e)*log(abs(2*c*x*e + c*d + b*e - abs(c*d - b*e))/abs(2*c*x*e
+ c*d + b*e + abs(c*d - b*e)))/(b*d*abs(c*d - b*e)) - 1/2*log(abs(c*x^2*e +
 c*d*x + b*x*e + b*d))/(b*d) + log(abs(x))/(b*d)
```

3.265 $\int \frac{1}{(d+ex)^2(bx+cx^2)}\,dx$

Optimal. Leaf size=87

$$-\frac{c^2\log(b+cx)}{b(cd-be)^2}+\frac{e(2cd-be)\log(d+ex)}{d^2(cd-be)^2}-\frac{e}{d(d+ex)(cd-be)}+\frac{\log(x)}{bd^2}$$

```
[Out] -(e/(d*(c*d - b*e)*(d + e*x))) + Log[x]/(b*d^2) - (c^2*Log[b + c*x])/(b*(c*
d - b*e)^2) + (e*(2*c*d - b*e)*Log[d + e*x])/(d^2*(c*d - b*e)^2)
```

Rubi [A] time = 0.0776652, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$-\frac{c^2\log(b+cx)}{b(cd-be)^2}+\frac{e(2cd-be)\log(d+ex)}{d^2(cd-be)^2}-\frac{e}{d(d+ex)(cd-be)}+\frac{\log(x)}{bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(b*x + c*x^2)),x]
```

```
[Out] -(e/(d*(c*d - b*e)*(d + e*x))) + Log[x]/(b*d^2) - (c^2*Log[b + c*x])/(b*(c*
d - b*e)^2) + (e*(2*c*d - b*e)*Log[d + e*x])/(d^2*(c*d - b*e)^2)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)^2\left(bx+cx^2\right)}\,dx &= \int \left(\frac{1}{bd^2x}-\frac{c^3}{b(-cd+be)^2(b+cx)}+\frac{e^2}{d(cd-be)(d+ex)^2}+\frac{e^2(2cd-be)}{d^2(cd-be)^2(d+ex)}\right)dx\\ &= -\frac{e}{d(cd-be)(d+ex)}+\frac{\log(x)}{bd^2}-\frac{c^2\log(b+cx)}{b(cd-be)^2}+\frac{e(2cd-be)\log(d+ex)}{d^2(cd-be)^2}\end{aligned}$$

Mathematica [A] time = 0.108466, size = 83, normalized size = 0.95

$$\frac{\frac{be((d+ex)(2cd-be)\log(d+ex)+d(be-cd))-c^2d^2(d+ex)\log(b+cx)}{(d+ex)(cd-be)^2}+\log(x)}{bd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(b*x + c*x^2)),x]
```

```
[Out] (Log[x] + (-(c^2*d^2*(d + e*x)*Log[b + c*x]) + b*e*(d*(-(c*d) + b*e) + (2*c
*d - b*e)*(d + e*x)*Log[d + e*x]))/((c*d - b*e)^2*(d + e*x)))/(b*d^2)
```

Maple [A] time = 0.085, size = 105, normalized size = 1.2

$$\frac{\ln(x)}{d^2b} - \frac{c^2\ln(cx+b)}{(be-cd)^2 b} + \frac{e}{d(be-cd)(ex+d)} - \frac{e^2\ln(ex+d)b}{d^2(be-cd)^2} + 2\frac{e\ln(ex+d)c}{d(be-cd)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^2+b*x),x)
```

```
[Out] ln(x)/b/d^2-c^2/(b*e-c*d)^2/b*ln(c*x+b)+e/d/(b*e-c*d)/(e*x+d)-e^2/d^2/(b*e-
c*d)^2*ln(e*x+d)*b+2*e/d/(b*e-c*d)^2*ln(e*x+d)*c
```

Maxima [A] time = 1.17196, size = 173, normalized size = 1.99

$$-\frac{c^2\log(cx+b)}{bc^2d^2-2b^2cde+b^3e^2} + \frac{(2cde-be^2)\log(ex+d)}{c^2d^4-2bcd^3e+b^2d^2e^2} - \frac{e}{cd^3-bd^2e+(cd^2e-bde^2)x} + \frac{\log(x)}{bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] -c^2*log(c*x + b)/(b*c^2*d^2 - 2*b^2*c*d*e + b^3*e^2) + (2*c*d*e - b*e^2)*l
og(e*x + d)/(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2) - e/(c*d^3 - b*d^2*e + (c
*d^2*e - b*d*e^2)*x) + log(x)/(b*d^2)
```

Fricas [B] time = 6.31435, size = 420, normalized size = 4.83

$$-\frac{bcd^2e-b^2de^2+(c^2d^2ex+c^2d^3)\log(cx+b)-(2bcd^2e-b^2de^2+(2bcde^2-b^2e^3)x)\log(ex+d)-(c^2d^3-2bcd^2e+b^2d}{bc^2d^5-2b^2cd^4e+b^3d^3e^2+(bc^2d^4e-2b^2cd^3e^2+b^3d^2e^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] -(b*c*d^2*e - b^2*d*e^2 + (c^2*d^2*e*x + c^2*d^3)*log(c*x + b) - (2*b*c*d^2
*e - b^2*d*e^2 + (2*b*c*d*e^2 - b^2*e^3)*x)*log(e*x + d) - (c^2*d^3 - 2*b*c
*d^2*e + b^2*d*e^2 + (c^2*d^2*e - 2*b*c*d*e^2 + b^2*e^3)*x)*log(x))/(b*c^2*
d^5 - 2*b^2*c*d^4*e + b^3*d^3*e^2 + (b*c^2*d^4*e - 2*b^2*c*d^3*e^2 + b^3*d^
2*e^3)*x)
```

Sympy [B] time = 161.156, size = 1238, normalized size = 14.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**2+b*x),x)
```

```
[Out] e/(b*d**2*e - c*d**3 + x*(b*d*e**2 - c*d**2*e)) - e*(b*e - 2*c*d)*log(x + (
-2*b**9*e**9*(b*e - 2*c*d)**2/(b*e - c*d)**4 + 13*b**8*c*d*e**8*(b*e - 2*c*
d)**2/(b*e - c*d)**4 - 35*b**7*c**2*d**2*e**7*(b*e - 2*c*d)**2/(b*e - c*d)*
*4 + b**7*c*d*e**7*(b*e - 2*c*d)/(b*e - c*d)**2 + 2*b**7*e**7 + 52*b**6*c**
3*d**3*e**6*(b*e - 2*c*d)**2/(b*e - c*d)**4 - 8*b**6*c**2*d**2*e**6*(b*e -
2*c*d)/(b*e - c*d)**2 - 12*b**6*c*d*e**6 - 48*b**5*c**4*d**4*e**5*(b*e - 2*
c*d)**2/(b*e - c*d)**4 + 23*b**5*c**3*d**3*e**5*(b*e - 2*c*d)/(b*e - c*d)**
2 + 27*b**5*c**2*d**2*e**5 + 29*b**4*c**5*d**5*e**4*(b*e - 2*c*d)**2/(b*e -
 c*d)**4 - 31*b**4*c**4*d**4*e**4*(b*e - 2*c*d)/(b*e - c*d)**2 - 29*b**4*c*
*3*d**3*e**4 - 11*b**3*c**6*d**6*e**3*(b*e - 2*c*d)**2/(b*e - c*d)**4 + 20*
b**3*c**5*d**5*e**3*(b*e - 2*c*d)/(b*e - c*d)**2 + 17*b**3*c**4*d**4*e**3 +
 2*b**2*c**7*d**7*e**2*(b*e - 2*c*d)**2/(b*e - c*d)**4 - 5*b**2*c**6*d**6*e
**2*(b*e - 2*c*d)/(b*e - c*d)**2 - 9*b**2*c**5*d**5*e**2 + 6*b*c**6*d**6*e
- 2*c**7*d**7)/(2*b**6*c*e**7 - 12*b**5*c**2*d*e**6 + 27*b**4*c**3*d**2*e**
5 - 28*b**3*c**4*d**3*e**4 + 9*b**2*c**5*d**4*e**3 + 6*b*c**6*d**5*e**2 - 2
*c**7*d**6*e))/(d**2*(b*e - c*d)**2) - c**2*log(x + (-2*b**7*c**4*d**4*e**7
/(b*e - c*d)**4 + 2*b**7*e**7 + 13*b**6*c**5*d**5*e**6/(b*e - c*d)**4 + b**
6*c**3*d**3*e**6/(b*e - c*d)**2 - 12*b**6*c*d*e**6 - 35*b**5*c**6*d**6*e**5
/(b*e - c*d)**4 - 8*b**5*c**4*d**4*e**5/(b*e - c*d)**2 + 27*b**5*c**2*d**2*
e**5 + 52*b**4*c**7*d**7*e**4/(b*e - c*d)**4 + 23*b**4*c**5*d**5*e**4/(b*e
- c*d)**2 - 29*b**4*c**3*d**3*e**4 - 48*b**3*c**8*d**8*e**3/(b*e - c*d)**4
- 31*b**3*c**6*d**6*e**3/(b*e - c*d)**2 + 17*b**3*c**4*d**4*e**3 + 29*b**2*
c**9*d**9*e**2/(b*e - c*d)**4 + 20*b**2*c**7*d**7*e**2/(b*e - c*d)**2 - 9*b
**2*c**5*d**5*e**2 - 11*b*c**10*d**10*e/(b*e - c*d)**4 - 5*b*c**8*d**8*e/(b
*e - c*d)**2 + 6*b*c**6*d**6*e + 2*c**11*d**11/(b*e - c*d)**4 - 2*c**7*d**7
)/(2*b**6*c*e**7 - 12*b**5*c**2*d*e**6 + 27*b**4*c**3*d**2*e**5 - 28*b**3*c
**4*d**3*e**4 + 9*b**2*c**5*d**4*e**3 + 6*b*c**6*d**5*e**2 - 2*c**7*d**6*e)
)/(b*(b*e - c*d)**2) + log(x)/(b*d**2)
```

Giac [B] time = 1.36252, size = 389, normalized size = 4.47

$$-\frac{\left(2\,c^2d^2e^2-2\,bcde^3+b^2e^4\right)e^{(-2)}\log\left(\frac{\left|-2\,cde+\frac{2\,cd^2e}{xe+d}+be^2-\frac{2\,bde^2}{xe+d}-|b|e^2\right|}{\left|-2\,cde+\frac{2\,cd^2e}{xe+d}+be^2-\frac{2\,bde^2}{xe+d}+|b|e^2\right|}\right)}{2\left(c^2d^4-2\,bcd^3e+b^2d^2e^2\right)|b|}-\frac{\left(2\,cde-be^2\right)\log\left(\left|-c+\frac{2\,cd}{xe+d}-\frac{cd^2}{(xe+d)^2}-\frac{be}{xe+d}+\frac{bd}{(xe+}\right.\right.}{2\left(c^2d^4-2\,bcd^3e+b^2d^2e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] -1/2*(2*c^2*d^2*e^2 - 2*b*c*d*e^3 + b^2*e^4)*e^(-2)*log(abs(-2*c*d*e + 2*c*
d^2*e/(x*e + d) + b*e^2 - 2*b*d*e^2/(x*e + d) - abs(b)*e^2)/abs(-2*c*d*e +
2*c*d^2*e/(x*e + d) + b*e^2 - 2*b*d*e^2/(x*e + d) + abs(b)*e^2))/((c^2*d^4
- 2*b*c*d^3*e + b^2*d^2*e^2)*abs(b)) - 1/2*(2*c*d*e - b*e^2)*log(abs(-c + 2
*c*d/(x*e + d) - c*d^2/(x*e + d)^2 - b*e/(x*e + d) + b*d*e/(x*e + d)^2))/(c
^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2) - e^3/((c*d^2*e^2 - b*d*e^3)*(x*e + d))
```

3.266 $\int \frac{1}{(d+ex)^3(bx+cx^2)}\,dx$

Optimal. Leaf size=134

$$\frac{e\left(b^2e^2-3bcde+3c^2d^2\right)\log(d+ex)}{d^3(cd-be)^3}-\frac{c^3\log(b+cx)}{b(cd-be)^3}-\frac{e(2cd-be)}{d^2(d+ex)(cd-be)^2}-\frac{e}{2d(d+ex)^2(cd-be)}+\frac{\log(x)}{bd^3}$$

```
[Out] -e/(2*d*(c*d - b*e)*(d + e*x)^2) - (e*(2*c*d - b*e))/(d^2*(c*d - b*e)^2*(d
+ e*x)) + Log[x]/(b*d^3) - (c^3*Log[b + c*x])/(b*(c*d - b*e)^3) + (e*(3*c^2
*d^2 - 3*b*c*d*e + b^2*e^2)*Log[d + e*x])/(d^3*(c*d - b*e)^3)
```

Rubi [A] time = 0.119824, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$\frac{e\left(b^2e^2-3bcde+3c^2d^2\right)\log(d+ex)}{d^3(cd-be)^3}-\frac{c^3\log(b+cx)}{b(cd-be)^3}-\frac{e(2cd-be)}{d^2(d+ex)(cd-be)^2}-\frac{e}{2d(d+ex)^2(cd-be)}+\frac{\log(x)}{bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(b*x + c*x^2)),x]
```

```
[Out] -e/(2*d*(c*d - b*e)*(d + e*x)^2) - (e*(2*c*d - b*e))/(d^2*(c*d - b*e)^2*(d
+ e*x)) + Log[x]/(b*d^3) - (c^3*Log[b + c*x])/(b*(c*d - b*e)^3) + (e*(3*c^2
*d^2 - 3*b*c*d*e + b^2*e^2)*Log[d + e*x])/(d^3*(c*d - b*e)^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(bx+cx^2\right)}\,dx=\int\left(\frac{1}{bd^3x}+\frac{c^4}{b(-cd+be)^3(b+cx)}+\frac{e^2}{d(cd-be)(d+ex)^3}+\frac{e^2(2cd-be)}{d^2(cd-be)^2(d+ex)^2}+\frac{e^2\left(3c^2d^2\right.}{d^3(cd-}\right.$$

$$=-\frac{e}{2d(cd-be)(d+ex)^2}-\frac{e(2cd-be)}{d^2(cd-be)^2(d+ex)}+\frac{\log(x)}{bd^3}-\frac{c^3\log(b+cx)}{b(cd-be)^3}+\frac{e\left(3c^2d^2-3bcde\right.}{d^3(cd}$$

Mathematica [A] time = 0.257857, size = 116, normalized size = 0.87

$$\frac{\frac{e\left(\frac{d(cd-be)(cd(5d+4ex)-be(3d+2ex))}{(d+ex)^2}-2\left(b^2e^2-3bcde+3c^2d^2\right)\log(d+ex)\right)}{d^3}+\frac{2c^3\log(b+cx)}{b}}{2(be-cd)^3}+\frac{\log(x)}{bd^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(b*x + c*x^2)),x]
```

```
[Out] Log[x]/(b*d^3) + ((2*c^3*Log[b + c*x])/b + (e*((d*(c*d - b*e)*(-(b*e*(3*d +
 2*e*x)) + c*d*(5*d + 4*e*x)))/(d + e*x)^2 - 2*(3*c^2*d^2 - 3*b*c*d*e + b^2
*e^2)*Log[d + e*x]))/d^3)/(2*(-(c*d) + b*e)^3)
```

Maple [A] time = 0.056, size = 184, normalized size = 1.4

$$\frac{\ln(x)}{d^3b} + \frac{c^3\ln(cx+b)}{(be-cd)^3 b} + \frac{e}{2d(be-cd)(ex+d)^2} + \frac{e^2b}{d^2(be-cd)^2(ex+d)} - 2\frac{ce}{d(be-cd)^2(ex+d)} - \frac{e^3\ln(ex+d)b^2}{d^3(be-cd)^3} + 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*x^2+b*x),x)
```

```
[Out] ln(x)/b/d^3+c^3/(b*e-c*d)^3/b*ln(c*x+b)+1/2*e/d/(b*e-c*d)/(e*x+d)^2+e^2/d^2
/(b*e-c*d)^2/(e*x+d)*b-2*e/d/(b*e-c*d)^2/(e*x+d)*c-e^3/d^3/(b*e-c*d)^3*ln(e
*x+d)*b^2+3*e^2/d^2/(b*e-c*d)^3*ln(e*x+d)*b*c-3*e/d/(b*e-c*d)^3*ln(e*x+d)*c
^2
```

Maxima [B] time = 1.15808, size = 359, normalized size = 2.68

$$-\frac{c^3\log(cx+b)}{bc^3d^3 - 3b^2c^2d^2e + 3b^3cde^2 - b^4e^3} + \frac{\left(3c^2d^2e - 3bcde^2 + b^2e^3\right)\log(ex+d)}{c^3d^6 - 3bc^2d^5e + 3b^2cd^4e^2 - b^3d^3e^3} - \frac{5cd^2e -}{2\left(c^2d^6 - 2bcd^5e + b^2d^4e^2 + \left(c^2d^4e^2 -\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] -c^3*log(c*x + b)/(b*c^3*d^3 - 3*b^2*c^2*d^2*e + 3*b^3*c*d*e^2 - b^4*e^3) +
 (3*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*log(e*x + d)/(c^3*d^6 - 3*b*c^2*d^5*
e + 3*b^2*c*d^4*e^2 - b^3*d^3*e^3) - 1/2*(5*c*d^2*e - 3*b*d*e^2 + 2*(2*c*d*
e^2 - b*e^3)*x)/(c^2*d^6 - 2*b*c*d^5*e + b^2*d^4*e^2 + (c^2*d^4*e^2 - 2*b*c
*d^3*e^3 + b^2*d^2*e^4)*x^2 + 2*(c^2*d^5*e - 2*b*c*d^4*e^2 + b^2*d^3*e^3)*x
) + log(x)/(b*d^3)
```

Fricas [B] time = 45.3123, size = 1008, normalized size = 7.52

$$-\frac{5bc^2d^4e - 8b^2cd^3e^2 + 3b^3d^2e^3 + 2\left(2bc^2d^3e^2 - 3b^2cd^2e^3 + b^3de^4\right)x + 2\left(c^3d^3e^2x^2 + 2c^3d^4ex + c^3d^5\right)\log(cx+b) - 2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] -1/2*(5*b*c^2*d^4*e - 8*b^2*c*d^3*e^2 + 3*b^3*d^2*e^3 + 2*(2*b*c^2*d^3*e^2
- 3*b^2*c*d^2*e^3 + b^3*d*e^4)*x + 2*(c^3*d^3*e^2*x^2 + 2*c^3*d^4*e*x + c^3
*d^5)*log(c*x + b) - 2*(3*b*c^2*d^4*e - 3*b^2*c*d^3*e^2 + b^3*d^2*e^3 + (3*
b*c^2*d^2*e^3 - 3*b^2*c*d*e^4 + b^3*e^5)*x^2 + 2*(3*b*c^2*d^3*e^2 - 3*b^2*c
*d^2*e^3 + b^3*d*e^4)*x)*log(e*x + d) - 2*(c^3*d^5 - 3*b*c^2*d^4*e + 3*b^2*
c*d^3*e^2 - b^3*d^2*e^3 + (c^3*d^3*e^2 - 3*b*c^2*d^2*e^3 + 3*b^2*c*d*e^4 -
b^3*e^5)*x^2 + 2*(c^3*d^4*e - 3*b*c^2*d^3*e^2 + 3*b^2*c*d^2*e^3 - b^3*d*e^4
)*x)*log(x))/(b*c^3*d^8 - 3*b^2*c^2*d^7*e + 3*b^3*c*d^6*e^2 - b^4*d^5*e^3 +
```

```
(b*c^3*d^6*e^2 - 3*b^2*c^2*d^5*e^3 + 3*b^3*c*d^4*e^4 - b^4*d^3*e^5)*x^2 +
2*(b*c^3*d^7*e - 3*b^2*c^2*d^6*e^2 + 3*b^3*c*d^5*e^3 - b^4*d^4*e^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**2+b*x),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27047, size = 306, normalized size = 2.28

$$-\frac{c^4\log(|cx+b|)}{bc^4d^3-3b^2c^3d^2e+3b^3c^2de^2-b^4ce^3}+\frac{\left(3c^2d^2e^2-3bcde^3+b^2e^4\right)\log(|xe+d|)}{c^3d^6e-3bc^2d^5e^2+3b^2cd^4e^3-b^3d^3e^4}+\frac{\log(|x|)}{bd^3}-\frac{5c^2d^4e-8bcd^3e^2+3b^2d}{2(cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] -c^4*log(abs(c*x + b))/(b*c^4*d^3 - 3*b^2*c^3*d^2*e + 3*b^3*c^2*d*e^2 - b^4
*c*e^3) + (3*c^2*d^2*e^2 - 3*b*c*d*e^3 + b^2*e^4)*log(abs(x*e + d))/(c^3*d^
6*e - 3*b*c^2*d^5*e^2 + 3*b^2*c*d^4*e^3 - b^3*d^3*e^4) + log(abs(x))/(b*d^3
) - 1/2*(5*c^2*d^4*e - 8*b*c*d^3*e^2 + 3*b^2*d^2*e^3 + 2*(2*c^2*d^3*e^2 - 3
*b*c*d^2*e^3 + b^2*d*e^4)*x)/((c*d - b*e)^3*(x*e + d)^2*d^3)
```

3.267 $\int \frac{(d+ex)^5}{\left(bx+cx^2\right)^2} dx$

Optimal. Leaf size=118

$$-\frac{(cd-be)^5}{b^2c^4(b+cx)}+\frac{(cd-be)^4(3be+2cd)\log(b+cx)}{b^3c^4}-\frac{d^4\log(x)(2cd-5be)}{b^3}-\frac{d^5}{b^2x}+\frac{e^4x(5cd-2be)}{c^3}+\frac{e^5x^2}{2c^2}$$

```
[Out] -(d^5/(b^2*x)) + (e^4*(5*c*d - 2*b*e)*x)/c^3 + (e^5*x^2)/(2*c^2) - (c*d - b
*e)^5/(b^2*c^4*(b + c*x)) - (d^4*(2*c*d - 5*b*e)*Log[x])/b^3 + ((c*d - b*e)
^4*(2*c*d + 3*b*e)*Log[b + c*x])/(b^3*c^4)
```

Rubi [A] time = 0.13772, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$-\frac{(cd-be)^5}{b^2c^4(b+cx)}+\frac{(cd-be)^4(3be+2cd)\log(b+cx)}{b^3c^4}-\frac{d^4\log(x)(2cd-5be)}{b^3}-\frac{d^5}{b^2x}+\frac{e^4x(5cd-2be)}{c^3}+\frac{e^5x^2}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5/(b*x + c*x^2)^2,x]
```

```
[Out] -(d^5/(b^2*x)) + (e^4*(5*c*d - 2*b*e)*x)/c^3 + (e^5*x^2)/(2*c^2) - (c*d - b
*e)^5/(b^2*c^4*(b + c*x)) - (d^4*(2*c*d - 5*b*e)*Log[x])/b^3 + ((c*d - b*e)
^4*(2*c*d + 3*b*e)*Log[b + c*x])/(b^3*c^4)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^5}{\left(bx+cx^2\right)^2}dx &= \int \left(\frac{e^4(5cd-2be)}{c^3}+\frac{d^5}{b^2x^2}+\frac{d^4(-2cd+5be)}{b^3x}+\frac{e^5x}{c^2}-\frac{(-cd+be)^5}{b^2c^3(b+cx)^2}+\frac{(-cd+be)^4(2cd+3be)}{b^3c^3(b+cx)}\right)dx\\ &= -\frac{d^5}{b^2x}+\frac{e^4(5cd-2be)x}{c^3}+\frac{e^5x^2}{2c^2}-\frac{(cd-be)^5}{b^2c^4(b+cx)}-\frac{d^4(2cd-5be)\log(x)}{b^3}+\frac{(cd-be)^4(2cd+3be)\log}{b^3c^4}\end{aligned}$$

Mathematica [A] time = 0.0568775, size = 116, normalized size = 0.98

$$\frac{(be-cd)^5}{b^2c^4(b+cx)}+\frac{(cd-be)^4(3be+2cd)\log(b+cx)}{b^3c^4}+\frac{d^4\log(x)(5be-2cd)}{b^3}-\frac{d^5}{b^2x}+\frac{e^4x(5cd-2be)}{c^3}+\frac{e^5x^2}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5/(b*x + c*x^2)^2,x]
```

```
[Out] -(d^5/(b^2*x)) + (e^4*(5*c*d - 2*b*e)*x)/c^3 + (e^5*x^2)/(2*c^2) + (-(c*d)
+ b*e)^5/(b^2*c^4*(b + c*x)) + (d^4*(-2*c*d + 5*b*e)*Log[x])/b^3 + ((c*d -
```

```
b*e)^4*(2*c*d + 3*b*e)*Log[b + c*x])/(b^3*c^4)
```

Maple [B] time = 0.061, size = 251, normalized size = 2.1

$$\frac{e^5x^2}{2c^2}-2\frac{e^5xb}{c^3}+5\frac{de^4x}{c^2}-\frac{d^5}{b^2x}+5\frac{d^4\ln(x)e}{b^2}-2\frac{d^5\ln(x)c}{b^3}+3\frac{b^2\ln(cx+b)e^5}{c^4}-10\frac{b\ln(cx+b)de^4}{c^3}+10\frac{\ln(cx+b)d^2e}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5/(c*x^2+b*x)^2,x)
```

```
[Out] 1/2*e^5*x^2/c^2-2*e^5/c^3*x*b+5*e^4/c^2*d*x-d^5/b^2/x+5*d^4/b^2*ln(x)*e-2*d
^5/b^3*ln(x)*c+3/c^4*b^2*ln(c*x+b)*e^5-10/c^3*b*ln(c*x+b)*d*e^4+10/c^2*ln(c
*x+b)*d^2*e^3-5/b^2*ln(c*x+b)*d^4*e+2*c/b^3*ln(c*x+b)*d^5+b^3/c^4/(c*x+b)*e
^5-5*b^2/c^3/(c*x+b)*d*e^4+10*b/c^2/(c*x+b)*d^2*e^3-10/c/(c*x+b)*d^3*e^2+5/
b/(c*x+b)*d^4*e-1/b^2*c/(c*x+b)*d^5
```

Maxima [A] time = 0.997527, size = 292, normalized size = 2.47

$$-\frac{bc^4d^5+\left(2c^5d^5-5bc^4d^4e+10b^2c^3d^3e^2-10b^3c^2d^2e^3+5b^4cde^4-b^5e^5\right)x}{b^2c^5x^2+b^3c^4x}-\frac{\left(2cd^5-5bd^4e\right)\log(x)}{b^3}+\frac{ce^5x^2+2\left(5cde^4\right.}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*x^2+b*x)^2,x, algorithm="maxima")
```

```
[Out] -(b*c^4*d^5 + (2*c^5*d^5 - 5*b*c^4*d^4*e + 10*b^2*c^3*d^3*e^2 - 10*b^3*c^2*
d^2*e^3 + 5*b^4*c*d*e^4 - b^5*e^5)*x)/(b^2*c^5*x^2 + b^3*c^4*x) - (2*c*d^5
- 5*b*d^4*e)*log(x)/b^3 + 1/2*(c*e^5*x^2 + 2*(5*c*d*e^4 - 2*b*e^5)*x)/c^3 +
 (2*c^5*d^5 - 5*b*c^4*d^4*e + 10*b^3*c^2*d^2*e^3 - 10*b^4*c*d*e^4 + 3*b^5*e
^5)*log(c*x + b)/(b^3*c^4)
```

Fricas [B] time = 1.75372, size = 707, normalized size = 5.99

$$\frac{b^3c^3e^5x^4-2b^2c^4d^5+\left(10b^3c^3de^4-3b^4c^2e^5\right)x^3+2\left(5b^4c^2de^4-2b^5ce^5\right)x^2-2\left(2bc^5d^5-5b^2c^4d^4e+10b^3c^3d^3e^2-10b^4c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*x^2+b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*c^3*e^5*x^4 - 2*b^2*c^4*d^5 + (10*b^3*c^3*d*e^4 - 3*b^4*c^2*e^5)*x
^3 + 2*(5*b^4*c^2*d*e^4 - 2*b^5*c*e^5)*x^2 - 2*(2*b*c^5*d^5 - 5*b^2*c^4*d^4
*e + 10*b^3*c^3*d^3*e^2 - 10*b^4*c^2*d^2*e^3 + 5*b^5*c*d*e^4 - b^6*e^5)*x +
 2*((2*c^6*d^5 - 5*b*c^5*d^4*e + 10*b^3*c^3*d^2*e^3 - 10*b^4*c^2*d*e^4 + 3*
b^5*c*e^5)*x^2 + (2*b*c^5*d^5 - 5*b^2*c^4*d^4*e + 10*b^4*c^2*d^2*e^3 - 10*b
^5*c*d*e^4 + 3*b^6*e^5)*x)*log(c*x + b) - 2*((2*c^6*d^5 - 5*b*c^5*d^4*e)*x^
2 + (2*b*c^5*d^5 - 5*b^2*c^4*d^4*e)*x)*log(x))/(b^3*c^5*x^2 + b^4*c^4*x)
```

Sympy [B] time = 8.45732, size = 379, normalized size = 3.21

$$\frac{-bc^4d^5+x\left(b^5e^5-5b^4cde^4+10b^3c^2d^2e^3-10b^2c^3d^3e^2+5bc^4d^4e-2c^5d^5\right)}{b^3c^4x+b^2c^5x^2}+\frac{e^5x^2}{2c^2}-\frac{x\left(2be^5-5cde^4\right)}{c^3}+\frac{d^4\left(5be-2cd\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5/(c*x**2+b*x)**2,x)
```

```
[Out] (-b*c**4*d**5 + x*(b**5*e**5 - 5*b**4*c*d*e**4 + 10*b**3*c**2*d**2*e**3 - 1
0*b**2*c**3*d**3*e**2 + 5*b*c**4*d**4*e - 2*c**5*d**5))/(b**3*c**4*x + b**2
*c**5*x**2) + e**5*x**2/(2*c**2) - x*(2*b*e**5 - 5*c*d*e**4)/c**3 + d**4*(5
*b*e - 2*c*d)*log(x + (-5*b**2*c**3*d**4*e + 2*b*c**4*d**5 + b*c**3*d**4*(5
*b*e - 2*c*d))/(3*b**5*e**5 - 10*b**4*c*d*e**4 + 10*b**3*c**2*d**2*e**3 - 1
0*b*c**4*d**4*e + 4*c**5*d**5))/b**3 + (b*e - c*d)**4*(3*b*e + 2*c*d)*log(x
 + (-5*b**2*c**3*d**4*e + 2*b*c**4*d**5 + b*(b*e - c*d)**4*(3*b*e + 2*c*d)/
c)/(3*b**5*e**5 - 10*b**4*c*d*e**4 + 10*b**3*c**2*d**2*e**3 - 10*b*c**4*d**
4*e + 4*c**5*d**5))/(b**3*c**4)
```

Giac [A] time = 1.30331, size = 282, normalized size = 2.39

$$-\frac{\left(2\,cd^5-5\,bd^4e\right)\log\left(|x|\right)}{b^3}+\frac{c^2x^2e^5+10\,c^2dxe^4-4\,bcxe^5}{2\,c^4}+\frac{\left(2\,c^5d^5-5\,bc^4d^4e+10\,b^3c^2d^2e^3-10\,b^4cde^4+3\,b^5e^5\right)\log(}{b^3c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*x^2+b*x)^2,x, algorithm="giac")
```

```
[Out] -(2*c*d^5 - 5*b*d^4*e)*log(abs(x))/b^3 + 1/2*(c^2*x^2*e^5 + 10*c^2*d*x*e^4
- 4*b*c*x*e^5)/c^4 + (2*c^5*d^5 - 5*b*c^4*d^4*e + 10*b^3*c^2*d^2*e^3 - 10*b
^4*c*d*e^4 + 3*b^5*e^5)*log(abs(c*x + b))/(b^3*c^4) - (b*c^4*d^5 + (2*c^5*d
^5 - 5*b*c^4*d^4*e + 10*b^2*c^3*d^3*e^2 - 10*b^3*c^2*d^2*e^3 + 5*b^4*c*d*e^
4 - b^5*e^5)*x)/((c*x + b)*b^2*c^4*x)
```

3.268 $\int \frac{(d+ex)^4}{(bx+cx^2)^2}\,dx$

Optimal. Leaf size=94

$$-\frac{(cd-be)^4}{b^2c^3(b+cx)}+\frac{2(cd-be)^3(be+cd)\log(b+cx)}{b^3c^3}-\frac{2d^3\log(x)(cd-2be)}{b^3}-\frac{d^4}{b^2x}+\frac{e^4x}{c^2}$$

```
[Out] -(d^4/(b^2*x)) + (e^4*x)/c^2 - (c*d - b*e)^4/(b^2*c^3*(b + c*x)) - (2*d^3*(
c*d - 2*b*e)*Log[x])/b^3 + (2*(c*d - b*e)^3*(c*d + b*e)*Log[b + c*x])/(b^3*
c^3)
```

Rubi [A] time = 0.100295, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$-\frac{(cd-be)^4}{b^2c^3(b+cx)}+\frac{2(cd-be)^3(be+cd)\log(b+cx)}{b^3c^3}-\frac{2d^3\log(x)(cd-2be)}{b^3}-\frac{d^4}{b^2x}+\frac{e^4x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(b*x + c*x^2)^2,x]
```

```
[Out] -(d^4/(b^2*x)) + (e^4*x)/c^2 - (c*d - b*e)^4/(b^2*c^3*(b + c*x)) - (2*d^3*(
c*d - 2*b*e)*Log[x])/b^3 + (2*(c*d - b*e)^3*(c*d + b*e)*Log[b + c*x])/(b^3*
c^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^4}{(bx+cx^2)^2}\,dx &= \int \left(\frac{e^4}{c^2}+\frac{d^4}{b^2x^2}+\frac{2d^3(-cd+2be)}{b^3x}+\frac{(-cd+be)^4}{b^2c^2(b+cx)^2}-\frac{2(-cd+be)^3(cd+be)}{b^3c^2(b+cx)}\right)dx\\ &= -\frac{d^4}{b^2x}+\frac{e^4x}{c^2}-\frac{(cd-be)^4}{b^2c^3(b+cx)}-\frac{2d^3(cd-2be)\log(x)}{b^3}+\frac{2(cd-be)^3(cd+be)\log(b+cx)}{b^3c^3}\end{aligned}$$

Mathematica [A] time = 0.0943638, size = 95, normalized size = 1.01

$$-\frac{(cd-be)^4}{b^2c^3(b+cx)}+\frac{2(cd-be)^3(be+cd)\log(b+cx)}{b^3c^3}+\frac{2d^3\log(x)(2be-cd)}{b^3}-\frac{d^4}{b^2x}+\frac{e^4x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(b*x + c*x^2)^2,x]
```

```
[Out] -(d^4/(b^2*x)) + (e^4*x)/c^2 - (c*d - b*e)^4/(b^2*c^3*(b + c*x)) + (2*d^3*(
-(c*d) + 2*b*e)*Log[x])/b^3 + (2*(c*d - b*e)^3*(c*d + b*e)*Log[b + c*x])/(b
```

```
^3*c^3)
```

Maple [A] time = 0.055, size = 188, normalized size = 2.

$$\frac{e^4x}{c^2}-\frac{d^4}{b^2x}+4\frac{d^3\ln(x)e}{b^2}-2\frac{d^4\ln(x)c}{b^3}-2\frac{b\ln(cx+b)e^4}{c^3}+4\frac{\ln(cx+b)de^3}{c^2}-4\frac{\ln(cx+b)d^3e}{b^2}+2\frac{c\ln(cx+b)d^4}{b^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(c*x^2+b*x)^2,x)
```

```
[Out] e^4*x/c^2-d^4/b^2/x+4*d^3/b^2*ln(x)*e-2*d^4/b^3*ln(x)*c-2/c^3*b*ln(c*x+b)*e
^4+4/c^2*ln(c*x+b)*d*e^3-4/b^2*ln(c*x+b)*d^3*e+2*c/b^3*ln(c*x+b)*d^4-1/c^3*
b^2/(c*x+b)*e^4+4/c^2*b/(c*x+b)*d*e^3-6/c/(c*x+b)*d^2*e^2+4/b/(c*x+b)*d^3*e
-c/b^2/(c*x+b)*d^4
```

Maxima [A] time = 1.12418, size = 220, normalized size = 2.34

$$\frac{e^4x}{c^2}-\frac{bc^3d^4+\left(2\,c^4d^4-4\,bc^3d^3e+6\,b^2c^2d^2e^2-4\,b^3cde^3+b^4e^4\right)x}{b^2c^4x^2+b^3c^3x}-\frac{2\left(cd^4-2\,bd^3e\right)\log(x)}{b^3}+\frac{2\left(c^4d^4-2\,bc^3d^3e+2\,b\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x)^2,x, algorithm="maxima")
```

```
[Out] e^4*x/c^2 - (b*c^3*d^4 + (2*c^4*d^4 - 4*b*c^3*d^3*e + 6*b^2*c^2*d^2*e^2 - 4
*b^3*c*d*e^3 + b^4*e^4)*x)/(b^2*c^4*x^2 + b^3*c^3*x) - 2*(c*d^4 - 2*b*d^3*e
)*log(x)/b^3 + 2*(c^4*d^4 - 2*b*c^3*d^3*e + 2*b^3*c*d*e^3 - b^4*e^4)*log(c*
x + b)/(b^3*c^3)
```

Fricas [B] time = 1.74641, size = 504, normalized size = 5.36

$$\frac{b^3c^2e^4x^3+b^4ce^4x^2-b^2c^3d^4-\left(2\,bc^4d^4-4\,b^2c^3d^3e+6\,b^3c^2d^2e^2-4\,b^4cde^3+b^5e^4\right)x+2\left(\left(c^5d^4-2\,bc^4d^3e+2\,b^3c^2de^3-\right.\right.}{b^3c^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x)^2,x, algorithm="fricas")
```

```
[Out] (b^3*c^2*e^4*x^3 + b^4*c*e^4*x^2 - b^2*c^3*d^4 - (2*b*c^4*d^4 - 4*b^2*c^3*d
^3*e + 6*b^3*c^2*d^2*e^2 - 4*b^4*c*d*e^3 + b^5*e^4)*x + 2*((c^5*d^4 - 2*b*c
^4*d^3*e + 2*b^3*c^2*d*e^3 - b^4*c*e^4)*x^2 + (b*c^4*d^4 - 2*b^2*c^3*d^3*e
+ 2*b^4*c*d*e^3 - b^5*e^4)*x)*log(c*x + b) - 2*((c^5*d^4 - 2*b*c^4*d^3*e)*x
^2 + (b*c^4*d^4 - 2*b^2*c^3*d^3*e)*x)*log(x))/(b^3*c^4*x^2 + b^4*c^3*x)
```

Sympy [B] time = 7.36321, size = 306, normalized size = 3.26

$$-\frac{bc^3d^4+x\left(b^4e^4-4b^3cde^3+6b^2c^2d^2e^2-4bc^3d^3e+2c^4d^4\right)}{b^3c^3x+b^2c^4x^2}+\frac{e^4x}{c^2}+\frac{2d^3\left(2be-cd\right)\log\left(x+\frac{4b^2c^2d^3e-2bc^3d^4-2bc^2d^3(2be-cd)}{2b^4e^4-4b^3cde^3+8bc^3d^3e-4c^4d^4}\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(c*x**2+b*x)**2,x)
```

```
[Out] -(b*c**3*d**4 + x*(b**4*e**4 - 4*b**3*c*d*e**3 + 6*b**2*c**2*d**2*e**2 - 4*
b*c**3*d**3*e + 2*c**4*d**4))/(b**3*c**3*x + b**2*c**4*x**2) + e**4*x/c**2
+ 2*d**3*(2*b*e - c*d)*log(x + (4*b**2*c**2*d**3*e - 2*b*c**3*d**4 - 2*b*c*
*2*d**3*(2*b*e - c*d))/(2*b**4*e**4 - 4*b**3*c*d*e**3 + 8*b*c**3*d**3*e - 4
*c**4*d**4))/b**3 - 2*(b*e - c*d)**3*(b*e + c*d)*log(x + (4*b**2*c**2*d**3*
e - 2*b*c**3*d**4 + 2*b*(b*e - c*d)**3*(b*e + c*d)/c)/(2*b**4*e**4 - 4*b**3
*c*d*e**3 + 8*b*c**3*d**3*e - 4*c**4*d**4))/(b**3*c**3)
```

Giac [A] time = 1.36, size = 216, normalized size = 2.3

$$\frac{xe^4}{c^2} - \frac{2\left(cd^4 - 2\,bd^3e\right)\log\left(|x|\right)}{b^3} + \frac{2\left(c^4d^4 - 2\,bc^3d^3e + 2\,b^3cde^3 - b^4e^4\right)\log\left(|cx+b|\right)}{b^3c^3} - \frac{bc^2d^4 + \frac{\left(2\,c^4d^4 - 4\,bc^3d^3e + 6\,b^2c^2d^2e^2 - 4\,b^3c}{c}}{(cx+b)b^2c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x)^2,x, algorithm="giac")
```

```
[Out] x*e^4/c^2 - 2*(c*d^4 - 2*b*d^3*e)*log(abs(x))/b^3 + 2*(c^4*d^4 - 2*b*c^3*d^
3*e + 2*b^3*c*d*e^3 - b^4*e^4)*log(abs(c*x + b))/(b^3*c^3) - (b*c^2*d^4 + (
2*c^4*d^4 - 4*b*c^3*d^3*e + 6*b^2*c^2*d^2*e^2 - 4*b^3*c*d*e^3 + b^4*e^4)*x/
c)/((c*x + b)*b^2*c^2*x)
```

3.269 $\int \frac{(d+ex)^3}{\left(bx+cx^2\right)^2} dx$

Optimal. Leaf size=87

$$-\frac{(cd-be)^3}{b^2c^2(b+cx)}+\frac{(cd-be)^2(be+2cd)\log(b+cx)}{b^3c^2}-\frac{d^2\log(x)(2cd-3be)}{b^3}-\frac{d^3}{b^2x}$$

```
[Out] -(d^3/(b^2*x)) - (c*d - b*e)^3/(b^2*c^2*(b + c*x)) - (d^2*(2*c*d - 3*b*e)*L
og[x])/b^3 + ((c*d - b*e)^2*(2*c*d + b*e)*Log[b + c*x])/(b^3*c^2)
```

Rubi [A] time = 0.081705, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$-\frac{(cd-be)^3}{b^2c^2(b+cx)}+\frac{(cd-be)^2(be+2cd)\log(b+cx)}{b^3c^2}-\frac{d^2\log(x)(2cd-3be)}{b^3}-\frac{d^3}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(b*x + c*x^2)^2,x]
```

```
[Out] -(d^3/(b^2*x)) - (c*d - b*e)^3/(b^2*c^2*(b + c*x)) - (d^2*(2*c*d - 3*b*e)*L
og[x])/b^3 + ((c*d - b*e)^2*(2*c*d + b*e)*Log[b + c*x])/(b^3*c^2)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^3}{\left(bx+cx^2\right)^2} dx &= \int \left(\frac{d^3}{b^2x^2}+\frac{d^2(-2cd+3be)}{b^3x}-\frac{(-cd+be)^3}{b^2c(b+cx)^2}+\frac{(-cd+be)^2(2cd+be)}{b^3c(b+cx)}\right) dx\\ &= -\frac{d^3}{b^2x}-\frac{(cd-be)^3}{b^2c^2(b+cx)}-\frac{d^2(2cd-3be)\log(x)}{b^3}+\frac{(cd-be)^2(2cd+be)\log(b+cx)}{b^3c^2}\end{aligned}$$

Mathematica [A] time = 0.0754549, size = 79, normalized size = 0.91

$$\frac{\frac{b(be-cd)^3}{c^2(b+cx)}+\frac{(cd-be)^2(be+2cd)\log(b+cx)}{c^2}+d^2\log(x)(3be-2cd)-\frac{bd^3}{x}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(b*x + c*x^2)^2,x]
```

```
[Out] (-((b*d^3)/x) + (b*(-(c*d) + b*e)^3)/(c^2*(b + c*x)) + d^2*(-2*c*d + 3*b*e)
*Log[x] + ((c*d - b*e)^2*(2*c*d + b*e)*Log[b + c*x])/c^2)/b^3
```

Maple [A] time = 0.057, size = 141, normalized size = 1.6

$$-\frac{d^3}{b^2x}+3\frac{d^2\ln(x)e}{b^2}-2\frac{d^3\ln(x)c}{b^3}+\frac{\ln(cx+b)e^3}{c^2}-3\frac{\ln(cx+b)d^2e}{b^2}+2\frac{c\ln(cx+b)d^3}{b^3}+\frac{be^3}{c^2(cx+b)}-3\frac{de^2}{c(cx+b)}+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^2+b*x)^2,x)
```

```
[Out] -d^3/b^2/x+3*d^2/b^2*ln(x)*e-2*d^3/b^3*ln(x)*c+1/c^2*ln(c*x+b)*e^3-3/b^2*ln
(c*x+b)*d^2*e+2/b^3*c*ln(c*x+b)*d^3+b/c^2/(c*x+b)*e^3-3/c/(c*x+b)*d*e^2+3/b
/(c*x+b)*d^2*e-1/b^2*c/(c*x+b)*d^3
```

Maxima [A] time = 1.14688, size = 178, normalized size = 2.05

$$-\frac{bc^2d^3+\left(2c^3d^3-3bc^2d^2e+3b^2cde^2-b^3e^3\right)x}{b^2c^3x^2+b^3c^2x}-\frac{\left(2cd^3-3bd^2e\right)\log(x)}{b^3}+\frac{\left(2c^3d^3-3bc^2d^2e+b^3e^3\right)\log(cx+b)}{b^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x)^2,x, algorithm="maxima")
```

```
[Out] -(b*c^2*d^3 + (2*c^3*d^3 - 3*b*c^2*d^2*e + 3*b^2*c*d*e^2 - b^3*e^3)*x)/(b^2
*c^3*x^2 + b^3*c^2*x) - (2*c*d^3 - 3*b*d^2*e)*log(x)/b^3 + (2*c^3*d^3 - 3*b
*c^2*d^2*e + b^3*e^3)*log(c*x + b)/(b^3*c^2)
```

Fricas [B] time = 1.72213, size = 392, normalized size = 4.51

$$-\frac{b^2c^2d^3+\left(2bc^3d^3-3b^2c^2d^2e+3b^3cde^2-b^4e^3\right)x-\left(\left(2c^4d^3-3bc^3d^2e+b^3ce^3\right)x^2+\left(2bc^3d^3-3b^2c^2d^2e+b^4e^3\right)x\right)\log(}{b^3c^3x^2+b^4c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x)^2,x, algorithm="fricas")
```

```
[Out] -(b^2*c^2*d^3 + (2*b*c^3*d^3 - 3*b^2*c^2*d^2*e + 3*b^3*c*d*e^2 - b^4*e^3)*x
 - ((2*c^4*d^3 - 3*b*c^3*d^2*e + b^3*c*e^3)*x^2 + (2*b*c^3*d^3 - 3*b^2*c^2*
d^2*e + b^4*e^3)*x)*log(c*x + b) + ((2*c^4*d^3 - 3*b*c^3*d^2*e)*x^2 + (2*b*
c^3*d^3 - 3*b^2*c^2*d^2*e)*x)*log(x))/(b^3*c^3*x^2 + b^4*c^2*x)
```

Sympy [B] time = 3.59428, size = 250, normalized size = 2.87

$$\frac{-bc^2d^3+x\left(b^3e^3-3b^2cde^2+3bc^2d^2e-2c^3d^3\right)}{b^3c^2x+b^2c^3x^2}+\frac{d^2(3be-2cd)\log\left(x+\frac{-3b^2cd^2e+2bc^2d^3+bcd^2(3be-2cd)}{b^3e^3-6bc^2d^2e+4c^3d^3}\right)}{b^3}+\frac{(be-cd)^2(be+2c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**2+b*x)**2,x)
```

```
[Out] (-b*c**2*d**3 + x*(b**3*e**3 - 3*b**2*c*d*e**2 + 3*b*c**2*d**2*e - 2*c**3*d
**3))/(b**3*c**2*x + b**2*c**3*x**2) + d**2*(3*b*e - 2*c*d)*log(x + (-3*b**
2*c*d**2*e + 2*b*c**2*d**3 + b*c*d**2*(3*b*e - 2*c*d))/(b**3*e**3 - 6*b*c**
2*d**2*e + 4*c**3*d**3))/b**3 + (b*e - c*d)**2*(b*e + 2*c*d)*log(x + (-3*b*
*2*c*d**2*e + 2*b*c**2*d**3 + b*(b*e - c*d)**2*(b*e + 2*c*d)/c)/(b**3*e**3
- 6*b*c**2*d**2*e + 4*c**3*d**3))/(b**3*c**2)
```

Giac [A] time = 1.37846, size = 174, normalized size = 2.

$$-\frac{\left(2\,cd^3-3\,bd^2e\right)\log\left(|x|\right)}{b^3}+\frac{\left(2\,c^3d^3-3\,bc^2d^2e+b^3e^3\right)\log\left(|cx+b|\right)}{b^3c^2}-\frac{bc^2d^3+\left(2\,c^3d^3-3\,bc^2d^2e+3\,b^2cde^2-b^3e^3\right)x}{(cx+b)b^2c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x)^2,x, algorithm="giac")
```

```
[Out] -(2*c*d^3 - 3*b*d^2*e)*log(abs(x))/b^3 + (2*c^3*d^3 - 3*b*c^2*d^2*e + b^3*e
^3)*log(abs(c*x + b))/(b^3*c^2) - (b*c^2*d^3 + (2*c^3*d^3 - 3*b*c^2*d^2*e +
 3*b^2*c*d*e^2 - b^3*e^3)*x)/((c*x + b)*b^2*c^2*x)
```

3.270 $\int \frac{(d+ex)^2}{(bx+cx^2)^2} dx$

Optimal. Leaf size=73

$$-\frac{(cd-be)^2}{b^2c(b+cx)}-\frac{2d\log(x)(cd-be)}{b^3}+\frac{2d(cd-be)\log(b+cx)}{b^3}-\frac{d^2}{b^2x}$$

```
[Out] -(d^2/(b^2*x)) - (c*d - b*e)^2/(b^2*c*(b + c*x)) - (2*d*(c*d - b*e)*Log[x])
/b^3 + (2*d*(c*d - b*e)*Log[b + c*x])/b^3
```

Rubi [A] time = 0.0581193, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$-\frac{(cd-be)^2}{b^2c(b+cx)}-\frac{2d\log(x)(cd-be)}{b^3}+\frac{2d(cd-be)\log(b+cx)}{b^3}-\frac{d^2}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(b*x + c*x^2)^2,x]
```

```
[Out] -(d^2/(b^2*x)) - (c*d - b*e)^2/(b^2*c*(b + c*x)) - (2*d*(c*d - b*e)*Log[x])
/b^3 + (2*d*(c*d - b*e)*Log[b + c*x])/b^3
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2}{(bx+cx^2)^2}\,dx &= \int \left(\frac{d^2}{b^2x^2}+\frac{2d(-cd+be)}{b^3x}+\frac{(-cd+be)^2}{b^2(b+cx)^2}-\frac{2cd(-cd+be)}{b^3(b+cx)}\right)dx\\ &= -\frac{d^2}{b^2x}-\frac{(cd-be)^2}{b^2c(b+cx)}-\frac{2d(cd-be)\log(x)}{b^3}+\frac{2d(cd-be)\log(b+cx)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.0769541, size = 67, normalized size = 0.92

$$\frac{-\frac{b(cd-be)^2}{c(b+cx)}+2d\log(x)(be-cd)+2d(cd-be)\log(b+cx)-\frac{bd^2}{x}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(b*x + c*x^2)^2,x]
```

```
[Out] (-((b*d^2)/x) - (b*(c*d - b*e)^2)/(c*(b + c*x)) + 2*d*(-(c*d) + b*e)*Log[x]
 + 2*d*(c*d - b*e)*Log[b + c*x])/b^3
```

Maple [A] time = 0.057, size = 106, normalized size = 1.5

$$-\frac{d^2}{b^2x}+2\frac{d\ln(x)e}{b^2}-2\frac{d^2\ln(x)c}{b^3}-\frac{e^2}{c(cx+b)}+2\frac{de}{b(cx+b)}-\frac{cd^2}{b^2(cx+b)}-2\frac{d\ln(cx+b)e}{b^2}+2\frac{d^2\ln(cx+b)c}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^2+b*x)^2,x)
```

```
[Out] -d^2/b^2/x+2*d/b^2*ln(x)*e-2*d^2/b^3*ln(x)*c-1/c/(c*x+b)*e^2+2/b/(c*x+b)*d*
e-1/b^2*c/(c*x+b)*d^2-2*d/b^2*ln(c*x+b)*e+2*d^2/b^3*ln(c*x+b)*c
```

Maxima [A] time = 1.14467, size = 126, normalized size = 1.73

$$-\frac{bcd^2+\left(2c^2d^2-2bcde+b^2e^2\right)x}{b^2c^2x^2+b^3cx}+\frac{2\left(cd^2-bde\right)\log(cx+b)}{b^3}-\frac{2\left(cd^2-bde\right)\log(x)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x)^2,x, algorithm="maxima")
```

```
[Out] -(b*c*d^2 + (2*c^2*d^2 - 2*b*c*d*e + b^2*e^2)*x)/(b^2*c^2*x^2 + b^3*c*x) +
2*(c*d^2 - b*d*e)*log(c*x + b)/b^3 - 2*(c*d^2 - b*d*e)*log(x)/b^3
```

Fricas [B] time = 1.74305, size = 297, normalized size = 4.07

$$-\frac{b^2cd^2+\left(2bc^2d^2-2b^2cde+b^3e^2\right)x-2\left(\left(c^3d^2-bc^2de\right)x^2+\left(bc^2d^2-b^2cde\right)x\right)\log(cx+b)+2\left(\left(c^3d^2-bc^2de\right)x^2+\left(\right.\right.}{b^3c^2x^2+b^4cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x)^2,x, algorithm="fricas")
```

```
[Out] -(b^2*c*d^2 + (2*b*c^2*d^2 - 2*b^2*c*d*e + b^3*e^2)*x - 2*((c^3*d^2 - b*c^2
*d*e)*x^2 + (b*c^2*d^2 - b^2*c*d*e)*x)*log(c*x + b) + 2*((c^3*d^2 - b*c^2*d
*e)*x^2 + (b*c^2*d^2 - b^2*c*d*e)*x)*log(x))/(b^3*c^2*x^2 + b^4*c*x)
```

Sympy [B] time = 1.60136, size = 173, normalized size = 2.37

$$-\frac{bcd^2+x\left(b^2e^2-2bcde+2c^2d^2\right)}{b^3cx+b^2c^2x^2}+\frac{2d(be-cd)\log\left(x+\frac{2b^2de-2bcd^2-2bd(be-cd)}{4bcde-4c^2d^2}\right)}{b^3}-\frac{2d(be-cd)\log\left(x+\frac{2b^2de-2bcd^2+2bd(be}{4bcde-4c^2d^2}\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**2+b*x)**2,x)
```

```
[Out] -(b*c*d**2 + x*(b**2*e**2 - 2*b*c*d*e + 2*c**2*d**2))/(b**3*c*x + b**2*c**2
*x**2) + 2*d*(b*e - c*d)*log(x + (2*b**2*d*e - 2*b*c*d**2 - 2*b*d*(b*e - c*
d))/(4*b*c*d*e - 4*c**2*d**2))/b**3 - 2*d*(b*e - c*d)*log(x + (2*b**2*d*e -
```

```
2*b*c*d**2 + 2*b*d*(b*e - c*d))/(4*b*c*d*e - 4*c**2*d**2))/b**3
```

Giac [A] time = 1.23792, size = 136, normalized size = 1.86

$$-\frac{2\left(cd^2-bde\right)\log\left(|x|\right)}{b^3}+\frac{2\left(c^2d^2-bcde\right)\log\left(|cx+b|\right)}{b^3c}-\frac{2\,c^2d^2x-2\,bcdxe+bcd^2+b^2xe^2}{\left(cx^2+bx\right)b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x)^2,x, algorithm="giac")
```

```
[Out] -2*(c*d^2 - b*d*e)*log(abs(x))/b^3 + 2*(c^2*d^2 - b*c*d*e)*log(abs(c*x + b)
)/(b^3*c) - (2*c^2*d^2*x - 2*b*c*d*x*e + b*c*d^2 + b^2*x*e^2)/((c*x^2 + b*x
)*b^2*c)
```

3.271 $\int \frac{d+ex}{\left(bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=65

$$-\frac{cd-be}{b^2(b+cx)}-\frac{\log(x)(2cd-be)}{b^3}+\frac{(2cd-be)\log(b+cx)}{b^3}-\frac{d}{b^2x}$$

```
[Out] -(d/(b^2*x)) - (c*d - b*e)/(b^2*(b + c*x)) - ((2*c*d - b*e)*Log[x])/b^3 + (
(2*c*d - b*e)*Log[b + c*x])/b^3
```

Rubi [A] time = 0.0503731, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {631}

$$-\frac{cd-be}{b^2(b+cx)}-\frac{\log(x)(2cd-be)}{b^3}+\frac{(2cd-be)\log(b+cx)}{b^3}-\frac{d}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(b*x + c*x^2)^2,x]
```

```
[Out] -(d/(b^2*x)) - (c*d - b*e)/(b^2*(b + c*x)) - ((2*c*d - b*e)*Log[x])/b^3 + (
(2*c*d - b*e)*Log[b + c*x])/b^3
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex}{\left(bx+cx^2\right)^2}\,dx &= \int \left(\frac{d}{b^2x^2}+\frac{-2cd+be}{b^3x}-\frac{c(-cd+be)}{b^2(b+cx)^2}-\frac{c(-2cd+be)}{b^3(b+cx)}\right)dx\\ &= -\frac{d}{b^2x}-\frac{cd-be}{b^2(b+cx)}-\frac{(2cd-be)\log(x)}{b^3}+\frac{(2cd-be)\log(b+cx)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.0396616, size = 56, normalized size = 0.86

$$\frac{\frac{b(be-cd)}{b+cx}+\log(x)(be-2cd)+(2cd-be)\log(b+cx)-\frac{bd}{x}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(b*x + c*x^2)^2,x]
```

```
[Out] (-((b*d)/x) + (b*(-(c*d) + b*e))/(b + c*x) + (-2*c*d + b*e)*Log[x] + (2*c*d
 - b*e)*Log[b + c*x])/b^3
```

Maple [A] time = 0.054, size = 78, normalized size = 1.2

$$-\frac{d}{b^{2}x}+\frac{\ln\left(x\right)e}{b^{2}}-2\,\frac{\ln\left(x\right)cd}{b^{3}}-\frac{\ln\left(cx+b\right)e}{b^{2}}+2\,\frac{\ln\left(cx+b\right)cd}{b^{3}}+\frac{e}{b\left(cx+b\right)}-\frac{cd}{b^{2}\left(cx+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^2+b*x)^2,x)
```

```
[Out] -d/b^2/x+1/b^2*ln(x)*e-2/b^3*ln(x)*c*d-1/b^2*ln(c*x+b)*e+2/b^3*ln(c*x+b)*c*
d+1/b/(c*x+b)*e-1/b^2/(c*x+b)*c*d
```

Maxima [A] time = 1.13112, size = 93, normalized size = 1.43

$$-\frac{bd+\left(2\,cd-be\right)x}{b^{2}cx^{2}+b^{3}x}+\frac{\left(2\,cd-be\right)\log\left(cx+b\right)}{b^{3}}-\frac{\left(2\,cd-be\right)\log\left(x\right)}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

```
[Out] -(b*d + (2*c*d - b*e)*x)/(b^2*c*x^2 + b^3*x) + (2*c*d - b*e)*log(c*x + b)/b
^3 - (2*c*d - b*e)*log(x)/b^3
```

Fricas [A] time = 1.68922, size = 227, normalized size = 3.49

$$-\frac{b^{2}d+\left(2\,bcd-b^{2}e\right)x-\left(\left(2\,c^{2}d-bce\right)x^{2}+\left(2\,bcd-b^{2}e\right)x\right)\log\left(cx+b\right)+\left(\left(2\,c^{2}d-bce\right)x^{2}+\left(2\,bcd-b^{2}e\right)x\right)\log\left(x\right)}{b^{3}cx^{2}+b^{4}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

```
[Out] -(b^2*d + (2*b*c*d - b^2*e)*x - ((2*c^2*d - b*c*e)*x^2 + (2*b*c*d - b^2*e)*
x)*log(c*x + b) + ((2*c^2*d - b*c*e)*x^2 + (2*b*c*d - b^2*e)*x)*log(x))/(b^
3*c*x^2 + b^4*x)
```

Sympy [B] time = 1.45253, size = 128, normalized size = 1.97

$$\frac{-bd+x\left(be-2cd\right)}{b^{3}x+b^{2}cx^{2}}+\frac{\left(be-2cd\right)\log\left(x+\frac{b^{2}e-2bcd-b(be-2cd)}{2bce-4c^{2}d}\right)}{b^{3}}-\frac{\left(be-2cd\right)\log\left(x+\frac{b^{2}e-2bcd+b(be-2cd)}{2bce-4c^{2}d}\right)}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**2+b*x)**2,x)
```

```
[Out] (-b*d + x*(b*e - 2*c*d))/(b**3*x + b**2*c*x**2) + (b*e - 2*c*d)*log(x + (b*
*2*e - 2*b*c*d - b*(b*e - 2*c*d))/(2*b*c*e - 4*c**2*d))/b**3 - (b*e - 2*c*d
)*log(x + (b**2*e - 2*b*c*d + b*(b*e - 2*c*d))/(2*b*c*e - 4*c**2*d))/b**3
```

Giac [A] time = 1.2503, size = 104, normalized size = 1.6

$$-\frac{(2\,cd-be)\log\left(|x|\right)}{b^3}-\frac{2\,cdx-bxe+bd}{\left(cx^2+bx\right)b^2}+\frac{\left(2\,c^2d-bce\right)\log\left(|cx+b|\right)}{b^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x)^2,x, algorithm="giac")
```

```
[Out] -(2*c*d - b*e)*log(abs(x))/b^3 - (2*c*d*x - b*x*e + b*d)/((c*x^2 + b*x)*b^2
) + (2*c^2*d - b*c*e)*log(abs(c*x + b))/(b^3*c)
```

3.272 $\int \frac{1}{(bx+cx^2)^2} dx$

Optimal. Leaf size=43

$$-\frac{b+2cx}{b^2\left(bx+cx^2\right)}-\frac{2c\log(x)}{b^3}+\frac{2c\log(b+cx)}{b^3}$$

```
[Out] -((b + 2*c*x)/(b^2*(b*x + c*x^2))) - (2*c*Log[x])/b^3 + (2*c*Log[b + c*x])/
b^3
```

Rubi [A] time = 0.009857, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {614, 615}

$$-\frac{b+2cx}{b^2\left(bx+cx^2\right)}-\frac{2c\log(x)}{b^3}+\frac{2c\log(b+cx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(-2),x]
```

```
[Out] -((b + 2*c*x)/(b^2*(b*x + c*x^2))) - (2*c*Log[x])/b^3 + (2*c*Log[b + c*x])/
b^3
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 615

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[Log[x]/b, x] - Simp
[Log[RemoveContent[b + c*x, x]]/b, x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(bx+cx^2\right)^2}\,dx &= -\frac{b+2cx}{b^2\left(bx+cx^2\right)}-\frac{(2c)\int \frac{1}{bx+cx^2}\,dx}{b^2}\\ &= -\frac{b+2cx}{b^2\left(bx+cx^2\right)}-\frac{2c\log(x)}{b^3}+\frac{2c\log(b+cx)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.0381159, size = 35, normalized size = 0.81

$$-\frac{b\left(\frac{c}{b+cx}+\frac{1}{x}\right)-2c\log(b+cx)+2c\log(x)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(-2),x]

[Out] -((b*(x^(-1) + c/(b + c*x)) + 2*c*Log[x] - 2*c*Log[b + c*x])/b^3)
```

Maple [A] time = 0.056, size = 43, normalized size = 1.

$$-\frac{1}{b^2x}-2\,\frac{c\ln(x)}{b^3}-\frac{c}{b^2\,(cx+b)}+2\,\frac{c\ln(cx+b)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^2,x)

[Out] -1/b^2/x-2*c*ln(x)/b^3-c/b^2/(c*x+b)+2*c*ln(c*x+b)/b^3
```

Maxima [A] time = 1.13675, size = 61, normalized size = 1.42

$$-\frac{2\,cx+b}{b^2cx^2+b^3x}+\frac{2\,c\log(cx+b)}{b^3}-\frac{2\,c\log(x)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^2,x, algorithm="maxima")

[Out] -(2*c*x + b)/(b^2*c*x^2 + b^3*x) + 2*c*log(c*x + b)/b^3 - 2*c*log(x)/b^3
```

Fricas [A] time = 1.71884, size = 138, normalized size = 3.21

$$-\frac{2\,bcx+b^2-2\left(c^2x^2+bcx\right)\log(cx+b)+2\left(c^2x^2+bcx\right)\log(x)}{b^3cx^2+b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^2,x, algorithm="fricas")

[Out] -(2*b*c*x + b^2 - 2*(c^2*x^2 + b*c*x)*log(c*x + b) + 2*(c^2*x^2 + b*c*x)*lo
g(x))/(b^3*c*x^2 + b^4*x)
```

Sympy [A] time = 1.22442, size = 36, normalized size = 0.84

$$-\frac{b+2cx}{b^3x+b^2cx^2}+\frac{2c\left(-\log(x)+\log\left(\frac{b}{c}+x\right)\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**2,x)

[Out] -(b + 2*c*x)/(b**3*x + b**2*c*x**2) + 2*c*(-log(x) + log(b/c + x))/b**3
```

Giac [A] time = 1.3025, size = 61, normalized size = 1.42

$$\frac{2\,c\log\left(|cx+b|\right)}{b^3}-\frac{2\,c\log\left(|x|\right)}{b^3}-\frac{2\,cx+b}{\left(cx^2+bx\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^2,x, algorithm="giac")
```

```
[Out] 2*c*log(abs(c*x + b))/b^3 - 2*c*log(abs(x))/b^3 - (2*c*x + b)/((c*x^2 + b*x
)*b^2)
```

3.273 $\int \frac{1}{(d+ex)\left(bx+cx^2\right)^2} \, dx$

Optimal. Leaf size=110

$$-\frac{c^2}{b^2(b+cx)(cd-be)}+\frac{c^2(2cd-3be)\log(b+cx)}{b^3(cd-be)^2}-\frac{\log(x)(be+2cd)}{b^3d^2}-\frac{1}{b^2dx}+\frac{e^3\log(d+ex)}{d^2(cd-be)^2}$$

```
[Out] -(1/(b^2*d*x)) - c^2/(b^2*(c*d - b*e)*(b + c*x)) - ((2*c*d + b*e)*Log[x])/(
b^3*d^2) + (c^2*(2*c*d - 3*b*e)*Log[b + c*x])/(b^3*(c*d - b*e)^2) + (e^3*Lo
g[d + e*x])/(d^2*(c*d - b*e)^2)
```

Rubi [A] time = 0.118041, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$-\frac{c^2}{b^2(b+cx)(cd-be)}+\frac{c^2(2cd-3be)\log(b+cx)}{b^3(cd-be)^2}-\frac{\log(x)(be+2cd)}{b^3d^2}-\frac{1}{b^2dx}+\frac{e^3\log(d+ex)}{d^2(cd-be)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(b*x + c*x^2)^2),x]
```

```
[Out] -(1/(b^2*d*x)) - c^2/(b^2*(c*d - b*e)*(b + c*x)) - ((2*c*d + b*e)*Log[x])/(
b^3*d^2) + (c^2*(2*c*d - 3*b*e)*Log[b + c*x])/(b^3*(c*d - b*e)^2) + (e^3*Lo
g[d + e*x])/(d^2*(c*d - b*e)^2)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)^2} \, dx = \int \left(\frac{1}{b^2dx^2}+\frac{-2cd-be}{b^3d^2x}-\frac{c^3}{b^2(-cd+be)(b+cx)^2}-\frac{c^3(-2cd+3be)}{b^3(-cd+be)^2(b+cx)}+\frac{e^4}{d^2(cd-be)^2(\ldots}\right.$$

$$=-\frac{1}{b^2dx}-\frac{c^2}{b^2(cd-be)(b+cx)}-\frac{(2cd+be)\log(x)}{b^3d^2}+\frac{c^2(2cd-3be)\log(b+cx)}{b^3(cd-be)^2}+\frac{e^3\log(d\ldots}{d^2(cd-\ldots}$$

Mathematica [A] time = 0.0960025, size = 111, normalized size = 1.01

$$\frac{c^2}{b^2(b+cx)(be-cd)}+\frac{\left(2c^3d-3bc^2e\right)\log(b+cx)}{b^3(be-cd)^2}+\frac{\log(x)(-be-2cd)}{b^3d^2}-\frac{1}{b^2dx}+\frac{e^3\log(d+ex)}{d^2(cd-be)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(b*x + c*x^2)^2),x]
```

```
[Out] -(1/(b^2*d*x)) + c^2/(b^2*(-(c*d) + b*e)*(b + c*x)) + ((-2*c*d - b*e)*Log[x
])/(b^3*d^2) + ((2*c^3*d - 3*b*c^2*e)*Log[b + c*x])/(b^3*(-(c*d) + b*e)^2)
```

```
+ (e^3*Log[d + e*x])/(d^2*(c*d - b*e)^2)
```

Maple [A] time = 0.105, size = 132, normalized size = 1.2

$$-\frac{1}{b^{2}dx}-\frac{\ln\left(x\right)e}{b^{2}d^{2}}-2\,\frac{\ln\left(x\right)c}{db^{3}}+\frac{c^{2}}{\left(be-cd\right)b^{2}\left(cx+b\right)}-3\,\frac{c^{2}\ln\left(cx+b\right)e}{\left(be-cd\right)^{2}b^{2}}+2\,\frac{c^{3}\ln\left(cx+b\right)d}{\left(be-cd\right)^{2}b^{3}}+\frac{e^{3}\ln\left(ex+d\right)}{d^{2}\left(be-cd\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+b*x)^2,x)
```

```
[Out] -1/b^2/d/x-1/d^2/b^2*ln(x)*e-2/d/b^3*ln(x)*c+c^2/(b*e-c*d)/b^2/(c*x+b)-3*c^
2/(b*e-c*d)^2/b^2*ln(c*x+b)*e+2*c^3/(b*e-c*d)^2/b^3*ln(c*x+b)*d+e^3/d^2/(b*
e-c*d)^2*ln(e*x+d)
```

Maxima [A] time = 1.17669, size = 239, normalized size = 2.17

$$\frac{e^{3}\log\left(ex+d\right)}{c^{2}d^{4}-2\,bcd^{3}e+b^{2}d^{2}e^{2}}+\frac{\left(2\,c^{3}d-3\,bc^{2}e\right)\log\left(cx+b\right)}{b^{3}c^{2}d^{2}-2\,b^{4}cde+b^{5}e^{2}}-\frac{bcd-b^{2}e+\left(2\,c^{2}d-bce\right)x}{\left(b^{2}c^{2}d^{2}-b^{3}cde\right)x^{2}+\left(b^{3}cd^{2}-b^{4}de\right)x}-\frac{\left(2\,cd+be\right)\log\left(x\right)}{b^{3}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

```
[Out] e^3*log(e*x + d)/(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2) + (2*c^3*d - 3*b*c^2
*e)*log(c*x + b)/(b^3*c^2*d^2 - 2*b^4*c*d*e + b^5*e^2) - (b*c*d - b^2*e + (
2*c^2*d - b*c*e)*x)/((b^2*c^2*d^2 - b^3*c*d*e)*x^2 + (b^3*c*d^2 - b^4*d*e)*
x) - (2*c*d + b*e)*log(x)/(b^3*d^2)
```

Fricas [B] time = 24.7447, size = 574, normalized size = 5.22

$$-\frac{b^{2}c^{2}d^{3}-2\,b^{3}cd^{2}e+b^{4}de^{2}+\left(2\,bc^{3}d^{3}-3\,b^{2}c^{2}d^{2}e+b^{3}cde^{2}\right)x-\left(\left(2\,c^{4}d^{3}-3\,bc^{3}d^{2}e\right)x^{2}+\left(2\,bc^{3}d^{3}-3\,b^{2}c^{2}d^{2}e\right)x\right)\log\left(cx+\right.}{\left(b^{3}c^{3}d^{4}-2\,b^{4}c^{2}d^{3}e+b^{5}cd^{2}e^{2}\right)x^{2}+\left(b^{4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

```
[Out] -(b^2*c^2*d^3 - 2*b^3*c*d^2*e + b^4*d*e^2 + (2*b*c^3*d^3 - 3*b^2*c^2*d^2*e
+ b^3*c*d*e^2)*x - ((2*c^4*d^3 - 3*b*c^3*d^2*e)*x^2 + (2*b*c^3*d^3 - 3*b^2*
c^2*d^2*e)*x)*log(c*x + b) - (b^3*c*e^3*x^2 + b^4*e^3*x)*log(e*x + d) + ((2
*c^4*d^3 - 3*b*c^3*d^2*e + b^3*c*e^3)*x^2 + (2*b*c^3*d^3 - 3*b^2*c^2*d^2*e
+ b^4*e^3)*x)*log(x))/((b^3*c^3*d^4 - 2*b^4*c^2*d^3*e + b^5*c*d^2*e^2)*x^2
+ (b^4*c^2*d^4 - 2*b^5*c*d^3*e + b^6*d^2*e^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+b*x)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29849, size = 273, normalized size = 2.48

$$\frac{\left(2\,c^4d-3\,bc^3e\right)\log\left(|cx+b|\right)}{b^3c^3d^2-2\,b^4c^2de+b^5ce^2}+\frac{e^4\log\left(|xe+d|\right)}{c^2d^4e-2\,bcd^3e^2+b^2d^2e^3}-\frac{(2\,cd+be)\log\left(|x|\right)}{b^3d^2}-\frac{bc^2d^3-2\,b^2cd^2e+b^3de^2+\left(2\,c^3d^3-\right.}{(cd-be)^2(cx+b)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x)^2,x, algorithm="giac")
```

```
[Out] (2*c^4*d - 3*b*c^3*e)*log(abs(c*x + b))/(b^3*c^3*d^2 - 2*b^4*c^2*d*e + b^5*
c*e^2) + e^4*log(abs(x*e + d))/(c^2*d^4*e - 2*b*c*d^3*e^2 + b^2*d^2*e^3) -
(2*c*d + b*e)*log(abs(x))/(b^3*d^2) - (b*c^2*d^3 - 2*b^2*c*d^2*e + b^3*d*e^
2 + (2*c^3*d^3 - 3*b*c^2*d^2*e + b^2*c*d*e^2)*x)/((c*d - b*e)^2*(c*x + b)*b
^2*d^2*x)
```

3.274 $\int \frac{1}{(d+ex)^2\left(bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=144

$$-\frac{c^3}{b^2(b+cx)(cd-be)^2}+\frac{2c^3(cd-2be)\log(b+cx)}{b^3(cd-be)^3}-\frac{2\log(x)(be+cd)}{b^3d^3}-\frac{1}{b^2d^2x}-\frac{e^3}{d^2(d+ex)(cd-be)^2}+\frac{2e^3(2cd-be)\log}{d^3(cd-be}$$

```
[Out] -(1/(b^2*d^2*x)) - c^3/(b^2*(c*d - b*e)^2*(b + c*x)) - e^3/(d^2*(c*d - b*e)
^2*(d + e*x)) - (2*(c*d + b*e)*Log[x])/(b^3*d^3) + (2*c^3*(c*d - 2*b*e)*Log
[b + c*x])/(b^3*(c*d - b*e)^3) + (2*e^3*(2*c*d - b*e)*Log[d + e*x])/(d^3*(c
*d - b*e)^3)
```

Rubi [A] time = 0.167268, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$-\frac{c^3}{b^2(b+cx)(cd-be)^2}+\frac{2c^3(cd-2be)\log(b+cx)}{b^3(cd-be)^3}-\frac{2\log(x)(be+cd)}{b^3d^3}-\frac{1}{b^2d^2x}-\frac{e^3}{d^2(d+ex)(cd-be)^2}+\frac{2e^3(2cd-be)\log}{d^3(cd-be}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(b*x + c*x^2)^2),x]
```

```
[Out] -(1/(b^2*d^2*x)) - c^3/(b^2*(c*d - b*e)^2*(b + c*x)) - e^3/(d^2*(c*d - b*e)
^2*(d + e*x)) - (2*(c*d + b*e)*Log[x])/(b^3*d^3) + (2*c^3*(c*d - 2*b*e)*Log
[b + c*x])/(b^3*(c*d - b*e)^3) + (2*e^3*(2*c*d - b*e)*Log[d + e*x])/(d^3*(c
*d - b*e)^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(bx+cx^2\right)^2}\,dx=\int\left(\frac{1}{b^2d^2x^2}-\frac{2(cd+be)}{b^3d^3x}+\frac{c^4}{b^2(-cd+be)^2(b+cx)^2}+\frac{2c^4(-cd+2be)}{b^3(-cd+be)^3(b+cx)}+\frac{e^4}{d^2(cd-be)^2(}\right.$$
$$=-\frac{1}{b^2d^2x}-\frac{c^3}{b^2(cd-be)^2(b+cx)}-\frac{e^3}{d^2(cd-be)^2(d+ex)}-\frac{2(cd+be)\log(x)}{b^3d^3}+\frac{2c^3(cd-2be)\,\mathrm{l}}{b^3(cd-}$$

Mathematica [A] time = 0.178529, size = 145, normalized size = 1.01

$$-\frac{c^3}{b^2(b+cx)(cd-be)^2}+\frac{2c^3(2be-cd)\log(b+cx)}{b^3(be-cd)^3}-\frac{2\log(x)(be+cd)}{b^3d^3}-\frac{1}{b^2d^2x}-\frac{e^3}{d^2(d+ex)(cd-be)^2}+\frac{2e^3(2cd-be)\log}{d^3(cd-be}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(b*x + c*x^2)^2),x]
```

```
[Out] -(1/(b^2*d^2*x)) - c^3/(b^2*(c*d - b*e)^2*(b + c*x)) - e^3/(d^2*(c*d - b*e)
^2*(d + e*x)) - (2*(c*d + b*e)*Log[x])/(b^3*d^3) + (2*c^3*(-(c*d) + 2*b*e)*
Log[b + c*x])/(b^3*(-(c*d) + b*e)^3) + (2*e^3*(2*c*d - b*e)*Log[d + e*x])/(
d^3*(c*d - b*e)^3)
```

Maple [A] time = 0.065, size = 185, normalized size = 1.3

$$-\frac{1}{b^2d^2x}-2\,\frac{\ln(x)\,e}{d^3b^2}-2\,\frac{\ln(x)\,c}{d^2b^3}-\frac{c^3}{(be-cd)^2\,b^2\,(cx+b)}+4\,\frac{c^3\ln(cx+b)\,e}{(be-cd)^3\,b^2}-2\,\frac{c^4\ln(cx+b)\,d}{(be-cd)^3\,b^3}-\frac{e^3}{d^2\,(be-cd)^2\,(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^2+b*x)^2,x)
```

```
[Out] -1/b^2/d^2/x-2/d^3/b^2*ln(x)*e-2/d^2/b^3*ln(x)*c-c^3/(b*e-c*d)^2/b^2/(c*x+b
)+4*c^3/(b*e-c*d)^3/b^2*ln(c*x+b)*e-2*c^4/(b*e-c*d)^3/b^3*ln(c*x+b)*d-e^3/d
^2/(b*e-c*d)^2/(e*x+d)+2*e^4/d^3/(b*e-c*d)^3*ln(e*x+d)*b-4*e^3/d^2/(b*e-c*d
)^3*ln(e*x+d)*c
```

Maxima [B] time = 1.2827, size = 504, normalized size = 3.5

$$\frac{2\left(c^4d-2\,bc^3e\right)\log(cx+b)}{b^3c^3d^3-3\,b^4c^2d^2e+3\,b^5cde^2-b^6e^3}+\frac{2\left(2\,cde^3-be^4\right)\log(ex+d)}{c^3d^6-3\,bc^2d^5e+3\,b^2cd^4e^2-b^3d^3e^3}-\frac{bc^2d^3-2\,b^2cd^2e+b^3de^2+2\,(c^3 \ldots}{\left(b^2c^3d^4e-2\,b^3c^2d^3e^2+b^4cd^2e^3\right)x^3+(b \ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x)^2,x, algorithm="maxima")
```

```
[Out] 2*(c^4*d - 2*b*c^3*e)*log(c*x + b)/(b^3*c^3*d^3 - 3*b^4*c^2*d^2*e + 3*b^5*c
*d*e^2 - b^6*e^3) + 2*(2*c*d*e^3 - b*e^4)*log(e*x + d)/(c^3*d^6 - 3*b*c^2*d
^5*e + 3*b^2*c*d^4*e^2 - b^3*d^3*e^3) - (b*c^2*d^3 - 2*b^2*c*d^2*e + b^3*d*
e^2 + 2*(c^3*d^2*e - b*c^2*d*e^2 + b^2*c*e^3)*x^2 + (2*c^3*d^3 - b*c^2*d^2*
e - b^2*c*d*e^2 + 2*b^3*e^3)*x)/((b^2*c^3*d^4*e - 2*b^3*c^2*d^3*e^2 + b^4*c
*d^2*e^3)*x^3 + (b^2*c^3*d^5 - b^3*c^2*d^4*e - b^4*c*d^3*e^2 + b^5*d^2*e^3)
*x^2 + (b^3*c^2*d^5 - 2*b^4*c*d^4*e + b^5*d^3*e^2)*x) - 2*(c*d + b*e)*log(x
)/(b^3*d^3)
```

Fricas [B] time = 85.7863, size = 1262, normalized size = 8.76

$$-\frac{b^2c^3d^5-3\,b^3c^2d^4e+3\,b^4cd^3e^2-b^5d^2e^3+2\left(bc^4d^4e-2\,b^2c^3d^3e^2+2\,b^3c^2d^2e^3-b^4cde^4\right)x^2+\left(2\,bc^4d^5-3\,b^2c^3d^4e+3 \ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x)^2,x, algorithm="fricas")
```

```
[Out] -(b^2*c^3*d^5 - 3*b^3*c^2*d^4*e + 3*b^4*c*d^3*e^2 - b^5*d^2*e^3 + 2*(b*c^4*
d^4*e - 2*b^2*c^3*d^3*e^2 + 2*b^3*c^2*d^2*e^3 - b^4*c*d*e^4)*x^2 + (2*b*c^4
*d^5 - 3*b^2*c^3*d^4*e + 3*b^4*c*d^2*e^3 - 2*b^5*d*e^4)*x - 2*((c^5*d^4*e -
 2*b*c^4*d^3*e^2)*x^3 + (c^5*d^5 - b*c^4*d^4*e - 2*b^2*c^3*d^3*e^2)*x^2 + (
b*c^4*d^5 - 2*b^2*c^3*d^4*e)*x)*log(c*x + b) - 2*((2*b^3*c^2*d*e^4 - b^4*c*
```

```
e^5)*x^3 + (2*b^3*c^2*d^2*e^3 + b^4*c*d*e^4 - b^5*e^5)*x^2 + (2*b^4*c*d^2*e
^3 - b^5*d*e^4)*x)*log(e*x + d) + 2*((c^5*d^4*e - 2*b*c^4*d^3*e^2 + 2*b^3*c
^2*d*e^4 - b^4*c*e^5)*x^3 + (c^5*d^5 - b*c^4*d^4*e - 2*b^2*c^3*d^3*e^2 + 2*
b^3*c^2*d^2*e^3 + b^4*c*d*e^4 - b^5*e^5)*x^2 + (b*c^4*d^5 - 2*b^2*c^3*d^4*e
 + 2*b^4*c*d^2*e^3 - b^5*d*e^4)*x)*log(x))/((b^3*c^4*d^6*e - 3*b^4*c^3*d^5*
e^2 + 3*b^5*c^2*d^4*e^3 - b^6*c*d^3*e^4)*x^3 + (b^3*c^4*d^7 - 2*b^4*c^3*d^6
*e + 2*b^6*c*d^4*e^3 - b^7*d^3*e^4)*x^2 + (b^4*c^3*d^7 - 3*b^5*c^2*d^6*e +
3*b^6*c*d^5*e^2 - b^7*d^4*e^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**2+b*x)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.84037, size = 749, normalized size = 5.2

$$\frac{\left(2\,c^4d^4e^2-4\,bc^3d^3e^3+2\,b^3cde^5-b^4e^6\right)e^{(-2)}\log\left(\frac{\left|-2\,cde+\frac{2\,cd^2e}{xe+d}+be^2-\frac{2\,bde^2}{xe+d}-|b|e^2\right|}{\left|-2\,cde+\frac{2\,cd^2e}{xe+d}+be^2-\frac{2\,bde^2}{xe+d}+|b|e^2\right|}\right)}{\left(b^2c^3d^6-3\,b^3c^2d^5e+3\,b^4cd^4e^2-b^5d^3e^3\right)|b|}-\frac{\left(2\,cde^3-be^4\right)\log\left(\left|-c+\frac{2\,cd}{xe+d}-\frac{cd^2}{(xe+d)^2}-\frac{b}{xe}\right.\right.}{c^3d^6-3\,bc^2d^5e+3\,b^2cd^4e^2-b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x)^2,x, algorithm="giac")
```

```
[Out] (2*c^4*d^4*e^2 - 4*b*c^3*d^3*e^3 + 2*b^3*c*d*e^5 - b^4*e^6)*e^(-2)*log(abs(
-2*c*d*e + 2*c*d^2*e/(x*e + d) + b*e^2 - 2*b*d*e^2/(x*e + d) - abs(b)*e^2)/
abs(-2*c*d*e + 2*c*d^2*e/(x*e + d) + b*e^2 - 2*b*d*e^2/(x*e + d) + abs(b)*e
^2))/((b^2*c^3*d^6 - 3*b^3*c^2*d^5*e + 3*b^4*c*d^4*e^2 - b^5*d^3*e^3)*abs(b
)) - (2*c*d*e^3 - b*e^4)*log(abs(-c + 2*c*d/(x*e + d) - c*d^2/(x*e + d)^2 -
 b*e/(x*e + d) + b*d*e/(x*e + d)^2))/(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4
*e^2 - b^3*d^3*e^3) - e^7/((c^2*d^4*e^4 - 2*b*c*d^3*e^5 + b^2*d^2*e^6)*(x*e
 + d)) - ((2*c^4*d^3*e - 3*b*c^3*d^2*e^2 + 3*b^2*c^2*d*e^3 - b^3*c*e^4)/(c*
d^2 - b*d*e) - (2*c^4*d^4*e^2 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*b^3
*c*d*e^5 + b^4*e^6)*e^(-1)/((c*d^2 - b*d*e)*(x*e + d)))/((c*d - b*e)^2*b^2*
(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + b*e/(x*e + d) - b*d*e/(x*e + d)^
2)*d^2)
```

3.275 $\int \frac{(d+ex)^7}{(bx+cx^2)^3} dx$

Optimal. Leaf size=203

$$\frac{3d^5\log(x)\left(7b^2e^2-7bcde+2c^2d^2\right)}{b^5}-\frac{3(cd-be)^5\left(2b^2e^2+3bcde+2c^2d^2\right)\log(b+cx)}{b^5c^5}+\frac{(cd-be)^6(4be+3cd)}{b^4c^5(b+cx)}+\frac{(cd}{2b^3c^5}$$

```
[Out] -d^7/(2*b^3*x^2) + (d^6*(3*c*d - 7*b*e))/(b^4*x) + (e^6*(7*c*d - 3*b*e)*x)/
c^4 + (e^7*x^2)/(2*c^3) + (c*d - b*e)^7/(2*b^3*c^5*(b + c*x)^2) + ((c*d - b
*e)^6*(3*c*d + 4*b*e))/(b^4*c^5*(b + c*x)) + (3*d^5*(2*c^2*d^2 - 7*b*c*d*e
+ 7*b^2*e^2)*Log[x])/b^5 - (3*(c*d - b*e)^5*(2*c^2*d^2 + 3*b*c*d*e + 2*b^2*
e^2)*Log[b + c*x])/(b^5*c^5)
```

Rubi [A] time = 0.268947, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$\frac{3d^5\log(x)\left(7b^2e^2-7bcde+2c^2d^2\right)}{b^5}-\frac{3(cd-be)^5\left(2b^2e^2+3bcde+2c^2d^2\right)\log(b+cx)}{b^5c^5}+\frac{(cd-be)^6(4be+3cd)}{b^4c^5(b+cx)}+\frac{(cd}{2b^3c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^7/(b*x + c*x^2)^3,x]
```

```
[Out] -d^7/(2*b^3*x^2) + (d^6*(3*c*d - 7*b*e))/(b^4*x) + (e^6*(7*c*d - 3*b*e)*x)/
c^4 + (e^7*x^2)/(2*c^3) + (c*d - b*e)^7/(2*b^3*c^5*(b + c*x)^2) + ((c*d - b
*e)^6*(3*c*d + 4*b*e))/(b^4*c^5*(b + c*x)) + (3*d^5*(2*c^2*d^2 - 7*b*c*d*e
+ 7*b^2*e^2)*Log[x])/b^5 - (3*(c*d - b*e)^5*(2*c^2*d^2 + 3*b*c*d*e + 2*b^2*
e^2)*Log[b + c*x])/(b^5*c^5)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(d+ex)^7}{\left(bx+cx^2\right)^3} dx = \int \left(\frac{e^6(7cd-3be)}{c^4}+\frac{d^7}{b^3x^3}+\frac{d^6(-3cd+7be)}{b^4x^2}+\frac{3d^5\left(2c^2d^2-7bcde+7b^2e^2\right)}{b^5x}+\frac{e^7x}{c^3}+\frac{(-cd+be)^7}{b^3c^4(b+cx)^3}\right.$$

$$= -\frac{d^7}{2b^3x^2}+\frac{d^6(3cd-7be)}{b^4x}+\frac{e^6(7cd-3be)x}{c^4}+\frac{e^7x^2}{2c^3}+\frac{(cd-be)^7}{2b^3c^5(b+cx)^2}+\frac{(cd-be)^6(3cd+4be)}{b^4c^5(b+cx)}+\frac{3d}{}$$

Mathematica [A] time = 0.123366, size = 202, normalized size = 1.

$$\frac{1}{2}\left(\frac{6d^5\log(x)\left(7b^2e^2-7bcde+2c^2d^2\right)}{b^5}+\frac{6(be-cd)^5\left(2b^2e^2+3bcde+2c^2d^2\right)\log(b+cx)}{b^5c^5}+\frac{2(cd-be)^6(4be+3cd)}{b^4c^5(b+cx)}+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^7/(b*x + c*x^2)^3,x]
```

```
[Out] (-(d^7/(b^3*x^2)) + (2*d^6*(3*c*d - 7*b*e))/(b^4*x) + (2*e^6*(7*c*d - 3*b*e
)*x)/c^4 + (e^7*x^2)/c^3 + (c*d - b*e)^7/(b^3*c^5*(b + c*x)^2) + (2*(c*d -
b*e)^6*(3*c*d + 4*b*e))/(b^4*c^5*(b + c*x)) + (6*d^5*(2*c^2*d^2 - 7*b*c*d*e
 + 7*b^2*e^2)*Log[x])/b^5 + (6*(-(c*d) + b*e)^5*(2*c^2*d^2 + 3*b*c*d*e + 2*
b^2*e^2)*Log[b + c*x])/(b^5*c^5))/2
```

Maple [B] time = 0.066, size = 481, normalized size = 2.4

$$-\frac{d^7}{2\,b^3x^2}+\frac{e^7x^2}{2\,c^3}-3\,\frac{e^7xb}{c^4}+7\,\frac{de^6x}{c^3}-7\,\frac{d^6e}{b^3x}+3\,\frac{d^7c}{b^4x}+21\,\frac{d^5\ln(x)\,e^2}{b^3}+6\,\frac{d^7\ln(x)\,c^2}{b^5}+6\,\frac{b^2\ln(cx+b)\,e^7}{c^5}+21\,\frac{\ln(cx+b)\,c}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^7/(c*x^2+b*x)^3,x)
```

```
[Out] -1/2*d^7/b^3/x^2+1/2*e^7*x^2/c^3-3*e^7/c^4*x*b+7*e^6/c^3*d*x-7*d^6/b^3/x*e+
3*d^7/b^4/x*c+21*d^5/b^3*ln(x)*e^2+6*d^7/b^5*ln(x)*c^2+6/c^5*b^2*ln(c*x+b)*
e^7+21/c^3*ln(c*x+b)*d^2*e^5-21/b^3*ln(c*x+b)*d^5*e^2-6*c^2/b^5*ln(c*x+b)*d
^7+4/c^5*b^3/(c*x+b)*e^7+3*c^2/b^4/(c*x+b)*d^7-1/2/c^5*b^4/(c*x+b)^2*e^7+1/
2*c^2/b^3/(c*x+b)^2*d^7-35/c^2/(c*x+b)*d^3*e^4+21/b^2/(c*x+b)*d^5*e^2-35/2/
c/(c*x+b)^2*d^4*e^3+21/2/b/(c*x+b)^2*d^5*e^2-21/2/c^3*b^2/(c*x+b)^2*d^2*e^5
+35/2/c^2*b/(c*x+b)^2*d^3*e^4-7/2*c/b^2/(c*x+b)^2*d^6*e-21/c^4*b*ln(c*x+b)*
d*e^6+21*c/b^4*ln(c*x+b)*d^6*e-21/c^4*b^2/(c*x+b)*d*e^6+42/c^3*b/(c*x+b)*d^
2*e^5-21*d^6/b^4*ln(x)*c*e-14*c/b^3/(c*x+b)*d^6*e+7/2/c^4*b^3/(c*x+b)^2*d*e
^6
```

Maxima [B] time = 1.17236, size = 551, normalized size = 2.71

$$-\frac{b^3c^5d^7-2\left(6\,c^8d^7-21\,bc^7d^6e+21\,b^2c^6d^5e^2-35\,b^4c^4d^3e^4+42\,b^5c^3d^2e^5-21\,b^6c^2de^6+4\,b^7ce^7\right)x^3-\left(18\,bc^7d^7-63\,b^2c^6\right.}{2\left(b^4c^7x^4+2\,b^5c^6x^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7/(c*x^2+b*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*c^5*d^7 - 2*(6*c^8*d^7 - 21*b*c^7*d^6*e + 21*b^2*c^6*d^5*e^2 - 35
*b^4*c^4*d^3*e^4 + 42*b^5*c^3*d^2*e^5 - 21*b^6*c^2*d*e^6 + 4*b^7*c*e^7)*x^3
 - (18*b*c^7*d^7 - 63*b^2*c^6*d^6*e + 63*b^3*c^5*d^5*e^2 - 35*b^4*c^4*d^4*e
^3 - 35*b^5*c^3*d^3*e^4 + 63*b^6*c^2*d^2*e^5 - 35*b^7*c*d*e^6 + 7*b^8*e^7)*
x^2 - 2*(2*b^2*c^6*d^7 - 7*b^3*c^5*d^6*e)*x)/(b^4*c^7*x^4 + 2*b^5*c^6*x^3 +
 b^6*c^5*x^2) + 1/2*(c*e^7*x^2 + 2*(7*c*d*e^6 - 3*b*e^7)*x)/c^4 + 3*(2*c^2*
d^7 - 7*b*c*d^6*e + 7*b^2*d^5*e^2)*log(x)/b^5 - 3*(2*c^7*d^7 - 7*b*c^6*d^6*
e + 7*b^2*c^5*d^5*e^2 - 7*b^5*c^2*d^2*e^5 + 7*b^6*c*d*e^6 - 2*b^7*e^7)*log(
c*x + b)/(b^5*c^5)
```

Fricas [B] time = 2.12712, size = 1384, normalized size = 6.82

$$\frac{b^5c^4e^7x^6-b^4c^5d^7+2\left(7\,b^5c^4de^6-2\,b^6c^3e^7\right)x^5+\left(28\,b^6c^3de^6-11\,b^7c^2e^7\right)x^4+2\left(6\,bc^8d^7-21\,b^2c^7d^6e+21\,b^3c^6d^5e^2-35\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7/(c*x^2+b*x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^5*c^4*e^7*x^6 - b^4*c^5*d^7 + 2*(7*b^5*c^4*d*e^6 - 2*b^6*c^3*e^7)*x^
5 + (28*b^6*c^3*d*e^6 - 11*b^7*c^2*e^7)*x^4 + 2*(6*b*c^8*d^7 - 21*b^2*c^7*d
^6*e + 21*b^3*c^6*d^5*e^2 - 35*b^5*c^4*d^3*e^4 + 42*b^6*c^3*d^2*e^5 - 14*b^
7*c^2*d*e^6 + b^8*c*e^7)*x^3 + (18*b^2*c^7*d^7 - 63*b^3*c^6*d^6*e + 63*b^4*
c^5*d^5*e^2 - 35*b^5*c^4*d^4*e^3 - 35*b^6*c^3*d^3*e^4 + 63*b^7*c^2*d^2*e^5
- 35*b^8*c*d*e^6 + 7*b^9*e^7)*x^2 + 2*(2*b^3*c^6*d^7 - 7*b^4*c^5*d^6*e)*x -
 6*((2*c^9*d^7 - 7*b*c^8*d^6*e + 7*b^2*c^7*d^5*e^2 - 7*b^5*c^4*d^2*e^5 + 7*
b^6*c^3*d*e^6 - 2*b^7*c^2*e^7)*x^4 + 2*(2*b*c^8*d^7 - 7*b^2*c^7*d^6*e + 7*b
^3*c^6*d^5*e^2 - 7*b^6*c^3*d^2*e^5 + 7*b^7*c^2*d*e^6 - 2*b^8*c*e^7)*x^3 + (
2*b^2*c^7*d^7 - 7*b^3*c^6*d^6*e + 7*b^4*c^5*d^5*e^2 - 7*b^7*c^2*d^2*e^5 + 7
*b^8*c*d*e^6 - 2*b^9*e^7)*x^2)*log(c*x + b) + 6*((2*c^9*d^7 - 7*b*c^8*d^6*e
 + 7*b^2*c^7*d^5*e^2)*x^4 + 2*(2*b*c^8*d^7 - 7*b^2*c^7*d^6*e + 7*b^3*c^6*d^
5*e^2)*x^3 + (2*b^2*c^7*d^7 - 7*b^3*c^6*d^6*e + 7*b^4*c^5*d^5*e^2)*x^2)*log
(x))/(b^5*c^7*x^4 + 2*b^6*c^6*x^3 + b^7*c^5*x^2)
```

Sympy [B] time = 61.7282, size = 685, normalized size = 3.37

$$\frac{-b^3c^5d^7 + x^3\left(8b^7ce^7 - 42b^6c^2de^6 + 84b^5c^3d^2e^5 - 70b^4c^4d^3e^4 + 42b^2c^6d^5e^2 - 42bc^7d^6e + 12c^8d^7\right) + x^2\left(7b^8e^7 - 35b^7cd\right.}{2b^6c^5x^2 + 4b^5c^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**7/(c*x**2+b*x)**3,x)
```

```
[Out] (-b**3*c**5*d**7 + x**3*(8*b**7*c*e**7 - 42*b**6*c**2*d*e**6 + 84*b**5*c**3
*d**2*e**5 - 70*b**4*c**4*d**3*e**4 + 42*b**2*c**6*d**5*e**2 - 42*b*c**7*d*
*6*e + 12*c**8*d**7) + x**2*(7*b**8*e**7 - 35*b**7*c*d*e**6 + 63*b**6*c**2*
d**2*e**5 - 35*b**5*c**3*d**3*e**4 - 35*b**4*c**4*d**4*e**3 + 63*b**3*c**5*
d**5*e**2 - 63*b**2*c**6*d**6*e + 18*b*c**7*d**7) + x*(-14*b**3*c**5*d**6*e
 + 4*b**2*c**6*d**7))/(2*b**6*c**5*x**2 + 4*b**5*c**6*x**3 + 2*b**4*c**7*x*
*4) + e**7*x**2/(2*c**3) - x*(3*b*e**7 - 7*c*d*e**6)/c**4 + 3*d**5*(7*b**2*
e**2 - 7*b*c*d*e + 2*c**2*d**2)*log(x + (-21*b**3*c**4*d**5*e**2 + 21*b**2*
c**5*d**6*e - 6*b*c**6*d**7 + 3*b*c**4*d**5*(7*b**2*e**2 - 7*b*c*d*e + 2*c*
*2*d**2))/(6*b**7*e**7 - 21*b**6*c*d*e**6 + 21*b**5*c**2*d**2*e**5 - 42*b**
2*c**5*d**5*e**2 + 42*b*c**6*d**6*e - 12*c**7*d**7))/b**5 + 3*(b*e - c*d)**
5*(2*b**2*e**2 + 3*b*c*d*e + 2*c**2*d**2)*log(x + (-21*b**3*c**4*d**5*e**2
+ 21*b**2*c**5*d**6*e - 6*b*c**6*d**7 + 3*b*(b*e - c*d)**5*(2*b**2*e**2 + 3
*b*c*d*e + 2*c**2*d**2)/c)/(6*b**7*e**7 - 21*b**6*c*d*e**6 + 21*b**5*c**2*d
**2*e**5 - 42*b**2*c**5*d**5*e**2 + 42*b*c**6*d**6*e - 12*c**7*d**7))/(b**5
*c**5)
```

Giac [A] time = 2.01136, size = 517, normalized size = 2.55

$$\frac{3\left(2c^2d^7 - 7bcd^6e + 7b^2d^5e^2\right)\log\left(|x|\right)}{b^5} + \frac{c^3x^2e^7 + 14c^3dxe^6 - 6bc^2xe^7}{2c^6} - \frac{3\left(2c^7d^7 - 7bc^6d^6e + 7b^2c^5d^5e^2 - 7b^5c^2d^2e\right.}{b^5c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7/(c*x^2+b*x)^3,x, algorithm="giac")
```

```
[Out] 3*(2*c^2*d^7 - 7*b*c*d^6*e + 7*b^2*d^5*e^2)*log(abs(x))/b^5 + 1/2*(c^3*x^2*
e^7 + 14*c^3*d*x*e^6 - 6*b*c^2*x*e^7)/c^6 - 3*(2*c^7*d^7 - 7*b*c^6*d^6*e +
7*b^2*c^5*d^5*e^2 - 7*b^5*c^2*d^2*e^5 + 7*b^6*c*d*e^6 - 2*b^7*e^7)*log(abs(
c*x + b))/(b^5*c^5) - 1/2*(b^3*c^5*d^7 - 2*(6*c^8*d^7 - 21*b*c^7*d^6*e + 21
*b^2*c^6*d^5*e^2 - 35*b^4*c^4*d^3*e^4 + 42*b^5*c^3*d^2*e^5 - 21*b^6*c^2*d*e
^6 + 4*b^7*c*e^7)*x^3 - (18*b*c^7*d^7 - 63*b^2*c^6*d^6*e + 63*b^3*c^5*d^5*e
^2 - 35*b^4*c^4*d^4*e^3 - 35*b^5*c^3*d^3*e^4 + 63*b^6*c^2*d^2*e^5 - 35*b^7*
c*d*e^6 + 7*b^8*e^7)*x^2 - 2*(2*b^2*c^6*d^7 - 7*b^3*c^5*d^6*e)*x)/((c*x + b
)^2*b^4*c^5*x^2)
```

3.276 $\int \frac{(d+ex)^6}{\left(bx+cx^2\right)^3}\,dx$

Optimal. Leaf size=179

$$\frac{3d^4\log(x)\left(5b^2e^2-6bcde+2c^2d^2\right)}{b^5}-\frac{3(cd-be)^4\left(b^2e^2+2bcde+2c^2d^2\right)\log(b+cx)}{b^5c^4}+\frac{3(cd-be)^5(be+cd)}{b^4c^4(b+cx)}+\frac{(cd-}{2b^3c^4(b}$$

```
[Out] -d^6/(2*b^3*x^2) + (3*d^5*(c*d - 2*b*e))/(b^4*x) + (e^6*x)/c^3 + (c*d - b*e
)^6/(2*b^3*c^4*(b + c*x)^2) + (3*(c*d - b*e)^5*(c*d + b*e))/(b^4*c^4*(b + c
*x)) + (3*d^4*(2*c^2*d^2 - 6*b*c*d*e + 5*b^2*e^2)*Log[x])/b^5 - (3*(c*d - b
*e)^4*(2*c^2*d^2 + 2*b*c*d*e + b^2*e^2)*Log[b + c*x])/(b^5*c^4)
```

Rubi [A] time = 0.222377, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$\frac{3d^4\log(x)\left(5b^2e^2-6bcde+2c^2d^2\right)}{b^5}-\frac{3(cd-be)^4\left(b^2e^2+2bcde+2c^2d^2\right)\log(b+cx)}{b^5c^4}+\frac{3(cd-be)^5(be+cd)}{b^4c^4(b+cx)}+\frac{(cd-}{2b^3c^4(b}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^6/(b*x + c*x^2)^3,x]
```

```
[Out] -d^6/(2*b^3*x^2) + (3*d^5*(c*d - 2*b*e))/(b^4*x) + (e^6*x)/c^3 + (c*d - b*e
)^6/(2*b^3*c^4*(b + c*x)^2) + (3*(c*d - b*e)^5*(c*d + b*e))/(b^4*c^4*(b + c
*x)) + (3*d^4*(2*c^2*d^2 - 6*b*c*d*e + 5*b^2*e^2)*Log[x])/b^5 - (3*(c*d - b
*e)^4*(2*c^2*d^2 + 2*b*c*d*e + b^2*e^2)*Log[b + c*x])/(b^5*c^4)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(d+ex)^6}{\left(bx+cx^2\right)^3}\,dx=\int \left(\frac{e^6}{c^3}+\frac{d^6}{b^3x^3}+\frac{3d^5(-cd+2be)}{b^4x^2}+\frac{3d^4\left(2c^2d^2-6bcde+5b^2e^2\right)}{b^5x}-\frac{(-cd+be)^6}{b^3c^3(b+cx)^3}+\frac{3(-cd+be)^5(}{b^4c^3(b+c}\right.$$

$$=-\frac{d^6}{2b^3x^2}+\frac{3d^5(cd-2be)}{b^4x}+\frac{e^6x}{c^3}+\frac{(cd-be)^6}{2b^3c^4(b+cx)^2}+\frac{3(cd-be)^5(cd+be)}{b^4c^4(b+cx)}+\frac{3d^4\left(2c^2d^2-6bcde+5\right.}{b^5}$$

Mathematica [A] time = 0.0874307, size = 179, normalized size = 1.

$$\frac{3d^4\log(x)\left(5b^2e^2-6bcde+2c^2d^2\right)}{b^5}-\frac{3(cd-be)^4\left(b^2e^2+2bcde+2c^2d^2\right)\log(b+cx)}{b^5c^4}+\frac{3(cd-be)^5(be+cd)}{b^4c^4(b+cx)}+\frac{(cd-}{2b^3c^4(b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^6/(b*x + c*x^2)^3,x]
```

```
[Out] -d^6/(2*b^3*x^2) + (3*d^5*(c*d - 2*b*e))/(b^4*x) + (e^6*x)/c^3 + (c*d - b*e
)^6/(2*b^3*c^4*(b + c*x)^2) + (3*(c*d - b*e)^5*(c*d + b*e))/(b^4*c^4*(b + c
*x)) + (3*d^4*(2*c^2*d^2 - 6*b*c*d*e + 5*b^2*e^2)*Log[x])/b^5 - (3*(c*d - b
*e)^4*(2*c^2*d^2 + 2*b*c*d*e + b^2*e^2)*Log[b + c*x])/(b^5*c^4)
```

Maple [B] time = 0.062, size = 396, normalized size = 2.2

$$-18\,\frac{d^5\ln(x)\,ce}{b^4}+18\,\frac{c\ln(cx+b)\,d^5e}{b^4}-\frac{d^6}{2\,b^3x^2}+\frac{e^6x}{c^3}+15\,\frac{d^4\ln(x)\,e^2}{b^3}+6\,\frac{d^6\ln(x)\,c^2}{b^5}-6\,\frac{d^5e}{b^3x}+3\,\frac{d^6c}{b^4x}-3\,\frac{b\ln(cx+b)\,e^6}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^6/(c*x^2+b*x)^3,x)
```

```
[Out] -18*d^5/b^4*ln(x)*c*e+18*c/b^4*ln(c*x+b)*d^5*e-1/2*d^6/b^3/x^2+e^6*x/c^3+15
*d^4/b^3*ln(x)*e^2+6*d^6/b^5*ln(x)*c^2-6*d^5/b^3/x*e+3*d^6/b^4/x*c-3/c^4*b*
ln(c*x+b)*e^6+6/c^3*ln(c*x+b)*d*e^5-15/b^3*ln(c*x+b)*d^4*e^2-6*c^2/b^5*ln(c
*x+b)*d^6-3/c^4*b^2/(c*x+b)*e^6+3*c^2/b^4/(c*x+b)*d^6+1/2/c^4*b^3/(c*x+b)^2
*e^6+1/2*c^2/b^3/(c*x+b)^2*d^6-10/c/(c*x+b)^2*d^3*e^3+15/2/b/(c*x+b)^2*d^4*
e^2-15/c^2/(c*x+b)*d^2*e^4+15/b^2/(c*x+b)*d^4*e^2+12/c^3*b/(c*x+b)*d*e^5-12
*c/b^3/(c*x+b)*d^5*e-3/c^3*b^2/(c*x+b)^2*d*e^5+15/2/c^2*b/(c*x+b)^2*d^2*e^4
-3*c/b^2/(c*x+b)^2*d^5*e
```

Maxima [A] time = 1.00397, size = 460, normalized size = 2.57

$$\frac{e^6x}{c^3}-\frac{b^3c^4d^6-6\left(2\,c^7d^6-6\,bc^6d^5e+5\,b^2c^5d^4e^2-5\,b^4c^3d^2e^4+4\,b^5c^2de^5-b^6ce^6\right)x^3-\left(18\,bc^6d^6-54\,b^2c^5d^5e+45\,b^3c^4d^4e\right.}{2\left(b^4c^6x^4+2\,b^5c^5x^3+b^6c^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(c*x^2+b*x)^3,x, algorithm="maxima")
```

```
[Out] e^6*x/c^3 - 1/2*(b^3*c^4*d^6 - 6*(2*c^7*d^6 - 6*b*c^6*d^5*e + 5*b^2*c^5*d^4
*e^2 - 5*b^4*c^3*d^2*e^4 + 4*b^5*c^2*d*e^5 - b^6*c*e^6)*x^3 - (18*b*c^6*d^6
 - 54*b^2*c^5*d^5*e + 45*b^3*c^4*d^4*e^2 - 20*b^4*c^3*d^3*e^3 - 15*b^5*c^2*
d^2*e^4 + 18*b^6*c*d*e^5 - 5*b^7*e^6)*x^2 - 4*(b^2*c^5*d^6 - 3*b^3*c^4*d^5*
e)*x)/(b^4*c^6*x^4 + 2*b^5*c^5*x^3 + b^6*c^4*x^2) + 3*(2*c^2*d^6 - 6*b*c*d^
5*e + 5*b^2*d^4*e^2)*log(x)/b^5 - 3*(2*c^6*d^6 - 6*b*c^5*d^5*e + 5*b^2*c^4*
d^4*e^2 - 2*b^5*c*d*e^5 + b^6*e^6)*log(c*x + b)/(b^5*c^4)
```

Fricas [B] time = 1.98504, size = 1154, normalized size = 6.45

$$\frac{2\,b^5c^3e^6x^5+4\,b^6c^2e^6x^4-b^4c^4d^6+2\left(6\,bc^7d^6-18\,b^2c^6d^5e+15\,b^3c^5d^4e^2-15\,b^5c^3d^2e^4+12\,b^6c^2de^5-2\,b^7ce^6\right)x^3+\left(18\,b^2c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(c*x^2+b*x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^5*c^3*e^6*x^5 + 4*b^6*c^2*e^6*x^4 - b^4*c^4*d^6 + 2*(6*b*c^7*d^6 -
 18*b^2*c^6*d^5*e + 15*b^3*c^5*d^4*e^2 - 15*b^5*c^3*d^2*e^4 + 12*b^6*c^2*d*
e^5 - 2*b^7*c*e^6)*x^3 + (18*b^2*c^6*d^6 - 54*b^3*c^5*d^5*e + 45*b^4*c^4*d^
4*e^2 - 20*b^5*c^3*d^3*e^3 - 15*b^6*c^2*d^2*e^4 + 18*b^7*c*d*e^5 - 5*b^8*e^
6)*x^2 + 4*(b^3*c^5*d^6 - 3*b^4*c^4*d^5*e)*x - 6*((2*c^8*d^6 - 6*b*c^7*d^5*
e + 5*b^2*c^6*d^4*e^2 - 2*b^5*c^3*d*e^5 + b^6*c^2*e^6)*x^4 + 2*(2*b*c^7*d^6
 - 6*b^2*c^6*d^5*e + 5*b^3*c^5*d^4*e^2 - 2*b^6*c^2*d*e^5 + b^7*c*e^6)*x^3 +
 (2*b^2*c^6*d^6 - 6*b^3*c^5*d^5*e + 5*b^4*c^4*d^4*e^2 - 2*b^7*c*d*e^5 + b^8
*e^6)*x^2)*log(c*x + b) + 6*((2*c^8*d^6 - 6*b*c^7*d^5*e + 5*b^2*c^6*d^4*e^2
)*x^4 + 2*(2*b*c^7*d^6 - 6*b^2*c^6*d^5*e + 5*b^3*c^5*d^4*e^2)*x^3 + (2*b^2*
c^6*d^6 - 6*b^3*c^5*d^5*e + 5*b^4*c^4*d^4*e^2)*x^2)*log(x))/(b^5*c^6*x^4 +
2*b^6*c^5*x^3 + b^7*c^4*x^2)
```

Sympy [B] time = 30.1229, size = 597, normalized size = 3.34

$$-\frac{b^3c^4d^6 + x^3\left(6b^6ce^6 - 24b^5c^2de^5 + 30b^4c^3d^2e^4 - 30b^2c^5d^4e^2 + 36bc^6d^5e - 12c^7d^6\right) + x^2\left(5b^7e^6 - 18b^6cde^5 + 15b^5c^2d^2 \cdots\right.}{2b^6c^4x^2 + 4b^5c^5x^3 + 2b^4c^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**6/(c*x**2+b*x)**3,x)
```

```
[Out] -(b**3*c**4*d**6 + x**3*(6*b**6*c*e**6 - 24*b**5*c**2*d*e**5 + 30*b**4*c**3
*d**2*e**4 - 30*b**2*c**5*d**4*e**2 + 36*b*c**6*d**5*e - 12*c**7*d**6) + x*
*2*(5*b**7*e**6 - 18*b**6*c*d*e**5 + 15*b**5*c**2*d**2*e**4 + 20*b**4*c**3*
d**3*e**3 - 45*b**3*c**4*d**4*e**2 + 54*b**2*c**5*d**5*e - 18*b*c**6*d**6)
+ x*(12*b**3*c**4*d**5*e - 4*b**2*c**5*d**6))/(2*b**6*c**4*x**2 + 4*b**5*c*
*5*x**3 + 2*b**4*c**6*x**4) + e**6*x/c**3 + 3*d**4*(5*b**2*e**2 - 6*b*c*d*e
 + 2*c**2*d**2)*log(x + (15*b**3*c**3*d**4*e**2 - 18*b**2*c**4*d**5*e + 6*b
*c**5*d**6 - 3*b*c**3*d**4*(5*b**2*e**2 - 6*b*c*d*e + 2*c**2*d**2))/(3*b**6
*e**6 - 6*b**5*c*d*e**5 + 30*b**2*c**4*d**4*e**2 - 36*b*c**5*d**5*e + 12*c*
*6*d**6))/b**5 - 3*(b*e - c*d)**4*(b**2*e**2 + 2*b*c*d*e + 2*c**2*d**2)*log
(x + (15*b**3*c**3*d**4*e**2 - 18*b**2*c**4*d**5*e + 6*b*c**5*d**6 + 3*b*(b
*e - c*d)**4*(b**2*e**2 + 2*b*c*d*e + 2*c**2*d**2)/c)/(3*b**6*e**6 - 6*b**5
*c*d*e**5 + 30*b**2*c**4*d**4*e**2 - 36*b*c**5*d**5*e + 12*c**6*d**6))/(b**
5*c**4)
```

Giac [A] time = 3.19695, size = 427, normalized size = 2.39

$$\frac{xe^6}{c^3} + \frac{3\left(2\,c^2d^6 - 6\,bcd^5e + 5\,b^2d^4e^2\right)\log\left(|x|\right)}{b^5} - \frac{3\left(2\,c^6d^6 - 6\,bc^5d^5e + 5\,b^2c^4d^4e^2 - 2\,b^5cde^5 + b^6e^6\right)\log\left(|cx + b|\right)}{b^5c^4} - \frac{b^3 \cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(c*x^2+b*x)^3,x, algorithm="giac")
```

```
[Out] x*e^6/c^3 + 3*(2*c^2*d^6 - 6*b*c*d^5*e + 5*b^2*d^4*e^2)*log(abs(x))/b^5 - 3
*(2*c^6*d^6 - 6*b*c^5*d^5*e + 5*b^2*c^4*d^4*e^2 - 2*b^5*c*d*e^5 + b^6*e^6)*
log(abs(c*x + b))/(b^5*c^4) - 1/2*(b^3*c^4*d^6 - 6*(2*c^7*d^6 - 6*b*c^6*d^5
*e + 5*b^2*c^5*d^4*e^2 - 5*b^4*c^3*d^2*e^4 + 4*b^5*c^2*d*e^5 - b^6*c*e^6)*x
^3 - (18*b*c^6*d^6 - 54*b^2*c^5*d^5*e + 45*b^3*c^4*d^4*e^2 - 20*b^4*c^3*d^3
*e^3 - 15*b^5*c^2*d^2*e^4 + 18*b^6*c*d*e^5 - 5*b^7*e^6)*x^2 - 4*(b^2*c^5*d^
6 - 3*b^3*c^4*d^5*e)*x)/((c*x + b)^2*b^4*c^4*x^2)
```

3.277 $\int \frac{(d+ex)^5}{(bx+cx^2)^3} dx$

Optimal. Leaf size=171

$$\frac{d^3 \log(x) \left(10b^2e^2 - 15bcde + 6c^2d^2\right)}{b^5} - \frac{(cd-be)^3 \left(b^2e^2 + 3bcde + 6c^2d^2\right) \log(b+cx)}{b^5c^3} + \frac{(cd-be)^4(2be+3cd)}{b^4c^3(b+cx)} + \frac{(cd-be}{2b^3c^3(b+}$$

```
[Out] -d^5/(2*b^3*x^2) + (d^4*(3*c*d - 5*b*e))/(b^4*x) + (c*d - b*e)^5/(2*b^3*c^3
*(b + c*x)^2) + ((c*d - b*e)^4*(3*c*d + 2*b*e))/(b^4*c^3*(b + c*x)) + (d^3*
(6*c^2*d^2 - 15*b*c*d*e + 10*b^2*e^2)*Log[x])/b^5 - ((c*d - b*e)^3*(6*c^2*d
^2 + 3*b*c*d*e + b^2*e^2)*Log[b + c*x])/(b^5*c^3)
```

Rubi [A] time = 0.178273, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$\frac{d^3 \log(x) \left(10b^2e^2 - 15bcde + 6c^2d^2\right)}{b^5} - \frac{(cd-be)^3 \left(b^2e^2 + 3bcde + 6c^2d^2\right) \log(b+cx)}{b^5c^3} + \frac{(cd-be)^4(2be+3cd)}{b^4c^3(b+cx)} + \frac{(cd-be}{2b^3c^3(b+}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5/(b*x + c*x^2)^3,x]
```

```
[Out] -d^5/(2*b^3*x^2) + (d^4*(3*c*d - 5*b*e))/(b^4*x) + (c*d - b*e)^5/(2*b^3*c^3
*(b + c*x)^2) + ((c*d - b*e)^4*(3*c*d + 2*b*e))/(b^4*c^3*(b + c*x)) + (d^3*
(6*c^2*d^2 - 15*b*c*d*e + 10*b^2*e^2)*Log[x])/b^5 - ((c*d - b*e)^3*(6*c^2*d
^2 + 3*b*c*d*e + b^2*e^2)*Log[b + c*x])/(b^5*c^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^5}{(bx+cx^2)^3} dx &= \int \left(\frac{d^5}{b^3x^3} + \frac{d^4(-3cd+5be)}{b^4x^2} + \frac{d^3\left(6c^2d^2-15bcde+10b^2e^2\right)}{b^5x} + \frac{(-cd+be)^5}{b^3c^2(b+cx)^3} - \frac{(-cd+be)^4(3cd+2be}{b^4c^2(b+cx)^2}\right. \\
&= -\frac{d^5}{2b^3x^2} + \frac{d^4(3cd-5be)}{b^4x} + \frac{(cd-be)^5}{2b^3c^3(b+cx)^2} + \frac{(cd-be)^4(3cd+2be)}{b^4c^3(b+cx)} + \frac{d^3\left(6c^2d^2-15bcde+10b^2e^2\right) \mathrm{lo}}{b^5}
\end{aligned}$$

Mathematica [A] time = 0.108152, size = 165, normalized size = 0.96

$$-\frac{-2d^3\log(x)\left(10b^2e^2-15bcde+6c^2d^2\right) + \frac{2(cd-be)^3\left(b^2e^2+3bcde+6c^2d^2\right)\log(b+cx)}{c^3} + \frac{b^2(be-cd)^5}{c^3(b+cx)^2} + \frac{b^2d^5}{x^2} - \frac{2b(cd-be)^4(2be+3cd)}{c^3(b+cx)} + \frac{2bd^4(5be-}{x}}{2b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5/(b*x + c*x^2)^3,x]
```

```
[Out] -((b^2*d^5)/x^2 + (2*b*d^4*(-3*c*d + 5*b*e))/x + (b^2*(-(c*d) + b*e)^5)/(c^
3*(b + c*x)^2) - (2*b*(c*d - b*e)^4*(3*c*d + 2*b*e))/(c^3*(b + c*x)) - 2*d^
3*(6*c^2*d^2 - 15*b*c*d*e + 10*b^2*e^2)*Log[x] + (2*(c*d - b*e)^3*(6*c^2*d^
2 + 3*b*c*d*e + b^2*e^2)*Log[b + c*x])/c^3)/(2*b^5)
```

Maple [A] time = 0.06, size = 329, normalized size = 1.9

$$-\frac{d^5}{2\,b^3x^2}+10\,\frac{d^3\ln(x)\,e^2}{b^3}-15\,\frac{d^4\ln(x)\,ce}{b^4}+6\,\frac{d^5\ln(x)\,c^2}{b^5}-5\,\frac{d^4e}{b^3x}+3\,\frac{d^5c}{b^4x}+\frac{\ln(cx+b)\,e^5}{c^3}-10\,\frac{\ln(cx+b)\,d^3e^2}{b^3}+15\,c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5/(c*x^2+b*x)^3,x)
```

```
[Out] -1/2*d^5/b^3/x^2+10*d^3/b^3*ln(x)*e^2-15*d^4/b^4*ln(x)*c*e+6*d^5/b^5*ln(x)*
c^2-5*d^4/b^3/x*e+3*d^5/b^4/x*c+1/c^3*ln(c*x+b)*e^5-10/b^3*ln(c*x+b)*d^3*e^
2+15/b^4*c*ln(c*x+b)*d^4*e-6/b^5*c^2*ln(c*x+b)*d^5+2/c^3*b/(c*x+b)*e^5-5/c^
2/(c*x+b)*d*e^4+10/b^2/(c*x+b)*d^3*e^2-10*c/b^3/(c*x+b)*d^4*e+3*c^2/b^4/(c*
x+b)*d^5-1/2/c^3*b^2/(c*x+b)^2*e^5+5/2/c^2*b/(c*x+b)^2*d*e^4-5/c/(c*x+b)^2*
d^2*e^3+5/b/(c*x+b)^2*d^3*e^2-5/2*c/b^2/(c*x+b)^2*d^4*e+1/2*c^2/b^3/(c*x+b)
^2*d^5
```

Maxima [A] time = 1.13554, size = 400, normalized size = 2.34

$$-\frac{b^3c^3d^5-2\left(6\,c^6d^5-15\,bc^5d^4e+10\,b^2c^4d^3e^2-5\,b^4c^2de^4+2\,b^5ce^5\right)x^3-\left(18\,bc^5d^5-45\,b^2c^4d^4e+30\,b^3c^3d^3e^2-10\,b^4c^2}{2\left(b^4c^5x^4+2\,b^5c^4x^3+b^6c^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*x^2+b*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*c^3*d^5 - 2*(6*c^6*d^5 - 15*b*c^5*d^4*e + 10*b^2*c^4*d^3*e^2 - 5*
b^4*c^2*d*e^4 + 2*b^5*c*e^5)*x^3 - (18*b*c^5*d^5 - 45*b^2*c^4*d^4*e + 30*b^
3*c^3*d^3*e^2 - 10*b^4*c^2*d^2*e^3 - 5*b^5*c*d*e^4 + 3*b^6*e^5)*x^2 - 2*(2*
b^2*c^4*d^5 - 5*b^3*c^3*d^4*e)*x)/(b^4*c^5*x^4 + 2*b^5*c^4*x^3 + b^6*c^3*x^
2) + (6*c^2*d^5 - 15*b*c*d^4*e + 10*b^2*d^3*e^2)*log(x)/b^5 - (6*c^5*d^5 -
15*b*c^4*d^4*e + 10*b^2*c^3*d^3*e^2 - b^5*e^5)*log(c*x + b)/(b^5*c^3)
```

Fricas [B] time = 2.01836, size = 991, normalized size = 5.8

$$-\frac{b^4c^3d^5-2\left(6\,bc^6d^5-15\,b^2c^5d^4e+10\,b^3c^4d^3e^2-5\,b^5c^2de^4+2\,b^6ce^5\right)x^3-\left(18\,b^2c^5d^5-45\,b^3c^4d^4e+30\,b^4c^3d^3e^2-10\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*x^2+b*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b^4*c^3*d^5 - 2*(6*b*c^6*d^5 - 15*b^2*c^5*d^4*e + 10*b^3*c^4*d^3*e^2
- 5*b^5*c^2*d*e^4 + 2*b^6*c*e^5)*x^3 - (18*b^2*c^5*d^5 - 45*b^3*c^4*d^4*e +
```

```
 30*b^4*c^3*d^3*e^2 - 10*b^5*c^2*d^2*e^3 - 5*b^6*c*d*e^4 + 3*b^7*e^5)*x^2 -
 2*(2*b^3*c^4*d^5 - 5*b^4*c^3*d^4*e)*x + 2*((6*c^7*d^5 - 15*b*c^6*d^4*e + 1
0*b^2*c^5*d^3*e^2 - b^5*c^2*e^5)*x^4 + 2*(6*b*c^6*d^5 - 15*b^2*c^5*d^4*e +
10*b^3*c^4*d^3*e^2 - b^6*c*e^5)*x^3 + (6*b^2*c^5*d^5 - 15*b^3*c^4*d^4*e + 1
0*b^4*c^3*d^3*e^2 - b^7*e^5)*x^2)*log(c*x + b) - 2*((6*c^7*d^5 - 15*b*c^6*d
^4*e + 10*b^2*c^5*d^3*e^2)*x^4 + 2*(6*b*c^6*d^5 - 15*b^2*c^5*d^4*e + 10*b^3
*c^4*d^3*e^2)*x^3 + (6*b^2*c^5*d^5 - 15*b^3*c^4*d^4*e + 10*b^4*c^3*d^3*e^2)
*x^2)*log(x))/(b^5*c^5*x^4 + 2*b^6*c^4*x^3 + b^7*c^3*x^2)
```

Sympy [B] time = 12.1009, size = 524, normalized size = 3.06

$$\frac{-b^3c^3d^5 + x^3\left(4b^5ce^5 - 10b^4c^2de^4 + 20b^2c^4d^3e^2 - 30bc^5d^4e + 12c^6d^5\right) + x^2\left(3b^6e^5 - 5b^5cde^4 - 10b^4c^2d^2e^3 + 30b^3c^3d^3e^2 - \cdots\right.}{2b^6c^3x^2 + 4b^5c^4x^3 + 2b^4c^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5/(c*x**2+b*x)**3,x)
```

```
[Out] (-b**3*c**3*d**5 + x**3*(4*b**5*c*e**5 - 10*b**4*c**2*d*e**4 + 20*b**2*c**4
*d**3*e**2 - 30*b*c**5*d**4*e + 12*c**6*d**5) + x**2*(3*b**6*e**5 - 5*b**5*
c*d*e**4 - 10*b**4*c**2*d**2*e**3 + 30*b**3*c**3*d**3*e**2 - 45*b**2*c**4*d
**4*e + 18*b*c**5*d**5) + x*(-10*b**3*c**3*d**4*e + 4*b**2*c**4*d**5))/(2*b
**6*c**3*x**2 + 4*b**5*c**4*x**3 + 2*b**4*c**5*x**4) + d**3*(10*b**2*e**2 -
 15*b*c*d*e + 6*c**2*d**2)*log(x + (-10*b**3*c**2*d**3*e**2 + 15*b**2*c**3*
d**4*e - 6*b*c**4*d**5 + b*c**2*d**3*(10*b**2*e**2 - 15*b*c*d*e + 6*c**2*d*
*2))/(b**5*e**5 - 20*b**2*c**3*d**3*e**2 + 30*b*c**4*d**4*e - 12*c**5*d**5)
)/b**5 + (b*e - c*d)**3*(b**2*e**2 + 3*b*c*d*e + 6*c**2*d**2)*log(x + (-10*
b**3*c**2*d**3*e**2 + 15*b**2*c**3*d**4*e - 6*b*c**4*d**5 + b*(b*e - c*d)**
3*(b**2*e**2 + 3*b*c*d*e + 6*c**2*d**2)/c)/(b**5*e**5 - 20*b**2*c**3*d**3*e
**2 + 30*b*c**4*d**4*e - 12*c**5*d**5))/(b**5*c**3)
```

Giac [A] time = 1.32663, size = 371, normalized size = 2.17

$$\frac{\left(6c^2d^5 - 15bcd^4e + 10b^2d^3e^2\right)\log\left(|x|\right)}{b^5} - \frac{\left(6c^5d^5 - 15bc^4d^4e + 10b^2c^3d^3e^2 - b^5e^5\right)\log\left(|cx + b|\right)}{b^5c^3} - \frac{b^3c^3d^5 - 2\left(6c^6d^5 - 15\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*x^2+b*x)^3,x, algorithm="giac")
```

```
[Out] (6*c^2*d^5 - 15*b*c*d^4*e + 10*b^2*d^3*e^2)*log(abs(x))/b^5 - (6*c^5*d^5 -
15*b*c^4*d^4*e + 10*b^2*c^3*d^3*e^2 - b^5*e^5)*log(abs(c*x + b))/(b^5*c^3)
- 1/2*(b^3*c^3*d^5 - 2*(6*c^6*d^5 - 15*b*c^5*d^4*e + 10*b^2*c^4*d^3*e^2 - 5
*b^4*c^2*d*e^4 + 2*b^5*c*e^5)*x^3 - (18*b*c^5*d^5 - 45*b^2*c^4*d^4*e + 30*b
^3*c^3*d^3*e^2 - 10*b^4*c^2*d^2*e^3 - 5*b^5*c*d*e^4 + 3*b^6*e^5)*x^2 - 2*(2
*b^2*c^4*d^5 - 5*b^3*c^3*d^4*e)*x)/((c*x + b)^2*b^4*c^3*x^2)
```

3.278 $\int \frac{(d+ex)^4}{(bx+cx^2)^3} dx$

Optimal. Leaf size=136

$$\frac{(cd-be)^3(be+3cd)}{b^4c^2(b+cx)}+\frac{(cd-be)^4}{2b^3c^2(b+cx)^2}+\frac{d^3(3cd-4be)}{b^4x}+\frac{6d^2\log(x)(cd-be)^2}{b^5}-\frac{6d^2(cd-be)^2\log(b+cx)}{b^5}-\frac{d^4}{2b^3x^2}$$

```
[Out] -d^4/(2*b^3*x^2) + (d^3*(3*c*d - 4*b*e))/(b^4*x) + (c*d - b*e)^4/(2*b^3*c^2
*(b + c*x)^2) + ((c*d - b*e)^3*(3*c*d + b*e))/(b^4*c^2*(b + c*x)) + (6*d^2*
(c*d - b*e)^2*Log[x])/b^5 - (6*d^2*(c*d - b*e)^2*Log[b + c*x])/b^5
```

Rubi [A] time = 0.151547, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$\frac{(cd-be)^3(be+3cd)}{b^4c^2(b+cx)}+\frac{(cd-be)^4}{2b^3c^2(b+cx)^2}+\frac{d^3(3cd-4be)}{b^4x}+\frac{6d^2\log(x)(cd-be)^2}{b^5}-\frac{6d^2(cd-be)^2\log(b+cx)}{b^5}-\frac{d^4}{2b^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(b*x + c*x^2)^3,x]
```

```
[Out] -d^4/(2*b^3*x^2) + (d^3*(3*c*d - 4*b*e))/(b^4*x) + (c*d - b*e)^4/(2*b^3*c^2
*(b + c*x)^2) + ((c*d - b*e)^3*(3*c*d + b*e))/(b^4*c^2*(b + c*x)) + (6*d^2*
(c*d - b*e)^2*Log[x])/b^5 - (6*d^2*(c*d - b*e)^2*Log[b + c*x])/b^5
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(d+ex)^4}{(bx+cx^2)^3} dx = \int \left(\frac{d^4}{b^3x^3}+\frac{d^3(-3cd+4be)}{b^4x^2}+\frac{6d^2(-cd+be)^2}{b^5x}-\frac{(-cd+be)^4}{b^3c(b+cx)^3}+\frac{(-cd+be)^3(3cd+be)}{b^4c(b+cx)^2}-\frac{6cd^2(-c\right.$$

$$= -\frac{d^4}{2b^3x^2}+\frac{d^3(3cd-4be)}{b^4x}+\frac{(cd-be)^4}{2b^3c^2(b+cx)^2}+\frac{(cd-be)^3(3cd+be)}{b^4c^2(b+cx)}+\frac{6d^2(cd-be)^2\log(x)}{b^5}-\frac{6d^2(c$$

Mathematica [A] time = 0.0835923, size = 130, normalized size = 0.96

$$-\frac{-\frac{b^2(cd-be)^4}{c^2(b+cx)^2}+\frac{b^2d^4}{x^2}+\frac{2b(be-cd)^3(be+3cd)}{c^2(b+cx)}+\frac{2bd^3(4be-3cd)}{x}-12d^2\log(x)(cd-be)^2+12d^2(cd-be)^2\log(b+cx)}{2b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(b*x + c*x^2)^3,x]
```

```
[Out] -((b^2*d^4)/x^2 + (2*b*d^3*(-3*c*d + 4*b*e))/x - (b^2*(c*d - b*e)^4)/(c^2*(
b + c*x)^2) + (2*b*(-(c*d) + b*e)^3*(3*c*d + b*e))/(c^2*(b + c*x)) - 12*d^2
*(c*d - b*e)^2*Log[x] + 12*d^2*(c*d - b*e)^2*Log[b + c*x])/(2*b^5)
```

Maple [B] time = 0.062, size = 278, normalized size = 2.

$$-\frac{d^4}{2\,b^3x^2}-4\,\frac{d^3e}{b^3x}+3\,\frac{d^4c}{b^4x}+6\,\frac{d^2\ln\left(x\right)e^2}{b^3}-12\,\frac{d^3\ln\left(x\right)ce}{b^4}+6\,\frac{d^4\ln\left(x\right)c^2}{b^5}-\frac{e^4}{c^2\left(cx+b\right)}+6\,\frac{d^2e^2}{b^2\left(cx+b\right)}-8\,\frac{d^3ec}{b^3\left(cx+b\right)}+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(c*x^2+b*x)^3,x)
```

```
[Out] -1/2*d^4/b^3/x^2-4*d^3/b^3/x*e+3*d^4/b^4/x*c+6*d^2/b^3*ln(x)*e^2-12*d^3/b^4
*ln(x)*c*e+6*d^4/b^5*ln(x)*c^2-1/c^2/(c*x+b)*e^4+6/b^2/(c*x+b)*d^2*e^2-8/b^
3*c/(c*x+b)*d^3*e+3/b^4*c^2/(c*x+b)*d^4+1/2*b/c^2/(c*x+b)^2*e^4-2/c/(c*x+b)
^2*d*e^3+3/b/(c*x+b)^2*d^2*e^2-2/b^2*c/(c*x+b)^2*d^3*e+1/2/b^3*c^2/(c*x+b)^
2*d^4-6*d^2/b^3*ln(c*x+b)*e^2+12*d^3/b^4*ln(c*x+b)*c*e-6*d^4/b^5*ln(c*x+b)*
c^2
```

Maxima [A] time = 1.02357, size = 338, normalized size = 2.49

$$-\frac{b^3c^2d^4-2\left(6\,c^5d^4-12\,bc^4d^3e+6\,b^2c^3d^2e^2-b^4ce^4\right)x^3-\left(18\,bc^4d^4-36\,b^2c^3d^3e+18\,b^3c^2d^2e^2-4\,b^4cde^3-b^5e^4\right)x^2-4\left(b\right.}{2\left(b^4c^4x^4+2\,b^5c^3x^3+b^6c^2x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*c^2*d^4 - 2*(6*c^5*d^4 - 12*b*c^4*d^3*e + 6*b^2*c^3*d^2*e^2 - b^4
*c*e^4)*x^3 - (18*b*c^4*d^4 - 36*b^2*c^3*d^3*e + 18*b^3*c^2*d^2*e^2 - 4*b^4
*c*d*e^3 - b^5*e^4)*x^2 - 4*(b^2*c^3*d^4 - 2*b^3*c^2*d^3*e)*x)/(b^4*c^4*x^4
 + 2*b^5*c^3*x^3 + b^6*c^2*x^2) - 6*(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2)*l
og(c*x + b)/b^5 + 6*(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2)*log(x)/b^5
```

Fricas [B] time = 1.8601, size = 834, normalized size = 6.13

$$-\frac{b^4c^2d^4-2\left(6\,bc^5d^4-12\,b^2c^4d^3e+6\,b^3c^3d^2e^2-b^5ce^4\right)x^3-\left(18\,b^2c^4d^4-36\,b^3c^3d^3e+18\,b^4c^2d^2e^2-4\,b^5cde^3-b^6e^4\right)x^2-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b^4*c^2*d^4 - 2*(6*b*c^5*d^4 - 12*b^2*c^4*d^3*e + 6*b^3*c^3*d^2*e^2 -
 b^5*c*e^4)*x^3 - (18*b^2*c^4*d^4 - 36*b^3*c^3*d^3*e + 18*b^4*c^2*d^2*e^2 -
 4*b^5*c*d*e^3 - b^6*e^4)*x^2 - 4*(b^3*c^3*d^4 - 2*b^4*c^2*d^3*e)*x + 12*((
c^6*d^4 - 2*b*c^5*d^3*e + b^2*c^4*d^2*e^2)*x^4 + 2*(b*c^5*d^4 - 2*b^2*c^4*d
^3*e + b^3*c^3*d^2*e^2)*x^3 + (b^2*c^4*d^4 - 2*b^3*c^3*d^3*e + b^4*c^2*d^2*
e^2)*x^2)*log(c*x + b) - 12*((c^6*d^4 - 2*b*c^5*d^3*e + b^2*c^4*d^2*e^2)*x^
4 + 2*(b*c^5*d^4 - 2*b^2*c^4*d^3*e + b^3*c^3*d^2*e^2)*x^3 + (b^2*c^4*d^4 -
```

```
2*b^3*c^3*d^3*e + b^4*c^2*d^2*e^2)*x^2)*log(x))/(b^5*c^4*x^4 + 2*b^6*c^3*x^
3 + b^7*c^2*x^2)
```

Sympy [B] time = 8.51847, size = 389, normalized size = 2.86

$$-\frac{b^3c^2d^4 + x^3\left(2b^4ce^4 - 12b^2c^3d^2e^2 + 24bc^4d^3e - 12c^5d^4\right) + x^2\left(b^5e^4 + 4b^4cde^3 - 18b^3c^2d^2e^2 + 36b^2c^3d^3e - 18bc^4d^4\right) +}{2b^6c^2x^2 + 4b^5c^3x^3 + 2b^4c^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(c*x**2+b*x)**3,x)
```

```
[Out] -(b**3*c**2*d**4 + x**3*(2*b**4*c*e**4 - 12*b**2*c**3*d**2*e**2 + 24*b*c**4
*d**3*e - 12*c**5*d**4) + x**2*(b**5*e**4 + 4*b**4*c*d*e**3 - 18*b**3*c**2*
d**2*e**2 + 36*b**2*c**3*d**3*e - 18*b*c**4*d**4) + x*(8*b**3*c**2*d**3*e -
 4*b**2*c**3*d**4))/(2*b**6*c**2*x**2 + 4*b**5*c**3*x**3 + 2*b**4*c**4*x**4
) + 6*d**2*(b*e - c*d)**2*log(x + (6*b**3*d**2*e**2 - 12*b**2*c*d**3*e + 6*
b*c**2*d**4 - 6*b*d**2*(b*e - c*d)**2)/(12*b**2*c*d**2*e**2 - 24*b*c**2*d**
3*e + 12*c**3*d**4))/b**5 - 6*d**2*(b*e - c*d)**2*log(x + (6*b**3*d**2*e**2
 - 12*b**2*c*d**3*e + 6*b*c**2*d**4 + 6*b*d**2*(b*e - c*d)**2)/(12*b**2*c*d
**2*e**2 - 24*b*c**2*d**3*e + 12*c**3*d**4))/b**5
```

Giac [A] time = 1.2758, size = 343, normalized size = 2.52

$$\frac{6\left(c^2d^4 - 2\,bcd^3e + b^2d^2e^2\right)\log\left(|x|\right)}{b^5} - \frac{6\left(c^3d^4 - 2\,bc^2d^3e + b^2cd^2e^2\right)\log\left(|cx + b|\right)}{b^5c} + \frac{12\,c^5d^4x^3 - 24\,bc^4d^3x^3e + 18\,bc^4d^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x)^3,x, algorithm="giac")
```

```
[Out] 6*(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2)*log(abs(x))/b^5 - 6*(c^3*d^4 - 2*b*
c^2*d^3*e + b^2*c*d^2*e^2)*log(abs(c*x + b))/(b^5*c) + 1/2*(12*c^5*d^4*x^3
- 24*b*c^4*d^3*x^3*e + 18*b*c^4*d^4*x^2 + 12*b^2*c^3*d^2*x^3*e^2 - 36*b^2*c
^3*d^3*x^2*e + 4*b^2*c^3*d^4*x + 18*b^3*c^2*d^2*x^2*e^2 - 8*b^3*c^2*d^3*x*e
 - b^3*c^2*d^4 - 2*b^4*c*x^3*e^4 - 4*b^4*c*d*x^2*e^3 - b^5*x^2*e^4)/((c*x^2
 + b*x)^2*b^4*c^2)
```

3.279 $\int \frac{(d+ex)^3}{\left(bx+cx^2\right)^3} dx$

Optimal. Leaf size=137

$$\frac{3d^2(cd-be)}{b^4x}+\frac{3d(cd-be)^2}{b^4(b+cx)}+\frac{(cd-be)^3}{2b^3c(b+cx)^2}+\frac{3d\log(x)(cd-be)(2cd-be)}{b^5}-\frac{3d(cd-be)(2cd-be)\log(b+cx)}{b^5}-\frac{d^3}{2b^3x^2}$$

```
[Out] -d^3/(2*b^3*x^2) + (3*d^2*(c*d - b*e))/(b^4*x) + (c*d - b*e)^3/(2*b^3*c*(b
+ c*x)^2) + (3*d*(c*d - b*e)^2)/(b^4*(b + c*x)) + (3*d*(c*d - b*e)*(2*c*d -
 b*e)*Log[x])/b^5 - (3*d*(c*d - b*e)*(2*c*d - b*e)*Log[b + c*x])/b^5
```

Rubi [A] time = 0.147119, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$\frac{3d^2(cd-be)}{b^4x}+\frac{3d(cd-be)^2}{b^4(b+cx)}+\frac{(cd-be)^3}{2b^3c(b+cx)^2}+\frac{3d\log(x)(cd-be)(2cd-be)}{b^5}-\frac{3d(cd-be)(2cd-be)\log(b+cx)}{b^5}-\frac{d^3}{2b^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(b*x + c*x^2)^3,x]
```

```
[Out] -d^3/(2*b^3*x^2) + (3*d^2*(c*d - b*e))/(b^4*x) + (c*d - b*e)^3/(2*b^3*c*(b
+ c*x)^2) + (3*d*(c*d - b*e)^2)/(b^4*(b + c*x)) + (3*d*(c*d - b*e)*(2*c*d -
 b*e)*Log[x])/b^5 - (3*d*(c*d - b*e)*(2*c*d - b*e)*Log[b + c*x])/b^5
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(bx+cx^2\right)^3} dx = \int \left(\frac{d^3}{b^3x^3}+\frac{3d^2(-cd+be)}{b^4x^2}+\frac{3d(cd-be)(2cd-be)}{b^5x}+\frac{(-cd+be)^3}{b^3(b+cx)^3}-\frac{3cd(-cd+be)^2}{b^4(b+cx)^2}+\frac{3cd(cd-be)(-}{b^5(b+}\right.$$

$$=-\frac{d^3}{2b^3x^2}+\frac{3d^2(cd-be)}{b^4x}+\frac{(cd-be)^3}{2b^3c(b+cx)^2}+\frac{3d(cd-be)^2}{b^4(b+cx)}+\frac{3d(cd-be)(2cd-be)\log(x)}{b^5}-\frac{3d(cd-be)}{}$$

Mathematica [A] time = 0.15537, size = 138, normalized size = 1.01

$$-\frac{-6d\log(x)\left(b^2e^2-3bcde+2c^2d^2\right)+6d\left(b^2e^2-3bcde+2c^2d^2\right)\log(b+cx)+\frac{b^2(be-cd)^3}{c(b+cx)^2}+\frac{b^2d^3}{x^2}+\frac{6bd^2(be-cd)}{x}-\frac{6bd(cd-be)^2}{b+cx}}{2b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(b*x + c*x^2)^3,x]
```

```
[Out] -((b^2*d^3)/x^2 + (6*b*d^2*(-(c*d) + b*e))/x + (b^2*(-(c*d) + b*e)^3)/(c*(b
 + c*x)^2) - (6*b*d*(c*d - b*e)^2)/(b + c*x) - 6*d*(2*c^2*d^2 - 3*b*c*d*e +
 b^2*e^2)*Log[x] + 6*d*(2*c^2*d^2 - 3*b*c*d*e + b^2*e^2)*Log[b + c*x])/(2*b
^5)
```

Maple [A] time = 0.058, size = 238, normalized size = 1.7

$$-\frac{d^3}{2\,b^3x^2}+3\,\frac{d\ln(x)\,e^2}{b^3}-9\,\frac{d^2\ln(x)\,ce}{b^4}+6\,\frac{d^3\ln(x)\,c^2}{b^5}-3\,\frac{d^2e}{b^3x}+3\,\frac{d^3c}{b^4x}-\frac{e^3}{2\,c\,(cx+b)^2}+\frac{3\,de^2}{2\,b\,(cx+b)^2}-\frac{3\,d^2ec}{2\,b^2\,(cx+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^2+b*x)^3,x)
```

```
[Out] -1/2*d^3/b^3/x^2+3*d/b^3*ln(x)*e^2-9*d^2/b^4*ln(x)*c*e+6*d^3/b^5*ln(x)*c^2-
3*d^2/b^3/x*e+3*d^3/b^4/x*c-1/2/c/(c*x+b)^2*e^3+3/2/b/(c*x+b)^2*d*e^2-3/2/b
^2*c/(c*x+b)^2*d^2*e+1/2/b^3*c^2/(c*x+b)^2*d^3-3*d/b^3*ln(c*x+b)*e^2+9*d^2/
b^4*ln(c*x+b)*c*e-6*d^3/b^5*ln(c*x+b)*c^2+3*d/b^2/(c*x+b)*e^2-6*d^2/b^3/(c*
x+b)*c*e+3*d^3/b^4/(c*x+b)*c^2
```

Maxima [A] time = 1.08297, size = 293, normalized size = 2.14

$$-\frac{b^3cd^3-6\left(2\,c^4d^3-3\,bc^3d^2e+b^2c^2de^2\right)x^3-\left(18\,bc^3d^3-27\,b^2c^2d^2e+9\,b^3cde^2-b^4e^3\right)x^2-2\left(2\,b^2c^2d^3-3\,b^3cd^2e\right)x}{2\left(b^4c^3x^4+2\,b^5c^2x^3+b^6cx^2\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*c*d^3 - 6*(2*c^4*d^3 - 3*b*c^3*d^2*e + b^2*c^2*d*e^2)*x^3 - (18*b
*c^3*d^3 - 27*b^2*c^2*d^2*e + 9*b^3*c*d*e^2 - b^4*e^3)*x^2 - 2*(2*b^2*c^2*d
^3 - 3*b^3*c*d^2*e)*x)/(b^4*c^3*x^4 + 2*b^5*c^2*x^3 + b^6*c*x^2) - 3*(2*c^2
*d^3 - 3*b*c*d^2*e + b^2*d*e^2)*log(c*x + b)/b^5 + 3*(2*c^2*d^3 - 3*b*c*d^2
*e + b^2*d*e^2)*log(x)/b^5
```

Fricas [B] time = 1.93519, size = 770, normalized size = 5.62

$$-\frac{b^4cd^3-6\left(2\,bc^4d^3-3\,b^2c^3d^2e+b^3c^2de^2\right)x^3-\left(18\,b^2c^3d^3-27\,b^3c^2d^2e+9\,b^4cde^2-b^5e^3\right)x^2-2\left(2\,b^3c^2d^3-3\,b^4cd^2e\right)x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b^4*c*d^3 - 6*(2*b*c^4*d^3 - 3*b^2*c^3*d^2*e + b^3*c^2*d*e^2)*x^3 - (
18*b^2*c^3*d^3 - 27*b^3*c^2*d^2*e + 9*b^4*c*d*e^2 - b^5*e^3)*x^2 - 2*(2*b^3
*c^2*d^3 - 3*b^4*c*d^2*e)*x + 6*((2*c^5*d^3 - 3*b*c^4*d^2*e + b^2*c^3*d*e^2
)*x^4 + 2*(2*b*c^4*d^3 - 3*b^2*c^3*d^2*e + b^3*c^2*d*e^2)*x^3 + (2*b^2*c^3*
d^3 - 3*b^3*c^2*d^2*e + b^4*c*d*e^2)*x^2)*log(c*x + b) - 6*((2*c^5*d^3 - 3*
b*c^4*d^2*e + b^2*c^3*d*e^2)*x^4 + 2*(2*b*c^4*d^3 - 3*b^2*c^3*d^2*e + b^3*c
^2*d*e^2)*x^3 + (2*b^2*c^3*d^3 - 3*b^3*c^2*d^2*e + b^4*c*d*e^2)*x^2)*log(x)
```

```
)/(b^5*c^3*x^4 + 2*b^6*c^2*x^3 + b^7*c*x^2)
```

Sympy [B] time = 4.0019, size = 371, normalized size = 2.71

$$\frac{-b^3cd^3 + x^3\left(6b^2c^2de^2 - 18bc^3d^2e + 12c^4d^3\right) + x^2\left(-b^4e^3 + 9b^3cde^2 - 27b^2c^2d^2e + 18bc^3d^3\right) + x\left(-6b^3cd^2e + 4b^2c^2d^3\right)}{2b^6cx^2 + 4b^5c^2x^3 + 2b^4c^3x^4} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**2+b*x)**3,x)
```

```
[Out] (-b**3*c*d**3 + x**3*(6*b**2*c**2*d*e**2 - 18*b*c**3*d**2*e + 12*c**4*d**3)
 + x**2*(-b**4*e**3 + 9*b**3*c*d*e**2 - 27*b**2*c**2*d**2*e + 18*b*c**3*d**
3) + x*(-6*b**3*c*d**2*e + 4*b**2*c**2*d**3))/(2*b**6*c*x**2 + 4*b**5*c**2*
x**3 + 2*b**4*c**3*x**4) + 3*d*(b*e - 2*c*d)*(b*e - c*d)*log(x + (3*b**3*d*
e**2 - 9*b**2*c*d**2*e + 6*b*c**2*d**3 - 3*b*d*(b*e - 2*c*d)*(b*e - c*d))/(
6*b**2*c*d*e**2 - 18*b*c**2*d**2*e + 12*c**3*d**3))/b**5 - 3*d*(b*e - 2*c*d
)*(b*e - c*d)*log(x + (3*b**3*d*e**2 - 9*b**2*c*d**2*e + 6*b*c**2*d**3 + 3*
b*d*(b*e - 2*c*d)*(b*e - c*d))/(6*b**2*c*d*e**2 - 18*b*c**2*d**2*e + 12*c**
3*d**3))/b**5
```

Giac [A] time = 1.26788, size = 296, normalized size = 2.16

$$\frac{3\left(2\,c^2d^3 - 3\,bcd^2e + b^2de^2\right)\log\left(|x|\right)}{b^5} - \frac{3\left(2\,c^3d^3 - 3\,bc^2d^2e + b^2cde^2\right)\log\left(|cx+b|\right)}{b^5c} + \frac{12\,c^4d^3x^3 - 18\,bc^3d^2x^3e + 18\,bc^3d^3x^2 \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x)^3,x, algorithm="giac")
```

```
[Out] 3*(2*c^2*d^3 - 3*b*c*d^2*e + b^2*d*e^2)*log(abs(x))/b^5 - 3*(2*c^3*d^3 - 3*
b*c^2*d^2*e + b^2*c*d*e^2)*log(abs(c*x + b))/(b^5*c) + 1/2*(12*c^4*d^3*x^3
- 18*b*c^3*d^2*x^3*e + 18*b*c^3*d^3*x^2 + 6*b^2*c^2*d*x^3*e^2 - 27*b^2*c^2*
d^2*x^2*e + 4*b^2*c^2*d^3*x + 9*b^3*c*d*x^2*e^2 - 6*b^3*c*d^2*x*e - b^3*c*d
^3 - b^4*x^2*e^3)/((c*x^2 + b*x)^2*b^4*c)
```

3.280 $\int \frac{(d+ex)^2}{\left(bx+cx^2\right)^3} dx$

Optimal. Leaf size=144

$$\frac{\log(x)\left(b^2e^2-6bcde+6c^2d^2\right)}{b^5}-\frac{\left(b^2e^2-6bcde+6c^2d^2\right)\log(b+cx)}{b^5}+\frac{d(3cd-2be)}{b^4x}+\frac{(cd-be)(3cd-be)}{b^4(b+cx)}+\frac{(cd-be)}{2b^3(b+c}$$

```
[Out] -d^2/(2*b^3*x^2) + (d*(3*c*d - 2*b*e))/(b^4*x) + (c*d - b*e)^2/(2*b^3*(b +
c*x)^2) + ((c*d - b*e)*(3*c*d - b*e))/(b^4*(b + c*x)) + ((6*c^2*d^2 - 6*b*c
*d*e + b^2*e^2)*Log[x])/b^5 - ((6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*Log[b + c*
x])/b^5
```

Rubi [A] time = 0.132759, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$\frac{\log(x)\left(b^2e^2-6bcde+6c^2d^2\right)}{b^5}-\frac{\left(b^2e^2-6bcde+6c^2d^2\right)\log(b+cx)}{b^5}+\frac{d(3cd-2be)}{b^4x}+\frac{(cd-be)(3cd-be)}{b^4(b+cx)}+\frac{(cd-be)}{2b^3(b+c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(b*x + c*x^2)^3,x]
```

```
[Out] -d^2/(2*b^3*x^2) + (d*(3*c*d - 2*b*e))/(b^4*x) + (c*d - b*e)^2/(2*b^3*(b +
c*x)^2) + ((c*d - b*e)*(3*c*d - b*e))/(b^4*(b + c*x)) + ((6*c^2*d^2 - 6*b*c
*d*e + b^2*e^2)*Log[x])/b^5 - ((6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*Log[b + c*
x])/b^5
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(bx+cx^2\right)^3} dx = \int \left(\frac{d^2}{b^3x^3}+\frac{d(-3cd+2be)}{b^4x^2}+\frac{6c^2d^2-6bcde+b^2e^2}{b^5x}-\frac{c(-cd+be)^2}{b^3(b+cx)^3}+\frac{c(cd-be)(-3cd+be)}{b^4(b+cx)^2}-\frac{c(6}{}\right.$$

$$= -\frac{d^2}{2b^3x^2}+\frac{d(3cd-2be)}{b^4x}+\frac{(cd-be)^2}{2b^3(b+cx)^2}+\frac{(cd-be)(3cd-be)}{b^4(b+cx)}+\frac{\left(6c^2d^2-6bcde+b^2e^2\right)\log(x)}{b^5}-$$

Mathematica [A] time = 0.0939355, size = 144, normalized size = 1.

$$\frac{\frac{2b\left(b^2e^2-4bcde+3c^2d^2\right)}{b+cx}+2\log(x)\left(b^2e^2-6bcde+6c^2d^2\right)-2\left(b^2e^2-6bcde+6c^2d^2\right)\log(b+cx)+\frac{b^2(cd-be)^2}{(b+cx)^2}-\frac{b^2d^2}{x^2}-\frac{2bd(2be-}{x}}{2b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(b*x + c*x^2)^3,x]
```

```
[Out] (-((b^2*d^2)/x^2) - (2*b*d*(-3*c*d + 2*b*e))/x + (b^2*(c*d - b*e)^2)/(b + c
*x)^2 + (2*b*(3*c^2*d^2 - 4*b*c*d*e + b^2*e^2))/(b + c*x) + 2*(6*c^2*d^2 -
6*b*c*d*e + b^2*e^2)*Log[x] - 2*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*Log[b + c
*x])/(2*b^5)
```

Maple [A] time = 0.056, size = 207, normalized size = 1.4

$$-\frac{d^2}{2\,b^3x^2}+\frac{\ln(x)\,e^2}{b^3}-6\,\frac{\ln(x)\,cde}{b^4}+6\,\frac{\ln(x)\,c^2d^2}{b^5}-2\,\frac{de}{b^3x}+3\,\frac{cd^2}{b^4x}-\frac{\ln(cx+b)\,e^2}{b^3}+6\,\frac{\ln(cx+b)\,cde}{b^4}-6\,\frac{\ln(cx+b)\,c^2d^2}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^2+b*x)^3,x)
```

```
[Out] -1/2*d^2/b^3/x^2+1/b^3*ln(x)*e^2-6/b^4*ln(x)*c*d*e+6/b^5*ln(x)*c^2*d^2-2*d/
b^3/x*e+3*d^2/b^4/x*c-1/b^3*ln(c*x+b)*e^2+6/b^4*ln(c*x+b)*c*d*e-6/b^5*ln(c*
x+b)*c^2*d^2+1/b^2/(c*x+b)*e^2-4/b^3/(c*x+b)*c*d*e+3/b^4/(c*x+b)*c^2*d^2+1/
2/b/(c*x+b)^2*e^2-1/b^2/(c*x+b)^2*c*d*e+1/2/b^3/(c*x+b)^2*c^2*d^2
```

Maxima [A] time = 1.11236, size = 243, normalized size = 1.69

$$-\frac{b^3d^2-2\left(6\,c^3d^2-6\,bc^2de+b^2ce^2\right)x^3-3\left(6\,bc^2d^2-6\,b^2cde+b^3e^2\right)x^2-4\left(b^2cd^2-b^3de\right)x}{2\left(b^4c^2x^4+2\,b^5cx^3+b^6x^2\right)}-\frac{\left(6\,c^2d^2-6\,bcde+b^2e^2\right)\log}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*d^2 - 2*(6*c^3*d^2 - 6*b*c^2*d*e + b^2*c*e^2)*x^3 - 3*(6*b*c^2*d^
2 - 6*b^2*c*d*e + b^3*e^2)*x^2 - 4*(b^2*c*d^2 - b^3*d*e)*x)/(b^4*c^2*x^4 +
2*b^5*c*x^3 + b^6*x^2) - (6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*log(c*x + b)/b^5
 + (6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*log(x)/b^5
```

Fricas [B] time = 1.80002, size = 664, normalized size = 4.61

$$-\frac{b^4d^2-2\left(6\,bc^3d^2-6\,b^2c^2de+b^3ce^2\right)x^3-3\left(6\,b^2c^2d^2-6\,b^3cde+b^4e^2\right)x^2-4\left(b^3cd^2-b^4de\right)x+2\left(\left(6\,c^4d^2-6\,bc^3de+b^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b^4*d^2 - 2*(6*b*c^3*d^2 - 6*b^2*c^2*d*e + b^3*c*e^2)*x^3 - 3*(6*b^2*
c^2*d^2 - 6*b^3*c*d*e + b^4*e^2)*x^2 - 4*(b^3*c*d^2 - b^4*d*e)*x + 2*((6*c^
4*d^2 - 6*b*c^3*d*e + b^2*c^2*e^2)*x^4 + 2*(6*b*c^3*d^2 - 6*b^2*c^2*d*e + b
^3*c*e^2)*x^3 + (6*b^2*c^2*d^2 - 6*b^3*c*d*e + b^4*e^2)*x^2)*log(c*x + b) -
 2*((6*c^4*d^2 - 6*b*c^3*d*e + b^2*c^2*e^2)*x^4 + 2*(6*b*c^3*d^2 - 6*b^2*c^
2*d*e + b^3*c*e^2)*x^3 + (6*b^2*c^2*d^2 - 6*b^3*c*d*e + b^4*e^2)*x^2)*log(x
```

```
))/(b^5*c^2*x^4 + 2*b^6*c*x^3 + b^7*x^2)
```

Sympy [B] time = 2.5886, size = 345, normalized size = 2.4

$$\frac{-b^3d^2 + x^3\left(2b^2ce^2 - 12bc^2de + 12c^3d^2\right) + x^2\left(3b^3e^2 - 18b^2cde + 18bc^2d^2\right) + x\left(-4b^3de + 4b^2cd^2\right)}{2b^6x^2 + 4b^5cx^3 + 2b^4c^2x^4} + \frac{\left(b^2e^2 - 6bcde + 6c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**2+b*x)**3,x)
```

```
[Out] (-b**3*d**2 + x**3*(2*b**2*c*e**2 - 12*b*c**2*d*e + 12*c**3*d**2) + x**2*(3
*b**3*e**2 - 18*b**2*c*d*e + 18*b*c**2*d**2) + x*(-4*b**3*d*e + 4*b**2*c*d*
*2))/(2*b**6*x**2 + 4*b**5*c*x**3 + 2*b**4*c**2*x**4) + (b**2*e**2 - 6*b*c*
d*e + 6*c**2*d**2)*log(x + (b**3*e**2 - 6*b**2*c*d*e + 6*b*c**2*d**2 - b*(b
**2*e**2 - 6*b*c*d*e + 6*c**2*d**2))/(2*b**2*c*e**2 - 12*b*c**2*d*e + 12*c*
*3*d**2))/b**5 - (b**2*e**2 - 6*b*c*d*e + 6*c**2*d**2)*log(x + (b**3*e**2 -
 6*b**2*c*d*e + 6*b*c**2*d**2 + b*(b**2*e**2 - 6*b*c*d*e + 6*c**2*d**2))/(2
*b**2*c*e**2 - 12*b*c**2*d*e + 12*c**3*d**2))/b**5
```

Giac [A] time = 1.26516, size = 246, normalized size = 1.71

$$\frac{\left(6\,c^2d^2 - 6\,bcde + b^2e^2\right)\log\left(|x|\right)}{b^5} - \frac{\left(6\,c^3d^2 - 6\,bc^2de + b^2ce^2\right)\log\left(|cx + b|\right)}{b^5c} + \frac{12\,c^3d^2x^3 - 12\,bc^2dx^3e + 18\,bc^2d^2x^2 + 2\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x)^3,x, algorithm="giac")
```

```
[Out] (6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*log(abs(x))/b^5 - (6*c^3*d^2 - 6*b*c^2*d*
e + b^2*c*e^2)*log(abs(c*x + b))/(b^5*c) + 1/2*(12*c^3*d^2*x^3 - 12*b*c^2*d
*x^3*e + 18*b*c^2*d^2*x^2 + 2*b^2*c*x^3*e^2 - 18*b^2*c*d*x^2*e + 4*b^2*c*d^
2*x + 3*b^3*x^2*e^2 - 4*b^3*d*x*e - b^3*d^2)/((c*x^2 + b*x)^2*b^4)
```

3.281 $\int \frac{d+ex}{(bx+cx^2)^3} dx$

Optimal. Leaf size=110

$$\frac{3cd-be}{b^4x}+\frac{c(3cd-2be)}{b^4(b+cx)}+\frac{c(cd-be)}{2b^3(b+cx)^2}+\frac{3c\log(x)(2cd-be)}{b^5}-\frac{3c(2cd-be)\log(b+cx)}{b^5}-\frac{d}{2b^3x^2}$$

```
[Out] -d/(2*b^3*x^2) + (3*c*d - b*e)/(b^4*x) + (c*(c*d - b*e))/(2*b^3*(b + c*x)^2
) + (c*(3*c*d - 2*b*e))/(b^4*(b + c*x)) + (3*c*(2*c*d - b*e)*Log[x])/b^5 -
(3*c*(2*c*d - b*e)*Log[b + c*x])/b^5
```

Rubi [A] time = 0.103272, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {631}

$$\frac{3cd-be}{b^4x}+\frac{c(3cd-2be)}{b^4(b+cx)}+\frac{c(cd-be)}{2b^3(b+cx)^2}+\frac{3c\log(x)(2cd-be)}{b^5}-\frac{3c(2cd-be)\log(b+cx)}{b^5}-\frac{d}{2b^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(b*x + c*x^2)^3,x]
```

```
[Out] -d/(2*b^3*x^2) + (3*c*d - b*e)/(b^4*x) + (c*(c*d - b*e))/(2*b^3*(b + c*x)^2
) + (c*(3*c*d - 2*b*e))/(b^4*(b + c*x)) + (3*c*(2*c*d - b*e)*Log[x])/b^5 -
(3*c*(2*c*d - b*e)*Log[b + c*x])/b^5
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex}{(bx+cx^2)^3}dx &= \int \left(\frac{d}{b^3x^3}+\frac{-3cd+be}{b^4x^2}-\frac{3c(-2cd+be)}{b^5x}+\frac{c^2(-cd+be)}{b^3(b+cx)^3}+\frac{c^2(-3cd+2be)}{b^4(b+cx)^2}+\frac{3c^2(-2cd+be)}{b^5(b+cx)}\right)dx\\ &= -\frac{d}{2b^3x^2}+\frac{3cd-be}{b^4x}+\frac{c(cd-be)}{2b^3(b+cx)^2}+\frac{c(3cd-2be)}{b^4(b+cx)}+\frac{3c(2cd-be)\log(x)}{b^5}-\frac{3c(2cd-be)\log(b+cx)}{b^5}\end{aligned}$$

Mathematica [A] time = 0.080662, size = 102, normalized size = 0.93

$$\frac{-\frac{b\left(b^2cx(9ex-4d)+b^3(d+2ex)+6bc^2x^2(ex-3d)-12c^3dx^3\right)}{x^2(b+cx)^2}+6c\log(x)(2cd-be)+6c(be-2cd)\log(b+cx)}{2b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(b*x + c*x^2)^3,x]
```

```
[Out] (-((b*(-12*c^3*d*x^3 + 6*b*c^2*x^2*(-3*d + e*x) + b^3*(d + 2*e*x) + b^2*c*x
*(-4*d + 9*e*x)))/(x^2*(b + c*x)^2)) + 6*c*(2*c*d - b*e)*Log[x] + 6*c*(-2*c
```

```
*d + b*e)*Log[b + c*x])/(2*b^5)
```

Maple [A] time = 0.056, size = 138, normalized size = 1.3

$$-\frac{d}{2\,b^{3}x^{2}}-\frac{e}{b^{3}x}+3\,\frac{cd}{b^{4}x}-3\,\frac{c\ln(x)\,e}{b^{4}}+6\,\frac{c^{2}\ln(x)\,d}{b^{5}}-2\,\frac{ce}{b^{3}(cx+b)}+3\,\frac{dc^{2}}{b^{4}(cx+b)}-\frac{ce}{2\,b^{2}(cx+b)^{2}}+\frac{dc^{2}}{2\,b^{3}(cx+b)^{2}}+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^2+b*x)^3,x)
```

```
[Out] -1/2*d/b^3/x^2-1/b^3/x*e+3/b^4/x*c*d-3*c/b^4*ln(x)*e+6*c^2/b^5*ln(x)*d-2*c/
b^3/(c*x+b)*e+3*c^2/b^4/(c*x+b)*d-1/2*c/b^2/(c*x+b)^2*e+1/2*c^2/b^3/(c*x+b)
^2*d+3*c/b^4*ln(c*x+b)*e-6*c^2/b^5*ln(c*x+b)*d
```

Maxima [A] time = 1.12258, size = 184, normalized size = 1.67

$$-\frac{b^{3}d-6\left(2\,c^{3}d-bc^{2}e\right)x^{3}-9\left(2\,bc^{2}d-b^{2}ce\right)x^{2}-2\left(2\,b^{2}cd-b^{3}e\right)x}{2\left(b^{4}c^{2}x^{4}+2\,b^{5}cx^{3}+b^{6}x^{2}\right)}-\frac{3\left(2\,c^{2}d-bce\right)\log(cx+b)}{b^{5}}+\frac{3\left(2\,c^{2}d-bce\right)\log}{b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*d - 6*(2*c^3*d - b*c^2*e)*x^3 - 9*(2*b*c^2*d - b^2*c*e)*x^2 - 2*(
2*b^2*c*d - b^3*e)*x)/(b^4*c^2*x^4 + 2*b^5*c*x^3 + b^6*x^2) - 3*(2*c^2*d -
b*c*e)*log(c*x + b)/b^5 + 3*(2*c^2*d - b*c*e)*log(x)/b^5
```

Fricas [B] time = 1.87802, size = 467, normalized size = 4.25

$$-\frac{b^{4}d-6\left(2\,bc^{3}d-b^{2}c^{2}e\right)x^{3}-9\left(2\,b^{2}c^{2}d-b^{3}ce\right)x^{2}-2\left(2\,b^{3}cd-b^{4}e\right)x+6\left(\left(2\,c^{4}d-bc^{3}e\right)x^{4}+2\left(2\,bc^{3}d-b^{2}c^{2}e\right)x^{3}+(2}{2\left(b^{5}c^{2}x^{4}+2\,b^{6}cx^{3}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b^4*d - 6*(2*b*c^3*d - b^2*c^2*e)*x^3 - 9*(2*b^2*c^2*d - b^3*c*e)*x^2
 - 2*(2*b^3*c*d - b^4*e)*x + 6*((2*c^4*d - b*c^3*e)*x^4 + 2*(2*b*c^3*d - b^
2*c^2*e)*x^3 + (2*b^2*c^2*d - b^3*c*e)*x^2)*log(c*x + b) - 6*((2*c^4*d - b*
c^3*e)*x^4 + 2*(2*b*c^3*d - b^2*c^2*e)*x^3 + (2*b^2*c^2*d - b^3*c*e)*x^2)*l
og(x))/(b^5*c^2*x^4 + 2*b^6*c*x^3 + b^7*x^2)
```

Sympy [B] time = 1.74491, size = 219, normalized size = 1.99

$$-\frac{b^{3}d+x^{3}\left(6bc^{2}e-12c^{3}d\right)+x^{2}\left(9b^{2}ce-18bc^{2}d\right)+x\left(2b^{3}e-4b^{2}cd\right)}{2b^{6}x^{2}+4b^{5}cx^{3}+2b^{4}c^{2}x^{4}}-\frac{3c\,(be-2cd)\log\left(x+\frac{3b^{2}ce-6bc^{2}d-3bc(be-2cd)}{6bc^{2}e-12c^{3}d}\right)}{b^{5}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**2+b*x)**3,x)
```

```
[Out] -(b**3*d + x**3*(6*b*c**2*e - 12*c**3*d) + x**2*(9*b**2*c*e - 18*b*c**2*d)
+ x*(2*b**3*e - 4*b**2*c*d))/(2*b**6*x**2 + 4*b**5*c*x**3 + 2*b**4*c**2*x**
4) - 3*c*(b*e - 2*c*d)*log(x + (3*b**2*c*e - 6*b*c**2*d - 3*b*c*(b*e - 2*c*
d))/(6*b*c**2*e - 12*c**3*d))/b**5 + 3*c*(b*e - 2*c*d)*log(x + (3*b**2*c*e
- 6*b*c**2*d + 3*b*c*(b*e - 2*c*d))/(6*b*c**2*e - 12*c**3*d))/b**5
```

Giac [A] time = 1.25839, size = 178, normalized size = 1.62

$$\frac{3\left(2\,c^2d-bce\right)\log\left(|x|\right)}{b^5}-\frac{3\left(2\,c^3d-bc^2e\right)\log\left(|cx+b|\right)}{b^5c}+\frac{12\,c^3dx^3-6\,bc^2x^3e+18\,bc^2dx^2-9\,b^2cx^2e+4\,b^2cdx-2\,b^3xe-}{2\left(cx^2+bx\right)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x)^3,x, algorithm="giac")
```

```
[Out] 3*(2*c^2*d - b*c*e)*log(abs(x))/b^5 - 3*(2*c^3*d - b*c^2*e)*log(abs(c*x + b
))/(b^5*c) + 1/2*(12*c^3*d*x^3 - 6*b*c^2*x^3*e + 18*b*c^2*d*x^2 - 9*b^2*c*x
^2*e + 4*b^2*c*d*x - 2*b^3*x*e - b^3*d)/((c*x^2 + b*x)^2*b^4)
```

3.282 $\int \frac{1}{\left(bx+cx^2\right)^3} dx$

Optimal. Leaf size=72

$$\frac{6c^2\log(x)}{b^5}-\frac{6c^2\log(b+cx)}{b^5}+\frac{3c(b+2cx)}{b^4\left(bx+cx^2\right)}-\frac{b+2cx}{2b^2\left(bx+cx^2\right)^2}$$

```
[Out] -(b + 2*c*x)/(2*b^2*(b*x + c*x^2)^2) + (3*c*(b + 2*c*x))/(b^4*(b*x + c*x^2)
) + (6*c^2*Log[x])/b^5 - (6*c^2*Log[b + c*x])/b^5
```

Rubi [A] time = 0.0192287, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {614, 615}

$$\frac{6c^2\log(x)}{b^5}-\frac{6c^2\log(b+cx)}{b^5}+\frac{3c(b+2cx)}{b^4\left(bx+cx^2\right)}-\frac{b+2cx}{2b^2\left(bx+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(-3),x]
```

```
[Out] -(b + 2*c*x)/(2*b^2*(b*x + c*x^2)^2) + (3*c*(b + 2*c*x))/(b^4*(b*x + c*x^2)
) + (6*c^2*Log[x])/b^5 - (6*c^2*Log[b + c*x])/b^5
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 615

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[Log[x]/b, x] - Simp
[Log[RemoveContent[b + c*x, x]]/b, x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(bx+cx^2\right)^3} dx &= -\frac{b+2cx}{2b^2\left(bx+cx^2\right)^2}-\frac{(3c)\int \frac{1}{\left(bx+cx^2\right)^2} dx}{b^2}\\ &= -\frac{b+2cx}{2b^2\left(bx+cx^2\right)^2}+\frac{3c(b+2cx)}{b^4\left(bx+cx^2\right)}+\frac{\left(6c^2\right)\int \frac{1}{bx+cx^2} dx}{b^4}\\ &= -\frac{b+2cx}{2b^2\left(bx+cx^2\right)^2}+\frac{3c(b+2cx)}{b^4\left(bx+cx^2\right)}+\frac{6c^2\log(x)}{b^5}-\frac{6c^2\log(b+cx)}{b^5}\end{aligned}$$

Mathematica [A] time = 0.0459729, size = 68, normalized size = 0.94

$$\frac{\frac{b\left(4b^2cx-b^3+18bc^2x^2+12c^3x^3\right)}{x^2(b+cx)^2}-12c^2\log(b+cx)+12c^2\log(x)}{2b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(-3),x]
```

```
[Out] ((b*(-b^3 + 4*b^2*c*x + 18*b*c^2*x^2 + 12*c^3*x^3))/(x^2*(b + c*x)^2) + 12*
c^2*Log[x] - 12*c^2*Log[b + c*x])/(2*b^5)
```

Maple [A] time = 0.051, size = 73, normalized size = 1.

$$-\frac{1}{2\,b^3x^2}+6\,\frac{c^2\ln(x)}{b^5}+3\,\frac{c}{b^4x}-6\,\frac{c^2\ln(cx+b)}{b^5}+3\,\frac{c^2}{b^4\,(cx+b)}+\frac{c^2}{2\,b^3\,(cx+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^3,x)
```

```
[Out] -1/2/b^3/x^2+6*c^2*ln(x)/b^5+3/b^4*c/x-6*c^2*ln(c*x+b)/b^5+3/b^4*c^2/(c*x+b
)+1/2*c^2/b^3/(c*x+b)^2
```

Maxima [A] time = 1.12503, size = 116, normalized size = 1.61

$$\frac{12\,c^3x^3+18\,bc^2x^2+4\,b^2cx-b^3}{2\left(b^4c^2x^4+2\,b^5cx^3+b^6x^2\right)}-\frac{6\,c^2\log(cx+b)}{b^5}+\frac{6\,c^2\log(x)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(12*c^3*x^3 + 18*b*c^2*x^2 + 4*b^2*c*x - b^3)/(b^4*c^2*x^4 + 2*b^5*c*x^
3 + b^6*x^2) - 6*c^2*log(c*x + b)/b^5 + 6*c^2*log(x)/b^5
```

Fricas [A] time = 1.69016, size = 269, normalized size = 3.74

$$\frac{12\,bc^3x^3+18\,b^2c^2x^2+4\,b^3cx-b^4-12\left(c^4x^4+2\,bc^3x^3+b^2c^2x^2\right)\log(cx+b)+12\left(c^4x^4+2\,bc^3x^3+b^2c^2x^2\right)\log(x)}{2\left(b^5c^2x^4+2\,b^6cx^3+b^7x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(12*b*c^3*x^3 + 18*b^2*c^2*x^2 + 4*b^3*c*x - b^4 - 12*(c^4*x^4 + 2*b*c^
3*x^3 + b^2*c^2*x^2)*log(c*x + b) + 12*(c^4*x^4 + 2*b*c^3*x^3 + b^2*c^2*x^2
)*log(x))/(b^5*c^2*x^4 + 2*b^6*c*x^3 + b^7*x^2)
```

Sympy [A] time = 1.50587, size = 78, normalized size = 1.08

$$\frac{-b^3+4b^2cx+18bc^2x^2+12c^3x^3}{2b^6x^2+4b^5cx^3+2b^4c^2x^4}+\frac{6c^2\left(\log(x)-\log\left(\frac{b}{c}+x\right)\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**3,x)
```

```
[Out] (-b**3 + 4*b**2*c*x + 18*b*c**2*x**2 + 12*c**3*x**3)/(2*b**6*x**2 + 4*b**5*
c*x**3 + 2*b**4*c**2*x**4) + 6*c**2*(log(x) - log(b/c + x))/b**5
```

Giac [A] time = 1.29595, size = 99, normalized size = 1.38

$$-\frac{6\,c^2\log\left(|cx+b|\right)}{b^5}+\frac{6\,c^2\log\left(|x|\right)}{b^5}+\frac{12\,c^3x^3+18\,bc^2x^2+4\,b^2cx-b^3}{2\left(cx^2+bx\right)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^3,x, algorithm="giac")
```

```
[Out] -6*c^2*log(abs(c*x + b))/b^5 + 6*c^2*log(abs(x))/b^5 + 1/2*(12*c^3*x^3 + 18
*b*c^2*x^2 + 4*b^2*c*x - b^3)/((c*x^2 + b*x)^2*b^4)
```

3.283 $\int \frac{1}{(d+ex)\left(bx+cx^2\right)^3} dx$

Optimal. Leaf size=193

$$\frac{\log(x)\left(b^2e^2+3bcde+6c^2d^2\right)}{b^5d^3}-\frac{c^3\left(10b^2e^2-15bcde+6c^2d^2\right)\log(b+cx)}{b^5(cd-be)^3}+\frac{c^3(3cd-4be)}{b^4(b+cx)(cd-be)^2}+\frac{c^3}{2b^3(b+cx)^2(cd-be)}$$

```
[Out] -1/(2*b^3*d*x^2) + (3*c*d + b*e)/(b^4*d^2*x) + c^3/(2*b^3*(c*d - b*e)*(b +
c*x)^2) + (c^3*(3*c*d - 4*b*e))/(b^4*(c*d - b*e)^2*(b + c*x)) + ((6*c^2*d^2
 + 3*b*c*d*e + b^2*e^2)*Log[x])/(b^5*d^3) - (c^3*(6*c^2*d^2 - 15*b*c*d*e +
10*b^2*e^2)*Log[b + c*x])/(b^5*(c*d - b*e)^3) + (e^5*Log[d + e*x])/(d^3*(c*
d - b*e)^3)
```

Rubi [A] time = 0.227096, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$\frac{\log(x)\left(b^2e^2+3bcde+6c^2d^2\right)}{b^5d^3}-\frac{c^3\left(10b^2e^2-15bcde+6c^2d^2\right)\log(b+cx)}{b^5(cd-be)^3}+\frac{c^3(3cd-4be)}{b^4(b+cx)(cd-be)^2}+\frac{c^3}{2b^3(b+cx)^2(cd-be)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(b*x + c*x^2)^3),x]
```

```
[Out] -1/(2*b^3*d*x^2) + (3*c*d + b*e)/(b^4*d^2*x) + c^3/(2*b^3*(c*d - b*e)*(b +
c*x)^2) + (c^3*(3*c*d - 4*b*e))/(b^4*(c*d - b*e)^2*(b + c*x)) + ((6*c^2*d^2
 + 3*b*c*d*e + b^2*e^2)*Log[x])/(b^5*d^3) - (c^3*(6*c^2*d^2 - 15*b*c*d*e +
10*b^2*e^2)*Log[b + c*x])/(b^5*(c*d - b*e)^3) + (e^5*Log[d + e*x])/(d^3*(c*
d - b*e)^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)^3}dx=\int \left(\frac{1}{b^3dx^3}+\frac{-3cd-be}{b^4d^2x^2}+\frac{6c^2d^2+3bcde+b^2e^2}{b^5d^3x}+\frac{c^4}{b^3(-cd+be)(b+cx)^3}+\frac{c^4(-3cd+4b}{b^4(-cd+be)^2(b}\right.$$

$$=-\frac{1}{2b^3dx^2}+\frac{3cd+be}{b^4d^2x}+\frac{c^3}{2b^3(cd-be)(b+cx)^2}+\frac{c^3(3cd-4be)}{b^4(cd-be)^2(b+cx)}+\frac{\left(6c^2d^2+3bcde+b^2e\right.}{b^5d^3}$$

Mathematica [A] time = 0.196233, size = 192, normalized size = 0.99

$$\frac{\log(x)\left(b^2e^2+3bcde+6c^2d^2\right)}{b^5d^3}+\frac{c^3\left(10b^2e^2-15bcde+6c^2d^2\right)\log(b+cx)}{b^5(be-cd)^3}+\frac{c^3(3cd-4be)}{b^4(b+cx)(cd-be)^2}-\frac{c^3}{2b^3(b+cx)^2(be-cd)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(b*x + c*x^2)^3),x]
```

```
[Out] -1/(2*b^3*d*x^2) + (3*c*d + b*e)/(b^4*d^2*x) - c^3/(2*b^3*(-(c*d) + b*e)*(b
 + c*x)^2) + (c^3*(3*c*d - 4*b*e))/(b^4*(c*d - b*e)^2*(b + c*x)) + ((6*c^2*
d^2 + 3*b*c*d*e + b^2*e^2)*Log[x])/(b^5*d^3) + (c^3*(6*c^2*d^2 - 15*b*c*d*e
 + 10*b^2*e^2)*Log[b + c*x])/(b^5*(-(c*d) + b*e)^3) + (e^5*Log[d + e*x])/(d
^3*(c*d - b*e)^3)
```

Maple [A] time = 0.061, size = 254, normalized size = 1.3

$$-\frac{1}{2\,db^3x^2}+\frac{e}{d^2b^3x}+3\,\frac{c}{db^4x}+\frac{\ln(x)\,e^2}{d^3b^3}+3\,\frac{\ln(x)\,ce}{d^2b^4}+6\,\frac{\ln(x)\,c^2}{db^5}-\frac{c^3}{(2\,be-2\,cd)\,b^3\,(cx+b)^2}-4\,\frac{c^3e}{(be-cd)^2\,b^3\,(cx+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+b*x)^3,x)
```

```
[Out] -1/2/b^3/d/x^2+1/d^2/b^3/x*e+3/d/b^4/x*c+1/d^3/b^3*ln(x)*e^2+3/d^2/b^4*ln(x
)*c*e+6/d/b^5*ln(x)*c^2-1/2*c^3/(b*e-c*d)/b^3/(c*x+b)^2-4*c^3/(b*e-c*d)^2/b
^3/(c*x+b)*e+3*c^4/(b*e-c*d)^2/b^4/(c*x+b)*d+10*c^3/(b*e-c*d)^3/b^3*ln(c*x+
b)*e^2-15*c^4/(b*e-c*d)^3/b^4*ln(c*x+b)*d*e+6*c^5/(b*e-c*d)^3/b^5*ln(c*x+b)
*d^2-e^5/d^3/(b*e-c*d)^3*ln(e*x+d)
```

Maxima [B] time = 1.23846, size = 593, normalized size = 3.07

$$\frac{e^5\log(ex+d)}{c^3d^6-3\,bc^2d^5e+3\,b^2cd^4e^2-b^3d^3e^3}-\frac{\left(6\,c^5d^2-15\,bc^4de+10\,b^2c^3e^2\right)\log(cx+b)}{b^5c^3d^3-3\,b^6c^2d^2e+3\,b^7cde^2-b^8e^3}-\frac{b^3c^2d^3-2\,b^4cd^2e+b^5de^2-2\left(6\,c\right.}{2\left(\left(b^4c^4d^4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

```
[Out] e^5*log(e*x + d)/(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 - b^3*d^3*e^3)
- (6*c^5*d^2 - 15*b*c^4*d*e + 10*b^2*c^3*e^2)*log(c*x + b)/(b^5*c^3*d^3 - 3
*b^6*c^2*d^2*e + 3*b^7*c*d*e^2 - b^8*e^3) - 1/2*(b^3*c^2*d^3 - 2*b^4*c*d^2*
e + b^5*d*e^2 - 2*(6*c^5*d^3 - 9*b*c^4*d^2*e + b^2*c^3*d*e^2 + b^3*c^2*e^3)
*x^3 - (18*b*c^4*d^3 - 27*b^2*c^3*d^2*e + 3*b^3*c^2*d*e^2 + 4*b^4*c*e^3)*x^
2 - 2*(2*b^2*c^3*d^3 - 3*b^3*c^2*d^2*e + b^5*e^3)*x)/((b^4*c^4*d^4 - 2*b^5*
c^3*d^3*e + b^6*c^2*d^2*e^2)*x^4 + 2*(b^5*c^3*d^4 - 2*b^6*c^2*d^3*e + b^7*c
*d^2*e^2)*x^3 + (b^6*c^2*d^4 - 2*b^7*c*d^3*e + b^8*d^2*e^2)*x^2) + (6*c^2*d
^2 + 3*b*c*d*e + b^2*e^2)*log(x)/(b^5*d^3)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+b*x)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.3336, size = 559, normalized size = 2.9

$$-\frac{\left(6\,c^6d^2-15\,bc^5de+10\,b^2c^4e^2\right)\log\left(|cx+b|\right)}{b^5c^4d^3-3\,b^6c^3d^2e+3\,b^7c^2de^2-b^8ce^3}+\frac{e^6\log\left(|xe+d|\right)}{c^3d^6e-3\,bc^2d^5e^2+3\,b^2cd^4e^3-b^3d^3e^4}+\frac{\left(6\,c^2d^2+3\,bcde+b^2e^2\right)\log\left(|x|\right)}{b^5d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x)^3,x, algorithm="giac")
```

```
[Out] -(6*c^6*d^2 - 15*b*c^5*d*e + 10*b^2*c^4*e^2)*log(abs(c*x + b))/(b^5*c^4*d^3
 - 3*b^6*c^3*d^2*e + 3*b^7*c^2*d*e^2 - b^8*c*e^3) + e^6*log(abs(x*e + d))/(
c^3*d^6*e - 3*b*c^2*d^5*e^2 + 3*b^2*c*d^4*e^3 - b^3*d^3*e^4) + (6*c^2*d^2 +
 3*b*c*d*e + b^2*e^2)*log(abs(x))/(b^5*d^3) - 1/2*(b^3*c^3*d^5 - 3*b^4*c^2*
d^4*e + 3*b^5*c*d^3*e^2 - b^6*d^2*e^3 - 2*(6*c^6*d^5 - 15*b*c^5*d^4*e + 10*
b^2*c^4*d^3*e^2 - b^4*c^2*d*e^4)*x^3 - (18*b*c^5*d^5 - 45*b^2*c^4*d^4*e + 3
0*b^3*c^3*d^3*e^2 + b^4*c^2*d^2*e^3 - 4*b^5*c*d*e^4)*x^2 - 2*(2*b^2*c^4*d^5
 - 5*b^3*c^3*d^4*e + 3*b^4*c^2*d^3*e^2 + b^5*c*d^2*e^3 - b^6*d*e^4)*x)/((c*
d - b*e)^3*(c*x + b)^2*b^4*d^3*x^2)
```

3.284 $\int \frac{1}{(d+ex)^2\left(bx+cx^2\right)^3}\,dx$

Optimal. Leaf size=230

$$\frac{3\log(x)\left(b^2e^2+2bcde+2c^2d^2\right)}{b^5d^4}-\frac{3c^4\left(5b^2e^2-6bcde+2c^2d^2\right)\log(b+cx)}{b^5(cd-be)^4}+\frac{c^4(3cd-5be)}{b^4(b+cx)(cd-be)^3}+\frac{c^4}{2b^3(b+cx)^2(cd-$$

```
[Out] -1/(2*b^3*d^2*x^2) + (3*c*d + 2*b*e)/(b^4*d^3*x) + c^4/(2*b^3*(c*d - b*e)^2
*(b + c*x)^2) + (c^4*(3*c*d - 5*b*e))/(b^4*(c*d - b*e)^3*(b + c*x)) - e^5/(
d^3*(c*d - b*e)^3*(d + e*x)) + (3*(2*c^2*d^2 + 2*b*c*d*e + b^2*e^2)*Log[x])
/(b^5*d^4) - (3*c^4*(2*c^2*d^2 - 6*b*c*d*e + 5*b^2*e^2)*Log[b + c*x])/(b^5*
(c*d - b*e)^4) + (3*e^5*(2*c*d - b*e)*Log[d + e*x])/(d^4*(c*d - b*e)^4)
```

Rubi [A] time = 0.33923, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$\frac{3\log(x)\left(b^2e^2+2bcde+2c^2d^2\right)}{b^5d^4}-\frac{3c^4\left(5b^2e^2-6bcde+2c^2d^2\right)\log(b+cx)}{b^5(cd-be)^4}+\frac{c^4(3cd-5be)}{b^4(b+cx)(cd-be)^3}+\frac{c^4}{2b^3(b+cx)^2(cd-$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(b*x + c*x^2)^3),x]
```

```
[Out] -1/(2*b^3*d^2*x^2) + (3*c*d + 2*b*e)/(b^4*d^3*x) + c^4/(2*b^3*(c*d - b*e)^2
*(b + c*x)^2) + (c^4*(3*c*d - 5*b*e))/(b^4*(c*d - b*e)^3*(b + c*x)) - e^5/(
d^3*(c*d - b*e)^3*(d + e*x)) + (3*(2*c^2*d^2 + 2*b*c*d*e + b^2*e^2)*Log[x])
/(b^5*d^4) - (3*c^4*(2*c^2*d^2 - 6*b*c*d*e + 5*b^2*e^2)*Log[b + c*x])/(b^5*
(c*d - b*e)^4) + (3*e^5*(2*c*d - b*e)*Log[d + e*x])/(d^4*(c*d - b*e)^4)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(bx+cx^2\right)^3}\,dx=\int\left(\frac{1}{b^3d^2x^3}+\frac{-3cd-2be}{b^4d^3x^2}+\frac{3\left(2c^2d^2+2bcde+b^2e^2\right)}{b^5d^4x}-\frac{c^5}{b^3(-cd+be)^2(b+cx)^3}-\frac{c^5}{b^4(-c$$

$$=-\frac{1}{2b^3d^2x^2}+\frac{3cd+2be}{b^4d^3x}+\frac{c^4}{2b^3(cd-be)^2(b+cx)^2}+\frac{c^4(3cd-5be)}{b^4(cd-be)^3(b+cx)}-\frac{e^5}{d^3(cd-be)^3(d$$

Mathematica [A] time = 0.29037, size = 230, normalized size = 1.

$$\frac{3\log(x)\left(b^2e^2+2bcde+2c^2d^2\right)}{b^5d^4}-\frac{3c^4\left(5b^2e^2-6bcde+2c^2d^2\right)\log(b+cx)}{b^5(cd-be)^4}+\frac{c^4(5be-3cd)}{b^4(b+cx)(be-cd)^3}+\frac{c^4}{2b^3(b+cx)^2(cd-$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(b*x + c*x^2)^3),x]
```

```
[Out] -1/(2*b^3*d^2*x^2) + (3*c*d + 2*b*e)/(b^4*d^3*x) + c^4/(2*b^3*(c*d - b*e)^2
*(b + c*x)^2) + (c^4*(-3*c*d + 5*b*e))/(b^4*(-(c*d) + b*e)^3*(b + c*x)) - e
^5/(d^3*(c*d - b*e)^3*(d + e*x)) + (3*(2*c^2*d^2 + 2*b*c*d*e + b^2*e^2)*Log
[x])/(b^5*d^4) - (3*c^4*(2*c^2*d^2 - 6*b*c*d*e + 5*b^2*e^2)*Log[b + c*x])/(
b^5*(c*d - b*e)^4) + (3*e^5*(2*c*d - b*e)*Log[d + e*x])/(d^4*(c*d - b*e)^4)
```

Maple [A] time = 0.069, size = 306, normalized size = 1.3

$$-\frac{1}{2\,d^2b^3x^2}+2\,\frac{e}{d^3b^3x}+3\,\frac{c}{d^2b^4x}+3\,\frac{\ln(x)\,e^2}{d^4b^3}+6\,\frac{\ln(x)\,ce}{d^3b^4}+6\,\frac{\ln(x)\,c^2}{d^2b^5}+\frac{c^4}{2\,(be-cd)^2\,b^3\,(cx+b)^2}+5\,\frac{c^4e}{(be-cd)^3\,b^3\,(cx+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^2+b*x)^3,x)
```

```
[Out] -1/2/b^3/d^2/x^2+2/d^3/b^3/x*e+3/d^2/b^4/x*c+3/d^4/b^3*ln(x)*e^2+6/d^3/b^4*
ln(x)*c*e+6/d^2/b^5*ln(x)*c^2+1/2*c^4/(b*e-c*d)^2/b^3/(c*x+b)^2+5*c^4/(b*e-
c*d)^3/b^3/(c*x+b)*e-3*c^5/(b*e-c*d)^3/b^4/(c*x+b)*d-15*c^4/(b*e-c*d)^4/b^3
*ln(c*x+b)*e^2+18*c^5/(b*e-c*d)^4/b^4*ln(c*x+b)*d*e-6*c^6/(b*e-c*d)^4/b^5*l
n(c*x+b)*d^2+e^5/d^3/(b*e-c*d)^3/(e*x+d)-3*e^6/d^4/(b*e-c*d)^4*ln(e*x+d)*b+
6*e^5/d^3/(b*e-c*d)^4*ln(e*x+d)*c
```

Maxima [B] time = 1.29149, size = 1015, normalized size = 4.41

$$-\frac{3\left(2\,c^6d^2-6\,bc^5de+5\,b^2c^4e^2\right)\log(cx+b)}{b^5c^4d^4-4\,b^6c^3d^3e+6\,b^7c^2d^2e^2-4\,b^8cde^3+b^9e^4}+\frac{3\left(2\,cde^5-be^6\right)\log(ex+d)}{c^4d^8-4\,bc^3d^7e+6\,b^2c^2d^6e^2-4\,b^3cd^5e^3+b^4d^4e^4}-\frac{b^3c^3d^5-3\,b^4c^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x)^3,x, algorithm="maxima")
```

```
[Out] -3*(2*c^6*d^2 - 6*b*c^5*d*e + 5*b^2*c^4*e^2)*log(c*x + b)/(b^5*c^4*d^4 - 4*
b^6*c^3*d^3*e + 6*b^7*c^2*d^2*e^2 - 4*b^8*c*d*e^3 + b^9*e^4) + 3*(2*c*d*e^5
 - b*e^6)*log(e*x + d)/(c^4*d^8 - 4*b*c^3*d^7*e + 6*b^2*c^2*d^6*e^2 - 4*b^3
*c*d^5*e^3 + b^4*d^4*e^4) - 1/2*(b^3*c^3*d^5 - 3*b^4*c^2*d^4*e + 3*b^5*c*d^
3*e^2 - b^6*d^2*e^3 - 6*(2*c^6*d^4*e - 4*b*c^5*d^3*e^2 + b^2*c^4*d^2*e^3 +
b^3*c^3*d*e^4 - b^4*c^2*e^5)*x^4 - 3*(4*c^6*d^5 - 2*b*c^5*d^4*e - 10*b^2*c^
4*d^3*e^2 + 5*b^3*c^3*d^2*e^3 + 3*b^4*c^2*d*e^4 - 4*b^5*c*e^5)*x^3 - (18*b*
c^5*d^5 - 32*b^2*c^4*d^4*e + b^3*c^3*d^3*e^2 + 13*b^4*c^2*d^2*e^3 - 6*b^6*e
^5)*x^2 - (4*b^2*c^4*d^5 - 9*b^3*c^3*d^4*e + 3*b^4*c^2*d^3*e^2 + 5*b^5*c*d^
2*e^3 - 3*b^6*d*e^4)*x)/((b^4*c^5*d^6*e - 3*b^5*c^4*d^5*e^2 + 3*b^6*c^3*d^4
*e^3 - b^7*c^2*d^3*e^4)*x^5 + (b^4*c^5*d^7 - b^5*c^4*d^6*e - 3*b^6*c^3*d^5*
e^2 + 5*b^7*c^2*d^4*e^3 - 2*b^8*c*d^3*e^4)*x^4 + (2*b^5*c^4*d^7 - 5*b^6*c^3
*d^6*e + 3*b^7*c^2*d^5*e^2 + b^8*c*d^4*e^3 - b^9*d^3*e^4)*x^3 + (b^6*c^3*d^
7 - 3*b^7*c^2*d^6*e + 3*b^8*c*d^5*e^2 - b^9*d^4*e^3)*x^2) + 3*(2*c^2*d^2 +
2*b*c*d*e + b^2*e^2)*log(x)/(b^5*d^4)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**2+b*x)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.35597, size = 1133, normalized size = 4.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x)^3,x, algorithm="giac")
```

```
[Out] -3/2*(4*c^6*d^6*e^2 - 12*b*c^5*d^5*e^3 + 10*b^2*c^4*d^4*e^4 - 2*b^5*c*d*e^7
 + b^6*e^8)*e^(-2)*log(abs(-2*c*d*e + 2*c*d^2*e/(x*e + d) + b*e^2 - 2*b*d*e
^2/(x*e + d) - abs(b)*e^2)/abs(-2*c*d*e + 2*c*d^2*e/(x*e + d) + b*e^2 - 2*b
*d*e^2/(x*e + d) + abs(b)*e^2))/((b^4*c^4*d^8 - 4*b^5*c^3*d^7*e + 6*b^6*c^2
*d^6*e^2 - 4*b^7*c*d^5*e^3 + b^8*d^4*e^4)*abs(b)) - 3/2*(2*c*d*e^5 - b*e^6)
*log(abs(-c + 2*c*d/(x*e + d) - c*d^2/(x*e + d)^2 - b*e/(x*e + d) + b*d*e/(
x*e + d)^2))/(c^4*d^8 - 4*b*c^3*d^7*e + 6*b^2*c^2*d^6*e^2 - 4*b^3*c*d^5*e^3
 + b^4*d^4*e^4) - e^11/((c^3*d^6*e^6 - 3*b*c^2*d^5*e^7 + 3*b^2*c*d^4*e^8 -
b^3*d^3*e^9)*(x*e + d)) + 1/2*(12*c^7*d^5*e - 30*b*c^6*d^4*e^2 + 16*b^2*c^5
*d^3*e^3 + 6*b^3*c^4*d^2*e^4 - 14*b^4*c^3*d*e^5 + 5*b^5*c^2*e^6 - 2*(18*c^7
*d^6*e^2 - 54*b*c^6*d^5*e^3 + 47*b^2*c^5*d^4*e^4 - 4*b^3*c^4*d^3*e^5 - 29*b
^4*c^3*d^2*e^6 + 22*b^5*c^2*d*e^7 - 5*b^6*c*e^8)*e^(-1)/(x*e + d) + (36*c^7
*d^7*e^3 - 126*b*c^6*d^6*e^4 + 144*b^2*c^5*d^5*e^5 - 45*b^3*c^4*d^4*e^6 - 7
0*b^4*c^3*d^3*e^7 + 87*b^5*c^2*d^2*e^8 - 36*b^6*c*d*e^9 + 5*b^7*e^10)*e^(-2
)/(x*e + d)^2 - 6*(2*c^7*d^8*e^4 - 8*b*c^6*d^7*e^5 + 11*b^2*c^5*d^6*e^6 - 5
*b^3*c^4*d^5*e^7 - 5*b^4*c^3*d^4*e^8 + 9*b^5*c^2*d^3*e^9 - 5*b^6*c*d^2*e^10
 + b^7*d*e^11)*e^(-3)/(x*e + d)^3)/((c*d - b*e)^4*b^4*(c - 2*c*d/(x*e + d)
+ c*d^2/(x*e + d)^2 + b*e/(x*e + d) - b*d*e/(x*e + d)^2)^2*d^4)
```

3.285 $\int (d+ex)^3\sqrt{bx+cx^2}\,dx$

Optimal. Leaf size=210

$$\frac{e\left(bx+cx^2\right)^{3/2}\left(35b^2e^2+42cex(2cd-be)-150bcde+192c^2d^2\right)}{240c^3}+\frac{(b+2cx)\sqrt{bx+cx^2}(2cd-be)\left(7b^2e^2-16bcde+16c^2d^2\right.}{128c^4}$$

```
[Out] ((2*c*d - b*e)*(16*c^2*d^2 - 16*b*c*d*e + 7*b^2*e^2)*(b + 2*c*x)*Sqrt[b*x +
 c*x^2])/(128*c^4) + (e*(d + e*x)^2*(b*x + c*x^2)^(3/2))/(5*c) + (e*(192*c^
2*d^2 - 150*b*c*d*e + 35*b^2*e^2 + 42*c*e*(2*c*d - b*e)*x)*(b*x + c*x^2)^(3
/2))/(240*c^3) - (b^2*(2*c*d - b*e)*(16*c^2*d^2 - 16*b*c*d*e + 7*b^2*e^2)*A
rcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(128*c^(9/2))
```

Rubi [A] time = 0.269013, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {742, 779, 612, 620, 206}

$$\frac{e\left(bx+cx^2\right)^{3/2}\left(35b^2e^2+42cex(2cd-be)-150bcde+192c^2d^2\right)}{240c^3}+\frac{(b+2cx)\sqrt{bx+cx^2}(2cd-be)\left(7b^2e^2-16bcde+16c^2d^2\right.}{128c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*Sqrt[b*x + c*x^2],x]
```

```
[Out] ((2*c*d - b*e)*(16*c^2*d^2 - 16*b*c*d*e + 7*b^2*e^2)*(b + 2*c*x)*Sqrt[b*x +
 c*x^2])/(128*c^4) + (e*(d + e*x)^2*(b*x + c*x^2)^(3/2))/(5*c) + (e*(192*c^
2*d^2 - 150*b*c*d*e + 35*b^2*e^2 + 42*c*e*(2*c*d - b*e)*x)*(b*x + c*x^2)^(3
/2))/(240*c^3) - (b^2*(2*c*d - b*e)*(16*c^2*d^2 - 16*b*c*d*e + 7*b^2*e^2)*A
rcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(128*c^(9/2))
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
```

```
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^3\sqrt{bx+cx^2}\,dx = \frac{e(d+ex)^2\left(bx+cx^2\right)^{3/2}}{5c}+\frac{\int (d+ex)\left(\frac{1}{2}d(10cd-3be)+\frac{7}{2}e(2cd-be)x\right)\sqrt{bx+cx^2}\,dx}{5c}$$

$$= \frac{e(d+ex)^2\left(bx+cx^2\right)^{3/2}}{5c}+\frac{e\left(192c^2d^2-150bcde+35b^2e^2+42ce(2cd-be)x\right)\left(bx+cx^2\right)^{3/2}}{240c^3}$$

$$= \frac{(2cd-be)\left(16c^2d^2-16bcde+7b^2e^2\right)(b+2cx)\sqrt{bx+cx^2}}{128c^4}+\frac{e(d+ex)^2\left(bx+cx^2\right)^{3/2}}{5c}+\frac{e(1}{}$$

$$= \frac{(2cd-be)\left(16c^2d^2-16bcde+7b^2e^2\right)(b+2cx)\sqrt{bx+cx^2}}{128c^4}+\frac{e(d+ex)^2\left(bx+cx^2\right)^{3/2}}{5c}+\frac{e(1}{}$$

$$= \frac{(2cd-be)\left(16c^2d^2-16bcde+7b^2e^2\right)(b+2cx)\sqrt{bx+cx^2}}{128c^4}+\frac{e(d+ex)^2\left(bx+cx^2\right)^{3/2}}{5c}+\frac{e(1}{}$$

Mathematica [A] time = 0.536631, size = 230, normalized size = 1.1

$$\frac{\sqrt{x(b+cx)}\left(\sqrt{c}\left(-4b^2c^2e\left(180d^2+75dex+14e^2x^2\right)+10b^3ce^2(45d+7ex)-105b^4e^3+48bc^3\left(10d^2ex+10d^3+5de^2x^2\right.\right.\right.}{1920c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*Sqrt[b*x + c*x^2],x]
```

```
[Out] (Sqrt[x*(b + c*x)]*(Sqrt[c]*(-105*b^4*e^3 + 10*b^3*c*e^2*(45*d + 7*e*x) - 4
*b^2*c^2*e*(180*d^2 + 75*d*e*x + 14*e^2*x^2) + 48*b*c^3*(10*d^3 + 10*d^2*e*
x + 5*d*e^2*x^2 + e^3*x^3) + 96*c^4*x*(10*d^3 + 20*d^2*e*x + 15*d*e^2*x^2 +
 4*e^3*x^3)) + (15*b^(3/2)*(-32*c^3*d^3 + 48*b*c^2*d^2*e - 30*b^2*c*d*e^2 +
 7*b^3*e^3)*ArcSinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(Sqrt[x]*Sqrt[1 + (c*x)/b])
))/(1920*c^(9/2))
```

Maple [B] time = 0.053, size = 444, normalized size = 2.1

$$\frac{e^3x^2}{5c}\left(cx^2+bx\right)^{\frac{3}{2}}-\frac{7be^3x}{40c^2}\left(cx^2+bx\right)^{\frac{3}{2}}+\frac{7e^3b^2}{48c^3}\left(cx^2+bx\right)^{\frac{3}{2}}-\frac{7b^3e^3x}{64c^3}\sqrt{cx^2+bx}-\frac{7e^3b^4}{128c^4}\sqrt{cx^2+bx}+\frac{7e^3b^5}{256}\ln\left(\left(\frac{b}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x)^(1/2),x)
```

```
[Out] 1/5*e^3*x^2*(c*x^2+b*x)^(3/2)/c-7/40*e^3*b/c^2*x*(c*x^2+b*x)^(3/2)+7/48*e^3
*b^2/c^3*(c*x^2+b*x)^(3/2)-7/64*e^3*b^3/c^3*x*(c*x^2+b*x)^(1/2)-7/128*e^3*b
^4/c^4*(c*x^2+b*x)^(1/2)+7/256*e^3*b^5/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^
2+b*x)^(1/2))+3/4*d*e^2*x*(c*x^2+b*x)^(3/2)/c-5/8*d*e^2*b/c^2*(c*x^2+b*x)^(
3/2)+15/32*d*e^2*b^2/c^2*x*(c*x^2+b*x)^(1/2)+15/64*d*e^2*b^3/c^3*(c*x^2+b*x
)^(1/2)-15/128*d*e^2*b^4/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))+
d^2*e*(c*x^2+b*x)^(3/2)/c-3/4*d^2*e*b/c*x*(c*x^2+b*x)^(1/2)-3/8*d^2*e*b^2/c
^2*(c*x^2+b*x)^(1/2)+3/16*d^2*e*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b
*x)^(1/2))+1/2*d^3*x*(c*x^2+b*x)^(1/2)+1/4*d^3/c*(c*x^2+b*x)^(1/2)*b-1/8*d^
3*b^2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.76282, size = 1112, normalized size = 5.3

$$\left[-\frac{15\left(32\,b^2c^3d^3-48\,b^3c^2d^2e+30\,b^4cde^2-7\,b^5e^3\right)\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\left(384\,c^5e^3x^4+480\,bc^4d^3-720\,b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/3840*(15*(32*b^2*c^3*d^3 - 48*b^3*c^2*d^2*e + 30*b^4*c*d*e^2 - 7*b^5*e^
3)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 2*(384*c^5*e^3*x^
4 + 480*b*c^4*d^3 - 720*b^2*c^3*d^2*e + 450*b^3*c^2*d*e^2 - 105*b^4*c*e^3 +
 48*(30*c^5*d*e^2 + b*c^4*e^3)*x^3 + 8*(240*c^5*d^2*e + 30*b*c^4*d*e^2 - 7*
b^2*c^3*e^3)*x^2 + 10*(96*c^5*d^3 + 48*b*c^4*d^2*e - 30*b^2*c^3*d*e^2 + 7*b
^3*c^2*e^3)*x)*sqrt(c*x^2 + b*x))/c^5, 1/1920*(15*(32*b^2*c^3*d^3 - 48*b^3*
c^2*d^2*e + 30*b^4*c*d*e^2 - 7*b^5*e^3)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*s
qrt(-c)/(c*x)) + (384*c^5*e^3*x^4 + 480*b*c^4*d^3 - 720*b^2*c^3*d^2*e + 450
*b^3*c^2*d*e^2 - 105*b^4*c*e^3 + 48*(30*c^5*d*e^2 + b*c^4*e^3)*x^3 + 8*(240
*c^5*d^2*e + 30*b*c^4*d*e^2 - 7*b^2*c^3*e^3)*x^2 + 10*(96*c^5*d^3 + 48*b*c^
4*d^2*e - 30*b^2*c^3*d*e^2 + 7*b^3*c^2*e^3)*x)*sqrt(c*x^2 + b*x))/c^5]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x\,(b+cx)}\,(d+ex)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x*(b + c*x))*(d + e*x)**3, x)
```

Giac [A] time = 1.31892, size = 338, normalized size = 1.61

$$\frac{1}{1920}\sqrt{cx^2+bx}\left(2\left(4\left(6\left(8\,xe^3+\frac{30\,c^4de^2+bc^3e^3}{c^4}\right)x+\frac{240\,c^4d^2e+30\,bc^3de^2-7\,b^2c^2e^3}{c^4}\right)x+\frac{5\left(96\,c^4d^3+48\,bc^3d^2e-\cdots\right.}{c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(c*x^2 + b*x)*(2*(4*(6*(8*x*e^3 + (30*c^4*d*e^2 + b*c^3*e^3)/c^4
)*x + (240*c^4*d^2*e + 30*b*c^3*d*e^2 - 7*b^2*c^2*e^3)/c^4)*x + 5*(96*c^4*d
^3 + 48*b*c^3*d^2*e - 30*b^2*c^2*d*e^2 + 7*b^3*c*e^3)/c^4)*x + 15*(32*b*c^3
*d^3 - 48*b^2*c^2*d^2*e + 30*b^3*c*d*e^2 - 7*b^4*e^3)/c^4) + 1/256*(32*b^2*
c^3*d^3 - 48*b^3*c^2*d^2*e + 30*b^4*c*d*e^2 - 7*b^5*e^3)*log(abs(-2*(sqrt(c
)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b))/c^(9/2)
```

3.286 $\int (d+ex)^2\sqrt{bx+cx^2}\,dx$

Optimal. Leaf size=162

$$\frac{(b+2cx)\sqrt{bx+cx^2}\left(5b^2e^2-16bcde+16c^2d^2\right)}{64c^3}-\frac{b^2\left(5b^2e^2-16bcde+16c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{64c^{7/2}}+\frac{5e\left(bx+cx^2\right)^{3/2}(2cd-$$

```
[Out] ((16*c^2*d^2 - 16*b*c*d*e + 5*b^2*e^2)*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(64*c
^3) + (5*e*(2*c*d - b*e)*(b*x + c*x^2)^(3/2))/(24*c^2) + (e*(d + e*x)*(b*x
+ c*x^2)^(3/2))/(4*c) - (b^2*(16*c^2*d^2 - 16*b*c*d*e + 5*b^2*e^2)*ArcTanh[
(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(64*c^(7/2))
```

Rubi [A] time = 0.12639, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {742, 640, 612, 620, 206}

$$\frac{(b+2cx)\sqrt{bx+cx^2}\left(5b^2e^2-16bcde+16c^2d^2\right)}{64c^3}-\frac{b^2\left(5b^2e^2-16bcde+16c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{64c^{7/2}}+\frac{5e\left(bx+cx^2\right)^{3/2}(2cd-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*Sqrt[b*x + c*x^2],x]
```



```
[Out] ((16*c^2*d^2 - 16*b*c*d*e + 5*b^2*e^2)*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(64*c
^3) + (5*e*(2*c*d - b*e)*(b*x + c*x^2)^(3/2))/(24*c^2) + (e*(d + e*x)*(b*x
+ c*x^2)^(3/2))/(4*c) - (b^2*(16*c^2*d^2 - 16*b*c*d*e + 5*b^2*e^2)*ArcTanh[
(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(64*c^(7/2))
```


Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^2\sqrt{bx+cx^2}\,dx = \frac{e(d+ex)\left(bx+cx^2\right)^{3/2}}{4c}+\frac{\int \left(\frac{1}{2}d(8cd-3be)+\frac{5}{2}e(2cd-be)x\right)\sqrt{bx+cx^2}\,dx}{4c}$$

$$=\frac{5e(2cd-be)\left(bx+cx^2\right)^{3/2}}{24c^2}+\frac{e(d+ex)\left(bx+cx^2\right)^{3/2}}{4c}+\frac{\left(cd(8cd-3be)-\frac{5}{2}be(2cd-be)\right)\int \sqrt{}}{8c^2}$$

$$=\frac{\left(16c^2d^2-16bcde+5b^2e^2\right)(b+2cx)\sqrt{bx+cx^2}}{64c^3}+\frac{5e(2cd-be)\left(bx+cx^2\right)^{3/2}}{24c^2}+\frac{e(d+ex)\left(b\right.}{4c}$$

$$=\frac{\left(16c^2d^2-16bcde+5b^2e^2\right)(b+2cx)\sqrt{bx+cx^2}}{64c^3}+\frac{5e(2cd-be)\left(bx+cx^2\right)^{3/2}}{24c^2}+\frac{e(d+ex)\left(b\right.}{4c}$$

$$=\frac{\left(16c^2d^2-16bcde+5b^2e^2\right)(b+2cx)\sqrt{bx+cx^2}}{64c^3}+\frac{5e(2cd-be)\left(bx+cx^2\right)^{3/2}}{24c^2}+\frac{e(d+ex)\left(b\right.}{4c}$$

Mathematica [A] time = 0.298318, size = 164, normalized size = 1.01

$$\frac{\sqrt{x(b+cx)}\left(\sqrt{c}\left(-2b^2ce(24d+5ex)+15b^3e^2+8bc^2\left(6d^2+4dex+e^2x^2\right)+16c^3x\left(6d^2+8dex+3e^2x^2\right)\right)-\frac{3b^{3/2}\left(5b^2e^2-16\right.}{}\right.}{192c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*Sqrt[b*x + c*x^2],x]
```

```
[Out] (Sqrt[x*(b + c*x)]*(Sqrt[c]*(15*b^3*e^2 - 2*b^2*c*e*(24*d + 5*e*x) + 8*b*c^
2*(6*d^2 + 4*d*e*x + e^2*x^2) + 16*c^3*x*(6*d^2 + 8*d*e*x + 3*e^2*x^2)) - (
3*b^(3/2)*(16*c^2*d^2 - 16*b*c*d*e + 5*b^2*e^2)*ArcSinh[(Sqrt[c]*Sqrt[x])/S
qrt[b]])/(Sqrt[x]*Sqrt[1 + (c*x)/b])))/(192*c^(7/2))
```

Maple [B] time = 0.052, size = 287, normalized size = 1.8

$$\frac{e^2x}{4c}\left(cx^2+bx\right)^{\frac{3}{2}}-\frac{5e^2b}{24c^2}\left(cx^2+bx\right)^{\frac{3}{2}}+\frac{5b^2e^2x}{32c^2}\sqrt{cx^2+bx}+\frac{5b^3e^2}{64c^3}\sqrt{cx^2+bx}-\frac{5e^2b^4}{128}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+b*x)^(1/2),x)
```

```
[Out] 1/4*e^2*x*(c*x^2+b*x)^(3/2)/c-5/24*e^2*b/c^2*(c*x^2+b*x)^(3/2)+5/32*e^2*b^2
/c^2*x*(c*x^2+b*x)^(1/2)+5/64*e^2*b^3/c^3*(c*x^2+b*x)^(1/2)-5/128*e^2*b^4/c
^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))+2/3*d*e*(c*x^2+b*x)^(3/2)/
```

```
c-1/2*d*e*b/c*x*(c*x^2+b*x)^(1/2)-1/4*d*e*b^2/c^2*(c*x^2+b*x)^(1/2)+1/8*d*e
*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))+1/2*d^2*x*(c*x^2+b*x
)^(1/2)+1/4*d^2/c*(c*x^2+b*x)^(1/2)*b-1/8*d^2*b^2/c^(3/2)*ln((1/2*b+c*x)/c^
(1/2)+(c*x^2+b*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.66614, size = 757, normalized size = 4.67

$$\left[\frac{3\left(16\,b^2c^2d^2-16\,b^3cde+5\,b^4e^2\right)\sqrt{c}\log\left(2\,cx+b-2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\left(48\,c^4e^2x^3+48\,bc^3d^2-48\,b^2c^2de+15\,b^3ce^2+8\right.}{384\,c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/384*(3*(16*b^2*c^2*d^2 - 16*b^3*c*d*e + 5*b^4*e^2)*sqrt(c)*log(2*c*x + b
 - 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*(48*c^4*e^2*x^3 + 48*b*c^3*d^2 - 48*b^2
*c^2*d*e + 15*b^3*c*e^2 + 8*(16*c^4*d*e + b*c^3*e^2)*x^2 + 2*(48*c^4*d^2 +
16*b*c^3*d*e - 5*b^2*c^2*e^2)*x)*sqrt(c*x^2 + b*x))/c^4, 1/192*(3*(16*b^2*c
^2*d^2 - 16*b^3*c*d*e + 5*b^4*e^2)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-
c)/(c*x)) + (48*c^4*e^2*x^3 + 48*b*c^3*d^2 - 48*b^2*c^2*d*e + 15*b^3*c*e^2
+ 8*(16*c^4*d*e + b*c^3*e^2)*x^2 + 2*(48*c^4*d^2 + 16*b*c^3*d*e - 5*b^2*c^2
*e^2)*x)*sqrt(c*x^2 + b*x))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x\left(b+cx\right)}\left(d+ex\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x*(b + c*x))*(d + e*x)**2, x)
```

Giac [A] time = 1.3201, size = 232, normalized size = 1.43

$$\frac{1}{192}\sqrt{cx^2+bx}\left(2\left(4\left(6\,xe^2+\frac{16\,c^3de+bc^2e^2}{c^3}\right)x+\frac{48\,c^3d^2+16\,bc^2de-5\,b^2ce^2}{c^3}\right)x+\frac{3\left(16\,bc^2d^2-16\,b^2cde+5\,b^3e^2\right)}{c^3}\right)+\frac{(1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x)*(2*(4*(6*x*e^2 + (16*c^3*d*e + b*c^2*e^2)/c^3)*x +
(48*c^3*d^2 + 16*b*c^2*d*e - 5*b^2*c*e^2)/c^3)*x + 3*(16*b*c^2*d^2 - 16*b^2
*c*d*e + 5*b^3*e^2)/c^3) + 1/128*(16*b^2*c^2*d^2 - 16*b^3*c*d*e + 5*b^4*e^2
)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b))/c^(7/2)
```

3.287 $\int (d+ex)\sqrt{bx+cx^2}\,dx$

Optimal. Leaf size=99

$$-\frac{b^2(2cd-be)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{8c^{5/2}}+\frac{(b+2cx)\sqrt{bx+cx^2}(2cd-be)}{8c^2}+\frac{e\left(bx+cx^2\right)^{3/2}}{3c}$$

```
[Out] ((2*c*d - b*e)*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(8*c^2) + (e*(b*x + c*x^2)^(3
/2))/(3*c) - (b^2*(2*c*d - b*e)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(8*
c^(5/2))
```

Rubi [A] time = 0.0358537, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {640, 612, 620, 206}

$$-\frac{b^2(2cd-be)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{8c^{5/2}}+\frac{(b+2cx)\sqrt{bx+cx^2}(2cd-be)}{8c^2}+\frac{e\left(bx+cx^2\right)^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*Sqrt[b*x + c*x^2],x]
```

```
[Out] ((2*c*d - b*e)*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(8*c^2) + (e*(b*x + c*x^2)^(3
/2))/(3*c) - (b^2*(2*c*d - b*e)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(8*
c^(5/2))
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)\sqrt{bx+cx^2}\,dx = \frac{e\left(bx+cx^2\right)^{3/2}}{3c} + \frac{(2cd-be)\int \sqrt{bx+cx^2}\,dx}{2c}$$
$$= \frac{(2cd-be)(b+2cx)\sqrt{bx+cx^2}}{8c^2} + \frac{e\left(bx+cx^2\right)^{3/2}}{3c} - \frac{\left(b^2(2cd-be)\right)\int \frac{1}{\sqrt{bx+cx^2}}\,dx}{16c^2}$$
$$= \frac{(2cd-be)(b+2cx)\sqrt{bx+cx^2}}{8c^2} + \frac{e\left(bx+cx^2\right)^{3/2}}{3c} - \frac{\left(b^2(2cd-be)\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{bx+c}}\right.}{8c^2}$$
$$= \frac{(2cd-be)(b+2cx)\sqrt{bx+cx^2}}{8c^2} + \frac{e\left(bx+cx^2\right)^{3/2}}{3c} - \frac{b^2(2cd-be)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{8c^{5/2}}$$

Mathematica [A] time = 0.190187, size = 108, normalized size = 1.09

$$\frac{\sqrt{x(b+cx)}\left(\sqrt{c}\left(-3b^2e+2bc(3d+ex)+4c^2x(3d+2ex)\right)+\frac{3b^{3/2}(be-2cd)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}\right)}{24c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Sqrt[b*x + c*x^2],x]
```

```
[Out] (Sqrt[x*(b + c*x)]*(Sqrt[c]*(-3*b^2*e + 2*b*c*(3*d + e*x) + 4*c^2*x*(3*d +
2*e*x)) + (3*b^(3/2)*(-2*c*d + b*e)*ArcSinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(Sq
rt[x]*Sqrt[1 + (c*x)/b])))/(24*c^(5/2))
```

Maple [A] time = 0.05, size = 157, normalized size = 1.6

$$\frac{e}{3c}\left(cx^2+bx\right)^{\frac{3}{2}} - \frac{bxe}{4c}\sqrt{cx^2+bx} - \frac{b^2e}{8c^2}\sqrt{cx^2+bx} + \frac{eb^3}{16}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}} + \sqrt{cx^2+bx}\right)c^{-\frac{5}{2}} + \frac{dx}{2}\sqrt{cx^2+bx} + \frac{bd}{4c}\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+b*x)^(1/2),x)
```

```
[Out] 1/3*e*(c*x^2+b*x)^(3/2)/c-1/4*e*b/c*x*(c*x^2+b*x)^(1/2)-1/8*e*b^2/c^2*(c*x^
2+b*x)^(1/2)+1/16*e*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))+1
/2*d*x*(c*x^2+b*x)^(1/2)+1/4*d/c*(c*x^2+b*x)^(1/2)*b-1/8*d*b^2/c^(3/2)*ln((
1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.48892, size = 467, normalized size = 4.72

$$\left[-\frac{3\left(2b^2cd-b^3e\right)\sqrt{c}\log\left(2cx+b+2\sqrt{cx^2+bx}\sqrt{c}\right)-2\left(8c^3ex^2+6bc^2d-3b^2ce+2\left(6c^3d+bc^2e\right)x\right)\sqrt{cx^2+bx}}{48c^3}, \frac{3\left(2b^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*(2*b^2*c*d - b^3*e)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*s
qrt(c)) - 2*(8*c^3*e*x^2 + 6*b*c^2*d - 3*b^2*c*e + 2*(6*c^3*d + b*c^2*e)*x)
*sqrt(c*x^2 + b*x))/c^3, 1/24*(3*(2*b^2*c*d - b^3*e)*sqrt(-c)*arctan(sqrt(c
*x^2 + b*x)*sqrt(-c)/(c*x)) + (8*c^3*e*x^2 + 6*b*c^2*d - 3*b^2*c*e + 2*(6*c
^3*d + b*c^2*e)*x)*sqrt(c*x^2 + b*x))/c^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x\,(b+cx)}\,(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x*(b + c*x))*(d + e*x), x)
```

Giac [A] time = 1.31288, size = 146, normalized size = 1.47

$$\frac{1}{24}\sqrt{cx^2+bx}\left(2\left(4xe+\frac{6c^2d+bce}{c^2}\right)x+\frac{3\left(2bcd-b^2e\right)}{c^2}\right)+\frac{\left(2b^2cd-b^3e\right)\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}-b\right|\right)}{16c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + b*x)*(2*(4*x*e + (6*c^2*d + b*c*e)/c^2)*x + 3*(2*b*c*d -
b^2*e)/c^2) + 1/16*(2*b^2*c*d - b^3*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 +
 b*x))*sqrt(c) - b))/c^(5/2)
```

3.288 $\int \sqrt{bx+cx^2}\,dx$

Optimal. Leaf size=60

$$\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}$$

```
[Out] ((b + 2*c*x)*Sqrt[b*x + c*x^2])/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x +
 c*x^2]])/(4*c^(3/2))
```

Rubi [A] time = 0.0152118, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {612, 620, 206}

$$\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x + c*x^2],x]
```

```
[Out] ((b + 2*c*x)*Sqrt[b*x + c*x^2])/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x +
 c*x^2]])/(4*c^(3/2))
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{bx+cx^2}\,dx &= \frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int \frac{1}{\sqrt{bx+cx^2}}\,dx}{8c}\\ &= \frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{bx+cx^2}}\right)}{4c}\\ &= \frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0992279, size = 74, normalized size = 1.23

$$\frac{\sqrt{x(b+cx)}\left(\sqrt{c}(b+2cx)-\frac{b^{3/2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}\right)}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x + c*x^2],x]
```

```
[Out] (Sqrt[x*(b + c*x)]*(Sqrt[c]*(b + 2*c*x) - (b^(3/2)*ArcSinh[(Sqrt[c]*Sqrt[x]
)/Sqrt[b]])/(Sqrt[x]*Sqrt[1 + (c*x)/b])))/(4*c^(3/2))
```

Maple [A] time = 0.045, size = 56, normalized size = 0.9

$$\frac{2\,cx+b}{4\,c}\sqrt{cx^2+bx}-\frac{b^2}{8}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx}\right)c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(1/2),x)
```

```
[Out] 1/4*(2*c*x+b)*(c*x^2+b*x)^(1/2)/c-1/8*b^2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c
*x^2+b*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.22897, size = 285, normalized size = 4.75

$$\left[\frac{b^2\sqrt{c}\log\left(2\,cx+b-2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\left(2\,c^2x+bc\right)\sqrt{cx^2+bx}}{8\,c^2},\ \frac{b^2\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx}\right)+\left(2\,c^2x+bc\right)\sqrt{cx^2+bx}}{4\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(b^2*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*(2*c^2*x
 + b*c)*sqrt(c*x^2 + b*x))/c^2, 1/4*(b^2*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*
sqrt(-c)/(c*x)) + (2*c^2*x + b*c)*sqrt(c*x^2 + b*x))/c^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx + cx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(sqrt(b*x + c*x**2), x)
```

Giac [A] time = 1.15665, size = 82, normalized size = 1.37

$$\frac{1}{4}\sqrt{cx^2 + bx}\left(2x + \frac{b}{c}\right) + \frac{b^2 \log\left(\left|-2\left(\sqrt{c}x - \sqrt{cx^2 + bx}\right)\sqrt{c} - b\right|\right)}{8c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^2 + b*x)*(2*x + b/c) + 1/8*b^2*log(abs(-2*(sqrt(c)*x - sqrt(c*
x^2 + b*x))*sqrt(c) - b))/c^(3/2)
```

3.289 $\int \frac{\sqrt{bx+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=129

$$-\frac{(2cd-be)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{\sqrt{c}e^2}+\frac{\sqrt{d}\sqrt{cd-be}\tanh^{-1}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{e^2}+\frac{\sqrt{bx+cx^2}}{e}$$

```
[Out] Sqrt[b*x + c*x^2]/e - ((2*c*d - b*e)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]]
)/(Sqrt[c]*e^2) + (Sqrt[d]*Sqrt[c*d - b*e]*ArcTanh[(b*d + (2*c*d - b*e)*x)/
(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/e^2
```

Rubi [A] time = 0.141262, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {734, 843, 620, 206, 724}

$$-\frac{(2cd-be)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{\sqrt{c}e^2}+\frac{\sqrt{d}\sqrt{cd-be}\tanh^{-1}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{e^2}+\frac{\sqrt{bx+cx^2}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x + c*x^2]/(d + e*x),x]
```

```
[Out] Sqrt[b*x + c*x^2]/e - ((2*c*d - b*e)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]]
)/(Sqrt[c]*e^2) + (Sqrt[d]*Sqrt[c*d - b*e]*ArcTanh[(b*d + (2*c*d - b*e)*x)/
(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/e^2
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{bx+cx^2}}{d+ex}\,dx = \frac{\sqrt{bx+cx^2}}{e} - \frac{\int \frac{bd+(2cd-be)x}{(d+ex)\sqrt{bx+cx^2}}\,dx}{2e}$$
$$= \frac{\sqrt{bx+cx^2}}{e} + \frac{(d(cd-be))\int \frac{1}{(d+ex)\sqrt{bx+cx^2}}\,dx}{e^2} - \frac{(2cd-be)\int \frac{1}{\sqrt{bx+cx^2}}\,dx}{2e^2}$$
$$= \frac{\sqrt{bx+cx^2}}{e} - \frac{(2d(cd-be))\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde-x^2}\,dx, x, \frac{-bd-(2cd-be)x}{\sqrt{bx+cx^2}}\right)}{e^2} - \frac{(2cd-be)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,d\right.}{e^2}$$
$$= \frac{\sqrt{bx+cx^2}}{e} - \frac{(2cd-be)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{\sqrt{c}e^2} + \frac{\sqrt{d}\sqrt{cd-be}\tanh^{-1}\left(\frac{bd+(2cd-be)x}{2\sqrt{d}\sqrt{cd-be}\sqrt{bx+cx^2}}\right)}{e^2}$$

Mathematica [A] time = 0.523773, size = 137, normalized size = 1.06

$$\frac{\sqrt{x(b+cx)}\left(-\frac{2\sqrt{d}\sqrt{be-cd}\tan^{-1}\left(\frac{\sqrt{x}\sqrt{be-cd}}{\sqrt{d}\sqrt{b+cx}}\right)}{\sqrt{b+cx}} + \frac{(be-2cd)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{c}\sqrt{\frac{cx}{b}+1}} + e\sqrt{x}\right)}{e^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x + c*x^2]/(d + e*x),x]
```

```
[Out] (Sqrt[x*(b + c*x)]*(e*Sqrt[x] + ((-2*c*d + b*e)*ArcSinh[(Sqrt[c]*Sqrt[x])/S
qrt[b]])/(Sqrt[b]*Sqrt[c]*Sqrt[1 + (c*x)/b]) - (2*Sqrt[d]*Sqrt[-(c*d) + b*e
]*ArcTan[(Sqrt[-(c*d) + b*e]*Sqrt[x])/(Sqrt[d]*Sqrt[b + c*x])])/Sqrt[b + c*
x]))/(e^2*Sqrt[x])
```

Maple [B] time = 0.249, size = 490, normalized size = 3.8

$$\frac{1}{e}\sqrt{c\left(\frac{d}{e}+x\right)^2 + \frac{be-2\,cd}{e}\left(\frac{d}{e}+x\right) - \frac{d\,(be-cd)}{e^2}} + \frac{b}{2\,e}\ln\left(\left(\frac{be-2\,cd}{2\,e} + \left(\frac{d}{e}+x\right)c\right)\frac{1}{\sqrt{c}} + \sqrt{c\left(\frac{d}{e}+x\right)^2 + \frac{be-2\,cd}{e}\left(\frac{d}{e}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(1/2)/(e*x+d),x)
```

```
[Out] 1/e*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)+1/2/e*ln((1/2
*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e
-c*d)/e^2)^(1/2))/c^(1/2)*b-1/e^2*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+
(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))*c^(1/2)*d+1/e^2*
d/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(
-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)
^(1/2))/(d/e+x))*b-1/e^3*d^2/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^
```

```
2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)
/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.59809, size = 1102, normalized size = 8.54

$$\left[\frac{2\sqrt{cx^2+bx}ce-(2\,cd-be)\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\,\sqrt{cd^2-bde}c\log\left(\frac{bd+(2\,cd-be)x+2\,\sqrt{cd^2-bde}\sqrt{cx^2+bx}}{ex+d}\right)}{2\,ce^2}, \frac{2\,\sqrt{cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(c*x^2 + b*x)*c*e - (2*c*d - b*e)*sqrt(c)*log(2*c*x + b + 2*sqr
t(c*x^2 + b*x)*sqrt(c)) + 2*sqrt(c*d^2 - b*d*e)*c*log((b*d + (2*c*d - b*e)*
x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*x + d)))/(c*e^2), 1/2*(2*sq
rt(c*x^2 + b*x)*c*e + 4*sqrt(-c*d^2 + b*d*e)*c*arctan(-sqrt(-c*d^2 + b*d*e)
*sqrt(c*x^2 + b*x)/((c*d - b*e)*x)) - (2*c*d - b*e)*sqrt(c)*log(2*c*x + b +
 2*sqrt(c*x^2 + b*x)*sqrt(c)))/(c*e^2), (sqrt(c*x^2 + b*x)*c*e + (2*c*d - b
*e)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)) + sqrt(c*d^2 - b*d*e)
*c*log((b*d + (2*c*d - b*e)*x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e
*x + d)))/(c*e^2), (sqrt(c*x^2 + b*x)*c*e + 2*sqrt(-c*d^2 + b*d*e)*c*arctan
(-sqrt(-c*d^2 + b*d*e)*sqrt(c*x^2 + b*x)/((c*d - b*e)*x)) + (2*c*d - b*e)*s
qrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)))/(c*e^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x\,(b+cx)}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(x*(b + c*x))/(d + e*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.290 $\int \frac{\sqrt{bx+cx^2}}{(d+ex)^2} dx$

Optimal. Leaf size=140

$$-\frac{(2cd-be)\tanh^{-1}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{2\sqrt{d}e^2\sqrt{cd-be}}-\frac{\sqrt{bx+cx^2}}{e(d+ex)}+\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{e^2}$$

```
[Out] -(Sqrt[b*x + c*x^2]/(e*(d + e*x))) + (2*Sqrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[b*
x + c*x^2]])/e^2 - ((2*c*d - b*e)*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d
]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/(2*Sqrt[d]*e^2*Sqrt[c*d - b*e])
```

Rubi [A] time = 0.110819, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {732, 843, 620, 206, 724}

$$-\frac{(2cd-be)\tanh^{-1}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{2\sqrt{d}e^2\sqrt{cd-be}}-\frac{\sqrt{bx+cx^2}}{e(d+ex)}+\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x + c*x^2]/(d + e*x)^2,x]
```

```
[Out] -(Sqrt[b*x + c*x^2]/(e*(d + e*x))) + (2*Sqrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[b*
x + c*x^2]])/e^2 - ((2*c*d - b*e)*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d
]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/(2*Sqrt[d]*e^2*Sqrt[c*d - b*e])
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx+cx^2}}{(d+ex)^2}\,dx &= -\frac{\sqrt{bx+cx^2}}{e(d+ex)}+\frac{\int \frac{b+2cx}{(d+ex)\sqrt{bx+cx^2}}\,dx}{2e}\\ &= -\frac{\sqrt{bx+cx^2}}{e(d+ex)}+\frac{c\int \frac{1}{\sqrt{bx+cx^2}}\,dx}{e^2}-\frac{(2cd-be)\int \frac{1}{(d+ex)\sqrt{bx+cx^2}}\,dx}{2e^2}\\ &= -\frac{\sqrt{bx+cx^2}}{e(d+ex)}+\frac{(2c)\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{bx+cx^2}}\right)}{e^2}+\frac{(2cd-be)\operatorname{Subst}\left(\int \frac{1}{4cd^2-4bde-x^2}\,dx,x,\frac{-bd-(2cd}{\sqrt{bx+c}}\right.}{e^2}\\ &= -\frac{\sqrt{bx+cx^2}}{e(d+ex)}+\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{e^2}-\frac{(2cd-be)\tanh^{-1}\left(\frac{bd+(2cd-be)x}{2\sqrt{d}\sqrt{cd-be}\sqrt{bx+cx^2}}\right)}{2\sqrt{d}e^2\sqrt{cd-be}}\end{aligned}$$

Mathematica [A] time = 0.601086, size = 147, normalized size = 1.05

$$\frac{\sqrt{x(b+cx)}\left(-\frac{(2cd-be)\tan^{-1}\left(\frac{\sqrt{x}\sqrt{be-cd}}{\sqrt{d}\sqrt{b+cx}}\right)}{\sqrt{d}\sqrt{b+cx}\sqrt{be-cd}}+\frac{2\sqrt{c}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{\frac{cx}{b}+1}}-\frac{e\sqrt{x}}{d+ex}\right)}{e^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x + c*x^2]/(d + e*x)^2,x]
```

```
[Out] (Sqrt[x*(b + c*x)]*(-((e*Sqrt[x])/(d + e*x)) + (2*Sqrt[c]*ArcSinh[(Sqrt[c]*
Sqrt[x])/Sqrt[b]])/(Sqrt[b]*Sqrt[1 + (c*x)/b]) - ((2*c*d - b*e)*ArcTan[(Sqr
t[-(c*d) + b*e]*Sqrt[x])/(Sqrt[d]*Sqrt[b + c*x])])/(Sqrt[d]*Sqrt[-(c*d) + b
*e]*Sqrt[b + c*x])))/(e^2*Sqrt[x])
```

Maple [B] time = 0.213, size = 885, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(1/2)/(e*x+d)^2,x)
```

```
[Out] 1/d/(b*e-c*d)/(d/e+x)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(
3/2)-1/d/(b*e-c*d)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2
)*b+1/e/(b*e-c*d)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)
*c+1/e/(b*e-c*d)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e
-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))*c^(1/2)*b-1/e^2*d/(b*e-c*d)*ln((1
/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b
*e-c*d)/e^2)^(1/2))*c^(3/2)-1/2/e/(b*e-c*d)/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2
*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x
```

```
)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*b^2+3/2/e^2*d/(b
*e-c*d)/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+
x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d
)/e^2)^(1/2))/(d/e+x))*b*c-1/e^3*d^2/(b*e-c*d)/(-d*(b*e-c*d)/e^2)^(1/2)*ln(
(-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/
e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*c^2-c/d/(b*e-
c*d)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.59493, size = 1775, normalized size = 12.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(c*d^3 - b*d^2*e + (c*d^2*e - b*d*e^2)*x)*sqrt(c)*log(2*c*x + b + 2
*sqrt(c*x^2 + b*x)*sqrt(c)) - (2*c*d^2 - b*d*e + (2*c*d*e - b*e^2)*x)*sqrt(
c*d^2 - b*d*e)*log((b*d + (2*c*d - b*e)*x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^
2 + b*x))/(e*x + d)) - 2*(c*d^2*e - b*d*e^2)*sqrt(c*x^2 + b*x))/(c*d^3*e^2
- b*d^2*e^3 + (c*d^2*e^3 - b*d*e^4)*x), -((2*c*d^2 - b*d*e + (2*c*d*e - b*e
^2)*x)*sqrt(-c*d^2 + b*d*e)*arctan(-sqrt(-c*d^2 + b*d*e)*sqrt(c*x^2 + b*x)/
((c*d - b*e)*x)) - (c*d^3 - b*d^2*e + (c*d^2*e - b*d*e^2)*x)*sqrt(c)*log(2*
c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) + (c*d^2*e - b*d*e^2)*sqrt(c*x^2 + b
*x))/(c*d^3*e^2 - b*d^2*e^3 + (c*d^2*e^3 - b*d*e^4)*x), -1/2*(4*(c*d^3 - b*
d^2*e + (c*d^2*e - b*d*e^2)*x)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(
c*x)) + (2*c*d^2 - b*d*e + (2*c*d*e - b*e^2)*x)*sqrt(c*d^2 - b*d*e)*log((b*
d + (2*c*d - b*e)*x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*x + d)) +
 2*(c*d^2*e - b*d*e^2)*sqrt(c*x^2 + b*x))/(c*d^3*e^2 - b*d^2*e^3 + (c*d^2*e
^3 - b*d*e^4)*x), -((2*c*d^2 - b*d*e + (2*c*d*e - b*e^2)*x)*sqrt(-c*d^2 + b
*d*e)*arctan(-sqrt(-c*d^2 + b*d*e)*sqrt(c*x^2 + b*x)/((c*d - b*e)*x)) + 2*(
c*d^3 - b*d^2*e + (c*d^2*e - b*d*e^2)*x)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*
sqrt(-c)/(c*x)) + (c*d^2*e - b*d*e^2)*sqrt(c*x^2 + b*x))/(c*d^3*e^2 - b*d^2
*e^3 + (c*d^2*e^3 - b*d*e^4)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x\left(b + cx\right)}}{\left(d + ex\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(1/2)/(e*x+d)**2,x)
```

```
[Out] Integral(sqrt(x*(b + c*x))/(d + e*x)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.291 $\int \frac{\sqrt{bx+cx^2}}{(d+ex)^3}\,dx$

Optimal. Leaf size=127

$$\frac{\sqrt{bx+cx^2}(x(2cd-be)+bd)}{4d(d+ex)^2(cd-be)}-\frac{b^2\tanh^{-1}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{8d^{3/2}(cd-be)^{3/2}}$$

```
[Out] ((b*d + (2*c*d - b*e)*x)*Sqrt[b*x + c*x^2])/(4*d*(c*d - b*e)*(d + e*x)^2) -
 (b^2*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x +
 c*x^2])])/(8*d^(3/2)*(c*d - b*e)^(3/2))
```

Rubi [A] time = 0.0876072, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {720, 724, 206}

$$\frac{\sqrt{bx+cx^2}(x(2cd-be)+bd)}{4d(d+ex)^2(cd-be)}-\frac{b^2\tanh^{-1}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{8d^{3/2}(cd-be)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x + c*x^2]/(d + e*x)^3,x]
```

```
[Out] ((b*d + (2*c*d - b*e)*x)*Sqrt[b*x + c*x^2])/(4*d*(c*d - b*e)*(d + e*x)^2) -
 (b^2*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x +
 c*x^2])])/(8*d^(3/2)*(c*d - b*e)^(3/2))
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^3}\,dx = \frac{(bd+(2cd-be)x)\sqrt{bx+cx^2}}{4d(cd-be)(d+ex)^2} - \frac{b^2\int \frac{1}{(d+ex)\sqrt{bx+cx^2}}\,dx}{8d(cd-be)}$$
$$= \frac{(bd+(2cd-be)x)\sqrt{bx+cx^2}}{4d(cd-be)(d+ex)^2} + \frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{4cd^2-4bde-x^2}\,dx, x, \frac{-bd-(2cd-be)x}{\sqrt{bx+cx^2}}\right)}{4d(cd-be)}$$
$$= \frac{(bd+(2cd-be)x)\sqrt{bx+cx^2}}{4d(cd-be)(d+ex)^2} - \frac{b^2\tanh^{-1}\left(\frac{bd+(2cd-be)x}{2\sqrt{d}\sqrt{cd-be}\sqrt{bx+cx^2}}\right)}{8d^{3/2}(cd-be)^{3/2}}$$

Mathematica [A] time = 0.216279, size = 121, normalized size = 0.95

$$\frac{\sqrt{x(b+cx)}\left(\frac{b^2\tan^{-1}\left(\frac{\sqrt{x}\sqrt{be-cd}}{\sqrt{d}\sqrt{b+cx}}\right)}{\sqrt{x}\sqrt{b+cx}(be-cd)^{3/2}} + \frac{\sqrt{d}(b(d-ex)+2cdx)}{(d+ex)^2(cd-be)}\right)}{4d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x + c*x^2]/(d + e*x)^3,x]
```

```
[Out] (Sqrt[x*(b + c*x)]*((Sqrt[d]*(2*c*d*x + b*(d - e*x)))/((c*d - b*e)*(d + e*x
)^2) + (b^2*ArcTan[(Sqrt[-(c*d) + b*e]*Sqrt[x])/(Sqrt[d]*Sqrt[b + c*x])])/(
(-(c*d) + b*e)^(3/2)*Sqrt[x]*Sqrt[b + c*x])))/(4*d^(3/2))
```

Maple [B] time = 0.23, size = 1963, normalized size = 15.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(1/2)/(e*x+d)^3,x)
```

```
[Out] 1/2/e/d/(b*e-c*d)/(d/e+x)^2*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/
e^2)^(3/2)+1/4*e/d^2/(b*e-c*d)^2/(d/e+x)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)
-d*(b*e-c*d)/e^2)^(3/2)*b-1/2/d/(b*e-c*d)^2/(d/e+x)*(c*(d/e+x)^2+(b*e-2*c*d
)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)*c-1/4*e/d^2/(b*e-c*d)^2*(c*(d/e+x)^2+(b*
e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*b^2+3/4/d/(b*e-c*d)^2*(c*(d/e+x)^
2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*b*c-1/2/e/(b*e-c*d)^2*(c*(d/
e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*c^2+1/4/d/(b*e-c*d)^2*l
n((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-
d*(b*e-c*d)/e^2)^(1/2))*c^(1/2)*b^2-3/4/e/(b*e-c*d)^2*ln((1/2*(b*e-2*c*d)/e
+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/
2))*c^(3/2)*b+1/2/e^2*d/(b*e-c*d)^2*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2
)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))*c^(5/2)-1/8/d/
(b*e-c*d)^2/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(
d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e
-c*d)/e^2)^(1/2))/(d/e+x))*b^3+5/8/e/(b*e-c*d)^2/(-d*(b*e-c*d)/e^2)^(1/2)*l
n((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(
d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*b^2*c-1/e^2
*d/(b*e-c*d)^2/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/
e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(
b*e-c*d)/e^2)^(1/2))/(d/e+x))*b*c^2+1/2/e^3*d^2/(b*e-c*d)^2/(-d*(b*e-c*d)/e
^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)
```

```
^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*
c^3-1/4*e/d^2/(b*e-c*d)^2*c*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/
e^2)^(1/2)*x*b+1/2/d/(b*e-c*d)^2*c^2*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(
b*e-c*d)/e^2)^(1/2)*x-1/2/e*c/d/(b*e-c*d)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x
)-d*(b*e-c*d)/e^2)^(1/2)-1/4/e*c^(1/2)/d/(b*e-c*d)*ln((1/2*(b*e-2*c*d)/e+(d
/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))
*b+1/2/e^2*c^(3/2)/(b*e-c*d)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d
/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))-1/2/e^2*c/(b*e-c*d)/(
-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*
(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1
/2))/(d/e+x))*b+1/2/e^3*c^2*d/(b*e-c*d)/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(
b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+
(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.27907, size = 977, normalized size = 7.69

$$\left[-\frac{\left(b^2e^2x^2+2\,b^2dex+b^2d^2\right)\sqrt{cd^2-bde}\log\left(\frac{bd+(2\,cd-be)x+2\,\sqrt{cd^2-bde}\sqrt{cx^2+bx}}{ex+d}\right)-2\left(bcd^3-b^2d^2e+\left(2\,c^2d^3-3\,bcd^2e+b^2de^2\right)x\right.}{8\left(c^2d^6-2\,bcd^5e+b^2d^4e^2+\left(c^2d^4e^2-2\,bcd^3e^3+b^2d^2e^4\right)x^2+2\left(c^2d^5e-2\,bcd^4e^2+b^2d^3e^3\right)x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*((b^2*e^2*x^2 + 2*b^2*d*e*x + b^2*d^2)*sqrt(c*d^2 - b*d*e)*log((b*d +
 (2*c*d - b*e)*x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*x + d)) - 2*
(b*c*d^3 - b^2*d^2*e + (2*c^2*d^3 - 3*b*c*d^2*e + b^2*d*e^2)*x)*sqrt(c*x^2
+ b*x))/(c^2*d^6 - 2*b*c*d^5*e + b^2*d^4*e^2 + (c^2*d^4*e^2 - 2*b*c*d^3*e^3
 + b^2*d^2*e^4)*x^2 + 2*(c^2*d^5*e - 2*b*c*d^4*e^2 + b^2*d^3*e^3)*x), -1/4*
((b^2*e^2*x^2 + 2*b^2*d*e*x + b^2*d^2)*sqrt(-c*d^2 + b*d*e)*arctan(-sqrt(-c
*d^2 + b*d*e)*sqrt(c*x^2 + b*x)/((c*d - b*e)*x)) - (b*c*d^3 - b^2*d^2*e + (
2*c^2*d^3 - 3*b*c*d^2*e + b^2*d*e^2)*x)*sqrt(c*x^2 + b*x))/(c^2*d^6 - 2*b*c
*d^5*e + b^2*d^4*e^2 + (c^2*d^4*e^2 - 2*b*c*d^3*e^3 + b^2*d^2*e^4)*x^2 + 2*
(c^2*d^5*e - 2*b*c*d^4*e^2 + b^2*d^3*e^3)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x\,(b+cx)}}{(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(1/2)/(e*x+d)**3,x)
```

```
[Out] Integral(sqrt(x*(b + c*x))/(d + e*x)**3, x)
```

Giac [B] time = 1.40329, size = 552, normalized size = 4.35

$$-\frac{b^2\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde}}\right)}{4\left(cd^2-bde\right)\sqrt{-cd^2+bde}}+\frac{8\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3c^2d^2e+8\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^2c^{\frac{5}{2}}d^3+8\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/4*b^2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x))*e + sqrt(c)*d)/sqrt(-c*d^
2 + b*d*e))/((c*d^2 - b*d*e)*sqrt(-c*d^2 + b*d*e)) + 1/4*(8*(sqrt(c)*x - sq
rt(c*x^2 + b*x))^3*c^2*d^2*e + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*c^(5/2)*
d^3 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b*c^2*d^3 - 8*(sqrt(c)*x - sqrt(c*x
^2 + b*x))^3*b*c*d*e^2 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b^2*c*d^2*e + 2*
b^2*c^(3/2)*d^3 - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*b^2*sqrt(c)*d*e^2 - b
^3*sqrt(c)*d^2*e + (sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b^2*e^3 - (sqrt(c)*x -
 sqrt(c*x^2 + b*x))*b^3*d*e^2)/((c*d^2*e^2 - b*d*e^3)*((sqrt(c)*x - sqrt(c*
x^2 + b*x))^2*e + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c)*d + b*d)^2)
```

3.292 $\int \frac{\sqrt{bx+cx^2}}{(d+ex)^4} dx$

Optimal. Leaf size=183

$$-\frac{b^2(2cd-be)\tanh^{-1}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{16d^{5/2}(cd-be)^{5/2}}+\frac{\sqrt{bx+cx^2}(2cd-be)(x(2cd-be)+bd)}{8d^2(d+ex)^2(cd-be)^2}-\frac{e\left(bx+cx^2\right)^{3/2}}{3d(d+ex)^3(cd-be)}$$

```
[Out] ((2*c*d - b*e)*(b*d + (2*c*d - b*e)*x)*Sqrt[b*x + c*x^2])/(8*d^2*(c*d - b*e
)^2*(d + e*x)^2) - (e*(b*x + c*x^2)^(3/2))/(3*d*(c*d - b*e)*(d + e*x)^3) -
(b^2*(2*c*d - b*e)*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*
e]*Sqrt[b*x + c*x^2])])/(16*d^(5/2)*(c*d - b*e)^(5/2))
```

Rubi [A] time = 0.138381, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {730, 720, 724, 206}

$$-\frac{b^2(2cd-be)\tanh^{-1}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{16d^{5/2}(cd-be)^{5/2}}+\frac{\sqrt{bx+cx^2}(2cd-be)(x(2cd-be)+bd)}{8d^2(d+ex)^2(cd-be)^2}-\frac{e\left(bx+cx^2\right)^{3/2}}{3d(d+ex)^3(cd-be)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x + c*x^2]/(d + e*x)^4,x]
```

```
[Out] ((2*c*d - b*e)*(b*d + (2*c*d - b*e)*x)*Sqrt[b*x + c*x^2])/(8*d^2*(c*d - b*e
)^2*(d + e*x)^2) - (e*(b*x + c*x^2)^(3/2))/(3*d*(c*d - b*e)*(d + e*x)^3) -
(b^2*(2*c*d - b*e)*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*
e]*Sqrt[b*x + c*x^2])])/(16*d^(5/2)*(c*d - b*e)^(5/2))
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^4}\,dx = -\frac{e\left(bx+cx^2\right)^{3/2}}{3d(cd-be)(d+ex)^3}+\frac{(2cd-be)\int \frac{\sqrt{bx+cx^2}}{(d+ex)^3}\,dx}{2d(cd-be)}$$

$$= \frac{(2cd-be)(bd+(2cd-be)x)\sqrt{bx+cx^2}}{8d^2(cd-be)^2(d+ex)^2}-\frac{e\left(bx+cx^2\right)^{3/2}}{3d(cd-be)(d+ex)^3}-\frac{\left(b^2(2cd-be)\right)\int \frac{1}{(d+ex)\sqrt{bx+cx^2}}\,dx}{16d^2(cd-be)^2}$$

$$= \frac{(2cd-be)(bd+(2cd-be)x)\sqrt{bx+cx^2}}{8d^2(cd-be)^2(d+ex)^2}-\frac{e\left(bx+cx^2\right)^{3/2}}{3d(cd-be)(d+ex)^3}+\frac{\left(b^2(2cd-be)\right)\mathrm{Subst}\left(\int \frac{1}{4cd^2-4bde\cdots}\right.}{8d^2(cd-be)\cdots}$$

$$= \frac{(2cd-be)(bd+(2cd-be)x)\sqrt{bx+cx^2}}{8d^2(cd-be)^2(d+ex)^2}-\frac{e\left(bx+cx^2\right)^{3/2}}{3d(cd-be)(d+ex)^3}-\frac{b^2(2cd-be)\tanh^{-1}\left(\frac{bd+(2cd-be\cdots}{2\sqrt{d}\sqrt{cd-be}\sqrt{b\cdots}}\right.}{16d^{5/2}(cd-be)^{5/2}}$$

Mathematica [A] time = 0.293452, size = 191, normalized size = 1.04

$$\frac{\sqrt{x(b+cx)}\left(\frac{3(2cd-be)\left(\sqrt{d}\sqrt{x}\sqrt{b+cx}\sqrt{be-cd}(b(d-ex)+2cdx)-b^2(d+ex)^2\tan^{-1}\left(\frac{\sqrt{x}\sqrt{be-cd}}{\sqrt{d}\sqrt{b+cx}}\right)\right)}{8d^{3/2}\sqrt{b+cx}(d+ex)^2(be-cd)^{3/2}}+\frac{ex^{3/2}(b+cx)}{(d+ex)^3}\right)}{3d\sqrt{x}(be-cd)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x + c*x^2]/(d + e*x)^4,x]
```

```
[Out] (Sqrt[x*(b + c*x)]*((e*x^(3/2)*(b + c*x))/(d + e*x)^3 + (3*(2*c*d - b*e)*(S
qrt[d]*Sqrt[-(c*d) + b*e]*Sqrt[x]*Sqrt[b + c*x]*(2*c*d*x + b*(d - e*x)) - b
^2*(d + e*x)^2*ArcTan[(Sqrt[-(c*d) + b*e]*Sqrt[x])/(Sqrt[d]*Sqrt[b + c*x])]
))/(8*d^(3/2)*(-(c*d) + b*e)^(3/2)*Sqrt[b + c*x]*(d + e*x)^2)))/(3*d*(-(c*d
) + b*e)*Sqrt[x])
```

Maple [B] time = 0.214, size = 2891, normalized size = 15.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(1/2)/(e*x+d)^4,x)
```

```
[Out] 5/4/e^2*d/(b*e-c*d)^3/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-
2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e
+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*b*c^3-1/2*e/d^2/(b*e-c*d)^3/(d/e+x)*(c
*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)*b*c-1/8*e^2/d^3/(b*
e-c*d)^3*c*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*x*b^2+
1/2*e/d^2/(b*e-c*d)^3*c^2*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^
2)^(1/2)*x*b-1/4/e/d/(b*e-c*d)^2*c/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c
*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-
```

```
2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*b^2-1/2/d/(b*e-c*d)^3*c^3
*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*x-5/8/d/(b*e-c*d
)^3*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/
e+x)-d*(b*e-c*d)/e^2)^(1/2))*c^(3/2)*b^2-1/4/d^2/(b*e-c*d)^2*c*(c*(d/e+x)^2
+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*b-1/8*e^2/d^3/(b*e-c*d)^3*(c*
(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*b^3+1/2/e/d/(b*e-c*d
)^2*c^2*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)-1/2/e^2*d
/(b*e-c*d)^3*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c
*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))*c^(7/2)+1/3/e^2/d/(b*e-c*d)/(d/e+x)^3
*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)-1/8/d^2/(b*e-c*d
)^2*c^(1/2)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*
d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))*b^2+1/4/d^2/(b*e-c*d)^2/(d/e+x)^2*(c*(
d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)*b+1/2/d/(b*e-c*d)^3/(
d/e+x)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)*c^2-1/d/(b
*e-c*d)^3*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*b*c^2+1
/e/(b*e-c*d)^3*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2
*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))*c^(5/2)*b+1/2/e/(b*e-c*d)^3*(c*(d/e
+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*c^3-1/2/e^2/(b*e-c*d)^2*
c^(5/2)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e
*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))-9/8/e/(b*e-c*d)^3/(-d*(b*e-c*d)/e^2)^(1/2)
*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c
*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*b^2*c^2+3
/4/e^2/(b*e-c*d)^2*c^2/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e
-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/
e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*b-1/2/e/d/(b*e-c*d)^2/(d/e+x)^2*(c*(d
/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)*c+1/8*e^2/d^3/(b*e-c*d
)^3/(d/e+x)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)*b^2+5
/8*e/d^2/(b*e-c*d)^3*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1
/2)*b^2*c+1/8*e/d^2/(b*e-c*d)^3*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c
*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))*c^(1/2)*b^3-1/16*e
/d^2/(b*e-c*d)^3/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d
)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d
*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*b^4+1/2/e/d/(b*e-c*d)^2*c^(3/2)*ln((1/2*(b*
e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d
)/e^2)^(1/2))*b+7/16/d/(b*e-c*d)^3/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c
*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-
2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*b^3*c-1/2/e^3*d/(b*e-c*d)
^2*c^3/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x
)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)
/e^2)^(1/2))/(d/e+x))-1/2/e^3*d^2/(b*e-c*d)^3/(-d*(b*e-c*d)/e^2)^(1/2)*ln((
-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e
+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*c^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.41711, size = 1925, normalized size = 10.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] [-1/48*(3*(2*b^2*c*d^4 - b^3*d^3*e + (2*b^2*c*d*e^3 - b^3*e^4)*x^3 + 3*(2*b
^2*c*d^2*e^2 - b^3*d*e^3)*x^2 + 3*(2*b^2*c*d^3*e - b^3*d^2*e^2)*x)*sqrt(c*d
^2 - b*d*e)*log((b*d + (2*c*d - b*e)*x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 +
 b*x))/(e*x + d)) - 2*(6*b*c^2*d^5 - 9*b^2*c*d^4*e + 3*b^3*d^3*e^2 + (4*c^3
*d^4*e - 8*b*c^2*d^3*e^2 + 7*b^2*c*d^2*e^3 - 3*b^3*d*e^4)*x^2 + 2*(6*c^3*d^
5 - 13*b*c^2*d^4*e + 11*b^2*c*d^3*e^2 - 4*b^3*d^2*e^3)*x)*sqrt(c*x^2 + b*x)
)/(c^3*d^9 - 3*b*c^2*d^8*e + 3*b^2*c*d^7*e^2 - b^3*d^6*e^3 + (c^3*d^6*e^3 -
 3*b*c^2*d^5*e^4 + 3*b^2*c*d^4*e^5 - b^3*d^3*e^6)*x^3 + 3*(c^3*d^7*e^2 - 3*
b*c^2*d^6*e^3 + 3*b^2*c*d^5*e^4 - b^3*d^4*e^5)*x^2 + 3*(c^3*d^8*e - 3*b*c^2
*d^7*e^2 + 3*b^2*c*d^6*e^3 - b^3*d^5*e^4)*x), -1/24*(3*(2*b^2*c*d^4 - b^3*d
^3*e + (2*b^2*c*d*e^3 - b^3*e^4)*x^3 + 3*(2*b^2*c*d^2*e^2 - b^3*d*e^3)*x^2
+ 3*(2*b^2*c*d^3*e - b^3*d^2*e^2)*x)*sqrt(-c*d^2 + b*d*e)*arctan(-sqrt(-c*d
^2 + b*d*e)*sqrt(c*x^2 + b*x)/((c*d - b*e)*x)) - (6*b*c^2*d^5 - 9*b^2*c*d^4
*e + 3*b^3*d^3*e^2 + (4*c^3*d^4*e - 8*b*c^2*d^3*e^2 + 7*b^2*c*d^2*e^3 - 3*b
^3*d*e^4)*x^2 + 2*(6*c^3*d^5 - 13*b*c^2*d^4*e + 11*b^2*c*d^3*e^2 - 4*b^3*d^
2*e^3)*x)*sqrt(c*x^2 + b*x))/(c^3*d^9 - 3*b*c^2*d^8*e + 3*b^2*c*d^7*e^2 - b
^3*d^6*e^3 + (c^3*d^6*e^3 - 3*b*c^2*d^5*e^4 + 3*b^2*c*d^4*e^5 - b^3*d^3*e^6
)*x^3 + 3*(c^3*d^7*e^2 - 3*b*c^2*d^6*e^3 + 3*b^2*c*d^5*e^4 - b^3*d^4*e^5)*x
^2 + 3*(c^3*d^8*e - 3*b*c^2*d^7*e^2 + 3*b^2*c*d^6*e^3 - b^3*d^5*e^4)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x\,(b+cx)}}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(sqrt(x*(b + c*x))/(d + e*x)**4, x)
```

Giac [B] time = 1.48826, size = 1114, normalized size = 6.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 1/8*(2*b^2*c*d - b^3*e)*arctan(((sqrt(c)*x - sqrt(c*x^2 + b*x))*e + sqrt(c)
*d)/sqrt(-c*d^2 + b*d*e))/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2)*sqrt(-c*d^
2 + b*d*e)) + 1/24*(48*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*c^(7/2)*d^4*e + 32
*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*c^4*d^5 + 16*(sqrt(c)*x - sqrt(c*x^2 + b
*x))^3*b*c^3*d^4*e + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*b*c^(7/2)*d^5 - 9
6*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*b*c^(5/2)*d^3*e^2 - 36*(sqrt(c)*x - sqr
t(c*x^2 + b*x))^2*b^2*c^(5/2)*d^4*e + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b^
2*c^3*d^5 - 84*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b^2*c^2*d^3*e^2 - 24*(sqrt
(c)*x - sqrt(c*x^2 + b*x))*b^3*c^2*d^4*e + 4*b^3*c^(5/2)*d^5 + 78*(sqrt(c)*
x - sqrt(c*x^2 + b*x))^4*b^2*c^(3/2)*d^2*e^3 - 6*(sqrt(c)*x - sqrt(c*x^2 +
b*x))^2*b^3*c^(3/2)*d^3*e^2 - 4*b^4*c^(3/2)*d^4*e + 6*(sqrt(c)*x - sqrt(c*x
```

```
^2 + b*x))^5*b^2*c*d*e^4 + 74*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b^3*c*d^2*e
^3 + 12*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b^4*c*d^3*e^2 - 15*(sqrt(c)*x - sqr
t(c*x^2 + b*x))^4*b^3*sqrt(c)*d*e^4 + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*
b^4*sqrt(c)*d^2*e^3 + 3*b^5*sqrt(c)*d^3*e^2 - 3*(sqrt(c)*x - sqrt(c*x^2 + b
*x))^5*b^3*e^5 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b^4*d*e^4 + 3*(sqrt(c)
*x - sqrt(c*x^2 + b*x))*b^5*d^2*e^3)/((c^2*d^4*e^2 - 2*b*c*d^3*e^3 + b^2*d^
2*e^4)*((sqrt(c)*x - sqrt(c*x^2 + b*x))^2*e + 2*(sqrt(c)*x - sqrt(c*x^2 + b
*x))*sqrt(c)*d + b*d)^3)
```

3.293 $\int \frac{\sqrt{bx+cx^2}}{(d+ex)^5} dx$

Optimal. Leaf size=258

$$\frac{\sqrt{bx+cx^2}\left(5b^2e^2-16bcde+16c^2d^2\right)(x(2cd-be)+bd)}{64d^3(d+ex)^2(cd-be)^3}-\frac{b^2\left(5b^2e^2-16bcde+16c^2d^2\right)\tanh^{-1}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{128d^{7/2}(cd-be)^{7/2}}-$$

```
[Out] ((16*c^2*d^2 - 16*b*c*d*e + 5*b^2*e^2)*(b*d + (2*c*d - b*e)*x)*Sqrt[b*x + c
*x^2])/(64*d^3*(c*d - b*e)^3*(d + e*x)^2) - (e*(b*x + c*x^2)^(3/2))/(4*d*(c
*d - b*e)*(d + e*x)^4) - (5*e*(2*c*d - b*e)*(b*x + c*x^2)^(3/2))/(24*d^2*(c
*d - b*e)^2*(d + e*x)^3) - (b^2*(16*c^2*d^2 - 16*b*c*d*e + 5*b^2*e^2)*ArcTa
nh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/
(128*d^(7/2)*(c*d - b*e)^(7/2))
```

Rubi [A] time = 0.319279, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {744, 806, 720, 724, 206}

$$\frac{\sqrt{bx+cx^2}\left(5b^2e^2-16bcde+16c^2d^2\right)(x(2cd-be)+bd)}{64d^3(d+ex)^2(cd-be)^3}-\frac{b^2\left(5b^2e^2-16bcde+16c^2d^2\right)\tanh^{-1}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{128d^{7/2}(cd-be)^{7/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x + c*x^2]/(d + e*x)^5,x]
```


```
[Out] ((16*c^2*d^2 - 16*b*c*d*e + 5*b^2*e^2)*(b*d + (2*c*d - b*e)*x)*Sqrt[b*x + c
*x^2])/(64*d^3*(c*d - b*e)^3*(d + e*x)^2) - (e*(b*x + c*x^2)^(3/2))/(4*d*(c
*d - b*e)*(d + e*x)^4) - (5*e*(2*c*d - b*e)*(b*x + c*x^2)^(3/2))/(24*d^2*(c
*d - b*e)^2*(d + e*x)^3) - (b^2*(16*c^2*d^2 - 16*b*c*d*e + 5*b^2*e^2)*ArcTa
nh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/
(128*d^(7/2)*(c*d - b*e)^(7/2))
```


Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^5}\,dx = -\frac{e\left(bx+cx^2\right)^{3/2}}{4d(cd-be)(d+ex)^4} - \frac{\int \frac{\left(\frac{1}{2}(-8cd+5be)+cex\right)\sqrt{bx+cx^2}}{(d+ex)^4}\,dx}{4d(cd-be)}$$

$$= -\frac{e\left(bx+cx^2\right)^{3/2}}{4d(cd-be)(d+ex)^4} - \frac{5e(2cd-be)\left(bx+cx^2\right)^{3/2}}{24d^2(cd-be)^2(d+ex)^3} + \frac{\left(16c^2d^2-16bcde+5b^2e^2\right)\int \frac{\sqrt{bx+cx^2}}{(d+ex)^3}\,dx}{16d^2(cd-be)^2}$$

$$= \frac{\left(16c^2d^2-16bcde+5b^2e^2\right)(bd+(2cd-be)x)\sqrt{bx+cx^2}}{64d^3(cd-be)^3(d+ex)^2} - \frac{e\left(bx+cx^2\right)^{3/2}}{4d(cd-be)(d+ex)^4} - \frac{5e(2cd-be)\left(bx+c\right.}{24d^2(cd-be)^2(d+}$$

$$= \frac{\left(16c^2d^2-16bcde+5b^2e^2\right)(bd+(2cd-be)x)\sqrt{bx+cx^2}}{64d^3(cd-be)^3(d+ex)^2} - \frac{e\left(bx+cx^2\right)^{3/2}}{4d(cd-be)(d+ex)^4} - \frac{5e(2cd-be)\left(bx+c\right.}{24d^2(cd-be)^2(d+}$$

$$= \frac{\left(16c^2d^2-16bcde+5b^2e^2\right)(bd+(2cd-be)x)\sqrt{bx+cx^2}}{64d^3(cd-be)^3(d+ex)^2} - \frac{e\left(bx+cx^2\right)^{3/2}}{4d(cd-be)(d+ex)^4} - \frac{5e(2cd-be)\left(bx+c\right.}{24d^2(cd-be)^2(d+}$$

Mathematica [A] time = 0.760626, size = 243, normalized size = 0.94

$$\frac{\sqrt{x(b+cx)}\left(\frac{3(d+ex)^2\left(5b^2e^2-16bcde+16c^2d^2\right)\left(b^2(d+ex)^2\tan^{-1}\left(\frac{\sqrt{x}\sqrt{be-cd}}{\sqrt{d}\sqrt{b+cx}}\right)+\sqrt{d}\sqrt{x}\sqrt{b+cx}\sqrt{be-cd}(-bd+bex-2cdx)\right)}{d^{5/2}\sqrt{b+cx}(be-cd)^{5/2}} + \frac{40ex^{3/2}(b+cx)(d+ex)(2cd-be)}{d(cd-be)} + 48e\right.}{192d\sqrt{x}(d+ex)^4(be-cd)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x + c*x^2]/(d + e*x)^5,x]
```

```
[Out] (Sqrt[x*(b + c*x)]*(48*e*x^(3/2)*(b + c*x) + (40*e*(2*c*d - b*e)*x^(3/2)*(b
 + c*x)*(d + e*x))/(d*(c*d - b*e)) + (3*(16*c^2*d^2 - 16*b*c*d*e + 5*b^2*e^
2)*(d + e*x)^2*(Sqrt[d]*Sqrt[-(c*d) + b*e]*Sqrt[x]*Sqrt[b + c*x]*(-(b*d) -
2*c*d*x + b*e*x) + b^2*(d + e*x)^2*ArcTan[(Sqrt[-(c*d) + b*e]*Sqrt[x])/(Sqr
t[d]*Sqrt[b + c*x])]))/(d^(5/2)*(-(c*d) + b*e)^(5/2)*Sqrt[b + c*x])))/(192*
d*(-(c*d) + b*e)*Sqrt[x]*(d + e*x)^4)
```

Maple [B] time = 0.222, size = 4819, normalized size = 18.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(1/2)/(e*x+d)^5,x)
```

```
[Out] 1/4/e^3/d/(b*e-c*d)/(d/e+x)^4*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d
)/e^2)^(3/2)-25/16/e/(b*e-c*d)^4*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(
c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))*c^(7/2)*b+1/8/e^3
*c^3/(b*e-c*d)^2/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d
)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d
*(b*e-c*d)/e^2)^(1/2))/(d/e+x))-5/64*e^3/d^4/(b*e-c*d)^4*(c*(d/e+x)^2+(b*e-
2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*b^4-3/4/e/d/(b*e-c*d)^3*c^3*(c*(d/e
+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)-1/8*c^2/d^2/(b*e-c*d)^3/
(d/e+x)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)+17/32/d^2
/(b*e-c*d)^3*c^(3/2)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+
(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))*b^2+45/32/d/(b*e-c*d)^4*ln((1
/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b
*e-c*d)/e^2)^(1/2))*c^(5/2)*b^2-5/8/d/(b*e-c*d)^4/(d/e+x)*(c*(d/e+x)^2+(b*e
-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)*c^3+1/8*c^3/d^2/(b*e-c*d)^3*(c*(d/
e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*x+25/16/d/(b*e-c*d)^4*(
c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*b*c^3+1/8/e^2*c^(5
/2)/d/(b*e-c*d)^2*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*
e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))+5/8/e^2*d/(b*e-c*d)^4*ln((1/2*(b
*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*
d)/e^2)^(1/2))*c^(9/2)+3/4/e^2/(b*e-c*d)^3*c^(7/2)*ln((1/2*(b*e-2*c*d)/e+(d
/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))
-5/8/e/(b*e-c*d)^4*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2
)*c^4+13/16/d^2/(b*e-c*d)^3*c^2*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c
*d)/e^2)^(1/2)*b-1/8/e*c^2/d^2/(b*e-c*d)^2*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+
x)-d*(b*e-c*d)/e^2)^(1/2)+5/8/d/(b*e-c*d)^4*c^4*(c*(d/e+x)^2+(b*e-2*c*d)/e*
(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*x+3/4/e^3*d/(b*e-c*d)^3*c^4/(-d*(b*e-c*d)/e^
2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^
(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))+4
5/128*e/d^2/(b*e-c*d)^4/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*
e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d
/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*b^4*c-15/16*e/d^2/(b*e-c*d)^4*c^3*(c
*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*x*b-15/8/e^2*d/(b*e
-c*d)^4/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+
x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d
)/e^2)^(1/2))/(d/e+x))*c^4*b+15/32*e^2/d^3/(b*e-c*d)^4*c^2*(c*(d/e+x)^2+(b*
e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*x*b^2-15/32*e^2/d^3/(b*e-c*d)^4/(
d/e+x)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)*b^2*c+15/1
6/e/d/(b*e-c*d)^3*c^2/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-
2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e
+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*b^2+15/16*e/d^2/(b*e-c*d)^4/(d/e+x)*(c
*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)*b*c^2-1/16*e*c^2/d^
3/(b*e-c*d)^3*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*x*b
-1/8/e^2*c^2/d/(b*e-c*d)^2/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+
(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e
*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*b+1/16*e*c/d^3/(b*e-c*d)^3/(d/e+x
)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)*b-5/64*e^3/d^4/
(b*e-c*d)^4*c*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*x*b
^3-5/64*e/d^3/(b*e-c*d)^3*c^(1/2)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+
(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))*b^3-5/4/d/(b*e-c
*d)^4/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)
```

```
+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/
e^2)^(1/2))/(d/e+x))*b^3*c^2-5/8/d^2/(b*e-c*d)^3/(d/e+x)^2*(c*(d/e+x)^2+(b*
e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)*b*c-3/16/d^2/(b*e-c*d)^3*c/(-d*(b
*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-
c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/
(d/e+x))*b^3-1/16/e*c^(3/2)/d^2/(b*e-c*d)^2*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c
)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))*b-45/3
2*e/d^2/(b*e-c*d)^4*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/
2)*b^2*c^2+1/8/e*c/d^2/(b*e-c*d)^2/(d/e+x)^2*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/
e+x)-d*(b*e-c*d)/e^2)^(3/2)-3/2/e^2/(b*e-c*d)^3*c^3/(-d*(b*e-c*d)/e^2)^(1/2
)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(
c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*b+35/16/
e/(b*e-c*d)^4/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e
*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b
*e-c*d)/e^2)^(1/2))/(d/e+x))*b^2*c^3+5/64*e^2/d^3/(b*e-c*d)^4*ln((1/2*(b*e-
2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/
e^2)^(1/2))*c^(1/2)*b^4-35/64*e/d^2/(b*e-c*d)^4*ln((1/2*(b*e-2*c*d)/e+(d/e+
x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))*c^
(3/2)*b^3+5/8/e^3*d^2/(b*e-c*d)^4/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*
d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2
*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*c^5-9/8/e/d/(b*e-c*d)^3*c^
(5/2)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(
d/e+x)-d*(b*e-c*d)/e^2)^(1/2))*b-5/128*e^2/d^3/(b*e-c*d)^4/(-d*(b*e-c*d)/e^
2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^
(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*b
^5-7/32*e/d^3/(b*e-c*d)^3*c*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/
e^2)^(1/2)*b^2+5/24/e/d^2/(b*e-c*d)^2/(d/e+x)^3*(c*(d/e+x)^2+(b*e-2*c*d)/e*
(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)*b-5/12/e^2/d/(b*e-c*d)^2/(d/e+x)^3*(c*(d/e+x
)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)*c+5/32*e/d^3/(b*e-c*d)^3/(
d/e+x)^2*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)*b^2+5/8/
e/d/(b*e-c*d)^3/(d/e+x)^2*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^
2)^(3/2)*c^2+5/64*e^3/d^4/(b*e-c*d)^4/(d/e+x)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d
/e+x)-d*(b*e-c*d)/e^2)^(3/2)*b^3+35/64*e^2/d^3/(b*e-c*d)^4*(c*(d/e+x)^2+(b*
e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*b^3*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/(e*x+d)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.55067, size = 3313, normalized size = 12.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/(e*x+d)^5,x, algorithm="fricas")
```

```
[Out] [-1/384*(3*(16*b^2*c^2*d^6 - 16*b^3*c*d^5*e + 5*b^4*d^4*e^2 + (16*b^2*c^2*d
^2*e^4 - 16*b^3*c*d*e^5 + 5*b^4*e^6)*x^4 + 4*(16*b^2*c^2*d^3*e^3 - 16*b^3*c
```

```
*d^2*e^4 + 5*b^4*d*e^5)*x^3 + 6*(16*b^2*c^2*d^4*e^2 - 16*b^3*c*d^3*e^3 + 5*
b^4*d^2*e^4)*x^2 + 4*(16*b^2*c^2*d^5*e - 16*b^3*c*d^4*e^2 + 5*b^4*d^3*e^3)*
x)*sqrt(c*d^2 - b*d*e)*log((b*d + (2*c*d - b*e)*x + 2*sqrt(c*d^2 - b*d*e)*s
qrt(c*x^2 + b*x))/(e*x + d)) - 2*(48*b*c^3*d^7 - 96*b^2*c^2*d^6*e + 63*b^3*
c*d^5*e^2 - 15*b^4*d^4*e^3 + (16*c^4*d^5*e^2 - 40*b*c^3*d^4*e^3 + 62*b^2*c^
2*d^3*e^4 - 53*b^3*c*d^2*e^5 + 15*b^4*d*e^6)*x^3 + (64*c^4*d^6*e - 168*b*c^
3*d^5*e^2 + 244*b^2*c^2*d^4*e^3 - 195*b^3*c*d^3*e^4 + 55*b^4*d^2*e^5)*x^2 +
 (96*c^4*d^7 - 272*b*c^3*d^6*e + 374*b^2*c^2*d^5*e^2 - 271*b^3*c*d^4*e^3 +
73*b^4*d^3*e^4)*x)*sqrt(c*x^2 + b*x))/(c^4*d^12 - 4*b*c^3*d^11*e + 6*b^2*c^
2*d^10*e^2 - 4*b^3*c*d^9*e^3 + b^4*d^8*e^4 + (c^4*d^8*e^4 - 4*b*c^3*d^7*e^5
 + 6*b^2*c^2*d^6*e^6 - 4*b^3*c*d^5*e^7 + b^4*d^4*e^8)*x^4 + 4*(c^4*d^9*e^3
- 4*b*c^3*d^8*e^4 + 6*b^2*c^2*d^7*e^5 - 4*b^3*c*d^6*e^6 + b^4*d^5*e^7)*x^3
+ 6*(c^4*d^10*e^2 - 4*b*c^3*d^9*e^3 + 6*b^2*c^2*d^8*e^4 - 4*b^3*c*d^7*e^5 +
 b^4*d^6*e^6)*x^2 + 4*(c^4*d^11*e - 4*b*c^3*d^10*e^2 + 6*b^2*c^2*d^9*e^3 -
4*b^3*c*d^8*e^4 + b^4*d^7*e^5)*x), -1/192*(3*(16*b^2*c^2*d^6 - 16*b^3*c*d^5
*e + 5*b^4*d^4*e^2 + (16*b^2*c^2*d^2*e^4 - 16*b^3*c*d*e^5 + 5*b^4*e^6)*x^4
+ 4*(16*b^2*c^2*d^3*e^3 - 16*b^3*c*d^2*e^4 + 5*b^4*d*e^5)*x^3 + 6*(16*b^2*c
^2*d^4*e^2 - 16*b^3*c*d^3*e^3 + 5*b^4*d^2*e^4)*x^2 + 4*(16*b^2*c^2*d^5*e -
16*b^3*c*d^4*e^2 + 5*b^4*d^3*e^3)*x)*sqrt(-c*d^2 + b*d*e)*arctan(-sqrt(-c*d
^2 + b*d*e)*sqrt(c*x^2 + b*x)/((c*d - b*e)*x)) - (48*b*c^3*d^7 - 96*b^2*c^2
*d^6*e + 63*b^3*c*d^5*e^2 - 15*b^4*d^4*e^3 + (16*c^4*d^5*e^2 - 40*b*c^3*d^4
*e^3 + 62*b^2*c^2*d^3*e^4 - 53*b^3*c*d^2*e^5 + 15*b^4*d*e^6)*x^3 + (64*c^4*
d^6*e - 168*b*c^3*d^5*e^2 + 244*b^2*c^2*d^4*e^3 - 195*b^3*c*d^3*e^4 + 55*b^
4*d^2*e^5)*x^2 + (96*c^4*d^7 - 272*b*c^3*d^6*e + 374*b^2*c^2*d^5*e^2 - 271*
b^3*c*d^4*e^3 + 73*b^4*d^3*e^4)*x)*sqrt(c*x^2 + b*x))/(c^4*d^12 - 4*b*c^3*d
^11*e + 6*b^2*c^2*d^10*e^2 - 4*b^3*c*d^9*e^3 + b^4*d^8*e^4 + (c^4*d^8*e^4 -
 4*b*c^3*d^7*e^5 + 6*b^2*c^2*d^6*e^6 - 4*b^3*c*d^5*e^7 + b^4*d^4*e^8)*x^4 +
 4*(c^4*d^9*e^3 - 4*b*c^3*d^8*e^4 + 6*b^2*c^2*d^7*e^5 - 4*b^3*c*d^6*e^6 + b
^4*d^5*e^7)*x^3 + 6*(c^4*d^10*e^2 - 4*b*c^3*d^9*e^3 + 6*b^2*c^2*d^8*e^4 - 4
*b^3*c*d^7*e^5 + b^4*d^6*e^6)*x^2 + 4*(c^4*d^11*e - 4*b*c^3*d^10*e^2 + 6*b^
2*c^2*d^9*e^3 - 4*b^3*c*d^8*e^4 + b^4*d^7*e^5)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(1/2)/(e*x+d)**5,x)

[Out] Timed out
```

Giac [B] time = 1.92823, size = 1661, normalized size = 6.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/(e*x+d)^5,x, algorithm="giac")

[Out] 1/384*(2*sqrt(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + b*e/(x*e + d) - b*d
*e/(x*e + d)^2)*(2*(4*((2*c^3*d^5*e^6*sgn(1/(x*e + d)) - 5*b*c^2*d^4*e^7*sg
n(1/(x*e + d)) + 4*b^2*c*d^3*e^8*sgn(1/(x*e + d)) - b^3*d^2*e^9*sgn(1/(x*e
+ d)))/(c^4*d^8*e^8 - 4*b*c^3*d^7*e^9 + 6*b^2*c^2*d^6*e^10 - 4*b^3*c*d^5*e^
```

```
11 + b^4*d^4*e^12) - 6*(c^3*d^6*e^7*sgn(1/(x*e + d)) - 3*b*c^2*d^5*e^8*sgn(
1/(x*e + d)) + 3*b^2*c*d^4*e^9*sgn(1/(x*e + d)) - b^3*d^3*e^10*sgn(1/(x*e +
 d)))*e^(-1)/((c^4*d^8*e^8 - 4*b*c^3*d^7*e^9 + 6*b^2*c^2*d^6*e^10 - 4*b^3*c
*d^5*e^11 + b^4*d^4*e^12)*(x*e + d)))*e^(-1)/(x*e + d) + (8*c^3*d^4*e^5*sgn
(1/(x*e + d)) - 16*b*c^2*d^3*e^6*sgn(1/(x*e + d)) + 13*b^2*c*d^2*e^7*sgn(1/
(x*e + d)) - 5*b^3*d*e^8*sgn(1/(x*e + d)))/(c^4*d^8*e^8 - 4*b*c^3*d^7*e^9 +
 6*b^2*c^2*d^6*e^10 - 4*b^3*c*d^5*e^11 + b^4*d^4*e^12))*e^(-1)/(x*e + d) +
(16*c^3*d^3*e^4*sgn(1/(x*e + d)) - 24*b*c^2*d^2*e^5*sgn(1/(x*e + d)) + 38*b
^2*c*d*e^6*sgn(1/(x*e + d)) - 15*b^3*e^7*sgn(1/(x*e + d)))/(c^4*d^8*e^8 - 4
*b*c^3*d^7*e^9 + 6*b^2*c^2*d^6*e^10 - 4*b^3*c*d^5*e^11 + b^4*d^4*e^12)) - (
48*b^2*c^2*d^2*e^2*log(abs(2*c*d - b*e - 2*sqrt(c*d^2 - b*d*e)*sqrt(c))) +
32*sqrt(c*d^2 - b*d*e)*c^(7/2)*d^3 - 48*sqrt(c*d^2 - b*d*e)*b*c^(5/2)*d^2*e
 - 48*b^3*c*d*e^3*log(abs(2*c*d - b*e - 2*sqrt(c*d^2 - b*d*e)*sqrt(c))) + 7
6*sqrt(c*d^2 - b*d*e)*b^2*c^(3/2)*d*e^2 + 15*b^4*e^4*log(abs(2*c*d - b*e -
2*sqrt(c*d^2 - b*d*e)*sqrt(c))) - 30*sqrt(c*d^2 - b*d*e)*b^3*sqrt(c)*e^3)*s
gn(1/(x*e + d))/(sqrt(c*d^2 - b*d*e)*c^4*d^8*e^4 - 4*sqrt(c*d^2 - b*d*e)*b*
c^3*d^7*e^5 + 6*sqrt(c*d^2 - b*d*e)*b^2*c^2*d^6*e^6 - 4*sqrt(c*d^2 - b*d*e)
*b^3*c*d^5*e^7 + sqrt(c*d^2 - b*d*e)*b^4*d^4*e^8) + 3*(16*b^2*c^2*d^2*sgn(1
/(x*e + d)) - 16*b^3*c*d*e*sgn(1/(x*e + d)) + 5*b^4*e^2*sgn(1/(x*e + d)))*l
og(abs(2*c*d - b*e - 2*sqrt(c*d^2 - b*d*e)*(sqrt(c - 2*c*d/(x*e + d) + c*d^
2/(x*e + d)^2 + b*e/(x*e + d) - b*d*e/(x*e + d)^2) + sqrt(c*d^2*e^2 - b*d*e
^3)*e^(-1)/(x*e + d))))/((c^4*d^8*e^2 - 4*b*c^3*d^7*e^3 + 6*b^2*c^2*d^6*e^4
 - 4*b^3*c*d^5*e^5 + b^4*d^4*e^6)*sqrt(c*d^2 - b*d*e)))*e^2
```

3.294 $\int \frac{\sqrt{bx+cx^2}}{(d+ex)^6} dx$

Optimal. Leaf size=337

$$-\frac{e\left(bx+cx^2\right)^{3/2}\left(35b^2e^2-108bcde+108c^2d^2\right)}{240d^3(d+ex)^3(cd-be)^3}+\frac{\sqrt{bx+cx^2}(2cd-be)\left(7b^2e^2-16bcde+16c^2d^2\right)(x(2cd-be)+bd)}{128d^4(d+ex)^2(cd-be)^4}-$$

```
[Out] ((2*c*d - b*e)*(16*c^2*d^2 - 16*b*c*d*e + 7*b^2*e^2)*(b*d + (2*c*d - b*e)*x
)*Sqrt[b*x + c*x^2])/(128*d^4*(c*d - b*e)^4*(d + e*x)^2) - (e*(b*x + c*x^2)
^(3/2))/(5*d*(c*d - b*e)*(d + e*x)^5) - (7*e*(2*c*d - b*e)*(b*x + c*x^2)^(3
/2))/(40*d^2*(c*d - b*e)^2*(d + e*x)^4) - (e*(108*c^2*d^2 - 108*b*c*d*e + 3
5*b^2*e^2)*(b*x + c*x^2)^(3/2))/(240*d^3*(c*d - b*e)^3*(d + e*x)^3) - (b^2*
(2*c*d - b*e)*(16*c^2*d^2 - 16*b*c*d*e + 7*b^2*e^2)*ArcTanh[(b*d + (2*c*d -
 b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/(256*d^(9/2)*(c*d
- b*e)^(9/2))
```

Rubi [A] time = 0.45576, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {744, 834, 806, 720, 724, 206}

$$-\frac{e\left(bx+cx^2\right)^{3/2}\left(35b^2e^2-108bcde+108c^2d^2\right)}{240d^3(d+ex)^3(cd-be)^3}+\frac{\sqrt{bx+cx^2}(2cd-be)\left(7b^2e^2-16bcde+16c^2d^2\right)(x(2cd-be)+bd)}{128d^4(d+ex)^2(cd-be)^4}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x + c*x^2]/(d + e*x)^6,x]
```


```
[Out] ((2*c*d - b*e)*(16*c^2*d^2 - 16*b*c*d*e + 7*b^2*e^2)*(b*d + (2*c*d - b*e)*x
)*Sqrt[b*x + c*x^2])/(128*d^4*(c*d - b*e)^4*(d + e*x)^2) - (e*(b*x + c*x^2)
^(3/2))/(5*d*(c*d - b*e)*(d + e*x)^5) - (7*e*(2*c*d - b*e)*(b*x + c*x^2)^(3
/2))/(40*d^2*(c*d - b*e)^2*(d + e*x)^4) - (e*(108*c^2*d^2 - 108*b*c*d*e + 3
5*b^2*e^2)*(b*x + c*x^2)^(3/2))/(240*d^3*(c*d - b*e)^3*(d + e*x)^3) - (b^2*
(2*c*d - b*e)*(16*c^2*d^2 - 16*b*c*d*e + 7*b^2*e^2)*ArcTanh[(b*d + (2*c*d -
 b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/(256*d^(9/2)*(c*d
- b*e)^(9/2))
```


Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
```

```
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{bx+cx^2}}{(d+ex)^6}\,dx &= -\frac{e\left(bx+cx^2\right)^{3/2}}{5d(cd-be)(d+ex)^5} - \frac{\int \frac{\left(\frac{1}{2}(-10cd+7be)+2cex\right)\sqrt{bx+cx^2}}{(d+ex)^5}\,dx}{5d(cd-be)}\\
&= -\frac{e\left(bx+cx^2\right)^{3/2}}{5d(cd-be)(d+ex)^5} - \frac{7e(2cd-be)\left(bx+cx^2\right)^{3/2}}{40d^2(cd-be)^2(d+ex)^4} + \frac{\int \frac{\left(\frac{1}{4}\left(80c^2d^2-94bcde+35b^2e^2\right)-\frac{7}{2}ce(2cd-be)x\right)\sqrt{bx+cx^2}}{(d+ex)^4}\,dx}{20d^2(cd-be)^2}\\
&= -\frac{e\left(bx+cx^2\right)^{3/2}}{5d(cd-be)(d+ex)^5} - \frac{7e(2cd-be)\left(bx+cx^2\right)^{3/2}}{40d^2(cd-be)^2(d+ex)^4} - \frac{e\left(108c^2d^2-108bcde+35b^2e^2\right)\left(bx+cx^2\right)^{3/2}}{240d^3(cd-be)^3(d+ex)^3} +\\
&= \frac{(2cd-be)\left(16c^2d^2-16bcde+7b^2e^2\right)(bd+(2cd-be)x)\sqrt{bx+cx^2}}{128d^4(cd-be)^4(d+ex)^2} - \frac{e\left(bx+cx^2\right)^{3/2}}{5d(cd-be)(d+ex)^5} - \frac{7e(2cd-}{40d^2(cd}\\
&= \frac{(2cd-be)\left(16c^2d^2-16bcde+7b^2e^2\right)(bd+(2cd-be)x)\sqrt{bx+cx^2}}{128d^4(cd-be)^4(d+ex)^2} - \frac{e\left(bx+cx^2\right)^{3/2}}{5d(cd-be)(d+ex)^5} - \frac{7e(2cd-}{40d^2(cd}\\
&= \frac{(2cd-be)\left(16c^2d^2-16bcde+7b^2e^2\right)(bd+(2cd-be)x)\sqrt{bx+cx^2}}{128d^4(cd-be)^4(d+ex)^2} - \frac{e\left(bx+cx^2\right)^{3/2}}{5d(cd-be)(d+ex)^5} - \frac{7e(2cd-}{40d^2(cd}
\end{aligned}
$$

Mathematica [A] time = 1.27726, size = 309, normalized size = 0.92

$$\frac{\sqrt{x(b+cx)}\left(\frac{(d+ex)^2\left(8ex^{3/2}(b+cx)\left(35b^2e^2-108bcde+108c^2d^2\right)+\frac{15(d+ex)(2cd-be)\left(7b^2e^2-16bcde+16c^2d^2\right)\left(\sqrt{d}\sqrt{x}\sqrt{b+cx}\sqrt{be-cd}(b(d-ex)+2cdx)-b^2(d+ex)^2\tan^{-1}\left(\frac{\sqrt{x}\sqrt{be-}}{\sqrt{d}\sqrt{b+}}\right.\right.}{d^{3/2}\sqrt{b+cx}(be-cd)^{3/2}}\right.}{d^2(cd-be)^2}\right.}{1920d\sqrt{x}(d+ex)^5(be-cd)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x + c*x^2]/(d + e*x)^6,x]
```

```
[Out] (Sqrt[x*(b + c*x)]*(384*e*x^(3/2)*(b + c*x) + (336*e*(2*c*d - b*e)*x^(3/2)*
(b + c*x)*(d + e*x))/(d*(c*d - b*e)) + ((d + e*x)^2*(8*e*(108*c^2*d^2 - 108
*b*c*d*e + 35*b^2*e^2)*x^(3/2)*(b + c*x) + (15*(2*c*d - b*e)*(16*c^2*d^2 -
16*b*c*d*e + 7*b^2*e^2)*(d + e*x)*(Sqrt[d]*Sqrt[-(c*d) + b*e]*Sqrt[x]*Sqrt[
b + c*x]*(2*c*d*x + b*(d - e*x)) - b^2*(d + e*x)^2*ArcTan[(Sqrt[-(c*d) + b*
e]*Sqrt[x])/(Sqrt[d]*Sqrt[b + c*x])]))/(d^(3/2)*(-(c*d) + b*e)^(3/2)*Sqrt[b
 + c*x])))/(d^2*(c*d - b*e)^2)))/(1920*d*(-(c*d) + b*e)*Sqrt[x]*(d + e*x)^5
)
```

Maple [B] time = 0.241, size = 6533, normalized size = 19.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(1/2)/(e*x+d)^6,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/(e*x+d)^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.78922, size = 5138, normalized size = 15.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/(e*x+d)^6,x, algorithm="fricas")
```

```
[Out] [-1/3840*(15*(32*b^2*c^3*d^8 - 48*b^3*c^2*d^7*e + 30*b^4*c*d^6*e^2 - 7*b^5*
d^5*e^3 + (32*b^2*c^3*d^3*e^5 - 48*b^3*c^2*d^2*e^6 + 30*b^4*c*d*e^7 - 7*b^5
*e^8)*x^5 + 5*(32*b^2*c^3*d^4*e^4 - 48*b^3*c^2*d^3*e^5 + 30*b^4*c*d^2*e^6 -
 7*b^5*d*e^7)*x^4 + 10*(32*b^2*c^3*d^5*e^3 - 48*b^3*c^2*d^4*e^4 + 30*b^4*c*
d^3*e^5 - 7*b^5*d^2*e^6)*x^3 + 10*(32*b^2*c^3*d^6*e^2 - 48*b^3*c^2*d^5*e^3
+ 30*b^4*c*d^4*e^4 - 7*b^5*d^3*e^5)*x^2 + 5*(32*b^2*c^3*d^7*e - 48*b^3*c^2*
d^6*e^2 + 30*b^4*c*d^5*e^3 - 7*b^5*d^4*e^4)*x)*sqrt(c*d^2 - b*d*e)*log((b*d
 + (2*c*d - b*e)*x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*x + d)) -
2*(480*b*c^4*d^9 - 1200*b^2*c^3*d^8*e + 1170*b^3*c^2*d^7*e^2 - 555*b^4*c*d^
6*e^3 + 105*b^5*d^5*e^4 + (96*c^5*d^6*e^3 - 288*b*c^4*d^5*e^4 + 668*b^2*c^3
*d^4*e^5 - 856*b^3*c^2*d^3*e^6 + 485*b^4*c*d^2*e^7 - 105*b^5*d*e^8)*x^4 + 2
*(240*c^5*d^7*e^2 - 744*b*c^4*d^6*e^3 + 1622*b^2*c^3*d^5*e^4 - 2007*b^3*c^2
*d^4*e^5 + 1134*b^4*c*d^3*e^6 - 245*b^5*d^2*e^7)*x^3 + 2*(480*c^5*d^8*e - 1
560*b*c^4*d^7*e^2 + 3178*b^2*c^3*d^6*e^3 - 3729*b^3*c^2*d^5*e^4 + 2079*b^4*
c*d^4*e^5 - 448*b^5*d^3*e^6)*x^2 + 10*(96*c^5*d^9 - 336*b*c^4*d^8*e + 642*b
^2*c^3*d^7*e^2 - 697*b^3*c^2*d^6*e^3 + 374*b^4*c*d^5*e^4 - 79*b^5*d^4*e^5)*
x)*sqrt(c*x^2 + b*x))/(c^5*d^15 - 5*b*c^4*d^14*e + 10*b^2*c^3*d^13*e^2 - 10
*b^3*c^2*d^12*e^3 + 5*b^4*c*d^11*e^4 - b^5*d^10*e^5 + (c^5*d^10*e^5 - 5*b*c
^4*d^9*e^6 + 10*b^2*c^3*d^8*e^7 - 10*b^3*c^2*d^7*e^8 + 5*b^4*c*d^6*e^9 - b^
5*d^5*e^10)*x^5 + 5*(c^5*d^11*e^4 - 5*b*c^4*d^10*e^5 + 10*b^2*c^3*d^9*e^6 -
 10*b^3*c^2*d^8*e^7 + 5*b^4*c*d^7*e^8 - b^5*d^6*e^9)*x^4 + 10*(c^5*d^12*e^3
 - 5*b*c^4*d^11*e^4 + 10*b^2*c^3*d^10*e^5 - 10*b^3*c^2*d^9*e^6 + 5*b^4*c*d^
8*e^7 - b^5*d^7*e^8)*x^3 + 10*(c^5*d^13*e^2 - 5*b*c^4*d^12*e^3 + 10*b^2*c^3
*d^11*e^4 - 10*b^3*c^2*d^10*e^5 + 5*b^4*c*d^9*e^6 - b^5*d^8*e^7)*x^2 + 5*(c
^5*d^14*e - 5*b*c^4*d^13*e^2 + 10*b^2*c^3*d^12*e^3 - 10*b^3*c^2*d^11*e^4 +
5*b^4*c*d^10*e^5 - b^5*d^9*e^6)*x), -1/1920*(15*(32*b^2*c^3*d^8 - 48*b^3*c^
2*d^7*e + 30*b^4*c*d^6*e^2 - 7*b^5*d^5*e^3 + (32*b^2*c^3*d^3*e^5 - 48*b^3*c
^2*d^2*e^6 + 30*b^4*c*d*e^7 - 7*b^5*e^8)*x^5 + 5*(32*b^2*c^3*d^4*e^4 - 48*b
^3*c^2*d^3*e^5 + 30*b^4*c*d^2*e^6 - 7*b^5*d*e^7)*x^4 + 10*(32*b^2*c^3*d^5*e
^3 - 48*b^3*c^2*d^4*e^4 + 30*b^4*c*d^3*e^5 - 7*b^5*d^2*e^6)*x^3 + 10*(32*b^
2*c^3*d^6*e^2 - 48*b^3*c^2*d^5*e^3 + 30*b^4*c*d^4*e^4 - 7*b^5*d^3*e^5)*x^2
+ 5*(32*b^2*c^3*d^7*e - 48*b^3*c^2*d^6*e^2 + 30*b^4*c*d^5*e^3 - 7*b^5*d^4*e
^4)*x)*sqrt(-c*d^2 + b*d*e)*arctan(-sqrt(-c*d^2 + b*d*e)*sqrt(c*x^2 + b*x)/
((c*d - b*e)*x)) - (480*b*c^4*d^9 - 1200*b^2*c^3*d^8*e + 1170*b^3*c^2*d^7*e
^2 - 555*b^4*c*d^6*e^3 + 105*b^5*d^5*e^4 + (96*c^5*d^6*e^3 - 288*b*c^4*d^5*
e^4 + 668*b^2*c^3*d^4*e^5 - 856*b^3*c^2*d^3*e^6 + 485*b^4*c*d^2*e^7 - 105*b
^5*d*e^8)*x^4 + 2*(240*c^5*d^7*e^2 - 744*b*c^4*d^6*e^3 + 1622*b^2*c^3*d^5*e
^4 - 2007*b^3*c^2*d^4*e^5 + 1134*b^4*c*d^3*e^6 - 245*b^5*d^2*e^7)*x^3 + 2*(
480*c^5*d^8*e - 1560*b*c^4*d^7*e^2 + 3178*b^2*c^3*d^6*e^3 - 3729*b^3*c^2*d^
5*e^4 + 2079*b^4*c*d^4*e^5 - 448*b^5*d^3*e^6)*x^2 + 10*(96*c^5*d^9 - 336*b*
c^4*d^8*e + 642*b^2*c^3*d^7*e^2 - 697*b^3*c^2*d^6*e^3 + 374*b^4*c*d^5*e^4 -
 79*b^5*d^4*e^5)*x)*sqrt(c*x^2 + b*x))/(c^5*d^15 - 5*b*c^4*d^14*e + 10*b^2*
c^3*d^13*e^2 - 10*b^3*c^2*d^12*e^3 + 5*b^4*c*d^11*e^4 - b^5*d^10*e^5 + (c^5
*d^10*e^5 - 5*b*c^4*d^9*e^6 + 10*b^2*c^3*d^8*e^7 - 10*b^3*c^2*d^7*e^8 + 5*b
^4*c*d^6*e^9 - b^5*d^5*e^10)*x^5 + 5*(c^5*d^11*e^4 - 5*b*c^4*d^10*e^5 + 10*
b^2*c^3*d^9*e^6 - 10*b^3*c^2*d^8*e^7 + 5*b^4*c*d^7*e^8 - b^5*d^6*e^9)*x^4 +
 10*(c^5*d^12*e^3 - 5*b*c^4*d^11*e^4 + 10*b^2*c^3*d^10*e^5 - 10*b^3*c^2*d^9
*e^6 + 5*b^4*c*d^8*e^7 - b^5*d^7*e^8)*x^3 + 10*(c^5*d^13*e^2 - 5*b*c^4*d^12
*e^3 + 10*b^2*c^3*d^11*e^4 - 10*b^3*c^2*d^10*e^5 + 5*b^4*c*d^9*e^6 - b^5*d^
8*e^7)*x^2 + 5*(c^5*d^14*e - 5*b*c^4*d^13*e^2 + 10*b^2*c^3*d^12*e^3 - 10*b^
3*c^2*d^11*e^4 + 5*b^4*c*d^10*e^5 - b^5*d^9*e^6)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(1/2)/(e*x+d)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.66808, size = 2836, normalized size = 8.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/(e*x+d)^6,x, algorithm="giac")
```

```
[Out] 1/128*(32*b^2*c^3*d^3 - 48*b^3*c^2*d^2*e + 30*b^4*c*d*e^2 - 7*b^5*e^3)*arct
an(((sqrt(c)*x - sqrt(c*x^2 + b*x))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e))/((
c^4*d^8 - 4*b*c^3*d^7*e + 6*b^2*c^2*d^6*e^2 - 4*b^3*c*d^5*e^3 + b^4*d^4*e^4
)*sqrt(-c*d^2 + b*d*e)) + 1/1920*(7680*(sqrt(c)*x - sqrt(c*x^2 + b*x))^6*c^
(13/2)*d^8*e + 3072*(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*c^7*d^9 + 9216*(sqrt(
c)*x - sqrt(c*x^2 + b*x))^5*b*c^6*d^8*e + 7680*(sqrt(c)*x - sqrt(c*x^2 + b*
x))^4*b*c^(13/2)*d^9 - 30720*(sqrt(c)*x - sqrt(c*x^2 + b*x))^6*b*c^(11/2)*d
^7*e^2 - 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*b^2*c^(11/2)*d^8*e + 7680*(
sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b^2*c^6*d^9 - 50048*(sqrt(c)*x - sqrt(c*x^
2 + b*x))^5*b^2*c^5*d^7*e^2 - 11520*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b^3*c
^5*d^8*e + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*b^3*c^(11/2)*d^9 + 70720*
(sqrt(c)*x - sqrt(c*x^2 + b*x))^6*b^2*c^(9/2)*d^6*e^3 - 17600*(sqrt(c)*x -
sqrt(c*x^2 + b*x))^4*b^3*c^(9/2)*d^7*e^2 - 7200*(sqrt(c)*x - sqrt(c*x^2 + b
*x))^2*b^4*c^(9/2)*d^8*e + 960*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b^4*c^5*d^9
+ 15040*(sqrt(c)*x - sqrt(c*x^2 + b*x))^7*b^2*c^4*d^5*e^4 + 129280*(sqrt(c)
*x - sqrt(c*x^2 + b*x))^5*b^3*c^4*d^6*e^3 + 14080*(sqrt(c)*x - sqrt(c*x^2 +
 b*x))^3*b^4*c^4*d^7*e^2 - 1920*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b^5*c^4*d^8
*e + 96*b^5*c^(9/2)*d^9 + 4320*(sqrt(c)*x - sqrt(c*x^2 + b*x))^8*b^2*c^(7/2
)*d^4*e^5 - 52000*(sqrt(c)*x - sqrt(c*x^2 + b*x))^6*b^3*c^(7/2)*d^5*e^4 + 8
1920*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*b^4*c^(7/2)*d^6*e^3 + 13760*(sqrt(c)
*x - sqrt(c*x^2 + b*x))^2*b^5*c^(7/2)*d^7*e^2 - 192*b^6*c^(7/2)*d^8*e + 480
*(sqrt(c)*x - sqrt(c*x^2 + b*x))^9*b^2*c^3*d^3*e^6 - 20320*(sqrt(c)*x - sqr
t(c*x^2 + b*x))^7*b^3*c^3*d^4*e^5 - 120680*(sqrt(c)*x - sqrt(c*x^2 + b*x))^
5*b^4*c^3*d^5*e^4 + 14080*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b^5*c^3*d^6*e^3
 + 4280*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b^6*c^3*d^7*e^2 - 6480*(sqrt(c)*x -
 sqrt(c*x^2 + b*x))^8*b^3*c^(5/2)*d^3*e^6 + 7260*(sqrt(c)*x - sqrt(c*x^2 +
b*x))^6*b^4*c^(5/2)*d^4*e^5 - 85780*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*b^5*c
^(5/2)*d^5*e^4 - 6340*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*b^6*c^(5/2)*d^6*e^3
 + 476*b^7*c^(5/2)*d^7*e^2 - 720*(sqrt(c)*x - sqrt(c*x^2 + b*x))^9*b^3*c^2*
d^2*e^7 + 10740*(sqrt(c)*x - sqrt(c*x^2 + b*x))^7*b^4*c^2*d^3*e^6 + 47944*(
sqrt(c)*x - sqrt(c*x^2 + b*x))^5*b^5*c^2*d^4*e^5 - 25220*(sqrt(c)*x - sqrt(
c*x^2 + b*x))^3*b^6*c^2*d^5*e^4 - 3080*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b^7*
c^2*d^6*e^3 + 4050*(sqrt(c)*x - sqrt(c*x^2 + b*x))^8*b^4*c^(3/2)*d^2*e^7 +
9310*(sqrt(c)*x - sqrt(c*x^2 + b*x))^6*b^5*c^(3/2)*d^3*e^6 + 35330*(sqrt(c)
*x - sqrt(c*x^2 + b*x))^4*b^6*c^(3/2)*d^4*e^5 - 1750*(sqrt(c)*x - sqrt(c*x^
2 + b*x))^2*b^7*c^(3/2)*d^5*e^4 - 380*b^8*c^(3/2)*d^6*e^3 + 450*(sqrt(c)*x
- sqrt(c*x^2 + b*x))^9*b^4*c*d*e^8 - 1190*(sqrt(c)*x - sqrt(c*x^2 + b*x))^7
*b^5*c*d^2*e^7 - 4658*(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*b^6*c*d^3*e^6 + 105
10*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b^7*c*d^4*e^5 + 600*(sqrt(c)*x - sqrt(
c*x^2 + b*x))*b^8*c*d^5*e^4 - 945*(sqrt(c)*x - sqrt(c*x^2 + b*x))^8*b^5*sqr
t(c)*d*e^8 - 3430*(sqrt(c)*x - sqrt(c*x^2 + b*x))^6*b^6*sqrt(c)*d^2*e^7 - 4
480*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*b^7*sqrt(c)*d^3*e^6 + 1470*(sqrt(c)*x
 - sqrt(c*x^2 + b*x))^2*b^8*sqrt(c)*d^4*e^5 + 105*b^9*sqrt(c)*d^5*e^4 - 105
*(sqrt(c)*x - sqrt(c*x^2 + b*x))^9*b^5*e^9 - 490*(sqrt(c)*x - sqrt(c*x^2 +
b*x))^7*b^6*d*e^8 - 896*(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*b^7*d^2*e^7 - 790
```

```
*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b^8*d^3*e^6 + 105*(sqrt(c)*x - sqrt(c*x^
2 + b*x))*b^9*d^4*e^5)/((c^4*d^8*e^2 - 4*b*c^3*d^7*e^3 + 6*b^2*c^2*d^6*e^4
- 4*b^3*c*d^5*e^5 + b^4*d^4*e^6)*((sqrt(c)*x - sqrt(c*x^2 + b*x))^2*e + 2*(
sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c)*d + b*d)^5)
```

3.295 $\int (d+ex)^3 \left(bx+cx^2\right)^{3/2} dx$

Optimal. Leaf size=271

$$-\frac{3b^2(b+2cx)\sqrt{bx+cx^2}(2cd-be)\left(3b^2e^2-8bcde+8c^2d^2\right)}{1024c^5}+\frac{e\left(bx+cx^2\right)^{5/2}\left(21b^2e^2+30cex(2cd-be)-98bcde+12\right.}{280c^3}$$

```
[Out] (-3*b^2*(2*c*d - b*e)*(8*c^2*d^2 - 8*b*c*d*e + 3*b^2*e^2)*(b + 2*c*x)*Sqrt[
b*x + c*x^2])/(1024*c^5) + ((2*c*d - b*e)*(8*c^2*d^2 - 8*b*c*d*e + 3*b^2*e^
2)*(b + 2*c*x)*(b*x + c*x^2)^(3/2))/(128*c^4) + (e*(d + e*x)^2*(b*x + c*x^2
)^(5/2))/(7*c) + (e*(128*c^2*d^2 - 98*b*c*d*e + 21*b^2*e^2 + 30*c*e*(2*c*d
- b*e)*x)*(b*x + c*x^2)^(5/2))/(280*c^3) + (3*b^4*(2*c*d - b*e)*(8*c^2*d^2
- 8*b*c*d*e + 3*b^2*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(1024*c^(1
1/2))
```

Rubi [A] time = 0.349665, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {742, 779, 612, 620, 206}

$$-\frac{3b^2(b+2cx)\sqrt{bx+cx^2}(2cd-be)\left(3b^2e^2-8bcde+8c^2d^2\right)}{1024c^5}+\frac{e\left(bx+cx^2\right)^{5/2}\left(21b^2e^2+30cex(2cd-be)-98bcde+12\right.}{280c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(b*x + c*x^2)^(3/2),x]
```

```
[Out] (-3*b^2*(2*c*d - b*e)*(8*c^2*d^2 - 8*b*c*d*e + 3*b^2*e^2)*(b + 2*c*x)*Sqrt[
b*x + c*x^2])/(1024*c^5) + ((2*c*d - b*e)*(8*c^2*d^2 - 8*b*c*d*e + 3*b^2*e^
2)*(b + 2*c*x)*(b*x + c*x^2)^(3/2))/(128*c^4) + (e*(d + e*x)^2*(b*x + c*x^2
)^(5/2))/(7*c) + (e*(128*c^2*d^2 - 98*b*c*d*e + 21*b^2*e^2 + 30*c*e*(2*c*d
- b*e)*x)*(b*x + c*x^2)^(5/2))/(280*c^3) + (3*b^4*(2*c*d - b*e)*(8*c^2*d^2
- 8*b*c*d*e + 3*b^2*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(1024*c^(1
1/2))
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^3 \left(bx+cx^2\right)^{3/2} dx = \frac{e(d+ex)^2 \left(bx+cx^2\right)^{5/2}}{7c} + \frac{\int (d+ex)\left(\frac{1}{2}d(14cd-5be)+\frac{9}{2}e(2cd-be)x\right)\left(bx+cx^2\right)^{3/2} dx}{7c}$$

$$= \frac{e(d+ex)^2 \left(bx+cx^2\right)^{5/2}}{7c} + \frac{e\left(128c^2d^2-98bcde+21b^2e^2+30ce(2cd-be)x\right)\left(bx+cx^2\right)^{5/2}}{280c^3} \ldots$$

$$= \frac{(2cd-be)\left(8c^2d^2-8bcde+3b^2e^2\right)(b+2cx)\left(bx+cx^2\right)^{3/2}}{128c^4} + \frac{e(d+ex)^2\left(bx+cx^2\right)^{5/2}}{7c} + \ldots$$

$$= -\frac{3b^2(2cd-be)\left(8c^2d^2-8bcde+3b^2e^2\right)(b+2cx)\sqrt{bx+cx^2}}{1024c^5} + \frac{(2cd-be)\left(8c^2d^2-8bcde+\ldots\right.}{\ldots} \ldots$$

$$= -\frac{3b^2(2cd-be)\left(8c^2d^2-8bcde+3b^2e^2\right)(b+2cx)\sqrt{bx+cx^2}}{1024c^5} + \frac{(2cd-be)\left(8c^2d^2-8bcde+\ldots\right.}{\ldots} \ldots$$

$$= -\frac{3b^2(2cd-be)\left(8c^2d^2-8bcde+3b^2e^2\right)(b+2cx)\sqrt{bx+cx^2}}{1024c^5} + \frac{(2cd-be)\left(8c^2d^2-8bcde+\ldots\right.}{\ldots} \ldots$$

Mathematica [A] time = 0.701945, size = 312, normalized size = 1.15

$$\frac{\sqrt{x(b+cx)}\left(\sqrt{c}\left(28b^4c^2e\left(90d^2+35dex+6e^2x^2\right)-16b^3c^3\left(105d^2ex+105d^3+49de^2x^2+9e^3x^3\right)+32b^2c^4x\left(42d^2ex+35d\ldots\right.\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(b*x + c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[x*(b + c*x)]*(Sqrt[c]*(315*b^6*e^3 - 210*b^5*c*e^2*(7*d + e*x) + 28*b
^4*c^2*e*(90*d^2 + 35*d*e*x + 6*e^2*x^2) + 32*b^2*c^4*x*(35*d^3 + 42*d^2*e*
x + 21*d*e^2*x^2 + 4*e^3*x^3) - 16*b^3*c^3*(105*d^3 + 105*d^2*e*x + 49*d*e^
2*x^2 + 9*e^3*x^3) + 256*c^6*x^3*(35*d^3 + 84*d^2*e*x + 70*d*e^2*x^2 + 20*e
^3*x^3) + 128*b*c^5*x^2*(105*d^3 + 231*d^2*e*x + 182*d*e^2*x^2 + 50*e^3*x^3
)) - (105*b^(7/2)*(-16*c^3*d^3 + 24*b*c^2*d^2*e - 14*b^2*c*d*e^2 + 3*b^3*e^
3)*ArcSinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(Sqrt[x]*Sqrt[1 + (c*x)/b])))/(35840
*c^(11/2))
```

Maple [B] time = 0.056, size = 629, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x)^(3/2),x)
```

```
[Out] -3/64*d^3*b^3/c^2*(c*x^2+b*x)^(1/2)+3/128*d^3*b^4/c^(5/2)*ln((1/2*b+c*x)/c^
(1/2)+(c*x^2+b*x)^(1/2))+1/7*e^3*x^2*(c*x^2+b*x)^(5/2)/c+3/40*e^3*b^2/c^3*(
c*x^2+b*x)^(5/2)-3/128*e^3*b^4/c^4*(c*x^2+b*x)^(3/2)+9/1024*e^3*b^6/c^5*(c*
x^2+b*x)^(1/2)-9/2048*e^3*b^7/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(
1/2))-21/512*d*e^2*b^5/c^4*(c*x^2+b*x)^(1/2)+21/1024*d*e^2*b^6/c^(9/2)*ln((
1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))-3/28*e^3*b/c^2*x*(c*x^2+b*x)^(5/2)-3/
64*e^3*b^3/c^3*x*(c*x^2+b*x)^(3/2)+9/512*e^3*b^5/c^4*(c*x^2+b*x)^(1/2)*x+7/
64*d*e^2*b^3/c^3*(c*x^2+b*x)^(3/2)-9/256*d^2*e*b^5/c^(7/2)*ln((1/2*b+c*x)/c
^(1/2)+(c*x^2+b*x)^(1/2))+1/2*d*e^2*x*(c*x^2+b*x)^(5/2)/c-7/20*d*e^2*b/c^2*
(c*x^2+b*x)^(5/2)+9/128*d^2*e*b^4/c^3*(c*x^2+b*x)^(1/2)+1/4*d^3*x*(c*x^2+b*
x)^(3/2)-3/32*d^3*b^2/c*(c*x^2+b*x)^(1/2)*x-3/16*d^2*e*b^2/c^2*(c*x^2+b*x)^
(3/2)+3/5*d^2*e*(c*x^2+b*x)^(5/2)/c+1/8*d^3/c*(c*x^2+b*x)^(3/2)*b+7/32*d*e^
2*b^2/c^2*x*(c*x^2+b*x)^(3/2)-21/256*d*e^2*b^4/c^3*(c*x^2+b*x)^(1/2)*x-3/8*
d^2*e*b/c*x*(c*x^2+b*x)^(3/2)+9/64*d^2*e*b^3/c^2*(c*x^2+b*x)^(1/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.40864, size = 1586, normalized size = 5.85

$$\left[-\frac{105\left(16\,b^4c^3d^3-24\,b^5c^2d^2e+14\,b^6cde^2-3\,b^7e^3\right)\sqrt{c}\log\left(2\,cx+b-2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\left(5120\,c^7e^3x^6-1680\,b^3c^4d^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/71680*(105*(16*b^4*c^3*d^3 - 24*b^5*c^2*d^2*e + 14*b^6*c*d*e^2 - 3*b^7*
e^3)*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 2*(5120*c^7*e^3
*x^6 - 1680*b^3*c^4*d^3 + 2520*b^4*c^3*d^2*e - 1470*b^5*c^2*d*e^2 + 315*b^6
*c*e^3 + 1280*(14*c^7*d*e^2 + 5*b*c^6*e^3)*x^5 + 128*(168*c^7*d^2*e + 182*b
*c^6*d*e^2 + b^2*c^5*e^3)*x^4 + 16*(560*c^7*d^3 + 1848*b*c^6*d^2*e + 42*b^2
*c^5*d*e^2 - 9*b^3*c^4*e^3)*x^3 + 56*(240*b*c^6*d^3 + 24*b^2*c^5*d^2*e - 14
*b^3*c^4*d*e^2 + 3*b^4*c^3*e^3)*x^2 + 70*(16*b^2*c^5*d^3 - 24*b^3*c^4*d^2*e
 + 14*b^4*c^3*d*e^2 - 3*b^5*c^2*e^3)*x)*sqrt(c*x^2 + b*x))/c^6, -1/35840*(1
05*(16*b^4*c^3*d^3 - 24*b^5*c^2*d^2*e + 14*b^6*c*d*e^2 - 3*b^7*e^3)*sqrt(-c
)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)) - (5120*c^7*e^3*x^6 - 1680*b^3*c
```

```
^4*d^3 + 2520*b^4*c^3*d^2*e - 1470*b^5*c^2*d*e^2 + 315*b^6*c*e^3 + 1280*(14
*c^7*d*e^2 + 5*b*c^6*e^3)*x^5 + 128*(168*c^7*d^2*e + 182*b*c^6*d*e^2 + b^2*
c^5*e^3)*x^4 + 16*(560*c^7*d^3 + 1848*b*c^6*d^2*e + 42*b^2*c^5*d*e^2 - 9*b^
3*c^4*e^3)*x^3 + 56*(240*b*c^6*d^3 + 24*b^2*c^5*d^2*e - 14*b^3*c^4*d*e^2 +
3*b^4*c^3*e^3)*x^2 + 70*(16*b^2*c^5*d^3 - 24*b^3*c^4*d^2*e + 14*b^4*c^3*d*e
^2 - 3*b^5*c^2*e^3)*x)*sqrt(c*x^2 + b*x))/c^6]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (x(b+cx))^{\frac{3}{2}} (d+ex)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral((x*(b + c*x))**(3/2)*(d + e*x)**3, x)
```

Giac [A] time = 1.34195, size = 493, normalized size = 1.82

$$\frac{1}{35840}\sqrt{cx^2+bx}\left(2\left(4\left(2\left(8\left(10\left(4\,cxe^3+\frac{14\,c^7de^2+5\,bc^6e^3}{c^6}\right)x+\frac{168\,c^7d^2e+182\,bc^6de^2+b^2c^5e^3}{c^6}\right)x+\frac{560\,c^7d^3+1848\,bc\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/35840*sqrt(c*x^2 + b*x)*(2*(4*(2*(8*(10*(4*c*x*e^3 + (14*c^7*d*e^2 + 5*b*
c^6*e^3)/c^6)*x + (168*c^7*d^2*e + 182*b*c^6*d*e^2 + b^2*c^5*e^3)/c^6)*x +
(560*c^7*d^3 + 1848*b*c^6*d^2*e + 42*b^2*c^5*d*e^2 - 9*b^3*c^4*e^3)/c^6)*x
+ 7*(240*b*c^6*d^3 + 24*b^2*c^5*d^2*e - 14*b^3*c^4*d*e^2 + 3*b^4*c^3*e^3)/c
^6)*x + 35*(16*b^2*c^5*d^3 - 24*b^3*c^4*d^2*e + 14*b^4*c^3*d*e^2 - 3*b^5*c^
2*e^3)/c^6)*x - 105*(16*b^3*c^4*d^3 - 24*b^4*c^3*d^2*e + 14*b^5*c^2*d*e^2 -
 3*b^6*c*e^3)/c^6) - 3/2048*(16*b^4*c^3*d^3 - 24*b^5*c^2*d^2*e + 14*b^6*c*d
*e^2 - 3*b^7*e^3)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b))/
c^(11/2)
```

3.296 $\int (d+ex)^2 \left(bx+cx^2\right)^{3/2} dx$

Optimal. Leaf size=214

$$-\frac{b^2(b+2cx)\sqrt{bx+cx^2}\left(7b^2e^2-24bcde+24c^2d^2\right)}{512c^4}+\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}\left(7b^2e^2-24bcde+24c^2d^2\right)}{192c^3}+\frac{b^4\left(7b^2e^2-\right.}{}$$

```
[Out] -(b^2*(24*c^2*d^2 - 24*b*c*d*e + 7*b^2*e^2)*(b + 2*c*x)*Sqrt[b*x + c*x^2])/
(512*c^4) + ((24*c^2*d^2 - 24*b*c*d*e + 7*b^2*e^2)*(b + 2*c*x)*(b*x + c*x^2
)^(3/2))/(192*c^3) + (7*e*(2*c*d - b*e)*(b*x + c*x^2)^(5/2))/(60*c^2) + (e*
(d + e*x)*(b*x + c*x^2)^(5/2))/(6*c) + (b^4*(24*c^2*d^2 - 24*b*c*d*e + 7*b^
2*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(512*c^(9/2))
```

Rubi [A] time = 0.179299, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {742, 640, 612, 620, 206}

$$-\frac{b^2(b+2cx)\sqrt{bx+cx^2}\left(7b^2e^2-24bcde+24c^2d^2\right)}{512c^4}+\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}\left(7b^2e^2-24bcde+24c^2d^2\right)}{192c^3}+\frac{b^4\left(7b^2e^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(b*x + c*x^2)^(3/2),x]
```


```
[Out] -(b^2*(24*c^2*d^2 - 24*b*c*d*e + 7*b^2*e^2)*(b + 2*c*x)*Sqrt[b*x + c*x^2])/
(512*c^4) + ((24*c^2*d^2 - 24*b*c*d*e + 7*b^2*e^2)*(b + 2*c*x)*(b*x + c*x^2
)^(3/2))/(192*c^3) + (7*e*(2*c*d - b*e)*(b*x + c*x^2)^(5/2))/(60*c^2) + (e*
(d + e*x)*(b*x + c*x^2)^(5/2))/(6*c) + (b^4*(24*c^2*d^2 - 24*b*c*d*e + 7*b^
2*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(512*c^(9/2))
```


Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^2\left(bx+cx^2\right)^{3/2}\,dx = \frac{e(d+ex)\left(bx+cx^2\right)^{5/2}}{6c} + \frac{\int \left(\frac{1}{2}d(12cd-5be)+\frac{7}{2}e(2cd-be)x\right)\left(bx+cx^2\right)^{3/2}\,dx}{6c}$$

$$= \frac{7e(2cd-be)\left(bx+cx^2\right)^{5/2}}{60c^2} + \frac{e(d+ex)\left(bx+cx^2\right)^{5/2}}{6c} + \frac{\left(cd(12cd-5be)-\frac{7}{2}be(2cd-be)\right)\int}{12c^2}$$

$$= \frac{\left(24c^2d^2-24bcde+7b^2e^2\right)(b+2cx)\left(bx+cx^2\right)^{3/2}}{192c^3} + \frac{7e(2cd-be)\left(bx+cx^2\right)^{5/2}}{60c^2} + \frac{e(d+ex}{}$$

$$= -\frac{b^2\left(24c^2d^2-24bcde+7b^2e^2\right)(b+2cx)\sqrt{bx+cx^2}}{512c^4} + \frac{\left(24c^2d^2-24bcde+7b^2e^2\right)(b+2cx)}{192c^3}$$

$$= -\frac{b^2\left(24c^2d^2-24bcde+7b^2e^2\right)(b+2cx)\sqrt{bx+cx^2}}{512c^4} + \frac{\left(24c^2d^2-24bcde+7b^2e^2\right)(b+2cx)}{192c^3}$$

$$= -\frac{b^2\left(24c^2d^2-24bcde+7b^2e^2\right)(b+2cx)\sqrt{bx+cx^2}}{512c^4} + \frac{\left(24c^2d^2-24bcde+7b^2e^2\right)(b+2cx)}{192c^3}$$

Mathematica [A] time = 0.464965, size = 197, normalized size = 0.92

$$\frac{(x(b+cx))^{3/2}\left(\frac{\left(7b^2e^2-24bcde+24c^2d^2\right)\left(\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\left(2b^2cx-3b^3+24bc^2x^2+16c^3x^3\right)+3b^{7/2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)\right)}{256c^{7/2}(b+cx)\sqrt{\frac{cx}{b}+1}} + \frac{7ex^{5/2}(b+cx)(2cd-be)}{10c} + ex^{5/2}(b+cx)(d\right.}{6cx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(b*x + c*x^2)^(3/2),x]
```

```
[Out] ((x*(b + c*x))^(3/2)*((7*e*(2*c*d - b*e)*x^(5/2)*(b + c*x))/(10*c) + e*x^(5
/2)*(b + c*x)*(d + e*x) + ((24*c^2*d^2 - 24*b*c*d*e + 7*b^2*e^2)*(Sqrt[c]*S
qrt[x]*Sqrt[1 + (c*x)/b]*(-3*b^3 + 2*b^2*c*x + 24*b*c^2*x^2 + 16*c^3*x^3) +
 3*b^(7/2)*ArcSinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]]))/(256*c^(7/2)*(b + c*x)*Sqrt
[1 + (c*x)/b])))/(6*c*x^(3/2))
```

Maple [B] time = 0.055, size = 420, normalized size = 2.

$$\frac{e^2x}{6\,c}\left(cx^2+bx\right)^{\frac{5}{2}} - \frac{7\,e^2b}{60\,c^2}\left(cx^2+bx\right)^{\frac{5}{2}} + \frac{7\,b^2e^2x}{96\,c^2}\left(cx^2+bx\right)^{\frac{3}{2}} + \frac{7\,b^3e^2}{192\,c^3}\left(cx^2+bx\right)^{\frac{3}{2}} - \frac{7\,e^2b^4x}{256\,c^3}\sqrt{cx^2+bx} - \frac{7\,e^2b^5}{512\,c^4}\sqrt{cx^2+bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+b*x)^(3/2),x)
```

```
[Out] 1/6*e^2*x*(c*x^2+b*x)^(5/2)/c-7/60*e^2*b/c^2*(c*x^2+b*x)^(5/2)+7/96*e^2*b^2
/c^2*x*(c*x^2+b*x)^(3/2)+7/192*e^2*b^3/c^3*(c*x^2+b*x)^(3/2)-7/256*e^2*b^4/
c^3*(c*x^2+b*x)^(1/2)*x-7/512*e^2*b^5/c^4*(c*x^2+b*x)^(1/2)+7/1024*e^2*b^6/
c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))+2/5*d*e*(c*x^2+b*x)^(5/2)
/c-1/4*d*e*b/c*x*(c*x^2+b*x)^(3/2)-1/8*d*e*b^2/c^2*(c*x^2+b*x)^(3/2)+3/32*d
*e*b^3/c^2*(c*x^2+b*x)^(1/2)*x+3/64*d*e*b^4/c^3*(c*x^2+b*x)^(1/2)-3/128*d*e
*b^5/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))+1/4*d^2*x*(c*x^2+b*x
)^(3/2)+1/8*d^2/c*(c*x^2+b*x)^(3/2)*b-3/32*d^2*b^2/c*(c*x^2+b*x)^(1/2)*x-3/
64*d^2*b^3/c^2*(c*x^2+b*x)^(1/2)+3/128*d^2*b^4/c^(5/2)*ln((1/2*b+c*x)/c^(1/
2)+(c*x^2+b*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.32955, size = 1108, normalized size = 5.18

$$\left[\frac{15\left(24\,b^4c^2d^2 - 24\,b^5cde + 7\,b^6e^2\right)\sqrt{c}\log\left(2\,cx + b + 2\,\sqrt{cx^2 + bx}\sqrt{c}\right) + 2\left(1280\,c^6e^2x^5 - 360\,b^3c^3d^2 + 360\,b^4c^2de - 10\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/15360*(15*(24*b^4*c^2*d^2 - 24*b^5*c*d*e + 7*b^6*e^2)*sqrt(c)*log(2*c*x
+ b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*(1280*c^6*e^2*x^5 - 360*b^3*c^3*d^2
+ 360*b^4*c^2*d*e - 105*b^5*c*e^2 + 128*(24*c^6*d*e + 13*b*c^5*e^2)*x^4 + 4
8*(40*c^6*d^2 + 88*b*c^5*d*e + b^2*c^4*e^2)*x^3 + 8*(360*b*c^5*d^2 + 24*b^2
*c^4*d*e - 7*b^3*c^3*e^2)*x^2 + 10*(24*b^2*c^4*d^2 - 24*b^3*c^3*d*e + 7*b^4
*c^2*e^2)*x)*sqrt(c*x^2 + b*x))/c^5, -1/7680*(15*(24*b^4*c^2*d^2 - 24*b^5*c
*d*e + 7*b^6*e^2)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)) - (1280
*c^6*e^2*x^5 - 360*b^3*c^3*d^2 + 360*b^4*c^2*d*e - 105*b^5*c*e^2 + 128*(24*
c^6*d*e + 13*b*c^5*e^2)*x^4 + 48*(40*c^6*d^2 + 88*b*c^5*d*e + b^2*c^4*e^2)*
x^3 + 8*(360*b*c^5*d^2 + 24*b^2*c^4*d*e - 7*b^3*c^3*e^2)*x^2 + 10*(24*b^2*c
^4*d^2 - 24*b^3*c^3*d*e + 7*b^4*c^2*e^2)*x)*sqrt(c*x^2 + b*x))/c^5]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (x(b+cx))^{\frac{3}{2}}(d+ex)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral((x*(b + c*x))**(3/2)*(d + e*x)**2, x)
```

Giac [A] time = 1.38887, size = 354, normalized size = 1.65

$$\frac{1}{7680}\sqrt{cx^2+bx}\left(2\left(4\left(2\left(8\left(10\,cxe^2+\frac{24\,c^6de+13\,bc^5e^2}{c^5}\right)x+\frac{3\left(40\,c^6d^2+88\,bc^5de+b^2c^4e^2\right)}{c^5}\right)x+\frac{360\,bc^5d^2+24\,b^2c^4de\cdot}{c^5}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x)*(2*(4*(2*(8*(10*c*x*e^2 + (24*c^6*d*e + 13*b*c^5*e
^2)/c^5)*x + 3*(40*c^6*d^2 + 88*b*c^5*d*e + b^2*c^4*e^2)/c^5)*x + (360*b*c^
5*d^2 + 24*b^2*c^4*d*e - 7*b^3*c^3*e^2)/c^5)*x + 5*(24*b^2*c^4*d^2 - 24*b^3
*c^3*d*e + 7*b^4*c^2*e^2)/c^5)*x - 15*(24*b^3*c^3*d^2 - 24*b^4*c^2*d*e + 7*
b^5*c*e^2)/c^5) - 1/1024*(24*b^4*c^2*d^2 - 24*b^5*c*d*e + 7*b^6*e^2)*log(ab
s(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b))/c^(9/2)
```

3.297 $\int (d+ex)\left(bx+cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=137

$$-\frac{3b^2(b+2cx)\sqrt{bx+cx^2}(2cd-be)}{128c^3}+\frac{3b^4(2cd-be)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{128c^{7/2}}+\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}(2cd-be)}{16c^2}+\frac{e\left(bx+cx\right.}{5c}$$

```
[Out] (-3*b^2*(2*c*d - b*e)*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(128*c^3) + ((2*c*d -
b*e)*(b + 2*c*x)*(b*x + c*x^2)^(3/2))/(16*c^2) + (e*(b*x + c*x^2)^(5/2))/(5
*c) + (3*b^4*(2*c*d - b*e)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(128*c^(
7/2))
```

Rubi [A] time = 0.0526371, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {640, 612, 620, 206}

$$-\frac{3b^2(b+2cx)\sqrt{bx+cx^2}(2cd-be)}{128c^3}+\frac{3b^4(2cd-be)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{128c^{7/2}}+\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}(2cd-be)}{16c^2}+\frac{e\left(bx+cx\right.}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(b*x + c*x^2)^(3/2),x]
```


```
[Out] (-3*b^2*(2*c*d - b*e)*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(128*c^3) + ((2*c*d -
b*e)*(b + 2*c*x)*(b*x + c*x^2)^(3/2))/(16*c^2) + (e*(b*x + c*x^2)^(5/2))/(5
*c) + (3*b^4*(2*c*d - b*e)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(128*c^(
7/2))
```


Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)\left(bx+cx^2\right)^{3/2}dx = \frac{e\left(bx+cx^2\right)^{5/2}}{5c} + \frac{(2cd-be)\int \left(bx+cx^2\right)^{3/2}dx}{2c}$$
$$= \frac{(2cd-be)(b+2cx)\left(bx+cx^2\right)^{3/2}}{16c^2} + \frac{e\left(bx+cx^2\right)^{5/2}}{5c} - \frac{\left(3b^2(2cd-be)\right)\int \sqrt{bx+cx^2}\,dx}{32c^2}$$
$$= -\frac{3b^2(2cd-be)(b+2cx)\sqrt{bx+cx^2}}{128c^3} + \frac{(2cd-be)(b+2cx)\left(bx+cx^2\right)^{3/2}}{16c^2} + \frac{e\left(bx+cx^2\right)^{5/2}}{5c} +$$
$$= -\frac{3b^2(2cd-be)(b+2cx)\sqrt{bx+cx^2}}{128c^3} + \frac{(2cd-be)(b+2cx)\left(bx+cx^2\right)^{3/2}}{16c^2} + \frac{e\left(bx+cx^2\right)^{5/2}}{5c} +$$
$$= -\frac{3b^2(2cd-be)(b+2cx)\sqrt{bx+cx^2}}{128c^3} + \frac{(2cd-be)(b+2cx)\left(bx+cx^2\right)^{3/2}}{16c^2} + \frac{e\left(bx+cx^2\right)^{5/2}}{5c} +$$

Mathematica [A] time = 0.265041, size = 146, normalized size = 1.07

$$\frac{\sqrt{x(b+cx)}\left(\sqrt{c}\left(4b^2c^2x(5d+2ex)-10b^3c(3d+ex)+15b^4e+16bc^3x^2(15d+11ex)+32c^4x^3(5d+4ex)\right)-\frac{15b^{7/2}(be-2cd)\sinh}{\sqrt{x}\sqrt{\frac{cx}{b}+}}\right.}{640c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(b*x + c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[x*(b + c*x)]*(Sqrt[c]*(15*b^4*e - 10*b^3*c*(3*d + e*x) + 4*b^2*c^2*x*
(5*d + 2*e*x) + 32*c^4*x^3*(5*d + 4*e*x) + 16*b*c^3*x^2*(15*d + 11*e*x)) -
(15*b^(7/2)*(-2*c*d + b*e)*ArcSinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(Sqrt[x]*Sqr
t[1 + (c*x)/b])))/(640*c^(7/2))
```

Maple [B] time = 0.049, size = 239, normalized size = 1.7

$$\frac{e}{5c}\left(cx^2+bx\right)^{\frac{5}{2}} - \frac{bxe}{8c}\left(cx^2+bx\right)^{\frac{3}{2}} - \frac{b^2e}{16c^2}\left(cx^2+bx\right)^{\frac{3}{2}} + \frac{3eb^3x}{64c^2}\sqrt{cx^2+bx} + \frac{3eb^4}{128c^3}\sqrt{cx^2+bx} - \frac{3eb^5}{256}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+b*x)^(3/2),x)
```

```
[Out] 1/5*e*(c*x^2+b*x)^(5/2)/c-1/8*e*b/c*x*(c*x^2+b*x)^(3/2)-1/16*e*b^2/c^2*(c*x
^2+b*x)^(3/2)+3/64*e*b^3/c^2*(c*x^2+b*x)^(1/2)*x+3/128*e*b^4/c^3*(c*x^2+b*x
)^(1/2)-3/256*e*b^5/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))+1/4*d
*x*(c*x^2+b*x)^(3/2)+1/8*d/c*(c*x^2+b*x)^(3/2)*b-3/32*d*b^2/c*(c*x^2+b*x)^(
1/2)*x-3/64*d*b^3/c^2*(c*x^2+b*x)^(1/2)+3/128*d*b^4/c^(5/2)*ln((1/2*b+c*x)/
c^(1/2)+(c*x^2+b*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.35333, size = 689, normalized size = 5.03

$$\left[-\frac{15\left(2\,b^4cd-b^5e\right)\sqrt{c}\log\left(2\,cx+b-2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\left(128\,c^5ex^4-30\,b^3c^2d+15\,b^4ce+16\left(10\,c^5d+11\,bc^4e\right)x^3+\right.}{1280\,c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/1280*(15*(2*b^4*c*d - b^5*e)*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 + b*x
)*sqrt(c)) - 2*(128*c^5*e*x^4 - 30*b^3*c^2*d + 15*b^4*c*e + 16*(10*c^5*d +
11*b*c^4*e)*x^3 + 8*(30*b*c^4*d + b^2*c^3*e)*x^2 + 10*(2*b^2*c^3*d - b^3*c^
2*e)*x)*sqrt(c*x^2 + b*x))/c^4, -1/640*(15*(2*b^4*c*d - b^5*e)*sqrt(-c)*arc
tan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)) - (128*c^5*e*x^4 - 30*b^3*c^2*d + 15*
b^4*c*e + 16*(10*c^5*d + 11*b*c^4*e)*x^3 + 8*(30*b*c^4*d + b^2*c^3*e)*x^2 +
 10*(2*b^2*c^3*d - b^3*c^2*e)*x)*sqrt(c*x^2 + b*x))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (x\,(b+cx))^{\frac{3}{2}}\,(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral((x*(b + c*x))**(3/2)*(d + e*x), x)
```

Giac [A] time = 1.27819, size = 231, normalized size = 1.69

$$\frac{1}{640}\sqrt{cx^2+bx}\left(2\left(4\left(2\left(8\,cxe+\frac{10\,c^5d+11\,bc^4e}{c^4}\right)x+\frac{30\,bc^4d+b^2c^3e}{c^4}\right)x+\frac{5\left(2\,b^2c^3d-b^3c^2e\right)}{c^4}\right)x-\frac{15\left(2\,b^3c^2d-b^4ce\right)}{c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/640*sqrt(c*x^2 + b*x)*(2*(4*(2*(8*c*x*e + (10*c^5*d + 11*b*c^4*e)/c^4)*x
+ (30*b*c^4*d + b^2*c^3*e)/c^4)*x + 5*(2*b^2*c^3*d - b^3*c^2*e)/c^4)*x - 15
*(2*b^3*c^2*d - b^4*c*e)/c^4) - 3/256*(2*b^4*c*d - b^5*e)*log(abs(-2*(sqrt(
c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b))/c^(7/2)
```

3.298 $\int \left(bx+cx^2\right)^{3/2} dx$

Optimal. Leaf size=89

$$-\frac{3b^2(b+2cx)\sqrt{bx+cx^2}}{64c^2}+\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{64c^{5/2}}+\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}$$

```
[Out] (-3*b^2*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(64*c^2) + ((b + 2*c*x)*(b*x + c*x^2
)^(3/2))/(8*c) + (3*b^4*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(64*c^(5/2)
)
```

Rubi [A] time = 0.0252396, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {612, 620, 206}

$$-\frac{3b^2(b+2cx)\sqrt{bx+cx^2}}{64c^2}+\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{64c^{5/2}}+\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(3/2),x]
```

```
[Out] (-3*b^2*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(64*c^2) + ((b + 2*c*x)*(b*x + c*x^2
)^(3/2))/(8*c) + (3*b^4*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(64*c^(5/2)
)
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(bx+cx^2\right)^{3/2} dx = \frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c} - \frac{\left(3b^2\right)\int \sqrt{bx+cx^2}\,dx}{16c}$$
$$= -\frac{3b^2(b+2cx)\sqrt{bx+cx^2}}{64c^2} + \frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c} + \frac{\left(3b^4\right)\int \frac{1}{\sqrt{bx+cx^2}}\,dx}{128c^2}$$
$$= -\frac{3b^2(b+2cx)\sqrt{bx+cx^2}}{64c^2} + \frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c} + \frac{\left(3b^4\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{bx+cx^2}}\right)}{64c^2}$$
$$= -\frac{3b^2(b+2cx)\sqrt{bx+cx^2}}{64c^2} + \frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c} + \frac{3b^4\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{64c^{5/2}}$$

Mathematica [A] time = 0.0876433, size = 98, normalized size = 1.1

$$\frac{\sqrt{x(b+cx)}\left(\sqrt{c}\left(2b^2cx-3b^3+24bc^2x^2+16c^3x^3\right)+\frac{3b^{7/2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}\right)}{64c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[x*(b + c*x)]*(Sqrt[c]*(-3*b^3 + 2*b^2*c*x + 24*b*c^2*x^2 + 16*c^3*x^3
) + (3*b^(7/2)*ArcSinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(Sqrt[x]*Sqrt[1 + (c*x)/
b])))/(64*c^(5/2))
```

Maple [A] time = 0.051, size = 95, normalized size = 1.1

$$\frac{2cx+b}{8c}\left(cx^2+bx\right)^{\frac{3}{2}} - \frac{3b^2x}{32c}\sqrt{cx^2+bx} - \frac{3b^3}{64c^2}\sqrt{cx^2+bx} + \frac{3b^4}{128}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx}\right)c^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(3/2),x)
```

```
[Out] 1/8*(2*c*x+b)*(c*x^2+b*x)^(3/2)/c-3/32*b^2/c*(c*x^2+b*x)^(1/2)*x-3/64*b^3/c
^2*(c*x^2+b*x)^(1/2)+3/128*b^4/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(
1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.36783, size = 393, normalized size = 4.42

$$\left[\frac{3\,b^4\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\left(16\,c^4x^3+24\,bc^3x^2+2\,b^2c^2x-3\,b^3c\right)\sqrt{cx^2+bx}}{128\,c^3},-\frac{3\,b^4\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{\ }}{cx}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/128*(3*b^4*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*(16*
c^4*x^3 + 24*b*c^3*x^2 + 2*b^2*c^2*x - 3*b^3*c)*sqrt(c*x^2 + b*x))/c^3, -1/
64*(3*b^4*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)) - (16*c^4*x^3 +
 24*b*c^3*x^2 + 2*b^2*c^2*x - 3*b^3*c)*sqrt(c*x^2 + b*x))/c^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx+cx^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral((b*x + c*x**2)**(3/2), x)
```

Giac [A] time = 1.28646, size = 112, normalized size = 1.26

$$-\frac{3\,b^4\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}-b\right|\right)}{128\,c^{\frac{5}{2}}}+\frac{1}{64}\,\sqrt{cx^2+bx}\left(2\left(4\,(2\,cx+3\,b)x+\frac{b^2}{c}\right)x-\frac{3\,b^3}{c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] -3/128*b^4*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b))/c^(5/2)
 + 1/64*sqrt(c*x^2 + b*x)*(2*(4*(2*c*x + 3*b)*x + b^2/c)*x - 3*b^3/c^2)
```

3.299 $\int \frac{(bx+cx^2)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=216

$$\frac{\sqrt{bx+cx^2}\left(b^2e^2-2cex(2cd-be)-10bcde+8c^2d^2\right)}{8ce^3}-\frac{(2cd-be)\left(-b^2e^2-8bcde+8c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{8c^{3/2}e^4}+\frac{d^{3/2}(cd}{}$$

```
[Out] ((8*c^2*d^2 - 10*b*c*d*e + b^2*e^2 - 2*c*e*(2*c*d - b*e)*x)*Sqrt[b*x + c*x^
2])/(8*c*e^3) + (b*x + c*x^2)^(3/2)/(3*e) - ((2*c*d - b*e)*(8*c^2*d^2 - 8*b
*c*d*e - b^2*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(8*c^(3/2)*e^4) +
 (d^(3/2)*(c*d - b*e)^(3/2)*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt
[c*d - b*e]*Sqrt[b*x + c*x^2])])/e^4
```

Rubi [A] time = 0.241744, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {734, 814, 843, 620, 206, 724}

$$\frac{\sqrt{bx+cx^2}\left(b^2e^2-2cex(2cd-be)-10bcde+8c^2d^2\right)}{8ce^3}-\frac{(2cd-be)\left(-b^2e^2-8bcde+8c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{8c^{3/2}e^4}+\frac{d^{3/2}(cd}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] ((8*c^2*d^2 - 10*b*c*d*e + b^2*e^2 - 2*c*e*(2*c*d - b*e)*x)*Sqrt[b*x + c*x^
2])/(8*c*e^3) + (b*x + c*x^2)^(3/2)/(3*e) - ((2*c*d - b*e)*(8*c^2*d^2 - 8*b
*c*d*e - b^2*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(8*c^(3/2)*e^4) +
 (d^(3/2)*(c*d - b*e)^(3/2)*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt
[c*d - b*e]*Sqrt[b*x + c*x^2])])/e^4
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(bx+cx^2\right)^{3/2}}{d+ex}\,dx = \frac{\left(bx+cx^2\right)^{3/2}}{3e} - \frac{\int \frac{(bd+(2cd-be)x)\sqrt{bx+cx^2}}{d+ex}\,dx}{2e}$$

$$= \frac{\left(8c^2d^2-10bcde+b^2e^2-2ce(2cd-be)x\right)\sqrt{bx+cx^2}}{8ce^3} + \frac{\left(bx+cx^2\right)^{3/2}}{3e} + \frac{\int \frac{-\frac{1}{2}bd\left(8c^2d^2-10bcde+b^2e^2\right)-\frac{1}{2}(2c}{(d+ex)\sqrt{bx}}}{8ce^3}$$

$$= \frac{\left(8c^2d^2-10bcde+b^2e^2-2ce(2cd-be)x\right)\sqrt{bx+cx^2}}{8ce^3} + \frac{\left(bx+cx^2\right)^{3/2}}{3e} + \frac{\left(d^2(cd-be)^2\right)\int \frac{1}{(d+ex)\sqrt{bx+cx}}}{e^4}$$

$$= \frac{\left(8c^2d^2-10bcde+b^2e^2-2ce(2cd-be)x\right)\sqrt{bx+cx^2}}{8ce^3} + \frac{\left(bx+cx^2\right)^{3/2}}{3e} - \frac{\left(2d^2(cd-be)^2\right)\mathrm{Subst}\left(\int \frac{}{4c}\right.}{e}$$

$$= \frac{\left(8c^2d^2-10bcde+b^2e^2-2ce(2cd-be)x\right)\sqrt{bx+cx^2}}{8ce^3} + \frac{\left(bx+cx^2\right)^{3/2}}{3e} - \frac{(2cd-be)\left(8c^2d^2-8bcde-\right.}{8c^{3/2}e}$$

Mathematica [A] time = 0.826096, size = 225, normalized size = 1.04

$$\frac{\sqrt{x(b+cx)}\left(\sqrt{c}e\sqrt{x}\left(3b^2e^2+2bce(7ex-15d)+4c^2\left(6d^2-3dex+2e^2x^2\right)\right)-\frac{3\left(6b^2cde^2+b^3e^3-24bc^2d^2e+16c^3d^3\right)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{\frac{cx}{b}+1}}+\frac{48c^3}{}\right.}{24c^{3/2}e^4\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] (Sqrt[x*(b + c*x)]*(Sqrt[c]*e*Sqrt[x]*(3*b^2*e^2 + 2*b*c*e*(-15*d + 7*e*x)
+ 4*c^2*(6*d^2 - 3*d*e*x + 2*e^2*x^2)) - (3*(16*c^3*d^3 - 24*b*c^2*d^2*e +
6*b^2*c*d*e^2 + b^3*e^3)*ArcSinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(Sqrt[b]*Sqrt[
```

```
1 + (c*x)/b]) + (48*c^(3/2)*d^(3/2)*(-(c*d) + b*e)^(3/2)*ArcTan[(Sqrt[-(c*d
) + b*e]*Sqrt[x])/(Sqrt[d]*Sqrt[b + c*x])])/Sqrt[b + c*x]))/(24*c^(3/2)*e^4
*Sqrt[x])
```

Maple [B] time = 0.209, size = 1090, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(3/2)/(e*x+d),x)
```

```
[Out] 1/3/e*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)+1/4/e*(c*(d
/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*x*b-1/2/e^2*(c*(d/e+x)
^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*x*c*d+1/8/e/c*(c*(d/e+x)^2+
(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*b^2-5/4/e^2*(c*(d/e+x)^2+(b*e-
2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*b*d-1/16/e/c^(3/2)*ln((1/2*(b*e-2*c
*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2
)^(1/2))*b^3-3/8/e^2*d*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^
2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/c^(1/2)*b^2+1/e^3*d^2*(c*(d
/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*c+3/2/e^3*d^2*ln((1/2*
(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-
c*d)/e^2)^(1/2))*c^(1/2)*b-1/e^4*d^3*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/
2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))*c^(3/2)-1/e^3
*d^2/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+
2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e
^2)^(1/2))/(d/e+x))*b^2+2/e^4*d^3/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*
d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2
*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*b*c-1/e^5*d^4/(-d*(b*e-c*d
)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e
^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x
))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 6.06685, size = 1979, normalized size = 9.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(16*c^3*d^3 - 24*b*c^2*d^2*e + 6*b^2*c*d*e^2 + b^3*e^3)*sqrt(c)*lo
g(2*c*x + b - 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 48*(c^3*d^2 - b*c^2*d*e)*sqrt(
```

```
c*d^2 - b*d*e)*log((b*d + (2*c*d - b*e)*x - 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^
2 + b*x))/(e*x + d)) + 2*(8*c^3*e^3*x^2 + 24*c^3*d^2*e - 30*b*c^2*d*e^2 + 3
*b^2*c*e^3 - 2*(6*c^3*d*e^2 - 7*b*c^2*e^3)*x)*sqrt(c*x^2 + b*x))/(c^2*e^4),
 1/48*(96*(c^3*d^2 - b*c^2*d*e)*sqrt(-c*d^2 + b*d*e)*arctan(-sqrt(-c*d^2 +
b*d*e)*sqrt(c*x^2 + b*x)/((c*d - b*e)*x)) + 3*(16*c^3*d^3 - 24*b*c^2*d^2*e
+ 6*b^2*c*d*e^2 + b^3*e^3)*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 + b*x)*sqrt
(c)) + 2*(8*c^3*e^3*x^2 + 24*c^3*d^2*e - 30*b*c^2*d*e^2 + 3*b^2*c*e^3 - 2*(
6*c^3*d*e^2 - 7*b*c^2*e^3)*x)*sqrt(c*x^2 + b*x))/(c^2*e^4), 1/24*(3*(16*c^3
*d^3 - 24*b*c^2*d^2*e + 6*b^2*c*d*e^2 + b^3*e^3)*sqrt(-c)*arctan(sqrt(c*x^2
 + b*x)*sqrt(-c)/(c*x)) - 24*(c^3*d^2 - b*c^2*d*e)*sqrt(c*d^2 - b*d*e)*log(
(b*d + (2*c*d - b*e)*x - 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*x + d)
) + (8*c^3*e^3*x^2 + 24*c^3*d^2*e - 30*b*c^2*d*e^2 + 3*b^2*c*e^3 - 2*(6*c^3
*d*e^2 - 7*b*c^2*e^3)*x)*sqrt(c*x^2 + b*x))/(c^2*e^4), 1/24*(48*(c^3*d^2 -
b*c^2*d*e)*sqrt(-c*d^2 + b*d*e)*arctan(-sqrt(-c*d^2 + b*d*e)*sqrt(c*x^2 + b
*x)/((c*d - b*e)*x)) + 3*(16*c^3*d^3 - 24*b*c^2*d^2*e + 6*b^2*c*d*e^2 + b^3
*e^3)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)) + (8*c^3*e^3*x^2 +
24*c^3*d^2*e - 30*b*c^2*d*e^2 + 3*b^2*c*e^3 - 2*(6*c^3*d*e^2 - 7*b*c^2*e^3)
*x)*sqrt(c*x^2 + b*x))/(c^2*e^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x(b+cx))^{\frac{3}{2}}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(3/2)/(e*x+d),x)
```

```
[Out] Integral((x*(b + c*x))**(3/2)/(d + e*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.300 $\int \frac{\left(bx+cx^2\right)^{3/2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=198

$$\frac{3\left(b^2e^2-8bcde+8c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4\sqrt{c}e^4}-\frac{3\sqrt{bx+cx^2}(-3be+4cd-2cex)}{4e^3}-\frac{3\sqrt{d}\sqrt{cd-be}(2cd-be)\tanh^{-1}\left(\frac{x(2}{2\sqrt{d}\sqrt{}}\right.}{2e^4}$$

```
[Out] (-3*(4*c*d - 3*b*e - 2*c*e*x)*Sqrt[b*x + c*x^2])/(4*e^3) - (b*x + c*x^2)^(3
/2)/(e*(d + e*x)) + (3*(8*c^2*d^2 - 8*b*c*d*e + b^2*e^2)*ArcTanh[(Sqrt[c]*x
)/Sqrt[b*x + c*x^2]])/(4*Sqrt[c]*e^4) - (3*Sqrt[d]*Sqrt[c*d - b*e]*(2*c*d -
 b*e)*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x +
 c*x^2])])/(2*e^4)
```

Rubi [A] time = 0.234135, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {732, 814, 843, 620, 206, 724}

$$\frac{3\left(b^2e^2-8bcde+8c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4\sqrt{c}e^4}-\frac{3\sqrt{bx+cx^2}(-3be+4cd-2cex)}{4e^3}-\frac{3\sqrt{d}\sqrt{cd-be}(2cd-be)\tanh^{-1}\left(\frac{x(2}{2\sqrt{d}\sqrt{}}\right.}{2e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(3/2)/(d + e*x)^2,x]
```

```
[Out] (-3*(4*c*d - 3*b*e - 2*c*e*x)*Sqrt[b*x + c*x^2])/(4*e^3) - (b*x + c*x^2)^(3
/2)/(e*(d + e*x)) + (3*(8*c^2*d^2 - 8*b*c*d*e + b^2*e^2)*ArcTanh[(Sqrt[c]*x
)/Sqrt[b*x + c*x^2]])/(4*Sqrt[c]*e^4) - (3*Sqrt[d]*Sqrt[c*d - b*e]*(2*c*d -
 b*e)*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x +
 c*x^2])])/(2*e^4)
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
```

```
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(bx+cx^2\right)^{3/2}}{(d+ex)^2}\,dx = -\frac{\left(bx+cx^2\right)^{3/2}}{e(d+ex)} + \frac{3\int \frac{(b+2cx)\sqrt{bx+cx^2}}{d+ex}\,dx}{2e}$$

$$= -\frac{3(4cd-3be-2cex)\sqrt{bx+cx^2}}{4e^3} - \frac{\left(bx+cx^2\right)^{3/2}}{e(d+ex)} - \frac{3\int \frac{-bcd(4cd-3be)-c\left(8c^2d^2-8bcde+b^2e^2\right)x}{(d+ex)\sqrt{bx+cx^2}}\,dx}{8ce^3}$$

$$= -\frac{3(4cd-3be-2cex)\sqrt{bx+cx^2}}{4e^3} - \frac{\left(bx+cx^2\right)^{3/2}}{e(d+ex)} - \frac{(3d(cd-be)(2cd-be))\int \frac{1}{(d+ex)\sqrt{bx+cx^2}}\,dx}{2e^4} + \frac{(3(8\cdots}{\cdots}$$

$$= -\frac{3(4cd-3be-2cex)\sqrt{bx+cx^2}}{4e^3} - \frac{\left(bx+cx^2\right)^{3/2}}{e(d+ex)} + \frac{(3d(cd-be)(2cd-be))\,\mathrm{Subst}\left(\int \frac{1}{4cd^2-4bde-x^2}\,dx,\cdots\right)}{e^4}\cdots$$

$$= -\frac{3(4cd-3be-2cex)\sqrt{bx+cx^2}}{4e^3} - \frac{\left(bx+cx^2\right)^{3/2}}{e(d+ex)} + \frac{3\left(8c^2d^2-8bcde+b^2e^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4\sqrt{c}e^4} - \frac{3\sqrt{\cdots}}{\cdots}$$

Mathematica [A] time = 1.04538, size = 218, normalized size = 1.1

$$\frac{\sqrt{x(b+cx)}\left(-\frac{12\sqrt{d}\left(b^2e^2-3bcde+2c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{x}\sqrt{be-cd}}{\sqrt{d}\sqrt{b+cx}}\right)}{\sqrt{b+cx}\sqrt{be-cd}} + \frac{3\left(b^2e^2-8bcde+8c^2d^2\right)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{c}\sqrt{\frac{cx}{b}+1}} + \frac{e\sqrt{x}\left(be(9d+5ex)-2c\left(6d^2+3dex-e^2x^2\right)\right)}{d+ex}\right)}{4e^4\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(3/2)/(d + e*x)^2,x]
```

```
[Out] (Sqrt[x*(b + c*x)]*((e*Sqrt[x]*(b*e*(9*d + 5*e*x) - 2*c*(6*d^2 + 3*d*e*x -
e^2*x^2)))/(d + e*x) + (3*(8*c^2*d^2 - 8*b*c*d*e + b^2*e^2)*ArcSinh[(Sqrt[c
]*Sqrt[x])/Sqrt[b]])/(Sqrt[b]*Sqrt[c]*Sqrt[1 + (c*x)/b]) - (12*Sqrt[d]*(2*c
^2*d^2 - 3*b*c*d*e + b^2*e^2)*ArcTan[(Sqrt[-(c*d) + b*e]*Sqrt[x])/(Sqrt[d]*
Sqrt[b + c*x])])/(Sqrt[-(c*d) + b*e]*Sqrt[b + c*x])))/(4*e^4*Sqrt[x])
```

Maple [B] time = 0.216, size = 1569, normalized size = 7.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(3/2)/(e*x+d)^2,x)
```

```
[Out] 1/d/(b*e-c*d)/(d/e+x)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(
5/2)-1/d/(b*e-c*d)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2
)*b+1/e/(b*e-c*d)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)
*c-27/8/e^2*d/(b*e-c*d)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)
^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))*c^(1/2)*b^2+6/e^3*d^2/(b*e
-c*d)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(
d/e+x)-d*(b*e-c*d)/e^2)^(1/2))*c^(3/2)*b-6/e^3*d^2/(b*e-c*d)/(-d*(b*e-c*d)/
e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2
)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))
*b^2*c+15/2/e^4*d^3/(b*e-c*d)/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e
^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d
)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*b*c^2+3/2/e/(b*e-c*d)*(c*(d/e+
x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*x*b*c-3/2/e^2*d/(b*e-c*d)
*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*x*c^2-21/4/e^2*d
/(b*e-c*d)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*b*c+3/
8/e/(b*e-c*d)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*
c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/c^(1/2)*b^3-3/e^4*d^3/(b*e-c*d)*ln((
1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(
b*e-c*d)/e^2)^(1/2))*c^(5/2)+3/2/e^2*d/(b*e-c*d)/(-d*(b*e-c*d)/e^2)^(1/2)*l
n((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(
d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*b^3-3/e^5*d
^4/(b*e-c*d)/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*
(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*
e-c*d)/e^2)^(1/2))/(d/e+x))*c^3+9/4/e/(b*e-c*d)*(c*(d/e+x)^2+(b*e-2*c*d)/e*
(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*b^2+3/e^3*d^2/(b*e-c*d)*(c*(d/e+x)^2+(b*e-2*
c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*c^2-c/d/(b*e-c*d)*(c*(d/e+x)^2+(b*e-2
*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.96756, size = 2187, normalized size = 11.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(3*(8*c^2*d^3 - 8*b*c*d^2*e + b^2*d*e^2 + (8*c^2*d^2*e - 8*b*c*d*e^2 +
 b^2*e^3)*x)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 12*(2*c
^2*d^2 - b*c*d*e + (2*c^2*d*e - b*c*e^2)*x)*sqrt(c*d^2 - b*d*e)*log((b*d +
(2*c*d - b*e)*x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*x + d)) + 2*(
2*c^2*e^3*x^2 - 12*c^2*d^2*e + 9*b*c*d*e^2 - (6*c^2*d*e^2 - 5*b*c*e^3)*x)*s
qrt(c*x^2 + b*x))/(c*e^5*x + c*d*e^4), -1/8*(24*(2*c^2*d^2 - b*c*d*e + (2*c
^2*d*e - b*c*e^2)*x)*sqrt(-c*d^2 + b*d*e)*arctan(-sqrt(-c*d^2 + b*d*e)*sqrt
(c*x^2 + b*x)/((c*d - b*e)*x)) - 3*(8*c^2*d^3 - 8*b*c*d^2*e + b^2*d*e^2 + (
8*c^2*d^2*e - 8*b*c*d*e^2 + b^2*e^3)*x)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^
2 + b*x)*sqrt(c)) - 2*(2*c^2*e^3*x^2 - 12*c^2*d^2*e + 9*b*c*d*e^2 - (6*c^2*
d*e^2 - 5*b*c*e^3)*x)*sqrt(c*x^2 + b*x))/(c*e^5*x + c*d*e^4), -1/4*(3*(8*c^
2*d^3 - 8*b*c*d^2*e + b^2*d*e^2 + (8*c^2*d^2*e - 8*b*c*d*e^2 + b^2*e^3)*x)*
sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)) + 6*(2*c^2*d^2 - b*c*d*e
+ (2*c^2*d*e - b*c*e^2)*x)*sqrt(c*d^2 - b*d*e)*log((b*d + (2*c*d - b*e)*x +
 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*x + d)) - (2*c^2*e^3*x^2 - 12*
c^2*d^2*e + 9*b*c*d*e^2 - (6*c^2*d*e^2 - 5*b*c*e^3)*x)*sqrt(c*x^2 + b*x))/(
c*e^5*x + c*d*e^4), -1/4*(12*(2*c^2*d^2 - b*c*d*e + (2*c^2*d*e - b*c*e^2)*x
)*sqrt(-c*d^2 + b*d*e)*arctan(-sqrt(-c*d^2 + b*d*e)*sqrt(c*x^2 + b*x)/((c*d
 - b*e)*x)) + 3*(8*c^2*d^3 - 8*b*c*d^2*e + b^2*d*e^2 + (8*c^2*d^2*e - 8*b*c
*d*e^2 + b^2*e^3)*x)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)) - (2
*c^2*e^3*x^2 - 12*c^2*d^2*e + 9*b*c*d*e^2 - (6*c^2*d*e^2 - 5*b*c*e^3)*x)*sq
rt(c*x^2 + b*x))/(c*e^5*x + c*d*e^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(3/2)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.301 $\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^3} dx$

Optimal. Leaf size=205

$$\frac{3(b^2e^2-8bcde+8c^2d^2)\tanh^{-1}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{8\sqrt{d}e^4\sqrt{cd-be}}+\frac{3\sqrt{bx+cx^2}(-be+4cd+2cex)}{4e^3(d+ex)}-\frac{3\sqrt{c}(2cd-be)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{e^4}$$

```
[Out] (3*(4*c*d - b*e + 2*c*e*x)*Sqrt[b*x + c*x^2])/(4*e^3*(d + e*x)) - (b*x + c*
x^2)^(3/2)/(2*e*(d + e*x)^2) - (3*Sqrt[c]*(2*c*d - b*e)*ArcTanh[(Sqrt[c]*x)
/Sqrt[b*x + c*x^2]])/e^4 + (3*(8*c^2*d^2 - 8*b*c*d*e + b^2*e^2)*ArcTanh[(b*
d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/(8*Sqr
t[d]*e^4*Sqrt[c*d - b*e])
```

Rubi [A] time = 0.197579, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {732, 812, 843, 620, 206, 724}

$$\frac{3(b^2e^2-8bcde+8c^2d^2)\tanh^{-1}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{8\sqrt{d}e^4\sqrt{cd-be}}+\frac{3\sqrt{bx+cx^2}(-be+4cd+2cex)}{4e^3(d+ex)}-\frac{3\sqrt{c}(2cd-be)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(3/2)/(d + e*x)^3,x]
```

```
[Out] (3*(4*c*d - b*e + 2*c*e*x)*Sqrt[b*x + c*x^2])/(4*e^3*(d + e*x)) - (b*x + c*
x^2)^(3/2)/(2*e*(d + e*x)^2) - (3*Sqrt[c]*(2*c*d - b*e)*ArcTanh[(Sqrt[c]*x)
/Sqrt[b*x + c*x^2]])/e^4 + (3*(8*c^2*d^2 - 8*b*c*d*e + b^2*e^2)*ArcTanh[(b*
d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/(8*Sqr
t[d]*e^4*Sqrt[c*d - b*e])
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(bx+cx^2\right)^{3/2}}{(d+ex)^3}\,dx = -\frac{\left(bx+cx^2\right)^{3/2}}{2e(d+ex)^2}+\frac{3\int \frac{(b+2cx)\sqrt{bx+cx^2}}{(d+ex)^2}\,dx}{4e}$$

$$= \frac{3(4cd-be+2cex)\sqrt{bx+cx^2}}{4e^3(d+ex)}-\frac{\left(bx+cx^2\right)^{3/2}}{2e(d+ex)^2}-\frac{3\int \frac{b(4cd-be)+4c(2cd-be)x}{(d+ex)\sqrt{bx+cx^2}}\,dx}{8e^3}$$

$$= \frac{3(4cd-be+2cex)\sqrt{bx+cx^2}}{4e^3(d+ex)}-\frac{\left(bx+cx^2\right)^{3/2}}{2e(d+ex)^2}-\frac{(3c(2cd-be))\int \frac{1}{\sqrt{bx+cx^2}}\,dx}{2e^4}+\frac{\left(3\left(8c^2d^2-8bcde+\right.\right.}{}$$

$$= \frac{3(4cd-be+2cex)\sqrt{bx+cx^2}}{4e^3(d+ex)}-\frac{\left(bx+cx^2\right)^{3/2}}{2e(d+ex)^2}-\frac{(3c(2cd-be))\,\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{bx+cx^2}}\right)}{e^4}-\frac{(3(}{}$$

$$= \frac{3(4cd-be+2cex)\sqrt{bx+cx^2}}{4e^3(d+ex)}-\frac{\left(bx+cx^2\right)^{3/2}}{2e(d+ex)^2}-\frac{3\sqrt{c}(2cd-be)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{e^4}+\frac{3\left(8c^2d^2-8bcd\right.}{}$$

Mathematica [A] time = 1.26578, size = 205, normalized size = 1.

$$\frac{\sqrt{x(b+cx)}\left(\frac{3\left(b^2e^2-8bcde+8c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{x}\sqrt{be-cd}}{\sqrt{d}\sqrt{b+cx}}\right)}{\sqrt{d}\sqrt{b+cx}\sqrt{be-cd}}+\frac{e\sqrt{x}\left(2c\left(6d^2+9dex+2e^2x^2\right)-be(3d+5ex)\right)}{(d+ex)^2}+\frac{12\sqrt{c}(be-2cd)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{\frac{cx}{b}+1}}\right)}{4e^4\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(3/2)/(d + e*x)^3,x]
```

```
[Out] (Sqrt[x*(b + c*x)]*((e*Sqrt[x]*(-(b*e*(3*d + 5*e*x)) + 2*c*(6*d^2 + 9*d*e*x
 + 2*e^2*x^2)))/(d + e*x)^2 + (12*Sqrt[c]*(-2*c*d + b*e)*ArcSinh[(Sqrt[c]*S
qrt[x])/Sqrt[b]])/(Sqrt[b]*Sqrt[1 + (c*x)/b]) + (3*(8*c^2*d^2 - 8*b*c*d*e +
 b^2*e^2)*ArcTan[(Sqrt[-(c*d) + b*e]*Sqrt[x])/(Sqrt[d]*Sqrt[b + c*x])]))/(Sq
```

```
rt[d]*Sqrt[-(c*d) + b*e]*Sqrt[b + c*x]))))/(4*e^4*Sqrt[x])
```

Maple [B] time = 0.235, size = 3466, normalized size = 16.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(3/2)/(e*x+d)^3,x)
```

```
[Out] 3/2/e^3*c*d/(b*e-c*d)/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-
2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e
+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*b^2+1/2/e/(b*e-c*d)^2*(c*(d/e+x)^2+(b*
e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)*c^2-9/16/d/(b*e-c*d)^2*(c*(d/e+x)
^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*b^3-3/16/e/d/(b*e-c*d)*(c*(
d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*b^2-9/4/e^3*c^(3/2)*d
/(b*e-c*d)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d
)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))*b+9/8/e/(b*e-c*d)^2*(c*(d/e+x)^2+(b*e-2
*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*x*b*c^2+3/32/e/c^(1/2)/d/(b*e-c*d)*l
n((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-
d*(b*e-c*d)/e^2)^(1/2))*b^3-1/4*e/d^2/(b*e-c*d)^2/(d/e+x)*(c*(d/e+x)^2+(b*e
-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(5/2)*b+3/2/e^5*c^3*d^3/(b*e-c*d)/(-d*(b
*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-
c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/
(d/e+x))-3/8/d/(b*e-c*d)^2*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e
^2)^(1/2)*x*b^2*c-3/4/e^2*d/(b*e-c*d)^2*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-
d*(b*e-c*d)/e^2)^(1/2)*x*c^3-27/8/e^2*d/(b*e-c*d)^2*(c*(d/e+x)^2+(b*e-2*c*d
)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*b*c^2-51/16/e^2*d/(b*e-c*d)^2*ln((1/2*(b
*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*
d)/e^2)^(1/2))*c^(3/2)*b^2+15/4/e^3*d^2/(b*e-c*d)^2*ln((1/2*(b*e-2*c*d)/e+(
d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)
)*c^(5/2)*b-3/2/e^5*d^4/(b*e-c*d)^2/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-
c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e
-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*c^4+33/32/e/(b*e-c*d)^2*
ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)
-d*(b*e-c*d)/e^2)^(1/2))*c^(1/2)*b^3-3/32/d/(b*e-c*d)^2*ln((1/2*(b*e-2*c*d)
/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(
1/2))/c^(1/2)*b^4-3/4/d/(b*e-c*d)^2*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b
*e-c*d)/e^2)^(3/2)*b*c+1/2/d/(b*e-c*d)^2/(d/e+x)*(c*(d/e+x)^2+(b*e-2*c*d)/e
*(d/e+x)-d*(b*e-c*d)/e^2)^(5/2)*c-1/2/d/(b*e-c*d)^2*c^2*(c*(d/e+x)^2+(b*e-2
*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)*x+15/8/e^2*c/(b*e-c*d)*(c*(d/e+x)^2+
(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*b+9/16/e^2*c^(1/2)/(b*e-c*d)*l
n((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-
d*(b*e-c*d)/e^2)^(1/2))*b^2+1/2/e/d/(b*e-c*d)/(d/e+x)^2*(c*(d/e+x)^2+(b*e-2
*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(5/2)-3/2/e^4*d^3/(b*e-c*d)^2*ln((1/2*(b*e
-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)
/e^2)^(1/2))*c^(7/2)+3/4/e^2*c^2/(b*e-c*d)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+
x)-d*(b*e-c*d)/e^2)^(1/2)*x-3/8/e/(b*e-c*d)^2/(-d*(b*e-c*d)/e^2)^(1/2)*ln((
-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e
+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*b^4+39/16/e/(b
*e-c*d)^2*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*b^2*c+3
/2/e^4*c^(5/2)*d^2/(b*e-c*d)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d
/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))-1/2/e*c/d/(b*e-c*d)*(
c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)-3/2/e^3*c^2*d/(b*e
-c*d)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)+1/4*e/d^2/(
b*e-c*d)^2*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)*b^2+3/
2/e^3*d^2/(b*e-c*d)^2*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(
```

```
1/2)*c^3-3/e^4*c^2*d^2/(b*e-c*d)/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d
)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*
c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*b-39/8/e^3*d^2/(b*e-c*d)^2/
(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d
*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(
1/2))/(d/e+x))*b^2*c^2+1/4*e/d^2/(b*e-c*d)^2*c*(c*(d/e+x)^2+(b*e-2*c*d)/e*(
d/e+x)-d*(b*e-c*d)/e^2)^(3/2)*x*b+9/2/e^4*d^3/(b*e-c*d)^2/(-d*(b*e-c*d)/e^2
)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(
1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*c^
3*b+9/4/e^2*d/(b*e-c*d)^2/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(
b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*
(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*b^3*c-3/8/e*c/d/(b*e-c*d)*(c*(d/e+
x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*x*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.89429, size = 3626, normalized size = 17.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(12*(2*c^2*d^5 - 3*b*c*d^4*e + b^2*d^3*e^2 + (2*c^2*d^3*e^2 - 3*b*c*d
^2*e^3 + b^2*d*e^4)*x^2 + 2*(2*c^2*d^4*e - 3*b*c*d^3*e^2 + b^2*d^2*e^3)*x)*
sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 3*(8*c^2*d^4 - 8*b*c
*d^3*e + b^2*d^2*e^2 + (8*c^2*d^2*e^2 - 8*b*c*d*e^3 + b^2*e^4)*x^2 + 2*(8*c
^2*d^3*e - 8*b*c*d^2*e^2 + b^2*d*e^3)*x)*sqrt(c*d^2 - b*d*e)*log((b*d + (2*
c*d - b*e)*x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*x + d)) - 2*(12*
c^2*d^4*e - 15*b*c*d^3*e^2 + 3*b^2*d^2*e^3 + 4*(c^2*d^2*e^3 - b*c*d*e^4)*x^
2 + (18*c^2*d^3*e^2 - 23*b*c*d^2*e^3 + 5*b^2*d*e^4)*x)*sqrt(c*x^2 + b*x))/(
c*d^4*e^4 - b*d^3*e^5 + (c*d^2*e^6 - b*d*e^7)*x^2 + 2*(c*d^3*e^5 - b*d^2*e^
6)*x), 1/4*(3*(8*c^2*d^4 - 8*b*c*d^3*e + b^2*d^2*e^2 + (8*c^2*d^2*e^2 - 8*b
*c*d*e^3 + b^2*e^4)*x^2 + 2*(8*c^2*d^3*e - 8*b*c*d^2*e^2 + b^2*d*e^3)*x)*sq
rt(-c*d^2 + b*d*e)*arctan(-sqrt(-c*d^2 + b*d*e)*sqrt(c*x^2 + b*x)/((c*d - b
*e)*x)) - 6*(2*c^2*d^5 - 3*b*c*d^4*e + b^2*d^3*e^2 + (2*c^2*d^3*e^2 - 3*b*c
*d^2*e^3 + b^2*d*e^4)*x^2 + 2*(2*c^2*d^4*e - 3*b*c*d^3*e^2 + b^2*d^2*e^3)*x
)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) + (12*c^2*d^4*e - 15
*b*c*d^3*e^2 + 3*b^2*d^2*e^3 + 4*(c^2*d^2*e^3 - b*c*d*e^4)*x^2 + (18*c^2*d^
3*e^2 - 23*b*c*d^2*e^3 + 5*b^2*d*e^4)*x)*sqrt(c*x^2 + b*x))/(c*d^4*e^4 - b*
d^3*e^5 + (c*d^2*e^6 - b*d*e^7)*x^2 + 2*(c*d^3*e^5 - b*d^2*e^6)*x), 1/8*(24
*(2*c^2*d^5 - 3*b*c*d^4*e + b^2*d^3*e^2 + (2*c^2*d^3*e^2 - 3*b*c*d^2*e^3 +
b^2*d*e^4)*x^2 + 2*(2*c^2*d^4*e - 3*b*c*d^3*e^2 + b^2*d^2*e^3)*x)*sqrt(-c)*
arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)) + 3*(8*c^2*d^4 - 8*b*c*d^3*e + b^2
*d^2*e^2 + (8*c^2*d^2*e^2 - 8*b*c*d*e^3 + b^2*e^4)*x^2 + 2*(8*c^2*d^3*e - 8
*b*c*d^2*e^2 + b^2*d*e^3)*x)*sqrt(c*d^2 - b*d*e)*log((b*d + (2*c*d - b*e)*x
```

```
 + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*x + d)) + 2*(12*c^2*d^4*e -
15*b*c*d^3*e^2 + 3*b^2*d^2*e^3 + 4*(c^2*d^2*e^3 - b*c*d*e^4)*x^2 + (18*c^2*
d^3*e^2 - 23*b*c*d^2*e^3 + 5*b^2*d*e^4)*x)*sqrt(c*x^2 + b*x))/(c*d^4*e^4 -
b*d^3*e^5 + (c*d^2*e^6 - b*d*e^7)*x^2 + 2*(c*d^3*e^5 - b*d^2*e^6)*x), 1/4*(
3*(8*c^2*d^4 - 8*b*c*d^3*e + b^2*d^2*e^2 + (8*c^2*d^2*e^2 - 8*b*c*d*e^3 + b
^2*e^4)*x^2 + 2*(8*c^2*d^3*e - 8*b*c*d^2*e^2 + b^2*d*e^3)*x)*sqrt(-c*d^2 +
b*d*e)*arctan(-sqrt(-c*d^2 + b*d*e)*sqrt(c*x^2 + b*x)/((c*d - b*e)*x)) + 12
*(2*c^2*d^5 - 3*b*c*d^4*e + b^2*d^3*e^2 + (2*c^2*d^3*e^2 - 3*b*c*d^2*e^3 +
b^2*d*e^4)*x^2 + 2*(2*c^2*d^4*e - 3*b*c*d^3*e^2 + b^2*d^2*e^3)*x)*sqrt(-c)*
arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)) + (12*c^2*d^4*e - 15*b*c*d^3*e^2 +
 3*b^2*d^2*e^3 + 4*(c^2*d^2*e^3 - b*c*d*e^4)*x^2 + (18*c^2*d^3*e^2 - 23*b*c
*d^2*e^3 + 5*b^2*d*e^4)*x)*sqrt(c*x^2 + b*x))/(c*d^4*e^4 - b*d^3*e^5 + (c*d
^2*e^6 - b*d*e^7)*x^2 + 2*(c*d^3*e^5 - b*d^2*e^6)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(3/2)/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.32572, size = 675, normalized size = 3.29

$$\sqrt{cx^2+bx}\,ce^{(-3)} + \frac{3\left(8\,c^2d^2 - 8\,bcde + b^2e^2\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde}}\right)e^{(-4)}}{4\,\sqrt{-cd^2+bde}} + \frac{3\left(2\,c^2d - bce\right)e^{(-4)}\log\left(\left|-2\left(\sqrt{c}x - \right.\right.\right.}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] sqrt(c*x^2 + b*x)*c*e^(-3) + 3/4*(8*c^2*d^2 - 8*b*c*d*e + b^2*e^2)*arctan(-
((sqrt(c)*x - sqrt(c*x^2 + b*x))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e))*e^(-4
)/sqrt(-c*d^2 + b*d*e) + 3/2*(2*c^2*d - b*c*e)*e^(-4)*log(abs(-2*(sqrt(c)*x
 - sqrt(c*x^2 + b*x))*sqrt(c) - b))/sqrt(c) + 1/4*(24*(sqrt(c)*x - sqrt(c*x
^2 + b*x))^3*c^2*d^2*e + 40*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*c^(5/2)*d^3 -
 24*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*b*c^(3/2)*d^2*e + 40*(sqrt(c)*x - sqr
t(c*x^2 + b*x))*b*c^2*d^3 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b*c*d*e^2
- 28*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b^2*c*d^2*e + 10*b^2*c^(3/2)*d^3 - (sq
rt(c)*x - sqrt(c*x^2 + b*x))^2*b^2*sqrt(c)*d*e^2 - 5*b^3*sqrt(c)*d^2*e + 5*
(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b^2*e^3 + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x
))*b^3*d*e^2)*e^(-4)/((sqrt(c)*x - sqrt(c*x^2 + b*x))^2*e + 2*(sqrt(c)*x -
sqrt(c*x^2 + b*x))*sqrt(c)*d + b*d)^2
```

3.302 $\int (d+ex)^3 \left(bx+cx^2\right)^{5/2} dx$

Optimal. Leaf size=332

$$\frac{5b^4(b+2cx)\sqrt{bx+cx^2}(2cd-be)\left(11b^2e^2-32bcde+32c^2d^2\right)}{32768c^6}-\frac{5b^2(b+2cx)\left(bx+cx^2\right)^{3/2}(2cd-be)\left(11b^2e^2-32bcde+3\right.}{12288c^5}$$

```
[Out] (5*b^4*(2*c*d - b*e)*(32*c^2*d^2 - 32*b*c*d*e + 11*b^2*e^2)*(b + 2*c*x)*Sqr
t[b*x + c*x^2])/(32768*c^6) - (5*b^2*(2*c*d - b*e)*(32*c^2*d^2 - 32*b*c*d*e
 + 11*b^2*e^2)*(b + 2*c*x)*(b*x + c*x^2)^(3/2))/(12288*c^5) + ((2*c*d - b*e
)*(32*c^2*d^2 - 32*b*c*d*e + 11*b^2*e^2)*(b + 2*c*x)*(b*x + c*x^2)^(5/2))/(
768*c^4) + (e*(d + e*x)^2*(b*x + c*x^2)^(7/2))/(9*c) + (e*(640*c^2*d^2 - 48
6*b*c*d*e + 99*b^2*e^2 + 154*c*e*(2*c*d - b*e)*x)*(b*x + c*x^2)^(7/2))/(201
6*c^3) - (5*b^6*(2*c*d - b*e)*(32*c^2*d^2 - 32*b*c*d*e + 11*b^2*e^2)*ArcTan
h[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(32768*c^(13/2))
```

Rubi [A] time = 0.454318, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {742, 779, 612, 620, 206}

$$\frac{5b^4(b+2cx)\sqrt{bx+cx^2}(2cd-be)\left(11b^2e^2-32bcde+32c^2d^2\right)}{32768c^6}-\frac{5b^2(b+2cx)\left(bx+cx^2\right)^{3/2}(2cd-be)\left(11b^2e^2-32bcde+3\right.}{12288c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(b*x + c*x^2)^(5/2),x]
```


```
[Out] (5*b^4*(2*c*d - b*e)*(32*c^2*d^2 - 32*b*c*d*e + 11*b^2*e^2)*(b + 2*c*x)*Sqr
t[b*x + c*x^2])/(32768*c^6) - (5*b^2*(2*c*d - b*e)*(32*c^2*d^2 - 32*b*c*d*e
 + 11*b^2*e^2)*(b + 2*c*x)*(b*x + c*x^2)^(3/2))/(12288*c^5) + ((2*c*d - b*e
)*(32*c^2*d^2 - 32*b*c*d*e + 11*b^2*e^2)*(b + 2*c*x)*(b*x + c*x^2)^(5/2))/(
768*c^4) + (e*(d + e*x)^2*(b*x + c*x^2)^(7/2))/(9*c) + (e*(640*c^2*d^2 - 48
6*b*c*d*e + 99*b^2*e^2 + 154*c*e*(2*c*d - b*e)*x)*(b*x + c*x^2)^(7/2))/(201
6*c^3) - (5*b^6*(2*c*d - b*e)*(32*c^2*d^2 - 32*b*c*d*e + 11*b^2*e^2)*ArcTan
h[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(32768*c^(13/2))
```


Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^3 \left(bx+cx^2\right)^{5/2} dx = \frac{e(d+ex)^2\left(bx+cx^2\right)^{7/2}}{9c} + \frac{\int (d+ex)\left(\frac{1}{2}d(18cd-7be)+\frac{11}{2}e(2cd-be)x\right)\left(bx+cx^2\right)^{5/2} d}{9c}$$

$$= \frac{e(d+ex)^2\left(bx+cx^2\right)^{7/2}}{9c} + \frac{e\left(640c^2d^2-486bcde+99b^2e^2+154ce(2cd-be)x\right)\left(bx+cx\right.}{2016c^3}$$

$$= \frac{(2cd-be)\left(32c^2d^2-32bcde+11b^2e^2\right)(b+2cx)\left(bx+cx^2\right)^{5/2}}{768c^4} + \frac{e(d+ex)^2\left(bx+cx^2\right)^{7/}}{9c}$$

$$= -\frac{5b^2(2cd-be)\left(32c^2d^2-32bcde+11b^2e^2\right)(b+2cx)\left(bx+cx^2\right)^{3/2}}{12288c^5} + \frac{(2cd-be)\left(32c^2d\right.}{}$$

$$= \frac{5b^4(2cd-be)\left(32c^2d^2-32bcde+11b^2e^2\right)(b+2cx)\sqrt{bx+cx^2}}{32768c^6} - \frac{5b^2(2cd-be)\left(32c^2d^2\right.}{}$$

$$= \frac{5b^4(2cd-be)\left(32c^2d^2-32bcde+11b^2e^2\right)(b+2cx)\sqrt{bx+cx^2}}{32768c^6} - \frac{5b^2(2cd-be)\left(32c^2d^2\right.}{}$$

$$= \frac{5b^4(2cd-be)\left(32c^2d^2-32bcde+11b^2e^2\right)(b+2cx)\sqrt{bx+cx^2}}{32768c^6} - \frac{5b^2(2cd-be)\left(32c^2d^2\right.}{}$$

Mathematica [A] time = 0.893081, size = 395, normalized size = 1.19

$$\frac{\sqrt{x(b+cx)}\left(\sqrt{c}\left(-84b^6c^2e\left(360d^2+135dex+22e^2x^2\right)+144b^5c^3\left(140d^2ex+140d^3+63de^2x^2+11e^3x^3\right)-32b^4c^4x\left(50\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(b*x + c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[x*(b + c*x)]*(Sqrt[c]*(-3465*b^8*e^3 + 210*b^7*c*e^2*(81*d + 11*e*x)
- 84*b^6*c^2*e*(360*d^2 + 135*d*e*x + 22*e^2*x^2) + 256*b^3*c^5*x^2*(42*d^3
 + 54*d^2*e*x + 27*d*e^2*x^2 + 5*e^3*x^3) + 144*b^5*c^3*(140*d^3 + 140*d^2*
e*x + 63*d*e^2*x^2 + 11*e^3*x^3) - 32*b^4*c^4*x*(420*d^3 + 504*d^2*e*x + 24
3*d*e^2*x^2 + 44*e^3*x^3) + 4096*c^8*x^5*(84*d^3 + 216*d^2*e*x + 189*d*e^2*
x^2 + 56*e^3*x^3) + 1536*b^2*c^6*x^3*(378*d^3 + 888*d^2*e*x + 729*d*e^2*x^2
 + 206*e^3*x^3) + 2048*b*c^7*x^4*(420*d^3 + 1044*d^2*e*x + 891*d*e^2*x^2 +
```

```
259*e^3*x^3)) + (315*b^(11/2)*(-64*c^3*d^3 + 96*b*c^2*d^2*e - 54*b^2*c*d*e^
2 + 11*b^3*e^3)*ArcSinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(Sqrt[x]*Sqrt[1 + (c*x)
/b])))/(2064384*c^(13/2))
```

Maple [B] time = 0.056, size = 813, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x)^(5/2),x)
```

```
[Out] 9/64*d*e^2*b^2/c^2*x*(c*x^2+b*x)^(5/2)+1/12*d^3/c*(c*x^2+b*x)^(5/2)*b-5/192
*d^3*b^3/c^2*(c*x^2+b*x)^(3/2)+5/512*d^3*b^5/c^3*(c*x^2+b*x)^(1/2)-5/1024*d
^3*b^6/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))+1/9*e^3*x^2*(c*x^2
+b*x)^(7/2)/c+11/224*e^3*b^2/c^3*(c*x^2+b*x)^(7/2)-11/768*e^3*b^4/c^4*(c*x^
2+b*x)^(5/2)+55/12288*e^3*b^6/c^5*(c*x^2+b*x)^(3/2)-55/32768*e^3*b^8/c^6*(c
*x^2+b*x)^(1/2)-45/1024*d*e^2*b^4/c^3*(c*x^2+b*x)^(3/2)*x+135/8192*d*e^2*b^
6/c^4*(c*x^2+b*x)^(1/2)*x-1/4*d^2*e*b/c*x*(c*x^2+b*x)^(5/2)+5/64*d^2*e*b^3/
c^2*(c*x^2+b*x)^(3/2)*x-15/512*d^2*e*b^5/c^3*(c*x^2+b*x)^(1/2)*x-15/1024*d^
2*e*b^6/c^4*(c*x^2+b*x)^(1/2)+15/2048*d^2*e*b^7/c^(9/2)*ln((1/2*b+c*x)/c^(1
/2)+(c*x^2+b*x)^(1/2))-5/96*d^3*b^2/c*(c*x^2+b*x)^(3/2)*x+5/256*d^3*b^4/c^2
*(c*x^2+b*x)^(1/2)*x+3/8*d*e^2*x*(c*x^2+b*x)^(7/2)/c-27/112*d*e^2*b/c^2*(c*
x^2+b*x)^(7/2)+9/128*d*e^2*b^3/c^3*(c*x^2+b*x)^(5/2)-45/2048*d*e^2*b^5/c^4*
(c*x^2+b*x)^(3/2)+135/16384*d*e^2*b^7/c^5*(c*x^2+b*x)^(1/2)-135/32768*d*e^2
*b^8/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))-1/8*d^2*e*b^2/c^2*(
c*x^2+b*x)^(5/2)+5/128*d^2*e*b^4/c^3*(c*x^2+b*x)^(3/2)-11/144*e^3*b/c^2*x*(
c*x^2+b*x)^(7/2)-11/384*e^3*b^3/c^3*x*(c*x^2+b*x)^(5/2)+55/65536*e^3*b^9/c^
(13/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))+3/7*d^2*e*(c*x^2+b*x)^(7/2
)/c+55/6144*e^3*b^5/c^4*(c*x^2+b*x)^(3/2)*x-55/16384*e^3*b^7/c^5*(c*x^2+b*x
)^(1/2)*x+1/6*d^3*x*(c*x^2+b*x)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.2217, size = 2109, normalized size = 6.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/4128768*(315*(64*b^6*c^3*d^3 - 96*b^7*c^2*d^2*e + 54*b^8*c*d*e^2 - 11*b
^9*e^3)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 2*(229376*c^
```

```
9*e^3*x^8 + 20160*b^5*c^4*d^3 - 30240*b^6*c^3*d^2*e + 17010*b^7*c^2*d*e^2 -
 3465*b^8*c*e^3 + 14336*(54*c^9*d*e^2 + 37*b*c^8*e^3)*x^7 + 3072*(288*c^9*d
^2*e + 594*b*c^8*d*e^2 + 103*b^2*c^7*e^3)*x^6 + 256*(1344*c^9*d^3 + 8352*b*
c^8*d^2*e + 4374*b^2*c^7*d*e^2 + 5*b^3*c^6*e^3)*x^5 + 128*(6720*b*c^8*d^3 +
 10656*b^2*c^7*d^2*e + 54*b^3*c^6*d*e^2 - 11*b^4*c^5*e^3)*x^4 + 144*(4032*b
^2*c^7*d^3 + 96*b^3*c^6*d^2*e - 54*b^4*c^5*d*e^2 + 11*b^5*c^4*e^3)*x^3 + 16
8*(64*b^3*c^6*d^3 - 96*b^4*c^5*d^2*e + 54*b^5*c^4*d*e^2 - 11*b^6*c^3*e^3)*x
^2 - 210*(64*b^4*c^5*d^3 - 96*b^5*c^4*d^2*e + 54*b^6*c^3*d*e^2 - 11*b^7*c^2
*e^3)*x)*sqrt(c*x^2 + b*x))/c^7, 1/2064384*(315*(64*b^6*c^3*d^3 - 96*b^7*c^
2*d^2*e + 54*b^8*c*d*e^2 - 11*b^9*e^3)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sq
rt(-c)/(c*x)) + (229376*c^9*e^3*x^8 + 20160*b^5*c^4*d^3 - 30240*b^6*c^3*d^2
*e + 17010*b^7*c^2*d*e^2 - 3465*b^8*c*e^3 + 14336*(54*c^9*d*e^2 + 37*b*c^8*
e^3)*x^7 + 3072*(288*c^9*d^2*e + 594*b*c^8*d*e^2 + 103*b^2*c^7*e^3)*x^6 + 2
56*(1344*c^9*d^3 + 8352*b*c^8*d^2*e + 4374*b^2*c^7*d*e^2 + 5*b^3*c^6*e^3)*x
^5 + 128*(6720*b*c^8*d^3 + 10656*b^2*c^7*d^2*e + 54*b^3*c^6*d*e^2 - 11*b^4*
c^5*e^3)*x^4 + 144*(4032*b^2*c^7*d^3 + 96*b^3*c^6*d^2*e - 54*b^4*c^5*d*e^2
+ 11*b^5*c^4*e^3)*x^3 + 168*(64*b^3*c^6*d^3 - 96*b^4*c^5*d^2*e + 54*b^5*c^4
*d*e^2 - 11*b^6*c^3*e^3)*x^2 - 210*(64*b^4*c^5*d^3 - 96*b^5*c^4*d^2*e + 54*
b^6*c^3*d*e^2 - 11*b^7*c^2*e^3)*x)*sqrt(c*x^2 + b*x))/c^7]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (x(b+cx))^{\frac{5}{2}}(d+ex)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x)**(5/2),x)
```

```
[Out] Integral((x*(b + c*x))**(5/2)*(d + e*x)**3, x)
```

Giac [A] time = 1.73549, size = 648, normalized size = 1.95

$$\frac{1}{2064384}\sqrt{cx^2+bx}\left(2\left(4\left(2\left(8\left(2\left(4\left(14\left(16c^2xe^3+\frac{54c^{10}de^2+37bc^9e^3}{c^8}\right)x+\frac{3\left(288c^{10}d^2e+594bc^9de^2+103b^2c^8e^3\right)}{c^8}\right.\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2064384*sqrt(c*x^2 + b*x)*(2*(4*(2*(8*(2*(4*(14*(16*c^2*x*e^3 + (54*c^10*
d*e^2 + 37*b*c^9*e^3)/c^8)*x + 3*(288*c^10*d^2*e + 594*b*c^9*d*e^2 + 103*b^
2*c^8*e^3)/c^8)*x + (1344*c^10*d^3 + 8352*b*c^9*d^2*e + 4374*b^2*c^8*d*e^2
+ 5*b^3*c^7*e^3)/c^8)*x + (6720*b*c^9*d^3 + 10656*b^2*c^8*d^2*e + 54*b^3*c^
7*d*e^2 - 11*b^4*c^6*e^3)/c^8)*x + 9*(4032*b^2*c^8*d^3 + 96*b^3*c^7*d^2*e -
 54*b^4*c^6*d*e^2 + 11*b^5*c^5*e^3)/c^8)*x + 21*(64*b^3*c^7*d^3 - 96*b^4*c^
6*d^2*e + 54*b^5*c^5*d*e^2 - 11*b^6*c^4*e^3)/c^8)*x - 105*(64*b^4*c^6*d^3 -
 96*b^5*c^5*d^2*e + 54*b^6*c^4*d*e^2 - 11*b^7*c^3*e^3)/c^8)*x + 315*(64*b^5
*c^5*d^3 - 96*b^6*c^4*d^2*e + 54*b^7*c^3*d*e^2 - 11*b^8*c^2*e^3)/c^8) + 5/6
5536*(64*b^6*c^3*d^3 - 96*b^7*c^2*d^2*e + 54*b^8*c*d*e^2 - 11*b^9*e^3)*log(
abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b))/c^(13/2)
```

3.303 $\int (d+ex)^2 \left(bx+cx^2\right)^{5/2} dx$

Optimal. Leaf size=266

$$\frac{5b^4(b+2cx)\sqrt{bx+cx^2}\left(9b^2e^2-32bcde+32c^2d^2\right)}{16384c^5}-\frac{5b^2(b+2cx)\left(bx+cx^2\right)^{3/2}\left(9b^2e^2-32bcde+32c^2d^2\right)}{6144c^4}+\frac{(b+2cx)\left(b\right.}{}$$

```
[Out] (5*b^4*(32*c^2*d^2 - 32*b*c*d*e + 9*b^2*e^2)*(b + 2*c*x)*Sqrt[b*x + c*x^2])
/(16384*c^5) - (5*b^2*(32*c^2*d^2 - 32*b*c*d*e + 9*b^2*e^2)*(b + 2*c*x)*(b*
x + c*x^2)^(3/2))/(6144*c^4) + ((32*c^2*d^2 - 32*b*c*d*e + 9*b^2*e^2)*(b +
2*c*x)*(b*x + c*x^2)^(5/2))/(384*c^3) + (9*e*(2*c*d - b*e)*(b*x + c*x^2)^(7
/2))/(112*c^2) + (e*(d + e*x)*(b*x + c*x^2)^(7/2))/(8*c) - (5*b^6*(32*c^2*d
^2 - 32*b*c*d*e + 9*b^2*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(16384
*c^(11/2))
```

Rubi [A] time = 0.220245, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {742, 640, 612, 620, 206}

$$\frac{5b^4(b+2cx)\sqrt{bx+cx^2}\left(9b^2e^2-32bcde+32c^2d^2\right)}{16384c^5}-\frac{5b^2(b+2cx)\left(bx+cx^2\right)^{3/2}\left(9b^2e^2-32bcde+32c^2d^2\right)}{6144c^4}+\frac{(b+2cx)\left(b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(b*x + c*x^2)^(5/2),x]
```

```
[Out] (5*b^4*(32*c^2*d^2 - 32*b*c*d*e + 9*b^2*e^2)*(b + 2*c*x)*Sqrt[b*x + c*x^2])
/(16384*c^5) - (5*b^2*(32*c^2*d^2 - 32*b*c*d*e + 9*b^2*e^2)*(b + 2*c*x)*(b*
x + c*x^2)^(3/2))/(6144*c^4) + ((32*c^2*d^2 - 32*b*c*d*e + 9*b^2*e^2)*(b +
2*c*x)*(b*x + c*x^2)^(5/2))/(384*c^3) + (9*e*(2*c*d - b*e)*(b*x + c*x^2)^(7
/2))/(112*c^2) + (e*(d + e*x)*(b*x + c*x^2)^(7/2))/(8*c) - (5*b^6*(32*c^2*d
^2 - 32*b*c*d*e + 9*b^2*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(16384
*c^(11/2))
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
```

```
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^2\left(bx+cx^2\right)^{5/2}dx = \frac{e(d+ex)\left(bx+cx^2\right)^{7/2}}{8c}+\frac{\int\left(\frac{1}{2}d(16cd-7be)+\frac{9}{2}e(2cd-be)x\right)\left(bx+cx^2\right)^{5/2}dx}{8c}$$

$$=\frac{9e(2cd-be)\left(bx+cx^2\right)^{7/2}}{112c^2}+\frac{e(d+ex)\left(bx+cx^2\right)^{7/2}}{8c}+\frac{\left(cd(16cd-7be)-\frac{9}{2}be(2cd-be)\right)}{16c^2}$$

$$=\frac{\left(32c^2d^2-32bcde+9b^2e^2\right)(b+2cx)\left(bx+cx^2\right)^{5/2}}{384c^3}+\frac{9e(2cd-be)\left(bx+cx^2\right)^{7/2}}{112c^2}+\frac{e(d+}{}$$

$$=-\frac{5b^2\left(32c^2d^2-32bcde+9b^2e^2\right)(b+2cx)\left(bx+cx^2\right)^{3/2}}{6144c^4}+\frac{\left(32c^2d^2-32bcde+9b^2e^2\right)(b}{384c^3}$$

$$=\frac{5b^4\left(32c^2d^2-32bcde+9b^2e^2\right)(b+2cx)\sqrt{bx+cx^2}}{16384c^5}-\frac{5b^2\left(32c^2d^2-32bcde+9b^2e^2\right)(b}{6144c^4}$$

$$=\frac{5b^4\left(32c^2d^2-32bcde+9b^2e^2\right)(b+2cx)\sqrt{bx+cx^2}}{16384c^5}-\frac{5b^2\left(32c^2d^2-32bcde+9b^2e^2\right)(b}{6144c^4}$$

$$=\frac{5b^4\left(32c^2d^2-32bcde+9b^2e^2\right)(b+2cx)\sqrt{bx+cx^2}}{16384c^5}-\frac{5b^2\left(32c^2d^2-32bcde+9b^2e^2\right)(b}{6144c^4}$$

Mathematica [A] time = 0.591327, size = 219, normalized size = 0.82

$$\frac{(x(b+cx))^{5/2}\left(\frac{\left(9b^2e^2-32bcde+32c^2d^2\right)\left(\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\left(8b^3c^2x^2+432b^2c^3x^3-10b^4cx+15b^5+640bc^4x^4+256c^5x^5\right)-15b^{11/2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)\right)}{6144c^{9/2}(b+cx)^2\sqrt{\frac{cx}{b}+1}}+\frac{9ex^{7/2}(b+cx)}{14c}\right.}{8cx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(b*x + c*x^2)^(5/2),x]
```

```
[Out] ((x*(b + c*x))^(5/2)*((9*e*(2*c*d - b*e)*x^(7/2)*(b + c*x))/(14*c) + e*x^(7
/2)*(b + c*x)*(d + e*x) + ((32*c^2*d^2 - 32*b*c*d*e + 9*b^2*e^2)*(Sqrt[c]*S
qrt[x]*Sqrt[1 + (c*x)/b]*(15*b^5 - 10*b^4*c*x + 8*b^3*c^2*x^2 + 432*b^2*c^3
*x^3 + 640*b*c^4*x^4 + 256*c^5*x^5) - 15*b^(11/2)*ArcSinh[(Sqrt[c]*Sqrt[x])
/Sqrt[b]]))/(6144*c^(9/2)*(b + c*x)^2*Sqrt[1 + (c*x)/b])))/(8*c*x^(5/2))
```

Maple [B] time = 0.052, size = 553, normalized size = 2.1

$$\frac{e^2x}{8c}\left(cx^2+bx\right)^{\frac{7}{2}}-\frac{9e^2b}{112c^2}\left(cx^2+bx\right)^{\frac{7}{2}}+\frac{3b^2e^2x}{64c^2}\left(cx^2+bx\right)^{\frac{5}{2}}+\frac{3b^3e^2}{128c^3}\left(cx^2+bx\right)^{\frac{5}{2}}-\frac{15e^2b^4x}{1024c^3}\left(cx^2+bx\right)^{\frac{3}{2}}-\frac{15e^2b^5}{2048c^4}($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+b*x)^(5/2),x)
```

```
[Out] 1/8*e^2*x*(c*x^2+b*x)^(7/2)/c-9/112*e^2*b/c^2*(c*x^2+b*x)^(7/2)+3/64*e^2*b^
2/c^2*x*(c*x^2+b*x)^(5/2)+3/128*e^2*b^3/c^3*(c*x^2+b*x)^(5/2)-15/1024*e^2*b
^4/c^3*(c*x^2+b*x)^(3/2)*x-15/2048*e^2*b^5/c^4*(c*x^2+b*x)^(3/2)+45/8192*e^
2*b^6/c^4*(c*x^2+b*x)^(1/2)*x+45/16384*e^2*b^7/c^5*(c*x^2+b*x)^(1/2)-45/327
68*e^2*b^8/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))+2/7*d*e*(c*x^
2+b*x)^(7/2)/c-1/6*d*e*b/c*x*(c*x^2+b*x)^(5/2)-1/12*d*e*b^2/c^2*(c*x^2+b*x)
^(5/2)+5/96*d*e*b^3/c^2*(c*x^2+b*x)^(3/2)*x+5/192*d*e*b^4/c^3*(c*x^2+b*x)^(
3/2)-5/256*d*e*b^5/c^3*(c*x^2+b*x)^(1/2)*x-5/512*d*e*b^6/c^4*(c*x^2+b*x)^(1
/2)+5/1024*d*e*b^7/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))+1/6*d^
2*x*(c*x^2+b*x)^(5/2)+1/12*d^2/c*(c*x^2+b*x)^(5/2)*b-5/96*d^2*b^2/c*(c*x^2+
b*x)^(3/2)*x-5/192*d^2*b^3/c^2*(c*x^2+b*x)^(3/2)+5/256*d^2*b^4/c^2*(c*x^2+b
*x)^(1/2)*x+5/512*d^2*b^5/c^3*(c*x^2+b*x)^(1/2)-5/1024*d^2*b^6/c^(7/2)*ln((
1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.16372, size = 1481, normalized size = 5.57

$$\left[\frac{105\left(32\,b^6c^2d^2 - 32\,b^7cde + 9\,b^8e^2\right)\sqrt{c}\log\left(2\,cx + b - 2\,\sqrt{cx^2 + bx}\sqrt{c}\right) + 2\left(43008\,c^8e^2x^7 + 3360\,b^5c^3d^2 - 3360\,b^6c^2de + \cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/688128*(105*(32*b^6*c^2*d^2 - 32*b^7*c*d*e + 9*b^8*e^2)*sqrt(c)*log(2*c*
x + b - 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*(43008*c^8*e^2*x^7 + 3360*b^5*c^3*
d^2 - 3360*b^6*c^2*d*e + 945*b^7*c*e^2 + 3072*(32*c^8*d*e + 33*b*c^7*e^2)*x
^6 + 256*(224*c^8*d^2 + 928*b*c^7*d*e + 243*b^2*c^6*e^2)*x^5 + 128*(1120*b*
c^7*d^2 + 1184*b^2*c^6*d*e + 3*b^3*c^5*e^2)*x^4 + 48*(2016*b^2*c^6*d^2 + 32
*b^3*c^5*d*e - 9*b^4*c^4*e^2)*x^3 + 56*(32*b^3*c^5*d^2 - 32*b^4*c^4*d*e + 9
*b^5*c^3*e^2)*x^2 - 70*(32*b^4*c^4*d^2 - 32*b^5*c^3*d*e + 9*b^6*c^2*e^2)*x)
*sqrt(c*x^2 + b*x))/c^6, 1/344064*(105*(32*b^6*c^2*d^2 - 32*b^7*c*d*e + 9*b
^8*e^2)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)) + (43008*c^8*e^2*
x^7 + 3360*b^5*c^3*d^2 - 3360*b^6*c^2*d*e + 945*b^7*c*e^2 + 3072*(32*c^8*d*
e + 33*b*c^7*e^2)*x^6 + 256*(224*c^8*d^2 + 928*b*c^7*d*e + 243*b^2*c^6*e^2)
*x^5 + 128*(1120*b*c^7*d^2 + 1184*b^2*c^6*d*e + 3*b^3*c^5*e^2)*x^4 + 48*(20
16*b^2*c^6*d^2 + 32*b^3*c^5*d*e - 9*b^4*c^4*e^2)*x^3 + 56*(32*b^3*c^5*d^2 -
 32*b^4*c^4*d*e + 9*b^5*c^3*e^2)*x^2 - 70*(32*b^4*c^4*d^2 - 32*b^5*c^3*d*e
+ 9*b^6*c^2*e^2)*x)*sqrt(c*x^2 + b*x))/c^6]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (x(b+cx))^{\frac{5}{2}} (d+ex)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+b*x)**(5/2),x)
```

```
[Out] Integral((x*(b + c*x))**(5/2)*(d + e*x)**2, x)
```

Giac [A] time = 1.89956, size = 473, normalized size = 1.78

$$\frac{1}{344064}\sqrt{cx^2+bx}\left(2\left(4\left(2\left(8\left(2\left(12\left(14\,c^2xe^2+\frac{32\,c^9de+33\,bc^8e^2}{c^7}\right)x+\frac{224\,c^9d^2+928\,bc^8de+243\,b^2c^7e^2}{c^7}\right)x+\frac{1120\,b\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/344064*sqrt(c*x^2 + b*x)*(2*(4*(2*(8*(2*(12*(14*c^2*x*e^2 + (32*c^9*d*e +
 33*b*c^8*e^2)/c^7)*x + (224*c^9*d^2 + 928*b*c^8*d*e + 243*b^2*c^7*e^2)/c^7
)*x + (1120*b*c^8*d^2 + 1184*b^2*c^7*d*e + 3*b^3*c^6*e^2)/c^7)*x + 3*(2016*
b^2*c^7*d^2 + 32*b^3*c^6*d*e - 9*b^4*c^5*e^2)/c^7)*x + 7*(32*b^3*c^6*d^2 -
32*b^4*c^5*d*e + 9*b^5*c^4*e^2)/c^7)*x - 35*(32*b^4*c^5*d^2 - 32*b^5*c^4*d*
e + 9*b^6*c^3*e^2)/c^7)*x + 105*(32*b^5*c^4*d^2 - 32*b^6*c^3*d*e + 9*b^7*c^
2*e^2)/c^7) + 5/32768*(32*b^6*c^2*d^2 - 32*b^7*c*d*e + 9*b^8*e^2)*log(abs(-
2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b))/c^(11/2)
```

3.304 $\int (d+ex)\left(bx+cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=175

$$\frac{5b^4(b+2cx)\sqrt{bx+cx^2}(2cd-be)}{1024c^4}-\frac{5b^2(b+2cx)\left(bx+cx^2\right)^{3/2}(2cd-be)}{384c^3}-\frac{5b^6(2cd-be)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{1024c^{9/2}}+\frac{(b+2cx)\left(b\right.}{}$$

```
[Out] (5*b^4*(2*c*d - b*e)*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(1024*c^4) - (5*b^2*(2*
c*d - b*e)*(b + 2*c*x)*(b*x + c*x^2)^(3/2))/(384*c^3) + ((2*c*d - b*e)*(b +
 2*c*x)*(b*x + c*x^2)^(5/2))/(24*c^2) + (e*(b*x + c*x^2)^(7/2))/(7*c) - (5*
b^6*(2*c*d - b*e)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(1024*c^(9/2))
```

Rubi [A] time = 0.0737285, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {640, 612, 620, 206}

$$\frac{5b^4(b+2cx)\sqrt{bx+cx^2}(2cd-be)}{1024c^4}-\frac{5b^2(b+2cx)\left(bx+cx^2\right)^{3/2}(2cd-be)}{384c^3}-\frac{5b^6(2cd-be)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{1024c^{9/2}}+\frac{(b+2cx)\left(b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(b*x + c*x^2)^(5/2),x]
```


```
[Out] (5*b^4*(2*c*d - b*e)*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(1024*c^4) - (5*b^2*(2*
c*d - b*e)*(b + 2*c*x)*(b*x + c*x^2)^(3/2))/(384*c^3) + ((2*c*d - b*e)*(b +
 2*c*x)*(b*x + c*x^2)^(5/2))/(24*c^2) + (e*(b*x + c*x^2)^(7/2))/(7*c) - (5*
b^6*(2*c*d - b*e)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(1024*c^(9/2))
```


Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)\left(bx+cx^2\right)^{5/2} dx = \frac{e\left(bx+cx^2\right)^{7/2}}{7c} + \frac{(2cd-be)\int \left(bx+cx^2\right)^{5/2} dx}{2c}$$
$$= \frac{(2cd-be)(b+2cx)\left(bx+cx^2\right)^{5/2}}{24c^2} + \frac{e\left(bx+cx^2\right)^{7/2}}{7c} - \frac{\left(5b^2(2cd-be)\right)\int \left(bx+cx^2\right)^{3/2} dx}{48c^2}$$
$$= -\frac{5b^2(2cd-be)(b+2cx)\left(bx+cx^2\right)^{3/2}}{384c^3} + \frac{(2cd-be)(b+2cx)\left(bx+cx^2\right)^{5/2}}{24c^2} + \frac{e\left(bx+cx^2\right.}{7c}$$
$$= \frac{5b^4(2cd-be)(b+2cx)\sqrt{bx+cx^2}}{1024c^4} - \frac{5b^2(2cd-be)(b+2cx)\left(bx+cx^2\right)^{3/2}}{384c^3} + \frac{(2cd-be)(b}{}$$
$$= \frac{5b^4(2cd-be)(b+2cx)\sqrt{bx+cx^2}}{1024c^4} - \frac{5b^2(2cd-be)(b+2cx)\left(bx+cx^2\right)^{3/2}}{384c^3} + \frac{(2cd-be)(b}{}$$
$$= \frac{5b^4(2cd-be)(b+2cx)\sqrt{bx+cx^2}}{1024c^4} - \frac{5b^2(2cd-be)(b+2cx)\left(bx+cx^2\right)^{3/2}}{384c^3} + \frac{(2cd-be)(b}{}$$

Mathematica [A] time = 0.316591, size = 171, normalized size = 0.98

$$\frac{(x(b+cx))^{7/2}\left(\frac{49(2cd-be)\left(\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\left(8b^3c^2x^2+432b^2c^3x^3-10b^4cx+15b^5+640bc^4x^4+256c^5x^5\right)-15b^{11/2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)\right)}{3072c^{7/2}x^{7/2}\sqrt{\frac{cx}{b}+1}}+7e(b+cx)^3\right)}{49c(b+cx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(b*x + c*x^2)^(5/2),x]
```

```
[Out] ((x*(b + c*x))^(7/2)*(7*e*(b + c*x)^3 + (49*(2*c*d - b*e)*(Sqrt[c]*Sqrt[x]*
Sqrt[1 + (c*x)/b]*(15*b^5 - 10*b^4*c*x + 8*b^3*c^2*x^2 + 432*b^2*c^3*x^3 +
640*b*c^4*x^4 + 256*c^5*x^5) - 15*b^(11/2)*ArcSinh[(Sqrt[c]*Sqrt[x])/Sqrt[b
]]))/(3072*c^(7/2)*x^(7/2)*Sqrt[1 + (c*x)/b])))/(49*c*(b + c*x)^3)
```

Maple [B] time = 0.049, size = 321, normalized size = 1.8

$$\frac{e}{7c}\left(cx^2+bx\right)^{\frac{7}{2}} - \frac{bxe}{12c}\left(cx^2+bx\right)^{\frac{5}{2}} - \frac{b^2e}{24c^2}\left(cx^2+bx\right)^{\frac{5}{2}} + \frac{5eb^3x}{192c^2}\left(cx^2+bx\right)^{\frac{3}{2}} + \frac{5eb^4}{384c^3}\left(cx^2+bx\right)^{\frac{3}{2}} - \frac{5eb^5x}{512c^3}\sqrt{cx^2+bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+b*x)^(5/2),x)
```

```
[Out] 1/7*e*(c*x^2+b*x)^(7/2)/c-1/12*e*b/c*x*(c*x^2+b*x)^(5/2)-1/24*e*b^2/c^2*(c*
x^2+b*x)^(5/2)+5/192*e*b^3/c^2*(c*x^2+b*x)^(3/2)*x+5/384*e*b^4/c^3*(c*x^2+b
*x)^(3/2)-5/512*e*b^5/c^3*(c*x^2+b*x)^(1/2)*x-5/1024*e*b^6/c^4*(c*x^2+b*x)^
(1/2)+5/2048*e*b^7/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))+1/6*d*
x*(c*x^2+b*x)^(5/2)+1/12*d/c*(c*x^2+b*x)^(5/2)*b-5/96*d*b^2/c*(c*x^2+b*x)^(
3/2)*x-5/192*d*b^3/c^2*(c*x^2+b*x)^(3/2)+5/256*d*b^4/c^2*(c*x^2+b*x)^(1/2)*
x+5/512*d*b^5/c^3*(c*x^2+b*x)^(1/2)-5/1024*d*b^6/c^(7/2)*ln((1/2*b+c*x)/c^(
1/2)+(c*x^2+b*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.07314, size = 913, normalized size = 5.22

$$\left[-\frac{105\left(2\,b^6cd-b^7e\right)\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\left(3072\,c^7ex^6+210\,b^5c^2d-105\,b^6ce+256\left(14\,c^7d+29\,bc^6e\right)x^5\right.}{4300}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/43008*(105*(2*b^6*c*d - b^7*e)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b
*x)*sqrt(c)) - 2*(3072*c^7*e*x^6 + 210*b^5*c^2*d - 105*b^6*c*e + 256*(14*c^
7*d + 29*b*c^6*e)*x^5 + 128*(70*b*c^6*d + 37*b^2*c^5*e)*x^4 + 48*(126*b^2*c
^5*d + b^3*c^4*e)*x^3 + 56*(2*b^3*c^4*d - b^4*c^3*e)*x^2 - 70*(2*b^4*c^3*d
- b^5*c^2*e)*x)*sqrt(c*x^2 + b*x))/c^5, 1/21504*(105*(2*b^6*c*d - b^7*e)*sq
rt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)) + (3072*c^7*e*x^6 + 210*b^5
*c^2*d - 105*b^6*c*e + 256*(14*c^7*d + 29*b*c^6*e)*x^5 + 128*(70*b*c^6*d +
37*b^2*c^5*e)*x^4 + 48*(126*b^2*c^5*d + b^3*c^4*e)*x^3 + 56*(2*b^3*c^4*d -
b^4*c^3*e)*x^2 - 70*(2*b^4*c^3*d - b^5*c^2*e)*x)*sqrt(c*x^2 + b*x))/c^5]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (x\,(b+cx))^{\frac{5}{2}}\,(d+ex)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+b*x)**(5/2),x)
```

```
[Out] Integral((x*(b + c*x))**(5/2)*(d + e*x), x)
```

Giac [A] time = 1.55569, size = 315, normalized size = 1.8

$$\frac{1}{21504}\,\sqrt{cx^2+bx}\left(2\left(4\left(2\left(8\left(2\left(12\,c^2xe+\frac{14\,c^8d+29\,bc^7e}{c^6}\right)x+\frac{70\,bc^7d+37\,b^2c^6e}{c^6}\right)x+\frac{3\left(126\,b^2c^6d+b^3c^5e\right)}{c^6}\right)x+\frac{7\left(2\,b^3\right.}{}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/21504*sqrt(c*x^2 + b*x)*(2*(4*(2*(8*(2*(12*c^2*x*e + (14*c^8*d + 29*b*c^7
*e)/c^6)*x + (70*b*c^7*d + 37*b^2*c^6*e)/c^6)*x + 3*(126*b^2*c^6*d + b^3*c^
5*e)/c^6)*x + 7*(2*b^3*c^5*d - b^4*c^4*e)/c^6)*x - 35*(2*b^4*c^4*d - b^5*c^
3*e)/c^6)*x + 105*(2*b^5*c^3*d - b^6*c^2*e)/c^6) + 5/2048*(2*b^6*c*d - b^7*
e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b))/c^(9/2)
```

3.305 $\int \left(bx+cx^2\right)^{5/2} dx$

Optimal. Leaf size=118

$$\frac{5b^4(b+2cx)\sqrt{bx+cx^2}}{512c^3}-\frac{5b^2(b+2cx)\left(bx+cx^2\right)^{3/2}}{192c^2}-\frac{5b^6\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{512c^{7/2}}+\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}$$

```
[Out] (5*b^4*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(512*c^3) - (5*b^2*(b + 2*c*x)*(b*x +
 c*x^2)^(3/2))/(192*c^2) + ((b + 2*c*x)*(b*x + c*x^2)^(5/2))/(12*c) - (5*b^
6*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(512*c^(7/2))
```

Rubi [A] time = 0.036886, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {612, 620, 206}

$$\frac{5b^4(b+2cx)\sqrt{bx+cx^2}}{512c^3}-\frac{5b^2(b+2cx)\left(bx+cx^2\right)^{3/2}}{192c^2}-\frac{5b^6\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{512c^{7/2}}+\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(5/2),x]
```

```
[Out] (5*b^4*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(512*c^3) - (5*b^2*(b + 2*c*x)*(b*x +
 c*x^2)^(3/2))/(192*c^2) + ((b + 2*c*x)*(b*x + c*x^2)^(5/2))/(12*c) - (5*b^
6*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(512*c^(7/2))
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(bx+cx^2\right)^{5/2} dx = \frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c} - \frac{\left(5b^2\right)\int \left(bx+cx^2\right)^{3/2} dx}{24c}$$
$$= -\frac{5b^2(b+2cx)\left(bx+cx^2\right)^{3/2}}{192c^2} + \frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c} + \frac{\left(5b^4\right)\int \sqrt{bx+cx^2}\,dx}{128c^2}$$
$$= \frac{5b^4(b+2cx)\sqrt{bx+cx^2}}{512c^3} - \frac{5b^2(b+2cx)\left(bx+cx^2\right)^{3/2}}{192c^2} + \frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c} - \frac{\left(5b^6\right)\int \frac{1}{\sqrt{bx+cx^2}} dx}{1024c^3}$$
$$= \frac{5b^4(b+2cx)\sqrt{bx+cx^2}}{512c^3} - \frac{5b^2(b+2cx)\left(bx+cx^2\right)^{3/2}}{192c^2} + \frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c} - \frac{\left(5b^6\right)\text{Subst}\left(\int \frac{}{1-} \right.}{512}$$
$$= \frac{5b^4(b+2cx)\sqrt{bx+cx^2}}{512c^3} - \frac{5b^2(b+2cx)\left(bx+cx^2\right)^{3/2}}{192c^2} + \frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c} - \frac{5b^6\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+c}}\right.}{512c^{7/2}}$$

Mathematica [A] time = 0.121959, size = 120, normalized size = 1.02

$$\frac{\sqrt{x(b+cx)}\left(\sqrt{c}\left(8b^3c^2x^2+432b^2c^3x^3-10b^4cx+15b^5+640bc^4x^4+256c^5x^5\right)-\frac{15b^{11/2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}\right)}{1536c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[x*(b + c*x)]*(Sqrt[c]*(15*b^5 - 10*b^4*c*x + 8*b^3*c^2*x^2 + 432*b^2*
c^3*x^3 + 640*b*c^4*x^4 + 256*c^5*x^5) - (15*b^(11/2)*ArcSinh[(Sqrt[c]*Sqrt
[x])/Sqrt[b]])/(Sqrt[x]*Sqrt[1 + (c*x)/b])))/(1536*c^(7/2))
```

Maple [A] time = 0.049, size = 134, normalized size = 1.1

$$\frac{2\,cx+b}{12\,c}\left(cx^2+bx\right)^{\frac{5}{2}} - \frac{5\,b^2x}{96\,c}\left(cx^2+bx\right)^{\frac{3}{2}} - \frac{5\,b^3}{192\,c^2}\left(cx^2+bx\right)^{\frac{3}{2}} + \frac{5\,b^4x}{256\,c^2}\sqrt{cx^2+bx} + \frac{5\,b^5}{512\,c^3}\sqrt{cx^2+bx} - \frac{5\,b^6}{1024}\ln\left(\left(\frac{b}{2}+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(5/2),x)
```

```
[Out] 1/12*(2*c*x+b)*(c*x^2+b*x)^(5/2)/c-5/96*b^2/c*(c*x^2+b*x)^(3/2)*x-5/192*b^3
/c^2*(c*x^2+b*x)^(3/2)+5/256*b^4/c^2*(c*x^2+b*x)^(1/2)*x+5/512*b^5/c^3*(c*x
^2+b*x)^(1/2)-5/1024*b^6/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.02157, size = 501, normalized size = 4.25

$$\left[\frac{15\,b^6\sqrt{c}\log\left(2\,cx+b-2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\left(256\,c^6x^5+640\,bc^5x^4+432\,b^2c^4x^3+8\,b^3c^3x^2-10\,b^4c^2x+15\,b^5c\right)\sqrt{cx^2}}{3072\,c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3072*(15*b^6*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*(2
56*c^6*x^5 + 640*b*c^5*x^4 + 432*b^2*c^4*x^3 + 8*b^3*c^3*x^2 - 10*b^4*c^2*x
 + 15*b^5*c)*sqrt(c*x^2 + b*x))/c^4, 1/1536*(15*b^6*sqrt(-c)*arctan(sqrt(c*
x^2 + b*x)*sqrt(-c)/(c*x)) + (256*c^6*x^5 + 640*b*c^5*x^4 + 432*b^2*c^4*x^3
 + 8*b^3*c^3*x^2 - 10*b^4*c^2*x + 15*b^5*c)*sqrt(c*x^2 + b*x))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx+cx^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(5/2),x)
```

```
[Out] Integral((b*x + c*x**2)**(5/2), x)
```

Giac [A] time = 1.35832, size = 144, normalized size = 1.22

$$\frac{5\,b^6\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}-b\right|\right)}{1024\,c^{\frac{7}{2}}}+\frac{1}{1536}\,\sqrt{cx^2+bx}\left(\frac{15\,b^5}{c^3}-2\left(\frac{5\,b^4}{c^2}-4\left(\frac{b^3}{c}+2\left(27\,b^2+8\left(2\,c^2x+5\,bc\right)x\right)x\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2),x, algorithm="giac")
```

```
[Out] 5/1024*b^6*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b))/c^(7/2)
 + 1/1536*sqrt(c*x^2 + b*x)*(15*b^5/c^3 - 2*(5*b^4/c^2 - 4*(b^3/c + 2*(27*b
^2 + 8*(2*c^2*x + 5*b*c)*x)*x)*x)*x)
```

3.306 $\int \frac{(bx+cx^2)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=356

$$\frac{(bx+cx^2)^{3/2}\left(3b^2e^2-6cex(2cd-be)-22bcde+16c^2d^2\right)}{48ce^3}+\frac{\sqrt{bx+cx^2}\left(-2cex(2cd-be)\left(-3b^2e^2-16bcde+16c^2d^2\right)+17\right.}{128c^2e^5}$$

```
[Out] ((128*c^4*d^4 - 288*b*c^3*d^3*e + 176*b^2*c^2*d^2*e^2 - 10*b^3*c*d*e^3 - 3*
b^4*e^4 - 2*c*e*(2*c*d - b*e)*(16*c^2*d^2 - 16*b*c*d*e - 3*b^2*e^2)*x)*Sqrt
[b*x + c*x^2])/(128*c^2*e^5) + ((16*c^2*d^2 - 22*b*c*d*e + 3*b^2*e^2 - 6*c*
e*(2*c*d - b*e)*x)*(b*x + c*x^2)^(3/2))/(48*c*e^3) + (b*x + c*x^2)^(5/2)/(5
*e) - ((2*c*d - b*e)*(128*c^4*d^4 - 256*b*c^3*d^3*e + 112*b^2*c^2*d^2*e^2 +
 16*b^3*c*d*e^3 + 3*b^4*e^4)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(128*c
^(5/2)*e^6) + (d^(5/2)*(c*d - b*e)^(5/2)*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2
*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/e^6
```

Rubi [A] time = 0.434445, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {734, 814, 843, 620, 206, 724}

$$\frac{(bx+cx^2)^{3/2}\left(3b^2e^2-6cex(2cd-be)-22bcde+16c^2d^2\right)}{48ce^3}+\frac{\sqrt{bx+cx^2}\left(-2cex(2cd-be)\left(-3b^2e^2-16bcde+16c^2d^2\right)+17\right.}{128c^2e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(5/2)/(d + e*x),x]
```



```
[Out] ((128*c^4*d^4 - 288*b*c^3*d^3*e + 176*b^2*c^2*d^2*e^2 - 10*b^3*c*d*e^3 - 3*
b^4*e^4 - 2*c*e*(2*c*d - b*e)*(16*c^2*d^2 - 16*b*c*d*e - 3*b^2*e^2)*x)*Sqrt
[b*x + c*x^2])/(128*c^2*e^5) + ((16*c^2*d^2 - 22*b*c*d*e + 3*b^2*e^2 - 6*c*
e*(2*c*d - b*e)*x)*(b*x + c*x^2)^(3/2))/(48*c*e^3) + (b*x + c*x^2)^(5/2)/(5
*e) - ((2*c*d - b*e)*(128*c^4*d^4 - 256*b*c^3*d^3*e + 112*b^2*c^2*d^2*e^2 +
 16*b^3*c*d*e^3 + 3*b^4*e^4)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(128*c
^(5/2)*e^6) + (d^(5/2)*(c*d - b*e)^(5/2)*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2
*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/e^6
```


Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
```

```
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(bx+cx^2\right)^{5/2}}{d+ex}\,dx = \frac{\left(bx+cx^2\right)^{5/2}}{5e} - \frac{\int \frac{(bd+(2cd-be)x)\left(bx+cx^2\right)^{3/2}}{d+ex}\,dx}{2e}$$

$$= \frac{\left(16c^2d^2-22bcde+3b^2e^2-6ce(2cd-be)x\right)\left(bx+cx^2\right)^{3/2}}{48ce^3} + \frac{\left(bx+cx^2\right)^{5/2}}{5e} + \frac{\int \frac{\left(-\frac{1}{2}bd\left(16c^2d^2-22bcde\right.\right.}{}}{}$$

$$= \frac{\left(128c^4d^4-288bc^3d^3e+176b^2c^2d^2e^2-10b^3cde^3-3b^4e^4-2ce(2cd-be)\left(16c^2d^2-16bcde-3b^2\right.\right.}{128c^2e^5}$$

$$= \frac{\left(128c^4d^4-288bc^3d^3e+176b^2c^2d^2e^2-10b^3cde^3-3b^4e^4-2ce(2cd-be)\left(16c^2d^2-16bcde-3b^2\right.\right.}{128c^2e^5}$$

$$= \frac{\left(128c^4d^4-288bc^3d^3e+176b^2c^2d^2e^2-10b^3cde^3-3b^4e^4-2ce(2cd-be)\left(16c^2d^2-16bcde-3b^2\right.\right.}{128c^2e^5}$$

$$= \frac{\left(128c^4d^4-288bc^3d^3e+176b^2c^2d^2e^2-10b^3cde^3-3b^4e^4-2ce(2cd-be)\left(16c^2d^2-16bcde-3b^2\right.\right.}{128c^2e^5}$$

Mathematica [B] time = 3.78271, size = 727, normalized size = 2.04

$$(x(b+cx))^{5/2}\left(3e^5(b+cx)^3\sqrt{be-cd}\left(bcx\sqrt{\frac{cx}{b}+1}\left(248b^2c^2x^2+10b^3cx-15b^4+336bc^3x^3+128c^4x^4\right)+15b^{11/2}\sqrt{c}\sqrt{x}\,\mathrm{s}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(5/2)/(d + e*x),x]
```

```
[Out] ((x*(b + c*x))^(5/2)*(3*e^5*Sqrt[-(c*d) + b*e]*(b + c*x)^3*(b*c*x*Sqrt[1 +
(c*x)/b]*(-15*b^4 + 10*b^3*c*x + 248*b^2*c^2*x^2 + 336*b*c^3*x^3 + 128*c^4*
x^4) + 15*b^(11/2)*Sqrt[c]*Sqrt[x]*ArcSinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]]) - 10
*c*d*(e^4*Sqrt[-(c*d) + b*e]*(b + c*x)^3*(b*c*x*Sqrt[1 + (c*x)/b]*(15*b^3 +
 118*b^2*c*x + 136*b*c^2*x^2 + 48*c^3*x^3) - 15*b^(9/2)*Sqrt[c]*Sqrt[x]*Arc
Sinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]]) - 8*b*c^(3/2)*d*Sqrt[x]*(b^2*e^3*Sqrt[-(c*
d) + b*e]*(b*Sqrt[c]*Sqrt[x]*(1 + (c*x)/b)^(7/2)*(33*b^2 + 26*b*c*x + 8*c^2
*x^2) + 15*Sqrt[b]*(b + c*x)^3*ArcSinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]]) - 6*Sqrt
[c]*Sqrt[d]*(b + c*x)*(b*Sqrt[c]*Sqrt[d]*e^2*Sqrt[-(c*d) + b*e]*(b*Sqrt[c]*
Sqrt[x]*(5*b + 2*c*x)*(1 + (c*x)/b)^(5/2) + 3*Sqrt[b]*(b + c*x)^2*ArcSinh[(
Sqrt[c]*Sqrt[x])/Sqrt[b]]) - 4*(c*d - b*e)*(Sqrt[c]*Sqrt[d]*e*Sqrt[-(c*d) +
 b*e]*(b + c*x)*(b*Sqrt[c]*Sqrt[x]*(1 + (c*x)/b)^(3/2) + Sqrt[b]*(b + c*x)*
ArcSinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]]) - 2*b*(c*d - b*e)*(1 + (c*x)/b)^(3/2)*(
Sqrt[b]*Sqrt[c]*Sqrt[d]*Sqrt[-(c*d) + b*e]*Sqrt[1 + (c*x)/b]*ArcSinh[(Sqrt[
c]*Sqrt[x])/Sqrt[b]] + (-(c*d) + b*e)*Sqrt[b + c*x]*ArcTan[(Sqrt[-(c*d) + b
*e]*Sqrt[x])/(Sqrt[d]*Sqrt[b + c*x])])))))))/(1920*b*c^3*e^6*Sqrt[-(c*d) +
b*e]*x^3*(b + c*x)^5*Sqrt[1 + (c*x)/b])
```

Maple [B] time = 0.233, size = 1932, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(5/2)/(e*x+d),x)
```

```
[Out] -1/e^7*d^6/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d
/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-
c*d)/e^2)^(1/2))/(d/e+x))*c^3+1/5/e*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b
*e-c*d)/e^2)^(5/2)-1/4/e^2*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e
^2)^(3/2)*x*c*d+5/128/e^2*d/c^(3/2)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2
)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))*b^4+5/16/e^3*d
^2*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e
+x)-d*(b*e-c*d)/e^2)^(1/2))/c^(1/2)*b^3-15/8/e^4*d^3*ln((1/2*(b*e-2*c*d)/e+
(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2
))*c^(1/2)*b^2+5/2/e^5*d^4*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e
+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))*c^(3/2)*b+1/e^4*d^3/(-d
*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b
*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2
))/(d/e+x))*b^3-5/32/e^2*b^2*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)
/e^2)^(1/2)*x*d-5/64/e^2/c*b^3*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*
d)/e^2)^(1/2)*d-1/2/e^4*d^3*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/
e^2)^(1/2)*x*c^2-9/4/e^4*d^3*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)
/e^2)^(1/2)*b*c-3/64/e/c*b^3*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)
/e^2)^(1/2)*x+3/4/e^3*d^2*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^
2)^(1/2)*x*b*c-3/e^5*d^4/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b
*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(
d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*b^2*c+3/e^6*d^5/(-d*(b*e-c*d)/e^2)^
(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/
2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*b*c^
2-3/128/e/c^2*b^4*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)
-1/e^6*d^5*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d
)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))*c^(5/2)+1/3/e^3*d^2*(c*(d/e+x)^2+(b*e-2
```

```
*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)*c+1/e^5*d^4*(c*(d/e+x)^2+(b*e-2*c*d)
/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*c^2+11/8/e^3*d^2*(c*(d/e+x)^2+(b*e-2*c*d)
/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*b^2+3/256/e/c^(5/2)*b^5*ln((1/2*(b*e-2*c*
d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)
^(1/2))+1/8/e*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)*x*b
+1/16/e/c*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)*b^2-11/
24/e^2*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)*b*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 31.8437, size = 3449, normalized size = 9.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/3840*(15*(256*c^5*d^5 - 640*b*c^4*d^4*e + 480*b^2*c^3*d^3*e^2 - 80*b^3*
c^2*d^2*e^3 - 10*b^4*c*d*e^4 - 3*b^5*e^5)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*
x^2 + b*x)*sqrt(c)) - 3840*(c^5*d^4 - 2*b*c^4*d^3*e + b^2*c^3*d^2*e^2)*sqrt
(c*d^2 - b*d*e)*log((b*d + (2*c*d - b*e)*x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x
^2 + b*x))/(e*x + d)) - 2*(384*c^5*e^5*x^4 + 1920*c^5*d^4*e - 4320*b*c^4*d^
3*e^2 + 2640*b^2*c^3*d^2*e^3 - 150*b^3*c^2*d*e^4 - 45*b^4*c*e^5 - 48*(10*c^
5*d*e^4 - 21*b*c^4*e^5)*x^3 + 8*(80*c^5*d^2*e^3 - 170*b*c^4*d*e^4 + 93*b^2*
c^3*e^5)*x^2 - 10*(96*c^5*d^3*e^2 - 208*b*c^4*d^2*e^3 + 118*b^2*c^3*d*e^4 -
 3*b^3*c^2*e^5)*x)*sqrt(c*x^2 + b*x))/(c^3*e^6), 1/3840*(7680*(c^5*d^4 - 2*
b*c^4*d^3*e + b^2*c^3*d^2*e^2)*sqrt(-c*d^2 + b*d*e)*arctan(-sqrt(-c*d^2 + b
*d*e)*sqrt(c*x^2 + b*x)/((c*d - b*e)*x)) - 15*(256*c^5*d^5 - 640*b*c^4*d^4*
e + 480*b^2*c^3*d^3*e^2 - 80*b^3*c^2*d^2*e^3 - 10*b^4*c*d*e^4 - 3*b^5*e^5)*
sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*(384*c^5*e^5*x^4 +
 1920*c^5*d^4*e - 4320*b*c^4*d^3*e^2 + 2640*b^2*c^3*d^2*e^3 - 150*b^3*c^2*d
*e^4 - 45*b^4*c*e^5 - 48*(10*c^5*d*e^4 - 21*b*c^4*e^5)*x^3 + 8*(80*c^5*d^2*
e^3 - 170*b*c^4*d*e^4 + 93*b^2*c^3*e^5)*x^2 - 10*(96*c^5*d^3*e^2 - 208*b*c^
4*d^2*e^3 + 118*b^2*c^3*d*e^4 - 3*b^3*c^2*e^5)*x)*sqrt(c*x^2 + b*x))/(c^3*e
^6), 1/1920*(15*(256*c^5*d^5 - 640*b*c^4*d^4*e + 480*b^2*c^3*d^3*e^2 - 80*b
^3*c^2*d^2*e^3 - 10*b^4*c*d*e^4 - 3*b^5*e^5)*sqrt(-c)*arctan(sqrt(c*x^2 + b
*x)*sqrt(-c)/(c*x)) + 1920*(c^5*d^4 - 2*b*c^4*d^3*e + b^2*c^3*d^2*e^2)*sqrt
(c*d^2 - b*d*e)*log((b*d + (2*c*d - b*e)*x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x
^2 + b*x))/(e*x + d)) + (384*c^5*e^5*x^4 + 1920*c^5*d^4*e - 4320*b*c^4*d^3*
e^2 + 2640*b^2*c^3*d^2*e^3 - 150*b^3*c^2*d*e^4 - 45*b^4*c*e^5 - 48*(10*c^5*
d*e^4 - 21*b*c^4*e^5)*x^3 + 8*(80*c^5*d^2*e^3 - 170*b*c^4*d*e^4 + 93*b^2*c^
3*e^5)*x^2 - 10*(96*c^5*d^3*e^2 - 208*b*c^4*d^2*e^3 + 118*b^2*c^3*d*e^4 - 3
*b^3*c^2*e^5)*x)*sqrt(c*x^2 + b*x))/(c^3*e^6), 1/1920*(3840*(c^5*d^4 - 2*b*
c^4*d^3*e + b^2*c^3*d^2*e^2)*sqrt(-c*d^2 + b*d*e)*arctan(-sqrt(-c*d^2 + b*d
*e)*sqrt(c*x^2 + b*x)/((c*d - b*e)*x)) + 15*(256*c^5*d^5 - 640*b*c^4*d^4*e
+ 480*b^2*c^3*d^3*e^2 - 80*b^3*c^2*d^2*e^3 - 10*b^4*c*d*e^4 - 3*b^5*e^5)*sq
```

```
rt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)) + (384*c^5*e^5*x^4 + 1920*c
^5*d^4*e - 4320*b*c^4*d^3*e^2 + 2640*b^2*c^3*d^2*e^3 - 150*b^3*c^2*d*e^4 -
45*b^4*c*e^5 - 48*(10*c^5*d*e^4 - 21*b*c^4*e^5)*x^3 + 8*(80*c^5*d^2*e^3 - 1
70*b*c^4*d*e^4 + 93*b^2*c^3*e^5)*x^2 - 10*(96*c^5*d^3*e^2 - 208*b*c^4*d^2*e
^3 + 118*b^2*c^3*d*e^4 - 3*b^3*c^2*e^5)*x)*sqrt(c*x^2 + b*x))/(c^3*e^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(5/2)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.307 $\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=314

$$-\frac{5\sqrt{bx+cx^2}\left(-2cex\left(b^2e^2-16bcde+16c^2d^2\right)+48b^2cde^2-b^3e^3-112bc^2d^2e+64c^3d^3\right)}{64ce^5}+\frac{5\left(144b^2c^2d^2e^2-16b^3cde^3\right.}{}$$

```
[Out] (-5*(64*c^3*d^3 - 112*b*c^2*d^2*e + 48*b^2*c*d*e^2 - b^3*e^3 - 2*c*e*(16*c^
2*d^2 - 16*b*c*d*e + b^2*e^2)*x)*Sqrt[b*x + c*x^2])/(64*c*e^5) - (5*(8*c*d
- 7*b*e - 6*c*e*x)*(b*x + c*x^2)^(3/2))/(24*e^3) - (b*x + c*x^2)^(5/2)/(e*(
d + e*x)) + (5*(128*c^4*d^4 - 256*b*c^3*d^3*e + 144*b^2*c^2*d^2*e^2 - 16*b^
3*c*d*e^3 - b^4*e^4)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(64*c^(3/2)*e^
6) - (5*d^(3/2)*(c*d - b*e)^(3/2)*(2*c*d - b*e)*ArcTanh[(b*d + (2*c*d - b*e
)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/(2*e^6)
```

Rubi [A] time = 0.393477, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {732, 814, 843, 620, 206, 724}

$$-\frac{5\sqrt{bx+cx^2}\left(-2cex\left(b^2e^2-16bcde+16c^2d^2\right)+48b^2cde^2-b^3e^3-112bc^2d^2e+64c^3d^3\right)}{64ce^5}+\frac{5\left(144b^2c^2d^2e^2-16b^3cde^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(5/2)/(d + e*x)^2,x]
```

```
[Out] (-5*(64*c^3*d^3 - 112*b*c^2*d^2*e + 48*b^2*c*d*e^2 - b^3*e^3 - 2*c*e*(16*c^
2*d^2 - 16*b*c*d*e + b^2*e^2)*x)*Sqrt[b*x + c*x^2])/(64*c*e^5) - (5*(8*c*d
- 7*b*e - 6*c*e*x)*(b*x + c*x^2)^(3/2))/(24*e^3) - (b*x + c*x^2)^(5/2)/(e*(
d + e*x)) + (5*(128*c^4*d^4 - 256*b*c^3*d^3*e + 144*b^2*c^2*d^2*e^2 - 16*b^
3*c*d*e^3 - b^4*e^4)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(64*c^(3/2)*e^
6) - (5*d^(3/2)*(c*d - b*e)^(3/2)*(2*c*d - b*e)*ArcTanh[(b*d + (2*c*d - b*e
)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/(2*e^6)
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
```

```
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(bx+cx^2\right)^{5/2}}{(d+ex)^2}\,dx = -\frac{\left(bx+cx^2\right)^{5/2}}{e(d+ex)} + \frac{5\int \frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{d+ex}\,dx}{2e}$$

$$= -\frac{5(8cd-7be-6cex)\left(bx+cx^2\right)^{3/2}}{24e^3} - \frac{\left(bx+cx^2\right)^{5/2}}{e(d+ex)} - \frac{5\int \frac{\left(-bcd(8cd-7be)-c\left(16c^2d^2-16bcde+b^2e^2\right)x\right)\sqrt{bx+cx^2}}{d+ex}\,d}{16ce^3}$$

$$= -\frac{5\left(64c^3d^3-112bc^2d^2e+48b^2cde^2-b^3e^3-2ce\left(16c^2d^2-16bcde+b^2e^2\right)x\right)\sqrt{bx+cx^2}}{64ce^5} - \frac{5(8cd-7}{}$$

$$= -\frac{5\left(64c^3d^3-112bc^2d^2e+48b^2cde^2-b^3e^3-2ce\left(16c^2d^2-16bcde+b^2e^2\right)x\right)\sqrt{bx+cx^2}}{64ce^5} - \frac{5(8cd-7}{}$$

$$= -\frac{5\left(64c^3d^3-112bc^2d^2e+48b^2cde^2-b^3e^3-2ce\left(16c^2d^2-16bcde+b^2e^2\right)x\right)\sqrt{bx+cx^2}}{64ce^5} - \frac{5(8cd-7}{}$$

$$= -\frac{5\left(64c^3d^3-112bc^2d^2e+48b^2cde^2-b^3e^3-2ce\left(16c^2d^2-16bcde+b^2e^2\right)x\right)\sqrt{bx+cx^2}}{64ce^5} - \frac{5(8cd-7}{}$$

Mathematica [A] time = 1.82675, size = 334, normalized size = 1.06

$$\frac{\sqrt{x(b+cx)}\left(\frac{\sqrt{c}e\sqrt{x}\left(2b^2ce^2\left(-360d^2-205dex+59e^2x^2\right)+15b^3e^3(d+ex)+8bc^2e\left(110d^2ex+210d^3-35de^2x^2+17e^3x^3\right)-16c^3\left(-10d^2e^2x^2+30d^3ex+60d^4+5de^3x^3-3e^4\right)\right)}{d+ex}\right)}{192c^{3/2}e^6\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(5/2)/(d + e*x)^2,x]
```

```
[Out] (Sqrt[x*(b + c*x)]*((Sqrt[c]*e*Sqrt[x]*(15*b^3*e^3*(d + e*x) + 2*b^2*c*e^2*
(-360*d^2 - 205*d*e*x + 59*e^2*x^2) + 8*b*c^2*e*(210*d^3 + 110*d^2*e*x - 35
*d*e^2*x^2 + 17*e^3*x^3) - 16*c^3*(60*d^4 + 30*d^3*e*x - 10*d^2*e^2*x^2 + 5
*d*e^3*x^3 - 3*e^4*x^4)))/(d + e*x) - (15*(-128*c^4*d^4 + 256*b*c^3*d^3*e -
 144*b^2*c^2*d^2*e^2 + 16*b^3*c*d*e^3 + b^4*e^4)*ArcSinh[(Sqrt[c]*Sqrt[x])/
Sqrt[b]])/(Sqrt[b]*Sqrt[1 + (c*x)/b]) - (960*c^(3/2)*d^(3/2)*(2*c*d - b*e)*
(-(c*d) + b*e)^(3/2)*ArcTan[(Sqrt[-(c*d) + b*e]*Sqrt[x])/(Sqrt[d]*Sqrt[b +
c*x])])/Sqrt[b + c*x]))/(192*c^(3/2)*e^6*Sqrt[x])
```

Maple [B] time = 0.272, size = 2534, normalized size = 8.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(5/2)/(e*x+d)^2,x)
```

```
[Out] 35/24/e/(b*e-c*d)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)
*b^2+1/e/(b*e-c*d)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(5/2
)*c-1/d/(b*e-c*d)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(5/2)
*b+1/d/(b*e-c*d)/(d/e+x)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2
)^(7/2)-5/128/e/(b*e-c*d)/c^(3/2)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+
(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))*b^5-85/32/e^2*d/
(b*e-c*d)*b^2*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*x*c
+5/e^3*d^2/(b*e-c*d)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1
/2)*x*c^2*b-45/2/e^5*d^4/(b*e-c*d)/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c
*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-
2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*b^2*c^2+25/2/e^4*d^3/(b*e
-c*d)/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)
+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/
e^2)^(1/2))/(d/e+x))*b^3*c+35/2/e^6*d^5/(b*e-c*d)/(-d*(b*e-c*d)/e^2)^(1/2)*
ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*
(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*c^3*b-c/d/
(b*e-c*d)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(5/2)*x-245/6
4/e^2*d/(b*e-c*d)*b^3*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(
1/2)+5/e^5*d^4/(b*e-c*d)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2
)^(1/2)*c^3+5/3/e^3*d^2/(b*e-c*d)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e
-c*d)/e^2)^(3/2)*c^2-5/e^6*d^5/(b*e-c*d)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c
^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))*c^(7/2)+5
/32/e/(b*e-c*d)*b^3*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/
2)*x+5/64/e/(b*e-c*d)/c*b^4*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/
e^2)^(1/2)-25/8/e^2*d/(b*e-c*d)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c
*d)/e^2)^(3/2)*b*c-5/2/e^4*d^3/(b*e-c*d)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)
-d*(b*e-c*d)/e^2)^(1/2)*x*c^3+25/2/e^3*d^2/(b*e-c*d)*(c*(d/e+x)^2+(b*e-2*c*
d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*b^2*c-55/4/e^4*d^3/(b*e-c*d)*(c*(d/e+x)
^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*b*c^2-75/128/e^2*d/(b*e-c*d
)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+
x)-d*(b*e-c*d)/e^2)^(1/2))/c^(1/2)*b^4+25/4/e^3*d^2/(b*e-c*d)*ln((1/2*(b*e-
2*c*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/
e^2)^(1/2))*c^(1/2)*b^3-5/2/e^3*d^2/(b*e-c*d)/(-d*(b*e-c*d)/e^2)^(1/2)*ln((
-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e
+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*b^4-5/e^7*d^6/
(b*e-c*d)/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/
```

```
e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c
*d)/e^2)^(1/2))/(d/e+x))*c^4-5/4/e^2*d/(b*e-c*d)*(c*(d/e+x)^2+(b*e-2*c*d)/e
*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)*x*c^2+15/e^5*d^4/(b*e-c*d)*ln((1/2*(b*e-2*c
*d)/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2
)^(1/2))*c^(5/2)*b+5/4/e/(b*e-c*d)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*
e-c*d)/e^2)^(3/2)*x*c*b-125/8/e^4*d^3/(b*e-c*d)*ln((1/2*(b*e-2*c*d)/e+(d/e+
x)*c)/c^(1/2)+(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))*c^
(3/2)*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 10.5872, size = 4049, normalized size = 12.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/384*(15*(128*c^4*d^5 - 256*b*c^3*d^4*e + 144*b^2*c^2*d^3*e^2 - 16*b^3*c
*d^2*e^3 - b^4*d*e^4 + (128*c^4*d^4*e - 256*b*c^3*d^3*e^2 + 144*b^2*c^2*d^2
*e^3 - 16*b^3*c*d*e^4 - b^4*e^5)*x)*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 +
b*x)*sqrt(c)) - 960*(2*c^4*d^4 - 3*b*c^3*d^3*e + b^2*c^2*d^2*e^2 + (2*c^4*d
^3*e - 3*b*c^3*d^2*e^2 + b^2*c^2*d*e^3)*x)*sqrt(c*d^2 - b*d*e)*log((b*d + (
2*c*d - b*e)*x - 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*x + d)) - 2*(4
8*c^4*e^5*x^4 - 960*c^4*d^4*e + 1680*b*c^3*d^3*e^2 - 720*b^2*c^2*d^2*e^3 +
15*b^3*c*d*e^4 - 8*(10*c^4*d*e^4 - 17*b*c^3*e^5)*x^3 + 2*(80*c^4*d^2*e^3 -
140*b*c^3*d*e^4 + 59*b^2*c^2*e^5)*x^2 - 5*(96*c^4*d^3*e^2 - 176*b*c^3*d^2*e
^3 + 82*b^2*c^2*d*e^4 - 3*b^3*c*e^5)*x)*sqrt(c*x^2 + b*x))/(c^2*e^7*x + c^2
*d*e^6), -1/384*(1920*(2*c^4*d^4 - 3*b*c^3*d^3*e + b^2*c^2*d^2*e^2 + (2*c^4
*d^3*e - 3*b*c^3*d^2*e^2 + b^2*c^2*d*e^3)*x)*sqrt(-c*d^2 + b*d*e)*arctan(-s
qrt(-c*d^2 + b*d*e)*sqrt(c*x^2 + b*x)/((c*d - b*e)*x)) + 15*(128*c^4*d^5 -
256*b*c^3*d^4*e + 144*b^2*c^2*d^3*e^2 - 16*b^3*c*d^2*e^3 - b^4*d*e^4 + (128
*c^4*d^4*e - 256*b*c^3*d^3*e^2 + 144*b^2*c^2*d^2*e^3 - 16*b^3*c*d*e^4 - b^4
*e^5)*x)*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 2*(48*c^4*e
^5*x^4 - 960*c^4*d^4*e + 1680*b*c^3*d^3*e^2 - 720*b^2*c^2*d^2*e^3 + 15*b^3*
c*d*e^4 - 8*(10*c^4*d*e^4 - 17*b*c^3*e^5)*x^3 + 2*(80*c^4*d^2*e^3 - 140*b*c
^3*d*e^4 + 59*b^2*c^2*e^5)*x^2 - 5*(96*c^4*d^3*e^2 - 176*b*c^3*d^2*e^3 + 82
*b^2*c^2*d*e^4 - 3*b^3*c*e^5)*x)*sqrt(c*x^2 + b*x))/(c^2*e^7*x + c^2*d*e^6)
, -1/192*(15*(128*c^4*d^5 - 256*b*c^3*d^4*e + 144*b^2*c^2*d^3*e^2 - 16*b^3*
c*d^2*e^3 - b^4*d*e^4 + (128*c^4*d^4*e - 256*b*c^3*d^3*e^2 + 144*b^2*c^2*d^
2*e^3 - 16*b^3*c*d*e^4 - b^4*e^5)*x)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt
(-c)/(c*x)) - 480*(2*c^4*d^4 - 3*b*c^3*d^3*e + b^2*c^2*d^2*e^2 + (2*c^4*d^3
*e - 3*b*c^3*d^2*e^2 + b^2*c^2*d*e^3)*x)*sqrt(c*d^2 - b*d*e)*log((b*d + (2*
c*d - b*e)*x - 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*x + d)) - (48*c^
4*e^5*x^4 - 960*c^4*d^4*e + 1680*b*c^3*d^3*e^2 - 720*b^2*c^2*d^2*e^3 + 15*b
^3*c*d*e^4 - 8*(10*c^4*d*e^4 - 17*b*c^3*e^5)*x^3 + 2*(80*c^4*d^2*e^3 - 140*
```

```
b*c^3*d*e^4 + 59*b^2*c^2*e^5)*x^2 - 5*(96*c^4*d^3*e^2 - 176*b*c^3*d^2*e^3 +
 82*b^2*c^2*d*e^4 - 3*b^3*c*e^5)*x)*sqrt(c*x^2 + b*x))/(c^2*e^7*x + c^2*d*e
^6), -1/192*(960*(2*c^4*d^4 - 3*b*c^3*d^3*e + b^2*c^2*d^2*e^2 + (2*c^4*d^3*
e - 3*b*c^3*d^2*e^2 + b^2*c^2*d*e^3)*x)*sqrt(-c*d^2 + b*d*e)*arctan(-sqrt(-
c*d^2 + b*d*e)*sqrt(c*x^2 + b*x)/((c*d - b*e)*x)) + 15*(128*c^4*d^5 - 256*b
*c^3*d^4*e + 144*b^2*c^2*d^3*e^2 - 16*b^3*c*d^2*e^3 - b^4*d*e^4 + (128*c^4*
d^4*e - 256*b*c^3*d^3*e^2 + 144*b^2*c^2*d^2*e^3 - 16*b^3*c*d*e^4 - b^4*e^5)
*x)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)) - (48*c^4*e^5*x^4 - 9
60*c^4*d^4*e + 1680*b*c^3*d^3*e^2 - 720*b^2*c^2*d^2*e^3 + 15*b^3*c*d*e^4 -
8*(10*c^4*d*e^4 - 17*b*c^3*e^5)*x^3 + 2*(80*c^4*d^2*e^3 - 140*b*c^3*d*e^4 +
 59*b^2*c^2*e^5)*x^2 - 5*(96*c^4*d^3*e^2 - 176*b*c^3*d^2*e^3 + 82*b^2*c^2*d
*e^4 - 3*b^3*c*e^5)*x)*sqrt(c*x^2 + b*x))/(c^2*e^7*x + c^2*d*e^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(5/2)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.308 $\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^3} dx$

Optimal. Leaf size=282

$$\frac{5\sqrt{bx+cx^2}\left(5b^2e^2-4cex(2cd-be)-20bcde+16c^2d^2\right)}{8e^5}-\frac{5(2cd-be)\left(b^2e^2-16bcde+16c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{8\sqrt{c}e^6}+\frac{5\left(bx\right.}{}$$

```
[Out] (5*(16*c^2*d^2 - 20*b*c*d*e + 5*b^2*e^2 - 4*c*e*(2*c*d - b*e)*x)*Sqrt[b*x +
 c*x^2])/(8*e^5) + (5*(8*c*d - 3*b*e + 2*c*e*x)*(b*x + c*x^2)^(3/2))/(12*e^
3*(d + e*x)) - (b*x + c*x^2)^(5/2)/(2*e*(d + e*x)^2) - (5*(2*c*d - b*e)*(16
*c^2*d^2 - 16*b*c*d*e + b^2*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(8
*Sqrt[c]*e^6) + (5*Sqrt[d]*(4*c*d - 3*b*e)*Sqrt[c*d - b*e]*(4*c*d - b*e)*Ar
cTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])
])/(8*e^6)
```

Rubi [A] time = 0.32579, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {732, 812, 814, 843, 620, 206, 724}

$$\frac{5\sqrt{bx+cx^2}\left(5b^2e^2-4cex(2cd-be)-20bcde+16c^2d^2\right)}{8e^5}-\frac{5(2cd-be)\left(b^2e^2-16bcde+16c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{8\sqrt{c}e^6}+\frac{5\left(bx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(5/2)/(d + e*x)^3,x]
```

```
[Out] (5*(16*c^2*d^2 - 20*b*c*d*e + 5*b^2*e^2 - 4*c*e*(2*c*d - b*e)*x)*Sqrt[b*x +
 c*x^2])/(8*e^5) + (5*(8*c*d - 3*b*e + 2*c*e*x)*(b*x + c*x^2)^(3/2))/(12*e^
3*(d + e*x)) - (b*x + c*x^2)^(5/2)/(2*e*(d + e*x)^2) - (5*(2*c*d - b*e)*(16
*c^2*d^2 - 16*b*c*d*e + b^2*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(8
*Sqrt[c]*e^6) + (5*Sqrt[d]*(4*c*d - 3*b*e)*Sqrt[c*d - b*e]*(4*c*d - b*e)*Ar
cTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])
])/(8*e^6)
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
```

```
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(bx+cx^2\right)^{5/2}}{(d+ex)^3}dx = -\frac{\left(bx+cx^2\right)^{5/2}}{2e(d+ex)^2} + \frac{5\int \frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{(d+ex)^2}dx}{4e}$$

$$= \frac{5(8cd-3be+2cex)\left(bx+cx^2\right)^{3/2}}{12e^3(d+ex)} - \frac{\left(bx+cx^2\right)^{5/2}}{2e(d+ex)^2} - \frac{5\int \frac{(b(8cd-3be)+8c(2cd-be)x)\sqrt{bx+cx^2}}{d+ex}dx}{8e^3}$$

$$= \frac{5\left(16c^2d^2-20bcde+5b^2e^2-4ce(2cd-be)x\right)\sqrt{bx+cx^2}}{8e^5} + \frac{5(8cd-3be+2cex)\left(bx+cx^2\right)^{3/2}}{12e^3(d+ex)} - \frac{(bx}{2e}$$

$$= \frac{5\left(16c^2d^2-20bcde+5b^2e^2-4ce(2cd-be)x\right)\sqrt{bx+cx^2}}{8e^5} + \frac{5(8cd-3be+2cex)\left(bx+cx^2\right)^{3/2}}{12e^3(d+ex)} - \frac{(bx}{2e}$$

$$= \frac{5\left(16c^2d^2-20bcde+5b^2e^2-4ce(2cd-be)x\right)\sqrt{bx+cx^2}}{8e^5} + \frac{5(8cd-3be+2cex)\left(bx+cx^2\right)^{3/2}}{12e^3(d+ex)} - \frac{(bx}{2e}$$

$$= \frac{5\left(16c^2d^2-20bcde+5b^2e^2-4ce(2cd-be)x\right)\sqrt{bx+cx^2}}{8e^5} + \frac{5(8cd-3be+2cex)\left(bx+cx^2\right)^{3/2}}{12e^3(d+ex)} - \frac{(bx}{2e}$$

Mathematica [A] time = 2.12393, size = 322, normalized size = 1.14

$$\frac{\sqrt{x(b+cx)}\left(\frac{e\sqrt{x}\left(3b^2e^2\left(25d^2+40dex+11e^2x^2\right)-2bce\left(230d^2ex+150d^3+55de^2x^2-13e^3x^3\right)+4c^2\left(20d^2e^2x^2+90d^3ex+60d^4-5de^3x^3+2e^4x^4\right)\right)}{(d+ex)^2} + \frac{30\sqrt{d}\left(19b^2cde^2-3\right.}{}\right.}{24e^6\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(5/2)/(d + e*x)^3,x]
```

```
[Out] (Sqrt[x*(b + c*x)]*((e*Sqrt[x]*(3*b^2*e^2*(25*d^2 + 40*d*e*x + 11*e^2*x^2)
- 2*b*c*e*(150*d^3 + 230*d^2*e*x + 55*d*e^2*x^2 - 13*e^3*x^3) + 4*c^2*(60*d
^4 + 90*d^3*e*x + 20*d^2*e^2*x^2 - 5*d*e^3*x^3 + 2*e^4*x^4)))/(d + e*x)^2 +
 (15*(-32*c^3*d^3 + 48*b*c^2*d^2*e - 18*b^2*c*d*e^2 + b^3*e^3)*ArcSinh[(Sqr
t[c]*Sqrt[x])/Sqrt[b]])/(Sqrt[b]*Sqrt[c]*Sqrt[1 + (c*x)/b]) + (30*Sqrt[d]*(
16*c^3*d^3 - 32*b*c^2*d^2*e + 19*b^2*c*d*e^2 - 3*b^3*e^3)*ArcTan[(Sqrt[-(c*
d) + b*e]*Sqrt[x])/(Sqrt[d]*Sqrt[b + c*x])])/(Sqrt[-(c*d) + b*e]*Sqrt[b + c
*x])))/(24*e^6*Sqrt[x])
```

Maple [B] time = 0.23, size = 5534, normalized size = 19.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(5/2)/(e*x+d)^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.16874, size = 4356, normalized size = 15.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/48*(15*(32*c^3*d^5 - 48*b*c^2*d^4*e + 18*b^2*c*d^3*e^2 - b^3*d^2*e^3 +
(32*c^3*d^3*e^2 - 48*b*c^2*d^2*e^3 + 18*b^2*c*d*e^4 - b^3*e^5)*x^2 + 2*(32*
c^3*d^4*e - 48*b*c^2*d^3*e^2 + 18*b^2*c*d^2*e^3 - b^3*d*e^4)*x)*sqrt(c)*log
(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 30*(16*c^3*d^4 - 16*b*c^2*d^3*e
 + 3*b^2*c*d^2*e^2 + (16*c^3*d^2*e^2 - 16*b*c^2*d*e^3 + 3*b^2*c*e^4)*x^2 +
2*(16*c^3*d^3*e - 16*b*c^2*d^2*e^2 + 3*b^2*c*d*e^3)*x)*sqrt(c*d^2 - b*d*e)*
log((b*d + (2*c*d - b*e)*x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*x
+ d)) - 2*(8*c^3*e^5*x^4 + 240*c^3*d^4*e - 300*b*c^2*d^3*e^2 + 75*b^2*c*d^2
*e^3 - 2*(10*c^3*d*e^4 - 13*b*c^2*e^5)*x^3 + (80*c^3*d^2*e^3 - 110*b*c^2*d*
e^4 + 33*b^2*c*e^5)*x^2 + 20*(18*c^3*d^3*e^2 - 23*b*c^2*d^2*e^3 + 6*b^2*c*d
*e^4)*x)*sqrt(c*x^2 + b*x))/(c*e^8*x^2 + 2*c*d*e^7*x + c*d^2*e^6), 1/48*(60
*(16*c^3*d^4 - 16*b*c^2*d^3*e + 3*b^2*c*d^2*e^2 + (16*c^3*d^2*e^2 - 16*b*c^
2*d*e^3 + 3*b^2*c*e^4)*x^2 + 2*(16*c^3*d^3*e - 16*b*c^2*d^2*e^2 + 3*b^2*c*d
*e^3)*x)*sqrt(-c*d^2 + b*d*e)*arctan(-sqrt(-c*d^2 + b*d*e)*sqrt(c*x^2 + b*x
)/((c*d - b*e)*x)) - 15*(32*c^3*d^5 - 48*b*c^2*d^4*e + 18*b^2*c*d^3*e^2 - b
^3*d^2*e^3 + (32*c^3*d^3*e^2 - 48*b*c^2*d^2*e^3 + 18*b^2*c*d*e^4 - b^3*e^5)
*x^2 + 2*(32*c^3*d^4*e - 48*b*c^2*d^3*e^2 + 18*b^2*c*d^2*e^3 - b^3*d*e^4)*x
)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*(8*c^3*e^5*x^4 +
 240*c^3*d^4*e - 300*b*c^2*d^3*e^2 + 75*b^2*c*d^2*e^3 - 2*(10*c^3*d*e^4 - 1
3*b*c^2*e^5)*x^3 + (80*c^3*d^2*e^3 - 110*b*c^2*d*e^4 + 33*b^2*c*e^5)*x^2 +
20*(18*c^3*d^3*e^2 - 23*b*c^2*d^2*e^3 + 6*b^2*c*d*e^4)*x)*sqrt(c*x^2 + b*x)
)/(c*e^8*x^2 + 2*c*d*e^7*x + c*d^2*e^6), 1/24*(15*(32*c^3*d^5 - 48*b*c^2*d^
4*e + 18*b^2*c*d^3*e^2 - b^3*d^2*e^3 + (32*c^3*d^3*e^2 - 48*b*c^2*d^2*e^3 +
 18*b^2*c*d*e^4 - b^3*e^5)*x^2 + 2*(32*c^3*d^4*e - 48*b*c^2*d^3*e^2 + 18*b^
2*c*d^2*e^3 - b^3*d*e^4)*x)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x
)) + 15*(16*c^3*d^4 - 16*b*c^2*d^3*e + 3*b^2*c*d^2*e^2 + (16*c^3*d^2*e^2 -
16*b*c^2*d*e^3 + 3*b^2*c*e^4)*x^2 + 2*(16*c^3*d^3*e - 16*b*c^2*d^2*e^2 + 3*
b^2*c*d*e^3)*x)*sqrt(c*d^2 - b*d*e)*log((b*d + (2*c*d - b*e)*x + 2*sqrt(c*d
^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*x + d)) + (8*c^3*e^5*x^4 + 240*c^3*d^4*e
 - 300*b*c^2*d^3*e^2 + 75*b^2*c*d^2*e^3 - 2*(10*c^3*d*e^4 - 13*b*c^2*e^5)*x^
3 + (80*c^3*d^2*e^3 - 110*b*c^2*d*e^4 + 33*b^2*c*e^5)*x^2 + 20*(18*c^3*d^3*
e^2 - 23*b*c^2*d^2*e^3 + 6*b^2*c*d*e^4)*x)*sqrt(c*x^2 + b*x))/(c*e^8*x^2 +
2*c*d*e^7*x + c*d^2*e^6), 1/24*(30*(16*c^3*d^4 - 16*b*c^2*d^3*e + 3*b^2*c*d
^2*e^2 + (16*c^3*d^2*e^2 - 16*b*c^2*d*e^3 + 3*b^2*c*e^4)*x^2 + 2*(16*c^3*d^
3*e - 16*b*c^2*d^2*e^2 + 3*b^2*c*d*e^3)*x)*sqrt(-c*d^2 + b*d*e)*arctan(-sqr
t(-c*d^2 + b*d*e)*sqrt(c*x^2 + b*x)/((c*d - b*e)*x)) + 15*(32*c^3*d^5 - 48*
b*c^2*d^4*e + 18*b^2*c*d^3*e^2 - b^3*d^2*e^3 + (32*c^3*d^3*e^2 - 48*b*c^2*d
^2*e^3 + 18*b^2*c*d*e^4 - b^3*e^5)*x^2 + 2*(32*c^3*d^4*e - 48*b*c^2*d^3*e^2
 + 18*b^2*c*d^2*e^3 - b^3*d*e^4)*x)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(
-c)/(c*x)) + (8*c^3*e^5*x^4 + 240*c^3*d^4*e - 300*b*c^2*d^3*e^2 + 75*b^2*c*
d^2*e^3 - 2*(10*c^3*d*e^4 - 13*b*c^2*e^5)*x^3 + (80*c^3*d^2*e^3 - 110*b*c^2
*d*e^4 + 33*b^2*c*e^5)*x^2 + 20*(18*c^3*d^3*e^2 - 23*b*c^2*d^2*e^3 + 6*b^2*
```

```
c*d*e^4)*x)*sqrt(c*x^2 + b*x))/(c*e^8*x^2 + 2*c*d*e^7*x + c*d^2*e^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(5/2)/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.66918, size = 988, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 5/4*(16*c^3*d^4 - 32*b*c^2*d^3*e + 19*b^2*c*d^2*e^2 - 3*b^3*d*e^3)*arctan(-
((sqrt(c)*x - sqrt(c*x^2 + b*x))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e))*e^(-6
)/sqrt(-c*d^2 + b*d*e) + 5/16*(32*c^3*d^3 - 48*b*c^2*d^2*e + 18*b^2*c*d*e^2
 - b^3*e^3)*e^(-6)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b))
/sqrt(c) + 1/24*sqrt(c*x^2 + b*x)*(2*(4*c^2*x*e^(-3) - (18*c^4*d*e^14 - 13*
b*c^3*e^15)*e^(-18)/c^2)*x + 3*(48*c^4*d^2*e^13 - 54*b*c^3*d*e^14 + 11*b^2*
c^2*e^15)*e^(-18)/c^2) + 1/4*(40*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*c^3*d^4*
e + 72*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*c^(7/2)*d^5 - 120*(sqrt(c)*x - sqr
t(c*x^2 + b*x))^2*b*c^(5/2)*d^4*e + 72*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b*c^
3*d^5 - 80*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b*c^2*d^3*e^2 - 124*(sqrt(c)*x
 - sqrt(c*x^2 + b*x))*b^2*c^2*d^4*e + 18*b^2*c^(5/2)*d^5 + 51*(sqrt(c)*x -
sqrt(c*x^2 + b*x))^2*b^2*c^(3/2)*d^3*e^2 - 27*b^3*c^(3/2)*d^4*e + 49*(sqrt(
c)*x - sqrt(c*x^2 + b*x))^3*b^2*c*d^2*e^3 + 59*(sqrt(c)*x - sqrt(c*x^2 + b*
x))*b^3*c*d^3*e^2 - 3*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*b^3*sqrt(c)*d^2*e^3
 + 9*b^4*sqrt(c)*d^3*e^2 - 9*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b^3*d*e^4 -
7*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b^4*d^2*e^3)*e^(-6)/((sqrt(c)*x - sqrt(c*
x^2 + b*x))^2*e + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c)*d + b*d)^2
```

3.309 $\int \frac{\sqrt{2x+x^2}}{1+x} dx$

Optimal. Leaf size=26

$$\sqrt{x^2+2x} - \tan^{-1}\left(\sqrt{x^2+2x}\right)$$

```
[Out] Sqrt[2*x + x^2] - ArcTan[Sqrt[2*x + x^2]]
```

Rubi [A] time = 0.0143339, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {685, 688, 203}

$$\sqrt{x^2+2x} - \tan^{-1}\left(\sqrt{x^2+2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2*x + x^2]/(1 + x),x]
```

```
[Out] Sqrt[2*x + x^2] - ArcTan[Sqrt[2*x + x^2]]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{2x+x^2}}{1+x}\,dx &= \sqrt{2x+x^2} - \int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx\\
&= \sqrt{2x+x^2} - 4\,\text{Subst}\left(\int \frac{1}{4+4x^2}\,dx, x, \sqrt{2x+x^2}\right)\\
&= \sqrt{2x+x^2} - \tan^{-1}\left(\sqrt{2x+x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0285005, size = 38, normalized size = 1.46

$$\sqrt{x(x+2)}\left(1-\frac{2\tan^{-1}\left(\sqrt{\frac{x}{x+2}}\right)}{\sqrt{x}\sqrt{x+2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[2*x + x^2]/(1 + x),x]
```

```
[Out] Sqrt[x*(2 + x)]*(1 - (2*ArcTan[Sqrt[x/(2 + x)]])/(Sqrt[x]*Sqrt[2 + x]))
```

Maple [A] time = 0.06, size = 21, normalized size = 0.8

$$\sqrt{(1+x)^2-1}+\arctan\left(\frac{1}{\sqrt{(1+x)^2-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x)^(1/2)/(1+x),x)
```

```
[Out] ((1+x)^2-1)^(1/2)+arctan(1/((1+x)^2-1)^(1/2))
```

Maxima [A] time = 1.72709, size = 23, normalized size = 0.88

$$\sqrt{x^2+2\,x}+\arcsin\left(\frac{1}{|x+1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)^(1/2)/(1+x),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 + 2*x) + arcsin(1/abs(x + 1))
```

Fricas [A] time = 2.09229, size = 73, normalized size = 2.81

$$\sqrt{x^2+2\,x}-2\,\arctan\left(-x+\sqrt{x^2+2\,x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)^(1/2)/(1+x),x, algorithm="fricas")
```

```
[Out] sqrt(x^2 + 2*x) - 2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x\,(x+2)}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x)**(1/2)/(1+x),x)
```

```
[Out] Integral(sqrt(x*(x + 2))/(x + 1), x)
```

Giac [A] time = 1.3209, size = 36, normalized size = 1.38

$$\sqrt{x^2+2x}-2\arctan\left(-x+\sqrt{x^2+2x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)^(1/2)/(1+x),x, algorithm="giac")
```

```
[Out] sqrt(x^2 + 2*x) - 2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

3.310 $\int \frac{(2x-x^2)^{3/2}}{2-2x} \, dx$

Optimal. Leaf size=53

$$-\frac{1}{6}\left(2x-x^2\right)^{3/2}-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)$$

```
[Out] -Sqrt[2*x - x^2]/2 - (2*x - x^2)^(3/2)/6 + ArcTanh[Sqrt[2*x - x^2]]/2
```

Rubi [A] time = 0.0241723, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {685, 688, 207}

$$-\frac{1}{6}\left(2x-x^2\right)^{3/2}-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*x - x^2)^(3/2)/(2 - 2*x),x]
```

```
[Out] -Sqrt[2*x - x^2]/2 - (2*x - x^2)^(3/2)/6 + ArcTanh[Sqrt[2*x - x^2]]/2
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(2x-x^2)^{3/2}}{2-2x}\,dx = -\frac{1}{6}\left(2x-x^2\right)^{3/2} + \int \frac{\sqrt{2x-x^2}}{2-2x}\,dx$$
$$= -\frac{1}{2}\sqrt{2x-x^2} - \frac{1}{6}\left(2x-x^2\right)^{3/2} + \int \frac{1}{(2-2x)\sqrt{2x-x^2}}\,dx$$
$$= -\frac{1}{2}\sqrt{2x-x^2} - \frac{1}{6}\left(2x-x^2\right)^{3/2} - 4\,\mathrm{Subst}\left(\int \frac{1}{-8+8x^2}\,dx, x, \sqrt{2x-x^2}\right)$$
$$= -\frac{1}{2}\sqrt{2x-x^2} - \frac{1}{6}\left(2x-x^2\right)^{3/2} + \frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)$$

Mathematica [A] time = 0.0600379, size = 48, normalized size = 0.91

$$\frac{1}{6}\sqrt{-(x-2)x}\left(x^2-2x+\frac{6\tan^{-1}\left(\sqrt{\frac{x-2}{x}}\right)}{\sqrt{x-2}\sqrt{x}}-3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2*x - x^2)^(3/2)/(2 - 2*x),x]
```

```
[Out] (Sqrt[-((-2 + x)*x)]*(-3 - 2*x + x^2 + (6*ArcTan[Sqrt[(-2 + x)/x]])/(Sqrt[-
2 + x]*Sqrt[x])))/6
```

Maple [A] time = 0.052, size = 42, normalized size = 0.8

$$-\frac{1}{6}\left(-(-1+x)^2+1\right)^{\frac{3}{2}} - \frac{1}{2}\sqrt{-(-1+x)^2+1} + \frac{1}{2}\mathrm{Artanh}\left(\frac{1}{\sqrt{-(-1+x)^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2*x)^(3/2)/(2-2*x),x)
```

```
[Out] -1/6*(-(-1+x)^2+1)^(3/2)-1/2*(-(-1+x)^2+1)^(1/2)+1/2*arctanh(1/(-(-1+x)^2+1
)^(1/2))
```

Maxima [A] time = 1.67784, size = 78, normalized size = 1.47

$$-\frac{1}{6}\left(-x^2+2x\right)^{\frac{3}{2}} - \frac{1}{2}\sqrt{-x^2+2x} + \frac{1}{2}\log\left(\frac{2\sqrt{-x^2+2x}}{|x-1|} + \frac{2}{|x-1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x)^(3/2)/(2-2*x),x, algorithm="maxima")
```

```
[Out] -1/6*(-x^2 + 2*x)^(3/2) - 1/2*sqrt(-x^2 + 2*x) + 1/2*log(2*sqrt(-x^2 + 2*x)
/abs(x - 1) + 2/abs(x - 1))
```

Fricas [A] time = 2.12967, size = 150, normalized size = 2.83

$$\frac{1}{6}\left(x^2-2x-3\right)\sqrt{-x^2+2x}+\frac{1}{2}\log\left(\frac{x+\sqrt{-x^2+2x}}{x}\right)-\frac{1}{2}\log\left(-\frac{x-\sqrt{-x^2+2x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x)^(3/2)/(2-2*x),x, algorithm="fricas")
```

```
[Out] 1/6*(x^2 - 2*x - 3)*sqrt(-x^2 + 2*x) + 1/2*log((x + sqrt(-x^2 + 2*x))/x) -
1/2*log(-(x - sqrt(-x^2 + 2*x))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{2x\sqrt{-x^2+2x}}{x-1}\,dx+\int -\frac{x^2\sqrt{-x^2+2x}}{x-1}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2*x)**(3/2)/(2-2*x),x)
```

```
[Out] -(Integral(2*x*sqrt(-x**2 + 2*x)/(x - 1), x) + Integral(-x**2*sqrt(-x**2 +
2*x)/(x - 1), x))/2
```

Giac [A] time = 1.31522, size = 63, normalized size = 1.19

$$\frac{1}{6}\left((x-2)x-3\right)\sqrt{-x^2+2x}-\frac{1}{2}\log\left(-\frac{2\left(\sqrt{-x^2+2x}-1\right)}{|-2x+2|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x)^(3/2)/(2-2*x),x, algorithm="giac")
```

```
[Out] 1/6*((x - 2)*x - 3)*sqrt(-x^2 + 2*x) - 1/2*log(-2*(sqrt(-x^2 + 2*x) - 1)/ab
s(-2*x + 2))
```

3.311 $\int \frac{\sqrt{2x-x^2}}{2-2x}\,dx$

Optimal. Leaf size=36

$$\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{1}{2}\sqrt{2x-x^2}$$

```
[Out] -Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2
```

Rubi [A] time = 0.0168601, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {685, 688, 207}

$$\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{1}{2}\sqrt{2x-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2*x - x^2]/(2 - 2*x),x]
```

```
[Out] -Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2x-x^2}}{2-2x}\,dx &= -\frac{1}{2}\sqrt{2x-x^2}+\int \frac{1}{(2-2x)\sqrt{2x-x^2}}\,dx\\ &= -\frac{1}{2}\sqrt{2x-x^2}-4\,\text{Subst}\left(\int \frac{1}{-8+8x^2}\,dx,x,\sqrt{2x-x^2}\right)\\ &= -\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.038693, size = 46, normalized size = 1.28

$$\frac{(x-2)x-2\sqrt{x-2}\sqrt{x}\tan^{-1}\left(\sqrt{\frac{x-2}{x}}\right)}{2\sqrt{-(x-2)x}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[2*x - x^2]/(2 - 2*x),x]
```

```
[Out] ((-2 + x)*x - 2*Sqrt[-2 + x]*Sqrt[x]*ArcTan[Sqrt[(-2 + x)/x]])/(2*Sqrt[-((-
2 + x)*x)])
```

Maple [A] time = 0.047, size = 29, normalized size = 0.8

$$-\frac{1}{2}\sqrt{-(-1+x)^2+1}+\frac{1}{2}\text{Artanh}\left(\frac{1}{\sqrt{-(-1+x)^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2*x)^(1/2)/(2-2*x),x)
```

```
[Out] -1/2*(-(-1+x)^2+1)^(1/2)+1/2*arctanh(1/(-(-1+x)^2+1)^(1/2))
```

Maxima [A] time = 1.6917, size = 61, normalized size = 1.69

$$-\frac{1}{2}\sqrt{-x^2+2x}+\frac{1}{2}\log\left(\frac{2\sqrt{-x^2+2x}}{|x-1|}+\frac{2}{|x-1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x)^(1/2)/(2-2*x),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-x^2 + 2*x) + 1/2*log(2*sqrt(-x^2 + 2*x)/abs(x - 1) + 2/abs(x - 1
))
```

Fricas [B] time = 2.21014, size = 130, normalized size = 3.61

$$-\frac{1}{2}\sqrt{-x^2+2x}+\frac{1}{2}\log\left(\frac{x+\sqrt{-x^2+2x}}{x}\right)-\frac{1}{2}\log\left(-\frac{x-\sqrt{-x^2+2x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x)^(1/2)/(2-2*x),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-x^2 + 2*x) + 1/2*log((x + sqrt(-x^2 + 2*x))/x) - 1/2*log(-(x - s
qrt(-x^2 + 2*x))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\sqrt{-x^2+2x}}{x-1}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2*x)**(1/2)/(2-2*x),x)
```

```
[Out] -Integral(sqrt(-x**2 + 2*x)/(x - 1), x)/2
```

Giac [A] time = 1.37813, size = 54, normalized size = 1.5

$$-\frac{1}{2}\sqrt{-x^2+2x}-\frac{1}{2}\log\left(-\frac{2\left(\sqrt{-x^2+2x}-1\right)}{|-2x+2|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x)^(1/2)/(2-2*x),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-x^2 + 2*x) - 1/2*log(-2*(sqrt(-x^2 + 2*x) - 1)/abs(-2*x + 2))
```

3.312 $\int \frac{1}{(2-2x)\sqrt{2x-x^2}}\,dx$

Optimal. Leaf size=18

$$\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)$$

```
[Out] ArcTanh[Sqrt[2*x - x^2]]/2
```

Rubi [A] time = 0.009766, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {688, 207}

$$\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 - 2*x)*Sqrt[2*x - x^2]),x]

[Out] ArcTanh[Sqrt[2*x - x^2]]/2
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(2-2x)\sqrt{2x-x^2}}\,dx &= -\left(4\,\text{Subst}\left(\int \frac{1}{-8+8x^2}\,dx, x, \sqrt{2x-x^2}\right)\right)\\ &= \frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)\end{aligned}$$

Mathematica [B] time = 0.0108599, size = 38, normalized size = 2.11

$$-\frac{\sqrt{x-2}\sqrt{x}\tan^{-1}\left(\frac{\sqrt{x-2}}{\sqrt{x}}\right)}{\sqrt{-(x-2)x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2 - 2*x)*Sqrt[2*x - x^2]),x]

[Out] -((Sqrt[-2 + x]*Sqrt[x]*ArcTan[Sqrt[-2 + x]/Sqrt[x]])/Sqrt[-((-2 + x)*x)])
```

Maple [A] time = 0.046, size = 15, normalized size = 0.8

$$\frac{1}{2}\operatorname{Artanh}\left(\frac{1}{\sqrt{-(-1+x)^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2-2*x)/(-x^2+2*x)^(1/2),x)
```

```
[Out] 1/2*arctanh(1/(-(-1+x)^2+1)^(1/2))
```

Maxima [B] time = 1.16931, size = 42, normalized size = 2.33

$$\frac{1}{2}\log\left(\frac{2\sqrt{-x^2+2x}}{|x-1|}+\frac{2}{|x-1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-2*x)/(-x^2+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*log(2*sqrt(-x^2 + 2*x)/abs(x - 1) + 2/abs(x - 1))
```

Fricas [B] time = 1.99834, size = 97, normalized size = 5.39

$$\frac{1}{2}\log\left(\frac{x+\sqrt{-x^2+2x}}{x}\right)-\frac{1}{2}\log\left(-\frac{x-\sqrt{-x^2+2x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-2*x)/(-x^2+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log((x + sqrt(-x^2 + 2*x))/x) - 1/2*log(-(x - sqrt(-x^2 + 2*x))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{1}{x\sqrt{-x^2+2x}-\sqrt{-x^2+2x}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-2*x)/(-x**2+2*x)**(1/2),x)
```

```
[Out] -Integral(1/(x*sqrt(-x**2 + 2*x) - sqrt(-x**2 + 2*x)), x)/2
```

Giac [A] time = 1.41763, size = 35, normalized size = 1.94

$$-\frac{1}{2}\log\left(-\frac{2\left(\sqrt{-x^2+2x}-1\right)}{|-2x+2|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-2*x)/(-x^2+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(-2*(sqrt(-x^2 + 2*x) - 1)/abs(-2*x + 2))
```

3.313 $\int \frac{1}{(2-2x)\left(2x-x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=36

$$\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{1}{2\sqrt{2x-x^2}}$$

```
[Out] -1/(2*Sqrt[2*x - x^2]) + ArcTanh[Sqrt[2*x - x^2]]/2
```

Rubi [A] time = 0.0169694, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {687, 688, 207}

$$\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{1}{2\sqrt{2x-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 - 2*x)*(2*x - x^2)^(3/2)),x]
```

```
[Out] -1/(2*Sqrt[2*x - x^2]) + ArcTanh[Sqrt[2*x - x^2]]/2
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(2-2x)\left(2x-x^2\right)^{3/2}}\,dx &= -\frac{1}{2\sqrt{2x-x^2}}+\int \frac{1}{(2-2x)\sqrt{2x-x^2}}\,dx\\ &= -\frac{1}{2\sqrt{2x-x^2}}-4\,\text{Subst}\left(\int \frac{1}{-8+8x^2}\,dx,x,\sqrt{2x-x^2}\right)\\ &= -\frac{1}{2\sqrt{2x-x^2}}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0203524, size = 42, normalized size = 1.17

$$-\frac{2\sqrt{x-2}\sqrt{x}\tan^{-1}\left(\sqrt{\frac{x-2}{x}}\right)+1}{2\sqrt{-(x-2)x}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 - 2*x)*(2*x - x^2)^(3/2)),x]
```

```
[Out] -(1 + 2*Sqrt[-2 + x]*Sqrt[x]*ArcTan[Sqrt[(-2 + x)/x]])/(2*Sqrt[-((-2 + x)*x
)])
```

Maple [A] time = 0.048, size = 29, normalized size = 0.8

$$-\frac{1}{2}\frac{1}{\sqrt{-(-1+x)^2+1}}+\frac{1}{2}\text{Artanh}\left(\frac{1}{\sqrt{-(-1+x)^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2-2*x)/(-x^2+2*x)^(3/2),x)
```

```
[Out] -1/2/(-(-1+x)^2+1)^(1/2)+1/2*arctanh(1/(-(-1+x)^2+1)^(1/2))
```

Maxima [A] time = 1.12103, size = 61, normalized size = 1.69

$$-\frac{1}{2\sqrt{-x^2+2x}}+\frac{1}{2}\log\left(\frac{2\sqrt{-x^2+2x}}{|x-1|}+\frac{2}{|x-1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-2*x)/(-x^2+2*x)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2/sqrt(-x^2 + 2*x) + 1/2*log(2*sqrt(-x^2 + 2*x)/abs(x - 1) + 2/abs(x - 1
))
```

Fricas [B] time = 2.01464, size = 169, normalized size = 4.69

$$\frac{\left(x^2-2x\right)\log\left(\frac{x+\sqrt{-x^2+2x}}{x}\right)-\left(x^2-2x\right)\log\left(-\frac{x-\sqrt{-x^2+2x}}{x}\right)+\sqrt{-x^2+2x}}{2\left(x^2-2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-2*x)/(-x^2+2*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 - 2*x)*log((x + sqrt(-x^2 + 2*x))/x) - (x^2 - 2*x)*log(-(x - sqrt
(-x^2 + 2*x))/x) + sqrt(-x^2 + 2*x))/(x^2 - 2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{1}{-x^3\sqrt{-x^2+2x}+3x^2\sqrt{-x^2+2x}-2x\sqrt{-x^2+2x}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-2*x)/(-x**2+2*x)**(3/2),x)
```

```
[Out] -Integral(1/(-x**3*sqrt(-x**2 + 2*x) + 3*x**2*sqrt(-x**2 + 2*x) - 2*x*sqrt(
-x**2 + 2*x)), x)/2
```

Giac [A] time = 1.35855, size = 66, normalized size = 1.83

$$\frac{\sqrt{-x^2+2\,x}}{2\left(x^2-2\,x\right)}-\frac{1}{2}\log\left(-\frac{2\left(\sqrt{-x^2+2\,x}-1\right)}{|-2\,x+2|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-2*x)/(-x^2+2*x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 2*x)/(x^2 - 2*x) - 1/2*log(-2*(sqrt(-x^2 + 2*x) - 1)/abs(-2
*x + 2))
```

3.314 $\int \frac{1}{(2-2x)(2x-x^2)^{5/2}}\,dx$

Optimal. Leaf size=53

$$-\frac{1}{2\sqrt{2x-x^2}}-\frac{1}{6\left(2x-x^2\right)^{3/2}}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)$$

```
[Out] -1/(6*(2*x - x^2)^(3/2)) - 1/(2*Sqrt[2*x - x^2]) + ArcTanh[Sqrt[2*x - x^2]]
/2
```

Rubi [A] time = 0.0244244, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {687, 688, 207}

$$-\frac{1}{2\sqrt{2x-x^2}}-\frac{1}{6\left(2x-x^2\right)^{3/2}}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 - 2*x)*(2*x - x^2)^(5/2)),x]
```

```
[Out] -1/(6*(2*x - x^2)^(3/2)) - 1/(2*Sqrt[2*x - x^2]) + ArcTanh[Sqrt[2*x - x^2]]
/2
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(2-2x)\left(2x-x^2\right)^{5/2}}\,dx = -\frac{1}{6\left(2x-x^2\right)^{3/2}} + \int \frac{1}{(2-2x)\left(2x-x^2\right)^{3/2}}\,dx$$
$$= -\frac{1}{6\left(2x-x^2\right)^{3/2}} - \frac{1}{2\sqrt{2x-x^2}} + \int \frac{1}{(2-2x)\sqrt{2x-x^2}}\,dx$$
$$= -\frac{1}{6\left(2x-x^2\right)^{3/2}} - \frac{1}{2\sqrt{2x-x^2}} - 4\,\text{Subst}\left(\int \frac{1}{-8+8x^2}\,dx, x, \sqrt{2x-x^2}\right)$$
$$= -\frac{1}{6\left(2x-x^2\right)^{3/2}} - \frac{1}{2\sqrt{2x-x^2}} + \frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)$$

Mathematica [A] time = 0.0427255, size = 50, normalized size = 0.94

$$\frac{3x^2+6(x-2)^{3/2}x^{3/2}\tan^{-1}\left(\sqrt{\frac{x-2}{x}}\right)-6x-1}{6(-(x-2)x)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 - 2*x)*(2*x - x^2)^(5/2)),x]
```

```
[Out] (-1 - 6*x + 3*x^2 + 6*(-2 + x)^(3/2)*x^(3/2)*ArcTan[Sqrt[(-2 + x)/x]])/(6*(
-((-2 + x)*x))^(3/2))
```

Maple [A] time = 0.049, size = 42, normalized size = 0.8

$$-\frac{1}{6}\left(-(-1+x)^2+1\right)^{-\frac{3}{2}}-\frac{1}{2}\frac{1}{\sqrt{-(-1+x)^2+1}}+\frac{1}{2}\text{Artanh}\left(\frac{1}{\sqrt{-(-1+x)^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2-2*x)/(-x^2+2*x)^(5/2),x)
```

```
[Out] -1/6/(-(-1+x)^2+1)^(3/2)-1/2/(-(-1+x)^2+1)^(1/2)+1/2*arctanh(1/(-(-1+x)^2+1
)^(1/2))
```

Maxima [A] time = 1.07213, size = 78, normalized size = 1.47

$$-\frac{1}{2\sqrt{-x^2+2x}}-\frac{1}{6\left(-x^2+2x\right)^{\frac{3}{2}}}+\frac{1}{2}\log\left(\frac{2\sqrt{-x^2+2x}}{|x-1|}+\frac{2}{|x-1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-2*x)/(-x^2+2*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/2/sqrt(-x^2 + 2*x) - 1/6/(-x^2 + 2*x)^(3/2) + 1/2*log(2*sqrt(-x^2 + 2*x)
/abs(x - 1) + 2/abs(x - 1))
```

Fricas [B] time = 1.88967, size = 239, normalized size = 4.51

$$\frac{3\left(x^4-4x^3+4x^2\right)\log\left(\frac{x+\sqrt{-x^2+2x}}{x}\right)-3\left(x^4-4x^3+4x^2\right)\log\left(-\frac{x-\sqrt{-x^2+2x}}{x}\right)+\left(3x^2-6x-1\right)\sqrt{-x^2+2x}}{6\left(x^4-4x^3+4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-2*x)/(-x^2+2*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(x^4 - 4*x^3 + 4*x^2)*log((x + sqrt(-x^2 + 2*x))/x) - 3*(x^4 - 4*x^3
 + 4*x^2)*log(-(x - sqrt(-x^2 + 2*x))/x) + (3*x^2 - 6*x - 1)*sqrt(-x^2 + 2*
x))/(x^4 - 4*x^3 + 4*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{1}{x^5\sqrt{-x^2+2x}-5x^4\sqrt{-x^2+2x}+8x^3\sqrt{-x^2+2x}-4x^2\sqrt{-x^2+2x}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-2*x)/(-x**2+2*x)**(5/2),x)
```

```
[Out] -Integral(1/(x**5*sqrt(-x**2 + 2*x) - 5*x**4*sqrt(-x**2 + 2*x) + 8*x**3*sqr
t(-x**2 + 2*x) - 4*x**2*sqrt(-x**2 + 2*x)), x)/2
```

Giac [A] time = 1.40564, size = 77, normalized size = 1.45

$$\frac{(3(x-2)x-1)\sqrt{-x^2+2x}}{6\left(x^2-2x\right)^2}-\frac{1}{2}\log\left(-\frac{2\left(\sqrt{-x^2+2x}-1\right)}{|-2x+2|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-2*x)/(-x^2+2*x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(3*(x - 2)*x - 1)*sqrt(-x^2 + 2*x)/(x^2 - 2*x)^2 - 1/2*log(-2*(sqrt(-x^
2 + 2*x) - 1)/abs(-2*x + 2))
```

3.315 $\int \frac{(d+ex)^3}{\sqrt{bx+cx^2}}\,dx$

Optimal. Leaf size=149

$$\frac{e\sqrt{bx+cx^2}\left(15b^2e^2+10cex(2cd-be)-54bcde+64c^2d^2\right)}{24c^3}+\frac{(2cd-be)\left(5b^2e^2-8bcde+8c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{8c^{7/2}}+\frac{e\cdots}{}$$

```
[Out] (e*(d + e*x)^2*Sqrt[b*x + c*x^2])/(3*c) + (e*(64*c^2*d^2 - 54*b*c*d*e + 15*
b^2*e^2 + 10*c*e*(2*c*d - b*e)*x)*Sqrt[b*x + c*x^2])/(24*c^3) + ((2*c*d - b
*e)*(8*c^2*d^2 - 8*b*c*d*e + 5*b^2*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^
2]])/(8*c^(7/2))
```

Rubi [A] time = 0.149362, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {742, 779, 620, 206}

$$\frac{e\sqrt{bx+cx^2}\left(15b^2e^2+10cex(2cd-be)-54bcde+64c^2d^2\right)}{24c^3}+\frac{(2cd-be)\left(5b^2e^2-8bcde+8c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{8c^{7/2}}+\frac{e\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/Sqrt[b*x + c*x^2],x]
```

```
[Out] (e*(d + e*x)^2*Sqrt[b*x + c*x^2])/(3*c) + (e*(64*c^2*d^2 - 54*b*c*d*e + 15*
b^2*e^2 + 10*c*e*(2*c*d - b*e)*x)*Sqrt[b*x + c*x^2])/(24*c^3) + ((2*c*d - b
*e)*(8*c^2*d^2 - 8*b*c*d*e + 5*b^2*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^
2]])/(8*c^(7/2))
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^3}{\sqrt{bx+cx^2}}\,dx = \frac{e(d+ex)^2\sqrt{bx+cx^2}}{3c} + \frac{\int \frac{(d+ex)\left(\frac{1}{2}d(6cd-be)+\frac{5}{2}e(2cd-be)x\right)}{\sqrt{bx+cx^2}}\,dx}{3c}$$

$$= \frac{e(d+ex)^2\sqrt{bx+cx^2}}{3c} + \frac{e\left(64c^2d^2-54bcde+15b^2e^2+10ce(2cd-be)x\right)\sqrt{bx+cx^2}}{24c^3} + \frac{\left((2cd-be)\left(8c^2\right.\right.}{}$$

$$= \frac{e(d+ex)^2\sqrt{bx+cx^2}}{3c} + \frac{e\left(64c^2d^2-54bcde+15b^2e^2+10ce(2cd-be)x\right)\sqrt{bx+cx^2}}{24c^3} + \frac{\left((2cd-be)\left(8c^2\right.\right.}{}$$

$$= \frac{e(d+ex)^2\sqrt{bx+cx^2}}{3c} + \frac{e\left(64c^2d^2-54bcde+15b^2e^2+10ce(2cd-be)x\right)\sqrt{bx+cx^2}}{24c^3} + \frac{(2cd-be)\left(8c^2d^2\right.}{}$$

Mathematica [A] time = 0.465465, size = 152, normalized size = 1.02

$$\frac{\sqrt{x(b+cx)}\left(\sqrt{c}e\left(15b^2e^2-2bce(27d+5ex)+4c^2\left(18d^2+9dex+2e^2x^2\right)\right)+\frac{3\left(18b^2cde^2-5b^3e^3-24bc^2d^2e+16c^3d^3\right)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{x}\sqrt{\frac{cx}{b}+1}}\right)}{24c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/Sqrt[b*x + c*x^2],x]
```

```
[Out] (Sqrt[x*(b + c*x)]*(Sqrt[c]*e*(15*b^2*e^2 - 2*b*c*e*(27*d + 5*e*x) + 4*c^2*
(18*d^2 + 9*d*e*x + 2*e^2*x^2)) + (3*(16*c^3*d^3 - 24*b*c^2*d^2*e + 18*b^2*
c*d*e^2 - 5*b^3*e^3)*ArcSinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(Sqrt[b]*Sqrt[x]*S
qrt[1 + (c*x)/b])))/(24*c^(7/2))
```

Maple [A] time = 0.057, size = 265, normalized size = 1.8

$$\frac{e^3x^2}{3c}\sqrt{cx^2+bx} - \frac{5be^3x}{12c^2}\sqrt{cx^2+bx} + \frac{5e^3b^2}{8c^3}\sqrt{cx^2+bx} - \frac{5b^3e^3}{16}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx}\right)c^{-\frac{7}{2}} + \frac{3de^2x}{2c}\sqrt{cx^2+bx} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^2+b*x)^(1/2),x)
```

```
[Out] 1/3*e^3*x^2/c*(c*x^2+b*x)^(1/2)-5/12*e^3*b/c^2*x*(c*x^2+b*x)^(1/2)+5/8*e^3*
b^2/c^3*(c*x^2+b*x)^(1/2)-5/16*e^3*b^3/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^
2+b*x)^(1/2))+3/2*d*e^2*x/c*(c*x^2+b*x)^(1/2)-9/4*d*e^2*b/c^2*(c*x^2+b*x)^(
1/2)+9/8*d*e^2*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))+3*d^2*
e/c*(c*x^2+b*x)^(1/2)-3/2*d^2*e*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x
)^(1/2))+d^3*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.03101, size = 666, normalized size = 4.47

$$\left[-\frac{3\left(16\,c^3d^3-24\,bc^2d^2e+18\,b^2cde^2-5\,b^3e^3\right)\sqrt{c}\log\left(2\,cx+b-2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\left(8\,c^3e^3x^2+72\,c^3d^2e-54\,bc^2de^2\right.}{48\,c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*(16*c^3*d^3 - 24*b*c^2*d^2*e + 18*b^2*c*d*e^2 - 5*b^3*e^3)*sqrt(c
)*log(2*c*x + b - 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 2*(8*c^3*e^3*x^2 + 72*c^3*
d^2*e - 54*b*c^2*d*e^2 + 15*b^2*c*e^3 + 2*(18*c^3*d*e^2 - 5*b*c^2*e^3)*x)*s
qrt(c*x^2 + b*x))/c^4, -1/24*(3*(16*c^3*d^3 - 24*b*c^2*d^2*e + 18*b^2*c*d*e
^2 - 5*b^3*e^3)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)) - (8*c^3*
e^3*x^2 + 72*c^3*d^2*e - 54*b*c^2*d*e^2 + 15*b^2*c*e^3 + 2*(18*c^3*d*e^2 -
5*b*c^2*e^3)*x)*sqrt(c*x^2 + b*x))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^3}{\sqrt{x\,(b+cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral((d + e*x)**3/sqrt(x*(b + c*x)), x)
```

Giac [A] time = 1.32395, size = 198, normalized size = 1.33

$$\frac{1}{24}\,\sqrt{cx^2+bx}\left(2\,x\left(\frac{4\,xe^3}{c}+\frac{18\,c^2de^2-5\,bce^3}{c^3}\right)+\frac{3\left(24\,c^2d^2e-18\,bcde^2+5\,b^2e^3\right)}{c^3}\right)-\frac{\left(16\,c^3d^3-24\,bc^2d^2e+18\,b^2cde^2-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + b*x)*(2*x*(4*x*e^3/c + (18*c^2*d*e^2 - 5*b*c*e^3)/c^3) +
3*(24*c^2*d^2*e - 18*b*c*d*e^2 + 5*b^2*e^3)/c^3) - 1/16*(16*c^3*d^3 - 24*b*
c^2*d^2*e + 18*b^2*c*d*e^2 - 5*b^3*e^3)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2
+ b*x))*sqrt(c) - b))/c^(7/2)
```

3.316 $\int \frac{(d+ex)^2}{\sqrt{bx+cx^2}} dx$

Optimal. Leaf size=110

$$\frac{(3b^2e^2 - 8bcde + 8c^2d^2)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{5/2}} + \frac{3e\sqrt{bx+cx^2}(2cd-be)}{4c^2} + \frac{e\sqrt{bx+cx^2}(d+ex)}{2c}$$

```
[Out] (3*e*(2*c*d - b*e)*Sqrt[b*x + c*x^2])/(4*c^2) + (e*(d + e*x)*Sqrt[b*x + c*x
^2])/(2*c) + ((8*c^2*d^2 - 8*b*c*d*e + 3*b^2*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[
b*x + c*x^2]])/(4*c^(5/2))
```

Rubi [A] time = 0.0773418, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {742, 640, 620, 206}

$$\frac{(3b^2e^2 - 8bcde + 8c^2d^2)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{5/2}} + \frac{3e\sqrt{bx+cx^2}(2cd-be)}{4c^2} + \frac{e\sqrt{bx+cx^2}(d+ex)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/Sqrt[b*x + c*x^2],x]
```

```
[Out] (3*e*(2*c*d - b*e)*Sqrt[b*x + c*x^2])/(4*c^2) + (e*(d + e*x)*Sqrt[b*x + c*x
^2])/(2*c) + ((8*c^2*d^2 - 8*b*c*d*e + 3*b^2*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[
b*x + c*x^2]])/(4*c^(5/2))
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^2}{\sqrt{bx+cx^2}}\,dx = \frac{e(d+ex)\sqrt{bx+cx^2}}{2c} + \frac{\int \frac{\frac{1}{2}d(4cd-be)+\frac{3}{2}e(2cd-be)x}{\sqrt{bx+cx^2}}\,dx}{2c}$$

$$= \frac{3e(2cd-be)\sqrt{bx+cx^2}}{4c^2} + \frac{e(d+ex)\sqrt{bx+cx^2}}{2c} + \frac{\left(-\frac{3}{2}be(2cd-be)+cd(4cd-be)\right)\int \frac{1}{\sqrt{bx+cx^2}}\,dx}{4c^2}$$

$$= \frac{3e(2cd-be)\sqrt{bx+cx^2}}{4c^2} + \frac{e(d+ex)\sqrt{bx+cx^2}}{2c} + \frac{\left(-\frac{3}{2}be(2cd-be)+cd(4cd-be)\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\,d\right.}{2c^2}$$

$$= \frac{3e(2cd-be)\sqrt{bx+cx^2}}{4c^2} + \frac{e(d+ex)\sqrt{bx+cx^2}}{2c} + \frac{\left(8c^2d^2-8bcde+3b^2e^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{5/2}}$$

Mathematica [A] time = 0.105496, size = 111, normalized size = 1.01

$$\frac{\sqrt{b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\left(3b^2e^2-8bcde+8c^2d^2\right)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)+\sqrt{c}ex(b+cx)(-3be+8cd+2cex)}{4c^{5/2}\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/Sqrt[b*x + c*x^2],x]
```

```
[Out] (Sqrt[c]*e*x*(b + c*x)*(8*c*d - 3*b*e + 2*c*e*x) + Sqrt[b]*(8*c^2*d^2 - 8*b
*c*d*e + 3*b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*ArcSinh[(Sqrt[c]*Sqrt[x])/Sqr
t[b]])/(4*c^(5/2)*Sqrt[x*(b + c*x)])
```

Maple [A] time = 0.053, size = 158, normalized size = 1.4

$$\frac{e^2x}{2c}\sqrt{cx^2+bx}-\frac{3e^2b}{4c^2}\sqrt{cx^2+bx}+\frac{3b^2e^2}{8}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx}\right)c^{-\frac{5}{2}}+2\frac{de\sqrt{cx^2+bx}}{c}-bde\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^2+b*x)^(1/2),x)
```

```
[Out] 1/2*e^2*x/c*(c*x^2+b*x)^(1/2)-3/4*e^2*b/c^2*(c*x^2+b*x)^(1/2)+3/8*e^2*b^2/c
^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))+2*d*e/c*(c*x^2+b*x)^(1/2)-
d*e*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))+d^2*ln((1/2*b+c*x)/
c^(1/2)+(c*x^2+b*x)^(1/2))/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.11732, size = 432, normalized size = 3.93

$$\left[\frac{\left(8\,c^2d^2 - 8\,bcde + 3\,b^2e^2\right)\sqrt{c}\log\left(2\,cx + b + 2\,\sqrt{cx^2 + bx}\sqrt{c}\right) + 2\left(2\,c^2e^2x + 8\,c^2de - 3\,bce^2\right)\sqrt{cx^2 + bx}}{8\,c^3}, -\frac{\left(8\,c^2d^2 - 8\,bcde\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((8*c^2*d^2 - 8*b*c*d*e + 3*b^2*e^2)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*
x^2 + b*x)*sqrt(c)) + 2*(2*c^2*e^2*x + 8*c^2*d*e - 3*b*c*e^2)*sqrt(c*x^2 +
b*x))/c^3, -1/4*((8*c^2*d^2 - 8*b*c*d*e + 3*b^2*e^2)*sqrt(-c)*arctan(sqrt(c
*x^2 + b*x)*sqrt(-c)/(c*x)) - (2*c^2*e^2*x + 8*c^2*d*e - 3*b*c*e^2)*sqrt(c*
x^2 + b*x))/c^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^2}{\sqrt{x(b + cx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral((d + e*x)**2/sqrt(x*(b + c*x)), x)
```

Giac [A] time = 1.44893, size = 131, normalized size = 1.19

$$\frac{1}{4}\sqrt{cx^2 + bx}\left(\frac{2\,xe^2}{c} + \frac{8\,cde - 3\,be^2}{c^2}\right) - \frac{\left(8\,c^2d^2 - 8\,bcde + 3\,b^2e^2\right)\log\left(\left|-2\left(\sqrt{c}x - \sqrt{cx^2 + bx}\right)\sqrt{c} - b\right|\right)}{8\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^2 + b*x)*(2*x*e^2/c + (8*c*d*e - 3*b*e^2)/c^2) - 1/8*(8*c^2*d^
2 - 8*b*c*d*e + 3*b^2*e^2)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(
c) - b))/c^(5/2)
```

3.317 $\int \frac{d+ex}{\sqrt{bx+cx^2}}\,dx$

Optimal. Leaf size=55

$$\frac{(2cd-be)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{3/2}}+\frac{e\sqrt{bx+cx^2}}{c}$$

```
[Out] (e*Sqrt[b*x + c*x^2])/c + ((2*c*d - b*e)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x
^2]])/c^(3/2)
```

Rubi [A] time = 0.0217423, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {640, 620, 206}

$$\frac{(2cd-be)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{3/2}}+\frac{e\sqrt{bx+cx^2}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/Sqrt[b*x + c*x^2],x]
```

```
[Out] (e*Sqrt[b*x + c*x^2])/c + ((2*c*d - b*e)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x
^2]])/c^(3/2)
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex}{\sqrt{bx+cx^2}}\,dx &= \frac{e\sqrt{bx+cx^2}}{c}+\frac{(2cd-be)\int \frac{1}{\sqrt{bx+cx^2}}\,dx}{2c}\\ &= \frac{e\sqrt{bx+cx^2}}{c}+\frac{(2cd-be)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{bx+cx^2}}\right)}{c}\\ &= \frac{e\sqrt{bx+cx^2}}{c}+\frac{(2cd-be)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0617841, size = 80, normalized size = 1.45

$$\frac{\sqrt{c}ex(b+cx)-\sqrt{b}\sqrt{x}\sqrt{\frac{cx}{b}+1}(be-2cd)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{c^{3/2}\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/Sqrt[b*x + c*x^2],x]
```

```
[Out] (Sqrt[c]*e*x*(b + c*x) - Sqrt[b]*(-2*c*d + b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*A
rcSinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(c^(3/2)*Sqrt[x*(b + c*x)])
```

Maple [A] time = 0.05, size = 78, normalized size = 1.4

$$\frac{e}{c}\sqrt{cx^2+bx}-\frac{be}{2}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx}\right)c^{-\frac{3}{2}}+d\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^2+b*x)^(1/2),x)
```

```
[Out] e*(c*x^2+b*x)^(1/2)/c-1/2*e*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1
/2))+d*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8986, size = 274, normalized size = 4.98

$$\left[\frac{2\sqrt{cx^2+bx}ce-(2\,cd-be)\sqrt{c}\log\left(2\,cx+b-2\sqrt{cx^2+bx}\sqrt{c}\right)}{2\,c^2}, \frac{\sqrt{cx^2+bx}ce-(2\,cd-be)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx}\right)}{c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(c*x^2 + b*x)*c*e - (2*c*d - b*e)*sqrt(c)*log(2*c*x + b - 2*sqr
t(c*x^2 + b*x)*sqrt(c)))/c^2, (sqrt(c*x^2 + b*x)*c*e - (2*c*d - b*e)*sqrt(-
c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)))/c^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d+ex}{\sqrt{x\left(b+cx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral((d + e*x)/sqrt(x*(b + c*x)), x)
```

Giac [A] time = 1.36431, size = 85, normalized size = 1.55

$$\frac{\sqrt{cx^2+bx}e}{c}-\frac{\left(2\,cd-be\right)\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}-b\right|\right)}{2\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(c*x^2 + b*x)*e/c - 1/2*(2*c*d - b*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^
2 + b*x))*sqrt(c) - b))/c^(3/2)
```

3.318 $\int \frac{1}{\sqrt{bx+cx^2}}\,dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{\sqrt{c}}$$

[Out] (2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/Sqrt[c]

Rubi [A] time = 0.0088108, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[b*x + c*x^2],x]

[Out] (2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/Sqrt[c]

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{bx+cx^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{bx+cx^2}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{\sqrt{c}}\end{aligned}$$

Mathematica [B] time = 0.0168189, size = 57, normalized size = 2.04

$$\frac{2\sqrt{b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{c}\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[b*x + c*x^2],x]

```
[Out] (2*Sqrt[b]*Sqrt[x]*Sqrt[1 + (c*x)/b]*ArcSinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(S
qrt[c]*Sqrt[x*(b + c*x)])
```

Maple [A] time = 0.05, size = 29, normalized size = 1.

$$\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^(1/2),x)
```

```
[Out] ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88861, size = 154, normalized size = 5.5

$$\left[\frac{\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{\sqrt{c}},-\frac{2\,\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx}\right)}{c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/sqrt(c), -2*sqrt(-c)*arctan(s
qrt(c*x^2 + b*x)*sqrt(-c)/(c*x))/c]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*x + c*x**2), x)
```

Giac [A] time = 1.22622, size = 47, normalized size = 1.68

$$-\frac{\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}-b\right|\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b))/sqrt(c)
```

3.319 $\int \frac{1}{(d+ex)\sqrt{bx+cx^2}}\,dx$

Optimal. Leaf size=68

$$\frac{\tanh^{-1}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{\sqrt{d}\sqrt{cd-be}}$$

```
[Out] ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2
])]/(Sqrt[d]*Sqrt[c*d - b*e])
```

Rubi [A] time = 0.0305797, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {724, 206}

$$\frac{\tanh^{-1}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{\sqrt{d}\sqrt{cd-be}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[b*x + c*x^2]),x]
```

```
[Out] ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2
])]/(Sqrt[d]*Sqrt[c*d - b*e])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)\sqrt{bx+cx^2}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde-x^2}\,dx, x, \frac{-bd-(2cd-be)x}{\sqrt{bx+cx^2}}\right)\right)\\ &= \frac{\tanh^{-1}\left(\frac{bd+(2cd-be)x}{2\sqrt{d}\sqrt{cd-be}\sqrt{bx+cx^2}}\right)}{\sqrt{d}\sqrt{cd-be}}\end{aligned}$$

Mathematica [A] time = 0.0258039, size = 77, normalized size = 1.13

$$\frac{2\sqrt{x}\sqrt{b+cx}\tan^{-1}\left(\frac{\sqrt{x}\sqrt{be-cd}}{\sqrt{d}\sqrt{b+cx}}\right)}{\sqrt{d}\sqrt{x(b+cx)}\sqrt{be-cd}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[b*x + c*x^2]),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[b + c*x]*ArcTan[(Sqrt[-(c*d) + b*e]*Sqrt[x])/(Sqrt[d]*Sqrt[
b + c*x])])/(Sqrt[d]*Sqrt[-(c*d) + b*e]*Sqrt[x*(b + c*x)])
```

Maple [B] time = 0.216, size = 132, normalized size = 1.9

$$-\frac{1}{e}\ln\left(\left(-2\,\frac{d\,(be-cd)}{e^2}+\frac{be-2\,cd}{e}\left(\frac{d}{e}+x\right)+2\,\sqrt{-\frac{d\,(be-cd)}{e^2}}\sqrt{c\left(\frac{d}{e}+x\right)^2+\frac{be-2\,cd}{e}\left(\frac{d}{e}+x\right)-\frac{d\,(be-cd)}{e^2}}\right)\left(\frac{d}{e}+x\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+b*x)^(1/2),x)
```

```
[Out] -1/e/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+
2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e
^2)^(1/2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.05929, size = 284, normalized size = 4.18

$$\left[\frac{\log\left(\frac{bd+(2\,cd-be)x+2\,\sqrt{cd^2-bde}\sqrt{cx^2+bx}}{ex+d}\right)}{\sqrt{cd^2-bde}},\ \frac{2\,\sqrt{-cd^2+bde}\arctan\left(-\frac{\sqrt{-cd^2+bde}\sqrt{cx^2+bx}}{(cd-be)x}\right)}{cd^2-bde}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*d + (2*c*d - b*e)*x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*x
 + d))/sqrt(c*d^2 - b*d*e), 2*sqrt(-c*d^2 + b*d*e)*arctan(-sqrt(-c*d^2 + b*
d*e)*sqrt(c*x^2 + b*x)/((c*d - b*e)*x))/(c*d^2 - b*d*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x\,(b+cx)}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x*(b + c*x))*(d + e*x)), x)
```

Giac [A] time = 1.29285, size = 82, normalized size = 1.21

$$\frac{2\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde}}\right)}{\sqrt{-cd^2+bde}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d
*e))/sqrt(-c*d^2 + b*d*e)
```

3.320 $\int \frac{1}{(d+ex)^2\sqrt{bx+cx^2}}\,dx$

Optimal. Leaf size=117

$$\frac{(2cd-be)\tanh^{-1}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{2d^{3/2}(cd-be)^{3/2}}-\frac{e\sqrt{bx+cx^2}}{d(d+ex)(cd-be)}$$

```
[Out] -((e*Sqrt[b*x + c*x^2])/(d*(c*d - b*e)*(d + e*x))) + ((2*c*d - b*e)*ArcTanh
[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/(2
*d^(3/2)*(c*d - b*e)^(3/2))
```

Rubi [A] time = 0.0746605, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {730, 724, 206}

$$\frac{(2cd-be)\tanh^{-1}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{2d^{3/2}(cd-be)^{3/2}}-\frac{e\sqrt{bx+cx^2}}{d(d+ex)(cd-be)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*Sqrt[b*x + c*x^2]),x]
```

```
[Out] -((e*Sqrt[b*x + c*x^2])/(d*(c*d - b*e)*(d + e*x))) + ((2*c*d - b*e)*ArcTanh
[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/(2
*d^(3/2)*(c*d - b*e)^(3/2))
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\sqrt{bx+cx^2}}\,dx = -\frac{e\sqrt{bx+cx^2}}{d(cd-be)(d+ex)} + \frac{(2cd-be)\int \frac{1}{(d+ex)\sqrt{bx+cx^2}}\,dx}{2d(cd-be)}$$
$$= -\frac{e\sqrt{bx+cx^2}}{d(cd-be)(d+ex)} - \frac{(2cd-be)\,\mathrm{Subst}\left(\int \frac{1}{4cd^2-4bde-x^2}\,dx, x, \frac{-bd-(2cd-be)x}{\sqrt{bx+cx^2}}\right)}{d(cd-be)}$$
$$= -\frac{e\sqrt{bx+cx^2}}{d(cd-be)(d+ex)} + \frac{(2cd-be)\tanh^{-1}\left(\frac{bd+(2cd-be)x}{2\sqrt{d}\sqrt{cd-be}\sqrt{bx+cx^2}}\right)}{2d^{3/2}(cd-be)^{3/2}}$$

Mathematica [A] time = 0.143699, size = 122, normalized size = 1.04

$$\frac{\sqrt{x}\left(\frac{\sqrt{d}e\sqrt{x}(b+cx)}{(d+ex)(be-cd)} - \frac{\sqrt{b+cx}(2cd-be)\tan^{-1}\left(\frac{\sqrt{x}\sqrt{be-cd}}{\sqrt{d}\sqrt{b+cx}}\right)}{(be-cd)^{3/2}}\right)}{d^{3/2}\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*Sqrt[b*x + c*x^2]),x]
```

```
[Out] (Sqrt[x]*((Sqrt[d]*e*Sqrt[x]*(b + c*x))/((-(c*d) + b*e)*(d + e*x)) - ((2*c*
d - b*e)*Sqrt[b + c*x]*ArcTan[(Sqrt[-(c*d) + b*e]*Sqrt[x])/(Sqrt[d]*Sqrt[b
+ c*x])])/(-(c*d) + b*e)^(3/2)))/(d^(3/2)*Sqrt[x*(b + c*x)])
```

Maple [B] time = 0.257, size = 355, normalized size = 3.

$$\frac{1}{d\,(be-cd)}\sqrt{c\left(\frac{d}{e}+x\right)^2+\frac{be-2\,cd}{e}\left(\frac{d}{e}+x\right)-\frac{d\,(be-cd)}{e^2}}\left(\frac{d}{e}+x\right)^{-1}-\frac{b}{2\,d\,(be-cd)}\ln\left(\left(-2\,\frac{d\,(be-cd)}{e^2}+\frac{be-2\,cd}{e}\left(\frac{d}{e}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^2+b*x)^(1/2),x)
```

```
[Out] 1/d/(b*e-c*d)/(d/e+x)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(
1/2)-1/2/d/(b*e-c*d)/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2
*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+
x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*b+1/e/(b*e-c*d)/(-d*(b*e-c*d)/e^2)^(1/2
)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(
c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.04603, size = 716, normalized size = 6.12

$$\left[\frac{\left(2\,cd^2-bde+\left(2\,cde-be^2\right)x\right)\sqrt{cd^2-bde}\log\left(\frac{bd+(2\,cd-be)x+2\,\sqrt{cd^2-bde}\sqrt{cx^2+bx}}{ex+d}\right)-2\left(cd^2e-bde^2\right)\sqrt{cx^2+bx}}{2\left(c^2d^5-2\,bcd^4e+b^2d^3e^2+\left(c^2d^4e-2\,bcd^3e^2+b^2d^2e^3\right)x\right)},\ \frac{\left(2\,cd^2-bde+\left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((2*c*d^2 - b*d*e + (2*c*d*e - b*e^2)*x)*sqrt(c*d^2 - b*d*e)*log((b*d
+ (2*c*d - b*e)*x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*x + d)) - 2
*(c*d^2*e - b*d*e^2)*sqrt(c*x^2 + b*x))/(c^2*d^5 - 2*b*c*d^4*e + b^2*d^3*e^
2 + (c^2*d^4*e - 2*b*c*d^3*e^2 + b^2*d^2*e^3)*x), ((2*c*d^2 - b*d*e + (2*c*
d*e - b*e^2)*x)*sqrt(-c*d^2 + b*d*e)*arctan(-sqrt(-c*d^2 + b*d*e)*sqrt(c*x^
2 + b*x)/((c*d - b*e)*x)) - (c*d^2*e - b*d*e^2)*sqrt(c*x^2 + b*x))/(c^2*d^5
 - 2*b*c*d^4*e + b^2*d^3*e^2 + (c^2*d^4*e - 2*b*c*d^3*e^2 + b^2*d^2*e^3)*x)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x\,(b+cx)}\,(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x*(b + c*x))*(d + e*x)**2), x)
```

Giac [B] time = 2.00405, size = 540, normalized size = 4.62

$$\frac{\left(2\,cd\log\left(\left|2\,cd-be-2\,\sqrt{cd^2-bde}\sqrt{c}\right|\right)-be\log\left(\left|2\,cd-be-2\,\sqrt{cd^2-bde}\sqrt{c}\right|\right)+2\,\sqrt{cd^2-bde}\sqrt{c}\right)\mathrm{sgn}\left(\frac{1}{xe+d}\right)}{2\left(\sqrt{cd^2-bde}cd^2-\sqrt{cd^2-bde}bde\right)}-\frac{\sqrt{c-\frac{2c}{xe}}}{cd^2\mathrm{sg}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*c*d*log(abs(2*c*d - b*e - 2*sqrt(c*d^2 - b*d*e)*sqrt(c))) - b*e*log(
abs(2*c*d - b*e - 2*sqrt(c*d^2 - b*d*e)*sqrt(c))) + 2*sqrt(c*d^2 - b*d*e)*s
qrt(c))*sgn(1/(x*e + d))/(sqrt(c*d^2 - b*d*e)*c*d^2 - sqrt(c*d^2 - b*d*e)*b
*d*e) - sqrt(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + b*e/(x*e + d) - b*d*
e/(x*e + d)^2)/(c*d^2*sgn(1/(x*e + d)) - b*d*e*sgn(1/(x*e + d))) - 1/2*(2*c
*d*e - b*e^2)*log(abs(2*c*d - b*e - 2*sqrt(c*d^2 - b*d*e)*(sqrt(c - 2*c*d/(
x*e + d) + c*d^2/(x*e + d)^2 + b*e/(x*e + d) - b*d*e/(x*e + d)^2) + sqrt(c*
d^2*e^2 - b*d*e^3)*e^(-1)/(x*e + d))))/((c*d^2*e - b*d*e^2)*sqrt(c*d^2 - b*
d*e)*sgn(1/(x*e + d)))
```

3.321 $\int \frac{1}{(d+ex)^3\sqrt{bx+cx^2}}\,dx$

Optimal. Leaf size=180

$$\frac{(3b^2e^2-8bcde+8c^2d^2)\tanh^{-1}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{8d^{5/2}(cd-be)^{5/2}}-\frac{3e\sqrt{bx+cx^2}(2cd-be)}{4d^2(d+ex)(cd-be)^2}-\frac{e\sqrt{bx+cx^2}}{2d(d+ex)^2(cd-be)}$$

```
[Out] -(e*Sqrt[b*x + c*x^2])/(2*d*(c*d - b*e)*(d + e*x)^2) - (3*e*(2*c*d - b*e)*S
qrt[b*x + c*x^2])/(4*d^2*(c*d - b*e)^2*(d + e*x)) + ((8*c^2*d^2 - 8*b*c*d*e
 + 3*b^2*e^2)*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sq
rt[b*x + c*x^2])])/(8*d^(5/2)*(c*d - b*e)^(5/2))
```

Rubi [A] time = 0.172979, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {744, 806, 724, 206}

$$\frac{(3b^2e^2-8bcde+8c^2d^2)\tanh^{-1}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{8d^{5/2}(cd-be)^{5/2}}-\frac{3e\sqrt{bx+cx^2}(2cd-be)}{4d^2(d+ex)(cd-be)^2}-\frac{e\sqrt{bx+cx^2}}{2d(d+ex)^2(cd-be)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*Sqrt[b*x + c*x^2]),x]
```


```
[Out] -(e*Sqrt[b*x + c*x^2])/(2*d*(c*d - b*e)*(d + e*x)^2) - (3*e*(2*c*d - b*e)*S
qrt[b*x + c*x^2])/(4*d^2*(c*d - b*e)^2*(d + e*x)) + ((8*c^2*d^2 - 8*b*c*d*e
 + 3*b^2*e^2)*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sq
rt[b*x + c*x^2])])/(8*d^(5/2)*(c*d - b*e)^(5/2))
```


Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\sqrt{bx+cx^2}}\,dx = -\frac{e\sqrt{bx+cx^2}}{2d(cd-be)(d+ex)^2} - \frac{\int \frac{\frac{1}{2}(-4cd+3be)+cex}{(d+ex)^2\sqrt{bx+cx^2}}\,dx}{2d(cd-be)}$$

$$= -\frac{e\sqrt{bx+cx^2}}{2d(cd-be)(d+ex)^2} - \frac{3e(2cd-be)\sqrt{bx+cx^2}}{4d^2(cd-be)^2(d+ex)} + \frac{\left(8c^2d^2-8bcde+3b^2e^2\right)\int \frac{1}{(d+ex)\sqrt{bx+cx^2}}\,dx}{8d^2(cd-be)^2}$$

$$= -\frac{e\sqrt{bx+cx^2}}{2d(cd-be)(d+ex)^2} - \frac{3e(2cd-be)\sqrt{bx+cx^2}}{4d^2(cd-be)^2(d+ex)} - \frac{\left(8c^2d^2-8bcde+3b^2e^2\right)\text{Subst}\left(\int \frac{1}{4cd^2-4bd}\right.}{4d^2(cd-be)^2}$$

$$= -\frac{e\sqrt{bx+cx^2}}{2d(cd-be)(d+ex)^2} - \frac{3e(2cd-be)\sqrt{bx+cx^2}}{4d^2(cd-be)^2(d+ex)} + \frac{\left(8c^2d^2-8bcde+3b^2e^2\right)\tanh^{-1}\left(\frac{bd+(2c}{2\sqrt{d}\sqrt{cd-b}}\right.}{8d^{5/2}(cd-be)^{5/2}}$$

Mathematica [A] time = 0.257207, size = 183, normalized size = 1.02

$$\frac{\sqrt{x}\left(\frac{\sqrt{b+cx}\left(3b^2e^2-8bcde+8c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{x}\sqrt{be-cd}}{\sqrt{d}\sqrt{b+cx}}\right)}{2d^{3/2}(be-cd)^{3/2}}+\frac{3e\sqrt{x}(b+cx)(2cd-be)}{2d(d+ex)(cd-be)}+\frac{e\sqrt{x}(b+cx)}{(d+ex)^2}\right)}{2d\sqrt{x(b+cx)}(be-cd)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*Sqrt[b*x + c*x^2]),x]
```

```
[Out] (Sqrt[x]*((e*Sqrt[x]*(b + c*x))/(d + e*x)^2 + (3*e*(2*c*d - b*e)*Sqrt[x]*(b
 + c*x))/(2*d*(c*d - b*e)*(d + e*x)) + ((8*c^2*d^2 - 8*b*c*d*e + 3*b^2*e^2)
*Sqrt[b + c*x]*ArcTan[(Sqrt[-(c*d) + b*e]*Sqrt[x])/(Sqrt[d]*Sqrt[b + c*x])]
)/(2*d^(3/2)*(-(c*d) + b*e)^(3/2))))/(2*d*(-(c*d) + b*e)*Sqrt[x*(b + c*x)])
```

Maple [B] time = 0.265, size = 798, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*x^2+b*x)^(1/2),x)
```

```
[Out] 1/2/e/d/(b*e-c*d)/(d/e+x)^2*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/
e^2)^(1/2)+3/4*e/d^2/(b*e-c*d)^2/(d/e+x)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)
-d*(b*e-c*d)/e^2)^(1/2)*b-3/2/d/(b*e-c*d)^2/(d/e+x)*(c*(d/e+x)^2+(b*e-2*c*d
)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*c-3/8*e/d^2/(b*e-c*d)^2/(-d*(b*e-c*d)/e^
2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^
(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*b
```

```
^2+3/2/d/(b*e-c*d)^2/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2
*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+
x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*b*c-3/2/e/(b*e-c*d)^2/(-d*(b*e-c*d)/e^2
)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(
1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*c^
2-1/2/e*c/d/(b*e-c*d)/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-
2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e
+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.03269, size = 1501, normalized size = 8.34

$$\left[\frac{\left(8\,c^2d^4-8\,bcd^3e+3\,b^2d^2e^2+\left(8\,c^2d^2e^2-8\,bcde^3+3\,b^2e^4\right)x^2+2\left(8\,c^2d^3e-8\,bcd^2e^2+3\,b^2de^3\right)x\right)\sqrt{cd^2-bde}\log\left(\frac{bd+\cdots}{\cdots}\right)\cdots}{8\left(c^3d^8-3\,bc^2d^7e+3\,b^2cd^6e^2-b^3d^5e^3+\left(c^3d^6e^2-3\,bc^2d^5e^3+3\,b^2cd^4e\cdots\right)\cdots\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((8*c^2*d^4 - 8*b*c*d^3*e + 3*b^2*d^2*e^2 + (8*c^2*d^2*e^2 - 8*b*c*d*e
^3 + 3*b^2*e^4)*x^2 + 2*(8*c^2*d^3*e - 8*b*c*d^2*e^2 + 3*b^2*d*e^3)*x)*sqrt
(c*d^2 - b*d*e)*log((b*d + (2*c*d - b*e)*x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x
^2 + b*x))/(e*x + d)) - 2*(8*c^2*d^4*e - 13*b*c*d^3*e^2 + 5*b^2*d^2*e^3 + 3
*(2*c^2*d^3*e^2 - 3*b*c*d^2*e^3 + b^2*d*e^4)*x)*sqrt(c*x^2 + b*x))/(c^3*d^8
 - 3*b*c^2*d^7*e + 3*b^2*c*d^6*e^2 - b^3*d^5*e^3 + (c^3*d^6*e^2 - 3*b*c^2*d
^5*e^3 + 3*b^2*c*d^4*e^4 - b^3*d^3*e^5)*x^2 + 2*(c^3*d^7*e - 3*b*c^2*d^6*e^
2 + 3*b^2*c*d^5*e^3 - b^3*d^4*e^4)*x), 1/4*((8*c^2*d^4 - 8*b*c*d^3*e + 3*b^
2*d^2*e^2 + (8*c^2*d^2*e^2 - 8*b*c*d*e^3 + 3*b^2*e^4)*x^2 + 2*(8*c^2*d^3*e
- 8*b*c*d^2*e^2 + 3*b^2*d*e^3)*x)*sqrt(-c*d^2 + b*d*e)*arctan(-sqrt(-c*d^2
+ b*d*e)*sqrt(c*x^2 + b*x)/((c*d - b*e)*x)) - (8*c^2*d^4*e - 13*b*c*d^3*e^2
 + 5*b^2*d^2*e^3 + 3*(2*c^2*d^3*e^2 - 3*b*c*d^2*e^3 + b^2*d*e^4)*x)*sqrt(c*
x^2 + b*x))/(c^3*d^8 - 3*b*c^2*d^7*e + 3*b^2*c*d^6*e^2 - b^3*d^5*e^3 + (c^3
*d^6*e^2 - 3*b*c^2*d^5*e^3 + 3*b^2*c*d^4*e^4 - b^3*d^3*e^5)*x^2 + 2*(c^3*d^
7*e - 3*b*c^2*d^6*e^2 + 3*b^2*c*d^5*e^3 - b^3*d^4*e^4)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x(b+cx)}\,(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x*(b + c*x))*(d + e*x)**3), x)
```

Giac [B] time = 1.3162, size = 657, normalized size = 3.65

$$-\frac{\left(8\,c^2d^2-8\,bcde+3\,b^2e^2\right)\arctan\left(\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde}}\right)}{4\left(c^2d^4-2\,bcd^3e+b^2d^2e^2\right)\sqrt{-cd^2+bde}}-\frac{8\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3c^2d^2e+24\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^2c^{\frac{5}{2}}d^3-24$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(8*c^2*d^2 - 8*b*c*d*e + 3*b^2*e^2)*arctan(((sqrt(c)*x - sqrt(c*x^2 +
b*x))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e))/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^
2*e^2)*sqrt(-c*d^2 + b*d*e)) - 1/4*(8*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*c^2
*d^2*e + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*c^(5/2)*d^3 - 24*(sqrt(c)*x -
 sqrt(c*x^2 + b*x))^2*b*c^(3/2)*d^2*e + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x))*
b*c^2*d^3 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b*c*d*e^2 - 20*(sqrt(c)*x -
 sqrt(c*x^2 + b*x))*b^2*c*d^2*e + 6*b^2*c^(3/2)*d^3 + 9*(sqrt(c)*x - sqrt(c
*x^2 + b*x))^2*b^2*sqrt(c)*d*e^2 - 3*b^3*sqrt(c)*d^2*e + 3*(sqrt(c)*x - sqr
t(c*x^2 + b*x))^3*b^2*e^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b^3*d*e^2)/((
c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2)*((sqrt(c)*x - sqrt(c*x^2 + b*x))^2*e +
 2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c)*d + b*d)^2)
```

3.322 $\int \frac{(d+ex)^3}{(bx+cx^2)^{3/2}} dx$

Optimal. Leaf size=139

$$\frac{e\sqrt{bx+cx^2}\left(3b^2e^2+2cex(2cd-be)-6bcde+8c^2d^2\right)}{b^2c^2}-\frac{2(d+ex)^2(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}+\frac{3e^2(2cd-be)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{5/2}}$$

```
[Out] (-2*(d + e*x)^2*(b*d + (2*c*d - b*e)*x))/(b^2*Sqrt[b*x + c*x^2]) + (e*(8*c^
2*d^2 - 6*b*c*d*e + 3*b^2*e^2 + 2*c*e*(2*c*d - b*e)*x)*Sqrt[b*x + c*x^2])/(
b^2*c^2) + (3*e^2*(2*c*d - b*e)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/c^(
5/2)
```

Rubi [A] time = 0.0931574, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {738, 779, 620, 206}

$$\frac{e\sqrt{bx+cx^2}\left(3b^2e^2+2cex(2cd-be)-6bcde+8c^2d^2\right)}{b^2c^2}-\frac{2(d+ex)^2(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}+\frac{3e^2(2cd-be)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*(d + e*x)^2*(b*d + (2*c*d - b*e)*x))/(b^2*Sqrt[b*x + c*x^2]) + (e*(8*c^
2*d^2 - 6*b*c*d*e + 3*b^2*e^2 + 2*c*e*(2*c*d - b*e)*x)*Sqrt[b*x + c*x^2])/(
b^2*c^2) + (3*e^2*(2*c*d - b*e)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/c^(
5/2)
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(bx+cx^2\right)^{3/2}}\,dx = -\frac{2(d+ex)^2(bd+(2cd-be)x)}{b^2\sqrt{bx+cx^2}} - \frac{2\int \frac{(d+ex)(-2bde-2e(2cd-be)x)}{\sqrt{bx+cx^2}}\,dx}{b^2}$$

$$= -\frac{2(d+ex)^2(bd+(2cd-be)x)}{b^2\sqrt{bx+cx^2}} + \frac{e\left(8c^2d^2-6bcde+3b^2e^2+2ce(2cd-be)x\right)\sqrt{bx+cx^2}}{b^2c^2} + \frac{\left(3e^2(2cd\right.}{}$$

$$= -\frac{2(d+ex)^2(bd+(2cd-be)x)}{b^2\sqrt{bx+cx^2}} + \frac{e\left(8c^2d^2-6bcde+3b^2e^2+2ce(2cd-be)x\right)\sqrt{bx+cx^2}}{b^2c^2} + \frac{\left(3e^2(2cd\right.}{}$$

$$= -\frac{2(d+ex)^2(bd+(2cd-be)x)}{b^2\sqrt{bx+cx^2}} + \frac{e\left(8c^2d^2-6bcde+3b^2e^2+2ce(2cd-be)x\right)\sqrt{bx+cx^2}}{b^2c^2} + \frac{3e^2(2cd-}{}$$

Mathematica [A] time = 0.112178, size = 129, normalized size = 0.93

$$\frac{\sqrt{c}\left(b^2ce^2x(ex-6d)+3b^3e^3x-2bc^2d^2(d-3ex)-4c^3d^3x\right)-3b^{5/2}e^2\sqrt{x}\sqrt{\frac{cx}{b}+1}(be-2cd)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{b^2c^{5/2}\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(b*x + c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[c]*(-4*c^3*d^3*x + 3*b^3*e^3*x - 2*b*c^2*d^2*(d - 3*e*x) + b^2*c*e^2*
x*(-6*d + e*x)) - 3*b^(5/2)*e^2*(-2*c*d + b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Ar
cSinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(b^2*c^(5/2)*Sqrt[x*(b + c*x)])
```

Maple [A] time = 0.054, size = 177, normalized size = 1.3

$$\frac{e^3x^2}{c}\frac{1}{\sqrt{cx^2+bx}}+3\frac{be^3x}{c^2\sqrt{cx^2+bx}}-\frac{3\,be^3}{2}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx}\right)c^{-\frac{5}{2}}-6\frac{de^2x}{c\sqrt{cx^2+bx}}+3\frac{de^2}{c^{3/2}}\ln\left(\frac{b/2+cx}{\sqrt{c}}+\sqrt{cx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^2+b*x)^(3/2),x)
```

```
[Out] e^3*x^2/c/(c*x^2+b*x)^(1/2)+3*e^3*b/c^2/(c*x^2+b*x)^(1/2)*x-3/2*e^3*b/c^(5/
2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))-6*d*e^2/c/(c*x^2+b*x)^(1/2)*x+
3*d*e^2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))+6*d^2*e/b/(c*x^2+
b*x)^(1/2)*x-2*d^3*(2*c*x+b)/b^2/(c*x^2+b*x)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.11231, size = 730, normalized size = 5.25

$$\left[-\frac{3\left(\left(2\,b^2c^2de^2-b^3ce^3\right)x^2+\left(2\,b^3cde^2-b^4e^3\right)x\right)\sqrt{c}\log\left(2\,cx+b-2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\left(b^2c^2e^3x^2-2\,bc^3d^3-\left(4\,c^4d^3-\right.\right.}{2\left(b^2c^4x^2+b^3c^3x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(3*((2*b^2*c^2*d*e^2 - b^3*c*e^3)*x^2 + (2*b^3*c*d*e^2 - b^4*e^3)*x)*
sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 2*(b^2*c^2*e^3*x^2 -
 2*b*c^3*d^3 - (4*c^4*d^3 - 6*b*c^3*d^2*e + 6*b^2*c^2*d*e^2 - 3*b^3*c*e^3)*
x)*sqrt(c*x^2 + b*x))/(b^2*c^4*x^2 + b^3*c^3*x), -(3*((2*b^2*c^2*d*e^2 - b^
3*c*e^3)*x^2 + (2*b^3*c*d*e^2 - b^4*e^3)*x)*sqrt(-c)*arctan(sqrt(c*x^2 + b*
x)*sqrt(-c)/(c*x)) - (b^2*c^2*e^3*x^2 - 2*b*c^3*d^3 - (4*c^4*d^3 - 6*b*c^3*
d^2*e + 6*b^2*c^2*d*e^2 - 3*b^3*c*e^3)*x)*sqrt(c*x^2 + b*x))/(b^2*c^4*x^2 +
 b^3*c^3*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^3}{(x\,(b+cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral((d + e*x)**3/(x*(b + c*x))**(3/2), x)
```

Giac [A] time = 1.32247, size = 169, normalized size = 1.22

$$-\frac{\frac{2\,d^3}{b}-x\left(\frac{xe^3}{c}-\frac{4\,c^3d^3-6\,bc^2d^2e+6\,b^2cde^2-3\,b^3e^3}{b^2c^2}\right)}{\sqrt{cx^2+bx}}-\frac{3\left(2\,cde^2-be^3\right)\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}-b\right|\right)}{2\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] -(2*d^3/b - x*(x*e^3/c - (4*c^3*d^3 - 6*b*c^2*d^2*e + 6*b^2*c*d*e^2 - 3*b^3
*e^3)/(b^2*c^2)))/sqrt(c*x^2 + b*x) - 3/2*(2*c*d*e^2 - b*e^3)*log(abs(-2*(s
qrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b))/c^(5/2)
```

3.323 $\int \frac{(d+ex)^2}{\left(bx+cx^2\right)^{3/2}} dx$

Optimal. Leaf size=101

$$\frac{2e\sqrt{bx+cx^2}(2cd-be)}{b^2c}-\frac{2(d+ex)(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}+\frac{2e^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{3/2}}$$

```
[Out] (-2*(d + e*x)*(b*d + (2*c*d - b*e)*x))/(b^2*Sqrt[b*x + c*x^2]) + (2*e*(2*c*
d - b*e)*Sqrt[b*x + c*x^2])/(b^2*c) + (2*e^2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x +
 c*x^2]])/c^(3/2)
```

Rubi [A] time = 0.061544, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {738, 640, 620, 206}

$$\frac{2e\sqrt{bx+cx^2}(2cd-be)}{b^2c}-\frac{2(d+ex)(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}+\frac{2e^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(b*x + c*x^2)^(3/2),x]
```


```
[Out] (-2*(d + e*x)*(b*d + (2*c*d - b*e)*x))/(b^2*Sqrt[b*x + c*x^2]) + (2*e*(2*c*
d - b*e)*Sqrt[b*x + c*x^2])/(b^2*c) + (2*e^2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x +
 c*x^2]])/c^(3/2)
```


Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(bx+cx^2\right)^{3/2}}\,dx = -\frac{2(d+ex)(bd+(2cd-be)x)}{b^2\sqrt{bx+cx^2}} - \frac{2\int \frac{-bde-e(2cd-be)x}{\sqrt{bx+cx^2}}\,dx}{b^2}$$

$$= -\frac{2(d+ex)(bd+(2cd-be)x)}{b^2\sqrt{bx+cx^2}} + \frac{2e(2cd-be)\sqrt{bx+cx^2}}{b^2c} + \frac{e^2\int \frac{1}{\sqrt{bx+cx^2}}\,dx}{c}$$

$$= -\frac{2(d+ex)(bd+(2cd-be)x)}{b^2\sqrt{bx+cx^2}} + \frac{2e(2cd-be)\sqrt{bx+cx^2}}{b^2c} + \frac{\left(2e^2\right)\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{bx+cx^2}}\right)}{c}$$

$$= -\frac{2(d+ex)(bd+(2cd-be)x)}{b^2\sqrt{bx+cx^2}} + \frac{2e(2cd-be)\sqrt{bx+cx^2}}{b^2c} + \frac{2e^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{3/2}}$$

Mathematica [A] time = 0.0960489, size = 100, normalized size = 0.99

$$\frac{2b^{5/2}e^2\sqrt{x}\sqrt{\frac{cx}{b}+1}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)-2\sqrt{c}\left(b^2e^2x+bcd(d-2ex)+2c^2d^2x\right)}{b^2c^{3/2}\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[c]*(2*c^2*d^2*x + b^2*e^2*x + b*c*d*(d - 2*e*x)) + 2*b^(5/2)*e^2*S
qrt[x]*Sqrt[1 + (c*x)/b]*ArcSinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(b^2*c^(3/2)*S
qrt[x*(b + c*x)])
```

Maple [A] time = 0.061, size = 97, normalized size = 1.

$$-2\,\frac{e^2x}{c\sqrt{cx^2+bx}}+e^2\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx}\right)c^{-\frac{3}{2}}+4\,\frac{dex}{b\sqrt{cx^2+bx}}-2\,\frac{d^2\,(2\,cx+b)}{b^2\sqrt{cx^2+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^2+b*x)^(3/2),x)
```

```
[Out] -2*e^2/c/(c*x^2+b*x)^(1/2)*x+e^2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)
^(1/2))+4*d*e/b/(c*x^2+b*x)^(1/2)*x-2*d^2*(2*c*x+b)/b^2/(c*x^2+b*x)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.05542, size = 505, normalized size = 5.

$$\left[\frac{\left(b^{2}ce^{2}x^{2}+b^{3}e^{2}x\right)\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^{2}+bx}\sqrt{c}\right)-2\left(bc^{2}d^{2}+\left(2\,c^{3}d^{2}-2\,bc^{2}de+b^{2}ce^{2}\right)x\right)\sqrt{cx^{2}+bx}}{b^{2}c^{3}x^{2}+b^{3}c^{2}x},-\frac{2\left(\left(b^{2}ce^{2}x^{2}+\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [((b^2*c*e^2*x^2 + b^3*e^2*x)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*s
qrt(c)) - 2*(b*c^2*d^2 + (2*c^3*d^2 - 2*b*c^2*d*e + b^2*c*e^2)*x)*sqrt(c*x^
2 + b*x))/(b^2*c^3*x^2 + b^3*c^2*x), -2*((b^2*c*e^2*x^2 + b^3*e^2*x)*sqrt(-
c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)) + (b*c^2*d^2 + (2*c^3*d^2 - 2*b
*c^2*d*e + b^2*c*e^2)*x)*sqrt(c*x^2 + b*x))/(b^2*c^3*x^2 + b^3*c^2*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^{2}}{(x\,(b+cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral((d + e*x)**2/(x*(b + c*x))**(3/2), x)
```

Giac [A] time = 1.26464, size = 120, normalized size = 1.19

$$-\frac{2\left(\frac{d^{2}}{b}+\frac{\left(2\,c^{2}d^{2}-2\,bcde+b^{2}e^{2}\right)x}{b^{2}c}\right)}{\sqrt{cx^{2}+bx}}-\frac{e^{2}\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^{2}+bx}\right)\sqrt{c}-b\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(d^2/b + (2*c^2*d^2 - 2*b*c*d*e + b^2*e^2)*x/(b^2*c))/sqrt(c*x^2 + b*x)
- e^2*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b))/c^(3/2)
```

3.324 $\int \frac{d+ex}{\left(bx+cx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=33

$$-\frac{2(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

```
[Out] (-2*(b*d + (2*c*d - b*e)*x))/(b^2*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.0103688, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {636}

$$-\frac{2(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*(b*d + (2*c*d - b*e)*x))/(b^2*Sqrt[b*x + c*x^2])
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{d+ex}{\left(bx+cx^2\right)^{3/2}} \, dx = -\frac{2(bd+(2cd-be)x)}{b^2\sqrt{bx+cx^2}}$$

Mathematica [A] time = 0.0118774, size = 30, normalized size = 0.91

$$\frac{2(-bd+bex-2cdx)}{b^2\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(-(b*d) - 2*c*d*x + b*e*x))/(b^2*Sqrt[x*(b + c*x)])
```

Maple [A] time = 0.047, size = 37, normalized size = 1.1

$$-2\,\frac{x\,(cx+b)\,(-bxe+2\,cdx+bd)}{b^2\left(cx^2+bx\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^2+b*x)^(3/2),x)
```

```
[Out] -2*x*(c*x+b)*(-b*e*x+2*c*d*x+b*d)/b^2/(c*x^2+b*x)^(3/2)
```

Maxima [A] time = 1.05328, size = 74, normalized size = 2.24

$$-\frac{4\,cdx}{\sqrt{cx^2+bx}b^2}+\frac{2\,ex}{\sqrt{cx^2+bx}b}-\frac{2\,d}{\sqrt{cx^2+bx}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] -4*c*d*x/(sqrt(c*x^2 + b*x)*b^2) + 2*e*x/(sqrt(c*x^2 + b*x)*b) - 2*d/(sqrt(
c*x^2 + b*x)*b)
```

Fricas [A] time = 1.96475, size = 89, normalized size = 2.7

$$-\frac{2\,\sqrt{cx^2+bx}(bd+(2\,cd-be)x)}{b^2cx^2+b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(c*x^2 + b*x)*(b*d + (2*c*d - b*e)*x)/(b^2*c*x^2 + b^3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{d+ex}{(x\,(b+cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral((d + e*x)/(x*(b + c*x))**(3/2), x)
```

Giac [A] time = 1.32997, size = 46, normalized size = 1.39

$$-\frac{2\left(\frac{d}{b}+\frac{(2\,cd-be)x}{b^2}\right)}{\sqrt{cx^2+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(d/b + (2*c*d - b*e)*x/b^2)/sqrt(c*x^2 + b*x)
```

3.325 $\int \frac{1}{(bx+cx^2)^{3/2}} dx$

Optimal. Leaf size=24

$$-\frac{2(b+2cx)}{b^2\sqrt{bx+cx^2}}$$

```
[Out] (-2*(b + 2*c*x))/(b^2*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.0036398, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {613}

$$-\frac{2(b+2cx)}{b^2\sqrt{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(-3/2),x]
```

```
[Out] (-2*(b + 2*c*x))/(b^2*Sqrt[b*x + c*x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{(bx+cx^2)^{3/2}} dx = -\frac{2(b+2cx)}{b^2\sqrt{bx+cx^2}}$$

Mathematica [A] time = 0.006469, size = 22, normalized size = 0.92

$$-\frac{2(b+2cx)}{b^2\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(-3/2),x]
```

```
[Out] (-2*(b + 2*c*x))/(b^2*Sqrt[x*(b + c*x)])
```

Maple [A] time = 0.044, size = 29, normalized size = 1.2

$$-2\,\frac{x(cx+b)(2\,cx+b)}{b^2\left(cx^2+bx\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^(3/2),x)
```

```
[Out] -2*x*(c*x+b)*(2*c*x+b)/b^2/(c*x^2+b*x)^(3/2)
```

Maxima [A] time = 1.12876, size = 47, normalized size = 1.96

$$-\frac{4\,cx}{\sqrt{cx^2+bx}b^2}-\frac{2}{\sqrt{cx^2+bx}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] -4*c*x/(sqrt(c*x^2 + b*x)*b^2) - 2/(sqrt(c*x^2 + b*x)*b)
```

Fricas [A] time = 1.81908, size = 73, normalized size = 3.04

$$-\frac{2\,\sqrt{cx^2+bx}(2\,cx+b)}{b^2cx^2+b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(c*x^2 + b*x)*(2*c*x + b)/(b^2*c*x^2 + b^3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral((b*x + c*x**2)**(-3/2), x)
```

Giac [A] time = 1.44096, size = 32, normalized size = 1.33

$$-\frac{2\left(\frac{2\,cx}{b^2}+\frac{1}{b}\right)}{\sqrt{cx^2+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(2*c*x/b^2 + 1/b)/sqrt(c*x^2 + b*x)
```

3.326 $\int \frac{1}{(d+ex)\left(bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=126

$$\frac{e^2\tanh^{-1}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{d^{3/2}(cd-be)^{3/2}}-\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(cd-be)}$$

```
[Out] (-2*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(b^2*d*(c*d - b*e)*Sqrt[b*x + c*x^
2]) + (e^2*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[
b*x + c*x^2])])/(d^(3/2)*(c*d - b*e)^(3/2))
```

Rubi [A] time = 0.0962518, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {740, 12, 724, 206}

$$\frac{e^2\tanh^{-1}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{d^{3/2}(cd-be)^{3/2}}-\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(cd-be)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(b^2*d*(c*d - b*e)*Sqrt[b*x + c*x^
2]) + (e^2*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[
b*x + c*x^2])])/(d^(3/2)*(c*d - b*e)^(3/2))
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)^{3/2}}\,dx = -\frac{2(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)\sqrt{bx+cx^2}} - \frac{2\int -\frac{b^2e^2}{2(d+ex)\sqrt{bx+cx^2}}\,dx}{b^2d(cd-be)}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)\sqrt{bx+cx^2}} + \frac{e^2\int \frac{1}{(d+ex)\sqrt{bx+cx^2}}\,dx}{d(cd-be)}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)\sqrt{bx+cx^2}} - \frac{\left(2e^2\right)\text{Subst}\left(\int \frac{1}{4cd^2-4bde-x^2}\,dx, x, \frac{-bd-(2cd-be)x}{\sqrt{bx+cx^2}}\right)}{d(cd-be)}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)\sqrt{bx+cx^2}} + \frac{e^2\tanh^{-1}\left(\frac{bd+(2cd-be)x}{2\sqrt{d}\sqrt{cd-be}\sqrt{bx+cx^2}}\right)}{d^{3/2}(cd-be)^{3/2}}$$

Mathematica [A] time = 0.101216, size = 129, normalized size = 1.02

$$-\frac{2\left(\sqrt{d}\sqrt{be-cd}\left(b^2e-bcd+bcex-2c^2dx\right)+b^2e^2\sqrt{x}\sqrt{b+cx}\tan^{-1}\left(\frac{\sqrt{x}\sqrt{be-cd}}{\sqrt{d}\sqrt{b+cx}}\right)\right)}{b^2d^{3/2}\sqrt{x(b+cx)}(be-cd)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(Sqrt[d]*Sqrt[-(c*d) + b*e]*(-(b*c*d) + b^2*e - 2*c^2*d*x + b*c*e*x) +
b^2*e^2*Sqrt[x]*Sqrt[b + c*x]*ArcTan[(Sqrt[-(c*d) + b*e]*Sqrt[x])/(Sqrt[d]*
Sqrt[b + c*x])]))/(b^2*d^(3/2)*(-(c*d) + b*e)^(3/2)*Sqrt[x*(b + c*x)])
```

Maple [B] time = 0.258, size = 403, normalized size = 3.2

$$-2\,\frac{e}{d\,(be-cd)}\frac{1}{\sqrt{c\left(\frac{d}{e}+x\right)^2+\frac{be-2\,cd}{e}\left(\frac{d}{e}+x\right)-\frac{d(be-cd)}{e^2}}}-2\,\frac{exc}{d\,(be-cd)\,b}\frac{1}{\sqrt{c\left(\frac{d}{e}+x\right)^2+\frac{be-2\,cd}{e}\left(\frac{d}{e}+x\right)-\frac{d(be-cd)}{e^2}}}+4\,\frac{c^2}{(be-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+b*x)^(3/2),x)
```

```
[Out] -2*e/d/(b*e-c*d)/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)-
2*e/d/(b*e-c*d)/b/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)
*x*c+4/(b*e-c*d)/b^2/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1
/2)*x*c^2+2/(b*e-c*d)/b/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)
^(1/2)*c+e/d/(b*e-c*d)/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e
-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/
e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.98713, size = 909, normalized size = 7.21

$$\left[-\frac{\left(b^2ce^2x^2+b^3e^2x\right)\sqrt{cd^2-bde}\log\left(\frac{bd+(2\,cd-be)x-2\,\sqrt{cd^2-bde}\sqrt{cx^2+bx}}{ex+d}\right)+2\left(bc^2d^3-2\,b^2cd^2e+b^3de^2+\left(2\,c^3d^3-3\,bc^2d^2e+\right.\right.}{\left(b^2c^3d^4-2\,b^3c^2d^3e+b^4cd^2e^2\right)x^2+\left(b^3c^2d^4-2\,b^4cd^3e+b^5d^2e^2\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-((b^2*c*e^2*x^2 + b^3*e^2*x)*sqrt(c*d^2 - b*d*e)*log((b*d + (2*c*d - b*e)
*x - 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*x + d)) + 2*(b*c^2*d^3 - 2
*b^2*c*d^2*e + b^3*d*e^2 + (2*c^3*d^3 - 3*b*c^2*d^2*e + b^2*c*d*e^2)*x)*sqr
t(c*x^2 + b*x))/((b^2*c^3*d^4 - 2*b^3*c^2*d^3*e + b^4*c*d^2*e^2)*x^2 + (b^3
*c^2*d^4 - 2*b^4*c*d^3*e + b^5*d^2*e^2)*x), 2*((b^2*c*e^2*x^2 + b^3*e^2*x)*
sqrt(-c*d^2 + b*d*e)*arctan(-sqrt(-c*d^2 + b*d*e)*sqrt(c*x^2 + b*x)/((c*d -
 b*e)*x)) - (b*c^2*d^3 - 2*b^2*c*d^2*e + b^3*d*e^2 + (2*c^3*d^3 - 3*b*c^2*d
^2*e + b^2*c*d*e^2)*x)*sqrt(c*x^2 + b*x))/((b^2*c^3*d^4 - 2*b^3*c^2*d^3*e +
 b^4*c*d^2*e^2)*x^2 + (b^3*c^2*d^4 - 2*b^4*c*d^3*e + b^5*d^2*e^2)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x\left(b+cx\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral(1/((x*(b + c*x))**(3/2)*(d + e*x)), x)
```

Giac [A] time = 1.40023, size = 225, normalized size = 1.79

$$-\frac{2\left(\frac{\left(2\,c^2d^2-bcde\right)x}{b^2cd^3-b^3d^2e}+\frac{bcd^2-b^2de}{b^2cd^3-b^3d^2e}\right)}{\sqrt{cx^2+bx}}-\frac{2\arctan\left(\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde}}\right)e^2}{\left(cd^2-bde\right)\sqrt{-cd^2+bde}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^2*d^2 - b*c*d*e)*x/(b^2*c*d^3 - b^3*d^2*e) + (b*c*d^2 - b^2*d*e)/(
b^2*c*d^3 - b^3*d^2*e))/sqrt(c*x^2 + b*x) - 2*arctan(((sqrt(c)*x - sqrt(c*x
^2 + b*x))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e))*e^2/((c*d^2 - b*d*e)*sqrt(-
c*d^2 + b*d*e))
```

3.327 $\int \frac{1}{(d+ex)^2\left(bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=207

$$-\frac{e\sqrt{bx+cx^2}\left(3b^2e^2-4bcde+4c^2d^2\right)}{b^2d^2(d+ex)(cd-be)^2}-\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(d+ex)(cd-be)}+\frac{3e^2(2cd-be)\tanh^{-1}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{2d^{5/2}(cd-be)^{5/2}}$$

```
[Out] (-2*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(b^2*d*(c*d - b*e)*(d + e*x)*Sqrt[
b*x + c*x^2]) - (e*(4*c^2*d^2 - 4*b*c*d*e + 3*b^2*e^2)*Sqrt[b*x + c*x^2])/(
b^2*d^2*(c*d - b*e)^2*(d + e*x)) + (3*e^2*(2*c*d - b*e)*ArcTanh[(b*d + (2*c
*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/(2*d^(5/2)*(c*
d - b*e)^(5/2))
```

Rubi [A] time = 0.235391, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {740, 806, 724, 206}

$$-\frac{e\sqrt{bx+cx^2}\left(3b^2e^2-4bcde+4c^2d^2\right)}{b^2d^2(d+ex)(cd-be)^2}-\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(d+ex)(cd-be)}+\frac{3e^2(2cd-be)\tanh^{-1}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{2d^{5/2}(cd-be)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(b^2*d*(c*d - b*e)*(d + e*x)*Sqrt[
b*x + c*x^2]) - (e*(4*c^2*d^2 - 4*b*c*d*e + 3*b^2*e^2)*Sqrt[b*x + c*x^2])/(
b^2*d^2*(c*d - b*e)^2*(d + e*x)) + (3*e^2*(2*c*d - b*e)*ArcTanh[(b*d + (2*c
*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/(2*d^(5/2)*(c*
d - b*e)^(5/2))
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(bx+cx^2\right)^{3/2}}\,dx = -\frac{2(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)(d+ex)\sqrt{bx+cx^2}} - \frac{2\int \frac{\frac{1}{2}be(2cd-3be)+ce(2cd-be)x}{(d+ex)^2\sqrt{bx+cx^2}}\,dx}{b^2d(cd-be)}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)(d+ex)\sqrt{bx+cx^2}} - \frac{e\left(4c^2d^2-4bcde+3b^2e^2\right)\sqrt{bx+cx^2}}{b^2d^2(cd-be)^2(d+ex)} + \frac{\left(3e^2(2cd-\right.}{2\cdots}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)(d+ex)\sqrt{bx+cx^2}} - \frac{e\left(4c^2d^2-4bcde+3b^2e^2\right)\sqrt{bx+cx^2}}{b^2d^2(cd-be)^2(d+ex)} - \frac{\left(3e^2(2cd-b\right.}{\cdots}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)(d+ex)\sqrt{bx+cx^2}} - \frac{e\left(4c^2d^2-4bcde+3b^2e^2\right)\sqrt{bx+cx^2}}{b^2d^2(cd-be)^2(d+ex)} + \frac{3e^2(2cd-b\cdots}{\cdots}$$

Mathematica [A] time = 0.188565, size = 206, normalized size = 1.

$$\frac{-\sqrt{d}\sqrt{be-cd}\left(b^2ce\left(-4d^2-2dex+3e^2x^2\right)+b^3e^2(2d+3ex)+2bc^2d\left(d^2-dex-2e^2x^2\right)+4c^3d^2x(d+ex)\right)-3b^2e^2\sqrt{x}\cdots}{b^2d^{5/2}\sqrt{x(b+cx)}(d+ex)(be-cd)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-(Sqrt[d]*Sqrt[-(c*d) + b*e]*(4*c^3*d^2*x*(d + e*x) + b^3*e^2*(2*d + 3*e*x
) + 2*b*c^2*d*(d^2 - d*e*x - 2*e^2*x^2) + b^2*c*e*(-4*d^2 - 2*d*e*x + 3*e^2
*x^2))) - 3*b^2*e^2*(-2*c*d + b*e)*Sqrt[x]*Sqrt[b + c*x]*(d + e*x)*ArcTan[(
Sqrt[-(c*d) + b*e]*Sqrt[x])/(Sqrt[d]*Sqrt[b + c*x])])/(b^2*d^(5/2)*(-(c*d)
+ b*e)^(5/2)*Sqrt[x*(b + c*x)]*(d + e*x))
```

Maple [B] time = 0.228, size = 893, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^2+b*x)^(3/2),x)
```

```
[Out] 1/d/(b*e-c*d)/(d/e+x)/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(
1/2)-3*e^2/d^2/(b*e-c*d)^2/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e
^2)^(1/2)*b+9*e/d/(b*e-c*d)^2/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d
```

```
)/e^2)^(1/2)*c-3*e^2/d^2/(b*e-c*d)^2/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(
b*e-c*d)/e^2)^(1/2)*x*c+12*e/d/(b*e-c*d)^2/b/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/
e+x)-d*(b*e-c*d)/e^2)^(1/2)*x*c^2-12/(b*e-c*d)^2/b^2/(c*(d/e+x)^2+(b*e-2*c*
d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*x*c^3-6/(b*e-c*d)^2/b/(c*(d/e+x)^2+(b*e
-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*c^2+3/2*e^2/d^2/(b*e-c*d)^2/(-d*(b
*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-
c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/
(d/e+x))*b-3*e/d/(b*e-c*d)^2/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^
2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)
/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*c-8*c^2/d/(b*e-c*d)/b^2/(c*(d/e
+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*x-4*c/d/(b*e-c*d)/b/(c*(
d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.13507, size = 1791, normalized size = 8.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(3*((2*b^2*c^2*d*e^3 - b^3*c*e^4)*x^3 + (2*b^2*c^2*d^2*e^2 + b^3*c*d*
e^3 - b^4*e^4)*x^2 + (2*b^3*c*d^2*e^2 - b^4*d*e^3)*x)*sqrt(c*d^2 - b*d*e)*l
og((b*d + (2*c*d - b*e)*x - 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*x +
 d)) + 2*(2*b*c^3*d^5 - 6*b^2*c^2*d^4*e + 6*b^3*c*d^3*e^2 - 2*b^4*d^2*e^3 +
 (4*c^4*d^4*e - 8*b*c^3*d^3*e^2 + 7*b^2*c^2*d^2*e^3 - 3*b^3*c*d*e^4)*x^2 +
(4*c^4*d^5 - 6*b*c^3*d^4*e + 5*b^3*c*d^2*e^3 - 3*b^4*d*e^4)*x)*sqrt(c*x^2 +
 b*x))/((b^2*c^4*d^6*e - 3*b^3*c^3*d^5*e^2 + 3*b^4*c^2*d^4*e^3 - b^5*c*d^3*
e^4)*x^3 + (b^2*c^4*d^7 - 2*b^3*c^3*d^6*e + 2*b^5*c*d^4*e^3 - b^6*d^3*e^4)*
x^2 + (b^3*c^3*d^7 - 3*b^4*c^2*d^6*e + 3*b^5*c*d^5*e^2 - b^6*d^4*e^3)*x), (
3*((2*b^2*c^2*d*e^3 - b^3*c*e^4)*x^3 + (2*b^2*c^2*d^2*e^2 + b^3*c*d*e^3 - b
^4*e^4)*x^2 + (2*b^3*c*d^2*e^2 - b^4*d*e^3)*x)*sqrt(-c*d^2 + b*d*e)*arctan(
-sqrt(-c*d^2 + b*d*e)*sqrt(c*x^2 + b*x)/((c*d - b*e)*x)) - (2*b*c^3*d^5 - 6
*b^2*c^2*d^4*e + 6*b^3*c*d^3*e^2 - 2*b^4*d^2*e^3 + (4*c^4*d^4*e - 8*b*c^3*d
^3*e^2 + 7*b^2*c^2*d^2*e^3 - 3*b^3*c*d*e^4)*x^2 + (4*c^4*d^5 - 6*b*c^3*d^4*
e + 5*b^3*c*d^2*e^3 - 3*b^4*d*e^4)*x)*sqrt(c*x^2 + b*x))/((b^2*c^4*d^6*e -
3*b^3*c^3*d^5*e^2 + 3*b^4*c^2*d^4*e^3 - b^5*c*d^3*e^4)*x^3 + (b^2*c^4*d^7 -
 2*b^3*c^3*d^6*e + 2*b^5*c*d^4*e^3 - b^6*d^3*e^4)*x^2 + (b^3*c^3*d^7 - 3*b^
4*c^2*d^6*e + 3*b^5*c*d^5*e^2 - b^6*d^4*e^3)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.328 $\int \frac{1}{(d+ex)^3\left(bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=296

$$-\frac{e\sqrt{bx+cx^2}(2cd-be)\left(15b^2e^2-8bcde+8c^2d^2\right)}{4b^2d^3(d+ex)(cd-be)^3}-\frac{e\sqrt{bx+cx^2}\left(5b^2e^2-8bcde+8c^2d^2\right)}{2b^2d^2(d+ex)^2(cd-be)^2}+\frac{3e^2\left(5b^2e^2-16bcde+16c^2d^2\right)}{8d^{7/2}(cd-}$$

```
[Out] (-2*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(b^2*d*(c*d - b*e)*(d + e*x)^2*Sqr
t[b*x + c*x^2]) - (e*(8*c^2*d^2 - 8*b*c*d*e + 5*b^2*e^2)*Sqrt[b*x + c*x^2])
/(2*b^2*d^2*(c*d - b*e)^2*(d + e*x)^2) - (e*(2*c*d - b*e)*(8*c^2*d^2 - 8*b*
c*d*e + 15*b^2*e^2)*Sqrt[b*x + c*x^2])/(4*b^2*d^3*(c*d - b*e)^3*(d + e*x))
+ (3*e^2*(16*c^2*d^2 - 16*b*c*d*e + 5*b^2*e^2)*ArcTanh[(b*d + (2*c*d - b*e)
*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/(8*d^(7/2)*(c*d - b*e)^
(7/2))
```

Rubi [A] time = 0.343582, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {740, 834, 806, 724, 206}

$$-\frac{e\sqrt{bx+cx^2}(2cd-be)\left(15b^2e^2-8bcde+8c^2d^2\right)}{4b^2d^3(d+ex)(cd-be)^3}-\frac{e\sqrt{bx+cx^2}\left(5b^2e^2-8bcde+8c^2d^2\right)}{2b^2d^2(d+ex)^2(cd-be)^2}+\frac{3e^2\left(5b^2e^2-16bcde+16c^2d^2\right)}{8d^{7/2}(cd-}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(b*x + c*x^2)^(3/2)),x]
```



```
[Out] (-2*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(b^2*d*(c*d - b*e)*(d + e*x)^2*Sqr
t[b*x + c*x^2]) - (e*(8*c^2*d^2 - 8*b*c*d*e + 5*b^2*e^2)*Sqrt[b*x + c*x^2])
/(2*b^2*d^2*(c*d - b*e)^2*(d + e*x)^2) - (e*(2*c*d - b*e)*(8*c^2*d^2 - 8*b*
c*d*e + 15*b^2*e^2)*Sqrt[b*x + c*x^2])/(4*b^2*d^3*(c*d - b*e)^3*(d + e*x))
+ (3*e^2*(16*c^2*d^2 - 16*b*c*d*e + 5*b^2*e^2)*ArcTanh[(b*d + (2*c*d - b*e)
*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/(8*d^(7/2)*(c*d - b*e)^
(7/2))
```


Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(bx+cx^2\right)^{3/2}}\,dx = -\frac{2(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)(d+ex)^2\sqrt{bx+cx^2}} - \frac{2\int \frac{\frac{1}{2}be(4cd-5be)+2ce(2cd-be)x}{(d+ex)^3\sqrt{bx+cx^2}}\,dx}{b^2d(cd-be)}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)(d+ex)^2\sqrt{bx+cx^2}} - \frac{e\left(8c^2d^2-8bcde+5b^2e^2\right)\sqrt{bx+cx^2}}{2b^2d^2(cd-be)^2(d+ex)^2} + \frac{\int \frac{-\frac{1}{4}be\left(8c^2d \ldots\right.}{\ldots}}{\ldots}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)(d+ex)^2\sqrt{bx+cx^2}} - \frac{e\left(8c^2d^2-8bcde+5b^2e^2\right)\sqrt{bx+cx^2}}{2b^2d^2(cd-be)^2(d+ex)^2} - \frac{e(2cd-be)\ldots}{\ldots}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)(d+ex)^2\sqrt{bx+cx^2}} - \frac{e\left(8c^2d^2-8bcde+5b^2e^2\right)\sqrt{bx+cx^2}}{2b^2d^2(cd-be)^2(d+ex)^2} - \frac{e(2cd-be)\ldots}{\ldots}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)(d+ex)^2\sqrt{bx+cx^2}} - \frac{e\left(8c^2d^2-8bcde+5b^2e^2\right)\sqrt{bx+cx^2}}{2b^2d^2(cd-be)^2(d+ex)^2} - \frac{e(2cd-be)\ldots}{\ldots}$$

Mathematica [A] time = 0.892341, size = 247, normalized size = 0.83

$$\frac{\frac{2b^2c^2de(19ex-18d)+b^3ce^2(43d-15ex)-15b^4e^3+8bc^3d^2(d-3ex)+16c^4d^3x}{b^2d^2(cd-be)^2} - \frac{3e^2\sqrt{x}\sqrt{b+cx}\left(5b^2e^2-16bcde+16c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{x}\sqrt{be-cd}}{\sqrt{d}\sqrt{b+cx}}\right)}{d^{5/2}(be-cd)^{5/2}} + \frac{5e(2cd-be)}{d(d+ex)(cd-be)} + \ldots}{4d\sqrt{x(b+cx)}(be-cd)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(b*x + c*x^2)^(3/2)),x]
```

```
[Out] ((2*e)/(d + e*x)^2 + (5*e*(2*c*d - b*e))/(d*(c*d - b*e)*(d + e*x)) + (-15*b
^4*e^3 + 16*c^4*d^3*x + b^3*c*e^2*(43*d - 15*e*x) + 8*b*c^3*d^2*(d - 3*e*x)
 + 2*b^2*c^2*d*e*(-18*d + 19*e*x))/(b^2*d^2*(c*d - b*e)^2) - (3*e^2*(16*c^2
*d^2 - 16*b*c*d*e + 5*b^2*e^2)*Sqrt[x]*Sqrt[b + c*x]*ArcTan[(Sqrt[-(c*d) +
b*e]*Sqrt[x])/(Sqrt[d]*Sqrt[b + c*x])])/(d^(5/2)*(-(c*d) + b*e)^(5/2)))/(4*
d*(-(c*d) + b*e)*Sqrt[x*(b + c*x)])
```

Maple [B] time = 0.244, size = 1612, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*x^2+b*x)^(3/2),x)
```

```
[Out] 1/2/e/d/(b*e-c*d)/(d/e+x)^2/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/
e^2)^(1/2)+5/4*e/d^2/(b*e-c*d)^2/(d/e+x)/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)
-d*(b*e-c*d)/e^2)^(1/2)*b-5/2/d/(b*e-c*d)^2/(d/e+x)/(c*(d/e+x)^2+(b*e-2*c*d
)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*c-15/4*e^3/d^3/(b*e-c*d)^3/(c*(d/e+x)^2+
(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*b^2+75/4*e^2/d^2/(b*e-c*d)^3/(
c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*b*c-30*e/d/(b*e-c*
d)^3/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*c^2-15/4*e^3
/d^3/(b*e-c*d)^3/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*
x*c*b+45/2*e^2/d^2/(b*e-c*d)^3/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*
d)/e^2)^(1/2)*x*c^2-45*e/d/(b*e-c*d)^3/b/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)
-d*(b*e-c*d)/e^2)^(1/2)*x*c^3+30/(b*e-c*d)^3/b^2/(c*(d/e+x)^2+(b*e-2*c*d)/e
*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*x*c^4+15/(b*e-c*d)^3/b/(c*(d/e+x)^2+(b*e-2*
c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*c^3+15/8*e^3/d^3/(b*e-c*d)^3/(-d*(b*e
-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*
d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d
/e+x))*b^2-15/2*e^2/d^2/(b*e-c*d)^3/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-
c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e
-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*b*c+15/2*e/d/(b*e-c*d)^3
/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-
d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^
(1/2))/(d/e+x))*c^2-13*e/d^2/(b*e-c*d)^2*c^2/b/(c*(d/e+x)^2+(b*e-2*c*d)/e*(
d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*x+26/d/(b*e-c*d)^2*c^3/b^2/(c*(d/e+x)^2+(b*e-
2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*x-8*e/d^2/(b*e-c*d)^2*c/(c*(d/e+x)^
2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)+13/d/(b*e-c*d)^2*c^2/b/(c*(d
/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)+3/2*e*c/d^2/(b*e-c*d)^
2/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(
-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)
^(1/2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.53763, size = 3347, normalized size = 11.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*((16*b^2*c^3*d^2*e^4 - 16*b^3*c^2*d*e^5 + 5*b^4*c*e^6)*x^4 + (32*b
^2*c^3*d^3*e^3 - 16*b^3*c^2*d^2*e^4 - 6*b^4*c*d*e^5 + 5*b^5*e^6)*x^3 + (16*
b^2*c^3*d^4*e^2 + 16*b^3*c^2*d^3*e^3 - 27*b^4*c*d^2*e^4 + 10*b^5*d*e^5)*x^2
 + (16*b^3*c^2*d^4*e^2 - 16*b^4*c*d^3*e^3 + 5*b^5*d^2*e^4)*x)*sqrt(c*d^2 -
b*d*e)*log((b*d + (2*c*d - b*e)*x - 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x)
)/(e*x + d)) + 2*(8*b*c^4*d^7 - 32*b^2*c^3*d^6*e + 48*b^3*c^2*d^5*e^2 - 32*
b^4*c*d^4*e^3 + 8*b^5*d^3*e^4 + (16*c^5*d^5*e^2 - 40*b*c^4*d^4*e^3 + 62*b^2
*c^3*d^3*e^4 - 53*b^3*c^2*d^2*e^5 + 15*b^4*c*d*e^6)*x^3 + (32*c^5*d^6*e - 7
2*b*c^4*d^5*e^2 + 80*b^2*c^3*d^4*e^3 - 27*b^3*c^2*d^3*e^4 - 28*b^4*c*d^2*e^
5 + 15*b^5*d*e^6)*x^2 + (16*c^5*d^7 - 24*b*c^4*d^6*e - 16*b^2*c^3*d^5*e^2 +
 80*b^3*c^2*d^4*e^3 - 81*b^4*c*d^3*e^4 + 25*b^5*d^2*e^5)*x)*sqrt(c*x^2 + b*
x))/((b^2*c^5*d^8*e^2 - 4*b^3*c^4*d^7*e^3 + 6*b^4*c^3*d^6*e^4 - 4*b^5*c^2*d
^5*e^5 + b^6*c*d^4*e^6)*x^4 + (2*b^2*c^5*d^9*e - 7*b^3*c^4*d^8*e^2 + 8*b^4*
c^3*d^7*e^3 - 2*b^5*c^2*d^6*e^4 - 2*b^6*c*d^5*e^5 + b^7*d^4*e^6)*x^3 + (b^2
*c^5*d^10 - 2*b^3*c^4*d^9*e - 2*b^4*c^3*d^8*e^2 + 8*b^5*c^2*d^7*e^3 - 7*b^6
*c*d^6*e^4 + 2*b^7*d^5*e^5)*x^2 + (b^3*c^4*d^10 - 4*b^4*c^3*d^9*e + 6*b^5*c
^2*d^8*e^2 - 4*b^6*c*d^7*e^3 + b^7*d^6*e^4)*x), 1/4*(3*((16*b^2*c^3*d^2*e^4
 - 16*b^3*c^2*d*e^5 + 5*b^4*c*e^6)*x^4 + (32*b^2*c^3*d^3*e^3 - 16*b^3*c^2*d
^2*e^4 - 6*b^4*c*d*e^5 + 5*b^5*e^6)*x^3 + (16*b^2*c^3*d^4*e^2 + 16*b^3*c^2*
d^3*e^3 - 27*b^4*c*d^2*e^4 + 10*b^5*d*e^5)*x^2 + (16*b^3*c^2*d^4*e^2 - 16*b
^4*c*d^3*e^3 + 5*b^5*d^2*e^4)*x)*sqrt(-c*d^2 + b*d*e)*arctan(-sqrt(-c*d^2 +
 b*d*e)*sqrt(c*x^2 + b*x)/((c*d - b*e)*x)) - (8*b*c^4*d^7 - 32*b^2*c^3*d^6*
e + 48*b^3*c^2*d^5*e^2 - 32*b^4*c*d^4*e^3 + 8*b^5*d^3*e^4 + (16*c^5*d^5*e^2
 - 40*b*c^4*d^4*e^3 + 62*b^2*c^3*d^3*e^4 - 53*b^3*c^2*d^2*e^5 + 15*b^4*c*d*
e^6)*x^3 + (32*c^5*d^6*e - 72*b*c^4*d^5*e^2 + 80*b^2*c^3*d^4*e^3 - 27*b^3*c
^2*d^3*e^4 - 28*b^4*c*d^2*e^5 + 15*b^5*d*e^6)*x^2 + (16*c^5*d^7 - 24*b*c^4*
d^6*e - 16*b^2*c^3*d^5*e^2 + 80*b^3*c^2*d^4*e^3 - 81*b^4*c*d^3*e^4 + 25*b^5
*d^2*e^5)*x)*sqrt(c*x^2 + b*x))/((b^2*c^5*d^8*e^2 - 4*b^3*c^4*d^7*e^3 + 6*b
^4*c^3*d^6*e^4 - 4*b^5*c^2*d^5*e^5 + b^6*c*d^4*e^6)*x^4 + (2*b^2*c^5*d^9*e
- 7*b^3*c^4*d^8*e^2 + 8*b^4*c^3*d^7*e^3 - 2*b^5*c^2*d^6*e^4 - 2*b^6*c*d^5*e
^5 + b^7*d^4*e^6)*x^3 + (b^2*c^5*d^10 - 2*b^3*c^4*d^9*e - 2*b^4*c^3*d^8*e^2
 + 8*b^5*c^2*d^7*e^3 - 7*b^6*c*d^6*e^4 + 2*b^7*d^5*e^5)*x^2 + (b^3*c^4*d^10
 - 4*b^4*c^3*d^9*e + 6*b^5*c^2*d^8*e^2 - 4*b^6*c*d^7*e^3 + b^7*d^6*e^4)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x(b+cx))^{\frac{3}{2}}(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral(1/((x*(b + c*x))**(3/2)*(d + e*x)**3), x)
```

Giac [B] time = 1.97243, size = 975, normalized size = 3.29

$$-\frac{2\left(\frac{\left(2c^4d^6-3bc^3d^5e+3b^2c^2d^4e^2-b^3cd^3e^3\right)x}{b^2c^3d^9-3b^3c^2d^8e+3b^4cd^7e^2-b^5d^6e^3}+\frac{bc^3d^6-3b^2c^2d^5e+3b^3cd^4e^2-b^4d^3e^3}{b^2c^3d^9-3b^3c^2d^8e+3b^4cd^7e^2-b^5d^6e^3}\right)}{\sqrt{cx^2+bx}}+\frac{3\left(16c^2d^2e^2-16bcde^3+5b^2e^4\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2\cdots}\right.}{\sqrt{-cd\cdots}}\right.}{4\left(c^3d^6-3bc^2d^5e+3b^2cd^4e^2-b^3d^3e^3\right)\sqrt{-cd^2\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^4*d^6 - 3*b*c^3*d^5*e + 3*b^2*c^2*d^4*e^2 - b^3*c*d^3*e^3)*x/(b^2*
c^3*d^9 - 3*b^3*c^2*d^8*e + 3*b^4*c*d^7*e^2 - b^5*d^6*e^3) + (b*c^3*d^6 - 3
*b^2*c^2*d^5*e + 3*b^3*c*d^4*e^2 - b^4*d^3*e^3)/(b^2*c^3*d^9 - 3*b^3*c^2*d^
8*e + 3*b^4*c*d^7*e^2 - b^5*d^6*e^3))/sqrt(c*x^2 + b*x) + 3/4*(16*c^2*d^2*e
^2 - 16*b*c*d*e^3 + 5*b^2*e^4)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x))*e +
 sqrt(c)*d)/sqrt(-c*d^2 + b*d*e))/((c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e
^2 - b^3*d^3*e^3)*sqrt(-c*d^2 + b*d*e)) - 1/4*(56*(sqrt(c)*x - sqrt(c*x^2 +
 b*x))^2*c^(5/2)*d^3*e^2 + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*c^2*d^2*e^3
 + 56*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b*c^2*d^3*e^2 - 48*(sqrt(c)*x - sqrt(
c*x^2 + b*x))^2*b*c^(3/2)*d^2*e^3 + 14*b^2*c^(3/2)*d^3*e^2 - 24*(sqrt(c)*x
- sqrt(c*x^2 + b*x))^3*b*c*d*e^4 - 44*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b^2*c
*d^2*e^3 + 13*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*b^2*sqrt(c)*d*e^4 - 7*b^3*s
qrt(c)*d^2*e^3 + 7*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b^2*e^5 + 9*(sqrt(c)*x
 - sqrt(c*x^2 + b*x))*b^3*d*e^4)/((c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^
2 - b^3*d^3*e^3)*((sqrt(c)*x - sqrt(c*x^2 + b*x))^2*e + 2*(sqrt(c)*x - sqrt
(c*x^2 + b*x))*sqrt(c)*d + b*d)^2)
```

3.329 $\int \frac{(d+ex)^4}{\left(bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=208

$$-\frac{2e\sqrt{bx+cx^2}(2cd-be)\left(-3b^2e^2-8bcde+8c^2d^2\right)}{3b^4c^2}+\frac{4(d+ex)\left(x(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)+bcd^2(4cd-5be\right.}{3b^4c\sqrt{bx+cx^2}}$$

```
[Out] (-2*(d + e*x)^3*(b*d + (2*c*d - b*e)*x))/(3*b^2*(b*x + c*x^2)^(3/2)) + (4*(
d + e*x)*(b*c*d^2*(4*c*d - 5*b*e) + (2*c*d - b*e)*(4*c^2*d^2 - 4*b*c*d*e -
b^2*e^2)*x))/(3*b^4*c*Sqrt[b*x + c*x^2]) - (2*e*(2*c*d - b*e)*(8*c^2*d^2 -
8*b*c*d*e - 3*b^2*e^2)*Sqrt[b*x + c*x^2])/(3*b^4*c^2) + (2*e^4*ArcTanh[(Sqr
t[c]*x)/Sqrt[b*x + c*x^2]])/c^(5/2)
```

Rubi [A] time = 0.198054, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {738, 818, 640, 620, 206}

$$-\frac{2e\sqrt{bx+cx^2}(2cd-be)\left(-3b^2e^2-8bcde+8c^2d^2\right)}{3b^4c^2}+\frac{4(d+ex)\left(x(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)+bcd^2(4cd-5be\right.}{3b^4c\sqrt{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^3*(b*d + (2*c*d - b*e)*x))/(3*b^2*(b*x + c*x^2)^(3/2)) + (4*(
d + e*x)*(b*c*d^2*(4*c*d - 5*b*e) + (2*c*d - b*e)*(4*c^2*d^2 - 4*b*c*d*e -
b^2*e^2)*x))/(3*b^4*c*Sqrt[b*x + c*x^2]) - (2*e*(2*c*d - b*e)*(8*c^2*d^2 -
8*b*c*d*e - 3*b^2*e^2)*Sqrt[b*x + c*x^2])/(3*b^4*c^2) + (2*e^4*ArcTanh[(Sqr
t[c]*x)/Sqrt[b*x + c*x^2]])/c^(5/2)
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
```

```
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^4}{\left(bx+cx^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^3(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}}-\frac{2\int \frac{(d+ex)^2(d(4cd-5be)-e(2cd-be)x)}{\left(bx+cx^2\right)^{3/2}}\,dx}{3b^2}$$

$$= -\frac{2(d+ex)^3(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}}+\frac{4(d+ex)\left(bcd^2(4cd-5be)+(2cd-be)\left(4c^2d^2-4bcde-b^2e^2\right)x\right)}{3b^4c\sqrt{bx+cx^2}}$$

$$= -\frac{2(d+ex)^3(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}}+\frac{4(d+ex)\left(bcd^2(4cd-5be)+(2cd-be)\left(4c^2d^2-4bcde-b^2e^2\right)x\right)}{3b^4c\sqrt{bx+cx^2}}$$

$$= -\frac{2(d+ex)^3(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}}+\frac{4(d+ex)\left(bcd^2(4cd-5be)+(2cd-be)\left(4c^2d^2-4bcde-b^2e^2\right)x\right)}{3b^4c\sqrt{bx+cx^2}}$$

$$= -\frac{2(d+ex)^3(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}}+\frac{4(d+ex)\left(bcd^2(4cd-5be)+(2cd-be)\left(4c^2d^2-4bcde-b^2e^2\right)x\right)}{3b^4c\sqrt{bx+cx^2}}$$

Mathematica [C] time = 2.01076, size = 389, normalized size = 1.87

$$-\frac{\sqrt{\frac{cx}{b}+1}\left(-33792b^2cx(d+ex)^4\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},2,2,2,\frac{7}{2}\right\},\left\{1,1,1,\frac{9}{2}\right\},-\frac{cx}{b}\right)-\frac{77\left(\sqrt{-\frac{cx(b+cx)}{b^2}}\left(2b^2cx\left(3810d^2e^2x^2+5\right.\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^4/(b*x + c*x^2)^(5/2),x]
```

```
[Out] -(Sqrt[1 + (c*x)/b]*((-77*(Sqrt[-((c*x*(b + c*x))/b^2)]*(-3*b^3*(1895*d^4 +
 5060*d^3*e*x + 3810*d^2*e^2*x^2 + 20*d*e^3*x^3 - 241*e^4*x^4) + 2*b^2*c*x*
(1895*d^4 + 5060*d^3*e*x + 3810*d^2*e^2*x^2 + 20*d*e^3*x^3 - 241*e^4*x^4) -
```

```
48*c^3*x^3*(-109*d^4 + 84*d^3*e*x + 138*d^2*e^2*x^2 + 100*d*e^3*x^3 + 27*e
^4*x^4) + 8*b*c^2*x^2*(-427*d^4 - 1588*d^3*e*x + 102*d^2*e^2*x^2 + 188*d*e^
3*x^3 + 77*e^4*x^4)) + 3*b^3*(1895*d^4 + 5060*d^3*e*x + 3810*d^2*e^2*x^2 +
20*d*e^3*x^3 - 241*e^4*x^4)*ArcSin[Sqrt[-((c*x)/b)]]))/(-((c*x)/b))^(5/2) +
 21504*c^3*d*x^3*(d + e*x)^3*Hypergeometric2F1[3/2, 11/2, 13/2, -((c*x)/b)]
 - 33792*b^2*c*x*(d + e*x)^4*HypergeometricPFQ[{-1/2, 2, 2, 2, 7/2}, {1, 1,
 1, 9/2}, -((c*x)/b)]))/(44352*b^5*x*Sqrt[x*(b + c*x)])
```

Maple [B] time = 0.055, size = 447, normalized size = 2.2

$$-\frac{e^4x^3}{3c}\left(cx^2+bx\right)^{-\frac{3}{2}}+\frac{e^4bx^2}{2c^2}\left(cx^2+bx\right)^{-\frac{3}{2}}+\frac{e^4b^2x}{6c^3}\left(cx^2+bx\right)^{-\frac{3}{2}}-\frac{7e^4x}{3c^2}\frac{1}{\sqrt{cx^2+bx}}-\frac{e^4b}{6c^3}\frac{1}{\sqrt{cx^2+bx}}+e^4\ln\left(\left(\frac{b}{2}+cx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(c*x^2+b*x)^(5/2),x)
```

```
[Out] -1/3*e^4*x^3/c/(c*x^2+b*x)^(3/2)+1/2*e^4*b/c^2*x^2/(c*x^2+b*x)^(3/2)+1/6*e^
4*b^2/c^3/(c*x^2+b*x)^(3/2)*x-7/3*e^4/c^2/(c*x^2+b*x)^(1/2)*x-1/6*e^4*b/c^3
/(c*x^2+b*x)^(1/2)+e^4/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))-4*
d*e^3*x^2/c/(c*x^2+b*x)^(3/2)-4/3*d*e^3*b/c^2/(c*x^2+b*x)^(3/2)*x+8/3*d*e^3
/b/c/(c*x^2+b*x)^(1/2)*x+4/3*d*e^3/c^2/(c*x^2+b*x)^(1/2)-4*d^2*e^2/c/(c*x^2
+b*x)^(3/2)*x+8*d^2*e^2/b^2/(c*x^2+b*x)^(1/2)*x+4*d^2*e^2/b/c/(c*x^2+b*x)^(
1/2)+8/3*d^3*e/b/(c*x^2+b*x)^(3/2)*x-64/3*d^3*e/b^3*c/(c*x^2+b*x)^(1/2)*x-3
2/3*d^3*e/b^2/(c*x^2+b*x)^(1/2)-4/3*d^4/b^2/(c*x^2+b*x)^(3/2)*x*c-2/3*d^4/b
/(c*x^2+b*x)^(3/2)+32/3*d^4*c^2/b^4/(c*x^2+b*x)^(1/2)*x+16/3*d^4*c/b^3/(c*x
^2+b*x)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.05773, size = 1050, normalized size = 5.05

$$\left[\frac{3\left(b^4c^2e^4x^4+2b^5ce^4x^3+b^6e^4x^2\right)\sqrt{c}\log\left(2cx+b+2\sqrt{cx^2+bx}\sqrt{c}\right)-2\left(b^3c^3d^4-4\left(4c^6d^4-8bc^5d^3e+3b^2c^4d^2e^2+\right.\right.}{3\left(b^4c^5x^4+2b^5c^4x^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b^4*c^2*e^4*x^4 + 2*b^5*c*e^4*x^3 + b^6*e^4*x^2)*sqrt(c)*log(2*c*x
 + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 2*(b^3*c^3*d^4 - 4*(4*c^6*d^4 - 8*b*c
```

```
^5*d^3*e + 3*b^2*c^4*d^2*e^2 + b^3*c^3*d*e^3 - b^4*c^2*e^4)*x^3 - 3*(8*b*c^
5*d^4 - 16*b^2*c^4*d^3*e + 6*b^3*c^3*d^2*e^2 - b^5*c*e^4)*x^2 - 6*(b^2*c^4*
d^4 - 2*b^3*c^3*d^3*e)*x)*sqrt(c*x^2 + b*x))/(b^4*c^5*x^4 + 2*b^5*c^4*x^3 +
 b^6*c^3*x^2), -2/3*(3*(b^4*c^2*e^4*x^4 + 2*b^5*c*e^4*x^3 + b^6*e^4*x^2)*sq
rt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)) + (b^3*c^3*d^4 - 4*(4*c^6*d
^4 - 8*b*c^5*d^3*e + 3*b^2*c^4*d^2*e^2 + b^3*c^3*d*e^3 - b^4*c^2*e^4)*x^3 -
 3*(8*b*c^5*d^4 - 16*b^2*c^4*d^3*e + 6*b^3*c^3*d^2*e^2 - b^5*c*e^4)*x^2 - 6
*(b^2*c^4*d^4 - 2*b^3*c^3*d^3*e)*x)*sqrt(c*x^2 + b*x))/(b^4*c^5*x^4 + 2*b^5
*c^4*x^3 + b^6*c^3*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^4}{(x(b+cx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(c*x**2+b*x)**(5/2),x)
```

```
[Out] Integral((d + e*x)**4/(x*(b + c*x))**(5/2), x)
```

Giac [A] time = 1.93715, size = 282, normalized size = 1.36

$$-\frac{2\left(\frac{d^4}{b}-\left(x\left(\frac{4\left(4c^5d^4-8bc^4d^3e+3b^2c^3d^2e^2+b^3c^2de^3-b^4ce^4\right)x}{b^4c^2}+\frac{3\left(8bc^4d^4-16b^2c^3d^3e+6b^3c^2d^2e^2-b^5e^4\right)}{b^4c^2}\right)+\frac{6\left(b^2c^3d^4-2b^3c^2d^3e\right)}{b^4c^2}\right)x\right)}{3\left(cx^2+bx\right)^{\frac{3}{2}}}-\frac{e^4\log\left(\left|-2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(d^4/b - (x*(4*(4*c^5*d^4 - 8*b*c^4*d^3*e + 3*b^2*c^3*d^2*e^2 + b^3*c^
2*d*e^3 - b^4*c*e^4)*x/(b^4*c^2) + 3*(8*b*c^4*d^4 - 16*b^2*c^3*d^3*e + 6*b^
3*c^2*d^2*e^2 - b^5*e^4)/(b^4*c^2)) + 6*(b^2*c^3*d^4 - 2*b^3*c^2*d^3*e)/(b^
4*c^2))*x)/(c*x^2 + b*x)^(3/2) - e^4*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b
*x))*sqrt(c) - b))/c^(5/2)
```

3.330 $\int \frac{(d+ex)^3}{\left(bx+cx^2\right)^{5/2}} \, dx$

Optimal. Leaf size=87

$$\frac{16d(cd-be)(x(2cd-be)+bd)}{3b^4\sqrt{bx+cx^2}}-\frac{2(d+ex)^2(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

```
[Out] (-2*(d + e*x)^2*(b*d + (2*c*d - b*e)*x))/(3*b^2*(b*x + c*x^2)^(3/2)) + (16*
d*(c*d - b*e)*(b*d + (2*c*d - b*e)*x))/(3*b^4*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.0364209, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {722, 636}

$$\frac{16d(cd-be)(x(2cd-be)+bd)}{3b^4\sqrt{bx+cx^2}}-\frac{2(d+ex)^2(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^2*(b*d + (2*c*d - b*e)*x))/(3*b^2*(b*x + c*x^2)^(3/2)) + (16*
d*(c*d - b*e)*(b*d + (2*c*d - b*e)*x))/(3*b^4*Sqrt[b*x + c*x^2])
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^3}{\left(bx+cx^2\right)^{5/2}} \, dx &= -\frac{2(d+ex)^2(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}}-\frac{(8d(cd-be))\int \frac{d+ex}{\left(bx+cx^2\right)^{3/2}} \, dx}{3b^2}\\ &= -\frac{2(d+ex)^2(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}}+\frac{16d(cd-be)(bd+(2cd-be)x)}{3b^4\sqrt{bx+cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0482613, size = 105, normalized size = 1.21

$$\frac{2\left(6b^2cdx\left(d^2-6dex+e^2x^2\right)+b^3\left(-9d^2ex-d^3+9de^2x^2+e^3x^3\right)+24bc^2d^2x^2(d-ex)+16c^3d^3x^3\right)}{3b^4(x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(16*c^3*d^3*x^3 + 24*b*c^2*d^2*x^2*(d - e*x) + 6*b^2*c*d*x*(d^2 - 6*d*e*
x + e^2*x^2) + b^3*(-d^3 - 9*d^2*e*x + 9*d*e^2*x^2 + e^3*x^3)))/(3*b^4*(x*(
b + c*x))^(3/2))
```

Maple [A] time = 0.047, size = 136, normalized size = 1.6

$$-\frac{2\,x\,(cx+b)\left(-b^3e^3x^3-6\,b^2cde^2x^3+24\,bc^2d^2ex^3-16\,c^3d^3x^3-9\,b^3de^2x^2+36\,b^2cd^2ex^2-24\,bc^2d^3x^2+9\,b^3d^2ex-6\,b^2cd\right.}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^2+b*x)^(5/2),x)
```

```
[Out] -2/3*x*(c*x+b)*(-b^3*e^3*x^3-6*b^2*c*d*e^2*x^3+24*b*c^2*d^2*e*x^3-16*c^3*d^
3*x^3-9*b^3*d*e^2*x^2+36*b^2*c*d^2*e*x^2-24*b*c^2*d^3*x^2+9*b^3*d^2*e*x-6*b
^2*c*d^3*x+b^3*d^3)/b^4/(c*x^2+b*x)^(5/2)
```

Maxima [B] time = 1.17347, size = 401, normalized size = 4.61

$$-\frac{e^3x^2}{\left(cx^2+bx\right)^{\frac{3}{2}}c}-\frac{4\,cd^3x}{3\left(cx^2+bx\right)^{\frac{3}{2}}b^2}+\frac{32\,c^2d^3x}{3\sqrt{cx^2+bx}b^4}+\frac{2\,d^2ex}{\left(cx^2+bx\right)^{\frac{3}{2}}b}-\frac{16\,cd^2ex}{\sqrt{cx^2+bx}b^3}+\frac{4\,de^2x}{\sqrt{cx^2+bx}b^2}-\frac{2\,de^2x}{\left(cx^2+bx\right)^{\frac{3}{2}}c}-\frac{}{3\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -e^3*x^2/((c*x^2 + b*x)^(3/2)*c) - 4/3*c*d^3*x/((c*x^2 + b*x)^(3/2)*b^2) +
32/3*c^2*d^3*x/(sqrt(c*x^2 + b*x)*b^4) + 2*d^2*e*x/((c*x^2 + b*x)^(3/2)*b)
- 16*c*d^2*e*x/(sqrt(c*x^2 + b*x)*b^3) + 4*d*e^2*x/(sqrt(c*x^2 + b*x)*b^2)
- 2*d*e^2*x/((c*x^2 + b*x)^(3/2)*c) - 1/3*b*e^3*x/((c*x^2 + b*x)^(3/2)*c^2)
 + 2/3*e^3*x/(sqrt(c*x^2 + b*x)*b*c) - 2/3*d^3/((c*x^2 + b*x)^(3/2)*b) + 16
/3*c*d^3/(sqrt(c*x^2 + b*x)*b^3) - 8*d^2*e/(sqrt(c*x^2 + b*x)*b^2) + 2*d*e^
2/(sqrt(c*x^2 + b*x)*b*c) + 1/3*e^3/(sqrt(c*x^2 + b*x)*c^2)
```

Fricas [A] time = 1.94477, size = 298, normalized size = 3.43

$$-\frac{2\left(b^3d^3-\left(16\,c^3d^3-24\,bc^2d^2e+6\,b^2cde^2+b^3e^3\right)x^3-3\left(8\,bc^2d^3-12\,b^2cd^2e+3\,b^3de^2\right)x^2-3\left(2\,b^2cd^3-3\,b^3d^2e\right)x\right)\sqrt{cx^2}}{3\left(b^4c^2x^4+2\,b^5cx^3+b^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(b^3*d^3 - (16*c^3*d^3 - 24*b*c^2*d^2*e + 6*b^2*c*d*e^2 + b^3*e^3)*x^3
 - 3*(8*b*c^2*d^3 - 12*b^2*c*d^2*e + 3*b^3*d*e^2)*x^2 - 3*(2*b^2*c*d^3 - 3*
b^3*d^2*e)*x)*sqrt(c*x^2 + b*x)/(b^4*c^2*x^4 + 2*b^5*c*x^3 + b^6*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^3}{(x(b+cx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**2+b*x)**(5/2),x)
```

```
[Out] Integral((d + e*x)**3/(x*(b + c*x))**(5/2), x)
```

Giac [A] time = 1.28533, size = 188, normalized size = 2.16

$$\frac{\left(x\left(x\left(\frac{\left(16\,c^3d^3-24\,bc^2d^2e+6\,b^2cde^2+b^3e^3\right)x}{b^4c^2}+\frac{3\left(8\,bc^2d^3-12\,b^2cd^2e+3\,b^3de^2\right)}{b^4c^2}\right)+\frac{3\left(2\,b^2cd^3-3\,b^3d^2e\right)}{b^4c^2}\right)x-\frac{d^3}{bc^2}\right)}{3\left(cx^2+bx\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((x*((16*c^3*d^3 - 24*b*c^2*d^2*e + 6*b^2*c*d*e^2 + b^3*e^3)*x/(b^4*c^2
) + 3*(8*b*c^2*d^3 - 12*b^2*c*d^2*e + 3*b^3*d*e^2)/(b^4*c^2)) + 3*(2*b^2*c*
d^3 - 3*b^3*d^2*e)/(b^4*c^2))*x - d^3/(b*c^2))/(c*x^2 + b*x)^(3/2)
```

3.331 $\int \frac{(d+ex)^2}{\left(bx+cx^2\right)^{5/2}} dx$

Optimal. Leaf size=78

$$\frac{8(2cd-be)(x(2cd-be)+bd)}{3b^4\sqrt{bx+cx^2}}-\frac{2(b+2cx)(d+ex)^2}{3b^2\left(bx+cx^2\right)^{3/2}}$$

```
[Out] (-2*(b + 2*c*x)*(d + e*x)^2)/(3*b^2*(b*x + c*x^2)^(3/2)) + (8*(2*c*d - b*e)
*(b*d + (2*c*d - b*e)*x))/(3*b^4*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.0326602, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {728, 636}

$$\frac{8(2cd-be)(x(2cd-be)+bd)}{3b^4\sqrt{bx+cx^2}}-\frac{2(b+2cx)(d+ex)^2}{3b^2\left(bx+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*(b + 2*c*x)*(d + e*x)^2)/(3*b^2*(b*x + c*x^2)^(3/2)) + (8*(2*c*d - b*e)
*(b*d + (2*c*d - b*e)*x))/(3*b^4*Sqrt[b*x + c*x^2])
```

Rule 728

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] + Dist[(m*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(
d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2}{\left(bx+cx^2\right)^{5/2}} dx &= -\frac{2(b+2cx)(d+ex)^2}{3b^2\left(bx+cx^2\right)^{3/2}}-\frac{(4(2cd-be))\int \frac{d+ex}{\left(bx+cx^2\right)^{3/2}} dx}{3b^2}\\ &= -\frac{2(b+2cx)(d+ex)^2}{3b^2\left(bx+cx^2\right)^{3/2}}+\frac{8(2cd-be)(bd+(2cd-be)x)}{3b^4\sqrt{bx+cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0399624, size = 95, normalized size = 1.22

$$\frac{4b^2cx\left(3d^2-12dex+e^2x^2\right)-2b^3\left(d^2+6dex-3e^2x^2\right)+16bc^2dx^2(3d-2ex)+32c^3d^2x^3}{3b^4(x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(b*x + c*x^2)^(5/2),x]
```

```
[Out] (32*c^3*d^2*x^3 + 16*b*c^2*d*x^2*(3*d - 2*e*x) - 2*b^3*(d^2 + 6*d*e*x - 3*e
^2*x^2) + 4*b^2*c*x*(3*d^2 - 12*d*e*x + e^2*x^2))/(3*b^4*(x*(b + c*x))^(3/2
))
```

Maple [A] time = 0.047, size = 117, normalized size = 1.5

$$-\frac{2x(cx+b)\left(-2b^2ce^2x^3+16bc^2dex^3-16c^3d^2x^3-3b^3e^2x^2+24b^2cdex^2-24bc^2d^2x^2+6b^3dex-6b^2cd^2x+d^2b^3\right)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^2+b*x)^(5/2),x)
```

```
[Out] -2/3*x*(c*x+b)*(-2*b^2*c*e^2*x^3+16*b*c^2*d*e*x^3-16*c^3*d^2*x^3-3*b^3*e^2*
x^2+24*b^2*c*d*e*x^2-24*b*c^2*d^2*x^2+6*b^3*d*e*x-6*b^2*c*d^2*x+b^3*d^2)/b^
4/(c*x^2+b*x)^(5/2)
```

Maxima [B] time = 1.07666, size = 274, normalized size = 3.51

$$-\frac{4cd^2x}{3(cx^2+bx)^{\frac{3}{2}}b^2}+\frac{32c^2d^2x}{3\sqrt{cx^2+bx}b^4}+\frac{4dex}{3(cx^2+bx)^{\frac{3}{2}}b}-\frac{32cdex}{3\sqrt{cx^2+bx}b^3}+\frac{4e^2x}{3\sqrt{cx^2+bx}b^2}-\frac{2e^2x}{3(cx^2+bx)^{\frac{3}{2}}c}-\frac{2d^2}{3(cx^2+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -4/3*c*d^2*x/((c*x^2 + b*x)^(3/2)*b^2) + 32/3*c^2*d^2*x/(sqrt(c*x^2 + b*x)*
b^4) + 4/3*d*e*x/((c*x^2 + b*x)^(3/2)*b) - 32/3*c*d*e*x/(sqrt(c*x^2 + b*x)*
b^3) + 4/3*e^2*x/(sqrt(c*x^2 + b*x)*b^2) - 2/3*e^2*x/((c*x^2 + b*x)^(3/2)*c
) - 2/3*d^2/((c*x^2 + b*x)^(3/2)*b) + 16/3*c*d^2/(sqrt(c*x^2 + b*x)*b^3) -
16/3*d*e/(sqrt(c*x^2 + b*x)*b^2) + 2/3*e^2/(sqrt(c*x^2 + b*x)*b*c)
```

Fricas [A] time = 2.00924, size = 259, normalized size = 3.32

$$-\frac{2\left(b^3d^2-2\left(8c^3d^2-8bc^2de+b^2ce^2\right)x^3-3\left(8bc^2d^2-8b^2cde+b^3e^2\right)x^2-6\left(b^2cd^2-b^3de\right)x\right)\sqrt{cx^2+bx}}{3\left(b^4c^2x^4+2b^5cx^3+b^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(b^3*d^2 - 2*(8*c^3*d^2 - 8*b*c^2*d*e + b^2*c*e^2)*x^3 - 3*(8*b*c^2*d^
2 - 8*b^2*c*d*e + b^3*e^2)*x^2 - 6*(b^2*c*d^2 - b^3*d*e)*x)*sqrt(c*x^2 + b*
x)/(b^4*c^2*x^4 + 2*b^5*c*x^3 + b^6*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^2}{(x(b+cx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**2+b*x)**(5/2),x)
```

```
[Out] Integral((d + e*x)**2/(x*(b + c*x))**(5/2), x)
```

Giac [A] time = 1.3299, size = 166, normalized size = 2.13

$$\frac{\left(x\left(\frac{2\left(8c^3d^2-8bc^2de+b^2ce^2\right)x}{b^4c^2}+\frac{3\left(8bc^2d^2-8b^2cde+b^3e^2\right)}{b^4c^2}\right)+\frac{6\left(b^2cd^2-b^3de\right)}{b^4c^2}\right)x-\frac{d^2}{bc^2}}{3\left(cx^2+bx\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((x*(2*(8*c^3*d^2 - 8*b*c^2*d*e + b^2*c*e^2)*x/(b^4*c^2) + 3*(8*b*c^2*d
^2 - 8*b^2*c*d*e + b^3*e^2)/(b^4*c^2)) + 6*(b^2*c*d^2 - b^3*d*e)/(b^4*c^2))
*x - d^2/(b*c^2))/(c*x^2 + b*x)^(3/2)
```

3.332 $\int \frac{d+ex}{\left(bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=71

$$\frac{8(b+2cx)(2cd-be)}{3b^4\sqrt{bx+cx^2}}-\frac{2(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

```
[Out] (-2*(b*d + (2*c*d - b*e)*x))/(3*b^2*(b*x + c*x^2)^(3/2)) + (8*(2*c*d - b*e)
*(b + 2*c*x))/(3*b^4*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.0181793, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {638, 613}

$$\frac{8(b+2cx)(2cd-be)}{3b^4\sqrt{bx+cx^2}}-\frac{2(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*(b*d + (2*c*d - b*e)*x))/(3*b^2*(b*x + c*x^2)^(3/2)) + (8*(2*c*d - b*e)
*(b + 2*c*x))/(3*b^4*Sqrt[b*x + c*x^2])
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex}{\left(bx+cx^2\right)^{5/2}}\,dx &= -\frac{2(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}}-\frac{(4(2cd-be))\int \frac{1}{\left(bx+cx^2\right)^{3/2}}\,dx}{3b^2}\\ &= -\frac{2(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}}+\frac{8(2cd-be)(b+2cx)}{3b^4\sqrt{bx+cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0244812, size = 67, normalized size = 0.94

$$-\frac{2\left(-6b^2cx(d-2ex)+b^3(d+3ex)+8bc^2x^2(ex-3d)-16c^3dx^3\right)}{3b^4(x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*(-16*c^3*d*x^3 - 6*b^2*c*x*(d - 2*e*x) + 8*b*c^2*x^2*(-3*d + e*x) + b^3
*(d + 3*e*x)))/(3*b^4*(x*(b + c*x))^(3/2))
```

Maple [A] time = 0.046, size = 83, normalized size = 1.2

$$-\frac{2x(cx+b)\left(8bc^2ex^3-16c^3dx^3+12b^2cex^2-24bc^2dx^2+3b^3ex-6b^2cdx+db^3\right)}{3b^4}\left(cx^2+bx\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^2+b*x)^(5/2),x)
```

```
[Out] -2/3*x*(c*x+b)*(8*b*c^2*e*x^3-16*c^3*d*x^3+12*b^2*c*e*x^2-24*b*c^2*d*x^2+3*
b^3*e*x-6*b^2*c*d*x+b^3*d)/b^4/(c*x^2+b*x)^(5/2)
```

Maxima [B] time = 1.14867, size = 176, normalized size = 2.48

$$-\frac{4\,cdx}{3\left(cx^2+bx\right)^{\frac{3}{2}}b^2}+\frac{32\,c^2dx}{3\sqrt{cx^2+bx}b^4}+\frac{2\,ex}{3\left(cx^2+bx\right)^{\frac{3}{2}}b}-\frac{16\,cex}{3\sqrt{cx^2+bx}b^3}-\frac{2\,d}{3\left(cx^2+bx\right)^{\frac{3}{2}}b}+\frac{16\,cd}{3\sqrt{cx^2+bx}b^3}-\frac{8\,e}{3\sqrt{cx^2+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -4/3*c*d*x/((c*x^2 + b*x)^(3/2)*b^2) + 32/3*c^2*d*x/(sqrt(c*x^2 + b*x)*b^4)
 + 2/3*e*x/((c*x^2 + b*x)^(3/2)*b) - 16/3*c*e*x/(sqrt(c*x^2 + b*x)*b^3) - 2
/3*d/((c*x^2 + b*x)^(3/2)*b) + 16/3*c*d/(sqrt(c*x^2 + b*x)*b^3) - 8/3*e/(sq
rt(c*x^2 + b*x)*b^2)
```

Fricas [A] time = 1.96854, size = 209, normalized size = 2.94

$$-\frac{2\left(b^3d-8\left(2c^3d-bc^2e\right)x^3-12\left(2bc^2d-b^2ce\right)x^2-3\left(2b^2cd-b^3e\right)x\right)\sqrt{cx^2+bx}}{3\left(b^4c^2x^4+2b^5cx^3+b^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(b^3*d - 8*(2*c^3*d - b*c^2*e)*x^3 - 12*(2*b*c^2*d - b^2*c*e)*x^2 - 3*
(2*b^2*c*d - b^3*e)*x)*sqrt(c*x^2 + b*x)/(b^4*c^2*x^4 + 2*b^5*c*x^3 + b^6*x
^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d+ex}{(x(b+cx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**2+b*x)**(5/2),x)
```

```
[Out] Integral((d + e*x)/(x*(b + c*x))**(5/2), x)
```

Giac [A] time = 1.39542, size = 136, normalized size = 1.92

$$\frac{\left(4\,x\left(\frac{2\left(2\,c^3d-bc^2e\right)x}{b^4c^2}+\frac{3\left(2\,bc^2d-b^2ce\right)}{b^4c^2}\right)+\frac{3\left(2\,b^2cd-b^3e\right)}{b^4c^2}\right)x-\frac{d}{bc^2}}{3\left(cx^2+bx\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((4*x*(2*(2*c^3*d - b*c^2*e)*x/(b^4*c^2) + 3*(2*b*c^2*d - b^2*c*e)/(b^4
*c^2)) + 3*(2*b^2*c*d - b^3*e)/(b^4*c^2))*x - d/(b*c^2))/(c*x^2 + b*x)^(3/2
)
```

3.333 $\int \frac{1}{(bx+cx^2)^{5/2}}\,dx$

Optimal. Leaf size=54

$$\frac{16c(b+2cx)}{3b^4\sqrt{bx+cx^2}}-\frac{2(b+2cx)}{3b^2(bx+cx^2)^{3/2}}$$

```
[Out] (-2*(b + 2*c*x))/(3*b^2*(b*x + c*x^2)^(3/2)) + (16*c*(b + 2*c*x))/(3*b^4*Sq
rt[b*x + c*x^2])
```

Rubi [A] time = 0.0101147, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {614, 613}

$$\frac{16c(b+2cx)}{3b^4\sqrt{bx+cx^2}}-\frac{2(b+2cx)}{3b^2(bx+cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(-5/2),x]
```

```
[Out] (-2*(b + 2*c*x))/(3*b^2*(b*x + c*x^2)^(3/2)) + (16*c*(b + 2*c*x))/(3*b^4*Sq
rt[b*x + c*x^2])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(bx+cx^2)^{5/2}}\,dx &= -\frac{2(b+2cx)}{3b^2(bx+cx^2)^{3/2}}-\frac{(8c)\int \frac{1}{(bx+cx^2)^{3/2}}\,dx}{3b^2}\\ &= -\frac{2(b+2cx)}{3b^2(bx+cx^2)^{3/2}}+\frac{16c(b+2cx)}{3b^4\sqrt{bx+cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0120318, size = 48, normalized size = 0.89

$$\frac{12b^2cx-2b^3+48bc^2x^2+32c^3x^3}{3b^4(x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(-5/2),x]
```

```
[Out] (-2*b^3 + 12*b^2*c*x + 48*b*c^2*x^2 + 32*c^3*x^3)/(3*b^4*(x*(b + c*x))^(3/2
))
```

Maple [A] time = 0.044, size = 51, normalized size = 0.9

$$-\frac{2x(cx+b)\left(-16x^3c^3-24bx^2c^2-6b^2xc+b^3\right)}{3b^4}\left(cx^2+bx\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^(5/2),x)
```

```
[Out] -2/3*x*(c*x+b)*(-16*c^3*x^3-24*b*c^2*x^2-6*b^2*c*x+b^3)/b^4/(c*x^2+b*x)^(5/
2)
```

Maxima [A] time = 1.10481, size = 97, normalized size = 1.8

$$-\frac{4cx}{3\left(cx^2+bx\right)^{\frac{3}{2}}b^2}+\frac{32c^2x}{3\sqrt{cx^2+bx}b^4}-\frac{2}{3\left(cx^2+bx\right)^{\frac{3}{2}}b}+\frac{16c}{3\sqrt{cx^2+bx}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -4/3*c*x/((c*x^2 + b*x)^(3/2)*b^2) + 32/3*c^2*x/(sqrt(c*x^2 + b*x)*b^4) - 2
/3/((c*x^2 + b*x)^(3/2)*b) + 16/3*c/(sqrt(c*x^2 + b*x)*b^3)
```

Fricas [A] time = 1.93749, size = 144, normalized size = 2.67

$$\frac{2\left(16c^3x^3+24bc^2x^2+6b^2cx-b^3\right)\sqrt{cx^2+bx}}{3\left(b^4c^2x^4+2b^5cx^3+b^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(16*c^3*x^3 + 24*b*c^2*x^2 + 6*b^2*c*x - b^3)*sqrt(c*x^2 + b*x)/(b^4*c^
2*x^4 + 2*b^5*c*x^3 + b^6*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(bx+cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(5/2),x)
```

```
[Out] Integral((b*x + c*x**2)**(-5/2), x)
```

Giac [A] time = 1.38635, size = 68, normalized size = 1.26

$$\frac{2\left(2\left(4x\left(\frac{2c^3x}{b^4}+\frac{3c^2}{b^3}\right)+\frac{3c}{b^2}\right)x-\frac{1}{b}\right)}{3\left(cx^2+bx\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(2*(4*x*(2*c^3*x/b^4 + 3*c^2/b^3) + 3*c/b^2)*x - 1/b)/(c*x^2 + b*x)^(3/
2)
```

3.334 $\int \frac{1}{(d+ex)\left(bx+cx^2\right)^{5/2}} dx$

Optimal. Leaf size=230

$$\frac{2\left(cx(2cd-be)\left(-3b^2e^2-8bcde+8c^2d^2\right)+b(cd-be)\left(-3b^2e^2-4bcde+8c^2d^2\right)\right)}{3b^4d^2\sqrt{bx+cx^2}(cd-be)^2}-\frac{2(cx(2cd-be)+b(cd-be))}{3b^2d\left(bx+cx^2\right)^{3/2}(cd-be)}+\frac{e^4 \text{t}}{}$$

```
[Out] (-2*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(3*b^2*d*(c*d - b*e)*(b*x + c*x^2)
^(3/2)) + (2*(b*(c*d - b*e)*(8*c^2*d^2 - 4*b*c*d*e - 3*b^2*e^2) + c*(2*c*d
- b*e)*(8*c^2*d^2 - 8*b*c*d*e - 3*b^2*e^2)*x))/(3*b^4*d^2*(c*d - b*e)^2*Sqr
t[b*x + c*x^2]) + (e^4*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d
- b*e]*Sqrt[b*x + c*x^2])])/(d^(5/2)*(c*d - b*e)^(5/2))
```

Rubi [A] time = 0.17944, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {740, 822, 12, 724, 206}

$$\frac{2\left(cx(2cd-be)\left(-3b^2e^2-8bcde+8c^2d^2\right)+b(cd-be)\left(-3b^2e^2-4bcde+8c^2d^2\right)\right)}{3b^4d^2\sqrt{bx+cx^2}(cd-be)^2}-\frac{2(cx(2cd-be)+b(cd-be))}{3b^2d\left(bx+cx^2\right)^{3/2}(cd-be)}+\frac{e^4 \text{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(b*x + c*x^2)^(5/2)),x]
```


```
[Out] (-2*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(3*b^2*d*(c*d - b*e)*(b*x + c*x^2)
^(3/2)) + (2*(b*(c*d - b*e)*(8*c^2*d^2 - 4*b*c*d*e - 3*b^2*e^2) + c*(2*c*d
- b*e)*(8*c^2*d^2 - 8*b*c*d*e - 3*b^2*e^2)*x))/(3*b^4*d^2*(c*d - b*e)^2*Sqr
t[b*x + c*x^2]) + (e^4*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d
- b*e]*Sqrt[b*x + c*x^2])])/(d^(5/2)*(c*d - b*e)^(5/2))
```


Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)^{5/2}}\,dx = -\frac{2(b(cd-be)+c(2cd-be)x)}{3b^2d(cd-be)\left(bx+cx^2\right)^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(8c^2d^2-4bcde-3b^2e^2\right)+2ce(2cd-be)x}{(d+ex)\left(bx+cx^2\right)^{3/2}}\,dx}{3b^2d(cd-be)}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{3b^2d(cd-be)\left(bx+cx^2\right)^{3/2}} + \frac{2\left(b(cd-be)\left(8c^2d^2-4bcde-3b^2e^2\right)+c(2cd-be)\left(8c^2d^2\right.\right.}{3b^4d^2(cd-be)^2\sqrt{bx+cx^2}}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{3b^2d(cd-be)\left(bx+cx^2\right)^{3/2}} + \frac{2\left(b(cd-be)\left(8c^2d^2-4bcde-3b^2e^2\right)+c(2cd-be)\left(8c^2d^2\right.\right.}{3b^4d^2(cd-be)^2\sqrt{bx+cx^2}}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{3b^2d(cd-be)\left(bx+cx^2\right)^{3/2}} + \frac{2\left(b(cd-be)\left(8c^2d^2-4bcde-3b^2e^2\right)+c(2cd-be)\left(8c^2d^2\right.\right.}{3b^4d^2(cd-be)^2\sqrt{bx+cx^2}}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{3b^2d(cd-be)\left(bx+cx^2\right)^{3/2}} + \frac{2\left(b(cd-be)\left(8c^2d^2-4bcde-3b^2e^2\right)+c(2cd-be)\left(8c^2d^2\right.\right.}{3b^4d^2(cd-be)^2\sqrt{bx+cx^2}}$$

Mathematica [A] time = 0.297749, size = 238, normalized size = 1.03

$$\frac{2\sqrt{d}\sqrt{be-cd}\left(-b^3c^2\left(9d^2ex+d^3-3de^2x^2-3e^3x^3\right)+2b^2c^3dx\left(3d^2-18dex+e^2x^2\right)+2b^4ce\left(d^2+3e^2x^2\right)+b^5e^2(3ex-d)\right.}{3b^4d^{5/2}(x(b+cx))^{3/2}(be-cd)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(b*x + c*x^2)^(5/2)),x]
```

```
[Out] (2*Sqrt[d]*Sqrt[-(c*d) + b*e]*(16*c^5*d^3*x^3 + 24*b*c^4*d^2*x^2*(d - e*x)
+ b^5*e^2*(-d + 3*e*x) + 2*b^2*c^3*d*x*(3*d^2 - 18*d*e*x + e^2*x^2) + 2*b^4
*c*e*(d^2 + 3*e^2*x^2) - b^3*c^2*(d^3 + 9*d^2*e*x - 3*d*e^2*x^2 - 3*e^3*x^3
)) + 6*b^4*e^4*x^(3/2)*(b + c*x)^(3/2)*ArcTan[(Sqrt[-(c*d) + b*e]*Sqrt[x])/
(Sqrt[d]*Sqrt[b + c*x])])/(3*b^4*d^(5/2)*(-(c*d) + b*e)^(5/2)*(x*(b + c*x))
```

```
^(3/2))
```

Maple [B] time = 0.219, size = 950, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+b*x)^(5/2),x)
```

```
[Out] -2/3*e/d/(b*e-c*d)/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2
)-2/3*e/d/(b*e-c*d)/b/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(
3/2)*x*c+4/3/(b*e-c*d)/b^2/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e
^2)^(3/2)*x*c^2+2/3/(b*e-c*d)/b/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c
*d)/e^2)^(3/2)*c+16/3*e/d/(b*e-c*d)*c^2/b^3/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e
+x)-d*(b*e-c*d)/e^2)^(1/2)*x-32/3/(b*e-c*d)*c^3/b^4/(c*(d/e+x)^2+(b*e-2*c*d
)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*x+8/3*e/d/(b*e-c*d)*c/b^2/(c*(d/e+x)^2+(
b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)-16/3/(b*e-c*d)*c^2/b^3/(c*(d/e+
x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)+2*e^3/d^2/(b*e-c*d)^2/(c*
(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)+2*e^3/d^2/(b*e-c*d)^
2/b/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*x*c-4*e^2/d/(
b*e-c*d)^2/b^2/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*x*
c^2-2*e^2/d/(b*e-c*d)^2/b/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^
2)^(1/2)*c-e^3/d^2/(b*e-c*d)^2/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/
e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*
d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.11545, size = 1993, normalized size = 8.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b^4*c^2*e^4*x^4 + 2*b^5*c*e^4*x^3 + b^6*e^4*x^2)*sqrt(c*d^2 - b*d*
e)*log((b*d + (2*c*d - b*e)*x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e
*x + d)) - 2*(b^3*c^3*d^5 - 3*b^4*c^2*d^4*e + 3*b^5*c*d^3*e^2 - b^6*d^2*e^3
 - (16*c^6*d^5 - 40*b*c^5*d^4*e + 26*b^2*c^4*d^3*e^2 + b^3*c^3*d^2*e^3 - 3*
b^4*c^2*d*e^4)*x^3 - 3*(8*b*c^5*d^5 - 20*b^2*c^4*d^4*e + 13*b^3*c^3*d^3*e^2
 + b^4*c^2*d^2*e^3 - 2*b^5*c*d*e^4)*x^2 - 3*(2*b^2*c^4*d^5 - 5*b^3*c^3*d^4*
e + 3*b^4*c^2*d^3*e^2 + b^5*c*d^2*e^3 - b^6*d*e^4)*x)*sqrt(c*x^2 + b*x))/((
```

```
b^4*c^5*d^6 - 3*b^5*c^4*d^5*e + 3*b^6*c^3*d^4*e^2 - b^7*c^2*d^3*e^3)*x^4 +
2*(b^5*c^4*d^6 - 3*b^6*c^3*d^5*e + 3*b^7*c^2*d^4*e^2 - b^8*c*d^3*e^3)*x^3 +
 (b^6*c^3*d^6 - 3*b^7*c^2*d^5*e + 3*b^8*c*d^4*e^2 - b^9*d^3*e^3)*x^2), 2/3*
(3*(b^4*c^2*e^4*x^4 + 2*b^5*c*e^4*x^3 + b^6*e^4*x^2)*sqrt(-c*d^2 + b*d*e)*a
rctan(-sqrt(-c*d^2 + b*d*e)*sqrt(c*x^2 + b*x)/((c*d - b*e)*x)) - (b^3*c^3*d
^5 - 3*b^4*c^2*d^4*e + 3*b^5*c*d^3*e^2 - b^6*d^2*e^3 - (16*c^6*d^5 - 40*b*c
^5*d^4*e + 26*b^2*c^4*d^3*e^2 + b^3*c^3*d^2*e^3 - 3*b^4*c^2*d*e^4)*x^3 - 3*
(8*b*c^5*d^5 - 20*b^2*c^4*d^4*e + 13*b^3*c^3*d^3*e^2 + b^4*c^2*d^2*e^3 - 2*
b^5*c*d*e^4)*x^2 - 3*(2*b^2*c^4*d^5 - 5*b^3*c^3*d^4*e + 3*b^4*c^2*d^3*e^2 +
 b^5*c*d^2*e^3 - b^6*d*e^4)*x)*sqrt(c*x^2 + b*x))/((b^4*c^5*d^6 - 3*b^5*c^4
*d^5*e + 3*b^6*c^3*d^4*e^2 - b^7*c^2*d^3*e^3)*x^4 + 2*(b^5*c^4*d^6 - 3*b^6*
c^3*d^5*e + 3*b^7*c^2*d^4*e^2 - b^8*c*d^3*e^3)*x^3 + (b^6*c^3*d^6 - 3*b^7*c
^2*d^5*e + 3*b^8*c*d^4*e^2 - b^9*d^3*e^3)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x(b+cx))^{\frac{5}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+b*x)**(5/2),x)
```

```
[Out] Integral(1/((x*(b + c*x))**(5/2)*(d + e*x)), x)
```

Giac [A] time = 1.41056, size = 562, normalized size = 2.44

$$-\frac{2\arctan\left(\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde}}\right)e^4}{\left(c^2d^4-2bcd^3e+b^2d^2e^2\right)\sqrt{-cd^2+bde}}-\frac{\left(x\left(\frac{\left(16c^7d^{10}-56bc^6d^9e+66b^2c^5d^8e^2-25b^3c^4d^7e^3-4b^4c^3d^6e^4+3b^5c^2d^5e^5\right)x}{b^4c^2}+\frac{3\left(8bc^6d^{10}-28b^2c^5d^9e\right.}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

```
[Out] -2*arctan(((sqrt(c)*x - sqrt(c*x^2 + b*x))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d
*e))*e^4/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2)*sqrt(-c*d^2 + b*d*e)) - 1/3
*((x*((16*c^7*d^10 - 56*b*c^6*d^9*e + 66*b^2*c^5*d^8*e^2 - 25*b^3*c^4*d^7*e
^3 - 4*b^4*c^3*d^6*e^4 + 3*b^5*c^2*d^5*e^5)*x/(b^4*c^2) + 3*(8*b*c^6*d^10 -
 28*b^2*c^5*d^9*e + 33*b^3*c^4*d^8*e^2 - 12*b^4*c^3*d^7*e^3 - 3*b^5*c^2*d^6
*e^4 + 2*b^6*c*d^5*e^5)/(b^4*c^2)) + 3*(2*b^2*c^5*d^10 - 7*b^3*c^4*d^9*e +
8*b^4*c^3*d^8*e^2 - 2*b^5*c^2*d^7*e^3 - 2*b^6*c*d^6*e^4 + b^7*d^5*e^5)/(b^4
*c^2))*x - (b^3*c^4*d^10 - 4*b^4*c^3*d^9*e + 6*b^5*c^2*d^8*e^2 - 4*b^6*c*d^
7*e^3 + b^7*d^6*e^4)/(b^4*c^2))/(c*x^2 + b*x)^(3/2)
```

3.335 $\int \frac{1}{(d+ex)^2\left(bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=348

$$\frac{e\sqrt{bx+cx^2}\left(12b^2c^2d^2e^2+20b^3cde^3-15b^4e^4-64bc^3d^3e+32c^4d^4\right)}{3b^4d^3(d+ex)(cd-be)^3}+\frac{2\left(cx(2cd-be)\left(-5b^2e^2-8bcde+8c^2d^2\right)+b(cd$$

```
[Out] (-2*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(3*b^2*d*(c*d - b*e)*(d + e*x)*(b*
x + c*x^2)^(3/2)) + (2*(b*(c*d - b*e)*(8*c^2*d^2 - 2*b*c*d*e - 5*b^2*e^2) +
 c*(2*c*d - b*e)*(8*c^2*d^2 - 8*b*c*d*e - 5*b^2*e^2)*x))/(3*b^4*d^2*(c*d -
b*e)^2*(d + e*x)*Sqrt[b*x + c*x^2]) + (e*(32*c^4*d^4 - 64*b*c^3*d^3*e + 12*
b^2*c^2*d^2*e^2 + 20*b^3*c*d*e^3 - 15*b^4*e^4)*Sqrt[b*x + c*x^2])/(3*b^4*d^
3*(c*d - b*e)^3*(d + e*x)) + (5*e^4*(2*c*d - b*e)*ArcTanh[(b*d + (2*c*d - b
*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/(2*d^(7/2)*(c*d - b*
e)^(7/2))
```

Rubi [A] time = 0.348716, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {740, 822, 806, 724, 206}

$$\frac{e\sqrt{bx+cx^2}\left(12b^2c^2d^2e^2+20b^3cde^3-15b^4e^4-64bc^3d^3e+32c^4d^4\right)}{3b^4d^3(d+ex)(cd-be)^3}+\frac{2\left(cx(2cd-be)\left(-5b^2e^2-8bcde+8c^2d^2\right)+b(cd$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(b*x + c*x^2)^(5/2)),x]
```

```
[Out] (-2*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(3*b^2*d*(c*d - b*e)*(d + e*x)*(b*
x + c*x^2)^(3/2)) + (2*(b*(c*d - b*e)*(8*c^2*d^2 - 2*b*c*d*e - 5*b^2*e^2) +
 c*(2*c*d - b*e)*(8*c^2*d^2 - 8*b*c*d*e - 5*b^2*e^2)*x))/(3*b^4*d^2*(c*d -
b*e)^2*(d + e*x)*Sqrt[b*x + c*x^2]) + (e*(32*c^4*d^4 - 64*b*c^3*d^3*e + 12*
b^2*c^2*d^2*e^2 + 20*b^3*c*d*e^3 - 15*b^4*e^4)*Sqrt[b*x + c*x^2])/(3*b^4*d^
3*(c*d - b*e)^3*(d + e*x)) + (5*e^4*(2*c*d - b*e)*ArcTanh[(b*d + (2*c*d - b
*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/(2*d^(7/2)*(c*d - b*
e)^(7/2))
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
```

```
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(bx+cx^2\right)^{5/2}}\,dx = -\frac{2(b(cd-be)+c(2cd-be)x)}{3b^2d(cd-be)(d+ex)\left(bx+cx^2\right)^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(8c^2d^2-2bcde-5b^2e^2\right)+3ce(2cd-be)x}{(d+ex)^2\left(bx+cx^2\right)^{3/2}}\,dx}{3b^2d(cd-be)}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{3b^2d(cd-be)(d+ex)\left(bx+cx^2\right)^{3/2}} + \frac{2\left(b(cd-be)\left(8c^2d^2-2bcde-5b^2e^2\right)+c(2cd-be\right.}{3b^4d^2(cd-be)^2(d+ex)\sqrt{bx}}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{3b^2d(cd-be)(d+ex)\left(bx+cx^2\right)^{3/2}} + \frac{2\left(b(cd-be)\left(8c^2d^2-2bcde-5b^2e^2\right)+c(2cd-be\right.}{3b^4d^2(cd-be)^2(d+ex)\sqrt{bx}}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{3b^2d(cd-be)(d+ex)\left(bx+cx^2\right)^{3/2}} + \frac{2\left(b(cd-be)\left(8c^2d^2-2bcde-5b^2e^2\right)+c(2cd-be\right.}{3b^4d^2(cd-be)^2(d+ex)\sqrt{bx}}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{3b^2d(cd-be)(d+ex)\left(bx+cx^2\right)^{3/2}} + \frac{2\left(b(cd-be)\left(8c^2d^2-2bcde-5b^2e^2\right)+c(2cd-be\right.}{3b^4d^2(cd-be)^2(d+ex)\sqrt{bx}}$$

Mathematica [A] time = 0.988586, size = 316, normalized size = 0.91

$$\frac{x\left(\frac{cx^2(b+cx)^2\left(-12b^2c^2d^2e^2-20b^3cde^3+15b^4e^4+64bc^3d^3e-32c^4d^4\right)}{b^4d^2(cd-be)^2}+\frac{cx^2(b+cx)\left(-10b^2cde^2+15b^3e^3-16bc^2d^2e+16c^3d^3\right)}{b^3d^2(be-cd)}+\frac{3x(b+cx)\left(5b^2e^2-4c^2d^2\right)}{b^2d^2}+\frac{15e^{\cdots}}{\cdots}\right.}{3d(x(b+cx))^{5/2}(be-cd)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(b*x + c*x^2)^(5/2)),x]
```

```
[Out] (x*(((2*c*d - 5*b*e)*(b + c*x))/(b*d) + (3*(-4*c^2*d^2 + 5*b^2*e^2)*x*(b +
c*x))/(b^2*d^2) + (c*(16*c^3*d^3 - 16*b*c^2*d^2*e - 10*b^2*c*d*e^2 + 15*b^3
*e^3)*x^2*(b + c*x))/(b^3*d^2*(-(c*d) + b*e)) + (c*(-32*c^4*d^4 + 64*b*c^3*
d^3*e - 12*b^2*c^2*d^2*e^2 - 20*b^3*c*d*e^3 + 15*b^4*e^4)*x^2*(b + c*x)^2)/
(b^4*d^2*(c*d - b*e)^2) + (3*e*(b + c*x))/(d + e*x) + (15*e^4*(-2*c*d + b*e
)*x^(3/2)*(b + c*x)^(5/2)*ArcTan[(Sqrt[-(c*d) + b*e]*Sqrt[x])/(Sqrt[d]*Sqrt
[b + c*x])])/(d^(5/2)*(-(c*d) + b*e)^(5/2))))/(3*d*(-(c*d) + b*e)*(x*(b + c
*x))^(5/2))
```

Maple [B] time = 0.219, size = 1857, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^2+b*x)^(5/2),x)
```

```
[Out] 80/3/(b*e-c*d)^2*c^3/b^3/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2
)^(1/2)+1/d/(b*e-c*d)/(d/e+x)/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d
)/e^2)^(3/2)-10/3/(b*e-c*d)^2/b/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c
*d)/e^2)^(3/2)*c^2+64/3*c^2/d/(b*e-c*d)/b^3/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e
+x)-d*(b*e-c*d)/e^2)^(1/2)-5/3*e^2/d^2/(b*e-c*d)^2/(c*(d/e+x)^2+(b*e-2*c*d)
/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)*b+5*e/d/(b*e-c*d)^2/(c*(d/e+x)^2+(b*e-2*c
*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)*c+5*e^4/d^3/(b*e-c*d)^3/(c*(d/e+x)^2+(
b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*b-15*e^3/d^2/(b*e-c*d)^3/(c*(d/
e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*c-20/3/(b*e-c*d)^2/b^2/
(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)*x*c^3+160/3/(b*e-
c*d)^2*c^4/b^4/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*x+
10*e^2/d/(b*e-c*d)^3/b/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^
(1/2)*c^2-5/2*e^4/d^3/(b*e-c*d)^3/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*
d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2
*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2))/(d/e+x))*b-8/3*c/d/(b*e-c*d)/b/(c*(
d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)-160/3*e/d/(b*e-c*d)^2
*c^3/b^3/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*x-20*e^3
/d^2/(b*e-c*d)^3/b/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2
)*x*c^2+20*e^2/d/(b*e-c*d)^3/b^2/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-
c*d)/e^2)^(1/2)*x*c^3+20/3*e/d/(b*e-c*d)^2/b/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/
e+x)-d*(b*e-c*d)/e^2)^(3/2)*x*c^2+40/3*e^2/d^2/(b*e-c*d)^2*c^2/b^2/(c*(d/e+
x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*x+5*e^3/d^2/(b*e-c*d)^3/(
-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(-d*
(b*e-c*d)/e^2)^(1/2)*(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1
/2))/(d/e+x))*c-80/3*e/d/(b*e-c*d)^2*c^2/b^2/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/
e+x)-d*(b*e-c*d)/e^2)^(1/2)-5/3*e^2/d^2/(b*e-c*d)^2/(c*(d/e+x)^2+(b*e-2*c*d
)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)*x*c+5*e^4/d^3/(b*e-c*d)^3/(c*(d/e+x)^2+(
b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)*x*c+20/3*e^2/d^2/(b*e-c*d)^2*c/
```

```
b/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1/2)-16/3*c^2/d/(b*e
-c*d)/b^2/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(3/2)*x+128/3
*c^3/d/(b*e-c*d)/b^4/(c*(d/e+x)^2+(b*e-2*c*d)/e*(d/e+x)-d*(b*e-c*d)/e^2)^(1
/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.74634, size = 3553, normalized size = 10.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*((2*b^4*c^3*d*e^5 - b^5*c^2*e^6)*x^5 + (2*b^4*c^3*d^2*e^4 + 3*b^5*
c^2*d*e^5 - 2*b^6*c*e^6)*x^4 + (4*b^5*c^2*d^2*e^4 - b^7*e^6)*x^3 + (2*b^6*c
*d^2*e^4 - b^7*d*e^5)*x^2)*sqrt(c*d^2 - b*d*e)*log((b*d + (2*c*d - b*e)*x +
 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*x + d)) - 2*(2*b^3*c^4*d^7 - 8
*b^4*c^3*d^6*e + 12*b^5*c^2*d^5*e^2 - 8*b^6*c*d^4*e^3 + 2*b^7*d^3*e^4 - (32
*c^7*d^6*e - 96*b*c^6*d^5*e^2 + 76*b^2*c^5*d^4*e^3 + 8*b^3*c^4*d^3*e^4 - 35
*b^4*c^3*d^2*e^5 + 15*b^5*c^2*d*e^6)*x^4 - 2*(16*c^7*d^7 - 24*b*c^6*d^6*e -
 34*b^2*c^5*d^5*e^2 + 61*b^3*c^4*d^4*e^3 - 4*b^4*c^3*d^3*e^4 - 30*b^5*c^2*d
^2*e^5 + 15*b^6*c*d*e^6)*x^3 - 3*(16*b*c^6*d^7 - 44*b^2*c^5*d^6*e + 26*b^3*
c^4*d^5*e^2 + 16*b^4*c^3*d^4*e^3 - 14*b^5*c^2*d^3*e^4 - 5*b^6*c*d^2*e^5 + 5
*b^7*d*e^6)*x^2 - 2*(6*b^2*c^5*d^7 - 19*b^3*c^4*d^6*e + 16*b^4*c^3*d^5*e^2
+ 6*b^5*c^2*d^4*e^3 - 14*b^6*c*d^3*e^4 + 5*b^7*d^2*e^5)*x)*sqrt(c*x^2 + b*x
))/((b^4*c^6*d^8*e - 4*b^5*c^5*d^7*e^2 + 6*b^6*c^4*d^6*e^3 - 4*b^7*c^3*d^5*
e^4 + b^8*c^2*d^4*e^5)*x^5 + (b^4*c^6*d^9 - 2*b^5*c^5*d^8*e - 2*b^6*c^4*d^7
*e^2 + 8*b^7*c^3*d^6*e^3 - 7*b^8*c^2*d^5*e^4 + 2*b^9*c*d^4*e^5)*x^4 + (2*b^
5*c^5*d^9 - 7*b^6*c^4*d^8*e + 8*b^7*c^3*d^7*e^2 - 2*b^8*c^2*d^6*e^3 - 2*b^9
*c*d^5*e^4 + b^10*d^4*e^5)*x^3 + (b^6*c^4*d^9 - 4*b^7*c^3*d^8*e + 6*b^8*c^2
*d^7*e^2 - 4*b^9*c*d^6*e^3 + b^10*d^5*e^4)*x^2), 1/3*(15*((2*b^4*c^3*d*e^5
- b^5*c^2*e^6)*x^5 + (2*b^4*c^3*d^2*e^4 + 3*b^5*c^2*d*e^5 - 2*b^6*c*e^6)*x^
4 + (4*b^5*c^2*d^2*e^4 - b^7*e^6)*x^3 + (2*b^6*c*d^2*e^4 - b^7*d*e^5)*x^2)*
sqrt(-c*d^2 + b*d*e)*arctan(-sqrt(-c*d^2 + b*d*e)*sqrt(c*x^2 + b*x)/((c*d -
 b*e)*x)) - (2*b^3*c^4*d^7 - 8*b^4*c^3*d^6*e + 12*b^5*c^2*d^5*e^2 - 8*b^6*c
*d^4*e^3 + 2*b^7*d^3*e^4 - (32*c^7*d^6*e - 96*b*c^6*d^5*e^2 + 76*b^2*c^5*d^
4*e^3 + 8*b^3*c^4*d^3*e^4 - 35*b^4*c^3*d^2*e^5 + 15*b^5*c^2*d*e^6)*x^4 - 2*
(16*c^7*d^7 - 24*b*c^6*d^6*e - 34*b^2*c^5*d^5*e^2 + 61*b^3*c^4*d^4*e^3 - 4*
b^4*c^3*d^3*e^4 - 30*b^5*c^2*d^2*e^5 + 15*b^6*c*d*e^6)*x^3 - 3*(16*b*c^6*d^
7 - 44*b^2*c^5*d^6*e + 26*b^3*c^4*d^5*e^2 + 16*b^4*c^3*d^4*e^3 - 14*b^5*c^2
*d^3*e^4 - 5*b^6*c*d^2*e^5 + 5*b^7*d*e^6)*x^2 - 2*(6*b^2*c^5*d^7 - 19*b^3*c
^4*d^6*e + 16*b^4*c^3*d^5*e^2 + 6*b^5*c^2*d^4*e^3 - 14*b^6*c*d^3*e^4 + 5*b^
7*d^2*e^5)*x)*sqrt(c*x^2 + b*x))/((b^4*c^6*d^8*e - 4*b^5*c^5*d^7*e^2 + 6*b^
6*c^4*d^6*e^3 - 4*b^7*c^3*d^5*e^4 + b^8*c^2*d^4*e^5)*x^5 + (b^4*c^6*d^9 - 2
*b^5*c^5*d^8*e - 2*b^6*c^4*d^7*e^2 + 8*b^7*c^3*d^6*e^3 - 7*b^8*c^2*d^5*e^4
```

```
+ 2*b^9*c*d^4*e^5)*x^4 + (2*b^5*c^5*d^9 - 7*b^6*c^4*d^8*e + 8*b^7*c^3*d^7*e
^2 - 2*b^8*c^2*d^6*e^3 - 2*b^9*c*d^5*e^4 + b^10*d^4*e^5)*x^3 + (b^6*c^4*d^9
 - 4*b^7*c^3*d^8*e + 6*b^8*c^2*d^7*e^2 - 4*b^9*c*d^6*e^3 + b^10*d^5*e^4)*x^
2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**2+b*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.336 $\int \frac{1}{(2+x)\sqrt{2x+x^2}}\,dx$

Optimal. Leaf size=17

$$\frac{\sqrt{x^2+2x}}{x+2}$$

[Out] Sqrt[2*x + x^2]/(2 + x)

Rubi [A] time = 0.0061051, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {650}

$$\frac{\sqrt{x^2+2x}}{x+2}$$

Antiderivative was successfully verified.

[In] Int[1/((2 + x)*Sqrt[2*x + x^2]),x]

[Out] Sqrt[2*x + x^2]/(2 + x)

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{1}{(2+x)\sqrt{2x+x^2}}\,dx = \frac{\sqrt{2x+x^2}}{2+x}$$

Mathematica [A] time = 0.0050184, size = 11, normalized size = 0.65

$$\frac{x}{\sqrt{x(x+2)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((2 + x)*Sqrt[2*x + x^2]),x]

[Out] x/Sqrt[x*(2 + x)]

Maple [A] time = 0.046, size = 12, normalized size = 0.7

$$x\frac{1}{\sqrt{x^2+2\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+x)/(x^2+2*x)^(1/2),x)
```

```
[Out] x/(x^2+2*x)^(1/2)
```

Maxima [A] time = 1.11167, size = 20, normalized size = 1.18

$$\frac{\sqrt{x^2+2x}}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)/(x^2+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 + 2*x)/(x + 2)
```

Fricas [A] time = 1.86742, size = 47, normalized size = 2.76

$$\frac{x+\sqrt{x^2+2x}+2}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)/(x^2+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] (x + sqrt(x^2 + 2*x) + 2)/(x + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x(x+2)}(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)/(x**2+2*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x*(x + 2))*(x + 2)), x)
```

Giac [A] time = 1.61689, size = 24, normalized size = 1.41

$$\frac{2}{x-\sqrt{x^2+2x}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)/(x^2+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/(x - sqrt(x^2 + 2*x) + 2)
```

3.337 $\int (d+ex)^{7/2} \left(bx+cx^2\right) dx$

Optimal. Leaf size=68

$$-\frac{2(d+ex)^{11/2}(2cd-be)}{11e^3}+\frac{2d(d+ex)^{9/2}(cd-be)}{9e^3}+\frac{2c(d+ex)^{13/2}}{13e^3}$$

```
[Out] (2*d*(c*d - b*e)*(d + e*x)^(9/2))/(9*e^3) - (2*(2*c*d - b*e)*(d + e*x)^(11/
2))/(11*e^3) + (2*c*(d + e*x)^(13/2))/(13*e^3)
```

Rubi [A] time = 0.0292805, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$-\frac{2(d+ex)^{11/2}(2cd-be)}{11e^3}+\frac{2d(d+ex)^{9/2}(cd-be)}{9e^3}+\frac{2c(d+ex)^{13/2}}{13e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)*(b*x + c*x^2),x]
```

```
[Out] (2*d*(c*d - b*e)*(d + e*x)^(9/2))/(9*e^3) - (2*(2*c*d - b*e)*(d + e*x)^(11/
2))/(11*e^3) + (2*c*(d + e*x)^(13/2))/(13*e^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int (d+ex)^{7/2}\left(bx+cx^2\right)dx &= \int \left(\frac{d(cd-be)(d+ex)^{7/2}}{e^2}+\frac{(-2cd+be)(d+ex)^{9/2}}{e^2}+\frac{c(d+ex)^{11/2}}{e^2}\right)dx\\ &= \frac{2d(cd-be)(d+ex)^{9/2}}{9e^3}-\frac{2(2cd-be)(d+ex)^{11/2}}{11e^3}+\frac{2c(d+ex)^{13/2}}{13e^3}\end{aligned}$$

Mathematica [A] time = 0.0443229, size = 50, normalized size = 0.74

$$\frac{2(d+ex)^{9/2}\left(13be(9ex-2d)+c\left(8d^2-36dex+99e^2x^2\right)\right)}{1287e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)*(b*x + c*x^2),x]
```

```
[Out] (2*(d + e*x)^(9/2)*(13*b*e*(-2*d + 9*e*x) + c*(8*d^2 - 36*d*e*x + 99*e^2*x^
2)))/(1287*e^3)
```

Maple [A] time = 0.046, size = 47, normalized size = 0.7

$$-\frac{-198\,ce^{2}x^{2}-234\,be^{2}x+72\,cdex+52\,bde-16\,cd^{2}}{1287\,e^{3}}(ex+d)^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)*(c*x^2+b*x),x)
```

```
[Out] -2/1287*(e*x+d)^(9/2)*(-99*c*e^2*x^2-117*b*e^2*x+36*c*d*e*x+26*b*d*e-8*c*d^
2)/e^3
```

Maxima [A] time = 1.05696, size = 73, normalized size = 1.07

$$\frac{2\left(99\,(ex+d)^{\frac{13}{2}}c-117\,(2\,cd-be)(ex+d)^{\frac{11}{2}}+143\left(cd^{2}-bde\right)(ex+d)^{\frac{9}{2}}\right)}{1287\,e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)*(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] 2/1287*(99*(e*x + d)^(13/2)*c - 117*(2*c*d - b*e)*(e*x + d)^(11/2) + 143*(c
*d^2 - b*d*e)*(e*x + d)^(9/2))/e^3
```

Fricas [B] time = 1.88319, size = 325, normalized size = 4.78

$$\frac{2\left(99\,ce^{6}x^{6}+8\,cd^{6}-26\,bd^{5}e+9\left(40\,cde^{5}+13\,be^{6}\right)x^{5}+2\left(229\,cd^{2}e^{4}+221\,bde^{5}\right)x^{4}+2\left(106\,cd^{3}e^{3}+299\,bd^{2}e^{4}\right)x^{3}+3\,(\cdots}{1287\,e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)*(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] 2/1287*(99*c*e^6*x^6 + 8*c*d^6 - 26*b*d^5*e + 9*(40*c*d*e^5 + 13*b*e^6)*x^5
 + 2*(229*c*d^2*e^4 + 221*b*d*e^5)*x^4 + 2*(106*c*d^3*e^3 + 299*b*d^2*e^4)*
x^3 + 3*(c*d^4*e^2 + 104*b*d^3*e^3)*x^2 - (4*c*d^5*e - 13*b*d^4*e^2)*x)*sqr
t(e*x + d)/e^3
```

Sympy [A] time = 15.1002, size = 292, normalized size = 4.29

$$\begin{cases}-\frac{4bd^{5}\sqrt{d+ex}}{99e^{2}}+\frac{2bd^{4}x\sqrt{d+ex}}{99e}+\frac{16bd^{3}x^{2}\sqrt{d+ex}}{33}+\frac{92bd^{2}ex^{3}\sqrt{d+ex}}{99}+\frac{68bde^{2}x^{4}\sqrt{d+ex}}{99}+\frac{2be^{3}x^{5}\sqrt{d+ex}}{11}+\frac{16cd^{6}\sqrt{d+ex}}{1287e^{3}}-\frac{8cd^{5}x\sqrt{d+ex}}{1287e^{2}}+\frac{2cd^{4}x^{2}\cdots}{429\cdots} \\ d^{\frac{7}{2}}\left(\frac{bx^{2}}{2}+\frac{cx^{3}}{3}\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)*(c*x**2+b*x),x)
```

```
[Out] Piecewise((-4*b*d**5*sqrt(d + e*x)/(99*e**2) + 2*b*d**4*x*sqrt(d + e*x)/(99
*e) + 16*b*d**3*x**2*sqrt(d + e*x)/33 + 92*b*d**2*e*x**3*sqrt(d + e*x)/99 +
 68*b*d*e**2*x**4*sqrt(d + e*x)/99 + 2*b*e**3*x**5*sqrt(d + e*x)/11 + 16*c*
```

```
d**6*sqrt(d + e*x)/(1287*e**3) - 8*c*d**5*x*sqrt(d + e*x)/(1287*e**2) + 2*c
*d**4*x**2*sqrt(d + e*x)/(429*e) + 424*c*d**3*x**3*sqrt(d + e*x)/1287 + 916
*c*d**2*e*x**4*sqrt(d + e*x)/1287 + 80*c*d*e**2*x**5*sqrt(d + e*x)/143 + 2*
c*e**3*x**6*sqrt(d + e*x)/13, Ne(e, 0)), (d**(7/2)*(b*x**2/2 + c*x**3/3), T
rue))
```

Giac [B] time = 1.46589, size = 598, normalized size = 8.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)*(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] 2/45045*(3003*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*b*d^3*e^(-1) + 429*
(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*c*d^3*
e^(-2) + 1287*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/
2)*d^2)*b*d^2*e^(-1) + 429*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 18
9*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*c*d^2*e^(-2) + 429*(35*(x*
e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e +
 d)^(3/2)*d^3)*b*d*e^(-1) + 39*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)
*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(
3/2)*d^4)*c*d*e^(-2) + 13*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d +
2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*
d^4)*b*e^(-1) + 5*(693*(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(
x*e + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 -
 3003*(x*e + d)^(3/2)*d^5)*c*e^(-2))*e^(-1)
```

3.338 $\int (d+ex)^{5/2} \left(bx+cx^2\right) dx$

Optimal. Leaf size=68

$$-\frac{2(d+ex)^{9/2}(2cd-be)}{9e^3}+\frac{2d(d+ex)^{7/2}(cd-be)}{7e^3}+\frac{2c(d+ex)^{11/2}}{11e^3}$$

```
[Out] (2*d*(c*d - b*e)*(d + e*x)^(7/2))/(7*e^3) - (2*(2*c*d - b*e)*(d + e*x)^(9/2
))/(9*e^3) + (2*c*(d + e*x)^(11/2))/(11*e^3)
```

Rubi [A] time = 0.0280261, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$-\frac{2(d+ex)^{9/2}(2cd-be)}{9e^3}+\frac{2d(d+ex)^{7/2}(cd-be)}{7e^3}+\frac{2c(d+ex)^{11/2}}{11e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)*(b*x + c*x^2),x]
```

```
[Out] (2*d*(c*d - b*e)*(d + e*x)^(7/2))/(7*e^3) - (2*(2*c*d - b*e)*(d + e*x)^(9/2
))/(9*e^3) + (2*c*(d + e*x)^(11/2))/(11*e^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int (d+ex)^{5/2}\left(bx+cx^2\right)dx &= \int \left(\frac{d(cd-be)(d+ex)^{5/2}}{e^2}+\frac{(-2cd+be)(d+ex)^{7/2}}{e^2}+\frac{c(d+ex)^{9/2}}{e^2}\right)dx\\ &= \frac{2d(cd-be)(d+ex)^{7/2}}{7e^3}-\frac{2(2cd-be)(d+ex)^{9/2}}{9e^3}+\frac{2c(d+ex)^{11/2}}{11e^3}\end{aligned}$$

Mathematica [A] time = 0.0369975, size = 50, normalized size = 0.74

$$\frac{2(d+ex)^{7/2}\left(11be(7ex-2d)+c\left(8d^2-28dex+63e^2x^2\right)\right)}{693e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)*(b*x + c*x^2),x]
```

```
[Out] (2*(d + e*x)^(7/2)*(11*b*e*(-2*d + 7*e*x) + c*(8*d^2 - 28*d*e*x + 63*e^2*x^
2)))/(693*e^3)
```

Maple [A] time = 0.046, size = 47, normalized size = 0.7

$$-\frac{-126\,c e^{2} x^{2}-154\,b e^{2} x+56\,c d e x+44\,b d e-16\,c d^{2}}{693\,e^{3}}\,(e x+d)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(c*x^2+b*x),x)
```

```
[Out] -2/693*(e*x+d)^(7/2)*(-63*c*e^2*x^2-77*b*e^2*x+28*c*d*e*x+22*b*d*e-8*c*d^2)
/e^3
```

Maxima [A] time = 1.10318, size = 73, normalized size = 1.07

$$\frac{2\left(63\,(e x+d)^{\frac{11}{2}} c-77\,(2\,c d-b e)(e x+d)^{\frac{9}{2}}+99\left(c d^{2}-b d e\right)(e x+d)^{\frac{7}{2}}\right)}{693\,e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] 2/693*(63*(e*x + d)^(11/2)*c - 77*(2*c*d - b*e)*(e*x + d)^(9/2) + 99*(c*d^2
 - b*d*e)*(e*x + d)^(7/2))/e^3
```

Fricas [B] time = 1.9966, size = 266, normalized size = 3.91

$$\frac{2\left(63\,c e^{5} x^{5}+8\,c d^{5}-22\,b d^{4} e+7\left(23\,c d e^{4}+11\,b e^{5}\right)x^{4}+\left(113\,c d^{2} e^{3}+209\,b d e^{4}\right)x^{3}+3\left(c d^{3} e^{2}+55\,b d^{2} e^{3}\right)x^{2}-\left(4\,c d^{4} e-11\,b\right.}{693\,e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] 2/693*(63*c*e^5*x^5 + 8*c*d^5 - 22*b*d^4*e + 7*(23*c*d*e^4 + 11*b*e^5)*x^4
+ (113*c*d^2*e^3 + 209*b*d*e^4)*x^3 + 3*(c*d^3*e^2 + 55*b*d^2*e^3)*x^2 - (4
*c*d^4*e - 11*b*d^3*e^2)*x)*sqrt(e*x + d)/e^3
```

Sympy [A] time = 4.95661, size = 245, normalized size = 3.6

$$\begin{cases} -\frac{4bd^4\sqrt{d+ex}}{63e^2}+\frac{2bd^3x\sqrt{d+ex}}{63e}+\frac{10bd^2x^2\sqrt{d+ex}}{21}+\frac{38bdex^3\sqrt{d+ex}}{63}+\frac{2be^2x^4\sqrt{d+ex}}{9}+\frac{16cd^5\sqrt{d+ex}}{693e^3}-\frac{8cd^4x\sqrt{d+ex}}{693e^2}+\frac{2cd^3x^2\sqrt{d+ex}}{231e}+\frac{226cd^2x^3\sqrt{d+ex}}{693} \\ d^{\frac{5}{2}}\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(c*x**2+b*x),x)
```

```
[Out] Piecewise((-4*b*d**4*sqrt(d + e*x)/(63*e**2) + 2*b*d**3*x*sqrt(d + e*x)/(63
*e) + 10*b*d**2*x**2*sqrt(d + e*x)/21 + 38*b*d*e*x**3*sqrt(d + e*x)/63 + 2*
b*e**2*x**4*sqrt(d + e*x)/9 + 16*c*d**5*sqrt(d + e*x)/(693*e**3) - 8*c*d**4
*x*sqrt(d + e*x)/(693*e**2) + 2*c*d**3*x**2*sqrt(d + e*x)/(231*e) + 226*c*d
```

```
**2*x**3*sqrt(d + e*x)/693 + 46*c*d*e*x**4*sqrt(d + e*x)/99 + 2*c*e**2*x**5
*sqrt(d + e*x)/11, Ne(e, 0)), (d**(5/2)*(b*x**2/2 + c*x**3/3), True))
```

Giac [B] time = 1.53346, size = 393, normalized size = 5.78

$$\frac{2}{3465}\left(231\left(3\,(xe+d)^{\frac{5}{2}}-5\,(xe+d)^{\frac{3}{2}}d\right)bd^{2}e^{(-1)}+33\left(15\,(xe+d)^{\frac{7}{2}}-42\,(xe+d)^{\frac{5}{2}}d+35\,(xe+d)^{\frac{3}{2}}d^{2}\right)cd^{2}e^{(-2)}+66\left(15\,(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] 2/3465*(231*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*b*d^2*e^(-1) + 33*(15
*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*c*d^2*e^(
-2) + 66*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^
2)*b*d*e^(-1) + 22*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e +
 d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*c*d*e^(-2) + 11*(35*(x*e + d)^(9/2
) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d
^3)*b*e^(-1) + (315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e +
 d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*c*e^(-
2))*e^(-1)
```

3.339 $\int (d+ex)^{3/2} \left(bx+cx^2\right) dx$

Optimal. Leaf size=68

$$-\frac{2(d+ex)^{7/2}(2cd-be)}{7e^3}+\frac{2d(d+ex)^{5/2}(cd-be)}{5e^3}+\frac{2c(d+ex)^{9/2}}{9e^3}$$

```
[Out] (2*d*(c*d - b*e)*(d + e*x)^(5/2))/(5*e^3) - (2*(2*c*d - b*e)*(d + e*x)^(7/2
))/(7*e^3) + (2*c*(d + e*x)^(9/2))/(9*e^3)
```

Rubi [A] time = 0.0283249, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$-\frac{2(d+ex)^{7/2}(2cd-be)}{7e^3}+\frac{2d(d+ex)^{5/2}(cd-be)}{5e^3}+\frac{2c(d+ex)^{9/2}}{9e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*(b*x + c*x^2),x]
```

```
[Out] (2*d*(c*d - b*e)*(d + e*x)^(5/2))/(5*e^3) - (2*(2*c*d - b*e)*(d + e*x)^(7/2
))/(7*e^3) + (2*c*(d + e*x)^(9/2))/(9*e^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int (d+ex)^{3/2}\left(bx+cx^2\right)dx &= \int \left(\frac{d(cd-be)(d+ex)^{3/2}}{e^2}+\frac{(-2cd+be)(d+ex)^{5/2}}{e^2}+\frac{c(d+ex)^{7/2}}{e^2}\right)dx\\ &= \frac{2d(cd-be)(d+ex)^{5/2}}{5e^3}-\frac{2(2cd-be)(d+ex)^{7/2}}{7e^3}+\frac{2c(d+ex)^{9/2}}{9e^3}\end{aligned}$$

Mathematica [A] time = 0.0331873, size = 50, normalized size = 0.74

$$\frac{2(d+ex)^{5/2}\left(9be(5ex-2d)+c\left(8d^2-20dex+35e^2x^2\right)\right)}{315e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)*(b*x + c*x^2),x]
```

```
[Out] (2*(d + e*x)^(5/2)*(9*b*e*(-2*d + 5*e*x) + c*(8*d^2 - 20*d*e*x + 35*e^2*x^2
)))/(315*e^3)
```

Maple [A] time = 0.043, size = 47, normalized size = 0.7

$$-\frac{-70\,ce^{2}x^{2}-90\,be^{2}x+40\,cdex+36\,bde-16\,cd^{2}}{315\,e^{3}}\left(ex+d\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(c*x^2+b*x),x)
```

```
[Out] -2/315*(e*x+d)^(5/2)*(-35*c*e^2*x^2-45*b*e^2*x+20*c*d*e*x+18*b*d*e-8*c*d^2)
/e^3
```

Maxima [A] time = 1.08007, size = 73, normalized size = 1.07

$$\frac{2\left(35\left(ex+d\right)^{\frac{9}{2}}c-45\left(2\,cd-be\right)\left(ex+d\right)^{\frac{7}{2}}+63\left(cd^{2}-bde\right)\left(ex+d\right)^{\frac{5}{2}}\right)}{315\,e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] 2/315*(35*(e*x + d)^(9/2)*c - 45*(2*c*d - b*e)*(e*x + d)^(7/2) + 63*(c*d^2
- b*d*e)*(e*x + d)^(5/2))/e^3
```

Fricas [A] time = 1.86388, size = 212, normalized size = 3.12

$$\frac{2\left(35\,ce^{4}x^{4}+8\,cd^{4}-18\,bd^{3}e+5\left(10\,cde^{3}+9\,be^{4}\right)x^{3}+3\left(cd^{2}e^{2}+24\,bde^{3}\right)x^{2}-\left(4\,cd^{3}e-9\,bd^{2}e^{2}\right)x\right)\sqrt{ex+d}}{315\,e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] 2/315*(35*c*e^4*x^4 + 8*c*d^4 - 18*b*d^3*e + 5*(10*c*d*e^3 + 9*b*e^4)*x^3 +
 3*(c*d^2*e^2 + 24*b*d*e^3)*x^2 - (4*c*d^3*e - 9*b*d^2*e^2)*x)*sqrt(e*x + d
)/e^3
```

Sympy [B] time = 9.54356, size = 178, normalized size = 2.62

$$\frac{2bd\left(-\frac{d(d+ex)^{\frac{3}{2}}}{3}+\frac{(d+ex)^{\frac{5}{2}}}{5}\right)}{e^{2}}+\frac{2b\left(\frac{d^{2}(d+ex)^{\frac{3}{2}}}{3}-\frac{2d(d+ex)^{\frac{5}{2}}}{5}+\frac{(d+ex)^{\frac{7}{2}}}{7}\right)}{e^{2}}+\frac{2cd\left(\frac{d^{2}(d+ex)^{\frac{3}{2}}}{3}-\frac{2d(d+ex)^{\frac{5}{2}}}{5}+\frac{(d+ex)^{\frac{7}{2}}}{7}\right)}{e^{3}}+\frac{2c\left(-\frac{d^{3}(d+ex)}{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(c*x**2+b*x),x)
```

```
[Out] 2*b*d*(-d*(d + e*x)**(3/2)/3 + (d + e*x)**(5/2)/5)/e**2 + 2*b*(d**2*(d + e*
x)**(3/2)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e**2 + 2*c*d*(d*
*2*(d + e*x)**(3/2)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e**3 +
 2*c*(-d**3*(d + e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d + e*x)*
```

```
*(7/2)/7 + (d + e*x)**(9/2)/9)/e**3
```

Giac [B] time = 2.41441, size = 224, normalized size = 3.29

$$\frac{2}{315}\left(21\left(3(xe+d)^{\frac{5}{2}}-5(xe+d)^{\frac{3}{2}}d\right)bde^{(-1)}+3\left(15(xe+d)^{\frac{7}{2}}-42(xe+d)^{\frac{5}{2}}d+35(xe+d)^{\frac{3}{2}}d^2\right)cde^{(-2)}+3\left(15(xe+d)^{\frac{7}{2}}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] 2/315*(21*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*b*d*e^(-1) + 3*(15*(x*e
 + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*c*d*e^(-2) + 3
*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*b*e^(
-1) + (35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2
 - 105*(x*e + d)^(3/2)*d^3)*c*e^(-2))*e^(-1)
```

3.340 $\int \sqrt{d+ex}\left(bx+cx^2\right)\,dx$

Optimal. Leaf size=68

$$-\frac{2(d+ex)^{5/2}(2cd-be)}{5e^3}+\frac{2d(d+ex)^{3/2}(cd-be)}{3e^3}+\frac{2c(d+ex)^{7/2}}{7e^3}$$

```
[Out] (2*d*(c*d - b*e)*(d + e*x)^(3/2))/(3*e^3) - (2*(2*c*d - b*e)*(d + e*x)^(5/2
))/(5*e^3) + (2*c*(d + e*x)^(7/2))/(7*e^3)
```

Rubi [A] time = 0.0261583, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$-\frac{2(d+ex)^{5/2}(2cd-be)}{5e^3}+\frac{2d(d+ex)^{3/2}(cd-be)}{3e^3}+\frac{2c(d+ex)^{7/2}}{7e^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(b*x + c*x^2),x]
```

```
[Out] (2*d*(c*d - b*e)*(d + e*x)^(3/2))/(3*e^3) - (2*(2*c*d - b*e)*(d + e*x)^(5/2
))/(5*e^3) + (2*c*(d + e*x)^(7/2))/(7*e^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \sqrt{d+ex}\left(bx+cx^2\right)\,dx &= \int \left(\frac{d(cd-be)\sqrt{d+ex}}{e^2}+\frac{(-2cd+be)(d+ex)^{3/2}}{e^2}+\frac{c(d+ex)^{5/2}}{e^2}\right)dx\\ &= \frac{2d(cd-be)(d+ex)^{3/2}}{3e^3}-\frac{2(2cd-be)(d+ex)^{5/2}}{5e^3}+\frac{2c(d+ex)^{7/2}}{7e^3}\end{aligned}$$

Mathematica [A] time = 0.0306111, size = 50, normalized size = 0.74

$$\frac{2(d+ex)^{3/2}\left(7be(3ex-2d)+c\left(8d^2-12dex+15e^2x^2\right)\right)}{105e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*(b*x + c*x^2),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(7*b*e*(-2*d + 3*e*x) + c*(8*d^2 - 12*d*e*x + 15*e^2*x^2
)))/(105*e^3)
```

Maple [A] time = 0.045, size = 47, normalized size = 0.7

$$-\frac{-30\,ce^{2}x^{2}-42\,be^{2}x+24\,cdex+28\,bde-16\,cd^{2}}{105\,e^{3}}\left(ex+d\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(c*x^2+b*x),x)
```

```
[Out] -2/105*(e*x+d)^(3/2)*(-15*c*e^2*x^2-21*b*e^2*x+12*c*d*e*x+14*b*d*e-8*c*d^2)
/e^3
```

Maxima [A] time = 1.05738, size = 73, normalized size = 1.07

$$\frac{2\left(15\,(ex+d)^{\frac{7}{2}}c-21\,(2\,cd-be)(ex+d)^{\frac{5}{2}}+35\left(cd^{2}-bde\right)(ex+d)^{\frac{3}{2}}\right)}{105\,e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] 2/105*(15*(e*x + d)^(7/2)*c - 21*(2*c*d - b*e)*(e*x + d)^(5/2) + 35*(c*d^2
- b*d*e)*(e*x + d)^(3/2))/e^3
```

Fricas [A] time = 1.87582, size = 161, normalized size = 2.37

$$\frac{2\left(15\,ce^{3}x^{3}+8\,cd^{3}-14\,bd^{2}e+3\left(cde^{2}+7\,be^{3}\right)x^{2}-\left(4\,cd^{2}e-7\,bde^{2}\right)x\right)\sqrt{ex+d}}{105\,e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] 2/105*(15*c*e^3*x^3 + 8*c*d^3 - 14*b*d^2*e + 3*(c*d*e^2 + 7*b*e^3)*x^2 - (4
*c*d^2*e - 7*b*d*e^2)*x)*sqrt(e*x + d)/e^3
```

Sympy [A] time = 3.14416, size = 66, normalized size = 0.97

$$\frac{2\left(\frac{c(d+ex)^{\frac{7}{2}}}{7e^{2}}+\frac{(d+ex)^{\frac{5}{2}}(be-2cd)}{5e^{2}}+\frac{(d+ex)^{\frac{3}{2}}\left(-bde+cd^{2}\right)}{3e^{2}}\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(c*x**2+b*x),x)
```

```
[Out] 2*(c*(d + e*x)**(7/2)/(7*e**2) + (d + e*x)**(5/2)*(b*e - 2*c*d)/(5*e**2) +
(d + e*x)**(3/2)*(-b*d*e + c*d**2)/(3*e**2))/e
```

Giac [A] time = 2.29449, size = 96, normalized size = 1.41

$$\frac{2}{105}\left(7\left(3\,(xe+d)^{\frac{5}{2}}-5\,(xe+d)^{\frac{3}{2}}d\right)be^{(-1)}+\left(15\,(xe+d)^{\frac{7}{2}}-42\,(xe+d)^{\frac{5}{2}}d+35\,(xe+d)^{\frac{3}{2}}d^2\right)ce^{(-2)}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] 2/105*(7*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*b*e^(-1) + (15*(x*e + d)
^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*c*e^(-2))*e^(-1)
```

3.341 $\int \frac{bx+cx^2}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=66

$$-\frac{2(d+ex)^{3/2}(2cd-be)}{3e^3}+\frac{2d\sqrt{d+ex}(cd-be)}{e^3}+\frac{2c(d+ex)^{5/2}}{5e^3}$$

```
[Out] (2*d*(c*d - b*e)*Sqrt[d + e*x])/e^3 - (2*(2*c*d - b*e)*(d + e*x)^(3/2))/(3*
e^3) + (2*c*(d + e*x)^(5/2))/(5*e^3)
```

Rubi [A] time = 0.0272946, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$-\frac{2(d+ex)^{3/2}(2cd-be)}{3e^3}+\frac{2d\sqrt{d+ex}(cd-be)}{e^3}+\frac{2c(d+ex)^{5/2}}{5e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)/Sqrt[d + e*x],x]
```

```
[Out] (2*d*(c*d - b*e)*Sqrt[d + e*x])/e^3 - (2*(2*c*d - b*e)*(d + e*x)^(3/2))/(3*
e^3) + (2*c*(d + e*x)^(5/2))/(5*e^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{bx+cx^2}{\sqrt{d+ex}}\,dx &= \int \left(\frac{d(cd-be)}{e^2\sqrt{d+ex}}+\frac{(-2cd+be)\sqrt{d+ex}}{e^2}+\frac{c(d+ex)^{3/2}}{e^2}\right)dx\\ &= \frac{2d(cd-be)\sqrt{d+ex}}{e^3}-\frac{2(2cd-be)(d+ex)^{3/2}}{3e^3}+\frac{2c(d+ex)^{5/2}}{5e^3}\end{aligned}$$

Mathematica [A] time = 0.0309657, size = 49, normalized size = 0.74

$$\frac{2\sqrt{d+ex}\left(5be(ex-2d)+c\left(8d^2-4dex+3e^2x^2\right)\right)}{15e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[d + e*x]*(5*b*e*(-2*d + e*x) + c*(8*d^2 - 4*d*e*x + 3*e^2*x^2)))/(1
5*e^3)
```

Maple [A] time = 0.048, size = 47, normalized size = 0.7

$$-\frac{-6\,c e^{2} x^{2}-10\,b e^{2} x+8\,c d e x+20\,b d e-16\,c d^{2}}{15\,e^{3}}\sqrt{e x+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)/(e*x+d)^(1/2),x)
```

```
[Out] -2/15*(-3*c*e^2*x^2-5*b*e^2*x+4*c*d*e*x+10*b*d*e-8*c*d^2)*(e*x+d)^(1/2)/e^3
```

Maxima [A] time = 1.0617, size = 90, normalized size = 1.36

$$\frac{2\left(\frac{5\left((ex+d)^{\frac{3}{2}}-3\,\sqrt{ex+d}d\right)b}{e}+\frac{\left(3\,(ex+d)^{\frac{5}{2}}-10\,(ex+d)^{\frac{3}{2}}d+15\,\sqrt{ex+d}d^{2}\right)c}{e^{2}}\right)}{15\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(5*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*b/e + (3*(e*x + d)^(5/2) - 10
*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*c/e^2)/e
```

Fricas [A] time = 1.82246, size = 112, normalized size = 1.7

$$\frac{2\left(3\,c e^{2} x^{2}+8\,c d^{2}-10\,b d e-\left(4\,c d e-5\,b e^{2}\right)x\right)\sqrt{ex+d}}{15\,e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*c*e^2*x^2 + 8*c*d^2 - 10*b*d*e - (4*c*d*e - 5*b*e^2)*x)*sqrt(e*x +
d)/e^3
```

Sympy [A] time = 11.9097, size = 182, normalized size = 2.76

$$\begin{cases} -\frac{\frac{2bd\left(-\frac{d}{\sqrt{d+ex}}-\sqrt{d+ex}\right)}{e}+\frac{2b\left(\frac{d^{2}}{\sqrt{d+ex}}+2d\sqrt{d+ex}-\frac{(d+ex)^{\frac{3}{2}}}{3}\right)}{e}+\frac{2cd\left(\frac{d^{2}}{\sqrt{d+ex}}+2d\sqrt{d+ex}-\frac{(d+ex)^{\frac{3}{2}}}{3}\right)}{e^{2}}+\frac{2c\left(-\frac{d^{3}}{\sqrt{d+ex}}-3d^{2}\sqrt{d+ex}+d(d+ex)^{\frac{3}{2}}-\frac{(d+ex)^{\frac{5}{2}}}{5}\right)}{e^{2}}}{e} & \text{for } e\neq 0 \\ \frac{\frac{bx^{2}}{2}+\frac{cx^{3}}{3}}{\sqrt{d}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)/(e*x+d)**(1/2),x)
```

```
[Out] Piecewise((-(2*b*d*(-d/sqrt(d + e*x) - sqrt(d + e*x))/e + 2*b*(d**2/sqrt(d
+ e*x) + 2*d*sqrt(d + e*x) - (d + e*x)**(3/2)/3)/e + 2*c*d*(d**2/sqrt(d + e
*x) + 2*d*sqrt(d + e*x) - (d + e*x)**(3/2)/3)/e**2 + 2*c*(-d**3/sqrt(d + e*
x) - 3*d**2*sqrt(d + e*x) + d*(d + e*x)**(3/2) - (d + e*x)**(5/2)/5)/e**2)/
e, Ne(e, 0)), ((b*x**2/2 + c*x**3/3)/sqrt(d), True))
```

Giac [A] time = 1.31709, size = 93, normalized size = 1.41

$$\frac{2}{15}\left(5\left((xe+d)^{\frac{3}{2}}-3\sqrt{xe+d}d\right)be^{(-1)}+\left(3(xe+d)^{\frac{5}{2}}-10(xe+d)^{\frac{3}{2}}d+15\sqrt{xe+d}d^{2}\right)ce^{(-2)}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(5*((x*e + d)^(3/2) - 3*sqrt(x*e + d)*d)*b*e^(-1) + (3*(x*e + d)^(5/2)
 - 10*(x*e + d)^(3/2)*d + 15*sqrt(x*e + d)*d^2)*c*e^(-2))*e^(-1)
```

3.342 $\int \frac{bx+cx^2}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=64

$$-\frac{2\sqrt{d+ex}(2cd-be)}{e^3}-\frac{2d(cd-be)}{e^3\sqrt{d+ex}}+\frac{2c(d+ex)^{3/2}}{3e^3}$$

```
[Out] (-2*d*(c*d - b*e))/(e^3*Sqrt[d + e*x]) - (2*(2*c*d - b*e)*Sqrt[d + e*x])/e^
3 + (2*c*(d + e*x)^(3/2))/(3*e^3)
```

Rubi [A] time = 0.0271376, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$-\frac{2\sqrt{d+ex}(2cd-be)}{e^3}-\frac{2d(cd-be)}{e^3\sqrt{d+ex}}+\frac{2c(d+ex)^{3/2}}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)/(d + e*x)^(3/2),x]
```

```
[Out] (-2*d*(c*d - b*e))/(e^3*Sqrt[d + e*x]) - (2*(2*c*d - b*e)*Sqrt[d + e*x])/e^
3 + (2*c*(d + e*x)^(3/2))/(3*e^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{bx+cx^2}{(d+ex)^{3/2}}\,dx &= \int \left(\frac{d(cd-be)}{e^2(d+ex)^{3/2}}+\frac{-2cd+be}{e^2\sqrt{d+ex}}+\frac{c\sqrt{d+ex}}{e^2}\right)dx\\ &= -\frac{2d(cd-be)}{e^3\sqrt{d+ex}}-\frac{2(2cd-be)\sqrt{d+ex}}{e^3}+\frac{2c(d+ex)^{3/2}}{3e^3}\end{aligned}$$

Mathematica [A] time = 0.0305917, size = 48, normalized size = 0.75

$$\frac{2\left(3be(2d+ex)+c\left(-8d^2-4dex+e^2x^2\right)\right)}{3e^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)/(d + e*x)^(3/2),x]
```

```
[Out] (2*(3*b*e*(2*d + e*x) + c*(-8*d^2 - 4*d*e*x + e^2*x^2)))/(3*e^3*Sqrt[d + e*
x])
```

Maple [A] time = 0.049, size = 46, normalized size = 0.7

$$\frac{2\,c e^{2} x^{2}+6\,b e^{2} x-8\,c d e x+12\,b d e-16\,c d^{2}}{3\,e^{3}}\frac{1}{\sqrt{ex+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)/(e*x+d)^(3/2),x)
```

```
[Out] 2/3*(c*e^2*x^2+3*b*e^2*x-4*c*d*e*x+6*b*d*e-8*c*d^2)/(e*x+d)^(1/2)/e^3
```

Maxima [A] time = 1.08546, size = 82, normalized size = 1.28

$$\frac{2\left(\frac{(ex+d)^{\frac{3}{2}}c-3\,(2\,cd-be)\sqrt{ex+d}}{e^{2}}-\frac{3\left(cd^{2}-bde\right)}{\sqrt{ex+d}e^{2}}\right)}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(((e*x + d)^(3/2)*c - 3*(2*c*d - b*e)*sqrt(e*x + d))/e^2 - 3*(c*d^2 - b
*d*e)/(sqrt(e*x + d)*e^2))/e
```

Fricas [A] time = 1.88869, size = 123, normalized size = 1.92

$$\frac{2\left(ce^{2}x^{2}-8\,cd^{2}+6\,bde-\left(4\,cde-3\,be^{2}\right)x\right)\sqrt{ex+d}}{3\left(e^{4}x+de^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(c*e^2*x^2 - 8*c*d^2 + 6*b*d*e - (4*c*d*e - 3*b*e^2)*x)*sqrt(e*x + d)/(
e^4*x + d*e^3)
```

Sympy [A] time = 10.5883, size = 60, normalized size = 0.94

$$\frac{2c\,(d+ex)^{\frac{3}{2}}}{3e^{3}}+\frac{2d\,(be-cd)}{e^{3}\sqrt{d+ex}}+\frac{\sqrt{d+ex}\,(2be-4cd)}{e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)/(e*x+d)**(3/2),x)
```

```
[Out] 2*c*(d + e*x)**(3/2)/(3*e**3) + 2*d*(b*e - c*d)/(e**3*sqrt(d + e*x)) + sqrt
(d + e*x)*(2*b*e - 4*c*d)/e**3
```

Giac [A] time = 1.28597, size = 93, normalized size = 1.45

$$\frac{2}{3}\left((xe+d)^{\frac{3}{2}}ce^6 - 6\sqrt{xe+d}cde^6 + 3\sqrt{xe+d}be^7\right)e^{(-9)} - \frac{2\left(cd^2 - bde\right)e^{(-3)}}{\sqrt{xe+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*((x*e + d)^(3/2)*c*e^6 - 6*sqrt(x*e + d)*c*d*e^6 + 3*sqrt(x*e + d)*b*e^
7)*e^(-9) - 2*(c*d^2 - b*d*e)*e^(-3)/sqrt(x*e + d)
```

3.343 $\int \frac{bx+cx^2}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=64

$$\frac{2(2cd-be)}{e^3\sqrt{d+ex}}-\frac{2d(cd-be)}{3e^3(d+ex)^{3/2}}+\frac{2c\sqrt{d+ex}}{e^3}$$

```
[Out] (-2*d*(c*d - b*e))/(3*e^3*(d + e*x)^(3/2)) + (2*(2*c*d - b*e))/(e^3*Sqrt[d + e*x]) + (2*c*Sqrt[d + e*x])/e^3
```

Rubi [A] time = 0.0277594, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$\frac{2(2cd-be)}{e^3\sqrt{d+ex}}-\frac{2d(cd-be)}{3e^3(d+ex)^{3/2}}+\frac{2c\sqrt{d+ex}}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)/(d + e*x)^(5/2),x]
```

```
[Out] (-2*d*(c*d - b*e))/(3*e^3*(d + e*x)^(3/2)) + (2*(2*c*d - b*e))/(e^3*Sqrt[d + e*x]) + (2*c*Sqrt[d + e*x])/e^3
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{bx+cx^2}{(d+ex)^{5/2}}\,dx&=\int \left(\frac{d(cd-be)}{e^2(d+ex)^{5/2}}+\frac{-2cd+be}{e^2(d+ex)^{3/2}}+\frac{c}{e^2\sqrt{d+ex}}\right)dx\\&=-\frac{2d(cd-be)}{3e^3(d+ex)^{3/2}}+\frac{2(2cd-be)}{e^3\sqrt{d+ex}}+\frac{2c\sqrt{d+ex}}{e^3}\end{aligned}$$

Mathematica [A] time = 0.0323933, size = 50, normalized size = 0.78

$$\frac{2\left(c\left(8d^2+12dex+3e^2x^2\right)-be(2d+3ex)\right)}{3e^3(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)/(d + e*x)^(5/2),x]
```

```
[Out] (2*(-(b*e*(2*d + 3*e*x)) + c*(8*d^2 + 12*d*e*x + 3*e^2*x^2)))/(3*e^3*(d + e*x)^(3/2))
```

Maple [A] time = 0.046, size = 47, normalized size = 0.7

$$-\frac{-6\,ce^{2}x^{2}+6\,be^{2}x-24\,cdex+4\,bde-16\,cd^{2}}{3\,e^{3}}(ex+d)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)/(e*x+d)^(5/2),x)
```

```
[Out] -2/3*(-3*c*e^2*x^2+3*b*e^2*x-12*c*d*e*x+2*b*d*e-8*c*d^2)/(e*x+d)^(3/2)/e^3
```

Maxima [A] time = 0.997937, size = 78, normalized size = 1.22

$$\frac{2\left(\frac{3\sqrt{ex+d}c}{e^{2}}-\frac{cd^{2}-bde-3\,(2\,cd-be)(ex+d)}{(ex+d)^{\frac{3}{2}}e^{2}}\right)}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(3*sqrt(e*x + d)*c/e^2 - (c*d^2 - b*d*e - 3*(2*c*d - b*e)*(e*x + d))/((
e*x + d)^(3/2)*e^2))/e
```

Fricas [A] time = 1.84956, size = 147, normalized size = 2.3

$$\frac{2\left(3\,ce^{2}x^{2}+8\,cd^{2}-2\,bde+3\left(4\,cde-be^{2}\right)x\right)\sqrt{ex+d}}{3\left(e^{5}x^{2}+2\,de^{4}x+d^{2}e^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*c*e^2*x^2 + 8*c*d^2 - 2*b*d*e + 3*(4*c*d*e - b*e^2)*x)*sqrt(e*x + d)
/(e^5*x^2 + 2*d*e^4*x + d^2*e^3)
```

Sympy [A] time = 1.59792, size = 211, normalized size = 3.3

$$\begin{cases} -\frac{4bde}{3de^{3}\sqrt{d+ex}+3e^{4}x\sqrt{d+ex}}-\frac{6be^{2}x}{3de^{3}\sqrt{d+ex}+3e^{4}x\sqrt{d+ex}}+\frac{16cd^{2}}{3de^{3}\sqrt{d+ex}+3e^{4}x\sqrt{d+ex}}+\frac{24cdex}{3de^{3}\sqrt{d+ex}+3e^{4}x\sqrt{d+ex}}+\frac{6ce^{2}x^{2}}{3de^{3}\sqrt{d+ex}+3e^{4}x\sqrt{d+ex}} & \text{for } e \neq \\ \frac{\frac{bx^{2}}{2}+\frac{cx^{3}}{3}}{d^{\frac{5}{2}}} & \text{otherw} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)/(e*x+d)**(5/2),x)
```

```
[Out] Piecewise((-4*b*d*e/(3*d*e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)) - 6*b
*e**2*x/(3*d*e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)) + 16*c*d**2/(3*d*
```

```
e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)) + 24*c*d*e*x/(3*d*e**3*sqrt(d
+ e*x) + 3*e**4*x*sqrt(d + e*x)) + 6*c*e**2*x**2/(3*d*e**3*sqrt(d + e*x) +
3*e**4*x*sqrt(d + e*x)), Ne(e, 0)), ((b*x**2/2 + c*x**3/3)/d**(5/2), True))
```

Giac [A] time = 1.34721, size = 80, normalized size = 1.25

$$2\sqrt{xe+d}ce^{(-3)} + \frac{2\left(6\,(xe+d)cd - cd^2 - 3\,(xe+d)be + bde\right)e^{(-3)}}{3\,(xe+d)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x*e + d)*c*e^(-3) + 2/3*(6*(x*e + d)*c*d - c*d^2 - 3*(x*e + d)*b*e +
 b*d*e)*e^(-3)/(x*e + d)^(3/2)
```

3.344 $\int \frac{bx+cx^2}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=66

$$\frac{2(2cd-be)}{3e^3(d+ex)^{3/2}}-\frac{2d(cd-be)}{5e^3(d+ex)^{5/2}}-\frac{2c}{e^3\sqrt{d+ex}}$$

```
[Out] (-2*d*(c*d - b*e))/(5*e^3*(d + e*x)^(5/2)) + (2*(2*c*d - b*e))/(3*e^3*(d +
e*x)^(3/2)) - (2*c)/(e^3*Sqrt[d + e*x])
```

Rubi [A] time = 0.0266006, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$\frac{2(2cd-be)}{3e^3(d+ex)^{3/2}}-\frac{2d(cd-be)}{5e^3(d+ex)^{5/2}}-\frac{2c}{e^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)/(d + e*x)^(7/2),x]
```

```
[Out] (-2*d*(c*d - b*e))/(5*e^3*(d + e*x)^(5/2)) + (2*(2*c*d - b*e))/(3*e^3*(d +
e*x)^(3/2)) - (2*c)/(e^3*Sqrt[d + e*x])
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{bx+cx^2}{(d+ex)^{7/2}}\,dx &= \int \left(\frac{d(cd-be)}{e^2(d+ex)^{7/2}}+\frac{-2cd+be}{e^2(d+ex)^{5/2}}+\frac{c}{e^2(d+ex)^{3/2}}\right)dx\\ &= -\frac{2d(cd-be)}{5e^3(d+ex)^{5/2}}+\frac{2(2cd-be)}{3e^3(d+ex)^{3/2}}-\frac{2c}{e^3\sqrt{d+ex}}\end{aligned}$$

Mathematica [A] time = 0.031262, size = 49, normalized size = 0.74

$$-\frac{2\left(be(2d+5ex)+c\left(8d^2+20dex+15e^2x^2\right)\right)}{15e^3(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)/(d + e*x)^(7/2),x]
```

```
[Out] (-2*(b*e*(2*d + 5*e*x) + c*(8*d^2 + 20*d*e*x + 15*e^2*x^2)))/(15*e^3*(d + e
*x)^(5/2))
```

Maple [A] time = 0.046, size = 47, normalized size = 0.7

$$-\frac{30\,c e^2 x^2+10\,b e^2 x+40\,cdex+4\,bde+16\,c d^2}{15\,e^3}\,(ex+d)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)/(e*x+d)^(7/2),x)
```

```
[Out] -2/15*(15*c*e^2*x^2+5*b*e^2*x+20*c*d*e*x+2*b*d*e+8*c*d^2)/(e*x+d)^(5/2)/e^3
```

Maxima [A] time = 1.00008, size = 68, normalized size = 1.03

$$-\frac{2\left(15\,(ex+d)^2 c+3\,c d^2-3\,bde-5\,(2\,cd-be)(ex+d)\right)}{15\,(ex+d)^{\frac{5}{2}}e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/15*(15*(e*x + d)^2*c + 3*c*d^2 - 3*b*d*e - 5*(2*c*d - b*e)*(e*x + d))/((
e*x + d)^(5/2)*e^3)
```

Fricas [A] time = 1.85042, size = 173, normalized size = 2.62

$$-\frac{2\left(15\,c e^2 x^2+8\,c d^2+2\,bde+5\left(4\,cde+b e^2\right)x\right)\sqrt{ex+d}}{15\left(e^6 x^3+3\,d e^5 x^2+3\,d^2 e^4 x+d^3 e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x+d)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/15*(15*c*e^2*x^2 + 8*c*d^2 + 2*b*d*e + 5*(4*c*d*e + b*e^2)*x)*sqrt(e*x +
 d)/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3)
```

Sympy [A] time = 4.1712, size = 314, normalized size = 4.76

$$\begin{cases} -\frac{4bde}{15d^2e^3\sqrt{d+ex}+30de^4x\sqrt{d+ex}+15e^5x^2\sqrt{d+ex}} - \frac{10be^2x}{15d^2e^3\sqrt{d+ex}+30de^4x\sqrt{d+ex}+15e^5x^2\sqrt{d+ex}} - \frac{16cd^2}{15d^2e^3\sqrt{d+ex}+30de^4x\sqrt{d+ex}+15e^5x^2\sqrt{d+ex}} - \frac{}{15d^2e^3\sqrt{d+ \\ \frac{\frac{bx^2}{2}+\frac{cx^3}{3}}{d^{\frac{7}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)/(e*x+d)**(7/2),x)
```

```
[Out] Piecewise((-4*b*d*e/(15*d**2*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d + e*x)
 + 15*e**5*x**2*sqrt(d + e*x)) - 10*b*e**2*x/(15*d**2*e**3*sqrt(d + e*x) +
30*d*e**4*x*sqrt(d + e*x) + 15*e**5*x**2*sqrt(d + e*x)) - 16*c*d**2/(15*d**
2*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d + e*x) + 15*e**5*x**2*sqrt(d + e*
```

```
x)) - 40*c*d*e*x/(15*d**2*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d + e*x) +
15*e**5*x**2*sqrt(d + e*x)) - 30*c*e**2*x**2/(15*d**2*e**3*sqrt(d + e*x) +
30*d*e**4*x*sqrt(d + e*x) + 15*e**5*x**2*sqrt(d + e*x)), Ne(e, 0)), ((b*x**
2/2 + c*x**3/3)/d**(7/2), True))
```

Giac [A] time = 1.23369, size = 77, normalized size = 1.17

$$-\frac{2\left(15\,(xe+d)^2c-10\,(xe+d)cd+3\,cd^2+5\,(xe+d)be-3\,bde\right)e^{(-3)}}{15\,(xe+d)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*(15*(x*e + d)^2*c - 10*(x*e + d)*c*d + 3*c*d^2 + 5*(x*e + d)*b*e - 3*
b*d*e)*e^(-3)/(x*e + d)^(5/2)
```

3.345 $\int (d+ex)^{7/2} \left(bx+cx^2\right)^2 \, dx$

Optimal. Leaf size=147

$$\frac{2(d+ex)^{13/2}\left(b^2e^2-6bcde+6c^2d^2\right)}{13e^5}+\frac{2d^2(d+ex)^{9/2}(cd-be)^2}{9e^5}-\frac{4c(d+ex)^{15/2}(2cd-be)}{15e^5}-\frac{4d(d+ex)^{11/2}(cd-be)(2cd-}{11e^5}$$

```
[Out] (2*d^2*(c*d - b*e)^2*(d + e*x)^(9/2))/(9*e^5) - (4*d*(c*d - b*e)*(2*c*d - b
*e)*(d + e*x)^(11/2))/(11*e^5) + (2*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(d +
e*x)^(13/2))/(13*e^5) - (4*c*(2*c*d - b*e)*(d + e*x)^(15/2))/(15*e^5) + (2*
c^2*(d + e*x)^(17/2))/(17*e^5)
```

Rubi [A] time = 0.0722376, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {698}

$$\frac{2(d+ex)^{13/2}\left(b^2e^2-6bcde+6c^2d^2\right)}{13e^5}+\frac{2d^2(d+ex)^{9/2}(cd-be)^2}{9e^5}-\frac{4c(d+ex)^{15/2}(2cd-be)}{15e^5}-\frac{4d(d+ex)^{11/2}(cd-be)(2cd-}{11e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)*(b*x + c*x^2)^2,x]
```

```
[Out] (2*d^2*(c*d - b*e)^2*(d + e*x)^(9/2))/(9*e^5) - (4*d*(c*d - b*e)*(2*c*d - b
*e)*(d + e*x)^(11/2))/(11*e^5) + (2*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(d +
e*x)^(13/2))/(13*e^5) - (4*c*(2*c*d - b*e)*(d + e*x)^(15/2))/(15*e^5) + (2*
c^2*(d + e*x)^(17/2))/(17*e^5)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^{7/2}\left(bx+cx^2\right)^2\,dx=\int \left(\frac{d^2(cd-be)^2(d+ex)^{7/2}}{e^4}+\frac{2d(cd-be)(-2cd+be)(d+ex)^{9/2}}{e^4}+\frac{\left(6c^2d^2-6bcde+b^2e^2\right)(}{e^4}\right.$$

$$=\frac{2d^2(cd-be)^2(d+ex)^{9/2}}{9e^5}-\frac{4d(cd-be)(2cd-be)(d+ex)^{11/2}}{11e^5}+\frac{2\left(6c^2d^2-6bcde+b^2e^2\right)(d}{13e^5}$$

Mathematica [A] time = 0.0960983, size = 124, normalized size = 0.84

$$\frac{2(d+ex)^{9/2}\left(85b^2e^2\left(8d^2-36dex+99e^2x^2\right)+34bce\left(72d^2ex-16d^3-198de^2x^2+429e^3x^3\right)+c^2\left(1584d^2e^2x^2-576d^3ex-\right.\right.}{109395e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)*(b*x + c*x^2)^2,x]
```

```
[Out] (2*(d + e*x)^(9/2)*(85*b^2*e^2*(8*d^2 - 36*d*e*x + 99*e^2*x^2) + 34*b*c*e*(
-16*d^3 + 72*d^2*e*x - 198*d*e^2*x^2 + 429*e^3*x^3) + c^2*(128*d^4 - 576*d^
3*e*x + 1584*d^2*e^2*x^2 - 3432*d*e^3*x^3 + 6435*e^4*x^4)))/(109395*e^5)
```

Maple [A] time = 0.048, size = 141, normalized size = 1.

$$\frac{12870\,c^2x^4e^4+29172\,bce^4x^3-6864\,c^2de^3x^3+16830\,b^2e^4x^2-13464\,bcde^3x^2+3168\,c^2d^2e^2x^2-6120\,b^2de^3x+4896\,bc}{109395\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)*(c*x^2+b*x)^2,x)
```

```
[Out] 2/109395*(e*x+d)^(9/2)*(6435*c^2*e^4*x^4+14586*b*c*e^4*x^3-3432*c^2*d*e^3*x
^3+8415*b^2*e^4*x^2-6732*b*c*d*e^3*x^2+1584*c^2*d^2*e^2*x^2-3060*b^2*d*e^3*
x+2448*b*c*d^2*e^2*x-576*c^2*d^3*e*x+680*b^2*d^2*e^2-544*b*c*d^3*e+128*c^2*
d^4)/e^5
```

Maxima [A] time = 1.09866, size = 188, normalized size = 1.28

$$\frac{2\left(6435\,(ex+d)^{\frac{17}{2}}c^2-14586\left(2\,c^2d-bce\right)(ex+d)^{\frac{15}{2}}+8415\left(6\,c^2d^2-6\,bcde+b^2e^2\right)(ex+d)^{\frac{13}{2}}-19890\left(2\,c^2d^3-3\,bcd\right.\right.}{109395\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)*(c*x^2+b*x)^2,x, algorithm="maxima")
```

```
[Out] 2/109395*(6435*(e*x + d)^(17/2)*c^2 - 14586*(2*c^2*d - b*c*e)*(e*x + d)^(15
/2) + 8415*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(e*x + d)^(13/2) - 19890*(2*c^
2*d^3 - 3*b*c*d^2*e + b^2*d*e^2)*(e*x + d)^(11/2) + 12155*(c^2*d^4 - 2*b*c*
d^3*e + b^2*d^2*e^2)*(e*x + d)^(9/2))/e^5
```

Fricas [B] time = 1.9592, size = 662, normalized size = 4.5

$$\frac{2\left(6435\,c^2e^8x^8+128\,c^2d^8-544\,bcd^7e+680\,b^2d^6e^2+858\left(26\,c^2de^7+17\,bce^8\right)x^7+33\left(802\,c^2d^2e^6+1564\,bcde^7+255\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)*(c*x^2+b*x)^2,x, algorithm="fricas")
```

```
[Out] 2/109395*(6435*c^2*e^8*x^8 + 128*c^2*d^8 - 544*b*c*d^7*e + 680*b^2*d^6*e^2
+ 858*(26*c^2*d*e^7 + 17*b*c*e^8)*x^7 + 33*(802*c^2*d^2*e^6 + 1564*b*c*d*e^
7 + 255*b^2*e^8)*x^6 + 36*(303*c^2*d^3*e^5 + 1751*b*c*d^2*e^6 + 850*b^2*d*e
^7)*x^5 + 5*(7*c^2*d^4*e^4 + 5440*b*c*d^3*e^5 + 7786*b^2*d^2*e^6)*x^4 - 10*
(4*c^2*d^5*e^3 - 17*b*c*d^4*e^4 - 1802*b^2*d^3*e^5)*x^3 + 3*(16*c^2*d^6*e^2
 - 68*b*c*d^5*e^3 + 85*b^2*d^4*e^4)*x^2 - 4*(16*c^2*d^7*e - 68*b*c*d^6*e^2
+ 85*b^2*d^5*e^3)*x)*sqrt(e*x + d)/e^5
```

Sympy [A] time = 21.8537, size = 590, normalized size = 4.01

$$\begin{cases} \frac{16b^2d^6\sqrt{d+ex}}{1287e^3} - \frac{8b^2d^5x\sqrt{d+ex}}{1287e^2} + \frac{2b^2d^4x^2\sqrt{d+ex}}{429e} + \frac{424b^2d^3x^3\sqrt{d+ex}}{1287} + \frac{916b^2d^2ex^4\sqrt{d+ex}}{1287} + \frac{80b^2de^2x^5\sqrt{d+ex}}{143} + \frac{2b^2e^3x^6\sqrt{d+ex}}{13} - \frac{64bcd^7\sqrt{d+ex}}{6435e^4} + 3 \\ d^{\frac{7}{2}}\left(\frac{b^2x^3}{3} + \frac{bcx^4}{2} + \frac{c^2x^5}{5}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)*(c*x**2+b*x)**2,x)
```

```
[Out] Piecewise((16*b**2*d**6*sqrt(d + e*x)/(1287*e**3) - 8*b**2*d**5*x*sqrt(d +
e*x)/(1287*e**2) + 2*b**2*d**4*x**2*sqrt(d + e*x)/(429*e) + 424*b**2*d**3*x
**3*sqrt(d + e*x)/1287 + 916*b**2*d**2*e*x**4*sqrt(d + e*x)/1287 + 80*b**2*
d*e**2*x**5*sqrt(d + e*x)/143 + 2*b**2*e**3*x**6*sqrt(d + e*x)/13 - 64*b*c*
d**7*sqrt(d + e*x)/(6435*e**4) + 32*b*c*d**6*x*sqrt(d + e*x)/(6435*e**3) -
8*b*c*d**5*x**2*sqrt(d + e*x)/(2145*e**2) + 4*b*c*d**4*x**3*sqrt(d + e*x)/(
1287*e) + 640*b*c*d**3*x**4*sqrt(d + e*x)/1287 + 824*b*c*d**2*e*x**5*sqrt(d
 + e*x)/715 + 184*b*c*d*e**2*x**6*sqrt(d + e*x)/195 + 4*b*c*e**3*x**7*sqrt(
d + e*x)/15 + 256*c**2*d**8*sqrt(d + e*x)/(109395*e**5) - 128*c**2*d**7*x*s
qrt(d + e*x)/(109395*e**4) + 32*c**2*d**6*x**2*sqrt(d + e*x)/(36465*e**3) -
 16*c**2*d**5*x**3*sqrt(d + e*x)/(21879*e**2) + 14*c**2*d**4*x**4*sqrt(d +
e*x)/(21879*e) + 2424*c**2*d**3*x**5*sqrt(d + e*x)/12155 + 1604*c**2*d**2*e
*x**6*sqrt(d + e*x)/3315 + 104*c**2*d*e**2*x**7*sqrt(d + e*x)/255 + 2*c**2*
e**3*x**8*sqrt(d + e*x)/17, Ne(e, 0)), (d**(7/2)*(b**2*x**3/3 + b*c*x**4/2
+ c**2*x**5/5), True))
```

Giac [B] time = 1.35583, size = 1237, normalized size = 8.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)*(c*x^2+b*x)^2,x, algorithm="giac")
```

```
[Out] 2/765765*(7293*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3
/2)*d^2)*b^2*d^3*e^(-2) + 4862*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d
+ 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*b*c*d^3*e^(-3) + 221*(
315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 -
2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*c^2*d^3*e^(-4) + 7293*
(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105
*(x*e + d)^(3/2)*d^3)*b^2*d^2*e^(-2) + 1326*(315*(x*e + d)^(11/2) - 1540*(x
*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 115
5*(x*e + d)^(3/2)*d^4)*b*c*d^2*e^(-3) + 255*(693*(x*e + d)^(13/2) - 4095*(x
*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 +
9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*c^2*d^2*e^(-4) + 663*(
315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 -
2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*b^2*d*e^(-2) + 510*(69
3*(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 -
12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)
*d^5)*b*c*d*e^(-3) + 51*(3003*(x*e + d)^(15/2) - 20790*(x*e + d)^(13/2)*d +
 61425*(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525*(x*e + d)^
(7/2)*d^4 - 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*c^2*d*e^
(-4) + 85*(693*(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)
^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x
*e + d)^(3/2)*d^5)*b^2*e^(-2) + 34*(3003*(x*e + d)^(15/2) - 20790*(x*e + d)
^(13/2)*d + 61425*(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525
```

```
*(x*e + d)^(7/2)*d^4 - 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^
6)*b*c*e^(-3) + 7*(6435*(x*e + d)^(17/2) - 51051*(x*e + d)^(15/2)*d + 17671
5*(x*e + d)^(13/2)*d^2 - 348075*(x*e + d)^(11/2)*d^3 + 425425*(x*e + d)^(9/
2)*d^4 - 328185*(x*e + d)^(7/2)*d^5 + 153153*(x*e + d)^(5/2)*d^6 - 36465*(x
*e + d)^(3/2)*d^7)*c^2*e^(-4))*e^(-1)
```

3.346 $\int (d+ex)^{5/2} \left(bx+cx^2\right)^2 dx$

Optimal. Leaf size=147

$$\frac{2(d+ex)^{11/2}\left(b^2e^2-6bcde+6c^2d^2\right)}{11e^5}+\frac{2d^2(d+ex)^{7/2}(cd-be)^2}{7e^5}-\frac{4c(d+ex)^{13/2}(2cd-be)}{13e^5}-\frac{4d(d+ex)^{9/2}(cd-be)(2cd-}{9e^5}$$

```
[Out] (2*d^2*(c*d - b*e)^2*(d + e*x)^(7/2))/(7*e^5) - (4*d*(c*d - b*e)*(2*c*d - b
*e)*(d + e*x)^(9/2))/(9*e^5) + (2*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(d + e*
x)^(11/2))/(11*e^5) - (4*c*(2*c*d - b*e)*(d + e*x)^(13/2))/(13*e^5) + (2*c^
2*(d + e*x)^(15/2))/(15*e^5)
```

Rubi [A] time = 0.0651057, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {698}

$$\frac{2(d+ex)^{11/2}\left(b^2e^2-6bcde+6c^2d^2\right)}{11e^5}+\frac{2d^2(d+ex)^{7/2}(cd-be)^2}{7e^5}-\frac{4c(d+ex)^{13/2}(2cd-be)}{13e^5}-\frac{4d(d+ex)^{9/2}(cd-be)(2cd-}{9e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)*(b*x + c*x^2)^2,x]
```

```
[Out] (2*d^2*(c*d - b*e)^2*(d + e*x)^(7/2))/(7*e^5) - (4*d*(c*d - b*e)*(2*c*d - b
*e)*(d + e*x)^(9/2))/(9*e^5) + (2*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(d + e*
x)^(11/2))/(11*e^5) - (4*c*(2*c*d - b*e)*(d + e*x)^(13/2))/(13*e^5) + (2*c^
2*(d + e*x)^(15/2))/(15*e^5)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int (d+ex)^{5/2}\left(bx+cx^2\right)^2 dx &= \int \left(\frac{d^2(cd-be)^2(d+ex)^{5/2}}{e^4}+\frac{2d(cd-be)(-2cd+be)(d+ex)^{7/2}}{e^4}+\frac{\left(6c^2d^2-6bcde+b^2e^2\right)(}{e^4}\right.\\ &= \frac{2d^2(cd-be)^2(d+ex)^{7/2}}{7e^5}-\frac{4d(cd-be)(2cd-be)(d+ex)^{9/2}}{9e^5}+\frac{2\left(6c^2d^2-6bcde+b^2e^2\right)(d-}{11e^5}\end{aligned}$$

Mathematica [A] time = 0.0776933, size = 124, normalized size = 0.84

$$\frac{2(d+ex)^{7/2}\left(65b^2e^2\left(8d^2-28dex+63e^2x^2\right)+30bce\left(56d^2ex-16d^3-126de^2x^2+231e^3x^3\right)+c^2\left(1008d^2e^2x^2-448d^3ex-\right.\right.}{45045e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)*(b*x + c*x^2)^2,x]
```

```
[Out] (2*(d + e*x)^(7/2)*(65*b^2*e^2*(8*d^2 - 28*d*e*x + 63*e^2*x^2) + 30*b*c*e*(
-16*d^3 + 56*d^2*e*x - 126*d*e^2*x^2 + 231*e^3*x^3) + c^2*(128*d^4 - 448*d^
3*e*x + 1008*d^2*e^2*x^2 - 1848*d*e^3*x^3 + 3003*e^4*x^4)))/(45045*e^5)
```

Maple [A] time = 0.049, size = 141, normalized size = 1.

$$\frac{6006\,c^2x^4e^4+13860\,bce^4x^3-3696\,c^2de^3x^3+8190\,b^2e^4x^2-7560\,bcde^3x^2+2016\,c^2d^2e^2x^2-3640\,b^2de^3x+3360\,bcd^2e}{45045\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(c*x^2+b*x)^2,x)
```

```
[Out] 2/45045*(e*x+d)^(7/2)*(3003*c^2*e^4*x^4+6930*b*c*e^4*x^3-1848*c^2*d*e^3*x^3
+4095*b^2*e^4*x^2-3780*b*c*d*e^3*x^2+1008*c^2*d^2*e^2*x^2-1820*b^2*d*e^3*x+
1680*b*c*d^2*e^2*x-448*c^2*d^3*e*x+520*b^2*d^2*e^2-480*b*c*d^3*e+128*c^2*d^
4)/e^5
```

Maxima [A] time = 1.13589, size = 188, normalized size = 1.28

$$\frac{2\left(3003\,(ex+d)^{\frac{15}{2}}c^2-6930\left(2\,c^2d-bce\right)(ex+d)^{\frac{13}{2}}+4095\left(6\,c^2d^2-6\,bcde+b^2e^2\right)(ex+d)^{\frac{11}{2}}-10010\left(2\,c^2d^3-3\,bcd^2\right.}{45045\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(c*x^2+b*x)^2,x, algorithm="maxima")
```

```
[Out] 2/45045*(3003*(e*x + d)^(15/2)*c^2 - 6930*(2*c^2*d - b*c*e)*(e*x + d)^(13/2
) + 4095*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(e*x + d)^(11/2) - 10010*(2*c^2*
d^3 - 3*b*c*d^2*e + b^2*d*e^2)*(e*x + d)^(9/2) + 6435*(c^2*d^4 - 2*b*c*d^3*
e + b^2*d^2*e^2)*(e*x + d)^(7/2))/e^5
```

Fricas [A] time = 1.90238, size = 564, normalized size = 3.84

$$\frac{2\left(3003\,c^2e^7x^7+128\,c^2d^7-480\,bcd^6e+520\,b^2d^5e^2+231\left(31\,c^2de^6+30\,bce^7\right)x^6+63\left(71\,c^2d^2e^5+270\,bcde^6+65\,b^2e^7\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(c*x^2+b*x)^2,x, algorithm="fricas")
```

```
[Out] 2/45045*(3003*c^2*e^7*x^7 + 128*c^2*d^7 - 480*b*c*d^6*e + 520*b^2*d^5*e^2 +
 231*(31*c^2*d*e^6 + 30*b*c*e^7)*x^6 + 63*(71*c^2*d^2*e^5 + 270*b*c*d*e^6 +
 65*b^2*e^7)*x^5 + 35*(c^2*d^3*e^4 + 318*b*c*d^2*e^5 + 299*b^2*d*e^6)*x^4 -
 5*(8*c^2*d^4*e^3 - 30*b*c*d^3*e^4 - 1469*b^2*d^2*e^5)*x^3 + 3*(16*c^2*d^5*
e^2 - 60*b*c*d^4*e^3 + 65*b^2*d^3*e^4)*x^2 - 4*(16*c^2*d^6*e - 60*b*c*d^5*e
^2 + 65*b^2*d^4*e^3)*x)*sqrt(e*x + d)/e^5
```

Sympy [B] time = 30.1276, size = 695, normalized size = 4.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(c*x**2+b*x)**2,x)
```

```
[Out] 2*b**2*d**2*(d**2*(d + e*x)**(3/2)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**
(7/2)/7)/e**3 + 4*b**2*d*(-d**3*(d + e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2
)/5 - 3*d*(d + e*x)**(7/2)/7 + (d + e*x)**(9/2)/9)/e**3 + 2*b**2*(d**4*(d +
 e*x)**(3/2)/3 - 4*d**3*(d + e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*
d*(d + e*x)**(9/2)/9 + (d + e*x)**(11/2)/11)/e**3 + 4*b*c*d**2*(-d**3*(d +
e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d + e*x)**(7/2)/7 + (d + e
*x)**(9/2)/9)/e**4 + 8*b*c*d*(d**4*(d + e*x)**(3/2)/3 - 4*d**3*(d + e*x)**(
5/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e*x)**(9/2)/9 + (d + e*x)**(1
1/2)/11)/e**4 + 4*b*c*(-d**5*(d + e*x)**(3/2)/3 + d**4*(d + e*x)**(5/2) - 1
0*d**3*(d + e*x)**(7/2)/7 + 10*d**2*(d + e*x)**(9/2)/9 - 5*d*(d + e*x)**(11
/2)/11 + (d + e*x)**(13/2)/13)/e**4 + 2*c**2*d**2*(d**4*(d + e*x)**(3/2)/3
- 4*d**3*(d + e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e*x)**(9
/2)/9 + (d + e*x)**(11/2)/11)/e**5 + 4*c**2*d*(-d**5*(d + e*x)**(3/2)/3 + d
**4*(d + e*x)**(5/2) - 10*d**3*(d + e*x)**(7/2)/7 + 10*d**2*(d + e*x)**(9/2
)/9 - 5*d*(d + e*x)**(11/2)/11 + (d + e*x)**(13/2)/13)/e**5 + 2*c**2*(d**6*
(d + e*x)**(3/2)/3 - 6*d**5*(d + e*x)**(5/2)/5 + 15*d**4*(d + e*x)**(7/2)/7
 - 20*d**3*(d + e*x)**(9/2)/9 + 15*d**2*(d + e*x)**(11/2)/11 - 6*d*(d + e*x
)**(13/2)/13 + (d + e*x)**(15/2)/15)/e**5
```

Giac [B] time = 1.35852, size = 844, normalized size = 5.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(c*x^2+b*x)^2,x, algorithm="giac")
```

```
[Out] 2/45045*(429*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2
)*d^2)*b^2*d^2*e^(-2) + 286*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 1
89*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*b*c*d^2*e^(-3) + 13*(315*
(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772
*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*c^2*d^2*e^(-4) + 286*(35*(
x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e
 + d)^(3/2)*d^3)*b^2*d*e^(-2) + 52*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(
9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e +
d)^(3/2)*d^4)*b*c*d*e^(-3) + 10*(693*(x*e + d)^(13/2) - 4095*(x*e + d)^(11/
2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e +
d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*c^2*d*e^(-4) + 13*(315*(x*e + d)^(
11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^
(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*b^2*e^(-2) + 10*(693*(x*e + d)^(13/2)
 - 4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12870*(x*e + d)^(7
/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*b*c*e^(-3) +
 (3003*(x*e + d)^(15/2) - 20790*(x*e + d)^(13/2)*d + 61425*(x*e + d)^(11/2)
*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525*(x*e + d)^(7/2)*d^4 - 54054*(x*e
+ d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*c^2*e^(-4))*e^(-1)
```

3.347 $\int (d+ex)^{3/2} \left(bx+cx^2\right)^2 \, dx$

Optimal. Leaf size=147

$$\frac{2(d+ex)^{9/2}\left(b^2e^2-6bcde+6c^2d^2\right)}{9e^5}+\frac{2d^2(d+ex)^{5/2}(cd-be)^2}{5e^5}-\frac{4c(d+ex)^{11/2}(2cd-be)}{11e^5}-\frac{4d(d+ex)^{7/2}(cd-be)(2cd}{7e^5}$$

```
[Out] (2*d^2*(c*d - b*e)^2*(d + e*x)^(5/2))/(5*e^5) - (4*d*(c*d - b*e)*(2*c*d - b
*e)*(d + e*x)^(7/2))/(7*e^5) + (2*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(d + e*
x)^(9/2))/(9*e^5) - (4*c*(2*c*d - b*e)*(d + e*x)^(11/2))/(11*e^5) + (2*c^2*
(d + e*x)^(13/2))/(13*e^5)
```

Rubi [A] time = 0.0606813, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {698}

$$\frac{2(d+ex)^{9/2}\left(b^2e^2-6bcde+6c^2d^2\right)}{9e^5}+\frac{2d^2(d+ex)^{5/2}(cd-be)^2}{5e^5}-\frac{4c(d+ex)^{11/2}(2cd-be)}{11e^5}-\frac{4d(d+ex)^{7/2}(cd-be)(2cd}{7e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*(b*x + c*x^2)^2,x]
```

```
[Out] (2*d^2*(c*d - b*e)^2*(d + e*x)^(5/2))/(5*e^5) - (4*d*(c*d - b*e)*(2*c*d - b
*e)*(d + e*x)^(7/2))/(7*e^5) + (2*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(d + e*
x)^(9/2))/(9*e^5) - (4*c*(2*c*d - b*e)*(d + e*x)^(11/2))/(11*e^5) + (2*c^2*
(d + e*x)^(13/2))/(13*e^5)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^{3/2}\left(bx+cx^2\right)^2\,dx=\int \left(\frac{d^2(cd-be)^2(d+ex)^{3/2}}{e^4}+\frac{2d(cd-be)(-2cd+be)(d+ex)^{5/2}}{e^4}+\frac{\left(6c^2d^2-6bcde+b^2e\right.}{e^4}\right.$$

$$=\frac{2d^2(cd-be)^2(d+ex)^{5/2}}{5e^5}-\frac{4d(cd-be)(2cd-be)(d+ex)^{7/2}}{7e^5}+\frac{2\left(6c^2d^2-6bcde+b^2e^2\right)}{9e^5}$$

Mathematica [A] time = 0.0775826, size = 125, normalized size = 0.85

$$\frac{2(d+ex)^{5/2}\left(143b^2e^2\left(8d^2-20dex+35e^2x^2\right)+78bce\left(40d^2ex-16d^3-70de^2x^2+105e^3x^3\right)+3c^2\left(560d^2e^2x^2-320d^3\right.\right.}{45045e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)*(b*x + c*x^2)^2,x]
```

```
[Out] (2*(d + e*x)^(5/2)*(143*b^2*e^2*(8*d^2 - 20*d*e*x + 35*e^2*x^2) + 78*b*c*e*
(-16*d^3 + 40*d^2*e*x - 70*d*e^2*x^2 + 105*e^3*x^3) + 3*c^2*(128*d^4 - 320*
d^3*e*x + 560*d^2*e^2*x^2 - 840*d*e^3*x^3 + 1155*e^4*x^4)))/(45045*e^5)
```

Maple [A] time = 0.05, size = 141, normalized size = 1.

$$\frac{6930\,c^2x^4e^4+16380\,bce^4x^3-5040\,c^2de^3x^3+10010\,b^2e^4x^2-10920\,bcde^3x^2+3360\,c^2d^2e^2x^2-5720\,b^2de^3x+6240\,bcd^2e^2}{45045\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(c*x^2+b*x)^2,x)
```

```
[Out] 2/45045*(e*x+d)^(5/2)*(3465*c^2*e^4*x^4+8190*b*c*e^4*x^3-2520*c^2*d*e^3*x^3
+5005*b^2*e^4*x^2-5460*b*c*d*e^3*x^2+1680*c^2*d^2*e^2*x^2-2860*b^2*d*e^3*x+
3120*b*c*d^2*e^2*x-960*c^2*d^3*e*x+1144*b^2*d^2*e^2-1248*b*c*d^3*e+384*c^2*
d^4)/e^5
```

Maxima [A] time = 1.218, size = 188, normalized size = 1.28

$$\frac{2\left(3465\,(ex+d)^{\frac{13}{2}}c^2-8190\left(2\,c^2d-bce\right)(ex+d)^{\frac{11}{2}}+5005\left(6\,c^2d^2-6\,bcde+b^2e^2\right)(ex+d)^{\frac{9}{2}}-12870\left(2\,c^2d^3-3\,bcd^2e+b\right.}{45045\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x)^2,x, algorithm="maxima")
```

```
[Out] 2/45045*(3465*(e*x + d)^(13/2)*c^2 - 8190*(2*c^2*d - b*c*e)*(e*x + d)^(11/2
) + 5005*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(e*x + d)^(9/2) - 12870*(2*c^2*d
^3 - 3*b*c*d^2*e + b^2*d*e^2)*(e*x + d)^(7/2) + 9009*(c^2*d^4 - 2*b*c*d^3*e
 + b^2*d^2*e^2)*(e*x + d)^(5/2))/e^5
```

Fricas [A] time = 1.89437, size = 493, normalized size = 3.35

$$2\left(3465\,c^2e^6x^6+384\,c^2d^6-1248\,bcd^5e+1144\,b^2d^4e^2+630\left(7\,c^2de^5+13\,bce^6\right)x^5+35\left(3\,c^2d^2e^4+312\,bcde^5+143\,b^2e^6\right)x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x)^2,x, algorithm="fricas")
```

```
[Out] 2/45045*(3465*c^2*e^6*x^6 + 384*c^2*d^6 - 1248*b*c*d^5*e + 1144*b^2*d^4*e^2
 + 630*(7*c^2*d*e^5 + 13*b*c*e^6)*x^5 + 35*(3*c^2*d^2*e^4 + 312*b*c*d*e^5 +
 143*b^2*e^6)*x^4 - 10*(12*c^2*d^3*e^3 - 39*b*c*d^2*e^4 - 715*b^2*d*e^5)*x^
3 + 3*(48*c^2*d^4*e^2 - 156*b*c*d^3*e^3 + 143*b^2*d^2*e^4)*x^2 - 4*(48*c^2*
d^5*e - 156*b*c*d^4*e^2 + 143*b^2*d^3*e^3)*x)*sqrt(e*x + d)/e^5
```

Sympy [B] time = 18.7187, size = 413, normalized size = 2.81

$$\frac{2b^2d\left(\frac{d^2(d+ex)^{\frac{3}{2}}}{3}-\frac{2d(d+ex)^{\frac{5}{2}}}{5}+\frac{(d+ex)^{\frac{7}{2}}}{7}\right)}{e^3}+\frac{2b^2\left(-\frac{d^3(d+ex)^{\frac{3}{2}}}{3}+\frac{3d^2(d+ex)^{\frac{5}{2}}}{5}-\frac{3d(d+ex)^{\frac{7}{2}}}{7}+\frac{(d+ex)^{\frac{9}{2}}}{9}\right)}{e^3}+\frac{4bcd\left(-\frac{d^3(d+ex)^{\frac{3}{2}}}{3}+\frac{3d^2(d+ex)^{\frac{5}{2}}}{5}\right.}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(c*x**2+b*x)**2,x)
```

```
[Out] 2*b**2*d*(d**2*(d + e*x)**(3/2)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/
2)/7)/e**3 + 2*b**2*(-d**3*(d + e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 -
 3*d*(d + e*x)**(7/2)/7 + (d + e*x)**(9/2)/9)/e**3 + 4*b*c*d*(-d**3*(d + e*
x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d + e*x)**(7/2)/7 + (d + e*x
)**(9/2)/9)/e**4 + 4*b*c*(d**4*(d + e*x)**(3/2)/3 - 4*d**3*(d + e*x)**(5/2)
/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e*x)**(9/2)/9 + (d + e*x)**(11/2)
/11)/e**4 + 2*c**2*d*(d**4*(d + e*x)**(3/2)/3 - 4*d**3*(d + e*x)**(5/2)/5 +
 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e*x)**(9/2)/9 + (d + e*x)**(11/2)/11)
/e**5 + 2*c**2*(-d**5*(d + e*x)**(3/2)/3 + d**4*(d + e*x)**(5/2) - 10*d**3*
(d + e*x)**(7/2)/7 + 10*d**2*(d + e*x)**(9/2)/9 - 5*d*(d + e*x)**(11/2)/11
+ (d + e*x)**(13/2)/13)/e**5
```

Giac [B] time = 1.42503, size = 506, normalized size = 3.44

$$\frac{2}{45045}\left(429\left(15\,(xe+d)^{\frac{7}{2}}-42\,(xe+d)^{\frac{5}{2}}d+35\,(xe+d)^{\frac{3}{2}}d^2\right)b^2de^{(-2)}+286\left(35\,(xe+d)^{\frac{9}{2}}-135\,(xe+d)^{\frac{7}{2}}d+189\,(xe+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x)^2,x, algorithm="giac")
```

```
[Out] 2/45045*(429*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2
)*d^2)*b^2*d*e^(-2) + 286*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189
*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*b*c*d*e^(-3) + 13*(315*(x*e
 + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*
e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*c^2*d*e^(-4) + 143*(35*(x*e +
d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^
(3/2)*d^3)*b^2*e^(-2) + 26*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d +
 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)
*d^4)*b*c*e^(-3) + 5*(693*(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 1001
0*(x*e + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^
4 - 3003*(x*e + d)^(3/2)*d^5)*c^2*e^(-4))*e^(-1)
```

3.348 $\int \sqrt{d+ex}\left(bx+cx^2\right)^2\,dx$

Optimal. Leaf size=147

$$\frac{2(d+ex)^{7/2}\left(b^2e^2-6bcde+6c^2d^2\right)}{7e^5}+\frac{2d^2(d+ex)^{3/2}(cd-be)^2}{3e^5}-\frac{4c(d+ex)^{9/2}(2cd-be)}{9e^5}-\frac{4d(d+ex)^{5/2}(cd-be)(2cd-be}{5e^5}$$

```
[Out] (2*d^2*(c*d - b*e)^2*(d + e*x)^(3/2))/(3*e^5) - (4*d*(c*d - b*e)*(2*c*d - b
*e)*(d + e*x)^(5/2))/(5*e^5) + (2*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(d + e*
x)^(7/2))/(7*e^5) - (4*c*(2*c*d - b*e)*(d + e*x)^(9/2))/(9*e^5) + (2*c^2*(d
 + e*x)^(11/2))/(11*e^5)
```

Rubi [A] time = 0.0586716, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {698}

$$\frac{2(d+ex)^{7/2}\left(b^2e^2-6bcde+6c^2d^2\right)}{7e^5}+\frac{2d^2(d+ex)^{3/2}(cd-be)^2}{3e^5}-\frac{4c(d+ex)^{9/2}(2cd-be)}{9e^5}-\frac{4d(d+ex)^{5/2}(cd-be)(2cd-be}{5e^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(b*x + c*x^2)^2,x]
```

```
[Out] (2*d^2*(c*d - b*e)^2*(d + e*x)^(3/2))/(3*e^5) - (4*d*(c*d - b*e)*(2*c*d - b
*e)*(d + e*x)^(5/2))/(5*e^5) + (2*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(d + e*
x)^(7/2))/(7*e^5) - (4*c*(2*c*d - b*e)*(d + e*x)^(9/2))/(9*e^5) + (2*c^2*(d
 + e*x)^(11/2))/(11*e^5)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^2\,dx=\int\left(\frac{d^2(cd-be)^2\sqrt{d+ex}}{e^4}+\frac{2d(cd-be)(-2cd+be)(d+ex)^{3/2}}{e^4}+\frac{\left(6c^2d^2-6bcde+b^2e^2\right)(d+}{e^4}\right.$$

$$=\frac{2d^2(cd-be)^2(d+ex)^{3/2}}{3e^5}-\frac{4d(cd-be)(2cd-be)(d+ex)^{5/2}}{5e^5}+\frac{2\left(6c^2d^2-6bcde+b^2e^2\right)(d+}{7e^5}$$

Mathematica [A] time = 0.0716285, size = 124, normalized size = 0.84

$$\frac{2(d+ex)^{3/2}\left(33b^2e^2\left(8d^2-12dex+15e^2x^2\right)+22bce\left(24d^2ex-16d^3-30de^2x^2+35e^3x^3\right)+c^2\left(240d^2e^2x^2-192d^3ex+12\right.\right.}{3465e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*(b*x + c*x^2)^2,x]
```

```
[Out] (2*(d + e*x)^(3/2)*(33*b^2*e^2*(8*d^2 - 12*d*e*x + 15*e^2*x^2) + 22*b*c*e*(
-16*d^3 + 24*d^2*e*x - 30*d*e^2*x^2 + 35*e^3*x^3) + c^2*(128*d^4 - 192*d^3*
e*x + 240*d^2*e^2*x^2 - 280*d*e^3*x^3 + 315*e^4*x^4)))/(3465*e^5)
```

Maple [A] time = 0.05, size = 141, normalized size = 1.

$$\frac{630\,c^2x^4e^4+1540\,bce^4x^3-560\,c^2de^3x^3+990\,b^2e^4x^2-1320\,bcde^3x^2+480\,c^2d^2e^2x^2-792\,b^2de^3x+1056\,bcd^2e^2x-38}{3465\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(c*x^2+b*x)^2,x)
```

```
[Out] 2/3465*(e*x+d)^(3/2)*(315*c^2*e^4*x^4+770*b*c*e^4*x^3-280*c^2*d*e^3*x^3+495
*b^2*e^4*x^2-660*b*c*d*e^3*x^2+240*c^2*d^2*e^2*x^2-396*b^2*d*e^3*x+528*b*c*
d^2*e^2*x-192*c^2*d^3*e*x+264*b^2*d^2*e^2-352*b*c*d^3*e+128*c^2*d^4)/e^5
```

Maxima [A] time = 1.11844, size = 188, normalized size = 1.28

$$\frac{2\left(315\,(ex+d)^{\frac{11}{2}}c^2-770\left(2\,c^2d-bce\right)(ex+d)^{\frac{9}{2}}+495\left(6\,c^2d^2-6\,bcde+b^2e^2\right)(ex+d)^{\frac{7}{2}}-1386\left(2\,c^2d^3-3\,bcd^2e+b^2c\right.}{3465\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x)^2,x, algorithm="maxima")
```

```
[Out] 2/3465*(315*(e*x + d)^(11/2)*c^2 - 770*(2*c^2*d - b*c*e)*(e*x + d)^(9/2) +
495*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(e*x + d)^(7/2) - 1386*(2*c^2*d^3 - 3
*b*c*d^2*e + b^2*d*e^2)*(e*x + d)^(5/2) + 1155*(c^2*d^4 - 2*b*c*d^3*e + b^2
*d^2*e^2)*(e*x + d)^(3/2))/e^5
```

Fricas [A] time = 1.9181, size = 392, normalized size = 2.67

$$\frac{2\left(315\,c^2e^5x^5+128\,c^2d^5-352\,bcd^4e+264\,b^2d^3e^2+35\left(c^2de^4+22\,bce^5\right)x^4-5\left(8\,c^2d^2e^3-22\,bcde^4-99\,b^2e^5\right)x^3+3\,(\right.}{3465\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x)^2,x, algorithm="fricas")
```

```
[Out] 2/3465*(315*c^2*e^5*x^5 + 128*c^2*d^5 - 352*b*c*d^4*e + 264*b^2*d^3*e^2 + 3
5*(c^2*d*e^4 + 22*b*c*e^5)*x^4 - 5*(8*c^2*d^2*e^3 - 22*b*c*d*e^4 - 99*b^2*e
^5)*x^3 + 3*(16*c^2*d^3*e^2 - 44*b*c*d^2*e^3 + 33*b^2*d*e^4)*x^2 - 4*(16*c^
2*d^4*e - 44*b*c*d^3*e^2 + 33*b^2*d^2*e^3)*x)*sqrt(e*x + d)/e^5
```

Sympy [A] time = 4.65131, size = 173, normalized size = 1.18

$$\frac{2\left(\frac{c^2(d+ex)^{\frac{11}{2}}}{11e^4}+\frac{(d+ex)^{\frac{9}{2}}\left(2bce-4c^2d\right)}{9e^4}+\frac{(d+ex)^{\frac{7}{2}}\left(b^2e^2-6bcde+6c^2d^2\right)}{7e^4}+\frac{(d+ex)^{\frac{5}{2}}\left(-2b^2de^2+6bcd^2e-4c^2d^3\right)}{5e^4}+\frac{(d+ex)^{\frac{3}{2}}\left(b^2d^2e^2-2bcd^3e+c^2d^4\right)}{3e^4}\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(c*x**2+b*x)**2,x)
```

```
[Out] 2*(c**2*(d + e*x)**(11/2)/(11*e**4) + (d + e*x)**(9/2)*(2*b*c*e - 4*c**2*d)
/(9*e**4) + (d + e*x)**(7/2)*(b**2*e**2 - 6*b*c*d*e + 6*c**2*d**2)/(7*e**4)
 + (d + e*x)**(5/2)*(-2*b**2*d*e**2 + 6*b*c*d**2*e - 4*c**2*d**3)/(5*e**4)
+ (d + e*x)**(3/2)*(b**2*d**2*e**2 - 2*b*c*d**3*e + c**2*d**4)/(3*e**4))/e
```

Giac [A] time = 1.28181, size = 227, normalized size = 1.54

$$\frac{2}{3465}\left(33\left(15\,(xe+d)^{\frac{7}{2}}-42\,(xe+d)^{\frac{5}{2}}d+35\,(xe+d)^{\frac{3}{2}}d^2\right)b^2e^{(-2)}+22\left(35\,(xe+d)^{\frac{9}{2}}-135\,(xe+d)^{\frac{7}{2}}d+189\,(xe+d)^{\frac{5}{2}}d^2-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x)^2,x, algorithm="giac")
```

```
[Out] 2/3465*(33*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*
d^2)*b^2*e^(-2) + 22*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e
 + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*b*c*e^(-3) + (315*(x*e + d)^(11/
2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/
2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*c^2*e^(-4))*e^(-1)
```

3.349 $\int \frac{(bx+cx^2)^2}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=145

$$\frac{2(d+ex)^{5/2}\left(b^2e^2-6bcde+6c^2d^2\right)}{5e^5}+\frac{2d^2\sqrt{d+ex}(cd-be)^2}{e^5}-\frac{4c(d+ex)^{7/2}(2cd-be)}{7e^5}-\frac{4d(d+ex)^{3/2}(cd-be)(2cd-b}{3e^5}$$

```
[Out] (2*d^2*(c*d - b*e)^2*Sqrt[d + e*x])/e^5 - (4*d*(c*d - b*e)*(2*c*d - b*e)*(d
 + e*x)^(3/2))/(3*e^5) + (2*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(d + e*x)^(5/
2))/(5*e^5) - (4*c*(2*c*d - b*e)*(d + e*x)^(7/2))/(7*e^5) + (2*c^2*(d + e*x
)^(9/2))/(9*e^5)
```

Rubi [A] time = 0.0589171, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {698}

$$\frac{2(d+ex)^{5/2}\left(b^2e^2-6bcde+6c^2d^2\right)}{5e^5}+\frac{2d^2\sqrt{d+ex}(cd-be)^2}{e^5}-\frac{4c(d+ex)^{7/2}(2cd-be)}{7e^5}-\frac{4d(d+ex)^{3/2}(cd-be)(2cd-b}{3e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^2/Sqrt[d + e*x],x]
```

```
[Out] (2*d^2*(c*d - b*e)^2*Sqrt[d + e*x])/e^5 - (4*d*(c*d - b*e)*(2*c*d - b*e)*(d
 + e*x)^(3/2))/(3*e^5) + (2*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(d + e*x)^(5/
2))/(5*e^5) - (4*c*(2*c*d - b*e)*(d + e*x)^(7/2))/(7*e^5) + (2*c^2*(d + e*x
)^(9/2))/(9*e^5)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(bx+cx^2)^2}{\sqrt{d+ex}}\,dx=\int\left(\frac{d^2(cd-be)^2}{e^4\sqrt{d+ex}}+\frac{2d(cd-be)(-2cd+be)\sqrt{d+ex}}{e^4}+\frac{\left(6c^2d^2-6bcde+b^2e^2\right)(d+ex)^{3/2}}{e^4}-\frac{2c(2cd$$
$$=\frac{2d^2(cd-be)^2\sqrt{d+ex}}{e^5}-\frac{4d(cd-be)(2cd-be)(d+ex)^{3/2}}{3e^5}+\frac{2\left(6c^2d^2-6bcde+b^2e^2\right)(d+ex)^{5/2}}{5e^5}-$$

Mathematica [A] time = 0.0760272, size = 124, normalized size = 0.86

$$\frac{2\sqrt{d+ex}\left(21b^2e^2\left(8d^2-4dex+3e^2x^2\right)+18bce\left(8d^2ex-16d^3-6de^2x^2+5e^3x^3\right)+c^2\left(48d^2e^2x^2-64d^3ex+128d^4-4\right.\right.}{315e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^2/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[d + e*x]*(21*b^2*e^2*(8*d^2 - 4*d*e*x + 3*e^2*x^2) + 18*b*c*e*(-16*
d^3 + 8*d^2*e*x - 6*d*e^2*x^2 + 5*e^3*x^3) + c^2*(128*d^4 - 64*d^3*e*x + 48
*d^2*e^2*x^2 - 40*d*e^3*x^3 + 35*e^4*x^4)))/(315*e^5)
```

Maple [A] time = 0.05, size = 141, normalized size = 1.

$$\frac{70\,c^2x^4e^4+180\,bce^4x^3-80\,c^2de^3x^3+126\,b^2e^4x^2-216\,bcde^3x^2+96\,c^2d^2e^2x^2-168\,b^2de^3x+288\,bcd^2e^2x-128\,c^2d^3ex+}{315\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^2/(e*x+d)^(1/2),x)
```

```
[Out] 2/315*(35*c^2*e^4*x^4+90*b*c*e^4*x^3-40*c^2*d*e^3*x^3+63*b^2*e^4*x^2-108*b*
c*d*e^3*x^2+48*c^2*d^2*e^2*x^2-84*b^2*d*e^3*x+144*b*c*d^2*e^2*x-64*c^2*d^3*
e*x+168*b^2*d^2*e^2-288*b*c*d^3*e+128*c^2*d^4)*(e*x+d)^(1/2)/e^5
```

Maxima [A] time = 1.17526, size = 216, normalized size = 1.49

$$\frac{2\left(\frac{21\left(3\,(ex+d)^{\frac{5}{2}}-10\,(ex+d)^{\frac{3}{2}}d+15\,\sqrt{ex+d}d^2\right)b^2}{e^2}+\frac{18\left(5\,(ex+d)^{\frac{7}{2}}-21\,(ex+d)^{\frac{5}{2}}d+35\,(ex+d)^{\frac{3}{2}}d^2-35\,\sqrt{ex+d}d^3\right)bc}{e^3}+\frac{\left(35\,(ex+d)^{\frac{9}{2}}-180\,(ex+d)^{\frac{7}{2}}d+378\,(ex+d)^{\frac{5}{2}}d^2\right.}{e^4}\right.}{315\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(21*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)
*b^2/e^2 + 18*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2
)*d^2 - 35*sqrt(e*x + d)*d^3)*b*c/e^3 + (35*(e*x + d)^(9/2) - 180*(e*x + d)
^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x
 + d)*d^4)*c^2/e^4)/e
```

Fricas [A] time = 1.91698, size = 312, normalized size = 2.15

$$\frac{2\left(35\,c^2e^4x^4+128\,c^2d^4-288\,bcd^3e+168\,b^2d^2e^2-10\left(4\,c^2de^3-9\,bce^4\right)x^3+3\left(16\,c^2d^2e^2-36\,bcde^3+21\,b^2e^4\right)x^2-4\left(16\,c\right.\right.}{315\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*c^2*e^4*x^4 + 128*c^2*d^4 - 288*b*c*d^3*e + 168*b^2*d^2*e^2 - 10*
(4*c^2*d*e^3 - 9*b*c*e^4)*x^3 + 3*(16*c^2*d^2*e^2 - 36*b*c*d*e^3 + 21*b^2*e
^4)*x^2 - 4*(16*c^2*d^3*e - 36*b*c*d^2*e^2 + 21*b^2*d*e^3)*x)*sqrt(e*x + d)
/e^5
```

Sympy [A] time = 62.1308, size = 418, normalized size = 2.88

$$\begin{cases} -\frac{\frac{2b^2d\left(\frac{d^2}{\sqrt{d+ex}}+2d\sqrt{d+ex}-\frac{(d+ex)^{\frac{3}{2}}}{3}\right)}{e^2}+\frac{2b^2\left(-\frac{d^3}{\sqrt{d+ex}}-3d^2\sqrt{d+ex}+d(d+ex)^{\frac{3}{2}}-\frac{(d+ex)^{\frac{5}{2}}}{5}\right)}{e^2}+\frac{4bcd\left(-\frac{d^3}{\sqrt{d+ex}}-3d^2\sqrt{d+ex}+d(d+ex)^{\frac{3}{2}}-\frac{(d+ex)^{\frac{5}{2}}}{5}\right)}{e^3}+\frac{4bc\left(\frac{d^4}{\sqrt{d+ex}}+4d^3\sqrt{d+ex}-2d^2(d+ex)^{\frac{3}{2}}+\right.}{e^3}}{e} \\ \frac{\frac{b^2x^3}{3}+\frac{bcx^4}{2}+\frac{c^2x^5}{5}}{\sqrt{d}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**2/(e*x+d)**(1/2),x)
```

```
[Out] Piecewise((-(2*b**2*d*(d**2/sqrt(d + e*x) + 2*d*sqrt(d + e*x) - (d + e*x)**
(3/2)/3)/e**2 + 2*b**2*(-d**3/sqrt(d + e*x) - 3*d**2*sqrt(d + e*x) + d*(d +
 e*x)**(3/2) - (d + e*x)**(5/2)/5)/e**2 + 4*b*c*d*(-d**3/sqrt(d + e*x) - 3*
d**2*sqrt(d + e*x) + d*(d + e*x)**(3/2) - (d + e*x)**(5/2)/5)/e**3 + 4*b*c*
(d**4/sqrt(d + e*x) + 4*d**3*sqrt(d + e*x) - 2*d**2*(d + e*x)**(3/2) + 4*d*
(d + e*x)**(5/2)/5 - (d + e*x)**(7/2)/7)/e**3 + 2*c**2*d*(d**4/sqrt(d + e*x
) + 4*d**3*sqrt(d + e*x) - 2*d**2*(d + e*x)**(3/2) + 4*d*(d + e*x)**(5/2)/5
 - (d + e*x)**(7/2)/7)/e**4 + 2*c**2*(-d**5/sqrt(d + e*x) - 5*d**4*sqrt(d +
 e*x) + 10*d**3*(d + e*x)**(3/2)/3 - 2*d**2*(d + e*x)**(5/2) + 5*d*(d + e*x
)**(7/2)/7 - (d + e*x)**(9/2)/9)/e**4)/e, Ne(e, 0)), ((b**2*x**3/3 + b*c*x*
*4/2 + c**2*x**5/5)/sqrt(d), True))
```

Giac [A] time = 1.27692, size = 227, normalized size = 1.57

$$\frac{2}{315}\left(21\left(3\,(xe+d)^{\frac{5}{2}}-10\,(xe+d)^{\frac{3}{2}}d+15\,\sqrt{xe+d}d^2\right)b^2e^{(-2)}+18\left(5\,(xe+d)^{\frac{7}{2}}-21\,(xe+d)^{\frac{5}{2}}d+35\,(xe+d)^{\frac{3}{2}}d^2-35\,\sqrt{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(21*(3*(x*e + d)^(5/2) - 10*(x*e + d)^(3/2)*d + 15*sqrt(x*e + d)*d^2)
*b^2*e^(-2) + 18*(5*(x*e + d)^(7/2) - 21*(x*e + d)^(5/2)*d + 35*(x*e + d)^(
3/2)*d^2 - 35*sqrt(x*e + d)*d^3)*b*c*e^(-3) + (35*(x*e + d)^(9/2) - 180*(x*
e + d)^(7/2)*d + 378*(x*e + d)^(5/2)*d^2 - 420*(x*e + d)^(3/2)*d^3 + 315*sq
rt(x*e + d)*d^4)*c^2*e^(-4))*e^(-1)
```

3.350 $\int \frac{(bx+cx^2)^2}{(d+ex)^{3/2}} \, dx$

Optimal. Leaf size=143

$$\frac{2(d+ex)^{3/2}\left(b^2e^2-6bcde+6c^2d^2\right)}{3e^5}-\frac{2d^2(cd-be)^2}{e^5\sqrt{d+ex}}-\frac{4c(d+ex)^{5/2}(2cd-be)}{5e^5}-\frac{4d\sqrt{d+ex}(cd-be)(2cd-be)}{e^5}+\frac{2c^2(d+ex)^{7/2}}{7e^5}$$

```
[Out] (-2*d^2*(c*d - b*e)^2)/(e^5*Sqrt[d + e*x]) - (4*d*(c*d - b*e)*(2*c*d - b*e)
*Sqrt[d + e*x])/e^5 + (2*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(d + e*x)^(3/2))
/(3*e^5) - (4*c*(2*c*d - b*e)*(d + e*x)^(5/2))/(5*e^5) + (2*c^2*(d + e*x)^(
7/2))/(7*e^5)
```

Rubi [A] time = 0.0582186, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {698}

$$\frac{2(d+ex)^{3/2}\left(b^2e^2-6bcde+6c^2d^2\right)}{3e^5}-\frac{2d^2(cd-be)^2}{e^5\sqrt{d+ex}}-\frac{4c(d+ex)^{5/2}(2cd-be)}{5e^5}-\frac{4d\sqrt{d+ex}(cd-be)(2cd-be)}{e^5}+\frac{2c^2(d+ex)^{7/2}}{7e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^2/(d + e*x)^(3/2),x]
```

```
[Out] (-2*d^2*(c*d - b*e)^2)/(e^5*Sqrt[d + e*x]) - (4*d*(c*d - b*e)*(2*c*d - b*e)
*Sqrt[d + e*x])/e^5 + (2*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(d + e*x)^(3/2))
/(3*e^5) - (4*c*(2*c*d - b*e)*(d + e*x)^(5/2))/(5*e^5) + (2*c^2*(d + e*x)^(
7/2))/(7*e^5)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(bx+cx^2)^2}{(d+ex)^{3/2}}\,dx = \int \left(\frac{d^2(cd-be)^2}{e^4(d+ex)^{3/2}}+\frac{2d(cd-be)(-2cd+be)}{e^4\sqrt{d+ex}}+\frac{\left(6c^2d^2-6bcde+b^2e^2\right)\sqrt{d+ex}}{e^4}-\frac{2c(2cd-be)(d+ex)^{3/2}}{e^4}+\dots\right)$$

$$= -\frac{2d^2(cd-be)^2}{e^5\sqrt{d+ex}}-\frac{4d(cd-be)(2cd-be)\sqrt{d+ex}}{e^5}+\frac{2\left(6c^2d^2-6bcde+b^2e^2\right)(d+ex)^{3/2}}{3e^5}-\frac{4c(2cd-be)\dots}{5\dots}$$

Mathematica [A] time = 0.0710242, size = 123, normalized size = 0.86

$$\frac{70b^2e^2\left(-8d^2-4dex+e^2x^2\right)+84bce\left(8d^2ex+16d^3-2de^2x^2+e^3x^3\right)-6c^2\left(-16d^2e^2x^2+64d^3ex+128d^4+8de^3x^3-5e^4x^4\right)}{105e^5\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^2/(d + e*x)^(3/2),x]
```

```
[Out] (70*b^2*e^2*(-8*d^2 - 4*d*e*x + e^2*x^2) + 84*b*c*e*(16*d^3 + 8*d^2*e*x - 2
*d*e^2*x^2 + e^3*x^3) - 6*c^2*(128*d^4 + 64*d^3*e*x - 16*d^2*e^2*x^2 + 8*d*
e^3*x^3 - 5*e^4*x^4))/(105*e^5*Sqrt[d + e*x])
```

Maple [A] time = 0.049, size = 141, normalized size = 1.

$$-\frac{-30\,c^{2}x^{4}e^{4}-84\,bce^{4}x^{3}+48\,c^{2}de^{3}x^{3}-70\,b^{2}e^{4}x^{2}+168\,bcde^{3}x^{2}-96\,c^{2}d^{2}e^{2}x^{2}+280\,b^{2}de^{3}x-672\,bcd^{2}e^{2}x+384\,c^{2}d^{3}e}{105\,e^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^2/(e*x+d)^(3/2),x)
```

```
[Out] -2/105*(-15*c^2*e^4*x^4-42*b*c*e^4*x^3+24*c^2*d*e^3*x^3-35*b^2*e^4*x^2+84*b
*c*d*e^3*x^2-48*c^2*d^2*e^2*x^2+140*b^2*d*e^3*x-336*b*c*d^2*e^2*x+192*c^2*d
^3*e*x+280*b^2*d^2*e^2-672*b*c*d^3*e+384*c^2*d^4)/(e*x+d)^(1/2)/e^5
```

Maxima [A] time = 1.19355, size = 198, normalized size = 1.38

$$\frac{2\left(\frac{15\,(ex+d)^{\frac{7}{2}}c^{2}-42\left(2\,c^{2}d-bce\right)(ex+d)^{\frac{5}{2}}+35\left(6\,c^{2}d^{2}-6\,bcde+b^{2}e^{2}\right)(ex+d)^{\frac{3}{2}}-210\left(2\,c^{2}d^{3}-3\,bcd^{2}e+b^{2}de^{2}\right)\sqrt{ex+d}}{e^{4}}-\frac{105\left(c^{2}d^{4}-2\,bcd^{3}e+b^{2}d^{2}e^{2}\right)}{\sqrt{ex+d}e^{4}}\right)}{105\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/105*((15*(e*x + d)^(7/2)*c^2 - 42*(2*c^2*d - b*c*e)*(e*x + d)^(5/2) + 35*
(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(e*x + d)^(3/2) - 210*(2*c^2*d^3 - 3*b*c*
d^2*e + b^2*d*e^2)*sqrt(e*x + d))/e^4 - 105*(c^2*d^4 - 2*b*c*d^3*e + b^2*d^
2*e^2)/(sqrt(e*x + d)*e^4))/e
```

Fricas [A] time = 1.81059, size = 324, normalized size = 2.27

$$\frac{2\left(15\,c^{2}e^{4}x^{4}-384\,c^{2}d^{4}+672\,bcd^{3}e-280\,b^{2}d^{2}e^{2}-6\left(4\,c^{2}de^{3}-7\,bce^{4}\right)x^{3}+\left(48\,c^{2}d^{2}e^{2}-84\,bcde^{3}+35\,b^{2}e^{4}\right)x^{2}-4\left(48\right.\right.}{105\left(e^{6}x+de^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*c^2*e^4*x^4 - 384*c^2*d^4 + 672*b*c*d^3*e - 280*b^2*d^2*e^2 - 6*(
4*c^2*d*e^3 - 7*b*c*e^4)*x^3 + (48*c^2*d^2*e^2 - 84*b*c*d*e^3 + 35*b^2*e^4)
*x^2 - 4*(48*c^2*d^3*e - 84*b*c*d^2*e^2 + 35*b^2*d*e^3)*x)*sqrt(e*x + d)/(e
^6*x + d*e^5)
```

Sympy [A] time = 24.154, size = 150, normalized size = 1.05

$$\frac{2c^2(d+ex)^{\frac{7}{2}}}{7e^5} - \frac{2d^2(be-cd)^2}{e^5\sqrt{d+ex}} + \frac{(d+ex)^{\frac{5}{2}}\left(4bce-8c^2d\right)}{5e^5} + \frac{(d+ex)^{\frac{3}{2}}\left(2b^2e^2-12bcde+12c^2d^2\right)}{3e^5} + \frac{\sqrt{d+ex}\left(-4b^2de^2+12\right.}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**2/(e*x+d)**(3/2),x)
```

```
[Out] 2*c**2*(d + e*x)**(7/2)/(7*e**5) - 2*d**2*(b*e - c*d)**2/(e**5*sqrt(d + e*x
)) + (d + e*x)**(5/2)*(4*b*c*e - 8*c**2*d)/(5*e**5) + (d + e*x)**(3/2)*(2*b
**2*e**2 - 12*b*c*d*e + 12*c**2*d**2)/(3*e**5) + sqrt(d + e*x)*(-4*b**2*d*e
**2 + 12*b*c*d**2*e - 8*c**2*d**3)/e**5
```

Giac [A] time = 1.28524, size = 254, normalized size = 1.78

$$\frac{2}{105}\left(15(xe+d)^{\frac{7}{2}}c^2e^{30} - 84(xe+d)^{\frac{5}{2}}c^2de^{30} + 210(xe+d)^{\frac{3}{2}}c^2d^2e^{30} - 420\sqrt{xe+d}c^2d^3e^{30} + 42(xe+d)^{\frac{5}{2}}bce^{31} - 210(xe+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] 2/105*(15*(x*e + d)^(7/2)*c^2*e^30 - 84*(x*e + d)^(5/2)*c^2*d*e^30 + 210*(x
*e + d)^(3/2)*c^2*d^2*e^30 - 420*sqrt(x*e + d)*c^2*d^3*e^30 + 42*(x*e + d)^
(5/2)*b*c*e^31 - 210*(x*e + d)^(3/2)*b*c*d*e^31 + 630*sqrt(x*e + d)*b*c*d^2
*e^31 + 35*(x*e + d)^(3/2)*b^2*e^32 - 210*sqrt(x*e + d)*b^2*d*e^32)*e^(-35)
 - 2*(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2)*e^(-5)/sqrt(x*e + d)
```

3.351 $\int \frac{(bx+cx^2)^2}{(d+ex)^{5/2}} \, dx$

Optimal. Leaf size=143

$$\frac{2\sqrt{d+ex}\left(b^2e^2-6bcde+6c^2d^2\right)}{e^5}-\frac{2d^2(cd-be)^2}{3e^5(d+ex)^{3/2}}-\frac{4c(d+ex)^{3/2}(2cd-be)}{3e^5}+\frac{4d(cd-be)(2cd-be)}{e^5\sqrt{d+ex}}+\frac{2c^2(d+ex)^{5/2}}{5e^5}$$

```
[Out] (-2*d^2*(c*d - b*e)^2)/(3*e^5*(d + e*x)^(3/2)) + (4*d*(c*d - b*e)*(2*c*d -
b*e))/(e^5*Sqrt[d + e*x]) + (2*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*Sqrt[d + e
*x])/e^5 - (4*c*(2*c*d - b*e)*(d + e*x)^(3/2))/(3*e^5) + (2*c^2*(d + e*x)^(
5/2))/(5*e^5)
```

Rubi [A] time = 0.0583636, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {698}

$$\frac{2\sqrt{d+ex}\left(b^2e^2-6bcde+6c^2d^2\right)}{e^5}-\frac{2d^2(cd-be)^2}{3e^5(d+ex)^{3/2}}-\frac{4c(d+ex)^{3/2}(2cd-be)}{3e^5}+\frac{4d(cd-be)(2cd-be)}{e^5\sqrt{d+ex}}+\frac{2c^2(d+ex)^{5/2}}{5e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^2/(d + e*x)^(5/2),x]
```

```
[Out] (-2*d^2*(c*d - b*e)^2)/(3*e^5*(d + e*x)^(3/2)) + (4*d*(c*d - b*e)*(2*c*d -
b*e))/(e^5*Sqrt[d + e*x]) + (2*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*Sqrt[d + e
*x])/e^5 - (4*c*(2*c*d - b*e)*(d + e*x)^(3/2))/(3*e^5) + (2*c^2*(d + e*x)^(
5/2))/(5*e^5)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(bx+cx^2)^2}{(d+ex)^{5/2}} \, dx = \int \left(\frac{d^2(cd-be)^2}{e^4(d+ex)^{5/2}}+\frac{2d(cd-be)(-2cd+be)}{e^4(d+ex)^{3/2}}+\frac{6c^2d^2-6bcde+b^2e^2}{e^4\sqrt{d+ex}}-\frac{2c(2cd-be)\sqrt{d+ex}}{e^4}+\frac{c^2(\right.$$

$$= -\frac{2d^2(cd-be)^2}{3e^5(d+ex)^{3/2}}+\frac{4d(cd-be)(2cd-be)}{e^5\sqrt{d+ex}}+\frac{2\left(6c^2d^2-6bcde+b^2e^2\right)\sqrt{d+ex}}{e^5}-\frac{4c(2cd-be)(d+e}{3e^5}$$

Mathematica [A] time = 0.0662151, size = 123, normalized size = 0.86

$$\frac{2\left(5b^2e^2\left(8d^2+12dex+3e^2x^2\right)+10bce\left(-24d^2ex-16d^3-6de^2x^2+e^3x^3\right)+c^2\left(48d^2e^2x^2+192d^3ex+128d^4-8de^3x^3\right.\right.}{15e^5(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^2/(d + e*x)^(5/2),x]
```

```
[Out] (2*(5*b^2*e^2*(8*d^2 + 12*d*e*x + 3*e^2*x^2) + 10*b*c*e*(-16*d^3 - 24*d^2*e
*x - 6*d*e^2*x^2 + e^3*x^3) + c^2*(128*d^4 + 192*d^3*e*x + 48*d^2*e^2*x^2 -
 8*d*e^3*x^3 + 3*e^4*x^4)))/(15*e^5*(d + e*x)^(3/2))
```

Maple [A] time = 0.049, size = 141, normalized size = 1.

$$\frac{6\,c^2x^4e^4+20\,bce^4x^3-16\,c^2de^3x^3+30\,b^2e^4x^2-120\,bcde^3x^2+96\,c^2d^2e^2x^2+120\,b^2de^3x-480\,bcd^2e^2x+384\,c^2d^3ex+80\ldots}{15\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^2/(e*x+d)^(5/2),x)
```

```
[Out] 2/15*(3*c^2*e^4*x^4+10*b*c*e^4*x^3-8*c^2*d*e^3*x^3+15*b^2*e^4*x^2-60*b*c*d*
e^3*x^2+48*c^2*d^2*e^2*x^2+60*b^2*d*e^3*x-240*b*c*d^2*e^2*x+192*c^2*d^3*e*x
+40*b^2*d^2*e^2-160*b*c*d^3*e+128*c^2*d^4)/(e*x+d)^(3/2)/e^5
```

Maxima [A] time = 1.03874, size = 196, normalized size = 1.37

$$\frac{2\left(\frac{3\,(ex+d)^{\frac{5}{2}}c^2-10\,\left(2\,c^2d-bce\right)(ex+d)^{\frac{3}{2}}+15\,\left(6\,c^2d^2-6\,bcde+b^2e^2\right)\sqrt{ex+d}}{e^4}-\frac{5\,\left(c^2d^4-2\,bcd^3e+b^2d^2e^2-6\,\left(2\,c^2d^3-3\,bcd^2e+b^2de^2\right)(ex+d)\right)}{(ex+d)^{\frac{3}{2}}e^4}\right)}{15\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*((3*(e*x + d)^(5/2)*c^2 - 10*(2*c^2*d - b*c*e)*(e*x + d)^(3/2) + 15*(6
*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*sqrt(e*x + d))/e^4 - 5*(c^2*d^4 - 2*b*c*d^3
*e + b^2*d^2*e^2 - 6*(2*c^2*d^3 - 3*b*c*d^2*e + b^2*d*e^2)*(e*x + d))/((e*x
 + d)^(3/2)*e^4))/e
```

Fricas [A] time = 1.94582, size = 343, normalized size = 2.4

$$\frac{2\left(3\,c^2e^4x^4+128\,c^2d^4-160\,bcd^3e+40\,b^2d^2e^2-2\left(4\,c^2de^3-5\,bce^4\right)x^3+3\left(16\,c^2d^2e^2-20\,bcde^3+5\,b^2e^4\right)x^2+12\left(16\,c^2d^3\ldots\right.\right.}{15\left(e^7x^2+2\,de^6x+d^2e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*c^2*e^4*x^4 + 128*c^2*d^4 - 160*b*c*d^3*e + 40*b^2*d^2*e^2 - 2*(4*c
^2*d*e^3 - 5*b*c*e^4)*x^3 + 3*(16*c^2*d^2*e^2 - 20*b*c*d*e^3 + 5*b^2*e^4)*x
^2 + 12*(16*c^2*d^3*e - 20*b*c*d^2*e^2 + 5*b^2*d*e^3)*x)*sqrt(e*x + d)/(e^7
*x^2 + 2*d*e^6*x + d^2*e^5)
```

Sympy [A] time = 41.188, size = 139, normalized size = 0.97

$$\frac{2c^2(d+ex)^{\frac{5}{2}}}{5e^5}-\frac{2d^2(be-cd)^2}{3e^5(d+ex)^{\frac{3}{2}}}+\frac{4d(be-2cd)(be-cd)}{e^5\sqrt{d+ex}}+\frac{(d+ex)^{\frac{3}{2}}(4bce-8c^2d)}{3e^5}+\frac{\sqrt{d+ex}\,(2b^2e^2-12bcde+12c^2d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**2/(e*x+d)**(5/2),x)
```

```
[Out] 2*c**2*(d + e*x)**(5/2)/(5*e**5) - 2*d**2*(b*e - c*d)**2/(3*e**5*(d + e*x)*
*(3/2)) + 4*d*(b*e - 2*c*d)*(b*e - c*d)/(e**5*sqrt(d + e*x)) + (d + e*x)**(
3/2)*(4*b*c*e - 8*c**2*d)/(3*e**5) + sqrt(d + e*x)*(2*b**2*e**2 - 12*b*c*d*
e + 12*c**2*d**2)/e**5
```

Giac [A] time = 1.3858, size = 246, normalized size = 1.72

$$\frac{2}{15}\Big(3(xe+d)^{\frac{5}{2}}c^2e^{20}-20(xe+d)^{\frac{3}{2}}c^2de^{20}+90\sqrt{xe+d}c^2d^2e^{20}+10(xe+d)^{\frac{3}{2}}bce^{21}-90\sqrt{xe+d}bcde^{21}+15\sqrt{xe+d}b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*(x*e + d)^(5/2)*c^2*e^20 - 20*(x*e + d)^(3/2)*c^2*d*e^20 + 90*sqrt(
x*e + d)*c^2*d^2*e^20 + 10*(x*e + d)^(3/2)*b*c*e^21 - 90*sqrt(x*e + d)*b*c*
d*e^21 + 15*sqrt(x*e + d)*b^2*e^22)*e^(-25) + 2/3*(12*(x*e + d)*c^2*d^3 - c
^2*d^4 - 18*(x*e + d)*b*c*d^2*e + 2*b*c*d^3*e + 6*(x*e + d)*b^2*d*e^2 - b^2
*d^2*e^2)*e^(-5)/(x*e + d)^(3/2)
```

3.352 $\int \frac{\left(bx+cx^2\right)^2}{(d+ex)^{7/2}} dx$

Optimal. Leaf size=143

$$-\frac{2\left(b^2e^2-6bcde+6c^2d^2\right)}{e^5\sqrt{d+ex}}-\frac{2d^2(cd-be)^2}{5e^5(d+ex)^{5/2}}-\frac{4c\sqrt{d+ex}(2cd-be)}{e^5}+\frac{4d(cd-be)(2cd-be)}{3e^5(d+ex)^{3/2}}+\frac{2c^2(d+ex)^{3/2}}{3e^5}$$

```
[Out] (-2*d^2*(c*d - b*e)^2)/(5*e^5*(d + e*x)^(5/2)) + (4*d*(c*d - b*e)*(2*c*d -
b*e))/(3*e^5*(d + e*x)^(3/2)) - (2*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2))/(e^5*
Sqrt[d + e*x]) - (4*c*(2*c*d - b*e)*Sqrt[d + e*x])/e^5 + (2*c^2*(d + e*x)^(
3/2))/(3*e^5)
```

Rubi [A] time = 0.0576748, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {698}

$$-\frac{2\left(b^2e^2-6bcde+6c^2d^2\right)}{e^5\sqrt{d+ex}}-\frac{2d^2(cd-be)^2}{5e^5(d+ex)^{5/2}}-\frac{4c\sqrt{d+ex}(2cd-be)}{e^5}+\frac{4d(cd-be)(2cd-be)}{3e^5(d+ex)^{3/2}}+\frac{2c^2(d+ex)^{3/2}}{3e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^2/(d + e*x)^(7/2),x]
```

```
[Out] (-2*d^2*(c*d - b*e)^2)/(5*e^5*(d + e*x)^(5/2)) + (4*d*(c*d - b*e)*(2*c*d -
b*e))/(3*e^5*(d + e*x)^(3/2)) - (2*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2))/(e^5*
Sqrt[d + e*x]) - (4*c*(2*c*d - b*e)*Sqrt[d + e*x])/e^5 + (2*c^2*(d + e*x)^(
3/2))/(3*e^5)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(bx+cx^2\right)^2}{(d+ex)^{7/2}} dx = \int \left(\frac{d^2(cd-be)^2}{e^4(d+ex)^{7/2}}+\frac{2d(cd-be)(-2cd+be)}{e^4(d+ex)^{5/2}}+\frac{6c^2d^2-6bcde+b^2e^2}{e^4(d+ex)^{3/2}}-\frac{2c(2cd-be)}{e^4\sqrt{d+ex}}+\frac{c^2\sqrt{d+ex}}{e^4}\right) dx$$
$$= -\frac{2d^2(cd-be)^2}{5e^5(d+ex)^{5/2}}+\frac{4d(cd-be)(2cd-be)}{3e^5(d+ex)^{3/2}}-\frac{2\left(6c^2d^2-6bcde+b^2e^2\right)}{e^5\sqrt{d+ex}}-\frac{4c(2cd-be)\sqrt{d+ex}}{e^5}+\frac{2c^2(d}{3}$$

Mathematica [A] time = 0.0663874, size = 123, normalized size = 0.86

$$-\frac{2\left(b^2e^2\left(8d^2+20dex+15e^2x^2\right)-6bce\left(40d^2ex+16d^3+30de^2x^2+5e^3x^3\right)+c^2\left(240d^2e^2x^2+320d^3ex+128d^4+40de^3x\right.\right.}{15e^5(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^2/(d + e*x)^(7/2),x]
```

```
[Out] (-2*(b^2*e^2*(8*d^2 + 20*d*e*x + 15*e^2*x^2) - 6*b*c*e*(16*d^3 + 40*d^2*e*x
 + 30*d*e^2*x^2 + 5*e^3*x^3) + c^2*(128*d^4 + 320*d^3*e*x + 240*d^2*e^2*x^2
 + 40*d*e^3*x^3 - 5*e^4*x^4)))/(15*e^5*(d + e*x)^(5/2))
```

Maple [A] time = 0.048, size = 141, normalized size = 1.

$$-\frac{-10c^2x^4e^4 - 60bce^4x^3 + 80c^2de^3x^3 + 30b^2e^4x^2 - 360bcde^3x^2 + 480c^2d^2e^2x^2 + 40b^2de^3x - 480bcd^2e^2x + 640c^2d^3 \cdots}{15e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^2/(e*x+d)^(7/2),x)
```

```
[Out] -2/15*(-5*c^2*e^4*x^4-30*b*c*e^4*x^3+40*c^2*d*e^3*x^3+15*b^2*e^4*x^2-180*b*
c*d*e^3*x^2+240*c^2*d^2*e^2*x^2+20*b^2*d*e^3*x-240*b*c*d^2*e^2*x+320*c^2*d^
3*e*x+8*b^2*d^2*e^2-96*b*c*d^3*e+128*c^2*d^4)/(e*x+d)^(5/2)/e^5
```

Maxima [A] time = 1.16068, size = 198, normalized size = 1.38

$$\frac{2\left(\frac{5\left((ex+d)^{\frac{3}{2}}c^2-6\left(2c^2d-bce\right)\sqrt{ex+d}\right)}{e^4}-\frac{3c^2d^4-6bcd^3e+3b^2d^2e^2+15\left(6c^2d^2-6bcde+b^2e^2\right)(ex+d)^2-10\left(2c^2d^3-3bcd^2e+b^2de^2\right)(ex+d)}{(ex+d)^{\frac{5}{2}}e^4}\right)}{15e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*(5*((e*x + d)^(3/2)*c^2 - 6*(2*c^2*d - b*c*e)*sqrt(e*x + d))/e^4 - (3*
c^2*d^4 - 6*b*c*d^3*e + 3*b^2*d^2*e^2 + 15*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2
)*(e*x + d)^2 - 10*(2*c^2*d^3 - 3*b*c*d^2*e + b^2*d*e^2)*(e*x + d))/((e*x +
 d)^(5/2)*e^4))/e
```

Fricas [A] time = 2.00331, size = 359, normalized size = 2.51

$$\frac{2\left(5c^2e^4x^4 - 128c^2d^4 + 96bcd^3e - 8b^2d^2e^2 - 10\left(4c^2de^3 - 3bce^4\right)x^3 - 15\left(16c^2d^2e^2 - 12bcde^3 + b^2e^4\right)x^2 - 20\left(16c^2d \cdots\right.\right.}{15\left(e^8x^3 + 3de^7x^2 + 3d^2e^6x + d^3e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(5*c^2*e^4*x^4 - 128*c^2*d^4 + 96*b*c*d^3*e - 8*b^2*d^2*e^2 - 10*(4*c^
2*d*e^3 - 3*b*c*e^4)*x^3 - 15*(16*c^2*d^2*e^2 - 12*b*c*d*e^3 + b^2*e^4)*x^2
 - 20*(16*c^2*d^3*e - 12*b*c*d^2*e^2 + b^2*d*e^3)*x)*sqrt(e*x + d)/(e^8*x^3
 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3*e^5)
```

Sympy [A] time = 4.84365, size = 787, normalized size = 5.5

$$\begin{cases} -\frac{16b^2d^2e^2}{15d^2e^5\sqrt{d+ex}+30de^6x\sqrt{d+ex}+15e^7x^2\sqrt{d+ex}} - \frac{40b^2de^3x}{15d^2e^5\sqrt{d+ex}+30de^6x\sqrt{d+ex}+15e^7x^2\sqrt{d+ex}} - \frac{30b^2e^4x^2}{15d^2e^5\sqrt{d+ex}+30de^6x\sqrt{d+ex}+15e^7x^2\sqrt{d+ex}} + \frac{\ }{15d^2e^5\sqrt{d+}} \\ \frac{\frac{b^2x^3}{3}+\frac{bcx^4}{2}+\frac{c^2x^5}{5}}{d^{\frac{7}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**2/(e*x+d)**(7/2),x)
```

```
[Out] Piecewise((-16*b**2*d**2*e**2/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqr
t(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 40*b**2*d*e**3*x/(15*d**2*e**5*s
qrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 30
*b**2*e**4*x**2/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 1
5*e**7*x**2*sqrt(d + e*x)) + 192*b*c*d**3*e/(15*d**2*e**5*sqrt(d + e*x) + 3
0*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) + 480*b*c*d**2*e**2*
x/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sq
rt(d + e*x)) + 360*b*c*d*e**3*x**2/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*
x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) + 60*b*c*e**4*x**3/(15*d**2*e
**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x))
 - 256*c**2*d**4/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) +
15*e**7*x**2*sqrt(d + e*x)) - 640*c**2*d**3*e*x/(15*d**2*e**5*sqrt(d + e*x)
 + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 480*c**2*d**2*
e**2*x**2/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7
*x**2*sqrt(d + e*x)) - 80*c**2*d*e**3*x**3/(15*d**2*e**5*sqrt(d + e*x) + 30
*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) + 10*c**2*e**4*x**4/(
15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(
d + e*x)), Ne(e, 0)), ((b**2*x**3/3 + b*c*x**4/2 + c**2*x**5/5)/d**(7/2), T
rue))
```

Giac [A] time = 1.34746, size = 242, normalized size = 1.69

$$\frac{2}{3}\left((xe+d)^{\frac{3}{2}}c^2e^{10} - 12\sqrt{xe+d}c^2de^{10} + 6\sqrt{xe+d}bce^{11}\right)e^{(-15)} - \frac{2\left(90(xe+d)^2c^2d^2 - 20(xe+d)c^2d^3 + 3c^2d^4 - 90(xe+d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^2/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] 2/3*((x*e + d)^(3/2)*c^2*e^10 - 12*sqrt(x*e + d)*c^2*d*e^10 + 6*sqrt(x*e +
d)*b*c*e^11)*e^(-15) - 2/15*(90*(x*e + d)^2*c^2*d^2 - 20*(x*e + d)*c^2*d^3
+ 3*c^2*d^4 - 90*(x*e + d)^2*b*c*d*e + 30*(x*e + d)*b*c*d^2*e - 6*b*c*d^3*e
 + 15*(x*e + d)^2*b^2*e^2 - 10*(x*e + d)*b^2*d*e^2 + 3*b^2*d^2*e^2)*e^(-5)/
(x*e + d)^(5/2)
```

3.353 $\int (d+ex)^{7/2} \left(bx+cx^2\right)^3 \, dx$

Optimal. Leaf size=248

$$\frac{6c(d+ex)^{17/2}\left(b^2e^2-5bcde+5c^2d^2\right)}{17e^7}-\frac{2(d+ex)^{15/2}(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{15e^7}+\frac{6d(d+ex)^{13/2}(cd-be)\left(b^2e\right.}{13e^7}$$

```
[Out] (2*d^3*(c*d - b*e)^3*(d + e*x)^(9/2))/(9*e^7) - (6*d^2*(c*d - b*e)^2*(2*c*d
 - b*e)*(d + e*x)^(11/2))/(11*e^7) + (6*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*
e + b^2*e^2)*(d + e*x)^(13/2))/(13*e^7) - (2*(2*c*d - b*e)*(10*c^2*d^2 - 10
*b*c*d*e + b^2*e^2)*(d + e*x)^(15/2))/(15*e^7) + (6*c*(5*c^2*d^2 - 5*b*c*d*
e + b^2*e^2)*(d + e*x)^(17/2))/(17*e^7) - (6*c^2*(2*c*d - b*e)*(d + e*x)^(1
9/2))/(19*e^7) + (2*c^3*(d + e*x)^(21/2))/(21*e^7)
```

Rubi [A] time = 0.139448, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {698}

$$\frac{6c(d+ex)^{17/2}\left(b^2e^2-5bcde+5c^2d^2\right)}{17e^7}-\frac{2(d+ex)^{15/2}(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{15e^7}+\frac{6d(d+ex)^{13/2}(cd-be)\left(b^2e\right.}{13e^7}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)*(b*x + c*x^2)^3,x]
```


```
[Out] (2*d^3*(c*d - b*e)^3*(d + e*x)^(9/2))/(9*e^7) - (6*d^2*(c*d - b*e)^2*(2*c*d
 - b*e)*(d + e*x)^(11/2))/(11*e^7) + (6*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*
e + b^2*e^2)*(d + e*x)^(13/2))/(13*e^7) - (2*(2*c*d - b*e)*(10*c^2*d^2 - 10
*b*c*d*e + b^2*e^2)*(d + e*x)^(15/2))/(15*e^7) + (6*c*(5*c^2*d^2 - 5*b*c*d*
e + b^2*e^2)*(d + e*x)^(17/2))/(17*e^7) - (6*c^2*(2*c*d - b*e)*(d + e*x)^(1
9/2))/(19*e^7) + (2*c^3*(d + e*x)^(21/2))/(21*e^7)
```


Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^{7/2}\left(bx+cx^2\right)^3\,dx=\int\left(\frac{d^3(cd-be)^3(d+ex)^{7/2}}{e^6}-\frac{3d^2(cd-be)^2(2cd-be)(d+ex)^{9/2}}{e^6}+\frac{3d(cd-be)\left(5c^2d^2-\right.}{}\right.$$

$$=\frac{2d^3(cd-be)^3(d+ex)^{9/2}}{9e^7}-\frac{6d^2(cd-be)^2(2cd-be)(d+ex)^{11/2}}{11e^7}+\frac{6d(cd-be)\left(5c^2d^2-5\right.}{1}$$

Mathematica [A] time = 0.196915, size = 206, normalized size = 0.83

$$\frac{2(d+ex)^{9/2}\left(2567565c(d+ex)^4\left(b^2e^2-5bcde+5c^2d^2\right)-969969(d+ex)^3(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)+33575\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)*(b*x + c*x^2)^3,x]
```

```
[Out] (2*(d + e*x)^(9/2)*(1616615*d^3*(c*d - b*e)^3 - 3968055*d^2*(c*d - b*e)^2*(
2*c*d - b*e)*(d + e*x) + 3357585*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2
*e^2)*(d + e*x)^2 - 969969*(2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e + b^2*e^2
)*(d + e*x)^3 + 2567565*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*(d + e*x)^4 - 2
297295*c^2*(2*c*d - b*e)*(d + e*x)^5 + 692835*c^3*(d + e*x)^6))/(14549535*e
^7)
```

Maple [A] time = 0.049, size = 286, normalized size = 1.2

$$-\frac{-1385670\,c^3x^6e^6 - 4594590\,bc^2e^6x^5 + 875160\,c^3de^5x^5 - 5135130\,b^2ce^6x^4 + 2702700\,bc^2de^5x^4 - 514800\,c^3d^2e^4x^4 - 193}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)*(c*x^2+b*x)^3,x)
```

```
[Out] -2/14549535*(e*x+d)^(9/2)*(-692835*c^3*e^6*x^6-2297295*b*c^2*e^6*x^5+437580
*c^3*d*e^5*x^5-2567565*b^2*c*e^6*x^4+1351350*b*c^2*d*e^5*x^4-257400*c^3*d^2
*e^4*x^4-969969*b^3*e^6*x^3+1369368*b^2*c*d*e^5*x^3-720720*b*c^2*d^2*e^4*x^
3+137280*c^3*d^3*e^3*x^3+447678*b^3*d*e^5*x^2-632016*b^2*c*d^2*e^4*x^2+3326
40*b*c^2*d^3*e^3*x^2-63360*c^3*d^4*e^2*x^2-162792*b^3*d^2*e^4*x+229824*b^2*
c*d^3*e^3*x-120960*b*c^2*d^4*e^2*x+23040*c^3*d^5*e*x+36176*b^3*d^3*e^3-5107
2*b^2*c*d^4*e^2+26880*b*c^2*d^5*e-5120*c^3*d^6)/e^7
```

Maxima [A] time = 1.06333, size = 366, normalized size = 1.48

$$\frac{2\left(692835\,(ex+d)^{\frac{21}{2}}c^3 - 2297295\left(2\,c^3d - bc^2e\right)(ex+d)^{\frac{19}{2}} + 2567565\left(5\,c^3d^2 - 5\,bc^2de + b^2ce^2\right)(ex+d)^{\frac{17}{2}} - 969969\left(20\,c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)*(c*x^2+b*x)^3,x, algorithm="maxima")
```

```
[Out] 2/14549535*(692835*(e*x + d)^(21/2)*c^3 - 2297295*(2*c^3*d - b*c^2*e)*(e*x
+ d)^(19/2) + 2567565*(5*c^3*d^2 - 5*b*c^2*d*e + b^2*c*e^2)*(e*x + d)^(17/2
) - 969969*(20*c^3*d^3 - 30*b*c^2*d^2*e + 12*b^2*c*d*e^2 - b^3*e^3)*(e*x +
d)^(15/2) + 3357585*(5*c^3*d^4 - 10*b*c^2*d^3*e + 6*b^2*c*d^2*e^2 - b^3*d*e
^3)*(e*x + d)^(13/2) - 3968055*(2*c^3*d^5 - 5*b*c^2*d^4*e + 4*b^2*c*d^3*e^2
 - b^3*d^2*e^3)*(e*x + d)^(11/2) + 1616615*(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2
*c*d^4*e^2 - b^3*d^3*e^3)*(e*x + d)^(9/2))/e^7
```

Fricas [B] time = 2.07016, size = 1141, normalized size = 4.6

$$\frac{2\left(692835\,c^3e^{10}x^{10} + 5120\,c^3d^{10} - 26880\,bc^2d^9e + 51072\,b^2cd^8e^2 - 36176\,b^3d^7e^3 + 36465\left(64\,c^3de^9 + 63\,bc^2e^{10}\right)x^9 + 1930\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)*(c*x^2+b*x)^3,x, algorithm="fricas")
```

```
[Out] 2/14549535*(692835*c^3*e^10*x^10 + 5120*c^3*d^10 - 26880*b*c^2*d^9*e + 5107
2*b^2*c*d^8*e^2 - 36176*b^3*d^7*e^3 + 36465*(64*c^3*d*e^9 + 63*b*c^2*e^10)*
x^9 + 19305*(138*c^3*d^2*e^8 + 406*b*c^2*d*e^9 + 133*b^2*c*e^10)*x^8 + 429*
(2420*c^3*d^3*e^7 + 21210*b*c^2*d^2*e^8 + 20748*b^2*c*d*e^9 + 2261*b^3*e^10
)*x^7 + 231*(5*c^3*d^4*e^6 + 15720*b*c^2*d^3*e^7 + 45714*b^2*c*d^2*e^8 + 14
858*b^3*d*e^9)*x^6 - 63*(20*c^3*d^5*e^5 - 105*b*c^2*d^4*e^6 - 69084*b^2*c*d
^3*e^7 - 66538*b^3*d^2*e^8)*x^5 + 35*(40*c^3*d^6*e^4 - 210*b*c^2*d^5*e^5 +
399*b^2*c*d^4*e^6 + 51680*b^3*d^3*e^7)*x^4 - 5*(320*c^3*d^7*e^3 - 1680*b*c^
2*d^6*e^4 + 3192*b^2*c*d^5*e^5 - 2261*b^3*d^4*e^6)*x^3 + 6*(320*c^3*d^8*e^2
 - 1680*b*c^2*d^7*e^3 + 3192*b^2*c*d^6*e^4 - 2261*b^3*d^5*e^5)*x^2 - 8*(320
*c^3*d^9*e - 1680*b*c^2*d^8*e^2 + 3192*b^2*c*d^7*e^3 - 2261*b^3*d^6*e^4)*x)
*sqrt(e*x + d)/e^7
```

Sympy [B] time = 83.6296, size = 1741, normalized size = 7.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)*(c*x**2+b*x)**3,x)
```

```
[Out] 2*b**3*d**3*(-d**3*(d + e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d
+ e*x)**(7/2)/7 + (d + e*x)**(9/2)/9)/e**4 + 6*b**3*d**2*(d**4*(d + e*x)**(
3/2)/3 - 4*d**3*(d + e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e
*x)**(9/2)/9 + (d + e*x)**(11/2)/11)/e**4 + 6*b**3*d*(-d**5*(d + e*x)**(3/2
)/3 + d**4*(d + e*x)**(5/2) - 10*d**3*(d + e*x)**(7/2)/7 + 10*d**2*(d + e*x
)**(9/2)/9 - 5*d*(d + e*x)**(11/2)/11 + (d + e*x)**(13/2)/13)/e**4 + 2*b**3
*(d**6*(d + e*x)**(3/2)/3 - 6*d**5*(d + e*x)**(5/2)/5 + 15*d**4*(d + e*x)**
(7/2)/7 - 20*d**3*(d + e*x)**(9/2)/9 + 15*d**2*(d + e*x)**(11/2)/11 - 6*d*(
d + e*x)**(13/2)/13 + (d + e*x)**(15/2)/15)/e**4 + 6*b**2*c*d**3*(d**4*(d +
 e*x)**(3/2)/3 - 4*d**3*(d + e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*
d*(d + e*x)**(9/2)/9 + (d + e*x)**(11/2)/11)/e**5 + 18*b**2*c*d**2*(-d**5*(
d + e*x)**(3/2)/3 + d**4*(d + e*x)**(5/2) - 10*d**3*(d + e*x)**(7/2)/7 + 10
*d**2*(d + e*x)**(9/2)/9 - 5*d*(d + e*x)**(11/2)/11 + (d + e*x)**(13/2)/13)
/e**5 + 18*b**2*c*d*(d**6*(d + e*x)**(3/2)/3 - 6*d**5*(d + e*x)**(5/2)/5 +
15*d**4*(d + e*x)**(7/2)/7 - 20*d**3*(d + e*x)**(9/2)/9 + 15*d**2*(d + e*x)
**(11/2)/11 - 6*d*(d + e*x)**(13/2)/13 + (d + e*x)**(15/2)/15)/e**5 + 6*b**
2*c*(-d**7*(d + e*x)**(3/2)/3 + 7*d**6*(d + e*x)**(5/2)/5 - 3*d**5*(d + e*x
)**(7/2) + 35*d**4*(d + e*x)**(9/2)/9 - 35*d**3*(d + e*x)**(11/2)/11 + 21*d
**2*(d + e*x)**(13/2)/13 - 7*d*(d + e*x)**(15/2)/15 + (d + e*x)**(17/2)/17)
/e**5 + 6*b*c**2*d**3*(-d**5*(d + e*x)**(3/2)/3 + d**4*(d + e*x)**(5/2) - 1
0*d**3*(d + e*x)**(7/2)/7 + 10*d**2*(d + e*x)**(9/2)/9 - 5*d*(d + e*x)**(11
/2)/11 + (d + e*x)**(13/2)/13)/e**6 + 18*b*c**2*d**2*(d**6*(d + e*x)**(3/2)
/3 - 6*d**5*(d + e*x)**(5/2)/5 + 15*d**4*(d + e*x)**(7/2)/7 - 20*d**3*(d +
e*x)**(9/2)/9 + 15*d**2*(d + e*x)**(11/2)/11 - 6*d*(d + e*x)**(13/2)/13 + (
d + e*x)**(15/2)/15)/e**6 + 18*b*c**2*d*(-d**7*(d + e*x)**(3/2)/3 + 7*d**6*
(d + e*x)**(5/2)/5 - 3*d**5*(d + e*x)**(7/2) + 35*d**4*(d + e*x)**(9/2)/9 -
 35*d**3*(d + e*x)**(11/2)/11 + 21*d**2*(d + e*x)**(13/2)/13 - 7*d*(d + e*x
)**(15/2)/15 + (d + e*x)**(17/2)/17)/e**6 + 6*b*c**2*(d**8*(d + e*x)**(3/2)
/3 - 8*d**7*(d + e*x)**(5/2)/5 + 4*d**6*(d + e*x)**(7/2) - 56*d**5*(d + e*x
)**(9/2)/9 + 70*d**4*(d + e*x)**(11/2)/11 - 56*d**3*(d + e*x)**(13/2)/13 +
28*d**2*(d + e*x)**(15/2)/15 - 8*d*(d + e*x)**(17/2)/17 + (d + e*x)**(19/2)
/19)/e**6 + 2*c**3*d**3*(d**6*(d + e*x)**(3/2)/3 - 6*d**5*(d + e*x)**(5/2)/
5 + 15*d**4*(d + e*x)**(7/2)/7 - 20*d**3*(d + e*x)**(9/2)/9 + 15*d**2*(d +
e*x)**(11/2)/11 - 6*d*(d + e*x)**(13/2)/13 + (d + e*x)**(15/2)/15)/e**7 + 6
*c**3*d**2*(-d**7*(d + e*x)**(3/2)/3 + 7*d**6*(d + e*x)**(5/2)/5 - 3*d**5*(
d + e*x)**(7/2) + 35*d**4*(d + e*x)**(9/2)/9 - 35*d**3*(d + e*x)**(11/2)/11
```

```
 + 21*d**2*(d + e*x)**(13/2)/13 - 7*d*(d + e*x)**(15/2)/15 + (d + e*x)**(17
/2)/17)/e**7 + 6*c**3*d*(d**8*(d + e*x)**(3/2)/3 - 8*d**7*(d + e*x)**(5/2)/
5 + 4*d**6*(d + e*x)**(7/2) - 56*d**5*(d + e*x)**(9/2)/9 + 70*d**4*(d + e*x
)**(11/2)/11 - 56*d**3*(d + e*x)**(13/2)/13 + 28*d**2*(d + e*x)**(15/2)/15
- 8*d*(d + e*x)**(17/2)/17 + (d + e*x)**(19/2)/19)/e**7 + 2*c**3*(-d**9*(d
+ e*x)**(3/2)/3 + 9*d**8*(d + e*x)**(5/2)/5 - 36*d**7*(d + e*x)**(7/2)/7 +
28*d**6*(d + e*x)**(9/2)/3 - 126*d**5*(d + e*x)**(11/2)/11 + 126*d**4*(d +
e*x)**(13/2)/13 - 28*d**3*(d + e*x)**(15/2)/5 + 36*d**2*(d + e*x)**(17/2)/1
7 - 9*d*(d + e*x)**(19/2)/19 + (d + e*x)**(21/2)/21)/e**7
```

Giac [B] time = 1.51417, size = 2086, normalized size = 8.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)*(c*x^2+b*x)^3,x, algorithm="giac")
```

```
[Out] 2/14549535*(46189*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e +
d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*b^3*d^3*e^(-3) + 12597*(315*(x*e +
d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e +
 d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*b^2*c*d^3*e^(-4) + 4845*(693*(x*e
 + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12870*
(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*
b*c^2*d^3*e^(-5) + 323*(3003*(x*e + d)^(15/2) - 20790*(x*e + d)^(13/2)*d +
61425*(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525*(x*e + d)^(
7/2)*d^4 - 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*c^3*d^3*e
^(-6) + 12597*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e +
d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*b^3*d^2
*e^(-3) + 14535*(693*(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*
e + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3
003*(x*e + d)^(3/2)*d^5)*b^2*c*d^2*e^(-4) + 2907*(3003*(x*e + d)^(15/2) - 2
0790*(x*e + d)^(13/2)*d + 61425*(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/
2)*d^3 + 96525*(x*e + d)^(7/2)*d^4 - 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*e
 + d)^(3/2)*d^6)*b*c^2*d^2*e^(-5) + 399*(6435*(x*e + d)^(17/2) - 51051*(x*e
 + d)^(15/2)*d + 176715*(x*e + d)^(13/2)*d^2 - 348075*(x*e + d)^(11/2)*d^3
+ 425425*(x*e + d)^(9/2)*d^4 - 328185*(x*e + d)^(7/2)*d^5 + 153153*(x*e + d
)^(5/2)*d^6 - 36465*(x*e + d)^(3/2)*d^7)*c^3*d^2*e^(-6) + 4845*(693*(x*e +
d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12870*(x*
e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*b^3
*d*e^(-3) + 2907*(3003*(x*e + d)^(15/2) - 20790*(x*e + d)^(13/2)*d + 61425*
(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525*(x*e + d)^(7/2)*d
^4 - 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*b^2*c*d*e^(-4)
+ 1197*(6435*(x*e + d)^(17/2) - 51051*(x*e + d)^(15/2)*d + 176715*(x*e + d)
^(13/2)*d^2 - 348075*(x*e + d)^(11/2)*d^3 + 425425*(x*e + d)^(9/2)*d^4 - 32
8185*(x*e + d)^(7/2)*d^5 + 153153*(x*e + d)^(5/2)*d^6 - 36465*(x*e + d)^(3/
2)*d^7)*b*c^2*d*e^(-5) + 21*(109395*(x*e + d)^(19/2) - 978120*(x*e + d)^(17
/2)*d + 3879876*(x*e + d)^(15/2)*d^2 - 8953560*(x*e + d)^(13/2)*d^3 + 13226
850*(x*e + d)^(11/2)*d^4 - 12932920*(x*e + d)^(9/2)*d^5 + 8314020*(x*e + d)
^(7/2)*d^6 - 3325608*(x*e + d)^(5/2)*d^7 + 692835*(x*e + d)^(3/2)*d^8)*c^3*
d*e^(-6) + 323*(3003*(x*e + d)^(15/2) - 20790*(x*e + d)^(13/2)*d + 61425*(x
*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525*(x*e + d)^(7/2)*d^4
 - 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*b^3*e^(-3) + 399*
(6435*(x*e + d)^(17/2) - 51051*(x*e + d)^(15/2)*d + 176715*(x*e + d)^(13/2)
*d^2 - 348075*(x*e + d)^(11/2)*d^3 + 425425*(x*e + d)^(9/2)*d^4 - 328185*(x
*e + d)^(7/2)*d^5 + 153153*(x*e + d)^(5/2)*d^6 - 36465*(x*e + d)^(3/2)*d^7)
*b^2*c*e^(-4) + 21*(109395*(x*e + d)^(19/2) - 978120*(x*e + d)^(17/2)*d + 3
```

```
879876*(x*e + d)^(15/2)*d^2 - 8953560*(x*e + d)^(13/2)*d^3 + 13226850*(x*e
+ d)^(11/2)*d^4 - 12932920*(x*e + d)^(9/2)*d^5 + 8314020*(x*e + d)^(7/2)*d^
6 - 3325608*(x*e + d)^(5/2)*d^7 + 692835*(x*e + d)^(3/2)*d^8)*b*c^2*e^(-5)
+ 3*(230945*(x*e + d)^(21/2) - 2297295*(x*e + d)^(19/2)*d + 10270260*(x*e +
 d)^(17/2)*d^2 - 27159132*(x*e + d)^(15/2)*d^3 + 47006190*(x*e + d)^(13/2)*
d^4 - 55552770*(x*e + d)^(11/2)*d^5 + 45265220*(x*e + d)^(9/2)*d^6 - 249420
60*(x*e + d)^(7/2)*d^7 + 8729721*(x*e + d)^(5/2)*d^8 - 1616615*(x*e + d)^(3
/2)*d^9)*c^3*e^(-6))*e^(-1)
```

3.354 $\int (d+ex)^{5/2} \left(bx+cx^2\right)^3 \, dx$

Optimal. Leaf size=248

$$\frac{2c(d+ex)^{15/2}\left(b^2e^2-5bcde+5c^2d^2\right)}{5e^7}-\frac{2(d+ex)^{13/2}(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{13e^7}+\frac{6d(d+ex)^{11/2}(cd-be)\left(b^2e^2-\right.}{11e^7}$$

```
[Out] (2*d^3*(c*d - b*e)^3*(d + e*x)^(7/2))/(7*e^7) - (2*d^2*(c*d - b*e)^2*(2*c*d
 - b*e)*(d + e*x)^(9/2))/(3*e^7) + (6*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e
+ b^2*e^2)*(d + e*x)^(11/2))/(11*e^7) - (2*(2*c*d - b*e)*(10*c^2*d^2 - 10*b
*c*d*e + b^2*e^2)*(d + e*x)^(13/2))/(13*e^7) + (2*c*(5*c^2*d^2 - 5*b*c*d*e
+ b^2*e^2)*(d + e*x)^(15/2))/(5*e^7) - (6*c^2*(2*c*d - b*e)*(d + e*x)^(17/2
))/(17*e^7) + (2*c^3*(d + e*x)^(19/2))/(19*e^7)
```

Rubi [A] time = 0.107545, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {698}

$$\frac{2c(d+ex)^{15/2}\left(b^2e^2-5bcde+5c^2d^2\right)}{5e^7}-\frac{2(d+ex)^{13/2}(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{13e^7}+\frac{6d(d+ex)^{11/2}(cd-be)\left(b^2e^2-\right.}{11e^7}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)*(b*x + c*x^2)^3,x]
```

```
[Out] (2*d^3*(c*d - b*e)^3*(d + e*x)^(7/2))/(7*e^7) - (2*d^2*(c*d - b*e)^2*(2*c*d
 - b*e)*(d + e*x)^(9/2))/(3*e^7) + (6*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e
+ b^2*e^2)*(d + e*x)^(11/2))/(11*e^7) - (2*(2*c*d - b*e)*(10*c^2*d^2 - 10*b
*c*d*e + b^2*e^2)*(d + e*x)^(13/2))/(13*e^7) + (2*c*(5*c^2*d^2 - 5*b*c*d*e
+ b^2*e^2)*(d + e*x)^(15/2))/(5*e^7) - (6*c^2*(2*c*d - b*e)*(d + e*x)^(17/2
))/(17*e^7) + (2*c^3*(d + e*x)^(19/2))/(19*e^7)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^{5/2}\left(bx+cx^2\right)^3\,dx=\int\left(\frac{d^3(cd-be)^3(d+ex)^{5/2}}{e^6}-\frac{3d^2(cd-be)^2(2cd-be)(d+ex)^{7/2}}{e^6}+\frac{3d(cd-be)\left(5c^2d^2-5bc\right.}{e^6}\right.$$

$$=\frac{2d^3(cd-be)^3(d+ex)^{7/2}}{7e^7}-\frac{2d^2(cd-be)^2(2cd-be)(d+ex)^{9/2}}{3e^7}+\frac{6d(cd-be)\left(5c^2d^2-5bcde\right.}{11e^7}$$

Mathematica [A] time = 0.160991, size = 206, normalized size = 0.83

$$\frac{2(d+ex)^{7/2}\left(969969c(d+ex)^4\left(b^2e^2-5bcde+5c^2d^2\right)-373065(d+ex)^3(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)+1322685d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)*(b*x + c*x^2)^3,x]
```

```
[Out] (2*(d + e*x)^(7/2)*(692835*d^3*(c*d - b*e)^3 - 1616615*d^2*(c*d - b*e)^2*(2
*c*d - b*e)*(d + e*x) + 1322685*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2*
e^2)*(d + e*x)^2 - 373065*(2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e + b^2*e^2)
*(d + e*x)^3 + 969969*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*(d + e*x)^4 - 855
855*c^2*(2*c*d - b*e)*(d + e*x)^5 + 255255*c^3*(d + e*x)^6))/(4849845*e^7)
```

Maple [A] time = 0.048, size = 286, normalized size = 1.2

$$-\frac{-510510\,c^3x^6e^6-1711710\,bc^2e^6x^5+360360\,c^3de^5x^5-1939938\,b^2ce^6x^4+1141140\,bc^2de^5x^4-240240\,c^3d^2e^4x^4-74\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(c*x^2+b*x)^3,x)
```

```
[Out] -2/4849845*(e*x+d)^(7/2)*(-255255*c^3*e^6*x^6-855855*b*c^2*e^6*x^5+180180*c
^3*d*e^5*x^5-969969*b^2*c*e^6*x^4+570570*b*c^2*d*e^5*x^4-120120*c^3*d^2*e^4
*x^4-373065*b^3*e^6*x^3+596904*b^2*c*d*e^5*x^3-351120*b*c^2*d^2*e^4*x^3+739
20*c^3*d^3*e^3*x^3+203490*b^3*d*e^5*x^2-325584*b^2*c*d^2*e^4*x^2+191520*b*c
^2*d^3*e^3*x^2-40320*c^3*d^4*e^2*x^2-90440*b^3*d^2*e^4*x+144704*b^2*c*d^3*e
^3*x-85120*b*c^2*d^4*e^2*x+17920*c^3*d^5*e*x+25840*b^3*d^3*e^3-41344*b^2*c*
d^4*e^2+24320*b*c^2*d^5*e-5120*c^3*d^6)/e^7
```

Maxima [A] time = 1.13156, size = 366, normalized size = 1.48

$$\frac{2\left(255255\,(ex+d)^{\frac{19}{2}}c^3-855855\left(2\,c^3d-bc^2e\right)(ex+d)^{\frac{17}{2}}+969969\left(5\,c^3d^2-5\,bc^2de+b^2ce^2\right)(ex+d)^{\frac{15}{2}}-373065\left(20\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(c*x^2+b*x)^3,x, algorithm="maxima")
```

```
[Out] 2/4849845*(255255*(e*x + d)^(19/2)*c^3 - 855855*(2*c^3*d - b*c^2*e)*(e*x +
d)^(17/2) + 969969*(5*c^3*d^2 - 5*b*c^2*d*e + b^2*c*e^2)*(e*x + d)^(15/2) -
 373065*(20*c^3*d^3 - 30*b*c^2*d^2*e + 12*b^2*c*d*e^2 - b^3*e^3)*(e*x + d)^
(13/2) + 1322685*(5*c^3*d^4 - 10*b*c^2*d^3*e + 6*b^2*c*d^2*e^2 - b^3*d*e^3)
*(e*x + d)^(11/2) - 1616615*(2*c^3*d^5 - 5*b*c^2*d^4*e + 4*b^2*c*d^3*e^2 -
b^3*d^2*e^3)*(e*x + d)^(9/2) + 692835*(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^
4*e^2 - b^3*d^3*e^3)*(e*x + d)^(7/2))/e^7
```

Fricas [A] time = 1.94932, size = 1002, normalized size = 4.04

$$\frac{2\left(255255\,c^3e^9x^9+5120\,c^3d^9-24320\,bc^2d^8e+41344\,b^2cd^7e^2-25840\,b^3d^6e^3+45045\left(13\,c^3de^8+19\,bc^2e^9\right)x^8+3003\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(c*x^2+b*x)^3,x, algorithm="fricas")
```

```
[Out] 2/4849845*(255255*c^3*e^9*x^9 + 5120*c^3*d^9 - 24320*b*c^2*d^8*e + 41344*b^
2*c*d^7*e^2 - 25840*b^3*d^6*e^3 + 45045*(13*c^3*d*e^8 + 19*b*c^2*e^9)*x^8 +
 3003*(115*c^3*d^2*e^7 + 665*b*c^2*d*e^8 + 323*b^2*c*e^9)*x^7 + 231*(5*c^3*
d^3*e^6 + 5225*b*c^2*d^2*e^7 + 10013*b^2*c*d*e^8 + 1615*b^3*e^9)*x^6 - 63*(
20*c^3*d^4*e^5 - 95*b*c^2*d^3*e^6 - 22933*b^2*c*d^2*e^7 - 14535*b^3*d*e^8)*
x^5 + 35*(40*c^3*d^5*e^4 - 190*b*c^2*d^4*e^5 + 323*b^2*c*d^3*e^6 + 17119*b^
3*d^2*e^7)*x^4 - 5*(320*c^3*d^6*e^3 - 1520*b*c^2*d^5*e^4 + 2584*b^2*c*d^4*e
^5 - 1615*b^3*d^3*e^6)*x^3 + 6*(320*c^3*d^7*e^2 - 1520*b*c^2*d^6*e^3 + 2584
*b^2*c*d^5*e^4 - 1615*b^3*d^4*e^5)*x^2 - 8*(320*c^3*d^8*e - 1520*b*c^2*d^7*
e^2 + 2584*b^2*c*d^6*e^3 - 1615*b^3*d^5*e^4)*x)*sqrt(e*x + d)/e^7
```

Sympy [B] time = 56.6808, size = 1207, normalized size = 4.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(c*x**2+b*x)**3,x)
```

```
[Out] 2*b**3*d**2*(-d**3*(d + e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d
+ e*x)**(7/2)/7 + (d + e*x)**(9/2)/9)/e**4 + 4*b**3*d*(d**4*(d + e*x)**(3/2
)/3 - 4*d**3*(d + e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e*x)
**(9/2)/9 + (d + e*x)**(11/2)/11)/e**4 + 2*b**3*(-d**5*(d + e*x)**(3/2)/3 +
 d**4*(d + e*x)**(5/2) - 10*d**3*(d + e*x)**(7/2)/7 + 10*d**2*(d + e*x)**(9
/2)/9 - 5*d*(d + e*x)**(11/2)/11 + (d + e*x)**(13/2)/13)/e**4 + 6*b**2*c*d*
*2*(d**4*(d + e*x)**(3/2)/3 - 4*d**3*(d + e*x)**(5/2)/5 + 6*d**2*(d + e*x)*
*(7/2)/7 - 4*d*(d + e*x)**(9/2)/9 + (d + e*x)**(11/2)/11)/e**5 + 12*b**2*c*
d*(-d**5*(d + e*x)**(3/2)/3 + d**4*(d + e*x)**(5/2) - 10*d**3*(d + e*x)**(7
/2)/7 + 10*d**2*(d + e*x)**(9/2)/9 - 5*d*(d + e*x)**(11/2)/11 + (d + e*x)**
(13/2)/13)/e**5 + 6*b**2*c*(d**6*(d + e*x)**(3/2)/3 - 6*d**5*(d + e*x)**(5/
2)/5 + 15*d**4*(d + e*x)**(7/2)/7 - 20*d**3*(d + e*x)**(9/2)/9 + 15*d**2*(d
 + e*x)**(11/2)/11 - 6*d*(d + e*x)**(13/2)/13 + (d + e*x)**(15/2)/15)/e**5
+ 6*b*c**2*d**2*(-d**5*(d + e*x)**(3/2)/3 + d**4*(d + e*x)**(5/2) - 10*d**3
*(d + e*x)**(7/2)/7 + 10*d**2*(d + e*x)**(9/2)/9 - 5*d*(d + e*x)**(11/2)/11
 + (d + e*x)**(13/2)/13)/e**6 + 12*b*c**2*d*(d**6*(d + e*x)**(3/2)/3 - 6*d*
*5*(d + e*x)**(5/2)/5 + 15*d**4*(d + e*x)**(7/2)/7 - 20*d**3*(d + e*x)**(9/
2)/9 + 15*d**2*(d + e*x)**(11/2)/11 - 6*d*(d + e*x)**(13/2)/13 + (d + e*x)*
*(15/2)/15)/e**6 + 6*b*c**2*(-d**7*(d + e*x)**(3/2)/3 + 7*d**6*(d + e*x)**(
5/2)/5 - 3*d**5*(d + e*x)**(7/2) + 35*d**4*(d + e*x)**(9/2)/9 - 35*d**3*(d
+ e*x)**(11/2)/11 + 21*d**2*(d + e*x)**(13/2)/13 - 7*d*(d + e*x)**(15/2)/15
 + (d + e*x)**(17/2)/17)/e**6 + 2*c**3*d**2*(d**6*(d + e*x)**(3/2)/3 - 6*d*
*5*(d + e*x)**(5/2)/5 + 15*d**4*(d + e*x)**(7/2)/7 - 20*d**3*(d + e*x)**(9/
2)/9 + 15*d**2*(d + e*x)**(11/2)/11 - 6*d*(d + e*x)**(13/2)/13 + (d + e*x)*
*(15/2)/15)/e**7 + 4*c**3*d*(-d**7*(d + e*x)**(3/2)/3 + 7*d**6*(d + e*x)**(
5/2)/5 - 3*d**5*(d + e*x)**(7/2) + 35*d**4*(d + e*x)**(9/2)/9 - 35*d**3*(d
+ e*x)**(11/2)/11 + 21*d**2*(d + e*x)**(13/2)/13 - 7*d*(d + e*x)**(15/2)/15
 + (d + e*x)**(17/2)/17)/e**7 + 2*c**3*(d**8*(d + e*x)**(3/2)/3 - 8*d**7*(d
 + e*x)**(5/2)/5 + 4*d**6*(d + e*x)**(7/2) - 56*d**5*(d + e*x)**(9/2)/9 + 7
0*d**4*(d + e*x)**(11/2)/11 - 56*d**3*(d + e*x)**(13/2)/13 + 28*d**2*(d + e
*x)**(15/2)/15 - 8*d*(d + e*x)**(17/2)/17 + (d + e*x)**(19/2)/19)/e**7
```

Giac [B] time = 1.46863, size = 1454, normalized size = 5.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(c*x^2+b*x)^3,x, algorithm="giac")
```

```
[Out] 2/14549535*(46189*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e +
d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*b^3*d^2*e^(-3) + 12597*(315*(x*e +
d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e +
 d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*b^2*c*d^2*e^(-4) + 4845*(693*(x*e
 + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12870*
(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*
b*c^2*d^2*e^(-5) + 323*(3003*(x*e + d)^(15/2) - 20790*(x*e + d)^(13/2)*d +
61425*(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525*(x*e + d)^(
7/2)*d^4 - 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*c^3*d^2*e
^(-6) + 8398*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d
)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*b^3*d*e^
(-3) + 9690*(693*(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e +
d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*
(x*e + d)^(3/2)*d^5)*b^2*c*d*e^(-4) + 1938*(3003*(x*e + d)^(15/2) - 20790*(
x*e + d)^(13/2)*d + 61425*(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3
 + 96525*(x*e + d)^(7/2)*d^4 - 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^
(3/2)*d^6)*b*c^2*d*e^(-5) + 266*(6435*(x*e + d)^(17/2) - 51051*(x*e + d)^(1
5/2)*d + 176715*(x*e + d)^(13/2)*d^2 - 348075*(x*e + d)^(11/2)*d^3 + 425425
*(x*e + d)^(9/2)*d^4 - 328185*(x*e + d)^(7/2)*d^5 + 153153*(x*e + d)^(5/2)*
d^6 - 36465*(x*e + d)^(3/2)*d^7)*c^3*d*e^(-6) + 1615*(693*(x*e + d)^(13/2)
- 4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/
2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*b^3*e^(-3) +
969*(3003*(x*e + d)^(15/2) - 20790*(x*e + d)^(13/2)*d + 61425*(x*e + d)^(11
/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525*(x*e + d)^(7/2)*d^4 - 54054*(x
*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*b^2*c*e^(-4) + 399*(6435*(x*
e + d)^(17/2) - 51051*(x*e + d)^(15/2)*d + 176715*(x*e + d)^(13/2)*d^2 - 34
8075*(x*e + d)^(11/2)*d^3 + 425425*(x*e + d)^(9/2)*d^4 - 328185*(x*e + d)^(
7/2)*d^5 + 153153*(x*e + d)^(5/2)*d^6 - 36465*(x*e + d)^(3/2)*d^7)*b*c^2*e^
(-5) + 7*(109395*(x*e + d)^(19/2) - 978120*(x*e + d)^(17/2)*d + 3879876*(x*
e + d)^(15/2)*d^2 - 8953560*(x*e + d)^(13/2)*d^3 + 13226850*(x*e + d)^(11/2
)*d^4 - 12932920*(x*e + d)^(9/2)*d^5 + 8314020*(x*e + d)^(7/2)*d^6 - 332560
8*(x*e + d)^(5/2)*d^7 + 692835*(x*e + d)^(3/2)*d^8)*c^3*e^(-6))*e^(-1)
```

3.355 $\int (d+ex)^{3/2} \left(bx+cx^2\right)^3 \, dx$

Optimal. Leaf size=248

$$\frac{6c(d+ex)^{13/2}\left(b^2e^2-5bcde+5c^2d^2\right)}{13e^7}-\frac{2(d+ex)^{11/2}(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{11e^7}+\frac{2d(d+ex)^{9/2}(cd-be)\left(b^2e^2-\right.}{3e^7}$$

```
[Out] (2*d^3*(c*d - b*e)^3*(d + e*x)^(5/2))/(5*e^7) - (6*d^2*(c*d - b*e)^2*(2*c*d
 - b*e)*(d + e*x)^(7/2))/(7*e^7) + (2*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e
+ b^2*e^2)*(d + e*x)^(9/2))/(3*e^7) - (2*(2*c*d - b*e)*(10*c^2*d^2 - 10*b*c
*d*e + b^2*e^2)*(d + e*x)^(11/2))/(11*e^7) + (6*c*(5*c^2*d^2 - 5*b*c*d*e +
b^2*e^2)*(d + e*x)^(13/2))/(13*e^7) - (2*c^2*(2*c*d - b*e)*(d + e*x)^(15/2)
)/(5*e^7) + (2*c^3*(d + e*x)^(17/2))/(17*e^7)
```

Rubi [A] time = 0.105789, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {698}

$$\frac{6c(d+ex)^{13/2}\left(b^2e^2-5bcde+5c^2d^2\right)}{13e^7}-\frac{2(d+ex)^{11/2}(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{11e^7}+\frac{2d(d+ex)^{9/2}(cd-be)\left(b^2e^2-\right.}{3e^7}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*(b*x + c*x^2)^3,x]
```

```
[Out] (2*d^3*(c*d - b*e)^3*(d + e*x)^(5/2))/(5*e^7) - (6*d^2*(c*d - b*e)^2*(2*c*d
 - b*e)*(d + e*x)^(7/2))/(7*e^7) + (2*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e
+ b^2*e^2)*(d + e*x)^(9/2))/(3*e^7) - (2*(2*c*d - b*e)*(10*c^2*d^2 - 10*b*c
*d*e + b^2*e^2)*(d + e*x)^(11/2))/(11*e^7) + (6*c*(5*c^2*d^2 - 5*b*c*d*e +
b^2*e^2)*(d + e*x)^(13/2))/(13*e^7) - (2*c^2*(2*c*d - b*e)*(d + e*x)^(15/2)
)/(5*e^7) + (2*c^3*(d + e*x)^(17/2))/(17*e^7)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^{3/2}\left(bx+cx^2\right)^3\,dx=\int\left(\frac{d^3(cd-be)^3(d+ex)^{3/2}}{e^6}-\frac{3d^2(cd-be)^2(2cd-be)(d+ex)^{5/2}}{e^6}+\frac{3d(cd-be)\left(5c^2d^2-5b\right.}{e^6}\right.$$

$$=\frac{2d^3(cd-be)^3(d+ex)^{5/2}}{5e^7}-\frac{6d^2(cd-be)^2(2cd-be)(d+ex)^{7/2}}{7e^7}+\frac{2d(cd-be)\left(5c^2d^2-5bcde\right.}{3e^7}$$

Mathematica [A] time = 0.161388, size = 206, normalized size = 0.83

$$\frac{2(d+ex)^{5/2}\left(58905c(d+ex)^4\left(b^2e^2-5bcde+5c^2d^2\right)-23205(d+ex)^3(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)+85085d(d+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)*(b*x + c*x^2)^3,x]
```

```
[Out] (2*(d + e*x)^(5/2)*(51051*d^3*(c*d - b*e)^3 - 109395*d^2*(c*d - b*e)^2*(2*c
*d - b*e)*(d + e*x) + 85085*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)
*(d + e*x)^2 - 23205*(2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e + b^2*e^2)*(d +
 e*x)^3 + 58905*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*(d + e*x)^4 - 51051*c^2
*(2*c*d - b*e)*(d + e*x)^5 + 15015*c^3*(d + e*x)^6))/(255255*e^7)
```

Maple [A] time = 0.049, size = 286, normalized size = 1.2

$$-\frac{-30030\,c^3x^6e^6-102102\,bc^2e^6x^5+24024\,c^3de^5x^5-117810\,b^2ce^6x^4+78540\,bc^2de^5x^4-18480\,c^3d^2e^4x^4-46410\,b^3e^6\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(c*x^2+b*x)^3,x)
```

```
[Out] -2/255255*(e*x+d)^(5/2)*(-15015*c^3*e^6*x^6-51051*b*c^2*e^6*x^5+12012*c^3*d
*e^5*x^5-58905*b^2*c*e^6*x^4+39270*b*c^2*d*e^5*x^4-9240*c^3*d^2*e^4*x^4-232
05*b^3*e^6*x^3+42840*b^2*c*d*e^5*x^3-28560*b*c^2*d^2*e^4*x^3+6720*c^3*d^3*e
^3*x^3+15470*b^3*d*e^5*x^2-28560*b^2*c*d^2*e^4*x^2+19040*b*c^2*d^3*e^3*x^2-
4480*c^3*d^4*e^2*x^2-8840*b^3*d^2*e^4*x+16320*b^2*c*d^3*e^3*x-10880*b*c^2*d
^4*e^2*x+2560*c^3*d^5*e*x+3536*b^3*d^3*e^3-6528*b^2*c*d^4*e^2+4352*b*c^2*d^
5*e-1024*c^3*d^6)/e^7
```

Maxima [A] time = 1.14933, size = 366, normalized size = 1.48

$$\frac{2\left(15015\,(ex+d)^{\frac{17}{2}}c^3-51051\left(2\,c^3d-bc^2e\right)(ex+d)^{\frac{15}{2}}+58905\left(5\,c^3d^2-5\,bc^2de+b^2ce^2\right)(ex+d)^{\frac{13}{2}}-23205\left(20\,c^3d^3\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x)^3,x, algorithm="maxima")
```

```
[Out] 2/255255*(15015*(e*x + d)^(17/2)*c^3 - 51051*(2*c^3*d - b*c^2*e)*(e*x + d)^
(15/2) + 58905*(5*c^3*d^2 - 5*b*c^2*d*e + b^2*c*e^2)*(e*x + d)^(13/2) - 232
05*(20*c^3*d^3 - 30*b*c^2*d^2*e + 12*b^2*c*d*e^2 - b^3*e^3)*(e*x + d)^(11/2
) + 85085*(5*c^3*d^4 - 10*b*c^2*d^3*e + 6*b^2*c*d^2*e^2 - b^3*d*e^3)*(e*x +
 d)^(9/2) - 109395*(2*c^3*d^5 - 5*b*c^2*d^4*e + 4*b^2*c*d^3*e^2 - b^3*d^2*e
^3)*(e*x + d)^(7/2) + 51051*(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 - b^
3*d^3*e^3)*(e*x + d)^(5/2))/e^7
```

Fricas [A] time = 2.03218, size = 841, normalized size = 3.39

$$\frac{2\left(15015\,c^3e^8x^8+1024\,c^3d^8-4352\,bc^2d^7e+6528\,b^2cd^6e^2-3536\,b^3d^5e^3+3003\left(6\,c^3de^7+17\,bc^2e^8\right)x^7+231\left(c^3d^2e^6\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x)^3,x, algorithm="fricas")
```

```
[Out] 2/255255*(15015*c^3*e^8*x^8 + 1024*c^3*d^8 - 4352*b*c^2*d^7*e + 6528*b^2*c*
d^6*e^2 - 3536*b^3*d^5*e^3 + 3003*(6*c^3*d*e^7 + 17*b*c^2*e^8)*x^7 + 231*(c
^3*d^2*e^6 + 272*b*c^2*d*e^7 + 255*b^2*c*e^8)*x^6 - 21*(12*c^3*d^3*e^5 - 51
*b*c^2*d^2*e^6 - 3570*b^2*c*d*e^7 - 1105*b^3*e^8)*x^5 + 35*(8*c^3*d^4*e^4 -
 34*b*c^2*d^3*e^5 + 51*b^2*c*d^2*e^6 + 884*b^3*d*e^7)*x^4 - 5*(64*c^3*d^5*e
^3 - 272*b*c^2*d^4*e^4 + 408*b^2*c*d^3*e^5 - 221*b^3*d^2*e^6)*x^3 + 6*(64*c
^3*d^6*e^2 - 272*b*c^2*d^5*e^3 + 408*b^2*c*d^4*e^4 - 221*b^3*d^3*e^5)*x^2 -
 8*(64*c^3*d^7*e - 272*b*c^2*d^6*e^2 + 408*b^2*c*d^5*e^3 - 221*b^3*d^4*e^4)
*x)*sqrt(e*x + d)/e^7
```

Sympy [B] time = 34.4452, size = 738, normalized size = 2.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(c*x**2+b*x)**3,x)
```

```
[Out] 2*b**3*d*(-d**3*(d + e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d + e
*x)**(7/2)/7 + (d + e*x)**(9/2)/9)/e**4 + 2*b**3*(d**4*(d + e*x)**(3/2)/3 -
 4*d**3*(d + e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e*x)**(9/
2)/9 + (d + e*x)**(11/2)/11)/e**4 + 6*b**2*c*d*(d**4*(d + e*x)**(3/2)/3 - 4
*d**3*(d + e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e*x)**(9/2)
/9 + (d + e*x)**(11/2)/11)/e**5 + 6*b**2*c*(-d**5*(d + e*x)**(3/2)/3 + d**4
*(d + e*x)**(5/2) - 10*d**3*(d + e*x)**(7/2)/7 + 10*d**2*(d + e*x)**(9/2)/9
 - 5*d*(d + e*x)**(11/2)/11 + (d + e*x)**(13/2)/13)/e**5 + 6*b*c**2*d*(-d**
5*(d + e*x)**(3/2)/3 + d**4*(d + e*x)**(5/2) - 10*d**3*(d + e*x)**(7/2)/7 +
 10*d**2*(d + e*x)**(9/2)/9 - 5*d*(d + e*x)**(11/2)/11 + (d + e*x)**(13/2)/
13)/e**6 + 6*b*c**2*(d**6*(d + e*x)**(3/2)/3 - 6*d**5*(d + e*x)**(5/2)/5 +
15*d**4*(d + e*x)**(7/2)/7 - 20*d**3*(d + e*x)**(9/2)/9 + 15*d**2*(d + e*x)
**(11/2)/11 - 6*d*(d + e*x)**(13/2)/13 + (d + e*x)**(15/2)/15)/e**6 + 2*c**
3*d*(d**6*(d + e*x)**(3/2)/3 - 6*d**5*(d + e*x)**(5/2)/5 + 15*d**4*(d + e*x
)**(7/2)/7 - 20*d**3*(d + e*x)**(9/2)/9 + 15*d**2*(d + e*x)**(11/2)/11 - 6*
d*(d + e*x)**(13/2)/13 + (d + e*x)**(15/2)/15)/e**7 + 2*c**3*(-d**7*(d + e*
x)**(3/2)/3 + 7*d**6*(d + e*x)**(5/2)/5 - 3*d**5*(d + e*x)**(7/2) + 35*d**4
*(d + e*x)**(9/2)/9 - 35*d**3*(d + e*x)**(11/2)/11 + 21*d**2*(d + e*x)**(13
/2)/13 - 7*d*(d + e*x)**(15/2)/15 + (d + e*x)**(17/2)/17)/e**7
```

Giac [B] time = 1.35408, size = 892, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x)^3,x, algorithm="giac")
```

```
[Out] 2/765765*(2431*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^
(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*b^3*d*e^(-3) + 663*(315*(x*e + d)^(11/
2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/
2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*b^2*c*d*e^(-4) + 255*(693*(x*e + d)^(13/
2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12870*(x*e + d)^
(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*b*c^2*d*e^
(-5) + 17*(3003*(x*e + d)^(15/2) - 20790*(x*e + d)^(13/2)*d + 61425*(x*e +
d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525*(x*e + d)^(7/2)*d^4 - 54
```

```
054*(x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*c^3*d*e^(-6) + 221*(31
5*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 27
72*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*b^3*e^(-3) + 255*(693*(x
*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 1287
0*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5
)*b^2*c*e^(-4) + 51*(3003*(x*e + d)^(15/2) - 20790*(x*e + d)^(13/2)*d + 614
25*(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525*(x*e + d)^(7/2
)*d^4 - 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*b*c^2*e^(-5)
 + 7*(6435*(x*e + d)^(17/2) - 51051*(x*e + d)^(15/2)*d + 176715*(x*e + d)^(
13/2)*d^2 - 348075*(x*e + d)^(11/2)*d^3 + 425425*(x*e + d)^(9/2)*d^4 - 3281
85*(x*e + d)^(7/2)*d^5 + 153153*(x*e + d)^(5/2)*d^6 - 36465*(x*e + d)^(3/2)
*d^7)*c^3*e^(-6))*e^(-1)
```

3.356 $\int \sqrt{d+ex}\left(bx+cx^2\right)^3\,dx$

Optimal. Leaf size=248

$$\frac{6c(d+ex)^{11/2}\left(b^2e^2-5bcde+5c^2d^2\right)}{11e^7}-\frac{2(d+ex)^{9/2}(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{9e^7}+\frac{6d(d+ex)^{7/2}(cd-be)\left(b^2e^2-5\right.}{7e^7}$$

```
[Out] (2*d^3*(c*d - b*e)^3*(d + e*x)^(3/2))/(3*e^7) - (6*d^2*(c*d - b*e)^2*(2*c*d
 - b*e)*(d + e*x)^(5/2))/(5*e^7) + (6*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e
+ b^2*e^2)*(d + e*x)^(7/2))/(7*e^7) - (2*(2*c*d - b*e)*(10*c^2*d^2 - 10*b*c
*d*e + b^2*e^2)*(d + e*x)^(9/2))/(9*e^7) + (6*c*(5*c^2*d^2 - 5*b*c*d*e + b^
2*e^2)*(d + e*x)^(11/2))/(11*e^7) - (6*c^2*(2*c*d - b*e)*(d + e*x)^(13/2))/
(13*e^7) + (2*c^3*(d + e*x)^(15/2))/(15*e^7)
```

Rubi [A] time = 0.105513, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {698}

$$\frac{6c(d+ex)^{11/2}\left(b^2e^2-5bcde+5c^2d^2\right)}{11e^7}-\frac{2(d+ex)^{9/2}(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{9e^7}+\frac{6d(d+ex)^{7/2}(cd-be)\left(b^2e^2-5\right.}{7e^7}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(b*x + c*x^2)^3,x]
```

```
[Out] (2*d^3*(c*d - b*e)^3*(d + e*x)^(3/2))/(3*e^7) - (6*d^2*(c*d - b*e)^2*(2*c*d
 - b*e)*(d + e*x)^(5/2))/(5*e^7) + (6*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e
+ b^2*e^2)*(d + e*x)^(7/2))/(7*e^7) - (2*(2*c*d - b*e)*(10*c^2*d^2 - 10*b*c
*d*e + b^2*e^2)*(d + e*x)^(9/2))/(9*e^7) + (6*c*(5*c^2*d^2 - 5*b*c*d*e + b^
2*e^2)*(d + e*x)^(11/2))/(11*e^7) - (6*c^2*(2*c*d - b*e)*(d + e*x)^(13/2))/
(13*e^7) + (2*c^3*(d + e*x)^(15/2))/(15*e^7)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^3\,dx=\int\left(\frac{d^3(cd-be)^3\sqrt{d+ex}}{e^6}-\frac{3d^2(cd-be)^2(2cd-be)(d+ex)^{3/2}}{e^6}+\frac{3d(cd-be)\left(5c^2d^2-5bcde\right.}{e^6}\right.$$

$$=\frac{2d^3(cd-be)^3(d+ex)^{3/2}}{3e^7}-\frac{6d^2(cd-be)^2(2cd-be)(d+ex)^{5/2}}{5e^7}+\frac{6d(cd-be)\left(5c^2d^2-5bcde+\right.}{7e^7}$$

Mathematica [A] time = 0.15012, size = 206, normalized size = 0.83

$$\frac{2(d+ex)^{3/2}\left(12285c(d+ex)^4\left(b^2e^2-5bcde+5c^2d^2\right)-5005(d+ex)^3(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)+19305d(d+ex\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*(b*x + c*x^2)^3,x]
```

```
[Out] (2*(d + e*x)^(3/2)*(15015*d^3*(c*d - b*e)^3 - 27027*d^2*(c*d - b*e)^2*(2*c*
d - b*e)*(d + e*x) + 19305*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*
(d + e*x)^2 - 5005*(2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e + b^2*e^2)*(d + e
*x)^3 + 12285*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*(d + e*x)^4 - 10395*c^2*(
2*c*d - b*e)*(d + e*x)^5 + 3003*c^3*(d + e*x)^6))/(45045*e^7)
```

Maple [A] time = 0.048, size = 286, normalized size = 1.2

$$-\frac{-6006\,c^3x^6e^6-20790\,bc^2e^6x^5+5544\,c^3de^5x^5-24570\,b^2ce^6x^4+18900\,bc^2de^5x^4-5040\,c^3d^2e^4x^4-10010\,b^3e^6x^3+2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(c*x^2+b*x)^3,x)
```

```
[Out] -2/45045*(e*x+d)^(3/2)*(-3003*c^3*e^6*x^6-10395*b*c^2*e^6*x^5+2772*c^3*d*e^
5*x^5-12285*b^2*c*e^6*x^4+9450*b*c^2*d*e^5*x^4-2520*c^3*d^2*e^4*x^4-5005*b^
3*e^6*x^3+10920*b^2*c*d*e^5*x^3-8400*b*c^2*d^2*e^4*x^3+2240*c^3*d^3*e^3*x^3
+4290*b^3*d*e^5*x^2-9360*b^2*c*d^2*e^4*x^2+7200*b*c^2*d^3*e^3*x^2-1920*c^3*
d^4*e^2*x^2-3432*b^3*d^2*e^4*x+7488*b^2*c*d^3*e^3*x-5760*b*c^2*d^4*e^2*x+15
36*c^3*d^5*e*x+2288*b^3*d^3*e^3-4992*b^2*c*d^4*e^2+3840*b*c^2*d^5*e-1024*c^
3*d^6)/e^7
```

Maxima [A] time = 1.13471, size = 366, normalized size = 1.48

$$\frac{2\left(3003\,(ex+d)^{\frac{15}{2}}c^3-10395\left(2\,c^3d-bc^2e\right)(ex+d)^{\frac{13}{2}}+12285\left(5\,c^3d^2-5\,bc^2de+b^2ce^2\right)(ex+d)^{\frac{11}{2}}-5005\left(20\,c^3d^3-3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x)^3,x, algorithm="maxima")
```

```
[Out] 2/45045*(3003*(e*x + d)^(15/2)*c^3 - 10395*(2*c^3*d - b*c^2*e)*(e*x + d)^(1
3/2) + 12285*(5*c^3*d^2 - 5*b*c^2*d*e + b^2*c*e^2)*(e*x + d)^(11/2) - 5005*
(20*c^3*d^3 - 30*b*c^2*d^2*e + 12*b^2*c*d*e^2 - b^3*e^3)*(e*x + d)^(9/2) +
19305*(5*c^3*d^4 - 10*b*c^2*d^3*e + 6*b^2*c*d^2*e^2 - b^3*d*e^3)*(e*x + d)^
(7/2) - 27027*(2*c^3*d^5 - 5*b*c^2*d^4*e + 4*b^2*c*d^3*e^2 - b^3*d^2*e^3)*(
e*x + d)^(5/2) + 15015*(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 - b^3*d^3
*e^3)*(e*x + d)^(3/2))/e^7
```

Fricas [A] time = 2.23182, size = 720, normalized size = 2.9

$$\frac{2\left(3003\,c^3e^7x^7+1024\,c^3d^7-3840\,bc^2d^6e+4992\,b^2cd^5e^2-2288\,b^3d^4e^3+231\left(c^3de^6+45\,bc^2e^7\right)x^6-63\left(4\,c^3d^2e^5-15\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x)^3,x, algorithm="fricas")
```

```
[Out] 2/45045*(3003*c^3*e^7*x^7 + 1024*c^3*d^7 - 3840*b*c^2*d^6*e + 4992*b^2*c*d^
5*e^2 - 2288*b^3*d^4*e^3 + 231*(c^3*d*e^6 + 45*b*c^2*e^7)*x^6 - 63*(4*c^3*d
^2*e^5 - 15*b*c^2*d*e^6 - 195*b^2*c*e^7)*x^5 + 35*(8*c^3*d^3*e^4 - 30*b*c^2
*d^2*e^5 + 39*b^2*c*d*e^6 + 143*b^3*e^7)*x^4 - 5*(64*c^3*d^4*e^3 - 240*b*c^
2*d^3*e^4 + 312*b^2*c*d^2*e^5 - 143*b^3*d*e^6)*x^3 + 6*(64*c^3*d^5*e^2 - 24
0*b*c^2*d^4*e^3 + 312*b^2*c*d^3*e^4 - 143*b^3*d^2*e^5)*x^2 - 8*(64*c^3*d^6*
e - 240*b*c^2*d^5*e^2 + 312*b^2*c*d^4*e^3 - 143*b^3*d^3*e^4)*x)*sqrt(e*x +
d)/e^7
```

Sympy [A] time = 6.26446, size = 326, normalized size = 1.31

$$\frac{2\left(\frac{c^3(d+ex)^{\frac{15}{2}}}{15e^6}+\frac{(d+ex)^{\frac{13}{2}}\left(3bc^2e-6c^3d\right)}{13e^6}+\frac{(d+ex)^{\frac{11}{2}}\left(3b^2ce^2-15bc^2de+15c^3d^2\right)}{11e^6}+\frac{(d+ex)^{\frac{9}{2}}\left(b^3e^3-12b^2cde^2+30bc^2d^2e-20c^3d^3\right)}{9e^6}+\frac{(d+ex)^{\frac{7}{2}}\left(-3b^3de^3+18b^2cd\cdots\right.}{7e^6}\cdots}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(c*x**2+b*x)**3,x)
```

```
[Out] 2*(c**3*(d + e*x)**(15/2)/(15*e**6) + (d + e*x)**(13/2)*(3*b*c**2*e - 6*c**
3*d)/(13*e**6) + (d + e*x)**(11/2)*(3*b**2*c*e**2 - 15*b*c**2*d*e + 15*c**3
*d**2)/(11*e**6) + (d + e*x)**(9/2)*(b**3*e**3 - 12*b**2*c*d*e**2 + 30*b*c*
*2*d**2*e - 20*c**3*d**3)/(9*e**6) + (d + e*x)**(7/2)*(-3*b**3*d*e**3 + 18*
b**2*c*d**2*e**2 - 30*b*c**2*d**3*e + 15*c**3*d**4)/(7*e**6) + (d + e*x)**(
5/2)*(3*b**3*d**2*e**3 - 12*b**2*c*d**3*e**2 + 15*b*c**2*d**4*e - 6*c**3*d*
*5)/(5*e**6) + (d + e*x)**(3/2)*(-b**3*d**3*e**3 + 3*b**2*c*d**4*e**2 - 3*b
*c**2*d**5*e + c**3*d**6)/(3*e**6))/e
```

Giac [A] time = 1.33636, size = 410, normalized size = 1.65

$$\frac{2}{45045}\left(143\left(35\,(xe+d)^{\frac{9}{2}}-135\,(xe+d)^{\frac{7}{2}}d+189\,(xe+d)^{\frac{5}{2}}d^2-105\,(xe+d)^{\frac{3}{2}}d^3\right)b^3e^{(-3)}+39\left(315\,(xe+d)^{\frac{11}{2}}-1540\,(xe+\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x)^3,x, algorithm="giac")
```

```
[Out] 2/45045*(143*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5
/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*b^3*e^(-3) + 39*(315*(x*e + d)^(11/2) -
1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^
3 + 1155*(x*e + d)^(3/2)*d^4)*b^2*c*e^(-4) + 15*(693*(x*e + d)^(13/2) - 409
5*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^
3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*b*c^2*e^(-5) + (30
03*(x*e + d)^(15/2) - 20790*(x*e + d)^(13/2)*d + 61425*(x*e + d)^(11/2)*d^2
 - 100100*(x*e + d)^(9/2)*d^3 + 96525*(x*e + d)^(7/2)*d^4 - 54054*(x*e + d)
^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*c^3*e^(-6))*e^(-1)
```

3.357 $\int \frac{(bx+cx^2)^3}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=244

$$\frac{2c(d+ex)^{9/2}\left(b^2e^2-5bcde+5c^2d^2\right)}{3e^7}-\frac{2(d+ex)^{7/2}(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{7e^7}+\frac{6d(d+ex)^{5/2}(cd-be)\left(b^2e^2-\right.}{5e^7}$$

```
[Out] (2*d^3*(c*d - b*e)^3*Sqrt[d + e*x])/e^7 - (2*d^2*(c*d - b*e)^2*(2*c*d - b*e
)*(d + e*x)^(3/2))/e^7 + (6*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)
*(d + e*x)^(5/2))/(5*e^7) - (2*(2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e + b^2
*e^2)*(d + e*x)^(7/2))/(7*e^7) + (2*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*(d
+ e*x)^(9/2))/(3*e^7) - (6*c^2*(2*c*d - b*e)*(d + e*x)^(11/2))/(11*e^7) + (
2*c^3*(d + e*x)^(13/2))/(13*e^7)
```

Rubi [A] time = 0.101605, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {698}

$$\frac{2c(d+ex)^{9/2}\left(b^2e^2-5bcde+5c^2d^2\right)}{3e^7}-\frac{2(d+ex)^{7/2}(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{7e^7}+\frac{6d(d+ex)^{5/2}(cd-be)\left(b^2e^2-\right.}{5e^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^3/Sqrt[d + e*x],x]
```



```
[Out] (2*d^3*(c*d - b*e)^3*Sqrt[d + e*x])/e^7 - (2*d^2*(c*d - b*e)^2*(2*c*d - b*e
)*(d + e*x)^(3/2))/e^7 + (6*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)
*(d + e*x)^(5/2))/(5*e^7) - (2*(2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e + b^2
*e^2)*(d + e*x)^(7/2))/(7*e^7) + (2*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*(d
+ e*x)^(9/2))/(3*e^7) - (6*c^2*(2*c*d - b*e)*(d + e*x)^(11/2))/(11*e^7) + (
2*c^3*(d + e*x)^(13/2))/(13*e^7)
```


Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(bx+cx^2\right)^3}{\sqrt{d+ex}}\,dx=\int \left(\frac{d^3(cd-be)^3}{e^6\sqrt{d+ex}}-\frac{3d^2(cd-be)^2(2cd-be)\sqrt{d+ex}}{e^6}+\frac{3d(cd-be)\left(5c^2d^2-5bcde+b^2e^2\right)(d+ex)^{3}}{e^6}\right.$$

$$=\frac{2d^3(cd-be)^3\sqrt{d+ex}}{e^7}-\frac{2d^2(cd-be)^2(2cd-be)(d+ex)^{3/2}}{e^7}+\frac{6d(cd-be)\left(5c^2d^2-5bcde+b^2e^2\right)(}{5e^7}$$

Mathematica [A] time = 0.135167, size = 206, normalized size = 0.84

$$2\sqrt{d+ex}\left(5005c(d+ex)^4\left(b^2e^2-5bcde+5c^2d^2\right)-2145(d+ex)^3(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)+9009d(d+ex)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^3/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[d + e*x]*(15015*d^3*(c*d - b*e)^3 - 15015*d^2*(c*d - b*e)^2*(2*c*d
- b*e)*(d + e*x) + 9009*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*(d
+ e*x)^2 - 2145*(2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e + b^2*e^2)*(d + e*x)
^3 + 5005*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*(d + e*x)^4 - 4095*c^2*(2*c*d
 - b*e)*(d + e*x)^5 + 1155*c^3*(d + e*x)^6))/(15015*e^7)
```

Maple [A] time = 0.053, size = 286, normalized size = 1.2

$$-\frac{-2310\,c^3x^6e^6-8190\,bc^2e^6x^5+2520\,c^3de^5x^5-10010\,b^2ce^6x^4+9100\,bc^2de^5x^4-2800\,c^3d^2e^4x^4-4290\,b^3e^6x^3+11440\,b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^3/(e*x+d)^(1/2),x)
```

```
[Out] -2/15015*(-1155*c^3*e^6*x^6-4095*b*c^2*e^6*x^5+1260*c^3*d*e^5*x^5-5005*b^2*
c*e^6*x^4+4550*b*c^2*d*e^5*x^4-1400*c^3*d^2*e^4*x^4-2145*b^3*e^6*x^3+5720*b
^2*c*d*e^5*x^3-5200*b*c^2*d^2*e^4*x^3+1600*c^3*d^3*e^3*x^3+2574*b^3*d*e^5*x
^2-6864*b^2*c*d^2*e^4*x^2+6240*b*c^2*d^3*e^3*x^2-1920*c^3*d^4*e^2*x^2-3432*
b^3*d^2*e^4*x+9152*b^2*c*d^3*e^3*x-8320*b*c^2*d^4*e^2*x+2560*c^3*d^5*e*x+68
64*b^3*d^3*e^3-18304*b^2*c*d^4*e^2+16640*b*c^2*d^5*e-5120*c^3*d^6)*(e*x+d)^
(1/2)/e^7
```

Maxima [A] time = 1.14775, size = 389, normalized size = 1.59

$$2\left(\frac{429\left(5\,(ex+d)^{\frac{7}{2}}-21\,(ex+d)^{\frac{5}{2}}d+35\,(ex+d)^{\frac{3}{2}}d^2-35\,\sqrt{ex+d}d^3\right)b^3}{e^3}+\frac{143\left(35\,(ex+d)^{\frac{9}{2}}-180\,(ex+d)^{\frac{7}{2}}d+378\,(ex+d)^{\frac{5}{2}}d^2-420\,(ex+d)^{\frac{3}{2}}d^3+315\,\sqrt{ex+d}d^4\right)b^2c}{e^4}+\frac{65\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15015*(429*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)
*d^2 - 35*sqrt(e*x + d)*d^3)*b^3/e^3 + 143*(35*(e*x + d)^(9/2) - 180*(e*x +
 d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(
e*x + d)*d^4)*b^2*c/e^4 + 65*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d +
 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*
d^4 - 693*sqrt(e*x + d)*d^5)*b*c^2/e^5 + 5*(231*(e*x + d)^(13/2) - 1638*(e*
x + d)^(11/2)*d + 5005*(e*x + d)^(9/2)*d^2 - 8580*(e*x + d)^(7/2)*d^3 + 900
9*(e*x + d)^(5/2)*d^4 - 6006*(e*x + d)^(3/2)*d^5 + 3003*sqrt(e*x + d)*d^6)*
c^3/e^6)/e
```

Fricas [A] time = 2.19685, size = 630, normalized size = 2.58

$$\frac{2\left(1155\,c^3e^6x^6+5120\,c^3d^6-16640\,bc^2d^5e+18304\,b^2cd^4e^2-6864\,b^3d^3e^3-315\left(4\,c^3de^5-13\,bc^2e^6\right)x^5+35\left(40\,c^3d^2e^4-1\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15015*(1155*c^3*e^6*x^6 + 5120*c^3*d^6 - 16640*b*c^2*d^5*e + 18304*b^2*c*
d^4*e^2 - 6864*b^3*d^3*e^3 - 315*(4*c^3*d*e^5 - 13*b*c^2*e^6)*x^5 + 35*(40*
c^3*d^2*e^4 - 130*b*c^2*d*e^5 + 143*b^2*c*e^6)*x^4 - 5*(320*c^3*d^3*e^3 - 1
040*b*c^2*d^2*e^4 + 1144*b^2*c*d*e^5 - 429*b^3*e^6)*x^3 + 6*(320*c^3*d^4*e^
2 - 1040*b*c^2*d^3*e^3 + 1144*b^2*c*d^2*e^4 - 429*b^3*d*e^5)*x^2 - 8*(320*c
^3*d^5*e - 1040*b*c^2*d^4*e^2 + 1144*b^2*c*d^3*e^3 - 429*b^3*d^2*e^4)*x)*sq
rt(e*x + d)/e^7
```

Sympy [A] time = 115.984, size = 745, normalized size = 3.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**3/(e*x+d)**(1/2),x)
```

```
[Out] Piecewise((-(2*b**3*d*(-d**3/sqrt(d + e*x) - 3*d**2*sqrt(d + e*x) + d*(d +
e*x)**(3/2) - (d + e*x)**(5/2)/5)/e**3 + 2*b**3*(d**4/sqrt(d + e*x) + 4*d**
3*sqrt(d + e*x) - 2*d**2*(d + e*x)**(3/2) + 4*d*(d + e*x)**(5/2)/5 - (d + e
*x)**(7/2)/7)/e**3 + 6*b**2*c*d*(d**4/sqrt(d + e*x) + 4*d**3*sqrt(d + e*x)
- 2*d**2*(d + e*x)**(3/2) + 4*d*(d + e*x)**(5/2)/5 - (d + e*x)**(7/2)/7)/e*
*4 + 6*b**2*c*(-d**5/sqrt(d + e*x) - 5*d**4*sqrt(d + e*x) + 10*d**3*(d + e*
x)**(3/2)/3 - 2*d**2*(d + e*x)**(5/2) + 5*d*(d + e*x)**(7/2)/7 - (d + e*x)*
*(9/2)/9)/e**4 + 6*b*c**2*d*(-d**5/sqrt(d + e*x) - 5*d**4*sqrt(d + e*x) + 1
0*d**3*(d + e*x)**(3/2)/3 - 2*d**2*(d + e*x)**(5/2) + 5*d*(d + e*x)**(7/2)/
7 - (d + e*x)**(9/2)/9)/e**5 + 6*b*c**2*(d**6/sqrt(d + e*x) + 6*d**5*sqrt(d
 + e*x) - 5*d**4*(d + e*x)**(3/2) + 4*d**3*(d + e*x)**(5/2) - 15*d**2*(d +
e*x)**(7/2)/7 + 2*d*(d + e*x)**(9/2)/3 - (d + e*x)**(11/2)/11)/e**5 + 2*c**
3*d*(d**6/sqrt(d + e*x) + 6*d**5*sqrt(d + e*x) - 5*d**4*(d + e*x)**(3/2) +
4*d**3*(d + e*x)**(5/2) - 15*d**2*(d + e*x)**(7/2)/7 + 2*d*(d + e*x)**(9/2)
/3 - (d + e*x)**(11/2)/11)/e**6 + 2*c**3*(-d**7/sqrt(d + e*x) - 7*d**6*sqrt
(d + e*x) + 7*d**5*(d + e*x)**(3/2) - 7*d**4*(d + e*x)**(5/2) + 5*d**3*(d +
 e*x)**(7/2) - 7*d**2*(d + e*x)**(9/2)/3 + 7*d*(d + e*x)**(11/2)/11 - (d +
e*x)**(13/2)/13)/e**6)/e, Ne(e, 0)), ((b**3*x**4/4 + 3*b**2*c*x**5/5 + b*c*
*2*x**6/2 + c**3*x**7/7)/sqrt(d), True))
```

Giac [A] time = 1.3159, size = 412, normalized size = 1.69

$$\frac{2}{15015}\left(429\left(5\,(xe+d)^{\frac{7}{2}}-21\,(xe+d)^{\frac{5}{2}}d+35\,(xe+d)^{\frac{3}{2}}d^2-35\,\sqrt{xe+d}d^3\right)b^3e^{(-3)}+143\left(35\,(xe+d)^{\frac{9}{2}}-180\,(xe+d)^{\frac{7}{2}}d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15015*(429*(5*(x*e + d)^(7/2) - 21*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)
*d^2 - 35*sqrt(x*e + d)*d^3)*b^3*e^(-3) + 143*(35*(x*e + d)^(9/2) - 180*(x*
e + d)^(7/2)*d + 378*(x*e + d)^(5/2)*d^2 - 420*(x*e + d)^(3/2)*d^3 + 315*sq
rt(x*e + d)*d^4)*b^2*c*e^(-4) + 65*(63*(x*e + d)^(11/2) - 385*(x*e + d)^(9/
2)*d + 990*(x*e + d)^(7/2)*d^2 - 1386*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^
```

```
(3/2)*d^4 - 693*sqrt(x*e + d)*d^5)*b*c^2*e^(-5) + 5*(231*(x*e + d)^(13/2) -
 1638*(x*e + d)^(11/2)*d + 5005*(x*e + d)^(9/2)*d^2 - 8580*(x*e + d)^(7/2)*
d^3 + 9009*(x*e + d)^(5/2)*d^4 - 6006*(x*e + d)^(3/2)*d^5 + 3003*sqrt(x*e +
 d)*d^6)*c^3*e^(-6))*e^(-1)
```

3.358 $\int \frac{(bx+cx^2)^3}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=242

$$\frac{6c(d+ex)^{7/2}\left(b^2e^2-5bcde+5c^2d^2\right)}{7e^7}-\frac{2(d+ex)^{5/2}(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{5e^7}+\frac{2d(d+ex)^{3/2}(cd-be)\left(b^2e^2-\right.}{e^7}$$

```
[Out] (-2*d^3*(c*d - b*e)^3)/(e^7*Sqrt[d + e*x]) - (6*d^2*(c*d - b*e)^2*(2*c*d -
b*e)*Sqrt[d + e*x])/e^7 + (2*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2
)*(d + e*x)^(3/2))/e^7 - (2*(2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e + b^2*e^
2)*(d + e*x)^(5/2))/(5*e^7) + (6*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*(d + e
*x)^(7/2))/(7*e^7) - (2*c^2*(2*c*d - b*e)*(d + e*x)^(9/2))/(3*e^7) + (2*c^3
*(d + e*x)^(11/2))/(11*e^7)
```

Rubi [A] time = 0.103651, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {698}

$$\frac{6c(d+ex)^{7/2}\left(b^2e^2-5bcde+5c^2d^2\right)}{7e^7}-\frac{2(d+ex)^{5/2}(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{5e^7}+\frac{2d(d+ex)^{3/2}(cd-be)\left(b^2e^2-\right.}{e^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^3/(d + e*x)^(3/2),x]
```

```
[Out] (-2*d^3*(c*d - b*e)^3)/(e^7*Sqrt[d + e*x]) - (6*d^2*(c*d - b*e)^2*(2*c*d -
b*e)*Sqrt[d + e*x])/e^7 + (2*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2
)*(d + e*x)^(3/2))/e^7 - (2*(2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e + b^2*e^
2)*(d + e*x)^(5/2))/(5*e^7) + (6*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*(d + e
*x)^(7/2))/(7*e^7) - (2*c^2*(2*c*d - b*e)*(d + e*x)^(9/2))/(3*e^7) + (2*c^3
*(d + e*x)^(11/2))/(11*e^7)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{3/2}}\,dx = \int \left(\frac{d^3(cd-be)^3}{e^6(d+ex)^{3/2}}-\frac{3d^2(cd-be)^2(2cd-be)}{e^6\sqrt{d+ex}}+\frac{3d(cd-be)\left(5c^2d^2-5bcde+b^2e^2\right)\sqrt{d+ex}}{e^6}+\frac{(2cd}{\ }\right.$$

$$=-\frac{2d^3(cd-be)^3}{e^7\sqrt{d+ex}}-\frac{6d^2(cd-be)^2(2cd-be)\sqrt{d+ex}}{e^7}+\frac{2d(cd-be)\left(5c^2d^2-5bcde+b^2e^2\right)(d+ex)^{3/}}{e^7}$$

Mathematica [A] time = 0.142685, size = 206, normalized size = 0.85

$$\frac{2\left(495c(d+ex)^4\left(b^2e^2-5bcde+5c^2d^2\right)-231(d+ex)^3(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)+1155d(d+ex)^2(cd-be)\left(\right.\right.}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^3/(d + e*x)^(3/2),x]
```

```
[Out] (2*(-1155*d^3*(c*d - b*e)^3 - 3465*d^2*(c*d - b*e)^2*(2*c*d - b*e)*(d + e*x
) + 1155*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*(d + e*x)^2 - 231*
(2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e + b^2*e^2)*(d + e*x)^3 + 495*c*(5*c^
2*d^2 - 5*b*c*d*e + b^2*e^2)*(d + e*x)^4 - 385*c^2*(2*c*d - b*e)*(d + e*x)^
5 + 105*c^3*(d + e*x)^6))/(1155*e^7*Sqrt[d + e*x])
```

Maple [A] time = 0.047, size = 286, normalized size = 1.2

$$210\,c^{3}x^{6}e^{6}+770\,bc^{2}e^{6}x^{5}-280\,c^{3}de^{5}x^{5}+990\,b^{2}ce^{6}x^{4}-1100\,bc^{2}de^{5}x^{4}+400\,c^{3}d^{2}e^{4}x^{4}+462\,b^{3}e^{6}x^{3}-1584\,b^{2}cde^{5}x^{3}+176$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^3/(e*x+d)^(3/2),x)
```

```
[Out] 2/1155*(105*c^3*e^6*x^6+385*b*c^2*e^6*x^5-140*c^3*d*e^5*x^5+495*b^2*c*e^6*x
^4-550*b*c^2*d*e^5*x^4+200*c^3*d^2*e^4*x^4+231*b^3*e^6*x^3-792*b^2*c*d*e^5*
x^3+880*b*c^2*d^2*e^4*x^3-320*c^3*d^3*e^3*x^3-462*b^3*d*e^5*x^2+1584*b^2*c*
d^2*e^4*x^2-1760*b*c^2*d^3*e^3*x^2+640*c^3*d^4*e^2*x^2+1848*b^3*d^2*e^4*x-6
336*b^2*c*d^3*e^3*x+7040*b*c^2*d^4*e^2*x-2560*c^3*d^5*e*x+3696*b^3*d^3*e^3-
12672*b^2*c*d^4*e^2+14080*b*c^2*d^5*e-5120*c^3*d^6)/(e*x+d)^(1/2)/e^7
```

Maxima [A] time = 1.15697, size = 377, normalized size = 1.56

$$\frac{2\left(\frac{105\,(ex+d)^{\frac{11}{2}}c^{3}-385\left(2\,c^{3}d-bc^{2}e\right)(ex+d)^{\frac{9}{2}}+495\left(5\,c^{3}d^{2}-5\,bc^{2}de+b^{2}ce^{2}\right)(ex+d)^{\frac{7}{2}}-231\left(20\,c^{3}d^{3}-30\,bc^{2}d^{2}e+12\,b^{2}cde^{2}-b^{3}e^{3}\right)(ex+d)^{\frac{5}{2}}+1155\left(5\,c^{3}d^{4}-10\,bc^{2}d^{3}e+6\right.}{e^{6}}}{1155\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/1155*((105*(e*x + d)^(11/2)*c^3 - 385*(2*c^3*d - b*c^2*e)*(e*x + d)^(9/2)
 + 495*(5*c^3*d^2 - 5*b*c^2*d*e + b^2*c*e^2)*(e*x + d)^(7/2) - 231*(20*c^3*
d^3 - 30*b*c^2*d^2*e + 12*b^2*c*d*e^2 - b^3*e^3)*(e*x + d)^(5/2) + 1155*(5*
c^3*d^4 - 10*b*c^2*d^3*e + 6*b^2*c*d^2*e^2 - b^3*d*e^3)*(e*x + d)^(3/2) - 3
465*(2*c^3*d^5 - 5*b*c^2*d^4*e + 4*b^2*c*d^3*e^2 - b^3*d^2*e^3)*sqrt(e*x +
d))/e^6 - 1155*(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 - b^3*d^3*e^3)/(s
qrt(e*x + d)*e^6))/e
```

Fricas [A] time = 2.30352, size = 629, normalized size = 2.6

$$2\left(105\,c^{3}e^{6}x^{6}-5120\,c^{3}d^{6}+14080\,bc^{2}d^{5}e-12672\,b^{2}cd^{4}e^{2}+3696\,b^{3}d^{3}e^{3}-35\left(4\,c^{3}de^{5}-11\,bc^{2}e^{6}\right)x^{5}+5\left(40\,c^{3}d^{2}e^{4}-110\,b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*c^3*e^6*x^6 - 5120*c^3*d^6 + 14080*b*c^2*d^5*e - 12672*b^2*c*d^
4*e^2 + 3696*b^3*d^3*e^3 - 35*(4*c^3*d*e^5 - 11*b*c^2*e^6)*x^5 + 5*(40*c^3*
d^2*e^4 - 110*b*c^2*d*e^5 + 99*b^2*c*e^6)*x^4 - (320*c^3*d^3*e^3 - 880*b*c^
2*d^2*e^4 + 792*b^2*c*d*e^5 - 231*b^3*e^6)*x^3 + 2*(320*c^3*d^4*e^2 - 880*b
*c^2*d^3*e^3 + 792*b^2*c*d^2*e^4 - 231*b^3*d*e^5)*x^2 - 8*(320*c^3*d^5*e -
880*b*c^2*d^4*e^2 + 792*b^2*c*d^3*e^3 - 231*b^3*d^2*e^4)*x)*sqrt(e*x + d)/(
e^8*x + d*e^7)
```

Sympy [A] time = 52.0278, size = 284, normalized size = 1.17

$$\frac{2c^3(d+ex)^{\frac{11}{2}}}{11e^7}+\frac{2d^3(be-cd)^3}{e^7\sqrt{d+ex}}+\frac{(d+ex)^{\frac{9}{2}}\left(6bc^2e-12c^3d\right)}{9e^7}+\frac{(d+ex)^{\frac{7}{2}}\left(6b^2ce^2-30bc^2de+30c^3d^2\right)}{7e^7}+\frac{(d+ex)^{\frac{5}{2}}\left(2b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**3/(e*x+d)**(3/2),x)
```

```
[Out] 2*c**3*(d + e*x)**(11/2)/(11*e**7) + 2*d**3*(b*e - c*d)**3/(e**7*sqrt(d + e
*x)) + (d + e*x)**(9/2)*(6*b*c**2*e - 12*c**3*d)/(9*e**7) + (d + e*x)**(7/2
)*(6*b**2*c*e**2 - 30*b*c**2*d*e + 30*c**3*d**2)/(7*e**7) + (d + e*x)**(5/2
)*(2*b**3*e**3 - 24*b**2*c*d*e**2 + 60*b*c**2*d**2*e - 40*c**3*d**3)/(5*e**
7) + (d + e*x)**(3/2)*(-6*b**3*d*e**3 + 36*b**2*c*d**2*e**2 - 60*b*c**2*d**
3*e + 30*c**3*d**4)/(3*e**7) + sqrt(d + e*x)*(6*b**3*d**2*e**3 - 24*b**2*c*
d**3*e**2 + 30*b*c**2*d**4*e - 12*c**3*d**5)/e**7
```

Giac [A] time = 1.36876, size = 501, normalized size = 2.07

$$\frac{2}{1155}\left(105(xe+d)^{\frac{11}{2}}c^3e^{70}-770(xe+d)^{\frac{9}{2}}c^3de^{70}+2475(xe+d)^{\frac{7}{2}}c^3d^2e^{70}-4620(xe+d)^{\frac{5}{2}}c^3d^3e^{70}+5775(xe+d)^{\frac{3}{2}}c^3d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] 2/1155*(105*(x*e + d)^(11/2)*c^3*e^70 - 770*(x*e + d)^(9/2)*c^3*d*e^70 + 24
75*(x*e + d)^(7/2)*c^3*d^2*e^70 - 4620*(x*e + d)^(5/2)*c^3*d^3*e^70 + 5775*
(x*e + d)^(3/2)*c^3*d^4*e^70 - 6930*sqrt(x*e + d)*c^3*d^5*e^70 + 385*(x*e +
 d)^(9/2)*b*c^2*e^71 - 2475*(x*e + d)^(7/2)*b*c^2*d*e^71 + 6930*(x*e + d)^(
5/2)*b*c^2*d^2*e^71 - 11550*(x*e + d)^(3/2)*b*c^2*d^3*e^71 + 17325*sqrt(x*e
 + d)*b*c^2*d^4*e^71 + 495*(x*e + d)^(7/2)*b^2*c*e^72 - 2772*(x*e + d)^(5/2
)*b^2*c*d*e^72 + 6930*(x*e + d)^(3/2)*b^2*c*d^2*e^72 - 13860*sqrt(x*e + d)*
b^2*c*d^3*e^72 + 231*(x*e + d)^(5/2)*b^3*e^73 - 1155*(x*e + d)^(3/2)*b^3*d*
e^73 + 3465*sqrt(x*e + d)*b^3*d^2*e^73)*e^(-77) - 2*(c^3*d^6 - 3*b*c^2*d^5*
e + 3*b^2*c*d^4*e^2 - b^3*d^3*e^3)*e^(-7)/sqrt(x*e + d)
```

3.359 $\int \frac{(bx+cx^2)^3}{(d+ex)^{5/2}} dx$

Optimal. Leaf size=244

$$\frac{6c(d+ex)^{5/2}\left(b^2e^2-5bcde+5c^2d^2\right)}{5e^7}-\frac{2(d+ex)^{3/2}(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{3e^7}+\frac{6d\sqrt{d+ex}(cd-be)\left(b^2e^2-5bcd\right.}{e^7}$$

```
[Out] (-2*d^3*(c*d - b*e)^3)/(3*e^7*(d + e*x)^(3/2)) + (6*d^2*(c*d - b*e)^2*(2*c*
d - b*e))/(e^7*Sqrt[d + e*x]) + (6*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b
^2*e^2)*Sqrt[d + e*x])/e^7 - (2*(2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e + b^
2*e^2)*(d + e*x)^(3/2))/(3*e^7) + (6*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*(d
 + e*x)^(5/2))/(5*e^7) - (6*c^2*(2*c*d - b*e)*(d + e*x)^(7/2))/(7*e^7) + (2
*c^3*(d + e*x)^(9/2))/(9*e^7)
```

Rubi [A] time = 0.100583, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {698}

$$\frac{6c(d+ex)^{5/2}\left(b^2e^2-5bcde+5c^2d^2\right)}{5e^7}-\frac{2(d+ex)^{3/2}(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{3e^7}+\frac{6d\sqrt{d+ex}(cd-be)\left(b^2e^2-5bcd\right.}{e^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^3/(d + e*x)^(5/2),x]
```


```
[Out] (-2*d^3*(c*d - b*e)^3)/(3*e^7*(d + e*x)^(3/2)) + (6*d^2*(c*d - b*e)^2*(2*c*
d - b*e))/(e^7*Sqrt[d + e*x]) + (6*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b
^2*e^2)*Sqrt[d + e*x])/e^7 - (2*(2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e + b^
2*e^2)*(d + e*x)^(3/2))/(3*e^7) + (6*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*(d
 + e*x)^(5/2))/(5*e^7) - (6*c^2*(2*c*d - b*e)*(d + e*x)^(7/2))/(7*e^7) + (2
*c^3*(d + e*x)^(9/2))/(9*e^7)
```


Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{5/2}} dx = \int \left(\frac{d^3(cd-be)^3}{e^6(d+ex)^{5/2}}-\frac{3d^2(cd-be)^2(2cd-be)}{e^6(d+ex)^{3/2}}+\frac{3d(cd-be)\left(5c^2d^2-5bcde+b^2e^2\right)}{e^6\sqrt{d+ex}}+\frac{(2cd-be)\left(-10c\right.}{}\right.$$

$$= -\frac{2d^3(cd-be)^3}{3e^7(d+ex)^{3/2}}+\frac{6d^2(cd-be)^2(2cd-be)}{e^7\sqrt{d+ex}}+\frac{6d(cd-be)\left(5c^2d^2-5bcde+b^2e^2\right)\sqrt{d+ex}}{e^7}-\frac{2(2cd-b}{}$$

Mathematica [A] time = 0.139984, size = 206, normalized size = 0.84

$$\frac{2\left(189c(d+ex)^4\left(b^2e^2-5bcde+5c^2d^2\right)-105(d+ex)^3(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)+945d(d+ex)^2(cd-be)\left(b^2e^2\right.\right.}{315e^7(d+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^3/(d + e*x)^(5/2),x]
```

```
[Out] (2*(-105*d^3*(c*d - b*e)^3 + 945*d^2*(c*d - b*e)^2*(2*c*d - b*e)*(d + e*x)
+ 945*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*(d + e*x)^2 - 105*(2*
c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e + b^2*e^2)*(d + e*x)^3 + 189*c*(5*c^2*d
^2 - 5*b*c*d*e + b^2*e^2)*(d + e*x)^4 - 135*c^2*(2*c*d - b*e)*(d + e*x)^5 +
 35*c^3*(d + e*x)^6))/(315*e^7*(d + e*x)^(3/2))
```

Maple [A] time = 0.048, size = 286, normalized size = 1.2

$$-\frac{-70\,c^3x^6e^6-270\,bc^2e^6x^5+120\,c^3de^5x^5-378\,b^2ce^6x^4+540\,bc^2de^5x^4-240\,c^3d^2e^4x^4-210\,b^3e^6x^3+1008\,b^2cde^5x^3-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^3/(e*x+d)^(5/2),x)
```

```
[Out] -2/315*(-35*c^3*e^6*x^6-135*b*c^2*e^6*x^5+60*c^3*d*e^5*x^5-189*b^2*c*e^6*x^
4+270*b*c^2*d*e^5*x^4-120*c^3*d^2*e^4*x^4-105*b^3*e^6*x^3+504*b^2*c*d*e^5*x
^3-720*b*c^2*d^2*e^4*x^3+320*c^3*d^3*e^3*x^3+630*b^3*d*e^5*x^2-3024*b^2*c*d
^2*e^4*x^2+4320*b*c^2*d^3*e^3*x^2-1920*c^3*d^4*e^2*x^2+2520*b^3*d^2*e^4*x-1
2096*b^2*c*d^3*e^3*x+17280*b*c^2*d^4*e^2*x-7680*c^3*d^5*e*x+1680*b^3*d^3*e^
3-8064*b^2*c*d^4*e^2+11520*b*c^2*d^5*e-5120*c^3*d^6)/(e*x+d)^(3/2)/e^7
```

Maxima [A] time = 1.15751, size = 374, normalized size = 1.53

$$\frac{2\left(\frac{35\,(ex+d)^{\frac{9}{2}}c^3-135\,(2\,c^3d-bc^2e)(ex+d)^{\frac{7}{2}}+189\,(5\,c^3d^2-5\,bc^2de+b^2ce^2)(ex+d)^{\frac{5}{2}}-105\,(20\,c^3d^3-30\,bc^2d^2e+12\,b^2cde^2-b^3e^3)(ex+d)^{\frac{3}{2}}+945\,(5\,c^3d^4-10\,bc^2d^3e+}{e^6}\right.}{315\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/315*((35*(e*x + d)^(9/2)*c^3 - 135*(2*c^3*d - b*c^2*e)*(e*x + d)^(7/2) +
189*(5*c^3*d^2 - 5*b*c^2*d*e + b^2*c*e^2)*(e*x + d)^(5/2) - 105*(20*c^3*d^3
 - 30*b*c^2*d^2*e + 12*b^2*c*d*e^2 - b^3*e^3)*(e*x + d)^(3/2) + 945*(5*c^3*
d^4 - 10*b*c^2*d^3*e + 6*b^2*c*d^2*e^2 - b^3*d*e^3)*sqrt(e*x + d))/e^6 - 10
5*(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 - b^3*d^3*e^3 - 9*(2*c^3*d^5 -
 5*b*c^2*d^4*e + 4*b^2*c*d^3*e^2 - b^3*d^2*e^3)*(e*x + d))/((e*x + d)^(3/2)
*e^6))/e
```

Fricas [A] time = 2.4127, size = 645, normalized size = 2.64

$$\frac{2\left(35\,c^3e^6x^6+5120\,c^3d^6-11520\,bc^2d^5e+8064\,b^2cd^4e^2-1680\,b^3d^3e^3-15\left(4\,c^3de^5-9\,bc^2e^6\right)x^5+3\left(40\,c^3d^2e^4-90\,bc\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*c^3*e^6*x^6 + 5120*c^3*d^6 - 11520*b*c^2*d^5*e + 8064*b^2*c*d^4*e
^2 - 1680*b^3*d^3*e^3 - 15*(4*c^3*d*e^5 - 9*b*c^2*e^6)*x^5 + 3*(40*c^3*d^2*
e^4 - 90*b*c^2*d*e^5 + 63*b^2*c*e^6)*x^4 - (320*c^3*d^3*e^3 - 720*b*c^2*d^2
*e^4 + 504*b^2*c*d*e^5 - 105*b^3*e^6)*x^3 + 6*(320*c^3*d^4*e^2 - 720*b*c^2*
d^3*e^3 + 504*b^2*c*d^2*e^4 - 105*b^3*d*e^5)*x^2 + 24*(320*c^3*d^5*e - 720*
b*c^2*d^4*e^2 + 504*b^2*c*d^3*e^3 - 105*b^3*d^2*e^4)*x)*sqrt(e*x + d)/(e^9*
x^2 + 2*d*e^8*x + d^2*e^7)
```

Sympy [A] time = 56.7304, size = 260, normalized size = 1.07

$$\frac{2c^3(d+ex)^{\frac{9}{2}}}{9e^7}+\frac{2d^3(be-cd)^3}{3e^7(d+ex)^{\frac{3}{2}}}-\frac{6d^2(be-2cd)(be-cd)^2}{e^7\sqrt{d+ex}}+\frac{(d+ex)^{\frac{7}{2}}\left(6bc^2e-12c^3d\right)}{7e^7}+\frac{(d+ex)^{\frac{5}{2}}\left(6b^2ce^2-30bc^2de+3\right.}{5e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**3/(e*x+d)**(5/2),x)
```

```
[Out] 2*c**3*(d + e*x)**(9/2)/(9*e**7) + 2*d**3*(b*e - c*d)**3/(3*e**7*(d + e*x)*
*(3/2)) - 6*d**2*(b*e - 2*c*d)*(b*e - c*d)**2/(e**7*sqrt(d + e*x)) + (d + e
*x)**(7/2)*(6*b*c**2*e - 12*c**3*d)/(7*e**7) + (d + e*x)**(5/2)*(6*b**2*c*e
**2 - 30*b*c**2*d*e + 30*c**3*d**2)/(5*e**7) + (d + e*x)**(3/2)*(2*b**3*e**
3 - 24*b**2*c*d*e**2 + 60*b*c**2*d**2*e - 40*c**3*d**3)/(3*e**7) + sqrt(d +
 e*x)*(-6*b**3*d*e**3 + 36*b**2*c*d**2*e**2 - 60*b*c**2*d**3*e + 30*c**3*d*
*4)/e**7
```

Giac [A] time = 1.39004, size = 487, normalized size = 2.

$$\frac{2}{315}\left(35(xe+d)^{\frac{9}{2}}c^3e^{56}-270(xe+d)^{\frac{7}{2}}c^3de^{56}+945(xe+d)^{\frac{5}{2}}c^3d^2e^{56}-2100(xe+d)^{\frac{3}{2}}c^3d^3e^{56}+4725\sqrt{xe+d}c^3d^4e^{56}+13\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] 2/315*(35*(x*e + d)^(9/2)*c^3*e^56 - 270*(x*e + d)^(7/2)*c^3*d*e^56 + 945*(
x*e + d)^(5/2)*c^3*d^2*e^56 - 2100*(x*e + d)^(3/2)*c^3*d^3*e^56 + 4725*sqrt
(x*e + d)*c^3*d^4*e^56 + 135*(x*e + d)^(7/2)*b*c^2*e^57 - 945*(x*e + d)^(5/
2)*b*c^2*d*e^57 + 3150*(x*e + d)^(3/2)*b*c^2*d^2*e^57 - 9450*sqrt(x*e + d)*
b*c^2*d^3*e^57 + 189*(x*e + d)^(5/2)*b^2*c*e^58 - 1260*(x*e + d)^(3/2)*b^2*
c*d*e^58 + 5670*sqrt(x*e + d)*b^2*c*d^2*e^58 + 105*(x*e + d)^(3/2)*b^3*e^59
 - 945*sqrt(x*e + d)*b^3*d*e^59)*e^(-63) + 2/3*(18*(x*e + d)*c^3*d^5 - c^3*
d^6 - 45*(x*e + d)*b*c^2*d^4*e + 3*b*c^2*d^5*e + 36*(x*e + d)*b^2*c*d^3*e^2
 - 3*b^2*c*d^4*e^2 - 9*(x*e + d)*b^3*d^2*e^3 + b^3*d^3*e^3)*e^(-7)/(x*e + d
)^(3/2)
```

3.360 $\int \frac{(bx+cx^2)^3}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=240

$$\frac{2c(d+ex)^{3/2}\left(b^2e^2-5bcde+5c^2d^2\right)}{e^7}-\frac{2\sqrt{d+ex}(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{e^7}-\frac{6d(cd-be)\left(b^2e^2-5bcde+5c^2\right.}{e^7\sqrt{d+ex}}$$

```
[Out] (-2*d^3*(c*d - b*e)^3)/(5*e^7*(d + e*x)^(5/2)) + (2*d^2*(c*d - b*e)^2*(2*c*
d - b*e))/(e^7*(d + e*x)^(3/2)) - (6*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e +
 b^2*e^2))/(e^7*Sqrt[d + e*x]) - (2*(2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e
+ b^2*e^2)*Sqrt[d + e*x])/e^7 + (2*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*(d +
 e*x)^(3/2))/e^7 - (6*c^2*(2*c*d - b*e)*(d + e*x)^(5/2))/(5*e^7) + (2*c^3*(
d + e*x)^(7/2))/(7*e^7)
```

Rubi [A] time = 0.106346, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {698}

$$\frac{2c(d+ex)^{3/2}\left(b^2e^2-5bcde+5c^2d^2\right)}{e^7}-\frac{2\sqrt{d+ex}(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{e^7}-\frac{6d(cd-be)\left(b^2e^2-5bcde+5c^2\right.}{e^7\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^3/(d + e*x)^(7/2),x]
```

```
[Out] (-2*d^3*(c*d - b*e)^3)/(5*e^7*(d + e*x)^(5/2)) + (2*d^2*(c*d - b*e)^2*(2*c*
d - b*e))/(e^7*(d + e*x)^(3/2)) - (6*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e +
 b^2*e^2))/(e^7*Sqrt[d + e*x]) - (2*(2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e
+ b^2*e^2)*Sqrt[d + e*x])/e^7 + (2*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*(d +
 e*x)^(3/2))/e^7 - (6*c^2*(2*c*d - b*e)*(d + e*x)^(5/2))/(5*e^7) + (2*c^3*(
d + e*x)^(7/2))/(7*e^7)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx+cx^2\right)^3}{(d+ex)^{7/2}}\,dx &= \int \left(\frac{d^3(cd-be)^3}{e^6(d+ex)^{7/2}}-\frac{3d^2(cd-be)^2(2cd-be)}{e^6(d+ex)^{5/2}}+\frac{3d(cd-be)\left(5c^2d^2-5bcde+b^2e^2\right)}{e^6(d+ex)^{3/2}}+\frac{(2cd-be)\left(-\right.}{}\right.\\ &= -\frac{2d^3(cd-be)^3}{5e^7(d+ex)^{5/2}}+\frac{2d^2(cd-be)^2(2cd-be)}{e^7(d+ex)^{3/2}}-\frac{6d(cd-be)\left(5c^2d^2-5bcde+b^2e^2\right)}{e^7\sqrt{d+ex}}-\frac{2(2cd-be)\left(10\right.}{}\end{aligned}$$

Mathematica [A] time = 0.139434, size = 206, normalized size = 0.86

$$\frac{2\left(35c(d+ex)^4\left(b^2e^2-5bcde+5c^2d^2\right)-35(d+ex)^3(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)-105d(d+ex)^2(cd-be)\left(b^2e^2\right.\right.}{35e^7(d+e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^3/(d + e*x)^(7/2),x]
```

```
[Out] (2*(-7*d^3*(c*d - b*e)^3 + 35*d^2*(c*d - b*e)^2*(2*c*d - b*e)*(d + e*x) - 1
05*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*(d + e*x)^2 - 35*(2*c*d
- b*e)*(10*c^2*d^2 - 10*b*c*d*e + b^2*e^2)*(d + e*x)^3 + 35*c*(5*c^2*d^2 -
5*b*c*d*e + b^2*e^2)*(d + e*x)^4 - 21*c^2*(2*c*d - b*e)*(d + e*x)^5 + 5*c^3
*(d + e*x)^6))/(35*e^7*(d + e*x)^(5/2))
```

Maple [A] time = 0.049, size = 286, normalized size = 1.2

$$\frac{10\,c^3x^6e^6+42\,bc^2e^6x^5-24\,c^3de^5x^5+70\,b^2ce^6x^4-140\,bc^2de^5x^4+80\,c^3d^2e^4x^4+70\,b^3e^6x^3-560\,b^2cde^5x^3+1120\,bc^2d^2e^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^3/(e*x+d)^(7/2),x)
```

```
[Out] 2/35*(5*c^3*e^6*x^6+21*b*c^2*e^6*x^5-12*c^3*d*e^5*x^5+35*b^2*c*e^6*x^4-70*b
*c^2*d*e^5*x^4+40*c^3*d^2*e^4*x^4+35*b^3*e^6*x^3-280*b^2*c*d*e^5*x^3+560*b*
c^2*d^2*e^4*x^3-320*c^3*d^3*e^3*x^3+210*b^3*d*e^5*x^2-1680*b^2*c*d^2*e^4*x^
2+3360*b*c^2*d^3*e^3*x^2-1920*c^3*d^4*e^2*x^2+280*b^3*d^2*e^4*x-2240*b^2*c*
d^3*e^3*x+4480*b*c^2*d^4*e^2*x-2560*c^3*d^5*e*x+112*b^3*d^3*e^3-896*b^2*c*d
^4*e^2+1792*b*c^2*d^5*e-1024*c^3*d^6)/(e*x+d)^(5/2)/e^7
```

Maxima [A] time = 1.12873, size = 374, normalized size = 1.56

$$\frac{2\left(\frac{5\,(ex+d)^{\frac{7}{2}}c^3-21\left(2\,c^3d-bc^2e\right)(ex+d)^{\frac{5}{2}}+35\left(5\,c^3d^2-5\,bc^2de+b^2ce^2\right)(ex+d)^{\frac{3}{2}}-35\left(20\,c^3d^3-30\,bc^2d^2e+12\,b^2cde^2-b^3e^3\right)\sqrt{ex+d}}{e^6}-\frac{7\left(c^3d^6-3\,bc^2d^5e+3\,b^2cd^4e^2-b^3c\right.}{}\right.}{35\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/35*((5*(e*x + d)^(7/2)*c^3 - 21*(2*c^3*d - b*c^2*e)*(e*x + d)^(5/2) + 35*
(5*c^3*d^2 - 5*b*c^2*d*e + b^2*c*e^2)*(e*x + d)^(3/2) - 35*(20*c^3*d^3 - 30
*b*c^2*d^2*e + 12*b^2*c*d*e^2 - b^3*e^3)*sqrt(e*x + d))/e^6 - 7*(c^3*d^6 -
3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 - b^3*d^3*e^3 + 15*(5*c^3*d^4 - 10*b*c^2*d^
3*e + 6*b^2*c*d^2*e^2 - b^3*d*e^3)*(e*x + d)^2 - 5*(2*c^3*d^5 - 5*b*c^2*d^4
*e + 4*b^2*c*d^3*e^2 - b^3*d^2*e^3)*(e*x + d))/((e*x + d)^(5/2)*e^6))/e
```

Fricas [A] time = 2.4311, size = 645, normalized size = 2.69

$$\frac{2\left(5\,c^3e^6x^6-1024\,c^3d^6+1792\,bc^2d^5e-896\,b^2cd^4e^2+112\,b^3d^3e^3-3\left(4\,c^3de^5-7\,bc^2e^6\right)x^5+5\left(8\,c^3d^2e^4-14\,bc^2de^5+7\,b^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*c^3*e^6*x^6 - 1024*c^3*d^6 + 1792*b*c^2*d^5*e - 896*b^2*c*d^4*e^2 +
 112*b^3*d^3*e^3 - 3*(4*c^3*d*e^5 - 7*b*c^2*e^6)*x^5 + 5*(8*c^3*d^2*e^4 - 1
4*b*c^2*d*e^5 + 7*b^2*c*e^6)*x^4 - 5*(64*c^3*d^3*e^3 - 112*b*c^2*d^2*e^4 +
56*b^2*c*d*e^5 - 7*b^3*e^6)*x^3 - 30*(64*c^3*d^4*e^2 - 112*b*c^2*d^3*e^3 +
56*b^2*c*d^2*e^4 - 7*b^3*d*e^5)*x^2 - 40*(64*c^3*d^5*e - 112*b*c^2*d^4*e^2
+ 56*b^2*c*d^3*e^3 - 7*b^3*d^2*e^4)*x)*sqrt(e*x + d)/(e^10*x^3 + 3*d*e^9*x^
2 + 3*d^2*e^8*x + d^3*e^7)
```

Sympy [A] time = 82.5956, size = 248, normalized size = 1.03

$$\frac{2c^3(d+ex)^{\frac{7}{2}}}{7e^7}+\frac{2d^3(be-cd)^3}{5e^7(d+ex)^{\frac{5}{2}}}-\frac{2d^2(be-2cd)(be-cd)^2}{e^7(d+ex)^{\frac{3}{2}}}+\frac{6d(be-cd)\left(b^2e^2-5bcde+5c^2d^2\right)}{e^7\sqrt{d+ex}}+\frac{(d+ex)^{\frac{5}{2}}\left(6bc^2e-1\right.}{5e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**3/(e*x+d)**(7/2),x)
```

```
[Out] 2*c**3*(d + e*x)**(7/2)/(7*e**7) + 2*d**3*(b*e - c*d)**3/(5*e**7*(d + e*x)*
*(5/2)) - 2*d**2*(b*e - 2*c*d)*(b*e - c*d)**2/(e**7*(d + e*x)**(3/2)) + 6*d
*(b*e - c*d)*(b**2*e**2 - 5*b*c*d*e + 5*c**2*d**2)/(e**7*sqrt(d + e*x)) + (
d + e*x)**(5/2)*(6*b*c**2*e - 12*c**3*d)/(5*e**7) + (d + e*x)**(3/2)*(6*b**
2*c*e**2 - 30*b*c**2*d*e + 30*c**3*d**2)/(3*e**7) + sqrt(d + e*x)*(2*b**3*e
**3 - 24*b**2*c*d*e**2 + 60*b*c**2*d**2*e - 40*c**3*d**3)/e**7
```

Giac [A] time = 1.30761, size = 485, normalized size = 2.02

$$\frac{2}{35}\left(5(xe+d)^{\frac{7}{2}}c^3e^{42}-42(xe+d)^{\frac{5}{2}}c^3de^{42}+175(xe+d)^{\frac{3}{2}}c^3d^2e^{42}-700\sqrt{xe+d}c^3d^3e^{42}+21(xe+d)^{\frac{5}{2}}bc^2e^{43}-175(xe\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^3/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] 2/35*(5*(x*e + d)^(7/2)*c^3*e^42 - 42*(x*e + d)^(5/2)*c^3*d*e^42 + 175*(x*e
 + d)^(3/2)*c^3*d^2*e^42 - 700*sqrt(x*e + d)*c^3*d^3*e^42 + 21*(x*e + d)^(5
/2)*b*c^2*e^43 - 175*(x*e + d)^(3/2)*b*c^2*d*e^43 + 1050*sqrt(x*e + d)*b*c^
2*d^2*e^43 + 35*(x*e + d)^(3/2)*b^2*c*e^44 - 420*sqrt(x*e + d)*b^2*c*d*e^44
 + 35*sqrt(x*e + d)*b^3*e^45)*e^(-49) - 2/5*(75*(x*e + d)^2*c^3*d^4 - 10*(x
*e + d)*c^3*d^5 + c^3*d^6 - 150*(x*e + d)^2*b*c^2*d^3*e + 25*(x*e + d)*b*c^
2*d^4*e - 3*b*c^2*d^5*e + 90*(x*e + d)^2*b^2*c*d^2*e^2 - 20*(x*e + d)*b^2*c
*d^3*e^2 + 3*b^2*c*d^4*e^2 - 15*(x*e + d)^2*b^3*d*e^3 + 5*(x*e + d)*b^3*d^2
*e^3 - b^3*d^3*e^3)*e^(-7)/(x*e + d)^(5/2)
```

3.361 $\int \frac{(d+ex)^{7/2}}{bx+cx^2} dx$

Optimal. Leaf size=157

$$\frac{2e\sqrt{d+ex}\left(b^2e^2-3bcde+3c^2d^2\right)}{c^3}+\frac{2e(d+ex)^{3/2}(2cd-be)}{3c^2}+\frac{2(cd-be)^{7/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{bc^{7/2}}-\frac{2d^{7/2}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b}+$$

```
[Out] (2*e*(3*c^2*d^2 - 3*b*c*d*e + b^2*e^2)*Sqrt[d + e*x])/c^3 + (2*e*(2*c*d - b
*e)*(d + e*x)^(3/2))/(3*c^2) + (2*e*(d + e*x)^(5/2))/(5*c) - (2*d^(7/2)*Arc
Tanh[Sqrt[d + e*x]/Sqrt[d]])/b + (2*(c*d - b*e)^(7/2)*ArcTanh[(Sqrt[c]*Sqrt
[d + e*x])/Sqrt[c*d - b*e]])/(b*c^(7/2))
```

Rubi [A] time = 0.387282, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {703, 824, 826, 1166, 208}

$$\frac{2e\sqrt{d+ex}\left(b^2e^2-3bcde+3c^2d^2\right)}{c^3}+\frac{2e(d+ex)^{3/2}(2cd-be)}{3c^2}+\frac{2(cd-be)^{7/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{bc^{7/2}}-\frac{2d^{7/2}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b}+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/(b*x + c*x^2),x]
```


```
[Out] (2*e*(3*c^2*d^2 - 3*b*c*d*e + b^2*e^2)*Sqrt[d + e*x])/c^3 + (2*e*(2*c*d - b
*e)*(d + e*x)^(3/2))/(3*c^2) + (2*e*(d + e*x)^(5/2))/(5*c) - (2*d^(7/2)*Arc
Tanh[Sqrt[d + e*x]/Sqrt[d]])/b + (2*(c*d - b*e)^(7/2)*ArcTanh[(Sqrt[c]*Sqrt
[d + e*x])/Sqrt[c*d - b*e]])/(b*c^(7/2))
```


Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{7/2}}{bx+cx^2}\,dx = \frac{2e(d+ex)^{5/2}}{5c} + \frac{\int \frac{(d+ex)^{3/2}\left(cd^2+e(2cd-be)x\right)}{bx+cx^2}\,dx}{c}$$

$$= \frac{2e(2cd-be)(d+ex)^{3/2}}{3c^2} + \frac{2e(d+ex)^{5/2}}{5c} + \frac{\int \frac{\sqrt{d+ex}\left(c^2d^3+e\left(3c^2d^2-3bcde+b^2e^2\right)x\right)}{bx+cx^2}\,dx}{c^2}$$

$$= \frac{2e\left(3c^2d^2-3bcde+b^2e^2\right)\sqrt{d+ex}}{c^3} + \frac{2e(2cd-be)(d+ex)^{3/2}}{3c^2} + \frac{2e(d+ex)^{5/2}}{5c} + \frac{\int \frac{c^3d^4+e(2cd-be)\left(2c^2d^2 \right.}{\sqrt{d+ex}\left(bx+ \right.}}{c^3}$$

$$= \frac{2e\left(3c^2d^2-3bcde+b^2e^2\right)\sqrt{d+ex}}{c^3} + \frac{2e(2cd-be)(d+ex)^{3/2}}{3c^2} + \frac{2e(d+ex)^{5/2}}{5c} + \frac{2\,\text{Subst}\left(\int \frac{c^3d^4e-de}{}\right.}{}$$

$$= \frac{2e\left(3c^2d^2-3bcde+b^2e^2\right)\sqrt{d+ex}}{c^3} + \frac{2e(2cd-be)(d+ex)^{3/2}}{3c^2} + \frac{2e(d+ex)^{5/2}}{5c} + \frac{\left(2cd^4\right)\text{Subst}\left(\int \right.}{}$$

$$= \frac{2e\left(3c^2d^2-3bcde+b^2e^2\right)\sqrt{d+ex}}{c^3} + \frac{2e(2cd-be)(d+ex)^{3/2}}{3c^2} + \frac{2e(d+ex)^{5/2}}{5c} - \frac{2d^{7/2}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right.}{b}$$

Mathematica [A] time = 0.206247, size = 138, normalized size = 0.88

$$\frac{2e\sqrt{d+ex}\left(15b^2e^2-5bce(10d+ex)+c^2\left(58d^2+16dex+3e^2x^2\right)\right)}{15c^3} + \frac{2(cd-be)^{7/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{bc^{7/2}} - \frac{2d^{7/2}\tanh^{-1}\left(\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)/(b*x + c*x^2),x]
```

```
[Out] (2*e*Sqrt[d + e*x]*(15*b^2*e^2 - 5*b*c*e*(10*d + e*x) + c^2*(58*d^2 + 16*d*
e*x + 3*e^2*x^2)))/(15*c^3) - (2*d^(7/2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/b
+ (2*(c*d - b*e)^(7/2)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(b
*c^(7/2))
```

Maple [B] time = 0.246, size = 336, normalized size = 2.1

$$\frac{2\,e}{5\,c}(ex+d)^{\frac{5}{2}} - \frac{2\,be^2}{3\,c^2}(ex+d)^{\frac{3}{2}} + \frac{4\,de}{3\,c}(ex+d)^{\frac{3}{2}} + 2\,\frac{e^3b^2\sqrt{ex+d}}{c^3} - 6\,\frac{bde^2\sqrt{ex+d}}{c^2} + 6\,\frac{ed^2\sqrt{ex+d}}{c} - 2\,\frac{b^3e^4}{c^3\sqrt{(be-cd)\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/(c*x^2+b*x),x)
```

```
[Out] 2/5*e*(e*x+d)^(5/2)/c-2/3/c^2*(e*x+d)^(3/2)*b*e^2+4/3*e/c*(e*x+d)^(3/2)*d+2
/c^3*b^2*e^3*(e*x+d)^(1/2)-6/c^2*b*d*e^2*(e*x+d)^(1/2)+6*e/c*d^2*(e*x+d)^(1
/2)-2/c^3*b^3*e^4/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^
(1/2))+8/c^2*b^2*e^3/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*
c)^(1/2))*d-12/c*b*e^2/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d
)*c)^(1/2))*d^2+8*e/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c
)^(1/2))*d^3-2*c/b/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)
^(1/2))*d^4-2*d^(7/2)*arctanh((e*x+d)^(1/2)/d^(1/2))/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 9.5173, size = 1820, normalized size = 11.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] [1/15*(15*c^3*d^(7/2)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) - 15*(c^
3*d^3 - 3*b*c^2*d^2*e + 3*b^2*c*d*e^2 - b^3*e^3)*sqrt((c*d - b*e)/c)*log((c
*e*x + 2*c*d - b*e - 2*sqrt(e*x + d)*c*sqrt((c*d - b*e)/c))/(c*x + b)) + 2*
(3*b*c^2*e^3*x^2 + 58*b*c^2*d^2*e - 50*b^2*c*d*e^2 + 15*b^3*e^3 + (16*b*c^2
*d*e^2 - 5*b^2*c*e^3)*x)*sqrt(e*x + d))/(b*c^3), 1/15*(15*c^3*d^(7/2)*log((
e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + 30*(c^3*d^3 - 3*b*c^2*d^2*e + 3*b
^2*c*d*e^2 - b^3*e^3)*sqrt(-(c*d - b*e)/c)*arctan(-sqrt(e*x + d)*c*sqrt(-(c
*d - b*e)/c)/(c*d - b*e)) + 2*(3*b*c^2*e^3*x^2 + 58*b*c^2*d^2*e - 50*b^2*c*
d*e^2 + 15*b^3*e^3 + (16*b*c^2*d*e^2 - 5*b^2*c*e^3)*x)*sqrt(e*x + d))/(b*c^
3), 1/15*(30*c^3*sqrt(-d)*d^3*arctan(sqrt(e*x + d)*sqrt(-d)/d) - 15*(c^3*d^
3 - 3*b*c^2*d^2*e + 3*b^2*c*d*e^2 - b^3*e^3)*sqrt((c*d - b*e)/c)*log((c*e*x
 + 2*c*d - b*e - 2*sqrt(e*x + d)*c*sqrt((c*d - b*e)/c))/(c*x + b)) + 2*(3*b
*c^2*e^3*x^2 + 58*b*c^2*d^2*e - 50*b^2*c*d*e^2 + 15*b^3*e^3 + (16*b*c^2*d*e
^2 - 5*b^2*c*e^3)*x)*sqrt(e*x + d))/(b*c^3), 2/15*(15*c^3*sqrt(-d)*d^3*arct
an(sqrt(e*x + d)*sqrt(-d)/d) + 15*(c^3*d^3 - 3*b*c^2*d^2*e + 3*b^2*c*d*e^2
- b^3*e^3)*sqrt(-(c*d - b*e)/c)*arctan(-sqrt(e*x + d)*c*sqrt(-(c*d - b*e)/c
)/(c*d - b*e)) + (3*b*c^2*e^3*x^2 + 58*b*c^2*d^2*e - 50*b^2*c*d*e^2 + 15*b^
3*e^3 + (16*b*c^2*d*e^2 - 5*b^2*c*e^3)*x)*sqrt(e*x + d))/(b*c^3)]
```

Sympy [A] time = 92.468, size = 162, normalized size = 1.03

$$\frac{2e(d+ex)^{\frac{5}{2}}}{5c} + \frac{(d+ex)^{\frac{3}{2}}\left(-2be^2+4cde\right)}{3c^2} + \frac{\sqrt{d+ex}\left(2b^2e^3-6bcde^2+6c^2d^2e\right)}{c^3} + \frac{2d^4\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-d}}\right)}{b\sqrt{-d}} - \frac{2(be-cd)^4\operatorname{atan}\left(\cdots\right.}{bc^4\sqrt{\frac{be-cd}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/(c*x**2+b*x),x)
```

```
[Out] 2*e*(d + e*x)**(5/2)/(5*c) + (d + e*x)**(3/2)*(-2*b*e**2 + 4*c*d*e)/(3*c**2
) + sqrt(d + e*x)*(2*b**2*e**3 - 6*b*c*d*e**2 + 6*c**2*d**2*e)/c**3 + 2*d**
4*atan(sqrt(d + e*x)/sqrt(-d))/(b*sqrt(-d)) - 2*(b*e - c*d)**4*atan(sqrt(d
+ e*x)/sqrt((b*e - c*d)/c))/(b*c**4*sqrt((b*e - c*d)/c))
```

Giac [A] time = 1.25215, size = 309, normalized size = 1.97

$$\frac{2\,d^4\arctan\left(\frac{\sqrt{xe+d}}{\sqrt{-d}}\right)}{b\sqrt{-d}}-\frac{2\left(c^4d^4-4\,bc^3d^3e+6\,b^2c^2d^2e^2-4\,b^3cde^3+b^4e^4\right)\arctan\left(\frac{\sqrt{xe+d}c}{\sqrt{-c^2d+bce}}\right)}{\sqrt{-c^2d+bce}bc^3}+\frac{2\left(3\,(xe+d)^{\frac{5}{2}}c^4e+10\,(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] 2*d^4*arctan(sqrt(x*e + d)/sqrt(-d))/(b*sqrt(-d)) - 2*(c^4*d^4 - 4*b*c^3*d^
3*e + 6*b^2*c^2*d^2*e^2 - 4*b^3*c*d*e^3 + b^4*e^4)*arctan(sqrt(x*e + d)*c/s
qrt(-c^2*d + b*c*e))/(sqrt(-c^2*d + b*c*e)*b*c^3) + 2/15*(3*(x*e + d)^(5/2)
*c^4*e + 10*(x*e + d)^(3/2)*c^4*d*e + 45*sqrt(x*e + d)*c^4*d^2*e - 5*(x*e +
 d)^(3/2)*b*c^3*e^2 - 45*sqrt(x*e + d)*b*c^3*d*e^2 + 15*sqrt(x*e + d)*b^2*c
^2*e^3)/c^5
```

3.362 $\int \frac{(d+ex)^{5/2}}{bx+cx^2}\,dx$

Optimal. Leaf size=118

$$\frac{2e\sqrt{d+ex}(2cd-be)}{c^2}+\frac{2(cd-be)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{bc^{5/2}}-\frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b}+\frac{2e(d+ex)^{3/2}}{3c}$$

```
[Out] (2*e*(2*c*d - b*e)*Sqrt[d + e*x])/c^2 + (2*e*(d + e*x)^(3/2))/(3*c) - (2*d^
(5/2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/b + (2*(c*d - b*e)^(5/2)*ArcTanh[(Sqr
t[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(b*c^(5/2))
```

Rubi [A] time = 0.227992, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {703, 824, 826, 1166, 208}

$$\frac{2e\sqrt{d+ex}(2cd-be)}{c^2}+\frac{2(cd-be)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{bc^{5/2}}-\frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b}+\frac{2e(d+ex)^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(b*x + c*x^2),x]
```

```
[Out] (2*e*(2*c*d - b*e)*Sqrt[d + e*x])/c^2 + (2*e*(d + e*x)^(3/2))/(3*c) - (2*d^
(5/2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/b + (2*(c*d - b*e)^(5/2)*ArcTanh[(Sqr
t[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(b*c^(5/2))
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{bx+cx^2}\,dx = \frac{2e(d+ex)^{3/2}}{3c} + \frac{\int \frac{\sqrt{d+ex}\left(cd^2+e(2cd-be)x\right)}{bx+cx^2}\,dx}{c}$$

$$= \frac{2e(2cd-be)\sqrt{d+ex}}{c^2} + \frac{2e(d+ex)^{3/2}}{3c} + \frac{\int \frac{c^2d^3+e\left(3c^2d^2-3bcde+b^2e^2\right)x}{\sqrt{d+ex}\left(bx+cx^2\right)}\,dx}{c^2}$$

$$= \frac{2e(2cd-be)\sqrt{d+ex}}{c^2} + \frac{2e(d+ex)^{3/2}}{3c} + \frac{2\,\text{Subst}\left(\int \frac{c^2d^3e-de\left(3c^2d^2-3bcde+b^2e^2\right)+e\left(3c^2d^2-3bcde+b^2e^2\right)x^2}{cd^2-bde+(-2cd+be)x^2+cx^4}\,dx, x$$

$$= \frac{2e(2cd-be)\sqrt{d+ex}}{c^2} + \frac{2e(d+ex)^{3/2}}{3c} + \frac{\left(2cd^3\right)\text{Subst}\left(\int \frac{1}{-\frac{be}{2}+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x, \sqrt{d+ex}\right)}{b} - \frac{\left(2(cd\right.}{}$$

$$= \frac{2e(2cd-be)\sqrt{d+ex}}{c^2} + \frac{2e(d+ex)^{3/2}}{3c} - \frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b} + \frac{2(cd-be)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{bc^{5/2}}$$

Mathematica [A] time = 0.123229, size = 107, normalized size = 0.91

$$\frac{2e\sqrt{d+ex}(-3be+7cd+cex)}{3c^2} + \frac{2(cd-be)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{bc^{5/2}} - \frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(b*x + c*x^2),x]
```

```
[Out] (2*e*Sqrt[d + e*x]*(7*c*d - 3*b*e + c*e*x))/(3*c^2) - (2*d^(5/2)*ArcTanh[Sq
rt[d + e*x]/Sqrt[d]])/b + (2*(c*d - b*e)^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[d + e*
x])/Sqrt[c*d - b*e]])/(b*c^(5/2))
```

Maple [B] time = 0.223, size = 237, normalized size = 2.

$$\frac{2e}{3c}(ex+d)^{\frac{3}{2}} - 2\frac{\sqrt{ex+d}be^2}{c^2} + 4\frac{de\sqrt{ex+d}}{c} + 2\frac{e^3b^2}{c^2\sqrt{(be-cd)c}}\arctan\left(\frac{\sqrt{ex+d}c}{\sqrt{(be-cd)c}}\right) - 6\frac{bde^2}{c\sqrt{(be-cd)c}}\arctan\left(\frac{\sqrt{e}}{\sqrt{(b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(c*x^2+b*x),x)
```

```
[Out] 2/3*e*(e*x+d)^(3/2)/c-2/c^2*(e*x+d)^(1/2)*b*e^2+4*d*e*(e*x+d)^(1/2)/c+2/c^2
*b^2*e^3/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))-6/
c*b*e^2/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))*d+6
*e/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))*d^2-2*c/
b/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))*d^3-2*d^(
```

```
5/2)*arctanh((e*x+d)^(1/2)/d^(1/2))/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.86695, size = 1350, normalized size = 11.44

$$\left[\frac{3\,c^2 d^{\frac{5}{2}}\log\left(\frac{ex-2\,\sqrt{ex+d}\sqrt{d}+2\,d}{x}\right)+3\left(c^2d^2-2\,bcde+b^2e^2\right)\sqrt{\frac{cd-be}{c}}\log\left(\frac{cex+2\,cd-be+2\,\sqrt{ex+d}c\sqrt{\frac{cd-be}{c}}}{cx+b}\right)+2\left(bce^2x+7\,bcde-3\,b^2e^2\right)}{3\,bc^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] [1/3*(3*c^2*d^(5/2)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + 3*(c^2*d
^2 - 2*b*c*d*e + b^2*e^2)*sqrt((c*d - b*e)/c)*log((c*e*x + 2*c*d - b*e + 2*
sqrt(e*x + d)*c*sqrt((c*d - b*e)/c))/(c*x + b)) + 2*(b*c*e^2*x + 7*b*c*d*e
- 3*b^2*e^2)*sqrt(e*x + d))/(b*c^2), 1/3*(3*c^2*d^(5/2)*log((e*x - 2*sqrt(e
*x + d)*sqrt(d) + 2*d)/x) + 6*(c^2*d^2 - 2*b*c*d*e + b^2*e^2)*sqrt(-(c*d -
b*e)/c)*arctan(-sqrt(e*x + d)*c*sqrt(-(c*d - b*e)/c)/(c*d - b*e)) + 2*(b*c*
e^2*x + 7*b*c*d*e - 3*b^2*e^2)*sqrt(e*x + d))/(b*c^2), 1/3*(6*c^2*sqrt(-d)*
d^2*arctan(sqrt(e*x + d)*sqrt(-d)/d) + 3*(c^2*d^2 - 2*b*c*d*e + b^2*e^2)*sq
rt((c*d - b*e)/c)*log((c*e*x + 2*c*d - b*e + 2*sqrt(e*x + d)*c*sqrt((c*d -
b*e)/c))/(c*x + b)) + 2*(b*c*e^2*x + 7*b*c*d*e - 3*b^2*e^2)*sqrt(e*x + d))/
(b*c^2), 2/3*(3*c^2*sqrt(-d)*d^2*arctan(sqrt(e*x + d)*sqrt(-d)/d) + 3*(c^2*
d^2 - 2*b*c*d*e + b^2*e^2)*sqrt(-(c*d - b*e)/c)*arctan(-sqrt(e*x + d)*c*sqr
t(-(c*d - b*e)/c)/(c*d - b*e)) + (b*c*e^2*x + 7*b*c*d*e - 3*b^2*e^2)*sqrt(e
*x + d))/(b*c^2)]
```

Sympy [A] time = 56.1243, size = 119, normalized size = 1.01

$$\frac{2e\,(d+ex)^{\frac{3}{2}}}{3c}+\frac{\sqrt{d+ex}\left(-2be^2+4cde\right)}{c^2}+\frac{2d^3\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-d}}\right)}{b\sqrt{-d}}+\frac{2\,(be-cd)^3\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{\frac{be-cd}{c}}}\right)}{bc^3\sqrt{\frac{be-cd}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(c*x**2+b*x),x)
```

```
[Out] 2*e*(d + e*x)**(3/2)/(3*c) + sqrt(d + e*x)*(-2*b*e**2 + 4*c*d*e)/c**2 + 2*d
**3*atan(sqrt(d + e*x)/sqrt(-d))/(b*sqrt(-d)) + 2*(b*e - c*d)**3*atan(sqrt(
```

```
d + e*x)/sqrt((b*e - c*d)/c))/(b*c**3*sqrt((b*e - c*d)/c))
```

Giac [A] time = 1.34448, size = 217, normalized size = 1.84

$$\frac{2\,d^3\arctan\left(\frac{\sqrt{xe+d}}{\sqrt{-d}}\right)}{b\sqrt{-d}}-\frac{2\left(c^3d^3-3\,bc^2d^2e+3\,b^2cde^2-b^3e^3\right)\arctan\left(\frac{\sqrt{xe+d}c}{\sqrt{-c^2d+bce}}\right)}{\sqrt{-c^2d+bce}bc^2}+\frac{2\left((xe+d)^{\frac{3}{2}}c^2e+6\sqrt{xe+d}c^2de-3\right.}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] 2*d^3*arctan(sqrt(x*e + d)/sqrt(-d))/(b*sqrt(-d)) - 2*(c^3*d^3 - 3*b*c^2*d^
2*e + 3*b^2*c*d*e^2 - b^3*e^3)*arctan(sqrt(x*e + d)*c/sqrt(-c^2*d + b*c*e))
/(sqrt(-c^2*d + b*c*e)*b*c^2) + 2/3*((x*e + d)^(3/2)*c^2*e + 6*sqrt(x*e + d
)*c^2*d*e - 3*sqrt(x*e + d)*b*c*e^2)/c^3
```

3.363 $\int \frac{(d+ex)^{3/2}}{bx+cx^2}\,dx$

Optimal. Leaf size=92

$$\frac{2(cd-be)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{bc^{3/2}}-\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b}+\frac{2e\sqrt{d+ex}}{c}$$

```
[Out] (2*e*Sqrt[d + e*x])/c - (2*d^(3/2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/b + (2*(
c*d - b*e)^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(b*c^(3/
2))
```

Rubi [A] time = 0.195558, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {703, 826, 1166, 208}

$$\frac{2(cd-be)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{bc^{3/2}}-\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b}+\frac{2e\sqrt{d+ex}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(b*x + c*x^2),x]
```

```
[Out] (2*e*Sqrt[d + e*x])/c - (2*d^(3/2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/b + (2*(
c*d - b*e)^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(b*c^(3/
2))
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{bx+cx^2}\,dx = \frac{2e\sqrt{d+ex}}{c} + \frac{\int \frac{cd^2+e(2cd-be)x}{\sqrt{d+ex}(bx+cx^2)}\,dx}{c}$$
$$= \frac{2e\sqrt{d+ex}}{c} + \frac{2\,\text{Subst}\left(\int \frac{cd^2e-de(2cd-be)+e(2cd-be)x^2}{cd^2-bde+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{c}$$
$$= \frac{2e\sqrt{d+ex}}{c} + \frac{(2cd^2)\,\text{Subst}\left(\int \frac{1}{-\frac{be}{2}+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x, \sqrt{d+ex}\right)}{b} - \frac{(2(cd-be)^2)\,\text{Subst}\left(\int \frac{1}{\frac{be}{2}+\frac{1}{2}(-2c}\right.}{bc}$$
$$= \frac{2e\sqrt{d+ex}}{c} - \frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b} + \frac{2(cd-be)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{bc^{3/2}}$$

Mathematica [A] time = 0.0605512, size = 97, normalized size = 1.05

$$\frac{2\left(b\sqrt{c}e\sqrt{d+ex}+(cd-be)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)-c^{3/2}d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\right)}{bc^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(b*x + c*x^2),x]
```

```
[Out] (2*(b*Sqrt[c]*e*Sqrt[d + e*x] - c^(3/2)*d^(3/2)*ArcTanh[Sqrt[d + e*x]/Sqrt[
d]] + (c*d - b*e)^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]]))/
(b*c^(3/2))
```

Maple [B] time = 0.223, size = 159, normalized size = 1.7

$$2\,\frac{e\sqrt{ex+d}}{c}-2\,\frac{be^2}{c\sqrt{(be-cd)\,c}}\arctan\left(\frac{\sqrt{ex+d}c}{\sqrt{(be-cd)\,c}}\right)+4\,\frac{de}{\sqrt{(be-cd)\,c}}\arctan\left(\frac{\sqrt{ex+d}c}{\sqrt{(be-cd)\,c}}\right)-2\,\frac{cd^2}{b\sqrt{(be-cd)\,c}}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*x^2+b*x),x)
```

```
[Out] 2*e*(e*x+d)^(1/2)/c-2*b/c*e^2/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((
b*e-c*d)*c)^(1/2))+4*e/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d
)*c)^(1/2))*d-2/b*c/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c
)^(1/2))*d^2-2*d^(3/2)*arctanh((e*x+d)^(1/2)/d^(1/2))/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.60947, size = 1007, normalized size = 10.95

$$\left[\frac{cd^{\frac{3}{2}}\log\left(\frac{ex-2\sqrt{ex+d}\sqrt{d}+2d}{x}\right)+2\sqrt{ex+d}be-(cd-be)\sqrt{\frac{cd-be}{c}}\log\left(\frac{cex+2cd-be-2\sqrt{ex+d}c\sqrt{\frac{cd-be}{c}}}{cx+b}\right)}{bc}, \frac{cd^{\frac{3}{2}}\log\left(\frac{ex-2\sqrt{ex+d}\sqrt{d}+2d}{x}\right)+2\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] [(c*d^(3/2)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + 2*sqrt(e*x + d)*
b*e - (c*d - b*e)*sqrt((c*d - b*e)/c)*log((c*e*x + 2*c*d - b*e - 2*sqrt(e*x
 + d)*c*sqrt((c*d - b*e)/c))/(c*x + b)))/(b*c), (c*d^(3/2)*log((e*x - 2*sqr
t(e*x + d)*sqrt(d) + 2*d)/x) + 2*sqrt(e*x + d)*b*e + 2*(c*d - b*e)*sqrt(-(c
*d - b*e)/c)*arctan(-sqrt(e*x + d)*c*sqrt(-(c*d - b*e)/c)/(c*d - b*e)))/(b*
c), (2*c*sqrt(-d)*d*arctan(sqrt(e*x + d)*sqrt(-d)/d) + 2*sqrt(e*x + d)*b*e
- (c*d - b*e)*sqrt((c*d - b*e)/c)*log((c*e*x + 2*c*d - b*e - 2*sqrt(e*x + d
)*c*sqrt((c*d - b*e)/c))/(c*x + b)))/(b*c), 2*(c*sqrt(-d)*d*arctan(sqrt(e*x
 + d)*sqrt(-d)/d) + sqrt(e*x + d)*b*e + (c*d - b*e)*sqrt(-(c*d - b*e)/c)*ar
ctan(-sqrt(e*x + d)*c*sqrt(-(c*d - b*e)/c)/(c*d - b*e)))/(b*c)]
```

Sympy [A] time = 33.7844, size = 92, normalized size = 1.

$$\frac{2e\sqrt{d+ex}}{c}+\frac{2d^2\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-d}}\right)}{b\sqrt{-d}}-\frac{2\,(be-cd)^2\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{\frac{be-cd}{c}}}\right)}{bc^2\sqrt{\frac{be-cd}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+b*x),x)
```

```
[Out] 2*e*sqrt(d + e*x)/c + 2*d**2*atan(sqrt(d + e*x)/sqrt(-d))/(b*sqrt(-d)) - 2*
(b*e - c*d)**2*atan(sqrt(d + e*x)/sqrt((b*e - c*d)/c))/(b*c**2*sqrt((b*e -
c*d)/c))
```

Giac [A] time = 1.37459, size = 151, normalized size = 1.64

$$\frac{2\,d^2\arctan\left(\frac{\sqrt{xe+d}}{\sqrt{-d}}\right)}{b\sqrt{-d}}+\frac{2\,\sqrt{xe+d}e}{c}-\frac{2\left(c^2d^2-2\,bcde+b^2e^2\right)\arctan\left(\frac{\sqrt{xe+d}c}{\sqrt{-c^2d+bce}}\right)}{\sqrt{-c^2d+bce}bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] 2*d^2*arctan(sqrt(x*e + d)/sqrt(-d))/(b*sqrt(-d)) + 2*sqrt(x*e + d)*e/c - 2
*(c^2*d^2 - 2*b*c*d*e + b^2*e^2)*arctan(sqrt(x*e + d)*c/sqrt(-c^2*d + b*c*e
))/(sqrt(-c^2*d + b*c*e)*b*c)
```

3.364 $\int \frac{\sqrt{d+ex}}{bx+cx^2} \, dx$

Optimal. Leaf size=77

$$\frac{2\sqrt{cd-be}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b\sqrt{c}}-\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b}$$

```
[Out] (-2*Sqrt[d]*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/b + (2*Sqrt[c*d - b*e]*ArcTanh[
(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(b*Sqrt[c])
```

Rubi [A] time = 0.0723879, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {699, 1130, 208}

$$\frac{2\sqrt{cd-be}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b\sqrt{c}}-\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(b*x + c*x^2),x]
```

```
[Out] (-2*Sqrt[d]*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/b + (2*Sqrt[c*d - b*e]*ArcTanh[
(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(b*Sqrt[c])
```

Rule 699

```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Dist[2*e, Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*
x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{bx+cx^2}\,dx = (2e)\,\mathrm{Subst}\left(\int \frac{x^2}{cd^2-bde+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)$$
$$= \frac{(2cd)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{be}{2}+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x, \sqrt{d+ex}\right)}{b} + \left(e\left(1+\frac{-2cd+be}{be}\right)\right)\mathrm{Subst}\left(\int \frac{1}{\frac{be}{2}+\frac{1}{2}(-2cd+be)+\right.$$
$$= -\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b} + \frac{2\sqrt{cd-be}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b\sqrt{c}}$$

Mathematica [A] time = 0.036094, size = 75, normalized size = 0.97

$$\frac{2\left(\frac{\sqrt{cd-be}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{\sqrt{c}}-\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(b*x + c*x^2),x]
```

```
[Out] (2*(-(Sqrt[d]*ArcTanh[Sqrt[d + e*x]/Sqrt[d]]) + (Sqrt[c*d - b*e]*ArcTanh[(S
qrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/Sqrt[c]))/b
```

Maple [A] time = 0.217, size = 100, normalized size = 1.3

$$2\,\frac{e}{\sqrt{(be-cd)\,c}}\arctan\left(\frac{\sqrt{ex+dc}}{\sqrt{(be-cd)\,c}}\right)-2\,\frac{cd}{b\sqrt{(be-cd)\,c}}\arctan\left(\frac{\sqrt{ex+dc}}{\sqrt{(be-cd)\,c}}\right)-2\,\frac{\sqrt{d}}{b}\mathrm{Artanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*x^2+b*x),x)
```

```
[Out] 2*e/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))-2/b/((b
*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))*c*d-2*arctanh(
(e*x+d)^(1/2)/d^(1/2))*d^(1/2)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.38493, size = 788, normalized size = 10.23

$$\left[\frac{\sqrt{\frac{cd-be}{c}}\log\left(\frac{cex+2\,cd-be+2\,\sqrt{ex+d}c\sqrt{\frac{cd-be}{c}}}{cx+b}\right)+\sqrt{d}\log\left(\frac{ex-2\,\sqrt{ex+d}\sqrt{d}+2\,d}{x}\right)}{b}, \frac{2\,\sqrt{-\frac{cd-be}{c}}\arctan\left(-\frac{\sqrt{ex+d}c\sqrt{-\frac{cd-be}{c}}}{cd-be}\right)+\sqrt{d}\log\left(\frac{ex-2}{}\right.}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] [(sqrt((c*d - b*e)/c)*log((c*e*x + 2*c*d - b*e + 2*sqrt(e*x + d)*c*sqrt((c*
d - b*e)/c))/(c*x + b)) + sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)
/x))/b, (2*sqrt(-(c*d - b*e)/c)*arctan(-sqrt(e*x + d)*c*sqrt(-(c*d - b*e)/c
)/(c*d - b*e)) + sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x))/b, (
2*sqrt(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d) + sqrt((c*d - b*e)/c)*log((c*e*
x + 2*c*d - b*e + 2*sqrt(e*x + d)*c*sqrt((c*d - b*e)/c))/(c*x + b)))/b, 2*(
sqrt(-(c*d - b*e)/c)*arctan(-sqrt(e*x + d)*c*sqrt(-(c*d - b*e)/c)/(c*d - b*
e)) + sqrt(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d))/b]
```

Sympy [A] time = 4.90654, size = 78, normalized size = 1.01

$$\frac{2\left(\frac{de\,\mathrm{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-d}}\right)}{b\sqrt{-d}}+\frac{e(be-cd)\,\mathrm{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{\frac{be-cd}{c}}}\right)}{bc\sqrt{\frac{be-cd}{c}}}\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+b*x),x)
```

```
[Out] 2*(d*e*atan(sqrt(d + e*x)/sqrt(-d))/(b*sqrt(-d)) + e*(b*e - c*d)*atan(sqrt(
d + e*x)/sqrt((b*e - c*d)/c))/(b*c*sqrt((b*e - c*d)/c)))/e
```

Giac [A] time = 1.17986, size = 108, normalized size = 1.4

$$-\frac{2\,(cd-be)\arctan\left(\frac{\sqrt{xe+d}c}{\sqrt{-c^2d+bce}}\right)}{\sqrt{-c^2d+bce}b}+\frac{2\,d\arctan\left(\frac{\sqrt{xe+d}}{\sqrt{-d}}\right)}{b\sqrt{-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] -2*(c*d - b*e)*arctan(sqrt(x*e + d)*c/sqrt(-c^2*d + b*c*e))/(sqrt(-c^2*d +
b*c*e)*b) + 2*d*arctan(sqrt(x*e + d)/sqrt(-d))/(b*sqrt(-d))
```

3.365 $\int \frac{1}{\sqrt{d+ex}(bx+cx^2)}\,dx$

Optimal. Leaf size=77

$$\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b\sqrt{cd-be}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b\sqrt{d}}$$

```
[Out] (-2*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(b*Sqrt[d]) + (2*Sqrt[c]*ArcTanh[(Sqrt[
c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(b*Sqrt[c*d - b*e])
```

Rubi [A] time = 0.0576517, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {707, 1093, 208}

$$\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b\sqrt{cd-be}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(b*x + c*x^2)),x]
```

```
[Out] (-2*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(b*Sqrt[d]) + (2*Sqrt[c]*ArcTanh[(Sqrt[
c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(b*Sqrt[c*d - b*e])
```

Rule 707

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Sym
bol] :> Dist[2*e, Subst[Int[1/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{d+ex}\left(bx+cx^2\right)}\,dx&=(2e)\,\text{Subst}\left(\int \frac{1}{cd^2-bde-(2cd-be)x^2+cx^4}\,dx,x,\sqrt{d+ex}\right)\\&=\frac{(2c)\,\text{Subst}\left(\int \frac{1}{-\frac{be}{2}+\frac{1}{2}(-2cd+be)+cx^2}\,dx,x,\sqrt{d+ex}\right)}{b}-\frac{(2c)\,\text{Subst}\left(\int \frac{1}{\frac{be}{2}+\frac{1}{2}(-2cd+be)+cx^2}\,dx,x,\sqrt{d+ex}\right)}{b}\\&=-\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b\sqrt{d}}+\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b\sqrt{cd-be}}\end{aligned}$$

Mathematica [A] time = 0.0841091, size = 75, normalized size = 0.97

$$\frac{\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{\sqrt{cd-be}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{d}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(b*x + c*x^2)),x]
```

```
[Out] ((-2*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/Sqrt[d] + (2*Sqrt[c]*ArcTanh[(Sqrt[c]*
Sqrt[d + e*x])/Sqrt[c*d - b*e]])/Sqrt[c*d - b*e])/b
```

Maple [A] time = 0.25, size = 62, normalized size = 0.8

$$-2\,\frac{c}{b\sqrt{(be-cd)\,c}}\arctan\left(\frac{\sqrt{ex+d}c}{\sqrt{(be-cd)\,c}}\right)-2\,\frac{1}{b\sqrt{d}}\mathrm{Artanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(c*x^2+b*x),x)
```

```
[Out] -2*c/b/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))-2*ar
ctanh((e*x+d)^(1/2)/d^(1/2))/b/d^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.33209, size = 880, normalized size = 11.43

$$\left[\frac{d\sqrt{\frac{c}{cd-be}}\log\left(\frac{cex+2\,cd-be+2\,(cd-be)\sqrt{ex+d}\sqrt{\frac{c}{cd-be}}}{cx+b}\right)+\sqrt{d}\log\left(\frac{ex-2\,\sqrt{ex+d}\sqrt{d}+2\,d}{x}\right)}{bd}, \frac{2\,d\sqrt{-\frac{c}{cd-be}}\arctan\left(-\frac{(cd-be)\sqrt{ex+d}\sqrt{-\frac{c}{cd-be}}}{cex+cd}\right)+ \cdots}{bd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] [(d*sqrt(c/(c*d - b*e))*log((c*e*x + 2*c*d - b*e + 2*(c*d - b*e)*sqrt(e*x +
 d)*sqrt(c/(c*d - b*e)))/(c*x + b)) + sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sq
rt(d) + 2*d)/x))/(b*d), (2*d*sqrt(-c/(c*d - b*e))*arctan(-(c*d - b*e)*sqrt(
e*x + d)*sqrt(-c/(c*d - b*e))/(c*e*x + c*d)) + sqrt(d)*log((e*x - 2*sqrt(e*
```

```
x + d)*sqrt(d) + 2*d)/x))/(b*d), (d*sqrt(c/(c*d - b*e))*log((c*e*x + 2*c*d
- b*e + 2*(c*d - b*e)*sqrt(e*x + d)*sqrt(c/(c*d - b*e)))/(c*x + b)) + 2*sqr
t(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d))/(b*d), 2*(d*sqrt(-c/(c*d - b*e))*ar
ctan(-(c*d - b*e)*sqrt(e*x + d)*sqrt(-c/(c*d - b*e))/(c*e*x + c*d)) + sqrt(
-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d))/(b*d)]
```

Sympy [A] time = 17.2026, size = 80, normalized size = 1.04

$$\frac{2c\operatorname{atan}\left(\frac{1}{\sqrt{\frac{c}{be-cd}}\sqrt{d+ex}}\right)}{b\sqrt{\frac{c}{be-cd}}\,(be-cd)}+\frac{2\operatorname{atan}\left(\frac{1}{\sqrt{-\frac{1}{d}}\sqrt{d+ex}}\right)}{bd\sqrt{-\frac{1}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(c*x**2+b*x),x)
```

```
[Out] 2*c*atan(1/(sqrt(c/(b*e - c*d))*sqrt(d + e*x)))/(b*sqrt(c/(b*e - c*d))*(b*e
 - c*d)) + 2*atan(1/(sqrt(-1/d)*sqrt(d + e*x)))/(b*d*sqrt(-1/d))
```

Giac [A] time = 1.17955, size = 96, normalized size = 1.25

$$-\frac{2\,c\arctan\left(\frac{\sqrt{xe+d}c}{\sqrt{-c^2d+bce}}\right)}{\sqrt{-c^2d+bce}b}+\frac{2\,\arctan\left(\frac{\sqrt{xe+d}}{\sqrt{-d}}\right)}{b\sqrt{-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] -2*c*arctan(sqrt(x*e + d)*c/sqrt(-c^2*d + b*c*e))/(sqrt(-c^2*d + b*c*e)*b)
+ 2*arctan(sqrt(x*e + d)/sqrt(-d))/(b*sqrt(-d))
```

3.366 $\int \frac{1}{(d+ex)^{3/2}(bx+cx^2)} dx$

Optimal. Leaf size=102

$$\frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b(cd-be)^{3/2}}-\frac{2e}{d\sqrt{d+ex}(cd-be)}-\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{bd^{3/2}}$$

```
[Out] (-2*e)/(d*(c*d - b*e)*Sqrt[d + e*x]) - (2*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(
b*d^(3/2)) + (2*c^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(
b*(c*d - b*e)^(3/2))
```

Rubi [A] time = 0.31775, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {709, 826, 1166, 208}

$$\frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b(cd-be)^{3/2}}-\frac{2e}{d\sqrt{d+ex}(cd-be)}-\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{bd^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*(b*x + c*x^2)),x]
```

```
[Out] (-2*e)/(d*(c*d - b*e)*Sqrt[d + e*x]) - (2*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(
b*d^(3/2)) + (2*c^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(
b*(c*d - b*e)^(3/2))
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)^{3/2}\left(bx+cx^2\right)}dx &= -\frac{2e}{d(cd-be)\sqrt{d+ex}}+\frac{\int \frac{cd-be-cex}{\sqrt{d+ex}\left(bx+cx^2\right)}dx}{d(cd-be)}\\ &= -\frac{2e}{d(cd-be)\sqrt{d+ex}}+\frac{2\,\mathrm{Subst}\left(\int \frac{cde+e(cd-be)-cex^2}{cd^2-bde+(-2cd+be)x^2+cx^4}dx,x,\sqrt{d+ex}\right)}{d(cd-be)}\\ &= -\frac{2e}{d(cd-be)\sqrt{d+ex}}+\frac{(2c)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{be}{2}+\frac{1}{2}(-2cd+be)+cx^2}dx,x,\sqrt{d+ex}\right)}{bd}-\frac{\left(2c^2\right)\mathrm{Subst}\left(\int \frac{}{}\right.}{}\\ &= -\frac{2e}{d(cd-be)\sqrt{d+ex}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{bd^{3/2}}+\frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b(cd-be)^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.0270691, size = 81, normalized size = 0.79

$$-\frac{2\left(cd\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{c(d+ex)}{cd-be}\right)+(be-cd)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{ex}{d}+1\right)\right)}{bd\sqrt{d+ex}(cd-be)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*(b*x + c*x^2)),x]
```

```
[Out] (-2*(c*d*Hypergeometric2F1[-1/2, 1, 1/2, (c*(d + e*x))/(c*d - b*e)] + (-(c*
d) + b*e)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (e*x)/d]))/(b*d*(c*d - b*e)*S
qrt[d + e*x])
```

Maple [A] time = 0.231, size = 97, normalized size = 1.

$$2\,\frac{c^2}{(be-cd)\,b\sqrt{(be-cd)\,c}}\arctan\left(\frac{\sqrt{ex+dc}}{\sqrt{(be-cd)\,c}}\right)-2\,\frac{1}{bd^{3/2}}\mathrm{Artanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)+2\,\frac{e}{d\,(be-cd)\,\sqrt{ex+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(c*x^2+b*x),x)
```

```
[Out] 2/(b*e-c*d)*c^2/b/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^
(1/2))-2*arctanh((e*x+d)^(1/2)/d^(1/2))/b/d^(3/2)+2*e/d/(b*e-c*d)/(e*x+d)^(
1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.73322, size = 1553, normalized size = 15.23

$$\left[-\frac{2\sqrt{ex+d}bde+\left(cd^2ex+cd^3\right)\sqrt{\frac{c}{cd-be}}\log\left(\frac{cex+2\,cd-be-2\,(cd-be)\sqrt{ex+d}\sqrt{\frac{c}{cd-be}}}{cx+b}\right)-\left(cd^2-bde+\left(cde-be^2\right)x\right)\sqrt{d}\log\left(\frac{ex-2\,\sqrt{ex}}{}\right.}{bcd^4-b^2d^3e+\left(bcd^3e-b^2d^2e^2\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] [-(2*sqrt(e*x + d)*b*d*e + (c*d^2*e*x + c*d^3)*sqrt(c/(c*d - b*e))*log((c*e
*x + 2*c*d - b*e - 2*(c*d - b*e)*sqrt(e*x + d)*sqrt(c/(c*d - b*e)))/(c*x +
b)) - (c*d^2 - b*d*e + (c*d*e - b*e^2)*x)*sqrt(d)*log((e*x - 2*sqrt(e*x + d
)*sqrt(d) + 2*d)/x))/(b*c*d^4 - b^2*d^3*e + (b*c*d^3*e - b^2*d^2*e^2)*x), -
(2*sqrt(e*x + d)*b*d*e - 2*(c*d^2*e*x + c*d^3)*sqrt(-c/(c*d - b*e))*arctan(
-(c*d - b*e)*sqrt(e*x + d)*sqrt(-c/(c*d - b*e))/(c*e*x + c*d)) - (c*d^2 - b
*d*e + (c*d*e - b*e^2)*x)*sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)
/x))/(b*c*d^4 - b^2*d^3*e + (b*c*d^3*e - b^2*d^2*e^2)*x), -(2*sqrt(e*x + d)
*b*d*e - 2*(c*d^2 - b*d*e + (c*d*e - b*e^2)*x)*sqrt(-d)*arctan(sqrt(e*x + d
)*sqrt(-d)/d) + (c*d^2*e*x + c*d^3)*sqrt(c/(c*d - b*e))*log((c*e*x + 2*c*d
- b*e - 2*(c*d - b*e)*sqrt(e*x + d)*sqrt(c/(c*d - b*e)))/(c*x + b)))/(b*c*d
^4 - b^2*d^3*e + (b*c*d^3*e - b^2*d^2*e^2)*x), -2*(sqrt(e*x + d)*b*d*e - (c
*d^2*e*x + c*d^3)*sqrt(-c/(c*d - b*e))*arctan(-(c*d - b*e)*sqrt(e*x + d)*sq
rt(-c/(c*d - b*e))/(c*e*x + c*d)) - (c*d^2 - b*d*e + (c*d*e - b*e^2)*x)*sqr
t(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d))/(b*c*d^4 - b^2*d^3*e + (b*c*d^3*e -
 b^2*d^2*e^2)*x)]
```

Sympy [A] time = 17.9726, size = 94, normalized size = 0.92

$$\frac{2e}{d\sqrt{d+ex}\,(be-cd)}+\frac{2c\,\mathrm{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{\frac{be-cd}{c}}}\right)}{b\sqrt{\frac{be-cd}{c}}\,(be-cd)}+\frac{2\,\mathrm{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-d}}\right)}{bd\sqrt{-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(c*x**2+b*x),x)
```

```
[Out] 2*e/(d*sqrt(d + e*x)*(b*e - c*d)) + 2*c*atan(sqrt(d + e*x)/sqrt((b*e - c*d)
/c))/(b*sqrt((b*e - c*d)/c)*(b*e - c*d)) + 2*atan(sqrt(d + e*x)/sqrt(-d))/(
b*d*sqrt(-d))
```

Giac [A] time = 1.25844, size = 153, normalized size = 1.5

$$-\frac{2\,c^2\arctan\left(\frac{\sqrt{xe+d}c}{\sqrt{-c^2d+bce}}\right)}{\left(bcd-b^2e\right)\sqrt{-c^2d+bce}}-\frac{2\,e}{\left(cd^2-bde\right)\sqrt{xe+d}}+\frac{2\,\arctan\left(\frac{\sqrt{xe+d}}{\sqrt{-d}}\right)}{b\sqrt{-d}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] -2*c^2*arctan(sqrt(x*e + d)*c/sqrt(-c^2*d + b*c*e))/((b*c*d - b^2*e)*sqrt(-
c^2*d + b*c*e)) - 2*e/((c*d^2 - b*d*e)*sqrt(x*e + d)) + 2*arctan(sqrt(x*e +
 d)/sqrt(-d))/(b*sqrt(-d)*d)
```

3.367 $\int \frac{1}{(d+ex)^{5/2}\left(bx+cx^2\right)}\,dx$

Optimal. Leaf size=138

$$\frac{2c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b(cd-be)^{5/2}}-\frac{2e(2cd-be)}{d^2\sqrt{d+ex}(cd-be)^2}-\frac{2e}{3d(d+ex)^{3/2}(cd-be)}-\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{bd^{5/2}}$$

```
[Out] (-2*e)/(3*d*(c*d - b*e)*(d + e*x)^(3/2)) - (2*e*(2*c*d - b*e))/(d^2*(c*d -
b*e)^2*Sqrt[d + e*x]) - (2*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(b*d^(5/2)) + (2
*c^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(b*(c*d - b*e)^(
5/2))
```

Rubi [A] time = 0.260793, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {709, 828, 826, 1166, 208}

$$\frac{2c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b(cd-be)^{5/2}}-\frac{2e(2cd-be)}{d^2\sqrt{d+ex}(cd-be)^2}-\frac{2e}{3d(d+ex)^{3/2}(cd-be)}-\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{bd^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(5/2)*(b*x + c*x^2)),x]
```

```
[Out] (-2*e)/(3*d*(c*d - b*e)*(d + e*x)^(3/2)) - (2*e*(2*c*d - b*e))/(d^2*(c*d -
b*e)^2*Sqrt[d + e*x]) - (2*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(b*d^(5/2)) + (2
*c^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(b*(c*d - b*e)^(
5/2))
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{5/2}\left(bx+cx^2\right)}dx = -\frac{2e}{3d(cd-be)(d+ex)^{3/2}} + \frac{\int \frac{cd-be-cex}{(d+ex)^{3/2}\left(bx+cx^2\right)}dx}{d(cd-be)}$$

$$= -\frac{2e}{3d(cd-be)(d+ex)^{3/2}} - \frac{2e(2cd-be)}{d^2(cd-be)^2\sqrt{d+ex}} + \frac{\int \frac{(cd-be)^2-ce(2cd-be)x}{\sqrt{d+ex}\left(bx+cx^2\right)}dx}{d^2(cd-be)^2}$$

$$= -\frac{2e}{3d(cd-be)(d+ex)^{3/2}} - \frac{2e(2cd-be)}{d^2(cd-be)^2\sqrt{d+ex}} + \frac{2\,\text{Subst}\left(\int \frac{e(cd-be)^2+cde(2cd-be)-ce(2cd-be)x^2}{cd^2-bde+(-2cd+be)x^2+cx^4}\right.}{d^2(cd-be)^2}$$

$$= -\frac{2e}{3d(cd-be)(d+ex)^{3/2}} - \frac{2e(2cd-be)}{d^2(cd-be)^2\sqrt{d+ex}} + \frac{(2c)\,\text{Subst}\left(\int \frac{1}{-\frac{be}{2}+\frac{1}{2}(-2cd+be)+cx^2}dx, x, \sqrt{\right.}{bd^2}$$

$$= -\frac{2e}{3d(cd-be)(d+ex)^{3/2}} - \frac{2e(2cd-be)}{d^2(cd-be)^2\sqrt{d+ex}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{bd^{5/2}} + \frac{2c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+}}{\sqrt{cd-b}}\right.}{b(cd-be)^{5/2}}$$

Mathematica [C] time = 0.0267906, size = 83, normalized size = 0.6

$$-\frac{2\left(cd\,_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{c(d+ex)}{cd-be}\right)+(be-cd)\,_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{ex}{d}+1\right)\right)}{3bd(d+ex)^{3/2}(cd-be)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(5/2)*(b*x + c*x^2)),x]
```

```
[Out] (-2*(c*d*Hypergeometric2F1[-3/2, 1, -1/2, (c*(d + e*x))/(c*d - b*e)] + (-(c
*d) + b*e)*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (e*x)/d]))/(3*b*d*(c*d - b*
e)*(d + e*x)^(3/2))
```

Maple [A] time = 0.224, size = 147, normalized size = 1.1

$$2\,\frac{be^2}{d^2\,(be-cd)^2\,\sqrt{ex+d}} - 4\,\frac{ce}{d\,(be-cd)^2\,\sqrt{ex+d}} + \frac{2\,e}{3\,d\,(be-cd)}\,(ex+d)^{-\frac{3}{2}} - 2\,\frac{c^3}{(be-cd)^2\,b\sqrt{(be-cd)\,c}}\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{(be-cd}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(5/2)/(c*x^2+b*x),x)
```

```
[Out] 2/d^2/(b*e-c*d)^2/(e*x+d)^(1/2)*b*e^2-4*e/d/(b*e-c*d)^2/(e*x+d)^(1/2)*c+2/3
*e/d/(b*e-c*d)/(e*x+d)^(3/2)-2/(b*e-c*d)^2*c^3/b/((b*e-c*d)*c)^(1/2)*arctan
((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))-2*arctanh((e*x+d)^(1/2)/d^(1/2))/b/d^
(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.59143, size = 3040, normalized size = 22.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(c^2*d^3*e^2*x^2 + 2*c^2*d^4*e*x + c^2*d^5)*sqrt(c/(c*d - b*e))*log
((c*e*x + 2*c*d - b*e + 2*(c*d - b*e)*sqrt(e*x + d)*sqrt(c/(c*d - b*e)))/(c
*x + b)) + 3*(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (c^2*d^2*e^2 - 2*b*c*d*
e^3 + b^2*e^4)*x^2 + 2*(c^2*d^3*e - 2*b*c*d^2*e^2 + b^2*d*e^3)*x)*sqrt(d)*l
og((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) - 2*(7*b*c*d^3*e - 4*b^2*d^2*e^
2 + 3*(2*b*c*d^2*e^2 - b^2*d*e^3)*x)*sqrt(e*x + d))/(b*c^2*d^7 - 2*b^2*c*d^
6*e + b^3*d^5*e^2 + (b*c^2*d^5*e^2 - 2*b^2*c*d^4*e^3 + b^3*d^3*e^4)*x^2 + 2
*(b*c^2*d^6*e - 2*b^2*c*d^5*e^2 + b^3*d^4*e^3)*x), 1/3*(6*(c^2*d^3*e^2*x^2
+ 2*c^2*d^4*e*x + c^2*d^5)*sqrt(-c/(c*d - b*e))*arctan(-(c*d - b*e)*sqrt(e*
x + d)*sqrt(-c/(c*d - b*e))/(c*e*x + c*d)) + 3*(c^2*d^4 - 2*b*c*d^3*e + b^2
*d^2*e^2 + (c^2*d^2*e^2 - 2*b*c*d*e^3 + b^2*e^4)*x^2 + 2*(c^2*d^3*e - 2*b*c
*d^2*e^2 + b^2*d*e^3)*x)*sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/
x) - 2*(7*b*c*d^3*e - 4*b^2*d^2*e^2 + 3*(2*b*c*d^2*e^2 - b^2*d*e^3)*x)*sqrt
(e*x + d))/(b*c^2*d^7 - 2*b^2*c*d^6*e + b^3*d^5*e^2 + (b*c^2*d^5*e^2 - 2*b^
2*c*d^4*e^3 + b^3*d^3*e^4)*x^2 + 2*(b*c^2*d^6*e - 2*b^2*c*d^5*e^2 + b^3*d^4
*e^3)*x), 1/3*(6*(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (c^2*d^2*e^2 - 2*b*
c*d*e^3 + b^2*e^4)*x^2 + 2*(c^2*d^3*e - 2*b*c*d^2*e^2 + b^2*d*e^3)*x)*sqrt(
-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d) + 3*(c^2*d^3*e^2*x^2 + 2*c^2*d^4*e*x +
 c^2*d^5)*sqrt(c/(c*d - b*e))*log((c*e*x + 2*c*d - b*e + 2*(c*d - b*e)*sqrt
(e*x + d)*sqrt(c/(c*d - b*e)))/(c*x + b)) - 2*(7*b*c*d^3*e - 4*b^2*d^2*e^2
+ 3*(2*b*c*d^2*e^2 - b^2*d*e^3)*x)*sqrt(e*x + d))/(b*c^2*d^7 - 2*b^2*c*d^6*
e + b^3*d^5*e^2 + (b*c^2*d^5*e^2 - 2*b^2*c*d^4*e^3 + b^3*d^3*e^4)*x^2 + 2*(
b*c^2*d^6*e - 2*b^2*c*d^5*e^2 + b^3*d^4*e^3)*x), 2/3*(3*(c^2*d^3*e^2*x^2 +
2*c^2*d^4*e*x + c^2*d^5)*sqrt(-c/(c*d - b*e))*arctan(-(c*d - b*e)*sqrt(e*x
+ d)*sqrt(-c/(c*d - b*e))/(c*e*x + c*d)) + 3*(c^2*d^4 - 2*b*c*d^3*e + b^2*d
^2*e^2 + (c^2*d^2*e^2 - 2*b*c*d*e^3 + b^2*e^4)*x^2 + 2*(c^2*d^3*e - 2*b*c*d
^2*e^2 + b^2*d*e^3)*x)*sqrt(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d) - (7*b*c*d
^3*e - 4*b^2*d^2*e^2 + 3*(2*b*c*d^2*e^2 - b^2*d*e^3)*x)*sqrt(e*x + d))/(b*c
^2*d^7 - 2*b^2*c*d^6*e + b^3*d^5*e^2 + (b*c^2*d^5*e^2 - 2*b^2*c*d^4*e^3 + b
^3*d^3*e^4)*x^2 + 2*(b*c^2*d^6*e - 2*b^2*c*d^5*e^2 + b^3*d^4*e^3)*x)]
```

Sympy [A] time = 21.7701, size = 133, normalized size = 0.96

$$\frac{2e}{3d(d+ex)^{\frac{3}{2}}(be-cd)}+\frac{2e(be-2cd)}{d^2\sqrt{d+ex}(be-cd)^2}-\frac{2c^2\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{\frac{be-cd}{c}}}\right)}{b\sqrt{\frac{be-cd}{c}}(be-cd)^2}+\frac{2\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-d}}\right)}{bd^2\sqrt{-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(5/2)/(c*x**2+b*x),x)
```

```
[Out] 2*e/(3*d*(d + e*x)**(3/2)*(b*e - c*d)) + 2*e*(b*e - 2*c*d)/(d**2*sqrt(d + e
*x)*(b*e - c*d)**2) - 2*c**2*atan(sqrt(d + e*x)/sqrt((b*e - c*d)/c))/(b*sqr
t((b*e - c*d)/c)*(b*e - c*d)**2) + 2*atan(sqrt(d + e*x)/sqrt(-d))/(b*d**2*s
qrt(-d))
```

Giac [A] time = 1.32073, size = 235, normalized size = 1.7

$$-\frac{2c^3\arctan\left(\frac{\sqrt{xe+d}c}{\sqrt{-c^2d+bce}}\right)}{\left(bc^2d^2-2b^2cde+b^3e^2\right)\sqrt{-c^2d+bce}}-\frac{2\left(6(xe+d)cde+cd^2e-3(xe+d)be^2-bde^2\right)}{3\left(c^2d^4-2bcd^3e+b^2d^2e^2\right)(xe+d)^{\frac{3}{2}}}+\frac{2\arctan\left(\frac{\sqrt{xe+d}}{\sqrt{-d}}\right)}{b\sqrt{-d}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] -2*c^3*arctan(sqrt(x*e + d)*c/sqrt(-c^2*d + b*c*e))/((b*c^2*d^2 - 2*b^2*c*d
*e + b^3*e^2)*sqrt(-c^2*d + b*c*e)) - 2/3*(6*(x*e + d)*c*d*e + c*d^2*e - 3*
(x*e + d)*b*e^2 - b*d*e^2)/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2)*(x*e + d)
^(3/2)) + 2*arctan(sqrt(x*e + d)/sqrt(-d))/(b*sqrt(-d)*d^2)
```

3.368 $\int \frac{1}{(d+ex)^{7/2}\left(bx+cx^2\right)}\,dx$

Optimal. Leaf size=187

$$-\frac{2e\left(b^2e^2-3bcde+3c^2d^2\right)}{d^3\sqrt{d+ex}(cd-be)^3}+\frac{2c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b(cd-be)^{7/2}}-\frac{2e(2cd-be)}{3d^2(d+ex)^{3/2}(cd-be)^2}-\frac{2e}{5d(d+ex)^{5/2}(cd-be)}-\frac{2\tanh^{-1}(}{bd^{7/}}$$

```
[Out] (-2*e)/(5*d*(c*d - b*e)*(d + e*x)^(5/2)) - (2*e*(2*c*d - b*e))/(3*d^2*(c*d
- b*e)^2*(d + e*x)^(3/2)) - (2*e*(3*c^2*d^2 - 3*b*c*d*e + b^2*e^2))/(d^3*(c
*d - b*e)^3*Sqrt[d + e*x]) - (2*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(b*d^(7/2))
 + (2*c^(7/2)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(b*(c*d - b
*e)^(7/2))
```

Rubi [A] time = 0.376799, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {709, 828, 826, 1166, 208}

$$-\frac{2e\left(b^2e^2-3bcde+3c^2d^2\right)}{d^3\sqrt{d+ex}(cd-be)^3}+\frac{2c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b(cd-be)^{7/2}}-\frac{2e(2cd-be)}{3d^2(d+ex)^{3/2}(cd-be)^2}-\frac{2e}{5d(d+ex)^{5/2}(cd-be)}-\frac{2\tanh^{-1}(}{bd^{7/}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(7/2)*(b*x + c*x^2)),x]
```


```
[Out] (-2*e)/(5*d*(c*d - b*e)*(d + e*x)^(5/2)) - (2*e*(2*c*d - b*e))/(3*d^2*(c*d
- b*e)^2*(d + e*x)^(3/2)) - (2*e*(3*c^2*d^2 - 3*b*c*d*e + b^2*e^2))/(d^3*(c
*d - b*e)^3*Sqrt[d + e*x]) - (2*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(b*d^(7/2))
 + (2*c^(7/2)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(b*(c*d - b
*e)^(7/2))
```


Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
```

```
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{7/2}\left(bx+cx^2\right)}\,dx = -\frac{2e}{5d(cd-be)(d+ex)^{5/2}}+\frac{\int \frac{cd-be-cex}{(d+ex)^{5/2}\left(bx+cx^2\right)}\,dx}{d(cd-be)}$$

$$= -\frac{2e}{5d(cd-be)(d+ex)^{5/2}}-\frac{2e(2cd-be)}{3d^2(cd-be)^2(d+ex)^{3/2}}+\frac{\int \frac{(cd-be)^2-ce(2cd-be)x}{(d+ex)^{3/2}\left(bx+cx^2\right)}\,dx}{d^2(cd-be)^2}$$

$$= -\frac{2e}{5d(cd-be)(d+ex)^{5/2}}-\frac{2e(2cd-be)}{3d^2(cd-be)^2(d+ex)^{3/2}}-\frac{2e\left(3c^2d^2-3bcde+b^2e^2\right)}{d^3(cd-be)^3\sqrt{d+ex}}+\frac{\int \frac{(cd-be)^3}{\cdots}}{\cdots}$$

$$= -\frac{2e}{5d(cd-be)(d+ex)^{5/2}}-\frac{2e(2cd-be)}{3d^2(cd-be)^2(d+ex)^{3/2}}-\frac{2e\left(3c^2d^2-3bcde+b^2e^2\right)}{d^3(cd-be)^3\sqrt{d+ex}}+\frac{2\,\mathrm{Subst}\cdots}{\cdots}$$

$$= -\frac{2e}{5d(cd-be)(d+ex)^{5/2}}-\frac{2e(2cd-be)}{3d^2(cd-be)^2(d+ex)^{3/2}}-\frac{2e\left(3c^2d^2-3bcde+b^2e^2\right)}{d^3(cd-be)^3\sqrt{d+ex}}+\frac{(2c)\,\mathrm{Sub}\cdots}{\cdots}$$

$$= -\frac{2e}{5d(cd-be)(d+ex)^{5/2}}-\frac{2e(2cd-be)}{3d^2(cd-be)^2(d+ex)^{3/2}}-\frac{2e\left(3c^2d^2-3bcde+b^2e^2\right)}{d^3(cd-be)^3\sqrt{d+ex}}-\frac{2\tanh^{-1}\cdots}{bd\cdots}$$

Mathematica [C] time = 0.0303762, size = 83, normalized size = 0.44

$$-\frac{2\left(cd\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\frac{c(d+ex)}{cd-be}\right)+(be-cd)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\frac{ex}{d}+1\right)\right)}{5bd(d+ex)^{5/2}(cd-be)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(7/2)*(b*x + c*x^2)),x]
```

```
[Out] (-2*(c*d*Hypergeometric2F1[-5/2, 1, -3/2, (c*(d + e*x))/(c*d - b*e)] + (-(c
*d) + b*e)*Hypergeometric2F1[-5/2, 1, -3/2, 1 + (e*x)/d]))/(5*b*d*(c*d - b*
e)*(d + e*x)^(5/2))
```

Maple [A] time = 0.235, size = 228, normalized size = 1.2

$$\frac{2\,be^2}{3\,d^2\,(be-cd)^2}(ex+d)^{-\frac{3}{2}}-\frac{4\,ce}{3\,d\,(be-cd)^2}(ex+d)^{-\frac{3}{2}}+2\,\frac{e^3b^2}{d^3\,(be-cd)^3\,\sqrt{ex+d}}-6\,\frac{bce^2}{d^2\,(be-cd)^3\,\sqrt{ex+d}}+6\,\frac{c^2e}{d\,(be-cd)^3}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(7/2)/(c*x^2+b*x),x)
```

```
[Out] 2/3/d^2/(b*e-c*d)^2/(e*x+d)^(3/2)*b*e^2-4/3*e/d/(b*e-c*d)^2/(e*x+d)^(3/2)*c
+2/d^3/(b*e-c*d)^3/(e*x+d)^(1/2)*b^2*e^3-6/d^2/(b*e-c*d)^3/(e*x+d)^(1/2)*b*
c*e^2+6*e/d/(b*e-c*d)^3/(e*x+d)^(1/2)*c^2+2/5*e/d/(b*e-c*d)/(e*x+d)^(5/2)+2
/(b*e-c*d)^3*c^4/b/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)
^(1/2))-2*arctanh((e*x+d)^(1/2)/d^(1/2))/b/d^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 8.15514, size = 5191, normalized size = 27.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] [-1/15*(15*(c^3*d^4*e^3*x^3 + 3*c^3*d^5*e^2*x^2 + 3*c^3*d^6*e*x + c^3*d^7)*
sqrt(c/(c*d - b*e))*log((c*e*x + 2*c*d - b*e - 2*(c*d - b*e)*sqrt(e*x + d)*
sqrt(c/(c*d - b*e)))/(c*x + b)) - 15*(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4
*e^2 - b^3*d^3*e^3 + (c^3*d^3*e^3 - 3*b*c^2*d^2*e^4 + 3*b^2*c*d*e^5 - b^3*e
^6)*x^3 + 3*(c^3*d^4*e^2 - 3*b*c^2*d^3*e^3 + 3*b^2*c*d^2*e^4 - b^3*d*e^5)*x
^2 + 3*(c^3*d^5*e - 3*b*c^2*d^4*e^2 + 3*b^2*c*d^3*e^3 - b^3*d^2*e^4)*x)*sqr
t(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + 2*(58*b*c^2*d^5*e - 66*
b^2*c*d^4*e^2 + 23*b^3*d^3*e^3 + 15*(3*b*c^2*d^3*e^3 - 3*b^2*c*d^2*e^4 + b^
3*d*e^5)*x^2 + 5*(20*b*c^2*d^4*e^2 - 21*b^2*c*d^3*e^3 + 7*b^3*d^2*e^4)*x)*s
qrt(e*x + d))/(b*c^3*d^10 - 3*b^2*c^2*d^9*e + 3*b^3*c*d^8*e^2 - b^4*d^7*e^3
 + (b*c^3*d^7*e^3 - 3*b^2*c^2*d^6*e^4 + 3*b^3*c*d^5*e^5 - b^4*d^4*e^6)*x^3
+ 3*(b*c^3*d^8*e^2 - 3*b^2*c^2*d^7*e^3 + 3*b^3*c*d^6*e^4 - b^4*d^5*e^5)*x^2
 + 3*(b*c^3*d^9*e - 3*b^2*c^2*d^8*e^2 + 3*b^3*c*d^7*e^3 - b^4*d^6*e^4)*x),
1/15*(30*(c^3*d^4*e^3*x^3 + 3*c^3*d^5*e^2*x^2 + 3*c^3*d^6*e*x + c^3*d^7)*sq
rt(-c/(c*d - b*e))*arctan(-(c*d - b*e)*sqrt(e*x + d)*sqrt(-c/(c*d - b*e))/(
c*e*x + c*d)) + 15*(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 - b^3*d^3*e^3
 + (c^3*d^3*e^3 - 3*b*c^2*d^2*e^4 + 3*b^2*c*d*e^5 - b^3*e^6)*x^3 + 3*(c^3*d
^4*e^2 - 3*b*c^2*d^3*e^3 + 3*b^2*c*d^2*e^4 - b^3*d*e^5)*x^2 + 3*(c^3*d^5*e
- 3*b*c^2*d^4*e^2 + 3*b^2*c*d^3*e^3 - b^3*d^2*e^4)*x)*sqrt(d)*log((e*x - 2*
sqrt(e*x + d)*sqrt(d) + 2*d)/x) - 2*(58*b*c^2*d^5*e - 66*b^2*c*d^4*e^2 + 23
*b^3*d^3*e^3 + 15*(3*b*c^2*d^3*e^3 - 3*b^2*c*d^2*e^4 + b^3*d*e^5)*x^2 + 5*(
20*b*c^2*d^4*e^2 - 21*b^2*c*d^3*e^3 + 7*b^3*d^2*e^4)*x)*sqrt(e*x + d))/(b*c
^3*d^10 - 3*b^2*c^2*d^9*e + 3*b^3*c*d^8*e^2 - b^4*d^7*e^3 + (b*c^3*d^7*e^3
- 3*b^2*c^2*d^6*e^4 + 3*b^3*c*d^5*e^5 - b^4*d^4*e^6)*x^3 + 3*(b*c^3*d^8*e^2
 - 3*b^2*c^2*d^7*e^3 + 3*b^3*c*d^6*e^4 - b^4*d^5*e^5)*x^2 + 3*(b*c^3*d^9*e
- 3*b^2*c^2*d^8*e^2 + 3*b^3*c*d^7*e^3 - b^4*d^6*e^4)*x), 1/15*(30*(c^3*d^6
```

```
- 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 - b^3*d^3*e^3 + (c^3*d^3*e^3 - 3*b*c^2*d^
2*e^4 + 3*b^2*c*d*e^5 - b^3*e^6)*x^3 + 3*(c^3*d^4*e^2 - 3*b*c^2*d^3*e^3 + 3
*b^2*c*d^2*e^4 - b^3*d*e^5)*x^2 + 3*(c^3*d^5*e - 3*b*c^2*d^4*e^2 + 3*b^2*c*
d^3*e^3 - b^3*d^2*e^4)*x)*sqrt(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d) - 15*(c
^3*d^4*e^3*x^3 + 3*c^3*d^5*e^2*x^2 + 3*c^3*d^6*e*x + c^3*d^7)*sqrt(c/(c*d -
 b*e))*log((c*e*x + 2*c*d - b*e - 2*(c*d - b*e)*sqrt(e*x + d)*sqrt(c/(c*d -
 b*e)))/(c*x + b)) - 2*(58*b*c^2*d^5*e - 66*b^2*c*d^4*e^2 + 23*b^3*d^3*e^3
+ 15*(3*b*c^2*d^3*e^3 - 3*b^2*c*d^2*e^4 + b^3*d*e^5)*x^2 + 5*(20*b*c^2*d^4*
e^2 - 21*b^2*c*d^3*e^3 + 7*b^3*d^2*e^4)*x)*sqrt(e*x + d))/(b*c^3*d^10 - 3*b
^2*c^2*d^9*e + 3*b^3*c*d^8*e^2 - b^4*d^7*e^3 + (b*c^3*d^7*e^3 - 3*b^2*c^2*d
^6*e^4 + 3*b^3*c*d^5*e^5 - b^4*d^4*e^6)*x^3 + 3*(b*c^3*d^8*e^2 - 3*b^2*c^2*
d^7*e^3 + 3*b^3*c*d^6*e^4 - b^4*d^5*e^5)*x^2 + 3*(b*c^3*d^9*e - 3*b^2*c^2*d
^8*e^2 + 3*b^3*c*d^7*e^3 - b^4*d^6*e^4)*x), 2/15*(15*(c^3*d^4*e^3*x^3 + 3*c
^3*d^5*e^2*x^2 + 3*c^3*d^6*e*x + c^3*d^7)*sqrt(-c/(c*d - b*e))*arctan(-(c*d
 - b*e)*sqrt(e*x + d)*sqrt(-c/(c*d - b*e))/(c*e*x + c*d)) + 15*(c^3*d^6 - 3
*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 - b^3*d^3*e^3 + (c^3*d^3*e^3 - 3*b*c^2*d^2*e
^4 + 3*b^2*c*d*e^5 - b^3*e^6)*x^3 + 3*(c^3*d^4*e^2 - 3*b*c^2*d^3*e^3 + 3*b^
2*c*d^2*e^4 - b^3*d*e^5)*x^2 + 3*(c^3*d^5*e - 3*b*c^2*d^4*e^2 + 3*b^2*c*d^3
*e^3 - b^3*d^2*e^4)*x)*sqrt(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d) - (58*b*c^
2*d^5*e - 66*b^2*c*d^4*e^2 + 23*b^3*d^3*e^3 + 15*(3*b*c^2*d^3*e^3 - 3*b^2*c
*d^2*e^4 + b^3*d*e^5)*x^2 + 5*(20*b*c^2*d^4*e^2 - 21*b^2*c*d^3*e^3 + 7*b^3*
d^2*e^4)*x)*sqrt(e*x + d))/(b*c^3*d^10 - 3*b^2*c^2*d^9*e + 3*b^3*c*d^8*e^2
- b^4*d^7*e^3 + (b*c^3*d^7*e^3 - 3*b^2*c^2*d^6*e^4 + 3*b^3*c*d^5*e^5 - b^4*
d^4*e^6)*x^3 + 3*(b*c^3*d^8*e^2 - 3*b^2*c^2*d^7*e^3 + 3*b^3*c*d^6*e^4 - b^4
*d^5*e^5)*x^2 + 3*(b*c^3*d^9*e - 3*b^2*c^2*d^8*e^2 + 3*b^3*c*d^7*e^3 - b^4*
d^6*e^4)*x)]
```

Sympy [A] time = 48.7899, size = 182, normalized size = 0.97

$$\frac{2e}{5d(d+ex)^{\frac{5}{2}}(be-cd)}+\frac{2e(be-2cd)}{3d^2(d+ex)^{\frac{3}{2}}(be-cd)^2}+\frac{2e\left(b^2e^2-3bcde+3c^2d^2\right)}{d^3\sqrt{d+ex}(be-cd)^3}+\frac{2c^3\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{\frac{be-cd}{c}}}\right)}{b\sqrt{\frac{be-cd}{c}}(be-cd)^3}+\frac{2\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-d}}\right)}{bd^3\sqrt{-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(7/2)/(c*x**2+b*x),x)
```

```
[Out] 2*e/(5*d*(d + e*x)**(5/2)*(b*e - c*d)) + 2*e*(b*e - 2*c*d)/(3*d**2*(d + e*x
)**(3/2)*(b*e - c*d)**2) + 2*e*(b**2*e**2 - 3*b*c*d*e + 3*c**2*d**2)/(d**3*
sqrt(d + e*x)*(b*e - c*d)**3) + 2*c**3*atan(sqrt(d + e*x)/sqrt((b*e - c*d)/
c))/(b*sqrt((b*e - c*d)/c)*(b*e - c*d)**3) + 2*atan(sqrt(d + e*x)/sqrt(-d))
/(b*d**3*sqrt(-d))
```

Giac [A] time = 1.30376, size = 389, normalized size = 2.08

$$-\frac{2c^4\arctan\left(\frac{\sqrt{xe+d}c}{\sqrt{-c^2d+bce}}\right)}{\left(bc^3d^3-3b^2c^2d^2e+3b^3cde^2-b^4e^3\right)\sqrt{-c^2d+bce}}-\frac{2\left(45(xe+d)^2c^2d^2e+10(xe+d)c^2d^3e+3c^2d^4e-45(xe+d)^2bcde\right.}{15\left(c^3d^6-3bc^2d^5e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] -2*c^4*arctan(sqrt(x*e + d)*c/sqrt(-c^2*d + b*c*e))/((b*c^3*d^3 - 3*b^2*c^2
*d^2*e + 3*b^3*c*d*e^2 - b^4*e^3)*sqrt(-c^2*d + b*c*e)) - 2/15*(45*(x*e + d
)^2*c^2*d^2*e + 10*(x*e + d)*c^2*d^3*e + 3*c^2*d^4*e - 45*(x*e + d)^2*b*c*d
*e^2 - 15*(x*e + d)*b*c*d^2*e^2 - 6*b*c*d^3*e^2 + 15*(x*e + d)^2*b^2*e^3 +
5*(x*e + d)*b^2*d*e^3 + 3*b^2*d^2*e^3)/((c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*
d^4*e^2 - b^3*d^3*e^3)*(x*e + d)^(5/2)) + 2*arctan(sqrt(x*e + d)/sqrt(-d))/
(b*sqrt(-d)*d^3)
```

3.369 $\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^2}\,dx$

Optimal. Leaf size=251

$$\frac{e(d+ex)^{3/2}\left(5b^2e^2-6bcde+6c^2d^2\right)}{3b^2c^2}+\frac{e\sqrt{d+ex}(2cd-be)\left(5b^2e^2-bcde+c^2d^2\right)}{b^2c^3}-\frac{(cd-be)^{7/2}(5be+4cd)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+}}{\sqrt{cd-b}}\right.}{b^3c^{7/2}}$$

```
[Out] (e*(2*c*d - b*e)*(c^2*d^2 - b*c*d*e + 5*b^2*e^2)*Sqrt[d + e*x])/(b^2*c^3) +
 (e*(6*c^2*d^2 - 6*b*c*d*e + 5*b^2*e^2)*(d + e*x)^(3/2))/(3*b^2*c^2) + (e*(
2*c*d - b*e)*(d + e*x)^(5/2))/(b^2*c) - ((d + e*x)^(7/2)*(b*d + (2*c*d - b*
e)*x))/(b^2*(b*x + c*x^2)) + (d^(7/2)*(4*c*d - 9*b*e)*ArcTanh[Sqrt[d + e*x]
/Sqrt[d]])/b^3 - ((c*d - b*e)^(7/2)*(4*c*d + 5*b*e)*ArcTanh[(Sqrt[c]*Sqrt[d
 + e*x])/Sqrt[c*d - b*e]])/(b^3*c^(7/2))
```

Rubi [A] time = 0.534316, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {738, 824, 826, 1166, 208}

$$\frac{e(d+ex)^{3/2}\left(5b^2e^2-6bcde+6c^2d^2\right)}{3b^2c^2}+\frac{e\sqrt{d+ex}(2cd-be)\left(5b^2e^2-bcde+c^2d^2\right)}{b^2c^3}-\frac{(cd-be)^{7/2}(5be+4cd)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+}}{\sqrt{cd-b}}\right.}{b^3c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(9/2)/(b*x + c*x^2)^2,x]
```


```
[Out] (e*(2*c*d - b*e)*(c^2*d^2 - b*c*d*e + 5*b^2*e^2)*Sqrt[d + e*x])/(b^2*c^3) +
 (e*(6*c^2*d^2 - 6*b*c*d*e + 5*b^2*e^2)*(d + e*x)^(3/2))/(3*b^2*c^2) + (e*(
2*c*d - b*e)*(d + e*x)^(5/2))/(b^2*c) - ((d + e*x)^(7/2)*(b*d + (2*c*d - b*
e)*x))/(b^2*(b*x + c*x^2)) + (d^(7/2)*(4*c*d - 9*b*e)*ArcTanh[Sqrt[d + e*x]
/Sqrt[d]])/b^3 - ((c*d - b*e)^(7/2)*(4*c*d + 5*b*e)*ArcTanh[(Sqrt[c]*Sqrt[d
 + e*x])/Sqrt[c*d - b*e]])/(b^3*c^(7/2))
```


Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{9/2}}{\left(bx+cx^2\right)^2}\,dx = -\frac{(d+ex)^{7/2}(bd+(2cd-be)x)}{b^2\left(bx+cx^2\right)} - \frac{\int \frac{(d+ex)^{5/2}\left(\frac{1}{2}d(4cd-9be)-\frac{5}{2}e(2cd-be)x\right)}{bx+cx^2}\,dx}{b^2}$$

$$= \frac{e(2cd-be)(d+ex)^{5/2}}{b^2c} - \frac{(d+ex)^{7/2}(bd+(2cd-be)x)}{b^2\left(bx+cx^2\right)} - \frac{\int \frac{(d+ex)^{3/2}\left(\frac{1}{2}cd^2(4cd-9be)-\frac{1}{2}e\left(6c^2d^2-6bcde+5b^2e^2\right)\right.}{bx+cx^2}}{b^2c}$$

$$= \frac{e\left(6c^2d^2-6bcde+5b^2e^2\right)(d+ex)^{3/2}}{3b^2c^2} + \frac{e(2cd-be)(d+ex)^{5/2}}{b^2c} - \frac{(d+ex)^{7/2}(bd+(2cd-be)x)}{b^2\left(bx+cx^2\right)} - \int$$

$$= \frac{e(2cd-be)\left(c^2d^2-bcde+5b^2e^2\right)\sqrt{d+ex}}{b^2c^3} + \frac{e\left(6c^2d^2-6bcde+5b^2e^2\right)(d+ex)^{3/2}}{3b^2c^2} + \frac{e(2cd-be)(d}{b^2c}$$

$$= \frac{e(2cd-be)\left(c^2d^2-bcde+5b^2e^2\right)\sqrt{d+ex}}{b^2c^3} + \frac{e\left(6c^2d^2-6bcde+5b^2e^2\right)(d+ex)^{3/2}}{3b^2c^2} + \frac{e(2cd-be)(d}{b^2c}$$

$$= \frac{e(2cd-be)\left(c^2d^2-bcde+5b^2e^2\right)\sqrt{d+ex}}{b^2c^3} + \frac{e\left(6c^2d^2-6bcde+5b^2e^2\right)(d+ex)^{3/2}}{3b^2c^2} + \frac{e(2cd-be)(d}{b^2c}$$

$$= \frac{e(2cd-be)\left(c^2d^2-bcde+5b^2e^2\right)\sqrt{d+ex}}{b^2c^3} + \frac{e\left(6c^2d^2-6bcde+5b^2e^2\right)(d+ex)^{3/2}}{3b^2c^2} + \frac{e(2cd-be)(d}{b^2c}$$

Mathematica [A] time = 0.395775, size = 202, normalized size = 0.8

$$\frac{\frac{b\sqrt{d+ex}\left(2b^2c^2e^2x\left(-9d^2+13dex+e^2x^2\right)+2b^3ce^3x(19d-5ex)-15b^4e^4x-3bc^3d^3(d-4ex)-6c^4d^4x\right)}{c^3x(b+cx)} - \frac{3(cd-be)^{7/2}(5be+4cd)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{c^{7/2}} + 3d^{7/2}(4cd-}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(9/2)/(b*x + c*x^2)^2,x]
```

```
[Out] ((b*Sqrt[d + e*x]*(-6*c^4*d^4*x - 15*b^4*e^4*x + 2*b^3*c*e^3*x*(19*d - 5*e*
x) - 3*b*c^3*d^3*(d - 4*e*x) + 2*b^2*c^2*e^2*x*(-9*d^2 + 13*d*e*x + e^2*x^2
)))/(c^3*x*(b + c*x)) + 3*d^(7/2)*(4*c*d - 9*b*e)*ArcTanh[Sqrt[d + e*x]/Sqr
t[d]] - (3*(c*d - b*e)^(7/2)*(4*c*d + 5*b*e)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x]
)/Sqrt[c*d - b*e]])/c^(7/2))/(3*b^3)
```

Maple [B] time = 0.28, size = 515, normalized size = 2.1

$$\frac{2e^3}{3c^2}(ex+d)^{\frac{3}{2}} - 4\frac{e^4\sqrt{ex+d}b}{c^3} + 8\frac{e^3d\sqrt{ex+d}}{c^2} - \frac{e^5b^2}{c^3(cex+be)}\sqrt{ex+d} + 4\frac{e^4\sqrt{ex+d}bd}{c^2(cex+be)} - 6\frac{e^3\sqrt{ex+d}d^2}{c(cex+be)} + 4\frac{e^2\sqrt{ex+d}d^3}{b(cex+be)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(9/2)/(c*x^2+b*x)^2,x)
```

```
[Out] 2/3*e^3/c^2*(e*x+d)^(3/2)-4*e^4/c^3*(e*x+d)^(1/2)*b+8*e^3/c^2*d*(e*x+d)^(1/
2)-e^5/c^3*b^2*(e*x+d)^(1/2)/(c*e*x+b*e)+4*e^4/c^2*b*(e*x+d)^(1/2)/(c*e*x+b
*e)*d-6*e^3/c*(e*x+d)^(1/2)/(c*e*x+b*e)*d^2+4*e^2/b*(e*x+d)^(1/2)/(c*e*x+b*
e)*d^3-e*c/b^2*(e*x+d)^(1/2)/(c*e*x+b*e)*d^4+5*e^5/c^3*b^2/((b*e-c*d)*c)^(1
/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))-16*e^4/c^2*b/((b*e-c*d)*c)^
(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))*d+14*e^3/c/((b*e-c*d)*c)^
(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))*d^2+4*e^2/b/((b*e-c*d)*c)
^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))*d^3-11*e*c/b^2/((b*e-c*d
)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))*d^4+4*c^2/b^3/((b*e-
c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))*d^5-d^4/b^2*(e*x+
d)^(1/2)/x-9*e*d^(7/2)/b^2*arctanh((e*x+d)^(1/2)/d^(1/2))+4*d^(9/2)/b^3*arc
tanh((e*x+d)^(1/2)/d^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 22.8602, size = 3251, normalized size = 12.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(3*((4*c^5*d^4 - 7*b*c^4*d^3*e - 3*b^2*c^3*d^2*e^2 + 11*b^3*c^2*d*e^3
 - 5*b^4*c*e^4)*x^2 + (4*b*c^4*d^4 - 7*b^2*c^3*d^3*e - 3*b^3*c^2*d^2*e^2 +
11*b^4*c*d*e^3 - 5*b^5*e^4)*x)*sqrt((c*d - b*e)/c)*log((c*e*x + 2*c*d - b*e
 + 2*sqrt(e*x + d)*c*sqrt((c*d - b*e)/c))/(c*x + b)) + 3*((4*c^5*d^4 - 9*b*
c^4*d^3*e)*x^2 + (4*b*c^4*d^4 - 9*b^2*c^3*d^3*e)*x)*sqrt(d)*log((e*x - 2*sq
rt(e*x + d)*sqrt(d) + 2*d)/x) - 2*(2*b^3*c^2*e^4*x^3 - 3*b^2*c^3*d^4 + 2*(1
```

```
3*b^3*c^2*d*e^3 - 5*b^4*c*e^4)*x^2 - (6*b*c^4*d^4 - 12*b^2*c^3*d^3*e + 18*b
^3*c^2*d^2*e^2 - 38*b^4*c*d*e^3 + 15*b^5*e^4)*x)*sqrt(e*x + d))/(b^3*c^4*x^
2 + b^4*c^3*x), -1/6*(6*((4*c^5*d^4 - 7*b*c^4*d^3*e - 3*b^2*c^3*d^2*e^2 + 1
1*b^3*c^2*d*e^3 - 5*b^4*c*e^4)*x^2 + (4*b*c^4*d^4 - 7*b^2*c^3*d^3*e - 3*b^3
*c^2*d^2*e^2 + 11*b^4*c*d*e^3 - 5*b^5*e^4)*x)*sqrt(-(c*d - b*e)/c)*arctan(-
sqrt(e*x + d)*c*sqrt(-(c*d - b*e)/c)/(c*d - b*e)) + 3*((4*c^5*d^4 - 9*b*c^4
*d^3*e)*x^2 + (4*b*c^4*d^4 - 9*b^2*c^3*d^3*e)*x)*sqrt(d)*log((e*x - 2*sqrt(
e*x + d)*sqrt(d) + 2*d)/x) - 2*(2*b^3*c^2*e^4*x^3 - 3*b^2*c^3*d^4 + 2*(13*b
^3*c^2*d*e^3 - 5*b^4*c*e^4)*x^2 - (6*b*c^4*d^4 - 12*b^2*c^3*d^3*e + 18*b^3*
c^2*d^2*e^2 - 38*b^4*c*d*e^3 + 15*b^5*e^4)*x)*sqrt(e*x + d))/(b^3*c^4*x^2 +
 b^4*c^3*x), -1/6*(6*((4*c^5*d^4 - 9*b*c^4*d^3*e)*x^2 + (4*b*c^4*d^4 - 9*b^
2*c^3*d^3*e)*x)*sqrt(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d) + 3*((4*c^5*d^4 -
 7*b*c^4*d^3*e - 3*b^2*c^3*d^2*e^2 + 11*b^3*c^2*d*e^3 - 5*b^4*c*e^4)*x^2 +
(4*b*c^4*d^4 - 7*b^2*c^3*d^3*e - 3*b^3*c^2*d^2*e^2 + 11*b^4*c*d*e^3 - 5*b^5
*e^4)*x)*sqrt((c*d - b*e)/c)*log((c*e*x + 2*c*d - b*e + 2*sqrt(e*x + d)*c*s
qrt((c*d - b*e)/c))/(c*x + b)) - 2*(2*b^3*c^2*e^4*x^3 - 3*b^2*c^3*d^4 + 2*(
13*b^3*c^2*d*e^3 - 5*b^4*c*e^4)*x^2 - (6*b*c^4*d^4 - 12*b^2*c^3*d^3*e + 18*
b^3*c^2*d^2*e^2 - 38*b^4*c*d*e^3 + 15*b^5*e^4)*x)*sqrt(e*x + d))/(b^3*c^4*x
^2 + b^4*c^3*x), -1/3*(3*((4*c^5*d^4 - 7*b*c^4*d^3*e - 3*b^2*c^3*d^2*e^2 +
11*b^3*c^2*d*e^3 - 5*b^4*c*e^4)*x^2 + (4*b*c^4*d^4 - 7*b^2*c^3*d^3*e - 3*b^
3*c^2*d^2*e^2 + 11*b^4*c*d*e^3 - 5*b^5*e^4)*x)*sqrt(-(c*d - b*e)/c)*arctan(
-sqrt(e*x + d)*c*sqrt(-(c*d - b*e)/c)/(c*d - b*e)) + 3*((4*c^5*d^4 - 9*b*c^
4*d^3*e)*x^2 + (4*b*c^4*d^4 - 9*b^2*c^3*d^3*e)*x)*sqrt(-d)*arctan(sqrt(e*x
+ d)*sqrt(-d)/d) - (2*b^3*c^2*e^4*x^3 - 3*b^2*c^3*d^4 + 2*(13*b^3*c^2*d*e^3
 - 5*b^4*c*e^4)*x^2 - (6*b*c^4*d^4 - 12*b^2*c^3*d^3*e + 18*b^3*c^2*d^2*e^2
- 38*b^4*c*d*e^3 + 15*b^5*e^4)*x)*sqrt(e*x + d))/(b^3*c^4*x^2 + b^4*c^3*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(9/2)/(c*x**2+b*x)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.43982, size = 589, normalized size = 2.35

$$-\frac{\left(4\,cd^5-9\,bd^4e\right)\arctan\left(\frac{\sqrt{xe+d}}{\sqrt{-d}}\right)}{b^3\sqrt{-d}}+\frac{\left(4\,c^5d^5-11\,bc^4d^4e+4\,b^2c^3d^3e^2+14\,b^3c^2d^2e^3-16\,b^4cde^4+5\,b^5e^5\right)\arctan\left(\frac{\sqrt{xe-}}{\sqrt{-c^2d}}\right.}{\sqrt{-c^2d+bce}b^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(c*x^2+b*x)^2,x, algorithm="giac")
```

```
[Out] -(4*c*d^5 - 9*b*d^4*e)*arctan(sqrt(x*e + d)/sqrt(-d))/(b^3*sqrt(-d)) + (4*c
^5*d^5 - 11*b*c^4*d^4*e + 4*b^2*c^3*d^3*e^2 + 14*b^3*c^2*d^2*e^3 - 16*b^4*c
*d*e^4 + 5*b^5*e^5)*arctan(sqrt(x*e + d)*c/sqrt(-c^2*d + b*c*e))/(sqrt(-c^2
*d + b*c*e)*b^3*c^3) + 2/3*((x*e + d)^(3/2)*c^4*e^3 + 12*sqrt(x*e + d)*c^4*
d*e^3 - 6*sqrt(x*e + d)*b*c^3*e^4)/c^6 - (2*(x*e + d)^(3/2)*c^4*d^4*e - 2*s
qrt(x*e + d)*c^4*d^5*e - 4*(x*e + d)^(3/2)*b*c^3*d^3*e^2 + 5*sqrt(x*e + d)*
b*c^3*d^4*e^2 + 6*(x*e + d)^(3/2)*b^2*c^2*d^2*e^3 - 6*sqrt(x*e + d)*b^2*c^2
```

```
*d^3*e^3 - 4*(x*e + d)^(3/2)*b^3*c*d*e^4 + 4*sqrt(x*e + d)*b^3*c*d^2*e^4 +
(x*e + d)^(3/2)*b^4*e^5 - sqrt(x*e + d)*b^4*d*e^5)/(((x*e + d)^2*c - 2*(x*e
 + d)*c*d + c*d^2 + (x*e + d)*b*e - b*d*e)*b^2*c^3)
```

3.370 $\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^2}\,dx$

Optimal. Leaf size=200

$$\frac{e\sqrt{d+ex}\left(3b^2e^2-2bcde+2c^2d^2\right)}{b^2c^2}-\frac{(cd-be)^{5/2}(3be+4cd)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b^3c^{5/2}}+\frac{d^{5/2}(4cd-7be)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b^3}-\frac{(d}{}$$

```
[Out] (e*(2*c^2*d^2 - 2*b*c*d*e + 3*b^2*e^2)*Sqrt[d + e*x])/(b^2*c^2) + (e*(2*c*d
 - b*e)*(d + e*x)^(3/2))/(b^2*c) - ((d + e*x)^(5/2)*(b*d + (2*c*d - b*e)*x)
)/(b^2*(b*x + c*x^2)) + (d^(5/2)*(4*c*d - 7*b*e)*ArcTanh[Sqrt[d + e*x]/Sqrt
[d]])/b^3 - ((c*d - b*e)^(5/2)*(4*c*d + 3*b*e)*ArcTanh[(Sqrt[c]*Sqrt[d + e*
x])/Sqrt[c*d - b*e]])/(b^3*c^(5/2))
```

Rubi [A] time = 0.395153, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {738, 824, 826, 1166, 208}

$$\frac{e\sqrt{d+ex}\left(3b^2e^2-2bcde+2c^2d^2\right)}{b^2c^2}-\frac{(cd-be)^{5/2}(3be+4cd)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b^3c^{5/2}}+\frac{d^{5/2}(4cd-7be)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b^3}-\frac{(d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/(b*x + c*x^2)^2,x]
```


```
[Out] (e*(2*c^2*d^2 - 2*b*c*d*e + 3*b^2*e^2)*Sqrt[d + e*x])/(b^2*c^2) + (e*(2*c*d
 - b*e)*(d + e*x)^(3/2))/(b^2*c) - ((d + e*x)^(5/2)*(b*d + (2*c*d - b*e)*x)
)/(b^2*(b*x + c*x^2)) + (d^(5/2)*(4*c*d - 7*b*e)*ArcTanh[Sqrt[d + e*x]/Sqrt
[d]])/b^3 - ((c*d - b*e)^(5/2)*(4*c*d + 3*b*e)*ArcTanh[(Sqrt[c]*Sqrt[d + e*
x])/Sqrt[c*d - b*e]])/(b^3*c^(5/2))
```


Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
```

```
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{7/2}}{\left(bx+cx^2\right)^2}\,dx = -\frac{(d+ex)^{5/2}(bd+(2cd-be)x)}{b^2\left(bx+cx^2\right)} - \frac{\int \frac{(d+ex)^{3/2}\left(\frac{1}{2}d(4cd-7be)-\frac{3}{2}e(2cd-be)x\right)}{bx+cx^2}\,dx}{b^2}$$

$$= \frac{e(2cd-be)(d+ex)^{3/2}}{b^2c} - \frac{(d+ex)^{5/2}(bd+(2cd-be)x)}{b^2\left(bx+cx^2\right)} - \frac{\int \frac{\sqrt{d+ex}\left(\frac{1}{2}cd^2(4cd-7be)-\frac{1}{2}e\left(2c^2d^2-2bcde+3b^2e^2\right)x\right)}{bx+cx^2}\,dx}{b^2c}$$

$$= \frac{e\left(2c^2d^2-2bcde+3b^2e^2\right)\sqrt{d+ex}}{b^2c^2} + \frac{e(2cd-be)(d+ex)^{3/2}}{b^2c} - \frac{(d+ex)^{5/2}(bd+(2cd-be)x)}{b^2\left(bx+cx^2\right)} - \frac{\int \frac{\frac{1}{2}c^2d^3}{}}{}$$

$$= \frac{e\left(2c^2d^2-2bcde+3b^2e^2\right)\sqrt{d+ex}}{b^2c^2} + \frac{e(2cd-be)(d+ex)^{3/2}}{b^2c} - \frac{(d+ex)^{5/2}(bd+(2cd-be)x)}{b^2\left(bx+cx^2\right)} - \frac{2\,\mathrm{Subs}}{}$$

$$= \frac{e\left(2c^2d^2-2bcde+3b^2e^2\right)\sqrt{d+ex}}{b^2c^2} + \frac{e(2cd-be)(d+ex)^{3/2}}{b^2c} - \frac{(d+ex)^{5/2}(bd+(2cd-be)x)}{b^2\left(bx+cx^2\right)} - \frac{\left(cd^3(4c\right.}{}$$

$$= \frac{e\left(2c^2d^2-2bcde+3b^2e^2\right)\sqrt{d+ex}}{b^2c^2} + \frac{e(2cd-be)(d+ex)^{3/2}}{b^2c} - \frac{(d+ex)^{5/2}(bd+(2cd-be)x)}{b^2\left(bx+cx^2\right)} + \frac{d^{5/2}(4c}{}$$

Mathematica [A] time = 0.311376, size = 167, normalized size = 0.84

$$\frac{\frac{b\sqrt{d+ex}\left(b^2ce^2x(2ex-3d)+3b^3e^3x-bc^2d^2(d-3ex)-2c^3d^3x\right)}{c^2x(b+cx)} - \frac{(cd-be)^{5/2}(3be+4cd)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{c^{5/2}} + d^{5/2}(4cd-7be)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)/(b*x + c*x^2)^2,x]
```

```
[Out] ((b*Sqrt[d + e*x]*(-2*c^3*d^3*x + 3*b^3*e^3*x - b*c^2*d^2*(d - 3*e*x) + b^2
*c*e^2*x*(-3*d + 2*e*x)))/(c^2*x*(b + c*x)) + d^(5/2)*(4*c*d - 7*b*e)*ArcTa
nh[Sqrt[d + e*x]/Sqrt[d]] - ((c*d - b*e)^(5/2)*(4*c*d + 3*b*e)*ArcTanh[(Sqr
t[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/c^(5/2))/b^3
```

Maple [B] time = 0.229, size = 403, normalized size = 2.

$$2\frac{e^3\sqrt{ex+d}}{c^2}+\frac{e^4b}{c^2(cex+be)}\sqrt{ex+d}-3\frac{e^3\sqrt{ex+d}d}{c(cex+be)}+3\frac{e^2\sqrt{ex+d}d^2}{b(cex+be)}-\frac{ced^3}{b^2(cex+be)}\sqrt{ex+d}-3\frac{e^4b}{c^2\sqrt{(be-cd)c}}\text{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/(c*x^2+b*x)^2,x)
```

```
[Out] 2*e^3/c^2*(e*x+d)^(1/2)+e^4/c^2*b*(e*x+d)^(1/2)/(c*e*x+b*e)-3*e^3/c*(e*x+d)
^(1/2)/(c*e*x+b*e)*d+3*e^2/b*(e*x+d)^(1/2)/(c*e*x+b*e)*d^2-e*c/b^2*(e*x+d)^
(1/2)/(c*e*x+b*e)*d^3-3*e^4/c^2*b/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*
c/((b*e-c*d)*c)^(1/2))+5*e^3/c/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/(
(b*e-c*d)*c)^(1/2))*d+3*e^2/b/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((
b*e-c*d)*c)^(1/2))*d^2-9*e*c/b^2/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c
/((b*e-c*d)*c)^(1/2))*d^3+4*c^2/b^3/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2
)*c/((b*e-c*d)*c)^(1/2))*d^4-d^3/b^2*(e*x+d)^(1/2)/x-7*e*d^(5/2)/b^2*arctan
h((e*x+d)^(1/2)/d^(1/2))+4*d^(7/2)/b^3*arctanh((e*x+d)^(1/2)/d^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 6.24772, size = 2628, normalized size = 13.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(((4*c^4*d^3 - 5*b*c^3*d^2*e - 2*b^2*c^2*d*e^2 + 3*b^3*c*e^3)*x^2 + (4
*b*c^3*d^3 - 5*b^2*c^2*d^2*e - 2*b^3*c*d*e^2 + 3*b^4*e^3)*x)*sqrt((c*d - b*
e)/c)*log((c*e*x + 2*c*d - b*e - 2*sqrt(e*x + d)*c*sqrt((c*d - b*e)/c))/(c*
x + b)) - ((4*c^4*d^3 - 7*b*c^3*d^2*e)*x^2 + (4*b*c^3*d^3 - 7*b^2*c^2*d^2*e
)*x)*sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + 2*(2*b^3*c*e^3*
x^2 - b^2*c^2*d^3 - (2*b*c^3*d^3 - 3*b^2*c^2*d^2*e + 3*b^3*c*d*e^2 - 3*b^4*
e^3)*x)*sqrt(e*x + d))/(b^3*c^3*x^2 + b^4*c^2*x), -1/2*(2*((4*c^4*d^3 - 5*b
*c^3*d^2*e - 2*b^2*c^2*d*e^2 + 3*b^3*c*e^3)*x^2 + (4*b*c^3*d^3 - 5*b^2*c^2*
d^2*e - 2*b^3*c*d*e^2 + 3*b^4*e^3)*x)*sqrt(-(c*d - b*e)/c)*arctan(-sqrt(e*x
 + d)*c*sqrt(-(c*d - b*e)/c)/(c*d - b*e)) + ((4*c^4*d^3 - 7*b*c^3*d^2*e)*x^
2 + (4*b*c^3*d^3 - 7*b^2*c^2*d^2*e)*x)*sqrt(d)*log((e*x - 2*sqrt(e*x + d)*s
qrt(d) + 2*d)/x) - 2*(2*b^3*c*e^3*x^2 - b^2*c^2*d^3 - (2*b*c^3*d^3 - 3*b^2*
c^2*d^2*e + 3*b^3*c*d*e^2 - 3*b^4*e^3)*x)*sqrt(e*x + d))/(b^3*c^3*x^2 + b^4
*c^2*x), -1/2*(2*((4*c^4*d^3 - 7*b*c^3*d^2*e)*x^2 + (4*b*c^3*d^3 - 7*b^2*c^
2*d^2*e)*x)*sqrt(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d) - ((4*c^4*d^3 - 5*b*c
```

```
^3*d^2*e - 2*b^2*c^2*d*e^2 + 3*b^3*c*e^3)*x^2 + (4*b*c^3*d^3 - 5*b^2*c^2*d^
2*e - 2*b^3*c*d*e^2 + 3*b^4*e^3)*x)*sqrt((c*d - b*e)/c)*log((c*e*x + 2*c*d
- b*e - 2*sqrt(e*x + d)*c*sqrt((c*d - b*e)/c))/(c*x + b)) - 2*(2*b^3*c*e^3*
x^2 - b^2*c^2*d^3 - (2*b*c^3*d^3 - 3*b^2*c^2*d^2*e + 3*b^3*c*d*e^2 - 3*b^4*
e^3)*x)*sqrt(e*x + d))/(b^3*c^3*x^2 + b^4*c^2*x), -(((4*c^4*d^3 - 5*b*c^3*d
^2*e - 2*b^2*c^2*d*e^2 + 3*b^3*c*e^3)*x^2 + (4*b*c^3*d^3 - 5*b^2*c^2*d^2*e
- 2*b^3*c*d*e^2 + 3*b^4*e^3)*x)*sqrt(-(c*d - b*e)/c)*arctan(-sqrt(e*x + d)*
c*sqrt(-(c*d - b*e)/c)/(c*d - b*e)) + ((4*c^4*d^3 - 7*b*c^3*d^2*e)*x^2 + (4
*b*c^3*d^3 - 7*b^2*c^2*d^2*e)*x)*sqrt(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d)
- (2*b^3*c*e^3*x^2 - b^2*c^2*d^3 - (2*b*c^3*d^3 - 3*b^2*c^2*d^2*e + 3*b^3*c
*d*e^2 - 3*b^4*e^3)*x)*sqrt(e*x + d))/(b^3*c^3*x^2 + b^4*c^2*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/(c*x**2+b*x)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38429, size = 464, normalized size = 2.32

$$\frac{2\sqrt{xe+d}e^3}{c^2} - \frac{\left(4\,cd^4 - 7\,bd^3e\right)\arctan\left(\frac{\sqrt{xe+d}}{\sqrt{-d}}\right)}{b^3\sqrt{-d}} + \frac{\left(4\,c^4d^4 - 9\,bc^3d^3e + 3\,b^2c^2d^2e^2 + 5\,b^3cde^3 - 3\,b^4e^4\right)\arctan\left(\frac{\sqrt{xe+d}c}{\sqrt{-c^2d+bce}}\right)}{\sqrt{-c^2d+bce}b^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x)^2,x, algorithm="giac")
```

```
[Out] 2*sqrt(x*e + d)*e^3/c^2 - (4*c*d^4 - 7*b*d^3*e)*arctan(sqrt(x*e + d)/sqrt(-
d))/(b^3*sqrt(-d)) + (4*c^4*d^4 - 9*b*c^3*d^3*e + 3*b^2*c^2*d^2*e^2 + 5*b^3
*c*d*e^3 - 3*b^4*e^4)*arctan(sqrt(x*e + d)*c/sqrt(-c^2*d + b*c*e))/(sqrt(-c
^2*d + b*c*e)*b^3*c^2) - (2*(x*e + d)^(3/2)*c^3*d^3*e - 2*sqrt(x*e + d)*c^3
*d^4*e - 3*(x*e + d)^(3/2)*b*c^2*d^2*e^2 + 4*sqrt(x*e + d)*b*c^2*d^3*e^2 +
3*(x*e + d)^(3/2)*b^2*c*d*e^3 - 3*sqrt(x*e + d)*b^2*c*d^2*e^3 - (x*e + d)^(
3/2)*b^3*e^4 + sqrt(x*e + d)*b^3*d*e^4)/(((x*e + d)^2*c - 2*(x*e + d)*c*d +
 c*d^2 + (x*e + d)*b*e - b*d*e)*b^2*c^2)
```

3.371 $\int \frac{(d+ex)^{5/2}}{\left(bx+cx^2\right)^2} dx$

Optimal. Leaf size=159

$$-\frac{(cd-be)^{3/2}(be+4cd)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b^3c^{3/2}}+\frac{d^{3/2}(4cd-5be)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b^3}-\frac{(d+ex)^{3/2}(x(2cd-be)+bd)}{b^2\left(bx+cx^2\right)}+\frac{e\sqrt{d+e}}{}$$

```
[Out] (e*(2*c*d - b*e)*Sqrt[d + e*x])/(b^2*c) - ((d + e*x)^(3/2)*(b*d + (2*c*d -
b*e)*x))/(b^2*(b*x + c*x^2)) + (d^(3/2)*(4*c*d - 5*b*e)*ArcTanh[Sqrt[d + e*
x]/Sqrt[d]])/b^3 - ((c*d - b*e)^(3/2)*(4*c*d + b*e)*ArcTanh[(Sqrt[c]*Sqrt[d
 + e*x])/Sqrt[c*d - b*e]])/(b^3*c^(3/2))
```

Rubi [A] time = 0.3019, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {738, 824, 826, 1166, 208}

$$-\frac{(cd-be)^{3/2}(be+4cd)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b^3c^{3/2}}+\frac{d^{3/2}(4cd-5be)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b^3}-\frac{(d+ex)^{3/2}(x(2cd-be)+bd)}{b^2\left(bx+cx^2\right)}+\frac{e\sqrt{d+e}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(b*x + c*x^2)^2,x]
```


```
[Out] (e*(2*c*d - b*e)*Sqrt[d + e*x])/(b^2*c) - ((d + e*x)^(3/2)*(b*d + (2*c*d -
b*e)*x))/(b^2*(b*x + c*x^2)) + (d^(3/2)*(4*c*d - 5*b*e)*ArcTanh[Sqrt[d + e*
x]/Sqrt[d]])/b^3 - ((c*d - b*e)^(3/2)*(4*c*d + b*e)*ArcTanh[(Sqrt[c]*Sqrt[d
 + e*x])/Sqrt[c*d - b*e]])/(b^3*c^(3/2))
```


Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
```

```
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\left(bx+cx^2\right)^2}\,dx = -\frac{(d+ex)^{3/2}(bd+(2cd-be)x)}{b^2\left(bx+cx^2\right)} - \frac{\int \frac{\sqrt{d+ex}\left(\frac{1}{2}d(4cd-5be)-\frac{1}{2}e(2cd-be)x\right)}{bx+cx^2}\,dx}{b^2}$$

$$= \frac{e(2cd-be)\sqrt{d+ex}}{b^2c} - \frac{(d+ex)^{3/2}(bd+(2cd-be)x)}{b^2\left(bx+cx^2\right)} - \frac{\int \frac{\frac{1}{2}cd^2(4cd-5be)+\frac{1}{2}e\left(2c^2d^2-2bcde-b^2e^2\right)x}{\sqrt{d+ex}\left(bx+cx^2\right)}\,dx}{b^2c}$$

$$= \frac{e(2cd-be)\sqrt{d+ex}}{b^2c} - \frac{(d+ex)^{3/2}(bd+(2cd-be)x)}{b^2\left(bx+cx^2\right)} - \frac{2\,\mathrm{Subst}\left(\int \frac{\frac{1}{2}cd^2e(4cd-5be)-\frac{1}{2}de\left(2c^2d^2-2bcde-b^2e^2\right)+\frac{1}{2}e}{cd^2-bde+(-2cd+be)x^2+c}\right.}{b^2c}$$

$$= \frac{e(2cd-be)\sqrt{d+ex}}{b^2c} - \frac{(d+ex)^{3/2}(bd+(2cd-be)x)}{b^2\left(bx+cx^2\right)} - \frac{\left(cd^2(4cd-5be)\right)\mathrm{Subst}\left(\int \frac{1}{-\frac{be}{2}+\frac{1}{2}(-2cd+be)+cx^2}\,d\right.}{b^3}$$

$$= \frac{e(2cd-be)\sqrt{d+ex}}{b^2c} - \frac{(d+ex)^{3/2}(bd+(2cd-be)x)}{b^2\left(bx+cx^2\right)} + \frac{d^{3/2}(4cd-5be)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b^3} - \frac{(cd-be)^{3/2}(}{}$$

Mathematica [A] time = 0.275935, size = 159, normalized size = 1.

$$\frac{-\frac{b\sqrt{d+ex}\left(b^2e^2x+bcd(d-2ex)+2c^2d^2x\right)}{cx(b+cx)}-\frac{\sqrt{cd-be}\left(-b^2e^2-3bcde+4c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{c^{3/2}}+d^{3/2}(4cd-5be)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(b*x + c*x^2)^2,x]
```

```
[Out] (-((b*Sqrt[d + e*x]*(2*c^2*d^2*x + b^2*e^2*x + b*c*d*(d - 2*e*x)))/(c*x*(b
+ c*x))) + d^(3/2)*(4*c*d - 5*b*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] - (Sqrt[c
*d - b*e]*(4*c^2*d^2 - 3*b*c*d*e - b^2*e^2)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])
/Sqrt[c*d - b*e]])/c^(3/2))/b^3
```

Maple [B] time = 0.242, size = 313, normalized size = 2.

$$-\frac{e^3}{c\,(cex+be)}\sqrt{ex+d}+2\,\frac{e^2\sqrt{ex+dd}}{b\,(cex+be)}-\frac{ced^2}{b^2\,(cex+be)}\sqrt{ex+d}+\frac{e^3}{c}\arctan\left(c\sqrt{ex+d}\frac{1}{\sqrt{(be-cd)\,c}}\right)\frac{1}{\sqrt{(be-cd)\,c}}+2\,\frac{}{b\sqrt{(b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(c*x^2+b*x)^2,x)
```

```
[Out] -e^3/c*(e*x+d)^(1/2)/(c*e*x+b*e)+2*e^2/b*(e*x+d)^(1/2)/(c*e*x+b*e)*d-e/b^2*
c*(e*x+d)^(1/2)/(c*e*x+b*e)*d^2+e^3/c/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1
/2)*c/((b*e-c*d)*c)^(1/2))+2*e^2/b/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)
*c/((b*e-c*d)*c)^(1/2))*d-7*e/b^2*c/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2
)*c/((b*e-c*d)*c)^(1/2))*d^2+4/b^3/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)
*c/((b*e-c*d)*c)^(1/2))*d^3*c^2-d^2/b^2*(e*x+d)^(1/2)/x-5*e*d^(3/2)/b^2*arc
tanh((e*x+d)^(1/2)/d^(1/2))+4*d^(5/2)/b^3*arctanh((e*x+d)^(1/2)/d^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.0379, size = 2122, normalized size = 13.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(((4*c^3*d^2 - 3*b*c^2*d*e - b^2*c*e^2)*x^2 + (4*b*c^2*d^2 - 3*b^2*c*
d*e - b^3*e^2)*x)*sqrt((c*d - b*e)/c)*log((c*e*x + 2*c*d - b*e + 2*sqrt(e*x
 + d)*c*sqrt((c*d - b*e)/c))/(c*x + b)) + ((4*c^3*d^2 - 5*b*c^2*d*e)*x^2 +
(4*b*c^2*d^2 - 5*b^2*c*d*e)*x)*sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) +
 2*d)/x) + 2*(b^2*c*d^2 + (2*b*c^2*d^2 - 2*b^2*c*d*e + b^3*e^2)*x)*sqrt(e*x
 + d))/(b^3*c^2*x^2 + b^4*c*x), -1/2*(2*((4*c^3*d^2 - 3*b*c^2*d*e - b^2*c*e
^2)*x^2 + (4*b*c^2*d^2 - 3*b^2*c*d*e - b^3*e^2)*x)*sqrt(-(c*d - b*e)/c)*arc
tan(-sqrt(e*x + d)*c*sqrt(-(c*d - b*e)/c)/(c*d - b*e)) + ((4*c^3*d^2 - 5*b*
c^2*d*e)*x^2 + (4*b*c^2*d^2 - 5*b^2*c*d*e)*x)*sqrt(d)*log((e*x - 2*sqrt(e*x
 + d)*sqrt(d) + 2*d)/x) + 2*(b^2*c*d^2 + (2*b*c^2*d^2 - 2*b^2*c*d*e + b^3*e
^2)*x)*sqrt(e*x + d))/(b^3*c^2*x^2 + b^4*c*x), -1/2*(2*((4*c^3*d^2 - 5*b*c^
2*d*e)*x^2 + (4*b*c^2*d^2 - 5*b^2*c*d*e)*x)*sqrt(-d)*arctan(sqrt(e*x + d)*s
qrt(-d)/d) + ((4*c^3*d^2 - 3*b*c^2*d*e - b^2*c*e^2)*x^2 + (4*b*c^2*d^2 - 3*
b^2*c*d*e - b^3*e^2)*x)*sqrt((c*d - b*e)/c)*log((c*e*x + 2*c*d - b*e + 2*sq
rt(e*x + d)*c*sqrt((c*d - b*e)/c))/(c*x + b)) + 2*(b^2*c*d^2 + (2*b*c^2*d^2
 - 2*b^2*c*d*e + b^3*e^2)*x)*sqrt(e*x + d))/(b^3*c^2*x^2 + b^4*c*x), -(((4*
c^3*d^2 - 3*b*c^2*d*e - b^2*c*e^2)*x^2 + (4*b*c^2*d^2 - 3*b^2*c*d*e - b^3*e
^2)*x)*sqrt(-(c*d - b*e)/c)*arctan(-sqrt(e*x + d)*c*sqrt(-(c*d - b*e)/c)/(c
*d - b*e)) + ((4*c^3*d^2 - 5*b*c^2*d*e)*x^2 + (4*b*c^2*d^2 - 5*b^2*c*d*e)*x
)*sqrt(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d) + (b^2*c*d^2 + (2*b*c^2*d^2 - 2
*b^2*c*d*e + b^3*e^2)*x)*sqrt(e*x + d))/(b^3*c^2*x^2 + b^4*c*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(c*x**2+b*x)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38802, size = 369, normalized size = 2.32

$$-\frac{\left(4\,cd^{3}-5\,bd^{2}e\right)\arctan\left(\frac{\sqrt{xe+d}}{\sqrt{-d}}\right)}{b^{3}\sqrt{-d}}+\frac{\left(4\,c^{3}d^{3}-7\,bc^{2}d^{2}e+2\,b^{2}cde^{2}+b^{3}e^{3}\right)\arctan\left(\frac{\sqrt{xe+d}c}{\sqrt{-c^{2}d+bce}}\right)}{\sqrt{-c^{2}d+bce}b^{3}c}-\frac{2\,(xe+d)^{\frac{3}{2}}c^{2}d^{2}e-2\,\sqrt{xe}}{(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x)^2,x, algorithm="giac")
```

```
[Out] -(4*c*d^3 - 5*b*d^2*e)*arctan(sqrt(x*e + d)/sqrt(-d))/(b^3*sqrt(-d)) + (4*c
^3*d^3 - 7*b*c^2*d^2*e + 2*b^2*c*d*e^2 + b^3*e^3)*arctan(sqrt(x*e + d)*c/sq
rt(-c^2*d + b*c*e))/(sqrt(-c^2*d + b*c*e)*b^3*c) - (2*(x*e + d)^(3/2)*c^2*d
^2*e - 2*sqrt(x*e + d)*c^2*d^3*e - 2*(x*e + d)^(3/2)*b*c*d*e^2 + 3*sqrt(x*e
 + d)*b*c*d^2*e^2 + (x*e + d)^(3/2)*b^2*e^3 - sqrt(x*e + d)*b^2*d*e^3)/(((x
*e + d)^2*c - 2*(x*e + d)*c*d + c*d^2 + (x*e + d)*b*e - b*d*e)*b^2*c)
```

3.372 $\int \frac{(d+ex)^{3/2}}{\left(bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=134

$$-\frac{\sqrt{d+ex}(x(2cd-be)+bd)}{b^2\left(bx+cx^2\right)}+\frac{\sqrt{d}(4cd-3be)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b^3}-\frac{\sqrt{cd-be}(4cd-be)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b^3\sqrt{c}}$$

```
[Out] -((Sqrt[d + e*x]*(b*d + (2*c*d - b*e)*x))/(b^2*(b*x + c*x^2))) + (Sqrt[d]*(
4*c*d - 3*b*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/b^3 - (Sqrt[c*d - b*e]*(4*c*
d - b*e)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(b^3*Sqrt[c])
```

Rubi [A] time = 0.183317, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {738, 826, 1166, 208}

$$-\frac{\sqrt{d+ex}(x(2cd-be)+bd)}{b^2\left(bx+cx^2\right)}+\frac{\sqrt{d}(4cd-3be)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b^3}-\frac{\sqrt{cd-be}(4cd-be)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b^3\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(b*x + c*x^2)^2,x]
```

```
[Out] -((Sqrt[d + e*x]*(b*d + (2*c*d - b*e)*x))/(b^2*(b*x + c*x^2))) + (Sqrt[d]*(
4*c*d - 3*b*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/b^3 - (Sqrt[c*d - b*e]*(4*c*
d - b*e)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(b^3*Sqrt[c])
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\left(bx+cx^2\right)^2}\,dx = -\frac{\sqrt{d+ex}(bd+(2cd-be)x)}{b^2\left(bx+cx^2\right)} - \frac{\int \frac{\frac{1}{2}d(4cd-3be)+\frac{1}{2}e(2cd-be)x}{\sqrt{d+ex}\left(bx+cx^2\right)}\,dx}{b^2}$$

$$= -\frac{\sqrt{d+ex}(bd+(2cd-be)x)}{b^2\left(bx+cx^2\right)} - \frac{2\,\text{Subst}\left(\int \frac{\frac{1}{2}de(4cd-3be)-\frac{1}{2}de(2cd-be)+\frac{1}{2}e(2cd-be)x^2}{cd^2-bde+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{b^2}$$

$$= -\frac{\sqrt{d+ex}(bd+(2cd-be)x)}{b^2\left(bx+cx^2\right)} - \frac{(cd(4cd-3be))\,\text{Subst}\left(\int \frac{1}{-\frac{be}{2}+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x, \sqrt{d+ex}\right)}{b^3} + \frac{((cd-b}{}$$

$$= -\frac{\sqrt{d+ex}(bd+(2cd-be)x)}{b^2\left(bx+cx^2\right)} + \frac{\sqrt{d}(4cd-3be)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b^3} - \frac{\sqrt{cd-be}(4cd-be)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b^3\sqrt{c}}$$

Mathematica [A] time = 0.214338, size = 127, normalized size = 0.95

$$\frac{\frac{b\sqrt{d+ex}(-bd+bex-2cdx)}{x(b+cx)} + \sqrt{d}(4cd-3be)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right) - \frac{\sqrt{cd-be}(4cd-be)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{\sqrt{c}}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(b*x + c*x^2)^2,x]
```

```
[Out] ((b*Sqrt[d + e*x]*(-(b*d) - 2*c*d*x + b*e*x))/(x*(b + c*x)) + Sqrt[d]*(4*c*
d - 3*b*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] - (Sqrt[c*d - b*e]*(4*c*d - b*e)*
ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/Sqrt[c])/b^3
```

Maple [B] time = 0.264, size = 237, normalized size = 1.8

$$\frac{e^2}{b\,(cex+be)}\sqrt{ex+d} - \frac{ced}{b^2\,(cex+be)}\sqrt{ex+d} + \frac{e^2}{b}\arctan\left(c\sqrt{ex+d}\frac{1}{\sqrt{(be-cd)\,c}}\right)\frac{1}{\sqrt{(be-cd)\,c}} - 5\,\frac{ced}{b^2\sqrt{(be-cd)\,c}}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*x^2+b*x)^2,x)
```

```
[Out] e^2/b*(e*x+d)^(1/2)/(c*e*x+b*e)-e/b^2*(e*x+d)^(1/2)/(c*e*x+b*e)*c*d+e^2/b/(
(b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))-5*e/b^2/((b*
e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))*c*d+4/b^3/((b*e
-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))*c^2*d^2-d/b^2*(e
*x+d)^(1/2)/x-3*e*d^(1/2)/b^2*arctanh((e*x+d)^(1/2)/d^(1/2))+4*d^(3/2)/b^3*
arctanh((e*x+d)^(1/2)/d^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.21837, size = 1679, normalized size = 12.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(((4*c^2*d - b*c*e)*x^2 + (4*b*c*d - b^2*e)*x)*sqrt((c*d - b*e)/c)*lo
g((c*e*x + 2*c*d - b*e + 2*sqrt(e*x + d)*c*sqrt((c*d - b*e)/c))/(c*x + b))
+ ((4*c^2*d - 3*b*c*e)*x^2 + (4*b*c*d - 3*b^2*e)*x)*sqrt(d)*log((e*x - 2*sq
rt(e*x + d)*sqrt(d) + 2*d)/x) + 2*(b^2*d + (2*b*c*d - b^2*e)*x)*sqrt(e*x +
d))/(b^3*c*x^2 + b^4*x), -1/2*(2*((4*c^2*d - b*c*e)*x^2 + (4*b*c*d - b^2*e)
*x)*sqrt(-(c*d - b*e)/c)*arctan(-sqrt(e*x + d)*c*sqrt(-(c*d - b*e)/c)/(c*d
- b*e)) + ((4*c^2*d - 3*b*c*e)*x^2 + (4*b*c*d - 3*b^2*e)*x)*sqrt(d)*log((e*
x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + 2*(b^2*d + (2*b*c*d - b^2*e)*x)*sqr
t(e*x + d))/(b^3*c*x^2 + b^4*x), -1/2*(2*((4*c^2*d - 3*b*c*e)*x^2 + (4*b*c*
d - 3*b^2*e)*x)*sqrt(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d) + ((4*c^2*d - b*c
*e)*x^2 + (4*b*c*d - b^2*e)*x)*sqrt((c*d - b*e)/c)*log((c*e*x + 2*c*d - b*e
 + 2*sqrt(e*x + d)*c*sqrt((c*d - b*e)/c))/(c*x + b)) + 2*(b^2*d + (2*b*c*d
- b^2*e)*x)*sqrt(e*x + d))/(b^3*c*x^2 + b^4*x), -(((4*c^2*d - b*c*e)*x^2 +
(4*b*c*d - b^2*e)*x)*sqrt(-(c*d - b*e)/c)*arctan(-sqrt(e*x + d)*c*sqrt(-(c*
d - b*e)/c)/(c*d - b*e)) + ((4*c^2*d - 3*b*c*e)*x^2 + (4*b*c*d - 3*b^2*e)*x
)*sqrt(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d) + (b^2*d + (2*b*c*d - b^2*e)*x)
*sqrt(e*x + d))/(b^3*c*x^2 + b^4*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+b*x)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38327, size = 285, normalized size = 2.13

$$\frac{\left(4\,c^2d^2 - 5\,bcde + b^2e^2\right)\arctan\left(\frac{\sqrt{xe+d}c}{\sqrt{-c^2d+bce}}\right)}{\sqrt{-c^2d+bce}b^3} - \frac{\left(4\,cd^2 - 3\,bde\right)\arctan\left(\frac{\sqrt{xe+d}}{\sqrt{-d}}\right)}{b^3\sqrt{-d}} - \frac{2\,(xe+d)^{\frac{3}{2}}cde - 2\,\sqrt{xe+d}cd^2e - (xe +}{\left((xe+d)^2c - 2\,(xe+d)cd + cd^2 +\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x)^2,x, algorithm="giac")
```

```
[Out] (4*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*arctan(sqrt(x*e + d)*c/sqrt(-c^2*d + b*c*
e))/(sqrt(-c^2*d + b*c*e)*b^3) - (4*c*d^2 - 3*b*d*e)*arctan(sqrt(x*e + d)/s
qrt(-d))/(b^3*sqrt(-d)) - (2*(x*e + d)^(3/2)*c*d*e - 2*sqrt(x*e + d)*c*d^2*
e - (x*e + d)^(3/2)*b*e^2 + 2*sqrt(x*e + d)*b*d*e^2)/(((x*e + d)^2*c - 2*(x
*e + d)*c*d + c*d^2 + (x*e + d)*b*e - b*d*e)*b^2)
```

3.373 $\int \frac{\sqrt{d+ex}}{\left(bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=125

$$-\frac{(b+2cx)\sqrt{d+ex}}{b^2\left(bx+cx^2\right)}+\frac{(4cd-be)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b^3\sqrt{d}}-\frac{\sqrt{c}(4cd-3be)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b^3\sqrt{cd-be}}$$

```
[Out] -(((b + 2*c*x)*Sqrt[d + e*x])/(b^2*(b*x + c*x^2))) + ((4*c*d - b*e)*ArcTanh
[Sqrt[d + e*x]/Sqrt[d]])/(b^3*Sqrt[d]) - (Sqrt[c]*(4*c*d - 3*b*e)*ArcTanh[(
Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(b^3*Sqrt[c*d - b*e])
```

Rubi [A] time = 0.233964, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {736, 826, 1166, 208}

$$-\frac{(b+2cx)\sqrt{d+ex}}{b^2\left(bx+cx^2\right)}+\frac{(4cd-be)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b^3\sqrt{d}}-\frac{\sqrt{c}(4cd-3be)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b^3\sqrt{cd-be}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(b*x + c*x^2)^2,x]
```

```
[Out] -(((b + 2*c*x)*Sqrt[d + e*x])/(b^2*(b*x + c*x^2))) + ((4*c*d - b*e)*ArcTanh
[Sqrt[d + e*x]/Sqrt[d]])/(b^3*Sqrt[d]) - (Sqrt[c]*(4*c*d - 3*b*e)*ArcTanh[(
Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(b^3*Sqrt[c*d - b*e])
```

Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] - Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)
*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p + 1
), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 0] && (L
tQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c,
d, e, m, p, x]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\left(bx+cx^2\right)^2}\,dx = -\frac{(b+2cx)\sqrt{d+ex}}{b^2\left(bx+cx^2\right)}+\frac{\int \frac{-2cd+\frac{be}{2}-cex}{\sqrt{d+ex}\left(bx+cx^2\right)}\,dx}{b^2}$$

$$= -\frac{(b+2cx)\sqrt{d+ex}}{b^2\left(bx+cx^2\right)}+\frac{2\,\text{Subst}\left(\int \frac{cde+e\left(-2cd+\frac{be}{2}\right)-cex^2}{cd^2-bde+(-2cd+be)x^2+cx^4}\,dx,x,\sqrt{d+ex}\right)}{b^2}$$

$$= -\frac{(b+2cx)\sqrt{d+ex}}{b^2\left(bx+cx^2\right)}+\frac{(c(4cd-3be))\,\text{Subst}\left(\int \frac{1}{\frac{be}{2}+\frac{1}{2}(-2cd+be)+cx^2}\,dx,x,\sqrt{d+ex}\right)}{b^3}-\frac{(c(4cd-be))\,\text{Subs}}{}$$

$$= -\frac{(b+2cx)\sqrt{d+ex}}{b^2\left(bx+cx^2\right)}+\frac{(4cd-be)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b^3\sqrt{d}}-\frac{\sqrt{c}(4cd-3be)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b^3\sqrt{cd-be}}$$

Mathematica [A] time = 0.216863, size = 169, normalized size = 1.35

$$\frac{\sqrt{d}\left(b(b+2cx)\sqrt{d+ex}(cd-be)+\sqrt{c}x(b+cx)(4cd-3be)\sqrt{cd-be}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)\right)-x(b+cx)\left(b^2e^2-5bcde+4c^2d^2\right)}{b^3\sqrt{d}x(b+cx)(be-cd)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(b*x + c*x^2)^2,x]
```

```
[Out] (-((4*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*x*(b + c*x)*ArcTanh[Sqrt[d + e*x]/Sqrt
[d]]) + Sqrt[d]*(b*(c*d - b*e)*(b + 2*c*x)*Sqrt[d + e*x] + Sqrt[c]*(4*c*d -
 3*b*e)*Sqrt[c*d - b*e]*x*(b + c*x)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*
d - b*e]]))/(b^3*Sqrt[d]*(-(c*d) + b*e)*x*(b + c*x))
```

Maple [A] time = 0.221, size = 167, normalized size = 1.3

$$-\frac{ce}{b^2\left(cex+be\right)}\sqrt{ex+d}-3\,\frac{ce}{b^2\sqrt{(be-cd)\,c}}\arctan\left(\frac{\sqrt{ex+d}c}{\sqrt{(be-cd)\,c}}\right)+4\,\frac{c^2d}{b^3\sqrt{(be-cd)\,c}}\arctan\left(\frac{\sqrt{ex+d}c}{\sqrt{(be-cd)\,c}}\right)-\frac{1}{b^2x}\sqrt{ex+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*x^2+b*x)^2,x)
```

```
[Out] -e/b^2*c*(e*x+d)^(1/2)/(c*e*x+b*e)-3*e/b^2*c/((b*e-c*d)*c)^(1/2)*arctan((e*
x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))+4/b^3*c^2/((b*e-c*d)*c)^(1/2)*arctan((e*x
+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))*d-1/b^2*(e*x+d)^(1/2)/x-e/b^2/d^(1/2)*arct
anh((e*x+d)^(1/2)/d^(1/2))+4/b^3*d^(1/2)*arctanh((e*x+d)^(1/2)/d^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.45067, size = 1750, normalized size = 14.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(((4*c^2*d^2 - 3*b*c*d*e)*x^2 + (4*b*c*d^2 - 3*b^2*d*e)*x)*sqrt(c/(c*
d - b*e))*log((c*e*x + 2*c*d - b*e + 2*(c*d - b*e)*sqrt(e*x + d)*sqrt(c/(c*
d - b*e)))/(c*x + b)) + ((4*c^2*d - b*c*e)*x^2 + (4*b*c*d - b^2*e)*x)*sqrt(
d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + 2*(2*b*c*d*x + b^2*d)*sqr
t(e*x + d))/(b^3*c*d*x^2 + b^4*d*x), -1/2*(2*((4*c^2*d^2 - 3*b*c*d*e)*x^2 +
 (4*b*c*d^2 - 3*b^2*d*e)*x)*sqrt(-c/(c*d - b*e))*arctan(-(c*d - b*e)*sqrt(e
*x + d)*sqrt(-c/(c*d - b*e))/(c*e*x + c*d)) + ((4*c^2*d - b*c*e)*x^2 + (4*b
*c*d - b^2*e)*x)*sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + 2*(
2*b*c*d*x + b^2*d)*sqrt(e*x + d))/(b^3*c*d*x^2 + b^4*d*x), -1/2*(2*((4*c^2*
d - b*c*e)*x^2 + (4*b*c*d - b^2*e)*x)*sqrt(-d)*arctan(sqrt(e*x + d)*sqrt(-d
)/d) + ((4*c^2*d^2 - 3*b*c*d*e)*x^2 + (4*b*c*d^2 - 3*b^2*d*e)*x)*sqrt(c/(c*
d - b*e))*log((c*e*x + 2*c*d - b*e + 2*(c*d - b*e)*sqrt(e*x + d)*sqrt(c/(c*
d - b*e)))/(c*x + b)) + 2*(2*b*c*d*x + b^2*d)*sqrt(e*x + d))/(b^3*c*d*x^2 +
 b^4*d*x), -(((4*c^2*d^2 - 3*b*c*d*e)*x^2 + (4*b*c*d^2 - 3*b^2*d*e)*x)*sqrt
(-c/(c*d - b*e))*arctan(-(c*d - b*e)*sqrt(e*x + d)*sqrt(-c/(c*d - b*e))/(c*
e*x + c*d)) + ((4*c^2*d - b*c*e)*x^2 + (4*b*c*d - b^2*e)*x)*sqrt(-d)*arctan
(sqrt(e*x + d)*sqrt(-d)/d) + (2*b*c*d*x + b^2*d)*sqrt(e*x + d))/(b^3*c*d*x^
2 + b^4*d*x)]
```

Sympy [B] time = 49.2941, size = 790, normalized size = 6.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+b*x)**2,x)
```

```
[Out] 2*c**2*d*e*sqrt(d + e*x)/(2*b**4*e**2 - 2*b**3*c*d*e + 2*b**3*c*e**2*x - 2*
b**2*c**2*d*e*x) - 2*c*e**2*sqrt(d + e*x)/(2*b**3*e**2 - 2*b**2*c*d*e + 2*b
**2*c*e**2*x - 2*b*c**2*d*e*x) + c*e**2*sqrt(-1/(c*(b*e - c*d)**3))*log(-b*
*2*e**2*sqrt(-1/(c*(b*e - c*d)**3)) + 2*b*c*d*e*sqrt(-1/(c*(b*e - c*d)**3))
 - c**2*d**2*sqrt(-1/(c*(b*e - c*d)**3)) + sqrt(d + e*x))/(2*b) - c*e**2*sq
rt(-1/(c*(b*e - c*d)**3))*log(b**2*e**2*sqrt(-1/(c*(b*e - c*d)**3)) - 2*b*c
*d*e*sqrt(-1/(c*(b*e - c*d)**3)) + c**2*d**2*sqrt(-1/(c*(b*e - c*d)**3)) +
sqrt(d + e*x))/(2*b) - c**2*d*e*sqrt(-1/(c*(b*e - c*d)**3))*log(-b**2*e**2*
sqrt(-1/(c*(b*e - c*d)**3)) + 2*b*c*d*e*sqrt(-1/(c*(b*e - c*d)**3)) - c**2*
```

```
d**2*sqrt(-1/(c*(b*e - c*d)**3)) + sqrt(d + e*x))/(2*b**2) + c**2*d*e*sqrt(
-1/(c*(b*e - c*d)**3))*log(b**2*e**2*sqrt(-1/(c*(b*e - c*d)**3)) - 2*b*c*d*
e*sqrt(-1/(c*(b*e - c*d)**3)) + c**2*d**2*sqrt(-1/(c*(b*e - c*d)**3)) + sqr
t(d + e*x))/(2*b**2) - d*e*sqrt(d**(-3))*log(-d**2*sqrt(d**(-3)) + sqrt(d +
 e*x))/(2*b**2) + d*e*sqrt(d**(-3))*log(d**2*sqrt(d**(-3)) + sqrt(d + e*x))
/(2*b**2) - 2*e*atan(sqrt(d + e*x)/sqrt(b*e/c - d))/(b**2*sqrt(b*e/c - d))
+ 2*e*atan(sqrt(d + e*x)/sqrt(-d))/(b**2*sqrt(-d)) - sqrt(d + e*x)/(b**2*x)
 + 4*c*d*atan(sqrt(d + e*x)/sqrt(b*e/c - d))/(b**3*sqrt(b*e/c - d)) - 4*c*d
*atan(sqrt(d + e*x)/sqrt(-d))/(b**3*sqrt(-d))
```

Giac [A] time = 1.32419, size = 244, normalized size = 1.95

$$\frac{\left(4\,c^2d - 3\,bce\right)\arctan\left(\frac{\sqrt{xe+d}c}{\sqrt{-c^2d+bce}}\right)}{\sqrt{-c^2d+bce}b^3} - \frac{(4\,cd - be)\arctan\left(\frac{\sqrt{xe+d}}{\sqrt{-d}}\right)}{b^3\sqrt{-d}} - \frac{2\,(xe+d)^{\frac{3}{2}}ce - 2\,\sqrt{xe+d}cde + \sqrt{xe+d}be^2}{\left((xe+d)^2c - 2\,(xe+d)cd + cd^2 + (xe+d)be - bde\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x)^2,x, algorithm="giac")
```

```
[Out] (4*c^2*d - 3*b*c*e)*arctan(sqrt(x*e + d)*c/sqrt(-c^2*d + b*c*e))/(sqrt(-c^2
*d + b*c*e)*b^3) - (4*c*d - b*e)*arctan(sqrt(x*e + d)/sqrt(-d))/(b^3*sqrt(-
d)) - (2*(x*e + d)^(3/2)*c*e - 2*sqrt(x*e + d)*c*d*e + sqrt(x*e + d)*b*e^2)
/(((x*e + d)^2*c - 2*(x*e + d)*c*d + c*d^2 + (x*e + d)*b*e - b*d*e)*b^2)
```

3.374 $\int \frac{1}{\sqrt{d+ex}\left(bx+cx^2\right)^2} dx$

Optimal. Leaf size=154

$$-\frac{c^{3/2}(4cd-5be)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b^3(cd-be)^{3/2}}+\frac{(be+4cd)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b^3d^{3/2}}-\frac{\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{b^2d\left(bx+cx^2\right)(cd-be)}$$

```
[Out] -((Sqrt[d + e*x]*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(b^2*d*(c*d - b*e)*(b
*x + c*x^2))) + ((4*c*d + b*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(b^3*d^(3/2)
) - (c^(3/2)*(4*c*d - 5*b*e)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e
]])/(b^3*(c*d - b*e)^(3/2))
```

Rubi [A] time = 0.260734, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {740, 826, 1166, 208}

$$-\frac{c^{3/2}(4cd-5be)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b^3(cd-be)^{3/2}}+\frac{(be+4cd)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b^3d^{3/2}}-\frac{\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{b^2d\left(bx+cx^2\right)(cd-be)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(b*x + c*x^2)^2),x]
```


```
[Out] -((Sqrt[d + e*x]*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(b^2*d*(c*d - b*e)*(b
*x + c*x^2))) + ((4*c*d + b*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(b^3*d^(3/2)
) - (c^(3/2)*(4*c*d - 5*b*e)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e
]])/(b^3*(c*d - b*e)^(3/2))
```


Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\left(bx+cx^2\right)^2}\,dx = -\frac{\sqrt{d+ex}(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)\left(bx+cx^2\right)} - \frac{\int \frac{\frac{1}{2}(cd-be)(4cd+be)+\frac{1}{2}ce(2cd-be)x}{\sqrt{d+ex}\left(bx+cx^2\right)}\,dx}{b^2d(cd-be)}$$

$$= -\frac{\sqrt{d+ex}(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)\left(bx+cx^2\right)} - \frac{2\,\text{Subst}\left(\int \frac{-\frac{1}{2}cde(2cd-be)+\frac{1}{2}e(cd-be)(4cd+be)+\frac{1}{2}ce(2cd-be)x^2}{cd^2-bde+(-2cd+be)x^2+cx^4}\,d\right.}{b^2d(cd-be)}$$

$$= -\frac{\sqrt{d+ex}(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)\left(bx+cx^2\right)} + \frac{\left(c^2(4cd-5be)\right)\text{Subst}\left(\int \frac{1}{\frac{be}{2}+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x, \sqrt{d}\right.}{b^3(cd-be)}$$

$$= -\frac{\sqrt{d+ex}(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)\left(bx+cx^2\right)} + \frac{(4cd+be)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b^3d^{3/2}} - \frac{c^{3/2}(4cd-5be)\tanh^{-1}}{b^3(cd-be)^{3/2}}$$

Mathematica [A] time = 0.499211, size = 148, normalized size = 0.96

$$\frac{\frac{b\sqrt{d+ex}\left(b^2e+bc(ex-d)-2c^2dx\right)}{x(b+cx)(cd-be)}+\frac{c^{3/2}d(5be-4cd)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{(cd-be)^{3/2}}+\frac{(be+4cd)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{d}}}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(b*x + c*x^2)^2),x]
```

```
[Out] ((b*Sqrt[d + e*x]*(b^2*e - 2*c^2*d*x + b*c*(-d + e*x)))/((c*d - b*e)*x*(b +
 c*x)) + ((4*c*d + b*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/Sqrt[d] + (c^(3/2)*
d*(-4*c*d + 5*b*e)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(c*d -
 b*e)^(3/2))/(b^3*d)
```

Maple [A] time = 0.227, size = 202, normalized size = 1.3

$$\frac{c^2e}{b^2\,(be-cd)\,(cex+be)}\sqrt{ex+d}+5\,\frac{c^2e}{b^2\,(be-cd)\,\sqrt{(be-cd)\,c}}\arctan\left(\frac{\sqrt{ex+dc}}{\sqrt{(be-cd)\,c}}\right)-4\,\frac{c^3d}{b^3\,(be-cd)\,\sqrt{(be-cd)\,c}}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(c*x^2+b*x)^2,x)
```

```
[Out] e*c^2/b^2/(b*e-c*d)*(e*x+d)^(1/2)/(c*e*x+b*e)+5*e*c^2/b^2/(b*e-c*d)/((b*e-c
*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))-4*c^3/b^3/(b*e-c*d
)/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))*d-1/b^2/d
*(e*x+d)^(1/2)/x+e/b^2/d^(3/2)*arctanh((e*x+d)^(1/2)/d^(1/2))+4/b^3/d^(1/2)
```

```
*arctanh((e*x+d)^(1/2)/d^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 7.38875, size = 2396, normalized size = 15.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(((4*c^3*d^3 - 5*b*c^2*d^2*e)*x^2 + (4*b*c^2*d^3 - 5*b^2*c*d^2*e)*x)*s
qrt(c/(c*d - b*e))*log((c*e*x + 2*c*d - b*e - 2*(c*d - b*e)*sqrt(e*x + d)*s
qrt(c/(c*d - b*e)))/(c*x + b)) + ((4*c^3*d^2 - 3*b*c^2*d*e - b^2*c*e^2)*x^2
 + (4*b*c^2*d^2 - 3*b^2*c*d*e - b^3*e^2)*x)*sqrt(d)*log((e*x + 2*sqrt(e*x +
 d)*sqrt(d) + 2*d)/x) - 2*(b^2*c*d^2 - b^3*d*e + (2*b*c^2*d^2 - b^2*c*d*e)*
x)*sqrt(e*x + d))/((b^3*c^2*d^3 - b^4*c*d^2*e)*x^2 + (b^4*c*d^3 - b^5*d^2*e
)*x), -1/2*(2*((4*c^3*d^3 - 5*b*c^2*d^2*e)*x^2 + (4*b*c^2*d^3 - 5*b^2*c*d^2
*e)*x)*sqrt(-c/(c*d - b*e))*arctan(-(c*d - b*e)*sqrt(e*x + d)*sqrt(-c/(c*d
- b*e))/(c*e*x + c*d)) - ((4*c^3*d^2 - 3*b*c^2*d*e - b^2*c*e^2)*x^2 + (4*b*
c^2*d^2 - 3*b^2*c*d*e - b^3*e^2)*x)*sqrt(d)*log((e*x + 2*sqrt(e*x + d)*sqrt
(d) + 2*d)/x) + 2*(b^2*c*d^2 - b^3*d*e + (2*b*c^2*d^2 - b^2*c*d*e)*x)*sqrt(
e*x + d))/((b^3*c^2*d^3 - b^4*c*d^2*e)*x^2 + (b^4*c*d^3 - b^5*d^2*e)*x), -1
/2*(2*((4*c^3*d^2 - 3*b*c^2*d*e - b^2*c*e^2)*x^2 + (4*b*c^2*d^2 - 3*b^2*c*d
*e - b^3*e^2)*x)*sqrt(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d) - ((4*c^3*d^3 -
5*b*c^2*d^2*e)*x^2 + (4*b*c^2*d^3 - 5*b^2*c*d^2*e)*x)*sqrt(c/(c*d - b*e))*l
og((c*e*x + 2*c*d - b*e - 2*(c*d - b*e)*sqrt(e*x + d)*sqrt(c/(c*d - b*e)))/
(c*x + b)) + 2*(b^2*c*d^2 - b^3*d*e + (2*b*c^2*d^2 - b^2*c*d*e)*x)*sqrt(e*x
 + d))/((b^3*c^2*d^3 - b^4*c*d^2*e)*x^2 + (b^4*c*d^3 - b^5*d^2*e)*x), -(((4
*c^3*d^3 - 5*b*c^2*d^2*e)*x^2 + (4*b*c^2*d^3 - 5*b^2*c*d^2*e)*x)*sqrt(-c/(c
*d - b*e))*arctan(-(c*d - b*e)*sqrt(e*x + d)*sqrt(-c/(c*d - b*e))/(c*e*x +
c*d)) + ((4*c^3*d^2 - 3*b*c^2*d*e - b^2*c*e^2)*x^2 + (4*b*c^2*d^2 - 3*b^2*c
*d*e - b^3*e^2)*x)*sqrt(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d) + (b^2*c*d^2 -
 b^3*d*e + (2*b*c^2*d^2 - b^2*c*d*e)*x)*sqrt(e*x + d))/((b^3*c^2*d^3 - b^4*
c*d^2*e)*x^2 + (b^4*c*d^3 - b^5*d^2*e)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (b + cx)^2 \sqrt{d + ex}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(c*x**2+b*x)**2,x)
```

```
[Out] Integral(1/(x**2*(b + c*x)**2*sqrt(d + e*x)), x)
```

Giac [A] time = 1.32119, size = 342, normalized size = 2.22

$$\frac{\left(4\,c^3d-5\,bc^2e\right)\arctan\left(\frac{\sqrt{xe+d}c}{\sqrt{-c^2d+bce}}\right)}{\left(b^3cd-b^4e\right)\sqrt{-c^2d+bce}}-\frac{2\,(xe+d)^{\frac{3}{2}}c^2de-2\,\sqrt{xe+d}c^2d^2e-(xe+d)^{\frac{3}{2}}bce^2+2\,\sqrt{xe+d}bcde^2-\sqrt{xe+d}b^2e^3}{\left(b^2cd^2-b^3de\right)\left((xe+d)^2c-2\,(xe+d)cd+cd^2+(xe+d)be-bde\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x)^2,x, algorithm="giac")
```

```
[Out] (4*c^3*d - 5*b*c^2*e)*arctan(sqrt(x*e + d)*c/sqrt(-c^2*d + b*c*e))/((b^3*c*
d - b^4*e)*sqrt(-c^2*d + b*c*e)) - (2*(x*e + d)^(3/2)*c^2*d*e - 2*sqrt(x*e
+ d)*c^2*d^2*e - (x*e + d)^(3/2)*b*c*e^2 + 2*sqrt(x*e + d)*b*c*d*e^2 - sqrt
(x*e + d)*b^2*e^3)/((b^2*c*d^2 - b^3*d*e)*((x*e + d)^2*c - 2*(x*e + d)*c*d
+ c*d^2 + (x*e + d)*b*e - b*d*e)) - (4*c*d + b*e)*arctan(sqrt(x*e + d)/sqrt
(-d))/(b^3*sqrt(-d)*d)
```

3.375 $\int \frac{1}{(d+ex)^{3/2}\left(bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=206

$$-\frac{e\left(3b^2e^2-2bcde+2c^2d^2\right)}{b^2d^2\sqrt{d+ex}(cd-be)^2}-\frac{c^{5/2}(4cd-7be)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b^3(cd-be)^{5/2}}+\frac{(3be+4cd)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b^3d^{5/2}}-\frac{cx(2cd-be)+b(cd-be)}{b^2d\left(bx+cx^2\right)\sqrt{d+ex}(cd-be)}$$

```
[Out] -((e*(2*c^2*d^2 - 2*b*c*d*e + 3*b^2*e^2))/(b^2*d^2*(c*d - b*e)^2*Sqrt[d + e
*x])) - (b*(c*d - b*e) + c*(2*c*d - b*e)*x)/(b^2*d*(c*d - b*e)*Sqrt[d + e*x
]*(b*x + c*x^2)) + ((4*c*d + 3*b*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(b^3*d^
(5/2)) - (c^(5/2)*(4*c*d - 7*b*e)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d
- b*e]])/(b^3*(c*d - b*e)^(5/2))
```

Rubi [A] time = 0.352746, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {740, 828, 826, 1166, 208}

$$-\frac{e\left(3b^2e^2-2bcde+2c^2d^2\right)}{b^2d^2\sqrt{d+ex}(cd-be)^2}-\frac{c^{5/2}(4cd-7be)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b^3(cd-be)^{5/2}}+\frac{(3be+4cd)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b^3d^{5/2}}-\frac{cx(2cd-be)+b(cd-be)}{b^2d\left(bx+cx^2\right)\sqrt{d+ex}(cd-be)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*(b*x + c*x^2)^2),x]
```

```
[Out] -((e*(2*c^2*d^2 - 2*b*c*d*e + 3*b^2*e^2))/(b^2*d^2*(c*d - b*e)^2*Sqrt[d + e
*x])) - (b*(c*d - b*e) + c*(2*c*d - b*e)*x)/(b^2*d*(c*d - b*e)*Sqrt[d + e*x
]*(b*x + c*x^2)) + ((4*c*d + 3*b*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(b^3*d^
(5/2)) - (c^(5/2)*(4*c*d - 7*b*e)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d
- b*e]])/(b^3*(c*d - b*e)^(5/2))
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\left(bx+cx^2\right)^2}\,dx = -\frac{b(cd-be)+c(2cd-be)x}{b^2d(cd-be)\sqrt{d+ex}\left(bx+cx^2\right)} - \frac{\int \frac{\frac{1}{2}(cd-be)(4cd+3be)+\frac{3}{2}ce(2cd-be)x}{(d+ex)^{3/2}\left(bx+cx^2\right)}\,dx}{b^2d(cd-be)}$$

$$= -\frac{e\left(2c^2d^2-2bcde+3b^2e^2\right)}{b^2d^2(cd-be)^2\sqrt{d+ex}} - \frac{b(cd-be)+c(2cd-be)x}{b^2d(cd-be)\sqrt{d+ex}\left(bx+cx^2\right)} - \frac{\int \frac{\frac{1}{2}(cd-be)^2(4cd+3be)+\frac{1}{2}ce\left(2c^2 \ldots\right.}{\sqrt{d+ex}\left(bx+cx \ldots\right.}}{b^2d^2(cd-b \ldots}$$

$$= -\frac{e\left(2c^2d^2-2bcde+3b^2e^2\right)}{b^2d^2(cd-be)^2\sqrt{d+ex}} - \frac{b(cd-be)+c(2cd-be)x}{b^2d(cd-be)\sqrt{d+ex}\left(bx+cx^2\right)} - \frac{2\,\text{Subst}\left(\int \frac{1}{2}e(cd-be)^2(4cd+ \ldots\right.}{\ldots}$$

$$= -\frac{e\left(2c^2d^2-2bcde+3b^2e^2\right)}{b^2d^2(cd-be)^2\sqrt{d+ex}} - \frac{b(cd-be)+c(2cd-be)x}{b^2d(cd-be)\sqrt{d+ex}\left(bx+cx^2\right)} + \frac{\left(c^3(4cd-7be)\right)\text{Subst}\left(\int \ldots\right.}{b^3 \ldots}$$

$$= -\frac{e\left(2c^2d^2-2bcde+3b^2e^2\right)}{b^2d^2(cd-be)^2\sqrt{d+ex}} - \frac{b(cd-be)+c(2cd-be)x}{b^2d(cd-be)\sqrt{d+ex}\left(bx+cx^2\right)} + \frac{(4cd+3be)\tanh^{-1}\left(\frac{\sqrt{d+ \ldots}}{\sqrt{d}}\right.}{b^3d^{5/2}}$$

Mathematica [C] time = 0.107133, size = 166, normalized size = 0.81

$$\frac{c^2d^2x(b+cx)(4cd-7be)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{c(d+ex)}{cd-be}\right)-(be-cd)\left(x(b+cx)\left(3b^2e^2+bcde-4c^2d^2\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{ex}{d}+1\right)+bd\left(b^2 \ldots\right.\right.}{b^3d^2x(b+cx)\sqrt{d+ex}(cd-be)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*(b*x + c*x^2)^2),x]
```

```
[Out] (c^2*d^2*(4*c*d - 7*b*e)*x*(b + c*x)*Hypergeometric2F1[-1/2, 1, 1/2, (c*(d
+ e*x))/(c*d - b*e)] - (-(c*d) + b*e)*(b*d*(b^2*e - 2*c^2*d*x + b*c*(-d + e
*x)) + (-4*c^2*d^2 + b*c*d*e + 3*b^2*e^2)*x*(b + c*x)*Hypergeometric2F1[-1/
2, 1, 1/2, 1 + (e*x)/d]))/(b^3*d^2*(c*d - b*e)^2*x*(b + c*x)*Sqrt[d + e*x])
```

Maple [A] time = 0.243, size = 229, normalized size = 1.1

$$-2\frac{e^3}{d^2(be-cd)^2\sqrt{ex+d}}-\frac{ec^3}{b^2(be-cd)^2(cex+be)}\sqrt{ex+d}-7\frac{ec^3}{b^2(be-cd)^2\sqrt{(be-cd)c}}\arctan\left(\frac{\sqrt{ex+d}c}{\sqrt{(be-cd)c}}\right)+4\frac{}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(c*x^2+b*x)^2,x)
```

```
[Out] -2*e^3/d^2/(b*e-c*d)^2/(e*x+d)^(1/2)-e*c^3/b^2/(b*e-c*d)^2*(e*x+d)^(1/2)/(c
*e*x+b*e)-7*e*c^3/b^2/(b*e-c*d)^2/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*
c/((b*e-c*d)*c)^(1/2))+4*c^4/b^3/(b*e-c*d)^2/((b*e-c*d)*c)^(1/2)*arctan((e*
x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))*d-1/b^2/d^2*(e*x+d)^(1/2)/x+3*e/b^2/d^(5/
2)*arctanh((e*x+d)^(1/2)/d^(1/2))+4/b^3/d^(3/2)*arctanh((e*x+d)^(1/2)/d^(1/
2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.29635, size = 4633, normalized size = 22.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(((4*c^4*d^4*e - 7*b*c^3*d^3*e^2)*x^3 + (4*c^4*d^5 - 3*b*c^3*d^4*e -
7*b^2*c^2*d^3*e^2)*x^2 + (4*b*c^3*d^5 - 7*b^2*c^2*d^4*e)*x)*sqrt(c/(c*d - b
*e))*log((c*e*x + 2*c*d - b*e + 2*(c*d - b*e)*sqrt(e*x + d)*sqrt(c/(c*d - b
*e)))/(c*x + b)) - ((4*c^4*d^3*e - 5*b*c^3*d^2*e^2 - 2*b^2*c^2*d*e^3 + 3*b^
3*c*e^4)*x^3 + (4*c^4*d^4 - b*c^3*d^3*e - 7*b^2*c^2*d^2*e^2 + b^3*c*d*e^3 +
 3*b^4*e^4)*x^2 + (4*b*c^3*d^4 - 5*b^2*c^2*d^3*e - 2*b^3*c*d^2*e^2 + 3*b^4*
d*e^3)*x)*sqrt(d)*log((e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + 2*(b^2*c^2
*d^4 - 2*b^3*c*d^3*e + b^4*d^2*e^2 + (2*b*c^3*d^3*e - 2*b^2*c^2*d^2*e^2 + 3
*b^3*c*d*e^3)*x^2 + (2*b*c^3*d^4 - b^2*c^2*d^3*e - b^3*c*d^2*e^2 + 3*b^4*d*
e^3)*x)*sqrt(e*x + d))/((b^3*c^3*d^5*e - 2*b^4*c^2*d^4*e^2 + b^5*c*d^3*e^3)
*x^3 + (b^3*c^3*d^6 - b^4*c^2*d^5*e - b^5*c*d^4*e^2 + b^6*d^3*e^3)*x^2 + (b
^4*c^2*d^6 - 2*b^5*c*d^5*e + b^6*d^4*e^2)*x), -1/2*(2*((4*c^4*d^4*e - 7*b*c
^3*d^3*e^2)*x^3 + (4*c^4*d^5 - 3*b*c^3*d^4*e - 7*b^2*c^2*d^3*e^2)*x^2 + (4*
b*c^3*d^5 - 7*b^2*c^2*d^4*e)*x)*sqrt(-c/(c*d - b*e))*arctan(-(c*d - b*e)*sq
rt(e*x + d)*sqrt(-c/(c*d - b*e))/(c*e*x + c*d)) - ((4*c^4*d^3*e - 5*b*c^3*d
^2*e^2 - 2*b^2*c^2*d*e^3 + 3*b^3*c*e^4)*x^3 + (4*c^4*d^4 - b*c^3*d^3*e - 7*
b^2*c^2*d^2*e^2 + b^3*c*d*e^3 + 3*b^4*e^4)*x^2 + (4*b*c^3*d^4 - 5*b^2*c^2*d
^3*e - 2*b^3*c*d^2*e^2 + 3*b^4*d*e^3)*x)*sqrt(d)*log((e*x + 2*sqrt(e*x + d)
*sqrt(d) + 2*d)/x) + 2*(b^2*c^2*d^4 - 2*b^3*c*d^3*e + b^4*d^2*e^2 + (2*b*c^
3*d^3*e - 2*b^2*c^2*d^2*e^2 + 3*b^3*c*d*e^3)*x^2 + (2*b*c^3*d^4 - b^2*c^2*d
```

```
^3*e - b^3*c*d^2*e^2 + 3*b^4*d*e^3)*x)*sqrt(e*x + d))/((b^3*c^3*d^5*e - 2*b
^4*c^2*d^4*e^2 + b^5*c*d^3*e^3)*x^3 + (b^3*c^3*d^6 - b^4*c^2*d^5*e - b^5*c*
d^4*e^2 + b^6*d^3*e^3)*x^2 + (b^4*c^2*d^6 - 2*b^5*c*d^5*e + b^6*d^4*e^2)*x)
, -1/2*(2*((4*c^4*d^3*e - 5*b*c^3*d^2*e^2 - 2*b^2*c^2*d*e^3 + 3*b^3*c*e^4)*
x^3 + (4*c^4*d^4 - b*c^3*d^3*e - 7*b^2*c^2*d^2*e^2 + b^3*c*d*e^3 + 3*b^4*e^
4)*x^2 + (4*b*c^3*d^4 - 5*b^2*c^2*d^3*e - 2*b^3*c*d^2*e^2 + 3*b^4*d*e^3)*x)
*sqrt(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d) + ((4*c^4*d^4*e - 7*b*c^3*d^3*e^
2)*x^3 + (4*c^4*d^5 - 3*b*c^3*d^4*e - 7*b^2*c^2*d^3*e^2)*x^2 + (4*b*c^3*d^5
 - 7*b^2*c^2*d^4*e)*x)*sqrt(c/(c*d - b*e))*log((c*e*x + 2*c*d - b*e + 2*(c*
d - b*e)*sqrt(e*x + d)*sqrt(c/(c*d - b*e)))/(c*x + b)) + 2*(b^2*c^2*d^4 - 2
*b^3*c*d^3*e + b^4*d^2*e^2 + (2*b*c^3*d^3*e - 2*b^2*c^2*d^2*e^2 + 3*b^3*c*d
*e^3)*x^2 + (2*b*c^3*d^4 - b^2*c^2*d^3*e - b^3*c*d^2*e^2 + 3*b^4*d*e^3)*x)*
sqrt(e*x + d))/((b^3*c^3*d^5*e - 2*b^4*c^2*d^4*e^2 + b^5*c*d^3*e^3)*x^3 + (
b^3*c^3*d^6 - b^4*c^2*d^5*e - b^5*c*d^4*e^2 + b^6*d^3*e^3)*x^2 + (b^4*c^2*d
^6 - 2*b^5*c*d^5*e + b^6*d^4*e^2)*x), -(((4*c^4*d^4*e - 7*b*c^3*d^3*e^2)*x^
3 + (4*c^4*d^5 - 3*b*c^3*d^4*e - 7*b^2*c^2*d^3*e^2)*x^2 + (4*b*c^3*d^5 - 7*
b^2*c^2*d^4*e)*x)*sqrt(-c/(c*d - b*e))*arctan(-(c*d - b*e)*sqrt(e*x + d)*sq
rt(-c/(c*d - b*e))/(c*e*x + c*d)) + ((4*c^4*d^3*e - 5*b*c^3*d^2*e^2 - 2*b^2
*c^2*d*e^3 + 3*b^3*c*e^4)*x^3 + (4*c^4*d^4 - b*c^3*d^3*e - 7*b^2*c^2*d^2*e^
2 + b^3*c*d*e^3 + 3*b^4*e^4)*x^2 + (4*b*c^3*d^4 - 5*b^2*c^2*d^3*e - 2*b^3*c
*d^2*e^2 + 3*b^4*d*e^3)*x)*sqrt(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d) + (b^2
*c^2*d^4 - 2*b^3*c*d^3*e + b^4*d^2*e^2 + (2*b*c^3*d^3*e - 2*b^2*c^2*d^2*e^2
 + 3*b^3*c*d*e^3)*x^2 + (2*b*c^3*d^4 - b^2*c^2*d^3*e - b^3*c*d^2*e^2 + 3*b^
4*d*e^3)*x)*sqrt(e*x + d))/((b^3*c^3*d^5*e - 2*b^4*c^2*d^4*e^2 + b^5*c*d^3*
e^3)*x^3 + (b^3*c^3*d^6 - b^4*c^2*d^5*e - b^5*c*d^4*e^2 + b^6*d^3*e^3)*x^2
+ (b^4*c^2*d^6 - 2*b^5*c*d^5*e + b^6*d^4*e^2)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(c*x**2+b*x)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36135, size = 477, normalized size = 2.32

$$\frac{\left(4\,c^4d-7\,bc^3e\right)\arctan\left(\frac{\sqrt{xe+dc}}{\sqrt{-c^2d+bce}}\right)}{\left(b^3c^2d^2-2\,b^4cde+b^5e^2\right)\sqrt{-c^2d+bce}}-\frac{2\,(xe+d)^2c^3d^2e-2\,(xe+d)c^3d^3e-2\,(xe+d)^2bc^2de^2+3\,(xe+d)bc^2d^2e^2+3\,(xe}{\left(b^2c^2d^4-2\,b^3cd^3e+b^4d^2e^2\right)\left((xe+d)^{\frac{5}{2}}c-2\,(xe+d)^{\frac{3}{2}}cd\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^2,x, algorithm="giac")
```

```
[Out] (4*c^4*d - 7*b*c^3*e)*arctan(sqrt(x*e + d)*c/sqrt(-c^2*d + b*c*e))/((b^3*c^
2*d^2 - 2*b^4*c*d*e + b^5*e^2)*sqrt(-c^2*d + b*c*e)) - (2*(x*e + d)^2*c^3*d
^2*e - 2*(x*e + d)*c^3*d^3*e - 2*(x*e + d)^2*b*c^2*d*e^2 + 3*(x*e + d)*b*c^
2*d^2*e^2 + 3*(x*e + d)^2*b^2*c*e^3 - 7*(x*e + d)*b^2*c*d*e^3 + 2*b^2*c*d^2
*e^3 + 3*(x*e + d)*b^3*e^4 - 2*b^3*d*e^4)/((b^2*c^2*d^4 - 2*b^3*c*d^3*e + b
^4*d^2*e^2)*((x*e + d)^(5/2)*c - 2*(x*e + d)^(3/2)*c*d + sqrt(x*e + d)*c*d^
2 + (x*e + d)^(3/2)*b*e - sqrt(x*e + d)*b*d*e)) - (4*c*d + 3*b*e)*arctan(sq
```

```
rt(x*e + d)/sqrt(-d))/(b^3*sqrt(-d)*d^2)
```

3.376 $\int \frac{1}{(d+ex)^{5/2}\left(bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=267

$$-\frac{e(2cd-be)\left(5b^2e^2-bcde+c^2d^2\right)}{b^2d^3\sqrt{d+ex}(cd-be)^3}-\frac{e\left(5b^2e^2-6bcde+6c^2d^2\right)}{3b^2d^2(d+ex)^{3/2}(cd-be)^2}-\frac{c^{7/2}(4cd-9be)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b^3(cd-be)^{7/2}}+\frac{(5be+4cd)\tanh^{-1}}{b^3d^{7/2}}$$

```
[Out] -(e*(6*c^2*d^2 - 6*b*c*d*e + 5*b^2*e^2))/(3*b^2*d^2*(c*d - b*e)^2*(d + e*x)
^(3/2)) - (e*(2*c*d - b*e)*(c^2*d^2 - b*c*d*e + 5*b^2*e^2))/(b^2*d^3*(c*d -
 b*e)^3*Sqrt[d + e*x]) - (b*(c*d - b*e) + c*(2*c*d - b*e)*x)/(b^2*d*(c*d -
b*e)*(d + e*x)^(3/2)*(b*x + c*x^2)) + ((4*c*d + 5*b*e)*ArcTanh[Sqrt[d + e*x
]/Sqrt[d]])/(b^3*d^(7/2)) - (c^(7/2)*(4*c*d - 9*b*e)*ArcTanh[(Sqrt[c]*Sqrt[
d + e*x])/Sqrt[c*d - b*e]])/(b^3*(c*d - b*e)^(7/2))
```

Rubi [A] time = 0.515051, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {740, 828, 826, 1166, 208}

$$-\frac{e(2cd-be)\left(5b^2e^2-bcde+c^2d^2\right)}{b^2d^3\sqrt{d+ex}(cd-be)^3}-\frac{e\left(5b^2e^2-6bcde+6c^2d^2\right)}{3b^2d^2(d+ex)^{3/2}(cd-be)^2}-\frac{c^{7/2}(4cd-9be)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b^3(cd-be)^{7/2}}+\frac{(5be+4cd)\tanh^{-1}}{b^3d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(5/2)*(b*x + c*x^2)^2),x]
```

```
[Out] -(e*(6*c^2*d^2 - 6*b*c*d*e + 5*b^2*e^2))/(3*b^2*d^2*(c*d - b*e)^2*(d + e*x)
^(3/2)) - (e*(2*c*d - b*e)*(c^2*d^2 - b*c*d*e + 5*b^2*e^2))/(b^2*d^3*(c*d -
 b*e)^3*Sqrt[d + e*x]) - (b*(c*d - b*e) + c*(2*c*d - b*e)*x)/(b^2*d*(c*d -
b*e)*(d + e*x)^(3/2)*(b*x + c*x^2)) + ((4*c*d + 5*b*e)*ArcTanh[Sqrt[d + e*x
]/Sqrt[d]])/(b^3*d^(7/2)) - (c^(7/2)*(4*c*d - 9*b*e)*ArcTanh[(Sqrt[c]*Sqrt[
d + e*x])/Sqrt[c*d - b*e]])/(b^3*(c*d - b*e)^(7/2))
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{5/2}\left(bx+cx^2\right)^2}\,dx = -\frac{b(cd-be)+c(2cd-be)x}{b^2d(cd-be)(d+ex)^{3/2}\left(bx+cx^2\right)} - \frac{\int \frac{\frac{1}{2}(cd-be)(4cd+5be)+\frac{5}{2}ce(2cd-be)x}{(d+ex)^{5/2}\left(bx+cx^2\right)}\,dx}{b^2d(cd-be)}$$

$$= -\frac{e\left(6c^2d^2-6bcde+5b^2e^2\right)}{3b^2d^2(cd-be)^2(d+ex)^{3/2}} - \frac{b(cd-be)+c(2cd-be)x}{b^2d(cd-be)(d+ex)^{3/2}\left(bx+cx^2\right)} - \frac{\int \frac{\frac{1}{2}(cd-be)^2(4cd+5be)+ \cdots}{(d+ex)^{3 \cdots}}}{b^2d^2(\cdots}$$

$$= -\frac{e\left(6c^2d^2-6bcde+5b^2e^2\right)}{3b^2d^2(cd-be)^2(d+ex)^{3/2}} - \frac{e(2cd-be)\left(c^2d^2-bcde+5b^2e^2\right)}{b^2d^3(cd-be)^3\sqrt{d+ex}} - \frac{b(cd-be)+c(2 \cdots}{b^2d(cd-be)(d+ex \cdots}$$

$$= -\frac{e\left(6c^2d^2-6bcde+5b^2e^2\right)}{3b^2d^2(cd-be)^2(d+ex)^{3/2}} - \frac{e(2cd-be)\left(c^2d^2-bcde+5b^2e^2\right)}{b^2d^3(cd-be)^3\sqrt{d+ex}} - \frac{b(cd-be)+c(2 \cdots}{b^2d(cd-be)(d+ex \cdots}$$

$$= -\frac{e\left(6c^2d^2-6bcde+5b^2e^2\right)}{3b^2d^2(cd-be)^2(d+ex)^{3/2}} - \frac{e(2cd-be)\left(c^2d^2-bcde+5b^2e^2\right)}{b^2d^3(cd-be)^3\sqrt{d+ex}} - \frac{b(cd-be)+c(2 \cdots}{b^2d(cd-be)(d+ex \cdots}$$

$$= -\frac{e\left(6c^2d^2-6bcde+5b^2e^2\right)}{3b^2d^2(cd-be)^2(d+ex)^{3/2}} - \frac{e(2cd-be)\left(c^2d^2-bcde+5b^2e^2\right)}{b^2d^3(cd-be)^3\sqrt{d+ex}} - \frac{b(cd-be)+c(2 \cdots}{b^2d(cd-be)(d+ex \cdots}$$

Mathematica [C] time = 0.11459, size = 170, normalized size = 0.64

$$\frac{c^2d^2x(b+cx)(4cd-9be)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{c(d+ex)}{cd-be}\right)-(cd-be)\left(x(b+cx)\left(-5b^2e^2+bcde+4c^2d^2\right)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{ex}{d}+1\right) \cdots\right.}{3b^3d^2x(b+cx)(d+ex)^{3/2}(cd-be)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(5/2)*(b*x + c*x^2)^2),x]
```

```
[Out] (c^2*d^2*(4*c*d - 9*b*e)*x*(b + c*x)*Hypergeometric2F1[-3/2, 1, -1/2, (c*(d
 + e*x))/(c*d - b*e)] - (c*d - b*e)*(-3*b*d*(b^2*e - 2*c^2*d*x + b*c*(-d +
```

```
e*x)) + (4*c^2*d^2 + b*c*d*e - 5*b^2*e^2)*x*(b + c*x)*Hypergeometric2F1[-3/
2, 1, -1/2, 1 + (e*x)/d]))/(3*b^3*d^2*(c*d - b*e)^2*x*(b + c*x)*(d + e*x)^(
3/2))
```

Maple [A] time = 0.254, size = 280, normalized size = 1.1

$$-\frac{2e^3}{3d^2(be-cd)^2}(ex+d)^{-\frac{3}{2}}-4\frac{e^4b}{d^3(be-cd)^3\sqrt{ex+d}}+8\frac{e^3c}{d^2(be-cd)^3\sqrt{ex+d}}+\frac{ec^4}{b^2(be-cd)^3(cex+be)}\sqrt{ex+d}+9\frac{}{b^2(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(5/2)/(c*x^2+b*x)^2,x)
```

```
[Out] -2/3*e^3/d^2/(b*e-c*d)^2/(e*x+d)^(3/2)-4*e^4/d^3/(b*e-c*d)^3/(e*x+d)^(1/2)*
b+8*e^3/d^2/(b*e-c*d)^3/(e*x+d)^(1/2)*c+e*c^4/b^2/(b*e-c*d)^3*(e*x+d)^(1/2)
/(c*e*x+b*e)+9*e*c^4/b^2/(b*e-c*d)^3/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/
2)*c/((b*e-c*d)*c)^(1/2))-4*c^5/b^3/(b*e-c*d)^3/((b*e-c*d)*c)^(1/2)*arctan(
(e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))*d-1/b^2/d^3*(e*x+d)^(1/2)/x+5*e/b^2/d^
(7/2)*arctanh((e*x+d)^(1/2)/d^(1/2))+4/b^3/d^(5/2)*arctanh((e*x+d)^(1/2)/d^
(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 18.6097, size = 7775, normalized size = 29.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(3*((4*c^5*d^5*e^2 - 9*b*c^4*d^4*e^3)*x^4 + (8*c^5*d^6*e - 14*b*c^4*d^
5*e^2 - 9*b^2*c^3*d^4*e^3)*x^3 + (4*c^5*d^7 - b*c^4*d^6*e - 18*b^2*c^3*d^5*
e^2)*x^2 + (4*b*c^4*d^7 - 9*b^2*c^3*d^6*e)*x)*sqrt(c/(c*d - b*e))*log((c*e*
x + 2*c*d - b*e - 2*(c*d - b*e)*sqrt(e*x + d)*sqrt(c/(c*d - b*e)))/(c*x + b
)) + 3*((4*c^5*d^4*e^2 - 7*b*c^4*d^3*e^3 - 3*b^2*c^3*d^2*e^4 + 11*b^3*c^2*d
*e^5 - 5*b^4*c*e^6)*x^4 + (8*c^5*d^5*e - 10*b*c^4*d^4*e^2 - 13*b^2*c^3*d^3*
e^3 + 19*b^3*c^2*d^2*e^4 + b^4*c*d*e^5 - 5*b^5*e^6)*x^3 + (4*c^5*d^6 + b*c^
4*d^5*e - 17*b^2*c^3*d^4*e^2 + 5*b^3*c^2*d^3*e^3 + 17*b^4*c*d^2*e^4 - 10*b^
5*d*e^5)*x^2 + (4*b*c^4*d^6 - 7*b^2*c^3*d^5*e - 3*b^3*c^2*d^4*e^2 + 11*b^4*
c*d^3*e^3 - 5*b^5*d^2*e^4)*x)*sqrt(d)*log((e*x + 2*sqrt(e*x + d)*sqrt(d) +
2*d)/x) - 2*(3*b^2*c^3*d^6 - 9*b^3*c^2*d^5*e + 9*b^4*c*d^4*e^2 - 3*b^5*d^3*
e^3 + 3*(2*b*c^4*d^4*e^2 - 3*b^2*c^3*d^3*e^3 + 11*b^3*c^2*d^2*e^4 - 5*b^4*c
*d*e^5)*x^3 + (12*b*c^4*d^5*e - 15*b^2*c^3*d^4*e^2 + 35*b^3*c^2*d^3*e^3 + 1
```

```
3*b^4*c*d^2*e^4 - 15*b^5*d*e^5)*x^2 + (6*b*c^4*d^6 - 3*b^2*c^3*d^5*e - 9*b^
3*c^2*d^4*e^2 + 41*b^4*c*d^3*e^3 - 20*b^5*d^2*e^4)*x)*sqrt(e*x + d))/((b^3*
c^4*d^7*e^2 - 3*b^4*c^3*d^6*e^3 + 3*b^5*c^2*d^5*e^4 - b^6*c*d^4*e^5)*x^4 +
(2*b^3*c^4*d^8*e - 5*b^4*c^3*d^7*e^2 + 3*b^5*c^2*d^6*e^3 + b^6*c*d^5*e^4 -
b^7*d^4*e^5)*x^3 + (b^3*c^4*d^9 - b^4*c^3*d^8*e - 3*b^5*c^2*d^7*e^2 + 5*b^6
*c*d^6*e^3 - 2*b^7*d^5*e^4)*x^2 + (b^4*c^3*d^9 - 3*b^5*c^2*d^8*e + 3*b^6*c*
d^7*e^2 - b^7*d^6*e^3)*x), -1/6*(6*((4*c^5*d^5*e^2 - 9*b*c^4*d^4*e^3)*x^4 +
 (8*c^5*d^6*e - 14*b*c^4*d^5*e^2 - 9*b^2*c^3*d^4*e^3)*x^3 + (4*c^5*d^7 - b*
c^4*d^6*e - 18*b^2*c^3*d^5*e^2)*x^2 + (4*b*c^4*d^7 - 9*b^2*c^3*d^6*e)*x)*sq
rt(-c/(c*d - b*e))*arctan(-(c*d - b*e)*sqrt(e*x + d)*sqrt(-c/(c*d - b*e))/(
c*e*x + c*d)) - 3*((4*c^5*d^4*e^2 - 7*b*c^4*d^3*e^3 - 3*b^2*c^3*d^2*e^4 + 1
1*b^3*c^2*d*e^5 - 5*b^4*c*e^6)*x^4 + (8*c^5*d^5*e - 10*b*c^4*d^4*e^2 - 13*b
^2*c^3*d^3*e^3 + 19*b^3*c^2*d^2*e^4 + b^4*c*d*e^5 - 5*b^5*e^6)*x^3 + (4*c^5
*d^6 + b*c^4*d^5*e - 17*b^2*c^3*d^4*e^2 + 5*b^3*c^2*d^3*e^3 + 17*b^4*c*d^2*
e^4 - 10*b^5*d*e^5)*x^2 + (4*b*c^4*d^6 - 7*b^2*c^3*d^5*e - 3*b^3*c^2*d^4*e^
2 + 11*b^4*c*d^3*e^3 - 5*b^5*d^2*e^4)*x)*sqrt(d)*log((e*x + 2*sqrt(e*x + d)
*sqrt(d) + 2*d)/x) + 2*(3*b^2*c^3*d^6 - 9*b^3*c^2*d^5*e + 9*b^4*c*d^4*e^2 -
 3*b^5*d^3*e^3 + 3*(2*b*c^4*d^4*e^2 - 3*b^2*c^3*d^3*e^3 + 11*b^3*c^2*d^2*e^
4 - 5*b^4*c*d*e^5)*x^3 + (12*b*c^4*d^5*e - 15*b^2*c^3*d^4*e^2 + 35*b^3*c^2*
d^3*e^3 + 13*b^4*c*d^2*e^4 - 15*b^5*d*e^5)*x^2 + (6*b*c^4*d^6 - 3*b^2*c^3*d
^5*e - 9*b^3*c^2*d^4*e^2 + 41*b^4*c*d^3*e^3 - 20*b^5*d^2*e^4)*x)*sqrt(e*x +
 d))/((b^3*c^4*d^7*e^2 - 3*b^4*c^3*d^6*e^3 + 3*b^5*c^2*d^5*e^4 - b^6*c*d^4*
e^5)*x^4 + (2*b^3*c^4*d^8*e - 5*b^4*c^3*d^7*e^2 + 3*b^5*c^2*d^6*e^3 + b^6*c
*d^5*e^4 - b^7*d^4*e^5)*x^3 + (b^3*c^4*d^9 - b^4*c^3*d^8*e - 3*b^5*c^2*d^7*
e^2 + 5*b^6*c*d^6*e^3 - 2*b^7*d^5*e^4)*x^2 + (b^4*c^3*d^9 - 3*b^5*c^2*d^8*e
 + 3*b^6*c*d^7*e^2 - b^7*d^6*e^3)*x), -1/6*(6*((4*c^5*d^4*e^2 - 7*b*c^4*d^3
*e^3 - 3*b^2*c^3*d^2*e^4 + 11*b^3*c^2*d*e^5 - 5*b^4*c*e^6)*x^4 + (8*c^5*d^5
*e - 10*b*c^4*d^4*e^2 - 13*b^2*c^3*d^3*e^3 + 19*b^3*c^2*d^2*e^4 + b^4*c*d*e
^5 - 5*b^5*e^6)*x^3 + (4*c^5*d^6 + b*c^4*d^5*e - 17*b^2*c^3*d^4*e^2 + 5*b^3
*c^2*d^3*e^3 + 17*b^4*c*d^2*e^4 - 10*b^5*d*e^5)*x^2 + (4*b*c^4*d^6 - 7*b^2*
c^3*d^5*e - 3*b^3*c^2*d^4*e^2 + 11*b^4*c*d^3*e^3 - 5*b^5*d^2*e^4)*x)*sqrt(-
d)*arctan(sqrt(e*x + d)*sqrt(-d)/d) - 3*((4*c^5*d^5*e^2 - 9*b*c^4*d^4*e^3)*
x^4 + (8*c^5*d^6*e - 14*b*c^4*d^5*e^2 - 9*b^2*c^3*d^4*e^3)*x^3 + (4*c^5*d^7
 - b*c^4*d^6*e - 18*b^2*c^3*d^5*e^2)*x^2 + (4*b*c^4*d^7 - 9*b^2*c^3*d^6*e)*
x)*sqrt(c/(c*d - b*e))*log((c*e*x + 2*c*d - b*e - 2*(c*d - b*e)*sqrt(e*x +
d)*sqrt(c/(c*d - b*e)))/(c*x + b)) + 2*(3*b^2*c^3*d^6 - 9*b^3*c^2*d^5*e + 9
*b^4*c*d^4*e^2 - 3*b^5*d^3*e^3 + 3*(2*b*c^4*d^4*e^2 - 3*b^2*c^3*d^3*e^3 + 1
1*b^3*c^2*d^2*e^4 - 5*b^4*c*d*e^5)*x^3 + (12*b*c^4*d^5*e - 15*b^2*c^3*d^4*e
^2 + 35*b^3*c^2*d^3*e^3 + 13*b^4*c*d^2*e^4 - 15*b^5*d*e^5)*x^2 + (6*b*c^4*d
^6 - 3*b^2*c^3*d^5*e - 9*b^3*c^2*d^4*e^2 + 41*b^4*c*d^3*e^3 - 20*b^5*d^2*e^
4)*x)*sqrt(e*x + d))/((b^3*c^4*d^7*e^2 - 3*b^4*c^3*d^6*e^3 + 3*b^5*c^2*d^5*
e^4 - b^6*c*d^4*e^5)*x^4 + (2*b^3*c^4*d^8*e - 5*b^4*c^3*d^7*e^2 + 3*b^5*c^2
*d^6*e^3 + b^6*c*d^5*e^4 - b^7*d^4*e^5)*x^3 + (b^3*c^4*d^9 - b^4*c^3*d^8*e
- 3*b^5*c^2*d^7*e^2 + 5*b^6*c*d^6*e^3 - 2*b^7*d^5*e^4)*x^2 + (b^4*c^3*d^9 -
 3*b^5*c^2*d^8*e + 3*b^6*c*d^7*e^2 - b^7*d^6*e^3)*x), -1/3*(3*((4*c^5*d^5*e
^2 - 9*b*c^4*d^4*e^3)*x^4 + (8*c^5*d^6*e - 14*b*c^4*d^5*e^2 - 9*b^2*c^3*d^4
*e^3)*x^3 + (4*c^5*d^7 - b*c^4*d^6*e - 18*b^2*c^3*d^5*e^2)*x^2 + (4*b*c^4*d
^7 - 9*b^2*c^3*d^6*e)*x)*sqrt(-c/(c*d - b*e))*arctan(-(c*d - b*e)*sqrt(e*x
+ d)*sqrt(-c/(c*d - b*e))/(c*e*x + c*d)) + 3*((4*c^5*d^4*e^2 - 7*b*c^4*d^3*
e^3 - 3*b^2*c^3*d^2*e^4 + 11*b^3*c^2*d*e^5 - 5*b^4*c*e^6)*x^4 + (8*c^5*d^5*
e - 10*b*c^4*d^4*e^2 - 13*b^2*c^3*d^3*e^3 + 19*b^3*c^2*d^2*e^4 + b^4*c*d*e^
5 - 5*b^5*e^6)*x^3 + (4*c^5*d^6 + b*c^4*d^5*e - 17*b^2*c^3*d^4*e^2 + 5*b^3*
c^2*d^3*e^3 + 17*b^4*c*d^2*e^4 - 10*b^5*d*e^5)*x^2 + (4*b*c^4*d^6 - 7*b^2*c
^3*d^5*e - 3*b^3*c^2*d^4*e^2 + 11*b^4*c*d^3*e^3 - 5*b^5*d^2*e^4)*x)*sqrt(-d
)*arctan(sqrt(e*x + d)*sqrt(-d)/d) + (3*b^2*c^3*d^6 - 9*b^3*c^2*d^5*e + 9*b
^4*c*d^4*e^2 - 3*b^5*d^3*e^3 + 3*(2*b*c^4*d^4*e^2 - 3*b^2*c^3*d^3*e^3 + 11*
b^3*c^2*d^2*e^4 - 5*b^4*c*d*e^5)*x^3 + (12*b*c^4*d^5*e - 15*b^2*c^3*d^4*e^2
 + 35*b^3*c^2*d^3*e^3 + 13*b^4*c*d^2*e^4 - 15*b^5*d*e^5)*x^2 + (6*b*c^4*d^6
 - 3*b^2*c^3*d^5*e - 9*b^3*c^2*d^4*e^2 + 41*b^4*c*d^3*e^3 - 20*b^5*d^2*e^4)
```

```
*x)*sqrt(e*x + d))/((b^3*c^4*d^7*e^2 - 3*b^4*c^3*d^6*e^3 + 3*b^5*c^2*d^5*e^
4 - b^6*c*d^4*e^5)*x^4 + (2*b^3*c^4*d^8*e - 5*b^4*c^3*d^7*e^2 + 3*b^5*c^2*d
^6*e^3 + b^6*c*d^5*e^4 - b^7*d^4*e^5)*x^3 + (b^3*c^4*d^9 - b^4*c^3*d^8*e -
3*b^5*c^2*d^7*e^2 + 5*b^6*c*d^6*e^3 - 2*b^7*d^5*e^4)*x^2 + (b^4*c^3*d^9 - 3
*b^5*c^2*d^8*e + 3*b^6*c*d^7*e^2 - b^7*d^6*e^3)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(5/2)/(c*x**2+b*x)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.44626, size = 649, normalized size = 2.43

$$\frac{\left(4\,c^5d-9\,bc^4e\right)\arctan\left(\frac{\sqrt{xe+d}c}{\sqrt{-c^2d+bce}}\right)}{\left(b^3c^3d^3-3\,b^4c^2d^2e+3\,b^5cde^2-b^6e^3\right)\sqrt{-c^2d+bce}}-\frac{2\,(xe+d)^{\frac{3}{2}}c^4d^3e-2\,\sqrt{xe+d}c^4d^4e-3\,(xe+d)^{\frac{3}{2}}bc^3d^2e^2+4\,\sqrt{xe+d}}{\left(b^2c^3d^6-3\,b^3c^2d^5e+3\,b^4cd^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x)^2,x, algorithm="giac")
```

```
[Out] (4*c^5*d - 9*b*c^4*e)*arctan(sqrt(x*e + d)*c/sqrt(-c^2*d + b*c*e))/((b^3*c^
3*d^3 - 3*b^4*c^2*d^2*e + 3*b^5*c*d*e^2 - b^6*e^3)*sqrt(-c^2*d + b*c*e)) -
(2*(x*e + d)^(3/2)*c^4*d^3*e - 2*sqrt(x*e + d)*c^4*d^4*e - 3*(x*e + d)^(3/2
)*b*c^3*d^2*e^2 + 4*sqrt(x*e + d)*b*c^3*d^3*e^2 + 3*(x*e + d)^(3/2)*b^2*c^2
*d*e^3 - 6*sqrt(x*e + d)*b^2*c^2*d^2*e^3 - (x*e + d)^(3/2)*b^3*c*e^4 + 4*sq
rt(x*e + d)*b^3*c*d*e^4 - sqrt(x*e + d)*b^4*e^5)/((b^2*c^3*d^6 - 3*b^3*c^2*
d^5*e + 3*b^4*c*d^4*e^2 - b^5*d^3*e^3)*((x*e + d)^2*c - 2*(x*e + d)*c*d + c
*d^2 + (x*e + d)*b*e - b*d*e)) - 2/3*(12*(x*e + d)*c*d*e^3 + c*d^2*e^3 - 6*
(x*e + d)*b*e^4 - b*d*e^4)/((c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 - b^
3*d^3*e^3)*(x*e + d)^(3/2)) - (4*c*d + 5*b*e)*arctan(sqrt(x*e + d)/sqrt(-d)
)/(b^3*sqrt(-d)*d^3)
```

3.377 $\int \frac{1}{(d+ex)^{7/2}\left(bx+cx^2\right)^2} dx$

Optimal. Leaf size=349

$$-\frac{e\left(26b^2c^2d^2e^2-24b^3cde^3+7b^4e^4-4bc^3d^3e+2c^4d^4\right)}{b^2d^4\sqrt{d+ex}(cd-be)^4}-\frac{e(2cd-be)\left(7b^2e^2-3bcde+3c^2d^2\right)}{3b^2d^3(d+ex)^{3/2}(cd-be)^3}-\frac{e\left(7b^2e^2-10bcde+10\right.}{5b^2d^2(d+ex)^{5/2}(cd-\right.}$$

```
[Out] -(e*(10*c^2*d^2 - 10*b*c*d*e + 7*b^2*e^2))/(5*b^2*d^2*(c*d - b*e)^2*(d + e*
x)^(5/2)) - (e*(2*c*d - b*e)*(3*c^2*d^2 - 3*b*c*d*e + 7*b^2*e^2))/(3*b^2*d^
3*(c*d - b*e)^3*(d + e*x)^(3/2)) - (e*(2*c^4*d^4 - 4*b*c^3*d^3*e + 26*b^2*c
^2*d^2*e^2 - 24*b^3*c*d*e^3 + 7*b^4*e^4))/(b^2*d^4*(c*d - b*e)^4*Sqrt[d + e
*x]) - (b*(c*d - b*e) + c*(2*c*d - b*e)*x)/(b^2*d*(c*d - b*e)*(d + e*x)^(5/
2)*(b*x + c*x^2)) + ((4*c*d + 7*b*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(b^3*d
^(9/2)) - (c^(9/2)*(4*c*d - 11*b*e)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*
d - b*e]])/(b^3*(c*d - b*e)^(9/2))
```

Rubi [A] time = 0.738855, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {740, 828, 826, 1166, 208}

$$-\frac{e\left(26b^2c^2d^2e^2-24b^3cde^3+7b^4e^4-4bc^3d^3e+2c^4d^4\right)}{b^2d^4\sqrt{d+ex}(cd-be)^4}-\frac{e(2cd-be)\left(7b^2e^2-3bcde+3c^2d^2\right)}{3b^2d^3(d+ex)^{3/2}(cd-be)^3}-\frac{e\left(7b^2e^2-10bcde+10\right.}{5b^2d^2(d+ex)^{5/2}(cd-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(7/2)*(b*x + c*x^2)^2),x]
```


```
[Out] -(e*(10*c^2*d^2 - 10*b*c*d*e + 7*b^2*e^2))/(5*b^2*d^2*(c*d - b*e)^2*(d + e*
x)^(5/2)) - (e*(2*c*d - b*e)*(3*c^2*d^2 - 3*b*c*d*e + 7*b^2*e^2))/(3*b^2*d^
3*(c*d - b*e)^3*(d + e*x)^(3/2)) - (e*(2*c^4*d^4 - 4*b*c^3*d^3*e + 26*b^2*c
^2*d^2*e^2 - 24*b^3*c*d*e^3 + 7*b^4*e^4))/(b^2*d^4*(c*d - b*e)^4*Sqrt[d + e
*x]) - (b*(c*d - b*e) + c*(2*c*d - b*e)*x)/(b^2*d*(c*d - b*e)*(d + e*x)^(5/
2)*(b*x + c*x^2)) + ((4*c*d + 7*b*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(b^3*d
^(9/2)) - (c^(9/2)*(4*c*d - 11*b*e)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*
d - b*e]])/(b^3*(c*d - b*e)^(9/2))
```


Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
```

```
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{7/2}\left(bx+cx^2\right)^2}\,dx = -\frac{b(cd-be)+c(2cd-be)x}{b^2d(cd-be)(d+ex)^{5/2}\left(bx+cx^2\right)} - \frac{\int \frac{\frac{1}{2}(cd-be)(4cd+7be)+\frac{7}{2}ce(2cd-be)x}{(d+ex)^{7/2}\left(bx+cx^2\right)}\,dx}{b^2d(cd-be)}$$

$$= -\frac{e\left(10c^2d^2-10bcde+7b^2e^2\right)}{5b^2d^2(cd-be)^2(d+ex)^{5/2}} - \frac{b(cd-be)+c(2cd-be)x}{b^2d(cd-be)(d+ex)^{5/2}\left(bx+cx^2\right)} - \frac{\int \frac{\frac{1}{2}(cd-be)^2(4cd+7be)+\frac{1}{2}c}{(d+ex)^{5/}}}{b^2d^2(}$$

$$= -\frac{e\left(10c^2d^2-10bcde+7b^2e^2\right)}{5b^2d^2(cd-be)^2(d+ex)^{5/2}} - \frac{e(2cd-be)\left(3c^2d^2-3bcde+7b^2e^2\right)}{3b^2d^3(cd-be)^3(d+ex)^{3/2}} - \frac{b(cd-be)+c(}{b^2d(cd-be)(d+e}$$

$$= -\frac{e\left(10c^2d^2-10bcde+7b^2e^2\right)}{5b^2d^2(cd-be)^2(d+ex)^{5/2}} - \frac{e(2cd-be)\left(3c^2d^2-3bcde+7b^2e^2\right)}{3b^2d^3(cd-be)^3(d+ex)^{3/2}} - \frac{e\left(2c^4d^4-4bc^3d^3e\right.}{b^2d}$$

$$= -\frac{e\left(10c^2d^2-10bcde+7b^2e^2\right)}{5b^2d^2(cd-be)^2(d+ex)^{5/2}} - \frac{e(2cd-be)\left(3c^2d^2-3bcde+7b^2e^2\right)}{3b^2d^3(cd-be)^3(d+ex)^{3/2}} - \frac{e\left(2c^4d^4-4bc^3d^3e\right.}{b^2d}$$

$$= -\frac{e\left(10c^2d^2-10bcde+7b^2e^2\right)}{5b^2d^2(cd-be)^2(d+ex)^{5/2}} - \frac{e(2cd-be)\left(3c^2d^2-3bcde+7b^2e^2\right)}{3b^2d^3(cd-be)^3(d+ex)^{3/2}} - \frac{e\left(2c^4d^4-4bc^3d^3e\right.}{b^2d}$$

$$= -\frac{e\left(10c^2d^2-10bcde+7b^2e^2\right)}{5b^2d^2(cd-be)^2(d+ex)^{5/2}} - \frac{e(2cd-be)\left(3c^2d^2-3bcde+7b^2e^2\right)}{3b^2d^3(cd-be)^3(d+ex)^{3/2}} - \frac{e\left(2c^4d^4-4bc^3d^3e\right.}{b^2d}$$

Mathematica [C] time = 0.117751, size = 171, normalized size = 0.49

$$\frac{c^2d^2x(b+cx)(4cd-11be)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\frac{c(d+ex)}{cd-be}\right)-(cd-be)\left(x(b+cx)\left(-7b^2e^2+3bcde+4c^2d^2\right)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\frac{ex}{d}+1\right)-\right.}{5b^3d^2x(b+cx)(d+ex)^{5/2}(cd-be)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(7/2)*(b*x + c*x^2)^2),x]
```

```
[Out] (c^2*d^2*(4*c*d - 11*b*e)*x*(b + c*x)*Hypergeometric2F1[-5/2, 1, -3/2, (c*(
d + e*x))/(c*d - b*e)] - (c*d - b*e)*(-5*b*d*(b^2*e - 2*c^2*d*x + b*c*(-d +
 e*x)) + (4*c^2*d^2 + 3*b*c*d*e - 7*b^2*e^2)*x*(b + c*x)*Hypergeometric2F1[
-5/2, 1, -3/2, 1 + (e*x)/d]))/(5*b^3*d^2*(c*d - b*e)^2*x*(b + c*x)*(d + e*x
)^(5/2))
```

Maple [A] time = 0.237, size = 364, normalized size = 1.

$$-\frac{2e^3}{5d^2(be-cd)^2}(ex+d)^{-\frac{5}{2}}-\frac{4e^4b}{3d^3(be-cd)^3}(ex+d)^{-\frac{3}{2}}+\frac{8e^3c}{3d^2(be-cd)^3}(ex+d)^{-\frac{3}{2}}-6\frac{e^5b^2}{d^4(be-cd)^4\sqrt{ex+d}}+20\frac{\ }{d^3(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(7/2)/(c*x^2+b*x)^2,x)
```

```
[Out] -2/5*e^3/d^2/(b*e-c*d)^2/(e*x+d)^(5/2)-4/3*e^4/d^3/(b*e-c*d)^3/(e*x+d)^(3/2
)*b+8/3*e^3/d^2/(b*e-c*d)^3/(e*x+d)^(3/2)*c-6*e^5/d^4/(b*e-c*d)^4/(e*x+d)^(
1/2)*b^2+20*e^4/d^3/(b*e-c*d)^4/(e*x+d)^(1/2)*b*c-20*e^3/d^2/(b*e-c*d)^4/(e
*x+d)^(1/2)*c^2-e*c^5/b^2/(b*e-c*d)^4*(e*x+d)^(1/2)/(c*e*x+b*e)-11*e*c^5/b^
2/(b*e-c*d)^4/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2
))+4*c^6/b^3/(b*e-c*d)^4/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c
*d)*c)^(1/2))*d-1/b^2/d^4*(e*x+d)^(1/2)/x+7*e/b^2/d^(9/2)*arctanh((e*x+d)^(
1/2)/d^(1/2))+4/b^3/d^(7/2)*arctanh((e*x+d)^(1/2)/d^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 45.9547, size = 11756, normalized size = 33.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

```
[Out] [-1/30*(15*((4*c^6*d^6*e^3 - 11*b*c^5*d^5*e^4)*x^5 + (12*c^6*d^7*e^2 - 29*b
*c^5*d^6*e^3 - 11*b^2*c^4*d^5*e^4)*x^4 + 3*(4*c^6*d^8*e - 7*b*c^5*d^7*e^2 -
 11*b^2*c^4*d^6*e^3)*x^3 + (4*c^6*d^9 + b*c^5*d^8*e - 33*b^2*c^4*d^7*e^2)*x
^2 + (4*b*c^5*d^9 - 11*b^2*c^4*d^8*e)*x)*sqrt(c/(c*d - b*e))*log((c*e*x + 2
*c*d - b*e + 2*(c*d - b*e)*sqrt(e*x + d)*sqrt(c/(c*d - b*e)))/(c*x + b)) -
```

15*((4*c^6*d^5*e^3 - 9*b*c^5*d^4*e^4 - 4*b^2*c^4*d^3*e^5 + 26*b^3*c^3*d^2*e^6 - 24*b^4*c^2*d*e^7 + 7*b^5*c*e^8)*x^5 + (12*c^6*d^6*e^2 - 23*b*c^5*d^5*e^3 - 21*b^2*c^4*d^4*e^4 + 74*b^3*c^3*d^3*e^5 - 46*b^4*c^2*d^2*e^6 - 3*b^5*c*d*e^7 + 7*b^6*e^8)*x^4 + 3*(4*c^6*d^7*e - 5*b*c^5*d^6*e^2 - 13*b^2*c^4*d^5*e^3 + 22*b^3*c^3*d^4*e^4 + 2*b^4*c^2*d^3*e^5 - 17*b^5*c*d^2*e^6 + 7*b^6*d*e^7)*x^3 + (4*c^6*d^8 + 3*b*c^5*d^7*e - 31*b^2*c^4*d^6*e^2 + 14*b^3*c^3*d^5*e^3 + 54*b^4*c^2*d^4*e^4 - 65*b^5*c*d^3*e^5 + 21*b^6*d^2*e^6)*x^2 + (4*b*c^5*d^8 - 9*b^2*c^4*d^7*e - 4*b^3*c^3*d^6*e^2 + 26*b^4*c^2*d^5*e^3 - 24*b^5*c*d^4*e^4 + 7*b^6*d^3*e^5)*x)*sqrt(d)*log((e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + 2*(15*b^2*c^4*d^8 - 60*b^3*c^3*d^7*e + 90*b^4*c^2*d^6*e^2 - 60*b^5*c*d^5*e^3 + 15*b^6*d^4*e^4 + 15*(2*b*c^5*d^5*e^3 - 4*b^2*c^4*d^4*e^4 + 26*b^3*c^3*d^3*e^5 - 24*b^4*c^2*d^2*e^6 + 7*b^5*c*d*e^7)*x^4 + 5*(18*b*c^5*d^6*e^2 - 33*b^2*c^4*d^5*e^3 + 170*b^3*c^3*d^4*e^4 - 90*b^4*c^2*d^3*e^5 - 23*b^5*c*d^2*e^6 + 21*b^6*d*e^7)*x^3 + (90*b*c^5*d^7*e - 135*b^2*c^4*d^6*e^2 + 436*b^3*c^3*d^5*e^3 + 358*b^4*c^2*d^4*e^4 - 679*b^5*c*d^3*e^5 + 245*b^6*d^2*e^6)*x^2 + (30*b*c^5*d^8 - 15*b^2*c^4*d^7*e - 90*b^3*c^3*d^6*e^2 + 556*b^4*c^2*d^5*e^3 - 537*b^5*c*d^4*e^4 + 161*b^6*d^3*e^5)*x)*sqrt(e*x + d))/((b^3*c^5*d^9*e^3 - 4*b^4*c^4*d^8*e^4 + 6*b^5*c^3*d^7*e^5 - 4*b^6*c^2*d^6*e^6 + b^7*c*d^5*e^7)*x^5 + (3*b^3*c^5*d^10*e^2 - 11*b^4*c^4*d^9*e^3 + 14*b^5*c^3*d^8*e^4 - 6*b^6*c^2*d^7*e^5 - b^7*c*d^6*e^6 + b^8*d^5*e^7)*x^4 + 3*(b^3*c^5*d^11*e - 3*b^4*c^4*d^10*e^2 + 2*b^5*c^3*d^9*e^3 + 2*b^6*c^2*d^8*e^4 - 3*b^7*c*d^7*e^5 + b^8*d^6*e^6)*x^3 + (b^3*c^5*d^12 - b^4*c^4*d^11*e - 6*b^5*c^3*d^10*e^2 + 14*b^6*c^2*d^9*e^3 - 11*b^7*c*d^8*e^4 + 3*b^8*d^7*e^5)*x^2 + (b^4*c^4*d^12 - 4*b^5*c^3*d^11*e + 6*b^6*c^2*d^10*e^2 - 4*b^7*c*d^9*e^3 + b^8*d^8*e^4)*x), -1/30*(30*((4*c^6*d^6*e^3 - 11*b*c^5*d^5*e^4)*x^5 + (12*c^6*d^7*e^2 - 29*b*c^5*d^6*e^3 - 11*b^2*c^4*d^5*e^4)*x^4 + 3*(4*c^6*d^8*e - 7*b*c^5*d^7*e^2 - 11*b^2*c^4*d^6*e^3)*x^3 + (4*c^6*d^9 + b*c^5*d^8*e - 33*b^2*c^4*d^7*e^2)*x^2 + (4*b*c^5*d^9 - 11*b^2*c^4*d^8*e)*x)*sqrt(-c/(c*d - b*e))*arctan(-(c*d - b*e)*sqrt(e*x + d)*sqrt(-c/(c*d - b*e))/(c*e*x + c*d)) - 15*((4*c^6*d^5*e^3 - 9*b*c^5*d^4*e^4 - 4*b^2*c^4*d^3*e^5 + 26*b^3*c^3*d^2*e^6 - 24*b^4*c^2*d*e^7 + 7*b^5*c*e^8)*x^5 + (12*c^6*d^6*e^2 - 23*b*c^5*d^5*e^3 - 21*b^2*c^4*d^4*e^4 + 74*b^3*c^3*d^3*e^5 - 46*b^4*c^2*d^2*e^6 - 3*b^5*c*d*e^7 + 7*b^6*e^8)*x^4 + 3*(4*c^6*d^7*e - 5*b*c^5*d^6*e^2 - 13*b^2*c^4*d^5*e^3 + 22*b^3*c^3*d^4*e^4 + 2*b^4*c^2*d^3*e^5 - 17*b^5*c*d^2*e^6 + 7*b^6*d*e^7)*x^3 + (4*c^6*d^8 + 3*b*c^5*d^7*e - 31*b^2*c^4*d^6*e^2 + 14*b^3*c^3*d^5*e^3 + 54*b^4*c^2*d^4*e^4 - 65*b^5*c*d^3*e^5 + 21*b^6*d^2*e^6)*x^2 + (4*b*c^5*d^8 - 9*b^2*c^4*d^7*e - 4*b^3*c^3*d^6*e^2 + 26*b^4*c^2*d^5*e^3 - 24*b^5*c*d^4*e^4 + 7*b^6*d^3*e^5)*x)*sqrt(d)*log((e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + 2*(15*b^2*c^4*d^8 - 60*b^3*c^3*d^7*e + 90*b^4*c^2*d^6*e^2 - 60*b^5*c*d^5*e^3 + 15*b^6*d^4*e^4 + 15*(2*b*c^5*d^5*e^3 - 4*b^2*c^4*d^4*e^4 + 26*b^3*c^3*d^3*e^5 - 24*b^4*c^2*d^2*e^6 + 7*b^5*c*d*e^7)*x^4 + 5*(18*b*c^5*d^6*e^2 - 33*b^2*c^4*d^5*e^3 + 170*b^3*c^3*d^4*e^4 - 90*b^4*c^2*d^3*e^5 - 23*b^5*c*d^2*e^6 + 21*b^6*d*e^7)*x^3 + (90*b*c^5*d^7*e - 135*b^2*c^4*d^6*e^2 + 436*b^3*c^3*d^5*e^3 + 358*b^4*c^2*d^4*e^4 - 679*b^5*c*d^3*e^5 + 245*b^6*d^2*e^6)*x^2 + (30*b*c^5*d^8 - 15*b^2*c^4*d^7*e - 90*b^3*c^3*d^6*e^2 + 556*b^4*c^2*d^5*e^3 - 537*b^5*c*d^4*e^4 + 161*b^6*d^3*e^5)*x)*sqrt(e*x + d))/((b^3*c^5*d^9*e^3 - 4*b^4*c^4*d^8*e^4 + 6*b^5*c^3*d^7*e^5 - 4*b^6*c^2*d^6*e^6 + b^7*c*d^5*e^7)*x^5 + (3*b^3*c^5*d^10*e^2 - 11*b^4*c^4*d^9*e^3 + 14*b^5*c^3*d^8*e^4 - 6*b^6*c^2*d^7*e^5 - b^7*c*d^6*e^6 + b^8*d^5*e^7)*x^4 + 3*(b^3*c^5*d^11*e - 3*b^4*c^4*d^10*e^2 + 2*b^5*c^3*d^9*e^3 + 2*b^6*c^2*d^8*e^4 - 3*b^7*c*d^7*e^5 + b^8*d^6*e^6)*x^3 + (b^3*c^5*d^12 - b^4*c^4*d^11*e - 6*b^5*c^3*d^10*e^2 + 14*b^6*c^2*d^9*e^3 - 11*b^7*c*d^8*e^4 + 3*b^8*d^7*e^5)*x^2 + (b^4*c^4*d^12 - 4*b^5*c^3*d^11*e + 6*b^6*c^2*d^10*e^2 - 4*b^7*c*d^9*e^3 + b^8*d^8*e^4)*x), -1/30*(30*((4*c^6*d^5*e^3 - 9*b*c^5*d^4*e^4 - 4*b^2*c^4*d^3*e^5 + 26*b^3*c^3*d^2*e^6 - 24*b^4*c^2*d*e^7 + 7*b^5*c*e^8)*x^5 + (12*c^6*d^6*e^2 - 23*b*c^5*d^5*e^3 - 21*b^2*c^4*d^4*e^4 + 74*b^3*c^3*d^3*e^5 - 46*b^4*c^2*d^2*e^6 - 3*b^5*c*d*e^7 + 7*b^6*e^8)*x^4 + 3*(4*c^6*d^7*e - 5*b*c^5*d^6*e^2 - 13*b^2*c^4*d^5*e^3 + 22*b^3*c^3*d^4*e^4 + 2*b^4*c^2*d^3*e^5 - 17*b^5*c*d^2*e^6 + 7*b^6*d*e^7)*x^3 + (4*c^6*d^8 + 3*b*c^5*d^7*e - 31*b^2*c^4*d^6*

```
e^2 + 14*b^3*c^3*d^5*e^3 + 54*b^4*c^2*d^4*e^4 - 65*b^5*c*d^3*e^5 + 21*b^6*d
^2*e^6)*x^2 + (4*b*c^5*d^8 - 9*b^2*c^4*d^7*e - 4*b^3*c^3*d^6*e^2 + 26*b^4*c
^2*d^5*e^3 - 24*b^5*c*d^4*e^4 + 7*b^6*d^3*e^5)*x)*sqrt(-d)*arctan(sqrt(e*x
+ d)*sqrt(-d)/d) + 15*((4*c^6*d^6*e^3 - 11*b*c^5*d^5*e^4)*x^5 + (12*c^6*d^7
*e^2 - 29*b*c^5*d^6*e^3 - 11*b^2*c^4*d^5*e^4)*x^4 + 3*(4*c^6*d^8*e - 7*b*c^
5*d^7*e^2 - 11*b^2*c^4*d^6*e^3)*x^3 + (4*c^6*d^9 + b*c^5*d^8*e - 33*b^2*c^4
*d^7*e^2)*x^2 + (4*b*c^5*d^9 - 11*b^2*c^4*d^8*e)*x)*sqrt(c/(c*d - b*e))*log
((c*e*x + 2*c*d - b*e + 2*(c*d - b*e)*sqrt(e*x + d)*sqrt(c/(c*d - b*e)))/(c
*x + b)) + 2*(15*b^2*c^4*d^8 - 60*b^3*c^3*d^7*e + 90*b^4*c^2*d^6*e^2 - 60*b
^5*c*d^5*e^3 + 15*b^6*d^4*e^4 + 15*(2*b*c^5*d^5*e^3 - 4*b^2*c^4*d^4*e^4 + 2
6*b^3*c^3*d^3*e^5 - 24*b^4*c^2*d^2*e^6 + 7*b^5*c*d*e^7)*x^4 + 5*(18*b*c^5*d
^6*e^2 - 33*b^2*c^4*d^5*e^3 + 170*b^3*c^3*d^4*e^4 - 90*b^4*c^2*d^3*e^5 - 23
*b^5*c*d^2*e^6 + 21*b^6*d*e^7)*x^3 + (90*b*c^5*d^7*e - 135*b^2*c^4*d^6*e^2
+ 436*b^3*c^3*d^5*e^3 + 358*b^4*c^2*d^4*e^4 - 679*b^5*c*d^3*e^5 + 245*b^6*d
^2*e^6)*x^2 + (30*b*c^5*d^8 - 15*b^2*c^4*d^7*e - 90*b^3*c^3*d^6*e^2 + 556*b
^4*c^2*d^5*e^3 - 537*b^5*c*d^4*e^4 + 161*b^6*d^3*e^5)*x)*sqrt(e*x + d))/((b
^3*c^5*d^9*e^3 - 4*b^4*c^4*d^8*e^4 + 6*b^5*c^3*d^7*e^5 - 4*b^6*c^2*d^6*e^6
+ b^7*c*d^5*e^7)*x^5 + (3*b^3*c^5*d^10*e^2 - 11*b^4*c^4*d^9*e^3 + 14*b^5*c^
3*d^8*e^4 - 6*b^6*c^2*d^7*e^5 - b^7*c*d^6*e^6 + b^8*d^5*e^7)*x^4 + 3*(b^3*c
^5*d^11*e - 3*b^4*c^4*d^10*e^2 + 2*b^5*c^3*d^9*e^3 + 2*b^6*c^2*d^8*e^4 - 3*
b^7*c*d^7*e^5 + b^8*d^6*e^6)*x^3 + (b^3*c^5*d^12 - b^4*c^4*d^11*e - 6*b^5*c
^3*d^10*e^2 + 14*b^6*c^2*d^9*e^3 - 11*b^7*c*d^8*e^4 + 3*b^8*d^7*e^5)*x^2 +
(b^4*c^4*d^12 - 4*b^5*c^3*d^11*e + 6*b^6*c^2*d^10*e^2 - 4*b^7*c*d^9*e^3 + b
^8*d^8*e^4)*x), -1/15*(15*((4*c^6*d^6*e^3 - 11*b*c^5*d^5*e^4)*x^5 + (12*c^6
*d^7*e^2 - 29*b*c^5*d^6*e^3 - 11*b^2*c^4*d^5*e^4)*x^4 + 3*(4*c^6*d^8*e - 7*
b*c^5*d^7*e^2 - 11*b^2*c^4*d^6*e^3)*x^3 + (4*c^6*d^9 + b*c^5*d^8*e - 33*b^2
*c^4*d^7*e^2)*x^2 + (4*b*c^5*d^9 - 11*b^2*c^4*d^8*e)*x)*sqrt(-c/(c*d - b*e)
)*arctan(-(c*d - b*e)*sqrt(e*x + d)*sqrt(-c/(c*d - b*e))/(c*e*x + c*d)) + 1
5*((4*c^6*d^5*e^3 - 9*b*c^5*d^4*e^4 - 4*b^2*c^4*d^3*e^5 + 26*b^3*c^3*d^2*e^
6 - 24*b^4*c^2*d*e^7 + 7*b^5*c*e^8)*x^5 + (12*c^6*d^6*e^2 - 23*b*c^5*d^5*e^
3 - 21*b^2*c^4*d^4*e^4 + 74*b^3*c^3*d^3*e^5 - 46*b^4*c^2*d^2*e^6 - 3*b^5*c*
d*e^7 + 7*b^6*e^8)*x^4 + 3*(4*c^6*d^7*e - 5*b*c^5*d^6*e^2 - 13*b^2*c^4*d^5*
e^3 + 22*b^3*c^3*d^4*e^4 + 2*b^4*c^2*d^3*e^5 - 17*b^5*c*d^2*e^6 + 7*b^6*d*e
^7)*x^3 + (4*c^6*d^8 + 3*b*c^5*d^7*e - 31*b^2*c^4*d^6*e^2 + 14*b^3*c^3*d^5*
e^3 + 54*b^4*c^2*d^4*e^4 - 65*b^5*c*d^3*e^5 + 21*b^6*d^2*e^6)*x^2 + (4*b*c^
5*d^8 - 9*b^2*c^4*d^7*e - 4*b^3*c^3*d^6*e^2 + 26*b^4*c^2*d^5*e^3 - 24*b^5*c
*d^4*e^4 + 7*b^6*d^3*e^5)*x)*sqrt(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d) + (1
5*b^2*c^4*d^8 - 60*b^3*c^3*d^7*e + 90*b^4*c^2*d^6*e^2 - 60*b^5*c*d^5*e^3 +
15*b^6*d^4*e^4 + 15*(2*b*c^5*d^5*e^3 - 4*b^2*c^4*d^4*e^4 + 26*b^3*c^3*d^3*e
^5 - 24*b^4*c^2*d^2*e^6 + 7*b^5*c*d*e^7)*x^4 + 5*(18*b*c^5*d^6*e^2 - 33*b^2
*c^4*d^5*e^3 + 170*b^3*c^3*d^4*e^4 - 90*b^4*c^2*d^3*e^5 - 23*b^5*c*d^2*e^6
+ 21*b^6*d*e^7)*x^3 + (90*b*c^5*d^7*e - 135*b^2*c^4*d^6*e^2 + 436*b^3*c^3*d
^5*e^3 + 358*b^4*c^2*d^4*e^4 - 679*b^5*c*d^3*e^5 + 245*b^6*d^2*e^6)*x^2 + (
30*b*c^5*d^8 - 15*b^2*c^4*d^7*e - 90*b^3*c^3*d^6*e^2 + 556*b^4*c^2*d^5*e^3
- 537*b^5*c*d^4*e^4 + 161*b^6*d^3*e^5)*x)*sqrt(e*x + d))/((b^3*c^5*d^9*e^3
- 4*b^4*c^4*d^8*e^4 + 6*b^5*c^3*d^7*e^5 - 4*b^6*c^2*d^6*e^6 + b^7*c*d^5*e^7
)*x^5 + (3*b^3*c^5*d^10*e^2 - 11*b^4*c^4*d^9*e^3 + 14*b^5*c^3*d^8*e^4 - 6*b
^6*c^2*d^7*e^5 - b^7*c*d^6*e^6 + b^8*d^5*e^7)*x^4 + 3*(b^3*c^5*d^11*e - 3*b
^4*c^4*d^10*e^2 + 2*b^5*c^3*d^9*e^3 + 2*b^6*c^2*d^8*e^4 - 3*b^7*c*d^7*e^5 +
 b^8*d^6*e^6)*x^3 + (b^3*c^5*d^12 - b^4*c^4*d^11*e - 6*b^5*c^3*d^10*e^2 + 1
4*b^6*c^2*d^9*e^3 - 11*b^7*c*d^8*e^4 + 3*b^8*d^7*e^5)*x^2 + (b^4*c^4*d^12 -
 4*b^5*c^3*d^11*e + 6*b^6*c^2*d^10*e^2 - 4*b^7*c*d^9*e^3 + b^8*d^8*e^4)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(7/2)/(c*x**2+b*x)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.40457, size = 868, normalized size = 2.49

$$\frac{\left(4\,c^6d - 11\,bc^5e\right)\arctan\left(\frac{\sqrt{xe+d}c}{\sqrt{-c^2d+bce}}\right)}{\left(b^3c^4d^4 - 4\,b^4c^3d^3e + 6\,b^5c^2d^2e^2 - 4\,b^6cde^3 + b^7e^4\right)\sqrt{-c^2d + bce}} - \frac{2\,(xe+d)^{\frac{3}{2}}c^5d^4e - 2\,\sqrt{xe+d}c^5d^5e - 4\,(xe+d)^{\frac{3}{2}}bc^4d^3e^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(c*x^2+b*x)^2,x, algorithm="giac")
```

```
[Out] (4*c^6*d - 11*b*c^5*e)*arctan(sqrt(x*e + d)*c/sqrt(-c^2*d + b*c*e))/((b^3*c
^4*d^4 - 4*b^4*c^3*d^3*e + 6*b^5*c^2*d^2*e^2 - 4*b^6*c*d*e^3 + b^7*e^4)*sqr
t(-c^2*d + b*c*e)) - (2*(x*e + d)^(3/2)*c^5*d^4*e - 2*sqrt(x*e + d)*c^5*d^5
*e - 4*(x*e + d)^(3/2)*b*c^4*d^3*e^2 + 5*sqrt(x*e + d)*b*c^4*d^4*e^2 + 6*(x
*e + d)^(3/2)*b^2*c^3*d^2*e^3 - 10*sqrt(x*e + d)*b^2*c^3*d^3*e^3 - 4*(x*e +
 d)^(3/2)*b^3*c^2*d*e^4 + 10*sqrt(x*e + d)*b^3*c^2*d^2*e^4 + (x*e + d)^(3/2
)*b^4*c*e^5 - 5*sqrt(x*e + d)*b^4*c*d*e^5 + sqrt(x*e + d)*b^5*e^6)/((b^2*c^
4*d^8 - 4*b^3*c^3*d^7*e + 6*b^4*c^2*d^6*e^2 - 4*b^5*c*d^5*e^3 + b^6*d^4*e^4
)*((x*e + d)^2*c - 2*(x*e + d)*c*d + c*d^2 + (x*e + d)*b*e - b*d*e)) - 2/15
*(150*(x*e + d)^2*c^2*d^2*e^3 + 20*(x*e + d)*c^2*d^3*e^3 + 3*c^2*d^4*e^3 -
150*(x*e + d)^2*b*c*d*e^4 - 30*(x*e + d)*b*c*d^2*e^4 - 6*b*c*d^3*e^4 + 45*(
x*e + d)^2*b^2*e^5 + 10*(x*e + d)*b^2*d*e^5 + 3*b^2*d^2*e^5)/((c^4*d^8 - 4*
b*c^3*d^7*e + 6*b^2*c^2*d^6*e^2 - 4*b^3*c*d^5*e^3 + b^4*d^4*e^4)*(x*e + d)^
(5/2)) - (4*c*d + 7*b*e)*arctan(sqrt(x*e + d)/sqrt(-d))/(b^3*sqrt(-d)*d^4)
```

3.378 $\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^3} dx$

Optimal. Leaf size=300

$$\frac{(d+ex)^{3/2}\left(x(2cd-be)\left(-b^2e^2-12bcde+12c^2d^2\right)+bcd^2(12cd-13be)\right)}{4b^4c\left(bx+cx^2\right)}-\frac{3e\sqrt{d+ex}(2cd-be)\left(-b^2e^2-4bcde+4c^2d\right.}{4b^4c^2}$$

```
[Out] (-3*e*(2*c*d - b*e)*(4*c^2*d^2 - 4*b*c*d*e - b^2*e^2)*Sqrt[d + e*x])/(4*b^4
*c^2) - ((d + e*x)^(7/2)*(b*d + (2*c*d - b*e)*x))/(2*b^2*(b*x + c*x^2)^2) +
 ((d + e*x)^(3/2)*(b*c*d^2*(12*c*d - 13*b*e) + (2*c*d - b*e)*(12*c^2*d^2 -
12*b*c*d*e - b^2*e^2)*x))/(4*b^4*c*(b*x + c*x^2)) - (3*d^(5/2)*(16*c^2*d^2
- 36*b*c*d*e + 21*b^2*e^2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(4*b^5) + (3*(c*
d - b*e)^(5/2)*(16*c^2*d^2 + 4*b*c*d*e + b^2*e^2)*ArcTanh[(Sqrt[c]*Sqrt[d +
 e*x])/Sqrt[c*d - b*e]])/(4*b^5*c^(5/2))
```

Rubi [A] time = 0.519851, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {738, 818, 824, 826, 1166, 208}

$$\frac{(d+ex)^{3/2}\left(x(2cd-be)\left(-b^2e^2-12bcde+12c^2d^2\right)+bcd^2(12cd-13be)\right)}{4b^4c\left(bx+cx^2\right)}-\frac{3e\sqrt{d+ex}(2cd-be)\left(-b^2e^2-4bcde+4c^2d\right.}{4b^4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(9/2)/(b*x + c*x^2)^3,x]
```


```
[Out] (-3*e*(2*c*d - b*e)*(4*c^2*d^2 - 4*b*c*d*e - b^2*e^2)*Sqrt[d + e*x])/(4*b^4
*c^2) - ((d + e*x)^(7/2)*(b*d + (2*c*d - b*e)*x))/(2*b^2*(b*x + c*x^2)^2) +
 ((d + e*x)^(3/2)*(b*c*d^2*(12*c*d - 13*b*e) + (2*c*d - b*e)*(12*c^2*d^2 -
12*b*c*d*e - b^2*e^2)*x))/(4*b^4*c*(b*x + c*x^2)) - (3*d^(5/2)*(16*c^2*d^2
- 36*b*c*d*e + 21*b^2*e^2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(4*b^5) + (3*(c*
d - b*e)^(5/2)*(16*c^2*d^2 + 4*b*c*d*e + b^2*e^2)*ArcTanh[(Sqrt[c]*Sqrt[d +
 e*x])/Sqrt[c*d - b*e]])/(4*b^5*c^(5/2))
```


Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
```

```
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^3}\,dx = -\frac{(d+ex)^{7/2}(bd+(2cd-be)x)}{2b^2(bx+cx^2)^2} - \frac{\int \frac{(d+ex)^{5/2}\left(\frac{1}{2}d(12cd-13be)-\frac{1}{2}e(2cd-be)x\right)}{(bx+cx^2)^2}\,dx}{2b^2}$$

$$= -\frac{(d+ex)^{7/2}(bd+(2cd-be)x)}{2b^2(bx+cx^2)^2} + \frac{(d+ex)^{3/2}\left(bcd^2(12cd-13be)+(2cd-be)\left(12c^2d^2-12bcde-b\right.\right.}{4b^4c(bx+cx^2)}$$

$$= -\frac{3e(2cd-be)\left(4c^2d^2-4bcde-b^2e^2\right)\sqrt{d+ex}}{4b^4c^2} - \frac{(d+ex)^{7/2}(bd+(2cd-be)x)}{2b^2(bx+cx^2)^2} + \frac{(d+ex)^{3/2}\left(bcd^2\right.}{}$$

$$= -\frac{3e(2cd-be)\left(4c^2d^2-4bcde-b^2e^2\right)\sqrt{d+ex}}{4b^4c^2} - \frac{(d+ex)^{7/2}(bd+(2cd-be)x)}{2b^2(bx+cx^2)^2} + \frac{(d+ex)^{3/2}\left(bcd^2\right.}{}$$

$$= -\frac{3e(2cd-be)\left(4c^2d^2-4bcde-b^2e^2\right)\sqrt{d+ex}}{4b^4c^2} - \frac{(d+ex)^{7/2}(bd+(2cd-be)x)}{2b^2(bx+cx^2)^2} + \frac{(d+ex)^{3/2}\left(bcd^2\right.}{}$$

$$= -\frac{3e(2cd-be)\left(4c^2d^2-4bcde-b^2e^2\right)\sqrt{d+ex}}{4b^4c^2} - \frac{(d+ex)^{7/2}(bd+(2cd-be)x)}{2b^2(bx+cx^2)^2} + \frac{(d+ex)^{3/2}\left(bcd^2\right.}{}$$

Mathematica [A] time = 0.641668, size = 275, normalized size = 0.92

$$\frac{\frac{b\sqrt{d+ex}\left(b^3c^2d\left(-17d^2ex-2d^3+33de^2x^2+3e^3x^3\right)+b^2c^3d^2x\left(8d^2-73dex+21e^2x^2\right)-5b^4ce^3x^2(d+ex)-3b^5e^4x^2+12bc^4d^3x^2(3d-4ex)+24c^5d^4x^3\right)}{c^2x^2(b+cx)^2} - 3d^{5/2}\left(21b^2e^2\right.}{4b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(9/2)/(b*x + c*x^2)^3,x]
```

```
[Out] ((b*Sqrt[d + e*x]*(-3*b^5*e^4*x^2 + 24*c^5*d^4*x^3 + 12*b*c^4*d^3*x^2*(3*d
- 4*e*x) - 5*b^4*c*e^3*x^2*(d + e*x) + b^2*c^3*d^2*x*(8*d^2 - 73*d*e*x + 21
*e^2*x^2) + b^3*c^2*d*(-2*d^3 - 17*d^2*e*x + 33*d*e^2*x^2 + 3*e^3*x^3)))/(c
^2*x^2*(b + c*x)^2) - 3*d^(5/2)*(16*c^2*d^2 - 36*b*c*d*e + 21*b^2*e^2)*ArcT
anh[Sqrt[d + e*x]/Sqrt[d]] + (3*(c*d - b*e)^(5/2)*(16*c^2*d^2 + 4*b*c*d*e +
 b^2*e^2)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/c^(5/2))/(4*b^5
)
```

Maple [B] time = 0.27, size = 703, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(9/2)/(c*x^2+b*x)^3,x)
```

```
[Out] -3*e/b^4/(c*e*x+b*e)^2*(e*x+d)^(1/2)*d^5*c^3+3/4*e^4/b/c/((b*e-c*d)*c)^(1/2
)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))*d+21/4*e^3/b^2/((b*e-c*d)*c)^
(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))*d^2-111/4*e^2/b^3*c/((b*e
```

```
-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))*d^3+33*e/b^4*c^2
/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))*d^4-12/b^5
/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))*d^5*c^3-31
/4*e^2/b^3/(c*e*x+b*e)^2*c^2*(e*x+d)^(3/2)*d^3+3*e/b^4/(c*e*x+b*e)^2*(e*x+d
)^(3/2)*d^4*c^3+15/2*e^4/b/(c*e*x+b*e)^2*(e*x+d)^(1/2)*d^2-15*e^3/b^2/(c*e*
x+b*e)^2*c*(e*x+d)^(1/2)*d^3+45/4*e^2/b^3/(c*e*x+b*e)^2*c^2*(e*x+d)^(1/2)*d
^4-5/4*e^5/(c*e*x+b*e)^2/c*(e*x+d)^(3/2)-3/4*e^6*b/(c*e*x+b*e)^2/c^2*(e*x+d
)^(1/2)+3/4*e^5/c^2/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c
)^(1/2))+3/4*e^4/b/(c*e*x+b*e)^2*(e*x+d)^(3/2)*d+21/4*e^3/b^2/(c*e*x+b*e)^2
*c*(e*x+d)^(3/2)*d^2-17/4*d^3/b^3/x^2*(e*x+d)^(3/2)+3/e*d^4/b^4/x^2*(e*x+d)
^(3/2)*c+15/4*d^4/b^3/x^2*(e*x+d)^(1/2)-3/e*d^5/b^4/x^2*(e*x+d)^(1/2)*c-63/
4*e^2*d^(5/2)/b^3*arctanh((e*x+d)^(1/2)/d^(1/2))+27*e*d^(7/2)/b^4*arctanh((
e*x+d)^(1/2)/d^(1/2))*c-12*d^(9/2)/b^5*arctanh((e*x+d)^(1/2)/d^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 12.6822, size = 4871, normalized size = 16.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*((16*c^6*d^4 - 28*b*c^5*d^3*e + 9*b^2*c^4*d^2*e^2 + 2*b^3*c^3*d*e^3
 + b^4*c^2*e^4)*x^4 + 2*(16*b*c^5*d^4 - 28*b^2*c^4*d^3*e + 9*b^3*c^3*d^2*e^
2 + 2*b^4*c^2*d*e^3 + b^5*c*e^4)*x^3 + (16*b^2*c^4*d^4 - 28*b^3*c^3*d^3*e +
 9*b^4*c^2*d^2*e^2 + 2*b^5*c*d*e^3 + b^6*e^4)*x^2)*sqrt((c*d - b*e)/c)*log(
(c*e*x + 2*c*d - b*e + 2*sqrt(e*x + d)*c*sqrt((c*d - b*e)/c))/(c*x + b)) +
3*((16*c^6*d^4 - 36*b*c^5*d^3*e + 21*b^2*c^4*d^2*e^2)*x^4 + 2*(16*b*c^5*d^4
 - 36*b^2*c^4*d^3*e + 21*b^3*c^3*d^2*e^2)*x^3 + (16*b^2*c^4*d^4 - 36*b^3*c^
3*d^3*e + 21*b^4*c^2*d^2*e^2)*x^2)*sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(
d) + 2*d)/x) - 2*(2*b^4*c^2*d^4 - (24*b*c^5*d^4 - 48*b^2*c^4*d^3*e + 21*b^3
*c^3*d^2*e^2 + 3*b^4*c^2*d*e^3 - 5*b^5*c*e^4)*x^3 - (36*b^2*c^4*d^4 - 73*b^
3*c^3*d^3*e + 33*b^4*c^2*d^2*e^2 - 5*b^5*c*d*e^3 - 3*b^6*e^4)*x^2 - (8*b^3*
c^3*d^4 - 17*b^4*c^2*d^3*e)*x)*sqrt(e*x + d))/(b^5*c^4*x^4 + 2*b^6*c^3*x^3
+ b^7*c^2*x^2), 1/8*(6*((16*c^6*d^4 - 28*b*c^5*d^3*e + 9*b^2*c^4*d^2*e^2 +
2*b^3*c^3*d*e^3 + b^4*c^2*e^4)*x^4 + 2*(16*b*c^5*d^4 - 28*b^2*c^4*d^3*e + 9
*b^3*c^3*d^2*e^2 + 2*b^4*c^2*d*e^3 + b^5*c*e^4)*x^3 + (16*b^2*c^4*d^4 - 28*
b^3*c^3*d^3*e + 9*b^4*c^2*d^2*e^2 + 2*b^5*c*d*e^3 + b^6*e^4)*x^2)*sqrt(-(c*
d - b*e)/c)*arctan(-sqrt(e*x + d)*c*sqrt(-(c*d - b*e)/c)/(c*d - b*e)) + 3*(
(16*c^6*d^4 - 36*b*c^5*d^3*e + 21*b^2*c^4*d^2*e^2)*x^4 + 2*(16*b*c^5*d^4 -
36*b^2*c^4*d^3*e + 21*b^3*c^3*d^2*e^2)*x^3 + (16*b^2*c^4*d^4 - 36*b^3*c^3*d
^3*e + 21*b^4*c^2*d^2*e^2)*x^2)*sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d)
+ 2*d)/x) - 2*(2*b^4*c^2*d^4 - (24*b*c^5*d^4 - 48*b^2*c^4*d^3*e + 21*b^3*c^
3*d^2*e^2 + 3*b^4*c^2*d*e^3 - 5*b^5*c*e^4)*x^3 - (36*b^2*c^4*d^4 - 73*b^3*c
^3*d^3*e + 33*b^4*c^2*d^2*e^2 - 5*b^5*c*d*e^3 - 3*b^6*e^4)*x^2 - (8*b^3*c^3
```

```
*d^4 - 17*b^4*c^2*d^3*e)*x)*sqrt(e*x + d))/(b^5*c^4*x^4 + 2*b^6*c^3*x^3 + b
^7*c^2*x^2), 1/8*(6*((16*c^6*d^4 - 36*b*c^5*d^3*e + 21*b^2*c^4*d^2*e^2)*x^4
 + 2*(16*b*c^5*d^4 - 36*b^2*c^4*d^3*e + 21*b^3*c^3*d^2*e^2)*x^3 + (16*b^2*c
^4*d^4 - 36*b^3*c^3*d^3*e + 21*b^4*c^2*d^2*e^2)*x^2)*sqrt(-d)*arctan(sqrt(e
*x + d)*sqrt(-d)/d) + 3*((16*c^6*d^4 - 28*b*c^5*d^3*e + 9*b^2*c^4*d^2*e^2 +
 2*b^3*c^3*d*e^3 + b^4*c^2*e^4)*x^4 + 2*(16*b*c^5*d^4 - 28*b^2*c^4*d^3*e +
9*b^3*c^3*d^2*e^2 + 2*b^4*c^2*d*e^3 + b^5*c*e^4)*x^3 + (16*b^2*c^4*d^4 - 28
*b^3*c^3*d^3*e + 9*b^4*c^2*d^2*e^2 + 2*b^5*c*d*e^3 + b^6*e^4)*x^2)*sqrt((c*
d - b*e)/c)*log((c*e*x + 2*c*d - b*e + 2*sqrt(e*x + d)*c*sqrt((c*d - b*e)/c
))/(c*x + b)) - 2*(2*b^4*c^2*d^4 - (24*b*c^5*d^4 - 48*b^2*c^4*d^3*e + 21*b^
3*c^3*d^2*e^2 + 3*b^4*c^2*d*e^3 - 5*b^5*c*e^4)*x^3 - (36*b^2*c^4*d^4 - 73*b
^3*c^3*d^3*e + 33*b^4*c^2*d^2*e^2 - 5*b^5*c*d*e^3 - 3*b^6*e^4)*x^2 - (8*b^3
*c^3*d^4 - 17*b^4*c^2*d^3*e)*x)*sqrt(e*x + d))/(b^5*c^4*x^4 + 2*b^6*c^3*x^3
 + b^7*c^2*x^2), 1/4*(3*((16*c^6*d^4 - 28*b*c^5*d^3*e + 9*b^2*c^4*d^2*e^2 +
 2*b^3*c^3*d*e^3 + b^4*c^2*e^4)*x^4 + 2*(16*b*c^5*d^4 - 28*b^2*c^4*d^3*e +
9*b^3*c^3*d^2*e^2 + 2*b^4*c^2*d*e^3 + b^5*c*e^4)*x^3 + (16*b^2*c^4*d^4 - 28
*b^3*c^3*d^3*e + 9*b^4*c^2*d^2*e^2 + 2*b^5*c*d*e^3 + b^6*e^4)*x^2)*sqrt(-(c
*d - b*e)/c)*arctan(-sqrt(e*x + d)*c*sqrt(-(c*d - b*e)/c)/(c*d - b*e)) + 3*
((16*c^6*d^4 - 36*b*c^5*d^3*e + 21*b^2*c^4*d^2*e^2)*x^4 + 2*(16*b*c^5*d^4 -
 36*b^2*c^4*d^3*e + 21*b^3*c^3*d^2*e^2)*x^3 + (16*b^2*c^4*d^4 - 36*b^3*c^3*
d^3*e + 21*b^4*c^2*d^2*e^2)*x^2)*sqrt(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d)
- (2*b^4*c^2*d^4 - (24*b*c^5*d^4 - 48*b^2*c^4*d^3*e + 21*b^3*c^3*d^2*e^2 +
3*b^4*c^2*d*e^3 - 5*b^5*c*e^4)*x^3 - (36*b^2*c^4*d^4 - 73*b^3*c^3*d^3*e + 3
3*b^4*c^2*d^2*e^2 - 5*b^5*c*d*e^3 - 3*b^6*e^4)*x^2 - (8*b^3*c^3*d^4 - 17*b^
4*c^2*d^3*e)*x)*sqrt(e*x + d))/(b^5*c^4*x^4 + 2*b^6*c^3*x^3 + b^7*c^2*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(9/2)/(c*x**2+b*x)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.36662, size = 852, normalized size = 2.84

$$\frac{3\left(16\,c^2d^5-36\,bcd^4e+21\,b^2d^3e^2\right)\arctan\left(\frac{\sqrt{xe+d}}{\sqrt{-d}}\right)}{4\,b^5\sqrt{-d}}-\frac{3\left(16\,c^5d^5-44\,bc^4d^4e+37\,b^2c^3d^3e^2-7\,b^3c^2d^2e^3-b^4cde^4-b^5e^5\right)}{4\,\sqrt{-c^2d+bce}b^5c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(c*x^2+b*x)^3,x, algorithm="giac")
```

```
[Out] 3/4*(16*c^2*d^5 - 36*b*c*d^4*e + 21*b^2*d^3*e^2)*arctan(sqrt(x*e + d)/sqrt(
-d))/(b^5*sqrt(-d)) - 3/4*(16*c^5*d^5 - 44*b*c^4*d^4*e + 37*b^2*c^3*d^3*e^2
 - 7*b^3*c^2*d^2*e^3 - b^4*c*d*e^4 - b^5*e^5)*arctan(sqrt(x*e + d)*c/sqrt(-
c^2*d + b*c*e))/(sqrt(-c^2*d + b*c*e)*b^5*c^2) + 1/4*(24*(x*e + d)^(7/2)*c^
5*d^4*e - 72*(x*e + d)^(5/2)*c^5*d^5*e + 72*(x*e + d)^(3/2)*c^5*d^6*e - 24*
sqrt(x*e + d)*c^5*d^7*e - 48*(x*e + d)^(7/2)*b*c^4*d^3*e^2 + 180*(x*e + d)^
(5/2)*b*c^4*d^4*e^2 - 216*(x*e + d)^(3/2)*b*c^4*d^5*e^2 + 84*sqrt(x*e + d)*
b*c^4*d^6*e^2 + 21*(x*e + d)^(7/2)*b^2*c^3*d^2*e^3 - 136*(x*e + d)^(5/2)*b^
```

```
2*c^3*d^3*e^3 + 217*(x*e + d)^(3/2)*b^2*c^3*d^4*e^3 - 102*sqrt(x*e + d)*b^2
*c^3*d^5*e^3 + 3*(x*e + d)^(7/2)*b^3*c^2*d*e^4 + 24*(x*e + d)^(5/2)*b^3*c^2
*d^2*e^4 - 74*(x*e + d)^(3/2)*b^3*c^2*d^3*e^4 + 45*sqrt(x*e + d)*b^3*c^2*d^
4*e^4 - 5*(x*e + d)^(7/2)*b^4*c*e^5 + 10*(x*e + d)^(5/2)*b^4*c*d*e^5 - 5*(x
*e + d)^(3/2)*b^4*c*d^2*e^5 - 3*(x*e + d)^(5/2)*b^5*e^6 + 6*(x*e + d)^(3/2)
*b^5*d*e^6 - 3*sqrt(x*e + d)*b^5*d^2*e^6)/(((x*e + d)^2*c - 2*(x*e + d)*c*d
 + c*d^2 + (x*e + d)*b*e - b*d*e)^2*b^4*c^2)
```

3.379 $\int \frac{(d+ex)^{7/2}}{\left(bx+cx^2\right)^3}\,dx$

Optimal. Leaf size=248

$$\frac{\sqrt{d+ex}\left(x(2cd-be)\left(b^2e^2-12bcde+12c^2d^2\right)+bcd^2(12cd-11be)\right)}{4b^4c\left(bx+cx^2\right)}-\frac{d^{3/2}\left(35b^2e^2-84bcde+48c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4b^5}$$

```
[Out] -((d + e*x)^(5/2)*(b*d + (2*c*d - b*e)*x))/(2*b^2*(b*x + c*x^2)^2) + (Sqrt[
d + e*x]*(b*c*d^2*(12*c*d - 11*b*e) + (2*c*d - b*e)*(12*c^2*d^2 - 12*b*c*d*
e + b^2*e^2)*x))/(4*b^4*c*(b*x + c*x^2)) - (d^(3/2)*(48*c^2*d^2 - 84*b*c*d*
e + 35*b^2*e^2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(4*b^5) + ((c*d - b*e)^(3/2
)*(48*c^2*d^2 - 12*b*c*d*e - b^2*e^2)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[
c*d - b*e]])/(4*b^5*c^(3/2))
```

Rubi [A] time = 0.386262, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {738, 818, 826, 1166, 208}

$$\frac{\sqrt{d+ex}\left(x(2cd-be)\left(b^2e^2-12bcde+12c^2d^2\right)+bcd^2(12cd-11be)\right)}{4b^4c\left(bx+cx^2\right)}-\frac{d^{3/2}\left(35b^2e^2-84bcde+48c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/(b*x + c*x^2)^3,x]
```


```
[Out] -((d + e*x)^(5/2)*(b*d + (2*c*d - b*e)*x))/(2*b^2*(b*x + c*x^2)^2) + (Sqrt[
d + e*x]*(b*c*d^2*(12*c*d - 11*b*e) + (2*c*d - b*e)*(12*c^2*d^2 - 12*b*c*d*
e + b^2*e^2)*x))/(4*b^4*c*(b*x + c*x^2)) - (d^(3/2)*(48*c^2*d^2 - 84*b*c*d*
e + 35*b^2*e^2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(4*b^5) + ((c*d - b*e)^(3/2
)*(48*c^2*d^2 - 12*b*c*d*e - b^2*e^2)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[
c*d - b*e]])/(4*b^5*c^(3/2))
```


Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
```

```
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{7/2}}{\left(bx+cx^2\right)^3}\,dx = -\frac{(d+ex)^{5/2}(bd+(2cd-be)x)}{2b^2\left(bx+cx^2\right)^2} - \frac{\int \frac{(d+ex)^{3/2}\left(\frac{1}{2}d(12cd-11be)+\frac{1}{2}e(2cd-be)x\right)}{\left(bx+cx^2\right)^2}\,dx}{2b^2}$$

$$= -\frac{(d+ex)^{5/2}(bd+(2cd-be)x)}{2b^2\left(bx+cx^2\right)^2} + \frac{\sqrt{d+ex}\left(bcd^2(12cd-11be)+(2cd-be)\left(12c^2d^2-12bcde+b^2e^2\right)x\right.}{4b^4c\left(bx+cx^2\right)}$$

$$= -\frac{(d+ex)^{5/2}(bd+(2cd-be)x)}{2b^2\left(bx+cx^2\right)^2} + \frac{\sqrt{d+ex}\left(bcd^2(12cd-11be)+(2cd-be)\left(12c^2d^2-12bcde+b^2e^2\right)x\right.}{4b^4c\left(bx+cx^2\right)}$$

$$= -\frac{(d+ex)^{5/2}(bd+(2cd-be)x)}{2b^2\left(bx+cx^2\right)^2} + \frac{\sqrt{d+ex}\left(bcd^2(12cd-11be)+(2cd-be)\left(12c^2d^2-12bcde+b^2e^2\right)x\right.}{4b^4c\left(bx+cx^2\right)}$$

$$= -\frac{(d+ex)^{5/2}(bd+(2cd-be)x)}{2b^2\left(bx+cx^2\right)^2} + \frac{\sqrt{d+ex}\left(bcd^2(12cd-11be)+(2cd-be)\left(12c^2d^2-12bcde+b^2e^2\right)x\right.}{4b^4c\left(bx+cx^2\right)}$$

Mathematica [A] time = 0.501917, size = 263, normalized size = 1.06

$$\frac{\frac{b\sqrt{d+ex}\left(b^2c^2dx\left(8d^2-55dex+10e^2x^2\right)+b^3c\left(-13d^2ex-2d^3+16de^2x^2+e^3x^3\right)+b^4\left(-e^3\right)x^2+36bc^3d^2x^2(d-ex)+24c^4d^3x^3\right)}{cx^2(b+cx)^2} + d^{3/2}\left(-\left(35b^2e^2-84bcde+48c^2\right.\right.}{4b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)/(b*x + c*x^2)^3,x]
```

```
[Out] ((b*Sqrt[d + e*x]*(-(b^4*e^3*x^2) + 24*c^4*d^3*x^3 + 36*b*c^3*d^2*x^2*(d -
e*x) + b^2*c^2*d*x*(8*d^2 - 55*d*e*x + 10*e^2*x^2) + b^3*c*(-2*d^3 - 13*d^2
*e*x + 16*d*e^2*x^2 + e^3*x^3)))/(c*x^2*(b + c*x)^2) - d^(3/2)*(48*c^2*d^2
- 84*b*c*d*e + 35*b^2*e^2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] + (Sqrt[c*d - b*e
]*(48*c^3*d^3 - 60*b*c^2*d^2*e + 11*b^2*c*d*e^2 + b^3*e^3)*ArcTanh[(Sqrt[c]
*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/c^(3/2))/(4*b^5)
```

Maple [B] time = 0.226, size = 627, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/(c*x^2+b*x)^3,x)
```

```
[Out] 1/4*e^4/b/(c*e*x+b*e)^2*(e*x+d)^(3/2)+5/2*e^3/b^2/(c*e*x+b*e)^2*(e*x+d)^(3/
2)*c*d-23/4*e^2/b^3/(c*e*x+b*e)^2*(e*x+d)^(3/2)*c^2*d^2+3*e/b^4/(c*e*x+b*e)
^2*(e*x+d)^(3/2)*c^3*d^3-1/4*e^5/(c*e*x+b*e)^2/c*(e*x+d)^(1/2)+15/4*e^4/b/(
c*e*x+b*e)^2*(e*x+d)^(1/2)*d-39/4*e^3/b^2/(c*e*x+b*e)^2*c*(e*x+d)^(1/2)*d^2
+37/4*e^2/b^3/(c*e*x+b*e)^2*(e*x+d)^(1/2)*d^3*c^2-3*e/b^4/(c*e*x+b*e)^2*c^3
*(e*x+d)^(1/2)*d^4+1/4*e^4/b/c/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/(
(b*e-c*d)*c)^(1/2))+5/2*e^3/b^2/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/
((b*e-c*d)*c)^(1/2))*d-71/4*e^2/b^3*c/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1
/2)*c/((b*e-c*d)*c)^(1/2))*d^2+27*e/b^4/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^
(1/2)*c/((b*e-c*d)*c)^(1/2))*d^3*c^2-12/b^5*c^3/((b*e-c*d)*c)^(1/2)*arctan(
(e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))*d^4-13/4*d^2/b^3/x^2*(e*x+d)^(3/2)+3/e
*d^3/b^4/x^2*(e*x+d)^(3/2)*c+11/4*d^3/b^3/x^2*(e*x+d)^(1/2)-3/e*d^4/b^4/x^2
*c*(e*x+d)^(1/2)-35/4*e^2*d^(3/2)/b^3*arctanh((e*x+d)^(1/2)/d^(1/2))+21*e*d
^(5/2)/b^4*arctanh((e*x+d)^(1/2)/d^(1/2))*c-12*d^(7/2)/b^5*arctanh((e*x+d)^
(1/2)/d^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.15726, size = 4234, normalized size = 17.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(((48*c^5*d^3 - 60*b*c^4*d^2*e + 11*b^2*c^3*d*e^2 + b^3*c^2*e^3)*x^4 +
 2*(48*b*c^4*d^3 - 60*b^2*c^3*d^2*e + 11*b^3*c^2*d*e^2 + b^4*c*e^3)*x^3 + (
48*b^2*c^3*d^3 - 60*b^3*c^2*d^2*e + 11*b^4*c*d*e^2 + b^5*e^3)*x^2)*sqrt((c*
d - b*e)/c)*log((c*e*x + 2*c*d - b*e + 2*sqrt(e*x + d)*c*sqrt((c*d - b*e)/c
))/(c*x + b)) + ((48*c^5*d^3 - 84*b*c^4*d^2*e + 35*b^2*c^3*d*e^2)*x^4 + 2*(
48*b*c^4*d^3 - 84*b^2*c^3*d^2*e + 35*b^3*c^2*d*e^2)*x^3 + (48*b^2*c^3*d^3 -
 84*b^3*c^2*d^2*e + 35*b^4*c*d*e^2)*x^2)*sqrt(d)*log((e*x - 2*sqrt(e*x + d)
*sqrt(d) + 2*d)/x) - 2*(2*b^4*c*d^3 - (24*b*c^4*d^3 - 36*b^2*c^3*d^2*e + 10
*b^3*c^2*d*e^2 + b^4*c*e^3)*x^3 - (36*b^2*c^3*d^3 - 55*b^3*c^2*d^2*e + 16*b
^4*c*d*e^2 - b^5*e^3)*x^2 - (8*b^3*c^2*d^3 - 13*b^4*c*d^2*e)*x)*sqrt(e*x +
d))/(b^5*c^3*x^4 + 2*b^6*c^2*x^3 + b^7*c*x^2), 1/8*(2*((48*c^5*d^3 - 60*b*c
^4*d^2*e + 11*b^2*c^3*d*e^2 + b^3*c^2*e^3)*x^4 + 2*(48*b*c^4*d^3 - 60*b^2*c
^3*d^2*e + 11*b^3*c^2*d*e^2 + b^4*c*e^3)*x^3 + (48*b^2*c^3*d^3 - 60*b^3*c^2
*d^2*e + 11*b^4*c*d*e^2 + b^5*e^3)*x^2)*sqrt(-(c*d - b*e)/c)*arctan(-sqrt(e
*x + d)*c*sqrt(-(c*d - b*e)/c)/(c*d - b*e)) + ((48*c^5*d^3 - 84*b*c^4*d^2*e
 + 35*b^2*c^3*d*e^2)*x^4 + 2*(48*b*c^4*d^3 - 84*b^2*c^3*d^2*e + 35*b^3*c^2*
d*e^2)*x^3 + (48*b^2*c^3*d^3 - 84*b^3*c^2*d^2*e + 35*b^4*c*d*e^2)*x^2)*sqrt
(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) - 2*(2*b^4*c*d^3 - (24*b*c
^4*d^3 - 36*b^2*c^3*d^2*e + 10*b^3*c^2*d*e^2 + b^4*c*e^3)*x^3 - (36*b^2*c^3
*d^3 - 55*b^3*c^2*d^2*e + 16*b^4*c*d*e^2 - b^5*e^3)*x^2 - (8*b^3*c^2*d^3 -
13*b^4*c*d^2*e)*x)*sqrt(e*x + d))/(b^5*c^3*x^4 + 2*b^6*c^2*x^3 + b^7*c*x^2)
, 1/8*(2*((48*c^5*d^3 - 84*b*c^4*d^2*e + 35*b^2*c^3*d*e^2)*x^4 + 2*(48*b*c^
4*d^3 - 84*b^2*c^3*d^2*e + 35*b^3*c^2*d*e^2)*x^3 + (48*b^2*c^3*d^3 - 84*b^3
*c^2*d^2*e + 35*b^4*c*d*e^2)*x^2)*sqrt(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d)
 + ((48*c^5*d^3 - 60*b*c^4*d^2*e + 11*b^2*c^3*d*e^2 + b^3*c^2*e^3)*x^4 + 2*
(48*b*c^4*d^3 - 60*b^2*c^3*d^2*e + 11*b^3*c^2*d*e^2 + b^4*c*e^3)*x^3 + (48*
b^2*c^3*d^3 - 60*b^3*c^2*d^2*e + 11*b^4*c*d*e^2 + b^5*e^3)*x^2)*sqrt((c*d -
 b*e)/c)*log((c*e*x + 2*c*d - b*e + 2*sqrt(e*x + d)*c*sqrt((c*d - b*e)/c))/
(c*x + b)) - 2*(2*b^4*c*d^3 - (24*b*c^4*d^3 - 36*b^2*c^3*d^2*e + 10*b^3*c^2
*d*e^2 + b^4*c*e^3)*x^3 - (36*b^2*c^3*d^3 - 55*b^3*c^2*d^2*e + 16*b^4*c*d*e
^2 - b^5*e^3)*x^2 - (8*b^3*c^2*d^3 - 13*b^4*c*d^2*e)*x)*sqrt(e*x + d))/(b^5
*c^3*x^4 + 2*b^6*c^2*x^3 + b^7*c*x^2), 1/4*(((48*c^5*d^3 - 60*b*c^4*d^2*e +
 11*b^2*c^3*d*e^2 + b^3*c^2*e^3)*x^4 + 2*(48*b*c^4*d^3 - 60*b^2*c^3*d^2*e +
 11*b^3*c^2*d*e^2 + b^4*c*e^3)*x^3 + (48*b^2*c^3*d^3 - 60*b^3*c^2*d^2*e + 1
1*b^4*c*d*e^2 + b^5*e^3)*x^2)*sqrt(-(c*d - b*e)/c)*arctan(-sqrt(e*x + d)*c*
sqrt(-(c*d - b*e)/c)/(c*d - b*e)) + ((48*c^5*d^3 - 84*b*c^4*d^2*e + 35*b^2*
c^3*d*e^2)*x^4 + 2*(48*b*c^4*d^3 - 84*b^2*c^3*d^2*e + 35*b^3*c^2*d*e^2)*x^3
 + (48*b^2*c^3*d^3 - 84*b^3*c^2*d^2*e + 35*b^4*c*d*e^2)*x^2)*sqrt(-d)*arcta
n(sqrt(e*x + d)*sqrt(-d)/d) - (2*b^4*c*d^3 - (24*b*c^4*d^3 - 36*b^2*c^3*d^2
*e + 10*b^3*c^2*d*e^2 + b^4*c*e^3)*x^3 - (36*b^2*c^3*d^3 - 55*b^3*c^2*d^2*e
 + 16*b^4*c*d*e^2 - b^5*e^3)*x^2 - (8*b^3*c^2*d^3 - 13*b^4*c*d^2*e)*x)*sqrt
(e*x + d))/(b^5*c^3*x^4 + 2*b^6*c^2*x^3 + b^7*c*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/(c*x**2+b*x)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.52494, size = 745, normalized size = 3.

$$\frac{\left(48\,c^2d^4-84\,bcd^3e+35\,b^2d^2e^2\right)\arctan\left(\frac{\sqrt{xe+d}}{\sqrt{-d}}\right)}{4\,b^5\sqrt{-d}}-\frac{\left(48\,c^4d^4-108\,bc^3d^3e+71\,b^2c^2d^2e^2-10\,b^3cde^3-b^4e^4\right)\arctan\left(\frac{\sqrt{}}{\sqrt{-}}\right.}{4\,\sqrt{-c^2d+bce}b^5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x)^3,x, algorithm="giac")
```

```
[Out] 1/4*(48*c^2*d^4 - 84*b*c*d^3*e + 35*b^2*d^2*e^2)*arctan(sqrt(x*e + d)/sqrt(
-d))/(b^5*sqrt(-d)) - 1/4*(48*c^4*d^4 - 108*b*c^3*d^3*e + 71*b^2*c^2*d^2*e^
2 - 10*b^3*c*d*e^3 - b^4*e^4)*arctan(sqrt(x*e + d)*c/sqrt(-c^2*d + b*c*e))/
(sqrt(-c^2*d + b*c*e)*b^5*c) + 1/4*(24*(x*e + d)^(7/2)*c^4*d^3*e - 72*(x*e
+ d)^(5/2)*c^4*d^4*e + 72*(x*e + d)^(3/2)*c^4*d^5*e - 24*sqrt(x*e + d)*c^4*
d^6*e - 36*(x*e + d)^(7/2)*b*c^3*d^2*e^2 + 144*(x*e + d)^(5/2)*b*c^3*d^3*e^
2 - 180*(x*e + d)^(3/2)*b*c^3*d^4*e^2 + 72*sqrt(x*e + d)*b*c^3*d^5*e^2 + 10
*(x*e + d)^(7/2)*b^2*c^2*d*e^3 - 85*(x*e + d)^(5/2)*b^2*c^2*d^2*e^3 + 148*(
x*e + d)^(3/2)*b^2*c^2*d^3*e^3 - 73*sqrt(x*e + d)*b^2*c^2*d^4*e^3 + (x*e +
d)^(7/2)*b^3*c*e^4 + 13*(x*e + d)^(5/2)*b^3*c*d*e^4 - 42*(x*e + d)^(3/2)*b^
3*c*d^2*e^4 + 26*sqrt(x*e + d)*b^3*c*d^3*e^4 - (x*e + d)^(5/2)*b^4*e^5 + 2*
(x*e + d)^(3/2)*b^4*d*e^5 - sqrt(x*e + d)*b^4*d^2*e^5)/(((x*e + d)^2*c - 2*
(x*e + d)*c*d + c*d^2 + (x*e + d)*b*e - b*d*e)^2*b^4*c)
```

3.380 $\int \frac{(d+ex)^{5/2}}{\left(bx+cx^2\right)^3} dx$

Optimal. Leaf size=232

$$\frac{3\sqrt{d+ex}\left(x\left(b^2e^2-8bcde+8c^2d^2\right)+bd(4cd-3be)\right)}{4b^4\left(bx+cx^2\right)}-\frac{3\sqrt{d}\left(5b^2e^2-20bcde+16c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4b^5}+\frac{3\sqrt{cd-be}\left(b^2e\right.}{}$$

```
[Out] -((d + e*x)^(3/2)*(b*d + (2*c*d - b*e)*x))/(2*b^2*(b*x + c*x^2)^2) + (3*Sqr
t[d + e*x]*(b*d*(4*c*d - 3*b*e) + (8*c^2*d^2 - 8*b*c*d*e + b^2*e^2)*x))/(4*
b^4*(b*x + c*x^2)) - (3*Sqrt[d]*(16*c^2*d^2 - 20*b*c*d*e + 5*b^2*e^2)*ArcTa
nh[Sqrt[d + e*x]/Sqrt[d]])/(4*b^5) + (3*Sqrt[c*d - b*e]*(16*c^2*d^2 - 12*b*
c*d*e + b^2*e^2)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(4*b^5*S
qrt[c])
```

Rubi [A] time = 0.307329, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {738, 820, 826, 1166, 208}

$$\frac{3\sqrt{d+ex}\left(x\left(b^2e^2-8bcde+8c^2d^2\right)+bd(4cd-3be)\right)}{4b^4\left(bx+cx^2\right)}-\frac{3\sqrt{d}\left(5b^2e^2-20bcde+16c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4b^5}+\frac{3\sqrt{cd-be}\left(b^2e\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(b*x + c*x^2)^3,x]
```


```
[Out] -((d + e*x)^(3/2)*(b*d + (2*c*d - b*e)*x))/(2*b^2*(b*x + c*x^2)^2) + (3*Sqr
t[d + e*x]*(b*d*(4*c*d - 3*b*e) + (8*c^2*d^2 - 8*b*c*d*e + b^2*e^2)*x))/(4*
b^4*(b*x + c*x^2)) - (3*Sqrt[d]*(16*c^2*d^2 - 20*b*c*d*e + 5*b^2*e^2)*ArcTa
nh[Sqrt[d + e*x]/Sqrt[d]])/(4*b^5) + (3*Sqrt[c*d - b*e]*(16*c^2*d^2 - 12*b*
c*d*e + b^2*e^2)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(4*b^5*S
qrt[c])
```


Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*
(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p +
 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*Simp[g*
(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(
m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (I
```

```
ntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\left(bx+cx^2\right)^3}\,dx = -\frac{(d+ex)^{3/2}(bd+(2cd-be)x)}{2b^2\left(bx+cx^2\right)^2} - \frac{\int \frac{\sqrt{d+ex}\left(\frac{3}{2}d(4cd-3be)+\frac{3}{2}e(2cd-be)x\right)}{\left(bx+cx^2\right)^2}\,dx}{2b^2}$$

$$= -\frac{(d+ex)^{3/2}(bd+(2cd-be)x)}{2b^2\left(bx+cx^2\right)^2} + \frac{3\sqrt{d+ex}\left(bd(4cd-3be)+\left(8c^2d^2-8bcde+b^2e^2\right)x\right)}{4b^4\left(bx+cx^2\right)} + \frac{\int \frac{\frac{3}{4}d(16c}{}}{}$$

$$= -\frac{(d+ex)^{3/2}(bd+(2cd-be)x)}{2b^2\left(bx+cx^2\right)^2} + \frac{3\sqrt{d+ex}\left(bd(4cd-3be)+\left(8c^2d^2-8bcde+b^2e^2\right)x\right)}{4b^4\left(bx+cx^2\right)} + \frac{\text{Subst}\Big(}{}$$

$$= -\frac{(d+ex)^{3/2}(bd+(2cd-be)x)}{2b^2\left(bx+cx^2\right)^2} + \frac{3\sqrt{d+ex}\left(bd(4cd-3be)+\left(8c^2d^2-8bcde+b^2e^2\right)x\right)}{4b^4\left(bx+cx^2\right)} - \frac{\big(3(cd-}{}$$

$$= -\frac{(d+ex)^{3/2}(bd+(2cd-be)x)}{2b^2\left(bx+cx^2\right)^2} + \frac{3\sqrt{d+ex}\left(bd(4cd-3be)+\left(8c^2d^2-8bcde+b^2e^2\right)x\right)}{4b^4\left(bx+cx^2\right)} - \frac{3\sqrt{d}\,(16}{}$$

Mathematica [A] time = 0.426926, size = 222, normalized size = 0.96

$$\frac{\frac{b\sqrt{d+ex}\left(b^2cx\left(8d^2-37dex+3e^2x^2\right)+b^3\left(-2d^2-9dex+5e^2x^2\right)+12bc^2dx^2(3d-2ex)+24c^3d^2x^3\right)}{x^2(b+cx)^2} - 3\sqrt{d}\left(5b^2e^2-20bcde+16c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(b*x + c*x^2)^3,x]
```

```
[Out] ((b*Sqrt[d + e*x]*(24*c^3*d^2*x^3 + 12*b*c^2*d*x^2*(3*d - 2*e*x) + b^2*c*x*
(8*d^2 - 37*d*e*x + 3*e^2*x^2) + b^3*(-2*d^2 - 9*d*e*x + 5*e^2*x^2)))/(x^2*
(b + c*x)^2) - 3*Sqrt[d]*(16*c^2*d^2 - 20*b*c*d*e + 5*b^2*e^2)*ArcTanh[Sqrt
[d + e*x]/Sqrt[d]] + (3*Sqrt[c*d - b*e]*(16*c^2*d^2 - 12*b*c*d*e + b^2*e^2)
*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/Sqrt[c])/(4*b^5)
```

Maple [B] time = 0.244, size = 521, normalized size = 2.3

$$\frac{3e^3c}{4b^2(cex+be)^2}(ex+d)^{\frac{3}{2}} - \frac{15e^2c^2d}{4b^3(cex+be)^2}(ex+d)^{\frac{3}{2}} + 3\frac{e(ex+d)^{3/2}d^2c^3}{b^4(cex+be)^2} + \frac{5e^4}{4b(cex+be)^2}\sqrt{ex+d} - \frac{11e^3cd}{2b^2(cex+be)^2}\sqrt{e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(c*x^2+b*x)^3,x)
```

```
[Out] 3/4*e^3/b^2/(c*e*x+b*e)^2*(e*x+d)^(3/2)*c-15/4*e^2/b^3/(c*e*x+b*e)^2*(e*x+d
)^(3/2)*c^2*d+3*e/b^4/(c*e*x+b*e)^2*(e*x+d)^(3/2)*d^2*c^3+5/4*e^4/b/(c*e*x+
b*e)^2*(e*x+d)^(1/2)-11/2*e^3/b^2/(c*e*x+b*e)^2*(e*x+d)^(1/2)*c*d+29/4*e^2/
b^3/(c*e*x+b*e)^2*(e*x+d)^(1/2)*c^2*d^2-3*e/b^4/(c*e*x+b*e)^2*(e*x+d)^(1/2)
*c^3*d^3+3/4*e^3/b^2/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*
c)^(1/2))-39/4*e^2/b^3/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d
)*c)^(1/2))*c*d+21*e/b^4/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c
*d)*c)^(1/2))*c^2*d^2-12/b^5/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b
*e-c*d)*c)^(1/2))*c^3*d^3-9/4*d/b^3/x^2*(e*x+d)^(3/2)+3/e*d^2/b^4/x^2*(e*x+
d)^(3/2)*c+7/4*d^2/b^3/x^2*(e*x+d)^(1/2)-3/e*d^3/b^4/x^2*(e*x+d)^(1/2)*c-15
/4*e^2*d^(1/2)/b^3*arctanh((e*x+d)^(1/2)/d^(1/2))+15*e*d^(3/2)/b^4*arctanh(
(e*x+d)^(1/2)/d^(1/2))*c-12*d^(5/2)/b^5*arctanh((e*x+d)^(1/2)/d^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.91168, size = 3510, normalized size = 15.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*((16*c^4*d^2 - 12*b*c^3*d*e + b^2*c^2*e^2)*x^4 + 2*(16*b*c^3*d^2 -
12*b^2*c^2*d*e + b^3*c*e^2)*x^3 + (16*b^2*c^2*d^2 - 12*b^3*c*d*e + b^4*e^2)
*x^2)*sqrt((c*d - b*e)/c)*log((c*e*x + 2*c*d - b*e + 2*sqrt(e*x + d)*c*sqrt
((c*d - b*e)/c))/(c*x + b)) + 3*((16*c^4*d^2 - 20*b*c^3*d*e + 5*b^2*c^2*e^2
)*x^4 + 2*(16*b*c^3*d^2 - 20*b^2*c^2*d*e + 5*b^3*c*e^2)*x^3 + (16*b^2*c^2*d
^2 - 20*b^3*c*d*e + 5*b^4*e^2)*x^2)*sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt
```

```
(d) + 2*d)/x) - 2*(2*b^4*d^2 - 3*(8*b*c^3*d^2 - 8*b^2*c^2*d*e + b^3*c*e^2)*
x^3 - (36*b^2*c^2*d^2 - 37*b^3*c*d*e + 5*b^4*e^2)*x^2 - (8*b^3*c*d^2 - 9*b^
4*d*e)*x)*sqrt(e*x + d))/(b^5*c^2*x^4 + 2*b^6*c*x^3 + b^7*x^2), 1/8*(6*((16
*c^4*d^2 - 12*b*c^3*d*e + b^2*c^2*e^2)*x^4 + 2*(16*b*c^3*d^2 - 12*b^2*c^2*d
*e + b^3*c*e^2)*x^3 + (16*b^2*c^2*d^2 - 12*b^3*c*d*e + b^4*e^2)*x^2)*sqrt(-
(c*d - b*e)/c)*arctan(-sqrt(e*x + d)*c*sqrt(-(c*d - b*e)/c)/(c*d - b*e)) +
3*((16*c^4*d^2 - 20*b*c^3*d*e + 5*b^2*c^2*e^2)*x^4 + 2*(16*b*c^3*d^2 - 20*b
^2*c^2*d*e + 5*b^3*c*e^2)*x^3 + (16*b^2*c^2*d^2 - 20*b^3*c*d*e + 5*b^4*e^2)
*x^2)*sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) - 2*(2*b^4*d^2 -
 3*(8*b*c^3*d^2 - 8*b^2*c^2*d*e + b^3*c*e^2)*x^3 - (36*b^2*c^2*d^2 - 37*b^3
*c*d*e + 5*b^4*e^2)*x^2 - (8*b^3*c*d^2 - 9*b^4*d*e)*x)*sqrt(e*x + d))/(b^5*
c^2*x^4 + 2*b^6*c*x^3 + b^7*x^2), 1/8*(6*((16*c^4*d^2 - 20*b*c^3*d*e + 5*b^
2*c^2*e^2)*x^4 + 2*(16*b*c^3*d^2 - 20*b^2*c^2*d*e + 5*b^3*c*e^2)*x^3 + (16*
b^2*c^2*d^2 - 20*b^3*c*d*e + 5*b^4*e^2)*x^2)*sqrt(-d)*arctan(sqrt(e*x + d)*
sqrt(-d)/d) + 3*((16*c^4*d^2 - 12*b*c^3*d*e + b^2*c^2*e^2)*x^4 + 2*(16*b*c^
3*d^2 - 12*b^2*c^2*d*e + b^3*c*e^2)*x^3 + (16*b^2*c^2*d^2 - 12*b^3*c*d*e +
b^4*e^2)*x^2)*sqrt((c*d - b*e)/c)*log((c*e*x + 2*c*d - b*e + 2*sqrt(e*x + d
)*c*sqrt((c*d - b*e)/c))/(c*x + b)) - 2*(2*b^4*d^2 - 3*(8*b*c^3*d^2 - 8*b^2
*c^2*d*e + b^3*c*e^2)*x^3 - (36*b^2*c^2*d^2 - 37*b^3*c*d*e + 5*b^4*e^2)*x^2
 - (8*b^3*c*d^2 - 9*b^4*d*e)*x)*sqrt(e*x + d))/(b^5*c^2*x^4 + 2*b^6*c*x^3 +
 b^7*x^2), 1/4*(3*((16*c^4*d^2 - 12*b*c^3*d*e + b^2*c^2*e^2)*x^4 + 2*(16*b*
c^3*d^2 - 12*b^2*c^2*d*e + b^3*c*e^2)*x^3 + (16*b^2*c^2*d^2 - 12*b^3*c*d*e
+ b^4*e^2)*x^2)*sqrt(-(c*d - b*e)/c)*arctan(-sqrt(e*x + d)*c*sqrt(-(c*d - b
*e)/c)/(c*d - b*e)) + 3*((16*c^4*d^2 - 20*b*c^3*d*e + 5*b^2*c^2*e^2)*x^4 +
2*(16*b*c^3*d^2 - 20*b^2*c^2*d*e + 5*b^3*c*e^2)*x^3 + (16*b^2*c^2*d^2 - 20*
b^3*c*d*e + 5*b^4*e^2)*x^2)*sqrt(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d) - (2*
b^4*d^2 - 3*(8*b*c^3*d^2 - 8*b^2*c^2*d*e + b^3*c*e^2)*x^3 - (36*b^2*c^2*d^2
 - 37*b^3*c*d*e + 5*b^4*e^2)*x^2 - (8*b^3*c*d^2 - 9*b^4*d*e)*x)*sqrt(e*x +
d))/(b^5*c^2*x^4 + 2*b^6*c*x^3 + b^7*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(c*x**2+b*x)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.44165, size = 605, normalized size = 2.61

$$-\frac{3\left(16\,c^3d^3-28\,bc^2d^2e+13\,b^2cde^2-b^3e^3\right)\arctan\left(\frac{\sqrt{xe+d}c}{\sqrt{-c^2d+bce}}\right)}{4\,\sqrt{-c^2d+bce}b^5}+\frac{3\left(16\,c^2d^3-20\,bcd^2e+5\,b^2de^2\right)\arctan\left(\frac{\sqrt{xe+d}}{\sqrt{-d}}\right)}{4\,b^5\sqrt{-d}}+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x)^3,x, algorithm="giac")
```

```
[Out] -3/4*(16*c^3*d^3 - 28*b*c^2*d^2*e + 13*b^2*c*d*e^2 - b^3*e^3)*arctan(sqrt(x
*e + d)*c/sqrt(-c^2*d + b*c*e))/(sqrt(-c^2*d + b*c*e)*b^5) + 3/4*(16*c^2*d^
3 - 20*b*c*d^2*e + 5*b^2*d*e^2)*arctan(sqrt(x*e + d)/sqrt(-d))/(b^5*sqrt(-d
)) + 1/4*(24*(x*e + d)^(7/2)*c^3*d^2*e - 72*(x*e + d)^(5/2)*c^3*d^3*e + 72*
```

```
(x*e + d)^(3/2)*c^3*d^4*e - 24*sqrt(x*e + d)*c^3*d^5*e - 24*(x*e + d)^(7/2)
*b*c^2*d*e^2 + 108*(x*e + d)^(5/2)*b*c^2*d^2*e^2 - 144*(x*e + d)^(3/2)*b*c^
2*d^3*e^2 + 60*sqrt(x*e + d)*b*c^2*d^4*e^2 + 3*(x*e + d)^(7/2)*b^2*c*e^3 -
46*(x*e + d)^(5/2)*b^2*c*d*e^3 + 91*(x*e + d)^(3/2)*b^2*c*d^2*e^3 - 48*sqrt
(x*e + d)*b^2*c*d^3*e^3 + 5*(x*e + d)^(5/2)*b^3*e^4 - 19*(x*e + d)^(3/2)*b^
3*d*e^4 + 12*sqrt(x*e + d)*b^3*d^2*e^4)/(((x*e + d)^2*c - 2*(x*e + d)*c*d +
 c*d^2 + (x*e + d)*b*e - b*d*e)^2*b^4)
```

3.381 $\int \frac{(d+ex)^{3/2}}{\left(bx+cx^2\right)^3} dx$

Optimal. Leaf size=246

$$-\frac{3\left(b^2e^2-12bcde+16c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4b^5\sqrt{d}}+\frac{3\sqrt{c}\left(5b^2e^2-20bcde+16c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{4b^5\sqrt{cd-be}}-\frac{\sqrt{d+ex}(x(2cd-b}{2b^2\left(bx+cx^2\right.}$$

```
[Out] -(Sqrt[d + e*x]*(b*d + (2*c*d - b*e)*x))/(2*b^2*(b*x + c*x^2)^2) + (Sqrt[d
+ e*x]*(b*(12*c*d - 7*b*e)*(c*d - b*e) + 12*c*(c*d - b*e)*(2*c*d - b*e)*x))
/(4*b^4*(c*d - b*e)*(b*x + c*x^2)) - (3*(16*c^2*d^2 - 12*b*c*d*e + b^2*e^2)
*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(4*b^5*Sqrt[d]) + (3*Sqrt[c]*(16*c^2*d^2 -
 20*b*c*d*e + 5*b^2*e^2)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/
(4*b^5*Sqrt[c*d - b*e])
```

Rubi [A] time = 0.464637, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {738, 822, 826, 1166, 208}

$$-\frac{3\left(b^2e^2-12bcde+16c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4b^5\sqrt{d}}+\frac{3\sqrt{c}\left(5b^2e^2-20bcde+16c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{4b^5\sqrt{cd-be}}-\frac{\sqrt{d+ex}(x(2cd-b}{2b^2\left(bx+cx^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(b*x + c*x^2)^3,x]
```

```
[Out] -(Sqrt[d + e*x]*(b*d + (2*c*d - b*e)*x))/(2*b^2*(b*x + c*x^2)^2) + (Sqrt[d
+ e*x]*(b*(12*c*d - 7*b*e)*(c*d - b*e) + 12*c*(c*d - b*e)*(2*c*d - b*e)*x))
/(4*b^4*(c*d - b*e)*(b*x + c*x^2)) - (3*(16*c^2*d^2 - 12*b*c*d*e + b^2*e^2)
*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(4*b^5*Sqrt[d]) + (3*Sqrt[c]*(16*c^2*d^2 -
 20*b*c*d*e + 5*b^2*e^2)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/
(4*b^5*Sqrt[c*d - b*e])
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
```

```
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\left(bx+cx^2\right)^3}\,dx = -\frac{\sqrt{d+ex}(bd+(2cd-be)x)}{2b^2\left(bx+cx^2\right)^2} - \frac{\int \frac{\frac{1}{2}d(12cd-7be)+\frac{5}{2}e(2cd-be)x}{\sqrt{d+ex}\left(bx+cx^2\right)^2}\,dx}{2b^2}$$

$$= -\frac{\sqrt{d+ex}(bd+(2cd-be)x)}{2b^2\left(bx+cx^2\right)^2} + \frac{\sqrt{d+ex}(b(12cd-7be)(cd-be)+12c(cd-be)(2cd-be)x)}{4b^4(cd-be)\left(bx+cx^2\right)} + \frac{\int \frac{3}{4}d(cd-\cdots}{\cdots}$$

$$= -\frac{\sqrt{d+ex}(bd+(2cd-be)x)}{2b^2\left(bx+cx^2\right)^2} + \frac{\sqrt{d+ex}(b(12cd-7be)(cd-be)+12c(cd-be)(2cd-be)x)}{4b^4(cd-be)\left(bx+cx^2\right)} + \frac{\mathrm{Subst}\left(\cdots\right.}{\cdots}$$

$$= -\frac{\sqrt{d+ex}(bd+(2cd-be)x)}{2b^2\left(bx+cx^2\right)^2} + \frac{\sqrt{d+ex}(b(12cd-7be)(cd-be)+12c(cd-be)(2cd-be)x)}{4b^4(cd-be)\left(bx+cx^2\right)} + \frac{\left(3c\left(16c\cdots\right.\right.}{\cdots}$$

$$= -\frac{\sqrt{d+ex}(bd+(2cd-be)x)}{2b^2\left(bx+cx^2\right)^2} + \frac{\sqrt{d+ex}(b(12cd-7be)(cd-be)+12c(cd-be)(2cd-be)x)}{4b^4(cd-be)\left(bx+cx^2\right)} - \frac{3\left(16c^2d\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.53347, size = 289, normalized size = 1.17

$$\frac{3x^2(b+cx)^2\left(13b^2cde^2-b^3e^3-28bc^2d^2e+16c^3d^3\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)-\sqrt{d}\left(b\sqrt{d+ex}\left(b^2c^2x\left(8d^2-55dex+12e^2x^2\right)+b^3c\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(b*x + c*x^2)^3,x]
```

```
[Out] (3*(16*c^3*d^3 - 28*b*c^2*d^2*e + 13*b^2*c*d*e^2 - b^3*e^3)*x^2*(b + c*x)^2
*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] - Sqrt[d]*(b*Sqrt[d + e*x]*(24*c^4*d^2*x^3
+ 36*b*c^3*d*x^2*(d - e*x) + b^4*e*(2*d + 5*e*x) + b^2*c^2*x*(8*d^2 - 55*d*
e*x + 12*e^2*x^2) + b^3*c*(-2*d^2 - 13*d*e*x + 19*e^2*x^2)) + 3*Sqrt[c]*Sqr
t[c*d - b*e]*(16*c^2*d^2 - 20*b*c*d*e + 5*b^2*e^2)*x^2*(b + c*x)^2*ArcTanh[
(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]]))/(4*b^5*Sqrt[d]*(-(c*d) + b*e)*x^
2*(b + c*x)^2)
```

Maple [A] time = 0.232, size = 414, normalized size = 1.7

$$-\frac{7\,e^2c^2}{4\,b^3\,(cex+be)^2}(ex+d)^{\frac{3}{2}}+3\,\frac{ec^3\,(ex+d)^{3/2}\,d}{b^4\,(cex+be)^2}-\frac{9\,e^3c}{4\,b^2\,(cex+be)^2}\sqrt{ex+d}+\frac{21\,e^2c^2d}{4\,b^3\,(cex+be)^2}\sqrt{ex+d}-3\,\frac{ec^3\sqrt{ex+d}d^2}{b^4\,(cex+be)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*x^2+b*x)^3,x)
```

```
[Out] -7/4*e^2*c^2/b^3/(c*e*x+b*e)^2*(e*x+d)^(3/2)+3*e*c^3/b^4/(c*e*x+b*e)^2*(e*x
+d)^(3/2)*d-9/4*e^3*c/b^2/(c*e*x+b*e)^2*(e*x+d)^(1/2)+21/4*e^2*c^2/b^3/(c*e
*x+b*e)^2*(e*x+d)^(1/2)*d-3*e*c^3/b^4/(c*e*x+b*e)^2*(e*x+d)^(1/2)*d^2-15/4*
e^2*c/b^3/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))+1
5*e*c^2/b^4/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))
*d-12*c^3/b^5/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2
))*d^2-5/4/b^3/x^2*(e*x+d)^(3/2)+3/e/b^4/x^2*(e*x+d)^(3/2)*c*d-3/e/b^4/x^2*
(e*x+d)^(1/2)*c*d^2+3/4/b^3/x^2*(e*x+d)^(1/2)*d-3/4*e^2/b^3/d^(1/2)*arctanh
((e*x+d)^(1/2)/d^(1/2))+9*e/b^4*d^(1/2)*arctanh((e*x+d)^(1/2)/d^(1/2))*c-12
/b^5*d^(3/2)*arctanh((e*x+d)^(1/2)/d^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.01614, size = 3532, normalized size = 14.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*((16*c^4*d^3 - 20*b*c^3*d^2*e + 5*b^2*c^2*d*e^2)*x^4 + 2*(16*b*c^3*
d^3 - 20*b^2*c^2*d^2*e + 5*b^3*c*d*e^2)*x^3 + (16*b^2*c^2*d^3 - 20*b^3*c*d^
2*e + 5*b^4*d*e^2)*x^2)*sqrt(c/(c*d - b*e))*log((c*e*x + 2*c*d - b*e + 2*(c
*d - b*e)*sqrt(e*x + d)*sqrt(c/(c*d - b*e)))/(c*x + b)) + 3*((16*c^4*d^2 -
12*b*c^3*d*e + b^2*c^2*e^2)*x^4 + 2*(16*b*c^3*d^2 - 12*b^2*c^2*d*e + b^3*c*
e^2)*x^3 + (16*b^2*c^2*d^2 - 12*b^3*c*d*e + b^4*e^2)*x^2)*sqrt(d)*log((e*x
```

```
- 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) - 2*(2*b^4*d^2 - 12*(2*b*c^3*d^2 - b^2*
c^2*d*e)*x^3 - (36*b^2*c^2*d^2 - 19*b^3*c*d*e)*x^2 - (8*b^3*c*d^2 - 5*b^4*d
*e)*x)*sqrt(e*x + d))/(b^5*c^2*d*x^4 + 2*b^6*c*d*x^3 + b^7*d*x^2), 1/8*(6*(
(16*c^4*d^3 - 20*b*c^3*d^2*e + 5*b^2*c^2*d*e^2)*x^4 + 2*(16*b*c^3*d^3 - 20*
b^2*c^2*d^2*e + 5*b^3*c*d*e^2)*x^3 + (16*b^2*c^2*d^3 - 20*b^3*c*d^2*e + 5*b
^4*d*e^2)*x^2)*sqrt(-c/(c*d - b*e))*arctan(-(c*d - b*e)*sqrt(e*x + d)*sqrt(
-c/(c*d - b*e))/(c*e*x + c*d)) + 3*((16*c^4*d^2 - 12*b*c^3*d*e + b^2*c^2*e^
2)*x^4 + 2*(16*b*c^3*d^2 - 12*b^2*c^2*d*e + b^3*c*e^2)*x^3 + (16*b^2*c^2*d^
2 - 12*b^3*c*d*e + b^4*e^2)*x^2)*sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d)
 + 2*d)/x) - 2*(2*b^4*d^2 - 12*(2*b*c^3*d^2 - b^2*c^2*d*e)*x^3 - (36*b^2*c^
2*d^2 - 19*b^3*c*d*e)*x^2 - (8*b^3*c*d^2 - 5*b^4*d*e)*x)*sqrt(e*x + d))/(b^
5*c^2*d*x^4 + 2*b^6*c*d*x^3 + b^7*d*x^2), 1/8*(6*((16*c^4*d^2 - 12*b*c^3*d*
e + b^2*c^2*e^2)*x^4 + 2*(16*b*c^3*d^2 - 12*b^2*c^2*d*e + b^3*c*e^2)*x^3 +
(16*b^2*c^2*d^2 - 12*b^3*c*d*e + b^4*e^2)*x^2)*sqrt(-d)*arctan(sqrt(e*x + d
)*sqrt(-d)/d) + 3*((16*c^4*d^3 - 20*b*c^3*d^2*e + 5*b^2*c^2*d*e^2)*x^4 + 2*
(16*b*c^3*d^3 - 20*b^2*c^2*d^2*e + 5*b^3*c*d*e^2)*x^3 + (16*b^2*c^2*d^3 - 2
0*b^3*c*d^2*e + 5*b^4*d*e^2)*x^2)*sqrt(c/(c*d - b*e))*log((c*e*x + 2*c*d -
b*e + 2*(c*d - b*e)*sqrt(e*x + d)*sqrt(c/(c*d - b*e)))/(c*x + b)) - 2*(2*b^
4*d^2 - 12*(2*b*c^3*d^2 - b^2*c^2*d*e)*x^3 - (36*b^2*c^2*d^2 - 19*b^3*c*d*e
)*x^2 - (8*b^3*c*d^2 - 5*b^4*d*e)*x)*sqrt(e*x + d))/(b^5*c^2*d*x^4 + 2*b^6*
c*d*x^3 + b^7*d*x^2), 1/4*(3*((16*c^4*d^3 - 20*b*c^3*d^2*e + 5*b^2*c^2*d*e^
2)*x^4 + 2*(16*b*c^3*d^3 - 20*b^2*c^2*d^2*e + 5*b^3*c*d*e^2)*x^3 + (16*b^2*
c^2*d^3 - 20*b^3*c*d^2*e + 5*b^4*d*e^2)*x^2)*sqrt(-c/(c*d - b*e))*arctan(-(
c*d - b*e)*sqrt(e*x + d)*sqrt(-c/(c*d - b*e))/(c*e*x + c*d)) + 3*((16*c^4*d
^2 - 12*b*c^3*d*e + b^2*c^2*e^2)*x^4 + 2*(16*b*c^3*d^2 - 12*b^2*c^2*d*e + b
^3*c*e^2)*x^3 + (16*b^2*c^2*d^2 - 12*b^3*c*d*e + b^4*e^2)*x^2)*sqrt(-d)*arc
tan(sqrt(e*x + d)*sqrt(-d)/d) - (2*b^4*d^2 - 12*(2*b*c^3*d^2 - b^2*c^2*d*e)
*x^3 - (36*b^2*c^2*d^2 - 19*b^3*c*d*e)*x^2 - (8*b^3*c*d^2 - 5*b^4*d*e)*x)*s
qrt(e*x + d))/(b^5*c^2*d*x^4 + 2*b^6*c*d*x^3 + b^7*d*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+b*x)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37057, size = 529, normalized size = 2.15

$$-\frac{3\left(16\,c^3d^2-20\,bc^2de+5\,b^2ce^2\right)\arctan\left(\frac{\sqrt{xe+d}c}{\sqrt{-c^2d+bce}}\right)}{4\,\sqrt{-c^2d+bce}b^5}+\frac{3\left(16\,c^2d^2-12\,bcde+b^2e^2\right)\arctan\left(\frac{\sqrt{xe+d}}{\sqrt{-d}}\right)}{4\,b^5\sqrt{-d}}+\frac{24\,(xe+d)^{\frac{7}{2}}c^3de-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x)^3,x, algorithm="giac")
```

```
[Out] -3/4*(16*c^3*d^2 - 20*b*c^2*d*e + 5*b^2*c*e^2)*arctan(sqrt(x*e + d)*c/sqrt(
-c^2*d + b*c*e))/(sqrt(-c^2*d + b*c*e)*b^5) + 3/4*(16*c^2*d^2 - 12*b*c*d*e
+ b^2*e^2)*arctan(sqrt(x*e + d)/sqrt(-d))/(b^5*sqrt(-d)) + 1/4*(24*(x*e + d
)^(7/2)*c^3*d*e - 72*(x*e + d)^(5/2)*c^3*d^2*e + 72*(x*e + d)^(3/2)*c^3*d^3
```

```
*e - 24*sqrt(x*e + d)*c^3*d^4*e - 12*(x*e + d)^(7/2)*b*c^2*e^2 + 72*(x*e +
d)^(5/2)*b*c^2*d*e^2 - 108*(x*e + d)^(3/2)*b*c^2*d^2*e^2 + 48*sqrt(x*e + d)
*b*c^2*d^3*e^2 - 19*(x*e + d)^(5/2)*b^2*c*e^3 + 46*(x*e + d)^(3/2)*b^2*c*d*
e^3 - 27*sqrt(x*e + d)*b^2*c*d^2*e^3 - 5*(x*e + d)^(3/2)*b^3*e^4 + 3*sqrt(x
*e + d)*b^3*d*e^4)/(((x*e + d)^2*c - 2*(x*e + d)*c*d + c*d^2 + (x*e + d)*b*
e - b*d*e)^2*b^4)
```

3.382 $\int \frac{\sqrt{d+ex}}{(bx+cx^2)^3}\,dx$

Optimal. Leaf size=245

$$\frac{\sqrt{d+ex}\left(cx\left(b^2e^2-24bcde+24c^2d^2\right)+b(cd-be)(12cd-be)\right)}{4b^4d\left(bx+cx^2\right)(cd-be)}+\frac{c^{3/2}\left(35b^2e^2-84bcde+48c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{4b^5(cd-be)^{3/2}}-\frac{(-$$

```
[Out] -((b + 2*c*x)*Sqrt[d + e*x])/(2*b^2*(b*x + c*x^2)^2) + (Sqrt[d + e*x]*(b*(c
*d - b*e)*(12*c*d - b*e) + c*(24*c^2*d^2 - 24*b*c*d*e + b^2*e^2)*x))/(4*b^4
*d*(c*d - b*e)*(b*x + c*x^2)) - ((48*c^2*d^2 - 12*b*c*d*e - b^2*e^2)*ArcTan
h[Sqrt[d + e*x]/Sqrt[d]])/(4*b^5*d^(3/2)) + (c^(3/2)*(48*c^2*d^2 - 84*b*c*d
*e + 35*b^2*e^2)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(4*b^5*(
c*d - b*e)^(3/2))
```

Rubi [A] time = 0.377742, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {736, 822, 826, 1166, 208}

$$\frac{\sqrt{d+ex}\left(cx\left(b^2e^2-24bcde+24c^2d^2\right)+b(cd-be)(12cd-be)\right)}{4b^4d\left(bx+cx^2\right)(cd-be)}+\frac{c^{3/2}\left(35b^2e^2-84bcde+48c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{4b^5(cd-be)^{3/2}}-\frac{(-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(b*x + c*x^2)^3,x]
```


```
[Out] -((b + 2*c*x)*Sqrt[d + e*x])/(2*b^2*(b*x + c*x^2)^2) + (Sqrt[d + e*x]*(b*(c
*d - b*e)*(12*c*d - b*e) + c*(24*c^2*d^2 - 24*b*c*d*e + b^2*e^2)*x))/(4*b^4
*d*(c*d - b*e)*(b*x + c*x^2)) - ((48*c^2*d^2 - 12*b*c*d*e - b^2*e^2)*ArcTan
h[Sqrt[d + e*x]/Sqrt[d]])/(4*b^5*d^(3/2)) + (c^(3/2)*(48*c^2*d^2 - 84*b*c*d
*e + 35*b^2*e^2)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(4*b^5*(
c*d - b*e)^(3/2))
```


Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] - Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)
*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p + 1
), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 0] && (L
tQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c,
d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
```

```
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\left(bx+cx^2\right)^3}\,dx = -\frac{(b+2cx)\sqrt{d+ex}}{2b^2\left(bx+cx^2\right)^2}+\frac{\int \frac{-6cd+\frac{be}{2}-5cex}{\sqrt{d+ex}\left(bx+cx^2\right)^2}\,dx}{2b^2}$$

$$= -\frac{(b+2cx)\sqrt{d+ex}}{2b^2\left(bx+cx^2\right)^2}+\frac{\sqrt{d+ex}\left(b(cd-be)(12cd-be)+c\left(24c^2d^2-24bcde+b^2e^2\right)x\right)}{4b^4d(cd-be)\left(bx+cx^2\right)}-\frac{\int \frac{-\frac{1}{4}(cd-b}{}}{}$$

$$= -\frac{(b+2cx)\sqrt{d+ex}}{2b^2\left(bx+cx^2\right)^2}+\frac{\sqrt{d+ex}\left(b(cd-be)(12cd-be)+c\left(24c^2d^2-24bcde+b^2e^2\right)x\right)}{4b^4d(cd-be)\left(bx+cx^2\right)}-\frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(b+2cx)\sqrt{d+ex}}{2b^2\left(bx+cx^2\right)^2}+\frac{\sqrt{d+ex}\left(b(cd-be)(12cd-be)+c\left(24c^2d^2-24bcde+b^2e^2\right)x\right)}{4b^4d(cd-be)\left(bx+cx^2\right)}+\frac{\left(c\left(48c^2d\right.\right.}{}$$

$$= -\frac{(b+2cx)\sqrt{d+ex}}{2b^2\left(bx+cx^2\right)^2}+\frac{\sqrt{d+ex}\left(b(cd-be)(12cd-be)+c\left(24c^2d^2-24bcde+b^2e^2\right)x\right)}{4b^4d(cd-be)\left(bx+cx^2\right)}-\frac{\left(48c^2d^2\right.}{}$$

Mathematica [A] time = 0.975623, size = 361, normalized size = 1.47

$$\frac{-\frac{2c(d+ex)^{3/2}\left(-b^2e^2-9bcde+12c^2d^2\right)}{b^2d(be-cd)}+\frac{(b+cx)\left(2bc^{5/2}(d+ex)^{3/2}\left(10b^2cde^2+b^3e^3-36bc^2d^2e+24c^3d^3\right)+(b+cx)\left(2c^{3/2}(cd-be)^2\left(-b^2e^2-12bcde+48c^2d^2\right)\left(\sqrt{d+ex}-\right.\right.\right.}{b^4c^{3/2}d(cd-be)^2}}{8bd(b+cx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(b*x + c*x^2)^3,x]
```

```
[Out] ((-2*c*(12*c^2*d^2 - 9*b*c*d*e - b^2*e^2)*(d + e*x)^(3/2))/(b^2*d*(-(c*d) +
 b*e)) - (4*(d + e*x)^(3/2))/x^2 + (2*(8*c*d + b*e)*(d + e*x)^(3/2))/(b*d*x
) + ((b + c*x)*(2*b*c^(5/2)*(24*c^3*d^3 - 36*b*c^2*d^2*e + 10*b^2*c*d*e^2 +
 b^3*e^3)*(d + e*x)^(3/2) + (b + c*x)*(2*c^(3/2)*(c*d - b*e)^2*(48*c^2*d^2
- 12*b*c*d*e - b^2*e^2)*(Sqrt[d + e*x] - Sqrt[d]*ArcTanh[Sqrt[d + e*x]/Sqrt
[d]]) - 2*c^3*d^2*(48*c^2*d^2 - 84*b*c*d*e + 35*b^2*e^2)*(Sqrt[c]*Sqrt[d +
e*x] - Sqrt[c*d - b*e]*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]]))))
/(b^4*c^(3/2)*d*(c*d - b*e)^2))/(8*b*d*(b + c*x)^2)
```

Maple [B] time = 0.234, size = 436, normalized size = 1.8

$$\frac{11\,e^2c^3}{4\,b^3\,(cex+be)^2\,(be-cd)}(ex+d)^{\frac{3}{2}}-3\,\frac{ec^4\,(ex+d)^{3/2}\,d}{b^4\,(cex+be)^2\,(be-cd)}+\frac{13\,e^2c^2}{4\,b^3\,(cex+be)^2}\sqrt{ex+d}-3\,\frac{ec^3\sqrt{ex+d}d}{b^4\,(cex+be)^2}+\frac{35\,e^2c^2}{4\,b^3\,(be-cd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*x^2+b*x)^3,x)
```

```
[Out] 11/4*e^2/b^3*c^3/(c*e*x+b*e)^2/(b*e-c*d)*(e*x+d)^(3/2)-3*e/b^4*c^4/(c*e*x+b
*e)^2/(b*e-c*d)*(e*x+d)^(3/2)*d+13/4*e^2/b^3*c^2/(c*e*x+b*e)^2*(e*x+d)^(1/2
)-3*e/b^4*c^3/(c*e*x+b*e)^2*(e*x+d)^(1/2)*d+35/4*e^2/b^3*c^2/(b*e-c*d)/((b*
e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))-21*e/b^4*c^3/(b
*e-c*d)/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))*d+1
2/b^5*c^4/(b*e-c*d)/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c
)^(1/2))*d^2-1/4/b^3/x^2/d*(e*x+d)^(3/2)+3/e/b^4/x^2*(e*x+d)^(3/2)*c-3/e/b^
4/x^2*(e*x+d)^(1/2)*c*d-1/4/b^3/x^2*(e*x+d)^(1/2)+1/4*e^2/b^3/d^(3/2)*arcta
nh((e*x+d)^(1/2)/d^(1/2))+3*e/b^4/d^(1/2)*arctanh((e*x+d)^(1/2)/d^(1/2))*c-
12/b^5*d^(1/2)*arctanh((e*x+d)^(1/2)/d^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.48442, size = 4645, normalized size = 18.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(((48*c^5*d^4 - 84*b*c^4*d^3*e + 35*b^2*c^3*d^2*e^2)*x^4 + 2*(48*b*c^
4*d^4 - 84*b^2*c^3*d^3*e + 35*b^3*c^2*d^2*e^2)*x^3 + (48*b^2*c^3*d^4 - 84*b
^3*c^2*d^3*e + 35*b^4*c*d^2*e^2)*x^2)*sqrt(c/(c*d - b*e))*log((c*e*x + 2*c*
d - b*e - 2*(c*d - b*e)*sqrt(e*x + d)*sqrt(c/(c*d - b*e)))/(c*x + b)) + ((4
8*c^5*d^3 - 60*b*c^4*d^2*e + 11*b^2*c^3*d*e^2 + b^3*c^2*e^3)*x^4 + 2*(48*b*
```

```
c^4*d^3 - 60*b^2*c^3*d^2*e + 11*b^3*c^2*d*e^2 + b^4*c*e^3)*x^3 + (48*b^2*c^
3*d^3 - 60*b^3*c^2*d^2*e + 11*b^4*c*d*e^2 + b^5*e^3)*x^2)*sqrt(d)*log((e*x
+ 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + 2*(2*b^4*c*d^3 - 2*b^5*d^2*e - (24*b*
c^4*d^3 - 24*b^2*c^3*d^2*e + b^3*c^2*d*e^2)*x^3 - (36*b^2*c^3*d^3 - 37*b^3*
c^2*d^2*e + 2*b^4*c*d*e^2)*x^2 - (8*b^3*c^2*d^3 - 9*b^4*c*d^2*e + b^5*d*e^2
)*x)*sqrt(e*x + d))/((b^5*c^3*d^3 - b^6*c^2*d^2*e)*x^4 + 2*(b^6*c^2*d^3 - b
^7*c*d^2*e)*x^3 + (b^7*c*d^3 - b^8*d^2*e)*x^2), 1/8*(2*((48*c^5*d^4 - 84*b*
c^4*d^3*e + 35*b^2*c^3*d^2*e^2)*x^4 + 2*(48*b*c^4*d^4 - 84*b^2*c^3*d^3*e +
35*b^3*c^2*d^2*e^2)*x^3 + (48*b^2*c^3*d^4 - 84*b^3*c^2*d^3*e + 35*b^4*c*d^2
*e^2)*x^2)*sqrt(-c/(c*d - b*e))*arctan(-(c*d - b*e)*sqrt(e*x + d)*sqrt(-c/(
c*d - b*e))/(c*e*x + c*d)) - ((48*c^5*d^3 - 60*b*c^4*d^2*e + 11*b^2*c^3*d*e
^2 + b^3*c^2*e^3)*x^4 + 2*(48*b*c^4*d^3 - 60*b^2*c^3*d^2*e + 11*b^3*c^2*d*e
^2 + b^4*c*e^3)*x^3 + (48*b^2*c^3*d^3 - 60*b^3*c^2*d^2*e + 11*b^4*c*d*e^2 +
 b^5*e^3)*x^2)*sqrt(d)*log((e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) - 2*(2*
b^4*c*d^3 - 2*b^5*d^2*e - (24*b*c^4*d^3 - 24*b^2*c^3*d^2*e + b^3*c^2*d*e^2)
*x^3 - (36*b^2*c^3*d^3 - 37*b^3*c^2*d^2*e + 2*b^4*c*d*e^2)*x^2 - (8*b^3*c^2
*d^3 - 9*b^4*c*d^2*e + b^5*d*e^2)*x)*sqrt(e*x + d))/((b^5*c^3*d^3 - b^6*c^2
*d^2*e)*x^4 + 2*(b^6*c^2*d^3 - b^7*c*d^2*e)*x^3 + (b^7*c*d^3 - b^8*d^2*e)*x
^2), 1/8*(2*((48*c^5*d^3 - 60*b*c^4*d^2*e + 11*b^2*c^3*d*e^2 + b^3*c^2*e^3)
*x^4 + 2*(48*b*c^4*d^3 - 60*b^2*c^3*d^2*e + 11*b^3*c^2*d*e^2 + b^4*c*e^3)*x
^3 + (48*b^2*c^3*d^3 - 60*b^3*c^2*d^2*e + 11*b^4*c*d*e^2 + b^5*e^3)*x^2)*sq
rt(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d) - ((48*c^5*d^4 - 84*b*c^4*d^3*e + 3
5*b^2*c^3*d^2*e^2)*x^4 + 2*(48*b*c^4*d^4 - 84*b^2*c^3*d^3*e + 35*b^3*c^2*d^
2*e^2)*x^3 + (48*b^2*c^3*d^4 - 84*b^3*c^2*d^3*e + 35*b^4*c*d^2*e^2)*x^2)*sq
rt(c/(c*d - b*e))*log((c*e*x + 2*c*d - b*e - 2*(c*d - b*e)*sqrt(e*x + d)*sq
rt(c/(c*d - b*e)))/(c*x + b)) - 2*(2*b^4*c*d^3 - 2*b^5*d^2*e - (24*b*c^4*d^
3 - 24*b^2*c^3*d^2*e + b^3*c^2*d*e^2)*x^3 - (36*b^2*c^3*d^3 - 37*b^3*c^2*d^
2*e + 2*b^4*c*d*e^2)*x^2 - (8*b^3*c^2*d^3 - 9*b^4*c*d^2*e + b^5*d*e^2)*x)*s
qrt(e*x + d))/((b^5*c^3*d^3 - b^6*c^2*d^2*e)*x^4 + 2*(b^6*c^2*d^3 - b^7*c*d
^2*e)*x^3 + (b^7*c*d^3 - b^8*d^2*e)*x^2), 1/4*(((48*c^5*d^4 - 84*b*c^4*d^3*
e + 35*b^2*c^3*d^2*e^2)*x^4 + 2*(48*b*c^4*d^4 - 84*b^2*c^3*d^3*e + 35*b^3*c
^2*d^2*e^2)*x^3 + (48*b^2*c^3*d^4 - 84*b^3*c^2*d^3*e + 35*b^4*c*d^2*e^2)*x^
2)*sqrt(-c/(c*d - b*e))*arctan(-(c*d - b*e)*sqrt(e*x + d)*sqrt(-c/(c*d - b*
e))/(c*e*x + c*d)) + ((48*c^5*d^3 - 60*b*c^4*d^2*e + 11*b^2*c^3*d*e^2 + b^3
*c^2*e^3)*x^4 + 2*(48*b*c^4*d^3 - 60*b^2*c^3*d^2*e + 11*b^3*c^2*d*e^2 + b^4
*c*e^3)*x^3 + (48*b^2*c^3*d^3 - 60*b^3*c^2*d^2*e + 11*b^4*c*d*e^2 + b^5*e^3
)*x^2)*sqrt(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d) - (2*b^4*c*d^3 - 2*b^5*d^2
*e - (24*b*c^4*d^3 - 24*b^2*c^3*d^2*e + b^3*c^2*d*e^2)*x^3 - (36*b^2*c^3*d^
3 - 37*b^3*c^2*d^2*e + 2*b^4*c*d*e^2)*x^2 - (8*b^3*c^2*d^3 - 9*b^4*c*d^2*e
+ b^5*d*e^2)*x)*sqrt(e*x + d))/((b^5*c^3*d^3 - b^6*c^2*d^2*e)*x^4 + 2*(b^6*
c^2*d^3 - b^7*c*d^2*e)*x^3 + (b^7*c*d^3 - b^8*d^2*e)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+b*x)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27475, size = 686, normalized size = 2.8

$$-\frac{\left(48\,c^4d^2-84\,bc^3de+35\,b^2c^2e^2\right)\arctan\left(\frac{\sqrt{xe+d}c}{\sqrt{-c^2d+bce}}\right)}{4\left(b^5cd-b^6e\right)\sqrt{-c^2d+bce}}+\frac{24\,(xe+d)^{\frac{7}{2}}c^4d^2e-72\,(xe+d)^{\frac{5}{2}}c^4d^3e+72\,(xe+d)^{\frac{3}{2}}c^4d^4e-24\,\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x)^3,x, algorithm="giac")
```

```
[Out] -1/4*(48*c^4*d^2 - 84*b*c^3*d*e + 35*b^2*c^2*e^2)*arctan(sqrt(x*e + d)*c/sq
rt(-c^2*d + b*c*e))/((b^5*c*d - b^6*e)*sqrt(-c^2*d + b*c*e)) + 1/4*(24*(x*e
 + d)^(7/2)*c^4*d^2*e - 72*(x*e + d)^(5/2)*c^4*d^3*e + 72*(x*e + d)^(3/2)*c
^4*d^4*e - 24*sqrt(x*e + d)*c^4*d^5*e - 24*(x*e + d)^(7/2)*b*c^3*d*e^2 + 10
8*(x*e + d)^(5/2)*b*c^3*d^2*e^2 - 144*(x*e + d)^(3/2)*b*c^3*d^3*e^2 + 60*sq
rt(x*e + d)*b*c^3*d^4*e^2 + (x*e + d)^(7/2)*b^2*c^2*e^3 - 40*(x*e + d)^(5/2
)*b^2*c^2*d*e^3 + 85*(x*e + d)^(3/2)*b^2*c^2*d^2*e^3 - 46*sqrt(x*e + d)*b^2
*c^2*d^3*e^3 + 2*(x*e + d)^(5/2)*b^3*c*e^4 - 13*(x*e + d)^(3/2)*b^3*c*d*e^4
 + 9*sqrt(x*e + d)*b^3*c*d^2*e^4 + (x*e + d)^(3/2)*b^4*e^5 + sqrt(x*e + d)*
b^4*d*e^5)/((b^4*c*d^2 - b^5*d*e)*((x*e + d)^2*c - 2*(x*e + d)*c*d + c*d^2
+ (x*e + d)*b*e - b*d*e)^2) + 1/4*(48*c^2*d^2 - 12*b*c*d*e - b^2*e^2)*arcta
n(sqrt(x*e + d)/sqrt(-d))/(b^5*sqrt(-d)*d)
```

3.383 $\int \frac{1}{\sqrt{d+ex}\left(bx+cx^2\right)^3}\,dx$

Optimal. Leaf size=299

$$\frac{\sqrt{d+ex}\left(3cx(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)+b(cd-be)\left(-3b^2e^2-7bcde+12c^2d^2\right)\right)}{4b^4d^2\left(bx+cx^2\right)(cd-be)^2}+\frac{3c^{5/2}\left(21b^2e^2-36bcde+1\right.}{4b^5(cd-}$$

```
[Out] -(Sqrt[d + e*x]*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(2*b^2*d*(c*d - b*e)*(
b*x + c*x^2)^2) + (Sqrt[d + e*x]*(b*(c*d - b*e)*(12*c^2*d^2 - 7*b*c*d*e - 3
*b^2*e^2) + 3*c*(2*c*d - b*e)*(4*c^2*d^2 - 4*b*c*d*e - b^2*e^2)*x))/(4*b^4*
d^2*(c*d - b*e)^2*(b*x + c*x^2)) - (3*(16*c^2*d^2 + 4*b*c*d*e + b^2*e^2)*Ar
cTanh[Sqrt[d + e*x]/Sqrt[d]])/(4*b^5*d^(5/2)) + (3*c^(5/2)*(16*c^2*d^2 - 36
*b*c*d*e + 21*b^2*e^2)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(4
*b^5*(c*d - b*e)^(5/2))
```

Rubi [A] time = 0.429534, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {740, 822, 826, 1166, 208}

$$\frac{\sqrt{d+ex}\left(3cx(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)+b(cd-be)\left(-3b^2e^2-7bcde+12c^2d^2\right)\right)}{4b^4d^2\left(bx+cx^2\right)(cd-be)^2}+\frac{3c^{5/2}\left(21b^2e^2-36bcde+1\right.}{4b^5(cd-}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(b*x + c*x^2)^3),x]
```

```
[Out] -(Sqrt[d + e*x]*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(2*b^2*d*(c*d - b*e)*(
b*x + c*x^2)^2) + (Sqrt[d + e*x]*(b*(c*d - b*e)*(12*c^2*d^2 - 7*b*c*d*e - 3
*b^2*e^2) + 3*c*(2*c*d - b*e)*(4*c^2*d^2 - 4*b*c*d*e - b^2*e^2)*x))/(4*b^4*
d^2*(c*d - b*e)^2*(b*x + c*x^2)) - (3*(16*c^2*d^2 + 4*b*c*d*e + b^2*e^2)*Ar
cTanh[Sqrt[d + e*x]/Sqrt[d]])/(4*b^5*d^(5/2)) + (3*c^(5/2)*(16*c^2*d^2 - 36
*b*c*d*e + 21*b^2*e^2)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(4
*b^5*(c*d - b*e)^(5/2))
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
```

```
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\left(bx+cx^2\right)^3}\,dx = -\frac{\sqrt{d+ex}(b(cd-be)+c(2cd-be)x)}{2b^2d(cd-be)\left(bx+cx^2\right)^2} - \frac{\int \frac{\frac{1}{2}\left(12c^2d^2-7bcde-3b^2e^2\right)+\frac{5}{2}ce(2cd-be)x}{\sqrt{d+ex}\left(bx+cx^2\right)^2}\,dx}{2b^2d(cd-be)}$$

$$= -\frac{\sqrt{d+ex}(b(cd-be)+c(2cd-be)x)}{2b^2d(cd-be)\left(bx+cx^2\right)^2} + \frac{\sqrt{d+ex}\left(b(cd-be)\left(12c^2d^2-7bcde-3b^2e^2\right)+3c(2\cdots\right.}{4b^4d^2(cd-be)^2\left(bx+c\cdots\right.}$$

$$= -\frac{\sqrt{d+ex}(b(cd-be)+c(2cd-be)x)}{2b^2d(cd-be)\left(bx+cx^2\right)^2} + \frac{\sqrt{d+ex}\left(b(cd-be)\left(12c^2d^2-7bcde-3b^2e^2\right)+3c(2\cdots\right.}{4b^4d^2(cd-be)^2\left(bx+c\cdots\right.}$$

$$= -\frac{\sqrt{d+ex}(b(cd-be)+c(2cd-be)x)}{2b^2d(cd-be)\left(bx+cx^2\right)^2} + \frac{\sqrt{d+ex}\left(b(cd-be)\left(12c^2d^2-7bcde-3b^2e^2\right)+3c(2\cdots\right.}{4b^4d^2(cd-be)^2\left(bx+c\cdots\right.}$$

$$= -\frac{\sqrt{d+ex}(b(cd-be)+c(2cd-be)x)}{2b^2d(cd-be)\left(bx+cx^2\right)^2} + \frac{\sqrt{d+ex}\left(b(cd-be)\left(12c^2d^2-7bcde-3b^2e^2\right)+3c(2\cdots\right.}{4b^4d^2(cd-be)^2\left(bx+c\cdots\right.}$$

Mathematica [A] time = 1.22247, size = 299, normalized size = 1.

$$\frac{\frac{b\sqrt{d}\sqrt{d+ex}\left(b^3c^2\left(-13d^2ex-2d^3+10de^2x^2+3e^3x^3\right)+b^2c^3dx\left(8d^2-55dex+6e^2x^2\right)+2b^4ce\left(2d^2+dex+3e^2x^2\right)+b^5e^2(3ex-2d)+36bc^4d^2x^2(d-ex)+24c^5d^3x^3\right)}{x^2(b+cx)^2(cd-be)^2}+\frac{3c^{5/2}d^{5/2}(\cdots}{\cdots}}{4b^5d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(b*x + c*x^2)^3),x]
```

```
[Out] ((b*Sqrt[d]*Sqrt[d + e*x]*(24*c^5*d^3*x^3 + 36*b*c^4*d^2*x^2*(d - e*x) + b^
5*e^2*(-2*d + 3*e*x) + 2*b^4*c*e*(2*d^2 + d*e*x + 3*e^2*x^2) + b^2*c^3*d*x*
(8*d^2 - 55*d*e*x + 6*e^2*x^2) + b^3*c^2*(-2*d^3 - 13*d^2*e*x + 10*d*e^2*x^
2 + 3*e^3*x^3)))/((c*d - b*e)^2*x^2*(b + c*x)^2) - 3*(16*c^2*d^2 + 4*b*c*d*
e + b^2*e^2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] + (3*c^(5/2)*d^(5/2)*(16*c^2*d^
2 - 36*b*c*d*e + 21*b^2*e^2)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e
]])/(c*d - b*e)^(5/2))/(4*b^5*d^(5/2))
```

Maple [A] time = 0.358, size = 526, normalized size = 1.8

$$-\frac{15\,e^2c^4}{4\,b^3\,(cex+be)^2\left(b^2e^2-2\,bcde+c^2d^2\right)}(ex+d)^{\frac{3}{2}}+3\,\frac{ec^5\,(ex+d)^{3/2}\,d}{b^4\,(cex+be)^2\left(b^2e^2-2\,bcde+c^2d^2\right)}-\frac{17\,e^2c^3}{4\,b^3\,(cex+be)^2\,(be-cd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(c*x^2+b*x)^3,x)
```

```
[Out] -15/4*e^2*c^4/b^3/(c*e*x+b*e)^2/(b^2*e^2-2*b*c*d*e+c^2*d^2)*(e*x+d)^(3/2)+3
*e*c^5/b^4/(c*e*x+b*e)^2/(b^2*e^2-2*b*c*d*e+c^2*d^2)*(e*x+d)^(3/2)*d-17/4*e
^2*c^3/b^3/(c*e*x+b*e)^2/(b*e-c*d)*(e*x+d)^(1/2)+3*e*c^4/b^4/(c*e*x+b*e)^2/
(b*e-c*d)*(e*x+d)^(1/2)*d-63/4*e^2*c^3/b^3/(b^2*e^2-2*b*c*d*e+c^2*d^2)/((b*
e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))+27*e*c^4/b^4/(b
^2*e^2-2*b*c*d*e+c^2*d^2)/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-
c*d)*c)^(1/2))*d-12*c^5/b^5/(b^2*e^2-2*b*c*d*e+c^2*d^2)/((b*e-c*d)*c)^(1/2)
*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))*d^2+3/4/b^3/x^2/d^2*(e*x+d)^(3
/2)+3/e/b^4/x^2/d*(e*x+d)^(3/2)*c-5/4/b^3/x^2/d*(e*x+d)^(1/2)-3/e/b^4/x^2*(
e*x+d)^(1/2)*c-3/4*e^2/b^3/d^(5/2)*arctanh((e*x+d)^(1/2)/d^(1/2))-3*e/b^4/d
^(3/2)*arctanh((e*x+d)^(1/2)/d^(1/2))*c-12/b^5/d^(1/2)*arctanh((e*x+d)^(1/2
)/d^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 11.389, size = 5762, normalized size = 19.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*((16*c^6*d^5 - 36*b*c^5*d^4*e + 21*b^2*c^4*d^3*e^2)*x^4 + 2*(16*b*c
^5*d^5 - 36*b^2*c^4*d^4*e + 21*b^3*c^3*d^3*e^2)*x^3 + (16*b^2*c^4*d^5 - 36*
```

```
b^3*c^3*d^4*e + 21*b^4*c^2*d^3*e^2)*x^2)*sqrt(c/(c*d - b*e))*log((c*e*x + 2
*c*d - b*e + 2*(c*d - b*e)*sqrt(e*x + d)*sqrt(c/(c*d - b*e)))/(c*x + b)) +
3*((16*c^6*d^4 - 28*b*c^5*d^3*e + 9*b^2*c^4*d^2*e^2 + 2*b^3*c^3*d*e^3 + b^4
*c^2*e^4)*x^4 + 2*(16*b*c^5*d^4 - 28*b^2*c^4*d^3*e + 9*b^3*c^3*d^2*e^2 + 2*
b^4*c^2*d*e^3 + b^5*c*e^4)*x^3 + (16*b^2*c^4*d^4 - 28*b^3*c^3*d^3*e + 9*b^4
*c^2*d^2*e^2 + 2*b^5*c*d*e^3 + b^6*e^4)*x^2)*sqrt(d)*log((e*x - 2*sqrt(e*x
+ d)*sqrt(d) + 2*d)/x) - 2*(2*b^4*c^2*d^4 - 4*b^5*c*d^3*e + 2*b^6*d^2*e^2 -
 3*(8*b*c^5*d^4 - 12*b^2*c^4*d^3*e + 2*b^3*c^3*d^2*e^2 + b^4*c^2*d*e^3)*x^3
 - (36*b^2*c^4*d^4 - 55*b^3*c^3*d^3*e + 10*b^4*c^2*d^2*e^2 + 6*b^5*c*d*e^3)
*x^2 - (8*b^3*c^3*d^4 - 13*b^4*c^2*d^3*e + 2*b^5*c*d^2*e^2 + 3*b^6*d*e^3)*x
)*sqrt(e*x + d))/((b^5*c^4*d^5 - 2*b^6*c^3*d^4*e + b^7*c^2*d^3*e^2)*x^4 + 2
*(b^6*c^3*d^5 - 2*b^7*c^2*d^4*e + b^8*c*d^3*e^2)*x^3 + (b^7*c^2*d^5 - 2*b^8
*c*d^4*e + b^9*d^3*e^2)*x^2), 1/8*(6*((16*c^6*d^5 - 36*b*c^5*d^4*e + 21*b^2
*c^4*d^3*e^2)*x^4 + 2*(16*b*c^5*d^5 - 36*b^2*c^4*d^4*e + 21*b^3*c^3*d^3*e^2
)*x^3 + (16*b^2*c^4*d^5 - 36*b^3*c^3*d^4*e + 21*b^4*c^2*d^3*e^2)*x^2)*sqrt(
-c/(c*d - b*e))*arctan(-(c*d - b*e)*sqrt(e*x + d)*sqrt(-c/(c*d - b*e))/(c*e
*x + c*d)) + 3*((16*c^6*d^4 - 28*b*c^5*d^3*e + 9*b^2*c^4*d^2*e^2 + 2*b^3*c^
3*d*e^3 + b^4*c^2*e^4)*x^4 + 2*(16*b*c^5*d^4 - 28*b^2*c^4*d^3*e + 9*b^3*c^3
*d^2*e^2 + 2*b^4*c^2*d*e^3 + b^5*c*e^4)*x^3 + (16*b^2*c^4*d^4 - 28*b^3*c^3*
d^3*e + 9*b^4*c^2*d^2*e^2 + 2*b^5*c*d*e^3 + b^6*e^4)*x^2)*sqrt(d)*log((e*x
- 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) - 2*(2*b^4*c^2*d^4 - 4*b^5*c*d^3*e + 2*
b^6*d^2*e^2 - 3*(8*b*c^5*d^4 - 12*b^2*c^4*d^3*e + 2*b^3*c^3*d^2*e^2 + b^4*c
^2*d*e^3)*x^3 - (36*b^2*c^4*d^4 - 55*b^3*c^3*d^3*e + 10*b^4*c^2*d^2*e^2 + 6
*b^5*c*d*e^3)*x^2 - (8*b^3*c^3*d^4 - 13*b^4*c^2*d^3*e + 2*b^5*c*d^2*e^2 + 3
*b^6*d*e^3)*x)*sqrt(e*x + d))/((b^5*c^4*d^5 - 2*b^6*c^3*d^4*e + b^7*c^2*d^3
*e^2)*x^4 + 2*(b^6*c^3*d^5 - 2*b^7*c^2*d^4*e + b^8*c*d^3*e^2)*x^3 + (b^7*c^
2*d^5 - 2*b^8*c*d^4*e + b^9*d^3*e^2)*x^2), 1/8*(6*((16*c^6*d^4 - 28*b*c^5*d
^3*e + 9*b^2*c^4*d^2*e^2 + 2*b^3*c^3*d*e^3 + b^4*c^2*e^4)*x^4 + 2*(16*b*c^5
*d^4 - 28*b^2*c^4*d^3*e + 9*b^3*c^3*d^2*e^2 + 2*b^4*c^2*d*e^3 + b^5*c*e^4)*
x^3 + (16*b^2*c^4*d^4 - 28*b^3*c^3*d^3*e + 9*b^4*c^2*d^2*e^2 + 2*b^5*c*d*e^
3 + b^6*e^4)*x^2)*sqrt(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d) + 3*((16*c^6*d^
5 - 36*b*c^5*d^4*e + 21*b^2*c^4*d^3*e^2)*x^4 + 2*(16*b*c^5*d^5 - 36*b^2*c^4
*d^4*e + 21*b^3*c^3*d^3*e^2)*x^3 + (16*b^2*c^4*d^5 - 36*b^3*c^3*d^4*e + 21*
b^4*c^2*d^3*e^2)*x^2)*sqrt(c/(c*d - b*e))*log((c*e*x + 2*c*d - b*e + 2*(c*d
 - b*e)*sqrt(e*x + d)*sqrt(c/(c*d - b*e)))/(c*x + b)) - 2*(2*b^4*c^2*d^4 -
4*b^5*c*d^3*e + 2*b^6*d^2*e^2 - 3*(8*b*c^5*d^4 - 12*b^2*c^4*d^3*e + 2*b^3*c
^3*d^2*e^2 + b^4*c^2*d*e^3)*x^3 - (36*b^2*c^4*d^4 - 55*b^3*c^3*d^3*e + 10*b
^4*c^2*d^2*e^2 + 6*b^5*c*d*e^3)*x^2 - (8*b^3*c^3*d^4 - 13*b^4*c^2*d^3*e + 2
*b^5*c*d^2*e^2 + 3*b^6*d*e^3)*x)*sqrt(e*x + d))/((b^5*c^4*d^5 - 2*b^6*c^3*d
^4*e + b^7*c^2*d^3*e^2)*x^4 + 2*(b^6*c^3*d^5 - 2*b^7*c^2*d^4*e + b^8*c*d^3*
e^2)*x^3 + (b^7*c^2*d^5 - 2*b^8*c*d^4*e + b^9*d^3*e^2)*x^2), 1/4*(3*((16*c^
6*d^5 - 36*b*c^5*d^4*e + 21*b^2*c^4*d^3*e^2)*x^4 + 2*(16*b*c^5*d^5 - 36*b^2
*c^4*d^4*e + 21*b^3*c^3*d^3*e^2)*x^3 + (16*b^2*c^4*d^5 - 36*b^3*c^3*d^4*e +
 21*b^4*c^2*d^3*e^2)*x^2)*sqrt(-c/(c*d - b*e))*arctan(-(c*d - b*e)*sqrt(e*x
 + d)*sqrt(-c/(c*d - b*e))/(c*e*x + c*d)) + 3*((16*c^6*d^4 - 28*b*c^5*d^3*e
 + 9*b^2*c^4*d^2*e^2 + 2*b^3*c^3*d*e^3 + b^4*c^2*e^4)*x^4 + 2*(16*b*c^5*d^4
 - 28*b^2*c^4*d^3*e + 9*b^3*c^3*d^2*e^2 + 2*b^4*c^2*d*e^3 + b^5*c*e^4)*x^3
+ (16*b^2*c^4*d^4 - 28*b^3*c^3*d^3*e + 9*b^4*c^2*d^2*e^2 + 2*b^5*c*d*e^3 +
b^6*e^4)*x^2)*sqrt(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d) - (2*b^4*c^2*d^4 -
4*b^5*c*d^3*e + 2*b^6*d^2*e^2 - 3*(8*b*c^5*d^4 - 12*b^2*c^4*d^3*e + 2*b^3*c
^3*d^2*e^2 + b^4*c^2*d*e^3)*x^3 - (36*b^2*c^4*d^4 - 55*b^3*c^3*d^3*e + 10*b
^4*c^2*d^2*e^2 + 6*b^5*c*d*e^3)*x^2 - (8*b^3*c^3*d^4 - 13*b^4*c^2*d^3*e + 2
*b^5*c*d^2*e^2 + 3*b^6*d*e^3)*x)*sqrt(e*x + d))/((b^5*c^4*d^5 - 2*b^6*c^3*d
^4*e + b^7*c^2*d^3*e^2)*x^4 + 2*(b^6*c^3*d^5 - 2*b^7*c^2*d^4*e + b^8*c*d^3*
e^2)*x^3 + (b^7*c^2*d^5 - 2*b^8*c*d^4*e + b^9*d^3*e^2)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(c*x**2+b*x)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28625, size = 836, normalized size = 2.8

$$-\frac{3\left(16\,c^5d^2-36\,bc^4de+21\,b^2c^3e^2\right)\arctan\left(\frac{\sqrt{xe+d}c}{\sqrt{-c^2d+bce}}\right)}{4\left(b^5c^2d^2-2\,b^6cde+b^7e^2\right)\sqrt{-c^2d+bce}}+\frac{24\,(xe+d)^{\frac{7}{2}}c^5d^3e-72\,(xe+d)^{\frac{5}{2}}c^5d^4e+72\,(xe+d)^{\frac{3}{2}}c^5d^5e-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x)^3,x, algorithm="giac")
```

```
[Out] -3/4*(16*c^5*d^2 - 36*b*c^4*d*e + 21*b^2*c^3*e^2)*arctan(sqrt(x*e + d)*c/sq
rt(-c^2*d + b*c*e))/((b^5*c^2*d^2 - 2*b^6*c*d*e + b^7*e^2)*sqrt(-c^2*d + b*
c*e)) + 1/4*(24*(x*e + d)^(7/2)*c^5*d^3*e - 72*(x*e + d)^(5/2)*c^5*d^4*e +
72*(x*e + d)^(3/2)*c^5*d^5*e - 24*sqrt(x*e + d)*c^5*d^6*e - 36*(x*e + d)^(7
/2)*b*c^4*d^2*e^2 + 144*(x*e + d)^(5/2)*b*c^4*d^3*e^2 - 180*(x*e + d)^(3/2)
*b*c^4*d^4*e^2 + 72*sqrt(x*e + d)*b*c^4*d^5*e^2 + 6*(x*e + d)^(7/2)*b^2*c^3
*d*e^3 - 73*(x*e + d)^(5/2)*b^2*c^3*d^2*e^3 + 136*(x*e + d)^(3/2)*b^2*c^3*d
^3*e^3 - 69*sqrt(x*e + d)*b^2*c^3*d^4*e^3 + 3*(x*e + d)^(7/2)*b^3*c^2*e^4 +
 (x*e + d)^(5/2)*b^3*c^2*d*e^4 - 24*(x*e + d)^(3/2)*b^3*c^2*d^2*e^4 + 18*sq
rt(x*e + d)*b^3*c^2*d^3*e^4 + 6*(x*e + d)^(5/2)*b^4*c*e^5 - 10*(x*e + d)^(3
/2)*b^4*c*d*e^5 + 8*sqrt(x*e + d)*b^4*c*d^2*e^5 + 3*(x*e + d)^(3/2)*b^5*e^6
 - 5*sqrt(x*e + d)*b^5*d*e^6)/((b^4*c^2*d^4 - 2*b^5*c*d^3*e + b^6*d^2*e^2)*
((x*e + d)^2*c - 2*(x*e + d)*c*d + c*d^2 + (x*e + d)*b*e - b*d*e)^2) + 3/4*
(16*c^2*d^2 + 4*b*c*d*e + b^2*e^2)*arctan(sqrt(x*e + d)/sqrt(-d))/(b^5*sqrt
(-d)*d^2)
```

3.384 $\int \frac{1}{(d+ex)^{3/2}(bx+cx^2)^3}\,dx$

Optimal. Leaf size=370

$$\frac{cx(2cd-be)\left(-5b^2e^2-12bcde+12c^2d^2\right)+b\left(5b^3e^3-17bc^2d^2e+12c^3d^3\right)}{4b^4d^2\left(bx+cx^2\right)\sqrt{d+ex}(cd-be)^2}+\frac{3e\left(-b^2e^2-bcde+c^2d^2\right)\left(5b^2e^2-8bcde+8c\right.}{4b^4d^3\sqrt{d+ex}(cd-be)^3}$$

```
[Out] (3*e*(c^2*d^2 - b*c*d*e - b^2*e^2)*(8*c^2*d^2 - 8*b*c*d*e + 5*b^2*e^2))/(4*
b^4*d^3*(c*d - b*e)^3*Sqrt[d + e*x]) - (b*(c*d - b*e) + c*(2*c*d - b*e)*x)/
(2*b^2*d*(c*d - b*e)*Sqrt[d + e*x]*(b*x + c*x^2)^2) + (b*(12*c^3*d^3 - 17*b
*c^2*d^2*e + 5*b^3*e^3) + c*(2*c*d - b*e)*(12*c^2*d^2 - 12*b*c*d*e - 5*b^2*
e^2)*x)/(4*b^4*d^2*(c*d - b*e)^2*Sqrt[d + e*x]*(b*x + c*x^2)) - (3*(16*c^2*
d^2 + 12*b*c*d*e + 5*b^2*e^2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(4*b^5*d^(7/2
)) + (3*c^(7/2)*(16*c^2*d^2 - 44*b*c*d*e + 33*b^2*e^2)*ArcTanh[(Sqrt[c]*Sqr
t[d + e*x])/Sqrt[c*d - b*e]])/(4*b^5*(c*d - b*e)^(7/2))
```

Rubi [A] time = 0.600412, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {740, 822, 828, 826, 1166, 208}

$$\frac{cx(2cd-be)\left(-5b^2e^2-12bcde+12c^2d^2\right)+b\left(5b^3e^3-17bc^2d^2e+12c^3d^3\right)}{4b^4d^2\left(bx+cx^2\right)\sqrt{d+ex}(cd-be)^2}+\frac{3e\left(-b^2e^2-bcde+c^2d^2\right)\left(5b^2e^2-8bcde+8c\right.}{4b^4d^3\sqrt{d+ex}(cd-be)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*(b*x + c*x^2)^3),x]
```


```
[Out] (3*e*(c^2*d^2 - b*c*d*e - b^2*e^2)*(8*c^2*d^2 - 8*b*c*d*e + 5*b^2*e^2))/(4*
b^4*d^3*(c*d - b*e)^3*Sqrt[d + e*x]) - (b*(c*d - b*e) + c*(2*c*d - b*e)*x)/
(2*b^2*d*(c*d - b*e)*Sqrt[d + e*x]*(b*x + c*x^2)^2) + (b*(12*c^3*d^3 - 17*b
*c^2*d^2*e + 5*b^3*e^3) + c*(2*c*d - b*e)*(12*c^2*d^2 - 12*b*c*d*e - 5*b^2*
e^2)*x)/(4*b^4*d^2*(c*d - b*e)^2*Sqrt[d + e*x]*(b*x + c*x^2)) - (3*(16*c^2*
d^2 + 12*b*c*d*e + 5*b^2*e^2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(4*b^5*d^(7/2
)) + (3*c^(7/2)*(16*c^2*d^2 - 44*b*c*d*e + 33*b^2*e^2)*ArcTanh[(Sqrt[c]*Sqr
t[d + e*x])/Sqrt[c*d - b*e]])/(4*b^5*(c*d - b*e)^(7/2))
```


Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
```

```
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\left(bx+cx^2\right)^3}\,dx = -\frac{b(cd-be)+c(2cd-be)x}{2b^2d(cd-be)\sqrt{d+ex}\left(bx+cx^2\right)^2} - \frac{\int \frac{\frac{1}{2}\left(12c^2d^2-5bcde-5b^2e^2\right)+\frac{7}{2}ce(2cd-be)x}{(d+ex)^{3/2}\left(bx+cx^2\right)^2}\,dx}{2b^2d(cd-be)}$$

$$= -\frac{b(cd-be)+c(2cd-be)x}{2b^2d(cd-be)\sqrt{d+ex}\left(bx+cx^2\right)^2} + \frac{b\left(12c^3d^3-17bc^2d^2e+5b^3e^3\right)+c(2cd-be)\left(12c^2d^2\right.}{4b^4d^2(cd-be)^2\sqrt{d+ex}\left(bx+cx\right.}$$

$$= \frac{3e\left(c^2d^2-bcde-b^2e^2\right)\left(8c^2d^2-8bcde+5b^2e^2\right)}{4b^4d^3(cd-be)^3\sqrt{d+ex}} - \frac{b(cd-be)+c(2cd-be)x}{2b^2d(cd-be)\sqrt{d+ex}\left(bx+cx^2\right)^2} + \frac{b(}{}$$

$$= \frac{3e\left(c^2d^2-bcde-b^2e^2\right)\left(8c^2d^2-8bcde+5b^2e^2\right)}{4b^4d^3(cd-be)^3\sqrt{d+ex}} - \frac{b(cd-be)+c(2cd-be)x}{2b^2d(cd-be)\sqrt{d+ex}\left(bx+cx^2\right)^2} + \frac{b(}{}$$

$$= \frac{3e\left(c^2d^2-bcde-b^2e^2\right)\left(8c^2d^2-8bcde+5b^2e^2\right)}{4b^4d^3(cd-be)^3\sqrt{d+ex}} - \frac{b(cd-be)+c(2cd-be)x}{2b^2d(cd-be)\sqrt{d+ex}\left(bx+cx^2\right)^2} + \frac{b(}{}$$

$$= \frac{3e\left(c^2d^2-bcde-b^2e^2\right)\left(8c^2d^2-8bcde+5b^2e^2\right)}{4b^4d^3(cd-be)^3\sqrt{d+ex}} - \frac{b(cd-be)+c(2cd-be)x}{2b^2d(cd-be)\sqrt{d+ex}\left(bx+cx^2\right)^2} + \frac{b(}{}$$

Mathematica [C] time = 0.2893, size = 299, normalized size = 0.81

$$\frac{-x^2\left((b+cx)\left(bcd(be-cd)\left(2b^2cde^2+5b^3e^3-36bc^2d^2e+24c^3d^3\right)-(b+cx)\left(3(cd-be)^3\left(5b^2e^2+12bcde+16c^2d^2\right){}_2F_1\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*(b*x + c*x^2)^3),x]
```

```
[Out] (2*b^4*d^2*(-(c*d) + b*e)^3 + b^3*d*(c*d - b*e)^3*(8*c*d + 5*b*e)*x - x^2*(
b^2*c*d*(c*d - b*e)^2*(-12*c^2*d^2 + 5*b*c*d*e + 5*b^2*e^2) + (b + c*x)*(b*
c*d*(-(c*d) + b*e)*(24*c^3*d^3 - 36*b*c^2*d^2*e + 2*b^2*c*d*e^2 + 5*b^3*e^3
) - (b + c*x)*(-3*c^3*d^3*(16*c^2*d^2 - 44*b*c*d*e + 33*b^2*e^2)*Hypergeome
tric2F1[-1/2, 1, 1/2, (c*(d + e*x))/(c*d - b*e)] + 3*(c*d - b*e)^3*(16*c^2*
d^2 + 12*b*c*d*e + 5*b^2*e^2)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (e*x)/d])
)))/(4*b^5*d^3*(c*d - b*e)^3*x^2*(b + c*x)^2*Sqrt[d + e*x])
```

Maple [A] time = 0.399, size = 530, normalized size = 1.4

$$2\,\frac{e^5}{d^3\,(be-cd)^3\,\sqrt{ex+d}} + \frac{19\,e^2c^5}{4\,b^3\,(be-cd)^3\,(cex+be)^2}\,(ex+d)^{\frac{3}{2}} - 3\,\frac{ec^6\,(ex+d)^{3/2}\,d}{b^4\,(be-cd)^3\,(cex+be)^2} + \frac{21\,e^3c^4}{4\,b^2\,(be-cd)^3\,(cex+be)^2}\sqrt{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(c*x^2+b*x)^3,x)
```

```
[Out] 2*e^5/d^3/(b*e-c*d)^3/(e*x+d)^(1/2)+19/4*e^2*c^5/b^3/(b*e-c*d)^3/(c*e*x+b*e
)^2*(e*x+d)^(3/2)-3*e*c^6/b^4/(b*e-c*d)^3/(c*e*x+b*e)^2*(e*x+d)^(3/2)*d+21/
```

```
4*e^3*c^4/b^2/(b*e-c*d)^3/(c*e*x+b*e)^2*(e*x+d)^(1/2)-33/4*e^2*c^5/b^3/(b*e
-c*d)^3/(c*e*x+b*e)^2*(e*x+d)^(1/2)*d+3*e*c^6/b^4/(b*e-c*d)^3/(c*e*x+b*e)^2
*(e*x+d)^(1/2)*d^2+99/4*e^2*c^4/b^3/(b*e-c*d)^3/((b*e-c*d)*c)^(1/2)*arctan(
(e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))-33*e*c^5/b^4/(b*e-c*d)^3/((b*e-c*d)*c)
^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))*d+12*c^6/b^5/(b*e-c*d)^3
/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))*d^2+7/4/d^
3/b^3/x^2*(e*x+d)^(3/2)+3/e/d^2/b^4/x^2*(e*x+d)^(3/2)*c-9/4/d^2/b^3/x^2*(e*
x+d)^(1/2)-3/e/d/b^4/x^2*(e*x+d)^(1/2)*c-15/4*e^2/d^(7/2)/b^3*arctanh((e*x+
d)^(1/2)/d^(1/2))-9*e/d^(5/2)/b^4*arctanh((e*x+d)^(1/2)/d^(1/2))*c-12/d^(3/
2)/b^5*arctanh((e*x+d)^(1/2)/d^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 39.5685, size = 9646, normalized size = 26.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(3*((16*c^7*d^6*e - 44*b*c^6*d^5*e^2 + 33*b^2*c^5*d^4*e^3)*x^5 + (16*
c^7*d^7 - 12*b*c^6*d^6*e - 55*b^2*c^5*d^5*e^2 + 66*b^3*c^4*d^4*e^3)*x^4 + (
32*b*c^6*d^7 - 72*b^2*c^5*d^6*e + 22*b^3*c^4*d^5*e^2 + 33*b^4*c^3*d^4*e^3)*
x^3 + (16*b^2*c^5*d^7 - 44*b^3*c^4*d^6*e + 33*b^4*c^3*d^5*e^2)*x^2)*sqrt(c/
(c*d - b*e))*log((c*e*x + 2*c*d - b*e - 2*(c*d - b*e)*sqrt(e*x + d)*sqrt(c/
(c*d - b*e)))/(c*x + b)) - 3*((16*c^7*d^5*e - 36*b*c^6*d^4*e^2 + 17*b^2*c^5
*d^3*e^3 + 5*b^3*c^4*d^2*e^4 + 3*b^4*c^3*d*e^5 - 5*b^5*c^2*e^6)*x^5 + (16*c
^7*d^6 - 4*b*c^6*d^5*e - 55*b^2*c^5*d^4*e^2 + 39*b^3*c^4*d^3*e^3 + 13*b^4*c
^3*d^2*e^4 + b^5*c^2*d*e^5 - 10*b^6*c*e^6)*x^4 + (32*b*c^6*d^6 - 56*b^2*c^5
*d^5*e - 2*b^3*c^4*d^4*e^2 + 27*b^4*c^3*d^3*e^3 + 11*b^5*c^2*d^2*e^4 - 7*b^
6*c*d*e^5 - 5*b^7*e^6)*x^3 + (16*b^2*c^5*d^6 - 36*b^3*c^4*d^5*e + 17*b^4*c^
3*d^4*e^2 + 5*b^5*c^2*d^3*e^3 + 3*b^6*c*d^2*e^4 - 5*b^7*d*e^5)*x^2)*sqrt(d)
*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + 2*(2*b^4*c^3*d^6 - 6*b^5*c^
2*d^5*e + 6*b^6*c*d^4*e^2 - 2*b^7*d^3*e^3 - 3*(8*b*c^6*d^5*e - 16*b^2*c^5*d
^4*e^2 + 5*b^3*c^4*d^3*e^3 + 3*b^4*c^3*d^2*e^4 - 5*b^5*c^2*d*e^5)*x^4 - (24
*b*c^6*d^6 - 12*b^2*c^5*d^5*e - 58*b^3*c^4*d^4*e^2 + 33*b^4*c^3*d^3*e^3 + 1
3*b^5*c^2*d^2*e^4 - 30*b^6*c*d*e^5)*x^3 - (36*b^2*c^5*d^6 - 65*b^3*c^4*d^5*
e + 7*b^4*c^3*d^4*e^2 + 23*b^5*c^2*d^3*e^3 - b^6*c*d^2*e^4 - 15*b^7*d*e^5)*
x^2 - (8*b^3*c^4*d^6 - 19*b^4*c^3*d^5*e + 9*b^5*c^2*d^4*e^2 + 7*b^6*c*d^3*e
^3 - 5*b^7*d^2*e^4)*x)*sqrt(e*x + d))/((b^5*c^5*d^7*e - 3*b^6*c^4*d^6*e^2 +
 3*b^7*c^3*d^5*e^3 - b^8*c^2*d^4*e^4)*x^5 + (b^5*c^5*d^8 - b^6*c^4*d^7*e -
3*b^7*c^3*d^6*e^2 + 5*b^8*c^2*d^5*e^3 - 2*b^9*c*d^4*e^4)*x^4 + (2*b^6*c^4*d
^8 - 5*b^7*c^3*d^7*e + 3*b^8*c^2*d^6*e^2 + b^9*c*d^5*e^3 - b^10*d^4*e^4)*x^
3 + (b^7*c^3*d^8 - 3*b^8*c^2*d^7*e + 3*b^9*c*d^6*e^2 - b^10*d^5*e^3)*x^2),
1/8*(6*((16*c^7*d^6*e - 44*b*c^6*d^5*e^2 + 33*b^2*c^5*d^4*e^3)*x^5 + (16*c^
7*d^7 - 12*b*c^6*d^6*e - 55*b^2*c^5*d^5*e^2 + 66*b^3*c^4*d^4*e^3)*x^4 + (32
```

```
*b*c^6*d^7 - 72*b^2*c^5*d^6*e + 22*b^3*c^4*d^5*e^2 + 33*b^4*c^3*d^4*e^3)*x^
3 + (16*b^2*c^5*d^7 - 44*b^3*c^4*d^6*e + 33*b^4*c^3*d^5*e^2)*x^2)*sqrt(-c/(
c*d - b*e))*arctan(-(c*d - b*e)*sqrt(e*x + d)*sqrt(-c/(c*d - b*e))/(c*e*x +
 c*d)) + 3*((16*c^7*d^5*e - 36*b*c^6*d^4*e^2 + 17*b^2*c^5*d^3*e^3 + 5*b^3*c
^4*d^2*e^4 + 3*b^4*c^3*d*e^5 - 5*b^5*c^2*e^6)*x^5 + (16*c^7*d^6 - 4*b*c^6*d
^5*e - 55*b^2*c^5*d^4*e^2 + 39*b^3*c^4*d^3*e^3 + 13*b^4*c^3*d^2*e^4 + b^5*c
^2*d*e^5 - 10*b^6*c*e^6)*x^4 + (32*b*c^6*d^6 - 56*b^2*c^5*d^5*e - 2*b^3*c^4
*d^4*e^2 + 27*b^4*c^3*d^3*e^3 + 11*b^5*c^2*d^2*e^4 - 7*b^6*c*d*e^5 - 5*b^7*
e^6)*x^3 + (16*b^2*c^5*d^6 - 36*b^3*c^4*d^5*e + 17*b^4*c^3*d^4*e^2 + 5*b^5*
c^2*d^3*e^3 + 3*b^6*c*d^2*e^4 - 5*b^7*d*e^5)*x^2)*sqrt(d)*log((e*x - 2*sqrt
(e*x + d)*sqrt(d) + 2*d)/x) - 2*(2*b^4*c^3*d^6 - 6*b^5*c^2*d^5*e + 6*b^6*c*
d^4*e^2 - 2*b^7*d^3*e^3 - 3*(8*b*c^6*d^5*e - 16*b^2*c^5*d^4*e^2 + 5*b^3*c^4
*d^3*e^3 + 3*b^4*c^3*d^2*e^4 - 5*b^5*c^2*d*e^5)*x^4 - (24*b*c^6*d^6 - 12*b^
2*c^5*d^5*e - 58*b^3*c^4*d^4*e^2 + 33*b^4*c^3*d^3*e^3 + 13*b^5*c^2*d^2*e^4
- 30*b^6*c*d*e^5)*x^3 - (36*b^2*c^5*d^6 - 65*b^3*c^4*d^5*e + 7*b^4*c^3*d^4*
e^2 + 23*b^5*c^2*d^3*e^3 - b^6*c*d^2*e^4 - 15*b^7*d*e^5)*x^2 - (8*b^3*c^4*d
^6 - 19*b^4*c^3*d^5*e + 9*b^5*c^2*d^4*e^2 + 7*b^6*c*d^3*e^3 - 5*b^7*d^2*e^4
)*x)*sqrt(e*x + d))/((b^5*c^5*d^7*e - 3*b^6*c^4*d^6*e^2 + 3*b^7*c^3*d^5*e^3
 - b^8*c^2*d^4*e^4)*x^5 + (b^5*c^5*d^8 - b^6*c^4*d^7*e - 3*b^7*c^3*d^6*e^2
+ 5*b^8*c^2*d^5*e^3 - 2*b^9*c*d^4*e^4)*x^4 + (2*b^6*c^4*d^8 - 5*b^7*c^3*d^7
*e + 3*b^8*c^2*d^6*e^2 + b^9*c*d^5*e^3 - b^10*d^4*e^4)*x^3 + (b^7*c^3*d^8 -
 3*b^8*c^2*d^7*e + 3*b^9*c*d^6*e^2 - b^10*d^5*e^3)*x^2), 1/8*(6*((16*c^7*d^
5*e - 36*b*c^6*d^4*e^2 + 17*b^2*c^5*d^3*e^3 + 5*b^3*c^4*d^2*e^4 + 3*b^4*c^3
*d*e^5 - 5*b^5*c^2*e^6)*x^5 + (16*c^7*d^6 - 4*b*c^6*d^5*e - 55*b^2*c^5*d^4*
e^2 + 39*b^3*c^4*d^3*e^3 + 13*b^4*c^3*d^2*e^4 + b^5*c^2*d*e^5 - 10*b^6*c*e^
6)*x^4 + (32*b*c^6*d^6 - 56*b^2*c^5*d^5*e - 2*b^3*c^4*d^4*e^2 + 27*b^4*c^3*
d^3*e^3 + 11*b^5*c^2*d^2*e^4 - 7*b^6*c*d*e^5 - 5*b^7*e^6)*x^3 + (16*b^2*c^5
*d^6 - 36*b^3*c^4*d^5*e + 17*b^4*c^3*d^4*e^2 + 5*b^5*c^2*d^3*e^3 + 3*b^6*c*
d^2*e^4 - 5*b^7*d*e^5)*x^2)*sqrt(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d) - 3*(
(16*c^7*d^6*e - 44*b*c^6*d^5*e^2 + 33*b^2*c^5*d^4*e^3)*x^5 + (16*c^7*d^7 -
12*b*c^6*d^6*e - 55*b^2*c^5*d^5*e^2 + 66*b^3*c^4*d^4*e^3)*x^4 + (32*b*c^6*d
^7 - 72*b^2*c^5*d^6*e + 22*b^3*c^4*d^5*e^2 + 33*b^4*c^3*d^4*e^3)*x^3 + (16*
b^2*c^5*d^7 - 44*b^3*c^4*d^6*e + 33*b^4*c^3*d^5*e^2)*x^2)*sqrt(c/(c*d - b*e
))*log((c*e*x + 2*c*d - b*e - 2*(c*d - b*e)*sqrt(e*x + d)*sqrt(c/(c*d - b*e
)))/(c*x + b)) - 2*(2*b^4*c^3*d^6 - 6*b^5*c^2*d^5*e + 6*b^6*c*d^4*e^2 - 2*b
^7*d^3*e^3 - 3*(8*b*c^6*d^5*e - 16*b^2*c^5*d^4*e^2 + 5*b^3*c^4*d^3*e^3 + 3*
b^4*c^3*d^2*e^4 - 5*b^5*c^2*d*e^5)*x^4 - (24*b*c^6*d^6 - 12*b^2*c^5*d^5*e -
 58*b^3*c^4*d^4*e^2 + 33*b^4*c^3*d^3*e^3 + 13*b^5*c^2*d^2*e^4 - 30*b^6*c*d*
e^5)*x^3 - (36*b^2*c^5*d^6 - 65*b^3*c^4*d^5*e + 7*b^4*c^3*d^4*e^2 + 23*b^5*
c^2*d^3*e^3 - b^6*c*d^2*e^4 - 15*b^7*d*e^5)*x^2 - (8*b^3*c^4*d^6 - 19*b^4*c
^3*d^5*e + 9*b^5*c^2*d^4*e^2 + 7*b^6*c*d^3*e^3 - 5*b^7*d^2*e^4)*x)*sqrt(e*x
 + d))/((b^5*c^5*d^7*e - 3*b^6*c^4*d^6*e^2 + 3*b^7*c^3*d^5*e^3 - b^8*c^2*d^
4*e^4)*x^5 + (b^5*c^5*d^8 - b^6*c^4*d^7*e - 3*b^7*c^3*d^6*e^2 + 5*b^8*c^2*d
^5*e^3 - 2*b^9*c*d^4*e^4)*x^4 + (2*b^6*c^4*d^8 - 5*b^7*c^3*d^7*e + 3*b^8*c^
2*d^6*e^2 + b^9*c*d^5*e^3 - b^10*d^4*e^4)*x^3 + (b^7*c^3*d^8 - 3*b^8*c^2*d^
7*e + 3*b^9*c*d^6*e^2 - b^10*d^5*e^3)*x^2), 1/4*(3*((16*c^7*d^6*e - 44*b*c^
6*d^5*e^2 + 33*b^2*c^5*d^4*e^3)*x^5 + (16*c^7*d^7 - 12*b*c^6*d^6*e - 55*b^2
*c^5*d^5*e^2 + 66*b^3*c^4*d^4*e^3)*x^4 + (32*b*c^6*d^7 - 72*b^2*c^5*d^6*e +
 22*b^3*c^4*d^5*e^2 + 33*b^4*c^3*d^4*e^3)*x^3 + (16*b^2*c^5*d^7 - 44*b^3*c^
4*d^6*e + 33*b^4*c^3*d^5*e^2)*x^2)*sqrt(-c/(c*d - b*e))*arctan(-(c*d - b*e)
*sqrt(e*x + d)*sqrt(-c/(c*d - b*e))/(c*e*x + c*d)) + 3*((16*c^7*d^5*e - 36*
b*c^6*d^4*e^2 + 17*b^2*c^5*d^3*e^3 + 5*b^3*c^4*d^2*e^4 + 3*b^4*c^3*d*e^5 -
5*b^5*c^2*e^6)*x^5 + (16*c^7*d^6 - 4*b*c^6*d^5*e - 55*b^2*c^5*d^4*e^2 + 39*
b^3*c^4*d^3*e^3 + 13*b^4*c^3*d^2*e^4 + b^5*c^2*d*e^5 - 10*b^6*c*e^6)*x^4 +
(32*b*c^6*d^6 - 56*b^2*c^5*d^5*e - 2*b^3*c^4*d^4*e^2 + 27*b^4*c^3*d^3*e^3 +
 11*b^5*c^2*d^2*e^4 - 7*b^6*c*d*e^5 - 5*b^7*e^6)*x^3 + (16*b^2*c^5*d^6 - 36
*b^3*c^4*d^5*e + 17*b^4*c^3*d^4*e^2 + 5*b^5*c^2*d^3*e^3 + 3*b^6*c*d^2*e^4 -
 5*b^7*d*e^5)*x^2)*sqrt(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d) - (2*b^4*c^3*d
^6 - 6*b^5*c^2*d^5*e + 6*b^6*c*d^4*e^2 - 2*b^7*d^3*e^3 - 3*(8*b*c^6*d^5*e -
```

```
16*b^2*c^5*d^4*e^2 + 5*b^3*c^4*d^3*e^3 + 3*b^4*c^3*d^2*e^4 - 5*b^5*c^2*d*e
^5)*x^4 - (24*b*c^6*d^6 - 12*b^2*c^5*d^5*e - 58*b^3*c^4*d^4*e^2 + 33*b^4*c^
3*d^3*e^3 + 13*b^5*c^2*d^2*e^4 - 30*b^6*c*d*e^5)*x^3 - (36*b^2*c^5*d^6 - 65
*b^3*c^4*d^5*e + 7*b^4*c^3*d^4*e^2 + 23*b^5*c^2*d^3*e^3 - b^6*c*d^2*e^4 - 1
5*b^7*d*e^5)*x^2 - (8*b^3*c^4*d^6 - 19*b^4*c^3*d^5*e + 9*b^5*c^2*d^4*e^2 +
7*b^6*c*d^3*e^3 - 5*b^7*d^2*e^4)*x)*sqrt(e*x + d))/((b^5*c^5*d^7*e - 3*b^6*
c^4*d^6*e^2 + 3*b^7*c^3*d^5*e^3 - b^8*c^2*d^4*e^4)*x^5 + (b^5*c^5*d^8 - b^6
*c^4*d^7*e - 3*b^7*c^3*d^6*e^2 + 5*b^8*c^2*d^5*e^3 - 2*b^9*c*d^4*e^4)*x^4 +
 (2*b^6*c^4*d^8 - 5*b^7*c^3*d^7*e + 3*b^8*c^2*d^6*e^2 + b^9*c*d^5*e^3 - b^1
0*d^4*e^4)*x^3 + (b^7*c^3*d^8 - 3*b^8*c^2*d^7*e + 3*b^9*c*d^6*e^2 - b^10*d^
5*e^3)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(c*x**2+b*x)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.44012, size = 1062, normalized size = 2.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^3,x, algorithm="giac")
```

```
[Out] -3/4*(16*c^6*d^2 - 44*b*c^5*d*e + 33*b^2*c^4*e^2)*arctan(sqrt(x*e + d)*c/sq
rt(-c^2*d + b*c*e))/((b^5*c^3*d^3 - 3*b^6*c^2*d^2*e + 3*b^7*c*d*e^2 - b^8*e
^3)*sqrt(-c^2*d + b*c*e)) - 2*e^5/((c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e
^2 - b^3*d^3*e^3)*sqrt(x*e + d)) + 1/4*(24*(x*e + d)^(7/2)*c^6*d^4*e - 72*(
x*e + d)^(5/2)*c^6*d^5*e + 72*(x*e + d)^(3/2)*c^6*d^6*e - 24*sqrt(x*e + d)*
c^6*d^7*e - 48*(x*e + d)^(7/2)*b*c^5*d^3*e^2 + 180*(x*e + d)^(5/2)*b*c^5*d^
4*e^2 - 216*(x*e + d)^(3/2)*b*c^5*d^5*e^2 + 84*sqrt(x*e + d)*b*c^5*d^6*e^2
+ 15*(x*e + d)^(7/2)*b^2*c^4*d^2*e^3 - 118*(x*e + d)^(5/2)*b^2*c^4*d^3*e^3
+ 199*(x*e + d)^(3/2)*b^2*c^4*d^4*e^3 - 96*sqrt(x*e + d)*b^2*c^4*d^5*e^3 +
9*(x*e + d)^(7/2)*b^3*c^3*d*e^4 - 3*(x*e + d)^(5/2)*b^3*c^3*d^2*e^4 - 38*(x
*e + d)^(3/2)*b^3*c^3*d^3*e^4 + 30*sqrt(x*e + d)*b^3*c^3*d^4*e^4 - 7*(x*e +
 d)^(7/2)*b^4*c^2*e^5 + 41*(x*e + d)^(5/2)*b^4*c^2*d*e^5 - 58*(x*e + d)^(3/
2)*b^4*c^2*d^2*e^5 + 30*sqrt(x*e + d)*b^4*c^2*d^3*e^5 - 14*(x*e + d)^(5/2)*
b^5*c*e^6 + 41*(x*e + d)^(3/2)*b^5*c*d*e^6 - 33*sqrt(x*e + d)*b^5*c*d^2*e^6
 - 7*(x*e + d)^(3/2)*b^6*e^7 + 9*sqrt(x*e + d)*b^6*d*e^7)/((b^4*c^3*d^6 - 3
*b^5*c^2*d^5*e + 3*b^6*c*d^4*e^2 - b^7*d^3*e^3)*((x*e + d)^2*c - 2*(x*e + d
)*c*d + c*d^2 + (x*e + d)*b*e - b*d*e)^2) + 3/4*(16*c^2*d^2 + 12*b*c*d*e +
5*b^2*e^2)*arctan(sqrt(x*e + d)/sqrt(-d))/(b^5*sqrt(-d)*d^3)
```

3.385 $\int \frac{1}{(d+ex)^{5/2}\left(bx+cx^2\right)^3}\,dx$

Optimal. Leaf size=470

$$\frac{cx(2cd-be)\left(-7b^2e^2-12bcde+12c^2d^2\right)+b(cd-be)\left(-7b^2e^2-3bcde+12c^2d^2\right)}{4b^4d^2\left(bx+cx^2\right)(d+ex)^{3/2}(cd-be)^2}+\frac{e(2cd-be)\left(2b^2c^2d^2e^2+10b^3cde^3-3\right.}{4b^4d^4\sqrt{d+ex}(cd}$$

```
[Out] (e*(72*c^4*d^4 - 144*b*c^3*d^3*e + 27*b^2*c^2*d^2*e^2 + 45*b^3*c*d*e^3 - 35
*b^4*e^4))/(12*b^4*d^3*(c*d - b*e)^3*(d + e*x)^(3/2)) + (e*(2*c*d - b*e)*(1
2*c^4*d^4 - 24*b*c^3*d^3*e + 2*b^2*c^2*d^2*e^2 + 10*b^3*c*d*e^3 - 35*b^4*e^
4))/(4*b^4*d^4*(c*d - b*e)^4*Sqrt[d + e*x]) - (b*(c*d - b*e) + c*(2*c*d - b
*e)*x)/(2*b^2*d*(c*d - b*e)*(d + e*x)^(3/2)*(b*x + c*x^2)^2) + (b*(c*d - b*
e)*(12*c^2*d^2 - 3*b*c*d*e - 7*b^2*e^2) + c*(2*c*d - b*e)*(12*c^2*d^2 - 12*
b*c*d*e - 7*b^2*e^2)*x)/(4*b^4*d^2*(c*d - b*e)^2*(d + e*x)^(3/2)*(b*x + c*x
^2)) - ((48*c^2*d^2 + 60*b*c*d*e + 35*b^2*e^2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d
]])/(4*b^5*d^(9/2)) + (c^(9/2)*(48*c^2*d^2 - 156*b*c*d*e + 143*b^2*e^2)*Arc
Tanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(4*b^5*(c*d - b*e)^(9/2))
```

Rubi [A] time = 0.878529, antiderivative size = 470, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {740, 822, 828, 826, 1166, 208}

$$\frac{cx(2cd-be)\left(-7b^2e^2-12bcde+12c^2d^2\right)+b(cd-be)\left(-7b^2e^2-3bcde+12c^2d^2\right)}{4b^4d^2\left(bx+cx^2\right)(d+ex)^{3/2}(cd-be)^2}+\frac{e(2cd-be)\left(2b^2c^2d^2e^2+10b^3cde^3-3\right.}{4b^4d^4\sqrt{d+ex}(cd}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(5/2)*(b*x + c*x^2)^3),x]
```


```
[Out] (e*(72*c^4*d^4 - 144*b*c^3*d^3*e + 27*b^2*c^2*d^2*e^2 + 45*b^3*c*d*e^3 - 35
*b^4*e^4))/(12*b^4*d^3*(c*d - b*e)^3*(d + e*x)^(3/2)) + (e*(2*c*d - b*e)*(1
2*c^4*d^4 - 24*b*c^3*d^3*e + 2*b^2*c^2*d^2*e^2 + 10*b^3*c*d*e^3 - 35*b^4*e^
4))/(4*b^4*d^4*(c*d - b*e)^4*Sqrt[d + e*x]) - (b*(c*d - b*e) + c*(2*c*d - b
*e)*x)/(2*b^2*d*(c*d - b*e)*(d + e*x)^(3/2)*(b*x + c*x^2)^2) + (b*(c*d - b*
e)*(12*c^2*d^2 - 3*b*c*d*e - 7*b^2*e^2) + c*(2*c*d - b*e)*(12*c^2*d^2 - 12*
b*c*d*e - 7*b^2*e^2)*x)/(4*b^4*d^2*(c*d - b*e)^2*(d + e*x)^(3/2)*(b*x + c*x
^2)) - ((48*c^2*d^2 + 60*b*c*d*e + 35*b^2*e^2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d
]])/(4*b^5*d^(9/2)) + (c^(9/2)*(48*c^2*d^2 - 156*b*c*d*e + 143*b^2*e^2)*Arc
Tanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(4*b^5*(c*d - b*e)^(9/2))
```


Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{5/2}\left(bx+cx^2\right)^3}\,dx = -\frac{b(cd-be)+c(2cd-be)x}{2b^2d(cd-be)(d+ex)^{3/2}\left(bx+cx^2\right)^2} - \frac{\int \frac{\frac{1}{2}\left(12c^2d^2-3bcde-7b^2e^2\right)+\frac{9}{2}ce(2cd-be)x}{(d+ex)^{5/2}\left(bx+cx^2\right)^2}dx}{2b^2d(cd-be)}$$

$$= -\frac{b(cd-be)+c(2cd-be)x}{2b^2d(cd-be)(d+ex)^{3/2}\left(bx+cx^2\right)^2} + \frac{b(cd-be)\left(12c^2d^2-3bcde-7b^2e^2\right)+c(2cd-be)}{4b^4d^2(cd-be)^2(d+ex)^{3/2}\left(b\right.}$$

$$= \frac{e\left(72c^4d^4-144bc^3d^3e+27b^2c^2d^2e^2+45b^3cde^3-35b^4e^4\right)}{12b^4d^3(cd-be)^3(d+ex)^{3/2}} - \frac{b(cd-be)+c(2cd-be}{2b^2d(cd-be)(d+ex)^{3/2}\left(bx\right.}$$

$$= \frac{e\left(72c^4d^4-144bc^3d^3e+27b^2c^2d^2e^2+45b^3cde^3-35b^4e^4\right)}{12b^4d^3(cd-be)^3(d+ex)^{3/2}} + \frac{e(2cd-be)\left(12c^4d^4-24bc^3d\right.}{4b^4d^4(c}$$

$$= \frac{e\left(72c^4d^4-144bc^3d^3e+27b^2c^2d^2e^2+45b^3cde^3-35b^4e^4\right)}{12b^4d^3(cd-be)^3(d+ex)^{3/2}} + \frac{e(2cd-be)\left(12c^4d^4-24bc^3d\right.}{4b^4d^4(c}$$

$$= \frac{e\left(72c^4d^4-144bc^3d^3e+27b^2c^2d^2e^2+45b^3cde^3-35b^4e^4\right)}{12b^4d^3(cd-be)^3(d+ex)^{3/2}} + \frac{e(2cd-be)\left(12c^4d^4-24bc^3d\right.}{4b^4d^4(c}$$

$$= \frac{e\left(72c^4d^4-144bc^3d^3e+27b^2c^2d^2e^2+45b^3cde^3-35b^4e^4\right)}{12b^4d^3(cd-be)^3(d+ex)^{3/2}} + \frac{e(2cd-be)\left(12c^4d^4-24bc^3d\right.}{4b^4d^4(c}$$

Mathematica [C] time = 0.293585, size = 301, normalized size = 0.64

$$\frac{-x^2\left((b+cx)\left(3bcd(be-cd)\left(-2b^2cde^2+7b^3e^3-36bc^2d^2e+24c^3d^3\right)-(b+cx)\left((cd-be)^3\left(35b^2e^2+60bcde+48c^2d^2\right)\,{}_2\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(5/2)*(b*x + c*x^2)^3),x]
```

```
[Out] (6*b^4*d^2*(-(c*d) + b*e)^3 + 3*b^3*d*(c*d - b*e)^3*(8*c*d + 7*b*e)*x - x^2
*(3*b^2*c*d*(c*d - b*e)^2*(-12*c^2*d^2 + 3*b*c*d*e + 7*b^2*e^2) + (b + c*x)
*(3*b*c*d*(-(c*d) + b*e)*(24*c^3*d^3 - 36*b*c^2*d^2*e - 2*b^2*c*d*e^2 + 7*b
^3*e^3) - (b + c*x)*(-(c^3*d^3*(48*c^2*d^2 - 156*b*c*d*e + 143*b^2*e^2)*Hyp
ergeometric2F1[-3/2, 1, -1/2, (c*(d + e*x))/(c*d - b*e)]) + (c*d - b*e)^3*(
48*c^2*d^2 + 60*b*c*d*e + 35*b^2*e^2)*Hypergeometric2F1[-3/2, 1, -1/2, 1 +
(e*x)/d]))))/(12*b^5*d^3*(c*d - b*e)^3*x^2*(b + c*x)^2*(d + e*x)^(3/2))
```

Maple [A] time = 0.253, size = 582, normalized size = 1.2

$$6\,\frac{e^6 b}{d^4\,(be-cd)^4\sqrt{ex+d}} - 12\,\frac{e^5 c}{d^3\,(be-cd)^4\sqrt{ex+d}} + \frac{2\,e^5}{3\,d^3\,(be-cd)^3}\,(ex+d)^{-\frac{3}{2}} - \frac{23\,e^2c^6}{4\,b^3\,(be-cd)^4\,(cex+be)^2}\,(ex+d)^{\frac{3}{2}} + 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(5/2)/(c*x^2+b*x)^3,x)
```

```
[Out] 6*e^6/d^4/(b*e-c*d)^4/(e*x+d)^(1/2)*b-12*e^5/d^3/(b*e-c*d)^4/(e*x+d)^(1/2)*
c+2/3*e^5/d^3/(b*e-c*d)^3/(e*x+d)^(3/2)-23/4*e^2*c^6/b^3/(b*e-c*d)^4/(c*e*x
+b*e)^2*(e*x+d)^(3/2)+3*e*c^7/b^4/(b*e-c*d)^4/(c*e*x+b*e)^2*(e*x+d)^(3/2)*d
-25/4*e^3*c^5/b^2/(b*e-c*d)^4/(c*e*x+b*e)^2*(e*x+d)^(1/2)+37/4*e^2*c^6/b^3/
(b*e-c*d)^4/(c*e*x+b*e)^2*(e*x+d)^(1/2)*d-3*e*c^7/b^4/(b*e-c*d)^4/(c*e*x+b*
e)^2*(e*x+d)^(1/2)*d^2-143/4*e^2*c^5/b^3/(b*e-c*d)^4/((b*e-c*d)*c)^(1/2)*ar
ctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))+39*e*c^6/b^4/(b*e-c*d)^4/((b*e-c*
d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))*d-12*c^7/b^5/(b*e-c
*d)^4/((b*e-c*d)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((b*e-c*d)*c)^(1/2))*d^2+1
1/4/b^3/d^4/x^2*(e*x+d)^(3/2)+3/e/b^4/d^3/x^2*(e*x+d)^(3/2)*c-13/4/b^3/d^3/
x^2*(e*x+d)^(1/2)-3/e/b^4/d^2/x^2*(e*x+d)^(1/2)*c-35/4*e^2/b^3/d^(9/2)*arct
anh((e*x+d)^(1/2)/d^(1/2))-15*e/b^4/d^(7/2)*arctanh((e*x+d)^(1/2)/d^(1/2))*
c-12/b^5/d^(5/2)*arctanh((e*x+d)^(1/2)/d^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 105.111, size = 14445, normalized size = 30.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(3*((48*c^8*d^7*e^2 - 156*b*c^7*d^6*e^3 + 143*b^2*c^6*d^5*e^4)*x^6 +
2*(48*c^8*d^8*e - 108*b*c^7*d^7*e^2 - 13*b^2*c^6*d^6*e^3 + 143*b^3*c^5*d^5*
e^4)*x^5 + (48*c^8*d^9 + 36*b*c^7*d^8*e - 433*b^2*c^6*d^7*e^2 + 416*b^3*c^5
*d^6*e^3 + 143*b^4*c^4*d^5*e^4)*x^4 + 2*(48*b*c^7*d^9 - 108*b^2*c^6*d^8*e -
 13*b^3*c^5*d^7*e^2 + 143*b^4*c^4*d^6*e^3)*x^3 + (48*b^2*c^6*d^9 - 156*b^3*
c^5*d^8*e + 143*b^4*c^4*d^7*e^2)*x^2)*sqrt(c/(c*d - b*e))*log((c*e*x + 2*c*
d - b*e + 2*(c*d - b*e)*sqrt(e*x + d)*sqrt(c/(c*d - b*e)))/(c*x + b)) + 3*(
(48*c^8*d^6*e^2 - 132*b*c^7*d^5*e^3 + 83*b^2*c^6*d^4*e^4 + 28*b^3*c^5*d^3*e
^5 + 18*b^4*c^4*d^2*e^6 - 80*b^5*c^3*d*e^7 + 35*b^6*c^2*e^8)*x^6 + 2*(48*c^
8*d^7*e - 84*b*c^7*d^6*e^2 - 49*b^2*c^6*d^5*e^3 + 111*b^3*c^5*d^4*e^4 + 46*
b^4*c^4*d^3*e^5 - 62*b^5*c^3*d^2*e^6 - 45*b^6*c^2*d*e^7 + 35*b^7*c*e^8)*x^5
 + (48*c^8*d^8 + 60*b*c^7*d^7*e - 397*b^2*c^6*d^6*e^2 + 228*b^3*c^5*d^5*e^3
 + 213*b^4*c^4*d^4*e^4 + 20*b^5*c^3*d^3*e^5 - 267*b^6*c^2*d^2*e^6 + 60*b^7*
c*d*e^7 + 35*b^8*e^8)*x^4 + 2*(48*b*c^7*d^8 - 84*b^2*c^6*d^7*e - 49*b^3*c^5
*d^6*e^2 + 111*b^4*c^4*d^5*e^3 + 46*b^5*c^3*d^4*e^4 - 62*b^6*c^2*d^3*e^5 -
45*b^7*c*d^2*e^6 + 35*b^8*d*e^7)*x^3 + (48*b^2*c^6*d^8 - 132*b^3*c^5*d^7*e
+ 83*b^4*c^4*d^6*e^2 + 28*b^5*c^3*d^5*e^3 + 18*b^6*c^2*d^4*e^4 - 80*b^7*c*d
^3*e^5 + 35*b^8*d^2*e^6)*x^2)*sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) +
2*d)/x) - 2*(6*b^4*c^4*d^8 - 24*b^5*c^3*d^7*e + 36*b^6*c^2*d^6*e^2 - 24*b^7
*c*d^5*e^3 + 6*b^8*d^4*e^4 - 3*(24*b*c^7*d^6*e^2 - 60*b^2*c^6*d^5*e^3 + 28*
b^3*c^5*d^4*e^4 + 18*b^4*c^4*d^3*e^5 - 80*b^5*c^3*d^2*e^6 + 35*b^6*c^2*d*e^
7)*x^5 - (144*b*c^7*d^7*e - 252*b^2*c^6*d^6*e^2 - 105*b^3*c^5*d^5*e^3 + 240
*b^4*c^4*d^4*e^4 - 212*b^5*c^3*d^3*e^5 - 340*b^6*c^2*d^2*e^6 + 210*b^7*c*d*
```

e^7)*x^4 - (72*b*c^7*d^8 + 36*b^2*c^6*d^7*e - 438*b^3*c^5*d^6*e^2 + 255*b^4
*c^4*d^5*e^3 + 180*b^5*c^3*d^4*e^4 - 565*b^6*c^2*d^3*e^5 + 40*b^7*c*d^2*e^6
 + 105*b^8*d*e^7)*x^3 - (108*b^2*c^6*d^8 - 225*b^3*c^5*d^7*e + 180*b^5*c^3*
d^5*e^3 - 30*b^6*c^2*d^4*e^4 - 278*b^7*c*d^3*e^5 + 140*b^8*d^2*e^6)*x^2 - 3
*(8*b^3*c^5*d^8 - 25*b^4*c^4*d^7*e + 20*b^5*c^3*d^6*e^2 + 10*b^6*c^2*d^5*e^
3 - 20*b^7*c*d^4*e^4 + 7*b^8*d^3*e^5)*x)*sqrt(e*x + d))/((b^5*c^6*d^9*e^2 -
 4*b^6*c^5*d^8*e^3 + 6*b^7*c^4*d^7*e^4 - 4*b^8*c^3*d^6*e^5 + b^9*c^2*d^5*e^
6)*x^6 + 2*(b^5*c^6*d^10*e - 3*b^6*c^5*d^9*e^2 + 2*b^7*c^4*d^8*e^3 + 2*b^8*
c^3*d^7*e^4 - 3*b^9*c^2*d^6*e^5 + b^10*c*d^5*e^6)*x^5 + (b^5*c^6*d^11 - 9*b
^7*c^4*d^9*e^2 + 16*b^8*c^3*d^8*e^3 - 9*b^9*c^2*d^7*e^4 + b^11*d^5*e^6)*x^4
 + 2*(b^6*c^5*d^11 - 3*b^7*c^4*d^10*e + 2*b^8*c^3*d^9*e^2 + 2*b^9*c^2*d^8*e
^3 - 3*b^10*c*d^7*e^4 + b^11*d^6*e^5)*x^3 + (b^7*c^4*d^11 - 4*b^8*c^3*d^10*
e + 6*b^9*c^2*d^9*e^2 - 4*b^10*c*d^8*e^3 + b^11*d^7*e^4)*x^2), 1/24*(6*((48
*c^8*d^7*e^2 - 156*b*c^7*d^6*e^3 + 143*b^2*c^6*d^5*e^4)*x^6 + 2*(48*c^8*d^8
*e - 108*b*c^7*d^7*e^2 - 13*b^2*c^6*d^6*e^3 + 143*b^3*c^5*d^5*e^4)*x^5 + (4
8*c^8*d^9 + 36*b*c^7*d^8*e - 433*b^2*c^6*d^7*e^2 + 416*b^3*c^5*d^6*e^3 + 14
3*b^4*c^4*d^5*e^4)*x^4 + 2*(48*b*c^7*d^9 - 108*b^2*c^6*d^8*e - 13*b^3*c^5*d
^7*e^2 + 143*b^4*c^4*d^6*e^3)*x^3 + (48*b^2*c^6*d^9 - 156*b^3*c^5*d^8*e + 1
43*b^4*c^4*d^7*e^2)*x^2)*sqrt(-c/(c*d - b*e))*arctan(-(c*d - b*e)*sqrt(e*x
+ d)*sqrt(-c/(c*d - b*e))/(c*e*x + c*d)) + 3*((48*c^8*d^6*e^2 - 132*b*c^7*d
^5*e^3 + 83*b^2*c^6*d^4*e^4 + 28*b^3*c^5*d^3*e^5 + 18*b^4*c^4*d^2*e^6 - 80*
b^5*c^3*d*e^7 + 35*b^6*c^2*e^8)*x^6 + 2*(48*c^8*d^7*e - 84*b*c^7*d^6*e^2 -
49*b^2*c^6*d^5*e^3 + 111*b^3*c^5*d^4*e^4 + 46*b^4*c^4*d^3*e^5 - 62*b^5*c^3*
d^2*e^6 - 45*b^6*c^2*d*e^7 + 35*b^7*c*e^8)*x^5 + (48*c^8*d^8 + 60*b*c^7*d^7
*e - 397*b^2*c^6*d^6*e^2 + 228*b^3*c^5*d^5*e^3 + 213*b^4*c^4*d^4*e^4 + 20*b
^5*c^3*d^3*e^5 - 267*b^6*c^2*d^2*e^6 + 60*b^7*c*d*e^7 + 35*b^8*e^8)*x^4 + 2
*(48*b*c^7*d^8 - 84*b^2*c^6*d^7*e - 49*b^3*c^5*d^6*e^2 + 111*b^4*c^4*d^5*e^
3 + 46*b^5*c^3*d^4*e^4 - 62*b^6*c^2*d^3*e^5 - 45*b^7*c*d^2*e^6 + 35*b^8*d*e
^7)*x^3 + (48*b^2*c^6*d^8 - 132*b^3*c^5*d^7*e + 83*b^4*c^4*d^6*e^2 + 28*b^5
*c^3*d^5*e^3 + 18*b^6*c^2*d^4*e^4 - 80*b^7*c*d^3*e^5 + 35*b^8*d^2*e^6)*x^2)
*sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) - 2*(6*b^4*c^4*d^8 -
24*b^5*c^3*d^7*e + 36*b^6*c^2*d^6*e^2 - 24*b^7*c*d^5*e^3 + 6*b^8*d^4*e^4 -
3*(24*b*c^7*d^6*e^2 - 60*b^2*c^6*d^5*e^3 + 28*b^3*c^5*d^4*e^4 + 18*b^4*c^4*
d^3*e^5 - 80*b^5*c^3*d^2*e^6 + 35*b^6*c^2*d*e^7)*x^5 - (144*b*c^7*d^7*e - 2
52*b^2*c^6*d^6*e^2 - 105*b^3*c^5*d^5*e^3 + 240*b^4*c^4*d^4*e^4 - 212*b^5*c^
3*d^3*e^5 - 340*b^6*c^2*d^2*e^6 + 210*b^7*c*d*e^7)*x^4 - (72*b*c^7*d^8 + 36
*b^2*c^6*d^7*e - 438*b^3*c^5*d^6*e^2 + 255*b^4*c^4*d^5*e^3 + 180*b^5*c^3*d^
4*e^4 - 565*b^6*c^2*d^3*e^5 + 40*b^7*c*d^2*e^6 + 105*b^8*d*e^7)*x^3 - (108*
b^2*c^6*d^8 - 225*b^3*c^5*d^7*e + 180*b^5*c^3*d^5*e^3 - 30*b^6*c^2*d^4*e^4
- 278*b^7*c*d^3*e^5 + 140*b^8*d^2*e^6)*x^2 - 3*(8*b^3*c^5*d^8 - 25*b^4*c^4*
d^7*e + 20*b^5*c^3*d^6*e^2 + 10*b^6*c^2*d^5*e^3 - 20*b^7*c*d^4*e^4 + 7*b^8*
d^3*e^5)*x)*sqrt(e*x + d))/((b^5*c^6*d^9*e^2 - 4*b^6*c^5*d^8*e^3 + 6*b^7*c^
4*d^7*e^4 - 4*b^8*c^3*d^6*e^5 + b^9*c^2*d^5*e^6)*x^6 + 2*(b^5*c^6*d^10*e -
3*b^6*c^5*d^9*e^2 + 2*b^7*c^4*d^8*e^3 + 2*b^8*c^3*d^7*e^4 - 3*b^9*c^2*d^6*e
^5 + b^10*c*d^5*e^6)*x^5 + (b^5*c^6*d^11 - 9*b^7*c^4*d^9*e^2 + 16*b^8*c^3*d
^8*e^3 - 9*b^9*c^2*d^7*e^4 + b^11*d^5*e^6)*x^4 + 2*(b^6*c^5*d^11 - 3*b^7*c^
4*d^10*e + 2*b^8*c^3*d^9*e^2 + 2*b^9*c^2*d^8*e^3 - 3*b^10*c*d^7*e^4 + b^11*
d^6*e^5)*x^3 + (b^7*c^4*d^11 - 4*b^8*c^3*d^10*e + 6*b^9*c^2*d^9*e^2 - 4*b^1
0*c*d^8*e^3 + b^11*d^7*e^4)*x^2), 1/24*(6*((48*c^8*d^6*e^2 - 132*b*c^7*d^5*
e^3 + 83*b^2*c^6*d^4*e^4 + 28*b^3*c^5*d^3*e^5 + 18*b^4*c^4*d^2*e^6 - 80*b^5
*c^3*d*e^7 + 35*b^6*c^2*e^8)*x^6 + 2*(48*c^8*d^7*e - 84*b*c^7*d^6*e^2 - 49*
b^2*c^6*d^5*e^3 + 111*b^3*c^5*d^4*e^4 + 46*b^4*c^4*d^3*e^5 - 62*b^5*c^3*d^2
*e^6 - 45*b^6*c^2*d*e^7 + 35*b^7*c*e^8)*x^5 + (48*c^8*d^8 + 60*b*c^7*d^7*e
- 397*b^2*c^6*d^6*e^2 + 228*b^3*c^5*d^5*e^3 + 213*b^4*c^4*d^4*e^4 + 20*b^5*
c^3*d^3*e^5 - 267*b^6*c^2*d^2*e^6 + 60*b^7*c*d*e^7 + 35*b^8*e^8)*x^4 + 2*(4
8*b*c^7*d^8 - 84*b^2*c^6*d^7*e - 49*b^3*c^5*d^6*e^2 + 111*b^4*c^4*d^5*e^3 +
 46*b^5*c^3*d^4*e^4 - 62*b^6*c^2*d^3*e^5 - 45*b^7*c*d^2*e^6 + 35*b^8*d*e^7)
*x^3 + (48*b^2*c^6*d^8 - 132*b^3*c^5*d^7*e + 83*b^4*c^4*d^6*e^2 + 28*b^5*c^
3*d^5*e^3 + 18*b^6*c^2*d^4*e^4 - 80*b^7*c*d^3*e^5 + 35*b^8*d^2*e^6)*x^2)*sq

rt(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d) + 3*((48*c^8*d^7*e^2 - 156*b*c^7*d^6*e^3 + 143*b^2*c^6*d^5*e^4)*x^6 + 2*(48*c^8*d^8*e - 108*b*c^7*d^7*e^2 - 13*b^2*c^6*d^6*e^3 + 143*b^3*c^5*d^5*e^4)*x^5 + (48*c^8*d^9 + 36*b*c^7*d^8*e - 433*b^2*c^6*d^7*e^2 + 416*b^3*c^5*d^6*e^3 + 143*b^4*c^4*d^5*e^4)*x^4 + 2*(48*b*c^7*d^9 - 108*b^2*c^6*d^8*e - 13*b^3*c^5*d^7*e^2 + 143*b^4*c^4*d^6*e^3)*x^3 + (48*b^2*c^6*d^9 - 156*b^3*c^5*d^8*e + 143*b^4*c^4*d^7*e^2)*x^2)*sqrt(c/(c*d - b*e))*log((c*e*x + 2*c*d - b*e + 2*(c*d - b*e)*sqrt(e*x + d)*sqrt(c/(c*d - b*e)))/(c*x + b)) - 2*(6*b^4*c^4*d^8 - 24*b^5*c^3*d^7*e + 36*b^6*c^2*d^6*e^2 - 24*b^7*c*d^5*e^3 + 6*b^8*d^4*e^4 - 3*(24*b*c^7*d^6*e^2 - 60*b^2*c^6*d^5*e^3 + 28*b^3*c^5*d^4*e^4 + 18*b^4*c^4*d^3*e^5 - 80*b^5*c^3*d^2*e^6 + 35*b^6*c^2*d*e^7)*x^5 - (144*b*c^7*d^7*e - 252*b^2*c^6*d^6*e^2 - 105*b^3*c^5*d^5*e^3 + 240*b^4*c^4*d^4*e^4 - 212*b^5*c^3*d^3*e^5 - 340*b^6*c^2*d^2*e^6 + 210*b^7*c*d*e^7)*x^4 - (72*b*c^7*d^8 + 36*b^2*c^6*d^7*e - 438*b^3*c^5*d^6*e^2 + 255*b^4*c^4*d^5*e^3 + 180*b^5*c^3*d^4*e^4 - 565*b^6*c^2*d^3*e^5 + 40*b^7*c*d^2*e^6 + 105*b^8*d*e^7)*x^3 - (108*b^2*c^6*d^8 - 225*b^3*c^5*d^7*e + 180*b^5*c^3*d^5*e^3 - 30*b^6*c^2*d^4*e^4 - 278*b^7*c*d^3*e^5 + 140*b^8*d^2*e^6)*x^2 - 3*(8*b^3*c^5*d^8 - 25*b^4*c^4*d^7*e + 20*b^5*c^3*d^6*e^2 + 10*b^6*c^2*d^5*e^3 - 20*b^7*c*d^4*e^4 + 7*b^8*d^3*e^5)*x)*sqrt(e*x + d))/((b^5*c^6*d^9*e^2 - 4*b^6*c^5*d^8*e^3 + 6*b^7*c^4*d^7*e^4 - 4*b^8*c^3*d^6*e^5 + b^9*c^2*d^5*e^6)*x^6 + 2*(b^5*c^6*d^10*e - 3*b^6*c^5*d^9*e^2 + 2*b^7*c^4*d^8*e^3 + 2*b^8*c^3*d^7*e^4 - 3*b^9*c^2*d^6*e^5 + b^10*c*d^5*e^6)*x^5 + (b^5*c^6*d^11 - 9*b^7*c^4*d^9*e^2 + 16*b^8*c^3*d^8*e^3 - 9*b^9*c^2*d^7*e^4 + b^11*d^5*e^6)*x^4 + 2*(b^6*c^5*d^11 - 3*b^7*c^4*d^10*e + 2*b^8*c^3*d^9*e^2 + 2*b^9*c^2*d^8*e^3 - 3*b^10*c*d^7*e^4 + b^11*d^6*e^5)*x^3 + (b^7*c^4*d^11 - 4*b^8*c^3*d^10*e + 6*b^9*c^2*d^9*e^2 - 4*b^10*c*d^8*e^3 + b^11*d^7*e^4)*x^2), 1/12*(3*((48*c^8*d^7*e^2 - 156*b*c^7*d^6*e^3 + 143*b^2*c^6*d^5*e^4)*x^6 + 2*(48*c^8*d^8*e - 108*b*c^7*d^7*e^2 - 13*b^2*c^6*d^6*e^3 + 143*b^3*c^5*d^5*e^4)*x^5 + (48*c^8*d^9 + 36*b*c^7*d^8*e - 433*b^2*c^6*d^7*e^2 + 416*b^3*c^5*d^6*e^3 + 143*b^4*c^4*d^5*e^4)*x^4 + 2*(48*b*c^7*d^9 - 108*b^2*c^6*d^8*e - 13*b^3*c^5*d^7*e^2 + 143*b^4*c^4*d^6*e^3)*x^3 + (48*b^2*c^6*d^9 - 156*b^3*c^5*d^8*e + 143*b^4*c^4*d^7*e^2)*x^2)*sqrt(-c/(c*d - b*e))*arctan(-(c*d - b*e)*sqrt(e*x + d)*sqrt(-c/(c*d - b*e))/(c*e*x + c*d)) + 3*((48*c^8*d^6*e^2 - 132*b*c^7*d^5*e^3 + 83*b^2*c^6*d^4*e^4 + 28*b^3*c^5*d^3*e^5 + 18*b^4*c^4*d^2*e^6 - 80*b^5*c^3*d*e^7 + 35*b^6*c^2*e^8)*x^6 + 2*(48*c^8*d^7*e - 84*b*c^7*d^6*e^2 - 49*b^2*c^6*d^5*e^3 + 111*b^3*c^5*d^4*e^4 + 46*b^4*c^4*d^3*e^5 - 62*b^5*c^3*d^2*e^6 - 45*b^6*c^2*d*e^7 + 35*b^7*c*e^8)*x^5 + (48*c^8*d^8 + 60*b*c^7*d^7*e - 397*b^2*c^6*d^6*e^2 + 228*b^3*c^5*d^5*e^3 + 213*b^4*c^4*d^4*e^4 + 20*b^5*c^3*d^3*e^5 - 267*b^6*c^2*d^2*e^6 + 60*b^7*c*d*e^7 + 35*b^8*e^8)*x^4 + 2*(48*b*c^7*d^8 - 84*b^2*c^6*d^7*e - 49*b^3*c^5*d^6*e^2 + 111*b^4*c^4*d^5*e^3 + 46*b^5*c^3*d^4*e^4 - 62*b^6*c^2*d^3*e^5 - 45*b^7*c*d^2*e^6 + 35*b^8*d*e^7)*x^3 + (48*b^2*c^6*d^8 - 132*b^3*c^5*d^7*e + 83*b^4*c^4*d^6*e^2 + 28*b^5*c^3*d^5*e^3 + 18*b^6*c^2*d^4*e^4 - 80*b^7*c*d^3*e^5 + 35*b^8*d^2*e^6)*x^2)*sqrt(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d) - (6*b^4*c^4*d^8 - 24*b^5*c^3*d^7*e + 36*b^6*c^2*d^6*e^2 - 24*b^7*c*d^5*e^3 + 6*b^8*d^4*e^4 - 3*(24*b*c^7*d^6*e^2 - 60*b^2*c^6*d^5*e^3 + 28*b^3*c^5*d^4*e^4 + 18*b^4*c^4*d^3*e^5 - 80*b^5*c^3*d^2*e^6 + 35*b^6*c^2*d*e^7)*x^5 - (144*b*c^7*d^7*e - 252*b^2*c^6*d^6*e^2 - 105*b^3*c^5*d^5*e^3 + 240*b^4*c^4*d^4*e^4 - 212*b^5*c^3*d^3*e^5 - 340*b^6*c^2*d^2*e^6 + 210*b^7*c*d*e^7)*x^4 - (72*b*c^7*d^8 + 36*b^2*c^6*d^7*e - 438*b^3*c^5*d^6*e^2 + 255*b^4*c^4*d^5*e^3 + 180*b^5*c^3*d^4*e^4 - 565*b^6*c^2*d^3*e^5 + 40*b^7*c*d^2*e^6 + 105*b^8*d*e^7)*x^3 - (108*b^2*c^6*d^8 - 225*b^3*c^5*d^7*e + 180*b^5*c^3*d^5*e^3 - 30*b^6*c^2*d^4*e^4 - 278*b^7*c*d^3*e^5 + 140*b^8*d^2*e^6)*x^2 - 3*(8*b^3*c^5*d^8 - 25*b^4*c^4*d^7*e + 20*b^5*c^3*d^6*e^2 + 10*b^6*c^2*d^5*e^3 - 20*b^7*c*d^4*e^4 + 7*b^8*d^3*e^5)*x)*sqrt(e*x + d))/((b^5*c^6*d^9*e^2 - 4*b^6*c^5*d^8*e^3 + 6*b^7*c^4*d^7*e^4 - 4*b^8*c^3*d^6*e^5 + b^9*c^2*d^5*e^6)*x^6 + 2*(b^5*c^6*d^10*e - 3*b^6*c^5*d^9*e^2 + 2*b^7*c^4*d^8*e^3 + 2*b^8*c^3*d^7*e^4 - 3*b^9*c^2*d^6*e^5 + b^10*c*d^5*e^6)*x^5 + (b^5*c^6*d^11 - 9*b^7*c^4*d^9*e^2 + 16*b^8*c^3*d^8*e^3 - 9*b^9*c^2*d^7*e^4 + b^11*d^5*e^6)*x^4 + 2*(b^6*c^5*d^11 - 3*b^7*c^4*d^10*e + 2*b^8*c^3*d^9*e^2 + 2*b^9*c^2*d^8*e^3 - 3*b^10*c*d^7*e^

```
4 + b^11*d^6*e^5)*x^3 + (b^7*c^4*d^11 - 4*b^8*c^3*d^10*e + 6*b^9*c^2*d^9*e^
2 - 4*b^10*c*d^8*e^3 + b^11*d^7*e^4)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(5/2)/(c*x**2+b*x)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.84377, size = 1272, normalized size = 2.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x)^3,x, algorithm="giac")
```

```
[Out] -1/4*(48*c^7*d^2 - 156*b*c^6*d*e + 143*b^2*c^5*e^2)*arctan(sqrt(x*e + d)*c/
sqrt(-c^2*d + b*c*e))/((b^5*c^4*d^4 - 4*b^6*c^3*d^3*e + 6*b^7*c^2*d^2*e^2 -
 4*b^8*c*d*e^3 + b^9*e^4)*sqrt(-c^2*d + b*c*e)) - 2/3*(18*(x*e + d)*c*d*e^5
 + c*d^2*e^5 - 9*(x*e + d)*b*e^6 - b*d*e^6)/((c^4*d^8 - 4*b*c^3*d^7*e + 6*b
^2*c^2*d^6*e^2 - 4*b^3*c*d^5*e^3 + b^4*d^4*e^4)*(x*e + d)^(3/2)) + 1/4*(24*
(x*e + d)^(7/2)*c^7*d^5*e - 72*(x*e + d)^(5/2)*c^7*d^6*e + 72*(x*e + d)^(3/
2)*c^7*d^7*e - 24*sqrt(x*e + d)*c^7*d^8*e - 60*(x*e + d)^(7/2)*b*c^6*d^4*e^
2 + 216*(x*e + d)^(5/2)*b*c^6*d^5*e^2 - 252*(x*e + d)^(3/2)*b*c^6*d^6*e^2 +
 96*sqrt(x*e + d)*b*c^6*d^7*e^2 + 28*(x*e + d)^(7/2)*b^2*c^5*d^3*e^3 - 175*
(x*e + d)^(5/2)*b^2*c^5*d^4*e^3 + 274*(x*e + d)^(3/2)*b^2*c^5*d^5*e^3 - 127
*sqrt(x*e + d)*b^2*c^5*d^6*e^3 + 18*(x*e + d)^(7/2)*b^3*c^4*d^2*e^4 - 10*(x
*e + d)^(5/2)*b^3*c^4*d^3*e^4 - 55*(x*e + d)^(3/2)*b^3*c^4*d^4*e^4 + 45*sqr
t(x*e + d)*b^3*c^4*d^5*e^4 - 32*(x*e + d)^(7/2)*b^4*c^3*d*e^5 + 140*(x*e +
d)^(5/2)*b^4*c^3*d^2*e^5 - 180*(x*e + d)^(3/2)*b^4*c^3*d^3*e^5 + 80*sqrt(x*
e + d)*b^4*c^3*d^4*e^5 + 11*(x*e + d)^(7/2)*b^5*c^2*e^6 - 99*(x*e + d)^(5/2
)*b^5*c^2*d*e^6 + 199*(x*e + d)^(3/2)*b^5*c^2*d^2*e^6 - 123*sqrt(x*e + d)*b
^5*c^2*d^3*e^6 + 22*(x*e + d)^(5/2)*b^6*c*e^7 - 80*(x*e + d)^(3/2)*b^6*c*d*
e^7 + 66*sqrt(x*e + d)*b^6*c*d^2*e^7 + 11*(x*e + d)^(3/2)*b^7*e^8 - 13*sqrt
(x*e + d)*b^7*d*e^8)/((b^4*c^4*d^8 - 4*b^5*c^3*d^7*e + 6*b^6*c^2*d^6*e^2 -
4*b^7*c*d^5*e^3 + b^8*d^4*e^4)*((x*e + d)^2*c - 2*(x*e + d)*c*d + c*d^2 + (
x*e + d)*b*e - b*d*e)^2) + 1/4*(48*c^2*d^2 + 60*b*c*d*e + 35*b^2*e^2)*arcta
n(sqrt(x*e + d)/sqrt(-d))/(b^5*sqrt(-d)*d^4)
```

3.386 $\int (d+ex)^{3/2}\sqrt{bx+cx^2}\,dx$

Optimal. Leaf size=362

$$\frac{4\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\left(2b^2e^2-3bcde+3c^2d^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{105c^{5/2}e^2\sqrt{bx+cx^2}\sqrt{d+ex}}+\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}\left(-4b^2\right.}{}$$

```
[Out] (2*Sqrt[d + e*x]*(3*c^2*d^2 + 9*b*c*d*e - 4*b^2*e^2 + 12*c*e*(2*c*d - b*e)*
x)*Sqrt[b*x + c*x^2])/(105*c^2*e) + (2*e*Sqrt[d + e*x]*(b*x + c*x^2)^(3/2))
/(7*c) - (2*Sqrt[-b]*(2*c*d - b*e)*(3*c^2*d^2 - 3*b*c*d*e + 8*b^2*e^2)*Sqrt
[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt
[-b]], (b*e)/(c*d)])/(105*c^(5/2)*e^2*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2])
+ (4*Sqrt[-b]*d*(c*d - b*e)*(3*c^2*d^2 - 3*b*c*d*e + 2*b^2*e^2)*Sqrt[x]*Sqr
t[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b
]], (b*e)/(c*d)])/(105*c^(5/2)*e^2*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.394124, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {742, 814, 843, 715, 112, 110, 117, 116}

$$\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}\left(-4b^2e^2+12cex(2cd-be)+9bcde+3c^2d^2\right)}{105c^2e}+\frac{4\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\left(2b^2e^2-3bcde\right.}{105c^{5/2}e^2\sqrt{bx+cx^2}\sqrt{d+}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*Sqrt[b*x + c*x^2],x]
```


```
[Out] (2*Sqrt[d + e*x]*(3*c^2*d^2 + 9*b*c*d*e - 4*b^2*e^2 + 12*c*e*(2*c*d - b*e)*
x)*Sqrt[b*x + c*x^2])/(105*c^2*e) + (2*e*Sqrt[d + e*x]*(b*x + c*x^2)^(3/2))
/(7*c) - (2*Sqrt[-b]*(2*c*d - b*e)*(3*c^2*d^2 - 3*b*c*d*e + 8*b^2*e^2)*Sqrt
[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt
[-b]], (b*e)/(c*d)])/(105*c^(5/2)*e^2*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2])
+ (4*Sqrt[-b]*d*(c*d - b*e)*(3*c^2*d^2 - 3*b*c*d*e + 2*b^2*e^2)*Sqrt[x]*Sqr
t[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b
]], (b*e)/(c*d)])/(105*c^(5/2)*e^2*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```


Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
```

```
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int (d+ex)^{3/2}\sqrt{bx+cx^2}\,dx = \frac{2e\sqrt{d+ex}\left(bx+cx^2\right)^{3/2}}{7c} + \frac{2\int \frac{\left(\frac{1}{2}d(7cd-3be)+2e(2cd-be)x\right)\sqrt{bx+cx^2}}{\sqrt{d+ex}}dx}{7c}$$

$$= \frac{2\sqrt{d+ex}\left(3c^2d^2+9bcde-4b^2e^2+12ce(2cd-be)x\right)\sqrt{bx+cx^2}}{105c^2e} + \frac{2e\sqrt{d+ex}\left(bx+cx^2\right)^{3}}{7c}$$

$$= \frac{2\sqrt{d+ex}\left(3c^2d^2+9bcde-4b^2e^2+12ce(2cd-be)x\right)\sqrt{bx+cx^2}}{105c^2e} + \frac{2e\sqrt{d+ex}\left(bx+cx^2\right)^{3}}{7c}$$

$$= \frac{2\sqrt{d+ex}\left(3c^2d^2+9bcde-4b^2e^2+12ce(2cd-be)x\right)\sqrt{bx+cx^2}}{105c^2e} + \frac{2e\sqrt{d+ex}\left(bx+cx^2\right)^{3}}{7c}$$

$$= \frac{2\sqrt{d+ex}\left(3c^2d^2+9bcde-4b^2e^2+12ce(2cd-be)x\right)\sqrt{bx+cx^2}}{105c^2e} + \frac{2e\sqrt{d+ex}\left(bx+cx^2\right)^{3}}{7c}$$

$$= \frac{2\sqrt{d+ex}\left(3c^2d^2+9bcde-4b^2e^2+12ce(2cd-be)x\right)\sqrt{bx+cx^2}}{105c^2e} + \frac{2e\sqrt{d+ex}\left(bx+cx^2\right)^{3}}{7c}$$

Mathematica [C] time = 1.76458, size = 372, normalized size = 1.03

$$2\left(bex(b+cx)(d+ex)\left(-4b^2e^2+3bce(3d+ex)+3c^2\left(d^2+8dex+5e^2x^2\right)\right)-\sqrt{\frac{b}{c}}\left(-ibex^{3/2}\sqrt{\frac{b}{cx}+1}\sqrt{\frac{d}{ex}+1}\left(23b^2cde^2\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)*Sqrt[b*x + c*x^2],x]
```

```
[Out] (2*(b*e*x*(b + c*x)*(d + e*x)*(-4*b^2*e^2 + 3*b*c*e*(3*d + e*x) + 3*c^2*(d^
2 + 8*d*e*x + 5*e^2*x^2)) - Sqrt[b/c]*(Sqrt[b/c]*(6*c^3*d^3 - 9*b*c^2*d^2*e
 + 19*b^2*c*d*e^2 - 8*b^3*e^3)*(b + c*x)*(d + e*x) + I*b*e*(6*c^3*d^3 - 9*b
*c^2*d^2*e + 19*b^2*c*d*e^2 - 8*b^3*e^3)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)
]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - I*b*e*(3*c
^3*d^3 - 18*b*c^2*d^2*e + 23*b^2*c*d*e^2 - 8*b^3*e^3)*Sqrt[1 + b/(c*x)]*Sqr
t[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)]
)))/(105*b*c^2*e^2*Sqrt[x*(b + c*x)]*Sqrt[d + e*x])
```

Maple [B] time = 0.343, size = 920, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(c*x^2+b*x)^(1/2),x)
```

```
[Out] -2/105*(e*x+d)^(1/2)*(x*(c*x+b))^(1/2)*(-15*x^5*c^5*e^4+4*((c*x+b)/b)^(1/2)
*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b
*e/(b*e-c*d))^(1/2))*b^4*c*d*e^3-10*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d)
)^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b
^3*c^2*d^2*e^2+12*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(
```

```
1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^2*c^3*d^3*e-6*((c
*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+
b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b*c^4*d^4+8*((c*x+b)/b)^(1/2)*(-(e*x+d)*
c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d
))^(1/2))*b^5*e^4-27*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b
)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^4*c*d*e^3+28*(
(c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*
x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^3*c^2*d^2*e^2-15*((c*x+b)/b)^(1/2)*(
-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e
/(b*e-c*d))^(1/2))*b^2*c^3*d^3*e+6*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))
^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b*
c^4*d^4-18*x^4*b*c^4*e^4-39*x^4*c^5*d*e^3+x^3*b^2*c^3*e^4-51*x^3*b*c^4*d*e^
3-27*x^3*c^5*d^2*e^2+4*x^2*b^3*c^2*e^4-8*x^2*b^2*c^3*d*e^3-36*x^2*b*c^4*d^2
*e^2-3*x^2*c^5*d^3*e+4*x*b^3*c^2*d*e^3-9*x*b^2*c^3*d^2*e^2-3*x*b*c^4*d^3*e)
/e^2/x/(c*e*x^2+b*e*x+c*d*x+b*d)/c^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2+bx}(ex+d)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x)*(e*x + d)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^2+bx}(ex+d)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x)*(e*x + d)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x\,(b+cx)}\,(d+ex)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x*(b + c*x))*(d + e*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2+bx}(ex+d)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x)*(e*x + d)^(3/2), x)
```

3.387 $\int \sqrt{d+ex}\sqrt{bx+cx^2}\,dx$

Optimal. Leaf size=308

$$\frac{2\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{15c^{3/2}e^2\sqrt{bx+cx^2}\sqrt{d+ex}}-\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-bcde+c^2d\right.}{15c^{3/2}e^2\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}}}$$

```
[Out] (-2*(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])/(15*c*e) + (2*(d + e*x)^
(3/2)*Sqrt[b*x + c*x^2])/(5*e) - (4*Sqrt[-b]*(c^2*d^2 - b*c*d*e + b^2*e^2)*
Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/
Sqrt[-b]], (b*e)/(c*d)])/(15*c^(3/2)*e^2*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2
]) + (2*Sqrt[-b]*d*(c*d - b*e)*(2*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt
[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(
15*c^(3/2)*e^2*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.341274, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {734, 832, 843, 715, 112, 110, 117, 116}

$$-\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-bcde+c^2d^2\right)E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{15c^{3/2}e^2\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}+\frac{2\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)F\left(\text{si}\right.}{15c^{3/2}e^2\sqrt{bx+cx^2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*Sqrt[b*x + c*x^2],x]
```

```
[Out] (-2*(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])/(15*c*e) + (2*(d + e*x)^
(3/2)*Sqrt[b*x + c*x^2])/(5*e) - (4*Sqrt[-b]*(c^2*d^2 - b*c*d*e + b^2*e^2)*
Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/
Sqrt[-b]], (b*e)/(c*d)])/(15*c^(3/2)*e^2*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2
]) + (2*Sqrt[-b]*d*(c*d - b*e)*(2*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt
[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(
15*c^(3/2)*e^2*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
```

```
|| IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] && !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] && !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \sqrt{d+ex}\sqrt{bx+cx^2}\,dx = \frac{2(d+ex)^{3/2}\sqrt{bx+cx^2}}{5e} - \frac{\int \frac{\sqrt{d+ex}(bd+(2cd-be)x)}{\sqrt{bx+cx^2}}dx}{5e}$$

$$= -\frac{2(2cd-be)\sqrt{d+ex}\sqrt{bx+cx^2}}{15ce} + \frac{2(d+ex)^{3/2}\sqrt{bx+cx^2}}{5e} - \frac{2\int \frac{\frac{1}{2}bd(cd+be)+\left(c^2d^2-bcde+b^2e^2\right)x}{\sqrt{d+ex}\sqrt{bx+cx^2}}dx}{15ce}$$

$$= -\frac{2(2cd-be)\sqrt{d+ex}\sqrt{bx+cx^2}}{15ce} + \frac{2(d+ex)^{3/2}\sqrt{bx+cx^2}}{5e} + \frac{(d(cd-be)(2cd-be))\int \frac{1}{\sqrt{d+ex}\sqrt{bx+}}}{15ce^2}$$

$$= -\frac{2(2cd-be)\sqrt{d+ex}\sqrt{bx+cx^2}}{15ce} + \frac{2(d+ex)^{3/2}\sqrt{bx+cx^2}}{5e} + \frac{\left(d(cd-be)(2cd-be)\sqrt{x}\sqrt{b+cx}\right)}{15ce^2\sqrt{bx+cx^2}}$$

$$= -\frac{2(2cd-be)\sqrt{d+ex}\sqrt{bx+cx^2}}{15ce} + \frac{2(d+ex)^{3/2}\sqrt{bx+cx^2}}{5e} - \frac{\left(2\left(c^2d^2-bcde+b^2e^2\right)\sqrt{x}\sqrt{1+\frac{c}{b}}\right.}{15ce^2\sqrt{1+\frac{ex}{d}}\sqrt{b}}$$

$$= -\frac{2(2cd-be)\sqrt{d+ex}\sqrt{bx+cx^2}}{15ce} + \frac{2(d+ex)^{3/2}\sqrt{bx+cx^2}}{5e} - \frac{4\sqrt{-b}\left(c^2d^2-bcde+b^2e^2\right)\sqrt{x}\sqrt{1}}{15c^{3/2}e^2\sqrt{1+}}$$

Mathematica [C] time = 1.1264, size = 294, normalized size = 0.95

$$\frac{2\left(bex(b+cx)(d+ex)(be+c(d+3ex))+\sqrt{\frac{b}{c}}\left(ibex^{3/2}\sqrt{\frac{b}{cx}+1}\sqrt{\frac{d}{ex}+1}\left(2b^2e^2-3bcde+c^2d^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right),\frac{c}{b}\right.\right.\right.}{15bce^2\sqrt{x(b+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*Sqrt[b*x + c*x^2],x]
```

```
[Out] (2*(b*e*x*(b + c*x)*(d + e*x)*(b*e + c*(d + 3*e*x)) + Sqrt[b/c]*(-2*Sqrt[b/
c]*(c^2*d^2 - b*c*d*e + b^2*e^2)*(b + c*x)*(d + e*x) - (2*I)*b*e*(c^2*d^2 -
 b*c*d*e + b^2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticE[I
*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] + I*b*e*(c^2*d^2 - 3*b*c*d*e + 2*
b^2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sq
rt[b/c]/Sqrt[x]], (c*d)/(b*e)])))/(15*b*c*e^2*Sqrt[x*(b + c*x)]*Sqrt[d + e*
x])
```

Maple [B] time = 0.289, size = 681, normalized size = 2.2

$$\frac{2}{15\,x\left(cex^2+bxe+cdx+bd\right)c^3e^2}\sqrt{ex+d}\sqrt{x(cx+b)}\left(\sqrt{-\frac{cx}{b}}\text{EllipticF}\left(\sqrt{\frac{cx+b}{b}},\sqrt{\frac{be}{be-cd}}\right)\sqrt{\frac{cx+b}{b}}\sqrt{-\frac{c(ex+d)}{be-cd}}b^3c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(c*x^2+b*x)^(1/2),x)
```

```
[Out] 2/15*(e*x+d)^(1/2)*(x*(c*x+b))^(1/2)*((-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^
(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)
*b^3*c*d*e^2-3*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(
1/2))*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*b^2*c^2*d^2*e+2*(-c*x/
b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/
```

```
2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*b*c^3*d^3+2*(-c*x/b)^(1/2)*EllipticE(((c*x+
b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))
^(1/2)*b^4*e^3-4*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))
^(1/2))*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*b^3*c*d*e^2+4*(-c*x/
b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/
2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*b^2*c^2*d^2*e-2*(-c*x/b)^(1/2)*EllipticE(((
c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c
*d))^(1/2)*b*c^3*d^3+3*x^4*c^4*e^3+4*x^3*b*c^3*e^3+4*x^3*c^4*d*e^2+x^2*b^2*
c^2*e^3+5*x^2*b*c^3*d*e^2+x^2*c^4*d^2*e+x*b^2*c^2*d*e^2+x*b*c^3*d^2*e)/x/(c
*e*x^2+b*e*x+c*d*x+b*d)/c^3/e^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2+bx}\sqrt{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x)*sqrt(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^2+bx}\sqrt{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x)*sqrt(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x\,(b+cx)}\sqrt{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x*(b + c*x))*sqrt(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2+bx}\sqrt{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x)*sqrt(e*x + d), x)
```

3.388 $\int \frac{\sqrt{bx+cx^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=246

$$\frac{4\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{3\sqrt{c}e^2\sqrt{bx+cx^2}\sqrt{d+ex}}-\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{3\sqrt{c}e^2\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}$$

```
[Out] (2*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])/(3*e) - (2*Sqrt[-b]*(2*c*d - b*e)*Sqrt[
x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[
-b]], (b*e)/(c*d)])/(3*Sqrt[c]*e^2*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) + (
4*Sqrt[-b]*d*(c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*Ellipt
icF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*Sqrt[c]*e^2*Sqrt[d
 + e*x]*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.189719, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {734, 843, 715, 112, 110, 117, 116}

$$\frac{4\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{3\sqrt{c}e^2\sqrt{bx+cx^2}\sqrt{d+ex}}-\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{3\sqrt{c}e^2\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}+\frac{2\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x + c*x^2]/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])/(3*e) - (2*Sqrt[-b]*(2*c*d - b*e)*Sqrt[
x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[
-b]], (b*e)/(c*d)])/(3*Sqrt[c]*e^2*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) + (
4*Sqrt[-b]*d*(c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*Ellipt
icF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*Sqrt[c]*e^2*Sqrt[d
 + e*x]*Sqrt[b*x + c*x^2])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
```

```
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx+cx^2}}{\sqrt{d+ex}}\,dx &= \frac{2\sqrt{d+ex}\sqrt{bx+cx^2}}{3e}-\frac{\int \frac{bd+(2cd-be)x}{\sqrt{d+ex}\sqrt{bx+cx^2}}\,dx}{3e}\\
&= \frac{2\sqrt{d+ex}\sqrt{bx+cx^2}}{3e}+\frac{(2d(cd-be))\int \frac{1}{\sqrt{d+ex}\sqrt{bx+cx^2}}\,dx}{3e^2}-\frac{(2cd-be)\int \frac{\sqrt{d+ex}}{\sqrt{bx+cx^2}}\,dx}{3e^2}\\
&= \frac{2\sqrt{d+ex}\sqrt{bx+cx^2}}{3e}+\frac{\left(2d(cd-be)\sqrt{x}\sqrt{b+cx}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}\,dx}{3e^2\sqrt{bx+cx^2}}-\frac{\left((2cd-be)\sqrt{x}\sqrt{b+cx}\right)\int \frac{\sqrt{\cdots}}{\sqrt{x}\cdots}}{3e^2\sqrt{bx+cx^2}}\\
&= \frac{2\sqrt{d+ex}\sqrt{bx+cx^2}}{3e}-\frac{\left((2cd-be)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{d+ex}\right)\int \frac{\sqrt{1+\frac{ex}{d}}}{\sqrt{x}\sqrt{1+\frac{cx}{b}}}\,dx}{3e^2\sqrt{1+\frac{ex}{d}}\sqrt{bx+cx^2}}+\frac{\left(2d(cd-be)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{\cdots}\right.}{3e^2\sqrt{d+e\cdots}}\\
&= \frac{2\sqrt{d+ex}\sqrt{bx+cx^2}}{3e}-\frac{2\sqrt{-b}(2cd-be)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{3\sqrt{c}e^2\sqrt{1+\frac{ex}{d}}\sqrt{bx+cx^2}}+\frac{4\sqrt{-b}d(cd-be)\cdots}{3\cdots}\end{aligned}$$

Mathematica [C] time = 1.39797, size = 226, normalized size = 0.92

$$\frac{2\left(-icex^{3/2}\sqrt{\frac{b}{c}}\sqrt{\frac{b}{cx}+1}\sqrt{\frac{d}{ex}+1}(be-cd)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right),\frac{cd}{be}\right)+icex^{3/2}\sqrt{\frac{b}{c}}\sqrt{\frac{b}{cx}+1}\sqrt{\frac{d}{ex}+1}(be-2cd)E\left(i\sinh^{-1}\cdots\right.\right.}{3ce^2\sqrt{x(b+cx)}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x + c*x^2]/Sqrt[d + e*x],x]
```

```
[Out] (2*((b + c*x)*(d + e*x)*(-2*c*d + b*e + c*e*x) + I*Sqrt[b/c]*c*e*(-2*c*d +
b*e)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[b
/c]/Sqrt[x]], (c*d)/(b*e)] - I*Sqrt[b/c]*c*e*(-(c*d) + b*e)*Sqrt[1 + b/(c*x
)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/
(b*e)]))/(3*c*e^2*Sqrt[x*(b + c*x)]*Sqrt[d + e*x])
```

Maple [B] time = 0.273, size = 467, normalized size = 1.9

$$-\frac{2}{3\,x\left(ce x^2+bxe+cdx+bd\right)e^2c^2}\sqrt{x\left(cx+b\right)}\sqrt{ex+d}\left(2\,b^2d\sqrt{\frac{cx+b}{b}}\sqrt{-\frac{c\left(ex+d\right)}{be-cd}}\sqrt{-\frac{cx}{b}}\mathrm{EllipticF}\left(\sqrt{\frac{cx+b}{b}},\sqrt{\frac{be}{be-cd}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -2/3*(x*(c*x+b))^(1/2)*(e*x+d)^(1/2)*(2*b^2*d*((c*x+b)/b)^(1/2)*(-(e*x+d)*c
/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d)
)^(1/2))*e*c-2*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2
)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b*c^2*d^2+((c*x+b)/b)^
(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/
2),(b*e/(b*e-c*d))^(1/2))*b^3*e^2-3*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d)
)^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b
^2*c*d*e+2*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*El
lipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b*c^2*d^2-x^3*c^3*e^2-x^2*
b*c^2*e^2-x^2*c^3*d*e-x*b*c^2*d*e)/x/(c*e*x^2+b*e*x+c*d*x+b*d)/e^2/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+bx}}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x)/sqrt(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^2+bx}}{\sqrt{ex+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x)/sqrt(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x\,(b+cx)}}{\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Integral(sqrt(x*(b + c*x))/sqrt(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+bx}}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x)/sqrt(e*x + d), x)
```

3.389 $\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=231

$$-\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(2cd-be)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{\sqrt{c}e^2\sqrt{bx+cx^2}\sqrt{d+ex}}+\frac{4\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{e^2\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-2$$

```
[Out] (-2*Sqrt[b*x + c*x^2])/(e*Sqrt[d + e*x]) + (4*Sqrt[-b]*Sqrt[c]*Sqrt[x]*Sqrt
[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (
b*e)/(c*d)])/(e^2*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) - (2*Sqrt[-b]*(2*c*d
 - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[
c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e^2*Sqrt[d + e*x]*Sqrt[b*x +
c*x^2])
```

Rubi [A] time = 0.15631, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {732, 843, 715, 112, 110, 117, 116}

$$-\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(2cd-be)F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e^2\sqrt{bx+cx^2}\sqrt{d+ex}}+\frac{4\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{e^2\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2\sqrt{bx+c}}{e\sqrt{d+e}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x + c*x^2]/(d + e*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[b*x + c*x^2])/(e*Sqrt[d + e*x]) + (4*Sqrt[-b]*Sqrt[c]*Sqrt[x]*Sqrt
[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (
b*e)/(c*d)])/(e^2*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) - (2*Sqrt[-b]*(2*c*d
 - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[
c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e^2*Sqrt[d + e*x]*Sqrt[b*x +
c*x^2])
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
```

```
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{3/2}}\,dx = -\frac{2\sqrt{bx+cx^2}}{e\sqrt{d+ex}} + \frac{\int \frac{b+2cx}{\sqrt{d+ex}\sqrt{bx+cx^2}}\,dx}{e}$$

$$= -\frac{2\sqrt{bx+cx^2}}{e\sqrt{d+ex}} + \frac{(2c)\int \frac{\sqrt{d+ex}}{\sqrt{bx+cx^2}}\,dx}{e^2} - \frac{(2cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{bx+cx^2}}\,dx}{e^2}$$

$$= -\frac{2\sqrt{bx+cx^2}}{e\sqrt{d+ex}} + \frac{\left(2c\sqrt{x}\sqrt{b+cx}\right)\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}\,dx}{e^2\sqrt{bx+cx^2}} - \frac{\left((2cd-be)\sqrt{x}\sqrt{b+cx}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}\,dx}{e^2\sqrt{bx+cx^2}}$$

$$= -\frac{2\sqrt{bx+cx^2}}{e\sqrt{d+ex}} + \frac{\left(2c\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{d+ex}\right)\int \frac{\sqrt{1+\frac{ex}{d}}}{\sqrt{x}\sqrt{1+\frac{cx}{b}}}\,dx}{e^2\sqrt{1+\frac{ex}{d}}\sqrt{bx+cx^2}} - \frac{\left((2cd-be)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{1+\frac{ex}{d}}\right)\int \frac{}{\sqrt{x}\sqrt{1+}}}{e^2\sqrt{d+ex}\sqrt{bx+cx^2}}$$

$$= -\frac{2\sqrt{bx+cx^2}}{e\sqrt{d+ex}} + \frac{4\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{e^2\sqrt{1+\frac{ex}{d}}\sqrt{bx+cx^2}} - \frac{2\sqrt{-b}(2cd-be)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{1}}{\sqrt{c}e^2\sqrt{d+ex}}$$

Mathematica [C] time = 0.561032, size = 195, normalized size = 0.84

$$\frac{2\left(-ibex^{3/2}\sqrt{\frac{b}{cx}+1}\sqrt{\frac{d}{ex}+1}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right),\frac{cd}{be}\right)+2ibex^{3/2}\sqrt{\frac{b}{cx}+1}\sqrt{\frac{d}{ex}+1}E\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right)|\frac{cd}{be}\right)+\sqrt{\frac{b}{c}}(b+c\right.}{e^2\sqrt{\frac{b}{c}}\sqrt{x(b+cx)}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x + c*x^2]/(d + e*x)^(3/2),x]
```

```
[Out] (2*(Sqrt[b/c]*(b + c*x)*(2*d + e*x) + (2*I)*b*e*Sqrt[1 + b/(c*x)]*Sqrt[1 +
d/(e*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - I*b
*e*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c
]/Sqrt[x]], (c*d)/(b*e)]))/(Sqrt[b/c]*e^2*Sqrt[x*(b + c*x)]*Sqrt[d + e*x])
```

Maple [A] time = 0.338, size = 353, normalized size = 1.5

$$2\,\frac{\sqrt{x\,(cx+b)}\sqrt{ex+d}}{x\left(cex^{2}+bxe+cdx+bd\right)e^{2}c}\left(\mathrm{EllipticF}\left(\sqrt{\frac{cx+b}{b}},\sqrt{\frac{be}{be-cd}}\right)b^{2}e\sqrt{\frac{cx+b}{b}}\sqrt{-\frac{c\,(ex+d)}{be-cd}}\sqrt{-\frac{cx}{b}}-2\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+b}{b}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(1/2)/(e*x+d)^(3/2),x)
```

```
[Out] 2*(x*(c*x+b))^(1/2)*(e*x+d)^(1/2)*(EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*
d))^(1/2))*b^2*e*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1
/2)-2*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b*c*d*((c*x+b)/b)^
(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-2*EllipticE(((c*x+b)/b)^(
1/2),(b*e/(b*e-c*d))^(1/2))*b^2*e*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^
(1/2)*(-c*x/b)^(1/2)+2*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b
*c*d*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-x^2*c^2*
e-x*b*c*e)/x/(c*e*x^2+b*e*x+c*d*x+b*d)/e^2/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^{2}+bx}}{(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x)/(e*x + d)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^{2}+bx}\sqrt{ex+d}}{e^{2}x^{2}+2\,dex+d^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x)*sqrt(e*x + d)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x(b+cx)}}{(d+ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(1/2)/(e*x+d)**(3/2),x)
```

```
[Out] Integral(sqrt(x*(b + c*x))/(d + e*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+bx}}{(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x)/(e*x + d)^(3/2), x)
```

3.390 $\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=301

$$\frac{4\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{3e^2\sqrt{bx+cx^2}\sqrt{d+ex}}-\frac{2\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{3de^2\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}(cd-be)}+$$

```
[Out] (-2*Sqrt[b*x + c*x^2])/(3*e*(d + e*x)^(3/2)) + (2*(2*c*d - b*e)*Sqrt[b*x +
c*x^2])/(3*d*e*(c*d - b*e)*Sqrt[d + e*x]) - (2*Sqrt[-b]*Sqrt[c]*(2*c*d - b*
e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x
])/Sqrt[-b]], (b*e)/(c*d)])/(3*d*e^2*(c*d - b*e)*Sqrt[1 + (e*x)/d]*Sqrt[b*x
 + c*x^2]) + (4*Sqrt[-b]*Sqrt[c]*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d
]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*e^2*Sqrt[d
 + e*x]*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.280329, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {732, 834, 843, 715, 112, 110, 117, 116}

$$\frac{4\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{3e^2\sqrt{bx+cx^2}\sqrt{d+ex}}-\frac{2\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{3de^2\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}(cd-be)}+\frac{2\sqrt{bx+}}{3de\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x + c*x^2]/(d + e*x)^(5/2),x]
```


```
[Out] (-2*Sqrt[b*x + c*x^2])/(3*e*(d + e*x)^(3/2)) + (2*(2*c*d - b*e)*Sqrt[b*x +
c*x^2])/(3*d*e*(c*d - b*e)*Sqrt[d + e*x]) - (2*Sqrt[-b]*Sqrt[c]*(2*c*d - b*
e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x
])/Sqrt[-b]], (b*e)/(c*d)])/(3*d*e^2*(c*d - b*e)*Sqrt[1 + (e*x)/d]*Sqrt[b*x
 + c*x^2]) + (4*Sqrt[-b]*Sqrt[c]*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d
]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*e^2*Sqrt[d
 + e*x]*Sqrt[b*x + c*x^2])
```


Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
```

```
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{5/2}}dx = -\frac{2\sqrt{bx+cx^2}}{3e(d+ex)^{3/2}} + \frac{\int \frac{b+2cx}{(d+ex)^{3/2}\sqrt{bx+cx^2}}dx}{3e}$$

$$= -\frac{2\sqrt{bx+cx^2}}{3e(d+ex)^{3/2}} + \frac{2(2cd-be)\sqrt{bx+cx^2}}{3de(cd-be)\sqrt{d+ex}} - \frac{2\int \frac{\frac{bcd}{2}+\frac{1}{2}c(2cd-be)x}{\sqrt{d+ex}\sqrt{bx+cx^2}}dx}{3de(cd-be)}$$

$$= -\frac{2\sqrt{bx+cx^2}}{3e(d+ex)^{3/2}} + \frac{2(2cd-be)\sqrt{bx+cx^2}}{3de(cd-be)\sqrt{d+ex}} + \frac{(2c)\int \frac{1}{\sqrt{d+ex}\sqrt{bx+cx^2}}dx}{3e^2} - \frac{(c(2cd-be))\int \frac{\sqrt{d+ex}}{\sqrt{bx+cx^2}}dx}{3de^2(cd-be)}$$

$$= -\frac{2\sqrt{bx+cx^2}}{3e(d+ex)^{3/2}} + \frac{2(2cd-be)\sqrt{bx+cx^2}}{3de(cd-be)\sqrt{d+ex}} + \frac{\left(2c\sqrt{x}\sqrt{b+cx}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{3e^2\sqrt{bx+cx^2}} - \frac{\left(c(2cd-be)\sqrt{x}\sqrt{b+c\right.}{3de^2(cd-be)\sqrt{}}$$

$$= -\frac{2\sqrt{bx+cx^2}}{3e(d+ex)^{3/2}} + \frac{2(2cd-be)\sqrt{bx+cx^2}}{3de(cd-be)\sqrt{d+ex}} - \frac{\left(c(2cd-be)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{d+ex}\right)\int \frac{\sqrt{1+\frac{ex}{d}}}{\sqrt{x}\sqrt{1+\frac{cx}{b}}}dx}{3de^2(cd-be)\sqrt{1+\frac{ex}{d}}\sqrt{bx+cx^2}} + \frac{\left(2c\sqrt{x}\sqrt{\right.}{}$$

$$= -\frac{2\sqrt{bx+cx^2}}{3e(d+ex)^{3/2}} + \frac{2(2cd-be)\sqrt{bx+cx^2}}{3de(cd-be)\sqrt{d+ex}} - \frac{2\sqrt{-b}\sqrt{c}(2cd-be)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{3de^2(cd-be)\sqrt{1+\frac{ex}{d}}\sqrt{bx+cx^2}}$$

Mathematica [C] time = 1.14323, size = 265, normalized size = 0.88

$$-\frac{2\left(ex(b+cx)\left(be^2x-cd(d+2ex)\right)+(d+ex)\left(icex^{3/2}\sqrt{\frac{b}{c}}\sqrt{\frac{b}{cx}+1}\sqrt{\frac{d}{ex}+1}(be-cd)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right),\frac{cd}{be}\right)-icex^{3/2}\right.\right.}{3de^2\sqrt{x(b+cx)}(d+ex)^{3/2}(cd-be)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x + c*x^2]/(d + e*x)^(5/2),x]
```

```
[Out] (-2*(e*x*(b + c*x)*(b*e^2*x - c*d*(d + 2*e*x)) + (d + e*x)*((2*c*d - b*e)*(
b + c*x)*(d + e*x) - I*Sqrt[b/c]*c*e*(-2*c*d + b*e)*Sqrt[1 + b/(c*x)]*Sqrt[
1 + d/(e*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] +
 I*Sqrt[b/c]*c*e*(-(c*d) + b*e)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)
*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])))/(3*d*e^2*(c*d - b*
e)*Sqrt[x*(b + c*x)]*(d + e*x)^(3/2))
```

Maple [B] time = 0.293, size = 887, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(1/2)/(e*x+d)^(5/2),x)
```

```
[Out] 2/3*(2*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^2*c*d*e^2*((c
*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-2*EllipticF(((c*
x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b*c^2*d^2*e*((c*x+b)/b)^(1/2)*(-(e*x
+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e
-c*d))^(1/2))*x*b^3*e^3*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*
x/b)^(1/2)-3*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^2*c*d*e
```

```
^2*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+2*Elliptic
E((((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b*c^2*d^2*e*((c*x+b)/b)^(1/2)*
(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+2*EllipticF(((c*x+b)/b)^(1/2),(
b*e/(b*e-c*d))^(1/2))*b^2*c*d^2*e*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^
(1/2)*(-c*x/b)^(1/2)-2*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b
*c^2*d^3*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+Elli
pticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^3*d*e^2*((c*x+b)/b)^(1/2)*
(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-3*EllipticE(((c*x+b)/b)^(1/2),(
b*e/(b*e-c*d))^(1/2))*b^2*c*d^2*e*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^
(1/2)*(-c*x/b)^(1/2)+2*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b
*c^2*d^3*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+x^3*
b*c^2*e^3-2*x^3*c^3*d*e^2+x^2*b^2*c*e^3-2*x^2*b*c^2*d*e^2-x^2*c^3*d^2*e-x*b
*c^2*d^2*e)*(x*(c*x+b))^(1/2)/c/(b*e-c*d)/d/(c*x+b)/x/e^2/(e*x+d)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx}}{(ex + d)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x)/(e*x + d)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2 + bx}\sqrt{ex + d}}{e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x)*sqrt(e*x + d)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x
 + d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x\,(b + cx)}}{(d + ex)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(1/2)/(e*x+d)**(5/2),x)
```

```
[Out] Integral(sqrt(x*(b + c*x))/(d + e*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx}}{(ex + d)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x)/(e*x + d)^(5/2), x)
```

3.391 $\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=398

$$\frac{2\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(2cd-be)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{15de^2\sqrt{bx+cx^2}\sqrt{d+ex}(cd-be)}+\frac{4\sqrt{bx+cx^2}\left(b^2e^2-bcde+c^2d^2\right)}{15d^2e\sqrt{d+ex}(cd-be)^2}-\frac{4\sqrt{-b}\sqrt{c}\sqrt{x}}{}$$

```
[Out] (-2*Sqrt[b*x + c*x^2])/(5*e*(d + e*x)^(5/2)) + (2*(2*c*d - b*e)*Sqrt[b*x +
c*x^2])/(15*d*e*(c*d - b*e)*(d + e*x)^(3/2)) + (4*(c^2*d^2 - b*c*d*e + b^2*
e^2)*Sqrt[b*x + c*x^2])/(15*d^2*e*(c*d - b*e)^2*Sqrt[d + e*x]) - (4*Sqrt[-b
]*Sqrt[c]*(c^2*d^2 - b*c*d*e + b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d +
e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(15*d^2*e^
2*(c*d - b*e)^2*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) + (2*Sqrt[-b]*Sqrt[c]*
(2*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[
(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(15*d*e^2*(c*d - b*e)*Sqrt[d + e
*x]*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.491153, antiderivative size = 398, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {732, 834, 843, 715, 112, 110, 117, 116}

$$\frac{4\sqrt{bx+cx^2}\left(b^2e^2-bcde+c^2d^2\right)}{15d^2e\sqrt{d+ex}(cd-be)^2}-\frac{4\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-bcde+c^2d^2\right)E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{15d^2e^2\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}(cd-be)^2}+\frac{2\sqrt{-b}\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x + c*x^2]/(d + e*x)^(7/2),x]
```


```
[Out] (-2*Sqrt[b*x + c*x^2])/(5*e*(d + e*x)^(5/2)) + (2*(2*c*d - b*e)*Sqrt[b*x +
c*x^2])/(15*d*e*(c*d - b*e)*(d + e*x)^(3/2)) + (4*(c^2*d^2 - b*c*d*e + b^2*
e^2)*Sqrt[b*x + c*x^2])/(15*d^2*e*(c*d - b*e)^2*Sqrt[d + e*x]) - (4*Sqrt[-b
]*Sqrt[c]*(c^2*d^2 - b*c*d*e + b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d +
e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(15*d^2*e^
2*(c*d - b*e)^2*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) + (2*Sqrt[-b]*Sqrt[c]*
(2*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[
(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(15*d*e^2*(c*d - b*e)*Sqrt[d + e
*x]*Sqrt[b*x + c*x^2])
```


Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
```

```
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{7/2}}\,dx = -\frac{2\sqrt{bx+cx^2}}{5e(d+ex)^{5/2}} + \frac{\int \frac{b+2cx}{(d+ex)^{5/2}\sqrt{bx+cx^2}}\,dx}{5e}$$

$$= -\frac{2\sqrt{bx+cx^2}}{5e(d+ex)^{5/2}} + \frac{2(2cd-be)\sqrt{bx+cx^2}}{15de(cd-be)(d+ex)^{3/2}} - \frac{2\int \frac{-\frac{1}{2}b(cd-2be)-\frac{1}{2}c(2cd-be)x}{(d+ex)^{3/2}\sqrt{bx+cx^2}}\,dx}{15de(cd-be)}$$

$$= -\frac{2\sqrt{bx+cx^2}}{5e(d+ex)^{5/2}} + \frac{2(2cd-be)\sqrt{bx+cx^2}}{15de(cd-be)(d+ex)^{3/2}} + \frac{4\left(c^2d^2-bcde+b^2e^2\right)\sqrt{bx+cx^2}}{15d^2e(cd-be)^2\sqrt{d+ex}} + \frac{4\int \frac{-\frac{1}{4}bcd(cd+be)-\frac{1}{2}c\cdots}{\sqrt{d+ex}\cdots}}{15d^2e(c\cdots}$$

$$= -\frac{2\sqrt{bx+cx^2}}{5e(d+ex)^{5/2}} + \frac{2(2cd-be)\sqrt{bx+cx^2}}{15de(cd-be)(d+ex)^{3/2}} + \frac{4\left(c^2d^2-bcde+b^2e^2\right)\sqrt{bx+cx^2}}{15d^2e(cd-be)^2\sqrt{d+ex}} + \frac{(c(2cd-be))\int \frac{\cdots}{\sqrt{d\cdots}}}{15de^2(cd\cdots}$$

$$= -\frac{2\sqrt{bx+cx^2}}{5e(d+ex)^{5/2}} + \frac{2(2cd-be)\sqrt{bx+cx^2}}{15de(cd-be)(d+ex)^{3/2}} + \frac{4\left(c^2d^2-bcde+b^2e^2\right)\sqrt{bx+cx^2}}{15d^2e(cd-be)^2\sqrt{d+ex}} + \frac{\left(c(2cd-be)\sqrt{x}\sqrt{b\cdots}\right.}{15de^2(cd\cdots}$$

$$= -\frac{2\sqrt{bx+cx^2}}{5e(d+ex)^{5/2}} + \frac{2(2cd-be)\sqrt{bx+cx^2}}{15de(cd-be)(d+ex)^{3/2}} + \frac{4\left(c^2d^2-bcde+b^2e^2\right)\sqrt{bx+cx^2}}{15d^2e(cd-be)^2\sqrt{d+ex}} - \frac{\left(2c\left(c^2d^2-bcde+\cdots\right.\right.}{15d^2e\cdots}$$

$$= -\frac{2\sqrt{bx+cx^2}}{5e(d+ex)^{5/2}} + \frac{2(2cd-be)\sqrt{bx+cx^2}}{15de(cd-be)(d+ex)^{3/2}} + \frac{4\left(c^2d^2-bcde+b^2e^2\right)\sqrt{bx+cx^2}}{15d^2e(cd-be)^2\sqrt{d+ex}} - \frac{4\sqrt{-b}\sqrt{c}\left(c^2d^2-b\cdots\right.}{1\cdots}$$

Mathematica [C] time = 1.31569, size = 362, normalized size = 0.91

$$-\frac{2\left(bex(b+cx)\left(-b^2e^3x(5d+2ex)+bcde\left(-d^2+7dex+2e^2x^2\right)-c^2d^2\left(d^2+6dex+2e^2x^2\right)\right)+c\sqrt{\frac{b}{c}}(d+ex)^2\left(-ibex^{3/2}\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x + c*x^2]/(d + e*x)^(7/2),x]
```

```
[Out] (-2*(b*e*x*(b + c*x)*(-(b^2*e^3*x*(5*d + 2*e*x)) - c^2*d^2*(d^2 + 6*d*e*x +
 2*e^2*x^2) + b*c*d*e*(-d^2 + 7*d*e*x + 2*e^2*x^2)) + Sqrt[b/c]*c*(d + e*x)
^2*(2*Sqrt[b/c]*(c^2*d^2 - b*c*d*e + b^2*e^2)*(b + c*x)*(d + e*x) + (2*I)*b
*e*(c^2*d^2 - b*c*d*e + b^2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2
)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - I*b*e*(c^2*d^2 - 3
*b*c*d*e + 2*b^2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF
[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])))/(15*b*d^2*e^2*(c*d - b*e)^2*
Sqrt[x*(b + c*x)]*(d + e*x)^(5/2))
```

Maple [B] time = 0.303, size = 1897, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(1/2)/(e*x+d)^(7/2),x)
```

```
[Out] 2/15*(3*x^3*b^2*c^2*d*e^4-5*x^3*b*c^3*d^2*e^3+5*x^2*b^3*c*d*e^4-7*x^2*b^2*c
^2*d^2*e^3+7*x^2*b*c^3*d^3*e^2+x*b^2*c^2*d^3*e^2+4*EllipticE(((c*x+b)/b)^(1
```

```
/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^2*c^2*d^2*e^3*(-c*x/b)^(1/2)*((c*x+b)/b)^(
1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)-2*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c
*d))^(1/2))*x^2*b*c^3*d^3*e^2*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/
(b*e-c*d))^(1/2)+2*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^3
*c*d^2*e^3*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)-6*
EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^2*c^2*d^3*e^2*(-c*x/
b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)+4*EllipticF(((c*x+b
)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b*c^3*d^4*e*(-c*x/b)^(1/2)*((c*x+b)/b)^
(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)-8*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-
c*d))^(1/2))*x*b^3*c*d^2*e^3*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(
b*e-c*d))^(1/2)+8*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^2*
c^2*d^3*e^2*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)-4
*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b*c^3*d^4*e*(-c*x/b)^
(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)+EllipticF(((c*x+b)/b)^
(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^3*c*d*e^4*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/
2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)-3*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d
))^(1/2))*x^2*b^2*c^2*d^2*e^3*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/
(b*e-c*d))^(1/2)+x*b*c^3*d^4*e+2*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d)
)^(1/2))*b^4*d^2*e^3*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d)
)^(1/2)-2*x^4*b*c^3*d*e^4+EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2)
)*b^3*c*d^3*e^2*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/
2)-3*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^2*c^2*d^4*e*(-c*x
/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)-4*EllipticE(((c*x+
b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^3*c*d^3*e^2*(-c*x/b)^(1/2)*((c*x+b)/b)
^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)+4*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e
-c*d))^(1/2))*b^2*c^2*d^4*e*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b
*e-c*d))^(1/2)+2*x^4*b^2*c^2*e^5+2*x^4*c^4*d^2*e^3+2*x^3*b^3*c*e^5+6*x^3*c^
4*d^3*e^2+x^2*c^4*d^4*e+4*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2)
)*x*b^4*d*e^4*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)
+2*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b*c^3*d^3*e^2*(-c
*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)-4*EllipticE(((c*
x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^3*c*d*e^4*(-c*x/b)^(1/2)*((c*x+b
)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)+2*EllipticE(((c*x+b)/b)^(1/2),(b*e/
(b*e-c*d))^(1/2))*x^2*b^4*e^5*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/
(b*e-c*d))^(1/2)+2*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b*c^3
*d^5*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)-2*Ellipt
icE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b*c^3*d^5*(-c*x/b)^(1/2)*((c*x
+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2))*(x*(c*x+b))^(1/2)/c/(b*e-c*d)^2/
d^2/(c*x+b)/x/e^2/(e*x+d)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx}}{(ex + d)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x)/(e*x + d)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2 + bx}\sqrt{ex + d}}{e^4x^4 + 4\,de^3x^3 + 6\,d^2e^2x^2 + 4\,d^3ex + d^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x)*sqrt(e*x + d)/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2
*x^2 + 4*d^3*e*x + d^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x\,(b+cx)}}{(d+ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(1/2)/(e*x+d)**(7/2),x)
```

```
[Out] Integral(sqrt(x*(b + c*x))/(d + e*x)**(7/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+bx}}{(ex+d)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/2)/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x)/(e*x + d)^(7/2), x)
```

3.392 $\int (d+ex)^{3/2} \left(bx+cx^2\right)^{3/2} dx$

Optimal. Leaf size=521

$$\frac{2\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\left(3b^2c^2d^2e^2+13b^3cde^3-8b^4e^4-32bc^3d^3e+16c^4d^4\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{1155c^{7/2}e^4\sqrt{bx+cx^2}\sqrt{d+ex}}+$$

```
[Out] (2*Sqrt[d + e*x]*(8*c^4*d^4 - 19*b*c^3*d^3*e + 6*b^2*c^2*d^2*e^2 - 19*b^3*c
*d*e^3 + 8*b^4*e^4 - 3*c*e*(2*c*d - b*e)*(c^2*d^2 - b*c*d*e + 8*b^2*e^2)*x)
*Sqrt[b*x + c*x^2])/(1155*c^3*e^3) + (2*Sqrt[d + e*x]*(c^2*d^2 + 13*b*c*d*e
 - 6*b^2*e^2 + 14*c*e*(2*c*d - b*e)*x)*(b*x + c*x^2)^(3/2))/(231*c^2*e) + (
2*e*Sqrt[d + e*x]*(b*x + c*x^2)^(5/2))/(11*c) - (16*Sqrt[-b]*(c*d - 2*b*e)*
(2*c*d - b*e)*(c*d + b*e)*(c^2*d^2 - b*c*d*e + b^2*e^2)*Sqrt[x]*Sqrt[1 + (c
*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c
*d)])/(1155*c^(7/2)*e^4*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) + (2*Sqrt[-b]*
d*(c*d - b*e)*(16*c^4*d^4 - 32*b*c^3*d^3*e + 3*b^2*c^2*d^2*e^2 + 13*b^3*c*d
*e^3 - 8*b^4*e^4)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[Arc
Sin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(1155*c^(7/2)*e^4*Sqrt[d + e
*x]*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.72893, antiderivative size = 521, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {742, 814, 843, 715, 112, 110, 117, 116}

$$\frac{2\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}\left(-6b^2e^2+14cex(2cd-be)+13bcde+c^2d^2\right)}{231c^2e}+\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}\left(-3cex(2cd-be)\left(8b^2e^2-bcd\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*(b*x + c*x^2)^(3/2),x]
```


```
[Out] (2*Sqrt[d + e*x]*(8*c^4*d^4 - 19*b*c^3*d^3*e + 6*b^2*c^2*d^2*e^2 - 19*b^3*c
*d*e^3 + 8*b^4*e^4 - 3*c*e*(2*c*d - b*e)*(c^2*d^2 - b*c*d*e + 8*b^2*e^2)*x)
*Sqrt[b*x + c*x^2])/(1155*c^3*e^3) + (2*Sqrt[d + e*x]*(c^2*d^2 + 13*b*c*d*e
 - 6*b^2*e^2 + 14*c*e*(2*c*d - b*e)*x)*(b*x + c*x^2)^(3/2))/(231*c^2*e) + (
2*e*Sqrt[d + e*x]*(b*x + c*x^2)^(5/2))/(11*c) - (16*Sqrt[-b]*(c*d - 2*b*e)*
(2*c*d - b*e)*(c*d + b*e)*(c^2*d^2 - b*c*d*e + b^2*e^2)*Sqrt[x]*Sqrt[1 + (c
*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c
*d)])/(1155*c^(7/2)*e^4*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) + (2*Sqrt[-b]*
d*(c*d - b*e)*(16*c^4*d^4 - 32*b*c^3*d^3*e + 3*b^2*c^2*d^2*e^2 + 13*b^3*c*d
*e^3 - 8*b^4*e^4)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[Arc
Sin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(1155*c^(7/2)*e^4*Sqrt[d + e
*x]*Sqrt[b*x + c*x^2])
```


Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
```

```
raticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
```

```
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int (d+ex)^{3/2}\left(bx+cx^2\right)^{3/2}\,dx=\frac{2e\sqrt{d+ex}\left(bx+cx^2\right)^{5/2}}{11c}+\frac{2\int\frac{\left(\frac{1}{2}d(11cd-5be)+3e(2cd-be)x\right)\left(bx+cx^2\right)^{3/2}}{\sqrt{d+ex}}\,dx}{11c}$$

$$=\frac{2\sqrt{d+ex}\left(c^2d^2+13bcde-6b^2e^2+14ce(2cd-be)x\right)\left(bx+cx^2\right)^{3/2}}{231c^2e}+\frac{2e\sqrt{d+ex}\left(bx+cx^2\right.}{11c}$$

$$=\frac{2\sqrt{d+ex}\left(8c^4d^4-19bc^3d^3e+6b^2c^2d^2e^2-19b^3cde^3+8b^4e^4-3ce(2cd-be)\left(c^2d^2-bcde\right.\right.}{1155c^3e^3}$$

$$=\frac{2\sqrt{d+ex}\left(8c^4d^4-19bc^3d^3e+6b^2c^2d^2e^2-19b^3cde^3+8b^4e^4-3ce(2cd-be)\left(c^2d^2-bcde\right.\right.}{1155c^3e^3}$$

$$=\frac{2\sqrt{d+ex}\left(8c^4d^4-19bc^3d^3e+6b^2c^2d^2e^2-19b^3cde^3+8b^4e^4-3ce(2cd-be)\left(c^2d^2-bcde\right.\right.}{1155c^3e^3}$$

$$=\frac{2\sqrt{d+ex}\left(8c^4d^4-19bc^3d^3e+6b^2c^2d^2e^2-19b^3cde^3+8b^4e^4-3ce(2cd-be)\left(c^2d^2-bcde\right.\right.}{1155c^3e^3}$$

$$=\frac{2\sqrt{d+ex}\left(8c^4d^4-19bc^3d^3e+6b^2c^2d^2e^2-19b^3cde^3+8b^4e^4-3ce(2cd-be)\left(c^2d^2-bcde\right.\right.}{1155c^3e^3}$$

Mathematica [C] time = 2.82639, size = 559, normalized size = 1.07

$$\frac{2(x(b+cx))^{3/2}\left(bex(b+cx)(d+ex)\left(b^2c^2e^2\left(6d^2+14dex+5e^2x^2\right)-b^3ce^3(19d+6ex)+8b^4e^4+bc^3e\left(14d^2ex-19d^3+20\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)*(b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(x*(b + c*x))^(3/2)*(b*e*x*(b + c*x)*(d + e*x)*(8*b^4*e^4 - b^3*c*e^3*(1
9*d + 6*e*x) + b^2*c^2*e^2*(6*d^2 + 14*d*e*x + 5*e^2*x^2) + b*c^3*e*(-19*d^
3 + 14*d^2*e*x + 205*d*e^2*x^2 + 140*e^3*x^3) + c^4*(8*d^4 - 6*d^3*e*x + 5*
d^2*e^2*x^2 + 140*d*e^3*x^3 + 105*e^4*x^4)) + Sqrt[b/c]*(-8*Sqrt[b/c]*(2*c^
5*d^5 - 5*b*c^4*d^4*e + 2*b^2*c^3*d^3*e^2 + 2*b^3*c^2*d^2*e^3 - 5*b^4*c*d*e
^4 + 2*b^5*e^5)*(b + c*x)*(d + e*x) - (8*I)*b*e*(2*c^5*d^5 - 5*b*c^4*d^4*e
+ 2*b^2*c^3*d^3*e^2 + 2*b^3*c^2*d^2*e^3 - 5*b^4*c*d*e^4 + 2*b^5*e^5)*Sqrt[1
 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]
], (c*d)/(b*e)] + I*b*e*(8*c^5*d^5 - 21*b*c^4*d^4*e + 10*b^2*c^3*d^3*e^2 +
35*b^3*c^2*d^2*e^3 - 48*b^4*c*d*e^4 + 16*b^5*e^5)*Sqrt[1 + b/(c*x)]*Sqrt[1
+ d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])))/
(1155*b*c^3*e^4*x^2*(b + c*x)^2*Sqrt[d + e*x])
```

Maple [B] time = 0.288, size = 1359, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(c*x^2+b*x)^(3/2),x)
```

```
[Out] 2/1155*(e*x+d)^(1/2)*(x*(c*x+b))^(1/2)*(245*x^6*b*c^6*e^6+245*x^6*c^7*d*e^5
+145*x^5*b^2*c^5*e^6+145*x^5*c^7*d^2*e^4-x^4*b^3*c^4*e^6-x^4*c^7*d^3*e^3+2*
x^3*b^4*c^3*e^6+2*x^3*c^7*d^4*e^2+8*x^2*b^5*c^2*e^6+8*x^2*c^7*d^5*e-17*x^2*
b*c^6*d^4*e^2+8*x*b^5*c^2*d*e^5-19*x*b^4*c^3*d^2*e^4+6*x*b^3*c^4*d^3*e^3-19
*x*b^2*c^5*d^4*e^2+8*x*b*c^6*d^5*e+590*x^5*b*c^6*d*e^5+364*x^4*b^2*c^5*d*e^
5+364*x^4*b*c^6*d^2*e^4-6*x^3*b^3*c^4*d*e^5+239*x^3*b^2*c^5*d^2*e^4-6*x^3*b
*c^6*d^3*e^3-17*x^2*b^4*c^3*d*e^5+x^2*b^3*c^4*d^2*e^4+16*((c*x+b)/b)^(1/2)*
(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*
e/(b*e-c*d))^(1/2))*b^7*e^6+16*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/
2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b*c^6*
d^6-56*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*Ellipt
icE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^6*c*d*e^5+105*x^7*c^7*e^6+x^
2*b^2*c^5*d^3*e^3-16*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b
)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b*c^6*d^6-48*((c
*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+
b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^2*c^5*d^5*e+35*((c*x+b)/b)^(1/2)*(-(e*
x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*
e-c*d))^(1/2))*b^3*c^4*d^4*e^2+8*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(
1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^6*
c*d*e^5-21*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*El
lipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^5*c^2*d^2*e^4+10*((c*x+b
)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b
)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^4*c^3*d^3*e^3+56*((c*x+b)/b)^(1/2)*(-(e*x+
d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-
c*d))^(1/2))*b^5*c^2*d^2*e^4-56*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1
/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^3*c
^4*d^4*e^2+56*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)
*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^2*c^5*d^5*e)/c^5/e^4/
x/(c*e*x^2+b*e*x+c*d*x+b*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx\right)^{\frac{3}{2}} (ex + d)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(3/2)*(e*x + d)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(cex^3 + bdx + (cd + be)x^2\right)\sqrt{cx^2 + bx}\sqrt{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*e*x^3 + b*d*x + (c*d + b*e)*x^2)*sqrt(c*x^2 + b*x)*sqrt(e*x + d
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (x(b+cx))^{\frac{3}{2}}(d+ex)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral((x*(b + c*x))**(3/2)*(d + e*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{\frac{3}{2}}(ex+d)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(3/2)*(e*x + d)^(3/2), x)
```

3.393 $\int \sqrt{d+ex}\left(bx+cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=457

$$\frac{8\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\left(-b^2e^2-2bcde+2c^2d^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{315c^{5/2}e^4\sqrt{bx+cx^2}\sqrt{d+ex}}+\frac{2\sqrt{bx+cx^2}\sqrt{d}}{}$$

```
[Out] (2*Sqrt[d + e*x]*(8*c^3*d^3 - 15*b*c^2*d^2*e + 3*b^2*c*d*e^2 - 4*b^3*e^3 -
6*c*e*(c^2*d^2 - b*c*d*e + 2*b^2*e^2)*x)*Sqrt[b*x + c*x^2])/(315*c^2*e^3) -
 (2*(2*c*d - b*e)*Sqrt[d + e*x]*(b*x + c*x^2)^(3/2))/(21*c*e) + (2*(d + e*x
)^(3/2)*(b*x + c*x^2)^(3/2))/(9*e) - (2*Sqrt[-b]*(16*c^4*d^4 - 32*b*c^3*d^3
*e + 9*b^2*c^2*d^2*e^2 + 7*b^3*c*d*e^3 - 8*b^4*e^4)*Sqrt[x]*Sqrt[1 + (c*x)/
b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)]
)/(315*c^(5/2)*e^4*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) + (8*Sqrt[-b]*d*(c*
d - b*e)*(2*c*d - b*e)*(2*c^2*d^2 - 2*b*c*d*e - b^2*e^2)*Sqrt[x]*Sqrt[1 + (
c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*
e)/(c*d)])/(315*c^(5/2)*e^4*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.577751, antiderivative size = 457, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {734, 832, 814, 843, 715, 112, 110, 117, 116}

$$\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}\left(-6cex\left(2b^2e^2-bcde+c^2d^2\right)+3b^2cde^2-4b^3e^3-15bc^2d^2e+8c^3d^3\right)}{315c^2e^3}+\frac{8\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(b*x + c*x^2)^(3/2),x]
```


```
[Out] (2*Sqrt[d + e*x]*(8*c^3*d^3 - 15*b*c^2*d^2*e + 3*b^2*c*d*e^2 - 4*b^3*e^3 -
6*c*e*(c^2*d^2 - b*c*d*e + 2*b^2*e^2)*x)*Sqrt[b*x + c*x^2])/(315*c^2*e^3) -
 (2*(2*c*d - b*e)*Sqrt[d + e*x]*(b*x + c*x^2)^(3/2))/(21*c*e) + (2*(d + e*x
)^(3/2)*(b*x + c*x^2)^(3/2))/(9*e) - (2*Sqrt[-b]*(16*c^4*d^4 - 32*b*c^3*d^3
*e + 9*b^2*c^2*d^2*e^2 + 7*b^3*c*d*e^3 - 8*b^4*e^4)*Sqrt[x]*Sqrt[1 + (c*x)/
b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)]
)/(315*c^(5/2)*e^4*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) + (8*Sqrt[-b]*d*(c*
d - b*e)*(2*c*d - b*e)*(2*c^2*d^2 - 2*b*c*d*e - b^2*e^2)*Sqrt[x]*Sqrt[1 + (
c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*
e)/(c*d)])/(315*c^(5/2)*e^4*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```


Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
```

```
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^{3/2}dx = \frac{2(d+ex)^{3/2}\left(bx+cx^2\right)^{3/2}}{9e} - \frac{\int \sqrt{d+ex}(bd+(2cd-be)x)\sqrt{bx+cx^2}\,dx}{3e}$$

$$= -\frac{2(2cd-be)\sqrt{d+ex}\left(bx+cx^2\right)^{3/2}}{21ce} + \frac{2(d+ex)^{3/2}\left(bx+cx^2\right)^{3/2}}{9e} - \frac{2\int \frac{\left(\frac{1}{2}bd(cd+3be)+\left(c^2d^2-\ldots\right.\right.}{\sqrt{d\ldots}}}{21\ldots}$$

$$= \frac{2\sqrt{d+ex}\left(8c^3d^3-15bc^2d^2e+3b^2cde^2-4b^3e^3-6ce\left(c^2d^2-bcde+2b^2e^2\right)x\right)\sqrt{bx+cx^2}\ldots}{315c^2e^3}$$

$$= \frac{2\sqrt{d+ex}\left(8c^3d^3-15bc^2d^2e+3b^2cde^2-4b^3e^3-6ce\left(c^2d^2-bcde+2b^2e^2\right)x\right)\sqrt{bx+cx^2}\ldots}{315c^2e^3}$$

$$= \frac{2\sqrt{d+ex}\left(8c^3d^3-15bc^2d^2e+3b^2cde^2-4b^3e^3-6ce\left(c^2d^2-bcde+2b^2e^2\right)x\right)\sqrt{bx+cx^2}\ldots}{315c^2e^3}$$

$$= \frac{2\sqrt{d+ex}\left(8c^3d^3-15bc^2d^2e+3b^2cde^2-4b^3e^3-6ce\left(c^2d^2-bcde+2b^2e^2\right)x\right)\sqrt{bx+cx^2}\ldots}{315c^2e^3}$$

$$= \frac{2\sqrt{d+ex}\left(8c^3d^3-15bc^2d^2e+3b^2cde^2-4b^3e^3-6ce\left(c^2d^2-bcde+2b^2e^2\right)x\right)\sqrt{bx+cx^2}\ldots}{315c^2e^3}$$

Mathematica [C] time = 2.36044, size = 463, normalized size = 1.01

$$\frac{2(x(b+cx))^{3/2}\left(bex(b+cx)(d+ex)\left(3b^2ce^2(d+ex)-4b^3e^3+bc^2e\left(-15d^2+11dex+50e^2x^2\right)+c^3\left(-6d^2ex+8d^3+5d\ldots\right.\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*(b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(x*(b + c*x))^(3/2)*(b*e*x*(b + c*x)*(d + e*x)*(-4*b^3*e^3 + 3*b^2*c*e^2
*(d + e*x) + b*c^2*e*(-15*d^2 + 11*d*e*x + 50*e^2*x^2) + c^3*(8*d^3 - 6*d^2
*e*x + 5*d*e^2*x^2 + 35*e^3*x^3)) - Sqrt[b/c]*(Sqrt[b/c]*(16*c^4*d^4 - 32*b
*c^3*d^3*e + 9*b^2*c^2*d^2*e^2 + 7*b^3*c*d*e^3 - 8*b^4*e^4)*(b + c*x)*(d +
e*x) + I*b*e*(16*c^4*d^4 - 32*b*c^3*d^3*e + 9*b^2*c^2*d^2*e^2 + 7*b^3*c*d*e
^3 - 8*b^4*e^4)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticE[I*Arc
Sinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - I*b*e*(8*c^4*d^4 - 17*b*c^3*d^3*e +
 6*b^2*c^2*d^2*e^2 + 11*b^3*c*d*e^3 - 8*b^4*e^4)*Sqrt[1 + b/(c*x)]*Sqrt[1 +
 d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])))/(
315*b*c^2*e^4*x^2*(b + c*x)^2*Sqrt[d + e*x])
```

Maple [B] time = 0.29, size = 1170, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(3/2)*(e*x+d)^(1/2),x)
```

```
[Out] -2/315*(x*(c*x+b))^(1/2)*(e*x+d)^(1/2)*(-85*x^5*b*c^5*e^5-40*x^5*c^6*d*e^4-
53*x^4*b^2*c^4*e^5+x^4*c^6*d^2*e^3+x^3*b^3*c^3*e^5-2*x^3*c^6*d^3*e^2+4*x^2*
b^4*c^2*e^5-8*x^2*c^6*d^4*e-16*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*
EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)*b*c^5*
d^5+16*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1
/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)*b*c^5*d^5+8*(-(e*x+d)*c/(b*e-c
*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2)
)*((c*x+b)/b)^(1/2)*b^6*e^5+41*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*
EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)*b^3*c^
3*d^3*e^2-2*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/
b)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)*b^4*c^2*d^2*e^3+40*(-(e*x
+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e
-c*d))^(1/2))*((c*x+b)/b)^(1/2)*b^2*c^4*d^4*e-15*(-(e*x+d)*c/(b*e-c*d))^(1/
2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+
b)/b)^(1/2)*b^5*c*d*e^4-24*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*Elli
pticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)*b^3*c^3*d^
3*e^2-4*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(
1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)*b^4*c^2*d^2*e^3+4*(-(e*x+d)*c
/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d)
)^(1/2))*((c*x+b)/b)^(1/2)*b^5*c*d*e^4-101*x^4*b*c^5*d*e^4-35*x^6*c^6*e^5-6
7*x^3*b^2*c^4*d*e^4+5*x^3*b*c^5*d^2*e^3-2*x^2*b^3*c^3*d*e^4+x^2*b^2*c^4*d^2
*e^3+13*x^2*b*c^5*d^3*e^2+4*x*b^4*c^2*d*e^4-3*x*b^3*c^3*d^2*e^3+15*x*b^2*c^
4*d^3*e^2-8*x*b*c^5*d^4*e-48*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*El
lipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)*b^2*c^4*
d^4*e)/c^4/x/(c*e*x^2+b*e*x+c*d*x+b*d)/e^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{\frac{3}{2}}\sqrt{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(3/2)*sqrt(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2+bx\right)^{\frac{3}{2}}\sqrt{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(3/2)*sqrt(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (x(b+cx))^{\frac{3}{2}}\sqrt{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(3/2)*(e*x+d)**(1/2),x)
```

```
[Out] Integral((x*(b + c*x))**(3/2)*sqrt(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{\frac{3}{2}}\sqrt{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(3/2)*sqrt(e*x + d), x)
```

3.394 $\int \frac{(bx+cx^2)^{3/2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=360

$$\frac{2\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\left(-b^2e^2-16bcde+16c^2d^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{35c^{3/2}e^4\sqrt{bx+cx^2}\sqrt{d+ex}}+\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}\left(b^2e^2\right.}{}$$

```
[Out] (2*Sqrt[d + e*x]*(8*c^2*d^2 - 11*b*c*d*e + b^2*e^2 - 3*c*e*(2*c*d - b*e)*x)
*Sqrt[b*x + c*x^2])/(35*c*e^3) + (2*Sqrt[d + e*x]*(b*x + c*x^2)^(3/2))/(7*e
) - (4*Sqrt[-b]*(2*c*d - b*e)*(4*c^2*d^2 - 4*b*c*d*e - b^2*e^2)*Sqrt[x]*Sqr
t[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]],
(b*e)/(c*d)])/(35*c^(3/2)*e^4*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) + (2*Sqr
t[-b]*d*(c*d - b*e)*(16*c^2*d^2 - 16*b*c*d*e - b^2*e^2)*Sqrt[x]*Sqrt[1 + (c
*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e
)/(c*d)])/(35*c^(3/2)*e^4*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.352649, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {734, 814, 843, 715, 112, 110, 117, 116}

$$\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}\left(b^2e^2-3cex(2cd-be)-11bcde+8c^2d^2\right)}{35ce^3}+\frac{2\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\left(-b^2e^2-16bcde+16\right.}{35c^{3/2}e^4\sqrt{bx+cx^2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(3/2)/Sqrt[d + e*x],x]
```


```
[Out] (2*Sqrt[d + e*x]*(8*c^2*d^2 - 11*b*c*d*e + b^2*e^2 - 3*c*e*(2*c*d - b*e)*x)
*Sqrt[b*x + c*x^2])/(35*c*e^3) + (2*Sqrt[d + e*x]*(b*x + c*x^2)^(3/2))/(7*e
) - (4*Sqrt[-b]*(2*c*d - b*e)*(4*c^2*d^2 - 4*b*c*d*e - b^2*e^2)*Sqrt[x]*Sqr
t[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]],
(b*e)/(c*d)])/(35*c^(3/2)*e^4*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) + (2*Sqr
t[-b]*d*(c*d - b*e)*(16*c^2*d^2 - 16*b*c*d*e - b^2*e^2)*Sqrt[x]*Sqrt[1 + (c
*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e
)/(c*d)])/(35*c^(3/2)*e^4*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```


Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
```

```
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{(bx+cx^2)^{3/2}}{\sqrt{d+ex}}dx = \frac{2\sqrt{d+ex}(bx+cx^2)^{3/2}}{7e} - \frac{3\int \frac{(bd+(2cd-be)x)\sqrt{bx+cx^2}}{\sqrt{d+ex}}dx}{7e}$$

$$= \frac{2\sqrt{d+ex}\left(8c^2d^2-11bcde+b^2e^2-3ce(2cd-be)x\right)\sqrt{bx+cx^2}}{35ce^3} + \frac{2\sqrt{d+ex}(bx+cx^2)^{3/2}}{7e} + \frac{2\int -\frac{1}{2}bc \ldots}{\ldots}$$

$$= \frac{2\sqrt{d+ex}\left(8c^2d^2-11bcde+b^2e^2-3ce(2cd-be)x\right)\sqrt{bx+cx^2}}{35ce^3} + \frac{2\sqrt{d+ex}(bx+cx^2)^{3/2}}{7e} + \frac{(d(cd- \ldots}{\ldots}$$

$$= \frac{2\sqrt{d+ex}\left(8c^2d^2-11bcde+b^2e^2-3ce(2cd-be)x\right)\sqrt{bx+cx^2}}{35ce^3} + \frac{2\sqrt{d+ex}(bx+cx^2)^{3/2}}{7e} + \frac{(d(cd- \ldots}{\ldots}$$

$$= \frac{2\sqrt{d+ex}\left(8c^2d^2-11bcde+b^2e^2-3ce(2cd-be)x\right)\sqrt{bx+cx^2}}{35ce^3} + \frac{2\sqrt{d+ex}(bx+cx^2)^{3/2}}{7e} - \frac{(2(2cd- \ldots}{\ldots}$$

$$= \frac{2\sqrt{d+ex}\left(8c^2d^2-11bcde+b^2e^2-3ce(2cd-be)x\right)\sqrt{bx+cx^2}}{35ce^3} + \frac{2\sqrt{d+ex}(bx+cx^2)^{3/2}}{7e} - \frac{4\sqrt{-b}(2 \ldots}{\ldots}$$

Mathematica [C] time = 2.02002, size = 380, normalized size = 1.06

$$\frac{2(x(b+cx))^{3/2}\left(bex(b+cx)(d+ex)\left(b^2e^2+bce(8ex-11d)+c^2\left(8d^2-6dex+5e^2x^2\right)\right)+\sqrt{\frac{b}{c}}\left(ibex^{3/2}\sqrt{\frac{b}{cx}+1}\sqrt{\frac{d}{ex}+1}\left(3b^2 \ldots\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(3/2)/Sqrt[d + e*x],x]
```

```
[Out] (2*(x*(b + c*x))^(3/2)*(b*e*x*(b + c*x)*(d + e*x)*(b^2*e^2 + b*c*e*(-11*d +
 8*e*x) + c^2*(8*d^2 - 6*d*e*x + 5*e^2*x^2)) + Sqrt[b/c]*(-2*Sqrt[b/c]*(8*c
^3*d^3 - 12*b*c^2*d^2*e + 2*b^2*c*d*e^2 + b^3*e^3)*(b + c*x)*(d + e*x) - (2
*I)*b*e*(8*c^3*d^3 - 12*b*c^2*d^2*e + 2*b^2*c*d*e^2 + b^3*e^3)*Sqrt[1 + b/(
c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*
d)/(b*e)] + I*b*e*(8*c^3*d^3 - 13*b*c^2*d^2*e + 3*b^2*c*d*e^2 + 2*b^3*e^3)*
Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/S
qrt[x]], (c*d)/(b*e)])))/(35*b*c*e^4*x^2*(b + c*x)^2*Sqrt[d + e*x])
```

Maple [B] time = 0.271, size = 918, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(3/2)/(e*x+d)^(1/2),x)
```

```
[Out] 2/35*(x*(c*x+b))^(1/2)*(e*x+d)^(1/2)*(5*x^5*c^5*e^4+2*((c*x+b)/b)^(1/2)*(-(
e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(
b*e-c*d))^(1/2))*b^5*e^4+2*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(
-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^4*c*d*e^
3-28*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*Elliptic
```

```
E(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^3*c^2*d^2*e^2+40*((c*x+b)/b)^(
1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2
),(b*e/(b*e-c*d))^(1/2))*b^2*c^3*d^3*e-16*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*
e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1
/2))*b*c^4*d^4+((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2
)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^4*c*d*e^3+15*((c*x+b
)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b
)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^3*c^2*d^2*e^2-32*((c*x+b)/b)^(1/2)*(-(e*x+
d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-
c*d))^(1/2))*b^2*c^3*d^3*e+16*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2
)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b*c^4*d
^4+13*x^4*b*c^4*e^4-x^4*c^5*d*e^3+9*x^3*b^2*c^3*e^4-4*x^3*b*c^4*d*e^3+2*x^3
*c^5*d^2*e^2+x^2*b^3*c^2*e^4-2*x^2*b^2*c^3*d*e^3-9*x^2*b*c^4*d^2*e^2+8*x^2*
c^5*d^3*e+x*b^3*c^2*d*e^3-11*x*b^2*c^3*d^2*e^2+8*x*b*c^4*d^3*e)/c^3/e^4/x/(
c*e*x^2+b*e*x+c*d*x+b*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2+bx)^{\frac{3}{2}}}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(3/2)/sqrt(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^2+bx)^{\frac{3}{2}}}{\sqrt{ex+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(3/2)/sqrt(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x(b+cx))^{\frac{3}{2}}}{\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(3/2)/(e*x+d)**(1/2),x)
```

```
[Out] Integral((x*(b + c*x))**(3/2)/sqrt(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx\right)^{\frac{3}{2}}}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(3/2)/sqrt(e*x + d), x)
```

3.395 $\int \frac{\left(bx+cx^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

<u>**Optimal**</u>. Leaf size=309

$$-\frac{16\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{5\sqrt{c}e^4\sqrt{bx+cx^2}\sqrt{d+ex}}+\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-16bc\right.}{5\sqrt{c}e^4\sqrt{bx+c}}$$

```
[Out] (-2*Sqrt[d + e*x]*(8*c*d - 7*b*e - 6*c*e*x)*Sqrt[b*x + c*x^2])/(5*e^3) - (2
*(b*x + c*x^2)^(3/2))/(e*Sqrt[d + e*x]) + (2*Sqrt[-b]*(16*c^2*d^2 - 16*b*c*
d*e + b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sq
rt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(5*Sqrt[c]*e^4*Sqrt[1 + (e*x)/d]*Sq
rt[b*x + c*x^2]) - (16*Sqrt[-b]*d*(c*d - b*e)*(2*c*d - b*e)*Sqrt[x]*Sqrt[1
+ (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]],
(b*e)/(c*d)])/(5*Sqrt[c]*e^4*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

<u>**Rubi**</u> **[A]** time = 0.338442, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {732, 814, 843, 715, 112, 110, 117, 116}

$$\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-16bcde+16c^2d^2\right)E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{5\sqrt{c}e^4\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}(-7be+8cd-6cex)}{5e^3}-$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(3/2)/(d + e*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(8*c*d - 7*b*e - 6*c*e*x)*Sqrt[b*x + c*x^2])/(5*e^3) - (2
*(b*x + c*x^2)^(3/2))/(e*Sqrt[d + e*x]) + (2*Sqrt[-b]*(16*c^2*d^2 - 16*b*c*
d*e + b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sq
rt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(5*Sqrt[c]*e^4*Sqrt[1 + (e*x)/d]*Sq
rt[b*x + c*x^2]) - (16*Sqrt[-b]*d*(c*d - b*e)*(2*c*d - b*e)*Sqrt[x]*Sqrt[1
+ (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]],
(b*e)/(c*d)])/(5*Sqrt[c]*e^4*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

<u>Rule 732</u>

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

<u>Rule 814</u>

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
```

```
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{\left(bx+cx^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = -\frac{2\left(bx+cx^2\right)^{3/2}}{e\sqrt{d+ex}}+\frac{3\int \frac{(b+2cx)\sqrt{bx+cx^2}}{\sqrt{d+ex}}dx}{e}$$

$$= -\frac{2\sqrt{d+ex}(8cd-7be-6cex)\sqrt{bx+cx^2}}{5e^3}-\frac{2\left(bx+cx^2\right)^{3/2}}{e\sqrt{d+ex}}-\frac{2\int \frac{-\frac{1}{2}bcd(8cd-7be)-\frac{1}{2}c\left(16c^2d^2-16bcde+b^2e\right.}{\sqrt{d+ex}\sqrt{bx+cx^2}}}{5ce^3}$$

$$= -\frac{2\sqrt{d+ex}(8cd-7be-6cex)\sqrt{bx+cx^2}}{5e^3}-\frac{2\left(bx+cx^2\right)^{3/2}}{e\sqrt{d+ex}}-\frac{(8d(cd-be)(2cd-be))\int \frac{1}{\sqrt{d+ex}\sqrt{bx+}}}{5e^4}$$

$$= -\frac{2\sqrt{d+ex}(8cd-7be-6cex)\sqrt{bx+cx^2}}{5e^3}-\frac{2\left(bx+cx^2\right)^{3/2}}{e\sqrt{d+ex}}-\frac{\left(8d(cd-be)(2cd-be)\sqrt{x}\sqrt{b+cx}\right)}{5e^4\sqrt{bx+cx^2}}$$

$$= -\frac{2\sqrt{d+ex}(8cd-7be-6cex)\sqrt{bx+cx^2}}{5e^3}-\frac{2\left(bx+cx^2\right)^{3/2}}{e\sqrt{d+ex}}+\frac{\left(\left(16c^2d^2-16bcde+b^2e^2\right)\sqrt{x}\sqrt{1+\right.}{5e^4\sqrt{1+\frac{ex}{d}}\sqrt{bx}}$$

$$= -\frac{2\sqrt{d+ex}(8cd-7be-6cex)\sqrt{bx+cx^2}}{5e^3}-\frac{2\left(bx+cx^2\right)^{3/2}}{e\sqrt{d+ex}}+\frac{2\sqrt{-b}\left(16c^2d^2-16bcde+b^2e^2\right)\sqrt{x}}{5\sqrt{c}e^4\sqrt{1+}}$$

Mathematica [C] time = 1.68754, size = 340, normalized size = 1.1

$$\frac{-2icex^{3/2}\sqrt{\frac{b}{c}}\sqrt{\frac{b}{cx}+1}\sqrt{\frac{d}{ex}+1}\left(b^2e^2-9bcde+8c^2d^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right),\frac{cd}{be}\right)+2\left(b^2ce\left(-16d^2-8dex+3e^2x^2\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(3/2)/(d + e*x)^(3/2),x]
```

```
[Out] (2*(b^3*e^2*(d + e*x) + b^2*c*e*(-16*d^2 - 8*d*e*x + 3*e^2*x^2) + c^3*x*(16
*d^3 + 8*d^2*e*x - 2*d*e^2*x^2 + e^3*x^3) + b*c^2*(16*d^3 - 8*d^2*e*x - 11*
d*e^2*x^2 + 3*e^3*x^3)) + (2*I)*Sqrt[b/c]*c*e*(16*c^2*d^2 - 16*b*c*d*e + b^
2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt
[b/c]/Sqrt[x]], (c*d)/(b*e)] - (2*I)*Sqrt[b/c]*c*e*(8*c^2*d^2 - 9*b*c*d*e +
 b^2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[S
qrt[b/c]/Sqrt[x]], (c*d)/(b*e)])/(5*c*e^4*Sqrt[x*(b + c*x)]*Sqrt[d + e*x])
```

Maple [B] time = 0.283, size = 685, normalized size = 2.2

$$-\frac{2}{5\,c^2x\left(cex^2+bxe+cdx+bd\right)e^4}\sqrt{x\,(cx+b)}\sqrt{ex+d}\left(8\sqrt{-\frac{cx}{b}}\text{EllipticF}\left(\sqrt{\frac{cx+b}{b}},\sqrt{\frac{be}{be-cd}}\right)\sqrt{\frac{cx+b}{b}}\sqrt{-\frac{c\,(ex+d)}{be-cd}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(3/2)/(e*x+d)^(3/2),x)
```

```
[Out] -2/5*(x*(c*x+b))^(1/2)*(e*x+d)^(1/2)*(8*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b
)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/
2)*b^3*c*d*e^2-24*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d)
)^(1/2))*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*b^2*c^2*d^2*e+16*(-
```

```
c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)
^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*b*c^3*d^3+(-c*x/b)^(1/2)*EllipticE(((c*
x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d
))^(1/2)*b^4*e^3-17*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*
d))^(1/2))*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*b^3*c*d*e^2+32*(-
c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)
^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*b^2*c^2*d^2*e-16*(-c*x/b)^(1/2)*Ellipti
cE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(
b*e-c*d))^(1/2)*b*c^3*d^3-x^4*c^4*e^3-3*x^3*b*c^3*e^3+2*x^3*c^4*d*e^2-2*x^2
*b^2*c^2*e^3-5*x^2*b*c^3*d*e^2+8*x^2*c^4*d^2*e-7*x*b^2*c^2*d*e^2+8*x*b*c^3*
d^2*e)/c^2/x/(c*e*x^2+b*e*x+c*d*x+b*d)/e^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx\right)^{\frac{3}{2}}}{\left(ex+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(3/2)/(e*x + d)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(cx^2+bx\right)^{\frac{3}{2}}\sqrt{ex+d}}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(3/2)*sqrt(e*x + d)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x\left(b+cx\right)\right)^{\frac{3}{2}}}{\left(d+ex\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(3/2)/(e*x+d)**(3/2),x)
```

```
[Out] Integral((x*(b + c*x))**(3/2)/(d + e*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx\right)^{\frac{3}{2}}}{\left(ex+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(3/2)/(e*x + d)^(3/2), x)
```

3.396 $\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^{5/2}} dx$

Optimal. Leaf size=298

$$\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(4cd-3be)(4cd-be)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{3\sqrt{c}e^4\sqrt{bx+cx^2}\sqrt{d+ex}}+\frac{2\sqrt{bx+cx^2}(-3be+8cd+2cex)}{3e^3\sqrt{d+ex}}-\frac{16\sqrt{-b}}{}$$

```
[Out] (2*(8*c*d - 3*b*e + 2*c*e*x)*Sqrt[b*x + c*x^2])/(3*e^3*Sqrt[d + e*x]) - (2*
(b*x + c*x^2)^(3/2))/(3*e*(d + e*x)^(3/2)) - (16*Sqrt[-b]*Sqrt[c]*(2*c*d -
b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt
[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*e^4*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) +
 (2*Sqrt[-b]*(4*c*d - 3*b*e)*(4*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1
 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*
Sqrt[c]*e^4*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.299729, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {732, 812, 843, 715, 112, 110, 117, 116}

$$\frac{2\sqrt{bx+cx^2}(-3be+8cd+2cex)}{3e^3\sqrt{d+ex}}+\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(4cd-3be)(4cd-be)F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{3\sqrt{c}e^4\sqrt{bx+cx^2}\sqrt{d+ex}}-\frac{16\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{c}{b}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(3/2)/(d + e*x)^(5/2),x]
```

```
[Out] (2*(8*c*d - 3*b*e + 2*c*e*x)*Sqrt[b*x + c*x^2])/(3*e^3*Sqrt[d + e*x]) - (2*
(b*x + c*x^2)^(3/2))/(3*e*(d + e*x)^(3/2)) - (16*Sqrt[-b]*Sqrt[c]*(2*c*d -
b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt
[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*e^4*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) +
 (2*Sqrt[-b]*(4*c*d - 3*b*e)*(4*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1
 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*
Sqrt[c]*e^4*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{\left(bx+cx^2\right)^{3/2}}{(d+ex)^{5/2}}\,dx = -\frac{2\left(bx+cx^2\right)^{3/2}}{3e(d+ex)^{3/2}} + \frac{\int \frac{(b+2cx)\sqrt{bx+cx^2}}{(d+ex)^{3/2}}dx}{e}$$

$$= \frac{2(8cd-3be+2cex)\sqrt{bx+cx^2}}{3e^3\sqrt{d+ex}} - \frac{2\left(bx+cx^2\right)^{3/2}}{3e(d+ex)^{3/2}} - \frac{2\int \frac{\frac{1}{2}b(8cd-3be)+4c(2cd-be)x}{\sqrt{d+ex}\sqrt{bx+cx^2}}dx}{3e^3}$$

$$= \frac{2(8cd-3be+2cex)\sqrt{bx+cx^2}}{3e^3\sqrt{d+ex}} - \frac{2\left(bx+cx^2\right)^{3/2}}{3e(d+ex)^{3/2}} - \frac{(8c(2cd-be))\int \frac{\sqrt{d+ex}}{\sqrt{bx+cx^2}}dx}{3e^4} + \frac{((4cd-3be)(4cd-$$

$$= \frac{2(8cd-3be+2cex)\sqrt{bx+cx^2}}{3e^3\sqrt{d+ex}} - \frac{2\left(bx+cx^2\right)^{3/2}}{3e(d+ex)^{3/2}} - \frac{\left(8c(2cd-be)\sqrt{x}\sqrt{b+cx}\right)\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{3e^4\sqrt{bx+cx^2}} + \frac{((4cd$$

$$= \frac{2(8cd-3be+2cex)\sqrt{bx+cx^2}}{3e^3\sqrt{d+ex}} - \frac{2\left(bx+cx^2\right)^{3/2}}{3e(d+ex)^{3/2}} - \frac{\left(8c(2cd-be)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{d+ex}\right)\int \frac{\sqrt{1+\frac{ex}{d}}}{\sqrt{x}\sqrt{1+\frac{cx}{b}}}dx}{3e^4\sqrt{1+\frac{ex}{d}}\sqrt{bx+cx^2}}$$

$$= \frac{2(8cd-3be+2cex)\sqrt{bx+cx^2}}{3e^3\sqrt{d+ex}} - \frac{2\left(bx+cx^2\right)^{3/2}}{3e(d+ex)^{3/2}} - \frac{16\sqrt{-b}\sqrt{c}(2cd-be)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{d+ex}E\left(\sin^{-1}\right.}{3e^4\sqrt{1+\frac{ex}{d}}\sqrt{bx+cx^2}}$$

Mathematica [C] time = 1.20718, size = 279, normalized size = 0.94

$$\frac{2(x(b+cx))^{3/2}\left(-icex^{3/2}\sqrt{\frac{b}{c}}\sqrt{\frac{b}{cx}+1}\sqrt{\frac{d}{ex}+1}(5be-8cd)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right),\frac{cd}{be}\right)+\frac{ex(b+cx)\left(c\left(8d^2+10dex+e^2x^2\right)-be(3d+4ex)\right)}{d+ex}\right.}{3e^4x^2(b+cx)^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(3/2)/(d + e*x)^(5/2),x]
```

```
[Out] (2*(x*(b + c*x))^(3/2)*(8*(-2*c*d + b*e)*(b + c*x)*(d + e*x) + (e*x*(b + c*
x)*(-(b*e*(3*d + 4*e*x)) + c*(8*d^2 + 10*d*e*x + e^2*x^2)))/(d + e*x) + (8*
I)*Sqrt[b/c]*c*e*(-2*c*d + b*e)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)
*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - I*Sqrt[b/c]*c*e*(-8
*c*d + 5*b*e)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSi
nh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)]))/(3*e^4*x^2*(b + c*x)^2*Sqrt[d + e*x])
```

Maple [B] time = 0.285, size = 1051, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(3/2)/(e*x+d)^(5/2),x)
```

```
[Out] 2/3*(x*(c*x+b))^(1/2)*(3*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))
*x*b^3*e^3*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-16
*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^2*c*d*e^2*((c*x+b)/
b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+16*EllipticF(((c*x+b)/
b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b*c^2*d^2*e*((c*x+b)/b)^(1/2)*(-(e*x+d)*c
/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-8*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*
```

```
d))^(1/2))*x*b^3*e^3*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b
)^(1/2)+24*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^2*c*d*e^2
*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-16*EllipticE
(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b*c^2*d^2*e*((c*x+b)/b)^(1/2)*(
-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+3*EllipticF(((c*x+b)/b)^(1/2),(b
*e/(b*e-c*d))^(1/2))*b^3*d*e^2*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/
2)*(-c*x/b)^(1/2)-16*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^2
*c*d^2*e*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+16*E
llipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b*c^2*d^3*((c*x+b)/b)^(1/
2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-8*EllipticE(((c*x+b)/b)^(1/2
),(b*e/(b*e-c*d))^(1/2))*b^3*d*e^2*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))
^(1/2)*(-c*x/b)^(1/2)+24*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))
*b^2*c*d^2*e*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-
16*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b*c^2*d^3*((c*x+b)/b)
^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+x^4*c^3*e^3-3*x^3*b*c^2*
e^3+10*x^3*c^3*d*e^2-4*x^2*b^2*c*e^3+7*x^2*b*c^2*d*e^2+8*x^2*c^3*d^2*e-3*x*
b^2*c*d*e^2+8*x*b*c^2*d^2*e)/(c*x+b)/x/(e*x+d)^(3/2)/c/e^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx\right)^{\frac{3}{2}}}{(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(3/2)/(e*x + d)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+bx\right)^{\frac{3}{2}}\sqrt{ex+d}}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(3/2)*sqrt(e*x + d)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e
*x + d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x\,(b+cx))^{\frac{3}{2}}}{(d+ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(3/2)/(e*x+d)**(5/2),x)
```

```
[Out] Integral((x*(b + c*x))**(3/2)/(d + e*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx\right)^{\frac{3}{2}}}{(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(3/2)/(e*x + d)^(5/2), x)
```

3.397 $\int \frac{\left(bx+cx^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=354

$$-\frac{16\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(2cd-be)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{5e^4\sqrt{bx+cx^2}\sqrt{d+ex}}-\frac{2\sqrt{bx+cx^2}\left(ex\left(b^2e^2-10bcde+10c^2d^2\right)+c\right.}{5de^3(d+ex)^{3/2}(cd-be)}$$

```
[Out] (-2*(c*d^2*(8*c*d - 7*b*e) + e*(10*c^2*d^2 - 10*b*c*d*e + b^2*e^2)*x)*Sqrt[
b*x + c*x^2])/(5*d*e^3*(c*d - b*e)*(d + e*x)^(3/2)) - (2*(b*x + c*x^2)^(3/2
))/(5*e*(d + e*x)^(5/2)) + (2*Sqrt[-b]*Sqrt[c]*(16*c^2*d^2 - 16*b*c*d*e + b
^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*S
qrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(5*d*e^4*(c*d - b*e)*Sqrt[1 + (e*x)/d]*Sqr
t[b*x + c*x^2]) - (16*Sqrt[-b]*Sqrt[c]*(2*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)
/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(
c*d)])/(5*e^4*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.373561, antiderivative size = 354, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {732, 810, 843, 715, 112, 110, 117, 116}

$$-\frac{2\sqrt{bx+cx^2}\left(ex\left(b^2e^2-10bcde+10c^2d^2\right)+cd^2(8cd-7be)\right)}{5de^3(d+ex)^{3/2}(cd-be)}+\frac{2\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-16bcde+16c^2d^2\right)}{5de^4\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}(cd-be)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(3/2)/(d + e*x)^(7/2),x]
```


```
[Out] (-2*(c*d^2*(8*c*d - 7*b*e) + e*(10*c^2*d^2 - 10*b*c*d*e + b^2*e^2)*x)*Sqrt[
b*x + c*x^2])/(5*d*e^3*(c*d - b*e)*(d + e*x)^(3/2)) - (2*(b*x + c*x^2)^(3/2
))/(5*e*(d + e*x)^(5/2)) + (2*Sqrt[-b]*Sqrt[c]*(16*c^2*d^2 - 16*b*c*d*e + b
^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*S
qrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(5*d*e^4*(c*d - b*e)*Sqrt[1 + (e*x)/d]*Sqr
t[b*x + c*x^2]) - (16*Sqrt[-b]*Sqrt[c]*(2*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)
/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(
c*d)])/(5*e^4*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```


Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
```

```
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{\left(bx+cx^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx = -\frac{2\left(bx+cx^2\right)^{3/2}}{5e(d+ex)^{5/2}} + \frac{3\int \frac{(b+2cx)\sqrt{bx+cx^2}}{(d+ex)^{5/2}}dx}{5e}$$

$$= -\frac{2\left(cd^2(8cd-7be)+e\left(10c^2d^2-10bcde+b^2e^2\right)x\right)\sqrt{bx+cx^2}}{5de^3(cd-be)(d+ex)^{3/2}} - \frac{2\left(bx+cx^2\right)^{3/2}}{5e(d+ex)^{5/2}} - \frac{2\int \frac{-\frac{1}{2}bcd(8cd-}{}}{}$$

$$= -\frac{2\left(cd^2(8cd-7be)+e\left(10c^2d^2-10bcde+b^2e^2\right)x\right)\sqrt{bx+cx^2}}{5de^3(cd-be)(d+ex)^{3/2}} - \frac{2\left(bx+cx^2\right)^{3/2}}{5e(d+ex)^{5/2}} - \frac{(8c(2cd-be)}{}$$

$$= -\frac{2\left(cd^2(8cd-7be)+e\left(10c^2d^2-10bcde+b^2e^2\right)x\right)\sqrt{bx+cx^2}}{5de^3(cd-be)(d+ex)^{3/2}} - \frac{2\left(bx+cx^2\right)^{3/2}}{5e(d+ex)^{5/2}} - \frac{\left(8c(2cd-be)\right.}{}$$

$$= -\frac{2\left(cd^2(8cd-7be)+e\left(10c^2d^2-10bcde+b^2e^2\right)x\right)\sqrt{bx+cx^2}}{5de^3(cd-be)(d+ex)^{3/2}} - \frac{2\left(bx+cx^2\right)^{3/2}}{5e(d+ex)^{5/2}} + \frac{\left(c\left(16c^2d^2-\right.\right.}{}$$

$$= -\frac{2\left(cd^2(8cd-7be)+e\left(10c^2d^2-10bcde+b^2e^2\right)x\right)\sqrt{bx+cx^2}}{5de^3(cd-be)(d+ex)^{3/2}} - \frac{2\left(bx+cx^2\right)^{3/2}}{5e(d+ex)^{5/2}} + \frac{2\sqrt{-b}\sqrt{c}\left(16c\right.}{}$$

Mathematica [C] time = 1.19756, size = 369, normalized size = 1.04

$$-\frac{2(x(b+cx))^{3/2}\left(bex(b+cx)\left(b^2e^4x^2-bcde\left(7d^2+16dex+11e^2x^2\right)+c^2d^2\left(8d^2+18dex+11e^2x^2\right)\right)-c\sqrt{\frac{b}{c}}(d+ex)^2\left(-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(3/2)/(d + e*x)^(7/2),x]
```

```
[Out] (-2*(x*(b + c*x))^(3/2)*(b*e*x*(b + c*x)*(b^2*e^4*x^2 - b*c*d*e*(7*d^2 + 16
*d*e*x + 11*e^2*x^2) + c^2*d^2*(8*d^2 + 18*d*e*x + 11*e^2*x^2)) - Sqrt[b/c]
*c*(d + e*x)^2*(Sqrt[b/c]*(16*c^2*d^2 - 16*b*c*d*e + b^2*e^2)*(b + c*x)*(d
+ e*x) + I*b*e*(16*c^2*d^2 - 16*b*c*d*e + b^2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1
 + d/(e*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] -
I*b*e*(8*c^2*d^2 - 9*b*c*d*e + b^2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]
*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])))/(5*b*d*e^4
*(c*d - b*e)*x^2*(b + c*x)^2*(d + e*x)^(5/2))
```

Maple [B] time = 0.302, size = 1885, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(3/2)/(e*x+d)^(7/2),x)
```

```
[Out] 2/5*(x*(c*x+b))^(1/2)*(-11*x^3*b^2*c^2*d*e^4-5*x^3*b*c^3*d^2*e^3-16*x^2*b^2
*c^2*d^2*e^3+11*x^2*b*c^3*d^3*e^2-7*x*b^2*c^2*d^3*e^2+32*EllipticE(((c*x+b)
/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^2*c^2*d^2*e^3*(-c*x/b)^(1/2)*((c*x+b
)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)-16*EllipticE(((c*x+b)/b)^(1/2),(b*e
/(b*e-c*d))^(1/2))*x^2*b*c^3*d^3*e^2*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*
```

```
x+d)*c/(b*e-c*d))^(1/2)+16*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2
))*x*b^3*c*d^2*e^3*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^
(1/2)-48*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^2*c^2*d^3*e
^2*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)+32*Ellipti
cF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b*c^3*d^4*e*(-c*x/b)^(1/2)*((
c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)-34*EllipticE(((c*x+b)/b)^(1/2)
,(b*e/(b*e-c*d))^(1/2))*x*b^3*c*d^2*e^3*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-
(e*x+d)*c/(b*e-c*d))^(1/2)+64*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(
1/2))*x*b^2*c^2*d^3*e^2*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c
*d))^(1/2)-32*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b*c^3*d^
4*e*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)+8*Ellipti
cF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^3*c*d*e^4*(-c*x/b)^(1/2)*
((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)-24*EllipticF(((c*x+b)/b)^(1/
2),(b*e/(b*e-c*d))^(1/2))*x^2*b^2*c^2*d^2*e^3*(-c*x/b)^(1/2)*((c*x+b)/b)^(1
/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)+8*x*b*c^3*d^4*e+EllipticE(((c*x+b)/b)^(1/2
),(b*e/(b*e-c*d))^(1/2))*b^4*d^2*e^3*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*
x+d)*c/(b*e-c*d))^(1/2)-11*x^4*b*c^3*d*e^4+8*EllipticF(((c*x+b)/b)^(1/2),(b
*e/(b*e-c*d))^(1/2))*b^3*c*d^3*e^2*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+
d)*c/(b*e-c*d))^(1/2)-24*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))
*b^2*c^2*d^4*e*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2
)-17*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^3*c*d^3*e^2*(-c*x
/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)+32*EllipticE(((c*x
+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^2*c^2*d^4*e*(-c*x/b)^(1/2)*((c*x+b)/b
)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)+x^4*b^2*c^2*e^5+11*x^4*c^4*d^2*e^3+x^3
*b^3*c*e^5+18*x^3*c^4*d^3*e^2+8*x^2*c^4*d^4*e+2*EllipticE(((c*x+b)/b)^(1/2)
,(b*e/(b*e-c*d))^(1/2))*x*b^4*d*e^4*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x
+d)*c/(b*e-c*d))^(1/2)+16*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2)
)*x^2*b*c^3*d^3*e^2*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))
^(1/2)-17*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^3*c*d*e^
4*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)+EllipticE((
(c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^4*e^5*(-c*x/b)^(1/2)*((c*x+b)
/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)+16*EllipticF(((c*x+b)/b)^(1/2),(b*e/
(b*e-c*d))^(1/2))*b*c^3*d^5*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b
*e-c*d))^(1/2)-16*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b*c^3*
d^5*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2))/(c*x+b)/
x/(b*e-c*d)/c/(e*x+d)^(5/2)/e^4/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx\right)^{\frac{3}{2}}}{\left(ex+d\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(3/2)/(e*x + d)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+bx\right)^{\frac{3}{2}}\sqrt{ex+d}}{e^4x^4+4\,de^3x^3+6\,d^2e^2x^2+4\,d^3ex+d^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(3/2)*sqrt(e*x + d)/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e
^2*x^2 + 4*d^3*e*x + d^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x(b+cx))^{\frac{3}{2}}}{(d+ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(3/2)/(e*x+d)**(7/2),x)
```

```
[Out] Integral((x*(b + c*x))**(3/2)/(d + e*x)**(7/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2+bx)^{\frac{3}{2}}}{(ex+d)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(3/2)/(e*x + d)^(7/2), x)
```

3.398 $\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^{9/2}}\,dx$

Optimal. Leaf size=476

$$\frac{2\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}\left(-b^2e^2-16bcde+16c^2d^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{35de^4\sqrt{bx+cx^2}\sqrt{d+ex}(cd-be)}-\frac{2\sqrt{bx+cx^2}\left(ex\left(b^2e^2-14bcde+\right.\right.}{35de^3(d+}$$

```
[Out] (4*(2*c*d - b*e)*(4*c^2*d^2 - 4*b*c*d*e - b^2*e^2)*Sqrt[b*x + c*x^2])/(35*d
^2*e^3*(c*d - b*e)^2*Sqrt[d + e*x]) - (2*(d*(8*c^2*d^2 - 5*b*c*d*e - 2*b^2*
e^2) + e*(14*c^2*d^2 - 14*b*c*d*e + b^2*e^2)*x)*Sqrt[b*x + c*x^2])/(35*d*e^
3*(c*d - b*e)*(d + e*x)^(5/2)) - (2*(b*x + c*x^2)^(3/2))/(7*e*(d + e*x)^(7/
2)) - (4*Sqrt[-b]*Sqrt[c]*(2*c*d - b*e)*(4*c^2*d^2 - 4*b*c*d*e - b^2*e^2)*S
qrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/S
qrt[-b]], (b*e)/(c*d)])/(35*d^2*e^4*(c*d - b*e)^2*Sqrt[1 + (e*x)/d]*Sqrt[b*
x + c*x^2]) + (2*Sqrt[-b]*Sqrt[c]*(16*c^2*d^2 - 16*b*c*d*e - b^2*e^2)*Sqrt[
x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/S
qrt[-b]], (b*e)/(c*d)])/(35*d*e^4*(c*d - b*e)*Sqrt[d + e*x]*Sqrt[b*x + c*x^
2])
```

Rubi [A] time = 0.588078, antiderivative size = 476, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {732, 810, 834, 843, 715, 112, 110, 117, 116}

$$-\frac{2\sqrt{bx+cx^2}\left(ex\left(b^2e^2-14bcde+14c^2d^2\right)+d\left(-2b^2e^2-5bcde+8c^2d^2\right)\right)}{35de^3(d+ex)^{5/2}(cd-be)}+\frac{4\sqrt{bx+cx^2}(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right.}{35d^2e^3\sqrt{d+ex}(cd-be)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(3/2)/(d + e*x)^(9/2),x]
```


```
[Out] (4*(2*c*d - b*e)*(4*c^2*d^2 - 4*b*c*d*e - b^2*e^2)*Sqrt[b*x + c*x^2])/(35*d
^2*e^3*(c*d - b*e)^2*Sqrt[d + e*x]) - (2*(d*(8*c^2*d^2 - 5*b*c*d*e - 2*b^2*
e^2) + e*(14*c^2*d^2 - 14*b*c*d*e + b^2*e^2)*x)*Sqrt[b*x + c*x^2])/(35*d*e^
3*(c*d - b*e)*(d + e*x)^(5/2)) - (2*(b*x + c*x^2)^(3/2))/(7*e*(d + e*x)^(7/
2)) - (4*Sqrt[-b]*Sqrt[c]*(2*c*d - b*e)*(4*c^2*d^2 - 4*b*c*d*e - b^2*e^2)*S
qrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/S
qrt[-b]], (b*e)/(c*d)])/(35*d^2*e^4*(c*d - b*e)^2*Sqrt[1 + (e*x)/d]*Sqrt[b*
x + c*x^2]) + (2*Sqrt[-b]*Sqrt[c]*(16*c^2*d^2 - 16*b*c*d*e - b^2*e^2)*Sqrt[
x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/S
qrt[-b]], (b*e)/(c*d)])/(35*d*e^4*(c*d - b*e)*Sqrt[d + e*x]*Sqrt[b*x + c*x^
2])
```


Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
```

```
FreeQ[{b, c, d, e, f}, x] && !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{\left(bx+cx^2\right)^{3/2}}{(d+ex)^{9/2}}\,dx = -\frac{2\left(bx+cx^2\right)^{3/2}}{7e(d+ex)^{7/2}} + \frac{3\int \frac{(b+2cx)\sqrt{bx+cx^2}}{(d+ex)^{7/2}}\,dx}{7e}$$

$$= -\frac{2\left(d\left(8c^2d^2-5bcde-2b^2e^2\right)+e\left(14c^2d^2-14bcde+b^2e^2\right)x\right)\sqrt{bx+cx^2}}{35de^3(cd-be)(d+ex)^{5/2}} - \frac{2\left(bx+cx^2\right)^{3/2}}{7e(d+ex)^{7/2}} - \frac{2\int -$$

$$= \frac{4(2cd-be)\left(4c^2d^2-4bcde-b^2e^2\right)\sqrt{bx+cx^2}}{35d^2e^3(cd-be)^2\sqrt{d+ex}} - \frac{2\left(d\left(8c^2d^2-5bcde-2b^2e^2\right)+e\left(14c^2d^2-14bcde\right.\right.}{35de^3(cd-be)(d+ex)^{5/2}}$$

$$= \frac{4(2cd-be)\left(4c^2d^2-4bcde-b^2e^2\right)\sqrt{bx+cx^2}}{35d^2e^3(cd-be)^2\sqrt{d+ex}} - \frac{2\left(d\left(8c^2d^2-5bcde-2b^2e^2\right)+e\left(14c^2d^2-14bcde\right.\right.}{35de^3(cd-be)(d+ex)^{5/2}}$$

$$= \frac{4(2cd-be)\left(4c^2d^2-4bcde-b^2e^2\right)\sqrt{bx+cx^2}}{35d^2e^3(cd-be)^2\sqrt{d+ex}} - \frac{2\left(d\left(8c^2d^2-5bcde-2b^2e^2\right)+e\left(14c^2d^2-14bcde\right.\right.}{35de^3(cd-be)(d+ex)^{5/2}}$$

$$= \frac{4(2cd-be)\left(4c^2d^2-4bcde-b^2e^2\right)\sqrt{bx+cx^2}}{35d^2e^3(cd-be)^2\sqrt{d+ex}} - \frac{2\left(d\left(8c^2d^2-5bcde-2b^2e^2\right)+e\left(14c^2d^2-14bcde\right.\right.}{35de^3(cd-be)(d+ex)^{5/2}}$$

$$= \frac{4(2cd-be)\left(4c^2d^2-4bcde-b^2e^2\right)\sqrt{bx+cx^2}}{35d^2e^3(cd-be)^2\sqrt{d+ex}} - \frac{2\left(d\left(8c^2d^2-5bcde-2b^2e^2\right)+e\left(14c^2d^2-14bcde\right.\right.}{35de^3(cd-be)(d+ex)^{5/2}}$$

Mathematica [C] time = 2.27756, size = 479, normalized size = 1.01

$$-\frac{2(x(b+cx))^{3/2}\left(bex(b+cx)\left(d(d+ex)^2\left(b^2e^2-19bcde+19c^2d^2\right)(cd-be)-2(d+ex)^3\left(2b^2cde^2+b^3e^3-12bc^2d^2e+8c^3\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(3/2)/(d + e*x)^(9/2),x]
```

```
[Out] (-2*(x*(b + c*x))^(3/2)*(b*e*x*(b + c*x)*(5*d^3*(c*d - b*e)^3 - 8*d^2*(c*d
- b*e)^2*(2*c*d - b*e)*(d + e*x) + d*(c*d - b*e)*(19*c^2*d^2 - 19*b*c*d*e +
 b^2*e^2)*(d + e*x)^2 - 2*(8*c^3*d^3 - 12*b*c^2*d^2*e + 2*b^2*c*d*e^2 + b^3
*e^3)*(d + e*x)^3) + Sqrt[b/c]*c*(d + e*x)^3*(2*Sqrt[b/c]*(8*c^3*d^3 - 12*b
*c^2*d^2*e + 2*b^2*c*d*e^2 + b^3*e^3)*(b + c*x)*(d + e*x) + (2*I)*b*e*(8*c^
3*d^3 - 12*b*c^2*d^2*e + 2*b^2*c*d*e^2 + b^3*e^3)*Sqrt[1 + b/(c*x)]*Sqrt[1
+ d/(e*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - I
*b*e*(8*c^3*d^3 - 13*b*c^2*d^2*e + 3*b^2*c*d*e^2 + 2*b^3*e^3)*Sqrt[1 + b/(c
*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d
```

```
)/(b*e)]))))/(35*b*d^2*e^4*(c*d - b*e)^2*x^2*(b + c*x)^2*(d + e*x)^(7/2))
```

Maple [B] time = 0.324, size = 3267, normalized size = 6.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(3/2)/(e*x+d)^(9/2),x)

[Out] 2/35*(x*(c*x+b))^(1/2)*(15*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2
))*x^3*b^3*c^2*d^2*e^5*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x
/b)^(1/2)+2*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^3*b^4*c*d*
e^6*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-28*Ellipt
icE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^3*b^3*c^2*d^2*e^5*((c*x+b)/b
)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+16*EllipticF(((c*x+b)/b
)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^3*b*c^4*d^4*e^3*((c*x+b)/b)^(1/2)*(-(e*x+d
)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+40*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*
e-c*d))^(1/2))*x^3*b^2*c^3*d^3*e^4*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))
^(1/2)*(-c*x/b)^(1/2)+6*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*
x^2*b^4*c*d^2*e^5*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(
1/2)-84*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^3*c^2*d^3*
e^4*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+120*Ellip
ticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^2*c^3*d^4*e^3*((c*x+b)/
b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-48*EllipticE(((c*x+b)/
b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b*c^4*d^5*e^2*((c*x+b)/b)^(1/2)*(-(e*x+
d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+3*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*
e-c*d))^(1/2))*x^2*b^4*c*d^2*e^5*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(
1/2)*(-c*x/b)^(1/2)-16*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x
^3*b*c^4*d^4*e^3*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1
/2)+45*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^3*c^2*d^3*e
^4*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-96*Ellipti
cF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^2*c^3*d^4*e^3*((c*x+b)/b)
^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+48*EllipticF(((c*x+b)/b)
^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b*c^4*d^5*e^2*((c*x+b)/b)^(1/2)*(-(e*x+d)
*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+6*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-
c*d))^(1/2))*x*b^4*c*d^3*e^4*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)
*(-c*x/b)^(1/2)-84*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^3
*c^2*d^4*e^3*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+
120*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^2*c^3*d^5*e^2*((
c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-48*EllipticE(((
c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b*c^4*d^6*e*((c*x+b)/b)^(1/2)*(-(e
*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+6*EllipticE(((c*x+b)/b)^(1/2),(b*e/
(b*e-c*d))^(1/2))*x*b^5*d^2*e^5*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1
/2)*(-c*x/b)^(1/2)+2*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^4
*c*d^4*e^3*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-28
*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^3*c^2*d^5*e^2*((c*x+b
)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+40*EllipticE(((c*x+b
)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^2*c^3*d^6*e*((c*x+b)/b)^(1/2)*(-(e*x+d)
*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*
d))^(1/2))*b^4*c*d^4*e^3*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c
*x/b)^(1/2)+15*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^3*c^2*d
^5*e^2*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-32*Ell
ipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^2*c^3*d^6*e*((c*x+b)/b)^(
1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-24*x^5*b*c^4*d^2*e^5+11*x^
4*b^3*c^2*d*e^6-32*x^4*b^2*c^3*d^2*e^5-18*x^4*b*c^4*d^3*e^4+7*x^3*b^4*c*d*e
^6-8*x^3*b^3*c^2*d^2*e^5-30*x^3*b^2*c^3*d^3*e^4-7*x^3*b*c^4*d^4*e^3+4*x^2*b
```

```
^3*c^2*d^3*e^4-35*x^2*b^2*c^3*d^4*e^3+15*x^2*b*c^4*d^5*e^2+x*b^3*c^2*d^4*e^
3-11*x*b^2*c^3*d^5*e^2+8*x*b*c^4*d^6*e+4*x^5*b^2*c^3*d*e^6+3*EllipticF(((c*
x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^4*c*d^3*e^4*((c*x+b)/b)^(1/2)*(-(e
*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+45*EllipticF(((c*x+b)/b)^(1/2),(b*e
/(b*e-c*d))^(1/2))*x*b^3*c^2*d^4*e^3*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d
))^(1/2)*(-c*x/b)^(1/2)-96*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2
))*x*b^2*c^3*d^5*e^2*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b
)^(1/2)+48*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b*c^4*d^6*e
*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+EllipticF(((
c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^3*b^4*c*d*e^6*((c*x+b)/b)^(1/2)*(-
(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-32*EllipticF(((c*x+b)/b)^(1/2),(b
*e/(b*e-c*d))^(1/2))*x^3*b^2*c^3*d^3*e^4*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e
-c*d))^(1/2)*(-c*x/b)^(1/2)+2*x^5*b^3*c^2*e^7+16*x^5*c^5*d^3*e^4+2*x^4*b^4*
c*e^7+29*x^4*c^5*d^4*e^3+26*x^3*c^5*d^5*e^2+8*x^2*c^5*d^6*e+2*EllipticE(((c
*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^3*b^5*e^7*((c*x+b)/b)^(1/2)*(-(e*x+
d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+2*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*
e-c*d))^(1/2))*b^5*d^3*e^4*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(
-c*x/b)^(1/2)-16*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b*c^4*d
^7*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+16*Ellipti
cF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b*c^4*d^7*((c*x+b)/b)^(1/2)*(-(
e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+6*EllipticE(((c*x+b)/b)^(1/2),(b*e
/(b*e-c*d))^(1/2))*x^2*b^5*d*e^6*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(
1/2)*(-c*x/b)^(1/2))/(c*x+b)/x/(b*e-c*d)^2/c/(e*x+d)^(7/2)/e^4/d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx\right)^{\frac{3}{2}}}{\left(ex+d\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(3/2)/(e*x + d)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+bx\right)^{\frac{3}{2}}\sqrt{ex+d}}{e^5x^5+5\,de^4x^4+10\,d^2e^3x^3+10\,d^3e^2x^2+5\,d^4ex+d^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(3/2)*sqrt(e*x + d)/(e^5*x^5 + 5*d*e^4*x^4 + 10*d^2*
e^3*x^3 + 10*d^3*e^2*x^2 + 5*d^4*e*x + d^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(3/2)/(e*x+d)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx\right)^{\frac{3}{2}}}{\left(ex+d\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(3/2)/(e*x + d)^(9/2), x)
```

3.399 $\int \sqrt{d+ex}\left(bx+cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=666

$$\frac{2\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\left(79b^2c^2d^2e^2+49b^3cde^3+24b^4e^4-256bc^3d^3e+128c^4d^4\right)\text{EllipticF}\left(\sin^{-1}\left(\right.\right.}{9009c^{7/2}e^6\sqrt{bx+cx^2}\sqrt{d+ex}}$$

```
[Out] (2*Sqrt[d + e*x]*(128*c^5*d^5 - 368*b*c^4*d^4*e + 303*b^2*c^3*d^3*e^2 - 22*
b^3*c^2*d^2*e^3 - 17*b^4*c*d*e^4 + 24*b^5*e^5 - 3*c*e*(32*c^4*d^4 - 64*b*c^
3*d^3*e + 21*b^2*c^2*d^2*e^2 + 11*b^3*c*d*e^3 - 24*b^4*e^4)*x)*Sqrt[b*x + c
*x^2])/(9009*c^3*e^5) + (10*Sqrt[d + e*x]*(16*c^3*d^3 - 31*b*c^2*d^2*e + 9*
b^2*c*d*e^2 - 18*b^3*e^3 - 14*c*e*(c^2*d^2 - b*c*d*e + 3*b^2*e^2)*x)*(b*x +
 c*x^2)^(3/2))/(9009*c^2*e^3) - (10*(2*c*d - b*e)*Sqrt[d + e*x]*(b*x + c*x^
2)^(5/2))/(143*c*e) + (2*(d + e*x)^(3/2)*(b*x + c*x^2)^(5/2))/(13*e) - (4*S
qrt[-b]*(128*c^6*d^6 - 384*b*c^5*d^5*e + 343*b^2*c^4*d^4*e^2 - 46*b^3*c^3*d
^3*e^3 - 21*b^4*c^2*d^2*e^4 - 20*b^5*c*d*e^5 + 24*b^6*e^6)*Sqrt[x]*Sqrt[1 +
 (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)
/(c*d)])/(9009*c^(7/2)*e^6*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) + (2*Sqrt[-
b]*d*(c*d - b*e)*(2*c*d - b*e)*(128*c^4*d^4 - 256*b*c^3*d^3*e + 79*b^2*c^2*
d^2*e^2 + 49*b^3*c*d*e^3 + 24*b^4*e^4)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (
e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(9009*c
^(7/2)*e^6*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.832554, antiderivative size = 666, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {734, 832, 814, 843, 715, 112, 110, 117, 116}

$$\frac{10\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}\left(-14cex\left(3b^2e^2-bcde+c^2d^2\right)+9b^2cde^2-18b^3e^3-31bc^2d^2e+16c^3d^3\right)}{9009c^2e^3}+\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(b*x + c*x^2)^(5/2),x]
```


```
[Out] (2*Sqrt[d + e*x]*(128*c^5*d^5 - 368*b*c^4*d^4*e + 303*b^2*c^3*d^3*e^2 - 22*
b^3*c^2*d^2*e^3 - 17*b^4*c*d*e^4 + 24*b^5*e^5 - 3*c*e*(32*c^4*d^4 - 64*b*c^
3*d^3*e + 21*b^2*c^2*d^2*e^2 + 11*b^3*c*d*e^3 - 24*b^4*e^4)*x)*Sqrt[b*x + c
*x^2])/(9009*c^3*e^5) + (10*Sqrt[d + e*x]*(16*c^3*d^3 - 31*b*c^2*d^2*e + 9*
b^2*c*d*e^2 - 18*b^3*e^3 - 14*c*e*(c^2*d^2 - b*c*d*e + 3*b^2*e^2)*x)*(b*x +
 c*x^2)^(3/2))/(9009*c^2*e^3) - (10*(2*c*d - b*e)*Sqrt[d + e*x]*(b*x + c*x^
2)^(5/2))/(143*c*e) + (2*(d + e*x)^(3/2)*(b*x + c*x^2)^(5/2))/(13*e) - (4*S
qrt[-b]*(128*c^6*d^6 - 384*b*c^5*d^5*e + 343*b^2*c^4*d^4*e^2 - 46*b^3*c^3*d
^3*e^3 - 21*b^4*c^2*d^2*e^4 - 20*b^5*c*d*e^5 + 24*b^6*e^6)*Sqrt[x]*Sqrt[1 +
 (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)
/(c*d)])/(9009*c^(7/2)*e^6*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) + (2*Sqrt[-
b]*d*(c*d - b*e)*(2*c*d - b*e)*(128*c^4*d^4 - 256*b*c^3*d^3*e + 79*b^2*c^2*
d^2*e^2 + 49*b^3*c*d*e^3 + 24*b^4*e^4)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (
e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(9009*c
^(7/2)*e^6*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```


Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
```

```
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
```

```
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^{5/2}\,dx = \frac{2(d+ex)^{3/2}\left(bx+cx^2\right)^{5/2}}{13e} - \frac{5\int \sqrt{d+ex}(bd+(2cd-be)x)\left(bx+cx^2\right)^{3/2}\,dx}{13e}$$

$$= -\frac{10(2cd-be)\sqrt{d+ex}\left(bx+cx^2\right)^{5/2}}{143ce} + \frac{2(d+ex)^{3/2}\left(bx+cx^2\right)^{5/2}}{13e} - \frac{10\int \frac{\left(\frac{1}{2}bd(cd+5be)+\left(c^2d^2-bc\right.\right.}{\sqrt{d}}}{143}$$

$$= \frac{10\sqrt{d+ex}\left(16c^3d^3-31bc^2d^2e+9b^2cde^2-18b^3e^3-14ce\left(c^2d^2-bcde+3b^2e^2\right)x\right)\left(bx+cx\right.}{9009c^2e^3}$$

$$= \frac{2\sqrt{d+ex}\left(128c^5d^5-368bc^4d^4e+303b^2c^3d^3e^2-22b^3c^2d^2e^3-17b^4cde^4+24b^5e^5-3ce\left(32\right.\right.}{9009c^3e^5}$$

$$= \frac{2\sqrt{d+ex}\left(128c^5d^5-368bc^4d^4e+303b^2c^3d^3e^2-22b^3c^2d^2e^3-17b^4cde^4+24b^5e^5-3ce\left(32\right.\right.}{9009c^3e^5}$$

$$= \frac{2\sqrt{d+ex}\left(128c^5d^5-368bc^4d^4e+303b^2c^3d^3e^2-22b^3c^2d^2e^3-17b^4cde^4+24b^5e^5-3ce\left(32\right.\right.}{9009c^3e^5}$$

$$= \frac{2\sqrt{d+ex}\left(128c^5d^5-368bc^4d^4e+303b^2c^3d^3e^2-22b^3c^2d^2e^3-17b^4cde^4+24b^5e^5-3ce\left(32\right.\right.}{9009c^3e^5}$$

$$= \frac{2\sqrt{d+ex}\left(128c^5d^5-368bc^4d^4e+303b^2c^3d^3e^2-22b^3c^2d^2e^3-17b^4cde^4+24b^5e^5-3ce\left(32\right.\right.}{9009c^3e^5}$$

Mathematica [C] time = 3.28284, size = 663, normalized size = 1.

$$\frac{2(x(b+cx))^{5/2}\left(bex(b+cx)(d+ex)\left(b^2c^3e^2\left(-218d^2ex+303d^3+178de^2x^2+1113e^3x^3\right)+b^3c^2e^3\left(-22d^2+12dex+15e^2x\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*(b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(x*(b + c*x))^(5/2)*(b*e*x*(b + c*x)*(d + e*x)*(24*b^5*e^5 - b^4*c*e^4*(
17*d + 18*e*x) + b^3*c^2*e^3*(-22*d^2 + 12*d*e*x + 15*e^2*x^2) + b^2*c^3*e^
2*(303*d^3 - 218*d^2*e*x + 178*d*e^2*x^2 + 1113*e^3*x^3) + b*c^4*e*(-368*d^
4 + 272*d^3*e*x - 225*d^2*e^2*x^2 + 196*d*e^3*x^3 + 1701*e^4*x^4) + c^5*(12
8*d^5 - 96*d^4*e*x + 80*d^3*e^2*x^2 - 70*d^2*e^3*x^3 + 63*d*e^4*x^4 + 693*e
^5*x^5)) + Sqrt[b/c]*(-2*Sqrt[b/c]*(128*c^6*d^6 - 384*b*c^5*d^5*e + 343*b^2
*c^4*d^4*e^2 - 46*b^3*c^3*d^3*e^3 - 21*b^4*c^2*d^2*e^4 - 20*b^5*c*d*e^5 + 2
4*b^6*e^6)*(b + c*x)*(d + e*x) - (2*I)*b*e*(128*c^6*d^6 - 384*b*c^5*d^5*e +
 343*b^2*c^4*d^4*e^2 - 46*b^3*c^3*d^3*e^3 - 21*b^4*c^2*d^2*e^4 - 20*b^5*c*d
*e^5 + 24*b^6*e^6)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticE[I*
ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] + I*b*e*(128*c^6*d^6 - 400*b*c^5*d
^5*e + 383*b^2*c^4*d^4*e^2 - 70*b^3*c^3*d^3*e^3 - 25*b^4*c^2*d^2*e^4 - 64*b
^5*c*d*e^5 + 48*b^6*e^6)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*Ellipt
icF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])))/(9009*b*c^3*e^6*x^3*(b +
c*x)^3*Sqrt[d + e*x])
```

Maple [B] time = 0.287, size = 1728, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(5/2)*(e*x+d)^(1/2),x)
```

```
[Out] 2/9009*(x*(c*x+b))^(1/2)*(e*x+d)^(1/2)*(-23*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(
b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^
(1/2))*b^6*c^2*d^2*e^5+303*x*b^3*c^5*d^4*e^3-368*x*b^2*c^6*d^5*e^2+128*x*b*
c^7*d^6*e+2653*x^6*b*c^7*d*e^6+48*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^
(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^8
*e^7+24*x*b^6*c^2*d*e^6-17*x*b^5*c^3*d^2*e^5+207*x^2*b^2*c^6*d^4*e^3+3188*x
^5*b^2*c^6*d*e^6-36*x^5*b*c^7*d^2*e^5+1318*x^4*b^3*c^5*d*e^6-69*x^4*b^2*c^6
*d^2*e^5+57*x^4*b*c^7*d^3*e^4-8*x^3*b^4*c^4*d*e^6-50*x^3*b^3*c^5*d^2*e^5+13
2*x^3*b^2*c^6*d^3*e^4-112*x^3*b*c^7*d^4*e^3-11*x^2*b^5*c^3*d*e^6-27*x^2*b^4
*c^4*d^2*e^5+63*x^2*b^3*c^5*d^3*e^4-336*x^2*b*c^7*d^5*e^2+778*((c*x+b)/b)^(
1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2
),(b*e/(b*e-c*d))^(1/2))*b^4*c^4*d^4*e^3-1454*((c*x+b)/b)^(1/2)*(-(e*x+d)*c
/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d)
)^(1/2))*b^3*c^5*d^5*e^2+1024*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2
)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^2*c^6
*d^6*e+24*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*Ell
ipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^7*c*d*e^6-20*((c*x+b)/b)^
(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/
2),(b*e/(b*e-c*d))^(1/2))*b^5*c^3*d^3*e^4-395*((c*x+b)/b)^(1/2)*(-(e*x+d)*c
/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d)
)^(1/2))*b^4*c^4*d^4*e^3+1054*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2
)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^3*c^5
*d^5*e^2-896*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*
EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^2*c^6*d^6*e-88*((c*x+b
)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b
)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^7*c*d*e^6-2*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/
(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))
^(1/2))*b^6*c^2*d^2*e^5-50*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(
-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^5*c^3*d^
3*e^4+693*x^8*c^8*e^7-22*x*b^4*c^4*d^3*e^4+2394*x^7*b*c^7*e^7+756*x^7*c^8*d
*e^6+2814*x^6*b^2*c^6*e^7-7*x^6*c^8*d^2*e^5+1128*x^5*b^3*c^5*e^7+10*x^5*c^8
*d^3*e^4-3*x^4*b^4*c^4*e^7-16*x^4*c^8*d^4*e^3+6*x^3*b^5*c^3*e^7+32*x^3*c^8*
d^5*e^2+24*x^2*b^6*c^2*e^7+128*x^2*c^8*d^6*e+256*((c*x+b)/b)^(1/2)*(-(e*x+d
```

```
)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c
*d))^(1/2))*b*c^7*d^7-256*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-
c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b*c^7*d^7)/
c^5/x/(c*e*x^2+b*e*x+c*d*x+b*d)/e^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx\right)^{\frac{5}{2}} \sqrt{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2)*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(5/2)*sqrt(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(c^2x^4 + 2\,bcx^3 + b^2x^2\right)\sqrt{cx^2 + bx}\sqrt{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + b^2*x^2)*sqrt(c*x^2 + b*x)*sqrt(e*x + d), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(5/2)*(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx\right)^{\frac{5}{2}} \sqrt{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2)*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(5/2)*sqrt(e*x + d), x)
```

3.400 $\int \frac{\left(bx+cx^2\right)^{5/2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=537

$$\frac{4\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\left(123b^2c^2d^2e^2+5b^3cde^3+2b^4e^4-256bc^3d^3e+128c^4d^4\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\right.}{693c^{5/2}e^6\sqrt{bx+cx^2}\sqrt{d+ex}}$$

```
[Out] (2*Sqrt[d + e*x]*(128*c^4*d^4 - 304*b*c^3*d^3*e + 195*b^2*c^2*d^2*e^2 - 7*b
^3*c*d*e^3 - 4*b^4*e^4 - 12*c*e*(2*c*d - b*e)*(4*c^2*d^2 - 4*b*c*d*e - b^2*
e^2)*x)*Sqrt[b*x + c*x^2])/(693*c^2*e^5) + (10*Sqrt[d + e*x]*(16*c^2*d^2 -
23*b*c*d*e + 3*b^2*e^2 - 7*c*e*(2*c*d - b*e)*x)*(b*x + c*x^2)^(3/2))/(693*c
*e^3) + (2*Sqrt[d + e*x]*(b*x + c*x^2)^(5/2))/(11*e) - (2*Sqrt[-b]*(2*c*d -
 b*e)*(128*c^4*d^4 - 256*b*c^3*d^3*e + 99*b^2*c^2*d^2*e^2 + 29*b^3*c*d*e^3
+ 8*b^4*e^4)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt
[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(693*c^(5/2)*e^6*Sqrt[1 + (e*x)/d]*Sq
rt[b*x + c*x^2]) + (4*Sqrt[-b]*d*(c*d - b*e)*(128*c^4*d^4 - 256*b*c^3*d^3*e
 + 123*b^2*c^2*d^2*e^2 + 5*b^3*c*d*e^3 + 2*b^4*e^4)*Sqrt[x]*Sqrt[1 + (c*x)/
b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c
*d)])/(693*c^(5/2)*e^6*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.603732, antiderivative size = 537, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {734, 814, 843, 715, 112, 110, 117, 116}

$$\frac{10\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}\left(3b^2e^2-7cex(2cd-be)-23bcde+16c^2d^2\right)}{693ce^3}+\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}\left(-12cex(2cd-be)\left(-b^2e^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(5/2)/Sqrt[d + e*x],x]
```


```
[Out] (2*Sqrt[d + e*x]*(128*c^4*d^4 - 304*b*c^3*d^3*e + 195*b^2*c^2*d^2*e^2 - 7*b
^3*c*d*e^3 - 4*b^4*e^4 - 12*c*e*(2*c*d - b*e)*(4*c^2*d^2 - 4*b*c*d*e - b^2*
e^2)*x)*Sqrt[b*x + c*x^2])/(693*c^2*e^5) + (10*Sqrt[d + e*x]*(16*c^2*d^2 -
23*b*c*d*e + 3*b^2*e^2 - 7*c*e*(2*c*d - b*e)*x)*(b*x + c*x^2)^(3/2))/(693*c
*e^3) + (2*Sqrt[d + e*x]*(b*x + c*x^2)^(5/2))/(11*e) - (2*Sqrt[-b]*(2*c*d -
 b*e)*(128*c^4*d^4 - 256*b*c^3*d^3*e + 99*b^2*c^2*d^2*e^2 + 29*b^3*c*d*e^3
+ 8*b^4*e^4)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt
[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(693*c^(5/2)*e^6*Sqrt[1 + (e*x)/d]*Sq
rt[b*x + c*x^2]) + (4*Sqrt[-b]*d*(c*d - b*e)*(128*c^4*d^4 - 256*b*c^3*d^3*e
 + 123*b^2*c^2*d^2*e^2 + 5*b^3*c*d*e^3 + 2*b^4*e^4)*Sqrt[x]*Sqrt[1 + (c*x)/
b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c
*d)])/(693*c^(5/2)*e^6*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```


Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
```

```
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
```

```
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{\left(bx+cx^2\right)^{5/2}}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{d+ex}\left(bx+cx^2\right)^{5/2}}{11e} - \frac{5\int \frac{(bd+(2cd-be)x)\left(bx+cx^2\right)^{3/2}}{\sqrt{d+ex}}\,dx}{11e}$$

$$= \frac{10\sqrt{d+ex}\left(16c^2d^2-23bcde+3b^2e^2-7ce(2cd-be)x\right)\left(bx+cx^2\right)^{3/2}}{693ce^3} + \frac{2\sqrt{d+ex}\left(bx+cx^2\right)^{5/2}}{11e} +$$

$$= \frac{2\sqrt{d+ex}\left(128c^4d^4-304bc^3d^3e+195b^2c^2d^2e^2-7b^3cde^3-4b^4e^4-12ce(2cd-be)\left(4c^2d^2-4bcd\right.\right.}{693c^2e^5}$$

$$= \frac{2\sqrt{d+ex}\left(128c^4d^4-304bc^3d^3e+195b^2c^2d^2e^2-7b^3cde^3-4b^4e^4-12ce(2cd-be)\left(4c^2d^2-4bcd\right.\right.}{693c^2e^5}$$

$$= \frac{2\sqrt{d+ex}\left(128c^4d^4-304bc^3d^3e+195b^2c^2d^2e^2-7b^3cde^3-4b^4e^4-12ce(2cd-be)\left(4c^2d^2-4bcd\right.\right.}{693c^2e^5}$$

$$= \frac{2\sqrt{d+ex}\left(128c^4d^4-304bc^3d^3e+195b^2c^2d^2e^2-7b^3cde^3-4b^4e^4-12ce(2cd-be)\left(4c^2d^2-4bcd\right.\right.}{693c^2e^5}$$

$$= \frac{2\sqrt{d+ex}\left(128c^4d^4-304bc^3d^3e+195b^2c^2d^2e^2-7b^3cde^3-4b^4e^4-12ce(2cd-be)\left(4c^2d^2-4bcd\right.\right.}{693c^2e^5}$$

Mathematica [C] time = 2.87561, size = 557, normalized size = 1.04

$$\frac{2(x(b+cx))^{5/2}\left(bex(b+cx)(d+ex)\left(b^2c^2e^2\left(195d^2-139dex+113e^2x^2\right)+b^3ce^3(3ex-7d)-4b^4e^4+bc^3e\left(224d^2ex-3\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(5/2)/Sqrt[d + e*x],x]
```

```
[Out] (2*(x*(b + c*x))^(5/2)*(b*e*x*(b + c*x)*(d + e*x)*(-4*b^4*e^4 + b^3*c*e^3*(
-7*d + 3*e*x) + b^2*c^2*e^2*(195*d^2 - 139*d*e*x + 113*e^2*x^2) + b*c^3*e*(
-304*d^3 + 224*d^2*e*x - 185*d*e^2*x^2 + 161*e^3*x^3) + c^4*(128*d^4 - 96*d
^3*e*x + 80*d^2*e^2*x^2 - 70*d*e^3*x^3 + 63*e^4*x^4)) + Sqrt[b/c]*(Sqrt[b/c
]*(-256*c^5*d^5 + 640*b*c^4*d^4*e - 454*b^2*c^3*d^3*e^2 + 41*b^3*c^2*d^2*e^
3 + 13*b^4*c*d*e^4 + 8*b^5*e^5)*(b + c*x)*(d + e*x) - I*b*e*(256*c^5*d^5 -
640*b*c^4*d^4*e + 454*b^2*c^3*d^3*e^2 - 41*b^3*c^2*d^2*e^3 - 13*b^4*c*d*e^4
 - 8*b^5*e^5)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticE[I*ArcSi
nh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] + I*b*e*(128*c^5*d^5 - 336*b*c^4*d^4*e
+ 259*b^2*c^3*d^3*e^2 - 34*b^3*c^2*d^2*e^3 - 9*b^4*c*d*e^4 - 8*b^5*e^5)*Sqr
t[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt
[x]], (c*d)/(b*e)])))/(693*b*c^2*e^6*x^3*(b + c*x)^3*Sqrt[d + e*x])
```

Maple [B] time = 0.279, size = 1441, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(5/2)/(e*x+d)^(1/2),x)
```

```
[Out] -2/693*(x*(c*x+b))^(1/2)*(e*x+d)^(1/2)*(-224*x^6*b*c^6*e^6+7*x^6*c^7*d*e^5-
274*x^5*b^2*c^5*e^6-10*x^5*c^7*d^2*e^4-116*x^4*b^3*c^4*e^6+16*x^4*c^7*d^3*e
^3+x^3*b^4*c^3*e^6-32*x^3*c^7*d^4*e^2+4*x^2*b^5*c^2*e^6-128*x^2*c^7*d^5*e+2
72*x^2*b*c^6*d^4*e^2+4*x*b^5*c^2*d*e^5+7*x*b^4*c^3*d^2*e^4-195*x*b^3*c^4*d^
3*e^3+304*x*b^2*c^5*d^4*e^2-128*x*b*c^6*d^5*e+31*x^5*b*c^6*d*e^5+50*x^4*b^2
*c^5*d*e^5-49*x^4*b*c^6*d^2*e^4+30*x^3*b^3*c^4*d*e^5-95*x^3*b^2*c^5*d^2*e^4
+96*x^3*b*c^6*d^3*e^3+8*x^2*b^4*c^3*d*e^5-49*x^2*b^3*c^4*d^2*e^4+8*((c*x+b)
/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)
^(1/2),(b*e/(b*e-c*d))^(1/2))*b^7*e^6-495*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*
e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1
/2))*b^4*c^3*d^3*e^3-256*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c
*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b*c^6*d^6+5*
((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c
*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^6*c*d*e^5-63*x^7*c^7*e^6-115*x^2*b^
2*c^5*d^3*e^3+256*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(
1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b*c^6*d^6+768*((c*x
+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)
/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^2*c^5*d^5*e-758*((c*x+b)/b)^(1/2)*(-(e*x
+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e
-c*d))^(1/2))*b^3*c^4*d^4*e^2+4*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1
/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^6*c
*d*e^5+6*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*Elli
pticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^5*c^2*d^2*e^4+236*((c*x+b)
/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)
^(1/2),(b*e/(b*e-c*d))^(1/2))*b^4*c^3*d^3*e^3+28*((c*x+b)/b)^(1/2)*(-(e*x+d
)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c
*d))^(1/2))*b^5*c^2*d^2*e^4+1094*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(
1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^3*
c^4*d^4*e^2-896*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/
2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^2*c^5*d^5*e)/c^4/e^
6/x/(c*e*x^2+b*e*x+c*d*x+b*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx)^{\frac{5}{2}}}{\sqrt{ex + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(5/2)/sqrt(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(c^2x^4 + 2\,bcx^3 + b^2x^2)\sqrt{cx^2 + bx}}{\sqrt{ex + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + b^2*x^2)*sqrt(c*x^2 + b*x)/sqrt(e*x + d), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(5/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx\right)^{\frac{5}{2}}}{\sqrt{ex + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(5/2)/sqrt(e*x + d), x)
```

3.401 $\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=457

$$-\frac{2\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\left(-b^2e^2-128bcde+128c^2d^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{63c^{3/2}e^6\sqrt{bx+cx^2}\sqrt{d+ex}}-\frac{2\sqrt{bx+cx^2}}{}$$

```
[Out] (-2*Sqrt[d + e*x]*(128*c^3*d^3 - 240*b*c^2*d^2*e + 111*b^2*c*d*e^2 - b^3*e^
3 - 3*c*e*(32*c^2*d^2 - 32*b*c*d*e + b^2*e^2)*x)*Sqrt[b*x + c*x^2])/(63*c*e
^5) - (10*Sqrt[d + e*x]*(16*c*d - 15*b*e - 14*c*e*x)*(b*x + c*x^2)^(3/2))/(
63*e^3) - (2*(b*x + c*x^2)^(5/2))/(e*Sqrt[d + e*x]) + (4*Sqrt[-b]*(128*c^4*
d^4 - 256*b*c^3*d^3*e + 135*b^2*c^2*d^2*e^2 - 7*b^3*c*d*e^3 - b^4*e^4)*Sqrt
[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt
[-b]], (b*e)/(c*d)])/(63*c^(3/2)*e^6*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) -
 (2*Sqrt[-b]*d*(c*d - b*e)*(2*c*d - b*e)*(128*c^2*d^2 - 128*b*c*d*e - b^2*e
^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*S
qrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(63*c^(3/2)*e^6*Sqrt[d + e*x]*Sqrt[b*x + c
*x^2])
```

Rubi [A] time = 0.588482, antiderivative size = 457, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {732, 814, 843, 715, 112, 110, 117, 116}

$$-\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}\left(-3cex\left(b^2e^2-32bcde+32c^2d^2\right)+111b^2cde^2-b^3e^3-240bc^2d^2e+128c^3d^3\right)}{63ce^5}-\frac{2\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(5/2)/(d + e*x)^(3/2),x]
```


```
[Out] (-2*Sqrt[d + e*x]*(128*c^3*d^3 - 240*b*c^2*d^2*e + 111*b^2*c*d*e^2 - b^3*e^
3 - 3*c*e*(32*c^2*d^2 - 32*b*c*d*e + b^2*e^2)*x)*Sqrt[b*x + c*x^2])/(63*c*e
^5) - (10*Sqrt[d + e*x]*(16*c*d - 15*b*e - 14*c*e*x)*(b*x + c*x^2)^(3/2))/(
63*e^3) - (2*(b*x + c*x^2)^(5/2))/(e*Sqrt[d + e*x]) + (4*Sqrt[-b]*(128*c^4*
d^4 - 256*b*c^3*d^3*e + 135*b^2*c^2*d^2*e^2 - 7*b^3*c*d*e^3 - b^4*e^4)*Sqrt
[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt
[-b]], (b*e)/(c*d)])/(63*c^(3/2)*e^6*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) -
 (2*Sqrt[-b]*d*(c*d - b*e)*(2*c*d - b*e)*(128*c^2*d^2 - 128*b*c*d*e - b^2*e
^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*S
qrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(63*c^(3/2)*e^6*Sqrt[d + e*x]*Sqrt[b*x + c
*x^2])
```


Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{\left(bx+cx^2\right)^{5/2}}{(d+ex)^{3/2}}\,dx = -\frac{2\left(bx+cx^2\right)^{5/2}}{e\sqrt{d+ex}} + \frac{5\int \frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{\sqrt{d+ex}}dx}{e}$$

$$= -\frac{10\sqrt{d+ex}(16cd-15be-14cex)\left(bx+cx^2\right)^{3/2}}{63e^3} - \frac{2\left(bx+cx^2\right)^{5/2}}{e\sqrt{d+ex}} - \frac{10\int \frac{\left(-\frac{1}{2}bcd(16cd-15be)-\frac{1}{2}c\left(32c^2d^2-3\right.}{\sqrt{d+ex}}}{21ce^3}$$

$$= -\frac{2\sqrt{d+ex}\left(128c^3d^3-240bc^2d^2e+111b^2cde^2-b^3e^3-3ce\left(32c^2d^2-32bcde+b^2e^2\right)x\right)\sqrt{bx+cx^2}}{63ce^5}$$

$$= -\frac{2\sqrt{d+ex}\left(128c^3d^3-240bc^2d^2e+111b^2cde^2-b^3e^3-3ce\left(32c^2d^2-32bcde+b^2e^2\right)x\right)\sqrt{bx+cx^2}}{63ce^5}$$

$$= -\frac{2\sqrt{d+ex}\left(128c^3d^3-240bc^2d^2e+111b^2cde^2-b^3e^3-3ce\left(32c^2d^2-32bcde+b^2e^2\right)x\right)\sqrt{bx+cx^2}}{63ce^5}$$

$$= -\frac{2\sqrt{d+ex}\left(128c^3d^3-240bc^2d^2e+111b^2cde^2-b^3e^3-3ce\left(32c^2d^2-32bcde+b^2e^2\right)x\right)\sqrt{bx+cx^2}}{63ce^5}$$

$$= -\frac{2\sqrt{d+ex}\left(128c^3d^3-240bc^2d^2e+111b^2cde^2-b^3e^3-3ce\left(32c^2d^2-32bcde+b^2e^2\right)x\right)\sqrt{bx+cx^2}}{63ce^5}$$

Mathematica [C] time = 2.41372, size = 498, normalized size = 1.09

$$\frac{2(x(b+cx))^{5/2}\left(iex\sqrt{\frac{b}{c}}\sqrt{\frac{b}{cx}+1}\sqrt{\frac{d}{ex}+1}\left(-159b^2c^2d^2e^2+13b^3cde^3+2b^4e^4+272bc^3d^3e-128c^4d^4\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(5/2)/(d + e*x)^(3/2),x]
```

```
[Out] (2*(x*(b + c*x))^(5/2)*((2*(128*c^4*d^4 - 256*b*c^3*d^3*e + 135*b^2*c^2*d^2
*e^2 - 7*b^3*c*d*e^3 - b^4*e^4)*(b + c*x)*(d + e*x))/(c*Sqrt[x]) - e*Sqrt[x
]*(b + c*x)*(-(b^3*e^3*(d + e*x)) + 3*b^2*c*e^2*(37*d^2 + 11*d*e*x - 5*e^2*
x^2) - b*c^2*e*(240*d^3 + 64*d^2*e*x - 31*d*e^2*x^2 + 19*e^3*x^3) + c^3*(12
8*d^4 + 32*d^3*e*x - 16*d^2*e^2*x^2 + 10*d*e^3*x^3 - 7*e^4*x^4)) - (2*I)*Sq
rt[b/c]*e*(-128*c^4*d^4 + 256*b*c^3*d^3*e - 135*b^2*c^2*d^2*e^2 + 7*b^3*c*d
*e^3 + b^4*e^4)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x*EllipticE[I*ArcSinh[S
qrt[b/c]/Sqrt[x]], (c*d)/(b*e)] + I*Sqrt[b/c]*e*(-128*c^4*d^4 + 272*b*c^3*d
^3*e - 159*b^2*c^2*d^2*e^2 + 13*b^3*c*d*e^3 + 2*b^4*e^4)*Sqrt[1 + b/(c*x)]*
Sqrt[1 + d/(e*x)]*x*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)]))/
(63*c*e^6*x^(5/2)*(b + c*x)^3*Sqrt[d + e*x])
```

Maple [B] time = 0.29, size = 1170, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(5/2)/(e*x+d)^(3/2),x)
```

```
[Out] 2/63*(x*(c*x+b))^(1/2)*(e*x+d)^(1/2)*(26*x^5*b*c^5*e^5-10*x^5*c^6*d*e^4+34*
x^4*b^2*c^4*e^5+16*x^4*c^6*d^2*e^3+16*x^3*b^3*c^3*e^5-32*x^3*c^6*d^3*e^2+x^
2*b^4*c^2*e^5-128*x^2*c^6*d^4*e-256*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(
1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)*b
*c^5*d^5+256*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)
/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)*b*c^5*d^5+2*(-(e*x+d)*c/
(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))
^(1/2))*((c*x+b)/b)^(1/2)*b^6*e^5+782*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)
^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)
*b^3*c^3*d^3*e^2-284*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(
((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)*b^4*c^2*d^2*e^3+
640*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2)
,(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)*b^2*c^4*d^4*e+12*(-(e*x+d)*c/(b*e
-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/
2))*((c*x+b)/b)^(1/2)*b^5*c*d*e^4-510*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)
^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)
*b^3*c^3*d^3*e^2+125*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(
((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)*b^4*c^2*d^2*e^3+
(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*
e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)*b^5*c*d*e^4-41*x^4*b*c^5*d*e^4+7*x^6*
c^6*e^5-64*x^3*b^2*c^4*d*e^4+80*x^3*b*c^5*d^2*e^3-32*x^2*b^3*c^3*d*e^4-47*x
^2*b^2*c^4*d^2*e^3+208*x^2*b*c^5*d^3*e^2+x*b^4*c^2*d*e^4-111*x*b^3*c^3*d^2*
e^3+240*x*b^2*c^4*d^3*e^2-128*x*b*c^5*d^4*e-768*(-(e*x+d)*c/(b*e-c*d))^(1/2
)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b
)/b)^(1/2)*b^2*c^4*d^4*e)/c^3/e^6/x/(c*e*x^2+b*e*x+c*d*x+b*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx\right)^{\frac{5}{2}}}{\left(ex+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(5/2)/(e*x + d)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(c^2x^4+2\,bcx^3+b^2x^2\right)\sqrt{cx^2+bx}\sqrt{ex+d}}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + b^2*x^2)*sqrt(c*x^2 + b*x)*sqrt(e*x + d)/(e
^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x(b+cx))^{\frac{5}{2}}}{(d+ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(5/2)/(e*x+d)**(3/2),x)
```

```
[Out] Integral((x*(b + c*x))**(5/2)/(d + e*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx\right)^{\frac{5}{2}}}{(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(5/2)/(e*x + d)^(3/2), x)
```

3.402 $\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=401

$$\frac{4\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\left(27b^2e^2-128bcde+128c^2d^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{21\sqrt{c}e^6\sqrt{bx+cx^2}\sqrt{d+ex}}+\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}}{}$$

```
[Out] (2*Sqrt[d + e*x]*(128*c^2*d^2 - 176*b*c*d*e + 51*b^2*e^2 - 48*c*e*(2*c*d -
b*e)*x)*Sqrt[b*x + c*x^2])/(21*e^5) + (10*(16*c*d - 7*b*e + 2*c*e*x)*(b*x +
 c*x^2)^(3/2))/(21*e^3*Sqrt[d + e*x]) - (2*(b*x + c*x^2)^(5/2))/(3*e*(d + e
*x)^(3/2)) - (2*Sqrt[-b]*(2*c*d - b*e)*(128*c^2*d^2 - 128*b*c*d*e + 3*b^2*e
^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[
x])/Sqrt[-b]], (b*e)/(c*d)])/(21*Sqrt[c]*e^6*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c
*x^2]) + (4*Sqrt[-b]*d*(c*d - b*e)*(128*c^2*d^2 - 128*b*c*d*e + 27*b^2*e^2)
*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt
[x])/Sqrt[-b]], (b*e)/(c*d)])/(21*Sqrt[c]*e^6*Sqrt[d + e*x]*Sqrt[b*x + c*x^
2])
```

Rubi [A] time = 0.476616, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {732, 812, 814, 843, 715, 112, 110, 117, 116}

$$\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}\left(51b^2e^2-48cex(2cd-be)-176bcde+128c^2d^2\right)}{21e^5}+\frac{4\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\left(27b^2e^2-\right.}{21\sqrt{c}e^6\sqrt{bx+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(5/2)/(d + e*x)^(5/2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(128*c^2*d^2 - 176*b*c*d*e + 51*b^2*e^2 - 48*c*e*(2*c*d -
b*e)*x)*Sqrt[b*x + c*x^2])/(21*e^5) + (10*(16*c*d - 7*b*e + 2*c*e*x)*(b*x +
 c*x^2)^(3/2))/(21*e^3*Sqrt[d + e*x]) - (2*(b*x + c*x^2)^(5/2))/(3*e*(d + e
*x)^(3/2)) - (2*Sqrt[-b]*(2*c*d - b*e)*(128*c^2*d^2 - 128*b*c*d*e + 3*b^2*e
^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[
x])/Sqrt[-b]], (b*e)/(c*d)])/(21*Sqrt[c]*e^6*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c
*x^2]) + (4*Sqrt[-b]*d*(c*d - b*e)*(128*c^2*d^2 - 128*b*c*d*e + 27*b^2*e^2)
*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt
[x])/Sqrt[-b]], (b*e)/(c*d)])/(21*Sqrt[c]*e^6*Sqrt[d + e*x]*Sqrt[b*x + c*x^
2])
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
```

```
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{\left(bx+cx^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = -\frac{2\left(bx+cx^2\right)^{5/2}}{3e(d+ex)^{3/2}}+\frac{5\int \frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx}{3e}$$

$$= \frac{10(16cd-7be+2cex)\left(bx+cx^2\right)^{3/2}}{21e^3\sqrt{d+ex}}-\frac{2\left(bx+cx^2\right)^{5/2}}{3e(d+ex)^{3/2}}-\frac{10\int \frac{\left(\frac{1}{2}b(16cd-7be)+8c(2cd-be)x\right)\sqrt{bx+cx^2}}{\sqrt{d+ex}}\,dx}{7e^3}$$

$$= \frac{2\sqrt{d+ex}\left(128c^2d^2-176bcde+51b^2e^2-48ce(2cd-be)x\right)\sqrt{bx+cx^2}}{21e^5}+\frac{10(16cd-7be+2cex)\left(\right.}{21e^3\sqrt{d+ex}}$$

$$= \frac{2\sqrt{d+ex}\left(128c^2d^2-176bcde+51b^2e^2-48ce(2cd-be)x\right)\sqrt{bx+cx^2}}{21e^5}+\frac{10(16cd-7be+2cex)\left(\right.}{21e^3\sqrt{d+ex}}$$

$$= \frac{2\sqrt{d+ex}\left(128c^2d^2-176bcde+51b^2e^2-48ce(2cd-be)x\right)\sqrt{bx+cx^2}}{21e^5}+\frac{10(16cd-7be+2cex)\left(\right.}{21e^3\sqrt{d+ex}}$$

$$= \frac{2\sqrt{d+ex}\left(128c^2d^2-176bcde+51b^2e^2-48ce(2cd-be)x\right)\sqrt{bx+cx^2}}{21e^5}+\frac{10(16cd-7be+2cex)\left(\right.}{21e^3\sqrt{d+ex}}$$

$$= \frac{2\sqrt{d+ex}\left(128c^2d^2-176bcde+51b^2e^2-48ce(2cd-be)x\right)\sqrt{bx+cx^2}}{21e^5}+\frac{10(16cd-7be+2cex)\left(\right.}{21e^3\sqrt{d+ex}}$$

Mathematica [C] time = 2.20526, size = 442, normalized size = 1.1

$$2(x(b+cx))^{5/2}\left(iex\sqrt{\frac{b}{c}}\sqrt{\frac{b}{cx}+1}\sqrt{\frac{d}{ex}+1}\left(83b^2cde^2-3b^3e^3-208bc^2d^2e+128c^3d^3\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right),\frac{cd}{be}\right)+\frac{e\sqrt{x}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(5/2)/(d + e*x)^(5/2),x]
```

```
[Out] (2*(x*(b + c*x))^(5/2)*(-(((256*c^3*d^3 - 384*b*c^2*d^2*e + 134*b^2*c*d*e^2
 - 3*b^3*e^3)*(b + c*x)*(d + e*x))/(c*Sqrt[x])) + (e*Sqrt[x]*(b + c*x)*(b^2
*e^2*(51*d^2 + 67*d*e*x + 9*e^2*x^2) + b*c*e*(-176*d^3 - 224*d^2*e*x - 25*d
*e^2*x^2 + 9*e^3*x^3) + c^2*(128*d^4 + 160*d^3*e*x + 16*d^2*e^2*x^2 - 6*d*e
^3*x^3 + 3*e^4*x^4)))/(d + e*x) + I*Sqrt[b/c]*e*(-256*c^3*d^3 + 384*b*c^2*d
^2*e - 134*b^2*c*d*e^2 + 3*b^3*e^3)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x*E
llipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] + I*Sqrt[b/c]*e*(128*c^
3*d^3 - 208*b*c^2*d^2*e + 83*b^2*c*d*e^2 - 3*b^3*e^3)*Sqrt[1 + b/(c*x)]*Sqr
t[1 + d/(e*x)]*x*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)]))/(21
```

```
*e^6*x^(5/2)*(b + c*x)^3*Sqrt[d + e*x])
```

Maple [B] time = 0.3, size = 1692, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(5/2)/(e*x+d)^(5/2),x)
```

```
[Out] -2/21*(x*(c*x+b))^(1/2)*(-640*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(
1/2))*b^2*c^3*d^4*e*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)
^(1/2)+54*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^4*c*d^2*e^3*
((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-310*EllipticF
(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^3*c^2*d^3*e^2*((c*x+b)/b)^(1/2)
*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+512*EllipticF(((c*x+b)/b)^(1/2
),(b*e/(b*e-c*d))^(1/2))*b^2*c^3*d^4*e*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c
*d))^(1/2)*(-c*x/b)^(1/2)-12*x^5*b*c^4*e^5+6*x^5*c^5*d*e^4-18*x^4*b^2*c^3*e
^5-16*x^4*c^5*d^2*e^3-9*x^3*b^3*c^2*e^5-160*x^3*c^5*d^3*e^2-128*x^2*c^5*d^4
*e-137*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^4*c*d^2*e^3*((c
*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-137*EllipticE(((
c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^4*c*d*e^4*((c*x+b)/b)^(1/2)*(-(e
*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+518*EllipticE(((c*x+b)/b)^(1/2),(b*
e/(b*e-c*d))^(1/2))*x*b^3*c^2*d^2*e^3*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*
d))^(1/2)*(-c*x/b)^(1/2)-640*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1
/2))*x*b^2*c^3*d^3*e^2*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x
/b)^(1/2)+256*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b*c^4*d^
4*e*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+54*Ellipt
icF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^4*c*d*e^4*((c*x+b)/b)^(1/2
)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-310*EllipticF(((c*x+b)/b)^(1/
2),(b*e/(b*e-c*d))^(1/2))*x*b^3*c^2*d^2*e^3*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(
b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+512*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*
d))^(1/2))*x*b^2*c^3*d^3*e^2*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)
*(-c*x/b)^(1/2)-256*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b*
c^4*d^4*e*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-128
*x*b*c^4*d^4*e+31*x^4*b*c^4*d*e^4-42*x^3*b^2*c^3*d*e^4+208*x^3*b*c^4*d^2*e^
3-67*x^2*b^3*c^2*d*e^4+173*x^2*b^2*c^3*d^2*e^3+16*x^2*b*c^4*d^3*e^2-51*x*b^
3*c^2*d^2*e^3+518*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^3*c^
2*d^3*e^2*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+176
*x*b^2*c^3*d^3*e^2-3*x^6*c^5*e^5+3*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*
d))^(1/2))*x*b^5*e^5*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b
)^(1/2)+3*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^5*d*e^4*((c*
x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+256*EllipticE(((c
*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b*c^4*d^5*((c*x+b)/b)^(1/2)*(-(e*x+d)
*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-256*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*
e-c*d))^(1/2))*b*c^4*d^5*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c
*x/b)^(1/2))/(c*x+b)/x/(e*x+d)^(3/2)/c^2/e^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx\right)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(5/2)/(e*x + d)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(c^{2} x^{4} + 2\, b c x^{3} + b^{2} x^{2}\right) \sqrt{c x^{2} + b x} \sqrt{e x + d}}{e^{3} x^{3} + 3\, d e^{2} x^{2} + 3\, d^{2} e x + d^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + b^2*x^2)*sqrt(c*x^2 + b*x)*sqrt(e*x + d)/(e
^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(5/2)/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c x^{2} + b x\right)^{\frac{5}{2}}}{\left(e x + d\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(5/2)/(e*x + d)^(5/2), x)
```

3.403 $\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=392

$$-\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(2cd-be)\left(15b^2e^2-128bcde+128c^2d^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{15\sqrt{c}e^6\sqrt{bx+cx^2}\sqrt{d+ex}}-\frac{2\sqrt{bx+cx^2}\left(15b^2e^2+\right.}{}$$

```
[Out] (-2*(128*c^2*d^2 - 112*b*c*d*e + 15*b^2*e^2 + 16*c*e*(2*c*d - b*e)*x)*Sqrt[
b*x + c*x^2])/(15*e^5*Sqrt[d + e*x]) + (2*(16*c*d - 5*b*e + 6*c*e*x)*(b*x +
 c*x^2)^(3/2))/(15*e^3*(d + e*x)^(3/2)) - (2*(b*x + c*x^2)^(5/2))/(5*e*(d +
 e*x)^(5/2)) + (4*Sqrt[-b]*Sqrt[c]*(128*c^2*d^2 - 128*b*c*d*e + 23*b^2*e^2)
*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])
/Sqrt[-b]], (b*e)/(c*d)])/(15*e^6*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) - (2
*Sqrt[-b]*(2*c*d - b*e)*(128*c^2*d^2 - 128*b*c*d*e + 15*b^2*e^2)*Sqrt[x]*Sq
rt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-
b]], (b*e)/(c*d)])/(15*Sqrt[c]*e^6*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.429493, antiderivative size = 392, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {732, 812, 843, 715, 112, 110, 117, 116}

$$-\frac{2\sqrt{bx+cx^2}\left(15b^2e^2+16cex(2cd-be)-112bcde+128c^2d^2\right)}{15e^5\sqrt{d+ex}}-\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(2cd-be)\left(15b^2e^2-128bcde+\right.}{15\sqrt{c}e^6\sqrt{bx+cx^2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(5/2)/(d + e*x)^(7/2),x]
```


```
[Out] (-2*(128*c^2*d^2 - 112*b*c*d*e + 15*b^2*e^2 + 16*c*e*(2*c*d - b*e)*x)*Sqrt[
b*x + c*x^2])/(15*e^5*Sqrt[d + e*x]) + (2*(16*c*d - 5*b*e + 6*c*e*x)*(b*x +
 c*x^2)^(3/2))/(15*e^3*(d + e*x)^(3/2)) - (2*(b*x + c*x^2)^(5/2))/(5*e*(d +
 e*x)^(5/2)) + (4*Sqrt[-b]*Sqrt[c]*(128*c^2*d^2 - 128*b*c*d*e + 23*b^2*e^2)
*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])
/Sqrt[-b]], (b*e)/(c*d)])/(15*e^6*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) - (2
*Sqrt[-b]*(2*c*d - b*e)*(128*c^2*d^2 - 128*b*c*d*e + 15*b^2*e^2)*Sqrt[x]*Sq
rt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-
b]], (b*e)/(c*d)])/(15*Sqrt[c]*e^6*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```


Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
```

```
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{\left(bx+cx^2\right)^{5/2}}{(d+ex)^{7/2}}\,dx = -\frac{2\left(bx+cx^2\right)^{5/2}}{5e(d+ex)^{5/2}} + \frac{\int \frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{(d+ex)^{5/2}}dx}{e}$$

$$= \frac{2(16cd-5be+6cex)\left(bx+cx^2\right)^{3/2}}{15e^3(d+ex)^{3/2}} - \frac{2\left(bx+cx^2\right)^{5/2}}{5e(d+ex)^{5/2}} - \frac{2\int \frac{\left(\frac{1}{2}b(16cd-5be)+8c(2cd-be)x\right)\sqrt{bx+cx^2}}{(d+ex)^{3/2}}dx}{5e^3}$$

$$= -\frac{2\left(128c^2d^2-112bcde+15b^2e^2+16ce(2cd-be)x\right)\sqrt{bx+cx^2}}{15e^5\sqrt{d+ex}} + \frac{2(16cd-5be+6cex)\left(bx+cx^2\right)^{3/}}{15e^3(d+ex)^{3/2}}$$

$$= -\frac{2\left(128c^2d^2-112bcde+15b^2e^2+16ce(2cd-be)x\right)\sqrt{bx+cx^2}}{15e^5\sqrt{d+ex}} + \frac{2(16cd-5be+6cex)\left(bx+cx^2\right)^{3/}}{15e^3(d+ex)^{3/2}}$$

$$= -\frac{2\left(128c^2d^2-112bcde+15b^2e^2+16ce(2cd-be)x\right)\sqrt{bx+cx^2}}{15e^5\sqrt{d+ex}} + \frac{2(16cd-5be+6cex)\left(bx+cx^2\right)^{3/}}{15e^3(d+ex)^{3/2}}$$

$$= -\frac{2\left(128c^2d^2-112bcde+15b^2e^2+16ce(2cd-be)x\right)\sqrt{bx+cx^2}}{15e^5\sqrt{d+ex}} + \frac{2(16cd-5be+6cex)\left(bx+cx^2\right)^{3/}}{15e^3(d+ex)^{3/2}}$$

$$= -\frac{2\left(128c^2d^2-112bcde+15b^2e^2+16ce(2cd-be)x\right)\sqrt{bx+cx^2}}{15e^5\sqrt{d+ex}} + \frac{2(16cd-5be+6cex)\left(bx+cx^2\right)^{3/}}{15e^3(d+ex)^{3/2}}$$

Mathematica [C] time = 1.62915, size = 401, normalized size = 1.02

$$2(x(b+cx))^{5/2}\left(-icex\sqrt{\frac{b}{c}}\sqrt{\frac{b}{cx}+1}\sqrt{\frac{d}{ex}+1}\left(31b^2e^2-144bcde+128c^2d^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right),\frac{cd}{be}\right)-\frac{e\sqrt{x}(b+cx)\left(b^2e^2(15d^2\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(5/2)/(d + e*x)^(7/2),x]
```

```
[Out] (2*(x*(b + c*x))^(5/2)*((2*(128*c^2*d^2 - 128*b*c*d*e + 23*b^2*e^2)*(b + c*
x)*(d + e*x))/Sqrt[x] - (e*Sqrt[x]*(b + c*x)*(b^2*e^2*(15*d^2 + 35*d*e*x +
23*e^2*x^2) - b*c*e*(112*d^3 + 256*d^2*e*x + 161*d*e^2*x^2 + 11*e^3*x^3) +
c^2*(128*d^4 + 288*d^3*e*x + 176*d^2*e^2*x^2 + 10*d*e^3*x^3 - 3*e^4*x^4)))/
(d + e*x)^2 + (2*I)*Sqrt[b/c]*c*e*(128*c^2*d^2 - 128*b*c*d*e + 23*b^2*e^2)*
Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]
], (c*d)/(b*e)] - I*Sqrt[b/c]*c*e*(128*c^2*d^2 - 144*b*c*d*e + 31*b^2*e^2)*
Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]
], (c*d)/(b*e)]))/(15*e^6*x^(5/2)*(b + c*x)^3*Sqrt[d + e*x])
```

Maple [B] time = 0.301, size = 2170, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(5/2)/(e*x+d)^(7/2),x)
```

```
[Out] 2/15*(x*(c*x+b))^(1/2)*(126*x^3*b^2*c^2*d*e^4+80*x^3*b*c^3*d^2*e^3-35*x^2*b
^3*c*d*e^4+241*x^2*b^2*c^2*d^2*e^3-176*x^2*b*c^3*d^3*e^2+112*x*b^2*c^2*d^3*
e^2-512*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^2*c^2*d^2*
e^3*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)+256*Ellip
ticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b*c^3*d^3*e^2*(-c*x/b)^(1
/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)-316*EllipticF(((c*x+b)/b
)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^3*c*d^2*e^3*(-c*x/b)^(1/2)*((c*x+b)/b)^(
1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)+768*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e
-c*d))^(1/2))*x*b^2*c^2*d^3*e^2*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*
c/(b*e-c*d))^(1/2)-512*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x
*b*c^3*d^4*e*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)+
604*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^3*c*d^2*e^3*(-c*
x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)-1024*EllipticE(((
c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^2*c^2*d^3*e^2*(-c*x/b)^(1/2)*((c
*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)+512*EllipticE(((c*x+b)/b)^(1/2)
,(b*e/(b*e-c*d))^(1/2))*x*b*c^3*d^4*e*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e
*x+d)*c/(b*e-c*d))^(1/2)-158*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1
/2))*x^2*b^3*c*d*e^4*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d)
)^(1/2)+384*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^2*c^2*
d^2*e^3*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)-15*x*
b^3*c*d^2*e^3-128*x*b*c^3*d^4*e-46*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*
d))^(1/2))*b^4*d^2*e^3*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*
d))^(1/2)+3*x^6*c^4*e^5+15*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2
))*x^2*b^4*e^5*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2
)+151*x^4*b*c^3*d*e^4-158*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2)
)*b^3*c*d^3*e^2*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/
2)+384*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^2*c^2*d^4*e*(-c
*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)+302*EllipticE(((
c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^3*c*d^3*e^2*(-c*x/b)^(1/2)*((c*x+b
)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)-512*EllipticE(((c*x+b)/b)^(1/2),(b*
e/(b*e-c*d))^(1/2))*b^2*c^2*d^4*e*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d
)*c/(b*e-c*d))^(1/2)-12*x^4*b^2*c^2*e^5-176*x^4*c^4*d^2*e^3-23*x^3*b^3*c*e^
5-288*x^3*c^4*d^3*e^2-128*x^2*c^4*d^4*e+15*EllipticF(((c*x+b)/b)^(1/2),(b*e
/(b*e-c*d))^(1/2))*b^4*d^2*e^3*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/
2)*(-c*x/b)^(1/2)-92*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b
^4*d*e^4*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)+14*x
^5*b*c^3*e^5-10*x^5*c^4*d*e^4-256*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d
))^(1/2))*x^2*b*c^3*d^3*e^2*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b
*e-c*d))^(1/2)+302*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b
^3*c*d*e^4*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)-46
*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^4*e^5*(-c*x/b)^(1
/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)-256*EllipticF(((c*x+b)/b
)^(1/2),(b*e/(b*e-c*d))^(1/2))*b*c^3*d^5*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(
-(e*x+d)*c/(b*e-c*d))^(1/2)+256*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))
^(1/2))*b*c^3*d^5*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(
1/2)+30*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^4*d*e^4*((c*
x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2))/(c*x+b)/x/(e*x+d
)^(5/2)/e^6/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx\right)^{\frac{5}{2}}}{\left(ex+d\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(5/2)/(e*x + d)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2x^4 + 2\,bcx^3 + b^2x^2\right)\sqrt{cx^2 + bx}\sqrt{ex + d}}{e^4x^4 + 4\,de^3x^3 + 6\,d^2e^2x^2 + 4\,d^3ex + d^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + b^2*x^2)*sqrt(c*x^2 + b*x)*sqrt(e*x + d)/(e
^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(5/2)/(e*x+d)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx\right)^{\frac{5}{2}}}{(ex + d)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(5/2)/(e*x + d)^(7/2), x)
```

3.404 $\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{9/2}}\,dx$

Optimal. Leaf size=474

$$\frac{4\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}\left(27b^2e^2-128bcde+128c^2d^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{21e^6\sqrt{bx+cx^2}\sqrt{d+ex}}-\frac{2\left(bx+cx^2\right)^{3/2}\left(ex\left(3b^2e^2-\right.\right.}{21de^3(}$$

```
[Out] (2*c*(d*(128*c^2*d^2 - 176*b*c*d*e + 51*b^2*e^2) + e*(32*c^2*d^2 - 32*b*c*d
*e + 3*b^2*e^2)*x)*Sqrt[b*x + c*x^2])/(21*d*e^5*(c*d - b*e)*Sqrt[d + e*x])
- (2*(c*d^2*(16*c*d - 13*b*e) + e*(22*c^2*d^2 - 22*b*c*d*e + 3*b^2*e^2)*x)*
(b*x + c*x^2)^(3/2))/(21*d*e^3*(c*d - b*e)*(d + e*x)^(5/2)) - (2*(b*x + c*x
^2)^(5/2))/(7*e*(d + e*x)^(7/2)) - (2*Sqrt[-b]*Sqrt[c]*(2*c*d - b*e)*(128*c
^2*d^2 - 128*b*c*d*e + 3*b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*E
llipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(21*d*e^6*(c*d -
 b*e)*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) + (4*Sqrt[-b]*Sqrt[c]*(128*c^2*d
^2 - 128*b*c*d*e + 27*b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*
EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(21*e^6*Sqrt[d
+ e*x]*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.561803, antiderivative size = 474, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {732, 810, 812, 843, 715, 112, 110, 117, 116}

$$-\frac{2\left(bx+cx^2\right)^{3/2}\left(ex\left(3b^2e^2-22bcde+22c^2d^2\right)+cd^2(16cd-13be)\right)}{21de^3(d+ex)^{5/2}(cd-be)}+\frac{2c\sqrt{bx+cx^2}\left(ex\left(3b^2e^2-32bcde+32c^2d^2\right)+d\right.}{21de^5\sqrt{d+ex}(cd-}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(5/2)/(d + e*x)^(9/2),x]
```

```
[Out] (2*c*(d*(128*c^2*d^2 - 176*b*c*d*e + 51*b^2*e^2) + e*(32*c^2*d^2 - 32*b*c*d
*e + 3*b^2*e^2)*x)*Sqrt[b*x + c*x^2])/(21*d*e^5*(c*d - b*e)*Sqrt[d + e*x])
- (2*(c*d^2*(16*c*d - 13*b*e) + e*(22*c^2*d^2 - 22*b*c*d*e + 3*b^2*e^2)*x)*
(b*x + c*x^2)^(3/2))/(21*d*e^3*(c*d - b*e)*(d + e*x)^(5/2)) - (2*(b*x + c*x
^2)^(5/2))/(7*e*(d + e*x)^(7/2)) - (2*Sqrt[-b]*Sqrt[c]*(2*c*d - b*e)*(128*c
^2*d^2 - 128*b*c*d*e + 3*b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*E
llipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(21*d*e^6*(c*d -
 b*e)*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) + (4*Sqrt[-b]*Sqrt[c]*(128*c^2*d
^2 - 128*b*c*d*e + 27*b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*
EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(21*e^6*Sqrt[d
+ e*x]*Sqrt[b*x + c*x^2])
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
```

```
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] && !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{\left(bx+cx^2\right)^{5/2}}{(d+ex)^{9/2}}\,dx = -\frac{2\left(bx+cx^2\right)^{5/2}}{7e(d+ex)^{7/2}}+\frac{5\int \frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx}{7e}$$

$$= -\frac{2\left(cd^2(16cd-13be)+e\left(22c^2d^2-22bcde+3b^2e^2\right)x\right)\left(bx+cx^2\right)^{3/2}}{21de^3(cd-be)(d+ex)^{5/2}}-\frac{2\left(bx+cx^2\right)^{5/2}}{7e(d+ex)^{7/2}}-\frac{2\int \frac{\left(-\frac{1}{2}\right.}{}}{}$$

$$= \frac{2c\left(d\left(128c^2d^2-176bcde+51b^2e^2\right)+e\left(32c^2d^2-32bcde+3b^2e^2\right)x\right)\sqrt{bx+cx^2}}{21de^5(cd-be)\sqrt{d+ex}}-\frac{2\left(cd^2(16cd-\right.}{}$$

$$= \frac{2c\left(d\left(128c^2d^2-176bcde+51b^2e^2\right)+e\left(32c^2d^2-32bcde+3b^2e^2\right)x\right)\sqrt{bx+cx^2}}{21de^5(cd-be)\sqrt{d+ex}}-\frac{2\left(cd^2(16cd-\right.}{}$$

$$= \frac{2c\left(d\left(128c^2d^2-176bcde+51b^2e^2\right)+e\left(32c^2d^2-32bcde+3b^2e^2\right)x\right)\sqrt{bx+cx^2}}{21de^5(cd-be)\sqrt{d+ex}}-\frac{2\left(cd^2(16cd-\right.}{}$$

$$= \frac{2c\left(d\left(128c^2d^2-176bcde+51b^2e^2\right)+e\left(32c^2d^2-32bcde+3b^2e^2\right)x\right)\sqrt{bx+cx^2}}{21de^5(cd-be)\sqrt{d+ex}}-\frac{2\left(cd^2(16cd-\right.}{}$$

$$= \frac{2c\left(d\left(128c^2d^2-176bcde+51b^2e^2\right)+e\left(32c^2d^2-32bcde+3b^2e^2\right)x\right)\sqrt{bx+cx^2}}{21de^5(cd-be)\sqrt{d+ex}}-\frac{2\left(cd^2(16cd-\right.}{}$$

Mathematica [C] time = 2.5287, size = 500, normalized size = 1.05

$$-\frac{2(x(b+cx))^{5/2}\left(bex(b+cx)\left(-b^2cde^2\left(169d^2ex+51d^3+194de^2x^2+85e^3x^3\right)+3b^3e^6x^3+bc^2de\left(649d^2e^2x^2+576d^3e\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(5/2)/(d + e*x)^(9/2),x]
```

```
[Out] (-2*(x*(b + c*x))^(5/2)*(b*e*x*(b + c*x)*(3*b^3*e^6*x^3 - b^2*c*d*e^2*(51*d
^3 + 169*d^2*e*x + 194*d*e^2*x^2 + 85*e^3*x^3) - c^3*d^2*(128*d^4 + 416*d^3
*e*x + 464*d^2*e^2*x^2 + 186*d*e^3*x^3 + 7*e^4*x^4) + b*c^2*d*e*(176*d^4 +
576*d^3*e*x + 649*d^2*e^2*x^2 + 265*d*e^3*x^3 + 7*e^4*x^4)) + Sqrt[b/c]*c*(
d + e*x)^3*(Sqrt[b/c]*(256*c^3*d^3 - 384*b*c^2*d^2*e + 134*b^2*c*d*e^2 - 3*
b^3*e^3)*(b + c*x)*(d + e*x) + I*b*e*(256*c^3*d^3 - 384*b*c^2*d^2*e + 134*b
^2*c*d*e^2 - 3*b^3*e^3)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*Ellipti
cE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - I*b*e*(128*c^3*d^3 - 208*b*
```

```
c^2*d^2*e + 83*b^2*c*d*e^2 - 3*b^3*e^3)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]
*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])))/(21*b*d*e^
6*(c*d - b*e)*x^3*(b + c*x)^3*(d + e*x)^(7/2))
```

Maple [B] time = 0.307, size = 3284, normalized size = 6.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(5/2)/(e*x+d)^(9/2),x)
```

```
[Out] 2/21*(x*(c*x+b))^(1/2)*(-310*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1
/2))*x^3*b^3*c^2*d^2*e^5*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c
*x/b)^(1/2)-137*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^3*b^4*
c*d*e^6*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+518*E
llipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^3*b^3*c^2*d^2*e^5*((c*x
+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-256*EllipticF(((c*
x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^3*b*c^4*d^4*e^3*((c*x+b)/b)^(1/2)*(-
(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-640*EllipticE(((c*x+b)/b)^(1/2),(
b*e/(b*e-c*d))^(1/2))*x^3*b^2*c^3*d^3*e^4*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*
e-c*d))^(1/2)*(-c*x/b)^(1/2)-411*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d)
)^(1/2))*x^2*b^4*c*d^2*e^5*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(
-c*x/b)^(1/2)+1554*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b
^3*c^2*d^3*e^4*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2
)-1920*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^2*c^3*d^4*e
^3*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+768*Ellipt
icE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b*c^4*d^5*e^2*((c*x+b)/b)^
(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+162*EllipticF(((c*x+b)/b)
^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^4*c*d^2*e^5*((c*x+b)/b)^(1/2)*(-(e*x+d)
*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+256*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*
e-c*d))^(1/2))*x^3*b*c^4*d^4*e^3*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(
1/2)*(-c*x/b)^(1/2)-930*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*
x^2*b^3*c^2*d^3*e^4*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)
^(1/2)+1536*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^2*c^3*
d^4*e^3*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-768*E
llipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b*c^4*d^5*e^2*((c*x+b
)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-411*EllipticE(((c*x+
b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^4*c*d^3*e^4*((c*x+b)/b)^(1/2)*(-(e*x
+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+1554*EllipticE(((c*x+b)/b)^(1/2),(b*e
/(b*e-c*d))^(1/2))*x*b^3*c^2*d^4*e^3*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d
))^(1/2)*(-c*x/b)^(1/2)-1920*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1
/2))*x*b^2*c^3*d^5*e^2*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x
/b)^(1/2)+768*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b*c^4*d^
6*e*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-7*x^6*c^5
*d^2*e^5+9*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^5*d^2*e^5
*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-137*Elliptic
E(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^4*c*d^4*e^3*((c*x+b)/b)^(1/2)*
(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+518*EllipticE(((c*x+b)/b)^(1/2)
,(b*e/(b*e-c*d))^(1/2))*b^3*c^2*d^5*e^2*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-
c*d))^(1/2)*(-c*x/b)^(1/2)-640*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^
(1/2))*b^2*c^3*d^6*e*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b
)^(1/2)+54*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^4*c*d^4*e^3
*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-310*Elliptic
F(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^3*c^2*d^5*e^2*((c*x+b)/b)^(1/2
)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+512*EllipticF(((c*x+b)/b)^(1/
2),(b*e/(b*e-c*d))^(1/2))*b^2*c^3*d^6*e*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-
```

```
c*d))^(1/2)*(-c*x/b)^(1/2)+258*x^5*b*c^4*d^2*e^5-85*x^4*b^3*c^2*d*e^6+71*x^
4*b^2*c^3*d^2*e^5+463*x^4*b*c^4*d^3*e^4-194*x^3*b^3*c^2*d^2*e^5+480*x^3*b^2
*c^3*d^3*e^4+112*x^3*b*c^4*d^4*e^3-169*x^2*b^3*c^2*d^3*e^4+525*x^2*b^2*c^3*
d^4*e^3-240*x^2*b*c^4*d^5*e^2-51*x*b^3*c^2*d^4*e^3+176*x*b^2*c^3*d^5*e^2-12
8*x*b*c^4*d^6*e-78*x^5*b^2*c^3*d*e^6+162*EllipticF(((c*x+b)/b)^(1/2),(b*e/(
b*e-c*d))^(1/2))*x*b^4*c*d^3*e^4*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(
1/2)*(-c*x/b)^(1/2)-930*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*
x*b^3*c^2*d^4*e^3*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(
1/2)+1536*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^2*c^3*d^5*
e^2*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-768*Ellip
ticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b*c^4*d^6*e*((c*x+b)/b)^(1/
2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+54*EllipticF(((c*x+b)/b)^(1/
2),(b*e/(b*e-c*d))^(1/2))*x^3*b^4*c*d*e^6*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*
e-c*d))^(1/2)*(-c*x/b)^(1/2)+512*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d)
)^(1/2))*x^3*b^2*c^3*d^3*e^4*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)
*(-c*x/b)^(1/2)+7*x^6*b*c^4*d*e^6+3*x^5*b^3*c^2*e^7-186*x^5*c^5*d^3*e^4+3*x
^4*b^4*c*e^7-464*x^4*c^5*d^4*e^3-416*x^3*c^5*d^5*e^2-128*x^2*c^5*d^6*e+3*El
lipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^3*b^5*e^7*((c*x+b)/b)^(1
/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+3*EllipticE(((c*x+b)/b)^(1/
2),(b*e/(b*e-c*d))^(1/2))*b^5*d^3*e^4*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*
d))^(1/2)*(-c*x/b)^(1/2)+256*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1
/2))*b*c^4*d^7*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2
)-256*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b*c^4*d^7*((c*x+b)
/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+9*EllipticE(((c*x+b)/
b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^5*d*e^6*((c*x+b)/b)^(1/2)*(-(e*x+d)*c
/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2))/(c*x+b)/x/(b*e-c*d)/(e*x+d)^(7/2)/d/e^6/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx\right)^{\frac{5}{2}}}{\left(ex+d\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(5/2)/(e*x + d)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2x^4+2\,bcx^3+b^2x^2\right)\sqrt{cx^2+bx}\sqrt{ex+d}}{e^5x^5+5\,de^4x^4+10\,d^2e^3x^3+10\,d^3e^2x^2+5\,d^4ex+d^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + b^2*x^2)*sqrt(c*x^2 + b*x)*sqrt(e*x + d)/(e
^5*x^5 + 5*d*e^4*x^4 + 10*d^2*e^3*x^3 + 10*d^3*e^2*x^2 + 5*d^4*e*x + d^5),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(5/2)/(e*x+d)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx)^{\frac{5}{2}}}{(ex + d)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(5/2)/(e*x + d)^(9/2), x)
```

3.405 $\int \frac{\left(bx+cx^2\right)^{5/2}}{(d+ex)^{11/2}}\,dx$

Optimal. Leaf size=570

$$-\frac{2\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(2cd-be)\left(-b^2e^2-128bcde+128c^2d^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{63de^6\sqrt{bx+cx^2}\sqrt{d+ex}(cd-be)}-\frac{2\left(bx+cx^2\right)^{3/2}(\cdots}{\cdots}$$

```
[Out] (-2*(c*d^2*(128*c^3*d^3 - 240*b*c^2*d^2*e + 111*b^2*c*d*e^2 - b^3*e^3) + e*
(160*c^4*d^4 - 320*b*c^3*d^3*e + 171*b^2*c^2*d^2*e^2 - 11*b^3*c*d*e^3 - 2*b
^4*e^4)*x)*Sqrt[b*x + c*x^2])/(63*d^2*e^5*(c*d - b*e)^2*(d + e*x)^(3/2)) -
(2*(d*(16*c^2*d^2 - 11*b*c*d*e - 2*b^2*e^2) + e*(26*c^2*d^2 - 26*b*c*d*e +
3*b^2*e^2)*x)*(b*x + c*x^2)^(3/2))/(63*d*e^3*(c*d - b*e)*(d + e*x)^(7/2)) -
 (2*(b*x + c*x^2)^(5/2))/(9*e*(d + e*x)^(9/2)) + (4*Sqrt[-b]*Sqrt[c]*(128*c
^4*d^4 - 256*b*c^3*d^3*e + 135*b^2*c^2*d^2*e^2 - 7*b^3*c*d*e^3 - b^4*e^4)*S
qrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/S
qrt[-b]], (b*e)/(c*d)])/(63*d^2*e^6*(c*d - b*e)^2*Sqrt[1 + (e*x)/d]*Sqrt[b*
x + c*x^2]) - (2*Sqrt[-b]*Sqrt[c]*(2*c*d - b*e)*(128*c^2*d^2 - 128*b*c*d*e
- b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sq
rt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(63*d*e^6*(c*d - b*e)*Sqrt[d + e*x]
*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.679917, antiderivative size = 570, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {732, 810, 843, 715, 112, 110, 117, 116}

$$-\frac{2\left(bx+cx^2\right)^{3/2}\left(ex\left(3b^2e^2-26bcde+26c^2d^2\right)+d\left(-2b^2e^2-11bcde+16c^2d^2\right)\right)}{63de^3(d+ex)^{7/2}(cd-be)}-\frac{2\sqrt{bx+cx^2}\left(ex\left(171b^2c^2d^2e^2-11b\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(5/2)/(d + e*x)^(11/2),x]
```


```
[Out] (-2*(c*d^2*(128*c^3*d^3 - 240*b*c^2*d^2*e + 111*b^2*c*d*e^2 - b^3*e^3) + e*
(160*c^4*d^4 - 320*b*c^3*d^3*e + 171*b^2*c^2*d^2*e^2 - 11*b^3*c*d*e^3 - 2*b
^4*e^4)*x)*Sqrt[b*x + c*x^2])/(63*d^2*e^5*(c*d - b*e)^2*(d + e*x)^(3/2)) -
(2*(d*(16*c^2*d^2 - 11*b*c*d*e - 2*b^2*e^2) + e*(26*c^2*d^2 - 26*b*c*d*e +
3*b^2*e^2)*x)*(b*x + c*x^2)^(3/2))/(63*d*e^3*(c*d - b*e)*(d + e*x)^(7/2)) -
 (2*(b*x + c*x^2)^(5/2))/(9*e*(d + e*x)^(9/2)) + (4*Sqrt[-b]*Sqrt[c]*(128*c
^4*d^4 - 256*b*c^3*d^3*e + 135*b^2*c^2*d^2*e^2 - 7*b^3*c*d*e^3 - b^4*e^4)*S
qrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/S
qrt[-b]], (b*e)/(c*d)])/(63*d^2*e^6*(c*d - b*e)^2*Sqrt[1 + (e*x)/d]*Sqrt[b*
x + c*x^2]) - (2*Sqrt[-b]*Sqrt[c]*(2*c*d - b*e)*(128*c^2*d^2 - 128*b*c*d*e
- b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sq
rt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(63*d*e^6*(c*d - b*e)*Sqrt[d + e*x]
*Sqrt[b*x + c*x^2])
```


Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
```

```
|| LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
```

```
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{\left(bx+cx^2\right)^{5/2}}{(d+ex)^{11/2}}\,dx = -\frac{2\left(bx+cx^2\right)^{5/2}}{9e(d+ex)^{9/2}} + \frac{5\int \frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{(d+ex)^{9/2}}\,dx}{9e}$$

$$= -\frac{2\left(d\left(16c^2d^2-11bcde-2b^2e^2\right)+e\left(26c^2d^2-26bcde+3b^2e^2\right)x\right)\left(bx+cx^2\right)^{3/2}}{63de^3(cd-be)(d+ex)^{7/2}} - \frac{2\left(bx+cx^2\right)^{5/}}{9e(d+ex)^{9/2}}$$

$$= -\frac{2\left(cd^2\left(128c^3d^3-240bc^2d^2e+111b^2cde^2-b^3e^3\right)+e\left(160c^4d^4-320bc^3d^3e+171b^2c^2d^2e^2-11\right.\right.}{63d^2e^5(cd-be)^2(d+ex)^{3/2}}$$

$$= -\frac{2\left(cd^2\left(128c^3d^3-240bc^2d^2e+111b^2cde^2-b^3e^3\right)+e\left(160c^4d^4-320bc^3d^3e+171b^2c^2d^2e^2-11\right.\right.}{63d^2e^5(cd-be)^2(d+ex)^{3/2}}$$

$$= -\frac{2\left(cd^2\left(128c^3d^3-240bc^2d^2e+111b^2cde^2-b^3e^3\right)+e\left(160c^4d^4-320bc^3d^3e+171b^2c^2d^2e^2-11\right.\right.}{63d^2e^5(cd-be)^2(d+ex)^{3/2}}$$

$$= -\frac{2\left(cd^2\left(128c^3d^3-240bc^2d^2e+111b^2cde^2-b^3e^3\right)+e\left(160c^4d^4-320bc^3d^3e+171b^2c^2d^2e^2-11\right.\right.}{63d^2e^5(cd-be)^2(d+ex)^{3/2}}$$

$$= -\frac{2\left(cd^2\left(128c^3d^3-240bc^2d^2e+111b^2cde^2-b^3e^3\right)+e\left(160c^4d^4-320bc^3d^3e+171b^2c^2d^2e^2-11\right.\right.}{63d^2e^5(cd-be)^2(d+ex)^{3/2}}$$

Mathematica [C] time = 3.5541, size = 610, normalized size = 1.07

$$-\frac{2(x(b+cx))^{5/2}\left(bex(b+cx)\left(d^2(d+ex)^2\left(15b^2e^2-88bcde+88c^2d^2\right)(cd-be)^2-19d^3(d+ex)\left(b^2e^2-3bcde+2c^2d^2\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(5/2)/(d + e*x)^(11/2),x]
```

```
[Out] (-2*(x*(b + c*x))^(5/2)*(b*e*x*(b + c*x)*(7*d^4*(c*d - b*e)^4 - 19*d^3*(c*d
 - b*e)^2*(2*c^2*d^2 - 3*b*c*d*e + b^2*e^2)*(d + e*x) + d^2*(c*d - b*e)^2*(
88*c^2*d^2 - 88*b*c*d*e + 15*b^2*e^2)*(d + e*x)^2 - d*(c*d - b*e)*(122*c^3*
d^3 - 183*b*c^2*d^2*e + 63*b^2*c*d*e^2 - b^3*e^3)*(d + e*x)^3 + (193*c^4*d^
4 - 386*b*c^3*d^3*e + 207*b^2*c^2*d^2*e^2 - 14*b^3*c*d*e^3 - 2*b^4*e^4)*(d
+ e*x)^4) - Sqrt[b/c]*c*(d + e*x)^4*(-2*Sqrt[b/c]*(-128*c^4*d^4 + 256*b*c^3
*d^3*e - 135*b^2*c^2*d^2*e^2 + 7*b^3*c*d*e^3 + b^4*e^4)*(b + c*x)*(d + e*x)
 + (2*I)*b*e*(128*c^4*d^4 - 256*b*c^3*d^3*e + 135*b^2*c^2*d^2*e^2 - 7*b^3*c
*d*e^3 - b^4*e^4)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticE[I*A
rcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - I*b*e*(128*c^4*d^4 - 272*b*c^3*d^
3*e + 159*b^2*c^2*d^2*e^2 - 13*b^3*c*d*e^3 - 2*b^4*e^4)*Sqrt[1 + b/(c*x)]*S
qrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e
)])))/(63*b*d^2*e^6*(c*d - b*e)^2*x^3*(b + c*x)^3*(d + e*x)^(9/2))
```

Maple [B] time = 0.337, size = 5005, normalized size = 8.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(5/2)/(e*x+d)^(11/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx\right)^{\frac{5}{2}}}{\left(ex+d\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(5/2)/(e*x + d)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c^2x^4+2\,bcx^3+b^2x^2\right)\sqrt{cx^2+bx}\sqrt{ex+d}}{e^6x^6+6\,de^5x^5+15\,d^2e^4x^4+20\,d^3e^3x^3+15\,d^4e^2x^2+6\,d^5ex+d^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(11/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + b^2*x^2)*sqrt(c*x^2 + b*x)*sqrt(e*x + d)/(e
^6*x^6 + 6*d*e^5*x^5 + 15*d^2*e^4*x^4 + 20*d^3*e^3*x^3 + 15*d^4*e^2*x^2 + 6
*d^5*e*x + d^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(5/2)/(e*x+d)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx\right)^{\frac{5}{2}}}{\left(ex+d\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(5/2)/(e*x + d)^(11/2), x)
```

3.406 $\int \frac{(d+ex)^{7/2}}{\sqrt{bx+cx^2}}\,dx$

Optimal. Leaf size=379

$$-\frac{2\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\left(24b^2e^2-71bcde+71c^2d^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{105c^{7/2}\sqrt{bx+cx^2}\sqrt{d+ex}}+\frac{2e\sqrt{bx+cx^2}\sqrt{d+ex}\,(2\cdots}{10\cdots}$$

```
[Out] (2*e*(71*c^2*d^2 - 71*b*c*d*e + 24*b^2*e^2)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2]
)/(105*c^3) + (12*e*(2*c*d - b*e)*(d + e*x)^(3/2)*Sqrt[b*x + c*x^2])/(35*c^
2) + (2*e*(d + e*x)^(5/2)*Sqrt[b*x + c*x^2])/(7*c) + (16*Sqrt[-b]*(2*c*d -
b*e)*(11*c^2*d^2 - 11*b*c*d*e + 6*b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d
 + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(105*c^
(7/2)*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) - (2*Sqrt[-b]*d*(c*d - b*e)*(71*
c^2*d^2 - 71*b*c*d*e + 24*b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)
/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(105*c^(7/2
)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.502728, antiderivative size = 379, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {742, 832, 843, 715, 112, 110, 117, 116}

$$\frac{2e\sqrt{bx+cx^2}\sqrt{d+ex}\left(24b^2e^2-71bcde+71c^2d^2\right)}{105c^3}-\frac{2\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\left(24b^2e^2-71bcde+71c^2d^2\right)F\left(\sin\cdots\right.}{105c^{7/2}\sqrt{bx+cx^2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/Sqrt[b*x + c*x^2],x]
```


```
[Out] (2*e*(71*c^2*d^2 - 71*b*c*d*e + 24*b^2*e^2)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2]
)/(105*c^3) + (12*e*(2*c*d - b*e)*(d + e*x)^(3/2)*Sqrt[b*x + c*x^2])/(35*c^
2) + (2*e*(d + e*x)^(5/2)*Sqrt[b*x + c*x^2])/(7*c) + (16*Sqrt[-b]*(2*c*d -
b*e)*(11*c^2*d^2 - 11*b*c*d*e + 6*b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d
 + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(105*c^
(7/2)*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) - (2*Sqrt[-b]*d*(c*d - b*e)*(71*
c^2*d^2 - 71*b*c*d*e + 24*b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)
/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(105*c^(7/2
)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```


Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
```

```
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{(d+ex)^{7/2}}{\sqrt{bx+cx^2}}\,dx = \frac{2e(d+ex)^{5/2}\sqrt{bx+cx^2}}{7c} + \frac{2\int \frac{(d+ex)^{3/2}\left(\frac{1}{2}d(7cd-be)+3e(2cd-be)x\right)}{\sqrt{bx+cx^2}}dx}{7c}$$

$$= \frac{12e(2cd-be)(d+ex)^{3/2}\sqrt{bx+cx^2}}{35c^2} + \frac{2e(d+ex)^{5/2}\sqrt{bx+cx^2}}{7c} + \frac{4\int \frac{\sqrt{d+ex}\left(\frac{1}{4}d\left(35c^2d^2-17bcde+6b^2e^2\right)+\frac{1}{4}e(71c^2}{\sqrt{bx+cx^2}}}{35c^2}$$

$$= \frac{2e\left(71c^2d^2-71bcde+24b^2e^2\right)\sqrt{d+ex}\sqrt{bx+cx^2}}{105c^3} + \frac{12e(2cd-be)(d+ex)^{3/2}\sqrt{bx+cx^2}}{35c^2} + \frac{2e(d+ex)^{5/}}{7}$$

$$= \frac{2e\left(71c^2d^2-71bcde+24b^2e^2\right)\sqrt{d+ex}\sqrt{bx+cx^2}}{105c^3} + \frac{12e(2cd-be)(d+ex)^{3/2}\sqrt{bx+cx^2}}{35c^2} + \frac{2e(d+ex)^{5/}}{7}$$

$$= \frac{2e\left(71c^2d^2-71bcde+24b^2e^2\right)\sqrt{d+ex}\sqrt{bx+cx^2}}{105c^3} + \frac{12e(2cd-be)(d+ex)^{3/2}\sqrt{bx+cx^2}}{35c^2} + \frac{2e(d+ex)^{5/}}{7}$$

$$= \frac{2e\left(71c^2d^2-71bcde+24b^2e^2\right)\sqrt{d+ex}\sqrt{bx+cx^2}}{105c^3} + \frac{12e(2cd-be)(d+ex)^{3/2}\sqrt{bx+cx^2}}{35c^2} + \frac{2e(d+ex)^{5/}}{7}$$

$$= \frac{2e\left(71c^2d^2-71bcde+24b^2e^2\right)\sqrt{d+ex}\sqrt{bx+cx^2}}{105c^3} + \frac{12e(2cd-be)(d+ex)^{3/2}\sqrt{bx+cx^2}}{35c^2} + \frac{2e(d+ex)^{5/}}{7}$$

Mathematica [C] time = 2.22875, size = 388, normalized size = 1.02

$$2\sqrt{x}\left(\frac{ix\sqrt{\frac{b}{c}}\sqrt{\frac{b}{cx}+1}\sqrt{\frac{d}{ex}+1}\left(353b^2c^2d^2e^2-208b^3cde^3+48b^4e^4-298bc^3d^3e+105c^4d^4\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right),\frac{cd}{be}\right)}{b} + e\sqrt{x}(b+cx)(d+ex)\left(24b^2e^2-bc\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)/Sqrt[b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[x]*((8*(22*c^3*d^3 - 33*b*c^2*d^2*e + 23*b^2*c*d*e^2 - 6*b^3*e^3)*(
b + c*x)*(d + e*x))/(c*Sqrt[x]) + e*Sqrt[x]*(b + c*x)*(d + e*x)*(24*b^2*e^2
 - b*c*e*(89*d + 18*e*x) + c^2*(122*d^2 + 66*d*e*x + 15*e^2*x^2)) + (8*I)*S
qrt[b/c]*e*(22*c^3*d^3 - 33*b*c^2*d^2*e + 23*b^2*c*d*e^2 - 6*b^3*e^3)*Sqrt[
1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c
*d)/(b*e)] + (I*Sqrt[b/c]*(105*c^4*d^4 - 298*b*c^3*d^3*e + 353*b^2*c^2*d^2*
e^2 - 208*b^3*c*d*e^3 + 48*b^4*e^4)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x*E
llipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])/b))/(105*c^3*Sqrt[x*(b
 + c*x)]*Sqrt[d + e*x])
```

Maple [B] time = 0.294, size = 918, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/(c*x^2+b*x)^(1/2),x)
```

```
[Out] 2/105*(e*x+d)^(1/2)*(x*(c*x+b))^(1/2)*(15*x^5*c^5*e^4+24*((c*x+b)/b)^(1/2)*
(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*
e/(b*e-c*d))^(1/2))*b^4*c*d*e^3-95*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))
^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^
3*c^2*d^2*e^2+142*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(
1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^2*c^3*d^3*e-71*((
c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x
+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b*c^4*d^4+48*((c*x+b)/b)^(1/2)*(-(e*x+d
)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c
*d))^(1/2))*b^5*e^4-232*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*
x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^4*c*d*e^3+4
48*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(
((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^3*c^2*d^2*e^2-440*((c*x+b)/b)^(1
/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2)
,(b*e/(b*e-c*d))^(1/2))*b^2*c^3*d^3*e+176*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*
e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1
/2))*b*c^4*d^4-3*x^4*b*c^4*e^4+81*x^4*c^5*d*e^3+6*x^3*b^2*c^3*e^4-26*x^3*b*
c^4*d*e^3+188*x^3*c^5*d^2*e^2+24*x^2*b^3*c^2*e^4-83*x^2*b^2*c^3*d*e^3+99*x^
2*b*c^4*d^2*e^2+122*x^2*c^5*d^3*e+24*x*b^3*c^2*d*e^3-89*x*b^2*c^3*d^2*e^2+1
22*x*b*c^4*d^3*e)/c^5/x/(c*e*x^2+b*e*x+c*d*x+b*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{7}{2}}}{\sqrt{cx^2+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(7/2)/sqrt(c*x^2 + b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\sqrt{ex+d}}{\sqrt{cx^2+bx}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(e*x + d)/sqrt(c*x^2
 + b*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{7}{2}}}{\sqrt{cx^2+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(7/2)/sqrt(c*x^2 + b*x), x)
```

3.407 $\int \frac{(d+ex)^{5/2}}{\sqrt{bx+cx^2}}\,dx$

Optimal. Leaf size=303

$$-\frac{8\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{15c^{5/2}\sqrt{bx+cx^2}\sqrt{d+ex}}+\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(8b^2e^2-23bc\right.}{15c^{5/2}\sqrt{bx+cx}}$$

```
[Out] (8*e*(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])/(15*c^2) + (2*e*(d + e*
x)^(3/2)*Sqrt[b*x + c*x^2])/(5*c) + (2*Sqrt[-b]*(23*c^2*d^2 - 23*b*c*d*e +
8*b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c
]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(15*c^(5/2)*Sqrt[1 + (e*x)/d]*Sqrt[b*x
+ c*x^2]) - (8*Sqrt[-b]*d*(c*d - b*e)*(2*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/
b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c
*d)])/(15*c^(5/2)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.330066, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {742, 832, 843, 715, 112, 110, 117, 116}

$$\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(8b^2e^2-23bcde+23c^2d^2\right)E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{15c^{5/2}\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}+\frac{8e\sqrt{bx+cx^2}\sqrt{d+ex}(2cd-be)}{15c^2}-\frac{8\sqrt{-b}d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/Sqrt[b*x + c*x^2],x]
```

```
[Out] (8*e*(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])/(15*c^2) + (2*e*(d + e*
x)^(3/2)*Sqrt[b*x + c*x^2])/(5*c) + (2*Sqrt[-b]*(23*c^2*d^2 - 23*b*c*d*e +
8*b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c
]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(15*c^(5/2)*Sqrt[1 + (e*x)/d]*Sqrt[b*x
+ c*x^2]) - (8*Sqrt[-b]*d*(c*d - b*e)*(2*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/
b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c
*d)])/(15*c^(5/2)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
```

```
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\sqrt{bx+cx^2}}\,dx = \frac{2e(d+ex)^{3/2}\sqrt{bx+cx^2}}{5c} + \frac{2\int \frac{\sqrt{d+ex}\left(\frac{1}{2}d(5cd-be)+2e(2cd-be)x\right)}{\sqrt{bx+cx^2}}dx}{5c}$$

$$= \frac{8e(2cd-be)\sqrt{d+ex}\sqrt{bx+cx^2}}{15c^2} + \frac{2e(d+ex)^{3/2}\sqrt{bx+cx^2}}{5c} + \frac{4\int \frac{\frac{1}{4}d(15c^2d^2-11bcde+4b^2e^2)+\frac{1}{4}e(23c^2d^2-23bc}{\sqrt{d+ex}\sqrt{bx+cx^2}}}{15c^2}$$

$$= \frac{8e(2cd-be)\sqrt{d+ex}\sqrt{bx+cx^2}}{15c^2} + \frac{2e(d+ex)^{3/2}\sqrt{bx+cx^2}}{5c} - \frac{(4d(cd-be)(2cd-be))\int \frac{1}{\sqrt{d+ex}\sqrt{bx+cx^2}}}{15c^2}$$

$$= \frac{8e(2cd-be)\sqrt{d+ex}\sqrt{bx+cx^2}}{15c^2} + \frac{2e(d+ex)^{3/2}\sqrt{bx+cx^2}}{5c} - \frac{\left(4d(cd-be)(2cd-be)\sqrt{x}\sqrt{b+cx}\right)\int}{15c^2\sqrt{bx+cx^2}}$$

$$= \frac{8e(2cd-be)\sqrt{d+ex}\sqrt{bx+cx^2}}{15c^2} + \frac{2e(d+ex)^{3/2}\sqrt{bx+cx^2}}{5c} + \frac{\left(\left(23c^2d^2-23bcde+8b^2e^2\right)\sqrt{x}\sqrt{1+}\right.}{15c^2\sqrt{1+\frac{ex}{d}}\sqrt{bx}}$$

$$= \frac{8e(2cd-be)\sqrt{d+ex}\sqrt{bx+cx^2}}{15c^2} + \frac{2e(d+ex)^{3/2}\sqrt{bx+cx^2}}{5c} + \frac{2\sqrt{-b}\left(23c^2d^2-23bcde+8b^2e^2\right)\sqrt{x}}{15c^{5/2}\sqrt{1+\frac{ex}{d}}}$$

Mathematica [C] time = 1.28467, size = 314, normalized size = 1.04

$$\frac{2\sqrt{x}\left(-\frac{ix\sqrt{\frac{b}{c}}\sqrt{\frac{b}{cx}+1}\sqrt{\frac{d}{ex}+1}\left(-27b^2cde^2+8b^3e^3+34bc^2d^2e-15c^3d^3\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right),\frac{cd}{be}\right)}{b}+\frac{(b+cx)(d+ex)\left(8b^2e^2-23bcde+23c^2d^2\right)}{c\sqrt{x}}+iex\sqrt{\frac{b}{c}}\sqrt{}\right.}{15c^2\sqrt{x(b+cx)}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/Sqrt[b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[x]*(((23*c^2*d^2 - 23*b*c*d*e + 8*b^2*e^2)*(b + c*x)*(d + e*x))/(c*
Sqrt[x]) + e*Sqrt[x]*(b + c*x)*(d + e*x)*(11*c*d - 4*b*e + 3*c*e*x) + I*Sqr
t[b/c]*e*(23*c^2*d^2 - 23*b*c*d*e + 8*b^2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d
/(e*x)]*x*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - (I*Sqrt[b/
c]*(-15*c^3*d^3 + 34*b*c^2*d^2*e - 27*b^2*c*d*e^2 + 8*b^3*e^3)*Sqrt[1 + b/(
c*x)]*Sqrt[1 + d/(e*x)]*x*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*
e)])/b))/(15*c^2*Sqrt[x*(b + c*x)]*Sqrt[d + e*x])
```

Maple [B] time = 0.272, size = 682, normalized size = 2.3

$$-\frac{2}{15\,x\left(cex^2+bxe+cdx+bd\right)c^4}\sqrt{ex+d}\sqrt{x\,(cx+b)}\left(4\sqrt{-\frac{cx}{b}}\text{EllipticF}\left(\sqrt{\frac{cx+b}{b}},\sqrt{\frac{be}{be-cd}}\right)\sqrt{\frac{cx+b}{b}}\sqrt{-\frac{c\,(ex+d}{be-cd}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(c*x^2+b*x)^(1/2),x)
```

```
[Out] -2/15*(e*x+d)^(1/2)*(x*(c*x+b))^(1/2)*(4*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/
b)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1
```

```
/2)*b^3*c*d*e^2-12*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d
))^(1/2))*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*b^2*c^2*d^2*e+8*(-
c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)
^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*b*c^3*d^3+8*(-c*x/b)^(1/2)*EllipticE(((
c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c
*d))^(1/2)*b^4*e^3-31*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-
c*d))^(1/2))*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*b^3*c*d*e^2+46*
(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/
b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*b^2*c^2*d^2*e-23*(-c*x/b)^(1/2)*Ellip
ticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)*(-(e*x+d)*c
/(b*e-c*d))^(1/2)*b*c^3*d^3-3*x^4*c^4*e^3+x^3*b*c^3*e^3-14*x^3*c^4*d*e^2+4*
x^2*b^2*c^2*e^3-10*x^2*b*c^3*d*e^2-11*x^2*c^4*d^2*e+4*x*b^2*c^2*d*e^2-11*x*
b*c^3*d^2*e)/x/(c*e*x^2+b*e*x+c*d*x+b*d)/c^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\sqrt{cx^2+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/sqrt(c*x^2 + b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(e^2x^2+2\,dex+d^2\right)\sqrt{ex+d}}{\sqrt{cx^2+bx}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*sqrt(e*x + d)/sqrt(c*x^2 + b*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^{\frac{5}{2}}}{\sqrt{x\,(b+cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral((d + e*x)**(5/2)/sqrt(x*(b + c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\sqrt{cx^2+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(5/2)/sqrt(c*x^2 + b*x), x)
```

3.408 $\int \frac{(d+ex)^{3/2}}{\sqrt{bx+cx^2}}\,dx$

Optimal. Leaf size=241

$$-\frac{2\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{3c^{3/2}\sqrt{bx+cx^2}\sqrt{d+ex}}+\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\right.}{3c^{3/2}\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}$$

```
[Out] (2*e*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])/(3*c) + (4*Sqrt[-b]*(2*c*d - b*e)*Sqr
t[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqr
t[-b]], (b*e)/(c*d)])/(3*c^(3/2)*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) - (2*
Sqrt[-b]*d*(c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*Elliptic
F[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*c^(3/2)*Sqrt[d + e*x
]*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.191716, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {742, 843, 715, 112, 110, 117, 116}

$$-\frac{2\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{3c^{3/2}\sqrt{bx+cx^2}\sqrt{d+ex}}+\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{3c^{3/2}\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}+\frac{2e}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/Sqrt[b*x + c*x^2],x]
```


```
[Out] (2*e*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])/(3*c) + (4*Sqrt[-b]*(2*c*d - b*e)*Sqr
t[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqr
t[-b]], (b*e)/(c*d)])/(3*c^(3/2)*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) - (2*
Sqrt[-b]*d*(c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*Elliptic
F[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*c^(3/2)*Sqrt[d + e*x
]*Sqrt[b*x + c*x^2])
```


Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\sqrt{bx+cx^2}}\,dx = \frac{2e\sqrt{d+ex}\sqrt{bx+cx^2}}{3c}+\frac{2\int \frac{\frac{1}{2}d(3cd-be)+e(2cd-be)x}{\sqrt{d+ex}\sqrt{bx+cx^2}}\,dx}{3c}$$

$$= \frac{2e\sqrt{d+ex}\sqrt{bx+cx^2}}{3c}-\frac{(d(cd-be))\int \frac{1}{\sqrt{d+ex}\sqrt{bx+cx^2}}\,dx}{3c}+\frac{(2(2cd-be))\int \frac{\sqrt{d+ex}}{\sqrt{bx+cx^2}}\,dx}{3c}$$

$$= \frac{2e\sqrt{d+ex}\sqrt{bx+cx^2}}{3c}-\frac{\left(d(cd-be)\sqrt{x}\sqrt{b+cx}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}\,dx}{3c\sqrt{bx+cx^2}}+\frac{\left(2(2cd-be)\sqrt{x}\sqrt{b+cx}\right)\int \cdots}{3c\sqrt{bx+cx^2}}$$

$$= \frac{2e\sqrt{d+ex}\sqrt{bx+cx^2}}{3c}+\frac{\left(2(2cd-be)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{d+ex}\right)\int \frac{\sqrt{1+\frac{ex}{d}}}{\sqrt{x}\sqrt{1+\frac{cx}{b}}}\,dx}{3c\sqrt{1+\frac{ex}{d}}\sqrt{bx+cx^2}}-\frac{\left(d(cd-be)\sqrt{x}\sqrt{1+\frac{cx}{b}}\cdots\right.}{3c\sqrt{d+\cdots}}$$

$$= \frac{2e\sqrt{d+ex}\sqrt{bx+cx^2}}{3c}+\frac{4\sqrt{-b}(2cd-be)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{3c^{3/2}\sqrt{1+\frac{ex}{d}}\sqrt{bx+cx^2}}-\frac{2\sqrt{-b}d(cd-be)\cdots}{\cdots}$$

Mathematica [C] time = 1.02004, size = 246, normalized size = 1.02

$$\frac{2icx^{3/2}\sqrt{\frac{b}{c}}\sqrt{\frac{b}{cx}+1}\sqrt{\frac{d}{ex}+1}\left(2b^2e^2-5bcde+3c^2d^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right),\frac{cd}{be}\right)-4ibcex^{3/2}\sqrt{\frac{b}{c}}\sqrt{\frac{b}{cx}+1}\sqrt{\frac{d}{ex}+1}(be-2cd}{3bc^2\sqrt{x(b+cx)}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/Sqrt[b*x + c*x^2],x]
```

```
[Out] (-2*b*(b + c*x)*(d + e*x)*(2*b*e - c*(4*d + e*x)) - (4*I)*b*Sqrt[b/c]*c*e*(
-2*c*d + b*e)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticE[I*ArcSi
nh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] + (2*I)*Sqrt[b/c]*c*(3*c^2*d^2 - 5*b*c*
d*e + 2*b^2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*Ar
cSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])/(3*b*c^2*Sqrt[x*(b + c*x)]*Sqrt[d +
 e*x])
```

Maple [B] time = 0.278, size = 460, normalized size = 1.9

$$\frac{2}{3x\left(cex^2+bxe+cdx+bd\right)c^3}\sqrt{ex+d}\sqrt{x(cx+b)}\left(b^2d\sqrt{\frac{cx+b}{b}}\sqrt{-\frac{c(ex+d)}{be-cd}}\sqrt{-\frac{cx}{b}}\text{EllipticF}\left(\sqrt{\frac{cx+b}{b}},\sqrt{\frac{be}{be-cd}}\right)ec-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*x^2+b*x)^(1/2),x)
```

```
[Out] 2/3*(e*x+d)^(1/2)*(x*(c*x+b))^(1/2)*(b^2*d*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b
*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(
1/2))*e*c-((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*Ell
ipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b*c^2*d^2+2*((c*x+b)/b)^(1/
2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),
(b*e/(b*e-c*d))^(1/2))*b^3*e^2-6*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(
1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^2*
c*d*e+4*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*Ellip
ticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b*c^2*d^2+x^3*c^3*e^2+x^2*b*c
^2*e^2+x^2*c^3*d*e+x*b*c^2*d*e)/x/(c*e*x^2+b*e*x+c*d*x+b*d)/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cx^2+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/sqrt(c*x^2 + b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cx^2+bx}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^(3/2)/sqrt(c*x^2 + b*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^{\frac{3}{2}}}{\sqrt{x(b+cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)/sqrt(x*(b + c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cx^2+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)/sqrt(c*x^2 + b*x), x)
```

3.409 $\int \frac{\sqrt{d+ex}}{\sqrt{bx+cx^2}} dx$

Optimal. Leaf size=94

$$\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}$$

```
[Out] (2*Sqrt[-b]*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[
c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*Sqrt[1 + (e*x)/d]*Sqrt[b*x +
c*x^2])
```

Rubi [A] time = 0.0374351, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {715, 112, 110}

$$\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/Sqrt[b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[-b]*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[
c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*Sqrt[1 + (e*x)/d]*Sqrt[b*x +
c*x^2])
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\sqrt{bx+cx^2}}\,dx = \frac{\left(\sqrt{x}\sqrt{b+cx}\right)\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}\,dx}{\sqrt{bx+cx^2}}$$
$$= \frac{\left(\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{d+ex}\right)\int \frac{\sqrt{1+\frac{ex}{d}}}{\sqrt{x}\sqrt{1+\frac{cx}{b}}}\,dx}{\sqrt{1+\frac{ex}{d}}\sqrt{bx+cx^2}}$$
$$= \frac{2\sqrt{-b}\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}\sqrt{1+\frac{ex}{d}}\sqrt{bx+cx^2}}$$

Mathematica [A] time = 0.563666, size = 121, normalized size = 1.29

$$-\frac{2\sqrt{x}\left(\frac{b}{x}+c\right)\sqrt{d+ex}\left(\frac{d\sqrt{\frac{d}{ex}+1}E\left(\sin^{-1}\left(\frac{\sqrt{-\frac{d}{e}}}{\sqrt{x}}\right)|\frac{be}{cd}\right)}{\sqrt{-\frac{d}{e}}\sqrt{\frac{b}{cx}+1}\left(\frac{d}{x}+e\right)}-\sqrt{x}\right)}{c\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/Sqrt[b*x + c*x^2],x]
```

```
[Out] (-2*(c + b/x)*Sqrt[x]*Sqrt[d + e*x]*(-Sqrt[x] + (d*Sqrt[1 + d/(e*x)]*Ellipt
icE[ArcSin[Sqrt[-(d/e)]/Sqrt[x]], (b*e)/(c*d)])/(Sqrt[-(d/e)]*Sqrt[1 + b/(c
*x)]*(e + d/x))))/(c*Sqrt[x*(b + c*x)])
```

Maple [A] time = 0.262, size = 121, normalized size = 1.3

$$-2\,\frac{\sqrt{ex+d}\sqrt{x\left(cx+b\right)}b\left(be-cd\right)}{c^2x\left(cex^2+bxe+cdx+bd\right)}\sqrt{\frac{cx+b}{b}}\sqrt{-\frac{c\left(ex+d\right)}{be-cd}}\sqrt{-\frac{cx}{b}}\mathrm{EllipticE}\left(\sqrt{\frac{cx+b}{b}},\sqrt{\frac{be}{be-cd}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*x^2+b*x)^(1/2),x)
```

```
[Out] -2*(e*x+d)^(1/2)*(x*(c*x+b))^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))
^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b*
(b*e-c*d)/c^2/x/(c*e*x^2+b*e*x+c*d*x+b*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{cx^2+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/sqrt(c*x^2 + b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex+d}}{\sqrt{cx^2+bx}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)/sqrt(c*x^2 + b*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}}{\sqrt{x(b+cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/sqrt(x*(b + c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{cx^2+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)/sqrt(c*x^2 + b*x), x)
```

3.410 $\int \frac{1}{\sqrt{d+ex}\sqrt{bx+cx^2}}\,dx$

Optimal. Leaf size=94

$$\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{\sqrt{c}\sqrt{bx+cx^2}\sqrt{d+ex}}$$

```
[Out] (2*Sqrt[-b]*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(S
qrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*Sqrt[d + e*x]*Sqrt[b*x +
c*x^2])
```

Rubi [A] time = 0.0404162, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {715, 117, 116}

$$\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}\sqrt{bx+cx^2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*Sqrt[b*x + c*x^2]),x]
```

```
[Out] (2*Sqrt[-b]*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(S
qrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*Sqrt[d + e*x]*Sqrt[b*x +
c*x^2])
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\sqrt{bx+cx^2}}\,dx = \frac{\left(\sqrt{x}\sqrt{b+cx}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}\,dx}{\sqrt{bx+cx^2}}$$
$$= \frac{\left(\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{1+\frac{ex}{d}}\right)\int \frac{1}{\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{1+\frac{ex}{d}}}\,dx}{\sqrt{d+ex}\sqrt{bx+cx^2}}$$
$$= \frac{2\sqrt{-b}\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{1+\frac{ex}{d}}F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}\sqrt{d+ex}\sqrt{bx+cx^2}}$$

Mathematica [A] time = 0.121854, size = 94, normalized size = 1.

$$-\frac{2x^{3/2}\sqrt{\frac{\frac{b}{x}+c}{c}}\sqrt{\frac{\frac{d}{x}+e}{e}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{-\frac{b}{c}}}{\sqrt{x}}\right),\frac{cd}{be}\right)}{\sqrt{-\frac{b}{c}}\sqrt{x(b+cx)}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*Sqrt[b*x + c*x^2]),x]
```

```
[Out] (-2*Sqrt[(c + b/x)/c]*Sqrt[(e + d/x)/e]*x^(3/2)*EllipticF[ArcSin[Sqrt[-(b/c
)]/Sqrt[x]], (c*d)/(b*e)])/(Sqrt[-(b/c)]*Sqrt[x*(b + c*x)]*Sqrt[d + e*x])
```

Maple [A] time = 0.268, size = 113, normalized size = 1.2

$$2\,\frac{b\sqrt{ex+d}\sqrt{x\,(cx+b)}}{cx\left(cex^2+bxe+cdx+bd\right)}\text{EllipticF}\left(\sqrt{\frac{cx+b}{b}},\sqrt{\frac{be}{be-cd}}\right)\sqrt{-\frac{cx}{b}}\sqrt{-\frac{c\,(ex+d)}{be-cd}}\sqrt{\frac{cx+b}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(c*x^2+b*x)^(1/2),x)
```

```
[Out] 2*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*(-c*x/b)^(1/2)*(-(e*x+
d)*c/(b*e-c*d))^(1/2)*((c*x+b)/b)^(1/2)*b*(e*x+d)^(1/2)*(x*(c*x+b))^(1/2)/c
/x/(c*e*x^2+b*e*x+c*d*x+b*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+bx}\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x)*sqrt(e*x + d)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+bx}\sqrt{ex+d}}{cex^3+bdx+(cd+be)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x)*sqrt(e*x + d)/(c*e*x^3 + b*d*x + (c*d + b*e)*x^2
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{x\,(b+cx)}\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x*(b + c*x))*sqrt(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{cx^2+bx}\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x)*sqrt(e*x + d)), x)
```

3.411 $\int \frac{1}{(d+ex)^{3/2}\sqrt{bx+cx^2}}\,dx$

Optimal. Leaf size=146

$$\frac{2\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{d\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}(cd-be)}-\frac{2e\sqrt{bx+cx^2}}{d\sqrt{d+ex}(cd-be)}$$

```
[Out] (-2*e*Sqrt[b*x + c*x^2])/(d*(c*d - b*e)*Sqrt[d + e*x]) + (2*Sqrt[-b]*Sqrt[c
]*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x]
)/Sqrt[-b]], (b*e)/(c*d)])/(d*(c*d - b*e)*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^
2])
```

Rubi [A] time = 0.0806936, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {744, 21, 715, 112, 110}

$$\frac{2\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{d\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}(cd-be)}-\frac{2e\sqrt{bx+cx^2}}{d\sqrt{d+ex}(cd-be)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*Sqrt[b*x + c*x^2]),x]
```


```
[Out] (-2*e*Sqrt[b*x + c*x^2])/(d*(c*d - b*e)*Sqrt[d + e*x]) + (2*Sqrt[-b]*Sqrt[c
]*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x]
)/Sqrt[-b]], (b*e)/(c*d)])/(d*(c*d - b*e)*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^
2])
```


Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(d+ex)^{3/2}\sqrt{bx+cx^2}}\,dx &= -\frac{2e\sqrt{bx+cx^2}}{d(cd-be)\sqrt{d+ex}}-\frac{2\int \frac{-\frac{cd}{2}-\frac{cex}{2}}{\sqrt{d+ex}\sqrt{bx+cx^2}}\,dx}{d(cd-be)}\\
&= -\frac{2e\sqrt{bx+cx^2}}{d(cd-be)\sqrt{d+ex}}+\frac{c\int \frac{\sqrt{d+ex}}{\sqrt{bx+cx^2}}\,dx}{d(cd-be)}\\
&= -\frac{2e\sqrt{bx+cx^2}}{d(cd-be)\sqrt{d+ex}}+\frac{\left(c\sqrt{x}\sqrt{b+cx}\right)\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}\,dx}{d(cd-be)\sqrt{bx+cx^2}}\\
&= -\frac{2e\sqrt{bx+cx^2}}{d(cd-be)\sqrt{d+ex}}+\frac{\left(c\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{d+ex}\right)\int \frac{\sqrt{1+\frac{ex}{d}}}{\sqrt{x}\sqrt{1+\frac{cx}{b}}}\,dx}{d(cd-be)\sqrt{1+\frac{ex}{d}}\sqrt{bx+cx^2}}\\
&= -\frac{2e\sqrt{bx+cx^2}}{d(cd-be)\sqrt{d+ex}}+\frac{2\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{d(cd-be)\sqrt{1+\frac{ex}{d}}\sqrt{bx+cx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.161222, size = 127, normalized size = 0.87

$$
\frac{2\sqrt{x(b+cx)}\left(e\sqrt{x}\sqrt{-\frac{d}{e}}\sqrt{\frac{d}{ex}+1}E\left(\sin^{-1}\left(\frac{\sqrt{-\frac{d}{e}}}{\sqrt{x}}\right)|\frac{be}{cd}\right)+d\sqrt{\frac{b}{cx}+1}\right)}{dx\sqrt{\frac{b}{cx}+1}\sqrt{d+ex}(cd-be)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*Sqrt[b*x + c*x^2]),x]
```

```
[Out] (2*Sqrt[x*(b + c*x)]*(d*Sqrt[1 + b/(c*x)] + Sqrt[-(d/e)]*e*Sqrt[1 + d/(e*x)
]*Sqrt[x]*EllipticE[ArcSin[Sqrt[-(d/e)]/Sqrt[x]], (b*e)/(c*d)]))/(d*(c*d -
b*e)*Sqrt[1 + b/(c*x)]*x*Sqrt[d + e*x])
```

Maple [A] time = 0.291, size = 216, normalized size = 1.5

$$
2\,\frac{\sqrt{x\,(cx+b)}\sqrt{ex+d}}{(be-cd)\,cdx\left(cex^2+bxe+cdx+bd\right)}\left(\mathrm{EllipticE}\left(\sqrt{\frac{cx+b}{b}},\sqrt{\frac{be}{be-cd}}\right)b^2e\sqrt{\frac{cx+b}{b}}\sqrt{-\frac{c\,(ex+d)}{be-cd}}\sqrt{-\frac{cx}{b}}-\mathrm{Elliptic}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(c*x^2+b*x)^(1/2),x)
```

```
[Out] 2*(EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^2*e*((c*x+b)/b)^(1/
2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-EllipticE(((c*x+b)/b)^(1/2),
(b*e/(b*e-c*d))^(1/2))*b*c*d*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)
*(-c*x/b)^(1/2)+x^2*c^2*e+x*b*c*e)*(x*(c*x+b))^(1/2)*(e*x+d)^(1/2)/d/c/(b*e
-c*d)/x/(c*e*x^2+b*e*x+c*d*x+b*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx}(ex + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x)*(e*x + d)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2 + bx}\sqrt{ex + d}}{ce^2x^4 + bd^2x + \left(2\,cde + be^2\right)x^3 + \left(cd^2 + 2\,bde\right)x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x)*sqrt(e*x + d)/(c*e^2*x^4 + b*d^2*x + (2*c*d*e +
b*e^2)*x^3 + (c*d^2 + 2*b*d*e)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x\,(b + cx)}\,(d + ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x*(b + c*x))*(d + e*x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx}(ex + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x)*(e*x + d)^(3/2)), x)
```

3.412 $\int \frac{1}{(d+ex)^{5/2}\sqrt{bx+cx^2}}\,dx$

Optimal. Leaf size=317

$$-\frac{2\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{3d\sqrt{bx+cx^2}\sqrt{d+ex}(cd-be)}-\frac{4e\sqrt{bx+cx^2}(2cd-be)}{3d^2\sqrt{d+ex}(cd-be)^2}+\frac{4\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd}{3d^2\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+}}$$

```
[Out] (-2*e*Sqrt[b*x + c*x^2])/(3*d*(c*d - b*e)*(d + e*x)^(3/2)) - (4*e*(2*c*d -
b*e)*Sqrt[b*x + c*x^2])/(3*d^2*(c*d - b*e)^2*Sqrt[d + e*x]) + (4*Sqrt[-b]*S
qrt[c]*(2*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcS
in[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*d^2*(c*d - b*e)^2*Sqrt[1 +
 (e*x)/d]*Sqrt[b*x + c*x^2]) - (2*Sqrt[-b]*Sqrt[c]*Sqrt[x]*Sqrt[1 + (c*x)/b
]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*
d)])/(3*d*(c*d - b*e)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.287765, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {744, 834, 843, 715, 112, 110, 117, 116}

$$-\frac{4e\sqrt{bx+cx^2}(2cd-be)}{3d^2\sqrt{d+ex}(cd-be)^2}+\frac{4\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{3d^2\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}(cd-be)^2}-\frac{2e\sqrt{bx+cx^2}}{3d(d+ex)^{3/2}(cd-be)}-\frac{2\sqrt{-b}\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(5/2)*Sqrt[b*x + c*x^2]),x]
```

```
[Out] (-2*e*Sqrt[b*x + c*x^2])/(3*d*(c*d - b*e)*(d + e*x)^(3/2)) - (4*e*(2*c*d -
b*e)*Sqrt[b*x + c*x^2])/(3*d^2*(c*d - b*e)^2*Sqrt[d + e*x]) + (4*Sqrt[-b]*S
qrt[c]*(2*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcS
in[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*d^2*(c*d - b*e)^2*Sqrt[1 +
 (e*x)/d]*Sqrt[b*x + c*x^2]) - (2*Sqrt[-b]*Sqrt[c]*Sqrt[x]*Sqrt[1 + (c*x)/b
]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*
d)])/(3*d*(c*d - b*e)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{bx+cx^2}}\,dx = -\frac{2e\sqrt{bx+cx^2}}{3d(cd-be)(d+ex)^{3/2}} - \frac{2\int \frac{\frac{1}{2}(-3cd+2be)+\frac{cex}{2}}{(d+ex)^{3/2}\sqrt{bx+cx^2}}dx}{3d(cd-be)}$$

$$= -\frac{2e\sqrt{bx+cx^2}}{3d(cd-be)(d+ex)^{3/2}} - \frac{4e(2cd-be)\sqrt{bx+cx^2}}{3d^2(cd-be)^2\sqrt{d+ex}} + \frac{4\int \frac{\frac{1}{4}cd(3cd-be)+\frac{1}{2}ce(2cd-be)x}{\sqrt{d+ex}\sqrt{bx+cx^2}}dx}{3d^2(cd-be)^2}$$

$$= -\frac{2e\sqrt{bx+cx^2}}{3d(cd-be)(d+ex)^{3/2}} - \frac{4e(2cd-be)\sqrt{bx+cx^2}}{3d^2(cd-be)^2\sqrt{d+ex}} - \frac{c\int \frac{1}{\sqrt{d+ex}\sqrt{bx+cx^2}}dx}{3d(cd-be)} + \frac{(2c(2cd-be))\int}{3d^2(cd-b}$$

$$= -\frac{2e\sqrt{bx+cx^2}}{3d(cd-be)(d+ex)^{3/2}} - \frac{4e(2cd-be)\sqrt{bx+cx^2}}{3d^2(cd-be)^2\sqrt{d+ex}} - \frac{\left(c\sqrt{x}\sqrt{b+cx}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{3d(cd-be)\sqrt{bx+cx^2}} + \frac{(2c}{}$$

$$= -\frac{2e\sqrt{bx+cx^2}}{3d(cd-be)(d+ex)^{3/2}} - \frac{4e(2cd-be)\sqrt{bx+cx^2}}{3d^2(cd-be)^2\sqrt{d+ex}} + \frac{\left(2c(2cd-be)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{d+ex}\right)\int}{3d^2(cd-be)^2\sqrt{1+\frac{ex}{d}}\sqrt{bx+cx}}$$

$$= -\frac{2e\sqrt{bx+cx^2}}{3d(cd-be)(d+ex)^{3/2}} - \frac{4e(2cd-be)\sqrt{bx+cx^2}}{3d^2(cd-be)^2\sqrt{d+ex}} + \frac{4\sqrt{-b}\sqrt{c}(2cd-be)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{d+ex}}{3d^2(cd-be)^2\sqrt{1+\frac{ex}{d}}\sqrt{bx}}$$

Mathematica [C] time = 1.18275, size = 290, normalized size = 0.91

$$-\frac{2\left(-bex(b+cx)(be(3d+2ex)-cd(5d+4ex))-c\sqrt{\frac{b}{c}}(d+ex)\left(ix^{3/2}\sqrt{\frac{b}{cx}+1}\sqrt{\frac{d}{ex}+1}\left(2b^2e^2-5bcde+3c^2d^2\right)\text{EllipticF}\left(i\right.\right.\right.}{3bd^2\sqrt{x(b+cx)}(d+e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(5/2)*Sqrt[b*x + c*x^2]),x]
```

```
[Out] (-2*(-(b*e*x*(b + c*x)*(b*e*(3*d + 2*e*x) - c*d*(5*d + 4*e*x))) - Sqrt[b/c]
*c*(d + e*x)*(-2*Sqrt[b/c]*(-2*c*d + b*e)*(b + c*x)*(d + e*x) + (2*I)*b*e*(
2*c*d - b*e)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticE[I*ArcSin
h[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] + I*(3*c^2*d^2 - 5*b*c*d*e + 2*b^2*e^2)*
Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/S
qrt[x]], (c*d)/(b*e)])))/(3*b*d^2*(c*d - b*e)^2*Sqrt[x*(b + c*x)]*(d + e*x)
^(3/2))
```

Maple [B] time = 0.319, size = 897, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(5/2)/(c*x^2+b*x)^(1/2),x)
```

```
[Out] 2/3*(x*(c*x+b))^(1/2)*(EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x
*b^2*c*d*e^2*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-
EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b*c^2*d^2*e*((c*x+b)/b
)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+2*EllipticE(((c*x+b)/b)
^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^3*e^3*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-
```

```
c*d))^(1/2)*(-c*x/b)^(1/2)-6*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1
/2))*x*b^2*c*d*e^2*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^
(1/2)+4*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b*c^2*d^2*e*((
c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+EllipticF(((c*x
+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^2*c*d^2*e*((c*x+b)/b)^(1/2)*(-(e*x+d)
*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*
d))^(1/2))*b*c^2*d^3*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b
)^(1/2)+2*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^3*d*e^2*((c*
x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-6*EllipticE(((c*x
+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^2*c*d^2*e*((c*x+b)/b)^(1/2)*(-(e*x+d)
*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+4*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-
c*d))^(1/2))*b*c^2*d^3*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x
/b)^(1/2)+2*x^3*b*c^2*e^3-4*x^3*c^3*d*e^2+2*x^2*b^2*c*e^3-x^2*b*c^2*d*e^2-5
*x^2*c^3*d^2*e+3*x*b^2*c*d*e^2-5*x*b*c^2*d^2*e)/(c*x+b)/x/(b*e-c*d)^2/(e*x+
d)^(3/2)/c/d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+bx}(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x)*(e*x + d)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+bx}\sqrt{ex+d}}{ce^3x^5+bd^3x+\left(3\,cde^2+be^3\right)x^4+3\left(cd^2e+bde^2\right)x^3+\left(cd^3+3\,bd^2e\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x)*sqrt(e*x + d)/(c*e^3*x^5 + b*d^3*x + (3*c*d*e^2
+ b*e^3)*x^4 + 3*(c*d^2*e + b*d*e^2)*x^3 + (c*d^3 + 3*b*d^2*e)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x\left(b+cx\right)}\left(d+ex\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(5/2)/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x*(b + c*x))*(d + e*x)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx}(ex + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x)*(e*x + d)^(5/2)), x)
```

3.413 $\int \frac{1}{(d+ex)^{7/2}\sqrt{bx+cx^2}}\,dx$

Optimal. Leaf size=403

$$-\frac{8\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(2cd-be)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{15d^2\sqrt{bx+cx^2}\sqrt{d+ex}(cd-be)^2}-\frac{2e\sqrt{bx+cx^2}\left(8b^2e^2-23bcde+23c^2d^2\right)}{15d^3\sqrt{d+ex}(cd-be)^3}+\frac{2\sqrt{}}{}$$

```
[Out] (-2*e*Sqrt[b*x + c*x^2])/(5*d*(c*d - b*e)*(d + e*x)^(5/2)) - (8*e*(2*c*d -
b*e)*Sqrt[b*x + c*x^2])/(15*d^2*(c*d - b*e)^2*(d + e*x)^(3/2)) - (2*e*(23*c
^2*d^2 - 23*b*c*d*e + 8*b^2*e^2)*Sqrt[b*x + c*x^2])/(15*d^3*(c*d - b*e)^3*S
qrt[d + e*x]) + (2*Sqrt[-b]*Sqrt[c]*(23*c^2*d^2 - 23*b*c*d*e + 8*b^2*e^2)*S
qrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/S
qrt[-b]], (b*e)/(c*d)])/(15*d^3*(c*d - b*e)^3*Sqrt[1 + (e*x)/d]*Sqrt[b*x +
c*x^2]) - (8*Sqrt[-b]*Sqrt[c]*(2*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[
1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(1
5*d^2*(c*d - b*e)^2*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.465188, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {744, 834, 843, 715, 112, 110, 117, 116}

$$-\frac{2e\sqrt{bx+cx^2}\left(8b^2e^2-23bcde+23c^2d^2\right)}{15d^3\sqrt{d+ex}(cd-be)^3}+\frac{2\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(8b^2e^2-23bcde+23c^2d^2\right)E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{b}{c}\right.}{15d^3\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}(cd-be)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(7/2)*Sqrt[b*x + c*x^2]),x]
```


```
[Out] (-2*e*Sqrt[b*x + c*x^2])/(5*d*(c*d - b*e)*(d + e*x)^(5/2)) - (8*e*(2*c*d -
b*e)*Sqrt[b*x + c*x^2])/(15*d^2*(c*d - b*e)^2*(d + e*x)^(3/2)) - (2*e*(23*c
^2*d^2 - 23*b*c*d*e + 8*b^2*e^2)*Sqrt[b*x + c*x^2])/(15*d^3*(c*d - b*e)^3*S
qrt[d + e*x]) + (2*Sqrt[-b]*Sqrt[c]*(23*c^2*d^2 - 23*b*c*d*e + 8*b^2*e^2)*S
qrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/S
qrt[-b]], (b*e)/(c*d)])/(15*d^3*(c*d - b*e)^3*Sqrt[1 + (e*x)/d]*Sqrt[b*x +
c*x^2]) - (8*Sqrt[-b]*Sqrt[c]*(2*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[
1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(1
5*d^2*(c*d - b*e)^2*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```


Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
```

```
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{1}{(d+ex)^{7/2}\sqrt{bx+cx^2}}\,dx = -\frac{2e\sqrt{bx+cx^2}}{5d(cd-be)(d+ex)^{5/2}} - \frac{2\int \frac{\frac{1}{2}(-5cd+4be)+\frac{3cex}{2}}{(d+ex)^{5/2}\sqrt{bx+cx^2}}dx}{5d(cd-be)}$$

$$= -\frac{2e\sqrt{bx+cx^2}}{5d(cd-be)(d+ex)^{5/2}} - \frac{8e(2cd-be)\sqrt{bx+cx^2}}{15d^2(cd-be)^2(d+ex)^{3/2}} + \frac{4\int \frac{\frac{1}{4}(15c^2d^2-19bcde+8b^2e^2)-ce(2cd-be)x}{(d+ex)^{3/2}\sqrt{bx+cx^2}}}{15d^2(cd-be)^2}$$

$$= -\frac{2e\sqrt{bx+cx^2}}{5d(cd-be)(d+ex)^{5/2}} - \frac{8e(2cd-be)\sqrt{bx+cx^2}}{15d^2(cd-be)^2(d+ex)^{3/2}} - \frac{2e\left(23c^2d^2-23bcde+8b^2e^2\right)\sqrt{bx}}{15d^3(cd-be)^3\sqrt{d+ex}}$$

$$= -\frac{2e\sqrt{bx+cx^2}}{5d(cd-be)(d+ex)^{5/2}} - \frac{8e(2cd-be)\sqrt{bx+cx^2}}{15d^2(cd-be)^2(d+ex)^{3/2}} - \frac{2e\left(23c^2d^2-23bcde+8b^2e^2\right)\sqrt{bx}}{15d^3(cd-be)^3\sqrt{d+ex}}$$

$$= -\frac{2e\sqrt{bx+cx^2}}{5d(cd-be)(d+ex)^{5/2}} - \frac{8e(2cd-be)\sqrt{bx+cx^2}}{15d^2(cd-be)^2(d+ex)^{3/2}} - \frac{2e\left(23c^2d^2-23bcde+8b^2e^2\right)\sqrt{bx}}{15d^3(cd-be)^3\sqrt{d+ex}}$$

$$= -\frac{2e\sqrt{bx+cx^2}}{5d(cd-be)(d+ex)^{5/2}} - \frac{8e(2cd-be)\sqrt{bx+cx^2}}{15d^2(cd-be)^2(d+ex)^{3/2}} - \frac{2e\left(23c^2d^2-23bcde+8b^2e^2\right)\sqrt{bx}}{15d^3(cd-be)^3\sqrt{d+ex}}$$

$$= -\frac{2e\sqrt{bx+cx^2}}{5d(cd-be)(d+ex)^{5/2}} - \frac{8e(2cd-be)\sqrt{bx+cx^2}}{15d^2(cd-be)^2(d+ex)^{3/2}} - \frac{2e\left(23c^2d^2-23bcde+8b^2e^2\right)\sqrt{bx}}{15d^3(cd-be)^3\sqrt{d+ex}}$$

Mathematica [C] time = 1.28309, size = 381, normalized size = 0.95

$$-\frac{2\left(bex(b+cx)\left((d+ex)^2\left(8b^2e^2-23bcde+23c^2d^2\right)+3d^2(cd-be)^2+4d(d+ex)(2cd-be)(cd-be)\right)-c\sqrt{\frac{b}{c}}(d+ex)^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(7/2)*Sqrt[b*x + c*x^2]),x]
```

```
[Out] (-2*(b*e*x*(b + c*x)*(3*d^2*(c*d - b*e)^2 + 4*d*(c*d - b*e)*(2*c*d - b*e)*(
d + e*x) + (23*c^2*d^2 - 23*b*c*d*e + 8*b^2*e^2)*(d + e*x)^2) - Sqrt[b/c]*c
*(d + e*x)^2*(Sqrt[b/c]*(23*c^2*d^2 - 23*b*c*d*e + 8*b^2*e^2)*(b + c*x)*(d
+ e*x) + I*b*e*(23*c^2*d^2 - 23*b*c*d*e + 8*b^2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt
[1 + d/(e*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)]
+ I*(15*c^3*d^3 - 34*b*c^2*d^2*e + 27*b^2*c*d*e^2 - 8*b^3*e^3)*Sqrt[1 + b/(
c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*
d)/(b*e)])))/(15*b*d^3*(c*d - b*e)^3*Sqrt[x*(b + c*x)]*(d + e*x)^(5/2))
```

Maple [B] time = 0.317, size = 1912, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(7/2)/(c*x^2+b*x)^(1/2),x)
```

```
[Out] 2/15*(x*(c*x+b))^(1/2)*(-3*x^3*b^2*c^2*d*e^4-35*x^3*b*c^3*d^2*e^3+20*x^2*b^
3*c*d*e^4-43*x^2*b^2*c^2*d^2*e^3+13*x^2*b*c^3*d^3*e^2-41*x*b^2*c^2*d^3*e^2+
46*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^2*c^2*d^2*e^3*(
-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)-23*EllipticE((
(c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b*c^3*d^3*e^2*(-c*x/b)^(1/2)*((
c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)+8*EllipticF(((c*x+b)/b)^(1/2),
(b*e/(b*e-c*d))^(1/2))*x*b^3*c*d^2*e^3*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(
e*x+d)*c/(b*e-c*d))^(1/2)-24*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1
/2))*x*b^2*c^2*d^3*e^2*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*
d))^(1/2)+16*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b*c^3*d^4
*e*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)-62*Ellipti
cE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^3*c*d^2*e^3*(-c*x/b)^(1/2)*
((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)+92*EllipticE(((c*x+b)/b)^(1/
2),(b*e/(b*e-c*d))^(1/2))*x*b^2*c^2*d^3*e^2*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2
)*(-(e*x+d)*c/(b*e-c*d))^(1/2)-46*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d
))^(1/2))*x*b*c^3*d^4*e*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c
*d))^(1/2)+4*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^3*c*d
*e^4*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)-12*Ellip
ticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^2*c^2*d^2*e^3*(-c*x/b)^
(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)+15*x*b^3*c*d^2*e^3+34*
x*b*c^3*d^4*e+8*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^4*d^2*
e^3*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)-23*x^4*b*
c^3*d*e^4+4*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^3*c*d^3*e^
2*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)-12*Elliptic
F(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^2*c^2*d^4*e*(-c*x/b)^(1/2)*((c
*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)-31*EllipticE(((c*x+b)/b)^(1/2),
(b*e/(b*e-c*d))^(1/2))*b^3*c*d^3*e^2*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*
x+d)*c/(b*e-c*d))^(1/2)+46*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2
))*b^2*c^2*d^4*e*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1
/2)+8*x^4*b^2*c^2*e^5+23*x^4*c^4*d^2*e^3+8*x^3*b^3*c*e^5+54*x^3*c^4*d^3*e^2
+34*x^2*c^4*d^4*e+16*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b
^4*d*e^4*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)+8*El
lipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b*c^3*d^3*e^2*(-c*x/b)
^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)-31*EllipticE(((c*x+b)
/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^3*c*d*e^4*(-c*x/b)^(1/2)*((c*x+b)/b)
^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)+8*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e
-c*d))^(1/2))*x^2*b^4*e^5*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e
-c*d))^(1/2)+8*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b*c^3*d^5
*(-c*x/b)^(1/2)*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)-23*EllipticE
(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b*c^3*d^5*(-c*x/b)^(1/2)*((c*x+b)
/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2))/(c*x+b)/x/(b*e-c*d)^3/(e*x+d)^(5/2)
/c/d^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+bx}(ex+d)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x)*(e*x + d)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+bx}\sqrt{ex+d}}{ce^4x^6+bd^4x+\left(4\,cde^3+be^4\right)x^5+2\left(3\,cd^2e^2+2\,bde^3\right)x^4+2\left(2\,cd^3e+3\,bd^2e^2\right)x^3+\left(cd^4+4\,bd^3e\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x)*sqrt(e*x + d)/(c*e^4*x^6 + b*d^4*x + (4*c*d*e^3
+ b*e^4)*x^5 + 2*(3*c*d^2*e^2 + 2*b*d*e^3)*x^4 + 2*(2*c*d^3*e + 3*b*d^2*e^2
)*x^3 + (c*d^4 + 4*b*d^3*e)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x\left(b+cx\right)}\left(d+ex\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(7/2)/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x*(b + c*x))*(d + e*x)**(7/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+bx}(ex+d)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x)*(e*x + d)^(7/2)), x)
```

3.414 $\int \frac{(d+ex)^{7/2}}{\left(bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=395

$$-\frac{4d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\left(2b^2e^2-3bcde+3c^2d^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{3(-b)^{3/2}c^{5/2}\sqrt{bx+cx^2}\sqrt{d+ex}}+\frac{4e\sqrt{bx+cx^2}\sqrt{d+ex}\left(2b^2e^2-3\right.}{3b^2c^2}$$

```
[Out] (-2*(d + e*x)^(5/2)*(b*d + (2*c*d - b*e)*x))/(b^2*Sqrt[b*x + c*x^2]) + (4*e
*(3*c^2*d^2 - 3*b*c*d*e + 2*b^2*e^2)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])/(3*b^
2*c^2) + (2*e*(2*c*d - b*e)*(d + e*x)^(3/2)*Sqrt[b*x + c*x^2])/(b^2*c) + (2
*(2*c*d - b*e)*(3*c^2*d^2 - 3*b*c*d*e + 8*b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b
]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])
/(3*(-b)^(3/2)*c^(5/2)*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) - (4*d*(c*d - b
*e)*(3*c^2*d^2 - 3*b*c*d*e + 2*b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 +
(e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*(-b
)^(3/2)*c^(5/2)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.513945, antiderivative size = 395, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {738, 832, 843, 715, 112, 110, 117, 116}

$$\frac{4e\sqrt{bx+cx^2}\sqrt{d+ex}\left(2b^2e^2-3bcde+3c^2d^2\right)}{3b^2c^2}-\frac{4d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\left(2b^2e^2-3bcde+3c^2d^2\right)F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)\right|}{3(-b)^{3/2}c^{5/2}\sqrt{bx+cx^2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/(b*x + c*x^2)^(3/2),x]
```


```
[Out] (-2*(d + e*x)^(5/2)*(b*d + (2*c*d - b*e)*x))/(b^2*Sqrt[b*x + c*x^2]) + (4*e
*(3*c^2*d^2 - 3*b*c*d*e + 2*b^2*e^2)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])/(3*b^
2*c^2) + (2*e*(2*c*d - b*e)*(d + e*x)^(3/2)*Sqrt[b*x + c*x^2])/(b^2*c) + (2
*(2*c*d - b*e)*(3*c^2*d^2 - 3*b*c*d*e + 8*b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b
]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])
/(3*(-b)^(3/2)*c^(5/2)*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) - (4*d*(c*d - b
*e)*(3*c^2*d^2 - 3*b*c*d*e + 2*b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 +
(e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*(-b
)^(3/2)*c^(5/2)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```


Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
```

```
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{(d+ex)^{7/2}}{\left(bx+cx^2\right)^{3/2}}\,dx = -\frac{2(d+ex)^{5/2}(bd+(2cd-be)x)}{b^2\sqrt{bx+cx^2}} - \frac{2\int \frac{(d+ex)^{3/2}\left(-\frac{5}{2}bde-\frac{5}{2}e(2cd-be)x\right)}{\sqrt{bx+cx^2}}dx}{b^2}$$

$$= -\frac{2(d+ex)^{5/2}(bd+(2cd-be)x)}{b^2\sqrt{bx+cx^2}} + \frac{2e(2cd-be)(d+ex)^{3/2}\sqrt{bx+cx^2}}{b^2c} - \frac{4\int \frac{\sqrt{d+ex}\left(-\frac{5}{4}bde(3cd+be)-\frac{5}{2}e(3c^2}{\sqrt{bx+cx^2}}}{5b^2c}$$

$$= -\frac{2(d+ex)^{5/2}(bd+(2cd-be)x)}{b^2\sqrt{bx+cx^2}} + \frac{4e\left(3c^2d^2-3bcde+2b^2e^2\right)\sqrt{d+ex}\sqrt{bx+cx^2}}{3b^2c^2} + \frac{2e(2cd-be)(d}{}$$

$$= -\frac{2(d+ex)^{5/2}(bd+(2cd-be)x)}{b^2\sqrt{bx+cx^2}} + \frac{4e\left(3c^2d^2-3bcde+2b^2e^2\right)\sqrt{d+ex}\sqrt{bx+cx^2}}{3b^2c^2} + \frac{2e(2cd-be)(d}{}$$

$$= -\frac{2(d+ex)^{5/2}(bd+(2cd-be)x)}{b^2\sqrt{bx+cx^2}} + \frac{4e\left(3c^2d^2-3bcde+2b^2e^2\right)\sqrt{d+ex}\sqrt{bx+cx^2}}{3b^2c^2} + \frac{2e(2cd-be)(d}{}$$

$$= -\frac{2(d+ex)^{5/2}(bd+(2cd-be)x)}{b^2\sqrt{bx+cx^2}} + \frac{4e\left(3c^2d^2-3bcde+2b^2e^2\right)\sqrt{d+ex}\sqrt{bx+cx^2}}{3b^2c^2} + \frac{2e(2cd-be)(d}{}$$

$$= -\frac{2(d+ex)^{5/2}(bd+(2cd-be)x)}{b^2\sqrt{bx+cx^2}} + \frac{4e\left(3c^2d^2-3bcde+2b^2e^2\right)\sqrt{d+ex}\sqrt{bx+cx^2}}{3b^2c^2} + \frac{2e(2cd-be)(d}{}$$

Mathematica [C] time = 1.24455, size = 360, normalized size = 0.91

$$-\frac{2\left(b(d+ex)\left(-b^2e^3x(b+cx)+3c^2d^3(b+cx)+3x(cd-be)^3\right)-\sqrt{\frac{b}{c}}\left(-ibex^{3/2}\sqrt{\frac{b}{cx}+1}\sqrt{\frac{d}{ex}+1}\left(23b^2cde^2-8b^3e^3-18bc^2d\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)/(b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*(b*(d + e*x)*(3*(c*d - b*e)^3*x + 3*c^2*d^3*(b + c*x) - b^2*e^3*x*(b +
c*x)) - Sqrt[b/c]*(Sqrt[b/c]*(6*c^3*d^3 - 9*b*c^2*d^2*e + 19*b^2*c*d*e^2 -
8*b^3*e^3)*(b + c*x)*(d + e*x) + I*b*e*(6*c^3*d^3 - 9*b*c^2*d^2*e + 19*b^2*
c*d*e^2 - 8*b^3*e^3)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticE[
I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - I*b*e*(3*c^3*d^3 - 18*b*c^2*d^
2*e + 23*b^2*c*d*e^2 - 8*b^3*e^3)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/
2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])))/(3*b^3*c^2*Sqrt[
x*(b + c*x)]*Sqrt[d + e*x])
```

Maple [B] time = 0.361, size = 863, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/(c*x^2+b*x)^(3/2),x)
```

```
[Out] 2/3*(4*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*Ellipt
icF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^4*c*d*e^3-10*((c*x+b)/b)^(1/
2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),
(b*e/(b*e-c*d))^(1/2))*b^3*c^2*d^2*e^2+12*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*
e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1
/2))*b^2*c^3*d^3*e-6*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b
)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b*c^4*d^4+8*((c*
x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b
)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^5*e^4-27*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/
(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))
^(1/2))*b^4*c*d*e^3+28*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x
/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^3*c^2*d^2*e^
2-15*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*Elliptic
E(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^2*c^3*d^3*e+6*((c*x+b)/b)^(1/2
)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(
b*e/(b*e-c*d))^(1/2))*b*c^4*d^4+x^3*b^2*c^3*e^4+4*x^2*b^3*c^2*e^4-8*x^2*b^2
*c^3*d*e^3+9*x^2*b*c^4*d^2*e^2-6*x^2*c^5*d^3*e+4*x*b^3*c^2*d*e^3-9*x*b^2*c^
3*d^2*e^2+6*x*b*c^4*d^3*e-6*x*c^5*d^4-3*b*c^4*d^4)/x*(x*(c*x+b))^(1/2)/(c*x
+b)/c^4/b^2/(e*x+d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{7}{2}}}{\left(cx^2+bx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(7/2)/(c*x^2 + b*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\sqrt{cx^2+bx}\sqrt{ex+d}}{c^2x^4+2\,bcx^3+b^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(c*x^2 + b*x)*sqrt(e
*x + d)/(c^2*x^4 + 2*b*c*x^3 + b^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{7}{2}}}{\left(cx^2+bx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(7/2)/(c*x^2 + b*x)^(3/2), x)
```

3.415 $\int \frac{(d+ex)^{5/2}}{\left(bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=310

$$-\frac{2d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{(-b)^{3/2}c^{3/2}\sqrt{bx+cx^2}\sqrt{d+ex}}+\frac{4\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-bcde+c^2d^2\right)E}{(-b)^{3/2}c^{3/2}\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+}}$$

```
[Out] (-2*(d + e*x)^(3/2)*(b*d + (2*c*d - b*e)*x))/(b^2*Sqrt[b*x + c*x^2]) + (2*e
*(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])/(b^2*c) + (4*(c^2*d^2 - b*c
*d*e + b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(S
qrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/((-b)^(3/2)*c^(3/2)*Sqrt[1 + (e*x)
/d]*Sqrt[b*x + c*x^2]) - (2*d*(c*d - b*e)*(2*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c
*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e
)/(c*d)])/((-b)^(3/2)*c^(3/2)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.33904, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {738, 832, 843, 715, 112, 110, 117, 116}

$$\frac{4\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-bcde+c^2d^2\right)E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{(-b)^{3/2}c^{3/2}\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2(d+ex)^{3/2}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}+\frac{2e\sqrt{bx+cx^2}\sqrt{d+e}}{b^2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(b*x + c*x^2)^(3/2),x]
```



```
[Out] (-2*(d + e*x)^(3/2)*(b*d + (2*c*d - b*e)*x))/(b^2*Sqrt[b*x + c*x^2]) + (2*e
*(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])/(b^2*c) + (4*(c^2*d^2 - b*c
*d*e + b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(S
qrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/((-b)^(3/2)*c^(3/2)*Sqrt[1 + (e*x)
/d]*Sqrt[b*x + c*x^2]) - (2*d*(c*d - b*e)*(2*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c
*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e
)/(c*d)])/((-b)^(3/2)*c^(3/2)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```


Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
```

```
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\left(bx+cx^2\right)^{3/2}}\,dx = -\frac{2(d+ex)^{3/2}(bd+(2cd-be)x)}{b^2\sqrt{bx+cx^2}} - \frac{2\int \frac{\sqrt{d+ex}\left(-\frac{3}{2}bde-\frac{3}{2}e(2cd-be)x\right)}{\sqrt{bx+cx^2}}dx}{b^2}$$

$$= -\frac{2(d+ex)^{3/2}(bd+(2cd-be)x)}{b^2\sqrt{bx+cx^2}} + \frac{2e(2cd-be)\sqrt{d+ex}\sqrt{bx+cx^2}}{b^2c} - \frac{4\int \frac{-\frac{3}{4}bde(cd+be)-\frac{3}{2}e\left(c^2d^2-bcde+\right.}{\sqrt{d+ex}\sqrt{bx+cx^2}}}{3b^2c}$$

$$= -\frac{2(d+ex)^{3/2}(bd+(2cd-be)x)}{b^2\sqrt{bx+cx^2}} + \frac{2e(2cd-be)\sqrt{d+ex}\sqrt{bx+cx^2}}{b^2c} - \frac{(d(cd-be)(2cd-be))\int \frac{}{\sqrt{d}}}{b^2c}$$

$$= -\frac{2(d+ex)^{3/2}(bd+(2cd-be)x)}{b^2\sqrt{bx+cx^2}} + \frac{2e(2cd-be)\sqrt{d+ex}\sqrt{bx+cx^2}}{b^2c} - \frac{\left(d(cd-be)(2cd-be)\sqrt{x}\sqrt{b}\right.}{b^2c\sqrt{bx}}$$

$$= -\frac{2(d+ex)^{3/2}(bd+(2cd-be)x)}{b^2\sqrt{bx+cx^2}} + \frac{2e(2cd-be)\sqrt{d+ex}\sqrt{bx+cx^2}}{b^2c} + \frac{\left(2\left(c^2d^2-bcde+b^2e^2\right)\sqrt{x}\right.}{b^2c\sqrt{1+}}$$

$$= -\frac{2(d+ex)^{3/2}(bd+(2cd-be)x)}{b^2\sqrt{bx+cx^2}} + \frac{2e(2cd-be)\sqrt{d+ex}\sqrt{bx+cx^2}}{b^2c} + \frac{4\left(c^2d^2-bcde+b^2e^2\right)\sqrt{x}}{(-b)^{3/2}c^{3/2}}$$

Mathematica [C] time = 1.16548, size = 262, normalized size = 0.85

$$\frac{-2icex^{3/2}\sqrt{\frac{b}{c}}\sqrt{\frac{b}{cx}+1}\sqrt{\frac{d}{ex}+1}\left(2b^2e^2-3bcde+c^2d^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right),\frac{cd}{be}\right)+4icex^{3/2}\sqrt{\frac{b}{c}}\sqrt{\frac{b}{cx}+1}\sqrt{\frac{d}{ex}+1}\left(b^2e\right.}{b^2c^2\sqrt{x(b+cx)}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*b*(d + e*x)*(c^2*d^2 + 2*b^2*e^2 + b*c*e*(-2*d + e*x)) + (4*I)*Sqrt[b/c]
*c*e*(c^2*d^2 - b*c*d*e + b^2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3
/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - (2*I)*Sqrt[b/c]*
c*e*(c^2*d^2 - 3*b*c*d*e + 2*b^2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x
^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])/(b^2*c^2*Sqrt[
x*(b + c*x)]*Sqrt[d + e*x])
```

Maple [B] time = 0.31, size = 652, normalized size = 2.1

$$-2\,\frac{\sqrt{x\,(cx+b)}}{x\,(cx+b)\,c^3b^2\sqrt{ex+d}}\left(\sqrt{-\frac{cx}{b}}\text{EllipticF}\left(\sqrt{\frac{cx+b}{b}},\sqrt{\frac{be}{be-cd}}\right)\sqrt{\frac{cx+b}{b}}\sqrt{-\frac{c\,(ex+d)}{be-cd}}b^3cde^2-3\sqrt{-\frac{cx}{b}}\text{EllipticF}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(c*x^2+b*x)^(3/2),x)
```

```
[Out] -2*((-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x
+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*b^3*c*d*e^2-3*(-c*x/b)^(1/2)*Elli
pticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)*(-(e*x+d)*
c/(b*e-c*d))^(1/2)*b^2*c^2*d^2*e+2*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/
```

```
2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*b*
c^3*d^3+2*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))
*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*b^4*e^3-4*(-c*x/b)^(1/2)*El
lipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)*(-(e*x+d
)*c/(b*e-c*d))^(1/2)*b^3*c*d*e^2+4*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/
2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*b^
2*c^2*d^2*e-2*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1
/2))*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*b*c^3*d^3+x^2*b^2*c^2*e
^3-2*x^2*b*c^3*d*e^2+2*x^2*c^4*d^2*e+x*b^2*c^2*d*e^2-x*b*c^3*d^2*e+2*x*c^4*
d^3+b*c^3*d^3)/x*(x*(c*x+b))^(1/2)/(c*x+b)/c^3/b^2/(e*x+d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\left(cx^2+bx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/(c*x^2 + b*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e^2x^2+2\,dex+d^2\right)\sqrt{cx^2+bx}\sqrt{ex+d}}{c^2x^4+2\,bcx^3+b^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*sqrt(c*x^2 + b*x)*sqrt(e*x + d)/(c^2*x^4
 + 2*b*c*x^3 + b^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\left(cx^2+bx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(5/2)/(c*x^2 + b*x)^(3/2), x)
```

3.416 $\int \frac{(d+ex)^{3/2}}{\left(bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=249

$$-\frac{4d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{(-b)^{3/2}\sqrt{c}\sqrt{bx+cx^2}\sqrt{d+ex}}-\frac{2\sqrt{d+ex}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}+\frac{2\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd}{(-b)^{3/2}\sqrt{c}\sqrt{bx}}$$

```
[Out] (-2*Sqrt[d + e*x]*(b*d + (2*c*d - b*e)*x))/(b^2*Sqrt[b*x + c*x^2]) + (2*(2*
c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c
]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/((-b)^(3/2)*Sqrt[c]*Sqrt[1 + (e*x)/d]*S
qrt[b*x + c*x^2]) - (4*d*(c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*
x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/((-b)^(3/
2)*Sqrt[c]*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.190674, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {738, 843, 715, 112, 110, 117, 116}

$$-\frac{2\sqrt{d+ex}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}-\frac{4d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{(-b)^{3/2}\sqrt{c}\sqrt{bx+cx^2}\sqrt{d+ex}}+\frac{2\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)E\left(\text{s}\right.}{(-b)^{3/2}\sqrt{c}\sqrt{bx+cx^2}\sqrt{\frac{e}{d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(b*d + (2*c*d - b*e)*x))/(b^2*Sqrt[b*x + c*x^2]) + (2*(2*
c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c
]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/((-b)^(3/2)*Sqrt[c]*Sqrt[1 + (e*x)/d]*S
qrt[b*x + c*x^2]) - (4*d*(c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*
x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/((-b)^(3/
2)*Sqrt[c]*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\left(bx+cx^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}(bd+(2cd-be)x)}{b^2\sqrt{bx+cx^2}} - \frac{2\int \frac{-\frac{1}{2}bde-\frac{1}{2}e(2cd-be)x}{\sqrt{d+ex}\sqrt{bx+cx^2}}\,dx}{b^2}$$

$$= -\frac{2\sqrt{d+ex}(bd+(2cd-be)x)}{b^2\sqrt{bx+cx^2}} - \frac{(2d(cd-be))\int \frac{1}{\sqrt{d+ex}\sqrt{bx+cx^2}}\,dx}{b^2} + \frac{(2cd-be)\int \frac{\sqrt{d+ex}}{\sqrt{bx+cx^2}}\,dx}{b^2}$$

$$= -\frac{2\sqrt{d+ex}(bd+(2cd-be)x)}{b^2\sqrt{bx+cx^2}} - \frac{\left(2d(cd-be)\sqrt{x}\sqrt{b+cx}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}\,dx}{b^2\sqrt{bx+cx^2}} + \frac{\left((2cd-be)\sqrt{x}\sqrt{b \cdots\right.}{b^2\sqrt{b \cdots}}$$

$$= -\frac{2\sqrt{d+ex}(bd+(2cd-be)x)}{b^2\sqrt{bx+cx^2}} + \frac{\left((2cd-be)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{d+ex}\right)\int \frac{\sqrt{1+\frac{ex}{d}}}{\sqrt{x}\sqrt{1+\frac{cx}{b}}}\,dx}{b^2\sqrt{1+\frac{ex}{d}}\sqrt{bx+cx^2}} - \frac{\left(2d(cd-be)\sqrt{\cdots\right.}{\cdots}$$

$$= -\frac{2\sqrt{d+ex}(bd+(2cd-be)x)}{b^2\sqrt{bx+cx^2}} + \frac{2(2cd-be)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{(-b)^{3/2}\sqrt{c}\sqrt{1+\frac{ex}{d}}\sqrt{bx+cx^2}} - \frac{4d(cd-b \cdots}{\cdots}$$

Mathematica [C] time = 1.1047, size = 210, normalized size = 0.84

$$\frac{2(cd-be)\left(b(d+ex)-icex^{3/2}\sqrt{\frac{b}{c}}\sqrt{\frac{b}{cx}+1}\sqrt{\frac{d}{ex}+1}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right),\frac{cd}{be}\right)\right)-2icex^{3/2}\sqrt{\frac{b}{c}}\sqrt{\frac{b}{cx}+1}\sqrt{\frac{d}{ex}+1}(be-2cd)I}{b^2c\sqrt{x(b+cx)}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(b*x + c*x^2)^(3/2),x]
```

```
[Out] ((-2*I)*Sqrt[b/c]*c*e*(-2*c*d + b*e)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^
(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] + 2*(c*d - b*e)*
(b*(d + e*x) - I*Sqrt[b/c]*c*e*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*
EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)]))/(b^2*c*Sqrt[x*(b + c
*x)]*Sqrt[d + e*x])
```

Maple [B] time = 0.298, size = 451, normalized size = 1.8

$$2\frac{\sqrt{x(cx+b)}}{x(cx+b)b^2c^2\sqrt{ex+d}}\left(2b^2d\sqrt{\frac{cx+b}{b}}\sqrt{-\frac{c(ex+d)}{be-cd}}\sqrt{-\frac{cx}{b}}\text{EllipticF}\left(\sqrt{\frac{cx+b}{b}},\sqrt{\frac{be}{be-cd}}\right)ec-2\sqrt{\frac{cx+b}{b}}\sqrt{-\frac{c(ex+}{be-c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*x^2+b*x)^(3/2),x)
```

```
[Out] 2*(2*b^2*d*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*El
lipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*e*c-2*((c*x+b)/b)^(1/2)*(-
(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/
(b*e-c*d))^(1/2))*b*c^2*d^2+((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*
(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^3*e^2-3
*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((
c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^2*c*d*e+2*((c*x+b)/b)^(1/2)*(-(e*x
+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e
-c*d))^(1/2))*b*c^2*d^2+x^2*b*c^2*e^2-2*x^2*c^3*d*e-2*x*c^3*d^2-b*c^2*d^2)/
x*(x*(c*x+b))^(1/2)/(c*x+b)/b^2/c^2/(e*x+d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cx^2+bx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(c*x^2 + b*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+bx}(ex+d)^{\frac{3}{2}}}{c^2x^4+2\,bcx^3+b^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x)*(e*x + d)^(3/2)/(c^2*x^4 + 2*b*c*x^3 + b^2*x^2),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^{\frac{3}{2}}}{(x(b+cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)/(x*(b + c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cx^2+bx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)/(c*x^2 + b*x)^(3/2), x)
```

3.417 $\int \frac{\sqrt{d+ex}}{(bx+cx^2)^{3/2}} dx$

Optimal. Leaf size=231

$$-\frac{2\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(2cd-be)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{(-b)^{3/2}\sqrt{c}\sqrt{bx+cx^2}\sqrt{d+ex}}-\frac{2(b+2cx)\sqrt{d+ex}}{b^2\sqrt{bx+cx^2}}+\frac{4\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)\right.}{(-b)^{3/2}\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}$$

```
[Out] (-2*(b + 2*c*x)*Sqrt[d + e*x])/(b^2*Sqrt[b*x + c*x^2]) + (4*Sqrt[c]*Sqrt[x]
*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b
]], (b*e)/(c*d)])/((-b)^(3/2)*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) - (2*(2*
c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sq
rt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/((-b)^(3/2)*Sqrt[c]*Sqrt[d + e*x]*S
qrt[b*x + c*x^2])
```

Rubi [A] time = 0.154094, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {736, 843, 715, 112, 110, 117, 116}

$$-\frac{2(b+2cx)\sqrt{d+ex}}{b^2\sqrt{bx+cx^2}}-\frac{2\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(2cd-be)F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{(-b)^{3/2}\sqrt{c}\sqrt{bx+cx^2}\sqrt{d+ex}}+\frac{4\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{(-b)^{3/2}\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(b*x + c*x^2)^(3/2),x]
```


```
[Out] (-2*(b + 2*c*x)*Sqrt[d + e*x])/(b^2*Sqrt[b*x + c*x^2]) + (4*Sqrt[c]*Sqrt[x]
*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b
]], (b*e)/(c*d)])/((-b)^(3/2)*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) - (2*(2*
c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sq
rt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/((-b)^(3/2)*Sqrt[c]*Sqrt[d + e*x]*S
qrt[b*x + c*x^2])
```


Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] - Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)
*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p + 1
), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 0] && (L
tQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c,
d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\left(bx+cx^2\right)^{3/2}}\,dx = -\frac{2(b+2cx)\sqrt{d+ex}}{b^2\sqrt{bx+cx^2}} + \frac{2\int \frac{\frac{be}{2}+cex}{\sqrt{d+ex}\sqrt{bx+cx^2}}\,dx}{b^2}$$

$$= -\frac{2(b+2cx)\sqrt{d+ex}}{b^2\sqrt{bx+cx^2}} + \frac{(2c)\int \frac{\sqrt{d+ex}}{\sqrt{bx+cx^2}}\,dx}{b^2} - \frac{(2cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{bx+cx^2}}\,dx}{b^2}$$

$$= -\frac{2(b+2cx)\sqrt{d+ex}}{b^2\sqrt{bx+cx^2}} + \frac{\left(2c\sqrt{x}\sqrt{b+cx}\right)\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}\,dx}{b^2\sqrt{bx+cx^2}} - \frac{\left((2cd-be)\sqrt{x}\sqrt{b+cx}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}\,d}{b^2\sqrt{bx+cx^2}}$$

$$= -\frac{2(b+2cx)\sqrt{d+ex}}{b^2\sqrt{bx+cx^2}} + \frac{\left(2c\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{d+ex}\right)\int \frac{\sqrt{1+\frac{ex}{d}}}{\sqrt{x}\sqrt{1+\frac{cx}{b}}}\,dx}{b^2\sqrt{1+\frac{ex}{d}}\sqrt{bx+cx^2}} - \frac{\left((2cd-be)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{1+\frac{ex}{d}}\right.}{b^2\sqrt{d+ex}\sqrt{bx}}$$

$$= -\frac{2(b+2cx)\sqrt{d+ex}}{b^2\sqrt{bx+cx^2}} + \frac{4\sqrt{c}\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{(-b)^{3/2}\sqrt{1+\frac{ex}{d}}\sqrt{bx+cx^2}} - \frac{2(2cd-be)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{}}{(-b)^{3/2}\sqrt{c}\sqrt{d}}$$

Mathematica [C] time = 0.41625, size = 186, normalized size = 0.81

$$\frac{-2iex^{3/2}\sqrt{\frac{b}{cx}+1}\sqrt{\frac{d}{ex}+1}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right),\frac{cd}{be}\right)+4iex^{3/2}\sqrt{\frac{b}{cx}+1}\sqrt{\frac{d}{ex}+1}E\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right)|\frac{cd}{be}\right)+2\sqrt{\frac{b}{c}}(d+ex)}{b\sqrt{\frac{b}{c}}\sqrt{x(b+cx)}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[b/c]*(d + e*x) + (4*I)*e*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2
)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - (2*I)*e*Sqrt[1 + b
/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (
c*d)/(b*e)])/(b*Sqrt[b/c]*Sqrt[x*(b + c*x)]*Sqrt[d + e*x])
```

Maple [A] time = 0.286, size = 352, normalized size = 1.5

$$2\frac{\sqrt{x(cx+b)}}{x(cx+b)b^2c\sqrt{ex+d}}\left(\text{EllipticF}\left(\sqrt{\frac{cx+b}{b}},\sqrt{\frac{be}{be-cd}}\right)b^2e\sqrt{\frac{cx+b}{b}}\sqrt{-\frac{c(ex+d)}{be-cd}}\sqrt{-\frac{cx}{b}}-2\,\text{EllipticF}\left(\sqrt{\frac{cx+b}{b}},\sqrt{b\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*x^2+b*x)^(3/2),x)
```

```
[Out] 2*(EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^2*e*((c*x+b)/b)^(1/
2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-2*EllipticF(((c*x+b)/b)^(1/2
),(b*e/(b*e-c*d))^(1/2))*b*c*d*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/
2)*(-c*x/b)^(1/2)-2*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^2*
e*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+2*EllipticE
(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b*c*d*((c*x+b)/b)^(1/2)*(-(e*x+d)
*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-2*x^2*c^2*e-x*b*c*e-2*x*c^2*d-b*c*d)/x*(
x*(c*x+b))^(1/2)/(c*x+b)/b^2/c/(e*x+d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{ex+d}}{\left(cx^2+bx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(c*x^2 + b*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+bx}\sqrt{ex+d}}{c^2x^4+2\,bcx^3+b^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x)*sqrt(e*x + d)/(c^2*x^4 + 2*b*c*x^3 + b^2*x^2), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}}{(x(b+cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/(x*(b + c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\left(cx^2+bx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)/(c*x^2 + b*x)^(3/2), x)
```

3.418 $\int \frac{1}{\sqrt{d+ex}\left(bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=274

$$-\frac{4\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{(-b)^{3/2}\sqrt{bx+cx^2}\sqrt{d+ex}}-\frac{2\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(cd-be)}+\frac{2\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(}{(-b)^{3/2}d\sqrt{bx+}}$$

```
[Out] (-2*Sqrt[d + e*x]*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(b^2*d*(c*d - b*e)*S
qrt[b*x + c*x^2]) + (2*Sqrt[c]*(2*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt
[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/((-b)
^(3/2)*d*(c*d - b*e)*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) - (4*Sqrt[c]*Sqrt
[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/
Sqrt[-b]], (b*e)/(c*d)])/((-b)^(3/2)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.219433, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {740, 843, 715, 112, 110, 117, 116}

$$-\frac{2\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(cd-be)}-\frac{4\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{(-b)^{3/2}\sqrt{bx+cx^2}\sqrt{d+ex}}+\frac{2\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be}{(-b)^{3/2}d\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(b^2*d*(c*d - b*e)*S
qrt[b*x + c*x^2]) + (2*Sqrt[c]*(2*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt
[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/((-b)
^(3/2)*d*(c*d - b*e)*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) - (4*Sqrt[c]*Sqrt
[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/
Sqrt[-b]], (b*e)/(c*d)])/((-b)^(3/2)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\left(bx+cx^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)\sqrt{bx+cx^2}} - \frac{2\int \frac{-\frac{1}{2}bcde-\frac{1}{2}ce(2cd-be)x}{\sqrt{d+ex}\sqrt{bx+cx^2}}\,dx}{b^2d(cd-be)}$$

$$= -\frac{2\sqrt{d+ex}(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)\sqrt{bx+cx^2}} - \frac{(2c)\int \frac{1}{\sqrt{d+ex}\sqrt{bx+cx^2}}\,dx}{b^2} + \frac{(c(2cd-be))\int \frac{\sqrt{d+ex}}{\sqrt{bx+cx^2}}}{b^2d(cd-be)}$$

$$= -\frac{2\sqrt{d+ex}(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)\sqrt{bx+cx^2}} - \frac{\left(2c\sqrt{x}\sqrt{b+cx}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}\,dx}{b^2\sqrt{bx+cx^2}} + \frac{(c(2cd-}{b}$$

$$= -\frac{2\sqrt{d+ex}(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)\sqrt{bx+cx^2}} + \frac{\left(c(2cd-be)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{d+ex}\right)\int \frac{\sqrt{1+\frac{ex}{d}}}{\sqrt{x}\sqrt{1+\frac{cx}{b}}}\,dx}{b^2d(cd-be)\sqrt{1+\frac{ex}{d}}\sqrt{bx+cx^2}}$$

$$= -\frac{2\sqrt{d+ex}(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)\sqrt{bx+cx^2}} + \frac{2\sqrt{c}(2cd-be)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{c}}{\sqrt{}}\right.\right.}{(-b)^{3/2}d(cd-be)\sqrt{1+\frac{ex}{d}}\sqrt{bx+cx^2}}$$

Mathematica [C] time = 0.591614, size = 220, normalized size = 0.8

$$\frac{-2icex^{3/2}\sqrt{\frac{b}{c}}\sqrt{\frac{b}{cx}+1}\sqrt{\frac{d}{ex}+1}(be-cd)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right),\frac{cd}{be}\right)+2icex^{3/2}\sqrt{\frac{b}{c}}\sqrt{\frac{b}{cx}+1}\sqrt{\frac{d}{ex}+1}(be-2cd)E\left(i\sinh^{-1}\left(\right.\right.}{b^2d\sqrt{x(b+cx)}\sqrt{d+ex}(be-cd)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*b*c*d*(d + e*x) + (2*I)*Sqrt[b/c]*c*e*(-2*c*d + b*e)*Sqrt[1 + b/(c*x)]*
Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*
e)] - (2*I)*Sqrt[b/c]*c*e*(-(c*d) + b*e)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)
]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])/(b^2*d*(-(c
*d) + b*e)*Sqrt[x*(b + c*x)]*Sqrt[d + e*x])
```

Maple [B] time = 0.317, size = 480, normalized size = 1.8

$$-2\frac{\sqrt{x(cx+b)}}{x(cx+b)(be-cd)cb^2d\sqrt{ex+d}}\left(2b^2d\sqrt{\frac{cx+b}{b}}\sqrt{-\frac{c(ex+d)}{be-cd}}\sqrt{-\frac{cx}{b}}\text{EllipticF}\left(\sqrt{\frac{cx+b}{b}},\sqrt{\frac{be}{be-cd}}\right)ec-2\sqrt{\frac{cx+b}{b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^(3/2)/(e*x+d)^(1/2),x)
```

```
[Out] -2/x*(2*b^2*d*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)
*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*e*c-2*((c*x+b)/b)^(1/2)
*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b
*e/(b*e-c*d))^(1/2))*b*c^2*d^2+((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/
2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^3*e^
2-3*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE
(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^2*c*d*e+2*((c*x+b)/b)^(1/2)*(-(
e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(
b*e-c*d))^(1/2))*b*c^2*d^2+x^2*b*c^2*e^2-2*x^2*c^3*d*e+x*b^2*c*e^2-2*x*c^3*
d^2+b^2*c*d*e-b*c^2*d^2)*(x*(c*x+b))^(1/2)/(c*x+b)/(b*e-c*d)/c/b^2/d/(e*x+d
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx\right)^{\frac{3}{2}}\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(3/2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x)^(3/2)*sqrt(e*x + d)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+bx}\sqrt{ex+d}}{c^2ex^5+b^2dx^2+\left(c^2d+2\,bce\right)x^4+\left(2\,bcd+b^2e\right)x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(3/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x)*sqrt(e*x + d)/(c^2*e*x^5 + b^2*d*x^2 + (c^2*d +
2*b*c*e)*x^4 + (2*b*c*d + b^2*e)*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x(b+cx))^{\frac{3}{2}}\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(3/2)/(e*x+d)**(1/2),x)
```

```
[Out] Integral(1/((x*(b + c*x))**(3/2)*sqrt(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx\right)^{\frac{3}{2}}\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(3/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^2 + b*x)^(3/2)*sqrt(e*x + d)), x)
```

3.419 $\int \frac{1}{(d+ex)^{3/2}\left(bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=370

$$-\frac{2\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(2cd-be)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{(-b)^{3/2}d\sqrt{bx+cx^2}\sqrt{d+ex}(cd-be)}-\frac{4e\sqrt{bx+cx^2}\left(b^2e^2-bcde+c^2d^2\right)}{b^2d^2\sqrt{d+ex}(cd-be)^2}+\frac{4\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\cdots}}{(-b\cdots}$$

```
[Out] (-2*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(b^2*d*(c*d - b*e)*Sqrt[d + e*x]*S
qrt[b*x + c*x^2]) - (4*e*(c^2*d^2 - b*c*d*e + b^2*e^2)*Sqrt[b*x + c*x^2])/(
b^2*d^2*(c*d - b*e)^2*Sqrt[d + e*x]) + (4*Sqrt[c]*(c^2*d^2 - b*c*d*e + b^2*
e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt
[x])/Sqrt[-b]], (b*e)/(c*d)])/((-b)^(3/2)*d^2*(c*d - b*e)^2*Sqrt[1 + (e*x)/
d]*Sqrt[b*x + c*x^2]) - (2*Sqrt[c]*(2*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*
Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)
])/((-b)^(3/2)*d*(c*d - b*e)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.387177, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {740, 834, 843, 715, 112, 110, 117, 116}

$$-\frac{4e\sqrt{bx+cx^2}\left(b^2e^2-bcde+c^2d^2\right)}{b^2d^2\sqrt{d+ex}(cd-be)^2}+\frac{4\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-bcde+c^2d^2\right)E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{(-b)^{3/2}d^2\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}(cd-be)^2}-\frac{2(cx(2cd-be\cdots}{b^2d\sqrt{bx+cx^2}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*(b*x + c*x^2)^(3/2)),x]
```


```
[Out] (-2*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(b^2*d*(c*d - b*e)*Sqrt[d + e*x]*S
qrt[b*x + c*x^2]) - (4*e*(c^2*d^2 - b*c*d*e + b^2*e^2)*Sqrt[b*x + c*x^2])/(
b^2*d^2*(c*d - b*e)^2*Sqrt[d + e*x]) + (4*Sqrt[c]*(c^2*d^2 - b*c*d*e + b^2*
e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt
[x])/Sqrt[-b]], (b*e)/(c*d)])/((-b)^(3/2)*d^2*(c*d - b*e)^2*Sqrt[1 + (e*x)/
d]*Sqrt[b*x + c*x^2]) - (2*Sqrt[c]*(2*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*
Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)
])/((-b)^(3/2)*d*(c*d - b*e)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```


Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
```

```
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\left(bx+cx^2\right)^{3/2}}\,dx = -\frac{2(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)\sqrt{d+ex}\sqrt{bx+cx^2}} - \frac{2\int \frac{\frac{1}{2}be(cd-2be)+\frac{1}{2}ce(2cd-be)x}{(d+ex)^{3/2}\sqrt{bx+cx^2}}dx}{b^2d(cd-be)}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)\sqrt{d+ex}\sqrt{bx+cx^2}} - \frac{4e\left(c^2d^2-bcde+b^2e^2\right)\sqrt{bx+cx^2}}{b^2d^2(cd-be)^2\sqrt{d+ex}} + \frac{4\int \frac{\frac{1}{4}bcde(cd+be)+}{\sqrt{d+}}}{b^2d^2}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)\sqrt{d+ex}\sqrt{bx+cx^2}} - \frac{4e\left(c^2d^2-bcde+b^2e^2\right)\sqrt{bx+cx^2}}{b^2d^2(cd-be)^2\sqrt{d+ex}} - \frac{(c(2cd-be))\int}{b^2d(cd}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)\sqrt{d+ex}\sqrt{bx+cx^2}} - \frac{4e\left(c^2d^2-bcde+b^2e^2\right)\sqrt{bx+cx^2}}{b^2d^2(cd-be)^2\sqrt{d+ex}} - \frac{(c(2cd-be)\sqrt{x}}{b^2d(c}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)\sqrt{d+ex}\sqrt{bx+cx^2}} - \frac{4e\left(c^2d^2-bcde+b^2e^2\right)\sqrt{bx+cx^2}}{b^2d^2(cd-be)^2\sqrt{d+ex}} + \frac{\left(2c\left(c^2d^2-bcd\right.\right.}{b^2}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)\sqrt{d+ex}\sqrt{bx+cx^2}} - \frac{4e\left(c^2d^2-bcde+b^2e^2\right)\sqrt{bx+cx^2}}{b^2d^2(cd-be)^2\sqrt{d+ex}} + \frac{4\sqrt{c}\left(c^2d^2-bc\right.}{(-}$$

Mathematica [C] time = 0.994208, size = 266, normalized size = 0.72

$$\frac{-2icex^{3/2}\sqrt{\frac{b}{c}}\sqrt{\frac{b}{cx}+1}\sqrt{\frac{d}{ex}+1}\left(2b^2e^2-3bcde+c^2d^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right),\frac{cd}{be}\right)+4icex^{3/2}\sqrt{\frac{b}{c}}\sqrt{\frac{b}{cx}+1}\sqrt{\frac{d}{ex}+1}\left(b^2e^2-\right.}{b^2d^2\sqrt{x(b+cx)}\sqrt{d+ex}(cd-be)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*(b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*b*d*(b^2*e^2 + b*c*e^2*x + c^2*d*(d + e*x)) + (4*I)*Sqrt[b/c]*c*e*(c^2*d
^2 - b*c*d*e + b^2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*Ellipti
cE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - (2*I)*Sqrt[b/c]*c*e*(c^2*d^
2 - 3*b*c*d*e + 2*b^2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*Elli
pticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])/(b^2*d^2*(c*d - b*e)^2*Sq
rt[x*(b + c*x)]*Sqrt[d + e*x])
```

Maple [B] time = 0.296, size = 698, normalized size = 1.9

$$-2\,\frac{\sqrt{x\,(cx+b)}}{x\,(cx+b)\,(be-cd)^2\,cb^2d^2\sqrt{ex+d}}\left(2\sqrt{-\frac{cx}{b}}\text{EllipticE}\left(\sqrt{\frac{cx+b}{b}},\sqrt{\frac{be}{be-cd}}\right)\sqrt{\frac{cx+b}{b}}\sqrt{-\frac{c\,(ex+d)}{be-cd}}b^4e^3-4\sqrt{-\frac{cx}{b}}\text{E}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(c*x^2+b*x)^(3/2),x)
```

```
[Out] -2/x*(2*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*(
(c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*b^4*e^3-4*(-c*x/b)^(1/2)*Elli
pticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)*(-(e*x+d)*
c/(b*e-c*d))^(1/2)*b^3*c*d*e^2+4*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2)
```

```
,(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*b^2*
c^2*d^2*e-2*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2
))*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*b*c^3*d^3+(-c*x/b)^(1/2)*
EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)*(-(e*x
+d)*c/(b*e-c*d))^(1/2)*b^3*c*d*e^2-3*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(
1/2),(b*e/(b*e-c*d))^(1/2))*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*
b^2*c^2*d^2*e+2*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^
(1/2))*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*b*c^3*d^3+2*x^2*b^2*c
^2*e^3-2*x^2*b*c^3*d*e^2+2*x^2*c^4*d^2*e+2*x*b^3*c*e^3-x*b^2*c^2*d*e^2-x*b*
c^3*d^2*e+2*x*c^4*d^3+b^3*c*d*e^2-2*b^2*c^2*d^2*e+b*c^3*d^3)*(x*(c*x+b))^(1
/2)/(c*x+b)/(b*e-c*d)^2/c/b^2/d^2/(e*x+d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx\right)^{\frac{3}{2}}(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x)^(3/2)*(e*x + d)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{cx^2+bx}\sqrt{ex+d}}{c^2e^2x^6+b^2d^2x^2+2\left(c^2de+bce^2\right)x^5+\left(c^2d^2+4\,bcde+b^2e^2\right)x^4+2\left(bcd^2+b^2de\right)x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x)*sqrt(e*x + d)/(c^2*e^2*x^6 + b^2*d^2*x^2 + 2*(c^
2*d*e + b*c*e^2)*x^5 + (c^2*d^2 + 4*b*c*d*e + b^2*e^2)*x^4 + 2*(b*c*d^2 + b
^2*d*e)*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x\left(b+cx\right)\right)^{\frac{3}{2}}\left(d+ex\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral(1/((x*(b + c*x))**(3/2)*(d + e*x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2 + bx\right)^{\frac{3}{2}} (ex + d)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^2 + b*x)^(3/2)*(e*x + d)^(3/2)), x)
```

3.420 $\int \frac{1}{(d+ex)^{5/2}\left(bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=478

$$-\frac{4\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}\left(2b^2e^2-3bcde+3c^2d^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{3(-b)^{3/2}d^2\sqrt{bx+cx^2}\sqrt{d+ex}(cd-be)^2}-\frac{4e\sqrt{bx+cx^2}\left(2b^2e^2-3bcde+3c^2d^2\right.}{3b^2d^2(d+ex)^{3/2}(cd-be)^2}$$

```
[Out] (-2*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(b^2*d*(c*d - b*e)*(d + e*x)^(3/2)
*Sqrt[b*x + c*x^2]) - (4*e*(3*c^2*d^2 - 3*b*c*d*e + 2*b^2*e^2)*Sqrt[b*x + c
*x^2])/(3*b^2*d^2*(c*d - b*e)^2*(d + e*x)^(3/2)) - (2*e*(2*c*d - b*e)*(3*c^
2*d^2 - 3*b*c*d*e + 8*b^2*e^2)*Sqrt[b*x + c*x^2])/(3*b^2*d^3*(c*d - b*e)^3*
Sqrt[d + e*x]) + (2*Sqrt[c]*(2*c*d - b*e)*(3*c^2*d^2 - 3*b*c*d*e + 8*b^2*e^
2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x
])/Sqrt[-b]], (b*e)/(c*d)])/(3*(-b)^(3/2)*d^3*(c*d - b*e)^3*Sqrt[1 + (e*x)/
d]*Sqrt[b*x + c*x^2]) - (4*Sqrt[c]*(3*c^2*d^2 - 3*b*c*d*e + 2*b^2*e^2)*Sqrt
[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x]])/
Sqrt[-b]], (b*e)/(c*d)])/(3*(-b)^(3/2)*d^2*(c*d - b*e)^2*Sqrt[d + e*x]*Sqrt
[b*x + c*x^2])
```

Rubi [A] time = 0.58809, antiderivative size = 478, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {740, 834, 843, 715, 112, 110, 117, 116}

$$-\frac{4e\sqrt{bx+cx^2}\left(2b^2e^2-3bcde+3c^2d^2\right)}{3b^2d^2(d+ex)^{3/2}(cd-be)^2}-\frac{2e\sqrt{bx+cx^2}(2cd-be)\left(8b^2e^2-3bcde+3c^2d^2\right)}{3b^2d^3\sqrt{d+ex}(cd-be)^3}-\frac{4\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}\,(2}{3(-b)^{3/2}d^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(5/2)*(b*x + c*x^2)^(3/2)),x]
```


```
[Out] (-2*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(b^2*d*(c*d - b*e)*(d + e*x)^(3/2)
*Sqrt[b*x + c*x^2]) - (4*e*(3*c^2*d^2 - 3*b*c*d*e + 2*b^2*e^2)*Sqrt[b*x + c
*x^2])/(3*b^2*d^2*(c*d - b*e)^2*(d + e*x)^(3/2)) - (2*e*(2*c*d - b*e)*(3*c^
2*d^2 - 3*b*c*d*e + 8*b^2*e^2)*Sqrt[b*x + c*x^2])/(3*b^2*d^3*(c*d - b*e)^3*
Sqrt[d + e*x]) + (2*Sqrt[c]*(2*c*d - b*e)*(3*c^2*d^2 - 3*b*c*d*e + 8*b^2*e^
2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x
])/Sqrt[-b]], (b*e)/(c*d)])/(3*(-b)^(3/2)*d^3*(c*d - b*e)^3*Sqrt[1 + (e*x)/
d]*Sqrt[b*x + c*x^2]) - (4*Sqrt[c]*(3*c^2*d^2 - 3*b*c*d*e + 2*b^2*e^2)*Sqrt
[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x]])/
Sqrt[-b]], (b*e)/(c*d)])/(3*(-b)^(3/2)*d^2*(c*d - b*e)^2*Sqrt[d + e*x]*Sqrt
[b*x + c*x^2])
```


Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
```

```
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{1}{(d+ex)^{5/2}\left(bx+cx^2\right)^{3/2}}\,dx = -\frac{2(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)(d+ex)^{3/2}\sqrt{bx+cx^2}} - \frac{2\int \frac{\frac{1}{2}be(3cd-4be)+\frac{3}{2}ce(2cd-be)x}{(d+ex)^{5/2}\sqrt{bx+cx^2}}\,dx}{b^2d(cd-be)}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)(d+ex)^{3/2}\sqrt{bx+cx^2}} - \frac{4e\left(3c^2d^2-3bcde+2b^2e^2\right)\sqrt{bx+cx^2}}{3b^2d^2(cd-be)^2(d+ex)^{3/2}} + \frac{4\int -\frac{1}{4}}{}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)(d+ex)^{3/2}\sqrt{bx+cx^2}} - \frac{4e\left(3c^2d^2-3bcde+2b^2e^2\right)\sqrt{bx+cx^2}}{3b^2d^2(cd-be)^2(d+ex)^{3/2}} - \frac{2e(2cd}{}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)(d+ex)^{3/2}\sqrt{bx+cx^2}} - \frac{4e\left(3c^2d^2-3bcde+2b^2e^2\right)\sqrt{bx+cx^2}}{3b^2d^2(cd-be)^2(d+ex)^{3/2}} - \frac{2e(2cd}{}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)(d+ex)^{3/2}\sqrt{bx+cx^2}} - \frac{4e\left(3c^2d^2-3bcde+2b^2e^2\right)\sqrt{bx+cx^2}}{3b^2d^2(cd-be)^2(d+ex)^{3/2}} - \frac{2e(2cd}{}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)(d+ex)^{3/2}\sqrt{bx+cx^2}} - \frac{4e\left(3c^2d^2-3bcde+2b^2e^2\right)\sqrt{bx+cx^2}}{3b^2d^2(cd-be)^2(d+ex)^{3/2}} - \frac{2e(2cd}{}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)(d+ex)^{3/2}\sqrt{bx+cx^2}} - \frac{4e\left(3c^2d^2-3bcde+2b^2e^2\right)\sqrt{bx+cx^2}}{3b^2d^2(cd-be)^2(d+ex)^{3/2}} - \frac{2e(2cd}{}$$

Mathematica [C] time = 1.63144, size = 420, normalized size = 0.88

$$-\frac{2\left(b\left(b^2de^3x(b+cx)(cd-be)-5b^2e^3x(b+cx)(d+ex)(be-2cd)+3(b+cx)(d+ex)^2(cd-be)^3+3c^4d^3x(d+ex)^2\right)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(5/2)*(b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(b*(b^2*d*e^3*(c*d - b*e)*x*(b + c*x) - 5*b^2*e^3*(-2*c*d + b*e)*x*(b +
 c*x)*(d + e*x) + 3*c^4*d^3*x*(d + e*x)^2 + 3*(c*d - b*e)^3*(b + c*x)*(d +
e*x)^2) - Sqrt[b/c]*c*(d + e*x)*(Sqrt[b/c]*(6*c^3*d^3 - 9*b*c^2*d^2*e + 19*
b^2*c*d*e^2 - 8*b^3*e^3)*(b + c*x)*(d + e*x) + I*b*e*(6*c^3*d^3 - 9*b*c^2*d
^2*e + 19*b^2*c*d*e^2 - 8*b^3*e^3)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3
/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - I*b*e*(3*c^3*d^3
 - 18*b*c^2*d^2*e + 23*b^2*c*d*e^2 - 8*b^3*e^3)*Sqrt[1 + b/(c*x)]*Sqrt[1 +
d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])))/(3
*b^3*d^3*(c*d - b*e)^3*Sqrt[x*(b + c*x)]*(d + e*x)^(3/2))
```

Maple [B] time = 0.313, size = 1708, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(5/2)/(c*x^2+b*x)^(3/2),x)
```

```
[Out] -2/3*(x*(c*x+b))^(1/2)/x*(-3*b*c^4*d^5+12*x*b^4*c*d*e^4-15*EllipticE(((c*x+
b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^2*c^3*d^4*e*((c*x+b)/b)^(1/2)*(-(e*x+d
)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+4*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e
-c*d))^(1/2))*b^4*c*d^2*e^3*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*
(-c*x/b)^(1/2)-10*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^3*c^
2*d^3*e^2*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+12*
EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^2*c^3*d^4*e*((c*x+b)/b
)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+8*x^3*b^3*c^2*e^5-6*x^3
*c^5*d^3*e^2-12*x^2*c^5*d^4*e-27*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d)
)^(1/2))*b^4*c*d^2*e^3*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x
/b)^(1/2)-27*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^4*c*d*e
^4*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+28*Ellipti
cE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^3*c^2*d^2*e^3*((c*x+b)/b)^(
1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-15*EllipticE(((c*x+b)/b)^(
1/2),(b*e/(b*e-c*d))^(1/2))*x*b^2*c^3*d^3*e^2*((c*x+b)/b)^(1/2)*(-(e*x+d)*c
/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+6*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*
d))^(1/2))*x*b*c^4*d^4*e*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c
*x/b)^(1/2)+4*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^4*c*d*
e^4*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-10*Ellipt
icF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^3*c^2*d^2*e^3*((c*x+b)/b)^
(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+12*EllipticF(((c*x+b)/b)^
(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^2*c^3*d^3*e^2*((c*x+b)/b)^(1/2)*(-(e*x+d)*
c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)-6*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c
*d))^(1/2))*x*b*c^4*d^4*e*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-
c*x/b)^(1/2)+3*x*b*c^4*d^4*e-19*x^3*b^2*c^3*d*e^4+9*x^3*b*c^4*d^2*e^3-7*x^2
*b^3*c^2*d*e^4-20*x^2*b^2*c^3*d^2*e^3+15*x^2*b*c^4*d^3*e^2-26*x*b^3*c^2*d^2
*e^3-6*x*c^5*d^5+28*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^3*
c^2*d^3*e^2*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+9
*x*b^2*c^3*d^3*e^2+8*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b
^5*e^5*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+8*Elli
pticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^5*d*e^4*((c*x+b)/b)^(1/2)*
(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+6*EllipticE(((c*x+b)/b)^(1/2),(
b*e/(b*e-c*d))^(1/2))*b*c^4*d^5*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1
/2)*(-c*x/b)^(1/2)-6*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b*c
^4*d^5*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)+8*x^2*
b^4*c*e^5+3*b^4*c*d^2*e^3-9*b^3*c^2*d^3*e^2+9*b^2*c^3*d^4*e)/b^2/d^3/c/(e*x
+d)^(3/2)/(b*e-c*d)^3/(c*x+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx\right)^{\frac{3}{2}}(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x)^(3/2)*(e*x + d)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+bx}\sqrt{ex+d}}{c^2e^3x^7+b^2d^3x^2+\left(3\,c^2de^2+2\,bce^3\right)x^6+\left(3\,c^2d^2e+6\,bcde^2+b^2e^3\right)x^5+\left(c^2d^3+6\,bcd^2e+3\,b^2de^2\right)x^4+\left(2\,bcd^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x)*sqrt(e*x + d)/(c^2*e^3*x^7 + b^2*d^3*x^2 + (3*c^
2*d*e^2 + 2*b*c*e^3)*x^6 + (3*c^2*d^2*e + 6*b*c*d*e^2 + b^2*e^3)*x^5 + (c^2
*d^3 + 6*b*c*d^2*e + 3*b^2*d*e^2)*x^4 + (2*b*c*d^3 + 3*b^2*d^2*e)*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x(b+cx))^{\frac{3}{2}}(d+ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(5/2)/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral(1/((x*(b + c*x))**(3/2)*(d + e*x)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx\right)^{\frac{3}{2}}(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^2 + b*x)^(3/2)*(e*x + d)^(5/2)), x)
```

3.421 $\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^{5/2}}\,dx$

Optimal. Leaf size=470

$$\frac{8d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\left(-b^2e^2-2bcde+2c^2d^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{3(-b)^{7/2}c^{5/2}\sqrt{bx+cx^2}\sqrt{d+ex}}+\frac{2(d+ex)^{3/2}\left(x(2cd-be\right.}{}$$

```
[Out] (-2*(d + e*x)^(7/2)*(b*d + (2*c*d - b*e)*x))/(3*b^2*(b*x + c*x^2)^(3/2)) +
(2*(d + e*x)^(3/2)*(b*c*d^2*(8*c*d - 11*b*e) + (2*c*d - b*e)*(8*c^2*d^2 - 8
*b*c*d*e - 3*b^2*e^2)*x))/(3*b^4*c*Sqrt[b*x + c*x^2]) - (8*e*(4*c^3*d^3 - 6
*b*c^2*d^2*e + b^3*e^3)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])/(3*b^4*c^2) - (2*(
16*c^4*d^4 - 32*b*c^3*d^3*e + 9*b^2*c^2*d^2*e^2 + 7*b^3*c*d*e^3 - 8*b^4*e^4
)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x]
)/Sqrt[-b]], (b*e)/(c*d)])/(3*(-b)^(7/2)*c^(5/2)*Sqrt[1 + (e*x)/d]*Sqrt[b*x
 + c*x^2]) + (8*d*(c*d - b*e)*(2*c*d - b*e)*(2*c^2*d^2 - 2*b*c*d*e - b^2*e^
2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sq
rt[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*(-b)^(7/2)*c^(5/2)*Sqrt[d + e*x]*Sqrt[b*
x + c*x^2])
```

Rubi [A] time = 0.589989, antiderivative size = 470, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {738, 818, 832, 843, 715, 112, 110, 117, 116}

$$\frac{2(d+ex)^{3/2}\left(x(2cd-be)\left(-3b^2e^2-8bcde+8c^2d^2\right)+bcd^2(8cd-11be)\right)}{3b^4c\sqrt{bx+cx^2}}-\frac{8e\sqrt{bx+cx^2}\sqrt{d+ex}\left(b^3e^3-6bc^2d^2e+4c^3d^3\right)}{3b^4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(9/2)/(b*x + c*x^2)^(5/2),x]
```


```
[Out] (-2*(d + e*x)^(7/2)*(b*d + (2*c*d - b*e)*x))/(3*b^2*(b*x + c*x^2)^(3/2)) +
(2*(d + e*x)^(3/2)*(b*c*d^2*(8*c*d - 11*b*e) + (2*c*d - b*e)*(8*c^2*d^2 - 8
*b*c*d*e - 3*b^2*e^2)*x))/(3*b^4*c*Sqrt[b*x + c*x^2]) - (8*e*(4*c^3*d^3 - 6
*b*c^2*d^2*e + b^3*e^3)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])/(3*b^4*c^2) - (2*(
16*c^4*d^4 - 32*b*c^3*d^3*e + 9*b^2*c^2*d^2*e^2 + 7*b^3*c*d*e^3 - 8*b^4*e^4
)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x]
)/Sqrt[-b]], (b*e)/(c*d)])/(3*(-b)^(7/2)*c^(5/2)*Sqrt[1 + (e*x)/d]*Sqrt[b*x
 + c*x^2]) + (8*d*(c*d - b*e)*(2*c*d - b*e)*(2*c^2*d^2 - 2*b*c*d*e - b^2*e^
2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sq
rt[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*(-b)^(7/2)*c^(5/2)*Sqrt[d + e*x]*Sqrt[b*
x + c*x^2])
```


Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
```

```
FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{(d+ex)^{9/2}}{\left(bx+cx^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{7/2}(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}} - \frac{2\int \frac{(d+ex)^{5/2}\left(\frac{1}{2}d(8cd-11be)-\frac{3}{2}e(2cd-be)x\right)}{\left(bx+cx^2\right)^{3/2}}\,dx}{3b^2}$$

$$= -\frac{2(d+ex)^{7/2}(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}} + \frac{2(d+ex)^{3/2}\left(bcd^2(8cd-11be)+(2cd-be)\left(8c^2d^2-8bcde-3b^2e\right.\right.}{3b^4c\sqrt{bx+cx^2}}$$

$$= -\frac{2(d+ex)^{7/2}(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}} + \frac{2(d+ex)^{3/2}\left(bcd^2(8cd-11be)+(2cd-be)\left(8c^2d^2-8bcde-3b^2e\right.\right.}{3b^4c\sqrt{bx+cx^2}}$$

$$= -\frac{2(d+ex)^{7/2}(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}} + \frac{2(d+ex)^{3/2}\left(bcd^2(8cd-11be)+(2cd-be)\left(8c^2d^2-8bcde-3b^2e\right.\right.}{3b^4c\sqrt{bx+cx^2}}$$

$$= -\frac{2(d+ex)^{7/2}(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}} + \frac{2(d+ex)^{3/2}\left(bcd^2(8cd-11be)+(2cd-be)\left(8c^2d^2-8bcde-3b^2e\right.\right.}{3b^4c\sqrt{bx+cx^2}}$$

$$= -\frac{2(d+ex)^{7/2}(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}} + \frac{2(d+ex)^{3/2}\left(bcd^2(8cd-11be)+(2cd-be)\left(8c^2d^2-8bcde-3b^2e\right.\right.}{3b^4c\sqrt{bx+cx^2}}$$

$$= -\frac{2(d+ex)^{7/2}(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}} + \frac{2(d+ex)^{3/2}\left(bcd^2(8cd-11be)+(2cd-be)\left(8c^2d^2-8bcde-3b^2e\right.\right.}{3b^4c\sqrt{bx+cx^2}}$$

Mathematica [C] time = 2.71415, size = 451, normalized size = 0.96

$$\frac{2\left(b(d+ex)\left(c^2d^3x(b+cx)^2(8cd-13be)-bc^2d^4(b+cx)^2+bx^2(cd-be)^4+x^2(b+cx)(cd-be)^3(5be+8cd)\right)-x\sqrt{\frac{b}{c}}(b+c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(9/2)/(b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(b*(d + e*x)*(b*(c*d - b*e)^4*x^2 + (c*d - b*e)^3*(8*c*d + 5*b*e)*x^2*(b
 + c*x) - b*c^2*d^4*(b + c*x)^2 + c^2*d^3*(8*c*d - 13*b*e)*x*(b + c*x)^2) -
 Sqrt[b/c]*x*(b + c*x)*(Sqrt[b/c]*(16*c^4*d^4 - 32*b*c^3*d^3*e + 9*b^2*c^2*
d^2*e^2 + 7*b^3*c*d*e^3 - 8*b^4*e^4)*(b + c*x)*(d + e*x) + I*b*e*(16*c^4*d^
4 - 32*b*c^3*d^3*e + 9*b^2*c^2*d^2*e^2 + 7*b^3*c*d*e^3 - 8*b^4*e^4)*Sqrt[1
+ b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]]
, (c*d)/(b*e)] - I*b*e*(8*c^4*d^4 - 17*b*c^3*d^3*e + 6*b^2*c^2*d^2*e^2 + 11
```

```
*b^3*c*d*e^3 - 8*b^4*e^4)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*Ellip
ticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)]])))/(3*b^5*c^2*(x*(b + c*x))
^(3/2)*Sqrt[d + e*x])
```

Maple [B] time = 0.349, size = 2086, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(9/2)/(c*x^2+b*x)^(5/2),x)
```

```
[Out] -2/3*(-3*x^2*b^4*c^3*d^2*e^3-2*x^2*b^3*c^4*d^3*e^2+43*x^2*b^2*c^5*d^4*e+14*
x*b^3*c^4*d^4*e-7*x^4*b^3*c^4*d*e^4-9*x^4*b^2*c^5*d^2*e^3+8*((c*x+b)/b)^(1/
2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),
(b*e/(b*e-c*d))^(1/2))*x^2*b^6*c*e^5+8*x^3*b*c^6*d^4*e+2*x^3*b^4*c^3*d*e^4-
22*x^3*b^3*c^4*d^2*e^3+40*x^3*b^2*c^5*d^3*e^2+4*x^2*b^5*c^2*d*e^4-2*((c*x+b
)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b
)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^4*c^3*d^2*e^3-4*((c*x+b)/b)^(1/2)*(-(e
*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b
*e-c*d))^(1/2))*x*b^5*c^2*d^2*e^3-24*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d
))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*
x*b^4*c^3*d^3*e^2+4*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)
^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^5*c^2*d*e^4
-4*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(
((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^4*c^3*d^2*e^3-24*((c*x+b)/b)
^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1
/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^3*c^4*d^3*e^2+40*((c*x+b)/b)^(1/2)*(-(e*x+
d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-
c*d))^(1/2))*x^2*b^2*c^5*d^4*e-16*x^4*c^7*d^4*e+4*x^3*b^5*c^2*e^5+5*x^4*b^4
*c^3*e^5-6*x*b^2*c^5*d^5-24*x^2*b*c^6*d^5+32*x^4*b*c^6*d^3*e^2-16*x^3*c^7*d
^5-15*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*Ellipti
cE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^6*c*d*e^4-48*((c*x+b)/b)^(1
/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2)
,(b*e/(b*e-c*d))^(1/2))*x*b^3*c^4*d^4*e+4*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*
e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1
/2))*x*b^6*c*d*e^4-15*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/
b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^5*c^2*d*e
^4+40*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*Ellipti
cF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^3*c^4*d^4*e+41*((c*x+b)/b)^
(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/
2),(b*e/(b*e-c*d))^(1/2))*x^2*b^3*c^4*d^3*e^2-48*((c*x+b)/b)^(1/2)*(-(e*x+d
)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c
*d))^(1/2))*x^2*b^2*c^5*d^4*e-2*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1
/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^5
*c^2*d^2*e^3+41*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/
2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^4*c^3*d^3*e^2+8*(
(c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*
x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^7*e^5+b^3*c^4*d^5+16*((c*x+b)/b)^(
1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2
),(b*e/(b*e-c*d))^(1/2))*x^2*b*c^6*d^5-16*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*
e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1
/2))*x^2*b*c^6*d^5+16*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/
b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^2*c^5*d^5-1
6*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF((
(c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^2*c^5*d^5)/x^2*(x*(c*x+b))^(1/2
)/b^4/(c*x+b)^2/c^4/(e*x+d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{9}{2}}}{(cx^2+bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(9/2)/(c*x^2 + b*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e^4x^4+4\,de^3x^3+6\,d^2e^2x^2+4\,d^3ex+d^4\right)\sqrt{cx^2+bx}\sqrt{ex+d}}{c^3x^6+3\,bc^2x^5+3\,b^2cx^4+b^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4)*sqrt(c*x
^2 + b*x)*sqrt(e*x + d)/(c^3*x^6 + 3*b*c^2*x^5 + 3*b^2*c*x^4 + b^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(9/2)/(c*x**2+b*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{9}{2}}}{(cx^2+bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(9/2)/(c*x^2 + b*x)^(5/2), x)
```

3.422 $\int \frac{(d+ex)^{7/2}}{\left(bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=383

$$\frac{2d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\left(-b^2e^2-16bcde+16c^2d^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{3(-b)^{7/2}c^{3/2}\sqrt{bx+cx^2}\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(x(2cd-be)\left(-b^2\right.\right.}{3b}$$

```
[Out] (-2*(d + e*x)^(5/2)*(b*d + (2*c*d - b*e)*x))/(3*b^2*(b*x + c*x^2)^(3/2)) +
(2*Sqrt[d + e*x]*(b*c*d^2*(8*c*d - 9*b*e) + (2*c*d - b*e)*(8*c^2*d^2 - 8*b*
c*d*e - b^2*e^2)*x))/(3*b^4*c*Sqrt[b*x + c*x^2]) - (4*(2*c*d - b*e)*(4*c^2*
d^2 - 4*b*c*d*e - b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*Elliptic
E[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*(-b)^(7/2)*c^(3/2)*S
qrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) + (2*d*(c*d - b*e)*(16*c^2*d^2 - 16*b*c
*d*e - b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSi
n[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*(-b)^(7/2)*c^(3/2)*Sqrt[d +
 e*x]*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.391554, antiderivative size = 383, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {738, 818, 843, 715, 112, 110, 117, 116}

$$\frac{2\sqrt{d+ex}\left(x(2cd-be)\left(-b^2e^2-8bcde+8c^2d^2\right)+bcd^2(8cd-9be)\right)}{3b^4c\sqrt{bx+cx^2}}+\frac{2d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\left(-b^2e^2-16bcde\right.}{3(-b)^{7/2}c^{3/2}\sqrt{bx+cx^2}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/(b*x + c*x^2)^(5/2),x]
```


```
[Out] (-2*(d + e*x)^(5/2)*(b*d + (2*c*d - b*e)*x))/(3*b^2*(b*x + c*x^2)^(3/2)) +
(2*Sqrt[d + e*x]*(b*c*d^2*(8*c*d - 9*b*e) + (2*c*d - b*e)*(8*c^2*d^2 - 8*b*
c*d*e - b^2*e^2)*x))/(3*b^4*c*Sqrt[b*x + c*x^2]) - (4*(2*c*d - b*e)*(4*c^2*
d^2 - 4*b*c*d*e - b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*Elliptic
E[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*(-b)^(7/2)*c^(3/2)*S
qrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) + (2*d*(c*d - b*e)*(16*c^2*d^2 - 16*b*c
*d*e - b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSi
n[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*(-b)^(7/2)*c^(3/2)*Sqrt[d +
 e*x]*Sqrt[b*x + c*x^2])
```


Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
```

```
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{(d+ex)^{7/2}}{\left(bx+cx^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{5/2}(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}} - \frac{2\int \frac{(d+ex)^{3/2}\left(\frac{1}{2}d(8cd-9be)-\frac{1}{2}e(2cd-be)x\right)}{\left(bx+cx^2\right)^{3/2}}dx}{3b^2}$$

$$= -\frac{2(d+ex)^{5/2}(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}\left(bcd^2(8cd-9be)+(2cd-be)\left(8c^2d^2-8bcde-b^2e^2\right.\right.}{3b^4c\sqrt{bx+cx^2}}$$

$$= -\frac{2(d+ex)^{5/2}(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}\left(bcd^2(8cd-9be)+(2cd-be)\left(8c^2d^2-8bcde-b^2e^2\right.\right.}{3b^4c\sqrt{bx+cx^2}}$$

$$= -\frac{2(d+ex)^{5/2}(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}\left(bcd^2(8cd-9be)+(2cd-be)\left(8c^2d^2-8bcde-b^2e^2\right.\right.}{3b^4c\sqrt{bx+cx^2}}$$

$$= -\frac{2(d+ex)^{5/2}(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}\left(bcd^2(8cd-9be)+(2cd-be)\left(8c^2d^2-8bcde-b^2e^2\right.\right.}{3b^4c\sqrt{bx+cx^2}}$$

$$= -\frac{2(d+ex)^{5/2}(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}\left(bcd^2(8cd-9be)+(2cd-be)\left(8c^2d^2-8bcde-b^2e^2\right.\right.}{3b^4c\sqrt{bx+cx^2}}$$

Mathematica [C] time = 1.993, size = 405, normalized size = 1.06

$$2\left(b(d+ex)\left(2cd^2x(b+cx)^2(4cd-5be)-bcd^3(b+cx)^2+bx^2(cd-be)^3+2x^2(b+cx)(cd-be)^2(be+4cd)\right)-x\sqrt{\frac{b}{c}}(b+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)/(b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(b*(d + e*x)*(b*(c*d - b*e)^3*x^2 + 2*(c*d - b*e)^2*(4*c*d + b*e)*x^2*(b
 + c*x) - b*c*d^3*(b + c*x)^2 + 2*c*d^2*(4*c*d - 5*b*e)*x*(b + c*x)^2) - Sq
rt[b/c]*x*(b + c*x)*(2*Sqrt[b/c]*(8*c^3*d^3 - 12*b*c^2*d^2*e + 2*b^2*c*d*e^
2 + b^3*e^3)*(b + c*x)*(d + e*x) + (2*I)*b*e*(8*c^3*d^3 - 12*b*c^2*d^2*e +
2*b^2*c*d*e^2 + b^3*e^3)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*Ellipt
icE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - I*b*e*(8*c^3*d^3 - 13*b*c^
2*d^2*e + 3*b^2*c*d*e^2 + 2*b^3*e^3)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^
(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])))/(3*b^5*c*(x*(
b + c*x))^(3/2)*Sqrt[d + e*x])
```

Maple [B] time = 0.318, size = 1687, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/(c*x^2+b*x)^(5/2),x)
```

```
[Out] 2/3*(2*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*Ellipt
icE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^4*c^2*d*e^3+2*((c*x+b)/b
```

```
)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(
1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^5*c*e^4+2*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(
b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^
(1/2))*x*b^6*e^4-b^3*c^3*d^4+9*x^3*b^3*c^3*d*e^3-33*x^3*b^2*c^4*d^2*e^2+x^2
*b^4*c^2*d*e^3-3*x^2*b^3*c^3*d^2*e^2-31*x^2*b^2*c^4*d^3*e-11*x*b^3*c^3*d^3*
e-24*x^4*b*c^5*d^2*e^2-28*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-
c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^3*c^3
*d^2*e^2+40*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*E
llipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^2*c^4*d^3*e+((c*x+b
)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b
)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^4*c^2*d*e^3+15*((c*x+b)/b)^(1/2)*(-(e*
x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*
e-c*d))^(1/2))*x^2*b^3*c^3*d^2*e^2-32*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*
d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))
*x^2*b^2*c^4*d^3*e+2*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b
)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^5*c*d*e^3-28
*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((
c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^4*c^2*d^2*e^2+40*((c*x+b)/b)^(1/
2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),
(b*e/(b*e-c*d))^(1/2))*x*b^3*c^3*d^3*e+((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c
*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2)
)*x*b^5*c*d*e^3+15*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^
(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^4*c^2*d^2*e^2-
32*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(
((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^3*c^3*d^3*e+4*x^4*b^2*c^4*d*e^
3+2*x^4*b^3*c^3*e^4+24*x^2*b*c^5*d^4+x^3*b^4*c^2*e^4+6*x*b^2*c^4*d^4+16*x^4
*c^6*d^3*e+16*x^3*c^6*d^4-16*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)
*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b*c^
5*d^4+16*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*Elli
pticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b*c^5*d^4-16*((c*x+b)/b)
^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1
/2),(b*e/(b*e-c*d))^(1/2))*x*b^2*c^4*d^4+16*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(
b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^
(1/2))*x*b^2*c^4*d^4)/x^2*(x*(c*x+b))^(1/2)/b^4/(c*x+b)^2/c^3/(e*x+d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{7}{2}}}{\left(cx^2+bx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(7/2)/(c*x^2 + b*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\sqrt{cx^2+bx}\sqrt{ex+d}}{c^3x^6+3\,bc^2x^5+3\,b^2cx^4+b^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(c*x^2 + b*x)*sqrt(e
*x + d)/(c^3*x^6 + 3*b*c^2*x^5 + 3*b^2*c*x^4 + b^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/(c*x**2+b*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{7}{2}}}{(cx^2+bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(7/2)/(c*x^2 + b*x)^(5/2), x)
```

3.423 $\int \frac{(d+ex)^{5/2}}{\left(bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=343

$$\frac{16d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{3(-b)^{7/2}\sqrt{c}\sqrt{bx+cx^2}\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(x\left(b^2e^2-16bcde+16c^2d^2\right)+bd(8cd\right.}{3b^4\sqrt{bx+cx^2}}$$

```
[Out] (-2*(d + e*x)^(3/2)*(b*d + (2*c*d - b*e)*x))/(3*b^2*(b*x + c*x^2)^(3/2)) +
(2*Sqrt[d + e*x]*(b*d*(8*c*d - 7*b*e) + (16*c^2*d^2 - 16*b*c*d*e + b^2*e^2)
*x))/(3*b^4*Sqrt[b*x + c*x^2]) - (2*(16*c^2*d^2 - 16*b*c*d*e + b^2*e^2)*Sqr
t[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqr
t[-b]], (b*e)/(c*d)])/(3*(-b)^(7/2)*Sqrt[c]*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*
x^2]) + (16*d*(c*d - b*e)*(2*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 +
(e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*(-b
)^(7/2)*Sqrt[c]*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.356276, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {738, 820, 843, 715, 112, 110, 117, 116}

$$\frac{2\sqrt{d+ex}\left(x\left(b^2e^2-16bcde+16c^2d^2\right)+bd(8cd-7be)\right)}{3b^4\sqrt{bx+cx^2}}-\frac{2\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-16bcde+16c^2d^2\right)E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)\right.}{3(-b)^{7/2}\sqrt{c}\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(b*x + c*x^2)^(5/2),x]
```


```
[Out] (-2*(d + e*x)^(3/2)*(b*d + (2*c*d - b*e)*x))/(3*b^2*(b*x + c*x^2)^(3/2)) +
(2*Sqrt[d + e*x]*(b*d*(8*c*d - 7*b*e) + (16*c^2*d^2 - 16*b*c*d*e + b^2*e^2)
*x))/(3*b^4*Sqrt[b*x + c*x^2]) - (2*(16*c^2*d^2 - 16*b*c*d*e + b^2*e^2)*Sqr
t[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqr
t[-b]], (b*e)/(c*d)])/(3*(-b)^(7/2)*Sqrt[c]*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*
x^2]) + (16*d*(c*d - b*e)*(2*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 +
(e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*(-b
)^(7/2)*Sqrt[c]*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```


Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*
(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p +
 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*Simp[g*
```

```
(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(
m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (I
ntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\left(bx+cx^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}} - \frac{2\int \frac{\sqrt{d+ex}\left(\frac{1}{2}d(8cd-7be)+\frac{1}{2}e(2cd-be)x\right)}{\left(bx+cx^2\right)^{3/2}}dx}{3b^2}$$

$$= -\frac{2(d+ex)^{3/2}(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}\left(bd(8cd-7be)+\left(16c^2d^2-16bcde+b^2e^2\right)x\right)}{3b^4\sqrt{bx+cx^2}} + \frac{4\int \frac{-\frac{1}{4}\cdots}{\cdots}}{\cdots}$$

$$= -\frac{2(d+ex)^{3/2}(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}\left(bd(8cd-7be)+\left(16c^2d^2-16bcde+b^2e^2\right)x\right)}{3b^4\sqrt{bx+cx^2}} + \frac{(8d(c\cdots}{\cdots}$$

$$= -\frac{2(d+ex)^{3/2}(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}\left(bd(8cd-7be)+\left(16c^2d^2-16bcde+b^2e^2\right)x\right)}{3b^4\sqrt{bx+cx^2}} + \frac{\left(8d(c\cdots\right.}{\cdots}$$

$$= -\frac{2(d+ex)^{3/2}(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}\left(bd(8cd-7be)+\left(16c^2d^2-16bcde+b^2e^2\right)x\right)}{3b^4\sqrt{bx+cx^2}} - \frac{\left(\left(16c^2\cdots\right.\right.}{\cdots}$$

$$= -\frac{2(d+ex)^{3/2}(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}\left(bd(8cd-7be)+\left(16c^2d^2-16bcde+b^2e^2\right)x\right)}{3b^4\sqrt{bx+cx^2}} - \frac{2\left(16c\cdots\right.}{\cdots}$$

Mathematica [C] time = 1.10205, size = 353, normalized size = 1.03

$$\frac{2\left(b(d+ex)\left(b^2cx\left(6d^2-25dex+e^2x^2\right)+b^3\left(-\left(d^2+7dex-2e^2x^2\right)\right)+8bc^2dx^2(3d-2ex)+16c^3d^2x^3\right)-x\sqrt{\frac{b}{c}}(b+cx)\left(-i\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(b*(d + e*x)*(16*c^3*d^2*x^3 + 8*b*c^2*d*x^2*(3*d - 2*e*x) - b^3*(d^2 +
7*d*e*x - 2*e^2*x^2) + b^2*c*x*(6*d^2 - 25*d*e*x + e^2*x^2)) - Sqrt[b/c]*x*
(b + c*x)*(Sqrt[b/c]*(16*c^2*d^2 - 16*b*c*d*e + b^2*e^2)*(b + c*x)*(d + e*x
) + I*b*e*(16*c^2*d^2 - 16*b*c*d*e + b^2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/
(e*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - I*b*e
*(8*c^2*d^2 - 9*b*c*d*e + b^2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3
/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])))/(3*b^5*(x*(b +
c*x))^(3/2)*Sqrt[d + e*x])
```

Maple [B] time = 0.336, size = 1318, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(c*x^2+b*x)^(5/2),x)
```

```
[Out] 2/3*(-17*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*Elli
pticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^4*c*d*e^2+32*((c*x+b)/b)
```

```
^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1
/2),(b*e/(b*e-c*d))^(1/2))*x*b^3*c^2*d^2*e+8*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/
(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))
^(1/2))*x*b^4*c*d*e^2-24*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c
*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^3*c^2*d^
2*e+8*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*Ellipti
cF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^3*c^2*d*e^2-24*((c*x+b)/b
)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(
1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^2*c^3*d^2*e-24*x^3*b^2*c^3*d*e^2-5*x^2*b^
3*c^2*d*e^2-19*x^2*b^2*c^3*d^2*e-16*x^4*b*c^4*d*e^2+8*x^3*b*c^4*d^2*e+x^4*b
^2*c^3*e^3+2*x^3*b^3*c^2*e^3+24*x^2*b*c^4*d^3+16*x^4*c^5*d^2*e+6*x*b^2*c^3*
d^3-17*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*Ellipt
icE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^3*c^2*d*e^2+32*((c*x+b)/
b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^
(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^2*c^3*d^2*e-16*((c*x+b)/b)^(1/2)*(-(e*x+
d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-
c*d))^(1/2))*x^2*b*c^4*d^3+16*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2
)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b*c
^4*d^3-16*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*Ell
ipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^2*c^3*d^3+16*((c*x+b)/b
)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(
1/2),(b*e/(b*e-c*d))^(1/2))*x*b^2*c^3*d^3-8*x*b^3*c^2*d^2*e+((c*x+b)/b)^(1/
2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),
(b*e/(b*e-c*d))^(1/2))*x*b^5*e^3-b^3*c^2*d^3+16*x^3*c^5*d^3+((c*x+b)/b)^(1/
2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),
(b*e/(b*e-c*d))^(1/2))*x^2*b^4*c*e^3)/x^2*(x*(c*x+b))^(1/2)/b^4/c^2/(c*x+b)
^2/(e*x+d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\left(cx^2+bx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/(c*x^2 + b*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e^2x^2+2\,dex+d^2\right)\sqrt{cx^2+bx}\sqrt{ex+d}}{c^3x^6+3\,bc^2x^5+3\,b^2cx^4+b^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*sqrt(c*x^2 + b*x)*sqrt(e*x + d)/(c^3*x^6
 + 3*b*c^2*x^5 + 3*b^2*c*x^4 + b^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(c*x**2+b*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{(cx^2+bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(5/2)/(c*x^2 + b*x)^(5/2), x)
```

3.424 $\int \frac{(d+ex)^{3/2}}{\left(bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=344

$$\frac{2\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(4cd-3be)(4cd-be)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{3(-b)^{7/2}\sqrt{c}\sqrt{bx+cx^2}\sqrt{d+ex}}-\frac{2\sqrt{d+ex}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}+\frac{2\sqrt{d+ex}(8c$$

```
[Out] (-2*Sqrt[d + e*x]*(b*d + (2*c*d - b*e)*x))/(3*b^2*(b*x + c*x^2)^(3/2)) + (2
*Sqrt[d + e*x]*(b*(8*c*d - 5*b*e)*(c*d - b*e) + 8*c*(c*d - b*e)*(2*c*d - b*
e)*x))/(3*b^4*(c*d - b*e)*Sqrt[b*x + c*x^2]) - (16*Sqrt[c]*(2*c*d - b*e)*Sq
rt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sq
rt[-b]], (b*e)/(c*d)])/(3*(-b)^(7/2)*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) +
 (2*(4*c*d - 3*b*e)*(4*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/
d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*(-b)^(7/2
)*Sqrt[c]*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.416195, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {738, 822, 843, 715, 112, 110, 117, 116}

$$-\frac{2\sqrt{d+ex}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}+\frac{2\sqrt{d+ex}(8cx(2cd-be)(cd-be)+b(8cd-5be)(cd-be))}{3b^4\sqrt{bx+cx^2}(cd-be)}+\frac{2\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(4cd}{3(-b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(b*x + c*x^2)^(5/2),x]
```


```
[Out] (-2*Sqrt[d + e*x]*(b*d + (2*c*d - b*e)*x))/(3*b^2*(b*x + c*x^2)^(3/2)) + (2
*Sqrt[d + e*x]*(b*(8*c*d - 5*b*e)*(c*d - b*e) + 8*c*(c*d - b*e)*(2*c*d - b*
e)*x))/(3*b^4*(c*d - b*e)*Sqrt[b*x + c*x^2]) - (16*Sqrt[c]*(2*c*d - b*e)*Sq
rt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sq
rt[-b]], (b*e)/(c*d)])/(3*(-b)^(7/2)*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) +
 (2*(4*c*d - 3*b*e)*(4*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/
d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*(-b)^(7/2
)*Sqrt[c]*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```


Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
```

```
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\left(bx+cx^2\right)^{5/2}}\,dx = -\frac{2\sqrt{d+ex}(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}} - \frac{2\int \frac{\frac{1}{2}d(8cd-5be)+\frac{3}{2}e(2cd-be)x}{\sqrt{d+ex}\left(bx+cx^2\right)^{3/2}}dx}{3b^2}$$

$$= -\frac{2\sqrt{d+ex}(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}(b(8cd-5be)(cd-be)+8c(cd-be)(2cd-be)x)}{3b^4(cd-be)\sqrt{bx+cx^2}} + \frac{4\int}{}$$

$$= -\frac{2\sqrt{d+ex}(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}(b(8cd-5be)(cd-be)+8c(cd-be)(2cd-be)x)}{3b^4(cd-be)\sqrt{bx+cx^2}} - \frac{(8c}{}$$

$$= -\frac{2\sqrt{d+ex}(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}(b(8cd-5be)(cd-be)+8c(cd-be)(2cd-be)x)}{3b^4(cd-be)\sqrt{bx+cx^2}} - \frac{(8c}{}$$

$$= -\frac{2\sqrt{d+ex}(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}(b(8cd-5be)(cd-be)+8c(cd-be)(2cd-be)x)}{3b^4(cd-be)\sqrt{bx+cx^2}} - \frac{(8c}{}$$

$$= -\frac{2\sqrt{d+ex}(bd+(2cd-be)x)}{3b^2\left(bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}(b(8cd-5be)(cd-be)+8c(cd-be)(2cd-be)x)}{3b^4(cd-be)\sqrt{bx+cx^2}} - \frac{16}{}$$

Mathematica [C] time = 1.15048, size = 290, normalized size = 0.84

$$\frac{2\left(icex^{5/2}\sqrt{\frac{b}{c}}\sqrt{\frac{b}{cx}+1}\sqrt{\frac{d}{ex}+1}(8cd-5be)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right),\frac{cd}{be}\right)+\frac{(d+ex)\left(b^2cx(6d-13ex)+b^3(-(d+4ex))-8bc^2x^2(ex-3d)+16c^3d\right.}{b+cx}\right.}{3b^4x\sqrt{x(b+cx)}\sqrt{d+e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(-8*(2*c*d - b*e)*x*(b + c*x)*(d + e*x) + ((d + e*x)*(16*c^3*d*x^3 + b^2
*c*x*(6*d - 13*e*x) - 8*b*c^2*x^2*(-3*d + e*x) - b^3*(d + 4*e*x)))/(b + c*x
) + (8*I)*Sqrt[b/c]*c*e*(-2*c*d + b*e)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*
x^(5/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] + I*Sqrt[b/c]*
c*e*(8*c*d - 5*b*e)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(5/2)*EllipticF[I
*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)]))/(3*b^4*x*Sqrt[x*(b + c*x)]*Sqrt
[d + e*x])
```

Maple [B] time = 0.313, size = 1099, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*x^2+b*x)^(5/2),x)
```

```
[Out] 2/3*(3*x^2*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*El
lipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^3*c*e^2-16*x^2*((c*x+b)/
b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^
```

```
(1/2),(b*e/(b*e-c*d))^(1/2))*b^2*c^2*d*e+16*x^2*((c*x+b)/b)^(1/2)*(-(e*x+d)
*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*
d))^(1/2))*b*c^3*d^2-8*x^2*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(
-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^3*c*e^2+
24*x^2*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*Ellipt
icE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^2*c^2*d*e-16*x^2*((c*x+b)/b)
^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1
/2),(b*e/(b*e-c*d))^(1/2))*b*c^3*d^2+3*x*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e
-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/
2))*b^4*e^2-16*x*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1
/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^3*c*d*e+16*x*((c*x
+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)
/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^2*c^2*d^2-8*x*((c*x+b)/b)^(1/2)*(-(e*x+d
)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c
*d))^(1/2))*b^4*e^2+24*x*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c
*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^3*c*d*e-16
*x*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(
((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*b^2*c^2*d^2-8*x^4*b*c^3*e^2+16*x^4
*c^4*d*e-13*x^3*b^2*c^2*e^2+16*x^3*b*c^3*d*e+16*x^3*c^4*d^2-4*x^2*b^3*c*e^2
-7*x^2*b^2*c^2*d*e+24*x^2*b*c^3*d^2-5*x*b^3*c*d*e+6*x*b^2*c^2*d^2-b^3*c*d^2
)/x^2*(x*(c*x+b))^(1/2)/b^4/c/(c*x+b)^2/(e*x+d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cx^2+bx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(c*x^2 + b*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^2+bx}(ex+d)^{\frac{3}{2}}}{c^3x^6+3\,bc^2x^5+3\,b^2cx^4+b^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x)*(e*x + d)^(3/2)/(c^3*x^6 + 3*b*c^2*x^5 + 3*b^2*c
*x^4 + b^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+b*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cx^2+bx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)/(c*x^2 + b*x)^(5/2), x)
```

3.425 $\int \frac{\sqrt{d+ex}}{\left(bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=359

$$\frac{16\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(2cd-be)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{3(-b)^{7/2}\sqrt{bx+cx^2}\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(cx\left(b^2e^2-16bcde+16c^2d^2\right)+b(cd-be)(8c\right.}{3b^4d\sqrt{bx+cx^2}(cd-be)}$$

```
[Out] (-2*(b + 2*c*x)*Sqrt[d + e*x])/(3*b^2*(b*x + c*x^2)^(3/2)) + (2*Sqrt[d + e*
x]*(b*(c*d - b*e)*(8*c*d - b*e) + c*(16*c^2*d^2 - 16*b*c*d*e + b^2*e^2)*x))
/(3*b^4*d*(c*d - b*e)*Sqrt[b*x + c*x^2]) - (2*Sqrt[c]*(16*c^2*d^2 - 16*b*c*
d*e + b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sq
rt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*(-b)^(7/2)*d*(c*d - b*e)*Sqrt[1
+ (e*x)/d]*Sqrt[b*x + c*x^2]) + (16*Sqrt[c]*(2*c*d - b*e)*Sqrt[x]*Sqrt[1 +
(c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b
*e)/(c*d)])/(3*(-b)^(7/2)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.398242, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {736, 822, 843, 715, 112, 110, 117, 116}

$$\frac{2\sqrt{d+ex}\left(cx\left(b^2e^2-16bcde+16c^2d^2\right)+b(cd-be)(8cd-be)\right)}{3b^4d\sqrt{bx+cx^2}(cd-be)}-\frac{2\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-16bcde+16c^2d^2\right)E\left(\sin\right.}{3(-b)^{7/2}d\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}(cd-be)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(b*x + c*x^2)^(5/2),x]
```


```
[Out] (-2*(b + 2*c*x)*Sqrt[d + e*x])/(3*b^2*(b*x + c*x^2)^(3/2)) + (2*Sqrt[d + e*
x]*(b*(c*d - b*e)*(8*c*d - b*e) + c*(16*c^2*d^2 - 16*b*c*d*e + b^2*e^2)*x))
/(3*b^4*d*(c*d - b*e)*Sqrt[b*x + c*x^2]) - (2*Sqrt[c]*(16*c^2*d^2 - 16*b*c*
d*e + b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sq
rt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*(-b)^(7/2)*d*(c*d - b*e)*Sqrt[1
+ (e*x)/d]*Sqrt[b*x + c*x^2]) + (16*Sqrt[c]*(2*c*d - b*e)*Sqrt[x]*Sqrt[1 +
(c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b
*e)/(c*d)])/(3*(-b)^(7/2)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```


Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] - Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)
*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p + 1
), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 0] && (L
tQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c,
d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
```

```
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\left(bx+cx^2\right)^{5/2}}\,dx = -\frac{2(b+2cx)\sqrt{d+ex}}{3b^2\left(bx+cx^2\right)^{3/2}} + \frac{2\int \frac{-4cd+\frac{be}{2}-3cex}{\sqrt{d+ex}\left(bx+cx^2\right)^{3/2}}dx}{3b^2}$$

$$= -\frac{2(b+2cx)\sqrt{d+ex}}{3b^2\left(bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}\left(b(cd-be)(8cd-be)+c\left(16c^2d^2-16bcde+b^2e^2\right)x\right)}{3b^4d(cd-be)\sqrt{bx+cx^2}} - \frac{4\int \frac{\frac{1}{4}bcde(8}{}}{}$$

$$= -\frac{2(b+2cx)\sqrt{d+ex}}{3b^2\left(bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}\left(b(cd-be)(8cd-be)+c\left(16c^2d^2-16bcde+b^2e^2\right)x\right)}{3b^4d(cd-be)\sqrt{bx+cx^2}} + \frac{(8c(2cd-}{}$$

$$= -\frac{2(b+2cx)\sqrt{d+ex}}{3b^2\left(bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}\left(b(cd-be)(8cd-be)+c\left(16c^2d^2-16bcde+b^2e^2\right)x\right)}{3b^4d(cd-be)\sqrt{bx+cx^2}} + \frac{\left(8c(2cd-\right.}{}$$

$$= -\frac{2(b+2cx)\sqrt{d+ex}}{3b^2\left(bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}\left(b(cd-be)(8cd-be)+c\left(16c^2d^2-16bcde+b^2e^2\right)x\right)}{3b^4d(cd-be)\sqrt{bx+cx^2}} - \frac{\left(c\left(16c^2d^2\right.\right.}{}$$

$$= -\frac{2(b+2cx)\sqrt{d+ex}}{3b^2\left(bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}\left(b(cd-be)(8cd-be)+c\left(16c^2d^2-16bcde+b^2e^2\right)x\right)}{3b^4d(cd-be)\sqrt{bx+cx^2}} - \frac{2\sqrt{c}\left(16c^2\right.}{}$$

Mathematica [C] time = 1.0706, size = 375, normalized size = 1.04

$$\frac{2\left(b(d+ex)\left(bc^2dx^2(cd-be)+c^2dx^2(b+cx)(8cd-7be)+x(b+cx)^2(cd-be)(8cd-be)+bd(b+cx)^2(be-cd)\right)-cx\sqrt{\frac{b}{c}}(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(b*(d + e*x)*(b*c^2*d*(c*d - b*e)*x^2 + c^2*d*(8*c*d - 7*b*e)*x^2*(b + c
*x) + b*d*(-(c*d) + b*e)*(b + c*x)^2 + (c*d - b*e)*(8*c*d - b*e)*x*(b + c*x
)^2) - Sqrt[b/c]*c*x*(b + c*x)*(Sqrt[b/c]*(16*c^2*d^2 - 16*b*c*d*e + b^2*e^
2)*(b + c*x)*(d + e*x) + I*b*e*(16*c^2*d^2 - 16*b*c*d*e + b^2*e^2)*Sqrt[1 +
 b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]],
 (c*d)/(b*e)] - I*b*e*(8*c^2*d^2 - 9*b*c*d*e + b^2*e^2)*Sqrt[1 + b/(c*x)]*S
qrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e
)])))/(3*b^5*d*(c*d - b*e)*(x*(b + c*x))^(3/2)*Sqrt[d + e*x])
```

Maple [B] time = 0.297, size = 1362, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*x^2+b*x)^(5/2),x)
```

```
[Out] -2/3/x^2*(2*x*b^4*c*d*e^2+x^2*b^4*c*e^3-17*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b
*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(
```

```
1/2))*x*b^4*c*d*e^2+32*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x
/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^3*c^2*d^2*
e+8*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF
(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^4*c*d*e^2-24*((c*x+b)/b)^(1/2
)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(
b*e/(b*e-c*d))^(1/2))*x*b^3*c^2*d^2*e+8*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-
c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2
))*x^2*b^3*c^2*d*e^2-24*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*
x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^2*c^3*d
^2*e-24*x^3*b^2*c^3*d*e^2-5*x^2*b^3*c^2*d*e^2-19*x^2*b^2*c^3*d^2*e-16*x^4*b
*c^4*d*e^2+8*x^3*b*c^4*d^2*e+x^4*b^2*c^3*e^3+2*x^3*b^3*c^2*e^3+24*x^2*b*c^4
*d^3+16*x^4*c^5*d^2*e+6*x*b^2*c^3*d^3-17*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e
-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/
2))*x^2*b^3*c^2*d*e^2+32*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c
*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^2*c^3*
d^2*e-16*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*Elli
pticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b*c^4*d^3+16*((c*x+b)/b)
^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1
/2),(b*e/(b*e-c*d))^(1/2))*x^2*b*c^4*d^3-16*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(
b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^
(1/2))*x*b^2*c^3*d^3+16*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*
x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^2*c^3*d^3
-8*x*b^3*c^2*d^2*e+((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^
(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^5*e^3-b^3*c^2*
d^3+16*x^3*c^5*d^3+((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^
(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^4*c*e^3+b^4*
c*d^2*e)*(x*(c*x+b))^(1/2)/b^4/c/(b*e-c*d)/d/(c*x+b)^2/(e*x+d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\left(cx^2+bx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(c*x^2 + b*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+bx}\sqrt{ex+d}}{c^3x^6+3\,bc^2x^5+3\,b^2cx^4+b^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x)*sqrt(e*x + d)/(c^3*x^6 + 3*b*c^2*x^5 + 3*b^2*c*x
^4 + b^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+b*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\left(cx^2+bx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)/(c*x^2 + b*x)^(5/2), x)
```

3.426 $\int \frac{1}{\sqrt{d+ex}\left(bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=451

$$\frac{2\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}\left(-b^2e^2-16bcde+16c^2d^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{3(-b)^{7/2}d\sqrt{bx+cx^2}\sqrt{d+ex}(cd-be)}+\frac{2\sqrt{d+ex}\left(2cx(2cd-be)\left(-b^2e^2-4\right.\right.}{3b^4}$$

```
[Out] (-2*Sqrt[d + e*x]*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(3*b^2*d*(c*d - b*e)
*(b*x + c*x^2)^(3/2)) + (2*Sqrt[d + e*x]*(b*(c*d - b*e)*(8*c^2*d^2 - 5*b*c*
d*e - 2*b^2*e^2) + 2*c*(2*c*d - b*e)*(4*c^2*d^2 - 4*b*c*d*e - b^2*e^2)*x))/
(3*b^4*d^2*(c*d - b*e)^2*Sqrt[b*x + c*x^2]) - (4*Sqrt[c]*(2*c*d - b*e)*(4*c
^2*d^2 - 4*b*c*d*e - b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*Ellip
ticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*(-b)^(7/2)*d^2*(c
*d - b*e)^2*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) + (2*Sqrt[c]*(16*c^2*d^2 -
 16*b*c*d*e - b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*Elliptic
F[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*(-b)^(7/2)*d*(c*d -
b*e)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.432825, antiderivative size = 451, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {740, 822, 843, 715, 112, 110, 117, 116}

$$\frac{2\sqrt{d+ex}\left(2cx(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)+b(cd-be)\left(-2b^2e^2-5bcde+8c^2d^2\right)\right)}{3b^4d^2\sqrt{bx+cx^2}(cd-be)^2}+\frac{2\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}\left(\right.}{3(-b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(b*x + c*x^2)^(5/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(3*b^2*d*(c*d - b*e)
*(b*x + c*x^2)^(3/2)) + (2*Sqrt[d + e*x]*(b*(c*d - b*e)*(8*c^2*d^2 - 5*b*c*
d*e - 2*b^2*e^2) + 2*c*(2*c*d - b*e)*(4*c^2*d^2 - 4*b*c*d*e - b^2*e^2)*x))/
(3*b^4*d^2*(c*d - b*e)^2*Sqrt[b*x + c*x^2]) - (4*Sqrt[c]*(2*c*d - b*e)*(4*c
^2*d^2 - 4*b*c*d*e - b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*Ellip
ticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*(-b)^(7/2)*d^2*(c
*d - b*e)^2*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) + (2*Sqrt[c]*(16*c^2*d^2 -
 16*b*c*d*e - b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*Elliptic
F[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*(-b)^(7/2)*d*(c*d -
b*e)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\left(bx+cx^2\right)^{5/2}}\,dx = -\frac{2\sqrt{d+ex}(b(cd-be)+c(2cd-be)x)}{3b^2d(cd-be)\left(bx+cx^2\right)^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(8c^2d^2-5bcde-2b^2e^2\right)+\frac{3}{2}ce(2cd-be)x}{\sqrt{d+ex}\left(bx+cx^2\right)^{3/2}}\,dx}{3b^2d(cd-be)}$$

$$= -\frac{2\sqrt{d+ex}(b(cd-be)+c(2cd-be)x)}{3b^2d(cd-be)\left(bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}\left(b(cd-be)\left(8c^2d^2-5bcde-2b^2e^2\right)\right.}{3b^4d^2(cd-be)^2}$$

$$= -\frac{2\sqrt{d+ex}(b(cd-be)+c(2cd-be)x)}{3b^2d(cd-be)\left(bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}\left(b(cd-be)\left(8c^2d^2-5bcde-2b^2e^2\right)\right.}{3b^4d^2(cd-be)^2}$$

$$= -\frac{2\sqrt{d+ex}(b(cd-be)+c(2cd-be)x)}{3b^2d(cd-be)\left(bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}\left(b(cd-be)\left(8c^2d^2-5bcde-2b^2e^2\right)\right.}{3b^4d^2(cd-be)^2}$$

$$= -\frac{2\sqrt{d+ex}(b(cd-be)+c(2cd-be)x)}{3b^2d(cd-be)\left(bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}\left(b(cd-be)\left(8c^2d^2-5bcde-2b^2e^2\right)\right.}{3b^4d^2(cd-be)^2}$$

$$= -\frac{2\sqrt{d+ex}(b(cd-be)+c(2cd-be)x)}{3b^2d(cd-be)\left(bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}\left(b(cd-be)\left(8c^2d^2-5bcde-2b^2e^2\right)\right.}{3b^4d^2(cd-be)^2}$$

Mathematica [C] time = 1.43565, size = 429, normalized size = 0.95

$$2\left(b(d+ex)\left(bc^3d^2x^2(cd-be)+2c^3d^2x^2(b+cx)(4cd-5be)-bd(b+cx)^2(cd-be)^2+2x(b+cx)^2(cd-be)^2(be+4cd)\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(b*x + c*x^2)^(5/2)),x]
```

```
[Out] (2*(b*(d + e*x)*(b*c^3*d^2*(c*d - b*e)*x^2 + 2*c^3*d^2*(4*c*d - 5*b*e)*x^2*
(b + c*x) - b*d*(c*d - b*e)^2*(b + c*x)^2 + 2*(c*d - b*e)^2*(4*c*d + b*e)*x
*(b + c*x)^2) - Sqrt[b/c]*c*x*(b + c*x)*(2*Sqrt[b/c]*(8*c^3*d^3 - 12*b*c^2*
d^2*e + 2*b^2*c*d*e^2 + b^3*e^3)*(b + c*x)*(d + e*x) + (2*I)*b*e*(8*c^3*d^3
 - 12*b*c^2*d^2*e + 2*b^2*c*d*e^2 + b^3*e^3)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(
e*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - I*b*e*
(8*c^3*d^3 - 13*b*c^2*d^2*e + 3*b^2*c*d*e^2 + 2*b^3*e^3)*Sqrt[1 + b/(c*x)]*
Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*
e)])))/(3*b^5*d^2*(c*d - b*e)^2*(x*(b + c*x))^(3/2)*Sqrt[d + e*x])
```

Maple [B] time = 0.323, size = 1763, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^(5/2)/(e*x+d)^(1/2),x)
```

```
[Out] 2/3*(2*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*Ellipt
icE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^4*c^2*d*e^3+2*((c*x+b)/b
```

```
)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(
1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^5*c*e^4+2*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(
b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^
(1/2))*x*b^6*e^4-b^3*c^3*d^4+9*x^3*b^3*c^3*d*e^3-33*x^3*b^2*c^4*d^2*e^2+6*x
^2*b^4*c^2*d*e^3-3*x^2*b^3*c^3*d^2*e^2-31*x^2*b^2*c^4*d^3*e-11*x*b^3*c^3*d^
3*e-24*x^4*b*c^5*d^2*e^2-28*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*
(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^3*c
^3*d^2*e^2+40*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)
*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^2*c^4*d^3*e+((c*x
+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)
/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^4*c^2*d*e^3+15*((c*x+b)/b)^(1/2)*(-(
e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(
b*e-c*d))^(1/2))*x^2*b^3*c^3*d^2*e^2-32*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-
c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2
))*x^2*b^2*c^4*d^3*e+2*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x
/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^5*c*d*e^3-
28*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(
((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^4*c^2*d^2*e^2+40*((c*x+b)/b)^(
1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2
),(b*e/(b*e-c*d))^(1/2))*x*b^3*c^3*d^3*e+((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e
-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/
2))*x*b^5*c*d*e^3+15*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b
)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^4*c^2*d^2*e^
2-32*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*Elliptic
F(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^3*c^3*d^3*e+4*x*b^4*c^2*d^2*
e^2+x*b^5*c*d*e^3+4*x^4*b^2*c^4*d*e^3+2*x^4*b^3*c^3*e^4+24*x^2*b*c^5*d^4+4*
x^3*b^4*c^2*e^4+6*x*b^2*c^4*d^4+16*x^4*c^6*d^3*e+16*x^3*c^6*d^4+2*x^2*b^5*c
*e^4-16*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*Ellip
ticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b*c^5*d^4+16*((c*x+b)/b)^
(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/
2),(b*e/(b*e-c*d))^(1/2))*x^2*b*c^5*d^4-16*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b
*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(
1/2))*x*b^2*c^4*d^4+16*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x
/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^2*c^4*d^4-
b^5*c*d^2*e^2+2*b^4*c^2*d^3*e)/x^2*(x*(c*x+b))^(1/2)/d^2/b^4/c/(c*x+b)^2/(b
*e-c*d)^2/(e*x+d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx\right)^{\frac{5}{2}}\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(5/2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x)^(5/2)*sqrt(e*x + d)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+bx}\sqrt{ex+d}}{c^3ex^7+b^3dx^3+\left(c^3d+3\,bc^2e\right)x^6+3\left(bc^2d+b^2ce\right)x^5+\left(3\,b^2cd+b^3e\right)x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(5/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x)*sqrt(e*x + d)/(c^3*e*x^7 + b^3*d*x^3 + (c^3*d +
3*b*c^2*e)*x^6 + 3*(b*c^2*d + b^2*c*e)*x^5 + (3*b^2*c*d + b^3*e)*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x(b+cx))^{\frac{5}{2}}\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(5/2)/(e*x+d)**(1/2),x)
```

```
[Out] Integral(1/((x*(b + c*x))**(5/2)*sqrt(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx\right)^{\frac{5}{2}}\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(5/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^2 + b*x)^(5/2)*sqrt(e*x + d)), x)
```

3.427 $\int \frac{1}{(d+ex)^{3/2}\left(bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=567

$$\frac{8\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(2cd-be)\left(-b^2e^2-2bcde+2c^2d^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{3(-b)^{7/2}d^2\sqrt{bx+cx^2}\sqrt{d+ex}(cd-be)^2}+\frac{2e\sqrt{bx+cx^2}\left(9b^2c^2d^2e^2+7b^3\ldots\right.}{3b^4d^3\sqrt{d\ldots}}$$

```
[Out] (-2*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(3*b^2*d*(c*d - b*e)*Sqrt[d + e*x]
*(b*x + c*x^2)^(3/2)) + (2*(b*(c*d - b*e)*(8*c^2*d^2 - 3*b*c*d*e - 4*b^2*e^
2) + 4*c*(4*c^3*d^3 - 6*b*c^2*d^2*e + b^3*e^3)*x))/(3*b^4*d^2*(c*d - b*e)^2
*Sqrt[d + e*x]*Sqrt[b*x + c*x^2]) + (2*e*(16*c^4*d^4 - 32*b*c^3*d^3*e + 9*b
^2*c^2*d^2*e^2 + 7*b^3*c*d*e^3 - 8*b^4*e^4)*Sqrt[b*x + c*x^2])/(3*b^4*d^3*(
c*d - b*e)^3*Sqrt[d + e*x]) - (2*Sqrt[c]*(16*c^4*d^4 - 32*b*c^3*d^3*e + 9*b
^2*c^2*d^2*e^2 + 7*b^3*c*d*e^3 - 8*b^4*e^4)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[
d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*(-b
)^(7/2)*d^3*(c*d - b*e)^3*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) + (8*Sqrt[c]
*(2*c*d - b*e)*(2*c^2*d^2 - 2*b*c*d*e - b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*
Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)
])/(3*(-b)^(7/2)*d^2*(c*d - b*e)^2*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.645148, antiderivative size = 567, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {740, 822, 834, 843, 715, 112, 110, 117, 116}

$$\frac{2e\sqrt{bx+cx^2}\left(9b^2c^2d^2e^2+7b^3cde^3-8b^4e^4-32bc^3d^3e+16c^4d^4\right)}{3b^4d^3\sqrt{d+ex}(cd-be)^3}+\frac{2\left(4cx\left(b^3e^3-6bc^2d^2e+4c^3d^3\right)+b(cd-be)\left(-4b^2e^2\ldots\right.\right.}{3b^4d^2\sqrt{bx+cx^2}\sqrt{d+ex}(cd-be)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*(b*x + c*x^2)^(5/2)),x]
```


```
[Out] (-2*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(3*b^2*d*(c*d - b*e)*Sqrt[d + e*x]
*(b*x + c*x^2)^(3/2)) + (2*(b*(c*d - b*e)*(8*c^2*d^2 - 3*b*c*d*e - 4*b^2*e^
2) + 4*c*(4*c^3*d^3 - 6*b*c^2*d^2*e + b^3*e^3)*x))/(3*b^4*d^2*(c*d - b*e)^2
*Sqrt[d + e*x]*Sqrt[b*x + c*x^2]) + (2*e*(16*c^4*d^4 - 32*b*c^3*d^3*e + 9*b
^2*c^2*d^2*e^2 + 7*b^3*c*d*e^3 - 8*b^4*e^4)*Sqrt[b*x + c*x^2])/(3*b^4*d^3*(
c*d - b*e)^3*Sqrt[d + e*x]) - (2*Sqrt[c]*(16*c^4*d^4 - 32*b*c^3*d^3*e + 9*b
^2*c^2*d^2*e^2 + 7*b^3*c*d*e^3 - 8*b^4*e^4)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[
d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(3*(-b
)^(7/2)*d^3*(c*d - b*e)^3*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) + (8*Sqrt[c]
*(2*c*d - b*e)*(2*c^2*d^2 - 2*b*c*d*e - b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*
Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)
])/(3*(-b)^(7/2)*d^2*(c*d - b*e)^2*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])
```


Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
```

```
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Dist[(Sqrt[x]*Sqrt[b + c*x])/Sqrt[b*x + c*x^2], Int[(d + e*x)^m/(Sqrt[x]*S
qrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && NeQ
[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\left(bx+cx^2\right)^{5/2}}\,dx = -\frac{2(b(cd-be)+c(2cd-be)x)}{3b^2d(cd-be)\sqrt{d+ex}\left(bx+cx^2\right)^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(8c^2d^2-3bcde-4b^2e^2\right)+\frac{5}{2}ce(2cd-be)x}{(d+ex)^{3/2}\left(bx+cx^2\right)^{3/2}}\,dx}{3b^2d(cd-be)}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{3b^2d(cd-be)\sqrt{d+ex}\left(bx+cx^2\right)^{3/2}} + \frac{2\left(b(cd-be)\left(8c^2d^2-3bcde-4b^2e^2\right)+4c\left(4c^3d^3\right.\right.}{3b^4d^2(cd-be)^2\sqrt{d+ex}\sqrt{bx+}}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{3b^2d(cd-be)\sqrt{d+ex}\left(bx+cx^2\right)^{3/2}} + \frac{2\left(b(cd-be)\left(8c^2d^2-3bcde-4b^2e^2\right)+4c\left(4c^3d^3\right.\right.}{3b^4d^2(cd-be)^2\sqrt{d+ex}\sqrt{bx+}}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{3b^2d(cd-be)\sqrt{d+ex}\left(bx+cx^2\right)^{3/2}} + \frac{2\left(b(cd-be)\left(8c^2d^2-3bcde-4b^2e^2\right)+4c\left(4c^3d^3\right.\right.}{3b^4d^2(cd-be)^2\sqrt{d+ex}\sqrt{bx+}}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{3b^2d(cd-be)\sqrt{d+ex}\left(bx+cx^2\right)^{3/2}} + \frac{2\left(b(cd-be)\left(8c^2d^2-3bcde-4b^2e^2\right)+4c\left(4c^3d^3\right.\right.}{3b^4d^2(cd-be)^2\sqrt{d+ex}\sqrt{bx+}}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{3b^2d(cd-be)\sqrt{d+ex}\left(bx+cx^2\right)^{3/2}} + \frac{2\left(b(cd-be)\left(8c^2d^2-3bcde-4b^2e^2\right)+4c\left(4c^3d^3\right.\right.}{3b^4d^2(cd-be)^2\sqrt{d+ex}\sqrt{bx+}}$$

$$= -\frac{2(b(cd-be)+c(2cd-be)x)}{3b^2d(cd-be)\sqrt{d+ex}\left(bx+cx^2\right)^{3/2}} + \frac{2\left(b(cd-be)\left(8c^2d^2-3bcde-4b^2e^2\right)+4c\left(4c^3d^3\right.\right.}{3b^4d^2(cd-be)^2\sqrt{d+ex}\sqrt{bx+}}$$

Mathematica [C] time = 2.95524, size = 504, normalized size = 0.89

$$-\frac{2\left(b\left(3b^4e^5x^2(b+cx)^2+bc^4d^3x^2(d+ex)(be-cd)-c^4d^3x^2(b+cx)(d+ex)(8cd-13be)+bd(b+cx)^2(d+ex)(cd-be)^3-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*(b*x + c*x^2)^(5/2)),x]
```

```
[Out] (-2*(b*(3*b^4*e^5*x^2*(b + c*x)^2 + b*c^4*d^3*(-(c*d) + b*e)*x^2*(d + e*x)
- c^4*d^3*(8*c*d - 13*b*e)*x^2*(b + c*x)*(d + e*x) + b*d*(c*d - b*e)^3*(b +
 c*x)^2*(d + e*x) - (c*d - b*e)^3*(8*c*d + 5*b*e)*x*(b + c*x)^2*(d + e*x))
+ Sqrt[b/c]*c*x*(b + c*x)*(Sqrt[b/c]*(16*c^4*d^4 - 32*b*c^3*d^3*e + 9*b^2*c
```

```
^2*d^2*e^2 + 7*b^3*c*d*e^3 - 8*b^4*e^4)*(b + c*x)*(d + e*x) + I*b*e*(16*c^4
*d^4 - 32*b*c^3*d^3*e + 9*b^2*c^2*d^2*e^2 + 7*b^3*c*d*e^3 - 8*b^4*e^4)*Sqrt
[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[
x]], (c*d)/(b*e)] - I*b*e*(8*c^4*d^4 - 17*b*c^3*d^3*e + 6*b^2*c^2*d^2*e^2 +
 11*b^3*c*d*e^3 - 8*b^4*e^4)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*El
lipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])))/(3*b^5*d^3*(c*d - b*e
)^3*(x*(b + c*x))^(3/2)*Sqrt[d + e*x])
```

Maple [B] time = 0.315, size = 2189, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(c*x^2+b*x)^(5/2),x)
```

```
[Out] 2/3*(-18*x^2*b^4*c^3*d^2*e^3-2*x^2*b^3*c^4*d^3*e^2+43*x^2*b^2*c^5*d^4*e+14*
x*b^3*c^4*d^4*e-7*x^4*b^3*c^4*d*e^4-9*x^4*b^2*c^5*d^2*e^3+8*((c*x+b)/b)^(1/
2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),
(b*e/(b*e-c*d))^(1/2))*x^2*b^6*c*e^5+4*x*b^6*c*d*e^4-6*x*b^5*c^2*d^2*e^3-6*
x*b^4*c^3*d^3*e^2+8*x^3*b*c^6*d^4*e-10*x^3*b^4*c^3*d*e^4-22*x^3*b^3*c^4*d^2
*e^3+40*x^3*b^2*c^5*d^3*e^2+x^2*b^5*c^2*d*e^4+8*x^2*b^6*c*e^5-2*((c*x+b)/b)
^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1
/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^4*c^3*d^2*e^3-4*((c*x+b)/b)^(1/2)*(-(e*x+d
)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c
*d))^(1/2))*x*b^5*c^2*d^2*e^3-24*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(
1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^
4*c^3*d^3*e^2+4*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/
2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^5*c^2*d*e^4-4*(
(c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*
x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^4*c^3*d^2*e^3-24*((c*x+b)/b)^(1/
2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),
(b*e/(b*e-c*d))^(1/2))*x^2*b^3*c^4*d^3*e^2+40*((c*x+b)/b)^(1/2)*(-(e*x+d)*c
/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d)
)^(1/2))*x^2*b^2*c^5*d^4*e-16*x^4*c^7*d^4*e+16*x^3*b^5*c^2*e^5+8*x^4*b^4*c^
3*e^5-6*x*b^2*c^5*d^5-24*x^2*b*c^6*d^5+32*x^4*b*c^6*d^3*e^2-16*x^3*c^7*d^5-
15*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(
((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^6*c*d*e^4-48*((c*x+b)/b)^(1/2)
*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b
*e/(b*e-c*d))^(1/2))*x*b^3*c^4*d^4*e+4*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c
*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2)
)*x*b^6*c*d*e^4-15*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^
(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x^2*b^5*c^2*d*e^4+
40*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(
((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^3*c^4*d^4*e+41*((c*x+b)/b)^(1/
2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),
(b*e/(b*e-c*d))^(1/2))*x^2*b^3*c^4*d^3*e^2-48*((c*x+b)/b)^(1/2)*(-(e*x+d)*c
/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d)
)^(1/2))*x^2*b^2*c^5*d^4*e-2*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)
*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^5*c^
2*d^2*e^3+41*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*
EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^4*c^3*d^3*e^2+8*((c*
x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b
)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^7*e^5+b^3*c^4*d^5+16*((c*x+b)/b)^(1/2
)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticE(((c*x+b)/b)^(1/2),(
b*e/(b*e-c*d))^(1/2))*x^2*b*c^6*d^5-16*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c
*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2)
```

```
)*x^2*b*c^6*d^5+16*((c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^
(1/2)*EllipticE(((c*x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^2*c^5*d^5-16*(
(c*x+b)/b)^(1/2)*(-(e*x+d)*c/(b*e-c*d))^(1/2)*(-c*x/b)^(1/2)*EllipticF(((c*
x+b)/b)^(1/2),(b*e/(b*e-c*d))^(1/2))*x*b^2*c^5*d^5-3*b^4*c^3*d^4*e-b^6*c*d^
2*e^3+3*b^5*c^2*d^3*e^2)/x^2*(x*(c*x+b))^(1/2)/b^4/d^3/c/(c*x+b)^2/(b*e-c*d
)^3/(e*x+d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx\right)^{\frac{5}{2}}(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x)^(5/2)*(e*x + d)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^2+bx}\sqrt{ex+d}}{c^3e^2x^8+b^3d^2x^3+\left(2\,c^3de+3\,bc^2e^2\right)x^7+\left(c^3d^2+6\,bc^2de+3\,b^2ce^2\right)x^6+\left(3\,bc^2d^2+6\,b^2cde+b^3e^2\right)x^5+\left(3\,b^2cd^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x)*sqrt(e*x + d)/(c^3*e^2*x^8 + b^3*d^2*x^3 + (2*c^
3*d*e + 3*b*c^2*e^2)*x^7 + (c^3*d^2 + 6*b*c^2*d*e + 3*b^2*c*e^2)*x^6 + (3*b
*c^2*d^2 + 6*b^2*c*d*e + b^3*e^2)*x^5 + (3*b^2*c*d^2 + 2*b^3*d*e)*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x\left(b+cx\right)\right)^{\frac{5}{2}}\left(d+ex\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(c*x**2+b*x)**(5/2),x)
```

```
[Out] Integral(1/((x*(b + c*x))**(5/2)*(d + e*x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx\right)^{\frac{5}{2}}(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^2 + b*x)^(5/2)*(e*x + d)^(3/2)), x)
```

3.428 $\int \frac{\sqrt{d+ex}}{\sqrt{2x-3x^2}}\,dx$

Optimal. Leaf size=51

$$\frac{2\sqrt{d+ex}E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\sqrt{x}\right)|-\frac{2e}{3d}\right)}{\sqrt{3}\sqrt{\frac{ex}{d}+1}}$$

```
[Out] (2*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt[3/2]*Sqrt[x]], (-2*e)/(3*d)])/(Sqrt[
3]*Sqrt[1 + (e*x)/d])
```

Rubi [A] time = 0.0261794, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {714, 12, 112, 110}

$$\frac{2\sqrt{d+ex}E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\sqrt{x}\right)|-\frac{2e}{3d}\right)}{\sqrt{3}\sqrt{\frac{ex}{d}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/Sqrt[2*x - 3*x^2],x]
```

```
[Out] (2*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt[3/2]*Sqrt[x]], (-2*e)/(3*d)])/(Sqrt[
3]*Sqrt[1 + (e*x)/d])
```

Rule 714

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Int[(d + e*x)^m/(Sqrt[b*x]*Sqrt[1 + (c*x)/b]), x] /; FreeQ[{b, c, d, e}, x
] && NeQ[c*d - b*e, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4] && LtQ[c, 0]
 && RationalQ[b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\sqrt{2x-3x^2}}\,dx = \int \frac{\sqrt{d+ex}}{\sqrt{2}\sqrt{1-\frac{3x}{2}}\sqrt{x}}\,dx$$
$$= \frac{\int \frac{\sqrt{d+ex}}{\sqrt{1-\frac{3x}{2}}\sqrt{x}}\,dx}{\sqrt{2}}$$
$$= \frac{\sqrt{d+ex}\int \frac{\sqrt{1+\frac{ex}{d}}}{\sqrt{1-\frac{3x}{2}}\sqrt{x}}\,dx}{\sqrt{2}\sqrt{1+\frac{ex}{d}}}$$
$$= \frac{2\sqrt{d+ex}E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\sqrt{x}\right)|-\frac{2e}{3d}\right)}{\sqrt{3}\sqrt{1+\frac{ex}{d}}}$$

Mathematica [B] time = 0.314161, size = 117, normalized size = 2.29

$$\frac{2(3x-2)\sqrt{-\frac{d}{e}}(d+ex)-2d\sqrt{9-\frac{6}{x}}x^{3/2}\sqrt{\frac{d}{ex}+1}E\left(\sin^{-1}\left(\frac{\sqrt{-\frac{d}{e}}}{\sqrt{x}}\right)|-\frac{2e}{3d}\right)}{3\sqrt{-x(3x-2)}\sqrt{-\frac{d}{e}}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/Sqrt[2*x - 3*x^2],x]
```

```
[Out] (2*Sqrt[-(d/e)]*(-2 + 3*x)*(d + e*x) - 2*d*Sqrt[9 - 6/x]*Sqrt[1 + d/(e*x)]*
x^(3/2)*EllipticE[ArcSin[Sqrt[-(d/e)]/Sqrt[x]], (-2*e)/(3*d)])/(3*Sqrt[-(d/
e)]*Sqrt[-(x*(-2 + 3*x))]*Sqrt[d + e*x])
```

Maple [B] time = 0.2, size = 215, normalized size = 4.2

$$-\frac{2d}{3ex\left(3ex^2+3dx-2ex-2d\right)}\sqrt{ex+d}\sqrt{-x(-2+3x)}\sqrt{\frac{ex+d}{d}}\sqrt{-\frac{(-2+3x)e}{3d+2e}}\sqrt{-\frac{ex}{d}}\left(3d\text{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(-3*x^2+2*x)^(1/2),x)
```

```
[Out] -2/3*(e*x+d)^(1/2)*(-x*(-2+3*x))^(1/2)*d*((e*x+d)/d)^(1/2)*(-(-2+3*x)*e/(3*
d+2*e))^(1/2)*(-e*x/d)^(1/2)*(3*d*EllipticF(((e*x+d)/d)^(1/2),3^(1/2)*(d/(3
*d+2*e))^(1/2))+2*EllipticF(((e*x+d)/d)^(1/2),3^(1/2)*(d/(3*d+2*e))^(1/2))*
e-3*EllipticE(((e*x+d)/d)^(1/2),3^(1/2)*(d/(3*d+2*e))^(1/2))*d-2*EllipticE(
((e*x+d)/d)^(1/2),3^(1/2)*(d/(3*d+2*e))^(1/2))*e)/e/x/(3*e*x^2+3*d*x-2*e*x-
2*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{-3x^2+2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(-3*x^2+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/sqrt(-3*x^2 + 2*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{ex+d}\sqrt{-3\,x^2+2\,x}}{3\,x^2-2\,x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(-3*x^2+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*x + d)*sqrt(-3*x^2 + 2*x)/(3*x^2 - 2*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}}{\sqrt{-x\,(3x-2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(-3*x**2+2*x)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/sqrt(-x*(3*x - 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{-3\,x^2+2\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(-3*x^2+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)/sqrt(-3*x^2 + 2*x), x)
```

3.429 $\int \frac{1}{\sqrt{d+ex}\sqrt{2x-3x^2}}\,dx$

Optimal. Leaf size=51

$$\frac{2\sqrt{\frac{ex}{d}+1}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\sqrt{x}\right),-\frac{2e}{3d}\right)}{\sqrt{3}\sqrt{d+ex}}$$

```
[Out] (2*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[Sqrt[3/2]*Sqrt[x]], (-2*e)/(3*d)])/(S
qrt[3]*Sqrt[d + e*x])
```

Rubi [A] time = 0.0253953, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {714, 12, 117, 115}

$$\frac{2\sqrt{\frac{ex}{d}+1}F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\sqrt{x}\right)|-\frac{2e}{3d}\right)}{\sqrt{3}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*Sqrt[2*x - 3*x^2]),x]
```

```
[Out] (2*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[Sqrt[3/2]*Sqrt[x]], (-2*e)/(3*d)])/(S
qrt[3]*Sqrt[d + e*x])
```

Rule 714

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Int[(d + e*x)^m/(Sqrt[b*x]*Sqrt[1 + (c*x)/b]), x] /; FreeQ[{b, c, d, e}, x
] && NeQ[c*d - b*e, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4] && LtQ[c, 0]
 && RationalQ[b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 115

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (GtQ[-(b/d), 0] || LtQ[-(b/f), 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\sqrt{2x-3x^2}}dx = \int \frac{1}{\sqrt{2}\sqrt{1-\frac{3x}{2}}\sqrt{x}\sqrt{d+ex}}dx$$
$$= \frac{\int \frac{1}{\sqrt{1-\frac{3x}{2}}\sqrt{x}\sqrt{d+ex}}dx}{\sqrt{2}}$$
$$= \frac{\sqrt{1+\frac{ex}{d}}\int \frac{1}{\sqrt{1-\frac{3x}{2}}\sqrt{x}\sqrt{1+\frac{ex}{d}}}dx}{\sqrt{2}\sqrt{d+ex}}$$
$$= \frac{2\sqrt{1+\frac{ex}{d}}F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\sqrt{x}\right)|-\frac{2e}{3d}\right)}{\sqrt{3}\sqrt{d+ex}}$$

Mathematica [A] time = 0.120767, size = 76, normalized size = 1.49

$$-\frac{\sqrt{6-\frac{4}{x}}x^{3/2}\sqrt{\frac{d}{ex}+1}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right),-\frac{3d}{2e}\right)}{\sqrt{-x(3x-2)}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*Sqrt[2*x - 3*x^2]),x]
```

```
[Out] -((Sqrt[6 - 4/x]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[ArcSin[Sqrt[2/3]/Sqrt[
x]], (-3*d)/(2*e)])/(Sqrt[-(x*(-2 + 3*x))]*Sqrt[d + e*x]))
```

Maple [B] time = 0.207, size = 115, normalized size = 2.3

$$-2\,\frac{d\sqrt{ex+d}\sqrt{-x\,(-2+3\,x)}}{ex\left(3\,ex^2+3\,dx-2\,ex-2\,d\right)}\text{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{3}\sqrt{\frac{d}{3\,d+2\,e}}\right)\sqrt{-\frac{ex}{d}}\sqrt{-\frac{(-2+3\,x)\,e}{3\,d+2\,e}}\sqrt{\frac{ex+d}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(-3*x^2+2*x)^(1/2),x)
```

```
[Out] -2*EllipticF(((e*x+d)/d)^(1/2),3^(1/2)*(d/(3*d+2*e))^(1/2))*(-e*x/d)^(1/2)*
(-(-2+3*x)*e/(3*d+2*e))^(1/2)*((e*x+d)/d)^(1/2)*d*(e*x+d)^(1/2)*(-x*(-2+3*x
))^(1/2)/e/x/(3*e*x^2+3*d*x-2*e*x-2*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ex+d}\sqrt{-3\,x^2+2\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(-3*x^2+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(e*x + d)*sqrt(-3*x^2 + 2*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{ex+d}\sqrt{-3\,x^2+2\,x}}{3\,ex^3+(3\,d-2\,e)x^2-2\,dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(-3*x^2+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*x + d)*sqrt(-3*x^2 + 2*x)/(3*e*x^3 + (3*d - 2*e)*x^2 - 2*d
*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x\,(3x-2)}\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(-3*x**2+2*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-x*(3*x - 2))*sqrt(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ex+d}\sqrt{-3\,x^2+2\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(-3*x^2+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*x + d)*sqrt(-3*x^2 + 2*x)), x)
```

3.430 $\int \frac{\sqrt{d+ex}}{\sqrt{-2x-3x^2}} dx$

Optimal. Leaf size=53

$$-\frac{2\sqrt{d+ex}E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\sqrt{-x}\right)|\frac{2e}{3d}\right)}{\sqrt{3}\sqrt{\frac{ex}{d}+1}}$$

```
[Out] (-2*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt[3/2]*Sqrt[-x]], (2*e)/(3*d)])/(Sqrt
[3]*Sqrt[1 + (e*x)/d])
```

Rubi [A] time = 0.0259557, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {714, 12, 112, 110}

$$-\frac{2\sqrt{d+ex}E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\sqrt{-x}\right)|\frac{2e}{3d}\right)}{\sqrt{3}\sqrt{\frac{ex}{d}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/Sqrt[-2*x - 3*x^2],x]
```

```
[Out] (-2*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt[3/2]*Sqrt[-x]], (2*e)/(3*d)])/(Sqrt
[3]*Sqrt[1 + (e*x)/d])
```

Rule 714

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Int[(d + e*x)^m/(Sqrt[b*x]*Sqrt[1 + (c*x)/b]), x] /; FreeQ[{b, c, d, e}, x
] && NeQ[c*d - b*e, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4] && LtQ[c, 0]
 && RationalQ[b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 112

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Dist[(Sqrt[e + f*x]*Sqrt[1 + (d*x)/c])/(Sqrt[c + d*x]*Sqrt[1 + (f
*x)/e]), Int[Sqrt[1 + (f*x)/e]/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]), x], x] /; Fre
eQ[{b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\sqrt{-2x-3x^2}}\,dx = \int \frac{\sqrt{d+ex}}{\sqrt{2}\sqrt{-x}\sqrt{1+\frac{3x}{2}}}\,dx$$
$$= \frac{\int \frac{\sqrt{d+ex}}{\sqrt{-x}\sqrt{1+\frac{3x}{2}}}\,dx}{\sqrt{2}}$$
$$= \frac{\sqrt{d+ex}\int \frac{\sqrt{1+\frac{ex}{d}}}{\sqrt{-x}\sqrt{1+\frac{3x}{2}}}\,dx}{\sqrt{2}\sqrt{1+\frac{ex}{d}}}$$
$$= -\frac{2\sqrt{d+ex}E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\sqrt{-x}\right)|\frac{2e}{3d}\right)}{\sqrt{3}\sqrt{1+\frac{ex}{d}}}$$

Mathematica [B] time = 0.211225, size = 117, normalized size = 2.21

$$\frac{2(3x+2)\sqrt{-\frac{d}{e}}(d+ex)-2d\sqrt{\frac{6}{x}+9}x^{3/2}\sqrt{\frac{d}{ex}+1}E\left(\sin^{-1}\left(\frac{\sqrt{-\frac{d}{e}}}{\sqrt{x}}\right)|\frac{2e}{3d}\right)}{3\sqrt{-x(3x+2)}\sqrt{-\frac{d}{e}}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/Sqrt[-2*x - 3*x^2],x]
```

```
[Out] (2*Sqrt[-(d/e)]*(2 + 3*x)*(d + e*x) - 2*d*Sqrt[9 + 6/x]*Sqrt[1 + d/(e*x)]*x
^(3/2)*EllipticE[ArcSin[Sqrt[-(d/e)]/Sqrt[x]], (2*e)/(3*d)])/(3*Sqrt[-(d/e)
]*Sqrt[-(x*(2 + 3*x))]*Sqrt[d + e*x])
```

Maple [B] time = 0.245, size = 215, normalized size = 4.1

$$-\frac{2d}{3ex\left(3ex^2+3dx+2ex+2d\right)}\sqrt{ex+d}\sqrt{-x(2+3x)}\sqrt{\frac{ex+d}{d}}\sqrt{-\frac{(2+3x)e}{3d-2e}}\sqrt{-\frac{ex}{d}}\left(3d\text{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{3}\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(-3*x^2-2*x)^(1/2),x)
```

```
[Out] -2/3*(e*x+d)^(1/2)*(-x*(2+3*x))^(1/2)*d*((e*x+d)/d)^(1/2)*(-(2+3*x)*e/(3*d-
2*e))^(1/2)*(-e*x/d)^(1/2)*(3*d*EllipticF(((e*x+d)/d)^(1/2),3^(1/2)*(d/(3*d
-2*e))^(1/2))-2*EllipticF(((e*x+d)/d)^(1/2),3^(1/2)*(d/(3*d-2*e))^(1/2))*e-
3*EllipticE(((e*x+d)/d)^(1/2),3^(1/2)*(d/(3*d-2*e))^(1/2))*d+2*EllipticE(((
e*x+d)/d)^(1/2),3^(1/2)*(d/(3*d-2*e))^(1/2))*e)/e/x/(3*e*x^2+3*d*x+2*e*x+2*
d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{-3x^2-2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(-3*x^2-2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/sqrt(-3*x^2 - 2*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{ex+d}\sqrt{-3\,x^2-2\,x}}{3\,x^2+2\,x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(-3*x^2-2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*x + d)*sqrt(-3*x^2 - 2*x)/(3*x^2 + 2*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}}{\sqrt{-x\,(3x+2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(-3*x**2-2*x)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/sqrt(-x*(3*x + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{-3\,x^2-2\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(-3*x^2-2*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)/sqrt(-3*x^2 - 2*x), x)
```

3.431 $\int \frac{1}{\sqrt{d+ex}\sqrt{-2x-3x^2}}\,dx$

Optimal. Leaf size=53

$$-\frac{2\sqrt{\frac{ex}{d}+1}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\sqrt{-x}\right),\frac{2e}{3d}\right)}{\sqrt{3}\sqrt{d+ex}}$$

```
[Out] (-2*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[Sqrt[3/2]*Sqrt[-x]], (2*e)/(3*d)])/(
Sqrt[3]*Sqrt[d + e*x])
```

Rubi [A] time = 0.026315, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {714, 12, 117, 115}

$$-\frac{2\sqrt{\frac{ex}{d}+1}F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\sqrt{-x}\right)|\frac{2e}{3d}\right)}{\sqrt{3}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*Sqrt[-2*x - 3*x^2]),x]
```

```
[Out] (-2*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[Sqrt[3/2]*Sqrt[-x]], (2*e)/(3*d)])/(
Sqrt[3]*Sqrt[d + e*x])
```

Rule 714

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Int[(d + e*x)^m/(Sqrt[b*x]*Sqrt[1 + (c*x)/b]), x] /; FreeQ[{b, c, d, e}, x
] && NeQ[c*d - b*e, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4] && LtQ[c, 0]
 && RationalQ[b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 115

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (GtQ[-(b/d), 0] || LtQ[-(b/f), 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\sqrt{-2x-3x^2}}\,dx = \int \frac{1}{\sqrt{2}\sqrt{-x}\sqrt{1+\frac{3x}{2}}\sqrt{d+ex}}\,dx$$
$$= \frac{\int \frac{1}{\sqrt{-x}\sqrt{1+\frac{3x}{2}}\sqrt{d+ex}}\,dx}{\sqrt{2}}$$
$$= \frac{\sqrt{1+\frac{ex}{d}}\int \frac{1}{\sqrt{-x}\sqrt{1+\frac{3x}{2}}\sqrt{1+\frac{ex}{d}}}\,dx}{\sqrt{2}\sqrt{d+ex}}$$
$$= -\frac{2\sqrt{1+\frac{ex}{d}}F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\sqrt{-x}\right)|\frac{2e}{3d}\right)}{\sqrt{3}\sqrt{d+ex}}$$

Mathematica [C] time = 0.109033, size = 82, normalized size = 1.55

$$\frac{i\sqrt{\frac{4}{x}+6}x^{3/2}\sqrt{\frac{d}{ex}+1}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right),\frac{3d}{2e}\right)}{\sqrt{-x(3x+2)}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*Sqrt[-2*x - 3*x^2]),x]
```

```
[Out] (I*Sqrt[6 + 4/x]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[2/3]/Sq
rt[x]], (3*d)/(2*e)])/(Sqrt[-(x*(2 + 3*x))]*Sqrt[d + e*x])
```

Maple [B] time = 0.207, size = 115, normalized size = 2.2

$$-2\,\frac{d\sqrt{ex+d}\sqrt{-x\,(2+3\,x)}}{ex\left(3\,e{x}^{2}+3\,dx+2\,ex+2\,d\right)}\text{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{3}\sqrt{\frac{d}{3\,d-2\,e}}\right)\sqrt{-\frac{ex}{d}}\sqrt{-\frac{(2+3\,x)\,e}{3\,d-2\,e}}\sqrt{\frac{ex+d}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(-3*x^2-2*x)^(1/2),x)
```

```
[Out] -2*EllipticF(((e*x+d)/d)^(1/2),3^(1/2)*(d/(3*d-2*e))^(1/2))*(-e*x/d)^(1/2)*
(-(2+3*x)*e/(3*d-2*e))^(1/2)*((e*x+d)/d)^(1/2)*d*(e*x+d)^(1/2)*(-x*(2+3*x))
^(1/2)/e/x/(3*e*x^2+3*d*x+2*e*x+2*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ex+d}\sqrt{-3\,x^2-2\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(-3*x^2-2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(e*x + d)*sqrt(-3*x^2 - 2*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{ex+d}\sqrt{-3\,x^2-2\,x}}{3\,ex^3+(3\,d+2\,e)x^2+2\,dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(-3*x^2-2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*x + d)*sqrt(-3*x^2 - 2*x)/(3*e*x^3 + (3*d + 2*e)*x^2 + 2*d
*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-x\,(3x+2)}\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(-3*x**2-2*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-x*(3*x + 2))*sqrt(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{ex+d}\sqrt{-3\,x^2-2\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(-3*x^2-2*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*x + d)*sqrt(-3*x^2 - 2*x)), x)
```

3.432 $\int \frac{\sqrt{1-x}}{\sqrt{-x}\sqrt{1+x}} \, dx$

Optimal. Leaf size=12

$$-2E\left(\sin^{-1}\left(\sqrt{-x}\right)\middle|-1\right)$$

```
[Out] -2*EllipticE[ArcSin[Sqrt[-x]], -1]
```

Rubi [A] time = 0.0040121, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {110}

$$-2E\left(\sin^{-1}\left(\sqrt{-x}\right)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]/(Sqrt[-x]*Sqrt[1 + x]),x]
```

```
[Out] -2*EllipticE[ArcSin[Sqrt[-x]], -1]
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rubi steps

$$\int \frac{\sqrt{1-x}}{\sqrt{-x}\sqrt{1+x}} \, dx = -2E\left(\sin^{-1}\left(\sqrt{-x}\right)\middle|-1\right)$$

Mathematica [C] time = 0.0261283, size = 66, normalized size = 5.5

$$-\frac{2x\sqrt{1-x^2}\left(x\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};x^2\right)-3\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};x^2\right)\right)}{3\sqrt{1-x}\sqrt{-x(x+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]/(Sqrt[-x]*Sqrt[1 + x]),x]
```

```
[Out] (-2*x*Sqrt[1 - x^2]*(-3*Hypergeometric2F1[1/4, 1/2, 5/4, x^2] + x*Hypergeom
etric2F1[1/2, 3/4, 7/4, x^2]))/(3*Sqrt[1 - x]*Sqrt[-(x*(1 + x))])
```

Maple [A] time = 0.097, size = 17, normalized size = 1.4

$$2\sqrt{2}\text{EllipticE}\left(\sqrt{1+x},1/2\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2)/(-x)^(1/2)/(1+x)^(1/2),x)
```

```
[Out] 2*2^(1/2)*EllipticE((1+x)^(1/2),1/2*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x+1}}{\sqrt{-x}\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(-x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x + 1)/(sqrt(-x)*sqrt(x + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x}\sqrt{x+1}\sqrt{-x+1}}{x^2+x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(-x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x)*sqrt(x + 1)*sqrt(-x + 1)/(x^2 + x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{1-x}}{\sqrt{-x}\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)/(-x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Integral(sqrt(1 - x)/(sqrt(-x)*sqrt(x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x+1}}{\sqrt{-x}\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(-x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x + 1)/(sqrt(-x)*sqrt(x + 1)), x)
```

3.433 $\int \frac{\sqrt{1-x}}{\sqrt{-x-x^2}} dx$

Optimal. Leaf size=12

$$-2E\left(\sin^{-1}\left(\sqrt{-x}\right)\middle|-1\right)$$

```
[Out] -2*EllipticE[ArcSin[Sqrt[-x]], -1]
```

Rubi [A] time = 0.0104788, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {714, 110}

$$-2E\left(\sin^{-1}\left(\sqrt{-x}\right)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]/Sqrt[-x - x^2],x]
```

```
[Out] -2*EllipticE[ArcSin[Sqrt[-x]], -1]
```

Rule 714

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Int[(d + e*x)^m/(Sqrt[b*x]*Sqrt[1 + (c*x)/b]), x] /; FreeQ[{b, c, d, e}, x
] && NeQ[c*d - b*e, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4] && LtQ[c, 0]
 && RationalQ[b]
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] && !LtQ[-(b/d), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-x}}{\sqrt{-x-x^2}} dx &= \int \frac{\sqrt{1-x}}{\sqrt{-x}\sqrt{1+x}} dx\\ &= -2E\left(\sin^{-1}\left(\sqrt{-x}\right)\middle|-1\right)\end{aligned}$$

Mathematica [C] time = 0.0115723, size = 66, normalized size = 5.5

$$-\frac{2x\sqrt{1-x^2}\left(x\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};x^2\right)-3\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};x^2\right)\right)}{3\sqrt{1-x}\sqrt{-x(x+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]/Sqrt[-x - x^2],x]
```

```
[Out] (-2*x*Sqrt[1 - x^2]*(-3*Hypergeometric2F1[1/4, 1/2, 5/4, x^2] + x*Hypergeom
etric2F1[1/2, 3/4, 7/4, x^2]))/(3*Sqrt[1 - x]*Sqrt[-(x*(1 + x))])
```

Maple [B] time = 0.105, size = 38, normalized size = 3.2

$$-2\frac{\text{EllipticE}\left(\sqrt{1+x}, 1/2\sqrt{2}\right)\sqrt{-x}\sqrt{2}\sqrt{-x(1+x)}}{\sqrt{1+x}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2)/(-x^2-x)^(1/2),x)
```

```
[Out] -2*EllipticE((1+x)^(1/2),1/2*2^(1/2))*(-x)^(1/2)*2^(1/2)/(1+x)^(1/2)*(-x*(1
+x))^(1/2)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x+1}}{\sqrt{-x^2-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(-x^2-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x + 1)/sqrt(-x^2 - x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^2-x}\sqrt{-x+1}}{x^2+x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(-x^2-x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^2 - x)*sqrt(-x + 1)/(x^2 + x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{1-x}}{\sqrt{-x(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)/(-x**2-x)**(1/2),x)
```

```
[Out] Integral(sqrt(1 - x)/sqrt(-x*(x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x+1}}{\sqrt{-x^2-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(-x^2-x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x + 1)/sqrt(-x^2 - x), x)
```

3.434 $\int (d+ex)^m \left(cdx+cex^2\right)^3 \, dx$

Optimal. Leaf size=95

$$-\frac{c^3d^3(d+ex)^{m+4}}{e^4(m+4)}+\frac{3c^3d^2(d+ex)^{m+5}}{e^4(m+5)}-\frac{3c^3d(d+ex)^{m+6}}{e^4(m+6)}+\frac{c^3(d+ex)^{m+7}}{e^4(m+7)}$$

```
[Out] -((c^3*d^3*(d + e*x)^(4 + m))/(e^4*(4 + m))) + (3*c^3*d^2*(d + e*x)^(5 + m)
)/(e^4*(5 + m)) - (3*c^3*d*(d + e*x)^(6 + m))/(e^4*(6 + m)) + (c^3*(d + e*x
)^(7 + m))/(e^4*(7 + m))
```

Rubi [A] time = 0.0735214, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {626, 12, 43}

$$-\frac{c^3d^3(d+ex)^{m+4}}{e^4(m+4)}+\frac{3c^3d^2(d+ex)^{m+5}}{e^4(m+5)}-\frac{3c^3d(d+ex)^{m+6}}{e^4(m+6)}+\frac{c^3(d+ex)^{m+7}}{e^4(m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(c*d*x + c*e*x^2)^3,x]
```

```
[Out] -((c^3*d^3*(d + e*x)^(4 + m))/(e^4*(4 + m))) + (3*c^3*d^2*(d + e*x)^(5 + m)
)/(e^4*(5 + m)) - (3*c^3*d*(d + e*x)^(6 + m))/(e^4*(6 + m)) + (c^3*(d + e*x
)^(7 + m))/(e^4*(7 + m))
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^m \left(cdx+cex^2\right)^3 \, dx &= \int c^3x^3(d+ex)^{3+m} \, dx\\
&= c^3 \int x^3(d+ex)^{3+m} \, dx\\
&= c^3 \int \left(-\frac{d^3(d+ex)^{3+m}}{e^3}+\frac{3d^2(d+ex)^{4+m}}{e^3}-\frac{3d(d+ex)^{5+m}}{e^3}+\frac{(d+ex)^{6+m}}{e^3}\right) dx\\
&= -\frac{c^3d^3(d+ex)^{4+m}}{e^4(4+m)}+\frac{3c^3d^2(d+ex)^{5+m}}{e^4(5+m)}-\frac{3c^3d(d+ex)^{6+m}}{e^4(6+m)}+\frac{c^3(d+ex)^{7+m}}{e^4(7+m)}
\end{aligned}$$

Mathematica [A] time = 0.0592135, size = 70, normalized size = 0.74

$$\frac{c^3(d+ex)^{m+4}\left(\frac{3d^2(d+ex)}{m+5}-\frac{d^3}{m+4}-\frac{3d(d+ex)^2}{m+6}+\frac{(d+ex)^3}{m+7}\right)}{e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(c*d*x + c*e*x^2)^3,x]
```

```
[Out] (c^3*(d + e*x)^(4 + m)*(-(d^3/(4 + m)) + (3*d^2*(d + e*x))/(5 + m) - (3*d*(
d + e*x)^2)/(6 + m) + (d + e*x)^3/(7 + m)))/e^4
```

Maple [A] time = 0.046, size = 129, normalized size = 1.4

$$-\frac{c^3\left(ex+d\right)^{4+m}\left(-e^3m^3x^3-15\,e^3m^2x^3+3\,de^2m^2x^2-74\,e^3mx^3+27\,de^2mx^2-120\,x^3e^3-6\,d^2emx+60\,dx^2e^2-24\,d^2xe+\right.}{e^4\left(m^4+22\,m^3+179\,m^2+638\,m+840\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*e*x^2+c*d*x)^3,x)
```

```
[Out] -c^3*(e*x+d)^(4+m)*(-e^3*m^3*x^3-15*e^3*m^2*x^3+3*d*e^2*m^2*x^2-74*e^3*m*x^
3+27*d*e^2*m*x^2-120*e^3*x^3-6*d^2*e*m*x+60*d*e^2*x^2-24*d^2*e*x+6*d^3)/e^4
/(m^4+22*m^3+179*m^2+638*m+840)
```

Maxima [B] time = 1.37479, size = 910, normalized size = 9.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e*x^2+c*d*x)^3,x, algorithm="maxima")
```

```
[Out] ((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2
+ m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*c^3*d^3/((m^4 + 10*m^3
+ 35*m^2 + 50*m + 24)*e^4) + 3*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5
 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x
^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m*c^3*d^2/
((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^4) + 3*((m^5 + 15*m^4 +
85*m^3 + 225*m^2 + 274*m + 120)*e^6*x^6 + (m^5 + 10*m^4 + 35*m^3 + 50*m^2 +
 24*m)*d*e^5*x^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*e^4*x^4 + 20*(m^3 + 3
*m^2 + 2*m)*d^3*e^3*x^3 - 60*(m^2 + m)*d^4*e^2*x^2 + 120*d^5*e*m*x - 120*d^
6)*(e*x + d)^m*c^3*d/((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m
 + 720)*e^4) + ((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720
)*e^7*x^7 + (m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d*e^6*x^6 -
 6*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^2*e^5*x^5 + 30*(m^4 + 6*m^3 +
11*m^2 + 6*m)*d^3*e^4*x^4 - 120*(m^3 + 3*m^2 + 2*m)*d^4*e^3*x^3 + 360*(m^2
+ m)*d^5*e^2*x^2 - 720*d^6*e*m*x + 720*d^7)*(e*x + d)^m*c^3/((m^7 + 28*m^6
+ 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^4)
```

Fricas [B] time = 2.06555, size = 699, normalized size = 7.36

$$\frac{\left(6\,c^{3}d^{6}emx-6\,c^{3}d^{7}+\left(c^{3}e^{7}m^{3}+15\,c^{3}e^{7}m^{2}+74\,c^{3}e^{7}m+120\,c^{3}e^{7}\right)x^{7}+\left(4\,c^{3}de^{6}m^{3}+57\,c^{3}de^{6}m^{2}+269\,c^{3}de^{6}m+420\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e*x^2+c*d*x)^3,x, algorithm="fricas")
```

```
[Out] (6*c^3*d^6*e*m*x - 6*c^3*d^7 + (c^3*e^7*m^3 + 15*c^3*e^7*m^2 + 74*c^3*e^7*m
 + 120*c^3*e^7)*x^7 + (4*c^3*d*e^6*m^3 + 57*c^3*d*e^6*m^2 + 269*c^3*d*e^6*m
 + 420*c^3*d*e^6)*x^6 + 6*(c^3*d^2*e^5*m^3 + 13*c^3*d^2*e^5*m^2 + 57*c^3*d^
2*e^5*m + 84*c^3*d^2*e^5)*x^5 + 2*(2*c^3*d^3*e^4*m^3 + 21*c^3*d^3*e^4*m^2 +
 79*c^3*d^3*e^4*m + 105*c^3*d^3*e^4)*x^4 + (c^3*d^4*e^3*m^3 + 3*c^3*d^4*e^3
*m^2 + 2*c^3*d^4*e^3*m)*x^3 - 3*(c^3*d^5*e^2*m^2 + c^3*d^5*e^2*m)*x^2)*(e*x
 + d)^m/(e^4*m^4 + 22*e^4*m^3 + 179*e^4*m^2 + 638*e^4*m + 840*e^4)
```

Sympy [A] time = 8.11399, size = 2270, normalized size = 23.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*e*x**2+c*d*x)**3,x)
```

```
[Out] Piecewise((c**3*d**3*d**m*x**4/4, Eq(e, 0)), (30*c**3*d**3*log(d/e + x)/(30
*d**3*e**4 + 90*d**2*e**5*x + 90*d*e**6*x**2 + 30*e**7*x**3) + 11*c**3*d**3
/(30*d**3*e**4 + 90*d**2*e**5*x + 90*d*e**6*x**2 + 30*e**7*x**3) + 90*c**3*
d**2*e*x*log(d/e + x)/(30*d**3*e**4 + 90*d**2*e**5*x + 90*d*e**6*x**2 + 30*
e**7*x**3) + 3*c**3*d**2*e*x/(30*d**3*e**4 + 90*d**2*e**5*x + 90*d*e**6*x**
2 + 30*e**7*x**3) + 90*c**3*d*e**2*x**2*log(d/e + x)/(30*d**3*e**4 + 90*d**
2*e**5*x + 90*d*e**6*x**2 + 30*e**7*x**3) - 42*c**3*d*e**2*x**2/(30*d**3*e*
*4 + 90*d**2*e**5*x + 90*d*e**6*x**2 + 30*e**7*x**3) + 30*c**3*e**3*x**3*lo
g(d/e + x)/(30*d**3*e**4 + 90*d**2*e**5*x + 90*d*e**6*x**2 + 30*e**7*x**3)
- 44*c**3*e**3*x**3/(30*d**3*e**4 + 90*d**2*e**5*x + 90*d*e**6*x**2 + 30*e*
*7*x**3), Eq(m, -7)), (-60*c**3*d**3*log(d/e + x)/(20*d**2*e**4 + 40*d*e**5
*x + 20*e**6*x**2) - 27*c**3*d**3/(20*d**2*e**4 + 40*d*e**5*x + 20*e**6*x**
2) - 120*c**3*d**2*e*x*log(d/e + x)/(20*d**2*e**4 + 40*d*e**5*x + 20*e**6*x
**2) + 6*c**3*d**2*e*x/(20*d**2*e**4 + 40*d*e**5*x + 20*e**6*x**2) - 60*c**
3*d*e**2*x**2*log(d/e + x)/(20*d**2*e**4 + 40*d*e**5*x + 20*e**6*x**2) + 63
*c**3*d*e**2*x**2/(20*d**2*e**4 + 40*d*e**5*x + 20*e**6*x**2) + 20*c**3*e**
3*x**3/(20*d**2*e**4 + 40*d*e**5*x + 20*e**6*x**2), Eq(m, -6)), (12*c**3*d*
*3*log(d/e + x)/(4*d*e**4 + 4*e**5*x) + 7*c**3*d**3/(4*d*e**4 + 4*e**5*x) +
 12*c**3*d**2*e*x*log(d/e + x)/(4*d*e**4 + 4*e**5*x) - 5*c**3*d**2*e*x/(4*d
*e**4 + 4*e**5*x) - 6*c**3*d*e**2*x**2/(4*d*e**4 + 4*e**5*x) + 2*c**3*e**3*
x**3/(4*d*e**4 + 4*e**5*x), Eq(m, -5)), (-c**3*d**3*log(d/e + x)/e**4 + c**
3*d**2*x/e**3 - c**3*d*x**2/(2*e**2) + c**3*x**3/(3*e), Eq(m, -4)), (-6*c**
3*d**7*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m
+ 840*e**4) + 6*c**3*d**6*e*m*x*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 17
9*e**4*m**2 + 638*e**4*m + 840*e**4) - 3*c**3*d**5*e**2*m**2*x**2*(d + e*x)
**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) - 3*
c**3*d**5*e**2*m*x**2*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**
2 + 638*e**4*m + 840*e**4) + c**3*d**4*e**3*m**3*x**3*(d + e*x)**m/(e**4*m*
*4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 3*c**3*d**4*e*
*3*m**2*x**3*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e
**4*m + 840*e**4) + 2*c**3*d**4*e**3*m*x**3*(d + e*x)**m/(e**4*m**4 + 22*e*
```

```
*4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 4*c**3*d**3*e**4*m**3*x*
*4*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 84
0*e**4) + 42*c**3*d**3*e**4*m**2*x**4*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**
3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 158*c**3*d**3*e**4*m*x**4*(d +
 e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4)
 + 210*c**3*d**3*e**4*x**4*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**
4*m**2 + 638*e**4*m + 840*e**4) + 6*c**3*d**2*e**5*m**3*x**5*(d + e*x)**m/(
e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 78*c**3
*d**2*e**5*m**2*x**5*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2
 + 638*e**4*m + 840*e**4) + 342*c**3*d**2*e**5*m*x**5*(d + e*x)**m/(e**4*m*
*4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 504*c**3*d**2*
e**5*x**5*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4
*m + 840*e**4) + 4*c**3*d*e**6*m**3*x**6*(d + e*x)**m/(e**4*m**4 + 22*e**4*
m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 57*c**3*d*e**6*m**2*x**6*(d
 + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**
4) + 269*c**3*d*e**6*m*x**6*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e*
*4*m**2 + 638*e**4*m + 840*e**4) + 420*c**3*d*e**6*x**6*(d + e*x)**m/(e**4*
m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + c**3*e**7*m*
*3*x**7*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m
 + 840*e**4) + 15*c**3*e**7*m**2*x**7*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**
3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 74*c**3*e**7*m*x**7*(d + e*x)*
*m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 120
*c**3*e**7*x**7*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 63
8*e**4*m + 840*e**4), True))
```

Giac [B] time = 1.25299, size = 713, normalized size = 7.51

$$\frac{(xe+d)^m c^3 m^3 x^7 e^7 + 4\,(xe+d)^m c^3 d m^3 x^6 e^6 + 6\,(xe+d)^m c^3 d^2 m^3 x^5 e^5 + 4\,(xe+d)^m c^3 d^3 m^3 x^4 e^4 + (xe+d)^m c^3 d^4 m^3 x^3 e^3 + 1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e*x^2+c*d*x)^3,x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*c^3*m^3*x^7*e^7 + 4*(x*e + d)^m*c^3*d*m^3*x^6*e^6 + 6*(x*e + d
)^m*c^3*d^2*m^3*x^5*e^5 + 4*(x*e + d)^m*c^3*d^3*m^3*x^4*e^4 + (x*e + d)^m*c
^3*d^4*m^3*x^3*e^3 + 15*(x*e + d)^m*c^3*m^2*x^7*e^7 + 57*(x*e + d)^m*c^3*d*
m^2*x^6*e^6 + 78*(x*e + d)^m*c^3*d^2*m^2*x^5*e^5 + 42*(x*e + d)^m*c^3*d^3*m
^2*x^4*e^4 + 3*(x*e + d)^m*c^3*d^4*m^2*x^3*e^3 - 3*(x*e + d)^m*c^3*d^5*m^2*
x^2*e^2 + 74*(x*e + d)^m*c^3*m*x^7*e^7 + 269*(x*e + d)^m*c^3*d*m*x^6*e^6 +
342*(x*e + d)^m*c^3*d^2*m*x^5*e^5 + 158*(x*e + d)^m*c^3*d^3*m*x^4*e^4 + 2*(
x*e + d)^m*c^3*d^4*m*x^3*e^3 - 3*(x*e + d)^m*c^3*d^5*m*x^2*e^2 + 6*(x*e + d
)^m*c^3*d^6*m*x*e + 120*(x*e + d)^m*c^3*x^7*e^7 + 420*(x*e + d)^m*c^3*d*x^6
*e^6 + 504*(x*e + d)^m*c^3*d^2*x^5*e^5 + 210*(x*e + d)^m*c^3*d^3*x^4*e^4 -
6*(x*e + d)^m*c^3*d^7)/(m^4*e^4 + 22*m^3*e^4 + 179*m^2*e^4 + 638*m*e^4 + 84
0*e^4)
```

3.435 $\int (d+ex)^m \left(cdx+cex^2\right)^2 \, dx$

Optimal. Leaf size=69

$$\frac{c^2d^2(d+ex)^{m+3}}{e^3(m+3)}-\frac{2c^2d(d+ex)^{m+4}}{e^3(m+4)}+\frac{c^2(d+ex)^{m+5}}{e^3(m+5)}$$

```
[Out] (c^2*d^2*(d + e*x)^(3 + m))/(e^3*(3 + m)) - (2*c^2*d*(d + e*x)^(4 + m))/(e^
3*(4 + m)) + (c^2*(d + e*x)^(5 + m))/(e^3*(5 + m))
```

Rubi [A] time = 0.0435253, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {626, 12, 43}

$$\frac{c^2d^2(d+ex)^{m+3}}{e^3(m+3)}-\frac{2c^2d(d+ex)^{m+4}}{e^3(m+4)}+\frac{c^2(d+ex)^{m+5}}{e^3(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(c*d*x + c*e*x^2)^2,x]
```

```
[Out] (c^2*d^2*(d + e*x)^(3 + m))/(e^3*(3 + m)) - (2*c^2*d*(d + e*x)^(4 + m))/(e^
3*(4 + m)) + (c^2*(d + e*x)^(5 + m))/(e^3*(5 + m))
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^m \left(cdx+cex^2\right)^2 \, dx &= \int c^2x^2(d+ex)^{2+m} \, dx\\
&= c^2 \int x^2(d+ex)^{2+m} \, dx\\
&= c^2 \int \left(\frac{d^2(d+ex)^{2+m}}{e^2}-\frac{2d(d+ex)^{3+m}}{e^2}+\frac{(d+ex)^{4+m}}{e^2}\right) dx\\
&= \frac{c^2d^2(d+ex)^{3+m}}{e^3(3+m)}-\frac{2c^2d(d+ex)^{4+m}}{e^3(4+m)}+\frac{c^2(d+ex)^{5+m}}{e^3(5+m)}
\end{aligned}$$

Mathematica [A] time = 0.030712, size = 60, normalized size = 0.87

$$\frac{c^2(d+ex)^{m+3}\left(2d^2-2de(m+3)x+e^2\left(m^2+7m+12\right)x^2\right)}{e^3(m+3)(m+4)(m+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(c*d*x + c*e*x^2)^2,x]
```

```
[Out] (c^2*(d + e*x)^(3 + m)*(2*d^2 - 2*d*e*(3 + m)*x + e^2*(12 + 7*m + m^2)*x^2)
)/(e^3*(3 + m)*(4 + m)*(5 + m))
```

Maple [A] time = 0.047, size = 76, normalized size = 1.1

$$\frac{(ex+d)^{3+m}\left(e^2m^2x^2+7\,e^2mx^2-2\,demx+12\,x^2e^2-6\,dxe+2\,d^2\right)c^2}{e^3\left(m^3+12\,m^2+47\,m+60\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*e*x^2+c*d*x)^2,x)
```

```
[Out] (e*x+d)^(3+m)*(e^2*m^2*x^2+7*e^2*m*x^2-2*d*e*m*x+12*e^2*x^2-6*d*e*x+2*d^2)*
c^2/e^3/(m^3+12*m^2+47*m+60)
```

Maxima [B] time = 1.30397, size = 436, normalized size = 6.32

$$\frac{\left(\left(m^2+3\,m+2\right)e^3x^3+\left(m^2+m\right)de^2x^2-2\,d^2emx+2\,d^3\right)(ex+d)^m c^2d^2}{\left(m^3+6\,m^2+11\,m+6\right)e^3}+\frac{2\left(\left(m^3+6\,m^2+11\,m+6\right)e^4x^4+\left(m^3+3\,m^2+\right.\right.}{\left(m^4+10\,n\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e*x^2+c*d*x)^2,x, algorithm="maxima")
```

```
[Out] ((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x
+ d)^m*c^2*d^2/((m^3 + 6*m^2 + 11*m + 6)*e^3) + 2*((m^3 + 6*m^2 + 11*m + 6)
*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*
e*m*x - 6*d^4)*(e*x + d)^m*c^2*d/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^3)
+ ((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*
m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2
 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m*c^2/((m^5 + 15*m^4 + 85*m^3 + 225*m^2
 + 274*m + 120)*e^3)
```

Fricas [B] time = 2.09147, size = 394, normalized size = 5.71

$$-\frac{\left(2\,c^2d^4emx-2\,c^2d^5-\left(c^2e^5m^2+7\,c^2e^5m+12\,c^2e^5\right)x^5-\left(3\,c^2de^4m^2+19\,c^2de^4m+30\,c^2de^4\right)x^4-\left(3\,c^2d^2e^3m^2+15\,c^2d^2e\right.\right.}{e^3m^3+12\,e^3m^2+47\,e^3m+60\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e*x^2+c*d*x)^2,x, algorithm="fricas")
```

```
[Out] -(2*c^2*d^4*e*m*x - 2*c^2*d^5 - (c^2*e^5*m^2 + 7*c^2*e^5*m + 12*c^2*e^5)*x^
5 - (3*c^2*d*e^4*m^2 + 19*c^2*d*e^4*m + 30*c^2*d*e^4)*x^4 - (3*c^2*d^2*e^3*
m^2 + 15*c^2*d^2*e^3*m + 20*c^2*d^2*e^3)*x^3 - (c^2*d^3*e^2*m^2 + c^2*d^3*e
^2*m)*x^2)*(e*x + d)^m/(e^3*m^3 + 12*e^3*m^2 + 47*e^3*m + 60*e^3)
```

Sympy [A] time = 3.87346, size = 1047, normalized size = 15.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*e*x**2+c*d*x)**2,x)
```

```
[Out] Piecewise((c**2*d**2*d**m*x**3/3, Eq(e, 0)), (12*c**2*d**2*log(d/e + x)/(12
*d**2*e**3 + 24*d*e**4*x + 12*e**5*x**2) + 7*c**2*d**2/(12*d**2*e**3 + 24*d
*e**4*x + 12*e**5*x**2) + 24*c**2*d*e*x*log(d/e + x)/(12*d**2*e**3 + 24*d*e
**4*x + 12*e**5*x**2) + 2*c**2*d*e*x/(12*d**2*e**3 + 24*d*e**4*x + 12*e**5*
x**2) + 12*c**2*e**2*x**2*log(d/e + x)/(12*d**2*e**3 + 24*d*e**4*x + 12*e**
5*x**2) - 11*c**2*e**2*x**2/(12*d**2*e**3 + 24*d*e**4*x + 12*e**5*x**2), Eq
(m, -5)), (-6*c**2*d**2*log(d/e + x)/(3*d*e**3 + 3*e**4*x) - 5*c**2*d**2/(3
*d*e**3 + 3*e**4*x) - 6*c**2*d*e*x*log(d/e + x)/(3*d*e**3 + 3*e**4*x) + c**
2*d*e*x/(3*d*e**3 + 3*e**4*x) + 3*c**2*e**2*x**2/(3*d*e**3 + 3*e**4*x), Eq(
m, -4)), (c**2*d**2*log(d/e + x)/e**3 - c**2*d*x/e**2 + c**2*x**2/(2*e), Eq
(m, -3)), (2*c**2*d**5*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*m +
 60*e**3) - 2*c**2*d**4*e*m*x*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e
**3*m + 60*e**3) + c**2*d**3*e**2*m**2*x**2*(d + e*x)**m/(e**3*m**3 + 12*e*
*3*m**2 + 47*e**3*m + 60*e**3) + c**2*d**3*e**2*m*x**2*(d + e*x)**m/(e**3*m
**3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3) + 3*c**2*d**2*e**3*m**2*x**3*(d +
 e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3) + 15*c**2*d**2*e*
*3*m*x**3*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3) + 2
0*c**2*d**2*e**3*x**3*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*m +
60*e**3) + 3*c**2*d*e**4*m**2*x**4*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**2 +
 47*e**3*m + 60*e**3) + 19*c**2*d*e**4*m*x**4*(d + e*x)**m/(e**3*m**3 + 12*
e**3*m**2 + 47*e**3*m + 60*e**3) + 30*c**2*d*e**4*x**4*(d + e*x)**m/(e**3*m
**3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3) + c**2*e**5*m**2*x**5*(d + e*x)**
m/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3) + 7*c**2*e**5*m*x**5*(d
+ e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3) + 12*c**2*e**5*x
**5*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3), True))
```

Giac [B] time = 1.22437, size = 394, normalized size = 5.71

$$\frac{(xe+d)^m c^2 m^2 x^5 e^5 + 3\,(xe+d)^m c^2 d m^2 x^4 e^4 + 3\,(xe+d)^m c^2 d^2 m^2 x^3 e^3 + (xe+d)^m c^2 d^3 m^2 x^2 e^2 + 7\,(xe+d)^m c^2 m x^5 e^5 + 1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e*x^2+c*d*x)^2,x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*c^2*m^2*x^5*e^5 + 3*(x*e + d)^m*c^2*d*m^2*x^4*e^4 + 3*(x*e + d
)^m*c^2*d^2*m^2*x^3*e^3 + (x*e + d)^m*c^2*d^3*m^2*x^2*e^2 + 7*(x*e + d)^m*c
^2*m*x^5*e^5 + 19*(x*e + d)^m*c^2*d*m*x^4*e^4 + 15*(x*e + d)^m*c^2*d^2*m*x^
3*e^3 + (x*e + d)^m*c^2*d^3*m*x^2*e^2 - 2*(x*e + d)^m*c^2*d^4*m*x*e + 12*(x
*e + d)^m*c^2*x^5*e^5 + 30*(x*e + d)^m*c^2*d*x^4*e^4 + 20*(x*e + d)^m*c^2*d
^2*x^3*e^3 + 2*(x*e + d)^m*c^2*d^5)/(m^3*e^3 + 12*m^2*e^3 + 47*m*e^3 + 60*e
^3)
```

3.436 $\int (d+ex)^m \left(cdx+cex^2\right) dx$

Optimal. Leaf size=41

$$\frac{c(d+ex)^{m+3}}{e^2(m+3)}-\frac{cd(d+ex)^{m+2}}{e^2(m+2)}$$

```
[Out] -((c*d*(d + e*x)^(2 + m))/(e^2*(2 + m))) + (c*(d + e*x)^(3 + m))/(e^2*(3 +
m))
```

Rubi [A] time = 0.0221012, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {626, 12, 43}

$$\frac{c(d+ex)^{m+3}}{e^2(m+3)}-\frac{cd(d+ex)^{m+2}}{e^2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(c*d*x + c*e*x^2),x]
```

```
[Out] -((c*d*(d + e*x)^(2 + m))/(e^2*(2 + m))) + (c*(d + e*x)^(3 + m))/(e^2*(3 +
m))
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^m \left(cdx+cex^2\right) dx &= \int cx(d+ex)^{1+m}\, dx \\
&= c\int x(d+ex)^{1+m}\, dx \\
&= c\int \left(-\frac{d(d+ex)^{1+m}}{e}+\frac{(d+ex)^{2+m}}{e}\right) dx \\
&= -\frac{cd(d+ex)^{2+m}}{e^2(2+m)}+\frac{c(d+ex)^{3+m}}{e^2(3+m)}
\end{aligned}$$

Mathematica [A] time = 0.0199035, size = 34, normalized size = 0.83

$$\frac{c(d+ex)^{m+2}(e(m+2)x-d)}{e^2(m+2)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(c*d*x + c*e*x^2),x]
```

```
[Out] (c*(d + e*x)^(2 + m)*(-d + e*(2 + m)*x))/(e^2*(2 + m)*(3 + m))
```

Maple [A] time = 0.044, size = 37, normalized size = 0.9

$$-\frac{c\,(ex+d)^{2+m}\,(-mex-2\,ex+d)}{e^2\left(m^2+5\,m+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*e*x^2+c*d*x),x)
```

```
[Out] -c*(e*x+d)^(2+m)*(-e*m*x-2*e*x+d)/e^2/(m^2+5*m+6)
```

Maxima [B] time = 1.23387, size = 154, normalized size = 3.76

$$\frac{\left(e^2(m+1)x^2+demx-d^2\right)(ex+d)^m cd}{\left(m^2+3\,m+2\right)e^2}+\frac{\left(\left(m^2+3\,m+2\right)e^3x^3+\left(m^2+m\right)de^2x^2-2\,d^2emx+2\,d^3\right)(ex+d)^m c}{\left(m^3+6\,m^2+11\,m+6\right)e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e*x^2+c*d*x),x, algorithm="maxima")
```

```
[Out] (e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*c*d/((m^2 + 3*m + 2)*e^2) + (
(m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x +
 d)^m*c/((m^3 + 6*m^2 + 11*m + 6)*e^2)
```

Fricas [A] time = 2.08918, size = 163, normalized size = 3.98

$$\frac{\left(cd^2emx-cd^3+\left(ce^3m+2\,ce^3\right)x^3+\left(2\,cde^2m+3\,cde^2\right)x^2\right)(ex+d)^m}{e^2m^2+5\,e^2m+6\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e*x^2+c*d*x),x, algorithm="fricas")
```

```
[Out] (c*d^2*e*m*x - c*d^3 + (c*e^3*m + 2*c*e^3)*x^3 + (2*c*d*e^2*m + 3*c*d*e^2)*
x^2)*(e*x + d)^m/(e^2*m^2 + 5*e^2*m + 6*e^2)
```

Sympy [A] time = 1.50261, size = 299, normalized size = 7.29

$$\begin{cases} \frac{cdd^m x^2}{2} & \text{for } e = 0 \\ \frac{cd \log\left(\frac{d}{e}+x\right)}{de^2+e^3x} + \frac{cd}{de^2+e^3x} + \frac{cex \log\left(\frac{d}{e}+x\right)}{de^2+e^3x} & \text{for } m = -3 \\ -\frac{cd \log\left(\frac{d}{e}+x\right)}{e^2} + \frac{cx}{e} & \text{for } m = -2 \\ -\frac{cd^3(d+ex)^m}{e^2m^2+5e^2m+6e^2} + \frac{cd^2emx(d+ex)^m}{e^2m^2+5e^2m+6e^2} + \frac{2cde^2mx^2(d+ex)^m}{e^2m^2+5e^2m+6e^2} + \frac{3cde^2x^2(d+ex)^m}{e^2m^2+5e^2m+6e^2} + \frac{ce^3mx^3(d+ex)^m}{e^2m^2+5e^2m+6e^2} + \frac{2ce^3x^3(d+ex)^m}{e^2m^2+5e^2m+6e^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*e*x**2+c*d*x),x)
```

```
[Out] Piecewise((c*d*d**m*x**2/2, Eq(e, 0)), (c*d*log(d/e + x)/(d*e**2 + e**3*x)
+ c*d/(d*e**2 + e**3*x) + c*e*x*log(d/e + x)/(d*e**2 + e**3*x), Eq(m, -3)),
 (-c*d*log(d/e + x)/e**2 + c*x/e, Eq(m, -2)), (-c*d**3*(d + e*x)**m/(e**2*m
**2 + 5*e**2*m + 6*e**2) + c*d**2*e*m*x*(d + e*x)**m/(e**2*m**2 + 5*e**2*m
+ 6*e**2) + 2*c*d*e**2*m*x**2*(d + e*x)**m/(e**2*m**2 + 5*e**2*m + 6*e**2)
+ 3*c*d*e**2*x**2*(d + e*x)**m/(e**2*m**2 + 5*e**2*m + 6*e**2) + c*e**3*m*x
**3*(d + e*x)**m/(e**2*m**2 + 5*e**2*m + 6*e**2) + 2*c*e**3*x**3*(d + e*x)*
*m/(e**2*m**2 + 5*e**2*m + 6*e**2), True))
```

Giac [B] time = 1.27748, size = 159, normalized size = 3.88

$$\frac{(xe+d)^m cmx^3e^3 + 2\,(xe+d)^m cdmx^2e^2 + (xe+d)^m cd^2mxe + 2\,(xe+d)^m cx^3e^3 + 3\,(xe+d)^m cdx^2e^2 - (xe+d)^m cd^3}{m^2e^2 + 5\,me^2 + 6\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e*x^2+c*d*x),x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*c*m*x^3*e^3 + 2*(x*e + d)^m*c*d*m*x^2*e^2 + (x*e + d)^m*c*d^2*
m*x*e + 2*(x*e + d)^m*c*x^3*e^3 + 3*(x*e + d)^m*c*d*x^2*e^2 - (x*e + d)^m*c
*d^3)/(m^2*e^2 + 5*m*e^2 + 6*e^2)
```

3.437 $\int (d+ex)^m\,dx$

Optimal. Leaf size=18

$$\frac{(d+ex)^{m+1}}{e(m+1)}$$

```
[Out] (d + e*x)^(1 + m)/(e*(1 + m))
```

Rubi [A] time = 0.0031172, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(d+ex)^{m+1}}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m,x]
```

```
[Out] (d + e*x)^(1 + m)/(e*(1 + m))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^m\,dx=\frac{(d+ex)^{1+m}}{e(1+m)}$$

Mathematica [A] time = 0.0092179, size = 17, normalized size = 0.94

$$\frac{(d+ex)^{m+1}}{em+e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m,x]
```

```
[Out] (d + e*x)^(1 + m)/(e + e*m)
```

Maple [A] time = 0.043, size = 19, normalized size = 1.1

$$\frac{(ex+d)^{1+m}}{e\,(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m,x)
```

```
[Out] (e*x+d)^(1+m)/e/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.9878, size = 45, normalized size = 2.5

$$\frac{(ex+d)(ex+d)^m}{em+e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m,x, algorithm="fricas")
```

```
[Out] (e*x + d)*(e*x + d)^m/(e*m + e)
```

Sympy [A] time = 0.06186, size = 20, normalized size = 1.11

$$\frac{\begin{cases} \frac{(d+ex)^{m+1}}{m+1} & \text{for } m \neq -1 \\ \log(d+ex) & \text{otherwise} \end{cases}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m,x)
```

```
[Out] Piecewise(((d + e*x)**(m + 1)/(m + 1), Ne(m, -1)), (log(d + e*x), True))/e
```

Giac [A] time = 1.23295, size = 24, normalized size = 1.33

$$\frac{(xe+d)^{m+1}e^{(-1)}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m,x, algorithm="giac")
```

```
[Out] (x*e + d)^(m + 1)*e^(-1)/(m + 1)
```

3.438 $\int \frac{(d+ex)^m}{cdx+cex^2} dx$

Optimal. Leaf size=32

$$-\frac{(d+ex)^m {}_2F_1\left(1,m;m+1;\frac{ex}{d}+1\right)}{cdm}$$

```
[Out] -(((d + e*x)^m*Hypergeometric2F1[1, m, 1 + m, 1 + (e*x)/d])/(c*d*m))
```

Rubi [A] time = 0.0151232, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {626, 12, 65}

$$-\frac{(d+ex)^m {}_2F_1\left(1,m;m+1;\frac{ex}{d}+1\right)}{cdm}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(c*d*x + c*e*x^2),x]
```

```
[Out] -(((d + e*x)^m*Hypergeometric2F1[1, m, 1 + m, 1 + (e*x)/d])/(c*d*m))
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^m}{cdx+cex^2} dx &= \int \frac{(d+ex)^{-1+m}}{cx} dx\\ &= \frac{\int \frac{(d+ex)^{-1+m}}{x} dx}{c}\\ &= -\frac{(d+ex)^m {}_2F_1\left(1,m;1+m;1+\frac{ex}{d}\right)}{cdm}\end{aligned}$$

Mathematica [A] time = 0.0084562, size = 32, normalized size = 1.

$$-\frac{(d+ex)^m {}_2F_1\left(1,m;m+1;\frac{ex}{d}+1\right)}{cdm}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(c*d*x + c*e*x^2),x]
```

```
[Out] -(((d + e*x)^m*Hypergeometric2F1[1, m, 1 + m, 1 + (e*x)/d])/(c*d*m))
```

Maple [F] time = 0.547, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{ce\,x^2+cdx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(c*e*x^2+c*d*x),x)
```

```
[Out] int((e*x+d)^m/(c*e*x^2+c*d*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{ce\,x^2+cdx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*e*x^2+c*d*x),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/(c*e*x^2 + c*d*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^m}{ce\,x^2+cdx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*e*x^2+c*d*x),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m/(c*e*x^2 + c*d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{(d+ex)^m}{dx+ex^2}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(c*e*x**2+c*d*x),x)
```

```
[Out] Integral((d + e*x)**m/(d*x + e*x**2), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{cex^2+cdx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*e*x^2+c*d*x),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(c*e*x^2 + c*d*x), x)
```

3.439 $\int \frac{(d+ex)^m}{\left(cdx+cex^2\right)^2}\,dx$

Optimal. Leaf size=39

$$-\frac{e(d+ex)^{m-1}\,{}_2F_1\left(2,m-1;m;\frac{ex}{d}+1\right)}{c^2d^2(1-m)}$$

```
[Out] -((e*(d + e*x)^(-1 + m)*Hypergeometric2F1[2, -1 + m, m, 1 + (e*x)/d])/(c^2*
d^2*(1 - m)))
```

Rubi [A] time = 0.02125, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {626, 12, 65}

$$-\frac{e(d+ex)^{m-1}\,{}_2F_1\left(2,m-1;m;\frac{ex}{d}+1\right)}{c^2d^2(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(c*d*x + c*e*x^2)^2,x]
```

```
[Out] -((e*(d + e*x)^(-1 + m)*Hypergeometric2F1[2, -1 + m, m, 1 + (e*x)/d])/(c^2*
d^2*(1 - m)))
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^m}{\left(cdx+cex^2\right)^2}\,dx &= \int \frac{(d+ex)^{-2+m}}{c^2x^2}\,dx\\ &= \frac{\int \frac{(d+ex)^{-2+m}}{x^2}\,dx}{c^2}\\ &= -\frac{e(d+ex)^{-1+m}\,{}_2F_1\left(2,-1+m;m;1+\frac{ex}{d}\right)}{c^2d^2(1-m)}\end{aligned}$$

Mathematica [A] time = 0.0126374, size = 36, normalized size = 0.92

$$\frac{e(d+ex)^{m-1}\,{}_2F_1\left(2,m-1;m;\frac{ex}{d}+1\right)}{c^2d^2(m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(c*d*x + c*e*x^2)^2,x]
```

```
[Out] (e*(d + e*x)^(-1 + m)*Hypergeometric2F1[2, -1 + m, m, 1 + (e*x)/d])/(c^2*d^
2*(-1 + m))
```

Maple [F] time = 0.563, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(cex^2+cdx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(c*e*x^2+c*d*x)^2,x)
```

```
[Out] int((e*x+d)^m/(c*e*x^2+c*d*x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(cex^2+cdx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*e*x^2+c*d*x)^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/(c*e*x^2 + c*d*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(ex+d)^m}{c^2e^2x^4+2\,c^2dex^3+c^2d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*e*x^2+c*d*x)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m/(c^2*e^2*x^4 + 2*c^2*d*e*x^3 + c^2*d^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{(d+ex)^m}{d^2x^2+2dex^3+e^2x^4}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(c*e*x**2+c*d*x)**2,x)
```

```
[Out] Integral((d + e*x)**m/(d**2*x**2 + 2*d*e*x**3 + e**2*x**4), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(cex^2+cdx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*e*x^2+c*d*x)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(c*e*x^2 + c*d*x)^2, x)
```

3.440 $\int (d+ex)^m \left(bx+cx^2\right)^3 \, dx$

Optimal. Leaf size=267

$$\frac{3d(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)(d+ex)^{m+3}}{e^7(m+3)}-\frac{(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)(d+ex)^{m+4}}{e^7(m+4)}+\frac{3c\left(b^2e^2-5bcde+\right.}{e^7(r}$$

```
[Out] (d^3*(c*d - b*e)^3*(d + e*x)^(1 + m))/(e^7*(1 + m)) - (3*d^2*(c*d - b*e)^2*
(2*c*d - b*e)*(d + e*x)^(2 + m))/(e^7*(2 + m)) + (3*d*(c*d - b*e)*(5*c^2*d^
2 - 5*b*c*d*e + b^2*e^2)*(d + e*x)^(3 + m))/(e^7*(3 + m)) - ((2*c*d - b*e)*
(10*c^2*d^2 - 10*b*c*d*e + b^2*e^2)*(d + e*x)^(4 + m))/(e^7*(4 + m)) + (3*c
*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*(d + e*x)^(5 + m))/(e^7*(5 + m)) - (3*c^
2*(2*c*d - b*e)*(d + e*x)^(6 + m))/(e^7*(6 + m)) + (c^3*(d + e*x)^(7 + m))/
(e^7*(7 + m))
```

Rubi [A] time = 0.165443, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$\frac{3d(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)(d+ex)^{m+3}}{e^7(m+3)}-\frac{(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)(d+ex)^{m+4}}{e^7(m+4)}+\frac{3c\left(b^2e^2-5bcde+\right.}{e^7(r}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(b*x + c*x^2)^3,x]
```

```
[Out] (d^3*(c*d - b*e)^3*(d + e*x)^(1 + m))/(e^7*(1 + m)) - (3*d^2*(c*d - b*e)^2*
(2*c*d - b*e)*(d + e*x)^(2 + m))/(e^7*(2 + m)) + (3*d*(c*d - b*e)*(5*c^2*d^
2 - 5*b*c*d*e + b^2*e^2)*(d + e*x)^(3 + m))/(e^7*(3 + m)) - ((2*c*d - b*e)*
(10*c^2*d^2 - 10*b*c*d*e + b^2*e^2)*(d + e*x)^(4 + m))/(e^7*(4 + m)) + (3*c
*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*(d + e*x)^(5 + m))/(e^7*(5 + m)) - (3*c^
2*(2*c*d - b*e)*(d + e*x)^(6 + m))/(e^7*(6 + m)) + (c^3*(d + e*x)^(7 + m))/
(e^7*(7 + m))
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^m \left(bx+cx^2\right)^3 \, dx = \int \left(\frac{d^3(cd-be)^3(d+ex)^m}{e^6}-\frac{3d^2(cd-be)^2(2cd-be)(d+ex)^{1+m}}{e^6}+\frac{3d(cd-be)\left(5c^2d^2-\right.}{}\right.$$

$$=\frac{d^3(cd-be)^3(d+ex)^{1+m}}{e^7(1+m)}-\frac{3d^2(cd-be)^2(2cd-be)(d+ex)^{2+m}}{e^7(2+m)}+\frac{3d(cd-be)\left(5c^2d^2-5b\right.}{e^7(3}$$

Mathematica [A] time = 0.206048, size = 236, normalized size = 0.88

$$\frac{(d+ex)^{m+1}\left(\frac{3c(d+ex)^4\left(b^2e^2-5bcde+5c^2d^2\right)}{m+5}-\frac{(d+ex)^3(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{m+4}+\frac{3d(d+ex)^2(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)}{m+3}-\frac{3c^2(d+ex)^5(2cd-be}{m+6}\right.}{e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(b*x + c*x^2)^3,x]
```

```
[Out] ((d + e*x)^(1 + m)*((d^3*(c*d - b*e)^3)/(1 + m) - (3*d^2*(c*d - b*e)^2*(2*c
*d - b*e)*(d + e*x))/(2 + m) + (3*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^
2*e^2)*(d + e*x)^2)/(3 + m) - ((2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e + b^2
*e^2)*(d + e*x)^3)/(4 + m) + (3*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*(d + e*
x)^4)/(5 + m) - (3*c^2*(2*c*d - b*e)*(d + e*x)^5)/(6 + m) + (c^3*(d + e*x)^
6)/(7 + m)))/e^7
```

Maple [B] time = 0.055, size = 1528, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x)^3,x)
```

```
[Out] -(e*x+d)^(1+m)*(-c^3*e^6*m^6*x^6-3*b*c^2*e^6*m^6*x^5-21*c^3*e^6*m^5*x^6-3*b
^2*c*e^6*m^6*x^4-66*b*c^2*e^6*m^5*x^5+6*c^3*d*e^5*m^5*x^5-175*c^3*e^6*m^4*x
^6-b^3*e^6*m^6*x^3-69*b^2*c*e^6*m^5*x^4+15*b*c^2*d*e^5*m^5*x^4-570*b*c^2*e^
6*m^4*x^5+90*c^3*d*e^5*m^4*x^5-735*c^3*e^6*m^3*x^6-24*b^3*e^6*m^5*x^3+12*b^
2*c*d*e^5*m^5*x^3-621*b^2*c*e^6*m^4*x^4+255*b*c^2*d*e^5*m^4*x^4-2460*b*c^2*
e^6*m^3*x^5-30*c^3*d^2*e^4*m^4*x^4+510*c^3*d*e^5*m^3*x^5-1624*c^3*e^6*m^2*x
^6+3*b^3*d*e^5*m^5*x^2-226*b^3*e^6*m^4*x^3+228*b^2*c*d*e^5*m^4*x^3-2775*b^2
*c*e^6*m^3*x^4-60*b*c^2*d^2*e^4*m^4*x^3+1575*b*c^2*d*e^5*m^3*x^4-5547*b*c^2
*e^6*m^2*x^5-300*c^3*d^2*e^4*m^3*x^4+1350*c^3*d*e^5*m^2*x^5-1764*c^3*e^6*m*
x^6+63*b^3*d*e^5*m^4*x^2-1056*b^3*e^6*m^3*x^3-36*b^2*c*d^2*e^4*m^4*x^2+1572
*b^2*c*d*e^5*m^3*x^3-6432*b^2*c*e^6*m^2*x^4-780*b*c^2*d^2*e^4*m^3*x^3+4425*
b*c^2*d*e^5*m^2*x^4-6114*b*c^2*e^6*m*x^5+120*c^3*d^3*e^3*m^3*x^3-1050*c^3*d
^2*e^4*m^2*x^4+1644*c^3*d*e^5*m*x^5-720*c^3*e^6*x^6-6*b^3*d^2*e^4*m^4*x+489
*b^3*d*e^5*m^3*x^2-2545*b^3*e^6*m^2*x^3-576*b^2*c*d^2*e^4*m^3*x^2+4812*b^2*
c*d*e^5*m^2*x^3-7236*b^2*c*e^6*m*x^4+180*b*c^2*d^3*e^3*m^3*x^2-3180*b*c^2*d
^2*e^4*m^2*x^3+5610*b*c^2*d*e^5*m*x^4-2520*b*c^2*e^6*x^5+720*c^3*d^3*e^3*m^
2*x^3-1500*c^3*d^2*e^4*m*x^4+720*c^3*d*e^5*x^5-114*b^3*d^2*e^4*m^3*x+1701*b
^3*d*e^5*m^2*x^2-2952*b^3*e^6*m*x^3+72*b^2*c*d^3*e^3*m^3*x-2988*b^2*c*d^2*e
^4*m^2*x^2+6480*b^2*c*d*e^5*m*x^3-3024*b^2*c*e^6*x^4+1800*b*c^2*d^3*e^3*m^2
*x^2-4980*b*c^2*d^2*e^4*m*x^3+2520*b*c^2*d*e^5*x^4-360*c^3*d^4*e^2*m^2*x^2+
1320*c^3*d^3*e^3*m*x^3-720*c^3*d^2*e^4*x^4+6*b^3*d^3*e^3*m^3-750*b^3*d^2*e^
4*m^2*x+2532*b^3*d*e^5*m*x^2-1260*b^3*e^6*x^3+1008*b^2*c*d^3*e^3*m^2*x-5472
*b^2*c*d^2*e^4*m*x^2+3024*b^2*c*d*e^5*x^3-360*b*c^2*d^4*e^2*m^2*x+4140*b*c^
2*d^3*e^3*m*x^2-2520*b*c^2*d^2*e^4*x^3-1080*c^3*d^4*e^2*m*x^2+720*c^3*d^3*e
^3*x^3+108*b^3*d^3*e^3*m^2-1902*b^3*d^2*e^4*m*x+1260*b^3*d*e^5*x^2-72*b^2*c
*d^4*e^2*m^2+3960*b^2*c*d^3*e^3*m*x-3024*b^2*c*d^2*e^4*x^2-2880*b*c^2*d^4*e
^2*m*x+2520*b*c^2*d^3*e^3*x^2+720*c^3*d^5*e*m*x-720*c^3*d^4*e^2*x^2+642*b^3
*d^3*e^3*m-1260*b^3*d^2*e^4*x-936*b^2*c*d^4*e^2*m+3024*b^2*c*d^3*e^3*x+360*
b*c^2*d^5*e*m-2520*b*c^2*d^4*e^2*x+720*c^3*d^5*e*x+1260*b^3*d^3*e^3-3024*b^
2*c*d^4*e^2+2520*b*c^2*d^5*e-720*c^3*d^6)/e^7/(m^7+28*m^6+322*m^5+1960*m^4+
6769*m^3+13132*m^2+13068*m+5040)
```

Maxima [B] time = 1.32643, size = 903, normalized size = 3.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x)^3,x, algorithm="maxima")
```

```
[Out] ((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2
+ m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*b^3/((m^4 + 10*m^3 + 35
*m^2 + 50*m + 24)*e^4) + 3*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (
m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 +
 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m*b^2*c/((m^5
+ 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) + 3*((m^5 + 15*m^4 + 85*m^3
 + 225*m^2 + 274*m + 120)*e^6*x^6 + (m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)
*d*e^5*x^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*e^4*x^4 + 20*(m^3 + 3*m^2 +
 2*m)*d^3*e^3*x^3 - 60*(m^2 + m)*d^4*e^2*x^2 + 120*d^5*e*m*x - 120*d^6)*(e*
x + d)^m*b*c^2/((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720
)*e^6) + ((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^7*
x^7 + (m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d*e^6*x^6 - 6*(m^
5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^2*e^5*x^5 + 30*(m^4 + 6*m^3 + 11*m^2
 + 6*m)*d^3*e^4*x^4 - 120*(m^3 + 3*m^2 + 2*m)*d^4*e^3*x^3 + 360*(m^2 + m)*d
^5*e^2*x^2 - 720*d^6*e*m*x + 720*d^7)*(e*x + d)^m*c^3/((m^7 + 28*m^6 + 322*
m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^7)
```

Fricas [B] time = 2.18289, size = 3106, normalized size = 11.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x)^3,x, algorithm="fricas")
```

```
[Out] -(6*b^3*d^4*e^3*m^3 - 720*c^3*d^7 + 2520*b*c^2*d^6*e - 3024*b^2*c*d^5*e^2 +
 1260*b^3*d^4*e^3 - (c^3*e^7*m^6 + 21*c^3*e^7*m^5 + 175*c^3*e^7*m^4 + 735*c
^3*e^7*m^3 + 1624*c^3*e^7*m^2 + 1764*c^3*e^7*m + 720*c^3*e^7)*x^7 - (2520*b
*c^2*e^7 + (c^3*d*e^6 + 3*b*c^2*e^7)*m^6 + 3*(5*c^3*d*e^6 + 22*b*c^2*e^7)*m
^5 + 5*(17*c^3*d*e^6 + 114*b*c^2*e^7)*m^4 + 15*(15*c^3*d*e^6 + 164*b*c^2*e^
7)*m^3 + (274*c^3*d*e^6 + 5547*b*c^2*e^7)*m^2 + 6*(20*c^3*d*e^6 + 1019*b*c^
2*e^7)*m)*x^6 - 3*(1008*b^2*c*e^7 + (b*c^2*d*e^6 + b^2*c*e^7)*m^6 - (2*c^3*
d^2*e^5 - 17*b*c^2*d*e^6 - 23*b^2*c*e^7)*m^5 - (20*c^3*d^2*e^5 - 105*b*c^2*
d*e^6 - 207*b^2*c*e^7)*m^4 - 5*(14*c^3*d^2*e^5 - 59*b*c^2*d*e^6 - 185*b^2*c
*e^7)*m^3 - 2*(50*c^3*d^2*e^5 - 187*b*c^2*d*e^6 - 1072*b^2*c*e^7)*m^2 - 12*
(4*c^3*d^2*e^5 - 14*b*c^2*d*e^6 - 201*b^2*c*e^7)*m)*x^5 - (1260*b^3*e^7 + (
3*b^2*c*d*e^6 + b^3*e^7)*m^6 - 3*(5*b*c^2*d^2*e^5 - 19*b^2*c*d*e^6 - 8*b^3*
e^7)*m^5 + (30*c^3*d^3*e^4 - 195*b*c^2*d^2*e^5 + 393*b^2*c*d*e^6 + 226*b^3*
e^7)*m^4 + 3*(60*c^3*d^3*e^4 - 265*b*c^2*d^2*e^5 + 401*b^2*c*d*e^6 + 352*b^
3*e^7)*m^3 + 5*(66*c^3*d^3*e^4 - 249*b*c^2*d^2*e^5 + 324*b^2*c*d*e^6 + 509*
b^3*e^7)*m^2 + 18*(10*c^3*d^3*e^4 - 35*b*c^2*d^2*e^5 + 42*b^2*c*d*e^6 + 164
*b^3*e^7)*m)*x^4 - (b^3*d*e^6*m^6 - 3*(4*b^2*c*d^2*e^5 - 7*b^3*d*e^6)*m^5 +
 (60*b*c^2*d^3*e^4 - 192*b^2*c*d^2*e^5 + 163*b^3*d*e^6)*m^4 - 3*(40*c^3*d^4
*e^3 - 200*b*c^2*d^3*e^4 + 332*b^2*c*d^2*e^5 - 189*b^3*d*e^6)*m^3 - 4*(90*c
^3*d^4*e^3 - 345*b*c^2*d^3*e^4 + 456*b^2*c*d^2*e^5 - 211*b^3*d*e^6)*m^2 - 1
2*(20*c^3*d^4*e^3 - 70*b*c^2*d^3*e^4 + 84*b^2*c*d^2*e^5 - 35*b^3*d*e^6)*m)*
x^3 - 36*(2*b^2*c*d^5*e^2 - 3*b^3*d^4*e^3)*m^2 + 3*(b^3*d^2*e^5*m^5 - (12*b
^2*c*d^3*e^4 - 19*b^3*d^2*e^5)*m^4 + (60*b*c^2*d^4*e^3 - 168*b^2*c*d^3*e^4
+ 125*b^3*d^2*e^5)*m^3 - (120*c^3*d^5*e^2 - 480*b*c^2*d^4*e^3 + 660*b^2*c*d
^3*e^4 - 317*b^3*d^2*e^5)*m^2 - 6*(20*c^3*d^5*e^2 - 70*b*c^2*d^4*e^3 + 84*b
^2*c*d^3*e^4 - 35*b^3*d^2*e^5)*m)*x^2 + 6*(60*b*c^2*d^6*e - 156*b^2*c*d^5*e
^2 + 107*b^3*d^4*e^3)*m - 6*(b^3*d^3*e^4*m^4 - 6*(2*b^2*c*d^4*e^3 - 3*b^3*d
^3*e^4)*m^3 + (60*b*c^2*d^5*e^2 - 156*b^2*c*d^4*e^3 + 107*b^3*d^3*e^4)*m^2
```

```
- 6*(20*c^3*d^6*e - 70*b*c^2*d^5*e^2 + 84*b^2*c*d^4*e^3 - 35*b^3*d^3*e^4)*m
)*x)*(e*x + d)^m/(e^7*m^7 + 28*e^7*m^6 + 322*e^7*m^5 + 1960*e^7*m^4 + 6769*
e^7*m^3 + 13132*e^7*m^2 + 13068*e^7*m + 5040*e^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.38683, size = 3426, normalized size = 12.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x)^3,x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*c^3*m^6*x^7*e^7 + (x*e + d)^m*c^3*d*m^6*x^6*e^6 + 3*(x*e + d)^
m*b*c^2*m^6*x^6*e^7 + 21*(x*e + d)^m*c^3*m^5*x^7*e^7 + 3*(x*e + d)^m*b*c^2*
d*m^6*x^5*e^6 + 15*(x*e + d)^m*c^3*d*m^5*x^6*e^6 - 6*(x*e + d)^m*c^3*d^2*m^
5*x^5*e^5 + 3*(x*e + d)^m*b^2*c*m^6*x^5*e^7 + 66*(x*e + d)^m*b*c^2*m^5*x^6*
e^7 + 175*(x*e + d)^m*c^3*m^4*x^7*e^7 + 3*(x*e + d)^m*b^2*c*d*m^6*x^4*e^6 +
 51*(x*e + d)^m*b*c^2*d*m^5*x^5*e^6 + 85*(x*e + d)^m*c^3*d*m^4*x^6*e^6 - 15
*(x*e + d)^m*b*c^2*d^2*m^5*x^4*e^5 - 60*(x*e + d)^m*c^3*d^2*m^4*x^5*e^5 + 3
0*(x*e + d)^m*c^3*d^3*m^4*x^4*e^4 + (x*e + d)^m*b^3*m^6*x^4*e^7 + 69*(x*e +
 d)^m*b^2*c*m^5*x^5*e^7 + 570*(x*e + d)^m*b*c^2*m^4*x^6*e^7 + 735*(x*e + d)
^m*c^3*m^3*x^7*e^7 + (x*e + d)^m*b^3*d*m^6*x^3*e^6 + 57*(x*e + d)^m*b^2*c*d
*m^5*x^4*e^6 + 315*(x*e + d)^m*b*c^2*d*m^4*x^5*e^6 + 225*(x*e + d)^m*c^3*d*
m^3*x^6*e^6 - 12*(x*e + d)^m*b^2*c*d^2*m^5*x^3*e^5 - 195*(x*e + d)^m*b*c^2*
d^2*m^4*x^4*e^5 - 210*(x*e + d)^m*c^3*d^2*m^3*x^5*e^5 + 60*(x*e + d)^m*b*c^
2*d^3*m^4*x^3*e^4 + 180*(x*e + d)^m*c^3*d^3*m^3*x^4*e^4 - 120*(x*e + d)^m*c
^3*d^4*m^3*x^3*e^3 + 24*(x*e + d)^m*b^3*m^5*x^4*e^7 + 621*(x*e + d)^m*b^2*c
*m^4*x^5*e^7 + 2460*(x*e + d)^m*b*c^2*m^3*x^6*e^7 + 1624*(x*e + d)^m*c^3*m^
2*x^7*e^7 + 21*(x*e + d)^m*b^3*d*m^5*x^3*e^6 + 393*(x*e + d)^m*b^2*c*d*m^4*
x^4*e^6 + 885*(x*e + d)^m*b*c^2*d*m^3*x^5*e^6 + 274*(x*e + d)^m*c^3*d*m^2*x
^6*e^6 - 3*(x*e + d)^m*b^3*d^2*m^5*x^2*e^5 - 192*(x*e + d)^m*b^2*c*d^2*m^4*
x^3*e^5 - 795*(x*e + d)^m*b*c^2*d^2*m^3*x^4*e^5 - 300*(x*e + d)^m*c^3*d^2*m
^2*x^5*e^5 + 36*(x*e + d)^m*b^2*c*d^3*m^4*x^2*e^4 + 600*(x*e + d)^m*b*c^2*d
^3*m^3*x^3*e^4 + 330*(x*e + d)^m*c^3*d^3*m^2*x^4*e^4 - 180*(x*e + d)^m*b*c^
2*d^4*m^3*x^2*e^3 - 360*(x*e + d)^m*c^3*d^4*m^2*x^3*e^3 + 360*(x*e + d)^m*c
^3*d^5*m^2*x^2*e^2 + 226*(x*e + d)^m*b^3*m^4*x^4*e^7 + 2775*(x*e + d)^m*b^2
*c*m^3*x^5*e^7 + 5547*(x*e + d)^m*b*c^2*m^2*x^6*e^7 + 1764*(x*e + d)^m*c^3*
m*x^7*e^7 + 163*(x*e + d)^m*b^3*d*m^4*x^3*e^6 + 1203*(x*e + d)^m*b^2*c*d*m^
3*x^4*e^6 + 1122*(x*e + d)^m*b*c^2*d*m^2*x^5*e^6 + 120*(x*e + d)^m*c^3*d*m*
x^6*e^6 - 57*(x*e + d)^m*b^3*d^2*m^4*x^2*e^5 - 996*(x*e + d)^m*b^2*c*d^2*m^
3*x^3*e^5 - 1245*(x*e + d)^m*b*c^2*d^2*m^2*x^4*e^5 - 144*(x*e + d)^m*c^3*d^
2*m*x^5*e^5 + 6*(x*e + d)^m*b^3*d^3*m^4*x*e^4 + 504*(x*e + d)^m*b^2*c*d^3*m
^3*x^2*e^4 + 1380*(x*e + d)^m*b*c^2*d^3*m^2*x^3*e^4 + 180*(x*e + d)^m*c^3*d
^3*m*x^4*e^4 - 72*(x*e + d)^m*b^2*c*d^4*m^3*x*e^3 - 1440*(x*e + d)^m*b*c^2*
```

```
d^4*m^2*x^2*e^3 - 240*(x*e + d)^m*c^3*d^4*m*x^3*e^3 + 360*(x*e + d)^m*b*c^2
*d^5*m^2*x*e^2 + 360*(x*e + d)^m*c^3*d^5*m*x^2*e^2 - 720*(x*e + d)^m*c^3*d^
6*m*x*e + 1056*(x*e + d)^m*b^3*m^3*x^4*e^7 + 6432*(x*e + d)^m*b^2*c*m^2*x^5
*e^7 + 6114*(x*e + d)^m*b*c^2*m*x^6*e^7 + 720*(x*e + d)^m*c^3*x^7*e^7 + 567
*(x*e + d)^m*b^3*d*m^3*x^3*e^6 + 1620*(x*e + d)^m*b^2*c*d*m^2*x^4*e^6 + 504
*(x*e + d)^m*b*c^2*d*m*x^5*e^6 - 375*(x*e + d)^m*b^3*d^2*m^3*x^2*e^5 - 1824
*(x*e + d)^m*b^2*c*d^2*m^2*x^3*e^5 - 630*(x*e + d)^m*b*c^2*d^2*m*x^4*e^5 +
108*(x*e + d)^m*b^3*d^3*m^3*x*e^4 + 1980*(x*e + d)^m*b^2*c*d^3*m^2*x^2*e^4
+ 840*(x*e + d)^m*b*c^2*d^3*m*x^3*e^4 - 6*(x*e + d)^m*b^3*d^4*m^3*e^3 - 936
*(x*e + d)^m*b^2*c*d^4*m^2*x*e^3 - 1260*(x*e + d)^m*b*c^2*d^4*m*x^2*e^3 + 7
2*(x*e + d)^m*b^2*c*d^5*m^2*e^2 + 2520*(x*e + d)^m*b*c^2*d^5*m*x*e^2 - 360*
(x*e + d)^m*b*c^2*d^6*m*e + 720*(x*e + d)^m*c^3*d^7 + 2545*(x*e + d)^m*b^3*
m^2*x^4*e^7 + 7236*(x*e + d)^m*b^2*c*m*x^5*e^7 + 2520*(x*e + d)^m*b*c^2*x^6
*e^7 + 844*(x*e + d)^m*b^3*d*m^2*x^3*e^6 + 756*(x*e + d)^m*b^2*c*d*m*x^4*e^
6 - 951*(x*e + d)^m*b^3*d^2*m^2*x^2*e^5 - 1008*(x*e + d)^m*b^2*c*d^2*m*x^3*
e^5 + 642*(x*e + d)^m*b^3*d^3*m^2*x*e^4 + 1512*(x*e + d)^m*b^2*c*d^3*m*x^2*
e^4 - 108*(x*e + d)^m*b^3*d^4*m^2*e^3 - 3024*(x*e + d)^m*b^2*c*d^4*m*x*e^3
+ 936*(x*e + d)^m*b^2*c*d^5*m*e^2 - 2520*(x*e + d)^m*b*c^2*d^6*e + 2952*(x*
e + d)^m*b^3*m*x^4*e^7 + 3024*(x*e + d)^m*b^2*c*x^5*e^7 + 420*(x*e + d)^m*b
^3*d*m*x^3*e^6 - 630*(x*e + d)^m*b^3*d^2*m*x^2*e^5 + 1260*(x*e + d)^m*b^3*d
^3*m*x*e^4 - 642*(x*e + d)^m*b^3*d^4*m*e^3 + 3024*(x*e + d)^m*b^2*c*d^5*e^2
 + 1260*(x*e + d)^m*b^3*x^4*e^7 - 1260*(x*e + d)^m*b^3*d^4*e^3)/(m^7*e^7 +
28*m^6*e^7 + 322*m^5*e^7 + 1960*m^4*e^7 + 6769*m^3*e^7 + 13132*m^2*e^7 + 13
068*m*e^7 + 5040*e^7)
```

3.441 $\int (d+ex)^m \left(bx+cx^2\right)^2 \, dx$

Optimal. Leaf size=159

$$\frac{\left(b^2e^2-6bcde+6c^2d^2\right)(d+ex)^{m+3}}{e^5(m+3)}+\frac{d^2(cd-be)^2(d+ex)^{m+1}}{e^5(m+1)}-\frac{2d(cd-be)(2cd-be)(d+ex)^{m+2}}{e^5(m+2)}-\frac{2c(2cd-be)(d+ex)^{m+4}}{e^5(m+4)}$$

```
[Out] (d^2*(c*d - b*e)^2*(d + e*x)^(1 + m))/(e^5*(1 + m)) - (2*d*(c*d - b*e)*(2*c
*d - b*e)*(d + e*x)^(2 + m))/(e^5*(2 + m)) + ((6*c^2*d^2 - 6*b*c*d*e + b^2*
e^2)*(d + e*x)^(3 + m))/(e^5*(3 + m)) - (2*c*(2*c*d - b*e)*(d + e*x)^(4 + m
))/(e^5*(4 + m)) + (c^2*(d + e*x)^(5 + m))/(e^5*(5 + m))
```

Rubi [A] time = 0.0825785, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {698}

$$\frac{\left(b^2e^2-6bcde+6c^2d^2\right)(d+ex)^{m+3}}{e^5(m+3)}+\frac{d^2(cd-be)^2(d+ex)^{m+1}}{e^5(m+1)}-\frac{2d(cd-be)(2cd-be)(d+ex)^{m+2}}{e^5(m+2)}-\frac{2c(2cd-be)(d+ex)^{m+4}}{e^5(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(b*x + c*x^2)^2,x]
```


```
[Out] (d^2*(c*d - b*e)^2*(d + e*x)^(1 + m))/(e^5*(1 + m)) - (2*d*(c*d - b*e)*(2*c
*d - b*e)*(d + e*x)^(2 + m))/(e^5*(2 + m)) + ((6*c^2*d^2 - 6*b*c*d*e + b^2*
e^2)*(d + e*x)^(3 + m))/(e^5*(3 + m)) - (2*c*(2*c*d - b*e)*(d + e*x)^(4 + m
))/(e^5*(4 + m)) + (c^2*(d + e*x)^(5 + m))/(e^5*(5 + m))
```


Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^m \left(bx+cx^2\right)^2 \, dx = \int \left(\frac{d^2(cd-be)^2(d+ex)^m}{e^4}+\frac{2d(cd-be)(-2cd+be)(d+ex)^{1+m}}{e^4}+\frac{\left(6c^2d^2-6bcde+b^2e^2\right)(d+ex)^{2+m}}{e^4}\right.$$

$$=\frac{d^2(cd-be)^2(d+ex)^{1+m}}{e^5(1+m)}-\frac{2d(cd-be)(2cd-be)(d+ex)^{2+m}}{e^5(2+m)}+\frac{\left(6c^2d^2-6bcde+b^2e^2\right)(d+ex)^{3+m}}{e^5(3+m)}$$

Mathematica [A] time = 0.0946224, size = 138, normalized size = 0.87

$$\frac{(d+ex)^{m+1}\left(\frac{(d+ex)^2\left(b^2e^2-6bcde+6c^2d^2\right)}{m+3}+\frac{d^2(cd-be)^2}{m+1}-\frac{2c(d+ex)^3(2cd-be)}{m+4}-\frac{2d(d+ex)(cd-be)(2cd-be)}{m+2}+\frac{c^2(d+ex)^4}{m+5}\right)}{e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(b*x + c*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + m)*((d^2*(c*d - b*e)^2)/(1 + m) - (2*d*(c*d - b*e)*(2*c*d -
 b*e)*(d + e*x))/(2 + m) + ((6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(d + e*x)^2)/
(3 + m) - (2*c*(2*c*d - b*e)*(d + e*x)^3)/(4 + m) + (c^2*(d + e*x)^4)/(5 +
m)))/e^5
```

Maple [B] time = 0.057, size = 547, normalized size = 3.4

$$\frac{(ex+d)^{1+m}\left(c^2e^4m^4x^4+2\,bce^4m^4x^3+10\,c^2e^4m^3x^4+b^2e^4m^4x^2+22\,bce^4m^3x^3-4\,c^2de^3m^3x^3+35\,c^2e^4m^2x^4+12\,b^2e^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x)^2,x)
```

```
[Out] (e*x+d)^(1+m)*(c^2*e^4*m^4*x^4+2*b*c*e^4*m^4*x^3+10*c^2*e^4*m^3*x^4+b^2*e^4
*m^4*x^2+22*b*c*e^4*m^3*x^3-4*c^2*d*e^3*m^3*x^3+35*c^2*e^4*m^2*x^4+12*b^2*e
^4*m^3*x^2-6*b*c*d*e^3*m^3*x^2+82*b*c*e^4*m^2*x^3-24*c^2*d*e^3*m^2*x^3+50*c
^2*e^4*m*x^4-2*b^2*d*e^3*m^3*x+49*b^2*e^4*m^2*x^2-48*b*c*d*e^3*m^2*x^2+122*
b*c*e^4*m*x^3+12*c^2*d^2*e^2*m^2*x^2-44*c^2*d*e^3*m*x^3+24*c^2*e^4*x^4-20*b
^2*d*e^3*m^2*x+78*b^2*e^4*m*x^2+12*b*c*d^2*e^2*m^2*x-102*b*c*d*e^3*m*x^2+60
*b*c*e^4*x^3+36*c^2*d^2*e^2*m*x^2-24*c^2*d*e^3*x^3+2*b^2*d^2*e^2*m^2-58*b^2
*d*e^3*m*x+40*b^2*e^4*x^2+72*b*c*d^2*e^2*m*x-60*b*c*d*e^3*x^2-24*c^2*d^3*e*
m*x+24*c^2*d^2*e^2*x^2+18*b^2*d^2*e^2*m-40*b^2*d*e^3*x-12*b*c*d^3*e*m+60*b*
c*d^2*e^2*x-24*c^2*d^3*e*x+40*b^2*d^2*e^2-60*b*c*d^3*e+24*c^2*d^4)/e^5/(m^5
+15*m^4+85*m^3+225*m^2+274*m+120)
```

Maxima [A] time = 1.25286, size = 429, normalized size = 2.7

$$\frac{\left(\left(m^2+3\,m+2\right)e^3x^3+\left(m^2+m\right)de^2x^2-2\,d^2emx+2\,d^3\right)(ex+d)^m b^2}{\left(m^3+6\,m^2+11\,m+6\right)e^3}+\frac{2\left(\left(m^3+6\,m^2+11\,m+6\right)e^4x^4+\left(m^3+3\,m^2\right.\right.}{\left(m^4+10\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x)^2,x, algorithm="maxima")
```

```
[Out] ((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x
+ d)^m*b^2/((m^3 + 6*m^2 + 11*m + 6)*e^3) + 2*((m^3 + 6*m^2 + 11*m + 6)*e^4
*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*
x - 6*d^4)*(e*x + d)^m*b*c/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) + ((m^
4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e
^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*
d^4*e*m*x + 24*d^5)*(e*x + d)^m*c^2/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274
*m + 120)*e^5)
```

Fricas [B] time = 1.97357, size = 1220, normalized size = 7.67

$$\frac{\left(2\,b^2d^3e^2m^2+24\,c^2d^5-60\,bcd^4e+40\,b^2d^3e^2+\left(c^2e^5m^4+10\,c^2e^5m^3+35\,c^2e^5m^2+50\,c^2e^5m+24\,c^2e^5\right)x^5+\left(60\,bce^5\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x)^2,x, algorithm="fricas")
```

```
[Out] (2*b^2*d^3*e^2*m^2 + 24*c^2*d^5 - 60*b*c*d^4*e + 40*b^2*d^3*e^2 + (c^2*e^5*
m^4 + 10*c^2*e^5*m^3 + 35*c^2*e^5*m^2 + 50*c^2*e^5*m + 24*c^2*e^5)*x^5 + (6
0*b*c*e^5 + (c^2*d*e^4 + 2*b*c*e^5)*m^4 + 2*(3*c^2*d*e^4 + 11*b*c*e^5)*m^3
+ (11*c^2*d*e^4 + 82*b*c*e^5)*m^2 + 2*(3*c^2*d*e^4 + 61*b*c*e^5)*m)*x^4 + (
40*b^2*e^5 + (2*b*c*d*e^4 + b^2*e^5)*m^4 - 4*(c^2*d^2*e^3 - 4*b*c*d*e^4 - 3
*b^2*e^5)*m^3 - (12*c^2*d^2*e^3 - 34*b*c*d*e^4 - 49*b^2*e^5)*m^2 - 2*(4*c^2
*d^2*e^3 - 10*b*c*d*e^4 - 39*b^2*e^5)*m)*x^3 + (b^2*d*e^4*m^4 - 2*(3*b*c*d^
2*e^3 - 5*b^2*d*e^4)*m^3 + (12*c^2*d^3*e^2 - 36*b*c*d^2*e^3 + 29*b^2*d*e^4)
*m^2 + 2*(6*c^2*d^3*e^2 - 15*b*c*d^2*e^3 + 10*b^2*d*e^4)*m)*x^2 - 6*(2*b*c*
d^4*e - 3*b^2*d^3*e^2)*m - 2*(b^2*d^2*e^3*m^3 - 3*(2*b*c*d^3*e^2 - 3*b^2*d^
2*e^3)*m^2 + 2*(6*c^2*d^4*e - 15*b*c*d^3*e^2 + 10*b^2*d^2*e^3)*m)*x)*(e*x +
 d)^m/(e^5*m^5 + 15*e^5*m^4 + 85*e^5*m^3 + 225*e^5*m^2 + 274*e^5*m + 120*e^
5)
```

Sympy [A] time = 7.87121, size = 6205, normalized size = 39.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x)**2,x)
```

```
[Out] Piecewise((d**m*(b**2*x**3/3 + b*c*x**4/2 + c**2*x**5/5), Eq(e, 0)), (4*b**
2*d*e**5*x**3/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*
e**8*x**3 + 12*d**2*e**9*x**4) + b**2*e**6*x**4/(12*d**6*e**5 + 48*d**5*e**
6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4) + 6*b*c*d*
e**5*x**4/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8
*x**3 + 12*d**2*e**9*x**4) + 12*c**2*d**6*log(d/e + x)/(12*d**6*e**5 + 48*d
**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4) + 7
*c**2*d**6/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**
8*x**3 + 12*d**2*e**9*x**4) + 48*c**2*d**5*e*x*log(d/e + x)/(12*d**6*e**5 +
 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4
) + 16*c**2*d**5*e*x/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 4
8*d**3*e**8*x**3 + 12*d**2*e**9*x**4) + 72*c**2*d**4*e**2*x**2*log(d/e + x)
/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 1
2*d**2*e**9*x**4) + 48*c**2*d**3*e**3*x**3*log(d/e + x)/(12*d**6*e**5 + 48*
d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4) -
24*c**2*d**3*e**3*x**3/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 +
 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4) + 12*c**2*d**2*e**4*x**4*log(d/e +
x)/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 +
 12*d**2*e**9*x**4) - 18*c**2*d**2*e**4*x**4/(12*d**6*e**5 + 48*d**5*e**6*x
 + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4), Eq(m, -5)),
(b**2*e**5*x**3/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*
x**3) + 6*b*c*d**4*e*log(d/e + x)/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**
7*x**2 + 3*d*e**8*x**3) + 5*b*c*d**4*e/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**
2*e**7*x**2 + 3*d*e**8*x**3) + 18*b*c*d**3*e**2*x*log(d/e + x)/(3*d**4*e**5
 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3) + 9*b*c*d**3*e**2*x/(3
*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3) + 18*b*c*d**
2*e**3*x**2*log(d/e + x)/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 +
3*d*e**8*x**3) + 6*b*c*d*e**4*x**3*log(d/e + x)/(3*d**4*e**5 + 9*d**3*e**6*
x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3) - 6*b*c*d*e**4*x**3/(3*d**4*e**5 + 9*
d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3) - 12*c**2*d**5*log(d/e + x)
/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3) - 10*c**2
*d**5/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3) - 36
*c**2*d**4*e*x*log(d/e + x)/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2
```

```
 + 3*d*e**8*x**3) - 18*c**2*d**4*e*x/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*
e**7*x**2 + 3*d*e**8*x**3) - 36*c**2*d**3*e**2*x**2*log(d/e + x)/(3*d**4*e*
*5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3) - 12*c**2*d**2*e**3*
x**3*log(d/e + x)/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**
8*x**3) + 12*c**2*d**2*e**3*x**3/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7
*x**2 + 3*d*e**8*x**3) + 3*c**2*d*e**4*x**4/(3*d**4*e**5 + 9*d**3*e**6*x +
9*d**2*e**7*x**2 + 3*d*e**8*x**3), Eq(m, -4)), (2*b**2*d**2*e**2*log(d/e +
x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 3*b**2*d**2*e**2/(2*d**2*e**5
 + 4*d*e**6*x + 2*e**7*x**2) + 4*b**2*d*e**3*x*log(d/e + x)/(2*d**2*e**5 +
4*d*e**6*x + 2*e**7*x**2) + 4*b**2*d*e**3*x/(2*d**2*e**5 + 4*d*e**6*x + 2*e
**7*x**2) + 2*b**2*e**4*x**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**
7*x**2) - 12*b*c*d**3*e*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**
2) - 18*b*c*d**3*e/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 24*b*c*d**2*e
**2*x*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 24*b*c*d**2*e
**2*x/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 12*b*c*d*e**3*x**2*log(d/e
 + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 4*b*c*e**4*x**3/(2*d**2*e*
*5 + 4*d*e**6*x + 2*e**7*x**2) + 12*c**2*d**4*log(d/e + x)/(2*d**2*e**5 + 4
*d*e**6*x + 2*e**7*x**2) + 18*c**2*d**4/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*
x**2) + 24*c**2*d**3*e*x*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x*
*2) + 24*c**2*d**3*e*x/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 12*c**2*d
**2*e**2*x**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 4*c**
2*d*e**3*x**3/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + c**2*e**4*x**4/(2*
d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2), Eq(m, -3)), (-6*b**2*d**2*e**2*log(d
/e + x)/(3*d*e**5 + 3*e**6*x) - 6*b**2*d**2*e**2/(3*d*e**5 + 3*e**6*x) - 6*
b**2*d*e**3*x*log(d/e + x)/(3*d*e**5 + 3*e**6*x) + 3*b**2*e**4*x**2/(3*d*e*
*5 + 3*e**6*x) + 18*b*c*d**3*e*log(d/e + x)/(3*d*e**5 + 3*e**6*x) + 18*b*c*
d**3*e/(3*d*e**5 + 3*e**6*x) + 18*b*c*d**2*e**2*x*log(d/e + x)/(3*d*e**5 +
3*e**6*x) - 9*b*c*d*e**3*x**2/(3*d*e**5 + 3*e**6*x) + 3*b*c*e**4*x**3/(3*d*
e**5 + 3*e**6*x) - 12*c**2*d**4*log(d/e + x)/(3*d*e**5 + 3*e**6*x) - 12*c**
2*d**4/(3*d*e**5 + 3*e**6*x) - 12*c**2*d**3*e*x*log(d/e + x)/(3*d*e**5 + 3*
e**6*x) + 6*c**2*d**2*e**2*x**2/(3*d*e**5 + 3*e**6*x) - 2*c**2*d*e**3*x**3/
(3*d*e**5 + 3*e**6*x) + c**2*e**4*x**4/(3*d*e**5 + 3*e**6*x), Eq(m, -2)), (
b**2*d**2*log(d/e + x)/e**3 - b**2*d*x/e**2 + b**2*x**2/(2*e) - 2*b*c*d**3*
log(d/e + x)/e**4 + 2*b*c*d**2*x/e**3 - b*c*d*x**2/e**2 + 2*b*c*x**3/(3*e)
+ c**2*d**4*log(d/e + x)/e**5 - c**2*d**3*x/e**4 + c**2*d**2*x**2/(2*e**3)
- c**2*d*x**3/(3*e**2) + c**2*x**4/(4*e), Eq(m, -1)), (2*b**2*d**3*e**2*m**
2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 2
74*e**5*m + 120*e**5) + 18*b**2*d**3*e**2*m*(d + e*x)**m/(e**5*m**5 + 15*e*
*5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 40*b**2*d
**3*e**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m
**2 + 274*e**5*m + 120*e**5) - 2*b**2*d**2*e**3*m**3*x*(d + e*x)**m/(e**5*m
**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5)
- 18*b**2*d**2*e**3*m**2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5
*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 40*b**2*d**2*e**3*m*x*(d +
 e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**
5*m + 120*e**5) + b**2*d*e**4*m**4*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m
**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 10*b**2*d*e**
4*m**3*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**
5*m**2 + 274*e**5*m + 120*e**5) + 29*b**2*d*e**4*m**2*x**2*(d + e*x)**m/(e*
*5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e*
*5) + 20*b**2*d*e**4*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**
5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + b**2*e**5*m**4*x**3*(d +
e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5
*m + 120*e**5) + 12*b**2*e**5*m**3*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m
**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 49*b**2*e**5*
m**2*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*
m**2 + 274*e**5*m + 120*e**5) + 78*b**2*e**5*m*x**3*(d + e*x)**m/(e**5*m**5
 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 4
0*b**2*e**5*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 22
```

```
5*e**5*m**2 + 274*e**5*m + 120*e**5) - 12*b*c*d**4*e*m*(d + e*x)**m/(e**5*m
**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5)
- 60*b*c*d**4*e*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225
*e**5*m**2 + 274*e**5*m + 120*e**5) + 12*b*c*d**3*e**2*m**2*x*(d + e*x)**m/
(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120
*e**5) + 60*b*c*d**3*e**2*m*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e
**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 6*b*c*d**2*e**3*m**3*x*
*2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 +
274*e**5*m + 120*e**5) - 36*b*c*d**2*e**3*m**2*x**2*(d + e*x)**m/(e**5*m**5
 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 3
0*b*c*d**2*e**3*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**
3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 2*b*c*d*e**4*m**4*x**3*(d + e*
x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m
 + 120*e**5) + 16*b*c*d*e**4*m**3*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m*
*4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 34*b*c*d*e**4*
m**2*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*
m**2 + 274*e**5*m + 120*e**5) + 20*b*c*d*e**4*m*x**3*(d + e*x)**m/(e**5*m**
5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) +
2*b*c*e**5*m**4*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3
+ 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 22*b*c*e**5*m**3*x**4*(d + e*x)*
*m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m +
120*e**5) + 82*b*c*e**5*m**2*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 +
85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 122*b*c*e**5*m*x**4
*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 27
4*e**5*m + 120*e**5) + 60*b*c*e**5*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m
**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 24*c**2*d**5*
(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274
*e**5*m + 120*e**5) - 24*c**2*d**4*e*m*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*
m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 12*c**2*d**3
*e**2*m**2*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225
*e**5*m**2 + 274*e**5*m + 120*e**5) + 12*c**2*d**3*e**2*m*x**2*(d + e*x)**m
/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 12
0*e**5) - 4*c**2*d**2*e**3*m**3*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4
 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 12*c**2*d**2*e**
3*m**2*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**
5*m**2 + 274*e**5*m + 120*e**5) - 8*c**2*d**2*e**3*m*x**3*(d + e*x)**m/(e**
5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**
5) + c**2*d*e**4*m**4*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5
*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 6*c**2*d*e**4*m**3*x**4*(d
 + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e
**5*m + 120*e**5) + 11*c**2*d*e**4*m**2*x**4*(d + e*x)**m/(e**5*m**5 + 15*e
**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 6*c**2*d
*e**4*m*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e*
*5*m**2 + 274*e**5*m + 120*e**5) + c**2*e**5*m**4*x**5*(d + e*x)**m/(e**5*m
**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5)
+ 10*c**2*e**5*m**3*x**5*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m
**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 35*c**2*e**5*m**2*x**5*(d +
e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5
*m + 120*e**5) + 50*c**2*e**5*m*x**5*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4
 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 24*c**2*e**5*x**
5*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 2
74*e**5*m + 120*e**5), True))
```

Giac [B] time = 1.34414, size = 1353, normalized size = 8.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x)^2,x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*c^2*m^4*x^5*e^5 + (x*e + d)^m*c^2*d*m^4*x^4*e^4 + 2*(x*e + d)^
m*b*c*m^4*x^4*e^5 + 10*(x*e + d)^m*c^2*m^3*x^5*e^5 + 2*(x*e + d)^m*b*c*d*m^
4*x^3*e^4 + 6*(x*e + d)^m*c^2*d*m^3*x^4*e^4 - 4*(x*e + d)^m*c^2*d^2*m^3*x^3
*e^3 + (x*e + d)^m*b^2*m^4*x^3*e^5 + 22*(x*e + d)^m*b*c*m^3*x^4*e^5 + 35*(x
*e + d)^m*c^2*m^2*x^5*e^5 + (x*e + d)^m*b^2*d*m^4*x^2*e^4 + 16*(x*e + d)^m*
b*c*d*m^3*x^3*e^4 + 11*(x*e + d)^m*c^2*d*m^2*x^4*e^4 - 6*(x*e + d)^m*b*c*d^
2*m^3*x^2*e^3 - 12*(x*e + d)^m*c^2*d^2*m^2*x^3*e^3 + 12*(x*e + d)^m*c^2*d^3
*m^2*x^2*e^2 + 12*(x*e + d)^m*b^2*m^3*x^3*e^5 + 82*(x*e + d)^m*b*c*m^2*x^4*
e^5 + 50*(x*e + d)^m*c^2*m*x^5*e^5 + 10*(x*e + d)^m*b^2*d*m^3*x^2*e^4 + 34*
(x*e + d)^m*b*c*d*m^2*x^3*e^4 + 6*(x*e + d)^m*c^2*d*m*x^4*e^4 - 2*(x*e + d)
^m*b^2*d^2*m^3*x*e^3 - 36*(x*e + d)^m*b*c*d^2*m^2*x^2*e^3 - 8*(x*e + d)^m*c
^2*d^2*m*x^3*e^3 + 12*(x*e + d)^m*b*c*d^3*m^2*x*e^2 + 12*(x*e + d)^m*c^2*d^
3*m*x^2*e^2 - 24*(x*e + d)^m*c^2*d^4*m*x*e + 49*(x*e + d)^m*b^2*m^2*x^3*e^5
 + 122*(x*e + d)^m*b*c*m*x^4*e^5 + 24*(x*e + d)^m*c^2*x^5*e^5 + 29*(x*e + d
)^m*b^2*d*m^2*x^2*e^4 + 20*(x*e + d)^m*b*c*d*m*x^3*e^4 - 18*(x*e + d)^m*b^2
*d^2*m^2*x*e^3 - 30*(x*e + d)^m*b*c*d^2*m*x^2*e^3 + 2*(x*e + d)^m*b^2*d^3*m
^2*e^2 + 60*(x*e + d)^m*b*c*d^3*m*x*e^2 - 12*(x*e + d)^m*b*c*d^4*m*e + 24*(
x*e + d)^m*c^2*d^5 + 78*(x*e + d)^m*b^2*m*x^3*e^5 + 60*(x*e + d)^m*b*c*x^4*
e^5 + 20*(x*e + d)^m*b^2*d*m*x^2*e^4 - 40*(x*e + d)^m*b^2*d^2*m*x*e^3 + 18*
(x*e + d)^m*b^2*d^3*m*e^2 - 60*(x*e + d)^m*b*c*d^4*e + 40*(x*e + d)^m*b^2*x
^3*e^5 + 40*(x*e + d)^m*b^2*d^3*e^2)/(m^5*e^5 + 15*m^4*e^5 + 85*m^3*e^5 + 2
25*m^2*e^5 + 274*m*e^5 + 120*e^5)
```

3.442 $\int (d+ex)^m \left(bx+cx^2\right) dx$

Optimal. Leaf size=75

$$\frac{d(cd-be)(d+ex)^{m+1}}{e^3(m+1)}-\frac{(2cd-be)(d+ex)^{m+2}}{e^3(m+2)}+\frac{c(d+ex)^{m+3}}{e^3(m+3)}$$

```
[Out] (d*(c*d - b*e)*(d + e*x)^(1 + m))/(e^3*(1 + m)) - ((2*c*d - b*e)*(d + e*x)^
(2 + m))/(e^3*(2 + m)) + (c*(d + e*x)^(3 + m))/(e^3*(3 + m))
```

Rubi [A] time = 0.0356838, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {698}

$$\frac{d(cd-be)(d+ex)^{m+1}}{e^3(m+1)}-\frac{(2cd-be)(d+ex)^{m+2}}{e^3(m+2)}+\frac{c(d+ex)^{m+3}}{e^3(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(b*x + c*x^2),x]
```

```
[Out] (d*(c*d - b*e)*(d + e*x)^(1 + m))/(e^3*(1 + m)) - ((2*c*d - b*e)*(d + e*x)^
(2 + m))/(e^3*(2 + m)) + (c*(d + e*x)^(3 + m))/(e^3*(3 + m))
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int (d+ex)^m \left(bx+cx^2\right) dx &= \int \left(\frac{d(cd-be)(d+ex)^m}{e^2}+\frac{(-2cd+be)(d+ex)^{1+m}}{e^2}+\frac{c(d+ex)^{2+m}}{e^2}\right) dx\\ &= \frac{d(cd-be)(d+ex)^{1+m}}{e^3(1+m)}-\frac{(2cd-be)(d+ex)^{2+m}}{e^3(2+m)}+\frac{c(d+ex)^{3+m}}{e^3(3+m)}\end{aligned}$$

Mathematica [A] time = 0.0529326, size = 75, normalized size = 1.

$$\frac{d(cd-be)(d+ex)^{m+1}}{e^3(m+1)}-\frac{(2cd-be)(d+ex)^{m+2}}{e^3(m+2)}+\frac{c(d+ex)^{m+3}}{e^3(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(b*x + c*x^2),x]
```

```
[Out] (d*(c*d - b*e)*(d + e*x)^(1 + m))/(e^3*(1 + m)) - ((2*c*d - b*e)*(d + e*x)^
(2 + m))/(e^3*(2 + m)) + (c*(d + e*x)^(3 + m))/(e^3*(3 + m))
```

Maple [A] time = 0.053, size = 116, normalized size = 1.6

$$-\frac{(ex+d)^{1+m}\left(-ce^2m^2x^2-be^2m^2x-3\,ce^2mx^2-4\,be^2mx+2\,cdemx-2\,ce^2x^2+bdem-3\,be^2x+2\,cdex+3\,bde-2\,cd^2\right)}{e^3\left(m^3+6\,m^2+11\,m+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x),x)
```

```
[Out] -(e*x+d)^(1+m)*(-c*e^2*m^2*x^2-b*e^2*m^2*x-3*c*e^2*m*x^2-4*b*e^2*m*x+2*c*d*
e*m*x-2*c*e^2*x^2+b*d*e*m-3*b*e^2*x+2*c*d*e*x+3*b*d*e-2*c*d^2)/e^3/(m^3+6*m
^2+11*m+6)
```

Maxima [A] time = 1.21144, size = 153, normalized size = 2.04

$$\frac{\left(e^2(m+1)x^2+demx-d^2\right)(ex+d)^m b}{\left(m^2+3\,m+2\right)e^2}+\frac{\left(\left(m^2+3\,m+2\right)e^3x^3+\left(m^2+m\right)de^2x^2-2\,d^2emx+2\,d^3\right)(ex+d)^m c}{\left(m^3+6\,m^2+11\,m+6\right)e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] (e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*b/((m^2 + 3*m + 2)*e^2) + ((m
^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d
)^m*c/((m^3 + 6*m^2 + 11*m + 6)*e^3)
```

Fricas [B] time = 2.02048, size = 323, normalized size = 4.31

$$-\frac{\left(bd^2em-2\,cd^3+3\,bd^2e-\left(ce^3m^2+3\,ce^3m+2\,ce^3\right)x^3-\left(3\,be^3+\left(cde^2+be^3\right)m^2+\left(cde^2+4\,be^3\right)m\right)x^2-\left(bde^2m^2-\left(\right.\right.}{e^3m^3+6\,e^3m^2+11\,e^3m+6\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] -(b*d^2*e*m - 2*c*d^3 + 3*b*d^2*e - (c*e^3*m^2 + 3*c*e^3*m + 2*c*e^3)*x^3 -
 (3*b*e^3 + (c*d*e^2 + b*e^3)*m^2 + (c*d*e^2 + 4*b*e^3)*m)*x^2 - (b*d*e^2*m
^2 - (2*c*d^2*e - 3*b*d*e^2)*m)*x)*(e*x + d)^m/(e^3*m^3 + 6*e^3*m^2 + 11*e^
3*m + 6*e^3)
```

Sympy [A] time = 2.31757, size = 1095, normalized size = 14.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x),x)
```

```
[Out] Piecewise((d**m*(b*x**2/2 + c*x**3/3), Eq(e, 0)), (-b*d*e/(2*d**2*e**3 + 4*
d*e**4*x + 2*e**5*x**2) - 2*b*e**2*x/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**
```

```
2) + 2*c*d**2*log(d/e + x)/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2) + 3*c*d
**2/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2) + 4*c*d*e*x*log(d/e + x)/(2*d*
*2*e**3 + 4*d*e**4*x + 2*e**5*x**2) + 4*c*d*e*x/(2*d**2*e**3 + 4*d*e**4*x +
 2*e**5*x**2) + 2*c*e**2*x**2*log(d/e + x)/(2*d**2*e**3 + 4*d*e**4*x + 2*e*
*5*x**2), Eq(m, -3)), (b*d*e*log(d/e + x)/(d*e**3 + e**4*x) + b*d*e/(d*e**3
 + e**4*x) + b*e**2*x*log(d/e + x)/(d*e**3 + e**4*x) - 2*c*d**2*log(d/e + x
)/(d*e**3 + e**4*x) - 2*c*d**2/(d*e**3 + e**4*x) - 2*c*d*e*x*log(d/e + x)/(
d*e**3 + e**4*x) + c*e**2*x**2/(d*e**3 + e**4*x), Eq(m, -2)), (-b*d*log(d/e
 + x)/e**2 + b*x/e + c*d**2*log(d/e + x)/e**3 - c*d*x/e**2 + c*x**2/(2*e),
Eq(m, -1)), (-b*d**2*e*m*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m
+ 6*e**3) - 3*b*d**2*e*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m +
6*e**3) + b*d*e**2*m**2*x*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m
 + 6*e**3) + 3*b*d*e**2*m*x*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3
*m + 6*e**3) + b*e**3*m**2*x**2*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*
e**3*m + 6*e**3) + 4*b*e**3*m*x**2*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 +
11*e**3*m + 6*e**3) + 3*b*e**3*x**2*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 +
 11*e**3*m + 6*e**3) + 2*c*d**3*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*
e**3*m + 6*e**3) - 2*c*d**2*e*m*x*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 1
1*e**3*m + 6*e**3) + c*d*e**2*m**2*x**2*(d + e*x)**m/(e**3*m**3 + 6*e**3*m*
*2 + 11*e**3*m + 6*e**3) + c*d*e**2*m*x**2*(d + e*x)**m/(e**3*m**3 + 6*e**3
*m**2 + 11*e**3*m + 6*e**3) + c*e**3*m**2*x**3*(d + e*x)**m/(e**3*m**3 + 6*
e**3*m**2 + 11*e**3*m + 6*e**3) + 3*c*e**3*m*x**3*(d + e*x)**m/(e**3*m**3 +
 6*e**3*m**2 + 11*e**3*m + 6*e**3) + 2*c*e**3*x**3*(d + e*x)**m/(e**3*m**3
+ 6*e**3*m**2 + 11*e**3*m + 6*e**3), True))
```

Giac [B] time = 1.34015, size = 355, normalized size = 4.73

$$(xe+d)^m cm^2x^3e^3 + (xe+d)^m cdm^2x^2e^2 + (xe+d)^m bm^2x^2e^3 + 3\,(xe+d)^m cmx^3e^3 + (xe+d)^m bdm^2xe^2 + (xe+d)^m cdmx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*c*m^2*x^3*e^3 + (x*e + d)^m*c*d*m^2*x^2*e^2 + (x*e + d)^m*b*m^
2*x^2*e^3 + 3*(x*e + d)^m*c*m*x^3*e^3 + (x*e + d)^m*b*d*m^2*x*e^2 + (x*e +
d)^m*c*d*m*x^2*e^2 - 2*(x*e + d)^m*c*d^2*m*x*e + 4*(x*e + d)^m*b*m*x^2*e^3
+ 2*(x*e + d)^m*c*x^3*e^3 + 3*(x*e + d)^m*b*d*m*x*e^2 - (x*e + d)^m*b*d^2*m
*e + 2*(x*e + d)^m*c*d^3 + 3*(x*e + d)^m*b*x^2*e^3 - 3*(x*e + d)^m*b*d^2*e)
/(m^3*e^3 + 6*m^2*e^3 + 11*m*e^3 + 6*e^3)
```

3.443 $\int \frac{(d+ex)^m}{bx+cx^2} dx$

Optimal. Leaf size=93

$$\frac{c(d+ex)^{m+1} {}_2F_1\left(1,m+1;m+2;\frac{c(d+ex)}{cd-be}\right)}{b(m+1)(cd-be)} - \frac{(d+ex)^{m+1} {}_2F_1\left(1,m+1;m+2;\frac{ex}{d}+1\right)}{bd(m+1)}$$

```
[Out] (c*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (c*(d + e*x))/(c*d
- b*e)])/(b*(c*d - b*e)*(1 + m)) - ((d + e*x)^(1 + m)*Hypergeometric2F1[1,
1 + m, 2 + m, 1 + (e*x)/d])/(b*d*(1 + m))
```

Rubi [A] time = 0.0644153, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {711, 65, 68}

$$\frac{c(d+ex)^{m+1} {}_2F_1\left(1,m+1;m+2;\frac{c(d+ex)}{cd-be}\right)}{b(m+1)(cd-be)} - \frac{(d+ex)^{m+1} {}_2F_1\left(1,m+1;m+2;\frac{ex}{d}+1\right)}{bd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(b*x + c*x^2),x]
```

```
[Out] (c*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (c*(d + e*x))/(c*d
- b*e)])/(b*(c*d - b*e)*(1 + m)) - ((d + e*x)^(1 + m)*Hypergeometric2F1[1,
1 + m, 2 + m, 1 + (e*x)/d])/(b*d*(1 + m))
```

Rule 711

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[ExpandIntegrand[(d + e*x)^m, 1/(a + b*x + c*x^2), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[m]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(d+ex)^m}{bx+cx^2} dx = \int \left(\frac{(d+ex)^m}{bx} - \frac{c(d+ex)^m}{b(b+cx)}\right) dx$$
$$= \frac{\int \frac{(d+ex)^m}{x} dx}{b} - \frac{c\int \frac{(d+ex)^m}{b+cx} dx}{b}$$
$$= \frac{c(d+ex)^{1+m} {}_2F_1\left(1,1+m;2+m;\frac{c(d+ex)}{cd-be}\right)}{b(cd-be)(1+m)} - \frac{(d+ex)^{1+m} {}_2F_1\left(1,1+m;2+m;1+\frac{ex}{d}\right)}{bd(1+m)}$$

Mathematica [A] time = 0.0285884, size = 86, normalized size = 0.92

$$-\frac{(d+ex)^{m+1}\left(cd\,{}_2F_1\left(1,m+1;m+2;\frac{c(d+ex)}{cd-be}\right)+(be-cd)\,{}_2F_1\left(1,m+1;m+2;\frac{ex}{d}+1\right)\right)}{bd(m+1)(be-cd)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(b*x + c*x^2),x]
```

```
[Out] -((((d + e*x)^(1 + m)*(c*d*Hypergeometric2F1[1, 1 + m, 2 + m, (c*(d + e*x))/
(c*d - b*e)] + (-(c*d) + b*e)*Hypergeometric2F1[1, 1 + m, 2 + m, 1 + (e*x)/
d]))/(b*d*(-(c*d) + b*e)*(1 + m)))
```

Maple [F] time = 0.591, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{cx^2+bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(c*x^2+b*x),x)
```

```
[Out] int((e*x+d)^m/(c*x^2+b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{cx^2+bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/(c*x^2 + b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^m}{cx^2+bx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m/(c*x^2 + b*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^m}{x (b + cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(c*x**2+b*x),x)
```

```
[Out] Integral((d + e*x)**m/(x*(b + c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^m}{cx^2 + bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(c*x^2 + b*x), x)
```

3.444 $\int \frac{(d+ex)^m}{\left(bx+cx^2\right)^2} dx$

Optimal. Leaf size=180

$$-\frac{c^2(d+ex)^{m+1}(2cd-be(2-m))\,{}_2F_1\left(1,m+1;m+2;\frac{c(d+ex)}{cd-be}\right)}{b^3(m+1)(cd-be)^2}+\frac{(d+ex)^{m+1}(2cd-bem)\,{}_2F_1\left(1,m+1;m+2;\frac{ex}{d}+1\right)}{b^3d^2(m+1)}-\frac{(d}{}$$

```
[Out] -(((d + e*x)^(1 + m)*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(b^2*d*(c*d - b*e
)*(b*x + c*x^2))) - (c^2*(2*c*d - b*e*(2 - m))*(d + e*x)^(1 + m)*Hypergeome
tric2F1[1, 1 + m, 2 + m, (c*(d + e*x))/(c*d - b*e)])/(b^3*(c*d - b*e)^2*(1
+ m)) + ((2*c*d - b*e*m)*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 +
m, 1 + (e*x)/d])/(b^3*d^2*(1 + m))
```

Rubi [A] time = 0.18127, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {740, 830, 65, 68}

$$-\frac{c^2(d+ex)^{m+1}(2cd-be(2-m))\,{}_2F_1\left(1,m+1;m+2;\frac{c(d+ex)}{cd-be}\right)}{b^3(m+1)(cd-be)^2}+\frac{(d+ex)^{m+1}(2cd-bem)\,{}_2F_1\left(1,m+1;m+2;\frac{ex}{d}+1\right)}{b^3d^2(m+1)}-\frac{(d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(b*x + c*x^2)^2,x]
```

```
[Out] -(((d + e*x)^(1 + m)*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(b^2*d*(c*d - b*e
)*(b*x + c*x^2))) - (c^2*(2*c*d - b*e*(2 - m))*(d + e*x)^(1 + m)*Hypergeome
tric2F1[1, 1 + m, 2 + m, (c*(d + e*x))/(c*d - b*e)])/(b^3*(c*d - b*e)^2*(1
+ m)) + ((2*c*d - b*e*m)*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 +
m, 1 + (e*x)/d])/(b^3*d^2*(1 + m))
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
```

```
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(d+ex)^m}{\left(bx+cx^2\right)^2}\,dx = -\frac{(d+ex)^{1+m}(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)\left(bx+cx^2\right)} - \frac{\int \frac{(d+ex)^m((cd-be)(2cd-bem)-ce(2cd-be)mx)}{bx+cx^2}\,dx}{b^2d(cd-be)}$$

$$= -\frac{(d+ex)^{1+m}(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)\left(bx+cx^2\right)} - \frac{\int \left(\frac{(-cd+be)(-2cd+bem)(d+ex)^m}{bx} + \frac{c^2d(-2cd+be(2-m))(d+ex)^m}{b(b+cx)}\right) dx}{b^2d(cd-be)}$$

$$= -\frac{(d+ex)^{1+m}(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)\left(bx+cx^2\right)} + \frac{\left(c^2(2cd-be(2-m))\right)\int \frac{(d+ex)^m}{b+cx}\,dx}{b^3(cd-be)} - \frac{(2cd-bem)\int \frac{(d+\cdots}{\cdots}}{b^3d}$$

$$= -\frac{(d+ex)^{1+m}(b(cd-be)+c(2cd-be)x)}{b^2d(cd-be)\left(bx+cx^2\right)} - \frac{c^2(2cd-be(2-m))(d+ex)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{c(\cdots}{c\cdots}\right.}{b^3(cd-be)^2(1+m)}$$

Mathematica [A] time = 0.147617, size = 174, normalized size = 0.97

$$-\frac{(d+ex)^{m+1}\left(b^2d(m+1)(cd-be)^2+x(b+cx)\left(c^2d^2(be(m-2)+2cd)\,{}_2F_1\left(1,m+1;m+2;\frac{c(d+ex)}{cd-be}\right)-(cd-be)^2(2cd-\cdots\right.\right.}{b^3d^2(m+1)x(b+cx)(cd-be)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(b*x + c*x^2)^2,x]
```

```
[Out] -(((d + e*x)^(1 + m)*(b^2*d*(c*d - b*e)^2*(1 + m) + b*c*d*(-2*c*d + b*e)*(-
(c*d) + b*e)*(1 + m)*x + x*(b + c*x)*(c^2*d^2*(2*c*d + b*e*(-2 + m))*Hyperg
eometric2F1[1, 1 + m, 2 + m, (c*(d + e*x))/(c*d - b*e)] - (c*d - b*e)^2*(2*
c*d - b*e*m)*Hypergeometric2F1[1, 1 + m, 2 + m, 1 + (e*x)/d])))/(b^3*d^2*(c
*d - b*e)^2*(1 + m)*x*(b + c*x)))
```

Maple [F] time = 0.61, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(cx^2+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(c*x^2+b*x)^2,x)
```

```
[Out] int((e*x+d)^m/(c*x^2+b*x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{(cx^2+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/(c*x^2 + b*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^m}{c^2x^4+2\,bcx^3+b^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m/(c^2*x^4 + 2*b*c*x^3 + b^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(c*x**2+b*x)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{(cx^2+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(c*x^2 + b*x)^2, x)
```

3.445 $\int \frac{(d+ex)^m}{\left(bx+cx^2\right)^3}\,dx$

Optimal. Leaf size=350

$$\frac{c^3(d+ex)^{m+1}\left(b^2e^2\left(m^2-7m+12\right)-6bcde(4-m)+12c^2d^2\right)\,{}_2F_1\left(1,m+1;m+2;\frac{c(d+ex)}{cd-be}\right)}{2b^5(m+1)(cd-be)^3}-\frac{(d+ex)^{m+1}\left(-b^2e^2(1-\right.}{}$$

```
[Out] -((d + e*x)^(1 + m)*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(2*b^2*d*(c*d - b*
e)*(b*x + c*x^2)^2) + ((d + e*x)^(1 + m)*(b*(c*d - b*e)*(6*c^2*d^2 - b^2*e^
2*(1 - m) - b*c*d*e*(4 + m)) + c*(2*c*d - b*e)*(6*c^2*d^2 - 6*b*c*d*e - b^2
*e^2*(1 - m))*x))/(2*b^4*d^2*(c*d - b*e)^2*(b*x + c*x^2)) + (c^3*(12*c^2*d^
2 - 6*b*c*d*e*(4 - m) + b^2*e^2*(12 - 7*m + m^2))*(d + e*x)^(1 + m)*Hyperge
ometric2F1[1, 1 + m, 2 + m, (c*(d + e*x))/(c*d - b*e)])/(2*b^5*(c*d - b*e)^
3*(1 + m)) - ((12*c^2*d^2 - 6*b*c*d*e*m - b^2*e^2*(1 - m)*m)*(d + e*x)^(1 +
 m)*Hypergeometric2F1[1, 1 + m, 2 + m, 1 + (e*x)/d])/(2*b^5*d^3*(1 + m))
```

Rubi [A] time = 0.427836, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {740, 822, 830, 65, 68}

$$\frac{c^3(d+ex)^{m+1}\left(b^2e^2\left(m^2-7m+12\right)-6bcde(4-m)+12c^2d^2\right)\,{}_2F_1\left(1,m+1;m+2;\frac{c(d+ex)}{cd-be}\right)}{2b^5(m+1)(cd-be)^3}-\frac{(d+ex)^{m+1}\left(-b^2e^2(1-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(b*x + c*x^2)^3,x]
```


```
[Out] -((d + e*x)^(1 + m)*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(2*b^2*d*(c*d - b*
e)*(b*x + c*x^2)^2) + ((d + e*x)^(1 + m)*(b*(c*d - b*e)*(6*c^2*d^2 - b^2*e^
2*(1 - m) - b*c*d*e*(4 + m)) + c*(2*c*d - b*e)*(6*c^2*d^2 - 6*b*c*d*e - b^2
*e^2*(1 - m))*x))/(2*b^4*d^2*(c*d - b*e)^2*(b*x + c*x^2)) + (c^3*(12*c^2*d^
2 - 6*b*c*d*e*(4 - m) + b^2*e^2*(12 - 7*m + m^2))*(d + e*x)^(1 + m)*Hyperge
ometric2F1[1, 1 + m, 2 + m, (c*(d + e*x))/(c*d - b*e)])/(2*b^5*(c*d - b*e)^
3*(1 + m)) - ((12*c^2*d^2 - 6*b*c*d*e*m - b^2*e^2*(1 - m)*m)*(d + e*x)^(1 +
 m)*Hypergeometric2F1[1, 1 + m, 2 + m, 1 + (e*x)/d])/(2*b^5*d^3*(1 + m))
```


Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
```

```
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(d+ex)^m}{\left(bx+cx^2\right)^3}\,dx = -\frac{(d+ex)^{1+m}(b(cd-be)+c(2cd-be)x)}{2b^2d(cd-be)\left(bx+cx^2\right)^2} - \frac{\int \frac{(d+ex)^m\left(6c^2d^2-b^2e^2(1-m)-bcde(4+m)+ce(2cd-be)(2-m)x\right)}{\left(bx+cx^2\right)^2}\,dx}{2b^2d(cd-be)}$$

$$= -\frac{(d+ex)^{1+m}(b(cd-be)+c(2cd-be)x)}{2b^2d(cd-be)\left(bx+cx^2\right)^2} + \frac{(d+ex)^{1+m}\left(b(cd-be)\left(6c^2d^2-b^2e^2(1-m)-bcde(4+m\right.\right.}{2b^4d^2(cd-be)^2\left(\right.}$$

$$= -\frac{(d+ex)^{1+m}(b(cd-be)+c(2cd-be)x)}{2b^2d(cd-be)\left(bx+cx^2\right)^2} + \frac{(d+ex)^{1+m}\left(b(cd-be)\left(6c^2d^2-b^2e^2(1-m)-bcde(4+m\right.\right.}{2b^4d^2(cd-be)^2\left(\right.}$$

$$= -\frac{(d+ex)^{1+m}(b(cd-be)+c(2cd-be)x)}{2b^2d(cd-be)\left(bx+cx^2\right)^2} + \frac{(d+ex)^{1+m}\left(b(cd-be)\left(6c^2d^2-b^2e^2(1-m)-bcde(4+m\right.\right.}{2b^4d^2(cd-be)^2\left(\right.}$$

$$= -\frac{(d+ex)^{1+m}(b(cd-be)+c(2cd-be)x)}{2b^2d(cd-be)\left(bx+cx^2\right)^2} + \frac{(d+ex)^{1+m}\left(b(cd-be)\left(6c^2d^2-b^2e^2(1-m)-bcde(4+m\right.\right.}{2b^4d^2(cd-be)^2\left(\right.}$$

Mathematica [A] time = 0.531144, size = 337, normalized size = 0.96

$$-\frac{(d+ex)^{m+1}\left(x^2\left(-(b+cx)\left((b+cx)\left(2c^3d^3\left(b^2e^2\left(m^2-7m+12\right)+6bcde(m-4)+12c^2d^2\right)\,{}_2F_1\left(1,m+1;m+2;\frac{c(d+ex)}{cd-be}\right.\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(b*x + c*x^2)^3,x]
```

```
[Out] -((d + e*x)^(1 + m)*(2*b^4*d^2*(c*d - b*e)^3*(1 + m) - 2*b^3*d*(c*d - b*e)^
3*(4*c*d - b*e*(-1 + m))*(1 + m)*x + x^2*(-2*b^2*c*d*(c*d - b*e)^2*(1 + m)*
(6*c^2*d^2 + b^2*e^2*(-1 + m) - b*c*d*e*(4 + m)) - (b + c*x)*(-2*b*c*d*(2*c
*d - b*e)*(-(c*d) + b*e)*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2*(-1 + m))*(1 + m)
 + (b + c*x)*(2*c^3*d^3*(12*c^2*d^2 + 6*b*c*d*e*(-4 + m) + b^2*e^2*(12 - 7*
m + m^2))*Hypergeometric2F1[1, 1 + m, 2 + m, (c*(d + e*x))/(c*d - b*e)] - 2
*(c*d - b*e)^3*(12*c^2*d^2 - 6*b*c*d*e*m + b^2*e^2*(-1 + m)*m)*Hypergeometr
ic2F1[1, 1 + m, 2 + m, 1 + (e*x)/d])))))/(4*b^5*d^3*(c*d - b*e)^3*(1 + m)*x
^2*(b + c*x)^2)
```

Maple [F] time = 0.71, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{(cx^2+bx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(c*x^2+b*x)^3,x)
```

```
[Out] int((e*x+d)^m/(c*x^2+b*x)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{(cx^2+bx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x)^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/(c*x^2 + b*x)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(ex+d)^m}{c^3x^6+3\,bc^2x^5+3\,b^2cx^4+b^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x)^3,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m/(c^3*x^6 + 3*b*c^2*x^5 + 3*b^2*c*x^4 + b^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(c*x**2+b*x)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{(cx^2+bx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x)^3,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(c*x^2 + b*x)^3, x)
```

3.446 $\int (d+ex)^m \left(bx+cx^2\right)^{3/2} dx$

Optimal. Leaf size=105

$$\frac{\left(bx+cx^2\right)^{3/2}(d+ex)^{m+1}F_1\left(m+1;-\frac{3}{2},-\frac{3}{2};m+2;\frac{d+ex}{d},\frac{c(d+ex)}{cd-be}\right)}{e(m+1)\left(-\frac{ex}{d}\right)^{3/2}\left(1-\frac{c(d+ex)}{cd-be}\right)^{3/2}}$$

```
[Out] ((d + e*x)^(1 + m)*(b*x + c*x^2)^(3/2)*AppellF1[1 + m, -3/2, -3/2, 2 + m, (
d + e*x)/d, (c*(d + e*x))/(c*d - b*e)])/(e*(1 + m)*(-((e*x)/d))^(3/2)*(1 -
(c*(d + e*x))/(c*d - b*e))^(3/2))
```

Rubi [A] time = 0.0590141, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {759, 133}

$$\frac{\left(bx+cx^2\right)^{3/2}(d+ex)^{m+1}F_1\left(m+1;-\frac{3}{2},-\frac{3}{2};m+2;\frac{d+ex}{d},\frac{c(d+ex)}{cd-be}\right)}{e(m+1)\left(-\frac{ex}{d}\right)^{3/2}\left(1-\frac{c(d+ex)}{cd-be}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(b*x + c*x^2)^(3/2),x]
```

```
[Out] ((d + e*x)^(1 + m)*(b*x + c*x^2)^(3/2)*AppellF1[1 + m, -3/2, -3/2, 2 + m, (
d + e*x)/d, (c*(d + e*x))/(c*d - b*e)])/(e*(1 + m)*(-((e*x)/d))^(3/2)*(1 -
(c*(d + e*x))/(c*d - b*e))^(3/2))
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (d+ex)^m \left(bx+cx^2\right)^{3/2} dx = \frac{\left(bx+cx^2\right)^{3/2} \text{Subst}\left(\int x^m \left(1-\frac{x}{d}\right)^{3/2} \left(1-\frac{cx}{cd-be}\right)^{3/2} dx, x, d+ex\right)}{e\left(1-\frac{d+ex}{d}\right)^{3/2} \left(1-\frac{d+ex}{d-\frac{be}{c}}\right)^{3/2}}$$

$$= \frac{(d+ex)^{1+m} \left(bx+cx^2\right)^{3/2} F_1\left(1+m;-\frac{3}{2},-\frac{3}{2};2+m;\frac{d+ex}{d},\frac{c(d+ex)}{cd-be}\right)}{e(1+m)\left(-\frac{ex}{d}\right)^{3/2} \left(1-\frac{c(d+ex)}{cd-be}\right)^{3/2}}$$

Mathematica [A] time = 0.0874568, size = 111, normalized size = 1.06

$$\frac{2x^2\sqrt{x(b+cx)}(d+ex)^m \left(\frac{ex}{d}+1\right)^{-m} \left(7bF_1\left(\frac{5}{2};-\frac{1}{2},-m;\frac{7}{2};-\frac{cx}{b},-\frac{ex}{d}\right)+5cxF_1\left(\frac{7}{2};-\frac{1}{2},-m;\frac{9}{2};-\frac{cx}{b},-\frac{ex}{d}\right)\right)}{35\sqrt{\frac{cx}{b}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^m*(b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*x^2*Sqrt[x*(b + c*x)]*(d + e*x)^m*(7*b*AppellF1[5/2, -1/2, -m, 7/2, -((c
*x)/b), -((e*x)/d)] + 5*c*x*AppellF1[7/2, -1/2, -m, 9/2, -((c*x)/b), -((e*x
)/d)]))/(35*Sqrt[1 + (c*x)/b]*(1 + (e*x)/d)^m)
```

Maple [F] time = 0.655, size = 0, normalized size = 0.

$$\int (ex+d)^m \left(cx^2+bx\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x)^(3/2),x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{\frac{3}{2}} (ex+d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(3/2)*(e*x + d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2+bx\right)^{\frac{3}{2}} (ex+d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(3/2)*(e*x + d)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx\right)^{\frac{3}{2}} (ex + d)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(3/2)*(e*x + d)^m, x)
```

3.447 $\int (d+ex)^m \sqrt{bx+cx^2}\,dx$

Optimal. Leaf size=105

$$\frac{\sqrt{bx+cx^2}(d+ex)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{d+ex}{d},\frac{c(d+ex)}{cd-be}\right)}{e(m+1)\sqrt{-\frac{ex}{d}}\sqrt{1-\frac{c(d+ex)}{cd-be}}}$$

```
[Out] ((d + e*x)^(1 + m)*Sqrt[b*x + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m, (d
+ e*x)/d, (c*(d + e*x))/(c*d - b*e)])/(e*(1 + m)*Sqrt[-((e*x)/d)]*Sqrt[1 -
(c*(d + e*x))/(c*d - b*e)])
```

Rubi [A] time = 0.0493167, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {759, 133}

$$\frac{\sqrt{bx+cx^2}(d+ex)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{d+ex}{d},\frac{c(d+ex)}{cd-be}\right)}{e(m+1)\sqrt{-\frac{ex}{d}}\sqrt{1-\frac{c(d+ex)}{cd-be}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*Sqrt[b*x + c*x^2],x]
```

```
[Out] ((d + e*x)^(1 + m)*Sqrt[b*x + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m, (d
+ e*x)/d, (c*(d + e*x))/(c*d - b*e)])/(e*(1 + m)*Sqrt[-((e*x)/d)]*Sqrt[1 -
(c*(d + e*x))/(c*d - b*e)])
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^m\sqrt{bx+cx^2}\,dx &= \frac{\sqrt{bx+cx^2}\,\text{Subst}\left(\int x^m\sqrt{1-\frac{x}{d}}\sqrt{1-\frac{cx}{cd-be}}\,dx,x,d+ex\right)}{e\sqrt{1-\frac{d+ex}{d}}\sqrt{1-\frac{d+ex}{d-\frac{be}{c}}}}\\ &= \frac{(d+ex)^{1+m}\sqrt{bx+cx^2}F_1\left(1+m;-\frac{1}{2},-\frac{1}{2};2+m;\frac{d+ex}{d},\frac{c(d+ex)}{cd-be}\right)}{e(1+m)\sqrt{-\frac{ex}{d}}\sqrt{1-\frac{c(d+ex)}{cd-be}}}\end{aligned}$$

Mathematica [A] time = 0.0352619, size = 76, normalized size = 0.72

$$\frac{2x\sqrt{x(b+cx)}(d+ex)^m\left(\frac{d+ex}{d}\right)^{-m}F_1\left(\frac{3}{2};-\frac{1}{2},-m;\frac{5}{2};-\frac{cx}{b},-\frac{ex}{d}\right)}{3\sqrt{\frac{b+cx}{b}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^m*Sqrt[b*x + c*x^2],x]
```

```
[Out] (2*x*Sqrt[x*(b + c*x)]*(d + e*x)^m*AppellF1[3/2, -1/2, -m, 5/2, -((c*x)/b),
 -((e*x)/d)])/(3*Sqrt[(b + c*x)/b]*((d + e*x)/d)^m)
```

Maple [F] time = 0.614, size = 0, normalized size = 0.

$$\int (ex+d)^m\sqrt{cx^2+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x)^(1/2),x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2+bx}(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x)*(e*x + d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^2+bx}(ex+d)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x)*(e*x + d)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x\,(b+cx)}\,(d+ex)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x*(b + c*x))*(d + e*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + bx}(ex + d)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x)*(e*x + d)^m, x)
```

3.448 $\int \frac{(d+ex)^m}{\sqrt{bx+cx^2}} dx$

Optimal. Leaf size=105

$$\frac{\sqrt{-\frac{ex}{d}}(d+ex)^{m+1}\sqrt{1-\frac{c(d+ex)}{cd-be}}F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{d+ex}{d},\frac{c(d+ex)}{cd-be}\right)}{e(m+1)\sqrt{bx+cx^2}}$$

```
[Out] (Sqrt[-((e*x)/d)]*(d + e*x)^(1 + m)*Sqrt[1 - (c*(d + e*x))/(c*d - b*e)]*App
ellF1[1 + m, 1/2, 1/2, 2 + m, (d + e*x)/d, (c*(d + e*x))/(c*d - b*e)])/(e*(
1 + m)*Sqrt[b*x + c*x^2])
```

Rubi [A] time = 0.0510385, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {759, 133}

$$\frac{\sqrt{-\frac{ex}{d}}(d+ex)^{m+1}\sqrt{1-\frac{c(d+ex)}{cd-be}}F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{d+ex}{d},\frac{c(d+ex)}{cd-be}\right)}{e(m+1)\sqrt{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/Sqrt[b*x + c*x^2],x]
```

```
[Out] (Sqrt[-((e*x)/d)]*(d + e*x)^(1 + m)*Sqrt[1 - (c*(d + e*x))/(c*d - b*e)]*App
ellF1[1 + m, 1/2, 1/2, 2 + m, (d + e*x)/d, (c*(d + e*x))/(c*d - b*e)])/(e*(
1 + m)*Sqrt[b*x + c*x^2])
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^m}{\sqrt{bx+cx^2}}\,dx &= \frac{\left(\sqrt{1-\frac{d+ex}{d}}\sqrt{1-\frac{d+ex}{d-\frac{be}{c}}}\right)\text{Subst}\left(\int \frac{x^m}{\sqrt{1-\frac{x}{d}}\sqrt{1-\frac{cx}{cd-be}}}\,dx,x,d+ex\right)}{e\sqrt{bx+cx^2}}\\ &= \frac{\sqrt{-\frac{ex}{d}}(d+ex)^{1+m}\sqrt{1-\frac{c(d+ex)}{cd-be}}F_1\left(1+m;\frac{1}{2},\frac{1}{2};2+m;\frac{d+ex}{d},\frac{c(d+ex)}{cd-be}\right)}{e(1+m)\sqrt{bx+cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0402688, size = 74, normalized size = 0.7

$$\frac{2x\sqrt{\frac{b+cx}{b}}(d+ex)^m\left(\frac{d+ex}{d}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};-\frac{cx}{b},-\frac{ex}{d}\right)}{\sqrt{x(b+cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^m/Sqrt[b*x + c*x^2],x]
```

```
[Out] (2*x*Sqrt[(b + c*x)/b]*(d + e*x)^m*AppellF1[1/2, 1/2, -m, 3/2, -((c*x)/b),
-((e*x)/d)])/(Sqrt[x*(b + c*x)]*((d + e*x)/d)^m)
```

Maple [F] time = 0.613, size = 0, normalized size = 0.

$$\int (ex+d)^m \frac{1}{\sqrt{cx^2+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(c*x^2+b*x)^(1/2),x)
```

```
[Out] int((e*x+d)^m/(c*x^2+b*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\sqrt{cx^2+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/sqrt(c*x^2 + b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^m}{\sqrt{cx^2+bx}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m/sqrt(c*x^2 + b*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m}{\sqrt{x(b+cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(c*x**2+b*x)**(1/2),x)
```

```
[Out] Integral((d + e*x)**m/sqrt(x*(b + c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\sqrt{cx^2+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/sqrt(c*x^2 + b*x), x)
```

3.449 $\int \frac{(d+ex)^m}{\left(bx+cx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=105

$$\frac{\left(-\frac{ex}{d}\right)^{3/2} (d+ex)^{m+1} \left(1-\frac{c(d+ex)}{cd-be}\right)^{3/2} F_1\left(m+1;\frac{3}{2},\frac{3}{2};m+2;\frac{d+ex}{d},\frac{c(d+ex)}{cd-be}\right)}{e(m+1)\left(bx+cx^2\right)^{3/2}}$$

```
[Out] ((-((e*x)/d))^(3/2)*(d + e*x)^(1 + m)*(1 - (c*(d + e*x))/(c*d - b*e))^(3/2)
*AppellF1[1 + m, 3/2, 3/2, 2 + m, (d + e*x)/d, (c*(d + e*x))/(c*d - b*e)])/
(e*(1 + m)*(b*x + c*x^2)^(3/2))
```

Rubi [A] time = 0.0480589, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {759, 133}

$$\frac{\left(-\frac{ex}{d}\right)^{3/2} (d+ex)^{m+1} \left(1-\frac{c(d+ex)}{cd-be}\right)^{3/2} F_1\left(m+1;\frac{3}{2},\frac{3}{2};m+2;\frac{d+ex}{d},\frac{c(d+ex)}{cd-be}\right)}{e(m+1)\left(bx+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(b*x + c*x^2)^(3/2),x]
```

```
[Out] ((-((e*x)/d))^(3/2)*(d + e*x)^(1 + m)*(1 - (c*(d + e*x))/(c*d - b*e))^(3/2)
*AppellF1[1 + m, 3/2, 3/2, 2 + m, (d + e*x)/d, (c*(d + e*x))/(c*d - b*e)])/
(e*(1 + m)*(b*x + c*x^2)^(3/2))
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] && !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{(d+ex)^m}{\left(bx+cx^2\right)^{3/2}}\,dx = \frac{\left(\left(1-\frac{d+ex}{d}\right)^{3/2}\left(1-\frac{d+ex}{d-\frac{be}{c}}\right)^{3/2}\right)\text{Subst}\left(\int \frac{x^m}{\left(1-\frac{x}{d}\right)^{3/2}\left(1-\frac{cx}{cd-be}\right)^{3/2}}\,dx, x, d+ex\right)}{e\left(bx+cx^2\right)^{3/2}}$$
$$= \frac{\left(-\frac{ex}{d}\right)^{3/2}(d+ex)^{1+m}\left(1-\frac{c(d+ex)}{cd-be}\right)^{3/2} F_1\left(1+m;\frac{3}{2},\frac{3}{2};2+m;\frac{d+ex}{d},\frac{c(d+ex)}{cd-be}\right)}{e(1+m)\left(bx+cx^2\right)^{3/2}}$$

Mathematica [A] time = 0.133985, size = 138, normalized size = 1.31

$$-\frac{2\sqrt{x(b+cx)}(d+ex)^m\left(\frac{ex}{d}+1\right)^{-m}\left(bF_1\left(-\frac{1}{2};-\frac{1}{2},-m;\frac{1}{2};-\frac{cx}{b},-\frac{ex}{d}\right)+cx\left(F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};-\frac{cx}{b},-\frac{ex}{d}\right)+F_1\left(\frac{1}{2};\frac{3}{2},-m;\frac{3}{2};-\right.\right.\right.}{b^3x\sqrt{\frac{cx}{b}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^m/(b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[x*(b + c*x)]*(d + e*x)^m*(b*AppellF1[-1/2, -1/2, -m, 1/2, -((c*x)/
b), -((e*x)/d)] + c*x*(AppellF1[1/2, 1/2, -m, 3/2, -((c*x)/b), -((e*x)/d)]
+ AppellF1[1/2, 3/2, -m, 3/2, -((c*x)/b), -((e*x)/d)])))/(b^3*x*Sqrt[1 + (c
*x)/b]*(1 + (e*x)/d)^m)
```

Maple [F] time = 0.613, size = 0, normalized size = 0.

$$\int (ex+d)^m \left(cx^2+bx\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(c*x^2+b*x)^(3/2),x)
```

```
[Out] int((e*x+d)^m/(c*x^2+b*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(cx^2+bx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/(c*x^2 + b*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+bx}(ex+d)^m}{c^2x^4+2\,bcx^3+b^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x)*(e*x + d)^m/(c^2*x^4 + 2*b*c*x^3 + b^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m}{(x(b+cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(c*x**2+b*x)**(3/2),x)
```

```
[Out] Integral((d + e*x)**m/(x*(b + c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(cx^2+bx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(c*x^2 + b*x)^(3/2), x)
```

3.450 $\int (d+ex)^m \left(bx+cx^2\right)^p \, dx$

Optimal. Leaf size=103

$$\frac{\left(bx+cx^2\right)^p (d+ex)^{m+1} \left(-\frac{ex}{d}\right)^{-p} \left(1-\frac{c(d+ex)}{cd-be}\right)^{-p} F_1\left(m+1;-p,-p;m+2;\frac{d+ex}{d},\frac{c(d+ex)}{cd-be}\right)}{e(m+1)}$$

```
[Out] ((d + e*x)^(1 + m)*(b*x + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (d + e*x)
/d, (c*(d + e*x))/(c*d - b*e)])/(e*(1 + m)*(-((e*x)/d))^p*(1 - (c*(d + e*x)
)/(c*d - b*e))^p)
```

Rubi [A] time = 0.0491622, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {759, 133}

$$\frac{\left(bx+cx^2\right)^p (d+ex)^{m+1} \left(-\frac{ex}{d}\right)^{-p} \left(1-\frac{c(d+ex)}{cd-be}\right)^{-p} F_1\left(m+1;-p,-p;m+2;\frac{d+ex}{d},\frac{c(d+ex)}{cd-be}\right)}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(b*x + c*x^2)^p,x]
```


```
[Out] ((d + e*x)^(1 + m)*(b*x + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (d + e*x)
/d, (c*(d + e*x))/(c*d - b*e)])/(e*(1 + m)*(-((e*x)/d))^p*(1 - (c*(d + e*x)
)/(c*d - b*e))^p)
```


Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^m \left(bx+cx^2\right)^p \, dx &= \frac{\left(\left(bx+cx^2\right)^p \left(1-\frac{d+ex}{d}\right)^{-p} \left(1-\frac{d+ex}{d-\frac{be}{c}}\right)^{-p}\right) \operatorname{Subst}\left(\int x^m \left(1-\frac{x}{d}\right)^p \left(1-\frac{cx}{cd-be}\right)^p \, dx, x, d+ex\right)}{e}\\ &= \frac{\left(-\frac{ex}{d}\right)^{-p} (d+ex)^{1+m} \left(bx+cx^2\right)^p \left(1-\frac{c(d+ex)}{cd-be}\right)^{-p} F_1\left(1+m;-p,-p;2+m;\frac{d+ex}{d},\frac{c(d+ex)}{cd-be}\right)}{e(1+m)}\end{aligned}$$

Mathematica [A] time = 0.101776, size = 76, normalized size = 0.74

$$\frac{x\left(\frac{b+cx}{b}\right)^{-p}(x(b+cx))^p(d+ex)^m\left(\frac{d+ex}{d}\right)^{-m}F_1\left(p+1;-p,-m;p+2;-\frac{cx}{b},-\frac{ex}{d}\right)}{p+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^m*(b*x + c*x^2)^p,x]
```

```
[Out] (x*(x*(b + c*x))^p*(d + e*x)^m*AppellF1[1 + p, -p, -m, 2 + p, -((c*x)/b), -
((e*x)/d)])/((1 + p)*((b + c*x)/b)^p*((d + e*x)/d)^m)
```

Maple [F] time = 0.664, size = 0, normalized size = 0.

$$\int (ex+d)^m \left(cx^2+bx\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x)^p,x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^p (ex+d)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^p*(e*x + d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(cx^2+bx\right)^p (ex+d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^p*(e*x + d)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx\right)^p (ex + d)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^p*(e*x + d)^m, x)
```

3.451 $\int (d+ex)^4 \left(a+cx^2\right) dx$

Optimal. Leaf size=57

$$\frac{(d+ex)^5 \left(ae^2+cd^2\right)}{5e^3}+\frac{c(d+ex)^7}{7e^3}-\frac{cd(d+ex)^6}{3e^3}$$

```
[Out] ((c*d^2 + a*e^2)*(d + e*x)^5)/(5*e^3) - (c*d*(d + e*x)^6)/(3*e^3) + (c*(d +
 e*x)^7)/(7*e^3)
```

Rubi [A] time = 0.0632473, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {697}

$$\frac{(d+ex)^5 \left(ae^2+cd^2\right)}{5e^3}+\frac{c(d+ex)^7}{7e^3}-\frac{cd(d+ex)^6}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4*(a + c*x^2),x]
```

```
[Out] ((c*d^2 + a*e^2)*(d + e*x)^5)/(5*e^3) - (c*d*(d + e*x)^6)/(3*e^3) + (c*(d +
 e*x)^7)/(7*e^3)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^4 \left(a+cx^2\right) dx &= \int \left(\frac{\left(cd^2+ae^2\right)(d+ex)^4}{e^2}-\frac{2cd(d+ex)^5}{e^2}+\frac{c(d+ex)^6}{e^2}\right) dx\\ &= \frac{\left(cd^2+ae^2\right)(d+ex)^5}{5e^3}-\frac{cd(d+ex)^6}{3e^3}+\frac{c(d+ex)^7}{7e^3}\end{aligned}$$

Mathematica [A] time = 0.0161587, size = 101, normalized size = 1.77

$$\frac{1}{5}e^2x^5\left(ae^2+6cd^2\right)+dex^4\left(ae^2+cd^2\right)+\frac{1}{3}d^2x^3\left(6ae^2+cd^2\right)+2ad^3ex^2+ad^4x+\frac{2}{3}cde^3x^6+\frac{1}{7}ce^4x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4*(a + c*x^2),x]
```

```
[Out] a*d^4*x + 2*a*d^3*e*x^2 + (d^2*(c*d^2 + 6*a*e^2)*x^3)/3 + d*e*(c*d^2 + a*e^
2)*x^4 + (e^2*(6*c*d^2 + a*e^2)*x^5)/5 + (2*c*d*e^3*x^6)/3 + (c*e^4*x^7)/7
```

Maple [A] time = 0.042, size = 97, normalized size = 1.7

$$\frac{e^4cx^7}{7}+\frac{2\,de^3cx^6}{3}+\frac{\left(e^4a+6\,d^2e^2c\right)x^5}{5}+\frac{\left(4\,de^3a+4\,d^3ec\right)x^4}{4}+\frac{\left(6\,d^2e^2a+d^4c\right)x^3}{3}+2\,d^3eax^2+d^4ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(c*x^2+a),x)
```

```
[Out] 1/7*e^4*c*x^7+2/3*d*e^3*c*x^6+1/5*(a*e^4+6*c*d^2*e^2)*x^5+1/4*(4*a*d*e^3+4*
c*d^3*e)*x^4+1/3*(6*a*d^2*e^2+c*d^4)*x^3+2*d^3*e*a*x^2+d^4*a*x
```

Maxima [A] time = 1.1672, size = 126, normalized size = 2.21

$$\frac{1}{7}\,ce^4x^7+\frac{2}{3}\,cde^3x^6+2\,ad^3ex^2+ad^4x+\frac{1}{5}\left(6\,cd^2e^2+ae^4\right)x^5+\left(cd^3e+ade^3\right)x^4+\frac{1}{3}\left(cd^4+6\,ad^2e^2\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/7*c*e^4*x^7 + 2/3*c*d*e^3*x^6 + 2*a*d^3*e*x^2 + a*d^4*x + 1/5*(6*c*d^2*e^
2 + a*e^4)*x^5 + (c*d^3*e + a*d*e^3)*x^4 + 1/3*(c*d^4 + 6*a*d^2*e^2)*x^3
```

Fricas [A] time = 1.64037, size = 212, normalized size = 3.72

$$\frac{1}{7}x^7e^4c+\frac{2}{3}x^6e^3dc+\frac{6}{5}x^5e^2d^2c+\frac{1}{5}x^5e^4a+x^4ed^3c+x^4e^3da+\frac{1}{3}x^3d^4c+2x^3e^2d^2a+2x^2ed^3a+xd^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*e^4*c + 2/3*x^6*e^3*d*c + 6/5*x^5*e^2*d^2*c + 1/5*x^5*e^4*a + x^4*e
*d^3*c + x^4*e^3*d*a + 1/3*x^3*d^4*c + 2*x^3*e^2*d^2*a + 2*x^2*e*d^3*a + x*
d^4*a
```

Sympy [A] time = 0.127347, size = 100, normalized size = 1.75

$$ad^4x+2ad^3ex^2+\frac{2cde^3x^6}{3}+\frac{ce^4x^7}{7}+x^5\left(\frac{ae^4}{5}+\frac{6cd^2e^2}{5}\right)+x^4\left(ade^3+cd^3e\right)+x^3\left(2ad^2e^2+\frac{cd^4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(c*x**2+a),x)
```

```
[Out] a*d**4*x + 2*a*d**3*e*x**2 + 2*c*d*e**3*x**6/3 + c*e**4*x**7/7 + x**5*(a*e*
*4/5 + 6*c*d**2*e**2/5) + x**4*(a*d*e**3 + c*d**3*e) + x**3*(2*a*d**2*e**2
+ c*d**4/3)
```

Giac [A] time = 1.17834, size = 124, normalized size = 2.18

$$\frac{1}{7}\,cx^7e^4 + \frac{2}{3}\,cdx^6e^3 + \frac{6}{5}\,cd^2x^5e^2 + cd^3x^4e + \frac{1}{3}\,cd^4x^3 + \frac{1}{5}\,ax^5e^4 + adx^4e^3 + 2\,ad^2x^3e^2 + 2\,ad^3x^2e + ad^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+a),x, algorithm="giac")
```

```
[Out] 1/7*c*x^7*e^4 + 2/3*c*d*x^6*e^3 + 6/5*c*d^2*x^5*e^2 + c*d^3*x^4*e + 1/3*c*d
^4*x^3 + 1/5*a*x^5*e^4 + a*d*x^4*e^3 + 2*a*d^2*x^3*e^2 + 2*a*d^3*x^2*e + a*
d^4*x
```

3.452 $\int (d+ex)^3 \left(a+cx^2\right) dx$

Optimal. Leaf size=57

$$\frac{(d+ex)^4 \left(ae^2+cd^2\right)}{4e^3}+\frac{c(d+ex)^6}{6e^3}-\frac{2cd(d+ex)^5}{5e^3}$$

```
[Out] ((c*d^2 + a*e^2)*(d + e*x)^4)/(4*e^3) - (2*c*d*(d + e*x)^5)/(5*e^3) + (c*(d
 + e*x)^6)/(6*e^3)
```

Rubi [A] time = 0.0455637, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {697}

$$\frac{(d+ex)^4 \left(ae^2+cd^2\right)}{4e^3}+\frac{c(d+ex)^6}{6e^3}-\frac{2cd(d+ex)^5}{5e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + c*x^2),x]
```

```
[Out] ((c*d^2 + a*e^2)*(d + e*x)^4)/(4*e^3) - (2*c*d*(d + e*x)^5)/(5*e^3) + (c*(d
 + e*x)^6)/(6*e^3)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^3 \left(a+cx^2\right) dx &= \int \left(\frac{\left(cd^2+ae^2\right)(d+ex)^3}{e^2}-\frac{2cd(d+ex)^4}{e^2}+\frac{c(d+ex)^5}{e^2}\right) dx\\ &= \frac{\left(cd^2+ae^2\right)(d+ex)^4}{4e^3}-\frac{2cd(d+ex)^5}{5e^3}+\frac{c(d+ex)^6}{6e^3}\end{aligned}$$

Mathematica [A] time = 0.013662, size = 74, normalized size = 1.3

$$\frac{1}{4}ax\left(6d^2ex+4d^3+4de^2x^2+e^3x^3\right)+\frac{1}{60}cx^3\left(45d^2ex+20d^3+36de^2x^2+10e^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + c*x^2),x]
```

```
[Out] (a*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3))/4 + (c*x^3*(20*d^3 + 45*d
^2*e*x + 36*d*e^2*x^2 + 10*e^3*x^3))/60
```

Maple [A] time = 0.04, size = 73, normalized size = 1.3

$$\frac{e^{3} c x^{6}}{6}+\frac{3 d e^{2} c x^{5}}{5}+\frac{\left(e^{3} a+3 d^{2} e c\right) x^{4}}{4}+\frac{\left(3 d e^{2} a+d^{3} c\right) x^{3}}{3}+\frac{3 d^{2} e a x^{2}}{2}+d^{3} a x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+a),x)
```

```
[Out] 1/6*e^3*c*x^6+3/5*d*e^2*c*x^5+1/4*(a*e^3+3*c*d^2*e)*x^4+1/3*(3*a*d*e^2+c*d^
3)*x^3+3/2*d^2*e*a*x^2+d^3*a*x
```

Maxima [A] time = 1.15385, size = 97, normalized size = 1.7

$$\frac{1}{6}\, c e^{3} x^{6}+\frac{3}{5}\, c d e^{2} x^{5}+\frac{3}{2}\, a d^{2} e x^{2}+a d^{3} x+\frac{1}{4}\left(3\, c d^{2} e+a e^{3}\right) x^{4}+\frac{1}{3}\left(c d^{3}+3\, a d e^{2}\right) x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/6*c*e^3*x^6 + 3/5*c*d*e^2*x^5 + 3/2*a*d^2*e*x^2 + a*d^3*x + 1/4*(3*c*d^2*
e + a*e^3)*x^4 + 1/3*(c*d^3 + 3*a*d*e^2)*x^3
```

Fricas [A] time = 1.60084, size = 169, normalized size = 2.96

$$\frac{1}{6} x^{6} e^{3} c+\frac{3}{5} x^{5} e^{2} d c+\frac{3}{4} x^{4} e d^{2} c+\frac{1}{4} x^{4} e^{3} a+\frac{1}{3} x^{3} d^{3} c+x^{3} e^{2} d a+\frac{3}{2} x^{2} e d^{2} a+x d^{3} a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*e^3*c + 3/5*x^5*e^2*d*c + 3/4*x^4*e*d^2*c + 1/4*x^4*e^3*a + 1/3*x^3
*d^3*c + x^3*e^2*d*a + 3/2*x^2*e*d^2*a + x*d^3*a
```

Sympy [A] time = 0.125414, size = 80, normalized size = 1.4

$$a d^{3} x+\frac{3 a d^{2} e x^{2}}{2}+\frac{3 c d e^{2} x^{5}}{5}+\frac{c e^{3} x^{6}}{6}+x^{4}\left(\frac{a e^{3}}{4}+\frac{3 c d^{2} e}{4}\right)+x^{3}\left(a d e^{2}+\frac{c d^{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a),x)
```

```
[Out] a*d**3*x + 3*a*d**2*e*x**2/2 + 3*c*d*e**2*x**5/5 + c*e**3*x**6/6 + x**4*(a*
e**3/4 + 3*c*d**2*e/4) + x**3*(a*d*e**2 + c*d**3/3)
```

Giac [A] time = 1.31064, size = 96, normalized size = 1.68

$$\frac{1}{6}\, c x^{6} e^{3}+\frac{3}{5}\, c d x^{5} e^{2}+\frac{3}{4}\, c d^{2} x^{4} e+\frac{1}{3}\, c d^{3} x^{3}+\frac{1}{4}\, a x^{4} e^{3}+a d x^{3} e^{2}+\frac{3}{2}\, a d^{2} x^{2} e+a d^{3} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a),x, algorithm="giac")
```

```
[Out] 1/6*c*x^6*e^3 + 3/5*c*d*x^5*e^2 + 3/4*c*d^2*x^4*e + 1/3*c*d^3*x^3 + 1/4*a*x
^4*e^3 + a*d*x^3*e^2 + 3/2*a*d^2*x^2*e + a*d^3*x
```

3.453 $\int (d+ex)^2 \left(a+cx^2\right) dx$

Optimal. Leaf size=57

$$\frac{(d+ex)^3\left(ae^2+cd^2\right)}{3e^3}+\frac{c(d+ex)^5}{5e^3}-\frac{cd(d+ex)^4}{2e^3}$$

```
[Out] ((c*d^2 + a*e^2)*(d + e*x)^3)/(3*e^3) - (c*d*(d + e*x)^4)/(2*e^3) + (c*(d +
 e*x)^5)/(5*e^3)
```

Rubi [A] time = 0.0320099, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {697}

$$\frac{(d+ex)^3\left(ae^2+cd^2\right)}{3e^3}+\frac{c(d+ex)^5}{5e^3}-\frac{cd(d+ex)^4}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + c*x^2),x]
```

```
[Out] ((c*d^2 + a*e^2)*(d + e*x)^3)/(3*e^3) - (c*d*(d + e*x)^4)/(2*e^3) + (c*(d +
 e*x)^5)/(5*e^3)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^2\left(a+cx^2\right)dx &= \int \left(\frac{\left(cd^2+ae^2\right)(d+ex)^2}{e^2}-\frac{2cd(d+ex)^3}{e^2}+\frac{c(d+ex)^4}{e^2}\right)dx\\ &= \frac{\left(cd^2+ae^2\right)(d+ex)^3}{3e^3}-\frac{cd(d+ex)^4}{2e^3}+\frac{c(d+ex)^5}{5e^3}\end{aligned}$$

Mathematica [A] time = 0.0083728, size = 53, normalized size = 0.93

$$\frac{1}{3}x^3\left(ae^2+cd^2\right)+ad^2x+adex^2+\frac{1}{2}cdex^4+\frac{1}{5}ce^2x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + c*x^2),x]
```

```
[Out] a*d^2*x + a*d*e*x^2 + ((c*d^2 + a*e^2)*x^3)/3 + (c*d*e*x^4)/2 + (c*e^2*x^5)
/5
```

Maple [A] time = 0.042, size = 48, normalized size = 0.8

$$\frac{ce^2x^5}{5}+\frac{cdex^4}{2}+\frac{(ae^2+cd^2)\,x^3}{3}+adex^2+ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a),x)
```

```
[Out] 1/5*c*e^2*x^5+1/2*c*d*e*x^4+1/3*(a*e^2+c*d^2)*x^3+a*d*e*x^2+a*d^2*x
```

Maxima [A] time = 1.29003, size = 63, normalized size = 1.11

$$\frac{1}{5}\,ce^2x^5+\frac{1}{2}\,cdex^4+adex^2+ad^2x+\frac{1}{3}\,(cd^2+ae^2)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/5*c*e^2*x^5 + 1/2*c*d*e*x^4 + a*d*e*x^2 + a*d^2*x + 1/3*(c*d^2 + a*e^2)*x
^3
```

Fricas [A] time = 1.71241, size = 115, normalized size = 2.02

$$\frac{1}{5}x^5e^2c+\frac{1}{2}x^4edc+\frac{1}{3}x^3d^2c+\frac{1}{3}x^3e^2a+x^2eda+xd^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/5*x^5*e^2*c + 1/2*x^4*e*d*c + 1/3*x^3*d^2*c + 1/3*x^3*e^2*a + x^2*e*d*a +
 x*d^2*a
```

Sympy [A] time = 0.109106, size = 51, normalized size = 0.89

$$ad^2x+adex^2+\frac{cdex^4}{2}+\frac{ce^2x^5}{5}+x^3\left(\frac{ae^2}{3}+\frac{cd^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+a),x)
```

```
[Out] a*d**2*x + a*d*e*x**2 + c*d*e*x**4/2 + c*e**2*x**5/5 + x**3*(a*e**2/3 + c*d
**2/3)
```

Giac [A] time = 1.25979, size = 66, normalized size = 1.16

$$\frac{1}{5}\,cx^5e^2+\frac{1}{2}\,cdx^4e+\frac{1}{3}\,cd^2x^3+\frac{1}{3}\,ax^3e^2+adx^2e+ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a),x, algorithm="giac")
```

```
[Out] 1/5*c*x^5*e^2 + 1/2*c*d*x^4*e + 1/3*c*d^2*x^3 + 1/3*a*x^3*e^2 + a*d*x^2*e +
 a*d^2*x
```

3.454 $\int (d+ex)(a+cx^2)\,dx$

Optimal. Leaf size=31

$$\frac{e\left(a+cx^2\right)^2}{4c}+adx+\frac{1}{3}cdx^3$$

```
[Out] a*d*x + (c*d*x^3)/3 + (e*(a + c*x^2)^2)/(4*c)
```

Rubi [A] time = 0.006554, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {641}

$$\frac{e\left(a+cx^2\right)^2}{4c}+adx+\frac{1}{3}cdx^3$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + c*x^2),x]
```

```
[Out] a*d*x + (c*d*x^3)/3 + (e*(a + c*x^2)^2)/(4*c)
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(a+cx^2\right)\,dx &= \frac{e\left(a+cx^2\right)^2}{4c}+d\int \left(a+cx^2\right)\,dx\\ &= adx+\frac{1}{3}cdx^3+\frac{e\left(a+cx^2\right)^2}{4c}\end{aligned}$$

Mathematica [A] time = 0.0013999, size = 32, normalized size = 1.03

$$adx+\frac{1}{2}aex^2+\frac{1}{3}cdx^3+\frac{1}{4}cex^4$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + c*x^2),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + (c*d*x^3)/3 + (c*e*x^4)/4
```

Maple [A] time = 0.042, size = 27, normalized size = 0.9

$$\frac{cex^4}{4}+\frac{cdx^3}{3}+\frac{aex^2}{2}+adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+a),x)
```

```
[Out] 1/4*c*e*x^4+1/3*c*d*x^3+1/2*a*e*x^2+a*d*x
```

Maxima [A] time = 1.12008, size = 35, normalized size = 1.13

$$\frac{1}{4}cex^4 + \frac{1}{3}cdx^3 + \frac{1}{2}aex^2 + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*c*e*x^4 + 1/3*c*d*x^3 + 1/2*a*e*x^2 + a*d*x
```

Fricas [A] time = 1.6751, size = 66, normalized size = 2.13

$$\frac{1}{4}x^4ec + \frac{1}{3}x^3dc + \frac{1}{2}x^2ea + xda$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*e*c + 1/3*x^3*d*c + 1/2*x^2*e*a + x*d*a
```

Sympy [A] time = 0.069461, size = 29, normalized size = 0.94

$$adx + \frac{aex^2}{2} + \frac{cdx^3}{3} + \frac{cex^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+a),x)
```

```
[Out] a*d*x + a*e*x**2/2 + c*d*x**3/3 + c*e*x**4/4
```

Giac [A] time = 1.26689, size = 38, normalized size = 1.23

$$\frac{1}{4}cx^4e + \frac{1}{3}cdx^3 + \frac{1}{2}ax^2e + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*c*x^4*e + 1/3*c*d*x^3 + 1/2*a*x^2*e + a*d*x
```

3.455 $\int \frac{a+cx^2}{d+ex}\,dx$

Optimal. Leaf size=41

$$\frac{\left(ae^2+cd^2\right)\log(d+ex)}{e^3}-\frac{cdx}{e^2}+\frac{cx^2}{2e}$$

```
[Out] -((c*d*x)/e^2) + (c*x^2)/(2*e) + ((c*d^2 + a*e^2)*Log[d + e*x])/e^3
```

Rubi [A] time = 0.0300263, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {697}

$$\frac{\left(ae^2+cd^2\right)\log(d+ex)}{e^3}-\frac{cdx}{e^2}+\frac{cx^2}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/(d + e*x),x]
```

```
[Out] -((c*d*x)/e^2) + (c*x^2)/(2*e) + ((c*d^2 + a*e^2)*Log[d + e*x])/e^3
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+cx^2}{d+ex}\,dx &= \int \left(-\frac{cd}{e^2}+\frac{cx}{e}+\frac{cd^2+ae^2}{e^2(d+ex)}\right)dx\\ &= -\frac{cdx}{e^2}+\frac{cx^2}{2e}+\frac{\left(cd^2+ae^2\right)\log(d+ex)}{e^3}\end{aligned}$$

Mathematica [A] time = 0.0112357, size = 38, normalized size = 0.93

$$\frac{2\left(ae^2+cd^2\right)\log(d+ex)+cex(ex-2d)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/(d + e*x),x]
```

```
[Out] (c*e*x*(-2*d + e*x) + 2*(c*d^2 + a*e^2)*Log[d + e*x])/(2*e^3)
```

Maple [A] time = 0.044, size = 44, normalized size = 1.1

$$\frac{cx^2}{2\,e}-\frac{cdx}{e^2}+\frac{\ln(ex+d)\,a}{e}+\frac{\ln(ex+d)\,cd^2}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d),x)
```

```
[Out] 1/2*c*x^2/e-c*d*x/e^2+1/e*ln(e*x+d)*a+1/e^3*ln(e*x+d)*c*d^2
```

Maxima [A] time = 1.1435, size = 53, normalized size = 1.29

$$\frac{cex^2 - 2\,cdx}{2\,e^2} + \frac{\left(cd^2 + ae^2\right)\log\left(ex + d\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*(c*e*x^2 - 2*c*d*x)/e^2 + (c*d^2 + a*e^2)*log(e*x + d)/e^3
```

Fricas [A] time = 2.12247, size = 89, normalized size = 2.17

$$\frac{ce^2x^2 - 2\,cdex + 2\left(cd^2 + ae^2\right)\log\left(ex + d\right)}{2\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/2*(c*e^2*x^2 - 2*c*d*e*x + 2*(c*d^2 + a*e^2)*log(e*x + d))/e^3
```

Sympy [A] time = 0.354507, size = 36, normalized size = 0.88

$$-\frac{cdx}{e^2} + \frac{cx^2}{2e} + \frac{\left(ae^2 + cd^2\right)\log\left(d + ex\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d),x)
```

```
[Out] -c*d*x/e**2 + c*x**2/(2*e) + (a*e**2 + c*d**2)*log(d + e*x)/e**3
```

Giac [A] time = 1.24533, size = 53, normalized size = 1.29

$$\left(cd^2 + ae^2\right)e^{(-3)}\log\left(|xe + d|\right) + \frac{1}{2}\left(cx^2e - 2\,cdx\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d),x, algorithm="giac")
```

```
[Out] (c*d^2 + a*e^2)*e^(-3)*log(abs(x*e + d)) + 1/2*(c*x^2*e - 2*c*d*x)*e^(-2)
```

3.456 $\int \frac{a+cx^2}{(d+ex)^2} \, dx$

Optimal. Leaf size=43

$$-\frac{ae^2+cd^2}{e^3(d+ex)}-\frac{2cd\log(d+ex)}{e^3}+\frac{cx}{e^2}$$

```
[Out] (c*x)/e^2 - (c*d^2 + a*e^2)/(e^3*(d + e*x)) - (2*c*d*Log[d + e*x])/e^3
```

Rubi [A] time = 0.0282407, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {697}

$$-\frac{ae^2+cd^2}{e^3(d+ex)}-\frac{2cd\log(d+ex)}{e^3}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/(d + e*x)^2,x]

[Out] (c*x)/e^2 - (c*d^2 + a*e^2)/(e^3*(d + e*x)) - (2*c*d*Log[d + e*x])/e^3
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{a+cx^2}{(d+ex)^2} \, dx &= \int \left(\frac{c}{e^2}+\frac{cd^2+ae^2}{e^2(d+ex)^2}-\frac{2cd}{e^2(d+ex)}\right) dx \\ &= \frac{cx}{e^2}-\frac{cd^2+ae^2}{e^3(d+ex)}-\frac{2cd\log(d+ex)}{e^3} \end{aligned}$$

Mathematica [A] time = 0.0208162, size = 39, normalized size = 0.91

$$\frac{-\frac{ae^2+cd^2}{d+ex}-2cd\log(d+ex)+cex}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/(d + e*x)^2,x]

[Out] (c*e*x - (c*d^2 + a*e^2)/(d + e*x) - 2*c*d*Log[d + e*x])/e^3
```

Maple [A] time = 0.054, size = 50, normalized size = 1.2

$$\frac{cx}{e^2}-2\,\frac{cd\ln(ex+d)}{e^3}-\frac{a}{e\,(ex+d)}-\frac{cd^2}{e^3\,(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^2,x)
```

```
[Out] c*x/e^2-2*c*d*ln(e*x+d)/e^3-1/e/(e*x+d)*a-1/e^3/(e*x+d)*c*d^2
```

Maxima [A] time = 1.17228, size = 62, normalized size = 1.44

$$-\frac{cd^2 + ae^2}{e^4x + de^3} + \frac{cx}{e^2} - \frac{2\,cd\log\left(ex + d\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(c*d^2 + a*e^2)/(e^4*x + d*e^3) + c*x/e^2 - 2*c*d*log(e*x + d)/e^3
```

Fricas [A] time = 1.80624, size = 122, normalized size = 2.84

$$\frac{ce^2x^2 + cdex - cd^2 - ae^2 - 2\left(cdex + cd^2\right)\log\left(ex + d\right)}{e^4x + de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] (c*e^2*x^2 + c*d*e*x - c*d^2 - a*e^2 - 2*(c*d*e*x + c*d^2)*log(e*x + d))/(e
^4*x + d*e^3)
```

Sympy [A] time = 0.516944, size = 41, normalized size = 0.95

$$-\frac{2cd\log\left(d + ex\right)}{e^3} + \frac{cx}{e^2} - \frac{ae^2 + cd^2}{de^3 + e^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**2,x)
```

```
[Out] -2*c*d*log(d + e*x)/e**3 + c*x/e**2 - (a*e**2 + c*d**2)/(d*e**3 + e**4*x)
```

Giac [A] time = 1.3839, size = 88, normalized size = 2.05

$$\left(2\,de^{(-3)}\log\left(\frac{|xe + d|e^{(-1)}}{(xe + d)^2}\right) + (xe + d)e^{(-3)} - \frac{d^2e^{(-3)}}{xe + d}\right)c - \frac{ae^{(-1)}}{xe + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] (2*d*e^(-3)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2) + (x*e + d)*e^(-3) - d^2*e
^(-3)/(x*e + d))*c - a*e^(-1)/(x*e + d)
```

3.457 $\int \frac{a+cx^2}{(d+ex)^3} dx$

Optimal. Leaf size=51

$$-\frac{ae^2+cd^2}{2e^3(d+ex)^2}+\frac{2cd}{e^3(d+ex)}+\frac{c\log(d+ex)}{e^3}$$

```
[Out] -(c*d^2 + a*e^2)/(2*e^3*(d + e*x)^2) + (2*c*d)/(e^3*(d + e*x)) + (c*Log[d +
 e*x])/e^3
```

Rubi [A] time = 0.0324775, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {697}

$$-\frac{ae^2+cd^2}{2e^3(d+ex)^2}+\frac{2cd}{e^3(d+ex)}+\frac{c\log(d+ex)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/(d + e*x)^3,x]
```

```
[Out] -(c*d^2 + a*e^2)/(2*e^3*(d + e*x)^2) + (2*c*d)/(e^3*(d + e*x)) + (c*Log[d +
 e*x])/e^3
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+cx^2}{(d+ex)^3}\,dx &= \int \left(\frac{cd^2+ae^2}{e^2(d+ex)^3}-\frac{2cd}{e^2(d+ex)^2}+\frac{c}{e^2(d+ex)}\right)dx\\ &= -\frac{cd^2+ae^2}{2e^3(d+ex)^2}+\frac{2cd}{e^3(d+ex)}+\frac{c\log(d+ex)}{e^3}\end{aligned}$$

Mathematica [A] time = 0.015273, size = 48, normalized size = 0.94

$$\frac{-ae^2+cd(3d+4ex)+2c(d+ex)^2\log(d+ex)}{2e^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/(d + e*x)^3,x]
```

```
[Out] (-(a*e^2) + c*d*(3*d + 4*e*x) + 2*c*(d + e*x)^2*Log[d + e*x])/(2*e^3*(d + e
*x)^2)
```

Maple [A] time = 0.052, size = 56, normalized size = 1.1

$$\frac{c\ln(ex+d)}{e^3}+2\frac{cd}{e^3(ex+d)}-\frac{a}{2e(ex+d)^2}-\frac{cd^2}{2e^3(ex+d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^3,x)
```

```
[Out] c*ln(e*x+d)/e^3+2*c*d/e^3/(e*x+d)-1/2/e/(e*x+d)^2*a-1/2/e^3/(e*x+d)^2*c*d^2
```

Maxima [A] time = 1.16519, size = 77, normalized size = 1.51

$$\frac{4\,cdex+3\,cd^2-ae^2}{2\left(e^5x^2+2\,de^4x+d^2e^3\right)}+\frac{c\log(ex+d)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(4*c*d*e*x + 3*c*d^2 - a*e^2)/(e^5*x^2 + 2*d*e^4*x + d^2*e^3) + c*log(e
*x + d)/e^3
```

Fricas [A] time = 1.78359, size = 157, normalized size = 3.08

$$\frac{4\,cdex+3\,cd^2-ae^2+2\left(ce^2x^2+2\,cdex+cd^2\right)\log(ex+d)}{2\left(e^5x^2+2\,de^4x+d^2e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*c*d*e*x + 3*c*d^2 - a*e^2 + 2*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*log(e*
x + d))/(e^5*x^2 + 2*d*e^4*x + d^2*e^3)
```

Sympy [A] time = 0.613881, size = 56, normalized size = 1.1

$$\frac{c\log(d+ex)}{e^3}+\frac{-ae^2+3cd^2+4cdex}{2d^2e^3+4de^4x+2e^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**3,x)
```

```
[Out] c*log(d + e*x)/e**3 + (-a*e**2 + 3*c*d**2 + 4*c*d*e*x)/(2*d**2*e**3 + 4*d*e
**4*x + 2*e**5*x**2)
```

Giac [A] time = 1.41966, size = 62, normalized size = 1.22

$$ce^{(-3)} \log\left(|xe + d|\right) + \frac{\left(4\,cdx + \left(3\,cd^2 - ae^2\right)e^{(-1)}\right)e^{(-2)}}{2\,(xe + d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] c*e^(-3)*log(abs(x*e + d)) + 1/2*(4*c*d*x + (3*c*d^2 - a*e^2)*e^(-1))*e^(-2
)/(x*e + d)^2
```

3.458 $\int \frac{a+cx^2}{(d+ex)^4} dx$

Optimal. Leaf size=52

$$-\frac{ae^2+cd^2}{3e^3(d+ex)^3}-\frac{c}{e^3(d+ex)}+\frac{cd}{e^3(d+ex)^2}$$

```
[Out] -(c*d^2 + a*e^2)/(3*e^3*(d + e*x)^3) + (c*d)/(e^3*(d + e*x)^2) - c/(e^3*(d
+ e*x))
```

Rubi [A] time = 0.0299414, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {697}

$$-\frac{ae^2+cd^2}{3e^3(d+ex)^3}-\frac{c}{e^3(d+ex)}+\frac{cd}{e^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/(d + e*x)^4,x]
```

```
[Out] -(c*d^2 + a*e^2)/(3*e^3*(d + e*x)^3) + (c*d)/(e^3*(d + e*x)^2) - c/(e^3*(d
+ e*x))
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+cx^2}{(d+ex)^4}\,dx &= \int \left(\frac{cd^2+ae^2}{e^2(d+ex)^4}-\frac{2cd}{e^2(d+ex)^3}+\frac{c}{e^2(d+ex)^2}\right) dx\\ &= -\frac{cd^2+ae^2}{3e^3(d+ex)^3}+\frac{cd}{e^3(d+ex)^2}-\frac{c}{e^3(d+ex)}\end{aligned}$$

Mathematica [A] time = 0.0134262, size = 39, normalized size = 0.75

$$-\frac{ae^2+c\left(d^2+3dex+3e^2x^2\right)}{3e^3(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/(d + e*x)^4,x]
```

```
[Out] -(a*e^2 + c*(d^2 + 3*d*e*x + 3*e^2*x^2))/(3*e^3*(d + e*x)^3)
```

Maple [A] time = 0.048, size = 51, normalized size = 1.

$$-\frac{ae^2+cd^2}{3\,e^3\,(ex+d)^3}-\frac{c}{e^3\,(ex+d)}+\frac{cd}{e^3\,(ex+d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^4,x)
```

```
[Out] -1/3*(a*e^2+c*d^2)/e^3/(e*x+d)^3-c/e^3/(e*x+d)+c*d/e^3/(e*x+d)^2
```

Maxima [A] time = 1.18936, size = 85, normalized size = 1.63

$$-\frac{3\,ce^2x^2 + 3\,cdex + cd^2 + ae^2}{3\left(e^6x^3 + 3\,de^5x^2 + 3\,d^2e^4x + d^3e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*c*e^2*x^2 + 3*c*d*e*x + c*d^2 + a*e^2)/(e^6*x^3 + 3*d*e^5*x^2 + 3*d
^2*e^4*x + d^3*e^3)
```

Fricas [A] time = 1.91301, size = 130, normalized size = 2.5

$$-\frac{3\,ce^2x^2 + 3\,cdex + cd^2 + ae^2}{3\left(e^6x^3 + 3\,de^5x^2 + 3\,d^2e^4x + d^3e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*c*e^2*x^2 + 3*c*d*e*x + c*d^2 + a*e^2)/(e^6*x^3 + 3*d*e^5*x^2 + 3*d
^2*e^4*x + d^3*e^3)
```

Sympy [A] time = 0.77414, size = 66, normalized size = 1.27

$$-\frac{ae^2 + cd^2 + 3cdex + 3ce^2x^2}{3d^3e^3 + 9d^2e^4x + 9de^5x^2 + 3e^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**4,x)
```

```
[Out] -(a*e**2 + c*d**2 + 3*c*d*e*x + 3*c*e**2*x**2)/(3*d**3*e**3 + 9*d**2*e**4*x
 + 9*d*e**5*x**2 + 3*e**6*x**3)
```

Giac [A] time = 1.28112, size = 50, normalized size = 0.96

$$-\frac{\left(3\,cx^2e^2 + 3\,cdxe + cd^2 + ae^2\right)e^{(-3)}}{3\,(xe + d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*c*x^2*e^2 + 3*c*d*x*e + c*d^2 + a*e^2)*e^(-3)/(x*e + d)^3
```

3.459 $\int \frac{a+cx^2}{(d+ex)^5} \, dx$

Optimal. Leaf size=57

$$-\frac{ae^2+cd^2}{4e^3(d+ex)^4}-\frac{c}{2e^3(d+ex)^2}+\frac{2cd}{3e^3(d+ex)^3}$$

```
[Out] -(c*d^2 + a*e^2)/(4*e^3*(d + e*x)^4) + (2*c*d)/(3*e^3*(d + e*x)^3) - c/(2*e
^3*(d + e*x)^2)
```

Rubi [A] time = 0.0302744, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {697}

$$-\frac{ae^2+cd^2}{4e^3(d+ex)^4}-\frac{c}{2e^3(d+ex)^2}+\frac{2cd}{3e^3(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/(d + e*x)^5,x]
```

```
[Out] -(c*d^2 + a*e^2)/(4*e^3*(d + e*x)^4) + (2*c*d)/(3*e^3*(d + e*x)^3) - c/(2*e
^3*(d + e*x)^2)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+cx^2}{(d+ex)^5}\,dx &= \int \left(\frac{cd^2+ae^2}{e^2(d+ex)^5}-\frac{2cd}{e^2(d+ex)^4}+\frac{c}{e^2(d+ex)^3}\right)dx\\ &= -\frac{cd^2+ae^2}{4e^3(d+ex)^4}+\frac{2cd}{3e^3(d+ex)^3}-\frac{c}{2e^3(d+ex)^2}\end{aligned}$$

Mathematica [A] time = 0.0144593, size = 40, normalized size = 0.7

$$-\frac{3ae^2+c\left(d^2+4dex+6e^2x^2\right)}{12e^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/(d + e*x)^5,x]
```

```
[Out] -(3*a*e^2 + c*(d^2 + 4*d*e*x + 6*e^2*x^2))/(12*e^3*(d + e*x)^4)
```

Maple [A] time = 0.046, size = 52, normalized size = 0.9

$$-\frac{ae^2+cd^2}{4\,e^3\,(ex+d)^4}+\frac{2\,cd}{3\,e^3\,(ex+d)^3}-\frac{c}{2\,e^3\,(ex+d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^5,x)
```

```
[Out] -1/4*(a*e^2+c*d^2)/e^3/(e*x+d)^4+2/3*c*d/e^3/(e*x+d)^3-1/2*c/e^3/(e*x+d)^2
```

Maxima [A] time = 1.25182, size = 101, normalized size = 1.77

$$-\frac{6\,ce^2x^2 + 4\,cdex + cd^2 + 3\,ae^2}{12\,\left(e^7x^4 + 4\,de^6x^3 + 6\,d^2e^5x^2 + 4\,d^3e^4x + d^4e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(6*c*e^2*x^2 + 4*c*d*e*x + c*d^2 + 3*a*e^2)/(e^7*x^4 + 4*d*e^6*x^3 +
6*d^2*e^5*x^2 + 4*d^3*e^4*x + d^4*e^3)
```

Fricas [A] time = 1.84587, size = 155, normalized size = 2.72

$$-\frac{6\,ce^2x^2 + 4\,cdex + cd^2 + 3\,ae^2}{12\,\left(e^7x^4 + 4\,de^6x^3 + 6\,d^2e^5x^2 + 4\,d^3e^4x + d^4e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^5,x, algorithm="fricas")
```

```
[Out] -1/12*(6*c*e^2*x^2 + 4*c*d*e*x + c*d^2 + 3*a*e^2)/(e^7*x^4 + 4*d*e^6*x^3 +
6*d^2*e^5*x^2 + 4*d^3*e^4*x + d^4*e^3)
```

Sympy [A] time = 0.914188, size = 80, normalized size = 1.4

$$-\frac{3ae^2 + cd^2 + 4cdex + 6ce^2x^2}{12d^4e^3 + 48d^3e^4x + 72d^2e^5x^2 + 48de^6x^3 + 12e^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**5,x)
```

```
[Out] -(3*a*e**2 + c*d**2 + 4*c*d*e*x + 6*c*e**2*x**2)/(12*d**4*e**3 + 48*d**3*e*
*4*x + 72*d**2*e**5*x**2 + 48*d*e**6*x**3 + 12*e**7*x**4)
```

Giac [A] time = 1.34556, size = 80, normalized size = 1.4

$$-\frac{1}{12}\left(\frac{6\,ce^{(-2)}}{(xe + d)^2} - \frac{8\,cde^{(-2)}}{(xe + d)^3} + \frac{3\,cd^2e^{(-2)}}{(xe + d)^4} + \frac{3\,a}{(xe + d)^4}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] -1/12*(6*c*e^(-2)/(x*e + d)^2 - 8*c*d*e^(-2)/(x*e + d)^3 + 3*c*d^2*e^(-2)/(
x*e + d)^4 + 3*a/(x*e + d)^4)*e^(-1)
```

3.460 $\int (d+ex)^4 \left(a+cx^2\right)^2 \, dx$

Optimal. Leaf size=117

$$\frac{2c(d+ex)^7\left(ae^2+3cd^2\right)}{7e^5}-\frac{2cd(d+ex)^6\left(ae^2+cd^2\right)}{3e^5}+\frac{(d+ex)^5\left(ae^2+cd^2\right)^2}{5e^5}+\frac{c^2(d+ex)^9}{9e^5}-\frac{c^2d(d+ex)^8}{2e^5}$$

```
[Out] ((c*d^2 + a*e^2)^2*(d + e*x)^5)/(5*e^5) - (2*c*d*(c*d^2 + a*e^2)*(d + e*x)^
6)/(3*e^5) + (2*c*(3*c*d^2 + a*e^2)*(d + e*x)^7)/(7*e^5) - (c^2*d*(d + e*x)
^8)/(2*e^5) + (c^2*(d + e*x)^9)/(9*e^5)
```

Rubi [A] time = 0.131277, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$\frac{2c(d+ex)^7\left(ae^2+3cd^2\right)}{7e^5}-\frac{2cd(d+ex)^6\left(ae^2+cd^2\right)}{3e^5}+\frac{(d+ex)^5\left(ae^2+cd^2\right)^2}{5e^5}+\frac{c^2(d+ex)^9}{9e^5}-\frac{c^2d(d+ex)^8}{2e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4*(a + c*x^2)^2,x]
```

```
[Out] ((c*d^2 + a*e^2)^2*(d + e*x)^5)/(5*e^5) - (2*c*d*(c*d^2 + a*e^2)*(d + e*x)^
6)/(3*e^5) + (2*c*(3*c*d^2 + a*e^2)*(d + e*x)^7)/(7*e^5) - (c^2*d*(d + e*x)
^8)/(2*e^5) + (c^2*(d + e*x)^9)/(9*e^5)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (d+ex)^4\left(a+cx^2\right)^2\,dx=\int\left(\frac{\left(cd^2+ae^2\right)^2(d+ex)^4}{e^4}-\frac{4cd\left(cd^2+ae^2\right)(d+ex)^5}{e^4}+\frac{2c\left(3cd^2+ae^2\right)(d+ex)^6}{e^4}-\frac{4c^2d(d+\right.$$
$$=\frac{\left(cd^2+ae^2\right)^2(d+ex)^5}{5e^5}-\frac{2cd\left(cd^2+ae^2\right)(d+ex)^6}{3e^5}+\frac{2c\left(3cd^2+ae^2\right)(d+ex)^7}{7e^5}-\frac{c^2d(d+ex)^8}{2e^5}$$

Mathematica [A] time = 0.0188699, size = 167, normalized size = 1.43

$$\frac{1}{5}x^5\left(a^2e^4+12acd^2e^2+c^2d^4\right)+2a^2d^3ex^2+a^2d^4x+\frac{2}{7}ce^2x^7\left(ae^2+3cd^2\right)+\frac{2}{3}cdex^6\left(2ae^2+cd^2\right)+adex^4\left(ae^2+2cd^2\right)+\frac{2}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4*(a + c*x^2)^2,x]
```

```
[Out] a^2*d^4*x + 2*a^2*d^3*e*x^2 + (2*a*d^2*(c*d^2 + 3*a*e^2)*x^3)/3 + a*d*e*(2*
c*d^2 + a*e^2)*x^4 + ((c^2*d^4 + 12*a*c*d^2*e^2 + a^2*e^4)*x^5)/5 + (2*c*d*
e*(c*d^2 + 2*a*e^2)*x^6)/3 + (2*c*e^2*(3*c*d^2 + a*e^2)*x^7)/7 + (c^2*d*e^3
```

```
*x^8)/2 + (c^2*e^4*x^9)/9
```

Maple [A] time = 0.044, size = 169, normalized size = 1.4

$$\frac{e^4c^2x^9}{9}+\frac{de^3c^2x^8}{2}+\frac{(2\,e^4ac+6\,d^2e^2c^2)\,x^7}{7}+\frac{(8\,de^3ac+4\,d^3ec^2)\,x^6}{6}+\frac{(a^2e^4+12\,acd^2e^2+c^2d^4)\,x^5}{5}+\frac{(4\,de^3a^2+8\,d^3ec}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(c*x^2+a)^2,x)
```

```
[Out] 1/9*e^4*c^2*x^9+1/2*d*e^3*c^2*x^8+1/7*(2*a*c*e^4+6*c^2*d^2*e^2)*x^7+1/6*(8*
a*c*d*e^3+4*c^2*d^3*e)*x^6+1/5*(a^2*e^4+12*a*c*d^2*e^2+c^2*d^4)*x^5+1/4*(4*
a^2*d*e^3+8*a*c*d^3*e)*x^4+1/3*(6*a^2*d^2*e^2+2*a*c*d^4)*x^3+2*d^3*e*a^2*x^
2+d^4*a^2*x
```

Maxima [A] time = 1.05288, size = 220, normalized size = 1.88

$$\frac{1}{9}\,c^2e^4x^9+\frac{1}{2}\,c^2de^3x^8+2\,a^2d^3ex^2+\frac{2}{7}\left(3\,c^2d^2e^2+ace^4\right)x^7+a^2d^4x+\frac{2}{3}\left(c^2d^3e+2\,acde^3\right)x^6+\frac{1}{5}\left(c^2d^4+12\,acd^2e^2+a^2e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/9*c^2*e^4*x^9 + 1/2*c^2*d*e^3*x^8 + 2*a^2*d^3*e*x^2 + 2/7*(3*c^2*d^2*e^2
+ a*c*e^4)*x^7 + a^2*d^4*x + 2/3*(c^2*d^3*e + 2*a*c*d*e^3)*x^6 + 1/5*(c^2*d
^4 + 12*a*c*d^2*e^2 + a^2*e^4)*x^5 + (2*a*c*d^3*e + a^2*d*e^3)*x^4 + 2/3*(a
*c*d^4 + 3*a^2*d^2*e^2)*x^3
```

Fricas [A] time = 1.60657, size = 375, normalized size = 3.21

$$\frac{1}{9}x^9e^4c^2+\frac{1}{2}x^8e^3dc^2+\frac{6}{7}x^7e^2d^2c^2+\frac{2}{7}x^7e^4ca+\frac{2}{3}x^6ed^3c^2+\frac{4}{3}x^6e^3dca+\frac{1}{5}x^5d^4c^2+\frac{12}{5}x^5e^2d^2ca+\frac{1}{5}x^5e^4a^2+2x^4ed^3ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*e^4*c^2 + 1/2*x^8*e^3*d*c^2 + 6/7*x^7*e^2*d^2*c^2 + 2/7*x^7*e^4*c*a
 + 2/3*x^6*e*d^3*c^2 + 4/3*x^6*e^3*d*c*a + 1/5*x^5*d^4*c^2 + 12/5*x^5*e^2*d
^2*c*a + 1/5*x^5*e^4*a^2 + 2*x^4*e*d^3*c*a + x^4*e^3*d*a^2 + 2/3*x^3*d^4*c*
a + 2*x^3*e^2*d^2*a^2 + 2*x^2*e*d^3*a^2 + x*d^4*a^2
```

Sympy [A] time = 0.138325, size = 182, normalized size = 1.56

$$a^2d^4x+2a^2d^3ex^2+\frac{c^2de^3x^8}{2}+\frac{c^2e^4x^9}{9}+x^7\left(\frac{2ace^4}{7}+\frac{6c^2d^2e^2}{7}\right)+x^6\left(\frac{4acde^3}{3}+\frac{2c^2d^3e}{3}\right)+x^5\left(\frac{a^2e^4}{5}+\frac{12acd^2e^2}{5}+\frac{c^2d^4}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(c*x**2+a)**2,x)
```

```
[Out] a**2*d**4*x + 2*a**2*d**3*e*x**2 + c**2*d*e**3*x**8/2 + c**2*e**4*x**9/9 +
x**7*(2*a*c*e**4/7 + 6*c**2*d**2*e**2/7) + x**6*(4*a*c*d*e**3/3 + 2*c**2*d*
*3*e/3) + x**5*(a**2*e**4/5 + 12*a*c*d**2*e**2/5 + c**2*d**4/5) + x**4*(a**
2*d*e**3 + 2*a*c*d**3*e) + x**3*(2*a**2*d**2*e**2 + 2*a*c*d**4/3)
```

Giac [A] time = 1.33699, size = 224, normalized size = 1.91

$$\frac{1}{9}c^2x^9e^4+\frac{1}{2}c^2dx^8e^3+\frac{6}{7}c^2d^2x^7e^2+\frac{2}{3}c^2d^3x^6e+\frac{1}{5}c^2d^4x^5+\frac{2}{7}acx^7e^4+\frac{4}{3}acdx^6e^3+\frac{12}{5}acd^2x^5e^2+2\,acd^3x^4e+\frac{2}{3}acd^4x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*c^2*x^9*e^4 + 1/2*c^2*d*x^8*e^3 + 6/7*c^2*d^2*x^7*e^2 + 2/3*c^2*d^3*x^6
*e + 1/5*c^2*d^4*x^5 + 2/7*a*c*x^7*e^4 + 4/3*a*c*d*x^6*e^3 + 12/5*a*c*d^2*x
^5*e^2 + 2*a*c*d^3*x^4*e + 2/3*a*c*d^4*x^3 + 1/5*a^2*x^5*e^4 + a^2*d*x^4*e^
3 + 2*a^2*d^2*x^3*e^2 + 2*a^2*d^3*x^2*e + a^2*d^4*x
```

3.461 $\int (d+ex)^3 \left(a+cx^2\right)^2 dx$

Optimal. Leaf size=117

$$\frac{c(d+ex)^6 \left(ae^2+3cd^2\right)}{3e^5}-\frac{4cd(d+ex)^5 \left(ae^2+cd^2\right)}{5e^5}+\frac{(d+ex)^4 \left(ae^2+cd^2\right)^2}{4e^5}+\frac{c^2(d+ex)^8}{8e^5}-\frac{4c^2d(d+ex)^7}{7e^5}$$

```
[Out] ((c*d^2 + a*e^2)^2*(d + e*x)^4)/(4*e^5) - (4*c*d*(c*d^2 + a*e^2)*(d + e*x)^
5)/(5*e^5) + (c*(3*c*d^2 + a*e^2)*(d + e*x)^6)/(3*e^5) - (4*c^2*d*(d + e*x)
^7)/(7*e^5) + (c^2*(d + e*x)^8)/(8*e^5)
```

Rubi [A] time = 0.102017, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$\frac{c(d+ex)^6 \left(ae^2+3cd^2\right)}{3e^5}-\frac{4cd(d+ex)^5 \left(ae^2+cd^2\right)}{5e^5}+\frac{(d+ex)^4 \left(ae^2+cd^2\right)^2}{4e^5}+\frac{c^2(d+ex)^8}{8e^5}-\frac{4c^2d(d+ex)^7}{7e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + c*x^2)^2,x]
```

```
[Out] ((c*d^2 + a*e^2)^2*(d + e*x)^4)/(4*e^5) - (4*c*d*(c*d^2 + a*e^2)*(d + e*x)^
5)/(5*e^5) + (c*(3*c*d^2 + a*e^2)*(d + e*x)^6)/(3*e^5) - (4*c^2*d*(d + e*x)
^7)/(7*e^5) + (c^2*(d + e*x)^8)/(8*e^5)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (d+ex)^3 \left(a+cx^2\right)^2 dx = \int \left(\frac{\left(cd^2+ae^2\right)^2 (d+ex)^3}{e^4}-\frac{4cd\left(cd^2+ae^2\right)(d+ex)^4}{e^4}+\frac{2c\left(3cd^2+ae^2\right)(d+ex)^5}{e^4}-\frac{4c^2d(\right.$$

$$=\frac{\left(cd^2+ae^2\right)^2 (d+ex)^4}{4e^5}-\frac{4cd\left(cd^2+ae^2\right)(d+ex)^5}{5e^5}+\frac{c\left(3cd^2+ae^2\right)(d+ex)^6}{3e^5}-\frac{4c^2d(d+e}{7e^5}$$

Mathematica [A] time = 0.0170777, size = 117, normalized size = 1.

$$\frac{1}{4}a^2x\left(6d^2ex+4d^3+4de^2x^2+e^3x^3\right)+\frac{1}{30}acx^3\left(45d^2ex+20d^3+36de^2x^2+10e^3x^3\right)+\frac{1}{280}c^2x^5\left(140d^2ex+56d^3+120\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + c*x^2)^2,x]
```

```
[Out] (a^2*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3))/4 + (a*c*x^3*(20*d^3 +
45*d^2*e*x + 36*d*e^2*x^2 + 10*e^3*x^3))/30 + (c^2*x^5*(56*d^3 + 140*d^2*e*
```

```
x + 120*d*e^2*x^2 + 35*e^3*x^3))/280
```

Maple [A] time = 0.043, size = 131, normalized size = 1.1

$$\frac{c^2e^3x^8}{8}+\frac{3\,de^2c^2x^7}{7}+\frac{\left(2\,e^3ac+3\,d^2ec^2\right)x^6}{6}+\frac{\left(6\,de^2ac+c^2d^3\right)x^5}{5}+\frac{\left(a^2e^3+6\,d^2eac\right)x^4}{4}+\frac{\left(3\,de^2a^2+2\,d^3ac\right)x^3}{3}+\frac{3\,d^2ea^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+a)^2,x)
```

```
[Out] 1/8*c^2*e^3*x^8+3/7*d*e^2*c^2*x^7+1/6*(2*a*c*e^3+3*c^2*d^2*e)*x^6+1/5*(6*a*
c*d*e^2+c^2*d^3)*x^5+1/4*(a^2*e^3+6*a*c*d^2*e)*x^4+1/3*(3*a^2*d*e^2+2*a*c*d
^3)*x^3+3/2*d^2*e*a^2*x^2+d^3*a^2*x
```

Maxima [A] time = 1.18425, size = 176, normalized size = 1.5

$$\frac{1}{8}\,c^2e^3x^8+\frac{3}{7}\,c^2de^2x^7+\frac{3}{2}\,a^2d^2ex^2+\frac{1}{6}\left(3\,c^2d^2e+2\,ace^3\right)x^6+a^2d^3x+\frac{1}{5}\left(c^2d^3+6\,acde^2\right)x^5+\frac{1}{4}\left(6\,acd^2e+a^2e^3\right)x^4+\frac{1}{3}\left(2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/8*c^2*e^3*x^8 + 3/7*c^2*d*e^2*x^7 + 3/2*a^2*d^2*e*x^2 + 1/6*(3*c^2*d^2*e
+ 2*a*c*e^3)*x^6 + a^2*d^3*x + 1/5*(c^2*d^3 + 6*a*c*d*e^2)*x^5 + 1/4*(6*a*c
*d^2*e + a^2*e^3)*x^4 + 1/3*(2*a*c*d^3 + 3*a^2*d*e^2)*x^3
```

Fricas [A] time = 1.73849, size = 293, normalized size = 2.5

$$\frac{1}{8}x^8e^3c^2+\frac{3}{7}x^7e^2dc^2+\frac{1}{2}x^6ed^2c^2+\frac{1}{3}x^6e^3ca+\frac{1}{5}x^5d^3c^2+\frac{6}{5}x^5e^2dca+\frac{3}{2}x^4ed^2ca+\frac{1}{4}x^4e^3a^2+\frac{2}{3}x^3d^3ca+x^3e^2da^2+\frac{3}{2}x^2ed^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*e^3*c^2 + 3/7*x^7*e^2*d*c^2 + 1/2*x^6*e*d^2*c^2 + 1/3*x^6*e^3*c*a +
 1/5*x^5*d^3*c^2 + 6/5*x^5*e^2*d*c*a + 3/2*x^4*e*d^2*c*a + 1/4*x^4*e^3*a^2
+ 2/3*x^3*d^3*c*a + x^3*e^2*d*a^2 + 3/2*x^2*e*d^2*a^2 + x*d^3*a^2
```

Sympy [A] time = 0.119764, size = 141, normalized size = 1.21

$$a^2d^3x+\frac{3a^2d^2ex^2}{2}+\frac{3c^2de^2x^7}{7}+\frac{c^2e^3x^8}{8}+x^6\left(\frac{ace^3}{3}+\frac{c^2d^2e}{2}\right)+x^5\left(\frac{6acde^2}{5}+\frac{c^2d^3}{5}\right)+x^4\left(\frac{a^2e^3}{4}+\frac{3acd^2e}{2}\right)+x^3\left(a^2de^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a)**2,x)
```

```
[Out] a**2*d**3*x + 3*a**2*d**2*e*x**2/2 + 3*c**2*d*e**2*x**7/7 + c**2*e**3*x**8/
8 + x**6*(a*c*e**3/3 + c**2*d**2*e/2) + x**5*(6*a*c*d*e**2/5 + c**2*d**3/5)
 + x**4*(a**2*e**3/4 + 3*a*c*d**2*e/2) + x**3*(a**2*d*e**2 + 2*a*c*d**3/3)
```

Giac [A] time = 1.37294, size = 173, normalized size = 1.48

$$\frac{1}{8}c^2x^8e^3 + \frac{3}{7}c^2dx^7e^2 + \frac{1}{2}c^2d^2x^6e + \frac{1}{5}c^2d^3x^5 + \frac{1}{3}acx^6e^3 + \frac{6}{5}acdx^5e^2 + \frac{3}{2}acd^2x^4e + \frac{2}{3}acd^3x^3 + \frac{1}{4}a^2x^4e^3 + a^2dx^3e^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*c^2*x^8*e^3 + 3/7*c^2*d*x^7*e^2 + 1/2*c^2*d^2*x^6*e + 1/5*c^2*d^3*x^5 +
 1/3*a*c*x^6*e^3 + 6/5*a*c*d*x^5*e^2 + 3/2*a*c*d^2*x^4*e + 2/3*a*c*d^3*x^3
+ 1/4*a^2*x^4*e^3 + a^2*d*x^3*e^2 + 3/2*a^2*d^2*x^2*e + a^2*d^3*x
```

3.462 $\int (d+ex)^2 \left(a+cx^2\right)^2 \, dx$

Optimal. Leaf size=80

$$a^2d^2x+\frac{1}{5}cx^5\left(2ae^2+cd^2\right)+\frac{1}{3}ax^3\left(ae^2+2cd^2\right)+\frac{de\left(a+cx^2\right)^3}{3c}+\frac{1}{7}c^2e^2x^7$$

```
[Out] a^2*d^2*x + (a*(2*c*d^2 + a*e^2)*x^3)/3 + (c*(c*d^2 + 2*a*e^2)*x^5)/5 + (c^
2*e^2*x^7)/7 + (d*e*(a + c*x^2)^3)/(3*c)
```

Rubi [A] time = 0.0461326, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {696, 1810}

$$a^2d^2x+\frac{1}{5}cx^5\left(2ae^2+cd^2\right)+\frac{1}{3}ax^3\left(ae^2+2cd^2\right)+\frac{de\left(a+cx^2\right)^3}{3c}+\frac{1}{7}c^2e^2x^7$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + c*x^2)^2,x]
```

```
[Out] a^2*d^2*x + (a*(2*c*d^2 + a*e^2)*x^3)/3 + (c*(c*d^2 + 2*a*e^2)*x^5)/5 + (c^
2*e^2*x^7)/7 + (d*e*(a + c*x^2)^3)/(3*c)
```

Rule 696

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*m*d^(m - 1)*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Int[((d + e*x)^m -
e*m*d^(m - 1)*x)*(a + c*x^2)^p, x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
 a*e^2, 0] && IGtQ[p, 1] && IGtQ[m, 0] && LeQ[m, p]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^2 \left(a+cx^2\right)^2 \, dx &= \frac{de\left(a+cx^2\right)^3}{3c}+\int \left(a+cx^2\right)^2 \left(-2dex+(d+ex)^2\right) \, dx\\ &= \frac{de\left(a+cx^2\right)^3}{3c}+\int \left(a^2d^2+a\left(2cd^2+ae^2\right)x^2+c\left(cd^2+2ae^2\right)x^4+c^2e^2x^6\right) \, dx\\ &= a^2d^2x+\frac{1}{3}a\left(2cd^2+ae^2\right)x^3+\frac{1}{5}c\left(cd^2+2ae^2\right)x^5+\frac{1}{7}c^2e^2x^7+\frac{de\left(a+cx^2\right)^3}{3c}\end{aligned}$$

Mathematica [A] time = 0.011535, size = 91, normalized size = 1.14

$$a^2d^2x+a^2dex^2+\frac{1}{5}cx^5\left(2ae^2+cd^2\right)+\frac{1}{3}ax^3\left(ae^2+2cd^2\right)+acdex^4+\frac{1}{3}c^2dex^6+\frac{1}{7}c^2e^2x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + c*x^2)^2,x]
```

```
[Out] a^2*d^2*x + a^2*d*e*x^2 + (a*(2*c*d^2 + a*e^2)*x^3)/3 + a*c*d*e*x^4 + (c*(c
*d^2 + 2*a*e^2)*x^5)/5 + (c^2*d*e*x^6)/3 + (c^2*e^2*x^7)/7
```

Maple [A] time = 0.048, size = 88, normalized size = 1.1

$$\frac{c^2e^2x^7}{7}+\frac{dec^2x^6}{3}+\frac{\left(2\,e^2ac+c^2d^2\right)x^5}{5}+acdex^4+\frac{\left(a^2e^2+2\,d^2ac\right)x^3}{3}+dea^2x^2+a^2d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a)^2,x)
```

```
[Out] 1/7*c^2*e^2*x^7+1/3*d*e*c^2*x^6+1/5*(2*a*c*e^2+c^2*d^2)*x^5+a*c*d*e*x^4+1/3
*(a^2*e^2+2*a*c*d^2)*x^3+d*e*a^2*x^2+a^2*d^2*x
```

Maxima [A] time = 1.15998, size = 117, normalized size = 1.46

$$\frac{1}{7}\,c^2e^2x^7+\frac{1}{3}\,c^2dex^6+acdex^4+a^2dex^2+\frac{1}{5}\left(c^2d^2+2\,ace^2\right)x^5+a^2d^2x+\frac{1}{3}\left(2\,acd^2+a^2e^2\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/7*c^2*e^2*x^7 + 1/3*c^2*d*e*x^6 + a*c*d*e*x^4 + a^2*d*e*x^2 + 1/5*(c^2*d^
2 + 2*a*c*e^2)*x^5 + a^2*d^2*x + 1/3*(2*a*c*d^2 + a^2*e^2)*x^3
```

Fricas [A] time = 1.66084, size = 198, normalized size = 2.48

$$\frac{1}{7}x^7e^2c^2+\frac{1}{3}x^6edc^2+\frac{1}{5}x^5d^2c^2+\frac{2}{5}x^5e^2ca+x^4edca+\frac{2}{3}x^3d^2ca+\frac{1}{3}x^3e^2a^2+x^2eda^2+xd^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*e^2*c^2 + 1/3*x^6*e*d*c^2 + 1/5*x^5*d^2*c^2 + 2/5*x^5*e^2*c*a + x^4
*e*d*c*a + 2/3*x^3*d^2*c*a + 1/3*x^3*e^2*a^2 + x^2*e*d*a^2 + x*d^2*a^2
```

Sympy [A] time = 0.122765, size = 95, normalized size = 1.19

$$a^2d^2x+a^2dex^2+acdex^4+\frac{c^2dex^6}{3}+\frac{c^2e^2x^7}{7}+x^5\left(\frac{2ace^2}{5}+\frac{c^2d^2}{5}\right)+x^3\left(\frac{a^2e^2}{3}+\frac{2acd^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+a)**2,x)
```

```
[Out] a**2*d**2*x + a**2*d*e*x**2 + a*c*d*e*x**4 + c**2*d*e*x**6/3 + c**2*e**2*x*
*7/7 + x**5*(2*a*c*e**2/5 + c**2*d**2/5) + x**3*(a**2*e**2/3 + 2*a*c*d**2/3
)
```

Giac [A] time = 1.35347, size = 120, normalized size = 1.5

$$\frac{1}{7}c^2x^7e^2 + \frac{1}{3}c^2dx^6e + \frac{1}{5}c^2d^2x^5 + \frac{2}{5}acx^5e^2 + acdx^4e + \frac{2}{3}acd^2x^3 + \frac{1}{3}a^2x^3e^2 + a^2dx^2e + a^2d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/7*c^2*x^7*e^2 + 1/3*c^2*d*x^6*e + 1/5*c^2*d^2*x^5 + 2/5*a*c*x^5*e^2 + a*c
*d*x^4*e + 2/3*a*c*d^2*x^3 + 1/3*a^2*x^3*e^2 + a^2*d*x^2*e + a^2*d^2*x
```

3.463 $\int (d+ex)\left(a+cx^2\right)^2 dx$

Optimal. Leaf size=45

$$a^2dx+\frac{2}{3}acdx^3+\frac{e\left(a+cx^2\right)^3}{6c}+\frac{1}{5}c^2dx^5$$

```
[Out] a^2*d*x + (2*a*c*d*x^3)/3 + (c^2*d*x^5)/5 + (e*(a + c*x^2)^3)/(6*c)
```

Rubi [A] time = 0.0152334, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {641, 194}

$$a^2dx+\frac{2}{3}acdx^3+\frac{e\left(a+cx^2\right)^3}{6c}+\frac{1}{5}c^2dx^5$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + c*x^2)^2,x]
```

```
[Out] a^2*d*x + (2*a*c*d*x^3)/3 + (c^2*d*x^5)/5 + (e*(a + c*x^2)^3)/(6*c)
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(a+cx^2\right)^2 dx &= \frac{e\left(a+cx^2\right)^3}{6c}+d\int \left(a+cx^2\right)^2 dx\\ &= \frac{e\left(a+cx^2\right)^3}{6c}+d\int \left(a^2+2acx^2+c^2x^4\right) dx\\ &= a^2dx+\frac{2}{3}acdx^3+\frac{1}{5}c^2dx^5+\frac{e\left(a+cx^2\right)^3}{6c}\end{aligned}$$

Mathematica [A] time = 0.0021371, size = 60, normalized size = 1.33

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}acdx^3+\frac{1}{2}acex^4+\frac{1}{5}c^2dx^5+\frac{1}{6}c^2ex^6$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + c*x^2)^2,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (2*a*c*d*x^3)/3 + (a*c*e*x^4)/2 + (c^2*d*x^5)/5 +
 (c^2*e*x^6)/6
```

Maple [A] time = 0.041, size = 51, normalized size = 1.1

$$\frac{c^2ex^6}{6}+\frac{c^2dx^5}{5}+\frac{acex^4}{2}+\frac{2\,acdx^3}{3}+\frac{a^2ex^2}{2}+a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+a)^2,x)
```

```
[Out] 1/6*c^2*e*x^6+1/5*c^2*d*x^5+1/2*a*c*e*x^4+2/3*a*c*d*x^3+1/2*a^2*e*x^2+a^2*d
*x
```

Maxima [A] time = 1.13861, size = 68, normalized size = 1.51

$$\frac{1}{6}\,c^2ex^6+\frac{1}{5}\,c^2dx^5+\frac{1}{2}\,acex^4+\frac{2}{3}\,acdx^3+\frac{1}{2}\,a^2ex^2+a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*c^2*e*x^6 + 1/5*c^2*d*x^5 + 1/2*a*c*e*x^4 + 2/3*a*c*d*x^3 + 1/2*a^2*e*x
^2 + a^2*d*x
```

Fricas [A] time = 1.57441, size = 120, normalized size = 2.67

$$\frac{1}{6}x^6ec^2+\frac{1}{5}x^5dc^2+\frac{1}{2}x^4eca+\frac{2}{3}x^3dca+\frac{1}{2}x^2ea^2+xda^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*x^6*e*c^2 + 1/5*x^5*d*c^2 + 1/2*x^4*e*c*a + 2/3*x^3*d*c*a + 1/2*x^2*e*a
^2 + x*d*a^2
```

Sympy [A] time = 0.123217, size = 58, normalized size = 1.29

$$a^2dx+\frac{a^2ex^2}{2}+\frac{2acdx^3}{3}+\frac{acex^4}{2}+\frac{c^2dx^5}{5}+\frac{c^2ex^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+a)**2,x)
```

```
[Out] a**2*d*x + a**2*e*x**2/2 + 2*a*c*d*x**3/3 + a*c*e*x**4/2 + c**2*d*x**5/5 +
c**2*e*x**6/6
```

Giac [A] time = 1.24112, size = 72, normalized size = 1.6

$$\frac{1}{6}c^2x^6e + \frac{1}{5}c^2dx^5 + \frac{1}{2}acx^4e + \frac{2}{3}acdx^3 + \frac{1}{2}a^2x^2e + a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*c^2*x^6*e + 1/5*c^2*d*x^5 + 1/2*a*c*x^4*e + 2/3*a*c*d*x^3 + 1/2*a^2*x^2
*e + a^2*d*x
```

3.464 $\int \frac{(a+cx^2)^2}{d+ex}\,dx$

Optimal. Leaf size=94

$$\frac{cx^2(2ae^2+cd^2)}{2e^3}-\frac{cdx(2ae^2+cd^2)}{e^4}+\frac{(ae^2+cd^2)^2\log(d+ex)}{e^5}-\frac{c^2dx^3}{3e^2}+\frac{c^2x^4}{4e}$$

```
[Out] -((c*d*(c*d^2 + 2*a*e^2)*x)/e^4) + (c*(c*d^2 + 2*a*e^2)*x^2)/(2*e^3) - (c^2
*d*x^3)/(3*e^2) + (c^2*x^4)/(4*e) + ((c*d^2 + a*e^2)^2*Log[d + e*x])/e^5
```

Rubi [A] time = 0.0753735, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$\frac{cx^2(2ae^2+cd^2)}{2e^3}-\frac{cdx(2ae^2+cd^2)}{e^4}+\frac{(ae^2+cd^2)^2\log(d+ex)}{e^5}-\frac{c^2dx^3}{3e^2}+\frac{c^2x^4}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^2/(d + e*x),x]
```

```
[Out] -((c*d*(c*d^2 + 2*a*e^2)*x)/e^4) + (c*(c*d^2 + 2*a*e^2)*x^2)/(2*e^3) - (c^2
*d*x^3)/(3*e^2) + (c^2*x^4)/(4*e) + ((c*d^2 + a*e^2)^2*Log[d + e*x])/e^5
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+cx^2)^2}{d+ex}\,dx &= \int \left(-\frac{cd(cd^2+2ae^2)}{e^4}+\frac{c(cd^2+2ae^2)x}{e^3}-\frac{c^2dx^2}{e^2}+\frac{c^2x^3}{e}+\frac{(cd^2+ae^2)^2}{e^4(d+ex)}\right)dx\\ &= -\frac{cd(cd^2+2ae^2)x}{e^4}+\frac{c(cd^2+2ae^2)x^2}{2e^3}-\frac{c^2dx^3}{3e^2}+\frac{c^2x^4}{4e}+\frac{(cd^2+ae^2)^2\log(d+ex)}{e^5}\end{aligned}$$

Mathematica [A] time = 0.0301811, size = 79, normalized size = 0.84

$$\frac{cex\left(12ae^2(ex-2d)+c\left(6d^2ex-12d^3-4de^2x^2+3e^3x^3\right)\right)+12\left(ae^2+cd^2\right)^2\log(d+ex)}{12e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^2/(d + e*x),x]
```

```
[Out] (c*e*x*(12*a*e^2*(-2*d + e*x) + c*(-12*d^3 + 6*d^2*e*x - 4*d*e^2*x^2 + 3*e^
3*x^3)) + 12*(c*d^2 + a*e^2)^2*Log[d + e*x])/(12*e^5)
```

Maple [A] time = 0.045, size = 114, normalized size = 1.2

$$\frac{c^{2}x^{4}}{4e}-\frac{c^{2}dx^{3}}{3e^{2}}+\frac{cx^{2}a}{e}+\frac{c^{2}x^{2}d^{2}}{2e^{3}}-2\frac{acdx}{e^{2}}-\frac{c^{2}d^{3}x}{e^{4}}+\frac{\ln(ex+d)a^{2}}{e}+2\frac{\ln(ex+d)acd^{2}}{e^{3}}+\frac{d^{4}\ln(ex+d)c^{2}}{e^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2/(e*x+d),x)
```

```
[Out] 1/4*c^2*x^4/e-1/3*c^2*d*x^3/e^2+c/e*x^2*a+1/2/e^3*x^2*c^2*d^2-2*c/e^2*a*d*x
-1/e^4*c^2*d^3*x+1/e*ln(e*x+d)*a^2+2/e^3*ln(e*x+d)*a*c*d^2+d^4/e^5*ln(e*x+d
)*c^2
```

Maxima [A] time = 1.18506, size = 142, normalized size = 1.51

$$\frac{3c^{2}e^{3}x^{4}-4c^{2}de^{2}x^{3}+6\left(c^{2}d^{2}e+2ace^{3}\right)x^{2}-12\left(c^{2}d^{3}+2acde^{2}\right)x}{12e^{4}}+\frac{\left(c^{2}d^{4}+2acd^{2}e^{2}+a^{2}e^{4}\right)\log(ex+d)}{e^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/12*(3*c^2*e^3*x^4 - 4*c^2*d*e^2*x^3 + 6*(c^2*d^2*e + 2*a*c*e^3)*x^2 - 12*
(c^2*d^3 + 2*a*c*d*e^2)*x)/e^4 + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*log(e*
x + d)/e^5
```

Fricas [A] time = 1.81788, size = 223, normalized size = 2.37

$$\frac{3c^{2}e^{4}x^{4}-4c^{2}de^{3}x^{3}+6\left(c^{2}d^{2}e^{2}+2ace^{4}\right)x^{2}-12\left(c^{2}d^{3}e+2acde^{3}\right)x+12\left(c^{2}d^{4}+2acd^{2}e^{2}+a^{2}e^{4}\right)\log(ex+d)}{12e^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/12*(3*c^2*e^4*x^4 - 4*c^2*d*e^3*x^3 + 6*(c^2*d^2*e^2 + 2*a*c*e^4)*x^2 - 1
2*(c^2*d^3*e + 2*a*c*d*e^3)*x + 12*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*log(
e*x + d))/e^5
```

Sympy [A] time = 0.579513, size = 90, normalized size = 0.96

$$-\frac{c^{2}dx^{3}}{3e^{2}}+\frac{c^{2}x^{4}}{4e}+\frac{x^{2}\left(2ace^{2}+c^{2}d^{2}\right)}{2e^{3}}-\frac{x\left(2acde^{2}+c^{2}d^{3}\right)}{e^{4}}+\frac{\left(ae^{2}+cd^{2}\right)^{2}\log(d+ex)}{e^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2/(e*x+d),x)
```

```
[Out] -c**2*d*x**3/(3*e**2) + c**2*x**4/(4*e) + x**2*(2*a*c*e**2 + c**2*d**2)/(2*
e**3) - x*(2*a*c*d*e**2 + c**2*d**3)/e**4 + (a*e**2 + c*d**2)**2*log(d + e*
```

```
x)/e**5
```

Giac [A] time = 1.2773, size = 135, normalized size = 1.44

$$\left(c^2d^4 + 2\,acd^2e^2 + a^2e^4\right)e^{(-5)}\log\left(|xe + d|\right) + \frac{1}{12}\left(3\,c^2x^4e^3 - 4\,c^2dx^3e^2 + 6\,c^2d^2x^2e - 12\,c^2d^3x + 12\,acx^2e^3 - 24\,acdxe^2\right)e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d),x, algorithm="giac")
```

```
[Out] (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*e^(-5)*log(abs(x*e + d)) + 1/12*(3*c^2*
x^4*e^3 - 4*c^2*d*x^3*e^2 + 6*c^2*d^2*x^2*e - 12*c^2*d^3*x + 12*a*c*x^2*e^3
 - 24*a*c*d*x*e^2)*e^(-4)
```

3.465 $\int \frac{(a+cx^2)^2}{(d+ex)^2} dx$

Optimal. Leaf size=94

$$\frac{cx(2ae^2+3cd^2)}{e^4}-\frac{(ae^2+cd^2)^2}{e^5(d+ex)}-\frac{4cd(ae^2+cd^2)\log(d+ex)}{e^5}-\frac{c^2dx^2}{e^3}+\frac{c^2x^3}{3e^2}$$

```
[Out] (c*(3*c*d^2 + 2*a*e^2)*x)/e^4 - (c^2*d*x^2)/e^3 + (c^2*x^3)/(3*e^2) - (c*d^
2 + a*e^2)^2/(e^5*(d + e*x)) - (4*c*d*(c*d^2 + a*e^2)*Log[d + e*x])/e^5
```

Rubi [A] time = 0.0756636, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$\frac{cx(2ae^2+3cd^2)}{e^4}-\frac{(ae^2+cd^2)^2}{e^5(d+ex)}-\frac{4cd(ae^2+cd^2)\log(d+ex)}{e^5}-\frac{c^2dx^2}{e^3}+\frac{c^2x^3}{3e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^2/(d + e*x)^2,x]
```

```
[Out] (c*(3*c*d^2 + 2*a*e^2)*x)/e^4 - (c^2*d*x^2)/e^3 + (c^2*x^3)/(3*e^2) - (c*d^
2 + a*e^2)^2/(e^5*(d + e*x)) - (4*c*d*(c*d^2 + a*e^2)*Log[d + e*x])/e^5
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+cx^2)^2}{(d+ex)^2}dx &= \int \left(\frac{c(3cd^2+2ae^2)}{e^4}-\frac{2c^2dx}{e^3}+\frac{c^2x^2}{e^2}+\frac{(cd^2+ae^2)^2}{e^4(d+ex)^2}-\frac{4cd(cd^2+ae^2)}{e^4(d+ex)}\right)dx\\ &= \frac{c(3cd^2+2ae^2)x}{e^4}-\frac{c^2dx^2}{e^3}+\frac{c^2x^3}{3e^2}-\frac{(cd^2+ae^2)^2}{e^5(d+ex)}-\frac{4cd(cd^2+ae^2)\log(d+ex)}{e^5}\end{aligned}$$

Mathematica [A] time = 0.0556821, size = 91, normalized size = 0.97

$$\frac{3cex(2ae^2+3cd^2)-\frac{3(ae^2+cd^2)^2}{d+ex}-12cd(ae^2+cd^2)\log(d+ex)-3c^2de^2x^2+c^2e^3x^3}{3e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^2/(d + e*x)^2,x]
```

```
[Out] (3*c*e*(3*c*d^2 + 2*a*e^2)*x - 3*c^2*d*e^2*x^2 + c^2*e^3*x^3 - (3*(c*d^2 +
a*e^2)^2)/(d + e*x) - 12*c*d*(c*d^2 + a*e^2)*Log[d + e*x])/(3*e^5)
```

Maple [A] time = 0.049, size = 126, normalized size = 1.3

$$\frac{c^2x^3}{3e^2} - \frac{c^2dx^2}{e^3} + 2\frac{acx}{e^2} + 3\frac{c^2d^2x}{e^4} - 4\frac{cd\ln(ex+d)a}{e^3} - 4\frac{c^2d^3\ln(ex+d)}{e^5} - \frac{a^2}{e(ex+d)} - 2\frac{acd^2}{e^3(ex+d)} - \frac{c^2d^4}{e^5(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2/(e*x+d)^2,x)
```

```
[Out] 1/3*c^2*x^3/e^2-c^2*d*x^2/e^3+2*c/e^2*a*x+3*c^2/e^4*d^2*x-4*c*d/e^3*ln(e*x+
d)*a-4*c^2*d^3/e^5*ln(e*x+d)-1/e/(e*x+d)*a^2-2/e^3/(e*x+d)*a*c*d^2-1/e^5/(e
*x+d)*c^2*d^4
```

Maxima [A] time = 1.17403, size = 151, normalized size = 1.61

$$-\frac{c^2d^4 + 2\,acd^2e^2 + a^2e^4}{e^6x + de^5} + \frac{c^2e^2x^3 - 3\,c^2dex^2 + 3\left(3\,c^2d^2 + 2\,ace^2\right)x}{3\,e^4} - \frac{4\left(c^2d^3 + acde^2\right)\log(ex+d)}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)/(e^6*x + d*e^5) + 1/3*(c^2*e^2*x^3 - 3
*c^2*d*e*x^2 + 3*(3*c^2*d^2 + 2*a*c*e^2)*x)/e^4 - 4*(c^2*d^3 + a*c*d*e^2)*l
og(e*x + d)/e^5
```

Fricas [A] time = 1.81917, size = 309, normalized size = 3.29

$$\frac{c^2e^4x^4 - 2\,c^2de^3x^3 - 3\,c^2d^4 - 6\,acd^2e^2 - 3\,a^2e^4 + 6\left(c^2d^2e^2 + ace^4\right)x^2 + 3\left(3\,c^2d^3e + 2\,acde^3\right)x - 12\left(c^2d^4 + acd^2e^2 + (c^2d^3 \cdots\right.}{3\left(e^6x + de^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(c^2*e^4*x^4 - 2*c^2*d*e^3*x^3 - 3*c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4
+ 6*(c^2*d^2*e^2 + a*c*e^4)*x^2 + 3*(3*c^2*d^3*e + 2*a*c*d*e^3)*x - 12*(c^2
*d^4 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)*log(e*x + d))/(e^6*x + d*e^
5)
```

Sympy [A] time = 0.720812, size = 105, normalized size = 1.12

$$-\frac{c^2dx^2}{e^3} + \frac{c^2x^3}{3e^2} - \frac{4cd\left(ae^2 + cd^2\right)\log(d+ex)}{e^5} - \frac{a^2e^4 + 2acd^2e^2 + c^2d^4}{de^5 + e^6x} + \frac{x\left(2ace^2 + 3c^2d^2\right)}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2/(e*x+d)**2,x)
```

```
[Out] -c**2*d*x**2/e**3 + c**2*x**3/(3*e**2) - 4*c*d*(a*e**2 + c*d**2)*log(d + e*
x)/e**5 - (a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4)/(d*e**5 + e**6*x) + x*(
2*a*c*e**2 + 3*c**2*d**2)/e**4
```

Giac [A] time = 1.27953, size = 201, normalized size = 2.14

$$\frac{1}{3}\left(c^2 - \frac{6\,c^2d}{xe+d} + \frac{6\left(3\,c^2d^2e^2 + ace^4\right)e^{(-2)}}{(xe+d)^2}\right)(xe+d)^3e^{(-5)} + 4\left(c^2d^3 + acde^2\right)e^{(-5)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^2}\right) - \left(\frac{c^2d^4e^3}{xe+d} + \frac{2\,acd^2e}{xe+d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/3*(c^2 - 6*c^2*d/(x*e + d) + 6*(3*c^2*d^2*e^2 + a*c*e^4)*e^(-2)/(x*e + d)
^2)*(x*e + d)^3*e^(-5) + 4*(c^2*d^3 + a*c*d*e^2)*e^(-5)*log(abs(x*e + d)*e^
(-1)/(x*e + d)^2) - (c^2*d^4*e^3/(x*e + d) + 2*a*c*d^2*e^5/(x*e + d) + a^2*
e^7/(x*e + d))*e^(-8)
```

3.466 $\int \frac{(a+cx^2)^2}{(d+ex)^3}\,dx$

Optimal. Leaf size=100

$$\frac{4cd(ae^2+cd^2)}{e^5(d+ex)}-\frac{(ae^2+cd^2)^2}{2e^5(d+ex)^2}+\frac{2c(ae^2+3cd^2)\log(d+ex)}{e^5}-\frac{3c^2dx}{e^4}+\frac{c^2x^2}{2e^3}$$

```
[Out] (-3*c^2*d*x)/e^4 + (c^2*x^2)/(2*e^3) - (c*d^2 + a*e^2)^2/(2*e^5*(d + e*x)^2
) + (4*c*d*(c*d^2 + a*e^2))/(e^5*(d + e*x)) + (2*c*(3*c*d^2 + a*e^2)*Log[d
+ e*x])/e^5
```

Rubi [A] time = 0.0810327, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$\frac{4cd(ae^2+cd^2)}{e^5(d+ex)}-\frac{(ae^2+cd^2)^2}{2e^5(d+ex)^2}+\frac{2c(ae^2+3cd^2)\log(d+ex)}{e^5}-\frac{3c^2dx}{e^4}+\frac{c^2x^2}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^2/(d + e*x)^3,x]
```



```
[Out] (-3*c^2*d*x)/e^4 + (c^2*x^2)/(2*e^3) - (c*d^2 + a*e^2)^2/(2*e^5*(d + e*x)^2
) + (4*c*d*(c*d^2 + a*e^2))/(e^5*(d + e*x)) + (2*c*(3*c*d^2 + a*e^2)*Log[d
+ e*x])/e^5
```


Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+cx^2)^2}{(d+ex)^3}\,dx &= \int \left(-\frac{3c^2d}{e^4}+\frac{c^2x}{e^3}+\frac{(cd^2+ae^2)^2}{e^4(d+ex)^3}-\frac{4cd(cd^2+ae^2)}{e^4(d+ex)^2}+\frac{2c(3cd^2+ae^2)}{e^4(d+ex)}\right)dx\\ &= -\frac{3c^2dx}{e^4}+\frac{c^2x^2}{2e^3}-\frac{(cd^2+ae^2)^2}{2e^5(d+ex)^2}+\frac{4cd(cd^2+ae^2)}{e^5(d+ex)}+\frac{2c(3cd^2+ae^2)\log(d+ex)}{e^5}\end{aligned}$$

Mathematica [A] time = 0.0374773, size = 111, normalized size = 1.11

$$\frac{-a^2e^4+4c(d+ex)^2(ae^2+3cd^2)\log(d+ex)+2acde^2(3d+4ex)+c^2(-11d^2e^2x^2+2d^3ex+7d^4-4de^3x^3+e^4x^4)}{2e^5(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^2/(d + e*x)^3,x]
```

```
[Out] (-(a^2*e^4) + 2*a*c*d*e^2*(3*d + 4*e*x) + c^2*(7*d^4 + 2*d^3*e*x - 11*d^2*e
^2*x^2 - 4*d*e^3*x^3 + e^4*x^4) + 4*c*(3*c*d^2 + a*e^2)*(d + e*x)^2*Log[d +
```

```
e*x])/(2*e^5*(d + e*x)^2)
```

Maple [A] time = 0.049, size = 136, normalized size = 1.4

$$\frac{c^2x^2}{2\,e^3}-3\,\frac{xc^2d}{e^4}+2\,\frac{c\ln(ex+d)\,a}{e^3}+6\,\frac{c^2\ln(ex+d)\,d^2}{e^5}+4\,\frac{acd}{e^3\,(ex+d)}+4\,\frac{c^2d^3}{e^5\,(ex+d)}-\frac{a^2}{2\,e\,(ex+d)^2}-\frac{acd^2}{e^3\,(ex+d)^2}-\frac{}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2/(e*x+d)^3,x)
```

```
[Out] 1/2*c^2*x^2/e^3-3*c^2*d*x/e^4+2*c/e^3*ln(e*x+d)*a+6*c^2/e^5*ln(e*x+d)*d^2+4
*c*d/e^3/(e*x+d)*a+4*c^2*d^3/e^5/(e*x+d)-1/2/e/(e*x+d)^2*a^2-1/e^3/(e*x+d)^
2*a*c*d^2-1/2/e^5/(e*x+d)^2*c^2*d^4
```

Maxima [A] time = 1.32838, size = 162, normalized size = 1.62

$$\frac{7\,c^2d^4+6\,acd^2e^2-a^2e^4+8\left(c^2d^3e+acde^3\right)x}{2\left(e^7x^2+2\,de^6x+d^2e^5\right)}+\frac{c^2ex^2-6\,c^2dx}{2\,e^4}+\frac{2\left(3\,c^2d^2+ace^2\right)\log(ex+d)}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(7*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4 + 8*(c^2*d^3*e + a*c*d*e^3)*x)/(e^
7*x^2 + 2*d*e^6*x + d^2*e^5) + 1/2*(c^2*e*x^2 - 6*c^2*d*x)/e^4 + 2*(3*c^2*d
^2 + a*c*e^2)*log(e*x + d)/e^5
```

Fricas [A] time = 1.93023, size = 360, normalized size = 3.6

$$\frac{c^2e^4x^4-4\,c^2de^3x^3-11\,c^2d^2e^2x^2+7\,c^2d^4+6\,acd^2e^2-a^2e^4+2\left(c^2d^3e+4\,acde^3\right)x+4\left(3\,c^2d^4+acd^2e^2+\left(3\,c^2d^2e^2+a\right.\right.}{2\left(e^7x^2+2\,de^6x+d^2e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(c^2*e^4*x^4 - 4*c^2*d*e^3*x^3 - 11*c^2*d^2*e^2*x^2 + 7*c^2*d^4 + 6*a*c
*d^2*e^2 - a^2*e^4 + 2*(c^2*d^3*e + 4*a*c*d*e^3)*x + 4*(3*c^2*d^4 + a*c*d^2
*e^2 + (3*c^2*d^2*e^2 + a*c*e^4)*x^2 + 2*(3*c^2*d^3*e + a*c*d*e^3)*x)*log(e
*x + d))/(e^7*x^2 + 2*d*e^6*x + d^2*e^5)
```

Sympy [A] time = 1.48875, size = 122, normalized size = 1.22

$$-\frac{3c^2dx}{e^4}+\frac{c^2x^2}{2e^3}+\frac{2c\left(ae^2+3cd^2\right)\log(d+ex)}{e^5}+\frac{-a^2e^4+6acd^2e^2+7c^2d^4+x\left(8acde^3+8c^2d^3e\right)}{2d^2e^5+4de^6x+2e^7x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2/(e*x+d)**3,x)
```

```
[Out] -3*c**2*d*x/e**4 + c**2*x**2/(2*e**3) + 2*c*(a*e**2 + 3*c*d**2)*log(d + e*x
)/e**5 + (-a**2*e**4 + 6*a*c*d**2*e**2 + 7*c**2*d**4 + x*(8*a*c*d*e**3 + 8*
c**2*d**3*e))/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2)
```

Giac [A] time = 1.30426, size = 143, normalized size = 1.43

$$2\left(3c^2d^2 + ace^2\right)e^{(-5)}\log\left(|xe + d|\right) + \frac{1}{2}\left(c^2x^2e^3 - 6c^2dxe^2\right)e^{(-6)} + \frac{\left(7c^2d^4 + 6acd^2e^2 - a^2e^4 + 8\left(c^2d^3e + acde^3\right)x\right)e^{(-5)}}{2(xe + d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 2*(3*c^2*d^2 + a*c*e^2)*e^(-5)*log(abs(x*e + d)) + 1/2*(c^2*x^2*e^3 - 6*c^2
*d*x*e^2)*e^(-6) + 1/2*(7*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4 + 8*(c^2*d^3*e
+ a*c*d*e^3)*x)*e^(-5)/(x*e + d)^2
```

3.467 $\int \frac{(a+cx^2)^2}{(d+ex)^4}\,dx$

Optimal. Leaf size=101

$$-\frac{2c\left(ae^2+3cd^2\right)}{e^5(d+ex)}+\frac{2cd\left(ae^2+cd^2\right)}{e^5(d+ex)^2}-\frac{\left(ae^2+cd^2\right)^2}{3e^5(d+ex)^3}-\frac{4c^2d\log(d+ex)}{e^5}+\frac{c^2x}{e^4}$$

```
[Out] (c^2*x)/e^4 - (c*d^2 + a*e^2)^2/(3*e^5*(d + e*x)^3) + (2*c*d*(c*d^2 + a*e^2
))/(e^5*(d + e*x)^2) - (2*c*(3*c*d^2 + a*e^2))/(e^5*(d + e*x)) - (4*c^2*d*L
og[d + e*x])/e^5
```

Rubi [A] time = 0.0746482, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$-\frac{2c\left(ae^2+3cd^2\right)}{e^5(d+ex)}+\frac{2cd\left(ae^2+cd^2\right)}{e^5(d+ex)^2}-\frac{\left(ae^2+cd^2\right)^2}{3e^5(d+ex)^3}-\frac{4c^2d\log(d+ex)}{e^5}+\frac{c^2x}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^2/(d + e*x)^4,x]
```

```
[Out] (c^2*x)/e^4 - (c*d^2 + a*e^2)^2/(3*e^5*(d + e*x)^3) + (2*c*d*(c*d^2 + a*e^2
))/(e^5*(d + e*x)^2) - (2*c*(3*c*d^2 + a*e^2))/(e^5*(d + e*x)) - (4*c^2*d*L
og[d + e*x])/e^5
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+cx^2\right)^2}{(d+ex)^4}\,dx &= \int \left(\frac{c^2}{e^4}+\frac{\left(cd^2+ae^2\right)^2}{e^4(d+ex)^4}-\frac{4cd\left(cd^2+ae^2\right)}{e^4(d+ex)^3}+\frac{2c\left(3cd^2+ae^2\right)}{e^4(d+ex)^2}-\frac{4c^2d}{e^4(d+ex)}\right)dx\\ &= \frac{c^2x}{e^4}-\frac{\left(cd^2+ae^2\right)^2}{3e^5(d+ex)^3}+\frac{2cd\left(cd^2+ae^2\right)}{e^5(d+ex)^2}-\frac{2c\left(3cd^2+ae^2\right)}{e^5(d+ex)}-\frac{4c^2d\log(d+ex)}{e^5}\end{aligned}$$

Mathematica [A] time = 0.0539365, size = 110, normalized size = 1.09

$$-\frac{a^2e^4+2ace^2\left(d^2+3dex+3e^2x^2\right)+c^2\left(9d^2e^2x^2+27d^3ex+13d^4-9de^3x^3-3e^4x^4\right)+12c^2d(d+ex)^3\log(d+ex)}{3e^5(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^2/(d + e*x)^4,x]
```

```
[Out] -(a^2*e^4 + 2*a*c*e^2*(d^2 + 3*d*e*x + 3*e^2*x^2) + c^2*(13*d^4 + 27*d^3*e*
x + 9*d^2*e^2*x^2 - 9*d*e^3*x^3 - 3*e^4*x^4) + 12*c^2*d*(d + e*x)^3*Log[d +
```

```
e*x])/(3*e^5*(d + e*x)^3)
```

Maple [A] time = 0.048, size = 140, normalized size = 1.4

$$\frac{c^2x}{e^4}-\frac{a^2}{3\,e\,(ex+d)^3}-\frac{2\,acd^2}{3\,e^3\,(ex+d)^3}-\frac{c^2d^4}{3\,e^5\,(ex+d)^3}-4\,\frac{c^2d\ln(ex+d)}{e^5}-2\,\frac{ac}{e^3\,(ex+d)}-6\,\frac{c^2d^2}{e^5\,(ex+d)}+2\,\frac{acd}{e^3\,(ex+d)^2}+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2/(e*x+d)^4,x)
```

```
[Out] c^2*x/e^4-1/3/e/(e*x+d)^3*a^2-2/3/e^3/(e*x+d)^3*a*c*d^2-1/3/e^5/(e*x+d)^3*c
^2*d^4-4*c^2*d*ln(e*x+d)/e^5-2*c/e^3/(e*x+d)*a-6*c^2/e^5/(e*x+d)*d^2+2*c*d/
e^3/(e*x+d)^2*a+2*c^2*d^3/e^5/(e*x+d)^2
```

Maxima [A] time = 1.29147, size = 176, normalized size = 1.74

$$-\frac{13\,c^2d^4+2\,acd^2e^2+a^2e^4+6\left(3\,c^2d^2e^2+ace^4\right)x^2+6\left(5\,c^2d^3e+acde^3\right)x}{3\left(e^8x^3+3\,de^7x^2+3\,d^2e^6x+d^3e^5\right)}+\frac{c^2x}{e^4}-\frac{4\,c^2d\log(ex+d)}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(13*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4 + 6*(3*c^2*d^2*e^2 + a*c*e^4)*x^
2 + 6*(5*c^2*d^3*e + a*c*d*e^3)*x)/(e^8*x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d
^3*e^5) + c^2*x/e^4 - 4*c^2*d*log(e*x + d)/e^5
```

Fricas [A] time = 1.94676, size = 373, normalized size = 3.69

$$\frac{3\,c^2e^4x^4+9\,c^2de^3x^3-13\,c^2d^4-2\,acd^2e^2-a^2e^4-3\left(3\,c^2d^2e^2+2\,ace^4\right)x^2-3\left(9\,c^2d^3e+2\,acde^3\right)x-12\left(c^2de^3x^3+3\,c^2d^2e^2\right.}{3\left(e^8x^3+3\,de^7x^2+3\,d^2e^6x+d^3e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*c^2*e^4*x^4 + 9*c^2*d*e^3*x^3 - 13*c^2*d^4 - 2*a*c*d^2*e^2 - a^2*e^4
 - 3*(3*c^2*d^2*e^2 + 2*a*c*e^4)*x^2 - 3*(9*c^2*d^3*e + 2*a*c*d*e^3)*x - 12
*(c^2*d*e^3*x^3 + 3*c^2*d^2*e^2*x^2 + 3*c^2*d^3*e*x + c^2*d^4)*log(e*x + d)
)/(e^8*x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3*e^5)
```

Sympy [A] time = 1.43739, size = 134, normalized size = 1.33

$$-\frac{4c^2d\log(d+ex)}{e^5}+\frac{c^2x}{e^4}-\frac{a^2e^4+2acd^2e^2+13c^2d^4+x^2\left(6ace^4+18c^2d^2e^2\right)+x\left(6acde^3+30c^2d^3e\right)}{3d^3e^5+9d^2e^6x+9de^7x^2+3e^8x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2/(e*x+d)**4,x)
```

```
[Out] -4*c**2*d*log(d + e*x)/e**5 + c**2*x/e**4 - (a**2*e**4 + 2*a*c*d**2*e**2 +
13*c**2*d**4 + x**2*(6*a*c*e**4 + 18*c**2*d**2*e**2) + x*(6*a*c*d*e**3 + 30
*c**2*d**3*e))/(3*d**3*e**5 + 9*d**2*e**6*x + 9*d*e**7*x**2 + 3*e**8*x**3)
```

Giac [A] time = 1.2176, size = 136, normalized size = 1.35

$$-4\,c^2de^{(-5)}\log\left(|xe+d|\right)+c^2xe^{(-4)}-\frac{\left(13\,c^2d^4+2\,acd^2e^2+6\left(3\,c^2d^2e^2+ace^4\right)x^2+a^2e^4+6\left(5\,c^2d^3e+acde^3\right)x\right)e^{(-5)}}{3\,(xe+d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -4*c^2*d*e^(-5)*log(abs(x*e + d)) + c^2*x*e^(-4) - 1/3*(13*c^2*d^4 + 2*a*c*
d^2*e^2 + 6*(3*c^2*d^2*e^2 + a*c*e^4)*x^2 + a^2*e^4 + 6*(5*c^2*d^3*e + a*c*
d*e^3)*x)*e^(-5)/(x*e + d)^3
```

3.468 $\int \frac{(a+cx^2)^2}{(d+ex)^5} dx$

Optimal. Leaf size=109

$$-\frac{c(ae^2+3cd^2)}{e^5(d+ex)^2}+\frac{4cd(ae^2+cd^2)}{3e^5(d+ex)^3}-\frac{(ae^2+cd^2)^2}{4e^5(d+ex)^4}+\frac{4c^2d}{e^5(d+ex)}+\frac{c^2\log(d+ex)}{e^5}$$

```
[Out] -(c*d^2 + a*e^2)^2/(4*e^5*(d + e*x)^4) + (4*c*d*(c*d^2 + a*e^2))/(3*e^5*(d
+ e*x)^3) - (c*(3*c*d^2 + a*e^2))/(e^5*(d + e*x)^2) + (4*c^2*d)/(e^5*(d + e
*x)) + (c^2*Log[d + e*x])/e^5
```

Rubi [A] time = 0.0747606, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$-\frac{c(ae^2+3cd^2)}{e^5(d+ex)^2}+\frac{4cd(ae^2+cd^2)}{3e^5(d+ex)^3}-\frac{(ae^2+cd^2)^2}{4e^5(d+ex)^4}+\frac{4c^2d}{e^5(d+ex)}+\frac{c^2\log(d+ex)}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^2/(d + e*x)^5,x]
```

```
[Out] -(c*d^2 + a*e^2)^2/(4*e^5*(d + e*x)^4) + (4*c*d*(c*d^2 + a*e^2))/(3*e^5*(d
+ e*x)^3) - (c*(3*c*d^2 + a*e^2))/(e^5*(d + e*x)^2) + (4*c^2*d)/(e^5*(d + e
*x)) + (c^2*Log[d + e*x])/e^5
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+cx^2)^2}{(d+ex)^5} dx &= \int \left(\frac{(cd^2+ae^2)^2}{e^4(d+ex)^5}-\frac{4cd(cd^2+ae^2)}{e^4(d+ex)^4}+\frac{2c(3cd^2+ae^2)}{e^4(d+ex)^3}-\frac{4c^2d}{e^4(d+ex)^2}+\frac{c^2}{e^4(d+ex)}\right) dx\\ &= -\frac{(cd^2+ae^2)^2}{4e^5(d+ex)^4}+\frac{4cd(cd^2+ae^2)}{3e^5(d+ex)^3}-\frac{c(3cd^2+ae^2)}{e^5(d+ex)^2}+\frac{4c^2d}{e^5(d+ex)}+\frac{c^2\log(d+ex)}{e^5}\end{aligned}$$

Mathematica [A] time = 0.0357054, size = 100, normalized size = 0.92

$$\frac{-3a^2e^4-2ace^2(d^2+4dex+6e^2x^2)+c^2d(88d^2ex+25d^3+108de^2x^2+48e^3x^3)+12c^2(d+ex)^4\log(d+ex)}{12e^5(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^2/(d + e*x)^5,x]
```

```
[Out] (-3*a^2*e^4 - 2*a*c*e^2*(d^2 + 4*d*e*x + 6*e^2*x^2) + c^2*d*(25*d^3 + 88*d^
2*e*x + 108*d*e^2*x^2 + 48*e^3*x^3) + 12*c^2*(d + e*x)^4*Log[d + e*x])/(12*
```

```
e^5*(d + e*x)^4)
```

Maple [A] time = 0.049, size = 146, normalized size = 1.3

$$-\frac{a^2}{4e(ex+d)^4}-\frac{acd^2}{2e^3(ex+d)^4}-\frac{c^2d^4}{4e^5(ex+d)^4}+\frac{4acd}{3e^3(ex+d)^3}+\frac{4c^2d^3}{3e^5(ex+d)^3}+\frac{c^2\ln(ex+d)}{e^5}+4\frac{c^2d}{e^5(ex+d)}-\frac{}{e^3(e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2/(e*x+d)^5,x)
```

```
[Out] -1/4/e/(e*x+d)^4*a^2-1/2/e^3/(e*x+d)^4*a*c*d^2-1/4/e^5/(e*x+d)^4*c^2*d^4+4/
3*c*d/e^3/(e*x+d)^3*a+4/3*c^2*d^3/e^5/(e*x+d)^3+c^2*ln(e*x+d)/e^5+4*c^2*d/e
^5/(e*x+d)-c/e^3/(e*x+d)^2*a-3*c^2/e^5/(e*x+d)^2*d^2
```

Maxima [A] time = 1.17105, size = 197, normalized size = 1.81

$$\frac{48\,c^2de^3x^3+25\,c^2d^4-2\,acd^2e^2-3\,a^2e^4+12\left(9\,c^2d^2e^2-ace^4\right)x^2+8\left(11\,c^2d^3e-acde^3\right)x}{12\left(e^9x^4+4\,de^8x^3+6\,d^2e^7x^2+4\,d^3e^6x+d^4e^5\right)}+\frac{c^2\log(ex+d)}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^5,x, algorithm="maxima")
```

```
[Out] 1/12*(48*c^2*d*e^3*x^3 + 25*c^2*d^4 - 2*a*c*d^2*e^2 - 3*a^2*e^4 + 12*(9*c^2
*d^2*e^2 - a*c*e^4)*x^2 + 8*(11*c^2*d^3*e - a*c*d*e^3)*x)/(e^9*x^4 + 4*d*e^
8*x^3 + 6*d^2*e^7*x^2 + 4*d^3*e^6*x + d^4*e^5) + c^2*log(e*x + d)/e^5
```

Fricas [A] time = 1.84814, size = 397, normalized size = 3.64

$$\frac{48\,c^2de^3x^3+25\,c^2d^4-2\,acd^2e^2-3\,a^2e^4+12\left(9\,c^2d^2e^2-ace^4\right)x^2+8\left(11\,c^2d^3e-acde^3\right)x+12\left(c^2e^4x^4+4\,c^2de^3x^3+6\,c\right.}{12\left(e^9x^4+4\,de^8x^3+6\,d^2e^7x^2+4\,d^3e^6x+d^4e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(48*c^2*d*e^3*x^3 + 25*c^2*d^4 - 2*a*c*d^2*e^2 - 3*a^2*e^4 + 12*(9*c^2
*d^2*e^2 - a*c*e^4)*x^2 + 8*(11*c^2*d^3*e - a*c*d*e^3)*x + 12*(c^2*e^4*x^4
+ 4*c^2*d*e^3*x^3 + 6*c^2*d^2*e^2*x^2 + 4*c^2*d^3*e*x + c^2*d^4)*log(e*x +
d))/(e^9*x^4 + 4*d*e^8*x^3 + 6*d^2*e^7*x^2 + 4*d^3*e^6*x + d^4*e^5)
```

Sympy [A] time = 1.84963, size = 150, normalized size = 1.38

$$\frac{c^2\log(d+ex)}{e^5}+\frac{-3a^2e^4-2acd^2e^2+25c^2d^4+48c^2de^3x^3+x^2\left(-12ace^4+108c^2d^2e^2\right)+x\left(-8acde^3+88c^2d^3e\right)}{12d^4e^5+48d^3e^6x+72d^2e^7x^2+48de^8x^3+12e^9x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2/(e*x+d)**5,x)
```

```
[Out] c**2*log(d + e*x)/e**5 + (-3*a**2*e**4 - 2*a*c*d**2*e**2 + 25*c**2*d**4 + 4
8*c**2*d*e**3*x**3 + x**2*(-12*a*c*e**4 + 108*c**2*d**2*e**2) + x*(-8*a*c*d
*e**3 + 88*c**2*d**3*e))/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2
 + 48*d*e**8*x**3 + 12*e**9*x**4)
```

Giac [A] time = 1.21402, size = 220, normalized size = 2.02

$$-c^2e^{(-5)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^2}\right)+\frac{1}{12}\left(\frac{48\,c^2de^{15}}{xe+d}-\frac{36\,c^2d^2e^{15}}{(xe+d)^2}+\frac{16\,c^2d^3e^{15}}{(xe+d)^3}-\frac{3\,c^2d^4e^{15}}{(xe+d)^4}-\frac{12\,ace^{17}}{(xe+d)^2}+\frac{16\,acde^{17}}{(xe+d)^3}-\frac{6\,acd^2e^{17}}{(xe+d)^4}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] -c^2*e^(-5)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2) + 1/12*(48*c^2*d*e^15/(x*e
 + d) - 36*c^2*d^2*e^15/(x*e + d)^2 + 16*c^2*d^3*e^15/(x*e + d)^3 - 3*c^2*d
^4*e^15/(x*e + d)^4 - 12*a*c*e^17/(x*e + d)^2 + 16*a*c*d*e^17/(x*e + d)^3 -
 6*a*c*d^2*e^17/(x*e + d)^4 - 3*a^2*e^19/(x*e + d)^4)*e^(-20)
```

3.469 $\int \frac{(a+cx^2)^2}{(d+ex)^6} \, dx$

Optimal. Leaf size=110

$$-\frac{2c\left(ae^2+3cd^2\right)}{3e^5(d+ex)^3}+\frac{cd\left(ae^2+cd^2\right)}{e^5(d+ex)^4}-\frac{\left(ae^2+cd^2\right)^2}{5e^5(d+ex)^5}-\frac{c^2}{e^5(d+ex)}+\frac{2c^2d}{e^5(d+ex)^2}$$

```
[Out] -(c*d^2 + a*e^2)^2/(5*e^5*(d + e*x)^5) + (c*d*(c*d^2 + a*e^2))/(e^5*(d + e*
x)^4) - (2*c*(3*c*d^2 + a*e^2))/(3*e^5*(d + e*x)^3) + (2*c^2*d)/(e^5*(d + e
*x)^2) - c^2/(e^5*(d + e*x))
```

Rubi [A] time = 0.0685907, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$-\frac{2c\left(ae^2+3cd^2\right)}{3e^5(d+ex)^3}+\frac{cd\left(ae^2+cd^2\right)}{e^5(d+ex)^4}-\frac{\left(ae^2+cd^2\right)^2}{5e^5(d+ex)^5}-\frac{c^2}{e^5(d+ex)}+\frac{2c^2d}{e^5(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^2/(d + e*x)^6,x]
```

```
[Out] -(c*d^2 + a*e^2)^2/(5*e^5*(d + e*x)^5) + (c*d*(c*d^2 + a*e^2))/(e^5*(d + e*
x)^4) - (2*c*(3*c*d^2 + a*e^2))/(3*e^5*(d + e*x)^3) + (2*c^2*d)/(e^5*(d + e
*x)^2) - c^2/(e^5*(d + e*x))
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+cx^2\right)^2}{(d+ex)^6}\,dx &= \int \left(\frac{\left(cd^2+ae^2\right)^2}{e^4(d+ex)^6}-\frac{4cd\left(cd^2+ae^2\right)}{e^4(d+ex)^5}+\frac{2c\left(3cd^2+ae^2\right)}{e^4(d+ex)^4}-\frac{4c^2d}{e^4(d+ex)^3}+\frac{c^2}{e^4(d+ex)^2}\right)dx\\ &= -\frac{\left(cd^2+ae^2\right)^2}{5e^5(d+ex)^5}+\frac{cd\left(cd^2+ae^2\right)}{e^5(d+ex)^4}-\frac{2c\left(3cd^2+ae^2\right)}{3e^5(d+ex)^3}+\frac{2c^2d}{e^5(d+ex)^2}-\frac{c^2}{e^5(d+ex)}\end{aligned}$$

Mathematica [A] time = 0.0374355, size = 90, normalized size = 0.82

$$-\frac{3a^2e^4+ace^2\left(d^2+5dex+10e^2x^2\right)+3c^2\left(10d^2e^2x^2+5d^3ex+d^4+10de^3x^3+5e^4x^4\right)}{15e^5(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^2/(d + e*x)^6,x]
```

```
[Out] -(3*a^2*e^4 + a*c*e^2*(d^2 + 5*d*e*x + 10*e^2*x^2) + 3*c^2*(d^4 + 5*d^3*e*x
 + 10*d^2*e^2*x^2 + 10*d*e^3*x^3 + 5*e^4*x^4))/(15*e^5*(d + e*x)^5)
```

Maple [A] time = 0.048, size = 119, normalized size = 1.1

$$\frac{cd\left(ae^2+cd^2\right)}{e^5\left(ex+d\right)^4}-\frac{2c\left(ae^2+3cd^2\right)}{3e^5\left(ex+d\right)^3}-\frac{a^2e^4+2acd^2e^2+c^2d^4}{5e^5\left(ex+d\right)^5}-\frac{c^2}{e^5\left(ex+d\right)}+2\frac{c^2d}{e^5\left(ex+d\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2/(e*x+d)^6,x)
```

```
[Out] c*d*(a*e^2+c*d^2)/e^5/(e*x+d)^4-2/3*c*(a*e^2+3*c*d^2)/e^5/(e*x+d)^3-1/5*(a^
2*e^4+2*a*c*d^2*e^2+c^2*d^4)/e^5/(e*x+d)^5-c^2/e^5/(e*x+d)+2*c^2*d/e^5/(e*x
+d)^2
```

Maxima [A] time = 1.33863, size = 204, normalized size = 1.85

$$-\frac{15\,c^2e^4x^4+30\,c^2de^3x^3+3\,c^2d^4+acd^2e^2+3\,a^2e^4+10\left(3\,c^2d^2e^2+ace^4\right)x^2+5\left(3\,c^2d^3e+acde^3\right)x}{15\left(e^{10}x^5+5\,de^9x^4+10\,d^2e^8x^3+10\,d^3e^7x^2+5\,d^4e^6x+d^5e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^6,x, algorithm="maxima")
```

```
[Out] -1/15*(15*c^2*e^4*x^4 + 30*c^2*d*e^3*x^3 + 3*c^2*d^4 + a*c*d^2*e^2 + 3*a^2*
e^4 + 10*(3*c^2*d^2*e^2 + a*c*e^4)*x^2 + 5*(3*c^2*d^3*e + a*c*d*e^3)*x)/(e^
10*x^5 + 5*d*e^9*x^4 + 10*d^2*e^8*x^3 + 10*d^3*e^7*x^2 + 5*d^4*e^6*x + d^5*
e^5)
```

Fricas [A] time = 1.80105, size = 312, normalized size = 2.84

$$-\frac{15\,c^2e^4x^4+30\,c^2de^3x^3+3\,c^2d^4+acd^2e^2+3\,a^2e^4+10\left(3\,c^2d^2e^2+ace^4\right)x^2+5\left(3\,c^2d^3e+acde^3\right)x}{15\left(e^{10}x^5+5\,de^9x^4+10\,d^2e^8x^3+10\,d^3e^7x^2+5\,d^4e^6x+d^5e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^6,x, algorithm="fricas")
```

```
[Out] -1/15*(15*c^2*e^4*x^4 + 30*c^2*d*e^3*x^3 + 3*c^2*d^4 + a*c*d^2*e^2 + 3*a^2*
e^4 + 10*(3*c^2*d^2*e^2 + a*c*e^4)*x^2 + 5*(3*c^2*d^3*e + a*c*d*e^3)*x)/(e^
10*x^5 + 5*d*e^9*x^4 + 10*d^2*e^8*x^3 + 10*d^3*e^7*x^2 + 5*d^4*e^6*x + d^5*
e^5)
```

Sympy [A] time = 2.3188, size = 160, normalized size = 1.45

$$-\frac{3a^2e^4+acd^2e^2+3c^2d^4+30c^2de^3x^3+15c^2e^4x^4+x^2\left(10ace^4+30c^2d^2e^2\right)+x\left(5acde^3+15c^2d^3e\right)}{15d^5e^5+75d^4e^6x+150d^3e^7x^2+150d^2e^8x^3+75de^9x^4+15e^{10}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2/(e*x+d)**6,x)
```

```
[Out] -(3*a**2*e**4 + a*c*d**2*e**2 + 3*c**2*d**4 + 30*c**2*d*e**3*x**3 + 15*c**2
*e**4*x**4 + x**2*(10*a*c*e**4 + 30*c**2*d**2*e**2) + x*(5*a*c*d*e**3 + 15*
c**2*d**3*e))/(15*d**5*e**5 + 75*d**4*e**6*x + 150*d**3*e**7*x**2 + 150*d**
2*e**8*x**3 + 75*d*e**9*x**4 + 15*e**10*x**5)
```

Giac [A] time = 1.29556, size = 132, normalized size = 1.2

$$-\frac{\left(15\,c^2x^4e^4 + 30\,c^2dx^3e^3 + 30\,c^2d^2x^2e^2 + 15\,c^2d^3xe + 3\,c^2d^4 + 10\,acx^2e^4 + 5\,acdxe^3 + acd^2e^2 + 3\,a^2e^4\right)e^{(-5)}}{15\,(xe + d)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^6,x, algorithm="giac")
```

```
[Out] -1/15*(15*c^2*x^4*e^4 + 30*c^2*d*x^3*e^3 + 30*c^2*d^2*x^2*e^2 + 15*c^2*d^3*
x*e + 3*c^2*d^4 + 10*a*c*x^2*e^4 + 5*a*c*d*x*e^3 + a*c*d^2*e^2 + 3*a^2*e^4)
*e^(-5)/(x*e + d)^5
```

3.470 $\int \frac{(a+cx^2)^2}{(d+ex)^7} dx$

Optimal. Leaf size=117

$$-\frac{c(ae^2+3cd^2)}{2e^5(d+ex)^4}+\frac{4cd(ae^2+cd^2)}{5e^5(d+ex)^5}-\frac{(ae^2+cd^2)^2}{6e^5(d+ex)^6}-\frac{c^2}{2e^5(d+ex)^2}+\frac{4c^2d}{3e^5(d+ex)^3}$$

```
[Out] -(c*d^2 + a*e^2)^2/(6*e^5*(d + e*x)^6) + (4*c*d*(c*d^2 + a*e^2))/(5*e^5*(d
+ e*x)^5) - (c*(3*c*d^2 + a*e^2))/(2*e^5*(d + e*x)^4) + (4*c^2*d)/(3*e^5*(d
 + e*x)^3) - c^2/(2*e^5*(d + e*x)^2)
```

Rubi [A] time = 0.0690073, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$-\frac{c(ae^2+3cd^2)}{2e^5(d+ex)^4}+\frac{4cd(ae^2+cd^2)}{5e^5(d+ex)^5}-\frac{(ae^2+cd^2)^2}{6e^5(d+ex)^6}-\frac{c^2}{2e^5(d+ex)^2}+\frac{4c^2d}{3e^5(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^2/(d + e*x)^7,x]
```

```
[Out] -(c*d^2 + a*e^2)^2/(6*e^5*(d + e*x)^6) + (4*c*d*(c*d^2 + a*e^2))/(5*e^5*(d
+ e*x)^5) - (c*(3*c*d^2 + a*e^2))/(2*e^5*(d + e*x)^4) + (4*c^2*d)/(3*e^5*(d
 + e*x)^3) - c^2/(2*e^5*(d + e*x)^2)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+cx^2)^2}{(d+ex)^7}dx &= \int \left(\frac{(cd^2+ae^2)^2}{e^4(d+ex)^7}-\frac{4cd(cd^2+ae^2)}{e^4(d+ex)^6}+\frac{2c(3cd^2+ae^2)}{e^4(d+ex)^5}-\frac{4c^2d}{e^4(d+ex)^4}+\frac{c^2}{e^4(d+ex)^3}\right)dx\\ &= -\frac{(cd^2+ae^2)^2}{6e^5(d+ex)^6}+\frac{4cd(cd^2+ae^2)}{5e^5(d+ex)^5}-\frac{c(3cd^2+ae^2)}{2e^5(d+ex)^4}+\frac{4c^2d}{3e^5(d+ex)^3}-\frac{c^2}{2e^5(d+ex)^2}\end{aligned}$$

Mathematica [A] time = 0.0302626, size = 89, normalized size = 0.76

$$-\frac{5a^2e^4+ace^2(d^2+6dex+15e^2x^2)+c^2(15d^2e^2x^2+6d^3ex+d^4+20de^3x^3+15e^4x^4)}{30e^5(d+ex)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^2/(d + e*x)^7,x]
```

```
[Out] -(5*a^2*e^4 + a*c*e^2*(d^2 + 6*d*e*x + 15*e^2*x^2) + c^2*(d^4 + 6*d^3*e*x +
 15*d^2*e^2*x^2 + 20*d*e^3*x^3 + 15*e^4*x^4))/(30*e^5*(d + e*x)^6)
```

Maple [A] time = 0.046, size = 120, normalized size = 1.

$$-\frac{c\left(ae^{2}+3\,cd^{2}\right)}{2\,e^{5}\left(ex+d\right)^{4}}-\frac{a^{2}e^{4}+2\,acd^{2}e^{2}+c^{2}d^{4}}{6\,e^{5}\left(ex+d\right)^{6}}+\frac{4\,c^{2}d}{3\,e^{5}\left(ex+d\right)^{3}}+\frac{4\,cd\left(ae^{2}+cd^{2}\right)}{5\,e^{5}\left(ex+d\right)^{5}}-\frac{c^{2}}{2\,e^{5}\left(ex+d\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2/(e*x+d)^7,x)
```

```
[Out] -1/2*c*(a*e^2+3*c*d^2)/e^5/(e*x+d)^4-1/6*(a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)/e^
5/(e*x+d)^6+4/3*c^2*d/e^5/(e*x+d)^3+4/5*c*d*(a*e^2+c*d^2)/e^5/(e*x+d)^5-1/2
*c^2/e^5/(e*x+d)^2
```

Maxima [A] time = 1.13421, size = 215, normalized size = 1.84

$$-\frac{15\,c^{2}e^{4}x^{4}+20\,c^{2}de^{3}x^{3}+c^{2}d^{4}+acd^{2}e^{2}+5\,a^{2}e^{4}+15\left(c^{2}d^{2}e^{2}+ace^{4}\right)x^{2}+6\left(c^{2}d^{3}e+acde^{3}\right)x}{30\left(e^{11}x^{6}+6\,de^{10}x^{5}+15\,d^{2}e^{9}x^{4}+20\,d^{3}e^{8}x^{3}+15\,d^{4}e^{7}x^{2}+6\,d^{5}e^{6}x+d^{6}e^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^7,x, algorithm="maxima")
```

```
[Out] -1/30*(15*c^2*e^4*x^4 + 20*c^2*d*e^3*x^3 + c^2*d^4 + a*c*d^2*e^2 + 5*a^2*e^
4 + 15*(c^2*d^2*e^2 + a*c*e^4)*x^2 + 6*(c^2*d^3*e + a*c*d*e^3)*x)/(e^11*x^6
 + 6*d*e^10*x^5 + 15*d^2*e^9*x^4 + 20*d^3*e^8*x^3 + 15*d^4*e^7*x^2 + 6*d^5*
e^6*x + d^6*e^5)
```

Fricas [A] time = 1.80798, size = 328, normalized size = 2.8

$$-\frac{15\,c^{2}e^{4}x^{4}+20\,c^{2}de^{3}x^{3}+c^{2}d^{4}+acd^{2}e^{2}+5\,a^{2}e^{4}+15\left(c^{2}d^{2}e^{2}+ace^{4}\right)x^{2}+6\left(c^{2}d^{3}e+acde^{3}\right)x}{30\left(e^{11}x^{6}+6\,de^{10}x^{5}+15\,d^{2}e^{9}x^{4}+20\,d^{3}e^{8}x^{3}+15\,d^{4}e^{7}x^{2}+6\,d^{5}e^{6}x+d^{6}e^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^7,x, algorithm="fricas")
```

```
[Out] -1/30*(15*c^2*e^4*x^4 + 20*c^2*d*e^3*x^3 + c^2*d^4 + a*c*d^2*e^2 + 5*a^2*e^
4 + 15*(c^2*d^2*e^2 + a*c*e^4)*x^2 + 6*(c^2*d^3*e + a*c*d*e^3)*x)/(e^11*x^6
 + 6*d*e^10*x^5 + 15*d^2*e^9*x^4 + 20*d^3*e^8*x^3 + 15*d^4*e^7*x^2 + 6*d^5*
e^6*x + d^6*e^5)
```

Sympy [A] time = 3.05701, size = 170, normalized size = 1.45

$$-\frac{5a^{2}e^{4}+acd^{2}e^{2}+c^{2}d^{4}+20c^{2}de^{3}x^{3}+15c^{2}e^{4}x^{4}+x^{2}\left(15ace^{4}+15c^{2}d^{2}e^{2}\right)+x\left(6acde^{3}+6c^{2}d^{3}e\right)}{30d^{6}e^{5}+180d^{5}e^{6}x+450d^{4}e^{7}x^{2}+600d^{3}e^{8}x^{3}+450d^{2}e^{9}x^{4}+180de^{10}x^{5}+30e^{11}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2/(e*x+d)**7,x)
```

```
[Out] -(5*a**2*e**4 + a*c*d**2*e**2 + c**2*d**4 + 20*c**2*d*e**3*x**3 + 15*c**2*e
**4*x**4 + x**2*(15*a*c*e**4 + 15*c**2*d**2*e**2) + x*(6*a*c*d*e**3 + 6*c**
2*d**3*e))/(30*d**6*e**5 + 180*d**5*e**6*x + 450*d**4*e**7*x**2 + 600*d**3*
e**8*x**3 + 450*d**2*e**9*x**4 + 180*d*e**10*x**5 + 30*e**11*x**6)
```

Giac [A] time = 1.29139, size = 131, normalized size = 1.12

$$-\frac{\left(15\,c^{2}x^{4}e^{4}+20\,c^{2}dx^{3}e^{3}+15\,c^{2}d^{2}x^{2}e^{2}+6\,c^{2}d^{3}xe+c^{2}d^{4}+15\,acx^{2}e^{4}+6\,acdxe^{3}+acd^{2}e^{2}+5\,a^{2}e^{4}\right)e^{(-5)}}{30\,(xe+d)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^7,x, algorithm="giac")
```

```
[Out] -1/30*(15*c^2*x^4*e^4 + 20*c^2*d*x^3*e^3 + 15*c^2*d^2*x^2*e^2 + 6*c^2*d^3*x
*e + c^2*d^4 + 15*a*c*x^2*e^4 + 6*a*c*d*x*e^3 + a*c*d^2*e^2 + 5*a^2*e^4)*e^
(-5)/(x*e + d)^6
```

3.471 $\int \frac{(a+cx^2)^2}{(d+ex)^8}\,dx$

Optimal. Leaf size=114

$$-\frac{2c\left(ae^2+3cd^2\right)}{5e^5(d+ex)^5}+\frac{2cd\left(ae^2+cd^2\right)}{3e^5(d+ex)^6}-\frac{\left(ae^2+cd^2\right)^2}{7e^5(d+ex)^7}-\frac{c^2}{3e^5(d+ex)^3}+\frac{c^2d}{e^5(d+ex)^4}$$

```
[Out] -(c*d^2 + a*e^2)^2/(7*e^5*(d + e*x)^7) + (2*c*d*(c*d^2 + a*e^2))/(3*e^5*(d
+ e*x)^6) - (2*c*(3*c*d^2 + a*e^2))/(5*e^5*(d + e*x)^5) + (c^2*d)/(e^5*(d +
 e*x)^4) - c^2/(3*e^5*(d + e*x)^3)
```

Rubi [A] time = 0.0695633, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$-\frac{2c\left(ae^2+3cd^2\right)}{5e^5(d+ex)^5}+\frac{2cd\left(ae^2+cd^2\right)}{3e^5(d+ex)^6}-\frac{\left(ae^2+cd^2\right)^2}{7e^5(d+ex)^7}-\frac{c^2}{3e^5(d+ex)^3}+\frac{c^2d}{e^5(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^2/(d + e*x)^8,x]
```

```
[Out] -(c*d^2 + a*e^2)^2/(7*e^5*(d + e*x)^7) + (2*c*d*(c*d^2 + a*e^2))/(3*e^5*(d
+ e*x)^6) - (2*c*(3*c*d^2 + a*e^2))/(5*e^5*(d + e*x)^5) + (c^2*d)/(e^5*(d +
 e*x)^4) - c^2/(3*e^5*(d + e*x)^3)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+cx^2\right)^2}{(d+ex)^8}\,dx &= \int \left(\frac{\left(cd^2+ae^2\right)^2}{e^4(d+ex)^8}-\frac{4cd\left(cd^2+ae^2\right)}{e^4(d+ex)^7}+\frac{2c\left(3cd^2+ae^2\right)}{e^4(d+ex)^6}-\frac{4c^2d}{e^4(d+ex)^5}+\frac{c^2}{e^4(d+ex)^4}\right)dx\\ &= -\frac{\left(cd^2+ae^2\right)^2}{7e^5(d+ex)^7}+\frac{2cd\left(cd^2+ae^2\right)}{3e^5(d+ex)^6}-\frac{2c\left(3cd^2+ae^2\right)}{5e^5(d+ex)^5}+\frac{c^2d}{e^5(d+ex)^4}-\frac{c^2}{3e^5(d+ex)^3}\end{aligned}$$

Mathematica [A] time = 0.035664, size = 90, normalized size = 0.79

$$-\frac{15a^2e^4+2ace^2\left(d^2+7dex+21e^2x^2\right)+c^2\left(21d^2e^2x^2+7d^3ex+d^4+35de^3x^3+35e^4x^4\right)}{105e^5(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^2/(d + e*x)^8,x]
```

```
[Out] -(15*a^2*e^4 + 2*a*c*e^2*(d^2 + 7*d*e*x + 21*e^2*x^2) + c^2*(d^4 + 7*d^3*e*
x + 21*d^2*e^2*x^2 + 35*d*e^3*x^3 + 35*e^4*x^4))/(105*e^5*(d + e*x)^7)
```

Maple [A] time = 0.046, size = 119, normalized size = 1.

$$\frac{c^2 d}{e^5 (ex+d)^4} + \frac{2\,cd\left(ae^2+cd^2\right)}{3\,e^5 (ex+d)^6} - \frac{c^2}{3\,e^5 (ex+d)^3} - \frac{a^2e^4+2\,acd^2e^2+c^2d^4}{7\,e^5 (ex+d)^7} - \frac{2\,c\left(ae^2+3\,cd^2\right)}{5\,e^5 (ex+d)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2/(e*x+d)^8,x)
```

```
[Out] c^2*d/e^5/(e*x+d)^4+2/3*c*d*(a*e^2+c*d^2)/e^5/(e*x+d)^6-1/3*c^2/e^5/(e*x+d)
^3-1/7*(a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)/e^5/(e*x+d)^7-2/5*c*(a*e^2+3*c*d^2)/
e^5/(e*x+d)^5
```

Maxima [A] time = 1.60838, size = 234, normalized size = 2.05

$$-\frac{35\,c^2e^4x^4+35\,c^2de^3x^3+c^2d^4+2\,acd^2e^2+15\,a^2e^4+21\left(c^2d^2e^2+2\,ace^4\right)x^2+7\left(c^2d^3e+2\,acde^3\right)x}{105\left(e^{12}x^7+7\,de^{11}x^6+21\,d^2e^{10}x^5+35\,d^3e^9x^4+35\,d^4e^8x^3+21\,d^5e^7x^2+7\,d^6e^6x+d^7e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^8,x, algorithm="maxima")
```

```
[Out] -1/105*(35*c^2*e^4*x^4 + 35*c^2*d*e^3*x^3 + c^2*d^4 + 2*a*c*d^2*e^2 + 15*a^
2*e^4 + 21*(c^2*d^2*e^2 + 2*a*c*e^4)*x^2 + 7*(c^2*d^3*e + 2*a*c*d*e^3)*x)/(
e^12*x^7 + 7*d*e^11*x^6 + 21*d^2*e^10*x^5 + 35*d^3*e^9*x^4 + 35*d^4*e^8*x^3
 + 21*d^5*e^7*x^2 + 7*d^6*e^6*x + d^7*e^5)
```

Fricas [A] time = 1.9446, size = 363, normalized size = 3.18

$$-\frac{35\,c^2e^4x^4+35\,c^2de^3x^3+c^2d^4+2\,acd^2e^2+15\,a^2e^4+21\left(c^2d^2e^2+2\,ace^4\right)x^2+7\left(c^2d^3e+2\,acde^3\right)x}{105\left(e^{12}x^7+7\,de^{11}x^6+21\,d^2e^{10}x^5+35\,d^3e^9x^4+35\,d^4e^8x^3+21\,d^5e^7x^2+7\,d^6e^6x+d^7e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^8,x, algorithm="fricas")
```

```
[Out] -1/105*(35*c^2*e^4*x^4 + 35*c^2*d*e^3*x^3 + c^2*d^4 + 2*a*c*d^2*e^2 + 15*a^
2*e^4 + 21*(c^2*d^2*e^2 + 2*a*c*e^4)*x^2 + 7*(c^2*d^3*e + 2*a*c*d*e^3)*x)/(
e^12*x^7 + 7*d*e^11*x^6 + 21*d^2*e^10*x^5 + 35*d^3*e^9*x^4 + 35*d^4*e^8*x^3
 + 21*d^5*e^7*x^2 + 7*d^6*e^6*x + d^7*e^5)
```

Sympy [A] time = 3.76125, size = 184, normalized size = 1.61

$$-\frac{15a^2e^4+2acd^2e^2+c^2d^4+35c^2de^3x^3+35c^2e^4x^4+x^2\left(42ace^4+21c^2d^2e^2\right)+x\left(14acde^3+7c^2d^3e\right)}{105d^7e^5+735d^6e^6x+2205d^5e^7x^2+3675d^4e^8x^3+3675d^3e^9x^4+2205d^2e^{10}x^5+735de^{11}x^6+105e^{12}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2/(e*x+d)**8,x)
```

```
[Out] -(15*a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4 + 35*c**2*d*e**3*x**3 + 35*c**
2*e**4*x**4 + x**2*(42*a*c*e**4 + 21*c**2*d**2*e**2) + x*(14*a*c*d*e**3 + 7
*c**2*d**3*e))/(105*d**7*e**5 + 735*d**6*e**6*x + 2205*d**5*e**7*x**2 + 367
5*d**4*e**8*x**3 + 3675*d**3*e**9*x**4 + 2205*d**2*e**10*x**5 + 735*d*e**11
*x**6 + 105*e**12*x**7)
```

Giac [A] time = 1.34554, size = 132, normalized size = 1.16

$$-\frac{\left(35\,c^2x^4e^4 + 35\,c^2dx^3e^3 + 21\,c^2d^2x^2e^2 + 7\,c^2d^3xe + c^2d^4 + 42\,acx^2e^4 + 14\,acdxe^3 + 2\,acd^2e^2 + 15\,a^2e^4\right)e^{(-5)}}{105\,(xe+d)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^8,x, algorithm="giac")
```

```
[Out] -1/105*(35*c^2*x^4*e^4 + 35*c^2*d*x^3*e^3 + 21*c^2*d^2*x^2*e^2 + 7*c^2*d^3*
x*e + c^2*d^4 + 42*a*c*x^2*e^4 + 14*a*c*d*x*e^3 + 2*a*c*d^2*e^2 + 15*a^2*e^
4)*e^(-5)/(x*e + d)^7
```

3.472 $\int (d+ex)^6 \left(a+cx^2\right)^3 \, dx$

Optimal. Leaf size=190

$$\frac{3c^2(d+ex)^{11}\left(ae^2+5cd^2\right)}{11e^7}-\frac{2c^2d(d+ex)^{10}\left(3ae^2+5cd^2\right)}{5e^7}+\frac{c(d+ex)^9\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{3e^7}-\frac{3cd(d+ex)^8\left(ae^2+cd\right.}{4e^7}$$

```
[Out] ((c*d^2 + a*e^2)^3*(d + e*x)^7)/(7*e^7) - (3*c*d*(c*d^2 + a*e^2)^2*(d + e*x
)^8)/(4*e^7) + (c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2)*(d + e*x)^9)/(3*e^7) -
(2*c^2*d*(5*c*d^2 + 3*a*e^2)*(d + e*x)^10)/(5*e^7) + (3*c^2*(5*c*d^2 + a*e^
2)*(d + e*x)^11)/(11*e^7) - (c^3*d*(d + e*x)^12)/(2*e^7) + (c^3*(d + e*x)^1
3)/(13*e^7)
```

Rubi [A] time = 0.329189, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$\frac{3c^2(d+ex)^{11}\left(ae^2+5cd^2\right)}{11e^7}-\frac{2c^2d(d+ex)^{10}\left(3ae^2+5cd^2\right)}{5e^7}+\frac{c(d+ex)^9\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{3e^7}-\frac{3cd(d+ex)^8\left(ae^2+cd\right.}{4e^7}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^6*(a + c*x^2)^3,x]
```

```
[Out] ((c*d^2 + a*e^2)^3*(d + e*x)^7)/(7*e^7) - (3*c*d*(c*d^2 + a*e^2)^2*(d + e*x
)^8)/(4*e^7) + (c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2)*(d + e*x)^9)/(3*e^7) -
(2*c^2*d*(5*c*d^2 + 3*a*e^2)*(d + e*x)^10)/(5*e^7) + (3*c^2*(5*c*d^2 + a*e^
2)*(d + e*x)^11)/(11*e^7) - (c^3*d*(d + e*x)^12)/(2*e^7) + (c^3*(d + e*x)^1
3)/(13*e^7)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (d+ex)^6\left(a+cx^2\right)^3\,dx=\int\left(\frac{\left(cd^2+ae^2\right)^3(d+ex)^6}{e^6}-\frac{6cd\left(cd^2+ae^2\right)^2(d+ex)^7}{e^6}+\frac{3c\left(cd^2+ae^2\right)\left(5cd^2+ae^2\right)(d+ex)}{e^6}\right.$$

$$=\frac{\left(cd^2+ae^2\right)^3(d+ex)^7}{7e^7}-\frac{3cd\left(cd^2+ae^2\right)^2(d+ex)^8}{4e^7}+\frac{c\left(cd^2+ae^2\right)\left(5cd^2+ae^2\right)(d+ex)^9}{3e^7}-$$

Mathematica [A] time = 0.0481254, size = 338, normalized size = 1.78

$$\frac{1}{3}ce^2x^9\left(a^2e^4+15acd^2e^2+5c^2d^4\right)+\frac{3}{4}cdex^8\left(3a^2e^4+10acd^2e^2+c^2d^4\right)+\frac{1}{7}x^7\left(45a^2cd^2e^4+a^3e^6+45ac^2d^4e^2+c^3d^6\right)+ad$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^6*(a + c*x^2)^3,x]
```

```
[Out] a^3*d^6*x + 3*a^3*d^5*e*x^2 + a^2*d^4*(c*d^2 + 5*a*e^2)*x^3 + (a^2*d^3*e*(9
*c*d^2 + 10*a*e^2)*x^4)/2 + (3*a*d^2*(c^2*d^4 + 15*a*c*d^2*e^2 + 5*a^2*e^4)
*x^5)/5 + a*d*e*(3*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4)*x^6 + ((c^3*d^6 + 45
*a*c^2*d^4*e^2 + 45*a^2*c*d^2*e^4 + a^3*e^6)*x^7)/7 + (3*c*d*e*(c^2*d^4 + 1
0*a*c*d^2*e^2 + 3*a^2*e^4)*x^8)/4 + (c*e^2*(5*c^2*d^4 + 15*a*c*d^2*e^2 + a^
2*e^4)*x^9)/3 + (c^2*d*e^3*(10*c*d^2 + 9*a*e^2)*x^10)/5 + (3*c^2*e^4*(5*c*d
^2 + a*e^2)*x^11)/11 + (c^3*d*e^5*x^12)/2 + (c^3*e^6*x^13)/13
```

Maple [A] time = 0.043, size = 345, normalized size = 1.8

$$\frac{e^6c^3x^{13}}{13}+\frac{de^5c^3x^{12}}{2}+\frac{\left(3\,e^6ac^2+15\,d^2e^4c^3\right)x^{11}}{11}+\frac{\left(18\,de^5ac^2+20\,d^3e^3c^3\right)x^{10}}{10}+\frac{\left(3\,e^6a^2c+45\,d^2e^4ac^2+15\,d^4e^2c^3\right)x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^6*(c*x^2+a)^3,x)
```

```
[Out] 1/13*e^6*c^3*x^13+1/2*d*e^5*c^3*x^12+1/11*(3*a*c^2*e^6+15*c^3*d^2*e^4)*x^11
+1/10*(18*a*c^2*d*e^5+20*c^3*d^3*e^3)*x^10+1/9*(3*a^2*c*e^6+45*a*c^2*d^2*e^
4+15*c^3*d^4*e^2)*x^9+1/8*(18*a^2*c*d*e^5+60*a*c^2*d^3*e^3+6*c^3*d^5*e)*x^8
+1/7*(a^3*e^6+45*a^2*c*d^2*e^4+45*a*c^2*d^4*e^2+c^3*d^6)*x^7+1/6*(6*a^3*d*e
^5+60*a^2*c*d^3*e^3+18*a*c^2*d^5*e)*x^6+1/5*(15*a^3*d^2*e^4+45*a^2*c*d^4*e^
2+3*a*c^2*d^6)*x^5+1/4*(20*a^3*d^3*e^3+18*a^2*c*d^5*e)*x^4+1/3*(15*a^3*d^4*
e^2+3*a^2*c*d^6)*x^3+3*d^5*e*a^3*x^2+d^6*a^3*x
```

Maxima [A] time = 1.17997, size = 454, normalized size = 2.39

$$\frac{1}{13}\,c^3e^6x^{13}+\frac{1}{2}\,c^3de^5x^{12}+\frac{3}{11}\left(5\,c^3d^2e^4+ac^2e^6\right)x^{11}+3\,a^3d^5ex^2+\frac{1}{5}\left(10\,c^3d^3e^3+9\,ac^2de^5\right)x^{10}+a^3d^6x+\frac{1}{3}\left(5\,c^3d^4e^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/13*c^3*e^6*x^13 + 1/2*c^3*d*e^5*x^12 + 3/11*(5*c^3*d^2*e^4 + a*c^2*e^6)*x
^11 + 3*a^3*d^5*e*x^2 + 1/5*(10*c^3*d^3*e^3 + 9*a*c^2*d*e^5)*x^10 + a^3*d^6
*x + 1/3*(5*c^3*d^4*e^2 + 15*a*c^2*d^2*e^4 + a^2*c*e^6)*x^9 + 3/4*(c^3*d^5*
e + 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*x^8 + 1/7*(c^3*d^6 + 45*a*c^2*d^4*e^2
 + 45*a^2*c*d^2*e^4 + a^3*e^6)*x^7 + (3*a*c^2*d^5*e + 10*a^2*c*d^3*e^3 + a^
3*d*e^5)*x^6 + 3/5*(a*c^2*d^6 + 15*a^2*c*d^4*e^2 + 5*a^3*d^2*e^4)*x^5 + 1/2
*(9*a^2*c*d^5*e + 10*a^3*d^3*e^3)*x^4 + (a^2*c*d^6 + 5*a^3*d^4*e^2)*x^3
```

Fricas [B] time = 1.77748, size = 779, normalized size = 4.1

$$\frac{1}{13}x^{13}e^6c^3+\frac{1}{2}x^{12}e^5dc^3+\frac{15}{11}x^{11}e^4d^2c^3+\frac{3}{11}x^{11}e^6c^2a+2x^{10}e^3d^3c^3+\frac{9}{5}x^{10}e^5dc^2a+\frac{5}{3}x^9e^2d^4c^3+5x^9e^4d^2c^2a+\frac{1}{3}x^9e^6ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*e^6*c^3 + 1/2*x^12*e^5*d*c^3 + 15/11*x^11*e^4*d^2*c^3 + 3/11*x^11
*e^6*c^2*a + 2*x^10*e^3*d^3*c^3 + 9/5*x^10*e^5*d*c^2*a + 5/3*x^9*e^2*d^4*c^
```

```
3 + 5*x^9*e^4*d^2*c^2*a + 1/3*x^9*e^6*c*a^2 + 3/4*x^8*e*d^5*c^3 + 15/2*x^8*
e^3*d^3*c^2*a + 9/4*x^8*e^5*d*c*a^2 + 1/7*x^7*d^6*c^3 + 45/7*x^7*e^2*d^4*c^
2*a + 45/7*x^7*e^4*d^2*c*a^2 + 1/7*x^7*e^6*a^3 + 3*x^6*e*d^5*c^2*a + 10*x^6
*e^3*d^3*c*a^2 + x^6*e^5*d*a^3 + 3/5*x^5*d^6*c^2*a + 9*x^5*e^2*d^4*c*a^2 +
3*x^5*e^4*d^2*a^3 + 9/2*x^4*e*d^5*c*a^2 + 5*x^4*e^3*d^3*a^3 + x^3*d^6*c*a^2
 + 5*x^3*e^2*d^4*a^3 + 3*x^2*e*d^5*a^3 + x*d^6*a^3
```

Sympy [B] time = 0.113902, size = 371, normalized size = 1.95

$$a^3d^6x + 3a^3d^5ex^2 + \frac{c^3de^5x^{12}}{2} + \frac{c^3e^6x^{13}}{13} + x^{11}\left(\frac{3ac^2e^6}{11} + \frac{15c^3d^2e^4}{11}\right) + x^{10}\left(\frac{9ac^2de^5}{5} + 2c^3d^3e^3\right) + x^9\left(\frac{a^2ce^6}{3} + 5ac^2d^2e^4 + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**6*(c*x**2+a)**3,x)
```

```
[Out] a**3*d**6*x + 3*a**3*d**5*e*x**2 + c**3*d*e**5*x**12/2 + c**3*e**6*x**13/13
 + x**11*(3*a*c**2*e**6/11 + 15*c**3*d**2*e**4/11) + x**10*(9*a*c**2*d*e**5
/5 + 2*c**3*d**3*e**3) + x**9*(a**2*c*e**6/3 + 5*a*c**2*d**2*e**4 + 5*c**3*
d**4*e**2/3) + x**8*(9*a**2*c*d*e**5/4 + 15*a*c**2*d**3*e**3/2 + 3*c**3*d**
5*e/4) + x**7*(a**3*e**6/7 + 45*a**2*c*d**2*e**4/7 + 45*a*c**2*d**4*e**2/7
+ c**3*d**6/7) + x**6*(a**3*d*e**5 + 10*a**2*c*d**3*e**3 + 3*a*c**2*d**5*e)
 + x**5*(3*a**3*d**2*e**4 + 9*a**2*c*d**4*e**2 + 3*a*c**2*d**6/5) + x**4*(5
*a**3*d**3*e**3 + 9*a**2*c*d**5*e/2) + x**3*(5*a**3*d**4*e**2 + a**2*c*d**6
)
```

Giac [A] time = 1.34576, size = 467, normalized size = 2.46

$$\frac{1}{13}c^3x^{13}e^6 + \frac{1}{2}c^3dx^{12}e^5 + \frac{15}{11}c^3d^2x^{11}e^4 + 2c^3d^3x^{10}e^3 + \frac{5}{3}c^3d^4x^9e^2 + \frac{3}{4}c^3d^5x^8e + \frac{1}{7}c^3d^6x^7 + \frac{3}{11}ac^2x^{11}e^6 + \frac{9}{5}ac^2dx^{10}e^5 -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/13*c^3*x^13*e^6 + 1/2*c^3*d*x^12*e^5 + 15/11*c^3*d^2*x^11*e^4 + 2*c^3*d^3
*x^10*e^3 + 5/3*c^3*d^4*x^9*e^2 + 3/4*c^3*d^5*x^8*e + 1/7*c^3*d^6*x^7 + 3/1
1*a*c^2*x^11*e^6 + 9/5*a*c^2*d*x^10*e^5 + 5*a*c^2*d^2*x^9*e^4 + 15/2*a*c^2*
d^3*x^8*e^3 + 45/7*a*c^2*d^4*x^7*e^2 + 3*a*c^2*d^5*x^6*e + 3/5*a*c^2*d^6*x^
5 + 1/3*a^2*c*x^9*e^6 + 9/4*a^2*c*d*x^8*e^5 + 45/7*a^2*c*d^2*x^7*e^4 + 10*a
^2*c*d^3*x^6*e^3 + 9*a^2*c*d^4*x^5*e^2 + 9/2*a^2*c*d^5*x^4*e + a^2*c*d^6*x^
3 + 1/7*a^3*x^7*e^6 + a^3*d*x^6*e^5 + 3*a^3*d^2*x^5*e^4 + 5*a^3*d^3*x^4*e^3
 + 5*a^3*d^4*x^3*e^2 + 3*a^3*d^5*x^2*e + a^3*d^6*x
```

3.473 $\int (d+ex)^5 \left(a+cx^2\right)^3 \, dx$

Optimal. Leaf size=190

$$\frac{3c^2(d+ex)^{10}\left(ae^2+5cd^2\right)}{10e^7}-\frac{4c^2d(d+ex)^9\left(3ae^2+5cd^2\right)}{9e^7}+\frac{3c(d+ex)^8\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{8e^7}-\frac{6cd(d+ex)^7\left(ae^2+\right.}{7e^7}$$

```
[Out] ((c*d^2 + a*e^2)^3*(d + e*x)^6)/(6*e^7) - (6*c*d*(c*d^2 + a*e^2)^2*(d + e*x
)^7)/(7*e^7) + (3*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2)*(d + e*x)^8)/(8*e^7)
- (4*c^2*d*(5*c*d^2 + 3*a*e^2)*(d + e*x)^9)/(9*e^7) + (3*c^2*(5*c*d^2 + a*e
^2)*(d + e*x)^10)/(10*e^7) - (6*c^3*d*(d + e*x)^11)/(11*e^7) + (c^3*(d + e*
x)^12)/(12*e^7)
```

Rubi [A] time = 0.260904, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$\frac{3c^2(d+ex)^{10}\left(ae^2+5cd^2\right)}{10e^7}-\frac{4c^2d(d+ex)^9\left(3ae^2+5cd^2\right)}{9e^7}+\frac{3c(d+ex)^8\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{8e^7}-\frac{6cd(d+ex)^7\left(ae^2+\right.}{7e^7}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5*(a + c*x^2)^3,x]
```

```
[Out] ((c*d^2 + a*e^2)^3*(d + e*x)^6)/(6*e^7) - (6*c*d*(c*d^2 + a*e^2)^2*(d + e*x
)^7)/(7*e^7) + (3*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2)*(d + e*x)^8)/(8*e^7)
- (4*c^2*d*(5*c*d^2 + 3*a*e^2)*(d + e*x)^9)/(9*e^7) + (3*c^2*(5*c*d^2 + a*e
^2)*(d + e*x)^10)/(10*e^7) - (6*c^3*d*(d + e*x)^11)/(11*e^7) + (c^3*(d + e*
x)^12)/(12*e^7)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (d+ex)^5\left(a+cx^2\right)^3 dx=\int \left(\frac{\left(cd^2+ae^2\right)^3(d+ex)^5}{e^6}-\frac{6cd\left(cd^2+ae^2\right)^2(d+ex)^6}{e^6}+\frac{3c\left(cd^2+ae^2\right)\left(5cd^2+ae^2\right)(d+}{e^6}\right.$$

$$=\frac{\left(cd^2+ae^2\right)^3(d+ex)^6}{6e^7}-\frac{6cd\left(cd^2+ae^2\right)^2(d+ex)^7}{7e^7}+\frac{3c\left(cd^2+ae^2\right)\left(5cd^2+ae^2\right)(d+ex)^8}{8e^7}$$

Mathematica [A] time = 0.0546938, size = 252, normalized size = 1.33

$$a^2c\left(5d^2e^3x^6+6d^3e^2x^5+\frac{15}{4}d^4ex^4+d^5x^3+\frac{15}{7}de^4x^7+\frac{3e^5x^8}{8}\right)+\frac{1}{6}a^3x\left(20d^3e^2x^2+15d^2e^3x^3+15d^4ex+6d^5+6de^4x^4\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5*(a + c*x^2)^3,x]
```

```
[Out] (a^3*x*(6*d^5 + 15*d^4*e*x + 20*d^3*e^2*x^2 + 15*d^2*e^3*x^3 + 6*d*e^4*x^4
+ e^5*x^5))/6 + (a*c^2*x^5*(252*d^5 + 1050*d^4*e*x + 1800*d^3*e^2*x^2 + 157
5*d^2*e^3*x^3 + 700*d*e^4*x^4 + 126*e^5*x^5))/420 + (c^3*x^7*(792*d^5 + 346
5*d^4*e*x + 6160*d^3*e^2*x^2 + 5544*d^2*e^3*x^3 + 2520*d*e^4*x^4 + 462*e^5*
x^5))/5544 + a^2*c*(d^5*x^3 + (15*d^4*e*x^4)/4 + 6*d^3*e^2*x^5 + 5*d^2*e^3*
x^6 + (15*d*e^4*x^7)/7 + (3*e^5*x^8)/8)
```

Maple [A] time = 0.043, size = 293, normalized size = 1.5

$$\frac{e^5c^3x^{12}}{12}+\frac{5\,de^4c^3x^{11}}{11}+\frac{\left(3\,e^5ac^2+10\,d^2e^3c^3\right)x^{10}}{10}+\frac{\left(15\,de^4ac^2+10\,d^3e^2c^3\right)x^9}{9}+\frac{\left(3\,e^5a^2c+30\,d^2e^3ac^2+5\,d^4ec^3\right)x^8}{8}+\frac{(15$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5*(c*x^2+a)^3,x)
```

```
[Out] 1/12*e^5*c^3*x^12+5/11*d*e^4*c^3*x^11+1/10*(3*a*c^2*e^5+10*c^3*d^2*e^3)*x^1
0+1/9*(15*a*c^2*d*e^4+10*c^3*d^3*e^2)*x^9+1/8*(3*a^2*c*e^5+30*a*c^2*d^2*e^3
+5*c^3*d^4*e)*x^8+1/7*(15*a^2*c*d*e^4+30*a*c^2*d^3*e^2+c^3*d^5)*x^7+1/6*(a^
3*e^5+30*a^2*c*d^2*e^3+15*a*c^2*d^4*e)*x^6+1/5*(5*a^3*d*e^4+30*a^2*c*d^3*e^
2+3*a*c^2*d^5)*x^5+1/4*(10*a^3*d^2*e^3+15*a^2*c*d^4*e)*x^4+1/3*(10*a^3*d^3*
e^2+3*a^2*c*d^5)*x^3+5/2*d^4*e*a^3*x^2+d^5*a^3*x
```

Maxima [A] time = 1.38474, size = 394, normalized size = 2.07

$$\frac{1}{12}\,c^3e^5x^{12}+\frac{5}{11}\,c^3de^4x^{11}+\frac{1}{10}\left(10\,c^3d^2e^3+3\,ac^2e^5\right)x^{10}+\frac{5}{2}\,a^3d^4ex^2+\frac{5}{9}\left(2\,c^3d^3e^2+3\,ac^2de^4\right)x^9+a^3d^5x+\frac{1}{8}\left(5\,c^3d^4e+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/12*c^3*e^5*x^12 + 5/11*c^3*d*e^4*x^11 + 1/10*(10*c^3*d^2*e^3 + 3*a*c^2*e^
5)*x^10 + 5/2*a^3*d^4*e*x^2 + 5/9*(2*c^3*d^3*e^2 + 3*a*c^2*d*e^4)*x^9 + a^3
*d^5*x + 1/8*(5*c^3*d^4*e + 30*a*c^2*d^2*e^3 + 3*a^2*c*e^5)*x^8 + 1/7*(c^3*
d^5 + 30*a*c^2*d^3*e^2 + 15*a^2*c*d*e^4)*x^7 + 1/6*(15*a*c^2*d^4*e + 30*a^2
*c*d^2*e^3 + a^3*e^5)*x^6 + 1/5*(3*a*c^2*d^5 + 30*a^2*c*d^3*e^2 + 5*a^3*d*e
^4)*x^5 + 5/4*(3*a^2*c*d^4*e + 2*a^3*d^2*e^3)*x^4 + 1/3*(3*a^2*c*d^5 + 10*a
^3*d^3*e^2)*x^3
```

Fricas [A] time = 1.62817, size = 667, normalized size = 3.51

$$\frac{1}{12}x^{12}e^5c^3+\frac{5}{11}x^{11}e^4dc^3+x^{10}e^3d^2c^3+\frac{3}{10}x^{10}e^5c^2a+\frac{10}{9}x^9e^2d^3c^3+\frac{5}{3}x^9e^4dc^2a+\frac{5}{8}x^8ed^4c^3+\frac{15}{4}x^8e^3d^2c^2a+\frac{3}{8}x^8e^5ca^2+\frac{1}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*e^5*c^3 + 5/11*x^11*e^4*d*c^3 + x^10*e^3*d^2*c^3 + 3/10*x^10*e^5*
c^2*a + 10/9*x^9*e^2*d^3*c^3 + 5/3*x^9*e^4*d*c^2*a + 5/8*x^8*e*d^4*c^3 + 15
/4*x^8*e^3*d^2*c^2*a + 3/8*x^8*e^5*c*a^2 + 1/7*x^7*d^5*c^3 + 30/7*x^7*e^2*d
^3*c^2*a + 15/7*x^7*e^4*d*c*a^2 + 5/2*x^6*e*d^4*c^2*a + 5*x^6*e^3*d^2*c*a^2
```

```
 + 1/6*x^6*e^5*a^3 + 3/5*x^5*d^5*c^2*a + 6*x^5*e^2*d^3*c*a^2 + x^5*e^4*d*a^
3 + 15/4*x^4*e*d^4*c*a^2 + 5/2*x^4*e^3*d^2*a^3 + x^3*d^5*c*a^2 + 10/3*x^3*e
^2*d^3*a^3 + 5/2*x^2*e*d^4*a^3 + x*d^5*a^3
```

Sympy [A] time = 0.124749, size = 321, normalized size = 1.69

$$a^3d^5x + \frac{5a^3d^4ex^2}{2} + \frac{5c^3de^4x^{11}}{11} + \frac{c^3e^5x^{12}}{12} + x^{10}\left(\frac{3ac^2e^5}{10} + c^3d^2e^3\right) + x^9\left(\frac{5ac^2de^4}{3} + \frac{10c^3d^3e^2}{9}\right) + x^8\left(\frac{3a^2ce^5}{8} + \frac{15ac^2d^2}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5*(c*x**2+a)**3,x)
```

```
[Out] a**3*d**5*x + 5*a**3*d**4*e*x**2/2 + 5*c**3*d*e**4*x**11/11 + c**3*e**5*x**
12/12 + x**10*(3*a*c**2*e**5/10 + c**3*d**2*e**3) + x**9*(5*a*c**2*d*e**4/3
 + 10*c**3*d**3*e**2/9) + x**8*(3*a**2*c*e**5/8 + 15*a*c**2*d**2*e**3/4 + 5
*c**3*d**4*e/8) + x**7*(15*a**2*c*d*e**4/7 + 30*a*c**2*d**3*e**2/7 + c**3*d
**5/7) + x**6*(a**3*e**5/6 + 5*a**2*c*d**2*e**3 + 5*a*c**2*d**4*e/2) + x**5
*(a**3*d*e**4 + 6*a**2*c*d**3*e**2 + 3*a*c**2*d**5/5) + x**4*(5*a**3*d**2*e
**3/2 + 15*a**2*c*d**4*e/4) + x**3*(10*a**3*d**3*e**2/3 + a**2*c*d**5)
```

Giac [A] time = 1.33593, size = 393, normalized size = 2.07

$$\frac{1}{12}c^3x^{12}e^5 + \frac{5}{11}c^3dx^{11}e^4 + c^3d^2x^{10}e^3 + \frac{10}{9}c^3d^3x^9e^2 + \frac{5}{8}c^3d^4x^8e + \frac{1}{7}c^3d^5x^7 + \frac{3}{10}ac^2x^{10}e^5 + \frac{5}{3}ac^2dx^9e^4 + \frac{15}{4}ac^2d^2x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/12*c^3*x^12*e^5 + 5/11*c^3*d*x^11*e^4 + c^3*d^2*x^10*e^3 + 10/9*c^3*d^3*x
^9*e^2 + 5/8*c^3*d^4*x^8*e + 1/7*c^3*d^5*x^7 + 3/10*a*c^2*x^10*e^5 + 5/3*a*
c^2*d*x^9*e^4 + 15/4*a*c^2*d^2*x^8*e^3 + 30/7*a*c^2*d^3*x^7*e^2 + 5/2*a*c^2
*d^4*x^6*e + 3/5*a*c^2*d^5*x^5 + 3/8*a^2*c*x^8*e^5 + 15/7*a^2*c*d*x^7*e^4 +
 5*a^2*c*d^2*x^6*e^3 + 6*a^2*c*d^3*x^5*e^2 + 15/4*a^2*c*d^4*x^4*e + a^2*c*d
^5*x^3 + 1/6*a^3*x^6*e^5 + a^3*d*x^5*e^4 + 5/2*a^3*d^2*x^4*e^3 + 10/3*a^3*d
^3*x^3*e^2 + 5/2*a^3*d^4*x^2*e + a^3*d^5*x
```

3.474 $\int (d+ex)^4 \left(a+cx^2\right)^3 dx$

Optimal. Leaf size=188

$$\frac{c^2(d+ex)^9\left(ae^2+5cd^2\right)}{3e^7}-\frac{c^2d(d+ex)^8\left(3ae^2+5cd^2\right)}{2e^7}+\frac{3c(d+ex)^7\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{7e^7}-\frac{cd(d+ex)^6\left(ae^2+cd^2\right)^2}{e^7}$$

```
[Out] ((c*d^2 + a*e^2)^3*(d + e*x)^5)/(5*e^7) - (c*d*(c*d^2 + a*e^2)^2*(d + e*x)^
6)/e^7 + (3*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2)*(d + e*x)^7)/(7*e^7) - (c^2
*d*(5*c*d^2 + 3*a*e^2)*(d + e*x)^8)/(2*e^7) + (c^2*(5*c*d^2 + a*e^2)*(d + e
*x)^9)/(3*e^7) - (3*c^3*d*(d + e*x)^10)/(5*e^7) + (c^3*(d + e*x)^11)/(11*e^
7)
```

Rubi [A] time = 0.233264, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$\frac{c^2(d+ex)^9\left(ae^2+5cd^2\right)}{3e^7}-\frac{c^2d(d+ex)^8\left(3ae^2+5cd^2\right)}{2e^7}+\frac{3c(d+ex)^7\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{7e^7}-\frac{cd(d+ex)^6\left(ae^2+cd^2\right)^2}{e^7}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4*(a + c*x^2)^3,x]
```

```
[Out] ((c*d^2 + a*e^2)^3*(d + e*x)^5)/(5*e^7) - (c*d*(c*d^2 + a*e^2)^2*(d + e*x)^
6)/e^7 + (3*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2)*(d + e*x)^7)/(7*e^7) - (c^2
*d*(5*c*d^2 + 3*a*e^2)*(d + e*x)^8)/(2*e^7) + (c^2*(5*c*d^2 + a*e^2)*(d + e
*x)^9)/(3*e^7) - (3*c^3*d*(d + e*x)^10)/(5*e^7) + (c^3*(d + e*x)^11)/(11*e^
7)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (d+ex)^4\left(a+cx^2\right)^3 dx=\int\left(\frac{\left(cd^2+ae^2\right)^3(d+ex)^4}{e^6}-\frac{6cd\left(cd^2+ae^2\right)^2(d+ex)^5}{e^6}+\frac{3c\left(cd^2+ae^2\right)\left(5cd^2+ae^2\right)(d+ex)}{e^6}\right.$$

$$=\frac{\left(cd^2+ae^2\right)^3(d+ex)^5}{5e^7}-\frac{cd\left(cd^2+ae^2\right)^2(d+ex)^6}{e^7}+\frac{3c\left(cd^2+ae^2\right)\left(5cd^2+ae^2\right)(d+ex)^7}{7e^7}-$$

Mathematica [A] time = 0.0489864, size = 206, normalized size = 1.1

$$a^2c\left(\frac{18}{5}d^2e^2x^5+3d^3ex^4+d^4x^3+2de^3x^6+\frac{3e^4x^7}{7}\right)+a^3\left(2d^2e^2x^3+2d^3ex^2+d^4x+de^3x^4+\frac{e^4x^5}{5}\right)+\frac{1}{210}ac^2x^5\left(540d^2e^2x^2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4*(a + c*x^2)^3,x]
```

```
[Out] (a*c^2*x^5*(126*d^4 + 420*d^3*e*x + 540*d^2*e^2*x^2 + 315*d*e^3*x^3 + 70*e^
4*x^4))/210 + (c^3*x^7*(330*d^4 + 1155*d^3*e*x + 1540*d^2*e^2*x^2 + 924*d*e
^3*x^3 + 210*e^4*x^4))/2310 + a^3*(d^4*x + 2*d^3*e*x^2 + 2*d^2*e^2*x^3 + d*
e^3*x^4 + (e^4*x^5)/5) + a^2*c*(d^4*x^3 + 3*d^3*e*x^4 + (18*d^2*e^2*x^5)/5
+ 2*d*e^3*x^6 + (3*e^4*x^7)/7)
```

Maple [A] time = 0.041, size = 241, normalized size = 1.3

$$\frac{c^3e^4x^{11}}{11}+\frac{2\,de^3c^3x^{10}}{5}+\frac{\left(3\,e^4ac^2+6\,d^2e^2c^3\right)x^9}{9}+\frac{\left(12\,de^3ac^2+4\,d^3ec^3\right)x^8}{8}+\frac{\left(3\,e^4a^2c+18\,d^2e^2ac^2+d^4c^3\right)x^7}{7}+\frac{(12\,de^{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(c*x^2+a)^3,x)
```

```
[Out] 1/11*c^3*e^4*x^11+2/5*d*e^3*c^3*x^10+1/9*(3*a*c^2*e^4+6*c^3*d^2*e^2)*x^9+1/
8*(12*a*c^2*d*e^3+4*c^3*d^3*e)*x^8+1/7*(3*a^2*c*e^4+18*a*c^2*d^2*e^2+c^3*d^
4)*x^7+1/6*(12*a^2*c*d*e^3+12*a*c^2*d^3*e)*x^6+1/5*(a^3*e^4+18*a^2*c*d^2*e^
2+3*a*c^2*d^4)*x^5+1/4*(4*a^3*d*e^3+12*a^2*c*d^3*e)*x^4+1/3*(6*a^3*d^2*e^2+
3*a^2*c*d^4)*x^3+2*d^3*e*a^3*x^2+d^4*a^3*x
```

Maxima [A] time = 1.87454, size = 313, normalized size = 1.66

$$\frac{1}{11}\,c^3e^4x^{11}+\frac{2}{5}\,c^3de^3x^{10}+\frac{1}{3}\left(2\,c^3d^2e^2+ac^2e^4\right)x^9+2\,a^3d^3ex^2+\frac{1}{2}\left(c^3d^3e+3\,ac^2de^3\right)x^8+a^3d^4x+\frac{1}{7}\left(c^3d^4+18\,ac^2d^2e^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/11*c^3*e^4*x^11 + 2/5*c^3*d*e^3*x^10 + 1/3*(2*c^3*d^2*e^2 + a*c^2*e^4)*x^
9 + 2*a^3*d^3*e*x^2 + 1/2*(c^3*d^3*e + 3*a*c^2*d*e^3)*x^8 + a^3*d^4*x + 1/7
*(c^3*d^4 + 18*a*c^2*d^2*e^2 + 3*a^2*c*e^4)*x^7 + 2*(a*c^2*d^3*e + a^2*c*d*
e^3)*x^6 + 1/5*(3*a*c^2*d^4 + 18*a^2*c*d^2*e^2 + a^3*e^4)*x^5 + (3*a^2*c*d^
3*e + a^3*d*e^3)*x^4 + (a^2*c*d^4 + 2*a^3*d^2*e^2)*x^3
```

Fricas [A] time = 1.70956, size = 529, normalized size = 2.81

$$\frac{1}{11}x^{11}e^4c^3+\frac{2}{5}x^{10}e^3dc^3+\frac{2}{3}x^9e^2d^2c^3+\frac{1}{3}x^9e^4c^2a+\frac{1}{2}x^8ed^3c^3+\frac{3}{2}x^8e^3dc^2a+\frac{1}{7}x^7d^4c^3+\frac{18}{7}x^7e^2d^2c^2a+\frac{3}{7}x^7e^4ca^2+2x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*e^4*c^3 + 2/5*x^10*e^3*d*c^3 + 2/3*x^9*e^2*d^2*c^3 + 1/3*x^9*e^4*
c^2*a + 1/2*x^8*e*d^3*c^3 + 3/2*x^8*e^3*d*c^2*a + 1/7*x^7*d^4*c^3 + 18/7*x^
7*e^2*d^2*c^2*a + 3/7*x^7*e^4*c*a^2 + 2*x^6*e*d^3*c^2*a + 2*x^6*e^3*d*c*a^2
 + 3/5*x^5*d^4*c^2*a + 18/5*x^5*e^2*d^2*c*a^2 + 1/5*x^5*e^4*a^3 + 3*x^4*e*d
^3*c*a^2 + x^4*e^3*d*a^3 + x^3*d^4*c*a^2 + 2*x^3*e^2*d^2*a^3 + 2*x^2*e*d^3*
a^3 + x*d^4*a^3
```

Sympy [A] time = 0.111197, size = 255, normalized size = 1.36

$$a^3d^4x + 2a^3d^3ex^2 + \frac{2c^3de^3x^{10}}{5} + \frac{c^3e^4x^{11}}{11} + x^9\left(\frac{ac^2e^4}{3} + \frac{2c^3d^2e^2}{3}\right) + x^8\left(\frac{3ac^2de^3}{2} + \frac{c^3d^3e}{2}\right) + x^7\left(\frac{3a^2ce^4}{7} + \frac{18ac^2d^2e^2}{7} + \frac{c^3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(c*x**2+a)**3,x)
```

```
[Out] a**3*d**4*x + 2*a**3*d**3*e*x**2 + 2*c**3*d*e**3*x**10/5 + c**3*e**4*x**11/
11 + x**9*(a*c**2*e**4/3 + 2*c**3*d**2*e**2/3) + x**8*(3*a*c**2*d*e**3/2 +
c**3*d**3*e/2) + x**7*(3*a**2*c*e**4/7 + 18*a*c**2*d**2*e**2/7 + c**3*d**4/
7) + x**6*(2*a**2*c*d*e**3 + 2*a*c**2*d**3*e) + x**5*(a**3*e**4/5 + 18*a**2
*c*d**2*e**2/5 + 3*a*c**2*d**4/5) + x**4*(a**3*d*e**3 + 3*a**2*c*d**3*e) +
x**3*(2*a**3*d**2*e**2 + a**2*c*d**4)
```

Giac [A] time = 1.32047, size = 321, normalized size = 1.71

$$\frac{1}{11}c^3x^{11}e^4 + \frac{2}{5}c^3dx^{10}e^3 + \frac{2}{3}c^3d^2x^9e^2 + \frac{1}{2}c^3d^3x^8e + \frac{1}{7}c^3d^4x^7 + \frac{1}{3}ac^2x^9e^4 + \frac{3}{2}ac^2dx^8e^3 + \frac{18}{7}ac^2d^2x^7e^2 + 2\,ac^2d^3x^6e + \frac{3}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/11*c^3*x^11*e^4 + 2/5*c^3*d*x^10*e^3 + 2/3*c^3*d^2*x^9*e^2 + 1/2*c^3*d^3*
x^8*e + 1/7*c^3*d^4*x^7 + 1/3*a*c^2*x^9*e^4 + 3/2*a*c^2*d*x^8*e^3 + 18/7*a*
c^2*d^2*x^7*e^2 + 2*a*c^2*d^3*x^6*e + 3/5*a*c^2*d^4*x^5 + 3/7*a^2*c*x^7*e^4
 + 2*a^2*c*d*x^6*e^3 + 18/5*a^2*c*d^2*x^5*e^2 + 3*a^2*c*d^3*x^4*e + a^2*c*d
^4*x^3 + 1/5*a^3*x^5*e^4 + a^3*d*x^4*e^3 + 2*a^3*d^2*x^3*e^2 + 2*a^3*d^3*x^
2*e + a^3*d^4*x
```

3.475 $\int (d+ex)^3 \left(a+cx^2\right)^3 \, dx$

Optimal. Leaf size=161

$$a^2dx^3\left(ae^2+cd^2\right)+\frac{1}{2}a^2ce^3x^6+a^3d^3x+\frac{1}{4}a^3e^3x^4+\frac{1}{7}c^2dx^7\left(9ae^2+cd^2\right)+\frac{3}{8}ac^2e^3x^8+\frac{3}{5}acdx^5\left(3ae^2+cd^2\right)+\frac{3d^2e\left(a\right.}{8}$$

```
[Out] a^3*d^3*x + a^2*d*(c*d^2 + a*e^2)*x^3 + (a^3*e^3*x^4)/4 + (3*a*c*d*(c*d^2 +
 3*a*e^2)*x^5)/5 + (a^2*c*e^3*x^6)/2 + (c^2*d*(c*d^2 + 9*a*e^2)*x^7)/7 + (3
*a*c^2*e^3*x^8)/8 + (c^3*d*e^2*x^9)/3 + (c^3*e^3*x^10)/10 + (3*d^2*e*(a + c
*x^2)^4)/(8*c)
```

Rubi [A] time = 0.140378, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {696, 1810}

$$a^2dx^3\left(ae^2+cd^2\right)+\frac{1}{2}a^2ce^3x^6+a^3d^3x+\frac{1}{4}a^3e^3x^4+\frac{1}{7}c^2dx^7\left(9ae^2+cd^2\right)+\frac{3}{8}ac^2e^3x^8+\frac{3}{5}acdx^5\left(3ae^2+cd^2\right)+\frac{3d^2e\left(a\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + c*x^2)^3,x]
```

```
[Out] a^3*d^3*x + a^2*d*(c*d^2 + a*e^2)*x^3 + (a^3*e^3*x^4)/4 + (3*a*c*d*(c*d^2 +
 3*a*e^2)*x^5)/5 + (a^2*c*e^3*x^6)/2 + (c^2*d*(c*d^2 + 9*a*e^2)*x^7)/7 + (3
*a*c^2*e^3*x^8)/8 + (c^3*d*e^2*x^9)/3 + (c^3*e^3*x^10)/10 + (3*d^2*e*(a + c
*x^2)^4)/(8*c)
```

Rule 696

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*m*d^(m - 1)*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Int[((d + e*x)^m -
e*m*d^(m - 1)*x)*(a + c*x^2)^p, x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
 a*e^2, 0] && IGtQ[p, 1] && IGtQ[m, 0] && LeQ[m, p]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^3\left(a+cx^2\right)^3\,dx &= \frac{3d^2e\left(a+cx^2\right)^4}{8c}+\int \left(a+cx^2\right)^3\left(-3d^2ex+(d+ex)^3\right)dx\\ &= \frac{3d^2e\left(a+cx^2\right)^4}{8c}+\int \left(a^3d^3+3a^2d\left(cd^2+ae^2\right)x^2+a^3e^3x^3+3acd\left(cd^2+3ae^2\right)x^4+3a^2ce\right.\\ &= a^3d^3x+a^2d\left(cd^2+ae^2\right)x^3+\frac{1}{4}a^3e^3x^4+\frac{3}{5}acd\left(cd^2+3ae^2\right)x^5+\frac{1}{2}a^2ce^3x^6+\frac{1}{7}c^2d\left(cd^2+9\right.\end{aligned}$$

Mathematica [A] time = 0.0475745, size = 155, normalized size = 0.96

$$\frac{1}{840}x\left(42a^2cx^2\left(45d^2ex+20d^3+36de^2x^2+10e^3x^3\right)+210a^3\left(6d^2ex+4d^3+4de^2x^2+e^3x^3\right)+9ac^2x^4\left(140d^2ex+56d^3\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + c*x^2)^3,x]
```

```
[Out] (x*(210*a^3*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) + 42*a^2*c*x^2*(20*
d^3 + 45*d^2*e*x + 36*d*e^2*x^2 + 10*e^3*x^3) + 9*a*c^2*x^4*(56*d^3 + 140*d
^2*e*x + 120*d*e^2*x^2 + 35*e^3*x^3) + c^3*x^6*(120*d^3 + 315*d^2*e*x + 280
*d*e^2*x^2 + 84*e^3*x^3)))/840
```

Maple [A] time = 0.039, size = 189, normalized size = 1.2

$$\frac{c^3e^3x^{10}}{10}+\frac{c^3de^2x^9}{3}+\frac{\left(3\,e^3ac^2+3\,d^2ec^3\right)x^8}{8}+\frac{\left(9\,de^2ac^2+c^3d^3\right)x^7}{7}+\frac{\left(3\,a^2ce^3+9\,d^2eac^2\right)x^6}{6}+\frac{\left(9\,de^2a^2c+3\,d^3ac^2\right)x^5}{5}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+a)^3,x)
```

```
[Out] 1/10*c^3*e^3*x^10+1/3*c^3*d*e^2*x^9+1/8*(3*a*c^2*e^3+3*c^3*d^2*e)*x^8+1/7*(
9*a*c^2*d*e^2+c^3*d^3)*x^7+1/6*(3*a^2*c*e^3+9*a*c^2*d^2*e)*x^6+1/5*(9*a^2*c
*d*e^2+3*a*c^2*d^3)*x^5+1/4*(a^3*e^3+9*a^2*c*d^2*e)*x^4+1/3*(3*a^3*d*e^2+3*
a^2*c*d^3)*x^3+3/2*d^2*e*a^3*x^2+a^3*d^3*x
```

Maxima [A] time = 1.16501, size = 244, normalized size = 1.52

$$\frac{1}{10}\,c^3e^3x^{10}+\frac{1}{3}\,c^3de^2x^9+\frac{3}{8}\,\left(c^3d^2e+ac^2e^3\right)x^8+\frac{3}{2}\,a^3d^2ex^2+\frac{1}{7}\,\left(c^3d^3+9\,ac^2de^2\right)x^7+a^3d^3x+\frac{1}{2}\,\left(3\,ac^2d^2e+a^2ce^3\right)x^6+\frac{3}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/10*c^3*e^3*x^10 + 1/3*c^3*d*e^2*x^9 + 3/8*(c^3*d^2*e + a*c^2*e^3)*x^8 + 3
/2*a^3*d^2*e*x^2 + 1/7*(c^3*d^3 + 9*a*c^2*d*e^2)*x^7 + a^3*d^3*x + 1/2*(3*a
*c^2*d^2*e + a^2*c*e^3)*x^6 + 3/5*(a*c^2*d^3 + 3*a^2*c*d*e^2)*x^5 + 1/4*(9*
a^2*c*d^2*e + a^3*e^3)*x^4 + (a^2*c*d^3 + a^3*d*e^2)*x^3
```

Fricas [A] time = 1.70597, size = 414, normalized size = 2.57

$$\frac{1}{10}x^{10}e^3c^3+\frac{1}{3}x^9e^2dc^3+\frac{3}{8}x^8ed^2c^3+\frac{3}{8}x^8e^3c^2a+\frac{1}{7}x^7d^3c^3+\frac{9}{7}x^7e^2dc^2a+\frac{3}{2}x^6ed^2c^2a+\frac{1}{2}x^6e^3ca^2+\frac{3}{5}x^5d^3c^2a+\frac{9}{5}x^5e^2dca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*e^3*c^3 + 1/3*x^9*e^2*d*c^3 + 3/8*x^8*e*d^2*c^3 + 3/8*x^8*e^3*c^2
*a + 1/7*x^7*d^3*c^3 + 9/7*x^7*e^2*d*c^2*a + 3/2*x^6*e*d^2*c^2*a + 1/2*x^6*
e^3*c*a^2 + 3/5*x^5*d^3*c^2*a + 9/5*x^5*e^2*d*c*a^2 + 9/4*x^4*e*d^2*c*a^2 +
 1/4*x^4*e^3*a^3 + x^3*d^3*c*a^2 + x^3*e^2*d*a^3 + 3/2*x^2*e*d^2*a^3 + x*d^
3*a^3
```

Sympy [A] time = 0.122793, size = 202, normalized size = 1.25

$$a^3d^3x + \frac{3a^3d^2ex^2}{2} + \frac{c^3de^2x^9}{3} + \frac{c^3e^3x^{10}}{10} + x^8\left(\frac{3ac^2e^3}{8} + \frac{3c^3d^2e}{8}\right) + x^7\left(\frac{9ac^2de^2}{7} + \frac{c^3d^3}{7}\right) + x^6\left(\frac{a^2ce^3}{2} + \frac{3ac^2d^2e}{2}\right) + x^5\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a)**3,x)
```

```
[Out] a**3*d**3*x + 3*a**3*d**2*e*x**2/2 + c**3*d*e**2*x**9/3 + c**3*e**3*x**10/1
0 + x**8*(3*a*c**2*e**3/8 + 3*c**3*d**2*e/8) + x**7*(9*a*c**2*d*e**2/7 + c*
*3*d**3/7) + x**6*(a**2*c*e**3/2 + 3*a*c**2*d**2*e/2) + x**5*(9*a**2*c*d*e*
*2/5 + 3*a*c**2*d**3/5) + x**4*(a**3*e**3/4 + 9*a**2*c*d**2*e/4) + x**3*(a*
*3*d*e**2 + a**2*c*d**3)
```

Giac [A] time = 1.31344, size = 248, normalized size = 1.54

$$\frac{1}{10}c^3x^{10}e^3 + \frac{1}{3}c^3dx^9e^2 + \frac{3}{8}c^3d^2x^8e + \frac{1}{7}c^3d^3x^7 + \frac{3}{8}ac^2x^8e^3 + \frac{9}{7}ac^2dx^7e^2 + \frac{3}{2}ac^2d^2x^6e + \frac{3}{5}ac^2d^3x^5 + \frac{1}{2}a^2cx^6e^3 + \frac{9}{5}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/10*c^3*x^10*e^3 + 1/3*c^3*d*x^9*e^2 + 3/8*c^3*d^2*x^8*e + 1/7*c^3*d^3*x^7
 + 3/8*a*c^2*x^8*e^3 + 9/7*a*c^2*d*x^7*e^2 + 3/2*a*c^2*d^2*x^6*e + 3/5*a*c^
2*d^3*x^5 + 1/2*a^2*c*x^6*e^3 + 9/5*a^2*c*d*x^5*e^2 + 9/4*a^2*c*d^2*x^4*e +
 a^2*c*d^3*x^3 + 1/4*a^3*x^4*e^3 + a^3*d*x^3*e^2 + 3/2*a^3*d^2*x^2*e + a^3*
d^3*x
```

3.476 $\int (d+ex)^2 \left(a+cx^2\right)^3 \, dx$

Optimal. Leaf size=104

$$\frac{1}{3}a^2x^3\left(ae^2+3cd^2\right)+a^3d^2x+\frac{1}{7}c^2x^7\left(3ae^2+cd^2\right)+\frac{3}{5}acx^5\left(ae^2+cd^2\right)+\frac{de\left(a+cx^2\right)^4}{4c}+\frac{1}{9}c^3e^2x^9$$

```
[Out] a^3*d^2*x + (a^2*(3*c*d^2 + a*e^2)*x^3)/3 + (3*a*c*(c*d^2 + a*e^2)*x^5)/5 +
 (c^2*(c*d^2 + 3*a*e^2)*x^7)/7 + (c^3*e^2*x^9)/9 + (d*e*(a + c*x^2)^4)/(4*c
)
```

Rubi [A] time = 0.0630388, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {696, 1810}

$$\frac{1}{3}a^2x^3\left(ae^2+3cd^2\right)+a^3d^2x+\frac{1}{7}c^2x^7\left(3ae^2+cd^2\right)+\frac{3}{5}acx^5\left(ae^2+cd^2\right)+\frac{de\left(a+cx^2\right)^4}{4c}+\frac{1}{9}c^3e^2x^9$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + c*x^2)^3,x]
```

```
[Out] a^3*d^2*x + (a^2*(3*c*d^2 + a*e^2)*x^3)/3 + (3*a*c*(c*d^2 + a*e^2)*x^5)/5 +
 (c^2*(c*d^2 + 3*a*e^2)*x^7)/7 + (c^3*e^2*x^9)/9 + (d*e*(a + c*x^2)^4)/(4*c
)
```

Rule 696

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*m*d^(m - 1)*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Int[((d + e*x)^m -
e*m*d^(m - 1)*x)*(a + c*x^2)^p, x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
 a*e^2, 0] && IGtQ[p, 1] && IGtQ[m, 0] && LeQ[m, p]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^2\left(a+cx^2\right)^3\,dx &= \frac{de\left(a+cx^2\right)^4}{4c}+\int \left(a+cx^2\right)^3\left(-2dex+(d+ex)^2\right)\,dx\\ &= \frac{de\left(a+cx^2\right)^4}{4c}+\int \left(a^3d^2+a^2\left(3cd^2+ae^2\right)x^2+3ac\left(cd^2+ae^2\right)x^4+c^2\left(cd^2+3ae^2\right)x^6+c^3e^2\right.\\ &= a^3d^2x+\frac{1}{3}a^2\left(3cd^2+ae^2\right)x^3+\frac{3}{5}ac\left(cd^2+ae^2\right)x^5+\frac{1}{7}c^2\left(cd^2+3ae^2\right)x^7+\frac{1}{9}c^3e^2x^9+\frac{de\left(a+\right.}{4c}\end{aligned}$$

Mathematica [A] time = 0.0231168, size = 116, normalized size = 1.12

$$\frac{1}{10}a^2cx^3\left(10d^2+15dex+6e^2x^2\right)+a^3\left(d^2x+dex^2+\frac{e^2x^3}{3}\right)+\frac{1}{35}ac^2x^5\left(21d^2+35dex+15e^2x^2\right)+\frac{1}{252}c^3x^7\left(36d^2+63dex\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + c*x^2)^3,x]
```

```
[Out] (a^2*c*x^3*(10*d^2 + 15*d*e*x + 6*e^2*x^2))/10 + (a*c^2*x^5*(21*d^2 + 35*d*
e*x + 15*e^2*x^2))/35 + (c^3*x^7*(36*d^2 + 63*d*e*x + 28*e^2*x^2))/252 + a^
3*(d^2*x + d*e*x^2 + (e^2*x^3)/3)
```

Maple [A] time = 0.043, size = 129, normalized size = 1.2

$$\frac{c^3e^2x^9}{9}+\frac{dec^3x^8}{4}+\frac{(3\,e^2ac^2+d^2c^3)\,x^7}{7}+ac^2dex^6+\frac{(3\,e^2a^2c+3\,d^2ac^2)\,x^5}{5}+\frac{3\,dea^2cx^4}{2}+\frac{(e^2a^3+3\,a^2cd^2)\,x^3}{3}+dea^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a)^3,x)
```

```
[Out] 1/9*c^3*e^2*x^9+1/4*d*e*c^3*x^8+1/7*(3*a*c^2*e^2+c^3*d^2)*x^7+a*c^2*d*e*x^6
+1/5*(3*a^2*c*e^2+3*a*c^2*d^2)*x^5+3/2*d*e*a^2*c*x^4+1/3*(a^3*e^2+3*a^2*c*d
^2)*x^3+d*e*a^3*x^2+a^3*d^2*x
```

Maxima [A] time = 1.14436, size = 170, normalized size = 1.63

$$\frac{1}{9}\,c^3e^2x^9+\frac{1}{4}\,c^3dex^8+ac^2dex^6+\frac{3}{2}\,a^2cdex^4+\frac{1}{7}\left(c^3d^2+3\,ac^2e^2\right)x^7+a^3dex^2+a^3d^2x+\frac{3}{5}\left(ac^2d^2+a^2ce^2\right)x^5+\frac{1}{3}\left(3\,a^2c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/9*c^3*e^2*x^9 + 1/4*c^3*d*e*x^8 + a*c^2*d*e*x^6 + 3/2*a^2*c*d*e*x^4 + 1/7
*(c^3*d^2 + 3*a*c^2*e^2)*x^7 + a^3*d*e*x^2 + a^3*d^2*x + 3/5*(a*c^2*d^2 + a
^2*c*e^2)*x^5 + 1/3*(3*a^2*c*d^2 + a^3*e^2)*x^3
```

Fricas [A] time = 1.60838, size = 282, normalized size = 2.71

$$\frac{1}{9}x^9e^2c^3+\frac{1}{4}x^8edc^3+\frac{1}{7}x^7d^2c^3+\frac{3}{7}x^7e^2c^2a+x^6edc^2a+\frac{3}{5}x^5d^2c^2a+\frac{3}{5}x^5e^2ca^2+\frac{3}{2}x^4edca^2+x^3d^2ca^2+\frac{1}{3}x^3e^2a^3+x^2ed$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*e^2*c^3 + 1/4*x^8*e*d*c^3 + 1/7*x^7*d^2*c^3 + 3/7*x^7*e^2*c^2*a + x
^6*e*d*c^2*a + 3/5*x^5*d^2*c^2*a + 3/5*x^5*e^2*c*a^2 + 3/2*x^4*e*d*c*a^2 +
x^3*d^2*c*a^2 + 1/3*x^3*e^2*a^3 + x^2*e*d*a^3 + x*d^2*a^3
```

Sympy [A] time = 0.09911, size = 139, normalized size = 1.34

$$a^3d^2x+a^3dex^2+\frac{3a^2cdex^4}{2}+ac^2dex^6+\frac{c^3dex^8}{4}+\frac{c^3e^2x^9}{9}+x^7\left(\frac{3ac^2e^2}{7}+\frac{c^3d^2}{7}\right)+x^5\left(\frac{3a^2ce^2}{5}+\frac{3ac^2d^2}{5}\right)+x^3\left(\frac{a^3e^2}{3}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+a)**3,x)
```

```
[Out] a**3*d**2*x + a**3*d*e*x**2 + 3*a**2*c*d*e*x**4/2 + a*c**2*d*e*x**6 + c**3*
d*e*x**8/4 + c**3*e**2*x**9/9 + x**7*(3*a*c**2*e**2/7 + c**3*d**2/7) + x**5
*(3*a**2*c*e**2/5 + 3*a*c**2*d**2/5) + x**3*(a**3*e**2/3 + a**2*c*d**2)
```

Giac [A] time = 1.28823, size = 174, normalized size = 1.67

$$\frac{1}{9}c^3x^9e^2 + \frac{1}{4}c^3dx^8e + \frac{1}{7}c^3d^2x^7 + \frac{3}{7}ac^2x^7e^2 + ac^2dx^6e + \frac{3}{5}ac^2d^2x^5 + \frac{3}{5}a^2cx^5e^2 + \frac{3}{2}a^2cdx^4e + a^2cd^2x^3 + \frac{1}{3}a^3x^3e^2 + a^3d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/9*c^3*x^9*e^2 + 1/4*c^3*d*x^8*e + 1/7*c^3*d^2*x^7 + 3/7*a*c^2*x^7*e^2 + a
*c^2*d*x^6*e + 3/5*a*c^2*d^2*x^5 + 3/5*a^2*c*x^5*e^2 + 3/2*a^2*c*d*x^4*e +
a^2*c*d^2*x^3 + 1/3*a^3*x^3*e^2 + a^3*d*x^2*e + a^3*d^2*x
```

3.477 $\int (d+ex)\left(a+cx^2\right)^3\,dx$

Optimal. Leaf size=56

$$a^2cdx^3+a^3dx+\frac{3}{5}ac^2dx^5+\frac{e\left(a+cx^2\right)^4}{8c}+\frac{1}{7}c^3dx^7$$

```
[Out] a^3*d*x + a^2*c*d*x^3 + (3*a*c^2*d*x^5)/5 + (c^3*d*x^7)/7 + (e*(a + c*x^2)^
4)/(8*c)
```

Rubi [A] time = 0.021189, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {641, 194}

$$a^2cdx^3+a^3dx+\frac{3}{5}ac^2dx^5+\frac{e\left(a+cx^2\right)^4}{8c}+\frac{1}{7}c^3dx^7$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + c*x^2)^3,x]
```

```
[Out] a^3*d*x + a^2*c*d*x^3 + (3*a*c^2*d*x^5)/5 + (c^3*d*x^7)/7 + (e*(a + c*x^2)^
4)/(8*c)
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(a+cx^2\right)^3\,dx&=\frac{e\left(a+cx^2\right)^4}{8c}+d\int \left(a+cx^2\right)^3\,dx\\&=\frac{e\left(a+cx^2\right)^4}{8c}+d\int \left(a^3+3a^2cx^2+3ac^2x^4+c^3x^6\right)dx\\&=a^3dx+a^2cdx^3+\frac{3}{5}ac^2dx^5+\frac{1}{7}c^3dx^7+\frac{e\left(a+cx^2\right)^4}{8c}\end{aligned}$$

Mathematica [A] time = 0.0024985, size = 85, normalized size = 1.52

$$a^2cdx^3+\frac{3}{4}a^2cex^4+a^3dx+\frac{1}{2}a^3ex^2+\frac{3}{5}ac^2dx^5+\frac{1}{2}ac^2ex^6+\frac{1}{7}c^3dx^7+\frac{1}{8}c^3ex^8$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + c*x^2)^3,x]
```

```
[Out] a^3*d*x + (a^3*e*x^2)/2 + a^2*c*d*x^3 + (3*a^2*c*e*x^4)/4 + (3*a*c^2*d*x^5)
/5 + (a*c^2*e*x^6)/2 + (c^3*d*x^7)/7 + (c^3*e*x^8)/8
```

Maple [A] time = 0.042, size = 74, normalized size = 1.3

$$\frac{c^3ex^8}{8}+\frac{c^3dx^7}{7}+\frac{eac^2x^6}{2}+\frac{3\,ac^2dx^5}{5}+\frac{3\,ea^2cx^4}{4}+a^2cdx^3+\frac{a^3ex^2}{2}+a^3dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+a)^3,x)
```

```
[Out] 1/8*c^3*e*x^8+1/7*c^3*d*x^7+1/2*e*a*c^2*x^6+3/5*a*c^2*d*x^5+3/4*e*a^2*c*x^4
+a^2*c*d*x^3+1/2*a^3*e*x^2+a^3*d*x
```

Maxima [A] time = 1.17, size = 99, normalized size = 1.77

$$\frac{1}{8}\,c^3ex^8+\frac{1}{7}\,c^3dx^7+\frac{1}{2}\,ac^2ex^6+\frac{3}{5}\,ac^2dx^5+\frac{3}{4}\,a^2cex^4+a^2cdx^3+\frac{1}{2}\,a^3ex^2+a^3dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*c^3*e*x^8 + 1/7*c^3*d*x^7 + 1/2*a*c^2*e*x^6 + 3/5*a*c^2*d*x^5 + 3/4*a^2
*c*e*x^4 + a^2*c*d*x^3 + 1/2*a^3*e*x^2 + a^3*d*x
```

Fricas [A] time = 1.81113, size = 169, normalized size = 3.02

$$\frac{1}{8}x^8ec^3+\frac{1}{7}x^7dc^3+\frac{1}{2}x^6ec^2a+\frac{3}{5}x^5dc^2a+\frac{3}{4}x^4eca^2+x^3dca^2+\frac{1}{2}x^2ea^3+xda^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*e*c^3 + 1/7*x^7*d*c^3 + 1/2*x^6*e*c^2*a + 3/5*x^5*d*c^2*a + 3/4*x^4
*e*c*a^2 + x^3*d*c*a^2 + 1/2*x^2*e*a^3 + x*d*a^3
```

Sympy [A] time = 0.087168, size = 85, normalized size = 1.52

$$a^3dx+\frac{a^3ex^2}{2}+a^2cdx^3+\frac{3a^2cex^4}{4}+\frac{3ac^2dx^5}{5}+\frac{ac^2ex^6}{2}+\frac{c^3dx^7}{7}+\frac{c^3ex^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+a)**3,x)
```

```
[Out] a**3*d*x + a**3*e*x**2/2 + a**2*c*d*x**3 + 3*a**2*c*e*x**4/4 + 3*a*c**2*d*x
**5/5 + a*c**2*e*x**6/2 + c**3*d*x**7/7 + c**3*e*x**8/8
```

Giac [A] time = 1.31346, size = 104, normalized size = 1.86

$$\frac{1}{8}\,c^3x^8e+\frac{1}{7}\,c^3dx^7+\frac{1}{2}\,ac^2x^6e+\frac{3}{5}\,ac^2dx^5+\frac{3}{4}\,a^2cx^4e+a^2cdx^3+\frac{1}{2}\,a^3x^2e+a^3dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*c^3*x^8*e + 1/7*c^3*d*x^7 + 1/2*a*c^2*x^6*e + 3/5*a*c^2*d*x^5 + 3/4*a^2
*c*x^4*e + a^2*c*d*x^3 + 1/2*a^3*x^2*e + a^3*d*x
```

3.478 $\int \frac{(a+cx^2)^3}{d+ex}\,dx$

Optimal. Leaf size=173

$$\frac{cx^2\left(3a^2e^4+3acd^2e^2+c^2d^4\right)}{2e^5}-\frac{cdx\left(3a^2e^4+3acd^2e^2+c^2d^4\right)}{e^6}+\frac{c^2x^4\left(3ae^2+cd^2\right)}{4e^3}-\frac{c^2dx^3\left(3ae^2+cd^2\right)}{3e^4}+\frac{\left(ae^2+cd^2\right)^3\log}{e^7}$$

```
[Out] -((c*d*(c^2*d^4 + 3*a*c*d^2*e^2 + 3*a^2*e^4)*x)/e^6) + (c*(c^2*d^4 + 3*a*c*
d^2*e^2 + 3*a^2*e^4)*x^2)/(2*e^5) - (c^2*d*(c*d^2 + 3*a*e^2)*x^3)/(3*e^4) +
 (c^2*(c*d^2 + 3*a*e^2)*x^4)/(4*e^3) - (c^3*d*x^5)/(5*e^2) + (c^3*x^6)/(6*e
) + ((c*d^2 + a*e^2)^3*Log[d + e*x])/e^7
```

Rubi [A] time = 0.144567, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$\frac{cx^2\left(3a^2e^4+3acd^2e^2+c^2d^4\right)}{2e^5}-\frac{cdx\left(3a^2e^4+3acd^2e^2+c^2d^4\right)}{e^6}+\frac{c^2x^4\left(3ae^2+cd^2\right)}{4e^3}-\frac{c^2dx^3\left(3ae^2+cd^2\right)}{3e^4}+\frac{\left(ae^2+cd^2\right)^3\log}{e^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^3/(d + e*x),x]
```

```
[Out] -((c*d*(c^2*d^4 + 3*a*c*d^2*e^2 + 3*a^2*e^4)*x)/e^6) + (c*(c^2*d^4 + 3*a*c*
d^2*e^2 + 3*a^2*e^4)*x^2)/(2*e^5) - (c^2*d*(c*d^2 + 3*a*e^2)*x^3)/(3*e^4) +
 (c^2*(c*d^2 + 3*a*e^2)*x^4)/(4*e^3) - (c^3*d*x^5)/(5*e^2) + (c^3*x^6)/(6*e
) + ((c*d^2 + a*e^2)^3*Log[d + e*x])/e^7
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^3}{d+ex}\,dx=\int \left(-\frac{cd\left(c^2d^4+3acd^2e^2+3a^2e^4\right)}{e^6}+\frac{c\left(c^2d^4+3acd^2e^2+3a^2e^4\right)x}{e^5}-\frac{c^2d\left(cd^2+3ae^2\right)x^2}{e^4}+\frac{c^2\left(cd^2+3a\right.}{e^3}\right.$$
$$=-\frac{cd\left(c^2d^4+3acd^2e^2+3a^2e^4\right)x}{e^6}+\frac{c\left(c^2d^4+3acd^2e^2+3a^2e^4\right)x^2}{2e^5}-\frac{c^2d\left(cd^2+3ae^2\right)x^3}{3e^4}+\frac{c^2\left(cd^2+3ae\right.}{4e^3}$$

Mathematica [A] time = 0.0539716, size = 142, normalized size = 0.82

$$\frac{cex\left(90a^2e^4(ex-2d)+15ace^2\left(6d^2ex-12d^3-4de^2x^2+3e^3x^3\right)+c^2\left(-20d^3e^2x^2+15d^2e^3x^3+30d^4ex-60d^5-12de^4x^4+\right.\right.}{60e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^3/(d + e*x),x]
```

```
[Out] (c*e*x*(90*a^2*e^4*(-2*d + e*x) + 15*a*c*e^2*(-12*d^3 + 6*d^2*e*x - 4*d*e^2
*x^2 + 3*e^3*x^3) + c^2*(-60*d^5 + 30*d^4*e*x - 20*d^3*e^2*x^2 + 15*d^2*e^3
*x^3 - 12*d*e^4*x^4 + 10*e^5*x^5)) + 60*(c*d^2 + a*e^2)^3*Log[d + e*x])/(60
*e^7)
```

Maple [A] time = 0.046, size = 220, normalized size = 1.3

$$\frac{c^3x^6}{6e}-\frac{c^3dx^5}{5e^2}+\frac{3c^2x^4a}{4e}+\frac{c^3x^4d^2}{4e^3}-\frac{c^2x^3ad}{e^2}-\frac{x^3c^3d^3}{3e^4}+\frac{3cx^2a^2}{2e}+\frac{3c^2x^2ad^2}{2e^3}+\frac{c^3x^2d^4}{2e^5}-3\frac{cda^2x}{e^2}-3\frac{ac^2d^3x}{e^4}-\frac{c^3d^5x}{e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3/(e*x+d),x)
```

```
[Out] 1/6*c^3*x^6/e-1/5*c^3*d*x^5/e^2+3/4*c^2/e*x^4*a+1/4*c^3/e^3*x^4*d^2-c^2/e^2
*x^3*a*d-1/3*c^3/e^4*x^3*d^3+3/2*c/e*x^2*a^2+3/2*c^2/e^3*x^2*a*d^2+1/2*c^3/
e^5*x^2*d^4-3*c/e^2*a^2*d*x-3*c^2/e^4*a*d^3*x-c^3/e^6*d^5*x+1/e*ln(e*x+d)*a
^3+3/e^3*ln(e*x+d)*a^2*c*d^2+3/e^5*ln(e*x+d)*d^4*a*c^2+1/e^7*ln(e*x+d)*d^6*
c^3
```

Maxima [A] time = 1.14583, size = 267, normalized size = 1.54

$$\frac{10\,c^3e^5x^6-12\,c^3de^4x^5+15\left(c^3d^2e^3+3\,ac^2e^5\right)x^4-20\left(c^3d^3e^2+3\,ac^2de^4\right)x^3+30\left(c^3d^4e+3\,ac^2d^2e^3+3\,a^2ce^5\right)x^2-60\,(}{60\,e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/60*(10*c^3*e^5*x^6 - 12*c^3*d*e^4*x^5 + 15*(c^3*d^2*e^3 + 3*a*c^2*e^5)*x^
4 - 20*(c^3*d^3*e^2 + 3*a*c^2*d*e^4)*x^3 + 30*(c^3*d^4*e + 3*a*c^2*d^2*e^3
+ 3*a^2*c*e^5)*x^2 - 60*(c^3*d^5 + 3*a*c^2*d^3*e^2 + 3*a^2*c*d*e^4)*x)/e^6
+ (c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)*log(e*x + d)/e^7
```

Fricas [A] time = 2.06353, size = 410, normalized size = 2.37

$$\frac{10\,c^3e^6x^6-12\,c^3de^5x^5+15\left(c^3d^2e^4+3\,ac^2e^6\right)x^4-20\left(c^3d^3e^3+3\,ac^2de^5\right)x^3+30\left(c^3d^4e^2+3\,ac^2d^2e^4+3\,a^2ce^6\right)x^2-60}{60\,e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/60*(10*c^3*e^6*x^6 - 12*c^3*d*e^5*x^5 + 15*(c^3*d^2*e^4 + 3*a*c^2*e^6)*x^
4 - 20*(c^3*d^3*e^3 + 3*a*c^2*d*e^5)*x^3 + 30*(c^3*d^4*e^2 + 3*a*c^2*d^2*e^
4 + 3*a^2*c*e^6)*x^2 - 60*(c^3*d^5*e + 3*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*x +
 60*(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)*log(e*x + d))/e
^7
```

Sympy [A] time = 0.719874, size = 173, normalized size = 1.

$$-\frac{c^3dx^5}{5e^2}+\frac{c^3x^6}{6e}+\frac{x^4\left(3ac^2e^2+c^3d^2\right)}{4e^3}-\frac{x^3\left(3ac^2de^2+c^3d^3\right)}{3e^4}+\frac{x^2\left(3a^2ce^4+3ac^2d^2e^2+c^3d^4\right)}{2e^5}-\frac{x\left(3a^2cde^4+3ac^2d^3e^2+c\right.}{e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3/(e*x+d),x)
```

```
[Out] -c**3*d*x**5/(5*e**2) + c**3*x**6/(6*e) + x**4*(3*a*c**2*e**2 + c**3*d**2)/
(4*e**3) - x**3*(3*a*c**2*d*e**2 + c**3*d**3)/(3*e**4) + x**2*(3*a**2*c*e**
4 + 3*a*c**2*d**2*e**2 + c**3*d**4)/(2*e**5) - x*(3*a**2*c*d*e**4 + 3*a*c**
2*d**3*e**2 + c**3*d**5)/e**6 + (a*e**2 + c*d**2)**3*log(d + e*x)/e**7
```

Giac [A] time = 1.32853, size = 259, normalized size = 1.5

$$\left(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6\right)e^{(-7)}\log\left(|xe+d|\right)+\frac{1}{60}\left(10\,c^3x^6e^5-12\,c^3dx^5e^4+15\,c^3d^2x^4e^3-20\,c^3d^3x^3e^2+30\,c^3d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d),x, algorithm="giac")
```

```
[Out] (c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)*e^(-7)*log(abs(x*e
+ d)) + 1/60*(10*c^3*x^6*e^5 - 12*c^3*d*x^5*e^4 + 15*c^3*d^2*x^4*e^3 - 20*c
^3*d^3*x^3*e^2 + 30*c^3*d^4*x^2*e - 60*c^3*d^5*x + 45*a*c^2*x^4*e^5 - 60*a*
c^2*d*x^3*e^4 + 90*a*c^2*d^2*x^2*e^3 - 180*a*c^2*d^3*x*e^2 + 90*a^2*c*x^2*e
^5 - 180*a^2*c*d*x*e^4)*e^(-6)
```

3.479 $\int \frac{(a+cx^2)^3}{(d+ex)^2} dx$

Optimal. Leaf size=158

$$\frac{cx(3a^2e^4+9acd^2e^2+5c^2d^4)}{e^6}+\frac{c^2x^3(ae^2+cd^2)}{e^4}-\frac{c^2dx^2(3ae^2+2cd^2)}{e^5}-\frac{(ae^2+cd^2)^3}{e^7(d+ex)}-\frac{6cd(ae^2+cd^2)^2\log(d+ex)}{e^7}$$

```
[Out] (c*(5*c^2*d^4 + 9*a*c*d^2*e^2 + 3*a^2*e^4)*x)/e^6 - (c^2*d*(2*c*d^2 + 3*a*e
^2)*x^2)/e^5 + (c^2*(c*d^2 + a*e^2)*x^3)/e^4 - (c^3*d*x^4)/(2*e^3) + (c^3*x
^5)/(5*e^2) - (c*d^2 + a*e^2)^3/(e^7*(d + e*x)) - (6*c*d*(c*d^2 + a*e^2)^2*
Log[d + e*x])/e^7
```

Rubi [A] time = 0.151524, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$\frac{cx(3a^2e^4+9acd^2e^2+5c^2d^4)}{e^6}+\frac{c^2x^3(ae^2+cd^2)}{e^4}-\frac{c^2dx^2(3ae^2+2cd^2)}{e^5}-\frac{(ae^2+cd^2)^3}{e^7(d+ex)}-\frac{6cd(ae^2+cd^2)^2\log(d+ex)}{e^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^3/(d + e*x)^2,x]
```


```
[Out] (c*(5*c^2*d^4 + 9*a*c*d^2*e^2 + 3*a^2*e^4)*x)/e^6 - (c^2*d*(2*c*d^2 + 3*a*e
^2)*x^2)/e^5 + (c^2*(c*d^2 + a*e^2)*x^3)/e^4 - (c^3*d*x^4)/(2*e^3) + (c^3*x
^5)/(5*e^2) - (c*d^2 + a*e^2)^3/(e^7*(d + e*x)) - (6*c*d*(c*d^2 + a*e^2)^2*
Log[d + e*x])/e^7
```


Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+cx^2)^3}{(d+ex)^2} dx = \int \left(\frac{c(5c^2d^4+9acd^2e^2+3a^2e^4)}{e^6}-\frac{2c^2d(2cd^2+3ae^2)x}{e^5}+\frac{3c^2(cd^2+ae^2)x^2}{e^4}-\frac{2c^3dx^3}{e^3}+\frac{c^3x^4}{e^2}+\ldots\right)$$

$$=\frac{c(5c^2d^4+9acd^2e^2+3a^2e^4)x}{e^6}-\frac{c^2d(2cd^2+3ae^2)x^2}{e^5}+\frac{c^2(cd^2+ae^2)x^3}{e^4}-\frac{c^3dx^4}{2e^3}+\frac{c^3x^5}{5e^2}-\frac{(cd^2+\ldots}{e^7(d\ldots}$$

Mathematica [A] time = 0.0583553, size = 193, normalized size = 1.22

$$\frac{30a^2ce^4(-d^2+dex+e^2x^2)-10a^3e^6+10ac^2e^2(6d^2e^2x^2+9d^3ex-3d^4-2de^3x^3+e^4x^4)-60cd(d+ex)(ae^2+cd^2)^2\,\mathrm{lo}\ldots}{10e^7(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^3/(d + e*x)^2,x]
```

```
[Out] (-10*a^3*e^6 + 30*a^2*c*e^4*(-d^2 + d*e*x + e^2*x^2) + 10*a*c^2*e^2*(-3*d^4
 + 9*d^3*e*x + 6*d^2*e^2*x^2 - 2*d*e^3*x^3 + e^4*x^4) + c^3*(-10*d^6 + 50*d
^5*e*x + 30*d^4*e^2*x^2 - 10*d^3*e^3*x^3 + 5*d^2*e^4*x^4 - 3*d*e^5*x^5 + 2*
e^6*x^6) - 60*c*d*(c*d^2 + a*e^2)^2*(d + e*x)*Log[d + e*x])/(10*e^7*(d + e*
x))
```

Maple [A] time = 0.05, size = 233, normalized size = 1.5

$$\frac{c^3x^5}{5e^2} - \frac{c^3dx^4}{2e^3} + \frac{c^2x^3a}{e^2} + \frac{x^3c^3d^2}{e^4} - 3\frac{c^2x^2ad}{e^3} - 2\frac{c^3x^2d^3}{e^5} + 3\frac{a^2cx}{e^2} + 9\frac{ac^2d^2x}{e^4} + 5\frac{c^3d^4x}{e^6} - 6\frac{cd\ln(ex+d)a^2}{e^3} - 12\frac{c^2d^3\ln(}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3/(e*x+d)^2,x)
```

```
[Out] 1/5*c^3*x^5/e^2-1/2*c^3*d*x^4/e^3+c^2/e^2*x^3*a+c^3/e^4*x^3*d^2-3*c^2/e^3*x
^2*a*d-2*c^3/e^5*x^2*d^3+3*c/e^2*a^2*x+9*c^2/e^4*a*d^2*x+5*c^3/e^6*d^4*x-6*
c*d/e^3*ln(e*x+d)*a^2-12*c^2*d^3/e^5*ln(e*x+d)*a-6*c^3*d^5/e^7*ln(e*x+d)-1/
e/(e*x+d)*a^3-3/e^3/(e*x+d)*a^2*c*d^2-3/e^5/(e*x+d)*d^4*a*c^2-1/e^7/(e*x+d)
*d^6*c^3
```

Maxima [A] time = 1.15595, size = 278, normalized size = 1.76

$$-\frac{c^3d^6 + 3ac^2d^4e^2 + 3a^2cd^2e^4 + a^3e^6}{e^8x + de^7} + \frac{2c^3e^4x^5 - 5c^3de^3x^4 + 10\left(c^3d^2e^2 + ac^2e^4\right)x^3 - 10\left(2c^3d^3e + 3ac^2de^3\right)x^2 + 10\left(5c^3\right.}{10e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)/(e^8*x + d*e^7) +
1/10*(2*c^3*e^4*x^5 - 5*c^3*d*e^3*x^4 + 10*(c^3*d^2*e^2 + a*c^2*e^4)*x^3 -
10*(2*c^3*d^3*e + 3*a*c^2*d*e^3)*x^2 + 10*(5*c^3*d^4 + 9*a*c^2*d^2*e^2 + 3*
a^2*c*e^4)*x)/e^6 - 6*(c^3*d^5 + 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*log(e*x + d
)/e^7
```

Fricas [A] time = 2.18339, size = 558, normalized size = 3.53

$$\frac{2c^3e^6x^6 - 3c^3de^5x^5 - 10c^3d^6 - 30ac^2d^4e^2 - 30a^2cd^2e^4 - 10a^3e^6 + 5\left(c^3d^2e^4 + 2ac^2e^6\right)x^4 - 10\left(c^3d^3e^3 + 2ac^2de^5\right)x^3 + 3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/10*(2*c^3*e^6*x^6 - 3*c^3*d*e^5*x^5 - 10*c^3*d^6 - 30*a*c^2*d^4*e^2 - 30*
a^2*c*d^2*e^4 - 10*a^3*e^6 + 5*(c^3*d^2*e^4 + 2*a*c^2*e^6)*x^4 - 10*(c^3*d^
3*e^3 + 2*a*c^2*d*e^5)*x^3 + 30*(c^3*d^4*e^2 + 2*a*c^2*d^2*e^4 + a^2*c*e^6)
*x^2 + 10*(5*c^3*d^5*e + 9*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*x - 60*(c^3*d^6 +
 2*a*c^2*d^4*e^2 + a^2*c*d^2*e^4 + (c^3*d^5*e + 2*a*c^2*d^3*e^3 + a^2*c*d*e
^5)*x)*log(e*x + d))/(e^8*x + d*e^7)
```

Sympy [A] time = 1.0412, size = 189, normalized size = 1.2

$$-\frac{c^3dx^4}{2e^3}+\frac{c^3x^5}{5e^2}-\frac{6cd\left(ae^2+cd^2\right)^2\log\left(d+ex\right)}{e^7}-\frac{a^3e^6+3a^2cd^2e^4+3ac^2d^4e^2+c^3d^6}{de^7+e^8x}+\frac{x^3\left(ac^2e^2+c^3d^2\right)}{e^4}-\frac{x^2\left(3ac^2de^2\right.}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3/(e*x+d)**2,x)
```

```
[Out] -c**3*d*x**4/(2*e**3) + c**3*x**5/(5*e**2) - 6*c*d*(a*e**2 + c*d**2)**2*log
(d + e*x)/e**7 - (a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**
3*d**6)/(d*e**7 + e**8*x) + x**3*(a*c**2*e**2 + c**3*d**2)/e**4 - x**2*(3*a
*c**2*d*e**2 + 2*c**3*d**3)/e**5 + x*(3*a**2*c*e**4 + 9*a*c**2*d**2*e**2 +
5*c**3*d**4)/e**6
```

Giac [A] time = 1.2953, size = 351, normalized size = 2.22

$$\frac{1}{10}\left(2\,c^3-\frac{15\,c^3d}{xe+d}+\frac{10\left(5\,c^3d^2e^2+ac^2e^4\right)e^{(-2)}}{(xe+d)^2}-\frac{20\left(5\,c^3d^3e^3+3\,ac^2de^5\right)e^{(-3)}}{(xe+d)^3}+\frac{30\left(5\,c^3d^4e^4+6\,ac^2d^2e^6+a^2ce^8\right)e^{(-4)}}{(xe+d)^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/10*(2*c^3 - 15*c^3*d/(x*e + d) + 10*(5*c^3*d^2*e^2 + a*c^2*e^4)*e^(-2)/(x
*e + d)^2 - 20*(5*c^3*d^3*e^3 + 3*a*c^2*d*e^5)*e^(-3)/(x*e + d)^3 + 30*(5*c
^3*d^4*e^4 + 6*a*c^2*d^2*e^6 + a^2*c*e^8)*e^(-4)/(x*e + d)^4)*(x*e + d)^5*e
^(-7) + 6*(c^3*d^5 + 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*e^(-7)*log(abs(x*e + d)
*e^(-1)/(x*e + d)^2) - (c^3*d^6*e^5/(x*e + d) + 3*a*c^2*d^4*e^7/(x*e + d) +
 3*a^2*c*d^2*e^9/(x*e + d) + a^3*e^11/(x*e + d))*e^(-12)
```

3.480 $\int \frac{(a+cx^2)^3}{(d+ex)^3} dx$

Optimal. Leaf size=163

$$\frac{3c^2x^2(ae^2+2cd^2)}{2e^5}-\frac{c^2dx(9ae^2+10cd^2)}{e^6}+\frac{6cd(ae^2+cd^2)^2}{e^7(d+ex)}-\frac{(ae^2+cd^2)^3}{2e^7(d+ex)^2}+\frac{3c(ae^2+cd^2)(ae^2+5cd^2)\log(d+ex)}{e^7}-$$

```
[Out] -((c^2*d*(10*c*d^2 + 9*a*e^2)*x)/e^6) + (3*c^2*(2*c*d^2 + a*e^2)*x^2)/(2*e^
5) - (c^3*d*x^3)/e^4 + (c^3*x^4)/(4*e^3) - (c*d^2 + a*e^2)^3/(2*e^7*(d + e*
x)^2) + (6*c*d*(c*d^2 + a*e^2)^2)/(e^7*(d + e*x)) + (3*c*(c*d^2 + a*e^2)*(5
*c*d^2 + a*e^2)*Log[d + e*x])/e^7
```

Rubi [A] time = 0.167547, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$\frac{3c^2x^2(ae^2+2cd^2)}{2e^5}-\frac{c^2dx(9ae^2+10cd^2)}{e^6}+\frac{6cd(ae^2+cd^2)^2}{e^7(d+ex)}-\frac{(ae^2+cd^2)^3}{2e^7(d+ex)^2}+\frac{3c(ae^2+cd^2)(ae^2+5cd^2)\log(d+ex)}{e^7}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^3/(d + e*x)^3,x]
```

```
[Out] -((c^2*d*(10*c*d^2 + 9*a*e^2)*x)/e^6) + (3*c^2*(2*c*d^2 + a*e^2)*x^2)/(2*e^
5) - (c^3*d*x^3)/e^4 + (c^3*x^4)/(4*e^3) - (c*d^2 + a*e^2)^3/(2*e^7*(d + e*
x)^2) + (6*c*d*(c*d^2 + a*e^2)^2)/(e^7*(d + e*x)) + (3*c*(c*d^2 + a*e^2)*(5
*c*d^2 + a*e^2)*Log[d + e*x])/e^7
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+cx^2)^3}{(d+ex)^3}dx=\int\left(-\frac{c^2d(10cd^2+9ae^2)}{e^6}+\frac{3c^2(2cd^2+ae^2)x}{e^5}-\frac{3c^3dx^2}{e^4}+\frac{c^3x^3}{e^3}+\frac{(cd^2+ae^2)^3}{e^6(d+ex)^3}-\frac{6cd(cd^2+ae^2)^2}{e^6(d+ex)^2}+\right.$$

$$=-\frac{c^2d(10cd^2+9ae^2)x}{e^6}+\frac{3c^2(2cd^2+ae^2)x^2}{2e^5}-\frac{c^3dx^3}{e^4}+\frac{c^3x^4}{4e^3}-\frac{(cd^2+ae^2)^3}{2e^7(d+ex)^2}+\frac{6cd(cd^2+ae^2)^2}{e^7(d+ex)}+\frac{3c}{}$$

Mathematica [A] time = 0.0690721, size = 198, normalized size = 1.21

$$\frac{12c(d+ex)^2(a^2e^4+6acd^2e^2+5c^2d^4)\log(d+ex)+6a^2cde^4(3d+4ex)-2a^3e^6+6ac^2e^2(-11d^2e^2x^2+2d^3ex+7d^4-4de^3}{4e^7(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^3/(d + e*x)^3,x]
```

```
[Out] (-2*a^3*e^6 + 6*a^2*c*d*e^4*(3*d + 4*e*x) + 6*a*c^2*e^2*(7*d^4 + 2*d^3*e*x
- 11*d^2*e^2*x^2 - 4*d*e^3*x^3 + e^4*x^4) + c^3*(22*d^6 - 16*d^5*e*x - 68*d
^4*e^2*x^2 - 20*d^3*e^3*x^3 + 5*d^2*e^4*x^4 - 2*d*e^5*x^5 + e^6*x^6) + 12*c
*(5*c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*(d + e*x)^2*Log[d + e*x])/(4*e^7*(d
+ e*x)^2)
```

Maple [A] time = 0.052, size = 249, normalized size = 1.5

$$\frac{c^3x^4}{4\,e^3}-\frac{c^3dx^3}{e^4}+\frac{3\,c^2x^2a}{2\,e^3}+3\,\frac{c^3x^2d^2}{e^5}-9\,\frac{ac^2dx}{e^4}-10\,\frac{c^3d^3x}{e^6}+3\,\frac{c\ln(ex+d)\,a^2}{e^3}+18\,\frac{c^2\ln(ex+d)\,ad^2}{e^5}+15\,\frac{c^3\ln(ex+d}{e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3/(e*x+d)^3,x)
```

```
[Out] 1/4*c^3*x^4/e^3-c^3*d*x^3/e^4+3/2*c^2/e^3*x^2*a+3*c^3/e^5*x^2*d^2-9*c^2/e^4
*a*d*x-10*c^3/e^6*d^3*x+3*c/e^3*ln(e*x+d)*a^2+18*c^2/e^5*ln(e*x+d)*a*d^2+15
*c^3/e^7*ln(e*x+d)*d^4+6*c*d/e^3/(e*x+d)*a^2+12*c^2*d^3/e^5/(e*x+d)*a+6*c^3
*d^5/e^7/(e*x+d)-1/2/e/(e*x+d)^2*a^3-3/2/e^3/(e*x+d)^2*d^2*a^2*c-3/2/e^5/(e
*x+d)^2*d^4*a*c^2-1/2/e^7/(e*x+d)^2*d^6*c^3
```

Maxima [A] time = 1.49776, size = 289, normalized size = 1.77

$$\frac{11\,c^3d^6+21\,ac^2d^4e^2+9\,a^2cd^2e^4-a^3e^6+12\left(c^3d^5e+2\,ac^2d^3e^3+a^2cde^5\right)x}{2\left(e^9x^2+2\,de^8x+d^2e^7\right)}+\frac{c^3e^3x^4-4\,c^3de^2x^3+6\left(2\,c^3d^2e+ac^2e^3\right)x}{4\,e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(11*c^3*d^6 + 21*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - a^3*e^6 + 12*(c^3*d^
5*e + 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x)/(e^9*x^2 + 2*d*e^8*x + d^2*e^7) + 1
/4*(c^3*e^3*x^4 - 4*c^3*d*e^2*x^3 + 6*(2*c^3*d^2*e + a*c^2*e^3)*x^2 - 4*(10
*c^3*d^3 + 9*a*c^2*d*e^2)*x)/e^6 + 3*(5*c^3*d^4 + 6*a*c^2*d^2*e^2 + a^2*c*e
^4)*log(e*x + d)/e^7
```

Fricas [B] time = 2.17337, size = 640, normalized size = 3.93

$$c^3e^6x^6-2\,c^3de^5x^5+22\,c^3d^6+42\,ac^2d^4e^2+18\,a^2cd^2e^4-2\,a^3e^6+\left(5\,c^3d^2e^4+6\,ac^2e^6\right)x^4-4\left(5\,c^3d^3e^3+6\,ac^2de^5\right)x^3-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(c^3*e^6*x^6 - 2*c^3*d*e^5*x^5 + 22*c^3*d^6 + 42*a*c^2*d^4*e^2 + 18*a^2
*c*d^2*e^4 - 2*a^3*e^6 + (5*c^3*d^2*e^4 + 6*a*c^2*e^6)*x^4 - 4*(5*c^3*d^3*e
^3 + 6*a*c^2*d*e^5)*x^3 - 2*(34*c^3*d^4*e^2 + 33*a*c^2*d^2*e^4)*x^2 - 4*(4*
c^3*d^5*e - 3*a*c^2*d^3*e^3 - 6*a^2*c*d*e^5)*x + 12*(5*c^3*d^6 + 6*a*c^2*d^
4*e^2 + a^2*c*d^2*e^4 + (5*c^3*d^4*e^2 + 6*a*c^2*d^2*e^4 + a^2*c*e^6)*x^2 +
 2*(5*c^3*d^5*e + 6*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x)*log(e*x + d))/(e^9*x^2
```

```
+ 2*d*e^8*x + d^2*e^7)
```

Sympy [A] time = 1.92448, size = 216, normalized size = 1.33

$$-\frac{c^3dx^3}{e^4}+\frac{c^3x^4}{4e^3}+\frac{3c\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)\log\left(d+ex\right)}{e^7}+\frac{-a^3e^6+9a^2cd^2e^4+21ac^2d^4e^2+11c^3d^6+x\left(12a^2cde^5+24ac\right.}{2d^2e^7+4de^8x+2e^9x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3/(e*x+d)**3,x)
```

```
[Out] -c**3*d*x**3/e**4 + c**3*x**4/(4*e**3) + 3*c*(a*e**2 + c*d**2)*(a*e**2 + 5*
c*d**2)*log(d + e*x)/e**7 + (-a**3*e**6 + 9*a**2*c*d**2*e**4 + 21*a*c**2*d*
*4*e**2 + 11*c**3*d**6 + x*(12*a**2*c*d*e**5 + 24*a*c**2*d**3*e**3 + 12*c**
3*d**5*e))/(2*d**2*e**7 + 4*d*e**8*x + 2*e**9*x**2) + x**2*(3*a*c**2*e**2 +
 6*c**3*d**2)/(2*e**5) - x*(9*a*c**2*d*e**2 + 10*c**3*d**3)/e**6
```

Giac [A] time = 1.22611, size = 259, normalized size = 1.59

$$3\left(5\,c^3d^4+6\,ac^2d^2e^2+a^2ce^4\right)e^{(-7)}\log\left(|xe+d|\right)+\frac{1}{4}\left(c^3x^4e^9-4\,c^3dx^3e^8+12\,c^3d^2x^2e^7-40\,c^3d^3xe^6+6\,ac^2x^2e^9-36\,ac^2dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 3*(5*c^3*d^4 + 6*a*c^2*d^2*e^2 + a^2*c*e^4)*e^(-7)*log(abs(x*e + d)) + 1/4*
(c^3*x^4*e^9 - 4*c^3*d*x^3*e^8 + 12*c^3*d^2*x^2*e^7 - 40*c^3*d^3*x*e^6 + 6*
a*c^2*x^2*e^9 - 36*a*c^2*d*x*e^8)*e^(-12) + 1/2*(11*c^3*d^6 + 21*a*c^2*d^4*
e^2 + 9*a^2*c*d^2*e^4 - a^3*e^6 + 12*(c^3*d^5*e + 2*a*c^2*d^3*e^3 + a^2*c*d
*e^5)*x)*e^(-7)/(x*e + d)^2
```

3.481 $\int \frac{(a+cx^2)^3}{(d+ex)^4}\,dx$

Optimal. Leaf size=165

$$\frac{c^2x(3ae^2+10cd^2)}{e^6}-\frac{4c^2d(3ae^2+5cd^2)\log(d+ex)}{e^7}-\frac{3c(ae^2+cd^2)(ae^2+5cd^2)}{e^7(d+ex)}+\frac{3cd(ae^2+cd^2)^2}{e^7(d+ex)^2}-\frac{(ae^2+cd^2)^3}{3e^7(d+ex)^3}$$

```
[Out] (c^2*(10*c*d^2 + 3*a*e^2)*x)/e^6 - (2*c^3*d*x^2)/e^5 + (c^3*x^3)/(3*e^4) -
(c*d^2 + a*e^2)^3/(3*e^7*(d + e*x)^3) + (3*c*d*(c*d^2 + a*e^2)^2)/(e^7*(d +
 e*x)^2) - (3*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2))/(e^7*(d + e*x)) - (4*c^2
*d*(5*c*d^2 + 3*a*e^2)*Log[d + e*x])/e^7
```

Rubi [A] time = 0.164905, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$\frac{c^2x(3ae^2+10cd^2)}{e^6}-\frac{4c^2d(3ae^2+5cd^2)\log(d+ex)}{e^7}-\frac{3c(ae^2+cd^2)(ae^2+5cd^2)}{e^7(d+ex)}+\frac{3cd(ae^2+cd^2)^2}{e^7(d+ex)^2}-\frac{(ae^2+cd^2)^3}{3e^7(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^3/(d + e*x)^4,x]
```

```
[Out] (c^2*(10*c*d^2 + 3*a*e^2)*x)/e^6 - (2*c^3*d*x^2)/e^5 + (c^3*x^3)/(3*e^4) -
(c*d^2 + a*e^2)^3/(3*e^7*(d + e*x)^3) + (3*c*d*(c*d^2 + a*e^2)^2)/(e^7*(d +
 e*x)^2) - (3*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2))/(e^7*(d + e*x)) - (4*c^2
*d*(5*c*d^2 + 3*a*e^2)*Log[d + e*x])/e^7
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int\frac{(a+cx^2)^3}{(d+ex)^4}\,dx=\int\left(\frac{c^2(10cd^2+3ae^2)}{e^6}-\frac{4c^3dx}{e^5}+\frac{c^3x^2}{e^4}+\frac{(cd^2+ae^2)^3}{e^6(d+ex)^4}-\frac{6cd(cd^2+ae^2)^2}{e^6(d+ex)^3}+\frac{3c(cd^2+ae^2)(5cd^2+\right.$$

$$=\frac{c^2(10cd^2+3ae^2)x}{e^6}-\frac{2c^3dx^2}{e^5}+\frac{c^3x^3}{3e^4}-\frac{(cd^2+ae^2)^3}{3e^7(d+ex)^3}+\frac{3cd(cd^2+ae^2)^2}{e^7(d+ex)^2}-\frac{3c(cd^2+ae^2)(5cd^2+a}{e^7(d+ex)}$$

Mathematica [A] time = 0.0657386, size = 197, normalized size = 1.19

$$\frac{-3a^2ce^4(d^2+3dex+3e^2x^2)-a^3e^6+3ac^2e^2(-9d^2e^2x^2-27d^3ex-13d^4+9de^3x^3+3e^4x^4)-12c^2d(d+ex)^3(3ae^2+5}{3e^7(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^3/(d + e*x)^4,x]
```

```
[Out] (-(a^3*e^6) - 3*a^2*c*e^4*(d^2 + 3*d*e*x + 3*e^2*x^2) + 3*a*c^2*e^2*(-13*d^
4 - 27*d^3*e*x - 9*d^2*e^2*x^2 + 9*d*e^3*x^3 + 3*e^4*x^4) + c^3*(-37*d^6 -
51*d^5*e*x + 39*d^4*e^2*x^2 + 73*d^3*e^3*x^3 + 15*d^2*e^4*x^4 - 3*d*e^5*x^5
 + e^6*x^6) - 12*c^2*d*(5*c*d^2 + 3*a*e^2)*(d + e*x)^3*Log[d + e*x])/(3*e^7
*(d + e*x)^3)
```

Maple [A] time = 0.056, size = 258, normalized size = 1.6

$$\frac{x^3c^3}{3\,e^4}-2\,\frac{c^3dx^2}{e^5}+3\,\frac{ac^2x}{e^4}+10\,\frac{xc^3d^2}{e^6}-\frac{a^3}{3\,e\,(ex+d)^3}-\frac{a^2cd^2}{e^3\,(ex+d)^3}-\frac{d^4ac^2}{e^5\,(ex+d)^3}-\frac{d^6c^3}{3\,e^7\,(ex+d)^3}-12\,\frac{c^2d\ln(ex+d)\,a}{e^5}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3/(e*x+d)^4,x)
```

```
[Out] 1/3*c^3*x^3/e^4-2*c^3*d*x^2/e^5+3*c^2/e^4*a*x+10*c^3/e^6*d^2*x-1/3/e/(e*x+d
)^3*a^3-1/e^3/(e*x+d)^3*d^2*a^2*c-1/e^5/(e*x+d)^3*d^4*a*c^2-1/3/e^7/(e*x+d)
^3*d^6*c^3-12*c^2*d/e^5*ln(e*x+d)*a-20*c^3*d^3/e^7*ln(e*x+d)-3*c/e^3/(e*x+d
)*a^2-18*c^2/e^5/(e*x+d)*a*d^2-15*c^3/e^7/(e*x+d)*d^4+3*c*d/e^3/(e*x+d)^2*a
^2+6*c^2*d^3/e^5/(e*x+d)^2*a+3*c^3*d^5/e^7/(e*x+d)^2
```

Maxima [A] time = 1.102, size = 305, normalized size = 1.85

$$-\frac{37\,c^3d^6+39\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6+9\left(5\,c^3d^4e^2+6\,ac^2d^2e^4+a^2ce^6\right)x^2+9\left(9\,c^3d^5e+10\,ac^2d^3e^3+a^2cde^5\right)x}{3\left(e^{10}x^3+3\,de^9x^2+3\,d^2e^8x+d^3e^7\right)}+\frac{c^3e^{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(37*c^3*d^6 + 39*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6 + 9*(5*c^3*
d^4*e^2 + 6*a*c^2*d^2*e^4 + a^2*c*e^6)*x^2 + 9*(9*c^3*d^5*e + 10*a*c^2*d^3*
e^3 + a^2*c*d*e^5)*x)/(e^10*x^3 + 3*d*e^9*x^2 + 3*d^2*e^8*x + d^3*e^7) + 1/
3*(c^3*e^2*x^3 - 6*c^3*d*e*x^2 + 3*(10*c^3*d^2 + 3*a*c^2*e^2)*x)/e^6 - 4*(5
*c^3*d^3 + 3*a*c^2*d*e^2)*log(e*x + d)/e^7
```

Fricas [B] time = 2.10702, size = 679, normalized size = 4.12

$$\frac{c^3e^6x^6-3\,c^3de^5x^5-37\,c^3d^6-39\,ac^2d^4e^2-3\,a^2cd^2e^4-a^3e^6+3\left(5\,c^3d^2e^4+3\,ac^2e^6\right)x^4+\left(73\,c^3d^3e^3+27\,ac^2de^5\right)x^3+3\,(1\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(c^3*e^6*x^6 - 3*c^3*d*e^5*x^5 - 37*c^3*d^6 - 39*a*c^2*d^4*e^2 - 3*a^2*
c*d^2*e^4 - a^3*e^6 + 3*(5*c^3*d^2*e^4 + 3*a*c^2*e^6)*x^4 + (73*c^3*d^3*e^3
 + 27*a*c^2*d*e^5)*x^3 + 3*(13*c^3*d^4*e^2 - 9*a*c^2*d^2*e^4 - 3*a^2*c*e^6)
*x^2 - 3*(17*c^3*d^5*e + 27*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*x - 12*(5*c^3*d^
6 + 3*a*c^2*d^4*e^2 + (5*c^3*d^3*e^3 + 3*a*c^2*d*e^5)*x^3 + 3*(5*c^3*d^4*e^
2 + 3*a*c^2*d^2*e^4)*x^2 + 3*(5*c^3*d^5*e + 3*a*c^2*d^3*e^3)*x)*log(e*x + d
```

```
))/(e^10*x^3 + 3*d*e^9*x^2 + 3*d^2*e^8*x + d^3*e^7)
```

Sympy [A] time = 2.88631, size = 235, normalized size = 1.42

$$-\frac{2c^3dx^2}{e^5}+\frac{c^3x^3}{3e^4}-\frac{4c^2d\left(3ae^2+5cd^2\right)\log\left(d+ex\right)}{e^7}-\frac{a^3e^6+3a^2cd^2e^4+39ac^2d^4e^2+37c^3d^6+x^2\left(9a^2ce^6+54ac^2d^2e^4\right.}{3d^3e^7+9d^2e^8x+9de^9x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3/(e*x+d)**4,x)
```

```
[Out] -2*c**3*d*x**2/e**5 + c**3*x**3/(3*e**4) - 4*c**2*d*(3*a*e**2 + 5*c*d**2)*l
og(d + e*x)/e**7 - (a**3*e**6 + 3*a**2*c*d**2*e**4 + 39*a*c**2*d**4*e**2 +
37*c**3*d**6 + x**2*(9*a**2*c*e**6 + 54*a*c**2*d**2*e**4 + 45*c**3*d**4*e**
2) + x*(9*a**2*c*d*e**5 + 90*a*c**2*d**3*e**3 + 81*c**3*d**5*e))/(3*d**3*e*
*7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + x*(3*a*c**2*e**2 + 10*
c**3*d**2)/e**6
```

Giac [A] time = 1.30372, size = 259, normalized size = 1.57

$$-4\left(5\,c^3d^3+3\,ac^2de^2\right)e^{(-7)}\log\left(|xe+d|\right)+\frac{1}{3}\left(c^3x^3e^8-6\,c^3dx^2e^7+30\,c^3d^2xe^6+9\,ac^2xe^8\right)e^{(-12)}-\frac{\left(37\,c^3d^6+39\,ac^2d^4e^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -4*(5*c^3*d^3 + 3*a*c^2*d*e^2)*e^(-7)*log(abs(x*e + d)) + 1/3*(c^3*x^3*e^8
- 6*c^3*d*x^2*e^7 + 30*c^3*d^2*x*e^6 + 9*a*c^2*x*e^8)*e^(-12) - 1/3*(37*c^3
*d^6 + 39*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6 + 9*(5*c^3*d^4*e^2 + 6*
a*c^2*d^2*e^4 + a^2*c*e^6)*x^2 + 9*(9*c^3*d^5*e + 10*a*c^2*d^3*e^3 + a^2*c*
d*e^5)*x)*e^(-7)/(x*e + d)^3
```

3.482 $\int \frac{(a+cx^2)^3}{(d+ex)^5} dx$

Optimal. Leaf size=171

$$\frac{4c^2d(3ae^2+5cd^2)}{e^7(d+ex)}+\frac{3c^2(ae^2+5cd^2)\log(d+ex)}{e^7}-\frac{3c(ae^2+cd^2)(ae^2+5cd^2)}{2e^7(d+ex)^2}+\frac{2cd(ae^2+cd^2)^2}{e^7(d+ex)^3}-\frac{(ae^2+cd^2)^3}{4e^7(d+ex)^4}-\frac{5c^3}{e}$$

```
[Out] (-5*c^3*d*x)/e^6 + (c^3*x^2)/(2*e^5) - (c*d^2 + a*e^2)^3/(4*e^7*(d + e*x)^4
) + (2*c*d*(c*d^2 + a*e^2)^2)/(e^7*(d + e*x)^3) - (3*c*(c*d^2 + a*e^2)*(5*c
*d^2 + a*e^2))/(2*e^7*(d + e*x)^2) + (4*c^2*d*(5*c*d^2 + 3*a*e^2))/(e^7*(d
+ e*x)) + (3*c^2*(5*c*d^2 + a*e^2)*Log[d + e*x])/e^7
```

Rubi [A] time = 0.157142, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$\frac{4c^2d(3ae^2+5cd^2)}{e^7(d+ex)}+\frac{3c^2(ae^2+5cd^2)\log(d+ex)}{e^7}-\frac{3c(ae^2+cd^2)(ae^2+5cd^2)}{2e^7(d+ex)^2}+\frac{2cd(ae^2+cd^2)^2}{e^7(d+ex)^3}-\frac{(ae^2+cd^2)^3}{4e^7(d+ex)^4}-\frac{5c^3}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^3/(d + e*x)^5,x]
```


```
[Out] (-5*c^3*d*x)/e^6 + (c^3*x^2)/(2*e^5) - (c*d^2 + a*e^2)^3/(4*e^7*(d + e*x)^4
) + (2*c*d*(c*d^2 + a*e^2)^2)/(e^7*(d + e*x)^3) - (3*c*(c*d^2 + a*e^2)*(5*c
*d^2 + a*e^2))/(2*e^7*(d + e*x)^2) + (4*c^2*d*(5*c*d^2 + 3*a*e^2))/(e^7*(d
+ e*x)) + (3*c^2*(5*c*d^2 + a*e^2)*Log[d + e*x])/e^7
```


Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+cx^2)^3}{(d+ex)^5}dx = \int \left(-\frac{5c^3d}{e^6}+\frac{c^3x}{e^5}+\frac{(cd^2+ae^2)^3}{e^6(d+ex)^5}-\frac{6cd(cd^2+ae^2)^2}{e^6(d+ex)^4}+\frac{3c(cd^2+ae^2)(5cd^2+ae^2)}{e^6(d+ex)^3}-\frac{4c^2d(5cd^2+3ae}{e^6(d+ex)^2}\right.$$

$$=-\frac{5c^3dx}{e^6}+\frac{c^3x^2}{2e^5}-\frac{(cd^2+ae^2)^3}{4e^7(d+ex)^4}+\frac{2cd(cd^2+ae^2)^2}{e^7(d+ex)^3}-\frac{3c(cd^2+ae^2)(5cd^2+ae^2)}{2e^7(d+ex)^2}+\frac{4c^2d(5cd^2+3ae^2)}{e^7(d+ex)}$$

Mathematica [A] time = 0.0647442, size = 185, normalized size = 1.08

$$\frac{-a^2ce^4(d^2+4dex+6e^2x^2)-a^3e^6+ac^2de^2(88d^2ex+25d^3+108de^2x^2+48e^3x^3)+12c^2(d+ex)^4(ae^2+5cd^2)\log(d+ex}{4e^7(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^3/(d + e*x)^5,x]
```

```
[Out] (-(a^3*e^6) - a^2*c*e^4*(d^2 + 4*d*e*x + 6*e^2*x^2) + a*c^2*d*e^2*(25*d^3 +
 88*d^2*e*x + 108*d*e^2*x^2 + 48*e^3*x^3) + c^3*(57*d^6 + 168*d^5*e*x + 132
*d^4*e^2*x^2 - 32*d^3*e^3*x^3 - 68*d^2*e^4*x^4 - 12*d*e^5*x^5 + 2*e^6*x^6)
+ 12*c^2*(5*c*d^2 + a*e^2)*(d + e*x)^4*Log[d + e*x])/(4*e^7*(d + e*x)^4)
```

Maple [A] time = 0.053, size = 268, normalized size = 1.6

$$\frac{c^3x^2}{2e^5}-5\frac{c^3dx}{e^6}-\frac{a^3}{4e(ex+d)^4}-\frac{3a^2cd^2}{4e^3(ex+d)^4}-\frac{3d^4ac^2}{4e^5(ex+d)^4}-\frac{d^6c^3}{4e^7(ex+d)^4}+2\frac{cda^2}{e^3(ex+d)^3}+4\frac{c^2d^3a}{e^5(ex+d)^3}+2\frac{}{e^7(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3/(e*x+d)^5,x)
```

```
[Out] 1/2*c^3*x^2/e^5-5*c^3*d*x/e^6-1/4/e/(e*x+d)^4*a^3-3/4/e^3/(e*x+d)^4*d^2*a^2
*c-3/4/e^5/(e*x+d)^4*d^4*a*c^2-1/4/e^7/(e*x+d)^4*d^6*c^3+2*c*d/e^3/(e*x+d)^
3*a^2+4*c^2*d^3/e^5/(e*x+d)^3*a+2*c^3*d^5/e^7/(e*x+d)^3+3*c^2/e^5*ln(e*x+d)
*a+15*c^3/e^7*ln(e*x+d)*d^2+12*c^2*d/e^5/(e*x+d)*a+20*c^3*d^3/e^7/(e*x+d)-3
/2*c/e^3/(e*x+d)^2*a^2-9*c^2/e^5/(e*x+d)^2*a*d^2-15/2*c^3/e^7/(e*x+d)^2*d^4
```

Maxima [A] time = 1.20839, size = 323, normalized size = 1.89

$$\frac{57c^3d^6+25ac^2d^4e^2-a^2cd^2e^4-a^3e^6+16\left(5c^3d^3e^3+3ac^2de^5\right)x^3+6\left(35c^3d^4e^2+18ac^2d^2e^4-a^2ce^6\right)x^2+4\left(47c^3d^5\right.}{4\left(e^{11}x^4+4de^{10}x^3+6d^2e^9x^2+4d^3e^8x+d^4e^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^5,x, algorithm="maxima")
```

```
[Out] 1/4*(57*c^3*d^6 + 25*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - a^3*e^6 + 16*(5*c^3*d^
3*e^3 + 3*a*c^2*d*e^5)*x^3 + 6*(35*c^3*d^4*e^2 + 18*a*c^2*d^2*e^4 - a^2*c*e
^6)*x^2 + 4*(47*c^3*d^5*e + 22*a*c^2*d^3*e^3 - a^2*c*d*e^5)*x)/(e^11*x^4 +
4*d*e^10*x^3 + 6*d^2*e^9*x^2 + 4*d^3*e^8*x + d^4*e^7) + 1/2*(c^3*e*x^2 - 10
*c^3*d*x)/e^6 + 3*(5*c^3*d^2 + a*c^2*e^2)*log(e*x + d)/e^7
```

Fricas [B] time = 2.08485, size = 716, normalized size = 4.19

$$\frac{2c^3e^6x^6-12c^3de^5x^5-68c^3d^2e^4x^4+57c^3d^6+25ac^2d^4e^2-a^2cd^2e^4-a^3e^6-16\left(2c^3d^3e^3-3ac^2de^5\right)x^3+6\left(22c^3d^4e^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^5,x, algorithm="fricas")
```

```
[Out] 1/4*(2*c^3*e^6*x^6 - 12*c^3*d*e^5*x^5 - 68*c^3*d^2*e^4*x^4 + 57*c^3*d^6 + 2
5*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - a^3*e^6 - 16*(2*c^3*d^3*e^3 - 3*a*c^2*d*e
^5)*x^3 + 6*(22*c^3*d^4*e^2 + 18*a*c^2*d^2*e^4 - a^2*c*e^6)*x^2 + 4*(42*c^3
*d^5*e + 22*a*c^2*d^3*e^3 - a^2*c*d*e^5)*x + 12*(5*c^3*d^6 + a*c^2*d^4*e^2
+ (5*c^3*d^2*e^4 + a*c^2*e^6)*x^4 + 4*(5*c^3*d^3*e^3 + a*c^2*d*e^5)*x^3 + 6
*(5*c^3*d^4*e^2 + a*c^2*d^2*e^4)*x^2 + 4*(5*c^3*d^5*e + a*c^2*d^3*e^3)*x)*l
og(e*x + d))/(e^11*x^4 + 4*d*e^10*x^3 + 6*d^2*e^9*x^2 + 4*d^3*e^8*x + d^4*e
```

```
^7)
```

Sympy [A] time = 4.59792, size = 243, normalized size = 1.42

$$-\frac{5c^3dx}{e^6}+\frac{c^3x^2}{2e^5}+\frac{3c^2\left(ae^2+5cd^2\right)\log\left(d+ex\right)}{e^7}+\frac{-a^3e^6-a^2cd^2e^4+25ac^2d^4e^2+57c^3d^6+x^3\left(48ac^2de^5+80c^3d^3e^3\right)+x^2}{4d^4e^7+16d^3e^8x+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3/(e*x+d)**5,x)
```

```
[Out] -5*c**3*d*x/e**6 + c**3*x**2/(2*e**5) + 3*c**2*(a*e**2 + 5*c*d**2)*log(d +
e*x)/e**7 + (-a**3*e**6 - a**2*c*d**2*e**4 + 25*a*c**2*d**4*e**2 + 57*c**3*
d**6 + x**3*(48*a*c**2*d*e**5 + 80*c**3*d**3*e**3) + x**2*(-6*a**2*c*e**6 +
 108*a*c**2*d**2*e**4 + 210*c**3*d**4*e**2) + x*(-4*a**2*c*d*e**5 + 88*a*c*
*2*d**3*e**3 + 188*c**3*d**5*e))/(4*d**4*e**7 + 16*d**3*e**8*x + 24*d**2*e*
*9*x**2 + 16*d*e**10*x**3 + 4*e**11*x**4)
```

Giac [A] time = 1.32245, size = 389, normalized size = 2.27

$$\frac{1}{2}\left(c^3-\frac{12\,c^3d}{xe+d}\right)(xe+d)^2e^{(-7)}-3\left(5\,c^3d^2+ac^2e^2\right)e^{(-7)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^2}\right)+\frac{1}{4}\left(\frac{80\,c^3d^3e^{29}}{xe+d}-\frac{30\,c^3d^4e^{29}}{(xe+d)^2}+\frac{8\,c^3d^5e^{29}}{(xe+d)^3}-\frac{c^3d}{(xe}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] 1/2*(c^3 - 12*c^3*d/(x*e + d))*(x*e + d)^2*e^(-7) - 3*(5*c^3*d^2 + a*c^2*e^
2)*e^(-7)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2) + 1/4*(80*c^3*d^3*e^29/(x*e
+ d) - 30*c^3*d^4*e^29/(x*e + d)^2 + 8*c^3*d^5*e^29/(x*e + d)^3 - c^3*d^6*e
^29/(x*e + d)^4 + 48*a*c^2*d*e^31/(x*e + d) - 36*a*c^2*d^2*e^31/(x*e + d)^2
 + 16*a*c^2*d^3*e^31/(x*e + d)^3 - 3*a*c^2*d^4*e^31/(x*e + d)^4 - 6*a^2*c*e
^33/(x*e + d)^2 + 8*a^2*c*d*e^33/(x*e + d)^3 - 3*a^2*c*d^2*e^33/(x*e + d)^4
 - a^3*e^35/(x*e + d)^4)*e^(-36)
```

3.483 $\int \frac{(a+cx^2)^3}{(d+ex)^6} \, dx$

Optimal. Leaf size=172

$$-\frac{3c^2(ae^2+5cd^2)}{e^7(d+ex)}+\frac{2c^2d(3ae^2+5cd^2)}{e^7(d+ex)^2}-\frac{c(ae^2+cd^2)(ae^2+5cd^2)}{e^7(d+ex)^3}+\frac{3cd(ae^2+cd^2)^2}{2e^7(d+ex)^4}-\frac{(ae^2+cd^2)^3}{5e^7(d+ex)^5}-\frac{6c^3d\log(d$$

```
[Out] (c^3*x)/e^6 - (c*d^2 + a*e^2)^3/(5*e^7*(d + e*x)^5) + (3*c*d*(c*d^2 + a*e^2
)^2)/(2*e^7*(d + e*x)^4) - (c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2))/(e^7*(d +
e*x)^3) + (2*c^2*d*(5*c*d^2 + 3*a*e^2))/(e^7*(d + e*x)^2) - (3*c^2*(5*c*d^2
 + a*e^2))/(e^7*(d + e*x)) - (6*c^3*d*Log[d + e*x])/e^7
```

Rubi [A] time = 0.149449, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$-\frac{3c^2(ae^2+5cd^2)}{e^7(d+ex)}+\frac{2c^2d(3ae^2+5cd^2)}{e^7(d+ex)^2}-\frac{c(ae^2+cd^2)(ae^2+5cd^2)}{e^7(d+ex)^3}+\frac{3cd(ae^2+cd^2)^2}{2e^7(d+ex)^4}-\frac{(ae^2+cd^2)^3}{5e^7(d+ex)^5}-\frac{6c^3d\log(d$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^3/(d + e*x)^6,x]
```

```
[Out] (c^3*x)/e^6 - (c*d^2 + a*e^2)^3/(5*e^7*(d + e*x)^5) + (3*c*d*(c*d^2 + a*e^2
)^2)/(2*e^7*(d + e*x)^4) - (c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2))/(e^7*(d +
e*x)^3) + (2*c^2*d*(5*c*d^2 + 3*a*e^2))/(e^7*(d + e*x)^2) - (3*c^2*(5*c*d^2
 + a*e^2))/(e^7*(d + e*x)) - (6*c^3*d*Log[d + e*x])/e^7
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+cx^2)^3}{(d+ex)^6}\,dx=\int\left(\frac{c^3}{e^6}+\frac{(cd^2+ae^2)^3}{e^6(d+ex)^6}-\frac{6cd(cd^2+ae^2)^2}{e^6(d+ex)^5}+\frac{3c(cd^2+ae^2)(5cd^2+ae^2)}{e^6(d+ex)^4}-\frac{4c^2d(5cd^2+3ae^2)}{e^6(d+ex)^3}+\frac{3c$$

$$=\frac{c^3x}{e^6}-\frac{(cd^2+ae^2)^3}{5e^7(d+ex)^5}+\frac{3cd(cd^2+ae^2)^2}{2e^7(d+ex)^4}-\frac{c(cd^2+ae^2)(5cd^2+ae^2)}{e^7(d+ex)^3}+\frac{2c^2d(5cd^2+3ae^2)}{e^7(d+ex)^2}-\frac{3c^2(5}{e^7(}$$

Mathematica [A] time = 0.0915828, size = 182, normalized size = 1.06

$$-\frac{a^2ce^4(d^2+5dex+10e^2x^2)+2a^3e^6+6ac^2e^2(10d^2e^2x^2+5d^3ex+d^4+10de^3x^3+5e^4x^4)+c^3(600d^4e^2x^2+400d^3e^3x}{10e^7(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^3/(d + e*x)^6,x]
```

```
[Out] -(2*a^3*e^6 + a^2*c*e^4*(d^2 + 5*d*e*x + 10*e^2*x^2) + 6*a*c^2*e^2*(d^4 + 5
*d^3*e*x + 10*d^2*e^2*x^2 + 10*d*e^3*x^3 + 5*e^4*x^4) + c^3*(87*d^6 + 375*d
^5*e*x + 600*d^4*e^2*x^2 + 400*d^3*e^3*x^3 + 50*d^2*e^4*x^4 - 50*d*e^5*x^5
- 10*e^6*x^6) + 60*c^3*d*(d + e*x)^5*Log[d + e*x])/(10*e^7*(d + e*x)^5)
```

Maple [A] time = 0.053, size = 272, normalized size = 1.6

$$\frac{c^3x}{e^6} + \frac{3\,cda^2}{2\,e^3\,(ex+d)^4} + 3\,\frac{c^2d^3a}{e^5\,(ex+d)^4} + \frac{3\,c^3d^5}{2\,e^7\,(ex+d)^4} - \frac{a^2c}{e^3\,(ex+d)^3} - 6\,\frac{ac^2d^2}{e^5\,(ex+d)^3} - 5\,\frac{c^3d^4}{e^7\,(ex+d)^3} - 6\,\frac{c^3d\ln(ex+d)}{e^7} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3/(e*x+d)^6,x)
```

```
[Out] c^3*x/e^6+3/2*c*d/e^3/(e*x+d)^4*a^2+3*c^2*d^3/e^5/(e*x+d)^4*a+3/2*d^5/e^7/(
e*x+d)^4*c^3-c/e^3/(e*x+d)^3*a^2-6*c^2/e^5/(e*x+d)^3*a*d^2-5*d^4/e^7/(e*x+d
)^3*c^3-6*c^3*d*ln(e*x+d)/e^7-1/5/e/(e*x+d)^5*a^3-3/5/e^3/(e*x+d)^5*d^2*a^2
*c-3/5/e^5/(e*x+d)^5*d^4*a*c^2-1/5*d^6/e^7/(e*x+d)^5*c^3-3*c^2/e^5/(e*x+d)*
a-15*c^3/e^7/(e*x+d)*d^2+6*c^2*d/e^5/(e*x+d)^2*a+10/e^7/(e*x+d)^2*c^3*d^3
```

Maxima [A] time = 1.17573, size = 332, normalized size = 1.93

$$-\frac{87\,c^3d^6 + 6\,ac^2d^4e^2 + a^2cd^2e^4 + 2\,a^3e^6 + 30\left(5\,c^3d^2e^4 + ac^2e^6\right)x^4 + 20\left(25\,c^3d^3e^3 + 3\,ac^2de^5\right)x^3 + 10\left(65\,c^3d^4e^2 + 6\,ac^2d^2\right.}{10\left(e^{12}x^5 + 5\,de^{11}x^4 + 10\,d^2e^{10}x^3 + 10\,d^3e^9x^2 + 5\,d^4e^8x + d^5e^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^6,x, algorithm="maxima")
```

```
[Out] -1/10*(87*c^3*d^6 + 6*a*c^2*d^4*e^2 + a^2*c*d^2*e^4 + 2*a^3*e^6 + 30*(5*c^3
*d^2*e^4 + a*c^2*e^6)*x^4 + 20*(25*c^3*d^3*e^3 + 3*a*c^2*d*e^5)*x^3 + 10*(6
5*c^3*d^4*e^2 + 6*a*c^2*d^2*e^4 + a^2*c*e^6)*x^2 + 5*(77*c^3*d^5*e + 6*a*c^
2*d^3*e^3 + a^2*c*d*e^5)*x)/(e^12*x^5 + 5*d*e^11*x^4 + 10*d^2*e^10*x^3 + 10
*d^3*e^9*x^2 + 5*d^4*e^8*x + d^5*e^7) + c^3*x/e^6 - 6*c^3*d*log(e*x + d)/e^
7
```

Fricas [A] time = 2.14604, size = 672, normalized size = 3.91

$$\frac{10\,c^3e^6x^6 + 50\,c^3de^5x^5 - 87\,c^3d^6 - 6\,ac^2d^4e^2 - a^2cd^2e^4 - 2\,a^3e^6 - 10\left(5\,c^3d^2e^4 + 3\,ac^2e^6\right)x^4 - 20\left(20\,c^3d^3e^3 + 3\,ac^2de^5\right)x^3 -}{10\left(e^{12}x^5 + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^6,x, algorithm="fricas")
```

```
[Out] 1/10*(10*c^3*e^6*x^6 + 50*c^3*d*e^5*x^5 - 87*c^3*d^6 - 6*a*c^2*d^4*e^2 - a^
2*c*d^2*e^4 - 2*a^3*e^6 - 10*(5*c^3*d^2*e^4 + 3*a*c^2*e^6)*x^4 - 20*(20*c^3
*d^3*e^3 + 3*a*c^2*d*e^5)*x^3 - 10*(60*c^3*d^4*e^2 + 6*a*c^2*d^2*e^4 + a^2*
c*e^6)*x^2 - 5*(75*c^3*d^5*e + 6*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x - 60*(c^3*d
*e^5*x^5 + 5*c^3*d^2*e^4*x^4 + 10*c^3*d^3*e^3*x^3 + 10*c^3*d^4*e^2*x^2 + 5*
c^3*d^5*e*x + c^3*d^6)*log(e*x + d))/(e^12*x^5 + 5*d*e^11*x^4 + 10*d^2*e^10
```

```
*x^3 + 10*d^3*e^9*x^2 + 5*d^4*e^8*x + d^5*e^7)
```

Sympy [A] time = 7.58859, size = 257, normalized size = 1.49

$$-\frac{6c^3d\log(d+ex)}{e^7}+\frac{c^3x}{e^6}-\frac{2a^3e^6+a^2cd^2e^4+6ac^2d^4e^2+87c^3d^6+x^4\left(30ac^2e^6+150c^3d^2e^4\right)+x^3\left(60ac^2de^5+500c^3d\right.}{10d^5e^7+50d^4e^8x+100d^3e^9x^2+100}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3/(e*x+d)**6,x)
```

```
[Out] -6*c**3*d*log(d + e*x)/e**7 + c**3*x/e**6 - (2*a**3*e**6 + a**2*c*d**2*e**4
 + 6*a*c**2*d**4*e**2 + 87*c**3*d**6 + x**4*(30*a*c**2*e**6 + 150*c**3*d**2
*e**4) + x**3*(60*a*c**2*d*e**5 + 500*c**3*d**3*e**3) + x**2*(10*a**2*c*e**
6 + 60*a*c**2*d**2*e**4 + 650*c**3*d**4*e**2) + x*(5*a**2*c*d*e**5 + 30*a*c
**2*d**3*e**3 + 385*c**3*d**5*e))/(10*d**5*e**7 + 50*d**4*e**8*x + 100*d**3
*e**9*x**2 + 100*d**2*e**10*x**3 + 50*d*e**11*x**4 + 10*e**12*x**5)
```

Giac [A] time = 1.30574, size = 254, normalized size = 1.48

$$-6c^3de^{(-7)}\log(|xe+d|)+c^3xe^{(-6)}-\frac{\left(87c^3d^6+6ac^2d^4e^2+a^2cd^2e^4+30\left(5c^3d^2e^4+ac^2e^6\right)x^4+20\left(25c^3d^3e^3+3ac^2d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^6,x, algorithm="giac")
```

```
[Out] -6*c^3*d*e^(-7)*log(abs(x*e + d)) + c^3*x*e^(-6) - 1/10*(87*c^3*d^6 + 6*a*c
^2*d^4*e^2 + a^2*c*d^2*e^4 + 30*(5*c^3*d^2*e^4 + a*c^2*e^6)*x^4 + 20*(25*c^
3*d^3*e^3 + 3*a*c^2*d*e^5)*x^3 + 2*a^3*e^6 + 10*(65*c^3*d^4*e^2 + 6*a*c^2*d
^2*e^4 + a^2*c*e^6)*x^2 + 5*(77*c^3*d^5*e + 6*a*c^2*d^3*e^3 + a^2*c*d*e^5)*
x)*e^(-7)/(x*e + d)^5
```

3.484 $\int \frac{(a+cx^2)^3}{(d+ex)^7} dx$

Optimal. Leaf size=184

$$-\frac{3c^2(ae^2+5cd^2)}{2e^7(d+ex)^2}+\frac{4c^2d(3ae^2+5cd^2)}{3e^7(d+ex)^3}-\frac{3c(ae^2+cd^2)(ae^2+5cd^2)}{4e^7(d+ex)^4}+\frac{6cd(ae^2+cd^2)^2}{5e^7(d+ex)^5}-\frac{(ae^2+cd^2)^3}{6e^7(d+ex)^6}+\frac{6c^3d}{e^7(d+ex)}+$$

```
[Out] -(c*d^2 + a*e^2)^3/(6*e^7*(d + e*x)^6) + (6*c*d*(c*d^2 + a*e^2)^2)/(5*e^7*(
d + e*x)^5) - (3*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2))/(4*e^7*(d + e*x)^4) +
 (4*c^2*d*(5*c*d^2 + 3*a*e^2))/(3*e^7*(d + e*x)^3) - (3*c^2*(5*c*d^2 + a*e^
2))/(2*e^7*(d + e*x)^2) + (6*c^3*d)/(e^7*(d + e*x)) + (c^3*Log[d + e*x])/e^
7
```

Rubi [A] time = 0.135293, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$-\frac{3c^2(ae^2+5cd^2)}{2e^7(d+ex)^2}+\frac{4c^2d(3ae^2+5cd^2)}{3e^7(d+ex)^3}-\frac{3c(ae^2+cd^2)(ae^2+5cd^2)}{4e^7(d+ex)^4}+\frac{6cd(ae^2+cd^2)^2}{5e^7(d+ex)^5}-\frac{(ae^2+cd^2)^3}{6e^7(d+ex)^6}+\frac{6c^3d}{e^7(d+ex)}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^3/(d + e*x)^7,x]
```

```
[Out] -(c*d^2 + a*e^2)^3/(6*e^7*(d + e*x)^6) + (6*c*d*(c*d^2 + a*e^2)^2)/(5*e^7*(
d + e*x)^5) - (3*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2))/(4*e^7*(d + e*x)^4) +
 (4*c^2*d*(5*c*d^2 + 3*a*e^2))/(3*e^7*(d + e*x)^3) - (3*c^2*(5*c*d^2 + a*e^
2))/(2*e^7*(d + e*x)^2) + (6*c^3*d)/(e^7*(d + e*x)) + (c^3*Log[d + e*x])/e^
7
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+cx^2)^3}{(d+ex)^7} dx = \int \left(\frac{(cd^2+ae^2)^3}{e^6(d+ex)^7}-\frac{6cd(cd^2+ae^2)^2}{e^6(d+ex)^6}+\frac{3c(cd^2+ae^2)(5cd^2+ae^2)}{e^6(d+ex)^5}-\frac{4c^2d(5cd^2+3ae^2)}{e^6(d+ex)^4}+\frac{3c^2(5cd^2-}{e^6(d+e}\right.$$

$$= -\frac{(cd^2+ae^2)^3}{6e^7(d+ex)^6}+\frac{6cd(cd^2+ae^2)^2}{5e^7(d+ex)^5}-\frac{3c(cd^2+ae^2)(5cd^2+ae^2)}{4e^7(d+ex)^4}+\frac{4c^2d(5cd^2+3ae^2)}{3e^7(d+ex)^3}-\frac{3c^2(5cd^2+a}{2e^7(d+ex}$$

Mathematica [A] time = 0.0623787, size = 172, normalized size = 0.93

$$\frac{-3a^2ce^4(d^2+6dex+15e^2x^2)-10a^3e^6-6ac^2e^2(15d^2e^2x^2+6d^3ex+d^4+20de^3x^3+15e^4x^4)+c^3d(1875d^3e^2x^2+2200d}{60e^7(d+ex)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^3/(d + e*x)^7,x]
```

```
[Out] (-10*a^3*e^6 - 3*a^2*c*e^4*(d^2 + 6*d*e*x + 15*e^2*x^2) - 6*a*c^2*e^2*(d^4
+ 6*d^3*e*x + 15*d^2*e^2*x^2 + 20*d*e^3*x^3 + 15*e^4*x^4) + c^3*d*(147*d^5
+ 822*d^4*e*x + 1875*d^3*e^2*x^2 + 2200*d^2*e^3*x^3 + 1350*d*e^4*x^4 + 360*
e^5*x^5) + 60*c^3*(d + e*x)^6*Log[d + e*x])/(60*e^7*(d + e*x)^6)
```

Maple [A] time = 0.049, size = 278, normalized size = 1.5

$$-\frac{3\,a^2c}{4\,e^3\,(ex+d)^4}-\frac{9\,ac^2d^2}{2\,e^5\,(ex+d)^4}-\frac{15\,c^3d^4}{4\,e^7\,(ex+d)^4}-\frac{a^3}{6\,e\,(ex+d)^6}-\frac{a^2cd^2}{2\,e^3\,(ex+d)^6}-\frac{ac^2d^4}{2\,e^5\,(ex+d)^6}-\frac{d^6c^3}{6\,e^7\,(ex+d)^6}+4\,\frac{}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3/(e*x+d)^7,x)
```

```
[Out] -3/4*c/e^3/(e*x+d)^4*a^2-9/2*c^2/e^5/(e*x+d)^4*a*d^2-15/4*c^3/e^7/(e*x+d)^4
*d^4-1/6/e/(e*x+d)^6*a^3-1/2/e^3/(e*x+d)^6*a^2*c*d^2-1/2/e^5/(e*x+d)^6*d^4*
a*c^2-1/6*d^6/e^7/(e*x+d)^6*c^3+4*c^2*d/e^5/(e*x+d)^3*a+20/3/e^7/(e*x+d)^3*
c^3*d^3+c^3*ln(e*x+d)/e^7+6/5*c*d/e^3/(e*x+d)^5*a^2+12/5*c^2*d^3/e^5/(e*x+d
)^5*a+6/5*c^3*d^5/e^7/(e*x+d)^5+6*c^3*d/e^7/(e*x+d)-3/2*c^2/e^5/(e*x+d)^2*a
-15/2*c^3/e^7/(e*x+d)^2*d^2
```

Maxima [A] time = 1.25224, size = 355, normalized size = 1.93

$$\frac{360\,c^3de^5x^5+147\,c^3d^6-6\,ac^2d^4e^2-3\,a^2cd^2e^4-10\,a^3e^6+90\left(15\,c^3d^2e^4-ac^2e^6\right)x^4+40\left(55\,c^3d^3e^3-3\,ac^2de^5\right)x^3+15}{60\left(e^{13}x^6+6\,de^{12}x^5+15\,d^2e^{11}x^4+20\,d^3e^{10}x^3+15\,d^4e^9x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^7,x, algorithm="maxima")
```

```
[Out] 1/60*(360*c^3*d*e^5*x^5 + 147*c^3*d^6 - 6*a*c^2*d^4*e^2 - 3*a^2*c*d^2*e^4 -
 10*a^3*e^6 + 90*(15*c^3*d^2*e^4 - a*c^2*e^6)*x^4 + 40*(55*c^3*d^3*e^3 - 3*
a*c^2*d*e^5)*x^3 + 15*(125*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 - 3*a^2*c*e^6)*x^2
 + 6*(137*c^3*d^5*e - 6*a*c^2*d^3*e^3 - 3*a^2*c*d*e^5)*x)/(e^13*x^6 + 6*d*e
^12*x^5 + 15*d^2*e^11*x^4 + 20*d^3*e^10*x^3 + 15*d^4*e^9*x^2 + 6*d^5*e^8*x
+ d^6*e^7) + c^3*log(e*x + d)/e^7
```

Fricas [A] time = 1.98179, size = 710, normalized size = 3.86

$$\frac{360\,c^3de^5x^5+147\,c^3d^6-6\,ac^2d^4e^2-3\,a^2cd^2e^4-10\,a^3e^6+90\left(15\,c^3d^2e^4-ac^2e^6\right)x^4+40\left(55\,c^3d^3e^3-3\,ac^2de^5\right)x^3+15}{60\left(e^{13}x^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^7,x, algorithm="fricas")
```

```
[Out] 1/60*(360*c^3*d*e^5*x^5 + 147*c^3*d^6 - 6*a*c^2*d^4*e^2 - 3*a^2*c*d^2*e^4 -
 10*a^3*e^6 + 90*(15*c^3*d^2*e^4 - a*c^2*e^6)*x^4 + 40*(55*c^3*d^3*e^3 - 3*
```

```
a*c^2*d*e^5)*x^3 + 15*(125*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 - 3*a^2*c*e^6)*x^2
 + 6*(137*c^3*d^5*e - 6*a*c^2*d^3*e^3 - 3*a^2*c*d*e^5)*x + 60*(c^3*e^6*x^6
+ 6*c^3*d*e^5*x^5 + 15*c^3*d^2*e^4*x^4 + 20*c^3*d^3*e^3*x^3 + 15*c^3*d^4*e^
2*x^2 + 6*c^3*d^5*e*x + c^3*d^6)*log(e*x + d))/(e^13*x^6 + 6*d*e^12*x^5 + 1
5*d^2*e^11*x^4 + 20*d^3*e^10*x^3 + 15*d^4*e^9*x^2 + 6*d^5*e^8*x + d^6*e^7)
```

Sympy [A] time = 11.546, size = 272, normalized size = 1.48

$$\frac{c^3\log(d+ex)}{e^7}+\frac{-10a^3e^6-3a^2cd^2e^4-6ac^2d^4e^2+147c^3d^6+360c^3de^5x^5+x^4\left(-90ac^2e^6+1350c^3d^2e^4\right)+x^3\left(-120ac^2de^5\right.}{60d^6e^7+360d^5e^8x+900d^4e^9x^2+1200d^3e^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3/(e*x+d)**7,x)
```

```
[Out] c**3*log(d + e*x)/e**7 + (-10*a**3*e**6 - 3*a**2*c*d**2*e**4 - 6*a*c**2*d**
4*e**2 + 147*c**3*d**6 + 360*c**3*d*e**5*x**5 + x**4*(-90*a*c**2*e**6 + 135
0*c**3*d**2*e**4) + x**3*(-120*a*c**2*d*e**5 + 2200*c**3*d**3*e**3) + x**2*
(-45*a**2*c*e**6 - 90*a*c**2*d**2*e**4 + 1875*c**3*d**4*e**2) + x*(-18*a**2
*c*d*e**5 - 36*a*c**2*d**3*e**3 + 822*c**3*d**5*e))/(60*d**6*e**7 + 360*d**
5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4
+ 360*d*e**12*x**5 + 60*e**13*x**6)
```

Giac [A] time = 1.29102, size = 265, normalized size = 1.44

$$c^3e^{(-7)}\log(|xe+d|)+\frac{\left(360\,c^3dx^5e^4+90\left(15\,c^3d^2e^3-ac^2e^5\right)x^4+40\left(55\,c^3d^3e^2-3\,ac^2de^4\right)x^3+15\left(125\,c^3d^4e-6\,ac^2d^2e^3-\right.\right.}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^7,x, algorithm="giac")
```

```
[Out] c^3*e^(-7)*log(abs(x*e + d)) + 1/60*(360*c^3*d*x^5*e^4 + 90*(15*c^3*d^2*e^3
 - a*c^2*e^5)*x^4 + 40*(55*c^3*d^3*e^2 - 3*a*c^2*d*e^4)*x^3 + 15*(125*c^3*d
^4*e - 6*a*c^2*d^2*e^3 - 3*a^2*c*e^5)*x^2 + 6*(137*c^3*d^5 - 6*a*c^2*d^3*e^
2 - 3*a^2*c*d*e^4)*x + (147*c^3*d^6 - 6*a*c^2*d^4*e^2 - 3*a^2*c*d^2*e^4 - 1
0*a^3*e^6)*e^(-1))*e^(-6)/(x*e + d)^6
```

3.485 $\int \frac{(a+cx^2)^3}{(d+ex)^8} \, dx$

Optimal. Leaf size=178

$$-\frac{c^2\left(ae^2+5cd^2\right)}{e^7(d+ex)^3}+\frac{c^2d\left(3ae^2+5cd^2\right)}{e^7(d+ex)^4}-\frac{3c\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{5e^7(d+ex)^5}+\frac{cd\left(ae^2+cd^2\right)^2}{e^7(d+ex)^6}-\frac{\left(ae^2+cd^2\right)^3}{7e^7(d+ex)^7}-\frac{c^3}{e^7(d+ex)}+$$

```
[Out] -(c*d^2 + a*e^2)^3/(7*e^7*(d + e*x)^7) + (c*d*(c*d^2 + a*e^2)^2)/(e^7*(d +
e*x)^6) - (3*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2))/(5*e^7*(d + e*x)^5) + (c^
2*d*(5*c*d^2 + 3*a*e^2))/(e^7*(d + e*x)^4) - (c^2*(5*c*d^2 + a*e^2))/(e^7*(
d + e*x)^3) + (3*c^3*d)/(e^7*(d + e*x)^2) - c^3/(e^7*(d + e*x))
```

Rubi [A] time = 0.121561, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$-\frac{c^2\left(ae^2+5cd^2\right)}{e^7(d+ex)^3}+\frac{c^2d\left(3ae^2+5cd^2\right)}{e^7(d+ex)^4}-\frac{3c\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{5e^7(d+ex)^5}+\frac{cd\left(ae^2+cd^2\right)^2}{e^7(d+ex)^6}-\frac{\left(ae^2+cd^2\right)^3}{7e^7(d+ex)^7}-\frac{c^3}{e^7(d+ex)}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^3/(d + e*x)^8,x]
```

```
[Out] -(c*d^2 + a*e^2)^3/(7*e^7*(d + e*x)^7) + (c*d*(c*d^2 + a*e^2)^2)/(e^7*(d +
e*x)^6) - (3*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2))/(5*e^7*(d + e*x)^5) + (c^
2*d*(5*c*d^2 + 3*a*e^2))/(e^7*(d + e*x)^4) - (c^2*(5*c*d^2 + a*e^2))/(e^7*(
d + e*x)^3) + (3*c^3*d)/(e^7*(d + e*x)^2) - c^3/(e^7*(d + e*x))
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+cx^2\right)^3}{(d+ex)^8}\,dx &= \int \left(\frac{\left(cd^2+ae^2\right)^3}{e^6(d+ex)^8}-\frac{6cd\left(cd^2+ae^2\right)^2}{e^6(d+ex)^7}+\frac{3c\left(cd^2+ae^2\right)\left(5cd^2+ae^2\right)}{e^6(d+ex)^6}-\frac{4c^2d\left(5cd^2+3ae^2\right)}{e^6(d+ex)^5}+\frac{3c^2\left(5c\right.}{e^6(d}\right.\\ &= -\frac{\left(cd^2+ae^2\right)^3}{7e^7(d+ex)^7}+\frac{cd\left(cd^2+ae^2\right)^2}{e^7(d+ex)^6}-\frac{3c\left(cd^2+ae^2\right)\left(5cd^2+ae^2\right)}{5e^7(d+ex)^5}+\frac{c^2d\left(5cd^2+3ae^2\right)}{e^7(d+ex)^4}-\frac{c^2\left(5cd^2+a\right.}{e^7(d+ex)}\end{aligned}$$

Mathematica [A] time = 0.052798, size = 161, normalized size = 0.9

$$-\frac{a^2ce^4\left(d^2+7dex+21e^2x^2\right)+5a^3e^6+ac^2e^2\left(21d^2e^2x^2+7d^3ex+d^4+35de^3x^3+35e^4x^4\right)+5c^3\left(21d^4e^2x^2+35d^3e^3x^3\right.}{35e^7(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^3/(d + e*x)^8,x]
```

```
[Out] -(5*a^3*e^6 + a^2*c*e^4*(d^2 + 7*d*e*x + 21*e^2*x^2) + a*c^2*e^2*(d^4 + 7*d
^3*e*x + 21*d^2*e^2*x^2 + 35*d*e^3*x^3 + 35*e^4*x^4) + 5*c^3*(d^6 + 7*d^5*e
*x + 21*d^4*e^2*x^2 + 35*d^3*e^3*x^3 + 35*d^2*e^4*x^4 + 21*d*e^5*x^5 + 7*e^
6*x^6))/(35*e^7*(d + e*x)^7)
```

Maple [A] time = 0.052, size = 216, normalized size = 1.2

$$\frac{c^2d\left(3\,ae^2+5\,cd^2\right)}{e^7\,(ex+d)^4}+\frac{cd\left(a^2e^4+2\,acd^2e^2+c^2d^4\right)}{e^7\,(ex+d)^6}-\frac{c^2\left(ae^2+5\,cd^2\right)}{e^7\,(ex+d)^3}-\frac{a^3e^6+3\,a^2cd^2e^4+3\,d^4e^2ac^2+d^6c^3}{7\,e^7\,(ex+d)^7}-\frac{3\,c\left(a^2e^4+6\right.}{5\,e^7\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3/(e*x+d)^8,x)
```

```
[Out] c^2*d*(3*a*e^2+5*c*d^2)/e^7/(e*x+d)^4+c*d*(a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)/e
^7/(e*x+d)^6-c^2*(a*e^2+5*c*d^2)/e^7/(e*x+d)^3-1/7*(a^3*e^6+3*a^2*c*d^2*e^4
+3*a*c^2*d^4*e^2+c^3*d^6)/e^7/(e*x+d)^7-3/5*c*(a^2*e^4+6*a*c*d^2*e^2+5*c^2*
d^4)/e^7/(e*x+d)^5-c^3/e^7/(e*x+d)+3*c^3*d/e^7/(e*x+d)^2
```

Maxima [A] time = 1.21563, size = 355, normalized size = 1.99

$$-\frac{35\,c^3e^6x^6+105\,c^3de^5x^5+5\,c^3d^6+ac^2d^4e^2+a^2cd^2e^4+5\,a^3e^6+35\left(5\,c^3d^2e^4+ac^2e^6\right)x^4+35\left(5\,c^3d^3e^3+ac^2de^5\right)x^3+21}{35\left(e^{14}x^7+7\,de^{13}x^6+21\,d^2e^{12}x^5+35\,d^3e^{11}x^4+35\,d^4e^{10}x^3+21\,d^5e^9x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^8,x, algorithm="maxima")
```

```
[Out] -1/35*(35*c^3*e^6*x^6 + 105*c^3*d*e^5*x^5 + 5*c^3*d^6 + a*c^2*d^4*e^2 + a^2
*c*d^2*e^4 + 5*a^3*e^6 + 35*(5*c^3*d^2*e^4 + a*c^2*e^6)*x^4 + 35*(5*c^3*d^3
*e^3 + a*c^2*d*e^5)*x^3 + 21*(5*c^3*d^4*e^2 + a*c^2*d^2*e^4 + a^2*c*e^6)*x^
2 + 7*(5*c^3*d^5*e + a*c^2*d^3*e^3 + a^2*c*d*e^5)*x)/(e^14*x^7 + 7*d*e^13*x
^6 + 21*d^2*e^12*x^5 + 35*d^3*e^11*x^4 + 35*d^4*e^10*x^3 + 21*d^5*e^9*x^2 +
 7*d^6*e^8*x + d^7*e^7)
```

Fricas [A] time = 2.0345, size = 540, normalized size = 3.03

$$-\frac{35\,c^3e^6x^6+105\,c^3de^5x^5+5\,c^3d^6+ac^2d^4e^2+a^2cd^2e^4+5\,a^3e^6+35\left(5\,c^3d^2e^4+ac^2e^6\right)x^4+35\left(5\,c^3d^3e^3+ac^2de^5\right)x^3+21}{35\left(e^{14}x^7+7\,de^{13}x^6+21\,d^2e^{12}x^5+35\,d^3e^{11}x^4+35\,d^4e^{10}x^3+21\,d^5e^9x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^8,x, algorithm="fricas")
```

```
[Out] -1/35*(35*c^3*e^6*x^6 + 105*c^3*d*e^5*x^5 + 5*c^3*d^6 + a*c^2*d^4*e^2 + a^2
*c*d^2*e^4 + 5*a^3*e^6 + 35*(5*c^3*d^2*e^4 + a*c^2*e^6)*x^4 + 35*(5*c^3*d^3
*e^3 + a*c^2*d*e^5)*x^3 + 21*(5*c^3*d^4*e^2 + a*c^2*d^2*e^4 + a^2*c*e^6)*x^
2 + 7*(5*c^3*d^5*e + a*c^2*d^3*e^3 + a^2*c*d*e^5)*x)/(e^14*x^7 + 7*d*e^13*x
^6 + 21*d^2*e^12*x^5 + 35*d^3*e^11*x^4 + 35*d^4*e^10*x^3 + 21*d^5*e^9*x^2 +
 7*d^6*e^8*x + d^7*e^7)
```

Sympy [A] time = 15.2541, size = 280, normalized size = 1.57

$$-\frac{5a^3e^6 + a^2cd^2e^4 + ac^2d^4e^2 + 5c^3d^6 + 105c^3de^5x^5 + 35c^3e^6x^6 + x^4\left(35ac^2e^6 + 175c^3d^2e^4\right) + x^3\left(35ac^2de^5 + 175c^3d^3e^3\right)}{35d^7e^7 + 245d^6e^8x + 735d^5e^9x^2 + 1225d^4e^{10}x^3 + 1225d^3e^{11}x^4 + 735d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3/(e*x+d)**8,x)
```

```
[Out] -(5*a**3*e**6 + a**2*c*d**2*e**4 + a*c**2*d**4*e**2 + 5*c**3*d**6 + 105*c**
3*d*e**5*x**5 + 35*c**3*e**6*x**6 + x**4*(35*a*c**2*e**6 + 175*c**3*d**2*e*
*4) + x**3*(35*a*c**2*d*e**5 + 175*c**3*d**3*e**3) + x**2*(21*a**2*c*e**6 +
 21*a*c**2*d**2*e**4 + 105*c**3*d**4*e**2) + x*(7*a**2*c*d*e**5 + 7*a*c**2*
d**3*e**3 + 35*c**3*d**5*e))/(35*d**7*e**7 + 245*d**6*e**8*x + 735*d**5*e**
9*x**2 + 1225*d**4*e**10*x**3 + 1225*d**3*e**11*x**4 + 735*d**2*e**12*x**5
+ 245*d*e**13*x**6 + 35*e**14*x**7)
```

Giac [A] time = 1.22163, size = 255, normalized size = 1.43

$$-\frac{\left(35\,c^3x^6e^6 + 105\,c^3dx^5e^5 + 175\,c^3d^2x^4e^4 + 175\,c^3d^3x^3e^3 + 105\,c^3d^4x^2e^2 + 35\,c^3d^5xe + 5\,c^3d^6 + 35\,ac^2x^4e^6 + 35\,ac^2dx}{35\,(xe + d)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^8,x, algorithm="giac")
```

```
[Out] -1/35*(35*c^3*x^6*e^6 + 105*c^3*d*x^5*e^5 + 175*c^3*d^2*x^4*e^4 + 175*c^3*d
^3*x^3*e^3 + 105*c^3*d^4*x^2*e^2 + 35*c^3*d^5*x*e + 5*c^3*d^6 + 35*a*c^2*x^
4*e^6 + 35*a*c^2*d*x^3*e^5 + 21*a*c^2*d^2*x^2*e^4 + 7*a*c^2*d^3*x*e^3 + a*c
^2*d^4*e^2 + 21*a^2*c*x^2*e^6 + 7*a^2*c*d*x*e^5 + a^2*c*d^2*e^4 + 5*a^3*e^6
)*e^(-7)/(x*e + d)^7
```

3.486 $\int \frac{(a+cx^2)^3}{(d+ex)^9}\,dx$

Optimal. Leaf size=188

$$-\frac{3c^2\left(ae^2+5cd^2\right)}{4e^7(d+ex)^4}+\frac{4c^2d\left(3ae^2+5cd^2\right)}{5e^7(d+ex)^5}-\frac{c\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{2e^7(d+ex)^6}+\frac{6cd\left(ae^2+cd^2\right)^2}{7e^7(d+ex)^7}-\frac{\left(ae^2+cd^2\right)^3}{8e^7(d+ex)^8}-\frac{c^3}{2e^7(d+ex)^2}+$$

```
[Out] -(c*d^2 + a*e^2)^3/(8*e^7*(d + e*x)^8) + (6*c*d*(c*d^2 + a*e^2)^2)/(7*e^7*(
d + e*x)^7) - (c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2))/(2*e^7*(d + e*x)^6) + (
4*c^2*d*(5*c*d^2 + 3*a*e^2))/(5*e^7*(d + e*x)^5) - (3*c^2*(5*c*d^2 + a*e^2)
)/(4*e^7*(d + e*x)^4) + (2*c^3*d)/(e^7*(d + e*x)^3) - c^3/(2*e^7*(d + e*x)^
2)
```

Rubi [A] time = 0.12541, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$-\frac{3c^2\left(ae^2+5cd^2\right)}{4e^7(d+ex)^4}+\frac{4c^2d\left(3ae^2+5cd^2\right)}{5e^7(d+ex)^5}-\frac{c\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{2e^7(d+ex)^6}+\frac{6cd\left(ae^2+cd^2\right)^2}{7e^7(d+ex)^7}-\frac{\left(ae^2+cd^2\right)^3}{8e^7(d+ex)^8}-\frac{c^3}{2e^7(d+ex)^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^3/(d + e*x)^9,x]
```

```
[Out] -(c*d^2 + a*e^2)^3/(8*e^7*(d + e*x)^8) + (6*c*d*(c*d^2 + a*e^2)^2)/(7*e^7*(
d + e*x)^7) - (c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2))/(2*e^7*(d + e*x)^6) + (
4*c^2*d*(5*c*d^2 + 3*a*e^2))/(5*e^7*(d + e*x)^5) - (3*c^2*(5*c*d^2 + a*e^2)
)/(4*e^7*(d + e*x)^4) + (2*c^3*d)/(e^7*(d + e*x)^3) - c^3/(2*e^7*(d + e*x)^
2)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+cx^2\right)^3}{(d+ex)^9}\,dx &= \int \left(\frac{\left(cd^2+ae^2\right)^3}{e^6(d+ex)^9}-\frac{6cd\left(cd^2+ae^2\right)^2}{e^6(d+ex)^8}+\frac{3c\left(cd^2+ae^2\right)\left(5cd^2+ae^2\right)}{e^6(d+ex)^7}-\frac{4c^2d\left(5cd^2+3ae^2\right)}{e^6(d+ex)^6}+\frac{3c^2\left(5cd^2\right.}{e^6(d+e}\right.\\ &= -\frac{\left(cd^2+ae^2\right)^3}{8e^7(d+ex)^8}+\frac{6cd\left(cd^2+ae^2\right)^2}{7e^7(d+ex)^7}-\frac{c\left(cd^2+ae^2\right)\left(5cd^2+ae^2\right)}{2e^7(d+ex)^6}+\frac{4c^2d\left(5cd^2+3ae^2\right)}{5e^7(d+ex)^5}-\frac{3c^2\left(5cd^2+ae\right.}{4e^7(d+ex)^4}\end{aligned}$$

Mathematica [A] time = 0.0616959, size = 163, normalized size = 0.87

$$-\frac{5a^2ce^4\left(d^2+8dex+28e^2x^2\right)+35a^3e^6+3ac^2e^2\left(28d^2e^2x^2+8d^3ex+d^4+56de^3x^3+70e^4x^4\right)+5c^3\left(28d^4e^2x^2+56d^3e^3x\right.}{280e^7(d+ex)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^3/(d + e*x)^9,x]
```

```
[Out] -(35*a^3*e^6 + 5*a^2*c*e^4*(d^2 + 8*d*e*x + 28*e^2*x^2) + 3*a*c^2*e^2*(d^4
+ 8*d^3*e*x + 28*d^2*e^2*x^2 + 56*d*e^3*x^3 + 70*e^4*x^4) + 5*c^3*(d^6 + 8*
d^5*e*x + 28*d^4*e^2*x^2 + 56*d^3*e^3*x^3 + 70*d^2*e^4*x^4 + 56*d*e^5*x^5 +
 28*e^6*x^6))/(280*e^7*(d + e*x)^8)
```

Maple [A] time = 0.05, size = 218, normalized size = 1.2

$$-\frac{3\,c^2\left(ae^2+5\,cd^2\right)}{4\,e^7\,(ex+d)^4}-\frac{c\left(a^2e^4+6\,acd^2e^2+5\,c^2d^4\right)}{2\,e^7\,(ex+d)^6}-\frac{a^3e^6+3\,a^2cd^2e^4+3\,d^4e^2ac^2+d^6c^3}{8\,e^7\,(ex+d)^8}+2\,\frac{c^3d}{e^7\,(ex+d)^3}+\frac{6\,cd\left(a^2e^4\right.}{7\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3/(e*x+d)^9,x)
```

```
[Out] -3/4*c^2*(a*e^2+5*c*d^2)/e^7/(e*x+d)^4-1/2*c*(a^2*e^4+6*a*c*d^2*e^2+5*c^2*d
^4)/e^7/(e*x+d)^6-1/8*(a^3*e^6+3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2+c^3*d^6)/e^7
/(e*x+d)^8+2*c^3*d/e^7/(e*x+d)^3+6/7*c*d*(a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)/e^
7/(e*x+d)^7+4/5*c^2*d*(3*a*e^2+5*c*d^2)/e^7/(e*x+d)^5-1/2*c^3/e^7/(e*x+d)^2
```

Maxima [A] time = 1.2149, size = 381, normalized size = 2.03

$$-\frac{140\,c^3e^6x^6+280\,c^3de^5x^5+5\,c^3d^6+3\,ac^2d^4e^2+5\,a^2cd^2e^4+35\,a^3e^6+70\left(5\,c^3d^2e^4+3\,ac^2e^6\right)x^4+56\left(5\,c^3d^3e^3+3\,ac\right.}{280\left(e^{15}x^8+8\,de^{14}x^7+28\,d^2e^{13}x^6+56\,d^3e^{12}x^5+70\,d^4e^{11}x^4+56\,d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^9,x, algorithm="maxima")
```

```
[Out] -1/280*(140*c^3*e^6*x^6 + 280*c^3*d*e^5*x^5 + 5*c^3*d^6 + 3*a*c^2*d^4*e^2 +
 5*a^2*c*d^2*e^4 + 35*a^3*e^6 + 70*(5*c^3*d^2*e^4 + 3*a*c^2*e^6)*x^4 + 56*(
5*c^3*d^3*e^3 + 3*a*c^2*d*e^5)*x^3 + 28*(5*c^3*d^4*e^2 + 3*a*c^2*d^2*e^4 +
5*a^2*c*e^6)*x^2 + 8*(5*c^3*d^5*e + 3*a*c^2*d^3*e^3 + 5*a^2*c*d*e^5)*x)/(e^
15*x^8 + 8*d*e^14*x^7 + 28*d^2*e^13*x^6 + 56*d^3*e^12*x^5 + 70*d^4*e^11*x^4
 + 56*d^5*e^10*x^3 + 28*d^6*e^9*x^2 + 8*d^7*e^8*x + d^8*e^7)
```

Fricas [A] time = 2.06227, size = 590, normalized size = 3.14

$$-\frac{140\,c^3e^6x^6+280\,c^3de^5x^5+5\,c^3d^6+3\,ac^2d^4e^2+5\,a^2cd^2e^4+35\,a^3e^6+70\left(5\,c^3d^2e^4+3\,ac^2e^6\right)x^4+56\left(5\,c^3d^3e^3+3\,ac\right.}{280\left(e^{15}x^8+8\,de^{14}x^7+28\,d^2e^{13}x^6+56\,d^3e^{12}x^5+70\,d^4e^{11}x^4+56\,d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^9,x, algorithm="fricas")
```

```
[Out] -1/280*(140*c^3*e^6*x^6 + 280*c^3*d*e^5*x^5 + 5*c^3*d^6 + 3*a*c^2*d^4*e^2 +
 5*a^2*c*d^2*e^4 + 35*a^3*e^6 + 70*(5*c^3*d^2*e^4 + 3*a*c^2*e^6)*x^4 + 56*(
5*c^3*d^3*e^3 + 3*a*c^2*d*e^5)*x^3 + 28*(5*c^3*d^4*e^2 + 3*a*c^2*d^2*e^4 +
5*a^2*c*e^6)*x^2 + 8*(5*c^3*d^5*e + 3*a*c^2*d^3*e^3 + 5*a^2*c*d*e^5)*x)/(e^
```

```
15*x^8 + 8*d*e^14*x^7 + 28*d^2*e^13*x^6 + 56*d^3*e^12*x^5 + 70*d^4*e^11*x^4
 + 56*d^5*e^10*x^3 + 28*d^6*e^9*x^2 + 8*d^7*e^8*x + d^8*e^7)
```

Sympy [A] time = 26.2735, size = 296, normalized size = 1.57

$$-\frac{35a^3e^6 + 5a^2cd^2e^4 + 3ac^2d^4e^2 + 5c^3d^6 + 280c^3de^5x^5 + 140c^3e^6x^6 + x^4\left(210ac^2e^6 + 350c^3d^2e^4\right) + x^3\left(168ac^2de^5 + 280c^3\right.}{280d^8e^7 + 2240d^7e^8x + 7840d^6e^9x^2 + 15680d^5e^{10}x^3 + 19600d^4e^{11}x^4 + 15680d^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3/(e*x+d)**9,x)
```

```
[Out] -(35*a**3*e**6 + 5*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + 5*c**3*d**6 + 28
0*c**3*d*e**5*x**5 + 140*c**3*e**6*x**6 + x**4*(210*a*c**2*e**6 + 350*c**3*
d**2*e**4) + x**3*(168*a*c**2*d*e**5 + 280*c**3*d**3*e**3) + x**2*(140*a**2
*c*e**6 + 84*a*c**2*d**2*e**4 + 140*c**3*d**4*e**2) + x*(40*a**2*c*d*e**5 +
 24*a*c**2*d**3*e**3 + 40*c**3*d**5*e))/(280*d**8*e**7 + 2240*d**7*e**8*x +
 7840*d**6*e**9*x**2 + 15680*d**5*e**10*x**3 + 19600*d**4*e**11*x**4 + 1568
0*d**3*e**12*x**5 + 7840*d**2*e**13*x**6 + 2240*d*e**14*x**7 + 280*e**15*x*
*8)
```

Giac [A] time = 1.20293, size = 258, normalized size = 1.37

$$-\frac{\left(140\,c^3x^6e^6 + 280\,c^3dx^5e^5 + 350\,c^3d^2x^4e^4 + 280\,c^3d^3x^3e^3 + 140\,c^3d^4x^2e^2 + 40\,c^3d^5xe + 5\,c^3d^6 + 210\,ac^2x^4e^6 + 168\,ac^2dx\right.}{280\,(xe+d)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^9,x, algorithm="giac")
```

```
[Out] -1/280*(140*c^3*x^6*e^6 + 280*c^3*d*x^5*e^5 + 350*c^3*d^2*x^4*e^4 + 280*c^3
*d^3*x^3*e^3 + 140*c^3*d^4*x^2*e^2 + 40*c^3*d^5*x*e + 5*c^3*d^6 + 210*a*c^2
*x^4*e^6 + 168*a*c^2*d*x^3*e^5 + 84*a*c^2*d^2*x^2*e^4 + 24*a*c^2*d^3*x*e^3
+ 3*a*c^2*d^4*e^2 + 140*a^2*c*x^2*e^6 + 40*a^2*c*d*x*e^5 + 5*a^2*c*d^2*e^4
+ 35*a^3*e^6)*e^(-7)/(x*e + d)^8
```

3.487 $\int \frac{(a+cx^2)^3}{(d+ex)^{10}}\,dx$

Optimal. Leaf size=190

$$-\frac{3c^2(ae^2+5cd^2)}{5e^7(d+ex)^5}+\frac{2c^2d(3ae^2+5cd^2)}{3e^7(d+ex)^6}-\frac{3c(ae^2+cd^2)(ae^2+5cd^2)}{7e^7(d+ex)^7}+\frac{3cd(ae^2+cd^2)^2}{4e^7(d+ex)^8}-\frac{(ae^2+cd^2)^3}{9e^7(d+ex)^9}-\frac{c^3}{3e^7(d+ex}$$

```
[Out] -(c*d^2 + a*e^2)^3/(9*e^7*(d + e*x)^9) + (3*c*d*(c*d^2 + a*e^2)^2)/(4*e^7*(
d + e*x)^8) - (3*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2))/(7*e^7*(d + e*x)^7) +
 (2*c^2*d*(5*c*d^2 + 3*a*e^2))/(3*e^7*(d + e*x)^6) - (3*c^2*(5*c*d^2 + a*e^
2))/(5*e^7*(d + e*x)^5) + (3*c^3*d)/(2*e^7*(d + e*x)^4) - c^3/(3*e^7*(d + e
*x)^3)
```

Rubi [A] time = 0.12076, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$-\frac{3c^2(ae^2+5cd^2)}{5e^7(d+ex)^5}+\frac{2c^2d(3ae^2+5cd^2)}{3e^7(d+ex)^6}-\frac{3c(ae^2+cd^2)(ae^2+5cd^2)}{7e^7(d+ex)^7}+\frac{3cd(ae^2+cd^2)^2}{4e^7(d+ex)^8}-\frac{(ae^2+cd^2)^3}{9e^7(d+ex)^9}-\frac{c^3}{3e^7(d+ex}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^3/(d + e*x)^10,x]
```


```
[Out] -(c*d^2 + a*e^2)^3/(9*e^7*(d + e*x)^9) + (3*c*d*(c*d^2 + a*e^2)^2)/(4*e^7*(
d + e*x)^8) - (3*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2))/(7*e^7*(d + e*x)^7) +
 (2*c^2*d*(5*c*d^2 + 3*a*e^2))/(3*e^7*(d + e*x)^6) - (3*c^2*(5*c*d^2 + a*e^
2))/(5*e^7*(d + e*x)^5) + (3*c^3*d)/(2*e^7*(d + e*x)^4) - c^3/(3*e^7*(d + e
*x)^3)
```


Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+cx^2)^3}{(d+ex)^{10}}\,dx=\int \left(\frac{(cd^2+ae^2)^3}{e^6(d+ex)^{10}}-\frac{6cd(cd^2+ae^2)^2}{e^6(d+ex)^9}+\frac{3c(cd^2+ae^2)(5cd^2+ae^2)}{e^6(d+ex)^8}-\frac{4c^2d(5cd^2+3ae^2)}{e^6(d+ex)^7}+\frac{3c^2(5c}{e^6(d}\right.$$

$$=-\frac{(cd^2+ae^2)^3}{9e^7(d+ex)^9}+\frac{3cd(cd^2+ae^2)^2}{4e^7(d+ex)^8}-\frac{3c(cd^2+ae^2)(5cd^2+ae^2)}{7e^7(d+ex)^7}+\frac{2c^2d(5cd^2+3ae^2)}{3e^7(d+ex)^6}-\frac{3c^2(5cd^2}{5e^7(d+}$$

Mathematica [A] time = 0.0537566, size = 163, normalized size = 0.86

$$-\frac{15a^2ce^4(d^2+9dex+36e^2x^2)+140a^3e^6+6ac^2e^2(36d^2e^2x^2+9d^3ex+d^4+84de^3x^3+126e^4x^4)+5c^3(36d^4e^2x^2+8}{1260e^7(d+ex)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^3/(d + e*x)^10,x]
```

```
[Out] -(140*a^3*e^6 + 15*a^2*c*e^4*(d^2 + 9*d*e*x + 36*e^2*x^2) + 6*a*c^2*e^2*(d^
4 + 9*d^3*e*x + 36*d^2*e^2*x^2 + 84*d*e^3*x^3 + 126*e^4*x^4) + 5*c^3*(d^6 +
 9*d^5*e*x + 36*d^4*e^2*x^2 + 84*d^3*e^3*x^3 + 126*d^2*e^4*x^4 + 126*d*e^5*
x^5 + 84*e^6*x^6))/(1260*e^7*(d + e*x)^9)
```

Maple [A] time = 0.049, size = 218, normalized size = 1.2

$$\frac{3\,c^3d}{2\,e^7\,(ex+d)^4}+\frac{2\,c^2d\left(3\,ae^2+5\,cd^2\right)}{3\,e^7\,(ex+d)^6}+\frac{3\,cd\left(a^2e^4+2\,acd^2e^2+c^2d^4\right)}{4\,e^7\,(ex+d)^8}-\frac{c^3}{3\,e^7\,(ex+d)^3}-\frac{a^3e^6+3\,a^2cd^2e^4+3\,d^4e^2ac^2+d^6c}{9\,e^7\,(ex+d)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3/(e*x+d)^10,x)
```

```
[Out] 3/2*c^3*d/e^7/(e*x+d)^4+2/3*c^2*d*(3*a*e^2+5*c*d^2)/e^7/(e*x+d)^6+3/4*c*d*(
a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)/e^7/(e*x+d)^8-1/3*c^3/e^7/(e*x+d)^3-1/9*(a^3
*e^6+3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2+c^3*d^6)/e^7/(e*x+d)^9-3/7*c*(a^2*e^4+
6*a*c*d^2*e^2+5*c^2*d^4)/e^7/(e*x+d)^7-3/5*c^2*(a*e^2+5*c*d^2)/e^7/(e*x+d)^
5
```

Maxima [A] time = 1.25438, size = 396, normalized size = 2.08

$$-\frac{420\,c^3e^6x^6+630\,c^3de^5x^5+5\,c^3d^6+6\,ac^2d^4e^2+15\,a^2cd^2e^4+140\,a^3e^6+126\left(5\,c^3d^2e^4+6\,ac^2e^6\right)x^4+84\left(5\,c^3d^3e^3+6\,ac\right.}{1260\left(e^{16}x^9+9\,de^{15}x^8+36\,d^2e^{14}x^7+84\,d^3e^{13}x^6+126\,d^4e^{12}x^5+126\,d^5e^{11}x^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^10,x, algorithm="maxima")
```

```
[Out] -1/1260*(420*c^3*e^6*x^6 + 630*c^3*d*e^5*x^5 + 5*c^3*d^6 + 6*a*c^2*d^4*e^2
+ 15*a^2*c*d^2*e^4 + 140*a^3*e^6 + 126*(5*c^3*d^2*e^4 + 6*a*c^2*e^6)*x^4 +
84*(5*c^3*d^3*e^3 + 6*a*c^2*d*e^5)*x^3 + 36*(5*c^3*d^4*e^2 + 6*a*c^2*d^2*e^
4 + 15*a^2*c*e^6)*x^2 + 9*(5*c^3*d^5*e + 6*a*c^2*d^3*e^3 + 15*a^2*c*d*e^5)*
x)/(e^16*x^9 + 9*d*e^15*x^8 + 36*d^2*e^14*x^7 + 84*d^3*e^13*x^6 + 126*d^4*e
^12*x^5 + 126*d^5*e^11*x^4 + 84*d^6*e^10*x^3 + 36*d^7*e^9*x^2 + 9*d^8*e^8*x
 + d^9*e^7)
```

Fricas [A] time = 2.20905, size = 625, normalized size = 3.29

$$-\frac{420\,c^3e^6x^6+630\,c^3de^5x^5+5\,c^3d^6+6\,ac^2d^4e^2+15\,a^2cd^2e^4+140\,a^3e^6+126\left(5\,c^3d^2e^4+6\,ac^2e^6\right)x^4+84\left(5\,c^3d^3e^3+6\,ac\right.}{1260\left(e^{16}x^9+9\,de^{15}x^8+36\,d^2e^{14}x^7+84\,d^3e^{13}x^6+126\,d^4e^{12}x^5+126\,d^5e^{11}x^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^10,x, algorithm="fricas")
```

```
[Out] -1/1260*(420*c^3*e^6*x^6 + 630*c^3*d*e^5*x^5 + 5*c^3*d^6 + 6*a*c^2*d^4*e^2
+ 15*a^2*c*d^2*e^4 + 140*a^3*e^6 + 126*(5*c^3*d^2*e^4 + 6*a*c^2*e^6)*x^4 +
```

```
84*(5*c^3*d^3*e^3 + 6*a*c^2*d*e^5)*x^3 + 36*(5*c^3*d^4*e^2 + 6*a*c^2*d^2*e^
4 + 15*a^2*c*e^6)*x^2 + 9*(5*c^3*d^5*e + 6*a*c^2*d^3*e^3 + 15*a^2*c*d*e^5)*
x)/(e^16*x^9 + 9*d*e^15*x^8 + 36*d^2*e^14*x^7 + 84*d^3*e^13*x^6 + 126*d^4*e
^12*x^5 + 126*d^5*e^11*x^4 + 84*d^6*e^10*x^3 + 36*d^7*e^9*x^2 + 9*d^8*e^8*x
 + d^9*e^7)
```

Sympy [A] time = 44.4539, size = 308, normalized size = 1.62

$$-\frac{140a^3e^6 + 15a^2cd^2e^4 + 6ac^2d^4e^2 + 5c^3d^6 + 630c^3de^5x^5 + 420c^3e^6x^6 + x^4\left(756ac^2e^6 + 630c^3d^2e^4\right) + x^3\left(504ac^2de^5 + \cdots\right.}{1260d^9e^7 + 11340d^8e^8x + 45360d^7e^9x^2 + 105840d^6e^{10}x^3 + 158760d^5e^{11}x^4 + 158760d^4e^{1\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3/(e*x+d)**10,x)
```

```
[Out] -(140*a**3*e**6 + 15*a**2*c*d**2*e**4 + 6*a*c**2*d**4*e**2 + 5*c**3*d**6 +
630*c**3*d*e**5*x**5 + 420*c**3*e**6*x**6 + x**4*(756*a*c**2*e**6 + 630*c**
3*d**2*e**4) + x**3*(504*a*c**2*d*e**5 + 420*c**3*d**3*e**3) + x**2*(540*a*
*2*c*e**6 + 216*a*c**2*d**2*e**4 + 180*c**3*d**4*e**2) + x*(135*a**2*c*d*e*
*5 + 54*a*c**2*d**3*e**3 + 45*c**3*d**5*e))/(1260*d**9*e**7 + 11340*d**8*e*
*8*x + 45360*d**7*e**9*x**2 + 105840*d**6*e**10*x**3 + 158760*d**5*e**11*x*
*4 + 158760*d**4*e**12*x**5 + 105840*d**3*e**13*x**6 + 45360*d**2*e**14*x**
7 + 11340*d*e**15*x**8 + 1260*e**16*x**9)
```

Giac [A] time = 1.33224, size = 258, normalized size = 1.36

$$-\frac{\left(420\,c^3x^6e^6 + 630\,c^3dx^5e^5 + 630\,c^3d^2x^4e^4 + 420\,c^3d^3x^3e^3 + 180\,c^3d^4x^2e^2 + 45\,c^3d^5xe + 5\,c^3d^6 + 756\,ac^2x^4e^6 + 504\,a\cdots\right.}{1260\,(xe + \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^10,x, algorithm="giac")
```

```
[Out] -1/1260*(420*c^3*x^6*e^6 + 630*c^3*d*x^5*e^5 + 630*c^3*d^2*x^4*e^4 + 420*c^
3*d^3*x^3*e^3 + 180*c^3*d^4*x^2*e^2 + 45*c^3*d^5*x*e + 5*c^3*d^6 + 756*a*c^
2*x^4*e^6 + 504*a*c^2*d*x^3*e^5 + 216*a*c^2*d^2*x^2*e^4 + 54*a*c^2*d^3*x*e^
3 + 6*a*c^2*d^4*e^2 + 540*a^2*c*x^2*e^6 + 135*a^2*c*d*x*e^5 + 15*a^2*c*d^2*
e^4 + 140*a^3*e^6)*e^(-7)/(x*e + d)^9
```

3.488 $\int (d+ex)^7 \left(a+cx^2\right)^4 \, dx$

Optimal. Leaf size=278

$$\frac{c^2(d+ex)^{12}\left(3a^2e^4+30acd^2e^2+35c^2d^4\right)}{6e^9}+\frac{2c^3(d+ex)^{14}\left(ae^2+7cd^2\right)}{7e^9}-\frac{8c^3d(d+ex)^{13}\left(3ae^2+7cd^2\right)}{13e^9}-\frac{8c^2d(d+ex)^{11}\left(\right.}{}$$

```
[Out] ((c*d^2 + a*e^2)^4*(d + e*x)^8)/(8*e^9) - (8*c*d*(c*d^2 + a*e^2)^3*(d + e*x
)^9)/(9*e^9) + (2*c*(c*d^2 + a*e^2)^2*(7*c*d^2 + a*e^2)*(d + e*x)^10)/(5*e^
9) - (8*c^2*d*(c*d^2 + a*e^2)*(7*c*d^2 + 3*a*e^2)*(d + e*x)^11)/(11*e^9) +
(c^2*(35*c^2*d^4 + 30*a*c*d^2*e^2 + 3*a^2*e^4)*(d + e*x)^12)/(6*e^9) - (8*c
^3*d*(7*c*d^2 + 3*a*e^2)*(d + e*x)^13)/(13*e^9) + (2*c^3*(7*c*d^2 + a*e^2)*
(d + e*x)^14)/(7*e^9) - (8*c^4*d*(d + e*x)^15)/(15*e^9) + (c^4*(d + e*x)^16
)/(16*e^9)
```

Rubi [A] time = 0.515855, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$\frac{c^2(d+ex)^{12}\left(3a^2e^4+30acd^2e^2+35c^2d^4\right)}{6e^9}+\frac{2c^3(d+ex)^{14}\left(ae^2+7cd^2\right)}{7e^9}-\frac{8c^3d(d+ex)^{13}\left(3ae^2+7cd^2\right)}{13e^9}-\frac{8c^2d(d+ex)^{11}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^7*(a + c*x^2)^4,x]
```


```
[Out] ((c*d^2 + a*e^2)^4*(d + e*x)^8)/(8*e^9) - (8*c*d*(c*d^2 + a*e^2)^3*(d + e*x
)^9)/(9*e^9) + (2*c*(c*d^2 + a*e^2)^2*(7*c*d^2 + a*e^2)*(d + e*x)^10)/(5*e^
9) - (8*c^2*d*(c*d^2 + a*e^2)*(7*c*d^2 + 3*a*e^2)*(d + e*x)^11)/(11*e^9) +
(c^2*(35*c^2*d^4 + 30*a*c*d^2*e^2 + 3*a^2*e^4)*(d + e*x)^12)/(6*e^9) - (8*c
^3*d*(7*c*d^2 + 3*a*e^2)*(d + e*x)^13)/(13*e^9) + (2*c^3*(7*c*d^2 + a*e^2)*
(d + e*x)^14)/(7*e^9) - (8*c^4*d*(d + e*x)^15)/(15*e^9) + (c^4*(d + e*x)^16
)/(16*e^9)
```


Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (d+ex)^7\left(a+cx^2\right)^4\,dx=\int \left(\frac{\left(cd^2+ae^2\right)^4(d+ex)^7}{e^8}-\frac{8cd\left(cd^2+ae^2\right)^3(d+ex)^8}{e^8}+\frac{4c\left(cd^2+ae^2\right)^2\left(7cd^2+ae^2\right)(d+ex}{e^8}\right.$$

$$=\frac{\left(cd^2+ae^2\right)^4(d+ex)^8}{8e^9}-\frac{8cd\left(cd^2+ae^2\right)^3(d+ex)^9}{9e^9}+\frac{2c\left(cd^2+ae^2\right)^2\left(7cd^2+ae^2\right)(d+ex)^{10}}{5e^9}$$

Mathematica [A] time = 0.083014, size = 423, normalized size = 1.52

$$\frac{1}{660}a^2c^2x^5\left(11880d^5e^2x^2+17325d^4e^3x^3+15400d^3e^4x^4+8316d^2e^5x^5+4620d^6ex+792d^7+2520de^6x^6+330e^7x^7\right)+\frac{1}{90}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^7*(a + c*x^2)^4,x]
```

```
[Out] (a^4*x*(8*d^7 + 28*d^6*e*x + 56*d^5*e^2*x^2 + 70*d^4*e^3*x^3 + 56*d^3*e^4*x
^4 + 28*d^2*e^5*x^5 + 8*d*e^6*x^6 + e^7*x^7))/8 + (a^3*c*x^3*(120*d^7 + 630
*d^6*e*x + 1512*d^5*e^2*x^2 + 2100*d^4*e^3*x^3 + 1800*d^3*e^4*x^4 + 945*d^2
*e^5*x^5 + 280*d*e^6*x^6 + 36*e^7*x^7))/90 + (a^2*c^2*x^5*(792*d^7 + 4620*d
^6*e*x + 11880*d^5*e^2*x^2 + 17325*d^4*e^3*x^3 + 15400*d^3*e^4*x^4 + 8316*d
^2*e^5*x^5 + 2520*d*e^6*x^6 + 330*e^7*x^7))/660 + (a*c^3*x^7*(3432*d^7 + 21
021*d^6*e*x + 56056*d^5*e^2*x^2 + 84084*d^4*e^3*x^3 + 76440*d^3*e^4*x^4 + 4
2042*d^2*e^5*x^5 + 12936*d*e^6*x^6 + 1716*e^7*x^7))/6006 + (c^4*x^9*(11440*
d^7 + 72072*d^6*e*x + 196560*d^5*e^2*x^2 + 300300*d^4*e^3*x^3 + 277200*d^3*
e^4*x^4 + 154440*d^2*e^5*x^5 + 48048*d*e^6*x^6 + 6435*e^7*x^7))/102960
```

Maple [A] time = 0.049, size = 511, normalized size = 1.8

$$\frac{e^7c^4x^{16}}{16}+\frac{7\,de^6c^4x^{15}}{15}+\frac{\left(4\,e^7ac^3+21\,d^2e^5c^4\right)x^{14}}{14}+\frac{\left(28\,de^6ac^3+35\,d^3e^4c^4\right)x^{13}}{13}+\frac{\left(6\,e^7a^2c^2+84\,d^2e^5ac^3+35\,d^4e^3c^4\right)x}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^7*(c*x^2+a)^4,x)
```

```
[Out] 1/16*e^7*c^4*x^16+7/15*d*e^6*c^4*x^15+1/14*(4*a*c^3*e^7+21*c^4*d^2*e^5)*x^1
4+1/13*(28*a*c^3*d*e^6+35*c^4*d^3*e^4)*x^13+1/12*(6*a^2*c^2*e^7+84*a*c^3*d^
2*e^5+35*c^4*d^4*e^3)*x^12+1/11*(42*a^2*c^2*d*e^6+140*a*c^3*d^3*e^4+21*c^4*
d^5*e^2)*x^11+1/10*(4*a^3*c*e^7+126*a^2*c^2*d^2*e^5+140*a*c^3*d^4*e^3+7*c^4
*d^6*e)*x^10+1/9*(28*a^3*c*d*e^6+210*a^2*c^2*d^3*e^4+84*a*c^3*d^5*e^2+c^4*d
^7)*x^9+1/8*(a^4*e^7+84*a^3*c*d^2*e^5+210*a^2*c^2*d^4*e^3+28*a*c^3*d^6*e)*x
^8+1/7*(7*a^4*d*e^6+140*a^3*c*d^3*e^4+126*a^2*c^2*d^5*e^2+4*a*c^3*d^7)*x^7+
1/6*(21*a^4*d^2*e^5+140*a^3*c*d^4*e^3+42*a^2*c^2*d^6*e)*x^6+1/5*(35*a^4*d^3
*e^4+84*a^3*c*d^5*e^2+6*a^2*c^2*d^7)*x^5+1/4*(35*a^4*d^4*e^3+28*a^3*c*d^6*e
)*x^4+1/3*(21*a^4*d^5*e^2+4*a^3*c*d^7)*x^3+7/2*d^6*e*a^4*x^2+d^7*a^4*x
```

Maxima [A] time = 1.1466, size = 689, normalized size = 2.48

$$\frac{1}{16}\,c^4e^7x^{16}+\frac{7}{15}\,c^4de^6x^{15}+\frac{1}{14}\left(21\,c^4d^2e^5+4\,ac^3e^7\right)x^{14}+\frac{7}{13}\left(5\,c^4d^3e^4+4\,ac^3de^6\right)x^{13}+\frac{7}{2}\,a^4d^6ex^2+\frac{1}{12}\left(35\,c^4d^4e^3+8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7*(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] 1/16*c^4*e^7*x^16 + 7/15*c^4*d*e^6*x^15 + 1/14*(21*c^4*d^2*e^5 + 4*a*c^3*e^
7)*x^14 + 7/13*(5*c^4*d^3*e^4 + 4*a*c^3*d*e^6)*x^13 + 7/2*a^4*d^6*e*x^2 + 1
/12*(35*c^4*d^4*e^3 + 84*a*c^3*d^2*e^5 + 6*a^2*c^2*e^7)*x^12 + a^4*d^7*x +
7/11*(3*c^4*d^5*e^2 + 20*a*c^3*d^3*e^4 + 6*a^2*c^2*d*e^6)*x^11 + 1/10*(7*c^
4*d^6*e + 140*a*c^3*d^4*e^3 + 126*a^2*c^2*d^2*e^5 + 4*a^3*c*e^7)*x^10 + 1/9
*(c^4*d^7 + 84*a*c^3*d^5*e^2 + 210*a^2*c^2*d^3*e^4 + 28*a^3*c*d*e^6)*x^9 +
1/8*(28*a*c^3*d^6*e + 210*a^2*c^2*d^4*e^3 + 84*a^3*c*d^2*e^5 + a^4*e^7)*x^8
 + 1/7*(4*a*c^3*d^7 + 126*a^2*c^2*d^5*e^2 + 140*a^3*c*d^3*e^4 + 7*a^4*d*e^6
)*x^7 + 7/6*(6*a^2*c^2*d^6*e + 20*a^3*c*d^4*e^3 + 3*a^4*d^2*e^5)*x^6 + 1/5*
(6*a^2*c^2*d^7 + 84*a^3*c*d^5*e^2 + 35*a^4*d^3*e^4)*x^5 + 7/4*(4*a^3*c*d^6*
```

```
e + 5*a^4*d^4*e^3)*x^4 + 1/3*(4*a^3*c*d^7 + 21*a^4*d^5*e^2)*x^3
```

Fricas [B] time = 1.86475, size = 1218, normalized size = 4.38

$$\frac{1}{16}x^{16}e^{7}c^{4} + \frac{7}{15}x^{15}e^{6}dc^{4} + \frac{3}{2}x^{14}e^{5}d^{2}c^{4} + \frac{2}{7}x^{14}e^{7}c^{3}a + \frac{35}{13}x^{13}e^{4}d^{3}c^{4} + \frac{28}{13}x^{13}e^{6}dc^{3}a + \frac{35}{12}x^{12}e^{3}d^{4}c^{4} + 7x^{12}e^{5}d^{2}c^{3}a + \frac{1}{2}x^{12}e^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7*(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*e^7*c^4 + 7/15*x^15*e^6*d*c^4 + 3/2*x^14*e^5*d^2*c^4 + 2/7*x^14*e
^7*c^3*a + 35/13*x^13*e^4*d^3*c^4 + 28/13*x^13*e^6*d*c^3*a + 35/12*x^12*e^3
*d^4*c^4 + 7*x^12*e^5*d^2*c^3*a + 1/2*x^12*e^7*c^2*a^2 + 21/11*x^11*e^2*d^5
*c^4 + 140/11*x^11*e^4*d^3*c^3*a + 42/11*x^11*e^6*d*c^2*a^2 + 7/10*x^10*e*d
^6*c^4 + 14*x^10*e^3*d^4*c^3*a + 63/5*x^10*e^5*d^2*c^2*a^2 + 2/5*x^10*e^7*c
*a^3 + 1/9*x^9*d^7*c^4 + 28/3*x^9*e^2*d^5*c^3*a + 70/3*x^9*e^4*d^3*c^2*a^2
+ 28/9*x^9*e^6*d*c*a^3 + 7/2*x^8*e*d^6*c^3*a + 105/4*x^8*e^3*d^4*c^2*a^2 +
21/2*x^8*e^5*d^2*c*a^3 + 1/8*x^8*e^7*a^4 + 4/7*x^7*d^7*c^3*a + 18*x^7*e^2*d
^5*c^2*a^2 + 20*x^7*e^4*d^3*c*a^3 + x^7*e^6*d*a^4 + 7*x^6*e*d^6*c^2*a^2 + 7
0/3*x^6*e^3*d^4*c*a^3 + 7/2*x^6*e^5*d^2*a^4 + 6/5*x^5*d^7*c^2*a^2 + 84/5*x^
5*e^2*d^5*c*a^3 + 7*x^5*e^4*d^3*a^4 + 7*x^4*e*d^6*c*a^3 + 35/4*x^4*e^3*d^4*
a^4 + 4/3*x^3*d^7*c*a^3 + 7*x^3*e^2*d^5*a^4 + 7/2*x^2*e*d^6*a^4 + x*d^7*a^4
```

Sympy [B] time = 0.137736, size = 571, normalized size = 2.05

$$a^{4}d^{7}x + \frac{7a^{4}d^{6}ex^{2}}{2} + \frac{7c^{4}de^{6}x^{15}}{15} + \frac{c^{4}e^{7}x^{16}}{16} + x^{14}\left(\frac{2ac^{3}e^{7}}{7} + \frac{3c^{4}d^{2}e^{5}}{2}\right) + x^{13}\left(\frac{28ac^{3}de^{6}}{13} + \frac{35c^{4}d^{3}e^{4}}{13}\right) + x^{12}\left(\frac{a^{2}c^{2}e^{7}}{2} + 7ac^{3}d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**7*(c*x**2+a)**4,x)
```

```
[Out] a**4*d**7*x + 7*a**4*d**6*e*x**2/2 + 7*c**4*d*e**6*x**15/15 + c**4*e**7*x**
16/16 + x**14*(2*a*c**3*e**7/7 + 3*c**4*d**2*e**5/2) + x**13*(28*a*c**3*d*e
**6/13 + 35*c**4*d**3*e**4/13) + x**12*(a**2*c**2*e**7/2 + 7*a*c**3*d**2*e*
*5 + 35*c**4*d**4*e**3/12) + x**11*(42*a**2*c**2*d*e**6/11 + 140*a*c**3*d**
3*e**4/11 + 21*c**4*d**5*e**2/11) + x**10*(2*a**3*c*e**7/5 + 63*a**2*c**2*d
**2*e**5/5 + 14*a*c**3*d**4*e**3 + 7*c**4*d**6*e/10) + x**9*(28*a**3*c*d*e*
*6/9 + 70*a**2*c**2*d**3*e**4/3 + 28*a*c**3*d**5*e**2/3 + c**4*d**7/9) + x*
*8*(a**4*e**7/8 + 21*a**3*c*d**2*e**5/2 + 105*a**2*c**2*d**4*e**3/4 + 7*a*c
**3*d**6*e/2) + x**7*(a**4*d*e**6 + 20*a**3*c*d**3*e**4 + 18*a**2*c**2*d**5
*e**2 + 4*a*c**3*d**7/7) + x**6*(7*a**4*d**2*e**5/2 + 70*a**3*c*d**4*e**3/3
 + 7*a**2*c**2*d**6*e) + x**5*(7*a**4*d**3*e**4 + 84*a**3*c*d**5*e**2/5 + 6
*a**2*c**2*d**7/5) + x**4*(35*a**4*d**4*e**3/4 + 7*a**3*c*d**6*e) + x**3*(7
*a**4*d**5*e**2 + 4*a**3*c*d**7/3)
```

Giac [B] time = 1.33366, size = 705, normalized size = 2.54

$$\frac{1}{16}c^{4}x^{16}e^{7} + \frac{7}{15}c^{4}dx^{15}e^{6} + \frac{3}{2}c^{4}d^{2}x^{14}e^{5} + \frac{35}{13}c^{4}d^{3}x^{13}e^{4} + \frac{35}{12}c^{4}d^{4}x^{12}e^{3} + \frac{21}{11}c^{4}d^{5}x^{11}e^{2} + \frac{7}{10}c^{4}d^{6}x^{10}e + \frac{1}{9}c^{4}d^{7}x^{9} + \frac{2}{7}ac^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7*(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*c^4*x^16*e^7 + 7/15*c^4*d*x^15*e^6 + 3/2*c^4*d^2*x^14*e^5 + 35/13*c^4*
d^3*x^13*e^4 + 35/12*c^4*d^4*x^12*e^3 + 21/11*c^4*d^5*x^11*e^2 + 7/10*c^4*d
^6*x^10*e + 1/9*c^4*d^7*x^9 + 2/7*a*c^3*x^14*e^7 + 28/13*a*c^3*d*x^13*e^6 +
 7*a*c^3*d^2*x^12*e^5 + 140/11*a*c^3*d^3*x^11*e^4 + 14*a*c^3*d^4*x^10*e^3 +
 28/3*a*c^3*d^5*x^9*e^2 + 7/2*a*c^3*d^6*x^8*e + 4/7*a*c^3*d^7*x^7 + 1/2*a^2
*c^2*x^12*e^7 + 42/11*a^2*c^2*d*x^11*e^6 + 63/5*a^2*c^2*d^2*x^10*e^5 + 70/3
*a^2*c^2*d^3*x^9*e^4 + 105/4*a^2*c^2*d^4*x^8*e^3 + 18*a^2*c^2*d^5*x^7*e^2 +
 7*a^2*c^2*d^6*x^6*e + 6/5*a^2*c^2*d^7*x^5 + 2/5*a^3*c*x^10*e^7 + 28/9*a^3*
c*d*x^9*e^6 + 21/2*a^3*c*d^2*x^8*e^5 + 20*a^3*c*d^3*x^7*e^4 + 70/3*a^3*c*d^
4*x^6*e^3 + 84/5*a^3*c*d^5*x^5*e^2 + 7*a^3*c*d^6*x^4*e + 4/3*a^3*c*d^7*x^3
+ 1/8*a^4*x^8*e^7 + a^4*d*x^7*e^6 + 7/2*a^4*d^2*x^6*e^5 + 7*a^4*d^3*x^5*e^4
 + 35/4*a^4*d^4*x^4*e^3 + 7*a^4*d^5*x^3*e^2 + 7/2*a^4*d^6*x^2*e + a^4*d^7*x
```

3.489 $\int (d+ex)^6 \left(a+cx^2\right)^4 \, dx$

Optimal. Leaf size=276

$$\frac{2c^2(d+ex)^{11}\left(3a^2e^4+30acd^2e^2+35c^2d^4\right)}{11e^9}+\frac{4c^3(d+ex)^{13}\left(ae^2+7cd^2\right)}{13e^9}-\frac{2c^3d(d+ex)^{12}\left(3ae^2+7cd^2\right)}{3e^9}-\frac{4c^2d(d+ex)^{10}}{}$$

```
[Out] ((c*d^2 + a*e^2)^4*(d + e*x)^7)/(7*e^9) - (c*d*(c*d^2 + a*e^2)^3*(d + e*x)^
8)/e^9 + (4*c*(c*d^2 + a*e^2)^2*(7*c*d^2 + a*e^2)*(d + e*x)^9)/(9*e^9) - (4
*c^2*d*(c*d^2 + a*e^2)*(7*c*d^2 + 3*a*e^2)*(d + e*x)^10)/(5*e^9) + (2*c^2*(
35*c^2*d^4 + 30*a*c*d^2*e^2 + 3*a^2*e^4)*(d + e*x)^11)/(11*e^9) - (2*c^3*d*
(7*c*d^2 + 3*a*e^2)*(d + e*x)^12)/(3*e^9) + (4*c^3*(7*c*d^2 + a*e^2)*(d + e
*x)^13)/(13*e^9) - (4*c^4*d*(d + e*x)^14)/(7*e^9) + (c^4*(d + e*x)^15)/(15*
e^9)
```

Rubi [A] time = 0.458363, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$\frac{2c^2(d+ex)^{11}\left(3a^2e^4+30acd^2e^2+35c^2d^4\right)}{11e^9}+\frac{4c^3(d+ex)^{13}\left(ae^2+7cd^2\right)}{13e^9}-\frac{2c^3d(d+ex)^{12}\left(3ae^2+7cd^2\right)}{3e^9}-\frac{4c^2d(d+ex)^{10}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^6*(a + c*x^2)^4,x]
```

```
[Out] ((c*d^2 + a*e^2)^4*(d + e*x)^7)/(7*e^9) - (c*d*(c*d^2 + a*e^2)^3*(d + e*x)^
8)/e^9 + (4*c*(c*d^2 + a*e^2)^2*(7*c*d^2 + a*e^2)*(d + e*x)^9)/(9*e^9) - (4
*c^2*d*(c*d^2 + a*e^2)*(7*c*d^2 + 3*a*e^2)*(d + e*x)^10)/(5*e^9) + (2*c^2*(
35*c^2*d^4 + 30*a*c*d^2*e^2 + 3*a^2*e^4)*(d + e*x)^11)/(11*e^9) - (2*c^3*d*
(7*c*d^2 + 3*a*e^2)*(d + e*x)^12)/(3*e^9) + (4*c^3*(7*c*d^2 + a*e^2)*(d + e
*x)^13)/(13*e^9) - (4*c^4*d*(d + e*x)^14)/(7*e^9) + (c^4*(d + e*x)^15)/(15*
e^9)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (d+ex)^6\left(a+cx^2\right)^4\,dx=\int \left(\frac{\left(cd^2+ae^2\right)^4(d+ex)^6}{e^8}-\frac{8cd\left(cd^2+ae^2\right)^3(d+ex)^7}{e^8}+\frac{4c\left(cd^2+ae^2\right)^2\left(7cd^2+ae^2\right)(d+ex)}{e^8}\right.$$

$$=\frac{\left(cd^2+ae^2\right)^4(d+ex)^7}{7e^9}-\frac{cd\left(cd^2+ae^2\right)^3(d+ex)^8}{e^9}+\frac{4c\left(cd^2+ae^2\right)^2\left(7cd^2+ae^2\right)(d+ex)^9}{9e^9}-$$

Mathematica [A] time = 0.122988, size = 361, normalized size = 1.31

$$117a^2c^2x^5\left(4950d^4e^2x^2+5775d^3e^3x^3+3850d^2e^4x^4+2310d^5ex+462d^6+1386de^5x^5+210e^6x^6\right)+715a^3cx^3\left(756d^4e^2x^2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^6*(a + c*x^2)^4,x]
```

```
[Out] (6435*a^4*x*(7*d^6 + 21*d^5*e*x + 35*d^4*e^2*x^2 + 35*d^3*e^3*x^3 + 21*d^2*
e^4*x^4 + 7*d*e^5*x^5 + e^6*x^6) + 715*a^3*c*x^3*(84*d^6 + 378*d^5*e*x + 75
6*d^4*e^2*x^2 + 840*d^3*e^3*x^3 + 540*d^2*e^4*x^4 + 189*d*e^5*x^5 + 28*e^6*
x^6) + 117*a^2*c^2*x^5*(462*d^6 + 2310*d^5*e*x + 4950*d^4*e^2*x^2 + 5775*d^
3*e^3*x^3 + 3850*d^2*e^4*x^4 + 1386*d*e^5*x^5 + 210*e^6*x^6) + 15*a*c^3*x^7
*(1716*d^6 + 9009*d^5*e*x + 20020*d^4*e^2*x^2 + 24024*d^3*e^3*x^3 + 16380*d
^2*e^4*x^4 + 6006*d*e^5*x^5 + 924*e^6*x^6) + c^4*x^9*(5005*d^6 + 27027*d^5*
e*x + 61425*d^4*e^2*x^2 + 75075*d^3*e^3*x^3 + 51975*d^2*e^4*x^4 + 19305*d*e
^5*x^5 + 3003*e^6*x^6))/45045
```

Maple [A] time = 0.043, size = 445, normalized size = 1.6

$$\frac{e^6c^4x^{15}}{15}+\frac{3\,de^5c^4x^{14}}{7}+\frac{\left(4\,e^6ac^3+15\,d^2e^4c^4\right)x^{13}}{13}+\frac{\left(24\,de^5ac^3+20\,d^3e^3c^4\right)x^{12}}{12}+\frac{\left(6\,e^6a^2c^2+60\,d^2e^4ac^3+15\,d^4e^2c^4\right)x}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^6*(c*x^2+a)^4,x)
```

```
[Out] 1/15*e^6*c^4*x^15+3/7*d*e^5*c^4*x^14+1/13*(4*a*c^3*e^6+15*c^4*d^2*e^4)*x^13
+1/12*(24*a*c^3*d*e^5+20*c^4*d^3*e^3)*x^12+1/11*(6*a^2*c^2*e^6+60*a*c^3*d^2
*e^4+15*c^4*d^4*e^2)*x^11+1/10*(36*a^2*c^2*d*e^5+80*a*c^3*d^3*e^3+6*c^4*d^5
*e)*x^10+1/9*(4*a^3*c*e^6+90*a^2*c^2*d^2*e^4+60*a*c^3*d^4*e^2+c^4*d^6)*x^9+
1/8*(24*a^3*c*d*e^5+120*a^2*c^2*d^3*e^3+24*a*c^3*d^5*e)*x^8+1/7*(a^4*e^6+60
*a^3*c*d^2*e^4+90*a^2*c^2*d^4*e^2+4*a*c^3*d^6)*x^7+1/6*(6*a^4*d*e^5+80*a^3*
c*d^3*e^3+36*a^2*c^2*d^5*e)*x^6+1/5*(15*a^4*d^2*e^4+60*a^3*c*d^4*e^2+6*a^2*
c^2*d^6)*x^5+1/4*(20*a^4*d^3*e^3+24*a^3*c*d^5*e)*x^4+1/3*(15*a^4*d^4*e^2+4*
a^3*c*d^6)*x^3+3*d^5*e*a^4*x^2+d^6*a^4*x
```

Maxima [A] time = 1.19209, size = 595, normalized size = 2.16

$$\frac{1}{15}\,c^4e^6x^{15}+\frac{3}{7}\,c^4de^5x^{14}+\frac{1}{13}\left(15\,c^4d^2e^4+4\,ac^3e^6\right)x^{13}+\frac{1}{3}\left(5\,c^4d^3e^3+6\,ac^3de^5\right)x^{12}+3\,a^4d^5ex^2+\frac{3}{11}\left(5\,c^4d^4e^2+20\,ac\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] 1/15*c^4*e^6*x^15 + 3/7*c^4*d*e^5*x^14 + 1/13*(15*c^4*d^2*e^4 + 4*a*c^3*e^6
)*x^13 + 1/3*(5*c^4*d^3*e^3 + 6*a*c^3*d*e^5)*x^12 + 3*a^4*d^5*e*x^2 + 3/11*
(5*c^4*d^4*e^2 + 20*a*c^3*d^2*e^4 + 2*a^2*c^2*e^6)*x^11 + a^4*d^6*x + 1/5*(
3*c^4*d^5*e + 40*a*c^3*d^3*e^3 + 18*a^2*c^2*d*e^5)*x^10 + 1/9*(c^4*d^6 + 60
*a*c^3*d^4*e^2 + 90*a^2*c^2*d^2*e^4 + 4*a^3*c*e^6)*x^9 + 3*(a*c^3*d^5*e + 5
*a^2*c^2*d^3*e^3 + a^3*c*d*e^5)*x^8 + 1/7*(4*a*c^3*d^6 + 90*a^2*c^2*d^4*e^2
 + 60*a^3*c*d^2*e^4 + a^4*e^6)*x^7 + 1/3*(18*a^2*c^2*d^5*e + 40*a^3*c*d^3*e
^3 + 3*a^4*d*e^5)*x^6 + 3/5*(2*a^2*c^2*d^6 + 20*a^3*c*d^4*e^2 + 5*a^4*d^2*e
^4)*x^5 + (6*a^3*c*d^5*e + 5*a^4*d^3*e^3)*x^4 + 1/3*(4*a^3*c*d^6 + 15*a^4*d
^4*e^2)*x^3
```

Fricas [A] time = 1.87541, size = 1023, normalized size = 3.71

$$\frac{1}{15}x^{15}e^6c^4 + \frac{3}{7}x^{14}e^5dc^4 + \frac{15}{13}x^{13}e^4d^2c^4 + \frac{4}{13}x^{13}e^6c^3a + \frac{5}{3}x^{12}e^3d^3c^4 + 2x^{12}e^5dc^3a + \frac{15}{11}x^{11}e^2d^4c^4 + \frac{60}{11}x^{11}e^4d^2c^3a + \frac{6}{11}x^{11}e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*e^6*c^4 + 3/7*x^14*e^5*d*c^4 + 15/13*x^13*e^4*d^2*c^4 + 4/13*x^13
*e^6*c^3*a + 5/3*x^12*e^3*d^3*c^4 + 2*x^12*e^5*d*c^3*a + 15/11*x^11*e^2*d^4
*c^4 + 60/11*x^11*e^4*d^2*c^3*a + 6/11*x^11*e^6*c^2*a^2 + 3/5*x^10*e*d^5*c^
4 + 8*x^10*e^3*d^3*c^3*a + 18/5*x^10*e^5*d*c^2*a^2 + 1/9*x^9*d^6*c^4 + 20/3
*x^9*e^2*d^4*c^3*a + 10*x^9*e^4*d^2*c^2*a^2 + 4/9*x^9*e^6*c*a^3 + 3*x^8*e*d
^5*c^3*a + 15*x^8*e^3*d^3*c^2*a^2 + 3*x^8*e^5*d*c*a^3 + 4/7*x^7*d^6*c^3*a +
 90/7*x^7*e^2*d^4*c^2*a^2 + 60/7*x^7*e^4*d^2*c*a^3 + 1/7*x^7*e^6*a^4 + 6*x^
6*e*d^5*c^2*a^2 + 40/3*x^6*e^3*d^3*c*a^3 + x^6*e^5*d*a^4 + 6/5*x^5*d^6*c^2*
a^2 + 12*x^5*e^2*d^4*c*a^3 + 3*x^5*e^4*d^2*a^4 + 6*x^4*e*d^5*c*a^3 + 5*x^4*
e^3*d^3*a^4 + 4/3*x^3*d^6*c*a^3 + 5*x^3*e^2*d^4*a^4 + 3*x^2*e*d^5*a^4 + x*d
^6*a^4
```

Sympy [A] time = 0.12627, size = 486, normalized size = 1.76

$$a^4d^6x + 3a^4d^5ex^2 + \frac{3c^4de^5x^{14}}{7} + \frac{c^4e^6x^{15}}{15} + x^{13}\left(\frac{4ac^3e^6}{13} + \frac{15c^4d^2e^4}{13}\right) + x^{12}\left(2ac^3de^5 + \frac{5c^4d^3e^3}{3}\right) + x^{11}\left(\frac{6a^2c^2e^6}{11} + \frac{60ac^3d}{11}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**6*(c*x**2+a)**4,x)
```

```
[Out] a**4*d**6*x + 3*a**4*d**5*e*x**2 + 3*c**4*d*e**5*x**14/7 + c**4*e**6*x**15/
15 + x**13*(4*a*c**3*e**6/13 + 15*c**4*d**2*e**4/13) + x**12*(2*a*c**3*d*e*
*5 + 5*c**4*d**3*e**3/3) + x**11*(6*a**2*c**2*e**6/11 + 60*a*c**3*d**2*e**4
/11 + 15*c**4*d**4*e**2/11) + x**10*(18*a**2*c**2*d*e**5/5 + 8*a*c**3*d**3*
e**3 + 3*c**4*d**5*e/5) + x**9*(4*a**3*c*e**6/9 + 10*a**2*c**2*d**2*e**4 +
20*a*c**3*d**4*e**2/3 + c**4*d**6/9) + x**8*(3*a**3*c*d*e**5 + 15*a**2*c**2
*d**3*e**3 + 3*a*c**3*d**5*e) + x**7*(a**4*e**6/7 + 60*a**3*c*d**2*e**4/7 +
 90*a**2*c**2*d**4*e**2/7 + 4*a*c**3*d**6/7) + x**6*(a**4*d*e**5 + 40*a**3*
c*d**3*e**3/3 + 6*a**2*c**2*d**5*e) + x**5*(3*a**4*d**2*e**4 + 12*a**3*c*d*
*4*e**2 + 6*a**2*c**2*d**6/5) + x**4*(5*a**4*d**3*e**3 + 6*a**3*c*d**5*e) +
 x**3*(5*a**4*d**4*e**2 + 4*a**3*c*d**6/3)
```

Giac [A] time = 1.28685, size = 610, normalized size = 2.21

$$\frac{1}{15}c^4x^{15}e^6 + \frac{3}{7}c^4dx^{14}e^5 + \frac{15}{13}c^4d^2x^{13}e^4 + \frac{5}{3}c^4d^3x^{12}e^3 + \frac{15}{11}c^4d^4x^{11}e^2 + \frac{3}{5}c^4d^5x^{10}e + \frac{1}{9}c^4d^6x^9 + \frac{4}{13}ac^3x^{13}e^6 + 2ac^3dx^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/15*c^4*x^15*e^6 + 3/7*c^4*d*x^14*e^5 + 15/13*c^4*d^2*x^13*e^4 + 5/3*c^4*d
^3*x^12*e^3 + 15/11*c^4*d^4*x^11*e^2 + 3/5*c^4*d^5*x^10*e + 1/9*c^4*d^6*x^9
 + 4/13*a*c^3*x^13*e^6 + 2*a*c^3*d*x^12*e^5 + 60/11*a*c^3*d^2*x^11*e^4 + 8*
a*c^3*d^3*x^10*e^3 + 20/3*a*c^3*d^4*x^9*e^2 + 3*a*c^3*d^5*x^8*e + 4/7*a*c^3
```

```
*d^6*x^7 + 6/11*a^2*c^2*x^11*e^6 + 18/5*a^2*c^2*d*x^10*e^5 + 10*a^2*c^2*d^2
*x^9*e^4 + 15*a^2*c^2*d^3*x^8*e^3 + 90/7*a^2*c^2*d^4*x^7*e^2 + 6*a^2*c^2*d^
5*x^6*e + 6/5*a^2*c^2*d^6*x^5 + 4/9*a^3*c*x^9*e^6 + 3*a^3*c*d*x^8*e^5 + 60/
7*a^3*c*d^2*x^7*e^4 + 40/3*a^3*c*d^3*x^6*e^3 + 12*a^3*c*d^4*x^5*e^2 + 6*a^3
*c*d^5*x^4*e + 4/3*a^3*c*d^6*x^3 + 1/7*a^4*x^7*e^6 + a^4*d*x^6*e^5 + 3*a^4*
d^2*x^5*e^4 + 5*a^4*d^3*x^4*e^3 + 5*a^4*d^4*x^3*e^2 + 3*a^4*d^5*x^2*e + a^4
*d^6*x
```

3.490 $\int (d+ex)^5 \left(a+cx^2\right)^4 \, dx$

Optimal. Leaf size=278

$$\frac{c^2(d+ex)^{10}\left(3a^2e^4+30acd^2e^2+35c^2d^4\right)}{5e^9}+\frac{c^3(d+ex)^{12}\left(ae^2+7cd^2\right)}{3e^9}-\frac{8c^3d(d+ex)^{11}\left(3ae^2+7cd^2\right)}{11e^9}-\frac{8c^2d(d+ex)^9\left(ae\right.}{}$$

```
[Out] ((c*d^2 + a*e^2)^4*(d + e*x)^6)/(6*e^9) - (8*c*d*(c*d^2 + a*e^2)^3*(d + e*x
)^7)/(7*e^9) + (c*(c*d^2 + a*e^2)^2*(7*c*d^2 + a*e^2)*(d + e*x)^8)/(2*e^9)
- (8*c^2*d*(c*d^2 + a*e^2)*(7*c*d^2 + 3*a*e^2)*(d + e*x)^9)/(9*e^9) + (c^2*
(35*c^2*d^4 + 30*a*c*d^2*e^2 + 3*a^2*e^4)*(d + e*x)^10)/(5*e^9) - (8*c^3*d*
(7*c*d^2 + 3*a*e^2)*(d + e*x)^11)/(11*e^9) + (c^3*(7*c*d^2 + a*e^2)*(d + e*
x)^12)/(3*e^9) - (8*c^4*d*(d + e*x)^13)/(13*e^9) + (c^4*(d + e*x)^14)/(14*e
^9)
```

Rubi [A] time = 0.406954, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$\frac{c^2(d+ex)^{10}\left(3a^2e^4+30acd^2e^2+35c^2d^4\right)}{5e^9}+\frac{c^3(d+ex)^{12}\left(ae^2+7cd^2\right)}{3e^9}-\frac{8c^3d(d+ex)^{11}\left(3ae^2+7cd^2\right)}{11e^9}-\frac{8c^2d(d+ex)^9\left(ae\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5*(a + c*x^2)^4,x]
```

```
[Out] ((c*d^2 + a*e^2)^4*(d + e*x)^6)/(6*e^9) - (8*c*d*(c*d^2 + a*e^2)^3*(d + e*x
)^7)/(7*e^9) + (c*(c*d^2 + a*e^2)^2*(7*c*d^2 + a*e^2)*(d + e*x)^8)/(2*e^9)
- (8*c^2*d*(c*d^2 + a*e^2)*(7*c*d^2 + 3*a*e^2)*(d + e*x)^9)/(9*e^9) + (c^2*
(35*c^2*d^4 + 30*a*c*d^2*e^2 + 3*a^2*e^4)*(d + e*x)^10)/(5*e^9) - (8*c^3*d*
(7*c*d^2 + 3*a*e^2)*(d + e*x)^11)/(11*e^9) + (c^3*(7*c*d^2 + a*e^2)*(d + e*
x)^12)/(3*e^9) - (8*c^4*d*(d + e*x)^13)/(13*e^9) + (c^4*(d + e*x)^14)/(14*e
^9)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (d+ex)^5\left(a+cx^2\right)^4\,dx=\int\left(\frac{\left(cd^2+ae^2\right)^4(d+ex)^5}{e^8}-\frac{8cd\left(cd^2+ae^2\right)^3(d+ex)^6}{e^8}+\frac{4c\left(cd^2+ae^2\right)^2\left(7cd^2+ae^2\right)(d+ex}{e^8}\right.$$

$$=\frac{\left(cd^2+ae^2\right)^4(d+ex)^6}{6e^9}-\frac{8cd\left(cd^2+ae^2\right)^3(d+ex)^7}{7e^9}+\frac{c\left(cd^2+ae^2\right)^2\left(7cd^2+ae^2\right)(d+ex)^8}{2e^9}-$$

Mathematica [A] time = 0.0963532, size = 307, normalized size = 1.1

$$\frac{x\left(429a^2c^2x^4\left(1800d^3e^2x^2+1575d^2e^3x^3+1050d^4ex+252d^5+700de^4x^4+126e^5x^5\right)+2145a^3cx^2\left(336d^3e^2x^2+280d^2e^3x\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5*(a + c*x^2)^4,x]
```

```
[Out] (x*(15015*a^4*(6*d^5 + 15*d^4*e*x + 20*d^3*e^2*x^2 + 15*d^2*e^3*x^3 + 6*d*e
^4*x^4 + e^5*x^5) + 2145*a^3*c*x^2*(56*d^5 + 210*d^4*e*x + 336*d^3*e^2*x^2
+ 280*d^2*e^3*x^3 + 120*d*e^4*x^4 + 21*e^5*x^5) + 429*a^2*c^2*x^4*(252*d^5
+ 1050*d^4*e*x + 1800*d^3*e^2*x^2 + 1575*d^2*e^3*x^3 + 700*d*e^4*x^4 + 126*
e^5*x^5) + 65*a*c^3*x^6*(792*d^5 + 3465*d^4*e*x + 6160*d^3*e^2*x^2 + 5544*d
^2*e^3*x^3 + 2520*d*e^4*x^4 + 462*e^5*x^5) + 5*c^4*x^8*(2002*d^5 + 9009*d^4
*e*x + 16380*d^3*e^2*x^2 + 15015*d^2*e^3*x^3 + 6930*d*e^4*x^4 + 1287*e^5*x^
5)))/90090
```

Maple [A] time = 0.05, size = 379, normalized size = 1.4

$$\frac{e^5c^4x^{14}}{14}+\frac{5\,de^4c^4x^{13}}{13}+\frac{\left(4\,e^5ac^3+10\,d^2e^3c^4\right)x^{12}}{12}+\frac{\left(20\,de^4ac^3+10\,d^3e^2c^4\right)x^{11}}{11}+\frac{\left(6\,e^5a^2c^2+40\,d^2e^3ac^3+5\,d^4ec^4\right)x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5*(c*x^2+a)^4,x)
```

```
[Out] 1/14*e^5*c^4*x^14+5/13*d*e^4*c^4*x^13+1/12*(4*a*c^3*e^5+10*c^4*d^2*e^3)*x^1
2+1/11*(20*a*c^3*d*e^4+10*c^4*d^3*e^2)*x^11+1/10*(6*a^2*c^2*e^5+40*a*c^3*d^
2*e^3+5*c^4*d^4*e)*x^10+1/9*(30*a^2*c^2*d*e^4+40*a*c^3*d^3*e^2+c^4*d^5)*x^9
+1/8*(4*a^3*c*e^5+60*a^2*c^2*d^2*e^3+20*a*c^3*d^4*e)*x^8+1/7*(20*a^3*c*d*e^
4+60*a^2*c^2*d^3*e^2+4*a*c^3*d^5)*x^7+1/6*(a^4*e^5+40*a^3*c*d^2*e^3+30*a^2*
c^2*d^4*e)*x^6+1/5*(5*a^4*d*e^4+40*a^3*c*d^3*e^2+6*a^2*c^2*d^5)*x^5+1/4*(10
*a^4*d^2*e^3+20*a^3*c*d^4*e)*x^4+1/3*(10*a^4*d^3*e^2+4*a^3*c*d^5)*x^3+5/2*d
^4*e*a^4*x^2+d^5*a^4*x
```

Maxima [A] time = 1.20822, size = 505, normalized size = 1.82

$$\frac{1}{14}\,c^4e^5x^{14}+\frac{5}{13}\,c^4de^4x^{13}+\frac{1}{6}\left(5\,c^4d^2e^3+2\,ac^3e^5\right)x^{12}+\frac{10}{11}\left(c^4d^3e^2+2\,ac^3de^4\right)x^{11}+\frac{5}{2}\,a^4d^4ex^2+\frac{1}{10}\left(5\,c^4d^4e+40\,ac^3d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] 1/14*c^4*e^5*x^14 + 5/13*c^4*d*e^4*x^13 + 1/6*(5*c^4*d^2*e^3 + 2*a*c^3*e^5)
*x^12 + 10/11*(c^4*d^3*e^2 + 2*a*c^3*d*e^4)*x^11 + 5/2*a^4*d^4*e*x^2 + 1/10
*(5*c^4*d^4*e + 40*a*c^3*d^2*e^3 + 6*a^2*c^2*e^5)*x^10 + a^4*d^5*x + 1/9*(c
^4*d^5 + 40*a*c^3*d^3*e^2 + 30*a^2*c^2*d*e^4)*x^9 + 1/2*(5*a*c^3*d^4*e + 15
*a^2*c^2*d^2*e^3 + a^3*c*e^5)*x^8 + 4/7*(a*c^3*d^5 + 15*a^2*c^2*d^3*e^2 + 5
*a^3*c*d*e^4)*x^7 + 1/6*(30*a^2*c^2*d^4*e + 40*a^3*c*d^2*e^3 + a^4*e^5)*x^6
 + 1/5*(6*a^2*c^2*d^5 + 40*a^3*c*d^3*e^2 + 5*a^4*d*e^4)*x^5 + 5/2*(2*a^3*c*
d^4*e + a^4*d^2*e^3)*x^4 + 2/3*(2*a^3*c*d^5 + 5*a^4*d^3*e^2)*x^3
```

Fricas [A] time = 1.75772, size = 879, normalized size = 3.16

$$\frac{1}{14}x^{14}e^5c^4+\frac{5}{13}x^{13}e^4dc^4+\frac{5}{6}x^{12}e^3d^2c^4+\frac{1}{3}x^{12}e^5c^3a+\frac{10}{11}x^{11}e^2d^3c^4+\frac{20}{11}x^{11}e^4dc^3a+\frac{1}{2}x^{10}ed^4c^4+4x^{10}e^3d^2c^3a+\frac{3}{5}x^{10}e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*e^5*c^4 + 5/13*x^13*e^4*d*c^4 + 5/6*x^12*e^3*d^2*c^4 + 1/3*x^12*e
^5*c^3*a + 10/11*x^11*e^2*d^3*c^4 + 20/11*x^11*e^4*d*c^3*a + 1/2*x^10*e*d^4
*c^4 + 4*x^10*e^3*d^2*c^3*a + 3/5*x^10*e^5*c^2*a^2 + 1/9*x^9*d^5*c^4 + 40/9
*x^9*e^2*d^3*c^3*a + 10/3*x^9*e^4*d*c^2*a^2 + 5/2*x^8*e*d^4*c^3*a + 15/2*x^
8*e^3*d^2*c^2*a^2 + 1/2*x^8*e^5*c*a^3 + 4/7*x^7*d^5*c^3*a + 60/7*x^7*e^2*d^
3*c^2*a^2 + 20/7*x^7*e^4*d*c*a^3 + 5*x^6*e*d^4*c^2*a^2 + 20/3*x^6*e^3*d^2*c
*a^3 + 1/6*x^6*e^5*a^4 + 6/5*x^5*d^5*c^2*a^2 + 8*x^5*e^2*d^3*c*a^3 + x^5*e^
4*d*a^4 + 5*x^4*e*d^4*c*a^3 + 5/2*x^4*e^3*d^2*a^4 + 4/3*x^3*d^5*c*a^3 + 10/
3*x^3*e^2*d^3*a^4 + 5/2*x^2*e*d^4*a^4 + x*d^5*a^4
```

Sympy [A] time = 0.124665, size = 418, normalized size = 1.5

$$a^4d^5x + \frac{5a^4d^4ex^2}{2} + \frac{5c^4de^4x^{13}}{13} + \frac{c^4e^5x^{14}}{14} + x^{12}\left(\frac{ac^3e^5}{3} + \frac{5c^4d^2e^3}{6}\right) + x^{11}\left(\frac{20ac^3de^4}{11} + \frac{10c^4d^3e^2}{11}\right) + x^{10}\left(\frac{3a^2c^2e^5}{5} + 4ac^3d^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5*(c*x**2+a)**4,x)
```

```
[Out] a**4*d**5*x + 5*a**4*d**4*e*x**2/2 + 5*c**4*d*e**4*x**13/13 + c**4*e**5*x**
14/14 + x**12*(a*c**3*e**5/3 + 5*c**4*d**2*e**3/6) + x**11*(20*a*c**3*d*e**
4/11 + 10*c**4*d**3*e**2/11) + x**10*(3*a**2*c**2*e**5/5 + 4*a*c**3*d**2*e*
*3 + c**4*d**4*e/2) + x**9*(10*a**2*c**2*d*e**4/3 + 40*a*c**3*d**3*e**2/9 +
 c**4*d**5/9) + x**8*(a**3*c*e**5/2 + 15*a**2*c**2*d**2*e**3/2 + 5*a*c**3*d
**4*e/2) + x**7*(20*a**3*c*d*e**4/7 + 60*a**2*c**2*d**3*e**2/7 + 4*a*c**3*d
**5/7) + x**6*(a**4*e**5/6 + 20*a**3*c*d**2*e**3/3 + 5*a**2*c**2*d**4*e) +
x**5*(a**4*d*e**4 + 8*a**3*c*d**3*e**2 + 6*a**2*c**2*d**5/5) + x**4*(5*a**4
*d**2*e**3/2 + 5*a**3*c*d**4*e) + x**3*(10*a**4*d**3*e**2/3 + 4*a**3*c*d**5
/3)
```

Giac [A] time = 1.2696, size = 516, normalized size = 1.86

$$\frac{1}{14}c^4x^{14}e^5 + \frac{5}{13}c^4dx^{13}e^4 + \frac{5}{6}c^4d^2x^{12}e^3 + \frac{10}{11}c^4d^3x^{11}e^2 + \frac{1}{2}c^4d^4x^{10}e + \frac{1}{9}c^4d^5x^9 + \frac{1}{3}ac^3x^{12}e^5 + \frac{20}{11}ac^3dx^{11}e^4 + 4\,ac^3d^2x^1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/14*c^4*x^14*e^5 + 5/13*c^4*d*x^13*e^4 + 5/6*c^4*d^2*x^12*e^3 + 10/11*c^4*
d^3*x^11*e^2 + 1/2*c^4*d^4*x^10*e + 1/9*c^4*d^5*x^9 + 1/3*a*c^3*x^12*e^5 +
20/11*a*c^3*d*x^11*e^4 + 4*a*c^3*d^2*x^10*e^3 + 40/9*a*c^3*d^3*x^9*e^2 + 5/
2*a*c^3*d^4*x^8*e + 4/7*a*c^3*d^5*x^7 + 3/5*a^2*c^2*x^10*e^5 + 10/3*a^2*c^2
*d*x^9*e^4 + 15/2*a^2*c^2*d^2*x^8*e^3 + 60/7*a^2*c^2*d^3*x^7*e^2 + 5*a^2*c^
2*d^4*x^6*e + 6/5*a^2*c^2*d^5*x^5 + 1/2*a^3*c*x^8*e^5 + 20/7*a^3*c*d*x^7*e^
4 + 20/3*a^3*c*d^2*x^6*e^3 + 8*a^3*c*d^3*x^5*e^2 + 5*a^3*c*d^4*x^4*e + 4/3*
a^3*c*d^5*x^3 + 1/6*a^4*x^6*e^5 + a^4*d*x^5*e^4 + 5/2*a^4*d^2*x^4*e^3 + 10/
3*a^4*d^3*x^3*e^2 + 5/2*a^4*d^4*x^2*e + a^4*d^5*x
```

3.491 $\int (d+ex)^4 \left(a+cx^2\right)^4 dx$

Optimal. Leaf size=270

$$\frac{1}{9}c^2x^9\left(6a^2e^4+24acd^2e^2+c^2d^4\right)+\frac{4}{7}acx^7\left(a^2e^4+9acd^2e^2+c^2d^4\right)+\frac{1}{5}a^2x^5\left(a^2e^4+24acd^2e^2+6c^2d^4\right)+3a^2c^2de^3x^8+$$

```
[Out] a^4*d^4*x + (2*a^3*d^2*(2*c*d^2 + 3*a*e^2)*x^3)/3 + a^4*d*e^3*x^4 + (a^2*(6
*c^2*d^4 + 24*a*c*d^2*e^2 + a^2*e^4)*x^5)/5 + (8*a^3*c*d*e^3*x^6)/3 + (4*a*
c*(c^2*d^4 + 9*a*c*d^2*e^2 + a^2*e^4)*x^7)/7 + 3*a^2*c^2*d*e^3*x^8 + (c^2*(
c^2*d^4 + 24*a*c*d^2*e^2 + 6*a^2*e^4)*x^9)/9 + (8*a*c^3*d*e^3*x^10)/5 + (2*
c^3*e^2*(3*c*d^2 + 2*a*e^2)*x^11)/11 + (c^4*d*e^3*x^12)/3 + (c^4*e^4*x^13)/
13 + (2*d^3*e*(a + c*x^2)^5)/(5*c)
```

Rubi [A] time = 0.252391, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {696, 1810}

$$\frac{1}{9}c^2x^9\left(6a^2e^4+24acd^2e^2+c^2d^4\right)+\frac{4}{7}acx^7\left(a^2e^4+9acd^2e^2+c^2d^4\right)+\frac{1}{5}a^2x^5\left(a^2e^4+24acd^2e^2+6c^2d^4\right)+3a^2c^2de^3x^8+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4*(a + c*x^2)^4,x]
```

```
[Out] a^4*d^4*x + (2*a^3*d^2*(2*c*d^2 + 3*a*e^2)*x^3)/3 + a^4*d*e^3*x^4 + (a^2*(6
*c^2*d^4 + 24*a*c*d^2*e^2 + a^2*e^4)*x^5)/5 + (8*a^3*c*d*e^3*x^6)/3 + (4*a*
c*(c^2*d^4 + 9*a*c*d^2*e^2 + a^2*e^4)*x^7)/7 + 3*a^2*c^2*d*e^3*x^8 + (c^2*(
c^2*d^4 + 24*a*c*d^2*e^2 + 6*a^2*e^4)*x^9)/9 + (8*a*c^3*d*e^3*x^10)/5 + (2*
c^3*e^2*(3*c*d^2 + 2*a*e^2)*x^11)/11 + (c^4*d*e^3*x^12)/3 + (c^4*e^4*x^13)/
13 + (2*d^3*e*(a + c*x^2)^5)/(5*c)
```

Rule 696

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*m*d^(m - 1)*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Int[((d + e*x)^m -
e*m*d^(m - 1)*x)*(a + c*x^2)^p, x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
 a*e^2, 0] && IGtQ[p, 1] && IGtQ[m, 0] && LeQ[m, p]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^4\left(a+cx^2\right)^4 dx &= \frac{2d^3e\left(a+cx^2\right)^5}{5c}+\int \left(a+cx^2\right)^4\left(-4d^3ex+(d+ex)^4\right) dx\\ &= \frac{2d^3e\left(a+cx^2\right)^5}{5c}+\int \left(a^4d^4+2a^3d^2\left(2cd^2+3ae^2\right)x^2+4a^4de^3x^3+a^2\left(6c^2d^4+24acd^2e^2+\right.\right.\\ &= a^4d^4x+\frac{2}{3}a^3d^2\left(2cd^2+3ae^2\right)x^3+a^4de^3x^4+\frac{1}{5}a^2\left(6c^2d^4+24acd^2e^2+a^2e^4\right)x^5+\frac{8}{3}a^3cde^3\end{aligned}$$

Mathematica [A] time = 0.0445337, size = 300, normalized size = 1.11

$$\frac{1}{9}c^2x^9\left(6a^2e^4+24acd^2e^2+c^2d^4\right)+\frac{4}{7}acx^7\left(a^2e^4+9acd^2e^2+c^2d^4\right)+\frac{1}{5}a^2x^5\left(a^2e^4+24acd^2e^2+6c^2d^4\right)+\frac{4}{3}a^2cdex^6\left(2ae^2+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4*(a + c*x^2)^4,x]
```

```
[Out] a^4*d^4*x + 2*a^4*d^3*e*x^2 + (2*a^3*d^2*(2*c*d^2 + 3*a*e^2)*x^3)/3 + a^3*d
*e*(4*c*d^2 + a*e^2)*x^4 + (a^2*(6*c^2*d^4 + 24*a*c*d^2*e^2 + a^2*e^4)*x^5)
/5 + (4*a^2*c*d*e*(3*c*d^2 + 2*a*e^2)*x^6)/3 + (4*a*c*(c^2*d^4 + 9*a*c*d^2*
e^2 + a^2*e^4)*x^7)/7 + a*c^2*d*e*(2*c*d^2 + 3*a*e^2)*x^8 + (c^2*(c^2*d^4 +
 24*a*c*d^2*e^2 + 6*a^2*e^4)*x^9)/9 + (2*c^3*d*e*(c*d^2 + 4*a*e^2)*x^10)/5
+ (2*c^3*e^2*(3*c*d^2 + 2*a*e^2)*x^11)/11 + (c^4*d*e^3*x^12)/3 + (c^4*e^4*x
^13)/13
```

Maple [A] time = 0.043, size = 313, normalized size = 1.2

$$\frac{c^4e^4x^{13}}{13}+\frac{c^4de^3x^{12}}{3}+\frac{\left(4\,e^4ac^3+6\,d^2e^2c^4\right)x^{11}}{11}+\frac{\left(16\,de^3ac^3+4\,d^3ec^4\right)x^{10}}{10}+\frac{\left(6\,e^4a^2c^2+24\,d^2e^2ac^3+c^4d^4\right)x^9}{9}+\frac{\left(24\,de^3a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(c*x^2+a)^4,x)
```

```
[Out] 1/13*c^4*e^4*x^13+1/3*c^4*d*e^3*x^12+1/11*(4*a*c^3*e^4+6*c^4*d^2*e^2)*x^11+
1/10*(16*a*c^3*d*e^3+4*c^4*d^3*e)*x^10+1/9*(6*a^2*c^2*e^4+24*a*c^3*d^2*e^2+
c^4*d^4)*x^9+1/8*(24*a^2*c^2*d*e^3+16*a*c^3*d^3*e)*x^8+1/7*(4*a^3*c*e^4+36*
a^2*c^2*d^2*e^2+4*a*c^3*d^4)*x^7+1/6*(16*a^3*c*d*e^3+24*a^2*c^2*d^3*e)*x^6+
1/5*(a^4*e^4+24*a^3*c*d^2*e^2+6*a^2*c^2*d^4)*x^5+1/4*(4*a^4*d*e^3+16*a^3*c*
d^3*e)*x^4+1/3*(6*a^4*d^2*e^2+4*a^3*c*d^4)*x^3+2*d^3*e*a^4*x^2+a^4*d^4*x
```

Maxima [A] time = 1.17414, size = 413, normalized size = 1.53

$$\frac{1}{13}\,c^4e^4x^{13}+\frac{1}{3}\,c^4de^3x^{12}+\frac{2}{11}\left(3\,c^4d^2e^2+2\,ac^3e^4\right)x^{11}+\frac{2}{5}\left(c^4d^3e+4\,ac^3de^3\right)x^{10}+2\,a^4d^3ex^2+\frac{1}{9}\left(c^4d^4+24\,ac^3d^2e^2+6\,a^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] 1/13*c^4*e^4*x^13 + 1/3*c^4*d*e^3*x^12 + 2/11*(3*c^4*d^2*e^2 + 2*a*c^3*e^4)
*x^11 + 2/5*(c^4*d^3*e + 4*a*c^3*d*e^3)*x^10 + 2*a^4*d^3*e*x^2 + 1/9*(c^4*d
^4 + 24*a*c^3*d^2*e^2 + 6*a^2*c^2*e^4)*x^9 + a^4*d^4*x + (2*a*c^3*d^3*e + 3
*a^2*c^2*d*e^3)*x^8 + 4/7*(a*c^3*d^4 + 9*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^7 +
 4/3*(3*a^2*c^2*d^3*e + 2*a^3*c*d*e^3)*x^6 + 1/5*(6*a^2*c^2*d^4 + 24*a^3*c*
d^2*e^2 + a^4*e^4)*x^5 + (4*a^3*c*d^3*e + a^4*d*e^3)*x^4 + 2/3*(2*a^3*c*d^4
 + 3*a^4*d^2*e^2)*x^3
```

Fricas [A] time = 1.67179, size = 699, normalized size = 2.59

$$\frac{1}{13}x^{13}e^4c^4+\frac{1}{3}x^{12}e^3dc^4+\frac{6}{11}x^{11}e^2d^2c^4+\frac{4}{11}x^{11}e^4c^3a+\frac{2}{5}x^{10}ed^3c^4+\frac{8}{5}x^{10}e^3dc^3a+\frac{1}{9}x^9d^4c^4+\frac{8}{3}x^9e^2d^2c^3a+\frac{2}{3}x^9e^4c^2a^2+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*e^4*c^4 + 1/3*x^12*e^3*d*c^4 + 6/11*x^11*e^2*d^2*c^4 + 4/11*x^11*
e^4*c^3*a + 2/5*x^10*e*d^3*c^4 + 8/5*x^10*e^3*d*c^3*a + 1/9*x^9*d^4*c^4 + 8
/3*x^9*e^2*d^2*c^3*a + 2/3*x^9*e^4*c^2*a^2 + 2*x^8*e*d^3*c^3*a + 3*x^8*e^3*
d*c^2*a^2 + 4/7*x^7*d^4*c^3*a + 36/7*x^7*e^2*d^2*c^2*a^2 + 4/7*x^7*e^4*c*a^
3 + 4*x^6*e*d^3*c^2*a^2 + 8/3*x^6*e^3*d*c*a^3 + 6/5*x^5*d^4*c^2*a^2 + 24/5*
x^5*e^2*d^2*c*a^3 + 1/5*x^5*e^4*a^4 + 4*x^4*e*d^3*c*a^3 + x^4*e^3*d*a^4 + 4
/3*x^3*d^4*c*a^3 + 2*x^3*e^2*d^2*a^4 + 2*x^2*e*d^3*a^4 + x*d^4*a^4
```

Sympy [A] time = 0.120948, size = 340, normalized size = 1.26

$$a^4d^4x + 2a^4d^3ex^2 + \frac{c^4de^3x^{12}}{3} + \frac{c^4e^4x^{13}}{13} + x^{11}\left(\frac{4ac^3e^4}{11} + \frac{6c^4d^2e^2}{11}\right) + x^{10}\left(\frac{8ac^3de^3}{5} + \frac{2c^4d^3e}{5}\right) + x^9\left(\frac{2a^2c^2e^4}{3} + \frac{8ac^3d^2e}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(c*x**2+a)**4,x)
```

```
[Out] a**4*d**4*x + 2*a**4*d**3*e*x**2 + c**4*d*e**3*x**12/3 + c**4*e**4*x**13/13
 + x**11*(4*a*c**3*e**4/11 + 6*c**4*d**2*e**2/11) + x**10*(8*a*c**3*d*e**3/
5 + 2*c**4*d**3*e/5) + x**9*(2*a**2*c**2*e**4/3 + 8*a*c**3*d**2*e**2/3 + c*
*4*d**4/9) + x**8*(3*a**2*c**2*d*e**3 + 2*a*c**3*d**3*e) + x**7*(4*a**3*c*e
**4/7 + 36*a**2*c**2*d**2*e**2/7 + 4*a*c**3*d**4/7) + x**6*(8*a**3*c*d*e**3
/3 + 4*a**2*c**2*d**3*e) + x**5*(a**4*e**4/5 + 24*a**3*c*d**2*e**2/5 + 6*a*
*2*c**2*d**4/5) + x**4*(a**4*d*e**3 + 4*a**3*c*d**3*e) + x**3*(2*a**4*d**2*
e**2 + 4*a**3*c*d**4/3)
```

Giac [A] time = 1.33728, size = 421, normalized size = 1.56

$$\frac{1}{13}c^4x^{13}e^4 + \frac{1}{3}c^4dx^{12}e^3 + \frac{6}{11}c^4d^2x^{11}e^2 + \frac{2}{5}c^4d^3x^{10}e + \frac{1}{9}c^4d^4x^9 + \frac{4}{11}ac^3x^{11}e^4 + \frac{8}{5}ac^3dx^{10}e^3 + \frac{8}{3}ac^3d^2x^9e^2 + 2\,ac^3d^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/13*c^4*x^13*e^4 + 1/3*c^4*d*x^12*e^3 + 6/11*c^4*d^2*x^11*e^2 + 2/5*c^4*d^
3*x^10*e + 1/9*c^4*d^4*x^9 + 4/11*a*c^3*x^11*e^4 + 8/5*a*c^3*d*x^10*e^3 + 8
/3*a*c^3*d^2*x^9*e^2 + 2*a*c^3*d^3*x^8*e + 4/7*a*c^3*d^4*x^7 + 2/3*a^2*c^2*
x^9*e^4 + 3*a^2*c^2*d*x^8*e^3 + 36/7*a^2*c^2*d^2*x^7*e^2 + 4*a^2*c^2*d^3*x^
6*e + 6/5*a^2*c^2*d^4*x^5 + 4/7*a^3*c*x^7*e^4 + 8/3*a^3*c*d*x^6*e^3 + 24/5*
a^3*c*d^2*x^5*e^2 + 4*a^3*c*d^3*x^4*e + 4/3*a^3*c*d^4*x^3 + 1/5*a^4*x^5*e^4
 + a^4*d*x^4*e^3 + 2*a^4*d^2*x^3*e^2 + 2*a^4*d^3*x^2*e + a^4*d^4*x
```

3.492 $\int (d+ex)^3 \left(a+cx^2\right)^4 \, dx$

Optimal. Leaf size=209

$$\frac{3}{4}a^2c^2e^3x^8+\frac{6}{5}a^2cdx^5\left(2ae^2+cd^2\right)+\frac{1}{3}a^3dx^3\left(3ae^2+4cd^2\right)+\frac{2}{3}a^3ce^3x^6+a^4d^3x+\frac{1}{4}a^4e^3x^4+\frac{1}{9}c^3dx^9\left(12ae^2+cd^2\right)+\frac{2}{7}ac$$

```
[Out] a^4*d^3*x + (a^3*d*(4*c*d^2 + 3*a*e^2)*x^3)/3 + (a^4*e^3*x^4)/4 + (6*a^2*c*
d*(c*d^2 + 2*a*e^2)*x^5)/5 + (2*a^3*c*e^3*x^6)/3 + (2*a*c^2*d*(2*c*d^2 + 9*
a*e^2)*x^7)/7 + (3*a^2*c^2*e^3*x^8)/4 + (c^3*d*(c*d^2 + 12*a*e^2)*x^9)/9 +
(2*a*c^3*e^3*x^10)/5 + (3*c^4*d*e^2*x^11)/11 + (c^4*e^3*x^12)/12 + (3*d^2*e
*(a + c*x^2)^5)/(10*c)
```

Rubi [A] time = 0.191729, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {696, 1810}

$$\frac{3}{4}a^2c^2e^3x^8+\frac{6}{5}a^2cdx^5\left(2ae^2+cd^2\right)+\frac{1}{3}a^3dx^3\left(3ae^2+4cd^2\right)+\frac{2}{3}a^3ce^3x^6+a^4d^3x+\frac{1}{4}a^4e^3x^4+\frac{1}{9}c^3dx^9\left(12ae^2+cd^2\right)+\frac{2}{7}ac$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + c*x^2)^4,x]
```


```
[Out] a^4*d^3*x + (a^3*d*(4*c*d^2 + 3*a*e^2)*x^3)/3 + (a^4*e^3*x^4)/4 + (6*a^2*c*
d*(c*d^2 + 2*a*e^2)*x^5)/5 + (2*a^3*c*e^3*x^6)/3 + (2*a*c^2*d*(2*c*d^2 + 9*
a*e^2)*x^7)/7 + (3*a^2*c^2*e^3*x^8)/4 + (c^3*d*(c*d^2 + 12*a*e^2)*x^9)/9 +
(2*a*c^3*e^3*x^10)/5 + (3*c^4*d*e^2*x^11)/11 + (c^4*e^3*x^12)/12 + (3*d^2*e
*(a + c*x^2)^5)/(10*c)
```


Rule 696

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*m*d^(m - 1)*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Int[((d + e*x)^m -
e*m*d^(m - 1)*x)*(a + c*x^2)^p, x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
 a*e^2, 0] && IGtQ[p, 1] && IGtQ[m, 0] && LeQ[m, p]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^3\left(a+cx^2\right)^4\,dx&=\frac{3d^2e\left(a+cx^2\right)^5}{10c}+\int \left(a+cx^2\right)^4\left(-3d^2ex+(d+ex)^3\right)dx\\&=\frac{3d^2e\left(a+cx^2\right)^5}{10c}+\int \left(a^4d^3+a^3d\left(4cd^2+3ae^2\right)x^2+a^4e^3x^3+6a^2cd\left(cd^2+2ae^2\right)x^4+4a^3ce^3\right.\\&=a^4d^3x+\frac{1}{3}a^3d\left(4cd^2+3ae^2\right)x^3+\frac{1}{4}a^4e^3x^4+\frac{6}{5}a^2cd\left(cd^2+2ae^2\right)x^5+\frac{2}{3}a^3ce^3x^6+\frac{2}{7}ac^2d\left(2cd\right.\end{aligned}$$

Mathematica [A] time = 0.064292, size = 197, normalized size = 0.94

$$x\left(297a^2c^2x^4\left(140d^2ex+56d^3+120de^2x^2+35e^3x^3\right)+924a^3cx^2\left(45d^2ex+20d^3+36de^2x^2+10e^3x^3\right)+3465a^4\left(6d^2e\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + c*x^2)^4,x]
```

```
[Out] (x*(3465*a^4*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) + 924*a^3*c*x^2*(2
0*d^3 + 45*d^2*e*x + 36*d*e^2*x^2 + 10*e^3*x^3) + 297*a^2*c^2*x^4*(56*d^3 +
 140*d^2*e*x + 120*d*e^2*x^2 + 35*e^3*x^3) + 66*a*c^3*x^6*(120*d^3 + 315*d^
2*e*x + 280*d*e^2*x^2 + 84*e^3*x^3) + 7*c^4*x^8*(220*d^3 + 594*d^2*e*x + 54
0*d*e^2*x^2 + 165*e^3*x^3)))/13860
```

Maple [A] time = 0.041, size = 247, normalized size = 1.2

$$\frac{c^4e^3x^{12}}{12}+\frac{3\,c^4de^2x^{11}}{11}+\frac{\left(4\,e^3ac^3+3\,d^2ec^4\right)x^{10}}{10}+\frac{\left(12\,de^2ac^3+d^3c^4\right)x^9}{9}+\frac{\left(6\,e^3a^2c^2+12\,d^2eac^3\right)x^8}{8}+\frac{\left(18\,de^2a^2c^2+4\,a\right.}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+a)^4,x)
```

```
[Out] 1/12*c^4*e^3*x^12+3/11*c^4*d*e^2*x^11+1/10*(4*a*c^3*e^3+3*c^4*d^2*e)*x^10+1
/9*(12*a*c^3*d*e^2+c^4*d^3)*x^9+1/8*(6*a^2*c^2*e^3+12*a*c^3*d^2*e)*x^8+1/7*
(18*a^2*c^2*d*e^2+4*a*c^3*d^3)*x^7+1/6*(4*a^3*c*e^3+18*a^2*c^2*d^2*e)*x^6+1
/5*(12*a^3*c*d*e^2+6*a^2*c^2*d^3)*x^5+1/4*(a^4*e^3+12*a^3*c*d^2*e)*x^4+1/3*
(3*a^4*d*e^2+4*a^3*c*d^3)*x^3+3/2*d^2*e*a^4*x^2+a^4*d^3*x
```

Maxima [A] time = 1.13363, size = 329, normalized size = 1.57

$$\frac{1}{12}\,c^4e^3x^{12}+\frac{3}{11}\,c^4de^2x^{11}+\frac{1}{10}\left(3\,c^4d^2e+4\,ac^3e^3\right)x^{10}+\frac{1}{9}\left(c^4d^3+12\,ac^3de^2\right)x^9+\frac{3}{2}\,a^4d^2ex^2+\frac{3}{4}\left(2\,ac^3d^2e+a^2c^2e^3\right)x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] 1/12*c^4*e^3*x^12 + 3/11*c^4*d*e^2*x^11 + 1/10*(3*c^4*d^2*e + 4*a*c^3*e^3)*
x^10 + 1/9*(c^4*d^3 + 12*a*c^3*d*e^2)*x^9 + 3/2*a^4*d^2*e*x^2 + 3/4*(2*a*c^
3*d^2*e + a^2*c^2*e^3)*x^8 + a^4*d^3*x + 2/7*(2*a*c^3*d^3 + 9*a^2*c^2*d*e^2
)*x^7 + 1/3*(9*a^2*c^2*d^2*e + 2*a^3*c*e^3)*x^6 + 6/5*(a^2*c^2*d^3 + 2*a^3*
c*d*e^2)*x^5 + 1/4*(12*a^3*c*d^2*e + a^4*e^3)*x^4 + 1/3*(4*a^3*c*d^3 + 3*a^
4*d*e^2)*x^3
```

Fricas [A] time = 1.53162, size = 548, normalized size = 2.62

$$\frac{1}{12}x^{12}e^3c^4+\frac{3}{11}x^{11}e^2dc^4+\frac{3}{10}x^{10}ed^2c^4+\frac{2}{5}x^{10}e^3c^3a+\frac{1}{9}x^9d^3c^4+\frac{4}{3}x^9e^2dc^3a+\frac{3}{2}x^8ed^2c^3a+\frac{3}{4}x^8e^3c^2a^2+\frac{4}{7}x^7d^3c^3a+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*e^3*c^4 + 3/11*x^11*e^2*d*c^4 + 3/10*x^10*e*d^2*c^4 + 2/5*x^10*e^
3*c^3*a + 1/9*x^9*d^3*c^4 + 4/3*x^9*e^2*d*c^3*a + 3/2*x^8*e*d^2*c^3*a + 3/4
*x^8*e^3*c^2*a^2 + 4/7*x^7*d^3*c^3*a + 18/7*x^7*e^2*d*c^2*a^2 + 3*x^6*e*d^2
*c^2*a^2 + 2/3*x^6*e^3*c*a^3 + 6/5*x^5*d^3*c^2*a^2 + 12/5*x^5*e^2*d*c*a^3 +
 3*x^4*e*d^2*c*a^3 + 1/4*x^4*e^3*a^4 + 4/3*x^3*d^3*c*a^3 + x^3*e^2*d*a^4 +
3/2*x^2*e*d^2*a^4 + x*d^3*a^4
```

Sympy [A] time = 0.112578, size = 270, normalized size = 1.29

$$a^4d^3x + \frac{3a^4d^2ex^2}{2} + \frac{3c^4de^2x^{11}}{11} + \frac{c^4e^3x^{12}}{12} + x^{10}\left(\frac{2ac^3e^3}{5} + \frac{3c^4d^2e}{10}\right) + x^9\left(\frac{4ac^3de^2}{3} + \frac{c^4d^3}{9}\right) + x^8\left(\frac{3a^2c^2e^3}{4} + \frac{3ac^3d^2e}{2}\right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a)**4,x)
```

```
[Out] a**4*d**3*x + 3*a**4*d**2*e*x**2/2 + 3*c**4*d*e**2*x**11/11 + c**4*e**3*x**
12/12 + x**10*(2*a*c**3*e**3/5 + 3*c**4*d**2*e/10) + x**9*(4*a*c**3*d*e**2/
3 + c**4*d**3/9) + x**8*(3*a**2*c**2*e**3/4 + 3*a*c**3*d**2*e/2) + x**7*(18
*a**2*c**2*d*e**2/7 + 4*a*c**3*d**3/7) + x**6*(2*a**3*c*e**3/3 + 3*a**2*c**
2*d**2*e) + x**5*(12*a**3*c*d*e**2/5 + 6*a**2*c**2*d**3/5) + x**4*(a**4*e**
3/4 + 3*a**3*c*d**2*e) + x**3*(a**4*d*e**2 + 4*a**3*c*d**3/3)
```

Giac [A] time = 1.35884, size = 327, normalized size = 1.56

$$\frac{1}{12}c^4x^{12}e^3 + \frac{3}{11}c^4dx^{11}e^2 + \frac{3}{10}c^4d^2x^{10}e + \frac{1}{9}c^4d^3x^9 + \frac{2}{5}ac^3x^{10}e^3 + \frac{4}{3}ac^3dx^9e^2 + \frac{3}{2}ac^3d^2x^8e + \frac{4}{7}ac^3d^3x^7 + \frac{3}{4}a^2c^2x^8e^3 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/12*c^4*x^12*e^3 + 3/11*c^4*d*x^11*e^2 + 3/10*c^4*d^2*x^10*e + 1/9*c^4*d^3
*x^9 + 2/5*a*c^3*x^10*e^3 + 4/3*a*c^3*d*x^9*e^2 + 3/2*a*c^3*d^2*x^8*e + 4/7
*a*c^3*d^3*x^7 + 3/4*a^2*c^2*x^8*e^3 + 18/7*a^2*c^2*d*x^7*e^2 + 3*a^2*c^2*d
^2*x^6*e + 6/5*a^2*c^2*d^3*x^5 + 2/3*a^3*c*x^6*e^3 + 12/5*a^3*c*d*x^5*e^2 +
 3*a^3*c*d^2*x^4*e + 4/3*a^3*c*d^3*x^3 + 1/4*a^4*x^4*e^3 + a^4*d*x^3*e^2 +
3/2*a^4*d^2*x^2*e + a^4*d^3*x
```

3.493 $\int (d+ex)^2 \left(a+cx^2\right)^4 \, dx$

Optimal. Leaf size=132

$$\frac{2}{5}a^2cx^5\left(2ae^2+3cd^2\right)+\frac{1}{3}a^3x^3\left(ae^2+4cd^2\right)+a^4d^2x+\frac{1}{9}c^3x^9\left(4ae^2+cd^2\right)+\frac{2}{7}ac^2x^7\left(3ae^2+2cd^2\right)+\frac{de\left(a+cx^2\right)^5}{5c}+\frac{1}{1}$$

```
[Out] a^4*d^2*x + (a^3*(4*c*d^2 + a*e^2)*x^3)/3 + (2*a^2*c*(3*c*d^2 + 2*a*e^2)*x^
5)/5 + (2*a*c^2*(2*c*d^2 + 3*a*e^2)*x^7)/7 + (c^3*(c*d^2 + 4*a*e^2)*x^9)/9
+ (c^4*e^2*x^11)/11 + (d*e*(a + c*x^2)^5)/(5*c)
```

Rubi [A] time = 0.0951265, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {696, 1810}

$$\frac{2}{5}a^2cx^5\left(2ae^2+3cd^2\right)+\frac{1}{3}a^3x^3\left(ae^2+4cd^2\right)+a^4d^2x+\frac{1}{9}c^3x^9\left(4ae^2+cd^2\right)+\frac{2}{7}ac^2x^7\left(3ae^2+2cd^2\right)+\frac{de\left(a+cx^2\right)^5}{5c}+\frac{1}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + c*x^2)^4,x]
```

```
[Out] a^4*d^2*x + (a^3*(4*c*d^2 + a*e^2)*x^3)/3 + (2*a^2*c*(3*c*d^2 + 2*a*e^2)*x^
5)/5 + (2*a*c^2*(2*c*d^2 + 3*a*e^2)*x^7)/7 + (c^3*(c*d^2 + 4*a*e^2)*x^9)/9
+ (c^4*e^2*x^11)/11 + (d*e*(a + c*x^2)^5)/(5*c)
```

Rule 696

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*m*d^(m - 1)*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Int[((d + e*x)^m -
e*m*d^(m - 1)*x)*(a + c*x^2)^p, x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
 a*e^2, 0] && IGtQ[p, 1] && IGtQ[m, 0] && LeQ[m, p]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^2\left(a+cx^2\right)^4\,dx &= \frac{de\left(a+cx^2\right)^5}{5c}+\int \left(a+cx^2\right)^4\left(-2dex+(d+ex)^2\right)dx\\
&= \frac{de\left(a+cx^2\right)^5}{5c}+\int \left(a^4d^2+a^3\left(4cd^2+ae^2\right)x^2+2a^2c\left(3cd^2+2ae^2\right)x^4+2ac^2\left(2cd^2+3ae^2\right.\right.\\
&= a^4d^2x+\frac{1}{3}a^3\left(4cd^2+ae^2\right)x^3+\frac{2}{5}a^2c\left(3cd^2+2ae^2\right)x^5+\frac{2}{7}ac^2\left(2cd^2+3ae^2\right)x^7+\frac{1}{9}c^3\left(cd^2\right.
\end{aligned}$$

Mathematica [A] time = 0.0300655, size = 148, normalized size = 1.12

$$\frac{2}{35}a^2c^2x^5\left(21d^2+35dex+15e^2x^2\right)+\frac{2}{15}a^3cx^3\left(10d^2+15dex+6e^2x^2\right)+a^4\left(d^2x+dex^2+\frac{e^2x^3}{3}\right)+\frac{1}{63}ac^3x^7\left(36d^2+63\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + c*x^2)^4,x]
```

```
[Out] (2*a^3*c*x^3*(10*d^2 + 15*d*e*x + 6*e^2*x^2))/15 + (2*a^2*c^2*x^5*(21*d^2 +
 35*d*e*x + 15*e^2*x^2))/35 + (a*c^3*x^7*(36*d^2 + 63*d*e*x + 28*e^2*x^2))/
63 + (c^4*x^9*(55*d^2 + 99*d*e*x + 45*e^2*x^2))/495 + a^4*(d^2*x + d*e*x^2
+ (e^2*x^3)/3)
```

Maple [A] time = 0.045, size = 170, normalized size = 1.3

$$\frac{c^4e^2x^{11}}{11}+\frac{dec^4x^{10}}{5}+\frac{\left(4\,e^2ac^3+d^2c^4\right)x^9}{9}+deac^3x^8+\frac{\left(6\,e^2a^2c^2+4\,d^2ac^3\right)x^7}{7}+2\,dea^2c^2x^6+\frac{\left(4\,e^2a^3c+6\,d^2a^2c^2\right)x^5}{5}+2\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a)^4,x)
```

```
[Out] 1/11*c^4*e^2*x^11+1/5*d*e*c^4*x^10+1/9*(4*a*c^3*e^2+c^4*d^2)*x^9+d*e*a*c^3*
x^8+1/7*(6*a^2*c^2*e^2+4*a*c^3*d^2)*x^7+2*d*e*a^2*c^2*x^6+1/5*(4*a^3*c*e^2+
6*a^2*c^2*d^2)*x^5+2*d*e*a^3*c*x^4+1/3*(a^4*e^2+4*a^3*c*d^2)*x^3+d*e*a^4*x^
2+a^4*d^2*x
```

Maxima [A] time = 1.16173, size = 228, normalized size = 1.73

$$\frac{1}{11}\,c^4e^2x^{11}+\frac{1}{5}\,c^4dex^{10}+ac^3dex^8+2\,a^2c^2dex^6+2\,a^3cdex^4+\frac{1}{9}\left(c^4d^2+4\,ac^3e^2\right)x^9+a^4dex^2+\frac{2}{7}\left(2\,ac^3d^2+3\,a^2c^2e^2\right)x^7+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] 1/11*c^4*e^2*x^11 + 1/5*c^4*d*e*x^10 + a*c^3*d*e*x^8 + 2*a^2*c^2*d*e*x^6 +
2*a^3*c*d*e*x^4 + 1/9*(c^4*d^2 + 4*a*c^3*e^2)*x^9 + a^4*d*e*x^2 + 2/7*(2*a*
c^3*d^2 + 3*a^2*c^2*e^2)*x^7 + a^4*d^2*x + 2/5*(3*a^2*c^2*d^2 + 2*a^3*c*e^2
)*x^5 + 1/3*(4*a^3*c*d^2 + a^4*e^2)*x^3
```

Fricas [A] time = 1.59751, size = 375, normalized size = 2.84

$$\frac{1}{11}x^{11}e^2c^4+\frac{1}{5}x^{10}edc^4+\frac{1}{9}x^9d^2c^4+\frac{4}{9}x^9e^2c^3a+x^8edc^3a+\frac{4}{7}x^7d^2c^3a+\frac{6}{7}x^7e^2c^2a^2+2x^6edc^2a^2+\frac{6}{5}x^5d^2c^2a^2+\frac{4}{5}x^5e^2ca^3+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*e^2*c^4 + 1/5*x^10*e*d*c^4 + 1/9*x^9*d^2*c^4 + 4/9*x^9*e^2*c^3*a
+ x^8*e*d*c^3*a + 4/7*x^7*d^2*c^3*a + 6/7*x^7*e^2*c^2*a^2 + 2*x^6*e*d*c^2*a
^2 + 6/5*x^5*d^2*c^2*a^2 + 4/5*x^5*e^2*c*a^3 + 2*x^4*e*d*c*a^3 + 4/3*x^3*d^
2*c*a^3 + 1/3*x^3*e^2*a^4 + x^2*e*d*a^4 + x*d^2*a^4
```

Sympy [A] time = 0.107205, size = 187, normalized size = 1.42

$$a^4d^2x + a^4dex^2 + 2a^3cdex^4 + 2a^2c^2dex^6 + ac^3dex^8 + \frac{c^4dex^{10}}{5} + \frac{c^4e^2x^{11}}{11} + x^9\left(\frac{4ac^3e^2}{9} + \frac{c^4d^2}{9}\right) + x^7\left(\frac{6a^2c^2e^2}{7} + \frac{4ac^3d^2}{7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+a)**4,x)
```

```
[Out] a**4*d**2*x + a**4*d*e*x**2 + 2*a**3*c*d*e*x**4 + 2*a**2*c**2*d*e*x**6 + a*
c**3*d*e*x**8 + c**4*d*e*x**10/5 + c**4*e**2*x**11/11 + x**9*(4*a*c**3*e**2
/9 + c**4*d**2/9) + x**7*(6*a**2*c**2*e**2/7 + 4*a*c**3*d**2/7) + x**5*(4*a
**3*c*e**2/5 + 6*a**2*c**2*d**2/5) + x**3*(a**4*e**2/3 + 4*a**3*c*d**2/3)
```

Giac [A] time = 1.3557, size = 231, normalized size = 1.75

$$\frac{1}{11}c^4x^{11}e^2 + \frac{1}{5}c^4dx^{10}e + \frac{1}{9}c^4d^2x^9 + \frac{4}{9}ac^3x^9e^2 + ac^3dx^8e + \frac{4}{7}ac^3d^2x^7 + \frac{6}{7}a^2c^2x^7e^2 + 2a^2c^2dx^6e + \frac{6}{5}a^2c^2d^2x^5 + \frac{4}{5}a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/11*c^4*x^11*e^2 + 1/5*c^4*d*x^10*e + 1/9*c^4*d^2*x^9 + 4/9*a*c^3*x^9*e^2
+ a*c^3*d*x^8*e + 4/7*a*c^3*d^2*x^7 + 6/7*a^2*c^2*x^7*e^2 + 2*a^2*c^2*d*x^6
*e + 6/5*a^2*c^2*d^2*x^5 + 4/5*a^3*c*x^5*e^2 + 2*a^3*c*d*x^4*e + 4/3*a^3*c*
d^2*x^3 + 1/3*a^4*x^3*e^2 + a^4*d*x^2*e + a^4*d^2*x
```

3.494 $\int (d+ex)\left(a+cx^2\right)^4\,dx$

Optimal. Leaf size=73

$$\frac{6}{5}a^2c^2dx^5+\frac{4}{3}a^3cdx^3+a^4dx+\frac{4}{7}ac^3dx^7+\frac{e\left(a+cx^2\right)^5}{10c}+\frac{1}{9}c^4dx^9$$

```
[Out] a^4*d*x + (4*a^3*c*d*x^3)/3 + (6*a^2*c^2*d*x^5)/5 + (4*a*c^3*d*x^7)/7 + (c^
4*d*x^9)/9 + (e*(a + c*x^2)^5)/(10*c)
```

Rubi [A] time = 0.0288876, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {641, 194}

$$\frac{6}{5}a^2c^2dx^5+\frac{4}{3}a^3cdx^3+a^4dx+\frac{4}{7}ac^3dx^7+\frac{e\left(a+cx^2\right)^5}{10c}+\frac{1}{9}c^4dx^9$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + c*x^2)^4,x]
```

```
[Out] a^4*d*x + (4*a^3*c*d*x^3)/3 + (6*a^2*c^2*d*x^5)/5 + (4*a*c^3*d*x^7)/7 + (c^
4*d*x^9)/9 + (e*(a + c*x^2)^5)/(10*c)
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(a+cx^2\right)^4\,dx &= \frac{e\left(a+cx^2\right)^5}{10c}+d\int \left(a+cx^2\right)^4\,dx\\ &= \frac{e\left(a+cx^2\right)^5}{10c}+d\int \left(a^4+4a^3cx^2+6a^2c^2x^4+4ac^3x^6+c^4x^8\right)\,dx\\ &= a^4dx+\frac{4}{3}a^3cdx^3+\frac{6}{5}a^2c^2dx^5+\frac{4}{7}ac^3dx^7+\frac{1}{9}c^4dx^9+\frac{e\left(a+cx^2\right)^5}{10c}\end{aligned}$$

Mathematica [A] time = 0.0029015, size = 110, normalized size = 1.51

$$\frac{6}{5}a^2c^2dx^5+a^2c^2ex^6+\frac{4}{3}a^3cdx^3+a^3cex^4+a^4dx+\frac{1}{2}a^4ex^2+\frac{4}{7}ac^3dx^7+\frac{1}{2}ac^3ex^8+\frac{1}{9}c^4dx^9+\frac{1}{10}c^4ex^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + c*x^2)^4,x]
```

```
[Out] a^4*d*x + (a^4*e*x^2)/2 + (4*a^3*c*d*x^3)/3 + a^3*c*e*x^4 + (6*a^2*c^2*d*x^
5)/5 + a^2*c^2*e*x^6 + (4*a*c^3*d*x^7)/7 + (a*c^3*e*x^8)/2 + (c^4*d*x^9)/9
+ (c^4*e*x^10)/10
```

Maple [A] time = 0.042, size = 97, normalized size = 1.3

$$\frac{c^4ex^{10}}{10}+\frac{c^4dx^9}{9}+\frac{eac^3x^8}{2}+\frac{4\,ac^3dx^7}{7}+ea^2c^2x^6+\frac{6\,a^2c^2dx^5}{5}+ea^3cx^4+\frac{4\,a^3cdx^3}{3}+\frac{ea^4x^2}{2}+a^4dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+a)^4,x)
```

```
[Out] 1/10*c^4*e*x^10+1/9*c^4*d*x^9+1/2*e*a*c^3*x^8+4/7*a*c^3*d*x^7+e*a^2*c^2*x^6
+6/5*a^2*c^2*d*x^5+e*a^3*c*x^4+4/3*a^3*c*d*x^3+1/2*e*a^4*x^2+a^4*d*x
```

Maxima [A] time = 1.20309, size = 130, normalized size = 1.78

$$\frac{1}{10}\,c^4ex^{10}+\frac{1}{9}\,c^4dx^9+\frac{1}{2}\,ac^3ex^8+\frac{4}{7}\,ac^3dx^7+a^2c^2ex^6+\frac{6}{5}\,a^2c^2dx^5+a^3cex^4+\frac{4}{3}\,a^3cdx^3+\frac{1}{2}\,a^4ex^2+a^4dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] 1/10*c^4*e*x^10 + 1/9*c^4*d*x^9 + 1/2*a*c^3*e*x^8 + 4/7*a*c^3*d*x^7 + a^2*c
^2*e*x^6 + 6/5*a^2*c^2*d*x^5 + a^3*c*e*x^4 + 4/3*a^3*c*d*x^3 + 1/2*a^4*e*x^
2 + a^4*d*x
```

Fricas [A] time = 1.71082, size = 220, normalized size = 3.01

$$\frac{1}{10}x^{10}ec^4+\frac{1}{9}x^9dc^4+\frac{1}{2}x^8ec^3a+\frac{4}{7}x^7dc^3a+x^6ec^2a^2+\frac{6}{5}x^5dc^2a^2+x^4eca^3+\frac{4}{3}x^3dca^3+\frac{1}{2}x^2ea^4+xda^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*e*c^4 + 1/9*x^9*d*c^4 + 1/2*x^8*e*c^3*a + 4/7*x^7*d*c^3*a + x^6*e
*c^2*a^2 + 6/5*x^5*d*c^2*a^2 + x^4*e*c*a^3 + 4/3*x^3*d*c*a^3 + 1/2*x^2*e*a^
4 + x*d*a^4
```

Sympy [A] time = 0.087271, size = 112, normalized size = 1.53

$$a^4dx+\frac{a^4ex^2}{2}+\frac{4a^3cdx^3}{3}+a^3cex^4+\frac{6a^2c^2dx^5}{5}+a^2c^2ex^6+\frac{4ac^3dx^7}{7}+\frac{ac^3ex^8}{2}+\frac{c^4dx^9}{9}+\frac{c^4ex^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+a)**4,x)
```

```
[Out] a**4*d*x + a**4*e*x**2/2 + 4*a**3*c*d*x**3/3 + a**3*c*e*x**4 + 6*a**2*c**2*
d*x**5/5 + a**2*c**2*e*x**6 + 4*a*c**3*d*x**7/7 + a*c**3*e*x**8/2 + c**4*d*
x**9/9 + c**4*e*x**10/10
```

Giac [A] time = 1.32896, size = 136, normalized size = 1.86

$$\frac{1}{10}c^4x^{10}e + \frac{1}{9}c^4dx^9 + \frac{1}{2}ac^3x^8e + \frac{4}{7}ac^3dx^7 + a^2c^2x^6e + \frac{6}{5}a^2c^2dx^5 + a^3cx^4e + \frac{4}{3}a^3cdx^3 + \frac{1}{2}a^4x^2e + a^4dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/10*c^4*x^10*e + 1/9*c^4*d*x^9 + 1/2*a*c^3*x^8*e + 4/7*a*c^3*d*x^7 + a^2*c
^2*x^6*e + 6/5*a^2*c^2*d*x^5 + a^3*c*x^4*e + 4/3*a^3*c*d*x^3 + 1/2*a^4*x^2*
e + a^4*d*x
```

3.495 $\int \frac{(a+cx^2)^4}{d+ex}\,dx$

Optimal. Leaf size=264

$$\frac{c^2(d+ex)^4\left(3a^2e^4+30acd^2e^2+35c^2d^4\right)}{2e^9}+\frac{2c^3(d+ex)^6\left(ae^2+7cd^2\right)}{3e^9}-\frac{8c^3d(d+ex)^5\left(3ae^2+7cd^2\right)}{5e^9}-\frac{8c^2d(d+ex)^3\left(\right.}{}$$

```
[Out] (-8*c*d*(c*d^2 + a*e^2)^3*x)/e^8 + (2*c*(c*d^2 + a*e^2)^2*(7*c*d^2 + a*e^2)
*(d + e*x)^2)/e^9 - (8*c^2*d*(c*d^2 + a*e^2)*(7*c*d^2 + 3*a*e^2)*(d + e*x)^
3)/(3*e^9) + (c^2*(35*c^2*d^4 + 30*a*c*d^2*e^2 + 3*a^2*e^4)*(d + e*x)^4)/(2
*e^9) - (8*c^3*d*(7*c*d^2 + 3*a*e^2)*(d + e*x)^5)/(5*e^9) + (2*c^3*(7*c*d^2
 + a*e^2)*(d + e*x)^6)/(3*e^9) - (8*c^4*d*(d + e*x)^7)/(7*e^9) + (c^4*(d +
e*x)^8)/(8*e^9) + ((c*d^2 + a*e^2)^4*Log[d + e*x])/e^9
```

Rubi [A] time = 0.280778, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$\frac{c^2(d+ex)^4\left(3a^2e^4+30acd^2e^2+35c^2d^4\right)}{2e^9}+\frac{2c^3(d+ex)^6\left(ae^2+7cd^2\right)}{3e^9}-\frac{8c^3d(d+ex)^5\left(3ae^2+7cd^2\right)}{5e^9}-\frac{8c^2d(d+ex)^3\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^4/(d + e*x),x]
```

```
[Out] (-8*c*d*(c*d^2 + a*e^2)^3*x)/e^8 + (2*c*(c*d^2 + a*e^2)^2*(7*c*d^2 + a*e^2)
*(d + e*x)^2)/e^9 - (8*c^2*d*(c*d^2 + a*e^2)*(7*c*d^2 + 3*a*e^2)*(d + e*x)^
3)/(3*e^9) + (c^2*(35*c^2*d^4 + 30*a*c*d^2*e^2 + 3*a^2*e^4)*(d + e*x)^4)/(2
*e^9) - (8*c^3*d*(7*c*d^2 + 3*a*e^2)*(d + e*x)^5)/(5*e^9) + (2*c^3*(7*c*d^2
 + a*e^2)*(d + e*x)^6)/(3*e^9) - (8*c^4*d*(d + e*x)^7)/(7*e^9) + (c^4*(d +
e*x)^8)/(8*e^9) + ((c*d^2 + a*e^2)^4*Log[d + e*x])/e^9
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^4}{d+ex}\,dx = \int \left(-\frac{8cd\left(cd^2+ae^2\right)^3}{e^8}+\frac{\left(cd^2+ae^2\right)^4}{e^8(d+ex)}+\frac{4c\left(cd^2+ae^2\right)^2\left(7cd^2+ae^2\right)(d+ex)}{e^8}+\frac{8c^2d\left(-7cd^2-3ae\right.}{}\right.$$

$$= -\frac{8cd\left(cd^2+ae^2\right)^3x}{e^8}+\frac{2c\left(cd^2+ae^2\right)^2\left(7cd^2+ae^2\right)(d+ex)^2}{e^9}-\frac{8c^2d\left(cd^2+ae^2\right)\left(7cd^2+3ae^2\right)(d+}{3e^9}$$

Mathematica [A] time = 0.105041, size = 227, normalized size = 0.86

$$\frac{cx\left(420a^2ce^4\left(6d^2ex-12d^3-4de^2x^2+3e^3x^3\right)+1680a^3e^6(ex-2d)+56ac^2e^2\left(-20d^3e^2x^2+15d^2e^3x^3+30d^4ex-60d^5\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^4/(d + e*x),x]
```

```
[Out] (c*x*(1680*a^3*e^6*(-2*d + e*x) + 420*a^2*c*e^4*(-12*d^3 + 6*d^2*e*x - 4*d*
e^2*x^2 + 3*e^3*x^3) + 56*a*c^2*e^2*(-60*d^5 + 30*d^4*e*x - 20*d^3*e^2*x^2
+ 15*d^2*e^3*x^3 - 12*d*e^4*x^4 + 10*e^5*x^5) + c^3*(-840*d^7 + 420*d^6*e*x
 - 280*d^5*e^2*x^2 + 210*d^4*e^3*x^3 - 168*d^3*e^4*x^4 + 140*d^2*e^5*x^5 -
120*d*e^6*x^6 + 105*e^7*x^7)))/(840*e^8) + ((c*d^2 + a*e^2)^4*Log[d + e*x])
/e^9
```

Maple [A] time = 0.046, size = 358, normalized size = 1.4

$$-\frac{4c^3x^5ad}{5e^2} + 4\frac{\ln(ex+d)a^3cd^2}{e^3} + \frac{3c^2x^4a^2}{2e} + \frac{c^4x^4d^4}{4e^5} - \frac{c^4x^3d^5}{3e^6} + \frac{\ln(ex+d)c^4d^8}{e^9} + 2\frac{cx^2a^3}{e} + \frac{c^4x^2d^6}{2e^7} - \frac{c^4d^7x}{e^8} - \frac{c^4dx^7}{7e^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^4/(e*x+d),x)
```

```
[Out] -4/5*c^3/e^2*x^5*a*d+4/e^3*ln(e*x+d)*a^3*c*d^2+3/2*c^2/e*x^4*a^2+1/4*c^4/e^
5*x^4*d^4-1/3*c^4/e^6*x^3*d^5+1/e^9*ln(e*x+d)*c^4*d^8+2*c/e*x^2*a^3+1/2*c^4
/e^7*x^2*d^6-c^4/e^8*d^7*x-1/7*c^4/e^2*d*x^7+2/3*c^3/e*x^6*a+1/6*c^4/e^3*x^
6*d^2+c^3/e^3*x^4*a*d^2+1/8*c^4/e*x^8+1/e*ln(e*x+d)*a^4-4*c^3/e^6*a*d^5*x-2
*c^2/e^2*x^3*a^2*d-4/3*c^3/e^4*x^3*a*d^3+3*c^2/e^3*x^2*a^2*d^2+2*c^3/e^5*x^
2*a*d^4-4*c/e^2*a^3*d*x-6*c^2/e^4*a^2*d^3*x+6/e^5*ln(e*x+d)*a^2*c^2*d^4+4/e
^7*ln(e*x+d)*a*c^3*d^6-1/5*c^4/e^4*x^5*d^3
```

Maxima [A] time = 1.18096, size = 431, normalized size = 1.63

$$\frac{105\,c^4e^7x^8 - 120\,c^4de^6x^7 + 140\left(c^4d^2e^5 + 4\,ac^3e^7\right)x^6 - 168\left(c^4d^3e^4 + 4\,ac^3de^6\right)x^5 + 210\left(c^4d^4e^3 + 4\,ac^3d^2e^5 + 6\,a^2c^2e^7\right)x^4 -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^4/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/840*(105*c^4*e^7*x^8 - 120*c^4*d*e^6*x^7 + 140*(c^4*d^2*e^5 + 4*a*c^3*e^7
)*x^6 - 168*(c^4*d^3*e^4 + 4*a*c^3*d*e^6)*x^5 + 210*(c^4*d^4*e^3 + 4*a*c^3*
d^2*e^5 + 6*a^2*c^2*e^7)*x^4 - 280*(c^4*d^5*e^2 + 4*a*c^3*d^3*e^4 + 6*a^2*c
^2*d*e^6)*x^3 + 420*(c^4*d^6*e + 4*a*c^3*d^4*e^3 + 6*a^2*c^2*d^2*e^5 + 4*a^
3*c*e^7)*x^2 - 840*(c^4*d^7 + 4*a*c^3*d^5*e^2 + 6*a^2*c^2*d^3*e^4 + 4*a^3*c
*d*e^6)*x)/e^8 + (c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d
^2*e^6 + a^4*e^8)*log(e*x + d)/e^9
```

Fricas [A] time = 1.88995, size = 662, normalized size = 2.51

$$\frac{105\,c^4e^8x^8 - 120\,c^4de^7x^7 + 140\left(c^4d^2e^6 + 4\,ac^3e^8\right)x^6 - 168\left(c^4d^3e^5 + 4\,ac^3de^7\right)x^5 + 210\left(c^4d^4e^4 + 4\,ac^3d^2e^6 + 6\,a^2c^2e^8\right)x^4 -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^4/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/840*(105*c^4*e^8*x^8 - 120*c^4*d*e^7*x^7 + 140*(c^4*d^2*e^6 + 4*a*c^3*e^8
)*x^6 - 168*(c^4*d^3*e^5 + 4*a*c^3*d*e^7)*x^5 + 210*(c^4*d^4*e^4 + 4*a*c^3*
d^2*e^6 + 6*a^2*c^2*e^8)*x^4 - 280*(c^4*d^5*e^3 + 4*a*c^3*d^3*e^5 + 6*a^2*c
^2*d*e^7)*x^3 + 420*(c^4*d^6*e^2 + 4*a*c^3*d^4*e^4 + 6*a^2*c^2*d^2*e^6 + 4*
a^3*c*e^8)*x^2 - 840*(c^4*d^7*e + 4*a*c^3*d^5*e^3 + 6*a^2*c^2*d^3*e^5 + 4*a
^3*c*d*e^7)*x + 840*(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*
c*d^2*e^6 + a^4*e^8)*log(e*x + d))/e^9
```

Sympy [A] time = 0.734259, size = 287, normalized size = 1.09

$$-\frac{c^4dx^7}{7e^2}+\frac{c^4x^8}{8e}+\frac{x^6\left(4ac^3e^2+c^4d^2\right)}{6e^3}-\frac{x^5\left(4ac^3de^2+c^4d^3\right)}{5e^4}+\frac{x^4\left(6a^2c^2e^4+4ac^3d^2e^2+c^4d^4\right)}{4e^5}-\frac{x^3\left(6a^2c^2de^4+4ac^3d^3\right.}{3e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**4/(e*x+d),x)
```

```
[Out] -c**4*d*x**7/(7*e**2) + c**4*x**8/(8*e) + x**6*(4*a*c**3*e**2 + c**4*d**2)/
(6*e**3) - x**5*(4*a*c**3*d*e**2 + c**4*d**3)/(5*e**4) + x**4*(6*a**2*c**2*
e**4 + 4*a*c**3*d**2*e**2 + c**4*d**4)/(4*e**5) - x**3*(6*a**2*c**2*d*e**4
+ 4*a*c**3*d**3*e**2 + c**4*d**5)/(3*e**6) + x**2*(4*a**3*c*e**6 + 6*a**2*c
**2*d**2*e**4 + 4*a*c**3*d**4*e**2 + c**4*d**6)/(2*e**7) - x*(4*a**3*c*d*e*
*6 + 6*a**2*c**2*d**3*e**4 + 4*a*c**3*d**5*e**2 + c**4*d**7)/e**8 + (a*e**2
 + c*d**2)**4*log(d + e*x)/e**9
```

Giac [A] time = 1.33633, size = 427, normalized size = 1.62

$$\left(c^4d^8+4\,ac^3d^6e^2+6\,a^2c^2d^4e^4+4\,a^3cd^2e^6+a^4e^8\right)e^{(-9)}\log\left(|xe+d|\right)+\frac{1}{840}\left(105\,c^4x^8e^7-120\,c^4dx^7e^6+140\,c^4d^2x^6e^5-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^4/(e*x+d),x, algorithm="giac")
```

```
[Out] (c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)
*e^(-9)*log(abs(x*e + d)) + 1/840*(105*c^4*x^8*e^7 - 120*c^4*d*x^7*e^6 + 14
0*c^4*d^2*x^6*e^5 - 168*c^4*d^3*x^5*e^4 + 210*c^4*d^4*x^4*e^3 - 280*c^4*d^5
*x^3*e^2 + 420*c^4*d^6*x^2*e - 840*c^4*d^7*x + 560*a*c^3*x^6*e^7 - 672*a*c^
3*d*x^5*e^6 + 840*a*c^3*d^2*x^4*e^5 - 1120*a*c^3*d^3*x^3*e^4 + 1680*a*c^3*d
^4*x^2*e^3 - 3360*a*c^3*d^5*x*e^2 + 1260*a^2*c^2*x^4*e^7 - 1680*a^2*c^2*d*x
^3*e^6 + 2520*a^2*c^2*d^2*x^2*e^5 - 5040*a^2*c^2*d^3*x*e^4 + 1680*a^3*c*x^2
*e^7 - 3360*a^3*c*d*x*e^6)*e^(-8)
```

3.496 $\int \frac{(a+cx^2)^4}{(d+ex)^2}\,dx$

Optimal. Leaf size=255

$$\frac{c^2x^3\left(6a^2e^4+12acd^2e^2+5c^2d^4\right)}{3e^6}-\frac{c^2dx^2\left(6a^2e^4+8acd^2e^2+3c^2d^4\right)}{e^7}+\frac{cx\left(18a^2cd^2e^4+4a^3e^6+20ac^2d^4e^2+7c^3d^6\right)}{e^8}+\frac{c^3x}{}$$

```
[Out] (c*(7*c^3*d^6 + 20*a*c^2*d^4*e^2 + 18*a^2*c*d^2*e^4 + 4*a^3*e^6)*x)/e^8 - (
c^2*d*(3*c^2*d^4 + 8*a*c*d^2*e^2 + 6*a^2*e^4)*x^2)/e^7 + (c^2*(5*c^2*d^4 +
12*a*c*d^2*e^2 + 6*a^2*e^4)*x^3)/(3*e^6) - (c^3*d*(c*d^2 + 2*a*e^2)*x^4)/e^
5 + (c^3*(3*c*d^2 + 4*a*e^2)*x^5)/(5*e^4) - (c^4*d*x^6)/(3*e^3) + (c^4*x^7)
/(7*e^2) - (c*d^2 + a*e^2)^4/(e^9*(d + e*x)) - (8*c*d*(c*d^2 + a*e^2)^3*Log
[d + e*x])/e^9
```

Rubi [A] time = 0.255321, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$\frac{c^2x^3\left(6a^2e^4+12acd^2e^2+5c^2d^4\right)}{3e^6}-\frac{c^2dx^2\left(6a^2e^4+8acd^2e^2+3c^2d^4\right)}{e^7}+\frac{cx\left(18a^2cd^2e^4+4a^3e^6+20ac^2d^4e^2+7c^3d^6\right)}{e^8}+\frac{c^3x}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^4/(d + e*x)^2,x]
```

```
[Out] (c*(7*c^3*d^6 + 20*a*c^2*d^4*e^2 + 18*a^2*c*d^2*e^4 + 4*a^3*e^6)*x)/e^8 - (
c^2*d*(3*c^2*d^4 + 8*a*c*d^2*e^2 + 6*a^2*e^4)*x^2)/e^7 + (c^2*(5*c^2*d^4 +
12*a*c*d^2*e^2 + 6*a^2*e^4)*x^3)/(3*e^6) - (c^3*d*(c*d^2 + 2*a*e^2)*x^4)/e^
5 + (c^3*(3*c*d^2 + 4*a*e^2)*x^5)/(5*e^4) - (c^4*d*x^6)/(3*e^3) + (c^4*x^7)
/(7*e^2) - (c*d^2 + a*e^2)^4/(e^9*(d + e*x)) - (8*c*d*(c*d^2 + a*e^2)^3*Log
[d + e*x])/e^9
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^4}{(d+ex)^2}\,dx=\int \left(\frac{c\left(7c^3d^6+20ac^2d^4e^2+18a^2cd^2e^4+4a^3e^6\right)}{e^8}-\frac{2c^2d\left(3c^2d^4+8acd^2e^2+6a^2e^4\right)x}{e^7}+\frac{c^2\left(5c^2d^4+12a\right.}{}\right.$$

$$=\frac{c\left(7c^3d^6+20ac^2d^4e^2+18a^2cd^2e^4+4a^3e^6\right)x}{e^8}-\frac{c^2d\left(3c^2d^4+8acd^2e^2+6a^2e^4\right)x^2}{e^7}+\frac{c^2\left(5c^2d^4+12acd\right.}{3e^6}$$

Mathematica [A] time = 0.0938536, size = 289, normalized size = 1.13

$$\frac{210a^2c^2e^4\left(6d^2e^2x^2+9d^3ex-3d^4-2de^3x^3+e^4x^4\right)+420a^3ce^6\left(-d^2+dex+e^2x^2\right)-105a^4e^8+42ac^3e^2\left(30d^4e^2x^2-10d^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^4/(d + e*x)^2,x]
```

```
[Out] (-105*a^4*e^8 + 420*a^3*c*e^6*(-d^2 + d*e*x + e^2*x^2) + 210*a^2*c^2*e^4*(-
3*d^4 + 9*d^3*e*x + 6*d^2*e^2*x^2 - 2*d*e^3*x^3 + e^4*x^4) + 42*a*c^3*e^2*(
-10*d^6 + 50*d^5*e*x + 30*d^4*e^2*x^2 - 10*d^3*e^3*x^3 + 5*d^2*e^4*x^4 - 3*
d*e^5*x^5 + 2*e^6*x^6) + c^4*(-105*d^8 + 735*d^7*e*x + 420*d^6*e^2*x^2 - 14
0*d^5*e^3*x^3 + 70*d^4*e^4*x^4 - 42*d^3*e^5*x^5 + 28*d^2*e^6*x^6 - 20*d*e^7
*x^7 + 15*e^8*x^8) - 840*c*d*(c*d^2 + a*e^2)^3*(d + e*x)*Log[d + e*x])/(105
*e^9*(d + e*x))
```

Maple [A] time = 0.052, size = 378, normalized size = 1.5

$$\frac{c^4x^7}{7e^2}-4\frac{ac^3d^6}{e^7(ex+d)}-6\frac{c^2x^2a^2d}{e^3}-8\frac{c^3x^2ad^3}{e^5}+18\frac{a^2c^2d^2x}{e^4}+20\frac{d^4ac^3x}{e^6}-8\frac{cd\ln(ex+d)a^3}{e^3}-24\frac{c^2d^3\ln(ex+d)a^2}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^4/(e*x+d)^2,x)
```

```
[Out] 1/7*c^4*x^7/e^2-4/e^7/(e*x+d)*a*c^3*d^6-6*c^2/e^3*x^2*a^2*d-8*c^3/e^5*x^2*a
*d^3+18*c^2/e^4*a^2*d^2*x+20*c^3/e^6*d^4*a*x-8*c*d/e^3*ln(e*x+d)*a^3-24*c^2
*d^3/e^5*ln(e*x+d)*a^2-24*c^3*d^5/e^7*ln(e*x+d)*a-4/e^3/(e*x+d)*a^3*c*d^2-6
/e^5/(e*x+d)*a^2*c^2*d^4-1/e/(e*x+d)*a^4-2*c^3/e^3*x^4*a*d+4*c^3/e^4*x^3*a*
d^2-c^4/e^5*x^4*d^3+2*c^2/e^2*x^3*a^2+5/3*c^4/e^6*x^3*d^4-3*c^4/e^7*x^2*d^5
-8*c^4*d^7/e^9*ln(e*x+d)-1/e^9/(e*x+d)*c^4*d^8+4*c/e^2*a^3*x+7*c^4/e^8*d^6*
x+4/5*c^3/e^2*x^5*a+3/5*c^4/e^4*x^5*d^2-1/3*c^4*d*x^6/e^3
```

Maxima [A] time = 1.14774, size = 446, normalized size = 1.75

$$-\frac{c^4d^8+4\,ac^3d^6e^2+6\,a^2c^2d^4e^4+4\,a^3cd^2e^6+a^4e^8}{e^{10}x+de^9}+\frac{15\,c^4e^6x^7-35\,c^4de^5x^6+21\left(3\,c^4d^2e^4+4\,ac^3e^6\right)x^5-105\left(c^4d^3e^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^4/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8
)/(e^10*x + d*e^9) + 1/105*(15*c^4*e^6*x^7 - 35*c^4*d*e^5*x^6 + 21*(3*c^4*d
^2*e^4 + 4*a*c^3*e^6)*x^5 - 105*(c^4*d^3*e^3 + 2*a*c^3*d*e^5)*x^4 + 35*(5*c
^4*d^4*e^2 + 12*a*c^3*d^2*e^4 + 6*a^2*c^2*e^6)*x^3 - 105*(3*c^4*d^5*e + 8*a
*c^3*d^3*e^3 + 6*a^2*c^2*d*e^5)*x^2 + 105*(7*c^4*d^6 + 20*a*c^3*d^4*e^2 + 1
8*a^2*c^2*d^2*e^4 + 4*a^3*c*e^6)*x)/e^8 - 8*(c^4*d^7 + 3*a*c^3*d^5*e^2 + 3*
a^2*c^2*d^3*e^4 + a^3*c*d*e^6)*log(e*x + d)/e^9
```

Fricas [A] time = 1.86017, size = 872, normalized size = 3.42

$$\frac{15\,c^4e^8x^8-20\,c^4de^7x^7-105\,c^4d^8-420\,ac^3d^6e^2-630\,a^2c^2d^4e^4-420\,a^3cd^2e^6-105\,a^4e^8+28\left(c^4d^2e^6+3\,ac^3e^8\right)x^6-4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^4/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/105*(15*c^4*e^8*x^8 - 20*c^4*d*e^7*x^7 - 105*c^4*d^8 - 420*a*c^3*d^6*e^2
- 630*a^2*c^2*d^4*e^4 - 420*a^3*c*d^2*e^6 - 105*a^4*e^8 + 28*(c^4*d^2*e^6 +
 3*a*c^3*e^8)*x^6 - 42*(c^4*d^3*e^5 + 3*a*c^3*d*e^7)*x^5 + 70*(c^4*d^4*e^4
+ 3*a*c^3*d^2*e^6 + 3*a^2*c^2*e^8)*x^4 - 140*(c^4*d^5*e^3 + 3*a*c^3*d^3*e^5
 + 3*a^2*c^2*d*e^7)*x^3 + 420*(c^4*d^6*e^2 + 3*a*c^3*d^4*e^4 + 3*a^2*c^2*d^
2*e^6 + a^3*c*e^8)*x^2 + 105*(7*c^4*d^7*e + 20*a*c^3*d^5*e^3 + 18*a^2*c^2*d
^3*e^5 + 4*a^3*c*d*e^7)*x - 840*(c^4*d^8 + 3*a*c^3*d^6*e^2 + 3*a^2*c^2*d^4*
e^4 + a^3*c*d^2*e^6 + (c^4*d^7*e + 3*a*c^3*d^5*e^3 + 3*a^2*c^2*d^3*e^5 + a^
3*c*d*e^7)*x)*log(e*x + d))/(e^10*x + d*e^9)
```

Sympy [A] time = 1.39615, size = 308, normalized size = 1.21

$$-\frac{c^4dx^6}{3e^3}+\frac{c^4x^7}{7e^2}-\frac{8cd\left(ae^2+cd^2\right)^3\log\left(d+ex\right)}{e^9}-\frac{a^4e^8+4a^3cd^2e^6+6a^2c^2d^4e^4+4ac^3d^6e^2+c^4d^8}{de^9+e^{10}x}+\frac{x^5\left(4ac^3e^2+3c^4d^2\right)}{5e^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**4/(e*x+d)**2,x)
```

```
[Out] -c**4*d*x**6/(3*e**3) + c**4*x**7/(7*e**2) - 8*c*d*(a*e**2 + c*d**2)**3*log
(d + e*x)/e**9 - (a**4*e**8 + 4*a**3*c*d**2*e**6 + 6*a**2*c**2*d**4*e**4 +
4*a*c**3*d**6*e**2 + c**4*d**8)/(d*e**9 + e**10*x) + x**5*(4*a*c**3*e**2 +
3*c**4*d**2)/(5*e**4) - x**4*(2*a*c**3*d*e**2 + c**4*d**3)/e**5 + x**3*(6*a
**2*c**2*e**4 + 12*a*c**3*d**2*e**2 + 5*c**4*d**4)/(3*e**6) - x**2*(6*a**2*
c**2*d*e**4 + 8*a*c**3*d**3*e**2 + 3*c**4*d**5)/e**7 + x*(4*a**3*c*e**6 + 1
8*a**2*c**2*d**2*e**4 + 20*a*c**3*d**4*e**2 + 7*c**4*d**6)/e**8
```

Giac [A] time = 1.29118, size = 535, normalized size = 2.1

$$\frac{1}{105}\left(15\,c^4-\frac{140\,c^4d}{xe+d}+\frac{84\left(7\,c^4d^2e^2+ac^3e^4\right)e^{(-2)}}{(xe+d)^2}-\frac{210\left(7\,c^4d^3e^3+3\,ac^3de^5\right)e^{(-3)}}{(xe+d)^3}+\frac{70\left(35\,c^4d^4e^4+30\,ac^3d^2e^6+3\,a^2c^2e^8\right.}{(xe+d)^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^4/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/105*(15*c^4 - 140*c^4*d/(x*e + d) + 84*(7*c^4*d^2*e^2 + a*c^3*e^4)*e^(-2)
/(x*e + d)^2 - 210*(7*c^4*d^3*e^3 + 3*a*c^3*d*e^5)*e^(-3)/(x*e + d)^3 + 70*
(35*c^4*d^4*e^4 + 30*a*c^3*d^2*e^6 + 3*a^2*c^2*e^8)*e^(-4)/(x*e + d)^4 - 42
0*(7*c^4*d^5*e^5 + 10*a*c^3*d^3*e^7 + 3*a^2*c^2*d*e^9)*e^(-5)/(x*e + d)^5 +
 420*(7*c^4*d^6*e^6 + 15*a*c^3*d^4*e^8 + 9*a^2*c^2*d^2*e^10 + a^3*c*e^12)*e
^(-6)/(x*e + d)^6)*(x*e + d)^7*e^(-9) + 8*(c^4*d^7 + 3*a*c^3*d^5*e^2 + 3*a^
2*c^2*d^3*e^4 + a^3*c*d*e^6)*e^(-9)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2) -
(c^4*d^8*e^7/(x*e + d) + 4*a*c^3*d^6*e^9/(x*e + d) + 6*a^2*c^2*d^4*e^11/(x*
e + d) + 4*a^3*c*d^2*e^13/(x*e + d) + a^4*e^15/(x*e + d))*e^(-16)
```

3.497 $\int (c+dx)\left(a+bx^2\right)^4\,dx$

Optimal. Leaf size=73

$$\frac{6}{5}a^2b^2cx^5+\frac{4}{3}a^3bcx^3+a^4cx+\frac{4}{7}ab^3cx^7+\frac{d\left(a+bx^2\right)^5}{10b}+\frac{1}{9}b^4cx^9$$

```
[Out] a^4*c*x + (4*a^3*b*c*x^3)/3 + (6*a^2*b^2*c*x^5)/5 + (4*a*b^3*c*x^7)/7 + (b^
4*c*x^9)/9 + (d*(a + b*x^2)^5)/(10*b)
```

Rubi [A] time = 0.0306908, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {641, 194}

$$\frac{6}{5}a^2b^2cx^5+\frac{4}{3}a^3bcx^3+a^4cx+\frac{4}{7}ab^3cx^7+\frac{d\left(a+bx^2\right)^5}{10b}+\frac{1}{9}b^4cx^9$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + b*x^2)^4,x]
```

```
[Out] a^4*c*x + (4*a^3*b*c*x^3)/3 + (6*a^2*b^2*c*x^5)/5 + (4*a*b^3*c*x^7)/7 + (b^
4*c*x^9)/9 + (d*(a + b*x^2)^5)/(10*b)
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)\left(a+bx^2\right)^4\,dx &= \frac{d\left(a+bx^2\right)^5}{10b}+c\int \left(a+bx^2\right)^4\,dx\\
&= \frac{d\left(a+bx^2\right)^5}{10b}+c\int \left(a^4+4a^3bx^2+6a^2b^2x^4+4ab^3x^6+b^4x^8\right)\,dx\\
&= a^4cx+\frac{4}{3}a^3bcx^3+\frac{6}{5}a^2b^2cx^5+\frac{4}{7}ab^3cx^7+\frac{1}{9}b^4cx^9+\frac{d\left(a+bx^2\right)^5}{10b}
\end{aligned}$$

Mathematica [A] time = 0.0036308, size = 110, normalized size = 1.51

$$\frac{6}{5}a^2b^2cx^5+a^2b^2dx^6+\frac{4}{3}a^3bcx^3+a^3bdx^4+a^4cx+\frac{1}{2}a^4dx^2+\frac{4}{7}ab^3cx^7+\frac{1}{2}ab^3dx^8+\frac{1}{9}b^4cx^9+\frac{1}{10}b^4dx^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + b*x^2)^4,x]
```

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + (4*a^3*b*c*x^3)/3 + a^3*b*d*x^4 + (6*a^2*b^2*c*x^
5)/5 + a^2*b^2*d*x^6 + (4*a*b^3*c*x^7)/7 + (a*b^3*d*x^8)/2 + (b^4*c*x^9)/9
+ (b^4*d*x^10)/10
```

Maple [A] time = 0.042, size = 97, normalized size = 1.3

$$\frac{db^4x^{10}}{10}+\frac{b^4cx^9}{9}+\frac{ab^3dx^8}{2}+\frac{4\,ab^3cx^7}{7}+a^2b^2dx^6+\frac{6\,a^2b^2cx^5}{5}+da^3bx^4+\frac{4\,a^3bcx^3}{3}+\frac{a^4dx^2}{2}+a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(b*x^2+a)^4,x)
```

```
[Out] 1/10*d*b^4*x^10+1/9*b^4*c*x^9+1/2*a*b^3*d*x^8+4/7*a*b^3*c*x^7+a^2*b^2*d*x^6
+6/5*a^2*b^2*c*x^5+d*a^3*b*x^4+4/3*a^3*b*c*x^3+1/2*a^4*d*x^2+a^4*c*x
```

Maxima [A] time = 1.06903, size = 130, normalized size = 1.78

$$\frac{1}{10}\,b^4dx^{10}+\frac{1}{9}\,b^4cx^9+\frac{1}{2}\,ab^3dx^8+\frac{4}{7}\,ab^3cx^7+a^2b^2dx^6+\frac{6}{5}\,a^2b^2cx^5+a^3bdx^4+\frac{4}{3}\,a^3bcx^3+\frac{1}{2}\,a^4dx^2+a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(b*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] 1/10*b^4*d*x^10 + 1/9*b^4*c*x^9 + 1/2*a*b^3*d*x^8 + 4/7*a*b^3*c*x^7 + a^2*b
^2*d*x^6 + 6/5*a^2*b^2*c*x^5 + a^3*b*d*x^4 + 4/3*a^3*b*c*x^3 + 1/2*a^4*d*x^
2 + a^4*c*x
```

Fricas [A] time = 1.62656, size = 220, normalized size = 3.01

$$\frac{1}{10}x^{10}db^4+\frac{1}{9}x^9cb^4+\frac{1}{2}x^8db^3a+\frac{4}{7}x^7cb^3a+x^6db^2a^2+\frac{6}{5}x^5cb^2a^2+x^4dba^3+\frac{4}{3}x^3cba^3+\frac{1}{2}x^2da^4+xca^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(b*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*d*b^4 + 1/9*x^9*c*b^4 + 1/2*x^8*d*b^3*a + 4/7*x^7*c*b^3*a + x^6*d
*b^2*a^2 + 6/5*x^5*c*b^2*a^2 + x^4*d*b*a^3 + 4/3*x^3*c*b*a^3 + 1/2*x^2*d*a^
4 + x*c*a^4
```

Sympy [A] time = 0.082166, size = 112, normalized size = 1.53

$$a^4cx+\frac{a^4dx^2}{2}+\frac{4a^3bcx^3}{3}+a^3bdx^4+\frac{6a^2b^2cx^5}{5}+a^2b^2dx^6+\frac{4ab^3cx^7}{7}+\frac{ab^3dx^8}{2}+\frac{b^4cx^9}{9}+\frac{b^4dx^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(b*x**2+a)**4,x)
```

```
[Out] a**4*c*x + a**4*d*x**2/2 + 4*a**3*b*c*x**3/3 + a**3*b*d*x**4 + 6*a**2*b**2*
c*x**5/5 + a**2*b**2*d*x**6 + 4*a*b**3*c*x**7/7 + a*b**3*d*x**8/2 + b**4*c*
x**9/9 + b**4*d*x**10/10
```

Giac [A] time = 1.27079, size = 130, normalized size = 1.78

$$\frac{1}{10}\,b^4dx^{10}+\frac{1}{9}\,b^4cx^9+\frac{1}{2}\,ab^3dx^8+\frac{4}{7}\,ab^3cx^7+a^2b^2dx^6+\frac{6}{5}\,a^2b^2cx^5+a^3bdx^4+\frac{4}{3}\,a^3bcx^3+\frac{1}{2}\,a^4dx^2+a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(b*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/10*b^4*d*x^10 + 1/9*b^4*c*x^9 + 1/2*a*b^3*d*x^8 + 4/7*a*b^3*c*x^7 + a^2*b
^2*d*x^6 + 6/5*a^2*b^2*c*x^5 + a^3*b*d*x^4 + 4/3*a^3*b*c*x^3 + 1/2*a^4*d*x^
2 + a^4*c*x
```

3.498 $\int \frac{(d+ex)^4}{a+cx^2}\,dx$

Optimal. Leaf size=123

$$\frac{\left(a^2e^4-6acd^2e^2+c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{5/2}}+\frac{2de\left(cd^2-ae^2\right)\log\left(a+cx^2\right)}{c^2}+\frac{e^2x\left(6cd^2-ae^2\right)}{c^2}+\frac{2de^3x^2}{c}+\frac{e^4x^3}{3c}$$

```
[Out] (e^2*(6*c*d^2 - a*e^2)*x)/c^2 + (2*d*e^3*x^2)/c + (e^4*x^3)/(3*c) + ((c^2*d
^4 - 6*a*c*d^2*e^2 + a^2*e^4)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(5/2)
) + (2*d*e*(c*d^2 - a*e^2)*Log[a + c*x^2])/c^2
```

Rubi [A] time = 0.0968408, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {702, 635, 205, 260}

$$\frac{\left(a^2e^4-6acd^2e^2+c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{5/2}}+\frac{2de\left(cd^2-ae^2\right)\log\left(a+cx^2\right)}{c^2}+\frac{e^2x\left(6cd^2-ae^2\right)}{c^2}+\frac{2de^3x^2}{c}+\frac{e^4x^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a + c*x^2),x]
```

```
[Out] (e^2*(6*c*d^2 - a*e^2)*x)/c^2 + (2*d*e^3*x^2)/c + (e^4*x^3)/(3*c) + ((c^2*d
^4 - 6*a*c*d^2*e^2 + a^2*e^4)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(5/2)
) + (2*d*e*(c*d^2 - a*e^2)*Log[a + c*x^2])/c^2
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{(d+ex)^4}{a+cx^2}\,dx = \int \left(\frac{e^2(6cd^2-ae^2)}{c^2} + \frac{4de^3x}{c} + \frac{e^4x^2}{c} + \frac{c^2d^4-6acd^2e^2+a^2e^4+4cde(cd^2-ae^2)x}{c^2(a+cx^2)}\right) dx$$
$$= \frac{e^2(6cd^2-ae^2)x}{c^2} + \frac{2de^3x^2}{c} + \frac{e^4x^3}{3c} + \frac{\int \frac{c^2d^4-6acd^2e^2+a^2e^4+4cde(cd^2-ae^2)x}{a+cx^2}dx}{c^2}$$
$$= \frac{e^2(6cd^2-ae^2)x}{c^2} + \frac{2de^3x^2}{c} + \frac{e^4x^3}{3c} + \frac{(4de(cd^2-ae^2))\int \frac{x}{a+cx^2}dx}{c} + \frac{(c^2d^4-6acd^2e^2+a^2e^4)\int \frac{1}{a+cx^2}$$
$$= \frac{e^2(6cd^2-ae^2)x}{c^2} + \frac{2de^3x^2}{c} + \frac{e^4x^3}{3c} + \frac{(c^2d^4-6acd^2e^2+a^2e^4)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{5/2}} + \frac{2de(cd^2-ae^2)\log(a}{c^2}$$

Mathematica [A] time = 0.0872038, size = 111, normalized size = 0.9

$$\frac{(a^2e^4-6acd^2e^2+c^2d^4)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{5/2}} + \frac{e(6(cd^3-ade^2)\log(a+cx^2)-3ae^3x+cex(18d^2+6dex+e^2x^2))}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a + c*x^2),x]
```

```
[Out] ((c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*
c^(5/2)) + (e*(-3*a*e^3*x + c*e*x*(18*d^2 + 6*d*e*x + e^2*x^2) + 6*(c*d^3 -
 a*d*e^2)*Log[a + c*x^2]))/(3*c^2)
```

Maple [A] time = 0.048, size = 150, normalized size = 1.2

$$\frac{e^4x^3}{3c} + 2\frac{de^3x^2}{c} - \frac{e^4ax}{c^2} + 6\frac{d^2e^2x}{c} - 2\frac{\ln(cx^2+a)ade^3}{c^2} + 2\frac{\ln(cx^2+a)d^3e}{c} + \frac{a^2e^4}{c^2}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}} - 6\frac{ad^2e^2}{c\sqrt{ac}}\text{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(c*x^2+a),x)
```

```
[Out] 1/3*e^4*x^3/c+2*d*e^3*x^2/c-e^4/c^2*a*x+6*e^2/c*d^2*x-2/c^2*ln(c*x^2+a)*a*d
*e^3+2/c*ln(c*x^2+a)*d^3*e+1/c^2/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*a^2*e^
4-6/c/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*a*d^2*e^2+1/(a*c)^(1/2)*arctan(x*
c/(a*c)^(1/2))*d^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.01712, size = 595, normalized size = 4.84

$$\left[\frac{2\,ac^{2}e^{4}x^{3}+12\,ac^{2}de^{3}x^{2}-3\,\left(c^{2}d^{4}-6\,acd^{2}e^{2}+a^{2}e^{4}\right)\sqrt{-ac}\log\left(\frac{cx^{2}-2\,\sqrt{-ac}x-a}{cx^{2}+a}\right)+6\,\left(6\,ac^{2}d^{2}e^{2}-a^{2}ce^{4}\right)x+12\,\left(ac^{2}d^{3}e-a^{2}cd\right.}{6\,ac^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*a*c^2*e^4*x^3 + 12*a*c^2*d*e^3*x^2 - 3*(c^2*d^4 - 6*a*c*d^2*e^2 + a
^2*e^4)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 6*(6*a*c
^2*d^2*e^2 - a^2*c*e^4)*x + 12*(a*c^2*d^3*e - a^2*c*d*e^3)*log(c*x^2 + a))/
(a*c^3), 1/3*(a*c^2*e^4*x^3 + 6*a*c^2*d*e^3*x^2 + 3*(c^2*d^4 - 6*a*c*d^2*e^
2 + a^2*e^4)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 3*(6*a*c^2*d^2*e^2 - a^2*c*e
^4)*x + 6*(a*c^2*d^3*e - a^2*c*d*e^3)*log(c*x^2 + a))/(a*c^3)]
```

Sympy [B] time = 1.24195, size = 401, normalized size = 3.26

$$\left(-\frac{2de\left(ae^{2}-cd^{2}\right)}{c^{2}}-\frac{\sqrt{-ac^{5}}\left(a^{2}e^{4}-6acd^{2}e^{2}+c^{2}d^{4}\right)}{2ac^{5}}\right)\log\left(x+\frac{4a^{2}de^{3}+2ac^{2}\left(-\frac{2de\left(ae^{2}-cd^{2}\right)}{c^{2}}-\frac{\sqrt{-ac^{5}}\left(a^{2}e^{4}-6acd^{2}e^{2}+c^{2}d^{4}\right)}{2ac^{5}}\right)-4ac\right.}{a^{2}e^{4}-6acd^{2}e^{2}+c^{2}d^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(c*x**2+a),x)
```

```
[Out] (-2*d*e*(a*e**2 - c*d**2)/c**2 - sqrt(-a*c**5)*(a**2*e**4 - 6*a*c*d**2*e**2
 + c**2*d**4)/(2*a*c**5))*log(x + (4*a**2*d*e**3 + 2*a*c**2*(-2*d*e*(a*e**2
 - c*d**2)/c**2 - sqrt(-a*c**5)*(a**2*e**4 - 6*a*c*d**2*e**2 + c**2*d**4)/(
2*a*c**5)) - 4*a*c*d**3*e)/(a**2*e**4 - 6*a*c*d**2*e**2 + c**2*d**4)) + (-2
*d*e*(a*e**2 - c*d**2)/c**2 + sqrt(-a*c**5)*(a**2*e**4 - 6*a*c*d**2*e**2 +
c**2*d**4)/(2*a*c**5))*log(x + (4*a**2*d*e**3 + 2*a*c**2*(-2*d*e*(a*e**2 -
c*d**2)/c**2 + sqrt(-a*c**5)*(a**2*e**4 - 6*a*c*d**2*e**2 + c**2*d**4)/(2*a
*c**5)) - 4*a*c*d**3*e)/(a**2*e**4 - 6*a*c*d**2*e**2 + c**2*d**4)) + 2*d*e*
*3*x**2/c + e**4*x**3/(3*c) - x*(a*e**4 - 6*c*d**2*e**2)/c**2
```

Giac [A] time = 1.25517, size = 153, normalized size = 1.24

$$\frac{2\left(cd^{3}e-ade^{3}\right)\log\left(cx^{2}+a\right)}{c^{2}}+\frac{\left(c^{2}d^{4}-6\,acd^{2}e^{2}+a^{2}e^{4}\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c^{2}}+\frac{c^{2}x^{3}e^{4}+6\,c^{2}dx^{2}e^{3}+18\,c^{2}d^{2}xe^{2}-3\,acxe^{4}}{3\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+a),x, algorithm="giac")
```

```
[Out] 2*(c*d^3*e - a*d*e^3)*log(c*x^2 + a)/c^2 + (c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e
^4)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c^2) + 1/3*(c^2*x^3*e^4 + 6*c^2*d*x^2*
e^3 + 18*c^2*d^2*x*e^2 - 3*a*c*x*e^4)/c^3
```

3.499 $\int \frac{(d+ex)^3}{a+cx^2} dx$

Optimal. Leaf size=90

$$\frac{e\left(3cd^2-ae^2\right)\log\left(a+cx^2\right)}{2c^2}+\frac{d\left(cd^2-3ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}}+\frac{3de^2x}{c}+\frac{e^3x^2}{2c}$$

```
[Out] (3*d*e^2*x)/c + (e^3*x^2)/(2*c) + (d*(c*d^2 - 3*a*e^2)*ArcTan[(Sqrt[c]*x)/S
qrt[a]])/(Sqrt[a]*c^(3/2)) + (e*(3*c*d^2 - a*e^2)*Log[a + c*x^2])/(2*c^2)
```

Rubi [A] time = 0.0745295, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {702, 635, 205, 260}

$$\frac{e\left(3cd^2-ae^2\right)\log\left(a+cx^2\right)}{2c^2}+\frac{d\left(cd^2-3ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}}+\frac{3de^2x}{c}+\frac{e^3x^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + c*x^2),x]
```

```
[Out] (3*d*e^2*x)/c + (e^3*x^2)/(2*c) + (d*(c*d^2 - 3*a*e^2)*ArcTan[(Sqrt[c]*x)/S
qrt[a]])/(Sqrt[a]*c^(3/2)) + (e*(3*c*d^2 - a*e^2)*Log[a + c*x^2])/(2*c^2)
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{(d+ex)^3}{a+cx^2}\,dx = \int \left(\frac{3de^2}{c}+\frac{e^3x}{c}+\frac{cd^3-3ade^2+e\left(3cd^2-ae^2\right)x}{c\left(a+cx^2\right)}\right)dx$$
$$= \frac{3de^2x}{c}+\frac{e^3x^2}{2c}+\frac{\int \frac{cd^3-3ade^2+e\left(3cd^2-ae^2\right)x}{a+cx^2}\,dx}{c}$$
$$= \frac{3de^2x}{c}+\frac{e^3x^2}{2c}+\frac{\left(d\left(cd^2-3ae^2\right)\right)\int \frac{1}{a+cx^2}\,dx}{c}+\frac{\left(e\left(3cd^2-ae^2\right)\right)\int \frac{x}{a+cx^2}\,dx}{c}$$
$$= \frac{3de^2x}{c}+\frac{e^3x^2}{2c}+\frac{d\left(cd^2-3ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}}+\frac{e\left(3cd^2-ae^2\right)\log\left(a+cx^2\right)}{2c^2}$$

Mathematica [A] time = 0.0565024, size = 80, normalized size = 0.89

$$\frac{e\left(\left(3cd^2-ae^2\right)\log\left(a+cx^2\right)+cex(6d+ex)\right)}{2c^2}+\frac{d\left(cd^2-3ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + c*x^2),x]
```

```
[Out] (d*(c*d^2 - 3*a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)) + (e*(c
*e*x*(6*d + e*x) + (3*c*d^2 - a*e^2)*Log[a + c*x^2]))/(2*c^2)
```

Maple [A] time = 0.045, size = 99, normalized size = 1.1

$$\frac{e^3x^2}{2c}+3\frac{de^2x}{c}-\frac{\ln\left(cx^2+a\right)ae^3}{2c^2}+\frac{3\ln\left(cx^2+a\right)d^2e}{2c}-3\frac{ade^2}{c\sqrt{ac}}\arctan\left(\frac{cx}{\sqrt{ac}}\right)+d^3\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^2+a),x)
```

```
[Out] 1/2*e^3*x^2/c+3*d*e^2*x/c-1/2/c^2*ln(c*x^2+a)*a*e^3+3/2/c*ln(c*x^2+a)*d^2*e
-3/c/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*a*d*e^2+1/(a*c)^(1/2)*arctan(x*c/(
a*c)^(1/2))*d^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77174, size = 414, normalized size = 4.6

$$\left[\frac{ace^3x^2 + 6\,acde^2x + \left(cd^3 - 3\,ade^2\right)\sqrt{-ac}\log\left(\frac{cx^2+2\,\sqrt{-ac}x-a}{cx^2+a}\right) + \left(3\,acd^2e - a^2e^3\right)\log\left(cx^2 + a\right)}{2\,ac^2}, \frac{ace^3x^2 + 6\,acde^2x + 2\left(c\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(a*c*e^3*x^2 + 6*a*c*d*e^2*x + (c*d^3 - 3*a*d*e^2)*sqrt(-a*c)*log((c*x
^2 + 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + (3*a*c*d^2*e - a^2*e^3)*log(c*x^2 +
 a))/(a*c^2), 1/2*(a*c*e^3*x^2 + 6*a*c*d*e^2*x + 2*(c*d^3 - 3*a*d*e^2)*sqrt
(a*c)*arctan(sqrt(a*c)*x/a) + (3*a*c*d^2*e - a^2*e^3)*log(c*x^2 + a))/(a*c^
2)]
```

Sympy [B] time = 0.969667, size = 308, normalized size = 3.42

$$\left(-\frac{e\left(ae^2 - 3cd^2\right)}{2c^2} - \frac{d\sqrt{-ac^5}\left(3ae^2 - cd^2\right)}{2ac^4}\right)\log\left(x + \frac{-a^2e^3 - 2ac^2\left(-\frac{e\left(ae^2-3cd^2\right)}{2c^2} - \frac{d\sqrt{-ac^5}\left(3ae^2-cd^2\right)}{2ac^4}\right) + 3acd^2e}{3acde^2 - c^2d^3}\right) + \left(-\frac{e\left(ae^2\right.}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**2+a),x)
```

```
[Out] (-e*(a*e**2 - 3*c*d**2)/(2*c**2) - d*sqrt(-a*c**5)*(3*a*e**2 - c*d**2)/(2*a
*c**4))*log(x + (-a**2*e**3 - 2*a*c**2*(-e*(a*e**2 - 3*c*d**2)/(2*c**2) - d
*sqrt(-a*c**5)*(3*a*e**2 - c*d**2)/(2*a*c**4)) + 3*a*c*d**2*e)/(3*a*c*d*e**
2 - c**2*d**3)) + (-e*(a*e**2 - 3*c*d**2)/(2*c**2) + d*sqrt(-a*c**5)*(3*a*e
**2 - c*d**2)/(2*a*c**4))*log(x + (-a**2*e**3 - 2*a*c**2*(-e*(a*e**2 - 3*c*
d**2)/(2*c**2) + d*sqrt(-a*c**5)*(3*a*e**2 - c*d**2)/(2*a*c**4)) + 3*a*c*d*
*2*e)/(3*a*c*d*e**2 - c**2*d**3)) + 3*d*e**2*x/c + e**3*x**2/(2*c)
```

Giac [A] time = 1.34387, size = 105, normalized size = 1.17

$$\frac{\left(cd^3 - 3\,ade^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c} + \frac{\left(3\,cd^2e - ae^3\right)\log\left(cx^2 + a\right)}{2\,c^2} + \frac{cx^2e^3 + 6\,cdxe^2}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+a),x, algorithm="giac")
```

```
[Out] (c*d^3 - 3*a*d*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c) + 1/2*(3*c*d^2*e -
a*e^3)*log(c*x^2 + a)/c^2 + 1/2*(c*x^2*e^3 + 6*c*d*x*e^2)/c^2
```

3.500 $\int \frac{(d+ex)^2}{a+cx^2}\,dx$

Optimal. Leaf size=59

$$\frac{(cd^2-ae^2)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}}+\frac{de\log(a+cx^2)}{c}+\frac{e^2x}{c}$$

```
[Out] (e^2*x)/c + ((c*d^2 - a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2))
 + (d*e*Log[a + c*x^2])/c
```

Rubi [A] time = 0.0509034, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {702, 635, 205, 260}

$$\frac{(cd^2-ae^2)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}}+\frac{de\log(a+cx^2)}{c}+\frac{e^2x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + c*x^2),x]
```

```
[Out] (e^2*x)/c + ((c*d^2 - a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2))
 + (d*e*Log[a + c*x^2])/c
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{(d+ex)^2}{a+cx^2}\,dx = \int \left(\frac{e^2}{c}+\frac{cd^2-ae^2+2cdex}{c\left(a+cx^2\right)}\right)dx$$
$$= \frac{e^2x}{c}+\frac{\int \frac{cd^2-ae^2+2cdex}{a+cx^2}\,dx}{c}$$
$$= \frac{e^2x}{c}+(2de)\int \frac{x}{a+cx^2}\,dx+\frac{\left(cd^2-ae^2\right)\int \frac{1}{a+cx^2}\,dx}{c}$$
$$= \frac{e^2x}{c}+\frac{\left(cd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}}+\frac{de\log\left(a+cx^2\right)}{c}$$

Mathematica [A] time = 0.0400094, size = 56, normalized size = 0.95

$$\frac{\left(cd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}}+\frac{e\left(d\log\left(a+cx^2\right)+ex\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + c*x^2),x]
```

```
[Out] ((c*d^2 - a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)) + (e*(e*x +
 d*Log[a + c*x^2]))/c
```

Maple [A] time = 0.046, size = 65, normalized size = 1.1

$$\frac{e^2x}{c}+\frac{de\ln\left(cx^2+a\right)}{c}-\frac{ae^2}{c}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}+d^2\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^2+a),x)
```

```
[Out] e^2*x/c+d*e*ln(c*x^2+a)/c-1/c/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*a*e^2+1/(
a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.91244, size = 309, normalized size = 5.24

$$\left[\frac{2\,ace^2x+2\,acde\log\left(cx^2+a\right)+\left(cd^2-ae^2\right)\sqrt{-ac}\log\left(\frac{cx^2+2\sqrt{-ac}x-a}{cx^2+a}\right)}{2\,ac^2},\frac{ace^2x+acde\log\left(cx^2+a\right)+\left(cd^2-ae^2\right)\sqrt{ac}\,\mathrm{arc}}{ac^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*c*e^2*x + 2*a*c*d*e*log(c*x^2 + a) + (c*d^2 - a*e^2)*sqrt(-a*c)*l
og((c*x^2 + 2*sqrt(-a*c)*x - a)/(c*x^2 + a)))/(a*c^2), (a*c*e^2*x + a*c*d*e
*log(c*x^2 + a) + (c*d^2 - a*e^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a))/(a*c^2)]
```

Sympy [B] time = 0.700622, size = 185, normalized size = 3.14

$$\left(\frac{de}{c} - \frac{\sqrt{-ac^3}\left(ae^2 - cd^2\right)}{2ac^3}\right)\log\left(x + \frac{-2ac\left(\frac{de}{c} - \frac{\sqrt{-ac^3}\left(ae^2-cd^2\right)}{2ac^3}\right) + 2ade}{ae^2 - cd^2}\right) + \left(\frac{de}{c} + \frac{\sqrt{-ac^3}\left(ae^2 - cd^2\right)}{2ac^3}\right)\log\left(x + \frac{-2ac\left(\frac{de}{c} + \cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**2+a),x)
```

```
[Out] (d*e/c - sqrt(-a*c**3)*(a*e**2 - c*d**2)/(2*a*c**3))*log(x + (-2*a*c*(d*e/c
 - sqrt(-a*c**3)*(a*e**2 - c*d**2)/(2*a*c**3)) + 2*a*d*e)/(a*e**2 - c*d**2)
) + (d*e/c + sqrt(-a*c**3)*(a*e**2 - c*d**2)/(2*a*c**3))*log(x + (-2*a*c*(d
*e/c + sqrt(-a*c**3)*(a*e**2 - c*d**2)/(2*a*c**3)) + 2*a*d*e)/(a*e**2 - c*d
**2)) + e**2*x/c
```

Giac [A] time = 1.42494, size = 70, normalized size = 1.19

$$\frac{de\log\left(cx^2 + a\right)}{c} + \frac{xe^2}{c} + \frac{\left(cd^2 - ae^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+a),x, algorithm="giac")
```

```
[Out] d*e*log(c*x^2 + a)/c + x*e^2/c + (c*d^2 - a*e^2)*arctan(c*x/sqrt(a*c))/(sqr
t(a*c)*c)
```

3.501 $\int \frac{d+ex}{a+cx^2} dx$

Optimal. Leaf size=42

$$\frac{d\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}+\frac{e\log\left(a+cx^2\right)}{2c}$$

```
[Out] (d*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) + (e*Log[a + c*x^2])/(2*c
)
```

Rubi [A] time = 0.0152003, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {635, 205, 260}

$$\frac{d\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}+\frac{e\log\left(a+cx^2\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + c*x^2),x]
```

```
[Out] (d*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) + (e*Log[a + c*x^2])/(2*c
)
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex}{a+cx^2}\,dx &= d\int \frac{1}{a+cx^2}\,dx+e\int \frac{x}{a+cx^2}\,dx\\ &= \frac{d\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}+\frac{e\log\left(a+cx^2\right)}{2c}\end{aligned}$$

Mathematica [A] time = 0.0131568, size = 42, normalized size = 1.

$$\frac{d\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}+\frac{e\log\left(a+cx^2\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + c*x^2),x]
```

```
[Out] (d*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) + (e*Log[a + c*x^2])/(2*c
)
```

Maple [A] time = 0.047, size = 32, normalized size = 0.8

$$\frac{e\ln\left(cx^2+a\right)}{2c}+d\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^2+a),x)
```

```
[Out] 1/2*e*ln(c*x^2+a)/c+d/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.91516, size = 225, normalized size = 5.36

$$\left[\frac{ae\log\left(cx^2+a\right)-\sqrt{-ac}d\log\left(\frac{cx^2-2\sqrt{-ac}x-a}{cx^2+a}\right)}{2ac},\frac{ae\log\left(cx^2+a\right)+2\sqrt{ac}d\arctan\left(\frac{\sqrt{ac}x}{a}\right)}{2ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(a*e*log(c*x^2 + a) - sqrt(-a*c)*d*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c
*x^2 + a)))/(a*c), 1/2*(a*e*log(c*x^2 + a) + 2*sqrt(a*c)*d*arctan(sqrt(a*c)
*x/a))/(a*c)]
```

Sympy [B] time = 0.278907, size = 124, normalized size = 2.95

$$\left(\frac{e}{2c}-\frac{d\sqrt{-ac^3}}{2ac^2}\right)\log\left(x+\frac{2ac\left(\frac{e}{2c}-\frac{d\sqrt{-ac^3}}{2ac^2}\right)-ae}{cd}\right)+\left(\frac{e}{2c}+\frac{d\sqrt{-ac^3}}{2ac^2}\right)\log\left(x+\frac{2ac\left(\frac{e}{2c}+\frac{d\sqrt{-ac^3}}{2ac^2}\right)-ae}{cd}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**2+a),x)
```

```
[Out] (e/(2*c) - d*sqrt(-a*c**3)/(2*a*c**2))*log(x + (2*a*c*(e/(2*c) - d*sqrt(-a*
c**3)/(2*a*c**2)) - a*e)/(c*d)) + (e/(2*c) + d*sqrt(-a*c**3)/(2*a*c**2))*lo
g(x + (2*a*c*(e/(2*c) + d*sqrt(-a*c**3)/(2*a*c**2)) - a*e)/(c*d))
```

Giac [A] time = 1.24502, size = 43, normalized size = 1.02

$$\frac{d\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}+\frac{e\log\left(cx^2+a\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] d*arctan(c*x/sqrt(a*c))/sqrt(a*c) + 1/2*e*log(c*x^2 + a)/c
```

3.502 $\int \frac{1}{(d+ex)(a+cx^2)} dx$

Optimal. Leaf size=86

$$-\frac{e\log\left(a+cx^2\right)}{2\left(ae^2+cd^2\right)}+\frac{e\log(d+ex)}{ae^2+cd^2}+\frac{\sqrt{c}d\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\left(ae^2+cd^2\right)}$$

```
[Out] (Sqrt[c]*d*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*(c*d^2 + a*e^2)) + (e*Log[
d + e*x])/(c*d^2 + a*e^2) - (e*Log[a + c*x^2])/(2*(c*d^2 + a*e^2))
```

Rubi [A] time = 0.0380176, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {706, 31, 635, 205, 260}

$$-\frac{e\log\left(a+cx^2\right)}{2\left(ae^2+cd^2\right)}+\frac{e\log(d+ex)}{ae^2+cd^2}+\frac{\sqrt{c}d\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + c*x^2)),x]
```

```
[Out] (Sqrt[c]*d*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*(c*d^2 + a*e^2)) + (e*Log[
d + e*x])/(c*d^2 + a*e^2) - (e*Log[a + c*x^2])/(2*(c*d^2 + a*e^2))
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+cx^2\right)}\,dx = \frac{\int \frac{cd-cex}{a+cx^2}\,dx}{cd^2+ae^2} + \frac{e^2\int \frac{1}{d+ex}\,dx}{cd^2+ae^2}$$
$$= \frac{e\log(d+ex)}{cd^2+ae^2} + \frac{(cd)\int \frac{1}{a+cx^2}\,dx}{cd^2+ae^2} - \frac{(ce)\int \frac{x}{a+cx^2}\,dx}{cd^2+ae^2}$$
$$= \frac{\sqrt{c}d\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\left(cd^2+ae^2\right)} + \frac{e\log(d+ex)}{cd^2+ae^2} - \frac{e\log\left(a+cx^2\right)}{2\left(cd^2+ae^2\right)}$$

Mathematica [A] time = 0.0354208, size = 63, normalized size = 0.73

$$\frac{\frac{2\sqrt{c}d\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}} - e\log\left(a+cx^2\right) + 2e\log(d+ex)}{2ae^2+2cd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + c*x^2)),x]
```

```
[Out] ((2*Sqrt[c]*d*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[a] + 2*e*Log[d + e*x] - e*L
og[a + c*x^2])/(2*c*d^2 + 2*a*e^2)
```

Maple [A] time = 0.049, size = 77, normalized size = 0.9

$$-\frac{e\ln\left(cx^2+a\right)}{2\,ae^2+2\,cd^2} + \frac{cd}{ae^2+cd^2}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}} + \frac{e\ln(ex+d)}{ae^2+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+a),x)
```

```
[Out] -1/2*e*ln(c*x^2+a)/(a*e^2+c*d^2)+c/(a*e^2+c*d^2)*d/(a*c)^(1/2)*arctan(x*c/(
a*c)^(1/2))+e*ln(e*x+d)/(a*e^2+c*d^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.14763, size = 298, normalized size = 3.47

$$\left[\frac{d\sqrt{-\frac{c}{a}}\log\left(\frac{cx^2+2\,ax\sqrt{-\frac{c}{a}}-a}{cx^2+a}\right) - e\log\left(cx^2+a\right) + 2\,e\log(ex+d)}{2\left(cd^2+ae^2\right)}, \frac{2\,d\sqrt{\frac{c}{a}}\arctan\left(x\sqrt{\frac{c}{a}}\right) - e\log\left(cx^2+a\right) + 2\,e\log(ex+d\right.}{2\left(cd^2+ae^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(d*sqrt(-c/a)*log((c*x^2 + 2*a*x*sqrt(-c/a) - a)/(c*x^2 + a)) - e*log(
c*x^2 + a) + 2*e*log(e*x + d))/(c*d^2 + a*e^2), 1/2*(2*d*sqrt(c/a)*arctan(x
*sqrt(c/a)) - e*log(c*x^2 + a) + 2*e*log(e*x + d))/(c*d^2 + a*e^2)]
```

Sympy [B] time = 3.80207, size = 1134, normalized size = 13.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+a),x)
```

```
[Out] e*log(x + (-12*a**4*e**8/(a*e**2 + c*d**2)**2 - 20*a**3*c*d**2*e**6/(a*e**2
 + c*d**2)**2 + 6*a**3*e**6/(a*e**2 + c*d**2) - 4*a**2*c**2*d**4*e**4/(a*e*
*2 + c*d**2)**2 + 12*a**2*c*d**2*e**4/(a*e**2 + c*d**2) + 6*a**2*e**4 + 4*a
*c**3*d**6*e**2/(a*e**2 + c*d**2)**2 + 6*a*c**2*d**4*e**2/(a*e**2 + c*d**2)
 - a*c*d**2*e**2 + c**2*d**4)/(9*a*c*d*e**3 + c**2*d**3*e))/(a*e**2 + c*d**
2) + (-e/(2*(a*e**2 + c*d**2)) - d*sqrt(-a*c)/(2*a*(a*e**2 + c*d**2)))*log(
x + (-12*a**4*e**6*(-e/(2*(a*e**2 + c*d**2)) - d*sqrt(-a*c)/(2*a*(a*e**2 +
c*d**2)))**2 - 20*a**3*c*d**2*e**4*(-e/(2*(a*e**2 + c*d**2)) - d*sqrt(-a*c)
/(2*a*(a*e**2 + c*d**2)))**2 + 6*a**3*e**5*(-e/(2*(a*e**2 + c*d**2)) - d*sq
rt(-a*c)/(2*a*(a*e**2 + c*d**2))) - 4*a**2*c**2*d**4*e**2*(-e/(2*(a*e**2 +
c*d**2)) - d*sqrt(-a*c)/(2*a*(a*e**2 + c*d**2)))**2 + 12*a**2*c*d**2*e**3*(
-e/(2*(a*e**2 + c*d**2)) - d*sqrt(-a*c)/(2*a*(a*e**2 + c*d**2))) + 6*a**2*e
**4 + 4*a*c**3*d**6*(-e/(2*(a*e**2 + c*d**2)) - d*sqrt(-a*c)/(2*a*(a*e**2 +
 c*d**2)))**2 + 6*a*c**2*d**4*e*(-e/(2*(a*e**2 + c*d**2)) - d*sqrt(-a*c)/(2
*a*(a*e**2 + c*d**2))) - a*c*d**2*e**2 + c**2*d**4)/(9*a*c*d*e**3 + c**2*d*
*3*e)) + (-e/(2*(a*e**2 + c*d**2)) + d*sqrt(-a*c)/(2*a*(a*e**2 + c*d**2)))*
log(x + (-12*a**4*e**6*(-e/(2*(a*e**2 + c*d**2)) + d*sqrt(-a*c)/(2*a*(a*e**
2 + c*d**2)))**2 - 20*a**3*c*d**2*e**4*(-e/(2*(a*e**2 + c*d**2)) + d*sqrt(-
a*c)/(2*a*(a*e**2 + c*d**2)))**2 + 6*a**3*e**5*(-e/(2*(a*e**2 + c*d**2)) +
d*sqrt(-a*c)/(2*a*(a*e**2 + c*d**2))) - 4*a**2*c**2*d**4*e**2*(-e/(2*(a*e**
2 + c*d**2)) + d*sqrt(-a*c)/(2*a*(a*e**2 + c*d**2)))**2 + 12*a**2*c*d**2*e*
*3*(-e/(2*(a*e**2 + c*d**2)) + d*sqrt(-a*c)/(2*a*(a*e**2 + c*d**2))) + 6*a*
*2*e**4 + 4*a*c**3*d**6*(-e/(2*(a*e**2 + c*d**2)) + d*sqrt(-a*c)/(2*a*(a*e*
*2 + c*d**2)))**2 + 6*a*c**2*d**4*e*(-e/(2*(a*e**2 + c*d**2)) + d*sqrt(-a*c
)/(2*a*(a*e**2 + c*d**2))) - a*c*d**2*e**2 + c**2*d**4)/(9*a*c*d*e**3 + c**
2*d**3*e))
```

Giac [A] time = 1.1845, size = 107, normalized size = 1.24

$$\frac{cd\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(cd^2+ae^2\right)\sqrt{ac}}-\frac{e\log\left(cx^2+a\right)}{2\left(cd^2+ae^2\right)}+\frac{e^2\log\left(|xe+d|\right)}{cd^2e+ae^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] c*d*arctan(c*x/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c)) - 1/2*e*log(c*x^2 + a
)/(c*d^2 + a*e^2) + e^2*log(abs(x*e + d))/(c*d^2*e + a*e^3)
```

3.503 $\int \frac{1}{(d+ex)^2(a+cx^2)}\,dx$

Optimal. Leaf size=123

$$-\frac{cde\log\left(a+cx^2\right)}{\left(ae^2+cd^2\right)^2}-\frac{e}{(d+ex)\left(ae^2+cd^2\right)}+\frac{2cde\log(d+ex)}{\left(ae^2+cd^2\right)^2}+\frac{\sqrt{c}\left(cd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\left(ae^2+cd^2\right)^2}$$

```
[Out] -(e/((c*d^2 + a*e^2)*(d + e*x))) + (Sqrt[c]*(c*d^2 - a*e^2)*ArcTan[(Sqrt[c]
*x)/Sqrt[a]])/(Sqrt[a]*(c*d^2 + a*e^2)^2) + (2*c*d*e*Log[d + e*x])/(c*d^2 +
 a*e^2)^2 - (c*d*e*Log[a + c*x^2])/(c*d^2 + a*e^2)^2
```

Rubi [A] time = 0.102623, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {710, 801, 635, 205, 260}

$$-\frac{cde\log\left(a+cx^2\right)}{\left(ae^2+cd^2\right)^2}-\frac{e}{(d+ex)\left(ae^2+cd^2\right)}+\frac{2cde\log(d+ex)}{\left(ae^2+cd^2\right)^2}+\frac{\sqrt{c}\left(cd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + c*x^2)),x]
```

```
[Out] -(e/((c*d^2 + a*e^2)*(d + e*x))) + (Sqrt[c]*(c*d^2 - a*e^2)*ArcTan[(Sqrt[c]
*x)/Sqrt[a]])/(Sqrt[a]*(c*d^2 + a*e^2)^2) + (2*c*d*e*Log[d + e*x])/(c*d^2 +
 a*e^2)^2 - (c*d*e*Log[a + c*x^2])/(c*d^2 + a*e^2)^2
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+cx^2\right)}\,dx = -\frac{e}{\left(cd^2+ae^2\right)(d+ex)} + \frac{c\int \frac{d-ex}{(d+ex)\left(a+cx^2\right)}\,dx}{cd^2+ae^2}$$

$$= -\frac{e}{\left(cd^2+ae^2\right)(d+ex)} + \frac{c\int \left(\frac{2de^2}{\left(cd^2+ae^2\right)(d+ex)} + \frac{cd^2-ae^2-2cdex}{\left(cd^2+ae^2\right)\left(a+cx^2\right)}\right)dx}{cd^2+ae^2}$$

$$= -\frac{e}{\left(cd^2+ae^2\right)(d+ex)} + \frac{2cde\log(d+ex)}{\left(cd^2+ae^2\right)^2} + \frac{c\int \frac{cd^2-ae^2-2cdex}{a+cx^2}\,dx}{\left(cd^2+ae^2\right)^2}$$

$$= -\frac{e}{\left(cd^2+ae^2\right)(d+ex)} + \frac{2cde\log(d+ex)}{\left(cd^2+ae^2\right)^2} - \frac{\left(2c^2de\right)\int \frac{x}{a+cx^2}\,dx}{\left(cd^2+ae^2\right)^2} + \frac{\left(c\left(cd^2-ae^2\right)\right)\int \frac{1}{a+cx^2}\,dx}{\left(cd^2+ae^2\right)^2}$$

$$= -\frac{e}{\left(cd^2+ae^2\right)(d+ex)} + \frac{\sqrt{c}\left(cd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\left(cd^2+ae^2\right)^2} + \frac{2cde\log(d+ex)}{\left(cd^2+ae^2\right)^2} - \frac{cde\log\left(a+cx^2\right)}{\left(cd^2+ae^2\right)^2}$$

Mathematica [A] time = 0.088362, size = 113, normalized size = 0.92

$$\frac{\sqrt{c}(d+ex)\left(cd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)-\sqrt{a}e\left(cd(d+ex)\log\left(a+cx^2\right)+ae^2+cd^2-2cd(d+ex)\log(d+ex)\right)}{\sqrt{a}(d+ex)\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + c*x^2)),x]
```

```
[Out] (Sqrt[c]*(c*d^2 - a*e^2)*(d + e*x)*ArcTan[(Sqrt[c]*x)/Sqrt[a]] - Sqrt[a]*e*
(c*d^2 + a*e^2 - 2*c*d*(d + e*x)*Log[d + e*x] + c*d*(d + e*x)*Log[a + c*x^2
]))/(Sqrt[a]*(c*d^2 + a*e^2)^2*(d + e*x))
```

Maple [A] time = 0.056, size = 143, normalized size = 1.2

$$-\frac{cde\ln\left(cx^2+a\right)}{\left(ae^2+cd^2\right)^2}-\frac{ace^2}{\left(ae^2+cd^2\right)^2}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}+\frac{c^2d^2}{\left(ae^2+cd^2\right)^2}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}-\frac{e}{\left(ae^2+cd^2\right)(ex+d)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^2+a),x)
```

```
[Out] -c*d*e*ln(c*x^2+a)/(a*e^2+c*d^2)^2-c/(a*e^2+c*d^2)^2/(a*c)^(1/2)*arctan(x*c
/(a*c)^(1/2))*a*e^2+c^2/(a*e^2+c*d^2)^2/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))
*d^2-e/(a*e^2+c*d^2)/(e*x+d)+2*c*d*e*ln(e*x+d)/(a*e^2+c*d^2)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.03712, size = 733, normalized size = 5.96

$$\left[-\frac{2\,cd^2e + 2\,ae^3 + \left(cd^3 - ade^2 + \left(cd^2e - ae^3\right)x\right)\sqrt{-\frac{c}{a}}\log\left(\frac{cx^2-2\,ax\sqrt{-\frac{c}{a}}-a}{cx^2+a}\right) + 2\left(cde^2x + cd^2e\right)\log\left(cx^2 + a\right) - 4\left(cde^2x + cd^2\right.}{2\left(c^2d^5 + 2\,acd^3e^2 + a^2de^4 + \left(c^2d^4e + 2\,acd^2e^3 + a^2e^5\right)x\right)} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*c*d^2*e + 2*a*e^3 + (c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)*sqrt(-
c/a)*log((c*x^2 - 2*a*x*sqrt(-c/a) - a)/(c*x^2 + a)) + 2*(c*d*e^2*x + c*d^2
*e)*log(c*x^2 + a) - 4*(c*d*e^2*x + c*d^2*e)*log(e*x + d))/(c^2*d^5 + 2*a*c
*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)*x), -(c*d^2*e
+ a*e^3 - (c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)*sqrt(c/a)*arctan(x*sqrt(c
/a)) + (c*d*e^2*x + c*d^2*e)*log(c*x^2 + a) - 2*(c*d*e^2*x + c*d^2*e)*log(e
*x + d))/(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3
+ a^2*e^5)*x)]
```

Sympy [B] time = 22.0573, size = 2508, normalized size = 20.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**2+a),x)
```

```
[Out] 2*c*d*e*log(x + (112*a**6*c**2*d**3*e**12/(a*e**2 + c*d**2)**4 + 432*a**5*c
**3*d**5*e**10/(a*e**2 + c*d**2)**4 - 4*a**5*c*d*e**10/(a*e**2 + c*d**2)**2
 + 608*a**4*c**4*d**7*e**8/(a*e**2 + c*d**2)**4 - 32*a**4*c**2*d**3*e**8/(a
*e**2 + c*d**2)**2 + 352*a**3*c**5*d**9*e**6/(a*e**2 + c*d**2)**4 - 72*a**3
*c**3*d**5*e**6/(a*e**2 + c*d**2)**2 + 5*a**3*c*d*e**6 + 48*a**2*c**6*d**11
*e**4/(a*e**2 + c*d**2)**4 - 64*a**2*c**4*d**7*e**4/(a*e**2 + c*d**2)**2 -
55*a**2*c**2*d**3*e**4 - 16*a*c**7*d**13*e**2/(a*e**2 + c*d**2)**4 - 20*a*c
**5*d**9*e**2/(a*e**2 + c*d**2)**2 + 3*a*c**3*d**5*e**2 - c**4*d**7)/(a**3*
c*e**7 + 33*a**2*c**2*d**2*e**5 - 33*a*c**3*d**4*e**3 - c**4*d**6*e))/(a*e*
*2 + c*d**2)**2 - e/(a*d*e**2 + c*d**3 + x*(a*e**3 + c*d**2*e)) + (-c*d*e/(
a*e**2 + c*d**2)**2 - sqrt(-a*c)*(a*e**2 - c*d**2)/(2*a*(a**2*e**4 + 2*a*c*
d**2*e**2 + c**2*d**4)))*log(x + (28*a**6*d*e**10*(-c*d*e/(a*e**2 + c*d**2)
**2 - sqrt(-a*c)*(a*e**2 - c*d**2)/(2*a*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2
*d**4)))**2 + 108*a**5*c*d**3*e**8*(-c*d*e/(a*e**2 + c*d**2)**2 - sqrt(-a*c
)*(a*e**2 - c*d**2)/(2*a*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4)))**2 - 2
```

```
*a**5*e**9*(-c*d*e/(a*e**2 + c*d**2)**2 - sqrt(-a*c)*(a*e**2 - c*d**2)/(2*a
*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4))) + 152*a**4*c**2*d**5*e**6*(-c*
d*e/(a*e**2 + c*d**2)**2 - sqrt(-a*c)*(a*e**2 - c*d**2)/(2*a*(a**2*e**4 + 2
*a*c*d**2*e**2 + c**2*d**4)))**2 - 16*a**4*c*d**2*e**7*(-c*d*e/(a*e**2 + c*
d**2)**2 - sqrt(-a*c)*(a*e**2 - c*d**2)/(2*a*(a**2*e**4 + 2*a*c*d**2*e**2 +
 c**2*d**4))) + 88*a**3*c**3*d**7*e**4*(-c*d*e/(a*e**2 + c*d**2)**2 - sqrt(
-a*c)*(a*e**2 - c*d**2)/(2*a*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4)))**2
 - 36*a**3*c**2*d**4*e**5*(-c*d*e/(a*e**2 + c*d**2)**2 - sqrt(-a*c)*(a*e**2
 - c*d**2)/(2*a*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4))) + 5*a**3*c*d*e*
*6 + 12*a**2*c**4*d**9*e**2*(-c*d*e/(a*e**2 + c*d**2)**2 - sqrt(-a*c)*(a*e*
*2 - c*d**2)/(2*a*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4)))**2 - 32*a**2*
c**3*d**6*e**3*(-c*d*e/(a*e**2 + c*d**2)**2 - sqrt(-a*c)*(a*e**2 - c*d**2)/
(2*a*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4))) - 55*a**2*c**2*d**3*e**4 -
 4*a*c**5*d**11*(-c*d*e/(a*e**2 + c*d**2)**2 - sqrt(-a*c)*(a*e**2 - c*d**2)
/(2*a*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4)))**2 - 10*a*c**4*d**8*e*(-c
*d*e/(a*e**2 + c*d**2)**2 - sqrt(-a*c)*(a*e**2 - c*d**2)/(2*a*(a**2*e**4 +
2*a*c*d**2*e**2 + c**2*d**4))) + 3*a*c**3*d**5*e**2 - c**4*d**7)/(a**3*c*e*
*7 + 33*a**2*c**2*d**2*e**5 - 33*a*c**3*d**4*e**3 - c**4*d**6*e)) + (-c*d*e
/(a*e**2 + c*d**2)**2 + sqrt(-a*c)*(a*e**2 - c*d**2)/(2*a*(a**2*e**4 + 2*a*
c*d**2*e**2 + c**2*d**4)))*log(x + (28*a**6*d*e**10*(-c*d*e/(a*e**2 + c*d**
2)**2 + sqrt(-a*c)*(a*e**2 - c*d**2)/(2*a*(a**2*e**4 + 2*a*c*d**2*e**2 + c*
*2*d**4)))**2 + 108*a**5*c*d**3*e**8*(-c*d*e/(a*e**2 + c*d**2)**2 + sqrt(-a
*c)*(a*e**2 - c*d**2)/(2*a*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4)))**2 -
 2*a**5*e**9*(-c*d*e/(a*e**2 + c*d**2)**2 + sqrt(-a*c)*(a*e**2 - c*d**2)/(2
*a*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4))) + 152*a**4*c**2*d**5*e**6*(-
c*d*e/(a*e**2 + c*d**2)**2 + sqrt(-a*c)*(a*e**2 - c*d**2)/(2*a*(a**2*e**4 +
 2*a*c*d**2*e**2 + c**2*d**4)))**2 - 16*a**4*c*d**2*e**7*(-c*d*e/(a*e**2 +
c*d**2)**2 + sqrt(-a*c)*(a*e**2 - c*d**2)/(2*a*(a**2*e**4 + 2*a*c*d**2*e**2
 + c**2*d**4))) + 88*a**3*c**3*d**7*e**4*(-c*d*e/(a*e**2 + c*d**2)**2 + sqr
t(-a*c)*(a*e**2 - c*d**2)/(2*a*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4)))*
*2 - 36*a**3*c**2*d**4*e**5*(-c*d*e/(a*e**2 + c*d**2)**2 + sqrt(-a*c)*(a*e*
*2 - c*d**2)/(2*a*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4))) + 5*a**3*c*d*
e**6 + 12*a**2*c**4*d**9*e**2*(-c*d*e/(a*e**2 + c*d**2)**2 + sqrt(-a*c)*(a*
e**2 - c*d**2)/(2*a*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4)))**2 - 32*a**
2*c**3*d**6*e**3*(-c*d*e/(a*e**2 + c*d**2)**2 + sqrt(-a*c)*(a*e**2 - c*d**2
)/(2*a*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4))) - 55*a**2*c**2*d**3*e**4
 - 4*a*c**5*d**11*(-c*d*e/(a*e**2 + c*d**2)**2 + sqrt(-a*c)*(a*e**2 - c*d**
2)/(2*a*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4)))**2 - 10*a*c**4*d**8*e*(
-c*d*e/(a*e**2 + c*d**2)**2 + sqrt(-a*c)*(a*e**2 - c*d**2)/(2*a*(a**2*e**4
+ 2*a*c*d**2*e**2 + c**2*d**4))) + 3*a*c**3*d**5*e**2 - c**4*d**7)/(a**3*c*
e**7 + 33*a**2*c**2*d**2*e**5 - 33*a*c**3*d**4*e**3 - c**4*d**6*e))
```

Giac [A] time = 1.29866, size = 252, normalized size = 2.05

$$-\frac{cde\log\left(c-\frac{2\,cd}{xe+d}+\frac{cd^2}{(xe+d)^2}+\frac{ae^2}{(xe+d)^2}\right)}{c^2d^4+2\,acd^2e^2+a^2e^4}+\frac{\left(c^2d^2e^2-ace^4\right)\arctan\left(\frac{\left(cd-\frac{cd^2}{xe+d}-\frac{ae^2}{xe+d}\right)e^{(-1)}}{\sqrt{ac}}\right)e^{(-2)}}{\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)\sqrt{ac}}-\frac{e^3}{\left(cd^2e^2+ae^4\right)(xe+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a),x, algorithm="giac")
```

```
[Out] -c*d*e*log(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(x*e + d)^2)/(c^
2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + (c^2*d^2*e^2 - a*c*e^4)*arctan((c*d - c*
d^2/(x*e + d) - a*e^2/(x*e + d))*e^(-1)/sqrt(a*c))*e^(-2)/((c^2*d^4 + 2*a*c
```

```
*d^2*e^2 + a^2*e^4)*sqrt(a*c)) - e^3/((c*d^2*e^2 + a*e^4)*(x*e + d))
```

3.504 $\int \frac{1}{(d+ex)^3(a+cx^2)}\,dx$

Optimal. Leaf size=176

$$\frac{c^{3/2}d\left(cd^2-3ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\left(ae^2+cd^2\right)^3}-\frac{ce\left(3cd^2-ae^2\right)\log\left(a+cx^2\right)}{2\left(ae^2+cd^2\right)^3}-\frac{2cde}{(d+ex)\left(ae^2+cd^2\right)^2}-\frac{e}{2(d+ex)^2\left(ae^2+cd^2\right)}+\frac{ce\,(3}{}$$

```
[Out] -e/(2*(c*d^2 + a*e^2)*(d + e*x)^2) - (2*c*d*e)/((c*d^2 + a*e^2)^2*(d + e*x)
) + (c^(3/2)*d*(c*d^2 - 3*a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*(c*d
^2 + a*e^2)^3) + (c*e*(3*c*d^2 - a*e^2)*Log[d + e*x])/(c*d^2 + a*e^2)^3 - (
c*e*(3*c*d^2 - a*e^2)*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^3)
```

Rubi [A] time = 0.166154, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {710, 801, 635, 205, 260}

$$\frac{c^{3/2}d\left(cd^2-3ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\left(ae^2+cd^2\right)^3}-\frac{ce\left(3cd^2-ae^2\right)\log\left(a+cx^2\right)}{2\left(ae^2+cd^2\right)^3}-\frac{2cde}{(d+ex)\left(ae^2+cd^2\right)^2}-\frac{e}{2(d+ex)^2\left(ae^2+cd^2\right)}+\frac{ce\,(3}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + c*x^2)),x]
```

```
[Out] -e/(2*(c*d^2 + a*e^2)*(d + e*x)^2) - (2*c*d*e)/((c*d^2 + a*e^2)^2*(d + e*x)
) + (c^(3/2)*d*(c*d^2 - 3*a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*(c*d
^2 + a*e^2)^3) + (c*e*(3*c*d^2 - a*e^2)*Log[d + e*x])/(c*d^2 + a*e^2)^3 - (
c*e*(3*c*d^2 - a*e^2)*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^3)
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3(a+cx^2)}\,dx = -\frac{e}{2(cd^2+ae^2)(d+ex)^2} + \frac{c\int \frac{d-ex}{(d+ex)^2(a+cx^2)}\,dx}{cd^2+ae^2}$$

$$= -\frac{e}{2(cd^2+ae^2)(d+ex)^2} + \frac{c\int \left(\frac{2de^2}{(cd^2+ae^2)(d+ex)^2} + \frac{3cd^2e^2-ae^4}{(cd^2+ae^2)^2(d+ex)} + \frac{c\left(d(cd^2-3ae^2)-e(3cd^2-ae^2)x\right)}{(cd^2+ae^2)^2(a+cx^2)}\right) dx}{cd^2+ae^2}$$

$$= -\frac{e}{2(cd^2+ae^2)(d+ex)^2} - \frac{2cde}{(cd^2+ae^2)^2(d+ex)} + \frac{ce(3cd^2-ae^2)\log(d+ex)}{(cd^2+ae^2)^3} + \frac{c^2\int \frac{d(cd^2-3ae}{\ldots}}{(cd^2}$$

$$= -\frac{e}{2(cd^2+ae^2)(d+ex)^2} - \frac{2cde}{(cd^2+ae^2)^2(d+ex)} + \frac{ce(3cd^2-ae^2)\log(d+ex)}{(cd^2+ae^2)^3} + \frac{(c^2d(cd^2-3}{(cd^2}$$

$$= -\frac{e}{2(cd^2+ae^2)(d+ex)^2} - \frac{2cde}{(cd^2+ae^2)^2(d+ex)} + \frac{c^{3/2}d(cd^2-3ae^2)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}(cd^2+ae^2)^3} + \frac{ce(3cd^2}{(c}$$

Mathematica [A] time = 0.289447, size = 140, normalized size = 0.8

$$\frac{\frac{2c^{3/2}d(cd^2-3ae^2)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}} + e\left(c(ae^2-3cd^2)\log(a+cx^2) - \frac{(ae^2+cd^2)(ae^2+cd(5d+4ex))}{(d+ex)^2} + 2c(3cd^2-ae^2)\log(d+ex)\right)}{2(ae^2+cd^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a + c*x^2)),x]
```

```
[Out] ((2*c^(3/2)*d*(c*d^2 - 3*a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[a] + e*(-
(((c*d^2 + a*e^2)*(a*e^2 + c*d*(5*d + 4*e*x)))/(d + e*x)^2) + 2*c*(3*c*d^2
- a*e^2)*Log[d + e*x] + c*(-3*c*d^2 + a*e^2)*Log[a + c*x^2]))/(2*(c*d^2 + a
*e^2)^3)
```

Maple [A] time = 0.062, size = 233, normalized size = 1.3

$$\frac{c\ln(cx^2+a)ae^3}{2(ae^2+cd^2)^3} - \frac{3c^2\ln(cx^2+a)d^2e}{2(ae^2+cd^2)^3} - 3\frac{ac^2de^2}{(ae^2+cd^2)^3\sqrt{ac}}\arctan\left(\frac{cx}{\sqrt{ac}}\right) + \frac{c^3d^3}{(ae^2+cd^2)^3}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}} - \frac{}{(2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*x^2+a),x)
```

```
[Out] 1/2*c/(a*e^2+c*d^2)^3*ln(c*x^2+a)*a*e^3-3/2*c^2/(a*e^2+c*d^2)^3*ln(c*x^2+a)
*d^2*e-3*c^2/(a*e^2+c*d^2)^3/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*a*d*e^2+c^
3/(a*e^2+c*d^2)^3/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*d^3-1/2*e/(a*e^2+c*d^
2)/(e*x+d)^2-2*c*d*e/(a*e^2+c*d^2)^2/(e*x+d)-e^3*c/(a*e^2+c*d^2)^3*ln(e*x+d
```

```
)*a+3*e*c^2/(a*e^2+c*d^2)^3*ln(e*x+d)*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 8.88927, size = 1716, normalized size = 9.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/2*(5*c^2*d^4*e + 6*a*c*d^2*e^3 + a^2*e^5 + (c^2*d^5 - 3*a*c*d^3*e^2 + (
c^2*d^3*e^2 - 3*a*c*d*e^4)*x^2 + 2*(c^2*d^4*e - 3*a*c*d^2*e^3)*x)*sqrt(-c/a
)*log((c*x^2 - 2*a*x*sqrt(-c/a) - a)/(c*x^2 + a)) + 4*(c^2*d^3*e^2 + a*c*d*
e^4)*x + (3*c^2*d^4*e - a*c*d^2*e^3 + (3*c^2*d^2*e^3 - a*c*e^5)*x^2 + 2*(3*
c^2*d^3*e^2 - a*c*d*e^4)*x)*log(c*x^2 + a) - 2*(3*c^2*d^4*e - a*c*d^2*e^3 +
 (3*c^2*d^2*e^3 - a*c*e^5)*x^2 + 2*(3*c^2*d^3*e^2 - a*c*d*e^4)*x)*log(e*x +
 d))/(c^3*d^8 + 3*a*c^2*d^6*e^2 + 3*a^2*c*d^4*e^4 + a^3*d^2*e^6 + (c^3*d^6*
e^2 + 3*a*c^2*d^4*e^4 + 3*a^2*c*d^2*e^6 + a^3*e^8)*x^2 + 2*(c^3*d^7*e + 3*a
*c^2*d^5*e^3 + 3*a^2*c*d^3*e^5 + a^3*d*e^7)*x), -1/2*(5*c^2*d^4*e + 6*a*c*d
^2*e^3 + a^2*e^5 - 2*(c^2*d^5 - 3*a*c*d^3*e^2 + (c^2*d^3*e^2 - 3*a*c*d*e^4)
*x^2 + 2*(c^2*d^4*e - 3*a*c*d^2*e^3)*x)*sqrt(c/a)*arctan(x*sqrt(c/a)) + 4*(
c^2*d^3*e^2 + a*c*d*e^4)*x + (3*c^2*d^4*e - a*c*d^2*e^3 + (3*c^2*d^2*e^3 -
a*c*e^5)*x^2 + 2*(3*c^2*d^3*e^2 - a*c*d*e^4)*x)*log(c*x^2 + a) - 2*(3*c^2*d
^4*e - a*c*d^2*e^3 + (3*c^2*d^2*e^3 - a*c*e^5)*x^2 + 2*(3*c^2*d^3*e^2 - a*c
*d*e^4)*x)*log(e*x + d))/(c^3*d^8 + 3*a*c^2*d^6*e^2 + 3*a^2*c*d^4*e^4 + a^3
*d^2*e^6 + (c^3*d^6*e^2 + 3*a*c^2*d^4*e^4 + 3*a^2*c*d^2*e^6 + a^3*e^8)*x^2
+ 2*(c^3*d^7*e + 3*a*c^2*d^5*e^3 + 3*a^2*c*d^3*e^5 + a^3*d*e^7)*x)]
```

Sympy [B] time = 116.536, size = 4996, normalized size = 28.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**2+a),x)
```

```
[Out] -c*e*(a*e**2 - 3*c*d**2)*log(x + (-12*a**9*c**2*e**18*(a*e**2 - 3*c*d**2)**
2/(a*e**2 + c*d**2)**6 - 24*a**8*c**3*d**2*e**16*(a*e**2 - 3*c*d**2)**2/(a*
e**2 + c*d**2)**6 + 104*a**7*c**4*d**4*e**14*(a*e**2 - 3*c*d**2)**2/(a*e**2
 + c*d**2)**6 + 6*a**7*c**2*e**14*(a*e**2 - 3*c*d**2)/(a*e**2 + c*d**2)**3
+ 456*a**6*c**5*d**6*e**12*(a*e**2 - 3*c*d**2)**2/(a*e**2 + c*d**2)**6 + 12
*a**6*c**3*d**2*e**12*(a*e**2 - 3*c*d**2)/(a*e**2 + c*d**2)**3 + 720*a**5*c
```

```
**6*d**8*e**10*(a*e**2 - 3*c*d**2)**2/(a*e**2 + c*d**2)**6 + 2*a**5*c**4*d*
*4*e**10*(a*e**2 - 3*c*d**2)/(a*e**2 + c*d**2)**3 + 6*a**5*c**2*e**10 + 568
*a**4*c**7*d**10*e**8*(a*e**2 - 3*c*d**2)**2/(a*e**2 + c*d**2)**6 + 8*a**4*
c**5*d**6*e**8*(a*e**2 - 3*c*d**2)/(a*e**2 + c*d**2)**3 - 69*a**4*c**3*d**2
*e**8 + 216*a**3*c**8*d**12*e**6*(a*e**2 - 3*c*d**2)**2/(a*e**2 + c*d**2)**
6 + 42*a**3*c**6*d**8*e**6*(a*e**2 - 3*c*d**2)/(a*e**2 + c*d**2)**3 + 236*a
**3*c**4*d**4*e**6 + 24*a**2*c**9*d**14*e**4*(a*e**2 - 3*c*d**2)**2/(a*e**2
 + c*d**2)**6 + 44*a**2*c**7*d**10*e**4*(a*e**2 - 3*c*d**2)/(a*e**2 + c*d**
2)**3 - 194*a**2*c**5*d**6*e**4 - 4*a*c**10*d**16*e**2*(a*e**2 - 3*c*d**2)*
*2/(a*e**2 + c*d**2)**6 + 14*a*c**8*d**12*e**2*(a*e**2 - 3*c*d**2)/(a*e**2
+ c*d**2)**3 + 6*a*c**6*d**8*e**2 - c**7*d**10)/(27*a**4*c**3*d*e**9 - 144*
a**3*c**4*d**3*e**7 + 270*a**2*c**5*d**5*e**5 - 72*a*c**6*d**7*e**3 - c**7*
d**9*e))/(a*e**2 + c*d**2)**3 + (c*e*(a*e**2 - 3*c*d**2)/(2*(a*e**2 + c*d**
2)**3) - d*sqrt(-a*c**3)*(3*a*e**2 - c*d**2)/(2*a*(a**3*e**6 + 3*a**2*c*d**
2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6)))*log(x + (-12*a**9*e**16*(c*e*(a*
e**2 - 3*c*d**2)/(2*(a*e**2 + c*d**2)**3) - d*sqrt(-a*c**3)*(3*a*e**2 - c*d
**2)/(2*a*(a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6)
))**2 - 24*a**8*c*d**2*e**14*(c*e*(a*e**2 - 3*c*d**2)/(2*(a*e**2 + c*d**2)*
*3) - d*sqrt(-a*c**3)*(3*a*e**2 - c*d**2)/(2*a*(a**3*e**6 + 3*a**2*c*d**2*e
**4 + 3*a*c**2*d**4*e**2 + c**3*d**6)))**2 + 104*a**7*c**2*d**4*e**12*(c*e*
(a*e**2 - 3*c*d**2)/(2*(a*e**2 + c*d**2)**3) - d*sqrt(-a*c**3)*(3*a*e**2 -
c*d**2)/(2*a*(a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d*
*6)))**2 - 6*a**7*c*e**13*(c*e*(a*e**2 - 3*c*d**2)/(2*(a*e**2 + c*d**2)**3)
 - d*sqrt(-a*c**3)*(3*a*e**2 - c*d**2)/(2*a*(a**3*e**6 + 3*a**2*c*d**2*e**4
 + 3*a*c**2*d**4*e**2 + c**3*d**6))) + 456*a**6*c**3*d**6*e**10*(c*e*(a*e**
2 - 3*c*d**2)/(2*(a*e**2 + c*d**2)**3) - d*sqrt(-a*c**3)*(3*a*e**2 - c*d**2
)/(2*a*(a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6)))*
*2 - 12*a**6*c**2*d**2*e**11*(c*e*(a*e**2 - 3*c*d**2)/(2*(a*e**2 + c*d**2)*
*3) - d*sqrt(-a*c**3)*(3*a*e**2 - c*d**2)/(2*a*(a**3*e**6 + 3*a**2*c*d**2*e
**4 + 3*a*c**2*d**4*e**2 + c**3*d**6))) + 720*a**5*c**4*d**8*e**8*(c*e*(a*e
**2 - 3*c*d**2)/(2*(a*e**2 + c*d**2)**3) - d*sqrt(-a*c**3)*(3*a*e**2 - c*d*
*2)/(2*a*(a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6))
)**2 - 2*a**5*c**3*d**4*e**9*(c*e*(a*e**2 - 3*c*d**2)/(2*(a*e**2 + c*d**2)*
*3) - d*sqrt(-a*c**3)*(3*a*e**2 - c*d**2)/(2*a*(a**3*e**6 + 3*a**2*c*d**2*e
**4 + 3*a*c**2*d**4*e**2 + c**3*d**6))) + 6*a**5*c**2*e**10 + 568*a**4*c**5
*d**10*e**6*(c*e*(a*e**2 - 3*c*d**2)/(2*(a*e**2 + c*d**2)**3) - d*sqrt(-a*c
**3)*(3*a*e**2 - c*d**2)/(2*a*(a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d*
*4*e**2 + c**3*d**6)))**2 - 8*a**4*c**4*d**6*e**7*(c*e*(a*e**2 - 3*c*d**2)/
(2*(a*e**2 + c*d**2)**3) - d*sqrt(-a*c**3)*(3*a*e**2 - c*d**2)/(2*a*(a**3*e
**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6))) - 69*a**4*c**3
*d**2*e**8 + 216*a**3*c**6*d**12*e**4*(c*e*(a*e**2 - 3*c*d**2)/(2*(a*e**2 +
 c*d**2)**3) - d*sqrt(-a*c**3)*(3*a*e**2 - c*d**2)/(2*a*(a**3*e**6 + 3*a**2
*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6)))**2 - 42*a**3*c**5*d**8*e**
5*(c*e*(a*e**2 - 3*c*d**2)/(2*(a*e**2 + c*d**2)**3) - d*sqrt(-a*c**3)*(3*a*
e**2 - c*d**2)/(2*a*(a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 +
c**3*d**6))) + 236*a**3*c**4*d**4*e**6 + 24*a**2*c**7*d**14*e**2*(c*e*(a*e*
*2 - 3*c*d**2)/(2*(a*e**2 + c*d**2)**3) - d*sqrt(-a*c**3)*(3*a*e**2 - c*d**
2)/(2*a*(a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6)))
**2 - 44*a**2*c**6*d**10*e**3*(c*e*(a*e**2 - 3*c*d**2)/(2*(a*e**2 + c*d**2)
**3) - d*sqrt(-a*c**3)*(3*a*e**2 - c*d**2)/(2*a*(a**3*e**6 + 3*a**2*c*d**2*
e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6))) - 194*a**2*c**5*d**6*e**4 - 4*a*c*
*8*d**16*(c*e*(a*e**2 - 3*c*d**2)/(2*(a*e**2 + c*d**2)**3) - d*sqrt(-a*c**3
)*(3*a*e**2 - c*d**2)/(2*a*(a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*
e**2 + c**3*d**6)))**2 - 14*a*c**7*d**12*e*(c*e*(a*e**2 - 3*c*d**2)/(2*(a*e
**2 + c*d**2)**3) - d*sqrt(-a*c**3)*(3*a*e**2 - c*d**2)/(2*a*(a**3*e**6 + 3
*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6))) + 6*a*c**6*d**8*e**2
- c**7*d**10)/(27*a**4*c**3*d*e**9 - 144*a**3*c**4*d**3*e**7 + 270*a**2*c**
5*d**5*e**5 - 72*a*c**6*d**7*e**3 - c**7*d**9*e)) + (c*e*(a*e**2 - 3*c*d**2
)/(2*(a*e**2 + c*d**2)**3) + d*sqrt(-a*c**3)*(3*a*e**2 - c*d**2)/(2*a*(a**3
```

```
*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6)))*log(x + (-12
*a**9*e**16*(c*e*(a*e**2 - 3*c*d**2)/(2*(a*e**2 + c*d**2)**3) + d*sqrt(-a*c
**3)*(3*a*e**2 - c*d**2)/(2*a*(a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d*
*4*e**2 + c**3*d**6)))**2 - 24*a**8*c*d**2*e**14*(c*e*(a*e**2 - 3*c*d**2)/(
2*(a*e**2 + c*d**2)**3) + d*sqrt(-a*c**3)*(3*a*e**2 - c*d**2)/(2*a*(a**3*e*
*6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6)))**2 + 104*a**7*c
**2*d**4*e**12*(c*e*(a*e**2 - 3*c*d**2)/(2*(a*e**2 + c*d**2)**3) + d*sqrt(-
a*c**3)*(3*a*e**2 - c*d**2)/(2*a*(a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2
*d**4*e**2 + c**3*d**6)))**2 - 6*a**7*c*e**13*(c*e*(a*e**2 - 3*c*d**2)/(2*(
a*e**2 + c*d**2)**3) + d*sqrt(-a*c**3)*(3*a*e**2 - c*d**2)/(2*a*(a**3*e**6
+ 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6))) + 456*a**6*c**3*d*
*6*e**10*(c*e*(a*e**2 - 3*c*d**2)/(2*(a*e**2 + c*d**2)**3) + d*sqrt(-a*c**3
)*(3*a*e**2 - c*d**2)/(2*a*(a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*
e**2 + c**3*d**6)))**2 - 12*a**6*c**2*d**2*e**11*(c*e*(a*e**2 - 3*c*d**2)/(
2*(a*e**2 + c*d**2)**3) + d*sqrt(-a*c**3)*(3*a*e**2 - c*d**2)/(2*a*(a**3*e*
*6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6))) + 720*a**5*c**4
*d**8*e**8*(c*e*(a*e**2 - 3*c*d**2)/(2*(a*e**2 + c*d**2)**3) + d*sqrt(-a*c*
*3)*(3*a*e**2 - c*d**2)/(2*a*(a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**
4*e**2 + c**3*d**6)))**2 - 2*a**5*c**3*d**4*e**9*(c*e*(a*e**2 - 3*c*d**2)/(
2*(a*e**2 + c*d**2)**3) + d*sqrt(-a*c**3)*(3*a*e**2 - c*d**2)/(2*a*(a**3*e*
*6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6))) + 6*a**5*c**2*e
**10 + 568*a**4*c**5*d**10*e**6*(c*e*(a*e**2 - 3*c*d**2)/(2*(a*e**2 + c*d**
2)**3) + d*sqrt(-a*c**3)*(3*a*e**2 - c*d**2)/(2*a*(a**3*e**6 + 3*a**2*c*d**
2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6)))**2 - 8*a**4*c**4*d**6*e**7*(c*e*
(a*e**2 - 3*c*d**2)/(2*(a*e**2 + c*d**2)**3) + d*sqrt(-a*c**3)*(3*a*e**2 -
c*d**2)/(2*a*(a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d*
*6))) - 69*a**4*c**3*d**2*e**8 + 216*a**3*c**6*d**12*e**4*(c*e*(a*e**2 - 3*
c*d**2)/(2*(a*e**2 + c*d**2)**3) + d*sqrt(-a*c**3)*(3*a*e**2 - c*d**2)/(2*a
*(a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6)))**2 - 4
2*a**3*c**5*d**8*e**5*(c*e*(a*e**2 - 3*c*d**2)/(2*(a*e**2 + c*d**2)**3) + d
*sqrt(-a*c**3)*(3*a*e**2 - c*d**2)/(2*a*(a**3*e**6 + 3*a**2*c*d**2*e**4 + 3
*a*c**2*d**4*e**2 + c**3*d**6))) + 236*a**3*c**4*d**4*e**6 + 24*a**2*c**7*d
**14*e**2*(c*e*(a*e**2 - 3*c*d**2)/(2*(a*e**2 + c*d**2)**3) + d*sqrt(-a*c**
3)*(3*a*e**2 - c*d**2)/(2*a*(a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4
*e**2 + c**3*d**6)))**2 - 44*a**2*c**6*d**10*e**3*(c*e*(a*e**2 - 3*c*d**2)/
(2*(a*e**2 + c*d**2)**3) + d*sqrt(-a*c**3)*(3*a*e**2 - c*d**2)/(2*a*(a**3*e
**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6))) - 194*a**2*c**
5*d**6*e**4 - 4*a*c**8*d**16*(c*e*(a*e**2 - 3*c*d**2)/(2*(a*e**2 + c*d**2)*
*3) + d*sqrt(-a*c**3)*(3*a*e**2 - c*d**2)/(2*a*(a**3*e**6 + 3*a**2*c*d**2*e
**4 + 3*a*c**2*d**4*e**2 + c**3*d**6)))**2 - 14*a*c**7*d**12*e*(c*e*(a*e**2
 - 3*c*d**2)/(2*(a*e**2 + c*d**2)**3) + d*sqrt(-a*c**3)*(3*a*e**2 - c*d**2)
/(2*a*(a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6))) +
 6*a*c**6*d**8*e**2 - c**7*d**10)/(27*a**4*c**3*d*e**9 - 144*a**3*c**4*d**3
*e**7 + 270*a**2*c**5*d**5*e**5 - 72*a*c**6*d**7*e**3 - c**7*d**9*e)) - (a*
e**3 + 5*c*d**2*e + 4*c*d*e**2*x)/(2*a**2*d**2*e**4 + 4*a*c*d**4*e**2 + 2*c
**2*d**6 + x**2*(2*a**2*e**6 + 4*a*c*d**2*e**4 + 2*c**2*d**4*e**2) + x*(4*a
**2*d*e**5 + 8*a*c*d**3*e**3 + 4*c**2*d**5*e))
```

Giac [A] time = 1.35745, size = 363, normalized size = 2.06

$$-\frac{\left(3\,c^2d^2e - ace^3\right)\log\left(cx^2 + a\right)}{2\left(c^3d^6 + 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 + a^3e^6\right)} + \frac{\left(3\,c^2d^2e^2 - ace^4\right)\log\left(|xe + d|\right)}{c^3d^6e + 3\,ac^2d^4e^3 + 3\,a^2cd^2e^5 + a^3e^7} + \frac{\left(c^3d^3 - 3\,ac^2de^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right.}{\left(c^3d^6 + 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 + a^3e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(3*c^2*d^2*e - a*c*e^3)*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*
a^2*c*d^2*e^4 + a^3*e^6) + (3*c^2*d^2*e^2 - a*c*e^4)*log(abs(x*e + d))/(c^3
*d^6*e + 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 + a^3*e^7) + (c^3*d^3 - 3*a*c^2*
d*e^2)*arctan(c*x/sqrt(a*c))/((c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4
+ a^3*e^6)*sqrt(a*c)) - 1/2*(5*c^2*d^4*e + 6*a*c*d^2*e^3 + a^2*e^5 + 4*(c^2
*d^3*e^2 + a*c*d*e^4)*x)/((c*d^2 + a*e^2)^3*(x*e + d)^2)
```

3.505 $\int \frac{(d+ex)^5}{\left(a+cx^2\right)^2} \, dx$

Optimal. Leaf size=190

$$\frac{d\left(-15a^2e^4+10acd^2e^2+c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{5/2}}-\frac{e^3x^2\left(2cd^2-ae^2\right)}{ac^2}+\frac{e^3\left(5cd^2-ae^2\right)\log\left(a+cx^2\right)}{c^3}-\frac{3de^2x\left(2cd^2-5ae^2\right)}{2ac^2}$$

```
[Out] (-3*d*e^2*(2*c*d^2 - 5*a*e^2)*x)/(2*a*c^2) - (e^3*(2*c*d^2 - a*e^2)*x^2)/(a
*c^2) - (d*e^4*x^3)/(2*a*c) - ((a*e - c*d*x)*(d + e*x)^4)/(2*a*c*(a + c*x^2
)) + (d*(c^2*d^4 + 10*a*c*d^2*e^2 - 15*a^2*e^4)*ArcTan[(Sqrt[c]*x)/Sqrt[a]]
)/(2*a^(3/2)*c^(5/2)) + (e^3*(5*c*d^2 - a*e^2)*Log[a + c*x^2])/c^3
```

Rubi [A] time = 0.176979, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {739, 801, 635, 205, 260}

$$\frac{d\left(-15a^2e^4+10acd^2e^2+c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{5/2}}-\frac{e^3x^2\left(2cd^2-ae^2\right)}{ac^2}+\frac{e^3\left(5cd^2-ae^2\right)\log\left(a+cx^2\right)}{c^3}-\frac{3de^2x\left(2cd^2-5ae^2\right)}{2ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5/(a + c*x^2)^2,x]
```

```
[Out] (-3*d*e^2*(2*c*d^2 - 5*a*e^2)*x)/(2*a*c^2) - (e^3*(2*c*d^2 - a*e^2)*x^2)/(a
*c^2) - (d*e^4*x^3)/(2*a*c) - ((a*e - c*d*x)*(d + e*x)^4)/(2*a*c*(a + c*x^2
)) + (d*(c^2*d^4 + 10*a*c*d^2*e^2 - 15*a^2*e^4)*ArcTan[(Sqrt[c]*x)/Sqrt[a]]
)/(2*a^(3/2)*c^(5/2)) + (e^3*(5*c*d^2 - a*e^2)*Log[a + c*x^2])/c^3
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{(d+ex)^5}{\left(a+cx^2\right)^2}\,dx = -\frac{(ae-cdx)(d+ex)^4}{2ac\left(a+cx^2\right)} + \frac{\int \frac{(d+ex)^3\left(cd^2+4ae^2-3cdex\right)}{a+cx^2}\,dx}{2ac}$$

$$= -\frac{(ae-cdx)(d+ex)^4}{2ac\left(a+cx^2\right)} + \frac{\int \left(-3de^2\left(2d^2-\frac{5ae^2}{c}\right) - \frac{4e^3\left(2cd^2-ae^2\right)x}{c} - 3de^4x^2 + \frac{c^2d^5+10acd^3e^2-15a^2de^4+4ae^3\left(5cd^2-\right.}{c\left(a+cx^2\right)}\right.}{2ac}$$

$$= -\frac{3de^2\left(2cd^2-5ae^2\right)x}{2ac^2} - \frac{e^3\left(2cd^2-ae^2\right)x^2}{ac^2} - \frac{de^4x^3}{2ac} - \frac{(ae-cdx)(d+ex)^4}{2ac\left(a+cx^2\right)} + \frac{\int \frac{c^2d^5+10acd^3e^2-15a^2de^4+4ae^3}{a+cx^2}}{2ac^2}$$

$$= -\frac{3de^2\left(2cd^2-5ae^2\right)x}{2ac^2} - \frac{e^3\left(2cd^2-ae^2\right)x^2}{ac^2} - \frac{de^4x^3}{2ac} - \frac{(ae-cdx)(d+ex)^4}{2ac\left(a+cx^2\right)} + \frac{\left(2e^3\left(5cd^2-ae^2\right)\right)\int \frac{x}{a+cx^2}}{c^2}$$

$$= -\frac{3de^2\left(2cd^2-5ae^2\right)x}{2ac^2} - \frac{e^3\left(2cd^2-ae^2\right)x^2}{ac^2} - \frac{de^4x^3}{2ac} - \frac{(ae-cdx)(d+ex)^4}{2ac\left(a+cx^2\right)} + \frac{d\left(c^2d^4+10acd^2e^2-15a\right.}{2a^{3/2}c^{5/2}}$$

Mathematica [A] time = 0.125413, size = 164, normalized size = 0.86

$$\frac{\frac{5a^2cde^3(2d+ex)-a^3e^5-5ac^2d^3e(d+2ex)+c^3d^5x}{a\left(a+cx^2\right)} + \frac{\sqrt{c}d\left(-15a^2e^4+10acd^2e^2+c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{a^{3/2}} + 2\left(5cd^2e^3-ae^5\right)\log\left(a+cx^2\right)+10cde^4x+ce^5x^2}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5/(a + c*x^2)^2,x]
```

```
[Out] (10*c*d*e^4*x + c*e^5*x^2 + (-(a^3*e^5) + c^3*d^5*x + 5*a^2*c*d*e^3*(2*d +
e*x) - 5*a*c^2*d^3*e*(d + 2*e*x))/(a*(a + c*x^2)) + (Sqrt[c]*d*(c^2*d^4 + 1
0*a*c*d^2*e^2 - 15*a^2*e^4)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(3/2) + 2*(5*c*d
^2*e^3 - a*e^5)*Log[a + c*x^2])/(2*c^3)
```

Maple [A] time = 0.056, size = 248, normalized size = 1.3

$$\frac{e^5x^2}{2c^2} + 5\frac{e^4xd}{c^2} + \frac{5\,adxe^4}{2c^2\left(cx^2+a\right)} - 5\frac{d^3xe^2}{c\left(cx^2+a\right)} + \frac{d^5x}{\left(2cx^2+2a\right)a} - \frac{a^2e^5}{2c^3\left(cx^2+a\right)} + 5\frac{e^3ad^2}{c^2\left(cx^2+a\right)} - \frac{5\,ed^4}{2c\left(cx^2+a\right)} - \frac{a\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5/(c*x^2+a)^2,x)
```

```
[Out] 1/2*e^5*x^2/c^2+5*e^4/c^2*x*d+5/2/c^2/(c*x^2+a)*d*a*x*e^4-5/c/(c*x^2+a)*d^3
*x*e^2+1/2/(c*x^2+a)*d^5/a*x-1/2/c^3/(c*x^2+a)*e^5*a^2+5/c^2/(c*x^2+a)*e^3*
a*d^2-5/2/c/(c*x^2+a)*e*d^4-1/c^3*a*ln(c*x^2+a)*e^5+5/c^2*ln(c*x^2+a)*d^2*e
^3-15/2/c^2*a/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*d*e^4+5/c/(a*c)^(1/2)*arc
tan(x*c/(a*c)^(1/2))*d^3*e^2+1/2/a/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*d^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.97456, size = 1142, normalized size = 6.01

$$\left[\frac{2\,a^2c^2e^5x^4 + 20\,a^2c^2de^4x^3 + 2\,a^3ce^5x^2 - 10\,a^2c^2d^4e + 20\,a^3cd^2e^3 - 2\,a^4e^5 + \left(ac^2d^5 + 10\,a^2cd^3e^2 - 15\,a^3de^4 + \left(c^3d^5 + 1\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a^2*c^2*e^5*x^4 + 20*a^2*c^2*d*e^4*x^3 + 2*a^3*c*e^5*x^2 - 10*a^2*c
^2*d^4*e + 20*a^3*c*d^2*e^3 - 2*a^4*e^5 + (a*c^2*d^5 + 10*a^2*c*d^3*e^2 - 1
5*a^3*d*e^4 + (c^3*d^5 + 10*a*c^2*d^3*e^2 - 15*a^2*c*d*e^4)*x^2)*sqrt(-a*c)
*log((c*x^2 + 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(a*c^3*d^5 - 10*a^2*c^2*
d^3*e^2 + 15*a^3*c*d*e^4)*x + 4*(5*a^3*c*d^2*e^3 - a^4*e^5 + (5*a^2*c^2*d^2
*e^3 - a^3*c*e^5)*x^2)*log(c*x^2 + a))/(a^2*c^4*x^2 + a^3*c^3), 1/2*(a^2*c^
2*e^5*x^4 + 10*a^2*c^2*d*e^4*x^3 + a^3*c*e^5*x^2 - 5*a^2*c^2*d^4*e + 10*a^3
*c*d^2*e^3 - a^4*e^5 + (a*c^2*d^5 + 10*a^2*c*d^3*e^2 - 15*a^3*d*e^4 + (c^3*
d^5 + 10*a*c^2*d^3*e^2 - 15*a^2*c*d*e^4)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/
a) + (a*c^3*d^5 - 10*a^2*c^2*d^3*e^2 + 15*a^3*c*d*e^4)*x + 2*(5*a^3*c*d^2*e
^3 - a^4*e^5 + (5*a^2*c^2*d^2*e^3 - a^3*c*e^5)*x^2)*log(c*x^2 + a))/(a^2*c^
4*x^2 + a^3*c^3)]
```

Sympy [B] time = 3.54667, size = 515, normalized size = 2.71

$$\left(-\frac{e^3\left(ae^2-5cd^2\right)}{c^3} - \frac{d\sqrt{-a^3c^7}\left(15a^2e^4-10acd^2e^2-c^2d^4\right)}{4a^3c^6}\right)\log\left(x + \frac{-4a^3e^5-4a^2c^3\left(-\frac{e^3\left(ae^2-5cd^2\right)}{c^3} - \frac{d\sqrt{-a^3c^7}\left(15a^2e^4-10acd^2\right.}{4a^3c^6}\right.}{15a^2cde^4-10ac^2d^3e^2-c^3d^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5/(c*x**2+a)**2,x)
```

```
[Out] (-e**3*(a*e**2 - 5*c*d**2)/c**3 - d*sqrt(-a**3*c**7)*(15*a**2*e**4 - 10*a*c
*d**2*e**2 - c**2*d**4)/(4*a**3*c**6))*log(x + (-4*a**3*e**5 - 4*a**2*c**3*
(-e**3*(a*e**2 - 5*c*d**2)/c**3 - d*sqrt(-a**3*c**7)*(15*a**2*e**4 - 10*a*c
*d**2*e**2 - c**2*d**4)/(4*a**3*c**6)) + 20*a**2*c*d**2*e**3)/(15*a**2*c*d*
e**4 - 10*a*c**2*d**3*e**2 - c**3*d**5)) + (-e**3*(a*e**2 - 5*c*d**2)/c**3
+ d*sqrt(-a**3*c**7)*(15*a**2*e**4 - 10*a*c*d**2*e**2 - c**2*d**4)/(4*a**3*
c**6))*log(x + (-4*a**3*e**5 - 4*a**2*c**3*(-e**3*(a*e**2 - 5*c*d**2)/c**3
+ d*sqrt(-a**3*c**7)*(15*a**2*e**4 - 10*a*c*d**2*e**2 - c**2*d**4)/(4*a**3*
```

```
c**6)) + 20*a**2*c*d**2*e**3)/(15*a**2*c*d*e**4 - 10*a*c**2*d**3*e**2 - c**
3*d**5)) + (-a**3*e**5 + 10*a**2*c*d**2*e**3 - 5*a*c**2*d**4*e + x*(5*a**2*
c*d*e**4 - 10*a*c**2*d**3*e**2 + c**3*d**5))/(2*a**2*c**3 + 2*a*c**4*x**2)
+ 5*d*e**4*x/c**2 + e**5*x**2/(2*c**2)
```

Giac [A] time = 1.31711, size = 236, normalized size = 1.24

$$\frac{\left(5\,cd^2e^3 - ae^5\right)\log\left(cx^2 + a\right)}{c^3} + \frac{\left(c^2d^5 + 10\,acd^3e^2 - 15\,a^2de^4\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac^2} + \frac{c^2x^2e^5 + 10\,c^2dxe^4}{2\,c^4} - \frac{5\,ac^2d^4e - 10\,a^2cd^2e^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] (5*c*d^2*e^3 - a*e^5)*log(c*x^2 + a)/c^3 + 1/2*(c^2*d^5 + 10*a*c*d^3*e^2 -
15*a^2*d*e^4)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c^2) + 1/2*(c^2*x^2*e^5 +
10*c^2*d*x*e^4)/c^4 - 1/2*(5*a*c^2*d^4*e - 10*a^2*c*d^2*e^3 + a^3*e^5 - (c^
3*d^5 - 10*a*c^2*d^3*e^2 + 5*a^2*c*d*e^4)*x)/((c*x^2 + a)*a*c^3)
```

3.506 $\int \frac{(d+ex)^4}{\left(a+cx^2\right)^2} dx$

Optimal. Leaf size=149

$$\frac{\left(-3a^2e^4+6acd^2e^2+c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{5/2}}-\frac{3e^2x\left(cd^2-ae^2\right)}{2ac^2}+\frac{2de^3\log\left(a+cx^2\right)}{c^2}-\frac{de^3x^2}{2ac}-\frac{(d+ex)^3(ae-cdx)}{2ac\left(a+cx^2\right)}$$

```
[Out] (-3*e^2*(c*d^2 - a*e^2)*x)/(2*a*c^2) - (d*e^3*x^2)/(2*a*c) - ((a*e - c*d*x)
*(d + e*x)^3)/(2*a*c*(a + c*x^2)) + ((c^2*d^4 + 6*a*c*d^2*e^2 - 3*a^2*e^4)*
ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(5/2)) + (2*d*e^3*Log[a + c*x^2])
/c^2
```

Rubi [A] time = 0.120247, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {739, 801, 635, 205, 260}

$$\frac{\left(-3a^2e^4+6acd^2e^2+c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{5/2}}-\frac{3e^2x\left(cd^2-ae^2\right)}{2ac^2}+\frac{2de^3\log\left(a+cx^2\right)}{c^2}-\frac{de^3x^2}{2ac}-\frac{(d+ex)^3(ae-cdx)}{2ac\left(a+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a + c*x^2)^2,x]
```

```
[Out] (-3*e^2*(c*d^2 - a*e^2)*x)/(2*a*c^2) - (d*e^3*x^2)/(2*a*c) - ((a*e - c*d*x)
*(d + e*x)^3)/(2*a*c*(a + c*x^2)) + ((c^2*d^4 + 6*a*c*d^2*e^2 - 3*a^2*e^4)*
ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(5/2)) + (2*d*e^3*Log[a + c*x^2])
/c^2
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{(d+ex)^4}{\left(a+cx^2\right)^2}\,dx = -\frac{(ae-cdx)(d+ex)^3}{2ac\left(a+cx^2\right)}+\frac{\int \frac{(d+ex)^2\left(cd^2+3ae^2-2cdex\right)}{a+cx^2}\,dx}{2ac}$$

$$= -\frac{(ae-cdx)(d+ex)^3}{2ac\left(a+cx^2\right)}+\frac{\int \left(-3e^2\left(d^2-\frac{ae^2}{c}\right)-2de^3x+\frac{c^2d^4+6acd^2e^2-3a^2e^4+8acde^3x}{c\left(a+cx^2\right)}\right)dx}{2ac}$$

$$= -\frac{3e^2\left(cd^2-ae^2\right)x}{2ac^2}-\frac{de^3x^2}{2ac}-\frac{(ae-cdx)(d+ex)^3}{2ac\left(a+cx^2\right)}+\frac{\int \frac{c^2d^4+6acd^2e^2-3a^2e^4+8acde^3x}{a+cx^2}\,dx}{2ac^2}$$

$$= -\frac{3e^2\left(cd^2-ae^2\right)x}{2ac^2}-\frac{de^3x^2}{2ac}-\frac{(ae-cdx)(d+ex)^3}{2ac\left(a+cx^2\right)}+\frac{\left(4de^3\right)\int \frac{x}{a+cx^2}\,dx}{c}+\frac{\left(c^2d^4+6acd^2e^2-3a^2e^4\right)\int \frac{}{a}$$

$$= -\frac{3e^2\left(cd^2-ae^2\right)x}{2ac^2}-\frac{de^3x^2}{2ac}-\frac{(ae-cdx)(d+ex)^3}{2ac\left(a+cx^2\right)}+\frac{\left(c^2d^4+6acd^2e^2-3a^2e^4\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{5/2}}+\frac{2de^3\log}{c}$$

Mathematica [A] time = 0.0832501, size = 137, normalized size = 0.92

$$\frac{a^2e^3(4d+ex)-2acd^2e(2d+3ex)+c^2d^4x}{2ac^2\left(a+cx^2\right)}+\frac{\left(-3a^2e^4+6acd^2e^2+c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{5/2}}+\frac{2de^3\log\left(a+cx^2\right)}{c^2}+\frac{e^4x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a + c*x^2)^2,x]
```

```
[Out] (e^4*x)/c^2 + (c^2*d^4*x + a^2*e^3*(4*d + e*x) - 2*a*c*d^2*e*(2*d + 3*e*x))
/(2*a*c^2*(a + c*x^2)) + ((c^2*d^4 + 6*a*c*d^2*e^2 - 3*a^2*e^4)*ArcTan[(Sqr
t[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(5/2)) + (2*d*e^3*Log[a + c*x^2])/c^2
```

Maple [A] time = 0.051, size = 192, normalized size = 1.3

$$\frac{e^4x}{c^2}+\frac{axe^4}{2\,c^2\left(cx^2+a\right)}-3\,\frac{xd^2e^2}{c\left(cx^2+a\right)}+\frac{xd^4}{\left(2\,cx^2+2\,a\right)a}+2\,\frac{ade^3}{c^2\left(cx^2+a\right)}-2\,\frac{d^3e}{c\left(cx^2+a\right)}+2\,\frac{de^3\ln\left(cx^2+a\right)}{c^2}-\frac{3\,ae^4}{2\,c^2}\,\mathrm{arctan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(c*x^2+a)^2,x)
```

```
[Out] e^4*x/c^2+1/2/c^2/(c*x^2+a)*x*a*e^4-3/c/(c*x^2+a)*x*d^2*e^2+1/2/(c*x^2+a)*x
/a*d^4+2/c^2/(c*x^2+a)*a*d*e^3-2/c/(c*x^2+a)*d^3*e+2*d*e^3*ln(c*x^2+a)/c^2-
3/2/c^2*a/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*e^4+3/c/(a*c)^(1/2)*arctan(x*
c/(a*c)^(1/2))*d^2*e^2+1/2/a/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*d^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.90149, size = 879, normalized size = 5.9

$$\left[\frac{4\,a^2c^2e^4x^3 - 8\,a^2c^2d^3e + 8\,a^3cde^3 + \left(ac^2d^4 + 6\,a^2cd^2e^2 - 3\,a^3e^4 + \left(c^3d^4 + 6\,ac^2d^2e^2 - 3\,a^2ce^4\right)x^2\right)\sqrt{-ac}\,\log\left(\frac{cx^2+2\sqrt{-ac}}{cx^2+a}\right.}{4\left(a^2c^4x^2 + a^3c^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^2*c^2*e^4*x^3 - 8*a^2*c^2*d^3*e + 8*a^3*c*d*e^3 + (a*c^2*d^4 + 6*
a^2*c*d^2*e^2 - 3*a^3*e^4 + (c^3*d^4 + 6*a*c^2*d^2*e^2 - 3*a^2*c*e^4)*x^2)*
sqrt(-a*c)*log((c*x^2 + 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(a*c^3*d^4 - 6
*a^2*c^2*d^2*e^2 + 3*a^3*c*e^4)*x + 8*(a^2*c^2*d*e^3*x^2 + a^3*c*d*e^3)*log
(c*x^2 + a))/(a^2*c^4*x^2 + a^3*c^3), 1/2*(2*a^2*c^2*e^4*x^3 - 4*a^2*c^2*d^
3*e + 4*a^3*c*d*e^3 + (a*c^2*d^4 + 6*a^2*c*d^2*e^2 - 3*a^3*e^4 + (c^3*d^4 +
 6*a*c^2*d^2*e^2 - 3*a^2*c*e^4)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + (a*c
^3*d^4 - 6*a^2*c^2*d^2*e^2 + 3*a^3*c*e^4)*x + 4*(a^2*c^2*d*e^3*x^2 + a^3*c*
d*e^3)*log(c*x^2 + a))/(a^2*c^4*x^2 + a^3*c^3)]
```

Sympy [B] time = 2.71385, size = 403, normalized size = 2.7

$$\left(\frac{2de^3}{c^2} - \frac{\sqrt{-a^3c^5}\left(3a^2e^4 - 6acd^2e^2 - c^2d^4\right)}{4a^3c^5}\right)\log\left(x + \frac{-4a^2c^2\left(\frac{2de^3}{c^2} - \frac{\sqrt{-a^3c^5}\left(3a^2e^4-6acd^2e^2-c^2d^4\right)}{4a^3c^5}\right) + 8a^2de^3}{3a^2e^4 - 6acd^2e^2 - c^2d^4}\right) + \left(\frac{2de^3}{c^2} + \frac{\sqrt{-}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(c*x**2+a)**2,x)
```

```
[Out] (2*d*e**3/c**2 - sqrt(-a**3*c**5)*(3*a**2*e**4 - 6*a*c*d**2*e**2 - c**2*d**
4)/(4*a**3*c**5))*log(x + (-4*a**2*c**2*(2*d*e**3/c**2 - sqrt(-a**3*c**5)*(
3*a**2*e**4 - 6*a*c*d**2*e**2 - c**2*d**4)/(4*a**3*c**5)) + 8*a**2*d*e**3)/
(3*a**2*e**4 - 6*a*c*d**2*e**2 - c**2*d**4)) + (2*d*e**3/c**2 + sqrt(-a**3*
c**5)*(3*a**2*e**4 - 6*a*c*d**2*e**2 - c**2*d**4)/(4*a**3*c**5))*log(x + (-
4*a**2*c**2*(2*d*e**3/c**2 + sqrt(-a**3*c**5)*(3*a**2*e**4 - 6*a*c*d**2*e**
2 - c**2*d**4)/(4*a**3*c**5)) + 8*a**2*d*e**3)/(3*a**2*e**4 - 6*a*c*d**2*e*
*2 - c**2*d**4)) + (4*a**2*d*e**3 - 4*a*c*d**3*e + x*(a**2*e**4 - 6*a*c*d**
2*e**2 + c**2*d**4))/(2*a**2*c**2 + 2*a*c**3*x**2) + e**4*x/c**2
```

Giac [A] time = 1.30165, size = 177, normalized size = 1.19

$$\frac{2\,de^3\log\left(cx^2+a\right)}{c^2}+\frac{xe^4}{c^2}+\frac{\left(c^2d^4+6\,acd^2e^2-3\,a^2e^4\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac^2}-\frac{4\,acd^3e-4\,a^2de^3-\left(c^2d^4-6\,acd^2e^2+a^2e^4\right)x}{2\,\left(cx^2+a\right)ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2*d*e^3*log(c*x^2 + a)/c^2 + x*e^4/c^2 + 1/2*(c^2*d^4 + 6*a*c*d^2*e^2 - 3*a
^2*e^4)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c^2) - 1/2*(4*a*c*d^3*e - 4*a^2*
d*e^3 - (c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4)*x)/((c*x^2 + a)*a*c^2)
```

3.507 $\int \frac{(d+ex)^3}{\left(a+cx^2\right)^2} dx$

Optimal. Leaf size=109

$$\frac{d\left(3ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}+\frac{e^3\log\left(a+cx^2\right)}{2c^2}-\frac{de^2x}{2ac}-\frac{(d+ex)^2(ae-cdx)}{2ac\left(a+cx^2\right)}$$

```
[Out] -(d*e^2*x)/(2*a*c) - ((a*e - c*d*x)*(d + e*x)^2)/(2*a*c*(a + c*x^2)) + (d*(
c*d^2 + 3*a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(3/2)) + (e^3*Lo
g[a + c*x^2])/(2*c^2)
```

Rubi [A] time = 0.080895, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {739, 774, 635, 205, 260}

$$\frac{d\left(3ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}+\frac{e^3\log\left(a+cx^2\right)}{2c^2}-\frac{de^2x}{2ac}-\frac{(d+ex)^2(ae-cdx)}{2ac\left(a+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + c*x^2)^2,x]
```

```
[Out] -(d*e^2*x)/(2*a*c) - ((a*e - c*d*x)*(d + e*x)^2)/(2*a*c*(a + c*x^2)) + (d*(
c*d^2 + 3*a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(3/2)) + (e^3*Lo
g[a + c*x^2])/(2*c^2)
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^3}{\left(a+cx^2\right)^2}\,dx &= -\frac{(ae-cdx)(d+ex)^2}{2ac\left(a+cx^2\right)}+\frac{\int \frac{(d+ex)\left(cd^2+2ae^2-cdex\right)}{a+cx^2}\,dx}{2ac}\\
&= -\frac{de^2x}{2ac}-\frac{(ae-cdx)(d+ex)^2}{2ac\left(a+cx^2\right)}+\frac{\int \frac{acde^2+cd\left(cd^2+2ae^2\right)+c\left(-cd^2e+e\left(cd^2+2ae^2\right)\right)x}{a+cx^2}\,dx}{2ac^2}\\
&= -\frac{de^2x}{2ac}-\frac{(ae-cdx)(d+ex)^2}{2ac\left(a+cx^2\right)}+\frac{e^3\int \frac{x}{a+cx^2}\,dx}{c}+\frac{\left(d\left(cd^2+3ae^2\right)\right)\int \frac{1}{a+cx^2}\,dx}{2ac}\\
&= -\frac{de^2x}{2ac}-\frac{(ae-cdx)(d+ex)^2}{2ac\left(a+cx^2\right)}+\frac{d\left(cd^2+3ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}+\frac{e^3\log\left(a+cx^2\right)}{2c^2}
\end{aligned}$$

Mathematica [A] time = 0.0785699, size = 107, normalized size = 0.98

$$\frac{\frac{\sqrt{a}\left(a^2e^3-3acde(d+ex)+ae^3\left(a+cx^2\right)\log\left(a+cx^2\right)+c^2d^3x\right)}{a+cx^2}+\sqrt{c}d\left(3ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + c*x^2)^2,x]
```

```
[Out] (Sqrt[c]*d*(c*d^2 + 3*a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]] + (Sqrt[a]*(a^2*e^
3 + c^2*d^3*x - 3*a*c*d*e*(d + e*x) + a*e^3*(a + c*x^2)*Log[a + c*x^2]))/(a
 + c*x^2))/(2*a^(3/2)*c^2)
```

Maple [A] time = 0.049, size = 115, normalized size = 1.1

$$\frac{1}{cx^2+a}\left(-\frac{d\left(3\,ae^2-cd^2\right)x}{2\,ac}+\frac{e\left(ae^2-3\,cd^2\right)}{2\,c^2}\right)+\frac{e^3\ln\left(cx^2+a\right)}{2\,c^2}+\frac{3\,de^2}{2\,c}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}+\frac{d^3}{2\,a}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^2+a)^2,x)
```

```
[Out] (-1/2*d*(3*a*e^2-c*d^2)/a/c*x+1/2*e*(a*e^2-3*c*d^2)/c^2)/(c*x^2+a)+1/2*e^3*
ln(c*x^2+a)/c^2+3/2/c/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*d*e^2+1/2/a/(a*c)
^(1/2)*arctan(x*c/(a*c)^(1/2))*d^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.92571, size = 639, normalized size = 5.86

$$\left[-\frac{6\,a^2cd^2e-2\,a^3e^3+\left(acd^3+3\,a^2de^2+\left(c^2d^3+3\,acde^2\right)x^2\right)\sqrt{-ac}\log\left(\frac{cx^2-2\,\sqrt{-ac}x-a}{cx^2+a}\right)-2\left(ac^2d^3-3\,a^2cde^2\right)x-2\left(a^2ce^3\right.}{4\left(a^2c^3x^2+a^3c^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*a^2*c*d^2*e - 2*a^3*e^3 + (a*c*d^3 + 3*a^2*d*e^2 + (c^2*d^3 + 3*a*
c*d*e^2)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*
(a*c^2*d^3 - 3*a^2*c*d*e^2)*x - 2*(a^2*c*e^3*x^2 + a^3*e^3)*log(c*x^2 + a))
/(a^2*c^3*x^2 + a^3*c^2), -1/2*(3*a^2*c*d^2*e - a^3*e^3 - (a*c*d^3 + 3*a^2*
d*e^2 + (c^2*d^3 + 3*a*c*d*e^2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) - (a*c
^2*d^3 - 3*a^2*c*d*e^2)*x - (a^2*c*e^3*x^2 + a^3*e^3)*log(c*x^2 + a))/(a^2*
c^3*x^2 + a^3*c^2)]
```

Sympy [B] time = 1.40224, size = 298, normalized size = 2.73

$$\left(\frac{e^3}{2c^2}-\frac{d\sqrt{-a^3c^5}\left(3ae^2+cd^2\right)}{4a^3c^4}\right)\log\left(x+\frac{4a^2c^2\left(\frac{e^3}{2c^2}-\frac{d\sqrt{-a^3c^5}\left(3ae^2+cd^2\right)}{4a^3c^4}\right)-2a^2e^3}{3acde^2+c^2d^3}\right)+\left(\frac{e^3}{2c^2}+\frac{d\sqrt{-a^3c^5}\left(3ae^2+cd^2\right)}{4a^3c^4}\right)\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**2+a)**2,x)
```

```
[Out] (e**3/(2*c**2) - d*sqrt(-a**3*c**5)*(3*a*e**2 + c*d**2)/(4*a**3*c**4))*log(
x + (4*a**2*c**2*(e**3/(2*c**2) - d*sqrt(-a**3*c**5)*(3*a*e**2 + c*d**2)/(4
*a**3*c**4)) - 2*a**2*e**3)/(3*a*c*d*e**2 + c**2*d**3)) + (e**3/(2*c**2) +
d*sqrt(-a**3*c**5)*(3*a*e**2 + c*d**2)/(4*a**3*c**4))*log(x + (4*a**2*c**2*
(e**3/(2*c**2) + d*sqrt(-a**3*c**5)*(3*a*e**2 + c*d**2)/(4*a**3*c**4)) - 2*
a**2*e**3)/(3*a*c*d*e**2 + c**2*d**3)) - (-a**2*e**3 + 3*a*c*d**2*e + x*(3*
a*c*d*e**2 - c**2*d**3))/(2*a**2*c**2 + 2*a*c**3*x**2)
```

Giac [A] time = 1.30426, size = 140, normalized size = 1.28

$$\frac{e^3\log\left(cx^2+a\right)}{2\,c^2}+\frac{\left(cd^3+3\,ade^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac}+\frac{\left(cd^3-3\,ade^2\right)x-\frac{3\,acd^2e-a^2e^3}{c}}{2\left(cx^2+a\right)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*e^3*log(c*x^2 + a)/c^2 + 1/2*(c*d^3 + 3*a*d*e^2)*arctan(c*x/sqrt(a*c))/
(sqrt(a*c)*a*c) + 1/2*((c*d^3 - 3*a*d*e^2)*x - (3*a*c*d^2*e - a^2*e^3)/c)/(
(c*x^2 + a)*a*c)
```

3.508 $\int \frac{(d+ex)^2}{\left(a+cx^2\right)^2} dx$

Optimal. Leaf size=72

$$\frac{\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}-\frac{(d+ex)(ae-cdx)}{2ac\left(a+cx^2\right)}$$

```
[Out] -((a*e - c*d*x)*(d + e*x))/(2*a*c*(a + c*x^2)) + ((c*d^2 + a*e^2)*ArcTan[(S
qrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(3/2))
```

Rubi [A] time = 0.0210555, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {723, 205}

$$\frac{\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}-\frac{(d+ex)(ae-cdx)}{2ac\left(a+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + c*x^2)^2,x]
```

```
[Out] -((a*e - c*d*x)*(d + e*x))/(2*a*c*(a + c*x^2)) + ((c*d^2 + a*e^2)*ArcTan[(S
qrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(3/2))
```

Rule 723

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[((2*p + 3)*(c*d^2 + a*e^2))/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2}{\left(a+cx^2\right)^2} dx &= -\frac{(ae-cdx)(d+ex)}{2ac\left(a+cx^2\right)}+\frac{\left(cd^2+ae^2\right)\int \frac{1}{a+cx^2} dx}{2ac}\\ &= -\frac{(ae-cdx)(d+ex)}{2ac\left(a+cx^2\right)}+\frac{\left(cd^2+ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0579996, size = 77, normalized size = 1.07

$$\frac{\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}+\frac{-2ade-ae^2x+cd^2x}{2ac\left(a+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + c*x^2)^2,x]
```

```
[Out] (-2*a*d*e + c*d^2*x - a*e^2*x)/(2*a*c*(a + c*x^2)) + ((c*d^2 + a*e^2)*ArcTa
n[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(3/2))
```

Maple [A] time = 0.05, size = 85, normalized size = 1.2

$$\frac{1}{cx^2+a}\left(-\frac{\left(ae^2-cd^2\right)x}{2\,ac}-\frac{de}{c}\right)+\frac{e^2}{2\,c}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}+\frac{d^2}{2\,a}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^2+a)^2,x)
```

```
[Out] (-1/2*(a*e^2-c*d^2)/a/c*x-d*e/c)/(c*x^2+a)+1/2/c/(a*c)^(1/2)*arctan(x*c/(a*
c)^(1/2))*e^2+1/2/a/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88351, size = 452, normalized size = 6.28

$$\left[-\frac{4\,a^2cde+\left(acd^2+a^2e^2+\left(c^2d^2+ace^2\right)x^2\right)\sqrt{-ac}\log\left(\frac{cx^2-2\,\sqrt{-ac}x-a}{cx^2+a}\right)-2\left(ac^2d^2-a^2ce^2\right)x}{4\left(a^2c^3x^2+a^3c^2\right)},-\frac{2\,a^2cde-\left(acd^2+a^2e^2+\left(c^2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^2*c*d*e + (a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)*sqrt(-a*
c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(a*c^2*d^2 - a^2*c*e^2
)*x)/(a^2*c^3*x^2 + a^3*c^2), -1/2*(2*a^2*c*d*e - (a*c*d^2 + a^2*e^2 + (c^2
*d^2 + a*c*e^2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) - (a*c^2*d^2 - a^2*c*e
^2)*x)/(a^2*c^3*x^2 + a^3*c^2)]
```

Sympy [B] time = 0.722852, size = 129, normalized size = 1.79

$$-\frac{\sqrt{-\frac{1}{a^3c^3}}\left(ae^2+cd^2\right)\log\left(-a^2c\sqrt{-\frac{1}{a^3c^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^3c^3}}\left(ae^2+cd^2\right)\log\left(a^2c\sqrt{-\frac{1}{a^3c^3}}+x\right)}{4}-\frac{2ade+x\left(ae^2-cd^2\right)}{2a^2c+2ac^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**3*c**3))*(a*e**2 + c*d**2)*log(-a**2*c*sqrt(-1/(a**3*c**3)) +
x)/4 + sqrt(-1/(a**3*c**3))*(a*e**2 + c*d**2)*log(a**2*c*sqrt(-1/(a**3*c**3
)) + x)/4 - (2*a*d*e + x*(a*e**2 - c*d**2))/(2*a**2*c + 2*a*c**2*x**2)
```

Giac [A] time = 1.31404, size = 93, normalized size = 1.29

$$\frac{\left(cd^2+ae^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}ac}+\frac{cd^2x-axe^2-2\,ade}{2\left(cx^2+a\right)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(c*d^2 + a*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c) + 1/2*(c*d^2*x -
a*x*e^2 - 2*a*d*e)/((c*x^2 + a)*a*c)
```

3.509 $\int \frac{d+ex}{\left(a+cx^2\right)^2} \, dx$

Optimal. Leaf size=57

$$\frac{d \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}} - \frac{ae - cdx}{2ac\left(a + cx^2\right)}$$

```
[Out] -(a*e - c*d*x)/(2*a*c*(a + c*x^2)) + (d*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(
3/2)*Sqrt[c])
```

Rubi [A] time = 0.0136079, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {639, 205}

$$\frac{d \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}} - \frac{ae - cdx}{2ac\left(a + cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + c*x^2)^2,x]
```

```
[Out] -(a*e - c*d*x)/(2*a*c*(a + c*x^2)) + (d*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(
3/2)*Sqrt[c])
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{d + ex}{\left(a + cx^2\right)^2} \, dx &= -\frac{ae - cdx}{2ac\left(a + cx^2\right)} + \frac{d \int \frac{1}{a+cx^2} \, dx}{2a} \\
&= -\frac{ae - cdx}{2ac\left(a + cx^2\right)} + \frac{d \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.0264744, size = 57, normalized size = 1.

$$\frac{d \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}} + \frac{cdx - ae}{2ac\left(a + cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + c*x^2)^2,x]
```

```
[Out] (-(a*e) + c*d*x)/(2*a*c*(a + c*x^2)) + (d*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a
^(3/2)*Sqrt[c])
```

Maple [A] time = 0.046, size = 49, normalized size = 0.9

$$\frac{2\,cdx-2\,ae}{4\,ac\left(cx^{2}+a\right)}+\frac{d}{2\,a}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^2+a)^2,x)
```

```
[Out] 1/4*(2*c*d*x-2*a*e)/a/c/(c*x^2+a)+1/2*d/a/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2
))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71684, size = 301, normalized size = 5.28

$$\left[\frac{2\,acdx-2\,a^{2}e-\left(cdx^{2}+ad\right)\sqrt{-ac}\log\left(\frac{cx^{2}-2\,\sqrt{-ac}x-a}{cx^{2}+a}\right)}{4\,\left(a^{2}c^{2}x^{2}+a^{3}c\right)},\ \frac{acdx-a^{2}e+\left(cdx^{2}+ad\right)\sqrt{ac}\arctan\left(\frac{\sqrt{ac}x}{a}\right)}{2\,\left(a^{2}c^{2}x^{2}+a^{3}c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*c*d*x - 2*a^2*e - (c*d*x^2 + a*d)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(
-a*c)*x - a)/(c*x^2 + a)))/(a^2*c^2*x^2 + a^3*c), 1/2*(a*c*d*x - a^2*e + (c
*d*x^2 + a*d)*sqrt(a*c)*arctan(sqrt(a*c)*x/a))/(a^2*c^2*x^2 + a^3*c)]
```

Sympy [A] time = 0.793173, size = 90, normalized size = 1.58

$$d\left(-\frac{\sqrt{-\frac{1}{a^{3}c}}\log\left(-a^{2}\sqrt{-\frac{1}{a^{3}c}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^{3}c}}\log\left(a^{2}\sqrt{-\frac{1}{a^{3}c}}+x\right)}{4}\right)+\frac{-ae+cdx}{2a^{2}c+2ac^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**2+a)**2,x)
```

```
[Out] d*(-sqrt(-1/(a**3*c))*log(-a**2*sqrt(-1/(a**3*c)) + x)/4 + sqrt(-1/(a**3*c)
)*log(a**2*sqrt(-1/(a**3*c)) + x)/4) + (-a*e + c*d*x)/(2*a**2*c + 2*a*c**2*
x**2)
```

Giac [A] time = 1.23295, size = 65, normalized size = 1.14

$$\frac{d \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}a} + \frac{cdx - ae}{2\left(cx^2 + a\right)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*d*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a) + 1/2*(c*d*x - a*e)/((c*x^2 + a)*
a*c)
```

3.510 $\int \frac{1}{(d+ex)(a+cx^2)^2} dx$

Optimal. Leaf size=142

$$\frac{\sqrt{c}d\left(3ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\left(ae^2+cd^2\right)^2}+\frac{ae+cdx}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)}-\frac{e^3\log\left(a+cx^2\right)}{2\left(ae^2+cd^2\right)^2}+\frac{e^3\log(d+ex)}{\left(ae^2+cd^2\right)^2}$$

```
[Out] (a*e + c*d*x)/(2*a*(c*d^2 + a*e^2)*(a + c*x^2)) + (Sqrt[c]*d*(c*d^2 + 3*a*e
^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*(c*d^2 + a*e^2)^2) + (e^3*Log[d
 + e*x])/(c*d^2 + a*e^2)^2 - (e^3*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^2)
```

Rubi [A] time = 0.125904, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {741, 801, 635, 205, 260}

$$\frac{\sqrt{c}d\left(3ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\left(ae^2+cd^2\right)^2}+\frac{ae+cdx}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)}-\frac{e^3\log\left(a+cx^2\right)}{2\left(ae^2+cd^2\right)^2}+\frac{e^3\log(d+ex)}{\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + c*x^2)^2),x]
```

```
[Out] (a*e + c*d*x)/(2*a*(c*d^2 + a*e^2)*(a + c*x^2)) + (Sqrt[c]*d*(c*d^2 + 3*a*e
^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*(c*d^2 + a*e^2)^2) + (e^3*Log[d
 + e*x])/(c*d^2 + a*e^2)^2 - (e^3*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^2)
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+cx^2\right)^2}\,dx = \frac{ae+cdx}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \frac{-cd^2-2ae^2-cdex}{(d+ex)\left(a+cx^2\right)}\,dx}{2a\left(cd^2+ae^2\right)}$$
$$= \frac{ae+cdx}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \left(-\frac{2ae^4}{\left(cd^2+ae^2\right)(d+ex)} - \frac{c\left(cd^3+3ade^2-2ae^3x\right)}{\left(cd^2+ae^2\right)\left(a+cx^2\right)}\right)dx}{2a\left(cd^2+ae^2\right)}$$
$$= \frac{ae+cdx}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{e^3\log(d+ex)}{\left(cd^2+ae^2\right)^2} + \frac{c\int \frac{cd^3+3ade^2-2ae^3x}{a+cx^2}\,dx}{2a\left(cd^2+ae^2\right)^2}$$
$$= \frac{ae+cdx}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{e^3\log(d+ex)}{\left(cd^2+ae^2\right)^2} - \frac{\left(ce^3\right)\int \frac{x}{a+cx^2}\,dx}{\left(cd^2+ae^2\right)^2} + \frac{\left(cd\left(cd^2+3ae^2\right)\right)\int \frac{1}{a+cx^2}\,dx}{2a\left(cd^2+ae^2\right)^2}$$
$$= \frac{ae+cdx}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{\sqrt{c}d\left(cd^2+3ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\left(cd^2+ae^2\right)^2} + \frac{e^3\log(d+ex)}{\left(cd^2+ae^2\right)^2} - \frac{e^3\log\left(a+cx^2\right)}{2\left(cd^2+ae^2\right)^2}$$

Mathematica [A] time = 0.100066, size = 138, normalized size = 0.97

$$\frac{\sqrt{a}\left(\left(ae^2+cd^2\right)(ae+cdx)+2ae^3\left(a+cx^2\right)\log(d+ex)-ae^3\left(a+cx^2\right)\log\left(a+cx^2\right)\right)+\sqrt{c}d\left(a+cx^2\right)\left(3ae^2+cd^2\right)\tan^{-1}}{2a^{3/2}\left(a+cx^2\right)\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + c*x^2)^2),x]
```

```
[Out] (Sqrt[c]*d*(c*d^2 + 3*a*e^2)*(a + c*x^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]] + Sqrt
[a]*((c*d^2 + a*e^2)*(a*e + c*d*x) + 2*a*e^3*(a + c*x^2)*Log[d + e*x] - a*e
^3*(a + c*x^2)*Log[a + c*x^2]))/(2*a^(3/2)*(c*d^2 + a*e^2)^2*(a + c*x^2))
```

Maple [A] time = 0.087, size = 244, normalized size = 1.7

$$\frac{cdxe^2}{2\left(ae^2+cd^2\right)^2\left(cx^2+a\right)} + \frac{c^2d^3x}{2\left(ae^2+cd^2\right)^2\left(cx^2+a\right)a} + \frac{ae^3}{2\left(ae^2+cd^2\right)^2\left(cx^2+a\right)} + \frac{ced^2}{2\left(ae^2+cd^2\right)^2\left(cx^2+a\right)} - \frac{e^3\ln\left(cx^2\right.}{2\left(ae^2+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+a)^2,x)
```

```
[Out] 1/2*c/(a*e^2+c*d^2)^2/(c*x^2+a)*d*x*e^2+1/2*c^2/(a*e^2+c*d^2)^2/(c*x^2+a)*d
^3/a*x+1/2/(a*e^2+c*d^2)^2/(c*x^2+a)*a*e^3+1/2*c/(a*e^2+c*d^2)^2/(c*x^2+a)*
e*d^2-1/2*e^3*ln(c*x^2+a)/(a*e^2+c*d^2)^2+3/2*c/(a*e^2+c*d^2)^2/(a*c)^(1/2)
*arctan(x*c/(a*c)^(1/2))*d*e^2+1/2*c^2/(a*e^2+c*d^2)^2/a/(a*c)^(1/2)*arctan
(x*c/(a*c)^(1/2))*d^3+e^3*ln(e*x+d)/(a*e^2+c*d^2)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.88093, size = 900, normalized size = 6.34

$$\left[\frac{2\,acd^2e+2\,a^2e^3+\left(acd^3+3\,a^2de^2+\left(c^2d^3+3\,acde^2\right)x^2\right)\sqrt{-\frac{c}{a}}\log\left(\frac{cx^2+2\,ax\sqrt{-\frac{c}{a}}-a}{cx^2+a}\right)+2\left(c^2d^3+acde^2\right)x-2\left(ace^3x^2+a\right.}{4\left(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4+\left(ac^3d^4+2\,a^2c^2d^2e^2+a^3ce^4\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*c*d^2*e + 2*a^2*e^3 + (a*c*d^3 + 3*a^2*d*e^2 + (c^2*d^3 + 3*a*c*d
*e^2)*x^2)*sqrt(-c/a)*log((c*x^2 + 2*a*x*sqrt(-c/a) - a)/(c*x^2 + a)) + 2*(
c^2*d^3 + a*c*d*e^2)*x - 2*(a*c*e^3*x^2 + a^2*e^3)*log(c*x^2 + a) + 4*(a*c*
e^3*x^2 + a^2*e^3)*log(e*x + d))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 +
 (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^2), 1/2*(a*c*d^2*e + a^2*e^3
 + (a*c*d^3 + 3*a^2*d*e^2 + (c^2*d^3 + 3*a*c*d*e^2)*x^2)*sqrt(c/a)*arctan(x
*sqrt(c/a)) + (c^2*d^3 + a*c*d*e^2)*x - (a*c*e^3*x^2 + a^2*e^3)*log(c*x^2 +
 a) + 2*(a*c*e^3*x^2 + a^2*e^3)*log(e*x + d))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^
2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^2)]
```

Sympy [B] time = 120.809, size = 3225, normalized size = 22.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+a)**2,x)
```

```
[Out] e**3*log(x + (-96*a**9*e**18/(a*e**2 + c*d**2)**4 - 336*a**8*c*d**2*e**16/(
a*e**2 + c*d**2)**4 - 368*a**7*c**2*d**4*e**14/(a*e**2 + c*d**2)**4 + 48*a*
*7*e**14/(a*e**2 + c*d**2)**2 - 32*a**6*c**3*d**6*e**12/(a*e**2 + c*d**2)**
4 + 180*a**6*c*d**2*e**12/(a*e**2 + c*d**2)**2 + 192*a**5*c**4*d**8*e**10/(
a*e**2 + c*d**2)**4 + 256*a**5*c**2*d**4*e**10/(a*e**2 + c*d**2)**2 + 48*a*
*5*e**10 + 112*a**4*c**5*d**10*e**8/(a*e**2 + c*d**2)**4 + 168*a**4*c**3*d*
*6*e**8/(a*e**2 + c*d**2)**2 - 24*a**4*c*d**2*e**8 + 16*a**3*c**6*d**12*e**
6/(a*e**2 + c*d**2)**4 + 48*a**3*c**4*d**8*e**6/(a*e**2 + c*d**2)**2 + 7*a*
*3*c**2*d**4*e**6 + 4*a**2*c**5*d**10*e**4/(a*e**2 + c*d**2)**2 + 23*a**2*c
**3*d**6*e**4 + 9*a*c**4*d**8*e**2 + c**5*d**10)/(108*a**4*c*d*e**9 + 63*a*
*3*c**2*d**3*e**7 + 27*a**2*c**3*d**5*e**5 + 9*a*c**4*d**7*e**3 + c**5*d**9
*e))/(a*e**2 + c*d**2)**2 + (a*e + c*d*x)/(2*a**3*e**2 + 2*a**2*c*d**2 + x*
*2*(2*a**2*c*e**2 + 2*a*c**2*d**2)) + (-e**3/(2*(a*e**2 + c*d**2)**2) - d*s
qrt(-a**3*c)*(3*a*e**2 + c*d**2)/(4*a**3*(a**2*e**4 + 2*a*c*d**2*e**2 + c**
```

```
2*d**4)))*log(x + (-96*a**9*e**12*(-e**3/(2*(a*e**2 + c*d**2)**2) - d*sqrt(
-a**3*c)*(3*a*e**2 + c*d**2)/(4*a**3*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d*
*4)))**2 - 336*a**8*c*d**2*e**10*(-e**3/(2*(a*e**2 + c*d**2)**2) - d*sqrt(-
a**3*c)*(3*a*e**2 + c*d**2)/(4*a**3*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**
4)))**2 - 368*a**7*c**2*d**4*e**8*(-e**3/(2*(a*e**2 + c*d**2)**2) - d*sqrt(
-a**3*c)*(3*a*e**2 + c*d**2)/(4*a**3*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d*
*4)))**2 + 48*a**7*e**11*(-e**3/(2*(a*e**2 + c*d**2)**2) - d*sqrt(-a**3*c)*
(3*a*e**2 + c*d**2)/(4*a**3*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4))) - 3
2*a**6*c**3*d**6*e**6*(-e**3/(2*(a*e**2 + c*d**2)**2) - d*sqrt(-a**3*c)*(3*
a*e**2 + c*d**2)/(4*a**3*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4)))**2 + 1
80*a**6*c*d**2*e**9*(-e**3/(2*(a*e**2 + c*d**2)**2) - d*sqrt(-a**3*c)*(3*a*
e**2 + c*d**2)/(4*a**3*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4))) + 192*a*
*5*c**4*d**8*e**4*(-e**3/(2*(a*e**2 + c*d**2)**2) - d*sqrt(-a**3*c)*(3*a*e*
*2 + c*d**2)/(4*a**3*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4)))**2 + 256*a
**5*c**2*d**4*e**7*(-e**3/(2*(a*e**2 + c*d**2)**2) - d*sqrt(-a**3*c)*(3*a*e
**2 + c*d**2)/(4*a**3*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4))) + 48*a**5
*e**10 + 112*a**4*c**5*d**10*e**2*(-e**3/(2*(a*e**2 + c*d**2)**2) - d*sqrt(
-a**3*c)*(3*a*e**2 + c*d**2)/(4*a**3*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d*
*4)))**2 + 168*a**4*c**3*d**6*e**5*(-e**3/(2*(a*e**2 + c*d**2)**2) - d*sqrt
(-a**3*c)*(3*a*e**2 + c*d**2)/(4*a**3*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d
**4))) - 24*a**4*c*d**2*e**8 + 16*a**3*c**6*d**12*(-e**3/(2*(a*e**2 + c*d**
2)**2) - d*sqrt(-a**3*c)*(3*a*e**2 + c*d**2)/(4*a**3*(a**2*e**4 + 2*a*c*d**
2*e**2 + c**2*d**4)))**2 + 48*a**3*c**4*d**8*e**3*(-e**3/(2*(a*e**2 + c*d**
2)**2) - d*sqrt(-a**3*c)*(3*a*e**2 + c*d**2)/(4*a**3*(a**2*e**4 + 2*a*c*d**
2*e**2 + c**2*d**4))) + 7*a**3*c**2*d**4*e**6 + 4*a**2*c**5*d**10*e*(-e**3/
(2*(a*e**2 + c*d**2)**2) - d*sqrt(-a**3*c)*(3*a*e**2 + c*d**2)/(4*a**3*(a**
2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4))) + 23*a**2*c**3*d**6*e**4 + 9*a*c**4
*d**8*e**2 + c**5*d**10)/(108*a**4*c*d*e**9 + 63*a**3*c**2*d**3*e**7 + 27*a
**2*c**3*d**5*e**5 + 9*a*c**4*d**7*e**3 + c**5*d**9*e)) + (-e**3/(2*(a*e**2
 + c*d**2)**2) + d*sqrt(-a**3*c)*(3*a*e**2 + c*d**2)/(4*a**3*(a**2*e**4 + 2
*a*c*d**2*e**2 + c**2*d**4)))*log(x + (-96*a**9*e**12*(-e**3/(2*(a*e**2 + c
*d**2)**2) + d*sqrt(-a**3*c)*(3*a*e**2 + c*d**2)/(4*a**3*(a**2*e**4 + 2*a*c
*d**2*e**2 + c**2*d**4)))**2 - 336*a**8*c*d**2*e**10*(-e**3/(2*(a*e**2 + c*
d**2)**2) + d*sqrt(-a**3*c)*(3*a*e**2 + c*d**2)/(4*a**3*(a**2*e**4 + 2*a*c*
d**2*e**2 + c**2*d**4)))**2 - 368*a**7*c**2*d**4*e**8*(-e**3/(2*(a*e**2 + c
*d**2)**2) + d*sqrt(-a**3*c)*(3*a*e**2 + c*d**2)/(4*a**3*(a**2*e**4 + 2*a*c
*d**2*e**2 + c**2*d**4)))**2 + 48*a**7*e**11*(-e**3/(2*(a*e**2 + c*d**2)**2
) + d*sqrt(-a**3*c)*(3*a*e**2 + c*d**2)/(4*a**3*(a**2*e**4 + 2*a*c*d**2*e**
2 + c**2*d**4))) - 32*a**6*c**3*d**6*e**6*(-e**3/(2*(a*e**2 + c*d**2)**2) +
 d*sqrt(-a**3*c)*(3*a*e**2 + c*d**2)/(4*a**3*(a**2*e**4 + 2*a*c*d**2*e**2 +
 c**2*d**4)))**2 + 180*a**6*c*d**2*e**9*(-e**3/(2*(a*e**2 + c*d**2)**2) + d
*sqrt(-a**3*c)*(3*a*e**2 + c*d**2)/(4*a**3*(a**2*e**4 + 2*a*c*d**2*e**2 + c
**2*d**4))) + 192*a**5*c**4*d**8*e**4*(-e**3/(2*(a*e**2 + c*d**2)**2) + d*s
qrt(-a**3*c)*(3*a*e**2 + c*d**2)/(4*a**3*(a**2*e**4 + 2*a*c*d**2*e**2 + c**
2*d**4)))**2 + 256*a**5*c**2*d**4*e**7*(-e**3/(2*(a*e**2 + c*d**2)**2) + d*
sqrt(-a**3*c)*(3*a*e**2 + c*d**2)/(4*a**3*(a**2*e**4 + 2*a*c*d**2*e**2 + c*
*2*d**4))) + 48*a**5*e**10 + 112*a**4*c**5*d**10*e**2*(-e**3/(2*(a*e**2 + c
*d**2)**2) + d*sqrt(-a**3*c)*(3*a*e**2 + c*d**2)/(4*a**3*(a**2*e**4 + 2*a*c
*d**2*e**2 + c**2*d**4)))**2 + 168*a**4*c**3*d**6*e**5*(-e**3/(2*(a*e**2 +
c*d**2)**2) + d*sqrt(-a**3*c)*(3*a*e**2 + c*d**2)/(4*a**3*(a**2*e**4 + 2*a*
c*d**2*e**2 + c**2*d**4))) - 24*a**4*c*d**2*e**8 + 16*a**3*c**6*d**12*(-e**
3/(2*(a*e**2 + c*d**2)**2) + d*sqrt(-a**3*c)*(3*a*e**2 + c*d**2)/(4*a**3*(a
**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4)))**2 + 48*a**3*c**4*d**8*e**3*(-e**
3/(2*(a*e**2 + c*d**2)**2) + d*sqrt(-a**3*c)*(3*a*e**2 + c*d**2)/(4*a**3*(a
**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4))) + 7*a**3*c**2*d**4*e**6 + 4*a**2*
c**5*d**10*e*(-e**3/(2*(a*e**2 + c*d**2)**2) + d*sqrt(-a**3*c)*(3*a*e**2 +
c*d**2)/(4*a**3*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4))) + 23*a**2*c**3*
d**6*e**4 + 9*a*c**4*d**8*e**2 + c**5*d**10)/(108*a**4*c*d*e**9 + 63*a**3*c
**2*d**3*e**7 + 27*a**2*c**3*d**5*e**5 + 9*a*c**4*d**7*e**3 + c**5*d**9*e))
```

Giac [A] time = 1.32524, size = 259, normalized size = 1.82

$$-\frac{e^3\log\left(cx^2+a\right)}{2\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}+\frac{e^4\log\left(|xe+d|\right)}{c^2d^4e+2\,acd^2e^3+a^2e^5}+\frac{\left(c^2d^3+3\,acde^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\left(ac^2d^4+2\,a^2cd^2e^2+a^3e^4\right)\sqrt{ac}}+\frac{acd^2e+a^2e^3+\left(c^2d^3+ac\right.}{2\left(cd^2+ae^2\right)^2\left(cx^2+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*e^3*log(c*x^2 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + e^4*log(abs(x
*e + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/2*(c^2*d^3 + 3*a*c*d*e^2
)*arctan(c*x/sqrt(a*c))/((a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(a*c))
 + 1/2*(a*c*d^2*e + a^2*e^3 + (c^2*d^3 + a*c*d*e^2)*x)/((c*d^2 + a*e^2)^2*(
c*x^2 + a)*a)
```

3.511 $\int \frac{1}{(d+ex)^2\left(a+cx^2\right)^2}\,dx$

Optimal. Leaf size=205

$$\frac{\sqrt{c}\left(-3a^2e^4+6acd^2e^2+c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\left(ae^2+cd^2\right)^3}+\frac{ae+cdx}{2a\left(a+cx^2\right)(d+ex)\left(ae^2+cd^2\right)}-\frac{2cde^3\log\left(a+cx^2\right)}{\left(ae^2+cd^2\right)^3}+\frac{e\left(cd^2-3ae^2\right.}{2a(d+ex)\left(ae^2+\right.}$$

```
[Out] (e*(c*d^2 - 3*a*e^2))/(2*a*(c*d^2 + a*e^2)^2*(d + e*x)) + (a*e + c*d*x)/(2*
a*(c*d^2 + a*e^2)*(d + e*x)*(a + c*x^2)) + (Sqrt[c]*(c^2*d^4 + 6*a*c*d^2*e^
2 - 3*a^2*e^4)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*(c*d^2 + a*e^2)^3) +
 (4*c*d*e^3*Log[d + e*x])/(c*d^2 + a*e^2)^3 - (2*c*d*e^3*Log[a + c*x^2])/(c
*d^2 + a*e^2)^3
```

Rubi [A] time = 0.189461, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {741, 801, 635, 205, 260}

$$\frac{\sqrt{c}\left(-3a^2e^4+6acd^2e^2+c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\left(ae^2+cd^2\right)^3}+\frac{ae+cdx}{2a\left(a+cx^2\right)(d+ex)\left(ae^2+cd^2\right)}-\frac{2cde^3\log\left(a+cx^2\right)}{\left(ae^2+cd^2\right)^3}+\frac{e\left(cd^2-3ae^2\right.}{2a(d+ex)\left(ae^2+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + c*x^2)^2),x]
```

```
[Out] (e*(c*d^2 - 3*a*e^2))/(2*a*(c*d^2 + a*e^2)^2*(d + e*x)) + (a*e + c*d*x)/(2*
a*(c*d^2 + a*e^2)*(d + e*x)*(a + c*x^2)) + (Sqrt[c]*(c^2*d^4 + 6*a*c*d^2*e^
2 - 3*a^2*e^4)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*(c*d^2 + a*e^2)^3) +
 (4*c*d*e^3*Log[d + e*x])/(c*d^2 + a*e^2)^3 - (2*c*d*e^3*Log[a + c*x^2])/(c
*d^2 + a*e^2)^3
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+cx^2\right)^2}\,dx = \frac{ae+cdx}{2a\left(cd^2+ae^2\right)(d+ex)\left(a+cx^2\right)} - \frac{\int \frac{-cd^2-3ae^2-2cdex}{(d+ex)^2\left(a+cx^2\right)}\,dx}{2a\left(cd^2+ae^2\right)}$$

$$= \frac{ae+cdx}{2a\left(cd^2+ae^2\right)(d+ex)\left(a+cx^2\right)} - \frac{\int \left(\frac{cd^2e^2-3ae^4}{\left(cd^2+ae^2\right)(d+ex)^2} - \frac{8acde^4}{\left(cd^2+ae^2\right)^2(d+ex)} - \frac{c\left(c^2d^4+6acd^2e^2-3a^2e^4\right.}{\left(cd^2+ae^2\right)^2\left(a+c\right.}\right.}{2a\left(cd^2+ae^2\right)}$$

$$= \frac{e\left(cd^2-3ae^2\right)}{2a\left(cd^2+ae^2\right)^2(d+ex)} + \frac{ae+cdx}{2a\left(cd^2+ae^2\right)(d+ex)\left(a+cx^2\right)} + \frac{4cde^3\log(d+ex)}{\left(cd^2+ae^2\right)^3} + \frac{c\int \frac{c^2d}{}}{}$$

$$= \frac{e\left(cd^2-3ae^2\right)}{2a\left(cd^2+ae^2\right)^2(d+ex)} + \frac{ae+cdx}{2a\left(cd^2+ae^2\right)(d+ex)\left(a+cx^2\right)} + \frac{4cde^3\log(d+ex)}{\left(cd^2+ae^2\right)^3} - \frac{\left(4c^2de\right.}{\left(cd\right.}$$

$$= \frac{e\left(cd^2-3ae^2\right)}{2a\left(cd^2+ae^2\right)^2(d+ex)} + \frac{ae+cdx}{2a\left(cd^2+ae^2\right)(d+ex)\left(a+cx^2\right)} + \frac{\sqrt{c}\left(c^2d^4+6acd^2e^2-3a^2e^4\right.}{2a^{3/2}\left(cd^2+ae^2\right.}$$

Mathematica [A] time = 0.213506, size = 162, normalized size = 0.79

$$\frac{\frac{\sqrt{c}\left(-3a^2e^4+6acd^2e^2+c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{a^{3/2}} + \frac{c\left(ae^2+cd^2\right)\left(ae(2d-ex)+cd^2x\right)}{a\left(a+cx^2\right)} - \frac{2e^3\left(ae^2+cd^2\right)}{d+ex} - 4cde^3\log\left(a+cx^2\right) + 8cde^3\log(d+ex)}{2\left(ae^2+cd^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + c*x^2)^2),x]
```

```
[Out] ((-2*e^3*(c*d^2 + a*e^2))/(d + e*x) + (c*(c*d^2 + a*e^2)*(c*d^2*x + a*e*(2*
d - e*x)))/(a*(a + c*x^2)) + (Sqrt[c]*(c^2*d^4 + 6*a*c*d^2*e^2 - 3*a^2*e^4)
*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(3/2) + 8*c*d*e^3*Log[d + e*x] - 4*c*d*e^3*
Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^3)
```

Maple [A] time = 0.064, size = 314, normalized size = 1.5

$$-\frac{acxe^4}{2\left(ae^2+cd^2\right)^3\left(cx^2+a\right)} + \frac{c^3xd^4}{2\left(ae^2+cd^2\right)^3\left(cx^2+a\right)a} + \frac{acde^3}{\left(ae^2+cd^2\right)^3\left(cx^2+a\right)} + \frac{c^2d^3e}{\left(ae^2+cd^2\right)^3\left(cx^2+a\right)} - 2\frac{de^3c\ln\left(\right.}{\left(ae^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^2+a)^2,x)
```

```
[Out] -1/2*c/(a*e^2+c*d^2)^3/(c*x^2+a)*x*a*e^4+1/2*c^3/(a*e^2+c*d^2)^3/(c*x^2+a)*
x/a*d^4+c/(a*e^2+c*d^2)^3/(c*x^2+a)*a*d*e^3+c^2/(a*e^2+c*d^2)^3/(c*x^2+a)*d
^3*e-2*c*d*e^3*ln(c*x^2+a)/(a*e^2+c*d^2)^3-3/2*c/(a*e^2+c*d^2)^3*a/(a*c)^(1
/2)*arctan(x*c/(a*c)^(1/2))*e^4+3*c^2/(a*e^2+c*d^2)^3/(a*c)^(1/2)*arctan(x*
c/(a*c)^(1/2))*d^2*e^2+1/2*c^3/(a*e^2+c*d^2)^3/a/(a*c)^(1/2)*arctan(x*c/(a*
c)^(1/2))*d^4-e^3/(a*e^2+c*d^2)^2/(e*x+d)+4*c*d*e^3*ln(e*x+d)/(a*e^2+c*d^2)
^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 16.3465, size = 2225, normalized size = 10.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*c^2*d^4*e - 4*a^3*e^5 + 2*(c^3*d^4*e - 2*a*c^2*d^2*e^3 - 3*a^2*c*
e^5)*x^2 - (a*c^2*d^5 + 6*a^2*c*d^3*e^2 - 3*a^3*d*e^4 + (c^3*d^4*e + 6*a*c^
2*d^2*e^3 - 3*a^2*c*e^5)*x^3 + (c^3*d^5 + 6*a*c^2*d^3*e^2 - 3*a^2*c*d*e^4)*
x^2 + (a*c^2*d^4*e + 6*a^2*c*d^2*e^3 - 3*a^3*e^5)*x)*sqrt(-c/a)*log((c*x^2
- 2*a*x*sqrt(-c/a) - a)/(c*x^2 + a)) + 2*(c^3*d^5 + 2*a*c^2*d^3*e^2 + a^2*c
*d*e^4)*x - 8*(a*c^2*d*e^4*x^3 + a*c^2*d^2*e^3*x^2 + a^2*c*d*e^4*x + a^2*c*
d^2*e^3)*log(c*x^2 + a) + 16*(a*c^2*d*e^4*x^3 + a*c^2*d^2*e^3*x^2 + a^2*c*d
*e^4*x + a^2*c*d^2*e^3)*log(e*x + d))/(a^2*c^3*d^7 + 3*a^3*c^2*d^5*e^2 + 3*
a^4*c*d^3*e^4 + a^5*d*e^6 + (a*c^4*d^6*e + 3*a^2*c^3*d^4*e^3 + 3*a^3*c^2*d^
2*e^5 + a^4*c*e^7)*x^3 + (a*c^4*d^7 + 3*a^2*c^3*d^5*e^2 + 3*a^3*c^2*d^3*e^4
 + a^4*c*d*e^6)*x^2 + (a^2*c^3*d^6*e + 3*a^3*c^2*d^4*e^3 + 3*a^4*c*d^2*e^5
+ a^5*e^7)*x), 1/2*(2*a*c^2*d^4*e - 2*a^3*e^5 + (c^3*d^4*e - 2*a*c^2*d^2*e^
3 - 3*a^2*c*e^5)*x^2 + (a*c^2*d^5 + 6*a^2*c*d^3*e^2 - 3*a^3*d*e^4 + (c^3*d^
4*e + 6*a*c^2*d^2*e^3 - 3*a^2*c*e^5)*x^3 + (c^3*d^5 + 6*a*c^2*d^3*e^2 - 3*a
^2*c*d*e^4)*x^2 + (a*c^2*d^4*e + 6*a^2*c*d^2*e^3 - 3*a^3*e^5)*x)*sqrt(c/a)*
arctan(x*sqrt(c/a)) + (c^3*d^5 + 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*x - 4*(a*c^
2*d*e^4*x^3 + a*c^2*d^2*e^3*x^2 + a^2*c*d*e^4*x + a^2*c*d^2*e^3)*log(c*x^2
+ a) + 8*(a*c^2*d*e^4*x^3 + a*c^2*d^2*e^3*x^2 + a^2*c*d*e^4*x + a^2*c*d^2*e
^3)*log(e*x + d))/(a^2*c^3*d^7 + 3*a^3*c^2*d^5*e^2 + 3*a^4*c*d^3*e^4 + a^5*
d*e^6 + (a*c^4*d^6*e + 3*a^2*c^3*d^4*e^3 + 3*a^3*c^2*d^2*e^5 + a^4*c*e^7)*x
^3 + (a*c^4*d^7 + 3*a^2*c^3*d^5*e^2 + 3*a^3*c^2*d^3*e^4 + a^4*c*d*e^6)*x^2
+ (a^2*c^3*d^6*e + 3*a^3*c^2*d^4*e^3 + 3*a^4*c*d^2*e^5 + a^5*e^7)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25612, size = 521, normalized size = 2.54

$$-\frac{2\,cde^3\log\left(c-\frac{2\,cd}{xe+d}+\frac{cd^2}{(xe+d)^2}+\frac{ae^2}{(xe+d)^2}\right)}{c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6}+\frac{\left(c^3d^4e^2+6\,ac^2d^2e^4-3\,a^2ce^6\right)\arctan\left(\frac{\left(cd-\frac{cd^2}{xe+d}-\frac{ae^2}{xe+d}\right)e^{(-1)}}{\sqrt{ac}}\right)e^{(-2)}}{2\left(ac^3d^6+3\,a^2c^2d^4e^2+3\,a^3cd^2e^4+a^4e^6\right)\sqrt{ac}}-\frac{}{\left(c^2d^4e^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -2*c*d*e^3*log(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(x*e + d)^2)
/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) + 1/2*(c^3*d^4*e^2
 + 6*a*c^2*d^2*e^4 - 3*a^2*c*e^6)*arctan((c*d - c*d^2/(x*e + d) - a*e^2/(x*
e + d))*e^(-1)/sqrt(a*c))*e^(-2)/((a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*
d^2*e^4 + a^4*e^6)*sqrt(a*c)) - e^7/((c^2*d^4*e^4 + 2*a*c*d^2*e^6 + a^2*e^8
)*(x*e + d)) + 1/2*((c^3*d^3*e - 3*a*c^2*d*e^3)/(c*d^2 + a*e^2) - (c^3*d^4*
e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)*e^(-1)/((c*d^2 + a*e^2)*(x*e + d)))/((c*
d^2 + a*e^2)^2*a*(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(x*e + d)
^2))
```

3.512 $\int \frac{(d+ex)^5}{(a+cx^2)^3} \, dx$

Optimal. Leaf size=198

$$-\frac{(d+ex)^2\left(2ae\left(2ae^2+cd^2\right)-cdx\left(5ae^2+3cd^2\right)\right)}{8a^2c^2\left(a+cx^2\right)}-\frac{de^2x\left(7ae^2+3cd^2\right)}{8a^2c^2}+\frac{d\left(15a^2e^4+10acd^2e^2+3c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{5/2}}+$$

```
[Out] -(d*e^2*(3*c*d^2 + 7*a*e^2)*x)/(8*a^2*c^2) - ((a*e - c*d*x)*(d + e*x)^4)/(4
*a*c*(a + c*x^2)^2) - ((d + e*x)^2*(2*a*e*(c*d^2 + 2*a*e^2) - c*d*(3*c*d^2
+ 5*a*e^2)*x))/(8*a^2*c^2*(a + c*x^2)) + (d*(3*c^2*d^4 + 10*a*c*d^2*e^2 + 1
5*a^2*e^4)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(5/2)) + (e^5*Log[a +
c*x^2])/(2*c^3)
```

Rubi [A] time = 0.185442, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {739, 819, 774, 635, 205, 260}

$$-\frac{(d+ex)^2\left(2ae\left(2ae^2+cd^2\right)-cdx\left(5ae^2+3cd^2\right)\right)}{8a^2c^2\left(a+cx^2\right)}-\frac{de^2x\left(7ae^2+3cd^2\right)}{8a^2c^2}+\frac{d\left(15a^2e^4+10acd^2e^2+3c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{5/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5/(a + c*x^2)^3,x]
```

```
[Out] -(d*e^2*(3*c*d^2 + 7*a*e^2)*x)/(8*a^2*c^2) - ((a*e - c*d*x)*(d + e*x)^4)/(4
*a*c*(a + c*x^2)^2) - ((d + e*x)^2*(2*a*e*(c*d^2 + 2*a*e^2) - c*d*(3*c*d^2
+ 5*a*e^2)*x))/(8*a^2*c^2*(a + c*x^2)) + (d*(3*c^2*d^4 + 10*a*c*d^2*e^2 + 1
5*a^2*e^4)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(5/2)) + (e^5*Log[a +
c*x^2])/(2*c^3)
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
```

```
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{(d+ex)^5}{\left(a+cx^2\right)^3}\,dx = -\frac{(ae-cdx)(d+ex)^4}{4ac\left(a+cx^2\right)^2} + \frac{\int \frac{(d+ex)^3\left(3cd^2+4ae^2-cdex\right)}{\left(a+cx^2\right)^2}\,dx}{4ac}$$

$$= -\frac{(ae-cdx)(d+ex)^4}{4ac\left(a+cx^2\right)^2} - \frac{(d+ex)^2\left(2ae\left(cd^2+2ae^2\right)-cd\left(3cd^2+5ae^2\right)x\right)}{8a^2c^2\left(a+cx^2\right)} + \frac{\int \frac{(d+ex)\left(3c^2d^4+7acd^2e^2+8a^2 \ldots\right.}{a+cx^2 \ldots}}{8a^2c \ldots}$$

$$= -\frac{de^2\left(3cd^2+7ae^2\right)x}{8a^2c^2} - \frac{(ae-cdx)(d+ex)^4}{4ac\left(a+cx^2\right)^2} - \frac{(d+ex)^2\left(2ae\left(cd^2+2ae^2\right)-cd\left(3cd^2+5ae^2\right)x\right)}{8a^2c^2\left(a+cx^2\right)} + \ldots$$

$$= -\frac{de^2\left(3cd^2+7ae^2\right)x}{8a^2c^2} - \frac{(ae-cdx)(d+ex)^4}{4ac\left(a+cx^2\right)^2} - \frac{(d+ex)^2\left(2ae\left(cd^2+2ae^2\right)-cd\left(3cd^2+5ae^2\right)x\right)}{8a^2c^2\left(a+cx^2\right)} + \ldots$$

$$= -\frac{de^2\left(3cd^2+7ae^2\right)x}{8a^2c^2} - \frac{(ae-cdx)(d+ex)^4}{4ac\left(a+cx^2\right)^2} - \frac{(d+ex)^2\left(2ae\left(cd^2+2ae^2\right)-cd\left(3cd^2+5ae^2\right)x\right)}{8a^2c^2\left(a+cx^2\right)} + \ldots$$

Mathematica [A] time = 0.152277, size = 199, normalized size = 1.01

$$\frac{-\frac{2\left(-5a^2cde^3(2d+ex)+a^3e^5+5ac^2d^3e(d+2ex)-c^3d^5x\right)}{a\left(a+cx^2\right)^2} + \frac{-5a^2cde^3(8d+5ex)+8a^3e^5+10ac^2d^3e^2x+3c^3d^5x}{a^2\left(a+cx^2\right)} + \frac{\sqrt{c}d\left(15a^2e^4+10acd^2e^2+3c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{a^{5/2}} + 4 \ldots}{8c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5/(a + c*x^2)^3,x]
```

```
[Out] ((-2*(a^3*e^5 - c^3*d^5*x - 5*a^2*c*d*e^3*(2*d + e*x) + 5*a*c^2*d^3*e*(d +
2*e*x)))/(a*(a + c*x^2)^2) + (8*a^3*e^5 + 3*c^3*d^5*x + 10*a*c^2*d^3*e^2*x
- 5*a^2*c*d*e^3*(8*d + 5*e*x))/(a^2*(a + c*x^2)) + (Sqrt[c]*d*(3*c^2*d^4 +
10*a*c*d^2*e^2 + 15*a^2*e^4)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(5/2) + 4*e^5*L
og[a + c*x^2])/(8*c^3)
```

Maple [A] time = 0.059, size = 233, normalized size = 1.2

$$\frac{1}{\left(cx^2+a\right)^2}\left(-\frac{d\left(25\,a^2e^4-10\,acd^2e^2-3\,c^2d^4\right)x^3}{8\,a^2c}+\frac{e^3\left(ae^2-5\,cd^2\right)x^2}{c^2}-\frac{5\,d\left(3\,a^2e^4+2\,acd^2e^2-c^2d^4\right)x}{8\,ac^2}+\frac{e\left(3\,a^2e^4-10\,a\right.}{4\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5/(c*x^2+a)^3,x)
```

```
[Out] (-1/8*d*(25*a^2*e^4-10*a*c*d^2*e^2-3*c^2*d^4)/a^2/c*x^3+e^3*(a*e^2-5*c*d^2)
/c^2*x^2-5/8*d*(3*a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/a/c^2*x+1/4*e*(3*a^2*e^4-1
0*a*c*d^2*e^2-5*c^2*d^4)/c^3)/(c*x^2+a)^2+1/2*e^5*ln(c*x^2+a)/c^3+15/8/c^2/
(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*d*e^4+5/4/a/c/(a*c)^(1/2)*arctan(x*c/(a
*c)^(1/2))*d^3*e^2+3/8/a^2/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*d^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.09844, size = 1443, normalized size = 7.29

$$\left[-\frac{20\,a^3c^2d^4e+40\,a^4cd^2e^3-12\,a^5e^5-2\left(3\,ac^4d^5+10\,a^2c^3d^3e^2-25\,a^3c^2de^4\right)x^3+16\left(5\,a^3c^2d^2e^3-a^4ce^5\right)x^2+\left(3\,a^2c^2d^5+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(20*a^3*c^2*d^4*e + 40*a^4*c*d^2*e^3 - 12*a^5*e^5 - 2*(3*a*c^4*d^5 +
 10*a^2*c^3*d^3*e^2 - 25*a^3*c^2*d*e^4)*x^3 + 16*(5*a^3*c^2*d^2*e^3 - a^4*c
*e^5)*x^2 + (3*a^2*c^2*d^5 + 10*a^3*c*d^3*e^2 + 15*a^4*d*e^4 + (3*c^4*d^5 +
 10*a*c^3*d^3*e^2 + 15*a^2*c^2*d*e^4)*x^4 + 2*(3*a*c^3*d^5 + 10*a^2*c^2*d^3
*e^2 + 15*a^3*c*d*e^4)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*
x^2 + a)) - 10*(a^2*c^3*d^5 - 2*a^3*c^2*d^3*e^2 - 3*a^4*c*d*e^4)*x - 8*(a^3
*c^2*e^5*x^4 + 2*a^4*c*e^5*x^2 + a^5*e^5)*log(c*x^2 + a))/(a^3*c^5*x^4 + 2*
a^4*c^4*x^2 + a^5*c^3), -1/8*(10*a^3*c^2*d^4*e + 20*a^4*c*d^2*e^3 - 6*a^5*e
^5 - (3*a*c^4*d^5 + 10*a^2*c^3*d^3*e^2 - 25*a^3*c^2*d*e^4)*x^3 + 8*(5*a^3*c
^2*d^2*e^3 - a^4*c*e^5)*x^2 - (3*a^2*c^2*d^5 + 10*a^3*c*d^3*e^2 + 15*a^4*d*
e^4 + (3*c^4*d^5 + 10*a*c^3*d^3*e^2 + 15*a^2*c^2*d*e^4)*x^4 + 2*(3*a*c^3*d^
5 + 10*a^2*c^2*d^3*e^2 + 15*a^3*c*d*e^4)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/
a) - 5*(a^2*c^3*d^5 - 2*a^3*c^2*d^3*e^2 - 3*a^4*c*d*e^4)*x - 4*(a^3*c^2*e^5
*x^4 + 2*a^4*c*e^5*x^2 + a^5*e^5)*log(c*x^2 + a))/(a^3*c^5*x^4 + 2*a^4*c^4*
x^2 + a^5*c^3)]
```

Sympy [B] time = 4.43216, size = 520, normalized size = 2.63

$$\left(\frac{e^5}{2c^3} - \frac{d\sqrt{-a^5c^7}\left(15a^2e^4 + 10acd^2e^2 + 3c^2d^4\right)}{16a^5c^6}\right)\log\left(x + \frac{16a^3c^3\left(\frac{e^5}{2c^3} - \frac{d\sqrt{-a^5c^7}(15a^2e^4+10acd^2e^2+3c^2d^4)}{16a^5c^6}\right) - 8a^3e^5}{15a^2cde^4 + 10ac^2d^3e^2 + 3c^3d^5}\right) + \left(\frac{e^5}{2c^3} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5/(c*x**2+a)**3,x)
```

```
[Out] (e**5/(2*c**3) - d*sqrt(-a**5*c**7)*(15*a**2*e**4 + 10*a*c*d**2*e**2 + 3*c*
*2*d**4)/(16*a**5*c**6))*log(x + (16*a**3*c**3*(e**5/(2*c**3) - d*sqrt(-a**
5*c**7)*(15*a**2*e**4 + 10*a*c*d**2*e**2 + 3*c**2*d**4)/(16*a**5*c**6)) - 8
*a**3*e**5)/(15*a**2*c*d*e**4 + 10*a*c**2*d**3*e**2 + 3*c**3*d**5)) + (e**5
/(2*c**3) + d*sqrt(-a**5*c**7)*(15*a**2*e**4 + 10*a*c*d**2*e**2 + 3*c**2*d*
*4)/(16*a**5*c**6))*log(x + (16*a**3*c**3*(e**5/(2*c**3) + d*sqrt(-a**5*c**
7)*(15*a**2*e**4 + 10*a*c*d**2*e**2 + 3*c**2*d**4)/(16*a**5*c**6)) - 8*a**3
*e**5)/(15*a**2*c*d*e**4 + 10*a*c**2*d**3*e**2 + 3*c**3*d**5)) - (-6*a**4*e
**5 + 20*a**3*c*d**2*e**3 + 10*a**2*c**2*d**4*e + x**3*(25*a**2*c**2*d*e**4
 - 10*a*c**3*d**3*e**2 - 3*c**4*d**5) + x**2*(-8*a**3*c*e**5 + 40*a**2*c**2
*d**2*e**3) + x*(15*a**3*c*d*e**4 + 10*a**2*c**2*d**3*e**2 - 5*a*c**3*d**5)
)/(8*a**4*c**3 + 16*a**3*c**4*x**2 + 8*a**2*c**5*x**4)
```

Giac [A] time = 1.26101, size = 279, normalized size = 1.41

$$\frac{e^5\log\left(cx^2 + a\right)}{2\,c^3} + \frac{\left(3\,c^2d^5 + 10\,acd^3e^2 + 15\,a^2de^4\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c^2} + \frac{\left(3\,c^3d^5 + 10\,ac^2d^3e^2 - 25\,a^2cde^4\right)x^3 - 8\left(5\,a^2cd^2e^3 - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*e^5*log(c*x^2 + a)/c^3 + 1/8*(3*c^2*d^5 + 10*a*c*d^3*e^2 + 15*a^2*d*e^4
)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c^2) + 1/8*((3*c^3*d^5 + 10*a*c^2*d^
3*e^2 - 25*a^2*c*d*e^4)*x^3 - 8*(5*a^2*c*d^2*e^3 - a^3*e^5)*x^2 + 5*(a*c^2*
d^5 - 2*a^2*c*d^3*e^2 - 3*a^3*d*e^4)*x - 2*(5*a^2*c^2*d^4*e + 10*a^3*c*d^2*
e^3 - 3*a^4*e^5)/c)/((c*x^2 + a)^2*a^2*c^2)
```

3.513 $\int \frac{(d+ex)^4}{\left(a+cx^2\right)^3}\,dx$

Optimal. Leaf size=120

$$-\frac{3(d+ex)\left(ae^2+cd^2\right)(ae-cdx)}{8a^2c^2\left(a+cx^2\right)}+\frac{3\left(ae^2+cd^2\right)^2\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{5/2}}-\frac{(d+ex)^3(ae-cdx)}{4ac\left(a+cx^2\right)^2}$$

```
[Out] -((a*e - c*d*x)*(d + e*x)^3)/(4*a*c*(a + c*x^2)^2) - (3*(c*d^2 + a*e^2)*(a*
e - c*d*x)*(d + e*x))/(8*a^2*c^2*(a + c*x^2)) + (3*(c*d^2 + a*e^2)^2*ArcTan
[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(5/2))
```

Rubi [A] time = 0.0479793, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {723, 205}

$$-\frac{3(d+ex)\left(ae^2+cd^2\right)(ae-cdx)}{8a^2c^2\left(a+cx^2\right)}+\frac{3\left(ae^2+cd^2\right)^2\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{5/2}}-\frac{(d+ex)^3(ae-cdx)}{4ac\left(a+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a + c*x^2)^3,x]
```

```
[Out] -((a*e - c*d*x)*(d + e*x)^3)/(4*a*c*(a + c*x^2)^2) - (3*(c*d^2 + a*e^2)*(a*
e - c*d*x)*(d + e*x))/(8*a^2*c^2*(a + c*x^2)) + (3*(c*d^2 + a*e^2)^2*ArcTan
[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(5/2))
```

Rule 723

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[((2*p + 3)*(c*d^2 + a*e^2))/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^4}{\left(a+cx^2\right)^3}\,dx &= -\frac{(ae-cdx)(d+ex)^3}{4ac\left(a+cx^2\right)^2}+\frac{\left(3\left(cd^2+ae^2\right)\right)\int \frac{(d+ex)^2}{\left(a+cx^2\right)^2}\,dx}{4ac}\\
&= -\frac{(ae-cdx)(d+ex)^3}{4ac\left(a+cx^2\right)^2}-\frac{3\left(cd^2+ae^2\right)(ae-cdx)(d+ex)}{8a^2c^2\left(a+cx^2\right)}+\frac{\left(3\left(cd^2+ae^2\right)^2\right)\int \frac{1}{a+cx^2}\,dx}{8a^2c^2}\\
&= -\frac{(ae-cdx)(d+ex)^3}{4ac\left(a+cx^2\right)^2}-\frac{3\left(cd^2+ae^2\right)(ae-cdx)(d+ex)}{8a^2c^2\left(a+cx^2\right)}+\frac{3\left(cd^2+ae^2\right)^2\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.101979, size = 148, normalized size = 1.23

$$\frac{-a^2ce\left(6d^2ex+8d^3+16de^2x^2+5e^3x^3\right)-a^3e^3(8d+3ex)+ac^2d^2x\left(5d^2+6e^2x^2\right)+3c^3d^4x^3}{8a^2c^2\left(a+cx^2\right)^2}+\frac{3\left(ae^2+cd^2\right)^2\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a + c*x^2)^3,x]
```

```
[Out] (3*c^3*d^4*x^3 - a^3*e^3*(8*d + 3*e*x) + a*c^2*d^2*x*(5*d^2 + 6*e^2*x^2) -
a^2*c*e*(8*d^3 + 6*d^2*e*x + 16*d*e^2*x^2 + 5*e^3*x^3))/(8*a^2*c^2*(a + c*x
^2)^2) + (3*(c*d^2 + a*e^2)^2*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(5/
2))
```

Maple [A] time = 0.051, size = 189, normalized size = 1.6

$$\frac{1}{\left(cx^2+a\right)^2}\left(-\frac{\left(5\,a^2e^4-6\,acd^2e^2-3\,c^2d^4\right)x^3}{8\,a^2c}-2\,\frac{de^3x^2}{c}-\frac{\left(3\,a^2e^4+6\,acd^2e^2-5\,c^2d^4\right)x}{8\,ac^2}-\frac{de\left(ae^2+cd^2\right)}{c^2}\right)+\frac{3\,e^4}{8\,c^2}\text{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(c*x^2+a)^3,x)
```

```
[Out] (-1/8*(5*a^2*e^4-6*a*c*d^2*e^2-3*c^2*d^4)/a^2/c*x^3-2*d*e^3*x^2/c-1/8*(3*a^
2*e^4+6*a*c*d^2*e^2-5*c^2*d^4)/a/c^2*x-d*e*(a*e^2+c*d^2)/c^2)/(c*x^2+a)^2+3
/8/c^2/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*e^4+3/4/a/c/(a*c)^(1/2)*arctan(x
*c/(a*c)^(1/2))*d^2*e^2+3/8/a^2/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*d^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.90785, size = 1104, normalized size = 9.2

$$\left[-\frac{32\,a^3c^2de^3x^2+16\,a^3c^2d^3e+16\,a^4cde^3-2\left(3\,ac^4d^4+6\,a^2c^3d^2e^2-5\,a^3c^2e^4\right)x^3+3\left(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4+\left(c^4d^4\right.\right.}{16\left(a^3c^5x^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(32*a^3*c^2*d*e^3*x^2 + 16*a^3*c^2*d^3*e + 16*a^4*c*d*e^3 - 2*(3*a*c
^4*d^4 + 6*a^2*c^3*d^2*e^2 - 5*a^3*c^2*e^4)*x^3 + 3*(a^2*c^2*d^4 + 2*a^3*c*
```

```
d^2*e^2 + a^4*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4 + 2*(a*c^
3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(
-a*c)*x - a)/(c*x^2 + a)) - 2*(5*a^2*c^3*d^4 - 6*a^3*c^2*d^2*e^2 - 3*a^4*c*
e^4)*x)/(a^3*c^5*x^4 + 2*a^4*c^4*x^2 + a^5*c^3), -1/8*(16*a^3*c^2*d*e^3*x^2
 + 8*a^3*c^2*d^3*e + 8*a^4*c*d*e^3 - (3*a*c^4*d^4 + 6*a^2*c^3*d^2*e^2 - 5*a
^3*c^2*e^4)*x^3 - 3*(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (c^4*d^4 + 2
*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4 + 2*(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*
c*e^4)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) - (5*a^2*c^3*d^4 - 6*a^3*c^2*d^
2*e^2 - 3*a^4*c*e^4)*x)/(a^3*c^5*x^4 + 2*a^4*c^4*x^2 + a^5*c^3)]
```

Sympy [B] time = 2.63037, size = 328, normalized size = 2.73

$$-\frac{3\sqrt{-\frac{1}{a^5c^5}}\left(ae^2+cd^2\right)^2\log\left(-\frac{3a^3c^2\sqrt{-\frac{1}{a^5c^5}}\left(ae^2+cd^2\right)^2}{3a^2e^4+6acd^2e^2+3c^2d^4}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{a^5c^5}}\left(ae^2+cd^2\right)^2\log\left(\frac{3a^3c^2\sqrt{-\frac{1}{a^5c^5}}\left(ae^2+cd^2\right)^2}{3a^2e^4+6acd^2e^2+3c^2d^4}+x\right)}{16}-\frac{8a^3de^3+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(c*x**2+a)**3,x)
```

```
[Out] -3*sqrt(-1/(a**5*c**5))*(a*e**2 + c*d**2)**2*log(-3*a**3*c**2*sqrt(-1/(a**5
*c**5))*(a*e**2 + c*d**2)**2/(3*a**2*e**4 + 6*a*c*d**2*e**2 + 3*c**2*d**4)
+ x)/16 + 3*sqrt(-1/(a**5*c**5))*(a*e**2 + c*d**2)**2*log(3*a**3*c**2*sqrt(
-1/(a**5*c**5))*(a*e**2 + c*d**2)**2/(3*a**2*e**4 + 6*a*c*d**2*e**2 + 3*c**
2*d**4) + x)/16 - (8*a**3*d*e**3 + 8*a**2*c*d**3*e + 16*a**2*c*d*e**3*x**2
+ x**3*(5*a**2*c*e**4 - 6*a*c**2*d**2*e**2 - 3*c**3*d**4) + x*(3*a**3*e**4
+ 6*a**2*c*d**2*e**2 - 5*a*c**2*d**4))/(8*a**4*c**2 + 16*a**3*c**3*x**2 + 8
*a**2*c**4*x**4)
```

Giac [A] time = 1.33361, size = 217, normalized size = 1.81

$$\frac{3\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}a^2c^2}+\frac{3\,c^3d^4x^3+6\,ac^2d^2x^3e^2+5\,ac^2d^4x-5\,a^2cx^3e^4-16\,a^2cdx^2e^3-6\,a^2cd^2xe^2-8\,a}{8\left(cx^2+a\right)^2a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/8*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^
2*c^2) + 1/8*(3*c^3*d^4*x^3 + 6*a*c^2*d^2*x^3*e^2 + 5*a*c^2*d^4*x - 5*a^2*c
*x^3*e^4 - 16*a^2*c*d*x^2*e^3 - 6*a^2*c*d^2*x*e^2 - 8*a^2*c*d^3*e - 3*a^3*x
*e^4 - 8*a^3*d*e^3)/((c*x^2 + a)^2*a^2*c^2)
```

3.514 $\int \frac{(d+ex)^3}{\left(a+cx^2\right)^3} \, dx$

Optimal. Leaf size=98

$$\frac{3d\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}-\frac{3d(d+ex)(ae-cdx)}{8a^2c\left(a+cx^2\right)}+\frac{x(d+ex)^3}{4a\left(a+cx^2\right)^2}$$

```
[Out] (x*(d + e*x)^3)/(4*a*(a + c*x^2)^2) - (3*d*(a*e - c*d*x)*(d + e*x))/(8*a^2*
c*(a + c*x^2)) + (3*d*(c*d^2 + a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/
2)*c^(3/2))
```

Rubi [A] time = 0.0380723, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {729, 723, 205}

$$\frac{3d\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}-\frac{3d(d+ex)(ae-cdx)}{8a^2c\left(a+cx^2\right)}+\frac{x(d+ex)^3}{4a\left(a+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + c*x^2)^3,x]
```

```
[Out] (x*(d + e*x)^3)/(4*a*(a + c*x^2)^2) - (3*d*(a*e - c*d*x)*(d + e*x))/(8*a^2*
c*(a + c*x^2)) + (3*d*(c*d^2 + a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/
2)*c^(3/2))
```

Rule 729

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^m*(2*c*x)*(a + c*x^2)^(p + 1))/(4*a*c*(p + 1)), x] - Dist[(m*(2
*c*d))/(4*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /
; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3,
 0] && LtQ[p, -1]
```

Rule 723

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[((2*p + 3)*(c*d^2 + a*e^2))/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(a+cx^2\right)^3}\,dx = \frac{x(d+ex)^3}{4a\left(a+cx^2\right)^2} + \frac{(3d)\int \frac{(d+ex)^2}{\left(a+cx^2\right)^2}\,dx}{4a}$$
$$= \frac{x(d+ex)^3}{4a\left(a+cx^2\right)^2} - \frac{3d(ae-cdx)(d+ex)}{8a^2c\left(a+cx^2\right)} + \frac{\left(3d\left(cd^2+ae^2\right)\right)\int \frac{1}{a+cx^2}\,dx}{8a^2c}$$
$$= \frac{x(d+ex)^3}{4a\left(a+cx^2\right)^2} - \frac{3d(ae-cdx)(d+ex)}{8a^2c\left(a+cx^2\right)} + \frac{3d\left(cd^2+ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}$$

Mathematica [A] time = 0.103615, size = 127, normalized size = 1.3

$$\frac{\frac{\sqrt{a}\left(-a^2ce\left(6d^2+3dex+4e^2x^2\right)-2a^3e^3+ac^2dx\left(5d^2+3e^2x^2\right)+3c^3d^3x^3\right)}{\left(a+cx^2\right)^2} + 3\sqrt{c}d\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + c*x^2)^3,x]
```

```
[Out] ((Sqrt[a]*(-2*a^3*e^3 + 3*c^3*d^3*x^3 + a*c^2*d*x*(5*d^2 + 3*e^2*x^2) - a^2
*c*e*(6*d^2 + 3*d*e*x + 4*e^2*x^2)))/(a + c*x^2)^2 + 3*Sqrt[c]*d*(c*d^2 + a
*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^2)
```

Maple [A] time = 0.051, size = 133, normalized size = 1.4

$$\frac{1}{\left(cx^2+a\right)^2}\left(\frac{3\,d\left(ae^2+cd^2\right)x^3}{8\,a^2} - \frac{e^3x^2}{2\,c} - \frac{d\left(3\,ae^2-5\,cd^2\right)x}{8\,ac} - \frac{e\left(ae^2+3\,cd^2\right)}{4\,c^2}\right) + \frac{3\,de^2}{8\,ac}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}} + \frac{3\,d^3}{8\,a^2}\mathrm{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^2+a)^3,x)
```

```
[Out] (3/8*d*(a*e^2+c*d^2)/a^2*x^3-1/2*e^3*x^2/c-1/8*d*(3*a*e^2-5*c*d^2)/a/c*x-1/
4*e*(a*e^2+3*c*d^2)/c^2)/(c*x^2+a)^2+3/8*d/a/c/(a*c)^(1/2)*arctan(x*c/(a*c)
^(1/2))*e^2+3/8*d^3/a^2/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.05173, size = 822, normalized size = 8.39

$$\left[-\frac{8\,a^3ce^3x^2+12\,a^3cd^2e+4\,a^4e^3-6\left(ac^3d^3+a^2c^2de^2\right)x^3+3\left(a^2cd^3+a^3de^2+\left(c^3d^3+ac^2de^2\right)x^4+2\left(ac^2d^3+a^2cde^2\right)\right.}{16\left(a^3c^4x^4+2\,a^4c^3x^2+a^5c^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(8*a^3*c*e^3*x^2 + 12*a^3*c*d^2*e + 4*a^4*e^3 - 6*(a*c^3*d^3 + a^2*c
^2*d*e^2)*x^3 + 3*(a^2*c*d^3 + a^3*d*e^2 + (c^3*d^3 + a*c^2*d*e^2)*x^4 + 2*
(a*c^2*d^3 + a^2*c*d*e^2)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/
(c*x^2 + a)) - 2*(5*a^2*c^2*d^3 - 3*a^3*c*d*e^2)*x)/(a^3*c^4*x^4 + 2*a^4*c^
3*x^2 + a^5*c^2), -1/8*(4*a^3*c*e^3*x^2 + 6*a^3*c*d^2*e + 2*a^4*e^3 - 3*(a*
c^3*d^3 + a^2*c^2*d*e^2)*x^3 - 3*(a^2*c*d^3 + a^3*d*e^2 + (c^3*d^3 + a*c^2*
d*e^2)*x^4 + 2*(a*c^2*d^3 + a^2*c*d*e^2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/
a) - (5*a^2*c^2*d^3 - 3*a^3*c*d*e^2)*x)/(a^3*c^4*x^4 + 2*a^4*c^3*x^2 + a^5*
c^2)]
```

Sympy [B] time = 1.72044, size = 272, normalized size = 2.78

$$-\frac{3d\sqrt{-\frac{1}{a^5c^3}}\left(ae^2+cd^2\right)\log\left(-\frac{3a^3cd\sqrt{-\frac{1}{a^5c^3}}\left(ae^2+cd^2\right)}{3ade^2+3cd^3}+x\right)}{16}+\frac{3d\sqrt{-\frac{1}{a^5c^3}}\left(ae^2+cd^2\right)\log\left(\frac{3a^3cd\sqrt{-\frac{1}{a^5c^3}}\left(ae^2+cd^2\right)}{3ade^2+3cd^3}+x\right)}{16}+\frac{-2a^3e^3\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**2+a)**3,x)
```

```
[Out] -3*d*sqrt(-1/(a**5*c**3))*(a*e**2 + c*d**2)*log(-3*a**3*c*d*sqrt(-1/(a**5*c
**3))*(a*e**2 + c*d**2)/(3*a*d*e**2 + 3*c*d**3) + x)/16 + 3*d*sqrt(-1/(a**5
*c**3))*(a*e**2 + c*d**2)*log(3*a**3*c*d*sqrt(-1/(a**5*c**3))*(a*e**2 + c*d
**2)/(3*a*d*e**2 + 3*c*d**3) + x)/16 + (-2*a**3*e**3 - 6*a**2*c*d**2*e - 4*
a**2*c*e**3*x**2 + x**3*(3*a*c**2*d*e**2 + 3*c**3*d**3) + x*(-3*a**2*c*d*e*
*2 + 5*a*c**2*d**3))/(8*a**4*c**2 + 16*a**3*c**3*x**2 + 8*a**2*c**4*x**4)
```

Giac [A] time = 1.33972, size = 167, normalized size = 1.7

$$\frac{3\left(cd^3+ade^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}a^2c}+\frac{3\,c^3d^3x^3+3\,ac^2dx^3e^2+5\,ac^2d^3x-4\,a^2cx^2e^3-3\,a^2cdxe^2-6\,a^2cd^2e-2\,a^3e^3}{8\left(cx^2+a\right)^2a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/8*(c*d^3 + a*d*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c) + 1/8*(3*c^3*
d^3*x^3 + 3*a*c^2*d*x^3*e^2 + 5*a*c^2*d^3*x - 4*a^2*c*x^2*e^3 - 3*a^2*c*d*x
*e^2 - 6*a^2*c*d^2*e - 2*a^3*e^3)/((c*x^2 + a)^2*a^2*c^2)
```

3.515 $\int \frac{(d+ex)^2}{(a+cx^2)^3}\,dx$

Optimal. Leaf size=113

$$\frac{(ae^2+3cd^2)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}-\frac{2ade-x(ae^2+3cd^2)}{8a^2c(a+cx^2)}-\frac{(d+ex)(ae-cdx)}{4ac(a+cx^2)^2}$$

```
[Out] -((a*e - c*d*x)*(d + e*x))/(4*a*c*(a + c*x^2)^2) - (2*a*d*e - (3*c*d^2 + a*
e^2)*x)/(8*a^2*c*(a + c*x^2)) + ((3*c*d^2 + a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[
a]])/(8*a^(5/2)*c^(3/2))
```

Rubi [A] time = 0.0493133, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {739, 639, 205}

$$\frac{(ae^2+3cd^2)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}-\frac{2ade-x(ae^2+3cd^2)}{8a^2c(a+cx^2)}-\frac{(d+ex)(ae-cdx)}{4ac(a+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + c*x^2)^3,x]
```

```
[Out] -((a*e - c*d*x)*(d + e*x))/(4*a*c*(a + c*x^2)^2) - (2*a*d*e - (3*c*d^2 + a*
e^2)*x)/(8*a^2*c*(a + c*x^2)) + ((3*c*d^2 + a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[
a]])/(8*a^(5/2)*c^(3/2))
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(a+cx^2\right)^3}\,dx = -\frac{(ae-cdx)(d+ex)}{4ac\left(a+cx^2\right)^2} + \frac{\int \frac{3cd^2+ae^2+2cdex}{\left(a+cx^2\right)^2}\,dx}{4ac}$$

$$= -\frac{(ae-cdx)(d+ex)}{4ac\left(a+cx^2\right)^2} - \frac{2ade-\left(3cd^2+ae^2\right)x}{8a^2c\left(a+cx^2\right)} + \frac{\left(3cd^2+ae^2\right)\int \frac{1}{a+cx^2}\,dx}{8a^2c}$$

$$= -\frac{(ae-cdx)(d+ex)}{4ac\left(a+cx^2\right)^2} - \frac{2ade-\left(3cd^2+ae^2\right)x}{8a^2c\left(a+cx^2\right)} + \frac{\left(3cd^2+ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}$$

Mathematica [A] time = 0.0703977, size = 101, normalized size = 0.89

$$\frac{-a^2e(4d+ex)+acx\left(5d^2+e^2x^2\right)+3c^2d^2x^3}{8a^2c\left(a+cx^2\right)^2} + \frac{\left(ae^2+3cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + c*x^2)^3,x]
```

```
[Out] (3*c^2*d^2*x^3 - a^2*e*(4*d + e*x) + a*c*x*(5*d^2 + e^2*x^2))/(8*a^2*c*(a +
 c*x^2)^2) + ((3*c*d^2 + a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(
3/2))
```

Maple [A] time = 0.047, size = 108, normalized size = 1.

$$\frac{1}{\left(cx^2+a\right)^2}\left(\frac{\left(ae^2+3\,cd^2\right)x^3}{8\,a^2} - \frac{\left(ae^2-5\,cd^2\right)x}{8\,ac} - \frac{de}{2\,c}\right) + \frac{e^2}{8\,ac}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}} + \frac{3\,d^2}{8\,a^2}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^2+a)^3,x)
```

```
[Out] (1/8*(a*e^2+3*c*d^2)/a^2*x^3-1/8*(a*e^2-5*c*d^2)/a/c*x-1/2*d*e/c)/(c*x^2+a)
^2+1/8/a/c/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*e^2+3/8/a^2/(a*c)^(1/2)*arct
an(x*c/(a*c)^(1/2))*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.98568, size = 716, normalized size = 6.34

$$\left[-\frac{8\,a^3cde-2\left(3\,ac^3d^2+a^2c^2e^2\right)x^3+\left(3\,a^2cd^2+a^3e^2+\left(3\,c^3d^2+ac^2e^2\right)x^4+2\left(3\,ac^2d^2+a^2ce^2\right)x^2\right)\sqrt{-ac}\log\left(\frac{cx^2-2\sqrt{-ac}x-a}{cx^2+a}\right.}{16\left(a^3c^4x^4+2\,a^4c^3x^2+a^5c^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(8*a^3*c*d*e - 2*(3*a*c^3*d^2 + a^2*c^2*e^2)*x^3 + (3*a^2*c*d^2 + a^
3*e^2 + (3*c^3*d^2 + a*c^2*e^2)*x^4 + 2*(3*a*c^2*d^2 + a^2*c*e^2)*x^2)*sqrt
(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(5*a^2*c^2*d^2 - a
^3*c*e^2)*x)/(a^3*c^4*x^4 + 2*a^4*c^3*x^2 + a^5*c^2), -1/8*(4*a^3*c*d*e - (
3*a*c^3*d^2 + a^2*c^2*e^2)*x^3 - (3*a^2*c*d^2 + a^3*e^2 + (3*c^3*d^2 + a*c^
2*e^2)*x^4 + 2*(3*a*c^2*d^2 + a^2*c*e^2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/
a) - (5*a^2*c^2*d^2 - a^3*c*e^2)*x)/(a^3*c^4*x^4 + 2*a^4*c^3*x^2 + a^5*c^2)
]
```

Sympy [A] time = 1.11785, size = 172, normalized size = 1.52

$$-\frac{\sqrt{-\frac{1}{a^5c^3}}\left(ae^2+3cd^2\right)\log\left(-a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^5c^3}}\left(ae^2+3cd^2\right)\log\left(a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16}+\frac{-4a^2de+x^3\left(ace^2+3c^2d^2\right)-}{8a^4c+16a^3c^2x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*c**3))*(a*e**2 + 3*c*d**2)*log(-a**3*c*sqrt(-1/(a**5*c**3))
+ x)/16 + sqrt(-1/(a**5*c**3))*(a*e**2 + 3*c*d**2)*log(a**3*c*sqrt(-1/(a**5
*c**3)) + x)/16 + (-4*a**2*d*e + x**3*(a*c*e**2 + 3*c**2*d**2) + x*(-a**2*e
**2 + 5*a*c*d**2))/(8*a**4*c + 16*a**3*c**2*x**2 + 8*a**2*c**3*x**4)
```

Giac [A] time = 1.25419, size = 128, normalized size = 1.13

$$\frac{\left(3\,cd^2+ae^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}a^2c}+\frac{3\,c^2d^2x^3+acx^3e^2+5\,acd^2x-a^2xe^2-4\,a^2de}{8\left(cx^2+a\right)^2a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*c*d^2 + a*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c) + 1/8*(3*c^2*
d^2*x^3 + a*c*x^3*e^2 + 5*a*c*d^2*x - a^2*x*e^2 - 4*a^2*d*e)/((c*x^2 + a)^2
*a^2*c)
```

3.516 $\int \frac{d+ex}{(a+cx^2)^3}\,dx$

Optimal. Leaf size=75

$$\frac{3dx}{8a^2\left(a+cx^2\right)}+\frac{3d\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}}-\frac{ae-cdx}{4ac\left(a+cx^2\right)^2}$$

```
[Out] -(a*e - c*d*x)/(4*a*c*(a + c*x^2)^2) + (3*d*x)/(8*a^2*(a + c*x^2)) + (3*d*A
rcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[c])
```

Rubi [A] time = 0.0197236, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {639, 199, 205}

$$\frac{3dx}{8a^2\left(a+cx^2\right)}+\frac{3d\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}}-\frac{ae-cdx}{4ac\left(a+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + c*x^2)^3,x]
```

```
[Out] -(a*e - c*d*x)/(4*a*c*(a + c*x^2)^2) + (3*d*x)/(8*a^2*(a + c*x^2)) + (3*d*A
rcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[c])
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{d+ex}{(a+cx^2)^3}\,dx = -\frac{ae-cdx}{4ac(a+cx^2)^2} + \frac{(3d)\int \frac{1}{(a+cx^2)^2}\,dx}{4a}$$
$$= -\frac{ae-cdx}{4ac(a+cx^2)^2} + \frac{3dx}{8a^2(a+cx^2)} + \frac{(3d)\int \frac{1}{a+cx^2}\,dx}{8a^2}$$
$$= -\frac{ae-cdx}{4ac(a+cx^2)^2} + \frac{3dx}{8a^2(a+cx^2)} + \frac{3d\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}}$$

Mathematica [A] time = 0.0486735, size = 71, normalized size = 0.95

$$\frac{\frac{\sqrt{a}(-2a^2e+5acdx+3c^2dx^3)}{(a+cx^2)^2} + 3\sqrt{c}d\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + c*x^2)^3,x]
```

```
[Out] ((Sqrt[a]*(-2*a^2*e + 5*a*c*d*x + 3*c^2*d*x^3))/(a + c*x^2)^2 + 3*Sqrt[c]*d
*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c)
```

Maple [A] time = 0.044, size = 65, normalized size = 0.9

$$\frac{2\,cdx - 2\,ae}{8\,ac(cx^2+a)^2} + \frac{3\,dx}{8\,a^2(cx^2+a)} + \frac{3\,d}{8\,a^2}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^2+a)^3,x)
```

```
[Out] 1/8*(2*c*d*x-2*a*e)/a/c/(c*x^2+a)^2+3/8*d*x/a^2/(c*x^2+a)+3/8*d/a^2/(a*c)^(
1/2)*arctan(x*c/(a*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94332, size = 455, normalized size = 6.07

$$\left[\frac{6\,ac^2dx^3 + 10\,a^2cdx - 4\,a^3e - 3\left(c^2dx^4 + 2\,acdx^2 + a^2d\right)\sqrt{-ac}\log\left(\frac{cx^2-2\sqrt{-ac}x-a}{cx^2+a}\right)}{16\left(a^3c^3x^4 + 2\,a^4c^2x^2 + a^5c\right)}, \frac{3\,ac^2dx^3 + 5\,a^2cdx - 2\,a^3e + 3\left(c^2dx^4 + \right.}{8\left(a^3c^3x^4 + 2\,a^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*a*c^2*d*x^3 + 10*a^2*c*d*x - 4*a^3*e - 3*(c^2*d*x^4 + 2*a*c*d*x^2
+ a^2*d)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)))/(a^3*c^3
*x^4 + 2*a^4*c^2*x^2 + a^5*c), 1/8*(3*a*c^2*d*x^3 + 5*a^2*c*d*x - 2*a^3*e +
 3*(c^2*d*x^4 + 2*a*c*d*x^2 + a^2*d)*sqrt(a*c)*arctan(sqrt(a*c)*x/a))/(a^3*
c^3*x^4 + 2*a^4*c^2*x^2 + a^5*c)]
```

Sympy [A] time = 0.658126, size = 124, normalized size = 1.65

$$d\left(-\frac{3\sqrt{-\frac{1}{a^5c}}\log\left(-a^3\sqrt{-\frac{1}{a^5c}}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{a^5c}}\log\left(a^3\sqrt{-\frac{1}{a^5c}}+x\right)}{16}\right)+\frac{-2a^2e+5acdx+3c^2dx^3}{8a^4c+16a^3c^2x^2+8a^2c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**2+a)**3,x)
```

```
[Out] d*(-3*sqrt(-1/(a**5*c))*log(-a**3*sqrt(-1/(a**5*c)) + x)/16 + 3*sqrt(-1/(a*
*5*c))*log(a**3*sqrt(-1/(a**5*c)) + x)/16) + (-2*a**2*e + 5*a*c*d*x + 3*c**
2*d*x**3)/(8*a**4*c + 16*a**3*c**2*x**2 + 8*a**2*c**3*x**4)
```

Giac [A] time = 1.31913, size = 82, normalized size = 1.09

$$\frac{3\,d\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2}+\frac{3\,c^2dx^3+5\,acdx-2\,a^2e}{8\,\left(cx^2+a\right)^2a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/8*d*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2) + 1/8*(3*c^2*d*x^3 + 5*a*c*d*x
- 2*a^2*e)/((c*x^2 + a)^2*a^2*c)
```

3.517 $\int \frac{1}{(d+ex)\left(a+cx^2\right)^3} dx$

Optimal. Leaf size=212

$$\frac{\sqrt{c}d\left(15a^2e^4+10acd^2e^2+3c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}\left(ae^2+cd^2\right)^3}+\frac{4a^2e^3+cdx\left(7ae^2+3cd^2\right)}{8a^2\left(a+cx^2\right)\left(ae^2+cd^2\right)^2}+\frac{ae+cdx}{4a\left(a+cx^2\right)^2\left(ae^2+cd^2\right)}-\frac{e^5\log\left(a+cx^2\right)}{2\left(ae^2+cd^2\right)^3}$$

```
[Out] (a*e + c*d*x)/(4*a*(c*d^2 + a*e^2)*(a + c*x^2)^2) + (4*a^2*e^3 + c*d*(3*c*d
^2 + 7*a*e^2)*x)/(8*a^2*(c*d^2 + a*e^2)^2*(a + c*x^2)) + (Sqrt[c]*d*(3*c^2*
d^4 + 10*a*c*d^2*e^2 + 15*a^2*e^4)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*
(c*d^2 + a*e^2)^3) + (e^5*Log[d + e*x])/(c*d^2 + a*e^2)^3 - (e^5*Log[a + c*
x^2])/(2*(c*d^2 + a*e^2)^3)
```

Rubi [A] time = 0.220795, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {741, 823, 801, 635, 205, 260}

$$\frac{\sqrt{c}d\left(15a^2e^4+10acd^2e^2+3c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}\left(ae^2+cd^2\right)^3}+\frac{4a^2e^3+cdx\left(7ae^2+3cd^2\right)}{8a^2\left(a+cx^2\right)\left(ae^2+cd^2\right)^2}+\frac{ae+cdx}{4a\left(a+cx^2\right)^2\left(ae^2+cd^2\right)}-\frac{e^5\log\left(a+cx^2\right)}{2\left(ae^2+cd^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + c*x^2)^3),x]
```


```
[Out] (a*e + c*d*x)/(4*a*(c*d^2 + a*e^2)*(a + c*x^2)^2) + (4*a^2*e^3 + c*d*(3*c*d
^2 + 7*a*e^2)*x)/(8*a^2*(c*d^2 + a*e^2)^2*(a + c*x^2)) + (Sqrt[c]*d*(3*c^2*
d^4 + 10*a*c*d^2*e^2 + 15*a^2*e^4)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*
(c*d^2 + a*e^2)^3) + (e^5*Log[d + e*x])/(c*d^2 + a*e^2)^3 - (e^5*Log[a + c*
x^2])/(2*(c*d^2 + a*e^2)^3)
```


Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+cx^2\right)^3}\,dx = \frac{ae+cdx}{4a\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} - \frac{\int \frac{-3cd^2-4ae^2-3cdex}{(d+ex)\left(a+cx^2\right)^2}\,dx}{4a\left(cd^2+ae^2\right)}$$

$$= \frac{ae+cdx}{4a\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{4a^2e^3+cd\left(3cd^2+7ae^2\right)x}{8a^2\left(cd^2+ae^2\right)^2\left(a+cx^2\right)} + \frac{\int \frac{c\left(3c^2d^4+7acd^2e^2+8a^2e^4\right)+c^2de\left(3cd^2+7\right.}{(d+ex)\left(a+cx^2\right)}}{8a^2c\left(cd^2+ae^2\right)^2}$$

$$= \frac{ae+cdx}{4a\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{4a^2e^3+cd\left(3cd^2+7ae^2\right)x}{8a^2\left(cd^2+ae^2\right)^2\left(a+cx^2\right)} + \frac{\int \left(\frac{8a^2ce^6}{\left(cd^2+ae^2\right)(d+ex)} + \frac{c^2\left(3c^2d^5+10acd^3\right.}{\left(cd^2+a\right.}\right.}{8a^2c\left(cd^2+ae\right.}$$

$$= \frac{ae+cdx}{4a\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{4a^2e^3+cd\left(3cd^2+7ae^2\right)x}{8a^2\left(cd^2+ae^2\right)^2\left(a+cx^2\right)} + \frac{e^5\log(d+ex)}{\left(cd^2+ae^2\right)^3} + \frac{c\int \frac{3c^2d^5+10acd^3}{a}}{8a^2\left(cd\right.}$$

$$= \frac{ae+cdx}{4a\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{4a^2e^3+cd\left(3cd^2+7ae^2\right)x}{8a^2\left(cd^2+ae^2\right)^2\left(a+cx^2\right)} + \frac{e^5\log(d+ex)}{\left(cd^2+ae^2\right)^3} - \frac{\left(ce^5\right)\int \frac{x}{a+cx^2}\,dx}{\left(cd^2+ae^2\right)^3}$$

$$= \frac{ae+cdx}{4a\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{4a^2e^3+cd\left(3cd^2+7ae^2\right)x}{8a^2\left(cd^2+ae^2\right)^2\left(a+cx^2\right)} + \frac{\sqrt{c}d\left(3c^2d^4+10acd^2e^2+15a^2e^4\right.}{8a^{5/2}\left(cd^2+ae^2\right)^3}$$

Mathematica [A] time = 0.146377, size = 180, normalized size = 0.85

$$\frac{\frac{\left(ae^2+cd^2\right)\left(4a^2e^3+7acde^2x+3c^2d^3x\right)}{a^2\left(a+cx^2\right)} + \frac{\sqrt{c}d\left(15a^2e^4+10acd^2e^2+3c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{a^{5/2}} + \frac{2\left(ae^2+cd^2\right)^2(ae+cdx)}{a\left(a+cx^2\right)^2} - 4e^5\log\left(a+cx^2\right) + 8e^5\log(d+ex}{8\left(ae^2+cd^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + c*x^2)^3),x]
```

```
[Out] ((2*(c*d^2 + a*e^2)^2*(a*e + c*d*x))/(a*(a + c*x^2)^2) + ((c*d^2 + a*e^2)*(
4*a^2*e^3 + 3*c^2*d^3*x + 7*a*c*d*e^2*x))/(a^2*(a + c*x^2)) + (Sqrt[c]*d*(3
*c^2*d^4 + 10*a*c*d^2*e^2 + 15*a^2*e^4)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(5/2
```

```
) + 8*e^5*Log[d + e*x] - 4*e^5*Log[a + c*x^2])/(8*(c*d^2 + a*e^2)^3)
```

Maple [B] time = 0.059, size = 532, normalized size = 2.5

$$\frac{7c^2dx^3e^4}{8\left(ae^2+cd^2\right)^3\left(cx^2+a\right)^2}+\frac{5c^3d^3x^3e^2}{4\left(ae^2+cd^2\right)^3\left(cx^2+a\right)^2a}+\frac{3c^4d^5x^3}{8\left(ae^2+cd^2\right)^3\left(cx^2+a\right)^2a^2}+\frac{cx^2ae^5}{2\left(ae^2+cd^2\right)^3\left(cx^2+a\right)^2}+\frac{}{2(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+a)^3,x)
```

```
[Out] 7/8*c^2/(a*e^2+c*d^2)^3/(c*x^2+a)^2*d*x^3*e^4+5/4*c^3/(a*e^2+c*d^2)^3/(c*x^
2+a)^2*d^3/a*x^3*e^2+3/8*c^4/(a*e^2+c*d^2)^3/(c*x^2+a)^2*d^5/a^2*x^3+1/2*c/
(a*e^2+c*d^2)^3/(c*x^2+a)^2*x^2*a*e^5+1/2*c^2/(a*e^2+c*d^2)^3/(c*x^2+a)^2*x
^2*d^2*e^3+9/8*c/(a*e^2+c*d^2)^3/(c*x^2+a)^2*d*a*x*e^4+7/4*c^2/(a*e^2+c*d^2
)^3/(c*x^2+a)^2*d^3*x*e^2+5/8*c^3/(a*e^2+c*d^2)^3/(c*x^2+a)^2*d^5/a*x+3/4/(
a*e^2+c*d^2)^3/(c*x^2+a)^2*a^2*e^5+c/(a*e^2+c*d^2)^3/(c*x^2+a)^2*e^3*a*d^2+
1/4*c^2/(a*e^2+c*d^2)^3/(c*x^2+a)^2*e*d^4-1/2*e^5*ln(c*x^2+a)/(a*e^2+c*d^2)
^3+15/8*c/(a*e^2+c*d^2)^3/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*d*e^4+5/4*c^2
/(a*e^2+c*d^2)^3/a/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*d^3*e^2+3/8*c^3/(a*e
^2+c*d^2)^3/a^2/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*d^5+e^5*ln(e*x+d)/(a*e^
2+c*d^2)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 24.1499, size = 2045, normalized size = 9.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*a^2*c^2*d^4*e + 16*a^3*c*d^2*e^3 + 12*a^4*e^5 + 2*(3*c^4*d^5 + 10*
a*c^3*d^3*e^2 + 7*a^2*c^2*d*e^4)*x^3 + 8*(a^2*c^2*d^2*e^3 + a^3*c*e^5)*x^2
+ (3*a^2*c^2*d^5 + 10*a^3*c*d^3*e^2 + 15*a^4*d*e^4 + (3*c^4*d^5 + 10*a*c^3*
d^3*e^2 + 15*a^2*c^2*d*e^4)*x^4 + 2*(3*a*c^3*d^5 + 10*a^2*c^2*d^3*e^2 + 15*
a^3*c*d*e^4)*x^2)*sqrt(-c/a)*log((c*x^2 + 2*a*x*sqrt(-c/a) - a)/(c*x^2 + a)
) + 2*(5*a*c^3*d^5 + 14*a^2*c^2*d^3*e^2 + 9*a^3*c*d*e^4)*x - 8*(a^2*c^2*e^5
*x^4 + 2*a^3*c*e^5*x^2 + a^4*e^5)*log(c*x^2 + a) + 16*(a^2*c^2*e^5*x^4 + 2*
a^3*c*e^5*x^2 + a^4*e^5)*log(e*x + d))/(a^4*c^3*d^6 + 3*a^5*c^2*d^4*e^2 + 3
*a^6*c*d^2*e^4 + a^7*e^6 + (a^2*c^5*d^6 + 3*a^3*c^4*d^4*e^2 + 3*a^4*c^3*d^2
*e^4 + a^5*c^2*e^6)*x^4 + 2*(a^3*c^4*d^6 + 3*a^4*c^3*d^4*e^2 + 3*a^5*c^2*d^
```

```
2*e^4 + a^6*c*e^6)*x^2), 1/8*(2*a^2*c^2*d^4*e + 8*a^3*c*d^2*e^3 + 6*a^4*e^5
 + (3*c^4*d^5 + 10*a*c^3*d^3*e^2 + 7*a^2*c^2*d*e^4)*x^3 + 4*(a^2*c^2*d^2*e^
3 + a^3*c*e^5)*x^2 + (3*a^2*c^2*d^5 + 10*a^3*c*d^3*e^2 + 15*a^4*d*e^4 + (3*
c^4*d^5 + 10*a*c^3*d^3*e^2 + 15*a^2*c^2*d*e^4)*x^4 + 2*(3*a*c^3*d^5 + 10*a^
2*c^2*d^3*e^2 + 15*a^3*c*d*e^4)*x^2)*sqrt(c/a)*arctan(x*sqrt(c/a)) + (5*a*c
^3*d^5 + 14*a^2*c^2*d^3*e^2 + 9*a^3*c*d*e^4)*x - 4*(a^2*c^2*e^5*x^4 + 2*a^3
*c*e^5*x^2 + a^4*e^5)*log(c*x^2 + a) + 8*(a^2*c^2*e^5*x^4 + 2*a^3*c*e^5*x^2
 + a^4*e^5)*log(e*x + d))/(a^4*c^3*d^6 + 3*a^5*c^2*d^4*e^2 + 3*a^6*c*d^2*e^
4 + a^7*e^6 + (a^2*c^5*d^6 + 3*a^3*c^4*d^4*e^2 + 3*a^4*c^3*d^2*e^4 + a^5*c^
2*e^6)*x^4 + 2*(a^3*c^4*d^6 + 3*a^4*c^3*d^4*e^2 + 3*a^5*c^2*d^2*e^4 + a^6*c
*e^6)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37224, size = 462, normalized size = 2.18

$$-\frac{e^5\log\left(cx^2+a\right)}{2\left(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6\right)}+\frac{e^6\log\left(|xe+d|\right)}{c^3d^6e+3\,ac^2d^4e^3+3\,a^2cd^2e^5+a^3e^7}+\frac{\left(3\,c^3d^5+10\,ac^2d^3e^2+15\,a^2cde^4\right)\mathrm{a}}{8\left(a^2c^3d^6+3\,a^3c^2d^4e^2+3\,a^4cd^2e^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*e^5*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*
e^6) + e^6*log(abs(x*e + d))/(c^3*d^6*e + 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5
 + a^3*e^7) + 1/8*(3*c^3*d^5 + 10*a*c^2*d^3*e^2 + 15*a^2*c*d*e^4)*arctan(c*
x/sqrt(a*c))/((a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 + a^5*e^6)
*sqrt(a*c)) + 1/8*(2*a^2*c^2*d^4*e + 8*a^3*c*d^2*e^3 + 6*a^4*e^5 + (3*c^4*d
^5 + 10*a*c^3*d^3*e^2 + 7*a^2*c^2*d*e^4)*x^3 + 4*(a^2*c^2*d^2*e^3 + a^3*c*e
^5)*x^2 + (5*a*c^3*d^5 + 14*a^2*c^2*d^3*e^2 + 9*a^3*c*d*e^4)*x)/((c*d^2 + a
*e^2)^3*(c*x^2 + a)^2*a^2)
```

3.518 $\int \frac{1}{(d+ex)^2(a+cx^2)^3}\,dx$

Optimal. Leaf size=300

$$\frac{3\sqrt{c}\left(15a^2cd^2e^4-5a^3e^6+5ac^2d^4e^2+c^3d^6\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}\left(ae^2+cd^2\right)^4}-\frac{ae\left(cd^2-5ae^2\right)-3cdx\left(3ae^2+cd^2\right)}{8a^2\left(a+cx^2\right)(d+ex)\left(ae^2+cd^2\right)^2}+\frac{3e\left(cd^2-ae^2\right)\left(5ae^2+c\right.}{8a^2(d+ex)\left(ae^2+cd^2\right.}$$

```
[Out] (3*e*(c*d^2 - a*e^2)*(c*d^2 + 5*a*e^2))/(8*a^2*(c*d^2 + a*e^2)^3*(d + e*x))
 + (a*e + c*d*x)/(4*a*(c*d^2 + a*e^2)*(d + e*x)*(a + c*x^2)^2) - (a*e*(c*d^
2 - 5*a*e^2) - 3*c*d*(c*d^2 + 3*a*e^2)*x)/(8*a^2*(c*d^2 + a*e^2)^2*(d + e*x
)*(a + c*x^2)) + (3*Sqrt[c]*(c^3*d^6 + 5*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 -
 5*a^3*e^6)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*(c*d^2 + a*e^2)^4) + (6
*c*d*e^5*Log[d + e*x])/(c*d^2 + a*e^2)^4 - (3*c*d*e^5*Log[a + c*x^2])/(c*d^
2 + a*e^2)^4
```

Rubi [A] time = 0.356834, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {741, 823, 801, 635, 205, 260}

$$\frac{3\sqrt{c}\left(15a^2cd^2e^4-5a^3e^6+5ac^2d^4e^2+c^3d^6\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}\left(ae^2+cd^2\right)^4}-\frac{ae\left(cd^2-5ae^2\right)-3cdx\left(3ae^2+cd^2\right)}{8a^2\left(a+cx^2\right)(d+ex)\left(ae^2+cd^2\right)^2}+\frac{3e\left(cd^2-ae^2\right)\left(5ae^2+c\right.}{8a^2(d+ex)\left(ae^2+cd^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + c*x^2)^3),x]
```

```
[Out] (3*e*(c*d^2 - a*e^2)*(c*d^2 + 5*a*e^2))/(8*a^2*(c*d^2 + a*e^2)^3*(d + e*x))
 + (a*e + c*d*x)/(4*a*(c*d^2 + a*e^2)*(d + e*x)*(a + c*x^2)^2) - (a*e*(c*d^
2 - 5*a*e^2) - 3*c*d*(c*d^2 + 3*a*e^2)*x)/(8*a^2*(c*d^2 + a*e^2)^2*(d + e*x
)*(a + c*x^2)) + (3*Sqrt[c]*(c^3*d^6 + 5*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 -
 5*a^3*e^6)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*(c*d^2 + a*e^2)^4) + (6
*c*d*e^5*Log[d + e*x])/(c*d^2 + a*e^2)^4 - (3*c*d*e^5*Log[a + c*x^2])/(c*d^
2 + a*e^2)^4
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+cx^2\right)^3}\,dx = \frac{ae+cdx}{4a\left(cd^2+ae^2\right)(d+ex)\left(a+cx^2\right)^2} - \frac{\int \frac{-3cd^2-5ae^2-4cdex}{(d+ex)^2\left(a+cx^2\right)^2}\,dx}{4a\left(cd^2+ae^2\right)}$$

$$= \frac{ae+cdx}{4a\left(cd^2+ae^2\right)(d+ex)\left(a+cx^2\right)^2} - \frac{ae\left(cd^2-5ae^2\right)-3cd\left(cd^2+3ae^2\right)x}{8a^2\left(cd^2+ae^2\right)^2(d+ex)\left(a+cx^2\right)} + \frac{\int \frac{3c\left(c^2d^4+2acd^2 \ldots\right.}{\ldots}}{8a \ldots}$$

$$= \frac{ae+cdx}{4a\left(cd^2+ae^2\right)(d+ex)\left(a+cx^2\right)^2} - \frac{ae\left(cd^2-5ae^2\right)-3cd\left(cd^2+3ae^2\right)x}{8a^2\left(cd^2+ae^2\right)^2(d+ex)\left(a+cx^2\right)} + \frac{\int \left(\frac{3ce^2\left(-cd^2-5 \ldots\right.}{\left(cd^2+ae^2 \ldots\right.} \ldots\right)}{\ldots}$$

$$= \frac{3e\left(cd^2-ae^2\right)\left(cd^2+5ae^2\right)}{8a^2\left(cd^2+ae^2\right)^3(d+ex)} + \frac{ae+cdx}{4a\left(cd^2+ae^2\right)(d+ex)\left(a+cx^2\right)^2} - \frac{ae\left(cd^2-5ae^2\right)-3cd\left(\ldots\right.}{8a^2\left(cd^2+ae^2\right)^2(d+ \ldots}$$

$$= \frac{3e\left(cd^2-ae^2\right)\left(cd^2+5ae^2\right)}{8a^2\left(cd^2+ae^2\right)^3(d+ex)} + \frac{ae+cdx}{4a\left(cd^2+ae^2\right)(d+ex)\left(a+cx^2\right)^2} - \frac{ae\left(cd^2-5ae^2\right)-3cd\left(\ldots\right.}{8a^2\left(cd^2+ae^2\right)^2(d+ \ldots}$$

$$= \frac{3e\left(cd^2-ae^2\right)\left(cd^2+5ae^2\right)}{8a^2\left(cd^2+ae^2\right)^3(d+ex)} + \frac{ae+cdx}{4a\left(cd^2+ae^2\right)(d+ex)\left(a+cx^2\right)^2} - \frac{ae\left(cd^2-5ae^2\right)-3cd\left(\ldots\right.}{8a^2\left(cd^2+ae^2\right)^2(d+ \ldots}$$

Mathematica [A] time = 0.388818, size = 241, normalized size = 0.8

$$\frac{\frac{c\left(ae^2+cd^2\right)\left(a^2e^3(16d-7ex)+12acd^2e^2x+3c^2d^4x\right)}{a^2\left(a+cx^2\right)} + \frac{3\sqrt{c}\left(15a^2cd^2e^4-5a^3e^6+5ac^2d^4e^2+c^3d^6\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{a^{5/2}} + \frac{2c\left(ae^2+cd^2\right)^2\left(ae(2d-ex)+cd^2x\right)}{a\left(a+cx^2\right)^2} - \frac{8e^5\left(ae^2+cd^2 \ldots\right.}{d+ex} \ldots}{8\left(ae^2+cd^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + c*x^2)^3),x]
```

```
[Out] ((-8*e^5*(c*d^2 + a*e^2))/(d + e*x) + (c*(c*d^2 + a*e^2)*(3*c^2*d^4*x + 12*
a*c*d^2*e^2*x + a^2*e^3*(16*d - 7*e*x)))/(a^2*(a + c*x^2)) + (2*c*(c*d^2 +
a*e^2)^2*(c*d^2*x + a*e*(2*d - e*x)))/(a*(a + c*x^2)^2) + (3*Sqrt[c]*(c^3*d
^6 + 5*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - 5*a^3*e^6)*ArcTan[(Sqrt[c]*x)/Sqr
t[a]])/a^(5/2) + 48*c*d*e^5*Log[d + e*x] - 24*c*d*e^5*Log[a + c*x^2])/(8*(c
*d^2 + a*e^2)^4)
```

Maple [B] time = 0.06, size = 680, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^2+a)^3,x)
```

```
[Out] -7/8*c^2/(a*e^2+c*d^2)^4/(c*x^2+a)^2*a*x^3*e^6+5/8*c^3/(a*e^2+c*d^2)^4/(c*x
^2+a)^2*x^3*d^2*e^4+15/8*c^4/(a*e^2+c*d^2)^4/(c*x^2+a)^2/a*x^3*d^4*e^2+3/8*
c^5/(a*e^2+c*d^2)^4/(c*x^2+a)^2/a^2*x^3*d^6+2*c^2/(a*e^2+c*d^2)^4/(c*x^2+a)
^2*x^2*a*d*e^5+2*c^3/(a*e^2+c*d^2)^4/(c*x^2+a)^2*x^2*d^3*e^3-9/8*c/(a*e^2+c
*d^2)^4/(c*x^2+a)^2*x*a^2*e^6+3/8*c^2/(a*e^2+c*d^2)^4/(c*x^2+a)^2*x*a*d^2*e
^4+17/8*c^3/(a*e^2+c*d^2)^4/(c*x^2+a)^2*x*d^4*e^2+5/8*c^4/(a*e^2+c*d^2)^4/(
c*x^2+a)^2*x/a*d^6+5/2*c/(a*e^2+c*d^2)^4/(c*x^2+a)^2*a^2*d*e^5+3*c^2/(a*e^2
+c*d^2)^4/(c*x^2+a)^2*a*d^3*e^3+1/2*c^3/(a*e^2+c*d^2)^4/(c*x^2+a)^2*d^5*e-3
*c*d*e^5*ln(c*x^2+a)/(a*e^2+c*d^2)^4-15/8*c/(a*e^2+c*d^2)^4*a/(a*c)^(1/2)*a
rctan(x*c/(a*c)^(1/2))*e^6+45/8*c^2/(a*e^2+c*d^2)^4/(a*c)^(1/2)*arctan(x*c/
(a*c)^(1/2))*d^2*e^4+15/8*c^3/(a*e^2+c*d^2)^4/a/(a*c)^(1/2)*arctan(x*c/(a*c
)^(1/2))*d^4*e^2+3/8*c^4/(a*e^2+c*d^2)^4/a^2/(a*c)^(1/2)*arctan(x*c/(a*c)^(
1/2))*d^6-e^5/(a*e^2+c*d^2)^3/(e*x+d)+6*c*d*e^5*ln(e*x+d)/(a*e^2+c*d^2)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 67.5698, size = 4594, normalized size = 15.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(8*a^2*c^3*d^6*e + 48*a^3*c^2*d^4*e^3 + 24*a^4*c*d^2*e^5 - 16*a^5*e^7
 + 6*(c^5*d^6*e + 5*a*c^4*d^4*e^3 - a^2*c^3*d^2*e^5 - 5*a^3*c^2*e^7)*x^4 +
6*(c^5*d^7 + 5*a*c^4*d^5*e^2 + 7*a^2*c^3*d^3*e^4 + 3*a^3*c^2*d*e^6)*x^3 + 2
*(5*a*c^4*d^6*e + 33*a^2*c^3*d^4*e^3 + 3*a^3*c^2*d^2*e^5 - 25*a^4*c*e^7)*x^
2 + 3*(a^2*c^3*d^7 + 5*a^3*c^2*d^5*e^2 + 15*a^4*c*d^3*e^4 - 5*a^5*d*e^6 + (
```

```
c^5*d^6*e + 5*a*c^4*d^4*e^3 + 15*a^2*c^3*d^2*e^5 - 5*a^3*c^2*e^7)*x^5 + (c^
5*d^7 + 5*a*c^4*d^5*e^2 + 15*a^2*c^3*d^3*e^4 - 5*a^3*c^2*d*e^6)*x^4 + 2*(a*
c^4*d^6*e + 5*a^2*c^3*d^4*e^3 + 15*a^3*c^2*d^2*e^5 - 5*a^4*c*e^7)*x^3 + 2*(
a*c^4*d^7 + 5*a^2*c^3*d^5*e^2 + 15*a^3*c^2*d^3*e^4 - 5*a^4*c*d*e^6)*x^2 + (
a^2*c^3*d^6*e + 5*a^3*c^2*d^4*e^3 + 15*a^4*c*d^2*e^5 - 5*a^5*e^7)*x)*sqrt(-
c/a)*log((c*x^2 + 2*a*x*sqrt(-c/a) - a)/(c*x^2 + a)) + 2*(5*a*c^4*d^7 + 21*
a^2*c^3*d^5*e^2 + 27*a^3*c^2*d^3*e^4 + 11*a^4*c*d*e^6)*x - 48*(a^2*c^3*d*e^
6*x^5 + a^2*c^3*d^2*e^5*x^4 + 2*a^3*c^2*d*e^6*x^3 + 2*a^3*c^2*d^2*e^5*x^2 +
 a^4*c*d*e^6*x + a^4*c*d^2*e^5)*log(c*x^2 + a) + 96*(a^2*c^3*d*e^6*x^5 + a^
2*c^3*d^2*e^5*x^4 + 2*a^3*c^2*d*e^6*x^3 + 2*a^3*c^2*d^2*e^5*x^2 + a^4*c*d*e
^6*x + a^4*c*d^2*e^5)*log(e*x + d))/(a^4*c^4*d^9 + 4*a^5*c^3*d^7*e^2 + 6*a^
6*c^2*d^5*e^4 + 4*a^7*c*d^3*e^6 + a^8*d*e^8 + (a^2*c^6*d^8*e + 4*a^3*c^5*d^
6*e^3 + 6*a^4*c^4*d^4*e^5 + 4*a^5*c^3*d^2*e^7 + a^6*c^2*e^9)*x^5 + (a^2*c^6
*d^9 + 4*a^3*c^5*d^7*e^2 + 6*a^4*c^4*d^5*e^4 + 4*a^5*c^3*d^3*e^6 + a^6*c^2*
d*e^8)*x^4 + 2*(a^3*c^5*d^8*e + 4*a^4*c^4*d^6*e^3 + 6*a^5*c^3*d^4*e^5 + 4*a
^6*c^2*d^2*e^7 + a^7*c*e^9)*x^3 + 2*(a^3*c^5*d^9 + 4*a^4*c^4*d^7*e^2 + 6*a^
5*c^3*d^5*e^4 + 4*a^6*c^2*d^3*e^6 + a^7*c*d*e^8)*x^2 + (a^4*c^4*d^8*e + 4*a
^5*c^3*d^6*e^3 + 6*a^6*c^2*d^4*e^5 + 4*a^7*c*d^2*e^7 + a^8*e^9)*x), 1/8*(4*
a^2*c^3*d^6*e + 24*a^3*c^2*d^4*e^3 + 12*a^4*c*d^2*e^5 - 8*a^5*e^7 + 3*(c^5*
d^6*e + 5*a*c^4*d^4*e^3 - a^2*c^3*d^2*e^5 - 5*a^3*c^2*e^7)*x^4 + 3*(c^5*d^7
 + 5*a*c^4*d^5*e^2 + 7*a^2*c^3*d^3*e^4 + 3*a^3*c^2*d*e^6)*x^3 + (5*a*c^4*d^
6*e + 33*a^2*c^3*d^4*e^3 + 3*a^3*c^2*d^2*e^5 - 25*a^4*c*e^7)*x^2 + 3*(a^2*c
^3*d^7 + 5*a^3*c^2*d^5*e^2 + 15*a^4*c*d^3*e^4 - 5*a^5*d*e^6 + (c^5*d^6*e +
5*a*c^4*d^4*e^3 + 15*a^2*c^3*d^2*e^5 - 5*a^3*c^2*e^7)*x^5 + (c^5*d^7 + 5*a*
c^4*d^5*e^2 + 15*a^2*c^3*d^3*e^4 - 5*a^3*c^2*d*e^6)*x^4 + 2*(a*c^4*d^6*e +
5*a^2*c^3*d^4*e^3 + 15*a^3*c^2*d^2*e^5 - 5*a^4*c*e^7)*x^3 + 2*(a*c^4*d^7 +
5*a^2*c^3*d^5*e^2 + 15*a^3*c^2*d^3*e^4 - 5*a^4*c*d*e^6)*x^2 + (a^2*c^3*d^6*
e + 5*a^3*c^2*d^4*e^3 + 15*a^4*c*d^2*e^5 - 5*a^5*e^7)*x)*sqrt(c/a)*arctan(x
*sqrt(c/a)) + (5*a*c^4*d^7 + 21*a^2*c^3*d^5*e^2 + 27*a^3*c^2*d^3*e^4 + 11*a
^4*c*d*e^6)*x - 24*(a^2*c^3*d*e^6*x^5 + a^2*c^3*d^2*e^5*x^4 + 2*a^3*c^2*d*e
^6*x^3 + 2*a^3*c^2*d^2*e^5*x^2 + a^4*c*d*e^6*x + a^4*c*d^2*e^5)*log(c*x^2 +
 a) + 48*(a^2*c^3*d*e^6*x^5 + a^2*c^3*d^2*e^5*x^4 + 2*a^3*c^2*d*e^6*x^3 + 2
*a^3*c^2*d^2*e^5*x^2 + a^4*c*d*e^6*x + a^4*c*d^2*e^5)*log(e*x + d))/(a^4*c^
4*d^9 + 4*a^5*c^3*d^7*e^2 + 6*a^6*c^2*d^5*e^4 + 4*a^7*c*d^3*e^6 + a^8*d*e^8
 + (a^2*c^6*d^8*e + 4*a^3*c^5*d^6*e^3 + 6*a^4*c^4*d^4*e^5 + 4*a^5*c^3*d^2*e
^7 + a^6*c^2*e^9)*x^5 + (a^2*c^6*d^9 + 4*a^3*c^5*d^7*e^2 + 6*a^4*c^4*d^5*e^
4 + 4*a^5*c^3*d^3*e^6 + a^6*c^2*d*e^8)*x^4 + 2*(a^3*c^5*d^8*e + 4*a^4*c^4*d
^6*e^3 + 6*a^5*c^3*d^4*e^5 + 4*a^6*c^2*d^2*e^7 + a^7*c*e^9)*x^3 + 2*(a^3*c^
5*d^9 + 4*a^4*c^4*d^7*e^2 + 6*a^5*c^3*d^5*e^4 + 4*a^6*c^2*d^3*e^6 + a^7*c*d
*e^8)*x^2 + (a^4*c^4*d^8*e + 4*a^5*c^3*d^6*e^3 + 6*a^6*c^2*d^4*e^5 + 4*a^7*
c*d^2*e^7 + a^8*e^9)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36292, size = 768, normalized size = 2.56

$$-\frac{3\,cde^5\log\left(c-\frac{2\,cd}{xe+d}+\frac{cd^2}{(xe+d)^2}+\frac{ae^2}{(xe+d)^2}\right)}{c^4d^8+4\,ac^3d^6e^2+6\,a^2c^2d^4e^4+4\,a^3cd^2e^6+a^4e^8}+\frac{3\left(c^4d^6e^2+5\,ac^3d^4e^4+15\,a^2c^2d^2e^6-5\,a^3ce^8\right)\arctan\left(\frac{\left(cd-\frac{cd^2}{xe+d}-\frac{ae^2}{xe+d}\right)e}{\sqrt{ac}}\right.}{8\left(a^2c^4d^8+4\,a^3c^3d^6e^2+6\,a^4c^2d^4e^4+4\,a^5cd^2e^6+a^6e^8\right)\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3*c*d*e^5*log(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(x*e + d)^2)
/(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8
) + 3/8*(c^4*d^6*e^2 + 5*a*c^3*d^4*e^4 + 15*a^2*c^2*d^2*e^6 - 5*a^3*c*e^8)*
arctan((c*d - c*d^2/(x*e + d) - a*e^2/(x*e + d))*e^(-1)/sqrt(a*c))*e^(-2)/(
(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^
6*e^8)*sqrt(a*c)) - e^11/((c^3*d^6*e^6 + 3*a*c^2*d^4*e^8 + 3*a^2*c*d^2*e^10
 + a^3*e^12)*(x*e + d)) + 1/8*(3*c^5*d^5*e + 14*a*c^4*d^3*e^3 - 29*a^2*c^3*
d*e^5 - (9*c^5*d^6*e^2 + 41*a*c^4*d^4*e^4 - 121*a^2*c^3*d^2*e^6 + 7*a^3*c^2
*e^8)*e^(-1)/(x*e + d) + (9*c^5*d^7*e^3 + 45*a*c^4*d^5*e^5 - 145*a^2*c^3*d^
3*e^7 - 21*a^3*c^2*d*e^9)*e^(-2)/(x*e + d)^2 - 3*(c^5*d^8*e^4 + 6*a*c^4*d^6
*e^6 - 20*a^2*c^3*d^4*e^8 - 22*a^3*c^2*d^2*e^10 + 3*a^4*c*e^12)*e^(-3)/(x*e
 + d)^3)/((c*d^2 + a*e^2)^4*a^2*(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 +
a*e^2/(x*e + d)^2)^2)
```

3.519 $\int \frac{(d+ex)^4}{\left(a+cx^2\right)^4} \, dx$

Optimal. Leaf size=155

$$-\frac{(d+ex)\left(ae^2+5cd^2\right)(ae-cdx)}{16a^3c^2\left(a+cx^2\right)}+\frac{\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{5/2}}-\frac{(d+ex)^3(ae-5cdx)}{24a^2c\left(a+cx^2\right)^2}+\frac{x(d+ex)^4}{6a\left(a+cx^2\right)^3}$$

```
[Out] (x*(d + e*x)^4)/(6*a*(a + c*x^2)^3) - ((a*e - 5*c*d*x)*(d + e*x)^3)/(24*a^2
*c*(a + c*x^2)^2) - ((5*c*d^2 + a*e^2)*(a*e - c*d*x)*(d + e*x))/(16*a^3*c^2
*(a + c*x^2)) + ((c*d^2 + a*e^2)*(5*c*d^2 + a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[
a]])/(16*a^(7/2)*c^(5/2))
```

Rubi [A] time = 0.0845514, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {737, 805, 723, 205}

$$-\frac{(d+ex)\left(ae^2+5cd^2\right)(ae-cdx)}{16a^3c^2\left(a+cx^2\right)}+\frac{\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{5/2}}-\frac{(d+ex)^3(ae-5cdx)}{24a^2c\left(a+cx^2\right)^2}+\frac{x(d+ex)^4}{6a\left(a+cx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a + c*x^2)^4,x]
```

```
[Out] (x*(d + e*x)^4)/(6*a*(a + c*x^2)^3) - ((a*e - 5*c*d*x)*(d + e*x)^3)/(24*a^2
*c*(a + c*x^2)^2) - ((5*c*d^2 + a*e^2)*(a*e - c*d*x)*(d + e*x))/(16*a^3*c^2
*(a + c*x^2)) + ((c*d^2 + a*e^2)*(5*c*d^2 + a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[
a]])/(16*a^(7/2)*c^(5/2))
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(x*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*(p + 1)), x] + Dist[1/(2*a*(p + 1
)), Int[(d + e*x)^(m - 1)*(d*(2*p + 3) + e*(m + 2*p + 3)*x)*(a + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -
1] && GtQ[m, 0] && (LtQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && In
tQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 805

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[(m*(c*d*f + a*e*g))/(2*a*c*(p + 1)), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 723

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[((2*p + 3)*(c*d^2 + a*e^2))/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^4}{\left(a+cx^2\right)^4}dx = \frac{x(d+ex)^4}{6a\left(a+cx^2\right)^3} - \frac{\int \frac{(-5d-ex)(d+ex)^3}{\left(a+cx^2\right)^3}dx}{6a}$$

$$= \frac{x(d+ex)^4}{6a\left(a+cx^2\right)^3} - \frac{(ae-5cdx)(d+ex)^3}{24a^2c\left(a+cx^2\right)^2} + \frac{\left(5cd^2+ae^2\right)\int \frac{(d+ex)^2}{\left(a+cx^2\right)^2}dx}{8a^2c}$$

$$= \frac{x(d+ex)^4}{6a\left(a+cx^2\right)^3} - \frac{(ae-5cdx)(d+ex)^3}{24a^2c\left(a+cx^2\right)^2} - \frac{\left(5cd^2+ae^2\right)(ae-cdx)(d+ex)}{16a^3c^2\left(a+cx^2\right)} + \frac{\left(\left(cd^2+ae^2\right)\left(5cd^2+ae^2\right)\right)\int}{16a^3c^2}$$

$$= \frac{x(d+ex)^4}{6a\left(a+cx^2\right)^3} - \frac{(ae-5cdx)(d+ex)^3}{24a^2c\left(a+cx^2\right)^2} - \frac{\left(5cd^2+ae^2\right)(ae-cdx)(d+ex)}{16a^3c^2\left(a+cx^2\right)} + \frac{\left(cd^2+ae^2\right)\left(5cd^2+ae^2\right)\tan}{16a^{7/2}c^{5/2}}$$

Mathematica [A] time = 0.146524, size = 197, normalized size = 1.27

$$\frac{3a^2c^2x\left(16d^2e^2x^2+11d^4+e^4x^4\right)-2a^3ce\left(9d^2ex+16d^3+24de^2x^2+4e^3x^3\right)-a^4e^3(16d+3ex)+2ac^3d^2x^3\left(20d^2+9e^2x^2\right)}{48a^3c^2\left(a+cx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a + c*x^2)^4,x]
```

```
[Out] (15*c^4*d^4*x^5 - a^4*e^3*(16*d + 3*e*x) + 2*a*c^3*d^2*x^3*(20*d^2 + 9*e^2*
x^2) - 2*a^3*c*e*(16*d^3 + 9*d^2*e*x + 24*d*e^2*x^2 + 4*e^3*x^3) + 3*a^2*c^
2*x*(11*d^4 + 16*d^2*e^2*x^2 + e^4*x^4))/(48*a^3*c^2*(a + c*x^2)^3) + ((5*c
^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*
c^(5/2))
```

Maple [A] time = 0.053, size = 225, normalized size = 1.5

$$\frac{1}{\left(cx^2+a\right)^3}\left(\frac{\left(a^2e^4+6\,acd^2e^2+5\,c^2d^4\right)x^5}{16\,a^3} - \frac{\left(a^2e^4-6\,acd^2e^2-5\,c^2d^4\right)x^3}{6\,a^2c} - \frac{de^3x^2}{c} - \frac{\left(a^2e^4+6\,acd^2e^2-11\,c^2d^4\right)x}{16\,ac^2} - \frac{de\left(ae\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(c*x^2+a)^4,x)
```

```
[Out] (1/16*(a^2*e^4+6*a*c*d^2*e^2+5*c^2*d^4)/a^3*x^5-1/6*(a^2*e^4-6*a*c*d^2*e^2-
5*c^2*d^4)/a^2/c*x^3-d*e^3*x^2/c-1/16*(a^2*e^4+6*a*c*d^2*e^2-11*c^2*d^4)/a/
c^2*x-1/3*d*e*(a*e^2+2*c*d^2)/c^2)/(c*x^2+a)^3+1/16/a/c^2/(a*c)^(1/2)*arcta
n(x*c/(a*c)^(1/2))*e^4+3/8/a^2/c/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*d^2*e^
2+5/16/a^3/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*d^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.30775, size = 1480, normalized size = 9.55

$$\left[-\frac{96\,a^4c^2de^3x^2+64\,a^4c^2d^3e+32\,a^5cde^3-6\left(5\,ac^5d^4+6\,a^2c^4d^2e^2+a^3c^3e^4\right)x^5-16\left(5\,a^2c^4d^4+6\,a^3c^3d^2e^2-a^4c^2e^4\right)x^3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(96*a^4*c^2*d*e^3*x^2 + 64*a^4*c^2*d^3*e + 32*a^5*c*d*e^3 - 6*(5*a*c
^5*d^4 + 6*a^2*c^4*d^2*e^2 + a^3*c^3*e^4)*x^5 - 16*(5*a^2*c^4*d^4 + 6*a^3*c
^3*d^2*e^2 - a^4*c^2*e^4)*x^3 + 3*(5*a^3*c^2*d^4 + 6*a^4*c*d^2*e^2 + a^5*e^
4 + (5*c^5*d^4 + 6*a*c^4*d^2*e^2 + a^2*c^3*e^4)*x^6 + 3*(5*a*c^4*d^4 + 6*a^
2*c^3*d^2*e^2 + a^3*c^2*e^4)*x^4 + 3*(5*a^2*c^3*d^4 + 6*a^3*c^2*d^2*e^2 + a
^4*c*e^4)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 6
*(11*a^3*c^3*d^4 - 6*a^4*c^2*d^2*e^2 - a^5*c*e^4)*x)/(a^4*c^6*x^6 + 3*a^5*c
^5*x^4 + 3*a^6*c^4*x^2 + a^7*c^3), -1/48*(48*a^4*c^2*d*e^3*x^2 + 32*a^4*c^2
*d^3*e + 16*a^5*c*d*e^3 - 3*(5*a*c^5*d^4 + 6*a^2*c^4*d^2*e^2 + a^3*c^3*e^4)
*x^5 - 8*(5*a^2*c^4*d^4 + 6*a^3*c^3*d^2*e^2 - a^4*c^2*e^4)*x^3 - 3*(5*a^3*c
^2*d^4 + 6*a^4*c*d^2*e^2 + a^5*e^4 + (5*c^5*d^4 + 6*a*c^4*d^2*e^2 + a^2*c^3
*e^4)*x^6 + 3*(5*a*c^4*d^4 + 6*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*x^4 + 3*(5*a^
2*c^3*d^4 + 6*a^3*c^2*d^2*e^2 + a^4*c*e^4)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*
x/a) - 3*(11*a^3*c^3*d^4 - 6*a^4*c^2*d^2*e^2 - a^5*c*e^4)*x)/(a^4*c^6*x^6 +
 3*a^5*c^5*x^4 + 3*a^6*c^4*x^2 + a^7*c^3)]
```

Sympy [B] time = 3.74925, size = 413, normalized size = 2.66

$$-\frac{\sqrt{-\frac{1}{a^7c^5}}\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)\log\left(-\frac{a^4c^2\sqrt{-\frac{1}{a^7c^5}}\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{a^2e^4+6acd^2e^2+5c^2d^4}+x\right)}{32}+\frac{\sqrt{-\frac{1}{a^7c^5}}\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)\log\left(\frac{a^4c^2\sqrt{}}{}\right.}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(c*x**2+a)**4,x)
```

```
[Out] -sqrt(-1/(a**7*c**5))*(a*e**2 + c*d**2)*(a*e**2 + 5*c*d**2)*log(-a**4*c**2*
sqrt(-1/(a**7*c**5))*(a*e**2 + c*d**2)*(a*e**2 + 5*c*d**2)/(a**2*e**4 + 6*a
*c*d**2*e**2 + 5*c**2*d**4) + x)/32 + sqrt(-1/(a**7*c**5))*(a*e**2 + c*d**2
)*(a*e**2 + 5*c*d**2)*log(a**4*c**2*sqrt(-1/(a**7*c**5))*(a*e**2 + c*d**2)*
(a*e**2 + 5*c*d**2)/(a**2*e**4 + 6*a*c*d**2*e**2 + 5*c**2*d**4) + x)/32 + (
```

```
-16*a**4*d*e**3 - 32*a**3*c*d**3*e - 48*a**3*c*d*e**3*x**2 + x**5*(3*a**2*c
**2*e**4 + 18*a*c**3*d**2*e**2 + 15*c**4*d**4) + x**3*(-8*a**3*c*e**4 + 48*
a**2*c**2*d**2*e**2 + 40*a*c**3*d**4) + x*(-3*a**4*e**4 - 18*a**3*c*d**2*e*
*2 + 33*a**2*c**2*d**4))/(48*a**6*c**2 + 144*a**5*c**3*x**2 + 144*a**4*c**4
*x**4 + 48*a**3*c**5*x**6)
```

Giac [A] time = 1.28245, size = 277, normalized size = 1.79

$$\frac{\left(5\,c^2d^4+6\,acd^2e^2+a^2e^4\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c^2}+\frac{15\,c^4d^4x^5+18\,ac^3d^2x^5e^2+40\,ac^3d^4x^3+3\,a^2c^2x^5e^4+48\,a^2c^2d^2x^3e^2+33\,a^2c^2d}{48\,\left(cx^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*(5*c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)
*a^3*c^2) + 1/48*(15*c^4*d^4*x^5 + 18*a*c^3*d^2*x^5*e^2 + 40*a*c^3*d^4*x^3
+ 3*a^2*c^2*x^5*e^4 + 48*a^2*c^2*d^2*x^3*e^2 + 33*a^2*c^2*d^4*x - 8*a^3*c*x
^3*e^4 - 48*a^3*c*d*x^2*e^3 - 18*a^3*c*d^2*x*e^2 - 32*a^3*c*d^3*e - 3*a^4*x
*e^4 - 16*a^4*d*e^3)/((c*x^2 + a)^3*a^3*c^2)
```

3.520 $\int \frac{(d+ex)^3}{\left(a+cx^2\right)^4} dx$

Optimal. Leaf size=156

$$-\frac{4ae\left(ae^2+5cd^2\right)-cdx\left(15cd^2-ae^2\right)}{48a^3c^2\left(a+cx^2\right)}+\frac{d\left(3ae^2+5cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}-\frac{(d+ex)^2(2ae-5cdx)}{24a^2c\left(a+cx^2\right)^2}+\frac{x(d+ex)^3}{6a\left(a+cx^2\right)^3}$$

```
[Out] (x*(d + e*x)^3)/(6*a*(a + c*x^2)^3) - ((2*a*e - 5*c*d*x)*(d + e*x)^2)/(24*a
^2*c*(a + c*x^2)^2) - (4*a*e*(5*c*d^2 + a*e^2) - c*d*(15*c*d^2 - a*e^2)*x)/
(48*a^3*c^2*(a + c*x^2)) + (d*(5*c*d^2 + 3*a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a
]])/(16*a^(7/2)*c^(3/2))
```

Rubi [A] time = 0.118263, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {737, 821, 778, 205}

$$-\frac{4ae\left(ae^2+5cd^2\right)-cdx\left(15cd^2-ae^2\right)}{48a^3c^2\left(a+cx^2\right)}+\frac{d\left(3ae^2+5cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}-\frac{(d+ex)^2(2ae-5cdx)}{24a^2c\left(a+cx^2\right)^2}+\frac{x(d+ex)^3}{6a\left(a+cx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + c*x^2)^4,x]
```

```
[Out] (x*(d + e*x)^3)/(6*a*(a + c*x^2)^3) - ((2*a*e - 5*c*d*x)*(d + e*x)^2)/(24*a
^2*c*(a + c*x^2)^2) - (4*a*e*(5*c*d^2 + a*e^2) - c*d*(15*c*d^2 - a*e^2)*x)/
(48*a^3*c^2*(a + c*x^2)) + (d*(5*c*d^2 + 3*a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a
]])/(16*a^(7/2)*c^(3/2))
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(x*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*(p + 1)), x] + Dist[1/(2*a*(p + 1
)), Int[(d + e*x)^(m - 1)*(d*(2*p + 3) + e*(m + 2*p + 3)*x)*(a + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -
1] && GtQ[m, 0] && (LtQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && In
tQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(a+cx^2\right)^4}\,dx = \frac{x(d+ex)^3}{6a\left(a+cx^2\right)^3} - \frac{\int \frac{(-5d-2ex)(d+ex)^2}{\left(a+cx^2\right)^3}\,dx}{6a}$$
$$= \frac{x(d+ex)^3}{6a\left(a+cx^2\right)^3} - \frac{(2ae-5cdx)(d+ex)^2}{24a^2c\left(a+cx^2\right)^2} - \frac{\int \frac{(d+ex)\left(-15cd^2-4ae^2-5cdex\right)}{\left(a+cx^2\right)^2}\,dx}{24a^2c}$$
$$= \frac{x(d+ex)^3}{6a\left(a+cx^2\right)^3} - \frac{(2ae-5cdx)(d+ex)^2}{24a^2c\left(a+cx^2\right)^2} - \frac{4ae\left(5cd^2+ae^2\right)-cd\left(15cd^2-ae^2\right)x}{48a^3c^2\left(a+cx^2\right)} + \frac{\left(d\left(5cd^2+3ae^2\right)\right)\int}{16a^3c}$$
$$= \frac{x(d+ex)^3}{6a\left(a+cx^2\right)^3} - \frac{(2ae-5cdx)(d+ex)^2}{24a^2c\left(a+cx^2\right)^2} - \frac{4ae\left(5cd^2+ae^2\right)-cd\left(15cd^2-ae^2\right)x}{48a^3c^2\left(a+cx^2\right)} + \frac{d\left(5cd^2+3ae^2\right)\tan^{-1}}{16a^{7/2}c^{3/2}}$$

Mathematica [A] time = 0.152602, size = 155, normalized size = 0.99

$$\frac{\frac{\sqrt{a}\left(3a^2c^2dx\left(11d^2+8e^2x^2\right)-3a^3ce\left(8d^2+3dex+4e^2x^2\right)-4a^4e^3+ac^3dx^3\left(40d^2+9e^2x^2\right)+15c^4d^3x^5\right)}{\left(a+cx^2\right)^3}+3\sqrt{c}d\left(3ae^2+5cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{48a^{7/2}c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + c*x^2)^4,x]
```

```
[Out] ((Sqrt[a]*(-4*a^4*e^3 + 15*c^4*d^3*x^5 - 3*a^3*c*e*(8*d^2 + 3*d*e*x + 4*e^2
*x^2) + 3*a^2*c^2*d*x*(11*d^2 + 8*e^2*x^2) + a*c^3*d*x^3*(40*d^2 + 9*e^2*x^
2)))/(a + c*x^2)^3 + 3*Sqrt[c]*d*(5*c*d^2 + 3*a*e^2)*ArcTan[(Sqrt[c]*x)/Sqr
t[a]])/(48*a^(7/2)*c^2)
```

Maple [A] time = 0.05, size = 158, normalized size = 1.

$$\frac{1}{\left(cx^2+a\right)^3}\left(\frac{d\left(3\,ae^2+5\,cd^2\right)cx^5}{16\,a^3}+\frac{d\left(3\,ae^2+5\,cd^2\right)x^3}{6\,a^2}-\frac{e^3x^2}{4\,c}-\frac{d\left(3\,ae^2-11\,cd^2\right)x}{16\,ac}-\frac{e\left(ae^2+6\,cd^2\right)}{12\,c^2}\right)+\frac{3\,de^2}{16\,a^2c}\arctan\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^2+a)^4,x)
```

```
[Out] (1/16*d*(3*a*e^2+5*c*d^2)/a^3*c*x^5+1/6/a^2*d*(3*a*e^2+5*c*d^2)*x^3-1/4*e^3
*x^2/c-1/16*d*(3*a*e^2-11*c*d^2)/a/c*x-1/12*e*(a*e^2+6*c*d^2)/c^2)/(c*x^2+a
)^3+3/16*d/a^2/c/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*e^2+5/16*d^3/a^3/(a*c)
^(1/2)*arctan(x*c/(a*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.20226, size = 1148, normalized size = 7.36

$$\left[-\frac{24\,a^4ce^3x^2 + 48\,a^4cd^2e + 8\,a^5e^3 - 6\left(5\,ac^4d^3 + 3\,a^2c^3de^2\right)x^5 - 16\left(5\,a^2c^3d^3 + 3\,a^3c^2de^2\right)x^3 + 3\left(5\,a^3cd^3 + 3\,a^4de^2 + (\right.}{96\left(a^4c^5x^6 + 3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(24*a^4*c*e^3*x^2 + 48*a^4*c*d^2*e + 8*a^5*e^3 - 6*(5*a*c^4*d^3 + 3*
a^2*c^3*d*e^2)*x^5 - 16*(5*a^2*c^3*d^3 + 3*a^3*c^2*d*e^2)*x^3 + 3*(5*a^3*c*
d^3 + 3*a^4*d*e^2 + (5*c^4*d^3 + 3*a*c^3*d*e^2)*x^6 + 3*(5*a*c^3*d^3 + 3*a^
2*c^2*d*e^2)*x^4 + 3*(5*a^2*c^2*d^3 + 3*a^3*c*d*e^2)*x^2)*sqrt(-a*c)*log((c
*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 6*(11*a^3*c^2*d^3 - 3*a^4*c*d*e^2
)*x)/(a^4*c^5*x^6 + 3*a^5*c^4*x^4 + 3*a^6*c^3*x^2 + a^7*c^2), -1/48*(12*a^4
*c*e^3*x^2 + 24*a^4*c*d^2*e + 4*a^5*e^3 - 3*(5*a*c^4*d^3 + 3*a^2*c^3*d*e^2)
*x^5 - 8*(5*a^2*c^3*d^3 + 3*a^3*c^2*d*e^2)*x^3 - 3*(5*a^3*c*d^3 + 3*a^4*d*e
^2 + (5*c^4*d^3 + 3*a*c^3*d*e^2)*x^6 + 3*(5*a*c^3*d^3 + 3*a^2*c^2*d*e^2)*x^
4 + 3*(5*a^2*c^2*d^3 + 3*a^3*c*d*e^2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a)
- 3*(11*a^3*c^2*d^3 - 3*a^4*c*d*e^2)*x)/(a^4*c^5*x^6 + 3*a^5*c^4*x^4 + 3*a^
6*c^3*x^2 + a^7*c^2)]
```

Sympy [B] time = 2.35928, size = 320, normalized size = 2.05

$$-\frac{d\sqrt{-\frac{1}{a^7c^3}}\left(3ae^2 + 5cd^2\right)\log\left(-\frac{a^4cd\sqrt{-\frac{1}{a^7c^3}}\left(3ae^2+5cd^2\right)}{3ade^2+5cd^3} + x\right)}{32} + \frac{d\sqrt{-\frac{1}{a^7c^3}}\left(3ae^2 + 5cd^2\right)\log\left(\frac{a^4cd\sqrt{-\frac{1}{a^7c^3}}\left(3ae^2+5cd^2\right)}{3ade^2+5cd^3} + x\right)}{32} + \frac{-4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**2+a)**4,x)
```

```
[Out] -d*sqrt(-1/(a**7*c**3))*(3*a*e**2 + 5*c*d**2)*log(-a**4*c*d*sqrt(-1/(a**7*c
**3))*(3*a*e**2 + 5*c*d**2)/(3*a*d*e**2 + 5*c*d**3) + x)/32 + d*sqrt(-1/(a*
*7*c**3))*(3*a*e**2 + 5*c*d**2)*log(a**4*c*d*sqrt(-1/(a**7*c**3))*(3*a*e**2
 + 5*c*d**2)/(3*a*d*e**2 + 5*c*d**3) + x)/32 + (-4*a**4*e**3 - 24*a**3*c*d*
*2*e - 12*a**3*c*e**3*x**2 + x**5*(9*a*c**3*d*e**2 + 15*c**4*d**3) + x**3*(
24*a**2*c**2*d*e**2 + 40*a*c**3*d**3) + x*(-9*a**3*c*d*e**2 + 33*a**2*c**2*
d**3))/(48*a**6*c**2 + 144*a**5*c**3*x**2 + 144*a**4*c**4*x**4 + 48*a**3*c*
*5*x**6)
```

Giac [A] time = 1.18849, size = 208, normalized size = 1.33

$$\frac{\left(5\,cd^3+3\,ade^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c}+\frac{15\,c^4d^3x^5+9\,ac^3dx^5e^2+40\,ac^3d^3x^3+24\,a^2c^2dx^3e^2+33\,a^2c^2d^3x-12\,a^3cx^2e^3-9\,a^3cdx}{48\,\left(cx^2+a\right)^3a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*(5*c*d^3 + 3*a*d*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c) + 1/48*(
15*c^4*d^3*x^5 + 9*a*c^3*d*x^5*e^2 + 40*a*c^3*d^3*x^3 + 24*a^2*c^2*d*x^3*e^
2 + 33*a^2*c^2*d^3*x - 12*a^3*c*x^2*e^3 - 9*a^3*c*d*x*e^2 - 24*a^3*c*d^2*e
- 4*a^4*e^3)/((c*x^2 + a)^3*a^3*c^2)
```

3.521 $\int \frac{(d+ex)^2}{\left(a+cx^2\right)^4} dx$

Optimal. Leaf size=145

$$\frac{\left(ae^2+5cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}+\frac{x\left(ae^2+5cd^2\right)}{16a^3c\left(a+cx^2\right)}-\frac{4ade-x\left(ae^2+5cd^2\right)}{24a^2c\left(a+cx^2\right)^2}-\frac{(d+ex)(ae-cdx)}{6ac\left(a+cx^2\right)^3}$$

```
[Out] -((a*e - c*d*x)*(d + e*x))/(6*a*c*(a + c*x^2)^3) - (4*a*d*e - (5*c*d^2 + a*
e^2)*x)/(24*a^2*c*(a + c*x^2)^2) + ((5*c*d^2 + a*e^2)*x)/(16*a^3*c*(a + c*x
^2)) + ((5*c*d^2 + a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*c^(3/2))
```

Rubi [A] time = 0.0640524, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {739, 639, 199, 205}

$$\frac{\left(ae^2+5cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}+\frac{x\left(ae^2+5cd^2\right)}{16a^3c\left(a+cx^2\right)}-\frac{4ade-x\left(ae^2+5cd^2\right)}{24a^2c\left(a+cx^2\right)^2}-\frac{(d+ex)(ae-cdx)}{6ac\left(a+cx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + c*x^2)^4,x]
```

```
[Out] -((a*e - c*d*x)*(d + e*x))/(6*a*c*(a + c*x^2)^3) - (4*a*d*e - (5*c*d^2 + a*
e^2)*x)/(24*a^2*c*(a + c*x^2)^2) + ((5*c*d^2 + a*e^2)*x)/(16*a^3*c*(a + c*x
^2)) + ((5*c*d^2 + a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*c^(3/2))
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(a+cx^2\right)^4}\,dx = -\frac{(ae-cdx)(d+ex)}{6ac\left(a+cx^2\right)^3}+\frac{\int \frac{5cd^2+ae^2+4cdex}{\left(a+cx^2\right)^3}\,dx}{6ac}$$

$$= -\frac{(ae-cdx)(d+ex)}{6ac\left(a+cx^2\right)^3}-\frac{4ade-\left(5cd^2+ae^2\right)x}{24a^2c\left(a+cx^2\right)^2}+\frac{\left(5cd^2+ae^2\right)\int \frac{1}{\left(a+cx^2\right)^2}\,dx}{8a^2c}$$

$$= -\frac{(ae-cdx)(d+ex)}{6ac\left(a+cx^2\right)^3}-\frac{4ade-\left(5cd^2+ae^2\right)x}{24a^2c\left(a+cx^2\right)^2}+\frac{\left(5cd^2+ae^2\right)x}{16a^3c\left(a+cx^2\right)}+\frac{\left(5cd^2+ae^2\right)\int \frac{1}{a+cx^2}\,dx}{16a^3c}$$

$$= -\frac{(ae-cdx)(d+ex)}{6ac\left(a+cx^2\right)^3}-\frac{4ade-\left(5cd^2+ae^2\right)x}{24a^2c\left(a+cx^2\right)^2}+\frac{\left(5cd^2+ae^2\right)x}{16a^3c\left(a+cx^2\right)}+\frac{\left(5cd^2+ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}$$

Mathematica [A] time = 0.0910285, size = 127, normalized size = 0.88

$$\frac{a^2cx\left(33d^2+8e^2x^2\right)-a^3e(16d+3ex)+ac^2x^3\left(40d^2+3e^2x^2\right)+15c^3d^2x^5}{48a^3c\left(a+cx^2\right)^3}+\frac{\left(ae^2+5cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + c*x^2)^4,x]
```

```
[Out] (15*c^3*d^2*x^5 - a^3*e*(16*d + 3*e*x) + a*c^2*x^3*(40*d^2 + 3*e^2*x^2) + a
^2*c*x*(33*d^2 + 8*e^2*x^2))/(48*a^3*c*(a + c*x^2)^3) + ((5*c*d^2 + a*e^2)*
ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*c^(3/2))
```

Maple [A] time = 0.049, size = 129, normalized size = 0.9

$$\frac{1}{\left(cx^2+a\right)^3}\left(\frac{\left(ae^2+5\,cd^2\right)cx^5}{16\,a^3}+\frac{\left(ae^2+5\,cd^2\right)x^3}{6\,a^2}-\frac{\left(ae^2-11\,cd^2\right)x}{16\,ac}-\frac{de}{3\,c}\right)+\frac{e^2}{16\,a^2c}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}+\frac{5\,d^2}{16\,a^3}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^2+a)^4,x)
```

```
[Out] (1/16*(a*e^2+5*c*d^2)/a^3*c*x^5+1/6/a^2*(a*e^2+5*c*d^2)*x^3-1/16*(a*e^2-11*
c*d^2)/a/c*x-1/3*d*e/c)/(c*x^2+a)^3+1/16/a^2/c/(a*c)^(1/2)*arctan(x*c/(a*c)
^(1/2))*e^2+5/16/a^3/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.1203, size = 983, normalized size = 6.78

$$\left[-\frac{32\,a^4cde - 6\left(5\,ac^4d^2 + a^2c^3e^2\right)x^5 - 16\left(5\,a^2c^3d^2 + a^3c^2e^2\right)x^3 + 3\left(\left(5\,c^4d^2 + ac^3e^2\right)x^6 + 5\,a^3cd^2 + a^4e^2 + 3\left(5\,ac^3d^2 + \right.\right.}{96\left(a^4c^5x^6 + 3\,a^5c^4x^4 + 3\,a^6c^3x^2 + \right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(32*a^4*c*d*e - 6*(5*a*c^4*d^2 + a^2*c^3*e^2)*x^5 - 16*(5*a^2*c^3*d^
2 + a^3*c^2*e^2)*x^3 + 3*((5*c^4*d^2 + a*c^3*e^2)*x^6 + 5*a^3*c*d^2 + a^4*e
^2 + 3*(5*a*c^3*d^2 + a^2*c^2*e^2)*x^4 + 3*(5*a^2*c^2*d^2 + a^3*c*e^2)*x^2)
*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 6*(11*a^3*c^2*d
^2 - a^4*c*e^2)*x)/(a^4*c^5*x^6 + 3*a^5*c^4*x^4 + 3*a^6*c^3*x^2 + a^7*c^2),
 -1/48*(16*a^4*c*d*e - 3*(5*a*c^4*d^2 + a^2*c^3*e^2)*x^5 - 8*(5*a^2*c^3*d^2
 + a^3*c^2*e^2)*x^3 - 3*((5*c^4*d^2 + a*c^3*e^2)*x^6 + 5*a^3*c*d^2 + a^4*e^
2 + 3*(5*a*c^3*d^2 + a^2*c^2*e^2)*x^4 + 3*(5*a^2*c^2*d^2 + a^3*c*e^2)*x^2)*
sqrt(a*c)*arctan(sqrt(a*c)*x/a) - 3*(11*a^3*c^2*d^2 - a^4*c*e^2)*x)/(a^4*c^
5*x^6 + 3*a^5*c^4*x^4 + 3*a^6*c^3*x^2 + a^7*c^2)]
```

Sympy [A] time = 1.4061, size = 214, normalized size = 1.48

$$-\frac{\sqrt{-\frac{1}{a^7c^3}}\left(ae^2 + 5cd^2\right)\log\left(-a^4c\sqrt{-\frac{1}{a^7c^3}} + x\right)}{32} + \frac{\sqrt{-\frac{1}{a^7c^3}}\left(ae^2 + 5cd^2\right)\log\left(a^4c\sqrt{-\frac{1}{a^7c^3}} + x\right)}{32} + \frac{-16a^3de + x^5\left(3ac^2e^2 + 1\right.}{48a^6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**2+a)**4,x)
```

```
[Out] -sqrt(-1/(a**7*c**3))*(a*e**2 + 5*c*d**2)*log(-a**4*c*sqrt(-1/(a**7*c**3))
+ x)/32 + sqrt(-1/(a**7*c**3))*(a*e**2 + 5*c*d**2)*log(a**4*c*sqrt(-1/(a**7
*c**3)) + x)/32 + (-16*a**3*d*e + x**5*(3*a*c**2*e**2 + 15*c**3*d**2) + x**
3*(8*a**2*c*e**2 + 40*a*c**2*d**2) + x*(-3*a**3*e**2 + 33*a**2*c*d**2))/(48
*a**6*c + 144*a**5*c**2*x**2 + 144*a**4*c**3*x**4 + 48*a**3*c**4*x**6)
```

Giac [A] time = 1.31693, size = 166, normalized size = 1.14

$$\frac{\left(5\,cd^2 + ae^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c} + \frac{15\,c^3d^2x^5 + 3\,ac^2x^5e^2 + 40\,ac^2d^2x^3 + 8\,a^2cx^3e^2 + 33\,a^2cd^2x - 3\,a^3xe^2 - 16\,a^3de}{48\left(cx^2 + a\right)^3a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*(5*c*d^2 + a*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c) + 1/48*(15*c
^3*d^2*x^5 + 3*a*c^2*x^5*e^2 + 40*a*c^2*d^2*x^3 + 8*a^2*c*x^3*e^2 + 33*a^2*
c*d^2*x - 3*a^3*x*e^2 - 16*a^3*d*e)/((c*x^2 + a)^3*a^3*c)
```

3.522 $\int \frac{d+ex}{(a+cx^2)^4} \, dx$

Optimal. Leaf size=93

$$\frac{5dx}{16a^3(a+cx^2)}+\frac{5dx}{24a^2(a+cx^2)^2}+\frac{5d\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}\sqrt{c}}-\frac{ae-cdx}{6ac(a+cx^2)^3}$$

```
[Out] -(a*e - c*d*x)/(6*a*c*(a + c*x^2)^3) + (5*d*x)/(24*a^2*(a + c*x^2)^2) + (5*
d*x)/(16*a^3*(a + c*x^2)) + (5*d*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*S
qrt[c])
```

Rubi [A] time = 0.0280046, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {639, 199, 205}

$$\frac{5dx}{16a^3(a+cx^2)}+\frac{5dx}{24a^2(a+cx^2)^2}+\frac{5d\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}\sqrt{c}}-\frac{ae-cdx}{6ac(a+cx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + c*x^2)^4,x]
```

```
[Out] -(a*e - c*d*x)/(6*a*c*(a + c*x^2)^3) + (5*d*x)/(24*a^2*(a + c*x^2)^2) + (5*
d*x)/(16*a^3*(a + c*x^2)) + (5*d*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*S
qrt[c])
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{d+ex}{(a+cx^2)^4}\,dx = -\frac{ae-cdx}{6ac(a+cx^2)^3} + \frac{(5d)\int \frac{1}{(a+cx^2)^3}\,dx}{6a}$$
$$= -\frac{ae-cdx}{6ac(a+cx^2)^3} + \frac{5dx}{24a^2(a+cx^2)^2} + \frac{(5d)\int \frac{1}{(a+cx^2)^2}\,dx}{8a^2}$$
$$= -\frac{ae-cdx}{6ac(a+cx^2)^3} + \frac{5dx}{24a^2(a+cx^2)^2} + \frac{5dx}{16a^3(a+cx^2)} + \frac{(5d)\int \frac{1}{a+cx^2}\,dx}{16a^3}$$
$$= -\frac{ae-cdx}{6ac(a+cx^2)^3} + \frac{5dx}{24a^2(a+cx^2)^2} + \frac{5dx}{16a^3(a+cx^2)} + \frac{5d\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}\sqrt{c}}$$

Mathematica [A] time = 0.0463111, size = 83, normalized size = 0.89

$$\frac{\frac{\sqrt{a}(33a^2cdx-8a^3e+40ac^2dx^3+15c^3dx^5)}{(a+cx^2)^3} + 15\sqrt{c}d\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{48a^{7/2}c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + c*x^2)^4,x]
```

```
[Out] ((Sqrt[a]*(-8*a^3*e + 33*a^2*c*d*x + 40*a*c^2*d*x^3 + 15*c^3*d*x^5))/(a + c
*x^2)^3 + 15*Sqrt[c]*d*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(48*a^(7/2)*c)
```

Maple [A] time = 0.044, size = 81, normalized size = 0.9

$$\frac{2\,cdx-2\,ae}{12\,ac(cx^2+a)^3} + \frac{5\,dx}{24\,a^2(cx^2+a)^2} + \frac{5\,dx}{16\,a^3(cx^2+a)} + \frac{5\,d}{16\,a^3}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^2+a)^4,x)
```

```
[Out] 1/12*(2*c*d*x-2*a*e)/a/c/(c*x^2+a)^3+5/24*d*x/a^2/(c*x^2+a)^2+5/16*d*x/a^3/
(c*x^2+a)+5/16*d/a^3/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.09654, size = 608, normalized size = 6.54

$$\left[\frac{30\,ac^3dx^5 + 80\,a^2c^2dx^3 + 66\,a^3cdx - 16\,a^4e - 15\left(c^3dx^6 + 3\,ac^2dx^4 + 3\,a^2cdx^2 + a^3d\right)\sqrt{-ac}\log\left(\frac{cx^2-2\,\sqrt{-ac}x-a}{cx^2+a}\right)}{96\left(a^4c^4x^6 + 3\,a^5c^3x^4 + 3\,a^6c^2x^2 + a^7c\right)}, \frac{15\,ac^3 \cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(30*a*c^3*d*x^5 + 80*a^2*c^2*d*x^3 + 66*a^3*c*d*x - 16*a^4*e - 15*(c^
3*d*x^6 + 3*a*c^2*d*x^4 + 3*a^2*c*d*x^2 + a^3*d)*sqrt(-a*c)*log((c*x^2 - 2*
sqrt(-a*c)*x - a)/(c*x^2 + a)))/(a^4*c^4*x^6 + 3*a^5*c^3*x^4 + 3*a^6*c^2*x^
2 + a^7*c), 1/48*(15*a*c^3*d*x^5 + 40*a^2*c^2*d*x^3 + 33*a^3*c*d*x - 8*a^4*
e + 15*(c^3*d*x^6 + 3*a*c^2*d*x^4 + 3*a^2*c*d*x^2 + a^3*d)*sqrt(a*c)*arctan
(sqrt(a*c)*x/a))/(a^4*c^4*x^6 + 3*a^5*c^3*x^4 + 3*a^6*c^2*x^2 + a^7*c)]
```

Sympy [A] time = 0.838742, size = 150, normalized size = 1.61

$$d\left(-\frac{5\sqrt{-\frac{1}{a^7c}}\log\left(-a^4\sqrt{-\frac{1}{a^7c}} + x\right)}{32} + \frac{5\sqrt{-\frac{1}{a^7c}}\log\left(a^4\sqrt{-\frac{1}{a^7c}} + x\right)}{32}\right) + \frac{-8a^3e + 33a^2cdx + 40ac^2dx^3 + 15c^3dx^5}{48a^6c + 144a^5c^2x^2 + 144a^4c^3x^4 + 48a^3c^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**2+a)**4,x)
```

```
[Out] d*(-5*sqrt(-1/(a**7*c))*log(-a**4*sqrt(-1/(a**7*c)) + x)/32 + 5*sqrt(-1/(a*
*7*c))*log(a**4*sqrt(-1/(a**7*c)) + x)/32) + (-8*a**3*e + 33*a**2*c*d*x + 4
0*a*c**2*d*x**3 + 15*c**3*d*x**5)/(48*a**6*c + 144*a**5*c**2*x**2 + 144*a**
4*c**3*x**4 + 48*a**3*c**4*x**6)
```

Giac [A] time = 1.3043, size = 99, normalized size = 1.06

$$\frac{5\,d\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3} + \frac{15\,c^3dx^5 + 40\,ac^2dx^3 + 33\,a^2cdx - 8\,a^3e}{48\left(cx^2 + a\right)^3a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 5/16*d*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3) + 1/48*(15*c^3*d*x^5 + 40*a*c^
2*d*x^3 + 33*a^2*c*d*x - 8*a^3*e)/((c*x^2 + a)^3*a^3*c)
```

3.523 $\int \frac{1}{(d+ex)\left(a+cx^2\right)^4}\,dx$

Optimal. Leaf size=295

$$\frac{cdx\left(19a^2e^4+16acd^2e^2+5c^2d^4\right)+8a^3e^5}{16a^3\left(a+cx^2\right)\left(ae^2+cd^2\right)^3}+\frac{\sqrt{c}d\left(35a^2cd^2e^4+35a^3e^6+21ac^2d^4e^2+5c^3d^6\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}\left(ae^2+cd^2\right)^4}+\frac{6a^2e^3+cdx\,(1}{24a^2\left(a+cx^2\right)}$$

```
[Out] (a*e + c*d*x)/(6*a*(c*d^2 + a*e^2)*(a + c*x^2)^3) + (6*a^2*e^3 + c*d*(5*c*d
^2 + 11*a*e^2)*x)/(24*a^2*(c*d^2 + a*e^2)^2*(a + c*x^2)^2) + (8*a^3*e^5 + c
*d*(5*c^2*d^4 + 16*a*c*d^2*e^2 + 19*a^2*e^4)*x)/(16*a^3*(c*d^2 + a*e^2)^3*(
a + c*x^2)) + (Sqrt[c]*d*(5*c^3*d^6 + 21*a*c^2*d^4*e^2 + 35*a^2*c*d^2*e^4 +
 35*a^3*e^6)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*(c*d^2 + a*e^2)^4) +
(e^7*Log[d + e*x])/(c*d^2 + a*e^2)^4 - (e^7*Log[a + c*x^2])/(2*(c*d^2 + a*e
^2)^4)
```

Rubi [A] time = 0.38205, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {741, 823, 801, 635, 205, 260}

$$\frac{cdx\left(19a^2e^4+16acd^2e^2+5c^2d^4\right)+8a^3e^5}{16a^3\left(a+cx^2\right)\left(ae^2+cd^2\right)^3}+\frac{\sqrt{c}d\left(35a^2cd^2e^4+35a^3e^6+21ac^2d^4e^2+5c^3d^6\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}\left(ae^2+cd^2\right)^4}+\frac{6a^2e^3+cdx\,(1}{24a^2\left(a+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + c*x^2)^4),x]
```

```
[Out] (a*e + c*d*x)/(6*a*(c*d^2 + a*e^2)*(a + c*x^2)^3) + (6*a^2*e^3 + c*d*(5*c*d
^2 + 11*a*e^2)*x)/(24*a^2*(c*d^2 + a*e^2)^2*(a + c*x^2)^2) + (8*a^3*e^5 + c
*d*(5*c^2*d^4 + 16*a*c*d^2*e^2 + 19*a^2*e^4)*x)/(16*a^3*(c*d^2 + a*e^2)^3*(
a + c*x^2)) + (Sqrt[c]*d*(5*c^3*d^6 + 21*a*c^2*d^4*e^2 + 35*a^2*c*d^2*e^4 +
 35*a^3*e^6)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*(c*d^2 + a*e^2)^4) +
(e^7*Log[d + e*x])/(c*d^2 + a*e^2)^4 - (e^7*Log[a + c*x^2])/(2*(c*d^2 + a*e
^2)^4)
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+cx^2\right)^4}\,dx = \frac{ae+cdx}{6a\left(cd^2+ae^2\right)\left(a+cx^2\right)^3} - \frac{\int \frac{-5cd^2-6ae^2-5cdex}{(d+ex)\left(a+cx^2\right)^3}\,dx}{6a\left(cd^2+ae^2\right)}$$

$$= \frac{ae+cdx}{6a\left(cd^2+ae^2\right)\left(a+cx^2\right)^3} + \frac{6a^2e^3+cd\left(5cd^2+11ae^2\right)x}{24a^2\left(cd^2+ae^2\right)^2\left(a+cx^2\right)^2} + \frac{\int \frac{3c\left(5c^2d^4+11acd^2e^2+8a^2e^4\right)+3c^2de(5\cdots}{(d+ex)\left(a+cx^2\right)^2}}{24a^2c\left(cd^2+ae^2\right)^2\cdots}$$

$$= \frac{ae+cdx}{6a\left(cd^2+ae^2\right)\left(a+cx^2\right)^3} + \frac{6a^2e^3+cd\left(5cd^2+11ae^2\right)x}{24a^2\left(cd^2+ae^2\right)^2\left(a+cx^2\right)^2} + \frac{8a^3e^5+cd\left(5c^2d^4+16acd^2e^2-\cdots\right.}{16a^3\left(cd^2+ae^2\right)^3(a+c\cdots}$$

$$= \frac{ae+cdx}{6a\left(cd^2+ae^2\right)\left(a+cx^2\right)^3} + \frac{6a^2e^3+cd\left(5cd^2+11ae^2\right)x}{24a^2\left(cd^2+ae^2\right)^2\left(a+cx^2\right)^2} + \frac{8a^3e^5+cd\left(5c^2d^4+16acd^2e^2-\cdots\right.}{16a^3\left(cd^2+ae^2\right)^3(a+c\cdots}$$

$$= \frac{ae+cdx}{6a\left(cd^2+ae^2\right)\left(a+cx^2\right)^3} + \frac{6a^2e^3+cd\left(5cd^2+11ae^2\right)x}{24a^2\left(cd^2+ae^2\right)^2\left(a+cx^2\right)^2} + \frac{8a^3e^5+cd\left(5c^2d^4+16acd^2e^2-\cdots\right.}{16a^3\left(cd^2+ae^2\right)^3(a+c\cdots}$$

$$= \frac{ae+cdx}{6a\left(cd^2+ae^2\right)\left(a+cx^2\right)^3} + \frac{6a^2e^3+cd\left(5cd^2+11ae^2\right)x}{24a^2\left(cd^2+ae^2\right)^2\left(a+cx^2\right)^2} + \frac{8a^3e^5+cd\left(5c^2d^4+16acd^2e^2-\cdots\right.}{16a^3\left(cd^2+ae^2\right)^3(a+c\cdots}$$

$$= \frac{ae+cdx}{6a\left(cd^2+ae^2\right)\left(a+cx^2\right)^3} + \frac{6a^2e^3+cd\left(5cd^2+11ae^2\right)x}{24a^2\left(cd^2+ae^2\right)^2\left(a+cx^2\right)^2} + \frac{8a^3e^5+cd\left(5c^2d^4+16acd^2e^2-\cdots\right.}{16a^3\left(cd^2+ae^2\right)^3(a+c\cdots}$$

Mathematica [A] time = 0.224831, size = 265, normalized size = 0.9

$$\frac{\frac{3\left(ae^2+cd^2\right)\left(19a^2cde^4x+8a^3e^5+16ac^2d^3e^2x+5c^3d^5x\right)}{a^3\left(a+cx^2\right)} + \frac{2\left(ae^2+cd^2\right)^2\left(6a^2e^3+11acde^2x+5c^2d^3x\right)}{a^2\left(a+cx^2\right)^2} + \frac{3\sqrt{c}d\left(35a^2cd^2e^4+35a^3e^6+21ac^2d^4e^2+5c^3d^6\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\cdots}{a^{7/2}}\cdots}{48\left(ae^2+cd^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + c*x^2)^4),x]
```

```
[Out] ((8*(c*d^2 + a*e^2)^3*(a*e + c*d*x))/(a*(a + c*x^2)^3) + (2*(c*d^2 + a*e^2)
^2*(6*a^2*e^3 + 5*c^2*d^3*x + 11*a*c*d*e^2*x))/(a^2*(a + c*x^2)^2) + (3*(c*
d^2 + a*e^2)*(8*a^3*e^5 + 5*c^3*d^5*x + 16*a*c^2*d^3*e^2*x + 19*a^2*c*d*e^4
*x))/(a^3*(a + c*x^2)) + (3*Sqrt[c]*d*(5*c^3*d^6 + 21*a*c^2*d^4*e^2 + 35*a^
2*c*d^2*e^4 + 35*a^3*e^6)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(7/2) + 48*e^7*Log
[d + e*x] - 24*e^7*Log[a + c*x^2])/(48*(c*d^2 + a*e^2)^4)
```

Maple [B] time = 0.062, size = 941, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+a)^4,x)
```

```
[Out] 3/2*c^2/(a*e^2+c*d^2)^4/(c*x^2+a)^3*x^2*a*d^2*e^5+61/16*c^2/(a*e^2+c*d^2)^4
/(c*x^2+a)^3*d^3*a*x*e^4+29/16*c/(a*e^2+c*d^2)^4/(c*x^2+a)^3*d*a^2*x*e^6+21
/16*c^3/(a*e^2+c*d^2)^4/a^2/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*d^5*e^2+35/
16*c^2/(a*e^2+c*d^2)^4/a/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*d^3*e^4+1/6*c^
3/(a*e^2+c*d^2)^4/(c*x^2+a)^3*e*d^6+11/12/(a*e^2+c*d^2)^4/(c*x^2+a)^3*a^3*e
^7+35/16*c^4/(a*e^2+c*d^2)^4/(c*x^2+a)^3*d^3/a*x^5*e^4+21/16*c^5/(a*e^2+c*d
^2)^4/(c*x^2+a)^3*d^5/a^2*x^5*e^2+17/6*c^2/(a*e^2+c*d^2)^4/(c*x^2+a)^3*d*a*
x^3*e^6+7/2*c^4/(a*e^2+c*d^2)^4/(c*x^2+a)^3*d^5/a*x^3*e^2+35/16*c/(a*e^2+c*
d^2)^4/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*d*e^6+5/16*c^4/(a*e^2+c*d^2)^4/a
^3/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*d^7+5/6*c^5/(a*e^2+c*d^2)^4/(c*x^2+a
)^3*d^7/a^2*x^3+1/4*c^3/(a*e^2+c*d^2)^4/(c*x^2+a)^3*x^2*d^4*e^3+43/16*c^3/(
a*e^2+c*d^2)^4/(c*x^2+a)^3*d^5*x*e^2+11/16*c^4/(a*e^2+c*d^2)^4/(c*x^2+a)^3*
d^7/a*x+3/4*c^2/(a*e^2+c*d^2)^4/(c*x^2+a)^3*e^3*d^4*a+19/16*c^3/(a*e^2+c*d^
2)^4/(c*x^2+a)^3*d*x^5*e^6+5/16*c^6/(a*e^2+c*d^2)^4/(c*x^2+a)^3*d^7/a^3*x^5
+1/2*c^2/(a*e^2+c*d^2)^4/(c*x^2+a)^3*x^4*a*e^7+1/2*c^3/(a*e^2+c*d^2)^4/(c*x
^2+a)^3*x^4*d^2*e^5+11/2*c^3/(a*e^2+c*d^2)^4/(c*x^2+a)^3*d^3*x^3*e^4+5/4*c/
(a*e^2+c*d^2)^4/(c*x^2+a)^3*x^2*a^2*e^7+3/2*c/(a*e^2+c*d^2)^4/(c*x^2+a)^3*a
^2*d^2*e^5+e^7*ln(e*x+d)/(a*e^2+c*d^2)^4-1/2*e^7*ln(c*x^2+a)/(a*e^2+c*d^2)^
4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 97.5941, size = 3580, normalized size = 12.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(16*a^3*c^3*d^6*e + 72*a^4*c^2*d^4*e^3 + 144*a^5*c*d^2*e^5 + 88*a^6*e
^7 + 6*(5*c^6*d^7 + 21*a*c^5*d^5*e^2 + 35*a^2*c^4*d^3*e^4 + 19*a^3*c^3*d*e^
6)*x^5 + 48*(a^3*c^3*d^2*e^5 + a^4*c^2*e^7)*x^4 + 16*(5*a*c^5*d^7 + 21*a^2*
c^4*d^5*e^2 + 33*a^3*c^3*d^3*e^4 + 17*a^4*c^2*d*e^6)*x^3 + 24*(a^3*c^3*d^4*
e^3 + 6*a^4*c^2*d^2*e^5 + 5*a^5*c*e^7)*x^2 + 3*(5*a^3*c^3*d^7 + 21*a^4*c^2*
d^5*e^2 + 35*a^5*c*d^3*e^4 + 35*a^6*d*e^6 + (5*c^6*d^7 + 21*a*c^5*d^5*e^2 +
 35*a^2*c^4*d^3*e^4 + 35*a^3*c^3*d*e^6)*x^6 + 3*(5*a*c^5*d^7 + 21*a^2*c^4*d
^5*e^2 + 35*a^3*c^3*d^3*e^4 + 35*a^4*c^2*d*e^6)*x^4 + 3*(5*a^2*c^4*d^7 + 21
*a^3*c^3*d^5*e^2 + 35*a^4*c^2*d^3*e^4 + 35*a^5*c*d*e^6)*x^2)*sqrt(-c/a)*log
((c*x^2 + 2*a*x*sqrt(-c/a) - a)/(c*x^2 + a)) + 6*(11*a^2*c^4*d^7 + 43*a^3*c
^3*d^5*e^2 + 61*a^4*c^2*d^3*e^4 + 29*a^5*c*d*e^6)*x - 48*(a^3*c^3*e^7*x^6 +
 3*a^4*c^2*e^7*x^4 + 3*a^5*c*e^7*x^2 + a^6*e^7)*log(c*x^2 + a) + 96*(a^3*c^
3*e^7*x^6 + 3*a^4*c^2*e^7*x^4 + 3*a^5*c*e^7*x^2 + a^6*e^7)*log(e*x + d))/(a
^6*c^4*d^8 + 4*a^7*c^3*d^6*e^2 + 6*a^8*c^2*d^4*e^4 + 4*a^9*c*d^2*e^6 + a^10
*e^8 + (a^3*c^7*d^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2
*e^6 + a^7*c^3*e^8)*x^6 + 3*(a^4*c^6*d^8 + 4*a^5*c^5*d^6*e^2 + 6*a^6*c^4*d^
4*e^4 + 4*a^7*c^3*d^2*e^6 + a^8*c^2*e^8)*x^4 + 3*(a^5*c^5*d^8 + 4*a^6*c^4*d
^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6 + a^9*c*e^8)*x^2), 1/48*(8*a
^3*c^3*d^6*e + 36*a^4*c^2*d^4*e^3 + 72*a^5*c*d^2*e^5 + 44*a^6*e^7 + 3*(5*c^
6*d^7 + 21*a*c^5*d^5*e^2 + 35*a^2*c^4*d^3*e^4 + 19*a^3*c^3*d*e^6)*x^5 + 24*
(a^3*c^3*d^2*e^5 + a^4*c^2*e^7)*x^4 + 8*(5*a*c^5*d^7 + 21*a^2*c^4*d^5*e^2 +
 33*a^3*c^3*d^3*e^4 + 17*a^4*c^2*d*e^6)*x^3 + 12*(a^3*c^3*d^4*e^3 + 6*a^4*c
^2*d^2*e^5 + 5*a^5*c*e^7)*x^2 + 3*(5*a^3*c^3*d^7 + 21*a^4*c^2*d^5*e^2 + 35*
a^5*c*d^3*e^4 + 35*a^6*d*e^6 + (5*c^6*d^7 + 21*a*c^5*d^5*e^2 + 35*a^2*c^4*d
^3*e^4 + 35*a^3*c^3*d*e^6)*x^6 + 3*(5*a*c^5*d^7 + 21*a^2*c^4*d^5*e^2 + 35*a
^3*c^3*d^3*e^4 + 35*a^4*c^2*d*e^6)*x^4 + 3*(5*a^2*c^4*d^7 + 21*a^3*c^3*d^5*
e^2 + 35*a^4*c^2*d^3*e^4 + 35*a^5*c*d*e^6)*x^2)*sqrt(c/a)*arctan(x*sqrt(c/a
)) + 3*(11*a^2*c^4*d^7 + 43*a^3*c^3*d^5*e^2 + 61*a^4*c^2*d^3*e^4 + 29*a^5*c
*d*e^6)*x - 24*(a^3*c^3*e^7*x^6 + 3*a^4*c^2*e^7*x^4 + 3*a^5*c*e^7*x^2 + a^6
*e^7)*log(c*x^2 + a) + 48*(a^3*c^3*e^7*x^6 + 3*a^4*c^2*e^7*x^4 + 3*a^5*c*e^
7*x^2 + a^6*e^7)*log(e*x + d))/(a^6*c^4*d^8 + 4*a^7*c^3*d^6*e^2 + 6*a^8*c^2
*d^4*e^4 + 4*a^9*c*d^2*e^6 + a^10*e^8 + (a^3*c^7*d^8 + 4*a^4*c^6*d^6*e^2 +
6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6 + a^7*c^3*e^8)*x^6 + 3*(a^4*c^6*d^8 +
 4*a^5*c^5*d^6*e^2 + 6*a^6*c^4*d^4*e^4 + 4*a^7*c^3*d^2*e^6 + a^8*c^2*e^8)*x
^4 + 3*(a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2
*e^6 + a^9*c*e^8)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37122, size = 716, normalized size = 2.43

$$-\frac{e^7 \log\left(cx^2+a\right)}{2\left(c^4d^8+4\,ac^3d^6e^2+6\,a^2c^2d^4e^4+4\,a^3cd^2e^6+a^4e^8\right)}+\frac{e^8 \log\left(|xe+d|\right)}{c^4d^8e+4\,ac^3d^6e^3+6\,a^2c^2d^4e^5+4\,a^3cd^2e^7+a^4e^9}+\frac{\left(5\,c^4d^7+\right.}{16\left(a^3c^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] -1/2*e^7*log(c*x^2 + a)/(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*
a^3*c*d^2*e^6 + a^4*e^8) + e^8*log(abs(x*e + d))/(c^4*d^8*e + 4*a*c^3*d^6*e
^3 + 6*a^2*c^2*d^4*e^5 + 4*a^3*c*d^2*e^7 + a^4*e^9) + 1/16*(5*c^4*d^7 + 21*
a*c^3*d^5*e^2 + 35*a^2*c^2*d^3*e^4 + 35*a^3*c*d*e^6)*arctan(c*x/sqrt(a*c))/
((a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a
^7*e^8)*sqrt(a*c)) + 1/48*(8*a^3*c^3*d^6*e + 36*a^4*c^2*d^4*e^3 + 72*a^5*c*
d^2*e^5 + 44*a^6*e^7 + 3*(5*c^6*d^7 + 21*a*c^5*d^5*e^2 + 35*a^2*c^4*d^3*e^4
 + 19*a^3*c^3*d*e^6)*x^5 + 24*(a^3*c^3*d^2*e^5 + a^4*c^2*e^7)*x^4 + 8*(5*a*
c^5*d^7 + 21*a^2*c^4*d^5*e^2 + 33*a^3*c^3*d^3*e^4 + 17*a^4*c^2*d*e^6)*x^3 +
 12*(a^3*c^3*d^4*e^3 + 6*a^4*c^2*d^2*e^5 + 5*a^5*c*e^7)*x^2 + 3*(11*a^2*c^4
*d^7 + 43*a^3*c^3*d^5*e^2 + 61*a^4*c^2*d^3*e^4 + 29*a^5*c*d*e^6)*x)/((c*d^2
 + a*e^2)^4*(c*x^2 + a)^3*a^3)
```

3.524 $\int \frac{1}{(d+ex)^2 (a+cx^2)^4} dx$

Optimal. Leaf size=430

$$-\frac{ae\left(5cd^2-7ae^2\right)\left(5ae^2+cd^2\right)-3cdx\left(29a^2e^4+18acd^2e^2+5c^2d^4\right)}{48a^3\left(a+cx^2\right)(d+ex)\left(ae^2+cd^2\right)^3}+\frac{e\left(47a^2cd^2e^4-35a^3e^6+23ac^2d^4e^2+5c^3d^6\right)}{16a^3(d+ex)\left(ae^2+cd^2\right)^4}+\cdots$$

```
[Out] (e*(5*c^3*d^6 + 23*a*c^2*d^4*e^2 + 47*a^2*c*d^2*e^4 - 35*a^3*e^6))/(16*a^3*
(c*d^2 + a*e^2)^4*(d + e*x)) + (a*e + c*d*x)/(6*a*(c*d^2 + a*e^2)*(d + e*x)
*(a + c*x^2)^3) - (a*e*(c*d^2 - 7*a*e^2) - c*d*(5*c*d^2 + 13*a*e^2)*x)/(24*
a^2*(c*d^2 + a*e^2)^2*(d + e*x)*(a + c*x^2)^2) - (a*e*(5*c*d^2 - 7*a*e^2)*(
c*d^2 + 5*a*e^2) - 3*c*d*(5*c^2*d^4 + 18*a*c*d^2*e^2 + 29*a^2*e^4)*x)/(48*a
^3*(c*d^2 + a*e^2)^3*(d + e*x)*(a + c*x^2)) + (Sqrt[c]*(5*c^4*d^8 + 28*a*c^
3*d^6*e^2 + 70*a^2*c^2*d^4*e^4 + 140*a^3*c*d^2*e^6 - 35*a^4*e^8)*ArcTan[(Sq
rt[c]*x)/Sqrt[a]])/(16*a^(7/2)*(c*d^2 + a*e^2)^5) + (8*c*d*e^7*Log[d + e*x]
)/(c*d^2 + a*e^2)^5 - (4*c*d*e^7*Log[a + c*x^2])/(c*d^2 + a*e^2)^5
```

Rubi [A] time = 0.533842, antiderivative size = 430, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {741, 823, 801, 635, 205, 260}

$$-\frac{ae\left(5cd^2-7ae^2\right)\left(5ae^2+cd^2\right)-3cdx\left(29a^2e^4+18acd^2e^2+5c^2d^4\right)}{48a^3\left(a+cx^2\right)(d+ex)\left(ae^2+cd^2\right)^3}+\frac{e\left(47a^2cd^2e^4-35a^3e^6+23ac^2d^4e^2+5c^3d^6\right)}{16a^3(d+ex)\left(ae^2+cd^2\right)^4}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + c*x^2)^4),x]
```

```
[Out] (e*(5*c^3*d^6 + 23*a*c^2*d^4*e^2 + 47*a^2*c*d^2*e^4 - 35*a^3*e^6))/(16*a^3*
(c*d^2 + a*e^2)^4*(d + e*x)) + (a*e + c*d*x)/(6*a*(c*d^2 + a*e^2)*(d + e*x)
*(a + c*x^2)^3) - (a*e*(c*d^2 - 7*a*e^2) - c*d*(5*c*d^2 + 13*a*e^2)*x)/(24*
a^2*(c*d^2 + a*e^2)^2*(d + e*x)*(a + c*x^2)^2) - (a*e*(5*c*d^2 - 7*a*e^2)*(
c*d^2 + 5*a*e^2) - 3*c*d*(5*c^2*d^4 + 18*a*c*d^2*e^2 + 29*a^2*e^4)*x)/(48*a
^3*(c*d^2 + a*e^2)^3*(d + e*x)*(a + c*x^2)) + (Sqrt[c]*(5*c^4*d^8 + 28*a*c^
3*d^6*e^2 + 70*a^2*c^2*d^4*e^4 + 140*a^3*c*d^2*e^6 - 35*a^4*e^8)*ArcTan[(Sq
rt[c]*x)/Sqrt[a]])/(16*a^(7/2)*(c*d^2 + a*e^2)^5) + (8*c*d*e^7*Log[d + e*x]
)/(c*d^2 + a*e^2)^5 - (4*c*d*e^7*Log[a + c*x^2])/(c*d^2 + a*e^2)^5
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
```

```
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+cx^2\right)^4}\,dx = \frac{ae+cdx}{6a\left(cd^2+ae^2\right)(d+ex)\left(a+cx^2\right)^3} - \frac{\int \frac{-5cd^2-7ae^2-6cdex}{(d+ex)^2\left(a+cx^2\right)^3}\,dx}{6a\left(cd^2+ae^2\right)}$$

$$= \frac{ae+cdx}{6a\left(cd^2+ae^2\right)(d+ex)\left(a+cx^2\right)^3} - \frac{ae\left(cd^2-7ae^2\right)-cd\left(5cd^2+13ae^2\right)x}{24a^2\left(cd^2+ae^2\right)^2(d+ex)\left(a+cx^2\right)^2} + \frac{\int \frac{c\left(15c^2d^4+34acd^2 \cdots\right.}{(\cdots}}{24a \cdots}$$

$$= \frac{ae+cdx}{6a\left(cd^2+ae^2\right)(d+ex)\left(a+cx^2\right)^3} - \frac{ae\left(cd^2-7ae^2\right)-cd\left(5cd^2+13ae^2\right)x}{24a^2\left(cd^2+ae^2\right)^2(d+ex)\left(a+cx^2\right)^2} - \frac{ae\left(5cd^2-7ae^2\right) \cdots}{4 \cdots}$$

$$= \frac{ae+cdx}{6a\left(cd^2+ae^2\right)(d+ex)\left(a+cx^2\right)^3} - \frac{ae\left(cd^2-7ae^2\right)-cd\left(5cd^2+13ae^2\right)x}{24a^2\left(cd^2+ae^2\right)^2(d+ex)\left(a+cx^2\right)^2} - \frac{ae\left(5cd^2-7ae^2\right) \cdots}{4 \cdots}$$

$$= \frac{e\left(5c^3d^6+23ac^2d^4e^2+47a^2cd^2e^4-35a^3e^6\right)}{16a^3\left(cd^2+ae^2\right)^4(d+ex)} + \frac{ae+cdx}{6a\left(cd^2+ae^2\right)(d+ex)\left(a+cx^2\right)^3} - \frac{ae\left(cd^2 - \cdots\right.}{24a^2\left(cd \cdots\right.}$$

$$= \frac{e\left(5c^3d^6+23ac^2d^4e^2+47a^2cd^2e^4-35a^3e^6\right)}{16a^3\left(cd^2+ae^2\right)^4(d+ex)} + \frac{ae+cdx}{6a\left(cd^2+ae^2\right)(d+ex)\left(a+cx^2\right)^3} - \frac{ae\left(cd^2 - \cdots\right.}{24a^2\left(cd \cdots\right.}$$

$$= \frac{e\left(5c^3d^6+23ac^2d^4e^2+47a^2cd^2e^4-35a^3e^6\right)}{16a^3\left(cd^2+ae^2\right)^4(d+ex)} + \frac{ae+cdx}{6a\left(cd^2+ae^2\right)(d+ex)\left(a+cx^2\right)^3} - \frac{ae\left(cd^2 - \cdots\right.}{24a^2\left(cd \cdots\right.}$$

Mathematica [A] time = 0.43234, size = 336, normalized size = 0.78

$$\frac{\frac{3c\left(ae^2+cd^2\right)\left(47a^2cd^2e^4x+a^3e^5(48d-19ex)+23ac^2d^4e^2x+5c^3d^6x\right)}{a^3\left(a+cx^2\right)}+\frac{2c\left(ae^2+cd^2\right)^2\left(a^2e^3(24d-11ex)+18acd^2e^2x+5c^2d^4x\right)}{a^2\left(a+cx^2\right)^2}+\frac{3\sqrt{c}\left(70a^2c^2d^4e^4+140a^3cd^2e^6-35\cdots\right.}{\cdots}}{48\left(ae^2+\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + c*x^2)^4),x]
```

```
[Out] ((-48*e^7*(c*d^2 + a*e^2))/(d + e*x) + (3*c*(c*d^2 + a*e^2)*(5*c^3*d^6*x +
23*a*c^2*d^4*e^2*x + 47*a^2*c*d^2*e^4*x + a^3*e^5*(48*d - 19*e*x)))/(a^3*(a
 + c*x^2)) + (2*c*(c*d^2 + a*e^2)^2*(5*c^2*d^4*x + 18*a*c*d^2*e^2*x + a^2*e
^3*(24*d - 11*e*x)))/(a^2*(a + c*x^2)^2) + (8*c*(c*d^2 + a*e^2)^3*(c*d^2*x
+ a*e*(2*d - e*x)))/(a*(a + c*x^2)^3) + (3*Sqrt[c]*(5*c^4*d^8 + 28*a*c^3*d^
6*e^2 + 70*a^2*c^2*d^4*e^4 + 140*a^3*c*d^2*e^6 - 35*a^4*e^8)*ArcTan[(Sqrt[c
]*x)/Sqrt[a]])/a^(7/2) + 384*c*d*e^7*Log[d + e*x] - 192*c*d*e^7*Log[a + c*x
^2])/(48*(c*d^2 + a*e^2)^5)
```

Maple [B] time = 0.063, size = 1126, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^2+a)^4,x)
```

```
[Out] 1/3*c^4/(a*e^2+c*d^2)^5/(c*x^2+a)^3*d^7*e-17/6*c^2/(a*e^2+c*d^2)^5/(c*x^2+a
)^3*a^2*x^3*e^8+10*c^4/(a*e^2+c*d^2)^5/(c*x^2+a)^3*x^3*d^4*e^4+5/6*c^6/(a*e
^2+c*d^2)^5/(c*x^2+a)^3/a^2*x^3*d^8+5/16*c^7/(a*e^2+c*d^2)^5/(c*x^2+a)^3/a^
3*x^5*d^8+3*c^4/(a*e^2+c*d^2)^5/(c*x^2+a)^3*x^4*d^3*e^5+45/8*c^3/(a*e^2+c*d
^2)^5/(c*x^2+a)^3*x*a*d^4*e^4+35/8*c^3/(a*e^2+c*d^2)^5/a/(a*c)^(1/2)*arctan
(x*c/(a*c)^(1/2))*d^4*e^4+7/4*c^4/(a*e^2+c*d^2)^5/a^2/(a*c)^(1/2)*arctan(x*
c/(a*c)^(1/2))*d^6*e^2+35/8*c^5/(a*e^2+c*d^2)^5/(c*x^2+a)^3/a*x^5*d^4*e^4+7
/4*c^6/(a*e^2+c*d^2)^5/(c*x^2+a)^3/a^2*x^5*d^6*e^2+3*c^3/(a*e^2+c*d^2)^5/(c
*x^2+a)^3*x^4*a*d*e^7+10/3*c^3/(a*e^2+c*d^2)^5/(c*x^2+a)^3*a*x^3*d^2*e^6+14
/3*c^5/(a*e^2+c*d^2)^5/(c*x^2+a)^3/a*x^3*d^6*e^2+7*c^2/(a*e^2+c*d^2)^5/(c*x
^2+a)^3*x^2*a^2*d*e^7+8*c^3/(a*e^2+c*d^2)^5/(c*x^2+a)^3*x^2*a*d^3*e^5+5/4*c
^2/(a*e^2+c*d^2)^5/(c*x^2+a)^3*x*a^2*d^2*e^6-e^7/(a*e^2+c*d^2)^4/(e*x+d)+c^
4/(a*e^2+c*d^2)^5/(c*x^2+a)^3*x^2*d^5*e^3+13/4*c^4/(a*e^2+c*d^2)^5/(c*x^2+a
)^3*x*d^6*e^2+11/16*c^5/(a*e^2+c*d^2)^5/(c*x^2+a)^3*x/a*d^8-19/16*c^3/(a*e^
2+c*d^2)^5/(c*x^2+a)^3*a*x^5*e^8+6*c^2/(a*e^2+c*d^2)^5/(c*x^2+a)^3*a^2*d^3*
e^5+2*c^3/(a*e^2+c*d^2)^5/(c*x^2+a)^3*a*d^5*e^3-29/16*c/(a*e^2+c*d^2)^5/(c*
x^2+a)^3*x*a^3*e^8+13/3*c/(a*e^2+c*d^2)^5/(c*x^2+a)^3*a^3*d*e^7+35/4*c^2/(a
*e^2+c*d^2)^5/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*d^2*e^6-35/16*c/(a*e^2+c*
d^2)^5*a/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*e^8+5/16*c^5/(a*e^2+c*d^2)^5/a
^3/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*d^8+7/4*c^4/(a*e^2+c*d^2)^5/(c*x^2+a
)^3*x^5*d^2*e^6+8*c*d*e^7*ln(e*x+d)/(a*e^2+c*d^2)^5-4*c*d*e^7*ln(c*x^2+a)/(
a*e^2+c*d^2)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**2+a)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.35452, size = 1156, normalized size = 2.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] -4*c*d*e^7*log(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(x*e + d)^2)
/(c^5*d^10 + 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 + 10*a^3*c^2*d^4*e^6 + 5*
a^4*c*d^2*e^8 + a^5*e^10) + 1/16*(5*c^5*d^8*e^2 + 28*a*c^4*d^6*e^4 + 70*a^2
*c^3*d^4*e^6 + 140*a^3*c^2*d^2*e^8 - 35*a^4*c*e^10)*arctan((c*d - c*d^2/(x*
e + d) - a*e^2/(x*e + d))*e^(-1)/sqrt(a*c))*e^(-2)/((a^3*c^5*d^10 + 5*a^4*c
^4*d^8*e^2 + 10*a^5*c^3*d^6*e^4 + 10*a^6*c^2*d^4*e^6 + 5*a^7*c*d^2*e^8 + a^
8*e^10)*sqrt(a*c)) - e^15/((c^4*d^8*e^8 + 4*a*c^3*d^6*e^10 + 6*a^2*c^2*d^4*
e^12 + 4*a^3*c*d^2*e^14 + a^4*e^16)*(x*e + d)) + 1/48*(15*c^7*d^7*e + 79*a*
c^6*d^5*e^3 + 185*a^2*c^5*d^3*e^5 - 295*a^3*c^4*d*e^7 - 3*(25*c^7*d^8*e^2 +
 130*a*c^6*d^6*e^4 + 300*a^2*c^5*d^4*e^6 - 618*a^3*c^4*d^2*e^8 + 19*a^4*c^3
*e^10)*e^(-1)/(x*e + d) + 6*(25*c^7*d^9*e^3 + 135*a*c^6*d^7*e^5 + 327*a^2*c
^5*d^5*e^7 - 691*a^3*c^4*d^3*e^9 - 76*a^4*c^3*d*e^11)*e^(-2)/(x*e + d)^2 -
2*(75*c^7*d^10*e^4 + 440*a*c^6*d^8*e^6 + 1162*a^2*c^5*d^6*e^8 - 2212*a^3*c^
4*d^4*e^10 - 1277*a^4*c^3*d^2*e^12 + 68*a^5*c^2*e^14)*e^(-3)/(x*e + d)^3 +
3*(25*c^7*d^11*e^5 + 165*a*c^6*d^9*e^7 + 490*a^2*c^5*d^7*e^9 - 742*a^3*c^4*
d^5*e^11 - 1139*a^4*c^3*d^3*e^13 - 47*a^5*c^2*d*e^15)*e^(-4)/(x*e + d)^4 -
3*(5*c^7*d^12*e^6 + 38*a*c^6*d^10*e^8 + 131*a^2*c^5*d^8*e^10 - 140*a^3*c^4*
```

```
d^6*e^12 - 517*a^4*c^3*d^4*e^14 - 250*a^5*c^2*d^2*e^16 + 29*a^6*c*e^18)*e^(
-5)/(x*e + d)^5)/((c*d^2 + a*e^2)^5*a^3*(c - 2*c*d/(x*e + d) + c*d^2/(x*e +
 d)^2 + a*e^2/(x*e + d)^2)^3)
```

3.525 $\int (d+ex)^4\sqrt{a+cx^2}\,dx$

Optimal. Leaf size=207

$$\frac{x\sqrt{a+cx^2}\left(a^2e^4-12acd^2e^2+8c^2d^4\right)}{16c^2}+\frac{a\left(a^2e^4-12acd^2e^2+8c^2d^4\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16c^{5/2}}+\frac{e\left(a+cx^2\right)^{3/2}\left(3ex\left(16cd^2-5ae^2\right)\right.}{120c^2}$$

```
[Out] ((8*c^2*d^4 - 12*a*c*d^2*e^2 + a^2*e^4)*x*Sqrt[a + c*x^2])/(16*c^2) + (3*d*
e*(d + e*x)^2*(a + c*x^2)^(3/2))/(10*c) + (e*(d + e*x)^3*(a + c*x^2)^(3/2))
/(6*c) + (e*(8*d*(13*c*d^2 - 8*a*e^2) + 3*e*(16*c*d^2 - 5*a*e^2)*x)*(a + c*
x^2)^(3/2))/(120*c^2) + (a*(8*c^2*d^4 - 12*a*c*d^2*e^2 + a^2*e^4)*ArcTanh[(
Sqrt[c]*x)/Sqrt[a + c*x^2]])/(16*c^(5/2))
```

Rubi [A] time = 0.208813, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {743, 833, 780, 195, 217, 206}

$$\frac{x\sqrt{a+cx^2}\left(a^2e^4-12acd^2e^2+8c^2d^4\right)}{16c^2}+\frac{a\left(a^2e^4-12acd^2e^2+8c^2d^4\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16c^{5/2}}+\frac{e\left(a+cx^2\right)^{3/2}\left(3ex\left(16cd^2-5ae^2\right)\right.}{120c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4*Sqrt[a + c*x^2],x]
```

```
[Out] ((8*c^2*d^4 - 12*a*c*d^2*e^2 + a^2*e^4)*x*Sqrt[a + c*x^2])/(16*c^2) + (3*d*
e*(d + e*x)^2*(a + c*x^2)^(3/2))/(10*c) + (e*(d + e*x)^3*(a + c*x^2)^(3/2))
/(6*c) + (e*(8*d*(13*c*d^2 - 8*a*e^2) + 3*e*(16*c*d^2 - 5*a*e^2)*x)*(a + c*
x^2)^(3/2))/(120*c^2) + (a*(8*c^2*d^4 - 12*a*c*d^2*e^2 + a^2*e^4)*ArcTanh[(
Sqrt[c]*x)/Sqrt[a + c*x^2]])/(16*c^(5/2))
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^4\sqrt{a+cx^2}\,dx = \frac{e(d+ex)^3\left(a+cx^2\right)^{3/2}}{6c}+\frac{\int (d+ex)^2\left(3\left(2cd^2-ae^2\right)+9cdex\right)\sqrt{a+cx^2}\,dx}{6c}$$
$$= \frac{3de(d+ex)^2\left(a+cx^2\right)^{3/2}}{10c}+\frac{e(d+ex)^3\left(a+cx^2\right)^{3/2}}{6c}+\frac{\int (d+ex)\left(3cd\left(10cd^2-11ae^2\right)+3ce\left(\right.\right.}{30c^2}$$
$$= \frac{3de(d+ex)^2\left(a+cx^2\right)^{3/2}}{10c}+\frac{e(d+ex)^3\left(a+cx^2\right)^{3/2}}{6c}+\frac{e\left(8d\left(13cd^2-8ae^2\right)+3e\left(16cd^2-5ae\right.\right.}{120c^2}$$
$$= \frac{\left(8c^2d^4-12acd^2e^2+a^2e^4\right)x\sqrt{a+cx^2}}{16c^2}+\frac{3de(d+ex)^2\left(a+cx^2\right)^{3/2}}{10c}+\frac{e(d+ex)^3\left(a+cx^2\right)^{3/2}}{6c}$$
$$= \frac{\left(8c^2d^4-12acd^2e^2+a^2e^4\right)x\sqrt{a+cx^2}}{16c^2}+\frac{3de(d+ex)^2\left(a+cx^2\right)^{3/2}}{10c}+\frac{e(d+ex)^3\left(a+cx^2\right)^{3/2}}{6c}$$
$$= \frac{\left(8c^2d^4-12acd^2e^2+a^2e^4\right)x\sqrt{a+cx^2}}{16c^2}+\frac{3de(d+ex)^2\left(a+cx^2\right)^{3/2}}{10c}+\frac{e(d+ex)^3\left(a+cx^2\right)^{3/2}}{6c}$$

Mathematica [A] time = 0.133395, size = 177, normalized size = 0.86

$$\frac{\sqrt{c}\sqrt{a+cx^2}\left(-a^2e^3(128d+15ex)+2ace\left(90d^2ex+160d^3+32de^2x^2+5e^3x^3\right)+8c^2x\left(45d^2e^2x^2+40d^3ex+15d^4+24\right.\right.}{240c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4*Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[a + c*x^2]*(-(a^2*e^3*(128*d + 15*e*x)) + 2*a*c*e*(160*d^3 +
90*d^2*e*x + 32*d*e^2*x^2 + 5*e^3*x^3) + 8*c^2*x*(15*d^4 + 40*d^3*e*x + 45*
d^2*e^2*x^2 + 24*d*e^3*x^3 + 5*e^4*x^4)) + 15*a*(8*c^2*d^4 - 12*a*c*d^2*e^2
 + a^2*e^4)*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/(240*c^(5/2))
```

Maple [A] time = 0.051, size = 260, normalized size = 1.3

$$\frac{e^4x^3}{6c}\left(cx^2+a\right)^{\frac{3}{2}}-\frac{e^4ax}{8c^2}\left(cx^2+a\right)^{\frac{3}{2}}+\frac{a^2e^4x}{16c^2}\sqrt{cx^2+a}+\frac{e^4a^3}{16}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)c^{-\frac{5}{2}}+\frac{4de^3x^2}{5c}\left(cx^2+a\right)^{\frac{3}{2}}-\frac{8de^3a}{15c^2}\left(cx^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(c*x^2+a)^(1/2),x)
```

```
[Out] 1/6*e^4*x^3*(c*x^2+a)^(3/2)/c-1/8*e^4*a/c^2*x*(c*x^2+a)^(3/2)+1/16*e^4*a^2/
c^2*x*(c*x^2+a)^(1/2)+1/16*e^4*a^3/c^(5/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))+4/
5*d*e^3*x^2*(c*x^2+a)^(3/2)/c-8/15*d*e^3*a/c^2*(c*x^2+a)^(3/2)+3/2*d^2*e^2*
x*(c*x^2+a)^(3/2)/c-3/4*d^2*e^2*a/c*x*(c*x^2+a)^(1/2)-3/4*d^2*e^2*a^2/c^(3/
2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))+4/3*d^3*e*(c*x^2+a)^(3/2)/c+1/2*d^4*x*(c*x
^2+a)^(1/2)+1/2*d^4*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.71369, size = 887, normalized size = 4.29

$$\left[\frac{15\left(8ac^2d^4-12a^2cd^2e^2+a^3e^4\right)\sqrt{c}\log\left(-2cx^2-2\sqrt{cx^2+a}\sqrt{c}x-a\right)+2\left(40c^3e^4x^5+192c^3de^3x^4+320ac^2d^3e-128a^2\right.}{480c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/480*(15*(8*a*c^2*d^4 - 12*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(c)*log(-2*c*x^2
- 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(40*c^3*e^4*x^5 + 192*c^3*d*e^3*x^4
+ 320*a*c^2*d^3*e - 128*a^2*c*d*e^3 + 10*(36*c^3*d^2*e^2 + a*c^2*e^4)*x^3 +
 64*(5*c^3*d^3*e + a*c^2*d*e^3)*x^2 + 15*(8*c^3*d^4 + 12*a*c^2*d^2*e^2 - a^
2*c*e^4)*x)*sqrt(c*x^2 + a))/c^3, -1/240*(15*(8*a*c^2*d^4 - 12*a^2*c*d^2*e^
2 + a^3*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (40*c^3*e^4*x^5
+ 192*c^3*d*e^3*x^4 + 320*a*c^2*d^3*e - 128*a^2*c*d*e^3 + 10*(36*c^3*d^2*e^
2 + a*c^2*e^4)*x^3 + 64*(5*c^3*d^3*e + a*c^2*d*e^3)*x^2 + 15*(8*c^3*d^4 + 1
2*a*c^2*d^2*e^2 - a^2*c*e^4)*x)*sqrt(c*x^2 + a))/c^3]
```

Sympy [A] time = 11.85, size = 411, normalized size = 1.99

$$-\frac{a^{\frac{5}{2}}e^4x}{16c^2\sqrt{1+\frac{cx^2}{a}}}+\frac{3a^{\frac{3}{2}}d^2e^2x}{4c\sqrt{1+\frac{cx^2}{a}}}-\frac{a^{\frac{3}{2}}e^4x^3}{48c\sqrt{1+\frac{cx^2}{a}}}+\frac{\sqrt{a}d^4x\sqrt{1+\frac{cx^2}{a}}}{2}+\frac{9\sqrt{a}d^2e^2x^3}{4\sqrt{1+\frac{cx^2}{a}}}+\frac{5\sqrt{a}e^4x^5}{24\sqrt{1+\frac{cx^2}{a}}}+\frac{a^3e^4\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16c^{\frac{5}{2}}}-\frac{3a^2d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(c*x**2+a)**(1/2),x)
```

```
[Out] -a**(5/2)*e**4*x/(16*c**2*sqrt(1 + c*x**2/a)) + 3*a**(3/2)*d**2*e**2*x/(4*c
*sqrt(1 + c*x**2/a)) - a**(3/2)*e**4*x**3/(48*c*sqrt(1 + c*x**2/a)) + sqrt(
a)*d**4*x*sqrt(1 + c*x**2/a)/2 + 9*sqrt(a)*d**2*e**2*x**3/(4*sqrt(1 + c*x**
2/a)) + 5*sqrt(a)*e**4*x**5/(24*sqrt(1 + c*x**2/a)) + a**3*e**4*asinh(sqrt(
c)*x/sqrt(a))/(16*c**(5/2)) - 3*a**2*d**2*e**2*asinh(sqrt(c)*x/sqrt(a))/(4*
c**(3/2)) + a*d**4*asinh(sqrt(c)*x/sqrt(a))/(2*sqrt(c)) + 4*d**3*e*Piecewis
e((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + 4*d*e**3
*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(1
5*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + 3*c*d*
*2*e**2*x**5/(2*sqrt(a)*sqrt(1 + c*x**2/a)) + c*e**4*x**7/(6*sqrt(a)*sqrt(1
 + c*x**2/a))
```

Giac [A] time = 1.36844, size = 266, normalized size = 1.29

$$\frac{1}{240}\sqrt{cx^2+a}\left(\left(2\left(\left(4\left(5xe^4+24de^3\right)x+\frac{5\left(36c^4d^2e^2+ac^3e^4\right)}{c^4}\right)x+\frac{32\left(5c^4d^3e+ac^3de^3\right)}{c^4}\right)x+\frac{15\left(8c^4d^4+12ac^3d^2e^2\right.}{c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/240*sqrt(c*x^2 + a)*((2*((4*(5*x*e^4 + 24*d*e^3)*x + 5*(36*c^4*d^2*e^2 +
a*c^3*e^4)/c^4)*x + 32*(5*c^4*d^3*e + a*c^3*d*e^3)/c^4)*x + 15*(8*c^4*d^4 +
 12*a*c^3*d^2*e^2 - a^2*c^2*e^4)/c^4)*x + 64*(5*a*c^3*d^3*e - 2*a^2*c^2*d*e
^3)/c^4) - 1/16*(8*a*c^2*d^4 - 12*a^2*c*d^2*e^2 + a^3*e^4)*log(abs(-sqrt(c)
*x + sqrt(c*x^2 + a)))/c^(5/2)
```

3.526 $\int (d+ex)^3\sqrt{a+cx^2}\,dx$

Optimal. Leaf size=144

$$\frac{e\left(a+cx^2\right)^{3/2}\left(8\left(6cd^2-ae^2\right)+21cdex\right)}{60c^2}+\frac{ad\left(4cd^2-3ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}}+\frac{dx\sqrt{a+cx^2}\left(4cd^2-3ae^2\right)}{8c}+\frac{e\left(a+cx^2\right)^{3/2}}{5c}$$

```
[Out] (d*(4*c*d^2 - 3*a*e^2)*x*Sqrt[a + c*x^2])/(8*c) + (e*(d + e*x)^2*(a + c*x^2
)^(3/2))/(5*c) + (e*(8*(6*c*d^2 - a*e^2) + 21*c*d*e*x)*(a + c*x^2)^(3/2))/(
60*c^2) + (a*d*(4*c*d^2 - 3*a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8
*c^(3/2))
```

Rubi [A] time = 0.114714, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {743, 780, 195, 217, 206}

$$\frac{e\left(a+cx^2\right)^{3/2}\left(8\left(6cd^2-ae^2\right)+21cdex\right)}{60c^2}+\frac{ad\left(4cd^2-3ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}}+\frac{dx\sqrt{a+cx^2}\left(4cd^2-3ae^2\right)}{8c}+\frac{e\left(a+cx^2\right)^{3/2}}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*Sqrt[a + c*x^2],x]
```

```
[Out] (d*(4*c*d^2 - 3*a*e^2)*x*Sqrt[a + c*x^2])/(8*c) + (e*(d + e*x)^2*(a + c*x^2
)^(3/2))/(5*c) + (e*(8*(6*c*d^2 - a*e^2) + 21*c*d*e*x)*(a + c*x^2)^(3/2))/(
60*c^2) + (a*d*(4*c*d^2 - 3*a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8
*c^(3/2))
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^3\sqrt{a+cx^2}\,dx = \frac{e(d+ex)^2\left(a+cx^2\right)^{3/2}}{5c} + \frac{\int (d+ex)\left(5cd^2-2ae^2+7cdex\right)\sqrt{a+cx^2}\,dx}{5c}$$

$$= \frac{e(d+ex)^2\left(a+cx^2\right)^{3/2}}{5c} + \frac{e\left(8\left(6cd^2-ae^2\right)+21cdex\right)\left(a+cx^2\right)^{3/2}}{60c^2} + \frac{\left(d\left(4cd^2-3ae^2\right)\right)\int \sqrt{a}}{4c}$$

$$= \frac{d\left(4cd^2-3ae^2\right)x\sqrt{a+cx^2}}{8c} + \frac{e(d+ex)^2\left(a+cx^2\right)^{3/2}}{5c} + \frac{e\left(8\left(6cd^2-ae^2\right)+21cdex\right)\left(a+cx^2\right)}{60c^2}$$

$$= \frac{d\left(4cd^2-3ae^2\right)x\sqrt{a+cx^2}}{8c} + \frac{e(d+ex)^2\left(a+cx^2\right)^{3/2}}{5c} + \frac{e\left(8\left(6cd^2-ae^2\right)+21cdex\right)\left(a+cx^2\right)}{60c^2}$$

$$= \frac{d\left(4cd^2-3ae^2\right)x\sqrt{a+cx^2}}{8c} + \frac{e(d+ex)^2\left(a+cx^2\right)^{3/2}}{5c} + \frac{e\left(8\left(6cd^2-ae^2\right)+21cdex\right)\left(a+cx^2\right)}{60c^2}$$

Mathematica [A] time = 0.0835308, size = 132, normalized size = 0.92

$$\frac{\sqrt{a+cx^2}\left(-16a^2e^3+ace\left(120d^2+45dex+8e^2x^2\right)+6c^2x\left(20d^2ex+10d^3+15de^2x^2+4e^3x^3\right)\right)+15a\sqrt{c}d\left(4cd^2-3ae^2\right.}{120c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[a + c*x^2]*(-16*a^2*e^3 + a*c*e*(120*d^2 + 45*d*e*x + 8*e^2*x^2) + 6*
c^2*x*(10*d^3 + 20*d^2*e*x + 15*d*e^2*x^2 + 4*e^3*x^3)) + 15*a*Sqrt[c]*d*(4
*c*d^2 - 3*a*e^2)*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/(120*c^2)
```

Maple [A] time = 0.051, size = 164, normalized size = 1.1

$$\frac{e^3x^2}{5c}\left(cx^2+a\right)^{\frac{3}{2}} - \frac{2ae^3}{15c^2}\left(cx^2+a\right)^{\frac{3}{2}} + \frac{3de^2x}{4c}\left(cx^2+a\right)^{\frac{3}{2}} - \frac{3ade^2x}{8c}\sqrt{cx^2+a} - \frac{3de^2a^2}{8}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)c^{-\frac{3}{2}} + \frac{d^2e}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+a)^(1/2),x)
```

```
[Out] 1/5*e^3*x^2*(c*x^2+a)^(3/2)/c-2/15*e^3*a/c^2*(c*x^2+a)^(3/2)+3/4*d*e^2*x*(c
*x^2+a)^(3/2)/c-3/8*d*e^2*a/c*x*(c*x^2+a)^(1/2)-3/8*d*e^2*a^2/c^(3/2)*ln(x*
c^(1/2)+(c*x^2+a)^(1/2))+d^2*e*(c*x^2+a)^(3/2)/c+1/2*d^3*x*(c*x^2+a)^(1/2)+
1/2*d^3*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.28943, size = 662, normalized size = 4.6

$$\left[-\frac{15\left(4\,acd^3-3\,a^2de^2\right)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)-2\left(24\,c^2e^3x^4+90\,c^2de^2x^3+120\,acd^2e-16\,a^2e^3+8\left(15\,c^2d\right.\right.}{240\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/240*(15*(4*a*c*d^3 - 3*a^2*d*e^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 +
 a)*sqrt(c)*x - a) - 2*(24*c^2*e^3*x^4 + 90*c^2*d*e^2*x^3 + 120*a*c*d^2*e -
 16*a^2*e^3 + 8*(15*c^2*d^2*e + a*c*e^3)*x^2 + 15*(4*c^2*d^3 + 3*a*c*d*e^2)
*x)*sqrt(c*x^2 + a))/c^2, -1/120*(15*(4*a*c*d^3 - 3*a^2*d*e^2)*sqrt(-c)*arc
tan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (24*c^2*e^3*x^4 + 90*c^2*d*e^2*x^3 + 120*
a*c*d^2*e - 16*a^2*e^3 + 8*(15*c^2*d^2*e + a*c*e^3)*x^2 + 15*(4*c^2*d^3 + 3
*a*c*d*e^2)*x)*sqrt(c*x^2 + a))/c^2]
```

Sympy [A] time = 6.52748, size = 265, normalized size = 1.84

$$\frac{3a^{\frac{3}{2}}de^2x}{8c\sqrt{1+\frac{cx^2}{a}}}+\frac{\sqrt{a}d^3x\sqrt{1+\frac{cx^2}{a}}}{2}+\frac{9\sqrt{a}de^2x^3}{8\sqrt{1+\frac{cx^2}{a}}}-\frac{3a^2de^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8c^{\frac{3}{2}}}+\frac{ad^3\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2\sqrt{c}}+3d^2e\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{for}\ c=0\\ \frac{\left(a+cx^2\right)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a)**(1/2),x)
```

```
[Out] 3*a**(3/2)*d*e**2*x/(8*c*sqrt(1 + c*x**2/a)) + sqrt(a)*d**3*x*sqrt(1 + c*x*
*2/a)/2 + 9*sqrt(a)*d*e**2*x**3/(8*sqrt(1 + c*x**2/a)) - 3*a**2*d*e**2*asin
h(sqrt(c)*x/sqrt(a))/(8*c**(3/2)) + a*d**3*asinh(sqrt(c)*x/sqrt(a))/(2*sqrt
(c)) + 3*d**2*e*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/
(3*c), True)) + e**3*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2
*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**
4/4, True)) + 3*c*d*e**2*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a))
```

Giac [A] time = 1.40938, size = 194, normalized size = 1.35

$$\frac{1}{120}\sqrt{cx^2+a}\left(\left(2\left(3\left(4\,xe^3+15\,de^2\right)x+\frac{4\left(15\,c^3d^2e+ac^2e^3\right)}{c^3}\right)x+\frac{15\left(4\,c^3d^3+3\,ac^2de^2\right)}{c^3}\right)x+\frac{8\left(15\,ac^2d^2e-2\,a^2ce^3\right)}{c^3}\right)-\frac{(4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/120*sqrt(c*x^2 + a)*((2*(3*(4*x*e^3 + 15*d*e^2)*x + 4*(15*c^3*d^2*e + a*c
^2*e^3)/c^3)*x + 15*(4*c^3*d^3 + 3*a*c^2*d*e^2)/c^3)*x + 8*(15*a*c^2*d^2*e
- 2*a^2*c*e^3)/c^3) - 1/8*(4*a*c*d^3 - 3*a^2*d*e^2)*log(abs(-sqrt(c)*x + sq
rt(c*x^2 + a)))/c^(3/2)
```

3.527 $\int (d+ex)^2\sqrt{a+cx^2}\,dx$

Optimal. Leaf size=119

$$\frac{a\left(4cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}}+\frac{x\sqrt{a+cx^2}\left(4cd^2-ae^2\right)}{8c}+\frac{5de\left(a+cx^2\right)^{3/2}}{12c}+\frac{e\left(a+cx^2\right)^{3/2}(d+ex)}{4c}$$

```
[Out] ((4*c*d^2 - a*e^2)*x*Sqrt[a + c*x^2])/(8*c) + (5*d*e*(a + c*x^2)^(3/2))/(12
*c) + (e*(d + e*x)*(a + c*x^2)^(3/2))/(4*c) + (a*(4*c*d^2 - a*e^2)*ArcTanh[
(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*c^(3/2))
```

Rubi [A] time = 0.0537477, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {743, 641, 195, 217, 206}

$$\frac{a\left(4cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}}+\frac{x\sqrt{a+cx^2}\left(4cd^2-ae^2\right)}{8c}+\frac{5de\left(a+cx^2\right)^{3/2}}{12c}+\frac{e\left(a+cx^2\right)^{3/2}(d+ex)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*Sqrt[a + c*x^2],x]
```

```
[Out] ((4*c*d^2 - a*e^2)*x*Sqrt[a + c*x^2])/(8*c) + (5*d*e*(a + c*x^2)^(3/2))/(12
*c) + (e*(d + e*x)*(a + c*x^2)^(3/2))/(4*c) + (a*(4*c*d^2 - a*e^2)*ArcTanh[
(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*c^(3/2))
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^2\sqrt{a+cx^2}\,dx = \frac{e(d+ex)\left(a+cx^2\right)^{3/2}}{4c} + \frac{\int \left(4cd^2-ae^2+5cdex\right)\sqrt{a+cx^2}\,dx}{4c}$$

$$= \frac{5de\left(a+cx^2\right)^{3/2}}{12c} + \frac{e(d+ex)\left(a+cx^2\right)^{3/2}}{4c} + \frac{\left(4cd^2-ae^2\right)\int \sqrt{a+cx^2}\,dx}{4c}$$

$$= \frac{\left(4cd^2-ae^2\right)x\sqrt{a+cx^2}}{8c} + \frac{5de\left(a+cx^2\right)^{3/2}}{12c} + \frac{e(d+ex)\left(a+cx^2\right)^{3/2}}{4c} + \frac{\left(a\left(4cd^2-ae^2\right)\right)\int \frac{}{\sqrt{a\cdots}}}{8c}$$

$$= \frac{\left(4cd^2-ae^2\right)x\sqrt{a+cx^2}}{8c} + \frac{5de\left(a+cx^2\right)^{3/2}}{12c} + \frac{e(d+ex)\left(a+cx^2\right)^{3/2}}{4c} + \frac{\left(a\left(4cd^2-ae^2\right)\right)\mathrm{Subs}\cdots}{}$$

$$= \frac{\left(4cd^2-ae^2\right)x\sqrt{a+cx^2}}{8c} + \frac{5de\left(a+cx^2\right)^{3/2}}{12c} + \frac{e(d+ex)\left(a+cx^2\right)^{3/2}}{4c} + \frac{a\left(4cd^2-ae^2\right)\tanh^{-1}\cdots}{8c^{3/2}}$$

Mathematica [A] time = 0.0637657, size = 99, normalized size = 0.83

$$\frac{\sqrt{c}\sqrt{a+cx^2}\left(ae(16d+3ex)+2cx\left(6d^2+8dex+3e^2x^2\right)\right)-3a\left(ae^2-4cd^2\right)\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)}{24c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[a + c*x^2]*(a*e*(16*d + 3*e*x) + 2*c*x*(6*d^2 + 8*d*e*x + 3*e
^2*x^2)) - 3*a*(-4*c*d^2 + a*e^2)*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/(24*c
^(3/2))
```

Maple [A] time = 0.048, size = 122, normalized size = 1.

$$\frac{e^2x}{4c}\left(cx^2+a\right)^{\frac{3}{2}} - \frac{ae^2x}{8c}\sqrt{cx^2+a} - \frac{a^2e^2}{8}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)c^{-\frac{3}{2}} + \frac{2\,de}{3\,c}\left(cx^2+a\right)^{\frac{3}{2}} + \frac{d^2x}{2}\sqrt{cx^2+a} + \frac{ad^2}{2}\ln\left(x\sqrt{c}+\sqrt{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a)^(1/2),x)
```

```
[Out] 1/4*e^2*x*(c*x^2+a)^(3/2)/c-1/8*e^2*a/c*x*(c*x^2+a)^(1/2)-1/8*e^2*a^2/c^(3/
2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))+2/3*d*e*(c*x^2+a)^(3/2)/c+1/2*d^2*x*(c*x^2
+a)^(1/2)+1/2*d^2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.51873, size = 491, normalized size = 4.13

$$\left[-\frac{3\left(4\,acd^2-a^2e^2\right)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)-2\left(6\,c^2e^2x^3+16\,c^2dex^2+16\,acde+3\left(4\,c^2d^2+ace^2\right)x\right)\sqrt{cx^2+}}{48\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*(4*a*c*d^2 - a^2*e^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sq
rt(c)*x - a) - 2*(6*c^2*e^2*x^3 + 16*c^2*d*e*x^2 + 16*a*c*d*e + 3*(4*c^2*d^
2 + a*c*e^2)*x)*sqrt(c*x^2 + a))/c^2, -1/24*(3*(4*a*c*d^2 - a^2*e^2)*sqrt(-
c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (6*c^2*e^2*x^3 + 16*c^2*d*e*x^2 + 1
6*a*c*d*e + 3*(4*c^2*d^2 + a*c*e^2)*x)*sqrt(c*x^2 + a))/c^2]
```

Sympy [A] time = 6.08902, size = 184, normalized size = 1.55

$$\frac{a^{\frac{3}{2}}e^2x}{8c\sqrt{1+\frac{cx^2}{a}}}+\frac{\sqrt{a}d^2x\sqrt{1+\frac{cx^2}{a}}}{2}+\frac{3\sqrt{a}e^2x^3}{8\sqrt{1+\frac{cx^2}{a}}}-\frac{a^2e^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8c^{\frac{3}{2}}}+\frac{ad^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2\sqrt{c}}+2de\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{for}\ c=0\\ \frac{\left(a+cx^2\right)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)+\frac{}{4\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+a)**(1/2),x)
```

```
[Out] a**(3/2)*e**2*x/(8*c*sqrt(1 + c*x**2/a)) + sqrt(a)*d**2*x*sqrt(1 + c*x**2/a
)/2 + 3*sqrt(a)*e**2*x**3/(8*sqrt(1 + c*x**2/a)) - a**2*e**2*asinh(sqrt(c)*
x/sqrt(a))/(8*c**(3/2)) + a*d**2*asinh(sqrt(c)*x/sqrt(a))/(2*sqrt(c)) + 2*d
*e*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True))
 + c*e**2*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a))
```

Giac [A] time = 1.30097, size = 130, normalized size = 1.09

$$\frac{1}{24}\sqrt{cx^2+a}\left(\left(2\left(3\,xe^2+8\,de\right)x+\frac{3\left(4\,c^2d^2+ace^2\right)}{c^2}\right)x+\frac{16\,ade}{c}\right)-\frac{\left(4\,acd^2-a^2e^2\right)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{8\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + a)*((2*(3*x*e^2 + 8*d*e)*x + 3*(4*c^2*d^2 + a*c*e^2)/c^2)
*x + 16*a*d*e/c) - 1/8*(4*a*c*d^2 - a^2*e^2)*log(abs(-sqrt(c)*x + sqrt(c*x^
2 + a)))/c^(3/2)
```

3.528 $\int (d+ex)\sqrt{a+cx^2}\,dx$

Optimal. Leaf size=67

$$\frac{1}{2}dx\sqrt{a+cx^2}+\frac{ad\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}}+\frac{e\left(a+cx^2\right)^{3/2}}{3c}$$

```
[Out] (d*x*Sqrt[a + c*x^2])/2 + (e*(a + c*x^2)^(3/2))/(3*c) + (a*d*ArcTanh[(Sqrt[
c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c])
```

Rubi [A] time = 0.0178534, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {641, 195, 217, 206}

$$\frac{1}{2}dx\sqrt{a+cx^2}+\frac{ad\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}}+\frac{e\left(a+cx^2\right)^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*Sqrt[a + c*x^2],x]
```

```
[Out] (d*x*Sqrt[a + c*x^2])/2 + (e*(a + c*x^2)^(3/2))/(3*c) + (a*d*ArcTanh[(Sqrt[
c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)\sqrt{a+cx^2}\,dx = \frac{e\left(a+cx^2\right)^{3/2}}{3c} + d\int \sqrt{a+cx^2}\,dx$$
$$= \frac{1}{2}dx\sqrt{a+cx^2} + \frac{e\left(a+cx^2\right)^{3/2}}{3c} + \frac{1}{2}(ad)\int \frac{1}{\sqrt{a+cx^2}}\,dx$$
$$= \frac{1}{2}dx\sqrt{a+cx^2} + \frac{e\left(a+cx^2\right)^{3/2}}{3c} + \frac{1}{2}(ad)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)$$
$$= \frac{1}{2}dx\sqrt{a+cx^2} + \frac{e\left(a+cx^2\right)^{3/2}}{3c} + \frac{ad\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}}$$

Mathematica [A] time = 0.034908, size = 67, normalized size = 1.

$$\frac{\sqrt{a+cx^2}(2ae+cx(3d+2ex))+3a\sqrt{c}d\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[a + c*x^2]*(2*a*e + c*x*(3*d + 2*e*x)) + 3*a*Sqrt[c]*d*Log[c*x + Sqrt
[c]*Sqrt[a + c*x^2]])/(6*c)
```

Maple [A] time = 0.043, size = 53, normalized size = 0.8

$$\frac{e}{3\,c}\left(cx^2+a\right)^{\frac{3}{2}}+\frac{dx}{2}\sqrt{cx^2+a}+\frac{ad}{2}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+a)^(1/2),x)
```

```
[Out] 1/3*e*(c*x^2+a)^(3/2)/c+1/2*d*x*(c*x^2+a)^(1/2)+1/2*d*a/c^(1/2)*ln(x*c^(1/2
)+(c*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.32861, size = 316, normalized size = 4.72

$$\left[\frac{3\,a\sqrt{c}d\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)+2\left(2\,cex^2+3\,cdx+2\,ae\right)\sqrt{cx^2+a}}{12\,c}, -\frac{3\,a\sqrt{-c}d\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)-\left(2\,cex^2+3\right.}{6\,c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a*sqrt(c)*d*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(2
*c*e*x^2 + 3*c*d*x + 2*a*e)*sqrt(c*x^2 + a))/c, -1/6*(3*a*sqrt(-c)*d*arctan
(sqrt(-c)*x/sqrt(c*x^2 + a)) - (2*c*e*x^2 + 3*c*d*x + 2*a*e)*sqrt(c*x^2 + a
))/c]
```

Sympy [A] time = 2.87237, size = 70, normalized size = 1.04

$$\frac{\sqrt{a}dx\sqrt{1+\frac{cx^2}{a}}}{2}+\frac{ad\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2\sqrt{c}}+e\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{for } c=0\\ \frac{\left(a+cx^2\right)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*d*x*sqrt(1 + c*x**2/a)/2 + a*d*asinh(sqrt(c)*x/sqrt(a))/(2*sqrt(c))
 + e*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True
))
```

Giac [A] time = 1.49764, size = 77, normalized size = 1.15

$$-\frac{ad\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{2\sqrt{c}}+\frac{1}{6}\sqrt{cx^2+a}\left((2xe+3d)x+\frac{2ae}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*a*d*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/sqrt(c) + 1/6*sqrt(c*x^2 +
a)*((2*x*e + 3*d)*x + 2*a*e/c)
```

3.529 $\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=103

$$-\frac{\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2}-\frac{\sqrt{c}d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e^2}+\frac{\sqrt{a+cx^2}}{e}$$

```
[Out] Sqrt[a + c*x^2]/e - (Sqrt[c]*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/e^2 -
(Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*
x^2])])/e^2
```

Rubi [A] time = 0.0814617, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {735, 844, 217, 206, 725}

$$-\frac{\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2}-\frac{\sqrt{c}d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e^2}+\frac{\sqrt{a+cx^2}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(d + e*x),x]
```

```
[Out] Sqrt[a + c*x^2]/e - (Sqrt[c]*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/e^2 -
(Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*
x^2])])/e^2
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx &= \frac{\sqrt{a+cx^2}}{e} + \frac{\int \frac{ae-cdx}{(d+ex)\sqrt{a+cx^2}}\,dx}{e} \\
&= \frac{\sqrt{a+cx^2}}{e} + \left(a+\frac{cd^2}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx - \frac{(cd)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e^2} \\
&= \frac{\sqrt{a+cx^2}}{e} + \left(-a-\frac{cd^2}{e^2}\right)\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right) - \frac{(cd)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+\ldots}}\right.}{e^2} \\
&= \frac{\sqrt{a+cx^2}}{e} - \frac{\sqrt{c}d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e^2} - \frac{\sqrt{cd^2+ae^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^2}
\end{aligned}
$$

Mathematica [A] time = 0.0478463, size = 99, normalized size = 0.96

$$
\frac{-\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)-\sqrt{c}d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)+e\sqrt{a+cx^2}}{e^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(d + e*x),x]
```

```
[Out] (e*Sqrt[a + c*x^2] - Sqrt[c]*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - Sqrt[
c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])]
)/e^2
```

Maple [B] time = 0.257, size = 381, normalized size = 3.7

$$
\frac{1}{e}\sqrt{c\left(\frac{d}{e}+x\right)^2-2\,\frac{cd}{e}\left(\frac{d}{e}+x\right)+\frac{ae^2+cd^2}{e^2}}-\frac{d}{e^2}\sqrt{c}\ln\left(\left(-\frac{cd}{e}+\left(\frac{d}{e}+x\right)c\right)\frac{1}{\sqrt{c}}+\sqrt{c\left(\frac{d}{e}+x\right)^2-2\,\frac{cd}{e}\left(\frac{d}{e}+x\right)+\frac{ae^2+\ldots}{e^2}}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/(e*x+d),x)
```

```
[Out] 1/e*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)-1/e^2*c^(1/2)*d*l
n((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2
)^(1/2))-1/e/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e
+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/
e^2)^(1/2))/(d/e+x))*a-1/e^3/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/
e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x
)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*c*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.98198, size = 1274, normalized size = 12.37

$$\left[\frac{\sqrt{c}d\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)+2\,\sqrt{cx^2+a}e+\sqrt{cd^2+ae^2}\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2-2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{c}}{e^2x^2+2\,dex+d^2}\right.}{2\,e^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c)*d*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*sqrt(c*
x^2 + a)*e + sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (
2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 +
 a))/(e^2*x^2 + 2*d*e*x + d^2)))/e^2, 1/2*(2*sqrt(-c)*d*arctan(sqrt(-c)*x/s
qrt(c*x^2 + a)) + 2*sqrt(c*x^2 + a)*e + sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*
x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)
*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)))/e^2, 1/2*(sqrt(
c)*d*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*sqrt(c*x^2 + a)*e
- 2*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x
^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)))/e^2, (sqrt(-c)*d*ar
ctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + sqrt(c*x^2 + a)*e - sqrt(-c*d^2 - a*e^2)
*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e
^2 + (c^2*d^2 + a*c*e^2)*x^2)))/e^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(d + e*x), x)
```

Giac [A] time = 1.38916, size = 147, normalized size = 1.43

$$\sqrt{c}de^{(-2)}\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)+\frac{2\left(cd^2+ae^2\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)e^{(-2)}}{\sqrt{-cd^2-ae^2}}+\sqrt{cx^2+a}e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] sqrt(c)*d*e^(-2)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a))) + 2*(c*d^2 + a*e^2)
*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2)
)*e^(-2)/sqrt(-c*d^2 - a*e^2) + sqrt(c*x^2 + a)*e^(-1)
```

3.530 $\int \frac{\sqrt{a+cx^2}}{(d+ex)^2} dx$

Optimal. Leaf size=110

$$\frac{cd\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2\sqrt{ae^2+cd^2}}-\frac{\sqrt{a+cx^2}}{e(d+ex)}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e^2}$$

```
[Out] -(Sqrt[a + c*x^2]/(e*(d + e*x))) + (Sqrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*
x^2]])/e^2 + (c*d*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2
])])/(e^2*Sqrt[c*d^2 + a*e^2])
```

Rubi [A] time = 0.0681209, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {733, 844, 217, 206, 725}

$$\frac{cd\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2\sqrt{ae^2+cd^2}}-\frac{\sqrt{a+cx^2}}{e(d+ex)}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(d + e*x)^2,x]
```

```
[Out] -(Sqrt[a + c*x^2]/(e*(d + e*x))) + (Sqrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*
x^2]])/e^2 + (c*d*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2
])])/(e^2*Sqrt[c*d^2 + a*e^2])
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+cx^2}}{(d+ex)^2}\,dx &= -\frac{\sqrt{a+cx^2}}{e(d+ex)}+\frac{c\int \frac{x}{(d+ex)\sqrt{a+cx^2}}\,dx}{e}\\
&= -\frac{\sqrt{a+cx^2}}{e(d+ex)}+\frac{c\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e^2}-\frac{(cd)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^2}\\
&= -\frac{\sqrt{a+cx^2}}{e(d+ex)}+\frac{c\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{e^2}+\frac{(cd)\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e^2}\\
&= -\frac{\sqrt{a+cx^2}}{e(d+ex)}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e^2}+\frac{cd\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^2\sqrt{cd^2+ae^2}}
\end{aligned}$$

Mathematica [A] time = 0.13688, size = 134, normalized size = 1.22

$$\frac{\frac{cd\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{\sqrt{ae^2+cd^2}}-\frac{cd\log(d+ex)}{\sqrt{ae^2+cd^2}}-\frac{e\sqrt{a+cx^2}}{d+ex}+\sqrt{c}\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(d + e*x)^2,x]
```

```
[Out] (-((e*Sqrt[a + c*x^2])/(d + e*x)) - (c*d*Log[d + e*x])/Sqrt[c*d^2 + a*e^2]
+ Sqrt[c]*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] + (c*d*Log[a*e - c*d*x + Sqrt[
c*d^2 + a*e^2]*Sqrt[a + c*x^2]])/Sqrt[c*d^2 + a*e^2])/e^2
```

Maple [B] time = 0.203, size = 649, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/(e*x+d)^2,x)
```

```
[Out] -1/(a*e^2+c*d^2)/(d/e+x)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3
/2)-1/e*c*d/(a*e^2+c*d^2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(
1/2)+1/e^2*c^(3/2)*d^2/(a*e^2+c*d^2)*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+
x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))+1/e*c*d/(a*e^2+c*d^2)/((a*e^
2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2
)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x)
)*a+1/e^3*c^2*d^3/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^
2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/
e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))+1/(a*e^2+c*d^2)*c*(c*(d/e+x)^2-2*c*
d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x+1/(a*e^2+c*d^2)*c^(1/2)*ln((-c*d/e+(
d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.25039, size = 1891, normalized size = 17.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [1/2*((c*d^3 + a*d*e^2 + (c*d^2*e + a*e^3)*x)*sqrt(c)*log(-2*c*x^2 - 2*sqrt
(c*x^2 + a)*sqrt(c)*x - a) + (c*d*e*x + c*d^2)*sqrt(c*d^2 + a*e^2)*log((2*a
*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 +
 a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(c*d^
2*e + a*e^3)*sqrt(c*x^2 + a))/(c*d^3*e^2 + a*d*e^4 + (c*d^2*e^3 + a*e^5)*x)
, 1/2*(2*(c*d*e*x + c*d^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)
*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2
)) + (c*d^3 + a*d*e^2 + (c*d^2*e + a*e^3)*x)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(
c*x^2 + a)*sqrt(c)*x - a) - 2*(c*d^2*e + a*e^3)*sqrt(c*x^2 + a))/(c*d^3*e^2
 + a*d*e^4 + (c*d^2*e^3 + a*e^5)*x), -1/2*(2*(c*d^3 + a*d*e^2 + (c*d^2*e +
a*e^3)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (c*d*e*x + c*d^2)*s
qrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*
c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2
+ 2*d*e*x + d^2)) + 2*(c*d^2*e + a*e^3)*sqrt(c*x^2 + a))/(c*d^3*e^2 + a*d*e
^4 + (c*d^2*e^3 + a*e^5)*x), ((c*d*e*x + c*d^2)*sqrt(-c*d^2 - a*e^2)*arctan
(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c
^2*d^2 + a*c*e^2)*x^2)) - (c*d^3 + a*d*e^2 + (c*d^2*e + a*e^3)*x)*sqrt(-c)*
arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (c*d^2*e + a*e^3)*sqrt(c*x^2 + a))/(c*
d^3*e^2 + a*d*e^4 + (c*d^2*e^3 + a*e^5)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^2}}{(d + ex)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/(e*x+d)**2,x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(d + e*x)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.531 $\int \frac{\sqrt{a+cx^2}}{(d+ex)^3} dx$

Optimal. Leaf size=103

$$-\frac{\sqrt{a+cx^2}(ae-cdx)}{2(d+ex)^2\left(ae^2+cd^2\right)}-\frac{ac\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{2\left(ae^2+cd^2\right)^{3/2}}$$

```
[Out] -((a*e - c*d*x)*Sqrt[a + c*x^2])/(2*(c*d^2 + a*e^2)*(d + e*x)^2) - (a*c*Arc
Tanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(2*(c*d^2 + a*e^
2)^(3/2))
```

Rubi [A] time = 0.0438794, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {721, 725, 206}

$$-\frac{\sqrt{a+cx^2}(ae-cdx)}{2(d+ex)^2\left(ae^2+cd^2\right)}-\frac{ac\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{2\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(d + e*x)^3,x]
```

```
[Out] -((a*e - c*d*x)*Sqrt[a + c*x^2])/(2*(c*d^2 + a*e^2)*(d + e*x)^2) - (a*c*Arc
Tanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(2*(c*d^2 + a*e^
2)^(3/2))
```

Rule 721

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(-2*a*e + (2*c*d)*x)*(a + c*x^2)^p)/(2*(m + 1)*(c*d^2 +
 a*e^2)), x] - Dist[(4*a*c*p)/(2*(m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m
 + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^3}\,dx = -\frac{(ae-cdx)\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2}+\frac{(ac)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{2\left(cd^2+ae^2\right)}$$

$$= -\frac{(ae-cdx)\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2}-\frac{(ac)\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{2\left(cd^2+ae^2\right)}$$

$$= -\frac{(ae-cdx)\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2}-\frac{ac\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{2\left(cd^2+ae^2\right)^{3/2}}$$

Mathematica [A] time = 0.104056, size = 127, normalized size = 1.23

$$\frac{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}(cdx-ae)-ac(d+ex)^2\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)+ac(d+ex)^2\log(d+ex)}{2(d+ex)^2\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(d + e*x)^3,x]
```

```
[Out] (Sqrt[c*d^2 + a*e^2]*(-(a*e) + c*d*x)*Sqrt[a + c*x^2] + a*c*(d + e*x)^2*Log
[d + e*x] - a*c*(d + e*x)^2*Log[a*e - c*d*x + Sqrt[c*d^2 + a*e^2]*Sqrt[a +
c*x^2]])/(2*(c*d^2 + a*e^2)^(3/2)*(d + e*x)^2)
```

Maple [B] time = 0.213, size = 1174, normalized size = 11.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/(e*x+d)^3,x)
```

```
[Out] -1/2/e/(a*e^2+c*d^2)/(d/e+x)^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e
^2)^(3/2)-1/2*c*d/(a*e^2+c*d^2)^2/(d/e+x)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e
^2+c*d^2)/e^2)^(3/2)-1/2/e*c^2*d^2/(a*e^2+c*d^2)^2*(c*(d/e+x)^2-2*c*d/e*(d/
e+x)+(a*e^2+c*d^2)/e^2)^(1/2)+1/2/e^2*c^(5/2)*d^3/(a*e^2+c*d^2)^2*ln((-c*d/
e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))
+1/2/e*c^2*d^2/(a*e^2+c*d^2)^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2
)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e
+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a+1/2/e^3*c^3*d^4/(a*e^2+c*d^2)^2/((
a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c
*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/
e+x))+1/2*c^2*d/(a*e^2+c*d^2)^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/
e^2)^(1/2)*x+1/2*c^(3/2)*d/(a*e^2+c*d^2)^2*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c
*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))*a+1/2/e/(a*e^2+c*d^2)*
c*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)-1/2/e^2/(a*e^2+c*d^
2)*c^(3/2)*d*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*
e^2+c*d^2)/e^2)^(1/2))-1/2/e/(a*e^2+c*d^2)*c/((a*e^2+c*d^2)/e^2)^(1/2)*ln((
2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^
2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a-1/2/e^3/(a*e^2+c*d^2
)*c^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*(
(a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1
```

```
/2))/(d/e+x))*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.5001, size = 1064, normalized size = 10.33

$$\left[\frac{\left(ace^2x^2 + 2\,acdex + acd^2\right)\sqrt{cd^2 + ae^2}\log\left(\frac{2\,acdex - acd^2 - 2\,a^2e^2 - \left(2\,c^2d^2 + ace^2\right)x^2 - 2\,\sqrt{cd^2 + ae^2}(cdx - ae)\sqrt{cx^2 + a}}{e^2x^2 + 2\,dex + d^2}\right) - 2\left(acd^2e + a^2e^3 - \left(c^2d^3\right.\right.}{4\left(c^2d^6 + 2\,acd^4e^2 + a^2d^2e^4 + \left(c^2d^4e^2 + 2\,acd^2e^4 + a^2e^6\right)x^2 + 2\left(c^2d^5e + 2\,acd^3e^3 + a^2de^5\right)x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] [1/4*((a*c*e^2*x^2 + 2*a*c*d*e*x + a*c*d^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*
d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*
e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(a*c*d^2
*e + a^2*e^3 - (c^2*d^3 + a*c*d*e^2)*x)*sqrt(c*x^2 + a))/(c^2*d^6 + 2*a*c*d
^4*e^2 + a^2*d^2*e^4 + (c^2*d^4*e^2 + 2*a*c*d^2*e^4 + a^2*e^6)*x^2 + 2*(c^2
*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x), -1/2*((a*c*e^2*x^2 + 2*a*c*d*e*x +
a*c*d^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqr
t(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (a*c*d^2*e +
a^2*e^3 - (c^2*d^3 + a*c*d*e^2)*x)*sqrt(c*x^2 + a))/(c^2*d^6 + 2*a*c*d^4*e^
2 + a^2*d^2*e^4 + (c^2*d^4*e^2 + 2*a*c*d^2*e^4 + a^2*e^6)*x^2 + 2*(c^2*d^5*
e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^2}}{(d + ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/(e*x+d)**3,x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(d + e*x)**3, x)
```

Giac [B] time = 1.22874, size = 421, normalized size = 4.09

$$-\frac{ac\arctan\left(\frac{\left(\sqrt{c}x - \sqrt{cx^2 + a}\right)e + \sqrt{c}d}{\sqrt{-cd^2 - ae^2}}\right)}{\left(cd^2 + ae^2\right)\sqrt{-cd^2 - ae^2}} + \frac{2\left(\sqrt{c}x - \sqrt{cx^2 + a}\right)^3c^2d^2e + 2\left(\sqrt{c}x - \sqrt{cx^2 + a}\right)^2c^{\frac{5}{2}}d^3 - 2\left(\sqrt{c}x - \sqrt{cx^2 + a}\right)ac^2d^2e - \left(\right.}{\left(cd^2e^2 + ae^4\right)\left(\left(\sqrt{c}x - \sqrt{cx^2 + a}\right)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] -a*c*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e
^2))/((c*d^2 + a*e^2)*sqrt(-c*d^2 - a*e^2)) + (2*(sqrt(c)*x - sqrt(c*x^2 +
a))^3*c^2*d^2*e + 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*d^3 - 2*(sqrt(c
)*x - sqrt(c*x^2 + a))*a*c^2*d^2*e - (sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3
/2)*d*e^2 + (sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*e^3 + a^2*c^(3/2)*d*e^2 + (
sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*e^3)/((c*d^2*e^2 + a*e^4)*((sqrt(c)*x -
sqrt(c*x^2 + a))^2*e + 2*(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*d - a*e)^2)
```

3.532 $\int \frac{\sqrt{a+cx^2}}{(d+ex)^4} dx$

Optimal. Leaf size=144

$$-\frac{ac^2d\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{2\left(ae^2+cd^2\right)^{5/2}}-\frac{cd\sqrt{a+cx^2}(ae-cdx)}{2(d+ex)^2\left(ae^2+cd^2\right)^2}-\frac{e\left(a+cx^2\right)^{3/2}}{3(d+ex)^3\left(ae^2+cd^2\right)}$$

```
[Out] -(c*d*(a*e - c*d*x)*Sqrt[a + c*x^2])/(2*(c*d^2 + a*e^2)^2*(d + e*x)^2) - (e
*(a + c*x^2)^(3/2))/(3*(c*d^2 + a*e^2)*(d + e*x)^3) - (a*c^2*d*ArcTanh[(a*e
 - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(2*(c*d^2 + a*e^2)^(5/2))
```

Rubi [A] time = 0.0863029, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {731, 721, 725, 206}

$$-\frac{ac^2d\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{2\left(ae^2+cd^2\right)^{5/2}}-\frac{cd\sqrt{a+cx^2}(ae-cdx)}{2(d+ex)^2\left(ae^2+cd^2\right)^2}-\frac{e\left(a+cx^2\right)^{3/2}}{3(d+ex)^3\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(d + e*x)^4,x]
```

```
[Out] -(c*d*(a*e - c*d*x)*Sqrt[a + c*x^2])/(2*(c*d^2 + a*e^2)^2*(d + e*x)^2) - (e
*(a + c*x^2)^(3/2))/(3*(c*d^2 + a*e^2)*(d + e*x)^3) - (a*c^2*d*ArcTanh[(a*e
 - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(2*(c*d^2 + a*e^2)^(5/2))
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 721

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(-2*a*e + (2*c*d)*x)*(a + c*x^2)^p)/(2*(m + 1)*(c*d^2 +
 a*e^2)), x] - Dist[(4*a*c*p)/(2*(m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m
 + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+cx^2}}{(d+ex)^4}\,dx &= -\frac{e\left(a+cx^2\right)^{3/2}}{3\left(cd^2+ae^2\right)(d+ex)^3}+\frac{(cd)\int \frac{\sqrt{a+cx^2}}{(d+ex)^3}\,dx}{cd^2+ae^2}\\
&= -\frac{cd(ae-cdx)\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)^2(d+ex)^2}-\frac{e\left(a+cx^2\right)^{3/2}}{3\left(cd^2+ae^2\right)(d+ex)^3}+\frac{\left(ac^2d\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{2\left(cd^2+ae^2\right)^2}\\
&= -\frac{cd(ae-cdx)\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)^2(d+ex)^2}-\frac{e\left(a+cx^2\right)^{3/2}}{3\left(cd^2+ae^2\right)(d+ex)^3}-\frac{\left(ac^2d\right)\operatorname{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{2\left(cd^2+ae^2\right)^2}\\
&= -\frac{cd(ae-cdx)\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)^2(d+ex)^2}-\frac{e\left(a+cx^2\right)^{3/2}}{3\left(cd^2+ae^2\right)(d+ex)^3}-\frac{ac^2d\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{2\left(cd^2+ae^2\right)^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.169825, size = 173, normalized size = 1.2

$$
\frac{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}\left(-2a^2e^3-ace\left(5d^2+3dex+2e^2x^2\right)+c^2d^2x(3d+ex)\right)-3ac^2d(d+ex)^3\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}\right.}{6(d+ex)^3\left(ae^2+cd^2\right)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(d + e*x)^4,x]
```

```
[Out] (Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2]*(-2*a^2*e^3 + c^2*d^2*x*(3*d + e*x) -
a*c*e*(5*d^2 + 3*d*e*x + 2*e^2*x^2)) + 3*a*c^2*d*(d + e*x)^3*Log[d + e*x] -
 3*a*c^2*d*(d + e*x)^3*Log[a*e - c*d*x + Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2
]])/(6*(c*d^2 + a*e^2)^(5/2)*(d + e*x)^3)
```

Maple [B] time = 0.207, size = 1262, normalized size = 8.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/(e*x+d)^4,x)
```

```
[Out] -1/3/e^2/(a*e^2+c*d^2)/(d/e+x)^3*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)
/e^2)^(3/2)-1/2/e*c*d/(a*e^2+c*d^2)^2/(d/e+x)^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x
)+(a*e^2+c*d^2)/e^2)^(3/2)-1/2*c^2*d^2/(a*e^2+c*d^2)^3/(d/e+x)*(c*(d/e+x)^2
-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)-1/2/e*c^3*d^3/(a*e^2+c*d^2)^3*(c*
(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)+1/2/e^2*c^(7/2)*d^4/(a*e
^2+c*d^2)^3*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e
^2+c*d^2)/e^2)^(1/2))+1/2/e*c^3*d^3/(a*e^2+c*d^2)^3/((a*e^2+c*d^2)/e^2)^(1/
2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(
d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a+1/2/e^3*c^4*d
^5/(a*e^2+c*d^2)^3/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/
e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c
*d^2)/e^2)^(1/2))/(d/e+x))+1/2*c^3*d^2/(a*e^2+c*d^2)^3*(c*(d/e+x)^2-2*c*d/e
*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x+1/2*c^(5/2)*d^2/(a*e^2+c*d^2)^3*ln((-c*
d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2
```

```
))*a+1/2/e*c^2*d/(a*e^2+c*d^2)^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)
/e^2)^(1/2)-1/2/e^2*c^(5/2)*d^2/(a*e^2+c*d^2)^2*ln((-c*d/e+(d/e+x)*c)/c^(1/
2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))-1/2/e*c^2*d/(a*e^
2+c*d^2)^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x
)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^
2)^(1/2))/(d/e+x))*a-1/2/e^3*c^3*d^3/(a*e^2+c*d^2)^2/((a*e^2+c*d^2)/e^2)^(1
/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*
(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 7.25644, size = 1719, normalized size = 11.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a*c^2*d*e^3*x^3 + 3*a*c^2*d^2*e^2*x^2 + 3*a*c^2*d^3*e*x + a*c^2*d
^4)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2
 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2
*x^2 + 2*d*e*x + d^2)) - 2*(5*a*c^2*d^4*e + 7*a^2*c*d^2*e^3 + 2*a^3*e^5 - (
c^3*d^4*e - a*c^2*d^2*e^3 - 2*a^2*c*e^5)*x^2 - 3*(c^3*d^5 - a^2*c*d*e^4)*x)
*sqrt(c*x^2 + a))/(c^3*d^9 + 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4 + a^3*d^3*e^
6 + (c^3*d^6*e^3 + 3*a*c^2*d^4*e^5 + 3*a^2*c*d^2*e^7 + a^3*e^9)*x^3 + 3*(c^
3*d^7*e^2 + 3*a*c^2*d^5*e^4 + 3*a^2*c*d^3*e^6 + a^3*d*e^8)*x^2 + 3*(c^3*d^8
*e + 3*a*c^2*d^6*e^3 + 3*a^2*c*d^4*e^5 + a^3*d^2*e^7)*x), -1/6*(3*(a*c^2*d*
e^3*x^3 + 3*a*c^2*d^2*e^2*x^2 + 3*a*c^2*d^3*e*x + a*c^2*d^4)*sqrt(-c*d^2 -
a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 +
 a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (5*a*c^2*d^4*e + 7*a^2*c*d^2*e^3 + 2
*a^3*e^5 - (c^3*d^4*e - a*c^2*d^2*e^3 - 2*a^2*c*e^5)*x^2 - 3*(c^3*d^5 - a^2
*c*d*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^9 + 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4
+ a^3*d^3*e^6 + (c^3*d^6*e^3 + 3*a*c^2*d^4*e^5 + 3*a^2*c*d^2*e^7 + a^3*e^9)
*x^3 + 3*(c^3*d^7*e^2 + 3*a*c^2*d^5*e^4 + 3*a^2*c*d^3*e^6 + a^3*d*e^8)*x^2
+ 3*(c^3*d^8*e + 3*a*c^2*d^6*e^3 + 3*a^2*c*d^4*e^5 + a^3*d^2*e^7)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(d + e*x)**4, x)
```

Giac [B] time = 1.41163, size = 699, normalized size = 4.85

$$-\frac{ac^2d\arctan\left(\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)\sqrt{-cd^2-ae^2}}+\frac{6\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^4c^{\frac{7}{2}}d^4e+4\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^3c^4d^5-6\left(\sqrt{c}x-\sqrt{cx^2+a}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -a*c^2*d*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 -
 a*e^2))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-c*d^2 - a*e^2)) + 1/3*(
6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*c^(7/2)*d^4*e + 4*(sqrt(c)*x - sqrt(c*x^2
 + a))^3*c^4*d^5 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(7/2)*d^4*e - 14*(
sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c^3*d^3*e^2 - 3*(sqrt(c)*x - sqrt(c*x^2 +
a))^4*a*c^(5/2)*d^2*e^3 - 3*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^2*d*e^4 + 6
*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c^3*d^3*e^2 + 24*(sqrt(c)*x - sqrt(c*x^2
 + a))^2*a^2*c^(5/2)*d^2*e^3 + 12*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^2*d
*e^4 + 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(3/2)*e^5 - a^3*c^(5/2)*d^2*
e^3 - 9*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2*d*e^4 + 2*a^4*c^(3/2)*e^5)/((
c^2*d^4*e^2 + 2*a*c*d^2*e^4 + a^2*e^6)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*e +
 2*(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*d - a*e)^3)
```

3.533 $\int \frac{\sqrt{a+cx^2}}{(d+ex)^5} \, dx$

Optimal. Leaf size=206

$$-\frac{ac^2\left(4cd^2-ae^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{8\left(ae^2+cd^2\right)^{7/2}}-\frac{5cde\left(a+cx^2\right)^{3/2}}{12(d+ex)^3\left(ae^2+cd^2\right)^2}-\frac{c\sqrt{a+cx^2}\left(4cd^2-ae^2\right)(ae-cdx)}{8(d+ex)^2\left(ae^2+cd^2\right)^3}-\frac{e\left(a+c\cdots\right.}{4(d+ex)^4\left(a\cdots\right.}$$

```
[Out] -(c*(4*c*d^2 - a*e^2)*(a*e - c*d*x)*Sqrt[a + c*x^2])/(8*(c*d^2 + a*e^2)^3*(
d + e*x)^2) - (e*(a + c*x^2)^(3/2))/(4*(c*d^2 + a*e^2)*(d + e*x)^4) - (5*c*
d*e*(a + c*x^2)^(3/2))/(12*(c*d^2 + a*e^2)^2*(d + e*x)^3) - (a*c^2*(4*c*d^2
 - a*e^2)*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(8*
(c*d^2 + a*e^2)^(7/2))
```

Rubi [A] time = 0.126301, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {745, 807, 721, 725, 206}

$$-\frac{ac^2\left(4cd^2-ae^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{8\left(ae^2+cd^2\right)^{7/2}}-\frac{5cde\left(a+cx^2\right)^{3/2}}{12(d+ex)^3\left(ae^2+cd^2\right)^2}-\frac{c\sqrt{a+cx^2}\left(4cd^2-ae^2\right)(ae-cdx)}{8(d+ex)^2\left(ae^2+cd^2\right)^3}-\frac{e\left(a+c\cdots\right.}{4(d+ex)^4\left(a\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(d + e*x)^5,x]
```


```
[Out] -(c*(4*c*d^2 - a*e^2)*(a*e - c*d*x)*Sqrt[a + c*x^2])/(8*(c*d^2 + a*e^2)^3*(
d + e*x)^2) - (e*(a + c*x^2)^(3/2))/(4*(c*d^2 + a*e^2)*(d + e*x)^4) - (5*c*
d*e*(a + c*x^2)^(3/2))/(12*(c*d^2 + a*e^2)^2*(d + e*x)^3) - (a*c^2*(4*c*d^2
 - a*e^2)*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(8*
(c*d^2 + a*e^2)^(7/2))
```


Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 721

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(-2*a*e + (2*c*d)*x)*(a + c*x^2)^p)/(2*(m + 1)*(c*d^2 +
 a*e^2)), x] - Dist[(4*a*c*p)/(2*(m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m
```

```
+ 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^5}\,dx = -\frac{e\left(a+cx^2\right)^{3/2}}{4\left(cd^2+ae^2\right)(d+ex)^4} - \frac{c\int \frac{(-4d+ex)\sqrt{a+cx^2}}{(d+ex)^4}\,dx}{4\left(cd^2+ae^2\right)}$$

$$= -\frac{e\left(a+cx^2\right)^{3/2}}{4\left(cd^2+ae^2\right)(d+ex)^4} - \frac{5cde\left(a+cx^2\right)^{3/2}}{12\left(cd^2+ae^2\right)^2(d+ex)^3} + \frac{\left(c\left(4cd^2-ae^2\right)\right)\int \frac{\sqrt{a+cx^2}}{(d+ex)^3}\,dx}{4\left(cd^2+ae^2\right)^2}$$

$$= -\frac{c\left(4cd^2-ae^2\right)(ae-cdx)\sqrt{a+cx^2}}{8\left(cd^2+ae^2\right)^3(d+ex)^2} - \frac{e\left(a+cx^2\right)^{3/2}}{4\left(cd^2+ae^2\right)(d+ex)^4} - \frac{5cde\left(a+cx^2\right)^{3/2}}{12\left(cd^2+ae^2\right)^2(d+ex)^3} + \frac{\left(ac^2\left(4c\right.\right.}{}$$

$$= -\frac{c\left(4cd^2-ae^2\right)(ae-cdx)\sqrt{a+cx^2}}{8\left(cd^2+ae^2\right)^3(d+ex)^2} - \frac{e\left(a+cx^2\right)^{3/2}}{4\left(cd^2+ae^2\right)(d+ex)^4} - \frac{5cde\left(a+cx^2\right)^{3/2}}{12\left(cd^2+ae^2\right)^2(d+ex)^3} - \frac{\left(ac^2\left(4c\right.\right.}{}$$

$$= -\frac{c\left(4cd^2-ae^2\right)(ae-cdx)\sqrt{a+cx^2}}{8\left(cd^2+ae^2\right)^3(d+ex)^2} - \frac{e\left(a+cx^2\right)^{3/2}}{4\left(cd^2+ae^2\right)(d+ex)^4} - \frac{5cde\left(a+cx^2\right)^{3/2}}{12\left(cd^2+ae^2\right)^2(d+ex)^3} - \frac{ac^2\left(4cd\right.}{}$$

Mathematica [A] time = 0.23641, size = 248, normalized size = 1.2

$$\frac{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}\left(c^2d(d+ex)^3\left(2cd^2-13ae^2\right)+2cd(d+ex)\left(ae^2+cd^2\right)^2+c(d+ex)^2\left(2cd^2-3ae^2\right)\left(ae^2+cd^2\right)-6\right.}{24e(d+ex}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(d + e*x)^5,x]
```

```
[Out] (Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2]*(-6*(c*d^2 + a*e^2)^3 + 2*c*d*(c*d^2 +
 a*e^2)^2*(d + e*x) + c*(2*c*d^2 - 3*a*e^2)*(c*d^2 + a*e^2)*(d + e*x)^2 + c
^2*d*(2*c*d^2 - 13*a*e^2)*(d + e*x)^3) - 3*a*c^2*e*(-4*c*d^2 + a*e^2)*(d +
e*x)^4*Log[d + e*x] + 3*a*c^2*e*(-4*c*d^2 + a*e^2)*(d + e*x)^4*Log[a*e - c*
d*x + Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2]])/(24*e*(c*d^2 + a*e^2)^(7/2)*(d
+ e*x)^4)
```

Maple [B] time = 0.205, size = 2073, normalized size = 10.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/(e*x+d)^5,x)
```

```
[Out] 5/8/e*c^4*d^4/(a*e^2+c*d^2)^4/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)
/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+
x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a+5/8*c^(7/2)*d^3/(a*e^2+c*d^2)^4*ln(
(-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^
(1/2))*a-3/4/e*c^3*d^2/(a*e^2+c*d^2)^3/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e
^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*
d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a+1/8/(a*e^2+c*d^2)^3*c^2*d/
(d/e+x)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)-1/8/(a*e^2+c*
d^2)^3*c^3*d*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x-1/4/e^
3/(a*e^2+c*d^2)/(d/e+x)^4*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(
3/2)-1/8/e/(a*e^2+c*d^2)^2*c^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e
^2)^(1/2)+5/8/e^3*c^5*d^6/(a*e^2+c*d^2)^4/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(
a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2
*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))+5/8*c^4*d^3/(a*e^2+c*d^2)
^4*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x-3/4/e^2*c^(7/2)*
d^3/(a*e^2+c*d^2)^3*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e
+x)+(a*e^2+c*d^2)/e^2)^(1/2))-3/4/e^3*c^4*d^4/(a*e^2+c*d^2)^3/((a*e^2+c*d^2
)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^
(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))-5/8/e
*c^4*d^4/(a*e^2+c*d^2)^4*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1
/2)+3/4/e*c^3*d^2/(a*e^2+c*d^2)^3*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2
)/e^2)^(1/2)-5/12/e^2*c*d/(a*e^2+c*d^2)^2/(d/e+x)^3*(c*(d/e+x)^2-2*c*d/e*(d
/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)-5/8/e*c^2*d^2/(a*e^2+c*d^2)^3/(d/e+x)^2*(c*(
d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)-5/8*c^3*d^3/(a*e^2+c*d^2)
^4/(d/e+x)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)+5/8/e^2*c^
(9/2)*d^5/(a*e^2+c*d^2)^4*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/
e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))+1/8/e/(a*e^2+c*d^2)^2*c/(d/e+x)^2*(c*(d
/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)-1/8/(a*e^2+c*d^2)^3*c^(5/2
)*d*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2
)/e^2)^(1/2))*a+1/8/e^2/(a*e^2+c*d^2)^2*c^(5/2)*d*ln((-c*d/e+(d/e+x)*c)/c^(
1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))+1/8/e/(a*e^2+c*
d^2)^2*c^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x
)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^
2)^(1/2))/(d/e+x))*a+1/8/e^3/(a*e^2+c*d^2)^2*c^3/((a*e^2+c*d^2)/e^2)^(1/2)*
ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e
+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 24.853, size = 2958, normalized size = 14.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^5,x, algorithm="fricas")
```

```
[Out] [-1/48*(3*(4*a*c^3*d^6 - a^2*c^2*d^4*e^2 + (4*a*c^3*d^2*e^4 - a^2*c^2*e^6)*
x^4 + 4*(4*a*c^3*d^3*e^3 - a^2*c^2*d*e^5)*x^3 + 6*(4*a*c^3*d^4*e^2 - a^2*c^
2*d^2*e^4)*x^2 + 4*(4*a*c^3*d^5*e - a^2*c^2*d^3*e^3)*x)*sqrt(c*d^2 + a*e^2)
*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqr
t(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2))
+ 2*(28*a*c^3*d^6*e + 47*a^2*c^2*d^4*e^3 + 25*a^3*c*d^2*e^5 + 6*a^4*e^7 - (
2*c^4*d^5*e^2 - 11*a*c^3*d^3*e^4 - 13*a^2*c^2*d*e^6)*x^3 - (8*c^4*d^6*e - 3
2*a*c^3*d^4*e^3 - 43*a^2*c^2*d^2*e^5 - 3*a^3*c*e^7)*x^2 - (12*c^4*d^7 - 25*
a*c^3*d^5*e^2 - 41*a^2*c^2*d^3*e^4 - 4*a^3*c*d*e^6)*x)*sqrt(c*x^2 + a))/(c^
4*d^12 + 4*a*c^3*d^10*e^2 + 6*a^2*c^2*d^8*e^4 + 4*a^3*c*d^6*e^6 + a^4*d^4*e
^8 + (c^4*d^8*e^4 + 4*a*c^3*d^6*e^6 + 6*a^2*c^2*d^4*e^8 + 4*a^3*c*d^2*e^10
+ a^4*e^12)*x^4 + 4*(c^4*d^9*e^3 + 4*a*c^3*d^7*e^5 + 6*a^2*c^2*d^5*e^7 + 4*
a^3*c*d^3*e^9 + a^4*d*e^11)*x^3 + 6*(c^4*d^10*e^2 + 4*a*c^3*d^8*e^4 + 6*a^2
*c^2*d^6*e^6 + 4*a^3*c*d^4*e^8 + a^4*d^2*e^10)*x^2 + 4*(c^4*d^11*e + 4*a*c^
3*d^9*e^3 + 6*a^2*c^2*d^7*e^5 + 4*a^3*c*d^5*e^7 + a^4*d^3*e^9)*x), -1/24*(3
*(4*a*c^3*d^6 - a^2*c^2*d^4*e^2 + (4*a*c^3*d^2*e^4 - a^2*c^2*e^6)*x^4 + 4*(
4*a*c^3*d^3*e^3 - a^2*c^2*d*e^5)*x^3 + 6*(4*a*c^3*d^4*e^2 - a^2*c^2*d^2*e^4
)*x^2 + 4*(4*a*c^3*d^5*e - a^2*c^2*d^3*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(
sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^
2*d^2 + a*c*e^2)*x^2)) + (28*a*c^3*d^6*e + 47*a^2*c^2*d^4*e^3 + 25*a^3*c*d^
2*e^5 + 6*a^4*e^7 - (2*c^4*d^5*e^2 - 11*a*c^3*d^3*e^4 - 13*a^2*c^2*d*e^6)*x
^3 - (8*c^4*d^6*e - 32*a*c^3*d^4*e^3 - 43*a^2*c^2*d^2*e^5 - 3*a^3*c*e^7)*x^
2 - (12*c^4*d^7 - 25*a*c^3*d^5*e^2 - 41*a^2*c^2*d^3*e^4 - 4*a^3*c*d*e^6)*x)
*sqrt(c*x^2 + a))/(c^4*d^12 + 4*a*c^3*d^10*e^2 + 6*a^2*c^2*d^8*e^4 + 4*a^3*
c*d^6*e^6 + a^4*d^4*e^8 + (c^4*d^8*e^4 + 4*a*c^3*d^6*e^6 + 6*a^2*c^2*d^4*e^
8 + 4*a^3*c*d^2*e^10 + a^4*e^12)*x^4 + 4*(c^4*d^9*e^3 + 4*a*c^3*d^7*e^5 + 6
*a^2*c^2*d^5*e^7 + 4*a^3*c*d^3*e^9 + a^4*d*e^11)*x^3 + 6*(c^4*d^10*e^2 + 4*
a*c^3*d^8*e^4 + 6*a^2*c^2*d^6*e^6 + 4*a^3*c*d^4*e^8 + a^4*d^2*e^10)*x^2 + 4
*(c^4*d^11*e + 4*a*c^3*d^9*e^3 + 6*a^2*c^2*d^7*e^5 + 4*a^3*c*d^5*e^7 + a^4*
d^3*e^9)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^2}}{(d + ex)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/(e*x+d)**5,x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(d + e*x)**5, x)
```

Giac [B] time = 4.21485, size = 1297, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] 1/192*(sqrt(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(x*e + d)^2)*((
2*((c^3*d^5*e^6*sgn(1/(x*e + d)) + 2*a*c^2*d^3*e^8*sgn(1/(x*e + d)) + a^2*c
*d*e^10*sgn(1/(x*e + d)))/(c^4*d^8*e^8 + 4*a*c^3*d^6*e^10 + 6*a^2*c^2*d^4*e
^12 + 4*a^3*c*d^2*e^14 + a^4*e^16) - 3*(c^3*d^6*e^7*sgn(1/(x*e + d)) + 3*a*
c^2*d^4*e^9*sgn(1/(x*e + d)) + 3*a^2*c*d^2*e^11*sgn(1/(x*e + d)) + a^3*e^13
*sgn(1/(x*e + d)))*e^(-1)/((c^4*d^8*e^8 + 4*a*c^3*d^6*e^10 + 6*a^2*c^2*d^4*
e^12 + 4*a^3*c*d^2*e^14 + a^4*e^16)*(x*e + d)))*e^(-1)/(x*e + d) + (2*c^3*d
^4*e^5*sgn(1/(x*e + d)) - a*c^2*d^2*e^7*sgn(1/(x*e + d)) - 3*a^2*c*e^9*sgn(
1/(x*e + d)))/(c^4*d^8*e^8 + 4*a*c^3*d^6*e^10 + 6*a^2*c^2*d^4*e^12 + 4*a^3*
c*d^2*e^14 + a^4*e^16))*e^(-1)/(x*e + d) + (2*c^3*d^3*e^4*sgn(1/(x*e + d))
- 13*a*c^2*d*e^6*sgn(1/(x*e + d)))/(c^4*d^8*e^8 + 4*a*c^3*d^6*e^10 + 6*a^2*
c^2*d^4*e^12 + 4*a^3*c*d^2*e^14 + a^4*e^16)) - 3*(4*a*c^3*d^2*sgn(1/(x*e +
d)) - a^2*c^2*e^2*sgn(1/(x*e + d)))*sqrt(c*d^2 + a*e^2)*log(abs(sqrt(c*d^2
+ a*e^2)*c*d - (c*d^2 + a*e^2)*(sqrt(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^
2 + a*e^2/(x*e + d)^2) + sqrt(c*d^2*e^2 + a*e^4)*e^(-1)/(x*e + d))))/(c^5*d
^10*e^2 + 5*a*c^4*d^8*e^4 + 10*a^2*c^3*d^6*e^6 + 10*a^3*c^2*d^4*e^8 + 5*a^4
*c*d^2*e^10 + a^5*e^12) - (2*c^(9/2)*d^5 - 11*a*c^(7/2)*d^3*e^2 - 12*sqrt(c
*d^2 + a*e^2)*a*c^3*d^2*e^2*log(abs(-c^(3/2)*d^2 + sqrt(c*d^2 + a*e^2)*c*d
- a*sqrt(c)*e^2)) - 13*a^2*c^(5/2)*d*e^4 + 3*sqrt(c*d^2 + a*e^2)*a^2*c^2*e^
4*log(abs(-c^(3/2)*d^2 + sqrt(c*d^2 + a*e^2)*c*d - a*sqrt(c)*e^2)))*sgn(1/(
x*e + d))/(c^5*d^10*e^4 + 5*a*c^4*d^8*e^6 + 10*a^2*c^3*d^6*e^8 + 10*a^3*c^2
*d^4*e^10 + 5*a^4*c*d^2*e^12 + a^5*e^14))*e^2
```

3.534 $\int (d+ex)^4 \left(a+cx^2\right)^{3/2} dx$

Optimal. Leaf size=255

$$\frac{x\left(a+cx^2\right)^{3/2}\left(a^2e^4-16acd^2e^2+16c^2d^4\right)}{64c^2}+\frac{3ax\sqrt{a+cx^2}\left(a^2e^4-16acd^2e^2+16c^2d^4\right)}{128c^2}+\frac{3a^2\left(a^2e^4-16acd^2e^2+16c^2d^4\right.}{128c^{5/2}}$$

```
[Out] (3*a*(16*c^2*d^4 - 16*a*c*d^2*e^2 + a^2*e^4)*x*Sqrt[a + c*x^2])/(128*c^2) +
 ((16*c^2*d^4 - 16*a*c*d^2*e^2 + a^2*e^4)*x*(a + c*x^2)^(3/2))/(64*c^2) + (
11*d*e*(d + e*x)^2*(a + c*x^2)^(5/2))/(56*c) + (e*(d + e*x)^3*(a + c*x^2)^(
5/2))/(8*c) + (e*(4*d*(67*c*d^2 - 32*a*e^2) + 5*e*(26*c*d^2 - 7*a*e^2)*x)*(
a + c*x^2)^(5/2))/(560*c^2) + (3*a^2*(16*c^2*d^4 - 16*a*c*d^2*e^2 + a^2*e^4
)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(128*c^(5/2))
```

Rubi [A] time = 0.250085, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {743, 833, 780, 195, 217, 206}

$$\frac{x\left(a+cx^2\right)^{3/2}\left(a^2e^4-16acd^2e^2+16c^2d^4\right)}{64c^2}+\frac{3ax\sqrt{a+cx^2}\left(a^2e^4-16acd^2e^2+16c^2d^4\right)}{128c^2}+\frac{3a^2\left(a^2e^4-16acd^2e^2+16c^2d^4\right.}{128c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4*(a + c*x^2)^(3/2),x]
```


```
[Out] (3*a*(16*c^2*d^4 - 16*a*c*d^2*e^2 + a^2*e^4)*x*Sqrt[a + c*x^2])/(128*c^2) +
 ((16*c^2*d^4 - 16*a*c*d^2*e^2 + a^2*e^4)*x*(a + c*x^2)^(3/2))/(64*c^2) + (
11*d*e*(d + e*x)^2*(a + c*x^2)^(5/2))/(56*c) + (e*(d + e*x)^3*(a + c*x^2)^(
5/2))/(8*c) + (e*(4*d*(67*c*d^2 - 32*a*e^2) + 5*e*(26*c*d^2 - 7*a*e^2)*x)*(
a + c*x^2)^(5/2))/(560*c^2) + (3*a^2*(16*c^2*d^4 - 16*a*c*d^2*e^2 + a^2*e^4
)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(128*c^(5/2))
```


Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
```

```
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^4\left(a+cx^2\right)^{3/2}dx = \frac{e(d+ex)^3\left(a+cx^2\right)^{5/2}}{8c}+\frac{\int (d+ex)^2\left(8cd^2-3ae^2+11cdex\right)\left(a+cx^2\right)^{3/2}dx}{8c}$$

$$= \frac{11de(d+ex)^2\left(a+cx^2\right)^{5/2}}{56c}+\frac{e(d+ex)^3\left(a+cx^2\right)^{5/2}}{8c}+\frac{\int (d+ex)\left(cd\left(56cd^2-43ae^2\right)+3ce\left(\right.\right.}{56c^2}$$

$$= \frac{11de(d+ex)^2\left(a+cx^2\right)^{5/2}}{56c}+\frac{e(d+ex)^3\left(a+cx^2\right)^{5/2}}{8c}+\frac{e\left(4d\left(67cd^2-32ae^2\right)+5e\left(26cd^2-\right.\right.}{560c^2}$$

$$= \frac{\left(16c^2d^4-16acd^2e^2+a^2e^4\right)x\left(a+cx^2\right)^{3/2}}{64c^2}+\frac{11de(d+ex)^2\left(a+cx^2\right)^{5/2}}{56c}+\frac{e(d+ex)^3\left(a+cx^2\right.}{8c}$$

$$= \frac{3a\left(16c^2d^4-16acd^2e^2+a^2e^4\right)x\sqrt{a+cx^2}}{128c^2}+\frac{\left(16c^2d^4-16acd^2e^2+a^2e^4\right)x\left(a+cx^2\right)^{3/2}}{64c^2}+\frac{11a}{}$$

$$= \frac{3a\left(16c^2d^4-16acd^2e^2+a^2e^4\right)x\sqrt{a+cx^2}}{128c^2}+\frac{\left(16c^2d^4-16acd^2e^2+a^2e^4\right)x\left(a+cx^2\right)^{3/2}}{64c^2}+\frac{11a}{}$$

$$= \frac{3a\left(16c^2d^4-16acd^2e^2+a^2e^4\right)x\sqrt{a+cx^2}}{128c^2}+\frac{\left(16c^2d^4-16acd^2e^2+a^2e^4\right)x\left(a+cx^2\right)^{3/2}}{64c^2}+\frac{11a}{}$$

Mathematica [A] time = 0.172729, size = 231, normalized size = 0.91

$$\frac{\sqrt{c}\sqrt{a+cx^2}\left(2a^2ce\left(840d^2ex+1792d^3+256de^2x^2+35e^3x^3\right)-a^3e^3(1024d+105ex)+8ac^2x\left(980d^2e^2x^2+896d^3ex+35\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4*(a + c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[c]*Sqrt[a + c*x^2]*(-(a^3*e^3*(1024*d + 105*e*x)) + 2*a^2*c*e*(1792*d
^3 + 840*d^2*e*x + 256*d*e^2*x^2 + 35*e^3*x^3) + 16*c^3*x^3*(70*d^4 + 224*d
^3*e*x + 280*d^2*e^2*x^2 + 160*d*e^3*x^3 + 35*e^4*x^4) + 8*a*c^2*x*(350*d^4
```

```
 + 896*d^3*e*x + 980*d^2*e^2*x^2 + 512*d*e^3*x^3 + 105*e^4*x^4)) + 105*a^2*
(16*c^2*d^4 - 16*a*c*d^2*e^2 + a^2*e^4)*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])
/(4480*c^(5/2))
```

Maple [A] time = 0.056, size = 322, normalized size = 1.3

$$\frac{e^4x^3}{8c}\left(cx^2+a\right)^{\frac{5}{2}}-\frac{e^4ax}{16c^2}\left(cx^2+a\right)^{\frac{5}{2}}+\frac{a^2e^4x}{64c^2}\left(cx^2+a\right)^{\frac{3}{2}}+\frac{3e^4a^3x}{128c^2}\sqrt{cx^2+a}+\frac{3e^4a^4}{128}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)c^{-\frac{5}{2}}+\frac{4de^3x}{7c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(c*x^2+a)^(3/2),x)
```

```
[Out] 1/8*e^4*x^3*(c*x^2+a)^(5/2)/c-1/16*e^4*a/c^2*x*(c*x^2+a)^(5/2)+1/64*e^4*a^2
/c^2*x*(c*x^2+a)^(3/2)+3/128*e^4*a^3/c^2*x*(c*x^2+a)^(1/2)+3/128*e^4*a^4/c^
(5/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))+4/7*d*e^3*x^2*(c*x^2+a)^(5/2)/c-8/35*d*
e^3*a/c^2*(c*x^2+a)^(5/2)+d^2*e^2*x*(c*x^2+a)^(5/2)/c-1/4*d^2*e^2*a/c*x*(c*
x^2+a)^(3/2)-3/8*d^2*e^2*a^2/c*x*(c*x^2+a)^(1/2)-3/8*d^2*e^2*a^3/c^(3/2)*ln
(x*c^(1/2)+(c*x^2+a)^(1/2))+4/5*d^3*e*(c*x^2+a)^(5/2)/c+1/4*d^4*x*(c*x^2+a)
^(3/2)+3/8*d^4*a*x*(c*x^2+a)^(1/2)+3/8*d^4*a^2/c^(1/2)*ln(x*c^(1/2)+(c*x^2+
a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.2831, size = 1219, normalized size = 4.78

$$\left[\frac{105\left(16a^2c^2d^4-16a^3cd^2e^2+a^4e^4\right)\sqrt{c}\log\left(-2cx^2-2\sqrt{cx^2+a}\sqrt{c}x-a\right)+2\left(560c^4e^4x^7+2560c^4de^3x^6+3584a^2c^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8960*(105*(16*a^2*c^2*d^4 - 16*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(c)*log(-2*c
*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(560*c^4*e^4*x^7 + 2560*c^4*d*e
^3*x^6 + 3584*a^2*c^2*d^3*e - 1024*a^3*c*d*e^3 + 280*(16*c^4*d^2*e^2 + 3*a*
c^3*e^4)*x^5 + 512*(7*c^4*d^3*e + 8*a*c^3*d*e^3)*x^4 + 70*(16*c^4*d^4 + 112
*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^3 + 512*(14*a*c^3*d^3*e + a^2*c^2*d*e^3)*x^
2 + 35*(80*a*c^3*d^4 + 48*a^2*c^2*d^2*e^2 - 3*a^3*c*e^4)*x)*sqrt(c*x^2 + a)
)/c^3, -1/4480*(105*(16*a^2*c^2*d^4 - 16*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-c)*
arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (560*c^4*e^4*x^7 + 2560*c^4*d*e^3*x^6
+ 3584*a^2*c^2*d^3*e - 1024*a^3*c*d*e^3 + 280*(16*c^4*d^2*e^2 + 3*a*c^3*e^4
```

```
)*x^5 + 512*(7*c^4*d^3*e + 8*a*c^3*d*e^3)*x^4 + 70*(16*c^4*d^4 + 112*a*c^3*
d^2*e^2 + a^2*c^2*e^4)*x^3 + 512*(14*a*c^3*d^3*e + a^2*c^2*d*e^3)*x^2 + 35*
(80*a*c^3*d^4 + 48*a^2*c^2*d^2*e^2 - 3*a^3*c*e^4)*x)*sqrt(c*x^2 + a))/c^3]
```

Sympy [A] time = 32.2539, size = 734, normalized size = 2.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(c*x**2+a)**(3/2),x)
```

```
[Out] -3*a**(7/2)*e**4*x/(128*c**2*sqrt(1 + c*x**2/a)) + 3*a**(5/2)*d**2*e**2*x/(
8*c*sqrt(1 + c*x**2/a)) - a**(5/2)*e**4*x**3/(128*c*sqrt(1 + c*x**2/a)) + a
**(3/2)*d**4*x*sqrt(1 + c*x**2/a)/2 + a**(3/2)*d**4*x/(8*sqrt(1 + c*x**2/a)
) + 17*a**(3/2)*d**2*e**2*x**3/(8*sqrt(1 + c*x**2/a)) + 13*a**(3/2)*e**4*x*
*5/(64*sqrt(1 + c*x**2/a)) + 3*sqrt(a)*c*d**4*x**3/(8*sqrt(1 + c*x**2/a)) +
 11*sqrt(a)*c*d**2*e**2*x**5/(4*sqrt(1 + c*x**2/a)) + 5*sqrt(a)*c*e**4*x**7
/(16*sqrt(1 + c*x**2/a)) + 3*a**4*e**4*asinh(sqrt(c)*x/sqrt(a))/(128*c**(5/
2)) - 3*a**3*d**2*e**2*asinh(sqrt(c)*x/sqrt(a))/(8*c**(3/2)) + 3*a**2*d**4*
asinh(sqrt(c)*x/sqrt(a))/(8*sqrt(c)) + 4*a*d**3*e*Piecewise((sqrt(a)*x**2/2
, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + 4*a*d*e**3*Piecewise((-2*
a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqr
t(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + 4*c*d**3*e*Piecewise(
(-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4
*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + 4*c*d*e**3*Piecew
ise((8*a**3*sqrt(a + c*x**2)/(105*c**3) - 4*a**2*x**2*sqrt(a + c*x**2)/(105
*c**2) + a*x**4*sqrt(a + c*x**2)/(35*c) + x**6*sqrt(a + c*x**2)/7, Ne(c, 0)
), (sqrt(a)*x**6/6, True)) + c**2*d**4*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a))
+ c**2*d**2*e**2*x**7/(sqrt(a)*sqrt(1 + c*x**2/a)) + c**2*e**4*x**9/(8*sqrt
(a)*sqrt(1 + c*x**2/a))
```

Giac [A] time = 1.31103, size = 374, normalized size = 1.47

$$\frac{1}{4480}\sqrt{cx^2+a}\left(\left(2\left(\left(4\left(5\left(2\left(7cxe^4+32cde^3\right)x+\frac{7\left(16c^7d^2e^2+3ac^6e^4\right)}{c^6}\right)x+\frac{64\left(7c^7d^3e+8ac^6de^3\right)}{c^6}\right)x+\frac{35\left(16c^7d^4+1\right.}{}\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4480*sqrt(c*x^2 + a)*((2*((4*(5*(2*(7*c*x*e^4 + 32*c*d*e^3)*x + 7*(16*c^7
*d^2*e^2 + 3*a*c^6*e^4)/c^6)*x + 64*(7*c^7*d^3*e + 8*a*c^6*d*e^3)/c^6)*x +
35*(16*c^7*d^4 + 112*a*c^6*d^2*e^2 + a^2*c^5*e^4)/c^6)*x + 256*(14*a*c^6*d^
3*e + a^2*c^5*d*e^3)/c^6)*x + 35*(80*a*c^6*d^4 + 48*a^2*c^5*d^2*e^2 - 3*a^3
*c^4*e^4)/c^6)*x + 512*(7*a^2*c^5*d^3*e - 2*a^3*c^4*d*e^3)/c^6) - 3/128*(16
*a^2*c^2*d^4 - 16*a^3*c*d^2*e^2 + a^4*e^4)*log(abs(-sqrt(c)*x + sqrt(c*x^2
+ a)))/c^(5/2)
```

3.535 $\int (d+ex)^3 \left(a+cx^2\right)^{3/2} dx$

Optimal. Leaf size=180

$$\frac{3a^2d\left(2cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16c^{3/2}}+\frac{e\left(a+cx^2\right)^{5/2}\left(4\left(8cd^2-ae^2\right)+15cdex\right)}{70c^2}+\frac{dx\left(a+cx^2\right)^{3/2}\left(2cd^2-ae^2\right)}{8c}+\frac{3adx\sqrt{}}{}$$

```
[Out] (3*a*d*(2*c*d^2 - a*e^2)*x*Sqrt[a + c*x^2])/(16*c) + (d*(2*c*d^2 - a*e^2)*x
*(a + c*x^2)^(3/2))/(8*c) + (e*(d + e*x)^2*(a + c*x^2)^(5/2))/(7*c) + (e*(4
*(8*c*d^2 - a*e^2) + 15*c*d*e*x)*(a + c*x^2)^(5/2))/(70*c^2) + (3*a^2*d*(2*
c*d^2 - a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(16*c^(3/2))
```

Rubi [A] time = 0.145263, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {743, 780, 195, 217, 206}

$$\frac{3a^2d\left(2cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16c^{3/2}}+\frac{e\left(a+cx^2\right)^{5/2}\left(4\left(8cd^2-ae^2\right)+15cdex\right)}{70c^2}+\frac{dx\left(a+cx^2\right)^{3/2}\left(2cd^2-ae^2\right)}{8c}+\frac{3adx\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + c*x^2)^(3/2),x]
```

```
[Out] (3*a*d*(2*c*d^2 - a*e^2)*x*Sqrt[a + c*x^2])/(16*c) + (d*(2*c*d^2 - a*e^2)*x
*(a + c*x^2)^(3/2))/(8*c) + (e*(d + e*x)^2*(a + c*x^2)^(5/2))/(7*c) + (e*(4
*(8*c*d^2 - a*e^2) + 15*c*d*e*x)*(a + c*x^2)^(5/2))/(70*c^2) + (3*a^2*d*(2*
c*d^2 - a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(16*c^(3/2))
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^3\left(a+cx^2\right)^{3/2}dx = \frac{e(d+ex)^2\left(a+cx^2\right)^{5/2}}{7c}+\frac{\int (d+ex)\left(7cd^2-2ae^2+9cdex\right)\left(a+cx^2\right)^{3/2}dx}{7c}$$

$$=\frac{e(d+ex)^2\left(a+cx^2\right)^{5/2}}{7c}+\frac{e\left(4\left(8cd^2-ae^2\right)+15cdex\right)\left(a+cx^2\right)^{5/2}}{70c^2}+\frac{\left(d\left(2cd^2-ae^2\right)\right)\int\left(a+\right.}{2c}$$

$$=\frac{d\left(2cd^2-ae^2\right)x\left(a+cx^2\right)^{3/2}}{8c}+\frac{e(d+ex)^2\left(a+cx^2\right)^{5/2}}{7c}+\frac{e\left(4\left(8cd^2-ae^2\right)+15cdex\right)\left(a+cx\right.}{70c^2}$$

$$=\frac{3ad\left(2cd^2-ae^2\right)x\sqrt{a+cx^2}}{16c}+\frac{d\left(2cd^2-ae^2\right)x\left(a+cx^2\right)^{3/2}}{8c}+\frac{e(d+ex)^2\left(a+cx^2\right)^{5/2}}{7c}+\frac{e\left(4\right.}{}$$

$$=\frac{3ad\left(2cd^2-ae^2\right)x\sqrt{a+cx^2}}{16c}+\frac{d\left(2cd^2-ae^2\right)x\left(a+cx^2\right)^{3/2}}{8c}+\frac{e(d+ex)^2\left(a+cx^2\right)^{5/2}}{7c}+\frac{e\left(4\right.}{}$$

$$=\frac{3ad\left(2cd^2-ae^2\right)x\sqrt{a+cx^2}}{16c}+\frac{d\left(2cd^2-ae^2\right)x\left(a+cx^2\right)^{3/2}}{8c}+\frac{e(d+ex)^2\left(a+cx^2\right)^{5/2}}{7c}+\frac{e\left(4\right.}{}$$

Mathematica [A] time = 0.121345, size = 174, normalized size = 0.97

$$\frac{\sqrt{a+cx^2}\left(a^2ce\left(336d^2+105dex+16e^2x^2\right)-32a^3e^3+2ac^2x\left(336d^2ex+175d^3+245de^2x^2+64e^3x^3\right)+4c^3x^3\left(84d^2ex+\right.\right.}{560c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(-32*a^3*e^3 + a^2*c*e*(336*d^2 + 105*d*e*x + 16*e^2*x^2)
+ 4*c^3*x^3*(35*d^3 + 84*d^2*e*x + 70*d*e^2*x^2 + 20*e^3*x^3) + 2*a*c^2*x*(
175*d^3 + 336*d^2*e*x + 245*d*e^2*x^2 + 64*e^3*x^3)) - 105*a^2*Sqrt[c]*d*(-
2*c*d^2 + a*e^2)*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/(560*c^2)
```

Maple [A] time = 0.052, size = 205, normalized size = 1.1

$$\frac{e^3x^2}{7c}\left(cx^2+a\right)^{\frac{5}{2}}-\frac{2ae^3}{35c^2}\left(cx^2+a\right)^{\frac{5}{2}}+\frac{de^2x}{2c}\left(cx^2+a\right)^{\frac{5}{2}}-\frac{ade^2x}{8c}\left(cx^2+a\right)^{\frac{3}{2}}-\frac{3de^2a^2x}{16c}\sqrt{cx^2+a}-\frac{3de^2a^3}{16}\ln\left(x\sqrt{c}+\sqrt{cx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+a)^(3/2),x)
```

```
[Out] 1/7*e^3*x^2*(c*x^2+a)^(5/2)/c-2/35*e^3*a/c^2*(c*x^2+a)^(5/2)+1/2*d*e^2*x*(c
*x^2+a)^(5/2)/c-1/8*d*e^2*a/c*x*(c*x^2+a)^(3/2)-3/16*d*e^2*a^2/c*x*(c*x^2+a
```

```
)^(1/2)-3/16*d*e^2*a^3/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))+3/5*d^2*e*(c*x
^2+a)^(5/2)/c+1/4*d^3*x*(c*x^2+a)^(3/2)+3/8*d^3*a*x*(c*x^2+a)^(1/2)+3/8*d^3
*a^2/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.27818, size = 902, normalized size = 5.01

$$\left[\frac{105\left(2a^2cd^3 - a^3de^2\right)\sqrt{c}\log\left(-2cx^2 - 2\sqrt{cx^2+a}\sqrt{c}x - a\right) + 2\left(80c^3e^3x^6 + 280c^3de^2x^5 + 336a^2cd^2e - 32a^3e^3 + 16\right.}{1120}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/1120*(105*(2*a^2*c*d^3 - a^3*d*e^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2
+ a)*sqrt(c)*x - a) + 2*(80*c^3*e^3*x^6 + 280*c^3*d*e^2*x^5 + 336*a^2*c*d^2
*e - 32*a^3*e^3 + 16*(21*c^3*d^2*e + 8*a*c^2*e^3)*x^4 + 70*(2*c^3*d^3 + 7*a
*c^2*d*e^2)*x^3 + 16*(42*a*c^2*d^2*e + a^2*c*e^3)*x^2 + 35*(10*a*c^2*d^3 +
3*a^2*c*d*e^2)*x)*sqrt(c*x^2 + a))/c^2, -1/560*(105*(2*a^2*c*d^3 - a^3*d*e^
2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (80*c^3*e^3*x^6 + 280*c^3*
d*e^2*x^5 + 336*a^2*c*d^2*e - 32*a^3*e^3 + 16*(21*c^3*d^2*e + 8*a*c^2*e^3)*
x^4 + 70*(2*c^3*d^3 + 7*a*c^2*d*e^2)*x^3 + 16*(42*a*c^2*d^2*e + a^2*c*e^3)*
x^2 + 35*(10*a*c^2*d^3 + 3*a^2*c*d*e^2)*x)*sqrt(c*x^2 + a))/c^2]
```

Sympy [A] time = 17.0949, size = 551, normalized size = 3.06

$$\frac{3a^{\frac{5}{2}}de^2x}{16c\sqrt{1+\frac{cx^2}{a}}} + \frac{a^{\frac{3}{2}}d^3x\sqrt{1+\frac{cx^2}{a}}}{2} + \frac{a^{\frac{3}{2}}d^3x}{8\sqrt{1+\frac{cx^2}{a}}} + \frac{17a^{\frac{3}{2}}de^2x^3}{16\sqrt{1+\frac{cx^2}{a}}} + \frac{3\sqrt{a}cd^3x^3}{8\sqrt{1+\frac{cx^2}{a}}} + \frac{11\sqrt{a}cde^2x^5}{8\sqrt{1+\frac{cx^2}{a}}} - \frac{3a^3de^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16c^{\frac{3}{2}}} + \frac{3a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a)**(3/2),x)
```

```
[Out] 3*a**(5/2)*d*e**2*x/(16*c*sqrt(1 + c*x**2/a)) + a**(3/2)*d**3*x*sqrt(1 + c*
x**2/a)/2 + a**(3/2)*d**3*x/(8*sqrt(1 + c*x**2/a)) + 17*a**(3/2)*d*e**2*x**
3/(16*sqrt(1 + c*x**2/a)) + 3*sqrt(a)*c*d**3*x**3/(8*sqrt(1 + c*x**2/a)) +
11*sqrt(a)*c*d*e**2*x**5/(8*sqrt(1 + c*x**2/a)) - 3*a**3*d*e**2*asinh(sqrt(
c)*x/sqrt(a))/(16*c**(3/2)) + 3*a**2*d**3*asinh(sqrt(c)*x/sqrt(a))/(8*sqrt(
c)) + 3*a*d**2*e*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)
/(3*c), True)) + a*e**3*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x
```

```
**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*
x**4/4, True)) + 3*c*d**2*e*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) +
 a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt
(a)*x**4/4, True)) + c*e**3*Piecewise((8*a**3*sqrt(a + c*x**2)/(105*c**3) -
 4*a**2*x**2*sqrt(a + c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x**2)/(35*c) +
 x**6*sqrt(a + c*x**2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) + c**2*d**3*x*
*5/(4*sqrt(a)*sqrt(1 + c*x**2/a)) + c**2*d*e**2*x**7/(2*sqrt(a)*sqrt(1 + c*
x**2/a))
```

Giac [A] time = 1.29844, size = 286, normalized size = 1.59

$$\frac{1}{560}\sqrt{cx^2+a}\left(\left(2\left(\left(4\left(5\left(2\,cxe^3+7\,cde^2\right)x+\frac{2\left(21\,c^6d^2e+8\,ac^5e^3\right)}{c^5}\right)x+\frac{35\left(2\,c^6d^3+7\,ac^5de^2\right)}{c^5}\right)x+\frac{8\left(42\,ac^5d^2e+a^2c^4e^3\right)}{c^5}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/560*sqrt(c*x^2 + a)*((2*((4*(5*(2*c*x*e^3 + 7*c*d*e^2)*x + 2*(21*c^6*d^2*
e + 8*a*c^5*e^3)/c^5)*x + 35*(2*c^6*d^3 + 7*a*c^5*d*e^2)/c^5)*x + 8*(42*a*c
^5*d^2*e + a^2*c^4*e^3)/c^5)*x + 35*(10*a*c^5*d^3 + 3*a^2*c^4*d*e^2)/c^5)*x
 + 16*(21*a^2*c^4*d^2*e - 2*a^3*c^3*e^3)/c^5) - 3/16*(2*a^2*c*d^3 - a^3*d*e
^2)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

3.536 $\int (d+ex)^2 \left(a+cx^2\right)^{3/2} dx$

Optimal. Leaf size=154

$$\frac{a^2\left(6cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16c^{3/2}}+\frac{x\left(a+cx^2\right)^{3/2}\left(6cd^2-ae^2\right)}{24c}+\frac{ax\sqrt{a+cx^2}\left(6cd^2-ae^2\right)}{16c}+\frac{7de\left(a+cx^2\right)^{5/2}}{30c}+\frac{e\left(a+\right.}{}$$

```
[Out] (a*(6*c*d^2 - a*e^2)*x*Sqrt[a + c*x^2])/(16*c) + ((6*c*d^2 - a*e^2)*x*(a +
c*x^2)^(3/2))/(24*c) + (7*d*e*(a + c*x^2)^(5/2))/(30*c) + (e*(d + e*x)*(a +
 c*x^2)^(5/2))/(6*c) + (a^2*(6*c*d^2 - a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a +
c*x^2]])/(16*c^(3/2))
```

Rubi [A] time = 0.0685449, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {743, 641, 195, 217, 206}

$$\frac{a^2\left(6cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16c^{3/2}}+\frac{x\left(a+cx^2\right)^{3/2}\left(6cd^2-ae^2\right)}{24c}+\frac{ax\sqrt{a+cx^2}\left(6cd^2-ae^2\right)}{16c}+\frac{7de\left(a+cx^2\right)^{5/2}}{30c}+\frac{e\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + c*x^2)^(3/2),x]
```


```
[Out] (a*(6*c*d^2 - a*e^2)*x*Sqrt[a + c*x^2])/(16*c) + ((6*c*d^2 - a*e^2)*x*(a +
c*x^2)^(3/2))/(24*c) + (7*d*e*(a + c*x^2)^(5/2))/(30*c) + (e*(d + e*x)*(a +
 c*x^2)^(5/2))/(6*c) + (a^2*(6*c*d^2 - a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a +
c*x^2]])/(16*c^(3/2))
```


Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^2\left(a+cx^2\right)^{3/2}dx &= \frac{e(d+ex)\left(a+cx^2\right)^{5/2}}{6c}+\frac{\int \left(6cd^2-ae^2+7cdex\right)\left(a+cx^2\right)^{3/2}dx}{6c}\\ &= \frac{7de\left(a+cx^2\right)^{5/2}}{30c}+\frac{e(d+ex)\left(a+cx^2\right)^{5/2}}{6c}+\frac{\left(6cd^2-ae^2\right)\int \left(a+cx^2\right)^{3/2}dx}{6c}\\ &= \frac{\left(6cd^2-ae^2\right)x\left(a+cx^2\right)^{3/2}}{24c}+\frac{7de\left(a+cx^2\right)^{5/2}}{30c}+\frac{e(d+ex)\left(a+cx^2\right)^{5/2}}{6c}+\frac{\left(a\left(6cd^2-ae^2\right)\right)\int}{8c}\\ &= \frac{a\left(6cd^2-ae^2\right)x\sqrt{a+cx^2}}{16c}+\frac{\left(6cd^2-ae^2\right)x\left(a+cx^2\right)^{3/2}}{24c}+\frac{7de\left(a+cx^2\right)^{5/2}}{30c}+\frac{e(d+ex)\left(a+\right.}{6c}\\ &= \frac{a\left(6cd^2-ae^2\right)x\sqrt{a+cx^2}}{16c}+\frac{\left(6cd^2-ae^2\right)x\left(a+cx^2\right)^{3/2}}{24c}+\frac{7de\left(a+cx^2\right)^{5/2}}{30c}+\frac{e(d+ex)\left(a+\right.}{6c}\\ &= \frac{a\left(6cd^2-ae^2\right)x\sqrt{a+cx^2}}{16c}+\frac{\left(6cd^2-ae^2\right)x\left(a+cx^2\right)^{3/2}}{24c}+\frac{7de\left(a+cx^2\right)^{5/2}}{30c}+\frac{e(d+ex)\left(a+\right.}{6c}\end{aligned}$$

Mathematica [A] time = 0.0930768, size = 132, normalized size = 0.86

$$\frac{\sqrt{c}\sqrt{a+cx^2}\left(3a^2e(32d+5ex)+2acx\left(75d^2+96dex+35e^2x^2\right)+4c^2x^3\left(15d^2+24dex+10e^2x^2\right)\right)-15a^2\left(ae^2-6cd^2\right)\log}{240c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[c]*Sqrt[a + c*x^2]*(3*a^2*e*(32*d + 5*e*x) + 4*c^2*x^3*(15*d^2 + 24*d
*e*x + 10*e^2*x^2) + 2*a*c*x*(75*d^2 + 96*d*e*x + 35*e^2*x^2)) - 15*a^2*(-6
*c*d^2 + a*e^2)*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/(240*c^(3/2))
```

Maple [A] time = 0.052, size = 161, normalized size = 1.1

$$\frac{e^2x}{6c}\left(cx^2+a\right)^{\frac{5}{2}}-\frac{ae^2x}{24c}\left(cx^2+a\right)^{\frac{3}{2}}-\frac{a^2e^2x}{16c}\sqrt{cx^2+a}-\frac{e^2a^3}{16}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)c^{-\frac{3}{2}}+\frac{2de}{5c}\left(cx^2+a\right)^{\frac{5}{2}}+\frac{d^2x}{4}\left(cx^2+a\right)^{\frac{3}{2}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a)^(3/2),x)
```

```
[Out] 1/6*e^2*x*(c*x^2+a)^(5/2)/c-1/24*e^2*a/c*x*(c*x^2+a)^(3/2)-1/16*e^2*a^2/c*x
*(c*x^2+a)^(1/2)-1/16*e^2*a^3/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))+2/5*d*e
*(c*x^2+a)^(5/2)/c+1/4*d^2*x*(c*x^2+a)^(3/2)+3/8*d^2*a*x*(c*x^2+a)^(1/2)+3/
8*d^2*a^2/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.13032, size = 675, normalized size = 4.38

$$\left[-\frac{15\left(6\,a^2cd^2-a^3e^2\right)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)-2\left(40\,c^3e^2x^5+96\,c^3dex^4+192\,ac^2dex^2+96\,a^2cde+10\,(\right.}{480\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/480*(15*(6*a^2*c*d^2 - a^3*e^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a
)*sqrt(c)*x - a) - 2*(40*c^3*e^2*x^5 + 96*c^3*d*e*x^4 + 192*a*c^2*d*e*x^2 +
 96*a^2*c*d*e + 10*(6*c^3*d^2 + 7*a*c^2*e^2)*x^3 + 15*(10*a*c^2*d^2 + a^2*c
*e^2)*x)*sqrt(c*x^2 + a))/c^2, -1/240*(15*(6*a^2*c*d^2 - a^3*e^2)*sqrt(-c)*
arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (40*c^3*e^2*x^5 + 96*c^3*d*e*x^4 + 192
*a*c^2*d*e*x^2 + 96*a^2*c*d*e + 10*(6*c^3*d^2 + 7*a*c^2*e^2)*x^3 + 15*(10*a
*c^2*d^2 + a^2*c*e^2)*x)*sqrt(c*x^2 + a))/c^2]
```

Sympy [A] time = 15.5335, size = 372, normalized size = 2.42

$$\frac{a^{\frac{5}{2}}e^2x}{16c\sqrt{1+\frac{cx^2}{a}}}+\frac{a^{\frac{3}{2}}d^2x\sqrt{1+\frac{cx^2}{a}}}{2}+\frac{a^{\frac{3}{2}}d^2x}{8\sqrt{1+\frac{cx^2}{a}}}+\frac{17a^{\frac{3}{2}}e^2x^3}{48\sqrt{1+\frac{cx^2}{a}}}+\frac{3\sqrt{a}cd^2x^3}{8\sqrt{1+\frac{cx^2}{a}}}+\frac{11\sqrt{a}ce^2x^5}{24\sqrt{1+\frac{cx^2}{a}}}-\frac{a^3e^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16c^{\frac{3}{2}}}+\frac{3a^2d^2\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+a)**(3/2),x)
```

```
[Out] a**(5/2)*e**2*x/(16*c*sqrt(1 + c*x**2/a)) + a**(3/2)*d**2*x*sqrt(1 + c*x**2
/a)/2 + a**(3/2)*d**2*x/(8*sqrt(1 + c*x**2/a)) + 17*a**(3/2)*e**2*x**3/(48*
sqrt(1 + c*x**2/a)) + 3*sqrt(a)*c*d**2*x**3/(8*sqrt(1 + c*x**2/a)) + 11*sqr
t(a)*c*e**2*x**5/(24*sqrt(1 + c*x**2/a)) - a**3*e**2*asinh(sqrt(c)*x/sqrt(a
))/(16*c**(3/2)) + 3*a**2*d**2*asinh(sqrt(c)*x/sqrt(a))/(8*sqrt(c)) + 2*a*d
*e*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True))
 + 2*c*d*e*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a +
c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)
) + c**2*d**2*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a)) + c**2*e**2*x**7/(6*sqrt(
a)*sqrt(1 + c*x**2/a))
```

Giac [A] time = 1.37037, size = 192, normalized size = 1.25

$$\frac{1}{240}\sqrt{cx^2+a}\left(\frac{96\,a^2de}{c}+\left(2\left(96\,ade+\left(4\left(5\,cxe^2+12\,cde\right)x+\frac{5\left(6\,c^5d^2+7\,ac^4e^2\right)}{c^4}\right)x\right)x+\frac{15\left(10\,ac^4d^2+a^2c^3e^2\right)}{c^4}\right)x\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/240*sqrt(c*x^2 + a)*(96*a^2*d*e/c + (2*(96*a*d*e + (4*(5*c*x*e^2 + 12*c*d
*e)*x + 5*(6*c^5*d^2 + 7*a*c^4*e^2)/c^4)*x)*x + 15*(10*a*c^4*d^2 + a^2*c^3*
e^2)/c^4)*x) - 1/16*(6*a^2*c*d^2 - a^3*e^2)*log(abs(-sqrt(c)*x + sqrt(c*x^2
 + a)))/c^(3/2)
```

3.537 $\int (d+ex)\left(a+cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=87

$$\frac{3a^2d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8\sqrt{c}}+\frac{1}{4}dx\left(a+cx^2\right)^{3/2}+\frac{3}{8}adx\sqrt{a+cx^2}+\frac{e\left(a+cx^2\right)^{5/2}}{5c}$$

```
[Out] (3*a*d*x*Sqrt[a + c*x^2])/8 + (d*x*(a + c*x^2)^(3/2))/4 + (e*(a + c*x^2)^(5
/2))/(5*c) + (3*a^2*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*Sqrt[c])
```

Rubi [A] time = 0.0253526, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {641, 195, 217, 206}

$$\frac{3a^2d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8\sqrt{c}}+\frac{1}{4}dx\left(a+cx^2\right)^{3/2}+\frac{3}{8}adx\sqrt{a+cx^2}+\frac{e\left(a+cx^2\right)^{5/2}}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + c*x^2)^(3/2),x]
```

```
[Out] (3*a*d*x*Sqrt[a + c*x^2])/8 + (d*x*(a + c*x^2)^(3/2))/4 + (e*(a + c*x^2)^(5
/2))/(5*c) + (3*a^2*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*Sqrt[c])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)\left(a+cx^2\right)^{3/2}\,dx = \frac{e\left(a+cx^2\right)^{5/2}}{5c} + d\int \left(a+cx^2\right)^{3/2}\,dx$$
$$= \frac{1}{4}dx\left(a+cx^2\right)^{3/2} + \frac{e\left(a+cx^2\right)^{5/2}}{5c} + \frac{1}{4}(3ad)\int \sqrt{a+cx^2}\,dx$$
$$= \frac{3}{8}adx\sqrt{a+cx^2} + \frac{1}{4}dx\left(a+cx^2\right)^{3/2} + \frac{e\left(a+cx^2\right)^{5/2}}{5c} + \frac{1}{8}\left(3a^2d\right)\int \frac{1}{\sqrt{a+cx^2}}\,dx$$
$$= \frac{3}{8}adx\sqrt{a+cx^2} + \frac{1}{4}dx\left(a+cx^2\right)^{3/2} + \frac{e\left(a+cx^2\right)^{5/2}}{5c} + \frac{1}{8}\left(3a^2d\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+}}\right.$$
$$= \frac{3}{8}adx\sqrt{a+cx^2} + \frac{1}{4}dx\left(a+cx^2\right)^{3/2} + \frac{e\left(a+cx^2\right)^{5/2}}{5c} + \frac{3a^2d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8\sqrt{c}}$$

Mathematica [A] time = 0.0515484, size = 88, normalized size = 1.01

$$\frac{\sqrt{a+cx^2}\left(8a^2e + acx(25d+16ex) + 2c^2x^3(5d+4ex)\right) + 15a^2\sqrt{c}d\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)}{40c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(8*a^2*e + 2*c^2*x^3*(5*d + 4*e*x) + a*c*x*(25*d + 16*e*x)
) + 15*a^2*Sqrt[c]*d*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/(40*c)
```

Maple [A] time = 0.046, size = 69, normalized size = 0.8

$$\frac{e}{5c}\left(cx^2+a\right)^{\frac{5}{2}} + \frac{dx}{4}\left(cx^2+a\right)^{\frac{3}{2}} + \frac{3\,adx}{8}\sqrt{cx^2+a} + \frac{3\,a^2d}{8}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+a)^(3/2),x)
```

```
[Out] 1/5*e*(c*x^2+a)^(5/2)/c+1/4*d*x*(c*x^2+a)^(3/2)+3/8*a*d*x*(c*x^2+a)^(1/2)+3
/8*d*a^2/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.15479, size = 425, normalized size = 4.89

$$\left[\frac{15\,a^{2}\sqrt{c}d\log\left(-2\,cx^{2} - 2\,\sqrt{cx^{2} + a}\sqrt{c}x - a\right) + 2\,\left(8\,c^{2}ex^{4} + 10\,c^{2}dx^{3} + 16\,acex^{2} + 25\,acdx + 8\,a^{2}e\right)\sqrt{cx^{2} + a}}{80\,c}, -\frac{15\,a^{2}\sqrt{-}\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/80*(15*a^2*sqrt(c)*d*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2
*(8*c^2*e*x^4 + 10*c^2*d*x^3 + 16*a*c*e*x^2 + 25*a*c*d*x + 8*a^2*e)*sqrt(c*
x^2 + a))/c, -1/40*(15*a^2*sqrt(-c)*d*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) -
(8*c^2*e*x^4 + 10*c^2*d*x^3 + 16*a*c*e*x^2 + 25*a*c*d*x + 8*a^2*e)*sqrt(c*x
^2 + a))/c]
```

Sympy [A] time = 6.85815, size = 219, normalized size = 2.52

$$\frac{a^{\frac{3}{2}}dx\sqrt{1 + \frac{cx^{2}}{a}}}{2} + \frac{a^{\frac{3}{2}}dx}{8\sqrt{1 + \frac{cx^{2}}{a}}} + \frac{3\sqrt{a}cdx^{3}}{8\sqrt{1 + \frac{cx^{2}}{a}}} + \frac{3a^{2}d\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8\sqrt{c}} + ae\left(\begin{cases} \frac{\sqrt{a}x^{2}}{2} & \text{for}\ c = 0 \\ \frac{\left(a+cx^{2}\right)^{\frac{3}{2}}}{3c} & \text{otherwise} \end{cases}\right) + ce\left(\begin{cases} -\frac{2a^{2}\sqrt{a+cx^{2}}}{15c^{2}} + \frac{ax^{2}\cdots}{} \\ \frac{\sqrt{a}x^{4}}{4} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+a)**(3/2),x)
```

```
[Out] a**(3/2)*d*x*sqrt(1 + c*x**2/a)/2 + a**(3/2)*d*x/(8*sqrt(1 + c*x**2/a)) + 3
*sqrt(a)*c*d*x**3/(8*sqrt(1 + c*x**2/a)) + 3*a**2*d*asinh(sqrt(c)*x/sqrt(a)
)/(8*sqrt(c)) + a*e*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3
/2)/(3*c), True)) + c*e*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x
**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*
x**4/4, True)) + c**2*d*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a))
```

Giac [A] time = 1.24093, size = 107, normalized size = 1.23

$$-\frac{3\,a^{2}d\log\left(\left|-\sqrt{c}x + \sqrt{cx^{2} + a}\right|\right)}{8\,\sqrt{c}} + \frac{1}{40}\,\sqrt{cx^{2} + a}\left((25\,ad + 2\,((4\,cxe + 5\,cd)x + 8\,ae)x)x + \frac{8\,a^{2}e}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -3/8*a^2*d*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/sqrt(c) + 1/40*sqrt(c*x^2
 + a)*((25*a*d + 2*((4*c*x*e + 5*c*d)*x + 8*a*e)*x)*x + 8*a^2*e/c)
```

3.538 $\int \frac{(a+cx^2)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=159

$$\frac{\sqrt{a+cx^2}\left(2\left(ae^2+cd^2\right)-cdex\right)}{2e^3}-\frac{\left(ae^2+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4}-\frac{\sqrt{c}d\left(3ae^2+2cd^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2e^4}+\frac{(a+c\cdots}{3e\cdots}$$

```
[Out] ((2*(c*d^2 + a*e^2) - c*d*e*x)*Sqrt[a + c*x^2])/(2*e^3) + (a + c*x^2)^(3/2)
/(3*e) - (Sqrt[c]*d*(2*c*d^2 + 3*a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]
])/(2*e^4) - ((c*d^2 + a*e^2)^(3/2)*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e
^2]*Sqrt[a + c*x^2])])/e^4
```

Rubi [A] time = 0.179466, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {735, 815, 844, 217, 206, 725}

$$\frac{\sqrt{a+cx^2}\left(2\left(ae^2+cd^2\right)-cdex\right)}{2e^3}-\frac{\left(ae^2+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4}-\frac{\sqrt{c}d\left(3ae^2+2cd^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2e^4}+\frac{(a+c\cdots}{3e\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] ((2*(c*d^2 + a*e^2) - c*d*e*x)*Sqrt[a + c*x^2])/(2*e^3) + (a + c*x^2)^(3/2)
/(3*e) - (Sqrt[c]*d*(2*c*d^2 + 3*a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]
])/(2*e^4) - ((c*d^2 + a*e^2)^(3/2)*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e
^2]*Sqrt[a + c*x^2])])/e^4
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+cx^2\right)^{3/2}}{d+ex}\,dx &= \frac{\left(a+cx^2\right)^{3/2}}{3e}+\frac{\int \frac{(ae-cdx)\sqrt{a+cx^2}}{d+ex}\,dx}{e}\\
&= \frac{\left(2\left(cd^2+ae^2\right)-cdex\right)\sqrt{a+cx^2}}{2e^3}+\frac{\left(a+cx^2\right)^{3/2}}{3e}+\frac{\int \frac{ace\left(cd^2+2ae^2\right)-c^2d\left(2cd^2+3ae^2\right)x}{(d+ex)\sqrt{a+cx^2}}\,dx}{2ce^3}\\
&= \frac{\left(2\left(cd^2+ae^2\right)-cdex\right)\sqrt{a+cx^2}}{2e^3}+\frac{\left(a+cx^2\right)^{3/2}}{3e}+\frac{\left(cd^2+ae^2\right)^2\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^4}-\frac{\left(cd\left(2cd^2+3a\right.\right.}{2}\\
&= \frac{\left(2\left(cd^2+ae^2\right)-cdex\right)\sqrt{a+cx^2}}{2e^3}+\frac{\left(a+cx^2\right)^{3/2}}{3e}-\frac{\left(cd^2+ae^2\right)^2\operatorname{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e^4}\\
&= \frac{\left(2\left(cd^2+ae^2\right)-cdex\right)\sqrt{a+cx^2}}{2e^3}+\frac{\left(a+cx^2\right)^{3/2}}{3e}-\frac{\sqrt{c}d\left(2cd^2+3ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2e^4}-\frac{\left(cd^2+a\right.}{}\end{aligned}$$

Mathematica [A] time = 0.46266, size = 195, normalized size = 1.23

$$\frac{e\sqrt{a+cx^2}\left(8ae^2+c\left(6d^2-3dex+2e^2x^2\right)\right)-6\sqrt{c}d\left(ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)-6\left(ae^2+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+c}}\right.}{6e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] (e*Sqrt[a + c*x^2]*(8*a*e^2 + c*(6*d^2 - 3*d*e*x + 2*e^2*x^2)) - (3*Sqrt[a]
*Sqrt[c]*d*e^2*Sqrt[a + c*x^2]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[1 + (c*x^
2)/a] - 6*Sqrt[c]*d*(c*d^2 + a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] -
6*(c*d^2 + a*e^2)^(3/2)*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a +
 c*x^2])])/(6*e^4)
```

Maple [B] time = 0.249, size = 745, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)/(e*x+d),x)
```

```
[Out] 1/3/e*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)-1/2/e^2*c*d*(c*
(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x-3/2/e^2*c^(1/2)*d*ln((
-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(
1/2))*a+1/e*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*a+1/e^3*(
c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*c*d^2-1/e^4*c^(3/2)*d^
3*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/
e^2)^(1/2))-1/e/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(
d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^
2)/e^2)^(1/2))/(d/e+x))*a^2-2/e^3/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*
d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(
d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a*c*d^2-1/e^5/((a*e^2+c*d^2)/e^2)
^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*
(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*c^2*d^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 16.4782, size = 1716, normalized size = 10.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(2*c*d^3 + 3*a*d*e^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqr
t(c)*x - a) + 6*(c*d^2 + a*e^2)^(3/2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^
2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*
x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(2*c*e^3*x^2 - 3*c*d*e^2*x + 6*c*d
^2*e + 8*a*e^3)*sqrt(c*x^2 + a))/e^4, 1/6*(3*(2*c*d^3 + 3*a*d*e^2)*sqrt(-c)
*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + 3*(c*d^2 + a*e^2)^(3/2)*log((2*a*c*d*
e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^
2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + (2*c*e^3*x^2
 - 3*c*d*e^2*x + 6*c*d^2*e + 8*a*e^3)*sqrt(c*x^2 + a))/e^4, -1/12*(12*(c*d^
2 + a*e^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*s
qrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(2*c*d^3
+ 3*a*d*e^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(2
*c*e^3*x^2 - 3*c*d*e^2*x + 6*c*d^2*e + 8*a*e^3)*sqrt(c*x^2 + a))/e^4, -1/6*
(6*(c*d^2 + a*e^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x
- a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(
2*c*d^3 + 3*a*d*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (2*c*e^3
*x^2 - 3*c*d*e^2*x + 6*c*d^2*e + 8*a*e^3)*sqrt(c*x^2 + a))/e^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + cx^2\right)^{\frac{3}{2}}}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)/(e*x+d),x)
```

```
[Out] Integral((a + c*x**2)**(3/2)/(d + e*x), x)
```

Giac [A] time = 1.33563, size = 238, normalized size = 1.5

$$\frac{1}{2}\left(2\,c^{\frac{3}{2}}d^3 + 3\,a\sqrt{c}de^2\right)e^{(-4)}\log\left(\left|-\sqrt{c}x + \sqrt{cx^2 + a}\right|\right) + \frac{2\left(c^2d^4 + 2\,acd^2e^2 + a^2e^4\right)\arctan\left(-\frac{\left(\sqrt{c}x - \sqrt{cx^2 + a}\right)e + \sqrt{c}d}{\sqrt{-cd^2 - ae^2}}\right)e^{(-4)}}{\sqrt{-cd^2 - ae^2}} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] 1/2*(2*c^(3/2)*d^3 + 3*a*sqrt(c)*d*e^2)*e^(-4)*log(abs(-sqrt(c)*x + sqrt(c*
x^2 + a))) + 2*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*arctan(-((sqrt(c)*x - sq
rt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2))*e^(-4)/sqrt(-c*d^2 - a*
e^2) + 1/6*sqrt(c*x^2 + a)*((2*c*x*e^(-1) - 3*c*d*e^(-2))*x + 2*(3*c^2*d^2*
e^7 + 4*a*c*e^9)*e^(-10)/c)
```

3.539 $\int \frac{(a+cx^2)^{3/2}}{(d+ex)^2} dx$

Optimal. Leaf size=153

$$\frac{3\sqrt{c}\left(ae^2+2cd^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2e^4}+\frac{3cd\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4}-\frac{3c\sqrt{a+cx^2}(2d-ex)}{2e^3}-\frac{\left(a+cx^2\right)^{3/2}}{e(d+ex)}$$

```
[Out] (-3*c*(2*d - e*x)*Sqrt[a + c*x^2])/(2*e^3) - (a + c*x^2)^(3/2)/(e*(d + e*x)
) + (3*Sqrt[c]*(2*c*d^2 + a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*e
^4) + (3*c*d*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]
*Sqrt[a + c*x^2])])/e^4
```

Rubi [A] time = 0.135716, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {733, 815, 844, 217, 206, 725}

$$\frac{3\sqrt{c}\left(ae^2+2cd^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2e^4}+\frac{3cd\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4}-\frac{3c\sqrt{a+cx^2}(2d-ex)}{2e^3}-\frac{\left(a+cx^2\right)^{3/2}}{e(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2)/(d + e*x)^2,x]
```


```
[Out] (-3*c*(2*d - e*x)*Sqrt[a + c*x^2])/(2*e^3) - (a + c*x^2)^(3/2)/(e*(d + e*x)
) + (3*Sqrt[c]*(2*c*d^2 + a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*e
^4) + (3*c*d*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]
*Sqrt[a + c*x^2])])/e^4
```


Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}}{(d+ex)^2}\,dx=-\frac{\left(a+cx^2\right)^{3/2}}{e(d+ex)}+\frac{(3c)\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx}{e}$$

$$=-\frac{3c(2d-ex)\sqrt{a+cx^2}}{2e^3}-\frac{\left(a+cx^2\right)^{3/2}}{e(d+ex)}+\frac{3\int \frac{-acde+c\left(2cd^2+ae^2\right)x}{(d+ex)\sqrt{a+cx^2}}\,dx}{2e^3}$$

$$=-\frac{3c(2d-ex)\sqrt{a+cx^2}}{2e^3}-\frac{\left(a+cx^2\right)^{3/2}}{e(d+ex)}-\frac{\left(3cd\left(cd^2+ae^2\right)\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^4}+\frac{\left(3c\left(2cd^2+ae^2\right)\right)\int}{2e^4}$$

$$=-\frac{3c(2d-ex)\sqrt{a+cx^2}}{2e^3}-\frac{\left(a+cx^2\right)^{3/2}}{e(d+ex)}+\frac{\left(3cd\left(cd^2+ae^2\right)\right)\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e^4}+\frac{(3}{}$$

$$=-\frac{3c(2d-ex)\sqrt{a+cx^2}}{2e^3}-\frac{\left(a+cx^2\right)^{3/2}}{e(d+ex)}+\frac{3\sqrt{c}\left(2cd^2+ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2e^4}+\frac{3cd\sqrt{cd^2+ae^2}\,\tan}{e}$$

Mathematica [A] time = 0.192158, size = 179, normalized size = 1.17

$$\frac{-\frac{e\sqrt{a+cx^2}\left(2ae^2+c\left(6d^2+3dex-e^2x^2\right)\right)}{d+ex}+3\sqrt{c}\left(ae^2+2cd^2\right)\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)+6cd\sqrt{ae^2+cd^2}\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+\right.}{2e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2)/(d + e*x)^2,x]
```

```
[Out] (-((e*Sqrt[a + c*x^2]*(2*a*e^2 + c*(6*d^2 + 3*d*e*x - e^2*x^2)))/(d + e*x))
 - 6*c*d*Sqrt[c*d^2 + a*e^2]*Log[d + e*x] + 3*Sqrt[c]*(2*c*d^2 + a*e^2)*Log
[c*x + Sqrt[c]*Sqrt[a + c*x^2]] + 6*c*d*Sqrt[c*d^2 + a*e^2]*Log[a*e - c*d*x
 + Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2]])/(2*e^4)
```

Maple [B] time = 0.223, size = 1154, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)/(e*x+d)^2,x)
```

```
[Out] -1/(a*e^2+c*d^2)/(d/e+x)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(5
/2)-1/e*c*d/(a*e^2+c*d^2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(
3/2)+3/2/e^2*c^2*d^2/(a*e^2+c*d^2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^
2)/e^2)^(1/2)*x+9/2/e^2*c^(3/2)*d^2/(a*e^2+c*d^2)*ln((-c*d/e+(d/e+x)*c)/c^(
1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))*a-3/e*c*d/(a*e^
2+c*d^2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*a-3/e^3*c^2*
d^3/(a*e^2+c*d^2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)+3/e
^4*c^(5/2)*d^4/(a*e^2+c*d^2)*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c
*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))+3/e*c*d/(a*e^2+c*d^2)/((a*e^2+c*d^2)
/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(
1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a^2+6/
e^3*c^2*d^3/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2
-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(
a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a+3/e^5*c^3*d^5/(a*e^2+c*d^2)/((a*e^2+c*d
^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2
)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))+1/(
a*e^2+c*d^2)*c*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)*x+3/2/
(a*e^2+c*d^2)*c*a*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x+3
/2/(a*e^2+c*d^2)*c^(1/2)*a^2*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c
*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.68226, size = 1933, normalized size = 12.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(3*(2*c*d^3 + a*d*e^2 + (2*c*d^2*e + a*e^3)*x)*sqrt(c)*log(-2*c*x^2 -
2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 6*(c*d*e*x + c*d^2)*sqrt(c*d^2 + a*e^2)*
log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt
(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) +
 2*(c*e^3*x^2 - 3*c*d*e^2*x - 6*c*d^2*e - 2*a*e^3)*sqrt(c*x^2 + a))/(e^5*x
+ d*e^4), 1/4*(12*(c*d*e*x + c*d^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2
 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c
*e^2)*x^2)) + 3*(2*c*d^3 + a*d*e^2 + (2*c*d^2*e + a*e^3)*x)*sqrt(c)*log(-2*
c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(c*e^3*x^2 - 3*c*d*e^2*x - 6*c
*d^2*e - 2*a*e^3)*sqrt(c*x^2 + a))/(e^5*x + d*e^4), -1/2*(3*(2*c*d^3 + a*d*
e^2 + (2*c*d^2*e + a*e^3)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) -
```

```
3*(c*d*e*x + c*d^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*
e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(
c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - (c*e^3*x^2 - 3*c*d*e^2*x - 6*c*d^2
*e - 2*a*e^3)*sqrt(c*x^2 + a))/(e^5*x + d*e^4), 1/2*(6*(c*d*e*x + c*d^2)*sq
rt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a
)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(2*c*d^3 + a*d*e^2 + (
2*c*d^2*e + a*e^3)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (c*e^3*
x^2 - 3*c*d*e^2*x - 6*c*d^2*e - 2*a*e^3)*sqrt(c*x^2 + a))/(e^5*x + d*e^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + cx^2\right)^{\frac{3}{2}}}{(d + ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)/(e*x+d)**2,x)
```

```
[Out] Integral((a + c*x**2)**(3/2)/(d + e*x)**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.540 $\int \frac{(a+cx^2)^{3/2}}{(d+ex)^3} dx$

Optimal. Leaf size=161

$$-\frac{3c^{3/2}d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e^4}-\frac{3c\left(ae^2+2cd^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{2e^4\sqrt{ae^2+cd^2}}+\frac{3c\sqrt{a+cx^2}(2d+ex)}{2e^3(d+ex)}-\frac{\left(a+cx^2\right)^{3/2}}{2e(d+ex)^2}$$

```
[Out] (3*c*(2*d + e*x)*Sqrt[a + c*x^2])/(2*e^3*(d + e*x)) - (a + c*x^2)^(3/2)/(2*
e*(d + e*x)^2) - (3*c^(3/2)*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/e^4 - (
3*c*(2*c*d^2 + a*e^2)*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c
*x^2])])/(2*e^4*Sqrt[c*d^2 + a*e^2])
```

Rubi [A] time = 0.123799, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {733, 813, 844, 217, 206, 725}

$$-\frac{3c^{3/2}d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e^4}-\frac{3c\left(ae^2+2cd^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{2e^4\sqrt{ae^2+cd^2}}+\frac{3c\sqrt{a+cx^2}(2d+ex)}{2e^3(d+ex)}-\frac{\left(a+cx^2\right)^{3/2}}{2e(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2)/(d + e*x)^3,x]
```

```
[Out] (3*c*(2*d + e*x)*Sqrt[a + c*x^2])/(2*e^3*(d + e*x)) - (a + c*x^2)^(3/2)/(2*
e*(d + e*x)^2) - (3*c^(3/2)*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/e^4 - (
3*c*(2*c*d^2 + a*e^2)*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c
*x^2])])/(2*e^4*Sqrt[c*d^2 + a*e^2])
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+cx^2\right)^{3/2}}{(d+ex)^3}\,dx &= -\frac{\left(a+cx^2\right)^{3/2}}{2e(d+ex)^2}+\frac{(3c)\int \frac{x\sqrt{a+cx^2}}{(d+ex)^2}\,dx}{2e}\\
&= \frac{3c(2d+ex)\sqrt{a+cx^2}}{2e^3(d+ex)}-\frac{\left(a+cx^2\right)^{3/2}}{2e(d+ex)^2}-\frac{(3c)\int \frac{-2ae+4cdx}{(d+ex)\sqrt{a+cx^2}}\,dx}{4e^3}\\
&= \frac{3c(2d+ex)\sqrt{a+cx^2}}{2e^3(d+ex)}-\frac{\left(a+cx^2\right)^{3/2}}{2e(d+ex)^2}-\frac{\left(3c^2d\right)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e^4}+\frac{\left(3c\left(2cd^2+ae^2\right)\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{2e^4}\\
&= \frac{3c(2d+ex)\sqrt{a+cx^2}}{2e^3(d+ex)}-\frac{\left(a+cx^2\right)^{3/2}}{2e(d+ex)^2}-\frac{\left(3c^2d\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{e^4}-\frac{\left(3c\left(2cd^2+ae^2\right)\right)\text{S}}{}\\
&= \frac{3c(2d+ex)\sqrt{a+cx^2}}{2e^3(d+ex)}-\frac{\left(a+cx^2\right)^{3/2}}{2e(d+ex)^2}-\frac{3c^{3/2}d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e^4}-\frac{3c\left(2cd^2+ae^2\right)\tanh^{-1}\left(\frac{ae-c}{\sqrt{cd^2+ae^2}}\right)}{2e^4\sqrt{cd^2+ae^2}}\end{aligned}$$

Mathematica [A] time = 0.219271, size = 189, normalized size = 1.17

$$\frac{-6c^{3/2}d\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)+\frac{e\sqrt{a+cx^2}\left(c\left(6d^2+9dex+2e^2x^2\right)-ae^2\right)}{(d+ex)^2}-\frac{3c\left(ae^2+2cd^2\right)\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{\sqrt{ae^2+cd^2}}+\frac{3c\left(ae^2+2cd^2\right)\log(d+e}{\sqrt{ae^2+cd^2}}}{2e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2)/(d + e*x)^3,x]
```

```
[Out] ((e*Sqrt[a + c*x^2]*(-(a*e^2) + c*(6*d^2 + 9*d*e*x + 2*e^2*x^2)))/(d + e*x)
^2 + (3*c*(2*c*d^2 + a*e^2)*Log[d + e*x])/Sqrt[c*d^2 + a*e^2] - 6*c^(3/2)*d
*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] - (3*c*(2*c*d^2 + a*e^2)*Log[a*e - c*d*
x + Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2]])/Sqrt[c*d^2 + a*e^2])/(2*e^4)
```

Maple [B] time = 0.208, size = 2117, normalized size = 13.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)/(e*x+d)^3,x)
```

```
[Out] -1/2/e/(a*e^2+c*d^2)/(d/e+x)^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e
^2)^(5/2)+1/2*c*d/(a*e^2+c*d^2)^2/(d/e+x)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e
^2+c*d^2)/e^2)^(5/2)+1/2/e*c^2*d^2/(a*e^2+c*d^2)^2*(c*(d/e+x)^2-2*c*d/e*(d/
e+x)+(a*e^2+c*d^2)/e^2)^(3/2)-3/4/e^2*c^3*d^3/(a*e^2+c*d^2)^2*(c*(d/e+x)^2-
2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x-9/4/e^2*c^(5/2)*d^3/(a*e^2+c*d^2
)^2*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2
)/e^2)^(1/2))*a+3/2/e*c^2*d^2/(a*e^2+c*d^2)^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+
(a*e^2+c*d^2)/e^2)^(1/2)*a+3/2/e^3*c^3*d^4/(a*e^2+c*d^2)^2*(c*(d/e+x)^2-2*c
*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)-3/2/e^4*c^(7/2)*d^5/(a*e^2+c*d^2)^2*l
n((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2
)^(1/2))-3/2/e*c^2*d^2/(a*e^2+c*d^2)^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e
^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*
d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a^2-3/e^3*c^3*d^4/(a*e^2+c*d
^2)^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*(
(a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1
/2))/(d/e+x))*a-3/2/e^5*c^4*d^6/(a*e^2+c*d^2)^2/((a*e^2+c*d^2)/e^2)^(1/2)*l
n((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+
x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))-1/2*c^2*d/(a*e^2+c*
d^2)^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)*x-3/4*c^2*d/(a
*e^2+c*d^2)^2*a*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x-3/4
*c^(3/2)*d/(a*e^2+c*d^2)^2*a^2*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2
*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))+1/2/e/(a*e^2+c*d^2)*c*(c*(d/e+x)^2
-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)-3/4/e^2/(a*e^2+c*d^2)*c^2*d*(c*(d
/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x-9/4/e^2/(a*e^2+c*d^2)*c^
(3/2)*d*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c
*d^2)/e^2)^(1/2))*a+3/2/e/(a*e^2+c*d^2)*c*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e
^2+c*d^2)/e^2)^(1/2)*a+3/2/e^3/(a*e^2+c*d^2)*c^2*(c*(d/e+x)^2-2*c*d/e*(d/e+
x)+(a*e^2+c*d^2)/e^2)^(1/2)*d^2-3/2/e^4/(a*e^2+c*d^2)*c^(5/2)*d^3*ln((-c*d/
e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))
-3/2/e/(a*e^2+c*d^2)*c/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*
c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e
^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a^2-3/e^3/(a*e^2+c*d^2)*c^2/((a*e^2+c*d^2)/e
^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/
2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a*d^2-3/
2/e^5/(a*e^2+c*d^2)*c^3/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2
*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*
e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*d^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 7.3965, size = 3198, normalized size = 19.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(6*(c^2*d^5 + a*c*d^3*e^2 + (c^2*d^3*e^2 + a*c*d*e^4)*x^2 + 2*(c^2*d^4
*e + a*c*d^2*e^3)*x)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a
) + 3*(2*c^2*d^4 + a*c*d^2*e^2 + (2*c^2*d^2*e^2 + a*c*e^4)*x^2 + 2*(2*c^2*d
^3*e + a*c*d*e^3)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2
*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt
(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(6*c^2*d^4*e + 5*a*c*d^2*e^3 -
a^2*e^5 + 2*(c^2*d^2*e^3 + a*c*e^5)*x^2 + 9*(c^2*d^3*e^2 + a*c*d*e^4)*x)*sq
rt(c*x^2 + a))/(c*d^4*e^4 + a*d^2*e^6 + (c*d^2*e^6 + a*e^8)*x^2 + 2*(c*d^3*
e^5 + a*d*e^7)*x), 1/4*(12*(c^2*d^5 + a*c*d^3*e^2 + (c^2*d^3*e^2 + a*c*d*e^
4)*x^2 + 2*(c^2*d^4*e + a*c*d^2*e^3)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x
^2 + a)) + 3*(2*c^2*d^4 + a*c*d^2*e^2 + (2*c^2*d^2*e^2 + a*c*e^4)*x^2 + 2*(
2*c^2*d^3*e + a*c*d*e^3)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2
- 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*
e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(6*c^2*d^4*e + 5*a*c*d^2
*e^3 - a^2*e^5 + 2*(c^2*d^2*e^3 + a*c*e^5)*x^2 + 9*(c^2*d^3*e^2 + a*c*d*e^4
)*x)*sqrt(c*x^2 + a))/(c*d^4*e^4 + a*d^2*e^6 + (c*d^2*e^6 + a*e^8)*x^2 + 2*
(c*d^3*e^5 + a*d*e^7)*x), -1/2*(3*(2*c^2*d^4 + a*c*d^2*e^2 + (2*c^2*d^2*e^2
 + a*c*e^4)*x^2 + 2*(2*c^2*d^3*e + a*c*d*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arcta
n(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (
c^2*d^2 + a*c*e^2)*x^2)) - 3*(c^2*d^5 + a*c*d^3*e^2 + (c^2*d^3*e^2 + a*c*d*
e^4)*x^2 + 2*(c^2*d^4*e + a*c*d^2*e^3)*x)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x
^2 + a)*sqrt(c)*x - a) - (6*c^2*d^4*e + 5*a*c*d^2*e^3 - a^2*e^5 + 2*(c^2*d^
2*e^3 + a*c*e^5)*x^2 + 9*(c^2*d^3*e^2 + a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(c*d
^4*e^4 + a*d^2*e^6 + (c*d^2*e^6 + a*e^8)*x^2 + 2*(c*d^3*e^5 + a*d*e^7)*x),
-1/2*(3*(2*c^2*d^4 + a*c*d^2*e^2 + (2*c^2*d^2*e^2 + a*c*e^4)*x^2 + 2*(2*c^2
*d^3*e + a*c*d*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*
d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) -
 6*(c^2*d^5 + a*c*d^3*e^2 + (c^2*d^3*e^2 + a*c*d*e^4)*x^2 + 2*(c^2*d^4*e +
a*c*d^2*e^3)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (6*c^2*d^4*e
+ 5*a*c*d^2*e^3 - a^2*e^5 + 2*(c^2*d^2*e^3 + a*c*e^5)*x^2 + 9*(c^2*d^3*e^2
+ a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(c*d^4*e^4 + a*d^2*e^6 + (c*d^2*e^6 + a*e^
8)*x^2 + 2*(c*d^3*e^5 + a*d*e^7)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + cx^2\right)^{\frac{3}{2}}}{(d + ex)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)/(e*x+d)**3,x)
```

```
[Out] Integral((a + c*x**2)**(3/2)/(d + e*x)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^3,x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.541 $\int \frac{(a+cx^2)^{3/2}}{(d+ex)^4} dx$

Optimal. Leaf size=200

$$\frac{c^2 d (3ae^2+2cd^2) \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{2e^4 (ae^2+cd^2)^{3/2}}+\frac{c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e^4}-\frac{c\sqrt{a+cx^2}\left(ex(2ae^2+3cd^2)+d(ae^2+2cd^2)\right)}{2e^3(d+ex)^2(ae^2+cd^2)}-$$

```
[Out] -(c*(d*(2*c*d^2 + a*e^2) + e*(3*c*d^2 + 2*a*e^2)*x)*Sqrt[a + c*x^2])/(2*e^3
*(c*d^2 + a*e^2)*(d + e*x)^2) - (a + c*x^2)^(3/2)/(3*e*(d + e*x)^3) + (c^(3
/2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/e^4 + (c^2*d*(2*c*d^2 + 3*a*e^2)*
ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(2*e^4*(c*d^2
 + a*e^2)^(3/2))
```

Rubi [A] time = 0.176935, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {733, 811, 844, 217, 206, 725}

$$\frac{c^2 d (3ae^2+2cd^2) \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{2e^4 (ae^2+cd^2)^{3/2}}+\frac{c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e^4}-\frac{c\sqrt{a+cx^2}\left(ex(2ae^2+3cd^2)+d(ae^2+2cd^2)\right)}{2e^3(d+ex)^2(ae^2+cd^2)}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2)/(d + e*x)^4,x]
```

```
[Out] -(c*(d*(2*c*d^2 + a*e^2) + e*(3*c*d^2 + 2*a*e^2)*x)*Sqrt[a + c*x^2])/(2*e^3
*(c*d^2 + a*e^2)*(d + e*x)^2) - (a + c*x^2)^(3/2)/(3*e*(d + e*x)^3) + (c^(3
/2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/e^4 + (c^2*d*(2*c*d^2 + 3*a*e^2)*
ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(2*e^4*(c*d^2
 + a*e^2)^(3/2))
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 && !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}}{(d+ex)^4}\,dx = -\frac{\left(a+cx^2\right)^{3/2}}{3e(d+ex)^3}+\frac{c\int \frac{x\sqrt{a+cx^2}}{(d+ex)^3}\,dx}{e}$$

$$= -\frac{c\left(d\left(2cd^2+ae^2\right)+e\left(3cd^2+2ae^2\right)x\right)\sqrt{a+cx^2}}{2e^3\left(cd^2+ae^2\right)(d+ex)^2}-\frac{\left(a+cx^2\right)^{3/2}}{3e(d+ex)^3}-\frac{c\int \frac{2acde-4c\left(cd^2+ae^2\right)x}{(d+ex)\sqrt{a+cx^2}}\,dx}{4e^3\left(cd^2+ae^2\right)}$$

$$= -\frac{c\left(d\left(2cd^2+ae^2\right)+e\left(3cd^2+2ae^2\right)x\right)\sqrt{a+cx^2}}{2e^3\left(cd^2+ae^2\right)(d+ex)^2}-\frac{\left(a+cx^2\right)^{3/2}}{3e(d+ex)^3}+\frac{c^2\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e^4}-\frac{\left(c^2d\left(2cd^2+3ae\right.\right.}{2e^4\left(c\right.}$$

$$= -\frac{c\left(d\left(2cd^2+ae^2\right)+e\left(3cd^2+2ae^2\right)x\right)\sqrt{a+cx^2}}{2e^3\left(cd^2+ae^2\right)(d+ex)^2}-\frac{\left(a+cx^2\right)^{3/2}}{3e(d+ex)^3}+\frac{c^2\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{e^4}+$$

$$= -\frac{c\left(d\left(2cd^2+ae^2\right)+e\left(3cd^2+2ae^2\right)x\right)\sqrt{a+cx^2}}{2e^3\left(cd^2+ae^2\right)(d+ex)^2}-\frac{\left(a+cx^2\right)^{3/2}}{3e(d+ex)^3}+\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e^4}+\frac{c^2d\left(2cd^2\right.}{}$$

Mathematica [A] time = 0.282589, size = 242, normalized size = 1.21

$$\frac{-\frac{e\sqrt{a+cx^2}\left(2a^2e^4+ace^2\left(5d^2+9dex+8e^2x^2\right)+c^2d^2\left(6d^2+15dex+11e^2x^2\right)\right)}{(d+ex)^3\left(ae^2+cd^2\right)}+\frac{3c^2d\left(3ae^2+2cd^2\right)\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{\left(ae^2+cd^2\right)^{3/2}}-\frac{3c^2d\left(3ae^2+2cd^2\right)\log(d+ex)}{\left(ae^2+cd^2\right)^{3/2}}+6}{6e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2)/(d + e*x)^4,x]
```

```
[Out] (-((e*Sqrt[a + c*x^2]*(2*a^2*e^4 + a*c*e^2*(5*d^2 + 9*d*e*x + 8*e^2*x^2) +
c^2*d^2*(6*d^2 + 15*d*e*x + 11*e^2*x^2)))/((c*d^2 + a*e^2)*(d + e*x)^3)) -
(3*c^2*d*(2*c*d^2 + 3*a*e^2)*Log[d + e*x])/(c*d^2 + a*e^2)^(3/2) + 6*c^(3/2
)*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] + (3*c^2*d*(2*c*d^2 + 3*a*e^2)*Log[a*e
 - c*d*x + Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2]])/(c*d^2 + a*e^2)^(3/2))/(6*
```

```
e^4)
```

Maple [B] time = 0.216, size = 2490, normalized size = 12.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)/(e*x+d)^4,x)
```

```
[Out] -1/3/e^2/(a*e^2+c*d^2)/(d/e+x)^3*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)
/e^2)^(5/2)+2/3/(a*e^2+c*d^2)^2*c^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d
^2)/e^2)^(3/2)*x-2/3/(a*e^2+c*d^2)^2*c/(d/e+x)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)
+(a*e^2+c*d^2)/e^2)^(5/2)+1/(a*e^2+c*d^2)^2*c^(3/2)*a^2*ln((-c*d/e+(d/e+x)*
c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))+1/(a*e^2+
c*d^2)^2*c^2*a*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x-3/4/
e^2*c^(7/2)*d^4/(a*e^2+c*d^2)^3*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-
2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))*a+9/4/e^2*c^(5/2)*d^2/(a*e^2+c*d^
2)^2*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^
2)/e^2)^(1/2))*a-1/2/e*c^3*d^3/(a*e^2+c*d^2)^3/((a*e^2+c*d^2)/e^2)^(1/2)*ln
((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x
)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a^2+3/2/e*c^2*d/(a*e
^2+c*d^2)^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+
x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e
^2)^(1/2))/(d/e+x))*a^2+3/e^3*c^3*d^3/(a*e^2+c*d^2)^2/((a*e^2+c*d^2)/e^2)^(
1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c
*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a-1/e^3*c^4*d
^5/(a*e^2+c*d^2)^3/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/
e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c
*d^2)/e^2)^(1/2))/(d/e+x))*a+3/4/e^2*c^3*d^2/(a*e^2+c*d^2)^2*(c*(d/e+x)^2-2
*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x-3/2/e*c^2*d/(a*e^2+c*d^2)^2*(c*(d
/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*a-1/6/e*c*d/(a*e^2+c*d^2)^
2/(d/e+x)^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(5/2)-1/4/e^2*c
^4*d^4/(a*e^2+c*d^2)^3*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2
)*x+1/2/e*c^3*d^3/(a*e^2+c*d^2)^3*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2
)/e^2)^(1/2)*a-1/2/e^5*c^5*d^7/(a*e^2+c*d^2)^3/((a*e^2+c*d^2)/e^2)^(1/2)*ln
((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x
)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))-1/4*c^(5/2)*d^2/(a*e
^2+c*d^2)^3*a^2*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+
(a*e^2+c*d^2)/e^2)^(1/2))-1/6*c^3*d^2/(a*e^2+c*d^2)^3*(c*(d/e+x)^2-2*c*d/e*
(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)*x+1/6*c^2*d^2/(a*e^2+c*d^2)^3/(d/e+x)*(c*(
d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(5/2)+1/6/e*c^3*d^3/(a*e^2+c*d^
2)^3*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)+1/2/e^3*c^4*d^5/
(a*e^2+c*d^2)^3*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)-1/2/e
*c^2*d/(a*e^2+c*d^2)^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2
)-3/2/e^3*c^3*d^3/(a*e^2+c*d^2)^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2
)/e^2)^(1/2)+3/2/e^4*c^(7/2)*d^4/(a*e^2+c*d^2)^2*ln((-c*d/e+(d/e+x)*c)/c^(1
/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))-1/2/e^4*c^(9/2)*
d^6/(a*e^2+c*d^2)^3*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e
+x)+(a*e^2+c*d^2)/e^2)^(1/2))+3/2/e^5*c^4*d^5/(a*e^2+c*d^2)^2/((a*e^2+c*d^2
)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^
(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))-1/4*c
^3*d^2/(a*e^2+c*d^2)^3*a*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1
/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 54.1702, size = 5117, normalized size = 25.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] [1/12*(6*(c^3*d^7 + 2*a*c^2*d^5*e^2 + a^2*c*d^3*e^4 + (c^3*d^4*e^3 + 2*a*c^
2*d^2*e^5 + a^2*c*e^7)*x^3 + 3*(c^3*d^5*e^2 + 2*a*c^2*d^3*e^4 + a^2*c*d*e^6
)*x^2 + 3*(c^3*d^6*e + 2*a*c^2*d^4*e^3 + a^2*c*d^2*e^5)*x)*sqrt(c)*log(-2*c
*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 3*(2*c^3*d^6 + 3*a*c^2*d^4*e^2 +
(2*c^3*d^3*e^3 + 3*a*c^2*d*e^5)*x^3 + 3*(2*c^3*d^4*e^2 + 3*a*c^2*d^2*e^4)*x
^2 + 3*(2*c^3*d^5*e + 3*a*c^2*d^3*e^3)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*
e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^
2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(6*c^3*d^6
*e + 11*a*c^2*d^4*e^3 + 7*a^2*c*d^2*e^5 + 2*a^3*e^7 + (11*c^3*d^4*e^3 + 19*
a*c^2*d^2*e^5 + 8*a^2*c*e^7)*x^2 + 3*(5*c^3*d^5*e^2 + 8*a*c^2*d^3*e^4 + 3*a
^2*c*d*e^6)*x)*sqrt(c*x^2 + a))/(c^2*d^7*e^4 + 2*a*c*d^5*e^6 + a^2*d^3*e^8
+ (c^2*d^4*e^7 + 2*a*c*d^2*e^9 + a^2*e^11)*x^3 + 3*(c^2*d^5*e^6 + 2*a*c*d^3
*e^8 + a^2*d*e^10)*x^2 + 3*(c^2*d^6*e^5 + 2*a*c*d^4*e^7 + a^2*d^2*e^9)*x),
1/6*(3*(2*c^3*d^6 + 3*a*c^2*d^4*e^2 + (2*c^3*d^3*e^3 + 3*a*c^2*d*e^5)*x^3 +
 3*(2*c^3*d^4*e^2 + 3*a*c^2*d^2*e^4)*x^2 + 3*(2*c^3*d^5*e + 3*a*c^2*d^3*e^3
)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*
x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + 3*(c^3*d^7 + 2*a*
c^2*d^5*e^2 + a^2*c*d^3*e^4 + (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^7)*x
^3 + 3*(c^3*d^5*e^2 + 2*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x^2 + 3*(c^3*d^6*e + 2
*a*c^2*d^4*e^3 + a^2*c*d^2*e^5)*x)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)
*sqrt(c)*x - a) - (6*c^3*d^6*e + 11*a*c^2*d^4*e^3 + 7*a^2*c*d^2*e^5 + 2*a^3
*e^7 + (11*c^3*d^4*e^3 + 19*a*c^2*d^2*e^5 + 8*a^2*c*e^7)*x^2 + 3*(5*c^3*d^5
*e^2 + 8*a*c^2*d^3*e^4 + 3*a^2*c*d*e^6)*x)*sqrt(c*x^2 + a))/(c^2*d^7*e^4 +
2*a*c*d^5*e^6 + a^2*d^3*e^8 + (c^2*d^4*e^7 + 2*a*c*d^2*e^9 + a^2*e^11)*x^3
+ 3*(c^2*d^5*e^6 + 2*a*c*d^3*e^8 + a^2*d*e^10)*x^2 + 3*(c^2*d^6*e^5 + 2*a*c
*d^4*e^7 + a^2*d^2*e^9)*x), -1/12*(12*(c^3*d^7 + 2*a*c^2*d^5*e^2 + a^2*c*d^
3*e^4 + (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^7)*x^3 + 3*(c^3*d^5*e^2 +
2*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x^2 + 3*(c^3*d^6*e + 2*a*c^2*d^4*e^3 + a^2*c
*d^2*e^5)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - 3*(2*c^3*d^6 + 3
*a*c^2*d^4*e^2 + (2*c^3*d^3*e^3 + 3*a*c^2*d*e^5)*x^3 + 3*(2*c^3*d^4*e^2 + 3
*a*c^2*d^2*e^4)*x^2 + 3*(2*c^3*d^5*e + 3*a*c^2*d^3*e^3)*x)*sqrt(c*d^2 + a*e
^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*
sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2
)) + 2*(6*c^3*d^6*e + 11*a*c^2*d^4*e^3 + 7*a^2*c*d^2*e^5 + 2*a^3*e^7 + (11*
c^3*d^4*e^3 + 19*a*c^2*d^2*e^5 + 8*a^2*c*e^7)*x^2 + 3*(5*c^3*d^5*e^2 + 8*a*
c^2*d^3*e^4 + 3*a^2*c*d*e^6)*x)*sqrt(c*x^2 + a))/(c^2*d^7*e^4 + 2*a*c*d^5*e
^6 + a^2*d^3*e^8 + (c^2*d^4*e^7 + 2*a*c*d^2*e^9 + a^2*e^11)*x^3 + 3*(c^2*d^
5*e^6 + 2*a*c*d^3*e^8 + a^2*d*e^10)*x^2 + 3*(c^2*d^6*e^5 + 2*a*c*d^4*e^7 +
a^2*d^2*e^9)*x), 1/6*(3*(2*c^3*d^6 + 3*a*c^2*d^4*e^2 + (2*c^3*d^3*e^3 + 3*a
```

```
*c^2*d*e^5)*x^3 + 3*(2*c^3*d^4*e^2 + 3*a*c^2*d^2*e^4)*x^2 + 3*(2*c^3*d^5*e
+ 3*a*c^2*d^3*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d
*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) -
6*(c^3*d^7 + 2*a*c^2*d^5*e^2 + a^2*c*d^3*e^4 + (c^3*d^4*e^3 + 2*a*c^2*d^2*e
^5 + a^2*c*e^7)*x^3 + 3*(c^3*d^5*e^2 + 2*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x^2 +
 3*(c^3*d^6*e + 2*a*c^2*d^4*e^3 + a^2*c*d^2*e^5)*x)*sqrt(-c)*arctan(sqrt(-c
)*x/sqrt(c*x^2 + a)) - (6*c^3*d^6*e + 11*a*c^2*d^4*e^3 + 7*a^2*c*d^2*e^5 +
2*a^3*e^7 + (11*c^3*d^4*e^3 + 19*a*c^2*d^2*e^5 + 8*a^2*c*e^7)*x^2 + 3*(5*c^
3*d^5*e^2 + 8*a*c^2*d^3*e^4 + 3*a^2*c*d*e^6)*x)*sqrt(c*x^2 + a))/(c^2*d^7*e
^4 + 2*a*c*d^5*e^6 + a^2*d^3*e^8 + (c^2*d^4*e^7 + 2*a*c*d^2*e^9 + a^2*e^11)
*x^3 + 3*(c^2*d^5*e^6 + 2*a*c*d^3*e^8 + a^2*d*e^10)*x^2 + 3*(c^2*d^6*e^5 +
2*a*c*d^4*e^7 + a^2*d^2*e^9)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + cx^2\right)^{\frac{3}{2}}}{\left(d + ex\right)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)/(e*x+d)**4,x)
```

```
[Out] Integral((a + c*x**2)**(3/2)/(d + e*x)**4, x)
```

Giac [B] time = 1.57564, size = 795, normalized size = 3.98

$$-c^{\frac{3}{2}}e^{(-4)}\log\left(\left|-\sqrt{c}x + \sqrt{cx^2 + a}\right|\right) + \frac{\left(2\,c^3d^3 + 3\,ac^2de^2\right)\arctan\left(\frac{\left(\sqrt{c}x - \sqrt{cx^2+a}\right)e + \sqrt{c}d}{\sqrt{-cd^2 - ae^2}}\right)}{\left(cd^2e^4 + ae^6\right)\sqrt{-cd^2 - ae^2}} - \frac{54\left(\sqrt{c}x - \sqrt{cx^2 + a}\right)^4 c^{\frac{7}{2}}d^4e + 4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -c^(3/2)*e^(-4)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a))) + (2*c^3*d^3 + 3*a*c
^2*d*e^2)*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2
- a*e^2))/((c*d^2*e^4 + a*e^6)*sqrt(-c*d^2 - a*e^2)) - 1/3*(54*(sqrt(c)*x -
 sqrt(c*x^2 + a))^4*c^(7/2)*d^4*e + 44*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^4*
d^5 + 18*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^3*d^3*e^2 - 78*(sqrt(c)*x - sqrt
(c*x^2 + a))^2*a*c^(7/2)*d^4*e - 34*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c^3*d
^3*e^2 + 27*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(5/2)*d^2*e^3 + 15*(sqrt(c)
*x - sqrt(c*x^2 + a))^5*a*c^2*d*e^4 + 48*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*
c^3*d^3*e^2 - 36*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*c^(5/2)*d^2*e^3 - 48*(
sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^2*d*e^4 - 12*(sqrt(c)*x - sqrt(c*x^2 +
 a))^4*a^2*c^(3/2)*e^5 - 11*a^3*c^(5/2)*d^2*e^3 + 33*(sqrt(c)*x - sqrt(c*x^
2 + a))*a^3*c^2*d*e^4 + 12*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^3*c^(3/2)*e^5
- 8*a^4*c^(3/2)*e^5)/((c*d^2*e^4 + a*e^6)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*
e + 2*(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*d - a*e)^3)
```

3.542 $\int \frac{\left(a+cx^2\right)^{3/2}}{(d+ex)^5}\,dx$

Optimal. Leaf size=153

$$-\frac{3a^2c^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{8\left(ae^2+cd^2\right)^{5/2}}-\frac{3ac\sqrt{a+cx^2}(ae-cdx)}{8(d+ex)^2\left(ae^2+cd^2\right)^2}-\frac{\left(a+cx^2\right)^{3/2}(ae-cdx)}{4(d+ex)^4\left(ae^2+cd^2\right)}$$

```
[Out] (-3*a*c*(a*e - c*d*x)*Sqrt[a + c*x^2])/(8*(c*d^2 + a*e^2)^2*(d + e*x)^2) -
((a*e - c*d*x)*(a + c*x^2)^(3/2))/(4*(c*d^2 + a*e^2)*(d + e*x)^4) - (3*a^2*
c^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(8*(c*d^2
 + a*e^2)^(5/2))
```

Rubi [A] time = 0.0628769, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {721, 725, 206}

$$-\frac{3a^2c^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{8\left(ae^2+cd^2\right)^{5/2}}-\frac{3ac\sqrt{a+cx^2}(ae-cdx)}{8(d+ex)^2\left(ae^2+cd^2\right)^2}-\frac{\left(a+cx^2\right)^{3/2}(ae-cdx)}{4(d+ex)^4\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2)/(d + e*x)^5,x]
```

```
[Out] (-3*a*c*(a*e - c*d*x)*Sqrt[a + c*x^2])/(8*(c*d^2 + a*e^2)^2*(d + e*x)^2) -
((a*e - c*d*x)*(a + c*x^2)^(3/2))/(4*(c*d^2 + a*e^2)*(d + e*x)^4) - (3*a^2*
c^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(8*(c*d^2
 + a*e^2)^(5/2))
```

Rule 721

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(-2*a*e + (2*c*d)*x)*(a + c*x^2)^p)/(2*(m + 1)*(c*d^2 +
 a*e^2)), x] - Dist[(4*a*c*p)/(2*(m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m
 + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+cx^2)^{3/2}}{(d+ex)^5}\,dx = -\frac{(ae-cdx)\left(a+cx^2\right)^{3/2}}{4\left(cd^2+ae^2\right)(d+ex)^4}+\frac{(3ac)\int \frac{\sqrt{a+cx^2}}{(d+ex)^3}\,dx}{4\left(cd^2+ae^2\right)}$$

$$= -\frac{3ac(ae-cdx)\sqrt{a+cx^2}}{8\left(cd^2+ae^2\right)^2(d+ex)^2}-\frac{(ae-cdx)\left(a+cx^2\right)^{3/2}}{4\left(cd^2+ae^2\right)(d+ex)^4}+\frac{\left(3a^2c^2\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{8\left(cd^2+ae^2\right)^2}$$

$$= -\frac{3ac(ae-cdx)\sqrt{a+cx^2}}{8\left(cd^2+ae^2\right)^2(d+ex)^2}-\frac{(ae-cdx)\left(a+cx^2\right)^{3/2}}{4\left(cd^2+ae^2\right)(d+ex)^4}-\frac{\left(3a^2c^2\right)\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{8\left(cd^2+ae^2\right)^2}$$

$$= -\frac{3ac(ae-cdx)\sqrt{a+cx^2}}{8\left(cd^2+ae^2\right)^2(d+ex)^2}-\frac{(ae-cdx)\left(a+cx^2\right)^{3/2}}{4\left(cd^2+ae^2\right)(d+ex)^4}-\frac{3a^2c^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{8\left(cd^2+ae^2\right)^{5/2}}$$

Mathematica [A] time = 0.277666, size = 198, normalized size = 1.29

$$\frac{1}{8}\left(\frac{\sqrt{a+cx^2}\left(-a^2ce\left(5d^2+4dex+5e^2x^2\right)-2a^3e^3+ac^2dx\left(5d^2+4dex+5e^2x^2\right)+2c^3d^3x^3\right)}{(d+ex)^4\left(ae^2+cd^2\right)^2}-\frac{3a^2c^2\log\left(\sqrt{a+cx^2}\sqrt{ae\right.}}{\left(ae^2+c\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2)/(d + e*x)^5,x]
```

```
[Out] ((Sqrt[a + c*x^2]*(-2*a^3*e^3 + 2*c^3*d^3*x^3 - a^2*c*e*(5*d^2 + 4*d*e*x +
5*e^2*x^2) + a*c^2*d*x*(5*d^2 + 4*d*e*x + 5*e^2*x^2)))/((c*d^2 + a*e^2)^2*(
d + e*x)^4) + (3*a^2*c^2*Log[d + e*x])/(c*d^2 + a*e^2)^(5/2) - (3*a^2*c^2*L
og[a*e - c*d*x + Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2]])/(c*d^2 + a*e^2)^(5/2
))/8
```

Maple [B] time = 0.24, size = 3528, normalized size = 23.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)/(e*x+d)^5,x)
```

```
[Out] 3/8*c^3*d/(a*e^2+c*d^2)^3*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(
3/2)*x-3/8/e^5*c^6*d^8/(a*e^2+c*d^2)^4/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e
^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*
d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))+3/8/e^2*c^4*d^3/(a*e^2+c*d^2
)^3*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x-1/8/e*c^2*d^2/(
a*e^2+c*d^2)^3/(d/e+x)^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(5
/2)-1/4/e^2*c*d/(a*e^2+c*d^2)^2/(d/e+x)^3*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e
^2+c*d^2)/e^2)^(5/2)-3/16*c^4*d^3/(a*e^2+c*d^2)^4*a*(c*(d/e+x)^2-2*c*d/e*(d
/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x+9/16*c^3*d/(a*e^2+c*d^2)^3*a*(c*(d/e+x)^2-
2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x-9/16/e^2*c^(9/2)*d^5/(a*e^2+c*d^
2)^4*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^
2)/e^2)^(1/2))*a-3/8/e/(a*e^2+c*d^2)^2*c^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*
(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-
2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a^2-3/8/e^5/(a*e^2+c*d^2
)^2*c^4/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2
```

```
*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^
(1/2))/(d/e+x))*d^4-3/16/e^2/(a*e^2+c*d^2)^2*c^3*d*(c*(d/e+x)^2-2*c*d/e*(d/
e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x-1/8/e/(a*e^2+c*d^2)^2*c/(d/e+x)^2*(c*(d/e+x
)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(5/2)-3/8/e^4*c^(11/2)*d^7/(a*e^2+c*
d^2)^4*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*
d^2)/e^2)^(1/2))-3/4/e^3*c^4*d^4/(a*e^2+c*d^2)^3*(c*(d/e+x)^2-2*c*d/e*(d/e+
x)+(a*e^2+c*d^2)/e^2)^(1/2)-1/8*c^4*d^3/(a*e^2+c*d^2)^4*(c*(d/e+x)^2-2*c*d/
e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)*x+1/8/e/(a*e^2+c*d^2)^2*c^2*(c*(d/e+x)^2
-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)-1/4/e^3/(a*e^2+c*d^2)/(d/e+x)^4*(
c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(5/2)-3/8*c^2*d/(a*e^2+c*d^2
)^3/(d/e+x)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(5/2)-1/4/e*c^3
*d^2/(a*e^2+c*d^2)^3*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)+
1/8*c^3*d^3/(a*e^2+c*d^2)^4/(d/e+x)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d
^2)/e^2)^(5/2)+3/8/e^3*c^5*d^6/(a*e^2+c*d^2)^4*(c*(d/e+x)^2-2*c*d/e*(d/e+x)
+(a*e^2+c*d^2)/e^2)^(1/2)-3/16*c^(7/2)*d^3/(a*e^2+c*d^2)^4*a^2*ln((-c*d/e+(
d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))+9/
16*c^(5/2)*d/(a*e^2+c*d^2)^3*a^2*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2
-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))+1/8/e*c^4*d^4/(a*e^2+c*d^2)^4*(c
*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)+3/8/e/(a*e^2+c*d^2)^2*c
^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*a+3/8/e^3/(a*e^2+c
*d^2)^2*c^3*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*d^2-3/8/e
^4/(a*e^2+c*d^2)^2*c^(7/2)*d^3*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2
*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))+3/4/e^4*c^(9/2)*d^5/(a*e^2+c*d^2)^
3*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/
e^2)^(1/2))+3/4/e*c^3*d^2/(a*e^2+c*d^2)^3/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(
a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2
*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a^2+3/2/e^3*c^4*d^4/(a*e^
2+c*d^2)^3/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x
)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^
2)^(1/2))/(d/e+x))*a-3/4/e^3/(a*e^2+c*d^2)^2*c^3/((a*e^2+c*d^2)/e^2)^(1/2)*
ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e
+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a*d^2-3/8/e*c^4*d^
4/(a*e^2+c*d^2)^4/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e
*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*
d^2)/e^2)^(1/2))/(d/e+x))*a^2-3/4/e^3*c^5*d^6/(a*e^2+c*d^2)^4/((a*e^2+c*d^2
)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^
(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a-3/4
/e*c^3*d^2/(a*e^2+c*d^2)^3*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^
(1/2)*a+3/4/e^5*c^5*d^6/(a*e^2+c*d^2)^3/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*
e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c
*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))+9/8/e^2*c^(7/2)*d^3/(a*e^2+
c*d^2)^3*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+
c*d^2)/e^2)^(1/2))*a-9/16/e^2/(a*e^2+c*d^2)^2*c^(5/2)*d*ln((-c*d/e+(d/e+x)*
c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))*a-3/16/e^
2*c^5*d^5/(a*e^2+c*d^2)^4*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(
1/2)*x+3/8/e*c^4*d^4/(a*e^2+c*d^2)^4*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*
d^2)/e^2)^(1/2)*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 20.2292, size = 2221, normalized size = 14.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^5,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(a^2*c^2*e^4*x^4 + 4*a^2*c^2*d*e^3*x^3 + 6*a^2*c^2*d^2*e^2*x^2 + 4
*a^2*c^2*d^3*e*x + a^2*c^2*d^4)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*
d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x
- a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(5*a^2*c^2*d^4*e + 7
*a^3*c*d^2*e^3 + 2*a^4*e^5 - (2*c^4*d^5 + 7*a*c^3*d^3*e^2 + 5*a^2*c^2*d*e^4
)*x^3 - (4*a*c^3*d^4*e - a^2*c^2*d^2*e^3 - 5*a^3*c*e^5)*x^2 - (5*a*c^3*d^5
+ a^2*c^2*d^3*e^2 - 4*a^3*c*d*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^10 + 3*a*c^2*
d^8*e^2 + 3*a^2*c*d^6*e^4 + a^3*d^4*e^6 + (c^3*d^6*e^4 + 3*a*c^2*d^4*e^6 +
3*a^2*c*d^2*e^8 + a^3*e^10)*x^4 + 4*(c^3*d^7*e^3 + 3*a*c^2*d^5*e^5 + 3*a^2*
c*d^3*e^7 + a^3*d*e^9)*x^3 + 6*(c^3*d^8*e^2 + 3*a*c^2*d^6*e^4 + 3*a^2*c*d^4
*e^6 + a^3*d^2*e^8)*x^2 + 4*(c^3*d^9*e + 3*a*c^2*d^7*e^3 + 3*a^2*c*d^5*e^5
+ a^3*d^3*e^7)*x), -1/8*(3*(a^2*c^2*e^4*x^4 + 4*a^2*c^2*d*e^3*x^3 + 6*a^2*c
^2*d^2*e^2*x^2 + 4*a^2*c^2*d^3*e*x + a^2*c^2*d^4)*sqrt(-c*d^2 - a*e^2)*arct
an(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 +
(c^2*d^2 + a*c*e^2)*x^2)) + (5*a^2*c^2*d^4*e + 7*a^3*c*d^2*e^3 + 2*a^4*e^5
- (2*c^4*d^5 + 7*a*c^3*d^3*e^2 + 5*a^2*c^2*d*e^4)*x^3 - (4*a*c^3*d^4*e - a^
2*c^2*d^2*e^3 - 5*a^3*c*e^5)*x^2 - (5*a*c^3*d^5 + a^2*c^2*d^3*e^2 - 4*a^3*c
*d*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^10 + 3*a*c^2*d^8*e^2 + 3*a^2*c*d^6*e^4 +
 a^3*d^4*e^6 + (c^3*d^6*e^4 + 3*a*c^2*d^4*e^6 + 3*a^2*c*d^2*e^8 + a^3*e^10)
*x^4 + 4*(c^3*d^7*e^3 + 3*a*c^2*d^5*e^5 + 3*a^2*c*d^3*e^7 + a^3*d*e^9)*x^3
+ 6*(c^3*d^8*e^2 + 3*a*c^2*d^6*e^4 + 3*a^2*c*d^4*e^6 + a^3*d^2*e^8)*x^2 + 4
*(c^3*d^9*e + 3*a*c^2*d^7*e^3 + 3*a^2*c*d^5*e^5 + a^3*d^3*e^7)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+cx^2\right)^{\frac{3}{2}}}{(d+ex)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)/(e*x+d)**5,x)
```

```
[Out] Integral((a + c*x**2)**(3/2)/(d + e*x)**5, x)
```

Giac [B] time = 4.53682, size = 1278, normalized size = 8.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] -1/64*(3*sqrt(c*d^2 + a*e^2)*a^2*c^2*e^14*log(abs(sqrt(c*d^2 + a*e^2)*c*d -
 (c*d^2 + a*e^2)*(sqrt(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(x*e
```

```
 + d)^2) + sqrt(c*d^2*e^2 + a*e^4)*e^(-1)/(x*e + d))))*sgn(1/(x*e + d))/(c^
4*d^8*e + 4*a*c^3*d^6*e^3 + 6*a^2*c^2*d^4*e^5 + 4*a^3*c*d^2*e^7 + a^4*e^9)
- sqrt(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(x*e + d)^2)*((2*(3*
(c^4*d^7*e^19*sgn(1/(x*e + d)) + 3*a*c^3*d^5*e^21*sgn(1/(x*e + d)) + 3*a^2*
c^2*d^3*e^23*sgn(1/(x*e + d)) + a^3*c*d*e^25*sgn(1/(x*e + d)))/(c^4*d^8*e^8
 + 4*a*c^3*d^6*e^10 + 6*a^2*c^2*d^4*e^12 + 4*a^3*c*d^2*e^14 + a^4*e^16) - (
c^4*d^8*e^20*sgn(1/(x*e + d)) + 4*a*c^3*d^6*e^22*sgn(1/(x*e + d)) + 6*a^2*c
^2*d^4*e^24*sgn(1/(x*e + d)) + 4*a^3*c*d^2*e^26*sgn(1/(x*e + d)) + a^4*e^28
*sgn(1/(x*e + d)))*e^(-1)/((c^4*d^8*e^8 + 4*a*c^3*d^6*e^10 + 6*a^2*c^2*d^4*
e^12 + 4*a^3*c*d^2*e^14 + a^4*e^16)*(x*e + d)))*e^(-1)/(x*e + d) - (6*c^4*d
^6*e^18*sgn(1/(x*e + d)) + 17*a*c^3*d^4*e^20*sgn(1/(x*e + d)) + 16*a^2*c^2*
d^2*e^22*sgn(1/(x*e + d)) + 5*a^3*c*e^24*sgn(1/(x*e + d)))/(c^4*d^8*e^8 + 4
*a*c^3*d^6*e^10 + 6*a^2*c^2*d^4*e^12 + 4*a^3*c*d^2*e^14 + a^4*e^16))*e^(-1)
/(x*e + d) + (2*c^4*d^5*e^17*sgn(1/(x*e + d)) + 7*a*c^3*d^3*e^19*sgn(1/(x*e
 + d)) + 5*a^2*c^2*d*e^21*sgn(1/(x*e + d)))/(c^4*d^8*e^8 + 4*a*c^3*d^6*e^10
 + 6*a^2*c^2*d^4*e^12 + 4*a^3*c*d^2*e^14 + a^4*e^16)) + (2*c^(9/2)*d^5*e^9
+ 7*a*c^(7/2)*d^3*e^11 + 5*a^2*c^(5/2)*d*e^13 - 3*sqrt(c*d^2 + a*e^2)*a^2*c
^2*e^13*log(abs(-c^(3/2)*d^2 + sqrt(c*d^2 + a*e^2)*c*d - a*sqrt(c)*e^2)))*s
gn(1/(x*e + d))/(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^
2*e^6 + a^4*e^8))*e^2
```

3.543 $\int \frac{(a+cx^2)^{3/2}}{(d+ex)^6} dx$

Optimal. Leaf size=195

$$-\frac{3a^2c^3d\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{8\left(ae^2+cd^2\right)^{7/2}}-\frac{3ac^2d\sqrt{a+cx^2}(ae-cdx)}{8(d+ex)^2\left(ae^2+cd^2\right)^3}-\frac{cd\left(a+cx^2\right)^{3/2}(ae-cdx)}{4(d+ex)^4\left(ae^2+cd^2\right)^2}-\frac{e\left(a+cx^2\right)^{5/2}}{5(d+ex)^5\left(ae^2+cd^2\right)}$$

```
[Out] (-3*a*c^2*d*(a*e - c*d*x)*Sqrt[a + c*x^2])/(8*(c*d^2 + a*e^2)^3*(d + e*x)^2
) - (c*d*(a*e - c*d*x)*(a + c*x^2)^(3/2))/(4*(c*d^2 + a*e^2)^2*(d + e*x)^4)
 - (e*(a + c*x^2)^(5/2))/(5*(c*d^2 + a*e^2)*(d + e*x)^5) - (3*a^2*c^3*d*Arc
Tanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(8*(c*d^2 + a*e^
2)^(7/2))
```

Rubi [A] time = 0.0950982, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {731, 721, 725, 206}

$$-\frac{3a^2c^3d\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{8\left(ae^2+cd^2\right)^{7/2}}-\frac{3ac^2d\sqrt{a+cx^2}(ae-cdx)}{8(d+ex)^2\left(ae^2+cd^2\right)^3}-\frac{cd\left(a+cx^2\right)^{3/2}(ae-cdx)}{4(d+ex)^4\left(ae^2+cd^2\right)^2}-\frac{e\left(a+cx^2\right)^{5/2}}{5(d+ex)^5\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2)/(d + e*x)^6,x]
```


```
[Out] (-3*a*c^2*d*(a*e - c*d*x)*Sqrt[a + c*x^2])/(8*(c*d^2 + a*e^2)^3*(d + e*x)^2
) - (c*d*(a*e - c*d*x)*(a + c*x^2)^(3/2))/(4*(c*d^2 + a*e^2)^2*(d + e*x)^4)
 - (e*(a + c*x^2)^(5/2))/(5*(c*d^2 + a*e^2)*(d + e*x)^5) - (3*a^2*c^3*d*Arc
Tanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(8*(c*d^2 + a*e^
2)^(7/2))
```


Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 721

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(-2*a*e + (2*c*d)*x)*(a + c*x^2)^p)/(2*(m + 1)*(c*d^2 +
 a*e^2)), x] - Dist[(4*a*c*p)/(2*(m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m
 + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}}{(d+ex)^6}\,dx = -\frac{e\left(a+cx^2\right)^{5/2}}{5\left(cd^2+ae^2\right)(d+ex)^5}+\frac{(cd)\int \frac{\left(a+cx^2\right)^{3/2}}{(d+ex)^5}\,dx}{cd^2+ae^2}$$

$$= -\frac{cd(ae-cdx)\left(a+cx^2\right)^{3/2}}{4\left(cd^2+ae^2\right)^2(d+ex)^4}-\frac{e\left(a+cx^2\right)^{5/2}}{5\left(cd^2+ae^2\right)(d+ex)^5}+\frac{\left(3ac^2d\right)\int \frac{\sqrt{a+cx^2}}{(d+ex)^3}\,dx}{4\left(cd^2+ae^2\right)^2}$$

$$= -\frac{3ac^2d(ae-cdx)\sqrt{a+cx^2}}{8\left(cd^2+ae^2\right)^3(d+ex)^2}-\frac{cd(ae-cdx)\left(a+cx^2\right)^{3/2}}{4\left(cd^2+ae^2\right)^2(d+ex)^4}-\frac{e\left(a+cx^2\right)^{5/2}}{5\left(cd^2+ae^2\right)(d+ex)^5}+\frac{\left(3a^2c^3d\right)\int \frac{1}{(d+ex)\sqrt{\ldots}}}{8\left(cd^2+ae^2\ldots\right)}$$

$$= -\frac{3ac^2d(ae-cdx)\sqrt{a+cx^2}}{8\left(cd^2+ae^2\right)^3(d+ex)^2}-\frac{cd(ae-cdx)\left(a+cx^2\right)^{3/2}}{4\left(cd^2+ae^2\right)^2(d+ex)^4}-\frac{e\left(a+cx^2\right)^{5/2}}{5\left(cd^2+ae^2\right)(d+ex)^5}-\frac{\left(3a^2c^3d\right)\text{Subst}\left(\ldots\right)}{8\left(\ldots\right)}$$

$$= -\frac{3ac^2d(ae-cdx)\sqrt{a+cx^2}}{8\left(cd^2+ae^2\right)^3(d+ex)^2}-\frac{cd(ae-cdx)\left(a+cx^2\right)^{3/2}}{4\left(cd^2+ae^2\right)^2(d+ex)^4}-\frac{e\left(a+cx^2\right)^{5/2}}{5\left(cd^2+ae^2\right)(d+ex)^5}-\frac{3a^2c^3d\tanh^{-1}\left(\ldots\right)}{8\left(cd^2+\ldots\right)}$$

Mathematica [A] time = 0.374942, size = 272, normalized size = 1.39

$$\frac{1}{40}\left(\frac{\sqrt{a+cx^2}\left(-a^2c^2e\left(77d^2e^2x^2+45d^3ex+33d^4+25de^3x^3+8e^4x^4\right)-2a^3ce^3\left(13d^2+5dex+8e^2x^2\right)-8a^4e^5+ac^3d^2x\left(\ldots\right.\right.}{(d+ex)^5\left(ae^2+cd^2\right)^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2)/(d + e*x)^6,x]
```

```
[Out] ((Sqrt[a + c*x^2]*(-8*a^4*e^5 + 2*c^4*d^4*x^3*(5*d + e*x) - 2*a^3*c*e^3*(13
*d^2 + 5*d*e*x + 8*e^2*x^2) + a*c^3*d^2*x*(25*d^3 + 29*d^2*e*x + 45*d*e^2*x
^2 + 9*e^3*x^3) - a^2*c^2*e*(33*d^4 + 45*d^3*e*x + 77*d^2*e^2*x^2 + 25*d*e^
3*x^3 + 8*e^4*x^4)))/((c*d^2 + a*e^2)^3*(d + e*x)^5) + (15*a^2*c^3*d*Log[d
+ e*x])/(c*d^2 + a*e^2)^(7/2) - (15*a^2*c^3*d*Log[a*e - c*d*x + Sqrt[c*d^2
+ a*e^2]*Sqrt[a + c*x^2]])/(c*d^2 + a*e^2)^(7/2))/40
```

Maple [B] time = 0.217, size = 3622, normalized size = 18.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)/(e*x+d)^6,x)
```

```
[Out] -1/8*c^5*d^4/(a*e^2+c*d^2)^5*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2
)^(3/2)*x+1/8*c^4*d^4/(a*e^2+c*d^2)^5/(d/e+x)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+
(a*e^2+c*d^2)/e^2)^(5/2)-3/16*c^(9/2)*d^4/(a*e^2+c*d^2)^5*a^2*ln((-c*d/e+(d
/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))+9/1
```

6*c^(7/2)*d^2/(a*e^2+c*d^2)^4*a^2*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^
2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))-3/8*c^3*d^2/(a*e^2+c*d^2)^4/(d/
e+x)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(5/2)-1/5/e^4/(a*e^2+c
d^2)/(d/e+x)^5(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(5/2)+1/8/e
*c^5*d^5/(a*e^2+c*d^2)^5*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3
/2)+3/8/e^3*c^6*d^7/(a*e^2+c*d^2)^5*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d
^2)/e^2)^(1/2)-3/8/e^4*c^(9/2)*d^4/(a*e^2+c*d^2)^3*ln((-c*d/e+(d/e+x)*c)/c^
(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))+3/4/e^4*c^(11/
2)*d^6/(a*e^2+c*d^2)^4*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(
d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))-3/8/e^4*c^(13/2)*d^8/(a*e^2+c*d^2)^5*ln((-
c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1
/2))+3/8*c^4*d^2/(a*e^2+c*d^2)^4*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)
/e^2)^(3/2)*x-3/4/e^3*c^5*d^5/(a*e^2+c*d^2)^4*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+
(a*e^2+c*d^2)/e^2)^(1/2)-1/4/e*c^4*d^3/(a*e^2+c*d^2)^4*(c*(d/e+x)^2-2*c*d/e
*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)+1/8/e*c^3*d/(a*e^2+c*d^2)^3*(c*(d/e+x)^2-
2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)+3/8/e^3*c^4*d^3/(a*e^2+c*d^2)^3*(c
*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)-3/8/e*c^3*d/(a*e^2+c*d^
2)^3/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((
a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/
2))/(d/e+x))*a^2-3/4/e^3*c^4*d^3/(a*e^2+c*d^2)^3/((a*e^2+c*d^2)/e^2)^(1/2)*
ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e
+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a-3/8/e*c^5*d^5/(a
*e^2+c*d^2)^5/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/
e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)
/e^2)^(1/2))/(d/e+x))*a^2-3/4/e^3*c^6*d^7/(a*e^2+c*d^2)^5/((a*e^2+c*d^2)/e^
2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2
)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a+3/4/e*c
^4*d^3/(a*e^2+c*d^2)^4/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*
c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e
^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a^2+3/2/e^3*c^5*d^5/(a*e^2+c*d^2)^4/((a*e^2+
c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/
e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*
a+3/4/e^5*c^6*d^7/(a*e^2+c*d^2)^4/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*
d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(
d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))-1/4/e^3*c*d/(a*e^2+c*d^2)^2/(d/e+
x)^4*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(5/2)-1/8/e*c^2*d/(a*e
^2+c*d^2)^3/(d/e+x)^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(5/2)
-3/16*c^5*d^4/(a*e^2+c*d^2)^5*a*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/
e^2)^(1/2)*x+9/16*c^4*d^2/(a*e^2+c*d^2)^4*a*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a
*e^2+c*d^2)/e^2)^(1/2)*x+3/8/e*c^3*d/(a*e^2+c*d^2)^3*(c*(d/e+x)^2-2*c*d/e*(
d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*a-1/4/e^2*c^2*d^2/(a*e^2+c*d^2)^3/(d/e+x)^3
(c(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(5/2)+3/8/e^2*c^5*d^4/(a*e
^2+c*d^2)^4*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x-3/4/e*c
^4*d^3/(a*e^2+c*d^2)^4*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2
)*a-1/8/e*c^3*d^3/(a*e^2+c*d^2)^4/(d/e+x)^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a
*e^2+c*d^2)/e^2)^(5/2)-9/16/e^2*c^(11/2)*d^6/(a*e^2+c*d^2)^5*ln((-c*d/e+(d/
e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))*a-9/
16/e^2*c^(7/2)*d^2/(a*e^2+c*d^2)^3*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)
^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))*a+9/8/e^2*c^(9/2)*d^4/(a*e^2+c
*d^2)^4*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c
*d^2)/e^2)^(1/2))*a-3/16/e^2*c^6*d^6/(a*e^2+c*d^2)^5*(c*(d/e+x)^2-2*c*d/e*(
d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x+3/8/e*c^5*d^5/(a*e^2+c*d^2)^5*(c*(d/e+x)^
2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*a-3/8/e^5*c^5*d^5/(a*e^2+c*d^2)^
3/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e
^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))
/(d/e+x))-3/16/e^2*c^4*d^2/(a*e^2+c*d^2)^3*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*
e^2+c*d^2)/e^2)^(1/2)*x-3/8/e^5*c^7*d^9/(a*e^2+c*d^2)^5/((a*e^2+c*d^2)/e^2)
^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*
(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 74.4061, size = 3316, normalized size = 17.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^6,x, algorithm="fricas")
```

```
[Out] [1/80*(15*(a^2*c^3*d*e^5*x^5 + 5*a^2*c^3*d^2*e^4*x^4 + 10*a^2*c^3*d^3*e^3*x
^3 + 10*a^2*c^3*d^4*e^2*x^2 + 5*a^2*c^3*d^5*e*x + a^2*c^3*d^6)*sqrt(c*d^2 +
 a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2
- 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x +
 d^2)) - 2*(33*a^2*c^3*d^6*e + 59*a^3*c^2*d^4*e^3 + 34*a^4*c*d^2*e^5 + 8*a^
5*e^7 - (2*c^5*d^6*e + 11*a*c^4*d^4*e^3 + a^2*c^3*d^2*e^5 - 8*a^3*c^2*e^7)*
x^4 - 5*(2*c^5*d^7 + 11*a*c^4*d^5*e^2 + 4*a^2*c^3*d^3*e^4 - 5*a^3*c^2*d*e^6
)*x^3 - (29*a*c^4*d^6*e - 48*a^2*c^3*d^4*e^3 - 93*a^3*c^2*d^2*e^5 - 16*a^4*
c*e^7)*x^2 - 5*(5*a*c^4*d^7 - 4*a^2*c^3*d^5*e^2 - 11*a^3*c^2*d^3*e^4 - 2*a^
4*c*d*e^6)*x)*sqrt(c*x^2 + a))/(c^4*d^13 + 4*a*c^3*d^11*e^2 + 6*a^2*c^2*d^9
*e^4 + 4*a^3*c*d^7*e^6 + a^4*d^5*e^8 + (c^4*d^8*e^5 + 4*a*c^3*d^6*e^7 + 6*a
^2*c^2*d^4*e^9 + 4*a^3*c*d^2*e^11 + a^4*e^13)*x^5 + 5*(c^4*d^9*e^4 + 4*a*c^
3*d^7*e^6 + 6*a^2*c^2*d^5*e^8 + 4*a^3*c*d^3*e^10 + a^4*d*e^12)*x^4 + 10*(c^
4*d^10*e^3 + 4*a*c^3*d^8*e^5 + 6*a^2*c^2*d^6*e^7 + 4*a^3*c*d^4*e^9 + a^4*d^
2*e^11)*x^3 + 10*(c^4*d^11*e^2 + 4*a*c^3*d^9*e^4 + 6*a^2*c^2*d^7*e^6 + 4*a^
3*c*d^5*e^8 + a^4*d^3*e^10)*x^2 + 5*(c^4*d^12*e + 4*a*c^3*d^10*e^3 + 6*a^2*
c^2*d^8*e^5 + 4*a^3*c*d^6*e^7 + a^4*d^4*e^9)*x), -1/40*(15*(a^2*c^3*d*e^5*x
^5 + 5*a^2*c^3*d^2*e^4*x^4 + 10*a^2*c^3*d^3*e^3*x^3 + 10*a^2*c^3*d^4*e^2*x^
2 + 5*a^2*c^3*d^5*e*x + a^2*c^3*d^6)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^
2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*
c*e^2)*x^2)) + (33*a^2*c^3*d^6*e + 59*a^3*c^2*d^4*e^3 + 34*a^4*c*d^2*e^5 +
8*a^5*e^7 - (2*c^5*d^6*e + 11*a*c^4*d^4*e^3 + a^2*c^3*d^2*e^5 - 8*a^3*c^2*e
^7)*x^4 - 5*(2*c^5*d^7 + 11*a*c^4*d^5*e^2 + 4*a^2*c^3*d^3*e^4 - 5*a^3*c^2*d
*e^6)*x^3 - (29*a*c^4*d^6*e - 48*a^2*c^3*d^4*e^3 - 93*a^3*c^2*d^2*e^5 - 16*
a^4*c*e^7)*x^2 - 5*(5*a*c^4*d^7 - 4*a^2*c^3*d^5*e^2 - 11*a^3*c^2*d^3*e^4 -
2*a^4*c*d*e^6)*x)*sqrt(c*x^2 + a))/(c^4*d^13 + 4*a*c^3*d^11*e^2 + 6*a^2*c^2
*d^9*e^4 + 4*a^3*c*d^7*e^6 + a^4*d^5*e^8 + (c^4*d^8*e^5 + 4*a*c^3*d^6*e^7 +
 6*a^2*c^2*d^4*e^9 + 4*a^3*c*d^2*e^11 + a^4*e^13)*x^5 + 5*(c^4*d^9*e^4 + 4*
a*c^3*d^7*e^6 + 6*a^2*c^2*d^5*e^8 + 4*a^3*c*d^3*e^10 + a^4*d*e^12)*x^4 + 10
*(c^4*d^10*e^3 + 4*a*c^3*d^8*e^5 + 6*a^2*c^2*d^6*e^7 + 4*a^3*c*d^4*e^9 + a^
4*d^2*e^11)*x^3 + 10*(c^4*d^11*e^2 + 4*a*c^3*d^9*e^4 + 6*a^2*c^2*d^7*e^6 +
4*a^3*c*d^5*e^8 + a^4*d^3*e^10)*x^2 + 5*(c^4*d^12*e + 4*a*c^3*d^10*e^3 + 6*
a^2*c^2*d^8*e^5 + 4*a^3*c*d^6*e^7 + a^4*d^4*e^9)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + cx^2)^{\frac{3}{2}}}{(d + ex)^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)/(e*x+d)**6,x)
```

```
[Out] Integral((a + c*x**2)**(3/2)/(d + e*x)**6, x)
```

Giac [B] time = 1.45803, size = 1682, normalized size = 8.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^6,x, algorithm="giac")
```

```
[Out] -3/4*a^2*c^3*d*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c
*d^2 - a*e^2))/((c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)*sqr
t(-c*d^2 - a*e^2)) + 1/20*(80*(sqrt(c)*x - sqrt(c*x^2 + a))^6*c^(13/2)*d^8*
e + 32*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^7*d^9 + 80*(sqrt(c)*x - sqrt(c*x^2
 + a))^7*c^6*d^7*e^2 + 40*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(11/2)*d^6*e^3
 - 80*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(13/2)*d^8*e - 16*(sqrt(c)*x - sqr
t(c*x^2 + a))^5*a*c^6*d^7*e^2 + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(11
/2)*d^6*e^3 + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^5*d^5*e^4 + 80*(sqrt(
c)*x - sqrt(c*x^2 + a))^3*a^2*c^6*d^7*e^2 + 120*(sqrt(c)*x - sqrt(c*x^2 + a
))^8*a*c^(9/2)*d^4*e^5 - 240*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(11/2)*d
^6*e^3 - 788*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^5*d^5*e^4 - 530*(sqrt(c)
*x - sqrt(c*x^2 + a))^6*a^2*c^(9/2)*d^4*e^5 - 40*(sqrt(c)*x - sqrt(c*x^2 +
a))^2*a^3*c^(11/2)*d^6*e^3 - 230*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^4*d^
3*e^6 + 400*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c^5*d^5*e^4 - 15*(sqrt(c)*x
 - sqrt(c*x^2 + a))^8*a^2*c^(7/2)*d^2*e^7 + 1170*(sqrt(c)*x - sqrt(c*x^2 +
a))^4*a^3*c^(9/2)*d^4*e^5 - 15*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^3*d*e^
8 + 910*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^4*d^3*e^6 + 20*(sqrt(c)*x - s
qrt(c*x^2 + a))*a^4*c^5*d^5*e^4 + 570*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c
^(7/2)*d^2*e^7 - 230*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c^(9/2)*d^4*e^5 +
150*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^3*d*e^8 - 770*(sqrt(c)*x - sqrt(c
*x^2 + a))^3*a^4*c^4*d^3*e^6 + 40*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(5/
2)*e^9 - 480*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(7/2)*d^2*e^7 - 2*a^5*c^
(9/2)*d^4*e^5 - 240*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^3*d*e^8 + 90*(sqr
t(c)*x - sqrt(c*x^2 + a))*a^5*c^4*d^3*e^6 + 350*(sqrt(c)*x - sqrt(c*x^2 + a
))^2*a^5*c^(7/2)*d^2*e^7 + 170*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^3*d*e^
8 + 80*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(5/2)*e^9 - 9*a^6*c^(7/2)*d^2*
e^7 - 65*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^3*d*e^8 + 8*a^7*c^(5/2)*e^9)/(
(c^3*d^6*e^4 + 3*a*c^2*d^4*e^6 + 3*a^2*c*d^2*e^8 + a^3*e^10)*((sqrt(c)*x -
sqrt(c*x^2 + a))^2*e + 2*(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*d - a*e)^5)
```

3.544 $\int \frac{(a+cx^2)^{3/2}}{(d+ex)^7}\,dx$

Optimal. Leaf size=269

$$-\frac{a^2c^3\left(6cd^2-ae^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{16\left(ae^2+cd^2\right)^{9/2}}-\frac{ac^2\sqrt{a+cx^2}\left(6cd^2-ae^2\right)(ae-cdx)}{16(d+ex)^2\left(ae^2+cd^2\right)^4}-\frac{7cde\left(a+cx^2\right)^{5/2}}{30(d+ex)^5\left(ae^2+cd^2\right)^2}-\frac{c\left(a+cx^2\right.}{24\cdots}\cdots$$

```
[Out] -(a*c^2*(6*c*d^2 - a*e^2)*(a*e - c*d*x)*Sqrt[a + c*x^2])/(16*(c*d^2 + a*e^2
)^4*(d + e*x)^2) - (c*(6*c*d^2 - a*e^2)*(a*e - c*d*x)*(a + c*x^2)^(3/2))/(2
4*(c*d^2 + a*e^2)^3*(d + e*x)^4) - (e*(a + c*x^2)^(5/2))/(6*(c*d^2 + a*e^2)
*(d + e*x)^6) - (7*c*d*e*(a + c*x^2)^(5/2))/(30*(c*d^2 + a*e^2)^2*(d + e*x)
^5) - (a^2*c^3*(6*c*d^2 - a*e^2)*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]
*Sqrt[a + c*x^2])])/(16*(c*d^2 + a*e^2)^(9/2))
```

Rubi [A] time = 0.183595, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {745, 807, 721, 725, 206}

$$-\frac{a^2c^3\left(6cd^2-ae^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{16\left(ae^2+cd^2\right)^{9/2}}-\frac{ac^2\sqrt{a+cx^2}\left(6cd^2-ae^2\right)(ae-cdx)}{16(d+ex)^2\left(ae^2+cd^2\right)^4}-\frac{7cde\left(a+cx^2\right)^{5/2}}{30(d+ex)^5\left(ae^2+cd^2\right)^2}-\frac{c\left(a+cx^2\right.}{24\cdots}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2)/(d + e*x)^7,x]
```


```
[Out] -(a*c^2*(6*c*d^2 - a*e^2)*(a*e - c*d*x)*Sqrt[a + c*x^2])/(16*(c*d^2 + a*e^2
)^4*(d + e*x)^2) - (c*(6*c*d^2 - a*e^2)*(a*e - c*d*x)*(a + c*x^2)^(3/2))/(2
4*(c*d^2 + a*e^2)^3*(d + e*x)^4) - (e*(a + c*x^2)^(5/2))/(6*(c*d^2 + a*e^2)
*(d + e*x)^6) - (7*c*d*e*(a + c*x^2)^(5/2))/(30*(c*d^2 + a*e^2)^2*(d + e*x)
^5) - (a^2*c^3*(6*c*d^2 - a*e^2)*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]
*Sqrt[a + c*x^2])])/(16*(c*d^2 + a*e^2)^(9/2))
```


Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 721

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(-2*a*e + (2*c*d)*x)*(a + c*x^2)^p)/(2*(m + 1)*(c*d^2 +
```

```
 a*e^2)), x] - Dist[(4*a*c*p)/(2*(m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m
 + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}}{(d+ex)^7}\,dx = -\frac{e\left(a+cx^2\right)^{5/2}}{6\left(cd^2+ae^2\right)(d+ex)^6} - \frac{c\int \frac{(-6d+ex)\left(a+cx^2\right)^{3/2}}{(d+ex)^6}\,dx}{6\left(cd^2+ae^2\right)}$$

$$= -\frac{e\left(a+cx^2\right)^{5/2}}{6\left(cd^2+ae^2\right)(d+ex)^6} - \frac{7cde\left(a+cx^2\right)^{5/2}}{30\left(cd^2+ae^2\right)^2(d+ex)^5} + \frac{\left(c\left(6cd^2-ae^2\right)\right)\int \frac{\left(a+cx^2\right)^{3/2}}{(d+ex)^5}\,dx}{6\left(cd^2+ae^2\right)^2}$$

$$= -\frac{c\left(6cd^2-ae^2\right)(ae-cdx)\left(a+cx^2\right)^{3/2}}{24\left(cd^2+ae^2\right)^3(d+ex)^4} - \frac{e\left(a+cx^2\right)^{5/2}}{6\left(cd^2+ae^2\right)(d+ex)^6} - \frac{7cde\left(a+cx^2\right)^{5/2}}{30\left(cd^2+ae^2\right)^2(d+ex)^5} + \ldots$$

$$= -\frac{ac^2\left(6cd^2-ae^2\right)(ae-cdx)\sqrt{a+cx^2}}{16\left(cd^2+ae^2\right)^4(d+ex)^2} - \frac{c\left(6cd^2-ae^2\right)(ae-cdx)\left(a+cx^2\right)^{3/2}}{24\left(cd^2+ae^2\right)^3(d+ex)^4} - \frac{e\left(a+cx^2\right)^{5/2}}{6\left(cd^2+ae^2\right)(d+\ldots}$$

$$= -\frac{ac^2\left(6cd^2-ae^2\right)(ae-cdx)\sqrt{a+cx^2}}{16\left(cd^2+ae^2\right)^4(d+ex)^2} - \frac{c\left(6cd^2-ae^2\right)(ae-cdx)\left(a+cx^2\right)^{3/2}}{24\left(cd^2+ae^2\right)^3(d+ex)^4} - \frac{e\left(a+cx^2\right)^{5/2}}{6\left(cd^2+ae^2\right)(d+\ldots}$$

$$= -\frac{ac^2\left(6cd^2-ae^2\right)(ae-cdx)\sqrt{a+cx^2}}{16\left(cd^2+ae^2\right)^4(d+ex)^2} - \frac{c\left(6cd^2-ae^2\right)(ae-cdx)\left(a+cx^2\right)^{3/2}}{24\left(cd^2+ae^2\right)^3(d+ex)^4} - \frac{e\left(a+cx^2\right)^{5/2}}{6\left(cd^2+ae^2\right)(d+\ldots}$$

Mathematica [A] time = 0.649625, size = 358, normalized size = 1.33

$$\frac{1}{240}\left(-\frac{\sqrt{a+cx^2}\left(-c^2(d+ex)^4\left(-15a^2e^4+24acd^2e^2+4c^2d^4\right)\left(ae^2+cd^2\right)-c^3d(d+ex)^5\left(-81a^2e^4+28acd^2e^2+4c^2d^4\right)\ldots}{\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2)/(d + e*x)^7,x]
```

```
[Out] (-((Sqrt[a + c*x^2]*(40*(c*d^2 + a*e^2)^5 - 104*c*d*(c*d^2 + a*e^2)^4*(d +
e*x) + 2*c*(c*d^2 + a*e^2)^3*(38*c*d^2 + 35*a*e^2)*(d + e*x)^2 - 2*c^2*d*(c
*d^2 + a*e^2)^2*(2*c*d^2 + 9*a*e^2)*(d + e*x)^3 - c^2*(c*d^2 + a*e^2)*(4*c^
2*d^4 + 24*a*c*d^2*e^2 - 15*a^2*e^4)*(d + e*x)^4 - c^3*d*(4*c^2*d^4 + 28*a*
c*d^2*e^2 - 81*a^2*e^4)*(d + e*x)^5))/(e^3*(c*d^2 + a*e^2)^4*(d + e*x)^6))
+ (15*a^2*c^3*(6*c*d^2 - a*e^2)*Log[d + e*x])/(c*d^2 + a*e^2)^(9/2) + (15*a
```

```
^2*c^3*(-6*c*d^2 + a*e^2)*Log[a*e - c*d*x + Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*
x^2]])/(c*d^2 + a*e^2)^(9/2))/240
```

Maple [B] time = 0.26, size = 5087, normalized size = 18.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)/(e*x+d)^7,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 157.252, size = 5085, normalized size = 18.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^7,x, algorithm="fricas")
```

```
[Out] [-1/480*(15*(6*a^2*c^4*d^8 - a^3*c^3*d^6*e^2 + (6*a^2*c^4*d^2*e^6 - a^3*c^3
*e^8)*x^6 + 6*(6*a^2*c^4*d^3*e^5 - a^3*c^3*d*e^7)*x^5 + 15*(6*a^2*c^4*d^4*e
^4 - a^3*c^3*d^2*e^6)*x^4 + 20*(6*a^2*c^4*d^5*e^3 - a^3*c^3*d^3*e^5)*x^3 +
15*(6*a^2*c^4*d^6*e^2 - a^3*c^3*d^4*e^4)*x^2 + 6*(6*a^2*c^4*d^7*e - a^3*c^3
*d^5*e^3)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (
2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 +
 a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(246*a^2*c^4*d^8*e + 513*a^3*c^3*d^6*e^
3 + 433*a^4*c^2*d^4*e^5 + 206*a^5*c*d^2*e^7 + 40*a^6*e^9 - (4*c^6*d^7*e^2 +
 32*a*c^5*d^5*e^4 - 53*a^2*c^4*d^3*e^6 - 81*a^3*c^3*d*e^8)*x^5 - 3*(8*c^6*d
^8*e + 64*a*c^5*d^6*e^3 - 76*a^2*c^4*d^4*e^5 - 137*a^3*c^3*d^2*e^7 - 5*a^4*
c^2*e^9)*x^4 - 2*(30*c^6*d^9 + 239*a*c^5*d^7*e^2 - 158*a^2*c^4*d^5*e^4 - 38
8*a^3*c^3*d^3*e^6 - 21*a^4*c^2*d*e^8)*x^3 - 2*(114*a*c^5*d^8*e - 423*a^2*c^
4*d^6*e^3 - 698*a^3*c^3*d^4*e^5 - 196*a^4*c^2*d^2*e^7 - 35*a^5*c*e^9)*x^2 -
 3*(50*a*c^5*d^9 - 117*a^2*c^4*d^7*e^2 - 221*a^3*c^3*d^5*e^4 - 66*a^4*c^2*d
^3*e^6 - 12*a^5*c*d*e^8)*x)*sqrt(c*x^2 + a))/(c^5*d^16 + 5*a*c^4*d^14*e^2 +
 10*a^2*c^3*d^12*e^4 + 10*a^3*c^2*d^10*e^6 + 5*a^4*c*d^8*e^8 + a^5*d^6*e^10
 + (c^5*d^10*e^6 + 5*a*c^4*d^8*e^8 + 10*a^2*c^3*d^6*e^10 + 10*a^3*c^2*d^4*e
^12 + 5*a^4*c*d^2*e^14 + a^5*e^16)*x^6 + 6*(c^5*d^11*e^5 + 5*a*c^4*d^9*e^7
+ 10*a^2*c^3*d^7*e^9 + 10*a^3*c^2*d^5*e^11 + 5*a^4*c*d^3*e^13 + a^5*d*e^15)
*x^5 + 15*(c^5*d^12*e^4 + 5*a*c^4*d^10*e^6 + 10*a^2*c^3*d^8*e^8 + 10*a^3*c^
2*d^6*e^10 + 5*a^4*c*d^4*e^12 + a^5*d^2*e^14)*x^4 + 20*(c^5*d^13*e^3 + 5*a*
```

```
c^4*d^11*e^5 + 10*a^2*c^3*d^9*e^7 + 10*a^3*c^2*d^7*e^9 + 5*a^4*c*d^5*e^11 +
 a^5*d^3*e^13)*x^3 + 15*(c^5*d^14*e^2 + 5*a*c^4*d^12*e^4 + 10*a^2*c^3*d^10*
e^6 + 10*a^3*c^2*d^8*e^8 + 5*a^4*c*d^6*e^10 + a^5*d^4*e^12)*x^2 + 6*(c^5*d^
15*e + 5*a*c^4*d^13*e^3 + 10*a^2*c^3*d^11*e^5 + 10*a^3*c^2*d^9*e^7 + 5*a^4*
c*d^7*e^9 + a^5*d^5*e^11)*x), -1/240*(15*(6*a^2*c^4*d^8 - a^3*c^3*d^6*e^2 +
 (6*a^2*c^4*d^2*e^6 - a^3*c^3*e^8)*x^6 + 6*(6*a^2*c^4*d^3*e^5 - a^3*c^3*d*e
^7)*x^5 + 15*(6*a^2*c^4*d^4*e^4 - a^3*c^3*d^2*e^6)*x^4 + 20*(6*a^2*c^4*d^5*
e^3 - a^3*c^3*d^3*e^5)*x^3 + 15*(6*a^2*c^4*d^6*e^2 - a^3*c^3*d^4*e^4)*x^2 +
 6*(6*a^2*c^4*d^7*e - a^3*c^3*d^5*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(
-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2
 + a*c*e^2)*x^2)) + (246*a^2*c^4*d^8*e + 513*a^3*c^3*d^6*e^3 + 433*a^4*c^2*
d^4*e^5 + 206*a^5*c*d^2*e^7 + 40*a^6*e^9 - (4*c^6*d^7*e^2 + 32*a*c^5*d^5*e^
4 - 53*a^2*c^4*d^3*e^6 - 81*a^3*c^3*d*e^8)*x^5 - 3*(8*c^6*d^8*e + 64*a*c^5*
d^6*e^3 - 76*a^2*c^4*d^4*e^5 - 137*a^3*c^3*d^2*e^7 - 5*a^4*c^2*e^9)*x^4 - 2
*(30*c^6*d^9 + 239*a*c^5*d^7*e^2 - 158*a^2*c^4*d^5*e^4 - 388*a^3*c^3*d^3*e^
6 - 21*a^4*c^2*d*e^8)*x^3 - 2*(114*a*c^5*d^8*e - 423*a^2*c^4*d^6*e^3 - 698*
a^3*c^3*d^4*e^5 - 196*a^4*c^2*d^2*e^7 - 35*a^5*c*e^9)*x^2 - 3*(50*a*c^5*d^9
 - 117*a^2*c^4*d^7*e^2 - 221*a^3*c^3*d^5*e^4 - 66*a^4*c^2*d^3*e^6 - 12*a^5*
c*d*e^8)*x)*sqrt(c*x^2 + a))/(c^5*d^16 + 5*a*c^4*d^14*e^2 + 10*a^2*c^3*d^12
*e^4 + 10*a^3*c^2*d^10*e^6 + 5*a^4*c*d^8*e^8 + a^5*d^6*e^10 + (c^5*d^10*e^6
 + 5*a*c^4*d^8*e^8 + 10*a^2*c^3*d^6*e^10 + 10*a^3*c^2*d^4*e^12 + 5*a^4*c*d^
2*e^14 + a^5*e^16)*x^6 + 6*(c^5*d^11*e^5 + 5*a*c^4*d^9*e^7 + 10*a^2*c^3*d^7
*e^9 + 10*a^3*c^2*d^5*e^11 + 5*a^4*c*d^3*e^13 + a^5*d*e^15)*x^5 + 15*(c^5*d
^12*e^4 + 5*a*c^4*d^10*e^6 + 10*a^2*c^3*d^8*e^8 + 10*a^3*c^2*d^6*e^10 + 5*a
^4*c*d^4*e^12 + a^5*d^2*e^14)*x^4 + 20*(c^5*d^13*e^3 + 5*a*c^4*d^11*e^5 + 1
0*a^2*c^3*d^9*e^7 + 10*a^3*c^2*d^7*e^9 + 5*a^4*c*d^5*e^11 + a^5*d^3*e^13)*x
^3 + 15*(c^5*d^14*e^2 + 5*a*c^4*d^12*e^4 + 10*a^2*c^3*d^10*e^6 + 10*a^3*c^2
*d^8*e^8 + 5*a^4*c*d^6*e^10 + a^5*d^4*e^12)*x^2 + 6*(c^5*d^15*e + 5*a*c^4*d
^13*e^3 + 10*a^2*c^3*d^11*e^5 + 10*a^3*c^2*d^9*e^7 + 5*a^4*c*d^7*e^9 + a^5*
d^5*e^11)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + cx^2\right)^{\frac{3}{2}}}{\left(d + ex\right)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)/(e*x+d)**7,x)
```

```
[Out] Integral((a + c*x**2)**(3/2)/(d + e*x)**7, x)
```

Giac [B] time = 1.66162, size = 2454, normalized size = 9.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^7,x, algorithm="giac")
```

```
[Out] -1/8*(6*a^2*c^4*d^2 - a^3*c^3*e^2)*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e
+ sqrt(c)*d)/sqrt(-c*d^2 - a*e^2))/((c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*
d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)*sqrt(-c*d^2 - a*e^2)) + 1/120*(384*(sq
rt(c)*x - sqrt(c*x^2 + a))^7*c^8*d^10*e + 128*(sqrt(c)*x - sqrt(c*x^2 + a))
```

^6*c^(17/2)*d^11 + 480*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(15/2)*d^9*e^2 + 320*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^7*d^8*e^3 - 384*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^8*d^10*e - 64*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(15/2)*d^9*e^2 + 1728*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^7*d^8*e^3 + 1920*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(13/2)*d^7*e^4 + 480*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(15/2)*d^9*e^2 + 1280*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^6*d^6*e^5 - 1728*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^7*d^8*e^3 - 8592*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(13/2)*d^7*e^4 - 9456*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^6*d^6*e^5 - 320*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c^7*d^8*e^3 - 7380*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(11/2)*d^5*e^6 + 3840*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(13/2)*d^7*e^4 - 2520*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^5*d^4*e^7 + 19056*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^6*d^6*e^5 - 990*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^2*c^(9/2)*d^3*e^8 + 24440*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(11/2)*d^5*e^6 + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c^(13/2)*d^7*e^4 - 90*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^2*c^4*d^2*e^9 + 20760*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^5*d^4*e^7 - 2960*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^6*d^6*e^5 + 8220*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(9/2)*d^3*e^8 - 18720*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(11/2)*d^5*e^6 + 2530*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*c^4*d^2*e^9 - 21480*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^5*d^4*e^7 - 48*(sqrt(c)*x - sqrt(c*x^2 + a))*a^5*c^6*d^6*e^5 + 165*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^3*c^(7/2)*d*e^10 - 14860*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(9/2)*d^3*e^8 + 1656*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(11/2)*d^5*e^6 + 15*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^3*c^3*e^11 - 2700*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4*c^4*d^2*e^9 + 12120*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^5*d^4*e^7 - 285*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^4*c^(7/2)*d*e^10 + 11640*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(9/2)*d^3*e^8 + 4*a^6*c^(11/2)*d^5*e^6 + 235*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^4*c^3*e^11 + 7020*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^5*c^4*d^2*e^9 - 336*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^5*d^4*e^7 + 810*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^5*c^(7/2)*d*e^10 - 4038*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^6*c^(9/2)*d^3*e^8 + 390*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^5*c^3*e^11 - 2330*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^4*d^2*e^9 - 930*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(7/2)*d*e^10 + 28*a^7*c^(9/2)*d^3*e^8 + 390*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^6*c^3*e^11 + 882*(sqrt(c)*x - sqrt(c*x^2 + a))*a^7*c^4*d^2*e^9 + 321*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^7*c^(7/2)*d*e^10 + 235*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^7*c^3*e^11 - 81*a^8*c^(7/2)*d*e^10 + 15*(sqrt(c)*x - sqrt(c*x^2 + a))*a^8*c^3*e^11)/((c^4*d^8*e^4 + 4*a*c^3*d^6*e^6 + 6*a^2*c^2*d^4*e^8 + 4*a^3*c*d^2*e^10 + a^4*e^12)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*e + 2*(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*d - a*e)^6)

3.545 $\int (d+ex)^4 \left(a+cx^2\right)^{5/2} dx$

Optimal. Leaf size=307

$$\frac{x\left(a+cx^2\right)^{5/2}\left(3a^2e^4-60acd^2e^2+80c^2d^4\right)}{480c^2}+\frac{ax\left(a+cx^2\right)^{3/2}\left(3a^2e^4-60acd^2e^2+80c^2d^4\right)}{384c^2}+\frac{a^2x\sqrt{a+cx^2}\left(3a^2e^4-60a\right.}{256c^2}$$

```
[Out] (a^2*(80*c^2*d^4 - 60*a*c*d^2*e^2 + 3*a^2*e^4)*x*Sqrt[a + c*x^2])/(256*c^2)
 + (a*(80*c^2*d^4 - 60*a*c*d^2*e^2 + 3*a^2*e^4)*x*(a + c*x^2)^(3/2))/(384*c
^2) + ((80*c^2*d^4 - 60*a*c*d^2*e^2 + 3*a^2*e^4)*x*(a + c*x^2)^(5/2))/(480*
c^2) + (13*d*e*(d + e*x)^2*(a + c*x^2)^(7/2))/(90*c) + (e*(d + e*x)^3*(a +
c*x^2)^(7/2))/(10*c) + (e*(16*d*(103*c*d^2 - 40*a*e^2) + 7*e*(116*c*d^2 - 2
7*a*e^2)*x)*(a + c*x^2)^(7/2))/(5040*c^2) + (a^3*(80*c^2*d^4 - 60*a*c*d^2*e
^2 + 3*a^2*e^4)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(256*c^(5/2))
```

Rubi [A] time = 0.293323, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {743, 833, 780, 195, 217, 206}

$$\frac{x\left(a+cx^2\right)^{5/2}\left(3a^2e^4-60acd^2e^2+80c^2d^4\right)}{480c^2}+\frac{ax\left(a+cx^2\right)^{3/2}\left(3a^2e^4-60acd^2e^2+80c^2d^4\right)}{384c^2}+\frac{a^2x\sqrt{a+cx^2}\left(3a^2e^4-60a\right.}{256c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4*(a + c*x^2)^(5/2),x]
```

```
[Out] (a^2*(80*c^2*d^4 - 60*a*c*d^2*e^2 + 3*a^2*e^4)*x*Sqrt[a + c*x^2])/(256*c^2)
 + (a*(80*c^2*d^4 - 60*a*c*d^2*e^2 + 3*a^2*e^4)*x*(a + c*x^2)^(3/2))/(384*c
^2) + ((80*c^2*d^4 - 60*a*c*d^2*e^2 + 3*a^2*e^4)*x*(a + c*x^2)^(5/2))/(480*
c^2) + (13*d*e*(d + e*x)^2*(a + c*x^2)^(7/2))/(90*c) + (e*(d + e*x)^3*(a +
c*x^2)^(7/2))/(10*c) + (e*(16*d*(103*c*d^2 - 40*a*e^2) + 7*e*(116*c*d^2 - 2
7*a*e^2)*x)*(a + c*x^2)^(7/2))/(5040*c^2) + (a^3*(80*c^2*d^4 - 60*a*c*d^2*e
^2 + 3*a^2*e^4)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(256*c^(5/2))
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^4\left(a+cx^2\right)^{5/2}dx=\frac{e(d+ex)^3\left(a+cx^2\right)^{7/2}}{10c}+\frac{\int (d+ex)^2\left(10cd^2-3ae^2+13cdex\right)\left(a+cx^2\right)^{5/2}dx}{10c}$$

$$=\frac{13de(d+ex)^2\left(a+cx^2\right)^{7/2}}{90c}+\frac{e(d+ex)^3\left(a+cx^2\right)^{7/2}}{10c}+\frac{\int (d+ex)\left(cd\left(90cd^2-53ae^2\right)+ce\left(1\ldots\right.\right.}{90c\ldots}$$

$$=\frac{13de(d+ex)^2\left(a+cx^2\right)^{7/2}}{90c}+\frac{e(d+ex)^3\left(a+cx^2\right)^{7/2}}{10c}+\frac{e\left(16d\left(103cd^2-40ae^2\right)+7e\left(116cd^2\ldots\right.\right.}{5040c^2}$$

$$=\frac{\left(80c^2d^4-60acd^2e^2+3a^2e^4\right)x\left(a+cx^2\right)^{5/2}}{480c^2}+\frac{13de(d+ex)^2\left(a+cx^2\right)^{7/2}}{90c}+\frac{e(d+ex)^3\left(a+c\ldots\right.}{10c}$$

$$=\frac{a\left(80c^2d^4-60acd^2e^2+3a^2e^4\right)x\left(a+cx^2\right)^{3/2}}{384c^2}+\frac{\left(80c^2d^4-60acd^2e^2+3a^2e^4\right)x\left(a+cx^2\right)^{5/2}}{480c^2}\ldots$$

$$=\frac{a^2\left(80c^2d^4-60acd^2e^2+3a^2e^4\right)x\sqrt{a+cx^2}}{256c^2}+\frac{a\left(80c^2d^4-60acd^2e^2+3a^2e^4\right)x\left(a+cx^2\right)^{3/2}}{384c^2}\ldots$$

$$=\frac{a^2\left(80c^2d^4-60acd^2e^2+3a^2e^4\right)x\sqrt{a+cx^2}}{256c^2}+\frac{a\left(80c^2d^4-60acd^2e^2+3a^2e^4\right)x\left(a+cx^2\right)^{3/2}}{384c^2}\ldots$$

$$=\frac{a^2\left(80c^2d^4-60acd^2e^2+3a^2e^4\right)x\sqrt{a+cx^2}}{256c^2}+\frac{a\left(80c^2d^4-60acd^2e^2+3a^2e^4\right)x\left(a+cx^2\right)^{3/2}}{384c^2}\ldots$$

Mathematica [A] time = 0.251842, size = 283, normalized size = 0.92

$$\frac{\sqrt{a+cx^2}\left(24a^2c^2x\left(6195d^2e^2x^2+5760d^3ex+2310d^4+3200de^3x^3+651e^4x^4\right)+10a^3ce\left(1890d^2ex+4608d^3+512de^2x^2\ldots\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4*(a + c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(-5*a^4*e^3*(2048*d + 189*e*x) + 10*a^3*c*e*(4608*d^3 + 18
90*d^2*e*x + 512*d*e^2*x^2 + 63*e^3*x^3) + 64*c^4*x^5*(210*d^4 + 720*d^3*e*
x + 945*d^2*e^2*x^2 + 560*d*e^3*x^3 + 126*e^4*x^4) + 24*a^2*c^2*x*(2310*d^4
 + 5760*d^3*e*x + 6195*d^2*e^2*x^2 + 3200*d*e^3*x^3 + 651*e^4*x^4) + 16*a*c
^3*x^3*(2730*d^4 + 8640*d^3*e*x + 10710*d^2*e^2*x^2 + 6080*d*e^3*x^3 + 1323
*e^4*x^4)))/(80640*c^2) + (a^3*(80*c^2*d^4 - 60*a*c*d^2*e^2 + 3*a^2*e^4)*Lo
g[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/(256*c^(5/2))
```

Maple [A] time = 0.054, size = 386, normalized size = 1.3

$$\frac{e^4x^3}{10c}\left(cx^2+a\right)^{\frac{7}{2}}-\frac{3\,e^4ax}{80\,c^2}\left(cx^2+a\right)^{\frac{7}{2}}+\frac{a^2e^4x}{160\,c^2}\left(cx^2+a\right)^{\frac{5}{2}}+\frac{e^4a^3x}{128\,c^2}\left(cx^2+a\right)^{\frac{3}{2}}+\frac{3\,e^4a^4x}{256\,c^2}\sqrt{cx^2+a}+\frac{3\,e^4a^5}{256}\ln\left(x\sqrt{c}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(c*x^2+a)^(5/2),x)
```

```
[Out] 1/10*e^4*x^3*(c*x^2+a)^(7/2)/c-3/80*e^4*a/c^2*x*(c*x^2+a)^(7/2)+1/160*e^4*a
^2/c^2*x*(c*x^2+a)^(5/2)+1/128*e^4*a^3/c^2*x*(c*x^2+a)^(3/2)+3/256*e^4*a^4/
c^2*x*(c*x^2+a)^(1/2)+3/256*e^4*a^5/c^(5/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))+4
/9*d*e^3*x^2*(c*x^2+a)^(7/2)/c-8/63*d*e^3*a/c^2*(c*x^2+a)^(7/2)+3/4*d^2*e^2
*x*(c*x^2+a)^(7/2)/c-1/8*d^2*e^2*a/c*x*(c*x^2+a)^(5/2)-5/32*d^2*e^2*a^2/c*x
*(c*x^2+a)^(3/2)-15/64*d^2*e^2*a^3/c*x*(c*x^2+a)^(1/2)-15/64*d^2*e^2*a^4/c^
(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))+4/7*d^3*e*(c*x^2+a)^(7/2)/c+1/6*d^4*x*(
c*x^2+a)^(5/2)+5/24*d^4*a*x*(c*x^2+a)^(3/2)+5/16*d^4*a^2*x*(c*x^2+a)^(1/2)+
5/16*d^4*a^3/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.97832, size = 1577, normalized size = 5.14

$$\left[\frac{315\left(80\,a^3c^2d^4-60\,a^4cd^2e^2+3\,a^5e^4\right)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)+2\left(8064\,c^5e^4x^9+35840\,c^5de^3x^8+4608\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/161280*(315*(80*a^3*c^2*d^4 - 60*a^4*c*d^2*e^2 + 3*a^5*e^4)*sqrt(c)*log(
-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(8064*c^5*e^4*x^9 + 35840*c
^5*d*e^3*x^8 + 46080*a^3*c^2*d^3*e - 10240*a^4*c*d*e^3 + 3024*(20*c^5*d^2*e
^2 + 7*a*c^4*e^4)*x^7 + 5120*(9*c^5*d^3*e + 19*a*c^4*d*e^3)*x^6 + 168*(80*c
```

```
^5*d^4 + 1020*a*c^4*d^2*e^2 + 93*a^2*c^3*e^4)*x^5 + 15360*(9*a*c^4*d^3*e +
5*a^2*c^3*d*e^3)*x^4 + 210*(208*a*c^4*d^4 + 708*a^2*c^3*d^2*e^2 + 3*a^3*c^2
*e^4)*x^3 + 5120*(27*a^2*c^3*d^3*e + a^3*c^2*d*e^3)*x^2 + 315*(176*a^2*c^3*
d^4 + 60*a^3*c^2*d^2*e^2 - 3*a^4*c*e^4)*x)*sqrt(c*x^2 + a))/c^3, -1/80640*(
315*(80*a^3*c^2*d^4 - 60*a^4*c*d^2*e^2 + 3*a^5*e^4)*sqrt(-c)*arctan(sqrt(-c
)*x/sqrt(c*x^2 + a)) - (8064*c^5*e^4*x^9 + 35840*c^5*d*e^3*x^8 + 46080*a^3*
c^2*d^3*e - 10240*a^4*c*d*e^3 + 3024*(20*c^5*d^2*e^2 + 7*a*c^4*e^4)*x^7 + 5
120*(9*c^5*d^3*e + 19*a*c^4*d*e^3)*x^6 + 168*(80*c^5*d^4 + 1020*a*c^4*d^2*e
^2 + 93*a^2*c^3*e^4)*x^5 + 15360*(9*a*c^4*d^3*e + 5*a^2*c^3*d*e^3)*x^4 + 21
0*(208*a*c^4*d^4 + 708*a^2*c^3*d^2*e^2 + 3*a^3*c^2*e^4)*x^3 + 5120*(27*a^2*
c^3*d^3*e + a^3*c^2*d*e^3)*x^2 + 315*(176*a^2*c^3*d^4 + 60*a^3*c^2*d^2*e^2
- 3*a^4*c*e^4)*x)*sqrt(c*x^2 + a))/c^3]
```

Sympy [A] time = 75.149, size = 1062, normalized size = 3.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(c*x**2+a)**(5/2),x)
```

```
[Out] -3*a**(9/2)*e**4*x/(256*c**2*sqrt(1 + c*x**2/a)) + 15*a**(7/2)*d**2*e**2*x/
(64*c*sqrt(1 + c*x**2/a)) - a**(7/2)*e**4*x**3/(256*c*sqrt(1 + c*x**2/a)) +
 a**(5/2)*d**4*x*sqrt(1 + c*x**2/a)/2 + 3*a**(5/2)*d**4*x/(16*sqrt(1 + c*x*
*2/a)) + 133*a**(5/2)*d**2*e**2*x**3/(64*sqrt(1 + c*x**2/a)) + 129*a**(5/2)
*e**4*x**5/(640*sqrt(1 + c*x**2/a)) + 35*a**(3/2)*c*d**4*x**3/(48*sqrt(1 +
c*x**2/a)) + 127*a**(3/2)*c*d**2*e**2*x**5/(32*sqrt(1 + c*x**2/a)) + 73*a**
(3/2)*c*e**4*x**7/(160*sqrt(1 + c*x**2/a)) + 17*sqrt(a)*c**2*d**4*x**5/(24*
sqrt(1 + c*x**2/a)) + 23*sqrt(a)*c**2*d**2*e**2*x**7/(8*sqrt(1 + c*x**2/a))
 + 29*sqrt(a)*c**2*e**4*x**9/(80*sqrt(1 + c*x**2/a)) + 3*a**5*e**4*asinh(sq
rt(c)*x/sqrt(a))/(256*c**(5/2)) - 15*a**4*d**2*e**2*asinh(sqrt(c)*x/sqrt(a)
)/(64*c**(3/2)) + 5*a**3*d**4*asinh(sqrt(c)*x/sqrt(a))/(16*sqrt(c)) + 4*a**
2*d**3*e*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c),
True)) + 4*a**2*d*e**3*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x*
*2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x
**4/4, True)) + 8*a*c*d**3*e*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2)
+ a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqr
t(a)*x**4/4, True)) + 8*a*c*d*e**3*Piecewise((8*a**3*sqrt(a + c*x**2)/(105*
c**3) - 4*a**2*x**2*sqrt(a + c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x**2)/(
35*c) + x**6*sqrt(a + c*x**2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) + 4*c**
2*d**3*e*Piecewise((8*a**3*sqrt(a + c*x**2)/(105*c**3) - 4*a**2*x**2*sqrt(a
 + c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x**2)/(35*c) + x**6*sqrt(a + c*x*
*2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) + 4*c**2*d*e**3*Piecewise((-16*a*
*4*sqrt(a + c*x**2)/(315*c**4) + 8*a**3*x**2*sqrt(a + c*x**2)/(315*c**3) -
2*a**2*x**4*sqrt(a + c*x**2)/(105*c**2) + a*x**6*sqrt(a + c*x**2)/(63*c) +
x**8*sqrt(a + c*x**2)/9, Ne(c, 0)), (sqrt(a)*x**8/8, True)) + c**3*d**4*x**
7/(6*sqrt(a)*sqrt(1 + c*x**2/a)) + 3*c**3*d**2*e**2*x**9/(4*sqrt(a)*sqrt(1
+ c*x**2/a)) + c**3*e**4*x**11/(10*sqrt(a)*sqrt(1 + c*x**2/a))
```

Giac [A] time = 1.38486, size = 486, normalized size = 1.58

$$\frac{1}{80640}\sqrt{cx^2+a}\left(\left(2\left(\left(4\left(\left(2\left(7\left(8\left(9\,c^2xe^4+40\,c^2de^3\right)x+\frac{27\left(20\,c^{10}d^2e^2+7\,ac^9e^4\right)}{c^8}\right)x+\frac{320\left(9\,c^{10}d^3e+19\,ac^9de^3\right)}{c^8}\right)x+\frac{21}{}\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/80640*sqrt(c*x^2 + a)*((2*((4*((2*(7*(8*(9*c^2*x*e^4 + 40*c^2*d*e^3)*x +
27*(20*c^10*d^2*e^2 + 7*a*c^9*e^4)/c^8)*x + 320*(9*c^10*d^3*e + 19*a*c^9*d*
e^3)/c^8)*x + 21*(80*c^10*d^4 + 1020*a*c^9*d^2*e^2 + 93*a^2*c^8*e^4)/c^8)*x
 + 1920*(9*a*c^9*d^3*e + 5*a^2*c^8*d*e^3)/c^8)*x + 105*(208*a*c^9*d^4 + 708
*a^2*c^8*d^2*e^2 + 3*a^3*c^7*e^4)/c^8)*x + 2560*(27*a^2*c^8*d^3*e + a^3*c^7
*d*e^3)/c^8)*x + 315*(176*a^2*c^8*d^4 + 60*a^3*c^7*d^2*e^2 - 3*a^4*c^6*e^4)
/c^8)*x + 5120*(9*a^3*c^7*d^3*e - 2*a^4*c^6*d*e^3)/c^8) - 1/256*(80*a^3*c^2
*d^4 - 60*a^4*c*d^2*e^2 + 3*a^5*e^4)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))
/c^(5/2)
```

3.546 $\int (d+ex)^3 \left(a+cx^2\right)^{5/2} dx$

Optimal. Leaf size=216

$$\frac{5a^3d\left(8cd^2-3ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{128c^{3/2}}+\frac{5a^2dx\sqrt{a+cx^2}\left(8cd^2-3ae^2\right)}{128c}+\frac{e\left(a+cx^2\right)^{7/2}\left(16\left(10cd^2-ae^2\right)+77cdex\right)}{504c^2}+\frac{dx\left(a\right.}{}$$

```
[Out] (5*a^2*d*(8*c*d^2 - 3*a*e^2)*x*Sqrt[a + c*x^2])/(128*c) + (5*a*d*(8*c*d^2 -
 3*a*e^2)*x*(a + c*x^2)^(3/2))/(192*c) + (d*(8*c*d^2 - 3*a*e^2)*x*(a + c*x^
2)^(5/2))/(48*c) + (e*(d + e*x)^2*(a + c*x^2)^(7/2))/(9*c) + (e*(16*(10*c*d
^2 - a*e^2) + 77*c*d*e*x)*(a + c*x^2)^(7/2))/(504*c^2) + (5*a^3*d*(8*c*d^2
- 3*a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(128*c^(3/2))
```

Rubi [A] time = 0.163836, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {743, 780, 195, 217, 206}

$$\frac{5a^3d\left(8cd^2-3ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{128c^{3/2}}+\frac{5a^2dx\sqrt{a+cx^2}\left(8cd^2-3ae^2\right)}{128c}+\frac{e\left(a+cx^2\right)^{7/2}\left(16\left(10cd^2-ae^2\right)+77cdex\right)}{504c^2}+\frac{dx\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + c*x^2)^(5/2),x]
```

```
[Out] (5*a^2*d*(8*c*d^2 - 3*a*e^2)*x*Sqrt[a + c*x^2])/(128*c) + (5*a*d*(8*c*d^2 -
 3*a*e^2)*x*(a + c*x^2)^(3/2))/(192*c) + (d*(8*c*d^2 - 3*a*e^2)*x*(a + c*x^
2)^(5/2))/(48*c) + (e*(d + e*x)^2*(a + c*x^2)^(7/2))/(9*c) + (e*(16*(10*c*d
^2 - a*e^2) + 77*c*d*e*x)*(a + c*x^2)^(7/2))/(504*c^2) + (5*a^3*d*(8*c*d^2
- 3*a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(128*c^(3/2))
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^3\left(a+cx^2\right)^{5/2}\,dx=\frac{e(d+ex)^2\left(a+cx^2\right)^{7/2}}{9c}+\frac{\int (d+ex)\left(9cd^2-2ae^2+11cdex\right)\left(a+cx^2\right)^{5/2}\,dx}{9c}$$

$$=\frac{e(d+ex)^2\left(a+cx^2\right)^{7/2}}{9c}+\frac{e\left(16\left(10cd^2-ae^2\right)+77cdex\right)\left(a+cx^2\right)^{7/2}}{504c^2}+\frac{\left(d\left(8cd^2-3ae^2\right)\right)}{8}$$

$$=\frac{d\left(8cd^2-3ae^2\right)x\left(a+cx^2\right)^{5/2}}{48c}+\frac{e(d+ex)^2\left(a+cx^2\right)^{7/2}}{9c}+\frac{e\left(16\left(10cd^2-ae^2\right)+77cdex\right)}{504c^2}$$

$$=\frac{5ad\left(8cd^2-3ae^2\right)x\left(a+cx^2\right)^{3/2}}{192c}+\frac{d\left(8cd^2-3ae^2\right)x\left(a+cx^2\right)^{5/2}}{48c}+\frac{e(d+ex)^2\left(a+cx^2\right)^{7/}}{9c}$$

$$=\frac{5a^2d\left(8cd^2-3ae^2\right)x\sqrt{a+cx^2}}{128c}+\frac{5ad\left(8cd^2-3ae^2\right)x\left(a+cx^2\right)^{3/2}}{192c}+\frac{d\left(8cd^2-3ae^2\right)x\left(a\right.}{48c}$$

$$=\frac{5a^2d\left(8cd^2-3ae^2\right)x\sqrt{a+cx^2}}{128c}+\frac{5ad\left(8cd^2-3ae^2\right)x\left(a+cx^2\right)^{3/2}}{192c}+\frac{d\left(8cd^2-3ae^2\right)x\left(a\right.}{48c}$$

$$=\frac{5a^2d\left(8cd^2-3ae^2\right)x\sqrt{a+cx^2}}{128c}+\frac{5ad\left(8cd^2-3ae^2\right)x\left(a+cx^2\right)^{3/2}}{192c}+\frac{d\left(8cd^2-3ae^2\right)x\left(a\right.}{48c}$$

Mathematica [A] time = 0.16549, size = 216, normalized size = 1.

$$\sqrt{a+cx^2}\left(6a^2c^2x\left(1728d^2ex+924d^3+1239de^2x^2+320e^3x^3\right)+a^3ce\left(3456d^2+945dex+128e^2x^2\right)-256a^4e^3+8ac^3x\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(-256*a^4*e^3 + a^3*c*e*(3456*d^2 + 945*d*e*x + 128*e^2*x^
2) + 16*c^4*x^5*(84*d^3 + 216*d^2*e*x + 189*d*e^2*x^2 + 56*e^3*x^3) + 8*a*c
^3*x^3*(546*d^3 + 1296*d^2*e*x + 1071*d*e^2*x^2 + 304*e^3*x^3) + 6*a^2*c^2*
x*(924*d^3 + 1728*d^2*e*x + 1239*d*e^2*x^2 + 320*e^3*x^3)) - 315*a^3*Sqrt[c
]*d*(-8*c*d^2 + 3*a*e^2)*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/(8064*c^2)
```

Maple [A] time = 0.053, size = 245, normalized size = 1.1

$$\frac{e^3x^2}{9c}\left(cx^2+a\right)^{\frac{7}{2}}-\frac{2ae^3}{63c^2}\left(cx^2+a\right)^{\frac{7}{2}}+\frac{3de^2x}{8c}\left(cx^2+a\right)^{\frac{7}{2}}-\frac{ade^2x}{16c}\left(cx^2+a\right)^{\frac{5}{2}}-\frac{5de^2a^2x}{64c}\left(cx^2+a\right)^{\frac{3}{2}}-\frac{15de^2a^3x}{128c}\sqrt{cx^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+a)^(5/2),x)
```

```
[Out] 1/9*e^3*x^2*(c*x^2+a)^(7/2)/c-2/63*e^3*a/c^2*(c*x^2+a)^(7/2)+3/8*d*e^2*x*(c
*x^2+a)^(7/2)/c-1/16*d*e^2*a/c*x*(c*x^2+a)^(5/2)-5/64*d*e^2*a^2/c*x*(c*x^2+
a)^(3/2)-15/128*d*e^2*a^3/c*x*(c*x^2+a)^(1/2)-15/128*d*e^2*a^4/c^(3/2)*ln(x
*c^(1/2)+(c*x^2+a)^(1/2))+3/7*d^2*e*(c*x^2+a)^(7/2)/c+1/6*d^3*x*(c*x^2+a)^(
5/2)+5/24*d^3*a*x*(c*x^2+a)^(3/2)+5/16*d^3*a^2*x*(c*x^2+a)^(1/2)+5/16*d^3*a
^3/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.99163, size = 1185, normalized size = 5.49

$$\left[\frac{315\left(8\,a^3cd^3-3\,a^4de^2\right)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)+2\left(896\,c^4e^3x^8+3024\,c^4de^2x^7+3456\,a^3cd^2e-256\,a^4e^3+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/16128*(315*(8*a^3*c*d^3 - 3*a^4*d*e^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x
^2 + a)*sqrt(c)*x - a) + 2*(896*c^4*e^3*x^8 + 3024*c^4*d*e^2*x^7 + 3456*a^3
*c*d^2*e - 256*a^4*e^3 + 128*(27*c^4*d^2*e + 19*a*c^3*e^3)*x^6 + 168*(8*c^4
*d^3 + 51*a*c^3*d*e^2)*x^5 + 384*(27*a*c^3*d^2*e + 5*a^2*c^2*e^3)*x^4 + 42*
(104*a*c^3*d^3 + 177*a^2*c^2*d*e^2)*x^3 + 128*(81*a^2*c^2*d^2*e + a^3*c*e^3
)*x^2 + 63*(88*a^2*c^2*d^3 + 15*a^3*c*d*e^2)*x)*sqrt(c*x^2 + a))/c^2, -1/80
64*(315*(8*a^3*c*d^3 - 3*a^4*d*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 +
 a)) - (896*c^4*e^3*x^8 + 3024*c^4*d*e^2*x^7 + 3456*a^3*c*d^2*e - 256*a^4*e
^3 + 128*(27*c^4*d^2*e + 19*a*c^3*e^3)*x^6 + 168*(8*c^4*d^3 + 51*a*c^3*d*e^
2)*x^5 + 384*(27*a*c^3*d^2*e + 5*a^2*c^2*e^3)*x^4 + 42*(104*a*c^3*d^3 + 177
*a^2*c^2*d*e^2)*x^3 + 128*(81*a^2*c^2*d^2*e + a^3*c*e^3)*x^2 + 63*(88*a^2*c
^2*d^3 + 15*a^3*c*d*e^2)*x)*sqrt(c*x^2 + a))/c^2]
```

Sympy [A] time = 42.5977, size = 843, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a)**(5/2),x)
```

```
[Out] 15*a**(7/2)*d*e**2*x/(128*c*sqrt(1 + c*x**2/a)) + a**(5/2)*d**3*x*sqrt(1 +
c*x**2/a)/2 + 3*a**(5/2)*d**3*x/(16*sqrt(1 + c*x**2/a)) + 133*a**(5/2)*d*e*
```

```
*2*x**3/(128*sqrt(1 + c*x**2/a)) + 35*a**(3/2)*c*d**3*x**3/(48*sqrt(1 + c*x
**2/a)) + 127*a**(3/2)*c*d*e**2*x**5/(64*sqrt(1 + c*x**2/a)) + 17*sqrt(a)*c
**2*d**3*x**5/(24*sqrt(1 + c*x**2/a)) + 23*sqrt(a)*c**2*d*e**2*x**7/(16*sqr
t(1 + c*x**2/a)) - 15*a**4*d*e**2*asinh(sqrt(c)*x/sqrt(a))/(128*c**(3/2)) +
 5*a**3*d**3*asinh(sqrt(c)*x/sqrt(a))/(16*sqrt(c)) + 3*a**2*d**2*e*Piecewis
e((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + a**2*e**
3*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(
15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + 6*a*c
*d**2*e*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x
**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) +
 2*a*c*e**3*Piecewise((8*a**3*sqrt(a + c*x**2)/(105*c**3) - 4*a**2*x**2*sqr
t(a + c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x**2)/(35*c) + x**6*sqrt(a + c
*x**2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) + 3*c**2*d**2*e*Piecewise((8*a
**3*sqrt(a + c*x**2)/(105*c**3) - 4*a**2*x**2*sqrt(a + c*x**2)/(105*c**2) +
 a*x**4*sqrt(a + c*x**2)/(35*c) + x**6*sqrt(a + c*x**2)/7, Ne(c, 0)), (sqrt
(a)*x**6/6, True)) + c**2*e**3*Piecewise((-16*a**4*sqrt(a + c*x**2)/(315*c*
*4) + 8*a**3*x**2*sqrt(a + c*x**2)/(315*c**3) - 2*a**2*x**4*sqrt(a + c*x**2
)/(105*c**2) + a*x**6*sqrt(a + c*x**2)/(63*c) + x**8*sqrt(a + c*x**2)/9, Ne
(c, 0)), (sqrt(a)*x**8/8, True)) + c**3*d**3*x**7/(6*sqrt(a)*sqrt(1 + c*x**
2/a)) + 3*c**3*d*e**2*x**9/(8*sqrt(a)*sqrt(1 + c*x**2/a))
```

Giac [A] time = 1.3225, size = 378, normalized size = 1.75

$$\frac{1}{8064}\sqrt{cx^2+a}\left(\left(2\left(\left(4\left(\left(2\left(7\left(8c^2xe^3+27c^2de^2\right)x+\frac{8\left(27c^9d^2e+19ac^8e^3\right)}{c^7}\right)x+\frac{21\left(8c^9d^3+51ac^8de^2\right)}{c^7}\right)x+\frac{48\left(27a\right.}{}\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/8064*sqrt(c*x^2 + a)*((2*((4*((2*(7*(8*c^2*x*e^3 + 27*c^2*d*e^2)*x + 8*(2
7*c^9*d^2*e + 19*a*c^8*e^3)/c^7)*x + 21*(8*c^9*d^3 + 51*a*c^8*d*e^2)/c^7)*x
 + 48*(27*a*c^8*d^2*e + 5*a^2*c^7*e^3)/c^7)*x + 21*(104*a*c^8*d^3 + 177*a^2
*c^7*d*e^2)/c^7)*x + 64*(81*a^2*c^7*d^2*e + a^3*c^6*e^3)/c^7)*x + 63*(88*a^
2*c^7*d^3 + 15*a^3*c^6*d*e^2)/c^7)*x + 128*(27*a^3*c^6*d^2*e - 2*a^4*c^5*e^
3)/c^7) - 5/128*(8*a^3*c*d^3 - 3*a^4*d*e^2)*log(abs(-sqrt(c)*x + sqrt(c*x^2
 + a)))/c^(3/2)
```

3.547 $\int (d+ex)^2 \left(a+cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=189

$$\frac{5a^3\left(8cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{128c^{3/2}}+\frac{5a^2x\sqrt{a+cx^2}\left(8cd^2-ae^2\right)}{128c}+\frac{x\left(a+cx^2\right)^{5/2}\left(8cd^2-ae^2\right)}{48c}+\frac{5ax\left(a+cx^2\right)^{3/2}\left(8cd^2-a\right.}{192c}$$

```
[Out] (5*a^2*(8*c*d^2 - a*e^2)*x*Sqrt[a + c*x^2])/(128*c) + (5*a*(8*c*d^2 - a*e^2
)*x*(a + c*x^2)^(3/2))/(192*c) + ((8*c*d^2 - a*e^2)*x*(a + c*x^2)^(5/2))/(4
8*c) + (9*d*e*(a + c*x^2)^(7/2))/(56*c) + (e*(d + e*x)*(a + c*x^2)^(7/2))/(
8*c) + (5*a^3*(8*c*d^2 - a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(128*
c^(3/2))
```

Rubi [A] time = 0.087727, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {743, 641, 195, 217, 206}

$$\frac{5a^3\left(8cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{128c^{3/2}}+\frac{5a^2x\sqrt{a+cx^2}\left(8cd^2-ae^2\right)}{128c}+\frac{x\left(a+cx^2\right)^{5/2}\left(8cd^2-ae^2\right)}{48c}+\frac{5ax\left(a+cx^2\right)^{3/2}\left(8cd^2-a\right.}{192c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + c*x^2)^(5/2),x]
```


```
[Out] (5*a^2*(8*c*d^2 - a*e^2)*x*Sqrt[a + c*x^2])/(128*c) + (5*a*(8*c*d^2 - a*e^2
)*x*(a + c*x^2)^(3/2))/(192*c) + ((8*c*d^2 - a*e^2)*x*(a + c*x^2)^(5/2))/(4
8*c) + (9*d*e*(a + c*x^2)^(7/2))/(56*c) + (e*(d + e*x)*(a + c*x^2)^(7/2))/(
8*c) + (5*a^3*(8*c*d^2 - a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(128*
c^(3/2))
```


Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^2\left(a+cx^2\right)^{5/2}dx = \frac{e(d+ex)\left(a+cx^2\right)^{7/2}}{8c}+\frac{\int\left(8cd^2-ae^2+9cdex\right)\left(a+cx^2\right)^{5/2}dx}{8c}$$
$$=\frac{9de\left(a+cx^2\right)^{7/2}}{56c}+\frac{e(d+ex)\left(a+cx^2\right)^{7/2}}{8c}+\frac{\left(8cd^2-ae^2\right)\int\left(a+cx^2\right)^{5/2}dx}{8c}$$
$$=\frac{\left(8cd^2-ae^2\right)x\left(a+cx^2\right)^{5/2}}{48c}+\frac{9de\left(a+cx^2\right)^{7/2}}{56c}+\frac{e(d+ex)\left(a+cx^2\right)^{7/2}}{8c}+\frac{\left(5a\left(8cd^2-ae^2\right.\right.}{}$$
$$=\frac{5a\left(8cd^2-ae^2\right)x\left(a+cx^2\right)^{3/2}}{192c}+\frac{\left(8cd^2-ae^2\right)x\left(a+cx^2\right)^{5/2}}{48c}+\frac{9de\left(a+cx^2\right)^{7/2}}{56c}+\frac{e(d+e}{}$$
$$=\frac{5a^2\left(8cd^2-ae^2\right)x\sqrt{a+cx^2}}{128c}+\frac{5a\left(8cd^2-ae^2\right)x\left(a+cx^2\right)^{3/2}}{192c}+\frac{\left(8cd^2-ae^2\right)x\left(a+cx^2\right)^{5/}}{48c}$$
$$=\frac{5a^2\left(8cd^2-ae^2\right)x\sqrt{a+cx^2}}{128c}+\frac{5a\left(8cd^2-ae^2\right)x\left(a+cx^2\right)^{3/2}}{192c}+\frac{\left(8cd^2-ae^2\right)x\left(a+cx^2\right)^{5/}}{48c}$$
$$=\frac{5a^2\left(8cd^2-ae^2\right)x\sqrt{a+cx^2}}{128c}+\frac{5a\left(8cd^2-ae^2\right)x\left(a+cx^2\right)^{3/2}}{192c}+\frac{\left(8cd^2-ae^2\right)x\left(a+cx^2\right)^{5/}}{48c}$$

Mathematica [A] time = 0.122061, size = 162, normalized size = 0.86

$$\frac{\sqrt{c}\sqrt{a+cx^2}\left(2a^2cx\left(924d^2+1152dex+413e^2x^2\right)+3a^3e(256d+35ex)+8ac^2x^3\left(182d^2+288dex+119e^2x^2\right)+16c^3x^5\right.}{2688c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[c]*Sqrt[a + c*x^2]*(3*a^3*e*(256*d + 35*e*x) + 16*c^3*x^5*(28*d^2 + 4
8*d*e*x + 21*e^2*x^2) + 8*a*c^2*x^3*(182*d^2 + 288*d*e*x + 119*e^2*x^2) + 2
*a^2*c*x*(924*d^2 + 1152*d*e*x + 413*e^2*x^2)) - 105*a^3*(-8*c*d^2 + a*e^2)
*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/(2688*c^(3/2))
```

Maple [A] time = 0.056, size = 200, normalized size = 1.1

$$\frac{e^2x}{8c}\left(cx^2+a\right)^{\frac{7}{2}}-\frac{ae^2x}{48c}\left(cx^2+a\right)^{\frac{5}{2}}-\frac{5a^2e^2x}{192c}\left(cx^2+a\right)^{\frac{3}{2}}-\frac{5e^2a^3x}{128c}\sqrt{cx^2+a}-\frac{5e^2a^4}{128}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)c^{-\frac{3}{2}}+\frac{2de}{7c}\left(c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a)^(5/2),x)
```

```
[Out] 1/8*e^2*x*(c*x^2+a)^(7/2)/c-1/48*e^2*a/c*x*(c*x^2+a)^(5/2)-5/192*e^2*a^2/c*
x*(c*x^2+a)^(3/2)-5/128*e^2*a^3/c*x*(c*x^2+a)^(1/2)-5/128*e^2*a^4/c^(3/2)*l
n(x*c^(1/2)+(c*x^2+a)^(1/2))+2/7*d*e*(c*x^2+a)^(7/2)/c+1/6*d^2*x*(c*x^2+a)^
(5/2)+5/24*d^2*a*x*(c*x^2+a)^(3/2)+5/16*d^2*a^2*x*(c*x^2+a)^(1/2)+5/16*d^2*
a^3/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83758, size = 879, normalized size = 4.65

$$\left[\frac{105\left(8a^3cd^2-a^4e^2\right)\sqrt{c}\log\left(-2cx^2-2\sqrt{cx^2+a}\sqrt{c}x-a\right)+2\left(336c^4e^2x^7+768c^4dex^6+2304ac^3dex^4+2304a^2c^2dex^2\right.}{5376c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/5376*(105*(8*a^3*c*d^2 - a^4*e^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 +
a)*sqrt(c)*x - a) + 2*(336*c^4*e^2*x^7 + 768*c^4*d*e*x^6 + 2304*a*c^3*d*e*x
^4 + 2304*a^2*c^2*d*e*x^2 + 768*a^3*c*d*e + 56*(8*c^4*d^2 + 17*a*c^3*e^2)*x
^5 + 14*(104*a*c^3*d^2 + 59*a^2*c^2*e^2)*x^3 + 21*(88*a^2*c^2*d^2 + 5*a^3*c
*e^2)*x)*sqrt(c*x^2 + a))/c^2, -1/2688*(105*(8*a^3*c*d^2 - a^4*e^2)*sqrt(-c
)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (336*c^4*e^2*x^7 + 768*c^4*d*e*x^6 +
 2304*a*c^3*d*e*x^4 + 2304*a^2*c^2*d*e*x^2 + 768*a^3*c*d*e + 56*(8*c^4*d^2
+ 17*a*c^3*e^2)*x^5 + 14*(104*a*c^3*d^2 + 59*a^2*c^2*e^2)*x^3 + 21*(88*a^2*
c^2*d^2 + 5*a^3*c*e^2)*x)*sqrt(c*x^2 + a))/c^2]
```

Sympy [A] time = 39.1034, size = 539, normalized size = 2.85

$$\frac{5a^{\frac{7}{2}}e^2x}{128c\sqrt{1+\frac{cx^2}{a}}}+\frac{a^{\frac{5}{2}}d^2x\sqrt{1+\frac{cx^2}{a}}}{2}+\frac{3a^{\frac{5}{2}}d^2x}{16\sqrt{1+\frac{cx^2}{a}}}+\frac{133a^{\frac{5}{2}}e^2x^3}{384\sqrt{1+\frac{cx^2}{a}}}+\frac{35a^{\frac{3}{2}}cd^2x^3}{48\sqrt{1+\frac{cx^2}{a}}}+\frac{127a^{\frac{3}{2}}ce^2x^5}{192\sqrt{1+\frac{cx^2}{a}}}+\frac{17\sqrt{a}c^2d^2x^5}{24\sqrt{1+\frac{cx^2}{a}}}+\frac{23\sqrt{a}c^2e}{48\sqrt{1+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+a)**(5/2),x)
```

```
[Out] 5*a**(7/2)*e**2*x/(128*c*sqrt(1 + c*x**2/a)) + a**(5/2)*d**2*x*sqrt(1 + c*x
**2/a)/2 + 3*a**(5/2)*d**2*x/(16*sqrt(1 + c*x**2/a)) + 133*a**(5/2)*e**2*x*
*3/(384*sqrt(1 + c*x**2/a)) + 35*a**(3/2)*c*d**2*x**3/(48*sqrt(1 + c*x**2/a
)) + 127*a**(3/2)*c*e**2*x**5/(192*sqrt(1 + c*x**2/a)) + 17*sqrt(a)*c**2*d*
*2*x**5/(24*sqrt(1 + c*x**2/a)) + 23*sqrt(a)*c**2*e**2*x**7/(48*sqrt(1 + c*
```

```
x**2/a)) - 5*a**4*e**2*asinh(sqrt(c)*x/sqrt(a))/(128*c**(3/2)) + 5*a**3*d**
2*asinh(sqrt(c)*x/sqrt(a))/(16*sqrt(c)) + 2*a**2*d*e*Piecewise((sqrt(a)*x**
2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + 4*a*c*d*e*Piecewise((-
2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*s
qrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + 2*c**2*d*e*Piecewis
e((8*a**3*sqrt(a + c*x**2)/(105*c**3) - 4*a**2*x**2*sqrt(a + c*x**2)/(105*c
**2) + a*x**4*sqrt(a + c*x**2)/(35*c) + x**6*sqrt(a + c*x**2)/7, Ne(c, 0)),
 (sqrt(a)*x**6/6, True)) + c**3*d**2*x**7/(6*sqrt(a)*sqrt(1 + c*x**2/a)) +
c**3*e**2*x**9/(8*sqrt(a)*sqrt(1 + c*x**2/a))
```

Giac [A] time = 1.32431, size = 257, normalized size = 1.36

$$\frac{1}{2688}\left(\frac{768\,a^3de}{c}+\left(2\left(1152\,a^2de+\left(4\left(288\,acde+\left(6\left(7\,c^2xe^2+16\,c^2de\right)x+\frac{7\left(8\,c^8d^2+17\,ac^7e^2\right)}{c^6}\right)x\right)x+\frac{7\left(104\,ac^7d^2+\right.}{c^6}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2688*(768*a^3*d*e/c + (2*(1152*a^2*d*e + (4*(288*a*c*d*e + (6*(7*c^2*x*e^
2 + 16*c^2*d*e)*x + 7*(8*c^8*d^2 + 17*a*c^7*e^2)/c^6)*x)*x + 7*(104*a*c^7*d
^2 + 59*a^2*c^6*e^2)/c^6)*x)*x + 21*(88*a^2*c^6*d^2 + 5*a^3*c^5*e^2)/c^6)*x
)*sqrt(c*x^2 + a) - 5/128*(8*a^3*c*d^2 - a^4*e^2)*log(abs(-sqrt(c)*x + sqrt
(c*x^2 + a)))/c^(3/2)
```

3.548 $\int (d+ex)\left(a+cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=107

$$\frac{5}{16}a^2dx\sqrt{a+cx^2}+\frac{5a^3d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16\sqrt{c}}+\frac{1}{6}dx\left(a+cx^2\right)^{5/2}+\frac{5}{24}adx\left(a+cx^2\right)^{3/2}+\frac{e\left(a+cx^2\right)^{7/2}}{7c}$$

```
[Out] (5*a^2*d*x*Sqrt[a + c*x^2])/16 + (5*a*d*x*(a + c*x^2)^(3/2))/24 + (d*x*(a +
 c*x^2)^(5/2))/6 + (e*(a + c*x^2)^(7/2))/(7*c) + (5*a^3*d*ArcTanh[(Sqrt[c]*
x)/Sqrt[a + c*x^2]])/(16*Sqrt[c])
```

Rubi [A] time = 0.0337156, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {641, 195, 217, 206}

$$\frac{5}{16}a^2dx\sqrt{a+cx^2}+\frac{5a^3d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16\sqrt{c}}+\frac{1}{6}dx\left(a+cx^2\right)^{5/2}+\frac{5}{24}adx\left(a+cx^2\right)^{3/2}+\frac{e\left(a+cx^2\right)^{7/2}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + c*x^2)^(5/2),x]
```

```
[Out] (5*a^2*d*x*Sqrt[a + c*x^2])/16 + (5*a*d*x*(a + c*x^2)^(3/2))/24 + (d*x*(a +
 c*x^2)^(5/2))/6 + (e*(a + c*x^2)^(7/2))/(7*c) + (5*a^3*d*ArcTanh[(Sqrt[c]*
x)/Sqrt[a + c*x^2]])/(16*Sqrt[c])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)\left(a+cx^2\right)^{5/2}dx = \frac{e\left(a+cx^2\right)^{7/2}}{7c} + d\int \left(a+cx^2\right)^{5/2}dx$$
$$= \frac{1}{6}dx\left(a+cx^2\right)^{5/2} + \frac{e\left(a+cx^2\right)^{7/2}}{7c} + \frac{1}{6}(5ad)\int \left(a+cx^2\right)^{3/2}dx$$
$$= \frac{5}{24}adx\left(a+cx^2\right)^{3/2} + \frac{1}{6}dx\left(a+cx^2\right)^{5/2} + \frac{e\left(a+cx^2\right)^{7/2}}{7c} + \frac{1}{8}\left(5a^2d\right)\int \sqrt{a+cx^2}\,dx$$
$$= \frac{5}{16}a^2dx\sqrt{a+cx^2} + \frac{5}{24}adx\left(a+cx^2\right)^{3/2} + \frac{1}{6}dx\left(a+cx^2\right)^{5/2} + \frac{e\left(a+cx^2\right)^{7/2}}{7c} + \frac{1}{16}\left(5a^3d\right)\int$$
$$= \frac{5}{16}a^2dx\sqrt{a+cx^2} + \frac{5}{24}adx\left(a+cx^2\right)^{3/2} + \frac{1}{6}dx\left(a+cx^2\right)^{5/2} + \frac{e\left(a+cx^2\right)^{7/2}}{7c} + \frac{1}{16}\left(5a^3d\right)\mathrm{Su}$$
$$= \frac{5}{16}a^2dx\sqrt{a+cx^2} + \frac{5}{24}adx\left(a+cx^2\right)^{3/2} + \frac{1}{6}dx\left(a+cx^2\right)^{5/2} + \frac{e\left(a+cx^2\right)^{7/2}}{7c} + \frac{5a^3d\tanh^{-1}}{16\sqrt{}}$$

Mathematica [A] time = 0.0702851, size = 108, normalized size = 1.01

$$\frac{\sqrt{a+cx^2}\left(3a^2cx(77d+48ex)+48a^3e+2ac^2x^3(91d+72ex)+8c^3x^5(7d+6ex)\right)+105a^3\sqrt{c}d\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)}{336c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(48*a^3*e + 8*c^3*x^5*(7*d + 6*e*x) + 3*a^2*c*x*(77*d + 48
*e*x) + 2*a*c^2*x^3*(91*d + 72*e*x)) + 105*a^3*Sqrt[c]*d*Log[c*x + Sqrt[c]*
Sqrt[a + c*x^2]])/(336*c)
```

Maple [A] time = 0.044, size = 85, normalized size = 0.8

$$\frac{e}{7c}\left(cx^2+a\right)^{\frac{7}{2}} + \frac{dx}{6}\left(cx^2+a\right)^{\frac{5}{2}} + \frac{5\,adx}{24}\left(cx^2+a\right)^{\frac{3}{2}} + \frac{5\,a^2dx}{16}\sqrt{cx^2+a} + \frac{5\,da^3}{16}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+a)^(5/2),x)
```

```
[Out] 1/7*e*(c*x^2+a)^(7/2)/c+1/6*d*x*(c*x^2+a)^(5/2)+5/24*a*d*x*(c*x^2+a)^(3/2)+
5/16*a^2*d*x*(c*x^2+a)^(1/2)+5/16*d*a^3/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2
))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.67628, size = 549, normalized size = 5.13

$$\left[\frac{105\,a^3\sqrt{c}d\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)+2\left(48\,c^3ex^6+56\,c^3dx^5+144\,ac^2ex^4+182\,ac^2dx^3+144\,a^2cex^2+231\,a^2cd\right.}{672\,c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/672*(105*a^3*sqrt(c)*d*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) +
 2*(48*c^3*e*x^6 + 56*c^3*d*x^5 + 144*a*c^2*e*x^4 + 182*a*c^2*d*x^3 + 144*a
^2*c*e*x^2 + 231*a^2*c*d*x + 48*a^3*e)*sqrt(c*x^2 + a))/c, -1/336*(105*a^3*
sqrt(-c)*d*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (48*c^3*e*x^6 + 56*c^3*d*x^
5 + 144*a*c^2*e*x^4 + 182*a*c^2*d*x^3 + 144*a^2*c*e*x^2 + 231*a^2*c*d*x + 4
8*a^3*e)*sqrt(c*x^2 + a))/c]
```

Sympy [A] time = 17.0691, size = 348, normalized size = 3.25

$$\frac{a^{\frac{5}{2}}dx\sqrt{1+\frac{cx^2}{a}}}{2}+\frac{3a^{\frac{5}{2}}dx}{16\sqrt{1+\frac{cx^2}{a}}}+\frac{35a^{\frac{3}{2}}cdx^3}{48\sqrt{1+\frac{cx^2}{a}}}+\frac{17\sqrt{a}c^2dx^5}{24\sqrt{1+\frac{cx^2}{a}}}+\frac{5a^3d\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16\sqrt{c}}+a^2e\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{for } c=0\\ \frac{(a+cx^2)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)+2ace\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+a)**(5/2),x)
```

```
[Out] a**(5/2)*d*x*sqrt(1 + c*x**2/a)/2 + 3*a**(5/2)*d*x/(16*sqrt(1 + c*x**2/a))
+ 35*a**(3/2)*c*d*x**3/(48*sqrt(1 + c*x**2/a)) + 17*sqrt(a)*c**2*d*x**5/(24
*sqrt(1 + c*x**2/a)) + 5*a**3*d*asinh(sqrt(c)*x/sqrt(a))/(16*sqrt(c)) + a**
2*e*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)
) + 2*a*c*e*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a +
 c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True
)) + c**2*e*Piecewise((8*a**3*sqrt(a + c*x**2)/(105*c**3) - 4*a**2*x**2*sqr
t(a + c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x**2)/(35*c) + x**6*sqrt(a + c
*x**2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) + c**3*d*x**7/(6*sqrt(a)*sqrt(
1 + c*x**2/a))
```

Giac [A] time = 1.30598, size = 142, normalized size = 1.33

$$-\frac{5\,a^3d\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{16\,\sqrt{c}}+\frac{1}{336}\,\sqrt{cx^2+a}\left(\frac{48\,a^3e}{c}+\left(231\,a^2d+2\left(72\,a^2e+\left(91\,acd+4\left(18\,ace+\left(6\,c^2xe+7\,c^2d\right)x\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -5/16*a^3*d*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/sqrt(c) + 1/336*sqrt(c*x
^2 + a)*(48*a^3*e/c + (231*a^2*d + 2*(72*a^2*e + (91*a*c*d + 4*(18*a*c*e +
(6*c^2*x*e + 7*c^2*d)*x)*x)*x)*x)*x)
```

3.549 $\int \frac{(a+cx^2)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=226

$$-\frac{\sqrt{c}d\left(15a^2e^4+20acd^2e^2+8c^2d^4\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8e^6}+\frac{\left(a+cx^2\right)^{3/2}\left(4\left(ae^2+cd^2\right)-3cdex\right)}{12e^3}+\frac{\sqrt{a+cx^2}\left(8\left(ae^2+cd^2\right)^2\right.}{8e}$$

```
[Out] ((8*(c*d^2 + a*e^2)^2 - c*d*e*(4*c*d^2 + 7*a*e^2)*x)*Sqrt[a + c*x^2])/(8*e^
5) + ((4*(c*d^2 + a*e^2) - 3*c*d*e*x)*(a + c*x^2)^(3/2))/(12*e^3) + (a + c*
x^2)^(5/2)/(5*e) - (Sqrt[c]*d*(8*c^2*d^4 + 20*a*c*d^2*e^2 + 15*a^2*e^4)*Arc
Tanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*e^6) - ((c*d^2 + a*e^2)^(5/2)*ArcTanh
[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/e^6
```

Rubi [A] time = 0.259348, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {735, 815, 844, 217, 206, 725}

$$-\frac{\sqrt{c}d\left(15a^2e^4+20acd^2e^2+8c^2d^4\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8e^6}+\frac{\left(a+cx^2\right)^{3/2}\left(4\left(ae^2+cd^2\right)-3cdex\right)}{12e^3}+\frac{\sqrt{a+cx^2}\left(8\left(ae^2+cd^2\right)^2\right.}{8e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(5/2)/(d + e*x),x]
```


```
[Out] ((8*(c*d^2 + a*e^2)^2 - c*d*e*(4*c*d^2 + 7*a*e^2)*x)*Sqrt[a + c*x^2])/(8*e^
5) + ((4*(c*d^2 + a*e^2) - 3*c*d*e*x)*(a + c*x^2)^(3/2))/(12*e^3) + (a + c*
x^2)^(5/2)/(5*e) - (Sqrt[c]*d*(8*c^2*d^4 + 20*a*c*d^2*e^2 + 15*a^2*e^4)*Arc
Tanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*e^6) - ((c*d^2 + a*e^2)^(5/2)*ArcTanh
[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/e^6
```


Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+cx^2\right)^{5/2}}{d+ex}\,dx &= \frac{\left(a+cx^2\right)^{5/2}}{5e}+\frac{\int \frac{(ae-cdx)\left(a+cx^2\right)^{3/2}}{d+ex}\,dx}{e}\\
&= \frac{\left(4\left(cd^2+ae^2\right)-3cdex\right)\left(a+cx^2\right)^{3/2}}{12e^3}+\frac{\left(a+cx^2\right)^{5/2}}{5e}+\frac{\int \frac{\left(ace\left(cd^2+4ae^2\right)-c^2d\left(4cd^2+7ae^2\right)x\right)\sqrt{a+cx^2}}{d+ex}\,dx}{4ce^3}\\
&= \frac{\left(8\left(cd^2+ae^2\right)^2-cde\left(4cd^2+7ae^2\right)x\right)\sqrt{a+cx^2}}{8e^5}+\frac{\left(4\left(cd^2+ae^2\right)-3cdex\right)\left(a+cx^2\right)^{3/2}}{12e^3}+\frac{\left(a+cx^2\right)^{5/}}{5e}\\
&= \frac{\left(8\left(cd^2+ae^2\right)^2-cde\left(4cd^2+7ae^2\right)x\right)\sqrt{a+cx^2}}{8e^5}+\frac{\left(4\left(cd^2+ae^2\right)-3cdex\right)\left(a+cx^2\right)^{3/2}}{12e^3}+\frac{\left(a+cx^2\right)^{5/}}{5e}\\
&= \frac{\left(8\left(cd^2+ae^2\right)^2-cde\left(4cd^2+7ae^2\right)x\right)\sqrt{a+cx^2}}{8e^5}+\frac{\left(4\left(cd^2+ae^2\right)-3cdex\right)\left(a+cx^2\right)^{3/2}}{12e^3}+\frac{\left(a+cx^2\right)^{5/}}{5e}\\
&= \frac{\left(8\left(cd^2+ae^2\right)^2-cde\left(4cd^2+7ae^2\right)x\right)\sqrt{a+cx^2}}{8e^5}+\frac{\left(4\left(cd^2+ae^2\right)-3cdex\right)\left(a+cx^2\right)^{3/2}}{12e^3}+\frac{\left(a+cx^2\right)^{5/}}{5e}
\end{aligned}$$

Mathematica [A] time = 0.967422, size = 332, normalized size = 1.47

$$\frac{-\frac{5\sqrt{c}d\sqrt{a+cx^2}\left(3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)+\sqrt{c}x\left(5a+2cx^2\right)\sqrt{\frac{cx^2}{a}+1}\right)}{8e\sqrt{\frac{cx^2}{a}+1}}-\frac{5\left(ae^2+cd^2\right)\left(\sqrt{\frac{cx^2}{a}+1}\left(-e\sqrt{a+cx^2}\left(8ae^2+6cd^2-3cdex+2ce^2x^2\right)+6\left(ae^2+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae}}\right.\right.\right.}{6e^5\sqrt{\frac{cx^2}{a}+1}}}{5e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(5/2)/(d + e*x),x]
```

```
[Out] ((a + c*x^2)^(5/2) - (5*Sqrt[c]*d*Sqrt[a + c*x^2]*(Sqrt[c]*x*(5*a + 2*c*x^2
)*Sqrt[1 + (c*x^2)/a] + 3*a^(3/2)*ArcSinh[(Sqrt[c]*x)/Sqrt[a]]))/(8*e*Sqrt[
1 + (c*x^2)/a]) - (5*(c*d^2 + a*e^2)*(3*Sqrt[a]*Sqrt[c]*d*e^2*Sqrt[a + c*x^
2]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]] + Sqrt[1 + (c*x^2)/a]*(-(e*Sqrt[a + c*x^2]*
(6*c*d^2 + 8*a*e^2 - 3*c*d*e*x + 2*c*e^2*x^2)) + 6*Sqrt[c]*d*(c*d^2 + a*e^2
```

```
)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + 6*(c*d^2 + a*e^2)^(3/2)*ArcTanh[(a
*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])))/(6*e^5*Sqrt[1 + (c*x^
2)/a]))/(5*e)
```

Maple [B] time = 0.191, size = 1225, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(5/2)/(e*x+d),x)
```

```
[Out] 1/5/e*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(5/2)-1/4/e^2*c*d*(c*
(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)*x-7/8/e^2*c*d*a*(c*(d/e+
x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x-15/8/e^2*c^(1/2)*d*ln((-c*d
/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)
)*a^2+1/3/e*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)*a+1/3/e^3
*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)*c*d^2-1/2/e^4*c^2*d^
3*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x-5/2/e^4*c^(3/2)*d
^3*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)
/e^2)^(1/2))*a+1/e*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*a^
2+2/e^3*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*a*c*d^2+1/e^5
*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*c^2*d^4-1/e^6*c^(5/2
)*d^5*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d
^2)/e^2)^(1/2))-1/e/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d
/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+
c*d^2)/e^2)^(1/2))/(d/e+x))*a^3-3/e^3/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^
2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d
/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a^2*c*d^2-3/e^5/((a*e^2+c*d^2
)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^
(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a*c^2
*d^4-1/e^7/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x
)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^
2)^(1/2))/(d/e+x))*c^3*d^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+cx^2)^{\frac{5}{2}}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(5/2)/(e*x+d),x)
```

```
[Out] Integral((a + c*x**2)**(5/2)/(d + e*x), x)
```

Giac [A] time = 1.35843, size = 381, normalized size = 1.69

$$\frac{1}{8}\left(8\,c^{\frac{5}{2}}d^5+20\,ac^{\frac{3}{2}}d^3e^2+15\,a^2\sqrt{c}de^4\right)e^{(-6)}\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)+\frac{2\left(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6\right)\arctan\left(-\right.}{\sqrt{-cd^2-ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] 1/8*(8*c^(5/2)*d^5 + 20*a*c^(3/2)*d^3*e^2 + 15*a^2*sqrt(c)*d*e^4)*e^(-6)*lo
g(abs(-sqrt(c)*x + sqrt(c*x^2 + a))) + 2*(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2
*c*d^2*e^4 + a^3*e^6)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)
/sqrt(-c*d^2 - a*e^2))*e^(-6)/sqrt(-c*d^2 - a*e^2) + 1/120*sqrt(c*x^2 + a)*
((2*(3*(4*c^2*x*e^(-1) - 5*c^2*d*e^(-2))*x + 4*(5*c^5*d^2*e^18 + 11*a*c^4*e
^20)*e^(-21)/c^3)*x - 15*(4*c^5*d^3*e^17 + 9*a*c^4*d*e^19)*e^(-21)/c^3)*x +
 8*(15*c^5*d^4*e^16 + 35*a*c^4*d^2*e^18 + 23*a^2*c^3*e^20)*e^(-21)/c^3)
```

3.550 $\int \frac{(a+cx^2)^{5/2}}{(d+ex)^2} dx$

Optimal. Leaf size=219

$$\frac{5\sqrt{c}\left(3a^2e^4+12acd^2e^2+8c^2d^4\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8e^6}-\frac{5c\sqrt{a+cx^2}\left(8d\left(ae^2+cd^2\right)-ex\left(3ae^2+4cd^2\right)\right)}{8e^5}+\frac{5cd\left(ae^2+cd^2\right)^{3/2}}{}$$

```
[Out] (-5*c*(8*d*(c*d^2 + a*e^2) - e*(4*c*d^2 + 3*a*e^2)*x)*Sqrt[a + c*x^2])/(8*e
^5) - (5*c*(4*d - 3*e*x)*(a + c*x^2)^(3/2))/(12*e^3) - (a + c*x^2)^(5/2)/(e
*(d + e*x)) + (5*Sqrt[c]*(8*c^2*d^4 + 12*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTanh[(
Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*e^6) + (5*c*d*(c*d^2 + a*e^2)^(3/2)*ArcTanh
[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/e^6
```

Rubi [A] time = 0.243901, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {733, 815, 844, 217, 206, 725}

$$\frac{5\sqrt{c}\left(3a^2e^4+12acd^2e^2+8c^2d^4\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8e^6}-\frac{5c\sqrt{a+cx^2}\left(8d\left(ae^2+cd^2\right)-ex\left(3ae^2+4cd^2\right)\right)}{8e^5}+\frac{5cd\left(ae^2+cd^2\right)^{3/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(5/2)/(d + e*x)^2,x]
```


```
[Out] (-5*c*(8*d*(c*d^2 + a*e^2) - e*(4*c*d^2 + 3*a*e^2)*x)*Sqrt[a + c*x^2])/(8*e
^5) - (5*c*(4*d - 3*e*x)*(a + c*x^2)^(3/2))/(12*e^3) - (a + c*x^2)^(5/2)/(e
*(d + e*x)) + (5*Sqrt[c]*(8*c^2*d^4 + 12*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTanh[(
Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*e^6) + (5*c*d*(c*d^2 + a*e^2)^(3/2)*ArcTanh
[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/e^6
```


Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{5/2}}{(d+ex)^2}\,dx = -\frac{\left(a+cx^2\right)^{5/2}}{e(d+ex)} + \frac{(5c)\int \frac{x\left(a+cx^2\right)^{3/2}}{d+ex}\,dx}{e}$$

$$= -\frac{5c(4d-3ex)\left(a+cx^2\right)^{3/2}}{12e^3} - \frac{\left(a+cx^2\right)^{5/2}}{e(d+ex)} + \frac{5\int \frac{\left(-acde+c\left(4cd^2+3ae^2\right)x\right)\sqrt{a+cx^2}}{d+ex}\,dx}{4e^3}$$

$$= -\frac{5c\left(8d\left(cd^2+ae^2\right)-e\left(4cd^2+3ae^2\right)x\right)\sqrt{a+cx^2}}{8e^5} - \frac{5c(4d-3ex)\left(a+cx^2\right)^{3/2}}{12e^3} - \frac{\left(a+cx^2\right)^{5/2}}{e(d+ex)} + \frac{5\int}{}$$

$$= -\frac{5c\left(8d\left(cd^2+ae^2\right)-e\left(4cd^2+3ae^2\right)x\right)\sqrt{a+cx^2}}{8e^5} - \frac{5c(4d-3ex)\left(a+cx^2\right)^{3/2}}{12e^3} - \frac{\left(a+cx^2\right)^{5/2}}{e(d+ex)} - \frac{(5cd}{}$$

$$= -\frac{5c\left(8d\left(cd^2+ae^2\right)-e\left(4cd^2+3ae^2\right)x\right)\sqrt{a+cx^2}}{8e^5} - \frac{5c(4d-3ex)\left(a+cx^2\right)^{3/2}}{12e^3} - \frac{\left(a+cx^2\right)^{5/2}}{e(d+ex)} + \frac{(5cd}{}$$

$$= -\frac{5c\left(8d\left(cd^2+ae^2\right)-e\left(4cd^2+3ae^2\right)x\right)\sqrt{a+cx^2}}{8e^5} - \frac{5c(4d-3ex)\left(a+cx^2\right)^{3/2}}{12e^3} - \frac{\left(a+cx^2\right)^{5/2}}{e(d+ex)} + \frac{5\sqrt{c}}{}$$

Mathematica [A] time = 0.24748, size = 239, normalized size = 1.09

$$\frac{15\sqrt{c}\left(3a^2e^4+12acd^2e^2+8c^2d^4\right)\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)+e\sqrt{a+cx^2}\left(9cex\left(3ae^2+4cd^2\right)-\frac{24\left(ae^2+cd^2\right)^2}{d+ex}-16cd\left(7ae^2+6c\right.\right.}{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(5/2)/(d + e*x)^2,x]
```

```
[Out] (e*Sqrt[a + c*x^2]*(-16*c*d*(6*c*d^2 + 7*a*e^2) + 9*c*e*(4*c*d^2 + 3*a*e^2)
*x - 16*c^2*d*e^2*x^2 + 6*c^2*e^3*x^3 - (24*(c*d^2 + a*e^2)^2)/(d + e*x)) -
 120*c*d*(c*d^2 + a*e^2)^(3/2)*Log[d + e*x] + 15*Sqrt[c]*(8*c^2*d^4 + 12*a*
c*d^2*e^2 + 3*a^2*e^4)*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] + 120*c*d*(c*d^2
+ a*e^2)^(3/2)*Log[a*e - c*d*x + Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2]])/(24*
```

e^6)

Maple [B] time = 0.194, size = 1796, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+a)^(5/2)/(e*x+d)^2,x)

[Out] -1/(a*e^2+c*d^2)/(d/e+x)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(7/2)-1/e*c*d/(a*e^2+c*d^2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(5/2)+5/4/e^2*c^2*d^2/(a*e^2+c*d^2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)*x+35/8/e^2*c^2*d^2/(a*e^2+c*d^2)*a*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x+75/8/e^2*c^(3/2)*d^2/(a*e^2+c*d^2)*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))*a^2-5/3/e*c*d/(a*e^2+c*d^2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)*a-5/3/e^3*c^2*d^3/(a*e^2+c*d^2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)+5/2/e^4*c^3*d^4/(a*e^2+c*d^2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x+25/2/e^4*c^(5/2)*d^4/(a*e^2+c*d^2)*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))*a-5/e*c*d/(a*e^2+c*d^2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*a^2-10/e^3*c^2*d^3/(a*e^2+c*d^2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*a-5/e^5*c^3*d^5/(a*e^2+c*d^2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)+5/e^6*c^(7/2)*d^6/(a*e^2+c*d^2)*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))+5/e*c*d/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a^3+15/e^3*c^2*d^3/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a^2+15/e^5*c^3*d^5/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a+5/e^7*c^4*d^7/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))+1/(a*e^2+c*d^2)*c*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(5/2)*x+5/4/(a*e^2+c*d^2)*c*a*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)*x+15/8/(a*e^2+c*d^2)*c*a^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x+15/8/(a*e^2+c*d^2)*c^(1/2)*a^3*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 89.5191, size = 2967, normalized size = 13.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [1/48*(15*(8*c^2*d^5 + 12*a*c*d^3*e^2 + 3*a^2*d*e^4 + (8*c^2*d^4*e + 12*a*c
*d^2*e^3 + 3*a^2*e^5)*x)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x
 - a) + 120*(c^2*d^4 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)*sqrt(c*d^2
+ a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2
 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x
+ d^2)) + 2*(6*c^2*e^5*x^4 - 10*c^2*d*e^4*x^3 - 120*c^2*d^4*e - 160*a*c*d^2
*e^3 - 24*a^2*e^5 + (20*c^2*d^2*e^3 + 27*a*c*e^5)*x^2 - 5*(12*c^2*d^3*e^2 +
 17*a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(e^7*x + d*e^6), 1/48*(240*(c^2*d^4 + a*
c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*
d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 +
a*c*e^2)*x^2)) + 15*(8*c^2*d^5 + 12*a*c*d^3*e^2 + 3*a^2*d*e^4 + (8*c^2*d^4*
e + 12*a*c*d^2*e^3 + 3*a^2*e^5)*x)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)
*sqrt(c)*x - a) + 2*(6*c^2*e^5*x^4 - 10*c^2*d*e^4*x^3 - 120*c^2*d^4*e - 160
*a*c*d^2*e^3 - 24*a^2*e^5 + (20*c^2*d^2*e^3 + 27*a*c*e^5)*x^2 - 5*(12*c^2*d
^3*e^2 + 17*a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(e^7*x + d*e^6), -1/24*(15*(8*c^
2*d^5 + 12*a*c*d^3*e^2 + 3*a^2*d*e^4 + (8*c^2*d^4*e + 12*a*c*d^2*e^3 + 3*a^
2*e^5)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - 60*(c^2*d^4 + a*c*d
^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x -
a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*
d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - (6*c^2*e^5*x^4 - 1
0*c^2*d*e^4*x^3 - 120*c^2*d^4*e - 160*a*c*d^2*e^3 - 24*a^2*e^5 + (20*c^2*d^
2*e^3 + 27*a*c*e^5)*x^2 - 5*(12*c^2*d^3*e^2 + 17*a*c*d*e^4)*x)*sqrt(c*x^2 +
 a))/(e^7*x + d*e^6), 1/24*(120*(c^2*d^4 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d
*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqr
t(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 15*(8*c^2*d^5
 + 12*a*c*d^3*e^2 + 3*a^2*d*e^4 + (8*c^2*d^4*e + 12*a*c*d^2*e^3 + 3*a^2*e^5
)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (6*c^2*e^5*x^4 - 10*c^2*
d*e^4*x^3 - 120*c^2*d^4*e - 160*a*c*d^2*e^3 - 24*a^2*e^5 + (20*c^2*d^2*e^3
+ 27*a*c*e^5)*x^2 - 5*(12*c^2*d^3*e^2 + 17*a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(
e^7*x + d*e^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + cx^2\right)^{\frac{5}{2}}}{\left(d + ex\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(5/2)/(e*x+d)**2,x)
```

```
[Out] Integral((a + c*x**2)**(5/2)/(d + e*x)**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.551 $\int \frac{(a+cx^2)^{5/2}}{(d+ex)^3} dx$

Optimal. Leaf size=213

$$-\frac{5c^{3/2}d\left(3ae^2+4cd^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2e^6}+\frac{5c\sqrt{a+cx^2}\left(ae^2+4cd^2-2cdex\right)}{2e^5}-\frac{5c\sqrt{ae^2+cd^2}\left(ae^2+4cd^2\right)\tanh^{-1}\left(\frac{ae-c}{\sqrt{a+cx^2}\sqrt{}}\right)}{2e^6}$$

```
[Out] (5*c*(4*c*d^2 + a*e^2 - 2*c*d*e*x)*Sqrt[a + c*x^2])/(2*e^5) + (5*c*(4*d + e
*x)*(a + c*x^2)^(3/2))/(6*e^3*(d + e*x)) - (a + c*x^2)^(5/2)/(2*e*(d + e*x)
^2) - (5*c^(3/2)*d*(4*c*d^2 + 3*a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]
)/(2*e^6) - (5*c*Sqrt[c*d^2 + a*e^2]*(4*c*d^2 + a*e^2)*ArcTanh[(a*e - c*d*x
)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(2*e^6)
```

Rubi [A] time = 0.238605, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {733, 813, 815, 844, 217, 206, 725}

$$-\frac{5c^{3/2}d\left(3ae^2+4cd^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2e^6}+\frac{5c\sqrt{a+cx^2}\left(ae^2+4cd^2-2cdex\right)}{2e^5}-\frac{5c\sqrt{ae^2+cd^2}\left(ae^2+4cd^2\right)\tanh^{-1}\left(\frac{ae-c}{\sqrt{a+cx^2}\sqrt{}}\right)}{2e^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(5/2)/(d + e*x)^3,x]
```

```
[Out] (5*c*(4*c*d^2 + a*e^2 - 2*c*d*e*x)*Sqrt[a + c*x^2])/(2*e^5) + (5*c*(4*d + e
*x)*(a + c*x^2)^(3/2))/(6*e^3*(d + e*x)) - (a + c*x^2)^(5/2)/(2*e*(d + e*x)
^2) - (5*c^(3/2)*d*(4*c*d^2 + 3*a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]
)/(2*e^6) - (5*c*Sqrt[c*d^2 + a*e^2]*(4*c*d^2 + a*e^2)*ArcTanh[(a*e - c*d*x
)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(2*e^6)
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !Rational
Q[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
```

```
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{5/2}}{(d+ex)^3}\,dx = -\frac{\left(a+cx^2\right)^{5/2}}{2e(d+ex)^2}+\frac{(5c)\int \frac{x\left(a+cx^2\right)^{3/2}}{(d+ex)^2}\,dx}{2e}$$

$$= \frac{5c(4d+ex)\left(a+cx^2\right)^{3/2}}{6e^3(d+ex)}-\frac{\left(a+cx^2\right)^{5/2}}{2e(d+ex)^2}-\frac{(5c)\int \frac{(-2ae+8cdx)\sqrt{a+cx^2}}{d+ex}\,dx}{4e^3}$$

$$= \frac{5c\left(4cd^2+ae^2-2cdex\right)\sqrt{a+cx^2}}{2e^5}+\frac{5c(4d+ex)\left(a+cx^2\right)^{3/2}}{6e^3(d+ex)}-\frac{\left(a+cx^2\right)^{5/2}}{2e(d+ex)^2}-\frac{5\int \frac{-4ace\left(2cd^2+ae^2\right)+4}{(d+ex)\sqrt{}}}{8e^5}$$

$$= \frac{5c\left(4cd^2+ae^2-2cdex\right)\sqrt{a+cx^2}}{2e^5}+\frac{5c(4d+ex)\left(a+cx^2\right)^{3/2}}{6e^3(d+ex)}-\frac{\left(a+cx^2\right)^{5/2}}{2e(d+ex)^2}+\frac{\left(5c\left(cd^2+ae^2\right)\left(4c\right.\right.}{}$$

$$= \frac{5c\left(4cd^2+ae^2-2cdex\right)\sqrt{a+cx^2}}{2e^5}+\frac{5c(4d+ex)\left(a+cx^2\right)^{3/2}}{6e^3(d+ex)}-\frac{\left(a+cx^2\right)^{5/2}}{2e(d+ex)^2}-\frac{\left(5c\left(cd^2+ae^2\right)\left(4c\right.\right.}{}$$

$$= \frac{5c\left(4cd^2+ae^2-2cdex\right)\sqrt{a+cx^2}}{2e^5}+\frac{5c(4d+ex)\left(a+cx^2\right)^{3/2}}{6e^3(d+ex)}-\frac{\left(a+cx^2\right)^{5/2}}{2e(d+ex)^2}-\frac{5c^{3/2}d\left(4cd^2+3ae^2\right.}{2e}$$

Mathematica [A] time = 0.302595, size = 281, normalized size = 1.32

$$\frac{\frac{e\sqrt{a+cx^2}\left(-3a^2e^4+ace^2\left(35d^2+55dex+14e^2x^2\right)+c^2\left(20d^2e^2x^2+90d^3ex+60d^4-5de^3x^3+2e^4x^4\right)\right)}{(d+ex)^2}-\frac{15c\left(a^2e^4+5acd^2e^2+4c^2d^4\right)\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{\sqrt{ae^2+cd^2}}+}{6e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(5/2)/(d + e*x)^3,x]
```

```
[Out] ((e*Sqrt[a + c*x^2]*(-3*a^2*e^4 + a*c*e^2*(35*d^2 + 55*d*e*x + 14*e^2*x^2)
+ c^2*(60*d^4 + 90*d^3*e*x + 20*d^2*e^2*x^2 - 5*d*e^3*x^3 + 2*e^4*x^4)))/(d
 + e*x)^2 + (15*c*(4*c^2*d^4 + 5*a*c*d^2*e^2 + a^2*e^4)*Log[d + e*x])/Sqrt[
c*d^2 + a*e^2] - 15*c^(3/2)*d*(4*c*d^2 + 3*a*e^2)*Log[c*x + Sqrt[c]*Sqrt[a
+ c*x^2]] - (15*c*(4*c^2*d^4 + 5*a*c*d^2*e^2 + a^2*e^4)*Log[a*e - c*d*x + S
qrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2]])/Sqrt[c*d^2 + a*e^2])/(6*e^6)
```

Maple [B] time = 0.195, size = 3342, normalized size = 15.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(5/2)/(e*x+d)^3,x)
```

```
[Out] 5/6/e^3/(a*e^2+c*d^2)*c^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(
3/2)*d^2+3/2/e*c^2*d^2/(a*e^2+c*d^2)^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+
c*d^2)/e^2)^(5/2)+5/2/e^3*c^3*d^4/(a*e^2+c*d^2)^2*(c*(d/e+x)^2-2*c*d/e*(d/e
+x)+(a*e^2+c*d^2)/e^2)^(3/2)+15/2/e^5*c^4*d^6/(a*e^2+c*d^2)^2*(c*(d/e+x)^2-
2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)-3/2*c^2*d/(a*e^2+c*d^2)^2*(c*(d/e+
x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(5/2)*x-45/16*c^(3/2)*d/(a*e^2+c*d^
2)^2*a^3*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+
c*d^2)/e^2)^(1/2))+5/2/e/(a*e^2+c*d^2)*c*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^
2+c*d^2)/e^2)^(1/2)*a^2+3/2*c*d/(a*e^2+c*d^2)^2/(d/e+x)*(c*(d/e+x)^2-2*c*d/
e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(7/2)+5/2/e^5/(a*e^2+c*d^2)*c^3*(c*(d/e+x)^2-2
*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*d^4-5/2/e^6/(a*e^2+c*d^2)*c^(7/2)*d
^5*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)
/e^2)^(1/2))-15/2/e^6*c^(9/2)*d^7/(a*e^2+c*d^2)^2*ln((-c*d/e+(d/e+x)*c)/c^(
1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))-25/4/e^4/(a*e^2
+c*d^2)*c^(5/2)*d^3*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e
+x)+(a*e^2+c*d^2)/e^2)^(1/2))*a-45/16*c^2*d/(a*e^2+c*d^2)^2*a^2*(c*(d/e+x)^
2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x+15/2/e*c^2*d^2/(a*e^2+c*d^2)^2
*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*a^2-15/4/e^4*c^4*d^5
/(a*e^2+c*d^2)^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x-15
/8*c^2*d/(a*e^2+c*d^2)^2*a*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^
(3/2)*x-225/16/e^2*c^(5/2)*d^3/(a*e^2+c*d^2)^2*ln((-c*d/e+(d/e+x)*c)/c^(1/2
)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))*a^2-75/4/e^4*c^(7/
2)*d^5/(a*e^2+c*d^2)^2*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(
d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))*a-15/8/e^2*c^3*d^3/(a*e^2+c*d^2)^2*(c*(d/e
+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)*x+5/2/e*c^2*d^2/(a*e^2+c*d^2
)^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)*a-75/16/e^2/(a*e^
2+c*d^2)*c^(3/2)*d*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+
x)+(a*e^2+c*d^2)/e^2)^(1/2))*a^2+15/e^3*c^3*d^4/(a*e^2+c*d^2)^2*(c*(d/e+x)^
2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*a-15/2/e^7*c^5*d^8/(a*e^2+c*d^2)
^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*
e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)
)/(d/e+x))+1/2/e/(a*e^2+c*d^2)*c*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)
/e^2)^(5/2)-1/2/e/(a*e^2+c*d^2)/(d/e+x)^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e
^2+c*d^2)/e^2)^(7/2)+5/6/e/(a*e^2+c*d^2)*c*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*
e^2+c*d^2)/e^2)^(3/2)*a+5/e^3/(a*e^2+c*d^2)*c^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x
)+(a*e^2+c*d^2)/e^2)^(1/2)*a*d^2-5/2/e/(a*e^2+c*d^2)*c/((a*e^2+c*d^2)/e^2)^
(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(
c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a^3-5/2/e^7/
```

```
(a*e^2+c*d^2)*c^4/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e
*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*
d^2)/e^2)^(1/2))/(d/e+x))*d^6-5/8/e^2/(a*e^2+c*d^2)*c^2*d*(c*(d/e+x)^2-2*c*
d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)*x-5/4/e^4/(a*e^2+c*d^2)*c^3*d^3*(c*(d/
e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x-105/16/e^2*c^3*d^3/(a*e^2
+c*d^2)^2*a*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x-15/2/e*
c^2*d^2/(a*e^2+c*d^2)^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2
*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*
e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a^3-45/2/e^3*c^3*d^4/(a*e^2+c*d^2)^2/((a*e^
2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2
)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x)
)*a^2-45/2/e^5*c^4*d^6/(a*e^2+c*d^2)^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e
^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*
d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a-35/16/e^2/(a*e^2+c*d^2)*c^
2*d*a*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x-15/2/e^3/(a*e
^2+c*d^2)*c^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/
e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)
/e^2)^(1/2))/(d/e+x))*a^2*d^2-15/2/e^5/(a*e^2+c*d^2)*c^3/((a*e^2+c*d^2)/e^2
)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)
*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a*d^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 15.4451, size = 3295, normalized size = 15.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] [1/12*(15*(4*c^2*d^5 + 3*a*c*d^3*e^2 + (4*c^2*d^3*e^2 + 3*a*c*d*e^4)*x^2 +
2*(4*c^2*d^4*e + 3*a*c*d^2*e^3)*x)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)
*sqrt(c)*x - a) + 15*(4*c^2*d^4 + a*c*d^2*e^2 + (4*c^2*d^2*e^2 + a*c*e^4)*x
^2 + 2*(4*c^2*d^3*e + a*c*d*e^3)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x -
a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*
d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(2*c^2*e^5*x^4 -
 5*c^2*d*e^4*x^3 + 60*c^2*d^4*e + 35*a*c*d^2*e^3 - 3*a^2*e^5 + 2*(10*c^2*d^
2*e^3 + 7*a*c*e^5)*x^2 + 5*(18*c^2*d^3*e^2 + 11*a*c*d*e^4)*x)*sqrt(c*x^2 +
a))/(e^8*x^2 + 2*d*e^7*x + d^2*e^6), 1/12*(30*(4*c^2*d^5 + 3*a*c*d^3*e^2 +
(4*c^2*d^3*e^2 + 3*a*c*d*e^4)*x^2 + 2*(4*c^2*d^4*e + 3*a*c*d^2*e^3)*x)*sqrt
(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + 15*(4*c^2*d^4 + a*c*d^2*e^2 + (4*
c^2*d^2*e^2 + a*c*e^4)*x^2 + 2*(4*c^2*d^3*e + a*c*d*e^3)*x)*sqrt(c*d^2 + a*
e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2
*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^
2)) + 2*(2*c^2*e^5*x^4 - 5*c^2*d*e^4*x^3 + 60*c^2*d^4*e + 35*a*c*d^2*e^3 -
3*a^2*e^5 + 2*(10*c^2*d^2*e^3 + 7*a*c*e^5)*x^2 + 5*(18*c^2*d^3*e^2 + 11*a*c
```

```
*d*e^4)*x)*sqrt(c*x^2 + a))/(e^8*x^2 + 2*d*e^7*x + d^2*e^6), -1/12*(30*(4*c
^2*d^4 + a*c*d^2*e^2 + (4*c^2*d^2*e^2 + a*c*e^4)*x^2 + 2*(4*c^2*d^3*e + a*c
*d*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*s
qrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 15*(4*c^2*d
^5 + 3*a*c*d^3*e^2 + (4*c^2*d^3*e^2 + 3*a*c*d*e^4)*x^2 + 2*(4*c^2*d^4*e + 3
*a*c*d^2*e^3)*x)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) -
2*(2*c^2*e^5*x^4 - 5*c^2*d*e^4*x^3 + 60*c^2*d^4*e + 35*a*c*d^2*e^3 - 3*a^2*
e^5 + 2*(10*c^2*d^2*e^3 + 7*a*c*e^5)*x^2 + 5*(18*c^2*d^3*e^2 + 11*a*c*d*e^4
)*x)*sqrt(c*x^2 + a))/(e^8*x^2 + 2*d*e^7*x + d^2*e^6), -1/6*(15*(4*c^2*d^4
+ a*c*d^2*e^2 + (4*c^2*d^2*e^2 + a*c*e^4)*x^2 + 2*(4*c^2*d^3*e + a*c*d*e^3)
*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x
^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 15*(4*c^2*d^5 + 3*
a*c*d^3*e^2 + (4*c^2*d^3*e^2 + 3*a*c*d*e^4)*x^2 + 2*(4*c^2*d^4*e + 3*a*c*d^
2*e^3)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (2*c^2*e^5*x^4 - 5*
c^2*d*e^4*x^3 + 60*c^2*d^4*e + 35*a*c*d^2*e^3 - 3*a^2*e^5 + 2*(10*c^2*d^2*e
^3 + 7*a*c*e^5)*x^2 + 5*(18*c^2*d^3*e^2 + 11*a*c*d*e^4)*x)*sqrt(c*x^2 + a))
/(e^8*x^2 + 2*d*e^7*x + d^2*e^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + cx^2\right)^{\frac{5}{2}}}{\left(d + ex\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(5/2)/(e*x+d)**3,x)
```

```
[Out] Integral((a + c*x**2)**(5/2)/(d + e*x)**3, x)
```

Giac [B] time = 2.05407, size = 703, normalized size = 3.3

$$\frac{5}{2}\left(4\,c^{\frac{5}{2}}d^3 + 3\,ac^{\frac{3}{2}}de^2\right)e^{(-6)}\log\left(\left|-\sqrt{c}x + \sqrt{cx^2 + a}\right|\right) + \frac{5\left(4\,c^3d^4 + 5\,ac^2d^2e^2 + a^2ce^4\right)\arctan\left(-\frac{\left(\sqrt{c}x - \sqrt{cx^2 + a}\right)e + \sqrt{c}d}{\sqrt{-cd^2 - ae^2}}\right)e^{(-6)}}{\sqrt{-cd^2 - ae^2}} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 5/2*(4*c^(5/2)*d^3 + 3*a*c^(3/2)*d*e^2)*e^(-6)*log(abs(-sqrt(c)*x + sqrt(c*
x^2 + a))) + 5*(4*c^3*d^4 + 5*a*c^2*d^2*e^2 + a^2*c*e^4)*arctan(-((sqrt(c)*
x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2))*e^(-6)/sqrt(-c*d^
2 - a*e^2) + 1/6*sqrt(c*x^2 + a)*((2*c^2*x*e^(-3) - 9*c^2*d*e^(-4))*x + 2*(
18*c^3*d^2*e^13 + 7*a*c^2*e^15)*e^(-18)/c) + (10*(sqrt(c)*x - sqrt(c*x^2 +
a))^3*c^3*d^4*e + 18*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(7/2)*d^5 - 26*(sqrt
(c)*x - sqrt(c*x^2 + a))*a*c^3*d^4*e + 9*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*
c^(5/2)*d^3*e^2 + 11*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c^2*d^2*e^3 + 9*a^2*
c^(5/2)*d^3*e^2 - 25*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c^2*d^2*e^3 - 9*(sqr
t(c)*x - sqrt(c*x^2 + a))^2*a^2*c^(3/2)*d*e^4 + (sqrt(c)*x - sqrt(c*x^2 + a
))^3*a^2*c*e^5 + 9*a^3*c^(3/2)*d*e^4 + (sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c*
e^5)*e^(-6)/((sqrt(c)*x - sqrt(c*x^2 + a))^2*e + 2*(sqrt(c)*x - sqrt(c*x^2
+ a))*sqrt(c)*d - a*e)^2
```

3.552 $\int \frac{(a+cx^2)^{5/2}}{(d+ex)^4} dx$

Optimal. Leaf size=222

$$\frac{5c^{3/2}\left(ae^2+4cd^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2e^6}+\frac{5c^2d\left(3ae^2+4cd^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{2e^6\sqrt{ae^2+cd^2}}-\frac{5c\sqrt{a+cx^2}\left(ae^2+4cd^2+2cdex\right)}{2e^5(d+ex)}$$

```
[Out] (-5*c*(4*c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a + c*x^2])/(2*e^5*(d + e*x)) + (5
*c*(2*d + e*x)*(a + c*x^2)^(3/2))/(6*e^3*(d + e*x)^2) - (a + c*x^2)^(5/2)/(
3*e*(d + e*x)^3) + (5*c^(3/2)*(4*c*d^2 + a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a
+ c*x^2]])/(2*e^6) + (5*c^2*d*(4*c*d^2 + 3*a*e^2)*ArcTanh[(a*e - c*d*x)/(Sq
rt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(2*e^6*Sqrt[c*d^2 + a*e^2])
```

Rubi [A] time = 0.217984, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {733, 813, 844, 217, 206, 725}

$$\frac{5c^{3/2}\left(ae^2+4cd^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2e^6}+\frac{5c^2d\left(3ae^2+4cd^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{2e^6\sqrt{ae^2+cd^2}}-\frac{5c\sqrt{a+cx^2}\left(ae^2+4cd^2+2cdex\right)}{2e^5(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(5/2)/(d + e*x)^4,x]
```


```
[Out] (-5*c*(4*c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a + c*x^2])/(2*e^5*(d + e*x)) + (5
*c*(2*d + e*x)*(a + c*x^2)^(3/2))/(6*e^3*(d + e*x)^2) - (a + c*x^2)^(5/2)/(
3*e*(d + e*x)^3) + (5*c^(3/2)*(4*c*d^2 + a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a
+ c*x^2]])/(2*e^6) + (5*c^2*d*(4*c*d^2 + 3*a*e^2)*ArcTanh[(a*e - c*d*x)/(Sq
rt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(2*e^6*Sqrt[c*d^2 + a*e^2])
```


Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{(a+cx^2)^{5/2}}{(d+ex)^4}\,dx = -\frac{(a+cx^2)^{5/2}}{3e(d+ex)^3} + \frac{(5c)\int \frac{x(a+cx^2)^{3/2}}{(d+ex)^3}\,dx}{3e}$$

$$= \frac{5c(2d+ex)(a+cx^2)^{3/2}}{6e^3(d+ex)^2} - \frac{(a+cx^2)^{5/2}}{3e(d+ex)^3} - \frac{(5c)\int \frac{(-4ae+8cdx)\sqrt{a+cx^2}}{(d+ex)^2}\,dx}{8e^3}$$

$$= -\frac{5c(4cd^2+ae^2+2cdex)\sqrt{a+cx^2}}{2e^5(d+ex)} + \frac{5c(2d+ex)(a+cx^2)^{3/2}}{6e^3(d+ex)^2} - \frac{(a+cx^2)^{5/2}}{3e(d+ex)^3} + \frac{(5c)\int \frac{-16acde+8c(4cd^2}{(d+ex)\sqrt{a+c}}}{16e^5}$$

$$= -\frac{5c(4cd^2+ae^2+2cdex)\sqrt{a+cx^2}}{2e^5(d+ex)} + \frac{5c(2d+ex)(a+cx^2)^{3/2}}{6e^3(d+ex)^2} - \frac{(a+cx^2)^{5/2}}{3e(d+ex)^3} + \frac{(5c^2(4cd^2+ae^2))\int}{2e^6}$$

$$= -\frac{5c(4cd^2+ae^2+2cdex)\sqrt{a+cx^2}}{2e^5(d+ex)} + \frac{5c(2d+ex)(a+cx^2)^{3/2}}{6e^3(d+ex)^2} - \frac{(a+cx^2)^{5/2}}{3e(d+ex)^3} + \frac{(5c^2(4cd^2+ae^2))\,\mathrm{S}}{}$$

$$= -\frac{5c(4cd^2+ae^2+2cdex)\sqrt{a+cx^2}}{2e^5(d+ex)} + \frac{5c(2d+ex)(a+cx^2)^{3/2}}{6e^3(d+ex)^2} - \frac{(a+cx^2)^{5/2}}{3e(d+ex)^3} + \frac{5c^{3/2}(4cd^2+ae^2)\,\mathrm{ta}}{2e^6}$$

Mathematica [A] time = 0.304965, size = 260, normalized size = 1.17

$$\frac{-\frac{e\sqrt{a+cx^2}\left(2a^2e^4+ace^2(5d^2+15dex+14e^2x^2)+c^2(110d^2e^2x^2+150d^3ex+60d^4+15de^3x^3-3e^4x^4)\right)}{(d+ex)^3} + \frac{15c^2d(3ae^2+4cd^2)\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{\sqrt{ae^2+cd^2}} + 15c^{3/2}}{6e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(5/2)/(d + e*x)^4,x]
```

```
[Out] (-((e*Sqrt[a + c*x^2]*(2*a^2*e^4 + a*c*e^2*(5*d^2 + 15*d*e*x + 14*e^2*x^2)
+ c^2*(60*d^4 + 150*d^3*e*x + 110*d^2*e^2*x^2 + 15*d*e^3*x^3 - 3*e^4*x^4)))
/(d + e*x)^3) - (15*c^2*d*(4*c*d^2 + 3*a*e^2)*Log[d + e*x])/Sqrt[c*d^2 + a*
e^2] + 15*c^(3/2)*(4*c*d^2 + a*e^2)*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] + (1
5*c^2*d*(4*c*d^2 + 3*a*e^2)*Log[a*e - c*d*x + Sqrt[c*d^2 + a*e^2]*Sqrt[a +
```

```
c*x^2]])/Sqrt[c*d^2 + a*e^2])/(6*e^6)
```

Maple [B] time = 0.196, size = 3789, normalized size = 17.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(5/2)/(e*x+d)^4,x)
```

```
[Out] 1/2*c^3*d^2/(a*e^2+c*d^2)^3*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)
^(5/2)*x+5/2/(a*e^2+c*d^2)^2*c^(3/2)*a^3*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(
d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))-4/3/(a*e^2+c*d^2)^2*c/(d
/e+x)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(7/2)+4/3/(a*e^2+c*d^
2)^2*c^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(5/2)*x-1/3/e^2/(a
*e^2+c*d^2)/(d/e+x)^3*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(7/2)
-15/2/e^5*c^4*d^5/(a*e^2+c*d^2)^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2
)/e^2)^(1/2)+15/2/e^6*c^(9/2)*d^6/(a*e^2+c*d^2)^2*ln((-c*d/e+(d/e+x)*c)/c^(
1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))+5/2/e^6*c^(11/2
)*d^8/(a*e^2+c*d^2)^3*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d
/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))-1/2/e*c^3*d^3/(a*e^2+c*d^2)^3*(c*(d/e+x)^2-
2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(5/2)-5/6/e^3*c^4*d^5/(a*e^2+c*d^2)^3*(c
*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)-5/2/e^5*c^5*d^7/(a*e^2+
c*d^2)^3*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)-3/2/e*c^2*d/
(a*e^2+c*d^2)^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(5/2)-5/2/e
^3*c^3*d^3/(a*e^2+c*d^2)^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^
(3/2)+15/16*c^(5/2)*d^2/(a*e^2+c*d^2)^3*a^3*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(
c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))+5/3/(a*e^2+c*d^2)^2*c
^2*a*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)*x+5/2/(a*e^2+c*d
^2)^2*c^2*a^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x-1/2*c
^2*d^2/(a*e^2+c*d^2)^3/(d/e+x)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e
^2)^(7/2)+35/16/e^2*c^4*d^4/(a*e^2+c*d^2)^3*a*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+
(a*e^2+c*d^2)/e^2)^(1/2)*x+5/2/e*c^3*d^3/(a*e^2+c*d^2)^3/((a*e^2+c*d^2)/e^2
)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)
*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a^3+15/2/e
^3*c^4*d^5/(a*e^2+c*d^2)^3/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^
2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+
(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a^2+15/2/e^5*c^5*d^7/(a*e^2+c*d^2)^3/((a
*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*
d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e
+x))*a+45/2/e^3*c^3*d^3/(a*e^2+c*d^2)^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*
e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c
*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a^2+45/2/e^5*c^4*d^5/(a*e^2
+c*d^2)^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)
+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2
)^(1/2))/(d/e+x))*a+15/2/e*c^2*d/(a*e^2+c*d^2)^2/((a*e^2+c*d^2)/e^2)^(1/2)*
ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e
+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a^3+105/16/e^2*c^3
*d^2/(a*e^2+c*d^2)^2*a*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2
)*x+25/4/e^4*c^(9/2)*d^6/(a*e^2+c*d^2)^3*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(
d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))*a+5/4/e^4*c^5*d^6/(a*e^2
+c*d^2)^3*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x-5/2/e*c^3
*d^3/(a*e^2+c*d^2)^3*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*
a^2-5/e^3*c^4*d^5/(a*e^2+c*d^2)^3*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2
)/e^2)^(1/2)*a+5/2/e^7*c^6*d^9/(a*e^2+c*d^2)^3/((a*e^2+c*d^2)/e^2)^(1/2)*ln
((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x
)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))+5/8*c^3*d^2/(a*e^2+c
```

```
*d^2)^3*a*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)*x+15/16*c^3
*d^2/(a*e^2+c*d^2)^3*a^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1
/2)*x+15/8/e^2*c^3*d^2/(a*e^2+c*d^2)^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+
c*d^2)/e^2)^(3/2)*x+75/16/e^2*c^(7/2)*d^4/(a*e^2+c*d^2)^3*ln((-c*d/e+(d/e+x
)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))*a^2+225
/16/e^2*c^(5/2)*d^2/(a*e^2+c*d^2)^2*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+(c*(d/e+x
)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))*a^2+5/8/e^2*c^4*d^4/(a*e^2+c*
d^2)^3*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)*x-5/6/e*c^3*d^
3/(a*e^2+c*d^2)^3*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)*a+1
5/4/e^4*c^4*d^4/(a*e^2+c*d^2)^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/
e^2)^(1/2)*x-15/e^3*c^3*d^3/(a*e^2+c*d^2)^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a
*e^2+c*d^2)/e^2)^(1/2)*a+1/6/e*c*d/(a*e^2+c*d^2)^2/(d/e+x)^2*(c*(d/e+x)^2-2
*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(7/2)-5/2/e*c^2*d/(a*e^2+c*d^2)^2*(c*(d/e
+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)*a-15/2/e*c^2*d/(a*e^2+c*d^2)
^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*a^2+15/2/e^7*c^5*d
^7/(a*e^2+c*d^2)^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/
e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c
*d^2)/e^2)^(1/2))/(d/e+x))+75/4/e^4*c^(7/2)*d^4/(a*e^2+c*d^2)^2*ln((-c*d/e+
(d/e+x)*c)/c^(1/2)+(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 26.5249, size = 5146, normalized size = 23.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] [1/12*(15*(4*c^3*d^7 + 5*a*c^2*d^5*e^2 + a^2*c*d^3*e^4 + (4*c^3*d^4*e^3 + 5
*a*c^2*d^2*e^5 + a^2*c*e^7)*x^3 + 3*(4*c^3*d^5*e^2 + 5*a*c^2*d^3*e^4 + a^2*
c*d*e^6)*x^2 + 3*(4*c^3*d^6*e + 5*a*c^2*d^4*e^3 + a^2*c*d^2*e^5)*x)*sqrt(c)
*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 15*(4*c^3*d^6 + 3*a*c^2*
d^4*e^2 + (4*c^3*d^3*e^3 + 3*a*c^2*d*e^5)*x^3 + 3*(4*c^3*d^4*e^2 + 3*a*c^2*
d^2*e^4)*x^2 + 3*(4*c^3*d^5*e + 3*a*c^2*d^3*e^3)*x)*sqrt(c*d^2 + a*e^2)*log
((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*
d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*
(60*c^3*d^6*e + 65*a*c^2*d^4*e^3 + 7*a^2*c*d^2*e^5 + 2*a^3*e^7 - 3*(c^3*d^2
*e^5 + a*c^2*e^7)*x^4 + 15*(c^3*d^3*e^4 + a*c^2*d*e^6)*x^3 + 2*(55*c^3*d^4*
e^3 + 62*a*c^2*d^2*e^5 + 7*a^2*c*e^7)*x^2 + 15*(10*c^3*d^5*e^2 + 11*a*c^2*d
^3*e^4 + a^2*c*d*e^6)*x)*sqrt(c*x^2 + a))/(c*d^5*e^6 + a*d^3*e^8 + (c*d^2*e
^9 + a*e^11)*x^3 + 3*(c*d^3*e^8 + a*d*e^10)*x^2 + 3*(c*d^4*e^7 + a*d^2*e^9)
*x), 1/12*(30*(4*c^3*d^6 + 3*a*c^2*d^4*e^2 + (4*c^3*d^3*e^3 + 3*a*c^2*d*e^5
)*x^3 + 3*(4*c^3*d^4*e^2 + 3*a*c^2*d^2*e^4)*x^2 + 3*(4*c^3*d^5*e + 3*a*c^2*
d^3*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*
sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + 15*(4*c^3*
```

```
d^7 + 5*a*c^2*d^5*e^2 + a^2*c*d^3*e^4 + (4*c^3*d^4*e^3 + 5*a*c^2*d^2*e^5 +
a^2*c*e^7)*x^3 + 3*(4*c^3*d^5*e^2 + 5*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x^2 + 3*
(4*c^3*d^6*e + 5*a*c^2*d^4*e^3 + a^2*c*d^2*e^5)*x)*sqrt(c)*log(-2*c*x^2 - 2
*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(60*c^3*d^6*e + 65*a*c^2*d^4*e^3 + 7*a^
2*c*d^2*e^5 + 2*a^3*e^7 - 3*(c^3*d^2*e^5 + a*c^2*e^7)*x^4 + 15*(c^3*d^3*e^4
 + a*c^2*d*e^6)*x^3 + 2*(55*c^3*d^4*e^3 + 62*a*c^2*d^2*e^5 + 7*a^2*c*e^7)*x
^2 + 15*(10*c^3*d^5*e^2 + 11*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x)*sqrt(c*x^2 + a
))/(c*d^5*e^6 + a*d^3*e^8 + (c*d^2*e^9 + a*e^11)*x^3 + 3*(c*d^3*e^8 + a*d*e
^10)*x^2 + 3*(c*d^4*e^7 + a*d^2*e^9)*x), -1/12*(30*(4*c^3*d^7 + 5*a*c^2*d^5
*e^2 + a^2*c*d^3*e^4 + (4*c^3*d^4*e^3 + 5*a*c^2*d^2*e^5 + a^2*c*e^7)*x^3 +
3*(4*c^3*d^5*e^2 + 5*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x^2 + 3*(4*c^3*d^6*e + 5*
a*c^2*d^4*e^3 + a^2*c*d^2*e^5)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a
)) - 15*(4*c^3*d^6 + 3*a*c^2*d^4*e^2 + (4*c^3*d^3*e^3 + 3*a*c^2*d*e^5)*x^3
+ 3*(4*c^3*d^4*e^2 + 3*a*c^2*d^2*e^4)*x^2 + 3*(4*c^3*d^5*e + 3*a*c^2*d^3*e^
3)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d
^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e
^2*x^2 + 2*d*e*x + d^2)) + 2*(60*c^3*d^6*e + 65*a*c^2*d^4*e^3 + 7*a^2*c*d^2
*e^5 + 2*a^3*e^7 - 3*(c^3*d^2*e^5 + a*c^2*e^7)*x^4 + 15*(c^3*d^3*e^4 + a*c^
2*d*e^6)*x^3 + 2*(55*c^3*d^4*e^3 + 62*a*c^2*d^2*e^5 + 7*a^2*c*e^7)*x^2 + 15
*(10*c^3*d^5*e^2 + 11*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x)*sqrt(c*x^2 + a))/(c*d
^5*e^6 + a*d^3*e^8 + (c*d^2*e^9 + a*e^11)*x^3 + 3*(c*d^3*e^8 + a*d*e^10)*x^
2 + 3*(c*d^4*e^7 + a*d^2*e^9)*x), 1/6*(15*(4*c^3*d^6 + 3*a*c^2*d^4*e^2 + (4
*c^3*d^3*e^3 + 3*a*c^2*d*e^5)*x^3 + 3*(4*c^3*d^4*e^2 + 3*a*c^2*d^2*e^4)*x^2
 + 3*(4*c^3*d^5*e + 3*a*c^2*d^3*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c
*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 +
 a*c*e^2)*x^2)) - 15*(4*c^3*d^7 + 5*a*c^2*d^5*e^2 + a^2*c*d^3*e^4 + (4*c^3*
d^4*e^3 + 5*a*c^2*d^2*e^5 + a^2*c*e^7)*x^3 + 3*(4*c^3*d^5*e^2 + 5*a*c^2*d^3
*e^4 + a^2*c*d*e^6)*x^2 + 3*(4*c^3*d^6*e + 5*a*c^2*d^4*e^3 + a^2*c*d^2*e^5)
*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (60*c^3*d^6*e + 65*a*c^2*
d^4*e^3 + 7*a^2*c*d^2*e^5 + 2*a^3*e^7 - 3*(c^3*d^2*e^5 + a*c^2*e^7)*x^4 + 1
5*(c^3*d^3*e^4 + a*c^2*d*e^6)*x^3 + 2*(55*c^3*d^4*e^3 + 62*a*c^2*d^2*e^5 +
7*a^2*c*e^7)*x^2 + 15*(10*c^3*d^5*e^2 + 11*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x)*
sqrt(c*x^2 + a))/(c*d^5*e^6 + a*d^3*e^8 + (c*d^2*e^9 + a*e^11)*x^3 + 3*(c*d
^3*e^8 + a*d*e^10)*x^2 + 3*(c*d^4*e^7 + a*d^2*e^9)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+cx^2\right)^{\frac{5}{2}}}{\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(5/2)/(e*x+d)**4,x)
```

```
[Out] Integral((a + c*x**2)**(5/2)/(d + e*x)**4, x)
```

Giac [B] time = 2.18146, size = 776, normalized size = 3.5

$$-\frac{5}{2}\left(4\,c^{\frac{5}{2}}d^2+ac^{\frac{3}{2}}e^2\right)e^{(-6)}\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)-\frac{5\left(4\,c^3d^3+3\,ac^2de^2\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)e^{(-6)}}{\sqrt{-cd^2-ae^2}}+\frac{1}{2}\left(c^2xe\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -5/2*(4*c^(5/2)*d^2 + a*c^(3/2)*e^2)*e^(-6)*log(abs(-sqrt(c)*x + sqrt(c*x^2
 + a))) - 5*(4*c^3*d^3 + 3*a*c^2*d*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 +
a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2))*e^(-6)/sqrt(-c*d^2 - a*e^2) + 1/2*
(c^2*x*e^(-4) - 8*c^2*d*e^(-5))*sqrt(c*x^2 + a) - 1/3*(210*(sqrt(c)*x - sqr
t(c*x^2 + a))^4*c^(7/2)*d^4*e + 188*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^4*d^5
 + 60*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^3*d^3*e^2 - 354*(sqrt(c)*x - sqrt(c
*x^2 + a))^2*a*c^(7/2)*d^4*e - 226*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c^3*d^
3*e^2 + 27*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(5/2)*d^2*e^3 + 27*(sqrt(c)*
x - sqrt(c*x^2 + a))^5*a*c^2*d*e^4 + 222*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*
c^3*d^3*e^2 - 84*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^2*d*e^4 - 18*(sqrt(c
)*x - sqrt(c*x^2 + a))^4*a^2*c^(3/2)*e^5 - 47*a^3*c^(5/2)*d^2*e^3 + 57*(sqr
t(c)*x - sqrt(c*x^2 + a))*a^3*c^2*d*e^4 + 24*(sqrt(c)*x - sqrt(c*x^2 + a))^
2*a^3*c^(3/2)*e^5 - 14*a^4*c^(3/2)*e^5)*e^(-6)/((sqrt(c)*x - sqrt(c*x^2 + a
))^2*e + 2*(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*d - a*e)^3
```

3.553 $\int \frac{\left(a+cx^2\right)^{5/2}}{(d+ex)^5}\,dx$

Optimal. Leaf size=287

$$-\frac{5c^2\left(3a^2e^4+12acd^2e^2+8c^2d^4\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{8e^6\left(ae^2+cd^2\right)^{3/2}}+\frac{5c^2\sqrt{a+cx^2}\left(ex\left(3ae^2+4cd^2\right)+8d\left(ae^2+cd^2\right)\right)}{8e^5(d+ex)\left(ae^2+cd^2\right)}-\frac{5c^{5/2}d}{}$$

```
[Out] (5*c^2*(8*d*(c*d^2 + a*e^2) + e*(4*c*d^2 + 3*a*e^2)*x)*Sqrt[a + c*x^2])/(8*
e^5*(c*d^2 + a*e^2)*(d + e*x)) - (5*c*(d*(4*c*d^2 + a*e^2) + 3*e*(2*c*d^2 +
 a*e^2)*x)*(a + c*x^2)^(3/2))/(24*e^3*(c*d^2 + a*e^2)*(d + e*x)^3) - (a + c
*x^2)^(5/2)/(4*e*(d + e*x)^4) - (5*c^(5/2)*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c
*x^2]])/e^6 - (5*c^2*(8*c^2*d^4 + 12*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTanh[(a*e
- c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(8*e^6*(c*d^2 + a*e^2)^(3/
2))
```

Rubi [A] time = 0.302573, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {733, 811, 813, 844, 217, 206, 725}

$$-\frac{5c^2\left(3a^2e^4+12acd^2e^2+8c^2d^4\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{8e^6\left(ae^2+cd^2\right)^{3/2}}+\frac{5c^2\sqrt{a+cx^2}\left(ex\left(3ae^2+4cd^2\right)+8d\left(ae^2+cd^2\right)\right)}{8e^5(d+ex)\left(ae^2+cd^2\right)}-\frac{5c^{5/2}d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(5/2)/(d + e*x)^5,x]
```

```
[Out] (5*c^2*(8*d*(c*d^2 + a*e^2) + e*(4*c*d^2 + 3*a*e^2)*x)*Sqrt[a + c*x^2])/(8*
e^5*(c*d^2 + a*e^2)*(d + e*x)) - (5*c*(d*(4*c*d^2 + a*e^2) + 3*e*(2*c*d^2 +
 a*e^2)*x)*(a + c*x^2)^(3/2))/(24*e^3*(c*d^2 + a*e^2)*(d + e*x)^3) - (a + c
*x^2)^(5/2)/(4*e*(d + e*x)^4) - (5*c^(5/2)*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c
*x^2]])/e^6 - (5*c^2*(8*c^2*d^4 + 12*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTanh[(a*e
- c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(8*e^6*(c*d^2 + a*e^2)^(3/
2))
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
```

```
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{5/2}}{(d+ex)^5}\,dx = -\frac{\left(a+cx^2\right)^{5/2}}{4e(d+ex)^4} + \frac{(5c)\int \frac{x\left(a+cx^2\right)^{3/2}}{(d+ex)^4}dx}{4e}$$

$$= -\frac{5c\left(d\left(4cd^2+ae^2\right)+3e\left(2cd^2+ae^2\right)x\right)\left(a+cx^2\right)^{3/2}}{24e^3\left(cd^2+ae^2\right)(d+ex)^3} - \frac{\left(a+cx^2\right)^{5/2}}{4e(d+ex)^4} - \frac{(5c)\int \frac{\left(4acde-2c\left(4cd^2+3ae^2\right)x\right)\sqrt{}}{(d+ex)^2}}{16e^3\left(cd^2+ae^2\right)}$$

$$= \frac{5c^2\left(8d\left(cd^2+ae^2\right)+e\left(4cd^2+3ae^2\right)x\right)\sqrt{a+cx^2}}{8e^5\left(cd^2+ae^2\right)(d+ex)} - \frac{5c\left(d\left(4cd^2+ae^2\right)+3e\left(2cd^2+ae^2\right)x\right)\left(a+c\right.}{24e^3\left(cd^2+ae^2\right)(d+ex)^3}$$

$$= \frac{5c^2\left(8d\left(cd^2+ae^2\right)+e\left(4cd^2+3ae^2\right)x\right)\sqrt{a+cx^2}}{8e^5\left(cd^2+ae^2\right)(d+ex)} - \frac{5c\left(d\left(4cd^2+ae^2\right)+3e\left(2cd^2+ae^2\right)x\right)\left(a+c\right.}{24e^3\left(cd^2+ae^2\right)(d+ex)^3}$$

$$= \frac{5c^2\left(8d\left(cd^2+ae^2\right)+e\left(4cd^2+3ae^2\right)x\right)\sqrt{a+cx^2}}{8e^5\left(cd^2+ae^2\right)(d+ex)} - \frac{5c\left(d\left(4cd^2+ae^2\right)+3e\left(2cd^2+ae^2\right)x\right)\left(a+c\right.}{24e^3\left(cd^2+ae^2\right)(d+ex)^3}$$

$$= \frac{5c^2\left(8d\left(cd^2+ae^2\right)+e\left(4cd^2+3ae^2\right)x\right)\sqrt{a+cx^2}}{8e^5\left(cd^2+ae^2\right)(d+ex)} - \frac{5c\left(d\left(4cd^2+ae^2\right)+3e\left(2cd^2+ae^2\right)x\right)\left(a+c\right.}{24e^3\left(cd^2+ae^2\right)(d+ex)^3}$$

Mathematica [A] time = 0.545202, size = 299, normalized size = 1.04

$$\frac{-\frac{15c^2\left(3a^2e^4+12acd^2e^2+8c^2d^4\right)\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{\left(ae^2+cd^2\right)^{3/2}} + \frac{15c^2\left(3a^2e^4+12acd^2e^2+8c^2d^4\right)\log(d+ex)}{\left(ae^2+cd^2\right)^{3/2}} + e\sqrt{a+cx^2}\left(\frac{c^2d\left(139ae^2+154cd^2\right)}{(d+ex)\left(ae^2+cd^2\right)} - \frac{c(2}{}\right.}{24e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(5/2)/(d + e*x)^5,x]
```

```
[Out] (e*Sqrt[a + c*x^2]*(24*c^2 - (6*(c*d^2 + a*e^2)^2)/(d + e*x)^4 + (34*c*d*(c
*d^2 + a*e^2))/(d + e*x)^3 - (c*(86*c*d^2 + 27*a*e^2))/(d + e*x)^2 + (c^2*d
*(154*c*d^2 + 139*a*e^2))/((c*d^2 + a*e^2)*(d + e*x))) + (15*c^2*(8*c^2*d^4
 + 12*a*c*d^2*e^2 + 3*a^2*e^4)*Log[d + e*x])/(c*d^2 + a*e^2)^(3/2) - 120*c^
(5/2)*d*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] - (15*c^2*(8*c^2*d^4 + 12*a*c*d^
2*e^2 + 3*a^2*e^4)*Log[a*e - c*d*x + Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2]])/
(c*d^2 + a*e^2)^(3/2))/(24*e^6)
```

Maple [B] time = 0.201, size = 5406, normalized size = 18.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(5/2)/(e*x+d)^5,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 103.694, size = 7640, normalized size = 26.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^5,x, algorithm="fricas")
```

```
[Out] [1/48*(120*(c^4*d^9 + 2*a*c^3*d^7*e^2 + a^2*c^2*d^5*e^4 + (c^4*d^5*e^4 + 2*
a*c^3*d^3*e^6 + a^2*c^2*d*e^8)*x^4 + 4*(c^4*d^6*e^3 + 2*a*c^3*d^4*e^5 + a^2
*c^2*d^2*e^7)*x^3 + 6*(c^4*d^7*e^2 + 2*a*c^3*d^5*e^4 + a^2*c^2*d^3*e^6)*x^2
 + 4*(c^4*d^8*e + 2*a*c^3*d^6*e^3 + a^2*c^2*d^4*e^5)*x)*sqrt(c)*log(-2*c*x^
2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 15*(8*c^4*d^8 + 12*a*c^3*d^6*e^2 + 3
*a^2*c^2*d^4*e^4 + (8*c^4*d^4*e^4 + 12*a*c^3*d^2*e^6 + 3*a^2*c^2*e^8)*x^4 +
 4*(8*c^4*d^5*e^3 + 12*a*c^3*d^3*e^5 + 3*a^2*c^2*d*e^7)*x^3 + 6*(8*c^4*d^6*
e^2 + 12*a*c^3*d^4*e^4 + 3*a^2*c^2*d^2*e^6)*x^2 + 4*(8*c^4*d^7*e + 12*a*c^3
*d^5*e^3 + 3*a^2*c^2*d^3*e^5)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c
*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x
 - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(120*c^4*d^8*e + 22
0*a*c^3*d^6*e^3 + 89*a^2*c^2*d^4*e^5 - 17*a^3*c*d^2*e^7 - 6*a^4*e^9 + 24*(c
^4*d^4*e^5 + 2*a*c^3*d^2*e^7 + a^2*c^2*e^9)*x^4 + 5*(50*c^4*d^5*e^4 + 97*a*
c^3*d^3*e^6 + 47*a^2*c^2*d*e^8)*x^3 + (520*c^4*d^6*e^3 + 968*a*c^3*d^4*e^5
+ 421*a^2*c^2*d^2*e^7 - 27*a^3*c*e^9)*x^2 + 5*(84*c^4*d^7*e^2 + 155*a*c^3*d
^5*e^4 + 67*a^2*c^2*d^3*e^6 - 4*a^3*c*d*e^8)*x)*sqrt(c*x^2 + a))/(c^2*d^8*e
^6 + 2*a*c*d^6*e^8 + a^2*d^4*e^10 + (c^2*d^4*e^10 + 2*a*c*d^2*e^12 + a^2*e^
14)*x^4 + 4*(c^2*d^5*e^9 + 2*a*c*d^3*e^11 + a^2*d*e^13)*x^3 + 6*(c^2*d^6*e^
8 + 2*a*c*d^4*e^10 + a^2*d^2*e^12)*x^2 + 4*(c^2*d^7*e^7 + 2*a*c*d^5*e^9 + a
^2*d^3*e^11)*x), 1/48*(240*(c^4*d^9 + 2*a*c^3*d^7*e^2 + a^2*c^2*d^5*e^4 + (
c^4*d^5*e^4 + 2*a*c^3*d^3*e^6 + a^2*c^2*d*e^8)*x^4 + 4*(c^4*d^6*e^3 + 2*a*c
^3*d^4*e^5 + a^2*c^2*d^2*e^7)*x^3 + 6*(c^4*d^7*e^2 + 2*a*c^3*d^5*e^4 + a^2*
c^2*d^3*e^6)*x^2 + 4*(c^4*d^8*e + 2*a*c^3*d^6*e^3 + a^2*c^2*d^4*e^5)*x)*sqr
t(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + 15*(8*c^4*d^8 + 12*a*c^3*d^6*e^2
 + 3*a^2*c^2*d^4*e^4 + (8*c^4*d^4*e^4 + 12*a*c^3*d^2*e^6 + 3*a^2*c^2*e^8)*x
^4 + 4*(8*c^4*d^5*e^3 + 12*a*c^3*d^3*e^5 + 3*a^2*c^2*d*e^7)*x^3 + 6*(8*c^4*
d^6*e^2 + 12*a*c^3*d^4*e^4 + 3*a^2*c^2*d^2*e^6)*x^2 + 4*(8*c^4*d^7*e + 12*a
*c^3*d^5*e^3 + 3*a^2*c^2*d^3*e^5)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x -
 a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c
*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(120*c^4*d^8*e
+ 220*a*c^3*d^6*e^3 + 89*a^2*c^2*d^4*e^5 - 17*a^3*c*d^2*e^7 - 6*a^4*e^9 + 2
4*(c^4*d^4*e^5 + 2*a*c^3*d^2*e^7 + a^2*c^2*e^9)*x^4 + 5*(50*c^4*d^5*e^4 + 9
7*a*c^3*d^3*e^6 + 47*a^2*c^2*d*e^8)*x^3 + (520*c^4*d^6*e^3 + 968*a*c^3*d^4*
e^5 + 421*a^2*c^2*d^2*e^7 - 27*a^3*c*e^9)*x^2 + 5*(84*c^4*d^7*e^2 + 155*a*c
^3*d^5*e^4 + 67*a^2*c^2*d^3*e^6 - 4*a^3*c*d*e^8)*x)*sqrt(c*x^2 + a))/(c^2*d
^8*e^6 + 2*a*c*d^6*e^8 + a^2*d^4*e^10 + (c^2*d^4*e^10 + 2*a*c*d^2*e^12 + a^
2*e^14)*x^4 + 4*(c^2*d^5*e^9 + 2*a*c*d^3*e^11 + a^2*d*e^13)*x^3 + 6*(c^2*d^
6*e^8 + 2*a*c*d^4*e^10 + a^2*d^2*e^12)*x^2 + 4*(c^2*d^7*e^7 + 2*a*c*d^5*e^9
 + a^2*d^3*e^11)*x), -1/24*(15*(8*c^4*d^8 + 12*a*c^3*d^6*e^2 + 3*a^2*c^2*d^
4*e^4 + (8*c^4*d^4*e^4 + 12*a*c^3*d^2*e^6 + 3*a^2*c^2*e^8)*x^4 + 4*(8*c^4*d
^5*e^3 + 12*a*c^3*d^3*e^5 + 3*a^2*c^2*d*e^7)*x^3 + 6*(8*c^4*d^6*e^2 + 12*a*
c^3*d^4*e^4 + 3*a^2*c^2*d^2*e^6)*x^2 + 4*(8*c^4*d^7*e + 12*a*c^3*d^5*e^3 +
3*a^2*c^2*d^3*e^5)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d
```

```
*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) -
60*(c^4*d^9 + 2*a*c^3*d^7*e^2 + a^2*c^2*d^5*e^4 + (c^4*d^5*e^4 + 2*a*c^3*d^
3*e^6 + a^2*c^2*d*e^8)*x^4 + 4*(c^4*d^6*e^3 + 2*a*c^3*d^4*e^5 + a^2*c^2*d^2
*e^7)*x^3 + 6*(c^4*d^7*e^2 + 2*a*c^3*d^5*e^4 + a^2*c^2*d^3*e^6)*x^2 + 4*(c^
4*d^8*e + 2*a*c^3*d^6*e^3 + a^2*c^2*d^4*e^5)*x)*sqrt(c)*log(-2*c*x^2 + 2*sq
rt(c*x^2 + a)*sqrt(c)*x - a) - (120*c^4*d^8*e + 220*a*c^3*d^6*e^3 + 89*a^2*
c^2*d^4*e^5 - 17*a^3*c*d^2*e^7 - 6*a^4*e^9 + 24*(c^4*d^4*e^5 + 2*a*c^3*d^2*
e^7 + a^2*c^2*e^9)*x^4 + 5*(50*c^4*d^5*e^4 + 97*a*c^3*d^3*e^6 + 47*a^2*c^2*
d*e^8)*x^3 + (520*c^4*d^6*e^3 + 968*a*c^3*d^4*e^5 + 421*a^2*c^2*d^2*e^7 - 2
7*a^3*c*e^9)*x^2 + 5*(84*c^4*d^7*e^2 + 155*a*c^3*d^5*e^4 + 67*a^2*c^2*d^3*e
^6 - 4*a^3*c*d*e^8)*x)*sqrt(c*x^2 + a))/(c^2*d^8*e^6 + 2*a*c*d^6*e^8 + a^2*
d^4*e^10 + (c^2*d^4*e^10 + 2*a*c*d^2*e^12 + a^2*e^14)*x^4 + 4*(c^2*d^5*e^9
+ 2*a*c*d^3*e^11 + a^2*d*e^13)*x^3 + 6*(c^2*d^6*e^8 + 2*a*c*d^4*e^10 + a^2*
d^2*e^12)*x^2 + 4*(c^2*d^7*e^7 + 2*a*c*d^5*e^9 + a^2*d^3*e^11)*x), -1/24*(1
5*(8*c^4*d^8 + 12*a*c^3*d^6*e^2 + 3*a^2*c^2*d^4*e^4 + (8*c^4*d^4*e^4 + 12*a
*c^3*d^2*e^6 + 3*a^2*c^2*e^8)*x^4 + 4*(8*c^4*d^5*e^3 + 12*a*c^3*d^3*e^5 + 3
*a^2*c^2*d*e^7)*x^3 + 6*(8*c^4*d^6*e^2 + 12*a*c^3*d^4*e^4 + 3*a^2*c^2*d^2*e
^6)*x^2 + 4*(8*c^4*d^7*e + 12*a*c^3*d^5*e^3 + 3*a^2*c^2*d^3*e^5)*x)*sqrt(-c
*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*
c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 120*(c^4*d^9 + 2*a*c^3*d^7*e^
2 + a^2*c^2*d^5*e^4 + (c^4*d^5*e^4 + 2*a*c^3*d^3*e^6 + a^2*c^2*d*e^8)*x^4 +
 4*(c^4*d^6*e^3 + 2*a*c^3*d^4*e^5 + a^2*c^2*d^2*e^7)*x^3 + 6*(c^4*d^7*e^2 +
 2*a*c^3*d^5*e^4 + a^2*c^2*d^3*e^6)*x^2 + 4*(c^4*d^8*e + 2*a*c^3*d^6*e^3 +
a^2*c^2*d^4*e^5)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (120*c^4*
d^8*e + 220*a*c^3*d^6*e^3 + 89*a^2*c^2*d^4*e^5 - 17*a^3*c*d^2*e^7 - 6*a^4*e
^9 + 24*(c^4*d^4*e^5 + 2*a*c^3*d^2*e^7 + a^2*c^2*e^9)*x^4 + 5*(50*c^4*d^5*e
^4 + 97*a*c^3*d^3*e^6 + 47*a^2*c^2*d*e^8)*x^3 + (520*c^4*d^6*e^3 + 968*a*c^
3*d^4*e^5 + 421*a^2*c^2*d^2*e^7 - 27*a^3*c*e^9)*x^2 + 5*(84*c^4*d^7*e^2 + 1
55*a*c^3*d^5*e^4 + 67*a^2*c^2*d^3*e^6 - 4*a^3*c*d*e^8)*x)*sqrt(c*x^2 + a))/
(c^2*d^8*e^6 + 2*a*c*d^6*e^8 + a^2*d^4*e^10 + (c^2*d^4*e^10 + 2*a*c*d^2*e^1
2 + a^2*e^14)*x^4 + 4*(c^2*d^5*e^9 + 2*a*c*d^3*e^11 + a^2*d*e^13)*x^3 + 6*(
c^2*d^6*e^8 + 2*a*c*d^4*e^10 + a^2*d^2*e^12)*x^2 + 4*(c^2*d^7*e^7 + 2*a*c*d
^5*e^9 + a^2*d^3*e^11)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+cx^2\right)^{\frac{5}{2}}}{\left(d+ex\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(5/2)/(e*x+d)**5,x)
```

```
[Out] Integral((a + c*x**2)**(5/2)/(d + e*x)**5, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.554 $\int \frac{\left(a+cx^2\right)^{5/2}}{(d+ex)^6}\,dx$

Optimal. Leaf size=314

$$-\frac{c^2\sqrt{a+cx^2}\left(ex\left(8a^2e^4+23acd^2e^2+12c^2d^4\right)+d\left(a^2e^4+12acd^2e^2+8c^2d^4\right)\right)}{8e^5(d+ex)^2\left(ae^2+cd^2\right)^2}+\frac{c^3d\left(15a^2e^4+20acd^2e^2+8c^2d^4\right)\tanh^{-1}}{8e^6\left(ae^2+cd^2\right)^{5/2}}$$

```
[Out] -(c^2*(d*(8*c^2*d^4 + 12*a*c*d^2*e^2 + a^2*e^4) + e*(12*c^2*d^4 + 23*a*c*d^
2*e^2 + 8*a^2*e^4)*x)*Sqrt[a + c*x^2])/(8*e^5*(c*d^2 + a*e^2)^2*(d + e*x)^2
) - (c*(d*(4*c*d^2 + a*e^2) + e*(7*c*d^2 + 4*a*e^2)*x)*(a + c*x^2)^(3/2))/(
12*e^3*(c*d^2 + a*e^2)*(d + e*x)^4) - (a + c*x^2)^(5/2)/(5*e*(d + e*x)^5) +
 (c^(5/2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/e^6 + (c^3*d*(8*c^2*d^4 + 2
0*a*c*d^2*e^2 + 15*a^2*e^4)*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt
[a + c*x^2])])/(8*e^6*(c*d^2 + a*e^2)^(5/2))
```

Rubi [A] time = 0.343272, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {733, 811, 844, 217, 206, 725}

$$-\frac{c^2\sqrt{a+cx^2}\left(ex\left(8a^2e^4+23acd^2e^2+12c^2d^4\right)+d\left(a^2e^4+12acd^2e^2+8c^2d^4\right)\right)}{8e^5(d+ex)^2\left(ae^2+cd^2\right)^2}+\frac{c^3d\left(15a^2e^4+20acd^2e^2+8c^2d^4\right)\tanh^{-1}}{8e^6\left(ae^2+cd^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(5/2)/(d + e*x)^6,x]
```

```
[Out] -(c^2*(d*(8*c^2*d^4 + 12*a*c*d^2*e^2 + a^2*e^4) + e*(12*c^2*d^4 + 23*a*c*d^
2*e^2 + 8*a^2*e^4)*x)*Sqrt[a + c*x^2])/(8*e^5*(c*d^2 + a*e^2)^2*(d + e*x)^2
) - (c*(d*(4*c*d^2 + a*e^2) + e*(7*c*d^2 + 4*a*e^2)*x)*(a + c*x^2)^(3/2))/(
12*e^3*(c*d^2 + a*e^2)*(d + e*x)^4) - (a + c*x^2)^(5/2)/(5*e*(d + e*x)^5) +
 (c^(5/2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/e^6 + (c^3*d*(8*c^2*d^4 + 2
0*a*c*d^2*e^2 + 15*a^2*e^4)*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt
[a + c*x^2])])/(8*e^6*(c*d^2 + a*e^2)^(5/2))
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
```

```
&& !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{5/2}}{(d+ex)^6}\,dx = -\frac{\left(a+cx^2\right)^{5/2}}{5e(d+ex)^5}+\frac{c\int \frac{x\left(a+cx^2\right)^{3/2}}{(d+ex)^5}\,dx}{e}$$

$$= -\frac{c\left(d\left(4cd^2+ae^2\right)+e\left(7cd^2+4ae^2\right)x\right)\left(a+cx^2\right)^{3/2}}{12e^3\left(cd^2+ae^2\right)(d+ex)^4}-\frac{\left(a+cx^2\right)^{5/2}}{5e(d+ex)^5}-\frac{c\int \frac{\left(6acde-8c\left(cd^2+ae^2\right)x\right)\sqrt{a+cx^2}}{(d+ex)^3}\, \ldots}{8e^3\left(cd^2+ae^2\right)}$$

$$= -\frac{c^2\left(d\left(8c^2d^4+12acd^2e^2+a^2e^4\right)+e\left(12c^2d^4+23acd^2e^2+8a^2e^4\right)x\right)\sqrt{a+cx^2}}{8e^5\left(cd^2+ae^2\right)^2(d+ex)^2}-\frac{c\left(d\left(4cd^2+ae^2\ldots\right.\right.}{12e\ldots}$$

$$= -\frac{c^2\left(d\left(8c^2d^4+12acd^2e^2+a^2e^4\right)+e\left(12c^2d^4+23acd^2e^2+8a^2e^4\right)x\right)\sqrt{a+cx^2}}{8e^5\left(cd^2+ae^2\right)^2(d+ex)^2}-\frac{c\left(d\left(4cd^2+ae^2\ldots\right.\right.}{12e\ldots}$$

$$= -\frac{c^2\left(d\left(8c^2d^4+12acd^2e^2+a^2e^4\right)+e\left(12c^2d^4+23acd^2e^2+8a^2e^4\right)x\right)\sqrt{a+cx^2}}{8e^5\left(cd^2+ae^2\right)^2(d+ex)^2}-\frac{c\left(d\left(4cd^2+ae^2\ldots\right.\right.}{12e\ldots}$$

$$= -\frac{c^2\left(d\left(8c^2d^4+12acd^2e^2+a^2e^4\right)+e\left(12c^2d^4+23acd^2e^2+8a^2e^4\right)x\right)\sqrt{a+cx^2}}{8e^5\left(cd^2+ae^2\right)^2(d+ex)^2}-\frac{c\left(d\left(4cd^2+ae^2\ldots\right.\right.}{12e\ldots}$$

Mathematica [A] time = 0.559167, size = 359, normalized size = 1.14

$$-\frac{e\sqrt{a+cx^2}\left(c^2(d+ex)^4\left(184a^2e^4+503acd^2e^2+274c^2d^4\right)-c^2d(d+ex)^3\left(311ae^2+326cd^2\right)\left(ae^2+cd^2\right)-126cd(d+ex)\left(ae^2+cd^2\right)^3+2c(d+ex)^2\left(44ae^2+137cd^2\right)\left(ae^2+cd^2\right)^2\ldots}{(d+ex)^5\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(5/2)/(d + e*x)^6,x]
```

```
[Out] (-((e*Sqrt[a + c*x^2]*(24*(c*d^2 + a*e^2)^4 - 126*c*d*(c*d^2 + a*e^2)^3*(d
+ e*x) + 2*c*(c*d^2 + a*e^2)^2*(137*c*d^2 + 44*a*e^2)*(d + e*x)^2 - c^2*d*(
c*d^2 + a*e^2)*(326*c*d^2 + 311*a*e^2)*(d + e*x)^3 + c^2*(274*c^2*d^4 + 503
*a*c*d^2*e^2 + 184*a^2*e^4)*(d + e*x)^4))/((c*d^2 + a*e^2)^2*(d + e*x)^5))
- (15*c^3*d*(8*c^2*d^4 + 20*a*c*d^2*e^2 + 15*a^2*e^4)*Log[d + e*x])/(c*d^2
+ a*e^2)^(5/2) + 120*c^(5/2)*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] + (15*c^3*d
*(8*c^2*d^4 + 20*a*c*d^2*e^2 + 15*a^2*e^4)*Log[a*e - c*d*x + Sqrt[c*d^2 + a
*e^2]*Sqrt[a + c*x^2]])/(c*d^2 + a*e^2)^(5/2))/(120*e^6)
```

Maple [B] time = 0.207, size = 5921, normalized size = 18.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(5/2)/(e*x+d)^6,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^6,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + cx^2\right)^{\frac{5}{2}}}{(d + ex)^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(5/2)/(e*x+d)**6,x)
```

```
[Out] Integral((a + c*x**2)**(5/2)/(d + e*x)**6, x)
```

Giac [B] time = 2.18228, size = 1875, normalized size = 5.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^6,x, algorithm="giac")
```

```
[Out] -c^(5/2)*e^(-6)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a))) - 1/4*(8*c^5*d^5 + 2
0*a*c^4*d^3*e^2 + 15*a^2*c^3*d*e^4)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*
e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2))/((c^2*d^4*e^6 + 2*a*c*d^2*e^8 + a^2*e^
10)*sqrt(-c*d^2 - a*e^2)) - 1/60*(10000*(sqrt(c)*x - sqrt(c*x^2 + a))^6*c^(
13/2)*d^8*e + 4384*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^7*d^9 + 8800*(sqrt(c)*
x - sqrt(c*x^2 + a))^7*c^6*d^7*e^2 + 3600*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c
^(11/2)*d^6*e^3 - 11920*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(13/2)*d^8*e +
600*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^5*d^5*e^4 - 13872*(sqrt(c)*x - sqrt(c
*x^2 + a))^5*a*c^6*d^7*e^2 + 3800*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(11/2
)*d^6*e^3 + 12200*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^5*d^5*e^4 + 13120*(sq
rt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^6*d^7*e^2 + 6300*(sqrt(c)*x - sqrt(c*x^2
 + a))^8*a*c^(9/2)*d^4*e^5 - 3560*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(11
/2)*d^6*e^3 + 1140*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^4*d^3*e^6 - 29076*(s
qrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^5*d^5*e^4 - 18950*(sqrt(c)*x - sqrt(c*x
^2 + a))^6*a^2*c^(9/2)*d^4*e^5 - 7360*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^3*c
^(11/2)*d^6*e^3 - 1250*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^4*d^3*e^6 + 17
080*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c^5*d^5*e^4 + 1935*(sqrt(c)*x - sqr
t(c*x^2 + a))^8*a^2*c^(7/2)*d^2*e^7 + 23950*(sqrt(c)*x - sqrt(c*x^2 + a))^4
*a^3*c^(9/2)*d^4*e^5 + 495*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^3*d*e^8 +
370*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^4*d^3*e^6 + 2140*(sqrt(c)*x - sqr
t(c*x^2 + a))*a^4*c^5*d^5*e^4 - 8250*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^
(7/2)*d^2*e^7 - 12450*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c^(9/2)*d^4*e^5 -
 3030*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^3*d*e^8 - 4150*(sqrt(c)*x - sqr
t(c*x^2 + a))^3*a^4*c^4*d^3*e^6 - 360*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c
^(5/2)*e^9 + 8800*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(7/2)*d^2*e^7 - 274
*a^5*c^(9/2)*d^4*e^5 + 5520*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^3*d*e^8 +
 3890*(sqrt(c)*x - sqrt(c*x^2 + a))*a^5*c^4*d^3*e^6 + 720*(sqrt(c)*x - sqrt
(c*x^2 + a))^6*a^4*c^(5/2)*e^9 - 2910*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c
^(7/2)*d^2*e^7 - 4330*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^3*d*e^8 - 1120*
(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(5/2)*e^9 - 503*a^6*c^(7/2)*d^2*e^7 +
 1345*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^3*d*e^8 + 560*(sqrt(c)*x - sqrt(c
*x^2 + a))^2*a^6*c^(5/2)*e^9 - 184*a^7*c^(5/2)*e^9)/((c^2*d^4*e^6 + 2*a*c*d
^2*e^8 + a^2*e^10)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*e + 2*(sqrt(c)*x - sqrt
(c*x^2 + a))*sqrt(c)*d - a*e)^5)
```

3.555 $\int \frac{\left(a+cx^2\right)^{5/2}}{(d+ex)^7}\,dx$

Optimal. Leaf size=203

$$-\frac{5a^2c^2\sqrt{a+cx^2}(ae-cdx)}{16(d+ex)^2\left(ae^2+cd^2\right)^3}-\frac{5a^3c^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{16\left(ae^2+cd^2\right)^{7/2}}-\frac{5ac\left(a+cx^2\right)^{3/2}(ae-cdx)}{24(d+ex)^4\left(ae^2+cd^2\right)^2}-\frac{\left(a+cx^2\right)^{5/2}(ae-cdx)}{6(d+ex)^6\left(ae^2+cd^2\right)}$$

```
[Out] (-5*a^2*c^2*(a*e - c*d*x)*Sqrt[a + c*x^2])/(16*(c*d^2 + a*e^2)^3*(d + e*x)^
2) - (5*a*c*(a*e - c*d*x)*(a + c*x^2)^(3/2))/(24*(c*d^2 + a*e^2)^2*(d + e*x
)^4) - ((a*e - c*d*x)*(a + c*x^2)^(5/2))/(6*(c*d^2 + a*e^2)*(d + e*x)^6) -
(5*a^3*c^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(1
6*(c*d^2 + a*e^2)^(7/2))
```

Rubi [A] time = 0.105414, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {721, 725, 206}

$$-\frac{5a^2c^2\sqrt{a+cx^2}(ae-cdx)}{16(d+ex)^2\left(ae^2+cd^2\right)^3}-\frac{5a^3c^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{16\left(ae^2+cd^2\right)^{7/2}}-\frac{5ac\left(a+cx^2\right)^{3/2}(ae-cdx)}{24(d+ex)^4\left(ae^2+cd^2\right)^2}-\frac{\left(a+cx^2\right)^{5/2}(ae-cdx)}{6(d+ex)^6\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(5/2)/(d + e*x)^7,x]
```


```
[Out] (-5*a^2*c^2*(a*e - c*d*x)*Sqrt[a + c*x^2])/(16*(c*d^2 + a*e^2)^3*(d + e*x)^
2) - (5*a*c*(a*e - c*d*x)*(a + c*x^2)^(3/2))/(24*(c*d^2 + a*e^2)^2*(d + e*x
)^4) - ((a*e - c*d*x)*(a + c*x^2)^(5/2))/(6*(c*d^2 + a*e^2)*(d + e*x)^6) -
(5*a^3*c^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(1
6*(c*d^2 + a*e^2)^(7/2))
```


Rule 721

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(-2*a*e + (2*c*d)*x)*(a + c*x^2)^p)/(2*(m + 1)*(c*d^2 +
 a*e^2)), x] - Dist[(4*a*c*p)/(2*(m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m
 + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{5/2}}{(d+ex)^7}\,dx = -\frac{(ae-cdx)\left(a+cx^2\right)^{5/2}}{6\left(cd^2+ae^2\right)(d+ex)^6} + \frac{(5ac)\int \frac{\left(a+cx^2\right)^{3/2}}{(d+ex)^5}\,dx}{6\left(cd^2+ae^2\right)}$$

$$= -\frac{5ac(ae-cdx)\left(a+cx^2\right)^{3/2}}{24\left(cd^2+ae^2\right)^2(d+ex)^4} - \frac{(ae-cdx)\left(a+cx^2\right)^{5/2}}{6\left(cd^2+ae^2\right)(d+ex)^6} + \frac{\left(5a^2c^2\right)\int \frac{\sqrt{a+cx^2}}{(d+ex)^3}\,dx}{8\left(cd^2+ae^2\right)^2}$$

$$= -\frac{5a^2c^2(ae-cdx)\sqrt{a+cx^2}}{16\left(cd^2+ae^2\right)^3(d+ex)^2} - \frac{5ac(ae-cdx)\left(a+cx^2\right)^{3/2}}{24\left(cd^2+ae^2\right)^2(d+ex)^4} - \frac{(ae-cdx)\left(a+cx^2\right)^{5/2}}{6\left(cd^2+ae^2\right)(d+ex)^6} + \frac{\left(5a^3c^3\right)\int \frac{}{(d+}}{16\left(cd^2 -\right.}$$

$$= -\frac{5a^2c^2(ae-cdx)\sqrt{a+cx^2}}{16\left(cd^2+ae^2\right)^3(d+ex)^2} - \frac{5ac(ae-cdx)\left(a+cx^2\right)^{3/2}}{24\left(cd^2+ae^2\right)^2(d+ex)^4} - \frac{(ae-cdx)\left(a+cx^2\right)^{5/2}}{6\left(cd^2+ae^2\right)(d+ex)^6} - \frac{\left(5a^3c^3\right)\text{Subs}}{}$$

$$= -\frac{5a^2c^2(ae-cdx)\sqrt{a+cx^2}}{16\left(cd^2+ae^2\right)^3(d+ex)^2} - \frac{5ac(ae-cdx)\left(a+cx^2\right)^{3/2}}{24\left(cd^2+ae^2\right)^2(d+ex)^4} - \frac{(ae-cdx)\left(a+cx^2\right)^{5/2}}{6\left(cd^2+ae^2\right)(d+ex)^6} - \frac{5a^3c^3\tanh^{-1}}{16\left(cd^2\right.}$$

Mathematica [A] time = 0.536996, size = 305, normalized size = 1.5

$$\frac{1}{48}\left(\frac{\sqrt{a+cx^2}\left(-a^3c^2e\left(122d^2e^2x^2+54d^3ex+33d^4+54de^3x^3+33e^4x^4\right)+a^2c^3dx\left(122d^2e^2x^2+54d^3ex+33d^4+54de\right.\right.}{(d+ex)^6\left(ae^2+\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(5/2)/(d + e*x)^7,x]
```

```
[Out] ((Sqrt[a + c*x^2]*(-8*a^5*e^5 + 8*c^5*d^5*x^5 - 2*a^4*c*e^3*(13*d^2 + 6*d*e
*x + 13*e^2*x^2) + 2*a*c^4*d^3*x^3*(13*d^2 + 6*d*e*x + 13*e^2*x^2) - a^3*c^
2*e*(33*d^4 + 54*d^3*e*x + 122*d^2*e^2*x^2 + 54*d*e^3*x^3 + 33*e^4*x^4) + a
^2*c^3*d*x*(33*d^4 + 54*d^3*e*x + 122*d^2*e^2*x^2 + 54*d*e^3*x^3 + 33*e^4*x
^4)))/((c*d^2 + a*e^2)^3*(d + e*x)^6) + (15*a^3*c^3*Log[d + e*x])/(c*d^2 +
a*e^2)^(7/2) - (15*a^3*c^3*Log[a*e - c*d*x + Sqrt[c*d^2 + a*e^2]*Sqrt[a + c
*x^2]])/(c*d^2 + a*e^2)^(7/2))/48
```

Maple [B] time = 0.214, size = 7616, normalized size = 37.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(5/2)/(e*x+d)^7,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^7,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^7,x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+cx^2\right)^{\frac{5}{2}}}{(d+ex)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(5/2)/(e*x+d)**7,x)

[Out] Integral((a + c*x**2)**(5/2)/(d + e*x)**7, x)
```

Giac [B] time = 2.34626, size = 2558, normalized size = 12.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^7,x, algorithm="giac")

[Out] 5/8*a^3*c^3*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d
^2 - a*e^2))/((c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)*sqrt(
-c*d^2 - a*e^2)) + 1/24*(768*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^8*d^10*e + 2
56*(sqrt(c)*x - sqrt(c*x^2 + a))^6*c^(17/2)*d^11 + 960*(sqrt(c)*x - sqrt(c*
x^2 + a))^8*c^(15/2)*d^9*e^2 + 640*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^7*d^8*
e^3 - 768*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^8*d^10*e + 240*(sqrt(c)*x - s
qrt(c*x^2 + a))^10*c^(13/2)*d^7*e^4 - 1088*(sqrt(c)*x - sqrt(c*x^2 + a))^6*
a*c^(15/2)*d^9*e^2 + 48*(sqrt(c)*x - sqrt(c*x^2 + a))^11*c^6*d^6*e^5 + 576*
(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^7*d^8*e^3 + 2160*(sqrt(c)*x - sqrt(c*x^
2 + a))^8*a*c^(13/2)*d^7*e^4 + 960*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(1
5/2)*d^9*e^2 + 1840*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^6*d^6*e^5 - 576*(sq
rt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^7*d^8*e^3 + 720*(sqrt(c)*x - sqrt(c*x^2
+ a))^10*a*c^(11/2)*d^5*e^6 - 3744*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(1
3/2)*d^7*e^4 + 144*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a*c^5*d^4*e^7 - 2592*(s
qrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^6*d^6*e^5 - 640*(sqrt(c)*x - sqrt(c*x^2
 + a))^3*a^3*c^7*d^8*e^3 + 720*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(11/2)
*d^5*e^6 + 2160*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(13/2)*d^7*e^4 + 1680
*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^5*d^4*e^7 + 2592*(sqrt(c)*x - sqrt(c
```

```
*x^2 + a))^5*a^3*c^6*d^6*e^5 + 720*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^2*c^(
9/2)*d^3*e^8 - 3320*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(11/2)*d^5*e^6 +
240*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c^(13/2)*d^7*e^4 + 144*(sqrt(c)*x -
 sqrt(c*x^2 + a))^11*a^2*c^4*d^2*e^9 - 5640*(sqrt(c)*x - sqrt(c*x^2 + a))^7
*a^3*c^5*d^4*e^7 - 1840*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^6*d^6*e^5 - 2
910*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(9/2)*d^3*e^8 + 1080*(sqrt(c)*x -
 sqrt(c*x^2 + a))^4*a^4*c^(11/2)*d^5*e^6 - 340*(sqrt(c)*x - sqrt(c*x^2 + a)
)^9*a^3*c^4*d^2*e^9 + 7080*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^5*d^4*e^7
- 48*(sqrt(c)*x - sqrt(c*x^2 + a))*a^5*c^6*d^6*e^5 + 75*(sqrt(c)*x - sqrt(c
*x^2 + a))^10*a^3*c^(7/2)*d*e^10 + 5680*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4
*c^(9/2)*d^3*e^8 + 792*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(11/2)*d^5*e^6
 + 33*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^3*c^3*e^11 + 1800*(sqrt(c)*x - sqr
t(c*x^2 + a))^7*a^4*c^4*d^2*e^9 - 2040*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*
c^5*d^4*e^7 + 45*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^4*c^(7/2)*d*e^10 - 4620*
(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(9/2)*d^3*e^8 + 8*a^6*c^(11/2)*d^5*e^
6 + 5*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^4*c^3*e^11 - 2160*(sqrt(c)*x - sqrt
(c*x^2 + a))^5*a^5*c^4*d^2*e^9 - 168*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^5*
d^4*e^7 - 330*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^5*c^(7/2)*d*e^10 + 1104*(sq
rt(c)*x - sqrt(c*x^2 + a))^2*a^6*c^(9/2)*d^3*e^8 + 90*(sqrt(c)*x - sqrt(c*x
^2 + a))^7*a^5*c^3*e^11 + 1640*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^4*d^2*
e^9 + 450*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(7/2)*d*e^10 + 26*a^7*c^(9/
2)*d^3*e^8 + 90*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^6*c^3*e^11 - 252*(sqrt(c)
*x - sqrt(c*x^2 + a))*a^7*c^4*d^2*e^9 - 273*(sqrt(c)*x - sqrt(c*x^2 + a))^2
*a^7*c^(7/2)*d*e^10 + 5*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^7*c^3*e^11 + 33*a
^8*c^(7/2)*d*e^10 + 33*(sqrt(c)*x - sqrt(c*x^2 + a))*a^8*c^3*e^11)/((c^3*d^
6*e^6 + 3*a*c^2*d^4*e^8 + 3*a^2*c*d^2*e^10 + a^3*e^12)*((sqrt(c)*x - sqrt(c
*x^2 + a))^2*e + 2*(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*d - a*e)^6)
```

3.556 $\int \frac{(a+cx^2)^{5/2}}{(d+ex)^8} dx$

Optimal. Leaf size=246

$$-\frac{5a^2c^3d\sqrt{a+cx^2}(ae-cdx)}{16(d+ex)^2\left(ae^2+cd^2\right)^4}-\frac{5a^3c^4d\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{16\left(ae^2+cd^2\right)^{9/2}}-\frac{5ac^2d\left(a+cx^2\right)^{3/2}(ae-cdx)}{24(d+ex)^4\left(ae^2+cd^2\right)^3}-\frac{cd\left(a+cx^2\right)^{5/2}(ae-cdx)}{6(d+ex)^6\left(ae^2+cd^2\right)^2}$$

```
[Out] (-5*a^2*c^3*d*(a*e - c*d*x)*Sqrt[a + c*x^2])/(16*(c*d^2 + a*e^2)^4*(d + e*x
)^2) - (5*a*c^2*d*(a*e - c*d*x)*(a + c*x^2)^(3/2))/(24*(c*d^2 + a*e^2)^3*(d
 + e*x)^4) - (c*d*(a*e - c*d*x)*(a + c*x^2)^(5/2))/(6*(c*d^2 + a*e^2)^2*(d
+ e*x)^6) - (e*(a + c*x^2)^(7/2))/(7*(c*d^2 + a*e^2)*(d + e*x)^7) - (5*a^3*
c^4*d*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(16*(c*
d^2 + a*e^2)^(9/2))
```

Rubi [A] time = 0.124018, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {731, 721, 725, 206}

$$-\frac{5a^2c^3d\sqrt{a+cx^2}(ae-cdx)}{16(d+ex)^2\left(ae^2+cd^2\right)^4}-\frac{5a^3c^4d\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{16\left(ae^2+cd^2\right)^{9/2}}-\frac{5ac^2d\left(a+cx^2\right)^{3/2}(ae-cdx)}{24(d+ex)^4\left(ae^2+cd^2\right)^3}-\frac{cd\left(a+cx^2\right)^{5/2}(ae-cdx)}{6(d+ex)^6\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(5/2)/(d + e*x)^8,x]
```

```
[Out] (-5*a^2*c^3*d*(a*e - c*d*x)*Sqrt[a + c*x^2])/(16*(c*d^2 + a*e^2)^4*(d + e*x
)^2) - (5*a*c^2*d*(a*e - c*d*x)*(a + c*x^2)^(3/2))/(24*(c*d^2 + a*e^2)^3*(d
 + e*x)^4) - (c*d*(a*e - c*d*x)*(a + c*x^2)^(5/2))/(6*(c*d^2 + a*e^2)^2*(d
+ e*x)^6) - (e*(a + c*x^2)^(7/2))/(7*(c*d^2 + a*e^2)*(d + e*x)^7) - (5*a^3*
c^4*d*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(16*(c*
d^2 + a*e^2)^(9/2))
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 721

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(-2*a*e + (2*c*d)*x)*(a + c*x^2)^p)/(2*(m + 1)*(c*d^2 +
 a*e^2)), x] - Dist[(4*a*c*p)/(2*(m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m
 + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{5/2}}{(d+ex)^8}\,dx = -\frac{e\left(a+cx^2\right)^{7/2}}{7\left(cd^2+ae^2\right)(d+ex)^7} + \frac{(cd)\int \frac{\left(a+cx^2\right)^{5/2}}{(d+ex)^7}\,dx}{cd^2+ae^2}$$

$$= -\frac{cd(ae-cdx)\left(a+cx^2\right)^{5/2}}{6\left(cd^2+ae^2\right)^2(d+ex)^6} - \frac{e\left(a+cx^2\right)^{7/2}}{7\left(cd^2+ae^2\right)(d+ex)^7} + \frac{\left(5ac^2d\right)\int \frac{\left(a+cx^2\right)^{3/2}}{(d+ex)^5}\,dx}{6\left(cd^2+ae^2\right)^2}$$

$$= -\frac{5ac^2d(ae-cdx)\left(a+cx^2\right)^{3/2}}{24\left(cd^2+ae^2\right)^3(d+ex)^4} - \frac{cd(ae-cdx)\left(a+cx^2\right)^{5/2}}{6\left(cd^2+ae^2\right)^2(d+ex)^6} - \frac{e\left(a+cx^2\right)^{7/2}}{7\left(cd^2+ae^2\right)(d+ex)^7} + \frac{\left(5a^2c^3d\right)\int}{8\left(cd^2+\right.}$$

$$= -\frac{5a^2c^3d(ae-cdx)\sqrt{a+cx^2}}{16\left(cd^2+ae^2\right)^4(d+ex)^2} - \frac{5ac^2d(ae-cdx)\left(a+cx^2\right)^{3/2}}{24\left(cd^2+ae^2\right)^3(d+ex)^4} - \frac{cd(ae-cdx)\left(a+cx^2\right)^{5/2}}{6\left(cd^2+ae^2\right)^2(d+ex)^6} - \frac{e(}{7\left(cd^2+\right.}$$

$$= -\frac{5a^2c^3d(ae-cdx)\sqrt{a+cx^2}}{16\left(cd^2+ae^2\right)^4(d+ex)^2} - \frac{5ac^2d(ae-cdx)\left(a+cx^2\right)^{3/2}}{24\left(cd^2+ae^2\right)^3(d+ex)^4} - \frac{cd(ae-cdx)\left(a+cx^2\right)^{5/2}}{6\left(cd^2+ae^2\right)^2(d+ex)^6} - \frac{e(}{7\left(cd^2+\right.}$$

$$= -\frac{5a^2c^3d(ae-cdx)\sqrt{a+cx^2}}{16\left(cd^2+ae^2\right)^4(d+ex)^2} - \frac{5ac^2d(ae-cdx)\left(a+cx^2\right)^{3/2}}{24\left(cd^2+ae^2\right)^3(d+ex)^4} - \frac{cd(ae-cdx)\left(a+cx^2\right)^{5/2}}{6\left(cd^2+ae^2\right)^2(d+ex)^6} - \frac{e(}{7\left(cd^2+\right.}$$

Mathematica [A] time = 0.562648, size = 403, normalized size = 1.64

$$-\frac{\sqrt{a+cx^2}\left(2c^2(d+ex)^4\left(72a^2e^4+159acd^2e^2+80c^2d^4\right)\left(ae^2+cd^2\right)^2-c^3d(d+ex)^5\left(57a^2e^4+30acd^2e^2+8c^2d^4\right)\left(ae^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(5/2)/(d + e*x)^8,x]
```

```
[Out] -(Sqrt[a + c*x^2]*(48*(c*d^2 + a*e^2)^6 - 232*c*d*(c*d^2 + a*e^2)^5*(d + e*
x) + 8*c*(c*d^2 + a*e^2)^4*(55*c*d^2 + 18*a*e^2)*(d + e*x)^2 - 2*c^2*d*(c*d
^2 + a*e^2)^3*(200*c*d^2 + 197*a*e^2)*(d + e*x)^3 + 2*c^2*(c*d^2 + a*e^2)^2
*(80*c^2*d^4 + 159*a*c*d^2*e^2 + 72*a^2*e^4)*(d + e*x)^4 - c^3*d*(c*d^2 + a
*e^2)*(8*c^2*d^4 + 30*a*c*d^2*e^2 + 57*a^2*e^4)*(d + e*x)^5 - c^3*(8*c^3*d^
6 + 38*a*c^2*d^4*e^2 + 87*a^2*c*d^2*e^4 - 48*a^3*e^6)*(d + e*x)^6))/(336*e^
5*(c*d^2 + a*e^2)^4*(d + e*x)^7) + (5*a^3*c^4*d*Log[d + e*x])/(16*(c*d^2 +
a*e^2)^(9/2)) - (5*a^3*c^4*d*Log[a*e - c*d*x + Sqrt[c*d^2 + a*e^2]*Sqrt[a +
 c*x^2]])/(16*(c*d^2 + a*e^2)^(9/2))
```

Maple [B] time = 0.24, size = 7718, normalized size = 31.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(5/2)/(e*x+d)^8,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^8,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(5/2)/(e*x+d)**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.2945, size = 3178, normalized size = 12.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^8,x, algorithm="giac")
```

```
[Out] -5/8*a^3*c^4*d*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c
*d^2 - a*e^2))/((c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^
2*e^6 + a^4*e^8)*sqrt(-c*d^2 - a*e^2)) + 1/168*(1792*(sqrt(c)*x - sqrt(c*x^
2 + a))^8*c^(19/2)*d^12*e + 512*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^10*d^13 +
 2688*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^9*d^11*e^2 + 2240*(sqrt(c)*x - sqrt
(c*x^2 + a))^10*c^(17/2)*d^10*e^3 - 1792*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*
c^(19/2)*d^12*e + 1120*(sqrt(c)*x - sqrt(c*x^2 + a))^11*c^8*d^9*e^4 - 2944*
```

(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^9*d^11*e^2 + 336*(sqrt(c)*x - sqrt(c*x^2 + a))^12*c^(15/2)*d^8*e^5 + 1792*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(17/2)*d^10*e^3 + 8288*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^8*d^9*e^4 + 2688*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^9*d^11*e^2 + 8960*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a*c^(15/2)*d^8*e^5 - 1792*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(17/2)*d^10*e^3 + 4480*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a*c^7*d^7*e^6 - 13248*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^8*d^9*e^4 + 1344*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a*c^(13/2)*d^6*e^7 - 9072*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(15/2)*d^8*e^5 - 2240*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(17/2)*d^10*e^3 + 6272*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^7*d^7*e^6 + 8288*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^8*d^9*e^4 + 13440*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^2*c^(13/2)*d^6*e^7 + 9072*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(15/2)*d^8*e^5 + 6720*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^2*c^6*d^5*e^8 - 30736*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^7*d^7*e^6 + 1120*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^8*d^9*e^4 + 2016*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^2*c^(11/2)*d^4*e^9 - 55832*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(13/2)*d^6*e^7 - 8960*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(15/2)*d^8*e^5 - 42588*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*c^6*d^5*e^8 + 11312*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^7*d^7*e^6 - 13370*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^3*c^(11/2)*d^4*e^9 + 80192*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(13/2)*d^6*e^7 - 336*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(15/2)*d^8*e^5 - 3010*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^3*c^5*d^3*e^10 + 100016*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4*c^6*d^5*e^8 + 5488*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^7*d^7*e^6 - 21*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^3*c^(9/2)*d^2*e^11 + 70210*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^4*c^(11/2)*d^4*e^9 - 19488*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(13/2)*d^6*e^7 - 105*(sqrt(c)*x - sqrt(c*x^2 + a))^13*a^3*c^4*d*e^12 + 27370*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^4*c^5*d^3*e^10 - 79128*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^5*c^6*d^5*e^8 + 112*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^7*d^7*e^6 + 9940*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^4*c^(9/2)*d^2*e^11 - 82180*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^5*c^(11/2)*d^4*e^9 - 1792*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^6*c^(13/2)*d^6*e^7 + 1820*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^4*c^4*d*e^12 - 52500*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^5*c^5*d^3*e^10 + 14448*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^6*d^5*e^8 + 336*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^4*c^(7/2)*e^13 - 16485*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^5*c^(9/2)*d^2*e^11 + 49252*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(11/2)*d^4*e^9 - 8*a^7*c^(13/2)*d^6*e^7 - 4445*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^5*c^4*d*e^12 + 44660*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^6*c^5*d^3*e^10 + 532*(sqrt(c)*x - sqrt(c*x^2 + a))*a^7*c^6*d^5*e^8 + 26880*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^6*c^(9/2)*d^2*e^11 - 5026*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^7*c^(11/2)*d^4*e^9 + 6720*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^6*c^4*d*e^12 - 17738*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^7*c^5*d^3*e^10 + 1680*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^6*c^(7/2)*e^13 - 12047*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^7*c^(9/2)*d^2*e^11 - 38*a^8*c^(11/2)*d^4*e^9 - 5635*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^7*c^4*d*e^12 + 1218*(sqrt(c)*x - sqrt(c*x^2 + a))*a^8*c^5*d^3*e^10 + 4620*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^8*c^(9/2)*d^2*e^11 + 2212*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^8*c^4*d*e^12 + 1008*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^8*c^(7/2)*e^13 - 87*a^9*c^(9/2)*d^2*e^11 - 567*(sqrt(c)*x - sqrt(c*x^2 + a))*a^9*c^4*d*e^12 + 48*a^10*c^(7/2)*e^13)/((c^4*d^8*e^6 + 4*a*c^3*d^6*e^8 + 6*a^2*c^2*d^4*e^10 + 4*a^3*c*d^2*e^12 + a^4*e^14)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*e + 2*(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*d - a*e)^7)

3.557 $\int \frac{(a+cx^2)^{5/2}}{(d+ex)^9} dx$

Optimal. Leaf size=332

$$-\frac{5a^2c^3\sqrt{a+cx^2}\left(8cd^2-ae^2\right)(ae-cdx)}{128(d+ex)^2\left(ae^2+cd^2\right)^5}-\frac{5a^3c^4\left(8cd^2-ae^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{128\left(ae^2+cd^2\right)^{11/2}}-\frac{5ac^2\left(a+cx^2\right)^{3/2}\left(8cd^2-ae^2\right)(ae}{192(d+ex)^4\left(ae^2+cd^2\right)^4}$$

```
[Out] (-5*a^2*c^3*(8*c*d^2 - a*e^2)*(a*e - c*d*x)*Sqrt[a + c*x^2])/(128*(c*d^2 +
a*e^2)^5*(d + e*x)^2) - (5*a*c^2*(8*c*d^2 - a*e^2)*(a*e - c*d*x)*(a + c*x^2
)^(3/2))/(192*(c*d^2 + a*e^2)^4*(d + e*x)^4) - (c*(8*c*d^2 - a*e^2)*(a*e -
c*d*x)*(a + c*x^2)^(5/2))/(48*(c*d^2 + a*e^2)^3*(d + e*x)^6) - (e*(a + c*x^
2)^(7/2))/(8*(c*d^2 + a*e^2)*(d + e*x)^8) - (9*c*d*e*(a + c*x^2)^(7/2))/(56
*(c*d^2 + a*e^2)^2*(d + e*x)^7) - (5*a^3*c^4*(8*c*d^2 - a*e^2)*ArcTanh[(a*e
 - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(128*(c*d^2 + a*e^2)^(11/
2))
```

Rubi [A] time = 0.257773, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {745, 807, 721, 725, 206}

$$-\frac{5a^2c^3\sqrt{a+cx^2}\left(8cd^2-ae^2\right)(ae-cdx)}{128(d+ex)^2\left(ae^2+cd^2\right)^5}-\frac{5a^3c^4\left(8cd^2-ae^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{128\left(ae^2+cd^2\right)^{11/2}}-\frac{5ac^2\left(a+cx^2\right)^{3/2}\left(8cd^2-ae^2\right)(ae}{192(d+ex)^4\left(ae^2+cd^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(5/2)/(d + e*x)^9,x]
```


```
[Out] (-5*a^2*c^3*(8*c*d^2 - a*e^2)*(a*e - c*d*x)*Sqrt[a + c*x^2])/(128*(c*d^2 +
a*e^2)^5*(d + e*x)^2) - (5*a*c^2*(8*c*d^2 - a*e^2)*(a*e - c*d*x)*(a + c*x^2
)^(3/2))/(192*(c*d^2 + a*e^2)^4*(d + e*x)^4) - (c*(8*c*d^2 - a*e^2)*(a*e -
c*d*x)*(a + c*x^2)^(5/2))/(48*(c*d^2 + a*e^2)^3*(d + e*x)^6) - (e*(a + c*x^
2)^(7/2))/(8*(c*d^2 + a*e^2)*(d + e*x)^8) - (9*c*d*e*(a + c*x^2)^(7/2))/(56
*(c*d^2 + a*e^2)^2*(d + e*x)^7) - (5*a^3*c^4*(8*c*d^2 - a*e^2)*ArcTanh[(a*e
 - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(128*(c*d^2 + a*e^2)^(11/
2))
```


Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
```

```
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 721

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(-2*a*e + (2*c*d)*x)*(a + c*x^2)^p)/(2*(m + 1)*(c*d^2 +
 a*e^2)), x] - Dist[(4*a*c*p)/(2*(m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m
 + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{5/2}}{(d+ex)^9}\,dx = -\frac{e\left(a+cx^2\right)^{7/2}}{8\left(cd^2+ae^2\right)(d+ex)^8} - \frac{c\int \frac{(-8d+ex)\left(a+cx^2\right)^{5/2}}{(d+ex)^8}\,dx}{8\left(cd^2+ae^2\right)}$$

$$= -\frac{e\left(a+cx^2\right)^{7/2}}{8\left(cd^2+ae^2\right)(d+ex)^8} - \frac{9cde\left(a+cx^2\right)^{7/2}}{56\left(cd^2+ae^2\right)^2(d+ex)^7} + \frac{\left(c\left(8cd^2-ae^2\right)\right)\int \frac{\left(a+cx^2\right)^{5/2}}{(d+ex)^7}\,dx}{8\left(cd^2+ae^2\right)^2}$$

$$= -\frac{c\left(8cd^2-ae^2\right)(ae-cdx)\left(a+cx^2\right)^{5/2}}{48\left(cd^2+ae^2\right)^3(d+ex)^6} - \frac{e\left(a+cx^2\right)^{7/2}}{8\left(cd^2+ae^2\right)(d+ex)^8} - \frac{9cde\left(a+cx^2\right)^{7/2}}{56\left(cd^2+ae^2\right)^2(d+ex)^7} + \frac{(5\cdots}{\cdots}$$

$$= -\frac{5ac^2\left(8cd^2-ae^2\right)(ae-cdx)\left(a+cx^2\right)^{3/2}}{192\left(cd^2+ae^2\right)^4(d+ex)^4} - \frac{c\left(8cd^2-ae^2\right)(ae-cdx)\left(a+cx^2\right)^{5/2}}{48\left(cd^2+ae^2\right)^3(d+ex)^6} - \frac{e\left(a+cx\cdots}{8\left(cd^2+ae^2\right)\cdots}$$

$$= -\frac{5a^2c^3\left(8cd^2-ae^2\right)(ae-cdx)\sqrt{a+cx^2}}{128\left(cd^2+ae^2\right)^5(d+ex)^2} - \frac{5ac^2\left(8cd^2-ae^2\right)(ae-cdx)\left(a+cx^2\right)^{3/2}}{192\left(cd^2+ae^2\right)^4(d+ex)^4} - \frac{c\left(8cd^2-a\cdots}{48\left(c\cdots}$$

$$= -\frac{5a^2c^3\left(8cd^2-ae^2\right)(ae-cdx)\sqrt{a+cx^2}}{128\left(cd^2+ae^2\right)^5(d+ex)^2} - \frac{5ac^2\left(8cd^2-ae^2\right)(ae-cdx)\left(a+cx^2\right)^{3/2}}{192\left(cd^2+ae^2\right)^4(d+ex)^4} - \frac{c\left(8cd^2-a\cdots}{48\left(c\cdots}$$

$$= -\frac{5a^2c^3\left(8cd^2-ae^2\right)(ae-cdx)\sqrt{a+cx^2}}{128\left(cd^2+ae^2\right)^5(d+ex)^2} - \frac{5ac^2\left(8cd^2-ae^2\right)(ae-cdx)\left(a+cx^2\right)^{3/2}}{192\left(cd^2+ae^2\right)^4(d+ex)^4} - \frac{c\left(8cd^2-a\cdots}{48\left(c\cdots}$$

Mathematica [A] time = 1.05807, size = 489, normalized size = 1.47

$$-\frac{\sqrt{a+cx^2}\left(2c^2(d+ex)^4\left(413a^2e^4+880acd^2e^2+440c^2d^4\right)\left(ae^2+cd^2\right)^3-2c^3d(d+ex)^5\left(87a^2e^4+32acd^2e^2+8c^2d^4\right)\left(ae^2+cd^2\right)^2-c^3(d+ex)^6\left(282a^2cd^2e^4-105a^3e^6+88ac^2\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(5/2)/(d + e*x)^9,x]
```

```
[Out] (-((Sqrt[a + c*x^2]*(336*(c*d^2 + a*e^2)^7 - 1584*c*d*(c*d^2 + a*e^2)^6*(d
+ e*x) + 8*c*(c*d^2 + a*e^2)^5*(362*c*d^2 + 119*a*e^2)*(d + e*x)^2 - 8*c^2*
d*(c*d^2 + a*e^2)^4*(310*c*d^2 + 307*a*e^2)*(d + e*x)^3 + 2*c^2*(c*d^2 + a*
e^2)^3*(440*c^2*d^4 + 880*a*c*d^2*e^2 + 413*a^2*e^4)*(d + e*x)^4 - 2*c^3*d*
(c*d^2 + a*e^2)^2*(8*c^2*d^4 + 32*a*c*d^2*e^2 + 87*a^2*e^4)*(d + e*x)^5 - c
^3*(c*d^2 + a*e^2)*(16*c^3*d^6 + 88*a*c^2*d^4*e^2 + 282*a^2*c*d^2*e^4 - 105
*a^3*e^6)*(d + e*x)^6 - c^4*d*(16*c^3*d^6 + 104*a*c^2*d^4*e^2 + 370*a^2*c*d
^2*e^4 - 663*a^3*e^6)*(d + e*x)^7))/((c*d^2*e + a*e^3)^5*(d + e*x)^8)) + (1
05*a^3*c^4*(8*c*d^2 - a*e^2)*Log[d + e*x])/(c*d^2 + a*e^2)^(11/2) + (105*a^
3*c^4*(-8*c*d^2 + a*e^2)*Log[a*e - c*d*x + Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x
^2]])/(c*d^2 + a*e^2)^(11/2))/2688
```

Maple [B] time = 0.239, size = 9978, normalized size = 30.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(5/2)/(e*x+d)^9,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^9,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(5/2)/(e*x+d)**9,x)

[Out] Timed out
```

Giac [B] time = 3.04028, size = 4207, normalized size = 12.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^9,x, algorithm="giac")

[Out] 5/64*(8*a^3*c^5*d^2 - a^4*c^4*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e
 + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2))/((c^5*d^10 + 5*a*c^4*d^8*e^2 + 10*a^2*c
^3*d^6*e^4 + 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 + a^5*e^10)*sqrt(-c*d^2 -
 a*e^2)) + 1/1344*(8192*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^11*d^14*e + 2048*
(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(23/2)*d^15 + 14336*(sqrt(c)*x - sqrt(c*x
^2 + a))^10*c^(21/2)*d^13*e^2 + 14336*(sqrt(c)*x - sqrt(c*x^2 + a))^11*c^10
*d^12*e^3 - 8192*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^11*d^14*e + 8960*(sqrt
(c)*x - sqrt(c*x^2 + a))^12*c^(19/2)*d^11*e^4 - 15360*(sqrt(c)*x - sqrt(c*x
^2 + a))^8*a*c^(21/2)*d^13*e^2 + 3584*(sqrt(c)*x - sqrt(c*x^2 + a))^13*c^9*
d^10*e^5 + 10240*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^10*d^12*e^3 + 57344*(s
qrt(c)*x - sqrt(c*x^2 + a))^10*a*c^(19/2)*d^11*e^4 + 14336*(sqrt(c)*x - sqr
t(c*x^2 + a))^6*a^2*c^(21/2)*d^13*e^2 + 75264*(sqrt(c)*x - sqrt(c*x^2 + a))
^11*a*c^9*d^10*e^5 - 10240*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^10*d^12*e^
3 + 44800*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a*c^(17/2)*d^9*e^6 - 85248*(sqrt
(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(19/2)*d^11*e^4 + 17920*(sqrt(c)*x - sqrt(
c*x^2 + a))^13*a*c^8*d^8*e^7 - 54272*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^
9*d^10*e^5 - 14336*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^10*d^12*e^3 + 7168
0*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^2*c^(17/2)*d^9*e^6 + 57344*(sqrt(c)*x
- sqrt(c*x^2 + a))^6*a^3*c^(19/2)*d^11*e^4 + 161280*(sqrt(c)*x - sqrt(c*x^2
 + a))^11*a^2*c^8*d^8*e^7 + 54272*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^9*d
^10*e^5 + 89600*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^2*c^(15/2)*d^7*e^8 - 416
384*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(17/2)*d^9*e^6 + 8960*(sqrt(c)*x
- sqrt(c*x^2 + a))^4*a^4*c^(19/2)*d^11*e^4 + 35840*(sqrt(c)*x - sqrt(c*x^2
+ a))^13*a^2*c^7*d^6*e^9 - 877056*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*c^8*d
^8*e^7 - 75264*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^9*d^10*e^5 - 916608*(s
qrt(c)*x - sqrt(c*x^2 + a))^10*a^3*c^(15/2)*d^7*e^8 + 152320*(sqrt(c)*x - s
qrt(c*x^2 + a))^6*a^4*c^(17/2)*d^9*e^6 - 486528*(sqrt(c)*x - sqrt(c*x^2 + a
))^11*a^3*c^7*d^6*e^9 + 1334016*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4*c^8*d^8
*e^7 - 3584*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^9*d^10*e^5 - 208880*(sqrt
(c)*x - sqrt(c*x^2 + a))^12*a^3*c^(13/2)*d^5*e^10 + 2315376*(sqrt(c)*x - sq
rt(c*x^2 + a))^8*a^4*c^(15/2)*d^7*e^8 + 60928*(sqrt(c)*x - sqrt(c*x^2 + a))
^4*a^5*c^(17/2)*d^9*e^6 - 45920*(sqrt(c)*x - sqrt(c*x^2 + a))^13*a^3*c^6*d^
4*e^11 + 2366784*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^4*c^7*d^6*e^9 - 274176*(
sqrt(c)*x - sqrt(c*x^2 + a))^5*a^5*c^8*d^8*e^7 - 12600*(sqrt(c)*x - sqrt(c*
x^2 + a))^14*a^3*c^(11/2)*d^3*e^12 + 1412880*(sqrt(c)*x - sqrt(c*x^2 + a))^
10*a^4*c^(13/2)*d^5*e^10 - 1755264*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^5*c^(1
5/2)*d^7*e^8 + 1792*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^6*c^(17/2)*d^9*e^6 -
840*(sqrt(c)*x - sqrt(c*x^2 + a))^15*a^3*c^5*d^2*e^13 + 650160*(sqrt(c)*x -
 sqrt(c*x^2 + a))^11*a^4*c^6*d^4*e^11 - 2796864*(sqrt(c)*x - sqrt(c*x^2 + a
))^7*a^5*c^7*d^6*e^9 - 26880*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^8*d^8*e^
7 + 165830*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^4*c^(11/2)*d^3*e^12 - 2638440
*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^5*c^(13/2)*d^5*e^10 + 255360*(sqrt(c)*x
- sqrt(c*x^2 + a))^4*a^6*c^(15/2)*d^7*e^8 + 34580*(sqrt(c)*x - sqrt(c*x^2 +
 a))^13*a^4*c^5*d^2*e^13 - 1325520*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^5*c^6*
d^4*e^11 + 1495424*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^6*c^7*d^6*e^9 - 256*(s
```

```
qrt(c)*x - sqrt(c*x^2 + a))*a^7*c^8*d^8*e^7 + 1575*(sqrt(c)*x - sqrt(c*x^2
+ a))^14*a^4*c^(9/2)*d*e^14 - 464520*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^5*c
^(11/2)*d^3*e^12 + 2173136*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^6*c^(13/2)*d^5
*e^10 + 11520*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^7*c^(15/2)*d^7*e^8 + 105*(s
qrt(c)*x - sqrt(c*x^2 + a))^15*a^4*c^4*e^15 - 46620*(sqrt(c)*x - sqrt(c*x^2
 + a))^11*a^5*c^5*d^2*e^13 + 1851920*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^6*c^
6*d^4*e^11 - 118272*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^7*c^7*d^6*e^9 - 1505*
(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^5*c^(9/2)*d*e^14 + 755510*(sqrt(c)*x - s
qrt(c*x^2 + a))^8*a^6*c^(11/2)*d^3*e^12 - 779408*(sqrt(c)*x - sqrt(c*x^2 +
a))^4*a^7*c^(13/2)*d^5*e^10 + 16*a^8*c^(15/2)*d^7*e^8 + 2779*(sqrt(c)*x - s
qrt(c*x^2 + a))^13*a^5*c^4*e^15 + 229040*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^
6*c^5*d^2*e^13 - 959280*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^7*c^6*d^4*e^11 -
1664*(sqrt(c)*x - sqrt(c*x^2 + a))*a^8*c^7*d^6*e^9 + 15155*(sqrt(c)*x - sqr
t(c*x^2 + a))^10*a^6*c^(9/2)*d*e^14 - 670040*(sqrt(c)*x - sqrt(c*x^2 + a))^
6*a^7*c^(11/2)*d^3*e^12 + 40608*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^8*c^(13/2
)*d^5*e^10 + 6265*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^6*c^4*e^15 - 142240*(s
qrt(c)*x - sqrt(c*x^2 + a))^7*a^7*c^5*d^2*e^13 + 292544*(sqrt(c)*x - sqrt(c
*x^2 + a))^3*a^8*c^6*d^4*e^11 - 23205*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^7*c
^(9/2)*d*e^14 + 290066*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^8*c^(11/2)*d^3*e^1
2 + 104*a^9*c^(13/2)*d^5*e^10 + 12355*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^7*c
^4*e^15 + 176148*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^8*c^5*d^2*e^13 - 5920*(s
qrt(c)*x - sqrt(c*x^2 + a))*a^9*c^6*d^4*e^11 + 21973*(sqrt(c)*x - sqrt(c*x^
2 + a))^6*a^8*c^(9/2)*d*e^14 - 64616*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^9*c^
(11/2)*d^3*e^12 + 12355*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^8*c^4*e^15 - 3967
6*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^9*c^5*d^2*e^13 - 17059*(sqrt(c)*x - sqr
t(c*x^2 + a))^4*a^9*c^(9/2)*d*e^14 + 370*a^10*c^(11/2)*d^3*e^12 + 6265*(sqr
t(c)*x - sqrt(c*x^2 + a))^5*a^9*c^4*e^15 + 9768*(sqrt(c)*x - sqrt(c*x^2 + a
))*a^10*c^5*d^2*e^13 + 3729*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^10*c^(9/2)*d*
e^14 + 2779*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^10*c^4*e^15 - 663*a^11*c^(9/2
)*d*e^14 + 105*(sqrt(c)*x - sqrt(c*x^2 + a))*a^11*c^4*e^15)/((c^5*d^10*e^6
+ 5*a*c^4*d^8*e^8 + 10*a^2*c^3*d^6*e^10 + 10*a^3*c^2*d^4*e^12 + 5*a^4*c*d^2
*e^14 + a^5*e^16)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*e + 2*(sqrt(c)*x - sqrt(
c*x^2 + a))*sqrt(c)*d - a*e)^8)
```

3.558 $\int \frac{\sqrt{2+x^2}}{1+4x} dx$

Optimal. Leaf size=56

$$\frac{\sqrt{x^2+2}}{4}-\frac{1}{16}\sqrt{33}\tanh^{-1}\left(\frac{8-x}{\sqrt{33}\sqrt{x^2+2}}\right)-\frac{1}{16}\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] Sqrt[2 + x^2]/4 - ArcSinh[x/Sqrt[2]]/16 - (Sqrt[33]*ArcTanh[(8 - x)/(Sqrt[3
3]*Sqrt[2 + x^2])])/16
```

Rubi [A] time = 0.0327453, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {735, 844, 215, 725, 206}

$$\frac{\sqrt{x^2+2}}{4}-\frac{1}{16}\sqrt{33}\tanh^{-1}\left(\frac{8-x}{\sqrt{33}\sqrt{x^2+2}}\right)-\frac{1}{16}\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + x^2]/(1 + 4*x),x]
```

```
[Out] Sqrt[2 + x^2]/4 - ArcSinh[x/Sqrt[2]]/16 - (Sqrt[33]*ArcTanh[(8 - x)/(Sqrt[3
3]*Sqrt[2 + x^2])])/16
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2+x^2}}{1+4x}\,dx &= \frac{\sqrt{2+x^2}}{4}+\frac{1}{4}\int \frac{8-x}{(1+4x)\sqrt{2+x^2}}\,dx\\ &= \frac{\sqrt{2+x^2}}{4}-\frac{1}{16}\int \frac{1}{\sqrt{2+x^2}}\,dx+\frac{33}{16}\int \frac{1}{(1+4x)\sqrt{2+x^2}}\,dx\\ &= \frac{\sqrt{2+x^2}}{4}-\frac{1}{16}\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)-\frac{33}{16}\,\text{Subst}\left(\int \frac{1}{33-x^2}\,dx,x,\frac{8-x}{\sqrt{2+x^2}}\right)\\ &= \frac{\sqrt{2+x^2}}{4}-\frac{1}{16}\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{16}\sqrt{33}\tanh^{-1}\left(\frac{8-x}{\sqrt{33}\sqrt{2+x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0283808, size = 56, normalized size = 1.

$$\frac{\sqrt{x^2+2}}{4}-\frac{1}{16}\sqrt{33}\tanh^{-1}\left(\frac{8-x}{\sqrt{33}\sqrt{x^2+2}}\right)-\frac{1}{16}\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + x^2]/(1 + 4*x),x]
```

```
[Out] Sqrt[2 + x^2]/4 - ArcSinh[x/Sqrt[2]]/16 - (Sqrt[33]*ArcTanh[(8 - x)/(Sqrt[3
3]*Sqrt[2 + x^2])])/16
```

Maple [A] time = 0.043, size = 57, normalized size = 1.

$$\frac{1}{16}\sqrt{16\,(x+1/4)^2-8\,x+31}-\frac{1}{16}\text{Arcsinh}\left(\frac{x\sqrt{2}}{2}\right)-\frac{\sqrt{33}}{16}\text{Artanh}\left(\frac{8\sqrt{33}}{33}\left(4-\frac{x}{2}\right)\frac{1}{\sqrt{16\,(x+1/4)^2-8\,x+31}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2)^(1/2)/(4*x+1),x)
```

```
[Out] 1/16*(16*(x+1/4)^2-8*x+31)^(1/2)-1/16*arcsinh(1/2*x*2^(1/2))-1/16*33^(1/2)*
arctanh(8/33*(4-1/2*x)*33^(1/2)/(16*(x+1/4)^2-8*x+31)^(1/2))
```

Maxima [A] time = 1.53476, size = 72, normalized size = 1.29

$$\frac{1}{16}\sqrt{33}\,\text{arsinh}\left(\frac{\sqrt{2}x}{2\,|4\,x+1|}-\frac{4\sqrt{2}}{|4\,x+1|}\right)+\frac{1}{4}\sqrt{x^2+2}-\frac{1}{16}\,\text{arsinh}\left(\frac{1}{2}\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)^(1/2)/(1+4*x),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(33)*arcsinh(1/2*sqrt(2)*x/abs(4*x + 1) - 4*sqrt(2)/abs(4*x + 1))
+ 1/4*sqrt(x^2 + 2) - 1/16*arcsinh(1/2*sqrt(2)*x)
```

Fricas [A] time = 2.5118, size = 190, normalized size = 3.39

$$\frac{1}{16}\sqrt{33}\log\left(-\frac{\sqrt{33}(x-8)+\sqrt{x^2+2}\left(\sqrt{33}+33\right)+x-8}{4x+1}\right)+\frac{1}{4}\sqrt{x^2+2}+\frac{1}{16}\log\left(-x+\sqrt{x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)^(1/2)/(1+4*x),x, algorithm="fricas")
```

```
[Out] 1/16*sqrt(33)*log(-(sqrt(33)*(x - 8) + sqrt(x^2 + 2)*(sqrt(33) + 33) + x -
8)/(4*x + 1)) + 1/4*sqrt(x^2 + 2) + 1/16*log(-x + sqrt(x^2 + 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+2}}{4x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2)**(1/2)/(1+4*x),x)
```

```
[Out] Integral(sqrt(x**2 + 2)/(4*x + 1), x)
```

Giac [A] time = 1.48047, size = 96, normalized size = 1.71

$$\frac{1}{16}\sqrt{33}\log\left(\frac{\left|-4x-\sqrt{33}+4\sqrt{x^2+2}-1\right|}{\left|-4x+\sqrt{33}+4\sqrt{x^2+2}-1\right|}\right)+\frac{1}{4}\sqrt{x^2+2}+\frac{1}{16}\log\left(-x+\sqrt{x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)^(1/2)/(1+4*x),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(33)*log(abs(-4*x - sqrt(33) + 4*sqrt(x^2 + 2) - 1)/abs(-4*x + sqr
t(33) + 4*sqrt(x^2 + 2) - 1)) + 1/4*sqrt(x^2 + 2) + 1/16*log(-x + sqrt(x^2
+ 2))
```

3.559 $\int \frac{\sqrt{2+4x^2}}{5+4x}\,dx$

Optimal. Leaf size=67

$$\frac{\sqrt{2x^2+1}}{2\sqrt{2}}-\frac{1}{8}\sqrt{33}\tanh^{-1}\left(\frac{\sqrt{\frac{2}{33}}(2-5x)}{\sqrt{2x^2+1}}\right)-\frac{5}{8}\sinh^{-1}\left(\sqrt{2}x\right)$$

```
[Out] Sqrt[1 + 2*x^2]/(2*Sqrt[2]) - (5*ArcSinh[Sqrt[2]*x])/8 - (Sqrt[33]*ArcTanh[
(Sqrt[2/33]*(2 - 5*x))/Sqrt[1 + 2*x^2]])/8
```

Rubi [A] time = 0.0439505, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {735, 844, 215, 725, 206}

$$\frac{\sqrt{2x^2+1}}{2\sqrt{2}}-\frac{1}{8}\sqrt{33}\tanh^{-1}\left(\frac{\sqrt{\frac{2}{33}}(2-5x)}{\sqrt{2x^2+1}}\right)-\frac{5}{8}\sinh^{-1}\left(\sqrt{2}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 4*x^2]/(5 + 4*x),x]
```

```
[Out] Sqrt[1 + 2*x^2]/(2*Sqrt[2]) - (5*ArcSinh[Sqrt[2]*x])/8 - (Sqrt[33]*ArcTanh[
(Sqrt[2/33]*(2 - 5*x))/Sqrt[1 + 2*x^2]])/8
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2+4x^2}}{5+4x}\,dx &= \frac{\sqrt{1+2x^2}}{2\sqrt{2}}+\frac{1}{4}\int \frac{8-20x}{(5+4x)\sqrt{2+4x^2}}\,dx\\ &= \frac{\sqrt{1+2x^2}}{2\sqrt{2}}-\frac{5}{4}\int \frac{1}{\sqrt{2+4x^2}}\,dx+\frac{33}{4}\int \frac{1}{(5+4x)\sqrt{2+4x^2}}\,dx\\ &= \frac{\sqrt{1+2x^2}}{2\sqrt{2}}-\frac{5}{8}\sinh^{-1}\left(\sqrt{2}x\right)-\frac{33}{4}\text{Subst}\left(\int \frac{1}{132-x^2}\,dx,x,\frac{8-20x}{\sqrt{2+4x^2}}\right)\\ &= \frac{\sqrt{1+2x^2}}{2\sqrt{2}}-\frac{5}{8}\sinh^{-1}\left(\sqrt{2}x\right)-\frac{1}{8}\sqrt{33}\tanh^{-1}\left(\frac{\sqrt{\frac{2}{33}}(2-5x)}{\sqrt{1+2x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0450026, size = 57, normalized size = 0.85

$$\frac{1}{4}\sqrt{4x^2+2}-\frac{1}{8}\sqrt{33}\tanh^{-1}\left(\frac{2-5x}{\sqrt{33x^2+\frac{33}{2}}}\right)-\frac{5}{8}\sinh^{-1}\left(\sqrt{2}x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 4*x^2]/(5 + 4*x),x]
```

```
[Out] Sqrt[2 + 4*x^2]/4 - (5*ArcSinh[Sqrt[2]*x])/8 - (Sqrt[33]*ArcTanh[(2 - 5*x)/
Sqrt[33/2 + 33*x^2]])/8
```

Maple [A] time = 0.042, size = 56, normalized size = 0.8

$$\frac{1}{8}\sqrt{16\,(x+5/4)^2-40\,x-17}-\frac{5\,\text{Arcsinh}\left(x\sqrt{2}\right)}{8}-\frac{\sqrt{33}}{8}\text{Artanh}\left(\frac{(8-20\,x)\sqrt{33}}{33}\frac{1}{\sqrt{16\,(x+5/4)^2-40\,x-17}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+2)^(1/2)/(5+4*x),x)
```

```
[Out] 1/8*(16*(x+5/4)^2-40*x-17)^(1/2)-5/8*arcsinh(x*2^(1/2))-1/8*33^(1/2)*arctan
h(2/33*(4-10*x)*33^(1/2)/(16*(x+5/4)^2-40*x-17)^(1/2))
```

Maxima [A] time = 1.94091, size = 73, normalized size = 1.09

$$\frac{1}{8}\sqrt{33}\,\text{arsinh}\left(\frac{5\sqrt{2}x}{|4\,x+5|}-\frac{2\sqrt{2}}{|4\,x+5|}\right)+\frac{1}{4}\sqrt{4\,x^2+2}-\frac{5}{8}\,\text{arsinh}\left(\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+2)^(1/2)/(5+4*x),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(33)*arcsinh(5*sqrt(2)*x/abs(4*x + 5) - 2*sqrt(2)/abs(4*x + 5)) + 1
/4*sqrt(4*x^2 + 2) - 5/8*arcsinh(sqrt(2)*x)
```

Fricas [A] time = 2.47994, size = 212, normalized size = 3.16

$$\frac{1}{8}\sqrt{33}\log\left(-\frac{2\sqrt{33}(5x-2)+\sqrt{4x^2+2}\left(5\sqrt{33}+33\right)+50x-20}{4x+5}\right)+\frac{1}{4}\sqrt{4x^2+2}+\frac{5}{8}\log\left(-2x+\sqrt{4x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+2)^(1/2)/(5+4*x),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(33)*log(-(2*sqrt(33)*(5*x - 2) + sqrt(4*x^2 + 2)*(5*sqrt(33) + 33)
 + 50*x - 20)/(4*x + 5)) + 1/4*sqrt(4*x^2 + 2) + 5/8*log(-2*x + sqrt(4*x^2
+ 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\sqrt{2}\int\frac{\sqrt{2x^2+1}}{4x+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+2)**(1/2)/(5+4*x),x)
```

```
[Out] sqrt(2)*Integral(sqrt(2*x**2 + 1)/(4*x + 5), x)
```

Giac [B] time = 1.40396, size = 142, normalized size = 2.12

$$\frac{1}{16}\sqrt{2}\left(5\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{2x^2+1}\right)+\sqrt{66}\log\left(-\frac{\left|-4\sqrt{2}x-\sqrt{66}-5\sqrt{2}+4\sqrt{2x^2+1}\right|}{4\sqrt{2}x-\sqrt{66}+5\sqrt{2}-4\sqrt{2x^2+1}}\right)+4\sqrt{2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+2)^(1/2)/(5+4*x),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(5*sqrt(2)*log(-sqrt(2)*x + sqrt(2*x^2 + 1)) + sqrt(66)*log(-a
bs(-4*sqrt(2)*x - sqrt(66) - 5*sqrt(2) + 4*sqrt(2*x^2 + 1))/(4*sqrt(2)*x -
sqrt(66) + 5*sqrt(2) - 4*sqrt(2*x^2 + 1))) + 4*sqrt(2*x^2 + 1))
```

3.560 $\int (2+3x)\sqrt{-5+7x^2}\,dx$

Optimal. Leaf size=55

$$\frac{1}{7}\left(7x^2-5\right)^{3/2}+x\sqrt{7x^2-5}-\frac{5\tanh^{-1}\left(\frac{\sqrt{7}x}{\sqrt{7x^2-5}}\right)}{\sqrt{7}}$$

```
[Out] x*Sqrt[-5 + 7*x^2] + (-5 + 7*x^2)^(3/2)/7 - (5*ArcTanh[(Sqrt[7]*x)/Sqrt[-5
+ 7*x^2]])/Sqrt[7]
```

Rubi [A] time = 0.0152842, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {641, 195, 217, 206}

$$\frac{1}{7}\left(7x^2-5\right)^{3/2}+x\sqrt{7x^2-5}-\frac{5\tanh^{-1}\left(\frac{\sqrt{7}x}{\sqrt{7x^2-5}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)*Sqrt[-5 + 7*x^2],x]
```

```
[Out] x*Sqrt[-5 + 7*x^2] + (-5 + 7*x^2)^(3/2)/7 - (5*ArcTanh[(Sqrt[7]*x)/Sqrt[-5
+ 7*x^2]])/Sqrt[7]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int (2+3x)\sqrt{-5+7x^2}\,dx &= \frac{1}{7}\left(-5+7x^2\right)^{3/2}+2\int \sqrt{-5+7x^2}\,dx\\ &= x\sqrt{-5+7x^2}+\frac{1}{7}\left(-5+7x^2\right)^{3/2}-5\int \frac{1}{\sqrt{-5+7x^2}}\,dx\\ &= x\sqrt{-5+7x^2}+\frac{1}{7}\left(-5+7x^2\right)^{3/2}-5\,\text{Subst}\left(\int \frac{1}{1-7x^2}\,dx,x,\frac{x}{\sqrt{-5+7x^2}}\right)\\ &= x\sqrt{-5+7x^2}+\frac{1}{7}\left(-5+7x^2\right)^{3/2}-\frac{5\tanh^{-1}\left(\frac{\sqrt{7}x}{\sqrt{-5+7x^2}}\right)}{\sqrt{7}}\end{aligned}$$

Mathematica [A] time = 0.0401515, size = 50, normalized size = 0.91

$$\left(x^2+x-\frac{5}{7}\right)\sqrt{7x^2-5}-\frac{5\log\left(\sqrt{7}\sqrt{7x^2-5}+7x\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)*Sqrt[-5 + 7*x^2],x]
```

```
[Out] (-5/7 + x + x^2)*Sqrt[-5 + 7*x^2] - (5*Log[7*x + Sqrt[7]*Sqrt[-5 + 7*x^2]])
/Sqrt[7]
```

Maple [A] time = 0.041, size = 45, normalized size = 0.8

$$x\sqrt{7\,x^2-5}-\frac{5\,\sqrt{7}}{7}\ln\left(x\sqrt{7}+\sqrt{7\,x^2-5}\right)+\frac{1}{7}\left(7\,x^2-5\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)*(7*x^2-5)^(1/2),x)
```

```
[Out] x*(7*x^2-5)^(1/2)-5/7*ln(x*7^(1/2)+(7*x^2-5)^(1/2))*7^(1/2)+1/7*(7*x^2-5)^(
3/2)
```

Maxima [A] time = 1.8913, size = 63, normalized size = 1.15

$$\frac{1}{7}\left(7\,x^2-5\right)^{\frac{3}{2}}+\sqrt{7\,x^2-5}x-\frac{5}{7}\,\sqrt{7}\log\left(2\,\sqrt{7}\sqrt{7\,x^2-5}+14\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(7*x^2-5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*(7*x^2 - 5)^(3/2) + sqrt(7*x^2 - 5)*x - 5/7*sqrt(7)*log(2*sqrt(7)*sqrt(
7*x^2 - 5) + 14*x)
```

Fricas [A] time = 2.41585, size = 136, normalized size = 2.47

$$\frac{1}{7}\left(7\,x^2+7\,x-5\right)\sqrt{7\,x^2-5}+\frac{5}{14}\,\sqrt{7}\log\left(-2\,\sqrt{7}\sqrt{7\,x^2-5}x+14\,x^2-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(7*x^2-5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/7*(7*x^2 + 7*x - 5)*sqrt(7*x^2 - 5) + 5/14*sqrt(7)*log(-2*sqrt(7)*sqrt(7*
x^2 - 5)*x + 14*x^2 - 5)
```

Sympy [A] time = 0.416093, size = 56, normalized size = 1.02

$$x^2\sqrt{7x^2-5}+x\sqrt{7x^2-5}-\frac{5\sqrt{7x^2-5}}{7}-\frac{5\sqrt{7}\operatorname{acosh}\left(\frac{\sqrt{35}x}{5}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(7*x**2-5)**(1/2),x)
```

```
[Out] x**2*sqrt(7*x**2 - 5) + x*sqrt(7*x**2 - 5) - 5*sqrt(7*x**2 - 5)/7 - 5*sqrt(
7)*acosh(sqrt(35)*x/5)/7
```

Giac [A] time = 1.33174, size = 58, normalized size = 1.05

$$\frac{1}{7}\left(7\left(x+1\right)x-5\right)\sqrt{7\,x^2-5}+\frac{5}{7}\,\sqrt{7}\log\left(\left|-\sqrt{7}x+\sqrt{7\,x^2-5}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(7*x^2-5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/7*(7*(x + 1)*x - 5)*sqrt(7*x^2 - 5) + 5/7*sqrt(7)*log(abs(-sqrt(7)*x + sq
rt(7*x^2 - 5)))
```

3.561 $\int \frac{(d+ex)^4}{\sqrt{a+cx^2}} dx$

Optimal. Leaf size=161

$$\frac{(3a^2e^4 - 24acd^2e^2 + 8c^2d^4)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{5/2}} + \frac{e\sqrt{a+cx^2}\left(ex\left(26cd^2 - 9ae^2\right) + 4d\left(19cd^2 - 16ae^2\right)\right)}{24c^2} + \frac{e\sqrt{a+cx^2}(d+ex)^3}{4c}$$

```
[Out] (7*d*e*(d + e*x)^2*Sqrt[a + c*x^2])/(12*c) + (e*(d + e*x)^3*Sqrt[a + c*x^2]
)/(4*c) + (e*(4*d*(19*c*d^2 - 16*a*e^2) + e*(26*c*d^2 - 9*a*e^2)*x)*Sqrt[a
+ c*x^2])/(24*c^2) + ((8*c^2*d^4 - 24*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTanh[(Sqr
t[c]*x)/Sqrt[a + c*x^2]])/(8*c^(5/2))
```

Rubi [A] time = 0.161434, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {743, 833, 780, 217, 206}

$$\frac{(3a^2e^4 - 24acd^2e^2 + 8c^2d^4)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{5/2}} + \frac{e\sqrt{a+cx^2}\left(ex\left(26cd^2 - 9ae^2\right) + 4d\left(19cd^2 - 16ae^2\right)\right)}{24c^2} + \frac{e\sqrt{a+cx^2}(d+ex)^3}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/Sqrt[a + c*x^2],x]
```


```
[Out] (7*d*e*(d + e*x)^2*Sqrt[a + c*x^2])/(12*c) + (e*(d + e*x)^3*Sqrt[a + c*x^2]
)/(4*c) + (e*(4*d*(19*c*d^2 - 16*a*e^2) + e*(26*c*d^2 - 9*a*e^2)*x)*Sqrt[a
+ c*x^2])/(24*c^2) + ((8*c^2*d^4 - 24*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTanh[(Sqr
t[c]*x)/Sqrt[a + c*x^2]])/(8*c^(5/2))
```


Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^4}{\sqrt{a+cx^2}}dx = \frac{e(d+ex)^3\sqrt{a+cx^2}}{4c} + \frac{\int \frac{(d+ex)^2(4cd^2-3ae^2+7cdex)}{\sqrt{a+cx^2}}dx}{4c}$$

$$= \frac{7de(d+ex)^2\sqrt{a+cx^2}}{12c} + \frac{e(d+ex)^3\sqrt{a+cx^2}}{4c} + \frac{\int \frac{(d+ex)(cd(12cd^2-23ae^2)+ce(26cd^2-9ae^2)x)}{\sqrt{a+cx^2}}dx}{12c^2}$$

$$= \frac{7de(d+ex)^2\sqrt{a+cx^2}}{12c} + \frac{e(d+ex)^3\sqrt{a+cx^2}}{4c} + \frac{e\left(4d\left(19cd^2-16ae^2\right)+e\left(26cd^2-9ae^2\right)x\right)\sqrt{a+cx^2}}{24c^2}$$

$$= \frac{7de(d+ex)^2\sqrt{a+cx^2}}{12c} + \frac{e(d+ex)^3\sqrt{a+cx^2}}{4c} + \frac{e\left(4d\left(19cd^2-16ae^2\right)+e\left(26cd^2-9ae^2\right)x\right)\sqrt{a+cx^2}}{24c^2}$$

$$= \frac{7de(d+ex)^2\sqrt{a+cx^2}}{12c} + \frac{e(d+ex)^3\sqrt{a+cx^2}}{4c} + \frac{e\left(4d\left(19cd^2-16ae^2\right)+e\left(26cd^2-9ae^2\right)x\right)\sqrt{a+cx^2}}{24c^2}$$

Mathematica [A] time = 0.107137, size = 126, normalized size = 0.78

$$\frac{3\left(3a^2e^4-24acd^2e^2+8c^2d^4\right)\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)+\sqrt{c}e\sqrt{a+cx^2}\left(c\left(72d^2ex+96d^3+32de^2x^2+6e^3x^3\right)-ae^2(64d\right.}{24c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[c]*e*Sqrt[a + c*x^2]*(-(a*e^2*(64*d + 9*e*x)) + c*(96*d^3 + 72*d^2*e*
x + 32*d*e^2*x^2 + 6*e^3*x^3)) + 3*(8*c^2*d^4 - 24*a*c*d^2*e^2 + 3*a^2*e^4)
*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/(24*c^(5/2))
```

Maple [A] time = 0.049, size = 198, normalized size = 1.2

$$\frac{e^4x^3}{4c}\sqrt{cx^2+a}-\frac{3e^4ax}{8c^2}\sqrt{cx^2+a}+\frac{3a^2e^4}{8}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)c^{-\frac{5}{2}}+\frac{4de^3x^2}{3c}\sqrt{cx^2+a}-\frac{8de^3a}{3c^2}\sqrt{cx^2+a}+3\frac{d^2e^2x\sqrt{c}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/4*e^4*x^3/c*(c*x^2+a)^(1/2)-3/8*e^4*a/c^2*x*(c*x^2+a)^(1/2)+3/8*e^4*a^2/c
^(5/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))+4/3*d*e^3*x^2/c*(c*x^2+a)^(1/2)-8/3*d*
e^3*a/c^2*(c*x^2+a)^(1/2)+3*d^2*e^2*x/c*(c*x^2+a)^(1/2)-3*d^2*e^2*a/c^(3/2)
```

```
*ln(x*c^(1/2)+(c*x^2+a)^(1/2))+4*d^3*e/c*(c*x^2+a)^(1/2)+d^4*ln(x*c^(1/2)+(
c*x^2+a)^(1/2))/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94591, size = 603, normalized size = 3.75

$$\left[\frac{3\left(8\,c^2d^4 - 24\,acd^2e^2 + 3\,a^2e^4\right)\sqrt{c}\log\left(-2\,cx^2 - 2\,\sqrt{cx^2+a}\sqrt{c}x - a\right) + 2\left(6\,c^2e^4x^3 + 32\,c^2de^3x^2 + 96\,c^2d^3e - 64\,acde^3 + 9\right.}{48\,c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(8*c^2*d^4 - 24*a*c*d^2*e^2 + 3*a^2*e^4)*sqrt(c)*log(-2*c*x^2 - 2*
sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(6*c^2*e^4*x^3 + 32*c^2*d*e^3*x^2 + 96*c
^2*d^3*e - 64*a*c*d*e^3 + 9*(8*c^2*d^2*e^2 - a*c*e^4)*x)*sqrt(c*x^2 + a))/c
^3, -1/24*(3*(8*c^2*d^4 - 24*a*c*d^2*e^2 + 3*a^2*e^4)*sqrt(-c)*arctan(sqrt(
-c)*x/sqrt(c*x^2 + a)) - (6*c^2*e^4*x^3 + 32*c^2*d*e^3*x^2 + 96*c^2*d^3*e -
 64*a*c*d*e^3 + 9*(8*c^2*d^2*e^2 - a*c*e^4)*x)*sqrt(c*x^2 + a))/c^3]
```

Sympy [A] time = 8.7505, size = 330, normalized size = 2.05

$$-\frac{3a^{\frac{3}{2}}e^4x}{8c^2\sqrt{1+\frac{cx^2}{a}}} + \frac{3\sqrt{a}d^2e^2x\sqrt{1+\frac{cx^2}{a}}}{c} - \frac{\sqrt{a}e^4x^3}{8c\sqrt{1+\frac{cx^2}{a}}} + \frac{3a^2e^4\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8c^{\frac{5}{2}}} - \frac{3ad^2e^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{c^{\frac{3}{2}}} + d^4\left(\begin{cases}\frac{\sqrt{-\frac{a}{c}}\operatorname{asin}\left(x\sqrt{-\frac{c}{a}}\right)}{\sqrt{a}} \\ \frac{\sqrt{\frac{a}{c}}\operatorname{asinh}\left(x\sqrt{\frac{c}{a}}\right)}{\sqrt{a}} \\ \frac{\sqrt{-\frac{a}{c}}\operatorname{acosh}\left(x\sqrt{-\frac{c}{a}}\right)}{\sqrt{-a}}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(c*x**2+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*e**4*x/(8*c**2*sqrt(1 + c*x**2/a)) + 3*sqrt(a)*d**2*e**2*x*sqrt
(1 + c*x**2/a)/c - sqrt(a)*e**4*x**3/(8*c*sqrt(1 + c*x**2/a)) + 3*a**2*e**4
*asinh(sqrt(c)*x/sqrt(a))/(8*c**(5/2)) - 3*a*d**2*e**2*asinh(sqrt(c)*x/sqrt
(a))/c**(3/2) + d**4*Piecewise((sqrt(-a/c)*asin(x*sqrt(-c/a))/sqrt(a), (a >
 0) & (c < 0)), (sqrt(a/c)*asinh(x*sqrt(c/a))/sqrt(a), (a > 0) & (c > 0)),
(sqrt(-a/c)*acosh(x*sqrt(-c/a))/sqrt(-a), (c > 0) & (a < 0))) + 4*d**3*e*Pi
ecewise((x**2/(2*sqrt(a)), Eq(c, 0)), (sqrt(a + c*x**2)/c, True)) + 4*d*e**
3*Piecewise((-2*a*sqrt(a + c*x**2)/(3*c**2) + x**2*sqrt(a + c*x**2)/(3*c),
```

```
Ne(c, 0)), (x**4/(4*sqrt(a)), True)) + e**4*x**5/(4*sqrt(a)*sqrt(1 + c*x**2
/a))
```

Giac [A] time = 1.32206, size = 180, normalized size = 1.12

$$\frac{1}{24}\sqrt{cx^2+a}\left(\left(2x\left(\frac{3xe^4}{c}+\frac{16de^3}{c}\right)+\frac{9\left(8c^3d^2e^2-ac^2e^4\right)}{c^4}\right)x+\frac{32\left(3c^3d^3e-2ac^2de^3\right)}{c^4}\right)-\frac{\left(8c^2d^4-24acd^2e^2+3a^2e^4\right)}{8c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + a)*((2*x*(3*x*e^4/c + 16*d*e^3/c) + 9*(8*c^3*d^2*e^2 - a*
c^2*e^4)/c^4)*x + 32*(3*c^3*d^3*e - 2*a*c^2*d*e^3)/c^4) - 1/8*(8*c^2*d^4 -
24*a*c*d^2*e^2 + 3*a^2*e^4)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

3.562 $\int \frac{(d+ex)^3}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=110

$$\frac{e\sqrt{a+cx^2}\left(4\left(4cd^2-ae^2\right)+5cdex\right)}{6c^2}+\frac{d\left(2cd^2-3ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}}+\frac{e\sqrt{a+cx^2}(d+ex)^2}{3c}$$

```
[Out] (e*(d + e*x)^2*Sqrt[a + c*x^2])/(3*c) + (e*(4*(4*c*d^2 - a*e^2) + 5*c*d*e*x
)*Sqrt[a + c*x^2])/(6*c^2) + (d*(2*c*d^2 - 3*a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqr
t[a + c*x^2]])/(2*c^(3/2))
```

Rubi [A] time = 0.0753626, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {743, 780, 217, 206}

$$\frac{e\sqrt{a+cx^2}\left(4\left(4cd^2-ae^2\right)+5cdex\right)}{6c^2}+\frac{d\left(2cd^2-3ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}}+\frac{e\sqrt{a+cx^2}(d+ex)^2}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/Sqrt[a + c*x^2],x]
```

```
[Out] (e*(d + e*x)^2*Sqrt[a + c*x^2])/(3*c) + (e*(4*(4*c*d^2 - a*e^2) + 5*c*d*e*x
)*Sqrt[a + c*x^2])/(6*c^2) + (d*(2*c*d^2 - 3*a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqr
t[a + c*x^2]])/(2*c^(3/2))
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^2}}\,dx = \frac{e(d+ex)^2\sqrt{a+cx^2}}{3c} + \frac{\int \frac{(d+ex)\left(3cd^2-2ae^2+5cdex\right)}{\sqrt{a+cx^2}}dx}{3c}$$
$$= \frac{e(d+ex)^2\sqrt{a+cx^2}}{3c} + \frac{e\left(4\left(4cd^2-ae^2\right)+5cdex\right)\sqrt{a+cx^2}}{6c^2} + \frac{\left(d\left(2cd^2-3ae^2\right)\right)\int \frac{1}{\sqrt{a+cx^2}}dx}{2c}$$
$$= \frac{e(d+ex)^2\sqrt{a+cx^2}}{3c} + \frac{e\left(4\left(4cd^2-ae^2\right)+5cdex\right)\sqrt{a+cx^2}}{6c^2} + \frac{\left(d\left(2cd^2-3ae^2\right)\right)\text{Subst}\left(\int \frac{1}{1-cx^2}dx, x\right.}{2c}$$
$$= \frac{e(d+ex)^2\sqrt{a+cx^2}}{3c} + \frac{e\left(4\left(4cd^2-ae^2\right)+5cdex\right)\sqrt{a+cx^2}}{6c^2} + \frac{d\left(2cd^2-3ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}}$$

Mathematica [A] time = 0.0623585, size = 92, normalized size = 0.84

$$\frac{e\sqrt{a+cx^2}\left(c\left(18d^2+9dex+2e^2x^2\right)-4ae^2\right)+3\sqrt{c}d\left(2cd^2-3ae^2\right)\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/Sqrt[a + c*x^2],x]
```

```
[Out] (e*Sqrt[a + c*x^2]*(-4*a*e^2 + c*(18*d^2 + 9*d*e*x + 2*e^2*x^2)) + 3*Sqrt[c
]*d*(2*c*d^2 - 3*a*e^2)*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/(6*c^2)
```

Maple [A] time = 0.048, size = 126, normalized size = 1.2

$$\frac{e^3x^2}{3\,c}\sqrt{cx^2+a} - \frac{2\,ae^3}{3\,c^2}\sqrt{cx^2+a} + \frac{3\,de^2x}{2\,c}\sqrt{cx^2+a} - \frac{3\,ade^2}{2}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)c^{-\frac{3}{2}} + 3\,\frac{d^2e\sqrt{cx^2+a}}{c} + d^3\ln\left(x\sqrt{c}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/3*e^3*x^2/c*(c*x^2+a)^(1/2)-2/3*e^3*a/c^2*(c*x^2+a)^(1/2)+3/2*d*e^2*x/c*(
c*x^2+a)^(1/2)-3/2*d*e^2*a/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))+3*d^2*e/c*
(c*x^2+a)^(1/2)+d^3*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.90701, size = 428, normalized size = 3.89

$$\left[-\frac{3\left(2\,cd^3-3\,ade^2\right)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)-2\left(2\,ce^3x^2+9\,cde^2x+18\,cd^2e-4\,ae^3\right)\sqrt{cx^2+a}}{12\,c^2},-\frac{3\left(2\,cd^3-3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(2*c*d^3 - 3*a*d*e^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sq
rt(c)*x - a) - 2*(2*c*e^3*x^2 + 9*c*d*e^2*x + 18*c*d^2*e - 4*a*e^3)*sqrt(c*
x^2 + a))/c^2, -1/6*(3*(2*c*d^3 - 3*a*d*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqr
t(c*x^2 + a)) - (2*c*e^3*x^2 + 9*c*d*e^2*x + 18*c*d^2*e - 4*a*e^3)*sqrt(c*x
^2 + a))/c^2]
```

Sympy [A] time = 5.99054, size = 216, normalized size = 1.96

$$\frac{3\sqrt{a}de^2x\sqrt{1+\frac{cx^2}{a}}}{2c}-\frac{3ade^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2c^{\frac{3}{2}}}+d^3\left(\begin{cases}\frac{\sqrt{-\frac{a}{c}}\operatorname{asin}\left(x\sqrt{-\frac{c}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge c<0\\ \frac{\sqrt{\frac{a}{c}}\operatorname{asinh}\left(x\sqrt{\frac{c}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge c>0\\ \frac{\sqrt{-\frac{a}{c}}\operatorname{acosh}\left(x\sqrt{-\frac{c}{a}}\right)}{\sqrt{-a}} & \text{for}\ c>0\wedge a<0\end{cases}\right)+3d^2e\left(\begin{cases}\frac{x^2}{2\sqrt{a}} & \text{for}\ c=0\\ \frac{\sqrt{a+cx^2}}{c} & \text{otherwise}\end{cases}\right)+e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**2+a)**(1/2),x)
```

```
[Out] 3*sqrt(a)*d*e**2*x*sqrt(1 + c*x**2/a)/(2*c) - 3*a*d*e**2*asinh(sqrt(c)*x/sq
rt(a))/(2*c**(3/2)) + d**3*Piecewise((sqrt(-a/c)*asin(x*sqrt(-c/a))/sqrt(a)
, (a > 0) & (c < 0)), (sqrt(a/c)*asinh(x*sqrt(c/a))/sqrt(a), (a > 0) & (c >
 0)), (sqrt(-a/c)*acosh(x*sqrt(-c/a))/sqrt(-a), (c > 0) & (a < 0))) + 3*d**
2*e*Piecewise((x**2/(2*sqrt(a)), Eq(c, 0)), (sqrt(a + c*x**2)/c, True)) + e
**3*Piecewise((-2*a*sqrt(a + c*x**2)/(3*c**2) + x**2*sqrt(a + c*x**2)/(3*c)
, Ne(c, 0)), (x**4/(4*sqrt(a)), True))
```

Giac [A] time = 1.36014, size = 122, normalized size = 1.11

$$\frac{1}{6}\sqrt{cx^2+a}\left(x\left(\frac{2\,xe^3}{c}+\frac{9\,de^2}{c}\right)+\frac{2\left(9\,c^2d^2e-2\,ace^3\right)}{c^3}\right)-\frac{\left(2\,cd^3-3\,ade^2\right)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{2\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(c*x^2 + a)*(x*(2*x*e^3/c + 9*d*e^2/c) + 2*(9*c^2*d^2*e - 2*a*c*e^3
)/c^3) - 1/2*(2*c*d^3 - 3*a*d*e^2)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c
^(3/2)
```

3.563 $\int \frac{(d+ex)^2}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=86

$$\frac{\left(2cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}}+\frac{3de\sqrt{a+cx^2}}{2c}+\frac{e\sqrt{a+cx^2}(d+ex)}{2c}$$

```
[Out] (3*d*e*Sqrt[a + c*x^2])/(2*c) + (e*(d + e*x)*Sqrt[a + c*x^2])/(2*c) + ((2*c
*d^2 - a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2))
```

Rubi [A] time = 0.0402104, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {743, 641, 217, 206}

$$\frac{\left(2cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}}+\frac{3de\sqrt{a+cx^2}}{2c}+\frac{e\sqrt{a+cx^2}(d+ex)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/Sqrt[a + c*x^2],x]
```

```
[Out] (3*d*e*Sqrt[a + c*x^2])/(2*c) + (e*(d + e*x)*Sqrt[a + c*x^2])/(2*c) + ((2*c
*d^2 - a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2))
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^2}{\sqrt{a+cx^2}}\,dx = \frac{e(d+ex)\sqrt{a+cx^2}}{2c} + \frac{\int \frac{2cd^2-ae^2+3cdex}{\sqrt{a+cx^2}}\,dx}{2c}$$
$$= \frac{3de\sqrt{a+cx^2}}{2c} + \frac{e(d+ex)\sqrt{a+cx^2}}{2c} + \frac{\left(2cd^2-ae^2\right)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2c}$$
$$= \frac{3de\sqrt{a+cx^2}}{2c} + \frac{e(d+ex)\sqrt{a+cx^2}}{2c} + \frac{\left(2cd^2-ae^2\right)\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{2c}$$
$$= \frac{3de\sqrt{a+cx^2}}{2c} + \frac{e(d+ex)\sqrt{a+cx^2}}{2c} + \frac{\left(2cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}}$$

Mathematica [A] time = 0.0424227, size = 71, normalized size = 0.83

$$\frac{\left(2cd^2-ae^2\right)\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)+\sqrt{c}e\sqrt{a+cx^2}(4d+ex)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[c]*e*(4*d + e*x)*Sqrt[a + c*x^2] + (2*c*d^2 - a*e^2)*Log[c*x + Sqrt[c
]*Sqrt[a + c*x^2]])/(2*c^(3/2))
```

Maple [A] time = 0.047, size = 84, normalized size = 1.

$$\frac{e^2x}{2c}\sqrt{cx^2+a}-\frac{ae^2}{2}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)c^{-\frac{3}{2}}+2\frac{de\sqrt{cx^2+a}}{c}+d^2\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/2*e^2*x/c*(c*x^2+a)^(1/2)-1/2*e^2*a/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
+2*d*e*(c*x^2+a)^(1/2)/c+d^2*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.92376, size = 327, normalized size = 3.8

$$\left[-\frac{\left(2\,cd^2-ae^2\right)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)-2\left(ce^2x+4\,cde\right)\sqrt{cx^2+a}}{4\,c^2}, -\frac{\left(2\,cd^2-ae^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)-(\right.}{2\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*c*d^2 - a*e^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x
 - a) - 2*(c*e^2*x + 4*c*d*e)*sqrt(c*x^2 + a))/c^2, -1/2*((2*c*d^2 - a*e^2)
*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (c*e^2*x + 4*c*d*e)*sqrt(c*x
^2 + a))/c^2]
```

Sympy [A] time = 4.90411, size = 158, normalized size = 1.84

$$\frac{\sqrt{a}e^2x\sqrt{1+\frac{cx^2}{a}}}{2c}-\frac{ae^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2c^{\frac{3}{2}}}+d^2\left(\begin{cases}\frac{\sqrt{-\frac{a}{c}}\operatorname{asin}\left(x\sqrt{-\frac{c}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge c<0\\ \frac{\sqrt{\frac{a}{c}}\operatorname{asinh}\left(x\sqrt{\frac{c}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge c>0\\ \frac{\sqrt{-\frac{a}{c}}\operatorname{acosh}\left(x\sqrt{-\frac{c}{a}}\right)}{\sqrt{-a}} & \text{for } c>0\wedge a<0\end{cases}\right)+2de\left(\begin{cases}\frac{x^2}{2\sqrt{a}} & \text{for } c=0\\ \frac{\sqrt{a+cx^2}}{c} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*e**2*x*sqrt(1 + c*x**2/a)/(2*c) - a*e**2*asinh(sqrt(c)*x/sqrt(a))/(
2*c**(3/2)) + d**2*Piecewise((sqrt(-a/c)*asin(x*sqrt(-c/a))/sqrt(a), (a > 0
) & (c < 0)), (sqrt(a/c)*asinh(x*sqrt(c/a))/sqrt(a), (a > 0) & (c > 0)), (s
qrt(-a/c)*acosh(x*sqrt(-c/a))/sqrt(-a), (c > 0) & (a < 0))) + 2*d*e*Piecewi
se((x**2/(2*sqrt(a)), Eq(c, 0)), (sqrt(a + c*x**2)/c, True))
```

Giac [A] time = 1.37338, size = 85, normalized size = 0.99

$$\frac{1}{2}\sqrt{cx^2+a}\left(\frac{xe^2}{c}+\frac{4\,de}{c}\right)-\frac{\left(2\,cd^2-ae^2\right)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{2\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*(x*e^2/c + 4*d*e/c) - 1/2*(2*c*d^2 - a*e^2)*log(abs(-sq
rt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

3.564 $\int \frac{d+ex}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=43

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}}+\frac{e\sqrt{a+cx^2}}{c}$$

```
[Out] (e*Sqrt[a + c*x^2])/c + (d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/Sqrt[c]
```

Rubi [A] time = 0.0129593, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {641, 217, 206}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}}+\frac{e\sqrt{a+cx^2}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/Sqrt[a + c*x^2],x]
```

```
[Out] (e*Sqrt[a + c*x^2])/c + (d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/Sqrt[c]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex}{\sqrt{a+cx^2}}\,dx &= \frac{e\sqrt{a+cx^2}}{c}+d\int \frac{1}{\sqrt{a+cx^2}}\,dx\\
&= \frac{e\sqrt{a+cx^2}}{c}+d\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)\\
&= \frac{e\sqrt{a+cx^2}}{c}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.0212389, size = 46, normalized size = 1.07

$$\frac{d\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)}{\sqrt{c}}+\frac{e\sqrt{a+cx^2}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/Sqrt[a + c*x^2],x]
```

```
[Out] (e*Sqrt[a + c*x^2])/c + (d*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/Sqrt[c]
```

Maple [A] time = 0.042, size = 37, normalized size = 0.9

$$\frac{e}{c}\sqrt{cx^2+a}+d\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] e*(c*x^2+a)^(1/2)/c+d*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8446, size = 223, normalized size = 5.19

$$\left[\frac{\sqrt{c}d\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)+2\,\sqrt{cx^2+a}e}{2\,c},-\frac{\sqrt{-c}d\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)-\sqrt{cx^2+a}e}{c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c)*d*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*sqrt(c*
x^2 + a)*e)/c, -(sqrt(-c)*d*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - sqrt(c*x^2
 + a)*e)/c]
```

Sympy [B] time = 1.12576, size = 102, normalized size = 2.37

$$d\left(\begin{cases}\frac{\sqrt{-\frac{a}{c}}\operatorname{asin}\left(x\sqrt{-\frac{c}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge c<0\\ \frac{\sqrt{\frac{a}{c}}\operatorname{asinh}\left(x\sqrt{\frac{c}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge c>0\\ \frac{\sqrt{-\frac{a}{c}}\operatorname{acosh}\left(x\sqrt{-\frac{c}{a}}\right)}{\sqrt{-a}} & \text{for } c>0\wedge a<0\end{cases}\right)+e\left(\begin{cases}\frac{x^2}{2\sqrt{a}} & \text{for } c=0\\ \frac{\sqrt{a+cx^2}}{c} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] d*Piecewise((sqrt(-a/c)*asin(x*sqrt(-c/a))/sqrt(a), (a > 0) & (c < 0)), (sq
rt(a/c)*asinh(x*sqrt(c/a))/sqrt(a), (a > 0) & (c > 0)), (sqrt(-a/c)*acosh(x
*sqrt(-c/a))/sqrt(-a), (c > 0) & (a < 0))) + e*Piecewise((x**2/(2*sqrt(a)),
 Eq(c, 0)), (sqrt(a + c*x**2)/c, True))
```

Giac [A] time = 1.52074, size = 54, normalized size = 1.26

$$-\frac{d\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{\sqrt{c}}+\frac{\sqrt{cx^2+a}e}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -d*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/sqrt(c) + sqrt(c*x^2 + a)*e/c
```

3.565 $\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=54

$$-\frac{\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}}$$

```
[Out] -(ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])]/Sqrt[c*d^2 +
 a*e^2])
```

Rubi [A] time = 0.0166401, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {725, 206}

$$-\frac{\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] -(ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])]/Sqrt[c*d^2 +
 a*e^2])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx &= -\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\sqrt{cd^2+ae^2}}\end{aligned}$$

Mathematica [A] time = 0.0123467, size = 54, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] -(ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])]/Sqrt[c*d^2 +
 a*e^2])
```

Maple [B] time = 0.189, size = 127, normalized size = 2.4

$$-\frac{1}{e}\ln\left(\left(2\,\frac{ae^2+cd^2}{e^2}-2\,\frac{cd}{e}\left(\frac{d}{e}+x\right)+2\,\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{c\left(\frac{d}{e}+x\right)^2-2\,\frac{cd}{e}\left(\frac{d}{e}+x\right)+\frac{ae^2+cd^2}{e^2}}\right)\left(\frac{d}{e}+x\right)^{-1}\right)\frac{1}{\sqrt{\frac{ae^2+cd^2}{e^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] -1/e/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((
a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/
2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.98629, size = 435, normalized size = 8.06

$$\left[\frac{\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2-2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)}{2\,\sqrt{cd^2+ae^2}},-\frac{\sqrt{-cd^2-ae^2}\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+\left(c^2d^2+ace^2\right)x^2}\right)}{cd^2+ae^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2
*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^
2))/sqrt(c*d^2 + a*e^2), -sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*
(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)
)/(c*d^2 + a*e^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)), x)
```

Giac [A] time = 1.37275, size = 80, normalized size = 1.48

$$\frac{2\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2
))/sqrt(-c*d^2 - a*e^2)
```

3.566 $\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=91

$$-\frac{e\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)}-\frac{cd\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

```
[Out] -((e*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x))) - (c*d*ArcTanh[(a*e - c*
d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rubi [A] time = 0.0339196, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {731, 725, 206}

$$-\frac{e\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)}-\frac{cd\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -((e*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x))) - (c*d*ArcTanh[(a*e - c*
d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx = -\frac{e\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(d+ex)} + \frac{(cd)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$
$$= -\frac{e\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(d+ex)} - \frac{(cd)\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{cd^2+ae^2}$$
$$= -\frac{e\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(d+ex)} - \frac{cd\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{3/2}}$$

Mathematica [A] time = 0.101042, size = 115, normalized size = 1.26

$$-\frac{e\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)} - \frac{cd\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{\left(ae^2+cd^2\right)^{3/2}} + \frac{cd\log(d+ex)}{\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -((e*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x))) + (c*d*Log[d + e*x])/(c*
d^2 + a*e^2)^(3/2) - (c*d*Log[a*e - c*d*x + Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*
x^2]])/(c*d^2 + a*e^2)^(3/2)
```

Maple [B] time = 0.193, size = 210, normalized size = 2.3

$$-\frac{1}{ae^2+cd^2}\sqrt{c\left(\frac{d}{e}+x\right)^2-2\frac{cd}{e}\left(\frac{d}{e}+x\right)+\frac{ae^2+cd^2}{e^2}}\left(\frac{d}{e}+x\right)^{-1} - \frac{cd}{e\left(ae^2+cd^2\right)}\ln\left(\left(2\frac{ae^2+cd^2}{e^2}-2\frac{cd}{e}\left(\frac{d}{e}+x\right)+2\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] -1/(a*e^2+c*d^2)/(d/e+x)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1
/2)-1/e*c*d/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2
-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(
a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.49099, size = 783, normalized size = 8.6

$$\left[\frac{\left(cdex+cd^2\right)\sqrt{cd^2+ae^2}\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2-2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)-2\left(cd^2e+ae^3\right)\sqrt{cx^2+a}}{2\left(c^2d^5+2\,acd^3e^2+a^2de^4+\left(c^2d^4e+2\,acd^2e^3+a^2e^5\right)x\right)},-\frac{\left(cdex+cd\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c*d*e*x + c*d^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*
a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*s
qrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(c*d^2*e + a*e^3)*sqrt(c*x^2
 + a))/(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 +
a^2*e^5)*x), -((c*d*e*x + c*d^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 -
a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^
2)*x^2)) + (c*d^2*e + a*e^3)*sqrt(c*x^2 + a))/(c^2*d^5 + 2*a*c*d^3*e^2 + a^
2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

Giac [B] time = 5.03177, size = 439, normalized size = 4.82

$$-\frac{\sqrt{cd^2+ae^2}cde\log\left(\left|-\sqrt{cd^2+ae^2}cd+\left(cd^2+ae^2\right)\left(\sqrt{c-\frac{2\,cd}{xe+d}+\frac{cd^2}{(xe+d)^2}+\frac{ae^2}{(xe+d)^2}}+\frac{\sqrt{cd^2e^2+ae^4}e^{(-1)}}{xe+d}\right)\right|\right)}{\left(c^2d^4e+2\,acd^2e^3+a^2e^5\right)\mathrm{sgn}\left(\frac{1}{xe+d}\right)}+\frac{\left(c^{\frac{3}{2}}d^2+\sqrt{cd^2+ae\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(c*d^2 + a*e^2)*c*d*e*log(abs(-sqrt(c*d^2 + a*e^2)*c*d + (c*d^2 + a*e^
2)*(sqrt(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(x*e + d)^2) + sqr
t(c*d^2*e^2 + a*e^4)*e^(-1)/(x*e + d))))/((c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*
e^5)*sgn(1/(x*e + d))) + (c^(3/2)*d^2 + sqrt(c*d^2 + a*e^2)*c*d*log(abs(c^(
3/2)*d^2 - sqrt(c*d^2 + a*e^2)*c*d + a*sqrt(c)*e^2)) + a*sqrt(c)*e^2)*sgn(1
/(x*e + d))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - sqrt(c - 2*c*d/(x*e + d)
+ c*d^2/(x*e + d)^2 + a*e^2/(x*e + d)^2)/(c*d^2*sgn(1/(x*e + d)) + a*e^2*sg
n(1/(x*e + d)))
```

3.567 $\int \frac{1}{(d+ex)^3\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=145

$$-\frac{3cde\sqrt{a+cx^2}}{2(d+ex)\left(ae^2+cd^2\right)^2}-\frac{e\sqrt{a+cx^2}}{2(d+ex)^2\left(ae^2+cd^2\right)}-\frac{c\left(2cd^2-ae^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{2\left(ae^2+cd^2\right)^{5/2}}$$

```
[Out] -(e*Sqrt[a + c*x^2])/(2*(c*d^2 + a*e^2)*(d + e*x)^2) - (3*c*d*e*Sqrt[a + c*
x^2])/(2*(c*d^2 + a*e^2)^2*(d + e*x)) - (c*(2*c*d^2 - a*e^2)*ArcTanh[(a*e -
 c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(2*(c*d^2 + a*e^2)^(5/2))
```

Rubi [A] time = 0.0707614, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {745, 807, 725, 206}

$$-\frac{3cde\sqrt{a+cx^2}}{2(d+ex)\left(ae^2+cd^2\right)^2}-\frac{e\sqrt{a+cx^2}}{2(d+ex)^2\left(ae^2+cd^2\right)}-\frac{c\left(2cd^2-ae^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{2\left(ae^2+cd^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*Sqrt[a + c*x^2]),x]
```


```
[Out] -(e*Sqrt[a + c*x^2])/(2*(c*d^2 + a*e^2)*(d + e*x)^2) - (3*c*d*e*Sqrt[a + c*
x^2])/(2*(c*d^2 + a*e^2)^2*(d + e*x)) - (c*(2*c*d^2 - a*e^2)*ArcTanh[(a*e -
 c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(2*(c*d^2 + a*e^2)^(5/2))
```


Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\sqrt{a+cx^2}}\,dx = -\frac{e\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{c\int \frac{-2d+ex}{(d+ex)^2\sqrt{a+cx^2}}\,dx}{2\left(cd^2+ae^2\right)}$$
$$= -\frac{e\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{3cde\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)^2(d+ex)} + \frac{\left(c\left(2cd^2-ae^2\right)\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{2\left(cd^2+ae^2\right)^2}$$
$$= -\frac{e\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{3cde\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)^2(d+ex)} - \frac{\left(c\left(2cd^2-ae^2\right)\right)\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x\right.}{2\left(cd^2+ae^2\right)^2}$$
$$= -\frac{e\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{3cde\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)^2(d+ex)} - \frac{c\left(2cd^2-ae^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{2\left(cd^2+ae^2\right)^{5/2}}$$

Mathematica [A] time = 0.139823, size = 161, normalized size = 1.11

$$\frac{-e\sqrt{a+cx^2}\sqrt{ae^2+cd^2}\left(ae^2+cd(4d+3ex)\right)-c(d+ex)^2\left(2cd^2-ae^2\right)\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)+c(d+ex)^2\left(\right.}{2(d+ex)^2\left(ae^2+cd^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*Sqrt[a + c*x^2]),x]
```

```
[Out] (-(e*Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2]*(a*e^2 + c*d*(4*d + 3*e*x))) + c*(
2*c*d^2 - a*e^2)*(d + e*x)^2*Log[d + e*x] - c*(2*c*d^2 - a*e^2)*(d + e*x)^2
*Log[a*e - c*d*x + Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2]])/(2*(c*d^2 + a*e^2)
^(5/2)*(d + e*x)^2)
```

Maple [B] time = 0.193, size = 426, normalized size = 2.9

$$-\frac{1}{2e\left(ae^2+cd^2\right)}\sqrt{c\left(\frac{d}{e}+x\right)^2-2\frac{cd}{e}\left(\frac{d}{e}+x\right)+\frac{ae^2+cd^2}{e^2}}\left(\frac{d}{e}+x\right)^{-2}-\frac{3\,cd}{2\left(ae^2+cd^2\right)^2}\sqrt{c\left(\frac{d}{e}+x\right)^2-2\frac{cd}{e}\left(\frac{d}{e}+x\right)+\frac{ae^2}{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*x^2+a)^(1/2),x)
```

```
[Out] -1/2/e/(a*e^2+c*d^2)/(d/e+x)^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e
^2)^(1/2)-3/2*c*d/(a*e^2+c*d^2)^2/(d/e+x)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e
^2+c*d^2)/e^2)^(1/2)-3/2/e*c^2*d^2/(a*e^2+c*d^2)^2/((a*e^2+c*d^2)/e^2)^(1/2
)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d
/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))+1/2/e*c/(a*e^2+c
*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*(
(a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1
/2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.10872, size = 1432, normalized size = 9.88

$$\left[-\frac{\left(2\,c^2d^4 - acd^2e^2 + \left(2\,c^2d^2e^2 - ace^4\right)x^2 + 2\left(2\,c^2d^3e - acde^3\right)x\right)\sqrt{cd^2 + ae^2}\log\left(\frac{2\,acdex - acd^2 - 2\,a^2e^2 - \left(2\,c^2d^2 + ace^2\right)x^2 + 2\,\sqrt{cd^2 + a}}{e^2x^2 + 2\,dex + d^2}\right.}{4\left(c^3d^8 + 3\,ac^2d^6e^2 + 3\,a^2cd^4e^4 + a^3d^2e^6 + \left(c^3d^6e^2 + 3\,ac^2d^4e^4 + 3\,a^2cd^2e^6 + a^3e^8\right)x^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*c^2*d^4 - a*c*d^2*e^2 + (2*c^2*d^2*e^2 - a*c*e^4)*x^2 + 2*(2*c^2*
d^3*e - a*c*d*e^3)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^
2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqr
t(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(4*c^2*d^4*e + 5*a*c*d^2*e^3 +
 a^2*e^5 + 3*(c^2*d^3*e^2 + a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^8 + 3*a*c
^2*d^6*e^2 + 3*a^2*c*d^4*e^4 + a^3*d^2*e^6 + (c^3*d^6*e^2 + 3*a*c^2*d^4*e^4
 + 3*a^2*c*d^2*e^6 + a^3*e^8)*x^2 + 2*(c^3*d^7*e + 3*a*c^2*d^5*e^3 + 3*a^2*
c*d^3*e^5 + a^3*d*e^7)*x), -1/2*((2*c^2*d^4 - a*c*d^2*e^2 + (2*c^2*d^2*e^2
- a*c*e^4)*x^2 + 2*(2*c^2*d^3*e - a*c*d*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan
(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c
^2*d^2 + a*c*e^2)*x^2)) + (4*c^2*d^4*e + 5*a*c*d^2*e^3 + a^2*e^5 + 3*(c^2*d
^3*e^2 + a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^8 + 3*a*c^2*d^6*e^2 + 3*a^2*
c*d^4*e^4 + a^3*d^2*e^6 + (c^3*d^6*e^2 + 3*a*c^2*d^4*e^4 + 3*a^2*c*d^2*e^6
+ a^3*e^8)*x^2 + 2*(c^3*d^7*e + 3*a*c^2*d^5*e^3 + 3*a^2*c*d^3*e^5 + a^3*d*e
^7)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + cx^2}\,(d + ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)**3), x)
```

Giac [B] time = 1.42993, size = 466, normalized size = 3.21

$$-c\left(\frac{\left(2\,cd^2 - ae^2\right)\arctan\left(\frac{\left(\sqrt{c}x - \sqrt{cx^2 + a}\right)e + \sqrt{c}d}{\sqrt{-cd^2 - ae^2}}\right)}{\left(c^2d^4 + 2\,acd^2e^2 + a^2e^4\right)\sqrt{-cd^2 - ae^2}} + \frac{2\left(\sqrt{c}x - \sqrt{cx^2 + a}\right)^3 cd^2e + 6\left(\sqrt{c}x - \sqrt{cx^2 + a}\right)^2 c^{\frac{3}{2}}d^3 - 10\left(\sqrt{c}x - \sqrt{cx\right.}{\left(c^2d^4 + 2\,acd^2e^2 + a^2e\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -c*((2*c*d^2 - a*e^2)*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/
sqrt(-c*d^2 - a*e^2))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-c*d^2 - a*
e^2)) + (2*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c*d^2*e + 6*(sqrt(c)*x - sqrt(c*
x^2 + a))^2*c^(3/2)*d^3 - 10*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c*d^2*e - 3*(s
qrt(c)*x - sqrt(c*x^2 + a))^2*a*sqrt(c)*d*e^2 - (sqrt(c)*x - sqrt(c*x^2 + a
))^3*a*e^3 + 3*a^2*sqrt(c)*d*e^2 - (sqrt(c)*x - sqrt(c*x^2 + a))*a^2*e^3)/(
(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*e + 2*
(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*d - a*e)^2))
```

3.568 $\int \frac{1}{(d+ex)^4\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=198

$$-\frac{c^2d\left(2cd^2-3ae^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{2\left(ae^2+cd^2\right)^{7/2}}-\frac{ce\sqrt{a+cx^2}\left(11cd^2-4ae^2\right)}{6(d+ex)\left(ae^2+cd^2\right)^3}-\frac{5cde\sqrt{a+cx^2}}{6(d+ex)^2\left(ae^2+cd^2\right)^2}-\frac{e\sqrt{a+cx^2}}{3(d+ex)^3\left(ae^2+cd^2\right)}$$

```
[Out] -(e*Sqrt[a + c*x^2])/(3*(c*d^2 + a*e^2)*(d + e*x)^3) - (5*c*d*e*Sqrt[a + c*
x^2])/(6*(c*d^2 + a*e^2)^2*(d + e*x)^2) - (c*e*(11*c*d^2 - 4*a*e^2)*Sqrt[a
+ c*x^2])/(6*(c*d^2 + a*e^2)^3*(d + e*x)) - (c^2*d*(2*c*d^2 - 3*a*e^2)*ArcT
anh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(2*(c*d^2 + a*e^2
)^(7/2))
```

Rubi [A] time = 0.16382, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {745, 835, 807, 725, 206}

$$-\frac{c^2d\left(2cd^2-3ae^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{2\left(ae^2+cd^2\right)^{7/2}}-\frac{ce\sqrt{a+cx^2}\left(11cd^2-4ae^2\right)}{6(d+ex)\left(ae^2+cd^2\right)^3}-\frac{5cde\sqrt{a+cx^2}}{6(d+ex)^2\left(ae^2+cd^2\right)^2}-\frac{e\sqrt{a+cx^2}}{3(d+ex)^3\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^4*Sqrt[a + c*x^2]),x]
```

```
[Out] -(e*Sqrt[a + c*x^2])/(3*(c*d^2 + a*e^2)*(d + e*x)^3) - (5*c*d*e*Sqrt[a + c*
x^2])/(6*(c*d^2 + a*e^2)^2*(d + e*x)^2) - (c*e*(11*c*d^2 - 4*a*e^2)*Sqrt[a
+ c*x^2])/(6*(c*d^2 + a*e^2)^3*(d + e*x)) - (c^2*d*(2*c*d^2 - 3*a*e^2)*ArcT
anh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(2*(c*d^2 + a*e^2
)^(7/2))
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
```

```
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^4\sqrt{a+cx^2}}\,dx = -\frac{e\sqrt{a+cx^2}}{3\left(cd^2+ae^2\right)(d+ex)^3} - \frac{c\int \frac{-3d+2ex}{(d+ex)^3\sqrt{a+cx^2}}\,dx}{3\left(cd^2+ae^2\right)}$$

$$= -\frac{e\sqrt{a+cx^2}}{3\left(cd^2+ae^2\right)(d+ex)^3} - \frac{5cde\sqrt{a+cx^2}}{6\left(cd^2+ae^2\right)^2(d+ex)^2} + \frac{c\int \frac{2\left(3cd^2-2ae^2\right)-5cdex}{(d+ex)^2\sqrt{a+cx^2}}\,dx}{6\left(cd^2+ae^2\right)^2}$$

$$= -\frac{e\sqrt{a+cx^2}}{3\left(cd^2+ae^2\right)(d+ex)^3} - \frac{5cde\sqrt{a+cx^2}}{6\left(cd^2+ae^2\right)^2(d+ex)^2} - \frac{ce\left(11cd^2-4ae^2\right)\sqrt{a+cx^2}}{6\left(cd^2+ae^2\right)^3(d+ex)} + \frac{\left(c^2d\left(2cd^2\right.\right.}{}$$

$$= -\frac{e\sqrt{a+cx^2}}{3\left(cd^2+ae^2\right)(d+ex)^3} - \frac{5cde\sqrt{a+cx^2}}{6\left(cd^2+ae^2\right)^2(d+ex)^2} - \frac{ce\left(11cd^2-4ae^2\right)\sqrt{a+cx^2}}{6\left(cd^2+ae^2\right)^3(d+ex)} - \frac{\left(c^2d\left(2cd^2\right.\right.}{}$$

$$= -\frac{e\sqrt{a+cx^2}}{3\left(cd^2+ae^2\right)(d+ex)^3} - \frac{5cde\sqrt{a+cx^2}}{6\left(cd^2+ae^2\right)^2(d+ex)^2} - \frac{ce\left(11cd^2-4ae^2\right)\sqrt{a+cx^2}}{6\left(cd^2+ae^2\right)^3(d+ex)} - \frac{c^2d\left(2cd^2\right.}{}$$

Mathematica [A] time = 0.178611, size = 209, normalized size = 1.06

$$\frac{-3c^2d(d+ex)^3\left(2cd^2-3ae^2\right)\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)+3c^2d(d+ex)^3\left(2cd^2-3ae^2\right)\log(d+ex)-e\sqrt{a+cx^2}}{6(d+ex)^3\left(ae^2+cd^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^4*Sqrt[a + c*x^2]),x]
```

```
[Out] (-(e*Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2]*(2*(c*d^2 + a*e^2)^2 + 5*c*d*(c*d^
2 + a*e^2)*(d + e*x) + c*(11*c*d^2 - 4*a*e^2)*(d + e*x)^2)) + 3*c^2*d*(2*c*
d^2 - 3*a*e^2)*(d + e*x)^3*Log[d + e*x] - 3*c^2*d*(2*c*d^2 - 3*a*e^2)*(d +
e*x)^3*Log[a*e - c*d*x + Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2]])/(6*(c*d^2 +
a*e^2)^(7/2)*(d + e*x)^3)
```

Maple [B] time = 0.196, size = 573, normalized size = 2.9

$$-\frac{1}{3 e^{2} \left(a e^{2}+c d^{2}\right)} \sqrt{c \left(\frac{d}{e}+x\right)^{2}-2 \frac{c d}{e} \left(\frac{d}{e}+x\right)+\frac{a e^{2}+c d^{2}}{e^{2}}} \left(\frac{d}{e}+x\right)^{-3}-\frac{5 c d}{6 e \left(a e^{2}+c d^{2}\right)^{2}} \sqrt{c \left(\frac{d}{e}+x\right)^{2}-2 \frac{c d}{e} \left(\frac{d}{e}+x\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^4/(c*x^2+a)^(1/2),x)
```

```
[Out] -1/3/e^2/(a*e^2+c*d^2)/(d/e+x)^3*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)
/e^2)^(1/2)-5/6/e*c*d/(a*e^2+c*d^2)^2/(d/e+x)^2*(c*(d/e+x)^2-2*c*d/e*(d/e+x
)+(a*e^2+c*d^2)/e^2)^(1/2)-5/2*c^2*d^2/(a*e^2+c*d^2)^3/(d/e+x)*(c*(d/e+x)^2
-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)-5/2/e*c^3*d^3/(a*e^2+c*d^2)^3/((a
*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*
d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e
+x))+3/2/e*c^2*d/(a*e^2+c*d^2)^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d
^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d
/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))+2/3/(a*e^2+c*d^2)^2*c/(d/e+x)*(c*(
d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 8.50196, size = 2290, normalized size = 11.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(2*c^3*d^6 - 3*a*c^2*d^4*e^2 + (2*c^3*d^3*e^3 - 3*a*c^2*d*e^5)*x^
3 + 3*(2*c^3*d^4*e^2 - 3*a*c^2*d^2*e^4)*x^2 + 3*(2*c^3*d^5*e - 3*a*c^2*d^3*
e^3)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2
*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/
(e^2*x^2 + 2*d*e*x + d^2)) + 2*(18*c^3*d^6*e + 23*a*c^2*d^4*e^3 + 7*a^2*c*d
^2*e^5 + 2*a^3*e^7 + (11*c^3*d^4*e^3 + 7*a*c^2*d^2*e^5 - 4*a^2*c*e^7)*x^2 +
 3*(9*c^3*d^5*e^2 + 8*a*c^2*d^3*e^4 - a^2*c*d*e^6)*x)*sqrt(c*x^2 + a))/(c^4
*d^11 + 4*a*c^3*d^9*e^2 + 6*a^2*c^2*d^7*e^4 + 4*a^3*c*d^5*e^6 + a^4*d^3*e^8
 + (c^4*d^8*e^3 + 4*a*c^3*d^6*e^5 + 6*a^2*c^2*d^4*e^7 + 4*a^3*c*d^2*e^9 + a
^4*e^11)*x^3 + 3*(c^4*d^9*e^2 + 4*a*c^3*d^7*e^4 + 6*a^2*c^2*d^5*e^6 + 4*a^3
*c*d^3*e^8 + a^4*d*e^10)*x^2 + 3*(c^4*d^10*e + 4*a*c^3*d^8*e^3 + 6*a^2*c^2*
d^6*e^5 + 4*a^3*c*d^4*e^7 + a^4*d^2*e^9)*x), -1/6*(3*(2*c^3*d^6 - 3*a*c^2*d
^4*e^2 + (2*c^3*d^3*e^3 - 3*a*c^2*d*e^5)*x^3 + 3*(2*c^3*d^4*e^2 - 3*a*c^2*d
^2*e^4)*x^2 + 3*(2*c^3*d^5*e - 3*a*c^2*d^3*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arc
tan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 +
```

```
 (c^2*d^2 + a*c*e^2)*x^2)) + (18*c^3*d^6*e + 23*a*c^2*d^4*e^3 + 7*a^2*c*d^2
*e^5 + 2*a^3*e^7 + (11*c^3*d^4*e^3 + 7*a*c^2*d^2*e^5 - 4*a^2*c*e^7)*x^2 + 3
*(9*c^3*d^5*e^2 + 8*a*c^2*d^3*e^4 - a^2*c*d*e^6)*x)*sqrt(c*x^2 + a))/(c^4*d
^11 + 4*a*c^3*d^9*e^2 + 6*a^2*c^2*d^7*e^4 + 4*a^3*c*d^5*e^6 + a^4*d^3*e^8 +
 (c^4*d^8*e^3 + 4*a*c^3*d^6*e^5 + 6*a^2*c^2*d^4*e^7 + 4*a^3*c*d^2*e^9 + a^4
*e^11)*x^3 + 3*(c^4*d^9*e^2 + 4*a*c^3*d^7*e^4 + 6*a^2*c^2*d^5*e^6 + 4*a^3*c
*d^3*e^8 + a^4*d*e^10)*x^2 + 3*(c^4*d^10*e + 4*a*c^3*d^8*e^3 + 6*a^2*c^2*d^
6*e^5 + 4*a^3*c*d^4*e^7 + a^4*d^2*e^9)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + cx^2}\,(d + ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**4/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)**4), x)
```

Giac [B] time = 1.37986, size = 780, normalized size = 3.94

$$\frac{1}{3}c^{\frac{3}{2}}\left(\frac{3\left(2c^{\frac{3}{2}}d^3 - 3a\sqrt{c}de^2\right)\arctan\left(-\frac{\left(\sqrt{c}x - \sqrt{cx^2+a}\right)e + \sqrt{c}d}{\sqrt{-cd^2 - ae^2}}\right)}{\left(c^3d^6 + 3ac^2d^4e^2 + 3a^2cd^2e^4 + a^3e^6\right)\sqrt{-cd^2 - ae^2}} - \frac{30\left(\sqrt{c}x - \sqrt{cx^2 + a}\right)^4 c^2d^4e + 44\left(\sqrt{c}x - \sqrt{cx^2 + a}\right)^3 c^{\frac{5}{2}}d^5 + 6}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*c^(3/2)*(3*(2*c^(3/2)*d^3 - 3*a*sqrt(c)*d*e^2)*arctan(-((sqrt(c)*x - sq
rt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2))/((c^3*d^6 + 3*a*c^2*d^4
*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)*sqrt(-c*d^2 - a*e^2)) - (30*(sqrt(c)*x -
sqrt(c*x^2 + a))^4*c^2*d^4*e + 44*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^(5/2)*d
^5 + 6*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^(3/2)*d^3*e^2 - 102*(sqrt(c)*x - s
qrt(c*x^2 + a))^2*a*c^2*d^4*e - 82*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c^(3/2
)*d^3*e^2 - 45*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c*d^2*e^3 - 9*(sqrt(c)*x -
 sqrt(c*x^2 + a))^5*a*sqrt(c)*d*e^4 + 60*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*
c^(3/2)*d^3*e^2 + 36*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*c*d^2*e^3 + 24*(sq
rt(c)*x - sqrt(c*x^2 + a))^3*a^2*sqrt(c)*d*e^4 - 11*a^3*c*d^2*e^3 - 15*(sqr
t(c)*x - sqrt(c*x^2 + a))*a^3*sqrt(c)*d*e^4 - 12*(sqrt(c)*x - sqrt(c*x^2 +
a))^2*a^3*e^5 + 4*a^4*e^5)/((c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 +
a^3*e^6)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*e + 2*(sqrt(c)*x - sqrt(c*x^2 + a
))*sqrt(c)*d - a*e)^3))
```

3.569 $\int \frac{(d+ex)^4}{\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=162

$$-\frac{e\sqrt{a+cx^2}\left(ex\left(2cd^2-3ae^2\right)+4d\left(cd^2-4ae^2\right)\right)}{2ac^2}+\frac{3e^2\left(4cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{5/2}}-\frac{(d+ex)^3(ae-cdx)}{ac\sqrt{a+cx^2}}-\frac{de\sqrt{a+}}{}$$

```
[Out] -(((a*e - c*d*x)*(d + e*x)^3)/(a*c*Sqrt[a + c*x^2])) - (d*e*(d + e*x)^2*Sqr
t[a + c*x^2])/(a*c) - (e*(4*d*(c*d^2 - 4*a*e^2) + e*(2*c*d^2 - 3*a*e^2)*x)*
Sqrt[a + c*x^2])/(2*a*c^2) + (3*e^2*(4*c*d^2 - a*e^2)*ArcTanh[(Sqrt[c]*x)/S
qrt[a + c*x^2]])/(2*c^(5/2))
```

Rubi [A] time = 0.147554, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {739, 833, 780, 217, 206}

$$-\frac{e\sqrt{a+cx^2}\left(ex\left(2cd^2-3ae^2\right)+4d\left(cd^2-4ae^2\right)\right)}{2ac^2}+\frac{3e^2\left(4cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{5/2}}-\frac{(d+ex)^3(ae-cdx)}{ac\sqrt{a+cx^2}}-\frac{de\sqrt{a+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a + c*x^2)^(3/2),x]
```

```
[Out] -(((a*e - c*d*x)*(d + e*x)^3)/(a*c*Sqrt[a + c*x^2])) - (d*e*(d + e*x)^2*Sqr
t[a + c*x^2])/(a*c) - (e*(4*d*(c*d^2 - 4*a*e^2) + e*(2*c*d^2 - 3*a*e^2)*x)*
Sqrt[a + c*x^2])/(2*a*c^2) + (3*e^2*(4*c*d^2 - a*e^2)*ArcTanh[(Sqrt[c]*x)/S
qrt[a + c*x^2]])/(2*c^(5/2))
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^4}{\left(a+cx^2\right)^{3/2}}\,dx = -\frac{(ae-cdx)(d+ex)^3}{ac\sqrt{a+cx^2}} + \frac{\int \frac{(d+ex)^2\left(3ae^2-3cdex\right)}{\sqrt{a+cx^2}}\,dx}{ac}$$

$$= -\frac{(ae-cdx)(d+ex)^3}{ac\sqrt{a+cx^2}} - \frac{de(d+ex)^2\sqrt{a+cx^2}}{ac} + \frac{\int \frac{(d+ex)\left(15acde^2-3ce\left(2cd^2-3ae^2\right)x\right)}{\sqrt{a+cx^2}}\,dx}{3ac^2}$$

$$= -\frac{(ae-cdx)(d+ex)^3}{ac\sqrt{a+cx^2}} - \frac{de(d+ex)^2\sqrt{a+cx^2}}{ac} - \frac{e\left(4d\left(cd^2-4ae^2\right)+e\left(2cd^2-3ae^2\right)x\right)\sqrt{a+cx^2}}{2ac^2} + \frac{(3 \cdots}{\cdots}$$

$$= -\frac{(ae-cdx)(d+ex)^3}{ac\sqrt{a+cx^2}} - \frac{de(d+ex)^2\sqrt{a+cx^2}}{ac} - \frac{e\left(4d\left(cd^2-4ae^2\right)+e\left(2cd^2-3ae^2\right)x\right)\sqrt{a+cx^2}}{2ac^2} + \frac{(3 \cdots}{\cdots}$$

$$= -\frac{(ae-cdx)(d+ex)^3}{ac\sqrt{a+cx^2}} - \frac{de(d+ex)^2\sqrt{a+cx^2}}{ac} - \frac{e\left(4d\left(cd^2-4ae^2\right)+e\left(2cd^2-3ae^2\right)x\right)\sqrt{a+cx^2}}{2ac^2} + \frac{3e \cdots}{\cdots}$$

Mathematica [A] time = 0.141832, size = 127, normalized size = 0.78

$$\frac{a^2e^3(16d+3ex)+ace\left(-12d^2ex-8d^3+8de^2x^2+e^3x^3\right)+2c^2d^4x}{2ac^2\sqrt{a+cx^2}} + \frac{3\left(4cd^2e^2-ae^4\right)\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)}{2c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a + c*x^2)^(3/2),x]
```

```
[Out] (2*c^2*d^4*x + a^2*e^3*(16*d + 3*e*x) + a*c*e*(-8*d^3 - 12*d^2*e*x + 8*d*e^
2*x^2 + e^3*x^3))/(2*a*c^2*Sqrt[a + c*x^2]) + (3*(4*c*d^2*e^2 - a*e^4)*Log[
c*x + Sqrt[c]*Sqrt[a + c*x^2]])/(2*c^(5/2))
```

Maple [A] time = 0.049, size = 189, normalized size = 1.2

$$\frac{e^4x^3}{2c}\frac{1}{\sqrt{cx^2+a}} + \frac{3\,ae^4x}{2c^2}\frac{1}{\sqrt{cx^2+a}} - \frac{3\,ae^4}{2}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)c^{-\frac{5}{2}} + 4\frac{de^3x^2}{c\sqrt{cx^2+a}} + 8\frac{de^3a}{c^2\sqrt{cx^2+a}} - 6\frac{d^2e^2x}{c\sqrt{cx^2+a}} + 6\frac{d^2e \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(c*x^2+a)^(3/2),x)
```

```
[Out] 1/2*e^4*x^3/c/(c*x^2+a)^(1/2)+3/2*e^4*a/c^2*x/(c*x^2+a)^(1/2)-3/2*e^4*a/c^(
5/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))+4*d*e^3*x^2/c/(c*x^2+a)^(1/2)+8*d*e^3*a/
c^2/(c*x^2+a)^(1/2)-6*d^2*e^2*x/c/(c*x^2+a)^(1/2)+6*d^2*e^2/c^(3/2)*ln(x*c^
(1/2)+(c*x^2+a)^(1/2))-4*d^3*e/c/(c*x^2+a)^(1/2)+d^4*x/a/(c*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.96434, size = 768, normalized size = 4.74

$$\left[\frac{3\left(4\,a^2cd^2e^2 - a^3e^4 + \left(4\,ac^2d^2e^2 - a^2ce^4\right)x^2\right)\sqrt{c}\log\left(-2\,cx^2 - 2\,\sqrt{cx^2 + a}\sqrt{c}x - a\right) + 2\left(ac^2e^4x^3 + 8\,ac^2de^3x^2 - 8\,ac^2d^3\right.}{4\left(ac^4x^2 + a^2c^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(4*a^2*c*d^2*e^2 - a^3*e^4 + (4*a*c^2*d^2*e^2 - a^2*c*e^4)*x^2)*sqr
t(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(a*c^2*e^4*x^3 + 8
*a*c^2*d*e^3*x^2 - 8*a*c^2*d^3*e + 16*a^2*c*d*e^3 + (2*c^3*d^4 - 12*a*c^2*d
^2*e^2 + 3*a^2*c*e^4)*x)*sqrt(c*x^2 + a))/(a*c^4*x^2 + a^2*c^3), -1/2*(3*(4
*a^2*c*d^2*e^2 - a^3*e^4 + (4*a*c^2*d^2*e^2 - a^2*c*e^4)*x^2)*sqrt(-c)*arct
an(sqrt(-c)*x/sqrt(c*x^2 + a)) - (a*c^2*e^4*x^3 + 8*a*c^2*d*e^3*x^2 - 8*a*c
^2*d^3*e + 16*a^2*c*d*e^3 + (2*c^3*d^4 - 12*a*c^2*d^2*e^2 + 3*a^2*c*e^4)*x)
*sqrt(c*x^2 + a))/(a*c^4*x^2 + a^2*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^4}{\left(a + cx^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral((d + e*x)**4/(a + c*x**2)**(3/2), x)
```

Giac [A] time = 1.34381, size = 186, normalized size = 1.15

$$\frac{\left(x\left(\frac{xe^4}{c} + \frac{8\,de^3}{c}\right) + \frac{2\,c^4d^4 - 12\,ac^3d^2e^2 + 3\,a^2c^2e^4}{ac^4}\right)x - \frac{8\left(ac^3d^3e - 2\,a^2c^2de^3\right)}{ac^4}}{2\,\sqrt{cx^2 + a}} - \frac{3\left(4\,cd^2e^2 - ae^4\right)\log\left(\left|-\sqrt{c}x + \sqrt{cx^2 + a}\right|\right)}{2\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*((x*(x*e^4/c + 8*d*e^3/c) + (2*c^4*d^4 - 12*a*c^3*d^2*e^2 + 3*a^2*c^2*e
^4)/(a*c^4))*x - 8*(a*c^3*d^3*e - 2*a^2*c^2*d*e^3)/(a*c^4))/sqrt(c*x^2 + a)
 - 3/2*(4*c*d^2*e^2 - a*e^4)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

3.570 $\int \frac{(d+ex)^3}{\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=106

$$-\frac{e\sqrt{a+cx^2}\left(2\left(cd^2-ae^2\right)+cdex\right)}{ac^2}+\frac{3de^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}}-\frac{(d+ex)^2(ae-cdx)}{ac\sqrt{a+cx^2}}$$

```
[Out] -(((a*e - c*d*x)*(d + e*x)^2)/(a*c*Sqrt[a + c*x^2])) - (e*(2*(c*d^2 - a*e^2
) + c*d*e*x)*Sqrt[a + c*x^2])/(a*c^2) + (3*d*e^2*ArcTanh[(Sqrt[c]*x)/Sqrt[a
 + c*x^2]])/c^(3/2)
```

Rubi [A] time = 0.0574783, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {739, 780, 217, 206}

$$-\frac{e\sqrt{a+cx^2}\left(2\left(cd^2-ae^2\right)+cdex\right)}{ac^2}+\frac{3de^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}}-\frac{(d+ex)^2(ae-cdx)}{ac\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + c*x^2)^(3/2),x]
```

```
[Out] -(((a*e - c*d*x)*(d + e*x)^2)/(a*c*Sqrt[a + c*x^2])) - (e*(2*(c*d^2 - a*e^2
) + c*d*e*x)*Sqrt[a + c*x^2])/(a*c^2) + (3*d*e^2*ArcTanh[(Sqrt[c]*x)/Sqrt[a
 + c*x^2]])/c^(3/2)
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(a+cx^2\right)^{3/2}}\,dx = -\frac{(ae-cdx)(d+ex)^2}{ac\sqrt{a+cx^2}} + \frac{\int \frac{(d+ex)\left(2ae^2-2cdex\right)}{\sqrt{a+cx^2}}\,dx}{ac}$$

$$= -\frac{(ae-cdx)(d+ex)^2}{ac\sqrt{a+cx^2}} - \frac{e\left(2\left(cd^2-ae^2\right)+cdex\right)\sqrt{a+cx^2}}{ac^2} + \frac{\left(3de^2\right)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{c}$$

$$= -\frac{(ae-cdx)(d+ex)^2}{ac\sqrt{a+cx^2}} - \frac{e\left(2\left(cd^2-ae^2\right)+cdex\right)\sqrt{a+cx^2}}{ac^2} + \frac{\left(3de^2\right)\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{c}$$

$$= -\frac{(ae-cdx)(d+ex)^2}{ac\sqrt{a+cx^2}} - \frac{e\left(2\left(cd^2-ae^2\right)+cdex\right)\sqrt{a+cx^2}}{ac^2} + \frac{3de^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}}$$

Mathematica [A] time = 0.10542, size = 91, normalized size = 0.86

$$\frac{2a^2e^3+ace\left(-3d^2-3dex+e^2x^2\right)+c^2d^3x}{ac^2\sqrt{a+cx^2}} + \frac{3de^2\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + c*x^2)^(3/2),x]
```

```
[Out] (2*a^2*e^3 + c^2*d^3*x + a*c*e*(-3*d^2 - 3*d*e*x + e^2*x^2))/(a*c^2*Sqrt[a
+ c*x^2]) + (3*d*e^2*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/c^(3/2)
```

Maple [A] time = 0.049, size = 118, normalized size = 1.1

$$\frac{e^3x^2}{c}\frac{1}{\sqrt{cx^2+a}} + 2\frac{ae^3}{c^2\sqrt{cx^2+a}} - 3\frac{de^2x}{c\sqrt{cx^2+a}} + 3\frac{de^2\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)}{c^{3/2}} - 3\frac{d^2e}{c\sqrt{cx^2+a}} + \frac{d^3x}{a}\frac{1}{\sqrt{cx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^2+a)^(3/2),x)
```

```
[Out] e^3*x^2/c/(c*x^2+a)^(1/2)+2*e^3*a/c^2/(c*x^2+a)^(1/2)-3*d*e^2*x/c/(c*x^2+a)
^(1/2)+3*d*e^2/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))-3*d^2*e/c/(c*x^2+a)^(1
/2)+d^3*x/a/(c*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.96924, size = 531, normalized size = 5.01

$$\left[\frac{3\left(acde^2x^2 + a^2de^2\right)\sqrt{c}\log\left(-2\,cx^2 - 2\,\sqrt{cx^2+a}\sqrt{c}x - a\right) + 2\left(ace^3x^2 - 3\,acd^2e + 2\,a^2e^3 + \left(c^2d^3 - 3\,acde^2\right)x\right)\sqrt{cx^2+a}}{2\left(ac^3x^2 + a^2c^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(a*c*d*e^2*x^2 + a^2*d*e^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a
)*sqrt(c)*x - a) + 2*(a*c*e^3*x^2 - 3*a*c*d^2*e + 2*a^2*e^3 + (c^2*d^3 - 3*
a*c*d*e^2)*x)*sqrt(c*x^2 + a))/(a*c^3*x^2 + a^2*c^2), -(3*(a*c*d*e^2*x^2 +
a^2*d*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (a*c*e^3*x^2 - 3*a
*c*d^2*e + 2*a^2*e^3 + (c^2*d^3 - 3*a*c*d*e^2)*x)*sqrt(c*x^2 + a))/(a*c^3*x
^2 + a^2*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^3}{\left(a+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral((d + e*x)**3/(a + c*x**2)**(3/2), x)
```

Giac [A] time = 1.55404, size = 135, normalized size = 1.27

$$-\frac{3\,de^2\log\left(\left|-\sqrt{c}x + \sqrt{cx^2+a}\right|\right)}{c^{\frac{3}{2}}} + \frac{x\left(\frac{xe^3}{c} + \frac{c^3d^3 - 3\,ac^2de^2}{ac^3}\right) - \frac{3\,ac^2d^2e - 2\,a^2ce^3}{ac^3}}{\sqrt{cx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -3*d*e^2*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2) + (x*(x*e^3/c + (c^
3*d^3 - 3*a*c^2*d*e^2)/(a*c^3)) - (3*a*c^2*d^2*e - 2*a^2*c*e^3)/(a*c^3))/sq
rt(c*x^2 + a)
```

3.571 $\int \frac{(d+ex)^2}{\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=83

$$\frac{e^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}}-\frac{de\sqrt{a+cx^2}}{ac}-\frac{(d+ex)(ae-cdx)}{ac\sqrt{a+cx^2}}$$

```
[Out] -(((a*e - c*d*x)*(d + e*x))/(a*c*Sqrt[a + c*x^2])) - (d*e*Sqrt[a + c*x^2])/
(a*c) + (e^2*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/c^(3/2)
```

Rubi [A] time = 0.0345132, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {739, 641, 217, 206}

$$\frac{e^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}}-\frac{de\sqrt{a+cx^2}}{ac}-\frac{(d+ex)(ae-cdx)}{ac\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + c*x^2)^(3/2),x]
```



```
[Out] -(((a*e - c*d*x)*(d + e*x))/(a*c*Sqrt[a + c*x^2])) - (d*e*Sqrt[a + c*x^2])/
(a*c) + (e^2*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/c^(3/2)
```


Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(a+cx^2\right)^{3/2}}\,dx = -\frac{(ae-cdx)(d+ex)}{ac\sqrt{a+cx^2}} + \frac{\int \frac{ae^2-cdex}{\sqrt{a+cx^2}}\,dx}{ac}$$

$$= -\frac{(ae-cdx)(d+ex)}{ac\sqrt{a+cx^2}} - \frac{de\sqrt{a+cx^2}}{ac} + \frac{e^2\int \frac{1}{\sqrt{a+cx^2}}\,dx}{c}$$

$$= -\frac{(ae-cdx)(d+ex)}{ac\sqrt{a+cx^2}} - \frac{de\sqrt{a+cx^2}}{ac} + \frac{e^2\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{c}$$

$$= -\frac{(ae-cdx)(d+ex)}{ac\sqrt{a+cx^2}} - \frac{de\sqrt{a+cx^2}}{ac} + \frac{e^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}}$$

Mathematica [A] time = 0.0697005, size = 69, normalized size = 0.83

$$\frac{e^2\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)}{c^{3/2}} + \frac{-2ade-ae^2x+cd^2x}{ac\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + c*x^2)^(3/2),x]
```

```
[Out] (-2*a*d*e + c*d^2*x - a*e^2*x)/(a*c*Sqrt[a + c*x^2]) + (e^2*Log[c*x + Sqrt[
c]*Sqrt[a + c*x^2]])/c^(3/2)
```

Maple [A] time = 0.046, size = 76, normalized size = 0.9

$$-\frac{e^2x}{c}\frac{1}{\sqrt{cx^2+a}} + e^2\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)c^{-\frac{3}{2}} - 2\frac{de}{c\sqrt{cx^2+a}} + \frac{d^2x}{a}\frac{1}{\sqrt{cx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^2+a)^(3/2),x)
```

```
[Out] -e^2*x/c/(c*x^2+a)^(1/2)+e^2/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))-2*d*e/c/
(c*x^2+a)^(1/2)+d^2*x/a/(c*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.99505, size = 428, normalized size = 5.16

$$\left[\frac{\left(ace^2x^2+a^2e^2\right)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)-2\left(2\,acde-\left(c^2d^2-ace^2\right)x\right)\sqrt{cx^2+a}}{2\left(ac^3x^2+a^2c^2\right)},-\frac{\left(ace^2x^2+a^2e^2\right)\sqrt{-c}\,\mathrm{arcta}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*c*e^2*x^2 + a^2*e^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt
(c)*x - a) - 2*(2*a*c*d*e - (c^2*d^2 - a*c*e^2)*x)*sqrt(c*x^2 + a))/(a*c^3*
x^2 + a^2*c^2), -((a*c*e^2*x^2 + a^2*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c
*x^2 + a)) + (2*a*c*d*e - (c^2*d^2 - a*c*e^2)*x)*sqrt(c*x^2 + a))/(a*c^3*x^
2 + a^2*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(d+ex)^2}{\left(a+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral((d + e*x)**2/(a + c*x**2)**(3/2), x)
```

Giac [A] time = 1.39254, size = 93, normalized size = 1.12

$$-\frac{\frac{2\,de}{c}-\frac{\left(c^2d^2-ace^2\right)x}{ac^2}}{\sqrt{cx^2+a}}-\frac{e^2\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(2*d*e/c - (c^2*d^2 - a*c*e^2)*x/(a*c^2))/sqrt(c*x^2 + a) - e^2*log(abs(-s
qrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

3.572 $\int \frac{d+ex}{(a+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=28

$$-\frac{ae-cdx}{ac\sqrt{a+cx^2}}$$

```
[Out] -((a*e - c*d*x)/(a*c*Sqrt[a + c*x^2]))
```

Rubi [A] time = 0.0070311, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {637}

$$-\frac{ae-cdx}{ac\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + c*x^2)^(3/2),x]
```

```
[Out] -((a*e - c*d*x)/(a*c*Sqrt[a + c*x^2]))
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{d+ex}{\left(a+cx^2\right)^{3/2}}\,dx = -\frac{ae-cdx}{ac\sqrt{a+cx^2}}$$

Mathematica [A] time = 0.0137637, size = 27, normalized size = 0.96

$$\frac{cdx-ae}{ac\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + c*x^2)^(3/2),x]
```

```
[Out] (-(a*e) + c*d*x)/(a*c*Sqrt[a + c*x^2])
```

Maple [A] time = 0.042, size = 27, normalized size = 1.

$$-\frac{-cdx+ae}{ac}\frac{1}{\sqrt{cx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] -(-c*d*x+a*e)/(c*x^2+a)^(1/2)/a/c
```

Maxima [A] time = 1.12443, size = 42, normalized size = 1.5

$$\frac{dx}{\sqrt{cx^2 + a}a} - \frac{e}{\sqrt{cx^2 + a}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] d*x/(sqrt(c*x^2 + a)*a) - e/(sqrt(c*x^2 + a)*c)
```

Fricas [A] time = 1.82502, size = 69, normalized size = 2.46

$$\frac{(cdx - ae)\sqrt{cx^2 + a}}{ac^2x^2 + a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] (c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c^2*x^2 + a^2*c)
```

Sympy [A] time = 3.98878, size = 46, normalized size = 1.64

$$e\left(\begin{cases} -\frac{1}{c\sqrt{a+cx^2}} & \text{for}\ c \neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{dx}{a^{\frac{3}{2}}\sqrt{1 + \frac{cx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] e*Piecewise((-1/(c*sqrt(a + c*x**2)), Ne(c, 0)), (x**2/(2*a**(3/2)), True))
 + d*x/(a**(3/2)*sqrt(1 + c*x**2/a))
```

Giac [A] time = 1.34222, size = 32, normalized size = 1.14

$$\frac{\frac{dx}{a} - \frac{e}{c}}{\sqrt{cx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] (d*x/a - e/c)/sqrt(c*x^2 + a)
```

3.573 $\int \frac{1}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{ae+cdx}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

```
[Out] (a*e + c*d*x)/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) - (e^2*ArcTanh[(a*e - c*d
*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rubi [A] time = 0.0584725, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {741, 12, 725, 206}

$$\frac{ae+cdx}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*e + c*d*x)/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) - (e^2*ArcTanh[(a*e - c*d
*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\int \frac{ae^2}{(d+ex)\sqrt{a+cx^2}}\,dx}{a\left(cd^2+ae^2\right)}$$

$$= \frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{e^2\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$

$$= \frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{e^2\,\mathrm{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{cd^2+ae^2}$$

$$= \frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{3/2}}$$

Mathematica [A] time = 0.0582265, size = 94, normalized size = 1.

$$\frac{ae+cdx}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)} - \frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*e + c*d*x)/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) - (e^2*ArcTanh[(a*e - c*d
*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Maple [B] time = 0.19, size = 260, normalized size = 2.8

$$\frac{e}{ae^2+cd^2}\frac{1}{\sqrt{c\left(\frac{d}{e}+x\right)^2-2\frac{cd}{e}\left(\frac{d}{e}+x\right)+\frac{ae^2+cd^2}{e^2}}} + \frac{cdx}{\left(ae^2+cd^2\right)a}\frac{1}{\sqrt{c\left(\frac{d}{e}+x\right)^2-2\frac{cd}{e}\left(\frac{d}{e}+x\right)+\frac{ae^2+cd^2}{e^2}}} - \frac{e}{ae^2+cd^2}\ln\left(\left(2\frac{ae}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] e/(a*e^2+c*d^2)/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)+d/(a*
e^2+c*d^2)/a/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x*c-e/(a
*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+
x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e
^2)^(1/2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.75014, size = 913, normalized size = 9.71

$$\left[\frac{\left(ace^2x^2+a^2e^2\right)\sqrt{cd^2+ae^2}\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2-2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+2\left(acd^2e+a^2e^3+\left(c^2d^3+acd\right.\right.}{2\left(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4+\left(ac^3d^4+2\,a^2c^2d^2e^2+a^3ce^4\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*c*e^2*x^2 + a^2*e^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^
2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x -
a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(a*c*d^2*e + a^2*e^3 +
 (c^2*d^3 + a*c*d*e^2)*x)*sqrt(c*x^2 + a))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 +
 a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^2), -((a*c*e^2*x^2
 + a^2*e^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*
sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - (a*c*d^2*e
 + a^2*e^3 + (c^2*d^3 + a*c*d*e^2)*x)*sqrt(c*x^2 + a))/(a^2*c^2*d^4 + 2*a^3
*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(a+cx^2\right)^{\frac{3}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/((a + c*x**2)**(3/2)*(d + e*x)), x)
```

Giac [A] time = 1.40884, size = 232, normalized size = 2.47

$$\frac{\frac{\left(c^2d^3+acde^2\right)x}{ac^2d^4+2\,a^2cd^2e^2+a^3e^4}+\frac{acd^2e+a^2e^3}{ac^2d^4+2\,a^2cd^2e^2+a^3e^4}}{\sqrt{cx^2+a}}-\frac{2\,\arctan\left(\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)e^2}{\left(cd^2+ae^2\right)\sqrt{-cd^2-ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((c^2*d^3 + a*c*d*e^2)*x/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4) + (a*c*d^2
*e + a^2*e^3)/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))/sqrt(c*x^2 + a) - 2*
arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2))*
e^2/((c*d^2 + a*e^2)*sqrt(-c*d^2 - a*e^2))
```

3.574 $\int \frac{1}{(d+ex)^2\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=151

$$\frac{e\sqrt{a+cx^2}\left(cd^2-2ae^2\right)}{a(d+ex)\left(ae^2+cd^2\right)^2}+\frac{ae+cdx}{a\sqrt{a+cx^2}(d+ex)\left(ae^2+cd^2\right)}-\frac{3cde^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{5/2}}$$

```
[Out] (a*e + c*d*x)/(a*(c*d^2 + a*e^2)*(d + e*x)*Sqrt[a + c*x^2]) + (e*(c*d^2 - 2
*a*e^2)*Sqrt[a + c*x^2])/(a*(c*d^2 + a*e^2)^2*(d + e*x)) - (3*c*d*e^2*ArcTa
nh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(5
/2)
```

Rubi [A] time = 0.0844861, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {741, 807, 725, 206}

$$\frac{e\sqrt{a+cx^2}\left(cd^2-2ae^2\right)}{a(d+ex)\left(ae^2+cd^2\right)^2}+\frac{ae+cdx}{a\sqrt{a+cx^2}(d+ex)\left(ae^2+cd^2\right)}-\frac{3cde^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*e + c*d*x)/(a*(c*d^2 + a*e^2)*(d + e*x)*Sqrt[a + c*x^2]) + (e*(c*d^2 - 2
*a*e^2)*Sqrt[a + c*x^2])/(a*(c*d^2 + a*e^2)^2*(d + e*x)) - (3*c*d*e^2*ArcTa
nh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(5
/2)
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)^2\left(a+cx^2\right)^{3/2}}\,dx &= \frac{ae+cdx}{a\left(cd^2+ae^2\right)(d+ex)\sqrt{a+cx^2}} - \frac{\int \frac{-2ae^2-cdex}{(d+ex)^2\sqrt{a+cx^2}}\,dx}{a\left(cd^2+ae^2\right)}\\
&= \frac{ae+cdx}{a\left(cd^2+ae^2\right)(d+ex)\sqrt{a+cx^2}} + \frac{e\left(cd^2-2ae^2\right)\sqrt{a+cx^2}}{a\left(cd^2+ae^2\right)^2(d+ex)} + \frac{\left(3cde^2\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{\left(cd^2+ae^2\right)^2}\\
&= \frac{ae+cdx}{a\left(cd^2+ae^2\right)(d+ex)\sqrt{a+cx^2}} + \frac{e\left(cd^2-2ae^2\right)\sqrt{a+cx^2}}{a\left(cd^2+ae^2\right)^2(d+ex)} - \frac{\left(3cde^2\right)\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\right.}{\left(cd^2+ae^2\right)^2}\\
&= \frac{ae+cdx}{a\left(cd^2+ae^2\right)(d+ex)\sqrt{a+cx^2}} + \frac{e\left(cd^2-2ae^2\right)\sqrt{a+cx^2}}{a\left(cd^2+ae^2\right)^2(d+ex)} - \frac{3cde^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+c}}\right.}{\left(cd^2+ae^2\right)^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.125062, size = 139, normalized size = 0.92

$$\frac{-a^2e^3+ace\left(2d^2+dex-2e^2x^2\right)+c^2d^2x(d+ex)}{a\sqrt{a+cx^2}(d+ex)\left(ae^2+cd^2\right)^2} - \frac{3cde^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-(a^2*e^3) + c^2*d^2*x*(d + e*x) + a*c*e*(2*d^2 + d*e*x - 2*e^2*x^2))/(a*(
c*d^2 + a*e^2)^2*(d + e*x)*Sqrt[a + c*x^2]) - (3*c*d*e^2*ArcTanh[(a*e - c*d
*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(5/2)
```

Maple [B] time = 0.194, size = 400, normalized size = 2.7

$$-\frac{1}{ae^2+cd^2}\left(\frac{d}{e}+x\right)^{-1}\frac{1}{\sqrt{c\left(\frac{d}{e}+x\right)^2-2\frac{cd}{e}\left(\frac{d}{e}+x\right)+\frac{ae^2+cd^2}{e^2}}}+3\frac{ced}{\left(ae^2+cd^2\right)^2}\frac{1}{\sqrt{c\left(\frac{d}{e}+x\right)^2-2\frac{cd}{e}\left(\frac{d}{e}+x\right)+\frac{ae^2+cd^2}{e^2}}}+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^2+a)^(3/2),x)
```

```
[Out] -1/(a*e^2+c*d^2)/(d/e+x)/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1
/2)+3*e*c*d/(a*e^2+c*d^2)^2/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)
^(1/2)+3*c^2*d^2/(a*e^2+c*d^2)^2/a/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^
2)/e^2)^(1/2)*x-3*e*c*d/(a*e^2+c*d^2)^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*
e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c
*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))-2/(a*e^2+c*d^2)/a/(c*(d/e+x
)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.41863, size = 1777, normalized size = 11.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(a*c^2*d*e^3*x^3 + a*c^2*d^2*e^2*x^2 + a^2*c*d*e^3*x + a^2*c*d^2*e^
2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2
+ a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*
x^2 + 2*d*e*x + d^2)) + 2*(2*a*c^2*d^4*e + a^2*c*d^2*e^3 - a^3*e^5 + (c^3*d
^4*e - a*c^2*d^2*e^3 - 2*a^2*c*e^5)*x^2 + (c^3*d^5 + 2*a*c^2*d^3*e^2 + a^2*
c*d*e^4)*x)*sqrt(c*x^2 + a))/(a^2*c^3*d^7 + 3*a^3*c^2*d^5*e^2 + 3*a^4*c*d^3
*e^4 + a^5*d*e^6 + (a*c^4*d^6*e + 3*a^2*c^3*d^4*e^3 + 3*a^3*c^2*d^2*e^5 + a
^4*c*e^7)*x^3 + (a*c^4*d^7 + 3*a^2*c^3*d^5*e^2 + 3*a^3*c^2*d^3*e^4 + a^4*c*
d*e^6)*x^2 + (a^2*c^3*d^6*e + 3*a^3*c^2*d^4*e^3 + 3*a^4*c*d^2*e^5 + a^5*e^7
)*x), -(3*(a*c^2*d*e^3*x^3 + a*c^2*d^2*e^2*x^2 + a^2*c*d*e^3*x + a^2*c*d^2*
e^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*
x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - (2*a*c^2*d^4*e +
a^2*c*d^2*e^3 - a^3*e^5 + (c^3*d^4*e - a*c^2*d^2*e^3 - 2*a^2*c*e^5)*x^2 + (
c^3*d^5 + 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*x)*sqrt(c*x^2 + a))/(a^2*c^3*d^7 +
 3*a^3*c^2*d^5*e^2 + 3*a^4*c*d^3*e^4 + a^5*d*e^6 + (a*c^4*d^6*e + 3*a^2*c^3
*d^4*e^3 + 3*a^3*c^2*d^2*e^5 + a^4*c*e^7)*x^3 + (a*c^4*d^7 + 3*a^2*c^3*d^5*
e^2 + 3*a^3*c^2*d^3*e^4 + a^4*c*d*e^6)*x^2 + (a^2*c^3*d^6*e + 3*a^3*c^2*d^4
*e^3 + 3*a^4*c*d^2*e^5 + a^5*e^7)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + cx^2\right)^{\frac{3}{2}} (d + ex)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/((a + c*x**2)**(3/2)*(d + e*x)**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.575 $\int \frac{1}{(d+ex)^3(a+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=223

$$\frac{cde\sqrt{a+cx^2}\left(2cd^2-13ae^2\right)}{2a(d+ex)\left(ae^2+cd^2\right)^3}+\frac{e\sqrt{a+cx^2}\left(2cd^2-3ae^2\right)}{2a(d+ex)^2\left(ae^2+cd^2\right)^2}+\frac{ae+cdx}{a\sqrt{a+cx^2}(d+ex)^2\left(ae^2+cd^2\right)}-\frac{3ce^2\left(4cd^2-ae^2\right)\tanh^{-1}\left(\frac{}{\sqrt{a}}\right.}{2\left(ae^2+cd^2\right)^{7/2}}$$

```
[Out] (a*e + c*d*x)/(a*(c*d^2 + a*e^2)*(d + e*x)^2*Sqrt[a + c*x^2]) + (e*(2*c*d^2
 - 3*a*e^2)*Sqrt[a + c*x^2])/(2*a*(c*d^2 + a*e^2)^2*(d + e*x)^2) + (c*d*e*(
2*c*d^2 - 13*a*e^2)*Sqrt[a + c*x^2])/(2*a*(c*d^2 + a*e^2)^3*(d + e*x)) - (3
*c*e^2*(4*c*d^2 - a*e^2)*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a
+ c*x^2])])/(2*(c*d^2 + a*e^2)^(7/2))
```

Rubi [A] time = 0.198464, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {741, 835, 807, 725, 206}

$$\frac{cde\sqrt{a+cx^2}\left(2cd^2-13ae^2\right)}{2a(d+ex)\left(ae^2+cd^2\right)^3}+\frac{e\sqrt{a+cx^2}\left(2cd^2-3ae^2\right)}{2a(d+ex)^2\left(ae^2+cd^2\right)^2}+\frac{ae+cdx}{a\sqrt{a+cx^2}(d+ex)^2\left(ae^2+cd^2\right)}-\frac{3ce^2\left(4cd^2-ae^2\right)\tanh^{-1}\left(\frac{}{\sqrt{a}}\right.}{2\left(ae^2+cd^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*e + c*d*x)/(a*(c*d^2 + a*e^2)*(d + e*x)^2*Sqrt[a + c*x^2]) + (e*(2*c*d^2
 - 3*a*e^2)*Sqrt[a + c*x^2])/(2*a*(c*d^2 + a*e^2)^2*(d + e*x)^2) + (c*d*e*(
2*c*d^2 - 13*a*e^2)*Sqrt[a + c*x^2])/(2*a*(c*d^2 + a*e^2)^3*(d + e*x)) - (3
*c*e^2*(4*c*d^2 - a*e^2)*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a
+ c*x^2])])/(2*(c*d^2 + a*e^2)^(7/2))
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
```

```
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(a+cx^2\right)^{3/2}}\,dx = \frac{ae+cdx}{a\left(cd^2+ae^2\right)(d+ex)^2\sqrt{a+cx^2}} - \frac{\int \frac{-3ae^2-2cdex}{(d+ex)^3\sqrt{a+cx^2}}\,dx}{a\left(cd^2+ae^2\right)}$$

$$= \frac{ae+cdx}{a\left(cd^2+ae^2\right)(d+ex)^2\sqrt{a+cx^2}} + \frac{e\left(2cd^2-3ae^2\right)\sqrt{a+cx^2}}{2a\left(cd^2+ae^2\right)^2(d+ex)^2} + \frac{\int \frac{10acde^2+ce\left(2cd^2-3ae^2\right)x}{(d+ex)^2\sqrt{a+cx^2}}\,dx}{2a\left(cd^2+ae^2\right)^2}$$

$$= \frac{ae+cdx}{a\left(cd^2+ae^2\right)(d+ex)^2\sqrt{a+cx^2}} + \frac{e\left(2cd^2-3ae^2\right)\sqrt{a+cx^2}}{2a\left(cd^2+ae^2\right)^2(d+ex)^2} + \frac{cde\left(2cd^2-13ae^2\right)\sqrt{a+}}{2a\left(cd^2+ae^2\right)^3(d+ex}$$

$$= \frac{ae+cdx}{a\left(cd^2+ae^2\right)(d+ex)^2\sqrt{a+cx^2}} + \frac{e\left(2cd^2-3ae^2\right)\sqrt{a+cx^2}}{2a\left(cd^2+ae^2\right)^2(d+ex)^2} + \frac{cde\left(2cd^2-13ae^2\right)\sqrt{a+}}{2a\left(cd^2+ae^2\right)^3(d+ex}$$

$$= \frac{ae+cdx}{a\left(cd^2+ae^2\right)(d+ex)^2\sqrt{a+cx^2}} + \frac{e\left(2cd^2-3ae^2\right)\sqrt{a+cx^2}}{2a\left(cd^2+ae^2\right)^2(d+ex)^2} + \frac{cde\left(2cd^2-13ae^2\right)\sqrt{a+}}{2a\left(cd^2+ae^2\right)^3(d+ex}$$

Mathematica [A] time = 0.399432, size = 240, normalized size = 1.08

$$\frac{1}{2}\left(\frac{-a^2ce^3\left(10d^2+11dex+3e^2x^2\right)-a^3e^5+ac^2de\left(6d^2ex+6d^3-14de^2x^2-13e^3x^3\right)+2c^3d^3x(d+ex)^2}{a\sqrt{a+cx^2}(d+ex)^2\left(ae^2+cd^2\right)^3} + \frac{3ce^2\left(ae^2-4\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a + c*x^2)^(3/2)),x]
```

```
[Out] ((-(a^3*e^5) + 2*c^3*d^3*x*(d + e*x)^2 - a^2*c*e^3*(10*d^2 + 11*d*e*x + 3*e
^2*x^2) + a*c^2*d*e*(6*d^3 + 6*d^2*e*x - 14*d*e^2*x^2 - 13*e^3*x^3))/(a*(c*
d^2 + a*e^2)^3*(d + e*x)^2*Sqrt[a + c*x^2]) + (3*c*e^2*(4*c*d^2 - a*e^2)*Lo
g[d + e*x])/(c*d^2 + a*e^2)^(7/2) + (3*c*e^2*(-4*c*d^2 + a*e^2)*Log[a*e - c
*d*x + Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2]])/(c*d^2 + a*e^2)^(7/2))/2
```

Maple [B] time = 0.193, size = 681, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*x^2+a)^(3/2),x)
```

```
[Out] -1/2/e/(a*e^2+c*d^2)/(d/e+x)^2/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e
^2)^(1/2)-5/2*c*d/(a*e^2+c*d^2)^2/(d/e+x)/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e
^2+c*d^2)/e^2)^(1/2)+15/2*e*c^2*d^2/(a*e^2+c*d^2)^3/(c*(d/e+x)^2-2*c*d/e*(d
/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)+15/2*c^3*d^3/(a*e^2+c*d^2)^3/a/(c*(d/e+x)^2-
2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x-15/2*e*c^2*d^2/(a*e^2+c*d^2)^3/(
(a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+
c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d
/e+x))-13/2*c^2*d/(a*e^2+c*d^2)^2/a/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d
^2)/e^2)^(1/2)*x-3/2*e/(a*e^2+c*d^2)^2*c/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^
2+c*d^2)/e^2)^(1/2)+3/2*e/(a*e^2+c*d^2)^2*c/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2
*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2
-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 7.5163, size = 3078, normalized size = 13.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(4*a^2*c^2*d^4*e^2 - a^3*c*d^2*e^4 + (4*a*c^3*d^2*e^4 - a^2*c^2*e^6
)*x^4 + 2*(4*a*c^3*d^3*e^3 - a^2*c^2*d*e^5)*x^3 + (4*a*c^3*d^4*e^2 + 3*a^2*
c^2*d^2*e^4 - a^3*c*e^6)*x^2 + 2*(4*a^2*c^2*d^3*e^3 - a^3*c*d*e^5)*x)*sqrt(
c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^
2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*
d*e*x + d^2)) + 2*(6*a*c^3*d^6*e - 4*a^2*c^2*d^4*e^3 - 11*a^3*c*d^2*e^5 - a
^4*e^7 + (2*c^4*d^5*e^2 - 11*a*c^3*d^3*e^4 - 13*a^2*c^2*d*e^6)*x^3 + (4*c^4
*d^6*e - 10*a*c^3*d^4*e^3 - 17*a^2*c^2*d^2*e^5 - 3*a^3*c*e^7)*x^2 + (2*c^4*
d^7 + 8*a*c^3*d^5*e^2 - 5*a^2*c^2*d^3*e^4 - 11*a^3*c*d*e^6)*x)*sqrt(c*x^2 +
 a))/(a^2*c^4*d^10 + 4*a^3*c^3*d^8*e^2 + 6*a^4*c^2*d^6*e^4 + 4*a^5*c*d^4*e^
6 + a^6*d^2*e^8 + (a*c^5*d^8*e^2 + 4*a^2*c^4*d^6*e^4 + 6*a^3*c^3*d^4*e^6 +
4*a^4*c^2*d^2*e^8 + a^5*c*e^10)*x^4 + 2*(a*c^5*d^9*e + 4*a^2*c^4*d^7*e^3 +
6*a^3*c^3*d^5*e^5 + 4*a^4*c^2*d^3*e^7 + a^5*c*d*e^9)*x^3 + (a*c^5*d^10 + 5*
a^2*c^4*d^8*e^2 + 10*a^3*c^3*d^6*e^4 + 10*a^4*c^2*d^4*e^6 + 5*a^5*c*d^2*e^8
 + a^6*e^10)*x^2 + 2*(a^2*c^4*d^9*e + 4*a^3*c^3*d^7*e^3 + 6*a^4*c^2*d^5*e^5
 + 4*a^5*c*d^3*e^7 + a^6*d*e^9)*x), -1/2*(3*(4*a^2*c^2*d^4*e^2 - a^3*c*d^2*
e^4 + (4*a*c^3*d^2*e^4 - a^2*c^2*e^6)*x^4 + 2*(4*a*c^3*d^3*e^3 - a^2*c^2*d*
e^5)*x^3 + (4*a*c^3*d^4*e^2 + 3*a^2*c^2*d^2*e^4 - a^3*c*e^6)*x^2 + 2*(4*a^2
*c^2*d^3*e^3 - a^3*c*d*e^5)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*
```

```
e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)
*x^2)) - (6*a*c^3*d^6*e - 4*a^2*c^2*d^4*e^3 - 11*a^3*c*d^2*e^5 - a^4*e^7 +
(2*c^4*d^5*e^2 - 11*a*c^3*d^3*e^4 - 13*a^2*c^2*d*e^6)*x^3 + (4*c^4*d^6*e -
10*a*c^3*d^4*e^3 - 17*a^2*c^2*d^2*e^5 - 3*a^3*c*e^7)*x^2 + (2*c^4*d^7 + 8*a
*c^3*d^5*e^2 - 5*a^2*c^2*d^3*e^4 - 11*a^3*c*d*e^6)*x)*sqrt(c*x^2 + a))/(a^2
*c^4*d^10 + 4*a^3*c^3*d^8*e^2 + 6*a^4*c^2*d^6*e^4 + 4*a^5*c*d^4*e^6 + a^6*d
^2*e^8 + (a*c^5*d^8*e^2 + 4*a^2*c^4*d^6*e^4 + 6*a^3*c^3*d^4*e^6 + 4*a^4*c^2
*d^2*e^8 + a^5*c*e^10)*x^4 + 2*(a*c^5*d^9*e + 4*a^2*c^4*d^7*e^3 + 6*a^3*c^3
*d^5*e^5 + 4*a^4*c^2*d^3*e^7 + a^5*c*d*e^9)*x^3 + (a*c^5*d^10 + 5*a^2*c^4*d
^8*e^2 + 10*a^3*c^3*d^6*e^4 + 10*a^4*c^2*d^4*e^6 + 5*a^5*c*d^2*e^8 + a^6*e^
10)*x^2 + 2*(a^2*c^4*d^9*e + 4*a^3*c^3*d^7*e^3 + 6*a^4*c^2*d^5*e^5 + 4*a^5*
c*d^3*e^7 + a^6*d*e^9)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + cx^2\right)^{\frac{3}{2}} (d + ex)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/((a + c*x**2)**(3/2)*(d + e*x)**3), x)
```

Giac [B] time = 1.69494, size = 876, normalized size = 3.93

$$\frac{\frac{\left(c^6 d^9 - 6\,a^2 c^4 d^5 e^4 - 8\,a^3 c^3 d^3 e^6 - 3\,a^4 c^2 d e^8\right)x}{a c^6 d^{12} + 6\,a^2 c^5 d^{10} e^2 + 15\,a^3 c^4 d^8 e^4 + 20\,a^4 c^3 d^6 e^6 + 15\,a^5 c^2 d^4 e^8 + 6\,a^6 c d^2 e^{10} + a^7 e^{12}} + \frac{3\,a c^5 d^8 e + 8\,a^2 c^4 d^6 e^3 + 6\,a^3 c^3 d^4 e^5 - a^5 c e^9}{a c^6 d^{12} + 6\,a^2 c^5 d^{10} e^2 + 15\,a^3 c^4 d^8 e^4 + 20\,a^4 c^3 d^6 e^6 + 15\,a^5 c^2 d^4 e^8 + 6\,a^6 c d^2 e^{10} + a^7}}{\sqrt{cx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((c^6*d^9 - 6*a^2*c^4*d^5*e^4 - 8*a^3*c^3*d^3*e^6 - 3*a^4*c^2*d*e^8)*x/(a*c
^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15
*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12) + (3*a*c^5*d^8*e + 8*a^2*c^
4*d^6*e^3 + 6*a^3*c^3*d^4*e^5 - a^5*c*e^9)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2
 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d
^2*e^10 + a^7*e^12))/sqrt(c*x^2 + a) + 3*(4*c^2*d^2*e^2 - a*c*e^4)*arctan(-
((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2))/((c^3*d
^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)*sqrt(-c*d^2 - a*e^2)) - (
14*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*d^3*e^2 + 6*(sqrt(c)*x - sqrt(c*
x^2 + a))^3*c^2*d^2*e^3 - 22*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*d^2*e^3 -
7*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*d*e^4 - (sqrt(c)*x - sqrt(c*x^2
 + a))^3*a*c*e^5 + 7*a^2*c^(3/2)*d*e^4 - (sqrt(c)*x - sqrt(c*x^2 + a))*a^2*
c*e^5)/((c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)*((sqrt(c)*x
 - sqrt(c*x^2 + a))^2*e + 2*(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*d - a*e)^
2)
```

3.576 $\int \frac{1}{(d+ex)^4\left(a+cx^2\right)^{3/2}} dx$

Optimal. Leaf size=293

$$\frac{ce\sqrt{a+cx^2}\left(16a^2e^4-83acd^2e^2+6c^2d^4\right)}{6a(d+ex)\left(ae^2+cd^2\right)^4}-\frac{5c^2de^2\left(4cd^2-3ae^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{2\left(ae^2+cd^2\right)^{9/2}}+\frac{cde\sqrt{a+cx^2}\left(6cd^2-29ae^2\right)}{6a(d+ex)^2\left(ae^2+cd^2\right)^3}+$$

```
[Out] (a*e + c*d*x)/(a*(c*d^2 + a*e^2)*(d + e*x)^3*Sqrt[a + c*x^2]) + (e*(3*c*d^2
 - 4*a*e^2)*Sqrt[a + c*x^2])/(3*a*(c*d^2 + a*e^2)^2*(d + e*x)^3) + (c*d*e*(
6*c*d^2 - 29*a*e^2)*Sqrt[a + c*x^2])/(6*a*(c*d^2 + a*e^2)^3*(d + e*x)^2) +
(c*e*(6*c^2*d^4 - 83*a*c*d^2*e^2 + 16*a^2*e^4)*Sqrt[a + c*x^2])/(6*a*(c*d^2
 + a*e^2)^4*(d + e*x)) - (5*c^2*d*e^2*(4*c*d^2 - 3*a*e^2)*ArcTanh[(a*e - c*
d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(2*(c*d^2 + a*e^2)^(9/2))
```

Rubi [A] time = 0.373861, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {741, 835, 807, 725, 206}

$$\frac{ce\sqrt{a+cx^2}\left(16a^2e^4-83acd^2e^2+6c^2d^4\right)}{6a(d+ex)\left(ae^2+cd^2\right)^4}-\frac{5c^2de^2\left(4cd^2-3ae^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{2\left(ae^2+cd^2\right)^{9/2}}+\frac{cde\sqrt{a+cx^2}\left(6cd^2-29ae^2\right)}{6a(d+ex)^2\left(ae^2+cd^2\right)^3}+$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^4*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*e + c*d*x)/(a*(c*d^2 + a*e^2)*(d + e*x)^3*Sqrt[a + c*x^2]) + (e*(3*c*d^2
 - 4*a*e^2)*Sqrt[a + c*x^2])/(3*a*(c*d^2 + a*e^2)^2*(d + e*x)^3) + (c*d*e*(
6*c*d^2 - 29*a*e^2)*Sqrt[a + c*x^2])/(6*a*(c*d^2 + a*e^2)^3*(d + e*x)^2) +
(c*e*(6*c^2*d^4 - 83*a*c*d^2*e^2 + 16*a^2*e^4)*Sqrt[a + c*x^2])/(6*a*(c*d^2
 + a*e^2)^4*(d + e*x)) - (5*c^2*d*e^2*(4*c*d^2 - 3*a*e^2)*ArcTanh[(a*e - c*
d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(2*(c*d^2 + a*e^2)^(9/2))
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^4\left(a+cx^2\right)^{3/2}}\,dx = \frac{ae+cdx}{a\left(cd^2+ae^2\right)(d+ex)^3\sqrt{a+cx^2}} - \frac{\int \frac{-4ae^2-3cdex}{(d+ex)^4\sqrt{a+cx^2}}\,dx}{a\left(cd^2+ae^2\right)}$$

$$= \frac{ae+cdx}{a\left(cd^2+ae^2\right)(d+ex)^3\sqrt{a+cx^2}} + \frac{e\left(3cd^2-4ae^2\right)\sqrt{a+cx^2}}{3a\left(cd^2+ae^2\right)^2(d+ex)^3} + \frac{\int \frac{21acde^2+2ce\left(3cd^2-4ae^2\right)x}{(d+ex)^3\sqrt{a+cx^2}}\,dx}{3a\left(cd^2+ae^2\right)^2}$$

$$= \frac{ae+cdx}{a\left(cd^2+ae^2\right)(d+ex)^3\sqrt{a+cx^2}} + \frac{e\left(3cd^2-4ae^2\right)\sqrt{a+cx^2}}{3a\left(cd^2+ae^2\right)^2(d+ex)^3} + \frac{cde\left(6cd^2-29ae^2\right)\sqrt{a+}}{6a\left(cd^2+ae^2\right)^3(d+ex}$$

$$= \frac{ae+cdx}{a\left(cd^2+ae^2\right)(d+ex)^3\sqrt{a+cx^2}} + \frac{e\left(3cd^2-4ae^2\right)\sqrt{a+cx^2}}{3a\left(cd^2+ae^2\right)^2(d+ex)^3} + \frac{cde\left(6cd^2-29ae^2\right)\sqrt{a+}}{6a\left(cd^2+ae^2\right)^3(d+ex}$$

$$= \frac{ae+cdx}{a\left(cd^2+ae^2\right)(d+ex)^3\sqrt{a+cx^2}} + \frac{e\left(3cd^2-4ae^2\right)\sqrt{a+cx^2}}{3a\left(cd^2+ae^2\right)^2(d+ex)^3} + \frac{cde\left(6cd^2-29ae^2\right)\sqrt{a+}}{6a\left(cd^2+ae^2\right)^3(d+ex}$$

$$= \frac{ae+cdx}{a\left(cd^2+ae^2\right)(d+ex)^3\sqrt{a+cx^2}} + \frac{e\left(3cd^2-4ae^2\right)\sqrt{a+cx^2}}{3a\left(cd^2+ae^2\right)^2(d+ex)^3} + \frac{cde\left(6cd^2-29ae^2\right)\sqrt{a+}}{6a\left(cd^2+ae^2\right)^3(d+ex}$$

Mathematica [A] time = 0.651784, size = 279, normalized size = 0.95

$$\frac{1}{6}\left(\frac{\sqrt{a+cx^2}\left(\frac{6c^2\left(a^2e^3(ex-4d)+2acd^2e(2d-3ex)+c^2d^4x\right)}{a\left(a+cx^2\right)}+\frac{ce^3\left(10ae^2-47cd^2\right)}{d+ex}-\frac{11cde^3\left(ae^2+cd^2\right)}{(d+ex)^2}-\frac{2e^3\left(ae^2+cd^2\right)^2}{(d+ex)^3}\right)}{\left(ae^2+cd^2\right)^4} - \frac{15c^2de^2\left(4cd^2-3ae^2\right)}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^4*(a + c*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[a + c*x^2]*((-2*e^3*(c*d^2 + a*e^2)^2)/(d + e*x)^3 - (11*c*d*e^3*(c*
d^2 + a*e^2))/(d + e*x)^2 + (c*e^3*(-47*c*d^2 + 10*a*e^2))/(d + e*x) + (6*c
^2*(c^2*d^4*x + 2*a*c*d^2*e*(2*d - 3*e*x) + a^2*e^3*(-4*d + e*x)))/(a*(a +
```

```
c*x^2))))/(c*d^2 + a*e^2)^4 + (15*c^2*d*e^2*(4*c*d^2 - 3*a*e^2)*Log[d + e*x
])/(c*d^2 + a*e^2)^(9/2) - (15*c^2*d*e^2*(4*c*d^2 - 3*a*e^2)*Log[a*e - c*d*
x + Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2]])/(c*d^2 + a*e^2)^(9/2))/6
```

Maple [B] time = 0.201, size = 898, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^4/(c*x^2+a)^(3/2),x)
```

```
[Out] -1/3/e^2/(a*e^2+c*d^2)/(d/e+x)^3/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)
/e^2)^(1/2)-7/6/e*c*d/(a*e^2+c*d^2)^2/(d/e+x)^2/(c*(d/e+x)^2-2*c*d/e*(d/e+x
)+(a*e^2+c*d^2)/e^2)^(1/2)-35/6*c^2*d^2/(a*e^2+c*d^2)^3/(d/e+x)/(c*(d/e+x)^
2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)+35/2*e*c^3*d^3/(a*e^2+c*d^2)^4/(
c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)+35/2*c^4*d^4/(a*e^2+c*
d^2)^4/a/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x-35/2*e*c^3
*d^3/(a*e^2+c*d^2)^4/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*
d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2
+c*d^2)/e^2)^(1/2))/(d/e+x))-115/6*c^3*d^2/(a*e^2+c*d^2)^3/a/(c*(d/e+x)^2-2
*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x-15/2*e*c^2*d/(a*e^2+c*d^2)^3/(c*(
d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)+15/2*e*c^2*d/(a*e^2+c*d^2
)^3/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a
*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2
))/(d/e+x))+4/3/(a*e^2+c*d^2)^2*c/(d/e+x)/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e
^2+c*d^2)/e^2)^(1/2)+8/3/(a*e^2+c*d^2)^2*c^2/a/(c*(d/e+x)^2-2*c*d/e*(d/e+x)
+(a*e^2+c*d^2)/e^2)^(1/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 18.5289, size = 4508, normalized size = 15.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/12*(15*(4*a^2*c^3*d^6*e^2 - 3*a^3*c^2*d^4*e^4 + (4*a*c^4*d^3*e^5 - 3*a^2
*c^3*d*e^7)*x^5 + 3*(4*a*c^4*d^4*e^4 - 3*a^2*c^3*d^2*e^6)*x^4 + (12*a*c^4*d
^5*e^3 - 5*a^2*c^3*d^3*e^5 - 3*a^3*c^2*d*e^7)*x^3 + (4*a*c^4*d^6*e^2 + 9*a^
2*c^3*d^4*e^4 - 9*a^3*c^2*d^2*e^6)*x^2 + 3*(4*a^2*c^3*d^5*e^3 - 3*a^3*c^2*d
^3*e^5)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*
```

```
c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a
))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(24*a*c^4*d^8*e - 60*a^2*c^3*d^6*e^3 - 89
*a^3*c^2*d^4*e^5 - 7*a^4*c*d^2*e^7 - 2*a^5*e^9 + (6*c^5*d^6*e^3 - 77*a*c^4*
d^4*e^5 - 67*a^2*c^3*d^2*e^7 + 16*a^3*c^2*e^9)*x^4 + 3*(6*c^5*d^7*e^2 - 57*
a*c^4*d^5*e^4 - 62*a^2*c^3*d^3*e^6 + a^3*c^2*d*e^8)*x^3 + 2*(9*c^5*d^8*e -
39*a*c^4*d^6*e^3 - 101*a^2*c^3*d^4*e^5 - 49*a^3*c^2*d^2*e^7 + 4*a^4*c*e^9)*
x^2 + 3*(2*c^5*d^9 + 14*a*c^4*d^7*e^2 - 45*a^2*c^3*d^5*e^4 - 54*a^3*c^2*d^3
*e^6 + 3*a^4*c*d*e^8)*x)*sqrt(c*x^2 + a))/(a^2*c^5*d^13 + 5*a^3*c^4*d^11*e^
2 + 10*a^4*c^3*d^9*e^4 + 10*a^5*c^2*d^7*e^6 + 5*a^6*c*d^5*e^8 + a^7*d^3*e^1
0 + (a*c^6*d^10*e^3 + 5*a^2*c^5*d^8*e^5 + 10*a^3*c^4*d^6*e^7 + 10*a^4*c^3*d
^4*e^9 + 5*a^5*c^2*d^2*e^11 + a^6*c*e^13)*x^5 + 3*(a*c^6*d^11*e^2 + 5*a^2*c
^5*d^9*e^4 + 10*a^3*c^4*d^7*e^6 + 10*a^4*c^3*d^5*e^8 + 5*a^5*c^2*d^3*e^10 +
 a^6*c*d*e^12)*x^4 + (3*a*c^6*d^12*e + 16*a^2*c^5*d^10*e^3 + 35*a^3*c^4*d^8
*e^5 + 40*a^4*c^3*d^6*e^7 + 25*a^5*c^2*d^4*e^9 + 8*a^6*c*d^2*e^11 + a^7*e^1
3)*x^3 + (a*c^6*d^13 + 8*a^2*c^5*d^11*e^2 + 25*a^3*c^4*d^9*e^4 + 40*a^4*c^3
*d^7*e^6 + 35*a^5*c^2*d^5*e^8 + 16*a^6*c*d^3*e^10 + 3*a^7*d*e^12)*x^2 + 3*(
a^2*c^5*d^12*e + 5*a^3*c^4*d^10*e^3 + 10*a^4*c^3*d^8*e^5 + 10*a^5*c^2*d^6*e
^7 + 5*a^6*c*d^4*e^9 + a^7*d^2*e^11)*x), -1/6*(15*(4*a^2*c^3*d^6*e^2 - 3*a^
3*c^2*d^4*e^4 + (4*a*c^4*d^3*e^5 - 3*a^2*c^3*d*e^7)*x^5 + 3*(4*a*c^4*d^4*e^
4 - 3*a^2*c^3*d^2*e^6)*x^4 + (12*a*c^4*d^5*e^3 - 5*a^2*c^3*d^3*e^5 - 3*a^3*
c^2*d*e^7)*x^3 + (4*a*c^4*d^6*e^2 + 9*a^2*c^3*d^4*e^4 - 9*a^3*c^2*d^2*e^6)*
x^2 + 3*(4*a^2*c^3*d^5*e^3 - 3*a^3*c^2*d^3*e^5)*x)*sqrt(-c*d^2 - a*e^2)*arc
tan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 +
 (c^2*d^2 + a*c*e^2)*x^2)) - (24*a*c^4*d^8*e - 60*a^2*c^3*d^6*e^3 - 89*a^3*
c^2*d^4*e^5 - 7*a^4*c*d^2*e^7 - 2*a^5*e^9 + (6*c^5*d^6*e^3 - 77*a*c^4*d^4*e
^5 - 67*a^2*c^3*d^2*e^7 + 16*a^3*c^2*e^9)*x^4 + 3*(6*c^5*d^7*e^2 - 57*a*c^4
*d^5*e^4 - 62*a^2*c^3*d^3*e^6 + a^3*c^2*d*e^8)*x^3 + 2*(9*c^5*d^8*e - 39*a*
c^4*d^6*e^3 - 101*a^2*c^3*d^4*e^5 - 49*a^3*c^2*d^2*e^7 + 4*a^4*c*e^9)*x^2 +
 3*(2*c^5*d^9 + 14*a*c^4*d^7*e^2 - 45*a^2*c^3*d^5*e^4 - 54*a^3*c^2*d^3*e^6
+ 3*a^4*c*d*e^8)*x)*sqrt(c*x^2 + a))/(a^2*c^5*d^13 + 5*a^3*c^4*d^11*e^2 + 1
0*a^4*c^3*d^9*e^4 + 10*a^5*c^2*d^7*e^6 + 5*a^6*c*d^5*e^8 + a^7*d^3*e^10 + (
a*c^6*d^10*e^3 + 5*a^2*c^5*d^8*e^5 + 10*a^3*c^4*d^6*e^7 + 10*a^4*c^3*d^4*e^
9 + 5*a^5*c^2*d^2*e^11 + a^6*c*e^13)*x^5 + 3*(a*c^6*d^11*e^2 + 5*a^2*c^5*d^
9*e^4 + 10*a^3*c^4*d^7*e^6 + 10*a^4*c^3*d^5*e^8 + 5*a^5*c^2*d^3*e^10 + a^6*
c*d*e^12)*x^4 + (3*a*c^6*d^12*e + 16*a^2*c^5*d^10*e^3 + 35*a^3*c^4*d^8*e^5
+ 40*a^4*c^3*d^6*e^7 + 25*a^5*c^2*d^4*e^9 + 8*a^6*c*d^2*e^11 + a^7*e^13)*x^
3 + (a*c^6*d^13 + 8*a^2*c^5*d^11*e^2 + 25*a^3*c^4*d^9*e^4 + 40*a^4*c^3*d^7*
e^6 + 35*a^5*c^2*d^5*e^8 + 16*a^6*c*d^3*e^10 + 3*a^7*d*e^12)*x^2 + 3*(a^2*c
^5*d^12*e + 5*a^3*c^4*d^10*e^3 + 10*a^4*c^3*d^8*e^5 + 10*a^5*c^2*d^6*e^7 +
5*a^6*c*d^4*e^9 + a^7*d^2*e^11)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + cx^2\right)^{\frac{3}{2}} (d + ex)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**4/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/((a + c*x**2)**(3/2)*(d + e*x)**4), x)
```

Giac [B] time = 1.96978, size = 1384, normalized size = 4.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((c^8*d^12 - 2*a*c^7*d^10*e^2 - 17*a^2*c^6*d^8*e^4 - 28*a^3*c^5*d^6*e^6 - 1
7*a^4*c^4*d^4*e^8 - 2*a^5*c^3*d^2*e^10 + a^6*c^2*e^12)*x/(a*c^8*d^16 + 8*a^
2*c^7*d^14*e^2 + 28*a^3*c^6*d^12*e^4 + 56*a^4*c^5*d^10*e^6 + 70*a^5*c^4*d^8
*e^8 + 56*a^6*c^3*d^6*e^10 + 28*a^7*c^2*d^4*e^12 + 8*a^8*c*d^2*e^14 + a^9*e
^16) + 4*(a*c^7*d^11*e + 3*a^2*c^6*d^9*e^3 + 2*a^3*c^5*d^7*e^5 - 2*a^4*c^4*
d^5*e^7 - 3*a^5*c^3*d^3*e^9 - a^6*c^2*d*e^11)/(a*c^8*d^16 + 8*a^2*c^7*d^14*
e^2 + 28*a^3*c^6*d^12*e^4 + 56*a^4*c^5*d^10*e^6 + 70*a^5*c^4*d^8*e^8 + 56*a
^6*c^3*d^6*e^10 + 28*a^7*c^2*d^4*e^12 + 8*a^8*c*d^2*e^14 + a^9*e^16))/sqrt(
c*x^2 + a) + 5*(4*c^3*d^3*e^2 - 3*a*c^2*d*e^4)*arctan(-((sqrt(c)*x - sqrt(c
*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2))/((c^4*d^8 + 4*a*c^3*d^6*e^2
 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)*sqrt(-c*d^2 - a*e^2)) - 1
/3*(188*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^4*d^5*e^2 + 162*(sqrt(c)*x - sqrt
(c*x^2 + a))^4*c^(7/2)*d^4*e^3 + 36*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^3*d^3
*e^4 - 402*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(7/2)*d^4*e^3 - 322*(sqrt(c)
*x - sqrt(c*x^2 + a))^3*a*c^3*d^3*e^4 - 117*(sqrt(c)*x - sqrt(c*x^2 + a))^4
*a*c^(5/2)*d^2*e^5 - 21*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^2*d*e^6 + 246*(
sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c^3*d^3*e^4 + 144*(sqrt(c)*x - sqrt(c*x^2
+ a))^2*a^2*c^(5/2)*d^2*e^5 + 60*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^2*d*
e^6 + 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(3/2)*e^7 - 47*a^3*c^(5/2)*d^
2*e^5 - 39*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2*d*e^6 - 24*(sqrt(c)*x - sq
rt(c*x^2 + a))^2*a^3*c^(3/2)*e^7 + 10*a^4*c^(3/2)*e^7)/((c^4*d^8 + 4*a*c^3*
d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)*((sqrt(c)*x - sqrt
(c*x^2 + a))^2*e + 2*(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*d - a*e)^3)
```

3.577 $\int \frac{(d+ex)^5}{\left(a+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=191

$$-\frac{e\sqrt{a+cx^2}\left(4\left(-2a^2e^4+4acd^2e^2+c^2d^4\right)+cdex\left(7ae^2+2cd^2\right)\right)}{3a^2c^3}-\frac{2(d+ex)^2\left(2a^2e^3-cdx\left(3ae^2+cd^2\right)\right)}{3a^2c^2\sqrt{a+cx^2}}+\frac{5de^4\tanh}{c^5}$$

```
[Out] -((a*e - c*d*x)*(d + e*x)^4)/(3*a*c*(a + c*x^2)^(3/2)) - (2*(d + e*x)^2*(2*
a^2*e^3 - c*d*(c*d^2 + 3*a*e^2)*x))/(3*a^2*c^2*Sqrt[a + c*x^2]) - (e*(4*(c^
2*d^4 + 4*a*c*d^2*e^2 - 2*a^2*e^4) + c*d*e*(2*c*d^2 + 7*a*e^2)*x)*Sqrt[a +
c*x^2])/(3*a^2*c^3) + (5*d*e^4*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/c^(5/2
)
```

Rubi [A] time = 0.163214, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {739, 819, 780, 217, 206}

$$-\frac{e\sqrt{a+cx^2}\left(4\left(-2a^2e^4+4acd^2e^2+c^2d^4\right)+cdex\left(7ae^2+2cd^2\right)\right)}{3a^2c^3}-\frac{2(d+ex)^2\left(2a^2e^3-cdx\left(3ae^2+cd^2\right)\right)}{3a^2c^2\sqrt{a+cx^2}}+\frac{5de^4\tanh}{c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5/(a + c*x^2)^(5/2),x]
```

```
[Out] -((a*e - c*d*x)*(d + e*x)^4)/(3*a*c*(a + c*x^2)^(3/2)) - (2*(d + e*x)^2*(2*
a^2*e^3 - c*d*(c*d^2 + 3*a*e^2)*x))/(3*a^2*c^2*Sqrt[a + c*x^2]) - (e*(4*(c^
2*d^4 + 4*a*c*d^2*e^2 - 2*a^2*e^4) + c*d*e*(2*c*d^2 + 7*a*e^2)*x)*Sqrt[a +
c*x^2])/(3*a^2*c^3) + (5*d*e^4*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/c^(5/2
)
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
```

```
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^5}{\left(a+cx^2\right)^{5/2}}\,dx = -\frac{(ae-cdx)(d+ex)^4}{3ac\left(a+cx^2\right)^{3/2}}+\frac{\int \frac{(d+ex)^3\left(2\left(cd^2+2ae^2\right)-2cdex\right)}{\left(a+cx^2\right)^{3/2}}\,dx}{3ac}$$

$$= -\frac{(ae-cdx)(d+ex)^4}{3ac\left(a+cx^2\right)^{3/2}}-\frac{2(d+ex)^2\left(2a^2e^3-cd\left(cd^2+3ae^2\right)x\right)}{3a^2c^2\sqrt{a+cx^2}}+\frac{\int \frac{(d+ex)\left(-2ae^2\left(cd^2-4ae^2\right)-2cde\left(2cd^2+7ae^2\right)x\right.}{\sqrt{a+cx^2}}}{3a^2c^2}$$

$$= -\frac{(ae-cdx)(d+ex)^4}{3ac\left(a+cx^2\right)^{3/2}}-\frac{2(d+ex)^2\left(2a^2e^3-cd\left(cd^2+3ae^2\right)x\right)}{3a^2c^2\sqrt{a+cx^2}}-\frac{e\left(4\left(c^2d^4+4acd^2e^2-2a^2e^4\right)+cde\right.}{3a^2c^3}$$

$$= -\frac{(ae-cdx)(d+ex)^4}{3ac\left(a+cx^2\right)^{3/2}}-\frac{2(d+ex)^2\left(2a^2e^3-cd\left(cd^2+3ae^2\right)x\right)}{3a^2c^2\sqrt{a+cx^2}}-\frac{e\left(4\left(c^2d^4+4acd^2e^2-2a^2e^4\right)+cde\right.}{3a^2c^3}$$

$$= -\frac{(ae-cdx)(d+ex)^4}{3ac\left(a+cx^2\right)^{3/2}}-\frac{2(d+ex)^2\left(2a^2e^3-cd\left(cd^2+3ae^2\right)x\right)}{3a^2c^2\sqrt{a+cx^2}}-\frac{e\left(4\left(c^2d^4+4acd^2e^2-2a^2e^4\right)+cde\right.}{3a^2c^3}$$

Mathematica [A] time = 0.245504, size = 167, normalized size = 0.87

$$\frac{a^2c^2e\left(-30d^2e^2x^2-5d^4-20de^3x^3+3e^4x^4\right)+a^3ce^3\left(-20d^2-15dex+12e^2x^2\right)+8a^4e^5+ac^3d^3x\left(3d^2+10e^2x^2\right)+2c^4d^5x^3}{3a^2c^3\left(a+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5/(a + c*x^2)^(5/2),x]
```

```
[Out] (8*a^4*e^5 + 2*c^4*d^5*x^3 + a*c^3*d^3*x*(3*d^2 + 10*e^2*x^2) + a^3*c*e^3*(
-20*d^2 - 15*d*e*x + 12*e^2*x^2) + a^2*c^2*e*(-5*d^4 - 30*d^2*e^2*x^2 - 20*
d*e^3*x^3 + 3*e^4*x^4))/(3*a^2*c^3*(a + c*x^2)^(3/2)) + (5*d*e^4*Log[c*x +
Sqrt[c]*Sqrt[a + c*x^2]])/c^(5/2)
```

Maple [A] time = 0.053, size = 270, normalized size = 1.4

$$\frac{e^5x^4}{c}\left(cx^2+a\right)^{-\frac{3}{2}}+4\,\frac{ae^5x^2}{c^2\left(cx^2+a\right)^{3/2}}+\frac{8\,e^5a^2}{3\,c^3}\left(cx^2+a\right)^{-\frac{3}{2}}-\frac{5\,de^4x^3}{3\,c}\left(cx^2+a\right)^{-\frac{3}{2}}-5\,\frac{de^4x}{c^2\sqrt{cx^2+a}}+5\,\frac{de^4\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right.}{c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5/(c*x^2+a)^(5/2),x)
```

```
[Out] e^5*x^4/c/(c*x^2+a)^(3/2)+4*e^5*a/c^2*x^2/(c*x^2+a)^(3/2)+8/3*e^5*a^2/c^3/(
c*x^2+a)^(3/2)-5/3*d*e^4*x^3/c/(c*x^2+a)^(3/2)-5*d*e^4/c^2*x/(c*x^2+a)^(1/2
)+5*d*e^4/c^(5/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))-10*d^2*e^3*x^2/c/(c*x^2+a)^
(3/2)-20/3*d^2*e^3*a/c^2/(c*x^2+a)^(3/2)-10/3*d^3*e^2/c*x/(c*x^2+a)^(3/2)+1
0/3*d^3*e^2/a/c*x/(c*x^2+a)^(1/2)-5/3*d^4*e/c/(c*x^2+a)^(3/2)+1/3*d^5*x/a/(
c*x^2+a)^(3/2)+2/3*d^5/a^2*x/(c*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.4707, size = 1008, normalized size = 5.28

$$\left[\frac{15\left(a^2c^2de^4x^4+2\,a^3cde^4x^2+a^4de^4\right)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)+2\left(3\,a^2c^2e^5x^4-5\,a^2c^2d^4e-20\,a^3cd^2e^3+\right.}{6\left(a^2c^5x^4+2\,a^3c^4x^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a^2*c^2*d*e^4*x^4 + 2*a^3*c*d*e^4*x^2 + a^4*d*e^4)*sqrt(c)*log(-2
*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(3*a^2*c^2*e^5*x^4 - 5*a^2*c^
2*d^4*e - 20*a^3*c*d^2*e^3 + 8*a^4*e^5 + 2*(c^4*d^5 + 5*a*c^3*d^3*e^2 - 10*
a^2*c^2*d*e^4)*x^3 - 6*(5*a^2*c^2*d^2*e^3 - 2*a^3*c*e^5)*x^2 + 3*(a*c^3*d^5
 - 5*a^3*c*d*e^4)*x)*sqrt(c*x^2 + a))/(a^2*c^5*x^4 + 2*a^3*c^4*x^2 + a^4*c^
3), -1/3*(15*(a^2*c^2*d*e^4*x^4 + 2*a^3*c*d*e^4*x^2 + a^4*d*e^4)*sqrt(-c)*a
rctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (3*a^2*c^2*e^5*x^4 - 5*a^2*c^2*d^4*e -
20*a^3*c*d^2*e^3 + 8*a^4*e^5 + 2*(c^4*d^5 + 5*a*c^3*d^3*e^2 - 10*a^2*c^2*d*
e^4)*x^3 - 6*(5*a^2*c^2*d^2*e^3 - 2*a^3*c*e^5)*x^2 + 3*(a*c^3*d^5 - 5*a^3*c
*d*e^4)*x)*sqrt(c*x^2 + a))/(a^2*c^5*x^4 + 2*a^3*c^4*x^2 + a^4*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^5}{\left(a+cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5/(c*x**2+a)**(5/2),x)
```

```
[Out] Integral((d + e*x)**5/(a + c*x**2)**(5/2), x)
```

Giac [A] time = 1.3285, size = 269, normalized size = 1.41

$$-\frac{5\,de^4\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{c^{\frac{5}{2}}}+\frac{\left(\left(x\left(\frac{3\,xe^5}{c}+\frac{2\left(c^6d^5+5\,ac^5d^3e^2-10\,a^2c^4de^4\right)}{a^2c^5}\right)-\frac{6\left(5\,a^2c^4d^2e^3-2\,a^3c^3e^5\right)}{a^2c^5}\right)x+\frac{3\left(ac^5d^5-5\,a^3c^3de^4\right)}{a^2c^5}\right)x-\frac{5\,a^2}{}}{3\left(cx^2+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -5*d*e^4*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2) + 1/3*(((x*(3*x*e^5
/c + 2*(c^6*d^5 + 5*a*c^5*d^3*e^2 - 10*a^2*c^4*d*e^4)/(a^2*c^5)) - 6*(5*a^2
*c^4*d^2*e^3 - 2*a^3*c^3*e^5)/(a^2*c^5))*x + 3*(a*c^5*d^5 - 5*a^3*c^3*d*e^4
)/(a^2*c^5))*x - (5*a^2*c^4*d^4*e + 20*a^3*c^3*d^2*e^3 - 8*a^4*c^2*e^5)/(a^
2*c^5))/(c*x^2 + a)^(3/2)
```

3.578 $\int \frac{(d+ex)^4}{\left(a+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=161

$$-\frac{de\sqrt{a+cx^2}\left(5ae^2+2cd^2\right)}{3a^2c^2}-\frac{(d+ex)\left(ae\left(3ae^2+cd^2\right)-2cdx\left(2ae^2+cd^2\right)\right)}{3a^2c^2\sqrt{a+cx^2}}+\frac{e^4\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{5/2}}-\frac{(d+ex)^3(ae-cd\cdots}{3ac\left(a+cx^2\right)^{3/\cdots}}$$

```
[Out] -((a*e - c*d*x)*(d + e*x)^3)/(3*a*c*(a + c*x^2)^(3/2)) - ((d + e*x)*(a*e*(c
*d^2 + 3*a*e^2) - 2*c*d*(c*d^2 + 2*a*e^2)*x))/(3*a^2*c^2*Sqrt[a + c*x^2]) -
 (d*e*(2*c*d^2 + 5*a*e^2)*Sqrt[a + c*x^2])/(3*a^2*c^2) + (e^4*ArcTanh[(Sqrt
[c]*x)/Sqrt[a + c*x^2]])/c^(5/2)
```

Rubi [A] time = 0.118167, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {739, 819, 641, 217, 206}

$$-\frac{de\sqrt{a+cx^2}\left(5ae^2+2cd^2\right)}{3a^2c^2}-\frac{(d+ex)\left(ae\left(3ae^2+cd^2\right)-2cdx\left(2ae^2+cd^2\right)\right)}{3a^2c^2\sqrt{a+cx^2}}+\frac{e^4\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{5/2}}-\frac{(d+ex)^3(ae-cd\cdots}{3ac\left(a+cx^2\right)^{3/\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a + c*x^2)^(5/2),x]
```

```
[Out] -((a*e - c*d*x)*(d + e*x)^3)/(3*a*c*(a + c*x^2)^(3/2)) - ((d + e*x)*(a*e*(c
*d^2 + 3*a*e^2) - 2*c*d*(c*d^2 + 2*a*e^2)*x))/(3*a^2*c^2*Sqrt[a + c*x^2]) -
 (d*e*(2*c*d^2 + 5*a*e^2)*Sqrt[a + c*x^2])/(3*a^2*c^2) + (e^4*ArcTanh[(Sqrt
[c]*x)/Sqrt[a + c*x^2]])/c^(5/2)
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^4}{\left(a+cx^2\right)^{5/2}}\,dx &= -\frac{(ae-cdx)(d+ex)^3}{3ac\left(a+cx^2\right)^{3/2}}+\frac{\int \frac{(d+ex)^2\left(2cd^2+3ae^2-cdex\right)}{\left(a+cx^2\right)^{3/2}}\,dx}{3ac}\\
&= -\frac{(ae-cdx)(d+ex)^3}{3ac\left(a+cx^2\right)^{3/2}}-\frac{(d+ex)\left(ae\left(cd^2+3ae^2\right)-2cd\left(cd^2+2ae^2\right)x\right)}{3a^2c^2\sqrt{a+cx^2}}+\frac{\int \frac{3a^2e^4-cde\left(2cd^2+5ae^2\right)x}{\sqrt{a+cx^2}}\,dx}{3a^2c^2}\\
&= -\frac{(ae-cdx)(d+ex)^3}{3ac\left(a+cx^2\right)^{3/2}}-\frac{(d+ex)\left(ae\left(cd^2+3ae^2\right)-2cd\left(cd^2+2ae^2\right)x\right)}{3a^2c^2\sqrt{a+cx^2}}-\frac{de\left(2cd^2+5ae^2\right)\sqrt{a+cx^2}}{3a^2c^2}\\
&= -\frac{(ae-cdx)(d+ex)^3}{3ac\left(a+cx^2\right)^{3/2}}-\frac{(d+ex)\left(ae\left(cd^2+3ae^2\right)-2cd\left(cd^2+2ae^2\right)x\right)}{3a^2c^2\sqrt{a+cx^2}}-\frac{de\left(2cd^2+5ae^2\right)\sqrt{a+cx^2}}{3a^2c^2}\\
&= -\frac{(ae-cdx)(d+ex)^3}{3ac\left(a+cx^2\right)^{3/2}}-\frac{(d+ex)\left(ae\left(cd^2+3ae^2\right)-2cd\left(cd^2+2ae^2\right)x\right)}{3a^2c^2\sqrt{a+cx^2}}-\frac{de\left(2cd^2+5ae^2\right)\sqrt{a+cx^2}}{3a^2c^2}
\end{aligned}$$

Mathematica [A] time = 0.212186, size = 130, normalized size = 0.81

$$\frac{-4a^2ce\left(d^3+3de^2x^2+e^3x^3\right)-a^3e^3(8d+3ex)+3ac^2d^2x\left(d^2+2e^2x^2\right)+2c^3d^4x^3}{3a^2c^2\left(a+cx^2\right)^{3/2}}+\frac{e^4\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)}{c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a + c*x^2)^(5/2),x]
```

```
[Out] (2*c^3*d^4*x^3 - a^3*e^3*(8*d + 3*e*x) + 3*a*c^2*d^2*x*(d^2 + 2*e^2*x^2) -
4*a^2*c*e*(d^3 + 3*d*e^2*x^2 + e^3*x^3))/(3*a^2*c^2*(a + c*x^2)^(3/2)) + (e
^4*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/c^(5/2)
```

Maple [A] time = 0.053, size = 202, normalized size = 1.3

$$-\frac{e^4x^3}{3c}\left(cx^2+a\right)^{-\frac{3}{2}}-\frac{e^4x}{c^2}\frac{1}{\sqrt{cx^2+a}}+e^4\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)c^{-\frac{5}{2}}-4\frac{de^3x^2}{c\left(cx^2+a\right)^{3/2}}-\frac{8\,de^3a}{3\,c^2}\left(cx^2+a\right)^{-\frac{3}{2}}-2\frac{d^2e^2x}{c\left(cx^2+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(c*x^2+a)^(5/2),x)
```

```
[Out] -1/3*e^4*x^3/c/(c*x^2+a)^(3/2)-e^4/c^2*x/(c*x^2+a)^(1/2)+e^4/c^(5/2)*ln(x*c
^(1/2)+(c*x^2+a)^(1/2))-4*d*e^3*x^2/c/(c*x^2+a)^(3/2)-8/3*d*e^3*a/c^2/(c*x^
2+a)^(3/2)-2*d^2*e^2/c*x/(c*x^2+a)^(3/2)+2*d^2*e^2/a/c*x/(c*x^2+a)^(1/2)-4/
3*d^3*e/c/(c*x^2+a)^(3/2)+1/3*d^4*x/a/(c*x^2+a)^(3/2)+2/3*d^4/a^2*x/(c*x^2+
a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.3249, size = 825, normalized size = 5.12

$$\left[\frac{3\left(a^2c^2e^4x^4+2a^3ce^4x^2+a^4e^4\right)\sqrt{c}\log\left(-2cx^2-2\sqrt{cx^2+a}\sqrt{c}x-a\right)-2\left(12a^2c^2de^3x^2+4a^2c^2d^3e+8a^3cde^3-2\left(c^4d\right.\right.}{6\left(a^2c^5x^4+2a^3c^4x^2+a^4c^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^2*c^2*e^4*x^4 + 2*a^3*c*e^4*x^2 + a^4*e^4)*sqrt(c)*log(-2*c*x^2
- 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(12*a^2*c^2*d*e^3*x^2 + 4*a^2*c^2*d^
3*e + 8*a^3*c*d*e^3 - 2*(c^4*d^4 + 3*a*c^3*d^2*e^2 - 2*a^2*c^2*e^4)*x^3 - 3
*(a*c^3*d^4 - a^3*c*e^4)*x)*sqrt(c*x^2 + a))/(a^2*c^5*x^4 + 2*a^3*c^4*x^2 +
 a^4*c^3), -1/3*(3*(a^2*c^2*e^4*x^4 + 2*a^3*c*e^4*x^2 + a^4*e^4)*sqrt(-c)*a
rctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (12*a^2*c^2*d*e^3*x^2 + 4*a^2*c^2*d^3*e
 + 8*a^3*c*d*e^3 - 2*(c^4*d^4 + 3*a*c^3*d^2*e^2 - 2*a^2*c^2*e^4)*x^3 - 3*(a
*c^3*d^4 - a^3*c*e^4)*x)*sqrt(c*x^2 + a))/(a^2*c^5*x^4 + 2*a^3*c^4*x^2 + a^
4*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^4}{\left(a+cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(c*x**2+a)**(5/2),x)
```

```
[Out] Integral((d + e*x)**4/(a + c*x**2)**(5/2), x)
```

Giac [A] time = 1.32629, size = 203, normalized size = 1.26

$$-\frac{\left(2\,x\left(\frac{6\,de^3}{c}-\frac{\left(c^5d^4+3\,ac^4d^2e^2-2\,a^2c^3e^4\right)x}{a^2c^4}\right)-\frac{3\left(ac^4d^4-a^3c^2e^4\right)}{a^2c^4}\right)x+\frac{4\left(a^2c^3d^3e+2\,a^3c^2de^3\right)}{a^2c^4}}{3\left(cx^2+a\right)^{\frac{3}{2}}}-\frac{e^4\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*((2*x*(6*d*e^3/c - (c^5*d^4 + 3*a*c^4*d^2*e^2 - 2*a^2*c^3*e^4)*x/(a^2*
c^4)) - 3*(a*c^4*d^4 - a^3*c^2*e^4)/(a^2*c^4))*x + 4*(a^2*c^3*d^3*e + 2*a^3
*c^2*d*e^3)/(a^2*c^4))/(c*x^2 + a)^(3/2) - e^4*log(abs(-sqrt(c)*x + sqrt(c*
x^2 + a)))/c^(5/2)
```

3.579 $\int \frac{(d+ex)^3}{\left(a+cx^2\right)^{5/2}} dx$

Optimal. Leaf size=79

$$-\frac{2\left(ae^2+cd^2\right)(ae-cdx)}{3a^2c^2\sqrt{a+cx^2}}-\frac{(d+ex)^2(ae-cdx)}{3ac\left(a+cx^2\right)^{3/2}}$$

```
[Out] -((a*e - c*d*x)*(d + e*x)^2)/(3*a*c*(a + c*x^2)^(3/2)) - (2*(c*d^2 + a*e^2)
*(a*e - c*d*x))/(3*a^2*c^2*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.0267251, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {723, 637}

$$-\frac{2\left(ae^2+cd^2\right)(ae-cdx)}{3a^2c^2\sqrt{a+cx^2}}-\frac{(d+ex)^2(ae-cdx)}{3ac\left(a+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + c*x^2)^(5/2),x]
```

```
[Out] -((a*e - c*d*x)*(d + e*x)^2)/(3*a*c*(a + c*x^2)^(3/2)) - (2*(c*d^2 + a*e^2)
*(a*e - c*d*x))/(3*a^2*c^2*Sqrt[a + c*x^2])
```

Rule 723

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[((2*p + 3)*(c*d^2 + a*e^2))/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^3}{\left(a+cx^2\right)^{5/2}} dx &= -\frac{(ae-cdx)(d+ex)^2}{3ac\left(a+cx^2\right)^{3/2}}+\frac{\left(2\left(cd^2+ae^2\right)\right)\int \frac{d+ex}{\left(a+cx^2\right)^{3/2}} dx}{3ac}\\
&= -\frac{(ae-cdx)(d+ex)^2}{3ac\left(a+cx^2\right)^{3/2}}-\frac{2\left(cd^2+ae^2\right)(ae-cdx)}{3a^2c^2\sqrt{a+cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.104651, size = 78, normalized size = 0.99

$$\frac{-3a^2ce\left(d^2+e^2x^2\right)-2a^3e^3+3ac^2dx\left(d^2+e^2x^2\right)+2c^3d^3x^3}{3a^2c^2\left(a+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + c*x^2)^(5/2),x]
```

```
[Out] (-2*a^3*e^3 + 2*c^3*d^3*x^3 - 3*a^2*c*e*(d^2 + e^2*x^2) + 3*a*c^2*d*x*(d^2
+ e^2*x^2))/(3*a^2*c^2*(a + c*x^2)^(3/2))
```

Maple [A] time = 0.045, size = 83, normalized size = 1.1

$$-\frac{-3\,ac^2de^2x^3-2\,c^3d^3x^3+3\,e^3x^2a^2c-3\,d^3xac^2+2\,a^3e^3+3\,a^2cd^2e}{3\,a^2c^2}\left(cx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^2+a)^(5/2),x)
```

```
[Out] -1/3*(-3*a*c^2*d*e^2*x^3-2*c^3*d^3*x^3+3*a^2*c*e^3*x^2-3*a*c^2*d^3*x+2*a^3*
e^3+3*a^2*c*d^2*e)/(c*x^2+a)^(3/2)/a^2/c^2
```

Maxima [A] time = 1.70726, size = 180, normalized size = 2.28

$$-\frac{e^3x^2}{\left(cx^2+a\right)^{\frac{3}{2}}c}+\frac{2\,d^3x}{3\,\sqrt{cx^2+a}a^2}+\frac{d^3x}{3\left(cx^2+a\right)^{\frac{3}{2}}a}-\frac{de^2x}{\left(cx^2+a\right)^{\frac{3}{2}}c}+\frac{de^2x}{\sqrt{cx^2+a}ac}-\frac{d^2e}{\left(cx^2+a\right)^{\frac{3}{2}}c}-\frac{2\,ae^3}{3\left(cx^2+a\right)^{\frac{3}{2}}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -e^3*x^2/((c*x^2 + a)^(3/2)*c) + 2/3*d^3*x/(sqrt(c*x^2 + a)*a^2) + 1/3*d^3*
x/((c*x^2 + a)^(3/2)*a) - d*e^2*x/((c*x^2 + a)^(3/2)*c) + d*e^2*x/(sqrt(c*x
^2 + a)*a*c) - d^2*e/((c*x^2 + a)^(3/2)*c) - 2/3*a*e^3/((c*x^2 + a)^(3/2)*c
^2)
```

Fricas [A] time = 2.25374, size = 213, normalized size = 2.7

$$-\frac{\left(3\,a^2ce^3x^2-3\,ac^2d^3x+3\,a^2cd^2e+2\,a^3e^3-\left(2\,c^3d^3+3\,ac^2de^2\right)x^3\right)\sqrt{cx^2+a}}{3\left(a^2c^4x^4+2\,a^3c^3x^2+a^4c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^2*c*e^3*x^2 - 3*a*c^2*d^3*x + 3*a^2*c*d^2*e + 2*a^3*e^3 - (2*c^3*
d^3 + 3*a*c^2*d*e^2)*x^3)*sqrt(c*x^2 + a)/(a^2*c^4*x^4 + 2*a^3*c^3*x^2 + a^
4*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^3}{\left(a+cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**2+a)**(5/2),x)
```

```
[Out] Integral((d + e*x)**3/(a + c*x**2)**(5/2), x)
```

Giac [A] time = 1.27188, size = 119, normalized size = 1.51

$$\frac{\left(\frac{3d^3}{a} - x\left(\frac{3e^3}{c} - \frac{(2c^3d^3+3ac^2de^2)x}{a^2c^2}\right)\right)x - \frac{3a^2cd^2e+2a^3e^3}{a^2c^2}}{3\left(cx^2+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((3*d^3/a - x*(3*e^3/c - (2*c^3*d^3 + 3*a*c^2*d*e^2)*x/(a^2*c^2)))*x -
(3*a^2*c*d^2*e + 2*a^3*e^3)/(a^2*c^2))/(c*x^2 + a)^(3/2)
```

3.580 $\int \frac{(d+ex)^2}{\left(a+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=58

$$\frac{x(d+ex)^2}{3a\left(a+cx^2\right)^{3/2}}-\frac{2d(ae-cdx)}{3a^2c\sqrt{a+cx^2}}$$

```
[Out] (x*(d + e*x)^2)/(3*a*(a + c*x^2)^(3/2)) - (2*d*(a*e - c*d*x))/(3*a^2*c*Sqrt
[a + c*x^2])
```

Rubi [A] time = 0.0202496, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {729, 637}

$$\frac{x(d+ex)^2}{3a\left(a+cx^2\right)^{3/2}}-\frac{2d(ae-cdx)}{3a^2c\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + c*x^2)^(5/2),x]
```

```
[Out] (x*(d + e*x)^2)/(3*a*(a + c*x^2)^(3/2)) - (2*d*(a*e - c*d*x))/(3*a^2*c*Sqrt
[a + c*x^2])
```

Rule 729

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^m*(2*c*x)*(a + c*x^2)^(p + 1))/(4*a*c*(p + 1)), x] - Dist[(m*(2
*c*d))/(4*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /
; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3,
 0] && LtQ[p, -1]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2}{\left(a+cx^2\right)^{5/2}}\,dx &= \frac{x(d+ex)^2}{3a\left(a+cx^2\right)^{3/2}}+\frac{(2d)\int \frac{d+ex}{\left(a+cx^2\right)^{3/2}}\,dx}{3a}\\ &= \frac{x(d+ex)^2}{3a\left(a+cx^2\right)^{3/2}}-\frac{2d(ae-cdx)}{3a^2c\sqrt{a+cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0801214, size = 57, normalized size = 0.98

$$\frac{-2a^2de+acx\left(3d^2+e^2x^2\right)+2c^2d^2x^3}{3a^2c\left(a+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + c*x^2)^(5/2),x]
```

```
[Out] (-2*a^2*d*e + 2*c^2*d^2*x^3 + a*c*x*(3*d^2 + e^2*x^2))/(3*a^2*c*(a + c*x^2)
^(3/2))
```

Maple [A] time = 0.043, size = 55, normalized size = 1.

$$-\frac{-ace^{2}x^{3}-2\,c^{2}d^{2}x^{3}-3\,d^{2}xac+2\,dea^{2}}{3\,a^{2}c}\left(cx^{2}+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^2+a)^(5/2),x)
```

```
[Out] -1/3*(-a*c*e^2*x^3-2*c^2*d^2*x^3-3*a*c*d^2*x+2*a^2*d*e)/(c*x^2+a)^(3/2)/a^2
/c
```

Maxima [A] time = 1.17244, size = 124, normalized size = 2.14

$$\frac{2\,d^{2}x}{3\,\sqrt{cx^{2}+a}a^{2}}+\frac{d^{2}x}{3\,\left(cx^{2}+a\right)^{\frac{3}{2}}a}-\frac{e^{2}x}{3\,\left(cx^{2}+a\right)^{\frac{3}{2}}c}+\frac{e^{2}x}{3\,\sqrt{cx^{2}+a}ac}-\frac{2\,de}{3\,\left(cx^{2}+a\right)^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*d^2*x/(sqrt(c*x^2 + a)*a^2) + 1/3*d^2*x/((c*x^2 + a)^(3/2)*a) - 1/3*e^2
*x/((c*x^2 + a)^(3/2)*c) + 1/3*e^2*x/(sqrt(c*x^2 + a)*a*c) - 2/3*d*e/((c*x^
2 + a)^(3/2)*c)
```

Fricas [A] time = 2.16358, size = 153, normalized size = 2.64

$$\frac{\left(3\,acd^{2}x-2\,a^{2}de+\left(2\,c^{2}d^{2}+ace^{2}\right)x^{3}\right)\sqrt{cx^{2}+a}}{3\,\left(a^{2}c^{3}x^{4}+2\,a^{3}c^{2}x^{2}+a^{4}c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*a*c*d^2*x - 2*a^2*d*e + (2*c^2*d^2 + a*c*e^2)*x^3)*sqrt(c*x^2 + a)/(
a^2*c^3*x^4 + 2*a^3*c^2*x^2 + a^4*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(d+ex\right)^{2}}{\left(a+cx^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**2+a)**(5/2),x)
```

```
[Out] Integral((d + e*x)**2/(a + c*x**2)**(5/2), x)
```

Giac [A] time = 1.3501, size = 74, normalized size = 1.28

$$\frac{\left(\frac{3\,d^2}{a}+\frac{\left(2\,c^2d^2+ace^2\right)x^2}{a^2c}\right)x-\frac{2\,de}{c}}{3\left(cx^2+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((3*d^2/a + (2*c^2*d^2 + a*c*e^2)*x^2/(a^2*c))*x - 2*d*e/c)/(c*x^2 + a)
^(3/2)
```

3.581 $\int \frac{d+ex}{\left(a+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=51

$$\frac{2dx}{3a^2\sqrt{a+cx^2}}-\frac{ae-cdx}{3ac\left(a+cx^2\right)^{3/2}}$$

```
[Out] -(a*e - c*d*x)/(3*a*c*(a + c*x^2)^(3/2)) + (2*d*x)/(3*a^2*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.0114975, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {639, 191}

$$\frac{2dx}{3a^2\sqrt{a+cx^2}}-\frac{ae-cdx}{3ac\left(a+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + c*x^2)^(5/2),x]
```

```
[Out] -(a*e - c*d*x)/(3*a*c*(a + c*x^2)^(3/2)) + (2*d*x)/(3*a^2*Sqrt[a + c*x^2])
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex}{\left(a+cx^2\right)^{5/2}}\,dx &= -\frac{ae-cdx}{3ac\left(a+cx^2\right)^{3/2}}+\frac{(2d)\int \frac{1}{\left(a+cx^2\right)^{3/2}}\,dx}{3a}\\ &= -\frac{ae-cdx}{3ac\left(a+cx^2\right)^{3/2}}+\frac{2dx}{3a^2\sqrt{a+cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0182051, size = 43, normalized size = 0.84

$$\frac{-a^2e+3acdx+2c^2dx^3}{3a^2c\left(a+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + c*x^2)^(5/2),x]
```

```
[Out] (-(a^2*e) + 3*a*c*d*x + 2*c^2*d*x^3)/(3*a^2*c*(a + c*x^2)^(3/2))
```

Maple [A] time = 0.043, size = 39, normalized size = 0.8

$$-\frac{-2\,c^{2}dx^{3}-3\,dxac+a^{2}e}{3\,a^{2}c}\left(cx^{2}+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^2+a)^(5/2),x)
```

```
[Out] -1/3*(-2*c^2*d*x^3-3*a*c*d*x+a^2*e)/(c*x^2+a)^(3/2)/a^2/c
```

Maxima [A] time = 1.17246, size = 65, normalized size = 1.27

$$\frac{2\,dx}{3\,\sqrt{cx^{2}+a}a^{2}}+\frac{dx}{3\,\left(cx^{2}+a\right)^{\frac{3}{2}}a}-\frac{e}{3\,\left(cx^{2}+a\right)^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*d*x/(sqrt(c*x^2 + a)*a^2) + 1/3*d*x/((c*x^2 + a)^(3/2)*a) - 1/3*e/((c*x
^2 + a)^(3/2)*c)
```

Fricas [A] time = 2.18817, size = 126, normalized size = 2.47

$$\frac{\left(2\,c^{2}dx^{3}+3\,acdx-a^{2}e\right)\sqrt{cx^{2}+a}}{3\,\left(a^{2}c^{3}x^{4}+2\,a^{3}c^{2}x^{2}+a^{4}c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*c^2*d*x^3 + 3*a*c*d*x - a^2*e)*sqrt(c*x^2 + a)/(a^2*c^3*x^4 + 2*a^3*
c^2*x^2 + a^4*c)
```

Sympy [B] time = 13.3911, size = 146, normalized size = 2.86

$$d\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{cx^{2}}{a}}+3a^{\frac{5}{2}}cx^{2}\sqrt{1+\frac{cx^{2}}{a}}}+\frac{2cx^{3}}{3a^{\frac{7}{2}}\sqrt{1+\frac{cx^{2}}{a}}+3a^{\frac{5}{2}}cx^{2}\sqrt{1+\frac{cx^{2}}{a}}}\right)+e\left(\begin{cases}-\frac{1}{3ac\sqrt{a+cx^{2}}+3c^{2}x^{2}\sqrt{a+cx^{2}}} & \text{for}\ c\neq 0\\ \frac{x^{2}}{2a^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**2+a)**(5/2),x)
```

```
[Out] d*(3*a*x/(3*a**(7/2)*sqrt(1 + c*x**2/a) + 3*a**(5/2)*c*x**2*sqrt(1 + c*x**2
/a)) + 2*c*x**3/(3*a**(7/2)*sqrt(1 + c*x**2/a) + 3*a**(5/2)*c*x**2*sqrt(1 +
```

```
c*x**2/a))) + e*Piecewise((-1/(3*a*c*sqrt(a + c*x**2) + 3*c**2*x**2*sqrt(a
+ c*x**2)), Ne(c, 0)), (x**2/(2*a**(5/2)), True))
```

Giac [A] time = 1.6705, size = 51, normalized size = 1.

$$\frac{\left(\frac{2\,cdx^2}{a^2} + \frac{3\,d}{a}\right)x - \frac{e}{c}}{3\left(cx^2 + a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((2*c*d*x^2/a^2 + 3*d/a)*x - e/c)/(c*x^2 + a)^(3/2)
```

3.582 $\int \frac{1}{(d+ex)\left(a+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=154

$$\frac{3a^2e^3+cdx\left(5ae^2+2cd^2\right)}{3a^2\sqrt{a+cx^2}\left(ae^2+cd^2\right)^2}+\frac{ae+cdx}{3a\left(a+cx^2\right)^{3/2}\left(ae^2+cd^2\right)}-\frac{e^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{5/2}}$$

```
[Out] (a*e + c*d*x)/(3*a*(c*d^2 + a*e^2)*(a + c*x^2)^(3/2)) + (3*a^2*e^3 + c*d*(2
*c*d^2 + 5*a*e^2)*x)/(3*a^2*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^2]) - (e^4*ArcTa
nh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(5
/2)
```

Rubi [A] time = 0.126313, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {741, 823, 12, 725, 206}

$$\frac{3a^2e^3+cdx\left(5ae^2+2cd^2\right)}{3a^2\sqrt{a+cx^2}\left(ae^2+cd^2\right)^2}+\frac{ae+cdx}{3a\left(a+cx^2\right)^{3/2}\left(ae^2+cd^2\right)}-\frac{e^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + c*x^2)^(5/2)),x]
```

```
[Out] (a*e + c*d*x)/(3*a*(c*d^2 + a*e^2)*(a + c*x^2)^(3/2)) + (3*a^2*e^3 + c*d*(2
*c*d^2 + 5*a*e^2)*x)/(3*a^2*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^2]) - (e^4*ArcTa
nh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(5
/2)
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+cx^2\right)^{5/2}}\,dx = \frac{ae+cdx}{3a\left(cd^2+ae^2\right)\left(a+cx^2\right)^{3/2}} - \frac{\int \frac{-2cd^2-3ae^2-2cdex}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx}{3a\left(cd^2+ae^2\right)}$$

$$= \frac{ae+cdx}{3a\left(cd^2+ae^2\right)\left(a+cx^2\right)^{3/2}} + \frac{3a^2e^3+cd\left(2cd^2+5ae^2\right)x}{3a^2\left(cd^2+ae^2\right)^2\sqrt{a+cx^2}} + \frac{\int \frac{3a^2ce^4}{(d+ex)\sqrt{a+cx^2}}\,dx}{3a^2c\left(cd^2+ae^2\right)^2}$$

$$= \frac{ae+cdx}{3a\left(cd^2+ae^2\right)\left(a+cx^2\right)^{3/2}} + \frac{3a^2e^3+cd\left(2cd^2+5ae^2\right)x}{3a^2\left(cd^2+ae^2\right)^2\sqrt{a+cx^2}} + \frac{e^4\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{\left(cd^2+ae^2\right)^2}$$

$$= \frac{ae+cdx}{3a\left(cd^2+ae^2\right)\left(a+cx^2\right)^{3/2}} + \frac{3a^2e^3+cd\left(2cd^2+5ae^2\right)x}{3a^2\left(cd^2+ae^2\right)^2\sqrt{a+cx^2}} - \frac{e^4\,\mathrm{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \ldots\right.}{\left(cd^2+ae^2\right)^2}$$

$$= \frac{ae+cdx}{3a\left(cd^2+ae^2\right)\left(a+cx^2\right)^{3/2}} + \frac{3a^2e^3+cd\left(2cd^2+5ae^2\right)x}{3a^2\left(cd^2+ae^2\right)^2\sqrt{a+cx^2}} - \frac{e^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{5/2}}$$

Mathematica [A] time = 0.149545, size = 153, normalized size = 0.99

$$\frac{a^2ce\left(d^2+6dex+3e^2x^2\right)+4a^3e^3+ac^2dx\left(3d^2+5e^2x^2\right)+2c^3d^3x^3}{3a^2\left(a+cx^2\right)^{3/2}\left(ae^2+cd^2\right)^2} - \frac{e^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + c*x^2)^(5/2)),x]
```

```
[Out] (4*a^3*e^3 + 2*c^3*d^3*x^3 + a^2*c*e*(d^2 + 6*d*e*x + 3*e^2*x^2) + a*c^2*d*
x*(3*d^2 + 5*e^2*x^2))/(3*a^2*(c*d^2 + a*e^2)^2*(a + c*x^2)^(3/2)) - (e^4*A
rcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2
)^(5/2)
```

Maple [B] time = 0.192, size = 454, normalized size = 3.

$$\frac{e}{3ae^2+3cd^2}\left(c\left(\frac{d}{e}+x\right)^2-2\frac{cd}{e}\left(\frac{d}{e}+x\right)+\frac{ae^2+cd^2}{e^2}\right)^{-\frac{3}{2}} + \frac{cdx}{\left(3ae^2+3cd^2\right)a}\left(c\left(\frac{d}{e}+x\right)^2-2\frac{cd}{e}\left(\frac{d}{e}+x\right)+\frac{ae^2+cd^2}{e^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+a)^(5/2),x)
```

```
[Out] 1/3*e/(a*e^2+c*d^2)/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)+1
/3*c*d/(a*e^2+c*d^2)/a/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2
)*x+2/3*c*d/(a*e^2+c*d^2)/a^2/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^
2)^(1/2)*x+e^3/(a*e^2+c*d^2)^2/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e
^2)^(1/2)+e^2/(a*e^2+c*d^2)^2*d/a/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2
)/e^2)^(1/2)*x*c-e^3/(a*e^2+c*d^2)^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2
+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/
e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.05014, size = 1690, normalized size = 10.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^2*c^2*e^4*x^4 + 2*a^3*c*e^4*x^2 + a^4*e^4)*sqrt(c*d^2 + a*e^2)*l
og((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(
c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) +
2*(a^2*c^2*d^4*e + 5*a^3*c*d^2*e^3 + 4*a^4*e^5 + (2*c^4*d^5 + 7*a*c^3*d^3*e
^2 + 5*a^2*c^2*d*e^4)*x^3 + 3*(a^2*c^2*d^2*e^3 + a^3*c*e^5)*x^2 + 3*(a*c^3*
d^5 + 3*a^2*c^2*d^3*e^2 + 2*a^3*c*d*e^4)*x)*sqrt(c*x^2 + a))/(a^4*c^3*d^6 +
 3*a^5*c^2*d^4*e^2 + 3*a^6*c*d^2*e^4 + a^7*e^6 + (a^2*c^5*d^6 + 3*a^3*c^4*d
^4*e^2 + 3*a^4*c^3*d^2*e^4 + a^5*c^2*e^6)*x^4 + 2*(a^3*c^4*d^6 + 3*a^4*c^3*
d^4*e^2 + 3*a^5*c^2*d^2*e^4 + a^6*c*e^6)*x^2), -1/3*(3*(a^2*c^2*e^4*x^4 + 2
*a^3*c*e^4*x^2 + a^4*e^4)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*
(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)
) - (a^2*c^2*d^4*e + 5*a^3*c*d^2*e^3 + 4*a^4*e^5 + (2*c^4*d^5 + 7*a*c^3*d^3
*e^2 + 5*a^2*c^2*d*e^4)*x^3 + 3*(a^2*c^2*d^2*e^3 + a^3*c*e^5)*x^2 + 3*(a*c^
3*d^5 + 3*a^2*c^2*d^3*e^2 + 2*a^3*c*d*e^4)*x)*sqrt(c*x^2 + a))/(a^4*c^3*d^6
 + 3*a^5*c^2*d^4*e^2 + 3*a^6*c*d^2*e^4 + a^7*e^6 + (a^2*c^5*d^6 + 3*a^3*c^4
*d^4*e^2 + 3*a^4*c^3*d^2*e^4 + a^5*c^2*e^6)*x^4 + 2*(a^3*c^4*d^6 + 3*a^4*c^
3*d^4*e^2 + 3*a^5*c^2*d^2*e^4 + a^6*c*e^6)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + cx^2\right)^{\frac{5}{2}} (d + ex)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+a)**(5/2),x)
```

```
[Out] Integral(1/((a + c*x**2)**(5/2)*(d + e*x)), x)
```

Giac [B] time = 1.35178, size = 1262, normalized size = 8.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2
))*e^4/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-c*d^2 - a*e^2)) + 1/3*(((
(2*c^10*d^15 + 17*a*c^9*d^13*e^2 + 60*a^2*c^8*d^11*e^4 + 115*a^3*c^7*d^9*e^
6 + 130*a^4*c^6*d^7*e^8 + 87*a^5*c^5*d^5*e^10 + 32*a^6*c^4*d^3*e^12 + 5*a^7
*c^3*d*e^14)*x/(a^2*c^9*d^16 + 8*a^3*c^8*d^14*e^2 + 28*a^4*c^7*d^12*e^4 + 5
6*a^5*c^6*d^10*e^6 + 70*a^6*c^5*d^8*e^8 + 56*a^7*c^4*d^6*e^10 + 28*a^8*c^3*
d^4*e^12 + 8*a^9*c^2*d^2*e^14 + a^10*c*e^16) + 3*(a^2*c^8*d^12*e^3 + 6*a^3*
c^7*d^10*e^5 + 15*a^4*c^6*d^8*e^7 + 20*a^5*c^5*d^6*e^9 + 15*a^6*c^4*d^4*e^1
1 + 6*a^7*c^3*d^2*e^13 + a^8*c^2*e^15)/(a^2*c^9*d^16 + 8*a^3*c^8*d^14*e^2 +
 28*a^4*c^7*d^12*e^4 + 56*a^5*c^6*d^10*e^6 + 70*a^6*c^5*d^8*e^8 + 56*a^7*c^
4*d^6*e^10 + 28*a^8*c^3*d^4*e^12 + 8*a^9*c^2*d^2*e^14 + a^10*c*e^16))*x + 3
*(a*c^9*d^15 + 8*a^2*c^8*d^13*e^2 + 27*a^3*c^7*d^11*e^4 + 50*a^4*c^6*d^9*e^
6 + 55*a^5*c^5*d^7*e^8 + 36*a^6*c^4*d^5*e^10 + 13*a^7*c^3*d^3*e^12 + 2*a^8*
c^2*d*e^14)/(a^2*c^9*d^16 + 8*a^3*c^8*d^14*e^2 + 28*a^4*c^7*d^12*e^4 + 56*a
^5*c^6*d^10*e^6 + 70*a^6*c^5*d^8*e^8 + 56*a^7*c^4*d^6*e^10 + 28*a^8*c^3*d^4
*e^12 + 8*a^9*c^2*d^2*e^14 + a^10*c*e^16))*x + (a^2*c^8*d^14*e + 10*a^3*c^7
*d^12*e^3 + 39*a^4*c^6*d^10*e^5 + 80*a^5*c^5*d^8*e^7 + 95*a^6*c^4*d^6*e^9 +
 66*a^7*c^3*d^4*e^11 + 25*a^8*c^2*d^2*e^13 + 4*a^9*c*e^15)/(a^2*c^9*d^16 +
8*a^3*c^8*d^14*e^2 + 28*a^4*c^7*d^12*e^4 + 56*a^5*c^6*d^10*e^6 + 70*a^6*c^5
*d^8*e^8 + 56*a^7*c^4*d^6*e^10 + 28*a^8*c^3*d^4*e^12 + 8*a^9*c^2*d^2*e^14 +
 a^10*c*e^16))/(c*x^2 + a)^(3/2)
```

3.583 $\int \frac{1}{(d+ex)^2\left(a+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=244

$$\frac{e\sqrt{a+cx^2}\left(-8a^2e^4+9acd^2e^2+2c^2d^4\right)}{3a^2(d+ex)\left(ae^2+cd^2\right)^3}-\frac{ae\left(cd^2-4ae^2\right)-cdx\left(7ae^2+2cd^2\right)}{3a^2\sqrt{a+cx^2}(d+ex)\left(ae^2+cd^2\right)^2}+\frac{ae+cdx}{3a\left(a+cx^2\right)^{3/2}(d+ex)\left(ae^2+cd^2\right)}-\frac{5ca}{}$$

```
[Out] (a*e + c*d*x)/(3*a*(c*d^2 + a*e^2)*(d + e*x)*(a + c*x^2)^(3/2)) - (a*e*(c*d
^2 - 4*a*e^2) - c*d*(2*c*d^2 + 7*a*e^2)*x)/(3*a^2*(c*d^2 + a*e^2)^2*(d + e*
x)*Sqrt[a + c*x^2]) + (e*(2*c^2*d^4 + 9*a*c*d^2*e^2 - 8*a^2*e^4)*Sqrt[a + c
*x^2])/(3*a^2*(c*d^2 + a*e^2)^3*(d + e*x)) - (5*c*d*e^4*ArcTanh[(a*e - c*d*
x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(7/2)
```

Rubi [A] time = 0.233254, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {741, 823, 807, 725, 206}

$$\frac{e\sqrt{a+cx^2}\left(-8a^2e^4+9acd^2e^2+2c^2d^4\right)}{3a^2(d+ex)\left(ae^2+cd^2\right)^3}-\frac{ae\left(cd^2-4ae^2\right)-cdx\left(7ae^2+2cd^2\right)}{3a^2\sqrt{a+cx^2}(d+ex)\left(ae^2+cd^2\right)^2}+\frac{ae+cdx}{3a\left(a+cx^2\right)^{3/2}(d+ex)\left(ae^2+cd^2\right)}-\frac{5ca}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + c*x^2)^(5/2)),x]
```

```
[Out] (a*e + c*d*x)/(3*a*(c*d^2 + a*e^2)*(d + e*x)*(a + c*x^2)^(3/2)) - (a*e*(c*d
^2 - 4*a*e^2) - c*d*(2*c*d^2 + 7*a*e^2)*x)/(3*a^2*(c*d^2 + a*e^2)^2*(d + e*
x)*Sqrt[a + c*x^2]) + (e*(2*c^2*d^4 + 9*a*c*d^2*e^2 - 8*a^2*e^4)*Sqrt[a + c
*x^2])/(3*a^2*(c*d^2 + a*e^2)^3*(d + e*x)) - (5*c*d*e^4*ArcTanh[(a*e - c*d*
x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(7/2)
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
```

```
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+cx^2\right)^{5/2}}\,dx = \frac{ae+cdx}{3a\left(cd^2+ae^2\right)(d+ex)\left(a+cx^2\right)^{3/2}} - \frac{\int \frac{-2\left(cd^2+2ae^2\right)-3cdex}{(d+ex)^2\left(a+cx^2\right)^{3/2}}\,dx}{3a\left(cd^2+ae^2\right)}$$

$$= \frac{ae+cdx}{3a\left(cd^2+ae^2\right)(d+ex)\left(a+cx^2\right)^{3/2}} - \frac{ae\left(cd^2-4ae^2\right)-cd\left(2cd^2+7ae^2\right)x}{3a^2\left(cd^2+ae^2\right)^2(d+ex)\sqrt{a+cx^2}} + \frac{\int \frac{-2ace^2\left(cd^2 \cdots\right.}{\cdots}}{3a \cdots}$$

$$= \frac{ae+cdx}{3a\left(cd^2+ae^2\right)(d+ex)\left(a+cx^2\right)^{3/2}} - \frac{ae\left(cd^2-4ae^2\right)-cd\left(2cd^2+7ae^2\right)x}{3a^2\left(cd^2+ae^2\right)^2(d+ex)\sqrt{a+cx^2}} + \frac{e\left(2c^2d^4+\cdots\right.}{3a^2\left(\cdots\right.}$$

$$= \frac{ae+cdx}{3a\left(cd^2+ae^2\right)(d+ex)\left(a+cx^2\right)^{3/2}} - \frac{ae\left(cd^2-4ae^2\right)-cd\left(2cd^2+7ae^2\right)x}{3a^2\left(cd^2+ae^2\right)^2(d+ex)\sqrt{a+cx^2}} + \frac{e\left(2c^2d^4+\cdots\right.}{3a^2\left(\cdots\right.}$$

$$= \frac{ae+cdx}{3a\left(cd^2+ae^2\right)(d+ex)\left(a+cx^2\right)^{3/2}} - \frac{ae\left(cd^2-4ae^2\right)-cd\left(2cd^2+7ae^2\right)x}{3a^2\left(cd^2+ae^2\right)^2(d+ex)\sqrt{a+cx^2}} + \frac{e\left(2c^2d^4+\cdots\right.}{3a^2\left(\cdots\right.}$$

Mathematica [A] time = 0.298793, size = 237, normalized size = 0.97

$$\frac{a^2c^2e\left(21d^2e^2x^2+11d^3ex+2d^4+7de^3x^3-8e^4x^4\right)+2a^3ce^3\left(7d^2+4dex-6e^2x^2\right)-3a^4e^5+3ac^3d^2x\left(d^2ex+d^3+3de^2\cdots\right.}{3a^2\left(a+cx^2\right)^{3/2}(d+ex)\left(ae^2+cd^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + c*x^2)^(5/2)),x]
```

```
[Out] (-3*a^4*e^5 + 2*c^4*d^4*x^3*(d + e*x) + 2*a^3*c*e^3*(7*d^2 + 4*d*e*x - 6*e^
2*x^2) + 3*a*c^3*d^2*x*(d^3 + d^2*e*x + 3*d*e^2*x^2 + 3*e^3*x^3) + a^2*c^2*
e*(2*d^4 + 11*d^3*e*x + 21*d^2*e^2*x^2 + 7*d*e^3*x^3 - 8*e^4*x^4))/(3*a^2*(
c*d^2 + a*e^2)^3*(d + e*x)*(a + c*x^2)^(3/2)) - (5*c*d*e^4*ArcTanh[(a*e - c
*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(7/2)
```

Maple [B] time = 0.2, size = 667, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^2+a)^(5/2),x)
```

```
[Out] -1/(a*e^2+c*d^2)/(d/e+x)/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3
/2)+5/3*e*c*d/(a*e^2+c*d^2)^2/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^
2)^(3/2)+5/3*c^2*d^2/(a*e^2+c*d^2)^2/a/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+
c*d^2)/e^2)^(3/2)*x+10/3*c^2*d^2/(a*e^2+c*d^2)^2/a^2/(c*(d/e+x)^2-2*c*d/e*(
d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x+5*e^3*c*d/(a*e^2+c*d^2)^3/(c*(d/e+x)^2-2*
c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)+5*e^2*c^2*d^2/(a*e^2+c*d^2)^3/a/(c*(
d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x-5*e^3*c*d/(a*e^2+c*d^2)
^3/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*
e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)
)/(d/e+x))-4/3/(a*e^2+c*d^2)*c/a/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)
/e^2)^(3/2)*x-8/3/(a*e^2+c*d^2)*c/a^2/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c
*d^2)/e^2)^(1/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.18523, size = 3310, normalized size = 13.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a^2*c^3*d*e^5*x^5 + a^2*c^3*d^2*e^4*x^4 + 2*a^3*c^2*d*e^5*x^3 + 2
*a^3*c^2*d^2*e^4*x^2 + a^4*c*d*e^5*x + a^4*c*d^2*e^4)*sqrt(c*d^2 + a*e^2)*l
og((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(
c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) +
2*(2*a^2*c^3*d^6*e + 16*a^3*c^2*d^4*e^3 + 11*a^4*c*d^2*e^5 - 3*a^5*e^7 + (2
*c^5*d^6*e + 11*a*c^4*d^4*e^3 + a^2*c^3*d^2*e^5 - 8*a^3*c^2*e^7)*x^4 + (2*c
^5*d^7 + 11*a*c^4*d^5*e^2 + 16*a^2*c^3*d^3*e^4 + 7*a^3*c^2*d*e^6)*x^3 + 3*(
a*c^4*d^6*e + 8*a^2*c^3*d^4*e^3 + 3*a^3*c^2*d^2*e^5 - 4*a^4*c*e^7)*x^2 + (3
*a*c^4*d^7 + 14*a^2*c^3*d^5*e^2 + 19*a^3*c^2*d^3*e^4 + 8*a^4*c*d*e^6)*x)*sq
rt(c*x^2 + a))/(a^4*c^4*d^9 + 4*a^5*c^3*d^7*e^2 + 6*a^6*c^2*d^5*e^4 + 4*a^7
*c*d^3*e^6 + a^8*d*e^8 + (a^2*c^6*d^8*e + 4*a^3*c^5*d^6*e^3 + 6*a^4*c^4*d^4
*e^5 + 4*a^5*c^3*d^2*e^7 + a^6*c^2*e^9)*x^5 + (a^2*c^6*d^9 + 4*a^3*c^5*d^7*
e^2 + 6*a^4*c^4*d^5*e^4 + 4*a^5*c^3*d^3*e^6 + a^6*c^2*d*e^8)*x^4 + 2*(a^3*c
^5*d^8*e + 4*a^4*c^4*d^6*e^3 + 6*a^5*c^3*d^4*e^5 + 4*a^6*c^2*d^2*e^7 + a^7*
c*e^9)*x^3 + 2*(a^3*c^5*d^9 + 4*a^4*c^4*d^7*e^2 + 6*a^5*c^3*d^5*e^4 + 4*a^6
```

```
*c^2*d^3*e^6 + a^7*c*d*e^8)*x^2 + (a^4*c^4*d^8*e + 4*a^5*c^3*d^6*e^3 + 6*a^
6*c^2*d^4*e^5 + 4*a^7*c*d^2*e^7 + a^8*e^9)*x), -1/3*(15*(a^2*c^3*d*e^5*x^5
+ a^2*c^3*d^2*e^4*x^4 + 2*a^3*c^2*d*e^5*x^3 + 2*a^3*c^2*d^2*e^4*x^2 + a^4*c
*d*e^5*x + a^4*c*d^2*e^4)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*
(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)
) - (2*a^2*c^3*d^6*e + 16*a^3*c^2*d^4*e^3 + 11*a^4*c*d^2*e^5 - 3*a^5*e^7 +
(2*c^5*d^6*e + 11*a*c^4*d^4*e^3 + a^2*c^3*d^2*e^5 - 8*a^3*c^2*e^7)*x^4 + (2
*c^5*d^7 + 11*a*c^4*d^5*e^2 + 16*a^2*c^3*d^3*e^4 + 7*a^3*c^2*d*e^6)*x^3 + 3
*(a*c^4*d^6*e + 8*a^2*c^3*d^4*e^3 + 3*a^3*c^2*d^2*e^5 - 4*a^4*c*e^7)*x^2 +
(3*a*c^4*d^7 + 14*a^2*c^3*d^5*e^2 + 19*a^3*c^2*d^3*e^4 + 8*a^4*c*d*e^6)*x)*
sqrt(c*x^2 + a))/(a^4*c^4*d^9 + 4*a^5*c^3*d^7*e^2 + 6*a^6*c^2*d^5*e^4 + 4*a
^7*c*d^3*e^6 + a^8*d*e^8 + (a^2*c^6*d^8*e + 4*a^3*c^5*d^6*e^3 + 6*a^4*c^4*d
^4*e^5 + 4*a^5*c^3*d^2*e^7 + a^6*c^2*e^9)*x^5 + (a^2*c^6*d^9 + 4*a^3*c^5*d^
7*e^2 + 6*a^4*c^4*d^5*e^4 + 4*a^5*c^3*d^3*e^6 + a^6*c^2*d*e^8)*x^4 + 2*(a^3
*c^5*d^8*e + 4*a^4*c^4*d^6*e^3 + 6*a^5*c^3*d^4*e^5 + 4*a^6*c^2*d^2*e^7 + a^
7*c*e^9)*x^3 + 2*(a^3*c^5*d^9 + 4*a^4*c^4*d^7*e^2 + 6*a^5*c^3*d^5*e^4 + 4*a
^6*c^2*d^3*e^6 + a^7*c*d*e^8)*x^2 + (a^4*c^4*d^8*e + 4*a^5*c^3*d^6*e^3 + 6*
a^6*c^2*d^4*e^5 + 4*a^7*c*d^2*e^7 + a^8*e^9)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+cx^2\right)^{\frac{5}{2}}(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**2+a)**(5/2),x)
```

```
[Out] Integral(1/((a + c*x**2)**(5/2)*(d + e*x)**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.584 $\int \frac{1}{(d+ex)^3(a+cx^2)^{5/2}}\,dx$

Optimal. Leaf size=327

$$\frac{cde\sqrt{a+cx^2}\left(-81a^2e^4+28acd^2e^2+4c^2d^4\right)}{6a^2(d+ex)\left(ae^2+cd^2\right)^4}+\frac{e\sqrt{a+cx^2}\left(-15a^2e^4+24acd^2e^2+4c^2d^4\right)}{6a^2(d+ex)^2\left(ae^2+cd^2\right)^3}-\frac{ae\left(2cd^2-5ae^2\right)-cdx\left(9ae^2+\right.}{3a^2\sqrt{a+cx^2}(d+ex)^2\left(ae^2+c\right.}$$

```
[Out] (a*e + c*d*x)/(3*a*(c*d^2 + a*e^2)*(d + e*x)^2*(a + c*x^2)^(3/2)) - (a*e*(2
*c*d^2 - 5*a*e^2) - c*d*(2*c*d^2 + 9*a*e^2)*x)/(3*a^2*(c*d^2 + a*e^2)^2*(d
+ e*x)^2*Sqrt[a + c*x^2]) + (e*(4*c^2*d^4 + 24*a*c*d^2*e^2 - 15*a^2*e^4)*Sq
rt[a + c*x^2])/(6*a^2*(c*d^2 + a*e^2)^3*(d + e*x)^2) + (c*d*e*(4*c^2*d^4 +
28*a*c*d^2*e^2 - 81*a^2*e^4)*Sqrt[a + c*x^2])/(6*a^2*(c*d^2 + a*e^2)^4*(d +
 e*x)) - (5*c*e^4*(6*c*d^2 - a*e^2)*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e
^2]*Sqrt[a + c*x^2])])/(2*(c*d^2 + a*e^2)^(9/2))
```

Rubi [A] time = 0.371794, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {741, 823, 835, 807, 725, 206}

$$\frac{cde\sqrt{a+cx^2}\left(-81a^2e^4+28acd^2e^2+4c^2d^4\right)}{6a^2(d+ex)\left(ae^2+cd^2\right)^4}+\frac{e\sqrt{a+cx^2}\left(-15a^2e^4+24acd^2e^2+4c^2d^4\right)}{6a^2(d+ex)^2\left(ae^2+cd^2\right)^3}-\frac{ae\left(2cd^2-5ae^2\right)-cdx\left(9ae^2+\right.}{3a^2\sqrt{a+cx^2}(d+ex)^2\left(ae^2+c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + c*x^2)^(5/2)),x]
```


```
[Out] (a*e + c*d*x)/(3*a*(c*d^2 + a*e^2)*(d + e*x)^2*(a + c*x^2)^(3/2)) - (a*e*(2
*c*d^2 - 5*a*e^2) - c*d*(2*c*d^2 + 9*a*e^2)*x)/(3*a^2*(c*d^2 + a*e^2)^2*(d
+ e*x)^2*Sqrt[a + c*x^2]) + (e*(4*c^2*d^4 + 24*a*c*d^2*e^2 - 15*a^2*e^4)*Sq
rt[a + c*x^2])/(6*a^2*(c*d^2 + a*e^2)^3*(d + e*x)^2) + (c*d*e*(4*c^2*d^4 +
28*a*c*d^2*e^2 - 81*a^2*e^4)*Sqrt[a + c*x^2])/(6*a^2*(c*d^2 + a*e^2)^4*(d +
 e*x)) - (5*c*e^4*(6*c*d^2 - a*e^2)*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e
^2]*Sqrt[a + c*x^2])])/(2*(c*d^2 + a*e^2)^(9/2))
```


Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(a+cx^2\right)^{5/2}}\,dx = \frac{ae+cdx}{3a\left(cd^2+ae^2\right)(d+ex)^2\left(a+cx^2\right)^{3/2}} - \frac{\int \frac{-2cd^2-5ae^2-4cdex}{(d+ex)^3\left(a+cx^2\right)^{3/2}}\,dx}{3a\left(cd^2+ae^2\right)}$$

$$= \frac{ae+cdx}{3a\left(cd^2+ae^2\right)(d+ex)^2\left(a+cx^2\right)^{3/2}} - \frac{ae\left(2cd^2-5ae^2\right)-cd\left(2cd^2+9ae^2\right)x}{3a^2\left(cd^2+ae^2\right)^2(d+ex)^2\sqrt{a+cx^2}} + \int \frac{-3ace^2(}{}$$

$$= \frac{ae+cdx}{3a\left(cd^2+ae^2\right)(d+ex)^2\left(a+cx^2\right)^{3/2}} - \frac{ae\left(2cd^2-5ae^2\right)-cd\left(2cd^2+9ae^2\right)x}{3a^2\left(cd^2+ae^2\right)^2(d+ex)^2\sqrt{a+cx^2}} + \frac{e\left(4c^2d^4\right.}{6}$$

$$= \frac{ae+cdx}{3a\left(cd^2+ae^2\right)(d+ex)^2\left(a+cx^2\right)^{3/2}} - \frac{ae\left(2cd^2-5ae^2\right)-cd\left(2cd^2+9ae^2\right)x}{3a^2\left(cd^2+ae^2\right)^2(d+ex)^2\sqrt{a+cx^2}} + \frac{e\left(4c^2d^4\right.}{6}$$

$$= \frac{ae+cdx}{3a\left(cd^2+ae^2\right)(d+ex)^2\left(a+cx^2\right)^{3/2}} - \frac{ae\left(2cd^2-5ae^2\right)-cd\left(2cd^2+9ae^2\right)x}{3a^2\left(cd^2+ae^2\right)^2(d+ex)^2\sqrt{a+cx^2}} + \frac{e\left(4c^2d^4\right.}{6}$$

$$= \frac{ae+cdx}{3a\left(cd^2+ae^2\right)(d+ex)^2\left(a+cx^2\right)^{3/2}} - \frac{ae\left(2cd^2-5ae^2\right)-cd\left(2cd^2+9ae^2\right)x}{3a^2\left(cd^2+ae^2\right)^2(d+ex)^2\sqrt{a+cx^2}} + \frac{e\left(4c^2d^4\right.}{6}$$

Mathematica [A] time = 0.901353, size = 296, normalized size = 0.91

$$\frac{1}{6}\left(\frac{\sqrt{a+cx^2}\left(\frac{4c\left(3a^2cde^3(5d-4ex)-3a^3e^5+7ac^2d^3e^2x+c^3d^5x\right)}{a^2\left(a+cx^2\right)}+\frac{2c\left(ae^2+cd^2\right)\left(-a^2e^3+3acde(d-ex)+c^2d^3x\right)}{a\left(a+cx^2\right)^2}-\frac{3e^5\left(ae^2+cd^2\right)}{(d+ex)^2}-\frac{33cde^5}{d+ex}\right)}{\left(ae^2+cd^2\right)^4}+\frac{15ce^4\left(ae^2-\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a + c*x^2)^(5/2)),x]
```

```
[Out] ((Sqrt[a + c*x^2]*((-3*e^5*(c*d^2 + a*e^2))/(d + e*x)^2 - (33*c*d*e^5)/(d +
 e*x) + (4*c*(-3*a^3*e^5 + c^3*d^5*x + 7*a*c^2*d^3*e^2*x + 3*a^2*c*d*e^3*(5
*d - 4*e*x)))/(a^2*(a + c*x^2)) + (2*c*(c*d^2 + a*e^2)*(-(a^2*e^3) + c^2*d^
3*x + 3*a*c*d*e*(d - e*x)))/(a*(a + c*x^2)^2)))/(c*d^2 + a*e^2)^4 + (15*c*e
^4*(6*c*d^2 - a*e^2)*Log[d + e*x])/(c*d^2 + a*e^2)^(9/2) + (15*c*e^4*(-6*c*
d^2 + a*e^2)*Log[a*e - c*d*x + Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2]])/(c*d^2
 + a*e^2)^(9/2))/6
```

Maple [B] time = 0.199, size = 1088, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*x^2+a)^(5/2),x)
```

```
[Out] -1/2/e/(a*e^2+c*d^2)/(d/e+x)^2/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e
^2)^(3/2)-7/2*c*d/(a*e^2+c*d^2)^2/(d/e+x)/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e
^2+c*d^2)/e^2)^(3/2)+35/6*e*c^2*d^2/(a*e^2+c*d^2)^3/(c*(d/e+x)^2-2*c*d/e*(d
/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)+35/6*c^3*d^3/(a*e^2+c*d^2)^3/a/(c*(d/e+x)^2-
2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)*x+35/3*c^3*d^3/(a*e^2+c*d^2)^3/a^2
/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x+35/2*e^3*c^2*d^2/(
a*e^2+c*d^2)^4/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)+35/2*e
^2*c^3*d^3/(a*e^2+c*d^2)^4/a/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2
)^(1/2)*x-35/2*e^3*c^2*d^2/(a*e^2+c*d^2)^4/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*
(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-
2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))-11/2*c^2*d/(a*e^2+c*d^2)
^2/a/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)*x-11*c^2*d/(a*e^
2+c*d^2)^2/a^2/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x-5/6*
e/(a*e^2+c*d^2)^2*c/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(3/2)-5
/2*e^3/(a*e^2+c*d^2)^3*c/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1
/2)-5/2*e^2/(a*e^2+c*d^2)^3*c^2*d/a/(c*(d/e+x)^2-2*c*d/e*(d/e+x)+(a*e^2+c*d
^2)/e^2)^(1/2)*x+5/2*e^3/(a*e^2+c*d^2)^3*c/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*
(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(d/e+x)^2-
2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 13.7516, size = 5269, normalized size = 16.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(15*(6*a^4*c^2*d^4*e^4 - a^5*c*d^2*e^6 + (6*a^2*c^4*d^2*e^6 - a^3*c^3
*e^8)*x^6 + 2*(6*a^2*c^4*d^3*e^5 - a^3*c^3*d*e^7)*x^5 + (6*a^2*c^4*d^4*e^4
+ 11*a^3*c^3*d^2*e^6 - 2*a^4*c^2*e^8)*x^4 + 4*(6*a^3*c^3*d^3*e^5 - a^4*c^2*
d*e^7)*x^3 + (12*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 - a^5*c*e^8)*x^2 + 2*(
6*a^4*c^2*d^3*e^5 - a^5*c*d*e^7)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x -
a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*
d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(6*a^2*c^4*d^8*e
 + 70*a^3*c^3*d^6*e^3 + 14*a^4*c^2*d^4*e^5 - 53*a^5*c*d^2*e^7 - 3*a^6*e^9 +
 (4*c^6*d^7*e^2 + 32*a*c^5*d^5*e^4 - 53*a^2*c^4*d^3*e^6 - 81*a^3*c^3*d*e^8)
*x^5 + (8*c^6*d^8*e + 64*a*c^5*d^6*e^3 - 16*a^2*c^4*d^4*e^5 - 87*a^3*c^3*d^
2*e^7 - 15*a^4*c^2*e^9)*x^4 + 2*(2*c^6*d^9 + 19*a*c^5*d^7*e^2 + 65*a^2*c^4*
d^5*e^4 - 24*a^3*c^3*d^3*e^6 - 72*a^4*c^2*d*e^8)*x^3 + 2*(6*a*c^5*d^8*e + 6
3*a^2*c^4*d^6*e^3 - 7*a^3*c^3*d^4*e^5 - 74*a^4*c^2*d^2*e^7 - 10*a^5*c*e^9)*
x^2 + (6*a*c^5*d^9 + 42*a^2*c^4*d^7*e^2 + 110*a^3*c^3*d^5*e^4 + 13*a^4*c^2*
d^3*e^6 - 61*a^5*c*d*e^8)*x)*sqrt(c*x^2 + a))/(a^4*c^5*d^12 + 5*a^5*c^4*d^1
0*e^2 + 10*a^6*c^3*d^8*e^4 + 10*a^7*c^2*d^6*e^6 + 5*a^8*c*d^4*e^8 + a^9*d^2
*e^10 + (a^2*c^7*d^10*e^2 + 5*a^3*c^6*d^8*e^4 + 10*a^4*c^5*d^6*e^6 + 10*a^5
*c^4*d^4*e^8 + 5*a^6*c^3*d^2*e^10 + a^7*c^2*e^12)*x^6 + 2*(a^2*c^7*d^11*e +
 5*a^3*c^6*d^9*e^3 + 10*a^4*c^5*d^7*e^5 + 10*a^5*c^4*d^5*e^7 + 5*a^6*c^3*d^
3*e^9 + a^7*c^2*d*e^11)*x^5 + (a^2*c^7*d^12 + 7*a^3*c^6*d^10*e^2 + 20*a^4*c
^5*d^8*e^4 + 30*a^5*c^4*d^6*e^6 + 25*a^6*c^3*d^4*e^8 + 11*a^7*c^2*d^2*e^10
+ 2*a^8*c*e^12)*x^4 + 4*(a^3*c^6*d^11*e + 5*a^4*c^5*d^9*e^3 + 10*a^5*c^4*d^
7*e^5 + 10*a^6*c^3*d^5*e^7 + 5*a^7*c^2*d^3*e^9 + a^8*c*d*e^11)*x^3 + (2*a^3
*c^6*d^12 + 11*a^4*c^5*d^10*e^2 + 25*a^5*c^4*d^8*e^4 + 30*a^6*c^3*d^6*e^6 +
 20*a^7*c^2*d^4*e^8 + 7*a^8*c*d^2*e^10 + a^9*e^12)*x^2 + 2*(a^4*c^5*d^11*e
+ 5*a^5*c^4*d^9*e^3 + 10*a^6*c^3*d^7*e^5 + 10*a^7*c^2*d^5*e^7 + 5*a^8*c*d^3
*e^9 + a^9*d*e^11)*x), -1/6*(15*(6*a^4*c^2*d^4*e^4 - a^5*c*d^2*e^6 + (6*a^2
*c^4*d^2*e^6 - a^3*c^3*e^8)*x^6 + 2*(6*a^2*c^4*d^3*e^5 - a^3*c^3*d*e^7)*x^5
 + (6*a^2*c^4*d^4*e^4 + 11*a^3*c^3*d^2*e^6 - 2*a^4*c^2*e^8)*x^4 + 4*(6*a^3*
c^3*d^3*e^5 - a^4*c^2*d*e^7)*x^3 + (12*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6
- a^5*c*e^8)*x^2 + 2*(6*a^4*c^2*d^3*e^5 - a^5*c*d*e^7)*x)*sqrt(-c*d^2 - a*e
^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^
2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - (6*a^2*c^4*d^8*e + 70*a^3*c^3*d^6*e^3 +
 14*a^4*c^2*d^4*e^5 - 53*a^5*c*d^2*e^7 - 3*a^6*e^9 + (4*c^6*d^7*e^2 + 32*a*
c^5*d^5*e^4 - 53*a^2*c^4*d^3*e^6 - 81*a^3*c^3*d*e^8)*x^5 + (8*c^6*d^8*e + 6
4*a*c^5*d^6*e^3 - 16*a^2*c^4*d^4*e^5 - 87*a^3*c^3*d^2*e^7 - 15*a^4*c^2*e^9)
*x^4 + 2*(2*c^6*d^9 + 19*a*c^5*d^7*e^2 + 65*a^2*c^4*d^5*e^4 - 24*a^3*c^3*d^
3*e^6 - 72*a^4*c^2*d*e^8)*x^3 + 2*(6*a*c^5*d^8*e + 63*a^2*c^4*d^6*e^3 - 7*a
^3*c^3*d^4*e^5 - 74*a^4*c^2*d^2*e^7 - 10*a^5*c*e^9)*x^2 + (6*a*c^5*d^9 + 42
*a^2*c^4*d^7*e^2 + 110*a^3*c^3*d^5*e^4 + 13*a^4*c^2*d^3*e^6 - 61*a^5*c*d*e^
8)*x)*sqrt(c*x^2 + a))/(a^4*c^5*d^12 + 5*a^5*c^4*d^10*e^2 + 10*a^6*c^3*d^8*
e^4 + 10*a^7*c^2*d^6*e^6 + 5*a^8*c*d^4*e^8 + a^9*d^2*e^10 + (a^2*c^7*d^10*e
^2 + 5*a^3*c^6*d^8*e^4 + 10*a^4*c^5*d^6*e^6 + 10*a^5*c^4*d^4*e^8 + 5*a^6*c^
3*d^2*e^10 + a^7*c^2*e^12)*x^6 + 2*(a^2*c^7*d^11*e + 5*a^3*c^6*d^9*e^3 + 10
*a^4*c^5*d^7*e^5 + 10*a^5*c^4*d^5*e^7 + 5*a^6*c^3*d^3*e^9 + a^7*c^2*d*e^11)
*x^5 + (a^2*c^7*d^12 + 7*a^3*c^6*d^10*e^2 + 20*a^4*c^5*d^8*e^4 + 30*a^5*c^4
*d^6*e^6 + 25*a^6*c^3*d^4*e^8 + 11*a^7*c^2*d^2*e^10 + 2*a^8*c*e^12)*x^4 + 4
```

```
*(a^3*c^6*d^11*e + 5*a^4*c^5*d^9*e^3 + 10*a^5*c^4*d^7*e^5 + 10*a^6*c^3*d^5*
e^7 + 5*a^7*c^2*d^3*e^9 + a^8*c*d*e^11)*x^3 + (2*a^3*c^6*d^12 + 11*a^4*c^5*
d^10*e^2 + 25*a^5*c^4*d^8*e^4 + 30*a^6*c^3*d^6*e^6 + 20*a^7*c^2*d^4*e^8 + 7
*a^8*c*d^2*e^10 + a^9*e^12)*x^2 + 2*(a^4*c^5*d^11*e + 5*a^5*c^4*d^9*e^3 + 1
0*a^6*c^3*d^7*e^5 + 10*a^7*c^2*d^5*e^7 + 5*a^8*c*d^3*e^9 + a^9*d*e^11)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+cx^2\right)^{\frac{5}{2}}(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**2+a)**(5/2),x)
```

```
[Out] Integral(1/((a + c*x**2)**(5/2)*(d + e*x)**3), x)
```

Giac [B] time = 2.57672, size = 2759, normalized size = 8.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 5*(6*c^2*d^2*e^4 - a*c*e^6)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt
(c)*d)/sqrt(-c*d^2 - a*e^2))/((c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^
4 + 4*a^3*c*d^2*e^6 + a^4*e^8)*sqrt(-c*d^2 - a*e^2)) + 1/3*((2*((c^18*d^29
+ 19*a*c^17*d^27*e^2 + 138*a^2*c^16*d^25*e^4 + 538*a^3*c^15*d^23*e^6 + 1243
*a^4*c^14*d^21*e^8 + 1617*a^5*c^13*d^19*e^10 + 528*a^6*c^12*d^17*e^12 - 224
4*a^7*c^11*d^15*e^14 - 5049*a^8*c^10*d^13*e^16 - 5819*a^9*c^9*d^11*e^18 - 4
334*a^10*c^8*d^9*e^20 - 2166*a^11*c^7*d^7*e^22 - 707*a^12*c^6*d^5*e^24 - 13
7*a^13*c^5*d^3*e^26 - 12*a^14*c^4*d*e^28)*x/(a^2*c^17*d^32 + 16*a^3*c^16*d^
30*e^2 + 120*a^4*c^15*d^28*e^4 + 560*a^5*c^14*d^26*e^6 + 1820*a^6*c^13*d^24
*e^8 + 4368*a^7*c^12*d^22*e^10 + 8008*a^8*c^11*d^20*e^12 + 11440*a^9*c^10*d
^18*e^14 + 12870*a^10*c^9*d^16*e^16 + 11440*a^11*c^8*d^14*e^18 + 8008*a^12*
c^7*d^12*e^20 + 4368*a^13*c^6*d^10*e^22 + 1820*a^14*c^5*d^8*e^24 + 560*a^15
*c^4*d^6*e^26 + 120*a^16*c^3*d^4*e^28 + 16*a^17*c^2*d^2*e^30 + a^18*c*e^32)
 + 3*(5*a^2*c^16*d^26*e^3 + 59*a^3*c^15*d^24*e^5 + 318*a^4*c^14*d^22*e^7 +
1034*a^5*c^13*d^20*e^9 + 2255*a^6*c^12*d^18*e^11 + 3465*a^7*c^11*d^16*e^13
+ 3828*a^8*c^10*d^14*e^15 + 3036*a^9*c^9*d^12*e^17 + 1683*a^10*c^8*d^10*e^1
9 + 605*a^11*c^7*d^8*e^21 + 110*a^12*c^6*d^6*e^23 - 6*a^13*c^5*d^4*e^25 - 7
*a^14*c^4*d^2*e^27 - a^15*c^3*e^29)/(a^2*c^17*d^32 + 16*a^3*c^16*d^30*e^2 +
 120*a^4*c^15*d^28*e^4 + 560*a^5*c^14*d^26*e^6 + 1820*a^6*c^13*d^24*e^8 + 4
368*a^7*c^12*d^22*e^10 + 8008*a^8*c^11*d^20*e^12 + 11440*a^9*c^10*d^18*e^14
 + 12870*a^10*c^9*d^16*e^16 + 11440*a^11*c^8*d^14*e^18 + 8008*a^12*c^7*d^12
*e^20 + 4368*a^13*c^6*d^10*e^22 + 1820*a^14*c^5*d^8*e^24 + 560*a^15*c^4*d^6
*e^26 + 120*a^16*c^3*d^4*e^28 + 16*a^17*c^2*d^2*e^30 + a^18*c*e^32))*x + 3*
(a*c^17*d^29 + 16*a^2*c^16*d^27*e^2 + 105*a^3*c^15*d^25*e^4 + 376*a^4*c^14*
d^23*e^6 + 781*a^5*c^13*d^21*e^8 + 792*a^6*c^12*d^19*e^10 - 363*a^7*c^11*d^
17*e^12 - 2640*a^8*c^10*d^15*e^14 - 4653*a^9*c^9*d^13*e^16 - 4928*a^10*c^8*
d^11*e^18 - 3509*a^11*c^7*d^9*e^20 - 1704*a^12*c^6*d^7*e^22 - 545*a^13*c^5*
d^5*e^24 - 104*a^14*c^4*d^3*e^26 - 9*a^15*c^3*d*e^28)/(a^2*c^17*d^32 + 16*a
^3*c^16*d^30*e^2 + 120*a^4*c^15*d^28*e^4 + 560*a^5*c^14*d^26*e^6 + 1820*a^6
```

```
*c^13*d^24*e^8 + 4368*a^7*c^12*d^22*e^10 + 8008*a^8*c^11*d^20*e^12 + 11440*
a^9*c^10*d^18*e^14 + 12870*a^10*c^9*d^16*e^16 + 11440*a^11*c^8*d^14*e^18 +
8008*a^12*c^7*d^12*e^20 + 4368*a^13*c^6*d^10*e^22 + 1820*a^14*c^5*d^8*e^24
+ 560*a^15*c^4*d^6*e^26 + 120*a^16*c^3*d^4*e^28 + 16*a^17*c^2*d^2*e^30 + a^
18*c*e^32))*x + (3*a^2*c^16*d^28*e + 68*a^3*c^15*d^26*e^3 + 575*a^4*c^14*d^
24*e^5 + 2688*a^5*c^13*d^22*e^7 + 8063*a^6*c^12*d^20*e^9 + 16676*a^7*c^11*d
^18*e^11 + 24651*a^8*c^10*d^16*e^13 + 26400*a^9*c^9*d^14*e^15 + 20361*a^10*
c^8*d^12*e^17 + 10956*a^11*c^7*d^10*e^19 + 3773*a^12*c^6*d^8*e^21 + 608*a^1
3*c^5*d^6*e^23 - 75*a^14*c^4*d^4*e^25 - 52*a^15*c^3*d^2*e^27 - 7*a^16*c^2*e
^29)/(a^2*c^17*d^32 + 16*a^3*c^16*d^30*e^2 + 120*a^4*c^15*d^28*e^4 + 560*a^
5*c^14*d^26*e^6 + 1820*a^6*c^13*d^24*e^8 + 4368*a^7*c^12*d^22*e^10 + 8008*a
^8*c^11*d^20*e^12 + 11440*a^9*c^10*d^18*e^14 + 12870*a^10*c^9*d^16*e^16 + 1
1440*a^11*c^8*d^14*e^18 + 8008*a^12*c^7*d^12*e^20 + 4368*a^13*c^6*d^10*e^22
 + 1820*a^14*c^5*d^8*e^24 + 560*a^15*c^4*d^6*e^26 + 120*a^16*c^3*d^4*e^28 +
 16*a^17*c^2*d^2*e^30 + a^18*c*e^32))/(c*x^2 + a)^(3/2) - (22*(sqrt(c)*x -
sqrt(c*x^2 + a))^2*c^(5/2)*d^3*e^4 + 10*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2
*d^2*e^5 - 34*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*d^2*e^5 - 11*(sqrt(c)*x -
 sqrt(c*x^2 + a))^2*a*c^(3/2)*d*e^6 - (sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*e
^7 + 11*a^2*c^(3/2)*d*e^6 - (sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*e^7)/((c^4*
d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)*((sq
rt(c)*x - sqrt(c*x^2 + a))^2*e + 2*(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*d
- a*e)^2)
```

3.585 $\int \frac{3+x}{\sqrt{1-x^2}} \, dx$

Optimal. Leaf size=18

$$3 \sin^{-1}(x) - \sqrt{1 - x^2}$$

```
[Out] -Sqrt[1 - x^2] + 3*ArcSin[x]
```

Rubi [A] time = 0.0049978, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {641, 216}

$$3 \sin^{-1}(x) - \sqrt{1 - x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + x)/Sqrt[1 - x^2],x]

[Out] -Sqrt[1 - x^2] + 3*ArcSin[x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned} \int \frac{3 + x}{\sqrt{1 - x^2}} \, dx &= -\sqrt{1 - x^2} + 3 \int \frac{1}{\sqrt{1 - x^2}} \, dx \\ &= -\sqrt{1 - x^2} + 3 \sin^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.0107118, size = 18, normalized size = 1.

$$3 \sin^{-1}(x) - \sqrt{1 - x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x)/Sqrt[1 - x^2],x]

[Out] -Sqrt[1 - x^2] + 3*ArcSin[x]
```

Maple [A] time = 0.043, size = 17, normalized size = 0.9

$$3 \arcsin(x) - \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+x)/(-x^2+1)^(1/2),x)
```

```
[Out] 3*arcsin(x)-(-x^2+1)^(1/2)
```

Maxima [A] time = 1.74777, size = 22, normalized size = 1.22

$$-\sqrt{-x^2+1}+3\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1) + 3*arcsin(x)
```

Fricas [A] time = 1.72906, size = 70, normalized size = 3.89

$$-\sqrt{-x^2+1}-6\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 + 1) - 6*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] time = 0.136026, size = 12, normalized size = 0.67

$$-\sqrt{1-x^2}+3\operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(-x**2+1)**(1/2),x)
```

```
[Out] -sqrt(1 - x**2) + 3*asin(x)
```

Giac [A] time = 1.38096, size = 22, normalized size = 1.22

$$-\sqrt{-x^2+1}+3\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 1) + 3*arcsin(x)
```

3.586 $\int \frac{1+x}{\sqrt{4-x^2}} \, dx$

Optimal. Leaf size=20

$$\sin^{-1}\left(\frac{x}{2}\right) - \sqrt{4-x^2}$$

```
[Out] -Sqrt[4 - x^2] + ArcSin[x/2]
```

Rubi [A] time = 0.0048059, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {641, 216}

$$\sin^{-1}\left(\frac{x}{2}\right) - \sqrt{4-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/Sqrt[4 - x^2],x]
```

```
[Out] -Sqrt[4 - x^2] + ArcSin[x/2]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{\sqrt{4-x^2}} \, dx &= -\sqrt{4-x^2} + \int \frac{1}{\sqrt{4-x^2}} \, dx \\ &= -\sqrt{4-x^2} + \sin^{-1}\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.011744, size = 20, normalized size = 1.

$$\sin^{-1}\left(\frac{x}{2}\right) - \sqrt{4-x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/Sqrt[4 - x^2],x]
```

```
[Out] -Sqrt[4 - x^2] + ArcSin[x/2]
```

Maple [A] time = 0.045, size = 17, normalized size = 0.9

$$\arcsin\left(\frac{x}{2}\right)-\sqrt{-x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-x^2+4)^(1/2),x)
```

```
[Out] arcsin(1/2*x)-(-x^2+4)^(1/2)
```

Maxima [A] time = 1.72742, size = 22, normalized size = 1.1

$$-\sqrt{-x^2+4}+\arcsin\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 4) + arcsin(1/2*x)
```

Fricas [A] time = 1.82944, size = 70, normalized size = 3.5

$$-\sqrt{-x^2+4}-2\,\arctan\left(\frac{\sqrt{-x^2+4}-2}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 + 4) - 2*arctan((sqrt(-x^2 + 4) - 2)/x)
```

Sympy [A] time = 0.144732, size = 12, normalized size = 0.6

$$-\sqrt{4-x^2}+\operatorname{asin}\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**2+4)**(1/2),x)
```

```
[Out] -sqrt(4 - x**2) + asin(x/2)
```

Giac [A] time = 1.68049, size = 22, normalized size = 1.1

$$-\sqrt{-x^2+4}+\arcsin\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 4) + arcsin(1/2*x)
```

3.587 $\int \frac{2+x}{\sqrt{9+x^2}} dx$

Optimal. Leaf size=18

$$\sqrt{x^2+9}+2\sinh^{-1}\left(\frac{x}{3}\right)$$

```
[Out] Sqrt[9 + x^2] + 2*ArcSinh[x/3]
```

Rubi [A] time = 0.0044369, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {641, 215}

$$\sqrt{x^2+9}+2\sinh^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/Sqrt[9 + x^2],x]
```

```
[Out] Sqrt[9 + x^2] + 2*ArcSinh[x/3]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x}{\sqrt{9+x^2}} dx &= \sqrt{9+x^2}+2\int \frac{1}{\sqrt{9+x^2}} dx\\ &= \sqrt{9+x^2}+2\sinh^{-1}\left(\frac{x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.0103785, size = 18, normalized size = 1.

$$\sqrt{x^2+9}+2\sinh^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/Sqrt[9 + x^2],x]
```

```
[Out] Sqrt[9 + x^2] + 2*ArcSinh[x/3]
```

Maple [A] time = 0.042, size = 15, normalized size = 0.8

$$2\,\mathrm{Arcsinh}\,(x/3) + \sqrt{x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(x^2+9)^(1/2),x)
```

```
[Out] 2*arcsinh(1/3*x)+(x^2+9)^(1/2)
```

Maxima [A] time = 1.98458, size = 19, normalized size = 1.06

$$\sqrt{x^2+9} + 2\,\mathrm{arsinh}\left(\frac{1}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 + 9) + 2*arcsinh(1/3*x)
```

Fricas [A] time = 1.73403, size = 58, normalized size = 3.22

$$\sqrt{x^2+9} - 2\,\log\left(-x + \sqrt{x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x^2 + 9) - 2*log(-x + sqrt(x^2 + 9))
```

Sympy [A] time = 0.143122, size = 14, normalized size = 0.78

$$\sqrt{x^2+9} + 2\,\mathrm{asinh}\left(\frac{x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x**2+9)**(1/2),x)
```

```
[Out] sqrt(x**2 + 9) + 2*asinh(x/3)
```

Giac [A] time = 2.184, size = 30, normalized size = 1.67

$$\sqrt{x^2+9} - 2\,\log\left(-x + \sqrt{x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x^2 + 9) - 2*log(-x + sqrt(x^2 + 9))
```

3.588 $\int \frac{(a+bx)^2}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=54

$$\frac{1}{2}\left(2a^2+b^2\right)\sin^{-1}(x)-\frac{3}{2}ab\sqrt{1-x^2}-\frac{1}{2}b\sqrt{1-x^2}(a+bx)$$

```
[Out] (-3*a*b*Sqrt[1 - x^2])/2 - (b*(a + b*x)*Sqrt[1 - x^2])/2 + ((2*a^2 + b^2)*A
rcSin[x])/2
```

Rubi [A] time = 0.023162, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {743, 641, 216}

$$\frac{1}{2}\left(2a^2+b^2\right)\sin^{-1}(x)-\frac{3}{2}ab\sqrt{1-x^2}-\frac{1}{2}b\sqrt{1-x^2}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/Sqrt[1 - x^2],x]
```

```
[Out] (-3*a*b*Sqrt[1 - x^2])/2 - (b*(a + b*x)*Sqrt[1 - x^2])/2 + ((2*a^2 + b^2)*A
rcSin[x])/2
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{\sqrt{1-x^2}}\,dx &= -\frac{1}{2}b(a+bx)\sqrt{1-x^2}-\frac{1}{2}\int \frac{-2a^2-b^2-3abx}{\sqrt{1-x^2}}\,dx\\ &= -\frac{3}{2}ab\sqrt{1-x^2}-\frac{1}{2}b(a+bx)\sqrt{1-x^2}-\frac{1}{2}\left(-2a^2-b^2\right)\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= -\frac{3}{2}ab\sqrt{1-x^2}-\frac{1}{2}b(a+bx)\sqrt{1-x^2}+\frac{1}{2}\left(2a^2+b^2\right)\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0322044, size = 38, normalized size = 0.7

$$\frac{1}{2}\left(\left(2a^2+b^2\right)\sin^{-1}(x)-b\sqrt{1-x^2}(4a+bx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/Sqrt[1 - x^2],x]
```

```
[Out] (-(b*(4*a + b*x)*Sqrt[1 - x^2]) + (2*a^2 + b^2)*ArcSin[x])/2
```

Maple [A] time = 0.046, size = 42, normalized size = 0.8

$$b^2\left(-\frac{x}{2}\sqrt{-x^2+1}+\frac{\arcsin(x)}{2}\right)-2\,ab\sqrt{-x^2+1}+a^2\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-x^2+1)^(1/2),x)
```

```
[Out] b^2*(-1/2*x*(-x^2+1)^(1/2)+1/2*arcsin(x))-2*a*b*(-x^2+1)^(1/2)+a^2*arcsin(x
)
```

Maxima [A] time = 1.75586, size = 57, normalized size = 1.06

$$-\frac{1}{2}\sqrt{-x^2+1}b^2x+a^2\arcsin(x)+\frac{1}{2}b^2\arcsin(x)-2\sqrt{-x^2+1}ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-x^2 + 1)*b^2*x + a^2*arcsin(x) + 1/2*b^2*arcsin(x) - 2*sqrt(-x^2
 + 1)*a*b
```

Fricas [A] time = 1.85792, size = 113, normalized size = 2.09

$$-\left(2\,a^2+b^2\right)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\frac{1}{2}\left(b^2x+4\,ab\right)\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*a^2 + b^2)*arctan((sqrt(-x^2 + 1) - 1)/x) - 1/2*(b^2*x + 4*a*b)*sqrt(-x
^2 + 1)
```

Sympy [A] time = 0.244047, size = 42, normalized size = 0.78

$$a^2\operatorname{asin}(x)-2ab\sqrt{1-x^2}-\frac{b^2x\sqrt{1-x^2}}{2}+\frac{b^2\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-x**2+1)**(1/2),x)
```

```
[Out] a**2*asin(x) - 2*a*b*sqrt(1 - x**2) - b**2*x*sqrt(1 - x**2)/2 + b**2*asin(x
)/2
```

Giac [A] time = 1.76367, size = 47, normalized size = 0.87

$$\frac{1}{2}\left(2\,a^2+b^2\right)\arcsin\left(x\right)-\frac{1}{2}\left(b^2x+4\,ab\right)\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2 + b^2)*arcsin(x) - 1/2*(b^2*x + 4*a*b)*sqrt(-x^2 + 1)
```

3.589 $\int \frac{(a+bx)^2}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=52

$$\frac{1}{2}\left(2a^2-b^2\right)\sinh^{-1}(x)+\frac{3}{2}ab\sqrt{x^2+1}+\frac{1}{2}b\sqrt{x^2+1}(a+bx)$$

```
[Out] (3*a*b*Sqrt[1 + x^2])/2 + (b*(a + b*x)*Sqrt[1 + x^2])/2 + ((2*a^2 - b^2)*Ar
cSinh[x])/2
```

Rubi [A] time = 0.0203178, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {743, 641, 215}

$$\frac{1}{2}\left(2a^2-b^2\right)\sinh^{-1}(x)+\frac{3}{2}ab\sqrt{x^2+1}+\frac{1}{2}b\sqrt{x^2+1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/Sqrt[1 + x^2],x]
```

```
[Out] (3*a*b*Sqrt[1 + x^2])/2 + (b*(a + b*x)*Sqrt[1 + x^2])/2 + ((2*a^2 - b^2)*Ar
cSinh[x])/2
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{\sqrt{1+x^2}}\,dx &= \frac{1}{2}b(a+bx)\sqrt{1+x^2}+\frac{1}{2}\int \frac{2a^2-b^2+3abx}{\sqrt{1+x^2}}\,dx\\ &= \frac{3}{2}ab\sqrt{1+x^2}+\frac{1}{2}b(a+bx)\sqrt{1+x^2}+\frac{1}{2}\left(2a^2-b^2\right)\int \frac{1}{\sqrt{1+x^2}}\,dx\\ &= \frac{3}{2}ab\sqrt{1+x^2}+\frac{1}{2}b(a+bx)\sqrt{1+x^2}+\frac{1}{2}\left(2a^2-b^2\right)\sinh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0259011, size = 36, normalized size = 0.69

$$\left(a^2-\frac{b^2}{2}\right)\sinh^{-1}(x)+\frac{1}{2}b\sqrt{x^2+1}(4a+bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/Sqrt[1 + x^2],x]
```

```
[Out] (b*(4*a + b*x)*Sqrt[1 + x^2])/2 + (a^2 - b^2/2)*ArcSinh[x]
```

Maple [A] time = 0.045, size = 38, normalized size = 0.7

$$b^2\left(\frac{x}{2}\sqrt{x^2+1}-\frac{\operatorname{Arcsinh}(x)}{2}\right)+2\,ab\sqrt{x^2+1}+a^2\operatorname{Arcsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(x^2+1)^(1/2),x)
```

```
[Out] b^2*(1/2*x*(x^2+1)^(1/2)-1/2*arcsinh(x))+2*a*b*(x^2+1)^(1/2)+a^2*arcsinh(x)
```

Maxima [A] time = 1.6898, size = 51, normalized size = 0.98

$$\frac{1}{2}\sqrt{x^2+1}b^2x+a^2\operatorname{arsinh}(x)-\frac{1}{2}b^2\operatorname{arsinh}(x)+2\sqrt{x^2+1}ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x^2 + 1)*b^2*x + a^2*arcsinh(x) - 1/2*b^2*arcsinh(x) + 2*sqrt(x^2
+ 1)*a*b
```

Fricas [A] time = 1.81245, size = 108, normalized size = 2.08

$$-\frac{1}{2}\left(2\,a^2-b^2\right)\log\left(-x+\sqrt{x^2+1}\right)+\frac{1}{2}\left(b^2x+4\,ab\right)\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2 - b^2)*log(-x + sqrt(x^2 + 1)) + 1/2*(b^2*x + 4*a*b)*sqrt(x^2 +
 1)
```

Sympy [A] time = 0.251236, size = 42, normalized size = 0.81

$$a^2\operatorname{asinh}(x)+2ab\sqrt{x^2+1}+\frac{b^2x\sqrt{x^2+1}}{2}-\frac{b^2\operatorname{asinh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(x**2+1)**(1/2),x)
```

```
[Out] a**2*asinh(x) + 2*a*b*sqrt(x**2 + 1) + b**2*x*sqrt(x**2 + 1)/2 - b**2*asinh
(x)/2
```

Giac [A] time = 1.50868, size = 61, normalized size = 1.17

$$-\frac{1}{2}\left(2\,a^2-b^2\right)\log\left(-x+\sqrt{x^2+1}\right)+\frac{1}{2}\left(b^2x+4\,ab\right)\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*a^2 - b^2)*log(-x + sqrt(x^2 + 1)) + 1/2*(b^2*x + 4*a*b)*sqrt(x^2 +
 1)
```

3.590 $\int \frac{2+3x}{(4+x^2)^{3/2}} \, dx$

Optimal. Leaf size=18

$$-\frac{6-x}{2\sqrt{x^2+4}}$$

```
[Out] -(6 - x)/(2*Sqrt[4 + x^2])
```

Rubi [A] time = 0.0040183, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {637}

$$-\frac{6-x}{2\sqrt{x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)/(4 + x^2)^(3/2),x]
```

```
[Out] -(6 - x)/(2*Sqrt[4 + x^2])
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{2+3x}{(4+x^2)^{3/2}} \, dx = -\frac{6-x}{2\sqrt{4+x^2}}$$

Mathematica [A] time = 0.0091034, size = 16, normalized size = 0.89

$$\frac{x-6}{2\sqrt{x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)/(4 + x^2)^(3/2),x]
```

```
[Out] (-6 + x)/(2*Sqrt[4 + x^2])
```

Maple [A] time = 0.039, size = 13, normalized size = 0.7

$$\frac{x-6}{2} \frac{1}{\sqrt{x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)/(x^2+4)^(3/2),x)

[Out] 1/2*(x-6)/(x^2+4)^(1/2)
```

Maxima [A] time = 1.20106, size = 27, normalized size = 1.5

$$\frac{x}{2\sqrt{x^2+4}} - \frac{3}{\sqrt{x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(x^2+4)^(3/2),x, algorithm="maxima")

[Out] 1/2*x/sqrt(x^2 + 4) - 3/sqrt(x^2 + 4)
```

Fricas [B] time = 1.80335, size = 66, normalized size = 3.67

$$\frac{x^2 + \sqrt{x^2+4}(x-6) + 4}{2\left(x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(x^2+4)^(3/2),x, algorithm="fricas")

[Out] 1/2*(x^2 + sqrt(x^2 + 4)*(x - 6) + 4)/(x^2 + 4)
```

Sympy [A] time = 2.66411, size = 20, normalized size = 1.11

$$\frac{x}{2\sqrt{x^2+4}} - \frac{3}{\sqrt{x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(x**2+4)**(3/2),x)

[Out] x/(2*sqrt(x**2 + 4)) - 3/sqrt(x**2 + 4)
```

Giac [A] time = 2.20806, size = 16, normalized size = 0.89

$$\frac{x-6}{2\sqrt{x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(x^2+4)^(3/2),x, algorithm="giac")

[Out] 1/2*(x - 6)/sqrt(x^2 + 4)
```

3.591 $\int (d+ex)^{5/2}\left(a+cx^2\right)\,dx$

Optimal. Leaf size=63

$$\frac{2(d+ex)^{7/2}\left(ae^2+cd^2\right)}{7e^3}+\frac{2c(d+ex)^{11/2}}{11e^3}-\frac{4cd(d+ex)^{9/2}}{9e^3}$$

```
[Out] (2*(c*d^2 + a*e^2)*(d + e*x)^(7/2))/(7*e^3) - (4*c*d*(d + e*x)^(9/2))/(9*e^
3) + (2*c*(d + e*x)^(11/2))/(11*e^3)
```

Rubi [A] time = 0.0238877, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$\frac{2(d+ex)^{7/2}\left(ae^2+cd^2\right)}{7e^3}+\frac{2c(d+ex)^{11/2}}{11e^3}-\frac{4cd(d+ex)^{9/2}}{9e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)*(a + c*x^2),x]
```

```
[Out] (2*(c*d^2 + a*e^2)*(d + e*x)^(7/2))/(7*e^3) - (4*c*d*(d + e*x)^(9/2))/(9*e^
3) + (2*c*(d + e*x)^(11/2))/(11*e^3)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^{5/2}\left(a+cx^2\right)\,dx &= \int \left(\frac{\left(cd^2+ae^2\right)(d+ex)^{5/2}}{e^2}-\frac{2cd(d+ex)^{7/2}}{e^2}+\frac{c(d+ex)^{9/2}}{e^2}\right)dx\\ &= \frac{2\left(cd^2+ae^2\right)(d+ex)^{7/2}}{7e^3}-\frac{4cd(d+ex)^{9/2}}{9e^3}+\frac{2c(d+ex)^{11/2}}{11e^3}\end{aligned}$$

Mathematica [A] time = 0.0491549, size = 44, normalized size = 0.7

$$\frac{2(d+ex)^{7/2}\left(99ae^2+c\left(8d^2-28dex+63e^2x^2\right)\right)}{693e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)*(a + c*x^2),x]
```

```
[Out] (2*(d + e*x)^(7/2)*(99*a*e^2 + c*(8*d^2 - 28*d*e*x + 63*e^2*x^2)))/(693*e^3
)
```

Maple [A] time = 0.043, size = 41, normalized size = 0.7

$$\frac{126\,c e^{2} x^{2}-56\,c d e x+198\,a e^{2}+16\,c d^{2}}{693\,e^{3}}\,(e x+d)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(c*x^2+a),x)
```

```
[Out] 2/693*(e*x+d)^(7/2)*(63*c*e^2*x^2-28*c*d*e*x+99*a*e^2+8*c*d^2)/e^3
```

Maxima [A] time = 1.15476, size = 63, normalized size = 1.

$$\frac{2\left(63\,(e x+d)^{\frac{11}{2}} c-154\,(e x+d)^{\frac{9}{2}} c d+99\left(c d^{2}+a e^{2}\right)(e x+d)^{\frac{7}{2}}\right)}{693\,e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/693*(63*(e*x + d)^(11/2)*c - 154*(e*x + d)^(9/2)*c*d + 99*(c*d^2 + a*e^2)
*(e*x + d)^(7/2))/e^3
```

Fricas [B] time = 1.91704, size = 244, normalized size = 3.87

$$\frac{2\left(63\,c e^{5} x^{5}+161\,c d e^{4} x^{4}+8\,c d^{5}+99\,a d^{3} e^{2}+\left(113\,c d^{2} e^{3}+99\,a e^{5}\right)x^{3}+3\left(c d^{3} e^{2}+99\,a d e^{4}\right)x^{2}-\left(4\,c d^{4} e-297\,a d^{2} e^{3}\right)x\right)\sqrt{ex}}{693\,e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(c*x^2+a),x, algorithm="fricas")
```

```
[Out] 2/693*(63*c*e^5*x^5 + 161*c*d*e^4*x^4 + 8*c*d^5 + 99*a*d^3*e^2 + (113*c*d^2
*e^3 + 99*a*e^5)*x^3 + 3*(c*d^3*e^2 + 99*a*d*e^4)*x^2 - (4*c*d^4*e - 297*a*
d^2*e^3)*x)*sqrt(e*x + d)/e^3
```

Sympy [A] time = 3.19168, size = 218, normalized size = 3.46

$$\begin{cases} \frac{2ad^{3}\sqrt{d+ex}}{7e}+\frac{6ad^{2}x\sqrt{d+ex}}{7}+\frac{6adex^{2}\sqrt{d+ex}}{7}+\frac{2ae^{2}x^{3}\sqrt{d+ex}}{7}+\frac{16cd^{5}\sqrt{d+ex}}{693e^{3}}-\frac{8cd^{4}x\sqrt{d+ex}}{693e^{2}}+\frac{2cd^{3}x^{2}\sqrt{d+ex}}{231e}+\frac{226cd^{2}x^{3}\sqrt{d+ex}}{693}+\frac{46cdex^{4}\sqrt{d+ex}}{99}+ \\ d^{\frac{5}{2}}\left(ax+\frac{cx^{3}}{3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(c*x**2+a),x)
```

```
[Out] Piecewise((2*a*d**3*sqrt(d + e*x)/(7*e) + 6*a*d**2*x*sqrt(d + e*x)/7 + 6*a*
d*e*x**2*sqrt(d + e*x)/7 + 2*a*e**2*x**3*sqrt(d + e*x)/7 + 16*c*d**5*sqrt(d
 + e*x)/(693*e**3) - 8*c*d**4*x*sqrt(d + e*x)/(693*e**2) + 2*c*d**3*x**2*sq
rt(d + e*x)/(231*e) + 226*c*d**2*x**3*sqrt(d + e*x)/693 + 46*c*d*e*x**4*sqr
t(d + e*x)/99 + 2*c*e**2*x**5*sqrt(d + e*x)/11, Ne(e, 0)), (d**(5/2)*(a*x +
```

```
c*x**3/3), True))
```

Giac [B] time = 1.40527, size = 331, normalized size = 5.25

$$\frac{2}{3465}\left(33\left(15\,(xe+d)^{\frac{7}{2}}-42\,(xe+d)^{\frac{5}{2}}d+35\,(xe+d)^{\frac{3}{2}}d^2\right)cd^2e^{(-2)}+1155\,(xe+d)^{\frac{3}{2}}ad^2+22\left(35\,(xe+d)^{\frac{9}{2}}-135\,(xe+d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(c*x^2+a),x, algorithm="giac")
```

```
[Out] 2/3465*(33*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*
d^2)*c*d^2*e^(-2) + 1155*(x*e + d)^(3/2)*a*d^2 + 22*(35*(x*e + d)^(9/2) - 1
35*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*c
*d*e^(-2) + 462*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a*d + (315*(x*e +
 d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e
+ d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*c*e^(-2) + 33*(15*(x*e + d)^(7/2
) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*a)*e^(-1)
```

3.592 $\int (d+ex)^{3/2} \left(a+cx^2\right) dx$

Optimal. Leaf size=63

$$\frac{2(d+ex)^{5/2}\left(ae^2+cd^2\right)}{5e^3}+\frac{2c(d+ex)^{9/2}}{9e^3}-\frac{4cd(d+ex)^{7/2}}{7e^3}$$

```
[Out] (2*(c*d^2 + a*e^2)*(d + e*x)^(5/2))/(5*e^3) - (4*c*d*(d + e*x)^(7/2))/(7*e^
3) + (2*c*(d + e*x)^(9/2))/(9*e^3)
```

Rubi [A] time = 0.0210846, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$\frac{2(d+ex)^{5/2}\left(ae^2+cd^2\right)}{5e^3}+\frac{2c(d+ex)^{9/2}}{9e^3}-\frac{4cd(d+ex)^{7/2}}{7e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*(a + c*x^2),x]
```

```
[Out] (2*(c*d^2 + a*e^2)*(d + e*x)^(5/2))/(5*e^3) - (4*c*d*(d + e*x)^(7/2))/(7*e^
3) + (2*c*(d + e*x)^(9/2))/(9*e^3)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^{3/2}\left(a+cx^2\right)dx &= \int \left(\frac{\left(cd^2+ae^2\right)(d+ex)^{3/2}}{e^2}-\frac{2cd(d+ex)^{5/2}}{e^2}+\frac{c(d+ex)^{7/2}}{e^2}\right)dx\\ &= \frac{2\left(cd^2+ae^2\right)(d+ex)^{5/2}}{5e^3}-\frac{4cd(d+ex)^{7/2}}{7e^3}+\frac{2c(d+ex)^{9/2}}{9e^3}\end{aligned}$$

Mathematica [A] time = 0.0370441, size = 44, normalized size = 0.7

$$\frac{2(d+ex)^{5/2}\left(63ae^2+c\left(8d^2-20dex+35e^2x^2\right)\right)}{315e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)*(a + c*x^2),x]
```

```
[Out] (2*(d + e*x)^(5/2)*(63*a*e^2 + c*(8*d^2 - 20*d*e*x + 35*e^2*x^2)))/(315*e^3
)
```

Maple [A] time = 0.043, size = 41, normalized size = 0.7

$$\frac{70\,c e^{2} x^{2}-40\,c d e x+126\,a e^{2}+16\,c d^{2}}{315\,e^{3}}\left(e x+d\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(c*x^2+a),x)
```

```
[Out] 2/315*(e*x+d)^(5/2)*(35*c*e^2*x^2-20*c*d*e*x+63*a*e^2+8*c*d^2)/e^3
```

Maxima [A] time = 1.19216, size = 63, normalized size = 1.

$$\frac{2\left(35\left(e x+d\right)^{\frac{9}{2}}c-90\left(e x+d\right)^{\frac{7}{2}}c d+63\left(c d^{2}+a e^{2}\right)\left(e x+d\right)^{\frac{5}{2}}\right)}{315\,e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/315*(35*(e*x + d)^(9/2)*c - 90*(e*x + d)^(7/2)*c*d + 63*(c*d^2 + a*e^2)*(
e*x + d)^(5/2))/e^3
```

Fricas [A] time = 1.71276, size = 194, normalized size = 3.08

$$\frac{2\left(35\,c e^{4} x^{4}+50\,c d e^{3} x^{3}+8\,c d^{4}+63\,a d^{2} e^{2}+3\left(c d^{2} e^{2}+21\,a e^{4}\right)x^{2}-2\left(2\,c d^{3} e-63\,a d e^{3}\right)x\right)\sqrt{e x+d}}{315\,e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+a),x, algorithm="fricas")
```

```
[Out] 2/315*(35*c*e^4*x^4 + 50*c*d*e^3*x^3 + 8*c*d^4 + 63*a*d^2*e^2 + 3*(c*d^2*e^
2 + 21*a*e^4)*x^2 - 2*(2*c*d^3*e - 63*a*d*e^3)*x)*sqrt(e*x + d)/e^3
```

Sympy [A] time = 5.60879, size = 155, normalized size = 2.46

$$ad\left(\begin{cases}\sqrt{d}x & \text{for}\: e=0\\ \frac{2(d+ex)^{\frac{3}{2}}}{3e} & \text{otherwise}\end{cases}\right)+\frac{2a\left(-\frac{d(d+ex)^{\frac{3}{2}}}{3}+\frac{(d+ex)^{\frac{5}{2}}}{5}\right)}{e}+\frac{2cd\left(\frac{d^{2}(d+ex)^{\frac{3}{2}}}{3}-\frac{2d(d+ex)^{\frac{5}{2}}}{5}+\frac{(d+ex)^{\frac{7}{2}}}{7}\right)}{e^{3}}+\frac{2c\left(-\frac{d^{3}(d+ex)^{\frac{3}{2}}}{3}+\frac{3d^{2}(d+ex)^{\frac{5}{2}}}{5}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(c*x**2+a),x)
```

```
[Out] a*d*Piecewise((sqrt(d)*x, Eq(e, 0)), (2*(d + e*x)**(3/2)/(3*e), True)) + 2*
a*(-d*(d + e*x)**(3/2)/3 + (d + e*x)**(5/2)/5)/e + 2*c*d*(d**2*(d + e*x)**(
3/2)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e**3 + 2*c*(-d**3*(d
+ e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d + e*x)**(7/2)/7 + (d +
```

```
e*x)**(9/2)/9)/e**3
```

Giac [B] time = 1.41944, size = 182, normalized size = 2.89

$$\frac{2}{315}\left(3\left(15\,(xe+d)^{\frac{7}{2}}-42\,(xe+d)^{\frac{5}{2}}d+35\,(xe+d)^{\frac{3}{2}}d^{2}\right)cde^{(-2)}+105\,(xe+d)^{\frac{3}{2}}ad+\left(35\,(xe+d)^{\frac{9}{2}}-135\,(xe+d)^{\frac{7}{2}}d+189\,(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+a),x, algorithm="giac")
```

```
[Out] 2/315*(3*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^
2)*c*d*e^(-2) + 105*(x*e + d)^(3/2)*a*d + (35*(x*e + d)^(9/2) - 135*(x*e +
d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*c*e^(-2) +
21*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a)*e^(-1)
```

3.593 $\int \sqrt{d+ex}\left(a+cx^2\right) dx$

Optimal. Leaf size=63

$$\frac{2(d+ex)^{3/2}\left(ae^2+cd^2\right)}{3e^3}+\frac{2c(d+ex)^{7/2}}{7e^3}-\frac{4cd(d+ex)^{5/2}}{5e^3}$$

```
[Out] (2*(c*d^2 + a*e^2)*(d + e*x)^(3/2))/(3*e^3) - (4*c*d*(d + e*x)^(5/2))/(5*e^
3) + (2*c*(d + e*x)^(7/2))/(7*e^3)
```

Rubi [A] time = 0.0207032, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$\frac{2(d+ex)^{3/2}\left(ae^2+cd^2\right)}{3e^3}+\frac{2c(d+ex)^{7/2}}{7e^3}-\frac{4cd(d+ex)^{5/2}}{5e^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(a + c*x^2),x]
```

```
[Out] (2*(c*d^2 + a*e^2)*(d + e*x)^(3/2))/(3*e^3) - (4*c*d*(d + e*x)^(5/2))/(5*e^
3) + (2*c*(d + e*x)^(7/2))/(7*e^3)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{d+ex}\left(a+cx^2\right) dx &= \int \left(\frac{\left(cd^2+ae^2\right)\sqrt{d+ex}}{e^2}-\frac{2cd(d+ex)^{3/2}}{e^2}+\frac{c(d+ex)^{5/2}}{e^2}\right) dx\\ &= \frac{2\left(cd^2+ae^2\right)(d+ex)^{3/2}}{3e^3}-\frac{4cd(d+ex)^{5/2}}{5e^3}+\frac{2c(d+ex)^{7/2}}{7e^3}\end{aligned}$$

Mathematica [A] time = 0.0310447, size = 44, normalized size = 0.7

$$\frac{2(d+ex)^{3/2}\left(35ae^2+c\left(8d^2-12dex+15e^2x^2\right)\right)}{105e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*(a + c*x^2),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(35*a*e^2 + c*(8*d^2 - 12*d*e*x + 15*e^2*x^2)))/(105*e^3
)
```

Maple [A] time = 0.042, size = 41, normalized size = 0.7

$$\frac{30\,c e^{2} x^{2}-24\,c d e x+70\,a e^{2}+16\,c d^{2}}{105\,e^{3}}\,(e x+d)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(e*x+d)^(1/2),x)
```

```
[Out] 2/105*(e*x+d)^(3/2)*(15*c*e^2*x^2-12*c*d*e*x+35*a*e^2+8*c*d^2)/e^3
```

Maxima [A] time = 1.12747, size = 63, normalized size = 1.

$$\frac{2\left(15\,(e x+d)^{\frac{7}{2}} c-42\,(e x+d)^{\frac{5}{2}} c d+35\left(c d^{2}+a e^{2}\right)(e x+d)^{\frac{3}{2}}\right)}{105\,e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*(e*x + d)^(7/2)*c - 42*(e*x + d)^(5/2)*c*d + 35*(c*d^2 + a*e^2)*(
e*x + d)^(3/2))/e^3
```

Fricas [A] time = 1.66208, size = 143, normalized size = 2.27

$$\frac{2\left(15\,c e^{3} x^{3}+3\,c d e^{2} x^{2}+8\,c d^{3}+35\,a d e^{2}-\left(4\,c d^{2} e-35\,a e^{3}\right) x\right)\sqrt{e x+d}}{105\,e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*c*e^3*x^3 + 3*c*d*e^2*x^2 + 8*c*d^3 + 35*a*d*e^2 - (4*c*d^2*e - 3
5*a*e^3)*x)*sqrt(e*x + d)/e^3
```

Sympy [A] time = 1.73479, size = 61, normalized size = 0.97

$$\frac{2\left(-\frac{2cd(d+ex)^{\frac{5}{2}}}{5e^{2}}+\frac{c(d+ex)^{\frac{7}{2}}}{7e^{2}}+\frac{(d+ex)^{\frac{3}{2}}\left(ae^{2}+cd^{2}\right)}{3e^{2}}\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(e*x+d)**(1/2),x)
```

```
[Out] 2*(-2*c*d*(d + e*x)**(5/2)/(5*e**2) + c*(d + e*x)**(7/2)/(7*e**2) + (d + e*
x)**(3/2)*(a*e**2 + c*d**2)/(3*e**2))/e
```

<u>Giac</u> [A] time = 1.32537, size = 74, normalized size = 1.17

$$\frac{2}{105}\left(\left(15\left(xe+d\right)^{\frac{7}{2}}-42\left(xe+d\right)^{\frac{5}{2}}d+35\left(xe+d\right)^{\frac{3}{2}}d^{2}\right)ce^{(-2)}+35\left(xe+d\right)^{\frac{3}{2}}a\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*((15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)
*c*e^(-2) + 35*(x*e + d)^(3/2)*a)*e^(-1)
```

3.594 $\int \frac{a+cx^2}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=61

$$\frac{2\sqrt{d+ex}\left(ae^2+cd^2\right)}{e^3}+\frac{2c(d+ex)^{5/2}}{5e^3}-\frac{4cd(d+ex)^{3/2}}{3e^3}$$

```
[Out] (2*(c*d^2 + a*e^2)*Sqrt[d + e*x])/e^3 - (4*c*d*(d + e*x)^(3/2))/(3*e^3) + (
2*c*(d + e*x)^(5/2))/(5*e^3)
```

Rubi [A] time = 0.0208208, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$\frac{2\sqrt{d+ex}\left(ae^2+cd^2\right)}{e^3}+\frac{2c(d+ex)^{5/2}}{5e^3}-\frac{4cd(d+ex)^{3/2}}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/Sqrt[d + e*x],x]
```

```
[Out] (2*(c*d^2 + a*e^2)*Sqrt[d + e*x])/e^3 - (4*c*d*(d + e*x)^(3/2))/(3*e^3) + (
2*c*(d + e*x)^(5/2))/(5*e^3)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+cx^2}{\sqrt{d+ex}}\,dx &= \int \left(\frac{cd^2+ae^2}{e^2\sqrt{d+ex}}-\frac{2cd\sqrt{d+ex}}{e^2}+\frac{c(d+ex)^{3/2}}{e^2}\right)dx\\ &= \frac{2\left(cd^2+ae^2\right)\sqrt{d+ex}}{e^3}-\frac{4cd(d+ex)^{3/2}}{3e^3}+\frac{2c(d+ex)^{5/2}}{5e^3}\end{aligned}$$

Mathematica [A] time = 0.0333469, size = 44, normalized size = 0.72

$$\frac{2\sqrt{d+ex}\left(15ae^2+c\left(8d^2-4dex+3e^2x^2\right)\right)}{15e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[d + e*x]*(15*a*e^2 + c*(8*d^2 - 4*d*e*x + 3*e^2*x^2)))/(15*e^3)
```

Maple [A] time = 0.04, size = 41, normalized size = 0.7

$$\frac{6 c e^2 x^2 - 8 c d e x + 30 a e^2 + 16 c d^2}{15 e^3} \sqrt{e x + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^(1/2),x)
```

```
[Out] 2/15*(e*x+d)^(1/2)*(3*c*e^2*x^2-4*c*d*e*x+15*a*e^2+8*c*d^2)/e^3
```

Maxima [A] time = 1.14683, size = 72, normalized size = 1.18

$$\frac{2 \left(15 \sqrt{e x + d} a + \frac{\left(3 (e x+d)^{\frac{5}{2}} - 10 (e x+d)^{\frac{3}{2}} d + 15 \sqrt{e x+d} d^2\right) c}{e^2}\right)}{15 e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(e*x + d)*a + (3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*s
qrt(e*x + d)*d^2)*c/e^2)/e
```

Fricas [A] time = 1.78838, size = 96, normalized size = 1.57

$$\frac{2 \left(3 c e^2 x^2 - 4 c d e x + 8 c d^2 + 15 a e^2\right) \sqrt{e x + d}}{15 e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*c*e^2*x^2 - 4*c*d*e*x + 8*c*d^2 + 15*a*e^2)*sqrt(e*x + d)/e^3
```

Sympy [A] time = 5.98915, size = 150, normalized size = 2.46

$$\begin{cases} - \frac{\frac{2ad}{\sqrt{d+ex}} + 2a\left(-\frac{d}{\sqrt{d+ex}} - \sqrt{d+ex}\right) + \frac{2cd\left(\frac{d^2}{\sqrt{d+ex}} + 2d\sqrt{d+ex} - \frac{(d+ex)^{\frac{3}{2}}}{3}\right)}{e^2} + \frac{2c\left(-\frac{d^3}{\sqrt{d+ex}} - 3d^2\sqrt{d+ex} + d(d+ex)^{\frac{3}{2}} - \frac{(d+ex)^{\frac{5}{2}}}{5}\right)}{e^2}}{e} & \text{for } e \neq 0 \\ \frac{ax + \frac{cx^3}{3}}{\sqrt{d}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**(1/2),x)
```

```
[Out] Piecewise((-(2*a*d/sqrt(d + e*x) + 2*a*(-d/sqrt(d + e*x) - sqrt(d + e*x)) +
 2*c*d*(d**2/sqrt(d + e*x) + 2*d*sqrt(d + e*x) - (d + e*x)**(3/2)/3)/e**2 +
 2*c*(-d**3/sqrt(d + e*x) - 3*d**2*sqrt(d + e*x) + d*(d + e*x)**(3/2) - (d
```

```
+ e*x)**(5/2)/5)/e**2)/e, Ne(e, 0)), ((a*x + c*x**3/3)/sqrt(d), True))
```

Giac [A] time = 1.38449, size = 74, normalized size = 1.21

$$\frac{2}{15}\left(\left(3\,(xe+d)^{\frac{5}{2}}-10\,(xe+d)^{\frac{3}{2}}d+15\,\sqrt{xe+d}d^{2}\right)ce^{(-2)}+15\,\sqrt{xe+d}a\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*((3*(x*e + d)^(5/2) - 10*(x*e + d)^(3/2)*d + 15*sqrt(x*e + d)*d^2)*c*e
^(-2) + 15*sqrt(x*e + d)*a)*e^(-1)
```

3.595 $\int \frac{a+cx^2}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=59

$$-\frac{2\left(ae^2+cd^2\right)}{e^3\sqrt{d+ex}}+\frac{2c(d+ex)^{3/2}}{3e^3}-\frac{4cd\sqrt{d+ex}}{e^3}$$

```
[Out] (-2*(c*d^2 + a*e^2))/(e^3*Sqrt[d + e*x]) - (4*c*d*Sqrt[d + e*x])/e^3 + (2*c
*(d + e*x)^(3/2))/(3*e^3)
```

Rubi [A] time = 0.0208537, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$-\frac{2\left(ae^2+cd^2\right)}{e^3\sqrt{d+ex}}+\frac{2c(d+ex)^{3/2}}{3e^3}-\frac{4cd\sqrt{d+ex}}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/(d + e*x)^(3/2),x]
```

```
[Out] (-2*(c*d^2 + a*e^2))/(e^3*Sqrt[d + e*x]) - (4*c*d*Sqrt[d + e*x])/e^3 + (2*c
*(d + e*x)^(3/2))/(3*e^3)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+cx^2}{(d+ex)^{3/2}}\,dx &= \int \left(\frac{cd^2+ae^2}{e^2(d+ex)^{3/2}}-\frac{2cd}{e^2\sqrt{d+ex}}+\frac{c\sqrt{d+ex}}{e^2}\right)dx\\ &= -\frac{2\left(cd^2+ae^2\right)}{e^3\sqrt{d+ex}}-\frac{4cd\sqrt{d+ex}}{e^3}+\frac{2c(d+ex)^{3/2}}{3e^3}\end{aligned}$$

Mathematica [A] time = 0.034477, size = 43, normalized size = 0.73

$$\frac{2\left(c\left(-8d^2-4dex+e^2x^2\right)-3ae^2\right)}{3e^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/(d + e*x)^(3/2),x]
```

```
[Out] (2*(-3*a*e^2 + c*(-8*d^2 - 4*d*e*x + e^2*x^2)))/(3*e^3*Sqrt[d + e*x])
```

Maple [A] time = 0.041, size = 41, normalized size = 0.7

$$-\frac{-2\,ce^{2}x^{2}+8\,cdex+6\,ae^{2}+16\,cd^{2}}{3\,e^{3}}\frac{1}{\sqrt{ex+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^(3/2),x)
```

```
[Out] -2/3/(e*x+d)^(1/2)*(-c*e^2*x^2+4*c*d*e*x+3*a*e^2+8*c*d^2)/e^3
```

Maxima [A] time = 1.12727, size = 73, normalized size = 1.24

$$\frac{2\left(\frac{(ex+d)^{\frac{3}{2}}c-6\,\sqrt{ex+d}cd}{e^{2}}-\frac{3\left(cd^{2}+ae^{2}\right)}{\sqrt{ex+d}e^{2}}\right)}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(((e*x + d)^(3/2)*c - 6*sqrt(e*x + d)*c*d)/e^2 - 3*(c*d^2 + a*e^2)/(sqr
t(e*x + d)*e^2))/e
```

Fricas [A] time = 1.78122, size = 107, normalized size = 1.81

$$\frac{2\left(ce^{2}x^{2}-4\,cdex-8\,cd^{2}-3\,ae^{2}\right)\sqrt{ex+d}}{3\left(e^{4}x+de^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(c*e^2*x^2 - 4*c*d*e*x - 8*c*d^2 - 3*a*e^2)*sqrt(e*x + d)/(e^4*x + d*e^
3)
```

Sympy [A] time = 5.83845, size = 58, normalized size = 0.98

$$-\frac{4cd\sqrt{d+ex}}{e^{3}}+\frac{2c\,(d+ex)^{\frac{3}{2}}}{3e^{3}}-\frac{2\left(ae^{2}+cd^{2}\right)}{e^{3}\sqrt{d+ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**(3/2),x)
```

```
[Out] -4*c*d*sqrt(d + e*x)/e**3 + 2*c*(d + e*x)**(3/2)/(3*e**3) - 2*(a*e**2 + c*d
**2)/(e**3*sqrt(d + e*x))
```

Giac [A] time = 1.72146, size = 73, normalized size = 1.24

$$\frac{2}{3}\left((xe+d)^{\frac{3}{2}}ce^6 - 6\sqrt{xe+d}\,cde^6\right)e^{(-9)} - \frac{2\left(cd^2+ae^2\right)e^{(-3)}}{\sqrt{xe+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*((x*e + d)^(3/2)*c*e^6 - 6*sqrt(x*e + d)*c*d*e^6)*e^(-9) - 2*(c*d^2 + a
*e^2)*e^(-3)/sqrt(x*e + d)
```

3.596 $\int \frac{a+cx^2}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=59

$$-\frac{2\left(ae^2+cd^2\right)}{3e^3(d+ex)^{3/2}}+\frac{2c\sqrt{d+ex}}{e^3}+\frac{4cd}{e^3\sqrt{d+ex}}$$

```
[Out] (-2*(c*d^2 + a*e^2))/(3*e^3*(d + e*x)^(3/2)) + (4*c*d)/(e^3*Sqrt[d + e*x])
+ (2*c*Sqrt[d + e*x])/e^3
```

Rubi [A] time = 0.0222372, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$-\frac{2\left(ae^2+cd^2\right)}{3e^3(d+ex)^{3/2}}+\frac{2c\sqrt{d+ex}}{e^3}+\frac{4cd}{e^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/(d + e*x)^(5/2),x]
```

```
[Out] (-2*(c*d^2 + a*e^2))/(3*e^3*(d + e*x)^(3/2)) + (4*c*d)/(e^3*Sqrt[d + e*x])
+ (2*c*Sqrt[d + e*x])/e^3
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+cx^2}{(d+ex)^{5/2}}\,dx &= \int \left(\frac{cd^2+ae^2}{e^2(d+ex)^{5/2}}-\frac{2cd}{e^2(d+ex)^{3/2}}+\frac{c}{e^2\sqrt{d+ex}}\right)dx\\ &= -\frac{2\left(cd^2+ae^2\right)}{3e^3(d+ex)^{3/2}}+\frac{4cd}{e^3\sqrt{d+ex}}+\frac{2c\sqrt{d+ex}}{e^3}\end{aligned}$$

Mathematica [A] time = 0.037179, size = 44, normalized size = 0.75

$$\frac{2\left(c\left(8d^2+12dex+3e^2x^2\right)-ae^2\right)}{3e^3(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/(d + e*x)^(5/2),x]
```

```
[Out] (2*(-(a*e^2) + c*(8*d^2 + 12*d*e*x + 3*e^2*x^2)))/(3*e^3*(d + e*x)^(3/2))
```

Maple [A] time = 0.041, size = 40, normalized size = 0.7

$$-\frac{-6\,c e^{2} x^{2}-24\,c d e x+2\,a e^{2}-16\,c d^{2}}{3\,e^{3}}\,(ex+d)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^(5/2),x)
```

```
[Out] -2/3/(e*x+d)^(3/2)*(-3*c*e^2*x^2-12*c*d*e*x+a*e^2-8*c*d^2)/e^3
```

Maxima [A] time = 1.11347, size = 70, normalized size = 1.19

$$\frac{2\left(\frac{3\,\sqrt{ex+d}c}{e^{2}}+\frac{6\,(ex+d)cd-cd^{2}-ae^{2}}{(ex+d)^{\frac{3}{2}}e^{2}}\right)}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(3*sqrt(e*x + d)*c/e^2 + (6*(e*x + d)*c*d - c*d^2 - a*e^2)/((e*x + d)^(
3/2)*e^2))/e
```

Fricas [A] time = 1.84766, size = 130, normalized size = 2.2

$$\frac{2\left(3\,c e^{2} x^{2}+12\,c d e x+8\,c d^{2}-a e^{2}\right)\sqrt{ex+d}}{3\left(e^{5} x^{2}+2\,d e^{4} x+d^{2} e^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*c*e^2*x^2 + 12*c*d*e*x + 8*c*d^2 - a*e^2)*sqrt(e*x + d)/(e^5*x^2 + 2
*d*e^4*x + d^2*e^3)
```

Sympy [A] time = 1.19617, size = 168, normalized size = 2.85

$$\begin{cases} -\frac{2ae^{2}}{3de^{3}\sqrt{d+ex}+3e^{4}x\sqrt{d+ex}}+\frac{16cd^{2}}{3de^{3}\sqrt{d+ex}+3e^{4}x\sqrt{d+ex}}+\frac{24cdex}{3de^{3}\sqrt{d+ex}+3e^{4}x\sqrt{d+ex}}+\frac{6ce^{2}x^{2}}{3de^{3}\sqrt{d+ex}+3e^{4}x\sqrt{d+ex}} & \text{for}\ e\neq 0 \\ \frac{ax+\frac{cx^{3}}{3}}{d^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**(5/2),x)
```

```
[Out] Piecewise((-2*a*e**2/(3*d*e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)) + 16
*c*d**2/(3*d*e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)) + 24*c*d*e*x/(3*d
*e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)) + 6*c*e**2*x**2/(3*d*e**3*sqr
t(d + e*x) + 3*e**4*x*sqrt(d + e*x)), Ne(e, 0)), ((a*x + c*x**3/3)/d**(5/2)
```

```
, True))
```

Giac [A] time = 1.31088, size = 65, normalized size = 1.1

$$2\sqrt{xe+d}ce^{(-3)} + \frac{2\left(6\,(xe+d)cd - cd^2 - ae^2\right)e^{(-3)}}{3\,(xe+d)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x*e + d)*c*e^(-3) + 2/3*(6*(x*e + d)*c*d - c*d^2 - a*e^2)*e^(-3)/(x*
e + d)^(3/2)
```

3.597 $\int \frac{a+cx^2}{(d+ex)^{7/2}} dx$

Optimal. Leaf size=61

$$-\frac{2\left(ae^2+cd^2\right)}{5e^3(d+ex)^{5/2}}-\frac{2c}{e^3\sqrt{d+ex}}+\frac{4cd}{3e^3(d+ex)^{3/2}}$$

```
[Out] (-2*(c*d^2 + a*e^2))/(5*e^3*(d + e*x)^(5/2)) + (4*c*d)/(3*e^3*(d + e*x)^(3/
2)) - (2*c)/(e^3*Sqrt[d + e*x])
```

Rubi [A] time = 0.0225897, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$-\frac{2\left(ae^2+cd^2\right)}{5e^3(d+ex)^{5/2}}-\frac{2c}{e^3\sqrt{d+ex}}+\frac{4cd}{3e^3(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/(d + e*x)^(7/2),x]
```

```
[Out] (-2*(c*d^2 + a*e^2))/(5*e^3*(d + e*x)^(5/2)) + (4*c*d)/(3*e^3*(d + e*x)^(3/
2)) - (2*c)/(e^3*Sqrt[d + e*x])
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+cx^2}{(d+ex)^{7/2}}\,dx &= \int \left(\frac{cd^2+ae^2}{e^2(d+ex)^{7/2}}-\frac{2cd}{e^2(d+ex)^{5/2}}+\frac{c}{e^2(d+ex)^{3/2}}\right)dx\\ &= -\frac{2\left(cd^2+ae^2\right)}{5e^3(d+ex)^{5/2}}+\frac{4cd}{3e^3(d+ex)^{3/2}}-\frac{2c}{e^3\sqrt{d+ex}}\end{aligned}$$

Mathematica [A] time = 0.0385796, size = 44, normalized size = 0.72

$$-\frac{2\left(3ae^2+c\left(8d^2+20dex+15e^2x^2\right)\right)}{15e^3(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/(d + e*x)^(7/2),x]
```

```
[Out] (-2*(3*a*e^2 + c*(8*d^2 + 20*d*e*x + 15*e^2*x^2)))/(15*e^3*(d + e*x)^(5/2))
```

Maple [A] time = 0.042, size = 41, normalized size = 0.7

$$-\frac{30\,ce^{2}x^{2}+40\,cdex+6\,ae^{2}+16\,cd^{2}}{15\,e^{3}}\left(ex+d\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^(7/2),x)
```

```
[Out] -2/15/(e*x+d)^(5/2)*(15*c*e^2*x^2+20*c*d*e*x+3*a*e^2+8*c*d^2)/e^3
```

Maxima [A] time = 1.09602, size = 59, normalized size = 0.97

$$-\frac{2\left(15\,(ex+d)^{2}c-10\,(ex+d)cd+3\,cd^{2}+3\,ae^{2}\right)}{15\,(ex+d)^{\frac{5}{2}}e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/15*(15*(e*x + d)^2*c - 10*(e*x + d)*c*d + 3*c*d^2 + 3*a*e^2)/((e*x + d)^
(5/2)*e^3)
```

Fricas [A] time = 1.82718, size = 158, normalized size = 2.59

$$-\frac{2\left(15\,ce^{2}x^{2}+20\,cdex+8\,cd^{2}+3\,ae^{2}\right)\sqrt{ex+d}}{15\left(e^{6}x^{3}+3\,de^{5}x^{2}+3\,d^{2}e^{4}x+d^{3}e^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/15*(15*c*e^2*x^2 + 20*c*d*e*x + 8*c*d^2 + 3*a*e^2)*sqrt(e*x + d)/(e^6*x^
3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3)
```

Sympy [A] time = 2.8407, size = 252, normalized size = 4.13

$$\begin{cases} -\frac{6ae^{2}}{15d^{2}e^{3}\sqrt{d+ex}+30de^{4}x\sqrt{d+ex}+15e^{5}x^{2}\sqrt{d+ex}} - \frac{16cd^{2}}{15d^{2}e^{3}\sqrt{d+ex}+30de^{4}x\sqrt{d+ex}+15e^{5}x^{2}\sqrt{d+ex}} - \frac{40cdex}{15d^{2}e^{3}\sqrt{d+ex}+30de^{4}x\sqrt{d+ex}+15e^{5}x^{2}\sqrt{d+ex}} - \frac{}{15d^{2}e^{3}\sqrt{d+} \\ \frac{ax+\frac{cx^{3}}{3}}{d^{\frac{7}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**(7/2),x)
```

```
[Out] Piecewise((-6*a*e**2/(15*d**2*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d + e*x
) + 15*e**5*x**2*sqrt(d + e*x)) - 16*c*d**2/(15*d**2*e**3*sqrt(d + e*x) + 3
0*d*e**4*x*sqrt(d + e*x) + 15*e**5*x**2*sqrt(d + e*x)) - 40*c*d*e*x/(15*d**
2*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d + e*x) + 15*e**5*x**2*sqrt(d + e*
```

```
x)) - 30*c*e**2*x**2/(15*d**2*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d + e*x
) + 15*e**5*x**2*sqrt(d + e*x)), Ne(e, 0)), ((a*x + c*x**3/3)/d**(7/2), Tru
e))
```

Giac [A] time = 1.35545, size = 61, normalized size = 1.

$$-\frac{2\left(15(xe+d)^2c-10(xe+d)cd+3cd^2+3ae^2\right)e^{(-3)}}{15(xe+d)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*(15*(x*e + d)^2*c - 10*(x*e + d)*c*d + 3*c*d^2 + 3*a*e^2)*e^(-3)/(x*e
 + d)^(5/2)
```

3.598 $\int (d+ex)^{5/2} \left(a+cx^2\right)^2 \, dx$

Optimal. Leaf size=127

$$\frac{4c(d+ex)^{11/2}\left(ae^2+3cd^2\right)}{11e^5}-\frac{8cd(d+ex)^{9/2}\left(ae^2+cd^2\right)}{9e^5}+\frac{2(d+ex)^{7/2}\left(ae^2+cd^2\right)^2}{7e^5}+\frac{2c^2(d+ex)^{15/2}}{15e^5}-\frac{8c^2d(d+ex)^{13/2}}{13e^5}$$

```
[Out] (2*(c*d^2 + a*e^2)^2*(d + e*x)^(7/2))/(7*e^5) - (8*c*d*(c*d^2 + a*e^2)*(d +
 e*x)^(9/2))/(9*e^5) + (4*c*(3*c*d^2 + a*e^2)*(d + e*x)^(11/2))/(11*e^5) -
(8*c^2*d*(d + e*x)^(13/2))/(13*e^5) + (2*c^2*(d + e*x)^(15/2))/(15*e^5)
```

Rubi [A] time = 0.0568408, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {697}

$$\frac{4c(d+ex)^{11/2}\left(ae^2+3cd^2\right)}{11e^5}-\frac{8cd(d+ex)^{9/2}\left(ae^2+cd^2\right)}{9e^5}+\frac{2(d+ex)^{7/2}\left(ae^2+cd^2\right)^2}{7e^5}+\frac{2c^2(d+ex)^{15/2}}{15e^5}-\frac{8c^2d(d+ex)^{13/2}}{13e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)*(a + c*x^2)^2,x]
```

```
[Out] (2*(c*d^2 + a*e^2)^2*(d + e*x)^(7/2))/(7*e^5) - (8*c*d*(c*d^2 + a*e^2)*(d +
 e*x)^(9/2))/(9*e^5) + (4*c*(3*c*d^2 + a*e^2)*(d + e*x)^(11/2))/(11*e^5) -
(8*c^2*d*(d + e*x)^(13/2))/(13*e^5) + (2*c^2*(d + e*x)^(15/2))/(15*e^5)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (d+ex)^{5/2}\left(a+cx^2\right)^2 dx = \int \left(\frac{\left(cd^2+ae^2\right)^2(d+ex)^{5/2}}{e^4}-\frac{4cd\left(cd^2+ae^2\right)(d+ex)^{7/2}}{e^4}+\frac{2c\left(3cd^2+ae^2\right)(d+ex)^{9/2}}{e^4}-\frac{4c\ldots}{}\right.$$

$$=\frac{2\left(cd^2+ae^2\right)^2(d+ex)^{7/2}}{7e^5}-\frac{8cd\left(cd^2+ae^2\right)(d+ex)^{9/2}}{9e^5}+\frac{4c\left(3cd^2+ae^2\right)(d+ex)^{11/2}}{11e^5}-\frac{8c^2\ldots}{}$$

Mathematica [A] time = 0.111509, size = 96, normalized size = 0.76

$$\frac{2(d+ex)^{7/2}\left(6435a^2e^4+130ace^2\left(8d^2-28dex+63e^2x^2\right)+c^2\left(1008d^2e^2x^2-448d^3ex+128d^4-1848de^3x^3+3003e^4x^4\right)\right)}{45045e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)*(a + c*x^2)^2,x]
```

```
[Out] (2*(d + e*x)^(7/2)*(6435*a^2*e^4 + 130*a*c*e^2*(8*d^2 - 28*d*e*x + 63*e^2*x
^2) + c^2*(128*d^4 - 448*d^3*e*x + 1008*d^2*e^2*x^2 - 1848*d*e^3*x^3 + 3003
```

```
*e^4*x^4)))/(45045*e^5)
```

Maple [A] time = 0.044, size = 106, normalized size = 0.8

$$\frac{6006\,c^2x^4e^4-3696\,c^2dx^3e^3+16380\,ace^4x^2+2016\,c^2d^2e^2x^2-7280\,acde^3x-896\,c^2d^3ex+12870\,a^2e^4+2080\,acd^2e^2+}{45045\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(c*x^2+a)^2,x)
```

```
[Out] 2/45045*(e*x+d)^(7/2)*(3003*c^2*e^4*x^4-1848*c^2*d*e^3*x^3+8190*a*c*e^4*x^2
+1008*c^2*d^2*e^2*x^2-3640*a*c*d*e^3*x-448*c^2*d^3*e*x+6435*a^2*e^4+1040*a*
c*d^2*e^2+128*c^2*d^4)/e^5
```

Maxima [A] time = 1.12371, size = 153, normalized size = 1.2

$$\frac{2\left(3003\,(ex+d)^{\frac{15}{2}}c^2-13860\,(ex+d)^{\frac{13}{2}}c^2d+8190\left(3\,c^2d^2+ace^2\right)(ex+d)^{\frac{11}{2}}-20020\left(c^2d^3+acde^2\right)(ex+d)^{\frac{9}{2}}+6435\,(}{45045\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 2/45045*(3003*(e*x + d)^(15/2)*c^2 - 13860*(e*x + d)^(13/2)*c^2*d + 8190*(3
*c^2*d^2 + a*c*e^2)*(e*x + d)^(11/2) - 20020*(c^2*d^3 + a*c*d*e^2)*(e*x + d
)^(9/2) + 6435*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*(e*x + d)^(7/2))/e^5
```

Fricas [B] time = 1.75913, size = 504, normalized size = 3.97

$$\frac{2\left(3003\,c^2e^7x^7+7161\,c^2de^6x^6+128\,c^2d^7+1040\,acd^5e^2+6435\,a^2d^3e^4+63\left(71\,c^2d^2e^5+130\,ace^7\right)x^5+35\left(c^2d^3e^4+59\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 2/45045*(3003*c^2*e^7*x^7 + 7161*c^2*d*e^6*x^6 + 128*c^2*d^7 + 1040*a*c*d^5
*e^2 + 6435*a^2*d^3*e^4 + 63*(71*c^2*d^2*e^5 + 130*a*c*e^7)*x^5 + 35*(c^2*d
^3*e^4 + 598*a*c*d*e^6)*x^4 - 5*(8*c^2*d^4*e^3 - 2938*a*c*d^2*e^5 - 1287*a^
2*e^7)*x^3 + 3*(16*c^2*d^5*e^2 + 130*a*c*d^3*e^4 + 6435*a^2*d*e^6)*x^2 - (6
4*c^2*d^6*e + 520*a*c*d^4*e^3 - 19305*a^2*d^2*e^5)*x)*sqrt(e*x + d)/e^5
```

Sympy [A] time = 17.5292, size = 566, normalized size = 4.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(c*x**2+a)**2,x)
```

```
[Out] a**2*d**2*Piecewise((sqrt(d)*x, Eq(e, 0)), (2*(d + e*x)**(3/2)/(3*e), True)
) + 4*a**2*d*(-d*(d + e*x)**(3/2)/3 + (d + e*x)**(5/2)/5)/e + 2*a**2*(d**2*
(d + e*x)**(3/2)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e + 4*a*c
*d**2*(d**2*(d + e*x)**(3/2)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/
7)/e**3 + 8*a*c*d*(-d**3*(d + e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3
*d*(d + e*x)**(7/2)/7 + (d + e*x)**(9/2)/9)/e**3 + 4*a*c*(d**4*(d + e*x)**(
3/2)/3 - 4*d**3*(d + e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e
*x)**(9/2)/9 + (d + e*x)**(11/2)/11)/e**3 + 2*c**2*d**2*(d**4*(d + e*x)**(3
/2)/3 - 4*d**3*(d + e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e*
x)**(9/2)/9 + (d + e*x)**(11/2)/11)/e**5 + 4*c**2*d*(-d**5*(d + e*x)**(3/2)
/3 + d**4*(d + e*x)**(5/2) - 10*d**3*(d + e*x)**(7/2)/7 + 10*d**2*(d + e*x)
**(9/2)/9 - 5*d*(d + e*x)**(11/2)/11 + (d + e*x)**(13/2)/13)/e**5 + 2*c**2*
(d**6*(d + e*x)**(3/2)/3 - 6*d**5*(d + e*x)**(5/2)/5 + 15*d**4*(d + e*x)**(
7/2)/7 - 20*d**3*(d + e*x)**(9/2)/9 + 15*d**2*(d + e*x)**(11/2)/11 - 6*d*(d
 + e*x)**(13/2)/13 + (d + e*x)**(15/2)/15)/e**5
```

Giac [B] time = 1.71189, size = 676, normalized size = 5.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2/45045*(858*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2
)*d^2)*a*c*d^2*e^(-2) + 13*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d +
 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)
*d^4)*c^2*d^2*e^(-4) + 15015*(x*e + d)^(3/2)*a^2*d^2 + 572*(35*(x*e + d)^(9
/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)
*d^3)*a*c*d*e^(-2) + 10*(693*(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 1
0010*(x*e + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)
*d^4 - 3003*(x*e + d)^(3/2)*d^5)*c^2*d*e^(-4) + 6006*(3*(x*e + d)^(5/2) - 5
*(x*e + d)^(3/2)*d)*a^2*d + 26*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)
*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(
3/2)*d^4)*a*c*e^(-2) + (3003*(x*e + d)^(15/2) - 20790*(x*e + d)^(13/2)*d +
61425*(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525*(x*e + d)^(
7/2)*d^4 - 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*c^2*e^(-4
) + 429*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2
)*a^2)*e^(-1)
```

3.599 $\int (d+ex)^{3/2} \left(a+cx^2\right)^2 \, dx$

Optimal. Leaf size=127

$$\frac{4c(d+ex)^{9/2}\left(ae^2+3cd^2\right)}{9e^5}-\frac{8cd(d+ex)^{7/2}\left(ae^2+cd^2\right)}{7e^5}+\frac{2(d+ex)^{5/2}\left(ae^2+cd^2\right)^2}{5e^5}+\frac{2c^2(d+ex)^{13/2}}{13e^5}-\frac{8c^2d(d+ex)^{11/}}{11e^5}$$

```
[Out] (2*(c*d^2 + a*e^2)^2*(d + e*x)^(5/2))/(5*e^5) - (8*c*d*(c*d^2 + a*e^2)*(d +
 e*x)^(7/2))/(7*e^5) + (4*c*(3*c*d^2 + a*e^2)*(d + e*x)^(9/2))/(9*e^5) - (8
*c^2*d*(d + e*x)^(11/2))/(11*e^5) + (2*c^2*(d + e*x)^(13/2))/(13*e^5)
```

Rubi [A] time = 0.0476634, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {697}

$$\frac{4c(d+ex)^{9/2}\left(ae^2+3cd^2\right)}{9e^5}-\frac{8cd(d+ex)^{7/2}\left(ae^2+cd^2\right)}{7e^5}+\frac{2(d+ex)^{5/2}\left(ae^2+cd^2\right)^2}{5e^5}+\frac{2c^2(d+ex)^{13/2}}{13e^5}-\frac{8c^2d(d+ex)^{11/}}{11e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*(a + c*x^2)^2,x]
```

```
[Out] (2*(c*d^2 + a*e^2)^2*(d + e*x)^(5/2))/(5*e^5) - (8*c*d*(c*d^2 + a*e^2)*(d +
 e*x)^(7/2))/(7*e^5) + (4*c*(3*c*d^2 + a*e^2)*(d + e*x)^(9/2))/(9*e^5) - (8
*c^2*d*(d + e*x)^(11/2))/(11*e^5) + (2*c^2*(d + e*x)^(13/2))/(13*e^5)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (d+ex)^{3/2}\left(a+cx^2\right)^2\,dx=\int \left(\frac{\left(cd^2+ae^2\right)^2(d+ex)^{3/2}}{e^4}-\frac{4cd\left(cd^2+ae^2\right)(d+ex)^{5/2}}{e^4}+\frac{2c\left(3cd^2+ae^2\right)(d+ex)^{7/2}}{e^4}-\right.$$

$$=\frac{2\left(cd^2+ae^2\right)^2(d+ex)^{5/2}}{5e^5}-\frac{8cd\left(cd^2+ae^2\right)(d+ex)^{7/2}}{7e^5}+\frac{4c\left(3cd^2+ae^2\right)(d+ex)^{9/2}}{9e^5}-\frac{8}{}$$

Mathematica [A] time = 0.0870545, size = 97, normalized size = 0.76

$$\frac{2(d+ex)^{5/2}\left(9009a^2e^4+286ace^2\left(8d^2-20dex+35e^2x^2\right)+3c^2\left(560d^2e^2x^2-320d^3ex+128d^4-840de^3x^3+1155e^4x^4\right.\right.}{45045e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)*(a + c*x^2)^2,x]
```

```
[Out] (2*(d + e*x)^(5/2)*(9009*a^2*e^4 + 286*a*c*e^2*(8*d^2 - 20*d*e*x + 35*e^2*x
^2) + 3*c^2*(128*d^4 - 320*d^3*e*x + 560*d^2*e^2*x^2 - 840*d*e^3*x^3 + 1155
```

```
*e^4*x^4)))/(45045*e^5)
```

Maple [A] time = 0.044, size = 106, normalized size = 0.8

$$\frac{6930\,c^2x^4e^4-5040\,c^2dx^3e^3+20020\,ace^4x^2+3360\,c^2d^2e^2x^2-11440\,acde^3x-1920\,c^2d^3ex+18018\,a^2e^4+4576\,acd^2e^2+}{45045\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(c*x^2+a)^2,x)
```

```
[Out] 2/45045*(e*x+d)^(5/2)*(3465*c^2*e^4*x^4-2520*c^2*d*e^3*x^3+10010*a*c*e^4*x^
2+1680*c^2*d^2*e^2*x^2-5720*a*c*d*e^3*x-960*c^2*d^3*e*x+9009*a^2*e^4+2288*a
*c*d^2*e^2+384*c^2*d^4)/e^5
```

Maxima [A] time = 1.14843, size = 153, normalized size = 1.2

$$\frac{2\left(3465\,(ex+d)^{\frac{13}{2}}c^2-16380\,(ex+d)^{\frac{11}{2}}c^2d+10010\left(3\,c^2d^2+ace^2\right)(ex+d)^{\frac{9}{2}}-25740\left(c^2d^3+acde^2\right)(ex+d)^{\frac{7}{2}}+9009\left(c^2d\right.\right.}{45045\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 2/45045*(3465*(e*x + d)^(13/2)*c^2 - 16380*(e*x + d)^(11/2)*c^2*d + 10010*(
3*c^2*d^2 + a*c*e^2)*(e*x + d)^(9/2) - 25740*(c^2*d^3 + a*c*d*e^2)*(e*x + d
)^(7/2) + 9009*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*(e*x + d)^(5/2))/e^5
```

Fricas [A] time = 1.81298, size = 423, normalized size = 3.33

$$\frac{2\left(3465\,c^2e^6x^6+4410\,c^2de^5x^5+384\,c^2d^6+2288\,acd^4e^2+9009\,a^2d^2e^4+35\left(3\,c^2d^2e^4+286\,ace^6\right)x^4-20\left(6\,c^2d^3e^3-715\,\right.\right.}{45045\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 2/45045*(3465*c^2*e^6*x^6 + 4410*c^2*d*e^5*x^5 + 384*c^2*d^6 + 2288*a*c*d^4
*e^2 + 9009*a^2*d^2*e^4 + 35*(3*c^2*d^2*e^4 + 286*a*c*e^6)*x^4 - 20*(6*c^2*
d^3*e^3 - 715*a*c*d*e^5)*x^3 + 3*(48*c^2*d^4*e^2 + 286*a*c*d^2*e^4 + 3003*a
^2*e^6)*x^2 - 2*(96*c^2*d^5*e + 572*a*c*d^3*e^3 - 9009*a^2*d*e^5)*x)*sqrt(e
*x + d)/e^5
```

Sympy [A] time = 10.6265, size = 328, normalized size = 2.58

$$a^2d\left(\begin{cases}\sqrt{d}x & \text{for } e=0\\ \frac{2(d+ex)^{\frac{3}{2}}}{3e} & \text{otherwise}\end{cases}\right)+\frac{2a^2\left(-\frac{d(d+ex)^{\frac{3}{2}}}{3}+\frac{(d+ex)^{\frac{5}{2}}}{5}\right)}{e}+\frac{4acd\left(\frac{d^2(d+ex)^{\frac{3}{2}}}{3}-\frac{2d(d+ex)^{\frac{5}{2}}}{5}+\frac{(d+ex)^{\frac{7}{2}}}{7}\right)}{e^3}+\frac{4ac\left(-\frac{d^3(d+ex)^{\frac{3}{2}}}{3}+\frac{3d^2}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(c*x**2+a)**2,x)
```

```
[Out] a**2*d*Piecewise((sqrt(d)*x, Eq(e, 0)), (2*(d + e*x)**(3/2)/(3*e), True)) +
 2*a**2*(-d*(d + e*x)**(3/2)/3 + (d + e*x)**(5/2)/5)/e + 4*a*c*d*(d**2*(d +
 e*x)**(3/2)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e**3 + 4*a*c*
(-d**3*(d + e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d + e*x)**(7/2
)/7 + (d + e*x)**(9/2)/9)/e**3 + 2*c**2*d*(d**4*(d + e*x)**(3/2)/3 - 4*d**3
*(d + e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e*x)**(9/2)/9 +
(d + e*x)**(11/2)/11)/e**5 + 2*c**2*(-d**5*(d + e*x)**(3/2)/3 + d**4*(d + e
*x)**(5/2) - 10*d**3*(d + e*x)**(7/2)/7 + 10*d**2*(d + e*x)**(9/2)/9 - 5*d*
(d + e*x)**(11/2)/11 + (d + e*x)**(13/2)/13)/e**5
```

Giac [B] time = 1.41704, size = 394, normalized size = 3.1

$$\frac{2}{45045}\left(858\left(15\,(xe+d)^{\frac{7}{2}}-42\,(xe+d)^{\frac{5}{2}}d+35\,(xe+d)^{\frac{3}{2}}d^{2}\right)acde^{(-2)}+13\left(315\,(xe+d)^{\frac{11}{2}}-1540\,(xe+d)^{\frac{9}{2}}d+2970\,(xe\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2/45045*(858*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2
)*d^2)*a*c*d*e^(-2) + 13*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2
970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d
^4)*c^2*d*e^(-4) + 15015*(x*e + d)^(3/2)*a^2*d + 286*(35*(x*e + d)^(9/2) -
135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*
a*c*e^(-2) + 5*(693*(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e
 + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 30
03*(x*e + d)^(3/2)*d^5)*c^2*e^(-4) + 3003*(3*(x*e + d)^(5/2) - 5*(x*e + d)^
(3/2)*d)*a^2)*e^(-1)
```

3.600 $\int \sqrt{d+ex}\left(a+cx^2\right)^2 dx$

Optimal. Leaf size=127

$$\frac{4c(d+ex)^{7/2}\left(ae^2+3cd^2\right)}{7e^5}-\frac{8cd(d+ex)^{5/2}\left(ae^2+cd^2\right)}{5e^5}+\frac{2(d+ex)^{3/2}\left(ae^2+cd^2\right)^2}{3e^5}+\frac{2c^2(d+ex)^{11/2}}{11e^5}-\frac{8c^2d(d+ex)^{9/2}}{9e^5}$$

```
[Out] (2*(c*d^2 + a*e^2)^2*(d + e*x)^(3/2))/(3*e^5) - (8*c*d*(c*d^2 + a*e^2)*(d +
 e*x)^(5/2))/(5*e^5) + (4*c*(3*c*d^2 + a*e^2)*(d + e*x)^(7/2))/(7*e^5) - (8
*c^2*d*(d + e*x)^(9/2))/(9*e^5) + (2*c^2*(d + e*x)^(11/2))/(11*e^5)
```

Rubi [A] time = 0.0460433, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {697}

$$\frac{4c(d+ex)^{7/2}\left(ae^2+3cd^2\right)}{7e^5}-\frac{8cd(d+ex)^{5/2}\left(ae^2+cd^2\right)}{5e^5}+\frac{2(d+ex)^{3/2}\left(ae^2+cd^2\right)^2}{3e^5}+\frac{2c^2(d+ex)^{11/2}}{11e^5}-\frac{8c^2d(d+ex)^{9/2}}{9e^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(a + c*x^2)^2,x]
```

```
[Out] (2*(c*d^2 + a*e^2)^2*(d + e*x)^(3/2))/(3*e^5) - (8*c*d*(c*d^2 + a*e^2)*(d +
 e*x)^(5/2))/(5*e^5) + (4*c*(3*c*d^2 + a*e^2)*(d + e*x)^(7/2))/(7*e^5) - (8
*c^2*d*(d + e*x)^(9/2))/(9*e^5) + (2*c^2*(d + e*x)^(11/2))/(11*e^5)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \sqrt{d+ex}\left(a+cx^2\right)^2 dx = \int \left(\frac{\left(cd^2+ae^2\right)^2\sqrt{d+ex}}{e^4}-\frac{4cd\left(cd^2+ae^2\right)(d+ex)^{3/2}}{e^4}+\frac{2c\left(3cd^2+ae^2\right)(d+ex)^{5/2}}{e^4}-\frac{4c^2d(d\right.$$

$$=\frac{2\left(cd^2+ae^2\right)^2(d+ex)^{3/2}}{3e^5}-\frac{8cd\left(cd^2+ae^2\right)(d+ex)^{5/2}}{5e^5}+\frac{4c\left(3cd^2+ae^2\right)(d+ex)^{7/2}}{7e^5}-\frac{8c^2d(d}{9}$$

Mathematica [A] time = 0.0612854, size = 96, normalized size = 0.76

$$\frac{2(d+ex)^{3/2}\left(1155a^2e^4+66ace^2\left(8d^2-12dex+15e^2x^2\right)+c^2\left(240d^2e^2x^2-192d^3ex+128d^4-280de^3x^3+315e^4x^4\right)\right)}{3465e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*(a + c*x^2)^2,x]
```

```
[Out] (2*(d + e*x)^(3/2)*(1155*a^2*e^4 + 66*a*c*e^2*(8*d^2 - 12*d*e*x + 15*e^2*x^
2) + c^2*(128*d^4 - 192*d^3*e*x + 240*d^2*e^2*x^2 - 280*d*e^3*x^3 + 315*e^4
```

```
*x^4)))/(3465*e^5)
```

Maple [A] time = 0.046, size = 106, normalized size = 0.8

$$\frac{630 c^2x^4e^4 - 560 c^2dx^3e^3 + 1980 ace^4x^2 + 480 c^2d^2e^2x^2 - 1584 acde^3x - 384 c^2d^3ex + 2310 a^2e^4 + 1056 acd^2e^2 + 256 c^2}{3465 e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2*(e*x+d)^(1/2),x)
```

```
[Out] 2/3465*(e*x+d)^(3/2)*(315*c^2*e^4*x^4-280*c^2*d*e^3*x^3+990*a*c*e^4*x^2+240
*c^2*d^2*e^2*x^2-792*a*c*d*e^3*x-192*c^2*d^3*e*x+1155*a^2*e^4+528*a*c*d^2*e
^2+128*c^2*d^4)/e^5
```

Maxima [A] time = 1.1602, size = 153, normalized size = 1.2

$$\frac{2\left(315 (ex + d)^{\frac{11}{2}} c^2 - 1540 (ex + d)^{\frac{9}{2}} c^2d + 990 \left(3 c^2d^2 + ace^2\right)(ex + d)^{\frac{7}{2}} - 2772 \left(c^2d^3 + acde^2\right)(ex + d)^{\frac{5}{2}} + 1155 \left(c^2d^4 +\right.\right.}{3465 e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3465*(315*(e*x + d)^(11/2)*c^2 - 1540*(e*x + d)^(9/2)*c^2*d + 990*(3*c^2*
d^2 + a*c*e^2)*(e*x + d)^(7/2) - 2772*(c^2*d^3 + a*c*d*e^2)*(e*x + d)^(5/2)
 + 1155*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*(e*x + d)^(3/2))/e^5
```

Fricas [A] time = 1.78311, size = 325, normalized size = 2.56

$$\frac{2\left(315 c^2e^5x^5 + 35 c^2de^4x^4 + 128 c^2d^5 + 528 acd^3e^2 + 1155 a^2de^4 - 10 \left(4 c^2d^2e^3 - 99 ace^5\right)x^3 + 6 \left(8 c^2d^3e^2 + 33 acde^4\right)x\right.}{3465 e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*c^2*e^5*x^5 + 35*c^2*d*e^4*x^4 + 128*c^2*d^5 + 528*a*c*d^3*e^2
+ 1155*a^2*d*e^4 - 10*(4*c^2*d^2*e^3 - 99*a*c*e^5)*x^3 + 6*(8*c^2*d^3*e^2 +
 33*a*c*d*e^4)*x^2 - (64*c^2*d^4*e + 264*a*c*d^2*e^3 - 1155*a^2*e^5)*x)*sqr
t(e*x + d)/e^5
```

Sympy [A] time = 2.56615, size = 148, normalized size = 1.17

$$\frac{2\left(-\frac{4c^2d(d+ex)^{\frac{9}{2}}}{9e^4} + \frac{c^2(d+ex)^{\frac{11}{2}}}{11e^4} + \frac{(d+ex)^{\frac{7}{2}}\left(2ace^2+6c^2d^2\right)}{7e^4} + \frac{(d+ex)^{\frac{5}{2}}\left(-4acde^2-4c^2d^3\right)}{5e^4} + \frac{(d+ex)^{\frac{3}{2}}\left(a^2e^4+2acd^2e^2+c^2d^4\right)}{3e^4}\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2*(e*x+d)**(1/2),x)
```

```
[Out] 2*(-4*c**2*d*(d + e*x)**(9/2)/(9*e**4) + c**2*(d + e*x)**(11/2)/(11*e**4) +
 (d + e*x)**(7/2)*(2*a*c*e**2 + 6*c**2*d**2)/(7*e**4) + (d + e*x)**(5/2)*(-
4*a*c*d*e**2 - 4*c**2*d**3)/(5*e**4) + (d + e*x)**(3/2)*(a**2*e**4 + 2*a*c*
d**2*e**2 + c**2*d**4)/(3*e**4))/e
```

Giac [A] time = 1.6526, size = 170, normalized size = 1.34

$$\frac{2}{3465}\left(66\left(15\,(xe+d)^{\frac{7}{2}}-42\,(xe+d)^{\frac{5}{2}}d+35\,(xe+d)^{\frac{3}{2}}d^2\right)ace^{(-2)}+\left(315\,(xe+d)^{\frac{11}{2}}-1540\,(xe+d)^{\frac{9}{2}}d+2970\,(xe+d)^{\frac{7}{2}}d^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3465*(66*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*
d^2)*a*c*e^(-2) + (315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*
e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*c^2
*e^(-4) + 1155*(x*e + d)^(3/2)*a^2)*e^(-1)
```

3.601 $\int \frac{(a+cx^2)^2}{\sqrt{d+ex}} \, dx$

Optimal. Leaf size=125

$$\frac{4c(d+ex)^{5/2}\left(ae^2+3cd^2\right)}{5e^5}-\frac{8cd(d+ex)^{3/2}\left(ae^2+cd^2\right)}{3e^5}+\frac{2\sqrt{d+ex}\left(ae^2+cd^2\right)^2}{e^5}+\frac{2c^2(d+ex)^{9/2}}{9e^5}-\frac{8c^2d(d+ex)^{7/2}}{7e^5}$$

```
[Out] (2*(c*d^2 + a*e^2)^2*Sqrt[d + e*x])/e^5 - (8*c*d*(c*d^2 + a*e^2)*(d + e*x)^
(3/2))/(3*e^5) + (4*c*(3*c*d^2 + a*e^2)*(d + e*x)^(5/2))/(5*e^5) - (8*c^2*d
*(d + e*x)^(7/2))/(7*e^5) + (2*c^2*(d + e*x)^(9/2))/(9*e^5)
```

Rubi [A] time = 0.0453817, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {697}

$$\frac{4c(d+ex)^{5/2}\left(ae^2+3cd^2\right)}{5e^5}-\frac{8cd(d+ex)^{3/2}\left(ae^2+cd^2\right)}{3e^5}+\frac{2\sqrt{d+ex}\left(ae^2+cd^2\right)^2}{e^5}+\frac{2c^2(d+ex)^{9/2}}{9e^5}-\frac{8c^2d(d+ex)^{7/2}}{7e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^2/Sqrt[d + e*x],x]
```


```
[Out] (2*(c*d^2 + a*e^2)^2*Sqrt[d + e*x])/e^5 - (8*c*d*(c*d^2 + a*e^2)*(d + e*x)^
(3/2))/(3*e^5) + (4*c*(3*c*d^2 + a*e^2)*(d + e*x)^(5/2))/(5*e^5) - (8*c^2*d
*(d + e*x)^(7/2))/(7*e^5) + (2*c^2*(d + e*x)^(9/2))/(9*e^5)
```


Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^2}{\sqrt{d+ex}}\,dx=\int \left(\frac{\left(cd^2+ae^2\right)^2}{e^4\sqrt{d+ex}}-\frac{4cd\left(cd^2+ae^2\right)\sqrt{d+ex}}{e^4}+\frac{2c\left(3cd^2+ae^2\right)(d+ex)^{3/2}}{e^4}-\frac{4c^2d(d+ex)^{5/2}}{e^4}+\frac{c^2(d$$

$$=\frac{2\left(cd^2+ae^2\right)^2\sqrt{d+ex}}{e^5}-\frac{8cd\left(cd^2+ae^2\right)(d+ex)^{3/2}}{3e^5}+\frac{4c\left(3cd^2+ae^2\right)(d+ex)^{5/2}}{5e^5}-\frac{8c^2d(d+ex)^{7/}}{7e^5}$$

Mathematica [A] time = 0.0610542, size = 96, normalized size = 0.77

$$\frac{2\sqrt{d+ex}\left(315a^2e^4+42ace^2\left(8d^2-4dex+3e^2x^2\right)+c^2\left(48d^2e^2x^2-64d^3ex+128d^4-40de^3x^3+35e^4x^4\right)\right)}{315e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^2/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[d + e*x]*(315*a^2*e^4 + 42*a*c*e^2*(8*d^2 - 4*d*e*x + 3*e^2*x^2) +
c^2*(128*d^4 - 64*d^3*e*x + 48*d^2*e^2*x^2 - 40*d*e^3*x^3 + 35*e^4*x^4)))/(
```

```
315*e^5)
```

Maple [A] time = 0.045, size = 106, normalized size = 0.9

$$\frac{70 c^2x^4e^4 - 80 c^2dx^3e^3 + 252 ace^4x^2 + 96 c^2d^2e^2x^2 - 336 acde^3x - 128 c^2d^3ex + 630 a^2e^4 + 672 acd^2e^2 + 256 c^2d^4}{315 e^5}\sqrt{ex + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2/(e*x+d)^(1/2),x)
```

```
[Out] 2/315*(e*x+d)^(1/2)*(35*c^2*e^4*x^4-40*c^2*d*e^3*x^3+126*a*c*e^4*x^2+48*c^2
*d^2*e^2*x^2-168*a*c*d*e^3*x-64*c^2*d^3*e*x+315*a^2*e^4+336*a*c*d^2*e^2+128
*c^2*d^4)/e^5
```

Maxima [A] time = 1.10894, size = 162, normalized size = 1.3

$$\frac{2\left(315\sqrt{ex+d}a^2 + \frac{42\left(3(ex+d)^{\frac{5}{2}} - 10(ex+d)^{\frac{3}{2}}d + 15\sqrt{ex+d}d^2\right)ac}{e^2} + \frac{\left(35(ex+d)^{\frac{9}{2}} - 180(ex+d)^{\frac{7}{2}}d + 378(ex+d)^{\frac{5}{2}}d^2 - 420(ex+d)^{\frac{3}{2}}d^3 + 315\sqrt{ex+d}d^4\right)c^2}{e^4}\right)}{315 e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(315*sqrt(e*x + d)*a^2 + 42*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d
 + 15*sqrt(e*x + d)*d^2)*a*c/e^2 + (35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2
)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)
*d^4)*c^2/e^4)/e
```

Fricas [A] time = 1.79864, size = 242, normalized size = 1.94

$$\frac{2\left(35 c^2e^4x^4 - 40 c^2de^3x^3 + 128 c^2d^4 + 336 acd^2e^2 + 315 a^2e^4 + 6\left(8 c^2d^2e^2 + 21 ace^4\right)x^2 - 8\left(8 c^2d^3e + 21 acde^3\right)x\right)\sqrt{ex + d}}{315 e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*c^2*e^4*x^4 - 40*c^2*d*e^3*x^3 + 128*c^2*d^4 + 336*a*c*d^2*e^2 +
315*a^2*e^4 + 6*(8*c^2*d^2*e^2 + 21*a*c*e^4)*x^2 - 8*(8*c^2*d^3*e + 21*a*c*
d*e^3)*x)*sqrt(e*x + d)/e^5
```

Sympy [A] time = 27.2914, size = 330, normalized size = 2.64

$$\begin{cases} -\frac{\frac{2a^2d}{\sqrt{d+ex}} + 2a^2\left(-\frac{d}{\sqrt{d+ex}} - \sqrt{d+ex}\right) + \frac{4acd\left(\frac{d^2}{\sqrt{d+ex}} + 2d\sqrt{d+ex} - \frac{(d+ex)^{\frac{3}{2}}}{3}\right)}{e^2} + \frac{4ac\left(-\frac{d^3}{\sqrt{d+ex}} - 3d^2\sqrt{d+ex} + d(d+ex)^{\frac{3}{2}} - \frac{(d+ex)^{\frac{5}{2}}}{5}\right)}{e^2} + \frac{2c^2d\left(\frac{d^4}{\sqrt{d+ex}} + 4d^3\sqrt{d+ex} - 2d^2(d+ex)^{\frac{3}{2}} + \frac{4d(d+ex)^{\frac{5}{2}}}{5} - \frac{(d+ex}{7}\right.}{e^4}}{e} \\ \frac{a^2x + \frac{2acx^3}{3} + \frac{c^2x^5}{5}}{\sqrt{d}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2/(e*x+d)**(1/2),x)
```

```
[Out] Piecewise((-(2*a**2*d/sqrt(d + e*x) + 2*a**2*(-d/sqrt(d + e*x) - sqrt(d + e
*x)) + 4*a*c*d*(d**2/sqrt(d + e*x) + 2*d*sqrt(d + e*x) - (d + e*x)**(3/2)/3
)/e**2 + 4*a*c*(-d**3/sqrt(d + e*x) - 3*d**2*sqrt(d + e*x) + d*(d + e*x)**(
3/2) - (d + e*x)**(5/2)/5)/e**2 + 2*c**2*d*(d**4/sqrt(d + e*x) + 4*d**3*sqr
t(d + e*x) - 2*d**2*(d + e*x)**(3/2) + 4*d*(d + e*x)**(5/2)/5 - (d + e*x)**
(7/2)/7)/e**4 + 2*c**2*(-d**5/sqrt(d + e*x) - 5*d**4*sqrt(d + e*x) + 10*d**
3*(d + e*x)**(3/2)/3 - 2*d**2*(d + e*x)**(5/2) + 5*d*(d + e*x)**(7/2)/7 - (
d + e*x)**(9/2)/9)/e**4)/e, Ne(e, 0)), ((a**2*x + 2*a*c*x**3/3 + c**2*x**5/
5)/sqrt(d), True))
```

Giac [A] time = 1.23855, size = 170, normalized size = 1.36

$$\frac{2}{315}\left(42\left(3\,(xe+d)^{\frac{5}{2}}-10\,(xe+d)^{\frac{3}{2}}d+15\,\sqrt{xe+d}d^2\right)ace^{(-2)}+\left(35\,(xe+d)^{\frac{9}{2}}-180\,(xe+d)^{\frac{7}{2}}d+378\,(xe+d)^{\frac{5}{2}}d^2-420\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(42*(3*(x*e + d)^(5/2) - 10*(x*e + d)^(3/2)*d + 15*sqrt(x*e + d)*d^2)
*a*c*e^(-2) + (35*(x*e + d)^(9/2) - 180*(x*e + d)^(7/2)*d + 378*(x*e + d)^(
5/2)*d^2 - 420*(x*e + d)^(3/2)*d^3 + 315*sqrt(x*e + d)*d^4)*c^2*e^(-4) + 31
5*sqrt(x*e + d)*a^2)*e^(-1)
```

3.602 $\int \frac{(a+cx^2)^2}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=123

$$\frac{4c(d+ex)^{3/2}(ae^2+3cd^2)}{3e^5}-\frac{8cd\sqrt{d+ex}(ae^2+cd^2)}{e^5}-\frac{2(ae^2+cd^2)^2}{e^5\sqrt{d+ex}}+\frac{2c^2(d+ex)^{7/2}}{7e^5}-\frac{8c^2d(d+ex)^{5/2}}{5e^5}$$

```
[Out] (-2*(c*d^2 + a*e^2)^2)/(e^5*Sqrt[d + e*x]) - (8*c*d*(c*d^2 + a*e^2)*Sqrt[d
+ e*x])/e^5 + (4*c*(3*c*d^2 + a*e^2)*(d + e*x)^(3/2))/(3*e^5) - (8*c^2*d*(d
 + e*x)^(5/2))/(5*e^5) + (2*c^2*(d + e*x)^(7/2))/(7*e^5)
```

Rubi [A] time = 0.0472858, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {697}

$$\frac{4c(d+ex)^{3/2}(ae^2+3cd^2)}{3e^5}-\frac{8cd\sqrt{d+ex}(ae^2+cd^2)}{e^5}-\frac{2(ae^2+cd^2)^2}{e^5\sqrt{d+ex}}+\frac{2c^2(d+ex)^{7/2}}{7e^5}-\frac{8c^2d(d+ex)^{5/2}}{5e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^2/(d + e*x)^(3/2),x]
```

```
[Out] (-2*(c*d^2 + a*e^2)^2)/(e^5*Sqrt[d + e*x]) - (8*c*d*(c*d^2 + a*e^2)*Sqrt[d
+ e*x])/e^5 + (4*c*(3*c*d^2 + a*e^2)*(d + e*x)^(3/2))/(3*e^5) - (8*c^2*d*(d
 + e*x)^(5/2))/(5*e^5) + (2*c^2*(d + e*x)^(7/2))/(7*e^5)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+cx^2)^2}{(d+ex)^{3/2}}\,dx &= \int \left(\frac{(cd^2+ae^2)^2}{e^4(d+ex)^{3/2}}-\frac{4cd(cd^2+ae^2)}{e^4\sqrt{d+ex}}+\frac{2c(3cd^2+ae^2)\sqrt{d+ex}}{e^4}-\frac{4c^2d(d+ex)^{3/2}}{e^4}+\frac{c^2(d+ex)^{5/2}}{e^4}\right)dx\\ &= -\frac{2(cd^2+ae^2)^2}{e^5\sqrt{d+ex}}-\frac{8cd(cd^2+ae^2)\sqrt{d+ex}}{e^5}+\frac{4c(3cd^2+ae^2)(d+ex)^{3/2}}{3e^5}-\frac{8c^2d(d+ex)^{5/2}}{5e^5}+\frac{2c^2(d+ex)^{7/2}}{7e^5}\end{aligned}$$

Mathematica [A] time = 0.0559045, size = 97, normalized size = 0.79

$$-\frac{2\left(105a^2e^4+70ace^2\left(8d^2+4dex-e^2x^2\right)+3c^2\left(-16d^2e^2x^2+64d^3ex+128d^4+8de^3x^3-5e^4x^4\right)\right)}{105e^5\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^2/(d + e*x)^(3/2),x]
```

```
[Out] (-2*(105*a^2*e^4 + 70*a*c*e^2*(8*d^2 + 4*d*e*x - e^2*x^2) + 3*c^2*(128*d^4
+ 64*d^3*e*x - 16*d^2*e^2*x^2 + 8*d*e^3*x^3 - 5*e^4*x^4)))/(105*e^5*Sqrt[d
+ e*x])
```

Maple [A] time = 0.043, size = 106, normalized size = 0.9

$$-\frac{-30\,c^2x^4e^4+48\,c^2dx^3e^3-140\,ace^4x^2-96\,c^2d^2e^2x^2+560\,acde^3x+384\,c^2d^3ex+210\,a^2e^4+1120\,acd^2e^2+768\,c^2d^4}{105\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2/(e*x+d)^(3/2),x)
```

```
[Out] -2/105/(e*x+d)^(1/2)*(-15*c^2*e^4*x^4+24*c^2*d*e^3*x^3-70*a*c*e^4*x^2-48*c^
2*d^2*e^2*x^2+280*a*c*d*e^3*x+192*c^2*d^3*e*x+105*a^2*e^4+560*a*c*d^2*e^2+3
84*c^2*d^4)/e^5
```

Maxima [A] time = 1.10377, size = 163, normalized size = 1.33

$$\frac{2\left(\frac{15\,(ex+d)^{\frac{7}{2}}c^2-84\,(ex+d)^{\frac{5}{2}}c^2d+70\left(3\,c^2d^2+ace^2\right)(ex+d)^{\frac{3}{2}}-420\left(c^2d^3+acde^2\right)\sqrt{ex+d}}{e^4}-\frac{105\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}{\sqrt{ex+d}e^4}\right)}{105\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/105*((15*(e*x + d)^(7/2)*c^2 - 84*(e*x + d)^(5/2)*c^2*d + 70*(3*c^2*d^2 +
 a*c*e^2)*(e*x + d)^(3/2) - 420*(c^2*d^3 + a*c*d*e^2)*sqrt(e*x + d))/e^4 -
105*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)/(sqrt(e*x + d)*e^4))/e
```

Fricas [A] time = 1.69958, size = 261, normalized size = 2.12

$$\frac{2\left(15\,c^2e^4x^4-24\,c^2de^3x^3-384\,c^2d^4-560\,acd^2e^2-105\,a^2e^4+2\left(24\,c^2d^2e^2+35\,ace^4\right)x^2-8\left(24\,c^2d^3e+35\,acde^3\right)x\right)\sqrt{}}{105\left(e^6x+de^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*c^2*e^4*x^4 - 24*c^2*d*e^3*x^3 - 384*c^2*d^4 - 560*a*c*d^2*e^2 -
105*a^2*e^4 + 2*(24*c^2*d^2*e^2 + 35*a*c*e^4)*x^2 - 8*(24*c^2*d^3*e + 35*a*
c*d*e^3)*x)*sqrt(e*x + d)/(e^6*x + d*e^5)
```

Sympy [A] time = 12.2785, size = 126, normalized size = 1.02

$$-\frac{8c^2d\,(d+ex)^{\frac{5}{2}}}{5e^5}+\frac{2c^2\,(d+ex)^{\frac{7}{2}}}{7e^5}+\frac{(d+ex)^{\frac{3}{2}}\left(4ace^2+12c^2d^2\right)}{3e^5}+\frac{\sqrt{d+ex}\left(-8acde^2-8c^2d^3\right)}{e^5}-\frac{2\left(ae^2+cd^2\right)^2}{e^5\sqrt{d+ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2/(e*x+d)**(3/2),x)
```

```
[Out] -8*c**2*d*(d + e*x)**(5/2)/(5*e**5) + 2*c**2*(d + e*x)**(7/2)/(7*e**5) + (d
 + e*x)**(3/2)*(4*a*c*e**2 + 12*c**2*d**2)/(3*e**5) + sqrt(d + e*x)*(-8*a*c
*d*e**2 - 8*c**2*d**3)/e**5 - 2*(a*e**2 + c*d**2)**2/(e**5*sqrt(d + e*x))
```

Giac [A] time = 1.30337, size = 185, normalized size = 1.5

$$\frac{2}{105}\left(15\,(xe+d)^{\frac{7}{2}}c^2e^{30} - 84\,(xe+d)^{\frac{5}{2}}c^2de^{30} + 210\,(xe+d)^{\frac{3}{2}}c^2d^2e^{30} - 420\,\sqrt{xe+d}c^2d^3e^{30} + 70\,(xe+d)^{\frac{3}{2}}ace^{32} - 420\,\sqrt{xe\cdot}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] 2/105*(15*(x*e + d)^(7/2)*c^2*e^30 - 84*(x*e + d)^(5/2)*c^2*d*e^30 + 210*(x
*e + d)^(3/2)*c^2*d^2*e^30 - 420*sqrt(x*e + d)*c^2*d^3*e^30 + 70*(x*e + d)^
(3/2)*a*c*e^32 - 420*sqrt(x*e + d)*a*c*d*e^32)*e^(-35) - 2*(c^2*d^4 + 2*a*c
*d^2*e^2 + a^2*e^4)*e^(-5)/sqrt(x*e + d)
```

3.603 $\int \frac{(a+cx^2)^2}{(d+ex)^{5/2}} dx$

Optimal. Leaf size=123

$$\frac{4c\sqrt{d+ex}\left(ae^2+3cd^2\right)}{e^5}+\frac{8cd\left(ae^2+cd^2\right)}{e^5\sqrt{d+ex}}-\frac{2\left(ae^2+cd^2\right)^2}{3e^5(d+ex)^{3/2}}+\frac{2c^2(d+ex)^{5/2}}{5e^5}-\frac{8c^2d(d+ex)^{3/2}}{3e^5}$$

```
[Out] (-2*(c*d^2 + a*e^2)^2)/(3*e^5*(d + e*x)^(3/2)) + (8*c*d*(c*d^2 + a*e^2))/(e
^5*Sqrt[d + e*x]) + (4*c*(3*c*d^2 + a*e^2)*Sqrt[d + e*x])/e^5 - (8*c^2*d*(d
 + e*x)^(3/2))/(3*e^5) + (2*c^2*(d + e*x)^(5/2))/(5*e^5)
```

Rubi [A] time = 0.0472387, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {697}

$$\frac{4c\sqrt{d+ex}\left(ae^2+3cd^2\right)}{e^5}+\frac{8cd\left(ae^2+cd^2\right)}{e^5\sqrt{d+ex}}-\frac{2\left(ae^2+cd^2\right)^2}{3e^5(d+ex)^{3/2}}+\frac{2c^2(d+ex)^{5/2}}{5e^5}-\frac{8c^2d(d+ex)^{3/2}}{3e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^2/(d + e*x)^(5/2),x]
```

```
[Out] (-2*(c*d^2 + a*e^2)^2)/(3*e^5*(d + e*x)^(3/2)) + (8*c*d*(c*d^2 + a*e^2))/(e
^5*Sqrt[d + e*x]) + (4*c*(3*c*d^2 + a*e^2)*Sqrt[d + e*x])/e^5 - (8*c^2*d*(d
 + e*x)^(3/2))/(3*e^5) + (2*c^2*(d + e*x)^(5/2))/(5*e^5)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+cx^2\right)^2}{(d+ex)^{5/2}}\,dx &= \int \left(\frac{\left(cd^2+ae^2\right)^2}{e^4(d+ex)^{5/2}}-\frac{4cd\left(cd^2+ae^2\right)}{e^4(d+ex)^{3/2}}+\frac{2c\left(3cd^2+ae^2\right)}{e^4\sqrt{d+ex}}-\frac{4c^2d\sqrt{d+ex}}{e^4}+\frac{c^2(d+ex)^{3/2}}{e^4}\right)dx\\ &= -\frac{2\left(cd^2+ae^2\right)^2}{3e^5(d+ex)^{3/2}}+\frac{8cd\left(cd^2+ae^2\right)}{e^5\sqrt{d+ex}}+\frac{4c\left(3cd^2+ae^2\right)\sqrt{d+ex}}{e^5}-\frac{8c^2d(d+ex)^{3/2}}{3e^5}+\frac{2c^2(d+ex)^{5/2}}{5e^5}\end{aligned}$$

Mathematica [A] time = 0.0571416, size = 96, normalized size = 0.78

$$\frac{2\left(-5a^2e^4+10ace^2\left(8d^2+12dex+3e^2x^2\right)+c^2\left(48d^2e^2x^2+192d^3ex+128d^4-8de^3x^3+3e^4x^4\right)\right)}{15e^5(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^2/(d + e*x)^(5/2),x]
```

```
[Out] (2*(-5*a^2*e^4 + 10*a*c*e^2*(8*d^2 + 12*d*e*x + 3*e^2*x^2) + c^2*(128*d^4 +
 192*d^3*e*x + 48*d^2*e^2*x^2 - 8*d*e^3*x^3 + 3*e^4*x^4)))/(15*e^5*(d + e*x
```

```
)^(3/2))
```

Maple [A] time = 0.044, size = 106, normalized size = 0.9

$$-\frac{-6c^2x^4e^4+16c^2dx^3e^3-60ace^4x^2-96c^2d^2e^2x^2-240acde^3x-384c^2d^3ex+10a^2e^4-160acd^2e^2-256c^2d^4}{15e^5}(ex+d)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2/(e*x+d)^(5/2),x)
```

```
[Out] -2/15/(e*x+d)^(3/2)*(-3*c^2*e^4*x^4+8*c^2*d*e^3*x^3-30*a*c*e^4*x^2-48*c^2*d
^2*e^2*x^2-120*a*c*d*e^3*x-192*c^2*d^3*e*x+5*a^2*e^4-80*a*c*d^2*e^2-128*c^2
*d^4)/e^5
```

Maxima [A] time = 1.18341, size = 161, normalized size = 1.31

$$\frac{2\left(\frac{3(ex+d)^{\frac{5}{2}}c^2-20(ex+d)^{\frac{3}{2}}c^2d+30\left(3c^2d^2+ace^2\right)\sqrt{ex+d}}{e^4}-\frac{5\left(c^2d^4+2acd^2e^2+a^2e^4-12\left(c^2d^3+acde^2\right)(ex+d)\right)}{(ex+d)^{\frac{3}{2}}e^4}\right)}{15e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*((3*(e*x + d)^(5/2)*c^2 - 20*(e*x + d)^(3/2)*c^2*d + 30*(3*c^2*d^2 + a
*c*e^2)*sqrt(e*x + d))/e^4 - 5*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4 - 12*(c^2
*d^3 + a*c*d*e^2)*(e*x + d))/((e*x + d)^(3/2)*e^4))/e
```

Fricas [A] time = 1.77229, size = 270, normalized size = 2.2

$$\frac{2\left(3c^2e^4x^4-8c^2de^3x^3+128c^2d^4+80acd^2e^2-5a^2e^4+6\left(8c^2d^2e^2+5ace^4\right)x^2+24\left(8c^2d^3e+5acde^3\right)x\right)\sqrt{ex+d}}{15\left(e^7x^2+2de^6x+d^2e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*c^2*e^4*x^4 - 8*c^2*d*e^3*x^3 + 128*c^2*d^4 + 80*a*c*d^2*e^2 - 5*a^
2*e^4 + 6*(8*c^2*d^2*e^2 + 5*a*c*e^4)*x^2 + 24*(8*c^2*d^3*e + 5*a*c*d*e^3)*
x)*sqrt(e*x + d)/(e^7*x^2 + 2*d*e^6*x + d^2*e^5)
```

Sympy [A] time = 19.936, size = 121, normalized size = 0.98

$$-\frac{8c^2d(d+ex)^{\frac{3}{2}}}{3e^5}+\frac{2c^2(d+ex)^{\frac{5}{2}}}{5e^5}+\frac{8cd\left(ae^2+cd^2\right)}{e^5\sqrt{d+ex}}+\frac{\sqrt{d+ex}\left(4ace^2+12c^2d^2\right)}{e^5}-\frac{2\left(ae^2+cd^2\right)^2}{3e^5(d+ex)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2/(e*x+d)**(5/2),x)
```

```
[Out] -8*c**2*d*(d + e*x)**(3/2)/(3*e**5) + 2*c**2*(d + e*x)**(5/2)/(5*e**5) + 8*
c*d*(a*e**2 + c*d**2)/(e**5*sqrt(d + e*x)) + sqrt(d + e*x)*(4*a*c*e**2 + 12
*c**2*d**2)/e**5 - 2*(a*e**2 + c*d**2)**2/(3*e**5*(d + e*x)**(3/2))
```

Giac [A] time = 1.32235, size = 180, normalized size = 1.46

$$\frac{2}{15}\left(3\,(xe+d)^{\frac{5}{2}}c^2e^{20} - 20\,(xe+d)^{\frac{3}{2}}c^2de^{20} + 90\,\sqrt{xe+d}c^2d^2e^{20} + 30\,\sqrt{xe+d}ace^{22}\right)e^{(-25)} + \frac{2\left(12\,(xe+d)c^2d^3 - c^2d^4 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*(x*e + d)^(5/2)*c^2*e^20 - 20*(x*e + d)^(3/2)*c^2*d*e^20 + 90*sqrt(
x*e + d)*c^2*d^2*e^20 + 30*sqrt(x*e + d)*a*c*e^22)*e^(-25) + 2/3*(12*(x*e +
 d)*c^2*d^3 - c^2*d^4 + 12*(x*e + d)*a*c*d*e^2 - 2*a*c*d^2*e^2 - a^2*e^4)*e
^(-5)/(x*e + d)^(3/2)
```

3.604 $\int \frac{(a+cx^2)^2}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{4c\left(ae^2+3cd^2\right)}{e^5\sqrt{d+ex}}+\frac{8cd\left(ae^2+cd^2\right)}{3e^5(d+ex)^{3/2}}-\frac{2\left(ae^2+cd^2\right)^2}{5e^5(d+ex)^{5/2}}+\frac{2c^2(d+ex)^{3/2}}{3e^5}-\frac{8c^2d\sqrt{d+ex}}{e^5}$$

```
[Out] (-2*(c*d^2 + a*e^2)^2)/(5*e^5*(d + e*x)^(5/2)) + (8*c*d*(c*d^2 + a*e^2))/(3
*e^5*(d + e*x)^(3/2)) - (4*c*(3*c*d^2 + a*e^2))/(e^5*Sqrt[d + e*x]) - (8*c^
2*d*Sqrt[d + e*x])/e^5 + (2*c^2*(d + e*x)^(3/2))/(3*e^5)
```

Rubi [A] time = 0.046406, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {697}

$$-\frac{4c\left(ae^2+3cd^2\right)}{e^5\sqrt{d+ex}}+\frac{8cd\left(ae^2+cd^2\right)}{3e^5(d+ex)^{3/2}}-\frac{2\left(ae^2+cd^2\right)^2}{5e^5(d+ex)^{5/2}}+\frac{2c^2(d+ex)^{3/2}}{3e^5}-\frac{8c^2d\sqrt{d+ex}}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^2/(d + e*x)^(7/2),x]
```

```
[Out] (-2*(c*d^2 + a*e^2)^2)/(5*e^5*(d + e*x)^(5/2)) + (8*c*d*(c*d^2 + a*e^2))/(3
*e^5*(d + e*x)^(3/2)) - (4*c*(3*c*d^2 + a*e^2))/(e^5*Sqrt[d + e*x]) - (8*c^
2*d*Sqrt[d + e*x])/e^5 + (2*c^2*(d + e*x)^(3/2))/(3*e^5)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+cx^2\right)^2}{(d+ex)^{7/2}}\,dx &= \int \left(\frac{\left(cd^2+ae^2\right)^2}{e^4(d+ex)^{7/2}}-\frac{4cd\left(cd^2+ae^2\right)}{e^4(d+ex)^{5/2}}+\frac{2c\left(3cd^2+ae^2\right)}{e^4(d+ex)^{3/2}}-\frac{4c^2d}{e^4\sqrt{d+ex}}+\frac{c^2\sqrt{d+ex}}{e^4}\right)dx\\ &= -\frac{2\left(cd^2+ae^2\right)^2}{5e^5(d+ex)^{5/2}}+\frac{8cd\left(cd^2+ae^2\right)}{3e^5(d+ex)^{3/2}}-\frac{4c\left(3cd^2+ae^2\right)}{e^5\sqrt{d+ex}}-\frac{8c^2d\sqrt{d+ex}}{e^5}+\frac{2c^2(d+ex)^{3/2}}{3e^5}\end{aligned}$$

Mathematica [A] time = 0.0636942, size = 96, normalized size = 0.78

$$-\frac{2\left(3a^2e^4+2ace^2\left(8d^2+20dex+15e^2x^2\right)+c^2\left(240d^2e^2x^2+320d^3ex+128d^4+40de^3x^3-5e^4x^4\right)\right)}{15e^5(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^2/(d + e*x)^(7/2),x]
```

```
[Out] (-2*(3*a^2*e^4 + 2*a*c*e^2*(8*d^2 + 20*d*e*x + 15*e^2*x^2) + c^2*(128*d^4 +
 320*d^3*e*x + 240*d^2*e^2*x^2 + 40*d*e^3*x^3 - 5*e^4*x^4)))/(15*e^5*(d + e
```

```
*x)^(5/2))
```

Maple [A] time = 0.044, size = 106, normalized size = 0.9

$$-\frac{-10\,c^2x^4e^4+80\,c^2dx^3e^3+60\,ace^4x^2+480\,c^2d^2e^2x^2+80\,acde^3x+640\,c^2d^3ex+6\,a^2e^4+32\,acd^2e^2+256\,c^2d^4}{15\,e^5}(ex+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2/(e*x+d)^(7/2),x)
```

```
[Out] -2/15/(e*x+d)^(5/2)*(-5*c^2*e^4*x^4+40*c^2*d*e^3*x^3+30*a*c*e^4*x^2+240*c^2
*d^2*e^2*x^2+40*a*c*d*e^3*x+320*c^2*d^3*e*x+3*a^2*e^4+16*a*c*d^2*e^2+128*c^
2*d^4)/e^5
```

Maxima [A] time = 1.1731, size = 163, normalized size = 1.33

$$\frac{2\left(\frac{5\left((ex+d)^{\frac{3}{2}}c^2-12\sqrt{ex+d}c^2d\right)}{e^4}-\frac{3\,c^2d^4+6\,acd^2e^2+3\,a^2e^4+30\left(3\,c^2d^2+ace^2\right)(ex+d)^2-20\left(c^2d^3+acde^2\right)(ex+d)}{(ex+d)^{\frac{5}{2}}e^4}\right)}{15\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*(5*((e*x + d)^(3/2)*c^2 - 12*sqrt(e*x + d)*c^2*d)/e^4 - (3*c^2*d^4 + 6
*a*c*d^2*e^2 + 3*a^2*e^4 + 30*(3*c^2*d^2 + a*c*e^2)*(e*x + d)^2 - 20*(c^2*d
^3 + a*c*d*e^2)*(e*x + d))/((e*x + d)^(5/2)*e^4))/e
```

Fricas [A] time = 1.86978, size = 289, normalized size = 2.35

$$\frac{2\left(5\,c^2e^4x^4-40\,c^2de^3x^3-128\,c^2d^4-16\,acd^2e^2-3\,a^2e^4-30\left(8\,c^2d^2e^2+ace^4\right)x^2-40\left(8\,c^2d^3e+acde^3\right)x\right)\sqrt{ex+d}}{15\left(e^8x^3+3\,de^7x^2+3\,d^2e^6x+d^3e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(5*c^2*e^4*x^4 - 40*c^2*d*e^3*x^3 - 128*c^2*d^4 - 16*a*c*d^2*e^2 - 3*a
^2*e^4 - 30*(8*c^2*d^2*e^2 + a*c*e^4)*x^2 - 40*(8*c^2*d^3*e + a*c*d*e^3)*x)
*sqrt(e*x + d)/(e^8*x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3*e^5)
```

Sympy [A] time = 3.14516, size = 592, normalized size = 4.81

$$\begin{cases} -\frac{6a^2e^4}{15d^2e^5\sqrt{d+ex}+30de^6x\sqrt{d+ex}+15e^7x^2\sqrt{d+ex}}-\frac{32acd^2e^2}{15d^2e^5\sqrt{d+ex}+30de^6x\sqrt{d+ex}+15e^7x^2\sqrt{d+ex}}-\frac{80acde^3x}{15d^2e^5\sqrt{d+ex}+30de^6x\sqrt{d+ex}+15e^7x^2\sqrt{d+ex}}-\frac{}{15d^2e^5} \\ \frac{a^2x+\frac{2acx^3}{3}+\frac{c^2x^5}{5}}{d^{\frac{7}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2/(e*x+d)**(7/2),x)
```

```
[Out] Piecewise((-6*a**2*e**4/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d +
e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 32*a*c*d**2*e**2/(15*d**2*e**5*sqrt(d
+ e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 80*a*c*d
*e**3*x/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x
**2*sqrt(d + e*x)) - 60*a*c*e**4*x**2/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e*
*6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 256*c**2*d**4/(15*d**2*e
**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x))
 - 640*c**2*d**3*e*x/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x
) + 15*e**7*x**2*sqrt(d + e*x)) - 480*c**2*d**2*e**2*x**2/(15*d**2*e**5*sqr
t(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 80*c
**2*d*e**3*x**3/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 1
5*e**7*x**2*sqrt(d + e*x)) + 10*c**2*e**4*x**4/(15*d**2*e**5*sqrt(d + e*x)
+ 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)), Ne(e, 0)), ((a**
2*x + 2*a*c*x**3/3 + c**2*x**5/5)/d**(7/2), True))
```

Giac [A] time = 2.10165, size = 176, normalized size = 1.43

$$\frac{2}{3}\left((xe+d)^{\frac{3}{2}}c^2e^{10}-12\sqrt{xe+d}c^2de^{10}\right)e^{(-15)}-\frac{2\left(90\,(xe+d)^2c^2d^2-20\,(xe+d)c^2d^3+3\,c^2d^4+30\,(xe+d)^2ace^2-20\,(xe+\right.}{15\,(xe+d)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] 2/3*((x*e + d)^(3/2)*c^2*e^10 - 12*sqrt(x*e + d)*c^2*d*e^10)*e^(-15) - 2/15
*(90*(x*e + d)^2*c^2*d^2 - 20*(x*e + d)*c^2*d^3 + 3*c^2*d^4 + 30*(x*e + d)^
2*a*c*e^2 - 20*(x*e + d)*a*c*d*e^2 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*e^(-5)/(x*e
 + d)^(5/2)
```

3.605 $\int (d+ex)^{5/2} \left(a+cx^2\right)^3 \, dx$

Optimal. Leaf size=204

$$\frac{2c^2(d+ex)^{15/2}\left(ae^2+5cd^2\right)}{5e^7}-\frac{8c^2d(d+ex)^{13/2}\left(3ae^2+5cd^2\right)}{13e^7}+\frac{6c(d+ex)^{11/2}\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{11e^7}-\frac{4cd(d+ex)^{9/...}}{3...}$$

```
[Out] (2*(c*d^2 + a*e^2)^3*(d + e*x)^(7/2))/(7*e^7) - (4*c*d*(c*d^2 + a*e^2)^2*(d
 + e*x)^(9/2))/(3*e^7) + (6*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2)*(d + e*x)^(
11/2))/(11*e^7) - (8*c^2*d*(5*c*d^2 + 3*a*e^2)*(d + e*x)^(13/2))/(13*e^7) +
 (2*c^2*(5*c*d^2 + a*e^2)*(d + e*x)^(15/2))/(5*e^7) - (12*c^3*d*(d + e*x)^(
17/2))/(17*e^7) + (2*c^3*(d + e*x)^(19/2))/(19*e^7)
```

Rubi [A] time = 0.102997, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {697}

$$\frac{2c^2(d+ex)^{15/2}\left(ae^2+5cd^2\right)}{5e^7}-\frac{8c^2d(d+ex)^{13/2}\left(3ae^2+5cd^2\right)}{13e^7}+\frac{6c(d+ex)^{11/2}\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{11e^7}-\frac{4cd(d+ex)^{9/...}}{3...}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)*(a + c*x^2)^3,x]
```

```
[Out] (2*(c*d^2 + a*e^2)^3*(d + e*x)^(7/2))/(7*e^7) - (4*c*d*(c*d^2 + a*e^2)^2*(d
 + e*x)^(9/2))/(3*e^7) + (6*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2)*(d + e*x)^(
11/2))/(11*e^7) - (8*c^2*d*(5*c*d^2 + 3*a*e^2)*(d + e*x)^(13/2))/(13*e^7) +
 (2*c^2*(5*c*d^2 + a*e^2)*(d + e*x)^(15/2))/(5*e^7) - (12*c^3*d*(d + e*x)^(
17/2))/(17*e^7) + (2*c^3*(d + e*x)^(19/2))/(19*e^7)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (d+ex)^{5/2}\left(a+cx^2\right)^3 dx = \int \left(\frac{\left(cd^2+ae^2\right)^3(d+ex)^{5/2}}{e^6}-\frac{6cd\left(cd^2+ae^2\right)^2(d+ex)^{7/2}}{e^6}+\frac{3c\left(cd^2+ae^2\right)\left(5cd^2+ae^2\right)...}{e^6}\right.$$

$$=\frac{2\left(cd^2+ae^2\right)^3(d+ex)^{7/2}}{7e^7}-\frac{4cd\left(cd^2+ae^2\right)^2(d+ex)^{9/2}}{3e^7}+\frac{6c\left(cd^2+ae^2\right)\left(5cd^2+ae^2\right)(d...}{11e^7}$$

Mathematica [A] time = 0.254191, size = 188, normalized size = 0.92

$$\frac{2\left(\frac{1}{5}c^2(d+ex)^{15/2}\left(ae^2+5cd^2\right)-\frac{4}{13}c^2d(d+ex)^{13/2}\left(3ae^2+5cd^2\right)+\frac{3}{11}c(d+ex)^{11/2}\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)-\frac{2}{3}cd(d+...}{e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)*(a + c*x^2)^3,x]
```

```
[Out] (2*(((c*d^2 + a*e^2)^3*(d + e*x)^(7/2))/7 - (2*c*d*(c*d^2 + a*e^2)^2*(d + e
*x)^(9/2))/3 + (3*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2)*(d + e*x)^(11/2))/11
- (4*c^2*d*(5*c*d^2 + 3*a*e^2)*(d + e*x)^(13/2))/13 + (c^2*(5*c*d^2 + a*e^2
)*(d + e*x)^(15/2))/5 - (6*c^3*d*(d + e*x)^(17/2))/17 + (c^3*(d + e*x)^(19/
2))/19))/e^7
```

Maple [A] time = 0.044, size = 205, normalized size = 1.

$$\frac{510510\,c^3x^6e^6 - 360360\,c^3dx^5e^5 + 1939938\,ac^2e^6x^4 + 240240\,c^3d^2e^4x^4 - 1193808\,ac^2de^5x^3 - 147840\,c^3d^3e^3x^3 + 2645370\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(c*x^2+a)^3,x)
```

```
[Out] 2/4849845*(e*x+d)^(7/2)*(255255*c^3*e^6*x^6-180180*c^3*d*e^5*x^5+969969*a*c
^2*e^6*x^4+120120*c^3*d^2*e^4*x^4-596904*a*c^2*d*e^5*x^3-73920*c^3*d^3*e^3*
x^3+1322685*a^2*c*e^6*x^2+325584*a*c^2*d^2*e^4*x^2+40320*c^3*d^4*e^2*x^2-58
7860*a^2*c*d*e^5*x-144704*a*c^2*d^3*e^3*x-17920*c^3*d^5*e*x+692835*a^3*e^6+
167960*a^2*c*d^2*e^4+41344*a*c^2*d^4*e^2+5120*c^3*d^6)/e^7
```

Maxima [A] time = 1.18694, size = 282, normalized size = 1.38

$$\frac{2\left(255255\,(ex+d)^{\frac{19}{2}}c^3 - 1711710\,(ex+d)^{\frac{17}{2}}c^3d + 969969\left(5\,c^3d^2 + ac^2e^2\right)(ex+d)^{\frac{15}{2}} - 1492260\left(5\,c^3d^3 + 3\,ac^2de^2\right)(ex+\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 2/4849845*(255255*(e*x + d)^(19/2)*c^3 - 1711710*(e*x + d)^(17/2)*c^3*d + 9
69969*(5*c^3*d^2 + a*c^2*e^2)*(e*x + d)^(15/2) - 1492260*(5*c^3*d^3 + 3*a*c
^2*d*e^2)*(e*x + d)^(13/2) + 1322685*(5*c^3*d^4 + 6*a*c^2*d^2*e^2 + a^2*c*e
^4)*(e*x + d)^(11/2) - 3233230*(c^3*d^5 + 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*(e
*x + d)^(9/2) + 692835*(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e
^6)*(e*x + d)^(7/2))/e^7
```

Fricas [B] time = 1.82979, size = 863, normalized size = 4.23

$$\frac{2\left(255255\,c^3e^9x^9 + 585585\,c^3de^8x^8 + 5120\,c^3d^9 + 41344\,ac^2d^7e^2 + 167960\,a^2cd^5e^4 + 692835\,a^3d^3e^6 + 3003\left(115\,c^3d^2e^7 + \cdots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 2/4849845*(255255*c^3*e^9*x^9 + 585585*c^3*d*e^8*x^8 + 5120*c^3*d^9 + 41344
*a*c^2*d^7*e^2 + 167960*a^2*c*d^5*e^4 + 692835*a^3*d^3*e^6 + 3003*(115*c^3*
d^2*e^7 + 323*a*c^2*e^9)*x^7 + 231*(5*c^3*d^3*e^6 + 10013*a*c^2*d*e^8)*x^6
- 63*(20*c^3*d^4*e^5 - 22933*a*c^2*d^2*e^7 - 20995*a^2*c*e^9)*x^5 + 35*(40*
c^3*d^5*e^4 + 323*a*c^2*d^3*e^6 + 96577*a^2*c*d*e^8)*x^4 - 5*(320*c^3*d^6*e
^3 + 2584*a*c^2*d^4*e^5 - 474487*a^2*c*d^2*e^7 - 138567*a^3*e^9)*x^3 + 3*(6
```

```
40*c^3*d^7*e^2 + 5168*a*c^2*d^5*e^4 + 20995*a^2*c*d^3*e^6 + 692835*a^3*d*e^
8)*x^2 - (2560*c^3*d^8*e + 20672*a*c^2*d^6*e^3 + 83980*a^2*c*d^4*e^5 - 2078
505*a^3*d^2*e^7)*x)*sqrt(e*x + d)/e^7
```

Sympy [A] time = 27.7549, size = 945, normalized size = 4.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(c*x**2+a)**3,x)
```

```
[Out] a**3*d**2*Piecewise((sqrt(d)*x, Eq(e, 0)), (2*(d + e*x)**(3/2)/(3*e), True)
) + 4*a**3*d*(-d*(d + e*x)**(3/2)/3 + (d + e*x)**(5/2)/5)/e + 2*a**3*(d**2*
(d + e*x)**(3/2)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e + 6*a**
2*c*d**2*(d**2*(d + e*x)**(3/2)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/
2)/7)/e**3 + 12*a**2*c*d*(-d**3*(d + e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2
)/5 - 3*d*(d + e*x)**(7/2)/7 + (d + e*x)**(9/2)/9)/e**3 + 6*a**2*c*(d**4*(d
 + e*x)**(3/2)/3 - 4*d**3*(d + e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 -
4*d*(d + e*x)**(9/2)/9 + (d + e*x)**(11/2)/11)/e**3 + 6*a*c**2*d**2*(d**4*(
d + e*x)**(3/2)/3 - 4*d**3*(d + e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 -
 4*d*(d + e*x)**(9/2)/9 + (d + e*x)**(11/2)/11)/e**5 + 12*a*c**2*d*(-d**5*(
d + e*x)**(3/2)/3 + d**4*(d + e*x)**(5/2) - 10*d**3*(d + e*x)**(7/2)/7 + 10
*d**2*(d + e*x)**(9/2)/9 - 5*d*(d + e*x)**(11/2)/11 + (d + e*x)**(13/2)/13)
/e**5 + 6*a*c**2*(d**6*(d + e*x)**(3/2)/3 - 6*d**5*(d + e*x)**(5/2)/5 + 15*
d**4*(d + e*x)**(7/2)/7 - 20*d**3*(d + e*x)**(9/2)/9 + 15*d**2*(d + e*x)**(
11/2)/11 - 6*d*(d + e*x)**(13/2)/13 + (d + e*x)**(15/2)/15)/e**5 + 2*c**3*d
**2*(d**6*(d + e*x)**(3/2)/3 - 6*d**5*(d + e*x)**(5/2)/5 + 15*d**4*(d + e*x
)**(7/2)/7 - 20*d**3*(d + e*x)**(9/2)/9 + 15*d**2*(d + e*x)**(11/2)/11 - 6*
d*(d + e*x)**(13/2)/13 + (d + e*x)**(15/2)/15)/e**7 + 4*c**3*d*(-d**7*(d +
e*x)**(3/2)/3 + 7*d**6*(d + e*x)**(5/2)/5 - 3*d**5*(d + e*x)**(7/2) + 35*d*
*4*(d + e*x)**(9/2)/9 - 35*d**3*(d + e*x)**(11/2)/11 + 21*d**2*(d + e*x)**(
13/2)/13 - 7*d*(d + e*x)**(15/2)/15 + (d + e*x)**(17/2)/17)/e**7 + 2*c**3*(
d**8*(d + e*x)**(3/2)/3 - 8*d**7*(d + e*x)**(5/2)/5 + 4*d**6*(d + e*x)**(7/
2) - 56*d**5*(d + e*x)**(9/2)/9 + 70*d**4*(d + e*x)**(11/2)/11 - 56*d**3*(d
 + e*x)**(13/2)/13 + 28*d**2*(d + e*x)**(15/2)/15 - 8*d*(d + e*x)**(17/2)/1
7 + (d + e*x)**(19/2)/19)/e**7
```

Giac [B] time = 1.43203, size = 1129, normalized size = 5.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 2/14549535*(415701*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d
)^(3/2)*d^2)*a^2*c*d^2*e^(-2) + 12597*(315*(x*e + d)^(11/2) - 1540*(x*e + d
)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e
 + d)^(3/2)*d^4)*a*c^2*d^2*e^(-4) + 323*(3003*(x*e + d)^(15/2) - 20790*(x*e
 + d)^(13/2)*d + 61425*(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 +
96525*(x*e + d)^(7/2)*d^4 - 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/
2)*d^6)*c^3*d^2*e^(-6) + 4849845*(x*e + d)^(3/2)*a^3*d^2 + 277134*(35*(x*e
+ d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d
```

```
)^(3/2)*d^3)*a^2*c*d*e^(-2) + 9690*(693*(x*e + d)^(13/2) - 4095*(x*e + d)^(
11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e
 + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*a*c^2*d*e^(-4) + 266*(6435*(x*e
 + d)^(17/2) - 51051*(x*e + d)^(15/2)*d + 176715*(x*e + d)^(13/2)*d^2 - 348
075*(x*e + d)^(11/2)*d^3 + 425425*(x*e + d)^(9/2)*d^4 - 328185*(x*e + d)^(7
/2)*d^5 + 153153*(x*e + d)^(5/2)*d^6 - 36465*(x*e + d)^(3/2)*d^7)*c^3*d*e^(
-6) + 1939938*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^3*d + 12597*(315*
(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772
*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*a^2*c*e^(-2) + 969*(3003*(
x*e + d)^(15/2) - 20790*(x*e + d)^(13/2)*d + 61425*(x*e + d)^(11/2)*d^2 - 1
00100*(x*e + d)^(9/2)*d^3 + 96525*(x*e + d)^(7/2)*d^4 - 54054*(x*e + d)^(5/
2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*a*c^2*e^(-4) + 7*(109395*(x*e + d)^(19/
2) - 978120*(x*e + d)^(17/2)*d + 3879876*(x*e + d)^(15/2)*d^2 - 8953560*(x*
e + d)^(13/2)*d^3 + 13226850*(x*e + d)^(11/2)*d^4 - 12932920*(x*e + d)^(9/2
)*d^5 + 8314020*(x*e + d)^(7/2)*d^6 - 3325608*(x*e + d)^(5/2)*d^7 + 692835*
(x*e + d)^(3/2)*d^8)*c^3*e^(-6) + 138567*(15*(x*e + d)^(7/2) - 42*(x*e + d)
^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*a^3)*e^(-1)
```

3.606 $\int (d+ex)^{3/2} \left(a+cx^2\right)^3 \, dx$

Optimal. Leaf size=204

$$\frac{6c^2(d+ex)^{13/2}\left(ae^2+5cd^2\right)}{13e^7}-\frac{8c^2d(d+ex)^{11/2}\left(3ae^2+5cd^2\right)}{11e^7}+\frac{2c(d+ex)^{9/2}\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{3e^7}-\frac{12cd(d+ex)^{7}}{7}$$

```
[Out] (2*(c*d^2 + a*e^2)^3*(d + e*x)^(5/2))/(5*e^7) - (12*c*d*(c*d^2 + a*e^2)^2*(
d + e*x)^(7/2))/(7*e^7) + (2*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2)*(d + e*x)^
(9/2))/(3*e^7) - (8*c^2*d*(5*c*d^2 + 3*a*e^2)*(d + e*x)^(11/2))/(11*e^7) +
(6*c^2*(5*c*d^2 + a*e^2)*(d + e*x)^(13/2))/(13*e^7) - (4*c^3*d*(d + e*x)^(1
5/2))/(5*e^7) + (2*c^3*(d + e*x)^(17/2))/(17*e^7)
```

Rubi [A] time = 0.0850612, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {697}

$$\frac{6c^2(d+ex)^{13/2}\left(ae^2+5cd^2\right)}{13e^7}-\frac{8c^2d(d+ex)^{11/2}\left(3ae^2+5cd^2\right)}{11e^7}+\frac{2c(d+ex)^{9/2}\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{3e^7}-\frac{12cd(d+ex)^{7}}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*(a + c*x^2)^3,x]
```



```
[Out] (2*(c*d^2 + a*e^2)^3*(d + e*x)^(5/2))/(5*e^7) - (12*c*d*(c*d^2 + a*e^2)^2*(
d + e*x)^(7/2))/(7*e^7) + (2*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2)*(d + e*x)^
(9/2))/(3*e^7) - (8*c^2*d*(5*c*d^2 + 3*a*e^2)*(d + e*x)^(11/2))/(11*e^7) +
(6*c^2*(5*c*d^2 + a*e^2)*(d + e*x)^(13/2))/(13*e^7) - (4*c^3*d*(d + e*x)^(1
5/2))/(5*e^7) + (2*c^3*(d + e*x)^(17/2))/(17*e^7)
```


Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (d+ex)^{3/2}\left(a+cx^2\right)^3 dx=\int \left(\frac{\left(cd^2+ae^2\right)^3(d+ex)^{3/2}}{e^6}-\frac{6cd\left(cd^2+ae^2\right)^2(d+ex)^{5/2}}{e^6}+\frac{3c\left(cd^2+ae^2\right)\left(5cd^2+ae^2\right)}{e^6}\right.$$

$$=\frac{2\left(cd^2+ae^2\right)^3(d+ex)^{5/2}}{5e^7}-\frac{12cd\left(cd^2+ae^2\right)^2(d+ex)^{7/2}}{7e^7}+\frac{2c\left(cd^2+ae^2\right)\left(5cd^2+ae^2\right)(}{3e^7}$$

Mathematica [A] time = 0.196466, size = 188, normalized size = 0.92

$$\frac{2\left(\frac{3}{13}c^2(d+ex)^{13/2}\left(ae^2+5cd^2\right)-\frac{4}{11}c^2d(d+ex)^{11/2}\left(3ae^2+5cd^2\right)+\frac{1}{3}c(d+ex)^{9/2}\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)-\frac{6}{7}cd(d+e\right.}{e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)*(a + c*x^2)^3,x]
```

```
[Out] (2*(((c*d^2 + a*e^2)^3*(d + e*x)^(5/2))/5 - (6*c*d*(c*d^2 + a*e^2)^2*(d + e
*x)^(7/2))/7 + (c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2)*(d + e*x)^(9/2))/3 - (4
*c^2*d*(5*c*d^2 + 3*a*e^2)*(d + e*x)^(11/2))/11 + (3*c^2*(5*c*d^2 + a*e^2)*
(d + e*x)^(13/2))/13 - (2*c^3*d*(d + e*x)^(15/2))/5 + (c^3*(d + e*x)^(17/2)
)/17))/e^7
```

Maple [A] time = 0.045, size = 205, normalized size = 1.

$$\frac{30030\,c^3x^6e^6 - 24024\,c^3dx^5e^5 + 117810\,ac^2e^6x^4 + 18480\,c^3d^2e^4x^4 - 85680\,ac^2de^5x^3 - 13440\,c^3d^3e^3x^3 + 170170\,a^2ce^6x^2 + \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(c*x^2+a)^3,x)
```

```
[Out] 2/255255*(e*x+d)^(5/2)*(15015*c^3*e^6*x^6-12012*c^3*d*e^5*x^5+58905*a*c^2*e
^6*x^4+9240*c^3*d^2*e^4*x^4-42840*a*c^2*d*e^5*x^3-6720*c^3*d^3*e^3*x^3+8508
5*a^2*c*e^6*x^2+28560*a*c^2*d^2*e^4*x^2+4480*c^3*d^4*e^2*x^2-48620*a^2*c*d*
e^5*x-16320*a*c^2*d^3*e^3*x-2560*c^3*d^5*e*x+51051*a^3*e^6+19448*a^2*c*d^2*
e^4+6528*a*c^2*d^4*e^2+1024*c^3*d^6)/e^7
```

Maxima [A] time = 1.17442, size = 282, normalized size = 1.38

$$\frac{2\left(15015\,(ex+d)^{\frac{17}{2}}c^3 - 102102\,(ex+d)^{\frac{15}{2}}c^3d + 58905\left(5\,c^3d^2 + ac^2e^2\right)(ex+d)^{\frac{13}{2}} - 92820\left(5\,c^3d^3 + 3\,ac^2de^2\right)(ex+d)^{\frac{11}{2}} + \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 2/255255*(15015*(e*x + d)^(17/2)*c^3 - 102102*(e*x + d)^(15/2)*c^3*d + 5890
5*(5*c^3*d^2 + a*c^2*e^2)*(e*x + d)^(13/2) - 92820*(5*c^3*d^3 + 3*a*c^2*d*e
^2)*(e*x + d)^(11/2) + 85085*(5*c^3*d^4 + 6*a*c^2*d^2*e^2 + a^2*c*e^4)*(e*x
 + d)^(9/2) - 218790*(c^3*d^5 + 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*(e*x + d)^(7
/2) + 51051*(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)*(e*x +
d)^(5/2))/e^7
```

Fricas [A] time = 1.82975, size = 707, normalized size = 3.47

$$\frac{2\left(15015\,c^3e^8x^8 + 18018\,c^3de^7x^7 + 1024\,c^3d^8 + 6528\,ac^2d^6e^2 + 19448\,a^2cd^4e^4 + 51051\,a^3d^2e^6 + 231\left(c^3d^2e^6 + 255\,ac^2e^8\right)x \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 2/255255*(15015*c^3*e^8*x^8 + 18018*c^3*d*e^7*x^7 + 1024*c^3*d^8 + 6528*a*c
^2*d^6*e^2 + 19448*a^2*c*d^4*e^4 + 51051*a^3*d^2*e^6 + 231*(c^3*d^2*e^6 + 2
55*a*c^2*e^8)*x^6 - 126*(2*c^3*d^3*e^5 - 595*a*c^2*d*e^7)*x^5 + 35*(8*c^3*d
^4*e^4 + 51*a*c^2*d^2*e^6 + 2431*a^2*c*e^8)*x^4 - 10*(32*c^3*d^5*e^3 + 204*
a*c^2*d^3*e^5 - 12155*a^2*c*d*e^7)*x^3 + 3*(128*c^3*d^6*e^2 + 816*a*c^2*d^4
*e^4 + 2431*a^2*c*d^2*e^6 + 17017*a^3*e^8)*x^2 - 2*(256*c^3*d^7*e + 1632*a*
```

```
c^2*d^5*e^3 + 4862*a^2*c*d^3*e^5 - 51051*a^3*d*e^7)*x)*sqrt(e*x + d)/e^7
```

Sympy [A] time = 16.6605, size = 564, normalized size = 2.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(c*x**2+a)**3,x)
```

```
[Out] a**3*d*Piecewise((sqrt(d)*x, Eq(e, 0)), (2*(d + e*x)**(3/2)/(3*e), True)) +
 2*a**3*(-d*(d + e*x)**(3/2)/3 + (d + e*x)**(5/2)/5)/e + 6*a**2*c*d*(d**2*(
d + e*x)**(3/2)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e**3 + 6*a
**2*c*(-d**3*(d + e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d + e*x)
**(7/2)/7 + (d + e*x)**(9/2)/9)/e**3 + 6*a*c**2*d*(d**4*(d + e*x)**(3/2)/3
- 4*d**3*(d + e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e*x)**(9
/2)/9 + (d + e*x)**(11/2)/11)/e**5 + 6*a*c**2*(-d**5*(d + e*x)**(3/2)/3 + d
**4*(d + e*x)**(5/2) - 10*d**3*(d + e*x)**(7/2)/7 + 10*d**2*(d + e*x)**(9/2
)/9 - 5*d*(d + e*x)**(11/2)/11 + (d + e*x)**(13/2)/13)/e**5 + 2*c**3*d*(d**
6*(d + e*x)**(3/2)/3 - 6*d**5*(d + e*x)**(5/2)/5 + 15*d**4*(d + e*x)**(7/2)
/7 - 20*d**3*(d + e*x)**(9/2)/9 + 15*d**2*(d + e*x)**(11/2)/11 - 6*d*(d + e
*x)**(13/2)/13 + (d + e*x)**(15/2)/15)/e**7 + 2*c**3*(-d**7*(d + e*x)**(3/2
)/3 + 7*d**6*(d + e*x)**(5/2)/5 - 3*d**5*(d + e*x)**(7/2) + 35*d**4*(d + e*
x)**(9/2)/9 - 35*d**3*(d + e*x)**(11/2)/11 + 21*d**2*(d + e*x)**(13/2)/13 -
 7*d*(d + e*x)**(15/2)/15 + (d + e*x)**(17/2)/17)/e**7
```

Giac [B] time = 1.35377, size = 675, normalized size = 3.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 2/765765*(21879*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(
3/2)*d^2)*a^2*c*d*e^(-2) + 663*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)
*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(
3/2)*d^4)*a*c^2*d*e^(-4) + 17*(3003*(x*e + d)^(15/2) - 20790*(x*e + d)^(13/
2)*d + 61425*(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525*(x*e
 + d)^(7/2)*d^4 - 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*c^
3*d*e^(-6) + 255255*(x*e + d)^(3/2)*a^3*d + 7293*(35*(x*e + d)^(9/2) - 135*
(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a^2*
c*e^(-2) + 255*(693*(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e
 + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 30
03*(x*e + d)^(3/2)*d^5)*a*c^2*e^(-4) + 7*(6435*(x*e + d)^(17/2) - 51051*(x*
e + d)^(15/2)*d + 176715*(x*e + d)^(13/2)*d^2 - 348075*(x*e + d)^(11/2)*d^3
 + 425425*(x*e + d)^(9/2)*d^4 - 328185*(x*e + d)^(7/2)*d^5 + 153153*(x*e +
d)^(5/2)*d^6 - 36465*(x*e + d)^(3/2)*d^7)*c^3*e^(-6) + 51051*(3*(x*e + d)^(
5/2) - 5*(x*e + d)^(3/2)*d)*a^3)*e^(-1)
```

3.607 $\int \sqrt{d+ex}\left(a+cx^2\right)^3 dx$

Optimal. Leaf size=204

$$\frac{6c^2(d+ex)^{11/2}\left(ae^2+5cd^2\right)}{11e^7}-\frac{8c^2d(d+ex)^{9/2}\left(3ae^2+5cd^2\right)}{9e^7}+\frac{6c(d+ex)^{7/2}\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{7e^7}-\frac{12cd(d+ex)^{5/2}\left(a\right.}{5e^7}$$

```
[Out] (2*(c*d^2 + a*e^2)^3*(d + e*x)^(3/2))/(3*e^7) - (12*c*d*(c*d^2 + a*e^2)^2*(
d + e*x)^(5/2))/(5*e^7) + (6*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2)*(d + e*x)^
(7/2))/(7*e^7) - (8*c^2*d*(5*c*d^2 + 3*a*e^2)*(d + e*x)^(9/2))/(9*e^7) + (6
*c^2*(5*c*d^2 + a*e^2)*(d + e*x)^(11/2))/(11*e^7) - (12*c^3*d*(d + e*x)^(13
/2))/(13*e^7) + (2*c^3*(d + e*x)^(15/2))/(15*e^7)
```

Rubi [A] time = 0.0826531, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {697}

$$\frac{6c^2(d+ex)^{11/2}\left(ae^2+5cd^2\right)}{11e^7}-\frac{8c^2d(d+ex)^{9/2}\left(3ae^2+5cd^2\right)}{9e^7}+\frac{6c(d+ex)^{7/2}\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{7e^7}-\frac{12cd(d+ex)^{5/2}\left(a\right.}{5e^7}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(a + c*x^2)^3,x]
```


```
[Out] (2*(c*d^2 + a*e^2)^3*(d + e*x)^(3/2))/(3*e^7) - (12*c*d*(c*d^2 + a*e^2)^2*(
d + e*x)^(5/2))/(5*e^7) + (6*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2)*(d + e*x)^
(7/2))/(7*e^7) - (8*c^2*d*(5*c*d^2 + 3*a*e^2)*(d + e*x)^(9/2))/(9*e^7) + (6
*c^2*(5*c*d^2 + a*e^2)*(d + e*x)^(11/2))/(11*e^7) - (12*c^3*d*(d + e*x)^(13
/2))/(13*e^7) + (2*c^3*(d + e*x)^(15/2))/(15*e^7)
```


Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \sqrt{d+ex}\left(a+cx^2\right)^3 dx = \int \left(\frac{\left(cd^2+ae^2\right)^3\sqrt{d+ex}}{e^6}-\frac{6cd\left(cd^2+ae^2\right)^2(d+ex)^{3/2}}{e^6}+\frac{3c\left(cd^2+ae^2\right)\left(5cd^2+ae^2\right)(d+ex\right.}{e^6}$$

$$=\frac{2\left(cd^2+ae^2\right)^3(d+ex)^{3/2}}{3e^7}-\frac{12cd\left(cd^2+ae^2\right)^2(d+ex)^{5/2}}{5e^7}+\frac{6c\left(cd^2+ae^2\right)\left(5cd^2+ae^2\right)(d+e\right.}{7e^7}$$

Mathematica [A] time = 0.144417, size = 170, normalized size = 0.83

$$\frac{2(d+ex)^{3/2}\left(1287a^2ce^4\left(8d^2-12dex+15e^2x^2\right)+15015a^3e^6+39ac^2e^2\left(240d^2e^2x^2-192d^3ex+128d^4-280de^3x^3+315e\right.\right.}{45045e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*(a + c*x^2)^3,x]
```

```
[Out] (2*(d + e*x)^(3/2)*(15015*a^3*e^6 + 1287*a^2*c*e^4*(8*d^2 - 12*d*e*x + 15*e
^2*x^2) + 39*a*c^2*e^2*(128*d^4 - 192*d^3*e*x + 240*d^2*e^2*x^2 - 280*d*e^3
*x^3 + 315*e^4*x^4) + c^3*(1024*d^6 - 1536*d^5*e*x + 1920*d^4*e^2*x^2 - 224
0*d^3*e^3*x^3 + 2520*d^2*e^4*x^4 - 2772*d*e^5*x^5 + 3003*e^6*x^6)))/(45045*
e^7)
```

Maple [A] time = 0.046, size = 205, normalized size = 1.

$$6006\,c^3x^6e^6 - 5544\,c^3dx^5e^5 + 24570\,ac^2e^6x^4 + 5040\,c^3d^2e^4x^4 - 21840\,ac^2de^5x^3 - 4480\,c^3d^3e^3x^3 + 38610\,a^2ce^6x^2 + 18$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3*(e*x+d)^(1/2),x)
```

```
[Out] 2/45045*(e*x+d)^(3/2)*(3003*c^3*e^6*x^6-2772*c^3*d*e^5*x^5+12285*a*c^2*e^6*
x^4+2520*c^3*d^2*e^4*x^4-10920*a*c^2*d*e^5*x^3-2240*c^3*d^3*e^3*x^3+19305*a
^2*c*e^6*x^2+9360*a*c^2*d^2*e^4*x^2+1920*c^3*d^4*e^2*x^2-15444*a^2*c*d*e^5*
x-7488*a*c^2*d^3*e^3*x-1536*c^3*d^5*e*x+15015*a^3*e^6+10296*a^2*c*d^2*e^4+4
992*a*c^2*d^4*e^2+1024*c^3*d^6)/e^7
```

Maxima [A] time = 1.14468, size = 282, normalized size = 1.38

$$2\left(3003\,(ex+d)^{\frac{15}{2}}c^3 - 20790\,(ex+d)^{\frac{13}{2}}c^3d + 12285\left(5\,c^3d^2 + ac^2e^2\right)(ex+d)^{\frac{11}{2}} - 20020\left(5\,c^3d^3 + 3\,ac^2de^2\right)(ex+d)^{\frac{9}{2}} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(3003*(e*x + d)^(15/2)*c^3 - 20790*(e*x + d)^(13/2)*c^3*d + 12285*(
5*c^3*d^2 + a*c^2*e^2)*(e*x + d)^(11/2) - 20020*(5*c^3*d^3 + 3*a*c^2*d*e^2)
*(e*x + d)^(9/2) + 19305*(5*c^3*d^4 + 6*a*c^2*d^2*e^2 + a^2*c*e^4)*(e*x + d
)^(7/2) - 54054*(c^3*d^5 + 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*(e*x + d)^(5/2) +
 15015*(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)*(e*x + d)^(3
/2))/e^7
```

Fricas [A] time = 1.8093, size = 581, normalized size = 2.85

$$2\left(3003\,c^3e^7x^7 + 231\,c^3de^6x^6 + 1024\,c^3d^7 + 4992\,ac^2d^5e^2 + 10296\,a^2cd^3e^4 + 15015\,a^3de^6 - 63\left(4\,c^3d^2e^5 - 195\,ac^2e^7\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3003*c^3*e^7*x^7 + 231*c^3*d*e^6*x^6 + 1024*c^3*d^7 + 4992*a*c^2*d
^5*e^2 + 10296*a^2*c*d^3*e^4 + 15015*a^3*d*e^6 - 63*(4*c^3*d^2*e^5 - 195*a*
c^2*e^7)*x^5 + 35*(8*c^3*d^3*e^4 + 39*a*c^2*d*e^6)*x^4 - 5*(64*c^3*d^4*e^3
+ 312*a*c^2*d^2*e^5 - 3861*a^2*c*e^7)*x^3 + 3*(128*c^3*d^5*e^2 + 624*a*c^2*
d^3*e^4 + 1287*a^2*c*d*e^6)*x^2 - (512*c^3*d^6*e + 2496*a*c^2*d^4*e^3 + 514
```

```
8*a^2*c*d^2*e^5 - 15015*a^3*e^7)*x)*sqrt(e*x + d)/e^7
```

Sympy [A] time = 4.06417, size = 265, normalized size = 1.3

$$\frac{2\left(-\frac{6c^3d(d+ex)^{\frac{13}{2}}}{13e^6}+\frac{c^3(d+ex)^{\frac{15}{2}}}{15e^6}+\frac{(d+ex)^{\frac{11}{2}}\left(3ac^2e^2+15c^3d^2\right)}{11e^6}+\frac{(d+ex)^{\frac{9}{2}}\left(-12ac^2de^2-20c^3d^3\right)}{9e^6}+\frac{(d+ex)^{\frac{7}{2}}\left(3a^2ce^4+18ac^2d^2e^2+15c^3d^4\right)}{7e^6}+\frac{(d+ex)^{\frac{5}{2}}\left(-6a^2c\right.}{}\right.}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3*(e*x+d)**(1/2),x)
```

```
[Out] 2*(-6*c**3*d*(d + e*x)**(13/2)/(13*e**6) + c**3*(d + e*x)**(15/2)/(15*e**6)
 + (d + e*x)**(11/2)*(3*a*c**2*e**2 + 15*c**3*d**2)/(11*e**6) + (d + e*x)**
(9/2)*(-12*a*c**2*d*e**2 - 20*c**3*d**3)/(9*e**6) + (d + e*x)**(7/2)*(3*a**
2*c*e**4 + 18*a*c**2*d**2*e**2 + 15*c**3*d**4)/(7*e**6) + (d + e*x)**(5/2)*
(-6*a**2*c*d*e**4 - 12*a*c**2*d**3*e**2 - 6*c**3*d**5)/(5*e**6) + (d + e*x)
**(3/2)*(a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6)/(
3*e**6))/e
```

Giac [A] time = 1.36843, size = 301, normalized size = 1.48

$$\frac{2}{45045}\left(1287\left(15(xe+d)^{\frac{7}{2}}-42(xe+d)^{\frac{5}{2}}d+35(xe+d)^{\frac{3}{2}}d^2\right)a^2ce^{(-2)}+39\left(315(xe+d)^{\frac{11}{2}}-1540(xe+d)^{\frac{9}{2}}d+2970(xe+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 2/45045*(1287*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/
2)*d^2)*a^2*c*e^(-2) + 39*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d +
2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*
d^4)*a*c^2*e^(-4) + (3003*(x*e + d)^(15/2) - 20790*(x*e + d)^(13/2)*d + 614
25*(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525*(x*e + d)^(7/2
)*d^4 - 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*c^3*e^(-6) +
 15015*(x*e + d)^(3/2)*a^3)*e^(-1)
```

3.608 $\int \frac{\left(a+cx^2\right)^3}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=200

$$\frac{2c^2(d+ex)^{9/2}\left(ae^2+5cd^2\right)}{3e^7}-\frac{8c^2d(d+ex)^{7/2}\left(3ae^2+5cd^2\right)}{7e^7}+\frac{6c(d+ex)^{5/2}\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{5e^7}-\frac{4cd(d+ex)^{3/2}\left(\right.}{e^7}$$

```
[Out] (2*(c*d^2 + a*e^2)^3*Sqrt[d + e*x])/e^7 - (4*c*d*(c*d^2 + a*e^2)^2*(d + e*x
)^(3/2))/e^7 + (6*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2)*(d + e*x)^(5/2))/(5*e
^7) - (8*c^2*d*(5*c*d^2 + 3*a*e^2)*(d + e*x)^(7/2))/(7*e^7) + (2*c^2*(5*c*d
^2 + a*e^2)*(d + e*x)^(9/2))/(3*e^7) - (12*c^3*d*(d + e*x)^(11/2))/(11*e^7)
 + (2*c^3*(d + e*x)^(13/2))/(13*e^7)
```

Rubi [A] time = 0.0822133, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {697}

$$\frac{2c^2(d+ex)^{9/2}\left(ae^2+5cd^2\right)}{3e^7}-\frac{8c^2d(d+ex)^{7/2}\left(3ae^2+5cd^2\right)}{7e^7}+\frac{6c(d+ex)^{5/2}\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{5e^7}-\frac{4cd(d+ex)^{3/2}\left(\right.}{e^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^3/Sqrt[d + e*x],x]
```

```
[Out] (2*(c*d^2 + a*e^2)^3*Sqrt[d + e*x])/e^7 - (4*c*d*(c*d^2 + a*e^2)^2*(d + e*x
)^(3/2))/e^7 + (6*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2)*(d + e*x)^(5/2))/(5*e
^7) - (8*c^2*d*(5*c*d^2 + 3*a*e^2)*(d + e*x)^(7/2))/(7*e^7) + (2*c^2*(5*c*d
^2 + a*e^2)*(d + e*x)^(9/2))/(3*e^7) - (12*c^3*d*(d + e*x)^(11/2))/(11*e^7)
 + (2*c^3*(d + e*x)^(13/2))/(13*e^7)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+cx^2\right)^3}{\sqrt{d+ex}}\,dx &= \int \left(\frac{\left(cd^2+ae^2\right)^3}{e^6\sqrt{d+ex}}-\frac{6cd\left(cd^2+ae^2\right)^2\sqrt{d+ex}}{e^6}+\frac{3c\left(cd^2+ae^2\right)\left(5cd^2+ae^2\right)(d+ex)^{3/2}}{e^6}-\frac{4c^2d\left(5cd^2\right.}{}\right.\\ &= \frac{2\left(cd^2+ae^2\right)^3\sqrt{d+ex}}{e^7}-\frac{4cd\left(cd^2+ae^2\right)^2(d+ex)^{3/2}}{e^7}+\frac{6c\left(cd^2+ae^2\right)\left(5cd^2+ae^2\right)(d+ex)^{5/2}}{5e^7}-\frac{8}{}\end{aligned}$$

Mathematica [A] time = 0.117646, size = 171, normalized size = 0.86

$$\frac{2\sqrt{d+ex}\left(3003a^2ce^4\left(8d^2-4dex+3e^2x^2\right)+15015a^3e^6+143ac^2e^2\left(48d^2e^2x^2-64d^3ex+128d^4-40de^3x^3+35e^4x^4\right)\right.}{15015e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^3/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[d + e*x]*(15015*a^3*e^6 + 3003*a^2*c*e^4*(8*d^2 - 4*d*e*x + 3*e^2*x
^2) + 143*a*c^2*e^2*(128*d^4 - 64*d^3*e*x + 48*d^2*e^2*x^2 - 40*d*e^3*x^3 +
 35*e^4*x^4) + 5*c^3*(1024*d^6 - 512*d^5*e*x + 384*d^4*e^2*x^2 - 320*d^3*e^
3*x^3 + 280*d^2*e^4*x^4 - 252*d*e^5*x^5 + 231*e^6*x^6)))/(15015*e^7)
```

Maple [A] time = 0.046, size = 205, normalized size = 1.

$$\frac{2310\,c^3x^6e^6 - 2520\,c^3dx^5e^5 + 10010\,ac^2e^6x^4 + 2800\,c^3d^2e^4x^4 - 11440\,ac^2de^5x^3 - 3200\,c^3d^3e^3x^3 + 18018\,a^2ce^6x^2 + 13728\,\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3/(e*x+d)^(1/2),x)
```

```
[Out] 2/15015*(e*x+d)^(1/2)*(1155*c^3*e^6*x^6-1260*c^3*d*e^5*x^5+5005*a*c^2*e^6*x
^4+1400*c^3*d^2*e^4*x^4-5720*a*c^2*d*e^5*x^3-1600*c^3*d^3*e^3*x^3+9009*a^2*
c*e^6*x^2+6864*a*c^2*d^2*e^4*x^2+1920*c^3*d^4*e^2*x^2-12012*a^2*c*d*e^5*x-9
152*a*c^2*d^3*e^3*x-2560*c^3*d^5*e*x+15015*a^3*e^6+24024*a^2*c*d^2*e^4+1830
4*a*c^2*d^4*e^2+5120*c^3*d^6)/e^7
```

Maxima [A] time = 1.47941, size = 286, normalized size = 1.43

$$\frac{2\left(15015\,\sqrt{ex+d}a^3 + \frac{3003\left(3\,(ex+d)^{\frac{5}{2}} - 10\,(ex+d)^{\frac{3}{2}}d + 15\,\sqrt{ex+d}d^2\right)a^2c}{e^2} + \frac{143\left(35\,(ex+d)^{\frac{9}{2}} - 180\,(ex+d)^{\frac{7}{2}}d + 378\,(ex+d)^{\frac{5}{2}}d^2 - 420\,(ex+d)^{\frac{3}{2}}d^3 + 315\,\sqrt{ex+d}d^4 \ldots}{e^4} \ldots\right)}{15015\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15015*(15015*sqrt(e*x + d)*a^3 + 3003*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(
3/2)*d + 15*sqrt(e*x + d)*d^2)*a^2*c/e^2 + 143*(35*(e*x + d)^(9/2) - 180*(e
*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*s
qrt(e*x + d)*d^4)*a*c^2/e^4 + 5*(231*(e*x + d)^(13/2) - 1638*(e*x + d)^(11/
2)*d + 5005*(e*x + d)^(9/2)*d^2 - 8580*(e*x + d)^(7/2)*d^3 + 9009*(e*x + d)
^(5/2)*d^4 - 6006*(e*x + d)^(3/2)*d^5 + 3003*sqrt(e*x + d)*d^6)*c^3/e^6)/e
```

Fricas [A] time = 1.86052, size = 478, normalized size = 2.39

$$\frac{2\left(1155\,c^3e^6x^6 - 1260\,c^3de^5x^5 + 5120\,c^3d^6 + 18304\,ac^2d^4e^2 + 24024\,a^2cd^2e^4 + 15015\,a^3e^6 + 35\left(40\,c^3d^2e^4 + 143\,ac^2e^6\right)x^4 \ldots\right)}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15015*(1155*c^3*e^6*x^6 - 1260*c^3*d*e^5*x^5 + 5120*c^3*d^6 + 18304*a*c^2
*d^4*e^2 + 24024*a^2*c*d^2*e^4 + 15015*a^3*e^6 + 35*(40*c^3*d^2*e^4 + 143*a
```

```
*c^2*e^6)*x^4 - 40*(40*c^3*d^3*e^3 + 143*a*c^2*d*e^5)*x^3 + 3*(640*c^3*d^4*
e^2 + 2288*a*c^2*d^2*e^4 + 3003*a^2*c*e^6)*x^2 - 4*(640*c^3*d^5*e + 2288*a*
c^2*d^3*e^3 + 3003*a^2*c*d*e^5)*x)*sqrt(e*x + d)/e^7
```

Sympy [A] time = 48.8004, size = 563, normalized size = 2.82

$$\begin{cases} -\frac{\frac{2a^3d}{\sqrt{d+ex}}+2a^3\left(-\frac{d}{\sqrt{d+ex}}-\sqrt{d+ex}\right)+\frac{6a^2cd\left(\frac{d^2}{\sqrt{d+ex}}+2d\sqrt{d+ex}-\frac{(d+ex)^{\frac{3}{2}}}{3}\right)}{e^2}+\frac{6a^2c\left(-\frac{d^3}{\sqrt{d+ex}}-3d^2\sqrt{d+ex}+d(d+ex)^{\frac{3}{2}}-\frac{(d+ex)^{\frac{5}{2}}}{5}\right)}{e^2}+\frac{6ac^2d\left(\frac{d^4}{\sqrt{d+ex}}+4d^3\sqrt{d+ex}-2d^2(d+ex)^{\frac{3}{2}}+\frac{4d(d+ex)^{\frac{5}{2}}}{5}\right.}{e^4}}{} \\ \frac{a^3x+a^2cx^3+\frac{3ac^2x^5}{5}+\frac{c^3x^7}{7}}{\sqrt{d}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3/(e*x+d)**(1/2),x)
```

```
[Out] Piecewise((-(2*a**3*d/sqrt(d + e*x) + 2*a**3*(-d/sqrt(d + e*x) - sqrt(d + e
*x)) + 6*a**2*c*d*(d**2/sqrt(d + e*x) + 2*d*sqrt(d + e*x) - (d + e*x)**(3/2
)/3)/e**2 + 6*a**2*c*(-d**3/sqrt(d + e*x) - 3*d**2*sqrt(d + e*x) + d*(d + e
*x)**(3/2) - (d + e*x)**(5/2)/5)/e**2 + 6*a*c**2*d*(d**4/sqrt(d + e*x) + 4*
d**3*sqrt(d + e*x) - 2*d**2*(d + e*x)**(3/2) + 4*d*(d + e*x)**(5/2)/5 - (d
+ e*x)**(7/2)/7)/e**4 + 6*a*c**2*(-d**5/sqrt(d + e*x) - 5*d**4*sqrt(d + e*x
) + 10*d**3*(d + e*x)**(3/2)/3 - 2*d**2*(d + e*x)**(5/2) + 5*d*(d + e*x)**(
7/2)/7 - (d + e*x)**(9/2)/9)/e**4 + 2*c**3*d*(d**6/sqrt(d + e*x) + 6*d**5*s
qrt(d + e*x) - 5*d**4*(d + e*x)**(3/2) + 4*d**3*(d + e*x)**(5/2) - 15*d**2*
(d + e*x)**(7/2)/7 + 2*d*(d + e*x)**(9/2)/3 - (d + e*x)**(11/2)/11)/e**6 +
2*c**3*(-d**7/sqrt(d + e*x) - 7*d**6*sqrt(d + e*x) + 7*d**5*(d + e*x)**(3/2
) - 7*d**4*(d + e*x)**(5/2) + 5*d**3*(d + e*x)**(7/2) - 7*d**2*(d + e*x)**(
9/2)/3 + 7*d*(d + e*x)**(11/2)/11 - (d + e*x)**(13/2)/13)/e**6)/e, Ne(e, 0)
), ((a**3*x + a**2*c*x**3 + 3*a*c**2*x**5/5 + c**3*x**7/7)/sqrt(d), True))
```

Giac [A] time = 1.34009, size = 302, normalized size = 1.51

$$\frac{2}{15015}\left(3003\left(3(xe+d)^{\frac{5}{2}}-10(xe+d)^{\frac{3}{2}}d+15\sqrt{xe+d}d^2\right)a^2ce^{(-2)}+143\left(35(xe+d)^{\frac{9}{2}}-180(xe+d)^{\frac{7}{2}}d+378(xe+d)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15015*(3003*(3*(x*e + d)^(5/2) - 10*(x*e + d)^(3/2)*d + 15*sqrt(x*e + d)*
d^2)*a^2*c*e^(-2) + 143*(35*(x*e + d)^(9/2) - 180*(x*e + d)^(7/2)*d + 378*(
x*e + d)^(5/2)*d^2 - 420*(x*e + d)^(3/2)*d^3 + 315*sqrt(x*e + d)*d^4)*a*c^2
*e^(-4) + 5*(231*(x*e + d)^(13/2) - 1638*(x*e + d)^(11/2)*d + 5005*(x*e + d
)^(9/2)*d^2 - 8580*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 6006*(x
*e + d)^(3/2)*d^5 + 3003*sqrt(x*e + d)*d^6)*c^3*e^(-6) + 15015*sqrt(x*e + d
)*a^3)*e^(-1)
```

3.609 $\int \frac{(a+cx^2)^3}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=198

$$\frac{6c^2(d+ex)^{7/2}\left(ae^2+5cd^2\right)}{7e^7}-\frac{8c^2d(d+ex)^{5/2}\left(3ae^2+5cd^2\right)}{5e^7}+\frac{2c(d+ex)^{3/2}\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{e^7}-\frac{12cd\sqrt{d+ex}\left(ae^2\right.}{e^7}$$

```
[Out] (-2*(c*d^2 + a*e^2)^3)/(e^7*Sqrt[d + e*x]) - (12*c*d*(c*d^2 + a*e^2)^2*Sqrt
[d + e*x])/e^7 + (2*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2)*(d + e*x)^(3/2))/e^
7 - (8*c^2*d*(5*c*d^2 + 3*a*e^2)*(d + e*x)^(5/2))/(5*e^7) + (6*c^2*(5*c*d^2
 + a*e^2)*(d + e*x)^(7/2))/(7*e^7) - (4*c^3*d*(d + e*x)^(9/2))/(3*e^7) + (2
*c^3*(d + e*x)^(11/2))/(11*e^7)
```

Rubi [A] time = 0.0834626, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {697}

$$\frac{6c^2(d+ex)^{7/2}\left(ae^2+5cd^2\right)}{7e^7}-\frac{8c^2d(d+ex)^{5/2}\left(3ae^2+5cd^2\right)}{5e^7}+\frac{2c(d+ex)^{3/2}\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{e^7}-\frac{12cd\sqrt{d+ex}\left(ae^2\right.}{e^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^3/(d + e*x)^(3/2),x]
```


```
[Out] (-2*(c*d^2 + a*e^2)^3)/(e^7*Sqrt[d + e*x]) - (12*c*d*(c*d^2 + a*e^2)^2*Sqrt
[d + e*x])/e^7 + (2*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2)*(d + e*x)^(3/2))/e^
7 - (8*c^2*d*(5*c*d^2 + 3*a*e^2)*(d + e*x)^(5/2))/(5*e^7) + (6*c^2*(5*c*d^2
 + a*e^2)*(d + e*x)^(7/2))/(7*e^7) - (4*c^3*d*(d + e*x)^(9/2))/(3*e^7) + (2
*c^3*(d + e*x)^(11/2))/(11*e^7)
```


Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^3}{(d+ex)^{3/2}}\,dx=\int \left(\frac{\left(cd^2+ae^2\right)^3}{e^6(d+ex)^{3/2}}-\frac{6cd\left(cd^2+ae^2\right)^2}{e^6\sqrt{d+ex}}+\frac{3c\left(cd^2+ae^2\right)\left(5cd^2+ae^2\right)\sqrt{d+ex}}{e^6}-\frac{4c^2d\left(5cd^2+3ae^2\right)(d+\ldots}{e^6}\right.$$

$$=-\frac{2\left(cd^2+ae^2\right)^3}{e^7\sqrt{d+ex}}-\frac{12cd\left(cd^2+ae^2\right)^2\sqrt{d+ex}}{e^7}+\frac{2c\left(cd^2+ae^2\right)\left(5cd^2+ae^2\right)(d+ex)^{3/2}}{e^7}-\frac{8c^2d\left(5cd^2+\ldots\right.}{}$$

Mathematica [A] time = 0.115819, size = 171, normalized size = 0.86

$$-\frac{2\left(1155a^2ce^4\left(8d^2+4dex-e^2x^2\right)+1155a^3e^6+99ac^2e^2\left(-16d^2e^2x^2+64d^3ex+128d^4+8de^3x^3-5e^4x^4\right)+5c^3\left(-128d^4\ldots\right.\right.}{1155e^7\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^3/(d + e*x)^(3/2),x]
```

```
[Out] (-2*(1155*a^3*e^6 + 1155*a^2*c*e^4*(8*d^2 + 4*d*e*x - e^2*x^2) + 99*a*c^2*e
^2*(128*d^4 + 64*d^3*e*x - 16*d^2*e^2*x^2 + 8*d*e^3*x^3 - 5*e^4*x^4) + 5*c^
3*(1024*d^6 + 512*d^5*e*x - 128*d^4*e^2*x^2 + 64*d^3*e^3*x^3 - 40*d^2*e^4*x
^4 + 28*d*e^5*x^5 - 21*e^6*x^6)))/(1155*e^7*Sqrt[d + e*x])
```

Maple [A] time = 0.045, size = 205, normalized size = 1.

$$-\frac{-210\,c^3x^6e^6+280\,c^3dx^5e^5-990\,ac^2e^6x^4-400\,c^3d^2e^4x^4+1584\,ac^2de^5x^3+640\,c^3d^3e^3x^3-2310\,a^2ce^6x^2-3168\,ac^2d^2\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3/(e*x+d)^(3/2),x)
```

```
[Out] -2/1155/(e*x+d)^(1/2)*(-105*c^3*e^6*x^6+140*c^3*d*e^5*x^5-495*a*c^2*e^6*x^4
-200*c^3*d^2*e^4*x^4+792*a*c^2*d*e^5*x^3+320*c^3*d^3*e^3*x^3-1155*a^2*c*e^6
*x^2-1584*a*c^2*d^2*e^4*x^2-640*c^3*d^4*e^2*x^2+4620*a^2*c*d*e^5*x+6336*a*c
^2*d^3*e^3*x+2560*c^3*d^5*e*x+1155*a^3*e^6+9240*a^2*c*d^2*e^4+12672*a*c^2*d
^4*e^2+5120*c^3*d^6)/e^7
```

Maxima [A] time = 1.53718, size = 293, normalized size = 1.48

$$\frac{2\left(\frac{105\,(ex+d)^{\frac{11}{2}}c^3-770\,(ex+d)^{\frac{9}{2}}c^3d+495\left(5\,c^3d^2+ac^2e^2\right)(ex+d)^{\frac{7}{2}}-924\left(5\,c^3d^3+3\,ac^2de^2\right)(ex+d)^{\frac{5}{2}}+1155\left(5\,c^3d^4+6\,ac^2d^2e^2+a^2ce^4\right)(ex+d)^{\frac{3}{2}}-6930\left(c^3d^5+2\,ac^2\cdots\right.}{e^6}\cdots\right)}{1155\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/1155*((105*(e*x + d)^(11/2)*c^3 - 770*(e*x + d)^(9/2)*c^3*d + 495*(5*c^3*
d^2 + a*c^2*e^2)*(e*x + d)^(7/2) - 924*(5*c^3*d^3 + 3*a*c^2*d*e^2)*(e*x + d
)^(5/2) + 1155*(5*c^3*d^4 + 6*a*c^2*d^2*e^2 + a^2*c*e^4)*(e*x + d)^(3/2) -
6930*(c^3*d^5 + 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*sqrt(e*x + d))/e^6 - 1155*(c
^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)/(sqrt(e*x + d)*e^6))/
e
```

Fricas [A] time = 1.87235, size = 479, normalized size = 2.42

$$\frac{2\left(105\,c^3e^6x^6-140\,c^3de^5x^5-5120\,c^3d^6-12672\,ac^2d^4e^2-9240\,a^2cd^2e^4-1155\,a^3e^6+5\left(40\,c^3d^2e^4+99\,ac^2e^6\right)x^4-8\left(\cdots\right.}{1155\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*c^3*e^6*x^6 - 140*c^3*d*e^5*x^5 - 5120*c^3*d^6 - 12672*a*c^2*d^
4*e^2 - 9240*a^2*c*d^2*e^4 - 1155*a^3*e^6 + 5*(40*c^3*d^2*e^4 + 99*a*c^2*e^
```

```
6)*x^4 - 8*(40*c^3*d^3*e^3 + 99*a*c^2*d*e^5)*x^3 + (640*c^3*d^4*e^2 + 1584*
a*c^2*d^2*e^4 + 1155*a^2*c*e^6)*x^2 - 4*(640*c^3*d^5*e + 1584*a*c^2*d^3*e^3
 + 1155*a^2*c*d*e^5)*x)*sqrt(e*x + d)/(e^8*x + d*e^7)
```

Sympy [A] time = 23.6801, size = 224, normalized size = 1.13

$$-\frac{4c^3d(d+ex)^{\frac{9}{2}}}{3e^7}+\frac{2c^3(d+ex)^{\frac{11}{2}}}{11e^7}+\frac{(d+ex)^{\frac{7}{2}}\left(6ac^2e^2+30c^3d^2\right)}{7e^7}+\frac{(d+ex)^{\frac{5}{2}}\left(-24ac^2de^2-40c^3d^3\right)}{5e^7}+\frac{(d+ex)^{\frac{3}{2}}\left(6a^2ce^4+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3/(e*x+d)**(3/2),x)
```

```
[Out] -4*c**3*d*(d + e*x)**(9/2)/(3*e**7) + 2*c**3*(d + e*x)**(11/2)/(11*e**7) +
(d + e*x)**(7/2)*(6*a*c**2*e**2 + 30*c**3*d**2)/(7*e**7) + (d + e*x)**(5/2)
*(-24*a*c**2*d*e**2 - 40*c**3*d**3)/(5*e**7) + (d + e*x)**(3/2)*(6*a**2*c*e
**4 + 36*a*c**2*d**2*e**2 + 30*c**3*d**4)/(3*e**7) + sqrt(d + e*x)*(-12*a**
2*c*d*e**4 - 24*a*c**2*d**3*e**2 - 12*c**3*d**5)/e**7 - 2*(a*e**2 + c*d**2)
**3/(e**7*sqrt(d + e*x))
```

Giac [A] time = 1.4838, size = 352, normalized size = 1.78

$$\frac{2}{1155}\left(105(xe+d)^{\frac{11}{2}}c^3e^{70}-770(xe+d)^{\frac{9}{2}}c^3de^{70}+2475(xe+d)^{\frac{7}{2}}c^3d^2e^{70}-4620(xe+d)^{\frac{5}{2}}c^3d^3e^{70}+5775(xe+d)^{\frac{3}{2}}c^3d^4e^{70}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] 2/1155*(105*(x*e + d)^(11/2)*c^3*e^70 - 770*(x*e + d)^(9/2)*c^3*d*e^70 + 24
75*(x*e + d)^(7/2)*c^3*d^2*e^70 - 4620*(x*e + d)^(5/2)*c^3*d^3*e^70 + 5775*
(x*e + d)^(3/2)*c^3*d^4*e^70 - 6930*sqrt(x*e + d)*c^3*d^5*e^70 + 495*(x*e +
 d)^(7/2)*a*c^2*e^72 - 2772*(x*e + d)^(5/2)*a*c^2*d*e^72 + 6930*(x*e + d)^(
3/2)*a*c^2*d^2*e^72 - 13860*sqrt(x*e + d)*a*c^2*d^3*e^72 + 1155*(x*e + d)^(
3/2)*a^2*c*e^74 - 6930*sqrt(x*e + d)*a^2*c*d*e^74)*e^(-77) - 2*(c^3*d^6 + 3
*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)*e^(-7)/sqrt(x*e + d)
```

3.610 $\int \frac{(a+cx^2)^3}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=200

$$\frac{6c^2(d+ex)^{5/2}\left(ae^2+5cd^2\right)}{5e^7}-\frac{8c^2d(d+ex)^{3/2}\left(3ae^2+5cd^2\right)}{3e^7}+\frac{6c\sqrt{d+ex}\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{e^7}+\frac{12cd\left(ae^2+cd^2\right)^2}{e^7\sqrt{d+ex}}$$

```
[Out] (-2*(c*d^2 + a*e^2)^3)/(3*e^7*(d + e*x)^(3/2)) + (12*c*d*(c*d^2 + a*e^2)^2)
/(e^7*Sqrt[d + e*x]) + (6*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2)*Sqrt[d + e*x]
)/e^7 - (8*c^2*d*(5*c*d^2 + 3*a*e^2)*(d + e*x)^(3/2))/(3*e^7) + (6*c^2*(5*c
*d^2 + a*e^2)*(d + e*x)^(5/2))/(5*e^7) - (12*c^3*d*(d + e*x)^(7/2))/(7*e^7)
 + (2*c^3*(d + e*x)^(9/2))/(9*e^7)
```

Rubi [A] time = 0.082786, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {697}

$$\frac{6c^2(d+ex)^{5/2}\left(ae^2+5cd^2\right)}{5e^7}-\frac{8c^2d(d+ex)^{3/2}\left(3ae^2+5cd^2\right)}{3e^7}+\frac{6c\sqrt{d+ex}\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{e^7}+\frac{12cd\left(ae^2+cd^2\right)^2}{e^7\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^3/(d + e*x)^(5/2),x]
```

```
[Out] (-2*(c*d^2 + a*e^2)^3)/(3*e^7*(d + e*x)^(3/2)) + (12*c*d*(c*d^2 + a*e^2)^2)
/(e^7*Sqrt[d + e*x]) + (6*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2)*Sqrt[d + e*x]
)/e^7 - (8*c^2*d*(5*c*d^2 + 3*a*e^2)*(d + e*x)^(3/2))/(3*e^7) + (6*c^2*(5*c
*d^2 + a*e^2)*(d + e*x)^(5/2))/(5*e^7) - (12*c^3*d*(d + e*x)^(7/2))/(7*e^7)
 + (2*c^3*(d + e*x)^(9/2))/(9*e^7)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^3}{(d+ex)^{5/2}}\,dx=\int \left(\frac{\left(cd^2+ae^2\right)^3}{e^6(d+ex)^{5/2}}-\frac{6cd\left(cd^2+ae^2\right)^2}{e^6(d+ex)^{3/2}}+\frac{3c\left(cd^2+ae^2\right)\left(5cd^2+ae^2\right)}{e^6\sqrt{d+ex}}-\frac{4c^2d\left(5cd^2+3ae^2\right)\sqrt{d+ex}}{e^6}+\right.$$

$$=-\frac{2\left(cd^2+ae^2\right)^3}{3e^7(d+ex)^{3/2}}+\frac{12cd\left(cd^2+ae^2\right)^2}{e^7\sqrt{d+ex}}+\frac{6c\left(cd^2+ae^2\right)\left(5cd^2+ae^2\right)\sqrt{d+ex}}{e^7}-\frac{8c^2d\left(5cd^2+3ae^2\right)(}{3e^7}$$

Mathematica [A] time = 0.10891, size = 171, normalized size = 0.86

$$\frac{2\left(315a^2ce^4\left(8d^2+12dex+3e^2x^2\right)-105a^3e^6+63ac^2e^2\left(48d^2e^2x^2+192d^3ex+128d^4-8de^3x^3+3e^4x^4\right)+5c^3\left(384d^4\right.\right.}{315e^7(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^2)^3/(d + e*x)^(5/2),x]

[Out] (2*(-105*a^3*e^6 + 315*a^2*c*e^4*(8*d^2 + 12*d*e*x + 3*e^2*x^2) + 63*a*c^2*
e^2*(128*d^4 + 192*d^3*e*x + 48*d^2*e^2*x^2 - 8*d*e^3*x^3 + 3*e^4*x^4) + 5*
c^3*(1024*d^6 + 1536*d^5*e*x + 384*d^4*e^2*x^2 - 64*d^3*e^3*x^3 + 24*d^2*e^
4*x^4 - 12*d*e^5*x^5 + 7*e^6*x^6)))/(315*e^7*(d + e*x)^(3/2))

Maple [A] time = 0.046, size = 205, normalized size = 1.

$$-\frac{-70\,c^3x^6e^6+120\,c^3dx^5e^5-378\,ac^2e^6x^4-240\,c^3d^2e^4x^4+1008\,ac^2de^5x^3+640\,c^3d^3e^3x^3-1890\,a^2ce^6x^2-6048\,ac^2d^2e^4x^2 \cdots}{3 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+a)^3/(e*x+d)^(5/2),x)

[Out] -2/315/(e*x+d)^(3/2)*(-35*c^3*e^6*x^6+60*c^3*d*e^5*x^5-189*a*c^2*e^6*x^4-12
0*c^3*d^2*e^4*x^4+504*a*c^2*d*e^5*x^3+320*c^3*d^3*e^3*x^3-945*a^2*c*e^6*x^2
-3024*a*c^2*d^2*e^4*x^2-1920*c^3*d^4*e^2*x^2-3780*a^2*c*d*e^5*x-12096*a*c^2
*d^3*e^3*x-7680*c^3*d^5*e*x+105*a^3*e^6-2520*a^2*c*d^2*e^4-8064*a*c^2*d^4*e
^2-5120*c^3*d^6)/e^7

Maxima [A] time = 1.60327, size = 290, normalized size = 1.45

$$\frac{2\left(\frac{35\,(ex+d)^{\frac{9}{2}}c^3-270\,(ex+d)^{\frac{7}{2}}c^3d+189\left(5\,c^3d^2+ac^2e^2\right)(ex+d)^{\frac{5}{2}}-420\left(5\,c^3d^3+3\,ac^2de^2\right)(ex+d)^{\frac{3}{2}}+945\left(5\,c^3d^4+6\,ac^2d^2e^2+a^2ce^4\right)\sqrt{ex+d}}{e^6}-\frac{105\left(c^3d^6+3\,ac^2d^4e^2+3 \cdots\right.}{\cdots}\right.}{315\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^3/(e*x+d)^(5/2),x, algorithm="maxima")

[Out] 2/315*((35*(e*x + d)^(9/2)*c^3 - 270*(e*x + d)^(7/2)*c^3*d + 189*(5*c^3*d^2
 + a*c^2*e^2)*(e*x + d)^(5/2) - 420*(5*c^3*d^3 + 3*a*c^2*d*e^2)*(e*x + d)^(
3/2) + 945*(5*c^3*d^4 + 6*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(e*x + d))/e^6 - 1
05*(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6 - 18*(c^3*d^5 + 2
*a*c^2*d^3*e^2 + a^2*c*d*e^4)*(e*x + d))/((e*x + d)^(3/2)*e^6))/e

---

Fricas [A] time = 1.84655, size = 495, normalized size = 2.48

$$\frac{2\left(35\,c^3e^6x^6-60\,c^3de^5x^5+5120\,c^3d^6+8064\,ac^2d^4e^2+2520\,a^2cd^2e^4-105\,a^3e^6+3\left(40\,c^3d^2e^4+63\,ac^2e^6\right)x^4-8\left(40\,c^3d^3 \cdots\right.\right.}{315\left(e^9x^2+2 \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^3/(e*x+d)^(5/2),x, algorithm="fricas")

[Out] 2/315*(35*c^3*e^6*x^6 - 60*c^3*d*e^5*x^5 + 5120*c^3*d^6 + 8064*a*c^2*d^4*e^
2 + 2520*a^2*c*d^2*e^4 - 105*a^3*e^6 + 3*(40*c^3*d^2*e^4 + 63*a*c^2*e^6)*x^
4 - 8*(40*c^3*d^3*e^3 + 63*a*c^2*d*e^5)*x^3 + 3*(640*c^3*d^4*e^2 + 1008*a*c
^2*d^2*e^4 + 315*a^2*c*e^6)*x^2 + 12*(640*c^3*d^5*e + 1008*a*c^2*d^3*e^3 +

```
315*a^2*c*d*e^5)*x)*sqrt(e*x + d)/(e^9*x^2 + 2*d*e^8*x + d^2*e^7)
```

Sympy [A] time = 31.1395, size = 206, normalized size = 1.03

$$-\frac{12c^3d\,(d+ex)^{\frac{7}{2}}}{7e^7}+\frac{2c^3\,(d+ex)^{\frac{9}{2}}}{9e^7}+\frac{12cd\left(ae^2+cd^2\right)^2}{e^7\sqrt{d+ex}}+\frac{(d+ex)^{\frac{5}{2}}\left(6ac^2e^2+30c^3d^2\right)}{5e^7}+\frac{(d+ex)^{\frac{3}{2}}\left(-24ac^2de^2-40c^3d\right.}{3e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3/(e*x+d)**(5/2),x)
```

```
[Out] -12*c**3*d*(d + e*x)**(7/2)/(7*e**7) + 2*c**3*(d + e*x)**(9/2)/(9*e**7) + 1
2*c*d*(a*e**2 + c*d**2)**2/(e**7*sqrt(d + e*x)) + (d + e*x)**(5/2)*(6*a*c**
2*e**2 + 30*c**3*d**2)/(5*e**7) + (d + e*x)**(3/2)*(-24*a*c**2*d*e**2 - 40*
c**3*d**3)/(3*e**7) + sqrt(d + e*x)*(6*a**2*c*e**4 + 36*a*c**2*d**2*e**2 +
30*c**3*d**4)/e**7 - 2*(a*e**2 + c*d**2)**3/(3*e**7*(d + e*x)**(3/2))
```

Giac [A] time = 1.40015, size = 344, normalized size = 1.72

$$\frac{2}{315}\left(35\,(xe+d)^{\frac{9}{2}}c^3e^{56}-270\,(xe+d)^{\frac{7}{2}}c^3de^{56}+945\,(xe+d)^{\frac{5}{2}}c^3d^2e^{56}-2100\,(xe+d)^{\frac{3}{2}}c^3d^3e^{56}+4725\,\sqrt{xe+d}c^3d^4e^{56}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] 2/315*(35*(x*e + d)^(9/2)*c^3*e^56 - 270*(x*e + d)^(7/2)*c^3*d*e^56 + 945*(
x*e + d)^(5/2)*c^3*d^2*e^56 - 2100*(x*e + d)^(3/2)*c^3*d^3*e^56 + 4725*sqrt
(x*e + d)*c^3*d^4*e^56 + 189*(x*e + d)^(5/2)*a*c^2*e^58 - 1260*(x*e + d)^(3
/2)*a*c^2*d*e^58 + 5670*sqrt(x*e + d)*a*c^2*d^2*e^58 + 945*sqrt(x*e + d)*a^
2*c*e^60)*e^(-63) + 2/3*(18*(x*e + d)*c^3*d^5 - c^3*d^6 + 36*(x*e + d)*a*c^
2*d^3*e^2 - 3*a*c^2*d^4*e^2 + 18*(x*e + d)*a^2*c*d*e^4 - 3*a^2*c*d^2*e^4 -
a^3*e^6)*e^(-7)/(x*e + d)^(3/2)
```

3.611 $\int \frac{(a+cx^2)^3}{(d+ex)^{7/2}} dx$

Optimal. Leaf size=196

$$\frac{2c^2(d+ex)^{3/2}(ae^2+5cd^2)}{e^7} - \frac{8c^2d\sqrt{d+ex}(3ae^2+5cd^2)}{e^7} - \frac{6c(ae^2+cd^2)(ae^2+5cd^2)}{e^7\sqrt{d+ex}} + \frac{4cd(ae^2+cd^2)^2}{e^7(d+ex)^{3/2}} - \frac{2(ae^2+cd^2 ...}{5e^7(d+ex)^{5 ...}}$$

```
[Out] (-2*(c*d^2 + a*e^2)^3)/(5*e^7*(d + e*x)^(5/2)) + (4*c*d*(c*d^2 + a*e^2)^2)/
(e^7*(d + e*x)^(3/2)) - (6*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2))/(e^7*Sqrt[d
 + e*x]) - (8*c^2*d*(5*c*d^2 + 3*a*e^2)*Sqrt[d + e*x])/e^7 + (2*c^2*(5*c*d^
2 + a*e^2)*(d + e*x)^(3/2))/e^7 - (12*c^3*d*(d + e*x)^(5/2))/(5*e^7) + (2*c
^3*(d + e*x)^(7/2))/(7*e^7)
```

Rubi [A] time = 0.0807837, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {697}

$$\frac{2c^2(d+ex)^{3/2}(ae^2+5cd^2)}{e^7} - \frac{8c^2d\sqrt{d+ex}(3ae^2+5cd^2)}{e^7} - \frac{6c(ae^2+cd^2)(ae^2+5cd^2)}{e^7\sqrt{d+ex}} + \frac{4cd(ae^2+cd^2)^2}{e^7(d+ex)^{3/2}} - \frac{2(ae^2+cd^2 ...}{5e^7(d+ex)^{5 ...}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^3/(d + e*x)^(7/2),x]
```


```
[Out] (-2*(c*d^2 + a*e^2)^3)/(5*e^7*(d + e*x)^(5/2)) + (4*c*d*(c*d^2 + a*e^2)^2)/
(e^7*(d + e*x)^(3/2)) - (6*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2))/(e^7*Sqrt[d
 + e*x]) - (8*c^2*d*(5*c*d^2 + 3*a*e^2)*Sqrt[d + e*x])/e^7 + (2*c^2*(5*c*d^
2 + a*e^2)*(d + e*x)^(3/2))/e^7 - (12*c^3*d*(d + e*x)^(5/2))/(5*e^7) + (2*c
^3*(d + e*x)^(7/2))/(7*e^7)
```


Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+cx^2)^3}{(d+ex)^{7/2}} dx &= \int \left(\frac{(cd^2+ae^2)^3}{e^6(d+ex)^{7/2}} - \frac{6cd(cd^2+ae^2)^2}{e^6(d+ex)^{5/2}} + \frac{3c(cd^2+ae^2)(5cd^2+ae^2)}{e^6(d+ex)^{3/2}} - \frac{4c^2d(5cd^2+3ae^2)}{e^6\sqrt{d+ex}} + \frac{3c^2(5cd^2 ...}{...}\right.\\ &= -\frac{2(cd^2+ae^2)^3}{5e^7(d+ex)^{5/2}} + \frac{4cd(cd^2+ae^2)^2}{e^7(d+ex)^{3/2}} - \frac{6c(cd^2+ae^2)(5cd^2+ae^2)}{e^7\sqrt{d+ex}} - \frac{8c^2d(5cd^2+3ae^2)\sqrt{d+ex}}{e^7} + \frac{2c ...}{...}\end{aligned}$$

Mathematica [A] time = 0.115347, size = 170, normalized size = 0.87

$$-\frac{2\left(7a^2ce^4(8d^2+20dex+15e^2x^2)+7a^3e^6+7ac^2e^2(240d^2e^2x^2+320d^3ex+128d^4+40de^3x^3-5e^4x^4)+c^3(1920d^4e^2x^2 ...\right.}{35e^7(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^3/(d + e*x)^(7/2),x]
```

```
[Out] (-2*(7*a^3*e^6 + 7*a^2*c*e^4*(8*d^2 + 20*d*e*x + 15*e^2*x^2) + 7*a*c^2*e^2*
(128*d^4 + 320*d^3*e*x + 240*d^2*e^2*x^2 + 40*d*e^3*x^3 - 5*e^4*x^4) + c^3*
(1024*d^6 + 2560*d^5*e*x + 1920*d^4*e^2*x^2 + 320*d^3*e^3*x^3 - 40*d^2*e^4*
x^4 + 12*d*e^5*x^5 - 5*e^6*x^6)))/(35*e^7*(d + e*x)^(5/2))
```

Maple [A] time = 0.044, size = 205, normalized size = 1.1

$$-\frac{-10\,c^3x^6e^6+24\,c^3dx^5e^5-70\,ac^2e^6x^4-80\,c^3d^2e^4x^4+560\,ac^2de^5x^3+640\,c^3d^3e^3x^3+210\,a^2ce^6x^2+3360\,ac^2d^2e^4x^2+}{35\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3/(e*x+d)^(7/2),x)
```

```
[Out] -2/35/(e*x+d)^(5/2)*(-5*c^3*e^6*x^6+12*c^3*d*e^5*x^5-35*a*c^2*e^6*x^4-40*c^
3*d^2*e^4*x^4+280*a*c^2*d*e^5*x^3+320*c^3*d^3*e^3*x^3+105*a^2*c*e^6*x^2+168
0*a*c^2*d^2*e^4*x^2+1920*c^3*d^4*e^2*x^2+140*a^2*c*d*e^5*x+2240*a*c^2*d^3*e
^3*x+2560*c^3*d^5*e*x+7*a^3*e^6+56*a^2*c*d^2*e^4+896*a*c^2*d^4*e^2+1024*c^3
*d^6)/e^7
```

Maxima [A] time = 1.61279, size = 290, normalized size = 1.48

$$\frac{2\left(\frac{5\,(ex+d)^{\frac{7}{2}}c^3-42\,(ex+d)^{\frac{5}{2}}c^3d+35\left(5\,c^3d^2+ac^2e^2\right)(ex+d)^{\frac{3}{2}}-140\left(5\,c^3d^3+3\,ac^2de^2\right)\sqrt{ex+d}}{e^6}-\frac{7\left(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6+15\left(5\,c^3d^4+6\,ac^2d^2e^2+a^2ce\right.\right.}{(ex+d)^{\frac{5}{2}}e^6}\right.}{35\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/35*((5*(e*x + d)^(7/2)*c^3 - 42*(e*x + d)^(5/2)*c^3*d + 35*(5*c^3*d^2 + a
*c^2*e^2)*(e*x + d)^(3/2) - 140*(5*c^3*d^3 + 3*a*c^2*d*e^2)*sqrt(e*x + d))/
e^6 - 7*(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6 + 15*(5*c^3*
d^4 + 6*a*c^2*d^2*e^2 + a^2*c*e^4)*(e*x + d)^2 - 10*(c^3*d^5 + 2*a*c^2*d^3*
e^2 + a^2*c*d*e^4)*(e*x + d))/((e*x + d)^(5/2)*e^6))/e
```

Fricas [A] time = 1.76616, size = 498, normalized size = 2.54

$$\frac{2\left(5\,c^3e^6x^6-12\,c^3de^5x^5-1024\,c^3d^6-896\,ac^2d^4e^2-56\,a^2cd^2e^4-7\,a^3e^6+5\left(8\,c^3d^2e^4+7\,ac^2e^6\right)x^4-40\left(8\,c^3d^3e^3+7\,a\right.\right.}{35\left(e^{10}x^3+3\,de^9x^2+3\,d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*c^3*e^6*x^6 - 12*c^3*d*e^5*x^5 - 1024*c^3*d^6 - 896*a*c^2*d^4*e^2 -
 56*a^2*c*d^2*e^4 - 7*a^3*e^6 + 5*(8*c^3*d^2*e^4 + 7*a*c^2*e^6)*x^4 - 40*(8
*c^3*d^3*e^3 + 7*a*c^2*d*e^5)*x^3 - 15*(128*c^3*d^4*e^2 + 112*a*c^2*d^2*e^4
 + 7*a^2*c*e^6)*x^2 - 20*(128*c^3*d^5*e + 112*a*c^2*d^3*e^3 + 7*a^2*c*d*e^5
```

```
)*x)*sqrt(e*x + d)/(e^10*x^3 + 3*d*e^9*x^2 + 3*d^2*e^8*x + d^3*e^7)
```

Sympy [A] time = 48.5068, size = 197, normalized size = 1.01

$$-\frac{12c^3d(d+ex)^{\frac{5}{2}}}{5e^7}+\frac{2c^3(d+ex)^{\frac{7}{2}}}{7e^7}+\frac{4cd\left(ae^2+cd^2\right)^2}{e^7(d+ex)^{\frac{3}{2}}}-\frac{6c\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{e^7\sqrt{d+ex}}+\frac{(d+ex)^{\frac{3}{2}}\left(6ac^2e^2+30c^3d^2\right)}{3e^7}+\frac{\sqrt{d}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3/(e*x+d)**(7/2),x)
```

```
[Out] -12*c**3*d*(d + e*x)**(5/2)/(5*e**7) + 2*c**3*(d + e*x)**(7/2)/(7*e**7) + 4
*c*d*(a*e**2 + c*d**2)**2/(e**7*(d + e*x)**(3/2)) - 6*c*(a*e**2 + c*d**2)*(
a*e**2 + 5*c*d**2)/(e**7*sqrt(d + e*x)) + (d + e*x)**(3/2)*(6*a*c**2*e**2 +
 30*c**3*d**2)/(3*e**7) + sqrt(d + e*x)*(-24*a*c**2*d*e**2 - 40*c**3*d**3)/
e**7 - 2*(a*e**2 + c*d**2)**3/(5*e**7*(d + e*x)**(5/2))
```

Giac [A] time = 1.34764, size = 339, normalized size = 1.73

$$\frac{2}{35}\left(5(xe+d)^{\frac{7}{2}}c^3e^{42}-42(xe+d)^{\frac{5}{2}}c^3de^{42}+175(xe+d)^{\frac{3}{2}}c^3d^2e^{42}-700\sqrt{xe+d}c^3d^3e^{42}+35(xe+d)^{\frac{3}{2}}ac^2e^{44}-420\sqrt{xe+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] 2/35*(5*(x*e + d)^(7/2)*c^3*e^42 - 42*(x*e + d)^(5/2)*c^3*d*e^42 + 175*(x*e
 + d)^(3/2)*c^3*d^2*e^42 - 700*sqrt(x*e + d)*c^3*d^3*e^42 + 35*(x*e + d)^(3
/2)*a*c^2*e^44 - 420*sqrt(x*e + d)*a*c^2*d*e^44)*e^(-49) - 2/5*(75*(x*e + d
)^2*c^3*d^4 - 10*(x*e + d)*c^3*d^5 + c^3*d^6 + 90*(x*e + d)^2*a*c^2*d^2*e^2
 - 20*(x*e + d)*a*c^2*d^3*e^2 + 3*a*c^2*d^4*e^2 + 15*(x*e + d)^2*a^2*c*e^4
- 10*(x*e + d)*a^2*c*d*e^4 + 3*a^2*c*d^2*e^4 + a^3*e^6)*e^(-7)/(x*e + d)^(5
/2)
```

3.612 $\int \frac{(d+ex)^{5/2}}{a-cx^2}\,dx$

Optimal. Leaf size=167

$$-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}c^{7/4}}+\frac{\left(\sqrt{a}e+\sqrt{c}d\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{\sqrt{a}c^{7/4}}-\frac{2e(d+ex)^{3/2}}{3c}-\frac{4de\sqrt{d+ex}}{c}$$

```
[Out] (-4*d*e*Sqrt[d + e*x])/c - (2*e*(d + e*x)^(3/2))/(3*c) - ((Sqrt[c]*d - Sqrt
[a]*e)^(5/2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/
(Sqrt[a]*c^(7/4)) + ((Sqrt[c]*d + Sqrt[a]*e)^(5/2)*ArcTanh[(c^(1/4)*Sqrt[d
+ e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(Sqrt[a]*c^(7/4))
```

Rubi [A] time = 0.397827, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {704, 825, 827, 1166, 208}

$$-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}c^{7/4}}+\frac{\left(\sqrt{a}e+\sqrt{c}d\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{\sqrt{a}c^{7/4}}-\frac{2e(d+ex)^{3/2}}{3c}-\frac{4de\sqrt{d+ex}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(a - c*x^2),x]
```


```
[Out] (-4*d*e*Sqrt[d + e*x])/c - (2*e*(d + e*x)^(3/2))/(3*c) - ((Sqrt[c]*d - Sqrt
[a]*e)^(5/2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/
(Sqrt[a]*c^(7/4)) + ((Sqrt[c]*d + Sqrt[a]*e)^(5/2)*ArcTanh[(c^(1/4)*Sqrt[d
+ e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(Sqrt[a]*c^(7/4))
```


Rule 704

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^(m - 2)*Simp[c*
d^2 - a*e^2 + 2*c*d*e*x, x])/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x]
&& NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 1]
```

Rule 825

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[((d + e*x)^(m
- 1)*Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x])/(a + c*x^2), x], x] /; Fre
eQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m
, 0]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{a-cx^2}\,dx = -\frac{2e(d+ex)^{3/2}}{3c} - \frac{\int \frac{\sqrt{d+ex}\left(-cd^2-ae^2-2cdex\right)}{a-cx^2}\,dx}{c}$$

$$= -\frac{4de\sqrt{d+ex}}{c} - \frac{2e(d+ex)^{3/2}}{3c} + \frac{\int \frac{cd\left(cd^2+3ae^2\right)+ce\left(3cd^2+ae^2\right)x}{\sqrt{d+ex}\left(a-cx^2\right)}\,dx}{c^2}$$

$$= -\frac{4de\sqrt{d+ex}}{c} - \frac{2e(d+ex)^{3/2}}{3c} + \frac{2\,\mathrm{Subst}\left(\int \frac{-cde\left(3cd^2+ae^2\right)+cde\left(cd^2+3ae^2\right)+ce\left(3cd^2+ae^2\right)x^2}{-cd^2+ae^2+2cdx^2-cx^4}\,dx, x, \sqrt{d+ex}\right)}{c^2}$$

$$= -\frac{4de\sqrt{d+ex}}{c} - \frac{2e(d+ex)^{3/2}}{3c} - \frac{\left(\sqrt{c}d-\sqrt{a}e\right)^3 \mathrm{Subst}\left(\int \frac{1}{cd-\sqrt{a}\sqrt{c}e-cx^2}\,dx, x, \sqrt{d+ex}\right)}{\sqrt{a}c} + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)^3}{}$$

$$= -\frac{4de\sqrt{d+ex}}{c} - \frac{2e(d+ex)^{3/2}}{3c} - \frac{\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}c^{7/4}} + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+}}{\sqrt{\sqrt{c}d+}}\right)}{\sqrt{a}c^{7/4}}$$

Mathematica [A] time = 0.259399, size = 158, normalized size = 0.95

$$\frac{-2\sqrt{a}c^{3/4}e\sqrt{d+ex}(7d+ex)-3\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)+3\left(\sqrt{a}e+\sqrt{c}d\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{3\sqrt{a}c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(a - c*x^2),x]
```

```
[Out] (-2*Sqrt[a]*c^(3/4)*e*Sqrt[d + e*x]*(7*d + e*x) - 3*(Sqrt[c]*d - Sqrt[a]*e)
^(5/2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]] + 3*(Sq
rt[c]*d + Sqrt[a]*e)^(5/2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d +
 Sqrt[a]*e]])/(3*Sqrt[a]*c^(7/4))
```

Maple [B] time = 0.267, size = 460, normalized size = 2.8

$$-\frac{2e}{3c}(ex+d)^{\frac{3}{2}} - 4\frac{de\sqrt{ex+d}}{c} + 3\frac{ade^3}{\sqrt{ace^2}\sqrt{\left(-cd+\sqrt{ace^2}\right)c}}\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ace^2}\right)c}}\right) + ced^3\arctan\left(c\sqrt{ex+d}\frac{}{\sqrt{(-c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(-c*x^2+a),x)
```

```
[Out] -2/3*e*(e*x+d)^(3/2)/c-4*d*e*(e*x+d)^(1/2)/c+3/(a*c*e^2)^(1/2)/((-c*d+(a*c*
e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2
))*a*d*e^3+e*c/(a*c*e^2)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan((e*x
+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d^3-1/c/((-c*d+(a*c*e^2)^(1/2
))*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))*a*e^3-
3*e/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2
)^(1/2))*c)^(1/2))*d^2+3/(a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*ar
ctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))*a*d*e^3+e*c/(a*c*e^2
)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*
e^2)^(1/2))*c)^(1/2))*d^3+1/c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+
d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))*a*e^3+3*e/((c*d+(a*c*e^2)^(1/2)
)*c)^(1/2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ex+d)^{\frac{5}{2}}}{cx^2-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(-c*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate((e*x + d)^(5/2)/(c*x^2 - a), x)
```

Fricas [B] time = 2.58692, size = 3356, normalized size = 20.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(-c*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/6*(3*c*sqrt((c^2*d^5 + 10*a*c*d^3*e^2 + 5*a^2*d*e^4 + a*c^3*sqrt((25*c^4
*d^8*e^2 + 100*a*c^3*d^6*e^4 + 110*a^2*c^2*d^4*e^6 + 20*a^3*c*d^2*e^8 + a^4
*e^10)/(a*c^7)))/(a*c^3))*log((5*c^4*d^8*e - 14*a^2*c^2*d^4*e^5 + 8*a^3*c*d
^2*e^7 + a^4*e^9)*sqrt(e*x + d) + (10*a*c^4*d^5*e^2 + 20*a^2*c^3*d^3*e^4 +
2*a^3*c^2*d*e^6 - (a*c^6*d^2 + a^2*c^5*e^2)*sqrt((25*c^4*d^8*e^2 + 100*a*c^
3*d^6*e^4 + 110*a^2*c^2*d^4*e^6 + 20*a^3*c*d^2*e^8 + a^4*e^10)/(a*c^7)))*sq
rt((c^2*d^5 + 10*a*c*d^3*e^2 + 5*a^2*d*e^4 + a*c^3*sqrt((25*c^4*d^8*e^2 + 1
00*a*c^3*d^6*e^4 + 110*a^2*c^2*d^4*e^6 + 20*a^3*c*d^2*e^8 + a^4*e^10)/(a*c^
7)))/(a*c^3))) - 3*c*sqrt((c^2*d^5 + 10*a*c*d^3*e^2 + 5*a^2*d*e^4 + a*c^3*s
qrt((25*c^4*d^8*e^2 + 100*a*c^3*d^6*e^4 + 110*a^2*c^2*d^4*e^6 + 20*a^3*c*d^
2*e^8 + a^4*e^10)/(a*c^7)))/(a*c^3))*log((5*c^4*d^8*e - 14*a^2*c^2*d^4*e^5
+ 8*a^3*c*d^2*e^7 + a^4*e^9)*sqrt(e*x + d) - (10*a*c^4*d^5*e^2 + 20*a^2*c^3
*d^3*e^4 + 2*a^3*c^2*d*e^6 - (a*c^6*d^2 + a^2*c^5*e^2)*sqrt((25*c^4*d^8*e^2
 + 100*a*c^3*d^6*e^4 + 110*a^2*c^2*d^4*e^6 + 20*a^3*c*d^2*e^8 + a^4*e^10)/(
a*c^7)))*sqrt((c^2*d^5 + 10*a*c*d^3*e^2 + 5*a^2*d*e^4 + a*c^3*sqrt((25*c^4*
d^8*e^2 + 100*a*c^3*d^6*e^4 + 110*a^2*c^2*d^4*e^6 + 20*a^3*c*d^2*e^8 + a^4*
e^10)/(a*c^7)))/(a*c^3))) + 3*c*sqrt((c^2*d^5 + 10*a*c*d^3*e^2 + 5*a^2*d*e^
4 - a*c^3*sqrt((25*c^4*d^8*e^2 + 100*a*c^3*d^6*e^4 + 110*a^2*c^2*d^4*e^6 +
20*a^3*c*d^2*e^8 + a^4*e^10)/(a*c^7)))/(a*c^3))*log((5*c^4*d^8*e - 14*a^2*c
^2*d^4*e^5 + 8*a^3*c*d^2*e^7 + a^4*e^9)*sqrt(e*x + d) + (10*a*c^4*d^5*e^2 +
 20*a^2*c^3*d^3*e^4 + 2*a^3*c^2*d*e^6 + (a*c^6*d^2 + a^2*c^5*e^2)*sqrt((25*
c^4*d^8*e^2 + 100*a*c^3*d^6*e^4 + 110*a^2*c^2*d^4*e^6 + 20*a^3*c*d^2*e^8 +
a^4*e^10)/(a*c^7)))*sqrt((c^2*d^5 + 10*a*c*d^3*e^2 + 5*a^2*d*e^4 - a*c^3*sq
```

```
rt((25*c^4*d^8*e^2 + 100*a*c^3*d^6*e^4 + 110*a^2*c^2*d^4*e^6 + 20*a^3*c*d^2
*e^8 + a^4*e^10)/(a*c^7)))/(a*c^3))) - 3*c*sqrt((c^2*d^5 + 10*a*c*d^3*e^2 +
 5*a^2*d*e^4 - a*c^3*sqrt((25*c^4*d^8*e^2 + 100*a*c^3*d^6*e^4 + 110*a^2*c^2
*d^4*e^6 + 20*a^3*c*d^2*e^8 + a^4*e^10)/(a*c^7)))/(a*c^3))*log((5*c^4*d^8*e
 - 14*a^2*c^2*d^4*e^5 + 8*a^3*c*d^2*e^7 + a^4*e^9)*sqrt(e*x + d) - (10*a*c^
4*d^5*e^2 + 20*a^2*c^3*d^3*e^4 + 2*a^3*c^2*d*e^6 + (a*c^6*d^2 + a^2*c^5*e^2
)*sqrt((25*c^4*d^8*e^2 + 100*a*c^3*d^6*e^4 + 110*a^2*c^2*d^4*e^6 + 20*a^3*c
*d^2*e^8 + a^4*e^10)/(a*c^7)))*sqrt((c^2*d^5 + 10*a*c*d^3*e^2 + 5*a^2*d*e^4
 - a*c^3*sqrt((25*c^4*d^8*e^2 + 100*a*c^3*d^6*e^4 + 110*a^2*c^2*d^4*e^6 + 2
0*a^3*c*d^2*e^8 + a^4*e^10)/(a*c^7)))/(a*c^3))) + 4*(e^2*x + 7*d*e)*sqrt(e*
x + d))/c
```

Sympy [B] time = 70.1881, size = 418, normalized size = 2.5

$$-\frac{4ade^3\operatorname{RootSum}\left(t^4\left(256a^3ce^6-256a^2c^2d^2e^4\right)+32t^2acde^2-1,\left(t\mapsto t\log\left(-64t^3a^2cde^4+64t^3ac^2d^3e^2-4tae^2-4tcd^2 \right.\right.\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(-c*x**2+a),x)
```

```
[Out] -4*a*d*e**3*RootSum(_t**4*(256*a**3*c*e**6 - 256*a**2*c**2*d**2*e**4) + 32*
_t**2*a*c*d*e**2 - 1, Lambda(_t, _t*log(-64*_t**3*a**2*c*d*e**4 + 64*_t**3*
a*c**2*d**3*e**2 - 4*_t*a*e**2 - 4*_t*c*d**2 + sqrt(d + e*x))))/c - 2*a*e**
3*RootSum(256*_t**4*a**2*c**3*e**4 - 32*_t**2*a*c**2*d*e**2 - a*e**2 + c*d*
*2, Lambda(_t, _t*log(-64*_t**3*a*c**2*e**2 + 4*_t*c*d + sqrt(d + e*x))))/c
 + 4*d**3*e*RootSum(_t**4*(256*a**3*c*e**6 - 256*a**2*c**2*d**2*e**4) + 32*
_t**2*a*c*d*e**2 - 1, Lambda(_t, _t*log(-64*_t**3*a**2*c*d*e**4 + 64*_t**3*
a*c**2*d**3*e**2 - 4*_t*a*e**2 - 4*_t*c*d**2 + sqrt(d + e*x)))) - 6*d**2*e*
RootSum(256*_t**4*a**2*c**3*e**4 - 32*_t**2*a*c**2*d*e**2 - a*e**2 + c*d**2
, Lambda(_t, _t*log(-64*_t**3*a*c**2*e**2 + 4*_t*c*d + sqrt(d + e*x)))) - 4
*d*e*sqrt(d + e*x)/c - 2*e*(d + e*x)**(3/2)/(3*c)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(-c*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.613 $\int \frac{(d+ex)^{3/2}}{a-cx^2}\,dx$

Optimal. Leaf size=149

$$-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}c^{5/4}}+\frac{\left(\sqrt{a}e+\sqrt{c}d\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{\sqrt{a}c^{5/4}}-\frac{2e\sqrt{d+ex}}{c}$$

```
[Out] (-2*e*Sqrt[d + e*x])/c - ((Sqrt[c]*d - Sqrt[a]*e)^(3/2)*ArcTanh[(c^(1/4)*Sq
rt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(Sqrt[a]*c^(5/4)) + ((Sqrt[c]*d
+ Sqrt[a]*e)^(3/2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]
*e]])/(Sqrt[a]*c^(5/4))
```

Rubi [A] time = 0.281862, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {704, 827, 1166, 208}

$$-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}c^{5/4}}+\frac{\left(\sqrt{a}e+\sqrt{c}d\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{\sqrt{a}c^{5/4}}-\frac{2e\sqrt{d+ex}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(a - c*x^2),x]
```

```
[Out] (-2*e*Sqrt[d + e*x])/c - ((Sqrt[c]*d - Sqrt[a]*e)^(3/2)*ArcTanh[(c^(1/4)*Sq
rt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(Sqrt[a]*c^(5/4)) + ((Sqrt[c]*d
+ Sqrt[a]*e)^(3/2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]
*e]])/(Sqrt[a]*c^(5/4))
```

Rule 704

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^(m - 2)*Simp[c*
d^2 - a*e^2 + 2*c*d*e*x, x])/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x]
&& NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 1]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^{3/2}}{a-cx^2}\,dx &= -\frac{2e\sqrt{d+ex}}{c}-\frac{\int \frac{-cd^2-ae^2-2cdex}{\sqrt{d+ex}(a-cx^2)}\,dx}{c}\\ &= -\frac{2e\sqrt{d+ex}}{c}-\frac{2\,\text{Subst}\left(\int \frac{2cd^2e+e(-cd^2-ae^2)-2cdex^2}{-cd^2+ae^2+2cdx^2-cx^4}\,dx,x,\sqrt{d+ex}\right)}{c}\\ &= -\frac{2e\sqrt{d+ex}}{c}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2\text{Subst}\left(\int \frac{1}{cd-\sqrt{a}\sqrt{c}e-cx^2}\,dx,x,\sqrt{d+ex}\right)}{\sqrt{a}\sqrt{c}}+\frac{\left(\sqrt{c}d+\sqrt{a}e\right)^2\text{Subst}\left(\int \frac{1}{cd+\sqrt{a}\sqrt{\ldots}}\right.}{\sqrt{a}\sqrt{c}}\\ &= -\frac{2e\sqrt{d+ex}}{c}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}c^{5/4}}+\frac{\left(\sqrt{c}d+\sqrt{a}e\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{\sqrt{a}c^{5/4}}\end{aligned}$$

Mathematica [A] time = 0.115088, size = 147, normalized size = 0.99

$$\frac{-2\sqrt{a}\sqrt[4]{c}e\sqrt{d+ex}+\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}\left(-\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)\right)+\left(\sqrt{a}e+\sqrt{c}d\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{\sqrt{a}c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(a - c*x^2),x]
```

```
[Out] (-2*Sqrt[a]*c^(1/4)*e*Sqrt[d + e*x] - (Sqrt[c]*d - Sqrt[a]*e)^(3/2)*ArcTanh
[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]] + (Sqrt[c]*d + Sqrt[a
]*e)^(3/2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(S
qrt[a]*c^(5/4))
```

Maple [B] time = 0.209, size = 335, normalized size = 2.3

$$-2\,\frac{e\sqrt{ex+d}}{c}+ae^3\arctan\left(c\sqrt{ex+d}\frac{1}{\sqrt{\left(-cd+\sqrt{ace^2}\right)c}}\right)\frac{1}{\sqrt{ace^2}}\frac{1}{\sqrt{\left(-cd+\sqrt{ace^2}\right)c}}+ced^2\arctan\left(c\sqrt{ex+d}\frac{1}{\sqrt{\left(-cd+\sqrt{\ldots}\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(-c*x^2+a),x)
```

```
[Out] -2*e*(e*x+d)^(1/2)/c+1/(a*c*e^2)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arc
tan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))*a*e^3+e/(a*c*e^2)^(1/
2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)
^(1/2))*c)^(1/2))*c*d^2-2*e/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)
^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d+1/(a*c*e^2)^(1/2)/((c*d+(a*c*e
^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)
)*a*e^3+e/(a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(
1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))*c*d^2+2*e/((c*d+(a*c*e^2)^(1/2))*c)
```

```
^(1/2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ex+d)^{\frac{3}{2}}}{cx^2-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(-c*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate((e*x + d)^(3/2)/(c*x^2 - a), x)
```

Fricas [B] time = 2.03652, size = 1970, normalized size = 13.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(-c*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(c*sqrt((c*d^3 + 3*a*d*e^2 + a*c^2*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4
+ a^2*e^6)/(a*c^5)))/(a*c^2))*log(-(3*c^2*d^4*e - 2*a*c*d^2*e^3 - a^2*e^5)*
sqrt(e*x + d) + (3*a*c^2*d^2*e^2 + a^2*c*e^4 - a*c^4*d*sqrt((9*c^2*d^4*e^2
+ 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^5)))*sqrt((c*d^3 + 3*a*d*e^2 + a*c^2*sqrt((
9*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^5)))/(a*c^2))) - c*sqrt((c*d^
3 + 3*a*d*e^2 + a*c^2*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^5
)))/(a*c^2))*log(-(3*c^2*d^4*e - 2*a*c*d^2*e^3 - a^2*e^5)*sqrt(e*x + d) - (
3*a*c^2*d^2*e^2 + a^2*c*e^4 - a*c^4*d*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4 +
 a^2*e^6)/(a*c^5)))*sqrt((c*d^3 + 3*a*d*e^2 + a*c^2*sqrt((9*c^2*d^4*e^2 + 6
*a*c*d^2*e^4 + a^2*e^6)/(a*c^5)))/(a*c^2))) + c*sqrt((c*d^3 + 3*a*d*e^2 - a
*c^2*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^5)))/(a*c^2))*log(
-(3*c^2*d^4*e - 2*a*c*d^2*e^3 - a^2*e^5)*sqrt(e*x + d) + (3*a*c^2*d^2*e^2 +
 a^2*c*e^4 + a*c^4*d*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^5)
))*sqrt((c*d^3 + 3*a*d*e^2 - a*c^2*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + a^
2*e^6)/(a*c^5)))/(a*c^2))) - c*sqrt((c*d^3 + 3*a*d*e^2 - a*c^2*sqrt((9*c^2*
d^4*e^2 + 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^5)))/(a*c^2))*log(-(3*c^2*d^4*e - 2
*a*c*d^2*e^3 - a^2*e^5)*sqrt(e*x + d) - (3*a*c^2*d^2*e^2 + a^2*c*e^4 + a*c^
4*d*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^5)))*sqrt((c*d^3 +
3*a*d*e^2 - a*c^2*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^5)))/
(a*c^2))) - 4*sqrt(e*x + d)*e)/c
```

Sympy [B] time = 37.8756, size = 316, normalized size = 2.12

$$-\frac{2ae^3\operatorname{RootSum}\left(t^4\left(256a^3ce^6-256a^2c^2d^2e^4\right)+32t^2acde^2-1,\left(t\mapsto t\log\left(-64t^3a^2cde^4+64t^3ac^2d^3e^2-4tae^2-4tcd\right.\right.\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(-c*x**2+a),x)
```

```
[Out] -2*a*e**3*RootSum(_t**4*(256*a**3*c*e**6 - 256*a**2*c**2*d**2*e**4) + 32*_t
**2*a*c*d*e**2 - 1, Lambda(_t, _t*log(-64*_t**3*a**2*c*d*e**4 + 64*_t**3*a*
c**2*d**3*e**2 - 4*_t*a*e**2 - 4*_t*c*d**2 + sqrt(d + e*x))))/c + 2*d**2*e*
RootSum(_t**4*(256*a**3*c*e**6 - 256*a**2*c**2*d**2*e**4) + 32*_t**2*a*c*d*
e**2 - 1, Lambda(_t, _t*log(-64*_t**3*a**2*c*d*e**4 + 64*_t**3*a*c**2*d**3*
e**2 - 4*_t*a*e**2 - 4*_t*c*d**2 + sqrt(d + e*x)))) - 4*d*e*RootSum(256*_t*
*4*a**2*c**3*e**4 - 32*_t**2*a*c**2*d*e**2 - a*e**2 + c*d**2, Lambda(_t, _t
*log(-64*_t**3*a*c**2*e**2 + 4*_t*c*d + sqrt(d + e*x)))) - 2*e*sqrt(d + e*x
)/c
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(-c*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.614 $\int \frac{\sqrt{d+ex}}{a-cx^2}\,dx$

Optimal. Leaf size=134

$$\frac{\sqrt{\sqrt{a}e+\sqrt{c}d}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{\sqrt{a}c^{3/4}}-\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}c^{3/4}}$$

```
[Out] -((Sqrt[Sqrt[c]*d - Sqrt[a]*e]*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]
*d - Sqrt[a]*e]])/(Sqrt[a]*c^(3/4))) + (Sqrt[Sqrt[c]*d + Sqrt[a]*e]*ArcTanh
[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(Sqrt[a]*c^(3/4))
```

Rubi [A] time = 0.108612, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {700, 1130, 208}

$$\frac{\sqrt{\sqrt{a}e+\sqrt{c}d}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{\sqrt{a}c^{3/4}}-\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(a - c*x^2),x]
```

```
[Out] -((Sqrt[Sqrt[c]*d - Sqrt[a]*e]*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]
*d - Sqrt[a]*e]])/(Sqrt[a]*c^(3/4))) + (Sqrt[Sqrt[c]*d + Sqrt[a]*e]*ArcTanh
[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(Sqrt[a]*c^(3/4))
```

Rule 700

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{a-cx^2}\,dx = (2e)\,\text{Subst}\left(\int \frac{x^2}{-cd^2+ae^2+2cdx^2-cx^4}\,dx, x, \sqrt{d+ex}\right)$$
$$= -\left(\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\text{Subst}\left(\int \frac{1}{cd-\sqrt{a}\sqrt{c}e-cx^2}\,dx, x, \sqrt{d+ex}\right)\right)+\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\text{Subst}\left(\int \frac{1}{cd+\sqrt{a}\sqrt{c}e-cx^2}\right.$$
$$= -\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}c^{3/4}}+\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{\sqrt{a}c^{3/4}}$$

Mathematica [A] time = 0.0605512, size = 125, normalized size = 0.93

$$\frac{\sqrt{\sqrt{a}e+\sqrt{c}d}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)-\sqrt{\sqrt{c}d-\sqrt{a}e}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(a - c*x^2),x]
```

```
[Out] (-(Sqrt[Sqrt[c]*d - Sqrt[a]*e]*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]
*d - Sqrt[a]*e]]) + Sqrt[Sqrt[c]*d + Sqrt[a]*e]*ArcTanh[(c^(1/4)*Sqrt[d + e
*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(Sqrt[a]*c^(3/4))
```

Maple [B] time = 0.207, size = 203, normalized size = 1.5

$$ced\arctan\left(c\sqrt{ex+d}\frac{1}{\sqrt{\left(-cd+\sqrt{ace^2}\right)c}}\right)\frac{1}{\sqrt{ace^2}}\frac{1}{\sqrt{\left(-cd+\sqrt{ace^2}\right)c}}-e\arctan\left(c\sqrt{ex+d}\frac{1}{\sqrt{\left(-cd+\sqrt{ace^2}\right)c}}\right)\frac{1}{\sqrt{\left(-cd+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(-c*x^2+a),x)
```

```
[Out] c*e/(a*c*e^2)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c
/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d-e/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arc
tan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))+c*e/(a*c*e^2)^(1/2)/(
(c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2
))*c)^(1/2))*d+e/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c/((
c*d+(a*c*e^2)^(1/2))*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{ex+d}}{cx^2-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(-c*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(e*x + d)/(c*x^2 - a), x)
```

Fricas [B] time = 1.9826, size = 725, normalized size = 5.41

$$\frac{1}{2}\sqrt{\frac{ac\sqrt{\frac{e^2}{ac^3}}+d}{ac}}\log\left(ac^2\sqrt{\frac{ac\sqrt{\frac{e^2}{ac^3}}+d}{ac}}\sqrt{\frac{e^2}{ac^3}}+\sqrt{ex+d}e\right)-\frac{1}{2}\sqrt{\frac{ac\sqrt{\frac{e^2}{ac^3}}+d}{ac}}\log\left(-ac^2\sqrt{\frac{ac\sqrt{\frac{e^2}{ac^3}}+d}{ac}}\sqrt{\frac{e^2}{ac^3}}+\sqrt{ex+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(-c*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt((a*c*sqrt(e^2/(a*c^3)) + d)/(a*c))*log(a*c^2*sqrt((a*c*sqrt(e^2/(a
*c^3)) + d)/(a*c))*sqrt(e^2/(a*c^3)) + sqrt(e*x + d)*e) - 1/2*sqrt((a*c*sqr
t(e^2/(a*c^3)) + d)/(a*c))*log(-a*c^2*sqrt((a*c*sqrt(e^2/(a*c^3)) + d)/(a*c
))*sqrt(e^2/(a*c^3)) + sqrt(e*x + d)*e) - 1/2*sqrt(-(a*c*sqrt(e^2/(a*c^3))
- d)/(a*c))*log(a*c^2*sqrt(-(a*c*sqrt(e^2/(a*c^3)) - d)/(a*c))*sqrt(e^2/(a*
c^3)) + sqrt(e*x + d)*e) + 1/2*sqrt(-(a*c*sqrt(e^2/(a*c^3)) - d)/(a*c))*log
(-a*c^2*sqrt(-(a*c*sqrt(e^2/(a*c^3)) - d)/(a*c))*sqrt(e^2/(a*c^3)) + sqrt(e
*x + d)*e)
```

Sympy [A] time = 5.73174, size = 76, normalized size = 0.57

$$-2e\,\mathrm{RootSum}\left(256t^4a^2c^3e^4-32t^2ac^2de^2-ae^2+cd^2,\left(t\mapsto t\log\left(-64t^3ac^2e^2+4tcd+\sqrt{d+ex}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(-c*x**2+a),x)
```

```
[Out] -2*e*RootSum(256*_t**4*a**2*c**3*e**4 - 32*_t**2*a*c**2*d*e**2 - a*e**2 + c
*d**2, Lambda(_t, _t*log(-64*_t**3*a*c**2*e**2 + 4*_t*c*d + sqrt(d + e*x)))
)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(-c*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.615 $\int \frac{1}{\sqrt{d+ex}\left(a-cx^2\right)}\,dx$

Optimal. Leaf size=134

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{\sqrt{a}\sqrt[4]{c}\sqrt{\sqrt{a}e+\sqrt{c}d}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}\sqrt[4]{c}\sqrt{\sqrt{c}d-\sqrt{a}e}}$$

```
[Out] -(ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]]/(Sqrt[a]*c^(
1/4)*Sqrt[Sqrt[c]*d - Sqrt[a]*e])) + ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[S
qrt[c]*d + Sqrt[a]*e]]/(Sqrt[a]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[a]*e])
```

Rubi [A] time = 0.113946, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {708, 1093, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{\sqrt{a}\sqrt[4]{c}\sqrt{\sqrt{a}e+\sqrt{c}d}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}\sqrt[4]{c}\sqrt{\sqrt{c}d-\sqrt{a}e}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(a - c*x^2)),x]
```


```
[Out] -(ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]]/(Sqrt[a]*c^(
1/4)*Sqrt[Sqrt[c]*d - Sqrt[a]*e])) + ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[S
qrt[c]*d + Sqrt[a]*e]]/(Sqrt[a]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[a]*e])
```


Rule 708

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\left(a-cx^2\right)}\,dx = (2e)\,\text{Subst}\left(\int \frac{1}{-cd^2+ae^2+2cdx^2-cx^4}\,dx, x, \sqrt{d+ex}\right)$$
$$= -\frac{\sqrt{c}\,\text{Subst}\left(\int \frac{1}{cd-\sqrt{a}\sqrt{c}e-cx^2}\,dx, x, \sqrt{d+ex}\right)}{\sqrt{a}} + \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{1}{cd+\sqrt{a}\sqrt{c}e-cx^2}\,dx, x, \sqrt{d+ex}\right)}{\sqrt{a}}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}\sqrt[4]{c}\sqrt{\sqrt{c}d-\sqrt{a}e}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{\sqrt{a}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{a}e}}$$

Mathematica [A] time = 0.0868462, size = 125, normalized size = 0.93

$$\frac{\frac{\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{\sqrt{\sqrt{a}e+\sqrt{c}d}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{\sqrt{c}d-\sqrt{a}e}}}{\sqrt{a}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(a - c*x^2)),x]
```

```
[Out] (-(ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]]/Sqrt[Sqrt[c
]*d - Sqrt[a]*e]) + ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a
]*e]]/Sqrt[Sqrt[c]*d + Sqrt[a]*e])/(Sqrt[a]*c^(1/4))
```

Maple [A] time = 0.205, size = 110, normalized size = 0.8

$$ce\arctan\left(c\sqrt{ex+d}\frac{1}{\sqrt{\left(-cd+\sqrt{ace^2}\right)c}}\right)\frac{1}{\sqrt{ace^2}}\frac{1}{\sqrt{\left(-cd+\sqrt{ace^2}\right)c}} + ce\text{Artanh}\left(c\sqrt{ex+d}\frac{1}{\sqrt{\left(cd+\sqrt{ace^2}\right)c}}\right)\frac{1}{\sqrt{ace^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c*x^2+a)/(e*x+d)^(1/2),x)
```

```
[Out] c*e/(a*c*e^2)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c
/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))+c*e/(a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/2
))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(cx^2-a\right)\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*x^2+a)/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((c*x^2 - a)*sqrt(e*x + d)), x)
```

Fricas [B] time = 1.94096, size = 1791, normalized size = 13.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*x^2+a)/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(((a*c*d^2 - a^2*e^2)*sqrt(e^2/(a*c^3*d^4 - 2*a^2*c^2*d^2*e^2 + a^3
*c*e^4)) + d)/(a*c*d^2 - a^2*e^2))*log(sqrt(e*x + d)*e + (a*e^2 - (a*c^2*d^
3 - a^2*c*d*e^2)*sqrt(e^2/(a*c^3*d^4 - 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))*sqr
t(((a*c*d^2 - a^2*e^2)*sqrt(e^2/(a*c^3*d^4 - 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)
) + d)/(a*c*d^2 - a^2*e^2))) - 1/2*sqrt(((a*c*d^2 - a^2*e^2)*sqrt(e^2/(a*c^
3*d^4 - 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)) + d)/(a*c*d^2 - a^2*e^2))*log(sqrt(
e*x + d)*e - (a*e^2 - (a*c^2*d^3 - a^2*c*d*e^2)*sqrt(e^2/(a*c^3*d^4 - 2*a^2
*c^2*d^2*e^2 + a^3*c*e^4)))*sqrt(((a*c*d^2 - a^2*e^2)*sqrt(e^2/(a*c^3*d^4 -
 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)) + d)/(a*c*d^2 - a^2*e^2))) + 1/2*sqrt(-((a
*c*d^2 - a^2*e^2)*sqrt(e^2/(a*c^3*d^4 - 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)) - d
)/(a*c*d^2 - a^2*e^2))*log(sqrt(e*x + d)*e + (a*e^2 + (a*c^2*d^3 - a^2*c*d*
e^2)*sqrt(e^2/(a*c^3*d^4 - 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))*sqrt(-((a*c*d^2
 - a^2*e^2)*sqrt(e^2/(a*c^3*d^4 - 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)) - d)/(a*c
*d^2 - a^2*e^2))) - 1/2*sqrt(-((a*c*d^2 - a^2*e^2)*sqrt(e^2/(a*c^3*d^4 - 2*
a^2*c^2*d^2*e^2 + a^3*c*e^4)) - d)/(a*c*d^2 - a^2*e^2))*log(sqrt(e*x + d)*e
 - (a*e^2 + (a*c^2*d^3 - a^2*c*d*e^2)*sqrt(e^2/(a*c^3*d^4 - 2*a^2*c^2*d^2*e
^2 + a^3*c*e^4)))*sqrt(-((a*c*d^2 - a^2*e^2)*sqrt(e^2/(a*c^3*d^4 - 2*a^2*c^
2*d^2*e^2 + a^3*c*e^4)) - d)/(a*c*d^2 - a^2*e^2)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-a\sqrt{d+ex}+cx^2\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*x**2+a)/(e*x+d)**(1/2),x)
```

```
[Out] -Integral(1/(-a*sqrt(d + e*x) + c*x**2*sqrt(d + e*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*x^2+a)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.616 $\int \frac{1}{(d+ex)^{3/2}(a-cx^2)}\,dx$

Optimal. Leaf size=160

$$\frac{2e}{\sqrt{d+ex}\,(cd^2-ae^2)}-\frac{\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}}+\frac{\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{\sqrt{a}\left(\sqrt{a}e+\sqrt{c}d\right)^{3/2}}$$

```
[Out] (2*e)/((c*d^2 - a*e^2)*Sqrt[d + e*x]) - (c^(1/4)*ArcTanh[(c^(1/4)*Sqrt[d +
e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(Sqrt[a]*(Sqrt[c]*d - Sqrt[a]*e)^(3/2))
 + (c^(1/4)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(
Sqrt[a]*(Sqrt[c]*d + Sqrt[a]*e)^(3/2))
```

Rubi [A] time = 0.208049, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {710, 827, 1166, 208}

$$\frac{2e}{\sqrt{d+ex}\,(cd^2-ae^2)}-\frac{\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}}+\frac{\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{\sqrt{a}\left(\sqrt{a}e+\sqrt{c}d\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*(a - c*x^2)),x]
```

```
[Out] (2*e)/((c*d^2 - a*e^2)*Sqrt[d + e*x]) - (c^(1/4)*ArcTanh[(c^(1/4)*Sqrt[d +
e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(Sqrt[a]*(Sqrt[c]*d - Sqrt[a]*e)^(3/2))
 + (c^(1/4)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(
Sqrt[a]*(Sqrt[c]*d + Sqrt[a]*e)^(3/2))
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\left(a-cx^2\right)}\,dx = \frac{2e}{\left(cd^2-ae^2\right)\sqrt{d+ex}}+\frac{c\int \frac{d-ex}{\sqrt{d+ex}\left(a-cx^2\right)}\,dx}{cd^2-ae^2}$$

$$= \frac{2e}{\left(cd^2-ae^2\right)\sqrt{d+ex}}+\frac{(2c)\,\text{Subst}\left(\int \frac{2de-ex^2}{-cd^2+ae^2+2cdx^2-cx^4}\,dx,x,\sqrt{d+ex}\right)}{cd^2-ae^2}$$

$$= \frac{2e}{\left(cd^2-ae^2\right)\sqrt{d+ex}}-\frac{c\,\text{Subst}\left(\int \frac{1}{cd-\sqrt{a}\sqrt{c}e-cx^2}\,dx,x,\sqrt{d+ex}\right)}{\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)}+\frac{c\,\text{Subst}\left(\int \frac{1}{cd+\sqrt{a}\sqrt{c}e-cx^2}\,dx\right.}{\sqrt{a}\left(\sqrt{c}d+\sqrt{ae}\right.}$$

$$= \frac{2e}{\left(cd^2-ae^2\right)\sqrt{d+ex}}-\frac{\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}}+\frac{\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{\sqrt{a}\left(\sqrt{c}d+\sqrt{a}e\right)^{3/2}}$$

Mathematica [C] time = 0.0861443, size = 136, normalized size = 0.85

$$\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{a}e}\right)+\left(\sqrt{a}e-\sqrt{c}d\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}\right)}{\sqrt{a}\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*(a - c*x^2)),x]
```

```
[Out] ((Sqrt[c]*d + Sqrt[a]*e)*Hypergeometric2F1[-1/2, 1, 1/2, (Sqrt[c]*(d + e*x)
)/(Sqrt[c]*d - Sqrt[a]*e)] + (-(Sqrt[c]*d) + Sqrt[a]*e)*Hypergeometric2F1[-
1/2, 1, 1/2, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)])/(Sqrt[a]*(c*d^2
- a*e^2)*Sqrt[d + e*x])
```

Maple [B] time = 0.209, size = 291, normalized size = 1.8

$$-\frac{c^2ed}{ae^2-cd^2}\arctan\left(c\sqrt{ex+d}\frac{1}{\sqrt{\left(-cd+\sqrt{ace^2}\right)c}}\right)\frac{1}{\sqrt{ace^2}}\frac{1}{\sqrt{\left(-cd+\sqrt{ace^2}\right)c}}-\frac{ce}{ae^2-cd^2}\arctan\left(c\sqrt{ex+d}\frac{1}{\sqrt{\left(-cd+\sqrt{ac\right.}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(-c*x^2+a),x)
```

```
[Out] -e*c^2/(a*e^2-c*d^2)/(a*c*e^2)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arcta
n((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d-e*c/(a*e^2-c*d^2)/((-
c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2)
)*c)^(1/2))-e*c^2/(a*e^2-c*d^2)/(a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(
1/2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d+e*c/(a*e^2-
c*d^2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e
^2)^(1/2))*c)^(1/2))-2*e/(a*e^2-c*d^2)/(e*x+d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(cx^2-a\right)(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(-c*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate(1/((c*x^2 - a)*(e*x + d)^(3/2)), x)
```

Fricas [B] time = 2.52326, size = 5617, normalized size = 35.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(-c*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*((c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)*sqrt((c^2*d^3 + 3*a*c*d*e^2 +
(a*c^3*d^6 - 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 - a^4*e^6)*sqrt((9*c^3*d^4
*e^2 + 6*a*c^2*d^2*e^4 + a^2*c*e^6)/(a*c^6*d^12 - 6*a^2*c^5*d^10*e^2 + 15*a
^3*c^4*d^8*e^4 - 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 - 6*a^6*c*d^2*e^10
 + a^7*e^12)))/(a*c^3*d^6 - 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 - a^4*e^6))
*log((3*c^2*d^2*e + a*c*e^3)*sqrt(e*x + d) + (6*a*c^2*d^3*e^2 + 2*a^2*c*d*e
^4 - (a*c^4*d^8 - 2*a^2*c^3*d^6*e^2 + 2*a^4*c*d^2*e^6 - a^5*e^8)*sqrt((9*c^
3*d^4*e^2 + 6*a*c^2*d^2*e^4 + a^2*c*e^6)/(a*c^6*d^12 - 6*a^2*c^5*d^10*e^2 +
 15*a^3*c^4*d^8*e^4 - 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 - 6*a^6*c*d^2
*e^10 + a^7*e^12)))*sqrt((c^2*d^3 + 3*a*c*d*e^2 + (a*c^3*d^6 - 3*a^2*c^2*d^
4*e^2 + 3*a^3*c*d^2*e^4 - a^4*e^6)*sqrt((9*c^3*d^4*e^2 + 6*a*c^2*d^2*e^4 +
a^2*c*e^6)/(a*c^6*d^12 - 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 - 20*a^4*c
^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 - 6*a^6*c*d^2*e^10 + a^7*e^12)))/(a*c^3*d^6
 - 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 - a^4*e^6))) - (c*d^3 - a*d*e^2 + (c
*d^2*e - a*e^3)*x)*sqrt((c^2*d^3 + 3*a*c*d*e^2 + (a*c^3*d^6 - 3*a^2*c^2*d^4
*e^2 + 3*a^3*c*d^2*e^4 - a^4*e^6)*sqrt((9*c^3*d^4*e^2 + 6*a*c^2*d^2*e^4 + a
^2*c*e^6)/(a*c^6*d^12 - 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 - 20*a^4*c^
3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 - 6*a^6*c*d^2*e^10 + a^7*e^12)))/(a*c^3*d^6
- 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 - a^4*e^6))*log((3*c^2*d^2*e + a*c*e^
3)*sqrt(e*x + d) - (6*a*c^2*d^3*e^2 + 2*a^2*c*d*e^4 - (a*c^4*d^8 - 2*a^2*c^
3*d^6*e^2 + 2*a^4*c*d^2*e^6 - a^5*e^8)*sqrt((9*c^3*d^4*e^2 + 6*a*c^2*d^2*e^
4 + a^2*c*e^6)/(a*c^6*d^12 - 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 - 20*a
^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 - 6*a^6*c*d^2*e^10 + a^7*e^12)))*sqrt((
c^2*d^3 + 3*a*c*d*e^2 + (a*c^3*d^6 - 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 -
a^4*e^6)*sqrt((9*c^3*d^4*e^2 + 6*a*c^2*d^2*e^4 + a^2*c*e^6)/(a*c^6*d^12 - 6
*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 - 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^
4*e^8 - 6*a^6*c*d^2*e^10 + a^7*e^12)))/(a*c^3*d^6 - 3*a^2*c^2*d^4*e^2 + 3*a
^3*c*d^2*e^4 - a^4*e^6))) + (c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)*sqrt((c
^2*d^3 + 3*a*c*d*e^2 - (a*c^3*d^6 - 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 - a
^4*e^6)*sqrt((9*c^3*d^4*e^2 + 6*a*c^2*d^2*e^4 + a^2*c*e^6)/(a*c^6*d^12 - 6*
a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 - 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4
*e^8 - 6*a^6*c*d^2*e^10 + a^7*e^12)))/(a*c^3*d^6 - 3*a^2*c^2*d^4*e^2 + 3*a^
3*c*d^2*e^4 - a^4*e^6))*log((3*c^2*d^2*e + a*c*e^3)*sqrt(e*x + d) + (6*a*c^
2*d^3*e^2 + 2*a^2*c*d*e^4 + (a*c^4*d^8 - 2*a^2*c^3*d^6*e^2 + 2*a^4*c*d^2*e^
6 - a^5*e^8)*sqrt((9*c^3*d^4*e^2 + 6*a*c^2*d^2*e^4 + a^2*c*e^6)/(a*c^6*d^12
 - 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 - 20*a^4*c^3*d^6*e^6 + 15*a^5*c^
```

```
2*d^4*e^8 - 6*a^6*c*d^2*e^10 + a^7*e^12)))*sqrt((c^2*d^3 + 3*a*c*d*e^2 - (a
*c^3*d^6 - 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 - a^4*e^6)*sqrt((9*c^3*d^4*e
^2 + 6*a*c^2*d^2*e^4 + a^2*c*e^6)/(a*c^6*d^12 - 6*a^2*c^5*d^10*e^2 + 15*a^3
*c^4*d^8*e^4 - 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 - 6*a^6*c*d^2*e^10 +
 a^7*e^12)))/(a*c^3*d^6 - 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 - a^4*e^6)))
- (c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)*sqrt((c^2*d^3 + 3*a*c*d*e^2 - (a*
c^3*d^6 - 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 - a^4*e^6)*sqrt((9*c^3*d^4*e^
2 + 6*a*c^2*d^2*e^4 + a^2*c*e^6)/(a*c^6*d^12 - 6*a^2*c^5*d^10*e^2 + 15*a^3*
c^4*d^8*e^4 - 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 - 6*a^6*c*d^2*e^10 +
a^7*e^12)))/(a*c^3*d^6 - 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 - a^4*e^6))*lo
g((3*c^2*d^2*e + a*c*e^3)*sqrt(e*x + d) - (6*a*c^2*d^3*e^2 + 2*a^2*c*d*e^4
+ (a*c^4*d^8 - 2*a^2*c^3*d^6*e^2 + 2*a^4*c*d^2*e^6 - a^5*e^8)*sqrt((9*c^3*d
^4*e^2 + 6*a*c^2*d^2*e^4 + a^2*c*e^6)/(a*c^6*d^12 - 6*a^2*c^5*d^10*e^2 + 15
*a^3*c^4*d^8*e^4 - 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 - 6*a^6*c*d^2*e^
10 + a^7*e^12)))*sqrt((c^2*d^3 + 3*a*c*d*e^2 - (a*c^3*d^6 - 3*a^2*c^2*d^4*e
^2 + 3*a^3*c*d^2*e^4 - a^4*e^6)*sqrt((9*c^3*d^4*e^2 + 6*a*c^2*d^2*e^4 + a^2
*c*e^6)/(a*c^6*d^12 - 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 - 20*a^4*c^3*
d^6*e^6 + 15*a^5*c^2*d^4*e^8 - 6*a^6*c*d^2*e^10 + a^7*e^12)))/(a*c^3*d^6 -
3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 - a^4*e^6))) + 4*sqrt(e*x + d)*e)/(c*d^
3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-ad\sqrt{d+ex} - aex\sqrt{d+ex} + cdx^2\sqrt{d+ex} + cex^3\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(-c*x**2+a),x)
```

```
[Out] -Integral(1/(-a*d*sqrt(d + e*x) - a*e*x*sqrt(d + e*x) + c*d*x**2*sqrt(d + e
*x) + c*e*x**3*sqrt(d + e*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(-c*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.617 $\int \frac{1}{(d+ex)^{5/2}(a-cx^2)}\,dx$

Optimal. Leaf size=190

$$-\frac{c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}}+\frac{c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{\sqrt{a}\left(\sqrt{a}e+\sqrt{c}d\right)^{5/2}}+\frac{4cde}{\sqrt{d+ex}\left(cd^2-ae^2\right)^2}+\frac{2e}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

```
[Out] (2*e)/(3*(c*d^2 - a*e^2)*(d + e*x)^(3/2)) + (4*c*d*e)/((c*d^2 - a*e^2)^2*Sq
rt[d + e*x]) - (c^(3/4)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sq
rt[a]*e]])/(Sqrt[a]*(Sqrt[c]*d - Sqrt[a]*e)^(5/2)) + (c^(3/4)*ArcTanh[(c^(1
/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(Sqrt[a]*(Sqrt[c]*d + Sqrt
[a]*e)^(5/2))
```

Rubi [A] time = 0.379885, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {710, 829, 827, 1166, 208}

$$-\frac{c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}}+\frac{c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{\sqrt{a}\left(\sqrt{a}e+\sqrt{c}d\right)^{5/2}}+\frac{4cde}{\sqrt{d+ex}\left(cd^2-ae^2\right)^2}+\frac{2e}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(5/2)*(a - c*x^2)),x]
```


```
[Out] (2*e)/(3*(c*d^2 - a*e^2)*(d + e*x)^(3/2)) + (4*c*d*e)/((c*d^2 - a*e^2)^2*Sq
rt[d + e*x]) - (c^(3/4)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sq
rt[a]*e]])/(Sqrt[a]*(Sqrt[c]*d - Sqrt[a]*e)^(5/2)) + (c^(3/4)*ArcTanh[(c^(1
/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(Sqrt[a]*(Sqrt[c]*d + Sqrt
[a]*e)^(5/2))
```


Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 829

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)
), x] + Dist[1/(c*d^2 + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d*f + a*e*g -
 c*(e*f - d*g)*x, x])/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x
] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{5/2}\left(a-cx^2\right)}dx = \frac{2e}{3\left(cd^2-ae^2\right)(d+ex)^{3/2}}+\frac{c\int \frac{d-ex}{(d+ex)^{3/2}\left(a-cx^2\right)}dx}{cd^2-ae^2}$$

$$= \frac{2e}{3\left(cd^2-ae^2\right)(d+ex)^{3/2}}+\frac{4cde}{\left(cd^2-ae^2\right)^2\sqrt{d+ex}}-\frac{c\int \frac{-cd^2-ae^2+2cdex}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{\left(cd^2-ae^2\right)^2}$$

$$= \frac{2e}{3\left(cd^2-ae^2\right)(d+ex)^{3/2}}+\frac{4cde}{\left(cd^2-ae^2\right)^2\sqrt{d+ex}}-\frac{(2c)\,\mathrm{Subst}\left(\int \frac{-2cd^2e+e\left(-cd^2-ae^2\right)+2cdex^2}{-cd^2+ae^2+2cdx^2-cx^4}dx,\right.}{\left(cd^2-ae^2\right)^2}$$

$$= \frac{2e}{3\left(cd^2-ae^2\right)(d+ex)^{3/2}}+\frac{4cde}{\left(cd^2-ae^2\right)^2\sqrt{d+ex}}-\frac{c^{3/2}\,\mathrm{Subst}\left(\int \frac{1}{cd-\sqrt{a}\sqrt{c}e-cx^2}dx,x,\sqrt{d+ex}\right)}{\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)^2}$$

$$= \frac{2e}{3\left(cd^2-ae^2\right)(d+ex)^{3/2}}+\frac{4cde}{\left(cd^2-ae^2\right)^2\sqrt{d+ex}}-\frac{c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}}+\frac{c^{3/4}\tanh^{-1}\left(\right.}{\sqrt{a}\left(\sqrt{c}d+\right.}$$

Mathematica [C] time = 0.0593212, size = 130, normalized size = 0.68

$$\frac{\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{a}e}\right)}{\sqrt{c}d-\sqrt{a}e}-\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}\right)}{\sqrt{a}e+\sqrt{c}d}}{3\sqrt{a}(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(5/2)*(a - c*x^2)),x]
```

```
[Out] (Hypergeometric2F1[-3/2, 1, -1/2, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[a]*
e)]/(Sqrt[c]*d - Sqrt[a]*e) - Hypergeometric2F1[-3/2, 1, -1/2, (Sqrt[c]*(d
+ e*x))/(Sqrt[c]*d + Sqrt[a]*e)]/(Sqrt[c]*d + Sqrt[a]*e))/(3*Sqrt[a]*(d + e
*x)^(3/2))
```

Maple [B] time = 0.211, size = 472, normalized size = 2.5

$$-\frac{2e}{3ae^2-3cd^2}(ex+d)^{-\frac{3}{2}}+4\frac{ced}{\left(ae^2-cd^2\right)^2\sqrt{ex+d}}+\frac{c^2ae^3}{\left(ae^2-cd^2\right)^2}\arctan\left(c\sqrt{ex+d}\frac{1}{\sqrt{\left(-cd+\sqrt{ace^2}\right)c}}\right)\frac{1}{\sqrt{ace^2}}\frac{1}{\sqrt{\left(-cd+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(5/2)/(-c*x^2+a),x)
```

```
[Out] -2/3*e/(a*e^2-c*d^2)/(e*x+d)^(3/2)+4*e*c*d/(a*e^2-c*d^2)^2/(e*x+d)^(1/2)+c^
2/(a*e^2-c*d^2)^2/(a*c*e^2)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan((
e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))*a*e^3+e*c^3/(a*e^2-c*d^2)^
2/(a*c*e^2)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c/(
(-c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d^2+2*e*c^2/(a*e^2-c*d^2)^2/((-c*d+(a*c*e^
2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))
*d+c^2/(a*e^2-c*d^2)^2/(a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arct
anh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))*a*e^3+e*c^3/(a*e^2-c*d
^2)^2/(a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)
*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d^2-2*e*c^2/(a*e^2-c*d^2)^2/((c*d+(a*c*
e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2
))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(cx^2-a\right)(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(-c*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate(1/((c*x^2 - a)*(e*x + d)^(5/2)), x)
```

Fricas [B] time = 3.03937, size = 10656, normalized size = 56.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(-c*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(c^2*d^6 - 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^4*e^2 - 2*a*c*d^2*e^
4 + a^2*e^6)*x^2 + 2*(c^2*d^5*e - 2*a*c*d^3*e^3 + a^2*d*e^5)*x)*sqrt((c^4*d
^5 + 10*a*c^3*d^3*e^2 + 5*a^2*c^2*d*e^4 + (a*c^5*d^10 - 5*a^2*c^4*d^8*e^2 +
 10*a^3*c^3*d^6*e^4 - 10*a^4*c^2*d^4*e^6 + 5*a^5*c*d^2*e^8 - a^6*e^10)*sqrt
((25*c^7*d^8*e^2 + 100*a*c^6*d^6*e^4 + 110*a^2*c^5*d^4*e^6 + 20*a^3*c^4*d^2
*e^8 + a^4*c^3*e^10)/(a*c^10*d^20 - 10*a^2*c^9*d^18*e^2 + 45*a^3*c^8*d^16*e
^4 - 120*a^4*c^7*d^14*e^6 + 210*a^5*c^6*d^12*e^8 - 252*a^6*c^5*d^10*e^10 +
210*a^7*c^4*d^8*e^12 - 120*a^8*c^3*d^6*e^14 + 45*a^9*c^2*d^4*e^16 - 10*a^10
*c*d^2*e^18 + a^11*e^20)))/(a*c^5*d^10 - 5*a^2*c^4*d^8*e^2 + 10*a^3*c^3*d^6
*e^4 - 10*a^4*c^2*d^4*e^6 + 5*a^5*c*d^2*e^8 - a^6*e^10))*log((5*c^4*d^4*e +
 10*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(e*x + d) + (15*a*c^4*d^6*e^2 + 35*a^2
*c^3*d^4*e^4 + 13*a^3*c^2*d^2*e^6 + a^4*c*e^8 - (a*c^6*d^13 - 2*a^2*c^5*d^1
1*e^2 - 5*a^3*c^4*d^9*e^4 + 20*a^4*c^3*d^7*e^6 - 25*a^5*c^2*d^5*e^8 + 14*a^
6*c*d^3*e^10 - 3*a^7*d*e^12)*sqrt((25*c^7*d^8*e^2 + 100*a*c^6*d^6*e^4 + 110
*a^2*c^5*d^4*e^6 + 20*a^3*c^4*d^2*e^8 + a^4*c^3*e^10)/(a*c^10*d^20 - 10*a^2
*c^9*d^18*e^2 + 45*a^3*c^8*d^16*e^4 - 120*a^4*c^7*d^14*e^6 + 210*a^5*c^6*d^
12*e^8 - 252*a^6*c^5*d^10*e^10 + 210*a^7*c^4*d^8*e^12 - 120*a^8*c^3*d^6*e^1
4 + 45*a^9*c^2*d^4*e^16 - 10*a^10*c*d^2*e^18 + a^11*e^20)))*sqrt((c^4*d^5 +
```

```
10*a*c^3*d^3*e^2 + 5*a^2*c^2*d*e^4 + (a*c^5*d^10 - 5*a^2*c^4*d^8*e^2 + 10*
a^3*c^3*d^6*e^4 - 10*a^4*c^2*d^4*e^6 + 5*a^5*c*d^2*e^8 - a^6*e^10)*sqrt((25
*c^7*d^8*e^2 + 100*a*c^6*d^6*e^4 + 110*a^2*c^5*d^4*e^6 + 20*a^3*c^4*d^2*e^8
 + a^4*c^3*e^10)/(a*c^10*d^20 - 10*a^2*c^9*d^18*e^2 + 45*a^3*c^8*d^16*e^4 -
 120*a^4*c^7*d^14*e^6 + 210*a^5*c^6*d^12*e^8 - 252*a^6*c^5*d^10*e^10 + 210*
a^7*c^4*d^8*e^12 - 120*a^8*c^3*d^6*e^14 + 45*a^9*c^2*d^4*e^16 - 10*a^10*c*d
^2*e^18 + a^11*e^20)))/(a*c^5*d^10 - 5*a^2*c^4*d^8*e^2 + 10*a^3*c^3*d^6*e^4
 - 10*a^4*c^2*d^4*e^6 + 5*a^5*c*d^2*e^8 - a^6*e^10))) - 3*(c^2*d^6 - 2*a*c*
d^4*e^2 + a^2*d^2*e^4 + (c^2*d^4*e^2 - 2*a*c*d^2*e^4 + a^2*e^6)*x^2 + 2*(c^
2*d^5*e - 2*a*c*d^3*e^3 + a^2*d*e^5)*x)*sqrt((c^4*d^5 + 10*a*c^3*d^3*e^2 +
5*a^2*c^2*d*e^4 + (a*c^5*d^10 - 5*a^2*c^4*d^8*e^2 + 10*a^3*c^3*d^6*e^4 - 10
*a^4*c^2*d^4*e^6 + 5*a^5*c*d^2*e^8 - a^6*e^10)*sqrt((25*c^7*d^8*e^2 + 100*a
*c^6*d^6*e^4 + 110*a^2*c^5*d^4*e^6 + 20*a^3*c^4*d^2*e^8 + a^4*c^3*e^10)/(a*
c^10*d^20 - 10*a^2*c^9*d^18*e^2 + 45*a^3*c^8*d^16*e^4 - 120*a^4*c^7*d^14*e^
6 + 210*a^5*c^6*d^12*e^8 - 252*a^6*c^5*d^10*e^10 + 210*a^7*c^4*d^8*e^12 - 1
20*a^8*c^3*d^6*e^14 + 45*a^9*c^2*d^4*e^16 - 10*a^10*c*d^2*e^18 + a^11*e^20)
))/(a*c^5*d^10 - 5*a^2*c^4*d^8*e^2 + 10*a^3*c^3*d^6*e^4 - 10*a^4*c^2*d^4*e^
6 + 5*a^5*c*d^2*e^8 - a^6*e^10))*log((5*c^4*d^4*e + 10*a*c^3*d^2*e^3 + a^2*
c^2*e^5)*sqrt(e*x + d) - (15*a*c^4*d^6*e^2 + 35*a^2*c^3*d^4*e^4 + 13*a^3*c^
2*d^2*e^6 + a^4*c*e^8 - (a*c^6*d^13 - 2*a^2*c^5*d^11*e^2 - 5*a^3*c^4*d^9*e^
4 + 20*a^4*c^3*d^7*e^6 - 25*a^5*c^2*d^5*e^8 + 14*a^6*c*d^3*e^10 - 3*a^7*d*e
^12)*sqrt((25*c^7*d^8*e^2 + 100*a*c^6*d^6*e^4 + 110*a^2*c^5*d^4*e^6 + 20*a^
3*c^4*d^2*e^8 + a^4*c^3*e^10)/(a*c^10*d^20 - 10*a^2*c^9*d^18*e^2 + 45*a^3*c
^8*d^16*e^4 - 120*a^4*c^7*d^14*e^6 + 210*a^5*c^6*d^12*e^8 - 252*a^6*c^5*d^1
0*e^10 + 210*a^7*c^4*d^8*e^12 - 120*a^8*c^3*d^6*e^14 + 45*a^9*c^2*d^4*e^16
- 10*a^10*c*d^2*e^18 + a^11*e^20)))*sqrt((c^4*d^5 + 10*a*c^3*d^3*e^2 + 5*a^
2*c^2*d*e^4 + (a*c^5*d^10 - 5*a^2*c^4*d^8*e^2 + 10*a^3*c^3*d^6*e^4 - 10*a^4
*c^2*d^4*e^6 + 5*a^5*c*d^2*e^8 - a^6*e^10)*sqrt((25*c^7*d^8*e^2 + 100*a*c^6
*d^6*e^4 + 110*a^2*c^5*d^4*e^6 + 20*a^3*c^4*d^2*e^8 + a^4*c^3*e^10)/(a*c^10
*d^20 - 10*a^2*c^9*d^18*e^2 + 45*a^3*c^8*d^16*e^4 - 120*a^4*c^7*d^14*e^6 +
210*a^5*c^6*d^12*e^8 - 252*a^6*c^5*d^10*e^10 + 210*a^7*c^4*d^8*e^12 - 120*a
^8*c^3*d^6*e^14 + 45*a^9*c^2*d^4*e^16 - 10*a^10*c*d^2*e^18 + a^11*e^20)))/(
a*c^5*d^10 - 5*a^2*c^4*d^8*e^2 + 10*a^3*c^3*d^6*e^4 - 10*a^4*c^2*d^4*e^6 +
5*a^5*c*d^2*e^8 - a^6*e^10))) + 3*(c^2*d^6 - 2*a*c*d^4*e^2 + a^2*d^2*e^4 +
(c^2*d^4*e^2 - 2*a*c*d^2*e^4 + a^2*e^6)*x^2 + 2*(c^2*d^5*e - 2*a*c*d^3*e^3
+ a^2*d*e^5)*x)*sqrt((c^4*d^5 + 10*a*c^3*d^3*e^2 + 5*a^2*c^2*d*e^4 - (a*c^5
*d^10 - 5*a^2*c^4*d^8*e^2 + 10*a^3*c^3*d^6*e^4 - 10*a^4*c^2*d^4*e^6 + 5*a^5
*c*d^2*e^8 - a^6*e^10)*sqrt((25*c^7*d^8*e^2 + 100*a*c^6*d^6*e^4 + 110*a^2*c
^5*d^4*e^6 + 20*a^3*c^4*d^2*e^8 + a^4*c^3*e^10)/(a*c^10*d^20 - 10*a^2*c^9*d
^18*e^2 + 45*a^3*c^8*d^16*e^4 - 120*a^4*c^7*d^14*e^6 + 210*a^5*c^6*d^12*e^8
 - 252*a^6*c^5*d^10*e^10 + 210*a^7*c^4*d^8*e^12 - 120*a^8*c^3*d^6*e^14 + 45
*a^9*c^2*d^4*e^16 - 10*a^10*c*d^2*e^18 + a^11*e^20)))/(a*c^5*d^10 - 5*a^2*c
^4*d^8*e^2 + 10*a^3*c^3*d^6*e^4 - 10*a^4*c^2*d^4*e^6 + 5*a^5*c*d^2*e^8 - a^
6*e^10))*log((5*c^4*d^4*e + 10*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(e*x + d) +
 (15*a*c^4*d^6*e^2 + 35*a^2*c^3*d^4*e^4 + 13*a^3*c^2*d^2*e^6 + a^4*c*e^8 +
(a*c^6*d^13 - 2*a^2*c^5*d^11*e^2 - 5*a^3*c^4*d^9*e^4 + 20*a^4*c^3*d^7*e^6 -
 25*a^5*c^2*d^5*e^8 + 14*a^6*c*d^3*e^10 - 3*a^7*d*e^12)*sqrt((25*c^7*d^8*e^
2 + 100*a*c^6*d^6*e^4 + 110*a^2*c^5*d^4*e^6 + 20*a^3*c^4*d^2*e^8 + a^4*c^3*
e^10)/(a*c^10*d^20 - 10*a^2*c^9*d^18*e^2 + 45*a^3*c^8*d^16*e^4 - 120*a^4*c^
7*d^14*e^6 + 210*a^5*c^6*d^12*e^8 - 252*a^6*c^5*d^10*e^10 + 210*a^7*c^4*d^8
*e^12 - 120*a^8*c^3*d^6*e^14 + 45*a^9*c^2*d^4*e^16 - 10*a^10*c*d^2*e^18 + a
^11*e^20)))*sqrt((c^4*d^5 + 10*a*c^3*d^3*e^2 + 5*a^2*c^2*d*e^4 - (a*c^5*d^1
0 - 5*a^2*c^4*d^8*e^2 + 10*a^3*c^3*d^6*e^4 - 10*a^4*c^2*d^4*e^6 + 5*a^5*c*d
^2*e^8 - a^6*e^10)*sqrt((25*c^7*d^8*e^2 + 100*a*c^6*d^6*e^4 + 110*a^2*c^5*d
^4*e^6 + 20*a^3*c^4*d^2*e^8 + a^4*c^3*e^10)/(a*c^10*d^20 - 10*a^2*c^9*d^18*
e^2 + 45*a^3*c^8*d^16*e^4 - 120*a^4*c^7*d^14*e^6 + 210*a^5*c^6*d^12*e^8 - 2
52*a^6*c^5*d^10*e^10 + 210*a^7*c^4*d^8*e^12 - 120*a^8*c^3*d^6*e^14 + 45*a^9
*c^2*d^4*e^16 - 10*a^10*c*d^2*e^18 + a^11*e^20)))/(a*c^5*d^10 - 5*a^2*c^4*d
^8*e^2 + 10*a^3*c^3*d^6*e^4 - 10*a^4*c^2*d^4*e^6 + 5*a^5*c*d^2*e^8 - a^6*e^
```

```
10))) - 3*(c^2*d^6 - 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^4*e^2 - 2*a*c*d^2
*e^4 + a^2*e^6)*x^2 + 2*(c^2*d^5*e - 2*a*c*d^3*e^3 + a^2*d*e^5)*x)*sqrt((c^
4*d^5 + 10*a*c^3*d^3*e^2 + 5*a^2*c^2*d*e^4 - (a*c^5*d^10 - 5*a^2*c^4*d^8*e^
2 + 10*a^3*c^3*d^6*e^4 - 10*a^4*c^2*d^4*e^6 + 5*a^5*c*d^2*e^8 - a^6*e^10)*s
qrt((25*c^7*d^8*e^2 + 100*a*c^6*d^6*e^4 + 110*a^2*c^5*d^4*e^6 + 20*a^3*c^4*
d^2*e^8 + a^4*c^3*e^10)/(a*c^10*d^20 - 10*a^2*c^9*d^18*e^2 + 45*a^3*c^8*d^1
6*e^4 - 120*a^4*c^7*d^14*e^6 + 210*a^5*c^6*d^12*e^8 - 252*a^6*c^5*d^10*e^10
 + 210*a^7*c^4*d^8*e^12 - 120*a^8*c^3*d^6*e^14 + 45*a^9*c^2*d^4*e^16 - 10*a
^10*c*d^2*e^18 + a^11*e^20)))/(a*c^5*d^10 - 5*a^2*c^4*d^8*e^2 + 10*a^3*c^3*
d^6*e^4 - 10*a^4*c^2*d^4*e^6 + 5*a^5*c*d^2*e^8 - a^6*e^10))*log((5*c^4*d^4*
e + 10*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(e*x + d) - (15*a*c^4*d^6*e^2 + 35*
a^2*c^3*d^4*e^4 + 13*a^3*c^2*d^2*e^6 + a^4*c*e^8 + (a*c^6*d^13 - 2*a^2*c^5*
d^11*e^2 - 5*a^3*c^4*d^9*e^4 + 20*a^4*c^3*d^7*e^6 - 25*a^5*c^2*d^5*e^8 + 14
*a^6*c*d^3*e^10 - 3*a^7*d*e^12)*sqrt((25*c^7*d^8*e^2 + 100*a*c^6*d^6*e^4 +
110*a^2*c^5*d^4*e^6 + 20*a^3*c^4*d^2*e^8 + a^4*c^3*e^10)/(a*c^10*d^20 - 10*
a^2*c^9*d^18*e^2 + 45*a^3*c^8*d^16*e^4 - 120*a^4*c^7*d^14*e^6 + 210*a^5*c^6
*d^12*e^8 - 252*a^6*c^5*d^10*e^10 + 210*a^7*c^4*d^8*e^12 - 120*a^8*c^3*d^6*
e^14 + 45*a^9*c^2*d^4*e^16 - 10*a^10*c*d^2*e^18 + a^11*e^20)))*sqrt((c^4*d^
5 + 10*a*c^3*d^3*e^2 + 5*a^2*c^2*d*e^4 - (a*c^5*d^10 - 5*a^2*c^4*d^8*e^2 +
10*a^3*c^3*d^6*e^4 - 10*a^4*c^2*d^4*e^6 + 5*a^5*c*d^2*e^8 - a^6*e^10)*sqrt(
(25*c^7*d^8*e^2 + 100*a*c^6*d^6*e^4 + 110*a^2*c^5*d^4*e^6 + 20*a^3*c^4*d^2*
e^8 + a^4*c^3*e^10)/(a*c^10*d^20 - 10*a^2*c^9*d^18*e^2 + 45*a^3*c^8*d^16*e^
4 - 120*a^4*c^7*d^14*e^6 + 210*a^5*c^6*d^12*e^8 - 252*a^6*c^5*d^10*e^10 + 2
10*a^7*c^4*d^8*e^12 - 120*a^8*c^3*d^6*e^14 + 45*a^9*c^2*d^4*e^16 - 10*a^10*
c*d^2*e^18 + a^11*e^20)))/(a*c^5*d^10 - 5*a^2*c^4*d^8*e^2 + 10*a^3*c^3*d^6*
e^4 - 10*a^4*c^2*d^4*e^6 + 5*a^5*c*d^2*e^8 - a^6*e^10))) + 4*(6*c*d*e^2*x +
 7*c*d^2*e - a*e^3)*sqrt(e*x + d))/(c^2*d^6 - 2*a*c*d^4*e^2 + a^2*d^2*e^4 +
 (c^2*d^4*e^2 - 2*a*c*d^2*e^4 + a^2*e^6)*x^2 + 2*(c^2*d^5*e - 2*a*c*d^3*e^3
 + a^2*d*e^5)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(5/2)/(-c*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(-c*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.618 $\int \frac{(d+ex)^{5/2}}{a+cx^2}\,dx$

Optimal. Leaf size=781

$$\frac{e\left(\left(3cd^2-ae^2\right)\sqrt{ae^2+cd^2}+2a\sqrt{c}de^2+2c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}+\sqrt{ae^2+cd^2}+\sqrt{c}(d+ex)\right)}{2\sqrt{2}c^{7/4}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}-$$

```
[Out] (4*d*e*Sqrt[d + e*x])/c + (2*e*(d + e*x)^(3/2))/(3*c) - (e*(2*c^(3/2)*d^3 +
 2*a*Sqrt[c]*d*e^2 - (3*c*d^2 - a*e^2)*Sqrt[c*d^2 + a*e^2])*ArcTanh[(Sqrt[S
qrt[c]*d + Sqrt[c*d^2 + a*e^2]] - Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[
c]*d - Sqrt[c*d^2 + a*e^2]]])/(Sqrt[2]*c^(7/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqr
t[c]*d - Sqrt[c*d^2 + a*e^2]]) + (e*(2*c^(3/2)*d^3 + 2*a*Sqrt[c]*d*e^2 - (3
*c*d^2 - a*e^2)*Sqrt[c*d^2 + a*e^2])*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 +
 a*e^2]] + Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e
^2]]])/(Sqrt[2]*c^(7/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a
*e^2]]) + (e*(2*c^(3/2)*d^3 + 2*a*Sqrt[c]*d*e^2 + (3*c*d^2 - a*e^2)*Sqrt[c*
d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sq
rt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(2*Sqrt[2]*c^(7/4)*S
qrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]) - (e*(2*c^(3/2)*d
^3 + 2*a*Sqrt[c]*d*e^2 + (3*c*d^2 - a*e^2)*Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*
d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d
 + e*x] + Sqrt[c]*(d + e*x)])/(2*Sqrt[2]*c^(7/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[S
qrt[c]*d + Sqrt[c*d^2 + a*e^2]])
```

Rubi [A] time = 3.04345, antiderivative size = 781, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {704, 825, 827, 1169, 634, 618, 206, 628}

$$\frac{e\left(\left(3cd^2-ae^2\right)\sqrt{ae^2+cd^2}+2a\sqrt{c}de^2+2c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}+\sqrt{ae^2+cd^2}+\sqrt{c}(d+ex)\right)}{2\sqrt{2}c^{7/4}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(a + c*x^2),x]
```

```
[Out] (4*d*e*Sqrt[d + e*x])/c + (2*e*(d + e*x)^(3/2))/(3*c) - (e*(2*c^(3/2)*d^3 +
 2*a*Sqrt[c]*d*e^2 - (3*c*d^2 - a*e^2)*Sqrt[c*d^2 + a*e^2])*ArcTanh[(Sqrt[S
qrt[c]*d + Sqrt[c*d^2 + a*e^2]] - Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[
c]*d - Sqrt[c*d^2 + a*e^2]]])/(Sqrt[2]*c^(7/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqr
t[c]*d - Sqrt[c*d^2 + a*e^2]]) + (e*(2*c^(3/2)*d^3 + 2*a*Sqrt[c]*d*e^2 - (3
*c*d^2 - a*e^2)*Sqrt[c*d^2 + a*e^2])*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 +
 a*e^2]] + Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e
^2]]])/(Sqrt[2]*c^(7/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a
*e^2]]) + (e*(2*c^(3/2)*d^3 + 2*a*Sqrt[c]*d*e^2 + (3*c*d^2 - a*e^2)*Sqrt[c*
d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sq
rt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(2*Sqrt[2]*c^(7/4)*S
qrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]) - (e*(2*c^(3/2)*d
^3 + 2*a*Sqrt[c]*d*e^2 + (3*c*d^2 - a*e^2)*Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*
d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d
 + e*x] + Sqrt[c]*(d + e*x)])/(2*Sqrt[2]*c^(7/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[S
qrt[c]*d + Sqrt[c*d^2 + a*e^2]])
```

Rule 704

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^(m - 2)*Simp[c*
d^2 - a*e^2 + 2*c*d*e*x, x])/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x]
&& NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 1]
```

Rule 825

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[((d + e*x)^(m
- 1)*Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x])/(a + c*x^2), x], x] /; Fre
eQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m
, 0]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{a+cx^2}\,dx = \frac{2e(d+ex)^{3/2}}{3c} + \frac{\int \frac{\sqrt{d+ex}\left(cd^2-ae^2+2cdex\right)}{a+cx^2}\,dx}{c}$$

$$= \frac{4de\sqrt{d+ex}}{c} + \frac{2e(d+ex)^{3/2}}{3c} + \frac{\int \frac{cd\left(cd^2-3ae^2\right)+ce\left(3cd^2-ae^2\right)x}{\sqrt{d+ex}\left(a+cx^2\right)}\,dx}{c^2}$$

$$= \frac{4de\sqrt{d+ex}}{c} + \frac{2e(d+ex)^{3/2}}{3c} + \frac{2\,\text{Subst}\left(\int \frac{cde\left(cd^2-3ae^2\right)-cde\left(3cd^2-ae^2\right)+ce\left(3cd^2-ae^2\right)x^2}{cd^2+ae^2-2cdx^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{c^2}$$

$$= \frac{4de\sqrt{d+ex}}{c} + \frac{2e(d+ex)^{3/2}}{3c} + \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\left(cde\left(cd^2-3ae^2\right)-cde\left(3cd^2-ae^2\right)\right)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}-\left(cde\left(cd^2-3ae^2\right)-cde\left(3cd^2-ae^2\right)-\cdots\right.}{\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}x}{\sqrt[4]{c}}+x^2}\cdots\right.}{\sqrt{2}c^{9/4}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+a\cdots}}}$$

$$= \frac{4de\sqrt{d+ex}}{c} + \frac{2e(d+ex)^{3/2}}{3c} - \frac{\left(e\left(2c^{3/2}d^3+2a\sqrt{c}de^2-\left(3cd^2-ae^2\right)\sqrt{cd^2+ae^2}\right)\right)\text{Subst}\left(\int \frac{\cdots}{\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\cdots}\cdots\right.}{2c^2\sqrt{cd^2+ae^2}}$$

$$= \frac{4de\sqrt{d+ex}}{c} + \frac{2e(d+ex)^{3/2}}{3c} + \frac{e\left(2c^{3/2}d^3+2a\sqrt{c}de^2+\left(3cd^2-ae^2\right)\sqrt{cd^2+ae^2}\right)\log\left(\sqrt{cd^2+ae^2}-\sqrt{2}\cdots\right.}{2\sqrt{2}c^{7/4}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{c\cdots}}}$$

$$= \frac{4de\sqrt{d+ex}}{c} + \frac{2e(d+ex)^{3/2}}{3c} - \frac{e\left(2c^{3/2}d^3+2a\sqrt{c}de^2-\left(3cd^2-ae^2\right)\sqrt{cd^2+ae^2}\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+\cdots}}}{\sqrt[4]{c}}\cdots\right)}{\sqrt{\sqrt{c}d-\sqrt{\cdots}}}\cdots\right.}{\sqrt{2}c^{7/4}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}$$

Mathematica [A] time = 0.348457, size = 226, normalized size = 0.29

$$\frac{2\sqrt{-a}c^{3/4}e\sqrt{d+ex}(7d+ex)+3\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(-2\sqrt{-a}\sqrt{c}de-ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)-3\sqrt{\sqrt{-a}e+\sqrt{c}d}\left(2\sqrt{-a}\cdots\right.}{3\sqrt{-a}c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(a + c*x^2),x]
```

```
[Out] (2*Sqrt[-a]*c^(3/4)*e*Sqrt[d + e*x]*(7*d + e*x) + 3*Sqrt[Sqrt[c]*d - Sqrt[-
a]*e]*(c*d^2 - 2*Sqrt[-a]*Sqrt[c]*d*e - a*e^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*
x])/Sqrt[Sqrt[c]*d - Sqrt[-a]*e]] - 3*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*(c*d^2 +
 2*Sqrt[-a]*Sqrt[c]*d*e - a*e^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[
c]*d + Sqrt[-a]*e]])/(3*Sqrt[-a]*c^(7/4))
```

Maple [B] time = 0.313, size = 3931, normalized size = 5.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(c*x^2+a),x)
```

```
[Out] 1/2*e/c^(3/2)/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*
d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^
(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2
))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d
)^(1/2)*d-1/2*e/c^(5/2)/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^
(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*d^2))^(1/
2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2
*c*d)^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(
1/2)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)+3/4/c^(3/2)/a/e*ln(-(e*x+d)*c^(1/
2)+(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)-(a*e^2+c*d^2)^(1/2
))*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)*d^2-3/4/
c^(3/2)/a/e*ln((e*x+d)*c^(1/2)+(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c
*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c
*e^2+c^2*d^2)^(1/2)*d^2-1/2/c/a/e*ln(-(e*x+d)*c^(1/2)+(e*x+d)^(1/2)*(2*(c*(
a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)-(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)^
(1/2)+2*c*d)^(1/2)*(a*e^2+c*d^2)^(1/2)*d^2+1/2/c/a/e*ln((e*x+d)*c^(1/2)+(e*
x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*
(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*e^2+c*d^2)^(1/2)*d^2-1/2*e/c^(3/2)/
(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arcta
n((-2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*
e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2*(c*(a*e
^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*d+1/2
*e/c^(5/2)/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^
(1/2)*arctan((-2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1
/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))
*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^
(1/2)*(a*c*e^2+c^2*d^2)^(1/2)+2/3*e*(e*x+d)^(3/2)/c-1/4*e/c^(5/2)*ln(-(e*x+
d)*c^(1/2)+(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)-(a*e^2+c*d
^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)+
1/4*e/c^(3/2)*ln(-(e*x+d)*c^(1/2)+(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+
2*c*d)^(1/2)-(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*d
-1/4*e/c^(3/2)*ln((e*x+d)*c^(1/2)+(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+
2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*d
+1/4*e/c^(5/2)*ln((e*x+d)*c^(1/2)+(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+
2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(
a*c*e^2+c^2*d^2)^(1/2)+1/c^2/a/e/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2
+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*
d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2)
)^(1/2)-2*c*d)^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c
^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*e^2+c*d^2)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)*d+1/
c/a/e/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)
*arctan((-2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/
(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2*(
c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)
*(a*e^2+c*d^2)^(1/2)*d^2+1/2/c^2/a/e*ln(-(e*x+d)*c^(1/2)+(e*x+d)^(1/2)*(2*(
c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)-(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^
2)^(1/2)+2*c*d)^(1/2)*(a*e^2+c*d^2)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)*d-1/2/c^2
/a/e*ln((e*x+d)*c^(1/2)+(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/
2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*e^2+c*d^
2)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)*d+3/2/c^(3/2)/a/e/(4*(a*e^2+c*d^2)^(1/2)*c
^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/
2)+(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-
2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(
1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)*d^2-3/
2/c^(3/2)/a/e/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*
d)^(1/2)*arctan((-2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)
^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/
```

```
2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*
d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)*d^2-1/c/a/e/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)
-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*
(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(
a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(
2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*e^2+c*d^2)^(1/2)*d^2+3/2/c^(1/2)/
a/e/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*a
rctan((-2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4
*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2*(c*
(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*d
^3-3/2/c^(1/2)/a/e/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)
-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*d^2))^(1/2)+2*
c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)
^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+
2*c*d)^(1/2)*d^3-1/c^2/a/e/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2
))^(1/2)-2*c*d)^(1/2)*arctan((-2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*d^2))
^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/
2)-2*c*d)^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^
2)^(1/2)+2*c*d)^(1/2)*(a*e^2+c*d^2)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)*d-3/4/c^(
1/2)/a/e*ln(-(e*x+d)*c^(1/2)+(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d
)^(1/2)-(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*d^3+4*
e/c/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*a
rctan((-2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4
*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(a*e^2
+c*d^2)^(1/2)*d-4*e/c/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1
/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*d^2))^(1/2)
+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c
*d)^(1/2))*(a*e^2+c*d^2)^(1/2)*d+3/4/c^(1/2)/a/e*ln((e*x+d)*c^(1/2)+(e*x+d)
^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c
*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*d^3+4*d*e*(e*x+d)^(1/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/(c*x^2 + a), x)
```

Fricas [B] time = 2.52642, size = 3383, normalized size = 4.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/6*(3*c*sqrt(-(c^2*d^5 - 10*a*c*d^3*e^2 + 5*a^2*d*e^4 + a*c^3*sqrt(-(25*c
^4*d^8*e^2 - 100*a*c^3*d^6*e^4 + 110*a^2*c^2*d^4*e^6 - 20*a^3*c*d^2*e^8 + a
^4*e^10)/(a*c^7)))/(a*c^3))*log((5*c^4*d^8*e - 14*a^2*c^2*d^4*e^5 - 8*a^3*c
*d^2*e^7 + a^4*e^9)*sqrt(e*x + d) + (10*a*c^4*d^5*e^2 - 20*a^2*c^3*d^3*e^4
+ 2*a^3*c^2*d*e^6 + (a*c^6*d^2 - a^2*c^5*e^2)*sqrt(-(25*c^4*d^8*e^2 - 100*a
```

```
*c^3*d^6*e^4 + 110*a^2*c^2*d^4*e^6 - 20*a^3*c*d^2*e^8 + a^4*e^10)/(a*c^7)))
*sqrt(-(c^2*d^5 - 10*a*c*d^3*e^2 + 5*a^2*d*e^4 + a*c^3*sqrt(-(25*c^4*d^8*e^
2 - 100*a*c^3*d^6*e^4 + 110*a^2*c^2*d^4*e^6 - 20*a^3*c*d^2*e^8 + a^4*e^10)/
(a*c^7)))/(a*c^3))) - 3*c*sqrt(-(c^2*d^5 - 10*a*c*d^3*e^2 + 5*a^2*d*e^4 + a
*c^3*sqrt(-(25*c^4*d^8*e^2 - 100*a*c^3*d^6*e^4 + 110*a^2*c^2*d^4*e^6 - 20*a
^3*c*d^2*e^8 + a^4*e^10)/(a*c^7)))/(a*c^3))*log((5*c^4*d^8*e - 14*a^2*c^2*d
^4*e^5 - 8*a^3*c*d^2*e^7 + a^4*e^9)*sqrt(e*x + d) - (10*a*c^4*d^5*e^2 - 20*
a^2*c^3*d^3*e^4 + 2*a^3*c^2*d*e^6 + (a*c^6*d^2 - a^2*c^5*e^2)*sqrt(-(25*c^4
*d^8*e^2 - 100*a*c^3*d^6*e^4 + 110*a^2*c^2*d^4*e^6 - 20*a^3*c*d^2*e^8 + a^4
*e^10)/(a*c^7)))*sqrt(-(c^2*d^5 - 10*a*c*d^3*e^2 + 5*a^2*d*e^4 + a*c^3*sqrt
(-(25*c^4*d^8*e^2 - 100*a*c^3*d^6*e^4 + 110*a^2*c^2*d^4*e^6 - 20*a^3*c*d^2*
e^8 + a^4*e^10)/(a*c^7)))/(a*c^3))) + 3*c*sqrt(-(c^2*d^5 - 10*a*c*d^3*e^2 +
 5*a^2*d*e^4 - a*c^3*sqrt(-(25*c^4*d^8*e^2 - 100*a*c^3*d^6*e^4 + 110*a^2*c^
2*d^4*e^6 - 20*a^3*c*d^2*e^8 + a^4*e^10)/(a*c^7)))/(a*c^3))*log((5*c^4*d^8*
e - 14*a^2*c^2*d^4*e^5 - 8*a^3*c*d^2*e^7 + a^4*e^9)*sqrt(e*x + d) + (10*a*c
^4*d^5*e^2 - 20*a^2*c^3*d^3*e^4 + 2*a^3*c^2*d*e^6 - (a*c^6*d^2 - a^2*c^5*e^
2)*sqrt(-(25*c^4*d^8*e^2 - 100*a*c^3*d^6*e^4 + 110*a^2*c^2*d^4*e^6 - 20*a^3
*c*d^2*e^8 + a^4*e^10)/(a*c^7)))*sqrt(-(c^2*d^5 - 10*a*c*d^3*e^2 + 5*a^2*d*
e^4 - a*c^3*sqrt(-(25*c^4*d^8*e^2 - 100*a*c^3*d^6*e^4 + 110*a^2*c^2*d^4*e^6
 - 20*a^3*c*d^2*e^8 + a^4*e^10)/(a*c^7)))/(a*c^3))) - 3*c*sqrt(-(c^2*d^5 -
10*a*c*d^3*e^2 + 5*a^2*d*e^4 - a*c^3*sqrt(-(25*c^4*d^8*e^2 - 100*a*c^3*d^6*
e^4 + 110*a^2*c^2*d^4*e^6 - 20*a^3*c*d^2*e^8 + a^4*e^10)/(a*c^7)))/(a*c^3))
*log((5*c^4*d^8*e - 14*a^2*c^2*d^4*e^5 - 8*a^3*c*d^2*e^7 + a^4*e^9)*sqrt(e*
x + d) - (10*a*c^4*d^5*e^2 - 20*a^2*c^3*d^3*e^4 + 2*a^3*c^2*d*e^6 - (a*c^6*
d^2 - a^2*c^5*e^2)*sqrt(-(25*c^4*d^8*e^2 - 100*a*c^3*d^6*e^4 + 110*a^2*c^2*
d^4*e^6 - 20*a^3*c*d^2*e^8 + a^4*e^10)/(a*c^7)))*sqrt(-(c^2*d^5 - 10*a*c*d^
3*e^2 + 5*a^2*d*e^4 - a*c^3*sqrt(-(25*c^4*d^8*e^2 - 100*a*c^3*d^6*e^4 + 110
*a^2*c^2*d^4*e^6 - 20*a^3*c*d^2*e^8 + a^4*e^10)/(a*c^7)))/(a*c^3))) - 4*(e^
2*x + 7*d*e)*sqrt(e*x + d))/c
```

Sympy [A] time = 66.9776, size = 418, normalized size = 0.54

$$-\frac{4ade^3 \operatorname{RootSum}\left(t^4\left(256a^3ce^6+256a^2c^2d^2e^4\right)+32t^2acde^2+1,\left(t\mapsto t\log\left(-64t^3a^2cde^4-64t^3ac^2d^3e^2+4tae^2-4tc\right.\right.\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(c*x**2+a),x)
```

```
[Out] -4*a*d*e**3*RootSum(_t**4*(256*a**3*c*e**6 + 256*a**2*c**2*d**2*e**4) + 32*
_t**2*a*c*d*e**2 + 1, Lambda(_t, _t*log(-64*_t**3*a**2*c*d*e**4 - 64*_t**3*
a*c**2*d**3*e**2 + 4*_t*a*e**2 - 4*_t*c*d**2 + sqrt(d + e*x))))/c - 2*a*e**
3*RootSum(256*_t**4*a**2*c**3*e**4 + 32*_t**2*a*c**2*d*e**2 + a*e**2 + c*d*
*2, Lambda(_t, _t*log(64*_t**3*a*c**2*e**2 + 4*_t*c*d + sqrt(d + e*x))))/c
- 4*d**3*e*RootSum(_t**4*(256*a**3*c*e**6 + 256*a**2*c**2*d**2*e**4) + 32*_
t**2*a*c*d*e**2 + 1, Lambda(_t, _t*log(-64*_t**3*a**2*c*d*e**4 - 64*_t**3*a
*c**2*d**3*e**2 + 4*_t*a*e**2 - 4*_t*c*d**2 + sqrt(d + e*x)))) + 6*d**2*e*R
ootSum(256*_t**4*a**2*c**3*e**4 + 32*_t**2*a*c**2*d*e**2 + a*e**2 + c*d**2,
 Lambda(_t, _t*log(64*_t**3*a*c**2*e**2 + 4*_t*c*d + sqrt(d + e*x)))) + 4*d
*e*sqrt(d + e*x)/c + 2*e*(d + e*x)**(3/2)/(3*c)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.619 $\int \frac{(d+ex)^{3/2}}{a+cx^2}\,dx$

Optimal. Leaf size=689

$$\frac{e\left(2\sqrt{c}d\sqrt{ae^2+cd^2}+ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}+\sqrt{ae^2+cd^2}+\sqrt{c}(d+ex)\right)}{2\sqrt{2}c^{5/4}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}-\frac{e\left(2\sqrt{c}d\sqrt{ae^2\right.}{}$$

```
[Out] (2*e*Sqrt[d + e*x])/c - (e*(c*d^2 + a*e^2 - 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2]
)*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] - Sqrt[2]*c^(1/4)*Sqrt[d +
 e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(Sqrt[2]*c^(5/4)*Sqrt[c*d^2
+ a*e^2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) + (e*(c*d^2 + a*e^2 - 2*Sqr
t[c]*d*Sqrt[c*d^2 + a*e^2])*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]
+ Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(S
qrt[2]*c^(5/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) +
 (e*(c*d^2 + a*e^2 + 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^
2] - Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] +
Sqrt[c]*(d + e*x)])/(2*Sqrt[2]*c^(5/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d +
 Sqrt[c*d^2 + a*e^2]]) - (e*(c*d^2 + a*e^2 + 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2
])*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 +
a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(2*Sqrt[2]*c^(5/4)*Sqrt[c*d^2 +
 a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])
```

Rubi [A] time = 1.4766, antiderivative size = 689, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {704, 827, 1169, 634, 618, 206, 628}

$$\frac{e\left(2\sqrt{c}d\sqrt{ae^2+cd^2}+ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}+\sqrt{ae^2+cd^2}+\sqrt{c}(d+ex)\right)}{2\sqrt{2}c^{5/4}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}-\frac{e\left(2\sqrt{c}d\sqrt{ae^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(a + c*x^2),x]
```

```
[Out] (2*e*Sqrt[d + e*x])/c - (e*(c*d^2 + a*e^2 - 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2]
)*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] - Sqrt[2]*c^(1/4)*Sqrt[d +
 e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(Sqrt[2]*c^(5/4)*Sqrt[c*d^2
+ a*e^2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) + (e*(c*d^2 + a*e^2 - 2*Sqr
t[c]*d*Sqrt[c*d^2 + a*e^2])*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]
+ Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(S
qrt[2]*c^(5/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) +
 (e*(c*d^2 + a*e^2 + 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^
2] - Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] +
Sqrt[c]*(d + e*x)])/(2*Sqrt[2]*c^(5/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d +
 Sqrt[c*d^2 + a*e^2]]) - (e*(c*d^2 + a*e^2 + 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2
])*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 +
a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(2*Sqrt[2]*c^(5/4)*Sqrt[c*d^2 +
 a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])
```

Rule 704

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^(m - 2)*Simp[c*
```

```
d^2 - a*e^2 + 2*c*d*e*x, x])/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x]
&& NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 1]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{a+cx^2}\,dx = \frac{2e\sqrt{d+ex}}{c} + \frac{\int \frac{cd^2-ae^2+2cdex}{\sqrt{d+ex}(a+cx^2)}\,dx}{c}$$

$$= \frac{2e\sqrt{d+ex}}{c} + \frac{2\,\text{Subst}\left(\int \frac{-2cd^2e+e(cd^2-ae^2)+2cdex^2}{cd^2+ae^2-2cdx^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{c}$$

$$= \frac{2e\sqrt{d+ex}}{c} + \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}(-2cd^2e+e(cd^2-ae^2))\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}-\left(-2cd^2e+e(cd^2-ae^2)-2\sqrt{c}de\sqrt{cd^2+ae^2}\right)x}{\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{d+ex}\right)}{\sqrt{2}c^{5/4}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}$$

$$= \frac{2e\sqrt{d+ex}}{c} - \frac{\left(e\left(cd^2+ae^2-2\sqrt{c}d\sqrt{cd^2+ae^2}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{d+ex}\right)}{2c^{3/2}\sqrt{cd^2+ae^2}}$$

$$= \frac{2e\sqrt{d+ex}}{c} + \frac{e\left(cd^2+ae^2+2\sqrt{c}d\sqrt{cd^2+ae^2}\right)\log\left(\sqrt{cd^2+ae^2}-\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+e\right.}{2\sqrt{2}c^{5/4}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}$$

$$= \frac{2e\sqrt{d+ex}}{c} - \frac{e\left(cd^2+ae^2-2\sqrt{c}d\sqrt{cd^2+ae^2}\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}-\sqrt{2}\sqrt{d+ex}\right)}{\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}\right)}{\sqrt{2}c^{5/4}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}} + \frac{e\left(cd^2+ae^2-\right.}{}$$

Mathematica [A] time = 0.154856, size = 159, normalized size = 0.23

$$\frac{2\sqrt{-a}\sqrt[4]{c}e\sqrt{d+ex}+\left(\sqrt{c}d-\sqrt{-a}e\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)-\left(\sqrt{-a}e+\sqrt{c}d\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(a + c*x^2),x]
```

```
[Out] (2*Sqrt[-a]*c^(1/4)*e*Sqrt[d + e*x] + (Sqrt[c]*d - Sqrt[-a]*e)^(3/2)*ArcTan
h[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[-a]*e]] - (Sqrt[c]*d + Sqrt
[-a]*e)^(3/2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[-a]*e]]
)/(Sqrt[-a]*c^(5/4))
```

Maple [B] time = 0.248, size = 2763, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*x^2+a),x)
```

```
[Out] 2*e*(e*x+d)^(1/2)/c-1/4/c/a/e*ln((e*x+d)*c^(1/2)-(e*x+d)^(1/2)*(2*(c*(a*e^2
+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)^(1/2)
+2*c*d)^(1/2)*(a*e^2+c*d^2)^(1/2)*d-1/2/c^(1/2)/a/e*ln((e*x+d)*c^(1/2)-(e*x
+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(
a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*d^2+1/4/c^2/a/e*ln((e*x+d)*c^(1/2)-(e*x
+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(
a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)*(a*e^2+c*d^2)^(
1/2)+1/2/c^(3/2)/a/e*ln((e*x+d)*c^(1/2)-(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^
(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(
1/2)*(a*c*e^2+c^2*d^2)^(1/2)*d-2*e/c/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a
*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)-(2*(c*(a*e^
2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*
d^2))^(1/2)-2*c*d)^(1/2))*(a*e^2+c*d^2)^(1/2)-1/2/c/a/e/(4*(a*e^2+c*d^2)^(1
/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d
)^(1/2)-(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(
1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c
*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*e^2+c*d^2)^(1/2)*d-1/c
^(1/2)/a/e/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^
(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)-(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/
2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*
(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(
1/2)*d^2+1/2/c^2/a/e/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/
2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)-(2*(c*(a*e^2+c*d^2))^(1/2)+
2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*
d)^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2
)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)*(a*e^2+c*d^2)^(1/2)+1/c^(3/2)/a/e/(4
*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan(
(2*c^(1/2)*(e*x+d)^(1/2)-(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2
+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2*(c*(a*e^2+
c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2
+c^2*d^2)^(1/2)*d+1/4/c/a/e*ln((e*x+d)*c^(1/2)+(e*x+d)^(1/2)*(2*(c*(a*e^2+c
*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)^(1/2)+2
*c*d)^(1/2)*(a*e^2+c*d^2)^(1/2)*d+1/2/c^(1/2)/a/e*ln((e*x+d)*c^(1/2)+(e*x+d
)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*
c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*d^2-1/4/c^2/a/e*ln((e*x+d)*c^(1/2)+(e*x+d
)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*
c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)*(a*e^2+c*d^2)^(1/
2)-1/2/c^(3/2)/a/e*ln((e*x+d)*c^(1/2)+(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1
/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/
2)*(a*c*e^2+c^2*d^2)^(1/2)*d-2*e/c/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e
^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+
c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^
2))^(1/2)-2*c*d)^(1/2))*(a*e^2+c*d^2)^(1/2)-1/2/c/a/e/(4*(a*e^2+c*d^2)^(1/2
)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^
(1/2)+(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/
2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d
)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*e^2+c*d^2)^(1/2)*d-1/c^(
1/2)/a/e/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1
/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)
)/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2
*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/
2)*d^2+1/2/c^2/a/e/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)
-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*d^2))^(1/2)+2*
c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)
^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+
2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)*(a*e^2+c*d^2)^(1/2)+1/c^(3/2)/a/e/(4*(
a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2
*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c
*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2*(c*(a*e^2+c*
```

```
d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2+c
^2*d^2)^(1/2)*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(c*x^2 + a), x)
```

Fricas [A] time = 2.09714, size = 1997, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(c*sqrt(-(c*d^3 - 3*a*d*e^2 + a*c^2*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*d^2*e^
4 + a^2*e^6)/(a*c^5)))/(a*c^2))*log(-(3*c^2*d^4*e + 2*a*c*d^2*e^3 - a^2*e^5
)*sqrt(e*x + d) + (3*a*c^2*d^2*e^2 - a^2*c*e^4 + a*c^4*d*sqrt(-(9*c^2*d^4*e
^2 - 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^5)))*sqrt(-(c*d^3 - 3*a*d*e^2 + a*c^2*sq
rt(-(9*c^2*d^4*e^2 - 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^5)))/(a*c^2))) - c*sqrt(
-(c*d^3 - 3*a*d*e^2 + a*c^2*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*d^2*e^4 + a^2*e^6)
/(a*c^5)))/(a*c^2))*log(-(3*c^2*d^4*e + 2*a*c*d^2*e^3 - a^2*e^5)*sqrt(e*x +
 d) - (3*a*c^2*d^2*e^2 - a^2*c*e^4 + a*c^4*d*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*d
^2*e^4 + a^2*e^6)/(a*c^5)))*sqrt(-(c*d^3 - 3*a*d*e^2 + a*c^2*sqrt(-(9*c^2*d
^4*e^2 - 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^5)))/(a*c^2))) + c*sqrt(-(c*d^3 - 3*
a*d*e^2 - a*c^2*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^5)))/(
a*c^2))*log(-(3*c^2*d^4*e + 2*a*c*d^2*e^3 - a^2*e^5)*sqrt(e*x + d) + (3*a*c
^2*d^2*e^2 - a^2*c*e^4 - a*c^4*d*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*d^2*e^4 + a^2
*e^6)/(a*c^5)))*sqrt(-(c*d^3 - 3*a*d*e^2 - a*c^2*sqrt(-(9*c^2*d^4*e^2 - 6*a
*c*d^2*e^4 + a^2*e^6)/(a*c^5)))/(a*c^2))) - c*sqrt(-(c*d^3 - 3*a*d*e^2 - a*
c^2*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^5)))/(a*c^2))*log(
-(3*c^2*d^4*e + 2*a*c*d^2*e^3 - a^2*e^5)*sqrt(e*x + d) - (3*a*c^2*d^2*e^2 -
 a^2*c*e^4 - a*c^4*d*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^5
)))*sqrt(-(c*d^3 - 3*a*d*e^2 - a*c^2*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*d^2*e^4 +
 a^2*e^6)/(a*c^5)))/(a*c^2))) + 4*sqrt(e*x + d)*e)/c
```

Sympy [A] time = 34.9838, size = 316, normalized size = 0.46

$$-\frac{2ae^3\operatorname{RootSum}\left(t^4\left(256a^3ce^6+256a^2c^2d^2e^4\right)+32t^2acde^2+1,\left(t\mapsto t\log\left(-64t^3a^2cde^4-64t^3ac^2d^3e^2+4tae^2-4tcd\right.\right.\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+a),x)
```

```
[Out] -2*a*e**3*RootSum(_t**4*(256*a**3*c*e**6 + 256*a**2*c**2*d**2*e**4) + 32*_t
**2*a*c*d*e**2 + 1, Lambda(_t, _t*log(-64*_t**3*a**2*c*d*e**4 - 64*_t**3*a*
c**2*d**3*e**2 + 4*_t*a*e**2 - 4*_t*c*d**2 + sqrt(d + e*x))))/c - 2*d**2*e*
RootSum(_t**4*(256*a**3*c*e**6 + 256*a**2*c**2*d**2*e**4) + 32*_t**2*a*c*d*
e**2 + 1, Lambda(_t, _t*log(-64*_t**3*a**2*c*d*e**4 - 64*_t**3*a*c**2*d**3*
e**2 + 4*_t*a*e**2 - 4*_t*c*d**2 + sqrt(d + e*x)))) + 4*d*e*RootSum(256*_t*
*4*a**2*c**3*e**4 + 32*_t**2*a*c**2*d*e**2 + a*e**2 + c*d**2, Lambda(_t, _t
*log(64*_t**3*a*c**2*e**2 + 4*_t*c*d + sqrt(d + e*x)))) + 2*e*sqrt(d + e*x)
/c
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.620 $\int \frac{\sqrt{d+ex}}{a+cx^2}\,dx$

Optimal. Leaf size=478

$$\frac{e\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}+\sqrt{ae^2+cd^2}+\sqrt{c}(d+ex)\right)}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}-\frac{e\log\left(\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}+\right.}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{ae^2+cd^2}+}}$$

```
[Out] (e*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] - Sqrt[2]*c^(1/4)*Sqrt[d
+ e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(Sqrt[2]*c^(3/4)*Sqrt[Sqrt[
c]*d - Sqrt[c*d^2 + a*e^2]]) - (e*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*
e^2]] + Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]
]])/(Sqrt[2]*c^(3/4)*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) + (e*Log[Sqrt[c
*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[
d + e*x] + Sqrt[c]*(d + e*x)])/(2*Sqrt[2]*c^(3/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d
^2 + a*e^2]]) - (e*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d
 + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(2*Sqrt[2]*c^(3
/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])
```

Rubi [A] time = 0.404863, antiderivative size = 478, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {700, 1129, 634, 618, 206, 628}

$$\frac{e\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}+\sqrt{ae^2+cd^2}+\sqrt{c}(d+ex)\right)}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}-\frac{e\log\left(\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}+\right.}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{ae^2+cd^2}+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(a + c*x^2),x]
```

```
[Out] (e*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] - Sqrt[2]*c^(1/4)*Sqrt[d
+ e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(Sqrt[2]*c^(3/4)*Sqrt[Sqrt[
c]*d - Sqrt[c*d^2 + a*e^2]]) - (e*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*
e^2]] + Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]
]])/(Sqrt[2]*c^(3/4)*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) + (e*Log[Sqrt[c
*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[
d + e*x] + Sqrt[c]*(d + e*x)])/(2*Sqrt[2]*c^(3/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d
^2 + a*e^2]]) - (e*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d
 + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(2*Sqrt[2]*c^(3
/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])
```

Rule 700

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1129

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q =
 Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m - 1)/(q
 - r*x + x^2), x], x] - Dist[1/(2*c*r), Int[x^(m - 1)/(q + r*x + x^2), x],
x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 1] && LtQ[m, 3
```

```
] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{a+cx^2}\,dx = (2e)\,\text{Subst}\left(\int \frac{x^2}{cd^2+ae^2-2cdx^2+cx^4}\,dx, x, \sqrt{d+ex}\right)$$

$$= \frac{e\,\text{Subst}\left(\int \frac{x}{\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{d+ex}\right)}{\sqrt{2}c^{3/4}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}} - \frac{e\,\text{Subst}\left(\int \frac{x}{\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{d\ldots}\right)}{\sqrt{2}c^{3/4}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{d+ex}\right)}{2c} + \frac{e\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{d\ldots}\right)}{2c}$$

$$= \frac{e\log\left(\sqrt{cd^2+ae^2}-\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}+\sqrt{c}(d+ex)\right)}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}} - \frac{e\log\left(\sqrt{cd^2+ae^2}+\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}\ldots}\right)}{2\sqrt{2}c^{3/4}\sqrt{\ldots}}$$

$$= \frac{e\tanh^{-1}\left(\frac{\sqrt[4]{c}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}-\sqrt{2}\sqrt{d+ex}\right)}{\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}\right)}{\sqrt{2}c^{3/4}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}} - \frac{e\tanh^{-1}\left(\frac{\sqrt[4]{c}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}+\sqrt{2}\sqrt{d+ex}\right)}{\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}\right)}{\sqrt{2}c^{3/4}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}} + \frac{e\log\left(\sqrt{cd^2+ae^2}-\sqrt{2}\sqrt[4]{\ldots}\right)}{2\sqrt{2}\ldots}$$

Mathematica [A] time = 0.0923689, size = 135, normalized size = 0.28

$$\frac{\sqrt{\sqrt{c}d-\sqrt{-a}e}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)-\sqrt{\sqrt{-a}e+\sqrt{c}d}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(a + c*x^2),x]
```

```
[Out] (Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*
d - Sqrt[-a]*e]] - Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*ArcTanh[(c^(1/4)*Sqrt[d + e
*x])/Sqrt[Sqrt[c]*d + Sqrt[-a]*e]])/(Sqrt[-a]*c^(3/4))
```

Maple [B] time = 0.245, size = 1176, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*x^2+a),x)
```

```
[Out] -1/4*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)/a/e/c^(1/2)*ln((e*x+d)*c^(1/2)
-(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))
*d+1/4*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)/a/c^(3/2)/e*ln((e*x+d)*c^(1/
2)-(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2
))*(a*c*e^2+c^2*d^2)^(1/2)-1/2*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)/a/e/
c^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)/(4*(a*e^2+c*d^2)^(1/2)*c^(1
/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)-
(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(
c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*d+1/2*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d
)^(1/2)/a/c^(3/2)/e*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)/(4*(a*e^2+c*d^2
)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e
*x+d)^(1/2)-(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)
*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(a*c*e^2+c^2*d^2)^(1/2)+1/
4*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)/a/e/c^(1/2)*ln((e*x+d)*c^(1/2)+(e
*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*d-
1/4*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)/a/c^(3/2)/e*ln((e*x+d)*c^(1/2)+
(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*
(a*c*e^2+c^2*d^2)^(1/2)-1/2*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)/a/e/c^(
1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)
-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*
(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(
a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*d+1/2*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(
1/2)/a/c^(3/2)/e*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)/(4*(a*e^2+c*d^2)^(
1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+
d)^(1/2)+(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^
(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(a*c*e^2+c^2*d^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(c*x^2 + a), x)
```

Fricas [A] time = 1.9146, size = 743, normalized size = 1.55

$$-\frac{1}{2}\sqrt{-\frac{ac\sqrt{-\frac{e^2}{ac^3}}+d}{ac}}\log\left(ac^2\sqrt{-\frac{ac\sqrt{-\frac{e^2}{ac^3}}+d}{ac}}\sqrt{-\frac{e^2}{ac^3}}+\sqrt{ex+d}e\right)+\frac{1}{2}\sqrt{-\frac{ac\sqrt{-\frac{e^2}{ac^3}}+d}{ac}}\log\left(-ac^2\sqrt{-\frac{ac\sqrt{-\frac{e^2}{ac^3}}+d}{ac}}\sqrt{-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-(a*c*sqrt(-e^2/(a*c^3)) + d)/(a*c))*log(a*c^2*sqrt(-(a*c*sqrt(-e
^2/(a*c^3)) + d)/(a*c))*sqrt(-e^2/(a*c^3)) + sqrt(e*x + d)*e) + 1/2*sqrt(-(
a*c*sqrt(-e^2/(a*c^3)) + d)/(a*c))*log(-a*c^2*sqrt(-(a*c*sqrt(-e^2/(a*c^3))
 + d)/(a*c))*sqrt(-e^2/(a*c^3)) + sqrt(e*x + d)*e) + 1/2*sqrt((a*c*sqrt(-e^
2/(a*c^3)) - d)/(a*c))*log(a*c^2*sqrt((a*c*sqrt(-e^2/(a*c^3)) - d)/(a*c))*s
qrt(-e^2/(a*c^3)) + sqrt(e*x + d)*e) - 1/2*sqrt((a*c*sqrt(-e^2/(a*c^3)) - d
)/(a*c))*log(-a*c^2*sqrt((a*c*sqrt(-e^2/(a*c^3)) - d)/(a*c))*sqrt(-e^2/(a*c
^3)) + sqrt(e*x + d)*e)
```

Sympy [A] time = 5.04361, size = 75, normalized size = 0.16

$$2e\,\mathrm{RootSum}\left(256t^4a^2c^3e^4+32t^2ac^2de^2+ae^2+cd^2,\left(t\mapsto t\log\left(64t^3ac^2e^2+4tcd+\sqrt{d+ex}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+a),x)
```

```
[Out] 2*e*RootSum(256*_t**4*a**2*c**3*e**4 + 32*_t**2*a*c**2*d*e**2 + a*e**2 + c*
d**2, Lambda(_t, _t*log(64*_t**3*a*c**2*e**2 + 4*_t*c*d + sqrt(d + e*x))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.621 $\int \frac{1}{\sqrt{d+ex}(a+cx^2)}\,dx$

Optimal. Leaf size=538

$$-\frac{e\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}+\sqrt{ae^2+cd^2}+\sqrt{c}(d+ex)\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{e\log\left(\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}\right.}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2}}}$$

```
[Out] (e*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] - Sqrt[2]*c^(1/4)*Sqrt[d
+ e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(Sqrt[2]*c^(1/4)*Sqrt[c*d^2
 + a*e^2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (e*ArcTanh[(Sqrt[Sqrt[c]
*d + Sqrt[c*d^2 + a*e^2]] + Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d -
 Sqrt[c*d^2 + a*e^2]]])/(Sqrt[2]*c^(1/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d
 - Sqrt[c*d^2 + a*e^2]]) - (e*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sqr
t[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(2*S
qrt[2]*c^(1/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]) +
 (e*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 +
 a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(2*Sqrt[2]*c^(1/4)*Sqrt[c*d^2
+ a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])
```

Rubi [A] time = 0.446833, antiderivative size = 538, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {708, 1094, 634, 618, 206, 628}

$$-\frac{e\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}+\sqrt{ae^2+cd^2}+\sqrt{c}(d+ex)\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{e\log\left(\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}\right.}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(a + c*x^2)),x]
```



```
[Out] (e*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] - Sqrt[2]*c^(1/4)*Sqrt[d
+ e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(Sqrt[2]*c^(1/4)*Sqrt[c*d^2
 + a*e^2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (e*ArcTanh[(Sqrt[Sqrt[c]
*d + Sqrt[c*d^2 + a*e^2]] + Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d -
 Sqrt[c*d^2 + a*e^2]]])/(Sqrt[2]*c^(1/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d
 - Sqrt[c*d^2 + a*e^2]]) - (e*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sqr
t[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(2*S
qrt[2]*c^(1/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]) +
 (e*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 +
 a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(2*Sqrt[2]*c^(1/4)*Sqrt[c*d^2
+ a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])
```


Rule 708

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
```

```
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\left(a+cx^2\right)}\,dx = (2e)\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-2cdx^2+cx^4}\,dx, x, \sqrt{d+ex}\right)$$

$$= \frac{e\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}-x}{\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{d+ex}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}} + \frac{e\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}+x}{\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}x}{\sqrt[4]{c}}+x^2} \cdots\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{c \cdots}}}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{d+ex}\right)}{2\sqrt{c}\sqrt{cd^2+ae^2}} + \frac{e\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}x}{\sqrt[4]{c}}+x^2} \cdots\right)}{2\sqrt{c}\sqrt{cd^2+ae^2}}$$

$$= -\frac{e\log\left(\sqrt{cd^2+ae^2}-\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}+\sqrt{c}(d+ex)\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}} + \frac{e\log\left(\sqrt{cd^2+ae^2}+ \cdots\right.}{2\sqrt{2} \cdots}$$

$$= \frac{e\tanh^{-1}\left(\frac{\sqrt[4]{c}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}-\sqrt{2}\sqrt{d+ex}\right)}{\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}} - \frac{e\tanh^{-1}\left(\frac{\sqrt[4]{c}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}+\sqrt{2}\sqrt{d+ex}\right)}{\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}} - \frac{e\log\left(\sqrt{cd^2 \cdots\right.}{\cdots}$$

Mathematica [A] time = 0.137005, size = 135, normalized size = 0.25

$$\frac{\frac{\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{\sqrt{c}d-\sqrt{-a}e}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{\sqrt{-a}e+\sqrt{c}d}}}{\sqrt{-a}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(a + c*x^2)),x]
```

```
[Out] (ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[-a]*e]]/Sqrt[Sqrt[c]
*d - Sqrt[-a]*e] - ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[-a
]*e]]/Sqrt[Sqrt[c]*d + Sqrt[-a]*e])/(Sqrt[-a]*c^(1/4))
```

Maple [B] time = 0.24, size = 1548, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a)/(e*x+d)^(1/2),x)
```

```
[Out] 1/4/(a*e^2+c*d^2)^(1/2)/a/e*ln((e*x+d)*c^(1/2)-(e*x+d)^(1/2)*(2*(c*(a*e^2+c
*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)^(1/2)+2
*c*d)^(1/2)*d-1/4/(a*e^2+c*d^2)^(1/2)/c/a/e*ln((e*x+d)*c^(1/2)-(e*x+d)^(1/2
)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+
c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)+2*e/(a*e^2+c*d^2)^(1/2)
/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arct
an((2*c^(1/2)*(e*x+d)^(1/2)-(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*
e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))+1/2/(a*e^2
+c*d^2)^(1/2)/a/e/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-
2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)-(2*(c*(a*e^2+c*d^2))^(1/2)+2*c
*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^
(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2
*c*d)^(1/2)*d-1/2/(a*e^2+c*d^2)^(1/2)/c/a/e/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-
2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)-(2*(
c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a
*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2
*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)-1/4/(a*e^2+c*
d^2)^(1/2)/a/e*ln((e*x+d)*c^(1/2)+(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+
2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*d
+1/4/(a*e^2+c*d^2)^(1/2)/c/a/e*ln((e*x+d)*c^(1/2)+(e*x+d)^(1/2)*(2*(c*(a*e^
2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)^(1/2
)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)+2*e/(a*e^2+c*d^2)^(1/2)/(4*(a*e^2+c*
d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)
*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1
/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))+1/2/(a*e^2+c*d^2)^(1/2)
/a/e/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*
arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4
*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2*(c*
(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*d
-1/2/(a*e^2+c*d^2)^(1/2)/c/a/e/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c
*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*d^
2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^
(1/2)-2*c*d)^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2
```

```
*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2+a)\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)*sqrt(e*x + d)), x)
```

Fricas [B] time = 2.04934, size = 1808, normalized size = 3.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-((a*c*d^2 + a^2*e^2)*sqrt(-e^2/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a
^3*c*e^4)) + d)/(a*c*d^2 + a^2*e^2))*log(sqrt(e*x + d)*e + (a*e^2 + (a*c^2*
d^3 + a^2*c*d*e^2)*sqrt(-e^2/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))*
sqrt(-((a*c*d^2 + a^2*e^2)*sqrt(-e^2/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c
*e^4)) + d)/(a*c*d^2 + a^2*e^2))) - 1/2*sqrt(-((a*c*d^2 + a^2*e^2)*sqrt(-e^
2/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)) + d)/(a*c*d^2 + a^2*e^2))*lo
g(sqrt(e*x + d)*e - (a*e^2 + (a*c^2*d^3 + a^2*c*d*e^2)*sqrt(-e^2/(a*c^3*d^4
 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))*sqrt(-((a*c*d^2 + a^2*e^2)*sqrt(-e^2/(a
*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)) + d)/(a*c*d^2 + a^2*e^2))) + 1/2
*sqrt(((a*c*d^2 + a^2*e^2)*sqrt(-e^2/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c
*e^4)) - d)/(a*c*d^2 + a^2*e^2))*log(sqrt(e*x + d)*e + (a*e^2 - (a*c^2*d^3
+ a^2*c*d*e^2)*sqrt(-e^2/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))*sqrt
(((a*c*d^2 + a^2*e^2)*sqrt(-e^2/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)
) - d)/(a*c*d^2 + a^2*e^2))) - 1/2*sqrt(((a*c*d^2 + a^2*e^2)*sqrt(-e^2/(a*c
^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)) - d)/(a*c*d^2 + a^2*e^2))*log(sqrt
(e*x + d)*e - (a*e^2 - (a*c^2*d^3 + a^2*c*d*e^2)*sqrt(-e^2/(a*c^3*d^4 + 2*a
^2*c^2*d^2*e^2 + a^3*c*e^4)))*sqrt(((a*c*d^2 + a^2*e^2)*sqrt(-e^2/(a*c^3*d^
4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)) - d)/(a*c*d^2 + a^2*e^2)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+cx^2)\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a)/(e*x+d)**(1/2),x)
```

```
[Out] Integral(1/((a + c*x**2)*sqrt(d + e*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.622 $\int \frac{1}{(d+ex)^{3/2}(a+cx^2)}\,dx$

Optimal. Leaf size=663

$$-\frac{2e}{\sqrt{d+ex}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{c}e\left(\sqrt{ae^2+cd^2}+2\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}+\sqrt{ae^2+cd^2}+\sqrt{c}(d+ex)\right)}{2\sqrt{2}\left(ae^2+cd^2\right)^{3/2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}$$

```
[Out] (-2*e)/((c*d^2 + a*e^2)*Sqrt[d + e*x]) + (c^(1/4)*e*(2*Sqrt[c]*d - Sqrt[c*d
^2 + a*e^2])*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] - Sqrt[2]*c^(1/
4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(Sqrt[2]*(c*d^2 +
 a*e^2)^(3/2)*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (c^(1/4)*e*(2*Sqrt[c
]*d - Sqrt[c*d^2 + a*e^2])*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] +
 Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(Sq
rt[2]*(c*d^2 + a*e^2)^(3/2)*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (c^(1/
4)*e*(2*Sqrt[c]*d + Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*
c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d +
e*x)])/(2*Sqrt[2]*(c*d^2 + a*e^2)^(3/2)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2
]]) + (c^(1/4)*e*(2*Sqrt[c]*d + Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^2
] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + S
qrt[c]*(d + e*x)])/(2*Sqrt[2]*(c*d^2 + a*e^2)^(3/2)*Sqrt[Sqrt[c]*d + Sqrt[c
*d^2 + a*e^2]])
```

Rubi [A] time = 0.968651, antiderivative size = 663, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {710, 827, 1169, 634, 618, 206, 628}

$$-\frac{2e}{\sqrt{d+ex}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{c}e\left(\sqrt{ae^2+cd^2}+2\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}+\sqrt{ae^2+cd^2}+\sqrt{c}(d+ex)\right)}{2\sqrt{2}\left(ae^2+cd^2\right)^{3/2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*(a + c*x^2)),x]
```


```
[Out] (-2*e)/((c*d^2 + a*e^2)*Sqrt[d + e*x]) + (c^(1/4)*e*(2*Sqrt[c]*d - Sqrt[c*d
^2 + a*e^2])*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] - Sqrt[2]*c^(1/
4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(Sqrt[2]*(c*d^2 +
 a*e^2)^(3/2)*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (c^(1/4)*e*(2*Sqrt[c
]*d - Sqrt[c*d^2 + a*e^2])*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] +
 Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(Sq
rt[2]*(c*d^2 + a*e^2)^(3/2)*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (c^(1/
4)*e*(2*Sqrt[c]*d + Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*
c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d +
e*x)])/(2*Sqrt[2]*(c*d^2 + a*e^2)^(3/2)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2
]]) + (c^(1/4)*e*(2*Sqrt[c]*d + Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^2
] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + S
qrt[c]*(d + e*x)])/(2*Sqrt[2]*(c*d^2 + a*e^2)^(3/2)*Sqrt[Sqrt[c]*d + Sqrt[c
*d^2 + a*e^2]])
```


Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
```

```
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\left(a+cx^2\right)}dx = -\frac{2e}{\left(cd^2+ae^2\right)\sqrt{d+ex}} + \frac{c\int \frac{d-ex}{\sqrt{d+ex}\left(a+cx^2\right)}dx}{cd^2+ae^2}$$

$$= -\frac{2e}{\left(cd^2+ae^2\right)\sqrt{d+ex}} + \frac{(2c)\,\text{Subst}\left(\int \frac{2de-ex^2}{cd^2+ae^2-2cdx^2+cx^4}dx, x, \sqrt{d+ex}\right)}{cd^2+ae^2}$$

$$= -\frac{2e}{\left(cd^2+ae^2\right)\sqrt{d+ex}} + \frac{c^{3/4}\,\text{Subst}\left(\int \frac{\frac{2\sqrt{2}de\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}} - \left(2de+\frac{e\sqrt{cd^2+ae^2}}{\sqrt{c}}\right)x}{\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}x}{\sqrt[4]{c}}+x^2}dx, x, \sqrt{d+ex}\right)}{\sqrt{2}\left(cd^2+ae^2\right)^{3/2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}} + \ldots$$

$$= -\frac{2e}{\left(cd^2+ae^2\right)\sqrt{d+ex}} + \frac{\left(e\left(2\sqrt{c}d-\sqrt{cd^2+ae^2}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}x}{\sqrt[4]{c}}+x^2}dx, x, \ldots\right)}{2\left(cd^2+ae^2\right)^{3/2}} \ldots$$

$$= -\frac{2e}{\left(cd^2+ae^2\right)\sqrt{d+ex}} - \frac{\sqrt[4]{c}e\left(2\sqrt{c}d+\sqrt{cd^2+ae^2}\right)\log\left(\sqrt{cd^2+ae^2}-\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+\ldots}}\right.}{2\sqrt{2}\left(cd^2+ae^2\right)^{3/2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}} \ldots$$

$$= -\frac{2e}{\left(cd^2+ae^2\right)\sqrt{d+ex}} + \frac{\sqrt[4]{c}e\left(2\sqrt{c}d-\sqrt{cd^2+ae^2}\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}-\sqrt{2}\sqrt{d+ex}\right)}{\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}\right)}{\sqrt{2}\left(cd^2+ae^2\right)^{3/2}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}} - \ldots$$

Mathematica [C] time = 0.13075, size = 132, normalized size = 0.2

$$\frac{\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{\sqrt{-a}\sqrt{c}d-ae} - \frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\sqrt{-a}\sqrt{c}d+ae}}{\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (-(Hypergeometric2F1[-1/2, 1, 1/2, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a
]*e)]/(Sqrt[-a]*Sqrt[c]*d + a*e)) + Hypergeometric2F1[-1/2, 1, 1/2, (Sqrt[c
]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]/(Sqrt[-a]*Sqrt[c]*d - a*e))/Sqrt[d +
 e*x]
```

Maple [B] time = 0.232, size = 5659, normalized size = 8.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(c*x^2+a),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2+a)(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)*(e*x + d)^(3/2)), x)
```

Fricas [B] time = 2.48171, size = 5651, normalized size = 8.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*((c*d^3 + a*d*e^2 + (c*d^2*e + a*e^3)*x)*sqrt(-(c^2*d^3 - 3*a*c*d*e^2
+ (a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sqrt(-(9*c^3*
d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 1
5*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e
^10 + a^7*e^12)))/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^
6))*log(-(3*c^2*d^2*e - a*c*e^3)*sqrt(e*x + d) + (6*a*c^2*d^3*e^2 - 2*a^2*c
*d*e^4 + (a*c^4*d^8 + 2*a^2*c^3*d^6*e^2 - 2*a^4*c*d^2*e^6 - a^5*e^8)*sqrt(-
(9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)/(a*c^6*d^12 + 6*a^2*c^5*d^10*
e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*
c*d^2*e^10 + a^7*e^12)))*sqrt(-(c^2*d^3 - 3*a*c*d*e^2 + (a*c^3*d^6 + 3*a^2*
c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sqrt(-(9*c^3*d^4*e^2 - 6*a*c^2*d^2
*e^4 + a^2*c*e^6)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 2
0*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))/(a*
c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6))) - (c*d^3 + a*d*e
^2 + (c*d^2*e + a*e^3)*x)*sqrt(-(c^2*d^3 - 3*a*c*d*e^2 + (a*c^3*d^6 + 3*a^2
*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sqrt(-(9*c^3*d^4*e^2 - 6*a*c^2*d^
2*e^4 + a^2*c*e^6)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 +
20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))/(a
*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6))*log(-(3*c^2*d^2*
e - a*c*e^3)*sqrt(e*x + d) - (6*a*c^2*d^3*e^2 - 2*a^2*c*d*e^4 + (a*c^4*d^8
+ 2*a^2*c^3*d^6*e^2 - 2*a^4*c*d^2*e^6 - a^5*e^8)*sqrt(-(9*c^3*d^4*e^2 - 6*a
*c^2*d^2*e^4 + a^2*c*e^6)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8
*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^1
2)))*sqrt(-(c^2*d^3 - 3*a*c*d*e^2 + (a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*
c*d^2*e^4 + a^4*e^6)*sqrt(-(9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)/(a
*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 +
15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))/(a*c^3*d^6 + 3*a^2*c^2*
d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6))) + (c*d^3 + a*d*e^2 + (c*d^2*e + a*e^
3)*x)*sqrt(-(c^2*d^3 - 3*a*c*d*e^2 - (a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3
*c*d^2*e^4 + a^4*e^6)*sqrt(-(9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)/(
a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 +
 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))/(a*c^3*d^6 + 3*a^2*c^2
*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6))*log(-(3*c^2*d^2*e - a*c*e^3)*sqrt(e*
```

```
x + d) + (6*a*c^2*d^3*e^2 - 2*a^2*c*d*e^4 - (a*c^4*d^8 + 2*a^2*c^3*d^6*e^2
- 2*a^4*c*d^2*e^6 - a^5*e^8)*sqrt(-(9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c
*e^6)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^
6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))*sqrt(-(c^2*d^3
- 3*a*c*d*e^2 - (a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)
*sqrt(-(9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)/(a*c^6*d^12 + 6*a^2*c^
5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 +
 6*a^6*c*d^2*e^10 + a^7*e^12)))/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^
2*e^4 + a^4*e^6))) - (c*d^3 + a*d*e^2 + (c*d^2*e + a*e^3)*x)*sqrt(-(c^2*d^3
 - 3*a*c*d*e^2 - (a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6
)*sqrt(-(9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)/(a*c^6*d^12 + 6*a^2*c
^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8
+ 6*a^6*c*d^2*e^10 + a^7*e^12)))/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d
^2*e^4 + a^4*e^6))*log(-(3*c^2*d^2*e - a*c*e^3)*sqrt(e*x + d) - (6*a*c^2*d^
3*e^2 - 2*a^2*c*d*e^4 - (a*c^4*d^8 + 2*a^2*c^3*d^6*e^2 - 2*a^4*c*d^2*e^6 -
a^5*e^8)*sqrt(-(9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)/(a*c^6*d^12 +
6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d
^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))*sqrt(-(c^2*d^3 - 3*a*c*d*e^2 - (a*c
^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sqrt(-(9*c^3*d^4*e^
2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*
c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 +
a^7*e^12)))/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6))) +
 4*sqrt(e*x + d)*e)/(c*d^3 + a*d*e^2 + (c*d^2*e + a*e^3)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + cx^2\right)(d + ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(c*x**2+a),x)
```

```
[Out] Integral(1/((a + c*x**2)*(d + e*x)**(3/2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.623 $\int \frac{1}{(d+ex)^{5/2}(a+cx^2)}\,dx$

Optimal. Leaf size=736

$$-\frac{c^{3/4}e\left(2\sqrt{c}d\sqrt{ae^2+cd^2}-ae^2+3cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}+\sqrt{ae^2+cd^2}+\sqrt{c}(d+ex)\right)}{2\sqrt{2}\left(ae^2+cd^2\right)^{5/2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{c^{3/4}e\left(\right.}{}$$

```
[Out] (-2*e)/(3*(c*d^2 + a*e^2)*(d + e*x)^(3/2)) - (4*c*d*e)/((c*d^2 + a*e^2)^2*S
qrt[d + e*x]) + (c^(3/4)*e*(3*c*d^2 - a*e^2 - 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^
2])*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] - Sqrt[2]*c^(1/4)*Sqrt[d
 + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(Sqrt[2]*(c*d^2 + a*e^2)^(
5/2)*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (c^(3/4)*e*(3*c*d^2 - a*e^2 -
 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*
e^2]] + Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]
]])/(Sqrt[2]*(c*d^2 + a*e^2)^(5/2)*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) -
 (c^(3/4)*e*(3*c*d^2 - a*e^2 + 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*
d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d
 + e*x] + Sqrt[c]*(d + e*x)])/(2*Sqrt[2]*(c*d^2 + a*e^2)^(5/2)*Sqrt[Sqrt[c]
*d + Sqrt[c*d^2 + a*e^2]]) + (c^(3/4)*e*(3*c*d^2 - a*e^2 + 2*Sqrt[c]*d*Sqrt
[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d +
 Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(2*Sqrt[2]*(c*d^2
 + a*e^2)^(5/2)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])
```

Rubi [A] time = 1.73181, antiderivative size = 736, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {710, 829, 827, 1169, 634, 618, 206, 628}

$$-\frac{c^{3/4}e\left(2\sqrt{c}d\sqrt{ae^2+cd^2}-ae^2+3cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}+\sqrt{ae^2+cd^2}+\sqrt{c}(d+ex)\right)}{2\sqrt{2}\left(ae^2+cd^2\right)^{5/2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{c^{3/4}e\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(5/2)*(a + c*x^2)),x]
```


```
[Out] (-2*e)/(3*(c*d^2 + a*e^2)*(d + e*x)^(3/2)) - (4*c*d*e)/((c*d^2 + a*e^2)^2*S
qrt[d + e*x]) + (c^(3/4)*e*(3*c*d^2 - a*e^2 - 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^
2])*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] - Sqrt[2]*c^(1/4)*Sqrt[d
 + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(Sqrt[2]*(c*d^2 + a*e^2)^(
5/2)*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (c^(3/4)*e*(3*c*d^2 - a*e^2 -
 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*
e^2]] + Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]
]])/(Sqrt[2]*(c*d^2 + a*e^2)^(5/2)*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) -
 (c^(3/4)*e*(3*c*d^2 - a*e^2 + 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*
d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d
 + e*x] + Sqrt[c]*(d + e*x)])/(2*Sqrt[2]*(c*d^2 + a*e^2)^(5/2)*Sqrt[Sqrt[c]
*d + Sqrt[c*d^2 + a*e^2]]) + (c^(3/4)*e*(3*c*d^2 - a*e^2 + 2*Sqrt[c]*d*Sqrt
[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d +
 Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(2*Sqrt[2]*(c*d^2
 + a*e^2)^(5/2)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])
```


Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 829

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)
), x] + Dist[1/(c*d^2 + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d*f + a*e*g -
 c*(e*f - d*g)*x, x])/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x
] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{5/2}\left(a+cx^2\right)}\,dx = -\frac{2e}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} + \frac{c\int \frac{d-ex}{(d+ex)^{3/2}\left(a+cx^2\right)}\,dx}{cd^2+ae^2}$$

$$= -\frac{2e}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} - \frac{4cde}{\left(cd^2+ae^2\right)^2\sqrt{d+ex}} + \frac{c\int \frac{cd^2-ae^2-2cdex}{\sqrt{d+ex}\left(a+cx^2\right)}\,dx}{\left(cd^2+ae^2\right)^2}$$

$$= -\frac{2e}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} - \frac{4cde}{\left(cd^2+ae^2\right)^2\sqrt{d+ex}} + \frac{(2c)\,\text{Subst}\left(\int \frac{2cd^2e+e\left(cd^2-ae^2\right)-2cdex^2}{cd^2+ae^2-2cdx^2+cx^4}\,c\right.}{\left(cd^2+ae^2\right)^2}$$

$$= -\frac{2e}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} - \frac{4cde}{\left(cd^2+ae^2\right)^2\sqrt{d+ex}} + \frac{c^{3/4}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\left(2cd^2e+e\left(cd^2-ae^2\right)\right)\sqrt{\sqrt{c}d+\sqrt{c}}}{\sqrt[4]{c}}}{\frac{\sqrt{cd^2+ae}}{\sqrt{c}}}\right.}{\sqrt{2}\left(cd^2+a\right.}$$

$$= -\frac{2e}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} - \frac{4cde}{\left(cd^2+ae^2\right)^2\sqrt{d+ex}} + \frac{\left(\sqrt{c}e\left(3cd^2-ae^2-2\sqrt{c}d\sqrt{cd^2+ae^2}\right.\right.}{}$$

$$= -\frac{2e}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} - \frac{4cde}{\left(cd^2+ae^2\right)^2\sqrt{d+ex}} - \frac{c^{3/4}e\left(3cd^2-ae^2+2\sqrt{c}d\sqrt{cd^2+ae^2}\right.}{2\sqrt{}}$$

$$= -\frac{2e}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} - \frac{4cde}{\left(cd^2+ae^2\right)^2\sqrt{d+ex}} + \frac{c^{3/4}e\left(3cd^2-ae^2-2\sqrt{c}d\sqrt{cd^2+ae^2}\right.}{\sqrt{2}\left(cd^2+ae^2\right)^{5/2}}$$

Mathematica [C] time = 0.0645314, size = 135, normalized size = 0.18

$$\frac{\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{\sqrt{-a}\sqrt{c}d-ae} - \frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\sqrt{-a}\sqrt{c}d+ae}}{3(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(5/2)*(a + c*x^2)),x]
```

```
[Out] (-(Hypergeometric2F1[-3/2, 1, -1/2, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-
a]*e)]/(Sqrt[-a]*Sqrt[c]*d + a*e)) + Hypergeometric2F1[-3/2, 1, -1/2, (Sqrt
[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]/(Sqrt[-a]*Sqrt[c]*d - a*e))/(3*(d
+ e*x)^(3/2))
```

Maple [B] time = 0.254, size = 7264, normalized size = 9.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(5/2)/(c*x^2+a),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2 + a\right)(ex + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)*(e*x + d)^(5/2)), x)
```

Fricas [B] time = 3.11072, size = 10683, normalized size = 14.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^4*e^2 + 2*a*c*d^2*e^
4 + a^2*e^6)*x^2 + 2*(c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x)*sqrt(-(c^4*
d^5 - 10*a*c^3*d^3*e^2 + 5*a^2*c^2*d*e^4 + (a*c^5*d^10 + 5*a^2*c^4*d^8*e^2
+ 10*a^3*c^3*d^6*e^4 + 10*a^4*c^2*d^4*e^6 + 5*a^5*c*d^2*e^8 + a^6*e^10)*sqr
t(-(25*c^7*d^8*e^2 - 100*a*c^6*d^6*e^4 + 110*a^2*c^5*d^4*e^6 - 20*a^3*c^4*d
^2*e^8 + a^4*c^3*e^10)/(a*c^10*d^20 + 10*a^2*c^9*d^18*e^2 + 45*a^3*c^8*d^16
*e^4 + 120*a^4*c^7*d^14*e^6 + 210*a^5*c^6*d^12*e^8 + 252*a^6*c^5*d^10*e^10
+ 210*a^7*c^4*d^8*e^12 + 120*a^8*c^3*d^6*e^14 + 45*a^9*c^2*d^4*e^16 + 10*a^
10*c*d^2*e^18 + a^11*e^20)))/(a*c^5*d^10 + 5*a^2*c^4*d^8*e^2 + 10*a^3*c^3*d
^6*e^4 + 10*a^4*c^2*d^4*e^6 + 5*a^5*c*d^2*e^8 + a^6*e^10))*log((5*c^4*d^4*e
 - 10*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(e*x + d) + (15*a*c^4*d^6*e^2 - 35*a
^2*c^3*d^4*e^4 + 13*a^3*c^2*d^2*e^6 - a^4*c*e^8 + (a*c^6*d^13 + 2*a^2*c^5*d
^11*e^2 - 5*a^3*c^4*d^9*e^4 - 20*a^4*c^3*d^7*e^6 - 25*a^5*c^2*d^5*e^8 - 14*
a^6*c*d^3*e^10 - 3*a^7*d*e^12)*sqrt(-(25*c^7*d^8*e^2 - 100*a*c^6*d^6*e^4 +
110*a^2*c^5*d^4*e^6 - 20*a^3*c^4*d^2*e^8 + a^4*c^3*e^10)/(a*c^10*d^20 + 10*
a^2*c^9*d^18*e^2 + 45*a^3*c^8*d^16*e^4 + 120*a^4*c^7*d^14*e^6 + 210*a^5*c^6
*d^12*e^8 + 252*a^6*c^5*d^10*e^10 + 210*a^7*c^4*d^8*e^12 + 120*a^8*c^3*d^6*
e^14 + 45*a^9*c^2*d^4*e^16 + 10*a^10*c*d^2*e^18 + a^11*e^20)))*sqrt(-(c^4*d
^5 - 10*a*c^3*d^3*e^2 + 5*a^2*c^2*d*e^4 + (a*c^5*d^10 + 5*a^2*c^4*d^8*e^2 +
 10*a^3*c^3*d^6*e^4 + 10*a^4*c^2*d^4*e^6 + 5*a^5*c*d^2*e^8 + a^6*e^10)*sqrt
(-(25*c^7*d^8*e^2 - 100*a*c^6*d^6*e^4 + 110*a^2*c^5*d^4*e^6 - 20*a^3*c^4*d^
2*e^8 + a^4*c^3*e^10)/(a*c^10*d^20 + 10*a^2*c^9*d^18*e^2 + 45*a^3*c^8*d^16*
e^4 + 120*a^4*c^7*d^14*e^6 + 210*a^5*c^6*d^12*e^8 + 252*a^6*c^5*d^10*e^10 +
 210*a^7*c^4*d^8*e^12 + 120*a^8*c^3*d^6*e^14 + 45*a^9*c^2*d^4*e^16 + 10*a^1
0*c*d^2*e^18 + a^11*e^20)))/(a*c^5*d^10 + 5*a^2*c^4*d^8*e^2 + 10*a^3*c^3*d^
6*e^4 + 10*a^4*c^2*d^4*e^6 + 5*a^5*c*d^2*e^8 + a^6*e^10))) - 3*(c^2*d^6 + 2
*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^4*e^2 + 2*a*c*d^2*e^4 + a^2*e^6)*x^2 +
2*(c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x)*sqrt(-(c^4*d^5 - 10*a*c^3*d^3*
e^2 + 5*a^2*c^2*d*e^4 + (a*c^5*d^10 + 5*a^2*c^4*d^8*e^2 + 10*a^3*c^3*d^6*e^
4 + 10*a^4*c^2*d^4*e^6 + 5*a^5*c*d^2*e^8 + a^6*e^10)*sqrt(-(25*c^7*d^8*e^2
- 100*a*c^6*d^6*e^4 + 110*a^2*c^5*d^4*e^6 - 20*a^3*c^4*d^2*e^8 + a^4*c^3*e^
```

```
10)/(a*c^10*d^20 + 10*a^2*c^9*d^18*e^2 + 45*a^3*c^8*d^16*e^4 + 120*a^4*c^7*
d^14*e^6 + 210*a^5*c^6*d^12*e^8 + 252*a^6*c^5*d^10*e^10 + 210*a^7*c^4*d^8*e
^12 + 120*a^8*c^3*d^6*e^14 + 45*a^9*c^2*d^4*e^16 + 10*a^10*c*d^2*e^18 + a^1
1*e^20)))/(a*c^5*d^10 + 5*a^2*c^4*d^8*e^2 + 10*a^3*c^3*d^6*e^4 + 10*a^4*c^2
*d^4*e^6 + 5*a^5*c*d^2*e^8 + a^6*e^10))*log((5*c^4*d^4*e - 10*a*c^3*d^2*e^3
 + a^2*c^2*e^5)*sqrt(e*x + d) - (15*a*c^4*d^6*e^2 - 35*a^2*c^3*d^4*e^4 + 13
*a^3*c^2*d^2*e^6 - a^4*c*e^8 + (a*c^6*d^13 + 2*a^2*c^5*d^11*e^2 - 5*a^3*c^4
*d^9*e^4 - 20*a^4*c^3*d^7*e^6 - 25*a^5*c^2*d^5*e^8 - 14*a^6*c*d^3*e^10 - 3*
a^7*d*e^12)*sqrt(-(25*c^7*d^8*e^2 - 100*a*c^6*d^6*e^4 + 110*a^2*c^5*d^4*e^6
 - 20*a^3*c^4*d^2*e^8 + a^4*c^3*e^10)/(a*c^10*d^20 + 10*a^2*c^9*d^18*e^2 +
45*a^3*c^8*d^16*e^4 + 120*a^4*c^7*d^14*e^6 + 210*a^5*c^6*d^12*e^8 + 252*a^6
*c^5*d^10*e^10 + 210*a^7*c^4*d^8*e^12 + 120*a^8*c^3*d^6*e^14 + 45*a^9*c^2*d
^4*e^16 + 10*a^10*c*d^2*e^18 + a^11*e^20)))*sqrt(-(c^4*d^5 - 10*a*c^3*d^3*e
^2 + 5*a^2*c^2*d*e^4 + (a*c^5*d^10 + 5*a^2*c^4*d^8*e^2 + 10*a^3*c^3*d^6*e^4
 + 10*a^4*c^2*d^4*e^6 + 5*a^5*c*d^2*e^8 + a^6*e^10)*sqrt(-(25*c^7*d^8*e^2 -
 100*a*c^6*d^6*e^4 + 110*a^2*c^5*d^4*e^6 - 20*a^3*c^4*d^2*e^8 + a^4*c^3*e^1
0)/(a*c^10*d^20 + 10*a^2*c^9*d^18*e^2 + 45*a^3*c^8*d^16*e^4 + 120*a^4*c^7*d
^14*e^6 + 210*a^5*c^6*d^12*e^8 + 252*a^6*c^5*d^10*e^10 + 210*a^7*c^4*d^8*e^
12 + 120*a^8*c^3*d^6*e^14 + 45*a^9*c^2*d^4*e^16 + 10*a^10*c*d^2*e^18 + a^11
*e^20)))/(a*c^5*d^10 + 5*a^2*c^4*d^8*e^2 + 10*a^3*c^3*d^6*e^4 + 10*a^4*c^2*
d^4*e^6 + 5*a^5*c*d^2*e^8 + a^6*e^10))) + 3*(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*
d^2*e^4 + (c^2*d^4*e^2 + 2*a*c*d^2*e^4 + a^2*e^6)*x^2 + 2*(c^2*d^5*e + 2*a*
c*d^3*e^3 + a^2*d*e^5)*x)*sqrt(-(c^4*d^5 - 10*a*c^3*d^3*e^2 + 5*a^2*c^2*d*e
^4 - (a*c^5*d^10 + 5*a^2*c^4*d^8*e^2 + 10*a^3*c^3*d^6*e^4 + 10*a^4*c^2*d^4*
e^6 + 5*a^5*c*d^2*e^8 + a^6*e^10)*sqrt(-(25*c^7*d^8*e^2 - 100*a*c^6*d^6*e^4
 + 110*a^2*c^5*d^4*e^6 - 20*a^3*c^4*d^2*e^8 + a^4*c^3*e^10)/(a*c^10*d^20 +
10*a^2*c^9*d^18*e^2 + 45*a^3*c^8*d^16*e^4 + 120*a^4*c^7*d^14*e^6 + 210*a^5*
c^6*d^12*e^8 + 252*a^6*c^5*d^10*e^10 + 210*a^7*c^4*d^8*e^12 + 120*a^8*c^3*d
^6*e^14 + 45*a^9*c^2*d^4*e^16 + 10*a^10*c*d^2*e^18 + a^11*e^20)))/(a*c^5*d^
10 + 5*a^2*c^4*d^8*e^2 + 10*a^3*c^3*d^6*e^4 + 10*a^4*c^2*d^4*e^6 + 5*a^5*c*
d^2*e^8 + a^6*e^10))*log((5*c^4*d^4*e - 10*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqr
t(e*x + d) + (15*a*c^4*d^6*e^2 - 35*a^2*c^3*d^4*e^4 + 13*a^3*c^2*d^2*e^6 -
a^4*c*e^8 - (a*c^6*d^13 + 2*a^2*c^5*d^11*e^2 - 5*a^3*c^4*d^9*e^4 - 20*a^4*c
^3*d^7*e^6 - 25*a^5*c^2*d^5*e^8 - 14*a^6*c*d^3*e^10 - 3*a^7*d*e^12)*sqrt(-(
25*c^7*d^8*e^2 - 100*a*c^6*d^6*e^4 + 110*a^2*c^5*d^4*e^6 - 20*a^3*c^4*d^2*e
^8 + a^4*c^3*e^10)/(a*c^10*d^20 + 10*a^2*c^9*d^18*e^2 + 45*a^3*c^8*d^16*e^4
 + 120*a^4*c^7*d^14*e^6 + 210*a^5*c^6*d^12*e^8 + 252*a^6*c^5*d^10*e^10 + 21
0*a^7*c^4*d^8*e^12 + 120*a^8*c^3*d^6*e^14 + 45*a^9*c^2*d^4*e^16 + 10*a^10*c
*d^2*e^18 + a^11*e^20)))*sqrt(-(c^4*d^5 - 10*a*c^3*d^3*e^2 + 5*a^2*c^2*d*e^
4 - (a*c^5*d^10 + 5*a^2*c^4*d^8*e^2 + 10*a^3*c^3*d^6*e^4 + 10*a^4*c^2*d^4*e
^6 + 5*a^5*c*d^2*e^8 + a^6*e^10)*sqrt(-(25*c^7*d^8*e^2 - 100*a*c^6*d^6*e^4
+ 110*a^2*c^5*d^4*e^6 - 20*a^3*c^4*d^2*e^8 + a^4*c^3*e^10)/(a*c^10*d^20 + 1
0*a^2*c^9*d^18*e^2 + 45*a^3*c^8*d^16*e^4 + 120*a^4*c^7*d^14*e^6 + 210*a^5*c
^6*d^12*e^8 + 252*a^6*c^5*d^10*e^10 + 210*a^7*c^4*d^8*e^12 + 120*a^8*c^3*d^
6*e^14 + 45*a^9*c^2*d^4*e^16 + 10*a^10*c*d^2*e^18 + a^11*e^20)))/(a*c^5*d^1
0 + 5*a^2*c^4*d^8*e^2 + 10*a^3*c^3*d^6*e^4 + 10*a^4*c^2*d^4*e^6 + 5*a^5*c*d
^2*e^8 + a^6*e^10))) - 3*(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^4*
e^2 + 2*a*c*d^2*e^4 + a^2*e^6)*x^2 + 2*(c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e
^5)*x)*sqrt(-(c^4*d^5 - 10*a*c^3*d^3*e^2 + 5*a^2*c^2*d*e^4 - (a*c^5*d^10 +
5*a^2*c^4*d^8*e^2 + 10*a^3*c^3*d^6*e^4 + 10*a^4*c^2*d^4*e^6 + 5*a^5*c*d^2*e
^8 + a^6*e^10)*sqrt(-(25*c^7*d^8*e^2 - 100*a*c^6*d^6*e^4 + 110*a^2*c^5*d^4*
e^6 - 20*a^3*c^4*d^2*e^8 + a^4*c^3*e^10)/(a*c^10*d^20 + 10*a^2*c^9*d^18*e^2
 + 45*a^3*c^8*d^16*e^4 + 120*a^4*c^7*d^14*e^6 + 210*a^5*c^6*d^12*e^8 + 252*
a^6*c^5*d^10*e^10 + 210*a^7*c^4*d^8*e^12 + 120*a^8*c^3*d^6*e^14 + 45*a^9*c^
2*d^4*e^16 + 10*a^10*c*d^2*e^18 + a^11*e^20)))/(a*c^5*d^10 + 5*a^2*c^4*d^8*
e^2 + 10*a^3*c^3*d^6*e^4 + 10*a^4*c^2*d^4*e^6 + 5*a^5*c*d^2*e^8 + a^6*e^10)
)*log((5*c^4*d^4*e - 10*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(e*x + d) - (15*a*
c^4*d^6*e^2 - 35*a^2*c^3*d^4*e^4 + 13*a^3*c^2*d^2*e^6 - a^4*c*e^8 - (a*c^6*
d^13 + 2*a^2*c^5*d^11*e^2 - 5*a^3*c^4*d^9*e^4 - 20*a^4*c^3*d^7*e^6 - 25*a^5
```

```
*c^2*d^5*e^8 - 14*a^6*c*d^3*e^10 - 3*a^7*d*e^12)*sqrt(-(25*c^7*d^8*e^2 - 10
0*a*c^6*d^6*e^4 + 110*a^2*c^5*d^4*e^6 - 20*a^3*c^4*d^2*e^8 + a^4*c^3*e^10)/
(a*c^10*d^20 + 10*a^2*c^9*d^18*e^2 + 45*a^3*c^8*d^16*e^4 + 120*a^4*c^7*d^14
*e^6 + 210*a^5*c^6*d^12*e^8 + 252*a^6*c^5*d^10*e^10 + 210*a^7*c^4*d^8*e^12
+ 120*a^8*c^3*d^6*e^14 + 45*a^9*c^2*d^4*e^16 + 10*a^10*c*d^2*e^18 + a^11*e^
20)))*sqrt(-(c^4*d^5 - 10*a*c^3*d^3*e^2 + 5*a^2*c^2*d*e^4 - (a*c^5*d^10 + 5
*a^2*c^4*d^8*e^2 + 10*a^3*c^3*d^6*e^4 + 10*a^4*c^2*d^4*e^6 + 5*a^5*c*d^2*e^
8 + a^6*e^10)*sqrt(-(25*c^7*d^8*e^2 - 100*a*c^6*d^6*e^4 + 110*a^2*c^5*d^4*e
^6 - 20*a^3*c^4*d^2*e^8 + a^4*c^3*e^10)/(a*c^10*d^20 + 10*a^2*c^9*d^18*e^2
+ 45*a^3*c^8*d^16*e^4 + 120*a^4*c^7*d^14*e^6 + 210*a^5*c^6*d^12*e^8 + 252*a
^6*c^5*d^10*e^10 + 210*a^7*c^4*d^8*e^12 + 120*a^8*c^3*d^6*e^14 + 45*a^9*c^2
*d^4*e^16 + 10*a^10*c*d^2*e^18 + a^11*e^20)))/(a*c^5*d^10 + 5*a^2*c^4*d^8*e
^2 + 10*a^3*c^3*d^6*e^4 + 10*a^4*c^2*d^4*e^6 + 5*a^5*c*d^2*e^8 + a^6*e^10))
) - 4*(6*c*d*e^2*x + 7*c*d^2*e + a*e^3)*sqrt(e*x + d))/(c^2*d^6 + 2*a*c*d^4
*e^2 + a^2*d^2*e^4 + (c^2*d^4*e^2 + 2*a*c*d^2*e^4 + a^2*e^6)*x^2 + 2*(c^2*d
^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+cx^2\right)(d+ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(5/2)/(c*x**2+a),x)
```

```
[Out] Integral(1/((a + c*x**2)*(d + e*x)**(5/2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.624 $\int \frac{(d+ex)^{7/2}}{\left(a-cx^2\right)^2}\,dx$

Optimal. Leaf size=263

$$-\frac{\left(5\sqrt{ae}+2\sqrt{cd}\right)\left(\sqrt{cd}-\sqrt{ae}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{cd}-\sqrt{ae}}}\right)}{4a^{3/2}c^{9/4}}+\frac{\left(2\sqrt{cd}-5\sqrt{ae}\right)\left(\sqrt{ae}+\sqrt{cd}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{ae}+\sqrt{cd}}}\right)}{4a^{3/2}c^{9/4}}+\frac{e\sqrt{d+e}}{}$$

```
[Out] (e*(c*d^2 + 5*a*e^2)*Sqrt[d + e*x])/(2*a*c^2) + (d*e*(d + e*x)^(3/2))/(2*a*
c) + ((a*e + c*d*x)*(d + e*x)^(5/2))/(2*a*c*(a - c*x^2)) - ((Sqrt[c]*d - Sq
rt[a]*e)^(5/2)*(2*Sqrt[c]*d + 5*Sqrt[a]*e)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/
Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(4*a^(3/2)*c^(9/4)) + ((2*Sqrt[c]*d - 5*Sqrt[
a]*e)*(Sqrt[c]*d + Sqrt[a]*e)^(5/2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sq
rt[c]*d + Sqrt[a]*e]])/(4*a^(3/2)*c^(9/4))
```

Rubi [A] time = 0.534155, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {739, 825, 827, 1166, 208}

$$-\frac{\left(5\sqrt{ae}+2\sqrt{cd}\right)\left(\sqrt{cd}-\sqrt{ae}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{cd}-\sqrt{ae}}}\right)}{4a^{3/2}c^{9/4}}+\frac{\left(2\sqrt{cd}-5\sqrt{ae}\right)\left(\sqrt{ae}+\sqrt{cd}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{ae}+\sqrt{cd}}}\right)}{4a^{3/2}c^{9/4}}+\frac{e\sqrt{d+e}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/(a - c*x^2)^2,x]
```

```
[Out] (e*(c*d^2 + 5*a*e^2)*Sqrt[d + e*x])/(2*a*c^2) + (d*e*(d + e*x)^(3/2))/(2*a*
c) + ((a*e + c*d*x)*(d + e*x)^(5/2))/(2*a*c*(a - c*x^2)) - ((Sqrt[c]*d - Sq
rt[a]*e)^(5/2)*(2*Sqrt[c]*d + 5*Sqrt[a]*e)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/
Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(4*a^(3/2)*c^(9/4)) + ((2*Sqrt[c]*d - 5*Sqrt[
a]*e)*(Sqrt[c]*d + Sqrt[a]*e)^(5/2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sq
rt[c]*d + Sqrt[a]*e]])/(4*a^(3/2)*c^(9/4))
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 825

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[((d + e*x)^(m
- 1)*Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x])/(a + c*x^2), x], x] /; Fre
eQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m
, 0]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{7/2}}{\left(a-cx^2\right)^2}\,dx = \frac{(ae+cdx)(d+ex)^{5/2}}{2ac\left(a-cx^2\right)} - \frac{\int \frac{(d+ex)^{3/2}\left(\frac{1}{2}\left(-2cd^2+5ae^2\right)+\frac{3}{2}cdex\right)}{a-cx^2}\,dx}{2ac}$$

$$= \frac{de(d+ex)^{3/2}}{2ac} + \frac{(ae+cdx)(d+ex)^{5/2}}{2ac\left(a-cx^2\right)} + \frac{\int \frac{\sqrt{d+ex}\left(cd\left(cd^2-4ae^2\right)-\frac{1}{2}ce\left(cd^2+5ae^2\right)x\right)}{a-cx^2}\,dx}{2ac^2}$$

$$= \frac{e\left(cd^2+5ae^2\right)\sqrt{d+ex}}{2ac^2} + \frac{de(d+ex)^{3/2}}{2ac} + \frac{(ae+cdx)(d+ex)^{5/2}}{2ac\left(a-cx^2\right)} - \frac{\int \frac{-\frac{1}{2}c\left(cd^2-5ae^2\right)\left(2cd^2+ae^2\right)-\frac{1}{2}c^2de\left(cd^2-13ae\ldots\right.}{\sqrt{d+ex}\left(a-cx^2\right)}}{2ac^3}$$

$$= \frac{e\left(cd^2+5ae^2\right)\sqrt{d+ex}}{2ac^2} + \frac{de(d+ex)^{3/2}}{2ac} + \frac{(ae+cdx)(d+ex)^{5/2}}{2ac\left(a-cx^2\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}c^2d^2e\left(cd^2-13ae^2\right)-\frac{1}{2}ce\left(cd^2-5ae\ldots\right.}{-cd^2+ae^2\ldots}\right.}{\ldots}$$

$$= \frac{e\left(cd^2+5ae^2\right)\sqrt{d+ex}}{2ac^2} + \frac{de(d+ex)^{3/2}}{2ac} + \frac{(ae+cdx)(d+ex)^{5/2}}{2ac\left(a-cx^2\right)} + \frac{\left(\left(2\sqrt{c}d-5\sqrt{a}e\right)\left(\sqrt{c}d+\sqrt{a}e\right)^3\right)\text{Sub}\ldots}{4a^{3/2}\ldots}$$

$$= \frac{e\left(cd^2+5ae^2\right)\sqrt{d+ex}}{2ac^2} + \frac{de(d+ex)^{3/2}}{2ac} + \frac{(ae+cdx)(d+ex)^{5/2}}{2ac\left(a-cx^2\right)} - \frac{\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}\left(2\sqrt{c}d+5\sqrt{a}e\right)\tanh\ldots}{4a^{3/2}c^{9/4}}$$

Mathematica [A] time = 0.512183, size = 251, normalized size = 0.95

$$\frac{2\sqrt{a}\sqrt[4]{c}\sqrt{d+ex}\left(5a^2e^3+ace\left(3d^2+3dex-4e^2x^2\right)+c^2d^3x\right)+\left(cx^2-a\right)\left(5\sqrt{a}e+2\sqrt{c}d\right)\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\ldots\right.}{4a^{3/2}c^{9/4}\left(a-cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)/(a - c*x^2)^2,x]
```

```
[Out] (2*Sqrt[a]*c^(1/4)*Sqrt[d + e*x]*(5*a^2*e^3 + c^2*d^3*x + a*c*e*(3*d^2 + 3*
d*e*x - 4*e^2*x^2)) + (Sqrt[c]*d - Sqrt[a]*e)^(5/2)*(2*Sqrt[c]*d + 5*Sqrt[a
]*e)*(-a + c*x^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*
```

```
e]] - (2*Sqrt[c]*d - 5*Sqrt[a]*e)*(Sqrt[c]*d + Sqrt[a]*e)^(5/2)*(-a + c*x^2
)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(4*a^(3/2)*
c^(9/4)*(a - c*x^2))
```

Maple [B] time = 0.226, size = 717, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/(-c*x^2+a)^2,x)
```

```
[Out] 2*e^3/c^2*(e*x+d)^(1/2)-3/2*e^3/c/(c*e^2*x^2-a*e^2)*d*(e*x+d)^(3/2)-1/2*e/(
c*e^2*x^2-a*e^2)*d^3/a*(e*x+d)^(3/2)-1/2*e^5/c^2/(c*e^2*x^2-a*e^2)*a*(e*x+d
)^(1/2)+1/2*e/(c*e^2*x^2-a*e^2)/a*(e*x+d)^(1/2)*d^4-5/4*e^5/c*a/(a*c*e^2)^(
1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^
2)^(1/2))*c)^(1/2))-9/4*e^3/(a*c*e^2)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2
)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d^2+1/2*e*c/a/(a
*c*e^2)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((-c*
d+(a*c*e^2)^(1/2))*c)^(1/2))*d^4+13/4*e^3/c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2
)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d-1/4*e/a/((-c*d
+(a*c*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c
)^(1/2))*d^3-5/4*e^5/c*a/(a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*ar
ctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))-9/4*e^3/(a*c*e^2)^(1
/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)
^(1/2))*c)^(1/2))*d^2+1/2*e*c/a/(a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(
1/2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d^4-13/4*e^3/
c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(
1/2))*c)^(1/2))*d+1/4*e/a/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(
1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{7}{2}}}{\left(cx^2-a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(-c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(7/2)/(c*x^2 - a)^2, x)
```

Fricas [B] time = 3.51031, size = 4626, normalized size = 17.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(-c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((a*c^3*x^2 - a^2*c^2)*sqrt((4*c^3*d^7 - 35*a*c^2*d^5*e^2 + 70*a^2*c*d^
3*e^4 + 105*a^3*d*e^6 + a^3*c^4*sqrt((1225*c^4*d^8*e^6 - 10780*a*c^3*d^6*e^
```

```
8 + 21966*a^2*c^2*d^4*e^10 + 7700*a^3*c*d^2*e^12 + 625*a^4*e^14)/(a^3*c^9))
)/(a^3*c^4))*log((140*c^5*d^10*e^3 - 1771*a*c^4*d^8*e^5 + 6872*a^2*c^3*d^6*
e^7 - 8366*a^3*c^2*d^4*e^9 + 2500*a^4*c*d^2*e^11 + 625*a^5*e^13)*sqrt(e*x +
 d) + (35*a^2*c^5*d^6*e^4 + 21*a^3*c^4*d^4*e^6 - 795*a^4*c^3*d^2*e^8 - 125*
a^5*c^2*e^10 - 2*(a^3*c^8*d^3 - 4*a^4*c^7*d*e^2)*sqrt((1225*c^4*d^8*e^6 - 1
0780*a*c^3*d^6*e^8 + 21966*a^2*c^2*d^4*e^10 + 7700*a^3*c*d^2*e^12 + 625*a^4
*e^14)/(a^3*c^9)))*sqrt((4*c^3*d^7 - 35*a*c^2*d^5*e^2 + 70*a^2*c*d^3*e^4 +
105*a^3*d*e^6 + a^3*c^4*sqrt((1225*c^4*d^8*e^6 - 10780*a*c^3*d^6*e^8 + 2196
6*a^2*c^2*d^4*e^10 + 7700*a^3*c*d^2*e^12 + 625*a^4*e^14)/(a^3*c^9)))/(a^3*c
^4))) - (a*c^3*x^2 - a^2*c^2)*sqrt((4*c^3*d^7 - 35*a*c^2*d^5*e^2 + 70*a^2*c
*d^3*e^4 + 105*a^3*d*e^6 + a^3*c^4*sqrt((1225*c^4*d^8*e^6 - 10780*a*c^3*d^6
*e^8 + 21966*a^2*c^2*d^4*e^10 + 7700*a^3*c*d^2*e^12 + 625*a^4*e^14)/(a^3*c^
9)))/(a^3*c^4))*log((140*c^5*d^10*e^3 - 1771*a*c^4*d^8*e^5 + 6872*a^2*c^3*d
^6*e^7 - 8366*a^3*c^2*d^4*e^9 + 2500*a^4*c*d^2*e^11 + 625*a^5*e^13)*sqrt(e*
x + d) - (35*a^2*c^5*d^6*e^4 + 21*a^3*c^4*d^4*e^6 - 795*a^4*c^3*d^2*e^8 - 1
25*a^5*c^2*e^10 - 2*(a^3*c^8*d^3 - 4*a^4*c^7*d*e^2)*sqrt((1225*c^4*d^8*e^6
- 10780*a*c^3*d^6*e^8 + 21966*a^2*c^2*d^4*e^10 + 7700*a^3*c*d^2*e^12 + 625*
a^4*e^14)/(a^3*c^9)))*sqrt((4*c^3*d^7 - 35*a*c^2*d^5*e^2 + 70*a^2*c*d^3*e^4
 + 105*a^3*d*e^6 + a^3*c^4*sqrt((1225*c^4*d^8*e^6 - 10780*a*c^3*d^6*e^8 + 2
1966*a^2*c^2*d^4*e^10 + 7700*a^3*c*d^2*e^12 + 625*a^4*e^14)/(a^3*c^9)))/(a^
3*c^4))) + (a*c^3*x^2 - a^2*c^2)*sqrt((4*c^3*d^7 - 35*a*c^2*d^5*e^2 + 70*a^
2*c*d^3*e^4 + 105*a^3*d*e^6 - a^3*c^4*sqrt((1225*c^4*d^8*e^6 - 10780*a*c^3*
d^6*e^8 + 21966*a^2*c^2*d^4*e^10 + 7700*a^3*c*d^2*e^12 + 625*a^4*e^14)/(a^3
*c^9)))/(a^3*c^4))*log((140*c^5*d^10*e^3 - 1771*a*c^4*d^8*e^5 + 6872*a^2*c^
3*d^6*e^7 - 8366*a^3*c^2*d^4*e^9 + 2500*a^4*c*d^2*e^11 + 625*a^5*e^13)*sqrt
(e*x + d) + (35*a^2*c^5*d^6*e^4 + 21*a^3*c^4*d^4*e^6 - 795*a^4*c^3*d^2*e^8
- 125*a^5*c^2*e^10 + 2*(a^3*c^8*d^3 - 4*a^4*c^7*d*e^2)*sqrt((1225*c^4*d^8*e
^6 - 10780*a*c^3*d^6*e^8 + 21966*a^2*c^2*d^4*e^10 + 7700*a^3*c*d^2*e^12 + 6
25*a^4*e^14)/(a^3*c^9)))*sqrt((4*c^3*d^7 - 35*a*c^2*d^5*e^2 + 70*a^2*c*d^3*
e^4 + 105*a^3*d*e^6 - a^3*c^4*sqrt((1225*c^4*d^8*e^6 - 10780*a*c^3*d^6*e^8
+ 21966*a^2*c^2*d^4*e^10 + 7700*a^3*c*d^2*e^12 + 625*a^4*e^14)/(a^3*c^9)))/
(a^3*c^4))) - (a*c^3*x^2 - a^2*c^2)*sqrt((4*c^3*d^7 - 35*a*c^2*d^5*e^2 + 70
*a^2*c*d^3*e^4 + 105*a^3*d*e^6 - a^3*c^4*sqrt((1225*c^4*d^8*e^6 - 10780*a*c
^3*d^6*e^8 + 21966*a^2*c^2*d^4*e^10 + 7700*a^3*c*d^2*e^12 + 625*a^4*e^14)/(
a^3*c^9)))/(a^3*c^4))*log((140*c^5*d^10*e^3 - 1771*a*c^4*d^8*e^5 + 6872*a^2
*c^3*d^6*e^7 - 8366*a^3*c^2*d^4*e^9 + 2500*a^4*c*d^2*e^11 + 625*a^5*e^13)*s
qrt(e*x + d) - (35*a^2*c^5*d^6*e^4 + 21*a^3*c^4*d^4*e^6 - 795*a^4*c^3*d^2*e
^8 - 125*a^5*c^2*e^10 + 2*(a^3*c^8*d^3 - 4*a^4*c^7*d*e^2)*sqrt((1225*c^4*d^
8*e^6 - 10780*a*c^3*d^6*e^8 + 21966*a^2*c^2*d^4*e^10 + 7700*a^3*c*d^2*e^12
+ 625*a^4*e^14)/(a^3*c^9)))*sqrt((4*c^3*d^7 - 35*a*c^2*d^5*e^2 + 70*a^2*c*d
^3*e^4 + 105*a^3*d*e^6 - a^3*c^4*sqrt((1225*c^4*d^8*e^6 - 10780*a*c^3*d^6*e
^8 + 21966*a^2*c^2*d^4*e^10 + 7700*a^3*c*d^2*e^12 + 625*a^4*e^14)/(a^3*c^9)
))/(a^3*c^4))) + 4*(4*a*c*e^3*x^2 - 3*a*c*d^2*e - 5*a^2*e^3 - (c^2*d^3 + 3*
a*c*d*e^2)*x)*sqrt(e*x + d))/(a*c^3*x^2 - a^2*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/(-c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(-c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.625 $\int \frac{(d+ex)^{5/2}}{\left(a-cx^2\right)^2} dx$

Optimal. Leaf size=231

$$-\frac{\left(3\sqrt{a}e+2\sqrt{c}d\right)\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{4a^{3/2}c^{7/4}}+\frac{\left(2\sqrt{c}d-3\sqrt{a}e\right)\left(\sqrt{a}e+\sqrt{c}d\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{4a^{3/2}c^{7/4}}+\frac{(d+ex)^{3/2}(}{2ac\left(a-\right.}$$

```
[Out] (d*e*Sqrt[d + e*x])/(2*a*c) + ((a*e + c*d*x)*(d + e*x)^(3/2))/(2*a*c*(a - c
*x^2)) - ((Sqrt[c]*d - Sqrt[a]*e)^(3/2)*(2*Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTanh
[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(4*a^(3/2)*c^(7/4))
+ ((2*Sqrt[c]*d - 3*Sqrt[a]*e)*(Sqrt[c]*d + Sqrt[a]*e)^(3/2)*ArcTanh[(c^(1/
4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(4*a^(3/2)*c^(7/4))
```

Rubi [A] time = 0.374772, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {739, 825, 827, 1166, 208}

$$-\frac{\left(3\sqrt{a}e+2\sqrt{c}d\right)\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{4a^{3/2}c^{7/4}}+\frac{\left(2\sqrt{c}d-3\sqrt{a}e\right)\left(\sqrt{a}e+\sqrt{c}d\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{4a^{3/2}c^{7/4}}+\frac{(d+ex)^{3/2}(}{2ac\left(a-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(a - c*x^2)^2,x]
```


```
[Out] (d*e*Sqrt[d + e*x])/(2*a*c) + ((a*e + c*d*x)*(d + e*x)^(3/2))/(2*a*c*(a - c
*x^2)) - ((Sqrt[c]*d - Sqrt[a]*e)^(3/2)*(2*Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTanh
[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(4*a^(3/2)*c^(7/4))
+ ((2*Sqrt[c]*d - 3*Sqrt[a]*e)*(Sqrt[c]*d + Sqrt[a]*e)^(3/2)*ArcTanh[(c^(1/
4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(4*a^(3/2)*c^(7/4))
```


Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 825

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[((d + e*x)^(m
- 1)*Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x])/(a + c*x^2), x], x] /; Fre
eQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m
, 0]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
```

```
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\left(a-cx^2\right)^2}\,dx = \frac{(ae+cdx)(d+ex)^{3/2}}{2ac\left(a-cx^2\right)} - \frac{\int \frac{\sqrt{d+ex}\left(\frac{1}{2}\left(-2cd^2+3ae^2\right)+\frac{1}{2}cdex\right)}{a-cx^2}\,dx}{2ac}$$

$$= \frac{de\sqrt{d+ex}}{2ac} + \frac{(ae+cdx)(d+ex)^{3/2}}{2ac\left(a-cx^2\right)} + \frac{\int \frac{cd\left(cd^2-2ae^2\right)+\frac{1}{2}ce\left(cd^2-3ae^2\right)x}{\sqrt{d+ex}\left(a-cx^2\right)}\,dx}{2ac^2}$$

$$= \frac{de\sqrt{d+ex}}{2ac} + \frac{(ae+cdx)(d+ex)^{3/2}}{2ac\left(a-cx^2\right)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}cde\left(cd^2-3ae^2\right)+cde\left(cd^2-2ae^2\right)+\frac{1}{2}ce\left(cd^2-3ae^2\right)x^2}{-cd^2+ae^2+2cdx^2-cx^4}\,dx, x, \sqrt{d-\right.}{ac^2}$$

$$= \frac{de\sqrt{d+ex}}{2ac} + \frac{(ae+cdx)(d+ex)^{3/2}}{2ac\left(a-cx^2\right)} + \frac{\left(\left(2\sqrt{c}d-3\sqrt{a}e\right)\left(\sqrt{c}d+\sqrt{a}e\right)^2\right)\text{Subst}\left(\int \frac{1}{cd+\sqrt{a}\sqrt{c}e-cx^2}\,dx, x,\right.}{4a^{3/2}c}$$

$$= \frac{de\sqrt{d+ex}}{2ac} + \frac{(ae+cdx)(d+ex)^{3/2}}{2ac\left(a-cx^2\right)} - \frac{\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}\left(2\sqrt{c}d+3\sqrt{a}e\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{4a^{3/2}c^{7/4}} + \frac{\left(2\sqrt{c}d\right.}{}$$

Mathematica [A] time = 0.348846, size = 247, normalized size = 1.07

$$\frac{2\sqrt{a}c^{3/4}\sqrt{d+ex}\left(ae(2d+ex)+cd^2x\right)+\left(cx^2-a\right)\sqrt{\sqrt{c}d-\sqrt{a}e}\left(\sqrt{a}\sqrt{c}de-3ae^2+2cd^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)-\left(cx^2-a\right.}{4a^{3/2}c^{7/4}\left(a-cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(a - c*x^2)^2,x]
```

```
[Out] (2*Sqrt[a]*c^(3/4)*Sqrt[d + e*x]*(c*d^2*x + a*e*(2*d + e*x)) + Sqrt[Sqrt[c]
*d - Sqrt[a]*e]*(2*c*d^2 + Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*(-a + c*x^2)*ArcT
anh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]] - Sqrt[Sqrt[c]*d +
 Sqrt[a]*e]*(2*c*d^2 - Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*(-a + c*x^2)*ArcTanh[
(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(4*a^(3/2)*c^(7/4)*(a
 - c*x^2))
```

Maple [B] time = 0.221, size = 575, normalized size = 2.5

$$-\frac{e^{3}}{\left(2\,c e^{2} x^{2}-2\,a e^{2}\right) c}\,(ex+d)^{\frac{3}{2}}-\frac{e d^{2}}{\left(2\,c e^{2} x^{2}-2\,a e^{2}\right) a}\,(ex+d)^{\frac{3}{2}}-\frac{e^{3} d}{\left(2\,c e^{2} x^{2}-2\,a e^{2}\right) c}\sqrt{ex+d}+\frac{e d^{3}}{\left(2\,c e^{2} x^{2}-2\,a e^{2}\right) a}\sqrt{ex+d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(-c*x^2+a)^2,x)
```

```
[Out] -1/2*e^3/(c*e^2*x^2-a*e^2)/c*(e*x+d)^(3/2)-1/2*e/(c*e^2*x^2-a*e^2)/a*(e*x+d
)^(3/2)*d^2-1/2*e^3/(c*e^2*x^2-a*e^2)*d/c*(e*x+d)^(1/2)+1/2*e/(c*e^2*x^2-a*
e^2)*d^3/a*(e*x+d)^(1/2)-e^3/(a*c*e^2)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/
2)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d+1/2*e/a*c/(a*
c*e^2)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((-c*d
+(a*c*e^2)^(1/2))*c)^(1/2))*d^3+3/4*e^3/c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*
arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))-1/4*e/a/((-c*d+(a*
c*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1
/2))*d^2-e^3/(a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d
)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d+1/2*e/a*c/(a*c*e^2)^(1/2)/((c*
d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*
c)^(1/2))*d^3-3/4*e^3/c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/
2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))+1/4*e/a/((c*d+(a*c*e^2)^(1/2))*c)^(1/
2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\left(cx^{2}-a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(-c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/(c*x^2 - a)^2, x)
```

Fricas [B] time = 2.34586, size = 2897, normalized size = 12.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(-c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((a*c^2*x^2 - a^2*c)*sqrt((4*c^2*d^5 - 15*a*c*d^3*e^2 + 15*a^2*d*e^4 +
 a^3*c^3*sqrt((25*c^2*d^4*e^6 - 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7)))/(
a^3*c^3))*log(-(20*c^3*d^6*e^3 - 101*a*c^2*d^4*e^5 + 162*a^2*c*d^2*e^7 - 81
*a^3*e^9)*sqrt(e*x + d) + (5*a^2*c^3*d^3*e^4 - 9*a^3*c^2*d*e^6 - (2*a^3*c^6
*d^2 - 3*a^4*c^5*e^2)*sqrt((25*c^2*d^4*e^6 - 90*a*c*d^2*e^8 + 81*a^2*e^10)/
(a^3*c^7)))*sqrt((4*c^2*d^5 - 15*a*c*d^3*e^2 + 15*a^2*d*e^4 + a^3*c^3*sqrt(
(25*c^2*d^4*e^6 - 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7)))/(a^3*c^3))) - (
a*c^2*x^2 - a^2*c)*sqrt((4*c^2*d^5 - 15*a*c*d^3*e^2 + 15*a^2*d*e^4 + a^3*c^
```

```
3*sqrt((25*c^2*d^4*e^6 - 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7)))/(a^3*c^3
))*log(-(20*c^3*d^6*e^3 - 101*a*c^2*d^4*e^5 + 162*a^2*c*d^2*e^7 - 81*a^3*e^
9)*sqrt(e*x + d) - (5*a^2*c^3*d^3*e^4 - 9*a^3*c^2*d*e^6 - (2*a^3*c^6*d^2 -
3*a^4*c^5*e^2)*sqrt((25*c^2*d^4*e^6 - 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^
7)))*sqrt((4*c^2*d^5 - 15*a*c*d^3*e^2 + 15*a^2*d*e^4 + a^3*c^3*sqrt((25*c^2
*d^4*e^6 - 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7)))/(a^3*c^3))) + (a*c^2*x
^2 - a^2*c)*sqrt((4*c^2*d^5 - 15*a*c*d^3*e^2 + 15*a^2*d*e^4 - a^3*c^3*sqrt(
(25*c^2*d^4*e^6 - 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7)))/(a^3*c^3))*log(
-(20*c^3*d^6*e^3 - 101*a*c^2*d^4*e^5 + 162*a^2*c*d^2*e^7 - 81*a^3*e^9)*sqrt
(e*x + d) + (5*a^2*c^3*d^3*e^4 - 9*a^3*c^2*d*e^6 + (2*a^3*c^6*d^2 - 3*a^4*c
^5*e^2)*sqrt((25*c^2*d^4*e^6 - 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7)))*sq
rt((4*c^2*d^5 - 15*a*c*d^3*e^2 + 15*a^2*d*e^4 - a^3*c^3*sqrt((25*c^2*d^4*e^
6 - 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7)))/(a^3*c^3))) - (a*c^2*x^2 - a^
2*c)*sqrt((4*c^2*d^5 - 15*a*c*d^3*e^2 + 15*a^2*d*e^4 - a^3*c^3*sqrt((25*c^2
*d^4*e^6 - 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7)))/(a^3*c^3))*log(-(20*c^
3*d^6*e^3 - 101*a*c^2*d^4*e^5 + 162*a^2*c*d^2*e^7 - 81*a^3*e^9)*sqrt(e*x +
d) - (5*a^2*c^3*d^3*e^4 - 9*a^3*c^2*d*e^6 + (2*a^3*c^6*d^2 - 3*a^4*c^5*e^2)
*sqrt((25*c^2*d^4*e^6 - 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7)))*sqrt((4*c
^2*d^5 - 15*a*c*d^3*e^2 + 15*a^2*d*e^4 - a^3*c^3*sqrt((25*c^2*d^4*e^6 - 90*
a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7)))/(a^3*c^3))) + 4*(2*a*d*e + (c*d^2 +
a*e^2)*x)*sqrt(e*x + d))/(a*c^2*x^2 - a^2*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(-c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(-c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.626 $\int \frac{(d+ex)^{3/2}}{(a-cx^2)^2} dx$

Optimal. Leaf size=209

$$-\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\left(\sqrt{a}e+2\sqrt{c}d\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{4a^{3/2}c^{5/4}}+\frac{\left(2\sqrt{c}d-\sqrt{a}e\right)\sqrt{\sqrt{a}e+\sqrt{c}d}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{4a^{3/2}c^{5/4}}+\frac{\sqrt{d+ex}(ae+cdx)}{2ac\left(a-cx^2\right)}$$

```
[Out] ((a*e + c*d*x)*Sqrt[d + e*x])/(2*a*c*(a - c*x^2)) - (Sqrt[Sqrt[c]*d - Sqrt[
a]*e]*(2*Sqrt[c]*d + Sqrt[a]*e)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c
]*d - Sqrt[a]*e]])/(4*a^(3/2)*c^(5/4)) + ((2*Sqrt[c]*d - Sqrt[a]*e)*Sqrt[Sq
rt[c]*d + Sqrt[a]*e]*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[
a]*e]])/(4*a^(3/2)*c^(5/4))
```

Rubi [A] time = 0.269905, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {739, 827, 1166, 208}

$$-\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\left(\sqrt{a}e+2\sqrt{c}d\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{4a^{3/2}c^{5/4}}+\frac{\left(2\sqrt{c}d-\sqrt{a}e\right)\sqrt{\sqrt{a}e+\sqrt{c}d}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{4a^{3/2}c^{5/4}}+\frac{\sqrt{d+ex}(ae+cdx)}{2ac\left(a-cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(a - c*x^2)^2,x]
```

```
[Out] ((a*e + c*d*x)*Sqrt[d + e*x])/(2*a*c*(a - c*x^2)) - (Sqrt[Sqrt[c]*d - Sqrt[
a]*e]*(2*Sqrt[c]*d + Sqrt[a]*e)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c
]*d - Sqrt[a]*e]])/(4*a^(3/2)*c^(5/4)) + ((2*Sqrt[c]*d - Sqrt[a]*e)*Sqrt[Sq
rt[c]*d + Sqrt[a]*e]*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[
a]*e]])/(4*a^(3/2)*c^(5/4))
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\left(a-cx^2\right)^2}\,dx = \frac{(ae+cdx)\sqrt{d+ex}}{2ac\left(a-cx^2\right)} - \frac{\int \frac{\frac{1}{2}\left(-2cd^2+ae^2\right)-\frac{1}{2}cdex}{\sqrt{d+ex}\left(a-cx^2\right)}\,dx}{2ac}$$

$$= \frac{(ae+cdx)\sqrt{d+ex}}{2ac\left(a-cx^2\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}cd^2e+\frac{1}{2}e\left(-2cd^2+ae^2\right)-\frac{1}{2}cdex^2}{-cd^2+ae^2+2cdx^2-cx^4}\,dx, x, \sqrt{d+ex}\right)}{ac}$$

$$= \frac{(ae+cdx)\sqrt{d+ex}}{2ac\left(a-cx^2\right)} - \frac{\left(2cd^2-\sqrt{a}\sqrt{c}de-ae^2\right)\text{Subst}\left(\int \frac{1}{cd-\sqrt{a}\sqrt{c}e-cx^2}\,dx, x, \sqrt{d+ex}\right)}{4a^{3/2}\sqrt{c}} + \frac{\left(2cd^2+\sqrt{a}\cdots\right.}{}$$

$$= \frac{(ae+cdx)\sqrt{d+ex}}{2ac\left(a-cx^2\right)} - \frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\left(2\sqrt{c}d+\sqrt{a}e\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{4a^{3/2}c^{5/4}} + \frac{\left(2\sqrt{c}d-\sqrt{a}e\right)\sqrt{\sqrt{c}d+\sqrt{\cdots}}}{4a^{3/2}c^{5\cdots}}$$

Mathematica [A] time = 0.245728, size = 218, normalized size = 1.04

$$\frac{\left(cx^2-a\right)\sqrt{\sqrt{c}d-\sqrt{a}e}\left(\sqrt{a}e+2\sqrt{c}d\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)-\left(cx^2-a\right)\left(2\sqrt{c}d-\sqrt{a}e\right)\sqrt{\sqrt{a}e+\sqrt{c}d}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)+\cdots}{4a^{3/2}c^{5/4}\left(a-cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(a - c*x^2)^2,x]
```

```
[Out] (2*Sqrt[a]*c^(1/4)*(a*e + c*d*x)*Sqrt[d + e*x] + Sqrt[Sqrt[c]*d - Sqrt[a]*e
]*(2*Sqrt[c]*d + Sqrt[a]*e)*(-a + c*x^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sq
rt[Sqrt[c]*d - Sqrt[a]*e]] - (2*Sqrt[c]*d - Sqrt[a]*e)*Sqrt[Sqrt[c]*d + Sqr
t[a]*e]*(-a + c*x^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[
a]*e]])/(4*a^(3/2)*c^(5/4)*(a - c*x^2))
```

Maple [B] time = 0.214, size = 432, normalized size = 2.1

$$-\frac{de}{\left(2ce^2x^2-2ae^2\right)a}(ex+d)^{\frac{3}{2}} - \frac{e^3}{\left(2ce^2x^2-2ae^2\right)c}\sqrt{ex+d} + \frac{ed^2}{\left(2ce^2x^2-2ae^2\right)a}\sqrt{ex+d} - \frac{e^3}{4}\arctan\left(c\sqrt{ex+d}\frac{\cdots}{\sqrt{\left(-c\cdots\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(-c*x^2+a)^2,x)
```

```
[Out] -1/2*e/(c*e^2*x^2-a*e^2)*d/a*(e*x+d)^(3/2)-1/2*e^3/(c*e^2*x^2-a*e^2)/c*(e*x
+d)^(1/2)+1/2*e/(c*e^2*x^2-a*e^2)/a*(e*x+d)^(1/2)*d^2-1/4*e^3/(a*c*e^2)^(1/
```

```
2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)
^(1/2))*c)^(1/2))+1/2*e/a/(a*c*e^2)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*
arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))*c*d^2-1/4*e/a/((-c
*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))
*c)^(1/2))*d-1/4*e^3/(a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan
h((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))+1/2*e/a/(a*c*e^2)^(1/2)/
((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/
2))*c)^(1/2))*c*d^2+1/4*e/a/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)
^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2-a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(-c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(c*x^2 - a)^2, x)
```

Fricas [B] time = 2.05388, size = 1423, normalized size = 6.81

$$-\frac{\left(ac^2x^2-a^2c\right)\sqrt{\frac{a^3c^2\sqrt{\frac{e^6}{a^3c^5}}+4\,cd^3-3\,ade^2}{a^3c^2}}\log\left(-\left(4\,cd^2e^3-ae^5\right)\sqrt{ex+d}+\left(2\,a^3c^4d\sqrt{\frac{e^6}{a^3c^5}}+a^2ce^4\right)\sqrt{\frac{a^3c^2\sqrt{\frac{e^6}{a^3c^5}}+4\,cd^3-3\,ade^2}{a^3c^2}}\right)-\left(a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(-c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((a*c^2*x^2 - a^2*c)*sqrt((a^3*c^2*sqrt(e^6/(a^3*c^5)) + 4*c*d^3 - 3*a
*d*e^2)/(a^3*c^2))*log(-(4*c*d^2*e^3 - a*e^5)*sqrt(e*x + d) + (2*a^3*c^4*d*
sqrt(e^6/(a^3*c^5)) + a^2*c*e^4)*sqrt((a^3*c^2*sqrt(e^6/(a^3*c^5)) + 4*c*d^
3 - 3*a*d*e^2)/(a^3*c^2))) - (a*c^2*x^2 - a^2*c)*sqrt((a^3*c^2*sqrt(e^6/(a^
3*c^5)) + 4*c*d^3 - 3*a*d*e^2)/(a^3*c^2))*log(-(4*c*d^2*e^3 - a*e^5)*sqrt(e
*x + d) - (2*a^3*c^4*d*sqrt(e^6/(a^3*c^5)) + a^2*c*e^4)*sqrt((a^3*c^2*sqrt(
e^6/(a^3*c^5)) + 4*c*d^3 - 3*a*d*e^2)/(a^3*c^2))) - (a*c^2*x^2 - a^2*c)*sqr
t(-(a^3*c^2*sqrt(e^6/(a^3*c^5)) - 4*c*d^3 + 3*a*d*e^2)/(a^3*c^2))*log(-(4*c
*d^2*e^3 - a*e^5)*sqrt(e*x + d) + (2*a^3*c^4*d*sqrt(e^6/(a^3*c^5)) - a^2*c*
e^4)*sqrt(-(a^3*c^2*sqrt(e^6/(a^3*c^5)) - 4*c*d^3 + 3*a*d*e^2)/(a^3*c^2)))
+ (a*c^2*x^2 - a^2*c)*sqrt(-(a^3*c^2*sqrt(e^6/(a^3*c^5)) - 4*c*d^3 + 3*a*d*
e^2)/(a^3*c^2))*log(-(4*c*d^2*e^3 - a*e^5)*sqrt(e*x + d) - (2*a^3*c^4*d*sqr
t(e^6/(a^3*c^5)) - a^2*c*e^4)*sqrt(-(a^3*c^2*sqrt(e^6/(a^3*c^5)) - 4*c*d^3
+ 3*a*d*e^2)/(a^3*c^2))) + 4*(c*d*x + a*e)*sqrt(e*x + d))/(a*c^2*x^2 - a^2*
c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(-c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(-c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.627 $\int \frac{\sqrt{d+ex}}{\left(a-cx^2\right)^2}\,dx$

Optimal. Leaf size=194

$$-\frac{\left(\frac{2\sqrt{c}d}{\sqrt{a}}-e\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{4ac^{3/4}\sqrt{\sqrt{c}d-\sqrt{a}e}}+\frac{\left(\frac{2\sqrt{c}d}{\sqrt{a}}+e\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{4ac^{3/4}\sqrt{\sqrt{a}e+\sqrt{c}d}}+\frac{x\sqrt{d+ex}}{2a\left(a-cx^2\right)}$$

```
[Out] (x*Sqrt[d + e*x])/(2*a*(a - c*x^2)) - (((2*Sqrt[c]*d)/Sqrt[a] - e)*ArcTanh[
(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(4*a*c^(3/4)*Sqrt[Sqr
t[c]*d - Sqrt[a]*e]) + (((2*Sqrt[c]*d)/Sqrt[a] + e)*ArcTanh[(c^(1/4)*Sqrt[d
 + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(4*a*c^(3/4)*Sqrt[Sqrt[c]*d + Sqrt[a
]*e])
```

Rubi [A] time = 0.186265, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {737, 827, 1166, 208}

$$-\frac{\left(\frac{2\sqrt{c}d}{\sqrt{a}}-e\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{4ac^{3/4}\sqrt{\sqrt{c}d-\sqrt{a}e}}+\frac{\left(\frac{2\sqrt{c}d}{\sqrt{a}}+e\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{4ac^{3/4}\sqrt{\sqrt{a}e+\sqrt{c}d}}+\frac{x\sqrt{d+ex}}{2a\left(a-cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(a - c*x^2)^2,x]
```

```
[Out] (x*Sqrt[d + e*x])/(2*a*(a - c*x^2)) - (((2*Sqrt[c]*d)/Sqrt[a] - e)*ArcTanh[
(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(4*a*c^(3/4)*Sqrt[Sqr
t[c]*d - Sqrt[a]*e]) + (((2*Sqrt[c]*d)/Sqrt[a] + e)*ArcTanh[(c^(1/4)*Sqrt[d
 + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(4*a*c^(3/4)*Sqrt[Sqrt[c]*d + Sqrt[a
]*e])
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(x*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*(p + 1)), x] + Dist[1/(2*a*(p + 1
)), Int[(d + e*x)^(m - 1)*(d*(2*p + 3) + e*(m + 2*p + 3)*x)*(a + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -
1] && GtQ[m, 0] && (LtQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && In
tQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\left(a-cx^2\right)^2}\,dx = \frac{x\sqrt{d+ex}}{2a\left(a-cx^2\right)} - \frac{\int \frac{-d-\frac{ex}{2}}{\sqrt{d+ex}\left(a-cx^2\right)}\,dx}{2a}$$

$$= \frac{x\sqrt{d+ex}}{2a\left(a-cx^2\right)} - \frac{\text{Subst}\left(\int \frac{-\frac{de}{2}-\frac{ex^2}{2}}{-cd^2+ae^2+2cdx^2-cx^4}\,dx, x, \sqrt{d+ex}\right)}{a}$$

$$= \frac{x\sqrt{d+ex}}{2a\left(a-cx^2\right)} - \frac{\left(\frac{2\sqrt{cd}}{\sqrt{a}}-e\right)\text{Subst}\left(\int \frac{1}{cd-\sqrt{a}\sqrt{ce}-cx^2}\,dx, x, \sqrt{d+ex}\right)}{4a} + \frac{\left(2\sqrt{cd}+\sqrt{ae}\right)\text{Subst}\left(\int \frac{1}{cd+\sqrt{a}\sqrt{}}\right.}{4a^{3/2}}$$

$$= \frac{x\sqrt{d+ex}}{2a\left(a-cx^2\right)} - \frac{\left(\frac{2\sqrt{cd}}{\sqrt{a}}-e\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{cd}-\sqrt{ae}}}\right)}{4ac^{3/4}\sqrt{\sqrt{cd}-\sqrt{ae}}} + \frac{\left(2\sqrt{cd}+\sqrt{ae}\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{cd}+\sqrt{ae}}}\right)}{4a^{3/2}c^{3/4}\sqrt{\sqrt{cd}+\sqrt{ae}}}$$

Mathematica [A] time = 0.308631, size = 267, normalized size = 1.38

$$\frac{\left(\sqrt{cd}-\sqrt{ae}\right)\left(\left(cx^2-a\right)\sqrt{\sqrt{ae}+\sqrt{cd}}\left(\sqrt{ae}+2\sqrt{cd}\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{ae}+\sqrt{cd}}}\right)-2\sqrt{a}c^{3/4}x\sqrt{d+ex}\left(\sqrt{ae}+\sqrt{cd}\right)\right)-\left(cx^2-a\right)}{4a^{3/2}c^{3/4}\left(a-cx^2\right)\left(ae^2-cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(a - c*x^2)^2,x]
```

```
[Out] (-(Sqrt[Sqrt[c]*d - Sqrt[a]*e]*(2*c*d^2 + Sqrt[a]*Sqrt[c]*d*e - a*e^2)*(-a
+ c*x^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]]) + (S
qrt[c]*d - Sqrt[a]*e)*(-2*Sqrt[a]*c^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)*x*Sqrt[d
+ e*x] + Sqrt[Sqrt[c]*d + Sqrt[a]*e]*(2*Sqrt[c]*d + Sqrt[a]*e)*(-a + c*x^2)
*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]]))/(4*a^(3/2)*
c^(3/4)*(-(c*d^2) + a*e^2)*(a - c*x^2))
```

Maple [B] time = 0.268, size = 287, normalized size = 1.5

$$-\frac{e}{4\,ac}\sqrt{ex+d}\left(ex+\frac{1}{c}\sqrt{ace^2}\right)^{-1}+\frac{ced}{2\,a}\arctan\left(c\sqrt{ex+d}\frac{1}{\sqrt{\left(-cd+\sqrt{ace^2}\right)c}}\right)\frac{1}{\sqrt{ace^2}}\frac{1}{\sqrt{\left(-cd+\sqrt{ace^2}\right)c}}-\frac{e}{4\,a}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(-c*x^2+a)^2,x)
```

```
[Out] -1/4*e/c/a*(e*x+d)^(1/2)/(e*x+(a*c*e^2)^(1/2)/c)+1/2*e*c/a/(a*c*e^2)^(1/2)/
((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1
/2))*c)^(1/2))*d-1/4*e/a/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1
/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))-1/4*e/c/a*(e*x+d)^(1/2)/(e*x-(a*c*e
^2)^(1/2)/c)+1/2*e*c/a/(a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arct
anh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d+1/4*e/a/((c*d+(a*c*e
^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\left(cx^2-a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(-c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(c*x^2 - a)^2, x)
```

Fricas [B] time = 2.06153, size = 2573, normalized size = 13.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(-c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((a*c*x^2 - a^2)*sqrt((4*c*d^3 - 3*a*d*e^2 + (a^3*c^2*d^2 - a^4*c*e^2)
*sqrt(e^6/(a^3*c^5*d^4 - 2*a^4*c^4*d^2*e^2 + a^5*c^3*e^4)))/(a^3*c^2*d^2 -
a^4*c*e^2))*log(-(4*c*d^2*e^3 - a*e^5)*sqrt(e*x + d) + (a^2*c*d*e^4 - (2*a^
3*c^4*d^4 - 3*a^4*c^3*d^2*e^2 + a^5*c^2*e^4)*sqrt(e^6/(a^3*c^5*d^4 - 2*a^4*
c^4*d^2*e^2 + a^5*c^3*e^4)))*sqrt((4*c*d^3 - 3*a*d*e^2 + (a^3*c^2*d^2 - a^4
*c*e^2)*sqrt(e^6/(a^3*c^5*d^4 - 2*a^4*c^4*d^2*e^2 + a^5*c^3*e^4)))/(a^3*c^2
*d^2 - a^4*c*e^2))) - (a*c*x^2 - a^2)*sqrt((4*c*d^3 - 3*a*d*e^2 + (a^3*c^2*
d^2 - a^4*c*e^2)*sqrt(e^6/(a^3*c^5*d^4 - 2*a^4*c^4*d^2*e^2 + a^5*c^3*e^4)))
/(a^3*c^2*d^2 - a^4*c*e^2))*log(-(4*c*d^2*e^3 - a*e^5)*sqrt(e*x + d) - (a^2
*c*d*e^4 - (2*a^3*c^4*d^4 - 3*a^4*c^3*d^2*e^2 + a^5*c^2*e^4)*sqrt(e^6/(a^3*
c^5*d^4 - 2*a^4*c^4*d^2*e^2 + a^5*c^3*e^4)))*sqrt((4*c*d^3 - 3*a*d*e^2 + (a
^3*c^2*d^2 - a^4*c*e^2)*sqrt(e^6/(a^3*c^5*d^4 - 2*a^4*c^4*d^2*e^2 + a^5*c^3
*e^4)))/(a^3*c^2*d^2 - a^4*c*e^2))) + (a*c*x^2 - a^2)*sqrt((4*c*d^3 - 3*a*d
*e^2 - (a^3*c^2*d^2 - a^4*c*e^2)*sqrt(e^6/(a^3*c^5*d^4 - 2*a^4*c^4*d^2*e^2
+ a^5*c^3*e^4)))/(a^3*c^2*d^2 - a^4*c*e^2))*log(-(4*c*d^2*e^3 - a*e^5)*sqrt
(e*x + d) + (a^2*c*d*e^4 + (2*a^3*c^4*d^4 - 3*a^4*c^3*d^2*e^2 + a^5*c^2*e^4
)*sqrt(e^6/(a^3*c^5*d^4 - 2*a^4*c^4*d^2*e^2 + a^5*c^3*e^4)))*sqrt((4*c*d^3
- 3*a*d*e^2 - (a^3*c^2*d^2 - a^4*c*e^2)*sqrt(e^6/(a^3*c^5*d^4 - 2*a^4*c^4*d
^2*e^2 + a^5*c^3*e^4)))/(a^3*c^2*d^2 - a^4*c*e^2))) - (a*c*x^2 - a^2)*sqrt(
(4*c*d^3 - 3*a*d*e^2 - (a^3*c^2*d^2 - a^4*c*e^2)*sqrt(e^6/(a^3*c^5*d^4 - 2*
a^4*c^4*d^2*e^2 + a^5*c^3*e^4)))/(a^3*c^2*d^2 - a^4*c*e^2))*log(-(4*c*d^2*e
^3 - a*e^5)*sqrt(e*x + d) - (a^2*c*d*e^4 + (2*a^3*c^4*d^4 - 3*a^4*c^3*d^2*e
^2 + a^5*c^2*e^4)*sqrt(e^6/(a^3*c^5*d^4 - 2*a^4*c^4*d^2*e^2 + a^5*c^3*e^4))
)*sqrt((4*c*d^3 - 3*a*d*e^2 - (a^3*c^2*d^2 - a^4*c*e^2)*sqrt(e^6/(a^3*c^5*d
^4 - 2*a^4*c^4*d^2*e^2 + a^5*c^3*e^4)))/(a^3*c^2*d^2 - a^4*c*e^2))) + 4*sqr
t(e*x + d)*x)/(a*c*x^2 - a^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(-c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(-c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.628 $\int \frac{1}{\sqrt{d+ex}\left(a-cx^2\right)^2} dx$

Optimal. Leaf size=222

$$-\frac{\left(2\sqrt{c}d-3\sqrt{a}e\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{4a^{3/2}\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}}+\frac{\left(3\sqrt{a}e+2\sqrt{c}d\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{4a^{3/2}\sqrt[4]{c}\left(\sqrt{a}e+\sqrt{c}d\right)^{3/2}}-\frac{\sqrt{d+ex}(ae-cdx)}{2a\left(a-cx^2\right)\left(cd^2-ae^2\right)}$$

```
[Out] -((a*e - c*d*x)*Sqrt[d + e*x])/(2*a*(c*d^2 - a*e^2)*(a - c*x^2)) - ((2*Sqrt
[c]*d - 3*Sqrt[a]*e)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[
a]*e]])/(4*a^(3/2)*c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)^(3/2)) + ((2*Sqrt[c]*d +
 3*Sqrt[a]*e)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])
/(4*a^(3/2)*c^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)^(3/2))
```

Rubi [A] time = 0.338558, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {741, 827, 1166, 208}

$$-\frac{\left(2\sqrt{c}d-3\sqrt{a}e\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{4a^{3/2}\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}}+\frac{\left(3\sqrt{a}e+2\sqrt{c}d\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{4a^{3/2}\sqrt[4]{c}\left(\sqrt{a}e+\sqrt{c}d\right)^{3/2}}-\frac{\sqrt{d+ex}(ae-cdx)}{2a\left(a-cx^2\right)\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(a - c*x^2)^2),x]
```

```
[Out] -((a*e - c*d*x)*Sqrt[d + e*x])/(2*a*(c*d^2 - a*e^2)*(a - c*x^2)) - ((2*Sqrt
[c]*d - 3*Sqrt[a]*e)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[
a]*e]])/(4*a^(3/2)*c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)^(3/2)) + ((2*Sqrt[c]*d +
 3*Sqrt[a]*e)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])
/(4*a^(3/2)*c^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)^(3/2))
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\left(a-cx^2\right)^2}\,dx = -\frac{(ae-cdx)\sqrt{d+ex}}{2a\left(cd^2-ae^2\right)\left(a-cx^2\right)}+\frac{\int \frac{\frac{1}{2}\left(2cd^2-3ae^2\right)+\frac{1}{2}cdex}{\sqrt{d+ex}\left(a-cx^2\right)}\,dx}{2a\left(cd^2-ae^2\right)}$$

$$= -\frac{(ae-cdx)\sqrt{d+ex}}{2a\left(cd^2-ae^2\right)\left(a-cx^2\right)}+\frac{\text{Subst}\left(\int \frac{-\frac{1}{2}cd^2e+\frac{1}{2}e\left(2cd^2-3ae^2\right)+\frac{1}{2}cdex^2}{-cd^2+ae^2+2cdx^2-cx^4}\,dx,x,\sqrt{d+ex}\right)}{a\left(cd^2-ae^2\right)}$$

$$= -\frac{(ae-cdx)\sqrt{d+ex}}{2a\left(cd^2-ae^2\right)\left(a-cx^2\right)}-\frac{\left(\sqrt{c}\left(2\sqrt{c}d-3\sqrt{a}e\right)\right)\text{Subst}\left(\int \frac{1}{cd-\sqrt{a}\sqrt{c}e-cx^2}\,dx,x,\sqrt{d+ex}\right)}{4a^{3/2}\left(\sqrt{c}d-\sqrt{a}e\right)}+$$

$$= -\frac{(ae-cdx)\sqrt{d+ex}}{2a\left(cd^2-ae^2\right)\left(a-cx^2\right)}-\frac{\left(2\sqrt{c}d-3\sqrt{a}e\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{4a^{3/2}\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}}+\frac{\left(2\sqrt{c}d+3\sqrt{a}e\right)\tanh^{-1}}{4a^{3/2}\sqrt[4]{c}\left(\sqrt{c}d+\sqrt{\right.}\right.}$$

Mathematica [A] time = 0.491688, size = 248, normalized size = 1.12

$$\frac{\frac{\frac{\left(cx^2-a\right)\left(\sqrt{a}\sqrt{c}de-3ae^2+2cd^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{\sqrt{a}\sqrt[4]{c}\sqrt{\sqrt{a}e+\sqrt{c}d}}+2\sqrt{d+ex}(ae-cdx)}{a-cx^2}+\frac{\left(-\sqrt{a}\sqrt{c}de-3ae^2+2cd^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}\sqrt[4]{c}\sqrt{\sqrt{c}d-\sqrt{a}e}}}{4a\left(ae^2-cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(a - c*x^2)^2),x]
```

```
[Out] (((2*c*d^2 - Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])
/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(Sqrt[a]*c^(1/4)*Sqrt[Sqrt[c]*d - Sqrt[a]*e]
) + (2*(a*e - c*d*x)*Sqrt[d + e*x] + ((2*c*d^2 + Sqrt[a]*Sqrt[c]*d*e - 3*a*
e^2)*(-a + c*x^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*
e]])/(Sqrt[a]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[a]*e]))/(a - c*x^2))/(4*a*(-(c*
d^2) + a*e^2))
```

Maple [B] time = 0.227, size = 375, normalized size = 1.7

$$-\frac{e}{4\,a}\sqrt{ex+d}\left(cd-\sqrt{ace^2}\right)^{-1}\left(ex+\frac{1}{c}\sqrt{ace^2}\right)^{-1}-\frac{c^2ed}{2\,a}\arctan\left(c\sqrt{ex+d}\frac{1}{\sqrt{\left(-cd+\sqrt{ace^2}\right)c}}\right)\frac{1}{\sqrt{ace^2}}\left(-cd+\sqrt{ace^2}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c*x^2+a)^2/(e*x+d)^(1/2),x)
```

```
[Out] -1/4*e/a/(c*d-(a*c*e^2)^(1/2))*(e*x+d)^(1/2)/(e*x+(a*c*e^2)^(1/2)/c)-1/2*e*
c^2/a/(a*c*e^2)^(1/2)/(-c*d+(a*c*e^2)^(1/2))/((-c*d+(a*c*e^2)^(1/2))*c)^(1/
2)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d+3/4*e*c/a/(-c
*d+(a*c*e^2)^(1/2))/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c
/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))-1/4*e/a/(c*d+(a*c*e^2)^(1/2))*(e*x+d)^(1
/2)/(e*x-(a*c*e^2)^(1/2)/c)+1/2*e*c^2/a/(a*c*e^2)^(1/2)/(c*d+(a*c*e^2)^(1/2
))/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^
(1/2))*c)^(1/2))*d+3/4*e*c/a/(c*d+(a*c*e^2)^(1/2))/((c*d+(a*c*e^2)^(1/2))*c
)^(1/2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2-a\right)^2\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*x^2+a)^2/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 - a)^2*sqrt(e*x + d)), x)
```

Fricas [B] time = 3.06926, size = 6498, normalized size = 29.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*x^2+a)^2/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*((a^2*c*d^2 - a^3*e^2 - (a*c^2*d^2 - a^2*c*e^2)*x^2)*sqrt((4*c^2*d^5 -
15*a*c*d^3*e^2 + 15*a^2*d*e^4 + (a^3*c^3*d^6 - 3*a^4*c^2*d^4*e^2 + 3*a^5*c*
d^2*e^4 - a^6*e^6)*sqrt((25*c^2*d^4*e^6 - 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^
3*c^7*d^12 - 6*a^4*c^6*d^10*e^2 + 15*a^5*c^5*d^8*e^4 - 20*a^6*c^4*d^6*e^6 +
 15*a^7*c^3*d^4*e^8 - 6*a^8*c^2*d^2*e^10 + a^9*c*e^12)))/(a^3*c^3*d^6 - 3*a
^4*c^2*d^4*e^2 + 3*a^5*c*d^2*e^4 - a^6*e^6))*log((20*c^2*d^4*e^3 - 81*a*c*d
^2*e^5 + 81*a^2*e^7)*sqrt(e*x + d) + (5*a^2*c^2*d^4*e^4 - 24*a^3*c*d^2*e^6
+ 27*a^4*e^8 + 2*(a^3*c^5*d^9 - 5*a^4*c^4*d^7*e^2 + 9*a^5*c^3*d^5*e^4 - 7*a
^6*c^2*d^3*e^6 + 2*a^7*c*d*e^8)*sqrt((25*c^2*d^4*e^6 - 90*a*c*d^2*e^8 + 81*
a^2*e^10)/(a^3*c^7*d^12 - 6*a^4*c^6*d^10*e^2 + 15*a^5*c^5*d^8*e^4 - 20*a^6*
c^4*d^6*e^6 + 15*a^7*c^3*d^4*e^8 - 6*a^8*c^2*d^2*e^10 + a^9*c*e^12)))*sqrt(
(4*c^2*d^5 - 15*a*c*d^3*e^2 + 15*a^2*d*e^4 + (a^3*c^3*d^6 - 3*a^4*c^2*d^4*e
^2 + 3*a^5*c*d^2*e^4 - a^6*e^6)*sqrt((25*c^2*d^4*e^6 - 90*a*c*d^2*e^8 + 81*
a^2*e^10)/(a^3*c^7*d^12 - 6*a^4*c^6*d^10*e^2 + 15*a^5*c^5*d^8*e^4 - 20*a^6*
c^4*d^6*e^6 + 15*a^7*c^3*d^4*e^8 - 6*a^8*c^2*d^2*e^10 + a^9*c*e^12)))/(a^3*
c^3*d^6 - 3*a^4*c^2*d^4*e^2 + 3*a^5*c*d^2*e^4 - a^6*e^6))) - (a^2*c*d^2 - a
^3*e^2 - (a*c^2*d^2 - a^2*c*e^2)*x^2)*sqrt((4*c^2*d^5 - 15*a*c*d^3*e^2 + 15
*a^2*d*e^4 + (a^3*c^3*d^6 - 3*a^4*c^2*d^4*e^2 + 3*a^5*c*d^2*e^4 - a^6*e^6)*
sqrt((25*c^2*d^4*e^6 - 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7*d^12 - 6*a^4*
c^6*d^10*e^2 + 15*a^5*c^5*d^8*e^4 - 20*a^6*c^4*d^6*e^6 + 15*a^7*c^3*d^4*e^8
 - 6*a^8*c^2*d^2*e^10 + a^9*c*e^12)))/(a^3*c^3*d^6 - 3*a^4*c^2*d^4*e^2 + 3*
a^5*c*d^2*e^4 - a^6*e^6))*log((20*c^2*d^4*e^3 - 81*a*c*d^2*e^5 + 81*a^2*e^7
)*sqrt(e*x + d) - (5*a^2*c^2*d^4*e^4 - 24*a^3*c*d^2*e^6 + 27*a^4*e^8 + 2*(a
```

```
^3*c^5*d^9 - 5*a^4*c^4*d^7*e^2 + 9*a^5*c^3*d^5*e^4 - 7*a^6*c^2*d^3*e^6 + 2*
a^7*c*d*e^8)*sqrt((25*c^2*d^4*e^6 - 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7*
d^12 - 6*a^4*c^6*d^10*e^2 + 15*a^5*c^5*d^8*e^4 - 20*a^6*c^4*d^6*e^6 + 15*a^
7*c^3*d^4*e^8 - 6*a^8*c^2*d^2*e^10 + a^9*c*e^12)))*sqrt((4*c^2*d^5 - 15*a*c
*d^3*e^2 + 15*a^2*d*e^4 + (a^3*c^3*d^6 - 3*a^4*c^2*d^4*e^2 + 3*a^5*c*d^2*e^
4 - a^6*e^6)*sqrt((25*c^2*d^4*e^6 - 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7*
d^12 - 6*a^4*c^6*d^10*e^2 + 15*a^5*c^5*d^8*e^4 - 20*a^6*c^4*d^6*e^6 + 15*a^
7*c^3*d^4*e^8 - 6*a^8*c^2*d^2*e^10 + a^9*c*e^12)))/(a^3*c^3*d^6 - 3*a^4*c^2
*d^4*e^2 + 3*a^5*c*d^2*e^4 - a^6*e^6))) + (a^2*c*d^2 - a^3*e^2 - (a*c^2*d^2
 - a^2*c*e^2)*x^2)*sqrt((4*c^2*d^5 - 15*a*c*d^3*e^2 + 15*a^2*d*e^4 - (a^3*c
^3*d^6 - 3*a^4*c^2*d^4*e^2 + 3*a^5*c*d^2*e^4 - a^6*e^6)*sqrt((25*c^2*d^4*e^
6 - 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7*d^12 - 6*a^4*c^6*d^10*e^2 + 15*a
^5*c^5*d^8*e^4 - 20*a^6*c^4*d^6*e^6 + 15*a^7*c^3*d^4*e^8 - 6*a^8*c^2*d^2*e^
10 + a^9*c*e^12)))/(a^3*c^3*d^6 - 3*a^4*c^2*d^4*e^2 + 3*a^5*c*d^2*e^4 - a^6
*e^6))*log((20*c^2*d^4*e^3 - 81*a*c*d^2*e^5 + 81*a^2*e^7)*sqrt(e*x + d) + (
5*a^2*c^2*d^4*e^4 - 24*a^3*c*d^2*e^6 + 27*a^4*e^8 - 2*(a^3*c^5*d^9 - 5*a^4*
c^4*d^7*e^2 + 9*a^5*c^3*d^5*e^4 - 7*a^6*c^2*d^3*e^6 + 2*a^7*c*d*e^8)*sqrt((
25*c^2*d^4*e^6 - 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7*d^12 - 6*a^4*c^6*d^
10*e^2 + 15*a^5*c^5*d^8*e^4 - 20*a^6*c^4*d^6*e^6 + 15*a^7*c^3*d^4*e^8 - 6*a
^8*c^2*d^2*e^10 + a^9*c*e^12)))*sqrt((4*c^2*d^5 - 15*a*c*d^3*e^2 + 15*a^2*d
*e^4 - (a^3*c^3*d^6 - 3*a^4*c^2*d^4*e^2 + 3*a^5*c*d^2*e^4 - a^6*e^6)*sqrt((
25*c^2*d^4*e^6 - 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7*d^12 - 6*a^4*c^6*d^
10*e^2 + 15*a^5*c^5*d^8*e^4 - 20*a^6*c^4*d^6*e^6 + 15*a^7*c^3*d^4*e^8 - 6*a
^8*c^2*d^2*e^10 + a^9*c*e^12)))/(a^3*c^3*d^6 - 3*a^4*c^2*d^4*e^2 + 3*a^5*c*
d^2*e^4 - a^6*e^6))) - (a^2*c*d^2 - a^3*e^2 - (a*c^2*d^2 - a^2*c*e^2)*x^2)*
sqrt((4*c^2*d^5 - 15*a*c*d^3*e^2 + 15*a^2*d*e^4 - (a^3*c^3*d^6 - 3*a^4*c^2*
d^4*e^2 + 3*a^5*c*d^2*e^4 - a^6*e^6)*sqrt((25*c^2*d^4*e^6 - 90*a*c*d^2*e^8
 + 81*a^2*e^10)/(a^3*c^7*d^12 - 6*a^4*c^6*d^10*e^2 + 15*a^5*c^5*d^8*e^4 - 20
*a^6*c^4*d^6*e^6 + 15*a^7*c^3*d^4*e^8 - 6*a^8*c^2*d^2*e^10 + a^9*c*e^12)))/
(a^3*c^3*d^6 - 3*a^4*c^2*d^4*e^2 + 3*a^5*c*d^2*e^4 - a^6*e^6))*log((20*c^2*
d^4*e^3 - 81*a*c*d^2*e^5 + 81*a^2*e^7)*sqrt(e*x + d) - (5*a^2*c^2*d^4*e^4 -
 24*a^3*c*d^2*e^6 + 27*a^4*e^8 - 2*(a^3*c^5*d^9 - 5*a^4*c^4*d^7*e^2 + 9*a^5
*c^3*d^5*e^4 - 7*a^6*c^2*d^3*e^6 + 2*a^7*c*d*e^8)*sqrt((25*c^2*d^4*e^6 - 90
*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7*d^12 - 6*a^4*c^6*d^10*e^2 + 15*a^5*c^5
*d^8*e^4 - 20*a^6*c^4*d^6*e^6 + 15*a^7*c^3*d^4*e^8 - 6*a^8*c^2*d^2*e^10 + a
^9*c*e^12)))*sqrt((4*c^2*d^5 - 15*a*c*d^3*e^2 + 15*a^2*d*e^4 - (a^3*c^3*d^6
 - 3*a^4*c^2*d^4*e^2 + 3*a^5*c*d^2*e^4 - a^6*e^6)*sqrt((25*c^2*d^4*e^6 - 90
*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7*d^12 - 6*a^4*c^6*d^10*e^2 + 15*a^5*c^5
*d^8*e^4 - 20*a^6*c^4*d^6*e^6 + 15*a^7*c^3*d^4*e^8 - 6*a^8*c^2*d^2*e^10 + a
^9*c*e^12)))/(a^3*c^3*d^6 - 3*a^4*c^2*d^4*e^2 + 3*a^5*c*d^2*e^4 - a^6*e^6))
) + 4*(c*d*x - a*e)*sqrt(e*x + d))/(a^2*c*d^2 - a^3*e^2 - (a*c^2*d^2 - a^2*
c*e^2)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*x**2+a)**2/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*x^2+a)^2/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.629 $\int \frac{1}{(d+ex)^{3/2}\left(a-cx^2\right)^2}\,dx$

Optimal. Leaf size=265

$$-\frac{\sqrt[4]{c}\left(2\sqrt{c}d-5\sqrt{a}e\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{4a^{3/2}\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}}+\frac{\sqrt[4]{c}\left(5\sqrt{a}e+2\sqrt{c}d\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{4a^{3/2}\left(\sqrt{a}e+\sqrt{c}d\right)^{5/2}}-\frac{ae-cdx}{2a\left(a-cx^2\right)\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

```
[Out] -(e*(c*d^2 + 5*a*e^2))/(2*a*(c*d^2 - a*e^2)^2*Sqrt[d + e*x]) - (a*e - c*d*x
)/(2*a*(c*d^2 - a*e^2)*Sqrt[d + e*x]*(a - c*x^2)) - (c^(1/4)*(2*Sqrt[c]*d -
 5*Sqrt[a]*e)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])
/(4*a^(3/2)*(Sqrt[c]*d - Sqrt[a]*e)^(5/2)) + (c^(1/4)*(2*Sqrt[c]*d + 5*Sqrt
[a]*e)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(4*a^(
3/2)*(Sqrt[c]*d + Sqrt[a]*e)^(5/2))
```

Rubi [A] time = 0.479425, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {741, 829, 827, 1166, 208}

$$-\frac{\sqrt[4]{c}\left(2\sqrt{c}d-5\sqrt{a}e\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{4a^{3/2}\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}}+\frac{\sqrt[4]{c}\left(5\sqrt{a}e+2\sqrt{c}d\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{4a^{3/2}\left(\sqrt{a}e+\sqrt{c}d\right)^{5/2}}-\frac{ae-cdx}{2a\left(a-cx^2\right)\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*(a - c*x^2)^2),x]
```


```
[Out] -(e*(c*d^2 + 5*a*e^2))/(2*a*(c*d^2 - a*e^2)^2*Sqrt[d + e*x]) - (a*e - c*d*x
)/(2*a*(c*d^2 - a*e^2)*Sqrt[d + e*x]*(a - c*x^2)) - (c^(1/4)*(2*Sqrt[c]*d -
 5*Sqrt[a]*e)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])
/(4*a^(3/2)*(Sqrt[c]*d - Sqrt[a]*e)^(5/2)) + (c^(1/4)*(2*Sqrt[c]*d + 5*Sqrt
[a]*e)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(4*a^(
3/2)*(Sqrt[c]*d + Sqrt[a]*e)^(5/2))
```


Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 829

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)
), x] + Dist[1/(c*d^2 + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d*f + a*e*g -
 c*(e*f - d*g)*x, x])/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x
] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\left(a-cx^2\right)^2}\,dx = -\frac{ae-cdx}{2a\left(cd^2-ae^2\right)\sqrt{d+ex}\left(a-cx^2\right)}+\frac{\int \frac{\frac{1}{2}\left(2cd^2-5ae^2\right)+\frac{3}{2}cdex}{(d+ex)^{3/2}\left(a-cx^2\right)}\,dx}{2a\left(cd^2-ae^2\right)}$$

$$= -\frac{e\left(cd^2+5ae^2\right)}{2a\left(cd^2-ae^2\right)^2\sqrt{d+ex}}-\frac{ae-cdx}{2a\left(cd^2-ae^2\right)\sqrt{d+ex}\left(a-cx^2\right)}-\frac{\int \frac{-cd\left(cd^2-4ae^2\right)-\frac{1}{2}ce\left(cd^2+5ae^2\right)x}{\sqrt{d+ex}\left(a-cx^2\right)}}{2a\left(cd^2-ae^2\right)^2}$$

$$= -\frac{e\left(cd^2+5ae^2\right)}{2a\left(cd^2-ae^2\right)^2\sqrt{d+ex}}-\frac{ae-cdx}{2a\left(cd^2-ae^2\right)\sqrt{d+ex}\left(a-cx^2\right)}-\frac{\text{Subst}\left(\int \frac{-cde\left(cd^2-4ae^2\right)+\frac{1}{2}cde}{-cd^2+a}\right.}{a}$$

$$= -\frac{e\left(cd^2+5ae^2\right)}{2a\left(cd^2-ae^2\right)^2\sqrt{d+ex}}-\frac{ae-cdx}{2a\left(cd^2-ae^2\right)\sqrt{d+ex}\left(a-cx^2\right)}-\frac{\left(c\left(2\sqrt{c}d-5\sqrt{a}e\right)\right)\text{Subst}\left(\right.}{4a^{3/2}\left(\sqrt{\right.}}$$

$$= -\frac{e\left(cd^2+5ae^2\right)}{2a\left(cd^2-ae^2\right)^2\sqrt{d+ex}}-\frac{ae-cdx}{2a\left(cd^2-ae^2\right)\sqrt{d+ex}\left(a-cx^2\right)}-\frac{\sqrt[4]{c}\left(2\sqrt{c}d-5\sqrt{a}e\right)\tanh^{-1}\left(\right.}{4a^{3/2}\left(\sqrt{c}d-\sqrt{a}e\right)}$$

Mathematica [C] time = 0.451095, size = 331, normalized size = 1.25

$$\frac{\frac{3c^{3/4}d\left(\frac{\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{\sqrt{c}d-\sqrt{a}e}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2\sqrt{a}}+\frac{\left(5ae^2+cd^2\right)\left(\left(\sqrt{a}e+\sqrt{c}d\right){}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{a}e}\right)+\left(\sqrt{a}e-\sqrt{c}d\right){}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}\right)\right)}{2\sqrt{a}\sqrt{d+ex}\left(cd^2-ae^2\right)}+\frac{ae-cdx}{\left(a-cx^2\right)\sqrt{d+ex}}}{2a\left(ae^2-cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*(a - c*x^2)^2),x]
```

```
[Out] ((a*e - c*d*x)/(Sqrt[d + e*x]*(a - c*x^2)) + (3*c^(3/4)*d*(ArcTanh[(c^(1/4)
*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]]/Sqrt[Sqrt[c]*d - Sqrt[a]*e] -
ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]]/Sqrt[Sqrt[c]*d
```

```
 + Sqrt[a]*e]))/(2*Sqrt[a]) + ((c*d^2 + 5*a*e^2)*((Sqrt[c]*d + Sqrt[a]*e)*H
ypergeometric2F1[-1/2, 1, 1/2, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[a]*e)]
 + (-(Sqrt[c]*d) + Sqrt[a]*e)*Hypergeometric2F1[-1/2, 1, 1/2, (Sqrt[c]*(d +
 e*x))/(Sqrt[c]*d + Sqrt[a]*e)]))/(2*Sqrt[a]*(c*d^2 - a*e^2)*Sqrt[d + e*x])
)/(2*a*(-(c*d^2) + a*e^2))
```

Maple [B] time = 0.249, size = 783, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(-c*x^2+a)^2,x)
```

```
[Out] -2*e^3/(a*e^2-c*d^2)^2/(e*x+d)^(1/2)-1/2*e^3*c/(a*e^2-c*d^2)^2/(c*e^2*x^2-a
*e^2)*(e*x+d)^(3/2)-1/2*e*c^2/(a*e^2-c*d^2)^2/(c*e^2*x^2-a*e^2)/a*(e*x+d)^(
3/2)*d^2+3/2*e^3*c/(a*e^2-c*d^2)^2/(c*e^2*x^2-a*e^2)*d*(e*x+d)^(1/2)+1/2*e*
c^2/(a*e^2-c*d^2)^2/(c*e^2*x^2-a*e^2)*d^3/a*(e*x+d)^(1/2)-2*e^3*c^2/(a*e^2-
c*d^2)^2/(a*c*e^2)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1
/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d+1/2*e*c^3/(a*e^2-c*d^2)^2/a/(a*c*
e^2)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((-c*d+(
a*c*e^2)^(1/2))*c)^(1/2))*d^3-5/4*e^3*c/(a*e^2-c*d^2)^2/((-c*d+(a*c*e^2)^(1
/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))-1/4*
e*c^2/(a*e^2-c*d^2)^2/a/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/
2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d^2-2*e^3*c^2/(a*e^2-c*d^2)^2/(a*c*e
^2)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*
c*e^2)^(1/2))*c)^(1/2))*d+1/2*e*c^3/(a*e^2-c*d^2)^2/a/(a*c*e^2)^(1/2)/((c*d
+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c
)^(1/2))*d^3+5/4*e^3*c/(a*e^2-c*d^2)^2/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arct
anh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))+1/4*e*c^2/(a*e^2-c*d^2
)^2/a/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^
2)^(1/2))*c)^(1/2))*d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2-a\right)^2(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(-c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 - a)^2*(e*x + d)^(3/2)), x)
```

Fricas [B] time = 6.31496, size = 12070, normalized size = 45.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(-c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((a^2*c^2*d^5 - 2*a^3*c*d^3*e^2 + a^4*d*e^4 - (a*c^3*d^4*e - 2*a^2*c^2*
d^2*e^3 + a^3*c*e^5)*x^3 - (a*c^3*d^5 - 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*x^
2 + (a^2*c^2*d^4*e - 2*a^3*c*d^2*e^3 + a^4*e^5)*x)*sqrt((4*c^4*d^7 - 35*a*c
^3*d^5*e^2 + 70*a^2*c^2*d^3*e^4 + 105*a^3*c*d*e^6 + (a^3*c^5*d^10 - 5*a^4*c
^4*d^8*e^2 + 10*a^5*c^3*d^6*e^4 - 10*a^6*c^2*d^4*e^6 + 5*a^7*c*d^2*e^8 - a^
8*e^10)*sqrt((1225*c^5*d^8*e^6 - 10780*a*c^4*d^6*e^8 + 21966*a^2*c^3*d^4*e^
10 + 7700*a^3*c^2*d^2*e^12 + 625*a^4*c*e^14)/(a^3*c^10*d^20 - 10*a^4*c^9*d^
18*e^2 + 45*a^5*c^8*d^16*e^4 - 120*a^6*c^7*d^14*e^6 + 210*a^7*c^6*d^12*e^8
- 252*a^8*c^5*d^10*e^10 + 210*a^9*c^4*d^8*e^12 - 120*a^10*c^3*d^6*e^14 + 45
*a^11*c^2*d^4*e^16 - 10*a^12*c*d^2*e^18 + a^13*e^20)))/(a^3*c^5*d^10 - 5*a^
4*c^4*d^8*e^2 + 10*a^5*c^3*d^6*e^4 - 10*a^6*c^2*d^4*e^6 + 5*a^7*c*d^2*e^8 -
 a^8*e^10))*log((140*c^4*d^6*e^3 - 1491*a*c^3*d^4*e^5 + 3750*a^2*c^2*d^2*e^
7 + 625*a^3*c*e^9)*sqrt(e*x + d) + (35*a^2*c^4*d^7*e^4 - 609*a^3*c^3*d^5*e^
6 + 1977*a^4*c^2*d^3*e^8 + 325*a^5*c*d*e^10 + (2*a^3*c^7*d^14 - 19*a^4*c^6*
d^12*e^2 + 60*a^5*c^5*d^10*e^4 - 85*a^6*c^4*d^8*e^6 + 50*a^7*c^3*d^6*e^8 +
3*a^8*c^2*d^4*e^10 - 16*a^9*c*d^2*e^12 + 5*a^10*e^14)*sqrt((1225*c^5*d^8*e^
6 - 10780*a*c^4*d^6*e^8 + 21966*a^2*c^3*d^4*e^10 + 7700*a^3*c^2*d^2*e^12 +
625*a^4*c*e^14)/(a^3*c^10*d^20 - 10*a^4*c^9*d^18*e^2 + 45*a^5*c^8*d^16*e^4
- 120*a^6*c^7*d^14*e^6 + 210*a^7*c^6*d^12*e^8 - 252*a^8*c^5*d^10*e^10 + 210
*a^9*c^4*d^8*e^12 - 120*a^10*c^3*d^6*e^14 + 45*a^11*c^2*d^4*e^16 - 10*a^12*
c*d^2*e^18 + a^13*e^20)))*sqrt((4*c^4*d^7 - 35*a*c^3*d^5*e^2 + 70*a^2*c^2*d
^3*e^4 + 105*a^3*c*d*e^6 + (a^3*c^5*d^10 - 5*a^4*c^4*d^8*e^2 + 10*a^5*c^3*d
^6*e^4 - 10*a^6*c^2*d^4*e^6 + 5*a^7*c*d^2*e^8 - a^8*e^10)*sqrt((1225*c^5*d^
8*e^6 - 10780*a*c^4*d^6*e^8 + 21966*a^2*c^3*d^4*e^10 + 7700*a^3*c^2*d^2*e^1
2 + 625*a^4*c*e^14)/(a^3*c^10*d^20 - 10*a^4*c^9*d^18*e^2 + 45*a^5*c^8*d^16*
e^4 - 120*a^6*c^7*d^14*e^6 + 210*a^7*c^6*d^12*e^8 - 252*a^8*c^5*d^10*e^10 +
 210*a^9*c^4*d^8*e^12 - 120*a^10*c^3*d^6*e^14 + 45*a^11*c^2*d^4*e^16 - 10*a
^12*c*d^2*e^18 + a^13*e^20)))/(a^3*c^5*d^10 - 5*a^4*c^4*d^8*e^2 + 10*a^5*c^
3*d^6*e^4 - 10*a^6*c^2*d^4*e^6 + 5*a^7*c*d^2*e^8 - a^8*e^10))) - (a^2*c^2*d
^5 - 2*a^3*c*d^3*e^2 + a^4*d*e^4 - (a*c^3*d^4*e - 2*a^2*c^2*d^2*e^3 + a^3*c
*e^5)*x^3 - (a*c^3*d^5 - 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*x^2 + (a^2*c^2*d^
4*e - 2*a^3*c*d^2*e^3 + a^4*e^5)*x)*sqrt((4*c^4*d^7 - 35*a*c^3*d^5*e^2 + 70
*a^2*c^2*d^3*e^4 + 105*a^3*c*d*e^6 + (a^3*c^5*d^10 - 5*a^4*c^4*d^8*e^2 + 10
*a^5*c^3*d^6*e^4 - 10*a^6*c^2*d^4*e^6 + 5*a^7*c*d^2*e^8 - a^8*e^10)*sqrt((1
225*c^5*d^8*e^6 - 10780*a*c^4*d^6*e^8 + 21966*a^2*c^3*d^4*e^10 + 7700*a^3*c
^2*d^2*e^12 + 625*a^4*c*e^14)/(a^3*c^10*d^20 - 10*a^4*c^9*d^18*e^2 + 45*a^5
*c^8*d^16*e^4 - 120*a^6*c^7*d^14*e^6 + 210*a^7*c^6*d^12*e^8 - 252*a^8*c^5*d
^10*e^10 + 210*a^9*c^4*d^8*e^12 - 120*a^10*c^3*d^6*e^14 + 45*a^11*c^2*d^4*e
^16 - 10*a^12*c*d^2*e^18 + a^13*e^20)))/(a^3*c^5*d^10 - 5*a^4*c^4*d^8*e^2 +
 10*a^5*c^3*d^6*e^4 - 10*a^6*c^2*d^4*e^6 + 5*a^7*c*d^2*e^8 - a^8*e^10))*log
((140*c^4*d^6*e^3 - 1491*a*c^3*d^4*e^5 + 3750*a^2*c^2*d^2*e^7 + 625*a^3*c*e
^9)*sqrt(e*x + d) - (35*a^2*c^4*d^7*e^4 - 609*a^3*c^3*d^5*e^6 + 1977*a^4*c^
2*d^3*e^8 + 325*a^5*c*d*e^10 + (2*a^3*c^7*d^14 - 19*a^4*c^6*d^12*e^2 + 60*a
^5*c^5*d^10*e^4 - 85*a^6*c^4*d^8*e^6 + 50*a^7*c^3*d^6*e^8 + 3*a^8*c^2*d^4*e
^10 - 16*a^9*c*d^2*e^12 + 5*a^10*e^14)*sqrt((1225*c^5*d^8*e^6 - 10780*a*c^4
*d^6*e^8 + 21966*a^2*c^3*d^4*e^10 + 7700*a^3*c^2*d^2*e^12 + 625*a^4*c*e^14)
/(a^3*c^10*d^20 - 10*a^4*c^9*d^18*e^2 + 45*a^5*c^8*d^16*e^4 - 120*a^6*c^7*d
^14*e^6 + 210*a^7*c^6*d^12*e^8 - 252*a^8*c^5*d^10*e^10 + 210*a^9*c^4*d^8*e^
12 - 120*a^10*c^3*d^6*e^14 + 45*a^11*c^2*d^4*e^16 - 10*a^12*c*d^2*e^18 + a^
13*e^20)))*sqrt((4*c^4*d^7 - 35*a*c^3*d^5*e^2 + 70*a^2*c^2*d^3*e^4 + 105*a^
3*c*d*e^6 + (a^3*c^5*d^10 - 5*a^4*c^4*d^8*e^2 + 10*a^5*c^3*d^6*e^4 - 10*a^6
*c^2*d^4*e^6 + 5*a^7*c*d^2*e^8 - a^8*e^10)*sqrt((1225*c^5*d^8*e^6 - 10780*a
*c^4*d^6*e^8 + 21966*a^2*c^3*d^4*e^10 + 7700*a^3*c^2*d^2*e^12 + 625*a^4*c*e
^14)/(a^3*c^10*d^20 - 10*a^4*c^9*d^18*e^2 + 45*a^5*c^8*d^16*e^4 - 120*a^6*c
^7*d^14*e^6 + 210*a^7*c^6*d^12*e^8 - 252*a^8*c^5*d^10*e^10 + 210*a^9*c^4*d^
8*e^12 - 120*a^10*c^3*d^6*e^14 + 45*a^11*c^2*d^4*e^16 - 10*a^12*c*d^2*e^18
+ a^13*e^20)))/(a^3*c^5*d^10 - 5*a^4*c^4*d^8*e^2 + 10*a^5*c^3*d^6*e^4 - 10*
a^6*c^2*d^4*e^6 + 5*a^7*c*d^2*e^8 - a^8*e^10))) + (a^2*c^2*d^5 - 2*a^3*c*d^
3*e^2 + a^4*d*e^4 - (a*c^3*d^4*e - 2*a^2*c^2*d^2*e^3 + a^3*c*e^5)*x^3 - (a*
```

c^3*d^5 - 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*x^2 + (a^2*c^2*d^4*e - 2*a^3*c*d
^2*e^3 + a^4*e^5)*x)*sqrt((4*c^4*d^7 - 35*a*c^3*d^5*e^2 + 70*a^2*c^2*d^3*e^
4 + 105*a^3*c*d*e^6 - (a^3*c^5*d^10 - 5*a^4*c^4*d^8*e^2 + 10*a^5*c^3*d^6*e^
4 - 10*a^6*c^2*d^4*e^6 + 5*a^7*c*d^2*e^8 - a^8*e^10)*sqrt((1225*c^5*d^8*e^6
 - 10780*a*c^4*d^6*e^8 + 21966*a^2*c^3*d^4*e^10 + 7700*a^3*c^2*d^2*e^12 + 6
25*a^4*c*e^14)/(a^3*c^10*d^20 - 10*a^4*c^9*d^18*e^2 + 45*a^5*c^8*d^16*e^4 -
 120*a^6*c^7*d^14*e^6 + 210*a^7*c^6*d^12*e^8 - 252*a^8*c^5*d^10*e^10 + 210*
a^9*c^4*d^8*e^12 - 120*a^10*c^3*d^6*e^14 + 45*a^11*c^2*d^4*e^16 - 10*a^12*c
*d^2*e^18 + a^13*e^20)))/(a^3*c^5*d^10 - 5*a^4*c^4*d^8*e^2 + 10*a^5*c^3*d^6
*e^4 - 10*a^6*c^2*d^4*e^6 + 5*a^7*c*d^2*e^8 - a^8*e^10))*log((140*c^4*d^6*e
^3 - 1491*a*c^3*d^4*e^5 + 3750*a^2*c^2*d^2*e^7 + 625*a^3*c*e^9)*sqrt(e*x +
d) + (35*a^2*c^4*d^7*e^4 - 609*a^3*c^3*d^5*e^6 + 1977*a^4*c^2*d^3*e^8 + 325
*a^5*c*d*e^10 - (2*a^3*c^7*d^14 - 19*a^4*c^6*d^12*e^2 + 60*a^5*c^5*d^10*e^4
 - 85*a^6*c^4*d^8*e^6 + 50*a^7*c^3*d^6*e^8 + 3*a^8*c^2*d^4*e^10 - 16*a^9*c*
d^2*e^12 + 5*a^10*e^14)*sqrt((1225*c^5*d^8*e^6 - 10780*a*c^4*d^6*e^8 + 2196
6*a^2*c^3*d^4*e^10 + 7700*a^3*c^2*d^2*e^12 + 625*a^4*c*e^14)/(a^3*c^10*d^20
 - 10*a^4*c^9*d^18*e^2 + 45*a^5*c^8*d^16*e^4 - 120*a^6*c^7*d^14*e^6 + 210*a
^7*c^6*d^12*e^8 - 252*a^8*c^5*d^10*e^10 + 210*a^9*c^4*d^8*e^12 - 120*a^10*c
^3*d^6*e^14 + 45*a^11*c^2*d^4*e^16 - 10*a^12*c*d^2*e^18 + a^13*e^20)))*sqrt
((4*c^4*d^7 - 35*a*c^3*d^5*e^2 + 70*a^2*c^2*d^3*e^4 + 105*a^3*c*d*e^6 - (a^
3*c^5*d^10 - 5*a^4*c^4*d^8*e^2 + 10*a^5*c^3*d^6*e^4 - 10*a^6*c^2*d^4*e^6 +
5*a^7*c*d^2*e^8 - a^8*e^10)*sqrt((1225*c^5*d^8*e^6 - 10780*a*c^4*d^6*e^8 +
21966*a^2*c^3*d^4*e^10 + 7700*a^3*c^2*d^2*e^12 + 625*a^4*c*e^14)/(a^3*c^10*
d^20 - 10*a^4*c^9*d^18*e^2 + 45*a^5*c^8*d^16*e^4 - 120*a^6*c^7*d^14*e^6 + 2
10*a^7*c^6*d^12*e^8 - 252*a^8*c^5*d^10*e^10 + 210*a^9*c^4*d^8*e^12 - 120*a^
10*c^3*d^6*e^14 + 45*a^11*c^2*d^4*e^16 - 10*a^12*c*d^2*e^18 + a^13*e^20)))/
(a^3*c^5*d^10 - 5*a^4*c^4*d^8*e^2 + 10*a^5*c^3*d^6*e^4 - 10*a^6*c^2*d^4*e^6
 + 5*a^7*c*d^2*e^8 - a^8*e^10))) - (a^2*c^2*d^5 - 2*a^3*c*d^3*e^2 + a^4*d*e
^4 - (a*c^3*d^4*e - 2*a^2*c^2*d^2*e^3 + a^3*c*e^5)*x^3 - (a*c^3*d^5 - 2*a^2
*c^2*d^3*e^2 + a^3*c*d*e^4)*x^2 + (a^2*c^2*d^4*e - 2*a^3*c*d^2*e^3 + a^4*e^
5)*x)*sqrt((4*c^4*d^7 - 35*a*c^3*d^5*e^2 + 70*a^2*c^2*d^3*e^4 + 105*a^3*c*d
*e^6 - (a^3*c^5*d^10 - 5*a^4*c^4*d^8*e^2 + 10*a^5*c^3*d^6*e^4 - 10*a^6*c^2*
d^4*e^6 + 5*a^7*c*d^2*e^8 - a^8*e^10)*sqrt((1225*c^5*d^8*e^6 - 10780*a*c^4*
d^6*e^8 + 21966*a^2*c^3*d^4*e^10 + 7700*a^3*c^2*d^2*e^12 + 625*a^4*c*e^14)/
(a^3*c^10*d^20 - 10*a^4*c^9*d^18*e^2 + 45*a^5*c^8*d^16*e^4 - 120*a^6*c^7*d^
14*e^6 + 210*a^7*c^6*d^12*e^8 - 252*a^8*c^5*d^10*e^10 + 210*a^9*c^4*d^8*e^1
2 - 120*a^10*c^3*d^6*e^14 + 45*a^11*c^2*d^4*e^16 - 10*a^12*c*d^2*e^18 + a^1
3*e^20)))/(a^3*c^5*d^10 - 5*a^4*c^4*d^8*e^2 + 10*a^5*c^3*d^6*e^4 - 10*a^6*c
^2*d^4*e^6 + 5*a^7*c*d^2*e^8 - a^8*e^10))*log((140*c^4*d^6*e^3 - 1491*a*c^3
*d^4*e^5 + 3750*a^2*c^2*d^2*e^7 + 625*a^3*c*e^9)*sqrt(e*x + d) - (35*a^2*c^
4*d^7*e^4 - 609*a^3*c^3*d^5*e^6 + 1977*a^4*c^2*d^3*e^8 + 325*a^5*c*d*e^10 -
 (2*a^3*c^7*d^14 - 19*a^4*c^6*d^12*e^2 + 60*a^5*c^5*d^10*e^4 - 85*a^6*c^4*d
^8*e^6 + 50*a^7*c^3*d^6*e^8 + 3*a^8*c^2*d^4*e^10 - 16*a^9*c*d^2*e^12 + 5*a^
10*e^14)*sqrt((1225*c^5*d^8*e^6 - 10780*a*c^4*d^6*e^8 + 21966*a^2*c^3*d^4*e
^10 + 7700*a^3*c^2*d^2*e^12 + 625*a^4*c*e^14)/(a^3*c^10*d^20 - 10*a^4*c^9*d
^18*e^2 + 45*a^5*c^8*d^16*e^4 - 120*a^6*c^7*d^14*e^6 + 210*a^7*c^6*d^12*e^8
 - 252*a^8*c^5*d^10*e^10 + 210*a^9*c^4*d^8*e^12 - 120*a^10*c^3*d^6*e^14 + 4
5*a^11*c^2*d^4*e^16 - 10*a^12*c*d^2*e^18 + a^13*e^20)))*sqrt((4*c^4*d^7 - 3
5*a*c^3*d^5*e^2 + 70*a^2*c^2*d^3*e^4 + 105*a^3*c*d*e^6 - (a^3*c^5*d^10 - 5*
a^4*c^4*d^8*e^2 + 10*a^5*c^3*d^6*e^4 - 10*a^6*c^2*d^4*e^6 + 5*a^7*c*d^2*e^8
 - a^8*e^10)*sqrt((1225*c^5*d^8*e^6 - 10780*a*c^4*d^6*e^8 + 21966*a^2*c^3*d
^4*e^10 + 7700*a^3*c^2*d^2*e^12 + 625*a^4*c*e^14)/(a^3*c^10*d^20 - 10*a^4*c
^9*d^18*e^2 + 45*a^5*c^8*d^16*e^4 - 120*a^6*c^7*d^14*e^6 + 210*a^7*c^6*d^12
*e^8 - 252*a^8*c^5*d^10*e^10 + 210*a^9*c^4*d^8*e^12 - 120*a^10*c^3*d^6*e^14
 + 45*a^11*c^2*d^4*e^16 - 10*a^12*c*d^2*e^18 + a^13*e^20)))/(a^3*c^5*d^10 -
 5*a^4*c^4*d^8*e^2 + 10*a^5*c^3*d^6*e^4 - 10*a^6*c^2*d^4*e^6 + 5*a^7*c*d^2*
e^8 - a^8*e^10))) - 4*(2*a*c*d^2*e + 4*a^2*e^3 - (c^2*d^2*e + 5*a*c*e^3)*x^
2 - (c^2*d^3 - a*c*d*e^2)*x)*sqrt(e*x + d))/(a^2*c^2*d^5 - 2*a^3*c*d^3*e^2
+ a^4*d*e^4 - (a*c^3*d^4*e - 2*a^2*c^2*d^2*e^3 + a^3*c*e^5)*x^3 - (a*c^3*d^

```
5 - 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*x^2 + (a^2*c^2*d^4*e - 2*a^3*c*d^2*e^3
 + a^4*e^5)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(-c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(-c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.630 $\int \frac{1}{(d+ex)^{5/2}\left(a-cx^2\right)^2}\,dx$

Optimal. Leaf size=311

$$-\frac{c^{3/4}\left(2\sqrt{c}d-7\sqrt{a}e\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{4a^{3/2}\left(\sqrt{c}d-\sqrt{a}e\right)^{7/2}}+\frac{c^{3/4}\left(7\sqrt{a}e+2\sqrt{c}d\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{4a^{3/2}\left(\sqrt{a}e+\sqrt{c}d\right)^{7/2}}-\frac{ae-cdx}{2a\left(a-cx^2\right)(d+ex)^{3/2}\left(cd^2-ae\right.}$$

```
[Out] -(e*(3*c*d^2 + 7*a*e^2))/(6*a*(c*d^2 - a*e^2)^2*(d + e*x)^(3/2)) - (c*d*e*(
c*d^2 + 19*a*e^2))/(2*a*(c*d^2 - a*e^2)^3*Sqrt[d + e*x]) - (a*e - c*d*x)/(2
*a*(c*d^2 - a*e^2)*(d + e*x)^(3/2)*(a - c*x^2)) - (c^(3/4)*(2*Sqrt[c]*d - 7
*Sqrt[a]*e)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(
4*a^(3/2)*(Sqrt[c]*d - Sqrt[a]*e)^(7/2)) + (c^(3/4)*(2*Sqrt[c]*d + 7*Sqrt[a
]*e)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(4*a^(3/
2)*(Sqrt[c]*d + Sqrt[a]*e)^(7/2))
```

Rubi [A] time = 0.722576, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {741, 829, 827, 1166, 208}

$$-\frac{c^{3/4}\left(2\sqrt{c}d-7\sqrt{a}e\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{4a^{3/2}\left(\sqrt{c}d-\sqrt{a}e\right)^{7/2}}+\frac{c^{3/4}\left(7\sqrt{a}e+2\sqrt{c}d\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{4a^{3/2}\left(\sqrt{a}e+\sqrt{c}d\right)^{7/2}}-\frac{ae-cdx}{2a\left(a-cx^2\right)(d+ex)^{3/2}\left(cd^2-ae\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(5/2)*(a - c*x^2)^2),x]
```

```
[Out] -(e*(3*c*d^2 + 7*a*e^2))/(6*a*(c*d^2 - a*e^2)^2*(d + e*x)^(3/2)) - (c*d*e*(
c*d^2 + 19*a*e^2))/(2*a*(c*d^2 - a*e^2)^3*Sqrt[d + e*x]) - (a*e - c*d*x)/(2
*a*(c*d^2 - a*e^2)*(d + e*x)^(3/2)*(a - c*x^2)) - (c^(3/4)*(2*Sqrt[c]*d - 7
*Sqrt[a]*e)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(
4*a^(3/2)*(Sqrt[c]*d - Sqrt[a]*e)^(7/2)) + (c^(3/4)*(2*Sqrt[c]*d + 7*Sqrt[a
]*e)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(4*a^(3/
2)*(Sqrt[c]*d + Sqrt[a]*e)^(7/2))
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 829

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)
), x] + Dist[1/(c*d^2 + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d*f + a*e*g -
 c*(e*f - d*g)*x, x])/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x
] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{5/2}\left(a-cx^2\right)^2}\,dx = -\frac{ae-cdx}{2a\left(cd^2-ae^2\right)(d+ex)^{3/2}\left(a-cx^2\right)}+\frac{\int \frac{\frac{1}{2}\left(2cd^2-7ae^2\right)+\frac{5}{2}cdex}{(d+ex)^{5/2}\left(a-cx^2\right)}\,dx}{2a\left(cd^2-ae^2\right)}$$

$$= -\frac{e\left(3cd^2+7ae^2\right)}{6a\left(cd^2-ae^2\right)^2(d+ex)^{3/2}}-\frac{ae-cdx}{2a\left(cd^2-ae^2\right)(d+ex)^{3/2}\left(a-cx^2\right)}-\frac{\int \frac{-cd\left(cd^2-6ae^2\right)-\frac{1}{2}ce\left(3cd^2+\right.}{(d+ex)^{3/2}\left(a-cx^2\right)}}{2a\left(cd^2-ae^2\right)^2}$$

$$= -\frac{e\left(3cd^2+7ae^2\right)}{6a\left(cd^2-ae^2\right)^2(d+ex)^{3/2}}-\frac{cde\left(cd^2+19ae^2\right)}{2a\left(cd^2-ae^2\right)^3\sqrt{d+ex}}-\frac{ae-cdx}{2a\left(cd^2-ae^2\right)(d+ex)^{3/2}\left(a-cx^2\right)}$$

$$= -\frac{e\left(3cd^2+7ae^2\right)}{6a\left(cd^2-ae^2\right)^2(d+ex)^{3/2}}-\frac{cde\left(cd^2+19ae^2\right)}{2a\left(cd^2-ae^2\right)^3\sqrt{d+ex}}-\frac{ae-cdx}{2a\left(cd^2-ae^2\right)(d+ex)^{3/2}\left(a-cx^2\right)}$$

$$= -\frac{e\left(3cd^2+7ae^2\right)}{6a\left(cd^2-ae^2\right)^2(d+ex)^{3/2}}-\frac{cde\left(cd^2+19ae^2\right)}{2a\left(cd^2-ae^2\right)^3\sqrt{d+ex}}-\frac{ae-cdx}{2a\left(cd^2-ae^2\right)(d+ex)^{3/2}\left(a-cx^2\right)}$$

$$= -\frac{e\left(3cd^2+7ae^2\right)}{6a\left(cd^2-ae^2\right)^2(d+ex)^{3/2}}-\frac{cde\left(cd^2+19ae^2\right)}{2a\left(cd^2-ae^2\right)^3\sqrt{d+ex}}-\frac{ae-cdx}{2a\left(cd^2-ae^2\right)(d+ex)^{3/2}\left(a-cx^2\right)}$$

Mathematica [C] time = 0.302566, size = 327, normalized size = 1.05

$$\frac{\frac{\left(7ae^2+3cd^2\right)\left(\left(\sqrt{a}e+\sqrt{c}d\right){}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{a}e}\right)+\left(\sqrt{a}e-\sqrt{c}d\right){}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}\right)\right)}{\sqrt{a}\left(cd^2-ae^2\right)}+\frac{15cd(d+ex)\left(\left(\sqrt{a}e+\sqrt{c}d\right){}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{a}e}\right)+\left(\sqrt{a}e-\sqrt{c}d\right){}_2F_1\left(-\frac{1}{2},1\right.\right.}{\sqrt{a}\left(ae^2-cd^2\right)}}{12a(d+ex)^{3/2}\left(ae^2-cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(5/2)*(a - c*x^2)^2),x]
```

```
[Out] ((6*a*e - 6*c*d*x)/(a - c*x^2) + ((3*c*d^2 + 7*a*e^2)*((Sqrt[c]*d + Sqrt[a]
*e)*Hypergeometric2F1[-3/2, 1, -1/2, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[
a]*e)] + (-(Sqrt[c]*d) + Sqrt[a]*e)*Hypergeometric2F1[-3/2, 1, -1/2, (Sqrt[
c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]))/(Sqrt[a]*(c*d^2 - a*e^2)) + (15*c*
d*(d + e*x)*((Sqrt[c]*d + Sqrt[a]*e)*Hypergeometric2F1[-1/2, 1, 1/2, (Sqrt[
c]*(d + e*x))/(Sqrt[c]*d - Sqrt[a]*e)] + (-(Sqrt[c]*d) + Sqrt[a]*e)*Hyperge
ometric2F1[-1/2, 1, 1/2, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]))/(Sq
rt[a]*(-(c*d^2) + a*e^2)))/(12*a*(-(c*d^2) + a*e^2)*(d + e*x)^(3/2))
```

Maple [B] time = 0.24, size = 1021, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(5/2)/(-c*x^2+a)^2,x)
```

```
[Out] -2/3*e^3/(a*e^2-c*d^2)^2/(e*x+d)^(3/2)+8*e^3/(a*e^2-c*d^2)^3*c*d/(e*x+d)^(1
/2)+3/2*e^3/(a*e^2-c*d^2)^3*c^2/(c*e^2*x^2-a*e^2)*d*(e*x+d)^(3/2)+1/2*e/(a*
e^2-c*d^2)^3*c^3/(c*e^2*x^2-a*e^2)*d^3/a*(e*x+d)^(3/2)-1/2*e^5/(a*e^2-c*d^2
)^3*c/(c*e^2*x^2-a*e^2)*a*(e*x+d)^(1/2)-3*e^3/(a*e^2-c*d^2)^3*c^2/(c*e^2*x^
2-a*e^2)*(e*x+d)^(1/2)*d^2-1/2*e/(a*e^2-c*d^2)^3*c^3/(c*e^2*x^2-a*e^2)/a*(e
*x+d)^(1/2)*d^4+7/4*e^5/(a*e^2-c*d^2)^3*c^2*a/(a*c*e^2)^(1/2)/((-c*d+(a*c*e
^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)
)+15/4*e^3/(a*e^2-c*d^2)^3*c^3/(a*c*e^2)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(
1/2)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d^2-1/2*e/(a*
e^2-c*d^2)^3*c^4/a/(a*c*e^2)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan(
(e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d^4+19/4*e^3/(a*e^2-c*d^2
)^3*c^2/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c
*e^2)^(1/2))*c)^(1/2))*d+1/4*e/(a*e^2-c*d^2)^3*c^3/a/((-c*d+(a*c*e^2)^(1/2)
)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d^3+7/4
*e^5/(a*e^2-c*d^2)^3*c^2*a/(a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*
arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))+15/4*e^3/(a*e^2-c*
d^2)^3*c^3/(a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^
(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d^2-1/2*e/(a*e^2-c*d^2)^3*c^4/a/(a
*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c/((c*d
+(a*c*e^2)^(1/2))*c)^(1/2))*d^4-19/4*e^3/(a*e^2-c*d^2)^3*c^2/((c*d+(a*c*e^2
)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))*
d-1/4*e/(a*e^2-c*d^2)^3*c^3/a/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+
d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2 - a\right)^2 (ex + d)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(-c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 - a)^2*(e*x + d)^(5/2)), x)
```

Fricas [B] time = 16.4105, size = 18263, normalized size = 58.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(-c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(a^2*c^3*d^8 - 3*a^3*c^2*d^6*e^2 + 3*a^4*c*d^4*e^4 - a^5*d^2*e^6 -
(a*c^4*d^6*e^2 - 3*a^2*c^3*d^4*e^4 + 3*a^3*c^2*d^2*e^6 - a^4*c*e^8)*x^4 - 2
*(a*c^4*d^7*e - 3*a^2*c^3*d^5*e^3 + 3*a^3*c^2*d^3*e^5 - a^4*c*d*e^7)*x^3 -
(a*c^4*d^8 - 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 - 4*a^4*c*d^2*e^6 + a^5*
e^8)*x^2 + 2*(a^2*c^3*d^7*e - 3*a^3*c^2*d^5*e^3 + 3*a^4*c*d^3*e^5 - a^5*d*e
^7)*x)*sqrt((4*c^6*d^9 - 63*a*c^5*d^7*e^2 + 189*a^2*c^4*d^5*e^4 + 1155*a^3*
c^3*d^3*e^6 + 315*a^4*c^2*d*e^8 + (a^3*c^7*d^14 - 7*a^4*c^6*d^12*e^2 + 21*a
^5*c^5*d^10*e^4 - 35*a^6*c^4*d^8*e^6 + 35*a^7*c^3*d^6*e^8 - 21*a^8*c^2*d^4*
e^10 + 7*a^9*c*d^2*e^12 - a^10*e^14)*sqrt((11025*c^9*d^12*e^6 - 171990*a*c^
8*d^10*e^8 + 494991*a^2*c^7*d^8*e^10 + 1360716*a^3*c^6*d^6*e^12 + 780831*a^
4*c^5*d^4*e^14 + 82026*a^5*c^4*d^2*e^16 + 2401*a^6*c^3*e^18)/(a^3*c^14*d^28
 - 14*a^4*c^13*d^26*e^2 + 91*a^5*c^12*d^24*e^4 - 364*a^6*c^11*d^22*e^6 + 10
01*a^7*c^10*d^20*e^8 - 2002*a^8*c^9*d^18*e^10 + 3003*a^9*c^8*d^16*e^12 - 34
32*a^10*c^7*d^14*e^14 + 3003*a^11*c^6*d^12*e^16 - 2002*a^12*c^5*d^10*e^18 +
 1001*a^13*c^4*d^8*e^20 - 364*a^14*c^3*d^6*e^22 + 91*a^15*c^2*d^4*e^24 - 14
*a^16*c*d^2*e^26 + a^17*e^28)))/(a^3*c^7*d^14 - 7*a^4*c^6*d^12*e^2 + 21*a^5
*c^5*d^10*e^4 - 35*a^6*c^4*d^8*e^6 + 35*a^7*c^3*d^6*e^8 - 21*a^8*c^2*d^4*e^
10 + 7*a^9*c*d^2*e^12 - a^10*e^14))*log((420*c^6*d^8*e^3 - 8421*a*c^5*d^6*e
^5 + 36783*a^2*c^4*d^4*e^7 + 40817*a^3*c^3*d^2*e^9 + 2401*a^4*c^2*e^11)*sqr
t(e*x + d) + (105*a^2*c^6*d^10*e^4 - 4389*a^3*c^5*d^8*e^6 + 26274*a^4*c^4*d
^6*e^8 + 34142*a^5*c^3*d^4*e^10 + 7525*a^6*c^2*d^2*e^12 + 343*a^7*c*e^14 +
2*(a^3*c^9*d^19 - 15*a^4*c^8*d^17*e^2 + 64*a^5*c^7*d^15*e^4 - 112*a^6*c^6*d
^13*e^6 + 42*a^7*c^5*d^11*e^8 + 154*a^8*c^4*d^9*e^10 - 280*a^9*c^3*d^7*e^12
 + 216*a^10*c^2*d^5*e^14 - 83*a^11*c*d^3*e^16 + 13*a^12*d*e^18)*sqrt((11025
*c^9*d^12*e^6 - 171990*a*c^8*d^10*e^8 + 494991*a^2*c^7*d^8*e^10 + 1360716*a
^3*c^6*d^6*e^12 + 780831*a^4*c^5*d^4*e^14 + 82026*a^5*c^4*d^2*e^16 + 2401*a
^6*c^3*e^18)/(a^3*c^14*d^28 - 14*a^4*c^13*d^26*e^2 + 91*a^5*c^12*d^24*e^4 -
 364*a^6*c^11*d^22*e^6 + 1001*a^7*c^10*d^20*e^8 - 2002*a^8*c^9*d^18*e^10 +
3003*a^9*c^8*d^16*e^12 - 3432*a^10*c^7*d^14*e^14 + 3003*a^11*c^6*d^12*e^16
- 2002*a^12*c^5*d^10*e^18 + 1001*a^13*c^4*d^8*e^20 - 364*a^14*c^3*d^6*e^22
+ 91*a^15*c^2*d^4*e^24 - 14*a^16*c*d^2*e^26 + a^17*e^28)))*sqrt((4*c^6*d^9
- 63*a*c^5*d^7*e^2 + 189*a^2*c^4*d^5*e^4 + 1155*a^3*c^3*d^3*e^6 + 315*a^4*c
^2*d*e^8 + (a^3*c^7*d^14 - 7*a^4*c^6*d^12*e^2 + 21*a^5*c^5*d^10*e^4 - 35*a^
6*c^4*d^8*e^6 + 35*a^7*c^3*d^6*e^8 - 21*a^8*c^2*d^4*e^10 + 7*a^9*c*d^2*e^12
 - a^10*e^14)*sqrt((11025*c^9*d^12*e^6 - 171990*a*c^8*d^10*e^8 + 494991*a^2
*c^7*d^8*e^10 + 1360716*a^3*c^6*d^6*e^12 + 780831*a^4*c^5*d^4*e^14 + 82026*
a^5*c^4*d^2*e^16 + 2401*a^6*c^3*e^18)/(a^3*c^14*d^28 - 14*a^4*c^13*d^26*e^2
 + 91*a^5*c^12*d^24*e^4 - 364*a^6*c^11*d^22*e^6 + 1001*a^7*c^10*d^20*e^8 -
2002*a^8*c^9*d^18*e^10 + 3003*a^9*c^8*d^16*e^12 - 3432*a^10*c^7*d^14*e^14 +
 3003*a^11*c^6*d^12*e^16 - 2002*a^12*c^5*d^10*e^18 + 1001*a^13*c^4*d^8*e^20
 - 364*a^14*c^3*d^6*e^22 + 91*a^15*c^2*d^4*e^24 - 14*a^16*c*d^2*e^26 + a^17
*e^28)))/(a^3*c^7*d^14 - 7*a^4*c^6*d^12*e^2 + 21*a^5*c^5*d^10*e^4 - 35*a^6*
c^4*d^8*e^6 + 35*a^7*c^3*d^6*e^8 - 21*a^8*c^2*d^4*e^10 + 7*a^9*c*d^2*e^12 -
 a^10*e^14))) - 3*(a^2*c^3*d^8 - 3*a^3*c^2*d^6*e^2 + 3*a^4*c*d^4*e^4 - a^5*
d^2*e^6 - (a*c^4*d^6*e^2 - 3*a^2*c^3*d^4*e^4 + 3*a^3*c^2*d^2*e^6 - a^4*c*e^
8)*x^4 - 2*(a*c^4*d^7*e - 3*a^2*c^3*d^5*e^3 + 3*a^3*c^2*d^3*e^5 - a^4*c*d*e
^7)*x^3 - (a*c^4*d^8 - 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 - 4*a^4*c*d^2*
e^6 + a^5*e^8)*x^2 + 2*(a^2*c^3*d^7*e - 3*a^3*c^2*d^5*e^3 + 3*a^4*c*d^3*e^5
 - a^5*d*e^7)*x)*sqrt((4*c^6*d^9 - 63*a*c^5*d^7*e^2 + 189*a^2*c^4*d^5*e^4 +
 1155*a^3*c^3*d^3*e^6 + 315*a^4*c^2*d*e^8 + (a^3*c^7*d^14 - 7*a^4*c^6*d^12*
e^2 + 21*a^5*c^5*d^10*e^4 - 35*a^6*c^4*d^8*e^6 + 35*a^7*c^3*d^6*e^8 - 21*a^
```

8*c^2*d^4*e^10 + 7*a^9*c*d^2*e^12 - a^10*e^14)*sqrt((11025*c^9*d^12*e^6 - 1
71990*a*c^8*d^10*e^8 + 494991*a^2*c^7*d^8*e^10 + 1360716*a^3*c^6*d^6*e^12 +
780831*a^4*c^5*d^4*e^14 + 82026*a^5*c^4*d^2*e^16 + 2401*a^6*c^3*e^18)/(a^3
*c^14*d^28 - 14*a^4*c^13*d^26*e^2 + 91*a^5*c^12*d^24*e^4 - 364*a^6*c^11*d^2
2*e^6 + 1001*a^7*c^10*d^20*e^8 - 2002*a^8*c^9*d^18*e^10 + 3003*a^9*c^8*d^16
*e^12 - 3432*a^10*c^7*d^14*e^14 + 3003*a^11*c^6*d^12*e^16 - 2002*a^12*c^5*d
^10*e^18 + 1001*a^13*c^4*d^8*e^20 - 364*a^14*c^3*d^6*e^22 + 91*a^15*c^2*d^4
*e^24 - 14*a^16*c*d^2*e^26 + a^17*e^28)))/(a^3*c^7*d^14 - 7*a^4*c^6*d^12*e^
2 + 21*a^5*c^5*d^10*e^4 - 35*a^6*c^4*d^8*e^6 + 35*a^7*c^3*d^6*e^8 - 21*a^8*
c^2*d^4*e^10 + 7*a^9*c*d^2*e^12 - a^10*e^14))*log((420*c^6*d^8*e^3 - 8421*a
*c^5*d^6*e^5 + 36783*a^2*c^4*d^4*e^7 + 40817*a^3*c^3*d^2*e^9 + 2401*a^4*c^2
*e^11)*sqrt(e*x + d) - (105*a^2*c^6*d^10*e^4 - 4389*a^3*c^5*d^8*e^6 + 26274
*a^4*c^4*d^6*e^8 + 34142*a^5*c^3*d^4*e^10 + 7525*a^6*c^2*d^2*e^12 + 343*a^7
*c*e^14 + 2*(a^3*c^9*d^19 - 15*a^4*c^8*d^17*e^2 + 64*a^5*c^7*d^15*e^4 - 112
*a^6*c^6*d^13*e^6 + 42*a^7*c^5*d^11*e^8 + 154*a^8*c^4*d^9*e^10 - 280*a^9*c^
3*d^7*e^12 + 216*a^10*c^2*d^5*e^14 - 83*a^11*c*d^3*e^16 + 13*a^12*d*e^18)*s
qrt((11025*c^9*d^12*e^6 - 171990*a*c^8*d^10*e^8 + 494991*a^2*c^7*d^8*e^10 +
1360716*a^3*c^6*d^6*e^12 + 780831*a^4*c^5*d^4*e^14 + 82026*a^5*c^4*d^2*e^1
6 + 2401*a^6*c^3*e^18)/(a^3*c^14*d^28 - 14*a^4*c^13*d^26*e^2 + 91*a^5*c^12*
d^24*e^4 - 364*a^6*c^11*d^22*e^6 + 1001*a^7*c^10*d^20*e^8 - 2002*a^8*c^9*d^
18*e^10 + 3003*a^9*c^8*d^16*e^12 - 3432*a^10*c^7*d^14*e^14 + 3003*a^11*c^6*
d^12*e^16 - 2002*a^12*c^5*d^10*e^18 + 1001*a^13*c^4*d^8*e^20 - 364*a^14*c^3
*d^6*e^22 + 91*a^15*c^2*d^4*e^24 - 14*a^16*c*d^2*e^26 + a^17*e^28)))*sqrt((
4*c^6*d^9 - 63*a*c^5*d^7*e^2 + 189*a^2*c^4*d^5*e^4 + 1155*a^3*c^3*d^3*e^6 +
315*a^4*c^2*d*e^8 + (a^3*c^7*d^14 - 7*a^4*c^6*d^12*e^2 + 21*a^5*c^5*d^10*e
^4 - 35*a^6*c^4*d^8*e^6 + 35*a^7*c^3*d^6*e^8 - 21*a^8*c^2*d^4*e^10 + 7*a^9*
c*d^2*e^12 - a^10*e^14)*sqrt((11025*c^9*d^12*e^6 - 171990*a*c^8*d^10*e^8 +
494991*a^2*c^7*d^8*e^10 + 1360716*a^3*c^6*d^6*e^12 + 780831*a^4*c^5*d^4*e^1
4 + 82026*a^5*c^4*d^2*e^16 + 2401*a^6*c^3*e^18)/(a^3*c^14*d^28 - 14*a^4*c^1
3*d^26*e^2 + 91*a^5*c^12*d^24*e^4 - 364*a^6*c^11*d^22*e^6 + 1001*a^7*c^10*d
^20*e^8 - 2002*a^8*c^9*d^18*e^10 + 3003*a^9*c^8*d^16*e^12 - 3432*a^10*c^7*d
^14*e^14 + 3003*a^11*c^6*d^12*e^16 - 2002*a^12*c^5*d^10*e^18 + 1001*a^13*c^
4*d^8*e^20 - 364*a^14*c^3*d^6*e^22 + 91*a^15*c^2*d^4*e^24 - 14*a^16*c*d^2*e
^26 + a^17*e^28)))/(a^3*c^7*d^14 - 7*a^4*c^6*d^12*e^2 + 21*a^5*c^5*d^10*e^4
- 35*a^6*c^4*d^8*e^6 + 35*a^7*c^3*d^6*e^8 - 21*a^8*c^2*d^4*e^10 + 7*a^9*c*
d^2*e^12 - a^10*e^14))) + 3*(a^2*c^3*d^8 - 3*a^3*c^2*d^6*e^2 + 3*a^4*c*d^4*
e^4 - a^5*d^2*e^6 - (a*c^4*d^6*e^2 - 3*a^2*c^3*d^4*e^4 + 3*a^3*c^2*d^2*e^6
- a^4*c*e^8)*x^4 - 2*(a*c^4*d^7*e - 3*a^2*c^3*d^5*e^3 + 3*a^3*c^2*d^3*e^5 -
a^4*c*d*e^7)*x^3 - (a*c^4*d^8 - 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 - 4*
a^4*c*d^2*e^6 + a^5*e^8)*x^2 + 2*(a^2*c^3*d^7*e - 3*a^3*c^2*d^5*e^3 + 3*a^4
*c*d^3*e^5 - a^5*d*e^7)*x)*sqrt((4*c^6*d^9 - 63*a*c^5*d^7*e^2 + 189*a^2*c^4
*d^5*e^4 + 1155*a^3*c^3*d^3*e^6 + 315*a^4*c^2*d*e^8 - (a^3*c^7*d^14 - 7*a^4
*c^6*d^12*e^2 + 21*a^5*c^5*d^10*e^4 - 35*a^6*c^4*d^8*e^6 + 35*a^7*c^3*d^6*e
^8 - 21*a^8*c^2*d^4*e^10 + 7*a^9*c*d^2*e^12 - a^10*e^14)*sqrt((11025*c^9*d^
12*e^6 - 171990*a*c^8*d^10*e^8 + 494991*a^2*c^7*d^8*e^10 + 1360716*a^3*c^6*
d^6*e^12 + 780831*a^4*c^5*d^4*e^14 + 82026*a^5*c^4*d^2*e^16 + 2401*a^6*c^3*
e^18)/(a^3*c^14*d^28 - 14*a^4*c^13*d^26*e^2 + 91*a^5*c^12*d^24*e^4 - 364*a^
6*c^11*d^22*e^6 + 1001*a^7*c^10*d^20*e^8 - 2002*a^8*c^9*d^18*e^10 + 3003*a^
9*c^8*d^16*e^12 - 3432*a^10*c^7*d^14*e^14 + 3003*a^11*c^6*d^12*e^16 - 2002*
a^12*c^5*d^10*e^18 + 1001*a^13*c^4*d^8*e^20 - 364*a^14*c^3*d^6*e^22 + 91*a^
15*c^2*d^4*e^24 - 14*a^16*c*d^2*e^26 + a^17*e^28)))/(a^3*c^7*d^14 - 7*a^4*c
^6*d^12*e^2 + 21*a^5*c^5*d^10*e^4 - 35*a^6*c^4*d^8*e^6 + 35*a^7*c^3*d^6*e^8
- 21*a^8*c^2*d^4*e^10 + 7*a^9*c*d^2*e^12 - a^10*e^14))*log((420*c^6*d^8*e^
3 - 8421*a*c^5*d^6*e^5 + 36783*a^2*c^4*d^4*e^7 + 40817*a^3*c^3*d^2*e^9 + 24
01*a^4*c^2*e^11)*sqrt(e*x + d) + (105*a^2*c^6*d^10*e^4 - 4389*a^3*c^5*d^8*e
^6 + 26274*a^4*c^4*d^6*e^8 + 34142*a^5*c^3*d^4*e^10 + 7525*a^6*c^2*d^2*e^12
+ 343*a^7*c*e^14 - 2*(a^3*c^9*d^19 - 15*a^4*c^8*d^17*e^2 + 64*a^5*c^7*d^15
*e^4 - 112*a^6*c^6*d^13*e^6 + 42*a^7*c^5*d^11*e^8 + 154*a^8*c^4*d^9*e^10 -
280*a^9*c^3*d^7*e^12 + 216*a^10*c^2*d^5*e^14 - 83*a^11*c*d^3*e^16 + 13*a^12

```
*d*e^18)*sqrt((11025*c^9*d^12*e^6 - 171990*a*c^8*d^10*e^8 + 494991*a^2*c^7*
d^8*e^10 + 1360716*a^3*c^6*d^6*e^12 + 780831*a^4*c^5*d^4*e^14 + 82026*a^5*c
^4*d^2*e^16 + 2401*a^6*c^3*e^18)/(a^3*c^14*d^28 - 14*a^4*c^13*d^26*e^2 + 91
*a^5*c^12*d^24*e^4 - 364*a^6*c^11*d^22*e^6 + 1001*a^7*c^10*d^20*e^8 - 2002*
a^8*c^9*d^18*e^10 + 3003*a^9*c^8*d^16*e^12 - 3432*a^10*c^7*d^14*e^14 + 3003
*a^11*c^6*d^12*e^16 - 2002*a^12*c^5*d^10*e^18 + 1001*a^13*c^4*d^8*e^20 - 36
4*a^14*c^3*d^6*e^22 + 91*a^15*c^2*d^4*e^24 - 14*a^16*c*d^2*e^26 + a^17*e^28
)))*sqrt((4*c^6*d^9 - 63*a*c^5*d^7*e^2 + 189*a^2*c^4*d^5*e^4 + 1155*a^3*c^3
*d^3*e^6 + 315*a^4*c^2*d*e^8 - (a^3*c^7*d^14 - 7*a^4*c^6*d^12*e^2 + 21*a^5*
c^5*d^10*e^4 - 35*a^6*c^4*d^8*e^6 + 35*a^7*c^3*d^6*e^8 - 21*a^8*c^2*d^4*e^1
0 + 7*a^9*c*d^2*e^12 - a^10*e^14)*sqrt((11025*c^9*d^12*e^6 - 171990*a*c^8*d
^10*e^8 + 494991*a^2*c^7*d^8*e^10 + 1360716*a^3*c^6*d^6*e^12 + 780831*a^4*c
^5*d^4*e^14 + 82026*a^5*c^4*d^2*e^16 + 2401*a^6*c^3*e^18)/(a^3*c^14*d^28 -
14*a^4*c^13*d^26*e^2 + 91*a^5*c^12*d^24*e^4 - 364*a^6*c^11*d^22*e^6 + 1001*
a^7*c^10*d^20*e^8 - 2002*a^8*c^9*d^18*e^10 + 3003*a^9*c^8*d^16*e^12 - 3432*
a^10*c^7*d^14*e^14 + 3003*a^11*c^6*d^12*e^16 - 2002*a^12*c^5*d^10*e^18 + 10
01*a^13*c^4*d^8*e^20 - 364*a^14*c^3*d^6*e^22 + 91*a^15*c^2*d^4*e^24 - 14*a^
16*c*d^2*e^26 + a^17*e^28)))/(a^3*c^7*d^14 - 7*a^4*c^6*d^12*e^2 + 21*a^5*c^
5*d^10*e^4 - 35*a^6*c^4*d^8*e^6 + 35*a^7*c^3*d^6*e^8 - 21*a^8*c^2*d^4*e^10
 + 7*a^9*c*d^2*e^12 - a^10*e^14))) - 3*(a^2*c^3*d^8 - 3*a^3*c^2*d^6*e^2 + 3*
a^4*c*d^4*e^4 - a^5*d^2*e^6 - (a*c^4*d^6*e^2 - 3*a^2*c^3*d^4*e^4 + 3*a^3*c^
2*d^2*e^6 - a^4*c*e^8)*x^4 - 2*(a*c^4*d^7*e - 3*a^2*c^3*d^5*e^3 + 3*a^3*c^2
*d^3*e^5 - a^4*c*d*e^7)*x^3 - (a*c^4*d^8 - 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^
4*e^4 - 4*a^4*c*d^2*e^6 + a^5*e^8)*x^2 + 2*(a^2*c^3*d^7*e - 3*a^3*c^2*d^5*e
^3 + 3*a^4*c*d^3*e^5 - a^5*d*e^7)*x)*sqrt((4*c^6*d^9 - 63*a*c^5*d^7*e^2 + 1
89*a^2*c^4*d^5*e^4 + 1155*a^3*c^3*d^3*e^6 + 315*a^4*c^2*d*e^8 - (a^3*c^7*d^
14 - 7*a^4*c^6*d^12*e^2 + 21*a^5*c^5*d^10*e^4 - 35*a^6*c^4*d^8*e^6 + 35*a^7
*c^3*d^6*e^8 - 21*a^8*c^2*d^4*e^10 + 7*a^9*c*d^2*e^12 - a^10*e^14)*sqrt((11
025*c^9*d^12*e^6 - 171990*a*c^8*d^10*e^8 + 494991*a^2*c^7*d^8*e^10 + 136071
6*a^3*c^6*d^6*e^12 + 780831*a^4*c^5*d^4*e^14 + 82026*a^5*c^4*d^2*e^16 + 240
1*a^6*c^3*e^18)/(a^3*c^14*d^28 - 14*a^4*c^13*d^26*e^2 + 91*a^5*c^12*d^24*e^
4 - 364*a^6*c^11*d^22*e^6 + 1001*a^7*c^10*d^20*e^8 - 2002*a^8*c^9*d^18*e^10
 + 3003*a^9*c^8*d^16*e^12 - 3432*a^10*c^7*d^14*e^14 + 3003*a^11*c^6*d^12*e^
16 - 2002*a^12*c^5*d^10*e^18 + 1001*a^13*c^4*d^8*e^20 - 364*a^14*c^3*d^6*e^
22 + 91*a^15*c^2*d^4*e^24 - 14*a^16*c*d^2*e^26 + a^17*e^28)))/(a^3*c^7*d^14
 - 7*a^4*c^6*d^12*e^2 + 21*a^5*c^5*d^10*e^4 - 35*a^6*c^4*d^8*e^6 + 35*a^7*c
^3*d^6*e^8 - 21*a^8*c^2*d^4*e^10 + 7*a^9*c*d^2*e^12 - a^10*e^14))*log((420*
c^6*d^8*e^3 - 8421*a*c^5*d^6*e^5 + 36783*a^2*c^4*d^4*e^7 + 40817*a^3*c^3*d^
2*e^9 + 2401*a^4*c^2*e^11)*sqrt(e*x + d) - (105*a^2*c^6*d^10*e^4 - 4389*a^3
*c^5*d^8*e^6 + 26274*a^4*c^4*d^6*e^8 + 34142*a^5*c^3*d^4*e^10 + 7525*a^6*c^
2*d^2*e^12 + 343*a^7*c*e^14 - 2*(a^3*c^9*d^19 - 15*a^4*c^8*d^17*e^2 + 64*a^
5*c^7*d^15*e^4 - 112*a^6*c^6*d^13*e^6 + 42*a^7*c^5*d^11*e^8 + 154*a^8*c^4*d
^9*e^10 - 280*a^9*c^3*d^7*e^12 + 216*a^10*c^2*d^5*e^14 - 83*a^11*c*d^3*e^16
 + 13*a^12*d*e^18)*sqrt((11025*c^9*d^12*e^6 - 171990*a*c^8*d^10*e^8 + 49499
1*a^2*c^7*d^8*e^10 + 1360716*a^3*c^6*d^6*e^12 + 780831*a^4*c^5*d^4*e^14 + 8
2026*a^5*c^4*d^2*e^16 + 2401*a^6*c^3*e^18)/(a^3*c^14*d^28 - 14*a^4*c^13*d^2
6*e^2 + 91*a^5*c^12*d^24*e^4 - 364*a^6*c^11*d^22*e^6 + 1001*a^7*c^10*d^20*e
^8 - 2002*a^8*c^9*d^18*e^10 + 3003*a^9*c^8*d^16*e^12 - 3432*a^10*c^7*d^14*e
^14 + 3003*a^11*c^6*d^12*e^16 - 2002*a^12*c^5*d^10*e^18 + 1001*a^13*c^4*d^8
*e^20 - 364*a^14*c^3*d^6*e^22 + 91*a^15*c^2*d^4*e^24 - 14*a^16*c*d^2*e^26 +
 a^17*e^28)))*sqrt((4*c^6*d^9 - 63*a*c^5*d^7*e^2 + 189*a^2*c^4*d^5*e^4 + 11
55*a^3*c^3*d^3*e^6 + 315*a^4*c^2*d*e^8 - (a^3*c^7*d^14 - 7*a^4*c^6*d^12*e^2
 + 21*a^5*c^5*d^10*e^4 - 35*a^6*c^4*d^8*e^6 + 35*a^7*c^3*d^6*e^8 - 21*a^8*c
^2*d^4*e^10 + 7*a^9*c*d^2*e^12 - a^10*e^14)*sqrt((11025*c^9*d^12*e^6 - 1719
90*a*c^8*d^10*e^8 + 494991*a^2*c^7*d^8*e^10 + 1360716*a^3*c^6*d^6*e^12 + 78
0831*a^4*c^5*d^4*e^14 + 82026*a^5*c^4*d^2*e^16 + 2401*a^6*c^3*e^18)/(a^3*c^
14*d^28 - 14*a^4*c^13*d^26*e^2 + 91*a^5*c^12*d^24*e^4 - 364*a^6*c^11*d^22*e
^6 + 1001*a^7*c^10*d^20*e^8 - 2002*a^8*c^9*d^18*e^10 + 3003*a^9*c^8*d^16*e^
12 - 3432*a^10*c^7*d^14*e^14 + 3003*a^11*c^6*d^12*e^16 - 2002*a^12*c^5*d^10
```

```
*e^18 + 1001*a^13*c^4*d^8*e^20 - 364*a^14*c^3*d^6*e^22 + 91*a^15*c^2*d^4*e^
24 - 14*a^16*c*d^2*e^26 + a^17*e^28)))/(a^3*c^7*d^14 - 7*a^4*c^6*d^12*e^2 +
 21*a^5*c^5*d^10*e^4 - 35*a^6*c^4*d^8*e^6 + 35*a^7*c^3*d^6*e^8 - 21*a^8*c^2
*d^4*e^10 + 7*a^9*c*d^2*e^12 - a^10*e^14))) - 4*(9*a*c^2*d^4*e + 55*a^2*c*d
^2*e^3 - 4*a^3*e^5 - 3*(c^3*d^3*e^2 + 19*a*c^2*d*e^4)*x^3 - (6*c^3*d^4*e +
61*a*c^2*d^2*e^3 - 7*a^2*c*e^5)*x^2 - 3*(c^3*d^5 - 3*a*c^2*d^3*e^2 - 18*a^2
*c*d*e^4)*x)*sqrt(e*x + d))/(a^2*c^3*d^8 - 3*a^3*c^2*d^6*e^2 + 3*a^4*c*d^4*
e^4 - a^5*d^2*e^6 - (a*c^4*d^6*e^2 - 3*a^2*c^3*d^4*e^4 + 3*a^3*c^2*d^2*e^6
- a^4*c*e^8)*x^4 - 2*(a*c^4*d^7*e - 3*a^2*c^3*d^5*e^3 + 3*a^3*c^2*d^3*e^5 -
 a^4*c*d*e^7)*x^3 - (a*c^4*d^8 - 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 - 4*
a^4*c*d^2*e^6 + a^5*e^8)*x^2 + 2*(a^2*c^3*d^7*e - 3*a^3*c^2*d^5*e^3 + 3*a^4
*c*d^3*e^5 - a^5*d*e^7)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(5/2)/(-c*x**2+a)**2,x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(-c*x^2+a)^2,x, algorithm="giac")

[Out] Timed out
```

3.631 $\int \frac{(d+ex)^{7/2}}{\left(a+cx^2\right)^2} dx$

Optimal. Leaf size=887

$$-\frac{(ae-cdx)(d+ex)^{5/2}}{2ac\left(cx^2+a\right)}-\frac{de(d+ex)^{3/2}}{2ac}-\frac{e\left(cd^2-5ae^2\right)\sqrt{d+ex}}{2ac^2}+\frac{e\left(c^2d^4-4ace^2d^2+\sqrt{c}\sqrt{cd^2+ae^2}\left(cd^2+13ae^2\right)d-5a\right.}{4\sqrt{2}ac^{9/4}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d-}}$$

```
[Out] -(e*(c*d^2 - 5*a*e^2)*Sqrt[d + e*x])/(2*a*c^2) - (d*e*(d + e*x)^(3/2))/(2*a
*c) - ((a*e - c*d*x)*(d + e*x)^(5/2))/(2*a*c*(a + c*x^2)) + (e*(c^2*d^4 - 4
*a*c*d^2*e^2 - 5*a^2*e^4 + Sqrt[c]*d*Sqrt[c*d^2 + a*e^2]*(c*d^2 + 13*a*e^2)
)*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] - Sqrt[2]*c^(1/4)*Sqrt[d +
 e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(4*Sqrt[2]*a*c^(9/4)*Sqrt[c*
d^2 + a*e^2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (e*(c^2*d^4 - 4*a*c*d
^2*e^2 - 5*a^2*e^4 + Sqrt[c]*d*Sqrt[c*d^2 + a*e^2]*(c*d^2 + 13*a*e^2))*ArcT
anh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] + Sqrt[2]*c^(1/4)*Sqrt[d + e*x])
/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(4*Sqrt[2]*a*c^(9/4)*Sqrt[c*d^2 +
a*e^2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (e*(c^2*d^4 - 4*a*c*d^2*e^2
 - 5*a^2*e^4 - Sqrt[c]*d*Sqrt[c*d^2 + a*e^2]*(c*d^2 + 13*a*e^2))*Log[Sqrt[c
*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[
d + e*x] + Sqrt[c]*(d + e*x)])/(8*Sqrt[2]*a*c^(9/4)*Sqrt[c*d^2 + a*e^2]*Sqr
t[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]) + (e*(c^2*d^4 - 4*a*c*d^2*e^2 - 5*a^2*e
^4 - Sqrt[c]*d*Sqrt[c*d^2 + a*e^2]*(c*d^2 + 13*a*e^2))*Log[Sqrt[c*d^2 + a*e
^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] +
 Sqrt[c]*(d + e*x)])/(8*Sqrt[2]*a*c^(9/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*
d + Sqrt[c*d^2 + a*e^2]])
```

Rubi [A] time = 6.44858, antiderivative size = 887, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {739, 825, 827, 1169, 634, 618, 206, 628}

$$-\frac{(ae-cdx)(d+ex)^{5/2}}{2ac\left(cx^2+a\right)}-\frac{de(d+ex)^{3/2}}{2ac}-\frac{e\left(cd^2-5ae^2\right)\sqrt{d+ex}}{2ac^2}+\frac{e\left(c^2d^4-4ace^2d^2+\sqrt{c}\sqrt{cd^2+ae^2}\left(cd^2+13ae^2\right)d-5a\right.}{4\sqrt{2}ac^{9/4}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d-}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/(a + c*x^2)^2,x]
```


```
[Out] -(e*(c*d^2 - 5*a*e^2)*Sqrt[d + e*x])/(2*a*c^2) - (d*e*(d + e*x)^(3/2))/(2*a
*c) - ((a*e - c*d*x)*(d + e*x)^(5/2))/(2*a*c*(a + c*x^2)) + (e*(c^2*d^4 - 4
*a*c*d^2*e^2 - 5*a^2*e^4 + Sqrt[c]*d*Sqrt[c*d^2 + a*e^2]*(c*d^2 + 13*a*e^2)
)*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] - Sqrt[2]*c^(1/4)*Sqrt[d +
 e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(4*Sqrt[2]*a*c^(9/4)*Sqrt[c*
d^2 + a*e^2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (e*(c^2*d^4 - 4*a*c*d
^2*e^2 - 5*a^2*e^4 + Sqrt[c]*d*Sqrt[c*d^2 + a*e^2]*(c*d^2 + 13*a*e^2))*ArcT
anh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] + Sqrt[2]*c^(1/4)*Sqrt[d + e*x])
/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(4*Sqrt[2]*a*c^(9/4)*Sqrt[c*d^2 +
a*e^2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (e*(c^2*d^4 - 4*a*c*d^2*e^2
 - 5*a^2*e^4 - Sqrt[c]*d*Sqrt[c*d^2 + a*e^2]*(c*d^2 + 13*a*e^2))*Log[Sqrt[c
*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[
d + e*x] + Sqrt[c]*(d + e*x)])/(8*Sqrt[2]*a*c^(9/4)*Sqrt[c*d^2 + a*e^2]*Sqr
t[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]) + (e*(c^2*d^4 - 4*a*c*d^2*e^2 - 5*a^2*e
```

```
^4 - Sqrt[c]*d*Sqrt[c*d^2 + a*e^2]*(c*d^2 + 13*a*e^2))*Log[Sqrt[c*d^2 + a*e
^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] +
 Sqrt[c]*(d + e*x)])/(8*Sqrt[2]*a*c^(9/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*
d + Sqrt[c*d^2 + a*e^2]])
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 825

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[((d + e*x)^(m
- 1)*Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x])/(a + c*x^2), x], x] /; Fre
eQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m
, 0]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
```

```
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{7/2}}{\left(a+cx^2\right)^2}\,dx = -\frac{(ae-cdx)(d+ex)^{5/2}}{2ac\left(a+cx^2\right)} + \frac{\int \frac{(d+ex)^{3/2}\left(\frac{1}{2}\left(2cd^2+5ae^2\right)-\frac{3}{2}cdex\right)}{a+cx^2}\,dx}{2ac}$$

$$= -\frac{de(d+ex)^{3/2}}{2ac} - \frac{(ae-cdx)(d+ex)^{5/2}}{2ac\left(a+cx^2\right)} + \frac{\int \frac{\sqrt{d+ex}\left(cd\left(cd^2+4ae^2\right)-\frac{1}{2}ce\left(cd^2-5ae^2\right)x\right)}{a+cx^2}\,dx}{2ac^2}$$

$$= -\frac{e\left(cd^2-5ae^2\right)\sqrt{d+ex}}{2ac^2} - \frac{de(d+ex)^{3/2}}{2ac} - \frac{(ae-cdx)(d+ex)^{5/2}}{2ac\left(a+cx^2\right)} + \frac{\int \frac{\frac{1}{2}c\left(2cd^2-ae^2\right)\left(cd^2+5ae^2\right)+\frac{1}{2}c^2de\left(cd^2+13ae \ldots\right.}{\sqrt{d+ex}\left(a+cx^2\right)}}{2ac^3}$$

$$= -\frac{e\left(cd^2-5ae^2\right)\sqrt{d+ex}}{2ac^2} - \frac{de(d+ex)^{3/2}}{2ac} - \frac{(ae-cdx)(d+ex)^{5/2}}{2ac\left(a+cx^2\right)} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}ce\left(2cd^2-ae^2\right)\left(cd^2+5ae^2\right)-\frac{1}{2}c^2d \ldots}{cd^2+ae^2 \ldots}\right.}{\ldots}$$

$$= -\frac{e\left(cd^2-5ae^2\right)\sqrt{d+ex}}{2ac^2} - \frac{de(d+ex)^{3/2}}{2ac} - \frac{(ae-cdx)(d+ex)^{5/2}}{2ac\left(a+cx^2\right)} + \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(\frac{1}{2}ce\left(2cd^2-ae^2\right)\right)\ldots}{\sqrt[4]{c}}}{\ldots}\right.}{\ldots}$$

$$= -\frac{e\left(cd^2-5ae^2\right)\sqrt{d+ex}}{2ac^2} - \frac{de(d+ex)^{3/2}}{2ac} - \frac{(ae-cdx)(d+ex)^{5/2}}{2ac\left(a+cx^2\right)} - \frac{\left(e\left(c^2d^4-4acd^2e^2-5a^2e^4-\sqrt{c}d\sqrt{\ldots}\right.\right.}{\ldots}$$

$$= -\frac{e\left(cd^2-5ae^2\right)\sqrt{d+ex}}{2ac^2} - \frac{de(d+ex)^{3/2}}{2ac} - \frac{(ae-cdx)(d+ex)^{5/2}}{2ac\left(a+cx^2\right)} - \frac{e\left(c^2d^4-4acd^2e^2-5a^2e^4-\sqrt{c}d\sqrt{c\ldots}\right.}{\ldots}$$

$$= -\frac{e\left(cd^2-5ae^2\right)\sqrt{d+ex}}{2ac^2} - \frac{de(d+ex)^{3/2}}{2ac} - \frac{(ae-cdx)(d+ex)^{5/2}}{2ac\left(a+cx^2\right)} + \frac{e\left(c^2d^4-4acd^2e^2-5a^2e^4+\sqrt{c}d\sqrt{c\ldots}\right.}{4\sqrt{2}ac^{9/4}\ldots}$$

Mathematica [A] time = 0.884938, size = 311, normalized size = 0.35

$$\frac{\frac{2\sqrt[4]{c}\sqrt{d+ex}\left(5a^2e^3+ace\left(-3d^2-3dex+4e^2x^2\right)+c^2d^3x\right)}{a\left(a+cx^2\right)} + \frac{a\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(\sqrt{-a}cd^2e+8a\sqrt{c}de^2+5(-a)^{3/2}e^3+2c^{3/2}d^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{(-a)^{5/2}} + \frac{\sqrt{\sqrt{-a}e+\sqrt{c}d}\left(-\sqrt{-a}cd^2\ldots\right.}{\ldots}}{4c^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)/(a + c*x^2)^2,x]
```

```
[Out] ((2*c^(1/4)*Sqrt[d + e*x]*(5*a^2*e^3 + c^2*d^3*x + a*c*e*(-3*d^2 - 3*d*e*x
+ 4*e^2*x^2)))/(a*(a + c*x^2)) + (a*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*(2*c^(3/2)
```

```
*d^3 + Sqrt[-a]*c*d^2*e + 8*a*Sqrt[c]*d*e^2 + 5*(-a)^(3/2)*e^3)*ArcTanh[(c^
(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[-a]*e]])/(-a)^(5/2) + (Sqrt[Sqrt
[c]*d + Sqrt[-a]*e]*(2*c^(3/2)*d^3 - Sqrt[-a]*c*d^2*e + 8*a*Sqrt[c]*d*e^2 +
 5*Sqrt[-a]*a*e^3)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[-a
]*e]])/(-a)^(3/2))/(4*c^(9/4))
```

Maple [B] time = 0.247, size = 5653, normalized size = 6.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/(c*x^2+a)^2,x)

[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{7}{2}}}{\left(cx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+a)^2,x, algorithm="maxima")

[Out] integrate((e*x + d)^(7/2)/(c*x^2 + a)^2, x)
```

Fricas [B] time = 4.16918, size = 4660, normalized size = 5.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+a)^2,x, algorithm="fricas")

[Out] -1/8*((a*c^3*x^2 + a^2*c^2)*sqrt(-(4*c^3*d^7 + 35*a*c^2*d^5*e^2 + 70*a^2*c*
d^3*e^4 - 105*a^3*d*e^6 + a^3*c^4*sqrt(-(1225*c^4*d^8*e^6 + 10780*a*c^3*d^6
*e^8 + 21966*a^2*c^2*d^4*e^10 - 7700*a^3*c*d^2*e^12 + 625*a^4*e^14)/(a^3*c^
9)))/(a^3*c^4))*log(-(140*c^5*d^10*e^3 + 1771*a*c^4*d^8*e^5 + 6872*a^2*c^3*
d^6*e^7 + 8366*a^3*c^2*d^4*e^9 + 2500*a^4*c*d^2*e^11 - 625*a^5*e^13)*sqrt(e
*x + d) + (35*a^2*c^5*d^6*e^4 - 21*a^3*c^4*d^4*e^6 - 795*a^4*c^3*d^2*e^8 +
125*a^5*c^2*e^10 - 2*(a^3*c^8*d^3 + 4*a^4*c^7*d*e^2)*sqrt(-(1225*c^4*d^8*e^
6 + 10780*a*c^3*d^6*e^8 + 21966*a^2*c^2*d^4*e^10 - 7700*a^3*c*d^2*e^12 + 62
5*a^4*e^14)/(a^3*c^9)))*sqrt(-(4*c^3*d^7 + 35*a*c^2*d^5*e^2 + 70*a^2*c*d^3*
e^4 - 105*a^3*d*e^6 + a^3*c^4*sqrt(-(1225*c^4*d^8*e^6 + 10780*a*c^3*d^6*e^8
 + 21966*a^2*c^2*d^4*e^10 - 7700*a^3*c*d^2*e^12 + 625*a^4*e^14)/(a^3*c^9)))
/(a^3*c^4))) - (a*c^3*x^2 + a^2*c^2)*sqrt(-(4*c^3*d^7 + 35*a*c^2*d^5*e^2 +
70*a^2*c*d^3*e^4 - 105*a^3*d*e^6 + a^3*c^4*sqrt(-(1225*c^4*d^8*e^6 + 10780*
a*c^3*d^6*e^8 + 21966*a^2*c^2*d^4*e^10 - 7700*a^3*c*d^2*e^12 + 625*a^4*e^14
)/(a^3*c^9)))/(a^3*c^4))*log(-(140*c^5*d^10*e^3 + 1771*a*c^4*d^8*e^5 + 6872
*a^2*c^3*d^6*e^7 + 8366*a^3*c^2*d^4*e^9 + 2500*a^4*c*d^2*e^11 - 625*a^5*e^1
```

```
3)*sqrt(e*x + d) - (35*a^2*c^5*d^6*e^4 - 21*a^3*c^4*d^4*e^6 - 795*a^4*c^3*d
^2*e^8 + 125*a^5*c^2*e^10 - 2*(a^3*c^8*d^3 + 4*a^4*c^7*d*e^2)*sqrt(-(1225*c
^4*d^8*e^6 + 10780*a*c^3*d^6*e^8 + 21966*a^2*c^2*d^4*e^10 - 7700*a^3*c*d^2*
e^12 + 625*a^4*e^14)/(a^3*c^9)))*sqrt(-(4*c^3*d^7 + 35*a*c^2*d^5*e^2 + 70*a
^2*c*d^3*e^4 - 105*a^3*d*e^6 + a^3*c^4*sqrt(-(1225*c^4*d^8*e^6 + 10780*a*c^
3*d^6*e^8 + 21966*a^2*c^2*d^4*e^10 - 7700*a^3*c*d^2*e^12 + 625*a^4*e^14)/(a
^3*c^9)))/(a^3*c^4))) + (a*c^3*x^2 + a^2*c^2)*sqrt(-(4*c^3*d^7 + 35*a*c^2*d
^5*e^2 + 70*a^2*c*d^3*e^4 - 105*a^3*d*e^6 - a^3*c^4*sqrt(-(1225*c^4*d^8*e^6
 + 10780*a*c^3*d^6*e^8 + 21966*a^2*c^2*d^4*e^10 - 7700*a^3*c*d^2*e^12 + 625
*a^4*e^14)/(a^3*c^9)))/(a^3*c^4))*log(-(140*c^5*d^10*e^3 + 1771*a*c^4*d^8*e
^5 + 6872*a^2*c^3*d^6*e^7 + 8366*a^3*c^2*d^4*e^9 + 2500*a^4*c*d^2*e^11 - 62
5*a^5*e^13)*sqrt(e*x + d) + (35*a^2*c^5*d^6*e^4 - 21*a^3*c^4*d^4*e^6 - 795*
a^4*c^3*d^2*e^8 + 125*a^5*c^2*e^10 + 2*(a^3*c^8*d^3 + 4*a^4*c^7*d*e^2)*sqrt
(-(1225*c^4*d^8*e^6 + 10780*a*c^3*d^6*e^8 + 21966*a^2*c^2*d^4*e^10 - 7700*a
^3*c*d^2*e^12 + 625*a^4*e^14)/(a^3*c^9)))*sqrt(-(4*c^3*d^7 + 35*a*c^2*d^5*e
^2 + 70*a^2*c*d^3*e^4 - 105*a^3*d*e^6 - a^3*c^4*sqrt(-(1225*c^4*d^8*e^6 + 1
0780*a*c^3*d^6*e^8 + 21966*a^2*c^2*d^4*e^10 - 7700*a^3*c*d^2*e^12 + 625*a^4
*e^14)/(a^3*c^9)))/(a^3*c^4))) - (a*c^3*x^2 + a^2*c^2)*sqrt(-(4*c^3*d^7 + 3
5*a*c^2*d^5*e^2 + 70*a^2*c*d^3*e^4 - 105*a^3*d*e^6 - a^3*c^4*sqrt(-(1225*c^
4*d^8*e^6 + 10780*a*c^3*d^6*e^8 + 21966*a^2*c^2*d^4*e^10 - 7700*a^3*c*d^2*e
^12 + 625*a^4*e^14)/(a^3*c^9)))/(a^3*c^4))*log(-(140*c^5*d^10*e^3 + 1771*a*
c^4*d^8*e^5 + 6872*a^2*c^3*d^6*e^7 + 8366*a^3*c^2*d^4*e^9 + 2500*a^4*c*d^2*
e^11 - 625*a^5*e^13)*sqrt(e*x + d) - (35*a^2*c^5*d^6*e^4 - 21*a^3*c^4*d^4*e
^6 - 795*a^4*c^3*d^2*e^8 + 125*a^5*c^2*e^10 + 2*(a^3*c^8*d^3 + 4*a^4*c^7*d*
e^2)*sqrt(-(1225*c^4*d^8*e^6 + 10780*a*c^3*d^6*e^8 + 21966*a^2*c^2*d^4*e^10
 - 7700*a^3*c*d^2*e^12 + 625*a^4*e^14)/(a^3*c^9)))*sqrt(-(4*c^3*d^7 + 35*a*
c^2*d^5*e^2 + 70*a^2*c*d^3*e^4 - 105*a^3*d*e^6 - a^3*c^4*sqrt(-(1225*c^4*d^
8*e^6 + 10780*a*c^3*d^6*e^8 + 21966*a^2*c^2*d^4*e^10 - 7700*a^3*c*d^2*e^12
+ 625*a^4*e^14)/(a^3*c^9)))/(a^3*c^4))) - 4*(4*a*c*e^3*x^2 - 3*a*c*d^2*e +
5*a^2*e^3 + (c^2*d^3 - 3*a*c*d*e^2)*x)*sqrt(e*x + d))/(a*c^3*x^2 + a^2*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.632 $\int \frac{(d+ex)^{5/2}}{\left(a+cx^2\right)^2}\,dx$

Optimal. Leaf size=811

$$-\frac{(ae-cdx)(d+ex)^{3/2}}{2ac\left(cx^2+a\right)}-\frac{de\sqrt{d+ex}}{2ac}+\frac{e\left(c^{3/2}d^3+a\sqrt{c}e^2d+\sqrt{cd^2+ae^2}\left(cd^2+3ae^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+e}}{\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}\right.}{4\sqrt{2}ac^{7/4}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}$$

```
[Out] -(d*e*Sqrt[d + e*x])/(2*a*c) - ((a*e - c*d*x)*(d + e*x)^(3/2))/(2*a*c*(a +
c*x^2)) + (e*(c^(3/2)*d^3 + a*Sqrt[c]*d*e^2 + Sqrt[c*d^2 + a*e^2]*(c*d^2 +
3*a*e^2))*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] - Sqrt[2]*c^(1/4)*
Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(4*Sqrt[2]*a*c^(7/4)
*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (e*(c^(3/2)*d
^3 + a*Sqrt[c]*d*e^2 + Sqrt[c*d^2 + a*e^2]*(c*d^2 + 3*a*e^2))*ArcTanh[(Sqrt
[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] + Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqr
t[c]*d - Sqrt[c*d^2 + a*e^2]]])/(4*Sqrt[2]*a*c^(7/4)*Sqrt[c*d^2 + a*e^2]*Sq
rt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (e*(c^(3/2)*d^3 + a*Sqrt[c]*d*e^2 -
Sqrt[c*d^2 + a*e^2]*(c*d^2 + 3*a*e^2))*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^
(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*
x)])/(8*Sqrt[2]*a*c^(7/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 +
 a*e^2]]) + (e*(c^(3/2)*d^3 + a*Sqrt[c]*d*e^2 - Sqrt[c*d^2 + a*e^2]*(c*d^2
+ 3*a*e^2))*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt
[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(8*Sqrt[2]*a*c^(7/4)*S
qrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])
```

Rubi [A] time = 2.83939, antiderivative size = 811, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {739, 825, 827, 1169, 634, 618, 206, 628}

$$-\frac{(ae-cdx)(d+ex)^{3/2}}{2ac\left(cx^2+a\right)}-\frac{de\sqrt{d+ex}}{2ac}+\frac{e\left(c^{3/2}d^3+a\sqrt{c}e^2d+\sqrt{cd^2+ae^2}\left(cd^2+3ae^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+e}}{\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}\right.}{4\sqrt{2}ac^{7/4}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(a + c*x^2)^2,x]
```

```
[Out] -(d*e*Sqrt[d + e*x])/(2*a*c) - ((a*e - c*d*x)*(d + e*x)^(3/2))/(2*a*c*(a +
c*x^2)) + (e*(c^(3/2)*d^3 + a*Sqrt[c]*d*e^2 + Sqrt[c*d^2 + a*e^2]*(c*d^2 +
3*a*e^2))*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] - Sqrt[2]*c^(1/4)*
Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(4*Sqrt[2]*a*c^(7/4)
*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (e*(c^(3/2)*d
^3 + a*Sqrt[c]*d*e^2 + Sqrt[c*d^2 + a*e^2]*(c*d^2 + 3*a*e^2))*ArcTanh[(Sqrt
[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] + Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqr
t[c]*d - Sqrt[c*d^2 + a*e^2]]])/(4*Sqrt[2]*a*c^(7/4)*Sqrt[c*d^2 + a*e^2]*Sq
rt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (e*(c^(3/2)*d^3 + a*Sqrt[c]*d*e^2 -
Sqrt[c*d^2 + a*e^2]*(c*d^2 + 3*a*e^2))*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^
(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*
x)])/(8*Sqrt[2]*a*c^(7/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 +
 a*e^2]]) + (e*(c^(3/2)*d^3 + a*Sqrt[c]*d*e^2 - Sqrt[c*d^2 + a*e^2]*(c*d^2
+ 3*a*e^2))*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt
[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(8*Sqrt[2]*a*c^(7/4)*S
```

```
qrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 825

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[((d + e*x)^(m
- 1)*Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x])/(a + c*x^2), x], x] /; Fre
eQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m
, 0]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\left(a+cx^2\right)^2}\,dx = -\frac{(ae-cdx)(d+ex)^{3/2}}{2ac\left(a+cx^2\right)} + \frac{\int \frac{\sqrt{d+ex}\left(\frac{1}{2}\left(2cd^2+3ae^2\right)-\frac{1}{2}cdex\right)}{a+cx^2}\,dx}{2ac}$$

$$= -\frac{de\sqrt{d+ex}}{2ac} - \frac{(ae-cdx)(d+ex)^{3/2}}{2ac\left(a+cx^2\right)} + \frac{\int \frac{cd\left(cd^2+2ae^2\right)+\frac{1}{2}ce\left(cd^2+3ae^2\right)x}{\sqrt{d+ex}\left(a+cx^2\right)}\,dx}{2ac^2}$$

$$= -\frac{de\sqrt{d+ex}}{2ac} - \frac{(ae-cdx)(d+ex)^{3/2}}{2ac\left(a+cx^2\right)} + \frac{\text{Subst}\left(\int \frac{cde\left(cd^2+2ae^2\right)-\frac{1}{2}cde\left(cd^2+3ae^2\right)+\frac{1}{2}ce\left(cd^2+3ae^2\right)x^2}{cd^2+ae^2-2cdx^2+cx^4}\,dx, x, \sqrt{d\ \cdots}\right.}{ac^2}$$

$$= -\frac{de\sqrt{d+ex}}{2ac} - \frac{(ae-cdx)(d+ex)^{3/2}}{2ac\left(a+cx^2\right)} + \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cde\left(cd^2+2ae^2\right)-\frac{1}{2}cde\left(cd^2+3ae^2\right)\right)}{\sqrt[4]{c}}-\left(cde\left(cd^2+2ae^2\right)-\cdots\right.}{\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+\cdots}}}{\sqrt[4]{c}}\cdots}\right.}{2\sqrt{2}ac^{9/4}\sqrt{cd^2+ae^2}\sqrt{\sqrt{\cdots}}}$$

$$= -\frac{de\sqrt{d+ex}}{2ac} - \frac{(ae-cdx)(d+ex)^{3/2}}{2ac\left(a+cx^2\right)} - \frac{\left(e\left(c^{3/2}d^3+a\sqrt{c}de^2-\sqrt{cd^2+ae^2}\left(cd^2+3ae^2\right)\right)\right)\text{Subst}\left(\int \cdots\right.}{8\sqrt{2}ac^{7/4}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\cdots}}$$

$$= -\frac{de\sqrt{d+ex}}{2ac} - \frac{(ae-cdx)(d+ex)^{3/2}}{2ac\left(a+cx^2\right)} - \frac{e\left(c^{3/2}d^3+a\sqrt{c}de^2-\sqrt{cd^2+ae^2}\left(cd^2+3ae^2\right)\right)\log\left(\sqrt{cd^2+\cdots}\right.}{8\sqrt{2}ac^{7/4}\sqrt{cd^2+ae^2}\sqrt{\cdots}}$$

$$= -\frac{de\sqrt{d+ex}}{2ac} - \frac{(ae-cdx)(d+ex)^{3/2}}{2ac\left(a+cx^2\right)} + \frac{e\left(c^{3/2}d^3+a\sqrt{c}de^2+\sqrt{cd^2+ae^2}\left(cd^2+3ae^2\right)\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\left(\cdots\right.}{\cdots}\right.}{4\sqrt{2}ac^{7/4}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d-\sqrt{cd^2+a\cdots}}}$$

Mathematica [A] time = 0.611967, size = 248, normalized size = 0.31

$$\frac{\frac{2c^{3/4}\sqrt{d+ex}\left(cd^2x-ae(2d+ex)\right)}{a\left(a+cx^2\right)} + \frac{a\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(\sqrt{-a}\sqrt{c}de+3ae^2+2cd^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{(-a)^{5/2}} + \frac{\sqrt{\sqrt{-a}e+\sqrt{c}d}\left(-\sqrt{-a}\sqrt{c}de+3ae^2+2cd^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{-a}e+\sqrt{c}d\cdots}}\right.}{(-a)^{3/2}}}{4c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(a + c*x^2)^2,x]
```

```
[Out] ((2*c^(3/4)*Sqrt[d + e*x]*(c*d^2*x - a*e*(2*d + e*x)))/(a*(a + c*x^2)) + (a
*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*(2*c*d^2 + Sqrt[-a]*Sqrt[c]*d*e + 3*a*e^2)*Ar
cTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[-a]*e]])/(-a)^(5/2) + (
Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*(2*c*d^2 - Sqrt[-a]*Sqrt[c]*d*e + 3*a*e^2)*Arc
Tanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[-a]*e]])/(-a)^(3/2))/(4*
c^(7/4))
```

Maple [B] time = 0.238, size = 4065, normalized size = 5.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(c*x^2+a)^2,x)
```

```
[Out] 3/8*e/a/c^(5/2)/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*
c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d
)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1
/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c
*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)-3/8*e/a/c^(3/2)/(4*(a*e^2+c*d^2)^(1/2)*c^
(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2
)-(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2
*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1
/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*d+3/8*e/a/c^(5/2)/(4*(a*e^2+c*d
^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*
(e*x+d)^(1/2)-(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/
2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/
2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(
1/2)-3/8*e/a/c^(3/2)/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/
2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*d^2))^(1/2)+
2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*
d)^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2
)+2*c*d)^(1/2)*d+1/16/e/a^2/c^2*ln((e*x+d)*c^(1/2)+(e*x+d)^(1/2)*(2*(c*(a*e
^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(a*e^2+c*d^2)^(1/2)*(2*(
a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)*d-1/16/e/a^2/c^
2*ln((e*x+d)*c^(1/2)-(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+
(a*e^2+c*d^2)^(1/2))*(a*e^2+c*d^2)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^
(1/2)*(a*c*e^2+c^2*d^2)^(1/2)*d-1/8/e/a^2/c^(1/2)/(4*(a*e^2+c*d^2)^(1/2)*c^
(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2
)-(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2
*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1
/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*d^3-1/8/e/a^2/c^(1/2)/(4*(a*e^2
+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1
/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)
^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2*(c*(a*e^2+c*d^2))
^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*d^3-1/2*e^3/(c*
e^2*x^2+a*e^2)/c*(e*x+d)^(3/2)+1/2*e/a/c/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(
c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(
a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^
2+c*d^2))^(1/2)-2*c*d)^(1/2))*(a*e^2+c*d^2)^(1/2)*d+1/16/e/a^2/c*ln((e*x+d)
*c^(1/2)-(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2
)^(1/2))*(a*e^2+c*d^2)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*d^2+1/
2*e/a/c/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/
2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)-(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))
/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(a*
e^2+c*d^2)^(1/2)*d-1/16/e/a^2/c*ln((e*x+d)*c^(1/2)+(e*x+d)^(1/2)*(2*(c*(a*e
^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(a*e^2+c*d^2)^(1/2)*(2*(
a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*d^2-1/16/e/a^2/c^(3/2)*ln((e*x+d)*c^(1/
2)+(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2
))*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)*d^2+1/16
/e/a^2/c^(3/2)*ln((e*x+d)*c^(1/2)-(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+
2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(
a*c*e^2+c^2*d^2)^(1/2)*d^2-1/8/e/a^2/c^2/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(
c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(
a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^
2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(a*e^
```

```
2+c*d^2)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1
/2)*d-1/8/e/a^2/c^2/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2
)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)-(2*(c*(a*e^2+c*d^2))^(1/2)+2
*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d
)^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(a*e^2+c*d^2)^(1/2)*(2*(a*
c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)*d+1/8/e/a^2/c/(4*
(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((
2*c^(1/2)*(e*x+d)^(1/2)-(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+
c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2*(c*(a*e^2+c
*d^2))^(1/2)+2*c*d)^(1/2)*(a*e^2+c*d^2)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*
c*d)^(1/2)*d^2+1/8/e/a^2/c^(3/2)/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2
+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)-(2*(c*(a*e^2+c*
d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2)
)^(1/2)-2*c*d)^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c
^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)*d^2+1/8/e/a^2/c^(3/2)/(4
*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan(
(2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2
+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2*(c*(a*e^2+
c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2
+c^2*d^2)^(1/2)*d^2+1/8/e/a^2/c/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+
c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*d
^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))
^(1/2)-2*c*d)^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(a*e^2+c*d^2)^
(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*d^2-1/2*e^3/(c*e^2*x^2+a*e^2)
*d/c*(e*x+d)^(1/2)+1/2*e/(c*e^2*x^2+a*e^2)/a*(e*x+d)^(3/2)*d^2-1/2*e/(c*e^2
*x^2+a*e^2)*d^3/a*(e*x+d)^(1/2)-1/16/e/a^2/c^(1/2)*ln((e*x+d)*c^(1/2)-(e*x+
d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a
*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*d^3+3/16*e/a/c^(5/2)*ln((e*x+d)*c^(1/2)-
(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*
(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)+3/16*e/a/c^
(3/2)*ln((e*x+d)*c^(1/2)+(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1
/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*d-3/16*e/a
/c^(5/2)*ln((e*x+d)*c^(1/2)+(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)
^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^
2+c^2*d^2)^(1/2)-3/16*e/a/c^(3/2)*ln((e*x+d)*c^(1/2)-(e*x+d)^(1/2)*(2*(c*(a
*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)^(
1/2)+2*c*d)^(1/2)*d+1/16/e/a^2/c^(1/2)*ln((e*x+d)*c^(1/2)+(e*x+d)^(1/2)*(2*
(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d
^2)^(1/2)+2*c*d)^(1/2)*d^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\left(cx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/(c*x^2 + a)^2, x)
```

Fricas [B] time = 2.83791, size = 2917, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((a*c^2*x^2 + a^2*c)*sqrt(-(4*c^2*d^5 + 15*a*c*d^3*e^2 + 15*a^2*d*e^4 +
 a^3*c^3*sqrt(-(25*c^2*d^4*e^6 + 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7)))/
(a^3*c^3))*log((20*c^3*d^6*e^3 + 101*a*c^2*d^4*e^5 + 162*a^2*c*d^2*e^7 + 81
*a^3*e^9)*sqrt(e*x + d) + (5*a^2*c^3*d^3*e^4 + 9*a^3*c^2*d*e^6 - (2*a^3*c^6
*d^2 + 3*a^4*c^5*e^2)*sqrt(-(25*c^2*d^4*e^6 + 90*a*c*d^2*e^8 + 81*a^2*e^10)
/(a^3*c^7)))*sqrt(-(4*c^2*d^5 + 15*a*c*d^3*e^2 + 15*a^2*d*e^4 + a^3*c^3*sqr
t(-(25*c^2*d^4*e^6 + 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7)))/(a^3*c^3)))
- (a*c^2*x^2 + a^2*c)*sqrt(-(4*c^2*d^5 + 15*a*c*d^3*e^2 + 15*a^2*d*e^4 + a^
3*c^3*sqrt(-(25*c^2*d^4*e^6 + 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7)))/(a^
3*c^3))*log((20*c^3*d^6*e^3 + 101*a*c^2*d^4*e^5 + 162*a^2*c*d^2*e^7 + 81*a^
3*e^9)*sqrt(e*x + d) - (5*a^2*c^3*d^3*e^4 + 9*a^3*c^2*d*e^6 - (2*a^3*c^6*d^
2 + 3*a^4*c^5*e^2)*sqrt(-(25*c^2*d^4*e^6 + 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a
^3*c^7)))*sqrt(-(4*c^2*d^5 + 15*a*c*d^3*e^2 + 15*a^2*d*e^4 + a^3*c^3*sqrt(-
(25*c^2*d^4*e^6 + 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7)))/(a^3*c^3))) + (
a*c^2*x^2 + a^2*c)*sqrt(-(4*c^2*d^5 + 15*a*c*d^3*e^2 + 15*a^2*d*e^4 - a^3*c
^3*sqrt(-(25*c^2*d^4*e^6 + 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7)))/(a^3*c
^3))*log((20*c^3*d^6*e^3 + 101*a*c^2*d^4*e^5 + 162*a^2*c*d^2*e^7 + 81*a^3*e
^9)*sqrt(e*x + d) + (5*a^2*c^3*d^3*e^4 + 9*a^3*c^2*d*e^6 + (2*a^3*c^6*d^2 +
 3*a^4*c^5*e^2)*sqrt(-(25*c^2*d^4*e^6 + 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*
c^7)))*sqrt(-(4*c^2*d^5 + 15*a*c*d^3*e^2 + 15*a^2*d*e^4 - a^3*c^3*sqrt(-(25
*c^2*d^4*e^6 + 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7)))/(a^3*c^3))) - (a*c
^2*x^2 + a^2*c)*sqrt(-(4*c^2*d^5 + 15*a*c*d^3*e^2 + 15*a^2*d*e^4 - a^3*c^3*
sqrt(-(25*c^2*d^4*e^6 + 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7)))/(a^3*c^3)
)*log((20*c^3*d^6*e^3 + 101*a*c^2*d^4*e^5 + 162*a^2*c*d^2*e^7 + 81*a^3*e^9)
*sqrt(e*x + d) - (5*a^2*c^3*d^3*e^4 + 9*a^3*c^2*d*e^6 + (2*a^3*c^6*d^2 + 3*
a^4*c^5*e^2)*sqrt(-(25*c^2*d^4*e^6 + 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7
)))*sqrt(-(4*c^2*d^5 + 15*a*c*d^3*e^2 + 15*a^2*d*e^4 - a^3*c^3*sqrt(-(25*c^
2*d^4*e^6 + 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7)))/(a^3*c^3))) - 4*(2*a*
d*e - (c*d^2 - a*e^2)*x)*sqrt(e*x + d))/(a*c^2*x^2 + a^2*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.633 $\int \frac{(d+ex)^{3/2}}{\left(a+cx^2\right)^2}\,dx$

Optimal. Leaf size=726

$$-\frac{e\left(-\sqrt{c}d\sqrt{ae^2+cd^2}+ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}+\sqrt{ae^2+cd^2}+\sqrt{c}(d+ex)\right)}{8\sqrt{2}ac^{5/4}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{e\left(-\sqrt{c}d\sqrt{\right.}}{}$$

```
[Out] -((a*e - c*d*x)*Sqrt[d + e*x])/(2*a*c*(a + c*x^2)) + (e*(c*d^2 + a*e^2 + Sq
rt[c]*d*Sqrt[c*d^2 + a*e^2])*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]
 - Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(
4*Sqrt[2]*a*c^(5/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2
]]) - (e*(c*d^2 + a*e^2 + Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*ArcTanh[(Sqrt[Sqrt
[c]*d + Sqrt[c*d^2 + a*e^2]] + Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*
d - Sqrt[c*d^2 + a*e^2]]])/(4*Sqrt[2]*a*c^(5/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sq
rt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (e*(c*d^2 + a*e^2 - Sqrt[c]*d*Sqrt[c*d^2
+ a*e^2])*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c
*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(8*Sqrt[2]*a*c^(5/4)*Sqr
t[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]) + (e*(c*d^2 + a*e^2
 - Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)
*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/
(8*Sqrt[2]*a*c^(5/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^
2]])
```

Rubi [A] time = 1.29799, antiderivative size = 726, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {739, 827, 1169, 634, 618, 206, 628}

$$-\frac{e\left(-\sqrt{c}d\sqrt{ae^2+cd^2}+ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}+\sqrt{ae^2+cd^2}+\sqrt{c}(d+ex)\right)}{8\sqrt{2}ac^{5/4}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{e\left(-\sqrt{c}d\sqrt{\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(a + c*x^2)^2,x]
```



```
[Out] -((a*e - c*d*x)*Sqrt[d + e*x])/(2*a*c*(a + c*x^2)) + (e*(c*d^2 + a*e^2 + Sq
rt[c]*d*Sqrt[c*d^2 + a*e^2])*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]
 - Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(
4*Sqrt[2]*a*c^(5/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2
]]) - (e*(c*d^2 + a*e^2 + Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*ArcTanh[(Sqrt[Sqrt
[c]*d + Sqrt[c*d^2 + a*e^2]] + Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*
d - Sqrt[c*d^2 + a*e^2]]])/(4*Sqrt[2]*a*c^(5/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sq
rt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (e*(c*d^2 + a*e^2 - Sqrt[c]*d*Sqrt[c*d^2
+ a*e^2])*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c
*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(8*Sqrt[2]*a*c^(5/4)*Sqr
t[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]) + (e*(c*d^2 + a*e^2
 - Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)
*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/
(8*Sqrt[2]*a*c^(5/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^
2]])
```


Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\left(a+cx^2\right)^2}\,dx = -\frac{(ae-cdx)\sqrt{d+ex}}{2ac\left(a+cx^2\right)} + \frac{\int \frac{\frac{1}{2}\left(2cd^2+ae^2\right)+\frac{1}{2}cdex}{\sqrt{d+ex}\left(a+cx^2\right)}dx}{2ac}$$

$$= -\frac{(ae-cdx)\sqrt{d+ex}}{2ac\left(a+cx^2\right)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}cd^2e+\frac{1}{2}e\left(2cd^2+ae^2\right)+\frac{1}{2}cdex^2}{cd^2+ae^2-2cdx^2+cx^4}dx, x, \sqrt{d+ex}\right)}{ac}$$

$$= -\frac{(ae-cdx)\sqrt{d+ex}}{2ac\left(a+cx^2\right)} + \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(-\frac{1}{2}cd^2e+\frac{1}{2}e\left(2cd^2+ae^2\right)\right)}{\sqrt[4]{c}}-\left(-\frac{1}{2}cd^2e-\frac{1}{2}\sqrt{c}de\sqrt{cd^2+ae^2}+\frac{1}{2}e\left(2cd^2+ae^2\right)\right)x}{\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}x}{\sqrt[4]{c}}+x^2}\right.}{2\sqrt{2}ac^{5/4}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}$$

$$= -\frac{(ae-cdx)\sqrt{d+ex}}{2ac\left(a+cx^2\right)} - \frac{\left(e\left(cd^2+ae^2-\sqrt{c}d\sqrt{cd^2+ae^2}\right)\right)\text{Subst}\left(\int \frac{-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}+2x}{\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}x}{\sqrt[4]{c}}+x^2}dx, x,\right.}{8\sqrt{2}ac^{5/4}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}$$

$$= -\frac{(ae-cdx)\sqrt{d+ex}}{2ac\left(a+cx^2\right)} - \frac{e\left(cd^2+ae^2-\sqrt{c}d\sqrt{cd^2+ae^2}\right)\log\left(\sqrt{cd^2+ae^2}-\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae}\right.}{8\sqrt{2}ac^{5/4}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}$$

$$= -\frac{(ae-cdx)\sqrt{d+ex}}{2ac\left(a+cx^2\right)} + \frac{e\left(cd^2+ae^2+\sqrt{c}d\sqrt{cd^2+ae^2}\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}-\sqrt{2}\sqrt{d+ex}\right)}{\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}\right)}{4\sqrt{2}ac^{5/4}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}} - \frac{e\left(cd^2\right.}{}$$

Mathematica [A] time = 0.583789, size = 208, normalized size = 0.29

$$\frac{\frac{2a\sqrt[4]{c}\sqrt{d+ex}(cdx-ae)}{a+cx^2}-\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(2\sqrt{-a}\sqrt{c}d-ae\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)+\sqrt{\sqrt{-a}e+\sqrt{c}d}\left(2\sqrt{-a}\sqrt{c}d+ae\right)\tanh^{-1}\left(\right.}{4a^2c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(a + c*x^2)^2,x]
```

```
[Out] ((2*a*c^(1/4)*(-(a*e) + c*d*x)*Sqrt[d + e*x])/(a + c*x^2) - Sqrt[Sqrt[c]*d
- Sqrt[-a]*e]*(2*Sqrt[-a]*Sqrt[c]*d - a*e)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/
Sqrt[Sqrt[c]*d - Sqrt[-a]*e]] + Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*(2*Sqrt[-a]*Sq
rt[c]*d + a*e)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[-a]*e]
])/(4*a^2*c^(5/4))
```

Maple [B] time = 0.23, size = 2851, normalized size = 3.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*x^2+a)^2,x)
```

[Out] 1/2*e/(c*e^2*x^2+a*e^2)*d/a*(e*x+d)^(3/2)-1/2*e^3/(c*e^2*x^2+a*e^2)/c*(e*x+d)^(1/2)-1/2*e/(c*e^2*x^2+a*e^2)/a*(e*x+d)^(1/2)*d^2+1/16/e/a^2/c*ln((e*x+d)*c^(1/2)-(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*e^2+c*d^2)^(1/2)*d-1/16/e/a^2/c^(1/2)*ln((e*x+d)*c^(1/2)-(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*d^2-1/16/e/a^2/c^2*ln((e*x+d)*c^(1/2)-(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)*(a*e^2+c*d^2)^(1/2)+1/16/e/a^2/c^(3/2)*ln((e*x+d)*c^(1/2)-(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)*d+1/2*e/a/c/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)-(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(a*e^2+c*d^2)^(1/2)+1/8/e/a^2/c/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)-(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*e^2+c*d^2)^(1/2)*d-1/8/e/a^2/c^(1/2)/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)-(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*d^2-1/8/e/a^2/c^2/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)-(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)*(a*e^2+c*d^2)^(1/2)+1/8/e/a^2/c^(3/2)/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)-(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)*d-1/16/e/a^2/c*ln((e*x+d)*c^(1/2)+(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*e^2+c*d^2)^(1/2)*d+1/16/e/a^2/c^(1/2)*ln((e*x+d)*c^(1/2)+(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*d^2+1/16/e/a^2/c^2*ln((e*x+d)*c^(1/2)+(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)*(a*e^2+c*d^2)^(1/2)-1/16/e/a^2/c^(3/2)*ln((e*x+d)*c^(1/2)+(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)*d+1/2*e/a/c/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(a*e^2+c*d^2)^(1/2)+1/8/e/a^2/c/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*e^2+c*d^2)^(1/2)*d-1/8/e/a^2/c^(1/2)/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*d^2-1/8/e/a^2/c^2/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)*(a*e^2+c*d^2)^(1/2)+1/8/e/a^2/c^(3/2)/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(

```
1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2
))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(
2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1
/2)*(a*c*e^2+c^2*d^2)^(1/2)*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(c*x^2 + a)^2, x)
```

Fricas [A] time = 2.3905, size = 1432, normalized size = 1.97

$$\left(ac^2x^2+a^2c\right)\sqrt{-\frac{a^3c^2\sqrt{-\frac{e^6}{a^3c^5}}+4\,cd^3+3\,ade^2}{a^3c^2}}\log\left(\left(4\,cd^2e^3+ae^5\right)\sqrt{ex+d}+\left(2\,a^3c^4d\sqrt{-\frac{e^6}{a^3c^5}}+a^2ce^4\right)\sqrt{-\frac{a^3c^2\sqrt{-\frac{e^6}{a^3c^5}}+4\,cd^3+3\,ade}{a^3c^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((a*c^2*x^2 + a^2*c)*sqrt(-(a^3*c^2*sqrt(-e^6/(a^3*c^5)) + 4*c*d^3 + 3*
a*d*e^2)/(a^3*c^2))*log((4*c*d^2*e^3 + a*e^5)*sqrt(e*x + d) + (2*a^3*c^4*d*
sqrt(-e^6/(a^3*c^5)) + a^2*c*e^4)*sqrt(-(a^3*c^2*sqrt(-e^6/(a^3*c^5)) + 4*c
*d^3 + 3*a*d*e^2)/(a^3*c^2))) - (a*c^2*x^2 + a^2*c)*sqrt(-(a^3*c^2*sqrt(-e^
6/(a^3*c^5)) + 4*c*d^3 + 3*a*d*e^2)/(a^3*c^2))*log((4*c*d^2*e^3 + a*e^5)*sq
rt(e*x + d) - (2*a^3*c^4*d*sqrt(-e^6/(a^3*c^5)) + a^2*c*e^4)*sqrt(-(a^3*c^2
*sqrt(-e^6/(a^3*c^5)) + 4*c*d^3 + 3*a*d*e^2)/(a^3*c^2))) - (a*c^2*x^2 + a^2
*c)*sqrt((a^3*c^2*sqrt(-e^6/(a^3*c^5)) - 4*c*d^3 - 3*a*d*e^2)/(a^3*c^2))*lo
g((4*c*d^2*e^3 + a*e^5)*sqrt(e*x + d) + (2*a^3*c^4*d*sqrt(-e^6/(a^3*c^5)) -
 a^2*c*e^4)*sqrt((a^3*c^2*sqrt(-e^6/(a^3*c^5)) - 4*c*d^3 - 3*a*d*e^2)/(a^3*
c^2))) + (a*c^2*x^2 + a^2*c)*sqrt((a^3*c^2*sqrt(-e^6/(a^3*c^5)) - 4*c*d^3 -
 3*a*d*e^2)/(a^3*c^2))*log((4*c*d^2*e^3 + a*e^5)*sqrt(e*x + d) - (2*a^3*c^4
*d*sqrt(-e^6/(a^3*c^5)) - a^2*c*e^4)*sqrt((a^3*c^2*sqrt(-e^6/(a^3*c^5)) - 4
*c*d^3 - 3*a*d*e^2)/(a^3*c^2))) + 4*(c*d*x - a*e)*sqrt(e*x + d))/(a*c^2*x^2
 + a^2*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.634 $\int \frac{\sqrt{d+ex}}{\left(a+cx^2\right)^2}\,dx$

Optimal. Leaf size=675

$$\frac{e\left(\sqrt{ae^2+cd^2}+\sqrt{c}d\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}}{\sqrt{\sqrt{c}d-\sqrt{ae^2+cd^2}}}\right)}{4\sqrt{2}ac^{3/4}\sqrt{ae^2+cd^2}\sqrt{\sqrt{c}d-\sqrt{ae^2+cd^2}}}-\frac{e\left(\sqrt{ae^2+cd^2}+\sqrt{c}d\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d+\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}}{\sqrt{\sqrt{c}d-\sqrt{ae^2+cd^2}}}\right)}{4\sqrt{2}ac^{3/4}\sqrt{ae^2+cd^2}\sqrt{\sqrt{c}d-\sqrt{ae^2+cd^2}}}$$

```
[Out] (x*Sqrt[d + e*x])/(2*a*(a + c*x^2)) + (e*(Sqrt[c]*d + Sqrt[c*d^2 + a*e^2])*
ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] - Sqrt[2]*c^(1/4)*Sqrt[d + e
*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(4*Sqrt[2]*a*c^(3/4)*Sqrt[c*d^
2 + a*e^2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (e*(Sqrt[c]*d + Sqrt[c*
d^2 + a*e^2])*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] + Sqrt[2]*c^(1
/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(4*Sqrt[2]*a*c^(
3/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (e*(d - S
qrt[c*d^2 + a*e^2]/Sqrt[c])*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sqrt[
Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(8*Sqr
t[2]*a*c^(1/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]) +
 (e*(d - Sqrt[c*d^2 + a*e^2]/Sqrt[c])*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(
1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x
)])/(8*Sqrt[2]*a*c^(1/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 +
a*e^2]])
```

Rubi [A] time = 1.01755, antiderivative size = 675, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {737, 827, 1169, 634, 618, 206, 628}

$$\frac{e\left(\sqrt{ae^2+cd^2}+\sqrt{c}d\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}}{\sqrt{\sqrt{c}d-\sqrt{ae^2+cd^2}}}\right)}{4\sqrt{2}ac^{3/4}\sqrt{ae^2+cd^2}\sqrt{\sqrt{c}d-\sqrt{ae^2+cd^2}}}-\frac{e\left(\sqrt{ae^2+cd^2}+\sqrt{c}d\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d+\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}}{\sqrt{\sqrt{c}d-\sqrt{ae^2+cd^2}}}\right)}{4\sqrt{2}ac^{3/4}\sqrt{ae^2+cd^2}\sqrt{\sqrt{c}d-\sqrt{ae^2+cd^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(a + c*x^2)^2,x]
```

```
[Out] (x*Sqrt[d + e*x])/(2*a*(a + c*x^2)) + (e*(Sqrt[c]*d + Sqrt[c*d^2 + a*e^2])*
ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] - Sqrt[2]*c^(1/4)*Sqrt[d + e
*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(4*Sqrt[2]*a*c^(3/4)*Sqrt[c*d^
2 + a*e^2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (e*(Sqrt[c]*d + Sqrt[c*
d^2 + a*e^2])*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] + Sqrt[2]*c^(1
/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(4*Sqrt[2]*a*c^(
3/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (e*(d - S
qrt[c*d^2 + a*e^2]/Sqrt[c])*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sqrt[
Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(8*Sqr
t[2]*a*c^(1/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]) +
 (e*(d - Sqrt[c*d^2 + a*e^2]/Sqrt[c])*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(
1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x
)])/(8*Sqrt[2]*a*c^(1/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 +
a*e^2]])
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(x*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*(p + 1)), x] + Dist[1/(2*a*(p + 1
```

```
)), Int[(d + e*x)^(m - 1)*(d*(2*p + 3) + e*(m + 2*p + 3)*x)*(a + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -
1] && GtQ[m, 0] && (LtQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && In
tQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\left(a+cx^2\right)^2}\,dx = \frac{x\sqrt{d+ex}}{2a\left(a+cx^2\right)} - \frac{\int \frac{-d-\frac{ex}{2}}{\sqrt{d+ex}\left(a+cx^2\right)}\,dx}{2a}$$

$$= \frac{x\sqrt{d+ex}}{2a\left(a+cx^2\right)} - \frac{\text{Subst}\left(\int \frac{-\frac{de}{2}-\frac{ex^2}{2}}{cd^2+ae^2-2cdx^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{a}$$

$$= \frac{x\sqrt{d+ex}}{2a\left(a+cx^2\right)} - \frac{\text{Subst}\left(\int \frac{-\frac{de\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt{2}\sqrt[4]{c}}-\left(-\frac{de}{2}+\frac{e\sqrt{cd^2+ae^2}}{2\sqrt{c}}\right)x}{\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{d+ex}\right)}{2\sqrt{2}a\sqrt[4]{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}} - \frac{\text{Subst}\left(\int \frac{-\frac{de\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt{2}\sqrt[4]{c}}}{\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\sqrt{2}\sqrt{}\cdots}\cdots\right)}{2\sqrt{2}a\sqrt[4]{c}\sqrt{cd^2+\cdots}}$$

$$= \frac{x\sqrt{d+ex}}{2a\left(a+cx^2\right)} + \frac{\left(e\left(1+\frac{\sqrt{c}d}{\sqrt{cd^2+ae^2}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{d+ex}\right)}{8ac} + \frac{\left(e\left(1+\frac{}{\sqrt{c}\cdots}\right.\right.\cdots}{\cdots}$$

$$= \frac{x\sqrt{d+ex}}{2a\left(a+cx^2\right)} - \frac{e\left(d-\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}\right)\log\left(\sqrt{cd^2+ae^2}-\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}+\sqrt{c}(d+ex)\right)}{8\sqrt{2}a\sqrt[4]{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\cdots$$

$$= \frac{x\sqrt{d+ex}}{2a\left(a+cx^2\right)} + \frac{e\left(1+\frac{\sqrt{c}d}{\sqrt{cd^2+ae^2}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}-\sqrt{2}\sqrt{d+ex}\right)}{\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}\right)}{4\sqrt{2}ac^{3/4}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}} - \frac{e\left(1+\frac{\sqrt{c}d}{\sqrt{cd^2+ae^2}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\left(\cdots\right.}{\cdots}\right)}{4\sqrt{2}ac^{3/4}\sqrt{\sqrt{c}d-\cdots}}$$

Mathematica [A] time = 0.588015, size = 265, normalized size = 0.39

$$\frac{-\frac{\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(2\sqrt{-a}cd^2-a\sqrt{c}de+\sqrt{-a}ae^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{2ac^{3/4}}+\frac{\sqrt{\sqrt{-a}e+\sqrt{c}d}\left(2\sqrt{-a}cd^2+a\sqrt{c}de+\sqrt{-a}ae^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{2ac^{3/4}}+\frac{x\sqrt{d+ex}\left(ae^2+cd^2\right)}{a+cx^2}}{2a\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(a + c*x^2)^2,x]
```

```
[Out] (((c*d^2 + a*e^2)*x*Sqrt[d + e*x])/(a + c*x^2) - (Sqrt[Sqrt[c]*d - Sqrt[-a]
*e]*(2*Sqrt[-a]*c*d^2 - a*Sqrt[c]*d*e + Sqrt[-a]*a*e^2)*ArcTanh[(c^(1/4)*Sq
rt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[-a]*e]])/(2*a*c^(3/4)) + (Sqrt[Sqrt[c]*d
 + Sqrt[-a]*e]*(2*Sqrt[-a]*c*d^2 + a*Sqrt[c]*d*e + Sqrt[-a]*a*e^2)*ArcTanh[
(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[-a]*e]])/(2*a*c^(3/4)))/(2*a*
(c*d^2 + a*e^2))
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+a\right)^2}\sqrt{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*x^2+a)^2,x)
```

```
[Out] int((e*x+d)^(1/2)/(c*x^2+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\left(cx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(c*x^2 + a)^2, x)
```

Fricas [B] time = 2.46788, size = 2593, normalized size = 3.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((a*c*x^2 + a^2)*sqrt(-(4*c*d^3 + 3*a*d*e^2 + (a^3*c^2*d^2 + a^4*c*e^2)
*sqrt(-e^6/(a^3*c^5*d^4 + 2*a^4*c^4*d^2*e^2 + a^5*c^3*e^4)))/(a^3*c^2*d^2 +
 a^4*c*e^2))*log((4*c*d^2*e^3 + a*e^5)*sqrt(e*x + d) + (a^2*c*d*e^4 - (2*a^
3*c^4*d^4 + 3*a^4*c^3*d^2*e^2 + a^5*c^2*e^4)*sqrt(-e^6/(a^3*c^5*d^4 + 2*a^4
*c^4*d^2*e^2 + a^5*c^3*e^4)))*sqrt(-(4*c*d^3 + 3*a*d*e^2 + (a^3*c^2*d^2 + a
^4*c*e^2)*sqrt(-e^6/(a^3*c^5*d^4 + 2*a^4*c^4*d^2*e^2 + a^5*c^3*e^4)))/(a^3*
c^2*d^2 + a^4*c*e^2))) - (a*c*x^2 + a^2)*sqrt(-(4*c*d^3 + 3*a*d*e^2 + (a^3*
c^2*d^2 + a^4*c*e^2)*sqrt(-e^6/(a^3*c^5*d^4 + 2*a^4*c^4*d^2*e^2 + a^5*c^3*e
^4)))/(a^3*c^2*d^2 + a^4*c*e^2))*log((4*c*d^2*e^3 + a*e^5)*sqrt(e*x + d) -
(a^2*c*d*e^4 - (2*a^3*c^4*d^4 + 3*a^4*c^3*d^2*e^2 + a^5*c^2*e^4)*sqrt(-e^6/
(a^3*c^5*d^4 + 2*a^4*c^4*d^2*e^2 + a^5*c^3*e^4)))*sqrt(-(4*c*d^3 + 3*a*d*e^
2 + (a^3*c^2*d^2 + a^4*c*e^2)*sqrt(-e^6/(a^3*c^5*d^4 + 2*a^4*c^4*d^2*e^2 +
a^5*c^3*e^4)))/(a^3*c^2*d^2 + a^4*c*e^2))) + (a*c*x^2 + a^2)*sqrt(-(4*c*d^3
 + 3*a*d*e^2 - (a^3*c^2*d^2 + a^4*c*e^2)*sqrt(-e^6/(a^3*c^5*d^4 + 2*a^4*c^4
*d^2*e^2 + a^5*c^3*e^4)))/(a^3*c^2*d^2 + a^4*c*e^2))*log((4*c*d^2*e^3 + a*e
^5)*sqrt(e*x + d) + (a^2*c*d*e^4 + (2*a^3*c^4*d^4 + 3*a^4*c^3*d^2*e^2 + a^5
*c^2*e^4)*sqrt(-e^6/(a^3*c^5*d^4 + 2*a^4*c^4*d^2*e^2 + a^5*c^3*e^4)))*sqrt(
-(4*c*d^3 + 3*a*d*e^2 - (a^3*c^2*d^2 + a^4*c*e^2)*sqrt(-e^6/(a^3*c^5*d^4 +
2*a^4*c^4*d^2*e^2 + a^5*c^3*e^4)))/(a^3*c^2*d^2 + a^4*c*e^2))) - (a*c*x^2 +
 a^2)*sqrt(-(4*c*d^3 + 3*a*d*e^2 - (a^3*c^2*d^2 + a^4*c*e^2)*sqrt(-e^6/(a^3
*c^5*d^4 + 2*a^4*c^4*d^2*e^2 + a^5*c^3*e^4)))/(a^3*c^2*d^2 + a^4*c*e^2))*lo
g((4*c*d^2*e^3 + a*e^5)*sqrt(e*x + d) - (a^2*c*d*e^4 + (2*a^3*c^4*d^4 + 3*a
^4*c^3*d^2*e^2 + a^5*c^2*e^4)*sqrt(-e^6/(a^3*c^5*d^4 + 2*a^4*c^4*d^2*e^2 +
a^5*c^3*e^4)))*sqrt(-(4*c*d^3 + 3*a*d*e^2 - (a^3*c^2*d^2 + a^4*c*e^2)*sqrt(
-e^6/(a^3*c^5*d^4 + 2*a^4*c^4*d^2*e^2 + a^5*c^3*e^4)))/(a^3*c^2*d^2 + a^4*c
*e^2))) + 4*sqrt(e*x + d)*x)/(a*c*x^2 + a^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.635 $\int \frac{1}{\sqrt{d+ex}\left(a+cx^2\right)^2}\,dx$

Optimal. Leaf size=739

$$\frac{\sqrt{d+ex}(ae+cdx)}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)}-\frac{e\left(-\sqrt{c}d\sqrt{ae^2+cd^2}+3ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}+\sqrt{ae^2+cd^2}+\sqrt{c}\right.}{8\sqrt{2}a\sqrt[4]{c}\left(ae^2+cd^2\right)^{3/2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}$$

```
[Out] ((a*e + c*d*x)*Sqrt[d + e*x])/(2*a*(c*d^2 + a*e^2)*(a + c*x^2)) + (e*(c*d^2
 + 3*a*e^2 + Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[
c*d^2 + a*e^2]] - Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^
2 + a*e^2]]])/(4*Sqrt[2]*a*c^(1/4)*(c*d^2 + a*e^2)^(3/2)*Sqrt[Sqrt[c]*d - S
qrt[c*d^2 + a*e^2]]) - (e*(c*d^2 + 3*a*e^2 + Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])
*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] + Sqrt[2]*c^(1/4)*Sqrt[d +
e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(4*Sqrt[2]*a*c^(1/4)*(c*d^2 +
 a*e^2)^(3/2)*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (e*(c*d^2 + 3*a*e^2
- Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*
Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(
8*Sqrt[2]*a*c^(1/4)*(c*d^2 + a*e^2)^(3/2)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e
^2]]) + (e*(c*d^2 + 3*a*e^2 - Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^2
 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d +
e*x] + Sqrt[c]*(d + e*x)])/(8*Sqrt[2]*a*c^(1/4)*(c*d^2 + a*e^2)^(3/2)*Sqrt[
Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])
```

Rubi [A] time = 1.49835, antiderivative size = 739, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {741, 827, 1169, 634, 618, 206, 628}

$$\frac{\sqrt{d+ex}(ae+cdx)}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)}-\frac{e\left(-\sqrt{c}d\sqrt{ae^2+cd^2}+3ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}+\sqrt{ae^2+cd^2}+\sqrt{c}\right.}{8\sqrt{2}a\sqrt[4]{c}\left(ae^2+cd^2\right)^{3/2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(a + c*x^2)^2),x]
```

```
[Out] ((a*e + c*d*x)*Sqrt[d + e*x])/(2*a*(c*d^2 + a*e^2)*(a + c*x^2)) + (e*(c*d^2
 + 3*a*e^2 + Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[
c*d^2 + a*e^2]] - Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^
2 + a*e^2]]])/(4*Sqrt[2]*a*c^(1/4)*(c*d^2 + a*e^2)^(3/2)*Sqrt[Sqrt[c]*d - S
qrt[c*d^2 + a*e^2]]) - (e*(c*d^2 + 3*a*e^2 + Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])
*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] + Sqrt[2]*c^(1/4)*Sqrt[d +
e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(4*Sqrt[2]*a*c^(1/4)*(c*d^2 +
 a*e^2)^(3/2)*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (e*(c*d^2 + 3*a*e^2
- Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*
Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(
8*Sqrt[2]*a*c^(1/4)*(c*d^2 + a*e^2)^(3/2)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e
^2]]) + (e*(c*d^2 + 3*a*e^2 - Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^2
 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d +
e*x] + Sqrt[c]*(d + e*x)])/(8*Sqrt[2]*a*c^(1/4)*(c*d^2 + a*e^2)^(3/2)*Sqrt[
Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\left(a+cx^2\right)^2}dx = \frac{(ae+cdx)\sqrt{d+ex}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \frac{\frac{1}{2}\left(-2cd^2-3ae^2\right)-\frac{1}{2}cdex}{\sqrt{d+ex}\left(a+cx^2\right)}dx}{2a\left(cd^2+ae^2\right)}$$

$$= \frac{(ae+cdx)\sqrt{d+ex}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}cd^2e+\frac{1}{2}e\left(-2cd^2-3ae^2\right)-\frac{1}{2}cdex^2}{cd^2+ae^2-2cdx^2+cx^4}dx, x, \sqrt{d+ex}\right)}{a\left(cd^2+ae^2\right)}$$

$$= \frac{(ae+cdx)\sqrt{d+ex}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\left(\frac{1}{2}cd^2e+\frac{1}{2}e\left(-2cd^2-3ae^2\right)\right)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}-\left(\frac{1}{2}cd^2e+\frac{1}{2}e\left(-2cd^2-3ae^2\right)+\frac{1}{2}\sqrt{c}\right.}{\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}x}{\sqrt[4]{c}}+x^2}\right.}{2\sqrt{2}a\sqrt[4]{c}\left(cd^2+ae^2\right)^{3/2}\sqrt{\sqrt{c}d+\sqrt{cd^2+a}}}$$

$$= \frac{(ae+cdx)\sqrt{d+ex}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\left(e\left(cd^2+3ae^2-\sqrt{c}d\sqrt{cd^2+ae^2}\right)\right)\text{Subst}\left(\int \frac{-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}+2}{\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}}\right.}{8\sqrt{2}a\sqrt[4]{c}\left(cd^2+ae^2\right)^{3/2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}$$

$$= \frac{(ae+cdx)\sqrt{d+ex}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{e\left(cd^2+3ae^2-\sqrt{c}d\sqrt{cd^2+ae^2}\right)\log\left(\sqrt{cd^2+ae^2}-\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\right.}{8\sqrt{2}a\sqrt[4]{c}\left(cd^2+ae^2\right)^{3/2}\sqrt{\sqrt{c}d+\sqrt{cd^2}}}$$

$$= \frac{(ae+cdx)\sqrt{d+ex}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{e\left(cd^2+3ae^2+\sqrt{c}d\sqrt{cd^2+ae^2}\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}-\sqrt{2}\sqrt{d+ex}\right.}{\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}\right.}{4\sqrt{2}a\sqrt[4]{c}\left(cd^2+ae^2\right)^{3/2}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}$$

Mathematica [A] time = 0.600634, size = 255, normalized size = 0.35

$$\frac{\frac{\left(-\sqrt{-a}\sqrt{c}de+3ae^2+2cd^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt[4]{c}\sqrt{\sqrt{c}d-\sqrt{-a}e}}+\frac{\left(2\sqrt{-a}cd^2-a\sqrt{c}de+3\sqrt{-a}ae^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{a\sqrt[4]{c}\sqrt{\sqrt{-a}e+\sqrt{c}d}}+\frac{2\sqrt{d+ex}(ae+cdx)}{a+cx^2}}{4a\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(a + c*x^2)^2),x]
```

```
[Out] ((2*(a*e + c*d*x)*Sqrt[d + e*x])/(a + c*x^2) + ((2*c*d^2 - Sqrt[-a]*Sqrt[c]
*d*e + 3*a*e^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[-a]*e
]])/(Sqrt[-a]*c^(1/4)*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]) + ((2*Sqrt[-a]*c*d^2 -
a*Sqrt[c]*d*e + 3*Sqrt[-a]*a*e^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt
[c]*d + Sqrt[-a]*e]])/(a*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]))/(4*a*(c*d^2
 + a*e^2))
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+a\right)^2}\frac{1}{\sqrt{ex+d}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a)^2/(e*x+d)^(1/2),x)
```

```
[Out] int(1/(c*x^2+a)^2/(e*x+d)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+a\right)^2\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^2/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)^2*sqrt(e*x + d)), x)
```

Fricas [B] time = 3.67187, size = 6525, normalized size = 8.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^2/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*((a^2*c*d^2 + a^3*e^2 + (a*c^2*d^2 + a^2*c*e^2)*x^2)*sqrt(-(4*c^2*d^5 +
 15*a*c*d^3*e^2 + 15*a^2*d*e^4 + (a^3*c^3*d^6 + 3*a^4*c^2*d^4*e^2 + 3*a^5*c
*d^2*e^4 + a^6*e^6)*sqrt(-(25*c^2*d^4*e^6 + 90*a*c*d^2*e^8 + 81*a^2*e^10)/(
a^3*c^7*d^12 + 6*a^4*c^6*d^10*e^2 + 15*a^5*c^5*d^8*e^4 + 20*a^6*c^4*d^6*e^6
 + 15*a^7*c^3*d^4*e^8 + 6*a^8*c^2*d^2*e^10 + a^9*c*e^12)))/(a^3*c^3*d^6 + 3
*a^4*c^2*d^4*e^2 + 3*a^5*c*d^2*e^4 + a^6*e^6))*log((20*c^2*d^4*e^3 + 81*a*c
*d^2*e^5 + 81*a^2*e^7)*sqrt(e*x + d) + (5*a^2*c^2*d^4*e^4 + 24*a^3*c*d^2*e^
6 + 27*a^4*e^8 + 2*(a^3*c^5*d^9 + 5*a^4*c^4*d^7*e^2 + 9*a^5*c^3*d^5*e^4 + 7
*a^6*c^2*d^3*e^6 + 2*a^7*c*d*e^8)*sqrt(-(25*c^2*d^4*e^6 + 90*a*c*d^2*e^8 +
81*a^2*e^10)/(a^3*c^7*d^12 + 6*a^4*c^6*d^10*e^2 + 15*a^5*c^5*d^8*e^4 + 20*a
^6*c^4*d^6*e^6 + 15*a^7*c^3*d^4*e^8 + 6*a^8*c^2*d^2*e^10 + a^9*c*e^12)))*sq
rt(-(4*c^2*d^5 + 15*a*c*d^3*e^2 + 15*a^2*d*e^4 + (a^3*c^3*d^6 + 3*a^4*c^2*d
^4*e^2 + 3*a^5*c*d^2*e^4 + a^6*e^6)*sqrt(-(25*c^2*d^4*e^6 + 90*a*c*d^2*e^8
+ 81*a^2*e^10)/(a^3*c^7*d^12 + 6*a^4*c^6*d^10*e^2 + 15*a^5*c^5*d^8*e^4 + 20
*a^6*c^4*d^6*e^6 + 15*a^7*c^3*d^4*e^8 + 6*a^8*c^2*d^2*e^10 + a^9*c*e^12)))/
(a^3*c^3*d^6 + 3*a^4*c^2*d^4*e^2 + 3*a^5*c*d^2*e^4 + a^6*e^6))) - (a^2*c*d^
2 + a^3*e^2 + (a*c^2*d^2 + a^2*c*e^2)*x^2)*sqrt(-(4*c^2*d^5 + 15*a*c*d^3*e^
2 + 15*a^2*d*e^4 + (a^3*c^3*d^6 + 3*a^4*c^2*d^4*e^2 + 3*a^5*c*d^2*e^4 + a^6
*e^6)*sqrt(-(25*c^2*d^4*e^6 + 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7*d^12 +
 6*a^4*c^6*d^10*e^2 + 15*a^5*c^5*d^8*e^4 + 20*a^6*c^4*d^6*e^6 + 15*a^7*c^3*
d^4*e^8 + 6*a^8*c^2*d^2*e^10 + a^9*c*e^12)))/(a^3*c^3*d^6 + 3*a^4*c^2*d^4*e
^2 + 3*a^5*c*d^2*e^4 + a^6*e^6))*log((20*c^2*d^4*e^3 + 81*a*c*d^2*e^5 + 81*
a^2*e^7)*sqrt(e*x + d) - (5*a^2*c^2*d^4*e^4 + 24*a^3*c*d^2*e^6 + 27*a^4*e^8
 + 2*(a^3*c^5*d^9 + 5*a^4*c^4*d^7*e^2 + 9*a^5*c^3*d^5*e^4 + 7*a^6*c^2*d^3*e
^6 + 2*a^7*c*d*e^8)*sqrt(-(25*c^2*d^4*e^6 + 90*a*c*d^2*e^8 + 81*a^2*e^10)/(
a^3*c^7*d^12 + 6*a^4*c^6*d^10*e^2 + 15*a^5*c^5*d^8*e^4 + 20*a^6*c^4*d^6*e^6
 + 15*a^7*c^3*d^4*e^8 + 6*a^8*c^2*d^2*e^10 + a^9*c*e^12)))*sqrt(-(4*c^2*d^5
 + 15*a*c*d^3*e^2 + 15*a^2*d*e^4 + (a^3*c^3*d^6 + 3*a^4*c^2*d^4*e^2 + 3*a^5
*c*d^2*e^4 + a^6*e^6)*sqrt(-(25*c^2*d^4*e^6 + 90*a*c*d^2*e^8 + 81*a^2*e^10)
```

```
/(a^3*c^7*d^12 + 6*a^4*c^6*d^10*e^2 + 15*a^5*c^5*d^8*e^4 + 20*a^6*c^4*d^6*e
^6 + 15*a^7*c^3*d^4*e^8 + 6*a^8*c^2*d^2*e^10 + a^9*c*e^12)))/(a^3*c^3*d^6 +
 3*a^4*c^2*d^4*e^2 + 3*a^5*c*d^2*e^4 + a^6*e^6))) + (a^2*c*d^2 + a^3*e^2 +
(a*c^2*d^2 + a^2*c*e^2)*x^2)*sqrt(-(4*c^2*d^5 + 15*a*c*d^3*e^2 + 15*a^2*d*e
^4 - (a^3*c^3*d^6 + 3*a^4*c^2*d^4*e^2 + 3*a^5*c*d^2*e^4 + a^6*e^6)*sqrt(-(2
5*c^2*d^4*e^6 + 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7*d^12 + 6*a^4*c^6*d^1
0*e^2 + 15*a^5*c^5*d^8*e^4 + 20*a^6*c^4*d^6*e^6 + 15*a^7*c^3*d^4*e^8 + 6*a^
8*c^2*d^2*e^10 + a^9*c*e^12)))/(a^3*c^3*d^6 + 3*a^4*c^2*d^4*e^2 + 3*a^5*c*d
^2*e^4 + a^6*e^6))*log((20*c^2*d^4*e^3 + 81*a*c*d^2*e^5 + 81*a^2*e^7)*sqrt(
e*x + d) + (5*a^2*c^2*d^4*e^4 + 24*a^3*c*d^2*e^6 + 27*a^4*e^8 - 2*(a^3*c^5*
d^9 + 5*a^4*c^4*d^7*e^2 + 9*a^5*c^3*d^5*e^4 + 7*a^6*c^2*d^3*e^6 + 2*a^7*c*d
*e^8)*sqrt(-(25*c^2*d^4*e^6 + 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7*d^12 +
 6*a^4*c^6*d^10*e^2 + 15*a^5*c^5*d^8*e^4 + 20*a^6*c^4*d^6*e^6 + 15*a^7*c^3*
d^4*e^8 + 6*a^8*c^2*d^2*e^10 + a^9*c*e^12)))*sqrt(-(4*c^2*d^5 + 15*a*c*d^3*
e^2 + 15*a^2*d*e^4 - (a^3*c^3*d^6 + 3*a^4*c^2*d^4*e^2 + 3*a^5*c*d^2*e^4 + a
^6*e^6)*sqrt(-(25*c^2*d^4*e^6 + 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7*d^12
 + 6*a^4*c^6*d^10*e^2 + 15*a^5*c^5*d^8*e^4 + 20*a^6*c^4*d^6*e^6 + 15*a^7*c^
3*d^4*e^8 + 6*a^8*c^2*d^2*e^10 + a^9*c*e^12)))/(a^3*c^3*d^6 + 3*a^4*c^2*d^4
*e^2 + 3*a^5*c*d^2*e^4 + a^6*e^6))) - (a^2*c*d^2 + a^3*e^2 + (a*c^2*d^2 + a
^2*c*e^2)*x^2)*sqrt(-(4*c^2*d^5 + 15*a*c*d^3*e^2 + 15*a^2*d*e^4 - (a^3*c^3*
d^6 + 3*a^4*c^2*d^4*e^2 + 3*a^5*c*d^2*e^4 + a^6*e^6)*sqrt(-(25*c^2*d^4*e^6
+ 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7*d^12 + 6*a^4*c^6*d^10*e^2 + 15*a^5
*c^5*d^8*e^4 + 20*a^6*c^4*d^6*e^6 + 15*a^7*c^3*d^4*e^8 + 6*a^8*c^2*d^2*e^10
 + a^9*c*e^12)))/(a^3*c^3*d^6 + 3*a^4*c^2*d^4*e^2 + 3*a^5*c*d^2*e^4 + a^6*e
^6))*log((20*c^2*d^4*e^3 + 81*a*c*d^2*e^5 + 81*a^2*e^7)*sqrt(e*x + d) - (5*
a^2*c^2*d^4*e^4 + 24*a^3*c*d^2*e^6 + 27*a^4*e^8 - 2*(a^3*c^5*d^9 + 5*a^4*c^
4*d^7*e^2 + 9*a^5*c^3*d^5*e^4 + 7*a^6*c^2*d^3*e^6 + 2*a^7*c*d*e^8)*sqrt(-(2
5*c^2*d^4*e^6 + 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7*d^12 + 6*a^4*c^6*d^1
0*e^2 + 15*a^5*c^5*d^8*e^4 + 20*a^6*c^4*d^6*e^6 + 15*a^7*c^3*d^4*e^8 + 6*a^
8*c^2*d^2*e^10 + a^9*c*e^12)))*sqrt(-(4*c^2*d^5 + 15*a*c*d^3*e^2 + 15*a^2*d
*e^4 - (a^3*c^3*d^6 + 3*a^4*c^2*d^4*e^2 + 3*a^5*c*d^2*e^4 + a^6*e^6)*sqrt(-
(25*c^2*d^4*e^6 + 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7*d^12 + 6*a^4*c^6*d
^10*e^2 + 15*a^5*c^5*d^8*e^4 + 20*a^6*c^4*d^6*e^6 + 15*a^7*c^3*d^4*e^8 + 6*
a^8*c^2*d^2*e^10 + a^9*c*e^12)))/(a^3*c^3*d^6 + 3*a^4*c^2*d^4*e^2 + 3*a^5*c
*d^2*e^4 + a^6*e^6))) + 4*(c*d*x + a*e)*sqrt(e*x + d))/(a^2*c*d^2 + a^3*e^2
 + (a*c^2*d^2 + a^2*c*e^2)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a)**2/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^2/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.636 $\int \frac{1}{(d+ex)^{3/2}\left(a+cx^2\right)^2}\,dx$

Optimal. Leaf size=845

$$\frac{e\left(cd^2-5ae^2\right)}{2a\left(cd^2+ae^2\right)^2\sqrt{d+ex}}+\frac{\sqrt[4]{c}e\left(c^{3/2}d^3+13a\sqrt{c}e^2d+\left(cd^2-5ae^2\right)\sqrt{cd^2+ae^2}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}\right)}{4\sqrt{2}a\left(cd^2+ae^2\right)^{5/2}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}-\frac{\sqrt[4]{c}e}{\ }$$

```
[Out] (e*(c*d^2 - 5*a*e^2))/(2*a*(c*d^2 + a*e^2)^2*Sqrt[d + e*x]) + (a*e + c*d*x)
/(2*a*(c*d^2 + a*e^2)*Sqrt[d + e*x]*(a + c*x^2)) + (c^(1/4)*e*(c^(3/2)*d^3
+ 13*a*Sqrt[c]*d*e^2 + (c*d^2 - 5*a*e^2)*Sqrt[c*d^2 + a*e^2])*ArcTanh[(Sqrt
[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] - Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqr
t[c]*d - Sqrt[c*d^2 + a*e^2]]])/(4*Sqrt[2]*a*(c*d^2 + a*e^2)^(5/2)*Sqrt[Sqr
t[c]*d - Sqrt[c*d^2 + a*e^2]]) - (c^(1/4)*e*(c^(3/2)*d^3 + 13*a*Sqrt[c]*d*e
^2 + (c*d^2 - 5*a*e^2)*Sqrt[c*d^2 + a*e^2])*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[
c*d^2 + a*e^2]] + Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^
2 + a*e^2]]])/(4*Sqrt[2]*a*(c*d^2 + a*e^2)^(5/2)*Sqrt[Sqrt[c]*d - Sqrt[c*d^
2 + a*e^2]]) - (c^(1/4)*e*(c^(3/2)*d^3 + 13*a*Sqrt[c]*d*e^2 - (c*d^2 - 5*a*
e^2)*Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sqrt[Sq
rt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(8*Sqrt[
2]*a*(c*d^2 + a*e^2)^(5/2)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]) + (c^(1/4
)*e*(c^(3/2)*d^3 + 13*a*Sqrt[c]*d*e^2 - (c*d^2 - 5*a*e^2)*Sqrt[c*d^2 + a*e^
2])*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 +
 a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(8*Sqrt[2]*a*(c*d^2 + a*e^2)^(
5/2)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])
```

Rubi [A] time = 3.34351, antiderivative size = 845, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {741, 829, 827, 1169, 634, 618, 206, 628}

$$\frac{e\left(cd^2-5ae^2\right)}{2a\left(cd^2+ae^2\right)^2\sqrt{d+ex}}+\frac{\sqrt[4]{c}e\left(c^{3/2}d^3+13a\sqrt{c}e^2d+\left(cd^2-5ae^2\right)\sqrt{cd^2+ae^2}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}\right)}{4\sqrt{2}a\left(cd^2+ae^2\right)^{5/2}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}-\frac{\sqrt[4]{c}e}{\ }$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*(a + c*x^2)^2),x]
```

```
[Out] (e*(c*d^2 - 5*a*e^2))/(2*a*(c*d^2 + a*e^2)^2*Sqrt[d + e*x]) + (a*e + c*d*x)
/(2*a*(c*d^2 + a*e^2)*Sqrt[d + e*x]*(a + c*x^2)) + (c^(1/4)*e*(c^(3/2)*d^3
+ 13*a*Sqrt[c]*d*e^2 + (c*d^2 - 5*a*e^2)*Sqrt[c*d^2 + a*e^2])*ArcTanh[(Sqrt
[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] - Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqr
t[c]*d - Sqrt[c*d^2 + a*e^2]]])/(4*Sqrt[2]*a*(c*d^2 + a*e^2)^(5/2)*Sqrt[Sqr
t[c]*d - Sqrt[c*d^2 + a*e^2]]) - (c^(1/4)*e*(c^(3/2)*d^3 + 13*a*Sqrt[c]*d*e
^2 + (c*d^2 - 5*a*e^2)*Sqrt[c*d^2 + a*e^2])*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[
c*d^2 + a*e^2]] + Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^
2 + a*e^2]]])/(4*Sqrt[2]*a*(c*d^2 + a*e^2)^(5/2)*Sqrt[Sqrt[c]*d - Sqrt[c*d^
2 + a*e^2]]) - (c^(1/4)*e*(c^(3/2)*d^3 + 13*a*Sqrt[c]*d*e^2 - (c*d^2 - 5*a*
e^2)*Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sqrt[Sq
rt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(8*Sqrt[
2]*a*(c*d^2 + a*e^2)^(5/2)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]) + (c^(1/4
)*e*(c^(3/2)*d^3 + 13*a*Sqrt[c]*d*e^2 - (c*d^2 - 5*a*e^2)*Sqrt[c*d^2 + a*e^
2])*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 +
```

```
 a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(8*Sqrt[2]*a*(c*d^2 + a*e^2)^(
5/2)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 829

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)
), x] + Dist[1/(c*d^2 + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d*f + a*e*g -
 c*(e*f - d*g)*x, x])/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x
] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\left(a+cx^2\right)^2}\,dx = \frac{ae+cdx}{2a\left(cd^2+ae^2\right)\sqrt{d+ex}\left(a+cx^2\right)} - \frac{\int \frac{\frac{1}{2}\left(-2cd^2-5ae^2\right)-\frac{3}{2}cdex}{(d+ex)^{3/2}\left(a+cx^2\right)}\,dx}{2a\left(cd^2+ae^2\right)}$$

$$= \frac{e\left(cd^2-5ae^2\right)}{2a\left(cd^2+ae^2\right)^2\sqrt{d+ex}} + \frac{ae+cdx}{2a\left(cd^2+ae^2\right)\sqrt{d+ex}\left(a+cx^2\right)} - \frac{\int \frac{-cd\left(cd^2+4ae^2\right)-\frac{1}{2}ce\left(cd^2-5ae^2\right)x}{\sqrt{d+ex}\left(a+cx^2\right)}\,d}{2a\left(cd^2+ae^2\right)^2}$$

$$= \frac{e\left(cd^2-5ae^2\right)}{2a\left(cd^2+ae^2\right)^2\sqrt{d+ex}} + \frac{ae+cdx}{2a\left(cd^2+ae^2\right)\sqrt{d+ex}\left(a+cx^2\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}cde\left(cd^2-5ae^2\right)-cde(c}{cd^2+ae^2}\right.}{a\,(}$$

$$= \frac{e\left(cd^2-5ae^2\right)}{2a\left(cd^2+ae^2\right)^2\sqrt{d+ex}} + \frac{ae+cdx}{2a\left(cd^2+ae^2\right)\sqrt{d+ex}\left(a+cx^2\right)} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(\frac{1}{2}cde(\right.}{4}\right.}{}$$

$$= \frac{e\left(cd^2-5ae^2\right)}{2a\left(cd^2+ae^2\right)^2\sqrt{d+ex}} + \frac{ae+cdx}{2a\left(cd^2+ae^2\right)\sqrt{d+ex}\left(a+cx^2\right)} - \frac{\left(\sqrt[4]{c}e\left(c^{3/2}d^3+13a\sqrt{c}de^2-(\right.\right.}{}$$

$$= \frac{e\left(cd^2-5ae^2\right)}{2a\left(cd^2+ae^2\right)^2\sqrt{d+ex}} + \frac{ae+cdx}{2a\left(cd^2+ae^2\right)\sqrt{d+ex}\left(a+cx^2\right)} - \frac{\sqrt[4]{c}e\left(c^{3/2}d^3+13a\sqrt{c}de^2-(c\right.}{}$$

$$= \frac{e\left(cd^2-5ae^2\right)}{2a\left(cd^2+ae^2\right)^2\sqrt{d+ex}} + \frac{ae+cdx}{2a\left(cd^2+ae^2\right)\sqrt{d+ex}\left(a+cx^2\right)} + \frac{\sqrt[4]{c}e\left(c^{3/2}d^3+13a\sqrt{c}de^2+(c\right.}{4\sqrt{2}a}$$

Mathematica [C] time = 0.558981, size = 346, normalized size = 0.41

$$\frac{\frac{3c^{3/4}d\left(\frac{\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{\sqrt{c}d-\sqrt{-a}e}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{2\sqrt{-a}}-\frac{\left(5ae^2-cd^2\right)\left(\left(ae-\sqrt{-a}\sqrt{c}d\right){}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)+\left(\sqrt{-a}\sqrt{c}d+ae\right){}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)\right)}{2a\sqrt{d+ex}\left(ae^2+cd^2\right)}+\frac{ae+cd}{\left(a+cx^2\right)\sqrt{}}}{2a\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*(a + c*x^2)^2),x]
```

```
[Out] ((a*e + c*d*x)/(Sqrt[d + e*x]*(a + c*x^2)) + (3*c^(3/4)*d*(ArcTanh[(c^(1/4)
*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[-a]*e]]/Sqrt[Sqrt[c]*d - Sqrt[-a]*e]
- ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[-a]*e]]/Sqrt[Sqrt[c
]*d + Sqrt[-a]*e]))/(2*Sqrt[-a]) - ((-(c*d^2) + 5*a*e^2)*((-(Sqrt[-a]*Sqrt[
```

```
c]*d) + a*e)*Hypergeometric2F1[-1/2, 1, 1/2, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d
 - Sqrt[-a]*e)] + (Sqrt[-a]*Sqrt[c]*d + a*e)*Hypergeometric2F1[-1/2, 1, 1/2
, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]))/(2*a*(c*d^2 + a*e^2)*Sqrt
[d + e*x]))/(2*a*(c*d^2 + a*e^2))
```

Maple [B] time = 0.281, size = 8744, normalized size = 10.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(c*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+a\right)^2 (ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)^2*(e*x + d)^(3/2)), x)
```

Fricas [B] time = 7.46447, size = 12104, normalized size = 14.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((a^2*c^2*d^5 + 2*a^3*c*d^3*e^2 + a^4*d*e^4 + (a*c^3*d^4*e + 2*a^2*c^2
*d^2*e^3 + a^3*c*e^5)*x^3 + (a*c^3*d^5 + 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*x
^2 + (a^2*c^2*d^4*e + 2*a^3*c*d^2*e^3 + a^4*e^5)*x)*sqrt(-(4*c^4*d^7 + 35*a
*c^3*d^5*e^2 + 70*a^2*c^2*d^3*e^4 - 105*a^3*c*d*e^6 + (a^3*c^5*d^10 + 5*a^4
*c^4*d^8*e^2 + 10*a^5*c^3*d^6*e^4 + 10*a^6*c^2*d^4*e^6 + 5*a^7*c*d^2*e^8 +
a^8*e^10)*sqrt(-(1225*c^5*d^8*e^6 + 10780*a*c^4*d^6*e^8 + 21966*a^2*c^3*d^4
*e^10 - 7700*a^3*c^2*d^2*e^12 + 625*a^4*c*e^14)/(a^3*c^10*d^20 + 10*a^4*c^9
*d^18*e^2 + 45*a^5*c^8*d^16*e^4 + 120*a^6*c^7*d^14*e^6 + 210*a^7*c^6*d^12*e
^8 + 252*a^8*c^5*d^10*e^10 + 210*a^9*c^4*d^8*e^12 + 120*a^10*c^3*d^6*e^14 +
 45*a^11*c^2*d^4*e^16 + 10*a^12*c*d^2*e^18 + a^13*e^20)))/(a^3*c^5*d^10 + 5
*a^4*c^4*d^8*e^2 + 10*a^5*c^3*d^6*e^4 + 10*a^6*c^2*d^4*e^6 + 5*a^7*c*d^2*e^
8 + a^8*e^10))*log(-(140*c^4*d^6*e^3 + 1491*a*c^3*d^4*e^5 + 3750*a^2*c^2*d^
2*e^7 - 625*a^3*c*e^9)*sqrt(e*x + d) + (35*a^2*c^4*d^7*e^4 + 609*a^3*c^3*d^
5*e^6 + 1977*a^4*c^2*d^3*e^8 - 325*a^5*c*d*e^10 + (2*a^3*c^7*d^14 + 19*a^4*
c^6*d^12*e^2 + 60*a^5*c^5*d^10*e^4 + 85*a^6*c^4*d^8*e^6 + 50*a^7*c^3*d^6*e^
8 - 3*a^8*c^2*d^4*e^10 - 16*a^9*c*d^2*e^12 - 5*a^10*e^14)*sqrt(-(1225*c^5*d
^8*e^6 + 10780*a*c^4*d^6*e^8 + 21966*a^2*c^3*d^4*e^10 - 7700*a^3*c^2*d^2*e^
```

12 + 625*a^4*c*e^14)/(a^3*c^10*d^20 + 10*a^4*c^9*d^18*e^2 + 45*a^5*c^8*d^16
*e^4 + 120*a^6*c^7*d^14*e^6 + 210*a^7*c^6*d^12*e^8 + 252*a^8*c^5*d^10*e^10
+ 210*a^9*c^4*d^8*e^12 + 120*a^10*c^3*d^6*e^14 + 45*a^11*c^2*d^4*e^16 + 10*
a^12*c*d^2*e^18 + a^13*e^20)))*sqrt(-(4*c^4*d^7 + 35*a*c^3*d^5*e^2 + 70*a^2
*c^2*d^3*e^4 - 105*a^3*c*d*e^6 + (a^3*c^5*d^10 + 5*a^4*c^4*d^8*e^2 + 10*a^5
*c^3*d^6*e^4 + 10*a^6*c^2*d^4*e^6 + 5*a^7*c*d^2*e^8 + a^8*e^10)*sqrt(-(1225
*c^5*d^8*e^6 + 10780*a*c^4*d^6*e^8 + 21966*a^2*c^3*d^4*e^10 - 7700*a^3*c^2*
d^2*e^12 + 625*a^4*c*e^14)/(a^3*c^10*d^20 + 10*a^4*c^9*d^18*e^2 + 45*a^5*c^
8*d^16*e^4 + 120*a^6*c^7*d^14*e^6 + 210*a^7*c^6*d^12*e^8 + 252*a^8*c^5*d^10
*e^10 + 210*a^9*c^4*d^8*e^12 + 120*a^10*c^3*d^6*e^14 + 45*a^11*c^2*d^4*e^16
 + 10*a^12*c*d^2*e^18 + a^13*e^20)))/(a^3*c^5*d^10 + 5*a^4*c^4*d^8*e^2 + 10
*a^5*c^3*d^6*e^4 + 10*a^6*c^2*d^4*e^6 + 5*a^7*c*d^2*e^8 + a^8*e^10))) - (a^
2*c^2*d^5 + 2*a^3*c*d^3*e^2 + a^4*d*e^4 + (a*c^3*d^4*e + 2*a^2*c^2*d^2*e^3
+ a^3*c*e^5)*x^3 + (a*c^3*d^5 + 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*x^2 + (a^2
*c^2*d^4*e + 2*a^3*c*d^2*e^3 + a^4*e^5)*x)*sqrt(-(4*c^4*d^7 + 35*a*c^3*d^5*
e^2 + 70*a^2*c^2*d^3*e^4 - 105*a^3*c*d*e^6 + (a^3*c^5*d^10 + 5*a^4*c^4*d^8*
e^2 + 10*a^5*c^3*d^6*e^4 + 10*a^6*c^2*d^4*e^6 + 5*a^7*c*d^2*e^8 + a^8*e^10)
*sqrt(-(1225*c^5*d^8*e^6 + 10780*a*c^4*d^6*e^8 + 21966*a^2*c^3*d^4*e^10 - 7
700*a^3*c^2*d^2*e^12 + 625*a^4*c*e^14)/(a^3*c^10*d^20 + 10*a^4*c^9*d^18*e^2
 + 45*a^5*c^8*d^16*e^4 + 120*a^6*c^7*d^14*e^6 + 210*a^7*c^6*d^12*e^8 + 252*
a^8*c^5*d^10*e^10 + 210*a^9*c^4*d^8*e^12 + 120*a^10*c^3*d^6*e^14 + 45*a^11*
c^2*d^4*e^16 + 10*a^12*c*d^2*e^18 + a^13*e^20)))/(a^3*c^5*d^10 + 5*a^4*c^4*
d^8*e^2 + 10*a^5*c^3*d^6*e^4 + 10*a^6*c^2*d^4*e^6 + 5*a^7*c*d^2*e^8 + a^8*e
^10))*log(-(140*c^4*d^6*e^3 + 1491*a*c^3*d^4*e^5 + 3750*a^2*c^2*d^2*e^7 - 6
25*a^3*c*e^9)*sqrt(e*x + d) - (35*a^2*c^4*d^7*e^4 + 609*a^3*c^3*d^5*e^6 + 1
977*a^4*c^2*d^3*e^8 - 325*a^5*c*d*e^10 + (2*a^3*c^7*d^14 + 19*a^4*c^6*d^12*
e^2 + 60*a^5*c^5*d^10*e^4 + 85*a^6*c^4*d^8*e^6 + 50*a^7*c^3*d^6*e^8 - 3*a^8
*c^2*d^4*e^10 - 16*a^9*c*d^2*e^12 - 5*a^10*e^14)*sqrt(-(1225*c^5*d^8*e^6 +
10780*a*c^4*d^6*e^8 + 21966*a^2*c^3*d^4*e^10 - 7700*a^3*c^2*d^2*e^12 + 625*
a^4*c*e^14)/(a^3*c^10*d^20 + 10*a^4*c^9*d^18*e^2 + 45*a^5*c^8*d^16*e^4 + 12
0*a^6*c^7*d^14*e^6 + 210*a^7*c^6*d^12*e^8 + 252*a^8*c^5*d^10*e^10 + 210*a^9
*c^4*d^8*e^12 + 120*a^10*c^3*d^6*e^14 + 45*a^11*c^2*d^4*e^16 + 10*a^12*c*d^
2*e^18 + a^13*e^20)))*sqrt(-(4*c^4*d^7 + 35*a*c^3*d^5*e^2 + 70*a^2*c^2*d^3*
e^4 - 105*a^3*c*d*e^6 + (a^3*c^5*d^10 + 5*a^4*c^4*d^8*e^2 + 10*a^5*c^3*d^6*
e^4 + 10*a^6*c^2*d^4*e^6 + 5*a^7*c*d^2*e^8 + a^8*e^10)*sqrt(-(1225*c^5*d^8*
e^6 + 10780*a*c^4*d^6*e^8 + 21966*a^2*c^3*d^4*e^10 - 7700*a^3*c^2*d^2*e^12
+ 625*a^4*c*e^14)/(a^3*c^10*d^20 + 10*a^4*c^9*d^18*e^2 + 45*a^5*c^8*d^16*e^
4 + 120*a^6*c^7*d^14*e^6 + 210*a^7*c^6*d^12*e^8 + 252*a^8*c^5*d^10*e^10 + 2
10*a^9*c^4*d^8*e^12 + 120*a^10*c^3*d^6*e^14 + 45*a^11*c^2*d^4*e^16 + 10*a^1
2*c*d^2*e^18 + a^13*e^20)))/(a^3*c^5*d^10 + 5*a^4*c^4*d^8*e^2 + 10*a^5*c^3*
d^6*e^4 + 10*a^6*c^2*d^4*e^6 + 5*a^7*c*d^2*e^8 + a^8*e^10))) + (a^2*c^2*d^5
 + 2*a^3*c*d^3*e^2 + a^4*d*e^4 + (a*c^3*d^4*e + 2*a^2*c^2*d^2*e^3 + a^3*c*e
^5)*x^3 + (a*c^3*d^5 + 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*x^2 + (a^2*c^2*d^4*
e + 2*a^3*c*d^2*e^3 + a^4*e^5)*x)*sqrt(-(4*c^4*d^7 + 35*a*c^3*d^5*e^2 + 70*
a^2*c^2*d^3*e^4 - 105*a^3*c*d*e^6 - (a^3*c^5*d^10 + 5*a^4*c^4*d^8*e^2 + 10*
a^5*c^3*d^6*e^4 + 10*a^6*c^2*d^4*e^6 + 5*a^7*c*d^2*e^8 + a^8*e^10)*sqrt(-(1
225*c^5*d^8*e^6 + 10780*a*c^4*d^6*e^8 + 21966*a^2*c^3*d^4*e^10 - 7700*a^3*c
^2*d^2*e^12 + 625*a^4*c*e^14)/(a^3*c^10*d^20 + 10*a^4*c^9*d^18*e^2 + 45*a^5
*c^8*d^16*e^4 + 120*a^6*c^7*d^14*e^6 + 210*a^7*c^6*d^12*e^8 + 252*a^8*c^5*d
^10*e^10 + 210*a^9*c^4*d^8*e^12 + 120*a^10*c^3*d^6*e^14 + 45*a^11*c^2*d^4*e
^16 + 10*a^12*c*d^2*e^18 + a^13*e^20)))/(a^3*c^5*d^10 + 5*a^4*c^4*d^8*e^2 +
 10*a^5*c^3*d^6*e^4 + 10*a^6*c^2*d^4*e^6 + 5*a^7*c*d^2*e^8 + a^8*e^10))*log
(-(140*c^4*d^6*e^3 + 1491*a*c^3*d^4*e^5 + 3750*a^2*c^2*d^2*e^7 - 625*a^3*c*
e^9)*sqrt(e*x + d) + (35*a^2*c^4*d^7*e^4 + 609*a^3*c^3*d^5*e^6 + 1977*a^4*c
^2*d^3*e^8 - 325*a^5*c*d*e^10 - (2*a^3*c^7*d^14 + 19*a^4*c^6*d^12*e^2 + 60*
a^5*c^5*d^10*e^4 + 85*a^6*c^4*d^8*e^6 + 50*a^7*c^3*d^6*e^8 - 3*a^8*c^2*d^4*
e^10 - 16*a^9*c*d^2*e^12 - 5*a^10*e^14)*sqrt(-(1225*c^5*d^8*e^6 + 10780*a*c
^4*d^6*e^8 + 21966*a^2*c^3*d^4*e^10 - 7700*a^3*c^2*d^2*e^12 + 625*a^4*c*e^1
4)/(a^3*c^10*d^20 + 10*a^4*c^9*d^18*e^2 + 45*a^5*c^8*d^16*e^4 + 120*a^6*c^7

```
*d^14*e^6 + 210*a^7*c^6*d^12*e^8 + 252*a^8*c^5*d^10*e^10 + 210*a^9*c^4*d^8*
e^12 + 120*a^10*c^3*d^6*e^14 + 45*a^11*c^2*d^4*e^16 + 10*a^12*c*d^2*e^18 +
a^13*e^20)))*sqrt(-(4*c^4*d^7 + 35*a*c^3*d^5*e^2 + 70*a^2*c^2*d^3*e^4 - 105
*a^3*c*d*e^6 - (a^3*c^5*d^10 + 5*a^4*c^4*d^8*e^2 + 10*a^5*c^3*d^6*e^4 + 10*
a^6*c^2*d^4*e^6 + 5*a^7*c*d^2*e^8 + a^8*e^10)*sqrt(-(1225*c^5*d^8*e^6 + 107
80*a*c^4*d^6*e^8 + 21966*a^2*c^3*d^4*e^10 - 7700*a^3*c^2*d^2*e^12 + 625*a^4
*c*e^14)/(a^3*c^10*d^20 + 10*a^4*c^9*d^18*e^2 + 45*a^5*c^8*d^16*e^4 + 120*a
^6*c^7*d^14*e^6 + 210*a^7*c^6*d^12*e^8 + 252*a^8*c^5*d^10*e^10 + 210*a^9*c^
4*d^8*e^12 + 120*a^10*c^3*d^6*e^14 + 45*a^11*c^2*d^4*e^16 + 10*a^12*c*d^2*e
^18 + a^13*e^20)))/(a^3*c^5*d^10 + 5*a^4*c^4*d^8*e^2 + 10*a^5*c^3*d^6*e^4 +
 10*a^6*c^2*d^4*e^6 + 5*a^7*c*d^2*e^8 + a^8*e^10))) - (a^2*c^2*d^5 + 2*a^3*
c*d^3*e^2 + a^4*d*e^4 + (a*c^3*d^4*e + 2*a^2*c^2*d^2*e^3 + a^3*c*e^5)*x^3 +
 (a*c^3*d^5 + 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*x^2 + (a^2*c^2*d^4*e + 2*a^3
*c*d^2*e^3 + a^4*e^5)*x)*sqrt(-(4*c^4*d^7 + 35*a*c^3*d^5*e^2 + 70*a^2*c^2*d
^3*e^4 - 105*a^3*c*d*e^6 - (a^3*c^5*d^10 + 5*a^4*c^4*d^8*e^2 + 10*a^5*c^3*d
^6*e^4 + 10*a^6*c^2*d^4*e^6 + 5*a^7*c*d^2*e^8 + a^8*e^10)*sqrt(-(1225*c^5*d
^8*e^6 + 10780*a*c^4*d^6*e^8 + 21966*a^2*c^3*d^4*e^10 - 7700*a^3*c^2*d^2*e^
12 + 625*a^4*c*e^14)/(a^3*c^10*d^20 + 10*a^4*c^9*d^18*e^2 + 45*a^5*c^8*d^16
*e^4 + 120*a^6*c^7*d^14*e^6 + 210*a^7*c^6*d^12*e^8 + 252*a^8*c^5*d^10*e^10
+ 210*a^9*c^4*d^8*e^12 + 120*a^10*c^3*d^6*e^14 + 45*a^11*c^2*d^4*e^16 + 10*
a^12*c*d^2*e^18 + a^13*e^20)))/(a^3*c^5*d^10 + 5*a^4*c^4*d^8*e^2 + 10*a^5*c
^3*d^6*e^4 + 10*a^6*c^2*d^4*e^6 + 5*a^7*c*d^2*e^8 + a^8*e^10))*log(-(140*c^
4*d^6*e^3 + 1491*a*c^3*d^4*e^5 + 3750*a^2*c^2*d^2*e^7 - 625*a^3*c*e^9)*sqrt
(e*x + d) - (35*a^2*c^4*d^7*e^4 + 609*a^3*c^3*d^5*e^6 + 1977*a^4*c^2*d^3*e^
8 - 325*a^5*c*d*e^10 - (2*a^3*c^7*d^14 + 19*a^4*c^6*d^12*e^2 + 60*a^5*c^5*d
^10*e^4 + 85*a^6*c^4*d^8*e^6 + 50*a^7*c^3*d^6*e^8 - 3*a^8*c^2*d^4*e^10 - 16
*a^9*c*d^2*e^12 - 5*a^10*e^14)*sqrt(-(1225*c^5*d^8*e^6 + 10780*a*c^4*d^6*e^
8 + 21966*a^2*c^3*d^4*e^10 - 7700*a^3*c^2*d^2*e^12 + 625*a^4*c*e^14)/(a^3*c
^10*d^20 + 10*a^4*c^9*d^18*e^2 + 45*a^5*c^8*d^16*e^4 + 120*a^6*c^7*d^14*e^6
 + 210*a^7*c^6*d^12*e^8 + 252*a^8*c^5*d^10*e^10 + 210*a^9*c^4*d^8*e^12 + 12
0*a^10*c^3*d^6*e^14 + 45*a^11*c^2*d^4*e^16 + 10*a^12*c*d^2*e^18 + a^13*e^20
)))*sqrt(-(4*c^4*d^7 + 35*a*c^3*d^5*e^2 + 70*a^2*c^2*d^3*e^4 - 105*a^3*c*d*
e^6 - (a^3*c^5*d^10 + 5*a^4*c^4*d^8*e^2 + 10*a^5*c^3*d^6*e^4 + 10*a^6*c^2*d
^4*e^6 + 5*a^7*c*d^2*e^8 + a^8*e^10)*sqrt(-(1225*c^5*d^8*e^6 + 10780*a*c^4*
d^6*e^8 + 21966*a^2*c^3*d^4*e^10 - 7700*a^3*c^2*d^2*e^12 + 625*a^4*c*e^14)/
(a^3*c^10*d^20 + 10*a^4*c^9*d^18*e^2 + 45*a^5*c^8*d^16*e^4 + 120*a^6*c^7*d^
14*e^6 + 210*a^7*c^6*d^12*e^8 + 252*a^8*c^5*d^10*e^10 + 210*a^9*c^4*d^8*e^1
2 + 120*a^10*c^3*d^6*e^14 + 45*a^11*c^2*d^4*e^16 + 10*a^12*c*d^2*e^18 + a^1
3*e^20)))/(a^3*c^5*d^10 + 5*a^4*c^4*d^8*e^2 + 10*a^5*c^3*d^6*e^4 + 10*a^6*c
^2*d^4*e^6 + 5*a^7*c*d^2*e^8 + a^8*e^10))) - 4*(2*a*c*d^2*e - 4*a^2*e^3 + (
c^2*d^2*e - 5*a*c*e^3)*x^2 + (c^2*d^3 + a*c*d*e^2)*x)*sqrt(e*x + d))/(a^2*c
^2*d^5 + 2*a^3*c*d^3*e^2 + a^4*d*e^4 + (a*c^3*d^4*e + 2*a^2*c^2*d^2*e^3 + a
^3*c*e^5)*x^3 + (a*c^3*d^5 + 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*x^2 + (a^2*c^
2*d^4*e + 2*a^3*c*d^2*e^3 + a^4*e^5)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+cx^2\right)^2 (d+ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(c*x**2+a)**2,x)
```

```
[Out] Integral(1/((a + c*x**2)**2*(d + e*x)**(3/2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.637 $\int \frac{1}{(d+ex)^{5/2}\left(a+cx^2\right)^2}\,dx$

Optimal. Leaf size=930

$$\frac{cde\left(cd^2-19ae^2\right)}{2a\left(cd^2+ae^2\right)^3\sqrt{d+ex}}+\frac{c^{3/4}e\left(c^2d^4+34ace^2d^2+\sqrt{c}\left(cd^2-19ae^2\right)\sqrt{cd^2+ae^2}d-7a^2e^4\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\cdots}{\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}\cdots}\right.}{4\sqrt{2}a\left(cd^2+ae^2\right)^{7/2}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}$$

```
[Out] (e*(3*c*d^2 - 7*a*e^2))/(6*a*(c*d^2 + a*e^2)^2*(d + e*x)^(3/2)) + (c*d*e*(c
*d^2 - 19*a*e^2))/(2*a*(c*d^2 + a*e^2)^3*Sqrt[d + e*x]) + (a*e + c*d*x)/(2*
a*(c*d^2 + a*e^2)*(d + e*x)^(3/2)*(a + c*x^2)) + (c^(3/4)*e*(c^2*d^4 + 34*a
*c*d^2*e^2 - 7*a^2*e^4 + Sqrt[c]*d*(c*d^2 - 19*a*e^2)*Sqrt[c*d^2 + a*e^2])*
ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] - Sqrt[2]*c^(1/4)*Sqrt[d + e
*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(4*Sqrt[2]*a*(c*d^2 + a*e^2)^(
7/2)*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (c^(3/4)*e*(c^2*d^4 + 34*a*c*
d^2*e^2 - 7*a^2*e^4 + Sqrt[c]*d*(c*d^2 - 19*a*e^2)*Sqrt[c*d^2 + a*e^2])*Arc
Tanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] + Sqrt[2]*c^(1/4)*Sqrt[d + e*x]
)/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(4*Sqrt[2]*a*(c*d^2 + a*e^2)^(7/2
)*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (c^(3/4)*e*(c^2*d^4 + 34*a*c*d^2
*e^2 - 7*a^2*e^4 - Sqrt[c]*d*(c*d^2 - 19*a*e^2)*Sqrt[c*d^2 + a*e^2])*Log[Sq
rt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*S
qrt[d + e*x] + Sqrt[c]*(d + e*x)])/(8*Sqrt[2]*a*(c*d^2 + a*e^2)^(7/2)*Sqrt[
Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]) + (c^(3/4)*e*(c^2*d^4 + 34*a*c*d^2*e^2 -
7*a^2*e^4 - Sqrt[c]*d*(c*d^2 - 19*a*e^2)*Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^
2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d +
 e*x] + Sqrt[c]*(d + e*x)])/(8*Sqrt[2]*a*(c*d^2 + a*e^2)^(7/2)*Sqrt[Sqrt[c]
*d + Sqrt[c*d^2 + a*e^2]])
```

Rubi [A] time = 6.48863, antiderivative size = 930, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {741, 829, 827, 1169, 634, 618, 206, 628}

$$\frac{cde\left(cd^2-19ae^2\right)}{2a\left(cd^2+ae^2\right)^3\sqrt{d+ex}}+\frac{c^{3/4}e\left(c^2d^4+34ace^2d^2+\sqrt{c}\left(cd^2-19ae^2\right)\sqrt{cd^2+ae^2}d-7a^2e^4\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\cdots}{\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}\cdots}\right.}{4\sqrt{2}a\left(cd^2+ae^2\right)^{7/2}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(5/2)*(a + c*x^2)^2),x]
```

```
[Out] (e*(3*c*d^2 - 7*a*e^2))/(6*a*(c*d^2 + a*e^2)^2*(d + e*x)^(3/2)) + (c*d*e*(c
*d^2 - 19*a*e^2))/(2*a*(c*d^2 + a*e^2)^3*Sqrt[d + e*x]) + (a*e + c*d*x)/(2*
a*(c*d^2 + a*e^2)*(d + e*x)^(3/2)*(a + c*x^2)) + (c^(3/4)*e*(c^2*d^4 + 34*a
*c*d^2*e^2 - 7*a^2*e^4 + Sqrt[c]*d*(c*d^2 - 19*a*e^2)*Sqrt[c*d^2 + a*e^2])*
ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] - Sqrt[2]*c^(1/4)*Sqrt[d + e
*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(4*Sqrt[2]*a*(c*d^2 + a*e^2)^(
7/2)*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (c^(3/4)*e*(c^2*d^4 + 34*a*c*
d^2*e^2 - 7*a^2*e^4 + Sqrt[c]*d*(c*d^2 - 19*a*e^2)*Sqrt[c*d^2 + a*e^2])*Arc
Tanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] + Sqrt[2]*c^(1/4)*Sqrt[d + e*x]
)/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(4*Sqrt[2]*a*(c*d^2 + a*e^2)^(7/2
)*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (c^(3/4)*e*(c^2*d^4 + 34*a*c*d^2
*e^2 - 7*a^2*e^4 - Sqrt[c]*d*(c*d^2 - 19*a*e^2)*Sqrt[c*d^2 + a*e^2])*Log[Sq
rt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*S
```

```
qrt[d + e*x] + Sqrt[c]*(d + e*x)])/(8*Sqrt[2]*a*(c*d^2 + a*e^2)^(7/2)*Sqrt[
Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]) + (c^(3/4)*e*(c^2*d^4 + 34*a*c*d^2*e^2 -
7*a^2*e^4 - Sqrt[c]*d*(c*d^2 - 19*a*e^2)*Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^
2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d +
 e*x] + Sqrt[c]*(d + e*x)])/(8*Sqrt[2]*a*(c*d^2 + a*e^2)^(7/2)*Sqrt[Sqrt[c]
*d + Sqrt[c*d^2 + a*e^2]])
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 829

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)
), x] + Dist[1/(c*d^2 + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d*f + a*e*g -
 c*(e*f - d*g)*x, x])/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x
] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{5/2}\left(a+cx^2\right)^2}\,dx = \frac{ae+cdx}{2a\left(cd^2+ae^2\right)(d+ex)^{3/2}\left(a+cx^2\right)} - \frac{\int \frac{\frac{1}{2}\left(-2cd^2-7ae^2\right)-\frac{5}{2}cdex}{(d+ex)^{5/2}\left(a+cx^2\right)}\,dx}{2a\left(cd^2+ae^2\right)}$$

$$= \frac{e\left(3cd^2-7ae^2\right)}{6a\left(cd^2+ae^2\right)^2(d+ex)^{3/2}} + \frac{ae+cdx}{2a\left(cd^2+ae^2\right)(d+ex)^{3/2}\left(a+cx^2\right)} - \frac{\int \frac{-cd\left(cd^2+6ae^2\right)-\frac{1}{2}ce(3c\ldots}{(d+ex)^{3/2}\left(a+cx^2\right)}}{2a\left(cd^2+ae^2\ldots\right)}$$

$$= \frac{e\left(3cd^2-7ae^2\right)}{6a\left(cd^2+ae^2\right)^2(d+ex)^{3/2}} + \frac{cde\left(cd^2-19ae^2\right)}{2a\left(cd^2+ae^2\right)^3\sqrt{d+ex}} + \frac{ae+cdx}{2a\left(cd^2+ae^2\right)(d+ex)^{3/2}\left(a+c\ldots\right.}$$

$$= \frac{e\left(3cd^2-7ae^2\right)}{6a\left(cd^2+ae^2\right)^2(d+ex)^{3/2}} + \frac{cde\left(cd^2-19ae^2\right)}{2a\left(cd^2+ae^2\right)^3\sqrt{d+ex}} + \frac{ae+cdx}{2a\left(cd^2+ae^2\right)(d+ex)^{3/2}\left(a+c\ldots\right.}$$

$$= \frac{e\left(3cd^2-7ae^2\right)}{6a\left(cd^2+ae^2\right)^2(d+ex)^{3/2}} + \frac{cde\left(cd^2-19ae^2\right)}{2a\left(cd^2+ae^2\right)^3\sqrt{d+ex}} + \frac{ae+cdx}{2a\left(cd^2+ae^2\right)(d+ex)^{3/2}\left(a+c\ldots\right.}$$

$$= \frac{e\left(3cd^2-7ae^2\right)}{6a\left(cd^2+ae^2\right)^2(d+ex)^{3/2}} + \frac{cde\left(cd^2-19ae^2\right)}{2a\left(cd^2+ae^2\right)^3\sqrt{d+ex}} + \frac{ae+cdx}{2a\left(cd^2+ae^2\right)(d+ex)^{3/2}\left(a+c\ldots\right.}$$

$$= \frac{e\left(3cd^2-7ae^2\right)}{6a\left(cd^2+ae^2\right)^2(d+ex)^{3/2}} + \frac{cde\left(cd^2-19ae^2\right)}{2a\left(cd^2+ae^2\right)^3\sqrt{d+ex}} + \frac{ae+cdx}{2a\left(cd^2+ae^2\right)(d+ex)^{3/2}\left(a+c\ldots\right.}$$

$$= \frac{e\left(3cd^2-7ae^2\right)}{6a\left(cd^2+ae^2\right)^2(d+ex)^{3/2}} + \frac{cde\left(cd^2-19ae^2\right)}{2a\left(cd^2+ae^2\right)^3\sqrt{d+ex}} + \frac{ae+cdx}{2a\left(cd^2+ae^2\right)(d+ex)^{3/2}\left(a+c\ldots\right.}$$

Mathematica [C] time = 0.538045, size = 321, normalized size = 0.35

$$\frac{-\frac{\left(3cd^2e-7ae^3\right)\left(\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{\sqrt{-a}\sqrt{c}d-ae}-\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\sqrt{-a}\sqrt{c}d+ae}\right)}{e}+15cd(d+ex)\left(\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{\sqrt{-a}\sqrt{c}d-ae}-\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\sqrt{-a}\sqrt{c}d+ae}\right)+\frac{6(ae+cd\ldots}{a+cx^2}}{12a(d+ex)^{3/2}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(5/2)*(a + c*x^2)^2),x]
```

```
[Out] ((6*(a*e + c*d*x))/(a + c*x^2) - ((3*c*d^2*e - 7*a*e^3)*(-(Hypergeometric2F
1[-3/2, 1, -1/2, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)]/(Sqrt[-a]*Sq
rt[c]*d + a*e)) + Hypergeometric2F1[-3/2, 1, -1/2, (Sqrt[c]*(d + e*x))/(Sqr
t[c]*d + Sqrt[-a]*e)]/(Sqrt[-a]*Sqrt[c]*d - a*e)))/e + 15*c*d*(d + e*x)*(-(
Hypergeometric2F1[-1/2, 1, 1/2, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e
)]/(Sqrt[-a]*Sqrt[c]*d + a*e)) + Hypergeometric2F1[-1/2, 1, 1/2, (Sqrt[c]*(
d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]/(Sqrt[-a]*Sqrt[c]*d - a*e)))/(12*a*(c*d
^2 + a*e^2)*(d + e*x)^(3/2))
```

Maple [B] time = 0.268, size = 10403, normalized size = 11.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(5/2)/(c*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+a\right)^2 (ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)^2*(e*x + d)^(5/2)), x)
```

Fricas [B] time = 18.9744, size = 18290, normalized size = 19.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(a^2*c^3*d^8 + 3*a^3*c^2*d^6*e^2 + 3*a^4*c*d^4*e^4 + a^5*d^2*e^6 +
(a*c^4*d^6*e^2 + 3*a^2*c^3*d^4*e^4 + 3*a^3*c^2*d^2*e^6 + a^4*c*e^8)*x^4 + 2
*(a*c^4*d^7*e + 3*a^2*c^3*d^5*e^3 + 3*a^3*c^2*d^3*e^5 + a^4*c*d*e^7)*x^3 +
(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*
e^8)*x^2 + 2*(a^2*c^3*d^7*e + 3*a^3*c^2*d^5*e^3 + 3*a^4*c*d^3*e^5 + a^5*d*e
^7)*x)*sqrt(-(4*c^6*d^9 + 63*a*c^5*d^7*e^2 + 189*a^2*c^4*d^5*e^4 - 1155*a^3
*c^3*d^3*e^6 + 315*a^4*c^2*d*e^8 + (a^3*c^7*d^14 + 7*a^4*c^6*d^12*e^2 + 21*
a^5*c^5*d^10*e^4 + 35*a^6*c^4*d^8*e^6 + 35*a^7*c^3*d^6*e^8 + 21*a^8*c^2*d^4
*e^10 + 7*a^9*c*d^2*e^12 + a^10*e^14)*sqrt(-(11025*c^9*d^12*e^6 + 171990*a*
c^8*d^10*e^8 + 494991*a^2*c^7*d^8*e^10 - 1360716*a^3*c^6*d^6*e^12 + 780831*
a^4*c^5*d^4*e^14 - 82026*a^5*c^4*d^2*e^16 + 2401*a^6*c^3*e^18)/(a^3*c^14*d^
28 + 14*a^4*c^13*d^26*e^2 + 91*a^5*c^12*d^24*e^4 + 364*a^6*c^11*d^22*e^6 +
1001*a^7*c^10*d^20*e^8 + 2002*a^8*c^9*d^18*e^10 + 3003*a^9*c^8*d^16*e^12 +
```

3432*a^10*c^7*d^14*e^14 + 3003*a^11*c^6*d^12*e^16 + 2002*a^12*c^5*d^10*e^18 + 1001*a^13*c^4*d^8*e^20 + 364*a^14*c^3*d^6*e^22 + 91*a^15*c^2*d^4*e^24 + 14*a^16*c*d^2*e^26 + a^17*e^28)))/(a^3*c^7*d^14 + 7*a^4*c^6*d^12*e^2 + 21*a^5*c^5*d^10*e^4 + 35*a^6*c^4*d^8*e^6 + 35*a^7*c^3*d^6*e^8 + 21*a^8*c^2*d^4*e^10 + 7*a^9*c*d^2*e^12 + a^10*e^14))*log((420*c^6*d^8*e^3 + 8421*a*c^5*d^6*e^5 + 36783*a^2*c^4*d^4*e^7 - 40817*a^3*c^3*d^2*e^9 + 2401*a^4*c^2*e^11)*sqrt(e*x + d) + (105*a^2*c^6*d^10*e^4 + 4389*a^3*c^5*d^8*e^6 + 26274*a^4*c^4*d^6*e^8 - 34142*a^5*c^3*d^4*e^10 + 7525*a^6*c^2*d^2*e^12 - 343*a^7*c*e^14 + 2*(a^3*c^9*d^19 + 15*a^4*c^8*d^17*e^2 + 64*a^5*c^7*d^15*e^4 + 112*a^6*c^6*d^13*e^6 + 42*a^7*c^5*d^11*e^8 - 154*a^8*c^4*d^9*e^10 - 280*a^9*c^3*d^7*e^12 - 216*a^10*c^2*d^5*e^14 - 83*a^11*c*d^3*e^16 - 13*a^12*d*e^18)*sqrt(-(11025*c^9*d^12*e^6 + 171990*a*c^8*d^10*e^8 + 494991*a^2*c^7*d^8*e^10 - 1360716*a^3*c^6*d^6*e^12 + 780831*a^4*c^5*d^4*e^14 - 82026*a^5*c^4*d^2*e^16 + 2401*a^6*c^3*e^18)/(a^3*c^14*d^28 + 14*a^4*c^13*d^26*e^2 + 91*a^5*c^12*d^24*e^4 + 364*a^6*c^11*d^22*e^6 + 1001*a^7*c^10*d^20*e^8 + 2002*a^8*c^9*d^18*e^10 + 3003*a^9*c^8*d^16*e^12 + 3432*a^10*c^7*d^14*e^14 + 3003*a^11*c^6*d^12*e^16 + 2002*a^12*c^5*d^10*e^18 + 1001*a^13*c^4*d^8*e^20 + 364*a^14*c^3*d^6*e^22 + 91*a^15*c^2*d^4*e^24 + 14*a^16*c*d^2*e^26 + a^17*e^28)))*sqrt(-(4*c^6*d^9 + 63*a*c^5*d^7*e^2 + 189*a^2*c^4*d^5*e^4 - 1155*a^3*c^3*d^3*e^6 + 315*a^4*c^2*d*e^8 + (a^3*c^7*d^14 + 7*a^4*c^6*d^12*e^2 + 21*a^5*c^5*d^10*e^4 + 35*a^6*c^4*d^8*e^6 + 35*a^7*c^3*d^6*e^8 + 21*a^8*c^2*d^4*e^10 + 7*a^9*c*d^2*e^12 + a^10*e^14)*sqrt(-(11025*c^9*d^12*e^6 + 171990*a*c^8*d^10*e^8 + 494991*a^2*c^7*d^8*e^10 - 1360716*a^3*c^6*d^6*e^12 + 780831*a^4*c^5*d^4*e^14 - 82026*a^5*c^4*d^2*e^16 + 2401*a^6*c^3*e^18)/(a^3*c^14*d^28 + 14*a^4*c^13*d^26*e^2 + 91*a^5*c^12*d^24*e^4 + 364*a^6*c^11*d^22*e^6 + 1001*a^7*c^10*d^20*e^8 + 2002*a^8*c^9*d^18*e^10 + 3003*a^9*c^8*d^16*e^12 + 3432*a^10*c^7*d^14*e^14 + 3003*a^11*c^6*d^12*e^16 + 2002*a^12*c^5*d^10*e^18 + 1001*a^13*c^4*d^8*e^20 + 364*a^14*c^3*d^6*e^22 + 91*a^15*c^2*d^4*e^24 + 14*a^16*c*d^2*e^26 + a^17*e^28)))/(a^3*c^7*d^14 + 7*a^4*c^6*d^12*e^2 + 21*a^5*c^5*d^10*e^4 + 35*a^6*c^4*d^8*e^6 + 35*a^7*c^3*d^6*e^8 + 21*a^8*c^2*d^4*e^10 + 7*a^9*c*d^2*e^12 + a^10*e^14))) - 3*(a^2*c^3*d^8 + 3*a^3*c^2*d^6*e^2 + 3*a^4*c*d^4*e^4 + a^5*d^2*e^6 + (a*c^4*d^6*e^2 + 3*a^2*c^3*d^4*e^4 + 3*a^3*c^2*d^2*e^6 + a^4*c*e^8)*x^4 + 2*(a*c^4*d^7*e + 3*a^2*c^3*d^5*e^3 + 3*a^3*c^2*d^3*e^5 + a^4*c*d*e^7)*x^3 + (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*x^2 + 2*(a^2*c^3*d^7*e + 3*a^3*c^2*d^5*e^3 + 3*a^4*c*d^3*e^5 + a^5*d*e^7)*x)*sqrt(-(4*c^6*d^9 + 63*a*c^5*d^7*e^2 + 189*a^2*c^4*d^5*e^4 - 1155*a^3*c^3*d^3*e^6 + 315*a^4*c^2*d*e^8 + (a^3*c^7*d^14 + 7*a^4*c^6*d^12*e^2 + 21*a^5*c^5*d^10*e^4 + 35*a^6*c^4*d^8*e^6 + 35*a^7*c^3*d^6*e^8 + 21*a^8*c^2*d^4*e^10 + 7*a^9*c*d^2*e^12 + a^10*e^14)*sqrt(-(11025*c^9*d^12*e^6 + 171990*a*c^8*d^10*e^8 + 494991*a^2*c^7*d^8*e^10 - 1360716*a^3*c^6*d^6*e^12 + 780831*a^4*c^5*d^4*e^14 - 82026*a^5*c^4*d^2*e^16 + 2401*a^6*c^3*e^18)/(a^3*c^14*d^28 + 14*a^4*c^13*d^26*e^2 + 91*a^5*c^12*d^24*e^4 + 364*a^6*c^11*d^22*e^6 + 1001*a^7*c^10*d^20*e^8 + 2002*a^8*c^9*d^18*e^10 + 3003*a^9*c^8*d^16*e^12 + 3432*a^10*c^7*d^14*e^14 + 3003*a^11*c^6*d^12*e^16 + 2002*a^12*c^5*d^10*e^18 + 1001*a^13*c^4*d^8*e^20 + 364*a^14*c^3*d^6*e^22 + 91*a^15*c^2*d^4*e^24 + 14*a^16*c*d^2*e^26 + a^17*e^28)))/(a^3*c^7*d^14 + 7*a^4*c^6*d^12*e^2 + 21*a^5*c^5*d^10*e^4 + 35*a^6*c^4*d^8*e^6 + 35*a^7*c^3*d^6*e^8 + 21*a^8*c^2*d^4*e^10 + 7*a^9*c*d^2*e^12 + a^10*e^14))*log((420*c^6*d^8*e^3 + 8421*a*c^5*d^6*e^5 + 36783*a^2*c^4*d^4*e^7 - 40817*a^3*c^3*d^2*e^9 + 2401*a^4*c^2*e^11)*sqrt(e*x + d) - (105*a^2*c^6*d^10*e^4 + 4389*a^3*c^5*d^8*e^6 + 26274*a^4*c^4*d^6*e^8 - 34142*a^5*c^3*d^4*e^10 + 7525*a^6*c^2*d^2*e^12 - 343*a^7*c*e^14 + 2*(a^3*c^9*d^19 + 15*a^4*c^8*d^17*e^2 + 64*a^5*c^7*d^15*e^4 + 112*a^6*c^6*d^13*e^6 + 42*a^7*c^5*d^11*e^8 - 154*a^8*c^4*d^9*e^10 - 280*a^9*c^3*d^7*e^12 - 216*a^10*c^2*d^5*e^14 - 83*a^11*c*d^3*e^16 - 13*a^12*d*e^18)*sqrt(-(11025*c^9*d^12*e^6 + 171990*a*c^8*d^10*e^8 + 494991*a^2*c^7*d^8*e^10 - 1360716*a^3*c^6*d^6*e^12 + 780831*a^4*c^5*d^4*e^14 - 82026*a^5*c^4*d^2*e^16 + 2401*a^6*c^3*e^18)/(a^3*c^14*d^28 + 14*a^4*c^13*d^26*e^2 + 91*a^5*c^12*d^24*e^4 + 364*a^6*c^11*d^22*e^6 + 1001*a^7*c^10*d^20*e^8 + 2002*a^8*c^9*d^18*e^10 + 3003*a^9*c^8*d^16*e^12 + 3432*a^10*c^7*d^14*e^14 + 3003*a

```
^11*c^6*d^12*e^16 + 2002*a^12*c^5*d^10*e^18 + 1001*a^13*c^4*d^8*e^20 + 364*
a^14*c^3*d^6*e^22 + 91*a^15*c^2*d^4*e^24 + 14*a^16*c*d^2*e^26 + a^17*e^28))
)*sqrt(-(4*c^6*d^9 + 63*a*c^5*d^7*e^2 + 189*a^2*c^4*d^5*e^4 - 1155*a^3*c^3*
d^3*e^6 + 315*a^4*c^2*d*e^8 + (a^3*c^7*d^14 + 7*a^4*c^6*d^12*e^2 + 21*a^5*c
^5*d^10*e^4 + 35*a^6*c^4*d^8*e^6 + 35*a^7*c^3*d^6*e^8 + 21*a^8*c^2*d^4*e^10
 + 7*a^9*c*d^2*e^12 + a^10*e^14)*sqrt(-(11025*c^9*d^12*e^6 + 171990*a*c^8*d
^10*e^8 + 494991*a^2*c^7*d^8*e^10 - 1360716*a^3*c^6*d^6*e^12 + 780831*a^4*c
^5*d^4*e^14 - 82026*a^5*c^4*d^2*e^16 + 2401*a^6*c^3*e^18)/(a^3*c^14*d^28 +
14*a^4*c^13*d^26*e^2 + 91*a^5*c^12*d^24*e^4 + 364*a^6*c^11*d^22*e^6 + 1001*
a^7*c^10*d^20*e^8 + 2002*a^8*c^9*d^18*e^10 + 3003*a^9*c^8*d^16*e^12 + 3432*
a^10*c^7*d^14*e^14 + 3003*a^11*c^6*d^12*e^16 + 2002*a^12*c^5*d^10*e^18 + 10
01*a^13*c^4*d^8*e^20 + 364*a^14*c^3*d^6*e^22 + 91*a^15*c^2*d^4*e^24 + 14*a^
16*c*d^2*e^26 + a^17*e^28)))/(a^3*c^7*d^14 + 7*a^4*c^6*d^12*e^2 + 21*a^5*c^
5*d^10*e^4 + 35*a^6*c^4*d^8*e^6 + 35*a^7*c^3*d^6*e^8 + 21*a^8*c^2*d^4*e^10
+ 7*a^9*c*d^2*e^12 + a^10*e^14))) + 3*(a^2*c^3*d^8 + 3*a^3*c^2*d^6*e^2 + 3*
a^4*c*d^4*e^4 + a^5*d^2*e^6 + (a*c^4*d^6*e^2 + 3*a^2*c^3*d^4*e^4 + 3*a^3*c^
2*d^2*e^6 + a^4*c*e^8)*x^4 + 2*(a*c^4*d^7*e + 3*a^2*c^3*d^5*e^3 + 3*a^3*c^2
*d^3*e^5 + a^4*c*d*e^7)*x^3 + (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^
4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*x^2 + 2*(a^2*c^3*d^7*e + 3*a^3*c^2*d^5*e
^3 + 3*a^4*c*d^3*e^5 + a^5*d*e^7)*x)*sqrt(-(4*c^6*d^9 + 63*a*c^5*d^7*e^2 +
189*a^2*c^4*d^5*e^4 - 1155*a^3*c^3*d^3*e^6 + 315*a^4*c^2*d*e^8 - (a^3*c^7*d
^14 + 7*a^4*c^6*d^12*e^2 + 21*a^5*c^5*d^10*e^4 + 35*a^6*c^4*d^8*e^6 + 35*a^
7*c^3*d^6*e^8 + 21*a^8*c^2*d^4*e^10 + 7*a^9*c*d^2*e^12 + a^10*e^14)*sqrt(-(
11025*c^9*d^12*e^6 + 171990*a*c^8*d^10*e^8 + 494991*a^2*c^7*d^8*e^10 - 1360
716*a^3*c^6*d^6*e^12 + 780831*a^4*c^5*d^4*e^14 - 82026*a^5*c^4*d^2*e^16 + 2
401*a^6*c^3*e^18)/(a^3*c^14*d^28 + 14*a^4*c^13*d^26*e^2 + 91*a^5*c^12*d^24*
e^4 + 364*a^6*c^11*d^22*e^6 + 1001*a^7*c^10*d^20*e^8 + 2002*a^8*c^9*d^18*e^
10 + 3003*a^9*c^8*d^16*e^12 + 3432*a^10*c^7*d^14*e^14 + 3003*a^11*c^6*d^12*
e^16 + 2002*a^12*c^5*d^10*e^18 + 1001*a^13*c^4*d^8*e^20 + 364*a^14*c^3*d^6*
e^22 + 91*a^15*c^2*d^4*e^24 + 14*a^16*c*d^2*e^26 + a^17*e^28)))/(a^3*c^7*d^
14 + 7*a^4*c^6*d^12*e^2 + 21*a^5*c^5*d^10*e^4 + 35*a^6*c^4*d^8*e^6 + 35*a^7
*c^3*d^6*e^8 + 21*a^8*c^2*d^4*e^10 + 7*a^9*c*d^2*e^12 + a^10*e^14))*log((42
0*c^6*d^8*e^3 + 8421*a*c^5*d^6*e^5 + 36783*a^2*c^4*d^4*e^7 - 40817*a^3*c^3*
d^2*e^9 + 2401*a^4*c^2*e^11)*sqrt(e*x + d) + (105*a^2*c^6*d^10*e^4 + 4389*a
^3*c^5*d^8*e^6 + 26274*a^4*c^4*d^6*e^8 - 34142*a^5*c^3*d^4*e^10 + 7525*a^6*
c^2*d^2*e^12 - 343*a^7*c*e^14 - 2*(a^3*c^9*d^19 + 15*a^4*c^8*d^17*e^2 + 64*
a^5*c^7*d^15*e^4 + 112*a^6*c^6*d^13*e^6 + 42*a^7*c^5*d^11*e^8 - 154*a^8*c^4
*d^9*e^10 - 280*a^9*c^3*d^7*e^12 - 216*a^10*c^2*d^5*e^14 - 83*a^11*c*d^3*e^
16 - 13*a^12*d*e^18)*sqrt(-(11025*c^9*d^12*e^6 + 171990*a*c^8*d^10*e^8 + 49
4991*a^2*c^7*d^8*e^10 - 1360716*a^3*c^6*d^6*e^12 + 780831*a^4*c^5*d^4*e^14
- 82026*a^5*c^4*d^2*e^16 + 2401*a^6*c^3*e^18)/(a^3*c^14*d^28 + 14*a^4*c^13*
d^26*e^2 + 91*a^5*c^12*d^24*e^4 + 364*a^6*c^11*d^22*e^6 + 1001*a^7*c^10*d^2
0*e^8 + 2002*a^8*c^9*d^18*e^10 + 3003*a^9*c^8*d^16*e^12 + 3432*a^10*c^7*d^1
4*e^14 + 3003*a^11*c^6*d^12*e^16 + 2002*a^12*c^5*d^10*e^18 + 1001*a^13*c^4*
d^8*e^20 + 364*a^14*c^3*d^6*e^22 + 91*a^15*c^2*d^4*e^24 + 14*a^16*c*d^2*e^2
6 + a^17*e^28)))*sqrt(-(4*c^6*d^9 + 63*a*c^5*d^7*e^2 + 189*a^2*c^4*d^5*e^4
- 1155*a^3*c^3*d^3*e^6 + 315*a^4*c^2*d*e^8 - (a^3*c^7*d^14 + 7*a^4*c^6*d^12
*e^2 + 21*a^5*c^5*d^10*e^4 + 35*a^6*c^4*d^8*e^6 + 35*a^7*c^3*d^6*e^8 + 21*a
^8*c^2*d^4*e^10 + 7*a^9*c*d^2*e^12 + a^10*e^14)*sqrt(-(11025*c^9*d^12*e^6 +
 171990*a*c^8*d^10*e^8 + 494991*a^2*c^7*d^8*e^10 - 1360716*a^3*c^6*d^6*e^12
 + 780831*a^4*c^5*d^4*e^14 - 82026*a^5*c^4*d^2*e^16 + 2401*a^6*c^3*e^18)/(a
^3*c^14*d^28 + 14*a^4*c^13*d^26*e^2 + 91*a^5*c^12*d^24*e^4 + 364*a^6*c^11*d
^22*e^6 + 1001*a^7*c^10*d^20*e^8 + 2002*a^8*c^9*d^18*e^10 + 3003*a^9*c^8*d^
16*e^12 + 3432*a^10*c^7*d^14*e^14 + 3003*a^11*c^6*d^12*e^16 + 2002*a^12*c^5
*d^10*e^18 + 1001*a^13*c^4*d^8*e^20 + 364*a^14*c^3*d^6*e^22 + 91*a^15*c^2*d
^4*e^24 + 14*a^16*c*d^2*e^26 + a^17*e^28)))/(a^3*c^7*d^14 + 7*a^4*c^6*d^12*
e^2 + 21*a^5*c^5*d^10*e^4 + 35*a^6*c^4*d^8*e^6 + 35*a^7*c^3*d^6*e^8 + 21*a^
8*c^2*d^4*e^10 + 7*a^9*c*d^2*e^12 + a^10*e^14))) - 3*(a^2*c^3*d^8 + 3*a^3*c
^2*d^6*e^2 + 3*a^4*c*d^4*e^4 + a^5*d^2*e^6 + (a*c^4*d^6*e^2 + 3*a^2*c^3*d^4
```

```
*e^4 + 3*a^3*c^2*d^2*e^6 + a^4*c*e^8)*x^4 + 2*(a*c^4*d^7*e + 3*a^2*c^3*d^5*
e^3 + 3*a^3*c^2*d^3*e^5 + a^4*c*d*e^7)*x^3 + (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2
 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*x^2 + 2*(a^2*c^3*d^7*e +
3*a^3*c^2*d^5*e^3 + 3*a^4*c*d^3*e^5 + a^5*d*e^7)*x)*sqrt(-(4*c^6*d^9 + 63*a
*c^5*d^7*e^2 + 189*a^2*c^4*d^5*e^4 - 1155*a^3*c^3*d^3*e^6 + 315*a^4*c^2*d*e
^8 - (a^3*c^7*d^14 + 7*a^4*c^6*d^12*e^2 + 21*a^5*c^5*d^10*e^4 + 35*a^6*c^4*
d^8*e^6 + 35*a^7*c^3*d^6*e^8 + 21*a^8*c^2*d^4*e^10 + 7*a^9*c*d^2*e^12 + a^1
0*e^14)*sqrt(-(11025*c^9*d^12*e^6 + 171990*a*c^8*d^10*e^8 + 494991*a^2*c^7*
d^8*e^10 - 1360716*a^3*c^6*d^6*e^12 + 780831*a^4*c^5*d^4*e^14 - 82026*a^5*c
^4*d^2*e^16 + 2401*a^6*c^3*e^18)/(a^3*c^14*d^28 + 14*a^4*c^13*d^26*e^2 + 91
*a^5*c^12*d^24*e^4 + 364*a^6*c^11*d^22*e^6 + 1001*a^7*c^10*d^20*e^8 + 2002*
a^8*c^9*d^18*e^10 + 3003*a^9*c^8*d^16*e^12 + 3432*a^10*c^7*d^14*e^14 + 3003
*a^11*c^6*d^12*e^16 + 2002*a^12*c^5*d^10*e^18 + 1001*a^13*c^4*d^8*e^20 + 36
4*a^14*c^3*d^6*e^22 + 91*a^15*c^2*d^4*e^24 + 14*a^16*c*d^2*e^26 + a^17*e^28
)))/(a^3*c^7*d^14 + 7*a^4*c^6*d^12*e^2 + 21*a^5*c^5*d^10*e^4 + 35*a^6*c^4*d
^8*e^6 + 35*a^7*c^3*d^6*e^8 + 21*a^8*c^2*d^4*e^10 + 7*a^9*c*d^2*e^12 + a^10
*e^14))*log((420*c^6*d^8*e^3 + 8421*a*c^5*d^6*e^5 + 36783*a^2*c^4*d^4*e^7 -
 40817*a^3*c^3*d^2*e^9 + 2401*a^4*c^2*e^11)*sqrt(e*x + d) - (105*a^2*c^6*d^
10*e^4 + 4389*a^3*c^5*d^8*e^6 + 26274*a^4*c^4*d^6*e^8 - 34142*a^5*c^3*d^4*e
^10 + 7525*a^6*c^2*d^2*e^12 - 343*a^7*c*e^14 - 2*(a^3*c^9*d^19 + 15*a^4*c^8
*d^17*e^2 + 64*a^5*c^7*d^15*e^4 + 112*a^6*c^6*d^13*e^6 + 42*a^7*c^5*d^11*e^
8 - 154*a^8*c^4*d^9*e^10 - 280*a^9*c^3*d^7*e^12 - 216*a^10*c^2*d^5*e^14 - 8
3*a^11*c*d^3*e^16 - 13*a^12*d*e^18)*sqrt(-(11025*c^9*d^12*e^6 + 171990*a*c^
8*d^10*e^8 + 494991*a^2*c^7*d^8*e^10 - 1360716*a^3*c^6*d^6*e^12 + 780831*a^
4*c^5*d^4*e^14 - 82026*a^5*c^4*d^2*e^16 + 2401*a^6*c^3*e^18)/(a^3*c^14*d^28
 + 14*a^4*c^13*d^26*e^2 + 91*a^5*c^12*d^24*e^4 + 364*a^6*c^11*d^22*e^6 + 10
01*a^7*c^10*d^20*e^8 + 2002*a^8*c^9*d^18*e^10 + 3003*a^9*c^8*d^16*e^12 + 34
32*a^10*c^7*d^14*e^14 + 3003*a^11*c^6*d^12*e^16 + 2002*a^12*c^5*d^10*e^18 +
 1001*a^13*c^4*d^8*e^20 + 364*a^14*c^3*d^6*e^22 + 91*a^15*c^2*d^4*e^24 + 14
*a^16*c*d^2*e^26 + a^17*e^28)))*sqrt(-(4*c^6*d^9 + 63*a*c^5*d^7*e^2 + 189*a
^2*c^4*d^5*e^4 - 1155*a^3*c^3*d^3*e^6 + 315*a^4*c^2*d*e^8 - (a^3*c^7*d^14 +
 7*a^4*c^6*d^12*e^2 + 21*a^5*c^5*d^10*e^4 + 35*a^6*c^4*d^8*e^6 + 35*a^7*c^3
*d^6*e^8 + 21*a^8*c^2*d^4*e^10 + 7*a^9*c*d^2*e^12 + a^10*e^14)*sqrt(-(11025
*c^9*d^12*e^6 + 171990*a*c^8*d^10*e^8 + 494991*a^2*c^7*d^8*e^10 - 1360716*a
^3*c^6*d^6*e^12 + 780831*a^4*c^5*d^4*e^14 - 82026*a^5*c^4*d^2*e^16 + 2401*a
^6*c^3*e^18)/(a^3*c^14*d^28 + 14*a^4*c^13*d^26*e^2 + 91*a^5*c^12*d^24*e^4 +
 364*a^6*c^11*d^22*e^6 + 1001*a^7*c^10*d^20*e^8 + 2002*a^8*c^9*d^18*e^10 +
3003*a^9*c^8*d^16*e^12 + 3432*a^10*c^7*d^14*e^14 + 3003*a^11*c^6*d^12*e^16
+ 2002*a^12*c^5*d^10*e^18 + 1001*a^13*c^4*d^8*e^20 + 364*a^14*c^3*d^6*e^22
+ 91*a^15*c^2*d^4*e^24 + 14*a^16*c*d^2*e^26 + a^17*e^28)))/(a^3*c^7*d^14 +
7*a^4*c^6*d^12*e^2 + 21*a^5*c^5*d^10*e^4 + 35*a^6*c^4*d^8*e^6 + 35*a^7*c^3*
d^6*e^8 + 21*a^8*c^2*d^4*e^10 + 7*a^9*c*d^2*e^12 + a^10*e^14))) + 4*(9*a*c^
2*d^4*e - 55*a^2*c*d^2*e^3 - 4*a^3*e^5 + 3*(c^3*d^3*e^2 - 19*a*c^2*d*e^4)*x
^3 + (6*c^3*d^4*e - 61*a*c^2*d^2*e^3 - 7*a^2*c*e^5)*x^2 + 3*(c^3*d^5 + 3*a*
c^2*d^3*e^2 - 18*a^2*c*d*e^4)*x)*sqrt(e*x + d))/(a^2*c^3*d^8 + 3*a^3*c^2*d^
6*e^2 + 3*a^4*c*d^4*e^4 + a^5*d^2*e^6 + (a*c^4*d^6*e^2 + 3*a^2*c^3*d^4*e^4
+ 3*a^3*c^2*d^2*e^6 + a^4*c*e^8)*x^4 + 2*(a*c^4*d^7*e + 3*a^2*c^3*d^5*e^3 +
 3*a^3*c^2*d^3*e^5 + a^4*c*d*e^7)*x^3 + (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*
a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*x^2 + 2*(a^2*c^3*d^7*e + 3*a^3
*c^2*d^5*e^3 + 3*a^4*c*d^3*e^5 + a^5*d*e^7)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(5/2)/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.638 $\int \frac{(d+ex)^{7/2}}{\left(a-cx^2\right)^3}\,dx$

Optimal. Leaf size=294

$$\frac{\sqrt{d+ex}\left(2cdx\left(3cd^2-2ae^2\right)+ae\left(7cd^2-5ae^2\right)\right)}{16a^2c^2\left(a-cx^2\right)}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}\left(18\sqrt{a}\sqrt{c}de+5ae^2+12cd^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{32a^{5/2}c^{9/4}}+$$

```
[Out] ((a*e + c*d*x)*(d + e*x)^(5/2))/(4*a*c*(a - c*x^2)^2) + (Sqrt[d + e*x]*(a*e
*(7*c*d^2 - 5*a*e^2) + 2*c*d*(3*c*d^2 - 2*a*e^2)*x))/(16*a^2*c^2*(a - c*x^2
)) - ((Sqrt[c]*d - Sqrt[a]*e)^(3/2)*(12*c*d^2 + 18*Sqrt[a]*Sqrt[c]*d*e + 5*
a*e^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(32*a^
(5/2)*c^(9/4)) + ((Sqrt[c]*d + Sqrt[a]*e)^(3/2)*(12*c*d^2 - 18*Sqrt[a]*Sqrt
[c]*d*e + 5*a*e^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]
*e]])/(32*a^(5/2)*c^(9/4))
```

Rubi [A] time = 0.524751, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {739, 819, 827, 1166, 208}

$$\frac{\sqrt{d+ex}\left(2cdx\left(3cd^2-2ae^2\right)+ae\left(7cd^2-5ae^2\right)\right)}{16a^2c^2\left(a-cx^2\right)}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}\left(18\sqrt{a}\sqrt{c}de+5ae^2+12cd^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{32a^{5/2}c^{9/4}}+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/(a - c*x^2)^3,x]
```

```
[Out] ((a*e + c*d*x)*(d + e*x)^(5/2))/(4*a*c*(a - c*x^2)^2) + (Sqrt[d + e*x]*(a*e
*(7*c*d^2 - 5*a*e^2) + 2*c*d*(3*c*d^2 - 2*a*e^2)*x))/(16*a^2*c^2*(a - c*x^2
)) - ((Sqrt[c]*d - Sqrt[a]*e)^(3/2)*(12*c*d^2 + 18*Sqrt[a]*Sqrt[c]*d*e + 5*
a*e^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(32*a^
(5/2)*c^(9/4)) + ((Sqrt[c]*d + Sqrt[a]*e)^(3/2)*(12*c*d^2 - 18*Sqrt[a]*Sqrt
[c]*d*e + 5*a*e^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]
*e]])/(32*a^(5/2)*c^(9/4))
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
```

```
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{7/2}}{\left(a-cx^2\right)^3}\,dx = \frac{(ae+cdx)(d+ex)^{5/2}}{4ac\left(a-cx^2\right)^2} - \frac{\int \frac{(d+ex)^{3/2}\left(\frac{1}{2}\left(-6cd^2+5ae^2\right)-\frac{1}{2}cdex\right)}{\left(a-cx^2\right)^2}\,dx}{4ac}$$

$$= \frac{(ae+cdx)(d+ex)^{5/2}}{4ac\left(a-cx^2\right)^2} + \frac{\sqrt{d+ex}\left(ae\left(7cd^2-5ae^2\right)+2cd\left(3cd^2-2ae^2\right)x\right)}{16a^2c^2\left(a-cx^2\right)} + \frac{\int \frac{\frac{1}{4}\left(4cd^2-5ae^2\right)\left(3cd^2-ae^2\right)+\frac{1}{2}cd\ldots}{\sqrt{d+ex}\left(a-cx^2\right)}\ldots}{8a^2c^2}$$

$$= \frac{(ae+cdx)(d+ex)^{5/2}}{4ac\left(a-cx^2\right)^2} + \frac{\sqrt{d+ex}\left(ae\left(7cd^2-5ae^2\right)+2cd\left(3cd^2-2ae^2\right)x\right)}{16a^2c^2\left(a-cx^2\right)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}cd^2e\left(3cd^2-4ae^2\right)\ldots}{\ldots}\right)}{\ldots}$$

$$= \frac{(ae+cdx)(d+ex)^{5/2}}{4ac\left(a-cx^2\right)^2} + \frac{\sqrt{d+ex}\left(ae\left(7cd^2-5ae^2\right)+2cd\left(3cd^2-2ae^2\right)x\right)}{16a^2c^2\left(a-cx^2\right)} + \frac{\left(\left(\sqrt{c}d+\sqrt{a}e\right)^2\left(12cd^2-1\ldots\right.\right.}{\ldots}$$

$$= \frac{(ae+cdx)(d+ex)^{5/2}}{4ac\left(a-cx^2\right)^2} + \frac{\sqrt{d+ex}\left(ae\left(7cd^2-5ae^2\right)+2cd\left(3cd^2-2ae^2\right)x\right)}{16a^2c^2\left(a-cx^2\right)} - \frac{\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}\left(12cd^2+\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.710775, size = 319, normalized size = 1.09

$$\frac{\frac{2\sqrt{a}\sqrt[4]{c}\sqrt{d+ex}\left(a^2ce\left(11d^2+4dex+9e^2x^2\right)-5a^3e^3+ac^2dx\left(10d^2+dex+8e^2x^2\right)-6c^3d^3x^3\right)}{\left(a-cx^2\right)^2} - \sqrt{\sqrt{c}d-\sqrt{a}e}\left(-5a^{3/2}e^3+6\sqrt{a}cd^2e-13a\sqrt{c}de^2+12c^{3/2}\ldots\right.}{32a^{5/2}c^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)/(a - c*x^2)^3,x]
```

```
[Out] ((2*Sqrt[a]*c^(1/4)*Sqrt[d + e*x]*(-5*a^3*e^3 - 6*c^3*d^3*x^3 + a*c^2*d*x*(
10*d^2 + d*e*x + 8*e^2*x^2) + a^2*c*e*(11*d^2 + 4*d*e*x + 9*e^2*x^2)))/(a -
 c*x^2)^2 - Sqrt[Sqrt[c]*d - Sqrt[a]*e]*(12*c^(3/2)*d^3 + 6*Sqrt[a]*c*d^2*e
 - 13*a*Sqrt[c]*d*e^2 - 5*a^(3/2)*e^3)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt
[Sqrt[c]*d - Sqrt[a]*e]] + Sqrt[Sqrt[c]*d + Sqrt[a]*e]*(12*c^(3/2)*d^3 - 6*
Sqrt[a]*c*d^2*e - 13*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*ArcTanh[(c^(1/4)*Sqrt
[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(32*a^(5/2)*c^(9/4))
```

Maple [B] time = 0.234, size = 986, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/(-c*x^2+a)^3,x)
```

```
[Out] 1/2*e^3/(c*e^2*x^2-a*e^2)^2*d/a*(e*x+d)^(7/2)-3/8*e/(c*e^2*x^2-a*e^2)^2*d^3
/a^2*(e*x+d)^(7/2)*c+9/16*e^5/(c*e^2*x^2-a*e^2)^2/c*(e*x+d)^(5/2)-23/16*e^3
/(c*e^2*x^2-a*e^2)^2/a*(e*x+d)^(5/2)*d^2+9/8*e/(c*e^2*x^2-a*e^2)^2/a^2*c*(e
*x+d)^(5/2)*d^4-7/8*e^5/(c*e^2*x^2-a*e^2)^2*d/c*(e*x+d)^(3/2)+2*e^3/(c*e^2*
x^2-a*e^2)^2*d^3/a*(e*x+d)^(3/2)-9/8*e/(c*e^2*x^2-a*e^2)^2*d^5/a^2*c*(e*x+d
)^(3/2)-5/16*e^7/(c*e^2*x^2-a*e^2)^2/c^2*a*(e*x+d)^(1/2)+e^5/(c*e^2*x^2-a*e
^2)^2/c*(e*x+d)^(1/2)*d^2-17/16*e^3/(c*e^2*x^2-a*e^2)^2/a*(e*x+d)^(1/2)*d^4
+3/8*e/(c*e^2*x^2-a*e^2)^2*c/a^2*(e*x+d)^(1/2)*d^6+5/32*e^5/c/(a*c*e^2)^(1/
2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)
^(1/2))*c)^(1/2))-19/32*e^3/a/(a*c*e^2)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1
/2)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d^2+3/8*e/a^2*
c/(a*c*e^2)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c/(
(-c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d^4+1/4*e^3/a/c/((-c*d+(a*c*e^2)^(1/2))*c)
^(1/2)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d-3/16*e/a^
2/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^
(1/2))*c)^(1/2))*d^3+5/32*e^5/c/(a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(
1/2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))-19/32*e^3/a/(
a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c/((c*
d+(a*c*e^2)^(1/2))*c)^(1/2))*d^2+3/8*e/a^2*c/(a*c*e^2)^(1/2)/((c*d+(a*c*e^2
)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))*
d^4-1/4*e^3/a/c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c/((c
*d+(a*c*e^2)^(1/2))*c)^(1/2))*d+3/16*e/a^2/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*
arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ex+d)^{\frac{7}{2}}}{\left(cx^2-a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(-c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -integrate((e*x + d)^(7/2)/(c*x^2 - a)^3, x)
```

Fricas [B] time = 3.28605, size = 3876, normalized size = 13.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(-c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*((a^2*c^4*x^4 - 2*a^3*c^3*x^2 + a^4*c^2)*sqrt((144*c^3*d^7 - 420*a*c^2
*d^5*e^2 + 385*a^2*c*d^3*e^4 - 105*a^3*d*e^6 + a^5*c^4*sqrt((441*c^2*d^4*e^
10 - 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))/(a^5*c^4))*log((3024*c^4
*d^8*e^5 - 10908*a*c^3*d^6*e^7 + 13509*a^2*c^2*d^4*e^9 - 6250*a^3*c*d^2*e^1
1 + 625*a^4*e^13)*sqrt(e*x + d) + (126*a^3*c^4*d^4*e^6 - 255*a^4*c^3*d^2*e^
8 + 125*a^5*c^2*e^10 - (12*a^5*c^8*d^3 - 13*a^6*c^7*d*e^2)*sqrt((441*c^2*d^
4*e^10 - 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))*sqrt((144*c^3*d^7 -
420*a*c^2*d^5*e^2 + 385*a^2*c*d^3*e^4 - 105*a^3*d*e^6 + a^5*c^4*sqrt((441*c
^2*d^4*e^10 - 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))/(a^5*c^4))) - (
a^2*c^4*x^4 - 2*a^3*c^3*x^2 + a^4*c^2)*sqrt((144*c^3*d^7 - 420*a*c^2*d^5*e^
2 + 385*a^2*c*d^3*e^4 - 105*a^3*d*e^6 + a^5*c^4*sqrt((441*c^2*d^4*e^10 - 10
50*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))/(a^5*c^4))*log((3024*c^4*d^8*e^
5 - 10908*a*c^3*d^6*e^7 + 13509*a^2*c^2*d^4*e^9 - 6250*a^3*c*d^2*e^11 + 625
*a^4*e^13)*sqrt(e*x + d) - (126*a^3*c^4*d^4*e^6 - 255*a^4*c^3*d^2*e^8 + 125
*a^5*c^2*e^10 - (12*a^5*c^8*d^3 - 13*a^6*c^7*d*e^2)*sqrt((441*c^2*d^4*e^10
- 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))*sqrt((144*c^3*d^7 - 420*a*c
^2*d^5*e^2 + 385*a^2*c*d^3*e^4 - 105*a^3*d*e^6 + a^5*c^4*sqrt((441*c^2*d^4*
e^10 - 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))/(a^5*c^4))) + (a^2*c^4
*x^4 - 2*a^3*c^3*x^2 + a^4*c^2)*sqrt((144*c^3*d^7 - 420*a*c^2*d^5*e^2 + 385
*a^2*c*d^3*e^4 - 105*a^3*d*e^6 - a^5*c^4*sqrt((441*c^2*d^4*e^10 - 1050*a*c*
d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))/(a^5*c^4))*log((3024*c^4*d^8*e^5 - 109
08*a*c^3*d^6*e^7 + 13509*a^2*c^2*d^4*e^9 - 6250*a^3*c*d^2*e^11 + 625*a^4*e^
13)*sqrt(e*x + d) + (126*a^3*c^4*d^4*e^6 - 255*a^4*c^3*d^2*e^8 + 125*a^5*c^
2*e^10 + (12*a^5*c^8*d^3 - 13*a^6*c^7*d*e^2)*sqrt((441*c^2*d^4*e^10 - 1050*
a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))*sqrt((144*c^3*d^7 - 420*a*c^2*d^5*
e^2 + 385*a^2*c*d^3*e^4 - 105*a^3*d*e^6 - a^5*c^4*sqrt((441*c^2*d^4*e^10 -
1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))/(a^5*c^4))) - (a^2*c^4*x^4 -
2*a^3*c^3*x^2 + a^4*c^2)*sqrt((144*c^3*d^7 - 420*a*c^2*d^5*e^2 + 385*a^2*c*
d^3*e^4 - 105*a^3*d*e^6 - a^5*c^4*sqrt((441*c^2*d^4*e^10 - 1050*a*c*d^2*e^1
2 + 625*a^2*e^14)/(a^5*c^9)))/(a^5*c^4))*log((3024*c^4*d^8*e^5 - 10908*a*c^
3*d^6*e^7 + 13509*a^2*c^2*d^4*e^9 - 6250*a^3*c*d^2*e^11 + 625*a^4*e^13)*sqr
t(e*x + d) - (126*a^3*c^4*d^4*e^6 - 255*a^4*c^3*d^2*e^8 + 125*a^5*c^2*e^10
+ (12*a^5*c^8*d^3 - 13*a^6*c^7*d*e^2)*sqrt((441*c^2*d^4*e^10 - 1050*a*c*d^2
*e^12 + 625*a^2*e^14)/(a^5*c^9)))*sqrt((144*c^3*d^7 - 420*a*c^2*d^5*e^2 + 3
85*a^2*c*d^3*e^4 - 105*a^3*d*e^6 - a^5*c^4*sqrt((441*c^2*d^4*e^10 - 1050*a*
c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))/(a^5*c^4))) + 4*(11*a^2*c*d^2*e - 5*
a^3*e^3 - 2*(3*c^3*d^3 - 4*a*c^2*d*e^2)*x^3 + (a*c^2*d^2*e + 9*a^2*c*e^3)*x
^2 + 2*(5*a*c^2*d^3 + 2*a^2*c*d*e^2)*x)*sqrt(e*x + d))/(a^2*c^4*x^4 - 2*a^3
*c^3*x^2 + a^4*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/(-c*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(-c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.639 $\int \frac{(d+ex)^{5/2}}{\left(a-cx^2\right)^3}\,dx$

Optimal. Leaf size=279

$$-\frac{3\sqrt{\sqrt{c}d-\sqrt{a}e}\left(2\sqrt{a}\sqrt{c}de-ae^2+4cd^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{32a^{5/2}c^{7/4}}+\frac{3\sqrt{\sqrt{a}e+\sqrt{c}d}\left(-2\sqrt{a}\sqrt{c}de-ae^2+4cd^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{32a^{5/2}c^{7/4}}$$

```
[Out] ((a*e + c*d*x)*(d + e*x)^(3/2))/(4*a*c*(a - c*x^2)^2) + (3*Sqrt[d + e*x]*(a
*d*e + (2*c*d^2 - a*e^2)*x))/(16*a^2*c*(a - c*x^2)) - (3*Sqrt[Sqrt[c]*d - S
qrt[a]*e]*(4*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTanh[(c^(1/4)*Sqrt[d
 + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(32*a^(5/2)*c^(7/4)) + (3*Sqrt[Sqrt[
c]*d + Sqrt[a]*e]*(4*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTanh[(c^(1/4
)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(32*a^(5/2)*c^(7/4))
```

Rubi [A] time = 0.458589, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {739, 821, 827, 1166, 208}

$$-\frac{3\sqrt{\sqrt{c}d-\sqrt{a}e}\left(2\sqrt{a}\sqrt{c}de-ae^2+4cd^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{32a^{5/2}c^{7/4}}+\frac{3\sqrt{\sqrt{a}e+\sqrt{c}d}\left(-2\sqrt{a}\sqrt{c}de-ae^2+4cd^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{32a^{5/2}c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(a - c*x^2)^3,x]
```

```
[Out] ((a*e + c*d*x)*(d + e*x)^(3/2))/(4*a*c*(a - c*x^2)^2) + (3*Sqrt[d + e*x]*(a
*d*e + (2*c*d^2 - a*e^2)*x))/(16*a^2*c*(a - c*x^2)) - (3*Sqrt[Sqrt[c]*d - S
qrt[a]*e]*(4*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTanh[(c^(1/4)*Sqrt[d
 + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(32*a^(5/2)*c^(7/4)) + (3*Sqrt[Sqrt[
c]*d + Sqrt[a]*e]*(4*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTanh[(c^(1/4
)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(32*a^(5/2)*c^(7/4))
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\left(a-cx^2\right)^3}\,dx = \frac{(ae+cdx)(d+ex)^{3/2}}{4ac\left(a-cx^2\right)^2} - \frac{\int \frac{\sqrt{d+ex}\left(-\frac{3}{2}\left(2cd^2-ae^2\right)-\frac{3}{2}cdex\right)}{\left(a-cx^2\right)^2}\,dx}{4ac}$$

$$= \frac{(ae+cdx)(d+ex)^{3/2}}{4ac\left(a-cx^2\right)^2} + \frac{3\sqrt{d+ex}\left(ade+\left(2cd^2-ae^2\right)x\right)}{16a^2c\left(a-cx^2\right)} + \frac{\int \frac{\frac{3}{4}cd\left(4cd^2-3ae^2\right)+\frac{3}{4}ce\left(2cd^2-ae^2\right)x}{\sqrt{d+ex}\left(a-cx^2\right)}\,dx}{8a^2c^2}$$

$$= \frac{(ae+cdx)(d+ex)^{3/2}}{4ac\left(a-cx^2\right)^2} + \frac{3\sqrt{d+ex}\left(ade+\left(2cd^2-ae^2\right)x\right)}{16a^2c\left(a-cx^2\right)} + \frac{\text{Subst}\left(\int \frac{\frac{3}{4}cde\left(4cd^2-3ae^2\right)-\frac{3}{4}cde\left(2cd^2-ae^2\right)+\frac{3}{4}}{-cd^2+ae^2+2cdx^2-cx^4}\right.}{4a^2c^2}$$

$$= \frac{(ae+cdx)(d+ex)^{3/2}}{4ac\left(a-cx^2\right)^2} + \frac{3\sqrt{d+ex}\left(ade+\left(2cd^2-ae^2\right)x\right)}{16a^2c\left(a-cx^2\right)} + \frac{\left(3\left(\sqrt{c}d+\sqrt{a}e\right)\left(4cd^2-2\sqrt{a}\sqrt{c}de-ae\right.\right.}{32}$$

$$= \frac{(ae+cdx)(d+ex)^{3/2}}{4ac\left(a-cx^2\right)^2} + \frac{3\sqrt{d+ex}\left(ade+\left(2cd^2-ae^2\right)x\right)}{16a^2c\left(a-cx^2\right)} - \frac{3\sqrt{\sqrt{c}d-\sqrt{a}e}\left(4cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)}{32a^{5/2}c^{7/4}}$$

Mathematica [A] time = 0.520435, size = 260, normalized size = 0.93

$$\frac{\frac{2\sqrt{a}c^{3/4}\sqrt{d+ex}\left(a^2e(7d+ex)+acx\left(10d^2+dex+3e^2x^2\right)-6c^2d^2x^3\right)}{\left(a-cx^2\right)^2} - 3\sqrt{\sqrt{c}d-\sqrt{a}e}\left(2\sqrt{a}\sqrt{c}de-ae^2+4cd^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right) + 3\sqrt{\sqrt{ae}}}{32a^{5/2}c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(a - c*x^2)^3,x]
```

```
[Out] ((2*Sqrt[a]*c^(3/4)*Sqrt[d + e*x]*(-6*c^2*d^2*x^3 + a^2*e*(7*d + e*x) + a*c
*x*(10*d^2 + d*e*x + 3*e^2*x^2)))/(a - c*x^2)^2 - 3*Sqrt[Sqrt[c]*d - Sqrt[a
]*e]*(4*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*
x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]] + 3*Sqrt[Sqrt[c]*d + Sqrt[a]*e]*(4*c*d^2 -
 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c
```

```
]*d + Sqrt[a]*e]])/(32*a^(5/2)*c^(7/4))
```

Maple [B] time = 0.226, size = 792, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(-c*x^2+a)^3,x)
```

```
[Out] 3/16*e^3/(c*e^2*x^2-a*e^2)^2/a*(e*x+d)^(7/2)-3/8*e/(c*e^2*x^2-a*e^2)^2/a^2*
(e*x+d)^(7/2)*c*d^2-1/2*e^3/(c*e^2*x^2-a*e^2)^2*d/a*(e*x+d)^(5/2)+9/8*e/(c*
e^2*x^2-a*e^2)^2*d^3/a^2*(e*x+d)^(5/2)*c+1/16*e^5/(c*e^2*x^2-a*e^2)^2/c*(e*
x+d)^(3/2)+17/16*e^3/(c*e^2*x^2-a*e^2)^2/a*(e*x+d)^(3/2)*d^2-9/8*e/(c*e^2*x
^2-a*e^2)^2/a^2*c*(e*x+d)^(3/2)*d^4+3/8*e^5/(c*e^2*x^2-a*e^2)^2*d/c*(e*x+d)
^(1/2)-3/4*e^3/(c*e^2*x^2-a*e^2)^2*d^3/a*(e*x+d)^(1/2)+3/8*e/(c*e^2*x^2-a*e
^2)^2*d^5/a^2*c*(e*x+d)^(1/2)-9/32*e^3/a/(a*c*e^2)^(1/2)/((-c*d+(a*c*e^2)^(
1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d+3
/8*e/a^2*c/(a*c*e^2)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^
(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d^3+3/32*e^3/a/c/((-c*d+(a*c*e^2)
^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))-3
/16*e/a^2/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((-c*d+(a
*c*e^2)^(1/2))*c)^(1/2))*d^2-9/32*e^3/a/(a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/
2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d+3/8
*e/a^2*c/(a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1
/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d^3-3/32*e^3/a/c/((c*d+(a*c*e^2)^(1/
2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))+3/16*
e/a^2/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^
2)^(1/2))*c)^(1/2))*d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ex+d)^{\frac{5}{2}}}{\left(cx^2-a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(-c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -integrate((e*x + d)^(5/2)/(c*x^2 - a)^3, x)
```

Fricas [B] time = 2.56309, size = 2178, normalized size = 7.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(-c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(3*(a^2*c^3*x^4 - 2*a^3*c^2*x^2 + a^4*c)*sqrt((a^5*c^3*sqrt(e^10/(a^5*
c^7)) + 16*c^2*d^5 - 20*a*c*d^3*e^2 + 5*a^2*d*e^4)/(a^5*c^3))*log(27*(16*c^
```

```
2*d^4*e^5 - 12*a*c*d^2*e^7 + a^2*e^9)*sqrt(e*x + d) + 27*(2*a^3*c^2*d*e^6 +
 (4*a^5*c^6*d^2 - a^6*c^5*e^2)*sqrt(e^10/(a^5*c^7)))*sqrt((a^5*c^3*sqrt(e^1
0/(a^5*c^7)) + 16*c^2*d^5 - 20*a*c*d^3*e^2 + 5*a^2*d*e^4)/(a^5*c^3))) - 3*(
a^2*c^3*x^4 - 2*a^3*c^2*x^2 + a^4*c)*sqrt((a^5*c^3*sqrt(e^10/(a^5*c^7)) + 1
6*c^2*d^5 - 20*a*c*d^3*e^2 + 5*a^2*d*e^4)/(a^5*c^3))*log(27*(16*c^2*d^4*e^5
 - 12*a*c*d^2*e^7 + a^2*e^9)*sqrt(e*x + d) - 27*(2*a^3*c^2*d*e^6 + (4*a^5*c
^6*d^2 - a^6*c^5*e^2)*sqrt(e^10/(a^5*c^7)))*sqrt((a^5*c^3*sqrt(e^10/(a^5*c^
7)) + 16*c^2*d^5 - 20*a*c*d^3*e^2 + 5*a^2*d*e^4)/(a^5*c^3))) + 3*(a^2*c^3*x
^4 - 2*a^3*c^2*x^2 + a^4*c)*sqrt(-(a^5*c^3*sqrt(e^10/(a^5*c^7)) - 16*c^2*d^
5 + 20*a*c*d^3*e^2 - 5*a^2*d*e^4)/(a^5*c^3))*log(27*(16*c^2*d^4*e^5 - 12*a*
c*d^2*e^7 + a^2*e^9)*sqrt(e*x + d) + 27*(2*a^3*c^2*d*e^6 - (4*a^5*c^6*d^2 -
 a^6*c^5*e^2)*sqrt(e^10/(a^5*c^7)))*sqrt(-(a^5*c^3*sqrt(e^10/(a^5*c^7)) - 1
6*c^2*d^5 + 20*a*c*d^3*e^2 - 5*a^2*d*e^4)/(a^5*c^3))) - 3*(a^2*c^3*x^4 - 2*
a^3*c^2*x^2 + a^4*c)*sqrt(-(a^5*c^3*sqrt(e^10/(a^5*c^7)) - 16*c^2*d^5 + 20*
a*c*d^3*e^2 - 5*a^2*d*e^4)/(a^5*c^3))*log(27*(16*c^2*d^4*e^5 - 12*a*c*d^2*e
^7 + a^2*e^9)*sqrt(e*x + d) - 27*(2*a^3*c^2*d*e^6 - (4*a^5*c^6*d^2 - a^6*c^
5*e^2)*sqrt(e^10/(a^5*c^7)))*sqrt(-(a^5*c^3*sqrt(e^10/(a^5*c^7)) - 16*c^2*d
^5 + 20*a*c*d^3*e^2 - 5*a^2*d*e^4)/(a^5*c^3))) + 4*(a*c*d*e*x^2 + 7*a^2*d*e
 - 3*(2*c^2*d^2 - a*c*e^2)*x^3 + (10*a*c*d^2 + a^2*e^2)*x)*sqrt(e*x + d))/(
a^2*c^3*x^4 - 2*a^3*c^2*x^2 + a^4*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(-c*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(-c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.640 $\int \frac{(d+ex)^{3/2}}{\left(a-cx^2\right)^3}\,dx$

Optimal. Leaf size=268

$$-\frac{3\left(-2\sqrt{a}\sqrt{c}de-ae^2+4cd^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{32a^{5/2}c^{5/4}\sqrt{\sqrt{c}d-\sqrt{a}e}}+\frac{3\left(2\sqrt{a}\sqrt{c}de-ae^2+4cd^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{32a^{5/2}c^{5/4}\sqrt{\sqrt{a}e+\sqrt{c}d}}-\frac{\sqrt{d+ex}(ae-6cdx)}{16a^2c\left(a-cx^2\right)}$$

```
[Out] ((a*e + c*d*x)*Sqrt[d + e*x])/(4*a*c*(a - c*x^2)^2) - ((a*e - 6*c*d*x)*Sqrt
[d + e*x])/(16*a^2*c*(a - c*x^2)) - (3*(4*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a
*e^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(32*a^(
5/2)*c^(5/4)*Sqrt[Sqrt[c]*d - Sqrt[a]*e]) + (3*(4*c*d^2 + 2*Sqrt[a]*Sqrt[c]
*d*e - a*e^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])
/(32*a^(5/2)*c^(5/4)*Sqrt[Sqrt[c]*d + Sqrt[a]*e])
```

Rubi [A] time = 0.505557, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {739, 823, 827, 1166, 208}

$$-\frac{3\left(-2\sqrt{a}\sqrt{c}de-ae^2+4cd^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{32a^{5/2}c^{5/4}\sqrt{\sqrt{c}d-\sqrt{a}e}}+\frac{3\left(2\sqrt{a}\sqrt{c}de-ae^2+4cd^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{32a^{5/2}c^{5/4}\sqrt{\sqrt{a}e+\sqrt{c}d}}-\frac{\sqrt{d+ex}(ae-6cdx)}{16a^2c\left(a-cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(a - c*x^2)^3,x]
```

```
[Out] ((a*e + c*d*x)*Sqrt[d + e*x])/(4*a*c*(a - c*x^2)^2) - ((a*e - 6*c*d*x)*Sqrt
[d + e*x])/(16*a^2*c*(a - c*x^2)) - (3*(4*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a
*e^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(32*a^(
5/2)*c^(5/4)*Sqrt[Sqrt[c]*d - Sqrt[a]*e]) + (3*(4*c*d^2 + 2*Sqrt[a]*Sqrt[c]
*d*e - a*e^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])
/(32*a^(5/2)*c^(5/4)*Sqrt[Sqrt[c]*d + Sqrt[a]*e])
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
```

```
*m, 2*p])
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\left(a-cx^2\right)^3}\,dx = \frac{(ae+cdx)\sqrt{d+ex}}{4ac\left(a-cx^2\right)^2}-\frac{\int \frac{\frac{1}{2}\left(-6cd^2+ae^2\right)-\frac{5}{2}cdex}{\sqrt{d+ex}\left(a-cx^2\right)^2}\,dx}{4ac}$$

$$=\frac{(ae+cdx)\sqrt{d+ex}}{4ac\left(a-cx^2\right)^2}-\frac{(ae-6cdx)\sqrt{d+ex}}{16a^2c\left(a-cx^2\right)}+\frac{\int \frac{\frac{3}{4}c\left(cd^2-ae^2\right)\left(4cd^2-ae^2\right)+\frac{3}{2}c^2de\left(cd^2-ae^2\right)x}{\sqrt{d+ex}\left(a-cx^2\right)}\,dx}{8a^2c^2\left(cd^2-ae^2\right)}$$

$$=\frac{(ae+cdx)\sqrt{d+ex}}{4ac\left(a-cx^2\right)^2}-\frac{(ae-6cdx)\sqrt{d+ex}}{16a^2c\left(a-cx^2\right)}+\frac{\text{Subst}\left(\int \frac{-\frac{3}{2}c^2d^2e\left(cd^2-ae^2\right)+\frac{3}{4}ce\left(cd^2-ae^2\right)\left(4cd^2-ae^2\right)+\frac{3}{2}c^2de\left(cd^2-\right.}{-cd^2+ae^2+2cdx^2-cx^4}\right.}{4a^2c^2\left(cd^2-ae^2\right)}$$

$$=\frac{(ae+cdx)\sqrt{d+ex}}{4ac\left(a-cx^2\right)^2}-\frac{(ae-6cdx)\sqrt{d+ex}}{16a^2c\left(a-cx^2\right)}-\frac{\left(3\left(4cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\right)\text{Subst}\left(\int \frac{1}{cd-\sqrt{a}\sqrt{c}e-cx^2}\,dx,\right.}{32a^{5/2}\sqrt{c}}$$

$$=\frac{(ae+cdx)\sqrt{d+ex}}{4ac\left(a-cx^2\right)^2}-\frac{(ae-6cdx)\sqrt{d+ex}}{16a^2c\left(a-cx^2\right)}-\frac{3\left(4cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{32a^{5/2}c^{5/4}\sqrt{\sqrt{c}d-\sqrt{a}e}}+\frac{3\left(4c\right.}{}$$

Mathematica [A] time = 0.994475, size = 428, normalized size = 1.6

$$\frac{\frac{2(d+ex)^{5/2}\left(3a^2e^3-acde(5d+4ex)+6c^2d^3x\right)}{a-cx^2}+\frac{2\sqrt{a}\sqrt[4]{c}e\sqrt{d+ex}\left(3a^2e^4-acde^2(13d+4ex)+6c^2d^3(2d+ex)\right)+3\sqrt{\sqrt{a}e+\sqrt{c}d}\left(-a^{3/2}e^3+6\sqrt{a}cd^2e+a\sqrt{c}de^2+4c^{3/2}d^3\right)\left(\sqrt{c}\right.}{\sqrt{a}}}{32a^2\left(cd^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(a - c*x^2)^3,x]
```

```
[Out] ((8*a*(c*d^2 - a*e^2)*(-(a*e) + c*d*x)*(d + e*x)^(5/2))/(a - c*x^2)^2 + (2*
(d + e*x)^(5/2)*(3*a^2*e^3 + 6*c^2*d^3*x - a*c*d*e*(5*d + 4*e*x)))/(a - c*x
^2) + (2*Sqrt[a]*c^(1/4)*e*Sqrt[d + e*x]*(3*a^2*e^4 + 6*c^2*d^3*(2*d + e*x)
 - a*c*d*e^2*(13*d + 4*e*x)) - 3*Sqrt[Sqrt[c]*d - Sqrt[a]*e]*(Sqrt[c]*d + S
qrt[a]*e)^2*(4*c^(3/2)*d^3 - 6*Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*e^2 + a^(3/2)*
e^3)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]] + 3*(Sqrt
[c]*d - Sqrt[a]*e)^2*Sqrt[Sqrt[c]*d + Sqrt[a]*e]*(4*c^(3/2)*d^3 + 6*Sqrt[a]
*c*d^2*e + a*Sqrt[c]*d*e^2 - a^(3/2)*e^3)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/S
qrt[Sqrt[c]*d + Sqrt[a]*e]])/(Sqrt[a]*c^(5/4)))/(32*a^2*(c*d^2 - a*e^2)^2)
```

Maple [B] time = 0.218, size = 608, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(-c*x^2+a)^3,x)
```

```
[Out] -3/8*e/(c*e^2*x^2-a*e^2)^2*c*d/a^2*(e*x+d)^(7/2)+1/16*e^3/(c*e^2*x^2-a*e^2)
^2/a*(e*x+d)^(5/2)+9/8*e/(c*e^2*x^2-a*e^2)^2/a^2*(e*x+d)^(5/2)*c*d^2+1/2*e^
3/(c*e^2*x^2-a*e^2)^2*d/a*(e*x+d)^(3/2)-9/8*e/(c*e^2*x^2-a*e^2)^2*d^3/a^2*(
e*x+d)^(3/2)*c+3/16*e^5/(c*e^2*x^2-a*e^2)^2/c*(e*x+d)^(1/2)-9/16*e^3/(c*e^2
*x^2-a*e^2)^2/a*(e*x+d)^(1/2)*d^2+3/8*e/(c*e^2*x^2-a*e^2)^2/a^2*c*(e*x+d)^(
1/2)*d^4-3/32*e^3/a/(a*c*e^2)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan
((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))+3/8*e/a^2/(a*c*e^2)^(1/2
)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^
(1/2))*c)^(1/2))*c*d^2-3/16*e/a^2/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan((
e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d-3/32*e^3/a/(a*c*e^2)^(1/
2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^
(1/2))*c)^(1/2))+3/8*e/a^2/(a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*
arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))*c*d^2+3/16*e/a^2/(
(c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2
))*c)^(1/2))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cx^2-a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(-c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -integrate((e*x + d)^(3/2)/(c*x^2 - a)^3, x)
```

Fricas [B] time = 2.71421, size = 3357, normalized size = 12.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(-c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(3*(a^2*c^3*x^4 - 2*a^3*c^2*x^2 + a^4*c)*sqrt((16*c^2*d^5 - 20*a*c*d^3
*e^2 + 5*a^2*d*e^4 + (a^5*c^3*d^2 - a^6*c^2*e^2)*sqrt(e^10/(a^5*c^7*d^4 - 2
*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))/(a^5*c^3*d^2 - a^6*c^2*e^2))*log(27*(16*c
^2*d^4*e^5 - 12*a*c*d^2*e^7 + a^2*e^9)*sqrt(e*x + d) + 27*(2*a^3*c^2*d^2*e^
6 - a^4*c*e^8 - (4*a^5*c^6*d^5 - 7*a^6*c^5*d^3*e^2 + 3*a^7*c^4*d*e^4)*sqrt(
e^10/(a^5*c^7*d^4 - 2*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))*sqrt((16*c^2*d^5 - 2
0*a*c*d^3*e^2 + 5*a^2*d*e^4 + (a^5*c^3*d^2 - a^6*c^2*e^2)*sqrt(e^10/(a^5*c^
7*d^4 - 2*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))/(a^5*c^3*d^2 - a^6*c^2*e^2))) -
3*(a^2*c^3*x^4 - 2*a^3*c^2*x^2 + a^4*c)*sqrt((16*c^2*d^5 - 20*a*c*d^3*e^2 +
 5*a^2*d*e^4 + (a^5*c^3*d^2 - a^6*c^2*e^2)*sqrt(e^10/(a^5*c^7*d^4 - 2*a^6*c
^6*d^2*e^2 + a^7*c^5*e^4)))/(a^5*c^3*d^2 - a^6*c^2*e^2))*log(27*(16*c^2*d^4
*e^5 - 12*a*c*d^2*e^7 + a^2*e^9)*sqrt(e*x + d) - 27*(2*a^3*c^2*d^2*e^6 - a^
4*c*e^8 - (4*a^5*c^6*d^5 - 7*a^6*c^5*d^3*e^2 + 3*a^7*c^4*d*e^4)*sqrt(e^10/(
a^5*c^7*d^4 - 2*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))*sqrt((16*c^2*d^5 - 20*a*c*
d^3*e^2 + 5*a^2*d*e^4 + (a^5*c^3*d^2 - a^6*c^2*e^2)*sqrt(e^10/(a^5*c^7*d^4
- 2*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))/(a^5*c^3*d^2 - a^6*c^2*e^2))) + 3*(a^2
*c^3*x^4 - 2*a^3*c^2*x^2 + a^4*c)*sqrt((16*c^2*d^5 - 20*a*c*d^3*e^2 + 5*a^2
*d*e^4 - (a^5*c^3*d^2 - a^6*c^2*e^2)*sqrt(e^10/(a^5*c^7*d^4 - 2*a^6*c^6*d^2
*e^2 + a^7*c^5*e^4)))/(a^5*c^3*d^2 - a^6*c^2*e^2))*log(27*(16*c^2*d^4*e^5 -
 12*a*c*d^2*e^7 + a^2*e^9)*sqrt(e*x + d) + 27*(2*a^3*c^2*d^2*e^6 - a^4*c*e^
8 + (4*a^5*c^6*d^5 - 7*a^6*c^5*d^3*e^2 + 3*a^7*c^4*d*e^4)*sqrt(e^10/(a^5*c^
7*d^4 - 2*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))*sqrt((16*c^2*d^5 - 20*a*c*d^3*e^
2 + 5*a^2*d*e^4 - (a^5*c^3*d^2 - a^6*c^2*e^2)*sqrt(e^10/(a^5*c^7*d^4 - 2*a^
6*c^6*d^2*e^2 + a^7*c^5*e^4)))/(a^5*c^3*d^2 - a^6*c^2*e^2))) - 3*(a^2*c^3*x
^4 - 2*a^3*c^2*x^2 + a^4*c)*sqrt((16*c^2*d^5 - 20*a*c*d^3*e^2 + 5*a^2*d*e^4
 - (a^5*c^3*d^2 - a^6*c^2*e^2)*sqrt(e^10/(a^5*c^7*d^4 - 2*a^6*c^6*d^2*e^2 +
 a^7*c^5*e^4)))/(a^5*c^3*d^2 - a^6*c^2*e^2))*log(27*(16*c^2*d^4*e^5 - 12*a*
c*d^2*e^7 + a^2*e^9)*sqrt(e*x + d) - 27*(2*a^3*c^2*d^2*e^6 - a^4*c*e^8 + (4
*a^5*c^6*d^5 - 7*a^6*c^5*d^3*e^2 + 3*a^7*c^4*d*e^4)*sqrt(e^10/(a^5*c^7*d^4
- 2*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))*sqrt((16*c^2*d^5 - 20*a*c*d^3*e^2 + 5*
a^2*d*e^4 - (a^5*c^3*d^2 - a^6*c^2*e^2)*sqrt(e^10/(a^5*c^7*d^4 - 2*a^6*c^6*
d^2*e^2 + a^7*c^5*e^4)))/(a^5*c^3*d^2 - a^6*c^2*e^2))) - 4*(6*c^2*d*x^3 - a
*c*e*x^2 - 10*a*c*d*x - 3*a^2*e)*sqrt(e*x + d))/(a^2*c^3*x^4 - 2*a^3*c^2*x^
2 + a^4*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(-c*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(-c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.641 $\int \frac{\sqrt{d+ex}}{\left(a-cx^2\right)^3}\,dx$

Optimal. Leaf size=281

$$-\frac{\left(-18\sqrt{a}\sqrt{c}de+5ae^2+12cd^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{32a^{5/2}c^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}}+\frac{\left(18\sqrt{a}\sqrt{c}de+5ae^2+12cd^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{32a^{5/2}c^{3/4}\left(\sqrt{a}e+\sqrt{c}d\right)^{3/2}}-\frac{\sqrt{d+ex}\left(ade\ldots\right.}{16a^2\left(a-\ldots\right.}$$

```
[Out] (x*Sqrt[d + e*x])/(4*a*(a - c*x^2)^2) - (Sqrt[d + e*x]*(a*d*e - (6*c*d^2 -
5*a*e^2)*x))/(16*a^2*(c*d^2 - a*e^2)*(a - c*x^2)) - ((12*c*d^2 - 18*Sqrt[a]
*Sqrt[c]*d*e + 5*a*e^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sq
rt[a]*e]])/(32*a^(5/2)*c^(3/4)*(Sqrt[c]*d - Sqrt[a]*e)^(3/2)) + ((12*c*d^2
+ 18*Sqrt[a]*Sqrt[c]*d*e + 5*a*e^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sq
rt[c]*d + Sqrt[a]*e]])/(32*a^(5/2)*c^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)^(3/2))
```

Rubi [A] time = 0.490152, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {737, 823, 827, 1166, 208}

$$-\frac{\left(-18\sqrt{a}\sqrt{c}de+5ae^2+12cd^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{32a^{5/2}c^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}}+\frac{\left(18\sqrt{a}\sqrt{c}de+5ae^2+12cd^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{32a^{5/2}c^{3/4}\left(\sqrt{a}e+\sqrt{c}d\right)^{3/2}}-\frac{\sqrt{d+ex}\left(ade\ldots\right.}{16a^2\left(a-\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(a - c*x^2)^3,x]
```

```
[Out] (x*Sqrt[d + e*x])/(4*a*(a - c*x^2)^2) - (Sqrt[d + e*x]*(a*d*e - (6*c*d^2 -
5*a*e^2)*x))/(16*a^2*(c*d^2 - a*e^2)*(a - c*x^2)) - ((12*c*d^2 - 18*Sqrt[a]
*Sqrt[c]*d*e + 5*a*e^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sq
rt[a]*e]])/(32*a^(5/2)*c^(3/4)*(Sqrt[c]*d - Sqrt[a]*e)^(3/2)) + ((12*c*d^2
+ 18*Sqrt[a]*Sqrt[c]*d*e + 5*a*e^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sq
rt[c]*d + Sqrt[a]*e]])/(32*a^(5/2)*c^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)^(3/2))
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(x*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*(p + 1)), x] + Dist[1/(2*a*(p + 1
)), Int[(d + e*x)^(m - 1)*(d*(2*p + 3) + e*(m + 2*p + 3)*x)*(a + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -
1] && GtQ[m, 0] && (LtQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && In
tQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\left(a-cx^2\right)^3}\,dx = \frac{x\sqrt{d+ex}}{4a\left(a-cx^2\right)^2} - \frac{\int \frac{-3d-\frac{5ex}{2}}{\sqrt{d+ex}\left(a-cx^2\right)^2}\,dx}{4a}$$

$$= \frac{x\sqrt{d+ex}}{4a\left(a-cx^2\right)^2} - \frac{\sqrt{d+ex}\left(ade-\left(6cd^2-5ae^2\right)x\right)}{16a^2\left(cd^2-ae^2\right)\left(a-cx^2\right)} + \frac{\int \frac{\frac{1}{4}cd\left(12cd^2-13ae^2\right)+\frac{1}{4}ce\left(6cd^2-5ae^2\right)x}{\sqrt{d+ex}\left(a-cx^2\right)}\,dx}{8a^2c\left(cd^2-ae^2\right)}$$

$$= \frac{x\sqrt{d+ex}}{4a\left(a-cx^2\right)^2} - \frac{\sqrt{d+ex}\left(ade-\left(6cd^2-5ae^2\right)x\right)}{16a^2\left(cd^2-ae^2\right)\left(a-cx^2\right)} + \frac{\text{Subst}\left(\int \frac{\frac{1}{4}cde\left(12cd^2-13ae^2\right)-\frac{1}{4}cde\left(6cd^2-5ae^2\right)+\frac{1}{4}ce\left(6cd^2-5a\right.}{-cd^2+ae^2+2cdx^2-cx^4}\right.}{4a^2c\left(cd^2-ae^2\right)}$$

$$= \frac{x\sqrt{d+ex}}{4a\left(a-cx^2\right)^2} - \frac{\sqrt{d+ex}\left(ade-\left(6cd^2-5ae^2\right)x\right)}{16a^2\left(cd^2-ae^2\right)\left(a-cx^2\right)} - \frac{\left(12cd^2-18\sqrt{a}\sqrt{c}de+5ae^2\right)\text{Subst}\left(\int \frac{1}{cd-\sqrt{a}\sqrt{c}e-cx^2}\right.}{32a^{5/2}\left(\sqrt{c}d-\sqrt{a}e\right)}$$

$$= \frac{x\sqrt{d+ex}}{4a\left(a-cx^2\right)^2} - \frac{\sqrt{d+ex}\left(ade-\left(6cd^2-5ae^2\right)x\right)}{16a^2\left(cd^2-ae^2\right)\left(a-cx^2\right)} - \frac{\left(12cd^2-18\sqrt{a}\sqrt{c}de+5ae^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{32a^{5/2}c^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}} +$$

Mathematica [A] time = 0.735417, size = 368, normalized size = 1.31

$$\frac{\frac{2(d+ex)^{3/2}\left(5a^2e^3-acde(3d+8ex)+6c^2d^3x\right)}{a-cx^2} + \frac{4\sqrt{a}c^{3/4}de\sqrt{d+ex}\left(3cd^2-4ae^2\right)+\sqrt{\sqrt{a}e+\sqrt{c}d}\left(18\sqrt{a}\sqrt{c}de+5ae^2+12cd^2\right)\left(\sqrt{c}d-\sqrt{a}e\right)^2\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)-\left(\sqrt{a}e+\sqrt{c}d\right)^2}{\sqrt{a}c^{3/4}}}{32a^2\left(cd^2-ae^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(a - c*x^2)^3,x]
```

```
[Out] ((8*a*(c*d^2 - a*e^2)*(-(a*e) + c*d*x)*(d + e*x)^(3/2))/(a - c*x^2)^2 + (2*
(d + e*x)^(3/2)*(5*a^2*e^3 + 6*c^2*d^3*x - a*c*d*e*(3*d + 8*e*x)))/(a - c*x
^2) + (4*Sqrt[a]*c^(3/4)*d*e*(3*c*d^2 - 4*a*e^2)*Sqrt[d + e*x] - Sqrt[Sqrt[
```

```
c]*d - Sqrt[a]*e]*(Sqrt[c]*d + Sqrt[a]*e)^2*(12*c*d^2 - 18*Sqrt[a]*Sqrt[c]*
d*e + 5*a*e^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]]
 + (Sqrt[c]*d - Sqrt[a]*e)^2*Sqrt[Sqrt[c]*d + Sqrt[a]*e]*(12*c*d^2 + 18*Sqr
t[a]*Sqrt[c]*d*e + 5*a*e^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d
+ Sqrt[a]*e]])/(Sqrt[a]*c^(3/4)))/(32*a^2*(c*d^2 - a*e^2)^2)
```

Maple [B] time = 0.276, size = 803, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(-c*x^2+a)^3,x)
```

```
[Out] -3/16*e/a^2/(e*x+(a*c*e^2)^(1/2)/c)^2/(c*d-(a*c*e^2)^(1/2))*(e*x+d)^(3/2)*d
+5/32*e/c*(a*c*e^2)^(1/2)/a^2/(e*x+(a*c*e^2)^(1/2)/c)^2/(c*d-(a*c*e^2)^(1/2
))*(e*x+d)^(3/2)+3/16*e/c/a^2/(e*x+(a*c*e^2)^(1/2)/c)^2*(e*x+d)^(1/2)*d-7/3
2*e/c^2*(a*c*e^2)^(1/2)/a^2/(e*x+(a*c*e^2)^(1/2)/c)^2*(e*x+d)^(1/2)-5/32*e^
3*c/(a*c*e^2)^(1/2)/a/(-c*d+(a*c*e^2)^(1/2))/((-c*d+(a*c*e^2)^(1/2))*c)^(1/
2)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))-3/8*e*c^2/(a*c*
e^2)^(1/2)/a^2/(-c*d+(a*c*e^2)^(1/2))/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arct
an((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d^2+9/16*e*c/a^2/(-c*d
+(a*c*e^2)^(1/2))/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c/(
(-c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d-3/16*e/a^2/(e*x-(a*c*e^2)^(1/2)/c)^2/(c*
d+(a*c*e^2)^(1/2))*(e*x+d)^(3/2)*d-5/32*e/c*(a*c*e^2)^(1/2)/a^2/(e*x-(a*c*e
^2)^(1/2)/c)^2/(c*d+(a*c*e^2)^(1/2))*(e*x+d)^(3/2)+3/16*e/c/a^2/(e*x-(a*c*e
^2)^(1/2)/c)^2*(e*x+d)^(1/2)*d+7/32*e/c^2*(a*c*e^2)^(1/2)/a^2/(e*x-(a*c*e^2
)^(1/2)/c)^2*(e*x+d)^(1/2)+5/32*e^3*c/(a*c*e^2)^(1/2)/a/(c*d+(a*c*e^2)^(1/2
))/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^
(1/2))*c)^(1/2))+3/8*e*c^2/(a*c*e^2)^(1/2)/a^2/(c*d+(a*c*e^2)^(1/2))/((c*d+
(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)
^(1/2))*d^2+9/16*e*c/a^2/(c*d+(a*c*e^2)^(1/2))/((c*d+(a*c*e^2)^(1/2))*c)^(1
/2)*arctanh((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{ex+d}}{\left(cx^2-a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(-c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(e*x + d)/(c*x^2 - a)^3, x)
```

Fricas [B] time = 4.57217, size = 7752, normalized size = 27.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(-c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/64*((a^4*c*d^2 - a^5*e^2 + (a^2*c^3*d^2 - a^3*c^2*e^2)*x^4 - 2*(a^3*c^2*
d^2 - a^4*c*e^2)*x^2)*sqrt((144*c^3*d^7 - 420*a*c^2*d^5*e^2 + 385*a^2*c*d^3
*e^4 - 105*a^3*d*e^6 + (a^5*c^4*d^6 - 3*a^6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4
 - a^8*c*e^6)*sqrt((441*c^2*d^4*e^10 - 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a
^5*c^9*d^12 - 6*a^6*c^8*d^10*e^2 + 15*a^7*c^7*d^8*e^4 - 20*a^8*c^6*d^6*e^6
+ 15*a^9*c^5*d^4*e^8 - 6*a^10*c^4*d^2*e^10 + a^11*c^3*e^12)))/(a^5*c^4*d^6
- 3*a^6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4 - a^8*c*e^6))*log(-(3024*c^3*d^6*e^
5 - 7884*a*c^2*d^4*e^7 + 5625*a^2*c*d^2*e^9 - 625*a^3*e^11)*sqrt(e*x + d) +
 (126*a^3*c^3*d^5*e^6 - 318*a^4*c^2*d^3*e^8 + 200*a^5*c*d*e^10 + (12*a^5*c^
7*d^10 - 55*a^6*c^6*d^8*e^2 + 98*a^7*c^5*d^6*e^4 - 84*a^8*c^4*d^4*e^6 + 34*
a^9*c^3*d^2*e^8 - 5*a^10*c^2*e^10)*sqrt((441*c^2*d^4*e^10 - 1050*a*c*d^2*e^
12 + 625*a^2*e^14)/(a^5*c^9*d^12 - 6*a^6*c^8*d^10*e^2 + 15*a^7*c^7*d^8*e^4
- 20*a^8*c^6*d^6*e^6 + 15*a^9*c^5*d^4*e^8 - 6*a^10*c^4*d^2*e^10 + a^11*c^3*
e^12)))*sqrt((144*c^3*d^7 - 420*a*c^2*d^5*e^2 + 385*a^2*c*d^3*e^4 - 105*a^3
*d*e^6 + (a^5*c^4*d^6 - 3*a^6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4 - a^8*c*e^6)*
sqrt((441*c^2*d^4*e^10 - 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9*d^12 -
6*a^6*c^8*d^10*e^2 + 15*a^7*c^7*d^8*e^4 - 20*a^8*c^6*d^6*e^6 + 15*a^9*c^5*d
^4*e^8 - 6*a^10*c^4*d^2*e^10 + a^11*c^3*e^12)))/(a^5*c^4*d^6 - 3*a^6*c^3*d^
4*e^2 + 3*a^7*c^2*d^2*e^4 - a^8*c*e^6))) - (a^4*c*d^2 - a^5*e^2 + (a^2*c^3*
d^2 - a^3*c^2*e^2)*x^4 - 2*(a^3*c^2*d^2 - a^4*c*e^2)*x^2)*sqrt((144*c^3*d^7
 - 420*a*c^2*d^5*e^2 + 385*a^2*c*d^3*e^4 - 105*a^3*d*e^6 + (a^5*c^4*d^6 - 3
*a^6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4 - a^8*c*e^6)*sqrt((441*c^2*d^4*e^10 -
1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9*d^12 - 6*a^6*c^8*d^10*e^2 + 15*a
^7*c^7*d^8*e^4 - 20*a^8*c^6*d^6*e^6 + 15*a^9*c^5*d^4*e^8 - 6*a^10*c^4*d^2*e
^10 + a^11*c^3*e^12)))/(a^5*c^4*d^6 - 3*a^6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4
 - a^8*c*e^6))*log(-(3024*c^3*d^6*e^5 - 7884*a*c^2*d^4*e^7 + 5625*a^2*c*d^2
*e^9 - 625*a^3*e^11)*sqrt(e*x + d) - (126*a^3*c^3*d^5*e^6 - 318*a^4*c^2*d^3
*e^8 + 200*a^5*c*d*e^10 + (12*a^5*c^7*d^10 - 55*a^6*c^6*d^8*e^2 + 98*a^7*c^
5*d^6*e^4 - 84*a^8*c^4*d^4*e^6 + 34*a^9*c^3*d^2*e^8 - 5*a^10*c^2*e^10)*sqrt
((441*c^2*d^4*e^10 - 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9*d^12 - 6*a^
6*c^8*d^10*e^2 + 15*a^7*c^7*d^8*e^4 - 20*a^8*c^6*d^6*e^6 + 15*a^9*c^5*d^4*e
^8 - 6*a^10*c^4*d^2*e^10 + a^11*c^3*e^12)))*sqrt((144*c^3*d^7 - 420*a*c^2*d
^5*e^2 + 385*a^2*c*d^3*e^4 - 105*a^3*d*e^6 + (a^5*c^4*d^6 - 3*a^6*c^3*d^4*e
^2 + 3*a^7*c^2*d^2*e^4 - a^8*c*e^6)*sqrt((441*c^2*d^4*e^10 - 1050*a*c*d^2*e
^12 + 625*a^2*e^14)/(a^5*c^9*d^12 - 6*a^6*c^8*d^10*e^2 + 15*a^7*c^7*d^8*e^4
 - 20*a^8*c^6*d^6*e^6 + 15*a^9*c^5*d^4*e^8 - 6*a^10*c^4*d^2*e^10 + a^11*c^3
*e^12)))/(a^5*c^4*d^6 - 3*a^6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4 - a^8*c*e^6))
) + (a^4*c*d^2 - a^5*e^2 + (a^2*c^3*d^2 - a^3*c^2*e^2)*x^4 - 2*(a^3*c^2*d^2
 - a^4*c*e^2)*x^2)*sqrt((144*c^3*d^7 - 420*a*c^2*d^5*e^2 + 385*a^2*c*d^3*e^
4 - 105*a^3*d*e^6 - (a^5*c^4*d^6 - 3*a^6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4 -
a^8*c*e^6)*sqrt((441*c^2*d^4*e^10 - 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*
c^9*d^12 - 6*a^6*c^8*d^10*e^2 + 15*a^7*c^7*d^8*e^4 - 20*a^8*c^6*d^6*e^6 + 1
5*a^9*c^5*d^4*e^8 - 6*a^10*c^4*d^2*e^10 + a^11*c^3*e^12)))/(a^5*c^4*d^6 - 3
*a^6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4 - a^8*c*e^6))*log(-(3024*c^3*d^6*e^5 -
 7884*a*c^2*d^4*e^7 + 5625*a^2*c*d^2*e^9 - 625*a^3*e^11)*sqrt(e*x + d) + (1
26*a^3*c^3*d^5*e^6 - 318*a^4*c^2*d^3*e^8 + 200*a^5*c*d*e^10 - (12*a^5*c^7*d
^10 - 55*a^6*c^6*d^8*e^2 + 98*a^7*c^5*d^6*e^4 - 84*a^8*c^4*d^4*e^6 + 34*a^9
*c^3*d^2*e^8 - 5*a^10*c^2*e^10)*sqrt((441*c^2*d^4*e^10 - 1050*a*c*d^2*e^12
+ 625*a^2*e^14)/(a^5*c^9*d^12 - 6*a^6*c^8*d^10*e^2 + 15*a^7*c^7*d^8*e^4 - 2
0*a^8*c^6*d^6*e^6 + 15*a^9*c^5*d^4*e^8 - 6*a^10*c^4*d^2*e^10 + a^11*c^3*e^1
2)))*sqrt((144*c^3*d^7 - 420*a*c^2*d^5*e^2 + 385*a^2*c*d^3*e^4 - 105*a^3*d*
e^6 - (a^5*c^4*d^6 - 3*a^6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4 - a^8*c*e^6)*sqr
t((441*c^2*d^4*e^10 - 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9*d^12 - 6*a
^6*c^8*d^10*e^2 + 15*a^7*c^7*d^8*e^4 - 20*a^8*c^6*d^6*e^6 + 15*a^9*c^5*d^4*
e^8 - 6*a^10*c^4*d^2*e^10 + a^11*c^3*e^12)))/(a^5*c^4*d^6 - 3*a^6*c^3*d^4*e
^2 + 3*a^7*c^2*d^2*e^4 - a^8*c*e^6))) - (a^4*c*d^2 - a^5*e^2 + (a^2*c^3*d^2
 - a^3*c^2*e^2)*x^4 - 2*(a^3*c^2*d^2 - a^4*c*e^2)*x^2)*sqrt((144*c^3*d^7 -
420*a*c^2*d^5*e^2 + 385*a^2*c*d^3*e^4 - 105*a^3*d*e^6 - (a^5*c^4*d^6 - 3*a^
6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4 - a^8*c*e^6)*sqrt((441*c^2*d^4*e^10 - 105
```

```
0*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9*d^12 - 6*a^6*c^8*d^10*e^2 + 15*a^7*
c^7*d^8*e^4 - 20*a^8*c^6*d^6*e^6 + 15*a^9*c^5*d^4*e^8 - 6*a^10*c^4*d^2*e^10
 + a^11*c^3*e^12)))/(a^5*c^4*d^6 - 3*a^6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4 -
a^8*c*e^6))*log(-(3024*c^3*d^6*e^5 - 7884*a*c^2*d^4*e^7 + 5625*a^2*c*d^2*e^
9 - 625*a^3*e^11)*sqrt(e*x + d) - (126*a^3*c^3*d^5*e^6 - 318*a^4*c^2*d^3*e^
8 + 200*a^5*c*d*e^10 - (12*a^5*c^7*d^10 - 55*a^6*c^6*d^8*e^2 + 98*a^7*c^5*d
^6*e^4 - 84*a^8*c^4*d^4*e^6 + 34*a^9*c^3*d^2*e^8 - 5*a^10*c^2*e^10)*sqrt((4
41*c^2*d^4*e^10 - 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9*d^12 - 6*a^6*c
^8*d^10*e^2 + 15*a^7*c^7*d^8*e^4 - 20*a^8*c^6*d^6*e^6 + 15*a^9*c^5*d^4*e^8
- 6*a^10*c^4*d^2*e^10 + a^11*c^3*e^12)))*sqrt((144*c^3*d^7 - 420*a*c^2*d^5*
e^2 + 385*a^2*c*d^3*e^4 - 105*a^3*d*e^6 - (a^5*c^4*d^6 - 3*a^6*c^3*d^4*e^2
+ 3*a^7*c^2*d^2*e^4 - a^8*c*e^6)*sqrt((441*c^2*d^4*e^10 - 1050*a*c*d^2*e^12
 + 625*a^2*e^14)/(a^5*c^9*d^12 - 6*a^6*c^8*d^10*e^2 + 15*a^7*c^7*d^8*e^4 -
20*a^8*c^6*d^6*e^6 + 15*a^9*c^5*d^4*e^8 - 6*a^10*c^4*d^2*e^10 + a^11*c^3*e^
12)))/(a^5*c^4*d^6 - 3*a^6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4 - a^8*c*e^6))) -
 4*(a*c*d*e*x^2 - a^2*d*e - (6*c^2*d^2 - 5*a*c*e^2)*x^3 + (10*a*c*d^2 - 9*a
^2*e^2)*x)*sqrt(e*x + d))/(a^4*c*d^2 - a^5*e^2 + (a^2*c^3*d^2 - a^3*c^2*e^2
)*x^4 - 2*(a^3*c^2*d^2 - a^4*c*e^2)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(-c*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(-c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.642 $\int \frac{1}{\sqrt{d+ex}\left(a-cx^2\right)^3}\,dx$

Optimal. Leaf size=315

$$-\frac{\sqrt{d+ex}\left(ae\left(cd^2-7ae^2\right)-6cdx\left(cd^2-2ae^2\right)\right)}{16a^2\left(a-cx^2\right)\left(cd^2-ae^2\right)^2}-\frac{3\left(-10\sqrt{a}\sqrt{c}de+7ae^2+4cd^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{32a^{5/2}\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}}+\frac{3\left(10\sqrt{a}\sqrt{c}de+7\ldots\right.}{32a^{5/2}\ldots}$$

```
[Out] -((a*e - c*d*x)*Sqrt[d + e*x])/(4*a*(c*d^2 - a*e^2)*(a - c*x^2)^2) - (Sqrt[
d + e*x]*(a*e*(c*d^2 - 7*a*e^2) - 6*c*d*(c*d^2 - 2*a*e^2)*x))/(16*a^2*(c*d^
2 - a*e^2)^2*(a - c*x^2)) - (3*(4*c*d^2 - 10*Sqrt[a]*Sqrt[c]*d*e + 7*a*e^2)
*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(32*a^(5/2)*
c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)^(5/2)) + (3*(4*c*d^2 + 10*Sqrt[a]*Sqrt[c]*d
*e + 7*a*e^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])
/(32*a^(5/2)*c^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)^(5/2))
```

Rubi [A] time = 0.634021, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {741, 823, 827, 1166, 208}

$$-\frac{\sqrt{d+ex}\left(ae\left(cd^2-7ae^2\right)-6cdx\left(cd^2-2ae^2\right)\right)}{16a^2\left(a-cx^2\right)\left(cd^2-ae^2\right)^2}-\frac{3\left(-10\sqrt{a}\sqrt{c}de+7ae^2+4cd^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{32a^{5/2}\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}}+\frac{3\left(10\sqrt{a}\sqrt{c}de+7\ldots\right.}{32a^{5/2}\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(a - c*x^2)^3),x]
```


```
[Out] -((a*e - c*d*x)*Sqrt[d + e*x])/(4*a*(c*d^2 - a*e^2)*(a - c*x^2)^2) - (Sqrt[
d + e*x]*(a*e*(c*d^2 - 7*a*e^2) - 6*c*d*(c*d^2 - 2*a*e^2)*x))/(16*a^2*(c*d^
2 - a*e^2)^2*(a - c*x^2)) - (3*(4*c*d^2 - 10*Sqrt[a]*Sqrt[c]*d*e + 7*a*e^2)
*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(32*a^(5/2)*
c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)^(5/2)) + (3*(4*c*d^2 + 10*Sqrt[a]*Sqrt[c]*d
*e + 7*a*e^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])
/(32*a^(5/2)*c^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)^(5/2))
```


Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
```

```
*m, 2*p])
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\left(a-cx^2\right)^3}\,dx = -\frac{(ae-cdx)\sqrt{d+ex}}{4a\left(cd^2-ae^2\right)\left(a-cx^2\right)^2}+\frac{\int \frac{\frac{1}{2}\left(6cd^2-7ae^2\right)+\frac{5}{2}cdex}{\sqrt{d+ex}\left(a-cx^2\right)^2}\,dx}{4a\left(cd^2-ae^2\right)}$$

$$= -\frac{(ae-cdx)\sqrt{d+ex}}{4a\left(cd^2-ae^2\right)\left(a-cx^2\right)^2}-\frac{\sqrt{d+ex}\left(ae\left(cd^2-7ae^2\right)-6cd\left(cd^2-2ae^2\right)x\right)}{16a^2\left(cd^2-ae^2\right)^2\left(a-cx^2\right)}-\frac{\int \frac{-\frac{3}{4}c\left(4c^2d^4- \right.}{}}{}$$

$$= -\frac{(ae-cdx)\sqrt{d+ex}}{4a\left(cd^2-ae^2\right)\left(a-cx^2\right)^2}-\frac{\sqrt{d+ex}\left(ae\left(cd^2-7ae^2\right)-6cd\left(cd^2-2ae^2\right)x\right)}{16a^2\left(cd^2-ae^2\right)^2\left(a-cx^2\right)}-\frac{\text{Subst}\left(\int \frac{3}{2} \right.}{}$$

$$= -\frac{(ae-cdx)\sqrt{d+ex}}{4a\left(cd^2-ae^2\right)\left(a-cx^2\right)^2}-\frac{\sqrt{d+ex}\left(ae\left(cd^2-7ae^2\right)-6cd\left(cd^2-2ae^2\right)x\right)}{16a^2\left(cd^2-ae^2\right)^2\left(a-cx^2\right)}-\frac{\left(3\sqrt{c}\left(4cd^2 \right.\right.}{}$$

$$= -\frac{(ae-cdx)\sqrt{d+ex}}{4a\left(cd^2-ae^2\right)\left(a-cx^2\right)^2}-\frac{\sqrt{d+ex}\left(ae\left(cd^2-7ae^2\right)-6cd\left(cd^2-2ae^2\right)x\right)}{16a^2\left(cd^2-ae^2\right)^2\left(a-cx^2\right)}-\frac{3\left(4cd^2-1 \right.}{}$$

Mathematica [A] time = 0.70031, size = 335, normalized size = 1.06

$$\frac{\frac{2\sqrt{d+ex}\left(7a^2e^3-acde(d+12ex)+6c^2d^3x\right)}{a-cx^2}+\frac{8a\sqrt{d+ex}\left(cd^2-ae^2\right)(cdx-ae)}{\left(a-cx^2\right)^2}+\frac{3\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}\left(10\sqrt{a}\sqrt{c}de+7ae^2+4cd^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)-3\left(\sqrt{a}e+\sqrt{c}d\right)^{5/2}\left(-1 \right.}{\sqrt{a}\sqrt[4]{c}\sqrt{\sqrt{c}d-\sqrt{a}e}\sqrt{\sqrt{a}e+\sqrt{c}d}}}{32a^2\left(cd^2-ae^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(a - c*x^2)^3),x]
```

```
[Out] ((8*a*(c*d^2 - a*e^2)*(-(a*e) + c*d*x)*Sqrt[d + e*x])/(a - c*x^2)^2 + (2*Sq
rt[d + e*x]*(7*a^2*e^3 + 6*c^2*d^3*x - a*c*d*e*(d + 12*e*x)))/(a - c*x^2) +
 (-3*(Sqrt[c]*d + Sqrt[a]*e)^(5/2)*(4*c*d^2 - 10*Sqrt[a]*Sqrt[c]*d*e + 7*a*
e^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]] + 3*(Sqrt
[c]*d - Sqrt[a]*e)^(5/2)*(4*c*d^2 + 10*Sqrt[a]*Sqrt[c]*d*e + 7*a*e^2)*ArcTa
nh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(Sqrt[a]*c^(1/4)*S
qrt[Sqrt[c]*d - Sqrt[a]*e]*Sqrt[Sqrt[c]*d + Sqrt[a]*e]))/(32*a^2*(c*d^2 - a
*e^2)^2)
```

Maple [B] time = 0.267, size = 956, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c*x^2+a)^3/(e*x+d)^(1/2),x)
```

```
[Out] -3/16*e/a^2/(e*x+(a*c*e^2)^(1/2)/c)^2/(a*e^2+c*d^2-2*(a*c*e^2)^(1/2)*d)*(e*
x+d)^(3/2)*d+9/32*e/c*(a*c*e^2)^(1/2)/a^2/(e*x+(a*c*e^2)^(1/2)/c)^2/(a*e^2+
c*d^2-2*(a*c*e^2)^(1/2)*d)*(e*x+d)^(3/2)+3/16*e/a^2/(e*x+(a*c*e^2)^(1/2)/c)
^2/(c*d-(a*c*e^2)^(1/2))*(e*x+d)^(1/2)*d-11/32*e/c*(a*c*e^2)^(1/2)/a^2/(e*x
+(a*c*e^2)^(1/2)/c)^2/(c*d-(a*c*e^2)^(1/2))*(e*x+d)^(1/2)-21/32*e^3*c/(a*c*
e^2)^(1/2)/a/(-a*e^2-c*d^2+2*(a*c*e^2)^(1/2)*d)/((-c*d+(a*c*e^2)^(1/2))*c)^
(1/2)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))-3/8*e*c^2/(a
*c*e^2)^(1/2)/a^2/(-a*e^2-c*d^2+2*(a*c*e^2)^(1/2)*d)/((-c*d+(a*c*e^2)^(1/2)
)*c)^(1/2)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d^2+15/
16*e*c/a^2/(-a*e^2-c*d^2+2*(a*c*e^2)^(1/2)*d)/((-c*d+(a*c*e^2)^(1/2))*c)^(1
/2)*arctan((e*x+d)^(1/2)*c/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d-3/16*e/a^2/(
e*x-(a*c*e^2)^(1/2)/c)^2/(a*e^2+c*d^2+2*(a*c*e^2)^(1/2)*d)*(e*x+d)^(3/2)*d-
9/32*e/c*(a*c*e^2)^(1/2)/a^2/(e*x-(a*c*e^2)^(1/2)/c)^2/(a*e^2+c*d^2+2*(a*c*
e^2)^(1/2)*d)*(e*x+d)^(3/2)+3/16*e/a^2/(e*x-(a*c*e^2)^(1/2)/c)^2/(c*d+(a*c*
e^2)^(1/2))*(e*x+d)^(1/2)*d+11/32*e/c*(a*c*e^2)^(1/2)/a^2/(e*x-(a*c*e^2)^(1
/2)/c)^2/(c*d+(a*c*e^2)^(1/2))*(e*x+d)^(1/2)+21/32*e^3*c/(a*c*e^2)^(1/2)/a/
(a*e^2+c*d^2+2*(a*c*e^2)^(1/2)*d)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((
e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))+3/8*e*c^2/(a*c*e^2)^(1/2)/a
^2/(a*e^2+c*d^2+2*(a*c*e^2)^(1/2)*d)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan
h((e*x+d)^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d^2+15/16*e*c/a^2/(a*e^2
+c*d^2+2*(a*c*e^2)^(1/2)*d)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)
^(1/2)*c/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(cx^2 - a\right)^3 \sqrt{ex + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*x^2+a)^3/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((c*x^2 - a)^3*sqrt(e*x + d)), x)
```

Fricas [B] time = 13.2157, size = 12446, normalized size = 39.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*x^2+a)^3/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/64*(3*(a^4*c^2*d^4 - 2*a^5*c*d^2*e^2 + a^6*e^4 + (a^2*c^4*d^4 - 2*a^3*c^3
*d^2*e^2 + a^4*c^2*e^4)*x^4 - 2*(a^3*c^3*d^4 - 2*a^4*c^2*d^2*e^2 + a^5*c*e^
4)*x^2)*sqrt((16*c^4*d^9 - 84*a*c^3*d^7*e^2 + 189*a^2*c^2*d^5*e^4 - 210*a^3
*c*d^3*e^6 + 105*a^4*d*e^8 + (a^5*c^5*d^10 - 5*a^6*c^4*d^8*e^2 + 10*a^7*c^3
*d^6*e^4 - 10*a^8*c^2*d^4*e^6 + 5*a^9*c*d^2*e^8 - a^10*e^10)*sqrt((441*c^4*
d^8*e^10 - 2268*a*c^3*d^6*e^12 + 4974*a^2*c^2*d^4*e^14 - 5292*a^3*c*d^2*e^1
6 + 2401*a^4*e^18)/(a^5*c^11*d^20 - 10*a^6*c^10*d^18*e^2 + 45*a^7*c^9*d^16*
e^4 - 120*a^8*c^8*d^14*e^6 + 210*a^9*c^7*d^12*e^8 - 252*a^10*c^6*d^10*e^10
+ 210*a^11*c^5*d^8*e^12 - 120*a^12*c^4*d^6*e^14 + 45*a^13*c^3*d^4*e^16 - 10
*a^14*c^2*d^2*e^18 + a^15*c*e^20)))/(a^5*c^5*d^10 - 5*a^6*c^4*d^8*e^2 + 10*
a^7*c^3*d^6*e^4 - 10*a^8*c^2*d^4*e^6 + 5*a^9*c*d^2*e^8 - a^10*e^10))*log(27
*(336*c^4*d^8*e^5 - 1788*a*c^3*d^6*e^7 + 4189*a^2*c^2*d^4*e^9 - 4802*a^3*c*
d^2*e^11 + 2401*a^4*e^13)*sqrt(e*x + d) + 27*(42*a^3*c^4*d^8*e^6 - 213*a^4*
c^3*d^6*e^8 + 515*a^5*c^2*d^4*e^10 - 623*a^6*c*d^2*e^12 + 343*a^7*e^14 - (4
*a^5*c^8*d^15 - 31*a^6*c^7*d^13*e^2 + 106*a^7*c^6*d^11*e^4 - 205*a^8*c^5*d^
9*e^6 + 240*a^9*c^4*d^7*e^8 - 169*a^10*c^3*d^5*e^10 + 66*a^11*c^2*d^3*e^12
- 11*a^12*c*d*e^14)*sqrt((441*c^4*d^8*e^10 - 2268*a*c^3*d^6*e^12 + 4974*a^2
*c^2*d^4*e^14 - 5292*a^3*c*d^2*e^16 + 2401*a^4*e^18)/(a^5*c^11*d^20 - 10*a^
6*c^10*d^18*e^2 + 45*a^7*c^9*d^16*e^4 - 120*a^8*c^8*d^14*e^6 + 210*a^9*c^7*
d^12*e^8 - 252*a^10*c^6*d^10*e^10 + 210*a^11*c^5*d^8*e^12 - 120*a^12*c^4*d^
6*e^14 + 45*a^13*c^3*d^4*e^16 - 10*a^14*c^2*d^2*e^18 + a^15*c*e^20)))*sqrt(
(16*c^4*d^9 - 84*a*c^3*d^7*e^2 + 189*a^2*c^2*d^5*e^4 - 210*a^3*c*d^3*e^6 +
105*a^4*d*e^8 + (a^5*c^5*d^10 - 5*a^6*c^4*d^8*e^2 + 10*a^7*c^3*d^6*e^4 - 10
*a^8*c^2*d^4*e^6 + 5*a^9*c*d^2*e^8 - a^10*e^10)*sqrt((441*c^4*d^8*e^10 - 22
68*a*c^3*d^6*e^12 + 4974*a^2*c^2*d^4*e^14 - 5292*a^3*c*d^2*e^16 + 2401*a^4*
e^18)/(a^5*c^11*d^20 - 10*a^6*c^10*d^18*e^2 + 45*a^7*c^9*d^16*e^4 - 120*a^8
*c^8*d^14*e^6 + 210*a^9*c^7*d^12*e^8 - 252*a^10*c^6*d^10*e^10 + 210*a^11*c^
5*d^8*e^12 - 120*a^12*c^4*d^6*e^14 + 45*a^13*c^3*d^4*e^16 - 10*a^14*c^2*d^2
*e^18 + a^15*c*e^20)))/(a^5*c^5*d^10 - 5*a^6*c^4*d^8*e^2 + 10*a^7*c^3*d^6*e
^4 - 10*a^8*c^2*d^4*e^6 + 5*a^9*c*d^2*e^8 - a^10*e^10))) - 3*(a^4*c^2*d^4 -
 2*a^5*c*d^2*e^2 + a^6*e^4 + (a^2*c^4*d^4 - 2*a^3*c^3*d^2*e^2 + a^4*c^2*e^4
)*x^4 - 2*(a^3*c^3*d^4 - 2*a^4*c^2*d^2*e^2 + a^5*c*e^4)*x^2)*sqrt((16*c^4*d
^9 - 84*a*c^3*d^7*e^2 + 189*a^2*c^2*d^5*e^4 - 210*a^3*c*d^3*e^6 + 105*a^4*d
*e^8 + (a^5*c^5*d^10 - 5*a^6*c^4*d^8*e^2 + 10*a^7*c^3*d^6*e^4 - 10*a^8*c^2*
d^4*e^6 + 5*a^9*c*d^2*e^8 - a^10*e^10)*sqrt((441*c^4*d^8*e^10 - 2268*a*c^3*
d^6*e^12 + 4974*a^2*c^2*d^4*e^14 - 5292*a^3*c*d^2*e^16 + 2401*a^4*e^18)/(a^
5*c^11*d^20 - 10*a^6*c^10*d^18*e^2 + 45*a^7*c^9*d^16*e^4 - 120*a^8*c^8*d^14
*e^6 + 210*a^9*c^7*d^12*e^8 - 252*a^10*c^6*d^10*e^10 + 210*a^11*c^5*d^8*e^1
2 - 120*a^12*c^4*d^6*e^14 + 45*a^13*c^3*d^4*e^16 - 10*a^14*c^2*d^2*e^18 + a
^15*c*e^20)))/(a^5*c^5*d^10 - 5*a^6*c^4*d^8*e^2 + 10*a^7*c^3*d^6*e^4 - 10*a
^8*c^2*d^4*e^6 + 5*a^9*c*d^2*e^8 - a^10*e^10))*log(27*(336*c^4*d^8*e^5 - 17
88*a*c^3*d^6*e^7 + 4189*a^2*c^2*d^4*e^9 - 4802*a^3*c*d^2*e^11 + 2401*a^4*e^
13)*sqrt(e*x + d) - 27*(42*a^3*c^4*d^8*e^6 - 213*a^4*c^3*d^6*e^8 + 515*a^5*
c^2*d^4*e^10 - 623*a^6*c*d^2*e^12 + 343*a^7*e^14 - (4*a^5*c^8*d^15 - 31*a^6
*c^7*d^13*e^2 + 106*a^7*c^6*d^11*e^4 - 205*a^8*c^5*d^9*e^6 + 240*a^9*c^4*d^
7*e^8 - 169*a^10*c^3*d^5*e^10 + 66*a^11*c^2*d^3*e^12 - 11*a^12*c*d*e^14)*sq
rt((441*c^4*d^8*e^10 - 2268*a*c^3*d^6*e^12 + 4974*a^2*c^2*d^4*e^14 - 5292*a
^3*c*d^2*e^16 + 2401*a^4*e^18)/(a^5*c^11*d^20 - 10*a^6*c^10*d^18*e^2 + 45*a
^7*c^9*d^16*e^4 - 120*a^8*c^8*d^14*e^6 + 210*a^9*c^7*d^12*e^8 - 252*a^10*c^
6*d^10*e^10 + 210*a^11*c^5*d^8*e^12 - 120*a^12*c^4*d^6*e^14 + 45*a^13*c^3*d
^4*e^16 - 10*a^14*c^2*d^2*e^18 + a^15*c*e^20)))*sqrt((16*c^4*d^9 - 84*a*c^3
```

*d^7*e^2 + 189*a^2*c^2*d^5*e^4 - 210*a^3*c*d^3*e^6 + 105*a^4*d*e^8 + (a^5*c
^5*d^10 - 5*a^6*c^4*d^8*e^2 + 10*a^7*c^3*d^6*e^4 - 10*a^8*c^2*d^4*e^6 + 5*a
^9*c*d^2*e^8 - a^10*e^10)*sqrt((441*c^4*d^8*e^10 - 2268*a*c^3*d^6*e^12 + 49
74*a^2*c^2*d^4*e^14 - 5292*a^3*c*d^2*e^16 + 2401*a^4*e^18)/(a^5*c^11*d^20 -
10*a^6*c^10*d^18*e^2 + 45*a^7*c^9*d^16*e^4 - 120*a^8*c^8*d^14*e^6 + 210*a^
9*c^7*d^12*e^8 - 252*a^10*c^6*d^10*e^10 + 210*a^11*c^5*d^8*e^12 - 120*a^12*
c^4*d^6*e^14 + 45*a^13*c^3*d^4*e^16 - 10*a^14*c^2*d^2*e^18 + a^15*c*e^20)))
/(a^5*c^5*d^10 - 5*a^6*c^4*d^8*e^2 + 10*a^7*c^3*d^6*e^4 - 10*a^8*c^2*d^4*e^
6 + 5*a^9*c*d^2*e^8 - a^10*e^10))) + 3*(a^4*c^2*d^4 - 2*a^5*c*d^2*e^2 + a^6
*e^4 + (a^2*c^4*d^4 - 2*a^3*c^3*d^2*e^2 + a^4*c^2*e^4)*x^4 - 2*(a^3*c^3*d^4
- 2*a^4*c^2*d^2*e^2 + a^5*c*e^4)*x^2)*sqrt((16*c^4*d^9 - 84*a*c^3*d^7*e^2
+ 189*a^2*c^2*d^5*e^4 - 210*a^3*c*d^3*e^6 + 105*a^4*d*e^8 - (a^5*c^5*d^10 -
5*a^6*c^4*d^8*e^2 + 10*a^7*c^3*d^6*e^4 - 10*a^8*c^2*d^4*e^6 + 5*a^9*c*d^2*
e^8 - a^10*e^10)*sqrt((441*c^4*d^8*e^10 - 2268*a*c^3*d^6*e^12 + 4974*a^2*c^
2*d^4*e^14 - 5292*a^3*c*d^2*e^16 + 2401*a^4*e^18)/(a^5*c^11*d^20 - 10*a^6*c
^10*d^18*e^2 + 45*a^7*c^9*d^16*e^4 - 120*a^8*c^8*d^14*e^6 + 210*a^9*c^7*d^1
2*e^8 - 252*a^10*c^6*d^10*e^10 + 210*a^11*c^5*d^8*e^12 - 120*a^12*c^4*d^6*e
^14 + 45*a^13*c^3*d^4*e^16 - 10*a^14*c^2*d^2*e^18 + a^15*c*e^20)))/(a^5*c^5
*d^10 - 5*a^6*c^4*d^8*e^2 + 10*a^7*c^3*d^6*e^4 - 10*a^8*c^2*d^4*e^6 + 5*a^9
*c*d^2*e^8 - a^10*e^10))*log(27*(336*c^4*d^8*e^5 - 1788*a*c^3*d^6*e^7 + 418
9*a^2*c^2*d^4*e^9 - 4802*a^3*c*d^2*e^11 + 2401*a^4*e^13)*sqrt(e*x + d) + 27
*(42*a^3*c^4*d^8*e^6 - 213*a^4*c^3*d^6*e^8 + 515*a^5*c^2*d^4*e^10 - 623*a^6
*c*d^2*e^12 + 343*a^7*e^14 + (4*a^5*c^8*d^15 - 31*a^6*c^7*d^13*e^2 + 106*a^
7*c^6*d^11*e^4 - 205*a^8*c^5*d^9*e^6 + 240*a^9*c^4*d^7*e^8 - 169*a^10*c^3*d
^5*e^10 + 66*a^11*c^2*d^3*e^12 - 11*a^12*c*d*e^14)*sqrt((441*c^4*d^8*e^10 -
2268*a*c^3*d^6*e^12 + 4974*a^2*c^2*d^4*e^14 - 5292*a^3*c*d^2*e^16 + 2401*a
^4*e^18)/(a^5*c^11*d^20 - 10*a^6*c^10*d^18*e^2 + 45*a^7*c^9*d^16*e^4 - 120*
a^8*c^8*d^14*e^6 + 210*a^9*c^7*d^12*e^8 - 252*a^10*c^6*d^10*e^10 + 210*a^11
*c^5*d^8*e^12 - 120*a^12*c^4*d^6*e^14 + 45*a^13*c^3*d^4*e^16 - 10*a^14*c^2*
d^2*e^18 + a^15*c*e^20)))*sqrt((16*c^4*d^9 - 84*a*c^3*d^7*e^2 + 189*a^2*c^2
*d^5*e^4 - 210*a^3*c*d^3*e^6 + 105*a^4*d*e^8 - (a^5*c^5*d^10 - 5*a^6*c^4*d^
8*e^2 + 10*a^7*c^3*d^6*e^4 - 10*a^8*c^2*d^4*e^6 + 5*a^9*c*d^2*e^8 - a^10*e^
10)*sqrt((441*c^4*d^8*e^10 - 2268*a*c^3*d^6*e^12 + 4974*a^2*c^2*d^4*e^14 -
5292*a^3*c*d^2*e^16 + 2401*a^4*e^18)/(a^5*c^11*d^20 - 10*a^6*c^10*d^18*e^2
+ 45*a^7*c^9*d^16*e^4 - 120*a^8*c^8*d^14*e^6 + 210*a^9*c^7*d^12*e^8 - 252*a
^10*c^6*d^10*e^10 + 210*a^11*c^5*d^8*e^12 - 120*a^12*c^4*d^6*e^14 + 45*a^13
*c^3*d^4*e^16 - 10*a^14*c^2*d^2*e^18 + a^15*c*e^20)))/(a^5*c^5*d^10 - 5*a^6
*c^4*d^8*e^2 + 10*a^7*c^3*d^6*e^4 - 10*a^8*c^2*d^4*e^6 + 5*a^9*c*d^2*e^8 -
a^10*e^10))) - 3*(a^4*c^2*d^4 - 2*a^5*c*d^2*e^2 + a^6*e^4 + (a^2*c^4*d^4 -
2*a^3*c^3*d^2*e^2 + a^4*c^2*e^4)*x^4 - 2*(a^3*c^3*d^4 - 2*a^4*c^2*d^2*e^2 +
a^5*c*e^4)*x^2)*sqrt((16*c^4*d^9 - 84*a*c^3*d^7*e^2 + 189*a^2*c^2*d^5*e^4
- 210*a^3*c*d^3*e^6 + 105*a^4*d*e^8 - (a^5*c^5*d^10 - 5*a^6*c^4*d^8*e^2 + 1
0*a^7*c^3*d^6*e^4 - 10*a^8*c^2*d^4*e^6 + 5*a^9*c*d^2*e^8 - a^10*e^10)*sqrt(
(441*c^4*d^8*e^10 - 2268*a*c^3*d^6*e^12 + 4974*a^2*c^2*d^4*e^14 - 5292*a^3*
c*d^2*e^16 + 2401*a^4*e^18)/(a^5*c^11*d^20 - 10*a^6*c^10*d^18*e^2 + 45*a^7*
c^9*d^16*e^4 - 120*a^8*c^8*d^14*e^6 + 210*a^9*c^7*d^12*e^8 - 252*a^10*c^6*d
^10*e^10 + 210*a^11*c^5*d^8*e^12 - 120*a^12*c^4*d^6*e^14 + 45*a^13*c^3*d^4*
e^16 - 10*a^14*c^2*d^2*e^18 + a^15*c*e^20)))/(a^5*c^5*d^10 - 5*a^6*c^4*d^8*
e^2 + 10*a^7*c^3*d^6*e^4 - 10*a^8*c^2*d^4*e^6 + 5*a^9*c*d^2*e^8 - a^10*e^10
))*log(27*(336*c^4*d^8*e^5 - 1788*a*c^3*d^6*e^7 + 4189*a^2*c^2*d^4*e^9 - 48
02*a^3*c*d^2*e^11 + 2401*a^4*e^13)*sqrt(e*x + d) - 27*(42*a^3*c^4*d^8*e^6 -
213*a^4*c^3*d^6*e^8 + 515*a^5*c^2*d^4*e^10 - 623*a^6*c*d^2*e^12 + 343*a^7*
e^14 + (4*a^5*c^8*d^15 - 31*a^6*c^7*d^13*e^2 + 106*a^7*c^6*d^11*e^4 - 205*a
^8*c^5*d^9*e^6 + 240*a^9*c^4*d^7*e^8 - 169*a^10*c^3*d^5*e^10 + 66*a^11*c^2*
d^3*e^12 - 11*a^12*c*d*e^14)*sqrt((441*c^4*d^8*e^10 - 2268*a*c^3*d^6*e^12 +
4974*a^2*c^2*d^4*e^14 - 5292*a^3*c*d^2*e^16 + 2401*a^4*e^18)/(a^5*c^11*d^2
0 - 10*a^6*c^10*d^18*e^2 + 45*a^7*c^9*d^16*e^4 - 120*a^8*c^8*d^14*e^6 + 210
*a^9*c^7*d^12*e^8 - 252*a^10*c^6*d^10*e^10 + 210*a^11*c^5*d^8*e^12 - 120*a^
12*c^4*d^6*e^14 + 45*a^13*c^3*d^4*e^16 - 10*a^14*c^2*d^2*e^18 + a^15*c*e^20

```
)))*sqrt((16*c^4*d^9 - 84*a*c^3*d^7*e^2 + 189*a^2*c^2*d^5*e^4 - 210*a^3*c*d
^3*e^6 + 105*a^4*d*e^8 - (a^5*c^5*d^10 - 5*a^6*c^4*d^8*e^2 + 10*a^7*c^3*d^6
*e^4 - 10*a^8*c^2*d^4*e^6 + 5*a^9*c*d^2*e^8 - a^10*e^10)*sqrt((441*c^4*d^8*
e^10 - 2268*a*c^3*d^6*e^12 + 4974*a^2*c^2*d^4*e^14 - 5292*a^3*c*d^2*e^16 +
2401*a^4*e^18)/(a^5*c^11*d^20 - 10*a^6*c^10*d^18*e^2 + 45*a^7*c^9*d^16*e^4
- 120*a^8*c^8*d^14*e^6 + 210*a^9*c^7*d^12*e^8 - 252*a^10*c^6*d^10*e^10 + 21
0*a^11*c^5*d^8*e^12 - 120*a^12*c^4*d^6*e^14 + 45*a^13*c^3*d^4*e^16 - 10*a^1
4*c^2*d^2*e^18 + a^15*c*e^20)))/(a^5*c^5*d^10 - 5*a^6*c^4*d^8*e^2 + 10*a^7*
c^3*d^6*e^4 - 10*a^8*c^2*d^4*e^6 + 5*a^9*c*d^2*e^8 - a^10*e^10))) - 4*(5*a^
2*c*d^2*e - 11*a^3*e^3 + 6*(c^3*d^3 - 2*a*c^2*d*e^2)*x^3 - (a*c^2*d^2*e - 7
*a^2*c*e^3)*x^2 - 2*(5*a*c^2*d^3 - 8*a^2*c*d*e^2)*x)*sqrt(e*x + d))/(a^4*c^
2*d^4 - 2*a^5*c*d^2*e^2 + a^6*e^4 + (a^2*c^4*d^4 - 2*a^3*c^3*d^2*e^2 + a^4*
c^2*e^4)*x^4 - 2*(a^3*c^3*d^4 - 2*a^4*c^2*d^2*e^2 + a^5*c*e^4)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*x**2+a)**3/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*x^2+a)^3/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.643 $\int \frac{(d+ex)^{7/2}}{\left(a+cx^2\right)^3}\,dx$

Optimal. Leaf size=905

$$-\frac{(ae-cdx)(d+ex)^{5/2}}{4ac\left(cx^2+a\right)^2}-\frac{\left(ae\left(7cd^2+5ae^2\right)-2cd\left(3cd^2+2ae^2\right)x\right)\sqrt{d+ex}}{16a^2c^2\left(cx^2+a\right)}+\frac{e\left(6c^2d^4+11ace^2d^2+\sqrt{c}\sqrt{cd^2+ae^2}\left(6cd^2+\right.\right.}{32\sqrt{2}a^2c^{9/4}\sqrt{cd^2}}$$

```
[Out] -((a*e - c*d*x)*(d + e*x)^(5/2))/(4*a*c*(a + c*x^2)^2) - (Sqrt[d + e*x]*(a*
e*(7*c*d^2 + 5*a*e^2) - 2*c*d*(3*c*d^2 + 2*a*e^2)*x))/(16*a^2*c^2*(a + c*x^
2)) + (e*(6*c^2*d^4 + 11*a*c*d^2*e^2 + 5*a^2*e^4 + Sqrt[c]*d*Sqrt[c*d^2 + a
*e^2]*(6*c*d^2 + 8*a*e^2))*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] -
 Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(32
*Sqrt[2]*a^2*c^(9/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^
2]]) - (e*(6*c^2*d^4 + 11*a*c*d^2*e^2 + 5*a^2*e^4 + Sqrt[c]*d*Sqrt[c*d^2 +
a*e^2]*(6*c*d^2 + 8*a*e^2))*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]
+ Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(3
2*Sqrt[2]*a^2*c^(9/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e
^2]]) - (e*(6*c^2*d^4 + 11*a*c*d^2*e^2 + 5*a^2*e^4 - 2*Sqrt[c]*d*Sqrt[c*d^2
 + a*e^2]*(3*c*d^2 + 4*a*e^2))*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sq
rt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(64
*Sqrt[2]*a^2*c^(9/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^
2]]) + (e*(6*c^2*d^4 + 11*a*c*d^2*e^2 + 5*a^2*e^4 - 2*Sqrt[c]*d*Sqrt[c*d^2
+ a*e^2]*(3*c*d^2 + 4*a*e^2))*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqr
t[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(64*
Sqrt[2]*a^2*c^(9/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2
]])
```

Rubi [A] time = 5.63542, antiderivative size = 905, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {739, 819, 827, 1169, 634, 618, 206, 628}

$$-\frac{(ae-cdx)(d+ex)^{5/2}}{4ac\left(cx^2+a\right)^2}-\frac{\left(ae\left(7cd^2+5ae^2\right)-2cd\left(3cd^2+2ae^2\right)x\right)\sqrt{d+ex}}{16a^2c^2\left(cx^2+a\right)}+\frac{e\left(6c^2d^4+11ace^2d^2+\sqrt{c}\sqrt{cd^2+ae^2}\left(6cd^2+\right.\right.}{32\sqrt{2}a^2c^{9/4}\sqrt{cd^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/(a + c*x^2)^3,x]
```


```
[Out] -((a*e - c*d*x)*(d + e*x)^(5/2))/(4*a*c*(a + c*x^2)^2) - (Sqrt[d + e*x]*(a*
e*(7*c*d^2 + 5*a*e^2) - 2*c*d*(3*c*d^2 + 2*a*e^2)*x))/(16*a^2*c^2*(a + c*x^
2)) + (e*(6*c^2*d^4 + 11*a*c*d^2*e^2 + 5*a^2*e^4 + Sqrt[c]*d*Sqrt[c*d^2 + a
*e^2]*(6*c*d^2 + 8*a*e^2))*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] -
 Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(32
*Sqrt[2]*a^2*c^(9/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^
2]]) - (e*(6*c^2*d^4 + 11*a*c*d^2*e^2 + 5*a^2*e^4 + Sqrt[c]*d*Sqrt[c*d^2 +
a*e^2]*(6*c*d^2 + 8*a*e^2))*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]
+ Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(3
2*Sqrt[2]*a^2*c^(9/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e
^2]]) - (e*(6*c^2*d^4 + 11*a*c*d^2*e^2 + 5*a^2*e^4 - 2*Sqrt[c]*d*Sqrt[c*d^2
 + a*e^2]*(3*c*d^2 + 4*a*e^2))*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sq
rt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(64
```

```
*Sqrt[2]*a^2*c^(9/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^
2]]) + (e*(6*c^2*d^4 + 11*a*c*d^2*e^2 + 5*a^2*e^4 - 2*Sqrt[c]*d*Sqrt[c*d^2
+ a*e^2]*(3*c*d^2 + 4*a*e^2))*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqr
t[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(64*
Sqrt[2]*a^2*c^(9/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2
]])
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{7/2}}{\left(a+cx^2\right)^3}\,dx = -\frac{(ae-cdx)(d+ex)^{5/2}}{4ac\left(a+cx^2\right)^2} + \frac{\int \frac{(d+ex)^{3/2}\left(\frac{1}{2}\left(6cd^2+5ae^2\right)+\frac{1}{2}cdex\right)}{\left(a+cx^2\right)^2}\,dx}{4ac}$$

$$= -\frac{(ae-cdx)(d+ex)^{5/2}}{4ac\left(a+cx^2\right)^2} - \frac{\sqrt{d+ex}\left(ae\left(7cd^2+5ae^2\right)-2cd\left(3cd^2+2ae^2\right)x\right)}{16a^2c^2\left(a+cx^2\right)} + \frac{\int \frac{\frac{1}{4}\left(3cd^2+ae^2\right)\left(4cd^2+5ae^2\right)+\frac{1}{2}\cdots}{\sqrt{d+ex}\left(a+cx^2\right)}\cdots}{8a^2c^2}$$

$$= -\frac{(ae-cdx)(d+ex)^{5/2}}{4ac\left(a+cx^2\right)^2} - \frac{\sqrt{d+ex}\left(ae\left(7cd^2+5ae^2\right)-2cd\left(3cd^2+2ae^2\right)x\right)}{16a^2c^2\left(a+cx^2\right)} + \frac{\operatorname{Subst}\left(\int \frac{-\frac{1}{2}cd^2e\left(3cd^2+4ae^2\right)\cdots}{\cdots}\cdots\right.}{\cdots}$$

$$= -\frac{(ae-cdx)(d+ex)^{5/2}}{4ac\left(a+cx^2\right)^2} - \frac{\sqrt{d+ex}\left(ae\left(7cd^2+5ae^2\right)-2cd\left(3cd^2+2ae^2\right)x\right)}{16a^2c^2\left(a+cx^2\right)} + \frac{\operatorname{Subst}\left(\int \frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\cdots}{\cdots}\cdots\right.}{\cdots}$$

$$= -\frac{(ae-cdx)(d+ex)^{5/2}}{4ac\left(a+cx^2\right)^2} - \frac{\sqrt{d+ex}\left(ae\left(7cd^2+5ae^2\right)-2cd\left(3cd^2+2ae^2\right)x\right)}{16a^2c^2\left(a+cx^2\right)} + \frac{\left(\frac{1}{2}cd^2e\left(3cd^2+4ae^2\right)+\cdots\right.}{\cdots}$$

$$= -\frac{(ae-cdx)(d+ex)^{5/2}}{4ac\left(a+cx^2\right)^2} - \frac{\sqrt{d+ex}\left(ae\left(7cd^2+5ae^2\right)-2cd\left(3cd^2+2ae^2\right)x\right)}{16a^2c^2\left(a+cx^2\right)} - \frac{e\left(6c^2d^4+11acd^2e^2+5a\cdots\right.}{\cdots}$$

$$= -\frac{(ae-cdx)(d+ex)^{5/2}}{4ac\left(a+cx^2\right)^2} - \frac{\sqrt{d+ex}\left(ae\left(7cd^2+5ae^2\right)-2cd\left(3cd^2+2ae^2\right)x\right)}{16a^2c^2\left(a+cx^2\right)} + \frac{e\left(6c^2d^4+11acd^2e^2+5a\cdots\right.}{\cdots}$$

Mathematica [A] time = 1.09997, size = 343, normalized size = 0.38

$$\frac{\frac{2\sqrt[4]{c}\sqrt{d+ex}\left(-a^2ce\left(11d^2+4dex+9e^2x^2\right)-5a^3e^3+ac^2dx\left(10d^2+dex+8e^2x^2\right)+6c^3d^3x^3\right)}{a^2\left(a+cx^2\right)^2} + \frac{\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(6\sqrt{-a}cd^2e+13a\sqrt{c}de^2+5\sqrt{-a}ae^3+12c^{3/2}d^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+e\cdots}}{\sqrt{\sqrt{c}d-\sqrt{\cdots}}}\right.}{(-a)^{5/2}}\cdots}{32c^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)/(a + c*x^2)^3,x]
```

```
[Out] ((2*c^(1/4)*Sqrt[d + e*x]*(-5*a^3*e^3 + 6*c^3*d^3*x^3 + a*c^2*d*x*(10*d^2 +
 d*e*x + 8*e^2*x^2) - a^2*c*e*(11*d^2 + 4*d*e*x + 9*e^2*x^2)))/(a^2*(a + c*
x^2)^2) + (Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*(12*c^(3/2)*d^3 + 6*Sqrt[-a]*c*d^2*
e + 13*a*Sqrt[c]*d*e^2 + 5*Sqrt[-a]*a*e^3)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/
Sqrt[Sqrt[c]*d - Sqrt[-a]*e]])/(-a)^(5/2) + (a*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]
*(12*c^(3/2)*d^3 - 6*Sqrt[-a]*c*d^2*e + 13*a*Sqrt[c]*d*e^2 + 5*(-a)^(3/2)*e
^3)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[-a]*e]])/(-a)^(7/
2))/(32*c^(9/4))
```

Maple [B] time = 0.247, size = 5915, normalized size = 6.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/(c*x^2+a)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{7}{2}}}{\left(cx^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(7/2)/(c*x^2 + a)^3, x)
```

Fricas [B] time = 3.29809, size = 3903, normalized size = 4.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*((a^2*c^4*x^4 + 2*a^3*c^3*x^2 + a^4*c^2)*sqrt(-(144*c^3*d^7 + 420*a*c^
2*d^5*e^2 + 385*a^2*c*d^3*e^4 + 105*a^3*d*e^6 + a^5*c^4*sqrt(-(441*c^2*d^4*
e^10 + 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))/(a^5*c^4))*log((3024*c
^4*d^8*e^5 + 10908*a*c^3*d^6*e^7 + 13509*a^2*c^2*d^4*e^9 + 6250*a^3*c*d^2*e
^11 + 625*a^4*e^13)*sqrt(e*x + d) + (126*a^3*c^4*d^4*e^6 + 255*a^4*c^3*d^2*
e^8 + 125*a^5*c^2*e^10 + (12*a^5*c^8*d^3 + 13*a^6*c^7*d*e^2)*sqrt(-(441*c^2
*d^4*e^10 + 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))*sqrt(-(144*c^3*d^
7 + 420*a*c^2*d^5*e^2 + 385*a^2*c*d^3*e^4 + 105*a^3*d*e^6 + a^5*c^4*sqrt(-(
441*c^2*d^4*e^10 + 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))/(a^5*c^4))
) - (a^2*c^4*x^4 + 2*a^3*c^3*x^2 + a^4*c^2)*sqrt(-(144*c^3*d^7 + 420*a*c^2*
d^5*e^2 + 385*a^2*c*d^3*e^4 + 105*a^3*d*e^6 + a^5*c^4*sqrt(-(441*c^2*d^4*e^
10 + 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))/(a^5*c^4))*log((3024*c^4
*d^8*e^5 + 10908*a*c^3*d^6*e^7 + 13509*a^2*c^2*d^4*e^9 + 6250*a^3*c*d^2*e^1
```

```
1 + 625*a^4*e^13)*sqrt(e*x + d) - (126*a^3*c^4*d^4*e^6 + 255*a^4*c^3*d^2*e^
8 + 125*a^5*c^2*e^10 + (12*a^5*c^8*d^3 + 13*a^6*c^7*d*e^2)*sqrt(-(441*c^2*d
^4*e^10 + 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))*sqrt(-(144*c^3*d^7
+ 420*a*c^2*d^5*e^2 + 385*a^2*c*d^3*e^4 + 105*a^3*d*e^6 + a^5*c^4*sqrt(-(44
1*c^2*d^4*e^10 + 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))/(a^5*c^4)))
+ (a^2*c^4*x^4 + 2*a^3*c^3*x^2 + a^4*c^2)*sqrt(-(144*c^3*d^7 + 420*a*c^2*d^
5*e^2 + 385*a^2*c*d^3*e^4 + 105*a^3*d*e^6 - a^5*c^4*sqrt(-(441*c^2*d^4*e^10
 + 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))/(a^5*c^4))*log((3024*c^4*d
^8*e^5 + 10908*a*c^3*d^6*e^7 + 13509*a^2*c^2*d^4*e^9 + 6250*a^3*c*d^2*e^11
+ 625*a^4*e^13)*sqrt(e*x + d) + (126*a^3*c^4*d^4*e^6 + 255*a^4*c^3*d^2*e^8
+ 125*a^5*c^2*e^10 - (12*a^5*c^8*d^3 + 13*a^6*c^7*d*e^2)*sqrt(-(441*c^2*d^4
*e^10 + 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))*sqrt(-(144*c^3*d^7 +
420*a*c^2*d^5*e^2 + 385*a^2*c*d^3*e^4 + 105*a^3*d*e^6 - a^5*c^4*sqrt(-(441*
c^2*d^4*e^10 + 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))/(a^5*c^4))) -
(a^2*c^4*x^4 + 2*a^3*c^3*x^2 + a^4*c^2)*sqrt(-(144*c^3*d^7 + 420*a*c^2*d^5*
e^2 + 385*a^2*c*d^3*e^4 + 105*a^3*d*e^6 - a^5*c^4*sqrt(-(441*c^2*d^4*e^10 +
 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))/(a^5*c^4))*log((3024*c^4*d^8
*e^5 + 10908*a*c^3*d^6*e^7 + 13509*a^2*c^2*d^4*e^9 + 6250*a^3*c*d^2*e^11 +
625*a^4*e^13)*sqrt(e*x + d) - (126*a^3*c^4*d^4*e^6 + 255*a^4*c^3*d^2*e^8 +
125*a^5*c^2*e^10 - (12*a^5*c^8*d^3 + 13*a^6*c^7*d*e^2)*sqrt(-(441*c^2*d^4*e
^10 + 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))*sqrt(-(144*c^3*d^7 + 42
0*a*c^2*d^5*e^2 + 385*a^2*c*d^3*e^4 + 105*a^3*d*e^6 - a^5*c^4*sqrt(-(441*c^
2*d^4*e^10 + 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))/(a^5*c^4))) - 4*
(11*a^2*c*d^2*e + 5*a^3*e^3 - 2*(3*c^3*d^3 + 4*a*c^2*d*e^2)*x^3 - (a*c^2*d^
2*e - 9*a^2*c*e^3)*x^2 - 2*(5*a*c^2*d^3 - 2*a^2*c*d*e^2)*x)*sqrt(e*x + d))/
(a^2*c^4*x^4 + 2*a^3*c^3*x^2 + a^4*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/(c*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.644 $\int \frac{(d+ex)^{5/2}}{(a+cx^2)^3} dx$

Optimal. Leaf size=846

$$-\frac{(ae-cdx)(d+ex)^{3/2}}{4ac\left(cx^2+a\right)^2}-\frac{3\left(ade-\left(2cd^2+ae^2\right)x\right)\sqrt{d+ex}}{16a^2c\left(cx^2+a\right)}+\frac{3e\left(2c^{3/2}d^3+2a\sqrt{c}e^2d+\sqrt{cd^2+ae^2}\left(2cd^2+ae^2\right)\right)\tanh^{-}}{32\sqrt{2}a^2c^{7/4}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d-\sqrt{cd}}}$$

```
[Out] -((a*e - c*d*x)*(d + e*x)^(3/2))/(4*a*c*(a + c*x^2)^2) - (3*Sqrt[d + e*x]*(
a*d*e - (2*c*d^2 + a*e^2)*x))/(16*a^2*c*(a + c*x^2)) + (3*e*(2*c^(3/2)*d^3
+ 2*a*Sqrt[c]*d*e^2 + Sqrt[c*d^2 + a*e^2]*(2*c*d^2 + a*e^2))*ArcTanh[(Sqrt[
Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] - Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt
[c]*d - Sqrt[c*d^2 + a*e^2]]])/(32*Sqrt[2]*a^2*c^(7/4)*Sqrt[c*d^2 + a*e^2]*
Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (3*e*(2*c^(3/2)*d^3 + 2*a*Sqrt[c]*
d*e^2 + Sqrt[c*d^2 + a*e^2]*(2*c*d^2 + a*e^2))*ArcTanh[(Sqrt[Sqrt[c]*d + Sq
rt[c*d^2 + a*e^2]] + Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c
*d^2 + a*e^2]]])/(32*Sqrt[2]*a^2*c^(7/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d
 - Sqrt[c*d^2 + a*e^2]]) - (3*e*(2*c^(3/2)*d^3 + 2*a*Sqrt[c]*d*e^2 - Sqrt[c
*d^2 + a*e^2]*(2*c*d^2 + a*e^2))*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*
Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(
64*Sqrt[2]*a^2*c^(7/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*
e^2]]) + (3*e*(2*c^(3/2)*d^3 + 2*a*Sqrt[c]*d*e^2 - Sqrt[c*d^2 + a*e^2]*(2*c
*d^2 + a*e^2))*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + S
qrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(64*Sqrt[2]*a^2*c^(
7/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])
```

Rubi [A] time = 3.39583, antiderivative size = 846, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {739, 821, 827, 1169, 634, 618, 206, 628}

$$-\frac{(ae-cdx)(d+ex)^{3/2}}{4ac\left(cx^2+a\right)^2}-\frac{3\left(ade-\left(2cd^2+ae^2\right)x\right)\sqrt{d+ex}}{16a^2c\left(cx^2+a\right)}+\frac{3e\left(2c^{3/2}d^3+2a\sqrt{c}e^2d+\sqrt{cd^2+ae^2}\left(2cd^2+ae^2\right)\right)\tanh^{-}}{32\sqrt{2}a^2c^{7/4}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d-\sqrt{cd}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(a + c*x^2)^3,x]
```


```
[Out] -((a*e - c*d*x)*(d + e*x)^(3/2))/(4*a*c*(a + c*x^2)^2) - (3*Sqrt[d + e*x]*(
a*d*e - (2*c*d^2 + a*e^2)*x))/(16*a^2*c*(a + c*x^2)) + (3*e*(2*c^(3/2)*d^3
+ 2*a*Sqrt[c]*d*e^2 + Sqrt[c*d^2 + a*e^2]*(2*c*d^2 + a*e^2))*ArcTanh[(Sqrt[
Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] - Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt
[c]*d - Sqrt[c*d^2 + a*e^2]]])/(32*Sqrt[2]*a^2*c^(7/4)*Sqrt[c*d^2 + a*e^2]*
Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (3*e*(2*c^(3/2)*d^3 + 2*a*Sqrt[c]*
d*e^2 + Sqrt[c*d^2 + a*e^2]*(2*c*d^2 + a*e^2))*ArcTanh[(Sqrt[Sqrt[c]*d + Sq
rt[c*d^2 + a*e^2]] + Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c
*d^2 + a*e^2]]])/(32*Sqrt[2]*a^2*c^(7/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d
 - Sqrt[c*d^2 + a*e^2]]) - (3*e*(2*c^(3/2)*d^3 + 2*a*Sqrt[c]*d*e^2 - Sqrt[c
*d^2 + a*e^2]*(2*c*d^2 + a*e^2))*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*
Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(
64*Sqrt[2]*a^2*c^(7/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*
e^2]]) + (3*e*(2*c^(3/2)*d^3 + 2*a*Sqrt[c]*d*e^2 - Sqrt[c*d^2 + a*e^2]*(2*c
*d^2 + a*e^2))*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + S
```

```
qrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(64*Sqrt[2]*a^2*c^(
7/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\left(a+cx^2\right)^3}\,dx = -\frac{(ae-cdx)(d+ex)^{3/2}}{4ac\left(a+cx^2\right)^2} + \frac{\int \frac{\sqrt{d+ex}\left(\frac{3}{2}\left(2cd^2+ae^2\right)+\frac{3}{2}cdex\right)}{\left(a+cx^2\right)^2}dx}{4ac}$$

$$= -\frac{(ae-cdx)(d+ex)^{3/2}}{4ac\left(a+cx^2\right)^2} - \frac{3\sqrt{d+ex}\left(ade-\left(2cd^2+ae^2\right)x\right)}{16a^2c\left(a+cx^2\right)} + \frac{\int \frac{\frac{3}{4}cd\left(4cd^2+3ae^2\right)+\frac{3}{4}ce\left(2cd^2+ae^2\right)x}{\sqrt{d+ex}\left(a+cx^2\right)}dx}{8a^2c^2}$$

$$= -\frac{(ae-cdx)(d+ex)^{3/2}}{4ac\left(a+cx^2\right)^2} - \frac{3\sqrt{d+ex}\left(ade-\left(2cd^2+ae^2\right)x\right)}{16a^2c\left(a+cx^2\right)} + \frac{\text{Subst}\left(\int \frac{-\frac{3}{4}cde\left(2cd^2+ae^2\right)+\frac{3}{4}cde\left(4cd^2+3ae^2\right)}{cd^2+ae^2-2cdx^2+cx}\right.}{4a^2c^2}$$

$$= -\frac{(ae-cdx)(d+ex)^{3/2}}{4ac\left(a+cx^2\right)^2} - \frac{3\sqrt{d+ex}\left(ade-\left(2cd^2+ae^2\right)x\right)}{16a^2c\left(a+cx^2\right)} + \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(-\frac{3}{4}cde\left(2cd^2+ae^2\right)+\frac{3}{4}\right.}{\sqrt[4]{c}}}{}\right.}{}$$

$$= -\frac{(ae-cdx)(d+ex)^{3/2}}{4ac\left(a+cx^2\right)^2} - \frac{3\sqrt{d+ex}\left(ade-\left(2cd^2+ae^2\right)x\right)}{16a^2c\left(a+cx^2\right)} - \frac{\left(3e\left(2c^{3/2}d^3+2a\sqrt{c}de^2-\sqrt{cd^2+ae^2}\right.\right.}{64\sqrt{2}a}$$

$$= -\frac{(ae-cdx)(d+ex)^{3/2}}{4ac\left(a+cx^2\right)^2} - \frac{3\sqrt{d+ex}\left(ade-\left(2cd^2+ae^2\right)x\right)}{16a^2c\left(a+cx^2\right)} - \frac{3e\left(2c^{3/2}d^3+2a\sqrt{c}de^2-\sqrt{cd^2+ae^2}\right.}{}$$

$$= -\frac{(ae-cdx)(d+ex)^{3/2}}{4ac\left(a+cx^2\right)^2} - \frac{3\sqrt{d+ex}\left(ade-\left(2cd^2+ae^2\right)x\right)}{16a^2c\left(a+cx^2\right)} + \frac{3e\left(2c^{3/2}d^3+2a\sqrt{c}de^2+\sqrt{cd^2+ae^2}\right.}{32\sqrt{2}a^2c^{7/4}\sqrt{cd^2}}$$

Mathematica [A] time = 0.725081, size = 277, normalized size = 0.33

$$\frac{\frac{2c^{3/4}\sqrt{d+ex}\left(-a^2e(7d+ex)+acx\left(10d^2+dex+3e^2x^2\right)+6c^2d^2x^3\right)}{a^2\left(a+cx^2\right)^2} + \frac{3\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(2\sqrt{-a}\sqrt{c}de+ae^2+4cd^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{(-a)^{5/2}} - \frac{3\sqrt{\sqrt{-a}e+\sqrt{c}d}\left(-2\sqrt{-a}\sqrt{c}de+\right.}{(-}}{32c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(a + c*x^2)^3,x]
```

```
[Out] ((2*c^(3/4)*Sqrt[d + e*x]*(6*c^2*d^2*x^3 - a^2*e*(7*d + e*x) + a*c*x*(10*d^
2 + d*e*x + 3*e^2*x^2)))/(a^2*(a + c*x^2)^2) + (3*Sqrt[Sqrt[c]*d - Sqrt[-a]
*e]*(4*c*d^2 + 2*Sqrt[-a]*Sqrt[c]*d*e + a*e^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*
x])/Sqrt[Sqrt[c]*d - Sqrt[-a]*e]])/(-a)^(5/2) - (3*Sqrt[Sqrt[c]*d + Sqrt[-a
]*e]*(4*c*d^2 - 2*Sqrt[-a]*Sqrt[c]*d*e + a*e^2)*ArcTanh[(c^(1/4)*Sqrt[d + e
```

```
*x])/Sqrt[Sqrt[c]*d + Sqrt[-a]*e]])/(-a)^(5/2))/(32*c^(7/4))
```

Maple [B] time = 0.245, size = 4264, normalized size = 5.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(c*x^2+a)^3,x)
```

```
[Out] -3/32/e/a^3/c^(1/2)/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2
)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)-(2*(c*(a*e^2+c*d^2))^(1/2)+2
*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d
)^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)
+2*c*d)^(1/2)*d^3+3/64*e/a^2/c^(5/2)/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a
*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^
2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*
d^2))^(1/2)-2*c*d)^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e
^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)-3/64*e/a^2/c^(3/2)/(
4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan
((2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^
2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2*(c*(a*e^2
+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*d-3/64/
e/a^3/c^2*ln((e*x+d)*c^(1/2)-(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d
)^(1/2)+(a*e^2+c*d^2)^(1/2))*(a*e^2+c*d^2)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)
+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)*d-1/2*e^3/(c*e^2*x^2+a*e^2)^2*d/a*(e*
x+d)^(5/2)+17/16*e^3/(c*e^2*x^2+a*e^2)^2/a*(e*x+d)^(3/2)*d^2-3/4*e^3/(c*e^2
*x^2+a*e^2)^2*d^3/a*(e*x+d)^(1/2)-3/8*e^5/(c*e^2*x^2+a*e^2)^2*d/c*(e*x+d)^(
1/2)-3/32/e/a^3/c^2/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2
)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)-(2*(c*(a*e^2+c*d^2))^(1/2)+2
*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d
)^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(a*e^2+c*d^2)^(1/2)*(2*(a*
c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)*d-3/32/e/a^3/c^2/
(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arcta
n((2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e
^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2*(c*(a*e^
2+c*d^2))^(1/2)+2*c*d)^(1/2)*(a*e^2+c*d^2)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)
+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)*d+3/64/e/a^3/c*ln((e*x+d)*c^(1/2)-(e*
x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(a*
e^2+c*d^2)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*d^2+3/64/e/a^3/c^(
3/2)*ln((e*x+d)*c^(1/2)-(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/
2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2+c^
2*d^2)^(1/2)*d^2-3/64/e/a^3/c^(3/2)*ln((e*x+d)*c^(1/2)+(e*x+d)^(1/2)*(2*(c*
(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)
^(1/2)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)*d^2-3/64/e/a^3/c*ln((e*x+d)*c^(
1/2)+(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1
/2))*(a*e^2+c*d^2)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*d^2+3/8*e/
a^2/c/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)
*arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(
4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(a*e^
2+c*d^2)^(1/2)*d+3/16*e^3/(c*e^2*x^2+a*e^2)^2/a*(e*x+d)^(7/2)-1/16*e^5/(c*e
^2*x^2+a*e^2)^2/c*(e*x+d)^(3/2)+3/32/e/a^3/c^(3/2)/(4*(a*e^2+c*d^2)^(1/2)*c
^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/
2)+(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-
2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(
1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)*d^2+3/
32/e/a^3/c^(3/2)/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2
```

```
*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)-(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*
d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(
1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*
c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)*d^2+3/32/e/a^3/c/(4*(a*e^2+c*d^2)^(1/2)*
c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1
/2)+(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)
-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^
(1/2)*(a*e^2+c*d^2)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*d^2+3/32/
e/a^3/c/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/
2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)-(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))
/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2*
(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(a*e^2+c*d^2)^(1/2)*(2*(a*c*e^2+c^2*d^
2)^(1/2)+2*c*d)^(1/2)*d^2+3/64/e/a^3/c^2*ln((e*x+d)*c^(1/2)+(e*x+d)^(1/2)*(
2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(a*e^2+c*d^2)^(
1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)*d-3/64
*e/a^2/c^(3/2)/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c
*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)-(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)
^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/
2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*
d)^(1/2)*d+3/64*e/a^2/c^(5/2)/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*
d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)-(2*(c*(a*e^2+c*d^2
))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(
1/2)-2*c*d)^(1/2))*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*
d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)-3/32/e/a^3/c^(1/2)/(4*(a*e^
2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arctan((2*c^(
1/2)*(e*x+d)^(1/2)+(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2
)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(2*(c*(a*e^2+c*d^2)
)^(1/2)+2*c*d)^(1/2)*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*d^3+3/8*e/a^2/
c/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2)*arc
tan((2*c^(1/2)*(e*x+d)^(1/2)-(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2))/(4*(a
*e^2+c*d^2)^(1/2)*c^(1/2)-2*(c*(a*e^2+c*d^2))^(1/2)-2*c*d)^(1/2))*(a*e^2+c*
d^2)^(1/2)*d+3/64/e/a^3/c^(1/2)*ln((e*x+d)*c^(1/2)+(e*x+d)^(1/2)*(2*(c*(a*e
^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)^(1/
2)+2*c*d)^(1/2)*d^3-3/64/e/a^3/c^(1/2)*ln((e*x+d)*c^(1/2)-(e*x+d)^(1/2)*(2*
(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d
^2)^(1/2)+2*c*d)^(1/2)*d^3+3/8*e/(c*e^2*x^2+a*e^2)^2/a^2*(e*x+d)^(7/2)*c*d^
2-9/8*e/(c*e^2*x^2+a*e^2)^2*d^3/a^2*(e*x+d)^(5/2)*c+9/8*e/(c*e^2*x^2+a*e^2)
^2/a^2*c*(e*x+d)^(3/2)*d^4-3/8*e/(c*e^2*x^2+a*e^2)^2*d^5/a^2*c*(e*x+d)^(1/2
)+3/128*e/a^2/c^(3/2)*ln((e*x+d)*c^(1/2)+(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))
^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^
(1/2)*d-3/128*e/a^2/c^(5/2)*ln((e*x+d)*c^(1/2)+(e*x+d)^(1/2)*(2*(c*(a*e^2+c
*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*(2*(a*c*e^2+c^2*d^2)^(1/2)+2
*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)-3/128*e/a^2/c^(3/2)*ln((e*x+d)*c^(1/2)-
(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))*
(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*d+3/128*e/a^2/c^(5/2)*ln((e*x+d)*c^
(1/2)-(e*x+d)^(1/2)*(2*(c*(a*e^2+c*d^2))^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(
1/2))*(2*(a*c*e^2+c^2*d^2)^(1/2)+2*c*d)^(1/2)*(a*c*e^2+c^2*d^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{(cx^2+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/(c*x^2 + a)^3, x)
```

Fricas [A] time = 2.52199, size = 2194, normalized size = 2.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(3*(a^2*c^3*x^4 + 2*a^3*c^2*x^2 + a^4*c)*sqrt(-(a^5*c^3*sqrt(-e^10/(a^
5*c^7)) + 16*c^2*d^5 + 20*a*c*d^3*e^2 + 5*a^2*d*e^4)/(a^5*c^3))*log(27*(16*
c^2*d^4*e^5 + 12*a*c*d^2*e^7 + a^2*e^9)*sqrt(e*x + d) + 27*(2*a^3*c^2*d*e^6
 - (4*a^5*c^6*d^2 + a^6*c^5*e^2)*sqrt(-e^10/(a^5*c^7)))*sqrt(-(a^5*c^3*sqrt
(-e^10/(a^5*c^7)) + 16*c^2*d^5 + 20*a*c*d^3*e^2 + 5*a^2*d*e^4)/(a^5*c^3)))
- 3*(a^2*c^3*x^4 + 2*a^3*c^2*x^2 + a^4*c)*sqrt(-(a^5*c^3*sqrt(-e^10/(a^5*c^
7)) + 16*c^2*d^5 + 20*a*c*d^3*e^2 + 5*a^2*d*e^4)/(a^5*c^3))*log(27*(16*c^2*
d^4*e^5 + 12*a*c*d^2*e^7 + a^2*e^9)*sqrt(e*x + d) - 27*(2*a^3*c^2*d*e^6 - (
4*a^5*c^6*d^2 + a^6*c^5*e^2)*sqrt(-e^10/(a^5*c^7)))*sqrt(-(a^5*c^3*sqrt(-e^
10/(a^5*c^7)) + 16*c^2*d^5 + 20*a*c*d^3*e^2 + 5*a^2*d*e^4)/(a^5*c^3))) + 3*
(a^2*c^3*x^4 + 2*a^3*c^2*x^2 + a^4*c)*sqrt((a^5*c^3*sqrt(-e^10/(a^5*c^7)) -
 16*c^2*d^5 - 20*a*c*d^3*e^2 - 5*a^2*d*e^4)/(a^5*c^3))*log(27*(16*c^2*d^4*e
^5 + 12*a*c*d^2*e^7 + a^2*e^9)*sqrt(e*x + d) + 27*(2*a^3*c^2*d*e^6 + (4*a^5
*c^6*d^2 + a^6*c^5*e^2)*sqrt(-e^10/(a^5*c^7)))*sqrt((a^5*c^3*sqrt(-e^10/(a^
5*c^7)) - 16*c^2*d^5 - 20*a*c*d^3*e^2 - 5*a^2*d*e^4)/(a^5*c^3))) - 3*(a^2*c
^3*x^4 + 2*a^3*c^2*x^2 + a^4*c)*sqrt((a^5*c^3*sqrt(-e^10/(a^5*c^7)) - 16*c^
2*d^5 - 20*a*c*d^3*e^2 - 5*a^2*d*e^4)/(a^5*c^3))*log(27*(16*c^2*d^4*e^5 + 1
2*a*c*d^2*e^7 + a^2*e^9)*sqrt(e*x + d) - 27*(2*a^3*c^2*d*e^6 + (4*a^5*c^6*d
^2 + a^6*c^5*e^2)*sqrt(-e^10/(a^5*c^7)))*sqrt((a^5*c^3*sqrt(-e^10/(a^5*c^7)
) - 16*c^2*d^5 - 20*a*c*d^3*e^2 - 5*a^2*d*e^4)/(a^5*c^3))) + 4*(a*c*d*e*x^2
 - 7*a^2*d*e + 3*(2*c^2*d^2 + a*c*e^2)*x^3 + (10*a*c*d^2 - a^2*e^2)*x)*sqrt
(e*x + d))/(a^2*c^3*x^4 + 2*a^3*c^2*x^2 + a^4*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(c*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.645 $\int \frac{(d+ex)^{3/2}}{\left(a+cx^2\right)^3}\,dx$

Optimal. Leaf size=769

$$-\frac{3e\left(-2\sqrt{c}d\sqrt{ae^2+cd^2}+ae^2+2cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}+\sqrt{ae^2+cd^2}+\sqrt{c}(d+ex)\right)}{64\sqrt{2}a^2c^{5/4}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{3e\left(-2\right.}{}$$

```
[Out] -((a*e - c*d*x)*Sqrt[d + e*x])/(4*a*c*(a + c*x^2)^2) + ((a*e + 6*c*d*x)*Sqr
t[d + e*x])/(16*a^2*c*(a + c*x^2)) + (3*e*(2*c*d^2 + a*e^2 + 2*Sqrt[c]*d*Sq
rt[c*d^2 + a*e^2])*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] - Sqrt[2]
*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(32*Sqrt[2]
*a^2*c^(5/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (
3*e*(2*c*d^2 + a*e^2 + 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*ArcTanh[(Sqrt[Sqrt[
c]*d + Sqrt[c*d^2 + a*e^2]] + Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d
 - Sqrt[c*d^2 + a*e^2]]])/(32*Sqrt[2]*a^2*c^(5/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[
Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (3*e*(2*c*d^2 + a*e^2 - 2*Sqrt[c]*d*Sqr
t[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d
+ Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(64*Sqrt[2]*a^2*
c^(5/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]) + (3*e*(
2*c*d^2 + a*e^2 - 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^2]
+ Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqr
t[c]*(d + e*x)])/(64*Sqrt[2]*a^2*c^(5/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d
 + Sqrt[c*d^2 + a*e^2]])
```

Rubi [A] time = 1.93555, antiderivative size = 769, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {739, 823, 827, 1169, 634, 618, 206, 628}

$$-\frac{3e\left(-2\sqrt{c}d\sqrt{ae^2+cd^2}+ae^2+2cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}+\sqrt{ae^2+cd^2}+\sqrt{c}(d+ex)\right)}{64\sqrt{2}a^2c^{5/4}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{3e\left(-2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(a + c*x^2)^3,x]
```

```
[Out] -((a*e - c*d*x)*Sqrt[d + e*x])/(4*a*c*(a + c*x^2)^2) + ((a*e + 6*c*d*x)*Sqr
t[d + e*x])/(16*a^2*c*(a + c*x^2)) + (3*e*(2*c*d^2 + a*e^2 + 2*Sqrt[c]*d*Sq
rt[c*d^2 + a*e^2])*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]] - Sqrt[2]
*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]])/(32*Sqrt[2]
*a^2*c^(5/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (
3*e*(2*c*d^2 + a*e^2 + 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*ArcTanh[(Sqrt[Sqrt[
c]*d + Sqrt[c*d^2 + a*e^2]] + Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d
 - Sqrt[c*d^2 + a*e^2]]])/(32*Sqrt[2]*a^2*c^(5/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[
Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (3*e*(2*c*d^2 + a*e^2 - 2*Sqrt[c]*d*Sqr
t[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d
+ Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(64*Sqrt[2]*a^2*
c^(5/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]) + (3*e*(
2*c*d^2 + a*e^2 - 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^2]
+ Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqr
t[c]*(d + e*x)])/(64*Sqrt[2]*a^2*c^(5/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d
```

```
 + Sqrt[c*d^2 + a*e^2]])
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
```

```
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\left(a+cx^2\right)^3}\,dx = -\frac{(ae-cdx)\sqrt{d+ex}}{4ac\left(a+cx^2\right)^2}+\frac{\int \frac{\frac{1}{2}\left(6cd^2+ae^2\right)+\frac{5}{2}cdex}{\sqrt{d+ex}\left(a+cx^2\right)^2}\,dx}{4ac}$$

$$= -\frac{(ae-cdx)\sqrt{d+ex}}{4ac\left(a+cx^2\right)^2}+\frac{(ae+6cdx)\sqrt{d+ex}}{16a^2c\left(a+cx^2\right)}-\frac{\int \frac{-\frac{3}{4}c\left(cd^2+ae^2\right)\left(4cd^2+ae^2\right)-\frac{3}{2}c^2de\left(cd^2+ae^2\right)x}{\sqrt{d+ex}\left(a+cx^2\right)}\,dx}{8a^2c^2\left(cd^2+ae^2\right)}$$

$$= -\frac{(ae-cdx)\sqrt{d+ex}}{4ac\left(a+cx^2\right)^2}+\frac{(ae+6cdx)\sqrt{d+ex}}{16a^2c\left(a+cx^2\right)}-\frac{\text{Subst}\left(\int \frac{\frac{3}{2}c^2d^2e\left(cd^2+ae^2\right)-\frac{3}{4}ce\left(cd^2+ae^2\right)\left(4cd^2+ae^2\right)-\frac{3}{2}c^2de\left(cd^2\ldots\right.}{cd^2+ae^2-2cdx^2+cx^4}\ldots\right)}{4a^2c^2\left(cd^2+ae^2\right)}$$

$$= -\frac{(ae-cdx)\sqrt{d+ex}}{4ac\left(a+cx^2\right)^2}+\frac{(ae+6cdx)\sqrt{d+ex}}{16a^2c\left(a+cx^2\right)}-\frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(\frac{3}{2}c^2d^2e\left(cd^2+ae^2\right)-\frac{3}{4}ce\left(cd^2+ae^2\right)\left(4cd^2+ae^2\ldots\right.\right.}{\sqrt[4]{c}}}{\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}\ldots}\ldots\right)}{8\sqrt{2}a^2c^{9/4}\left(cd\ldots\right.}$$

$$= -\frac{(ae-cdx)\sqrt{d+ex}}{4ac\left(a+cx^2\right)^2}+\frac{(ae+6cdx)\sqrt{d+ex}}{16a^2c\left(a+cx^2\right)}-\frac{\left(3e\left(2cd^2+ae^2-2\sqrt{c}d\sqrt{cd^2+ae^2}\right)\right)\text{Subst}\left(\int \frac{-\ldots}{\frac{\sqrt{cd^2+\ldots}}{\sqrt{c}}\ldots}\ldots\right)}{64\sqrt{2}a^2c^{5/4}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d\ldots}}$$

$$= -\frac{(ae-cdx)\sqrt{d+ex}}{4ac\left(a+cx^2\right)^2}+\frac{(ae+6cdx)\sqrt{d+ex}}{16a^2c\left(a+cx^2\right)}-\frac{3e\left(2cd^2+ae^2-2\sqrt{c}d\sqrt{cd^2+ae^2}\right)\log\left(\sqrt{cd^2+ae^2}-\ldots\right.}{64\sqrt{2}a^2c^{5/4}\sqrt{cd^2+ae^2}\ldots}$$

$$= -\frac{(ae-cdx)\sqrt{d+ex}}{4ac\left(a+cx^2\right)^2}+\frac{(ae+6cdx)\sqrt{d+ex}}{16a^2c\left(a+cx^2\right)}+\frac{3e\left(2cd^2+ae^2+2\sqrt{c}d\sqrt{cd^2+ae^2}\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\left(\sqrt{\sqrt{c}d+\ldots}\right.}{\sqrt{\ldots}}\right)}{32\sqrt{2}a^2c^{5/4}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d-\sqrt{cd^2+\ldots}}}$$

Mathematica [A] time = 1.22698, size = 430, normalized size = 0.56

$$\frac{\frac{2(d+ex)^{5/2}\left(3a^2e^3+acde(5d+4ex)+6c^2d^3x\right)}{a+cx^2}+\frac{-2\sqrt{-a}\sqrt[4]{c}e\sqrt{d+ex}\left(3a^2e^4+acde^2(13d+4ex)+6c^2d^3(2d+ex)\right)+3\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(ae^2+cd^2\right)\left(2\sqrt{-a}cd^2e+3a\sqrt{c}de^2+\sqrt{-a}\ldots\right.}{\ldots}}{32a^2\left(a\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(a + c*x^2)^3,x]
```

```
[Out] ((8*a*(c*d^2 + a*e^2)*(a*e + c*d*x)*(d + e*x)^(5/2))/(a + c*x^2)^2 + (2*(d
+ e*x)^(5/2)*(3*a^2*e^3 + 6*c^2*d^3*x + a*c*d*e*(5*d + 4*e*x)))/(a + c*x^2)
 + (-2*Sqrt[-a]*c^(1/4)*e*Sqrt[d + e*x]*(3*a^2*e^4 + 6*c^2*d^3*(2*d + e*x)
+ a*c*d*e^2*(13*d + 4*e*x)) + 3*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*(c*d^2 + a*e^2
```

```
)*(4*c^(3/2)*d^3 + 2*Sqrt[-a]*c*d^2*e + 3*a*Sqrt[c]*d*e^2 + Sqrt[-a]*a*e^3)
*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[-a]*e]] - 3*Sqrt[Sqr
t[c]*d + Sqrt[-a]*e]*(c*d^2 + a*e^2)*(4*c^(3/2)*d^3 - 2*Sqrt[-a]*c*d^2*e +
3*a*Sqrt[c]*d*e^2 + (-a)^(3/2)*e^3)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sq
rt[c]*d + Sqrt[-a]*e]])/(Sqrt[-a]*c^(5/4)))/(32*a^2*(c*d^2 + a*e^2)^2)
```

Maple [B] time = 0.254, size = 7538, normalized size = 9.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*x^2+a)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(c*x^2 + a)^3, x)
```

Fricas [B] time = 2.77599, size = 3384, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(3*(a^2*c^3*x^4 + 2*a^3*c^2*x^2 + a^4*c)*sqrt(-(16*c^2*d^5 + 20*a*c*d^
3*e^2 + 5*a^2*d*e^4 + (a^5*c^3*d^2 + a^6*c^2*e^2)*sqrt(-e^10/(a^5*c^7*d^4 +
 2*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))/(a^5*c^3*d^2 + a^6*c^2*e^2))*log(27*(16
*c^2*d^4*e^5 + 12*a*c*d^2*e^7 + a^2*e^9)*sqrt(e*x + d) + 27*(2*a^3*c^2*d^2*
e^6 + a^4*c*e^8 + (4*a^5*c^6*d^5 + 7*a^6*c^5*d^3*e^2 + 3*a^7*c^4*d*e^4)*sqr
t(-e^10/(a^5*c^7*d^4 + 2*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))*sqrt(-(16*c^2*d^5
 + 20*a*c*d^3*e^2 + 5*a^2*d*e^4 + (a^5*c^3*d^2 + a^6*c^2*e^2)*sqrt(-e^10/(a
^5*c^7*d^4 + 2*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))/(a^5*c^3*d^2 + a^6*c^2*e^2)
)) - 3*(a^2*c^3*x^4 + 2*a^3*c^2*x^2 + a^4*c)*sqrt(-(16*c^2*d^5 + 20*a*c*d^3
*e^2 + 5*a^2*d*e^4 + (a^5*c^3*d^2 + a^6*c^2*e^2)*sqrt(-e^10/(a^5*c^7*d^4 +
2*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))/(a^5*c^3*d^2 + a^6*c^2*e^2))*log(27*(16*
c^2*d^4*e^5 + 12*a*c*d^2*e^7 + a^2*e^9)*sqrt(e*x + d) - 27*(2*a^3*c^2*d^2*e
^6 + a^4*c*e^8 + (4*a^5*c^6*d^5 + 7*a^6*c^5*d^3*e^2 + 3*a^7*c^4*d*e^4)*sqrt
(-e^10/(a^5*c^7*d^4 + 2*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))*sqrt(-(16*c^2*d^5
+ 20*a*c*d^3*e^2 + 5*a^2*d*e^4 + (a^5*c^3*d^2 + a^6*c^2*e^2)*sqrt(-e^10/(a^
5*c^7*d^4 + 2*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))/(a^5*c^3*d^2 + a^6*c^2*e^2))
```

```
) + 3*(a^2*c^3*x^4 + 2*a^3*c^2*x^2 + a^4*c)*sqrt(-(16*c^2*d^5 + 20*a*c*d^3*
e^2 + 5*a^2*d*e^4 - (a^5*c^3*d^2 + a^6*c^2*e^2)*sqrt(-e^10/(a^5*c^7*d^4 + 2
*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))/(a^5*c^3*d^2 + a^6*c^2*e^2))*log(27*(16*c
^2*d^4*e^5 + 12*a*c*d^2*e^7 + a^2*e^9)*sqrt(e*x + d) + 27*(2*a^3*c^2*d^2*e^
6 + a^4*c*e^8 - (4*a^5*c^6*d^5 + 7*a^6*c^5*d^3*e^2 + 3*a^7*c^4*d*e^4)*sqrt(
-e^10/(a^5*c^7*d^4 + 2*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))*sqrt(-(16*c^2*d^5 +
 20*a*c*d^3*e^2 + 5*a^2*d*e^4 - (a^5*c^3*d^2 + a^6*c^2*e^2)*sqrt(-e^10/(a^5
*c^7*d^4 + 2*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))/(a^5*c^3*d^2 + a^6*c^2*e^2)))
 - 3*(a^2*c^3*x^4 + 2*a^3*c^2*x^2 + a^4*c)*sqrt(-(16*c^2*d^5 + 20*a*c*d^3*e
^2 + 5*a^2*d*e^4 - (a^5*c^3*d^2 + a^6*c^2*e^2)*sqrt(-e^10/(a^5*c^7*d^4 + 2*
a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))/(a^5*c^3*d^2 + a^6*c^2*e^2))*log(27*(16*c^
2*d^4*e^5 + 12*a*c*d^2*e^7 + a^2*e^9)*sqrt(e*x + d) - 27*(2*a^3*c^2*d^2*e^6
 + a^4*c*e^8 - (4*a^5*c^6*d^5 + 7*a^6*c^5*d^3*e^2 + 3*a^7*c^4*d*e^4)*sqrt(-
e^10/(a^5*c^7*d^4 + 2*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))*sqrt(-(16*c^2*d^5 +
20*a*c*d^3*e^2 + 5*a^2*d*e^4 - (a^5*c^3*d^2 + a^6*c^2*e^2)*sqrt(-e^10/(a^5*
c^7*d^4 + 2*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))/(a^5*c^3*d^2 + a^6*c^2*e^2)))
+ 4*(6*c^2*d*x^3 + a*c*e*x^2 + 10*a*c*d*x - 3*a^2*e)*sqrt(e*x + d))/(a^2*c^
3*x^4 + 2*a^3*c^2*x^2 + a^4*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+a)**3,x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a)^3,x, algorithm="giac")

[Out] Timed out
```

3.646 $\int \frac{\sqrt{d+ex}}{\left(a+cx^2\right)^3}\,dx$

Optimal. Leaf size=849

$$\frac{\sqrt{d+ex}x}{4a\left(cx^2+a\right)^2}+\frac{e\left(6c^{3/2}d^3+8a\sqrt{c}e^2d+\sqrt{cd^2+ae^2}\left(6cd^2+5ae^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}\right)}{32\sqrt{2}a^2c^{3/4}\left(cd^2+ae^2\right)^{3/2}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}-\frac{e\left(6c^{3/2}d^3+8a\sqrt{}\right.}{}$$

```
[Out] (x*Sqrt[d + e*x])/(4*a*(a + c*x^2)^2) + (Sqrt[d + e*x]*(a*d*e + (6*c*d^2 +
5*a*e^2)*x))/(16*a^2*(c*d^2 + a*e^2)*(a + c*x^2)) + (e*(6*c^(3/2)*d^3 + 8*a
*Sqrt[c]*d*e^2 + Sqrt[c*d^2 + a*e^2]*(6*c*d^2 + 5*a*e^2))*ArcTanh[(Sqrt[Sqr
t[c]*d + Sqrt[c*d^2 + a*e^2]] - Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]
*d - Sqrt[c*d^2 + a*e^2]]])/(32*Sqrt[2]*a^2*c^(3/4)*(c*d^2 + a*e^2)^(3/2)*S
qrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (e*(6*c^(3/2)*d^3 + 8*a*Sqrt[c]*d*e
^2 + Sqrt[c*d^2 + a*e^2]*(6*c*d^2 + 5*a*e^2))*ArcTanh[(Sqrt[Sqrt[c]*d + Sqr
t[c*d^2 + a*e^2]] + Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*
d^2 + a*e^2]]])/(32*Sqrt[2]*a^2*c^(3/4)*(c*d^2 + a*e^2)^(3/2)*Sqrt[Sqrt[c]*
d - Sqrt[c*d^2 + a*e^2]]) - (e*(6*c^(3/2)*d^3 + 8*a*Sqrt[c]*d*e^2 - Sqrt[c*
d^2 + a*e^2]*(6*c*d^2 + 5*a*e^2))*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)
*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/
(64*Sqrt[2]*a^2*c^(3/4)*(c*d^2 + a*e^2)^(3/2)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 +
 a*e^2]]) + (e*(6*c^(3/2)*d^3 + 8*a*Sqrt[c]*d*e^2 - Sqrt[c*d^2 + a*e^2]*(6*
c*d^2 + 5*a*e^2))*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d
+ Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(64*Sqrt[2]*a^2*
c^(3/4)*(c*d^2 + a*e^2)^(3/2)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])
```

Rubi [A] time = 2.88348, antiderivative size = 849, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {737, 823, 827, 1169, 634, 618, 206, 628}

$$\frac{\sqrt{d+ex}x}{4a\left(cx^2+a\right)^2}+\frac{e\left(6c^{3/2}d^3+8a\sqrt{c}e^2d+\sqrt{cd^2+ae^2}\left(6cd^2+5ae^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}\right)}{32\sqrt{2}a^2c^{3/4}\left(cd^2+ae^2\right)^{3/2}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}-\frac{e\left(6c^{3/2}d^3+8a\sqrt{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(a + c*x^2)^3,x]
```

```
[Out] (x*Sqrt[d + e*x])/(4*a*(a + c*x^2)^2) + (Sqrt[d + e*x]*(a*d*e + (6*c*d^2 +
5*a*e^2)*x))/(16*a^2*(c*d^2 + a*e^2)*(a + c*x^2)) + (e*(6*c^(3/2)*d^3 + 8*a
*Sqrt[c]*d*e^2 + Sqrt[c*d^2 + a*e^2]*(6*c*d^2 + 5*a*e^2))*ArcTanh[(Sqrt[Sqr
t[c]*d + Sqrt[c*d^2 + a*e^2]] - Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]
*d - Sqrt[c*d^2 + a*e^2]]])/(32*Sqrt[2]*a^2*c^(3/4)*(c*d^2 + a*e^2)^(3/2)*S
qrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (e*(6*c^(3/2)*d^3 + 8*a*Sqrt[c]*d*e
^2 + Sqrt[c*d^2 + a*e^2]*(6*c*d^2 + 5*a*e^2))*ArcTanh[(Sqrt[Sqrt[c]*d + Sqr
t[c*d^2 + a*e^2]] + Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[c*
d^2 + a*e^2]]])/(32*Sqrt[2]*a^2*c^(3/4)*(c*d^2 + a*e^2)^(3/2)*Sqrt[Sqrt[c]*
d - Sqrt[c*d^2 + a*e^2]]) - (e*(6*c^(3/2)*d^3 + 8*a*Sqrt[c]*d*e^2 - Sqrt[c*
d^2 + a*e^2]*(6*c*d^2 + 5*a*e^2))*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)
*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/
(64*Sqrt[2]*a^2*c^(3/4)*(c*d^2 + a*e^2)^(3/2)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 +
 a*e^2]]) + (e*(6*c^(3/2)*d^3 + 8*a*Sqrt[c]*d*e^2 - Sqrt[c*d^2 + a*e^2]*(6*
c*d^2 + 5*a*e^2))*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d
```

```
+ Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(64*Sqrt[2]*a^2*
c^(3/4)*(c*d^2 + a*e^2)^(3/2)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(x*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*(p + 1)), x] + Dist[1/(2*a*(p + 1
)), Int[(d + e*x)^(m - 1)*(d*(2*p + 3) + e*(m + 2*p + 3)*x)*(a + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -
1] && GtQ[m, 0] && (LtQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && In
tQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\left(a+cx^2\right)^3}\,dx = \frac{x\sqrt{d+ex}}{4a\left(a+cx^2\right)^2} - \frac{\int \frac{-3d-\frac{5ex}{2}}{\sqrt{d+ex}\left(a+cx^2\right)^2}\,dx}{4a}$$

$$= \frac{x\sqrt{d+ex}}{4a\left(a+cx^2\right)^2} + \frac{\sqrt{d+ex}\left(ade+\left(6cd^2+5ae^2\right)x\right)}{16a^2\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{\int \frac{\frac{1}{4}cd\left(12cd^2+13ae^2\right)+\frac{1}{4}ce\left(6cd^2+5ae^2\right)x}{\sqrt{d+ex}\left(a+cx^2\right)}\,dx}{8a^2c\left(cd^2+ae^2\right)}$$

$$= \frac{x\sqrt{d+ex}}{4a\left(a+cx^2\right)^2} + \frac{\sqrt{d+ex}\left(ade+\left(6cd^2+5ae^2\right)x\right)}{16a^2\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{4}cde\left(6cd^2+5ae^2\right)+\frac{1}{4}cde\left(12cd^2+13ae^2\right)+\frac{1}{4}ce\left(6cd^2+\cdots\right.}{cd^2+ae^2-2cdx^2+cx^4}\cdots\right)}{4a^2c\left(cd^2+ae^2\right)}$$

$$= \frac{x\sqrt{d+ex}}{4a\left(a+cx^2\right)^2} + \frac{\sqrt{d+ex}\left(ade+\left(6cd^2+5ae^2\right)x\right)}{16a^2\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(-\frac{1}{4}cde\left(6cd^2+5ae^2\right)+\frac{1}{4}cde\left(12cd^2+13ae^2\cdots\right.\right.}{\sqrt[4]{c}}}{\frac{\sqrt{cd^2\cdots}}{\cdots}}\cdots\right)}{8\sqrt{2}a^2c^{5/4}\cdots}$$

$$= \frac{x\sqrt{d+ex}}{4a\left(a+cx^2\right)^2} + \frac{\sqrt{d+ex}\left(ade+\left(6cd^2+5ae^2\right)x\right)}{16a^2\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\left(e\left(6c^{3/2}d^3+8a\sqrt{c}de^2-\sqrt{cd^2+ae^2}\left(6cd^2+5ae^2\right)\cdots\right.\right.}{64\sqrt{2}a^2c^{3/4}\left(cd^2+a\cdots\right.}$$

$$= \frac{x\sqrt{d+ex}}{4a\left(a+cx^2\right)^2} + \frac{\sqrt{d+ex}\left(ade+\left(6cd^2+5ae^2\right)x\right)}{16a^2\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{e\left(6c^{3/2}d^3+8a\sqrt{c}de^2-\sqrt{cd^2+ae^2}\left(6cd^2+5ae^2\right)\right)\cdots}{64\sqrt{2}a^2c^{3/4}\left(c\cdots\right.}$$

$$= \frac{x\sqrt{d+ex}}{4a\left(a+cx^2\right)^2} + \frac{\sqrt{d+ex}\left(ade+\left(6cd^2+5ae^2\right)x\right)}{16a^2\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{e\left(6c^{3/2}d^3+8a\sqrt{c}de^2+\sqrt{cd^2+ae^2}\left(6cd^2+5ae^2\right)\cdots\right.}{32\sqrt{2}a^2c^{3/4}\left(cd^2+ae^2\right)^{3/2}\sqrt{\sqrt{\cdots}}}$$

Mathematica [A] time = 0.870625, size = 412, normalized size = 0.49

$$\frac{\frac{2(d+ex)^{3/2}\left(5a^2e^3+acde(3d+8ex)+6c^2d^3x\right)}{a+cx^2} + \frac{\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(5a^2e^4+6\sqrt{-a}c^{3/2}d^3e+19acd^2e^2+8\sqrt{-a}a\sqrt{c}de^3+12c^2d^4\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)-\sqrt{\sqrt{-a}e+\sqrt{c}d}\left(5a^2e^4-6\cdots\right.}{\sqrt{-a}c^{3/4}}}{32a^2\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(a + c*x^2)^3,x]
```

```
[Out] ((8*a*(c*d^2 + a*e^2)*(a*e + c*d*x)*(d + e*x)^(3/2))/(a + c*x^2)^2 + (2*(d
+ e*x)^(3/2)*(5*a^2*e^3 + 6*c^2*d^3*x + a*c*d*e*(3*d + 8*e*x)))/(a + c*x^2)
 + (-4*Sqrt[-a]*c^(3/4)*d*e*(3*c*d^2 + 4*a*e^2)*Sqrt[d + e*x] + Sqrt[Sqrt[c
```

```
]*d - Sqrt[-a]*e]*(12*c^2*d^4 + 6*Sqrt[-a]*c^(3/2)*d^3*e + 19*a*c*d^2*e^2 +
 8*Sqrt[-a]*a*Sqrt[c]*d*e^3 + 5*a^2*e^4)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sq
rt[Sqrt[c]*d - Sqrt[-a]*e]] - Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*(12*c^2*d^4 - 6*
Sqrt[-a]*c^(3/2)*d^3*e + 19*a*c*d^2*e^2 + 8*(-a)^(3/2)*Sqrt[c]*d*e^3 + 5*a^
2*e^4)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[-a]*e]])/(Sqrt
[-a]*c^(3/4)))/(32*a^2*(c*d^2 + a*e^2)^2)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+a\right)^3}\sqrt{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*x^2+a)^3,x)

[Out] int((e*x+d)^(1/2)/(c*x^2+a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\left(cx^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a)^3,x, algorithm="maxima")

[Out] integrate(sqrt(e*x + d)/(c*x^2 + a)^3, x)
```

Fricas [B] time = 4.63693, size = 7772, normalized size = 9.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a)^3,x, algorithm="fricas")

[Out] 1/64*((a^4*c*d^2 + a^5*e^2 + (a^2*c^3*d^2 + a^3*c^2*e^2)*x^4 + 2*(a^3*c^2*d
^2 + a^4*c*e^2)*x^2)*sqrt(-(144*c^3*d^7 + 420*a*c^2*d^5*e^2 + 385*a^2*c*d^3
*e^4 + 105*a^3*d*e^6 + (a^5*c^4*d^6 + 3*a^6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4
 + a^8*c*e^6)*sqrt(-(441*c^2*d^4*e^10 + 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(
a^5*c^9*d^12 + 6*a^6*c^8*d^10*e^2 + 15*a^7*c^7*d^8*e^4 + 20*a^8*c^6*d^6*e^6
 + 15*a^9*c^5*d^4*e^8 + 6*a^10*c^4*d^2*e^10 + a^11*c^3*e^12)))/(a^5*c^4*d^6
 + 3*a^6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4 + a^8*c*e^6))*log((3024*c^3*d^6*e^
5 + 7884*a*c^2*d^4*e^7 + 5625*a^2*c*d^2*e^9 + 625*a^3*e^11)*sqrt(e*x + d) +
 (126*a^3*c^3*d^5*e^6 + 318*a^4*c^2*d^3*e^8 + 200*a^5*c*d*e^10 - (12*a^5*c^
7*d^10 + 55*a^6*c^6*d^8*e^2 + 98*a^7*c^5*d^6*e^4 + 84*a^8*c^4*d^4*e^6 + 34*
a^9*c^3*d^2*e^8 + 5*a^10*c^2*e^10)*sqrt(-(441*c^2*d^4*e^10 + 1050*a*c*d^2*e
^12 + 625*a^2*e^14)/(a^5*c^9*d^12 + 6*a^6*c^8*d^10*e^2 + 15*a^7*c^7*d^8*e^4
 + 20*a^8*c^6*d^6*e^6 + 15*a^9*c^5*d^4*e^8 + 6*a^10*c^4*d^2*e^10 + a^11*c^3
*e^12)))*sqrt(-(144*c^3*d^7 + 420*a*c^2*d^5*e^2 + 385*a^2*c*d^3*e^4 + 105*a
```

^3*d*e^6 + (a^5*c^4*d^6 + 3*a^6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4 + a^8*c*e^6
)*sqrt(-(441*c^2*d^4*e^10 + 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9*d^12
 + 6*a^6*c^8*d^10*e^2 + 15*a^7*c^7*d^8*e^4 + 20*a^8*c^6*d^6*e^6 + 15*a^9*c^
5*d^4*e^8 + 6*a^10*c^4*d^2*e^10 + a^11*c^3*e^12)))/(a^5*c^4*d^6 + 3*a^6*c^3
*d^4*e^2 + 3*a^7*c^2*d^2*e^4 + a^8*c*e^6))) - (a^4*c*d^2 + a^5*e^2 + (a^2*c
^3*d^2 + a^3*c^2*e^2)*x^4 + 2*(a^3*c^2*d^2 + a^4*c*e^2)*x^2)*sqrt(-(144*c^3
*d^7 + 420*a*c^2*d^5*e^2 + 385*a^2*c*d^3*e^4 + 105*a^3*d*e^6 + (a^5*c^4*d^6
 + 3*a^6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4 + a^8*c*e^6)*sqrt(-(441*c^2*d^4*e^
10 + 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9*d^12 + 6*a^6*c^8*d^10*e^2 +
 15*a^7*c^7*d^8*e^4 + 20*a^8*c^6*d^6*e^6 + 15*a^9*c^5*d^4*e^8 + 6*a^10*c^4*
d^2*e^10 + a^11*c^3*e^12)))/(a^5*c^4*d^6 + 3*a^6*c^3*d^4*e^2 + 3*a^7*c^2*d^
2*e^4 + a^8*c*e^6))*log((3024*c^3*d^6*e^5 + 7884*a*c^2*d^4*e^7 + 5625*a^2*c
*d^2*e^9 + 625*a^3*e^11)*sqrt(e*x + d) - (126*a^3*c^3*d^5*e^6 + 318*a^4*c^2
*d^3*e^8 + 200*a^5*c*d*e^10 - (12*a^5*c^7*d^10 + 55*a^6*c^6*d^8*e^2 + 98*a^
7*c^5*d^6*e^4 + 84*a^8*c^4*d^4*e^6 + 34*a^9*c^3*d^2*e^8 + 5*a^10*c^2*e^10)*
sqrt(-(441*c^2*d^4*e^10 + 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9*d^12 +
 6*a^6*c^8*d^10*e^2 + 15*a^7*c^7*d^8*e^4 + 20*a^8*c^6*d^6*e^6 + 15*a^9*c^5*
d^4*e^8 + 6*a^10*c^4*d^2*e^10 + a^11*c^3*e^12)))*sqrt(-(144*c^3*d^7 + 420*a
*c^2*d^5*e^2 + 385*a^2*c*d^3*e^4 + 105*a^3*d*e^6 + (a^5*c^4*d^6 + 3*a^6*c^3
*d^4*e^2 + 3*a^7*c^2*d^2*e^4 + a^8*c*e^6)*sqrt(-(441*c^2*d^4*e^10 + 1050*a*
c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9*d^12 + 6*a^6*c^8*d^10*e^2 + 15*a^7*c^7*
d^8*e^4 + 20*a^8*c^6*d^6*e^6 + 15*a^9*c^5*d^4*e^8 + 6*a^10*c^4*d^2*e^10 + a
^11*c^3*e^12)))/(a^5*c^4*d^6 + 3*a^6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4 + a^8*
c*e^6))) + (a^4*c*d^2 + a^5*e^2 + (a^2*c^3*d^2 + a^3*c^2*e^2)*x^4 + 2*(a^3*
c^2*d^2 + a^4*c*e^2)*x^2)*sqrt(-(144*c^3*d^7 + 420*a*c^2*d^5*e^2 + 385*a^2*
c*d^3*e^4 + 105*a^3*d*e^6 - (a^5*c^4*d^6 + 3*a^6*c^3*d^4*e^2 + 3*a^7*c^2*d^
2*e^4 + a^8*c*e^6)*sqrt(-(441*c^2*d^4*e^10 + 1050*a*c*d^2*e^12 + 625*a^2*e^
14)/(a^5*c^9*d^12 + 6*a^6*c^8*d^10*e^2 + 15*a^7*c^7*d^8*e^4 + 20*a^8*c^6*d^
6*e^6 + 15*a^9*c^5*d^4*e^8 + 6*a^10*c^4*d^2*e^10 + a^11*c^3*e^12)))/(a^5*c^
4*d^6 + 3*a^6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4 + a^8*c*e^6))*log((3024*c^3*d
^6*e^5 + 7884*a*c^2*d^4*e^7 + 5625*a^2*c*d^2*e^9 + 625*a^3*e^11)*sqrt(e*x +
 d) + (126*a^3*c^3*d^5*e^6 + 318*a^4*c^2*d^3*e^8 + 200*a^5*c*d*e^10 + (12*a
^5*c^7*d^10 + 55*a^6*c^6*d^8*e^2 + 98*a^7*c^5*d^6*e^4 + 84*a^8*c^4*d^4*e^6
+ 34*a^9*c^3*d^2*e^8 + 5*a^10*c^2*e^10)*sqrt(-(441*c^2*d^4*e^10 + 1050*a*c*
d^2*e^12 + 625*a^2*e^14)/(a^5*c^9*d^12 + 6*a^6*c^8*d^10*e^2 + 15*a^7*c^7*d^
8*e^4 + 20*a^8*c^6*d^6*e^6 + 15*a^9*c^5*d^4*e^8 + 6*a^10*c^4*d^2*e^10 + a^1
1*c^3*e^12)))*sqrt(-(144*c^3*d^7 + 420*a*c^2*d^5*e^2 + 385*a^2*c*d^3*e^4 +
105*a^3*d*e^6 - (a^5*c^4*d^6 + 3*a^6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4 + a^8*
c*e^6)*sqrt(-(441*c^2*d^4*e^10 + 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9
*d^12 + 6*a^6*c^8*d^10*e^2 + 15*a^7*c^7*d^8*e^4 + 20*a^8*c^6*d^6*e^6 + 15*a
^9*c^5*d^4*e^8 + 6*a^10*c^4*d^2*e^10 + a^11*c^3*e^12)))/(a^5*c^4*d^6 + 3*a^
6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4 + a^8*c*e^6))) - (a^4*c*d^2 + a^5*e^2 + (
a^2*c^3*d^2 + a^3*c^2*e^2)*x^4 + 2*(a^3*c^2*d^2 + a^4*c*e^2)*x^2)*sqrt(-(14
4*c^3*d^7 + 420*a*c^2*d^5*e^2 + 385*a^2*c*d^3*e^4 + 105*a^3*d*e^6 - (a^5*c^
4*d^6 + 3*a^6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4 + a^8*c*e^6)*sqrt(-(441*c^2*d
^4*e^10 + 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9*d^12 + 6*a^6*c^8*d^10*
e^2 + 15*a^7*c^7*d^8*e^4 + 20*a^8*c^6*d^6*e^6 + 15*a^9*c^5*d^4*e^8 + 6*a^10
*c^4*d^2*e^10 + a^11*c^3*e^12)))/(a^5*c^4*d^6 + 3*a^6*c^3*d^4*e^2 + 3*a^7*c
^2*d^2*e^4 + a^8*c*e^6))*log((3024*c^3*d^6*e^5 + 7884*a*c^2*d^4*e^7 + 5625*
a^2*c*d^2*e^9 + 625*a^3*e^11)*sqrt(e*x + d) - (126*a^3*c^3*d^5*e^6 + 318*a^
4*c^2*d^3*e^8 + 200*a^5*c*d*e^10 + (12*a^5*c^7*d^10 + 55*a^6*c^6*d^8*e^2 +
98*a^7*c^5*d^6*e^4 + 84*a^8*c^4*d^4*e^6 + 34*a^9*c^3*d^2*e^8 + 5*a^10*c^2*e
^10)*sqrt(-(441*c^2*d^4*e^10 + 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9*d
^12 + 6*a^6*c^8*d^10*e^2 + 15*a^7*c^7*d^8*e^4 + 20*a^8*c^6*d^6*e^6 + 15*a^9
*c^5*d^4*e^8 + 6*a^10*c^4*d^2*e^10 + a^11*c^3*e^12)))*sqrt(-(144*c^3*d^7 +
420*a*c^2*d^5*e^2 + 385*a^2*c*d^3*e^4 + 105*a^3*d*e^6 - (a^5*c^4*d^6 + 3*a^
6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4 + a^8*c*e^6)*sqrt(-(441*c^2*d^4*e^10 + 10
50*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9*d^12 + 6*a^6*c^8*d^10*e^2 + 15*a^7
*c^7*d^8*e^4 + 20*a^8*c^6*d^6*e^6 + 15*a^9*c^5*d^4*e^8 + 6*a^10*c^4*d^2*e^1

```
0 + a^11*c^3*e^12)))/(a^5*c^4*d^6 + 3*a^6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4 +
 a^8*c*e^6))) + 4*(a*c*d*e*x^2 + a^2*d*e + (6*c^2*d^2 + 5*a*c*e^2)*x^3 + (1
0*a*c*d^2 + 9*a^2*e^2)*x)*sqrt(e*x + d))/(a^4*c*d^2 + a^5*e^2 + (a^2*c^3*d^
2 + a^3*c^2*e^2)*x^4 + 2*(a^3*c^2*d^2 + a^4*c*e^2)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.647 $\int \frac{1}{\sqrt{d+ex}\left(a+cx^2\right)^3}\,dx$

Optimal. Leaf size=920

$$\frac{\sqrt{d+ex}(ae+cdx)}{4a\left(cd^2+ae^2\right)\left(cx^2+a\right)^2}+\frac{3e\left(2c^2d^4+5ace^2d^2+2\sqrt{c}\sqrt{cd^2+ae^2}\left(cd^2+2ae^2\right)d+7a^2e^4\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d}}{\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}\right.}{32\sqrt{2}a^2\sqrt[4]{c}\left(cd^2+ae^2\right)^{5/2}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}$$

```
[Out] ((a*e + c*d*x)*Sqrt[d + e*x])/(4*a*(c*d^2 + a*e^2)*(a + c*x^2)^2) + (Sqrt[d
 + e*x]*(a*e*(c*d^2 + 7*a*e^2) + 6*c*d*(c*d^2 + 2*a*e^2)*x))/(16*a^2*(c*d^2
 + a*e^2)^2*(a + c*x^2)) + (3*e*(2*c^2*d^4 + 5*a*c*d^2*e^2 + 7*a^2*e^4 + 2*
Sqrt[c]*d*Sqrt[c*d^2 + a*e^2]*(c*d^2 + 2*a*e^2))*ArcTanh[(Sqrt[Sqrt[c]*d +
Sqrt[c*d^2 + a*e^2]] - Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt
[c*d^2 + a*e^2]]])/(32*Sqrt[2]*a^2*c^(1/4)*(c*d^2 + a*e^2)^(5/2)*Sqrt[Sqrt[
c]*d - Sqrt[c*d^2 + a*e^2]]) - (3*e*(2*c^2*d^4 + 5*a*c*d^2*e^2 + 7*a^2*e^4
+ 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2]*(c*d^2 + 2*a*e^2))*ArcTanh[(Sqrt[Sqrt[c]*
d + Sqrt[c*d^2 + a*e^2]] + Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d -
Sqrt[c*d^2 + a*e^2]]])/(32*Sqrt[2]*a^2*c^(1/4)*(c*d^2 + a*e^2)^(5/2)*Sqrt[S
qrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (3*e*(2*c^2*d^4 + 5*a*c*d^2*e^2 + 7*a^2*
e^4 - 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2]*(c*d^2 + 2*a*e^2))*Log[Sqrt[c*d^2 + a
*e^2] - Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x]
 + Sqrt[c]*(d + e*x)])/(64*Sqrt[2]*a^2*c^(1/4)*(c*d^2 + a*e^2)^(5/2)*Sqrt[S
qrt[c]*d + Sqrt[c*d^2 + a*e^2]]) + (3*e*(2*c^2*d^4 + 5*a*c*d^2*e^2 + 7*a^2*
e^4 - 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2]*(c*d^2 + 2*a*e^2))*Log[Sqrt[c*d^2 + a
*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x]
 + Sqrt[c]*(d + e*x)])/(64*Sqrt[2]*a^2*c^(1/4)*(c*d^2 + a*e^2)^(5/2)*Sqrt[S
qrt[c]*d + Sqrt[c*d^2 + a*e^2]])
```

Rubi [A] time = 5.8513, antiderivative size = 920, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {741, 823, 827, 1169, 634, 618, 206, 628}

$$\frac{\sqrt{d+ex}(ae+cdx)}{4a\left(cd^2+ae^2\right)\left(cx^2+a\right)^2}+\frac{3e\left(2c^2d^4+5ace^2d^2+2\sqrt{c}\sqrt{cd^2+ae^2}\left(cd^2+2ae^2\right)d+7a^2e^4\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d}}{\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}\right.}{32\sqrt{2}a^2\sqrt[4]{c}\left(cd^2+ae^2\right)^{5/2}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(a + c*x^2)^3),x]
```

```
[Out] ((a*e + c*d*x)*Sqrt[d + e*x])/(4*a*(c*d^2 + a*e^2)*(a + c*x^2)^2) + (Sqrt[d
 + e*x]*(a*e*(c*d^2 + 7*a*e^2) + 6*c*d*(c*d^2 + 2*a*e^2)*x))/(16*a^2*(c*d^2
 + a*e^2)^2*(a + c*x^2)) + (3*e*(2*c^2*d^4 + 5*a*c*d^2*e^2 + 7*a^2*e^4 + 2*
Sqrt[c]*d*Sqrt[c*d^2 + a*e^2]*(c*d^2 + 2*a*e^2))*ArcTanh[(Sqrt[Sqrt[c]*d +
Sqrt[c*d^2 + a*e^2]] - Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt
[c*d^2 + a*e^2]]])/(32*Sqrt[2]*a^2*c^(1/4)*(c*d^2 + a*e^2)^(5/2)*Sqrt[Sqrt[
c]*d - Sqrt[c*d^2 + a*e^2]]) - (3*e*(2*c^2*d^4 + 5*a*c*d^2*e^2 + 7*a^2*e^4
+ 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2]*(c*d^2 + 2*a*e^2))*ArcTanh[(Sqrt[Sqrt[c]*
d + Sqrt[c*d^2 + a*e^2]] + Sqrt[2]*c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d -
Sqrt[c*d^2 + a*e^2]]])/(32*Sqrt[2]*a^2*c^(1/4)*(c*d^2 + a*e^2)^(5/2)*Sqrt[S
qrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (3*e*(2*c^2*d^4 + 5*a*c*d^2*e^2 + 7*a^2*
e^4 - 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2]*(c*d^2 + 2*a*e^2))*Log[Sqrt[c*d^2 + a
*e^2] - Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x]
```

```
 + Sqrt[c]*(d + e*x)])/(64*Sqrt[2]*a^2*c^(1/4)*(c*d^2 + a*e^2)^(5/2)*Sqrt[S
qrt[c]*d + Sqrt[c*d^2 + a*e^2]]) + (3*e*(2*c^2*d^4 + 5*a*c*d^2*e^2 + 7*a^2*
e^4 - 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2]*(c*d^2 + 2*a*e^2))*Log[Sqrt[c*d^2 + a
*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x]
 + Sqrt[c]*(d + e*x)])/(64*Sqrt[2]*a^2*c^(1/4)*(c*d^2 + a*e^2)^(5/2)*Sqrt[S
qrt[c]*d + Sqrt[c*d^2 + a*e^2]])
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\left(a+cx^2\right)^3}\,dx = \frac{(ae+cdx)\sqrt{d+ex}}{4a\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} - \frac{\int \frac{\frac{1}{2}\left(-6cd^2-7ae^2\right)-\frac{5}{2}cdex}{\sqrt{d+ex}\left(a+cx^2\right)^2}\,dx}{4a\left(cd^2+ae^2\right)}$$

$$= \frac{(ae+cdx)\sqrt{d+ex}}{4a\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{\sqrt{d+ex}\left(ae\left(cd^2+7ae^2\right)+6cd\left(cd^2+2ae^2\right)x\right)}{16a^2\left(cd^2+ae^2\right)^2\left(a+cx^2\right)} + \frac{\int \frac{\frac{3}{4}c\left(4c^2d^4+9acd^2\cdots\right.}{\cdots}}{8a^2\cdots}$$

$$= \frac{(ae+cdx)\sqrt{d+ex}}{4a\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{\sqrt{d+ex}\left(ae\left(cd^2+7ae^2\right)+6cd\left(cd^2+2ae^2\right)x\right)}{16a^2\left(cd^2+ae^2\right)^2\left(a+cx^2\right)} + \frac{\text{Subst}\left(\int \frac{-\frac{3}{2}c^2\cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{(ae+cdx)\sqrt{d+ex}}{4a\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{\sqrt{d+ex}\left(ae\left(cd^2+7ae^2\right)+6cd\left(cd^2+2ae^2\right)x\right)}{16a^2\left(cd^2+ae^2\right)^2\left(a+cx^2\right)} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{\cdots}}{\cdots}\right.}{\cdots}$$

$$= \frac{(ae+cdx)\sqrt{d+ex}}{4a\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{\sqrt{d+ex}\left(ae\left(cd^2+7ae^2\right)+6cd\left(cd^2+2ae^2\right)x\right)}{16a^2\left(cd^2+ae^2\right)^2\left(a+cx^2\right)} + \frac{\left(\frac{3}{2}c^2d^2e\left(cd^2+\cdots\right.\right.}{\cdots}$$

$$= \frac{(ae+cdx)\sqrt{d+ex}}{4a\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{\sqrt{d+ex}\left(ae\left(cd^2+7ae^2\right)+6cd\left(cd^2+2ae^2\right)x\right)}{16a^2\left(cd^2+ae^2\right)^2\left(a+cx^2\right)} - \frac{3e\left(2c^2d^4+5a\cdots\right.}{\cdots}$$

$$= \frac{(ae+cdx)\sqrt{d+ex}}{4a\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{\sqrt{d+ex}\left(ae\left(cd^2+7ae^2\right)+6cd\left(cd^2+2ae^2\right)x\right)}{16a^2\left(cd^2+ae^2\right)^2\left(a+cx^2\right)} + \frac{3e\left(2c^2d^4+5a\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.763325, size = 464, normalized size = 0.5

$$\frac{\frac{\sqrt{d+ex}\left(7a^2e^3+acde(d+12ex)+6c^2d^3x\right)}{2\left(a+cx^2\right)} + \frac{3\left(\frac{\left(7a^2e^4+5acd^2e^2+2c^2d^4\right)\left(\sqrt{\sqrt{-a}e+\sqrt{c}d}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)-\sqrt{\sqrt{c}d-\sqrt{-a}e}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)\right)}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{-a}e+\sqrt{c}d}}+2\sqrt{c}d\left(2ae^2+cd^2\right)\left(\sqrt{\sqrt{c}d-\sqrt{\cdots}}\cdots\right.\right.}{4\sqrt{-a}\sqrt[4]{c}}}{8a^2\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(a + c*x^2)^3),x]
```

```
[Out] ((2*a*(c*d^2 + a*e^2)*(a*e + c*d*x)*Sqrt[d + e*x])/(a + c*x^2)^2 + (Sqrt[d
+ e*x]*(7*a^2*e^3 + 6*c^2*d^3*x + a*c*d*e*(d + 12*e*x)))/(2*(a + c*x^2)) +
(3*(((2*c^2*d^4 + 5*a*c*d^2*e^2 + 7*a^2*e^4)*(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*
ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[-a]*e]] - Sqrt[Sqrt[c
]*d - Sqrt[-a]*e]*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[-a]
*e]]))/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]) + 2*Sqrt
[c]*d*(c*d^2 + 2*a*e^2)*(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*ArcTanh[(c^(1/4)*Sqrt
[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[-a]*e]] - Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Arc
Tanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[-a]*e]])))/(4*Sqrt[-a]*c
^(1/4)))/(8*a^2*(c*d^2 + a*e^2)^2)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+a\right)^3}\frac{1}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a)^3/(e*x+d)^(1/2),x)
```

```
[Out] int(1/(c*x^2+a)^3/(e*x+d)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+a\right)^3\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^3/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)^3*sqrt(e*x + d)), x)
```

Fricas [B] time = 13.3048, size = 12473, normalized size = 13.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^3/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/64*(3*(a^4*c^2*d^4 + 2*a^5*c*d^2*e^2 + a^6*e^4 + (a^2*c^4*d^4 + 2*a^3*c^3
*d^2*e^2 + a^4*c^2*e^4)*x^4 + 2*(a^3*c^3*d^4 + 2*a^4*c^2*d^2*e^2 + a^5*c*e^
4)*x^2)*sqrt(-(16*c^4*d^9 + 84*a*c^3*d^7*e^2 + 189*a^2*c^2*d^5*e^4 + 210*a^
3*c*d^3*e^6 + 105*a^4*d*e^8 + (a^5*c^5*d^10 + 5*a^6*c^4*d^8*e^2 + 10*a^7*c^
3*d^6*e^4 + 10*a^8*c^2*d^4*e^6 + 5*a^9*c*d^2*e^8 + a^10*e^10)*sqrt(-(441*c^
4*d^8*e^10 + 2268*a*c^3*d^6*e^12 + 4974*a^2*c^2*d^4*e^14 + 5292*a^3*c*d^2*e
^16 + 2401*a^4*e^18)/(a^5*c^11*d^20 + 10*a^6*c^10*d^18*e^2 + 45*a^7*c^9*d^1
6*e^4 + 120*a^8*c^8*d^14*e^6 + 210*a^9*c^7*d^12*e^8 + 252*a^10*c^6*d^10*e^1
```

0 + 210*a^11*c^5*d^8*e^12 + 120*a^12*c^4*d^6*e^14 + 45*a^13*c^3*d^4*e^16 + 10*a^14*c^2*d^2*e^18 + a^15*c*e^20)))/(a^5*c^5*d^10 + 5*a^6*c^4*d^8*e^2 + 10*a^7*c^3*d^6*e^4 + 10*a^8*c^2*d^4*e^6 + 5*a^9*c*d^2*e^8 + a^10*e^10))*log(27*(336*c^4*d^8*e^5 + 1788*a*c^3*d^6*e^7 + 4189*a^2*c^2*d^4*e^9 + 4802*a^3*c*d^2*e^11 + 2401*a^4*e^13)*sqrt(e*x + d) + 27*(42*a^3*c^4*d^8*e^6 + 213*a^4*c^3*d^6*e^8 + 515*a^5*c^2*d^4*e^10 + 623*a^6*c*d^2*e^12 + 343*a^7*e^14 + (4*a^5*c^8*d^15 + 31*a^6*c^7*d^13*e^2 + 106*a^7*c^6*d^11*e^4 + 205*a^8*c^5*d^9*e^6 + 240*a^9*c^4*d^7*e^8 + 169*a^10*c^3*d^5*e^10 + 66*a^11*c^2*d^3*e^12 + 11*a^12*c*d*e^14)*sqrt(-(441*c^4*d^8*e^10 + 2268*a*c^3*d^6*e^12 + 4974*a^2*c^2*d^4*e^14 + 5292*a^3*c*d^2*e^16 + 2401*a^4*e^18)/(a^5*c^11*d^20 + 10*a^6*c^10*d^18*e^2 + 45*a^7*c^9*d^16*e^4 + 120*a^8*c^8*d^14*e^6 + 210*a^9*c^7*d^12*e^8 + 252*a^10*c^6*d^10*e^10 + 210*a^11*c^5*d^8*e^12 + 120*a^12*c^4*d^6*e^14 + 45*a^13*c^3*d^4*e^16 + 10*a^14*c^2*d^2*e^18 + a^15*c*e^20)))*sqrt(-(16*c^4*d^9 + 84*a*c^3*d^7*e^2 + 189*a^2*c^2*d^5*e^4 + 210*a^3*c*d^3*e^6 + 105*a^4*d*e^8 + (a^5*c^5*d^10 + 5*a^6*c^4*d^8*e^2 + 10*a^7*c^3*d^6*e^4 + 10*a^8*c^2*d^4*e^6 + 5*a^9*c*d^2*e^8 + a^10*e^10)*sqrt(-(441*c^4*d^8*e^10 + 2268*a*c^3*d^6*e^12 + 4974*a^2*c^2*d^4*e^14 + 5292*a^3*c*d^2*e^16 + 2401*a^4*e^18)/(a^5*c^11*d^20 + 10*a^6*c^10*d^18*e^2 + 45*a^7*c^9*d^16*e^4 + 120*a^8*c^8*d^14*e^6 + 210*a^9*c^7*d^12*e^8 + 252*a^10*c^6*d^10*e^10 + 210*a^11*c^5*d^8*e^12 + 120*a^12*c^4*d^6*e^14 + 45*a^13*c^3*d^4*e^16 + 10*a^14*c^2*d^2*e^18 + a^15*c*e^20)))/(a^5*c^5*d^10 + 5*a^6*c^4*d^8*e^2 + 10*a^7*c^3*d^6*e^4 + 10*a^8*c^2*d^4*e^6 + 5*a^9*c*d^2*e^8 + a^10*e^10))) - 3*(a^4*c^2*d^4 + 2*a^5*c*d^2*e^2 + a^6*e^4 + (a^2*c^4*d^4 + 2*a^3*c^3*d^2*e^2 + a^4*c^2*e^4)*x^4 + 2*(a^3*c^3*d^4 + 2*a^4*c^2*d^2*e^2 + a^5*c*e^4)*x^2)*sqrt(-(16*c^4*d^9 + 84*a*c^3*d^7*e^2 + 189*a^2*c^2*d^5*e^4 + 210*a^3*c*d^3*e^6 + 105*a^4*d*e^8 + (a^5*c^5*d^10 + 5*a^6*c^4*d^8*e^2 + 10*a^7*c^3*d^6*e^4 + 10*a^8*c^2*d^4*e^6 + 5*a^9*c*d^2*e^8 + a^10*e^10)*sqrt(-(441*c^4*d^8*e^10 + 2268*a*c^3*d^6*e^12 + 4974*a^2*c^2*d^4*e^14 + 5292*a^3*c*d^2*e^16 + 2401*a^4*e^18)/(a^5*c^11*d^20 + 10*a^6*c^10*d^18*e^2 + 45*a^7*c^9*d^16*e^4 + 120*a^8*c^8*d^14*e^6 + 210*a^9*c^7*d^12*e^8 + 252*a^10*c^6*d^10*e^10 + 210*a^11*c^5*d^8*e^12 + 120*a^12*c^4*d^6*e^14 + 45*a^13*c^3*d^4*e^16 + 10*a^14*c^2*d^2*e^18 + a^15*c*e^20)))/(a^5*c^5*d^10 + 5*a^6*c^4*d^8*e^2 + 10*a^7*c^3*d^6*e^4 + 10*a^8*c^2*d^4*e^6 + 5*a^9*c*d^2*e^8 + a^10*e^10))*log(27*(336*c^4*d^8*e^5 + 1788*a*c^3*d^6*e^7 + 4189*a^2*c^2*d^4*e^9 + 4802*a^3*c*d^2*e^11 + 2401*a^4*e^13)*sqrt(e*x + d) - 27*(42*a^3*c^4*d^8*e^6 + 213*a^4*c^3*d^6*e^8 + 515*a^5*c^2*d^4*e^10 + 623*a^6*c*d^2*e^12 + 343*a^7*e^14 + (4*a^5*c^8*d^15 + 31*a^6*c^7*d^13*e^2 + 106*a^7*c^6*d^11*e^4 + 205*a^8*c^5*d^9*e^6 + 240*a^9*c^4*d^7*e^8 + 169*a^10*c^3*d^5*e^10 + 66*a^11*c^2*d^3*e^12 + 11*a^12*c*d*e^14)*sqrt(-(441*c^4*d^8*e^10 + 2268*a*c^3*d^6*e^12 + 4974*a^2*c^2*d^4*e^14 + 5292*a^3*c*d^2*e^16 + 2401*a^4*e^18)/(a^5*c^11*d^20 + 10*a^6*c^10*d^18*e^2 + 45*a^7*c^9*d^16*e^4 + 120*a^8*c^8*d^14*e^6 + 210*a^9*c^7*d^12*e^8 + 252*a^10*c^6*d^10*e^10 + 210*a^11*c^5*d^8*e^12 + 120*a^12*c^4*d^6*e^14 + 45*a^13*c^3*d^4*e^16 + 10*a^14*c^2*d^2*e^18 + a^15*c*e^20)))*sqrt(-(16*c^4*d^9 + 84*a*c^3*d^7*e^2 + 189*a^2*c^2*d^5*e^4 + 210*a^3*c*d^3*e^6 + 105*a^4*d*e^8 + (a^5*c^5*d^10 + 5*a^6*c^4*d^8*e^2 + 10*a^7*c^3*d^6*e^4 + 10*a^8*c^2*d^4*e^6 + 5*a^9*c*d^2*e^8 + a^10*e^10)*sqrt(-(441*c^4*d^8*e^10 + 2268*a*c^3*d^6*e^12 + 4974*a^2*c^2*d^4*e^14 + 5292*a^3*c*d^2*e^16 + 2401*a^4*e^18)/(a^5*c^11*d^20 + 10*a^6*c^10*d^18*e^2 + 45*a^7*c^9*d^16*e^4 + 120*a^8*c^8*d^14*e^6 + 210*a^9*c^7*d^12*e^8 + 252*a^10*c^6*d^10*e^10 + 210*a^11*c^5*d^8*e^12 + 120*a^12*c^4*d^6*e^14 + 45*a^13*c^3*d^4*e^16 + 10*a^14*c^2*d^2*e^18 + a^15*c*e^20)))/(a^5*c^5*d^10 + 5*a^6*c^4*d^8*e^2 + 10*a^7*c^3*d^6*e^4 + 10*a^8*c^2*d^4*e^6 + 5*a^9*c*d^2*e^8 + a^10*e^10))) + 3*(a^4*c^2*d^4 + 2*a^5*c*d^2*e^2 + a^6*e^4 + (a^2*c^4*d^4 + 2*a^3*c^3*d^2*e^2 + a^4*c^2*e^4)*x^4 + 2*(a^3*c^3*d^4 + 2*a^4*c^2*d^2*e^2 + a^5*c*e^4)*x^2)*sqrt(-(16*c^4*d^9 + 84*a*c^3*d^7*e^2 + 189*a^2*c^2*d^5*e^4 + 210*a^3*c*d^3*e^6 + 105*a^4*d*e^8 - (a^5*c^5*d^10 + 5*a^6*c^4*d^8*e^2 + 10*a^7*c^3*d^6*e^4 + 10*a^8*c^2*d^4*e^6 + 5*a^9*c*d^2*e^8 + a^10*e^10)*sqrt(-(441*c^4*d^8*e^10 + 2268*a*c^3*d^6*e^12 + 4974*a^2*c^2*d^4*e^14 + 5292*a^3*c*d^2*e^16 + 2401*a^4*e^18)/(a^5*c^11*d^20 + 10*a^6*c^10*d^18*e^2 + 45*a^7*c^9*d^16*e^4 + 120*a^8*c^8*d^14*e^6 + 210

```
*a^9*c^7*d^12*e^8 + 252*a^10*c^6*d^10*e^10 + 210*a^11*c^5*d^8*e^12 + 120*a^
12*c^4*d^6*e^14 + 45*a^13*c^3*d^4*e^16 + 10*a^14*c^2*d^2*e^18 + a^15*c*e^20
)))/(a^5*c^5*d^10 + 5*a^6*c^4*d^8*e^2 + 10*a^7*c^3*d^6*e^4 + 10*a^8*c^2*d^4
*e^6 + 5*a^9*c*d^2*e^8 + a^10*e^10))*log(27*(336*c^4*d^8*e^5 + 1788*a*c^3*d
^6*e^7 + 4189*a^2*c^2*d^4*e^9 + 4802*a^3*c*d^2*e^11 + 2401*a^4*e^13)*sqrt(e
*x + d) + 27*(42*a^3*c^4*d^8*e^6 + 213*a^4*c^3*d^6*e^8 + 515*a^5*c^2*d^4*e^
10 + 623*a^6*c*d^2*e^12 + 343*a^7*e^14 - (4*a^5*c^8*d^15 + 31*a^6*c^7*d^13*
e^2 + 106*a^7*c^6*d^11*e^4 + 205*a^8*c^5*d^9*e^6 + 240*a^9*c^4*d^7*e^8 + 16
9*a^10*c^3*d^5*e^10 + 66*a^11*c^2*d^3*e^12 + 11*a^12*c*d*e^14)*sqrt(-(441*c
^4*d^8*e^10 + 2268*a*c^3*d^6*e^12 + 4974*a^2*c^2*d^4*e^14 + 5292*a^3*c*d^2*
e^16 + 2401*a^4*e^18)/(a^5*c^11*d^20 + 10*a^6*c^10*d^18*e^2 + 45*a^7*c^9*d^
16*e^4 + 120*a^8*c^8*d^14*e^6 + 210*a^9*c^7*d^12*e^8 + 252*a^10*c^6*d^10*e^
10 + 210*a^11*c^5*d^8*e^12 + 120*a^12*c^4*d^6*e^14 + 45*a^13*c^3*d^4*e^16 +
 10*a^14*c^2*d^2*e^18 + a^15*c*e^20)))*sqrt(-(16*c^4*d^9 + 84*a*c^3*d^7*e^2
 + 189*a^2*c^2*d^5*e^4 + 210*a^3*c*d^3*e^6 + 105*a^4*d*e^8 - (a^5*c^5*d^10
+ 5*a^6*c^4*d^8*e^2 + 10*a^7*c^3*d^6*e^4 + 10*a^8*c^2*d^4*e^6 + 5*a^9*c*d^2
*e^8 + a^10*e^10)*sqrt(-(441*c^4*d^8*e^10 + 2268*a*c^3*d^6*e^12 + 4974*a^2*
c^2*d^4*e^14 + 5292*a^3*c*d^2*e^16 + 2401*a^4*e^18)/(a^5*c^11*d^20 + 10*a^6
*c^10*d^18*e^2 + 45*a^7*c^9*d^16*e^4 + 120*a^8*c^8*d^14*e^6 + 210*a^9*c^7*d
^12*e^8 + 252*a^10*c^6*d^10*e^10 + 210*a^11*c^5*d^8*e^12 + 120*a^12*c^4*d^6
*e^14 + 45*a^13*c^3*d^4*e^16 + 10*a^14*c^2*d^2*e^18 + a^15*c*e^20)))/(a^5*c
^5*d^10 + 5*a^6*c^4*d^8*e^2 + 10*a^7*c^3*d^6*e^4 + 10*a^8*c^2*d^4*e^6 + 5*a
^9*c*d^2*e^8 + a^10*e^10))) - 3*(a^4*c^2*d^4 + 2*a^5*c*d^2*e^2 + a^6*e^4 +
(a^2*c^4*d^4 + 2*a^3*c^3*d^2*e^2 + a^4*c^2*e^4)*x^4 + 2*(a^3*c^3*d^4 + 2*a^
4*c^2*d^2*e^2 + a^5*c*e^4)*x^2)*sqrt(-(16*c^4*d^9 + 84*a*c^3*d^7*e^2 + 189*
a^2*c^2*d^5*e^4 + 210*a^3*c*d^3*e^6 + 105*a^4*d*e^8 - (a^5*c^5*d^10 + 5*a^6
*c^4*d^8*e^2 + 10*a^7*c^3*d^6*e^4 + 10*a^8*c^2*d^4*e^6 + 5*a^9*c*d^2*e^8 +
a^10*e^10)*sqrt(-(441*c^4*d^8*e^10 + 2268*a*c^3*d^6*e^12 + 4974*a^2*c^2*d^4
*e^14 + 5292*a^3*c*d^2*e^16 + 2401*a^4*e^18)/(a^5*c^11*d^20 + 10*a^6*c^10*d
^18*e^2 + 45*a^7*c^9*d^16*e^4 + 120*a^8*c^8*d^14*e^6 + 210*a^9*c^7*d^12*e^8
 + 252*a^10*c^6*d^10*e^10 + 210*a^11*c^5*d^8*e^12 + 120*a^12*c^4*d^6*e^14 +
 45*a^13*c^3*d^4*e^16 + 10*a^14*c^2*d^2*e^18 + a^15*c*e^20)))/(a^5*c^5*d^10
 + 5*a^6*c^4*d^8*e^2 + 10*a^7*c^3*d^6*e^4 + 10*a^8*c^2*d^4*e^6 + 5*a^9*c*d^
2*e^8 + a^10*e^10))*log(27*(336*c^4*d^8*e^5 + 1788*a*c^3*d^6*e^7 + 4189*a^2
*c^2*d^4*e^9 + 4802*a^3*c*d^2*e^11 + 2401*a^4*e^13)*sqrt(e*x + d) - 27*(42*
a^3*c^4*d^8*e^6 + 213*a^4*c^3*d^6*e^8 + 515*a^5*c^2*d^4*e^10 + 623*a^6*c*d^
2*e^12 + 343*a^7*e^14 - (4*a^5*c^8*d^15 + 31*a^6*c^7*d^13*e^2 + 106*a^7*c^6
*d^11*e^4 + 205*a^8*c^5*d^9*e^6 + 240*a^9*c^4*d^7*e^8 + 169*a^10*c^3*d^5*e^
10 + 66*a^11*c^2*d^3*e^12 + 11*a^12*c*d*e^14)*sqrt(-(441*c^4*d^8*e^10 + 226
8*a*c^3*d^6*e^12 + 4974*a^2*c^2*d^4*e^14 + 5292*a^3*c*d^2*e^16 + 2401*a^4*e
^18)/(a^5*c^11*d^20 + 10*a^6*c^10*d^18*e^2 + 45*a^7*c^9*d^16*e^4 + 120*a^8*
c^8*d^14*e^6 + 210*a^9*c^7*d^12*e^8 + 252*a^10*c^6*d^10*e^10 + 210*a^11*c^5
*d^8*e^12 + 120*a^12*c^4*d^6*e^14 + 45*a^13*c^3*d^4*e^16 + 10*a^14*c^2*d^2*
e^18 + a^15*c*e^20)))*sqrt(-(16*c^4*d^9 + 84*a*c^3*d^7*e^2 + 189*a^2*c^2*d^
5*e^4 + 210*a^3*c*d^3*e^6 + 105*a^4*d*e^8 - (a^5*c^5*d^10 + 5*a^6*c^4*d^8*e
^2 + 10*a^7*c^3*d^6*e^4 + 10*a^8*c^2*d^4*e^6 + 5*a^9*c*d^2*e^8 + a^10*e^10)
*sqrt(-(441*c^4*d^8*e^10 + 2268*a*c^3*d^6*e^12 + 4974*a^2*c^2*d^4*e^14 + 52
92*a^3*c*d^2*e^16 + 2401*a^4*e^18)/(a^5*c^11*d^20 + 10*a^6*c^10*d^18*e^2 +
45*a^7*c^9*d^16*e^4 + 120*a^8*c^8*d^14*e^6 + 210*a^9*c^7*d^12*e^8 + 252*a^1
0*c^6*d^10*e^10 + 210*a^11*c^5*d^8*e^12 + 120*a^12*c^4*d^6*e^14 + 45*a^13*c
^3*d^4*e^16 + 10*a^14*c^2*d^2*e^18 + a^15*c*e^20)))/(a^5*c^5*d^10 + 5*a^6*c
^4*d^8*e^2 + 10*a^7*c^3*d^6*e^4 + 10*a^8*c^2*d^4*e^6 + 5*a^9*c*d^2*e^8 + a^
10*e^10))) + 4*(5*a^2*c*d^2*e + 11*a^3*e^3 + 6*(c^3*d^3 + 2*a*c^2*d*e^2)*x^
3 + (a*c^2*d^2*e + 7*a^2*c*e^3)*x^2 + 2*(5*a*c^2*d^3 + 8*a^2*c*d*e^2)*x)*sq
rt(e*x + d))/(a^4*c^2*d^4 + 2*a^5*c*d^2*e^2 + a^6*e^4 + (a^2*c^4*d^4 + 2*a^
3*c^3*d^2*e^2 + a^4*c^2*e^4)*x^4 + 2*(a^3*c^3*d^4 + 2*a^4*c^2*d^2*e^2 + a^5
*c*e^4)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a)**3/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^3/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.648 $\int \frac{\sqrt{2+3x}}{1+x^2} \, dx$

Optimal. Leaf size=214

$$\frac{3 \log \left(3x-\sqrt{2 \left(2+\sqrt{13}\right)}\sqrt{3x+2}+\sqrt{13}+2\right)}{2\sqrt{2\left(2+\sqrt{13}\right)}}-\frac{3 \log \left(3x+\sqrt{2 \left(2+\sqrt{13}\right)}\sqrt{3x+2}+\sqrt{13}+2\right)}{2\sqrt{2\left(2+\sqrt{13}\right)}}-\frac{3\tan^{-1}\left(\frac{\sqrt{2(2+\sqrt{13})}}{\sqrt{2(\sqrt{1}}}\right.}{\sqrt{2\left(\sqrt{13}-\right.}}$$

```
[Out] (-3*ArcTan[(Sqrt[2*(2 + Sqrt[13])] - 2*Sqrt[2 + 3*x])/Sqrt[2*(-2 + Sqrt[13]
)]])/Sqrt[2*(-2 + Sqrt[13])] + (3*ArcTan[(Sqrt[2*(2 + Sqrt[13])] + 2*Sqrt[2
 + 3*x])/Sqrt[2*(-2 + Sqrt[13])]])/Sqrt[2*(-2 + Sqrt[13])] + (3*Log[2 + Sqr
t[13] + 3*x - Sqrt[2*(2 + Sqrt[13])]*Sqrt[2 + 3*x]])/(2*Sqrt[2*(2 + Sqrt[13
])]) - (3*Log[2 + Sqrt[13] + 3*x + Sqrt[2*(2 + Sqrt[13])]*Sqrt[2 + 3*x]])/(
2*Sqrt[2*(2 + Sqrt[13])])
```

Rubi [A] time = 0.230092, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {700, 1127, 1161, 618, 204, 1164, 628}

$$\frac{3 \log \left(3x-\sqrt{2 \left(2+\sqrt{13}\right)}\sqrt{3x+2}+\sqrt{13}+2\right)}{2\sqrt{2\left(2+\sqrt{13}\right)}}-\frac{3 \log \left(3x+\sqrt{2 \left(2+\sqrt{13}\right)}\sqrt{3x+2}+\sqrt{13}+2\right)}{2\sqrt{2\left(2+\sqrt{13}\right)}}-\frac{3\tan^{-1}\left(\frac{\sqrt{2(2+\sqrt{13})}}{\sqrt{2(\sqrt{1}}}\right.}{\sqrt{2\left(\sqrt{13}-\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 3*x]/(1 + x^2),x]
```

```
[Out] (-3*ArcTan[(Sqrt[2*(2 + Sqrt[13])] - 2*Sqrt[2 + 3*x])/Sqrt[2*(-2 + Sqrt[13]
)]])/Sqrt[2*(-2 + Sqrt[13])] + (3*ArcTan[(Sqrt[2*(2 + Sqrt[13])] + 2*Sqrt[2
 + 3*x])/Sqrt[2*(-2 + Sqrt[13])]])/Sqrt[2*(-2 + Sqrt[13])] + (3*Log[2 + Sqr
t[13] + 3*x - Sqrt[2*(2 + Sqrt[13])]*Sqrt[2 + 3*x]])/(2*Sqrt[2*(2 + Sqrt[13
])]) - (3*Log[2 + Sqrt[13] + 3*x + Sqrt[2*(2 + Sqrt[13])]*Sqrt[2 + 3*x]])/(
2*Sqrt[2*(2 + Sqrt[13])])
```

Rule 700

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
```

```
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{2+3x}}{1+x^2}\,dx &= 6\,\text{Subst}\left(\int \frac{x^2}{13-4x^2+x^4}\,dx, x, \sqrt{2+3x}\right)\\
&= -\left(3\,\text{Subst}\left(\int \frac{\sqrt{13}-x^2}{13-4x^2+x^4}\,dx, x, \sqrt{2+3x}\right)\right)+3\,\text{Subst}\left(\int \frac{\sqrt{13}+x^2}{13-4x^2+x^4}\,dx, x, \sqrt{2+3x}\right)\\
&= \frac{3}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{13}-\sqrt{2\left(2+\sqrt{13}\right)}x+x^2}\,dx, x, \sqrt{2+3x}\right)+\frac{3}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{13}+\sqrt{2\left(2+\sqrt{13}\right)}x+x^2}\,dx\right.\\
&= \frac{3\log\left(2+\sqrt{13}+3x-\sqrt{2\left(2+\sqrt{13}\right)}\sqrt{2+3x}\right)}{2\sqrt{2\left(2+\sqrt{13}\right)}}-\frac{3\log\left(2+\sqrt{13}+3x+\sqrt{2\left(2+\sqrt{13}\right)}\sqrt{2+3x}\right)}{2\sqrt{2\left(2+\sqrt{13}\right)}}-3\,\text{Su}\\
&= -\frac{3\tan^{-1}\left(\frac{\sqrt{2\left(2+\sqrt{13}\right)}-2\sqrt{2+3x}}{\sqrt{2\left(-2+\sqrt{13}\right)}}\right)}{\sqrt{2\left(-2+\sqrt{13}\right)}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2\left(2+\sqrt{13}\right)}+2\sqrt{2+3x}}{\sqrt{2\left(-2+\sqrt{13}\right)}}\right)}{\sqrt{2\left(-2+\sqrt{13}\right)}}+\frac{3\log\left(2+\sqrt{13}+3x-\sqrt{2\left(2+\sqrt{13}\right)}\sqrt{2}\right.}{2\sqrt{2\left(2+\sqrt{13}\right)}}
\end{aligned}$$

Mathematica [C] time = 0.0302046, size = 59, normalized size = 0.28

$$i\sqrt{2+3i}\tanh^{-1}\left(\frac{\sqrt{3x+2}}{\sqrt{2+3i}}\right)-i\sqrt{2-3i}\tanh^{-1}\left(\frac{\sqrt{3x+2}}{\sqrt{2-3i}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 3*x]/(1 + x^2),x]
```

```
[Out] (-I)*Sqrt[2 - 3*I]*ArcTanh[Sqrt[2 + 3*x]/Sqrt[2 - 3*I]] + I*Sqrt[2 + 3*I]*A
rcTanh[Sqrt[2 + 3*x]/Sqrt[2 + 3*I]]
```

Maple [B] time = 0.173, size = 360, normalized size = 1.7

$$-\frac{\sqrt{4+2\sqrt{13}}}{6}\ln\left(2+3x+\sqrt{13}-\sqrt{2+3x}\sqrt{4+2\sqrt{13}}\right)-\frac{4+2\sqrt{13}}{3\sqrt{-4+2\sqrt{13}}}\arctan\left(\frac{1}{\sqrt{-4+2\sqrt{13}}}\left(2\sqrt{2+3x}-\sqrt{4\cdot\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^(1/2)/(x^2+1),x)
```

```
[Out] -1/6*(4+2*13^(1/2))^(1/2)*ln(2+3*x+13^(1/2)-(2+3*x)^(1/2)*(4+2*13^(1/2))^(1
/2))-1/3*(4+2*13^(1/2))/(-4+2*13^(1/2))^(1/2)*arctan((2*(2+3*x)^(1/2)-(4+2*
13^(1/2))^(1/2))/(-4+2*13^(1/2))^(1/2))+1/12*(4+2*13^(1/2))^(1/2)*13^(1/2)*
ln(2+3*x+13^(1/2)-(2+3*x)^(1/2)*(4+2*13^(1/2))^(1/2))+1/6*13^(1/2)*(4+2*13^
(1/2))/(-4+2*13^(1/2))^(1/2)*arctan((2*(2+3*x)^(1/2)-(4+2*13^(1/2))^(1/2))/
(-4+2*13^(1/2))^(1/2))+1/6*(4+2*13^(1/2))^(1/2)*ln(2+3*x+13^(1/2)+(2+3*x)^(
1/2)*(4+2*13^(1/2))^(1/2))-1/3*(4+2*13^(1/2))/(-4+2*13^(1/2))^(1/2)*arctan(
(2*(2+3*x)^(1/2)+(4+2*13^(1/2))^(1/2))/(-4+2*13^(1/2))^(1/2))-1/12*(4+2*13^
(1/2))^(1/2)*13^(1/2)*ln(2+3*x+13^(1/2)+(2+3*x)^(1/2)*(4+2*13^(1/2))^(1/2))
+1/6*13^(1/2)*(4+2*13^(1/2))/(-4+2*13^(1/2))^(1/2)*arctan((2*(2+3*x)^(1/2)+
(4+2*13^(1/2))^(1/2))/(-4+2*13^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{3x+2}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*x + 2)/(x^2 + 1), x)
```

Fricas [A] time = 2.62328, size = 990, normalized size = 4.63

$$\frac{1}{156}\cdot13^{\frac{1}{4}}\sqrt{4\sqrt{13}+26}\left(2\sqrt{13}-13\right)\log\left(\frac{1}{13}\cdot13^{\frac{3}{4}}\sqrt{3x+2}\sqrt{4\sqrt{13}+26}+3x+\sqrt{13}+2\right)-\frac{1}{156}\cdot13^{\frac{1}{4}}\sqrt{4\sqrt{13}+26}\left(2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/156*13^(1/4)*sqrt(4*sqrt(13) + 26)*(2*sqrt(13) - 13)*log(1/13*13^(3/4)*sq
rt(3*x + 2)*sqrt(4*sqrt(13) + 26) + 3*x + sqrt(13) + 2) - 1/156*13^(1/4)*sq
rt(4*sqrt(13) + 26)*(2*sqrt(13) - 13)*log(-1/13*13^(3/4)*sqrt(3*x + 2)*sqrt
(4*sqrt(13) + 26) + 3*x + sqrt(13) + 2) - 1/13*13^(3/4)*sqrt(4*sqrt(13) + 2
6)*arctan(-1/39*13^(3/4)*sqrt(3*x + 2)*sqrt(4*sqrt(13) + 26) + 1/39*13^(1/4
)*sqrt(13^(3/4)*sqrt(3*x + 2)*sqrt(4*sqrt(13) + 26) + 39*x + 13*sqrt(13) +
```

```
26)*sqrt(4*sqrt(13) + 26) - 1/3*sqrt(13) - 2/3) - 1/13*13^(3/4)*sqrt(4*sqrt
(13) + 26)*arctan(-1/39*13^(3/4)*sqrt(3*x + 2)*sqrt(4*sqrt(13) + 26) + 1/39
*13^(1/4)*sqrt(-13^(3/4)*sqrt(3*x + 2)*sqrt(4*sqrt(13) + 26) + 39*x + 13*sq
rt(13) + 26)*sqrt(4*sqrt(13) + 26) + 1/3*sqrt(13) + 2/3)
```

Sympy [A] time = 3.08808, size = 32, normalized size = 0.15

$$6\,\mathrm{RootSum}\left(20736t^4 + 576t^2 + 13, \left(t \mapsto t\log\left(576t^3 + 8t + \sqrt{3x+2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**(1/2)/(x**2+1),x)
```

```
[Out] 6*RootSum(20736*_t**4 + 576*_t**2 + 13, Lambda(_t, _t*log(576*_t**3 + 8*_t
+ sqrt(3*x + 2))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{3\,x+2}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(3*x + 2)/(x^2 + 1), x)
```

3.649 $\int \frac{\sqrt{c+dx}}{1+x^2}\,dx$

Optimal. Leaf size=316

$$\frac{d\log\left(-\sqrt{2}\sqrt{\sqrt{c^2+d^2}+c}\sqrt{c+dx}+\sqrt{c^2+d^2}+c+dx\right)}{2\sqrt{2}\sqrt{\sqrt{c^2+d^2}+c}}-\frac{d\log\left(\sqrt{2}\sqrt{\sqrt{c^2+d^2}+c}\sqrt{c+dx}+\sqrt{c^2+d^2}+c+dx\right)}{2\sqrt{2}\sqrt{\sqrt{c^2+d^2}+c}}+$$

```
[Out] (d*ArcTanh[(Sqrt[c + Sqrt[c^2 + d^2]] - Sqrt[2]*Sqrt[c + d*x])/Sqrt[c - Sqr
t[c^2 + d^2]]])/(Sqrt[2]*Sqrt[c - Sqrt[c^2 + d^2]]) - (d*ArcTanh[(Sqrt[c +
Sqrt[c^2 + d^2]] + Sqrt[2]*Sqrt[c + d*x])/Sqrt[c - Sqrt[c^2 + d^2]]])/(Sqrt
[2]*Sqrt[c - Sqrt[c^2 + d^2]]) + (d*Log[c + Sqrt[c^2 + d^2] + d*x - Sqrt[2]
*Sqrt[c + Sqrt[c^2 + d^2]]*Sqrt[c + d*x]])/(2*Sqrt[2]*Sqrt[c + Sqrt[c^2 + d
^2]]) - (d*Log[c + Sqrt[c^2 + d^2] + d*x + Sqrt[2]*Sqrt[c + Sqrt[c^2 + d^2]
]*Sqrt[c + d*x]])/(2*Sqrt[2]*Sqrt[c + Sqrt[c^2 + d^2]])
```

Rubi [A] time = 0.346685, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {700, 1129, 634, 618, 206, 628}

$$\frac{d\log\left(-\sqrt{2}\sqrt{\sqrt{c^2+d^2}+c}\sqrt{c+dx}+\sqrt{c^2+d^2}+c+dx\right)}{2\sqrt{2}\sqrt{\sqrt{c^2+d^2}+c}}-\frac{d\log\left(\sqrt{2}\sqrt{\sqrt{c^2+d^2}+c}\sqrt{c+dx}+\sqrt{c^2+d^2}+c+dx\right)}{2\sqrt{2}\sqrt{\sqrt{c^2+d^2}+c}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(1 + x^2),x]
```

```
[Out] (d*ArcTanh[(Sqrt[c + Sqrt[c^2 + d^2]] - Sqrt[2]*Sqrt[c + d*x])/Sqrt[c - Sqr
t[c^2 + d^2]]])/(Sqrt[2]*Sqrt[c - Sqrt[c^2 + d^2]]) - (d*ArcTanh[(Sqrt[c +
Sqrt[c^2 + d^2]] + Sqrt[2]*Sqrt[c + d*x])/Sqrt[c - Sqrt[c^2 + d^2]]])/(Sqrt
[2]*Sqrt[c - Sqrt[c^2 + d^2]]) + (d*Log[c + Sqrt[c^2 + d^2] + d*x - Sqrt[2]
*Sqrt[c + Sqrt[c^2 + d^2]]*Sqrt[c + d*x]])/(2*Sqrt[2]*Sqrt[c + Sqrt[c^2 + d
^2]]) - (d*Log[c + Sqrt[c^2 + d^2] + d*x + Sqrt[2]*Sqrt[c + Sqrt[c^2 + d^2]
]*Sqrt[c + d*x]])/(2*Sqrt[2]*Sqrt[c + Sqrt[c^2 + d^2]])
```

Rule 700

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1129

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q =
 Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m - 1)/(q
 - r*x + x^2), x], x] - Dist[1/(2*c*r), Int[x^(m - 1)/(q + r*x + x^2), x],
x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 1] && LtQ[m, 3
] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{1+x^2}\,dx = (2d)\,\text{Subst}\left(\int \frac{x^2}{c^2+d^2-2cx^2+x^4}\,dx, x, \sqrt{c+dx}\right)$$

$$= \frac{d\,\text{Subst}\left(\int \frac{x}{\sqrt{c^2+d^2}-\sqrt{2}\sqrt{c+\sqrt{c^2+d^2}}x+x^2}\,dx, x, \sqrt{c+dx}\right)}{\sqrt{2}\sqrt{c+\sqrt{c^2+d^2}}} - \frac{d\,\text{Subst}\left(\int \frac{x}{\sqrt{c^2+d^2}+\sqrt{2}\sqrt{c+\sqrt{c^2+d^2}}x+x^2}\,dx, x, \sqrt{c+dx}\right.}{\sqrt{2}\sqrt{c+\sqrt{c^2+d^2}}}$$

$$= \frac{1}{2}d\,\text{Subst}\left(\int \frac{1}{\sqrt{c^2+d^2}-\sqrt{2}\sqrt{c+\sqrt{c^2+d^2}}x+x^2}\,dx, x, \sqrt{c+dx}\right) + \frac{1}{2}d\,\text{Subst}\left(\int \frac{1}{\sqrt{c^2+d^2}+\sqrt{2}\sqrt{c+}\right.$$

$$= \frac{d\log\left(c+\sqrt{c^2+d^2}+dx-\sqrt{2}\sqrt{c+\sqrt{c^2+d^2}}\sqrt{c+dx}\right)}{2\sqrt{2}\sqrt{c+\sqrt{c^2+d^2}}} - \frac{d\log\left(c+\sqrt{c^2+d^2}+dx+\sqrt{2}\sqrt{c+\sqrt{c^2+d^2}}\right.}{2\sqrt{2}\sqrt{c+\sqrt{c^2+d^2}}}$$

$$= \frac{d\tanh^{-1}\left(\frac{\sqrt{c+\sqrt{c^2+d^2}}-\sqrt{2}\sqrt{c+dx}}{\sqrt{c-\sqrt{c^2+d^2}}}\right)}{\sqrt{2}\sqrt{c-\sqrt{c^2+d^2}}} - \frac{d\tanh^{-1}\left(\frac{\sqrt{c+\sqrt{c^2+d^2}}+\sqrt{2}\sqrt{c+dx}}{\sqrt{c-\sqrt{c^2+d^2}}}\right)}{\sqrt{2}\sqrt{c-\sqrt{c^2+d^2}}} + \frac{d\log\left(c+\sqrt{c^2+d^2}+dx-\sqrt{2}\sqrt{c+}\right.}{2\sqrt{2}\sqrt{c+\sqrt{c^2+d^2}}}$$

Mathematica [C] time = 0.0424746, size = 75, normalized size = 0.24

$$i\sqrt{c+id}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c+id}}\right)-i\sqrt{c-id}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c-id}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(1 + x^2),x]
```

```
[Out] (-I)*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*x]/Sqrt[c - I*d]] + I*Sqrt[c + I*d]*A
rcTanh[Sqrt[c + d*x]/Sqrt[c + I*d]]
```

Maple [B] time = 0.209, size = 570, normalized size = 1.8

$$\frac{c}{4d}\sqrt{2\sqrt{c^2+d^2}+2c}\ln\left(dx+c+\sqrt{dx+c}\sqrt{2\sqrt{c^2+d^2}+2c}+\sqrt{c^2+d^2}\right)-\frac{c^2}{d}\arctan\left(\left(2\sqrt{dx+c}+\sqrt{2\sqrt{c^2+d^2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(x^2+1),x)
```

```
[Out] 1/4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)/d*c*ln(d*x+c+(d*x+c)^(1/2)*(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))-1/d*c^2/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arct
an((2*(d*x+c)^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^
(1/2))-1/4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)/d*(c^2+d^2)^(1/2)*ln(d*x+c+(d*x+c)
^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+1/d*(c^2+d^2)/(2*(c^2
+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(d*x+c)^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2
))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))-1/4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)/d*c*ln(
(d*x+c)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*x-c-(c^2+d^2)^(1/2))+1/d*c^2/
(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(d*x+
c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))+1/4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)/
d*(c^2+d^2)^(1/2)*ln((d*x+c)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*x-c-(c^2
+d^2)^(1/2))-1/d*(c^2+d^2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^
2)^(1/2)+2*c)^(1/2)-2*(d*x+c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx+c}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x + c)/(x^2 + 1), x)
```

Fricas [B] time = 2.98742, size = 2176, normalized size = 6.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*(c^2 + d^2)^(3/4)*sqrt(d^2)*sqrt((c^2 + d^2 + sqrt(c^2 + d^
2)*c)/d^2)*arctan(-(sqrt(2)*(c^2 + d^2)^(3/4)*sqrt(d^2)*sqrt(d*x + c)*d*sqr
t((c^2 + d^2 + sqrt(c^2 + d^2)*c)/d^2) - sqrt(2)*(c^2 + d^2)^(3/4)*sqrt(d^2
)*sqrt((sqrt(2)*(c^2 + d^2)^(3/4)*sqrt(d*x + c)*d^3*sqrt((c^2 + d^2 + sqrt(
c^2 + d^2)*c)/d^2) + c^3*d^2 + c*d^4 + (c^2*d^3 + d^5)*x + (c^2*d^2 + d^4)*
sqrt(c^2 + d^2))/(c^2 + d^2))*sqrt((c^2 + d^2 + sqrt(c^2 + d^2)*c)/d^2) + (
c^2 + d^2)^(3/2)*sqrt(d^2) + (c^3 + c*d^2)*sqrt(d^2))/(c^2*d^2 + d^4)) + 4*
sqrt(2)*(c^2 + d^2)^(3/4)*sqrt(d^2)*sqrt((c^2 + d^2 + sqrt(c^2 + d^2)*c)/d^
2)*arctan(-(sqrt(2)*(c^2 + d^2)^(3/4)*sqrt(d^2)*sqrt(d*x + c)*d*sqrt((c^2 +
```

```
d^2 + sqrt(c^2 + d^2)*c)/d^2) - sqrt(2)*(c^2 + d^2)^(3/4)*sqrt(d^2)*sqrt(-
(sqrt(2)*(c^2 + d^2)^(3/4)*sqrt(d*x + c)*d^3*sqrt((c^2 + d^2 + sqrt(c^2 + d
^2)*c)/d^2) - c^3*d^2 - c*d^4 - (c^2*d^3 + d^5)*x - (c^2*d^2 + d^4)*sqrt(c^
2 + d^2))/(c^2 + d^2))*sqrt((c^2 + d^2 + sqrt(c^2 + d^2)*c)/d^2) - (c^2 + d
^2)^(3/2)*sqrt(d^2) - (c^3 + c*d^2)*sqrt(d^2))/(c^2*d^2 + d^4)) + sqrt(2)*(
c^2 + d^2 - sqrt(c^2 + d^2)*c)*(c^2 + d^2)^(1/4)*sqrt((c^2 + d^2 + sqrt(c^2
 + d^2)*c)/d^2)*log((sqrt(2)*(c^2 + d^2)^(3/4)*sqrt(d*x + c)*d^3*sqrt((c^2
+ d^2 + sqrt(c^2 + d^2)*c)/d^2) + c^3*d^2 + c*d^4 + (c^2*d^3 + d^5)*x + (c^
2*d^2 + d^4)*sqrt(c^2 + d^2))/(c^2 + d^2)) - sqrt(2)*(c^2 + d^2 - sqrt(c^2
+ d^2)*c)*(c^2 + d^2)^(1/4)*sqrt((c^2 + d^2 + sqrt(c^2 + d^2)*c)/d^2)*log(-
(sqrt(2)*(c^2 + d^2)^(3/4)*sqrt(d*x + c)*d^3*sqrt((c^2 + d^2 + sqrt(c^2 + d
^2)*c)/d^2) - c^3*d^2 - c*d^4 - (c^2*d^3 + d^5)*x - (c^2*d^2 + d^4)*sqrt(c^
2 + d^2))/(c^2 + d^2)))/(c^2 + d^2)
```

Sympy [A] time = 3.88021, size = 53, normalized size = 0.17

$$2d\,\mathrm{RootSum}\left(256t^4d^4 + 32t^2cd^2 + c^2 + d^2, \left(t \mapsto t\log\left(64t^3d^2 + 4tc + \sqrt{c + dx}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(x**2+1),x)
```

```
[Out] 2*d*RootSum(256*_t**4*d**4 + 32*_t**2*c*d**2 + c**2 + d**2, Lambda(_t, _t*l
og(64*_t**3*d**2 + 4*_t*c + sqrt(c + d*x))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx + c}}{x^2 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x + c)/(x^2 + 1), x)
```

3.650 $\int \frac{\sqrt{2+3x}}{1-x^2} \, dx$

Optimal. Leaf size=35

$$\sqrt{5} \tanh^{-1}\left(\frac{\sqrt{3x+2}}{\sqrt{5}}\right) - \tan^{-1}\left(\sqrt{3x+2}\right)$$

```
[Out] -ArcTan[Sqrt[2 + 3*x]] + Sqrt[5]*ArcTanh[Sqrt[2 + 3*x]/Sqrt[5]]
```

Rubi [A] time = 0.0203036, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {700, 1130, 206, 204}

$$\sqrt{5} \tanh^{-1}\left(\frac{\sqrt{3x+2}}{\sqrt{5}}\right) - \tan^{-1}\left(\sqrt{3x+2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 3*x]/(1 - x^2),x]
```

```
[Out] -ArcTan[Sqrt[2 + 3*x]] + Sqrt[5]*ArcTanh[Sqrt[2 + 3*x]/Sqrt[5]]
```

Rule 700

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{2+3x}}{1-x^2}\,dx = 6\,\text{Subst}\left(\int \frac{x^2}{5+4x^2-x^4}\,dx, x, \sqrt{2+3x}\right)$$
$$= 5\,\text{Subst}\left(\int \frac{1}{5-x^2}\,dx, x, \sqrt{2+3x}\right) + \text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{2+3x}\right)$$
$$= -\tan^{-1}\left(\sqrt{2+3x}\right) + \sqrt{5}\tanh^{-1}\left(\frac{\sqrt{2+3x}}{\sqrt{5}}\right)$$

Mathematica [A] time = 0.0151169, size = 35, normalized size = 1.

$$\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{3x+2}}{\sqrt{5}}\right) - \tan^{-1}\left(\sqrt{3x+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 3*x]/(1 - x^2),x]
```

```
[Out] -ArcTan[Sqrt[2 + 3*x]] + Sqrt[5]*ArcTanh[Sqrt[2 + 3*x]/Sqrt[5]]
```

Maple [A] time = 0.047, size = 29, normalized size = 0.8

$$-\arctan\left(\sqrt{2+3\,x}\right) + \text{Artanh}\left(\frac{\sqrt{5}}{5}\sqrt{2+3\,x}\right)\sqrt{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^(1/2)/(-x^2+1),x)
```

```
[Out] -arctan((2+3*x)^(1/2))+arctanh(1/5*(2+3*x)^(1/2)*5^(1/2))*5^(1/2)
```

Maxima [A] time = 1.65505, size = 61, normalized size = 1.74

$$-\frac{1}{2}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{3\,x+2}}{\sqrt{5}+\sqrt{3\,x+2}}\right) - \arctan\left(\sqrt{3\,x+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^(1/2)/(-x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(5)*log(-(sqrt(5) - sqrt(3*x + 2))/(sqrt(5) + sqrt(3*x + 2))) - ar
ctan(sqrt(3*x + 2))
```

Fricas [A] time = 2.32939, size = 116, normalized size = 3.31

$$\frac{1}{2}\sqrt{5}\log\left(\frac{2\,\sqrt{5}\sqrt{3\,x+2}+3\,x+7}{x-1}\right) - \arctan\left(\sqrt{3\,x+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^(1/2)/(-x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(5)*log((2*sqrt(5)*sqrt(3*x + 2) + 3*x + 7)/(x - 1)) - arctan(sqrt(
3*x + 2))
```

Sympy [A] time = 3.09664, size = 70, normalized size = 2.

$$-5\left(\begin{cases} -\frac{\sqrt{5}\operatorname{acoth}\left(\frac{\sqrt{5}\sqrt{3x+2}}{5}\right)}{5} & \text{for } 3x+2>5 \\ -\frac{\sqrt{5}\operatorname{atanh}\left(\frac{\sqrt{5}\sqrt{3x+2}}{5}\right)}{5} & \text{for } 3x+2<5 \end{cases}\right) - \operatorname{atan}\left(\sqrt{3x+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**(1/2)/(-x**2+1),x)
```

```
[Out] -5*Piecewise((-sqrt(5)*acoth(sqrt(5)*sqrt(3*x + 2)/5)/5, 3*x + 2 > 5), (-sq
rt(5)*atanh(sqrt(5)*sqrt(3*x + 2)/5)/5, 3*x + 2 < 5)) - atan(sqrt(3*x + 2))
```

Giac [A] time = 1.32168, size = 65, normalized size = 1.86

$$-\frac{1}{2}\sqrt{5}\log\left(\frac{\left|-2\sqrt{5}+2\sqrt{3x+2}\right|}{2\left(\sqrt{5}+\sqrt{3x+2}\right)}\right) - \arctan\left(\sqrt{3x+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^(1/2)/(-x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(5)*log(1/2*abs(-2*sqrt(5) + 2*sqrt(3*x + 2))/(sqrt(5) + sqrt(3*x
+ 2))) - arctan(sqrt(3*x + 2))
```

3.651 $\int \frac{\sqrt{c+dx}}{1-x^2} dx$

Optimal. Leaf size=58

$$\sqrt{c+d}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c+d}}\right)-\sqrt{c-d}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c-d}}\right)$$

```
[Out] -(Sqrt[c - d]*ArcTanh[Sqrt[c + d*x]/Sqrt[c - d]]) + Sqrt[c + d]*ArcTanh[Sqr
t[c + d*x]/Sqrt[c + d]]
```

Rubi [A] time = 0.0580653, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {700, 1130, 206}

$$\sqrt{c+d}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c+d}}\right)-\sqrt{c-d}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c-d}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(1 - x^2),x]
```

```
[Out] -(Sqrt[c - d]*ArcTanh[Sqrt[c + d*x]/Sqrt[c - d]]) + Sqrt[c + d]*ArcTanh[Sqr
t[c + d*x]/Sqrt[c + d]]
```

Rule 700

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx}}{1-x^2} dx &= (2d)\,\text{Subst}\left(\int \frac{x^2}{-c^2+d^2+2cx^2-x^4} dx, x, \sqrt{c+dx}\right)\\
&= -\left((c-d)\,\text{Subst}\left(\int \frac{1}{c-d-x^2} dx, x, \sqrt{c+dx}\right)\right)+(c+d)\,\text{Subst}\left(\int \frac{1}{c+d-x^2} dx, x, \sqrt{c+dx}\right)\\
&= -\sqrt{c-d}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c-d}}\right)+\sqrt{c+d}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c+d}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0346304, size = 58, normalized size = 1.

$$\sqrt{c+d}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c+d}}\right)-\sqrt{c-d}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c-d}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(1 - x^2),x]
```

```
[Out] -(Sqrt[c - d]*ArcTanh[Sqrt[c + d*x]/Sqrt[c - d]]) + Sqrt[c + d]*ArcTanh[Sqr
t[c + d*x]/Sqrt[c + d]]
```

Maple [A] time = 0.131, size = 47, normalized size = 0.8

$$\operatorname{Artanh}\left(\sqrt{dx+c}\frac{1}{\sqrt{c+d}}\right)\sqrt{c+d}-\sqrt{-c+d}\arctan\left(\sqrt{dx+c}\frac{1}{\sqrt{-c+d}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(-x^2+1),x)
```

```
[Out] arctanh((d*x+c)^(1/2)/(c+d)^(1/2))*(c+d)^(1/2)-(-c+d)^(1/2)*arctan((d*x+c)^
(1/2)/(-c+d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(-x^2+1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.40727, size = 741, normalized size = 12.78

$$\left[\frac{1}{2}\sqrt{c-d}\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c-d}+2c-d}{x+1}\right)+\frac{1}{2}\sqrt{c+d}\log\left(\frac{dx+2\sqrt{dx+c}\sqrt{c+d}+2c+d}{x-1}\right),-\sqrt{-c+d}\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(-x^2+1),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(c - d)*log((d*x - 2*sqrt(d*x + c)*sqrt(c - d) + 2*c - d)/(x + 1))
 + 1/2*sqrt(c + d)*log((d*x + 2*sqrt(d*x + c)*sqrt(c + d) + 2*c + d)/(x - 1
)), -sqrt(-c + d)*arctan(-sqrt(d*x + c)*sqrt(-c + d)/(c - d)) + 1/2*sqrt(c
+ d)*log((d*x + 2*sqrt(d*x + c)*sqrt(c + d) + 2*c + d)/(x - 1)), -sqrt(-c -
 d)*arctan(sqrt(d*x + c)*sqrt(-c - d)/(c + d)) + 1/2*sqrt(c - d)*log((d*x -
 2*sqrt(d*x + c)*sqrt(c - d) + 2*c - d)/(x + 1)), -sqrt(-c + d)*arctan(-sqr
t(d*x + c)*sqrt(-c + d)/(c - d)) - sqrt(-c - d)*arctan(sqrt(d*x + c)*sqrt(-
```

```
c - d)/(c + d))]
```

Sympy [A] time = 3.65656, size = 61, normalized size = 1.05

$$\frac{2\left(\frac{d(c-d)\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c+d}}\right)}{2\sqrt{-c+d}}-\frac{d(c+d)\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c-d}}\right)}{2\sqrt{-c-d}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(-x**2+1),x)
```

```
[Out] 2*(d*(c - d)*atan(sqrt(c + d*x)/sqrt(-c + d))/(2*sqrt(-c + d)) - d*(c + d)*
atan(sqrt(c + d*x)/sqrt(-c - d))/(2*sqrt(-c - d)))/d
```

Giac [A] time = 1.36741, size = 84, normalized size = 1.45

$$\frac{(c-d)\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c+d}}\right)}{\sqrt{-c+d}}-\frac{(c+d)\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c-d}}\right)}{\sqrt{-c-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(-x^2+1),x, algorithm="giac")
```

```
[Out] (c - d)*arctan(sqrt(d*x + c)/sqrt(-c + d))/sqrt(-c + d) - (c + d)*arctan(sq
rt(d*x + c)/sqrt(-c - d))/sqrt(-c - d)
```

3.652 $\int \frac{\sqrt{2+3x}}{a+bx^2}\,dx$

Optimal. Leaf size=427

$$\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt{3x+2}\sqrt{\sqrt{9a+4b}+2\sqrt{b}}+\sqrt{9a+4b}+\sqrt{b}(3x+2)\right)}{2\sqrt{2}b^{3/4}\sqrt{\sqrt{9a+4b}+2\sqrt{b}}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt{3x+2}\sqrt{\sqrt{9a+4b}+2\sqrt{b}}+\sqrt{9\right.}}{2\sqrt{2}b^{3/4}\sqrt{\sqrt{9a+4b}+2\sqrt{}}}$$

```
[Out] (3*ArcTanh[(Sqrt[2*Sqrt[b] + Sqrt[9*a + 4*b]] - Sqrt[2]*b^(1/4)*Sqrt[2 + 3*
x])/Sqrt[2*Sqrt[b] - Sqrt[9*a + 4*b]]])/(Sqrt[2]*b^(3/4)*Sqrt[2*Sqrt[b] - S
qrt[9*a + 4*b]]) - (3*ArcTanh[(Sqrt[2*Sqrt[b] + Sqrt[9*a + 4*b]] + Sqrt[2]*
b^(1/4)*Sqrt[2 + 3*x])/Sqrt[2*Sqrt[b] - Sqrt[9*a + 4*b]]])/(Sqrt[2]*b^(3/4)
*Sqrt[2*Sqrt[b] - Sqrt[9*a + 4*b]]) + (3*Log[Sqrt[9*a + 4*b] - Sqrt[2]*b^(1
/4)*Sqrt[2*Sqrt[b] + Sqrt[9*a + 4*b]]*Sqrt[2 + 3*x] + Sqrt[b]*(2 + 3*x)])/(
2*Sqrt[2]*b^(3/4)*Sqrt[2*Sqrt[b] + Sqrt[9*a + 4*b]]) - (3*Log[Sqrt[9*a + 4*
b] + Sqrt[2]*b^(1/4)*Sqrt[2*Sqrt[b] + Sqrt[9*a + 4*b]]*Sqrt[2 + 3*x] + Sqrt
[b]*(2 + 3*x)])/(2*Sqrt[2]*b^(3/4)*Sqrt[2*Sqrt[b] + Sqrt[9*a + 4*b]])
```

Rubi [A] time = 0.530846, antiderivative size = 427, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {700, 1129, 634, 618, 206, 628}

$$\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt{3x+2}\sqrt{\sqrt{9a+4b}+2\sqrt{b}}+\sqrt{9a+4b}+\sqrt{b}(3x+2)\right)}{2\sqrt{2}b^{3/4}\sqrt{\sqrt{9a+4b}+2\sqrt{b}}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt{3x+2}\sqrt{\sqrt{9a+4b}+2\sqrt{b}}+\sqrt{9\right.}}{2\sqrt{2}b^{3/4}\sqrt{\sqrt{9a+4b}+2\sqrt{}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 3*x]/(a + b*x^2),x]
```

```
[Out] (3*ArcTanh[(Sqrt[2*Sqrt[b] + Sqrt[9*a + 4*b]] - Sqrt[2]*b^(1/4)*Sqrt[2 + 3*
x])/Sqrt[2*Sqrt[b] - Sqrt[9*a + 4*b]]])/(Sqrt[2]*b^(3/4)*Sqrt[2*Sqrt[b] - S
qrt[9*a + 4*b]]) - (3*ArcTanh[(Sqrt[2*Sqrt[b] + Sqrt[9*a + 4*b]] + Sqrt[2]*
b^(1/4)*Sqrt[2 + 3*x])/Sqrt[2*Sqrt[b] - Sqrt[9*a + 4*b]]])/(Sqrt[2]*b^(3/4)
*Sqrt[2*Sqrt[b] - Sqrt[9*a + 4*b]]) + (3*Log[Sqrt[9*a + 4*b] - Sqrt[2]*b^(1
/4)*Sqrt[2*Sqrt[b] + Sqrt[9*a + 4*b]]*Sqrt[2 + 3*x] + Sqrt[b]*(2 + 3*x)])/(
2*Sqrt[2]*b^(3/4)*Sqrt[2*Sqrt[b] + Sqrt[9*a + 4*b]]) - (3*Log[Sqrt[9*a + 4*
b] + Sqrt[2]*b^(1/4)*Sqrt[2*Sqrt[b] + Sqrt[9*a + 4*b]]*Sqrt[2 + 3*x] + Sqrt
[b]*(2 + 3*x)])/(2*Sqrt[2]*b^(3/4)*Sqrt[2*Sqrt[b] + Sqrt[9*a + 4*b]])
```

Rule 700

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1129

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q =
 Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m - 1)/(q
 - r*x + x^2), x], x] - Dist[1/(2*c*r), Int[x^(m - 1)/(q + r*x + x^2), x],
x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 1] && LtQ[m, 3
] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{2+3x}}{a+bx^2}\,dx = 6\,\text{Subst}\left(\int \frac{x^2}{9a+4b-4bx^2+bx^4}\,dx, x, \sqrt{2+3x}\right)$$

$$= \frac{3\,\text{Subst}\left(\int \frac{x}{\frac{\sqrt{9a+4b}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{2\sqrt{b}+\sqrt{9a+4b}}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{2+3x}\right)}{\sqrt{2}b^{3/4}\sqrt{2\sqrt{b}+\sqrt{9a+4b}}} - \frac{3\,\text{Subst}\left(\int \frac{x}{\frac{\sqrt{9a+4b}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt{2\sqrt{b}+\sqrt{9a+4b}}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{2+3x}\right)}{\sqrt{2}b^{3/4}\sqrt{2\sqrt{b}+\sqrt{9a+4b}}}$$

$$= \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{9a+4b}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{2\sqrt{b}+\sqrt{9a+4b}}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{2+3x}\right)}{2b} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{9a+4b}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt{2\sqrt{b}+\sqrt{9a+4b}}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{2+3x}\right)}{2b}$$

$$= \frac{3\log\left(\sqrt{9a+4b}-\sqrt{2}\sqrt[4]{b}\sqrt{2\sqrt{b}+\sqrt{9a+4b}}\sqrt{2+3x}+\sqrt{b}(2+3x)\right)}{2\sqrt{2}b^{3/4}\sqrt{2\sqrt{b}+\sqrt{9a+4b}}} - \frac{3\log\left(\sqrt{9a+4b}+\sqrt{2}\sqrt[4]{b}\sqrt{2\sqrt{b}\cdots}\right.}{2\sqrt{2}b^{3/4}\sqrt{2\cdots}}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt[4]{b}\left(\frac{\sqrt{2\sqrt{b}+\sqrt{9a+4b}}}{\sqrt[4]{b}}-\sqrt{2}\sqrt{2+3x}\right)}{\sqrt{2\sqrt{b}-\sqrt{9a+4b}}}\right)}{\sqrt{2}b^{3/4}\sqrt{2\sqrt{b}-\sqrt{9a+4b}}} - \frac{3\tanh^{-1}\left(\frac{\sqrt[4]{b}\left(\frac{\sqrt{2\sqrt{b}+\sqrt{9a+4b}}}{\sqrt[4]{b}}+\sqrt{2}\sqrt{2+3x}\right)}{\sqrt{2\sqrt{b}-\sqrt{9a+4b}}}\right)}{\sqrt{2}b^{3/4}\sqrt{2\sqrt{b}-\sqrt{9a+4b}}} + \frac{3\log\left(\sqrt{9a+4b}-\sqrt{2}\sqrt[4]{b}\cdots\right.}{2\sqrt{2}b\cdots}$$

Mathematica [A] time = 0.139488, size = 133, normalized size = 0.31

$$\frac{\sqrt{3\sqrt{-a}-2\sqrt{b}}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{3x+2}}{\sqrt{3\sqrt{-a}-2\sqrt{b}}}\right)-\sqrt{3\sqrt{-a}+2\sqrt{b}}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{3x+2}}{\sqrt{3\sqrt{-a}+2\sqrt{b}}}\right)}{\sqrt{-a}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 3*x]/(a + b*x^2),x]
```

```
[Out] (Sqrt[3*Sqrt[-a] - 2*Sqrt[b]]*ArcTan[(b^(1/4)*Sqrt[2 + 3*x])/Sqrt[3*Sqrt[-a
] - 2*Sqrt[b]]] - Sqrt[3*Sqrt[-a] + 2*Sqrt[b]]*ArcTanh[(b^(1/4)*Sqrt[2 + 3*
x])/Sqrt[3*Sqrt[-a] + 2*Sqrt[b]]])/(Sqrt[-a]*b^(3/4))
```

Maple [B] time = 0.227, size = 944, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^(1/2)/(b*x^2+a),x)
```

```
[Out] 1/12*(2*(9*a*b+4*b^2)^(1/2)+4*b)^(1/2)/a/b^(3/2)*(9*a*b+4*b^2)^(1/2)*ln(-(2
+3*x)*b^(1/2)+(2+3*x)^(1/2)*(2*(b*(9*a+4*b))^(1/2)+4*b)^(1/2)-(9*a+4*b)^(1/
2))-1/6*(2*(9*a*b+4*b^2)^(1/2)+4*b)^(1/2)/a/b^(3/2)*(9*a*b+4*b^2)^(1/2)*(2*
(b*(9*a+4*b))^(1/2)+4*b)^(1/2)/(4*(9*a+4*b)^(1/2)*b^(1/2)-2*(b*(9*a+4*b))^(
1/2)-4*b)^(1/2)*arctan((-2*b^(1/2)*(2+3*x)^(1/2)+(2*(b*(9*a+4*b))^(1/2)+4*b
)^(1/2))/(4*(9*a+4*b)^(1/2)*b^(1/2)-2*(b*(9*a+4*b))^(1/2)-4*b)^(1/2))-1/6*(
2*(9*a*b+4*b^2)^(1/2)+4*b)^(1/2)/a/b^(1/2)*ln(-(2+3*x)*b^(1/2)+(2+3*x)^(1/2
)*(2*(b*(9*a+4*b))^(1/2)+4*b)^(1/2)-(9*a+4*b)^(1/2))+1/3*(2*(9*a*b+4*b^2)^(
1/2)+4*b)^(1/2)/a/b^(1/2)*(2*(b*(9*a+4*b))^(1/2)+4*b)^(1/2)/(4*(9*a+4*b)^(1
/2)*b^(1/2)-2*(b*(9*a+4*b))^(1/2)-4*b)^(1/2)*arctan((-2*b^(1/2)*(2+3*x)^(1/
2)+(2*(b*(9*a+4*b))^(1/2)+4*b)^(1/2))/(4*(9*a+4*b)^(1/2)*b^(1/2)-2*(b*(9*a+
4*b))^(1/2)-4*b)^(1/2))-1/12*(2*(9*a*b+4*b^2)^(1/2)+4*b)^(1/2)/a/b^(3/2)*(9
*a*b+4*b^2)^(1/2)*ln((2+3*x)*b^(1/2)+(2+3*x)^(1/2)*(2*(b*(9*a+4*b))^(1/2)+4
*b)^(1/2)+(9*a+4*b)^(1/2))+1/6*(2*(9*a*b+4*b^2)^(1/2)+4*b)^(1/2)/a/b^(3/2)*
(9*a*b+4*b^2)^(1/2)*(2*(b*(9*a+4*b))^(1/2)+4*b)^(1/2)/(4*(9*a+4*b)^(1/2)*b^
(1/2)-2*(b*(9*a+4*b))^(1/2)-4*b)^(1/2)*arctan((2*b^(1/2)*(2+3*x)^(1/2)+(2*(
b*(9*a+4*b))^(1/2)+4*b)^(1/2))/(4*(9*a+4*b)^(1/2)*b^(1/2)-2*(b*(9*a+4*b))^(
1/2)-4*b)^(1/2))+1/6*(2*(9*a*b+4*b^2)^(1/2)+4*b)^(1/2)/a/b^(1/2)*ln((2+3*x)
*b^(1/2)+(2+3*x)^(1/2)*(2*(b*(9*a+4*b))^(1/2)+4*b)^(1/2)+(9*a+4*b)^(1/2))-1
/3*(2*(9*a*b+4*b^2)^(1/2)+4*b)^(1/2)/a/b^(1/2)*(2*(b*(9*a+4*b))^(1/2)+4*b)^
(1/2)/(4*(9*a+4*b)^(1/2)*b^(1/2)-2*(b*(9*a+4*b))^(1/2)-4*b)^(1/2)*arctan((2
*b^(1/2)*(2+3*x)^(1/2)+(2*(b*(9*a+4*b))^(1/2)+4*b)^(1/2))/(4*(9*a+4*b)^(1/2
)*b^(1/2)-2*(b*(9*a+4*b))^(1/2)-4*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{3x+2}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*x + 2)/(b*x^2 + a), x)
```

Fricas [A] time = 2.23057, size = 721, normalized size = 1.69

$$-\frac{1}{2}\sqrt{-\frac{3\,ab\sqrt{-\frac{1}{ab^3}}+2}{ab}}\log\left(ab^2\sqrt{-\frac{3\,ab\sqrt{-\frac{1}{ab^3}}+2}{ab}}\sqrt{-\frac{1}{ab^3}}+\sqrt{3\,x+2}\right)+\frac{1}{2}\sqrt{-\frac{3\,ab\sqrt{-\frac{1}{ab^3}}+2}{ab}}\log\left(-ab^2\sqrt{-\frac{3\,ab\sqrt{-\frac{1}{ab^3}}}{ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-(3*a*b*sqrt(-1/(a*b^3)) + 2)/(a*b))*log(a*b^2*sqrt(-(3*a*b*sqrt(
-1/(a*b^3)) + 2)/(a*b))*sqrt(-1/(a*b^3)) + sqrt(3*x + 2)) + 1/2*sqrt(-(3*a*
b*sqrt(-1/(a*b^3)) + 2)/(a*b))*log(-a*b^2*sqrt(-(3*a*b*sqrt(-1/(a*b^3)) + 2
)/(a*b))*sqrt(-1/(a*b^3)) + sqrt(3*x + 2)) + 1/2*sqrt((3*a*b*sqrt(-1/(a*b^3
)) - 2)/(a*b))*log(a*b^2*sqrt((3*a*b*sqrt(-1/(a*b^3)) - 2)/(a*b))*sqrt(-1/(
a*b^3)) + sqrt(3*x + 2)) - 1/2*sqrt((3*a*b*sqrt(-1/(a*b^3)) - 2)/(a*b))*log
(-a*b^2*sqrt((3*a*b*sqrt(-1/(a*b^3)) - 2)/(a*b))*sqrt(-1/(a*b^3)) + sqrt(3*
x + 2))
```

Sympy [A] time = 4.2523, size = 56, normalized size = 0.13

$$6\,\mathrm{RootSum}\left(20736t^4a^2b^3+576t^2ab^2+9a+4b,\left(t\mapsto t\log\left(576t^3ab^2+8tb+\sqrt{3x+2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**(1/2)/(b*x**2+a),x)
```

```
[Out] 6*RootSum(20736*_t**4*a**2*b**3 + 576*_t**2*a*b**2 + 9*a + 4*b, Lambda(_t,
_t*log(576*_t**3*a*b**2 + 8*_t*b + sqrt(3*x + 2))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.653 $\int \frac{\sqrt{2+3x}}{a-bx^2} dx$

Optimal. Leaf size=132

$$\frac{\sqrt{3\sqrt{a}+2\sqrt{b}}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{3x+2}}{\sqrt{3\sqrt{a}+2\sqrt{b}}}\right)}{\sqrt{a}b^{3/4}}-\frac{\sqrt{3\sqrt{a}-2\sqrt{b}}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{3x+2}}{\sqrt{3\sqrt{a}-2\sqrt{b}}}\right)}{\sqrt{a}b^{3/4}}$$

```
[Out] -((Sqrt[3*Sqrt[a] - 2*Sqrt[b]]*ArcTan[(b^(1/4)*Sqrt[2 + 3*x])/Sqrt[3*Sqrt[a
] - 2*Sqrt[b]]])/(Sqrt[a]*b^(3/4))) + (Sqrt[3*Sqrt[a] + 2*Sqrt[b]]*ArcTanh[
(b^(1/4)*Sqrt[2 + 3*x])/Sqrt[3*Sqrt[a] + 2*Sqrt[b]]])/(Sqrt[a]*b^(3/4))
```

Rubi [A] time = 0.132167, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {700, 1130, 208, 205}

$$\frac{\sqrt{3\sqrt{a}+2\sqrt{b}}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{3x+2}}{\sqrt{3\sqrt{a}+2\sqrt{b}}}\right)}{\sqrt{a}b^{3/4}}-\frac{\sqrt{3\sqrt{a}-2\sqrt{b}}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{3x+2}}{\sqrt{3\sqrt{a}-2\sqrt{b}}}\right)}{\sqrt{a}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 3*x]/(a - b*x^2),x]
```

```
[Out] -((Sqrt[3*Sqrt[a] - 2*Sqrt[b]]*ArcTan[(b^(1/4)*Sqrt[2 + 3*x])/Sqrt[3*Sqrt[a
] - 2*Sqrt[b]]])/(Sqrt[a]*b^(3/4))) + (Sqrt[3*Sqrt[a] + 2*Sqrt[b]]*ArcTanh[
(b^(1/4)*Sqrt[2 + 3*x])/Sqrt[3*Sqrt[a] + 2*Sqrt[b]]])/(Sqrt[a]*b^(3/4))
```

Rule 700

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{2+3x}}{a-bx^2}\,dx = 6\,\text{Subst}\left(\int \frac{x^2}{9a-4b+4bx^2-bx^4}\,dx, x, \sqrt{2+3x}\right)$$
$$= \left(3-\frac{2\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{-3\sqrt{a}\sqrt{b}+2b-bx^2}\,dx, x, \sqrt{2+3x}\right)+\left(3+\frac{2\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{3\sqrt{a}\sqrt{b}+2b-bx^2}\right.$$
$$= -\frac{\sqrt{3\sqrt{a}-2\sqrt{b}}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{2+3x}}{\sqrt{3\sqrt{a}-2\sqrt{b}}}\right)}{\sqrt{a}b^{3/4}}+\frac{\sqrt{3\sqrt{a}+2\sqrt{b}}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{2+3x}}{\sqrt{3\sqrt{a}+2\sqrt{b}}}\right)}{\sqrt{a}b^{3/4}}$$

Mathematica [A] time = 0.0932695, size = 123, normalized size = 0.93

$$\frac{\sqrt{3\sqrt{a}+2\sqrt{b}}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{3x+2}}{\sqrt{3\sqrt{a}+2\sqrt{b}}}\right)-\sqrt{3\sqrt{a}-2\sqrt{b}}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{3x+2}}{\sqrt{3\sqrt{a}-2\sqrt{b}}}\right)}{\sqrt{a}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 3*x]/(a - b*x^2),x]
```

```
[Out] (-(Sqrt[3*Sqrt[a] - 2*Sqrt[b]]*ArcTan[(b^(1/4)*Sqrt[2 + 3*x])/Sqrt[3*Sqrt[a
] - 2*Sqrt[b]]]) + Sqrt[3*Sqrt[a] + 2*Sqrt[b]]*ArcTanh[(b^(1/4)*Sqrt[2 + 3*
x])/Sqrt[3*Sqrt[a] + 2*Sqrt[b]]])/(Sqrt[a]*b^(3/4))
```

Maple [A] time = 0.158, size = 182, normalized size = 1.4

$$3\frac{1}{\sqrt{\left(3\sqrt{ab}+2b\right)b}}\text{Artanh}\left(\frac{b\sqrt{2+3x}}{\sqrt{\left(3\sqrt{ab}+2b\right)b}}\right)+2\frac{b}{\sqrt{ab}\sqrt{\left(3\sqrt{ab}+2b\right)b}}\text{Artanh}\left(\frac{b\sqrt{2+3x}}{\sqrt{\left(3\sqrt{ab}+2b\right)b}}\right)-3\frac{1}{\sqrt{\left(3\sqrt{ab}-2\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^(1/2)/(-b*x^2+a),x)
```

```
[Out] 3/((3*(a*b)^(1/2)+2*b)*b)^(1/2)*arctanh(b*(2+3*x)^(1/2)/((3*(a*b)^(1/2)+2*b
)*b)^(1/2))+2*b/(a*b)^(1/2)/((3*(a*b)^(1/2)+2*b)*b)^(1/2)*arctanh(b*(2+3*x)
^(1/2)/((3*(a*b)^(1/2)+2*b)*b)^(1/2))-3/((3*(a*b)^(1/2)-2*b)*b)^(1/2)*arcta
n(b*(2+3*x)^(1/2)/((3*(a*b)^(1/2)-2*b)*b)^(1/2))+2*b/(a*b)^(1/2)/((3*(a*b)^
(1/2)-2*b)*b)^(1/2)*arctan(b*(2+3*x)^(1/2)/((3*(a*b)^(1/2)-2*b)*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{3x+2}}{bx^2-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^(1/2)/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(3*x + 2)/(b*x^2 - a), x)
```

Fricas [B] time = 2.2393, size = 703, normalized size = 5.33

$$\frac{1}{2}\sqrt{\frac{3\,ab\sqrt{\frac{1}{ab^3}}+2}{ab}}\log\left(ab^2\sqrt{\frac{3\,ab\sqrt{\frac{1}{ab^3}}+2}{ab}}\sqrt{\frac{1}{ab^3}}+\sqrt{3\,x+2}\right)-\frac{1}{2}\sqrt{\frac{3\,ab\sqrt{\frac{1}{ab^3}}+2}{ab}}\log\left(-ab^2\sqrt{\frac{3\,ab\sqrt{\frac{1}{ab^3}}+2}{ab}}\sqrt{\frac{1}{ab^3}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^(1/2)/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt((3*a*b*sqrt(1/(a*b^3)) + 2)/(a*b))*log(a*b^2*sqrt((3*a*b*sqrt(1/(a
*b^3)) + 2)/(a*b))*sqrt(1/(a*b^3)) + sqrt(3*x + 2)) - 1/2*sqrt((3*a*b*sqrt(
1/(a*b^3)) + 2)/(a*b))*log(-a*b^2*sqrt((3*a*b*sqrt(1/(a*b^3)) + 2)/(a*b))*s
qrt(1/(a*b^3)) + sqrt(3*x + 2)) - 1/2*sqrt(-(3*a*b*sqrt(1/(a*b^3)) - 2)/(a*
b))*log(a*b^2*sqrt(-(3*a*b*sqrt(1/(a*b^3)) - 2)/(a*b))*sqrt(1/(a*b^3)) + sq
rt(3*x + 2)) + 1/2*sqrt(-(3*a*b*sqrt(1/(a*b^3)) - 2)/(a*b))*log(-a*b^2*sqrt
(-(3*a*b*sqrt(1/(a*b^3)) - 2)/(a*b))*sqrt(1/(a*b^3)) + sqrt(3*x + 2))
```

Sympy [A] time = 4.56618, size = 58, normalized size = 0.44

$$-6\operatorname{RootSum}\left(20736t^4a^2b^3-576t^2ab^2-9a+4b,\left(t\mapsto t\log\left(-576t^3ab^2+8tb+\sqrt{3x+2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**(1/2)/(-b*x**2+a),x)
```

```
[Out] -6*RootSum(20736*_t**4*a**2*b**3 - 576*_t**2*a*b**2 - 9*a + 4*b, Lambda(_t,
 _t*log(-576*_t**3*a*b**2 + 8*_t*b + sqrt(3*x + 2))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^(1/2)/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.654 $\int \frac{\sqrt{1+x}}{1+x^2} \, dx$

Optimal. Leaf size=205

$$\frac{\log\left(x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}-\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}\cdots}{\sqrt{2\left(\sqrt{2}\cdots}}\right.$$

```
[Out] -(Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + x])/Sqrt
[2*(-1 + Sqrt[2])]]) + Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])]
+ 2*Sqrt[1 + x])/Sqrt[2*(-1 + Sqrt[2])]] + Log[1 + Sqrt[2] + x - Sqrt[2*(1
+ Sqrt[2])]*Sqrt[1 + x]]/(2*Sqrt[2*(1 + Sqrt[2])]) - Log[1 + Sqrt[2] + x +
Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + x]]/(2*Sqrt[2*(1 + Sqrt[2])])
```

Rubi [A] time = 0.223013, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {700, 1127, 1161, 618, 204, 1164, 628}

$$\frac{\log\left(x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}-\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}\cdots}{\sqrt{2\left(\sqrt{2}\cdots}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/(1 + x^2),x]
```

```
[Out] -(Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + x])/Sqrt
[2*(-1 + Sqrt[2])]]) + Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])]
+ 2*Sqrt[1 + x])/Sqrt[2*(-1 + Sqrt[2])]] + Log[1 + Sqrt[2] + x - Sqrt[2*(1
+ Sqrt[2])]*Sqrt[1 + x]]/(2*Sqrt[2*(1 + Sqrt[2])]) - Log[1 + Sqrt[2] + x +
Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + x]]/(2*Sqrt[2*(1 + Sqrt[2])])
```

Rule 700

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+x}}{1+x^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^2}{2-2x^2+x^4}\,dx, x, \sqrt{1+x}\right)$$
$$= -\,\text{Subst}\left(\int \frac{\sqrt{2}-x^2}{2-2x^2+x^4}\,dx, x, \sqrt{1+x}\right) + \text{Subst}\left(\int \frac{\sqrt{2}+x^2}{2-2x^2+x^4}\,dx, x, \sqrt{1+x}\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+x}\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x,$$
$$= \frac{\log\left(1+\sqrt{2}+x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+x}\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}} - \frac{\log\left(1+\sqrt{2}+x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+x}\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}} - \text{Subst}\left(\int \frac{}{2(1}\right.$$
$$= \frac{\tan^{-1}\left(\frac{-\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{\sqrt{2\left(-1+\sqrt{2}\right)}} + \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{\sqrt{2\left(-1+\sqrt{2}\right)}} + \frac{\log\left(1+\sqrt{2}+x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+x}\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}} - \text{log}$$

Mathematica [C] time = 0.025796, size = 55, normalized size = 0.27

$$i\sqrt{1+i}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1+i}}\right) - i\sqrt{1-i}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1-i}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 + x^2),x]
```

```
[Out] (-I)*Sqrt[1 - I]*ArcTanh[Sqrt[1 + x]/Sqrt[1 - I]] + I*Sqrt[1 + I]*ArcTanh[S
qrt[1 + x]/Sqrt[1 + I]]
```

Maple [B] time = 0.107, size = 336, normalized size = 1.6

$$-\frac{\sqrt{2+2\sqrt{2}}\sqrt{2}}{4}\ln\left(1+x+\sqrt{2}+\sqrt{1+x}\sqrt{2+2\sqrt{2}}\right)+\frac{\sqrt{2}\left(2+2\sqrt{2}\right)}{2\sqrt{-2+2\sqrt{2}}}\arctan\left(\frac{1}{\sqrt{-2+2\sqrt{2}}}\left(2\sqrt{1+x}+\sqrt{2+2\sqrt{2}}\right)\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(x^2+1),x)
```

```
[Out] -1/4*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(1+x+2^(1/2)+(1+x)^(1/2)*(2+2*2^(1/2))^(
1/2))+1/2*2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+x)^(1/2)+
(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))+1/4*(2+2*2^(1/2))^(1/2)*ln(1+x+2
^(1/2)+(1+x)^(1/2)*(2+2*2^(1/2))^(1/2))-1/2*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1
/2)*arctan((2*(1+x)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))+1/4*(2
+2*2^(1/2))^(1/2)*2^(1/2)*ln(1+x+2^(1/2)-(1+x)^(1/2)*(2+2*2^(1/2))^(1/2))+1
/2*2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+x)^(1/2)-(2+2*2^
(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))-1/4*(2+2*2^(1/2))^(1/2)*ln(1+x+2^(1/2)-
(1+x)^(1/2)*(2+2*2^(1/2))^(1/2))-1/2*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arc
tan((2*(1+x)^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+1}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + 1)/(x^2 + 1), x)
```

Fricas [A] time = 2.31457, size = 860, normalized size = 4.2

$$\frac{1}{8}\cdot 2^{\frac{1}{4}}\sqrt{2\sqrt{2}+4}\left(\sqrt{2}-2\right)\log\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}\sqrt{x+1}\sqrt{2\sqrt{2}+4}+x+\sqrt{2}+1\right)-\frac{1}{8}\cdot 2^{\frac{1}{4}}\sqrt{2\sqrt{2}+4}\left(\sqrt{2}-2\right)\log\left(-\frac{1}{2}\cdot 2^{\frac{3}{4}}\sqrt{x+1}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/8*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)*log(1/2*2^(3/4)*sqrt(x + 1)*s
qrt(2*sqrt(2) + 4) + x + sqrt(2) + 1) - 1/8*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sq
rt(2) - 2)*log(-1/2*2^(3/4)*sqrt(x + 1)*sqrt(2*sqrt(2) + 4) + x + sqrt(2) +
 1) - 1/2*2^(3/4)*sqrt(2*sqrt(2) + 4)*arctan(-1/2*2^(3/4)*sqrt(x + 1)*sqrt(
2*sqrt(2) + 4) + 1/2*2^(1/4)*sqrt(2^(3/4)*sqrt(x + 1)*sqrt(2*sqrt(2) + 4) +
 2*x + 2*sqrt(2) + 2)*sqrt(2*sqrt(2) + 4) - sqrt(2) - 1) - 1/2*2^(3/4)*sqrt
(2*sqrt(2) + 4)*arctan(-1/2*2^(3/4)*sqrt(x + 1)*sqrt(2*sqrt(2) + 4) + 1/2*2
^(1/4)*sqrt(-2^(3/4)*sqrt(x + 1)*sqrt(2*sqrt(2) + 4) + 2*x + 2*sqrt(2) + 2)
*sqrt(2*sqrt(2) + 4) + sqrt(2) + 1)
```

Sympy [A] time = 2.7738, size = 31, normalized size = 0.15

$$2\,\mathrm{RootSum}\left(128t^4+16t^2+1,\left(t\mapsto t\log\left(64t^3+4t+\sqrt{x+1}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(x**2+1),x)
```

```
[Out] 2*RootSum(128*_t**4 + 16*_t**2 + 1, Lambda(_t, _t*log(64*_t**3 + 4*_t + sqr
t(x + 1))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x+1}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x + 1)/(x^2 + 1), x)
```

3.655 $\int \frac{1}{\sqrt{1+x}(1+x^2)}\,dx$

Optimal. Leaf size=198

$$-\frac{\log\left(x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)}{4\sqrt{1+\sqrt{2}}}+\frac{\log\left(x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)}{4\sqrt{1+\sqrt{2}}}-\frac{1}{2}\sqrt{1+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-}{\sqrt{2\left(\sqrt{2}-\right.}}\right.$$

```
[Out] -(Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + x])/Sqrt[2*(
-1 + Sqrt[2])]])/2 + (Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*S
qrt[1 + x])/Sqrt[2*(-1 + Sqrt[2])]])/2 - Log[1 + Sqrt[2] + x - Sqrt[2*(1 +
Sqrt[2])]*Sqrt[1 + x]]/(4*Sqrt[1 + Sqrt[2]]) + Log[1 + Sqrt[2] + x + Sqrt[2
*(1 + Sqrt[2])]*Sqrt[1 + x]]/(4*Sqrt[1 + Sqrt[2]])
```

Rubi [A] time = 0.132041, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {708, 1094, 634, 618, 204, 628}

$$-\frac{\log\left(x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)}{4\sqrt{1+\sqrt{2}}}+\frac{\log\left(x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)}{4\sqrt{1+\sqrt{2}}}-\frac{1}{2}\sqrt{1+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-}{\sqrt{2\left(\sqrt{2}-\right.}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + x]*(1 + x^2)),x]
```

```
[Out] -(Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + x])/Sqrt[2*(
-1 + Sqrt[2])]])/2 + (Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*S
qrt[1 + x])/Sqrt[2*(-1 + Sqrt[2])]])/2 - Log[1 + Sqrt[2] + x - Sqrt[2*(1 +
Sqrt[2])]*Sqrt[1 + x]]/(4*Sqrt[1 + Sqrt[2]]) + Log[1 + Sqrt[2] + x + Sqrt[2
*(1 + Sqrt[2])]*Sqrt[1 + x]]/(4*Sqrt[1 + Sqrt[2]])
```

Rule 708

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x}\left(1+x^2\right)}\,dx = 2\,\text{Subst}\left(\int \frac{1}{2-2x^2+x^4}\,dx, x, \sqrt{1+x}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-x}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+x}\right)}{2\sqrt{1+\sqrt{2}}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}+x}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+x}\right)}{2\sqrt{1+\sqrt{2}}}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+x}\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+x}\right)}{2\sqrt{2}} - \text{Subs}$$

$$= -\frac{\log\left(1+\sqrt{2}+x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+x}\right)}{4\sqrt{1+\sqrt{2}}} + \frac{\log\left(1+\sqrt{2}+x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+x}\right)}{4\sqrt{1+\sqrt{2}}} - \text{Subs}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{-1+\sqrt{2}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{-1+\sqrt{2}}} - \frac{\log\left(1+\sqrt{2}+x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\right.}}{4\sqrt{1+\sqrt{2}}}$$

Mathematica [C] time = 0.0230761, size = 55, normalized size = 0.28

$$\frac{1}{2}(1-i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1-i}}\right)+\frac{1}{2}(1+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1+i}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + x]*(1 + x^2)),x]
```

```
[Out] ((1 - I)^(3/2)*ArcTanh[Sqrt[1 + x]/Sqrt[1 - I]])/2 + ((1 + I)^(3/2)*ArcTanh
[Sqrt[1 + x]/Sqrt[1 + I]])/2
```

Maple [B] time = 0.058, size = 420, normalized size = 2.1

$$\frac{\sqrt{2+2\sqrt{2}}}{4}\ln\left(1+x+\sqrt{2}+\sqrt{1+x}\sqrt{2+2\sqrt{2}}\right)-\frac{\sqrt{2+2\sqrt{2}}\sqrt{2}}{8}\ln\left(1+x+\sqrt{2}+\sqrt{1+x}\sqrt{2+2\sqrt{2}}\right)+\frac{\left(2+2\sqrt{2}\right)}{4\sqrt{-2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)/(1+x)^(1/2),x)
```

```
[Out] 1/4*(2+2*2^(1/2))^(1/2)*ln(1+x+2^(1/2)+(1+x)^(1/2)*(2+2*2^(1/2))^(1/2))-1/8
*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(1+x+2^(1/2)+(1+x)^(1/2)*(2+2*2^(1/2))^(1/2)
)+1/4*2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+x)^(1/2)+(2+2
*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))-1/2*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/
2)*arctan((2*(1+x)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))+1/(-2+2
*2^(1/2))^(1/2)*arctan((2*(1+x)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(
1/2))*2^(1/2)-1/4*(2+2*2^(1/2))^(1/2)*ln(1+x+2^(1/2)-(1+x)^(1/2)*(2+2*2^(1/
2))^(1/2))+1/8*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(1+x+2^(1/2)-(1+x)^(1/2)*(2+2*
2^(1/2))^(1/2))+1/4*2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1
+x)^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))-1/2*(2+2*2^(1/2))/(-2+
2*2^(1/2))^(1/2)*arctan((2*(1+x)^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^
(1/2))+1/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+x)^(1/2)-(2+2*2^(1/2))^(1/2))/(-
2+2*2^(1/2))^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+1\right)\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 1)*sqrt(x + 1)), x)
```

Fricas [A] time = 2.29097, size = 868, normalized size = 4.38

$$\frac{1}{8}\cdot 2^{\frac{1}{4}}\sqrt{2\sqrt{2}+4}\left(\sqrt{2}-1\right)\log\left(2\cdot 2^{\frac{3}{4}}\sqrt{x+1}\sqrt{2\sqrt{2}+4}+4x+4\sqrt{2}+4\right)-\frac{1}{8}\cdot 2^{\frac{1}{4}}\sqrt{2\sqrt{2}+4}\left(\sqrt{2}-1\right)\log\left(-2\cdot 2^{\frac{3}{4}}\sqrt{x+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 1)*log(2*2^(3/4)*sqrt(x + 1)*sqr
t(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4) - 1/8*2^(1/4)*sqrt(2*sqrt(2) + 4)*(
sqrt(2) - 1)*log(-2*2^(3/4)*sqrt(x + 1)*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(
2) + 4) - 1/2*2^(1/4)*sqrt(2*sqrt(2) + 4)*arctan(1/4*2^(3/4)*sqrt(2*2^(3/4)
*sqrt(x + 1)*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4)*sqrt(2*sqrt(2) + 4)
 - 1/2*2^(3/4)*sqrt(x + 1)*sqrt(2*sqrt(2) + 4) - sqrt(2) - 1) - 1/2*2^(1/4)
*sqrt(2*sqrt(2) + 4)*arctan(1/4*2^(3/4)*sqrt(-2*2^(3/4)*sqrt(x + 1)*sqrt(2*
sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4)*sqrt(2*sqrt(2) + 4) - 1/2*2^(3/4)*sqrt(
x + 1)*sqrt(2*sqrt(2) + 4) + sqrt(2) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x+1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)/(1+x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x + 1)*(x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+1)\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 1)*sqrt(x + 1)), x)
```

3.656 $\int \frac{\sqrt{-1+x}}{(1+x^2)^3} \, dx$

Optimal. Leaf size=272

$$-\frac{\sqrt{x-1}(1-11x)}{32\left(x^2+1\right)}+\frac{\sqrt{x-1}x}{4\left(x^2+1\right)^2}-\frac{1}{128}\sqrt{\frac{1}{2}\left(527+373\sqrt{2}\right)}\log\left(-x-\sqrt{2\left(\sqrt{2}-1\right)}\sqrt{x-1}-\sqrt{2}+1\right)+\frac{1}{128}\sqrt{\frac{1}{2}\left(527+3\right.}$$

```
[Out] (Sqrt[-1 + x]*x)/(4*(1 + x^2)^2) - ((1 - 11*x)*Sqrt[-1 + x])/(32*(1 + x^2))
 - (Sqrt[(-527 + 373*Sqrt[2])/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[2])] - 2*Sqrt[-1
 + x])/Sqrt[2*(1 + Sqrt[2])]])/64 + (Sqrt[(-527 + 373*Sqrt[2])/2]*ArcTan[(S
qrt[2*(-1 + Sqrt[2])] + 2*Sqrt[-1 + x])/Sqrt[2*(1 + Sqrt[2])]])/64 - (Sqrt[
(527 + 373*Sqrt[2])/2]*Log[1 - Sqrt[2] - Sqrt[2*(-1 + Sqrt[2])]*Sqrt[-1 + x
] - x])/128 + (Sqrt[(527 + 373*Sqrt[2])/2]*Log[1 - Sqrt[2] + Sqrt[2*(-1 + S
qrt[2])]*Sqrt[-1 + x] - x])/128
```

Rubi [A] time = 0.364043, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {737, 823, 827, 1169, 634, 618, 204, 628}

$$-\frac{\sqrt{x-1}(1-11x)}{32\left(x^2+1\right)}+\frac{\sqrt{x-1}x}{4\left(x^2+1\right)^2}-\frac{1}{128}\sqrt{\frac{1}{2}\left(527+373\sqrt{2}\right)}\log\left(-x-\sqrt{2\left(\sqrt{2}-1\right)}\sqrt{x-1}-\sqrt{2}+1\right)+\frac{1}{128}\sqrt{\frac{1}{2}\left(527+3\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x]/(1 + x^2)^3,x]
```

```
[Out] (Sqrt[-1 + x]*x)/(4*(1 + x^2)^2) - ((1 - 11*x)*Sqrt[-1 + x])/(32*(1 + x^2))
 - (Sqrt[(-527 + 373*Sqrt[2])/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[2])] - 2*Sqrt[-1
 + x])/Sqrt[2*(1 + Sqrt[2])]])/64 + (Sqrt[(-527 + 373*Sqrt[2])/2]*ArcTan[(S
qrt[2*(-1 + Sqrt[2])] + 2*Sqrt[-1 + x])/Sqrt[2*(1 + Sqrt[2])]])/64 - (Sqrt[
(527 + 373*Sqrt[2])/2]*Log[1 - Sqrt[2] - Sqrt[2*(-1 + Sqrt[2])]*Sqrt[-1 + x
] - x])/128 + (Sqrt[(527 + 373*Sqrt[2])/2]*Log[1 - Sqrt[2] + Sqrt[2*(-1 + S
qrt[2])]*Sqrt[-1 + x] - x])/128
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(x*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*(p + 1)), x] + Dist[1/(2*a*(p + 1
)), Int[(d + e*x)^(m - 1)*(d*(2*p + 3) + e*(m + 2*p + 3)*x)*(a + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -
1] && GtQ[m, 0] && (LtQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && In
tQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
```

```
*m, 2*p])
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{-1+x}}{(1+x^2)^3}\,dx = \frac{\sqrt{-1+x}x}{4(1+x^2)^2} - \frac{1}{4}\int \frac{3-\frac{5x}{2}}{\sqrt{-1+x}(1+x^2)^2}\,dx$$
$$= \frac{\sqrt{-1+x}x}{4(1+x^2)^2} - \frac{(1-11x)\sqrt{-1+x}}{32(1+x^2)} + \frac{1}{16}\int \frac{-\frac{25}{4}+\frac{11x}{4}}{\sqrt{-1+x}(1+x^2)}\,dx$$
$$= \frac{\sqrt{-1+x}x}{4(1+x^2)^2} - \frac{(1-11x)\sqrt{-1+x}}{32(1+x^2)} + \frac{1}{8}\operatorname{Subst}\left(\int \frac{-\frac{7}{2}+\frac{11x^2}{4}}{2+2x^2+x^4}\,dx, x, \sqrt{-1+x}\right)$$
$$= \frac{\sqrt{-1+x}x}{4(1+x^2)^2} - \frac{(1-11x)\sqrt{-1+x}}{32(1+x^2)} + \frac{\operatorname{Subst}\left(\int \frac{-7\sqrt{\frac{1}{2}(-1+\sqrt{2})}-\left(-\frac{7}{2}-\frac{11}{2\sqrt{2}}\right)x}{\sqrt{2}-\sqrt{2(-1+\sqrt{2})}x+x^2}\,dx, x, \sqrt{-1+x}\right)}{32\sqrt{-1+\sqrt{2}}} + \frac{\operatorname{Subst}\left(\int \frac{-7\sqrt{\frac{1}{2}(}{\sqrt{2}}\right.}{}$$
$$= \frac{\sqrt{-1+x}x}{4(1+x^2)^2} - \frac{(1-11x)\sqrt{-1+x}}{32(1+x^2)} + \frac{1}{128}\sqrt{219-154\sqrt{2}}\operatorname{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{2(-1+\sqrt{2})}x+x^2}\,dx, x, \sqrt{-1}\right.$$
$$= \frac{\sqrt{-1+x}x}{4(1+x^2)^2} - \frac{(1-11x)\sqrt{-1+x}}{32(1+x^2)} - \frac{1}{256}\sqrt{1054+746\sqrt{2}}\log\left(1-\sqrt{2}-\sqrt{2(-1+\sqrt{2})}\sqrt{-1+x}-x\right) + \frac{}{2}$$
$$= \frac{\sqrt{-1+x}x}{4(1+x^2)^2} - \frac{(1-11x)\sqrt{-1+x}}{32(1+x^2)} - \frac{1}{64}\sqrt{\frac{1}{2}(-527+373\sqrt{2})}\tan^{-1}\left(\frac{\sqrt{2(-1+\sqrt{2})}-2\sqrt{-1+x}}{\sqrt{2(1+\sqrt{2})}}\right) + \frac{1}{64}$$

Mathematica [C] time = 0.0941321, size = 90, normalized size = 0.33

$$\frac{1}{64}\left(\frac{2\sqrt{x-1}(11x^3-x^2+19x-1)}{(x^2+1)^2} - (7-18i)\sqrt{1-i}\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{1-i}}\right) - (7+18i)\sqrt{1+i}\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{1+i}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x]/(1 + x^2)^3,x]
```

```
[Out] ((2*Sqrt[-1 + x]*(-1 + 19*x - x^2 + 11*x^3))/(1 + x^2)^2 - (7 - 18*I)*Sqrt[
1 - I]*ArcTan[Sqrt[-1 + x]/Sqrt[1 - I]] - (7 + 18*I)*Sqrt[1 + I]*ArcTan[Sqr
t[-1 + x]/Sqrt[1 + I]])/64
```

Maple [B] time = 0.365, size = 639, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)^(1/2)/(x^2+1)^3,x)
```

```
[Out] -1/128*(-4/23*(-759-506*2^(1/2))/(-6-4*2^(1/2))*(-1+x)^(3/2)-1/23/(-6-4*2^(
1/2))*(-5336-3588*2^(1/2))*(-2+2*2^(1/2))^(1/2)*(-1+x)-2/23*(-2392*2^(1/2)-
3036)/(-6-4*2^(1/2))*(-1+x)^(1/2)-1/46*(-3312*2^(1/2)-4416)*(-2+2*2^(1/2))^
(1/2)/(-6-4*2^(1/2)))/(-1+x+(-1+x)^(1/2)*(-2+2*2^(1/2))^(1/2)+2^(1/2))^2-13
/32/(3+2*2^(1/2))*ln(-1+x+(-1+x)^(1/2)*(-2+2*2^(1/2))^(1/2)+2^(1/2))*2^(1/2
)*(-2+2*2^(1/2))^(1/2)-147/256/(3+2*2^(1/2))*ln(-1+x+(-1+x)^(1/2)*(-2+2*2^(
```

```
1/2))^(1/2)+2^(1/2))*(-2+2*2^(1/2))^(1/2)+1/64/(3+2*2^(1/2))/(2+2*2^(1/2))^
(1/2)*arctan((2*(-1+x)^(1/2)+(-2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2))*2^(
1/2)+5/64/(3+2*2^(1/2))/(2+2*2^(1/2))^(1/2)*arctan((2*(-1+x)^(1/2)+(-2+2*2^
(1/2))^(1/2))/(2+2*2^(1/2))^(1/2))+1/128*(4/23*(-759-506*2^(1/2))/(-6-4*2^(
1/2))*(-1+x)^(3/2)-1/23/(-6-4*2^(1/2))*(-5336-3588*2^(1/2))*(-2+2*2^(1/2))^
(1/2)*(-1+x)+2/23*(-2392*2^(1/2)-3036)/(-6-4*2^(1/2))*(-1+x)^(1/2)-1/46*(-3
312*2^(1/2)-4416)*(-2+2*2^(1/2))^(1/2)/(-6-4*2^(1/2)))/(-1+x-(-1+x)^(1/2)*(
-2+2*2^(1/2))^(1/2)+2^(1/2))^2+13/32/(3+2*2^(1/2))*ln(-1+x-(-1+x)^(1/2)*(-2
+2*2^(1/2))^(1/2)+2^(1/2))*2^(1/2)*(-2+2*2^(1/2))^(1/2)+147/256/(3+2*2^(1/2
))*ln(-1+x-(-1+x)^(1/2)*(-2+2*2^(1/2))^(1/2)+2^(1/2))*(-2+2*2^(1/2))^(1/2)+
1/64/(3+2*2^(1/2))/(2+2*2^(1/2))^(1/2)*arctan((2*(-1+x)^(1/2)-(-2+2*2^(1/2)
)^(1/2))/(2+2*2^(1/2))^(1/2))*2^(1/2)+5/64/(3+2*2^(1/2))/(2+2*2^(1/2))^(1/2
)*arctan((2*(-1+x)^(1/2)-(-2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x-1}}{\left(x^2+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x - 1)/(x^2 + 1)^3, x)
```

Fricas [B] time = 2.42514, size = 1778, normalized size = 6.54

$$-\frac{92 \cdot 278258^{\frac{1}{4}} \sqrt{2}\left(x^4+2\,x^2+1\right)\sqrt{-393142\,\sqrt{2}+556516}\arctan\left(\frac{1}{109810067572} \cdot 278258^{\frac{3}{4}}\sqrt{46}\sqrt{373 \cdot 278258^{\frac{1}{4}}\sqrt{x-1}\left(\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/4392448*(92*278258^(1/4)*sqrt(2)*(x^4 + 2*x^2 + 1)*sqrt(-393142*sqrt(2)
+ 556516)*arctan(1/109810067572*278258^(3/4)*sqrt(46)*sqrt(373*278258^(1/4)
*sqrt(x - 1)*(11*sqrt(2) + 14)*sqrt(-393142*sqrt(2) + 556516) + 6399934*x +
 6399934*sqrt(2) - 6399934)*(7*sqrt(2) + 11)*sqrt(-393142*sqrt(2) + 556516)
 - 1/6399934*278258^(3/4)*sqrt(x - 1)*(7*sqrt(2) + 11)*sqrt(-393142*sqrt(2)
 + 556516) - sqrt(2) + 1) + 92*278258^(1/4)*sqrt(2)*(x^4 + 2*x^2 + 1)*sqrt(
-393142*sqrt(2) + 556516)*arctan(1/109810067572*278258^(3/4)*sqrt(46)*sqrt(
-373*278258^(1/4)*sqrt(x - 1)*(11*sqrt(2) + 14)*sqrt(-393142*sqrt(2) + 5565
16) + 6399934*x + 6399934*sqrt(2) - 6399934)*(7*sqrt(2) + 11)*sqrt(-393142*
sqrt(2) + 556516) - 1/6399934*278258^(3/4)*sqrt(x - 1)*(7*sqrt(2) + 11)*sqr
t(-393142*sqrt(2) + 556516) + sqrt(2) - 1) + 278258^(1/4)*(746*x^4 + 1492*x
^2 + 527*sqrt(2)*(x^4 + 2*x^2 + 1) + 746)*sqrt(-393142*sqrt(2) + 556516)*lo
g(373/46*278258^(1/4)*sqrt(x - 1)*(11*sqrt(2) + 14)*sqrt(-393142*sqrt(2) +
556516) + 139129*x + 139129*sqrt(2) - 139129) - 278258^(1/4)*(746*x^4 + 149
2*x^2 + 527*sqrt(2)*(x^4 + 2*x^2 + 1) + 746)*sqrt(-393142*sqrt(2) + 556516)
*log(-373/46*278258^(1/4)*sqrt(x - 1)*(11*sqrt(2) + 14)*sqrt(-393142*sqrt(2
) + 556516) + 139129*x + 139129*sqrt(2) - 139129) - 137264*(11*x^3 - x^2 +
19*x - 1)*sqrt(x - 1))/(x^4 + 2*x^2 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**(1/2)/(x**2+1)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x-1}}{\left(x^2+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x - 1)/(x^2 + 1)^3, x)
```

3.657 $\int (d+ex)^{3/2}\sqrt{a+cx^2}\,dx$

Optimal. Leaf size=398

$$-\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(3cd^2-5ae^2\right)\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{105c^{3/2}e^2\sqrt{a+cx^2}\sqrt{d+ex}}+\frac{2\sqrt{a+cx^2}\sqrt{d+ex}\,(\cdots}{1\cdots}$$

```
[Out] (2*Sqrt[d + e*x]*(3*c*d^2 - 5*a*e^2 + 24*c*d*e*x)*Sqrt[a + c*x^2])/(105*c*e
) + (2*e*Sqrt[d + e*x]*(a + c*x^2)^(3/2))/(7*c) + (4*Sqrt[-a]*d*(3*c*d^2 -
29*a*e^2)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt
[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(105*Sqrt[
c]*e^2*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2])
- (4*Sqrt[-a]*(3*c*d^2 - 5*a*e^2)*(c*d^2 + a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/
(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sq
rt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(105*c^(
3/2)*e^2*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.441283, antiderivative size = 398, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {743, 815, 844, 719, 424, 419}

$$-\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(3cd^2-5ae^2\right)\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{105c^{3/2}e^2\sqrt{a+cx^2}\sqrt{d+ex}}+\frac{2\sqrt{a+cx^2}\sqrt{d+ex}\left(-5ae^2+\cdots\right.}{105ce}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*Sqrt[a + c*x^2],x]
```

```
[Out] (2*Sqrt[d + e*x]*(3*c*d^2 - 5*a*e^2 + 24*c*d*e*x)*Sqrt[a + c*x^2])/(105*c*e
) + (2*e*Sqrt[d + e*x]*(a + c*x^2)^(3/2))/(7*c) + (4*Sqrt[-a]*d*(3*c*d^2 -
29*a*e^2)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt
[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(105*Sqrt[
c]*e^2*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2])
- (4*Sqrt[-a]*(3*c*d^2 - 5*a*e^2)*(c*d^2 + a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/
(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sq
rt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(105*c^(
3/2)*e^2*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
```

```
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int (d+ex)^{3/2}\sqrt{a+cx^2}\,dx = \frac{2e\sqrt{d+ex}\left(a+cx^2\right)^{3/2}}{7c} + \frac{2\int \frac{\left(\frac{1}{2}\left(7cd^2-ae^2\right)+4cdex\right)\sqrt{a+cx^2}}{\sqrt{d+ex}}dx}{7c}$$

$$= \frac{2\sqrt{d+ex}\left(3cd^2-5ae^2+24cdex\right)\sqrt{a+cx^2}}{105ce} + \frac{2e\sqrt{d+ex}\left(a+cx^2\right)^{3/2}}{7c} + \frac{8\int \frac{\frac{1}{4}ace^2\left(27cd^2-5ae^2 \ldots\right.}{\sqrt{d+\ldots}}}{10\ldots}$$

$$= \frac{2\sqrt{d+ex}\left(3cd^2-5ae^2+24cdex\right)\sqrt{a+cx^2}}{105ce} + \frac{2e\sqrt{d+ex}\left(a+cx^2\right)^{3/2}}{7c} + \frac{1}{105}\left(2d\left(29a - \frac{3c\ldots}{e\ldots}\right.\right.$$

$$= \frac{2\sqrt{d+ex}\left(3cd^2-5ae^2+24cdex\right)\sqrt{a+cx^2}}{105ce} + \frac{2e\sqrt{d+ex}\left(a+cx^2\right)^{3/2}}{7c} + \frac{\left(4ad\left(29a-\frac{3cd^2}{e^2}\right) \ldots\right.}{\ldots}$$

$$= \frac{2\sqrt{d+ex}\left(3cd^2-5ae^2+24cdex\right)\sqrt{a+cx^2}}{105ce} + \frac{2e\sqrt{d+ex}\left(a+cx^2\right)^{3/2}}{7c} - \frac{4\sqrt{-a}d\left(29a-\frac{3cd^{\ldots}}{e^2}\right.\ldots}{\ldots}$$

Mathematica [C] time = 3.41283, size = 582, normalized size = 1.46

$$\frac{\sqrt{d+ex}\left(\frac{2\left(a+cx^2\right)\left(10ae^2+3c\left(d^2+8dex+5e^2x^2\right)\right)}{ce} + \frac{4\left(\sqrt{ae}(d+ex)^{3/2}\left(-5ia^{3/2}e^3+27i\sqrt{a}cd^2e-29a\sqrt{c}de^2+3c^{3/2}d^3\right)\sqrt{\frac{e\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{d+ex}}\sqrt{-\frac{-ex+\frac{i\sqrt{ae}}{\sqrt{c}}}{d+ex}}\text{EllipticF}\left(i\sinh^{\ldots}\right.\ldots\right.}{\ldots}\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)*Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[d + e*x]*((2*(a + c*x^2)*(10*a*e^2 + 3*c*(d^2 + 8*d*e*x + 5*e^2*x^2))
)/(c*e) + (4*(-(d*e^2*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*(-29*a^2*e^2 + 3*c^2
*d^2*x^2 + a*c*(3*d^2 - 29*e^2*x^2))) + Sqrt[c]*d*((3*I)*c^(3/2)*d^3 - 3*Sq
rt[a]*c*d^2*e - (29*I)*a*Sqrt[c]*d*e^2 + 29*a^(3/2)*e^3)*Sqrt[(e*((I*Sqrt[a
])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))
]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt
[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)] + Sqrt[a]*
e*(3*c^(3/2)*d^3 + (27*I)*Sqrt[a]*c*d^2*e - 29*a*Sqrt[c]*d*e^2 - (5*I)*a^(3
/2)*e^3)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e
)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d -
(I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d
 + I*Sqrt[a]*e)]))/(c*e^3*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*(d + e*x))))/(10
5*Sqrt[a + c*x^2])
```

Maple [B] time = 0.394, size = 1386, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(c*x^2+a)^(1/2),x)
```

```
[Out] 2/105*(e*x+d)^(1/2)*(c*x^2+a)^(1/2)*(15*x^5*c^3*e^5+10*(-a*c)^(1/2)*(-(e*x+
d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d
))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x
+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*
d))^(1/2))*a^2*e^5+4*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*(
(-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-
a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),
(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c*d^2*e^3-6*(-a*c)^(1
/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^
(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*Ellip
ticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)
^(1/2)*e+c*d))^(1/2))*c^2*d^4*e+48*a^2*c*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^
(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2)
)*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))
^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*d*e^4+48*a*c^2*(
-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)
*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF(
(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2
)*e+c*d))^(1/2))*d^3*e^2-58*a^2*c*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*(
(-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-
a*c)^(1/2)*e-c*d))^(1/2)*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),
(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*d*e^4-52*a*c^2*(-(e*x+d
)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d)
)^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticE((-(e*x+
d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d
))^(1/2))*d^3*e^2+6*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(
1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c
*d))^(1/2)*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2
)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*c^3*d^5+39*x^4*c^3*d*e^4+25*x^3*a*c^2
*e^5+27*x^3*c^3*d^2*e^3+49*x^2*a*c^2*d*e^4+3*x^2*c^3*d^3*e^2+10*x*a^2*c*e^5
+27*x*a*c^2*d^2*e^3+10*a^2*c*d*e^4+3*a*c^2*d^3*e^2)/e^3/(c*e*x^3+c*d*x^2+a*
e*x+a*d)/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + a}(ex + d)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^2 + a}(ex + d)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*(e*x + d)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + cx^2} (d + ex)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + a}(ex + d)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)^(3/2), x)
```

3.658 $\int \sqrt{d+ex}\sqrt{a+cx^2}\,dx$

Optimal. Leaf size=362

$$-\frac{4\sqrt{-a}d\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{15\sqrt{c}e^2\sqrt{a+cx^2}\sqrt{d+ex}}+\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}\left(cd^2-3ae^2\right)}{15\sqrt{c}e^2\sqrt{a+cx}}$$

```
[Out] (-4*d*Sqrt[d + e*x]*Sqrt[a + c*x^2])/(15*e) + (2*(d + e*x)^(3/2)*Sqrt[a + c
*x^2])/(5*e) + (4*Sqrt[-a]*(c*d^2 - 3*a*e^2)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)
/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqr
t[-a]*Sqrt[c]*d - a*e)])/(15*Sqrt[c]*e^2*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*
d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*d*(c*d^2 + a*e^2)*Sqrt[(Sqr
t[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[Arc
Sin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d -
 a*e)])/(15*Sqrt[c]*e^2*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.298611, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {735, 833, 844, 719, 424, 419}

$$-\frac{4\sqrt{-a}d\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{15\sqrt{c}e^2\sqrt{a+cx^2}\sqrt{d+ex}}+\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}\left(cd^2-3ae^2\right)E\left(\sin^{-1}\right.}{15\sqrt{c}e^2\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(d}{\sqrt{-ae}}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*Sqrt[a + c*x^2],x]
```

```
[Out] (-4*d*Sqrt[d + e*x]*Sqrt[a + c*x^2])/(15*e) + (2*(d + e*x)^(3/2)*Sqrt[a + c
*x^2])/(5*e) + (4*Sqrt[-a]*(c*d^2 - 3*a*e^2)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)
/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqr
t[-a]*Sqrt[c]*d - a*e)])/(15*Sqrt[c]*e^2*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*
d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*d*(c*d^2 + a*e^2)*Sqrt[(Sqr
t[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[Arc
Sin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d -
 a*e)])/(15*Sqrt[c]*e^2*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
```

```
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \sqrt{d+ex}\sqrt{a+cx^2}\,dx = \frac{2(d+ex)^{3/2}\sqrt{a+cx^2}}{5e} + \frac{2\int \frac{(ae-cdx)\sqrt{d+ex}}{\sqrt{a+cx^2}}\,dx}{5e}$$

$$= -\frac{4d\sqrt{d+ex}\sqrt{a+cx^2}}{15e} + \frac{2(d+ex)^{3/2}\sqrt{a+cx^2}}{5e} + \frac{4\int \frac{2acde-\frac{1}{2}c\left(cd^2-3ae^2\right)x}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx}{15ce}$$

$$= -\frac{4d\sqrt{d+ex}\sqrt{a+cx^2}}{15e} + \frac{2(d+ex)^{3/2}\sqrt{a+cx^2}}{5e} + \frac{1}{15}\left(2\left(3a-\frac{cd^2}{e^2}\right)\right)\int \frac{\sqrt{d+ex}}{\sqrt{a+cx^2}}\,dx + \frac{1}{15}\Big(2\ldots$$

$$= -\frac{4d\sqrt{d+ex}\sqrt{a+cx^2}}{15e} + \frac{2(d+ex)^{3/2}\sqrt{a+cx^2}}{5e} + \frac{\left(4a\left(3a-\frac{cd^2}{e^2}\right)\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}\right)\mathrm{Subst}\Big(\int\ldots}{15\sqrt{-a}\sqrt{c}\sqrt{\frac{c(d+ex)}{cd-\frac{a\sqrt{c}e}{\sqrt{-a}}}}\ldots}$$

$$= -\frac{4d\sqrt{d+ex}\sqrt{a+cx^2}}{15e} + \frac{2(d+ex)^{3/2}\sqrt{a+cx^2}}{5e} - \frac{4\sqrt{-a}\left(3a-\frac{cd^2}{e^2}\right)\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}\,E\Big(\sin^{-1}\ldots}{15\sqrt{c}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}}\sqrt{a+\ldots}}$$

Mathematica [C] time = 2.88824, size = 536, normalized size = 1.48

$$\frac{\sqrt{d+ex}\left(\frac{2\left(a+cx^2\right)(d+3ex)}{e}-\frac{4\left(-\sqrt{a}\sqrt{c}e(d+ex)^{3/2}\left(4i\sqrt{a}\sqrt{c}de-3ae^2+cd^2\right)\sqrt{\frac{e\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{d+ex}}\sqrt{-\frac{-ex+\frac{i\sqrt{a}e}{\sqrt{c}}}{d+ex}}\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}}{\sqrt{d+ex}}\right),\frac{\sqrt{c}d-i\sqrt{a}e}{\sqrt{c}d+i\sqrt{a}e}\right)+e^2\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}\left(-\cdots\right.\right.}{ce^3\cdots}\right)}{15\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[d + e*x]*((2*(d + 3*e*x)*(a + c*x^2))/e - (4*(e^2*Sqrt[-d - (I*Sqrt[a
]*e)/Sqrt[c]]*(-3*a^2*e^2 + c^2*d^2*x^2 + a*c*(d^2 - 3*e^2*x^2)) + Sqrt[c]*
((-I)*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + (3*I)*a*Sqrt[c]*d*e^2 - 3*a^(3/2)*e^3
)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[
c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d - (I*Sqrt
[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sq
rt[a]*e)] - Sqrt[a]*Sqrt[c]*e*(c*d^2 + (4*I)*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)
*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c
] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d - (I*Sqrt[
a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqr
t[a]*e)]))/(c*e^3*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*(d + e*x))))/(15*Sqrt[a
+ c*x^2])
```

Maple [B] time = 0.244, size = 1162, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(c*x^2+a)^(1/2),x)
```

```
[Out] 2/15*(e*x+d)^(1/2)*(c*x^2+a)^(1/2)*(6*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/
2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e
/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1
/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*e^4+6*(-(e*x+d)
*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))
^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d
)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d)
)^(1/2))*a*c*d^2*e^2-2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c
)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*
e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(
1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*(-a*c)^(1/2)*a*d*e^3-2*(-(e*x+d)*c
/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(
1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*
c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^
(1/2))*(-a*c)^(1/2)*c*d^3*e-6*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*
x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)
^(1/2)*e-c*d))^(1/2)*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((
-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*e^4-4*(-(e*x+d)*c/((-a*
c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*(
(c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticE((-(e*x+d)*c/((-a
*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))
```

```
*a*c*d^2*e^2+2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))
*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^
(1/2)*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c
*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*c^2*d^4+3*x^4*c^2*e^4+4*x^3*c^2*d*e^3+3*x^
2*a*c*e^4+x^2*c^2*d^2*e^2+4*x*a*c*d*e^3+a*c*d^2*e^2)/c/(c*e*x^3+c*d*x^2+a*e
*x+a*d)/e^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + a}\sqrt{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)*sqrt(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^2 + a}\sqrt{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*sqrt(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + cx^2}\sqrt{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)*sqrt(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + a}\sqrt{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)*sqrt(e*x + d), x)
```

3.659 $\int \frac{\sqrt{a+cx^2}}{\sqrt{d+ex}} \, dx$

Optimal. Leaf size=322

$$-\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{3\sqrt{c}e^2\sqrt{a+cx^2}\sqrt{d+ex}}+\frac{4\sqrt{-a}\sqrt{c}d\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}E\left(\sin^{-1}\left(\right.\right.}{3e^2\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(d}{\sqrt{-ae}}}}$$

```
[Out] (2*Sqrt[d + e*x]*Sqrt[a + c*x^2])/(3*e) + (4*Sqrt[-a]*Sqrt[c]*d*Sqrt[d + e*
x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt
[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(3*e^2*Sqrt[(Sqrt[c]*(d + e*x))
/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*(c*d^2 + a*e^2)*S
qrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*Ellip
ticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqr
t[c]*d - a*e)])/(3*Sqrt[c]*e^2*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.203577, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {735, 844, 719, 424, 419}

$$-\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{3\sqrt{c}e^2\sqrt{a+cx^2}\sqrt{d+ex}}+\frac{4\sqrt{-a}\sqrt{c}d\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right.\right.}{3e^2\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[d + e*x]*Sqrt[a + c*x^2])/(3*e) + (4*Sqrt[-a]*Sqrt[c]*d*Sqrt[d + e*
x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt
[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(3*e^2*Sqrt[(Sqrt[c]*(d + e*x))
/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*(c*d^2 + a*e^2)*S
qrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*Ellip
ticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqr
t[c]*d - a*e)])/(3*Sqrt[c]*e^2*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{d+ex}\sqrt{a+cx^2}}{3e} + \frac{2\int \frac{ae-cdx}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx}{3e}$$

$$= \frac{2\sqrt{d+ex}\sqrt{a+cx^2}}{3e} + \frac{1}{3}\left(2\left(a+\frac{cd^2}{e^2}\right)\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx - \frac{(2cd)\int \frac{\sqrt{d+ex}}{\sqrt{a+cx^2}}\,dx}{3e^2}$$

$$= \frac{2\sqrt{d+ex}\sqrt{a+cx^2}}{3e} - \frac{\left(4a\sqrt{c}d\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}ex^2}{\sqrt{-a}\left(cd-\frac{a\sqrt{c}e}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{3\sqrt{-a}e^2\sqrt{\frac{c(d+ex)}{cd-\frac{a\sqrt{c}e}{\sqrt{-a}}}}\sqrt{a+cx^2}} + \frac{\left(4a\left(a+\right.\right.}{}$$

$$= \frac{2\sqrt{d+ex}\sqrt{a+cx^2}}{3e} + \frac{4\sqrt{-a}\sqrt{c}d\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{3e^2\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}}\sqrt{a+cx^2}} - \frac{4\sqrt{-a}\left(a+\frac{cd^2}{e^2}\right.}{}$$

Mathematica [C] time = 1.98539, size = 456, normalized size = 1.42

$$\frac{2\sqrt{d+ex}\left(e^2\left(a+cx^2\right) - \frac{2\left(-\sqrt{a}e(d+ex)^{3/2}\left(\sqrt{c}d+i\sqrt{a}e\right)\sqrt{\frac{e\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{d+ex}}\sqrt{-\frac{-ex+\frac{i\sqrt{a}e}{\sqrt{c}}}{d+ex}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}}{\sqrt{d+ex}}\right), \frac{\sqrt{c}d-i\sqrt{a}e}{\sqrt{c}d+i\sqrt{a}e}\right)+de^2\left(a+cx^2\right)\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}+\right.}{(d+ex)\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}}\right.}{3e^3\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[d + e*x]*(e^2*(a + c*x^2) - (2*(d*e^2*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[
c]]*(a + c*x^2) + Sqrt[c]*d*((-I)*Sqrt[c]*d + Sqrt[a]*e)*Sqrt[(e*((I*Sqrt[a
])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))
]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt
[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)] - Sqrt[a]*
e*(Sqrt[c]*d + I*Sqrt[a]*e)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*S
qrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticF[I
*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sq
rt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)]))/(Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*(d
+ e*x))))/(3*e^3*Sqrt[a + c*x^2])
```

Maple [B] time = 0.26, size = 688, normalized size = 2.1

$$-\frac{2}{3c\left(cex^3+cdx^2+aex+ad\right)e^3}\sqrt{ex+d}\sqrt{cx^2+a}\left(2\sqrt{-\frac{c(ex+d)}{\sqrt{-ac}e-cd}}\sqrt{\frac{\left(-cx+\sqrt{-ac}\right)e}{\sqrt{-ac}e+cd}}\sqrt{\frac{\left(cx+\sqrt{-ac}\right)e}{\sqrt{-ac}e-cd}}\text{EllipticF}\left(\sqrt{-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -2/3*(c*x^2+a)^(1/2)*(e*x+d)^(1/2)*(2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/
2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e
/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1
/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*(-a*c)^(1/2)*a*e^3+
2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1
/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*Ellipti
cF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(
1/2)*e+c*d))^(1/2))*(-a*c)^(1/2)*c*d^2*e-2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d)
)^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/
2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d
))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c*d*e^2-2*(-
(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*
e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticE((
-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)
*e+c*d))^(1/2))*c^2*d^3-x^3*c^2*e^3-x^2*c^2*d*e^2-x*a*c*e^3-a*d*e^2*c)/c/(c
*e*x^3+c*d*x^2+a*e*x+a*d)/e^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+a}}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/sqrt(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+a}}{\sqrt{ex+d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)/sqrt(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/sqrt(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+a}}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)/sqrt(e*x + d), x)
```

3.660 $\int \frac{\sqrt{a+cx^2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=305

$$\frac{4\sqrt{-a}\sqrt{c}d\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{e^2\sqrt{a+cx^2}\sqrt{d+ex}}-\frac{4\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\right.}{e^2\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}}$$

```
[Out] (-2*Sqrt[a + c*x^2])/(e*Sqrt[d + e*x]) - (4*Sqrt[-a]*Sqrt[c]*Sqrt[d + e*x]*
Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]
], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(e^2*Sqrt[(Sqrt[c]*(d + e*x))/(Sqr
t[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) + (4*Sqrt[-a]*Sqrt[c]*d*Sqrt[(Sqrt[c
]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin
[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*
e)])/(e^2*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.183042, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {733, 844, 719, 424, 419}

$$\frac{4\sqrt{-a}\sqrt{c}d\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{e^2\sqrt{a+cx^2}\sqrt{d+ex}}-\frac{4\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d}\right.}{e^2\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(d + e*x)^(3/2),x]
```



```
[Out] (-2*Sqrt[a + c*x^2])/(e*Sqrt[d + e*x]) - (4*Sqrt[-a]*Sqrt[c]*Sqrt[d + e*x]*
Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]
], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(e^2*Sqrt[(Sqrt[c]*(d + e*x))/(Sqr
t[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) + (4*Sqrt[-a]*Sqrt[c]*d*Sqrt[(Sqrt[c
]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin
[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*
e)])/(e^2*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```


Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+cx^2}}{(d+ex)^{3/2}}\,dx &= -\frac{2\sqrt{a+cx^2}}{e\sqrt{d+ex}} + \frac{(2c)\int \frac{x}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx}{e} \\
&= -\frac{2\sqrt{a+cx^2}}{e\sqrt{d+ex}} + \frac{(2c)\int \frac{\sqrt{d+ex}}{\sqrt{a+cx^2}}\,dx}{e^2} - \frac{(2cd)\int \frac{1}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx}{e^2} \\
&= -\frac{2\sqrt{a+cx^2}}{e\sqrt{d+ex}} + \frac{\left(4a\sqrt{c}\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}ex^2}{\sqrt{-a}\left(cd-\frac{a\sqrt{c}e}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}e^2\sqrt{\frac{c(d+ex)}{cd-\frac{a\sqrt{c}e}{\sqrt{-a}}}}\sqrt{a+cx^2}} - \frac{\left(4a\sqrt{c}d\sqrt{\frac{c(d+\cdots}{cd-\frac{a\cdots}{\cdots}}}\cdots\right.}{\cdots} \\
&= -\frac{2\sqrt{a+cx^2}}{e\sqrt{d+ex}} - \frac{4\sqrt{-a}\sqrt{c}\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{e^2\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}}\sqrt{a+cx^2}} + \frac{4\sqrt{-a}\sqrt{c}d\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}\cdots}}\cdots}{\cdots}
\end{aligned}
$$

Mathematica [C] time = 1.30891, size = 419, normalized size = 1.37

$$
\frac{2\left(-\frac{2\sqrt{a}\sqrt{c}e(d+ex)^{3/2}\sqrt{\frac{e\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{d+ex}}\sqrt{-\frac{-ex+\frac{i\sqrt{a}e}{\sqrt{c}}}{d+ex}}\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}}{\sqrt{d+ex}}\right),\frac{\sqrt{c}d-i\sqrt{a}e}{\sqrt{c}d+i\sqrt{a}e}\right)}{\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}} + \frac{2\sqrt{c}(d+ex)^{3/2}\left(\sqrt{a}e-i\sqrt{c}d\right)\sqrt{\frac{e\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{d+ex}}\sqrt{-\frac{-ex+\frac{i\sqrt{a}e}{\sqrt{c}}}{d+ex}}\,E\left(i\sinh^{-1}\cdots\right.}{\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}}\cdots\right)}{e^3\sqrt{a+cx^2}\sqrt{d+ex}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(d + e*x)^(3/2),x]
```

[Out] (2*(e^2*(a + c*x^2) + (2*Sqrt[c]*((-I)*Sqrt[c]*d + Sqrt[a]*e)*Sqrt[(e*((I*S
qrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d +
e*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]
/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)])/Sqrt
[-d - (I*Sqrt[a]*e)/Sqrt[c]] - (2*Sqrt[a]*Sqrt[c]*e*Sqrt[(e*((I*Sqrt[a])/Sq
rt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d
+ e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d +
e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)])/Sqrt[-d - (I*S
qrt[a]*e)/Sqrt[c]]))/(e^3*Sqrt[d + e*x]*Sqrt[a + c*x^2])

Maple [B] time = 0.323, size = 646, normalized size = 2.1

$$-2\,\frac{\sqrt{ex+d}\sqrt{cx^2+a}}{\left(cex^3+cdx^2+aex+ad\right)e^3}\left(2\,\mathrm{EllipticE}\left(\sqrt{-\frac{c\,(ex+d)}{\sqrt{-ac}e-cd}},\sqrt{-\frac{\sqrt{-ac}e-cd}{\sqrt{-ac}e+cd}}\right)ae^2\sqrt{-\frac{c\,(ex+d)}{\sqrt{-ac}e-cd}}\sqrt{\frac{\left(-cx+\sqrt{-ac}\right)e}{\sqrt{-ac}e+cd}}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+a)^(1/2)/(e*x+d)^(3/2),x)

[Out] -2*(c*x^2+a)^(1/2)*(e*x+d)^(1/2)*(2*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c
*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*e^2*(-(e*x
+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*
d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+2*EllipticE((-(
e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e
+c*d))^(1/2))*c*d^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(
1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c
*d))^(1/2)-2*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1
/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c
d))^(1/2)((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^
(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-2*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*
e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*d*e*(-a*c
)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a
*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+c
*e^2*x^2+a*e^2)/(c*e*x^3+c*d*x^2+a*e*x+a*d)/e^3

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+a}}{(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="maxima")

[Out] integrate(sqrt(c*x^2 + a)/(e*x + d)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^2+a}\sqrt{ex+d}}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*sqrt(e*x + d)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/(e*x+d)**(3/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(d + e*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+a}}{(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)/(e*x + d)^(3/2), x)
```

3.661 $\int \frac{\sqrt{a+cx^2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=366

$$-\frac{4\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{3e^2\sqrt{a+cx^2}\sqrt{d+ex}}+\frac{4\sqrt{-a}c^{3/2}d\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\cdots\right.}{3e^2\sqrt{a+cx^2}\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}}$$

```
[Out] (-2*Sqrt[a + c*x^2])/(3*e*(d + e*x)^(3/2)) + (4*c*d*Sqrt[a + c*x^2])/(3*e*(
c*d^2 + a*e^2)*Sqrt[d + e*x]) + (4*Sqrt[-a]*c^(3/2)*d*Sqrt[d + e*x]*Sqrt[1
+ (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*
a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(3*e^2*(c*d^2 + a*e^2)*Sqrt[(Sqrt[c]*(d +
 e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*Sqrt[c]*Sqr
t[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*Ellipti
cF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[
c]*d - a*e)])/(3*e^2*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.272954, antiderivative size = 366, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {733, 835, 844, 719, 424, 419}

$$\frac{4\sqrt{-a}c^{3/2}d\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{3e^2\sqrt{a+cx^2}\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}}+\frac{4cd\sqrt{a+cx^2}}{3e\sqrt{d+ex}\left(ae^2+cd^2\right)}-\frac{4\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\,F\left(\cdots\right.}{3e^2\sqrt{a+cx^2}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(d + e*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[a + c*x^2])/(3*e*(d + e*x)^(3/2)) + (4*c*d*Sqrt[a + c*x^2])/(3*e*(
c*d^2 + a*e^2)*Sqrt[d + e*x]) + (4*Sqrt[-a]*c^(3/2)*d*Sqrt[d + e*x]*Sqrt[1
+ (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*
a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(3*e^2*(c*d^2 + a*e^2)*Sqrt[(Sqrt[c]*(d +
 e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*Sqrt[c]*Sqr
t[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*Ellipti
cF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[
c]*d - a*e)])/(3*e^2*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
```

```
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^{5/2}}\,dx = -\frac{2\sqrt{a+cx^2}}{3e(d+ex)^{3/2}} + \frac{(2c)\int \frac{x}{(d+ex)^{3/2}\sqrt{a+cx^2}}\,dx}{3e}$$

$$= -\frac{2\sqrt{a+cx^2}}{3e(d+ex)^{3/2}} + \frac{4cd\sqrt{a+cx^2}}{3e\left(cd^2+ae^2\right)\sqrt{d+ex}} - \frac{(4c)\int \frac{-\frac{ae}{2}+\frac{cdx}{2}}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx}{3e\left(cd^2+ae^2\right)}$$

$$= -\frac{2\sqrt{a+cx^2}}{3e(d+ex)^{3/2}} + \frac{4cd\sqrt{a+cx^2}}{3e\left(cd^2+ae^2\right)\sqrt{d+ex}} + \frac{(2c)\int \frac{1}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx}{3e^2} - \frac{\left(2c^2d\right)\int \frac{\sqrt{d+ex}}{\sqrt{a+cx^2}}\,dx}{3e^2\left(cd^2+ae^2\right)}$$

$$= -\frac{2\sqrt{a+cx^2}}{3e(d+ex)^{3/2}} + \frac{4cd\sqrt{a+cx^2}}{3e\left(cd^2+ae^2\right)\sqrt{d+ex}} - \frac{\left(4ac^{3/2}d\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}ex^2}{\sqrt{-a}\left(cd-\frac{a\sqrt{c}e}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx, x, \ldots\right)}{3\sqrt{-a}e^2\left(cd^2+ae^2\right)\sqrt{\frac{c(d+ex)}{cd-\frac{a\sqrt{c}e}{\sqrt{-a}}}}\sqrt{a+cx^2}}$$

$$= -\frac{2\sqrt{a+cx^2}}{3e(d+ex)^{3/2}} + \frac{4cd\sqrt{a+cx^2}}{3e\left(cd^2+ae^2\right)\sqrt{d+ex}} + \frac{4\sqrt{-a}c^{3/2}d\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-\ldots}\right)}{3e^2\left(cd^2+ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}}\sqrt{a+cx^2}}$$

Mathematica [C] time = 1.17454, size = 504, normalized size = 1.38

$$\frac{2\sqrt{a+cx^2}\left(cd(d+2ex)-ae^2\right)}{3(d+ex)^{3/2}\left(ae^3+cd^2e\right)} - \frac{4c\left(-\sqrt{a}e(d+ex)^{3/2}\left(\sqrt{c}d+i\sqrt{a}e\right)\sqrt{\frac{e\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{d+ex}}\sqrt{-\frac{-ex+\frac{i\sqrt{a}e}{\sqrt{c}}}{d+ex}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}}{\sqrt{d+ex}}\right), \ldots\right)\ldots\right)}{3e^3\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(d + e*x)^(5/2),x]
```

```
[Out] (2*Sqrt[a + c*x^2]*(-(a*e^2) + c*d*(d + 2*e*x)))/(3*(c*d^2*e + a*e^3)*(d +
e*x)^(3/2)) - (4*c*(d*e^2*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*(a + c*x^2) + Sq
rt[c]*d*((-I)*Sqrt[c]*d + Sqrt[a]*e)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d
+ e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*El
lipticE[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]
*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)] - Sqrt[a]*e*(Sqrt[c]*d + I*Sqr
t[a]*e)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)
/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d - (
I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d
+ I*Sqrt[a]*e)]))/(3*e^3*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*(c*d^2 + a*e^2)*S
qrt[d + e*x]*Sqrt[a + c*x^2])
```

Maple [B] time = 0.278, size = 1309, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/(e*x+d)^(5/2),x)
```

```
[Out] -2/3*(2*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e
-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*a*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d)
)^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/
2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*(-a*c)^(1/2)+2*EllipticF((-(e*x+d)*c/((-a
*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))
*x*c*d^2*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e
/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1
/2)*(-a*c)^(1/2)-2*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a
*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*a*c*d*e^3*(-(e*x+d)*c/((-a*
c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*(
(c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-2*EllipticE((-(e*x+d)*c/((
-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2
))*x*c^2*d^3*e*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))
*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^
(1/2)+2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a
*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*E
llipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-
a*c)^(1/2)*e+c*d))^(1/2))*(-a*c)^(1/2)*a*d*e^3+2*(-(e*x+d)*c/((-a*c)^(1/2)*
e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*
c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)
*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*(-a*c)^(
1/2)*c*d^3*e-2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))
*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^
(1/2)*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c
*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c*d^2*e^2-2*(-(e*x+d)*c/((-a*c)^(1/2)*e-
c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)
^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e
-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*c^2*d^4-2*
x^3*c^2*d*e^3+x^2*a*c*e^4-x^2*c^2*d^2*e^2-2*x*a*c*d*e^3+a^2*e^4-a*c*d^2*e^2
)/(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/e^3/(e*x+d)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + a}}{(ex + d)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/(e*x + d)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2 + a}\sqrt{ex + d}}{e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*sqrt(e*x + d)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x +
 d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^2}}{(d + ex)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/(e*x+d)**(5/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(d + e*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + a}}{(ex + d)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)/(e*x + d)^(5/2), x)
```

3.662 $\int \frac{\sqrt{a+cx^2}}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=444

$$-\frac{4\sqrt{-a}c^{3/2}d\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{15e^2\sqrt{a+cx^2}\sqrt{d+ex}\left(ae^2+cd^2\right)}+\frac{4\sqrt{-a}c^{3/2}\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}\left(cd^2-3ae^2\right)E}{15e^2\sqrt{a+cx^2}\left(ae^2+cd^2\right.}$$

```
[Out] (-2*Sqrt[a + c*x^2])/(5*e*(d + e*x)^(5/2)) + (4*c*d*Sqrt[a + c*x^2])/(15*e*
(c*d^2 + a*e^2)*(d + e*x)^(3/2)) + (4*c*(c*d^2 - 3*a*e^2)*Sqrt[a + c*x^2])/
(15*e*(c*d^2 + a*e^2)^2*Sqrt[d + e*x]) + (4*Sqrt[-a]*c^(3/2)*(c*d^2 - 3*a*e
^2)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)
/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(15*e^2*(c*d^2 +
 a*e^2)^2*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2
]) - (4*Sqrt[-a]*c^(3/2)*d*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e
)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt
[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(15*e^2*(c*d^2 + a*e^2)*Sqrt[d
+ e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.38184, antiderivative size = 444, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {733, 835, 844, 719, 424, 419}

$$-\frac{4\sqrt{-a}c^{3/2}d\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{15e^2\sqrt{a+cx^2}\sqrt{d+ex}\left(ae^2+cd^2\right)}+\frac{4\sqrt{-a}c^{3/2}\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}\left(cd^2-3ae^2\right)E\left(\sin^{-1}\right.}{15e^2\sqrt{a+cx^2}\left(ae^2+cd^2\right)^2\sqrt{\frac{\sqrt{c}}{\sqrt{-}}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(d + e*x)^(7/2),x]
```

```
[Out] (-2*Sqrt[a + c*x^2])/(5*e*(d + e*x)^(5/2)) + (4*c*d*Sqrt[a + c*x^2])/(15*e*
(c*d^2 + a*e^2)*(d + e*x)^(3/2)) + (4*c*(c*d^2 - 3*a*e^2)*Sqrt[a + c*x^2])/
(15*e*(c*d^2 + a*e^2)^2*Sqrt[d + e*x]) + (4*Sqrt[-a]*c^(3/2)*(c*d^2 - 3*a*e
^2)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)
/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(15*e^2*(c*d^2 +
 a*e^2)^2*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2
]) - (4*Sqrt[-a]*c^(3/2)*d*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e
)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt
[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(15*e^2*(c*d^2 + a*e^2)*Sqrt[d
+ e*x]*Sqrt[a + c*x^2])
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^{7/2}}\,dx = -\frac{2\sqrt{a+cx^2}}{5e(d+ex)^{5/2}} + \frac{(2c)\int \frac{x}{(d+ex)^{5/2}\sqrt{a+cx^2}}\,dx}{5e}$$

$$= -\frac{2\sqrt{a+cx^2}}{5e(d+ex)^{5/2}} + \frac{4cd\sqrt{a+cx^2}}{15e\left(cd^2+ae^2\right)(d+ex)^{3/2}} - \frac{(4c)\int \frac{-\frac{3ae}{2}-\frac{cdx}{2}}{(d+ex)^{3/2}\sqrt{a+cx^2}}\,dx}{15e\left(cd^2+ae^2\right)}$$

$$= -\frac{2\sqrt{a+cx^2}}{5e(d+ex)^{5/2}} + \frac{4cd\sqrt{a+cx^2}}{15e\left(cd^2+ae^2\right)(d+ex)^{3/2}} + \frac{4c\left(cd^2-3ae^2\right)\sqrt{a+cx^2}}{15e\left(cd^2+ae^2\right)^2\sqrt{d+ex}} + \frac{(8c)\int \frac{acde-\frac{1}{4}c\left(cd^2-3ae^2\right)x}{\sqrt{d+ex}\sqrt{a+cx^2}}}{15e\left(cd^2+ae^2\right)^2}$$

$$= -\frac{2\sqrt{a+cx^2}}{5e(d+ex)^{5/2}} + \frac{4cd\sqrt{a+cx^2}}{15e\left(cd^2+ae^2\right)(d+ex)^{3/2}} + \frac{4c\left(cd^2-3ae^2\right)\sqrt{a+cx^2}}{15e\left(cd^2+ae^2\right)^2\sqrt{d+ex}} - \frac{\left(2c^2\left(cd^2-3ae^2\right)\right)\int \frac{\sqrt{d}}{\sqrt{a}}}{15e^2\left(cd^2+ae^2\right)^2}$$

$$= -\frac{2\sqrt{a+cx^2}}{5e(d+ex)^{5/2}} + \frac{4cd\sqrt{a+cx^2}}{15e\left(cd^2+ae^2\right)(d+ex)^{3/2}} + \frac{4c\left(cd^2-3ae^2\right)\sqrt{a+cx^2}}{15e\left(cd^2+ae^2\right)^2\sqrt{d+ex}} - \frac{\left(4ac^{3/2}\left(cd^2-3ae^2\right)\sqrt{d}\right.}{15\sqrt{}}$$

$$= -\frac{2\sqrt{a+cx^2}}{5e(d+ex)^{5/2}} + \frac{4cd\sqrt{a+cx^2}}{15e\left(cd^2+ae^2\right)(d+ex)^{3/2}} + \frac{4c\left(cd^2-3ae^2\right)\sqrt{a+cx^2}}{15e\left(cd^2+ae^2\right)^2\sqrt{d+ex}} + \frac{4\sqrt{-a}c^{3/2}\left(cd^2-3ae^2\right)}{15e^2\left(\right.}$$

Mathematica [C] time = 2.13211, size = 602, normalized size = 1.36

$$2\left(-e^2\left(a+cx^2\right)\left(3a^2e^4+2ace^2\left(5d^2+5dex+3e^2x^2\right)-c^2d^2\left(d^2+6dex+2e^2x^2\right)\right)+\frac{2c(d+ex)^2\left(\sqrt{a}\sqrt{c}e(d+ex)^{3/2}\left(4i\sqrt{a}\sqrt{c}de-3ae^2+\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(d + e*x)^(7/2),x]
```

```
[Out] (2*(-(e^2*(a + c*x^2)*(3*a^2*e^4 - c^2*d^2*(d^2 + 6*d*e*x + 2*e^2*x^2) + 2*
a*c*e^2*(5*d^2 + 5*d*e*x + 3*e^2*x^2))) + (2*c*(d + e*x)^2*(-(e^2*Sqrt[-d -
 (I*Sqrt[a]*e)/Sqrt[c]]*(-3*a^2*e^2 + c^2*d^2*x^2 + a*c*(d^2 - 3*e^2*x^2)))
 + Sqrt[c]*(I*c^(3/2)*d^3 - Sqrt[a]*c*d^2*e - (3*I)*a*Sqrt[c]*d*e^2 + 3*a^(
3/2)*e^3)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*
e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d -
 (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*
d + I*Sqrt[a]*e)] + Sqrt[a]*Sqrt[c]*e*(c*d^2 + (4*I)*Sqrt[a]*Sqrt[c]*d*e -
3*a*e^2)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e
)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d -
(I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d
 + I*Sqrt[a]*e)]))/Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]))/(15*e^3*(c*d^2 + a*e^
2)^2*(d + e*x)^(5/2)*Sqrt[a + c*x^2])
```

Maple [B] time = 0.286, size = 3409, normalized size = 7.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/(e*x+d)^(7/2),x)
```

```
[Out] 2/15*(2*x^4*c^3*d^2*e^4+6*x^3*c^3*d^3*e^3-9*x^2*a^2*c*e^6+x^2*c^3*d^4*e^2-6
*x^4*a*c^2*e^6-8*x^2*a*c^2*d^2*e^4-10*x*a^2*c*d*e^5+6*x*a*c^2*d^3*e^3+2*Ell
ipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*
c)^(1/2)*e+c*d))^(1/2))*c^3*d^6*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-
c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*
c)^(1/2)*e-c*d))^(1/2)-2*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),
(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*c^2*d^5*e*(-a*c)^(1/2)*
(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2
)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-10*x^3*a*
c^2*d*e^5+12*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1
/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*a^2*c*d*e^5*(-(e*x+d)*c/((-a*c)^(
1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x
+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+12*EllipticF((-(e*x+d)*c/((-a*
c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*
x*a*c^2*d^3*e^3*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2)
)*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))
^(1/2)-4*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*
e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*c^2*d^4*e^2*(-a*c)^(1/2)*(-(e*x+d)*c/
((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1
/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-12*EllipticE((-(e*x+d
)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d)
)^(1/2))*x*a^2*c*d*e^5*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c
)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*
e-c*d))^(1/2)-8*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)
^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*a*c^2*d^3*e^3*(-(e*x+d)*c/((-a
*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*
((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-2*EllipticF((-(e*x+d)*c/(
(-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/
2))*a*c*d^3*e^3*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x
+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^
(1/2)*e-c*d))^(1/2)+6*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-(
(-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a*c^2*d^2*e^4*(-(e*x+d
)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d)
)^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-2*EllipticF((-(e*
x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c
*d))^(1/2))*x^2*c^2*d^3*e^3*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^
(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2)
)*e/((-a*c)^(1/2)*e-c*d))^(1/2)-4*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d
))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a*c^2*d^2*
e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^
(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-6*Ell
ipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*
c)^(1/2)*e+c*d))^(1/2))*a^2*c*d^2*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/
2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e
/((-a*c)^(1/2)*e-c*d))^(1/2)+6*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^
(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a^2*c*e^6*(-(
e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e
+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-4*EllipticE(
(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2
)*e+c*d))^(1/2))*a*c^2*d^4*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c
```

```
*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c
)^(1/2)*e-c*d))^(1/2)-6*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(
-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a^2*c*e^6*(-(e*x+d)*
c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^
(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+2*EllipticE((-(e*x+
d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d
))^(1/2))*x^2*c^3*d^4*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-
a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/
2)*e-c*d))^(1/2)+4*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a
*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*c^3*d^5*e*(-(e*x+d)*c/((-a*
c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*(
(c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+6*EllipticF((-(e*x+d)*c/((
-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2
))*a^2*c*d^2*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2
))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d)
)^(1/2)+6*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)
*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c^2*d^4*e^2*(-(e*x+d)*c/((-a*c)^(1/2
)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-
a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-2*EllipticF((-(e*x+d)*c/((-a*c)^(
1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*
a*c*d*e^5*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c
)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*
e-c*d))^(1/2)-4*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)
^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*a*c*d^2*e^4*(-a*c)^(1/2)*(-(e*
x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c
*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-3*a^3*e^6-10*a
^2*c*d^2*e^4+a*c^2*d^4*e^2)/(c*x^2+a)^(1/2)/(a*e^2+c*d^2)^2/e^3/(e*x+d)^(5/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+a}}{(ex+d)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/(e*x + d)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+a}\sqrt{ex+d}}{e^4x^4+4\,de^3x^3+6\,d^2e^2x^2+4\,d^3ex+d^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*sqrt(e*x + d)/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x
^2 + 4*d^3*e*x + d^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/(e*x+d)**(7/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(d + e*x)**(7/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+a}}{(ex+d)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)/(e*x + d)^(7/2), x)
```

3.663 $\int (d+ex)^{3/2}\left(a+cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=497

$$-\frac{8\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\left(-15a^2e^4+21acd^2e^2+4c^2d^4\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{1155c^{3/2}e^4\sqrt{a+cx^2}\sqrt{d+ex}}+\frac{4\sqrt{\cdots}}{}$$

```
[Out] (4*Sqrt[d + e*x]*(4*c^2*d^4 + 21*a*c*d^2*e^2 - 15*a^2*e^4 - 3*c*d*e*(c*d^2
- 31*a*e^2)*x)*Sqrt[a + c*x^2])/(1155*c*e^3) + (2*Sqrt[d + e*x]*(c*d^2 - 3*
a*e^2 + 28*c*d*e*x)*(a + c*x^2)^(3/2))/(231*c*e) + (2*e*Sqrt[d + e*x]*(a +
c*x^2)^(5/2))/(11*c) + (32*Sqrt[-a]*d*(c*d^2 - 3*a*e^2)*(c*d^2 + 9*a*e^2)*S
qrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt
[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(1155*Sqrt[c]*e^4*Sqr
t[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) - (8*Sqrt[
-a]*(c*d^2 + a*e^2)*(4*c^2*d^4 + 21*a*c*d^2*e^2 - 15*a^2*e^4)*Sqrt[(Sqrt[c]
*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[
Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e
)])/(1155*c^(3/2)*e^4*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.559344, antiderivative size = 497, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {743, 815, 844, 719, 424, 419}

$$\frac{4\sqrt{a+cx^2}\sqrt{d+ex}\left(-15a^2e^4-3cdex\left(cd^2-31ae^2\right)+21acd^2e^2+4c^2d^4\right)}{1155ce^3}-\frac{8\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\left(-15a^2e^4+21ac\cdots\right.}{1155c^3\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*(a + c*x^2)^(3/2),x]
```


```
[Out] (4*Sqrt[d + e*x]*(4*c^2*d^4 + 21*a*c*d^2*e^2 - 15*a^2*e^4 - 3*c*d*e*(c*d^2
- 31*a*e^2)*x)*Sqrt[a + c*x^2])/(1155*c*e^3) + (2*Sqrt[d + e*x]*(c*d^2 - 3*
a*e^2 + 28*c*d*e*x)*(a + c*x^2)^(3/2))/(231*c*e) + (2*e*Sqrt[d + e*x]*(a +
c*x^2)^(5/2))/(11*c) + (32*Sqrt[-a]*d*(c*d^2 - 3*a*e^2)*(c*d^2 + 9*a*e^2)*S
qrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt
[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(1155*Sqrt[c]*e^4*Sqr
t[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) - (8*Sqrt[
-a]*(c*d^2 + a*e^2)*(4*c^2*d^4 + 21*a*c*d^2*e^2 - 15*a^2*e^4)*Sqrt[(Sqrt[c]
*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[
Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e
)])/(1155*c^(3/2)*e^4*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```


Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
```

```
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int (d+ex)^{3/2}\left(a+cx^2\right)^{3/2} dx = \frac{2e\sqrt{d+ex}\left(a+cx^2\right)^{5/2}}{11c} + \frac{2\int \frac{\left(\frac{1}{2}\left(11cd^2-ae^2\right)+6cdex\right)\left(a+cx^2\right)^{3/2}}{\sqrt{d+ex}}dx}{11c}$$

$$= \frac{2\sqrt{d+ex}\left(cd^2-3ae^2+28cdex\right)\left(a+cx^2\right)^{3/2}}{231ce} + \frac{2e\sqrt{d+ex}\left(a+cx^2\right)^{5/2}}{11c} + \frac{8\int \frac{\left(\frac{3}{4}ace^2(29ca\right.}{}}{}$$

$$= \frac{4\sqrt{d+ex}\left(4c^2d^4+21acd^2e^2-15a^2e^4-3cde\left(cd^2-31ae^2\right)x\right)\sqrt{a+cx^2}}{1155ce^3} + \frac{2\sqrt{d+ex}\left(ca\right.}{}$$

$$= \frac{4\sqrt{d+ex}\left(4c^2d^4+21acd^2e^2-15a^2e^4-3cde\left(cd^2-31ae^2\right)x\right)\sqrt{a+cx^2}}{1155ce^3} + \frac{2\sqrt{d+ex}\left(ca\right.}{}$$

$$= \frac{4\sqrt{d+ex}\left(4c^2d^4+21acd^2e^2-15a^2e^4-3cde\left(cd^2-31ae^2\right)x\right)\sqrt{a+cx^2}}{1155ce^3} + \frac{2\sqrt{d+ex}\left(ca\right.}{}$$

$$= \frac{4\sqrt{d+ex}\left(4c^2d^4+21acd^2e^2-15a^2e^4-3cde\left(cd^2-31ae^2\right)x\right)\sqrt{a+cx^2}}{1155ce^3} + \frac{2\sqrt{d+ex}\left(ca\right.}{}$$

Mathematica [C] time = 3.98236, size = 695, normalized size = 1.4

$$2\sqrt{d+ex}\left(e^2\left(a+cx^2\right)\left(60a^2e^4+ace^2\left(47d^2+326dex+195e^2x^2\right)+c^2\left(5d^2e^2x^2-6d^3ex+8d^4+140de^3x^3+105e^4x^4\right)\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)*(a + c*x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(e^2*(a + c*x^2)*(60*a^2*e^4 + a*c*e^2*(47*d^2 + 326*d*e*x
 + 195*e^2*x^2) + c^2*(8*d^4 - 6*d^3*e*x + 5*d^2*e^2*x^2 + 140*d*e^3*x^3 +
105*e^4*x^4)) + (4*(-4*d*e^2*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*(c^2*d^4 + 6*
a*c*d^2*e^2 - 27*a^2*e^4)*(a + c*x^2) + 4*Sqrt[c]*d*(I*c^(5/2)*d^5 - Sqrt[a
]*c^2*d^4*e + (6*I)*a*c^(3/2)*d^3*e^2 - 6*a^(3/2)*c*d^2*e^3 - (27*I)*a^2*Sq
rt[c]*d*e^4 + 27*a^(5/2)*e^5)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]
*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticE
[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*
Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)] + Sqrt[a]*e*(4*c^(5/2)*d^5 + I*Sqrt[a
]*c^2*d^4*e + 24*a*c^(3/2)*d^3*e^2 + (114*I)*a^(3/2)*c*d^2*e^3 - 108*a^2*Sq
rt[c]*d*e^4 - (15*I)*a^(5/2)*e^5)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e
*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*Ellip
ticF[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d
- I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)]))/(Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c
]]*(d + e*x))))/(1155*c*e^5*Sqrt[a + c*x^2])
```

Maple [B] time = 0.274, size = 1970, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(c*x^2+a)^(3/2),x)
```

```
[Out] 2/1155*(e*x+d)^(1/2)*(c*x^2+a)^(1/2)*(8*a*c^3*d^5*e^2-24*(-(e*x+d)*c/((-a*c
)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((
c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*
c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*
(-a*c)^(1/2)*a^2*c*d^2*e^5-100*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c
*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c
)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-(
(-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*(-a*c)^(1/2)*a*c^2*d^4*e^3
+518*x^3*a*c^3*d^2*e^5+581*x^2*a^2*c^2*d*e^6+46*x^2*a*c^3*d^3*e^4+373*x*a^2
*c^2*d^2*e^5+2*x*a*c^3*d^4*e^3+766*x^4*a*c^3*d*e^6+60*(-(e*x+d)*c/((-a*c)^(
1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x
+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^
(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*(-a
*c)^(1/2)*a^3*e^7+245*x^6*c^4*d*e^6+300*x^5*a*c^3*e^7+145*x^5*c^4*d^2*e^5-x
^4*c^4*d^3*e^4+255*x^3*a^2*c^2*e^7+2*x^3*c^4*d^4*e^3+8*x^2*c^4*d^5*e^2+60*x
*a^3*c*e^7+105*x^7*c^4*e^7+112*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c
*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c
)^(1/2)*e-c*d))^(1/2)*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-(
(-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c^3*d^5*e^2-432*(-(e*x+d
)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d)
)^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticE((-(e*x+
d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d
))^(1/2))*a^3*c*d*e^6-336*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-
a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/
2)*e-c*d))^(1/2)*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c
)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*c^2*d^3*e^4-16*(-(e*x+d)*c/
((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1
/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c
/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(
1/2))*(-a*c)^(1/2)*c^3*d^6*e-12*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-
c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*
c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-
((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c^3*d^5*e^2+372*(-(e*x+
d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d
))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x
+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*
d))^(1/2))*a^3*c*d*e^6+360*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(
-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1
/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*
c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*c^2*d^3*e^4+16*(-(e*x+d)*c
/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(
1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticE((-(e*x+d)*
c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^
(1/2))*c^4*d^7+60*a^3*c*d*e^6+47*a^2*c^2*d^3*e^4)/c^2/e^5/(c*e*x^3+c*d*x^2+
a*e*x+a*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + a\right)^{\frac{3}{2}} (ex + d)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^(3/2)*(e*x + d)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cex^3 + cdx^2 + aex + ad\right)\sqrt{cx^2 + a}\sqrt{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*e*x^3 + c*d*x^2 + a*e*x + a*d)*sqrt(c*x^2 + a)*sqrt(e*x + d), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + cx^2\right)^{\frac{3}{2}} (d + ex)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(c*x**2+a)**(3/2),x)
```

```
[Out] Integral((a + c*x**2)**(3/2)*(d + e*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + a\right)^{\frac{3}{2}} (ex + d)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^(3/2)*(e*x + d)^(3/2), x)
```

3.664 $\int \sqrt{d+ex}\left(a+cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=448

$$-\frac{32\sqrt{-a}d\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\left(3ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{315\sqrt{c}e^4\sqrt{a+cx^2}\sqrt{d+ex}}+\frac{8\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{d+e\ldots}}{}$$

```
[Out] (4*Sqrt[d + e*x]*(4*d*(c*d^2 + 3*a*e^2) - 3*e*(c*d^2 - 7*a*e^2)*x)*Sqrt[a +
 c*x^2])/(315*e^3) - (4*d*Sqrt[d + e*x]*(a + c*x^2)^(3/2))/(21*e) + (2*(d +
 e*x)^(3/2)*(a + c*x^2)^(3/2))/(9*e) + (8*Sqrt[-a]*(4*c^2*d^4 + 15*a*c*d^2*
e^2 - 21*a^2*e^4)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1
 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(3
15*Sqrt[c]*e^4*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a +
c*x^2]) - (32*Sqrt[-a]*d*(c*d^2 + a*e^2)*(c*d^2 + 3*a*e^2)*Sqrt[(Sqrt[c]*(d
 + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqr
t[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])
/(315*Sqrt[c]*e^4*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.467111, antiderivative size = 448, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {735, 833, 815, 844, 719, 424, 419}

$$\frac{8\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}\left(-21a^2e^4+15acd^2e^2+4c^2d^4\right)E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{315\sqrt{c}e^4\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}}+\frac{4\sqrt{a+cx^2}\sqrt{d+ex}\left(4d\left(3ae^2+\ldots\right.\right.}{315e^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(a + c*x^2)^(3/2),x]
```


```
[Out] (4*Sqrt[d + e*x]*(4*d*(c*d^2 + 3*a*e^2) - 3*e*(c*d^2 - 7*a*e^2)*x)*Sqrt[a +
 c*x^2])/(315*e^3) - (4*d*Sqrt[d + e*x]*(a + c*x^2)^(3/2))/(21*e) + (2*(d +
 e*x)^(3/2)*(a + c*x^2)^(3/2))/(9*e) + (8*Sqrt[-a]*(4*c^2*d^4 + 15*a*c*d^2*
e^2 - 21*a^2*e^4)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1
 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(3
15*Sqrt[c]*e^4*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a +
c*x^2]) - (32*Sqrt[-a]*d*(c*d^2 + a*e^2)*(c*d^2 + 3*a*e^2)*Sqrt[(Sqrt[c]*(d
 + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqr
t[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])
/(315*Sqrt[c]*e^4*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```


Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \sqrt{d+ex}\left(a+cx^2\right)^{3/2} dx = \frac{2(d+ex)^{3/2}\left(a+cx^2\right)^{3/2}}{9e} + \frac{2\int (ae-cdx)\sqrt{d+ex}\sqrt{a+cx^2}\,dx}{3e}$$

$$= -\frac{4d\sqrt{d+ex}\left(a+cx^2\right)^{3/2}}{21e} + \frac{2(d+ex)^{3/2}\left(a+cx^2\right)^{3/2}}{9e} + \frac{4\int \frac{\left(4acde-\frac{1}{2}c\left(cd^2-7ae^2\right)x\right)\sqrt{a+cx^2}}{\sqrt{d+ex}}\,dx}{21ce}$$

$$= \frac{4\sqrt{d+ex}\left(4d\left(cd^2+3ae^2\right)-3e\left(cd^2-7ae^2\right)x\right)\sqrt{a+cx^2}}{315e^3} - \frac{4d\sqrt{d+ex}\left(a+cx^2\right)^{3/2}}{21e} + \frac{2(d+ \cdots}{\cdots}$$

$$= \frac{4\sqrt{d+ex}\left(4d\left(cd^2+3ae^2\right)-3e\left(cd^2-7ae^2\right)x\right)\sqrt{a+cx^2}}{315e^3} - \frac{4d\sqrt{d+ex}\left(a+cx^2\right)^{3/2}}{21e} + \frac{2(d+ \cdots}{\cdots}$$

$$= \frac{4\sqrt{d+ex}\left(4d\left(cd^2+3ae^2\right)-3e\left(cd^2-7ae^2\right)x\right)\sqrt{a+cx^2}}{315e^3} - \frac{4d\sqrt{d+ex}\left(a+cx^2\right)^{3/2}}{21e} + \frac{2(d+ \cdots}{\cdots}$$

$$= \frac{4\sqrt{d+ex}\left(4d\left(cd^2+3ae^2\right)-3e\left(cd^2-7ae^2\right)x\right)\sqrt{a+cx^2}}{315e^3} - \frac{4d\sqrt{d+ex}\left(a+cx^2\right)^{3/2}}{21e} + \frac{2(d+ \cdots}{\cdots}$$

Mathematica [C] time = 4.16675, size = 646, normalized size = 1.44

$$\frac{\sqrt{d+ex}\left(\frac{2\left(a+cx^2\right)\left(ae^2(29d+77ex)+c\left(-6d^2ex+8d^3+5de^2x^2+35e^3x^3\right)\right)}{e^3} + \frac{8\left(\sqrt{a}\sqrt{c}e(d+ex)^{3/2}\left(33ia^{3/2}\sqrt{c}de^3-21a^2e^4+i\sqrt{a}c^{3/2}d^3e+15acd^2e^2+4c^2d^4\right)\sqrt{\frac{e\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{d+ex}} \cdots\right.}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*(a + c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d + e*x]*((2*(a + c*x^2)*(a*e^2*(29*d + 77*e*x) + c*(8*d^3 - 6*d^2*e*
x + 5*d*e^2*x^2 + 35*e^3*x^3)))/e^3 + (8*(-(e^2*Sqrt[-d - (I*Sqrt[a]*e)/Sqr
t[c]]*(4*c^2*d^4 + 15*a*c*d^2*e^2 - 21*a^2*e^4)*(a + c*x^2)) + Sqrt[c]*((4*
I)*c^(5/2)*d^5 - 4*Sqrt[a]*c^2*d^4*e + (15*I)*a*c^(3/2)*d^3*e^2 - 15*a^(3/2
)*c*d^2*e^3 - (21*I)*a^2*Sqrt[c]*d*e^4 + 21*a^(5/2)*e^5)*Sqrt[(e*((I*Sqrt[a
])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))
]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt
[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)] + Sqrt[a]*
Sqrt[c]*e*(4*c^2*d^4 + I*Sqrt[a]*c^(3/2)*d^3*e + 15*a*c*d^2*e^2 + (33*I)*a^
(3/2)*Sqrt[c]*d*e^3 - 21*a^2*e^4)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e
*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*Ellip
ticF[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d
- I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)]))/(c*e^5*Sqrt[-d - (I*Sqrt[a]*e)/
Sqrt[c]]*(d + e*x))))/(315*Sqrt[a + c*x^2])
```

Maple [B] time = 0.25, size = 1731, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(e*x+d)^(1/2),x)
```

```
[Out] 2/315*(c*x^2+a)^(1/2)*(e*x+d)^(1/2)*(35*x^6*c^3*e^6+40*x^5*c^3*d*e^5+84*(-(
e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e
+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-
(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*
e+c*d))^(1/2))*a^3*e^6+72*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)
,(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*c*d^2*e^4*(-(e*x+d
)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d)
)^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-48*(-(e*x+d)*c/((
-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2
)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/(
(-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/
2))*(-a*c)^(1/2)*a^2*d*e^5-12*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(
1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c^2*d^4*e^2*(-(e
*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+
c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-64*EllipticF(
(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2
)*e+c*d))^(1/2))*a*c*d^3*e^3*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))
^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2
))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-16*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c
*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*c^2*d^5*e*(-
a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/(
(-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2
)-84*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)
^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*Elli
pticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c
)^(1/2)*e+c*d))^(1/2))*a^3*e^6-24*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d
))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*c*d^2*e^4*
(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2
)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+76*Ellipt
icE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^
(1/2)*e+c*d))^(1/2))*a*c^2*d^4*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*
((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((
-a*c)^(1/2)*e-c*d))^(1/2)+16*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1
/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*c^3*d^6*(-(e*x+d)*c
/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(
1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+112*x^4*a*c^2*e^6-x^
4*c^3*d^2*e^4+146*x^3*a*c^2*d*e^5+2*x^3*c^3*d^3*e^3+77*x^2*a^2*c*e^6+28*x^2
*a*c^2*d^2*e^4+8*x^2*c^3*d^4*e^2+106*x*a^2*c*d*e^5+2*x*a*c^2*d^3*e^3+29*a^2
*c*d^2*e^4+8*a*c^2*d^4*e^2)/c/(c*e*x^3+c*d*x^2+a*e*x+a*d)/e^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+a\right)^{\frac{3}{2}}\sqrt{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^(3/2)*sqrt(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2+a\right)^{\frac{3}{2}}\sqrt{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + a)^(3/2)*sqrt(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+cx^2\right)^{\frac{3}{2}}\sqrt{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(e*x+d)**(1/2),x)
```

```
[Out] Integral((a + c*x**2)**(3/2)*sqrt(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^2+a\right)^{\frac{3}{2}}\sqrt{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^(3/2)*sqrt(e*x + d), x)
```

3.665 $\int \frac{(a+cx^2)^{3/2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=393

$$-\frac{8\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\left(5ae^2+4cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{35\sqrt{c}e^4\sqrt{a+cx^2}\sqrt{d+ex}}+\frac{4\sqrt{a+cx^2}\sqrt{d+ex}\,(}{35}$$

```
[Out] (4*Sqrt[d + e*x]*(4*c*d^2 + 5*a*e^2 - 3*c*d*e*x)*Sqrt[a + c*x^2])/(35*e^3)
+ (2*Sqrt[d + e*x]*(a + c*x^2)^(3/2))/(7*e) + (32*Sqrt[-a]*Sqrt[c]*d*(c*d^2
 + 2*a*e^2)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sq
rt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(35*e^4*
Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) - (8*Sq
rt[-a]*(c*d^2 + a*e^2)*(4*c*d^2 + 5*a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c
]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x
)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(35*Sqrt[c]*e^4
*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.357265, antiderivative size = 393, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {735, 815, 844, 719, 424, 419}

$$\frac{4\sqrt{a+cx^2}\sqrt{d+ex}\left(5ae^2+4cd^2-3cdex\right)}{35e^3}-\frac{8\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\left(5ae^2+4cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\right.}{35\sqrt{c}e^4\sqrt{a+cx^2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2)/Sqrt[d + e*x],x]
```


```
[Out] (4*Sqrt[d + e*x]*(4*c*d^2 + 5*a*e^2 - 3*c*d*e*x)*Sqrt[a + c*x^2])/(35*e^3)
+ (2*Sqrt[d + e*x]*(a + c*x^2)^(3/2))/(7*e) + (32*Sqrt[-a]*Sqrt[c]*d*(c*d^2
 + 2*a*e^2)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sq
rt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(35*e^4*
Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) - (8*Sq
rt[-a]*(c*d^2 + a*e^2)*(4*c*d^2 + 5*a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c
]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x
)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(35*Sqrt[c]*e^4
*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```


Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(a+cx^2)^{3/2}}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{d+ex}\,(a+cx^2)^{3/2}}{7e} + \frac{6\int \frac{(ae-cdx)\sqrt{a+cx^2}}{\sqrt{d+ex}}\,dx}{7e}$$

$$= \frac{4\sqrt{d+ex}\,(4cd^2+5ae^2-3cdex)\sqrt{a+cx^2}}{35e^3} + \frac{2\sqrt{d+ex}\,(a+cx^2)^{3/2}}{7e} + \frac{8\int \frac{\frac{1}{2}ace(cd^2+5ae^2)-2c^2d(cd^2+2a}{\sqrt{d+ex}\sqrt{a+cx^2}}}{35ce^3}$$

$$= \frac{4\sqrt{d+ex}\,(4cd^2+5ae^2-3cdex)\sqrt{a+cx^2}}{35e^3} + \frac{2\sqrt{d+ex}\,(a+cx^2)^{3/2}}{7e} - \frac{(16cd(cd^2+2ae^2))\int \frac{\sqrt{d+e}}{\sqrt{a+c}}}{35e^4}$$

$$= \frac{4\sqrt{d+ex}\,(4cd^2+5ae^2-3cdex)\sqrt{a+cx^2}}{35e^3} + \frac{2\sqrt{d+ex}\,(a+cx^2)^{3/2}}{7e} - \frac{\left(32a\sqrt{c}d(cd^2+2ae^2)\sqrt{d+}\right.}{3}$$

$$= \frac{4\sqrt{d+ex}\,(4cd^2+5ae^2-3cdex)\sqrt{a+cx^2}}{35e^3} + \frac{2\sqrt{d+ex}\,(a+cx^2)^{3/2}}{7e} + \frac{32\sqrt{-a}\sqrt{c}d(cd^2+2ae^2)\sqrt{}}{35}$$

Mathematica [C] time = 3.24404, size = 575, normalized size = 1.46

$$\sqrt{d+ex}\left(\frac{2(a+cx^2)(15ae^2+c(8d^2-6dex+5e^2x^2))}{e^3} - \frac{8\left(-\sqrt{a}e(d+ex)^{3/2}(5ia^{3/2}e^3+i\sqrt{a}cd^2e+8a\sqrt{c}de^2+4c^{3/2}d^3)\sqrt{\frac{e\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{d+ex}}\sqrt{-\frac{-ex+\frac{i\sqrt{a}e}{\sqrt{c}}}{d+ex}}\,\text{EllipticF}\left(i\sinh^{-1}\left(\right.\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2)/Sqrt[d + e*x],x]
```

```
[Out] (Sqrt[d + e*x]*((2*(a + c*x^2)*(15*a*e^2 + c*(8*d^2 - 6*d*e*x + 5*e^2*x^2))
)/e^3 - (8*(4*d*e^2*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*(2*a^2*e^2 + c^2*d^2*x
^2 + a*c*(d^2 + 2*e^2*x^2)) + 4*Sqrt[c]*d*((-I)*c^(3/2)*d^3 + Sqrt[a]*c*d^2
*e - (2*I)*a*Sqrt[c]*d*e^2 + 2*a^(3/2)*e^3)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] +
x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(
3/2)*EllipticE[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (
Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)] - Sqrt[a]*e*(4*c^(3/2)*
d^3 + I*Sqrt[a]*c*d^2*e + 8*a*Sqrt[c]*d*e^2 + (5*I)*a^(3/2)*e^3)*Sqrt[(e*((
I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d
 + e*x))]*(d + e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[
c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)]))/
(e^5*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*(d + e*x))))/(35*Sqrt[a + c*x^2])
```

Maple [B] time = 0.25, size = 1385, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)/(e*x+d)^(1/2),x)
```

```
[Out] -2/35*(c*x^2+a)^(1/2)*(e*x+d)^(1/2)*(-5*x^5*c^3*e^5+20*(-a*c)^(1/2)*(-(e*x+
d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d
))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x
+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*
d))^(1/2))*a^2*e^5+36*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*
((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((
-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)
,(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c*d^2*e^3+16*(-a*c)^
(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c
)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*Ell
ipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*
c)^(1/2)*e+c*d))^(1/2))*c^2*d^4*e+12*a^2*c*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d)
)^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/
2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d
))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*d*e^4+12*a*c^2
*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/
2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*Elliptic
F((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1
/2)*e+c*d))^(1/2))*d^3*e^2-32*a^2*c*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)
*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/(
(-a*c)^(1/2)*e-c*d))^(1/2)*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2
),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*d*e^4-48*a*c^2*(-(e*x
+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*
d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticE((-(e*
x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c
*d))^(1/2))*d^3*e^2-16*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c
)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*
e-c*d))^(1/2)*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(
1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*c^3*d^5+x^4*c^3*d*e^4-20*x^3*a*c^2
*e^5-2*x^3*c^3*d^2*e^3-14*x^2*a*c^2*d*e^4-8*x^2*c^3*d^3*e^2-15*x*a^2*c*e^5-
2*x*a*c^2*d^2*e^3-15*a^2*c*d*e^4-8*a*c^2*d^3*e^2)/c/e^5/(c*e*x^3+c*d*x^2+a*
e*x+a*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + a)^{\frac{3}{2}}}{\sqrt{ex + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^(3/2)/sqrt(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^2 + a)^{\frac{3}{2}}}{\sqrt{ex + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + a)^(3/2)/sqrt(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+cx^2\right)^{\frac{3}{2}}}{\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)/(e*x+d)**(1/2),x)
```

```
[Out] Integral((a + c*x**2)**(3/2)/sqrt(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+a\right)^{\frac{3}{2}}}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^(3/2)/sqrt(e*x + d), x)
```

3.666 $\int \frac{(a+cx^2)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=369

$$\frac{32\sqrt{-a}\sqrt{c}d\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{5e^4\sqrt{a+cx^2}\sqrt{d+ex}}-\frac{8\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}\left(3ae^2+\right.}{5e^4\sqrt{a+}}$$

```
[Out] (-4*c*(4*d - 3*e*x)*Sqrt[d + e*x]*Sqrt[a + c*x^2])/(5*e^3) - (2*(a + c*x^2)
^(3/2))/(e*Sqrt[d + e*x]) - (8*Sqrt[-a]*Sqrt[c]*(4*c*d^2 + 3*a*e^2)*Sqrt[d
+ e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/
Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(5*e^4*Sqrt[(Sqrt[c]*(d + e
*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) + (32*Sqrt[-a]*Sqrt[c]*d*(c
*d^2 + a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (
c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e
)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(5*e^4*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.306889, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {733, 815, 844, 719, 424, 419}

$$\frac{32\sqrt{-a}\sqrt{c}d\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{5e^4\sqrt{a+cx^2}\sqrt{d+ex}}-\frac{8\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}\left(3ae^2+4cd^2\right)E}{5e^4\sqrt{a+cx^2}\sqrt{\frac{\sqrt{}}{\sqrt{}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2)/(d + e*x)^(3/2),x]
```


```
[Out] (-4*c*(4*d - 3*e*x)*Sqrt[d + e*x]*Sqrt[a + c*x^2])/(5*e^3) - (2*(a + c*x^2)
^(3/2))/(e*Sqrt[d + e*x]) - (8*Sqrt[-a]*Sqrt[c]*(4*c*d^2 + 3*a*e^2)*Sqrt[d
+ e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/
Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(5*e^4*Sqrt[(Sqrt[c]*(d + e
*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) + (32*Sqrt[-a]*Sqrt[c]*d*(c
*d^2 + a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (
c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e
)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(5*e^4*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```


Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
```

```
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(a+cx^2)^{3/2}}{(d+ex)^{3/2}}\,dx = -\frac{2(a+cx^2)^{3/2}}{e\sqrt{d+ex}} + \frac{(6c)\int \frac{x\sqrt{a+cx^2}}{\sqrt{d+ex}}\,dx}{e}$$

$$= -\frac{4c(4d-3ex)\sqrt{d+ex}\sqrt{a+cx^2}}{5e^3} - \frac{2(a+cx^2)^{3/2}}{e\sqrt{d+ex}} + \frac{8\int \frac{-\frac{1}{2}acde+\frac{1}{2}c(4cd^2+3ae^2)x}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx}{5e^3}$$

$$= -\frac{4c(4d-3ex)\sqrt{d+ex}\sqrt{a+cx^2}}{5e^3} - \frac{2(a+cx^2)^{3/2}}{e\sqrt{d+ex}} - \frac{(16cd(cd^2+ae^2))\int \frac{1}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx}{5e^4} + \frac{(4c(4cd^2 \cdots}{\cdots}$$

$$= -\frac{4c(4d-3ex)\sqrt{d+ex}\sqrt{a+cx^2}}{5e^3} - \frac{2(a+cx^2)^{3/2}}{e\sqrt{d+ex}} + \frac{\left(8a\sqrt{c}(4cd^2+3ae^2)\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \cdots\right.}{5\sqrt{-a}e^4\sqrt{\frac{c(d+ex)}{cd-\frac{a\sqrt{c}e}{\sqrt{-a}}}}\sqrt{a+\cdots}}$$

$$= -\frac{4c(4d-3ex)\sqrt{d+ex}\sqrt{a+cx^2}}{5e^3} - \frac{2(a+cx^2)^{3/2}}{e\sqrt{d+ex}} - \frac{8\sqrt{-a}\sqrt{c}(4cd^2+3ae^2)\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\cdots\right.}{5e^4\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}}\sqrt{a+cx^2}}$$

Mathematica [C] time = 3.00988, size = 565, normalized size = 1.53

$$\frac{2\sqrt{a+cx^2}\left(c\left(-8d^2-2dex+e^2x^2\right)-5ae^2\right)}{5e^3\sqrt{d+ex}} + \frac{8\left(-\sqrt{a}\sqrt{c}e(d+ex)^{3/2}\left(i\sqrt{a}\sqrt{c}de+3ae^2+4cd^2\right)\sqrt{\frac{e\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{d+ex}}\sqrt{-\frac{-ex+\frac{i\sqrt{a}e}{\sqrt{c}}}{d+ex}}\text{Ellip}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2)/(d + e*x)^(3/2),x]
```

```
[Out] (2*Sqrt[a + c*x^2]*(-5*a*e^2 + c*(-8*d^2 - 2*d*e*x + e^2*x^2)))/(5*e^3*Sqrt
[d + e*x]) + (8*(e^2*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*(3*a^2*e^2 + 4*c^2*d^
2*x^2 + a*c*(4*d^2 + 3*e^2*x^2)) + Sqrt[c]*((-4*I)*c^(3/2)*d^3 + 4*Sqrt[a]*
c*d^2*e - (3*I)*a*Sqrt[c]*d*e^2 + 3*a^(3/2)*e^3)*Sqrt[(e*((I*Sqrt[a])/Sqrt[
c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e
*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x
]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)] - Sqrt[a]*Sqrt[c]*
e*(4*c*d^2 + I*Sqrt[a]*Sqrt[c]*d*e + 3*a*e^2)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c]
+ x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)
^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]],
 (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)]))/(5*e^5*Sqrt[-d - (I
*Sqrt[a]*e)/Sqrt[c]]*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Maple [B] time = 0.257, size = 1168, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)/(e*x+d)^(3/2),x)
```

```
[Out] 2/5*(c*x^2+a)^(1/2)*(e*x+d)^(1/2)*(12*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/
2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e
/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1
/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*e^4+12*(-(e*x+d
)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d)
)^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+
d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d
))^(1/2))*a*c*d^2*e^2+16*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a
*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2
)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)
^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*(-a*c)^(1/2)*a*d*e^3+16*(-(e*x+d
)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d)
)^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+
d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d
))^(1/2))*(-a*c)^(1/2)*c*d^3*e-12*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*(
(-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-
a*c)^(1/2)*e-c*d))^(1/2)*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),
(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*e^4-28*(-(e*x+d)*c/
((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1
/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticE((-(e*x+d)*c
/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(
1/2))*a*c*d^2*e^2-16*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^
(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-
c*d))^(1/2)*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/
2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*c^2*d^4+x^4*c^2*e^4-2*x^3*c^2*d*e^3-
4*x^2*a*c*e^4-8*x^2*c^2*d^2*e^2-2*x*a*c*d*e^3-5*a^2*e^4-8*a*c*d^2*e^2)/(c*e
*x^3+c*d*x^2+a*e*x+a*d)/e^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+a\right)^{\frac{3}{2}}}{\left(ex+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^(3/2)/(e*x + d)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+a\right)^{\frac{3}{2}}\sqrt{ex+d}}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + a)^(3/2)*sqrt(e*x + d)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+cx^2\right)^{\frac{3}{2}}}{(d+ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)/(e*x+d)**(3/2),x)
```

```
[Out] Integral((a + c*x**2)**(3/2)/(d + e*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+a\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^(3/2)/(e*x + d)^(3/2), x)
```

3.667 $\int \frac{(a+cx^2)^{3/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=358

$$-\frac{8\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\left(ae^2+4cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{3e^4\sqrt{a+cx^2}\sqrt{d+ex}}+\frac{32\sqrt{-a}c^{3/2}d\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}E}{3e^4\sqrt{a+cx}}$$

```
[Out] (4*c*(4*d + e*x)*Sqrt[a + c*x^2])/(3*e^3*Sqrt[d + e*x]) - (2*(a + c*x^2)^(3
/2))/(3*e*(d + e*x)^(3/2)) + (32*Sqrt[-a]*c^(3/2)*d*Sqrt[d + e*x]*Sqrt[1 +
(c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*
e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(3*e^4*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d
+ Sqrt[-a]*e)]*Sqrt[a + c*x^2]) - (8*Sqrt[-a]*Sqrt[c]*(4*c*d^2 + a*e^2)*Sqr
t[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*Ellipti
cF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[
c]*d - a*e)])/(3*e^4*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.255492, antiderivative size = 358, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {733, 813, 844, 719, 424, 419}

$$\frac{32\sqrt{-a}c^{3/2}d\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{3e^4\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}}-\frac{8\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\left(ae^2+4cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}F\left(\sin^{-1}\left(\right.\right.}{3e^4\sqrt{a+cx^2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2)/(d + e*x)^(5/2),x]
```


```
[Out] (4*c*(4*d + e*x)*Sqrt[a + c*x^2])/(3*e^3*Sqrt[d + e*x]) - (2*(a + c*x^2)^(3
/2))/(3*e*(d + e*x)^(3/2)) + (32*Sqrt[-a]*c^(3/2)*d*Sqrt[d + e*x]*Sqrt[1 +
(c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*
e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(3*e^4*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d
+ Sqrt[-a]*e)]*Sqrt[a + c*x^2]) - (8*Sqrt[-a]*Sqrt[c]*(4*c*d^2 + a*e^2)*Sqr
t[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*Ellipti
cF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[
c]*d - a*e)])/(3*e^4*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```


Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
```

```
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(a+cx^2)^{3/2}}{(d+ex)^{5/2}}\,dx = -\frac{2(a+cx^2)^{3/2}}{3e(d+ex)^{3/2}} + \frac{(2c)\int \frac{x\sqrt{a+cx^2}}{(d+ex)^{3/2}}\,dx}{e}$$

$$= \frac{4c(4d+ex)\sqrt{a+cx^2}}{3e^3\sqrt{d+ex}} - \frac{2(a+cx^2)^{3/2}}{3e(d+ex)^{3/2}} - \frac{(4c)\int \frac{-ae+4cdx}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx}{3e^3}$$

$$= \frac{4c(4d+ex)\sqrt{a+cx^2}}{3e^3\sqrt{d+ex}} - \frac{2(a+cx^2)^{3/2}}{3e(d+ex)^{3/2}} - \frac{(16c^2d)\int \frac{\sqrt{d+ex}}{\sqrt{a+cx^2}}\,dx}{3e^4} + \frac{(4c(4cd^2+ae^2))\int \frac{1}{\sqrt{d+ex}\sqrt{a+cx^2}}\,d}{3e^4}$$

$$= \frac{4c(4d+ex)\sqrt{a+cx^2}}{3e^3\sqrt{d+ex}} - \frac{2(a+cx^2)^{3/2}}{3e(d+ex)^{3/2}} - \frac{\left(32ac^{3/2}d\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}ex^2}{\sqrt{-a}\left(cd-\frac{a\sqrt{ce}}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx\right.}{3\sqrt{-a}e^4\sqrt{\frac{c(d+ex)}{cd-\frac{a\sqrt{ce}}{\sqrt{-a}}}}\sqrt{a+cx^2}}$$

$$= \frac{4c(4d+ex)\sqrt{a+cx^2}}{3e^3\sqrt{d+ex}} - \frac{2(a+cx^2)^{3/2}}{3e(d+ex)^{3/2}} + \frac{32\sqrt{-a}c^{3/2}d\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2}{\sqrt{-a}\sqrt{}}\right.}{3e^4\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}}\sqrt{a+cx^2}}$$

Mathematica [C] time = 2.36962, size = 494, normalized size = 1.38

$$\frac{2\sqrt{a+cx^2}\left(c\left(8d^2+10dex+e^2x^2\right)-ae^2\right)}{3e^3(d+ex)^{3/2}} - \frac{8c\left(-\sqrt{a}e(d+ex)^{3/2}\left(4\sqrt{c}d+i\sqrt{a}e\right)\sqrt{\frac{e\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{d+ex}}\sqrt{-\frac{-ex+\frac{i\sqrt{a}e}{\sqrt{c}}}{d+ex}}\text{EllipticF}\left(i\sinh\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2)/(d + e*x)^(5/2),x]
```

```
[Out] (2*Sqrt[a + c*x^2]*(-(a*e^2) + c*(8*d^2 + 10*d*e*x + e^2*x^2)))/(3*e^3*(d +
 e*x)^(3/2)) - (8*c*(4*d*e^2*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*(a + c*x^2) +
 4*Sqrt[c]*d*((-I)*Sqrt[c]*d + Sqrt[a]*e)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x)
)/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/
2)*EllipticE[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sq
rt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)] - Sqrt[a]*e*(4*Sqrt[c]*d
+ I*Sqrt[a]*e)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqr
t[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt
[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqr
t[c]*d + I*Sqrt[a]*e)]))/(3*e^5*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*Sqrt[d + e
*x]*Sqrt[a + c*x^2])
```

Maple [B] time = 0.261, size = 1597, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)/(e*x+d)^(5/2),x)
```

```
[Out] -2/3*(12*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*
e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*a*c*d*e^3*(-(e*x+d)*c/((-a*c)^(1/2)*e
-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c
)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+4*EllipticF((-(e*x+d)*c/((-a*c)^(1/2
)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*a*e^4
*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/
2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*(-a*c)^(
1/2)+16*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e
-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*c*d^2*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-
c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)
^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*(-a*c)^(1/2)-16*EllipticE((-(e*x+d)*c
/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(
1/2))*x*a*c*d*e^3*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/
2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d
))^(1/2)-16*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/
2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*c^2*d^3*e*(-(e*x+d)*c/((-a*c)^(1/2
)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-
a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+12*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*
d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(
1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c
*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c*d^2*e^2+
4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1
/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*Ellipti
cF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(
1/2)*e+c*d))^(1/2))*(-a*c)^(1/2)*a*d*e^3+16*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d
))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1
/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*
d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*(-a*c)^(1/2)*
c*d^3*e-16*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/(
(-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2
)*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/
((-a*c)^(1/2)*e+c*d))^(1/2))*a*c*d^2*e^2-16*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d
))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1
/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*
d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*c^2*d^4-x^4*c
^2*e^4-10*x^3*c^2*d*e^3-8*x^2*c^2*d^2*e^2-10*x*a*c*d*e^3+a^2*e^4-8*a*c*d^2*
e^2)/(c*x^2+a)^(1/2)/e^5/(e*x+d)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+a\right)^{\frac{3}{2}}}{(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^(3/2)/(e*x + d)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+a\right)^{\frac{3}{2}}\sqrt{ex+d}}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + a)^(3/2)*sqrt(e*x + d)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x
 + d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+cx^2\right)^{\frac{3}{2}}}{\left(d+ex\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)/(e*x+d)**(5/2),x)
```

```
[Out] Integral((a + c*x**2)**(3/2)/(d + e*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+a\right)^{\frac{3}{2}}}{\left(ex+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^(3/2)/(e*x + d)^(5/2), x)
```

3.668 $\int \frac{(a+cx^2)^{3/2}}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=410

$$\frac{32\sqrt{-a}c^{3/2}d\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{5e^4\sqrt{a+cx^2}\sqrt{d+ex}}-\frac{8\sqrt{-a}c^{3/2}\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}\left(3ae^2+4cd^2\right)E\left(\text{s}\right.}{5e^4\sqrt{a+cx^2}\left(ae^2+cd^2\right)\sqrt{}}$$

```
[Out] (-4*c*(2*d*(2*c*d^2 + a*e^2) + e*(5*c*d^2 + 3*a*e^2)*x)*Sqrt[a + c*x^2])/(5
*e^3*(c*d^2 + a*e^2)*(d + e*x)^(3/2)) - (2*(a + c*x^2)^(3/2))/(5*e*(d + e*x
)^(5/2)) - (8*Sqrt[-a]*c^(3/2)*(4*c*d^2 + 3*a*e^2)*Sqrt[d + e*x]*Sqrt[1 + (
c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e
)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(5*e^4*(c*d^2 + a*e^2)*Sqrt[(Sqrt[c]*(d + e*
x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) + (32*Sqrt[-a]*c^(3/2)*d*Sqr
t[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*Ellipti
cF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[
c]*d - a*e)])/(5*e^4*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.314088, antiderivative size = 410, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {733, 811, 844, 719, 424, 419}

$$-\frac{8\sqrt{-a}c^{3/2}\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}\left(3ae^2+4cd^2\right)E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{5e^4\sqrt{a+cx^2}\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}}+\frac{32\sqrt{-a}c^{3/2}d\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}F\left(\sin^{-1}\left(\right.\right.}{5e^4\sqrt{a+cx^2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2)/(d + e*x)^(7/2),x]
```

```
[Out] (-4*c*(2*d*(2*c*d^2 + a*e^2) + e*(5*c*d^2 + 3*a*e^2)*x)*Sqrt[a + c*x^2])/(5
*e^3*(c*d^2 + a*e^2)*(d + e*x)^(3/2)) - (2*(a + c*x^2)^(3/2))/(5*e*(d + e*x
)^(5/2)) - (8*Sqrt[-a]*c^(3/2)*(4*c*d^2 + 3*a*e^2)*Sqrt[d + e*x]*Sqrt[1 + (
c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e
)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(5*e^4*(c*d^2 + a*e^2)*Sqrt[(Sqrt[c]*(d + e*
x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) + (32*Sqrt[-a]*c^(3/2)*d*Sqr
t[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*Ellipti
cF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[
c]*d - a*e)])/(5*e^4*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx = -\frac{2\left(a+cx^2\right)^{3/2}}{5e(d+ex)^{5/2}} + \frac{(6c)\int \frac{x\sqrt{a+cx^2}}{(d+ex)^{5/2}}\,dx}{5e}$$

$$= -\frac{4c\left(2d\left(2cd^2+ae^2\right)+e\left(5cd^2+3ae^2\right)x\right)\sqrt{a+cx^2}}{5e^3\left(cd^2+ae^2\right)(d+ex)^{3/2}} - \frac{2\left(a+cx^2\right)^{3/2}}{5e(d+ex)^{5/2}} - \frac{(4c)\int \frac{acde-c\left(4cd^2+3ae^2\right)x}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx}{5e^3\left(cd^2+ae^2\right)}$$

$$= -\frac{4c\left(2d\left(2cd^2+ae^2\right)+e\left(5cd^2+3ae^2\right)x\right)\sqrt{a+cx^2}}{5e^3\left(cd^2+ae^2\right)(d+ex)^{3/2}} - \frac{2\left(a+cx^2\right)^{3/2}}{5e(d+ex)^{5/2}} - \frac{\left(16c^2d\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx}{5e^4} + \ldots$$

$$= -\frac{4c\left(2d\left(2cd^2+ae^2\right)+e\left(5cd^2+3ae^2\right)x\right)\sqrt{a+cx^2}}{5e^3\left(cd^2+ae^2\right)(d+ex)^{3/2}} - \frac{2\left(a+cx^2\right)^{3/2}}{5e(d+ex)^{5/2}} + \frac{\left(8ac^{3/2}\left(4cd^2+3ae^2\right)\sqrt{d+ex}\ldots\right.}{5\sqrt{-a}e^4\ldots}$$

$$= -\frac{4c\left(2d\left(2cd^2+ae^2\right)+e\left(5cd^2+3ae^2\right)x\right)\sqrt{a+cx^2}}{5e^3\left(cd^2+ae^2\right)(d+ex)^{3/2}} - \frac{2\left(a+cx^2\right)^{3/2}}{5e(d+ex)^{5/2}} - \frac{8\sqrt{-a}c^{3/2}\left(4cd^2+3ae^2\right)\sqrt{d+\ldots}}{5e^4\left(cd^2+\ldots\right.}$$

Mathematica [C] time = 2.86323, size = 602, normalized size = 1.47

$$2\left(-e^2\left(a+cx^2\right)\left(-4cd(d+ex)\left(ae^2+cd^2\right)+c(d+ex)^2\left(7ae^2+11cd^2\right)+\left(ae^2+cd^2\right)^2\right)+\frac{4c(d+ex)^2\left(-\sqrt{a}\sqrt{c}e(d+ex)^{3/2}\left(i\sqrt{a}\sqrt{c}de+3a\ldots\right.\right.}{\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2)/(d + e*x)^(7/2),x]
```

```
[Out] (2*(-(e^2*(a + c*x^2)*((c*d^2 + a*e^2)^2 - 4*c*d*(c*d^2 + a*e^2)*(d + e*x)
+ c*(11*c*d^2 + 7*a*e^2)*(d + e*x)^2)) + (4*c*(d + e*x)^2*(e^2*Sqrt[-d - (I
*Sqrt[a]*e)/Sqrt[c]]*(3*a^2*e^2 + 4*c^2*d^2*x^2 + a*c*(4*d^2 + 3*e^2*x^2))
+ Sqrt[c]*((-4*I)*c^(3/2)*d^3 + 4*Sqrt[a]*c*d^2*e - (3*I)*a*Sqrt[c]*d*e^2 +
 3*a^(3/2)*e^3)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sq
rt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqr
t[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sq
rt[c]*d + I*Sqrt[a]*e)] - Sqrt[a]*Sqrt[c]*e*(4*c*d^2 + I*Sqrt[a]*Sqrt[c]*d*
e + 3*a*e^2)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[
a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-
d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[
c]*d + I*Sqrt[a]*e)]))/Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]))/(5*e^5*(c*d^2 + a
*e^2)*(d + e*x)^(5/2)*Sqrt[a + c*x^2])
```

Maple [B] time = 0.273, size = 3411, normalized size = 8.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)/(e*x+d)^(7/2),x)
```

```
[Out] 2/5*(-11*x^4*c^3*d^2*e^4-18*x^3*c^3*d^3*e^3-8*x^2*a^2*c*e^6-8*x^2*c^3*d^4*e
^2-7*x^4*a*c^2*e^6-16*x^2*a*c^2*d^2*e^4-10*x*a^2*c*d*e^5-18*x*a*c^2*d^3*e^3
-16*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d
)/((-a*c)^(1/2)*e+c*d))^(1/2))*c^3*d^6*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1
/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*
e/((-a*c)^(1/2)*e-c*d))^(1/2)+16*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d)
)^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*c^2*d^5*e*(-a*c
)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a
*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-1
0*x^3*a*c^2*d*e^5+24*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((
-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*a^2*c*d*e^5*(-(e*x+d)*c/(
(-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/
2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+24*EllipticF((-(e*x+d)
*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))
^(1/2))*x*a*c^2*d^3*e^3*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*
c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)
*e-c*d))^(1/2)+32*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*
c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*c^2*d^4*e^2*(-a*c)^(1/2)*(-(
e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e
+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-24*EllipticE
((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/
2)*e+c*d))^(1/2))*x*a^2*c*d*e^5*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-
c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*
c)^(1/2)*e-c*d))^(1/2)-56*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)
,(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*a*c^2*d^3*e^3*(-(e*x
+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*
d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+16*EllipticF((-
(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*
e+c*d))^(1/2))*a*c*d^3*e^3*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(
1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))
*e/((-a*c)^(1/2)*e-c*d))^(1/2)+12*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d
))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a*c^2*d^2*
e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^
(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+16*El
lipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a
*c)^(1/2)*e+c*d))^(1/2))*x^2*c^2*d^3*e^3*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(
1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x
+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-28*EllipticE((-(e*x+d)*c/((-a*
c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*
x^2*a*c^2*d^2*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/
2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d
))^(1/2)-12*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/
2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*c*d^2*e^4*(-(e*x+d)*c/((-a*c)^(1
/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+
(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+12*EllipticF((-(e*x+d)*c/((-a*c
)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x
^2*a^2*c*e^6*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e
/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1
/2)-28*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-
c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c^2*d^4*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e
-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c
)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-12*EllipticE((-(e*x+d)*c/((-a*c)^(1/
2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a^
2*c*e^6*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a
*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-1
```

```
6*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/
((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*c^3*d^4*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*
d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(
1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-32*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*
e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*c^3*d^5
*e*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(
1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+12*Ell
ipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*
c)^(1/2)*e+c*d))^(1/2))*a^2*c*d^2*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/
2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e
/((-a*c)^(1/2)*e-c*d))^(1/2)+12*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))
^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c^2*d^4*e^2*(-
(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*
e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+16*Elliptic
F((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1
/2)*e+c*d))^(1/2))*x^2*a*c*d*e^5*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c
*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^
(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+32*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)
*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*a*c*d^
2*e^4*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1
/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*
d))^(1/2)-a^3*e^6-5*a^2*c*d^2*e^4-8*a*c^2*d^4*e^2)/(c*x^2+a)^(1/2)/(a*e^2+c
*d^2)/(e*x+d)^(5/2)/e^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+a\right)^{\frac{3}{2}}}{(ex+d)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^(3/2)/(e*x + d)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(cx^2+a\right)^{\frac{3}{2}}\sqrt{ex+d}}{e^4x^4+4\,de^3x^3+6\,d^2e^2x^2+4\,d^3ex+d^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + a)^(3/2)*sqrt(e*x + d)/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2
*x^2 + 4*d^3*e*x + d^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+cx^2\right)^{\frac{3}{2}}}{(d+ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)/(e*x+d)**(7/2),x)
```

```
[Out] Integral((a + c*x**2)**(3/2)/(d + e*x)**(7/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.669 $\int \frac{(a+cx^2)^{3/2}}{(d+ex)^{9/2}}\,dx$

Optimal. Leaf size=491

$$-\frac{8\sqrt{-a}c^{3/2}\sqrt{\frac{cx^2}{a}+1}\left(5ae^2+4cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{35e^4\sqrt{a+cx^2}\sqrt{d+ex}\left(ae^2+cd^2\right)}+\frac{32c^2d\sqrt{a+cx^2}\left(2ae^2+cd^2\right)}{35e^3\sqrt{d+ex}\left(ae^2+cd^2\right)^2}+\cdots$$

```
[Out] (32*c^2*d*(c*d^2 + 2*a*e^2)*Sqrt[a + c*x^2])/(35*e^3*(c*d^2 + a*e^2)^2*Sqrt
[d + e*x]) - (4*c*(2*d*(2*c*d^2 + a*e^2) + e*(7*c*d^2 + 5*a*e^2)*x)*Sqrt[a
+ c*x^2])/(35*e^3*(c*d^2 + a*e^2)*(d + e*x)^(5/2)) - (2*(a + c*x^2)^(3/2))/
(7*e*(d + e*x)^(7/2)) + (32*Sqrt[-a]*c^(5/2)*d*(c*d^2 + 2*a*e^2)*Sqrt[d + e
*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqr
t[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(35*e^4*(c*d^2 + a*e^2)^2*Sqrt
[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) - (8*Sqrt[-
a]*c^(3/2)*(4*c*d^2 + 5*a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-
a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/
Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(35*e^4*(c*d^2 + a*e^2)*Sqr
t[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.475794, antiderivative size = 491, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {733, 811, 835, 844, 719, 424, 419}

$$\frac{32c^2d\sqrt{a+cx^2}\left(2ae^2+cd^2\right)}{35e^3\sqrt{d+ex}\left(ae^2+cd^2\right)^2}-\frac{8\sqrt{-a}c^{3/2}\sqrt{\frac{cx^2}{a}+1}\left(5ae^2+4cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{35e^4\sqrt{a+cx^2}\sqrt{d+ex}\left(ae^2+cd^2\right)}+\frac{32\sqrt{-a}c^{5/}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2)/(d + e*x)^(9/2),x]
```

```
[Out] (32*c^2*d*(c*d^2 + 2*a*e^2)*Sqrt[a + c*x^2])/(35*e^3*(c*d^2 + a*e^2)^2*Sqrt
[d + e*x]) - (4*c*(2*d*(2*c*d^2 + a*e^2) + e*(7*c*d^2 + 5*a*e^2)*x)*Sqrt[a
+ c*x^2])/(35*e^3*(c*d^2 + a*e^2)*(d + e*x)^(5/2)) - (2*(a + c*x^2)^(3/2))/
(7*e*(d + e*x)^(7/2)) + (32*Sqrt[-a]*c^(5/2)*d*(c*d^2 + 2*a*e^2)*Sqrt[d + e
*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqr
t[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(35*e^4*(c*d^2 + a*e^2)^2*Sqrt
[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) - (8*Sqrt[-
a]*c^(3/2)*(4*c*d^2 + 5*a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-
a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/
Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(35*e^4*(c*d^2 + a*e^2)*Sqr
t[d + e*x]*Sqrt[a + c*x^2])
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
```

```
m, p, x]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}}{(d+ex)^{9/2}}\,dx = -\frac{2\left(a+cx^2\right)^{3/2}}{7e(d+ex)^{7/2}} + \frac{(6c)\int \frac{x\sqrt{a+cx^2}}{(d+ex)^{7/2}}\,dx}{7e}$$

$$= -\frac{4c\left(2d\left(2cd^2+ae^2\right)+e\left(7cd^2+5ae^2\right)x\right)\sqrt{a+cx^2}}{35e^3\left(cd^2+ae^2\right)(d+ex)^{5/2}} - \frac{2\left(a+cx^2\right)^{3/2}}{7e(d+ex)^{7/2}} - \frac{(4c)\int \frac{3acde-c\left(4cd^2+5ae^2\right)x}{(d+ex)^{3/2}\sqrt{a+cx^2}}\,dx}{35e^3\left(cd^2+ae^2\right)}$$

$$= \frac{32c^2d\left(cd^2+2ae^2\right)\sqrt{a+cx^2}}{35e^3\left(cd^2+ae^2\right)^2\sqrt{d+ex}} - \frac{4c\left(2d\left(2cd^2+ae^2\right)+e\left(7cd^2+5ae^2\right)x\right)\sqrt{a+cx^2}}{35e^3\left(cd^2+ae^2\right)(d+ex)^{5/2}} - \frac{2\left(a+cx^2\right)^{3/2}}{7e(d+ex)^{7/2}} +$$

$$= \frac{32c^2d\left(cd^2+2ae^2\right)\sqrt{a+cx^2}}{35e^3\left(cd^2+ae^2\right)^2\sqrt{d+ex}} - \frac{4c\left(2d\left(2cd^2+ae^2\right)+e\left(7cd^2+5ae^2\right)x\right)\sqrt{a+cx^2}}{35e^3\left(cd^2+ae^2\right)(d+ex)^{5/2}} - \frac{2\left(a+cx^2\right)^{3/2}}{7e(d+ex)^{7/2}} -$$

$$= \frac{32c^2d\left(cd^2+2ae^2\right)\sqrt{a+cx^2}}{35e^3\left(cd^2+ae^2\right)^2\sqrt{d+ex}} - \frac{4c\left(2d\left(2cd^2+ae^2\right)+e\left(7cd^2+5ae^2\right)x\right)\sqrt{a+cx^2}}{35e^3\left(cd^2+ae^2\right)(d+ex)^{5/2}} - \frac{2\left(a+cx^2\right)^{3/2}}{7e(d+ex)^{7/2}} -$$

$$= \frac{32c^2d\left(cd^2+2ae^2\right)\sqrt{a+cx^2}}{35e^3\left(cd^2+ae^2\right)^2\sqrt{d+ex}} - \frac{4c\left(2d\left(2cd^2+ae^2\right)+e\left(7cd^2+5ae^2\right)x\right)\sqrt{a+cx^2}}{35e^3\left(cd^2+ae^2\right)(d+ex)^{5/2}} - \frac{2\left(a+cx^2\right)^{3/2}}{7e(d+ex)^{7/2}} +$$

Mathematica [C] time = 3.81197, size = 659, normalized size = 1.34

$$2\left(-e^2\left(a+cx^2\right)\left(-16c^2d(d+ex)^3\left(2ae^2+cd^2\right)-16cd(d+ex)\left(ae^2+cd^2\right)^2+c(d+ex)^2\left(15ae^2+19cd^2\right)\left(ae^2+cd^2\right)+5\left(\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2)/(d + e*x)^(9/2),x]
```

```
[Out] (2*(-(e^2*(a + c*x^2)*(5*(c*d^2 + a*e^2)^3 - 16*c*d*(c*d^2 + a*e^2)^2*(d +
e*x) + c*(c*d^2 + a*e^2)*(19*c*d^2 + 15*a*e^2)*(d + e*x)^2 - 16*c^2*d*(c*d^
2 + 2*a*e^2)*(d + e*x)^3)) - (4*c^2*(d + e*x)^3*(4*d*e^2*Sqrt[-d - (I*Sqrt[
a]*e)/Sqrt[c]]*(2*a^2*e^2 + c^2*d^2*x^2 + a*c*(d^2 + 2*e^2*x^2)) + 4*Sqrt[c
]*d*((-I)*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e - (2*I)*a*Sqrt[c]*d*e^2 + 2*a^(3/2)
*e^3)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/S
qrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d - (I*
Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d +
I*Sqrt[a]*e)] - Sqrt[a]*e*(4*c^(3/2)*d^3 + I*Sqrt[a]*c*d^2*e + 8*a*Sqrt[c]*
d*e^2 + (5*I)*a^(3/2)*e^3)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sq
rt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticF[I*
ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqr
t[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)]))/Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]))/(35
*e^5*(c*d^2 + a*e^2)^2*(d + e*x)^(7/2)*Sqrt[a + c*x^2])
```

Maple [B] time = 0.306, size = 5277, normalized size = 10.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)/(e*x+d)^(9/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+a\right)^{\frac{3}{2}}}{\left(ex+d\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^(3/2)/(e*x + d)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+a\right)^{\frac{3}{2}}\sqrt{ex+d}}{e^5x^5+5\,de^4x^4+10\,d^2e^3x^3+10\,d^3e^2x^2+5\,d^4ex+d^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + a)^(3/2)*sqrt(e*x + d)/(e^5*x^5 + 5*d*e^4*x^4 + 10*d^2*e^
3*x^3 + 10*d^3*e^2*x^2 + 5*d^4*e*x + d^5), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+cx^2\right)^{\frac{3}{2}}}{\left(d+ex\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)/(e*x+d)**(9/2),x)
```

```
[Out] Integral((a + c*x**2)**(3/2)/(d + e*x)**(9/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + a\right)^{\frac{3}{2}}}{(ex + d)^{\frac{9}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(e*x+d)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^(3/2)/(e*x + d)^(9/2), x)
```

3.670 $\int \sqrt{d+ex}\left(a+cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=566

$$-\frac{16\sqrt{-a}d\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\left(177a^2e^4+113acd^2e^2+32c^2d^4\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{9009\sqrt{c}e^6\sqrt{a+cx^2}\sqrt{d+ex}}+$$

```
[Out] (8*Sqrt[d + e*x]*(d*(32*c^2*d^4 + 113*a*c*d^2*e^2 + 177*a^2*e^4) - 3*e*(8*c
^2*d^4 + 27*a*c*d^2*e^2 - 77*a^2*e^4)*x)*Sqrt[a + c*x^2])/(9009*e^5) + (20*
Sqrt[d + e*x]*(4*d*(2*c*d^2 + 5*a*e^2) - 7*e*(c*d^2 - 11*a*e^2)*x)*(a + c*x
^2)^(3/2))/(9009*e^3) - (20*d*Sqrt[d + e*x]*(a + c*x^2)^(5/2))/(143*e) + (2
*(d + e*x)^(3/2)*(a + c*x^2)^(5/2))/(13*e) + (16*Sqrt[-a]*(32*c^3*d^6 + 137
*a*c^2*d^4*e^2 + 258*a^2*c*d^2*e^4 - 231*a^3*e^6)*Sqrt[d + e*x]*Sqrt[1 + (c
*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)
/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(9009*Sqrt[c]*e^6*Sqrt[(Sqrt[c]*(d + e*x))/(S
qrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) - (16*Sqrt[-a]*d*(c*d^2 + a*e^2)*(
32*c^2*d^4 + 113*a*c*d^2*e^2 + 177*a^2*e^4)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[
c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*
x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(9009*Sqrt[c]*
e^6*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.665535, antiderivative size = 566, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {735, 833, 815, 844, 719, 424, 419}

$$\frac{8\sqrt{a+cx^2}\sqrt{d+ex}\left(d\left(177a^2e^4+113acd^2e^2+32c^2d^4\right)-3ex\left(-77a^2e^4+27acd^2e^2+8c^2d^4\right)\right)}{9009e^5}-\frac{16\sqrt{-a}d\sqrt{\frac{cx^2}{a}+1}\left(ae^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(a + c*x^2)^(5/2),x]
```


```
[Out] (8*Sqrt[d + e*x]*(d*(32*c^2*d^4 + 113*a*c*d^2*e^2 + 177*a^2*e^4) - 3*e*(8*c
^2*d^4 + 27*a*c*d^2*e^2 - 77*a^2*e^4)*x)*Sqrt[a + c*x^2])/(9009*e^5) + (20*
Sqrt[d + e*x]*(4*d*(2*c*d^2 + 5*a*e^2) - 7*e*(c*d^2 - 11*a*e^2)*x)*(a + c*x
^2)^(3/2))/(9009*e^3) - (20*d*Sqrt[d + e*x]*(a + c*x^2)^(5/2))/(143*e) + (2
*(d + e*x)^(3/2)*(a + c*x^2)^(5/2))/(13*e) + (16*Sqrt[-a]*(32*c^3*d^6 + 137
*a*c^2*d^4*e^2 + 258*a^2*c*d^2*e^4 - 231*a^3*e^6)*Sqrt[d + e*x]*Sqrt[1 + (c
*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)
/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(9009*Sqrt[c]*e^6*Sqrt[(Sqrt[c]*(d + e*x))/(S
qrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) - (16*Sqrt[-a]*d*(c*d^2 + a*e^2)*(
32*c^2*d^4 + 113*a*c*d^2*e^2 + 177*a^2*e^4)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[
c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*
x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(9009*Sqrt[c]*
e^6*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```


Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
```

```
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) && !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \sqrt{d+ex}\left(a+cx^2\right)^{5/2}dx = \frac{2(d+ex)^{3/2}\left(a+cx^2\right)^{5/2}}{13e} + \frac{10\int (ae-cdx)\sqrt{d+ex}\left(a+cx^2\right)^{3/2}dx}{13e}$$

$$= -\frac{20d\sqrt{d+ex}\left(a+cx^2\right)^{5/2}}{143e} + \frac{2(d+ex)^{3/2}\left(a+cx^2\right)^{5/2}}{13e} + \frac{20\int \frac{\left(6acde-\frac{1}{2}c\left(cd^2-11ae^2\right)x\right)\left(a+cx^2\right)^{3/2}}{\sqrt{d+ex}}}{143ce}$$

$$= \frac{20\sqrt{d+ex}\left(4d\left(2cd^2+5ae^2\right)-7e\left(cd^2-11ae^2\right)x\right)\left(a+cx^2\right)^{3/2}}{9009e^3} - \frac{20d\sqrt{d+ex}\left(a+cx^2\right)^{5/}}{143e}$$

$$= \frac{8\sqrt{d+ex}\left(d\left(32c^2d^4+113acd^2e^2+177a^2e^4\right)-3e\left(8c^2d^4+27acd^2e^2-77a^2e^4\right)x\right)\sqrt{a+c}}{9009e^5}$$

$$= \frac{8\sqrt{d+ex}\left(d\left(32c^2d^4+113acd^2e^2+177a^2e^4\right)-3e\left(8c^2d^4+27acd^2e^2-77a^2e^4\right)x\right)\sqrt{a+c}}{9009e^5}$$

$$= \frac{8\sqrt{d+ex}\left(d\left(32c^2d^4+113acd^2e^2+177a^2e^4\right)-3e\left(8c^2d^4+27acd^2e^2-77a^2e^4\right)x\right)\sqrt{a+c}}{9009e^5}$$

$$= \frac{8\sqrt{d+ex}\left(d\left(32c^2d^4+113acd^2e^2+177a^2e^4\right)-3e\left(8c^2d^4+27acd^2e^2-77a^2e^4\right)x\right)\sqrt{a+c}}{9009e^5}$$

Mathematica [C] time = 4.92649, size = 790, normalized size = 1.4

$$2\sqrt{d+ex}\left(e^2\left(a+cx^2\right)\left(a^2e^4(971d+2387ex)+2ace^2\left(-197d^2ex+266d^3+163de^2x^2+1078e^3x^3\right)+c^2\left(80d^3e^2x^2-70\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*(a + c*x^2)^(5/2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(e^2*(a + c*x^2)*(a^2*e^4*(971*d + 2387*e*x) + 2*a*c*e^2*(
266*d^3 - 197*d^2*e*x + 163*d*e^2*x^2 + 1078*e^3*x^3) + c^2*(128*d^5 - 96*d
^4*e*x + 80*d^3*e^2*x^2 - 70*d^2*e^3*x^3 + 63*d*e^4*x^4 + 693*e^5*x^5)) + (
8*(-(e^2*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*(32*c^3*d^6 + 137*a*c^2*d^4*e^2 +
 258*a^2*c*d^2*e^4 - 231*a^3*e^6)*(a + c*x^2)) + Sqrt[c]*((32*I)*c^(7/2)*d^
7 - 32*Sqrt[a]*c^3*d^6*e + (137*I)*a*c^(5/2)*d^5*e^2 - 137*a^(3/2)*c^2*d^4*
e^3 + (258*I)*a^2*c^(3/2)*d^3*e^4 - 258*a^(5/2)*c*d^2*e^5 - (231*I)*a^3*Sqr
t[c]*d*e^6 + 231*a^(7/2)*e^7)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]
*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticE
[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*
Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)] + Sqrt[a]*Sqrt[c]*e*(32*c^3*d^6 + (8*
I)*Sqrt[a]*c^(5/2)*d^5*e + 137*a*c^2*d^4*e^2 + (32*I)*a^(3/2)*c^(3/2)*d^3*e
^3 + 258*a^2*c*d^2*e^4 + (408*I)*a^(5/2)*Sqrt[c]*d*e^5 - 231*a^3*e^6)*Sqrt[
```

```
(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*
x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/
Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e
)]))/(c*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*(d + e*x))))/(9009*e^7*Sqrt[a + c*
x^2])
```

Maple [B] time = 0.268, size = 2332, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(5/2)*(e*x+d)^(1/2),x)
```

```
[Out] 2/9009*(c*x^2+a)^(1/2)*(e*x+d)^(1/2)*(-1416*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d
))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1
/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*
d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*(-a*c)^(1/2)*
a^3*d*e^7+256*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(
1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*c^4*d^8*((-c*x+(-a*c)^(1/2))*e/((-
a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*
(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)+1848*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*
d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(
1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c
*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^4*e^8-1848
*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/
2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*Elliptic
E((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1
/2)*e+c*d))^(1/2))*a^4*e^8+3238*x^5*a*c^3*d*e^7-840*EllipticF((-(e*x+d)*c/(
(-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/
2))*a^2*c^2*d^4*e^4*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*
x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c
*d))^(1/2)-192*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^
(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c^3*d^6*e^2*((-c*x+(-a*c)^(1/2)
)*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))
^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)-256*EllipticF((-(e*x+d)*c/((
-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2
))*c^3*d^7*e*(-a*c)^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2
)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*(-(e*x+d)*c/((-a*c)^(1/
2)*e-c*d))^(1/2)+1352*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-(
(-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c^3*d^6*e^2*((-c*x+(-a*c
)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*
e-c*d))^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)+3160*EllipticE((-(e*x
+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*
d))^(1/2))*a^2*c^2*d^4*e^4*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/
2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*(-(e*x+d)*c/((-a*c)^(1
/2)*e-c*d))^(1/2)+1200*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c
)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*
e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(
1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^3*c*d^2*e^6+216*(-(e*x+d)*c/((-a
*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*
((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticE((-(e*x+d)*c/((-
a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2)
)*a^3*c*d^2*e^6+756*x^7*c^4*d*e^7+2849*x^6*a*c^3*e^8-7*x^6*c^4*d^2*e^6+693*
x^8*c^4*e^8+148*x^3*a*c^3*d^3*e^5+903*x^2*a^2*c^2*d^2*e^6+516*x^2*a*c^3*d^4
*e^4+3358*x*a^3*c*d*e^7+138*x*a^2*c^2*d^3*e^5+32*x*a*c^3*d^5*e^3+128*x^2*c^
4*d^6*e^2+10*x^5*c^4*d^3*e^5+4543*x^4*a^2*c^2*e^8-16*x^4*c^4*d^4*e^4+32*x^3
```

```
*c^4*d^5*e^3+2387*x^2*a^3*c*e^8-75*x^4*a*c^3*d^2*e^6+5840*x^3*a^2*c^2*d*e^7
+971*a^3*c*d^2*e^6+532*a^2*c^2*d^4*e^4+128*a*c^3*d^6*e^2-2320*EllipticF((-(
e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e
+c*d))^(1/2))*a^2*c*d^3*e^5*(-a*c)^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/
2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*(-(e*x+d
)*c/((-a*c)^(1/2)*e-c*d))^(1/2)-1160*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-
c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c^2*d^5*e
^3*(-a*c)^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-
a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))
^(1/2))/c/(c*e*x^3+c*d*x^2+a*e*x+a*d)/e^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+a\right)^{\frac{5}{2}}\sqrt{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^(5/2)*sqrt(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(c^2x^4+2\,acx^2+a^2\right)\sqrt{cx^2+a}\sqrt{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*a*c*x^2 + a^2)*sqrt(c*x^2 + a)*sqrt(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+cx^2\right)^{\frac{5}{2}}\sqrt{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(5/2)*(e*x+d)**(1/2),x)
```

```
[Out] Integral((a + c*x**2)**(5/2)*sqrt(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+a\right)^{\frac{5}{2}}\sqrt{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^(5/2)*sqrt(e*x + d), x)
```

3.671 $\int \frac{(a+cx^2)^{5/2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=494

$$-\frac{16\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\left(45a^2e^4+69acd^2e^2+32c^2d^4\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{693\sqrt{c}e^6\sqrt{a+cx^2}\sqrt{d+ex}}+\frac{8\sqrt{}\cdots}{\cdots}$$

```
[Out] (8*Sqrt[d + e*x]*(32*c^2*d^4 + 69*a*c*d^2*e^2 + 45*a^2*e^4 - 24*c*d*e*(c*d^
2 + 2*a*e^2)*x)*Sqrt[a + c*x^2])/(693*e^5) + (20*Sqrt[d + e*x]*(8*c*d^2 + 9
*a*e^2 - 7*c*d*e*x)*(a + c*x^2)^(3/2))/(693*e^3) + (2*Sqrt[d + e*x]*(a + c*
x^2)^(5/2))/(11*e) + (16*Sqrt[-a]*Sqrt[c]*d*(32*c^2*d^4 + 93*a*c*d^2*e^2 +
93*a^2*e^4)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sq
rt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(693*e^6
*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) - (16*
Sqrt[-a]*(c*d^2 + a*e^2)*(32*c^2*d^4 + 69*a*c*d^2*e^2 + 45*a^2*e^4)*Sqrt[(S
qrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[A
rcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d
 - a*e)])/(693*Sqrt[c]*e^6*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.481906, antiderivative size = 494, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {735, 815, 844, 719, 424, 419}

$$\frac{8\sqrt{a+cx^2}\sqrt{d+ex}\left(45a^2e^4-24cdex\left(2ae^2+cd^2\right)+69acd^2e^2+32c^2d^4\right)}{693e^5}-\frac{16\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\left(45a^2e^4+69ac\cdots\right.}{693\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(5/2)/Sqrt[d + e*x],x]
```


```
[Out] (8*Sqrt[d + e*x]*(32*c^2*d^4 + 69*a*c*d^2*e^2 + 45*a^2*e^4 - 24*c*d*e*(c*d^
2 + 2*a*e^2)*x)*Sqrt[a + c*x^2])/(693*e^5) + (20*Sqrt[d + e*x]*(8*c*d^2 + 9
*a*e^2 - 7*c*d*e*x)*(a + c*x^2)^(3/2))/(693*e^3) + (2*Sqrt[d + e*x]*(a + c*
x^2)^(5/2))/(11*e) + (16*Sqrt[-a]*Sqrt[c]*d*(32*c^2*d^4 + 93*a*c*d^2*e^2 +
93*a^2*e^4)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sq
rt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(693*e^6
*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) - (16*
Sqrt[-a]*(c*d^2 + a*e^2)*(32*c^2*d^4 + 69*a*c*d^2*e^2 + 45*a^2*e^4)*Sqrt[(S
qrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[A
rcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d
 - a*e)])/(693*Sqrt[c]*e^6*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```


Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
```

```
IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{5/2}}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{d+ex}\left(a+cx^2\right)^{5/2}}{11e} + \frac{10\int \frac{(ae-cdx)\left(a+cx^2\right)^{3/2}}{\sqrt{d+ex}}dx}{11e}$$

$$= \frac{20\sqrt{d+ex}\left(8cd^2+9ae^2-7cdex\right)\left(a+cx^2\right)^{3/2}}{693e^3} + \frac{2\sqrt{d+ex}\left(a+cx^2\right)^{5/2}}{11e} + \frac{40\int \frac{\left(\frac{1}{2}ace\left(cd^2+9ae^2\right)-4c^2d(\cdots}{\sqrt{d+\cdots}}}{231ce\cdots}$$

$$= \frac{8\sqrt{d+ex}\left(32c^2d^4+69acd^2e^2+45a^2e^4-24cde\left(cd^2+2ae^2\right)x\right)\sqrt{a+cx^2}}{693e^5} + \frac{20\sqrt{d+ex}\left(8cd^2+9\cdots\right.}{6\cdots}$$

$$= \frac{8\sqrt{d+ex}\left(32c^2d^4+69acd^2e^2+45a^2e^4-24cde\left(cd^2+2ae^2\right)x\right)\sqrt{a+cx^2}}{693e^5} + \frac{20\sqrt{d+ex}\left(8cd^2+9\cdots\right.}{6\cdots}$$

$$= \frac{8\sqrt{d+ex}\left(32c^2d^4+69acd^2e^2+45a^2e^4-24cde\left(cd^2+2ae^2\right)x\right)\sqrt{a+cx^2}}{693e^5} + \frac{20\sqrt{d+ex}\left(8cd^2+9\cdots\right.}{6\cdots}$$

$$= \frac{8\sqrt{d+ex}\left(32c^2d^4+69acd^2e^2+45a^2e^4-24cde\left(cd^2+2ae^2\right)x\right)\sqrt{a+cx^2}}{693e^5} + \frac{20\sqrt{d+ex}\left(8cd^2+9\cdots\right.}{6\cdots}$$

Mathematica [C] time = 3.67884, size = 634, normalized size = 1.28

$$2\sqrt{d+ex}\left(\frac{8\sqrt{ae}\sqrt{d+ex}\left(21ia^{3/2}cd^2e^3+93a^2\sqrt{c}de^4+45ia^{5/2}e^5+93ac^{3/2}d^3e^2+8i\sqrt{a}c^2d^4e+32c^{5/2}d^5\right)\sqrt{\frac{e\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{d+ex}}\sqrt{-\frac{-ex+\frac{i\sqrt{ae}}{\sqrt{c}}}{d+ex}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-d-\frac{i\sqrt{ae}}{\sqrt{c}}}}{\sqrt{d+ex}}\right),\cdots\right.}{\sqrt{-d-\frac{i\sqrt{ae}}{\sqrt{c}}}}\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(5/2)/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[d + e*x]*((-8*d*e^2*(32*c^2*d^4 + 93*a*c*d^2*e^2 + 93*a^2*e^4)*(a +
 c*x^2))/(d + e*x) + e^2*(a + c*x^2)*(333*a^2*e^4 + 2*a*c*e^2*(178*d^2 - 13
1*d*e*x + 108*e^2*x^2) + c^2*(128*d^4 - 96*d^3*e*x + 80*d^2*e^2*x^2 - 70*d*
e^3*x^3 + 63*e^4*x^4)) - (8*I)*c*d*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*(32*c^2
*d^4 + 93*a*c*d^2*e^2 + 93*a^2*e^4)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d +
 e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*Sqrt[d + e*x]*Ellip
ticE[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d
- I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)] + (8*Sqrt[a]*e*(32*c^(5/2)*d^5 +
(8*I)*Sqrt[a]*c^2*d^4*e + 93*a*c^(3/2)*d^3*e^2 + (21*I)*a^(3/2)*c*d^2*e^3 +
 93*a^2*Sqrt[c]*d*e^4 + (45*I)*a^(5/2)*e^5)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] +
x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*Sqrt[d + e*
x]*EllipticF[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sq
rt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)])/Sqrt[-d - (I*Sqrt[a]*e)/
Sqrt[c]]))/(693*e^7*Sqrt[a + c*x^2])
```

Maple [B] time = 0.258, size = 1970, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(5/2)/(e*x+d)^(1/2),x)
```

```
[Out] 2/693*(c*x^2+a)^(1/2)*(e*x+d)^(1/2)*(128*a*c^3*d^5*e^2-912*(-(e*x+d)*c/((-a
*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*
((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-
a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2)
)*(-a*c)^(1/2)*a^2*c*d^2*e^5-808*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((
-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a
*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(
-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*(-a*c)^(1/2)*a*c^2*d^4*e
^3+104*x^3*a*c^3*d^2*e^5+287*x^2*a^2*c^2*d*e^6+340*x^2*a*c^3*d^3*e^4+94*x*a
^2*c^2*d^2*e^5+32*x*a*c^3*d^4*e^3-53*x^4*a*c^3*d*e^6-360*(-(e*x+d)*c/((-a*c
)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((
c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*
c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*
(-a*c)^(1/2)*a^3*e^7-7*x^6*c^4*d*e^6+279*x^5*a*c^3*e^7+10*x^5*c^4*d^2*e^5-1
6*x^4*c^4*d^3*e^4+549*x^3*a^2*c^2*e^7+32*x^3*c^4*d^4*e^3+128*x^2*c^4*d^5*e^
2+333*x*a^3*c*e^7+63*x^7*c^4*e^7+1000*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/
2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e
/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1
/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c^3*d^5*e^2+744*(
-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)
*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticE(
(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2
)*e+c*d))^(1/2))*a^3*c*d*e^6+1488*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*(
(-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-
a*c)^(1/2)*e-c*d))^(1/2)*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),
(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*c^2*d^3*e^4-256*(-(
e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e
+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-
(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*
e+c*d))^(1/2))*(-a*c)^(1/2)*c^3*d^6*e-192*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))
^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2
))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d)
)^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c^3*d^5*e^2-3
84*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(
1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*Ellipt
icF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^
(1/2)*e+c*d))^(1/2))*a^3*c*d*e^6-576*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2
)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/
((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/
2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*c^2*d^3*e^4+256*
(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2
)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticE
((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/
2)*e+c*d))^(1/2))*c^4*d^7+333*a^3*c*d*e^6+356*a^2*c^2*d^3*e^4)/c/e^7/(c*e*x
^3+c*d*x^2+a*e*x+a*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+a\right)^{\frac{5}{2}}}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^(5/2)/sqrt(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2x^4+2\,acx^2+a^2\right)\sqrt{cx^2+a}}{\sqrt{ex+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*a*c*x^2 + a^2)*sqrt(c*x^2 + a)/sqrt(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+cx^2\right)^{\frac{5}{2}}}{\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(5/2)/(e*x+d)**(1/2),x)
```

```
[Out] Integral((a + c*x**2)**(5/2)/sqrt(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+a\right)^{\frac{5}{2}}}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^(5/2)/sqrt(e*x + d), x)
```

3.672 $\int \frac{(a+cx^2)^{5/2}}{(d+ex)^{3/2}} \, dx$

Optimal. Leaf size=457

$$\frac{16\sqrt{-a}\sqrt{c}d\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\left(33ae^2+32cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{63e^6\sqrt{a+cx^2}\sqrt{d+ex}}-\frac{16\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+\cdots}}{\cdots}$$

```
[Out] (-8*c*Sqrt[d + e*x]*(d*(32*c*d^2 + 33*a*e^2) - 3*e*(8*c*d^2 + 7*a*e^2)*x)*S
qrt[a + c*x^2])/(63*e^5) - (20*c*(8*d - 7*e*x)*Sqrt[d + e*x]*(a + c*x^2)^(3
/2))/(63*e^3) - (2*(a + c*x^2)^(5/2))/(e*Sqrt[d + e*x]) - (16*Sqrt[-a]*Sqrt
[c]*(32*c^2*d^4 + 57*a*c*d^2*e^2 + 21*a^2*e^4)*Sqrt[d + e*x]*Sqrt[1 + (c*x^
2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(S
qrt[-a]*Sqrt[c]*d - a*e)])/(63*e^6*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sq
rt[-a]*e)]*Sqrt[a + c*x^2]) + (16*Sqrt[-a]*Sqrt[c]*d*(c*d^2 + a*e^2)*(32*c*
d^2 + 33*a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 +
 (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a
*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(63*e^6*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.455461, antiderivative size = 457, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {733, 815, 844, 719, 424, 419}

$$-\frac{16\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}\left(21a^2e^4+57acd^2e^2+32c^2d^4\right)E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{63e^6\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}}-\frac{8c\sqrt{a+cx^2}\sqrt{d+ex}\left(d\left(3\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(5/2)/(d + e*x)^(3/2),x]
```


```
[Out] (-8*c*Sqrt[d + e*x]*(d*(32*c*d^2 + 33*a*e^2) - 3*e*(8*c*d^2 + 7*a*e^2)*x)*S
qrt[a + c*x^2])/(63*e^5) - (20*c*(8*d - 7*e*x)*Sqrt[d + e*x]*(a + c*x^2)^(3
/2))/(63*e^3) - (2*(a + c*x^2)^(5/2))/(e*Sqrt[d + e*x]) - (16*Sqrt[-a]*Sqrt
[c]*(32*c^2*d^4 + 57*a*c*d^2*e^2 + 21*a^2*e^4)*Sqrt[d + e*x]*Sqrt[1 + (c*x^
2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(S
qrt[-a]*Sqrt[c]*d - a*e)])/(63*e^6*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sq
rt[-a]*e)]*Sqrt[a + c*x^2]) + (16*Sqrt[-a]*Sqrt[c]*d*(c*d^2 + a*e^2)*(32*c*
d^2 + 33*a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 +
 (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a
*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(63*e^6*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```


Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(a+cx^2)^{5/2}}{(d+ex)^{3/2}}\,dx = -\frac{2(a+cx^2)^{5/2}}{e\sqrt{d+ex}} + \frac{(10c)\int \frac{x(a+cx^2)^{3/2}}{\sqrt{d+ex}}\,dx}{e}$$

$$= -\frac{20c(8d-7ex)\sqrt{d+ex}\,(a+cx^2)^{3/2}}{63e^3} - \frac{2(a+cx^2)^{5/2}}{e\sqrt{d+ex}} + \frac{40\int \frac{\left(-\frac{1}{2}acde+\frac{1}{2}c(8cd^2+7ae^2)x\right)\sqrt{a+cx^2}}{\sqrt{d+ex}}\,dx}{21e^3}$$

$$= -\frac{8c\sqrt{d+ex}\,\left(d\left(32cd^2+33ae^2\right)-3e\left(8cd^2+7ae^2\right)x\right)\sqrt{a+cx^2}}{63e^5} - \frac{20c(8d-7ex)\sqrt{d+ex}\,(a+cx^2)^{3/2}}{63e^3}$$

$$= -\frac{8c\sqrt{d+ex}\,\left(d\left(32cd^2+33ae^2\right)-3e\left(8cd^2+7ae^2\right)x\right)\sqrt{a+cx^2}}{63e^5} - \frac{20c(8d-7ex)\sqrt{d+ex}\,(a+cx^2)^{3/2}}{63e^3}$$

$$= -\frac{8c\sqrt{d+ex}\,\left(d\left(32cd^2+33ae^2\right)-3e\left(8cd^2+7ae^2\right)x\right)\sqrt{a+cx^2}}{63e^5} - \frac{20c(8d-7ex)\sqrt{d+ex}\,(a+cx^2)^{3/2}}{63e^3}$$

$$= -\frac{8c\sqrt{d+ex}\,\left(d\left(32cd^2+33ae^2\right)-3e\left(8cd^2+7ae^2\right)x\right)\sqrt{a+cx^2}}{63e^5} - \frac{20c(8d-7ex)\sqrt{d+ex}\,(a+cx^2)^{3/2}}{63e^3}$$

Mathematica [C] time = 3.9395, size = 684, normalized size = 1.5

$$\sqrt{d+ex}\left(-\frac{2(a+cx^2)\left(63a^2e^4+2ace^2\left(106d^2+29dex-14e^2x^2\right)+c^2\left(-16d^2e^2x^2+32d^3ex+128d^4+10de^3x^3-7e^4x^4\right)\right)}{e^5(d+ex)} + \frac{16\left(-\sqrt{a}\sqrt{c}e(d+ex)^{3/2}\left(12ia^{3/2}\sqrt{c}de^3+21a^2e^4\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(5/2)/(d + e*x)^(3/2),x]
```

```
[Out] (Sqrt[d + e*x]*((-2*(a + c*x^2)*(63*a^2*e^4 + 2*a*c*e^2*(106*d^2 + 29*d*e*x
 - 14*e^2*x^2) + c^2*(128*d^4 + 32*d^3*e*x - 16*d^2*e^2*x^2 + 10*d*e^3*x^3
- 7*e^4*x^4)))/(e^5*(d + e*x)) + (16*(e^2*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*
(32*c^2*d^4 + 57*a*c*d^2*e^2 + 21*a^2*e^4)*(a + c*x^2) + Sqrt[c]*((-32*I)*c
^(5/2)*d^5 + 32*Sqrt[a]*c^2*d^4*e - (57*I)*a*c^(3/2)*d^3*e^2 + 57*a^(3/2)*c
*d^2*e^3 - (21*I)*a^2*Sqrt[c]*d*e^4 + 21*a^(5/2)*e^5)*Sqrt[(e*((I*Sqrt[a])/
Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(
d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d
+ e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)] - Sqrt[a]*Sqr
t[c]*e*(32*c^2*d^4 + (8*I)*Sqrt[a]*c^(3/2)*d^3*e + 57*a*c*d^2*e^2 + (12*I)*
a^(3/2)*Sqrt[c]*d*e^3 + 21*a^2*e^4)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d +
 e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*Ell
ipticF[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*
d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)]))/(e^7*Sqrt[-d - (I*Sqrt[a]*e)/
Sqrt[c]]*(d + e*x))))/(63*Sqrt[a + c*x^2])
```

Maple [B] time = 0.287, size = 1736, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(5/2)/(e*x+d)^(3/2),x)
```

```
[Out] 2/63*(c*x^2+a)^(1/2)*(e*x+d)^(1/2)*(7*x^6*c^3*e^6-10*x^5*c^3*d*e^5+168*(-(e
*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+
c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(
e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e
+c*d))^(1/2))*a^3*e^6+360*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)
,(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*c*d^2*e^4*(-(e*x+d
)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d)
)^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+264*(-(e*x+d)*c/(
(-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/
2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/
((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1
/2))*(-a*c)^(1/2)*a^2*d*e^5+192*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))
^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c^2*d^4*e^2*(-
(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*
e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+520*Ellipti
cF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(
1/2)*e+c*d))^(1/2))*a*c*d^3*e^3*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*
d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(
1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+256*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)
*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*c^2*d^5*
e*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))
*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^
(1/2)-168*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((
-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)
*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/(
(-a*c)^(1/2)*e+c*d))^(1/2))*a^3*e^6-624*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)
*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*c*d^
2*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c
)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-712
*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/(
(-a*c)^(1/2)*e+c*d))^(1/2))*a*c^2*d^4*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))
^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2
))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-256*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-
c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*c^3*d^6*(-(
e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e
+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+35*x^4*a*c^2
*e^6+16*x^4*c^3*d^2*e^4-68*x^3*a*c^2*d*e^5-32*x^3*c^3*d^3*e^3-35*x^2*a^2*c*
e^6-196*x^2*a*c^2*d^2*e^4-128*x^2*c^3*d^4*e^2-58*x*a^2*c*d*e^5-32*x*a*c^2*d
^3*e^3-63*a^3*e^6-212*a^2*c*d^2*e^4-128*a*c^2*d^4*e^2)/e^7/(c*e*x^3+c*d*x^2
+a*e*x+a*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2+a)^{\frac{5}{2}}}{(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^(5/2)/(e*x + d)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2x^4 + 2\,acx^2 + a^2\right)\sqrt{cx^2 + a}\sqrt{ex + d}}{e^2x^2 + 2\,dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*a*c*x^2 + a^2)*sqrt(c*x^2 + a)*sqrt(e*x + d)/(e^2*x^2
 + 2*d*e*x + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + cx^2\right)^{\frac{5}{2}}}{(d + ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(5/2)/(e*x+d)**(3/2),x)
```

```
[Out] Integral((a + c*x**2)**(5/2)/(d + e*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + a\right)^{\frac{5}{2}}}{(ex + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^(5/2)/(e*x + d)^(3/2), x)
```

3.673 $\int \frac{(a+cx^2)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=430

$$-\frac{16\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\left(5ae^2+32cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{21e^6\sqrt{a+cx^2}\sqrt{d+ex}}+\frac{16\sqrt{-a}c^{3/2}d\sqrt{\cdots}}{\cdots}$$

```
[Out] (8*c*Sqrt[d + e*x]*(32*c*d^2 + 5*a*e^2 - 24*c*d*e*x)*Sqrt[a + c*x^2])/(21*e
^5) + (20*c*(8*d + e*x)*(a + c*x^2)^(3/2))/(21*e^3*Sqrt[d + e*x]) - (2*(a +
 c*x^2)^(5/2))/(3*e*(d + e*x)^(3/2)) + (16*Sqrt[-a]*c^(3/2)*d*(32*c*d^2 + 2
9*a*e^2)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[
c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(21*e^6*Sqr
t[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) - (16*Sqrt
[-a]*Sqrt[c]*(c*d^2 + a*e^2)*(32*c*d^2 + 5*a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/
(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sq
rt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(21*e^6*
Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.437941, antiderivative size = 430, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {733, 813, 815, 844, 719, 424, 419}

$$\frac{16\sqrt{-a}c^{3/2}d\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}\left(29ae^2+32cd^2\right)E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{21e^6\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}}+\frac{8c\sqrt{a+cx^2}\sqrt{d+ex}\left(5ae^2+32cd^2-\cdots\right)}{21e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(5/2)/(d + e*x)^(5/2),x]
```


```
[Out] (8*c*Sqrt[d + e*x]*(32*c*d^2 + 5*a*e^2 - 24*c*d*e*x)*Sqrt[a + c*x^2])/(21*e
^5) + (20*c*(8*d + e*x)*(a + c*x^2)^(3/2))/(21*e^3*Sqrt[d + e*x]) - (2*(a +
 c*x^2)^(5/2))/(3*e*(d + e*x)^(3/2)) + (16*Sqrt[-a]*c^(3/2)*d*(32*c*d^2 + 2
9*a*e^2)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[
c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(21*e^6*Sqr
t[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) - (16*Sqrt
[-a]*Sqrt[c]*(c*d^2 + a*e^2)*(32*c*d^2 + 5*a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/
(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sq
rt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(21*e^6*
Sqrt[d + e*x]*Sqrt[a + c*x^2])
```


Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = -\frac{2\left(a+cx^2\right)^{5/2}}{3e(d+ex)^{3/2}} + \frac{(10c)\int \frac{x\left(a+cx^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx}{3e}$$

$$= \frac{20c(8d+ex)\left(a+cx^2\right)^{3/2}}{21e^3\sqrt{d+ex}} - \frac{2\left(a+cx^2\right)^{5/2}}{3e(d+ex)^{3/2}} - \frac{(20c)\int \frac{(-ae+8cdx)\sqrt{a+cx^2}}{\sqrt{d+ex}}\,dx}{7e^3}$$

$$= \frac{8c\sqrt{d+ex}\left(32cd^2+5ae^2-24cdex\right)\sqrt{a+cx^2}}{21e^5} + \frac{20c(8d+ex)\left(a+cx^2\right)^{3/2}}{21e^3\sqrt{d+ex}} - \frac{2\left(a+cx^2\right)^{5/2}}{3e(d+ex)^{3/2}} - \frac{16\int}{}$$

$$= \frac{8c\sqrt{d+ex}\left(32cd^2+5ae^2-24cdex\right)\sqrt{a+cx^2}}{21e^5} + \frac{20c(8d+ex)\left(a+cx^2\right)^{3/2}}{21e^3\sqrt{d+ex}} - \frac{2\left(a+cx^2\right)^{5/2}}{3e(d+ex)^{3/2}} + \frac{(8c}{}$$

$$= \frac{8c\sqrt{d+ex}\left(32cd^2+5ae^2-24cdex\right)\sqrt{a+cx^2}}{21e^5} + \frac{20c(8d+ex)\left(a+cx^2\right)^{3/2}}{21e^3\sqrt{d+ex}} - \frac{2\left(a+cx^2\right)^{5/2}}{3e(d+ex)^{3/2}} - \frac{(16}{}$$

$$= \frac{8c\sqrt{d+ex}\left(32cd^2+5ae^2-24cdex\right)\sqrt{a+cx^2}}{21e^5} + \frac{20c(8d+ex)\left(a+cx^2\right)^{3/2}}{21e^3\sqrt{d+ex}} - \frac{2\left(a+cx^2\right)^{5/2}}{3e(d+ex)^{3/2}} + \frac{16\sqrt{}}{}$$

Mathematica [C] time = 3.57923, size = 637, normalized size = 1.48

$$\sqrt{d+ex}\left(\frac{2\left(a+cx^2\right)\left(-7a^2e^4+2ace^2\left(50d^2+65dex+8e^2x^2\right)+c^2\left(16d^2e^2x^2+160d^3ex+128d^4-6de^3x^3+3e^4x^4\right)\right)}{e^5(d+ex)^2} - \frac{16c\left(-\sqrt{a}e(d+ex)^{3/2}\left(5ia^{3/2}e^3+8i\sqrt{a}cd^2e+29a\sqrt{}\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(5/2)/(d + e*x)^(5/2),x]
```

```
[Out] (Sqrt[d + e*x]*((2*(a + c*x^2)*(-7*a^2*e^4 + 2*a*c*e^2*(50*d^2 + 65*d*e*x +
 8*e^2*x^2) + c^2*(128*d^4 + 160*d^3*e*x + 16*d^2*e^2*x^2 - 6*d*e^3*x^3 + 3
*e^4*x^4)))/(e^5*(d + e*x)^2) - (16*c*(d*e^2*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c
]]*(29*a^2*e^2 + 32*c^2*d^2*x^2 + a*c*(32*d^2 + 29*e^2*x^2)) + Sqrt[c]*d*((
-32*I)*c^(3/2)*d^3 + 32*Sqrt[a]*c*d^2*e - (29*I)*a*Sqrt[c]*d*e^2 + 29*a^(3/
2)*e^3)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)
/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d - (
I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d
+ I*Sqrt[a]*e)] - Sqrt[a]*e*(32*c^(3/2)*d^3 + (8*I)*Sqrt[a]*c*d^2*e + 29*a*
Sqrt[c]*d*e^2 + (5*I)*a^(3/2)*e^3)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d +
e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*Elli
pticF[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d
 - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)]))/(e^7*Sqrt[-d - (I*Sqrt[a]*e)/S
qrt[c]]*(d + e*x))))/(21*Sqrt[a + c*x^2])
```

Maple [B] time = 0.273, size = 2646, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(5/2)/(e*x+d)^(5/2),x)
```

```
[Out] 2/21*(-6*x^5*c^3*d*e^5+16*x^4*c^3*d^2*e^4+160*x^3*c^3*d^3*e^3+9*x^2*a^2*c*e
^6+128*x^2*c^3*d^4*e^2+19*x^4*a*c^2*e^6+3*x^6*c^3*e^6+116*x^2*a*c^2*d^2*e^4
+130*x*a^2*c*d*e^5+160*x*a*c^2*d^3*e^3+256*EllipticE((-(e*x+d)*c/((-a*c)^(1
/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*c^3*d
^6*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(
1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-256*El
lipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a
*c)^(1/2)*e+c*d))^(1/2))*c^2*d^5*e*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e
-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c
)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+124*x^3*a*c^2*d*e^5-192*EllipticF((-
(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*
e+c*d))^(1/2))*x*a^2*c*d*e^5*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x
+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^
(1/2)*e-c*d))^(1/2)-192*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(
-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*a*c^2*d^3*e^3*(-(e*x+d
)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d)
)^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-256*EllipticF((-(
e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e
+c*d))^(1/2))*x*c^2*d^4*e^2*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^
(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2)
)*e/((-a*c)^(1/2)*e-c*d))^(1/2)+232*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c
*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*a^2*c*d*e^
5*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1
/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+488*Ell
ipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*
c)^(1/2)*e+c*d))^(1/2))*x*a*c^2*d^3*e^3*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(
1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))
*e/((-a*c)^(1/2)*e-c*d))^(1/2)-296*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*
d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c*d^3*e^3*(
-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/
((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/
2)+232*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-
c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*c*d^2*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e
-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c
)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+488*EllipticE((-(e*x+d)*c/((-a*c)^(1
/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c^2
*d^4*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-
a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+
256*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d
)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*c^3*d^5*e*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d)
)^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/
2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-192*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e
-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*c*d^2*
e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^
(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-192*E
llipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-
a*c)^(1/2)*e+c*d))^(1/2))*a*c^2*d^4*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(
1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))
*e/((-a*c)^(1/2)*e-c*d))^(1/2)-40*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d
))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*a^2*e^6*(-a*
```

```
c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-
a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-
40*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(
1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*Ellipt
icF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^
(1/2)*e+c*d))^(1/2))*(-a*c)^(1/2)*a^2*d*e^5-296*EllipticF((-(e*x+d)*c/((-a*
c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*
x*a*c*d^2*e^4*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(
-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1
/2)*e-c*d))^(1/2)-7*a^3*e^6+100*a^2*c*d^2*e^4+128*a*c^2*d^4*e^2)/(c*x^2+a)^
(1/2)/(e*x+d)^(3/2)/e^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + a\right)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^(5/2)/(e*x + d)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2x^4 + 2\,acx^2 + a^2\right)\sqrt{cx^2 + a}\sqrt{ex + d}}{e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*a*c*x^2 + a^2)*sqrt(c*x^2 + a)*sqrt(e*x + d)/(e^3*x^3
 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + cx^2\right)^{\frac{5}{2}}}{(d + ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(5/2)/(e*x+d)**(5/2),x)
```

```
[Out] Integral((a + c*x**2)**(5/2)/(d + e*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+a\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^(5/2)/(e*x + d)^(5/2), x)
```

3.674 $\int \frac{(a+cx^2)^{5/2}}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=420

$$\frac{16\sqrt{-a}c^{3/2}d\sqrt{\frac{cx^2}{a}+1}\left(17ae^2+32cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{15e^6\sqrt{a+cx^2}\sqrt{d+ex}}-\frac{16\sqrt{-a}c^{3/2}\sqrt{\frac{cx^2}{a}+1}\sqrt{d+\cdots}}{\cdots}$$

```
[Out] (-8*c*(32*c*d^2 + 9*a*e^2 + 8*c*d*e*x)*Sqrt[a + c*x^2])/(15*e^5*Sqrt[d + e*
x]) + (4*c*(8*d + 3*e*x)*(a + c*x^2)^(3/2))/(15*e^3*(d + e*x)^(3/2)) - (2*(
a + c*x^2)^(5/2))/(5*e*(d + e*x)^(5/2)) - (16*Sqrt[-a]*c^(3/2)*(32*c*d^2 +
9*a*e^2)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[
c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(15*e^6*Sqr
t[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) + (16*Sqrt
[-a]*c^(3/2)*d*(32*c*d^2 + 17*a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d +
Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt
[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(15*e^6*Sqrt[d + e*x]
*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.362422, antiderivative size = 420, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {733, 813, 844, 719, 424, 419}

$$\frac{16\sqrt{-a}c^{3/2}d\sqrt{\frac{cx^2}{a}+1}\left(17ae^2+32cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{15e^6\sqrt{a+cx^2}\sqrt{d+ex}}-\frac{16\sqrt{-a}c^{3/2}\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}\left(9ae^2\cdots\right.}{15e^6\sqrt{a\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(5/2)/(d + e*x)^(7/2),x]
```


```
[Out] (-8*c*(32*c*d^2 + 9*a*e^2 + 8*c*d*e*x)*Sqrt[a + c*x^2])/(15*e^5*Sqrt[d + e*
x]) + (4*c*(8*d + 3*e*x)*(a + c*x^2)^(3/2))/(15*e^3*(d + e*x)^(3/2)) - (2*(
a + c*x^2)^(5/2))/(5*e*(d + e*x)^(5/2)) - (16*Sqrt[-a]*c^(3/2)*(32*c*d^2 +
9*a*e^2)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[
c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(15*e^6*Sqr
t[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) + (16*Sqrt
[-a]*c^(3/2)*d*(32*c*d^2 + 17*a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d +
Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt
[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(15*e^6*Sqrt[d + e*x]
*Sqrt[a + c*x^2])
```


Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{5/2}}{(d+ex)^{7/2}}\,dx = -\frac{2\left(a+cx^2\right)^{5/2}}{5e(d+ex)^{5/2}} + \frac{(2c)\int \frac{x\left(a+cx^2\right)^{3/2}}{(d+ex)^{5/2}}\,dx}{e}$$

$$= \frac{4c(8d+3ex)\left(a+cx^2\right)^{3/2}}{15e^3(d+ex)^{3/2}} - \frac{2\left(a+cx^2\right)^{5/2}}{5e(d+ex)^{5/2}} - \frac{(4c)\int \frac{(-3ae+8cdx)\sqrt{a+cx^2}}{(d+ex)^{3/2}}\,dx}{5e^3}$$

$$= -\frac{8c\left(32cd^2+9ae^2+8cdex\right)\sqrt{a+cx^2}}{15e^5\sqrt{d+ex}} + \frac{4c(8d+3ex)\left(a+cx^2\right)^{3/2}}{15e^3(d+ex)^{3/2}} - \frac{2\left(a+cx^2\right)^{5/2}}{5e(d+ex)^{5/2}} + \frac{(8c)\int \frac{-8acd}{\sqrt{\ }}}{}$$

$$= -\frac{8c\left(32cd^2+9ae^2+8cdex\right)\sqrt{a+cx^2}}{15e^5\sqrt{d+ex}} + \frac{4c(8d+3ex)\left(a+cx^2\right)^{3/2}}{15e^3(d+ex)^{3/2}} - \frac{2\left(a+cx^2\right)^{5/2}}{5e(d+ex)^{5/2}} + \frac{\left(8c^2\left(32cd^2\right.\right.}{}$$

$$= -\frac{8c\left(32cd^2+9ae^2+8cdex\right)\sqrt{a+cx^2}}{15e^5\sqrt{d+ex}} + \frac{4c(8d+3ex)\left(a+cx^2\right)^{3/2}}{15e^3(d+ex)^{3/2}} - \frac{2\left(a+cx^2\right)^{5/2}}{5e(d+ex)^{5/2}} + \frac{\left(16ac^{3/2}\left(32\right.\right.}{}$$

$$= -\frac{8c\left(32cd^2+9ae^2+8cdex\right)\sqrt{a+cx^2}}{15e^5\sqrt{d+ex}} + \frac{4c(8d+3ex)\left(a+cx^2\right)^{3/2}}{15e^3(d+ex)^{3/2}} - \frac{2\left(a+cx^2\right)^{5/2}}{5e(d+ex)^{5/2}} - \frac{16\sqrt{-a}c^{3/2}\left(\right.}{}$$

Mathematica [C] time = 3.41537, size = 613, normalized size = 1.46

$$2\left(-e^2\left(a+cx^2\right)\left(3a^2e^4+2ace^2\left(10d^2+25dex+18e^2x^2\right)+c^2\left(176d^2e^2x^2+288d^3ex+128d^4+10de^3x^3-3e^4x^4\right)\right)+\frac{8c}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(5/2)/(d + e*x)^(7/2),x]
```

```
[Out] (2*(-(e^2*(a + c*x^2)*(3*a^2*e^4 + 2*a*c*e^2*(10*d^2 + 25*d*e*x + 18*e^2*x^
2) + c^2*(128*d^4 + 288*d^3*e*x + 176*d^2*e^2*x^2 + 10*d*e^3*x^3 - 3*e^4*x^
4))) + (8*c*(d + e*x)^2*(e^2*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*(9*a^2*e^2 +
32*c^2*d^2*x^2 + a*c*(32*d^2 + 9*e^2*x^2)) + Sqrt[c]*((-32*I)*c^(3/2)*d^3 +
 32*Sqrt[a]*c*d^2*e - (9*I)*a*Sqrt[c]*d*e^2 + 9*a^(3/2)*e^3)*Sqrt[(e*((I*Sq
rt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e
*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/
Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)] - Sqrt
[a]*Sqrt[c]*e*(32*c*d^2 + (8*I)*Sqrt[a]*Sqrt[c]*d*e + 9*a*e^2)*Sqrt[(e*((I*
Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d +
 e*x))]*(d + e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]
]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)]))/Sq
rt[-d - (I*Sqrt[a]*e)/Sqrt[c]]))/(15*e^7*(d + e*x)^(5/2)*Sqrt[a + c*x^2])
```

Maple [B] time = 0.278, size = 3421, normalized size = 8.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(5/2)/(e*x+d)^(7/2),x)
```

```
[Out] -2/15*(10*x^5*c^3*d*e^5+176*x^4*c^3*d^2*e^4+288*x^3*c^3*d^3*e^3+39*x^2*a^2*
c*e^6+128*x^2*c^3*d^4*e^2+33*x^4*a*c^2*e^6-3*x^6*c^3*e^6+196*x^2*a*c^2*d^2*
e^4+50*x*a^2*c*d*e^5+288*x*a*c^2*d^3*e^3+256*EllipticE((-(e*x+d)*c/((-a*c)^
(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*c^3
*d^6*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)
^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-256*
EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((
-a*c)^(1/2)*e+c*d))^(1/2))*c^2*d^5*e*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)
*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a
*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+60*x^3*a*c^2*d*e^5-144*EllipticF((
-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)
*e+c*d))^(1/2))*x*a^2*c*d*e^5*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*
x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)
^(1/2)*e-c*d))^(1/2)-384*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),
(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*a*c^2*d^3*e^3*(-(e*x+
d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d
))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-512*EllipticF((-
(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*
e+c*d))^(1/2))*x*c^2*d^4*e^2*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))
^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2
))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+144*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-
c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*a^2*c*d*e
^5*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(
1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+656*El
lipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a
*c)^(1/2)*e+c*d))^(1/2))*x*a*c^2*d^3*e^3*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^
(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2)
)*e/((-a*c)^(1/2)*e-c*d))^(1/2)-136*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c
*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c*d^3*e^3*
(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e
/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1
/2)-192*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e
-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a*c^2*d^2*e^4*(-(e*x+d)*c/((-a*c)^(1
/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+
(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-256*EllipticF((-(e*x+d)*c/((-a*
c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*
x^2*c^2*d^3*e^3*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x
+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^
(1/2)*e-c*d))^(1/2)+328*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(
-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a*c^2*d^2*e^4*(-(e*x
+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*
d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+72*EllipticE((-
(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*
e+c*d))^(1/2))*a^2*c*d^2*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x
+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^
(1/2)*e-c*d))^(1/2)-72*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-
((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a^2*c*e^6*(-(e*x+d)*c
/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(
1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+328*EllipticE((-(e*x
+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*
d))^(1/2))*a*c^2*d^4*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a
```

```
*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2
)*e-c*d))^(1/2)+72*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a
*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a^2*c*e^6*(-(e*x+d)*c/((-
a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)
*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+256*EllipticE((-(e*x+d)*
c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^
(1/2))*x^2*c^3*d^4*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c
)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*
e-c*d))^(1/2)+512*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*
c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*c^3*d^5*e*(-(e*x+d)*c/((-a*c
)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((
c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-72*EllipticF((-(e*x+d)*c/((
-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2
))*a^2*c*d^2*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2
))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d)
)^(1/2)-192*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/
2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c^2*d^4*e^2*(-(e*x+d)*c/((-a*c)^(1
/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+
(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-136*EllipticF((-(e*x+d)*c/((-a*
c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*
x^2*a*c*d*e^5*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(
-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1
/2)*e-c*d))^(1/2)-272*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-(
(-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*a*c*d^2*e^4*(-a*c)^(1/2)
*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/
2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+3*a^3*e^
6+20*a^2*c*d^2*e^4+128*a*c^2*d^4*e^2)/(c*x^2+a)^(1/2)/(e*x+d)^(5/2)/e^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+a\right)^{\frac{5}{2}}}{(ex+d)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^(5/2)/(e*x + d)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2x^4+2\,acx^2+a^2\right)\sqrt{cx^2+a}\sqrt{ex+d}}{e^4x^4+4\,de^3x^3+6\,d^2e^2x^2+4\,d^3ex+d^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*a*c*x^2 + a^2)*sqrt(c*x^2 + a)*sqrt(e*x + d)/(e^4*x^4
 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + cx^2)^{\frac{5}{2}}}{(d + ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(5/2)/(e*x+d)**(7/2),x)
```

```
[Out] Integral((a + c*x**2)**(5/2)/(d + e*x)**(7/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + a)^{\frac{5}{2}}}{(ex + d)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^(5/2)/(e*x + d)^(7/2), x)
```

3.675 $\int \frac{(a+cx^2)^{5/2}}{(d+ex)^{9/2}}\,dx$

Optimal. Leaf size=498

$$-\frac{16\sqrt{-a}c^{3/2}\sqrt{\frac{cx^2}{a}+1}\left(5ae^2+32cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{21e^6\sqrt{a+cx^2}\sqrt{d+ex}}+\frac{8c^2\sqrt{a+cx^2}\left(ex\left(5ae^2+8\right.\right.}{21e^5\sqrt{d+ex}}$$

```
[Out] (8*c^2*(d*(32*c*d^2 + 29*a*e^2) + e*(8*c*d^2 + 5*a*e^2)*x)*Sqrt[a + c*x^2])
/(21*e^5*(c*d^2 + a*e^2)*Sqrt[d + e*x]) - (4*c*(2*d*(4*c*d^2 + a*e^2) + e*(
11*c*d^2 + 5*a*e^2)*x)*(a + c*x^2)^(3/2))/(21*e^3*(c*d^2 + a*e^2)*(d + e*x)
^(5/2)) - (2*(a + c*x^2)^(5/2))/(7*e*(d + e*x)^(7/2)) + (16*Sqrt[-a]*c^(5/2
)*d*(32*c*d^2 + 29*a*e^2)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSi
n[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a
*e)])/(21*e^6*(c*d^2 + a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a
]*e)]*Sqrt[a + c*x^2]) - (16*Sqrt[-a]*c^(3/2)*(32*c*d^2 + 5*a*e^2)*Sqrt[(Sq
rt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[Ar
cSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d
- a*e)])/(21*e^6*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.454597, antiderivative size = 498, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {733, 811, 813, 844, 719, 424, 419}

$$\frac{8c^2\sqrt{a+cx^2}\left(ex\left(5ae^2+8cd^2\right)+d\left(29ae^2+32cd^2\right)\right)}{21e^5\sqrt{d+ex}\left(ae^2+cd^2\right)}-\frac{16\sqrt{-a}c^{3/2}\sqrt{\frac{cx^2}{a}+1}\left(5ae^2+32cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right.\right.}{21e^6\sqrt{a+cx^2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(5/2)/(d + e*x)^(9/2),x]
```


```
[Out] (8*c^2*(d*(32*c*d^2 + 29*a*e^2) + e*(8*c*d^2 + 5*a*e^2)*x)*Sqrt[a + c*x^2])
/(21*e^5*(c*d^2 + a*e^2)*Sqrt[d + e*x]) - (4*c*(2*d*(4*c*d^2 + a*e^2) + e*(
11*c*d^2 + 5*a*e^2)*x)*(a + c*x^2)^(3/2))/(21*e^3*(c*d^2 + a*e^2)*(d + e*x)
^(5/2)) - (2*(a + c*x^2)^(5/2))/(7*e*(d + e*x)^(7/2)) + (16*Sqrt[-a]*c^(5/2
)*d*(32*c*d^2 + 29*a*e^2)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSi
n[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a
*e)])/(21*e^6*(c*d^2 + a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a
]*e)]*Sqrt[a + c*x^2]) - (16*Sqrt[-a]*c^(3/2)*(32*c*d^2 + 5*a*e^2)*Sqrt[(Sq
rt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[Ar
cSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d
- a*e)])/(21*e^6*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```


Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
```

m, p, x]

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{5/2}}{(d+ex)^{9/2}}\,dx = -\frac{2\left(a+cx^2\right)^{5/2}}{7e(d+ex)^{7/2}} + \frac{(10c)\int \frac{x\left(a+cx^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx}{7e}$$

$$= -\frac{4c\left(2d\left(4cd^2+ae^2\right)+e\left(11cd^2+5ae^2\right)x\right)\left(a+cx^2\right)^{3/2}}{21e^3\left(cd^2+ae^2\right)(d+ex)^{5/2}} - \frac{2\left(a+cx^2\right)^{5/2}}{7e(d+ex)^{7/2}} - \frac{(4c)\int \frac{\left(3acde-c\left(8cd^2+5ae^2\right)\right.}{(d+ex)^{3/2}}}{7e^3\left(cd^2+ae^2\right.}$$

$$= \frac{8c^2\left(d\left(32cd^2+29ae^2\right)+e\left(8cd^2+5ae^2\right)x\right)\sqrt{a+cx^2}}{21e^5\left(cd^2+ae^2\right)\sqrt{d+ex}} - \frac{4c\left(2d\left(4cd^2+ae^2\right)+e\left(11cd^2+5ae^2\right)x\right)}{21e^3\left(cd^2+ae^2\right)(d+ex)^{5/2}}$$

$$= \frac{8c^2\left(d\left(32cd^2+29ae^2\right)+e\left(8cd^2+5ae^2\right)x\right)\sqrt{a+cx^2}}{21e^5\left(cd^2+ae^2\right)\sqrt{d+ex}} - \frac{4c\left(2d\left(4cd^2+ae^2\right)+e\left(11cd^2+5ae^2\right)x\right)}{21e^3\left(cd^2+ae^2\right)(d+ex)^{5/2}}$$

$$= \frac{8c^2\left(d\left(32cd^2+29ae^2\right)+e\left(8cd^2+5ae^2\right)x\right)\sqrt{a+cx^2}}{21e^5\left(cd^2+ae^2\right)\sqrt{d+ex}} - \frac{4c\left(2d\left(4cd^2+ae^2\right)+e\left(11cd^2+5ae^2\right)x\right)}{21e^3\left(cd^2+ae^2\right)(d+ex)^{5/2}}$$

$$= \frac{8c^2\left(d\left(32cd^2+29ae^2\right)+e\left(8cd^2+5ae^2\right)x\right)\sqrt{a+cx^2}}{21e^5\left(cd^2+ae^2\right)\sqrt{d+ex}} - \frac{4c\left(2d\left(4cd^2+ae^2\right)+e\left(11cd^2+5ae^2\right)x\right)}{21e^3\left(cd^2+ae^2\right)(d+ex)^{5/2}}$$

Mathematica [C] time = 3.99691, size = 677, normalized size = 1.36

$$\frac{\sqrt{d+ex}\left(\frac{2\left(a+cx^2\right)\left(\frac{2c^2d\left(67ae^2+79cd^2\right)}{(d+ex)\left(ae^2+cd^2\right)}-\frac{4c\left(4ae^2+13cd^2\right)}{(d+ex)^2}+\frac{18cd\left(ae^2+cd^2\right)}{(d+ex)^3}-\frac{3\left(ae^2+cd^2\right)^2}{(d+ex)^4}+7c^2\right)}{e^5}-\frac{16c^2\left(-\sqrt{a}e(d+ex)^{3/2}\left(5ia^{3/2}e^3+8i\sqrt{a}cd^2e+29a\sqrt{c}de^2+32c^{3/2}d^3\right)\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(5/2)/(d + e*x)^(9/2),x]
```

```
[Out] (Sqrt[d + e*x]*((2*(a + c*x^2)*(7*c^2 - (3*(c*d^2 + a*e^2)^2)/(d + e*x)^4 +
 (18*c*d*(c*d^2 + a*e^2))/(d + e*x)^3 - (4*c*(13*c*d^2 + 4*a*e^2))/(d + e*x
)^2 + (2*c^2*d*(79*c*d^2 + 67*a*e^2))/((c*d^2 + a*e^2)*(d + e*x))))/e^5 - (
16*c^2*(d*e^2*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*(29*a^2*e^2 + 32*c^2*d^2*x^2
 + a*c*(32*d^2 + 29*e^2*x^2)) + Sqrt[c]*d*((-32*I)*c^(3/2)*d^3 + 32*Sqrt[a]
*c*d^2*e - (29*I)*a*Sqrt[c]*d*e^2 + 29*a^(3/2)*e^3)*Sqrt[(e*((I*Sqrt[a])/Sq
rt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d
+ e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d +
e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)] - Sqrt[a]*e*(32
*c^(3/2)*d^3 + (8*I)*Sqrt[a]*c*d^2*e + 29*a*Sqrt[c]*d*e^2 + (5*I)*a^(3/2)*e
^3)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqr
t[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d - (I*Sq
rt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*
Sqrt[a]*e)]))/(e^7*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*(c*d^2 + a*e^2)*(d + e*
```

```
x))))/(21*Sqrt[a + c*x^2])
```

Maple [B] time = 0.286, size = 5303, normalized size = 10.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(5/2)/(e*x+d)^(9/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + a\right)^{\frac{5}{2}}}{(ex + d)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^(5/2)/(e*x + d)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2x^4 + 2\,acx^2 + a^2\right)\sqrt{cx^2 + a}\sqrt{ex + d}}{e^5x^5 + 5\,de^4x^4 + 10\,d^2e^3x^3 + 10\,d^3e^2x^2 + 5\,d^4ex + d^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*a*c*x^2 + a^2)*sqrt(c*x^2 + a)*sqrt(e*x + d)/(e^5*x^5
 + 5*d*e^4*x^4 + 10*d^2*e^3*x^3 + 10*d^3*e^2*x^2 + 5*d^4*e*x + d^5), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + cx^2\right)^{\frac{5}{2}}}{(d + ex)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(5/2)/(e*x+d)**(9/2),x)
```

```
[Out] Integral((a + c*x**2)**(5/2)/(d + e*x)**(9/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+a\right)^{\frac{5}{2}}}{\left(ex+d\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^(5/2)/(e*x + d)^(9/2), x)
```

3.676 $\int \frac{(a+cx^2)^{5/2}}{(d+ex)^{11/2}}\,dx$

Optimal. Leaf size=553

$$\frac{16\sqrt{-a}c^{5/2}d\sqrt{\frac{cx^2}{a}+1}\left(33ae^2+32cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{63e^6\sqrt{a+cx^2}\sqrt{d+ex}\left(ae^2+cd^2\right)}-\frac{8c^2\sqrt{a+cx^2}\left(ex\left(21a^2e^4+6\ldots\right.\right.}{63\ldots}$$

```
[Out] (-8*c^2*(d*(32*c^2*d^4 + 49*a*c*d^2*e^2 + 9*a^2*e^4) + e*(40*c^2*d^4 + 69*a
*c*d^2*e^2 + 21*a^2*e^4)*x)*Sqrt[a + c*x^2])/(63*e^5*(c*d^2 + a*e^2)^2*(d +
 e*x)^(3/2)) - (4*c*(2*d*(4*c*d^2 + a*e^2) + e*(13*c*d^2 + 7*a*e^2)*x)*(a +
 c*x^2)^(3/2))/(63*e^3*(c*d^2 + a*e^2)*(d + e*x)^(7/2)) - (2*(a + c*x^2)^(5
/2))/(9*e*(d + e*x)^(9/2)) - (16*Sqrt[-a]*c^(5/2)*(32*c^2*d^4 + 57*a*c*d^2*
e^2 + 21*a^2*e^4)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1
 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(6
3*e^6*(c*d^2 + a*e^2)^2*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*
Sqrt[a + c*x^2]) + (16*Sqrt[-a]*c^(5/2)*d*(32*c*d^2 + 33*a*e^2)*Sqrt[(Sqrt[
c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSi
n[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a
*e)])/(63*e^6*(c*d^2 + a*e^2)*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.492112, antiderivative size = 553, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {733, 811, 844, 719, 424, 419}

$$-\frac{8c^2\sqrt{a+cx^2}\left(ex\left(21a^2e^4+69acd^2e^2+40c^2d^4\right)+d\left(9a^2e^4+49acd^2e^2+32c^2d^4\right)\right)}{63e^5(d+ex)^{3/2}\left(ae^2+cd^2\right)^2}-\frac{16\sqrt{-a}c^{5/2}\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}\left(21a^2e^4\ldots\right.}{63e^6\sqrt{a+\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(5/2)/(d + e*x)^(11/2),x]
```


```
[Out] (-8*c^2*(d*(32*c^2*d^4 + 49*a*c*d^2*e^2 + 9*a^2*e^4) + e*(40*c^2*d^4 + 69*a
*c*d^2*e^2 + 21*a^2*e^4)*x)*Sqrt[a + c*x^2])/(63*e^5*(c*d^2 + a*e^2)^2*(d +
 e*x)^(3/2)) - (4*c*(2*d*(4*c*d^2 + a*e^2) + e*(13*c*d^2 + 7*a*e^2)*x)*(a +
 c*x^2)^(3/2))/(63*e^3*(c*d^2 + a*e^2)*(d + e*x)^(7/2)) - (2*(a + c*x^2)^(5
/2))/(9*e*(d + e*x)^(9/2)) - (16*Sqrt[-a]*c^(5/2)*(32*c^2*d^4 + 57*a*c*d^2*
e^2 + 21*a^2*e^4)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1
 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(6
3*e^6*(c*d^2 + a*e^2)^2*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*
Sqrt[a + c*x^2]) + (16*Sqrt[-a]*c^(5/2)*d*(32*c*d^2 + 33*a*e^2)*Sqrt[(Sqrt[
c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSi
n[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a
*e)])/(63*e^6*(c*d^2 + a*e^2)*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```


Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
```

```
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(a+cx^2)^{5/2}}{(d+ex)^{11/2}}\,dx = -\frac{2(a+cx^2)^{5/2}}{9e(d+ex)^{9/2}} + \frac{(10c)\int \frac{x(a+cx^2)^{3/2}}{(d+ex)^{9/2}}\,dx}{9e}$$

$$= -\frac{4c\left(2d\left(4cd^2+ae^2\right)+e\left(13cd^2+7ae^2\right)x\right)(a+cx^2)^{3/2}}{63e^3\left(cd^2+ae^2\right)(d+ex)^{7/2}} - \frac{2(a+cx^2)^{5/2}}{9e(d+ex)^{9/2}} - \frac{(4c)\int \frac{\left(5acde-c\left(8cd^2+7ae^2\right)x\right)\sqrt{}}{(d+ex)^{5/2}}}{21e^3\left(cd^2+ae^2\right)}$$

$$= -\frac{8c^2\left(d\left(32c^2d^4+49acd^2e^2+9a^2e^4\right)+e\left(40c^2d^4+69acd^2e^2+21a^2e^4\right)x\right)\sqrt{a+cx^2}}{63e^5\left(cd^2+ae^2\right)^2(d+ex)^{3/2}} - \frac{4c\left(2d\left(4cd^2+\right.\right.}{6}$$

$$= -\frac{8c^2\left(d\left(32c^2d^4+49acd^2e^2+9a^2e^4\right)+e\left(40c^2d^4+69acd^2e^2+21a^2e^4\right)x\right)\sqrt{a+cx^2}}{63e^5\left(cd^2+ae^2\right)^2(d+ex)^{3/2}} - \frac{4c\left(2d\left(4cd^2+\right.\right.}{6}$$

$$= -\frac{8c^2\left(d\left(32c^2d^4+49acd^2e^2+9a^2e^4\right)+e\left(40c^2d^4+69acd^2e^2+21a^2e^4\right)x\right)\sqrt{a+cx^2}}{63e^5\left(cd^2+ae^2\right)^2(d+ex)^{3/2}} - \frac{4c\left(2d\left(4cd^2+\right.\right.}{6}$$

$$= -\frac{8c^2\left(d\left(32c^2d^4+49acd^2e^2+9a^2e^4\right)+e\left(40c^2d^4+69acd^2e^2+21a^2e^4\right)x\right)\sqrt{a+cx^2}}{63e^5\left(cd^2+ae^2\right)^2(d+ex)^{3/2}} - \frac{4c\left(2d\left(4cd^2+\right.\right.}{6}$$

Mathematica [C] time = 5.13879, size = 762, normalized size = 1.38

$$2\left(-e^2\left(a+cx^2\right)\left(c^2(d+ex)^4\left(105a^2e^4+330acd^2e^2+193c^2d^4\right)-2c^2d(d+ex)^3\left(57ae^2+61cd^2\right)\left(ae^2+cd^2\right)-38cd(d+ex\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(5/2)/(d + e*x)^(11/2),x]
```

```
[Out] (2*(-(e^2*(a + c*x^2)*(7*(c*d^2 + a*e^2)^4 - 38*c*d*(c*d^2 + a*e^2)^3*(d +
e*x) + 4*c*(c*d^2 + a*e^2)^2*(22*c*d^2 + 7*a*e^2)*(d + e*x)^2 - 2*c^2*d*(c*
d^2 + a*e^2)*(61*c*d^2 + 57*a*e^2)*(d + e*x)^3 + c^2*(193*c^2*d^4 + 330*a*c
*d^2*e^2 + 105*a^2*e^4)*(d + e*x)^4)) + (8*c^2*(d + e*x)^4*(e^2*Sqrt[-d - (
I*Sqrt[a]*e)/Sqrt[c]]*(32*c^2*d^4 + 57*a*c*d^2*e^2 + 21*a^2*e^4)*(a + c*x^2
) + Sqrt[c]*((-32*I)*c^(5/2)*d^5 + 32*Sqrt[a]*c^2*d^4*e - (57*I)*a*c^(3/2)*
d^3*e^2 + 57*a^(3/2)*c*d^2*e^3 - (21*I)*a^2*Sqrt[c]*d*e^4 + 21*a^(5/2)*e^5)
*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c
] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d - (I*Sqrt[
a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqr
t[a]*e)] - Sqrt[a]*Sqrt[c]*e*(32*c^2*d^4 + (8*I)*Sqrt[a]*c^(3/2)*d^3*e + 57
*a*c*d^2*e^2 + (12*I)*a^(3/2)*Sqrt[c]*d*e^3 + 21*a^2*e^4)*Sqrt[(e*((I*Sqrt[
a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x)
)]*(d + e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqr
```

```
t[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)]))/Sqrt[-d
 - (I*Sqrt[a]*e)/Sqrt[c]]))/(63*e^7*(c*d^2 + a*e^2)^2*(d + e*x)^(9/2)*Sqrt[
a + c*x^2])
```

Maple [B] time = 0.323, size = 8244, normalized size = 14.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(5/2)/(e*x+d)^(11/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+a\right)^{\frac{5}{2}}}{(ex+d)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^(5/2)/(e*x + d)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2x^4+2\,acx^2+a^2\right)\sqrt{cx^2+a}\sqrt{ex+d}}{e^6x^6+6\,de^5x^5+15\,d^2e^4x^4+20\,d^3e^3x^3+15\,d^4e^2x^2+6\,d^5ex+d^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^(11/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*a*c*x^2 + a^2)*sqrt(c*x^2 + a)*sqrt(e*x + d)/(e^6*x^6
 + 6*d*e^5*x^5 + 15*d^2*e^4*x^4 + 20*d^3*e^3*x^3 + 15*d^4*e^2*x^2 + 6*d^5*e
*x + d^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(5/2)/(e*x+d)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)/(e*x+d)^(11/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.677 $\int \frac{(d+ex)^{7/2}}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=413

$$\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(71cd^2-25ae^2\right)\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{105c^{5/2}\sqrt{a+cx^2}\sqrt{d+ex}}+\frac{2e\sqrt{a+cx^2}\sqrt{d+ex}}{105c}$$

```
[Out] (2*e*(71*c*d^2 - 25*a*e^2)*Sqrt[d + e*x]*Sqrt[a + c*x^2])/(105*c^2) + (24*d
*e*(d + e*x)^(3/2)*Sqrt[a + c*x^2])/(35*c) + (2*e*(d + e*x)^(5/2)*Sqrt[a +
c*x^2])/(7*c) - (32*Sqrt[-a]*d*(11*c*d^2 - 13*a*e^2)*Sqrt[d + e*x]*Sqrt[1 +
 (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a
*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(105*c^(3/2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqr
t[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) + (2*Sqrt[-a]*(71*c*d^2 - 25*a*e^2)*
(c*d^2 + a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 +
 (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a
*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(105*c^(5/2)*Sqrt[d + e*x]*Sqrt[a + c*x^2]
)
```

Rubi [A] time = 0.434713, antiderivative size = 413, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {743, 833, 844, 719, 424, 419}

$$\frac{2e\sqrt{a+cx^2}\sqrt{d+ex}\left(71cd^2-25ae^2\right)}{105c^2}+\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(71cd^2-25ae^2\right)\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\right.}{105c^{5/2}\sqrt{a+cx^2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/Sqrt[a + c*x^2],x]
```


```
[Out] (2*e*(71*c*d^2 - 25*a*e^2)*Sqrt[d + e*x]*Sqrt[a + c*x^2])/(105*c^2) + (24*d
*e*(d + e*x)^(3/2)*Sqrt[a + c*x^2])/(35*c) + (2*e*(d + e*x)^(5/2)*Sqrt[a +
c*x^2])/(7*c) - (32*Sqrt[-a]*d*(11*c*d^2 - 13*a*e^2)*Sqrt[d + e*x]*Sqrt[1 +
 (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a
*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(105*c^(3/2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqr
t[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) + (2*Sqrt[-a]*(71*c*d^2 - 25*a*e^2)*
(c*d^2 + a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 +
 (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a
*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(105*c^(5/2)*Sqrt[d + e*x]*Sqrt[a + c*x^2]
)
```


Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^{7/2}}{\sqrt{a+cx^2}}\,dx = \frac{2e(d+ex)^{5/2}\sqrt{a+cx^2}}{7c} + \frac{2\int \frac{(d+ex)^{3/2}\left(\frac{1}{2}\left(7cd^2-5ae^2\right)+6cdex\right)}{\sqrt{a+cx^2}}dx}{7c}$$

$$= \frac{24de(d+ex)^{3/2}\sqrt{a+cx^2}}{35c} + \frac{2e(d+ex)^{5/2}\sqrt{a+cx^2}}{7c} + \frac{4\int \frac{\sqrt{d+ex}\left(\frac{1}{4}cd\left(35cd^2-61ae^2\right)+\frac{1}{4}ce\left(71cd^2-25ae^2\right)x\right)}{\sqrt{a+cx^2}}dx}{35c^2}$$

$$= \frac{2e\left(71cd^2-25ae^2\right)\sqrt{d+ex}\sqrt{a+cx^2}}{105c^2} + \frac{24de(d+ex)^{3/2}\sqrt{a+cx^2}}{35c} + \frac{2e(d+ex)^{5/2}\sqrt{a+cx^2}}{7c} + \frac{8\int \frac{1}{8}\cdots}{\cdots}$$

$$= \frac{2e\left(71cd^2-25ae^2\right)\sqrt{d+ex}\sqrt{a+cx^2}}{105c^2} + \frac{24de(d+ex)^{3/2}\sqrt{a+cx^2}}{35c} + \frac{2e(d+ex)^{5/2}\sqrt{a+cx^2}}{7c} + \frac{(16d(\cdots}{\cdots}$$

$$= \frac{2e\left(71cd^2-25ae^2\right)\sqrt{d+ex}\sqrt{a+cx^2}}{105c^2} + \frac{24de(d+ex)^{3/2}\sqrt{a+cx^2}}{35c} + \frac{2e(d+ex)^{5/2}\sqrt{a+cx^2}}{7c} + \frac{(32a\cdots}{\cdots}$$

$$= \frac{2e\left(71cd^2-25ae^2\right)\sqrt{d+ex}\sqrt{a+cx^2}}{105c^2} + \frac{24de(d+ex)^{3/2}\sqrt{a+cx^2}}{35c} + \frac{2e(d+ex)^{5/2}\sqrt{a+cx^2}}{7c} - \frac{32\sqrt{-\cdots}}{\cdots}$$

Mathematica [C] time = 3.1169, size = 548, normalized size = 1.33

$$\frac{2\sqrt{d+ex}\left(\frac{\sqrt{d+ex}\left(208a^{3/2}\sqrt{c}de^3+25ia^2e^4-176\sqrt{a}c^{3/2}d^3e-254iacd^2e^2+105ic^2d^4\right)\sqrt{\frac{e\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{d+ex}}\sqrt{-\frac{-ex+\frac{i\sqrt{ae}}{\sqrt{c}}}{d+ex}}\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-d-\frac{i\sqrt{ae}}{\sqrt{c}}}}{\sqrt{d+ex}}\right),\frac{\sqrt{c}d-i\sqrt{a}e}{\sqrt{c}d+i\sqrt{a}e}\right)}{e\sqrt{-d-\frac{i\sqrt{ae}}{\sqrt{c}}}} + \cdots\right)}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)/Sqrt[a + c*x^2],x]
```

```
[Out] (2*Sqrt[d + e*x]*((16*d*e*(-13*a^2*e^2 + 11*c^2*d^2*x^2 + a*c*(11*d^2 - 13*
e^2*x^2)))/(d + e*x) + (a + c*x^2)*(-25*a*e^3 + c*e*(122*d^2 + 66*d*e*x + 1
5*e^2*x^2)) + ((16*I)*c*d*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*(11*c*d^2 - 13*a
*e^2)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/S
qrt[c] - e*x)/(d + e*x))]*Sqrt[d + e*x]*EllipticE[I*ArcSinh[Sqrt[-d - (I*Sq
rt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*
Sqrt[a]*e)])/e + (((105*I)*c^2*d^4 - 176*Sqrt[a]*c^(3/2)*d^3*e - (254*I)*a*
c*d^2*e^2 + 208*a^(3/2)*Sqrt[c]*d*e^3 + (25*I)*a^2*e^4)*Sqrt[(e*((I*Sqrt[a]
)/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]
*Sqrt[d + e*x]*EllipticF[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d
+ e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)])/(e*Sqrt[-d -
 (I*Sqrt[a]*e)/Sqrt[c]])))/(105*c^2*Sqrt[a + c*x^2])
```

Maple [B] time = 0.27, size = 1534, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/(c*x^2+a)^(1/2),x)
```

```
[Out] -2/105*(e*x+d)^(1/2)*(c*x^2+a)^(1/2)*(-15*x^5*c^3*e^5+25*(-a*c)^(1/2)*(-(e*
x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c
*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e
*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+
c*d))^(1/2))*a^2*e^5-46*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2
)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/
((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/
2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c*d^2*e^3-71*(-a*c
)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a
*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*E
llipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-
a*c)^(1/2)*e+c*d))^(1/2))*c^2*d^4*e+183*a^2*c*(-(e*x+d)*c/((-a*c)^(1/2)*e-c
*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^
(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-
c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*d*e^4+78*a*
c^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^
(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*Ellip
ticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)
^(1/2)*e+c*d))^(1/2))*d^3*e^2-105*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*(
(-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-
a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),
(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*c^3*d^5-208*a^2*c*(-(e*
x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c
*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticE((-(e
*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+
c*d))^(1/2))*d*e^4-32*a*c^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+
(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(
1/2)*e-c*d))^(1/2)*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a
*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*d^3*e^2+176*(-(e*x+d)*c/((-a*
c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*(
(c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticE((-(e*x+d)*c/((-a
*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))
*c^3*d^5-81*x^4*c^3*d*e^4+10*x^3*a*c^2*e^5-188*x^3*c^3*d^2*e^3-56*x^2*a*c^2
*d*e^4-122*x^2*c^3*d^3*e^2+25*x*a^2*c*e^5-188*x*a*c^2*d^2*e^3+25*a^2*c*d*e^
4-122*a*c^2*d^3*e^2)/e/(c*e*x^3+c*d*x^2+a*e*x+a*d)/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{7}{2}}}{\sqrt{cx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(7/2)/sqrt(c*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e^{3}x^{3}+3\,de^{2}x^{2}+3\,d^{2}ex+d^{3}\right)\sqrt{ex+d}}{\sqrt{cx^{2}+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(e*x + d)/sqrt(c*x^2
 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(d+ex)^{\frac{7}{2}}}{\sqrt{a+cx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**(7/2)/sqrt(a + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ex+d)^{\frac{7}{2}}}{\sqrt{cx^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(7/2)/sqrt(c*x^2 + a), x)
```

3.678 $\int \frac{(d+ex)^{5/2}}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=359

$$\frac{16\sqrt{-a}d\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{15c^{3/2}\sqrt{a+cx^2}\sqrt{d+ex}}-\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}\left(23cd^2-9ae^2\right.}{15c^{3/2}\sqrt{a+cx^2}}$$

```
[Out] (16*d*e*Sqrt[d + e*x]*Sqrt[a + c*x^2])/(15*c) + (2*e*(d + e*x)^(3/2)*Sqrt[a
 + c*x^2])/(5*c) - (2*Sqrt[-a]*(23*c*d^2 - 9*a*e^2)*Sqrt[d + e*x]*Sqrt[1 +
(c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*
e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(15*c^(3/2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[
c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) + (16*Sqrt[-a]*d*(c*d^2 + a*e^2)*Sqrt[
(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF
[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]
*d - a*e)])/(15*c^(3/2)*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.32498, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {743, 833, 844, 719, 424, 419}

$$\frac{16\sqrt{-a}d\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{15c^{3/2}\sqrt{a+cx^2}\sqrt{d+ex}}-\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}\left(23cd^2-9ae^2\right)E\left(\sin\right.}{15c^{3/2}\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(d}{\sqrt{-ae}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/Sqrt[a + c*x^2],x]
```

```
[Out] (16*d*e*Sqrt[d + e*x]*Sqrt[a + c*x^2])/(15*c) + (2*e*(d + e*x)^(3/2)*Sqrt[a
 + c*x^2])/(5*c) - (2*Sqrt[-a]*(23*c*d^2 - 9*a*e^2)*Sqrt[d + e*x]*Sqrt[1 +
(c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*
e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(15*c^(3/2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[
c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) + (16*Sqrt[-a]*d*(c*d^2 + a*e^2)*Sqrt[
(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF
[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]
*d - a*e)])/(15*c^(3/2)*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
```

```
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\sqrt{a+cx^2}}\,dx = \frac{2e(d+ex)^{3/2}\sqrt{a+cx^2}}{5c} + \frac{2\int \frac{\sqrt{d+ex}\left(\frac{1}{2}(5cd^2-3ae^2)+4cdex\right)}{\sqrt{a+cx^2}}dx}{5c}$$

$$= \frac{16de\sqrt{d+ex}\sqrt{a+cx^2}}{15c} + \frac{2e(d+ex)^{3/2}\sqrt{a+cx^2}}{5c} + \frac{4\int \frac{\frac{1}{4}cd(15cd^2-17ae^2)+\frac{1}{4}ce(23cd^2-9ae^2)x}{\sqrt{d+ex}\sqrt{a+cx^2}}dx}{15c^2}$$

$$= \frac{16de\sqrt{d+ex}\sqrt{a+cx^2}}{15c} + \frac{2e(d+ex)^{3/2}\sqrt{a+cx^2}}{5c} + \frac{(23cd^2-9ae^2)\int \frac{\sqrt{d+ex}}{\sqrt{a+cx^2}}dx}{15c} - \frac{\left(8d\left(cd^2+ae^2\right)\right)\int \frac{}{\sqrt{a}}}{15c}$$

$$= \frac{16de\sqrt{d+ex}\sqrt{a+cx^2}}{15c} + \frac{2e(d+ex)^{3/2}\sqrt{a+cx^2}}{5c} + \frac{\left(2a\left(23cd^2-9ae^2\right)\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1}}{}\right.}{15\sqrt{-a}c^{3/2}\sqrt{\frac{c(d+ex)}{cd-\frac{a\sqrt{c}e}{\sqrt{-a}}}}\sqrt{a+c}}$$

$$= \frac{16de\sqrt{d+ex}\sqrt{a+cx^2}}{15c} + \frac{2e(d+ex)^{3/2}\sqrt{a+cx^2}}{5c} - \frac{2\sqrt{-a}\left(23cd^2-9ae^2\right)\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\sqrt{}\right.\right.}{15c^{3/2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}}\sqrt{a+cx^2}}$$

Mathematica [C] time = 3.05718, size = 557, normalized size = 1.55

$$\frac{\sqrt{d+ex}\left(\frac{2e(a+cx^2)(11d+3ex)}{c} + \frac{2\left(\sqrt{c}(d+ex)^{3/2}\left(9a^{3/2}e^3-23\sqrt{a}cd^2e-17ia\sqrt{c}de^2+15ic^{3/2}d^3\right)\sqrt{\frac{e\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{d+ex}}\sqrt{-\frac{-ex+\frac{i\sqrt{a}e}{\sqrt{c}}}{d+ex}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}}{\sqrt{d+ex}}\right),\frac{\sqrt{c}d-i\sqrt{a}e}{\sqrt{c}d+i\sqrt{a}e}\right)\right.}{}\right)}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[d + e*x]*((2*e*(11*d + 3*e*x)*(a + c*x^2))/c + (2*(e^2*Sqrt[-d - (I*S
qrt[a]*e)/Sqrt[c]]*(-9*a^2*e^2 + 23*c^2*d^2*x^2 + a*c*(23*d^2 - 9*e^2*x^2))
 + Sqrt[c]*((-23*I)*c^(3/2)*d^3 + 23*Sqrt[a]*c*d^2*e + (9*I)*a*Sqrt[c]*d*e^
2 - 9*a^(3/2)*e^3)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I
*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[
Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/
(Sqrt[c]*d + I*Sqrt[a]*e)] + Sqrt[c]*((15*I)*c^(3/2)*d^3 - 23*Sqrt[a]*c*d^2
*e - (17*I)*a*Sqrt[c]*d*e^2 + 9*a^(3/2)*e^3)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] +
 x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^
(3/2)*EllipticF[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]],
(Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)]))/(c^2*e*Sqrt[-d - (I*
Sqrt[a]*e)/Sqrt[c]]*(d + e*x))))/(15*Sqrt[a + c*x^2])
```

Maple [B] time = 0.287, size = 1312, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(c*x^2+a)^(1/2),x)
```

```
[Out] -2/15*(e*x+d)^(1/2)*(c*x^2+a)^(1/2)*(9*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1
/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*
e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(
1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*e^4-6*(-(e*x+d
)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d)
)^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+
d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d
))^(1/2))*a*c*d^2*e^2-8*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*
c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)
*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^
(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*(-a*c)^(1/2)*a*d*e^3-15*(-(e*x+d)
*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))
^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d
)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d)
)^(1/2))*c^2*d^4-8*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1
/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*
d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)
*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*(-a*c)^(1/2)*c*d^3*e-9*(-(e*x+d)*c/((-
a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)
*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticE((-(e*x+d)*c/((
-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2
))*a^2*e^4+14*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*
e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(
1/2)*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*
d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c*d^2*e^2+23*(-(e*x+d)*c/((-a*c)^(1/2)*e-
c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)
^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e
-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*c^2*d^4-3*
x^4*c^2*e^4-14*x^3*c^2*d*e^3-3*x^2*a*c*e^4-11*x^2*c^2*d^2*e^2-14*x*a*c*d*e^
3-11*a*c*d^2*e^2)/c^2/e/(c*e*x^3+c*d*x^2+a*e*x+a*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\sqrt{cx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/sqrt(c*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(e^{2}x^{2}+2\,dex+d^{2}\right)\sqrt{ex+d}}{\sqrt{cx^{2}+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*sqrt(e*x + d)/sqrt(c*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^{\frac{5}{2}}}{\sqrt{a+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**(5/2)/sqrt(a + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\sqrt{cx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(5/2)/sqrt(c*x^2 + a), x)
```

3.679 $\int \frac{(d+ex)^{3/2}}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=317

$$\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{3c^{3/2}\sqrt{a+cx^2}\sqrt{d+ex}}+\frac{2e\sqrt{a+cx^2}\sqrt{d+ex}}{3c}-\frac{8\sqrt{-a}d\sqrt{\frac{cx^2}{a}}}{}$$

```
[Out] (2*e*Sqrt[d + e*x]*Sqrt[a + c*x^2])/(3*c) - (8*Sqrt[-a]*d*Sqrt[d + e*x]*Sqr
t[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]],
(-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(3*Sqrt[c]*Sqrt[(Sqrt[c]*(d + e*x))/(
Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) + (2*Sqrt[-a]*(c*d^2 + a*e^2)*Sqr
t[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*Ellipti
cF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[
c]*d - a*e)])/(3*c^(3/2)*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.212071, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {743, 844, 719, 424, 419}

$$\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{3c^{3/2}\sqrt{a+cx^2}\sqrt{d+ex}}+\frac{2e\sqrt{a+cx^2}\sqrt{d+ex}}{3c}-\frac{8\sqrt{-a}d\sqrt{\frac{cx^2}{a}+1}\sqrt{d}}{3\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/Sqrt[a + c*x^2],x]
```

```
[Out] (2*e*Sqrt[d + e*x]*Sqrt[a + c*x^2])/(3*c) - (8*Sqrt[-a]*d*Sqrt[d + e*x]*Sqr
t[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]],
(-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(3*Sqrt[c]*Sqrt[(Sqrt[c]*(d + e*x))/(
Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) + (2*Sqrt[-a]*(c*d^2 + a*e^2)*Sqr
t[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*Ellipti
cF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[
c]*d - a*e)])/(3*c^(3/2)*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\sqrt{a+cx^2}}\,dx = \frac{2e\sqrt{d+ex}\sqrt{a+cx^2}}{3c} + \frac{2\int \frac{\frac{1}{2}\left(3cd^2-ae^2\right)+2cdex}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx}{3c}$$

$$= \frac{2e\sqrt{d+ex}\sqrt{a+cx^2}}{3c} + \frac{1}{3}(4d)\int \frac{\sqrt{d+ex}}{\sqrt{a+cx^2}}\,dx - \frac{\left(cd^2+ae^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx}{3c}$$

$$= \frac{2e\sqrt{d+ex}\sqrt{a+cx^2}}{3c} + \frac{\left(8ad\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}ex^2}{\sqrt{-a}\left(cd-\frac{a\sqrt{c}e}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{3\sqrt{-a}\sqrt{c}\sqrt{\frac{c(d+ex)}{cd-\frac{a\sqrt{c}e}{\sqrt{-a}}}}\sqrt{a+cx^2}} - \frac{\left(2a\left(cd^2+\right.\right.}{}$$

$$= \frac{2e\sqrt{d+ex}\sqrt{a+cx^2}}{3c} - \frac{8\sqrt{-a}d\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle| -\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{3\sqrt{c}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}}\sqrt{a+cx^2}} + \frac{2\sqrt{-a}\left(cd^2+ae^2\right)}{}$$

Mathematica [C] time = 2.33899, size = 445, normalized size = 1.4

$$\frac{2\sqrt{d+ex}\left(\frac{i\sqrt{d+ex}\left(4i\sqrt{a}\sqrt{c}de-ae^2+3cd^2\right)\sqrt{\frac{e\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{d+ex}}\sqrt{-\frac{-ex+\frac{i\sqrt{a}e}{\sqrt{c}}}{d+ex}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}}{\sqrt{d+ex}}\right),\frac{\sqrt{c}d-i\sqrt{a}e}{\sqrt{c}d+i\sqrt{a}e}\right)}{\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}} + \frac{4de^2\left(a+cx^2\right)}{d+ex} + 4icd\sqrt{d+ex}\sqrt{-d-\right.}{3ce\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/Sqrt[a + c*x^2],x]
```

```
[Out] (2*Sqrt[d + e*x]*(e^2*(a + c*x^2) + (4*d*e^2*(a + c*x^2))/(d + e*x) + (4*I)
*c*d*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d
 + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*Sqrt[d + e*x]*Ell
ipticE[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*
d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)] + (I*(3*c*d^2 + (4*I)*Sqrt[a]*S
qrt[c]*d*e - a*e^2)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((
I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*Sqrt[d + e*x]*EllipticF[I*ArcSinh[S
qrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(
Sqrt[c]*d + I*Sqrt[a]*e)])/Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]))/(3*c*e*Sqrt[a
 + c*x^2])
```

Maple [B] time = 0.26, size = 978, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*x^2+a)^(1/2),x)
```

```
[Out] 2/3*(e*x+d)^(1/2)*(c*x^2+a)^(1/2)*((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*
((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((
-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)
,(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*(-a*c)^(1/2)*a*e^3+(-(
e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e
+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-
(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*
e+c*d))^(1/2))*(-a*c)^(1/2)*c*d^2*e+3*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/
2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e
/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1
/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c*d*e^2+3*(-(e*x+
d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d
))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x
+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*
d))^(1/2))*c^2*d^3-4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^
(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-
c*d))^(1/2)*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/
2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c*d*e^2-4*(-(e*x+d)*c/((-a*c)^(1/2
)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-
a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticE((-(e*x+d)*c/((-a*c)^(1/
2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*c^2*d^
3+x^3*c^2*e^3+x^2*c^2*d*e^2+x*a*c*e^3+a*d*e^2*c)/e/(c*e*x^3+c*d*x^2+a*e*x+a
*d)/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/sqrt(c*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^(3/2)/sqrt(c*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^{\frac{3}{2}}}{\sqrt{a+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)/sqrt(a + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)/sqrt(c*x^2 + a), x)
```

3.680 $\int \frac{\sqrt{d+ex}}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=136

$$-\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{\sqrt{c}\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}}$$

```
[Out] (-2*Sqrt[-a]*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (S
qrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(Sqrt[c
]*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.0531155, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {719, 424}

$$-\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{\sqrt{c}\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/Sqrt[a + c*x^2],x]
```


```
[Out] (-2*Sqrt[-a]*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (S
qrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(Sqrt[c
]*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2])
```


Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\sqrt{a+cx^2}}\,dx = \frac{\left(2a\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}ex^2}{\sqrt{-a}\left(cd-\frac{a\sqrt{c}e}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\frac{c(d+ex)}{cd-\frac{a\sqrt{c}e}{\sqrt{-a}}}}\sqrt{a+cx^2}}$$

$$= -\frac{2\sqrt{-a}\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{\sqrt{c}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}}\sqrt{a+cx^2}}$$

Mathematica [C] time = 0.471626, size = 294, normalized size = 2.16

$$\frac{2i\sqrt{d+ex}\left(\sqrt{c}d+i\sqrt{a}e\right)\sqrt{\frac{e\left(\sqrt{a}+i\sqrt{c}x\right)}{\sqrt{a}e-i\sqrt{c}d}}\left(E\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-i\sqrt{a}e}}\right)|\frac{\sqrt{c}d-i\sqrt{a}e}{\sqrt{c}d+i\sqrt{a}e}\right)-\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-i\sqrt{a}e}}\right),\frac{\sqrt{c}d-i\sqrt{a}e}{\sqrt{c}d+i\sqrt{a}e}\right)\right)}{\sqrt{c}e\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{e\left(\sqrt{c}x+i\sqrt{a}\right)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/Sqrt[a + c*x^2],x]
```

```
[Out] ((2*I)*(Sqrt[c]*d + I*Sqrt[a]*e)*Sqrt[(e*(Sqrt[a] + I*Sqrt[c]*x))/((-I)*Sqr
t[c]*d + Sqrt[a]*e)]*Sqrt[d + e*x]*(EllipticE[I*ArcSinh[Sqrt[-((Sqrt[c]*(d
+ e*x))/(Sqrt[c]*d - I*Sqrt[a]*e))]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d
+ I*Sqrt[a]*e)] - EllipticF[I*ArcSinh[Sqrt[-((Sqrt[c]*(d + e*x))/(Sqrt[c]*d
 - I*Sqrt[a]*e))]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)]))/
(Sqrt[c]*e*Sqrt[(Sqrt[c]*(d + e*x))/(e*(I*Sqrt[a] + Sqrt[c]*x))]*Sqrt[a + c
*x^2])
```

Maple [B] time = 0.263, size = 396, normalized size = 2.9

$$2\frac{\sqrt{ex+d}\sqrt{cx^2+a}\left(-\sqrt{-ace}+cd\right)}{e\left(cex^3+cdx^2+aex+ad\right)c^2}\sqrt{-\frac{c\left(ex+d\right)}{\sqrt{-ace}-cd}}\sqrt{\frac{\left(-cx+\sqrt{-ac}\right)e}{\sqrt{-ace}+cd}}\sqrt{\frac{\left(cx+\sqrt{-ac}\right)e}{\sqrt{-ace}-cd}}\left(\sqrt{-ac}\text{EllipticF}\left(\sqrt{-\frac{c\left(ex+d\right)}{\sqrt{-ace}-cd}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*x^2+a)^(1/2),x)
```

```
[Out] 2*(e*x+d)^(1/2)*(c*x^2+a)^(1/2)*(-(-a*c)^(1/2)*e+c*d)*(-(e*x+d)*c/((-a*c)^(
1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x
+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*((-a*c)^(1/2)*EllipticF((-(e*x
+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*
d))^(1/2))*e+d*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^
(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*c-EllipticE((-(e*x+d)*c/((-a*c)^(
1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*(-a*
c)^(1/2)*e-EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2
)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*c*d)/e/(c*e*x^3+c*d*x^2+a*e*x+a*d)/c^
2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{cx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/sqrt(c*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex+d}}{\sqrt{cx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)/sqrt(c*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}}{\sqrt{a+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/sqrt(a + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{cx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)/sqrt(c*x^2 + a), x)
```

3.681 $\int \frac{1}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=136

$$-\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{\sqrt{c}\sqrt{a+cx^2}\sqrt{d+ex}}$$

```
[Out] (-2*Sqrt[-a]*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c
*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)
/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(Sqrt[c]*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.0565435, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {719, 419}

$$-\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{\sqrt{c}\sqrt{a+cx^2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (-2*Sqrt[-a]*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c
*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)
/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(Sqrt[c]*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx = \frac{\left(2a\sqrt{\frac{c(d+ex)}{cd-\frac{a\sqrt{ce}}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}ex^2}{\sqrt{-a}\left(cd-\frac{a\sqrt{c}e}{\sqrt{-a}}\right)}}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{d+ex}\sqrt{a+cx^2}}$$

$$= -\frac{2\sqrt{-a}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{\sqrt{c}\sqrt{d+ex}\sqrt{a+cx^2}}$$

Mathematica [C] time = 0.243081, size = 186, normalized size = 1.37

$$\frac{2i(d+ex)\sqrt{\frac{e\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{d+ex}}\sqrt{-\frac{-ex+\frac{i\sqrt{a}e}{\sqrt{c}}}{d+ex}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}}{\sqrt{d+ex}}\right), \frac{\sqrt{c}d-i\sqrt{a}e}{\sqrt{c}d+i\sqrt{a}e}\right)}{e\sqrt{a+cx^2}\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] ((2*I)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/
Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)*EllipticF[I*ArcSinh[Sqrt[-d - (I*Sqrt[
a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqr
t[a]*e)])/(e*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*Sqrt[a + c*x^2])
```

Maple [A] time = 0.264, size = 200, normalized size = 1.5

$$2\frac{\left(-\sqrt{-ace}+cd\right)\sqrt{ex+d}\sqrt{cx^2+a}}{ce\left(cex^3+cdx^2+aex+ad\right)}\text{EllipticF}\left(\sqrt{-\frac{c(ex+d)}{\sqrt{-ace}-cd}}, \sqrt{-\frac{\sqrt{-ace}-cd}{\sqrt{-ace}+cd}}\right)\sqrt{\frac{\left(cx+\sqrt{-ac}\right)e}{\sqrt{-ace}-cd}}\sqrt{\frac{\left(-cx+\sqrt{-ac}\right)e}{\sqrt{-ace}+cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(c*x^2+a)^(1/2),x)
```

```
[Out] 2*(-(-a*c)^(1/2)*e+c*d)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(
-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*((c*x+(-a*c)^(1/2))*e/((
-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2
)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*(e*x+d)^(1/2)*(c*x^2+a)^(1/2)/e/c
/(c*e*x^3+c*d*x^2+a*e*x+a*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+a}\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*sqrt(e*x + d)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+a}\sqrt{ex+d}}{cex^3+cdx^2+aex+ad}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*sqrt(e*x + d)/(c*e*x^3 + c*d*x^2 + a*e*x + a*d), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+cx^2}\sqrt{d+ex}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**2)*sqrt(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+a}\sqrt{ex+d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*sqrt(e*x + d)), x)
```

3.682 $\int \frac{1}{(d+ex)^{3/2}\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=186

$$-\frac{2e\sqrt{a+cx^2}}{\sqrt{d+ex}\left(ae^2+cd^2\right)}-\frac{2\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{\sqrt{a+cx^2}\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}}$$

```
[Out] (-2*e*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*Sqrt[d + e*x]) - (2*Sqrt[-a]*Sqrt[c
]*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/S
qrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/((c*d^2 + a*e^2)*S
qrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.090379, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {745, 21, 719, 424}

$$-\frac{2e\sqrt{a+cx^2}}{\sqrt{d+ex}\left(ae^2+cd^2\right)}-\frac{2\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{\sqrt{a+cx^2}\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*Sqrt[a + c*x^2]),x]
```

```
[Out] (-2*e*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*Sqrt[d + e*x]) - (2*Sqrt[-a]*Sqrt[c
]*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/S
qrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/((c*d^2 + a*e^2)*S
qrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2])
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
```

```
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)^{3/2}\sqrt{a+cx^2}}\,dx &= -\frac{2e\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)\sqrt{d+ex}}-\frac{(2c)\int \frac{-\frac{d}{2}-\frac{ex}{2}}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}\\
&= -\frac{2e\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)\sqrt{d+ex}}+\frac{c\int \frac{\sqrt{d+ex}}{\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}\\
&= -\frac{2e\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)\sqrt{d+ex}}+\frac{\left(2a\sqrt{c}\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}ex^2}{\sqrt{-a}\left(cd-\frac{a\sqrt{c}e}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}\left(cd^2+ae^2\right)\sqrt{\frac{c(d+ex)}{cd-\frac{a\sqrt{c}e}{\sqrt{-a}}}}\sqrt{a+cx^2}}\\
&= -\frac{2e\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)\sqrt{d+ex}}-\frac{2\sqrt{-a}\sqrt{c}\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{\left(cd^2+ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}}\sqrt{a+cx^2}}
\end{aligned}$$

Mathematica [C] time = 0.38786, size = 331, normalized size = 1.78

$$-\frac{2e\sqrt{a+cx^2}}{\sqrt{d+ex}\left(ae^2+cd^2\right)}-\frac{2\sqrt{c}\sqrt{d+ex}\sqrt{\frac{e\left(\sqrt{a}+i\sqrt{c}x\right)}{\sqrt{a}e-i\sqrt{c}d}}\left(E\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-i\sqrt{a}e}}\right)|\frac{\sqrt{c}d-i\sqrt{a}e}{\sqrt{c}d+i\sqrt{a}e}\right)-\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-i\sqrt{a}}}\right.\right.\right.}{e\sqrt{a+cx^2}\left(\sqrt{a}e+i\sqrt{c}d\right)\sqrt{\frac{\sqrt{c}(d+ex)}{e\left(\sqrt{c}x+i\sqrt{a}\right)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*Sqrt[a + c*x^2]),x]
```

```
[Out] (-2*e*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*Sqrt[d + e*x]) - (2*Sqrt[c]*Sqrt[(e
*(Sqrt[a] + I*Sqrt[c]*x))/((-I)*Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[d + e*x]*(Elli
pticE[I*ArcSinh[Sqrt[-((Sqrt[c]*(d + e*x))/(Sqrt[c]*d - I*Sqrt[a]*e))]], (S
qrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)] - EllipticF[I*ArcSinh[Sq
rt[-((Sqrt[c]*(d + e*x))/(Sqrt[c]*d - I*Sqrt[a]*e))]], (Sqrt[c]*d - I*Sqrt[
a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)]))/(e*(I*Sqrt[c]*d + Sqrt[a]*e)*Sqrt[(Sqrt[
c]*(d + e*x))/(e*(I*Sqrt[a] + Sqrt[c]*x))]*Sqrt[a + c*x^2])
```

Maple [B] time = 0.273, size = 655, normalized size = 3.5

$$2\frac{\sqrt{ex+d}\sqrt{cx^2+a}}{(ae^2+cd^2)\,e\,(cex^3+cdx^2+aex+ad)}\left(\mathrm{EllipticF}\left(\sqrt{-\frac{c(ex+d)}{\sqrt{-ace}-cd}},\sqrt{-\frac{\sqrt{-ace}-cd}{\sqrt{-ace}+cd}}\right)ae^2\sqrt{-\frac{c(ex+d)}{\sqrt{-ace}-cd}}\sqrt{\frac{(-cx+\sqrt{}}{\sqrt{-ace}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(c*x^2+a)^(1/2),x)
```

```
[Out] 2*(EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)
/((-a*c)^(1/2)*e+c*d))^(1/2))*a*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)
*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/(
(-a*c)^(1/2)*e-c*d))^(1/2)+EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2
),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*c*d^2*(-(e*x+d)*c/((-
a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)
*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-EllipticE((-(e*x+d)*c/((
-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2
))*a*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-
a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-
EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((
-a*c)^(1/2)*e+c*d))^(1/2))*c*d^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((
-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a
*c)^(1/2)*e-c*d))^(1/2)-c*e^2*x^2-a*e^2)*(c*x^2+a)^(1/2)*(e*x+d)^(1/2)/e/(a
*e^2+c*d^2)/(c*e*x^3+c*d*x^2+a*e*x+a*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+a}(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^2+a}\sqrt{ex+d}}{ce^2x^4+2\,cdex^3+2\,adex+ad^2+(cd^2+ae^2)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*sqrt(e*x + d)/(c*e^2*x^4 + 2*c*d*e*x^3 + 2*a*d*e*x
 + a*d^2 + (c*d^2 + a*e^2)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+cx^2}\,(d+ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + a}(ex + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^(3/2)), x)
```

3.683 $\int \frac{1}{(d+ex)^{5/2}\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=382

$$\frac{2\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{3\sqrt{a+cx^2}\sqrt{d+ex}\left(ae^2+cd^2\right)}-\frac{8\sqrt{-a}c^{3/2}d\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\dots\right)}{3\sqrt{a+cx^2}\left(ae^2+cd^2\right)^2\sqrt{\frac{\sqrt{c}(d+\dots}{\sqrt{-a}e+\dots}}}$$

```
[Out] (-2*e*Sqrt[a + c*x^2])/(3*(c*d^2 + a*e^2)*(d + e*x)^(3/2)) - (8*c*d*e*Sqrt[
a + c*x^2])/(3*(c*d^2 + a*e^2)^2*Sqrt[d + e*x]) - (8*Sqrt[-a]*c^(3/2)*d*Sqr
t[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-
a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(3*(c*d^2 + a*e^2)^2*Sq
rt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) + (2*Sqrt
[-a]*Sqrt[c]*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c
*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)
/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(3*(c*d^2 + a*e^2)*Sqrt[d + e*x]*Sqrt[a + c*x
^2])
```

Rubi [A] time = 0.265522, antiderivative size = 382, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {745, 835, 844, 719, 424, 419}

$$-\frac{8\sqrt{-a}c^{3/2}d\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{3\sqrt{a+cx^2}\left(ae^2+cd^2\right)^2\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}}-\frac{8cde\sqrt{a+cx^2}}{3\sqrt{d+ex}\left(ae^2+cd^2\right)^2}-\frac{2e\sqrt{a+cx^2}}{3(d+ex)^{3/2}\left(ae^2+cd^2\right)}+\dots$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(5/2)*Sqrt[a + c*x^2]),x]
```

```
[Out] (-2*e*Sqrt[a + c*x^2])/(3*(c*d^2 + a*e^2)*(d + e*x)^(3/2)) - (8*c*d*e*Sqrt[
a + c*x^2])/(3*(c*d^2 + a*e^2)^2*Sqrt[d + e*x]) - (8*Sqrt[-a]*c^(3/2)*d*Sqr
t[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-
a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(3*(c*d^2 + a*e^2)^2*Sq
rt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) + (2*Sqrt
[-a]*Sqrt[c]*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c
*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)
/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(3*(c*d^2 + a*e^2)*Sqrt[d + e*x]*Sqrt[a + c*x
^2])
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{a+cx^2}}\,dx = -\frac{2e\sqrt{a+cx^2}}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} - \frac{(2c)\int \frac{-\frac{3d}{2}+\frac{ex}{2}}{(d+ex)^{3/2}\sqrt{a+cx^2}}\,dx}{3\left(cd^2+ae^2\right)}$$

$$= -\frac{2e\sqrt{a+cx^2}}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} - \frac{8cde\sqrt{a+cx^2}}{3\left(cd^2+ae^2\right)^2\sqrt{d+ex}} + \frac{(4c)\int \frac{\frac{1}{4}\left(3cd^2-ae^2\right)+cdex}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx}{3\left(cd^2+ae^2\right)^2}$$

$$= -\frac{2e\sqrt{a+cx^2}}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} - \frac{8cde\sqrt{a+cx^2}}{3\left(cd^2+ae^2\right)^2\sqrt{d+ex}} + \frac{\left(4c^2d\right)\int \frac{\sqrt{d+ex}}{\sqrt{a+cx^2}}\,dx}{3\left(cd^2+ae^2\right)^2} - \frac{c\int \frac{1}{\sqrt{d+ex}\sqrt{a+c}}}{3\left(cd^2+a\right.}$$

$$= -\frac{2e\sqrt{a+cx^2}}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} - \frac{8cde\sqrt{a+cx^2}}{3\left(cd^2+ae^2\right)^2\sqrt{d+ex}} + \frac{\left(8ac^{3/2}d\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}}{3\sqrt{-a}\left(cd^2+ae^2\right)^2}$$

$$= -\frac{2e\sqrt{a+cx^2}}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} - \frac{8cde\sqrt{a+cx^2}}{3\left(cd^2+ae^2\right)^2\sqrt{d+ex}} - \frac{8\sqrt{-a}c^{3/2}d\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}E\left(\sin\right.}{3\left(cd^2+ae^2\right)^2\sqrt{\frac{\sqrt{c}(d-}{\sqrt{c}d+}}}$$

Mathematica [C] time = 1.66069, size = 494, normalized size = 1.29

$$\frac{2\left(-e^2\left(a+cx^2\right)\left(ae^2+cd(5d+4ex)\right)+\frac{c(d+ex)\left(i(d+ex)^{3/2}\left(4i\sqrt{a}\sqrt{c}de-ae^2+3cd^2\right)\sqrt{\frac{e\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{d+ex}}\sqrt{-\frac{-ex+\frac{i\sqrt{a}e}{\sqrt{c}}}{d+ex}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}}{\sqrt{d+ex}}\right),\frac{\sqrt{c}d-}{\sqrt{c}d+}\right.\right.}{}\right)}{3e\sqrt{a+cx^2}(d+ex)^{3/2}\left(ae^2+c\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(5/2)*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*(-(e^2*(a + c*x^2)*(a*e^2 + c*d*(5*d + 4*e*x))) + (c*(d + e*x)*(4*d*e^2*
Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*(a + c*x^2) + 4*Sqrt[c]*d*((-I)*Sqrt[c]*d
+ Sqrt[a]*e)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[
a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-
d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[
c]*d + I*Sqrt[a]*e)] + I*(3*c*d^2 + (4*I)*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Sqrt
[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e
*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)
/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*
e)]))/Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]))/(3*e*(c*d^2 + a*e^2)^2*(d + e*x)^(
3/2)*Sqrt[a + c*x^2])
```

Maple [B] time = 0.284, size = 1904, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(5/2)/(c*x^2+a)^(1/2),x)
```

```
[Out] 2/3*(3*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-
c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*a*c*d*e^3*(-(e*x+d)*c/((-a*c)^(1/2)*e-c
*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^
(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-
c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*a*e^4*(-(
e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e
+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*(-a*c)^(1/2)
+3*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)
/((-a*c)^(1/2)*e+c*d))^(1/2))*x*c^2*d^3*e*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))
^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2
))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d)
)^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*c*d^2*e^2*(-(
e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e
+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*(-a*c)^(1/2)
-4*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)
/((-a*c)^(1/2)*e+c*d))^(1/2))*x*a*c*d*e^3*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))
^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2
))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-4*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*
d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*c^2*d^3*e*(
-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)
*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+3*(-(e*x+d
)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d)
)^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+
d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d
))^(1/2))*a*c*d^2*e^2+(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)
^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e
-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1
/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*(-a*c)^(1/2)*a*d*e^3+3*(-(e*x+d)*c/
((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1
/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c
/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(
1/2))*c^2*d^4+(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*
e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(
1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*
d)/((-a*c)^(1/2)*e+c*d))^(1/2))*(-a*c)^(1/2)*c*d^3*e-4*(-(e*x+d)*c/((-a*c)^
(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*
x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticE((-(e*x+d)*c/((-a*c)
^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*
c*d^2*e^2-4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/
((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/
2)*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)
/((-a*c)^(1/2)*e+c*d))^(1/2))*c^2*d^4-4*x^3*c^2*d*e^3-x^2*a*c*e^4-5*x^2*c^2
*d^2*e^2-4*x*a*c*d*e^3-a^2*e^4-5*a*c*d^2*e^2)/(c*x^2+a)^(1/2)/(a*e^2+c*d^2)
^2/(e*x+d)^(3/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + a}(ex + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+a}\sqrt{ex+d}}{ce^3x^5+3\,cde^2x^4+3\,ad^2ex+ad^3+\left(3\,cd^2e+ae^3\right)x^3+\left(cd^3+3\,ade^2\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*sqrt(e*x + d)/(c*e^3*x^5 + 3*c*d*e^2*x^4 + 3*a*d^2
*e*x + a*d^3 + (3*c*d^2*e + a*e^3)*x^3 + (c*d^3 + 3*a*d*e^2)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a+cx^2}\,(d+ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(5/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{cx^2+a}(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^(5/2)), x)
```

3.684 $\int \frac{1}{(d+ex)^{7/2}\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=447

$$\frac{16\sqrt{-a}c^{3/2}d\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{15\sqrt{a+cx^2}\sqrt{d+ex}\left(ae^2+cd^2\right)^2}-\frac{2\sqrt{-a}c^{3/2}\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}\left(23cd^2-9ae^2\right)E\left(\right.}{15\sqrt{a+cx^2}\left(ae^2+cd^2\right)^3\ \sqrt{}}$$

```
[Out] (-2*e*Sqrt[a + c*x^2])/(5*(c*d^2 + a*e^2)*(d + e*x)^(5/2)) - (16*c*d*e*Sqrt
[a + c*x^2])/(15*(c*d^2 + a*e^2)^2*(d + e*x)^(3/2)) - (2*c*e*(23*c*d^2 - 9*
a*e^2)*Sqrt[a + c*x^2])/(15*(c*d^2 + a*e^2)^3*Sqrt[d + e*x]) - (2*Sqrt[-a]*
c^(3/2)*(23*c*d^2 - 9*a*e^2)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[Ar
cSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d
- a*e)])/(15*(c*d^2 + a*e^2)^3*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-
a]*e)]*Sqrt[a + c*x^2]) + (16*Sqrt[-a]*c^(3/2)*d*Sqrt[(Sqrt[c]*(d + e*x))/(
Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqr
t[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(15*(c*d^
2 + a*e^2)^2*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.408262, antiderivative size = 447, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {745, 835, 844, 719, 424, 419}

$$\frac{16\sqrt{-a}c^{3/2}d\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{15\sqrt{a+cx^2}\sqrt{d+ex}\left(ae^2+cd^2\right)^2}-\frac{2\sqrt{-a}c^{3/2}\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}\left(23cd^2-9ae^2\right)E\left(\sin^{-1}\left(\right.\right.}{15\sqrt{a+cx^2}\left(ae^2+cd^2\right)^3\sqrt{\frac{\sqrt{c}(d+e}{\sqrt{-ae+}}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(7/2)*Sqrt[a + c*x^2]),x]
```


```
[Out] (-2*e*Sqrt[a + c*x^2])/(5*(c*d^2 + a*e^2)*(d + e*x)^(5/2)) - (16*c*d*e*Sqrt
[a + c*x^2])/(15*(c*d^2 + a*e^2)^2*(d + e*x)^(3/2)) - (2*c*e*(23*c*d^2 - 9*
a*e^2)*Sqrt[a + c*x^2])/(15*(c*d^2 + a*e^2)^3*Sqrt[d + e*x]) - (2*Sqrt[-a]*
c^(3/2)*(23*c*d^2 - 9*a*e^2)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[Ar
cSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d
- a*e)])/(15*(c*d^2 + a*e^2)^3*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-
a]*e)]*Sqrt[a + c*x^2]) + (16*Sqrt[-a]*c^(3/2)*d*Sqrt[(Sqrt[c]*(d + e*x))/(
Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqr
t[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(15*(c*d^
2 + a*e^2)^2*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```


Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(d+ex)^{7/2}\sqrt{a+cx^2}}\,dx = -\frac{2e\sqrt{a+cx^2}}{5\left(cd^2+ae^2\right)(d+ex)^{5/2}} - \frac{(2c)\int \frac{-\frac{5d}{2}+\frac{3ex}{2}}{(d+ex)^{5/2}\sqrt{a+cx^2}}\,dx}{5\left(cd^2+ae^2\right)}$$

$$= -\frac{2e\sqrt{a+cx^2}}{5\left(cd^2+ae^2\right)(d+ex)^{5/2}} - \frac{16cde\sqrt{a+cx^2}}{15\left(cd^2+ae^2\right)^2(d+ex)^{3/2}} + \frac{(4c)\int \frac{\frac{3}{4}\left(5cd^2-3ae^2\right)-2cdex}{(d+ex)^{3/2}\sqrt{a+cx^2}}\,dx}{15\left(cd^2+ae^2\right)^2}$$

$$= -\frac{2e\sqrt{a+cx^2}}{5\left(cd^2+ae^2\right)(d+ex)^{5/2}} - \frac{16cde\sqrt{a+cx^2}}{15\left(cd^2+ae^2\right)^2(d+ex)^{3/2}} - \frac{2ce\left(23cd^2-9ae^2\right)\sqrt{a+cx^2}}{15\left(cd^2+ae^2\right)^3\sqrt{d+ex}} - \frac{(8c)\ldots}{\ldots}$$

$$= -\frac{2e\sqrt{a+cx^2}}{5\left(cd^2+ae^2\right)(d+ex)^{5/2}} - \frac{16cde\sqrt{a+cx^2}}{15\left(cd^2+ae^2\right)^2(d+ex)^{3/2}} - \frac{2ce\left(23cd^2-9ae^2\right)\sqrt{a+cx^2}}{15\left(cd^2+ae^2\right)^3\sqrt{d+ex}} + \frac{(c^2\ldots}{\ldots}$$

$$= -\frac{2e\sqrt{a+cx^2}}{5\left(cd^2+ae^2\right)(d+ex)^{5/2}} - \frac{16cde\sqrt{a+cx^2}}{15\left(cd^2+ae^2\right)^2(d+ex)^{3/2}} - \frac{2ce\left(23cd^2-9ae^2\right)\sqrt{a+cx^2}}{15\left(cd^2+ae^2\right)^3\sqrt{d+ex}} + \frac{(2a\ldots}{\ldots}$$

$$= -\frac{2e\sqrt{a+cx^2}}{5\left(cd^2+ae^2\right)(d+ex)^{5/2}} - \frac{16cde\sqrt{a+cx^2}}{15\left(cd^2+ae^2\right)^2(d+ex)^{3/2}} - \frac{2ce\left(23cd^2-9ae^2\right)\sqrt{a+cx^2}}{15\left(cd^2+ae^2\right)^3\sqrt{d+ex}} - \frac{2\sqrt{\ldots}}{\ldots}$$

Mathematica [C] time = 3.25857, size = 618, normalized size = 1.38

$$\frac{2\left(e^2\left(-\left(a+cx^2\right)\right)\left(8cd(d+ex)\left(ae^2+cd^2\right)+c(d+ex)^2\left(23cd^2-9ae^2\right)+3\left(ae^2+cd^2\right)^2\right)-\frac{c(d+ex)^2\left(\sqrt{c}(d+ex)^{3/2}\left(-9a^{3/2}e^3+23\sqrt{a}c\ldots\right.\right.}{\ldots}\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(7/2)*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*(-(e^2*(a + c*x^2)*(3*(c*d^2 + a*e^2)^2 + 8*c*d*(c*d^2 + a*e^2)*(d + e*x
) + c*(23*c*d^2 - 9*a*e^2)*(d + e*x)^2)) - (c*(d + e*x)^2*(-(e^2*Sqrt[-d -
(I*Sqrt[a]*e)/Sqrt[c]]*(-9*a^2*e^2 + 23*c^2*d^2*x^2 + a*c*(23*d^2 - 9*e^2*x
^2))) + Sqrt[c]*((23*I)*c^(3/2)*d^3 - 23*Sqrt[a]*c*d^2*e - (9*I)*a*Sqrt[c]*
d*e^2 + 9*a^(3/2)*e^3)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-
(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcS
inh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]
*e)/(Sqrt[c]*d + I*Sqrt[a]*e)] + Sqrt[c]*((-15*I)*c^(3/2)*d^3 + 23*Sqrt[a]*
c*d^2*e + (17*I)*a*Sqrt[c]*d*e^2 - 9*a^(3/2)*e^3)*Sqrt[(e*((I*Sqrt[a])/Sqrt
[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d +
e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*
x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)]))/Sqrt[-d - (I*Sq
rt[a]*e)/Sqrt[c]]))/(15*e*(c*d^2 + a*e^2)^3*(d + e*x)^(5/2)*Sqrt[a + c*x^2]
```

)

Maple [B] time = 0.307, size = 3863, normalized size = 8.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(7/2)/(c*x^2+a)^(1/2),x)
```

```
[Out] -2/15*(23*x^4*c^3*d^2*e^4+54*x^3*c^3*d^3*e^3-6*x^2*a^2*c*e^6+34*x^2*c^3*d^4
*e^2-9*x^4*a*c^2*e^6-15*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(
-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*c^3*d^4*e^2*(-(e*x+d
)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d)
)^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+28*x^2*a*c^2*d^2*
e^4-10*x*a^2*c*d*e^5+54*x*a*c^2*d^3*e^3+23*EllipticE((-(e*x+d)*c/((-a*c)^(1
/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*c^3*d
^6*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(
1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-8*Elli
pticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c
)^(1/2)*e+c*d))^(1/2))*c^2*d^5*e*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c
*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^
(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-10*x^3*a*c^2*d*e^5+18*EllipticF((-(e*x
+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*
d))^(1/2))*x*a^2*c*d*e^5*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a
*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2
)*e-c*d))^(1/2)-12*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a
*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*a*c^2*d^3*e^3*(-(e*x+d)*c/(
(-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/
2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-16*EllipticF((-(e*x+d)
*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))
^(1/2))*x*c^2*d^4*e^2*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*
((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((
-a*c)^(1/2)*e-c*d))^(1/2)-18*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1
/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*a^2*c*d*e^5*(-(e*
x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c
*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+28*EllipticE((
-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)
*e+c*d))^(1/2))*x*a*c^2*d^3*e^3*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-
c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*
c)^(1/2)*e-c*d))^(1/2)-8*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),
(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c*d^3*e^3*(-a*c)^(1/2
)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1
/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-6*Ellip
ticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)
^(1/2)*e+c*d))^(1/2))*x^2*a*c^2*d^2*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(
1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))
*e/((-a*c)^(1/2)*e-c*d))^(1/2)-8*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d)
)^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*c^2*d^3*e^3
*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*
e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(
1/2)+14*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e
-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a*c^2*d^2*e^4*(-(e*x+d)*c/((-a*c)^(1
/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+
(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-9*EllipticE((-(e*x+d)*c/((-a*c)
^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^
2*c*d^2*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/
```

```
((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/
2)+9*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*
d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a^2*c*e^6*(-(e*x+d)*c/((-a*c)^(1/2)*e-c
*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^
(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+14*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)
*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c^2*d^
4*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c
)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-9*E
llipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-
a*c)^(1/2)*e+c*d))^(1/2))*x^2*a^2*c*e^6*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(
1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))
*e/((-a*c)^(1/2)*e-c*d))^(1/2)+23*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d
))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*c^3*d^4*e^
2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1
/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+46*Elli
pticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c
)^(1/2)*e+c*d))^(1/2))*x*c^3*d^5*e*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*
((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((
-a*c)^(1/2)*e-c*d))^(1/2)+9*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/
2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*c*d^2*e^4*(-(e*x
+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*
d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-6*EllipticF((-(
e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e
+c*d))^(1/2))*a*c^2*d^4*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+
(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(
1/2)*e-c*d))^(1/2)-30*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-(
(-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*c^3*d^5*e*(-(e*x+d)*c/((
-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2
)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-8*EllipticF((-(e*x+d)*c
/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(
1/2))*x^2*a*c*d*e^5*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((
-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a
*c)^(1/2)*e-c*d))^(1/2)-16*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2
),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*a*c*d^2*e^4*(-a*c)^
(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c
)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+3*a
^3*e^6-15*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)
*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*c^3*d^6*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*
d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(
1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+5*a^2*c*d^2*e^4+34*a*c^2*d^4*e^2)/(c*x^
2+a)^(1/2)/(a*e^2+c*d^2)^3/(e*x+d)^(5/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + a}(ex + d)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+a}\sqrt{ex+d}}{ce^4x^6+4\,cde^3x^5+4\,ad^3ex+ad^4+\left(6\,cd^2e^2+ae^4\right)x^4+4\,\left(cd^3e+ade^3\right)x^3+\left(cd^4+6\,ad^2e^2\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*sqrt(e*x + d)/(c*e^4*x^6 + 4*c*d*e^3*x^5 + 4*a*d^3
*e*x + a*d^4 + (6*c*d^2*e^2 + a*e^4)*x^4 + 4*(c*d^3*e + a*d*e^3)*x^3 + (c*d
^4 + 6*a*d^2*e^2)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+cx^2}\,(d+ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(7/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)**(7/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+a}(ex+d)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^(7/2)), x)
```

3.685 $\int \frac{(d+ex)^{7/2}}{\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=426

$$\frac{\sqrt{\frac{cx^2}{a}+1}\left(3cd^2-5ae^2\right)\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{3\sqrt{-a}c^{5/2}\sqrt{a+cx^2}\sqrt{d+ex}}-\frac{e\sqrt{a+cx^2}\sqrt{d+ex}\left(3cd^2-5ae^2\right.}{3ac^2}$$

```
[Out] -(((a*e - c*d*x)*(d + e*x)^(5/2))/(a*c*Sqrt[a + c*x^2])) - (e*(3*c*d^2 - 5*
a*e^2)*Sqrt[d + e*x]*Sqrt[a + c*x^2])/(3*a*c^2) - (d*e*(d + e*x)^(3/2)*Sqrt
[a + c*x^2])/(a*c) - (d*(3*c*d^2 - 29*a*e^2)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)
/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqr
t[-a]*Sqrt[c]*d - a*e)])/(3*Sqrt[-a]*c^(3/2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt
[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) + ((3*c*d^2 - 5*a*e^2)*(c*d^2 + a*e^2
)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*El
lipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*
Sqrt[c]*d - a*e)])/(3*Sqrt[-a]*c^(5/2)*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.469331, antiderivative size = 426, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {739, 833, 844, 719, 424, 419}

$$-\frac{e\sqrt{a+cx^2}\sqrt{d+ex}\left(3cd^2-5ae^2\right)}{3ac^2}+\frac{\sqrt{\frac{cx^2}{a}+1}\left(3cd^2-5ae^2\right)\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{3\sqrt{-a}c^{5/2}\sqrt{a+cx^2}\sqrt{d+ex}}-\frac{d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/(a + c*x^2)^(3/2),x]
```


```
[Out] -(((a*e - c*d*x)*(d + e*x)^(5/2))/(a*c*Sqrt[a + c*x^2])) - (e*(3*c*d^2 - 5*
a*e^2)*Sqrt[d + e*x]*Sqrt[a + c*x^2])/(3*a*c^2) - (d*e*(d + e*x)^(3/2)*Sqrt
[a + c*x^2])/(a*c) - (d*(3*c*d^2 - 29*a*e^2)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)
/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqr
t[-a]*Sqrt[c]*d - a*e)])/(3*Sqrt[-a]*c^(3/2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt
[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) + ((3*c*d^2 - 5*a*e^2)*(c*d^2 + a*e^2
)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*El
lipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*
Sqrt[c]*d - a*e)])/(3*Sqrt[-a]*c^(5/2)*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```


Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^{7/2}}{\left(a+cx^2\right)^{3/2}}\,dx = -\frac{(ae-cdx)(d+ex)^{5/2}}{ac\sqrt{a+cx^2}} + \frac{\int \frac{(d+ex)^{3/2}\left(\frac{5ae^2}{2}-\frac{5}{2}cdex\right)}{\sqrt{a+cx^2}}dx}{ac}$$

$$= -\frac{(ae-cdx)(d+ex)^{5/2}}{ac\sqrt{a+cx^2}} - \frac{de(d+ex)^{3/2}\sqrt{a+cx^2}}{ac} + \frac{2\int \frac{\sqrt{d+ex}\left(10acde^2-\frac{5}{4}ce\left(3cd^2-5ae^2\right)x\right)}{\sqrt{a+cx^2}}dx}{5ac^2}$$

$$= -\frac{(ae-cdx)(d+ex)^{5/2}}{ac\sqrt{a+cx^2}} - \frac{e\left(3cd^2-5ae^2\right)\sqrt{d+ex}\sqrt{a+cx^2}}{3ac^2} - \frac{de(d+ex)^{3/2}\sqrt{a+cx^2}}{ac} + \frac{4\int \frac{\frac{5}{8}ace^2\left(27cd^2 \cdots\right.}{\cdots}}{\cdots}$$

$$= -\frac{(ae-cdx)(d+ex)^{5/2}}{ac\sqrt{a+cx^2}} - \frac{e\left(3cd^2-5ae^2\right)\sqrt{d+ex}\sqrt{a+cx^2}}{3ac^2} - \frac{de(d+ex)^{3/2}\sqrt{a+cx^2}}{ac} - \frac{\left(d\left(3cd^2-29a \cdots\right.\right.}{6 \cdots}$$

$$= -\frac{(ae-cdx)(d+ex)^{5/2}}{ac\sqrt{a+cx^2}} - \frac{e\left(3cd^2-5ae^2\right)\sqrt{d+ex}\sqrt{a+cx^2}}{3ac^2} - \frac{de(d+ex)^{3/2}\sqrt{a+cx^2}}{ac} - \frac{\left(d\left(3cd^2-29a \cdots\right.\right.}{\cdots}$$

$$= -\frac{(ae-cdx)(d+ex)^{5/2}}{ac\sqrt{a+cx^2}} - \frac{e\left(3cd^2-5ae^2\right)\sqrt{d+ex}\sqrt{a+cx^2}}{3ac^2} - \frac{de(d+ex)^{3/2}\sqrt{a+cx^2}}{ac} - \frac{d\left(3cd^2-29ae \cdots\right.}{\cdots}$$

Mathematica [C] time = 3.47229, size = 586, normalized size = 1.38

$$\frac{\sqrt{d+ex}\left(\frac{2\left(\sqrt{a}e(d+ex)^{3/2}\left(-5ia^{3/2}e^3+27i\sqrt{a}cd^2e-29a\sqrt{c}de^2+3c^{3/2}d^3\right)\sqrt{\frac{e\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{d+ex}}\sqrt{-\frac{-ex+\frac{i\sqrt{a}e}{\sqrt{c}}}{d+ex}}\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}}{\sqrt{d+ex}}\right),\frac{\sqrt{c}d-i\sqrt{a}e}{\sqrt{c}d+i\sqrt{a}e}\right)-de^2\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}\left(-29a \cdots\right.\right.}{ac^2e(d+ex) \cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)/(a + c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d + e*x]*((10*a*e^3)/c^2 + (6*d^3*x)/a + (2*e*(-9*d^2 - 9*d*e*x + 2*e
^2*x^2))/c + (2*(-(d*e^2*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*(-29*a^2*e^2 + 3*
c^2*d^2*x^2 + a*c*(3*d^2 - 29*e^2*x^2))) + Sqrt[c]*d*((3*I)*c^(3/2)*d^3 - 3
*Sqrt[a]*c*d^2*e - (29*I)*a*Sqrt[c]*d*e^2 + 29*a^(3/2)*e^3)*Sqrt[(e*((I*Sqr
t[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*
x))]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/S
qrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)] + Sqrt[
a]*e*(3*c^(3/2)*d^3 + (27*I)*Sqrt[a]*c*d^2*e - 29*a*Sqrt[c]*d*e^2 - (5*I)*a
^(3/2)*e^3)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a
]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d
 - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c
]*d + I*Sqrt[a]*e)]))/(a*c^2*e*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*(d + e*x)))
)/(6*Sqrt[a + c*x^2])
```

Maple [B] time = 0.35, size = 1362, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/(c*x^2+a)^(3/2),x)
```

```
[Out] 1/3*(e*x+d)^(1/2)*(c*x^2+a)^(1/2)*(24*a^2*c*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d
))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1
/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*
d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*d*e^4+5*(-a*c
)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a
*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*E
llipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-
a*c)^(1/2)*e+c*d))^(1/2))*a^2*e^5+24*a*c^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d)
)^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/
2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d
))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*d^3*e^2+2*(-a*
c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-
a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*
EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((
-a*c)^(1/2)*e+c*d))^(1/2))*a*c*d^2*e^3-3*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(
1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x
+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^
(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*c^2
*d^4*e-29*a^2*c*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2)
)*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))
^(1/2)*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-
c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*d*e^4-26*a*c^2*(-(e*x+d)*c/((-a*c)^(1/2)*
e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*
c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)
*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*d^3*e^2+
3*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1
/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*Ellipti
cE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(
1/2)*e+c*d))^(1/2))*c^3*d^5+2*x^3*a*c^2*e^5-7*x^2*a*c^2*d*e^4+3*x^2*c^3*d^3
*e^2+5*x*a^2*c*e^5-18*x*a*c^2*d^2*e^3+3*x*c^3*d^4*e+5*a^2*c*d*e^4-9*a*c^2*d
^3*e^2)/e/(c*e*x^3+c*d*x^2+a*e*x+a*d)/c^3/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{7}{2}}}{\left(cx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(7/2)/(c*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\sqrt{cx^2+a}\sqrt{ex+d}}{c^2x^4+2\,acx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(c*x^2 + a)*sqrt(e*x
 + d)/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/(c*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{7}{2}}}{\left(cx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(7/2)/(c*x^2 + a)^(3/2), x)
```

3.686 $\int \frac{(d+ex)^{5/2}}{\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=363

$$\frac{d\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{\sqrt{-a}c^{3/2}\sqrt{a+cx^2}\sqrt{d+ex}}-\frac{\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}\left(cd^2-3ae^2\right)E\left(\sin^{-1}\left(\right.\right.}{\sqrt{-a}c^{3/2}\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(}{\sqrt{-a}}}}$$

```
[Out] -(((a*e - c*d*x)*(d + e*x)^(3/2))/(a*c*Sqrt[a + c*x^2])) - (d*e*Sqrt[d + e*
x]*Sqrt[a + c*x^2])/(a*c) - ((c*d^2 - 3*a*e^2)*Sqrt[d + e*x]*Sqrt[1 + (c*x^
2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(S
qrt[-a]*Sqrt[c]*d - a*e)])/(Sqrt[-a]*c^(3/2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt
[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) + (d*(c*d^2 + a*e^2)*Sqrt[(Sqrt[c]*(d
 + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqr
t[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])
/(Sqrt[-a]*c^(3/2)*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.314003, antiderivative size = 363, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {739, 833, 844, 719, 424, 419}

$$\frac{d\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{\sqrt{-a}c^{3/2}\sqrt{a+cx^2}\sqrt{d+ex}}-\frac{\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}\left(cd^2-3ae^2\right)E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right.\right.}{\sqrt{-a}c^{3/2}\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(a + c*x^2)^(3/2),x]
```

```
[Out] -(((a*e - c*d*x)*(d + e*x)^(3/2))/(a*c*Sqrt[a + c*x^2])) - (d*e*Sqrt[d + e*
x]*Sqrt[a + c*x^2])/(a*c) - ((c*d^2 - 3*a*e^2)*Sqrt[d + e*x]*Sqrt[1 + (c*x^
2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(S
qrt[-a]*Sqrt[c]*d - a*e)])/(Sqrt[-a]*c^(3/2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt
[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) + (d*(c*d^2 + a*e^2)*Sqrt[(Sqrt[c]*(d
 + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqr
t[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])
/(Sqrt[-a]*c^(3/2)*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
```

```
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\left(a+cx^2\right)^{3/2}}dx = -\frac{(ae-cdx)(d+ex)^{3/2}}{ac\sqrt{a+cx^2}} + \frac{\int \frac{\sqrt{d+ex}\left(\frac{3ae^2}{2}-\frac{3}{2}cdex\right)}{\sqrt{a+cx^2}}dx}{ac}$$

$$= -\frac{(ae-cdx)(d+ex)^{3/2}}{ac\sqrt{a+cx^2}} - \frac{de\sqrt{d+ex}\sqrt{a+cx^2}}{ac} + \frac{2\int \frac{3acde^2-\frac{3}{4}ce\left(cd^2-3ae^2\right)x}{\sqrt{d+ex}\sqrt{a+cx^2}}dx}{3ac^2}$$

$$= -\frac{(ae-cdx)(d+ex)^{3/2}}{ac\sqrt{a+cx^2}} - \frac{de\sqrt{d+ex}\sqrt{a+cx^2}}{ac} - \frac{\left(cd^2-3ae^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{a+cx^2}}dx}{2ac} + \frac{\left(d\left(cd^2+ae^2\right)\right)\int \frac{}{\sqrt{d+}}}{2ac}$$

$$= -\frac{(ae-cdx)(d+ex)^{3/2}}{ac\sqrt{a+cx^2}} - \frac{de\sqrt{d+ex}\sqrt{a+cx^2}}{ac} - \frac{\left(\left(cd^2-3ae^2\right)\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\cdots}}{\sqrt{\cdots}}\right.}{\sqrt{-a}c^{3/2}\sqrt{\frac{c(d+ex)}{cd-\frac{a\sqrt{c}e}{\sqrt{-a}}}}\sqrt{a+cx^2}}$$

$$= -\frac{(ae-cdx)(d+ex)^{3/2}}{ac\sqrt{a+cx^2}} - \frac{de\sqrt{d+ex}\sqrt{a+cx^2}}{ac} - \frac{\left(cd^2-3ae^2\right)\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\right.}{\sqrt{-a}c^{3/2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}}\sqrt{a+cx^2}}$$

Mathematica [C] time = 3.04731, size = 495, normalized size = 1.36

$$\frac{\sqrt{d+ex}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{d+ex}\left(4i\sqrt{a}\sqrt{c}de-3ae^2+cd^2\right)\sqrt{\frac{e\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{d+ex}}\sqrt{-\frac{-ex+\frac{i\sqrt{a}e}{\sqrt{c}}}{d+ex}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}}{\sqrt{d+ex}}\right),\frac{\sqrt{c}d-i\sqrt{a}e}{\sqrt{c}d+i\sqrt{a}e}\right)}{\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}} - \frac{e\left(-3a^2e^2+ac\left(d^2-3e^2x^2\right)+c^2d^2x^2\right)}{d+ex} - \cdots\right.}{ac^2\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(a + c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d + e*x]*(c*(c*d^2*x - a*e*(2*d + e*x)) - (e*(-3*a^2*e^2 + c^2*d^2*x^
2 + a*c*(d^2 - 3*e^2*x^2)))/(d + e*x) - (I*c*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c
]]*(c*d^2 - 3*a*e^2)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-((
(I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*Sqrt[d + e*x]*EllipticE[I*ArcSinh[
Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/
(Sqrt[c]*d + I*Sqrt[a]*e)])/e + (Sqrt[a]*Sqrt[c]*(c*d^2 + (4*I)*Sqrt[a]*Sqr
t[c]*d*e - 3*a*e^2)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((
I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*Sqrt[d + e*x]*EllipticF[I*ArcSinh[S
qrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(
Sqrt[c]*d + I*Sqrt[a]*e)])/Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]))/(a*c^2*Sqrt[a
 + c*x^2])
```

Maple [B] time = 0.27, size = 1150, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(c*x^2+a)^(3/2),x)
```

```
[Out] (e*x+d)^(1/2)*(c*x^2+a)^(1/2)*(3*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((
-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a
*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(
-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*e^4+3*(-(e*x+d)*c/((
-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2
)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/(
(-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/
2))*a*c*d^2*e^2-(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2)
)*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))
^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-
c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*(-a*c)^(1/2)*a*d*e^3-(-(e*x+d)*c/((-a*c)^
(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*
x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)
^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*(-
a*c)^(1/2)*c*d^3*e-3*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^
(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-
c*d))^(1/2)*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/
2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*e^4-2*(-(e*x+d)*c/((-a*c)^(1/2)*
e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*
c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)
*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c*d^2*
e^2+(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^
(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*Ellip
ticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)
^(1/2)*e+c*d))^(1/2))*c^2*d^4-x^2*a*c*e^4+x^2*c^2*d^2*e^2-3*x*a*c*d*e^3+x*c
^2*d^3*e-2*a*c*d^2*e^2)/c^2/e/(c*e*x^3+c*d*x^2+a*e*x+a*d)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\left(cx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/(c*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e^2x^2+2\,dex+d^2\right)\sqrt{cx^2+a}\sqrt{ex+d}}{c^2x^4+2\,acx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*sqrt(c*x^2 + a)*sqrt(e*x + d)/(c^2*x^4 +
 2*a*c*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(c*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\left(cx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(5/2)/(c*x^2 + a)^(3/2), x)
```

3.687 $\int \frac{(d+ex)^{3/2}}{\left(a+cx^2\right)^{3/2}} dx$

Optimal. Leaf size=321

$$\frac{\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{\sqrt{-a}c^{3/2}\sqrt{a+cx^2}\sqrt{d+ex}}-\frac{\sqrt{d+ex}(ae-cdx)}{ac\sqrt{a+cx^2}}-\frac{d\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}E\left(\text{si}\ldots\right.}{\sqrt{-a}\sqrt{c}\sqrt{a+\ldots}}$$

```
[Out] -(((a*e - c*d*x)*Sqrt[d + e*x])/(a*c*Sqrt[a + c*x^2])) - (d*Sqrt[d + e*x]*S
qrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]]
, (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(Sqrt[-a]*Sqrt[c]*Sqrt[(Sqrt[c]*(d
+ e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) + ((c*d^2 + a*e^2)*Sqrt[
(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF
[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]
*d - a*e)])/(Sqrt[-a]*c^(3/2)*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.212791, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {739, 844, 719, 424, 419}

$$\frac{\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{\sqrt{-a}c^{3/2}\sqrt{a+cx^2}\sqrt{d+ex}}-\frac{\sqrt{d+ex}(ae-cdx)}{ac\sqrt{a+cx^2}}-\frac{d\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{1\ldots}}{\ldots}\right.\right.}{\sqrt{-a}\sqrt{c}\sqrt{a+cx^2}\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(a + c*x^2)^(3/2),x]
```


```
[Out] -(((a*e - c*d*x)*Sqrt[d + e*x])/(a*c*Sqrt[a + c*x^2])) - (d*Sqrt[d + e*x]*S
qrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]]
, (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(Sqrt[-a]*Sqrt[c]*Sqrt[(Sqrt[c]*(d
+ e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) + ((c*d^2 + a*e^2)*Sqrt[
(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF
[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]
*d - a*e)])/(Sqrt[-a]*c^(3/2)*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```


Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\left(a+cx^2\right)^{3/2}}\,dx = -\frac{(ae-cdx)\sqrt{d+ex}}{ac\sqrt{a+cx^2}}+\frac{\int \frac{\frac{ae^2}{2}-\frac{1}{2}cdex}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx}{ac}$$

$$= -\frac{(ae-cdx)\sqrt{d+ex}}{ac\sqrt{a+cx^2}}-\frac{d\int \frac{\sqrt{d+ex}}{\sqrt{a+cx^2}}\,dx}{2a}+\frac{\left(cd^2+ae^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx}{2ac}$$

$$= -\frac{(ae-cdx)\sqrt{d+ex}}{ac\sqrt{a+cx^2}}-\frac{\left(d\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}ex^2}{\sqrt{-a}\left(cd-\frac{a\sqrt{c}e}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\frac{c(d+ex)}{cd-\frac{a\sqrt{c}e}{\sqrt{-a}}}}\sqrt{a+cx^2}}+\frac{\left(\left(cd^2+a\right.\right.}{}$$

$$= -\frac{(ae-cdx)\sqrt{d+ex}}{ac\sqrt{a+cx^2}}-\frac{d\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}}\sqrt{a+cx^2}}+\frac{\left(cd^2+ae^2\right)\sqrt{\frac{\sqrt{c}(d-}{\sqrt{c}d+}}}{}$$

Mathematica [C] time = 5.02757, size = 414, normalized size = 1.29

$$\frac{-\sqrt{a}\sqrt{c}e(d+ex)^{3/2}\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}\sqrt{\frac{e\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{d+ex}}\sqrt{-\frac{-ex+\frac{i\sqrt{a}e}{\sqrt{c}}}{d+ex}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}}{\sqrt{d+ex}}\right),\frac{\sqrt{c}d-i\sqrt{a}e}{\sqrt{c}d+i\sqrt{a}e}\right)+e\left(cd^2x-ae(2d+ex)\right.}{ace\sqrt{a+cx^2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(a + c*x^2)^(3/2),x]
```

```
[Out] (e*(c*d^2*x - a*e*(2*d + e*x)) - I*c*d*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*Sqr
t[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] -
e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e
)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]
*e)] - Sqrt[a]*Sqrt[c]*e*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*Sqrt[(e*((I*Sqrt[
a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x)
)]*(d + e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqr
t[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)])/(a*c*e*S
qrt[d + e*x]*Sqrt[a + c*x^2])
```

Maple [B] time = 0.266, size = 685, normalized size = 2.1

$$\frac{1}{e\left(cex^3+cdx^2+aex+ad\right)ac^2}\left(-\sqrt{-c\left(ex+d\right)\left(\sqrt{-ace}-cd\right)^{-1}}\sqrt{e\left(-cx+\sqrt{-ac}\right)\left(\sqrt{-ace}+cd\right)^{-1}}\sqrt{e\left(cx+\sqrt{-ac}\right)\left(\sqrt{-ace}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*x^2+a)^(3/2),x)
```

```
[Out] (-(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1
/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*Ellipti
cF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(
1/2)*e+c*d))^(1/2))*(-a*c)^(1/2)*a*e^3-(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1
/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*
e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(
1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*(-a*c)^(1/2)*c*d^2
*e+(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(
1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*Ellipt
icE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^
(1/2)*e+c*d))^(1/2))*a*c*d*e^2+(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c
*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c
)^(1/2)*e-c*d))^(1/2)*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-(
(-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*c^2*d^3+x^2*c^2*d*e^2-x*a*
c*e^3+x*c^2*d^2*e-a*d*e^2*c)*(e*x+d)^(1/2)*(c*x^2+a)^(1/2)/e/(c*e*x^3+c*d*x
^2+a*e*x+a*d)/a/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(c*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+a}(ex+d)^{\frac{3}{2}}}{c^2x^4+2\,acx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*(e*x + d)^(3/2)/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)/(c*x^2 + a)^(3/2), x)
```

3.688 $\int \frac{\sqrt{d+ex}}{(a+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=298

$$\frac{d\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{\sqrt{-a}\sqrt{c}\sqrt{a+cx^2}\sqrt{d+ex}}+\frac{x\sqrt{d+ex}}{a\sqrt{a+cx^2}}-\frac{\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\right.}{\sqrt{-a}\sqrt{c}\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}}$$

```
[Out] (x*Sqrt[d + e*x])/(a*Sqrt[a + c*x^2]) - (Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*
EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a
]*Sqrt[c]*d - a*e)])/(Sqrt[-a]*Sqrt[c]*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d
+ Sqrt[-a]*e)]*Sqrt[a + c*x^2]) + (d*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d +
Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt
[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(Sqrt[-a]*Sqrt[c]*Sqr
t[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.209321, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {737, 12, 844, 719, 424, 419}

$$\frac{x\sqrt{d+ex}}{a\sqrt{a+cx^2}}+\frac{d\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{\sqrt{-a}\sqrt{c}\sqrt{a+cx^2}\sqrt{d+ex}}-\frac{\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{\sqrt{-a}\sqrt{c}\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(a + c*x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[d + e*x])/(a*Sqrt[a + c*x^2]) - (Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*
EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a
]*Sqrt[c]*d - a*e)])/(Sqrt[-a]*Sqrt[c]*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d
+ Sqrt[-a]*e)]*Sqrt[a + c*x^2]) + (d*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d +
Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt
[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(Sqrt[-a]*Sqrt[c]*Sqr
t[d + e*x]*Sqrt[a + c*x^2])
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(x*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*(p + 1)), x] + Dist[1/(2*a*(p + 1
)), Int[(d + e*x)^(m - 1)*(d*(2*p + 3) + e*(m + 2*p + 3)*x)*(a + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -
1] && GtQ[m, 0] && (LtQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && In
tQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\left(a+cx^2\right)^{3/2}}\,dx = \frac{x\sqrt{d+ex}}{a\sqrt{a+cx^2}} - \frac{\int \frac{ex}{2\sqrt{d+ex}\sqrt{a+cx^2}}\,dx}{a}$$

$$= \frac{x\sqrt{d+ex}}{a\sqrt{a+cx^2}} - \frac{e\int \frac{x}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx}{2a}$$

$$= \frac{x\sqrt{d+ex}}{a\sqrt{a+cx^2}} - \frac{\int \frac{\sqrt{d+ex}}{\sqrt{a+cx^2}}\,dx}{2a} + \frac{d\int \frac{1}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx}{2a}$$

$$= \frac{x\sqrt{d+ex}}{a\sqrt{a+cx^2}} - \frac{\left(\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}ex^2}{\sqrt{-a}\left(cd-\frac{a\sqrt{c}e}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\frac{c(d+ex)}{cd-\frac{a\sqrt{c}e}{\sqrt{-a}}}}\sqrt{a+cx^2}} + \frac{(d\sqrt{\frac{c(d+ex)}{cd-\frac{a\sqrt{c}e}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}}{}$$

$$= \frac{x\sqrt{d+ex}}{a\sqrt{a+cx^2}} - \frac{\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}}\sqrt{a+cx^2}} + \frac{d\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}}\sqrt{1+\frac{cx^2}{a}}F(\sin^{-1}}{\sqrt{-a}\sqrt{c}\sqrt{d+e}}$$

Mathematica [C] time = 2.05554, size = 408, normalized size = 1.37

$$\frac{\sqrt{d+ex}\left(\frac{\sqrt{a}\sqrt{d+ex}\sqrt{\frac{e\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{d+ex}}\sqrt{-\frac{-ex+\frac{i\sqrt{ae}}{\sqrt{c}}}{d+ex}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-d-\frac{i\sqrt{ae}}{\sqrt{c}}}}{\sqrt{d+ex}}\right),\frac{\sqrt{c}d-i\sqrt{ae}}{\sqrt{c}d+i\sqrt{ae}}\right)}{\sqrt{c}\sqrt{-d-\frac{i\sqrt{ae}}{\sqrt{c}}}}-\frac{e\left(a+cx^2\right)}{c(d+ex)}-\frac{i\sqrt{d+ex}\sqrt{-d-\frac{i\sqrt{ae}}{\sqrt{c}}}\sqrt{\frac{e\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{d+ex}}\sqrt{-\frac{-ex+\frac{i\sqrt{ae}}{\sqrt{c}}}{d+ex}}E\left(i\sinh^{-}\right.}{e}\right.}{a\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(a + c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d + e*x]*(x - (e*(a + c*x^2))/(c*(d + e*x)) - (I*Sqrt[-d - (I*Sqrt[a]
*e)/Sqrt[c]]*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[
a]*e)/Sqrt[c] - e*x)/(d + e*x))]*Sqrt[d + e*x]*EllipticE[I*ArcSinh[Sqrt[-d
- (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]
*d + I*Sqrt[a]*e)])/e + (Sqrt[a]*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*
x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*Sqrt[d + e*x]*Elliptic
F[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I
*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)])/(Sqrt[c]*Sqrt[-d - (I*Sqrt[a]*e)/Sq
rt[c]])))/(a*Sqrt[a + c*x^2])
```

Maple [B] time = 0.268, size = 649, normalized size = 2.2

$$\frac{1}{ce\left(cex^3+cdx^2+aex+ad\right)a}\sqrt{ex+d}\sqrt{cx^2+a}\left(\text{EllipticE}\left(\sqrt{-c\left(ex+d\right)\left(\sqrt{-ac}e-cd\right)^{-1}},\sqrt{-\left(\sqrt{-ac}e-cd\right)\left(\sqrt{-ac}e+cd\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*x^2+a)^(3/2),x)
```

```
[Out] (e*x+d)^(1/2)*(c*x^2+a)^(1/2)*(EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^
(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*e^2*(-(e*x+d)*c
/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(
1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+EllipticE((-(e*x+d)*
c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^
(1/2))*c*d^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e
/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1
/2)-EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d
)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2
)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/
((-a*c)^(1/2)*e-c*d))^(1/2)-EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/
2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*d*e*(-a*c)^(1/2)*(-(
e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e
+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+c*e^2*x^2+c*
d*e*x)/c/e/(c*e*x^3+c*d*x^2+a*e*x+a*d)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{ex+d}}{\left(cx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(c*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+a}\sqrt{ex+d}}{c^2x^4+2\,acx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*sqrt(e*x + d)/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{d+ex}}{\left(a+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/(a + c*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{ex+d}}{\left(cx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)/(c*x^2 + a)^(3/2), x)
```

3.689 $\int \frac{1}{\sqrt{d+ex}\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=331

$$\frac{\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{\sqrt{-a}\sqrt{c}\sqrt{a+cx^2}\sqrt{d+ex}}+\frac{\sqrt{d+ex}(ae+cdx)}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{\sqrt{c}d\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}E\left(\sin^{-1}\left(\cdots\right.\right.}{\sqrt{-a}\sqrt{a+cx^2}\left(ae^2+cd\cdots\right.}$$

```
[Out] ((a*e + c*d*x)*Sqrt[d + e*x])/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) - (Sqrt[c
]*d*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)
/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(Sqrt[-a]*(c*d^2
 + a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2
]) + (Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a
]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[
-a]*Sqrt[c]*d - a*e)])/(Sqrt[-a]*Sqrt[c]*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.21975, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {741, 844, 719, 424, 419}

$$\frac{\sqrt{d+ex}(ae+cdx)}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{\sqrt{c}d\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{\sqrt{-a}\sqrt{a+cx^2}\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}}+\frac{\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right.\right.\cdots}{\sqrt{-a}\sqrt{c}\sqrt{a+cx^2}\sqrt{d+e\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(a + c*x^2)^(3/2)),x]
```


```
[Out] ((a*e + c*d*x)*Sqrt[d + e*x])/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) - (Sqrt[c
]*d*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)
/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(Sqrt[-a]*(c*d^2
 + a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2
]) + (Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a
]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[
-a]*Sqrt[c]*d - a*e)])/(Sqrt[-a]*Sqrt[c]*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```


Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\left(a+cx^2\right)^{3/2}}\,dx = \frac{(ae+cdx)\sqrt{d+ex}}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{\int \frac{-\frac{ae^2}{2}+\frac{1}{2}cdex}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx}{a\left(cd^2+ae^2\right)}$$

$$= \frac{(ae+cdx)\sqrt{d+ex}}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\int \frac{1}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx}{2a} - \frac{(cd)\int \frac{\sqrt{d+ex}}{\sqrt{a+cx^2}}\,dx}{2a\left(cd^2+ae^2\right)}$$

$$= \frac{(ae+cdx)\sqrt{d+ex}}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{\left(\sqrt{c}d\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}ex^2}{\sqrt{-a}\left(cd-\frac{a\sqrt{c}e}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right.}{\sqrt{-a}\left(cd^2+ae^2\right)\sqrt{\frac{c(d+ex)}{cd-\frac{a\sqrt{c}e}{\sqrt{-a}}}}\sqrt{a+cx^2}}$$

$$= \frac{(ae+cdx)\sqrt{d+ex}}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{\sqrt{c}d\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{\sqrt{-a}\left(cd^2+ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}}\sqrt{a+cx^2}} + \ldots$$

Mathematica [C] time = 0.974426, size = 430, normalized size = 1.3

$$\frac{\sqrt{a}e(d+ex)^{3/2}\sqrt{\frac{e\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{d+ex}}\sqrt{-\frac{-ex+\frac{i\sqrt{a}e}{\sqrt{c}}}{d+ex}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}}{\sqrt{d+ex}}\right),\frac{\sqrt{c}d-i\sqrt{a}e}{\sqrt{c}d+i\sqrt{a}e}\right)+ex\left(\sqrt{c}d-i\sqrt{a}e\right)\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}+i\sqrt{c}d(\ldots}{ae\sqrt{a+cx^2}\sqrt{d+ex}\left(\sqrt{c}d-i\sqrt{a}e\right)\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(a + c*x^2)^(3/2)),x]
```

```
[Out] (e*(Sqrt[c]*d - I*Sqrt[a]*e)*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*x + I*Sqrt[c]
*d*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt
[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d - (I*Sqr
t[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*S
qrt[a]*e)] + Sqrt[a]*e*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-
(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticF[I*ArcS
inh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]
*e)/(Sqrt[c]*d + I*Sqrt[a]*e)])/(a*e*(Sqrt[c]*d - I*Sqrt[a]*e)*Sqrt[-d - (I
*Sqrt[a]*e)/Sqrt[c]]*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Maple [B] time = 0.279, size = 696, normalized size = 2.1

$$\frac{1}{\left(ae^2+cd^2\right)ace\left(cex^3+cdx^2+aex+ad\right)}\left(-\sqrt{-c\,(ex+d)\left(\sqrt{-ac}e-cd\right)^{-1}}\sqrt{e\left(-cx+\sqrt{-ac}\right)\left(\sqrt{-ac}e+cd\right)^{-1}}\sqrt{e\left(cx+\sqrt{-a}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a)^(3/2)/(e*x+d)^(1/2),x)
```

```
[Out] (-(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1
/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*Ellipti
cF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(
1/2)*e+c*d))^(1/2))*(-a*c)^(1/2)*a*e^3-(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1
/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*
e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(
1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*(-a*c)^(1/2)*c*d^2
*e+(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(
1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*Ellipt
icE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^
(1/2)*e+c*d))^(1/2))*a*c*d*e^2+(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c
*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c
)^(1/2)*e-c*d))^(1/2)*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-(
(-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*c^2*d^3+x^2*c^2*d*e^2+x*a*
c*e^3+x*c^2*d^2*e+a*d*e^2*c)*(e*x+d)^(1/2)*(c*x^2+a)^(1/2)/e/c/a/(a*e^2+c*d
^2)/(c*e*x^3+c*d*x^2+a*e*x+a*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+a\right)^{\frac{3}{2}}\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(3/2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)^(3/2)*sqrt(e*x + d)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+a}\sqrt{ex+d}}{c^2ex^5+c^2dx^4+2\,acex^3+2\,acdx^2+a^2ex+a^2d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(3/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*sqrt(e*x + d)/(c^2*e*x^5 + c^2*d*x^4 + 2*a*c*e*x^3
 + 2*a*c*d*x^2 + a^2*e*x + a^2*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+cx^2\right)^{\frac{3}{2}}\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a)**(3/2)/(e*x+d)**(1/2),x)
```

```
[Out] Integral(1/((a + c*x**2)**(3/2)*sqrt(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+a\right)^{\frac{3}{2}}\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(3/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^2 + a)^(3/2)*sqrt(e*x + d)), x)
```

3.690 $\int \frac{1}{(d+ex)^{3/2}\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=406

$$\frac{\sqrt{c}d\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{\sqrt{-a}\sqrt{a+cx^2}\sqrt{d+ex}\left(ae^2+cd^2\right)}+\frac{e\sqrt{a+cx^2}\left(cd^2-3ae^2\right)}{a\sqrt{d+ex}\left(ae^2+cd^2\right)^2}+\frac{ae+cdx}{a\sqrt{a+cx^2}\sqrt{d+ex}\left(ae^2+c\ldots\right.}$$

```
[Out] (a*e + c*d*x)/(a*(c*d^2 + a*e^2)*Sqrt[d + e*x]*Sqrt[a + c*x^2]) + (e*(c*d^2
 - 3*a*e^2)*Sqrt[a + c*x^2])/(a*(c*d^2 + a*e^2)^2*Sqrt[d + e*x]) - (Sqrt[c]
*(c*d^2 - 3*a*e^2)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[
1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(
Sqrt[-a]*(c*d^2 + a*e^2)^2*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e
)]*Sqrt[a + c*x^2]) + (Sqrt[c]*d*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt
[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]
]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(Sqrt[-a]*(c*d^2 + a*e^2)
*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.303746, antiderivative size = 406, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {741, 835, 844, 719, 424, 419}

$$\frac{e\sqrt{a+cx^2}\left(cd^2-3ae^2\right)}{a\sqrt{d+ex}\left(ae^2+cd^2\right)^2}+\frac{ae+cdx}{a\sqrt{a+cx^2}\sqrt{d+ex}\left(ae^2+cd^2\right)}+\frac{\sqrt{c}d\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{\sqrt{-a}\sqrt{a+cx^2}\sqrt{d+ex}\left(ae^2+cd^2\right)}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*(a + c*x^2)^(3/2)),x]
```


```
[Out] (a*e + c*d*x)/(a*(c*d^2 + a*e^2)*Sqrt[d + e*x]*Sqrt[a + c*x^2]) + (e*(c*d^2
 - 3*a*e^2)*Sqrt[a + c*x^2])/(a*(c*d^2 + a*e^2)^2*Sqrt[d + e*x]) - (Sqrt[c]
*(c*d^2 - 3*a*e^2)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[
1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(
Sqrt[-a]*(c*d^2 + a*e^2)^2*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e
)]*Sqrt[a + c*x^2]) + (Sqrt[c]*d*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt
[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]
]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(Sqrt[-a]*(c*d^2 + a*e^2)
*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```


Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\left(a+cx^2\right)^{3/2}}\,dx = \frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{d+ex}\sqrt{a+cx^2}} - \frac{\int \frac{-\frac{3ae^2}{2}-\frac{1}{2}cdex}{(d+ex)^{3/2}\sqrt{a+cx^2}}\,dx}{a\left(cd^2+ae^2\right)}$$

$$= \frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{d+ex}\sqrt{a+cx^2}} + \frac{e\left(cd^2-3ae^2\right)\sqrt{a+cx^2}}{a\left(cd^2+ae^2\right)^2\sqrt{d+ex}} + \frac{2\int \frac{acde^2-\frac{1}{4}ce\left(cd^2-3ae^2\right)x}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx}{a\left(cd^2+ae^2\right)^2}$$

$$= \frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{d+ex}\sqrt{a+cx^2}} + \frac{e\left(cd^2-3ae^2\right)\sqrt{a+cx^2}}{a\left(cd^2+ae^2\right)^2\sqrt{d+ex}} - \frac{\left(c\left(cd^2-3ae^2\right)\right)\int \frac{\sqrt{d+ex}}{\sqrt{a+cx^2}}\,dx}{2a\left(cd^2+ae^2\right)^2}$$

$$= \frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{d+ex}\sqrt{a+cx^2}} + \frac{e\left(cd^2-3ae^2\right)\sqrt{a+cx^2}}{a\left(cd^2+ae^2\right)^2\sqrt{d+ex}} - \frac{\left(\sqrt{c}\left(cd^2-3ae^2\right)\sqrt{d+ex}\sqrt{1}\right.}{\sqrt{-a}\left(cd^{2}\right.}$$

$$= \frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{d+ex}\sqrt{a+cx^2}} + \frac{e\left(cd^2-3ae^2\right)\sqrt{a+cx^2}}{a\left(cd^2+ae^2\right)^2\sqrt{d+ex}} - \frac{\sqrt{c}\left(cd^2-3ae^2\right)\sqrt{d+ex}\sqrt{1-}}{\sqrt{-a}\left(cd^2+ae^{2}\right.}$$

Mathematica [C] time = 1.37857, size = 583, normalized size = 1.44

$$\frac{\sqrt{a}\sqrt{c}e(d+ex)^{3/2}\left(4i\sqrt{a}\sqrt{c}de-3ae^2+cd^2\right)\sqrt{\frac{e\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{d+ex}}\sqrt{-\frac{-ex+\frac{i\sqrt{ae}}{\sqrt{c}}}{d+ex}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-d-\frac{i\sqrt{ae}}{\sqrt{c}}}}{\sqrt{d+ex}}\right),\frac{\sqrt{c}d-i\sqrt{a}e}{\sqrt{c}d+i\sqrt{a}e}\right)+e^2\left(-\sqrt{-d-}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-(e^2*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*(-3*a^2*e^2 + c^2*d^2*x^2 + a*c*(d^
2 - 3*e^2*x^2))) + e*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*(-2*a*e^3*(a + c*x^2)
 + c*(d + e*x)*(c*d^2*x + a*e*(2*d - e*x))) + Sqrt[c]*(I*c^(3/2)*d^3 - Sqrt
[a]*c*d^2*e - (3*I)*a*Sqrt[c]*d*e^2 + 3*a^(3/2)*e^3)*Sqrt[(e*((I*Sqrt[a])/S
qrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d
 + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d +
 e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)] + Sqrt[a]*Sqrt
[c]*e*(c*d^2 + (4*I)*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*Sqrt[(e*((I*Sqrt[a])/Sq
rt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d
+ e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d +
e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)])/(a*e*Sqrt[-d -
 (I*Sqrt[a]*e)/Sqrt[c]]*(c*d^2 + a*e^2)^2*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Maple [B] time = 0.295, size = 1167, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(c*x^2+a)^(3/2),x)
```

```
[Out] (3*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(
1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*Ellipt
icF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^
(1/2)*e+c*d))^(1/2))*a^2*e^4+3*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c
*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c
)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-(
(-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c*d^2*e^2-(-(e*x+d)*c/((
-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2
)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/(
(-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/
2))*(-a*c)^(1/2)*a*d*e^3-(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a
*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2
)*e-c*d))^(1/2)*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)
^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*(-a*c)^(1/2)*c*d^3*e-3*(-(e*x+d)
*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))
^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticE((-(e*x+d
)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d)
)^(1/2))*a^2*e^4-2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1
/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*
d))^(1/2)*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)
*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c*d^2*e^2+(-(e*x+d)*c/((-a*c)^(1/2)*
e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*
c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)
*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*c^2*d^4-
3*x^2*a*c*e^4+x^2*c^2*d^2*e^2+x*a*c*d*e^3+x*c^2*d^3*e-2*a^2*e^4+2*a*c*d^2*e
^2)*(c*x^2+a)^(1/2)*(e*x+d)^(1/2)/e/a/(a*e^2+c*d^2)^2/(c*e*x^3+c*d*x^2+a*e*
x+a*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+a\right)^{\frac{3}{2}}(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)^(3/2)*(e*x + d)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+a}\sqrt{ex+d}}{c^2e^2x^6+2\,c^2dex^5+4\,acdex^3+2\,a^2dex+\left(c^2d^2+2\,ace^2\right)x^4+a^2d^2+\left(2\,acd^2+a^2e^2\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*sqrt(e*x + d)/(c^2*e^2*x^6 + 2*c^2*d*e*x^5 + 4*a*c
*d*e*x^3 + 2*a^2*d*e*x + (c^2*d^2 + 2*a*c*e^2)*x^4 + a^2*d^2 + (2*a*c*d^2 +
```

```
a^2*e^2)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+cx^2\right)^{\frac{3}{2}}(d+ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/((a + c*x**2)**(3/2)*(d + e*x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+a\right)^{\frac{3}{2}}(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^2 + a)^(3/2)*(e*x + d)^(3/2)), x)
```

3.691 $\int \frac{1}{(d+ex)^{5/2}\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=485

$$\frac{\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\left(3cd^2-5ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-ae}+\sqrt{c}d}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{3\sqrt{-a}\sqrt{a+cx^2}\sqrt{d+ex}\left(ae^2+cd^2\right)^2}-\frac{c^{3/2}d\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}\left(3cd^2-29ae^2\right)\cdots}{3\sqrt{-a}\sqrt{a+cx^2}\left(ae^2\cdots\right.}$$

```
[Out] (a*e + c*d*x)/(a*(c*d^2 + a*e^2)*(d + e*x)^(3/2)*Sqrt[a + c*x^2]) + (e*(3*c
*d^2 - 5*a*e^2)*Sqrt[a + c*x^2])/(3*a*(c*d^2 + a*e^2)^2*(d + e*x)^(3/2)) +
(c*d*e*(3*c*d^2 - 29*a*e^2)*Sqrt[a + c*x^2])/(3*a*(c*d^2 + a*e^2)^3*Sqrt[d
+ e*x]) - (c^(3/2)*d*(3*c*d^2 - 29*a*e^2)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]
*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-
a]*Sqrt[c]*d - a*e)])/(3*Sqrt[-a]*(c*d^2 + a*e^2)^3*Sqrt[(Sqrt[c]*(d + e*x)
)/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) + (Sqrt[c]*(3*c*d^2 - 5*a*e^2)
*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*Ell
ipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*S
qrt[c]*d - a*e)])/(3*Sqrt[-a]*(c*d^2 + a*e^2)^2*Sqrt[d + e*x]*Sqrt[a + c*x^
2])
```

Rubi [A] time = 0.494563, antiderivative size = 485, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {741, 835, 844, 719, 424, 419}

$$-\frac{c^{3/2}d\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}\left(3cd^2-29ae^2\right)E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{3\sqrt{-a}\sqrt{a+cx^2}\left(ae^2+cd^2\right)^3\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-ae}+\sqrt{c}d}}}+\frac{cde\sqrt{a+cx^2}\left(3cd^2-29ae^2\right)}{3a\sqrt{d+ex}\left(ae^2+cd^2\right)^3}+\frac{e\sqrt{a+cx^2}\cdots}{3a(d+ex)^{3/2}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(5/2)*(a + c*x^2)^(3/2)),x]
```


```
[Out] (a*e + c*d*x)/(a*(c*d^2 + a*e^2)*(d + e*x)^(3/2)*Sqrt[a + c*x^2]) + (e*(3*c
*d^2 - 5*a*e^2)*Sqrt[a + c*x^2])/(3*a*(c*d^2 + a*e^2)^2*(d + e*x)^(3/2)) +
(c*d*e*(3*c*d^2 - 29*a*e^2)*Sqrt[a + c*x^2])/(3*a*(c*d^2 + a*e^2)^3*Sqrt[d
+ e*x]) - (c^(3/2)*d*(3*c*d^2 - 29*a*e^2)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]
*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-
a]*Sqrt[c]*d - a*e)])/(3*Sqrt[-a]*(c*d^2 + a*e^2)^3*Sqrt[(Sqrt[c]*(d + e*x)
)/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) + (Sqrt[c]*(3*c*d^2 - 5*a*e^2)
*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*Ell
ipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*S
qrt[c]*d - a*e)])/(3*Sqrt[-a]*(c*d^2 + a*e^2)^2*Sqrt[d + e*x]*Sqrt[a + c*x^
2])
```


Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
```

```
LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(d+ex)^{5/2}\left(a+cx^2\right)^{3/2}}\,dx = \frac{ae+cdx}{a\left(cd^2+ae^2\right)(d+ex)^{3/2}\sqrt{a+cx^2}} - \frac{\int \frac{-\frac{5ae^2}{2}-\frac{3}{2}cdex}{(d+ex)^{5/2}\sqrt{a+cx^2}}\,dx}{a\left(cd^2+ae^2\right)}$$

$$= \frac{ae+cdx}{a\left(cd^2+ae^2\right)(d+ex)^{3/2}\sqrt{a+cx^2}} + \frac{e\left(3cd^2-5ae^2\right)\sqrt{a+cx^2}}{3a\left(cd^2+ae^2\right)^2(d+ex)^{3/2}} + \frac{2\int \frac{6acde^2+\frac{1}{4}ce\left(3cd^2-5ae\ldots\right)}{(d+ex)^{3/2}\sqrt{a+cx^2}}}{3a\left(cd^2+ae^2\right)^2}$$

$$= \frac{ae+cdx}{a\left(cd^2+ae^2\right)(d+ex)^{3/2}\sqrt{a+cx^2}} + \frac{e\left(3cd^2-5ae^2\right)\sqrt{a+cx^2}}{3a\left(cd^2+ae^2\right)^2(d+ex)^{3/2}} + \frac{cde\left(3cd^2-29ae^2\right)\sqrt{\ldots}}{3a\left(cd^2+ae^2\right)^3\sqrt{a\ldots}}$$

$$= \frac{ae+cdx}{a\left(cd^2+ae^2\right)(d+ex)^{3/2}\sqrt{a+cx^2}} + \frac{e\left(3cd^2-5ae^2\right)\sqrt{a+cx^2}}{3a\left(cd^2+ae^2\right)^2(d+ex)^{3/2}} + \frac{cde\left(3cd^2-29ae^2\right)\sqrt{\ldots}}{3a\left(cd^2+ae^2\right)^3\sqrt{a\ldots}}$$

$$= \frac{ae+cdx}{a\left(cd^2+ae^2\right)(d+ex)^{3/2}\sqrt{a+cx^2}} + \frac{e\left(3cd^2-5ae^2\right)\sqrt{a+cx^2}}{3a\left(cd^2+ae^2\right)^2(d+ex)^{3/2}} + \frac{cde\left(3cd^2-29ae^2\right)\sqrt{\ldots}}{3a\left(cd^2+ae^2\right)^3\sqrt{a\ldots}}$$

$$= \frac{ae+cdx}{a\left(cd^2+ae^2\right)(d+ex)^{3/2}\sqrt{a+cx^2}} + \frac{e\left(3cd^2-5ae^2\right)\sqrt{a+cx^2}}{3a\left(cd^2+ae^2\right)^2(d+ex)^{3/2}} + \frac{cde\left(3cd^2-29ae^2\right)\sqrt{\ldots}}{3a\left(cd^2+ae^2\right)^3\sqrt{a\ldots}}$$

Mathematica [C] time = 3.62985, size = 634, normalized size = 1.31

$$\frac{c(d+ex)\left(\sqrt{a}e(d+ex)^{3/2}\left(-5ia^{3/2}e^3+27i\sqrt{a}cd^2e-29a\sqrt{c}de^2+3c^{3/2}d^3\right)\sqrt{\frac{e\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{d+ex}}\sqrt{-\frac{-ex+\frac{i\sqrt{a}e}{\sqrt{c}}}{d+ex}}\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}}{\sqrt{d+ex}}\right),\frac{\sqrt{c}d-i\sqrt{a}e}{\sqrt{c}d+i\sqrt{a}e}\right)-de^2\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}\left(-29a^2e^{\ldots}\right.\right.}{e\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(5/2)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-2*a*e^3*(c*d^2 + a*e^2)*(a + c*x^2) - 20*a*c*d*e^3*(d + e*x)*(a + c*x^2)
+ 3*c*(d + e*x)^2*(-(a^2*e^3) + c^2*d^3*x + 3*a*c*d*e*(d - e*x)) + (c*(d +
e*x)*(-(d*e^2*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*(-29*a^2*e^2 + 3*c^2*d^2*x^2
 + a*c*(3*d^2 - 29*e^2*x^2))) + Sqrt[c]*d*((3*I)*c^(3/2)*d^3 - 3*Sqrt[a]*c*
d^2*e - (29*I)*a*Sqrt[c]*d*e^2 + 29*a^(3/2)*e^3)*Sqrt[(e*((I*Sqrt[a])/Sqrt[
c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e
*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x
]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)] + Sqrt[a]*e*(3*c^(
3/2)*d^3 + (27*I)*Sqrt[a]*c*d^2*e - 29*a*Sqrt[c]*d*e^2 - (5*I)*a^(3/2)*e^3)
*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c
] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d - (I*Sqrt[
a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqr
t[a]*e)]))/(e*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]))/(3*a*(c*d^2 + a*e^2)^3*(d
+ e*x)^(3/2)*Sqrt[a + c*x^2])
```

Maple [B] time = 0.305, size = 2623, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(5/2)/(c*x^2+a)^(3/2),x)
```

```
[Out] 1/3*(3*x^3*c^3*d^3*e^3-5*x^2*a^2*c*e^6+6*x^2*c^3*d^4*e^2-31*x^2*a*c^2*d^2*e
^4-26*x*a^2*c*d*e^5+9*x*a*c^2*d^3*e^3+3*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)
*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*c^3*d^6*
(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2
)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-3*Ellipti
cF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(
1/2)*e+c*d))^(1/2))*c^2*d^5*e*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d)
)^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/
2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-29*x^3*a*c^2*d*e^5+24*EllipticF((-(e*x+d)
*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))
^(1/2))*x*a^2*c*d*e^5*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)
^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e
-c*d))^(1/2)+24*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)
^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*a*c^2*d^3*e^3*(-(e*x+d)*c/((-a
*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*
((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-3*EllipticF((-(e*x+d)*c/(
(-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/
2))*x*c^2*d^4*e^2*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c
*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c
)^(1/2)*e-c*d))^(1/2)-29*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),
(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*a^2*c*d*e^5*(-(e*x+d)
*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))
^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-26*EllipticE((-(e*
x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c
*d))^(1/2))*x*a*c^2*d^3*e^3*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+
(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(
1/2)*e-c*d))^(1/2)+2*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((
-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c*d^3*e^3*(-a*c)^(1/2)*(-
(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*
e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-29*Elliptic
E((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1
/2)*e+c*d))^(1/2))*a^2*c*d^2*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((
-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a
*c)^(1/2)*e-c*d))^(1/2)-26*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2
),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c^2*d^4*e^2*(-(e*x+
d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d
))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+3*EllipticE((-(e
*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+
c*d))^(1/2))*x*c^3*d^5*e*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a
*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2
)*e-c*d))^(1/2)+24*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a
*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*c*d^2*e^4*(-(e*x+d)*c/((-
a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)
*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+24*EllipticF((-(e*x+d)*c
/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(
1/2))*a*c^2*d^4*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(
1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c
*d))^(1/2)+3*x*c^3*d^5*e+5*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2
),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*a^2*e^6*(-a*c)^(1/2
)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1
```

```
/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+5*(-(e*
x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c
*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)*EllipticF((-(e
*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+
c*d))^(1/2))*(-a*c)^(1/2)*a^2*d*e^5+2*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e
-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x*a*c*d^2*
e^4*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2
))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d)
)^(1/2)-2*a^3*e^6-25*a^2*c*d^2*e^4+9*a*c^2*d^4*e^2)/(c*x^2+a)^(1/2)/(a*e^2+
c*d^2)^3/a/(e*x+d)^(3/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+a\right)^{\frac{3}{2}}(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)^(3/2)*(e*x + d)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+a}\sqrt{ex+d}}{c^2e^3x^7+3\,c^2de^2x^6+3\,a^2d^2ex+\left(3\,c^2d^2e+2\,ace^3\right)x^5+a^2d^3+\left(c^2d^3+6\,acde^2\right)x^4+\left(6\,acd^2e+a^2e^3\right)x^3+\left(2\,a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*sqrt(e*x + d)/(c^2*e^3*x^7 + 3*c^2*d*e^2*x^6 + 3*a
^2*d^2*e*x + (3*c^2*d^2*e + 2*a*c*e^3)*x^5 + a^2*d^3 + (c^2*d^3 + 6*a*c*d*e
^2)*x^4 + (6*a*c*d^2*e + a^2*e^3)*x^3 + (2*a*c*d^3 + 3*a^2*d*e^2)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+cx^2\right)^{\frac{3}{2}}\,(d+ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(5/2)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/((a + c*x**2)**(3/2)*(d + e*x)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+a\right)^{\frac{3}{2}}(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^2 + a)^(3/2)*(e*x + d)^(5/2)), x)
```

3.692 $\int \frac{(d+ex)^{9/2}}{\left(a+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=475

$$-\frac{2d\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\left(3ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{3(-a)^{3/2}c^{5/2}\sqrt{a+cx^2}\sqrt{d+ex}}-\frac{(d+ex)^{3/2}\left(ae\left(7ae^2+cd\right.\right.}{6a^2c^2\sqrt{}}$$

```
[Out] -((a*e - c*d*x)*(d + e*x)^(7/2))/(3*a*c*(a + c*x^2)^(3/2)) - ((d + e*x)^(3/
2)*(a*e*(c*d^2 + 7*a*e^2) - 2*c*d*(2*c*d^2 + 5*a*e^2)*x))/(6*a^2*c^2*Sqrt[a
 + c*x^2]) - (2*d*e*(c*d^2 + 3*a*e^2)*Sqrt[d + e*x]*Sqrt[a + c*x^2])/(3*a^2
*c^2) + ((4*c^2*d^4 + 15*a*c*d^2*e^2 - 21*a^2*e^4)*Sqrt[d + e*x]*Sqrt[1 + (
c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e
)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(6*(-a)^(3/2)*c^(5/2)*Sqrt[(Sqrt[c]*(d + e*x
))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) - (2*d*(c*d^2 + a*e^2)*(c*d^2
 + 3*a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*
x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/
(Sqrt[-a]*Sqrt[c]*d - a*e)])/(3*(-a)^(3/2)*c^(5/2)*Sqrt[d + e*x]*Sqrt[a + c
*x^2])
```

Rubi [A] time = 0.504921, antiderivative size = 475, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {739, 819, 833, 844, 719, 424, 419}

$$-\frac{(d+ex)^{3/2}\left(ae\left(7ae^2+cd^2\right)-2cdx\left(5ae^2+2cd^2\right)\right)}{6a^2c^2\sqrt{a+cx^2}}-\frac{2de\sqrt{a+cx^2}\sqrt{d+ex}\left(3ae^2+cd^2\right)}{3a^2c^2}+\frac{\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}\left(-21a^2e^4\right.}{6(}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(9/2)/(a + c*x^2)^(5/2),x]
```


```
[Out] -((a*e - c*d*x)*(d + e*x)^(7/2))/(3*a*c*(a + c*x^2)^(3/2)) - ((d + e*x)^(3/
2)*(a*e*(c*d^2 + 7*a*e^2) - 2*c*d*(2*c*d^2 + 5*a*e^2)*x))/(6*a^2*c^2*Sqrt[a
 + c*x^2]) - (2*d*e*(c*d^2 + 3*a*e^2)*Sqrt[d + e*x]*Sqrt[a + c*x^2])/(3*a^2
*c^2) + ((4*c^2*d^4 + 15*a*c*d^2*e^2 - 21*a^2*e^4)*Sqrt[d + e*x]*Sqrt[1 + (
c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e
)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(6*(-a)^(3/2)*c^(5/2)*Sqrt[(Sqrt[c]*(d + e*x
))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) - (2*d*(c*d^2 + a*e^2)*(c*d^2
 + 3*a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*
x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/
(Sqrt[-a]*Sqrt[c]*d - a*e)])/(3*(-a)^(3/2)*c^(5/2)*Sqrt[d + e*x]*Sqrt[a + c
*x^2])
```


Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
```

```
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^{9/2}}{\left(a+cx^2\right)^{5/2}}\,dx = -\frac{(ae-cdx)(d+ex)^{7/2}}{3ac\left(a+cx^2\right)^{3/2}} + \frac{\int \frac{(d+ex)^{5/2}\left(\frac{1}{2}\left(4cd^2+7ae^2\right)-\frac{3}{2}cdex\right)}{\left(a+cx^2\right)^{3/2}}dx}{3ac}$$

$$= -\frac{(ae-cdx)(d+ex)^{7/2}}{3ac\left(a+cx^2\right)^{3/2}} - \frac{(d+ex)^{3/2}\left(ae\left(cd^2+7ae^2\right)-2cd\left(2cd^2+5ae^2\right)x\right)}{6a^2c^2\sqrt{a+cx^2}} + \frac{\int \frac{\sqrt{d+ex}\left(-\frac{3}{4}ae^2\left(cd^2-7\right.\right.}{\sqrt{a}}}{3}$$

$$= -\frac{(ae-cdx)(d+ex)^{7/2}}{3ac\left(a+cx^2\right)^{3/2}} - \frac{(d+ex)^{3/2}\left(ae\left(cd^2+7ae^2\right)-2cd\left(2cd^2+5ae^2\right)x\right)}{6a^2c^2\sqrt{a+cx^2}} - \frac{2de\left(cd^2+3ae^2\right)\sqrt{}}{3a^2c}$$

$$= -\frac{(ae-cdx)(d+ex)^{7/2}}{3ac\left(a+cx^2\right)^{3/2}} - \frac{(d+ex)^{3/2}\left(ae\left(cd^2+7ae^2\right)-2cd\left(2cd^2+5ae^2\right)x\right)}{6a^2c^2\sqrt{a+cx^2}} - \frac{2de\left(cd^2+3ae^2\right)\sqrt{}}{3a^2c}$$

$$= -\frac{(ae-cdx)(d+ex)^{7/2}}{3ac\left(a+cx^2\right)^{3/2}} - \frac{(d+ex)^{3/2}\left(ae\left(cd^2+7ae^2\right)-2cd\left(2cd^2+5ae^2\right)x\right)}{6a^2c^2\sqrt{a+cx^2}} - \frac{2de\left(cd^2+3ae^2\right)\sqrt{}}{3a^2c}$$

$$= -\frac{(ae-cdx)(d+ex)^{7/2}}{3ac\left(a+cx^2\right)^{3/2}} - \frac{(d+ex)^{3/2}\left(ae\left(cd^2+7ae^2\right)-2cd\left(2cd^2+5ae^2\right)x\right)}{6a^2c^2\sqrt{a+cx^2}} - \frac{2de\left(cd^2+3ae^2\right)\sqrt{}}{3a^2c}$$

Mathematica [C] time = 4.59405, size = 700, normalized size = 1.47

$$\sqrt{d+ex}\left(\frac{-2a^2ce\left(-3d^2ex+7d^3+27de^2x^2+9e^3x^3\right)-2a^3e^3(19d+7ex)+2ac^2d^2x\left(6d^2+dex+15e^2x^2\right)+8c^3d^4x^3}{a^2c^2\left(a+cx^2\right)} + \frac{2\left(\sqrt{a}\sqrt{c}e(d+ex)^{3/2}\left(33ia^{3/2}\sqrt{c}de^3-21a^2e^4+i\sqrt{a}\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(9/2)/(a + c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d + e*x]*((8*c^3*d^4*x^3 - 2*a^3*e^3*(19*d + 7*e*x) + 2*a*c^2*d^2*x*(
6*d^2 + d*e*x + 15*e^2*x^2) - 2*a^2*c*e*(7*d^3 - 3*d^2*e*x + 27*d*e^2*x^2 +
 9*e^3*x^3))/(a^2*c^2*(a + c*x^2)) + (2*(-(e^2*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt
[c]]*(4*c^2*d^4 + 15*a*c*d^2*e^2 - 21*a^2*e^4)*(a + c*x^2)) + Sqrt[c]*((4*I
)*c^(5/2)*d^5 - 4*Sqrt[a]*c^2*d^4*e + (15*I)*a*c^(3/2)*d^3*e^2 - 15*a^(3/2)
*c*d^2*e^3 - (21*I)*a^2*Sqrt[c]*d*e^4 + 21*a^(5/2)*e^5)*Sqrt[(e*((I*Sqrt[a]
)/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]
*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[
d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)] + Sqrt[a]*S
qrt[c]*e*(4*c^2*d^4 + I*Sqrt[a]*c^(3/2)*d^3*e + 15*a*c*d^2*e^2 + (33*I)*a^(
3/2)*Sqrt[c]*d*e^3 - 21*a^2*e^4)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*
x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*Ellipt
icF[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d -
```

```
I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)]))/(a^2*c^3*e*Sqrt[-d - (I*Sqrt[a]*
e)/Sqrt[c]]*(d + e*x))))/(12*Sqrt[a + c*x^2])
```

Maple [B] time = 0.329, size = 3304, normalized size = 7.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(9/2)/(c*x^2+a)^(5/2),x)
```

```
[Out] 1/6*(19*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e
-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*c^2*d^4*e^2*(-(e*x+d)*c/((-a*c)^(1/2
)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-
a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+21*EllipticF((-(e*x+d)*c/((-a*c)^
(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2
*a^3*c*e^6*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/(
(-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2
)-21*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*
d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a^3*c*e^6*(-(e*x+d)*c/((-a*c)^(1/2)*e-c
*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^
(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+18*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)
*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^3*c*d^
2*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c
)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-12*
EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((
-a*c)^(1/2)*e+c*d))^(1/2))*a^3*d*e^5*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)
*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a
*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+18*EllipticF((-(e*x+d)*c/((-a*c)^(
1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*
a^2*c^2*d^2*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2)
)*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))
^(1/2)-3*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*
e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a*c^3*d^4*e^2*(-(e*x+d)*c/((-a*c)^(
1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x
+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-4*EllipticF((-(e*x+d)*c/((-a*c
)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x
^2*c^3*d^5*e*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-
a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/
2)*e-c*d))^(1/2)-6*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a
*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a^2*c^2*d^2*e^4*(-(e*x+d)
*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))
^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+19*EllipticE((-(e*
x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c
*d))^(1/2))*x^2*a*c^3*d^4*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*
x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)
^(1/2)*e-c*d))^(1/2)-16*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(
-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*c*d^3*e^3*(-a*c)^(1/
2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(
1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-4*Elli
pticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c
)^(1/2)*e+c*d))^(1/2))*a*c^2*d^5*e*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e
-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c
)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-19*a^3*c*d^2*e^4-7*a^2*c^2*d^4*e^2+2
1*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/
((-a*c)^(1/2)*e+c*d))^(1/2))*a^4*e^6*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2
)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/
```

```
((-a*c)^(1/2)*e-c*d))^(1/2)+15*x^4*a*c^3*d^2*e^4-3*EllipticF((-(e*x+d)*c/((
-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2
))*a^2*c^2*d^4*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1
/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*
d))^(1/2)-24*x^2*a^2*c^2*d^2*e^4+7*x^2*a*c^3*d^4*e^2-36*x^3*a^2*c^2*d*e^5+1
6*x^3*a*c^3*d^3*e^3-26*x*a^3*c*d*e^5-4*x*a^2*c^2*d^3*e^3+6*x*a*c^3*d^5*e+4*
EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((
-a*c)^(1/2)*e+c*d))^(1/2))*x^2*c^4*d^6*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1
/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*
e/((-a*c)^(1/2)*e-c*d))^(1/2)+4*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))
^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c^3*d^6*(-(e*x
+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*
d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-9*x^4*a^2*c^2*e
^6+4*x^4*c^4*d^4*e^2-7*x^2*a^3*c*e^6+4*x^3*c^4*d^5*e-6*EllipticE((-(e*x+d)*
c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^
(1/2))*a^3*c*d^2*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^
(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-
c*d))^(1/2)-12*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^
(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a^2*c*d*e^5*(-a*c)^(1/2)*(-(e
*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+
c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-16*EllipticF(
(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2
)*e+c*d))^(1/2))*x^2*a*c^2*d^3*e^3*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e
-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c
)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-21*EllipticE((-(e*x+d)*c/((-a*c)^(1/
2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^4*e^
6*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1
/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2))/c^3/(e
*x+d)^(1/2)/a^2/e/(c*x^2+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{9}{2}}}{(cx^2+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(c*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(9/2)/(c*x^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e^4x^4 + 4\,de^3x^3 + 6\,d^2e^2x^2 + 4\,d^3ex + d^4\right)\sqrt{cx^2+a}\sqrt{ex+d}}{c^3x^6 + 3\,ac^2x^4 + 3\,a^2cx^2 + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4)*sqrt(c*x
^2 + a)*sqrt(e*x + d)/(c^3*x^6 + 3*a*c^2*x^4 + 3*a^2*c*x^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(9/2)/(c*x**2+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{9}{2}}}{\left(cx^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(c*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(9/2)/(c*x^2 + a)^(5/2), x)
```

3.693 $\int \frac{(d+ex)^{7/2}}{\left(a+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=418

$$-\frac{\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\left(5ae^2+4cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{6(-a)^{3/2}c^{5/2}\sqrt{a+cx^2}\sqrt{d+ex}}-\frac{\sqrt{d+ex}\left(ae\left(5ae^2+3cd^2\right)\right.}{6a^2c^2\sqrt{a}}$$

```
[Out] -((a*e - c*d*x)*(d + e*x)^(5/2))/(3*a*c*(a + c*x^2)^(3/2)) - (Sqrt[d + e*x]
*(a*e*(3*c*d^2 + 5*a*e^2) - 2*c*d*(2*c*d^2 + 3*a*e^2)*x))/(6*a^2*c^2*Sqrt[a
 + c*x^2]) + (2*d*(c*d^2 + 2*a*e^2)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*Ellip
ticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqr
t[c]*d - a*e)])/(3*(-a)^(3/2)*c^(3/2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d +
 Sqrt[-a]*e)]*Sqrt[a + c*x^2]) - ((c*d^2 + a*e^2)*(4*c*d^2 + 5*a*e^2)*Sqrt[
(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF
[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]
*d - a*e)])/(6*(-a)^(3/2)*c^(5/2)*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.379881, antiderivative size = 418, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {739, 819, 844, 719, 424, 419}

$$-\frac{\sqrt{d+ex}\left(ae\left(5ae^2+3cd^2\right)-2cdx\left(3ae^2+2cd^2\right)\right)}{6a^2c^2\sqrt{a+cx^2}}-\frac{\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\left(5ae^2+4cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right.\right.}{6(-a)^{3/2}c^{5/2}\sqrt{a+cx^2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/(a + c*x^2)^(5/2),x]
```

```
[Out] -((a*e - c*d*x)*(d + e*x)^(5/2))/(3*a*c*(a + c*x^2)^(3/2)) - (Sqrt[d + e*x]
*(a*e*(3*c*d^2 + 5*a*e^2) - 2*c*d*(2*c*d^2 + 3*a*e^2)*x))/(6*a^2*c^2*Sqrt[a
 + c*x^2]) + (2*d*(c*d^2 + 2*a*e^2)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*Ellip
ticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqr
t[c]*d - a*e)])/(3*(-a)^(3/2)*c^(3/2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d +
 Sqrt[-a]*e)]*Sqrt[a + c*x^2]) - ((c*d^2 + a*e^2)*(4*c*d^2 + 5*a*e^2)*Sqrt[
(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF
[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]
*d - a*e)])/(6*(-a)^(3/2)*c^(5/2)*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^{7/2}}{\left(a+cx^2\right)^{5/2}}\,dx = -\frac{(ae-cdx)(d+ex)^{5/2}}{3ac\left(a+cx^2\right)^{3/2}} + \frac{\int \frac{(d+ex)^{3/2}\left(\frac{1}{2}\left(4cd^2+5ae^2\right)-\frac{1}{2}cdex\right)}{\left(a+cx^2\right)^{3/2}}dx}{3ac}$$

$$= -\frac{(ae-cdx)(d+ex)^{5/2}}{3ac\left(a+cx^2\right)^{3/2}} - \frac{\sqrt{d+ex}\left(ae\left(3cd^2+5ae^2\right)-2cd\left(2cd^2+3ae^2\right)x\right)}{6a^2c^2\sqrt{a+cx^2}} + \frac{\int \frac{\frac{1}{4}ae^2\left(cd^2+5ae^2\right)-cde(c}{\sqrt{d+ex}\sqrt{a+cx}}}{3a^2c^2}$$

$$= -\frac{(ae-cdx)(d+ex)^{5/2}}{3ac\left(a+cx^2\right)^{3/2}} - \frac{\sqrt{d+ex}\left(ae\left(3cd^2+5ae^2\right)-2cd\left(2cd^2+3ae^2\right)x\right)}{6a^2c^2\sqrt{a+cx^2}} - \frac{\left(d\left(cd^2+2ae^2\right)\right)\int}{3a^2c}$$

$$= -\frac{(ae-cdx)(d+ex)^{5/2}}{3ac\left(a+cx^2\right)^{3/2}} - \frac{\sqrt{d+ex}\left(ae\left(3cd^2+5ae^2\right)-2cd\left(2cd^2+3ae^2\right)x\right)}{6a^2c^2\sqrt{a+cx^2}} - \frac{\left(2d\left(cd^2+2ae^2\right)\sqrt{\cdots}\right.}{}$$

$$= -\frac{(ae-cdx)(d+ex)^{5/2}}{3ac\left(a+cx^2\right)^{3/2}} - \frac{\sqrt{d+ex}\left(ae\left(3cd^2+5ae^2\right)-2cd\left(2cd^2+3ae^2\right)x\right)}{6a^2c^2\sqrt{a+cx^2}} + \frac{2d\left(cd^2+2ae^2\right)\sqrt{d}\cdots}{3(\cdots}$$

Mathematica [C] time = 3.67683, size = 627, normalized size = 1.5

$$\sqrt{d+ex}\left(\frac{a^2ce\left(-5d^2+2dex-7e^2x^2\right)-5a^3e^3+ac^2dx\left(6d^2+dex+8e^2x^2\right)+4c^3d^3x^3}{a^2c^2\left(a+cx^2\right)} - \frac{-\sqrt{a}e(d+ex)^{3/2}\left(5ia^{3/2}e^3+i\sqrt{a}cd^2e+8a\sqrt{c}de^2+4c^{3/2}d^3\right)\sqrt{\frac{e\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{d+ex}}\sqrt{-\frac{-ex+\cdots}{d+\cdots}}\cdots}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)/(a + c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d + e*x]*((-5*a^3*e^3 + 4*c^3*d^3*x^3 + a^2*c*e*(-5*d^2 + 2*d*e*x - 7
*e^2*x^2) + a*c^2*d*x*(6*d^2 + d*e*x + 8*e^2*x^2))/(a^2*c^2*(a + c*x^2)) -
(4*d*e^2*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*(2*a^2*e^2 + c^2*d^2*x^2 + a*c*(d
^2 + 2*e^2*x^2)) + 4*Sqrt[c]*d*((-I)*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e - (2*I)*
a*Sqrt[c]*d*e^2 + 2*a^(3/2)*e^3)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*
x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*Ellipt
icE[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d -
 I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)] - Sqrt[a]*e*(4*c^(3/2)*d^3 + I*Sqr
t[a]*c*d^2*e + 8*a*Sqrt[c]*d*e^2 + (5*I)*a^(3/2)*e^3)*Sqrt[(e*((I*Sqrt[a])/
Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(
d + e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d
+ e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)])/(a^2*c^2*e*S
qrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*(d + e*x))))/(6*Sqrt[a + c*x^2])
```

Maple [B] time = 0.3, size = 2660, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/(c*x^2+a)^(5/2),x)
```

```
[Out] -1/6*(3*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e
-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^3*c*d*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-
c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)
^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-9*x^3*a*c^3*d^2*e^3+3*x*a^2*c^2*d^2*e
^3-6*x*a*c^3*d^4*e-7*x^2*a*c^3*d^3*e^2+5*x^2*a^2*c^2*d*e^4-8*x^4*a*c^3*d*e^
4+9*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d
)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a*c^2*d^2*e^3*(-a*c)^(1/2)*(-(e*x+d)*c/(
(-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/
2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-12*EllipticE((-(e*x+d)
*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))
^(1/2))*a^2*c^2*d^3*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*
c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)
*e-c*d))^(1/2)-4*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c
)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*c^4*d^5*(-(e*x+d)*c/((-a*c)
^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c
*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+5*EllipticF((-(e*x+d)*c/((-a
*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))
*a^3*e^5*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)
^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e
-c*d))^(1/2)-4*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^
(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c^3*d^5*(-(e*x+d)*c/((-a*c)^(1/
2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(
-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-4*x^4*c^4*d^3*e^2+7*x^3*a^2*c^2*
e^5-4*x^3*c^4*d^4*e+5*x*a^3*c*e^5+5*a^3*c*d*e^4+5*a^2*c^2*d^3*e^2+4*Ellipti
cF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(
1/2)*e+c*d))^(1/2))*x^2*c^3*d^4*e*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-
c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)
^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-8*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)
*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a^2*
c^2*d*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((
-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)
-12*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d
)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a*c^3*d^3*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*
e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*
c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+9*EllipticF((-(e*x+d)*c/((-a*c)^(1/
2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*c*
d^2*e^3*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^
(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-
c*d))^(1/2)+4*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(
1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c^2*d^4*e*(-a*c)^(1/2)*(-(e*x+d)
*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))
^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+3*EllipticF((-(e*x
+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*
d))^(1/2))*x^2*a^2*c^2*d*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x
+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^
(1/2)*e-c*d))^(1/2)+5*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-(
(-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a^2*c*e^5*(-a*c)^(1/2)
*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/
2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+3*Ellipt
icF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^
(1/2)*e+c*d))^(1/2))*x^2*a*c^3*d^3*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1
```

```
/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*
e/((-a*c)^(1/2)*e-c*d))^(1/2)+3*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))
^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*c^2*d^3*e^2*
(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2
)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-8*Ellipti
cE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(
1/2)*e+c*d))^(1/2))*a^3*c*d*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-
c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*
c)^(1/2)*e-c*d))^(1/2))/(e*x+d)^(1/2)/a^2/e/c^3/(c*x^2+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{7}{2}}}{(cx^2+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(7/2)/(c*x^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3)\sqrt{cx^2+a}\sqrt{ex+d}}{c^3x^6+3\,ac^2x^4+3\,a^2cx^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(c*x^2 + a)*sqrt(e*x
 + d)/(c^3*x^6 + 3*a*c^2*x^4 + 3*a^2*c*x^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/(c*x**2+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{7}{2}}}{(cx^2+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(7/2)/(c*x^2 + a)^(5/2), x)
```

3.694 $\int \frac{(d+ex)^{5/2}}{\left(a+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=392

$$-\frac{2d\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{3(-a)^{3/2}c^{3/2}\sqrt{a+cx^2}\sqrt{d+ex}}-\frac{\sqrt{d+ex}\left(ade-x\left(3ae^2+4cd^2\right)\right)}{6a^2c\sqrt{a+cx^2}}+ \cdots$$

```
[Out] -((a*e - c*d*x)*(d + e*x)^(3/2))/(3*a*c*(a + c*x^2)^(3/2)) - (Sqrt[d + e*x]
*(a*d*e - (4*c*d^2 + 3*a*e^2)*x))/(6*a^2*c*Sqrt[a + c*x^2]) + ((4*c*d^2 + 3
*a*e^2)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c
]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(6*(-a)^(3/2
)*c^(3/2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2
]) - (2*d*(c*d^2 + a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)
]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[
2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(3*(-a)^(3/2)*c^(3/2)*Sqrt[d + e
*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.32543, antiderivative size = 392, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {739, 821, 844, 719, 424, 419}

$$-\frac{\sqrt{d+ex}\left(ade-x\left(3ae^2+4cd^2\right)\right)}{6a^2c\sqrt{a+cx^2}}-\frac{2d\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{3(-a)^{3/2}c^{3/2}\sqrt{a+cx^2}\sqrt{d+ex}}+\frac{\sqrt{\frac{cx^2}{a}+1}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(a + c*x^2)^(5/2),x]
```


```
[Out] -((a*e - c*d*x)*(d + e*x)^(3/2))/(3*a*c*(a + c*x^2)^(3/2)) - (Sqrt[d + e*x]
*(a*d*e - (4*c*d^2 + 3*a*e^2)*x))/(6*a^2*c*Sqrt[a + c*x^2]) + ((4*c*d^2 + 3
*a*e^2)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c
]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(6*(-a)^(3/2
)*c^(3/2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2
]) - (2*d*(c*d^2 + a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)
]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[
2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(3*(-a)^(3/2)*c^(3/2)*Sqrt[d + e
*x]*Sqrt[a + c*x^2])
```


Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\left(a+cx^2\right)^{5/2}}dx = -\frac{(ae-cdx)(d+ex)^{3/2}}{3ac\left(a+cx^2\right)^{3/2}} + \frac{\int \frac{\sqrt{d+ex}\left(\frac{1}{2}\left(4cd^2+3ae^2\right)+\frac{1}{2}cdex\right)}{\left(a+cx^2\right)^{3/2}}dx}{3ac}$$

$$= -\frac{(ae-cdx)(d+ex)^{3/2}}{3ac\left(a+cx^2\right)^{3/2}} - \frac{\sqrt{d+ex}\left(ade-\left(4cd^2+3ae^2\right)x\right)}{6a^2c\sqrt{a+cx^2}} + \frac{\int \frac{\frac{1}{4}acde^2-\frac{1}{4}ce\left(4cd^2+3ae^2\right)x}{\sqrt{d+ex}\sqrt{a+cx^2}}dx}{3a^2c^2}$$

$$= -\frac{(ae-cdx)(d+ex)^{3/2}}{3ac\left(a+cx^2\right)^{3/2}} - \frac{\sqrt{d+ex}\left(ade-\left(4cd^2+3ae^2\right)x\right)}{6a^2c\sqrt{a+cx^2}} + \frac{\left(d\left(cd^2+ae^2\right)\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a+cx^2}}dx}{3a^2c} - \cdots$$

$$= -\frac{(ae-cdx)(d+ex)^{3/2}}{3ac\left(a+cx^2\right)^{3/2}} - \frac{\sqrt{d+ex}\left(ade-\left(4cd^2+3ae^2\right)x\right)}{6a^2c\sqrt{a+cx^2}} - \frac{\left(\left(4cd^2+3ae^2\right)\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}\right)\text{Su}\cdots}{6\sqrt{-a}ac^{3/2}\sqrt{\cdots}}$$

$$= -\frac{(ae-cdx)(d+ex)^{3/2}}{3ac\left(a+cx^2\right)^{3/2}} - \frac{\sqrt{d+ex}\left(ade-\left(4cd^2+3ae^2\right)x\right)}{6a^2c\sqrt{a+cx^2}} + \frac{\left(4cd^2+3ae^2\right)\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}E\left(\text{si}\cdots\right)}{6(-a)^{3/2}c^{3/2}\sqrt{\frac{\sqrt{c}(d+\cdots}{\sqrt{c}d+\sqrt{\cdots}}}}$$

Mathematica [C] time = 3.50144, size = 597, normalized size = 1.52

$$\frac{\sqrt{d+ex}\left(\frac{2\left(a^2e(ex-3d)+acx\left(6d^2+dex+3e^2x^2\right)+4c^2d^2x^3\right)}{a^2c\left(a+cx^2\right)} + \frac{(d+ex)\left(\frac{2\sqrt{a}\sqrt{c}e\left(i\sqrt{a}\sqrt{c}de+3ae^2+4cd^2\right)\sqrt{\frac{e\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{d+ex}}\sqrt{-\frac{-ex+\frac{i\sqrt{a}e}{\sqrt{c}}}{d+ex}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}}{\sqrt{d+ex}}\right),\frac{\sqrt{c}d-i\cdots}{\sqrt{c}d+i\cdots}\right)}{\sqrt{d+ex}}\cdots\right)}{\cdots}\right)}{12\sqrt{a+c\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(a + c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d + e*x]*((2*(4*c^2*d^2*x^3 + a^2*e*(-3*d + e*x) + a*c*x*(6*d^2 + d*e
*x + 3*e^2*x^2)))/(a^2*c*(a + c*x^2)) + ((d + e*x)*((-2*e^2*Sqrt[-d - (I*Sq
rt[a]*e)/Sqrt[c]]*(3*a^2*e^2 + 4*c^2*d^2*x^2 + a*c*(4*d^2 + 3*e^2*x^2)))/(d
 + e*x)^2 + ((2*I)*Sqrt[c]*(4*c^(3/2)*d^3 + (4*I)*Sqrt[a]*c*d^2*e + 3*a*Sqr
t[c]*d*e^2 + (3*I)*a^(3/2)*e^3)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x
)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*EllipticE[I*ArcSinh[Sqr
t[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sq
rt[c]*d + I*Sqrt[a]*e)])/Sqrt[d + e*x] + (2*Sqrt[a]*Sqrt[c]*e*(4*c*d^2 + I*
Sqrt[a]*Sqrt[c]*d*e + 3*a*e^2)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)
]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*EllipticF[I*ArcSinh[Sqrt
[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqr
```

```
t[c]*d + I*Sqrt[a]*e)])/Sqrt[d + e*x]))/(a^2*c^2*e*Sqrt[-d - (I*Sqrt[a]*e)/
Sqrt[c]])))/(12*Sqrt[a + c*x^2])
```

Maple [B] time = 0.287, size = 2278, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(c*x^2+a)^(5/2),x)
```

```
[Out] 1/6*(3*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-
c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a^2*c*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e
-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c
)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+7*EllipticE((-(e*x+d)*c/((-a*c)^(1/2
)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a*c
^2*d^2*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/(
(-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2
)+4*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d
)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*c^3*d^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d)
)^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/
2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-3*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c
*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a^2*c*e^
4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1
/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-3*Ellip
ticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)
^(1/2)*e+c*d))^(1/2))*x^2*a*c^2*d^2*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(
1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))
*e/((-a*c)^(1/2)*e-c*d))^(1/2)-4*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d)
)^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a*c*d*e^3*(
-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/
((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/
2)-4*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*
d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*c^2*d^3*e*(-a*c)^(1/2)*(-(e*x+d)*c/((-a
*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*
((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+3*EllipticE((-(e*x+d)*c/(
(-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/
2))*a^3*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/
((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/
2)+7*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*
d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*c*d^2*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c
*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^
(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+4*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*
e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c^2*d^4
*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/
2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-3*Ellipt
icF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^
(1/2)*e+c*d))^(1/2))*a^3*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x
+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^
(1/2)*e-c*d))^(1/2)-3*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-(
(-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*c*d^2*e^2*(-(e*x+d)*c/
((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1
/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-4*EllipticF((-(e*x+d)
*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))
^(1/2))*a^2*d*e^3*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c
*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c
)^(1/2)*e-c*d))^(1/2)-4*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(
```

```
-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c*d^3*e*(-a*c)^(1/2)*(
-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)
*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+3*x^4*a*c^
2*e^4+4*x^4*c^3*d^2*e^2+4*x^3*a*c^2*d*e^3+4*x^3*c^3*d^3*e+x^2*a^2*c*e^4+7*x
^2*a*c^2*d^2*e^2-2*x*a^2*c*d*e^3+6*x*a*c^2*d^3*e-3*a^2*c*d^2*e^2)/c^2/(e*x+
d)^(1/2)/a^2/e/(c*x^2+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{(cx^2+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/(c*x^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(e^2x^2+2\,dex+d^2)\sqrt{cx^2+a}\sqrt{ex+d}}{c^3x^6+3\,ac^2x^4+3\,a^2cx^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*sqrt(c*x^2 + a)*sqrt(e*x + d)/(c^3*x^6 +
 3*a*c^2*x^4 + 3*a^2*c*x^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(c*x**2+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{(cx^2+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(5/2)/(c*x^2 + a)^(5/2), x)
```

3.695 $\int \frac{(d+ex)^{3/2}}{\left(a+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=368

$$-\frac{\sqrt{\frac{cx^2}{a}+1}\left(ae^2+4cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{6(-a)^{3/2}c^{3/2}\sqrt{a+cx^2}\sqrt{d+ex}}+\frac{\sqrt{d+ex}(ae+4cdx)}{6a^2c\sqrt{a+cx^2}}-\frac{\sqrt{d+ex}(ae-cd}{3ac\left(a+cx^2\right)^{3/}}$$

```
[Out] -((a*e - c*d*x)*Sqrt[d + e*x])/(3*a*c*(a + c*x^2)^(3/2)) + ((a*e + 4*c*d*x)
*Sqrt[d + e*x])/(6*a^2*c*Sqrt[a + c*x^2]) + (2*d*Sqrt[d + e*x]*Sqrt[1 + (c*
x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/
(Sqrt[-a]*Sqrt[c]*d - a*e)])/(3*(-a)^(3/2)*Sqrt[c]*Sqrt[(Sqrt[c]*(d + e*x))
/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) - ((4*c*d^2 + a*e^2)*Sqrt[(Sqrt
[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcS
in[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d -
a*e)])/(6*(-a)^(3/2)*c^(3/2)*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.351321, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {739, 823, 844, 719, 424, 419}

$$\frac{\sqrt{d+ex}(ae+4cdx)}{6a^2c\sqrt{a+cx^2}}-\frac{\sqrt{\frac{cx^2}{a}+1}\left(ae^2+4cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{6(-a)^{3/2}c^{3/2}\sqrt{a+cx^2}\sqrt{d+ex}}-\frac{\sqrt{d+ex}(ae-cdx)}{3ac\left(a+cx^2\right)^{3/2}}+\frac{2d\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(a + c*x^2)^(5/2),x]
```



```
[Out] -((a*e - c*d*x)*Sqrt[d + e*x])/(3*a*c*(a + c*x^2)^(3/2)) + ((a*e + 4*c*d*x)
*Sqrt[d + e*x])/(6*a^2*c*Sqrt[a + c*x^2]) + (2*d*Sqrt[d + e*x]*Sqrt[1 + (c*
x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/
(Sqrt[-a]*Sqrt[c]*d - a*e)])/(3*(-a)^(3/2)*Sqrt[c]*Sqrt[(Sqrt[c]*(d + e*x))
/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) - ((4*c*d^2 + a*e^2)*Sqrt[(Sqrt
[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcS
in[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d -
a*e)])/(6*(-a)^(3/2)*c^(3/2)*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```


Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
```

```
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\left(a+cx^2\right)^{5/2}}\,dx = -\frac{(ae-cdx)\sqrt{d+ex}}{3ac\left(a+cx^2\right)^{3/2}} + \frac{\int \frac{\frac{1}{2}\left(4cd^2+ae^2\right)+\frac{3}{2}cdex}{\sqrt{d+ex}\left(a+cx^2\right)^{3/2}}\,dx}{3ac}$$

$$= -\frac{(ae-cdx)\sqrt{d+ex}}{3ac\left(a+cx^2\right)^{3/2}} + \frac{(ae+4cdx)\sqrt{d+ex}}{6a^2c\sqrt{a+cx^2}} - \frac{\int \frac{-\frac{1}{4}ace^2\left(cd^2+ae^2\right)+c^2de\left(cd^2+ae^2\right)x}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx}{3a^2c^2\left(cd^2+ae^2\right)}$$

$$= -\frac{(ae-cdx)\sqrt{d+ex}}{3ac\left(a+cx^2\right)^{3/2}} + \frac{(ae+4cdx)\sqrt{d+ex}}{6a^2c\sqrt{a+cx^2}} - \frac{d\int \frac{\sqrt{d+ex}}{\sqrt{a+cx^2}}\,dx}{3a^2} + \frac{\left(4cd^2+ae^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx}{12a^2c}$$

$$= -\frac{(ae-cdx)\sqrt{d+ex}}{3ac\left(a+cx^2\right)^{3/2}} + \frac{(ae+4cdx)\sqrt{d+ex}}{6a^2c\sqrt{a+cx^2}} - \frac{\left(2d\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}ex^2}{\sqrt{-a}\left(cd-\frac{a\sqrt{c}e}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx, \right.}{3\sqrt{-a}a\sqrt{c}\sqrt{\frac{c(d+ex)}{cd-\frac{a\sqrt{c}e}{\sqrt{-a}}}}\sqrt{a+cx^2}}$$

$$= -\frac{(ae-cdx)\sqrt{d+ex}}{3ac\left(a+cx^2\right)^{3/2}} + \frac{(ae+4cdx)\sqrt{d+ex}}{6a^2c\sqrt{a+cx^2}} + \frac{2d\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-a\ldots}\right.}{3(-a)^{3/2}\sqrt{c}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}}\sqrt{a+cx^2}}$$

Mathematica [C] time = 3.49469, size = 504, normalized size = 1.37

$$\frac{\sqrt{d+ex}\left(\frac{-2a^2e+2acx(6d+ex)+8c^2dx^3}{a^2c\left(a+cx^2\right)} + \frac{(d+ex)\left(\frac{2\sqrt{a}e\left(4\sqrt{c}d+i\sqrt{a}e\right)\sqrt{\frac{e\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{d+ex}}\sqrt{-\frac{-ex+\frac{i\sqrt{a}e}{\sqrt{c}}}{d+ex}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}}{\sqrt{d+ex}}\right),\frac{\sqrt{c}d-i\sqrt{a}e}{\sqrt{c}d+i\sqrt{a}e}\right)}{\sqrt{d+ex}} - \frac{8de^2\left(a+cx^2\right)\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}}{(d+ex)^2} + \ldots\right)}{a^2ce\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}}\right)}{12\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(a + c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d + e*x]*((-2*a^2*e + 8*c^2*d*x^3 + 2*a*c*x*(6*d + e*x))/(a^2*c*(a +
c*x^2)) + ((d + e*x)*((-8*d*e^2*Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*(a + c*x^2
))/(d + e*x)^2 + ((8*I)*Sqrt[c]*d*(Sqrt[c]*d + I*Sqrt[a]*e)*Sqrt[(e*((I*Sqr
t[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*
x))]*EllipticE[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (
Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)])/Sqrt[d + e*x] + (2*Sqr
t[a]*e*(4*Sqrt[c]*d + I*Sqrt[a]*e)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d +
e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*EllipticF[I*ArcSinh[
Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/
(Sqrt[c]*d + I*Sqrt[a]*e)])/Sqrt[d + e*x]))/(a^2*c*e*Sqrt[-d - (I*Sqrt[a]*e
)/Sqrt[c]])))/(12*Sqrt[a + c*x^2])
```

Maple [B] time = 0.278, size = 1633, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*x^2+a)^(5/2),x)
```

```
[Out] -1/6*(3*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e
-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a*c^2*d*e^2*(-(e*x+d)*c/((-a*c)^(1/2
)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-
a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+EllipticF((-(e*x+d)*c/((-a*c)^(1/
2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a*
c*e^3*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1
/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*
d))^(1/2)+4*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/
2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*c^2*d^2*e*(-a*c)^(1/2)*(-(e*x+d)
*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))
^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-4*EllipticE((-(e*x
+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*
d))^(1/2))*x^2*a*c^2*d*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(
-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1
/2)*e-c*d))^(1/2)-4*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-
a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*c^3*d^3*(-(e*x+d)*c/((-a
*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*
((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+3*EllipticF((-(e*x+d)*c/(
(-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/
2))*a^2*c*d*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2)
)*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))
^(1/2)+EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-
c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*e^3*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^
(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*
x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+4*EllipticF((-(e*x+d)*c/((-a*
c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*
a*c*d^2*e*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c
)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*
e-c*d))^(1/2)-4*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)
^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*c*d*e^2*(-(e*x+d)*c/((-a*c)^
(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*
x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-4*EllipticE((-(e*x+d)*c/((-a*
c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*
a*c^2*d^3*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((
-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)
-4*x^4*c^3*d*e^2-x^3*a*c^2*e^3-4*x^3*c^3*d^2*e-7*x^2*a*c^2*d*e^2+x*a^2*c*e^
3-6*x*a*c^2*d^2*e+a^2*c*d*e^2)/(e*x+d)^(1/2)/a^2/e/c^2/(c*x^2+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(c*x^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+a}(ex+d)^{\frac{3}{2}}}{c^3x^6+3\,ac^2x^4+3\,a^2cx^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*(e*x + d)^(3/2)/(c^3*x^6 + 3*a*c^2*x^4 + 3*a^2*c*x
^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ex+d)^{\frac{3}{2}}}{(cx^2+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)/(c*x^2 + a)^(5/2), x)
```

3.696 $\int \frac{\sqrt{d+ex}}{\left(a+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=392

$$-\frac{2d\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{3(-a)^{3/2}\sqrt{c}\sqrt{a+cx^2}\sqrt{d+ex}}+\frac{\sqrt{d+ex}\left(x\left(3ae^2+4cd^2\right)+ade\right)}{6a^2\sqrt{a+cx^2}\left(ae^2+cd^2\right)}+\frac{\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}}{6(-a)^{3/}}$$

```
[Out] (x*Sqrt[d + e*x])/(3*a*(a + c*x^2)^(3/2)) + (Sqrt[d + e*x]*(a*d*e + (4*c*d^
2 + 3*a*e^2)*x))/(6*a^2*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) + ((4*c*d^2 + 3*a*
e^2)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x
)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(6*(-a)^(3/2)*S
qrt[c]*(c*d^2 + a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*S
qrt[a + c*x^2]) - (2*d*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*S
qrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]]
, (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(3*(-a)^(3/2)*Sqrt[c]*Sqrt[d + e*x]
*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.322009, antiderivative size = 392, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {737, 823, 844, 719, 424, 419}

$$\frac{\sqrt{d+ex}\left(x\left(3ae^2+4cd^2\right)+ade\right)}{6a^2\sqrt{a+cx^2}\left(ae^2+cd^2\right)}+\frac{\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}\left(3ae^2+4cd^2\right)E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{6(-a)^{3/2}\sqrt{c}\sqrt{a+cx^2}\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}}+\frac{x\sqrt{d+ex}}{3a\left(a+cx^2\right)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(a + c*x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[d + e*x])/(3*a*(a + c*x^2)^(3/2)) + (Sqrt[d + e*x]*(a*d*e + (4*c*d^
2 + 3*a*e^2)*x))/(6*a^2*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) + ((4*c*d^2 + 3*a*
e^2)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x
)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(6*(-a)^(3/2)*S
qrt[c]*(c*d^2 + a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*S
qrt[a + c*x^2]) - (2*d*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*S
qrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]]
, (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(3*(-a)^(3/2)*Sqrt[c]*Sqrt[d + e*x]
*Sqrt[a + c*x^2])
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(x*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*(p + 1)), x] + Dist[1/(2*a*(p + 1
)), Int[(d + e*x)^(m - 1)*(d*(2*p + 3) + e*(m + 2*p + 3)*x)*(a + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -
1] && GtQ[m, 0] && (LtQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && In
tQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\left(a+cx^2\right)^{5/2}}\,dx = \frac{x\sqrt{d+ex}}{3a\left(a+cx^2\right)^{3/2}} - \frac{\int \frac{-2d-\frac{3ex}{2}}{\sqrt{d+ex}\left(a+cx^2\right)^{3/2}}\,dx}{3a}$$

$$= \frac{x\sqrt{d+ex}}{3a\left(a+cx^2\right)^{3/2}} + \frac{\sqrt{d+ex}\left(ade+\left(4cd^2+3ae^2\right)x\right)}{6a^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\int \frac{\frac{1}{4}acde^2-\frac{1}{4}ce\left(4cd^2+3ae^2\right)x}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx}{3a^2c\left(cd^2+ae^2\right)}$$

$$= \frac{x\sqrt{d+ex}}{3a\left(a+cx^2\right)^{3/2}} + \frac{\sqrt{d+ex}\left(ade+\left(4cd^2+3ae^2\right)x\right)}{6a^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{d\int \frac{1}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx}{3a^2} - \frac{\left(4cd^2+3ae^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{a+cx^2}}\,dx}{12a^2\left(cd^2+ae^2\right)}$$

$$= \frac{x\sqrt{d+ex}}{3a\left(a+cx^2\right)^{3/2}} + \frac{\sqrt{d+ex}\left(ade+\left(4cd^2+3ae^2\right)x\right)}{6a^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{\left(\left(4cd^2+3ae^2\right)\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \ldots\right.}{6\sqrt{-a}a\sqrt{c}\left(cd^2+ae^2\right)\sqrt{\frac{c(d+ex)}{cd-\frac{a\sqrt{c}\ldots}{\sqrt{-\ldots}}}}\ldots}$$

$$= \frac{x\sqrt{d+ex}}{3a\left(a+cx^2\right)^{3/2}} + \frac{\sqrt{d+ex}\left(ade+\left(4cd^2+3ae^2\right)x\right)}{6a^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\left(4cd^2+3ae^2\right)\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\ldots}}{\sqrt{2}}\ldots\right.\right.}{6(-a)^{3/2}\sqrt{c}\left(cd^2+ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}}\ldots}$$

Mathematica [C] time = 2.11673, size = 619, normalized size = 1.58

$$\frac{\sqrt{d+ex}\left(\frac{2\left(a^2e(d+5ex)+acx\left(6d^2+dex+3e^2x^2\right)+4c^2d^2x^3\right)}{a^2\left(a+cx^2\right)\left(ae^2+cd^2\right)} + \frac{(d+ex)\left(\frac{2\sqrt{a}\sqrt{c}e\left(i\sqrt{a}\sqrt{c}de+3ae^2+4cd^2\right)\sqrt{\frac{e\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{d+ex}}\sqrt{-\frac{-ex+\frac{i\sqrt{a}e}{\sqrt{c}}}{d+ex}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}}{\sqrt{d+ex}}\right),\frac{\sqrt{c}d-i\sqrt{a}e}{\sqrt{c}d+i\sqrt{a}e}\right)}{\sqrt{d+ex}}\ldots\right.}{\ldots}\right)}{12\sqrt{a+cx^2}\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(a + c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d + e*x]*((2*(4*c^2*d^2*x^3 + a^2*e*(d + 5*e*x) + a*c*x*(6*d^2 + d*e*
x + 3*e^2*x^2)))/(a^2*(c*d^2 + a*e^2)*(a + c*x^2)) + ((d + e*x)*((-2*e^2*Sq
rt[-d - (I*Sqrt[a]*e)/Sqrt[c]]*(3*a^2*e^2 + 4*c^2*d^2*x^2 + a*c*(4*d^2 + 3*
e^2*x^2)))/(d + e*x)^2 + ((2*I)*Sqrt[c]*(4*c^(3/2)*d^3 + (4*I)*Sqrt[a]*c*d^
2*e + 3*a*Sqrt[c]*d*e^2 + (3*I)*a^(3/2)*e^3)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] +
 x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*EllipticE[
I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*S
qrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)])/Sqrt[d + e*x] + (2*Sqrt[a]*Sqrt[c]*e*
(4*c*d^2 + I*Sqrt[a]*Sqrt[c]*d*e + 3*a*e^2)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] +
x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*EllipticF[I
*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sq
```

```
rt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)])/Sqrt[d + e*x]))/(a^2*c*e*Sqrt[-d - (I*
Sqrt[a]*e)/Sqrt[c]]*(c*d^2 + a*e^2))))/(12*Sqrt[a + c*x^2])
```

Maple [B] time = 0.277, size = 2292, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*x^2+a)^(5/2),x)
```

```
[Out] -1/6*(3*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e
-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a^2*c*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*
e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*
c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+3*EllipticF((-(e*x+d)*c/((-a*c)^(1/
2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a*
c^2*d^2*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/
((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/
2)+4*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*
d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a*c*d*e^3*(-a*c)^(1/2)*(-(e*x+d)*c/((-a
*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*
((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+4*EllipticF((-(e*x+d)*c/(
(-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/
2))*x^2*c^2*d^3*e*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c
*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c
)^(1/2)*e-c*d))^(1/2)-3*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(
-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a^2*c*e^4*(-(e*x+d)*
c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^
(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-7*EllipticE((-(e*x+
d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d
))^(1/2))*x^2*a*c^2*d^2*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+
(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(
1/2)*e-c*d))^(1/2)-4*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((
-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*c^3*d^4*(-(e*x+d)*c/((-
a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)
*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+3*EllipticF((-(e*x+d)*c/
((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1
/2))*a^3*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e
/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1
/2)+3*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c
*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*c*d^2*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-
c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)
^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+4*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)
*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*d*e^
3*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))
*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^
(1/2)+4*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e
-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c*d^3*e*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*
c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*(
(c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-3*EllipticE((-(e*x+d)*c/((
-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2
))*a^3*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/(
(-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2
)-7*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d
)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*c*d^2*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*
d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(
1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-4*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e
```

```
-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c^2*d^4*
(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2
)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-3*x^4*a*c
^2*e^4-4*x^4*c^3*d^2*e^2-4*x^3*a*c^2*d*e^3-4*x^3*c^3*d^3*e-5*x^2*a^2*c*e^4-
7*x^2*a*c^2*d^2*e^2-6*x*a^2*c*d*e^3-6*x*a*c^2*d^3*e-a^2*c*d^2*e^2)/c/(e*x+d
)^(1/2)/(a*e^2+c*d^2)/a^2/e/(c*x^2+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\left(cx^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(c*x^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{cx^2+a}\sqrt{ex+d}}{c^3x^6+3\,ac^2x^4+3\,a^2cx^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*sqrt(e*x + d)/(c^3*x^6 + 3*a*c^2*x^4 + 3*a^2*c*x^2
 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\left(cx^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)/(c*x^2 + a)^(5/2), x)
```

3.697 $\int \frac{1}{\sqrt{d+ex}\left(a+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=450

$$-\frac{\sqrt{\frac{cx^2}{a}+1}\left(5ae^2+4cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{6(-a)^{3/2}\sqrt{c}\sqrt{a+cx^2}\sqrt{d+ex}\left(ae^2+cd^2\right)}+\frac{\sqrt{d+ex}\left(4cdx\left(2ae^2+cd^2\right)+ae\left(5ae^2+\right.\right.}{6a^2\sqrt{a+cx^2}\left(ae^2+cd^2\right)^2}$$

```
[Out] ((a*e + c*d*x)*Sqrt[d + e*x])/(3*a*(c*d^2 + a*e^2)*(a + c*x^2)^(3/2)) + (Sq
rt[d + e*x]*(a*e*(c*d^2 + 5*a*e^2) + 4*c*d*(c*d^2 + 2*a*e^2)*x))/(6*a^2*(c*
d^2 + a*e^2)^2*Sqrt[a + c*x^2]) + (2*Sqrt[c]*d*(c*d^2 + 2*a*e^2)*Sqrt[d + e
*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqr
t[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(3*(-a)^(3/2)*(c*d^2 + a*e^2)^
2*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) - ((4
*c*d^2 + 5*a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1
 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2
*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(6*(-a)^(3/2)*Sqrt[c]*(c*d^2 + a*e^2)*Sq
rt[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.384817, antiderivative size = 450, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {741, 823, 844, 719, 424, 419}

$$\frac{\sqrt{d+ex}\left(4cdx\left(2ae^2+cd^2\right)+ae\left(5ae^2+cd^2\right)\right)}{6a^2\sqrt{a+cx^2}\left(ae^2+cd^2\right)^2}+\frac{\sqrt{d+ex}(ae+cdx)}{3a\left(a+cx^2\right)^{3/2}\left(ae^2+cd^2\right)}-\frac{\sqrt{\frac{cx^2}{a}+1}\left(5ae^2+4cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}F\left(\sin^{-}\right.}{6(-a)^{3/2}\sqrt{c}\sqrt{a+cx^2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(a + c*x^2)^(5/2)),x]
```


```
[Out] ((a*e + c*d*x)*Sqrt[d + e*x])/(3*a*(c*d^2 + a*e^2)*(a + c*x^2)^(3/2)) + (Sq
rt[d + e*x]*(a*e*(c*d^2 + 5*a*e^2) + 4*c*d*(c*d^2 + 2*a*e^2)*x))/(6*a^2*(c*
d^2 + a*e^2)^2*Sqrt[a + c*x^2]) + (2*Sqrt[c]*d*(c*d^2 + 2*a*e^2)*Sqrt[d + e
*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqr
t[2]], (-2*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(3*(-a)^(3/2)*(c*d^2 + a*e^2)^
2*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) - ((4
*c*d^2 + 5*a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1
 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2
*a*e)/(Sqrt[-a]*Sqrt[c]*d - a*e)])/(6*(-a)^(3/2)*Sqrt[c]*(c*d^2 + a*e^2)*Sq
rt[d + e*x]*Sqrt[a + c*x^2])
```


Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\left(a+cx^2\right)^{5/2}}\,dx = \frac{(ae+cdx)\sqrt{d+ex}}{3a\left(cd^2+ae^2\right)\left(a+cx^2\right)^{3/2}} - \frac{\int \frac{\frac{1}{2}\left(-4cd^2-5ae^2\right)-\frac{3}{2}cdex}{\sqrt{d+ex}\left(a+cx^2\right)^{3/2}}dx}{3a\left(cd^2+ae^2\right)}$$

$$= \frac{(ae+cdx)\sqrt{d+ex}}{3a\left(cd^2+ae^2\right)\left(a+cx^2\right)^{3/2}} + \frac{\sqrt{d+ex}\left(ae\left(cd^2+5ae^2\right)+4cd\left(cd^2+2ae^2\right)x\right)}{6a^2\left(cd^2+ae^2\right)^2\sqrt{a+cx^2}} + \frac{\int \frac{\frac{1}{4}ace^2\left(cd^2+5\ldots\right.}{\sqrt{\ldots}}}{3a^2c\ldots}$$

$$= \frac{(ae+cdx)\sqrt{d+ex}}{3a\left(cd^2+ae^2\right)\left(a+cx^2\right)^{3/2}} + \frac{\sqrt{d+ex}\left(ae\left(cd^2+5ae^2\right)+4cd\left(cd^2+2ae^2\right)x\right)}{6a^2\left(cd^2+ae^2\right)^2\sqrt{a+cx^2}} - \frac{\left(cd\left(cd^2+2\ldots\right.\right.}{3a^2\left(\ldots\right.}$$

$$= \frac{(ae+cdx)\sqrt{d+ex}}{3a\left(cd^2+ae^2\right)\left(a+cx^2\right)^{3/2}} + \frac{\sqrt{d+ex}\left(ae\left(cd^2+5ae^2\right)+4cd\left(cd^2+2ae^2\right)x\right)}{6a^2\left(cd^2+ae^2\right)^2\sqrt{a+cx^2}} - \frac{\left(2\sqrt{c}d\left(cd^2\ldots\right.\right.}{\ldots}$$

$$= \frac{(ae+cdx)\sqrt{d+ex}}{3a\left(cd^2+ae^2\right)\left(a+cx^2\right)^{3/2}} + \frac{\sqrt{d+ex}\left(ae\left(cd^2+5ae^2\right)+4cd\left(cd^2+2ae^2\right)x\right)}{6a^2\left(cd^2+ae^2\right)^2\sqrt{a+cx^2}} + \frac{2\sqrt{c}d\left(cd^2+\ldots\right.}{\ldots}$$

Mathematica [C] time = 2.0383, size = 570, normalized size = 1.27

$$\frac{\sqrt{d+ex}\left(\frac{\sqrt{a}\sqrt{d+ex}\left(5ia^{3/2}e^3+i\sqrt{a}cd^2e+8a\sqrt{c}de^2+4c^{3/2}d^3\right)\sqrt{\frac{e\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{d+ex}}\sqrt{-\frac{-ex+\frac{i\sqrt{a}e}{\sqrt{c}}}{d+ex}}\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}}{\sqrt{d+ex}}\right),\frac{\sqrt{c}d-i\sqrt{a}e}{\sqrt{c}d+i\sqrt{a}e}\right)}{\sqrt{-d-\frac{i\sqrt{a}e}{\sqrt{c}}}} - \frac{4de\left(2a^2e^2+ac\left(d^2+2e^2x^2\right)+c^2\ldots\right.}{d+ex}\ldots\right.}{6a^2\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(a + c*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d + e*x]*(a*c*d^2*e + 5*a^2*e^3 + 4*c^2*d^3*x + 8*a*c*d*e^2*x + (2*a*
(c*d^2 + a*e^2)*(a*e + c*d*x))/(a + c*x^2) - (4*d*e*(2*a^2*e^2 + c^2*d^2*x^
2 + a*c*(d^2 + 2*e^2*x^2)))/(d + e*x) - ((4*I)*c*d*Sqrt[-d - (I*Sqrt[a]*e)/
Sqrt[c]]*(c*d^2 + 2*a*e^2)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))/(d + e*x)]*Sq
rt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*Sqrt[d + e*x]*EllipticE[I*Ar
cSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[
a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)])/e + (Sqrt[a]*(4*c^(3/2)*d^3 + I*Sqrt[a]*c
*d^2*e + 8*a*Sqrt[c]*d*e^2 + (5*I)*a^(3/2)*e^3)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c
] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*Sqrt[d
+ e*x]*EllipticF[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]],
 (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)])/Sqrt[-d - (I*Sqrt[a]
*e)/Sqrt[c]]))/(6*a^2*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^2])
```

Maple [B] time = 0.299, size = 2673, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a)^(5/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/6*(3*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e
-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^3*c*d*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-
c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)
^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-9*x^3*a*c^3*d^2*e^3-13*x*a^2*c^2*d^2*
e^3-6*x*a*c^3*d^4*e-7*x^2*a*c^3*d^3*e^2-15*x^2*a^2*c^2*d*e^4-8*x^4*a*c^3*d*
e^4+9*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c
*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a*c^2*d^2*e^3*(-a*c)^(1/2)*(-(e*x+d)*c
/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(
1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-12*EllipticE((-(e*x+
d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d
))^(1/2))*a^2*c^2*d^3*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-
a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/
2)*e-c*d))^(1/2)-4*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a
*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*c^4*d^5*(-(e*x+d)*c/((-a*
c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*(
(c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+5*EllipticF((-(e*x+d)*c/((
-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2
))*a^3*e^5*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*
c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)
*e-c*d))^(1/2)-4*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c
)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c^3*d^5*(-(e*x+d)*c/((-a*c)^(
1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x
+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-4*x^4*c^4*d^3*e^2-5*x^3*a^2*c^
2*e^5-4*x^3*c^4*d^4*e-7*x*a^3*c*e^5-7*a^3*c*d*e^4-3*a^2*c^2*d^3*e^2+4*Ellip
ticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)
^(1/2)*e+c*d))^(1/2))*x^2*c^3*d^4*e*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*
e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*
c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-8*EllipticE((-(e*x+d)*c/((-a*c)^(1/
2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a^
2*c^2*d*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/
((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/
2)-12*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c
*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a*c^3*d^3*e^2*(-(e*x+d)*c/((-a*c)^(1/2
)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-
a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+9*EllipticF((-(e*x+d)*c/((-a*c)^(
1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*
c*d^2*e^3*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c
)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*
e-c*d))^(1/2)+4*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)
^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*c^2*d^4*e*(-a*c)^(1/2)*(-(e*x+
d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d
))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+3*EllipticF((-(e
*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+
c*d))^(1/2))*x^2*a^2*c^2*d*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c
*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c
)^(1/2)*e-c*d))^(1/2)+5*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(
-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a^2*c*e^5*(-a*c)^(1/
2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(
1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+3*Elli
pticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c
)^(1/2)*e+c*d))^(1/2))*x^2*a*c^3*d^3*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^
(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2)
)*e/((-a*c)^(1/2)*e-c*d))^(1/2)+3*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d
))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*c^2*d^3*e^
2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1
```

```
/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-8*Ellip
ticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)
^(1/2)*e+c*d))^(1/2))*a^3*c*d*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*(
(-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-
a*c)^(1/2)*e-c*d))^(1/2))/(e*x+d)^(1/2)/(a*e^2+c*d^2)^2/a^2/(c*x^2+a)^(3/2)
/c/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+a\right)^{\frac{5}{2}}\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(5/2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)^(5/2)*sqrt(e*x + d)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+a}\sqrt{ex+d}}{c^3ex^7+c^3dx^6+3\,ac^2ex^5+3\,ac^2dx^4+3\,a^2cex^3+3\,a^2cdx^2+a^3ex+a^3d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(5/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*sqrt(e*x + d)/(c^3*e*x^7 + c^3*d*x^6 + 3*a*c^2*e*x
^5 + 3*a*c^2*d*x^4 + 3*a^2*c*e*x^3 + 3*a^2*c*d*x^2 + a^3*e*x + a^3*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+cx^2\right)^{\frac{5}{2}}\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a)**(5/2)/(e*x+d)**(1/2),x)
```

```
[Out] Integral(1/((a + c*x**2)**(5/2)*sqrt(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+a\right)^{\frac{5}{2}}\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(5/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^2 + a)^(5/2)*sqrt(e*x + d)), x)
```

3.698 $\int \frac{1}{(d+ex)^{3/2}\left(a+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=532

$$-\frac{2\sqrt{c}d\sqrt{\frac{cx^2}{a}+1}\left(3ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ae}{\sqrt{-a}\sqrt{c}d-ae}\right)}{3(-a)^{3/2}\sqrt{a+cx^2}\sqrt{d+ex}\left(ae^2+cd^2\right)^2}+\frac{e\sqrt{a+cx^2}\left(-21a^2e^4+15acd^2e^2+4c\right.}{6a^2\sqrt{d+ex}\left(ae^2+cd^2\right)^3}$$

```
[Out] (a*e + c*d*x)/(3*a*(c*d^2 + a*e^2)*Sqrt[d + e*x]*(a + c*x^2)^(3/2)) - (a*e*
(c*d^2 - 7*a*e^2) - 4*c*d*(c*d^2 + 3*a*e^2)*x)/(6*a^2*(c*d^2 + a*e^2)^2*Sqr
t[d + e*x]*Sqrt[a + c*x^2]) + (e*(4*c^2*d^4 + 15*a*c*d^2*e^2 - 21*a^2*e^4)*
Sqrt[a + c*x^2])/(6*a^2*(c*d^2 + a*e^2)^3*Sqrt[d + e*x]) + (Sqrt[c]*(4*c^2*
d^4 + 15*a*c*d^2*e^2 - 21*a^2*e^4)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*Ellipt
icE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt
[c]*d - a*e)])/(6*(-a)^(3/2)*(c*d^2 + a*e^2)^3*Sqrt[(Sqrt[c]*(d + e*x))/(Sq
rt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) - (2*Sqrt[c]*d*(c*d^2 + 3*a*e^2)*Sq
rt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*Ellipt
icF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt
[c]*d - a*e)])/(3*(-a)^(3/2)*(c*d^2 + a*e^2)^2*Sqrt[d + e*x]*Sqrt[a + c*x^2
])
```

Rubi [A] time = 0.530986, antiderivative size = 532, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {741, 823, 835, 844, 719, 424, 419}

$$\frac{e\sqrt{a+cx^2}\left(-21a^2e^4+15acd^2e^2+4c^2d^4\right)}{6a^2\sqrt{d+ex}\left(ae^2+cd^2\right)^3}+\frac{\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\sqrt{d+ex}\left(-21a^2e^4+15acd^2e^2+4c^2d^4\right)E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2}{\sqrt{-a}}\right.}{6(-a)^{3/2}\sqrt{a+cx^2}\left(ae^2+cd^2\right)^3\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{-a}e+\sqrt{c}d}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*(a + c*x^2)^(5/2)),x]
```


```
[Out] (a*e + c*d*x)/(3*a*(c*d^2 + a*e^2)*Sqrt[d + e*x]*(a + c*x^2)^(3/2)) - (a*e*
(c*d^2 - 7*a*e^2) - 4*c*d*(c*d^2 + 3*a*e^2)*x)/(6*a^2*(c*d^2 + a*e^2)^2*Sqr
t[d + e*x]*Sqrt[a + c*x^2]) + (e*(4*c^2*d^4 + 15*a*c*d^2*e^2 - 21*a^2*e^4)*
Sqrt[a + c*x^2])/(6*a^2*(c*d^2 + a*e^2)^3*Sqrt[d + e*x]) + (Sqrt[c]*(4*c^2*
d^4 + 15*a*c*d^2*e^2 - 21*a^2*e^4)*Sqrt[d + e*x]*Sqrt[1 + (c*x^2)/a]*Ellipt
icE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt
[c]*d - a*e)])/(6*(-a)^(3/2)*(c*d^2 + a*e^2)^3*Sqrt[(Sqrt[c]*(d + e*x))/(Sq
rt[c]*d + Sqrt[-a]*e)]*Sqrt[a + c*x^2]) - (2*Sqrt[c]*d*(c*d^2 + 3*a*e^2)*Sq
rt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]*Sqrt[1 + (c*x^2)/a]*Ellipt
icF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*e)/(Sqrt[-a]*Sqrt
[c]*d - a*e)])/(3*(-a)^(3/2)*(c*d^2 + a*e^2)^2*Sqrt[d + e*x]*Sqrt[a + c*x^2
])
```


Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
```

```
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\left(a+cx^2\right)^{5/2}}\,dx = \frac{ae+cdx}{3a\left(cd^2+ae^2\right)\sqrt{d+ex}\left(a+cx^2\right)^{3/2}} - \frac{\int \frac{\frac{1}{2}\left(-4cd^2-7ae^2\right)-\frac{5}{2}cdex}{(d+ex)^{3/2}\left(a+cx^2\right)^{3/2}}\,dx}{3a\left(cd^2+ae^2\right)}$$

$$= \frac{ae+cdx}{3a\left(cd^2+ae^2\right)\sqrt{d+ex}\left(a+cx^2\right)^{3/2}} - \frac{ae\left(cd^2-7ae^2\right)-4cd\left(cd^2+3ae^2\right)x}{6a^2\left(cd^2+ae^2\right)^2\sqrt{d+ex}\sqrt{a+cx^2}} + \frac{\int \frac{-\frac{3}{4}ace^2\left(cd^2-\right.}{(d+}}{3a^2c}$$

$$= \frac{ae+cdx}{3a\left(cd^2+ae^2\right)\sqrt{d+ex}\left(a+cx^2\right)^{3/2}} - \frac{ae\left(cd^2-7ae^2\right)-4cd\left(cd^2+3ae^2\right)x}{6a^2\left(cd^2+ae^2\right)^2\sqrt{d+ex}\sqrt{a+cx^2}} + \frac{e\left(4c^2d^4+15\right.}{6a^2\left(\right.}$$

$$= \frac{ae+cdx}{3a\left(cd^2+ae^2\right)\sqrt{d+ex}\left(a+cx^2\right)^{3/2}} - \frac{ae\left(cd^2-7ae^2\right)-4cd\left(cd^2+3ae^2\right)x}{6a^2\left(cd^2+ae^2\right)^2\sqrt{d+ex}\sqrt{a+cx^2}} + \frac{e\left(4c^2d^4+15\right.}{6a^2\left(\right.}$$

$$= \frac{ae+cdx}{3a\left(cd^2+ae^2\right)\sqrt{d+ex}\left(a+cx^2\right)^{3/2}} - \frac{ae\left(cd^2-7ae^2\right)-4cd\left(cd^2+3ae^2\right)x}{6a^2\left(cd^2+ae^2\right)^2\sqrt{d+ex}\sqrt{a+cx^2}} + \frac{e\left(4c^2d^4+15\right.}{6a^2\left(\right.}$$

$$= \frac{ae+cdx}{3a\left(cd^2+ae^2\right)\sqrt{d+ex}\left(a+cx^2\right)^{3/2}} - \frac{ae\left(cd^2-7ae^2\right)-4cd\left(cd^2+3ae^2\right)x}{6a^2\left(cd^2+ae^2\right)^2\sqrt{d+ex}\sqrt{a+cx^2}} + \frac{e\left(4c^2d^4+15\right.}{6a^2\left(\right.}$$

Mathematica [C] time = 4.00881, size = 669, normalized size = 1.26

$$\frac{\frac{\sqrt{a}\sqrt{c}(d+ex)^{3/2}\left(33ia^{3/2}\sqrt{c}de^3-21a^2e^4+i\sqrt{a}c^{3/2}d^3e+15acd^2e^2+4c^2d^4\right)\sqrt{\frac{e\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{d+ex}}\sqrt{-\frac{-ex+\frac{i\sqrt{ae}}{\sqrt{c}}}{d+ex}}\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-d-\frac{i\sqrt{ae}}{\sqrt{c}}}}{\sqrt{d+ex}}\right),\frac{\sqrt{c}d-i\sqrt{a}e}{\sqrt{c}d+i\sqrt{a}e}\right)}{\sqrt{-d-\frac{i\sqrt{ae}}{\sqrt{c}}}}+c(d+ex)\left(3a^2e^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*(a + c*x^2)^(5/2)),x]
```

```
[Out] (21*a^3*e^5 - 4*c^3*d^4*e*x^2 - 12*a^2*e^5*(a + c*x^2) + 3*a^2*c*e^3*(-5*d^
2 + 7*e^2*x^2) - a*c^2*d^2*e*(4*d^2 + 15*e^2*x^2) + (2*a*c*(c*d^2 + a*e^2)*
(d + e*x)*(c*d^2*x + a*e*(2*d - e*x)))/(a + c*x^2) + c*(d + e*x)*(4*c^2*d^4
*x + 3*a^2*e^3*(7*d - 3*e*x) + a*c*d^2*e*(d + 15*e*x)) - (I*c*Sqrt[-d - (I*
Sqrt[a]*e)/Sqrt[c]]*(4*c^2*d^4 + 15*a*c*d^2*e^2 - 21*a^2*e^4)*Sqrt[(e*((I*S
qrt[a])/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d +
e*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]
/Sqrt[d + e*x]], (Sqrt[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)])/e +
(Sqrt[a]*Sqrt[c]*(4*c^2*d^4 + I*Sqrt[a]*c^(3/2)*d^3*e + 15*a*c*d^2*e^2 + (3
3*I)*a^(3/2)*Sqrt[c]*d*e^3 - 21*a^2*e^4)*Sqrt[(e*((I*Sqrt[a])/Sqrt[c] + x))
/(d + e*x)]*Sqrt[-(((I*Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2
)*EllipticF[I*ArcSinh[Sqrt[-d - (I*Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqr
t[c]*d - I*Sqrt[a]*e)/(Sqrt[c]*d + I*Sqrt[a]*e)])/Sqrt[-d - (I*Sqrt[a]*e)/S
qrt[c]])/(6*a^2*(c*d^2 + a*e^2)^3*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Maple [B] time = 0.324, size = 3322, normalized size = 6.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(c*x^2+a)^(5/2),x)
```

```
[Out] -1/6*(-19*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)
*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*c^2*d^4*e^2*(-(e*x+d)*c/((-a*c)^(1
/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+
(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-21*EllipticF((-(e*x+d)*c/((-a*c
)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x
^2*a^3*c*e^6*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e
/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1
/2)+21*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-
c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a^3*c*e^6*(-(e*x+d)*c/((-a*c)^(1/2)*e
-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c
)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-18*EllipticF((-(e*x+d)*c/((-a*c)^(1/
2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^3*c*
d^2*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a
*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+1
2*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/
((-a*c)^(1/2)*e+c*d))^(1/2))*a^3*d*e^5*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/
2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(
-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-18*EllipticF((-(e*x+d)*c/((-a*c)
^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^
2*a^2*c^2*d^2*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/
2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d
))^(1/2)+3*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2
)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a*c^3*d^4*e^2*(-(e*x+d)*c/((-a*c)
^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c
*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+4*EllipticF((-(e*x+d)*c/((-a
*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))
*x^2*c^3*d^5*e*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+
(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(
1/2)*e-c*d))^(1/2)+6*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((
-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a^2*c^2*d^2*e^4*(-(e*x+
d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d
))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-19*EllipticE((-(
e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e
+c*d))^(1/2))*x^2*a*c^3*d^4*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-
c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*
c)^(1/2)*e-c*d))^(1/2)+16*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)
,(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^2*c*d^3*e^3*(-a*c)^(
1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)
^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+4*El
lipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a
*c)^(1/2)*e+c*d))^(1/2))*a*c^2*d^5*e*(-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)
*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a
*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-25*a^3*c*d^2*e^4-5*a^2*c^2*d^4*e^2
-21*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d
)/((-a*c)^(1/2)*e+c*d))^(1/2))*a^4*e^6*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1
/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*
e/((-a*c)^(1/2)*e-c*d))^(1/2)-15*x^4*a*c^3*d^2*e^4+12*a^4*e^6+3*EllipticF((
-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)
*e+c*d))^(1/2))*a^2*c^2*d^4*e^2*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-
c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*
```

```
c)^(1/2)*e-c*d))^(1/2)-36*x^2*a^2*c^2*d^2*e^4-7*x^2*a*c^3*d^4*e^2-12*x^3*a^
2*c^2*d*e^5-16*x^3*a*c^3*d^3*e^3-14*x*a^3*c*d*e^5-20*x*a^2*c^2*d^3*e^3-6*x*
a*c^3*d^5*e-4*EllipticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(
1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*c^4*d^6*(-(e*x+d)*c/((-a*c)^(1
/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+
(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)-4*EllipticE((-(e*x+d)*c/((-a*c)
^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*a*
c^3*d^6*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a
*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+2
1*x^4*a^2*c^2*e^6-4*x^4*c^4*d^4*e^2+35*x^2*a^3*c*e^6-4*x^3*c^4*d^5*e+6*Elli
pticE((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c
)^(1/2)*e+c*d))^(1/2))*a^3*c*d^2*e^4*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2
)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/
((-a*c)^(1/2)*e-c*d))^(1/2)+12*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^
(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a^2*c*d*e^5*(
-a*c)^(1/2)*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/
((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/
2)+16*EllipticF((-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c
*d)/((-a*c)^(1/2)*e+c*d))^(1/2))*x^2*a*c^2*d^3*e^3*(-a*c)^(1/2)*(-(e*x+d)*c
/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e+c*d))^(
1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d))^(1/2)+21*EllipticE((-(e*x+
d)*c/((-a*c)^(1/2)*e-c*d))^(1/2),(-((-a*c)^(1/2)*e-c*d)/((-a*c)^(1/2)*e+c*d
))^(1/2))*a^4*e^6*(-(e*x+d)*c/((-a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(-a*c)^(1/
2))*e/((-a*c)^(1/2)*e+c*d))^(1/2)*((c*x+(-a*c)^(1/2))*e/((-a*c)^(1/2)*e-c*d
))^(1/2))/(e*x+d)^(1/2)/(a*e^2+c*d^2)^3/a^2/(c*x^2+a)^(3/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+a\right)^{\frac{5}{2}}(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)^(5/2)*(e*x + d)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+a}\sqrt{ex+d}}{c^3e^2x^8+2\,c^3dex^7+6\,ac^2dex^5+6\,a^2cdex^3+\left(c^3d^2+3\,ac^2e^2\right)x^6+2\,a^3dex+a^3d^2+3\left(ac^2d^2+a^2ce^2\right)x^4+\left(3\,a^2c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*sqrt(e*x + d)/(c^3*e^2*x^8 + 2*c^3*d*e*x^7 + 6*a*c
^2*d*e*x^5 + 6*a^2*c*d*e*x^3 + (c^3*d^2 + 3*a*c^2*e^2)*x^6 + 2*a^3*d*e*x +
a^3*d^2 + 3*(a*c^2*d^2 + a^2*c*e^2)*x^4 + (3*a^2*c*d^2 + a^3*e^2)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+cx^2\right)^{\frac{5}{2}}(d+ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(c*x**2+a)**(5/2),x)
```

```
[Out] Integral(1/((a + c*x**2)**(5/2)*(d + e*x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+a\right)^{\frac{5}{2}}(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^2 + a)^(5/2)*(e*x + d)^(3/2)), x)
```

3.699 $\int \frac{1}{(d+ex)\sqrt[3]{d^2+3e^2x^2}}\,dx$

Optimal. Leaf size=151

$$\frac{\log\left(-3\sqrt[3]{2}\sqrt[3]{d}e^2\sqrt[3]{d^2+3e^2x^2}+3de^2-3e^3x\right)}{2\ 2^{2/3}d^{2/3}e}-\frac{\tan^{-1}\left(\frac{2^{2/3}(d-ex)}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{d^2+3e^2x^2}}+\frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}d^{2/3}e}-\frac{\log(d+ex)}{2\ 2^{2/3}d^{2/3}e}$$

```
[Out] -(ArcTan[1/Sqrt[3] + (2^(2/3)*(d - e*x))/(Sqrt[3]*d^(1/3)*(d^2 + 3*e^2*x^2)
^(1/3))]/(2^(2/3)*Sqrt[3]*d^(2/3)*e)) - Log[d + e*x]/(2*2^(2/3)*d^(2/3)*e)
+ Log[3*d*e^2 - 3*e^3*x - 3*2^(1/3)*d^(1/3)*e^2*(d^2 + 3*e^2*x^2)^(1/3)]/(2
*2^(2/3)*d^(2/3)*e)
```

Rubi [A] time = 0.0391441, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {751}

$$\frac{\log\left(-3\sqrt[3]{2}\sqrt[3]{d}e^2\sqrt[3]{d^2+3e^2x^2}+3de^2-3e^3x\right)}{2\ 2^{2/3}d^{2/3}e}-\frac{\tan^{-1}\left(\frac{2^{2/3}(d-ex)}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{d^2+3e^2x^2}}+\frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}d^{2/3}e}-\frac{\log(d+ex)}{2\ 2^{2/3}d^{2/3}e}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(d^2 + 3*e^2*x^2)^(1/3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] + (2^(2/3)*(d - e*x))/(Sqrt[3]*d^(1/3)*(d^2 + 3*e^2*x^2)
^(1/3))]/(2^(2/3)*Sqrt[3]*d^(2/3)*e)) - Log[d + e*x]/(2*2^(2/3)*d^(2/3)*e)
+ Log[3*d*e^2 - 3*e^3*x - 3*2^(1/3)*d^(1/3)*e^2*(d^2 + 3*e^2*x^2)^(1/3)]/(2
*2^(2/3)*d^(2/3)*e)
```

Rule 751

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/3)), x_Symbol] :> With[
{q = Rt[(6*c^2*e^2)/d^2, 3]}, -Simp[(Sqrt[3]*c*e*ArcTan[1/Sqrt[3] + (2*c*(d
 - e*x))/(Sqrt[3]*d*q*(a + c*x^2)^(1/3))])/(d^2*q^2), x] + (-Simp[(3*c*e*Lo
g[d + e*x])/(2*d^2*q^2), x] + Simp[(3*c*e*Log[c*d - c*e*x - d*q*(a + c*x^2)
^(1/3)])/(2*d^2*q^2), x])] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - 3*a*e^2
, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\sqrt[3]{d^2+3e^2x^2}}\,dx=-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2^{2/3}(d-ex)}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{d^2+3e^2x^2}}\right)}{2^{2/3}\sqrt{3}d^{2/3}e}-\frac{\log(d+ex)}{2\ 2^{2/3}d^{2/3}e}+\frac{\log\left(3de^2-3e^3x-3\sqrt[3]{2}\sqrt[3]{d}e^2\sqrt[3]{d^2+3e^2x^2}\right)}{2\ 2^{2/3}d^{2/3}e}$$

Mathematica [C] time = 0.134442, size = 176, normalized size = 1.17

$$-\frac{\sqrt[3]{\frac{e\left(\sqrt{3}\sqrt{-\frac{d^2}{e^2}}+3x\right)}{d+ex}}\sqrt[3]{\frac{e\left(9x-3\sqrt{3}\sqrt{-\frac{d^2}{e^2}}\right)}{d+ex}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{3d-\sqrt{3}\sqrt{-\frac{d^2}{e^2}}e}{3d+3ex},\frac{3d+\sqrt{3}\sqrt{-\frac{d^2}{e^2}}e}{3d+3ex}\right)}{2e\sqrt[3]{d^2+3e^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x)*(d^2 + 3*e^2*x^2)^(1/3)),x]
```

```
[Out] -(((e*(Sqrt[3]*Sqrt[-(d^2/e^2)] + 3*x))/(d + e*x))^(1/3)*((e*(-3*Sqrt[3]*Sq
rt[-(d^2/e^2)] + 9*x))/(d + e*x))^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, (3*d -
 Sqrt[3]*Sqrt[-(d^2/e^2)]*e)/(3*d + 3*e*x), (3*d + Sqrt[3]*Sqrt[-(d^2/e^2)]
*e)/(3*d + 3*e*x)])/(2*e*(d^2 + 3*e^2*x^2)^(1/3))
```

Maple [F] time = 0.542, size = 0, normalized size = 0.

$$\int \frac{1}{ex+d}\frac{1}{\sqrt[3]{3\,e^{2}x^{2}+d^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(3*e^2*x^2+d^2)^(1/3),x)
```

```
[Out] int(1/(e*x+d)/(3*e^2*x^2+d^2)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,e^{2}x^{2}+d^{2}\right)^{\frac{1}{3}}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(3*e^2*x^2+d^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*e^2*x^2 + d^2)^(1/3)*(e*x + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(3*e^2*x^2+d^2)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)\sqrt[3]{d^{2}+3e^{2}x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(3*e**2*x**2+d**2)**(1/3),x)
```

```
[Out] Integral(1/((d + e*x)*(d**2 + 3*e**2*x**2)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,e^2x^2+d^2\right)^{\frac{1}{3}}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(3*e^2*x^2+d^2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((3*e^2*x^2 + d^2)^(1/3)*(e*x + d)), x)
```

3.700 $\int \frac{(2+3x)^3}{\sqrt[3]{4+27x^2}}\,dx$

Optimal. Leaf size=558

$$-\frac{32\, 2^{5/6}\left(2^{2/3}-\sqrt[3]{27x^2+4}\right)\sqrt{\frac{\left(27x^2+4\right)^{2/3}+2^{2/3}\sqrt[3]{27x^2+4}+2\sqrt[3]{2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{27x^2+4}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{27x^2+4}}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{27x^2+4}}\right),4\sqrt{3}-7\right)}{63\sqrt[4]{3}x\sqrt{-\frac{2^{2/3}-\sqrt[3]{27x^2+4}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{27x^2+4}\right)^2}}}+\frac{1}{30}\left(27x^2\right.$$

```
[Out] ((2 + 3*x)^2*(4 + 27*x^2)^(2/3))/30 + (4*(7 + 4*x)*(4 + 27*x^2)^(2/3))/35 -
 (96*x)/(7*(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))) + (16*2^(1/3)*Sqrt
[2 + Sqrt[3]]*(2^(2/3) - (4 + 27*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 +
 27*x^2)^(1/3) + (4 + 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^
(1/3))^2]*EllipticE[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 + 27*x^2)^(1/3))/(2^
(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(21*3^(3/4)*x*
Sqrt[-((2^(2/3) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)
^(1/3))^2)]) - (32*2^(5/6)*(2^(2/3) - (4 + 27*x^2)^(1/3))*Sqrt[(2*2^(1/3) +
 2^(2/3)*(4 + 27*x^2)^(1/3) + (4 + 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3]) -
(4 + 27*x^2)^(1/3))^2]*EllipticF[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 + 27*x^
2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(
63*3^(1/4)*x*Sqrt[-((2^(2/3) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) -
 (4 + 27*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.41015, antiderivative size = 558, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {743, 780, 235, 304, 219, 1879}

$$\frac{1}{30}\left(27x^2+4\right)^{2/3}(3x+2)^2+\frac{4}{35}(4x+7)\left(27x^2+4\right)^{2/3}-\frac{96x}{7\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{27x^2+4}\right)}-\frac{32\, 2^{5/6}\left(2^{2/3}-\sqrt[3]{27x^2+4}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)^3/(4 + 27*x^2)^(1/3),x]
```


```
[Out] ((2 + 3*x)^2*(4 + 27*x^2)^(2/3))/30 + (4*(7 + 4*x)*(4 + 27*x^2)^(2/3))/35 -
 (96*x)/(7*(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))) + (16*2^(1/3)*Sqrt
[2 + Sqrt[3]]*(2^(2/3) - (4 + 27*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 +
 27*x^2)^(1/3) + (4 + 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^
(1/3))^2]*EllipticE[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 + 27*x^2)^(1/3))/(2^
(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(21*3^(3/4)*x*
Sqrt[-((2^(2/3) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)
^(1/3))^2)]) - (32*2^(5/6)*(2^(2/3) - (4 + 27*x^2)^(1/3))*Sqrt[(2*2^(1/3) +
 2^(2/3)*(4 + 27*x^2)^(1/3) + (4 + 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3]) -
(4 + 27*x^2)^(1/3))^2]*EllipticF[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 + 27*x^
2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(
63*3^(1/4)*x*Sqrt[-((2^(2/3) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) -
 (4 + 27*x^2)^(1/3))^2)])
```


Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
```

```
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(2+3x)^3}{\sqrt[3]{4+27x^2}}\,dx = \frac{1}{30}(2+3x)^2\left(4+27x^2\right)^{2/3} + \frac{1}{90}\int \frac{(2+3x)(288+864x)}{\sqrt[3]{4+27x^2}}\,dx$$

$$= \frac{1}{30}(2+3x)^2\left(4+27x^2\right)^{2/3} + \frac{4}{35}(7+4x)\left(4+27x^2\right)^{2/3} + \frac{32}{7}\int \frac{1}{\sqrt[3]{4+27x^2}}\,dx$$

$$= \frac{1}{30}(2+3x)^2\left(4+27x^2\right)^{2/3} + \frac{4}{35}(7+4x)\left(4+27x^2\right)^{2/3} + \frac{\left(16\sqrt{x^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-4+x^3}}\,dx, x, \sqrt[3]{4+27\cdots}\right.}{7\sqrt{3}x}$$

$$= \frac{1}{30}(2+3x)^2\left(4+27x^2\right)^{2/3} + \frac{4}{35}(7+4x)\left(4+27x^2\right)^{2/3} - \frac{\left(16\sqrt{x^2}\right)\text{Subst}\left(\int \frac{2^{2/3}(1+\sqrt{3})-x}{\sqrt{-4+x^3}}\,dx, x, \sqrt[3]{4-\cdots}\right.}{7\sqrt{3}x}$$

$$= \frac{1}{30}(2+3x)^2\left(4+27x^2\right)^{2/3} + \frac{4}{35}(7+4x)\left(4+27x^2\right)^{2/3} - \frac{96x}{7\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}\right)} + \frac{16\sqrt[3]{2}\sqrt{\cdots}}{\cdots}$$

Mathematica [C] time = 0.0332021, size = 53, normalized size = 0.09

$$\frac{16}{7}\sqrt[3]{2}x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{27x^2}{4}\right) + \frac{1}{210}\left(27x^2+4\right)^{2/3}\left(63x^2+180x+196\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)^3/(4 + 27*x^2)^(1/3),x]
```

```
[Out] ((4 + 27*x^2)^(2/3)*(196 + 180*x + 63*x^2))/210 + (16*2^(1/3)*x*Hypergeomet
ric2F1[1/3, 1/2, 3/2, (-27*x^2)/4])/7
```

Maple [C] time = 0.244, size = 40, normalized size = 0.1

$$\frac{63x^2+180x+196}{210}\left(27x^2+4\right)^{\frac{2}{3}} + \frac{16\sqrt[3]{2}x}{7}{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{27x^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^3/(27*x^2+4)^(1/3),x)
```

```
[Out] 1/210*(63*x^2+180*x+196)*(27*x^2+4)^(2/3)+16/7*2^(1/3)*x*hypergeom([1/3,1/2
],[3/2],-27/4*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x+2)^3}{\left(27x^2+4\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^3/(27*x^2+4)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((3*x + 2)^3/(27*x^2 + 4)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{27\,x^{3}+54\,x^{2}+36\,x+8}{\left(27\,x^{2}+4\right)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^3/(27*x^2+4)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((27*x^3 + 54*x^2 + 36*x + 8)/(27*x^2 + 4)^(1/3), x)
```

Sympy [A] time = 4.74001, size = 85, normalized size = 0.15

$$9\sqrt[3]{2}x^{3}{}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{27x^{2}e^{i\pi}}{4}\right)+\frac{3x^{2}\left(27x^{2}+4\right)^{\frac{2}{3}}}{10}+4\sqrt[3]{2}x\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{27x^{2}e^{i\pi}}{4}\right)+\frac{14\left(27x^{2}+4\right)^{\frac{2}{3}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**3/(27*x**2+4)**(1/3),x)
```

```
[Out] 9*2**(1/3)*x**3*hyper((1/3, 3/2), (5/2,), 27*x**2*exp_polar(I*pi)/4) + 3*x*
*2*(27*x**2 + 4)**(2/3)/10 + 4*2**(1/3)*x*hyper((1/3, 1/2), (3/2,), 27*x**2
*exp_polar(I*pi)/4) + 14*(27*x**2 + 4)**(2/3)/15
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3\,x+2)^{3}}{\left(27\,x^{2}+4\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^3/(27*x^2+4)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((3*x + 2)^3/(27*x^2 + 4)^(1/3), x)
```

3.701 $\int \frac{(2+3x)^2}{\sqrt[3]{4+27x^2}}\,dx$

Optimal. Leaf size=551

$$-\frac{8\,2^{5/6}\left(2^{2/3}-\sqrt[3]{27x^2+4}\right)\sqrt{\frac{\left(27x^2+4\right)^{2/3}+2^{2/3}\sqrt[3]{27x^2+4}+2\sqrt[3]{2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{27x^2+4}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{27x^2+4}}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{27x^2+4}}\right),4\sqrt{3}-7\right)}{21\sqrt[4]{3}\sqrt{-\frac{2^{2/3}-\sqrt[3]{27x^2+4}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{27x^2+4}\right)^2}}\,x}-\frac{\ldots}{7\left(2^{2/3}\left(1-\ldots\right.\right.}$$

```
[Out] (5*(4 + 27*x^2)^(2/3))/21 + ((2 + 3*x)*(4 + 27*x^2)^(2/3))/21 - (72*x)/(7*(
2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))) + (4*2^(1/3)*Sqrt[2 + Sqrt[3]]
*(2^(2/3) - (4 + 27*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 + 27*x^2)^(1/3
) + (4 + 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))^2]*Ell
ipticE[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sq
rt[3]) - (4 + 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*3^(3/4)*x*Sqrt[-((2^(2/3
) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))^2)]) -
 (8*2^(5/6)*(2^(2/3) - (4 + 27*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 + 2
7*x^2)^(1/3) + (4 + 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1
/3))^2]*EllipticF[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 + 27*x^2)^(1/3))/(2^(2
/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(21*3^(1/4)*x*Sq
rt[-((2^(2/3) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(
1/3))^2)])
```

Rubi [A] time = 0.312908, antiderivative size = 551, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {743, 641, 235, 304, 219, 1879}

$$-\frac{72x}{7\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{27x^2+4}\right)}+\frac{1}{21}(3x+2)\left(27x^2+4\right)^{2/3}+\frac{5}{21}\left(27x^2+4\right)^{2/3}-\frac{8\,2^{5/6}\left(2^{2/3}-\sqrt[3]{27x^2+4}\right)\sqrt{\frac{\left(27x^2+4\right)^{\ldots}}{\left(2^{2/3}\ldots\right.}}}{21\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)^2/(4 + 27*x^2)^(1/3),x]
```


```
[Out] (5*(4 + 27*x^2)^(2/3))/21 + ((2 + 3*x)*(4 + 27*x^2)^(2/3))/21 - (72*x)/(7*(
2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))) + (4*2^(1/3)*Sqrt[2 + Sqrt[3]]
*(2^(2/3) - (4 + 27*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 + 27*x^2)^(1/3
) + (4 + 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))^2]*Ell
ipticE[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sq
rt[3]) - (4 + 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*3^(3/4)*x*Sqrt[-((2^(2/3
) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))^2)]) -
 (8*2^(5/6)*(2^(2/3) - (4 + 27*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 + 2
7*x^2)^(1/3) + (4 + 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1
/3))^2]*EllipticF[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 + 27*x^2)^(1/3))/(2^(2
/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(21*3^(1/4)*x*Sq
rt[-((2^(2/3) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(
1/3))^2)])
```


Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
```

```
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(2+3x)^2}{\sqrt[3]{4+27x^2}}\,dx = \frac{1}{21}(2+3x)\left(4+27x^2\right)^{2/3}+\frac{1}{63}\int \frac{216+540x}{\sqrt[3]{4+27x^2}}\,dx$$
$$= \frac{5}{21}\left(4+27x^2\right)^{2/3}+\frac{1}{21}(2+3x)\left(4+27x^2\right)^{2/3}+\frac{24}{7}\int \frac{1}{\sqrt[3]{4+27x^2}}\,dx$$
$$= \frac{5}{21}\left(4+27x^2\right)^{2/3}+\frac{1}{21}(2+3x)\left(4+27x^2\right)^{2/3}+\frac{\left(4\sqrt{3}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-4+x^3}}\,dx,x,\sqrt[3]{4+27x^2}\right)}{7x}$$
$$= \frac{5}{21}\left(4+27x^2\right)^{2/3}+\frac{1}{21}(2+3x)\left(4+27x^2\right)^{2/3}-\frac{\left(4\sqrt{3}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{2^{2/3}(1+\sqrt{3})-x}{\sqrt{-4+x^3}}\,dx,x,\sqrt[3]{4+27x^2}\right)}{7x}$$
$$= \frac{5}{21}\left(4+27x^2\right)^{2/3}+\frac{1}{21}(2+3x)\left(4+27x^2\right)^{2/3}-\frac{72x}{7\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}\right)}+\frac{4\sqrt[3]{2}\sqrt{2+\sqrt{3}}\,(2^2}{}$$

Mathematica [C] time = 0.0251864, size = 47, normalized size = 0.09

$$\frac{1}{21}\left(36\sqrt[3]{2}x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{27x^2}{4}\right)+\left(27x^2+4\right)^{2/3}(3x+7)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)^2/(4 + 27*x^2)^(1/3),x]
```

```
[Out] ((7 + 3*x)*(4 + 27*x^2)^(2/3) + 36*2^(1/3)*x*Hypergeometric2F1[1/3, 1/2, 3/
2, (-27*x^2)/4])/21
```

Maple [C] time = 0.218, size = 35, normalized size = 0.1

$$\frac{7+3x}{21}\left(27x^2+4\right)^{\frac{2}{3}}+\frac{12\sqrt[3]{2}x}{7}{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{27x^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^2/(27*x^2+4)^(1/3),x)
```

```
[Out] 1/21*(7+3*x)*(27*x^2+4)^(2/3)+12/7*2^(1/3)*x*hypergeom([1/3,1/2],[3/2],-27/
4*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x+2)^2}{\left(27x^2+4\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2/(27*x^2+4)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((3*x + 2)^2/(27*x^2 + 4)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{9\,x^2+12\,x+4}{\left(27\,x^2+4\right)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2/(27*x^2+4)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((9*x^2 + 12*x + 4)/(27*x^2 + 4)^(1/3), x)
```

Sympy [A] time = 3.433, size = 68, normalized size = 0.12

$$\frac{3\sqrt[3]{2}x^3{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{27x^2e^{i\pi}}{4}\right)}{2}+2\sqrt[3]{2}x{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{27x^2e^{i\pi}}{4}\right)+\frac{\left(27x^2+4\right)^{\frac{2}{3}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**2/(27*x**2+4)**(1/3),x)
```

```
[Out] 3*2**(1/3)*x**3*hyper((1/3, 3/2), (5/2,), 27*x**2*exp_polar(I*pi)/4)/2 + 2*
2**(1/3)*x*hyper((1/3, 1/2), (3/2,), 27*x**2*exp_polar(I*pi)/4) + (27*x**2
+ 4)**(2/3)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3\,x+2)^2}{\left(27\,x^2+4\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2/(27*x^2+4)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((3*x + 2)^2/(27*x^2 + 4)^(1/3), x)
```

3.702 $\int \frac{2+3x}{\sqrt[3]{4+27x^2}}\,dx$

Optimal. Leaf size=529

$$-\frac{2\,2^{5/6}\left(2^{2/3}-\sqrt[3]{27x^2+4}\right)\sqrt{\frac{(27x^2+4)^{2/3}+2^{2/3}\sqrt[3]{27x^2+4}+2\sqrt[3]{2}}{\left(2^{2/3}(1-\sqrt{3})-\sqrt[3]{27x^2+4}\right)^2}}\,\mathrm{EllipticF}\left(\sin^{-1}\left(\frac{2^{2/3}(1+\sqrt{3})-\sqrt[3]{27x^2+4}}{2^{2/3}(1-\sqrt{3})-\sqrt[3]{27x^2+4}}\right),4\sqrt{3}-7\right)}{9\sqrt[4]{3}\sqrt{-\frac{2^{2/3}-\sqrt[3]{27x^2+4}}{\left(2^{2/3}(1-\sqrt{3})-\sqrt[3]{27x^2+4}\right)^2}}\,x}-\frac{\quad}{2^{2/3}\left(1-\sqrt{\right.}\right.}$$

```
[Out] (4 + 27*x^2)^(2/3)/12 - (6*x)/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))
+ (2^(1/3)*Sqrt[2 + Sqrt[3]]*(2^(2/3) - (4 + 27*x^2)^(1/3))*Sqrt[(2*2^(1/3)
 + 2^(2/3)*(4 + 27*x^2)^(1/3) + (4 + 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3])
- (4 + 27*x^2)^(1/3))^2]*EllipticE[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 + 27*
x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])
/(3*3^(3/4)*x*Sqrt[-((2^(2/3) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3])
- (4 + 27*x^2)^(1/3))^2)]) - (2*2^(5/6)*(2^(2/3) - (4 + 27*x^2)^(1/3))*Sqrt
[(2*2^(1/3) + 2^(2/3)*(4 + 27*x^2)^(1/3) + (4 + 27*x^2)^(2/3))/(2^(2/3)*(1
- Sqrt[3]) - (4 + 27*x^2)^(1/3))^2]*EllipticF[ArcSin[(2^(2/3)*(1 + Sqrt[3])
 - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))], -7 +
4*Sqrt[3]])/(9*3^(1/4)*x*Sqrt[-((2^(2/3) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1
- Sqrt[3]) - (4 + 27*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.265839, antiderivative size = 529, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {641, 235, 304, 219, 1879}

$$-\frac{6x}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{27x^2+4}}+\frac{1}{12}\left(27x^2+4\right)^{2/3}-\frac{2\,2^{5/6}\left(2^{2/3}-\sqrt[3]{27x^2+4}\right)\sqrt{\frac{(27x^2+4)^{2/3}+2^{2/3}\sqrt[3]{27x^2+4}+2\sqrt[3]{2}}{\left(2^{2/3}(1-\sqrt{3})-\sqrt[3]{27x^2+4}\right)^2}}\,F\left(\sin^{-1}\left(\frac{2^{2/3}(\cdots}{2^{2/3}(\cdots}\right.\right.}{9\sqrt[4]{3}\sqrt{-\frac{2^{2/3}-\sqrt[3]{27x^2+4}}{\left(2^{2/3}(1-\sqrt{3})-\sqrt[3]{27x^2+4}\right)^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)/(4 + 27*x^2)^(1/3),x]
```


```
[Out] (4 + 27*x^2)^(2/3)/12 - (6*x)/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))
+ (2^(1/3)*Sqrt[2 + Sqrt[3]]*(2^(2/3) - (4 + 27*x^2)^(1/3))*Sqrt[(2*2^(1/3)
 + 2^(2/3)*(4 + 27*x^2)^(1/3) + (4 + 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3])
- (4 + 27*x^2)^(1/3))^2]*EllipticE[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 + 27*
x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])
/(3*3^(3/4)*x*Sqrt[-((2^(2/3) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3])
- (4 + 27*x^2)^(1/3))^2)]) - (2*2^(5/6)*(2^(2/3) - (4 + 27*x^2)^(1/3))*Sqrt
[(2*2^(1/3) + 2^(2/3)*(4 + 27*x^2)^(1/3) + (4 + 27*x^2)^(2/3))/(2^(2/3)*(1
- Sqrt[3]) - (4 + 27*x^2)^(1/3))^2]*EllipticF[ArcSin[(2^(2/3)*(1 + Sqrt[3])
 - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))], -7 +
4*Sqrt[3]])/(9*3^(1/4)*x*Sqrt[-((2^(2/3) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1
- Sqrt[3]) - (4 + 27*x^2)^(1/3))^2)])
```


Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2+3x}{\sqrt[3]{4+27x^2}}\,dx &= \frac{1}{12}\left(4+27x^2\right)^{2/3}+2\int \frac{1}{\sqrt[3]{4+27x^2}}\,dx \\
&= \frac{1}{12}\left(4+27x^2\right)^{2/3}+\frac{\sqrt{x^2}\,\mathrm{Subst}\left(\int \frac{x}{\sqrt{-4+x^3}}\,dx,x,\sqrt[3]{4+27x^2}\right)}{\sqrt{3}x} \\
&= \frac{1}{12}\left(4+27x^2\right)^{2/3}-\frac{\sqrt{x^2}\,\mathrm{Subst}\left(\int \frac{2^{2/3}\left(1+\sqrt{3}\right)-x}{\sqrt{-4+x^3}}\,dx,x,\sqrt[3]{4+27x^2}\right)}{\sqrt{3}x}+\frac{\left(2\sqrt[6]{2}\sqrt{x^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-4+x^3}}\,dx,x,\right.}{\sqrt{3\left(2-\sqrt{3}\right)}x} \\
&= \frac{1}{12}\left(4+27x^2\right)^{2/3}-\frac{6x}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}}+\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(2^{2/3}-\sqrt[3]{4+27x^2}\right)\sqrt{\frac{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{4+27x^2}+(}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-}\right.}}}{3\,3^{3/4}x\sqrt{-\frac{2}{\left(2^{2/3}(\right.}}}
\end{aligned}
$$

Mathematica [C] time = 0.0156151, size = 40, normalized size = 0.08

$$\sqrt[3]{2}x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{27x^2}{4}\right)+\frac{1}{12}\left(27x^2+4\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)/(4 + 27*x^2)^(1/3),x]
```

```
[Out] (4 + 27*x^2)^(2/3)/12 + 2^(1/3)*x*Hypergeometric2F1[1/3, 1/2, 3/2, (-27*x^2
)/4]
```

Maple [C] time = 0.215, size = 29, normalized size = 0.1

$$\frac{1}{12}\left(27\,x^{2}+4\right)^{\frac{2}{3}}+\sqrt[3]{2}x{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{27\,x^{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)/(27*x^2+4)^(1/3),x)
```

```
[Out] 1/12*(27*x^2+4)^(2/3)+2^(1/3)*x*hypergeom([1/3,1/2],[3/2],-27/4*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x+2}{\left(27\,x^{2}+4\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(27*x^2+4)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((3*x + 2)/(27*x^2 + 4)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{3\,x+2}{\left(27\,x^{2}+4\right)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(27*x^2+4)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((3*x + 2)/(27*x^2 + 4)^(1/3), x)
```

Sympy [A] time = 2.35865, size = 36, normalized size = 0.07

$$\sqrt[3]{2}x{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{27x^{2}e^{i\pi}}{4}\right)+\frac{\left(27x^{2}+4\right)^{\frac{2}{3}}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(27*x**2+4)**(1/3),x)
```

```
[Out] 2**(1/3)*x*hyper((1/3, 1/2), (3/2,), 27*x**2*exp_polar(I*pi)/4) + (27*x**2
+ 4)**(2/3)/12
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x + 2}{\left(27x^2 + 4\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(27*x^2+4)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((3*x + 2)/(27*x^2 + 4)^(1/3), x)
```

3.703 $\int \frac{1}{(2+3x)\sqrt[3]{4+27x^2}}\,dx$

Optimal. Leaf size=97

$$\frac{\log\left(-27\ 2^{2/3}\sqrt[3]{27x^2+4}-81x+54\right)}{12\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2}(2-3x)}{\sqrt{3}\sqrt[3]{27x^2+4}}+\frac{1}{\sqrt{3}}\right)}{6\sqrt[3]{2}\sqrt{3}}-\frac{\log(3x+2)}{12\sqrt[3]{2}}$$

```
[Out] -ArcTan[1/Sqrt[3] + (2^(1/3)*(2 - 3*x))/(Sqrt[3]*(4 + 27*x^2)^(1/3))]/(6*2^
(1/3)*Sqrt[3]) - Log[2 + 3*x]/(12*2^(1/3)) + Log[54 - 81*x - 27*2^(2/3)*(4
+ 27*x^2)^(1/3)]/(12*2^(1/3))
```

Rubi [A] time = 0.0143885, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {751}

$$\frac{\log\left(-27\ 2^{2/3}\sqrt[3]{27x^2+4}-81x+54\right)}{12\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2}(2-3x)}{\sqrt{3}\sqrt[3]{27x^2+4}}+\frac{1}{\sqrt{3}}\right)}{6\sqrt[3]{2}\sqrt{3}}-\frac{\log(3x+2)}{12\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + 3*x)*(4 + 27*x^2)^(1/3)),x]
```

```
[Out] -ArcTan[1/Sqrt[3] + (2^(1/3)*(2 - 3*x))/(Sqrt[3]*(4 + 27*x^2)^(1/3))]/(6*2^
(1/3)*Sqrt[3]) - Log[2 + 3*x]/(12*2^(1/3)) + Log[54 - 81*x - 27*2^(2/3)*(4
+ 27*x^2)^(1/3)]/(12*2^(1/3))
```

Rule 751

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/3)), x_Symbol] :> With[
{q = Rt[(6*c^2*e^2)/d^2, 3]}, -Simp[(Sqrt[3]*c*e*ArcTan[1/Sqrt[3] + (2*c*(d
 - e*x))/(Sqrt[3]*d*q*(a + c*x^2)^(1/3))])/(d^2*q^2), x] + (-Simp[(3*c*e*Lo
g[d + e*x])/(2*d^2*q^2), x] + Simp[(3*c*e*Log[c*d - c*e*x - d*q*(a + c*x^2)
^(1/3)])/(2*d^2*q^2), x])] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - 3*a*e^2
, 0]
```

Rubi steps

$$\int \frac{1}{(2+3x)\sqrt[3]{4+27x^2}}\,dx=-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{\sqrt[3]{2}(2-3x)}{\sqrt{3}\sqrt[3]{4+27x^2}}\right)}{6\sqrt[3]{2}\sqrt{3}}-\frac{\log(2+3x)}{12\sqrt[3]{2}}+\frac{\log\left(54-81x-27\ 2^{2/3}\sqrt[3]{4+27x^2}\right)}{12\sqrt[3]{2}}$$

Mathematica [C] time = 0.0776071, size = 121, normalized size = 1.25

$$-\frac{\sqrt[3]{\frac{9x-2i\sqrt{3}}{3x+2}}\sqrt[3]{\frac{9x+2i\sqrt{3}}{3x+2}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{6-2i\sqrt{3}}{9x+6},\frac{6+2i\sqrt{3}}{9x+6}\right)}{2\ 3^{2/3}\sqrt[3]{27x^2+4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 + 3*x)*(4 + 27*x^2)^(1/3)),x]
```

```
[Out] -((((-2*I)*Sqrt[3] + 9*x)/(2 + 3*x))^(1/3)*(((2*I)*Sqrt[3] + 9*x)/(2 + 3*x)
)^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, (6 - (2*I)*Sqrt[3])/(6 + 9*x), (6 + (2
*I)*Sqrt[3])/(6 + 9*x)])/(2*3^(2/3)*(4 + 27*x^2)^(1/3))
```

Maple [F] time = 0.362, size = 0, normalized size = 0.

$$\int \frac{1}{2+3x}\frac{1}{\sqrt[3]{27x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*x)/(27*x^2+4)^(1/3),x)
```

```
[Out] int(1/(2+3*x)/(27*x^2+4)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(27x^2+4\right)^{\frac{1}{3}}(3x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(27*x^2+4)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((27*x^2 + 4)^(1/3)*(3*x + 2)), x)
```

Fricas [B] time = 28.3654, size = 571, normalized size = 5.89

$$-\frac{1}{36}\sqrt{6}2^{\frac{1}{6}}\arctan\left(\frac{2^{\frac{1}{6}}\left(4\sqrt{6}2^{\frac{2}{3}}\left(27x^2+4\right)^{\frac{2}{3}}(3x-2)+\sqrt{6}2^{\frac{1}{3}}\left(27x^3+54x^2+36x+8\right)+4\sqrt{6}\left(27x^2+4\right)^{\frac{1}{3}}\left(9x^2-12x+\right.\right.}{18\left(9x^3-54x^2+12x-8\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(27*x^2+4)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(6)*2^(1/6)*arctan(1/18*2^(1/6)*(4*sqrt(6)*2^(2/3)*(27*x^2 + 4)^(
2/3)*(3*x - 2) + sqrt(6)*2^(1/3)*(27*x^3 + 54*x^2 + 36*x + 8) + 4*sqrt(6)*(
27*x^2 + 4)^(1/3)*(9*x^2 - 12*x + 4))/(9*x^3 - 54*x^2 + 12*x - 8)) - 1/72*2
^(2/3)*log((2*2^(2/3)*(27*x^2 + 4)^(2/3) + 2^(1/3)*(9*x^2 - 12*x + 4) - 2*(
27*x^2 + 4)^(1/3)*(3*x - 2))/(9*x^2 + 12*x + 4)) + 1/36*2^(2/3)*log((2^(1/3
)*(3*x - 2) + 2*(27*x^2 + 4)^(1/3))/(3*x + 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x+2)\sqrt[3]{27x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(27*x**2+4)**(1/3),x)
```

```
[Out] Integral(1/((3*x + 2)*(27*x**2 + 4)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(27\,x^2+4\right)^{\frac{1}{3}}(3\,x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(27*x^2+4)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((27*x^2 + 4)^(1/3)*(3*x + 2)), x)
```

3.704 $\int \frac{1}{(2+3x)^2 \sqrt[3]{4+27x^2}}\, dx$

Optimal. Leaf size=634

$$-\frac{\left(2^{2/3}-\sqrt[3]{27x^2+4}\right)\sqrt{\frac{\left(27x^2+4\right)^{2/3}+2^{2/3}\sqrt[3]{27x^2+4}+2\sqrt[3]{2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{27x^2+4}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{27x^2+4}}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{27x^2+4}}\right),4\sqrt{3}-7\right)}{72\sqrt[6]{2}\sqrt[4]{3}\sqrt{-\frac{2^{2/3}-\sqrt[3]{27x^2+4}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{27x^2+4}\right)^2}}\,x}-\frac{3x}{16\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{\ldots}\right.}$$

```
[Out] -(4 + 27*x^2)^(2/3)/(48*(2 + 3*x)) - (3*x)/(16*(2^(2/3)*(1 - Sqrt[3]) - (4
+ 27*x^2)^(1/3))) - ArcTan[1/Sqrt[3] + (2^(1/3)*(2 - 3*x))/(Sqrt[3]*(4 + 27
*x^2)^(1/3))]/(24*2^(1/3)*Sqrt[3]) + (Sqrt[2 + Sqrt[3]]*(2^(2/3) - (4 + 27*
x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 + 27*x^2)^(1/3) + (4 + 27*x^2)^(2/
3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))^2]*EllipticE[ArcSin[(2^(2/
3)*(1 + Sqrt[3]) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2
)^(1/3))], -7 + 4*Sqrt[3]])/(48*2^(2/3)*3^(3/4)*x*Sqrt[-((2^(2/3) - (4 + 27
*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))^2)]) - ((2^(2/3)
- (4 + 27*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 + 27*x^2)^(1/3) + (4 + 2
7*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))^2]*EllipticF[Arc
Sin[(2^(2/3)*(1 + Sqrt[3]) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (
4 + 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(72*2^(1/6)*3^(1/4)*x*Sqrt[-((2^(2/3)
 - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))^2)]) -
Log[2 + 3*x]/(48*2^(1/3)) + Log[54 - 81*x - 27*2^(2/3)*(4 + 27*x^2)^(1/3)]/
(48*2^(1/3))
```

Rubi [A] time = 0.364388, antiderivative size = 634, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {745, 844, 235, 304, 219, 1879, 751}

$$-\frac{3x}{16\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{27x^2+4}\right)}-\frac{\left(27x^2+4\right)^{2/3}}{48(3x+2)}+\frac{\log\left(-27\, 2^{2/3}\sqrt[3]{27x^2+4}-81x+54\right)}{48\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2}(2-3x)}{\sqrt{3}\sqrt[3]{27x^2+4}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt[3]{2}\sqrt{3}}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + 3*x)^2*(4 + 27*x^2)^(1/3)),x]
```


```
[Out] -(4 + 27*x^2)^(2/3)/(48*(2 + 3*x)) - (3*x)/(16*(2^(2/3)*(1 - Sqrt[3]) - (4
+ 27*x^2)^(1/3))) - ArcTan[1/Sqrt[3] + (2^(1/3)*(2 - 3*x))/(Sqrt[3]*(4 + 27
*x^2)^(1/3))]/(24*2^(1/3)*Sqrt[3]) + (Sqrt[2 + Sqrt[3]]*(2^(2/3) - (4 + 27*
x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 + 27*x^2)^(1/3) + (4 + 27*x^2)^(2/
3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))^2]*EllipticE[ArcSin[(2^(2/
3)*(1 + Sqrt[3]) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2
)^(1/3))], -7 + 4*Sqrt[3]])/(48*2^(2/3)*3^(3/4)*x*Sqrt[-((2^(2/3) - (4 + 27
*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))^2)]) - ((2^(2/3)
- (4 + 27*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 + 27*x^2)^(1/3) + (4 + 2
7*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))^2]*EllipticF[Arc
Sin[(2^(2/3)*(1 + Sqrt[3]) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (
4 + 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(72*2^(1/6)*3^(1/4)*x*Sqrt[-((2^(2/3)
 - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))^2)]) -
Log[2 + 3*x]/(48*2^(1/3)) + Log[54 - 81*x - 27*2^(2/3)*(4 + 27*x^2)^(1/3)]/
```

(48*2^(1/3))

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 751

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/3)), x_Symbol] :> With[
{q = Rt[(6*c^2*e^2)/d^2, 3]}, -Simp[(Sqrt[3]*c*e*ArcTan[1/Sqrt[3] + (2*c*(d
 - e*x))/(Sqrt[3]*d*q*(a + c*x^2)^(1/3))])/(d^2*q^2), x] + (-Simp[(3*c*e*Lo
g[d + e*x])/(2*d^2*q^2), x] + Simp[(3*c*e*Log[c*d - c*e*x - d*q*(a + c*x^2)
^(1/3)])/(2*d^2*q^2), x])] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - 3*a*e^2
```

, 0]

Rubi steps

$$\begin{aligned}\int \frac{1}{(2+3x)^2\sqrt[3]{4+27x^2}}\,dx &= -\frac{\left(4+27x^2\right)^{2/3}}{48(2+3x)}-\frac{3}{16}\int \frac{-2-x}{(2+3x)\sqrt[3]{4+27x^2}}\,dx\\ &= -\frac{\left(4+27x^2\right)^{2/3}}{48(2+3x)}+\frac{1}{16}\int \frac{1}{\sqrt[3]{4+27x^2}}\,dx+\frac{1}{4}\int \frac{1}{(2+3x)\sqrt[3]{4+27x^2}}\,dx\\ &= -\frac{\left(4+27x^2\right)^{2/3}}{48(2+3x)}-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{\sqrt[3]{2}(2-3x)}{\sqrt{3}\sqrt[3]{4+27x^2}}\right)}{24\sqrt[3]{2}\sqrt{3}}-\frac{\log(2+3x)}{48\sqrt[3]{2}}+\frac{\log\left(54-81x-27\,2^{2/3}\sqrt[3]{4+27\right.}{48\sqrt[3]{2}}\\ &= -\frac{\left(4+27x^2\right)^{2/3}}{48(2+3x)}-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{\sqrt[3]{2}(2-3x)}{\sqrt{3}\sqrt[3]{4+27x^2}}\right)}{24\sqrt[3]{2}\sqrt{3}}-\frac{\log(2+3x)}{48\sqrt[3]{2}}+\frac{\log\left(54-81x-27\,2^{2/3}\sqrt[3]{4+27\right.}{48\sqrt[3]{2}}\\ &= -\frac{\left(4+27x^2\right)^{2/3}}{48(2+3x)}-\frac{3x}{16\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}\right)}-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{\sqrt[3]{2}(2-3x)}{\sqrt{3}\sqrt[3]{4+27x^2}}\right)}{24\sqrt[3]{2}\sqrt{3}}+\frac{\sqrt{2+\sqrt{3}}\left(2\right.}{}\end{aligned}$$

Mathematica [C] time = 0.242957, size = 211, normalized size = 0.33

$$\frac{-8\sqrt[3]{3}(3x+2)\sqrt[3]{\frac{9x-2i\sqrt{3}}{3x+2}}\sqrt[3]{\frac{9x+2i\sqrt{3}}{3x+2}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{6-2i\sqrt{3}}{9x+6},\frac{6+2i\sqrt{3}}{9x+6}\right)+\sqrt[3]{6}\sqrt[3]{2\sqrt{3}-9ix}(3x+2)\left(3\sqrt{3}x-2i\right)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};\frac{3}{4}i\sqrt{3}x\right.}{192(3x+2)\sqrt[3]{27x^2+4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 + 3*x)^2*(4 + 27*x^2)^(1/3)),x]
```

```
[Out] (-4*(4 + 27*x^2) - 8*3^(1/3)*(2 + 3*x)*(((-2*I)*Sqrt[3] + 9*x)/(2 + 3*x))^(
1/3)*(((2*I)*Sqrt[3] + 9*x)/(2 + 3*x))^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, (
6 - (2*I)*Sqrt[3])/(6 + 9*x), (6 + (2*I)*Sqrt[3])/(6 + 9*x)] + 6^(1/3)*(2*S
qrt[3] - (9*I)*x)^(1/3)*(2 + 3*x)*(-2*I + 3*Sqrt[3]*x)*Hypergeometric2F1[1/
3, 2/3, 5/3, 1/2 + ((3*I)/4)*Sqrt[3]*x])/(192*(2 + 3*x)*(4 + 27*x^2)^(1/3))
```

Maple [F] time = 0.378, size = 0, normalized size = 0.

$$\int \frac{1}{(2+3x)^2}\frac{1}{\sqrt[3]{27x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*x)^2/(27*x^2+4)^(1/3),x)
```

```
[Out] int(1/(2+3*x)^2/(27*x^2+4)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(27\,x^2+4\right)^{\frac{1}{3}}\left(3\,x+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)^2/(27*x^2+4)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((27*x^2 + 4)^(1/3)*(3*x + 2)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(27\,x^2+4\right)^{\frac{2}{3}}}{243\,x^4+324\,x^3+144\,x^2+48\,x+16},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)^2/(27*x^2+4)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((27*x^2 + 4)^(2/3)/(243*x^4 + 324*x^3 + 144*x^2 + 48*x + 16), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3x+2\right)^2\sqrt[3]{27x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)**2/(27*x**2+4)**(1/3),x)
```

```
[Out] Integral(1/((3*x + 2)**2*(27*x**2 + 4)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(27\,x^2+4\right)^{\frac{1}{3}}\left(3\,x+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)^2/(27*x^2+4)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((27*x^2 + 4)^(1/3)*(3*x + 2)^2), x)
```

3.705 $\int \frac{1}{(2+3x)^3 \sqrt[3]{4+27x^2}} dx$

Optimal. Leaf size=656

$$-\frac{\left(2^{2/3}-\sqrt[3]{27x^2+4}\right)\sqrt{\frac{(27x^2+4)^{2/3}+2^{2/3}\sqrt[3]{27x^2+4}+2\sqrt[3]{2}}{\left(2^{2/3}(1-\sqrt{3})-\sqrt[3]{27x^2+4}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{2^{2/3}(1+\sqrt{3})-\sqrt[3]{27x^2+4}}{2^{2/3}(1-\sqrt{3})-\sqrt[3]{27x^2+4}}\right),4\sqrt{3}-7\right)}{144\sqrt[6]{2}\sqrt[4]{3}\sqrt{-\frac{2^{2/3}-\sqrt[3]{27x^2+4}}{\left(2^{2/3}(1-\sqrt{3})-\sqrt[3]{27x^2+4}\right)^2}}x}-\frac{3x}{32\left(2^{2/3}\left(1-\sqrt{3}\right)-\right.}$$

```
[Out] -(4 + 27*x^2)^(2/3)/(96*(2 + 3*x)^2) - (4 + 27*x^2)^(2/3)/(96*(2 + 3*x)) -
(3*x)/(32*(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))) - ArcTan[1/Sqrt[3]
+ (2^(1/3)*(2 - 3*x))/(Sqrt[3]*(4 + 27*x^2)^(1/3))]/(96*2^(1/3)*Sqrt[3]) +
(Sqrt[2 + Sqrt[3]]*(2^(2/3) - (4 + 27*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)
*(4 + 27*x^2)^(1/3) + (4 + 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*
x^2)^(1/3))^2]*EllipticE[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 + 27*x^2)^(1/3)
)/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(96*2^(2/
3)*3^(3/4)*x*Sqrt[-((2^(2/3) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) -
 (4 + 27*x^2)^(1/3))^2)]) - ((2^(2/3) - (4 + 27*x^2)^(1/3))*Sqrt[(2*2^(1/3)
 + 2^(2/3)*(4 + 27*x^2)^(1/3) + (4 + 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3])
- (4 + 27*x^2)^(1/3))^2]*EllipticF[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 + 27*
x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])
/(144*2^(1/6)*3^(1/4)*x*Sqrt[-((2^(2/3) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 -
 Sqrt[3]) - (4 + 27*x^2)^(1/3))^2)]) - Log[2 + 3*x]/(192*2^(1/3)) + Log[54
- 81*x - 27*2^(2/3)*(4 + 27*x^2)^(1/3)]/(192*2^(1/3))
```

Rubi [A] time = 0.411907, antiderivative size = 656, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {745, 835, 844, 235, 304, 219, 1879, 751}

$$-\frac{3x}{32\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{27x^2+4}\right)}-\frac{\left(27x^2+4\right)^{2/3}}{96(3x+2)}-\frac{\left(27x^2+4\right)^{2/3}}{96(3x+2)^2}+\frac{\log\left(-27\ 2^{2/3}\sqrt[3]{27x^2+4}-81x+54\right)}{192\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2}(\ldots}{\sqrt{3}\sqrt[3]{2}\ldots}\right.}{96\sqrt[3]{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + 3*x)^3*(4 + 27*x^2)^(1/3)),x]
```


```
[Out] -(4 + 27*x^2)^(2/3)/(96*(2 + 3*x)^2) - (4 + 27*x^2)^(2/3)/(96*(2 + 3*x)) -
(3*x)/(32*(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))) - ArcTan[1/Sqrt[3]
+ (2^(1/3)*(2 - 3*x))/(Sqrt[3]*(4 + 27*x^2)^(1/3))]/(96*2^(1/3)*Sqrt[3]) +
(Sqrt[2 + Sqrt[3]]*(2^(2/3) - (4 + 27*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)
*(4 + 27*x^2)^(1/3) + (4 + 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*
x^2)^(1/3))^2]*EllipticE[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 + 27*x^2)^(1/3)
)/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(96*2^(2/
3)*3^(3/4)*x*Sqrt[-((2^(2/3) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) -
 (4 + 27*x^2)^(1/3))^2)]) - ((2^(2/3) - (4 + 27*x^2)^(1/3))*Sqrt[(2*2^(1/3)
 + 2^(2/3)*(4 + 27*x^2)^(1/3) + (4 + 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3])
- (4 + 27*x^2)^(1/3))^2]*EllipticF[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 + 27*
x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])
/(144*2^(1/6)*3^(1/4)*x*Sqrt[-((2^(2/3) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 -
 Sqrt[3]) - (4 + 27*x^2)^(1/3))^2)]) - Log[2 + 3*x]/(192*2^(1/3)) + Log[54
```

```
- 81*x - 27*2^(2/3)*(4 + 27*x^2)^(1/3)]/(192*2^(1/3))
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
```

```
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 751

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/3)), x_Symbol] :> With[
{q = Rt[(6*c^2*e^2)/d^2, 3]}, -Simp[(Sqrt[3]*c*e*ArcTan[1/Sqrt[3] + (2*c*(d
 - e*x))/(Sqrt[3]*d*q*(a + c*x^2)^(1/3))])/(d^2*q^2), x] + (-Simp[(3*c*e*Lo
g[d + e*x])/(2*d^2*q^2), x] + Simp[(3*c*e*Log[c*d - c*e*x - d*q*(a + c*x^2)
^(1/3)])/(2*d^2*q^2), x])] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - 3*a*e^2
, 0]
```

Rubi steps

$$\int \frac{1}{(2+3x)^3\sqrt[3]{4+27x^2}}\,dx = -\frac{\left(4+27x^2\right)^{2/3}}{96(2+3x)^2} - \frac{3}{32}\int \frac{-4+2x}{(2+3x)^2\sqrt[3]{4+27x^2}}\,dx$$

$$= -\frac{\left(4+27x^2\right)^{2/3}}{96(2+3x)^2} - \frac{\left(4+27x^2\right)^{2/3}}{96(2+3x)} + \frac{\int \frac{192+144x}{(2+3x)\sqrt[3]{4+27x^2}}\,dx}{1536}$$

$$= -\frac{\left(4+27x^2\right)^{2/3}}{96(2+3x)^2} - \frac{\left(4+27x^2\right)^{2/3}}{96(2+3x)} + \frac{1}{32}\int \frac{1}{\sqrt[3]{4+27x^2}}\,dx + \frac{1}{16}\int \frac{1}{(2+3x)\sqrt[3]{4+27x^2}}\,dx$$

$$= -\frac{\left(4+27x^2\right)^{2/3}}{96(2+3x)^2} - \frac{\left(4+27x^2\right)^{2/3}}{96(2+3x)} - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{\sqrt[3]{2}(2-3x)}{\sqrt{3}\sqrt[3]{4+27x^2}}\right)}{96\sqrt[3]{2}\sqrt{3}} - \frac{\log(2+3x)}{192\sqrt[3]{2}} + \frac{\log\left(54-81x-\right.}{1}$$

$$= -\frac{\left(4+27x^2\right)^{2/3}}{96(2+3x)^2} - \frac{\left(4+27x^2\right)^{2/3}}{96(2+3x)} - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{\sqrt[3]{2}(2-3x)}{\sqrt{3}\sqrt[3]{4+27x^2}}\right)}{96\sqrt[3]{2}\sqrt{3}} - \frac{\log(2+3x)}{192\sqrt[3]{2}} + \frac{\log\left(54-81x-\right.}{1}$$

$$= -\frac{\left(4+27x^2\right)^{2/3}}{96(2+3x)^2} - \frac{\left(4+27x^2\right)^{2/3}}{96(2+3x)} - \frac{3x}{32\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}\right)} - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{\sqrt[3]{2}(2-3x)}{\sqrt{3}\sqrt[3]{4+27x^2}}\right.}{96\sqrt[3]{2}\sqrt{3}}$$

Mathematica [C] time = 0.271006, size = 223, normalized size = 0.34

$$\frac{-4\sqrt[3]{3}\sqrt[3]{\frac{9x-2i\sqrt{3}}{3x+2}}\sqrt[3]{\frac{9x+2i\sqrt{3}}{3x+2}}(3x+2)^2F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{6-2i\sqrt{3}}{9x+6},\frac{6+2i\sqrt{3}}{9x+6}\right) + \sqrt[3]{6}\sqrt[3]{2\sqrt{3}-9ix}\left(3\sqrt{3}x-2i\right)(3x+2)^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};\frac{3}{4}i\sqrt{\right.}{384(3x+2)^2\sqrt[3]{27x^2+4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 + 3*x)^3*(4 + 27*x^2)^(1/3)),x]
```

```
[Out] (-12*(4 + 4*x + 27*x^2 + 27*x^3) - 4*3^(1/3)*(2 + 3*x)^2*(((-2*I)*Sqrt[3] +
 9*x)/(2 + 3*x))^(1/3)*(((2*I)*Sqrt[3] + 9*x)/(2 + 3*x))^(1/3)*AppellF1[2/3
, 1/3, 1/3, 5/3, (6 - (2*I)*Sqrt[3])/(6 + 9*x), (6 + (2*I)*Sqrt[3])/(6 + 9*
x)] + 6^(1/3)*(2*Sqrt[3] - (9*I)*x)^(1/3)*(2 + 3*x)^2*(-2*I + 3*Sqrt[3]*x)*
Hypergeometric2F1[1/3, 2/3, 5/3, 1/2 + ((3*I)/4)*Sqrt[3]*x])/(384*(2 + 3*x)
^2*(4 + 27*x^2)^(1/3))
```

Maple [F] time = 0.375, size = 0, normalized size = 0.

$$\int \frac{1}{(2+3x)^{3}} \frac{1}{\sqrt[3]{27x^{2}+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*x)^3/(27*x^2+4)^(1/3),x)
```

```
[Out] int(1/(2+3*x)^3/(27*x^2+4)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(27x^{2}+4\right)^{\frac{1}{3}}(3x+2)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)^3/(27*x^2+4)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((27*x^2 + 4)^(1/3)*(3*x + 2)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(27x^{2}+4\right)^{\frac{2}{3}}}{729x^{5}+1458x^{4}+1080x^{3}+432x^{2}+144x+32}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)^3/(27*x^2+4)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((27*x^2 + 4)^(2/3)/(729*x^5 + 1458*x^4 + 1080*x^3 + 432*x^2 + 144*
x + 32), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x+2)^{3}\sqrt[3]{27x^{2}+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)**3/(27*x**2+4)**(1/3),x)
```

```
[Out] Integral(1/((3*x + 2)**3*(27*x**2 + 4)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(27x^{2}+4\right)^{\frac{1}{3}}(3x+2)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)^3/(27*x^2+4)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((27*x^2 + 4)^(1/3)*(3*x + 2)^3), x)
```

3.706 $\int \frac{(2+3ix)^3}{\sqrt[3]{4-27x^2}}\,dx$

Optimal. Leaf size=564

$$\frac{32\,2^{5/6}\left(2^{2/3}-\sqrt[3]{4-27x^2}\right)\sqrt{\frac{\left(4-27x^2\right)^{2/3}+2^{2/3}\sqrt[3]{4-27x^2}+2\sqrt[3]{2}}{\left(2^{2/3}(1-\sqrt{3})-\sqrt[3]{4-27x^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{2^{2/3}(1+\sqrt{3})-\sqrt[3]{4-27x^2}}{2^{2/3}(1-\sqrt{3})-\sqrt[3]{4-27x^2}}\right),4\sqrt{3}-7\right)}{63\sqrt[4]{3}x\sqrt{-\frac{2^{2/3}-\sqrt[3]{4-27x^2}}{\left(2^{2/3}(1-\sqrt{3})-\sqrt[3]{4-27x^2}\right)^2}}}-\frac{1}{30}i\left(4-27x\right.$$

```
[Out] (-4*(7*I - 4*x)*(4 - 27*x^2)^(2/3))/35 - (I/30)*(2 + (3*I)*x)^2*(4 - 27*x^2
)^(2/3) - (96*x)/(7*(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))) - (16*2^(
1/3)*Sqrt[2 + Sqrt[3]]*(2^(2/3) - (4 - 27*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(
2/3)*(4 - 27*x^2)^(1/3) + (4 - 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 -
 27*x^2)^(1/3))^2]*EllipticE[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 - 27*x^2)^(
1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(21*3
^(3/4)*x*Sqrt[-((2^(2/3) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4
- 27*x^2)^(1/3))^2)]) + (32*2^(5/6)*(2^(2/3) - (4 - 27*x^2)^(1/3))*Sqrt[(2*
2^(1/3) + 2^(2/3)*(4 - 27*x^2)^(1/3) + (4 - 27*x^2)^(2/3))/(2^(2/3)*(1 - Sq
rt[3]) - (4 - 27*x^2)^(1/3))^2]*EllipticF[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (
4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))], -7 + 4*Sq
rt[3]])/(63*3^(1/4)*x*Sqrt[-((2^(2/3) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - S
qrt[3]) - (4 - 27*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.362809, antiderivative size = 564, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {743, 780, 235, 304, 219, 1879}

$$-\frac{1}{30}i\left(4-27x^2\right)^{2/3}(2+3ix)^2-\frac{4}{35}(-4x+7i)\left(4-27x^2\right)^{2/3}-\frac{96x}{7\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)}+\frac{32\,2^{5/6}\left(2^{2/3}-\sqrt[3]{4-27\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + (3*I)*x)^3/(4 - 27*x^2)^(1/3),x]
```


```
[Out] (-4*(7*I - 4*x)*(4 - 27*x^2)^(2/3))/35 - (I/30)*(2 + (3*I)*x)^2*(4 - 27*x^2
)^(2/3) - (96*x)/(7*(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))) - (16*2^(
1/3)*Sqrt[2 + Sqrt[3]]*(2^(2/3) - (4 - 27*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(
2/3)*(4 - 27*x^2)^(1/3) + (4 - 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 -
 27*x^2)^(1/3))^2]*EllipticE[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 - 27*x^2)^(
1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(21*3
^(3/4)*x*Sqrt[-((2^(2/3) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4
- 27*x^2)^(1/3))^2)]) + (32*2^(5/6)*(2^(2/3) - (4 - 27*x^2)^(1/3))*Sqrt[(2*
2^(1/3) + 2^(2/3)*(4 - 27*x^2)^(1/3) + (4 - 27*x^2)^(2/3))/(2^(2/3)*(1 - Sq
rt[3]) - (4 - 27*x^2)^(1/3))^2]*EllipticF[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (
4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))], -7 + 4*Sq
rt[3]])/(63*3^(1/4)*x*Sqrt[-((2^(2/3) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - S
qrt[3]) - (4 - 27*x^2)^(1/3))^2)])
```


Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
```

```
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(2+3ix)^3}{\sqrt[3]{4-27x^2}}\,dx = -\frac{1}{30}i(2+3ix)^2\left(4-27x^2\right)^{2/3} - \frac{1}{90}\int \frac{(2+3ix)(-288-864ix)}{\sqrt[3]{4-27x^2}}\,dx$$
$$= -\frac{4}{35}(7i-4x)\left(4-27x^2\right)^{2/3} - \frac{1}{30}i(2+3ix)^2\left(4-27x^2\right)^{2/3} + \frac{32}{7}\int \frac{1}{\sqrt[3]{4-27x^2}}\,dx$$
$$= -\frac{4}{35}(7i-4x)\left(4-27x^2\right)^{2/3} - \frac{1}{30}i(2+3ix)^2\left(4-27x^2\right)^{2/3} - \frac{\left(16\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-4+x^3}}\,dx, x, \sqrt[3]{4-\right.}{7\sqrt{3}x}$$
$$= -\frac{4}{35}(7i-4x)\left(4-27x^2\right)^{2/3} - \frac{1}{30}i(2+3ix)^2\left(4-27x^2\right)^{2/3} + \frac{\left(16\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{2^{2/3}(1+\sqrt{3})-x}{\sqrt{-4+x^3}}\,dx, x,\right.}{7\sqrt{3}x}$$
$$= -\frac{4}{35}(7i-4x)\left(4-27x^2\right)^{2/3} - \frac{1}{30}i(2+3ix)^2\left(4-27x^2\right)^{2/3} - \frac{96x}{7\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)} - \frac{16\sqrt[3]{2}}{}$$

Mathematica [C] time = 0.0351779, size = 60, normalized size = 0.11

$$\frac{16}{7}\sqrt[3]{2}x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{27x^2}{4}\right) + \left(4-27x^2\right)^{2/3}\left(\frac{3ix^2}{10}+\frac{6x}{7}-\frac{14i}{15}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + (3*I)*x)^3/(4 - 27*x^2)^(1/3),x]
```

```
[Out] (4 - 27*x^2)^(2/3)*((-14*I)/15 + (6*x)/7 + ((3*I)/10)*x^2) + (16*2^(1/3)*x*
Hypergeometric2F1[1/3, 1/2, 3/2, (27*x^2)/4])/7
```

Maple [C] time = 0.267, size = 49, normalized size = 0.1

$$-\frac{i}{210}\left(-180\,ix+63\,x^2-196\right)\left(27\,x^2-4\right)\frac{1}{\sqrt[3]{-27\,x^2+4}} + \frac{16\sqrt[3]{2}x}{7}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\,\frac{3}{2};\,\frac{27\,x^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*I*x)^3/(-27*x^2+4)^(1/3),x)
```

```
[Out] -1/210*I*(-180*I*x+63*x^2-196)*(27*x^2-4)/(-27*x^2+4)^(1/3)+16/7*2^(1/3)*x*
hypergeom([1/3,1/2],[3/2],27/4*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3i\,x+2)^3}{\left(-27\,x^2+4\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*I*x)^3/(-27*x^2+4)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((3*I*x + 2)^3/(-27*x^2 + 4)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{630\,x\mathrm{integral}\left(\frac{128\left(-27\,x^2+4\right)^{\frac{2}{3}}}{63\left(27\,x^4-4\,x^2\right)},x\right)+\left(189i\,x^3+540\,x^2-588i\,x-320\right)\left(-27\,x^2+4\right)^{\frac{2}{3}}}{630\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*I*x)^3/(-27*x^2+4)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/630*(630*x*integral(128/63*(-27*x^2 + 4)^(2/3)/(27*x^4 - 4*x^2), x) + (18
9*I*x^3 + 540*x^2 - 588*I*x - 320)*(-27*x^2 + 4)^(2/3))/x
```

Sympy [A] time = 4.99207, size = 150, normalized size = 0.27

$$-9\sqrt[3]{2}x^3{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\frac{27x^2e^{2i\pi}}{4}\right)+4\sqrt[3]{2}x{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{3}{2}\end{matrix}\middle|\frac{27x^2e^{2i\pi}}{4}\right)-i\left(4-27x^2\right)^{\frac{2}{3}}-27i\left(\begin{cases}\frac{x^2\left(27x^2-4\right)^{\frac{2}{3}}e^{-\frac{i\pi}{3}}}{90}+\frac{\left(27x^2-4\right)^{\frac{2}{3}}e^{-\frac{i\pi}{3}}}{405} & \text{for } \frac{27|x^2|}{4} \\ -\frac{x^2\left(4-27x^2\right)^{\frac{2}{3}}}{90}-\frac{\left(4-27x^2\right)^{\frac{2}{3}}}{405} & \text{otherwi}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*I*x)**3/(-27*x**2+4)**(1/3),x)
```

```
[Out] -9*2**(1/3)*x**3*hyper((1/3, 3/2), (5/2,), 27*x**2*exp_polar(2*I*pi)/4) + 4
*2**(1/3)*x*hyper((1/3, 1/2), (3/2,), 27*x**2*exp_polar(2*I*pi)/4) - I*(4 -
 27*x**2)**(2/3) - 27*I*Piecewise((x**2*(27*x**2 - 4)**(2/3)*exp(-I*pi/3)/9
0 + (27*x**2 - 4)**(2/3)*exp(-I*pi/3)/405, 27*Abs(x**2)/4 > 1), (-x**2*(4 -
 27*x**2)**(2/3)/90 - (4 - 27*x**2)**(2/3)/405, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(3i\,x+2)^3}{\left(-27\,x^2+4\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*I*x)^3/(-27*x^2+4)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((3*I*x + 2)^3/(-27*x^2 + 4)^(1/3), x)
```

3.707 $\int \frac{(2+3ix)^2}{\sqrt[3]{4-27x^2}}\,dx$

Optimal. Leaf size=557

$$\frac{8\,2^{5/6}\left(2^{2/3}-\sqrt[3]{4-27x^2}\right)\sqrt{\frac{\left(4-27x^2\right)^{2/3}+2^{2/3}\sqrt[3]{4-27x^2}+2\sqrt[3]{2}}{\left(2^{2/3}(1-\sqrt{3})-\sqrt[3]{4-27x^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{2^{2/3}(1+\sqrt{3})-\sqrt[3]{4-27x^2}}{2^{2/3}(1-\sqrt{3})-\sqrt[3]{4-27x^2}}\right),4\sqrt{3}-7\right)}{21\sqrt[4]{3}x\sqrt{-\frac{2^{2/3}-\sqrt[3]{4-27x^2}}{\left(2^{2/3}(1-\sqrt{3})-\sqrt[3]{4-27x^2}\right)^2}}}-\frac{1}{21}i\left(4-27x^2\right.$$

```
[Out] ((-5*I)/21)*(4 - 27*x^2)^(2/3) - (I/21)*(2 + (3*I)*x)*(4 - 27*x^2)^(2/3) -
(72*x)/(7*(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))) - (4*2^(1/3)*Sqrt[2
 + Sqrt[3]]*(2^(2/3) - (4 - 27*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 - 2
7*x^2)^(1/3) + (4 - 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1
/3))^2]*EllipticE[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 - 27*x^2)^(1/3))/(2^(2
/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*3^(3/4)*x*Sqr
t[-((2^(2/3) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1
/3))^2)]) + (8*2^(5/6)*(2^(2/3) - (4 - 27*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(
2/3)*(4 - 27*x^2)^(1/3) + (4 - 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 -
 27*x^2)^(1/3))^2]*EllipticF[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 - 27*x^2)^(
1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(21*3
^(1/4)*x*Sqrt[-((2^(2/3) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4
- 27*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.315223, antiderivative size = 557, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {743, 641, 235, 304, 219, 1879}

$$-\frac{1}{21}i\left(4-27x^2\right)^{2/3}(2+3ix)-\frac{5}{21}i\left(4-27x^2\right)^{2/3}-\frac{72x}{7\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)}+\frac{8\,2^{5/6}\left(2^{2/3}-\sqrt[3]{4-27x^2}\right)\sqrt{\frac{(4-27x}{(2^{2/}}}}{21}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + (3*I)*x)^2/(4 - 27*x^2)^(1/3),x]
```


```
[Out] ((-5*I)/21)*(4 - 27*x^2)^(2/3) - (I/21)*(2 + (3*I)*x)*(4 - 27*x^2)^(2/3) -
(72*x)/(7*(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))) - (4*2^(1/3)*Sqrt[2
 + Sqrt[3]]*(2^(2/3) - (4 - 27*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 - 2
7*x^2)^(1/3) + (4 - 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1
/3))^2]*EllipticE[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 - 27*x^2)^(1/3))/(2^(2
/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*3^(3/4)*x*Sqr
t[-((2^(2/3) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1
/3))^2)]) + (8*2^(5/6)*(2^(2/3) - (4 - 27*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(
2/3)*(4 - 27*x^2)^(1/3) + (4 - 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 -
 27*x^2)^(1/3))^2]*EllipticF[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 - 27*x^2)^(
1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(21*3
^(1/4)*x*Sqrt[-((2^(2/3) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4
- 27*x^2)^(1/3))^2)])
```


Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
```

```
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(2+3ix)^2}{\sqrt[3]{4-27x^2}}\,dx = -\frac{1}{21}i(2+3ix)\left(4-27x^2\right)^{2/3} - \frac{1}{63}\int \frac{-216-540ix}{\sqrt[3]{4-27x^2}}\,dx$$
$$= -\frac{5}{21}i\left(4-27x^2\right)^{2/3} - \frac{1}{21}i(2+3ix)\left(4-27x^2\right)^{2/3} + \frac{24}{7}\int \frac{1}{\sqrt[3]{4-27x^2}}\,dx$$
$$= -\frac{5}{21}i\left(4-27x^2\right)^{2/3} - \frac{1}{21}i(2+3ix)\left(4-27x^2\right)^{2/3} - \frac{\left(4\sqrt{3}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-4+x^3}}\,dx, x, \sqrt[3]{4-27x^2}\right)}{7x}$$
$$= -\frac{5}{21}i\left(4-27x^2\right)^{2/3} - \frac{1}{21}i(2+3ix)\left(4-27x^2\right)^{2/3} + \frac{\left(4\sqrt{3}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{2^{2/3}(1+\sqrt{3})-x}{\sqrt{-4+x^3}}\,dx, x, \sqrt[3]{4-2\ldots}\right.}{7x}$$
$$= -\frac{5}{21}i\left(4-27x^2\right)^{2/3} - \frac{1}{21}i(2+3ix)\left(4-27x^2\right)^{2/3} - \frac{72x}{7\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)} - \frac{4\sqrt[3]{2}\sqrt{2+\sqrt{3}}\,(\ldots}{\ldots}$$

Mathematica [C] time = 0.0313351, size = 51, normalized size = 0.09

$$\frac{12}{7}\sqrt[3]{2}x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{27x^2}{4}\right) + \left(4-27x^2\right)^{2/3}\left(\frac{x}{7}-\frac{i}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + (3*I)*x)^2/(4 - 27*x^2)^(1/3),x]
```

```
[Out] (-I/3 + x/7)*(4 - 27*x^2)^(2/3) + (12*2^(1/3)*x*Hypergeometric2F1[1/3, 1/2,
 3/2, (27*x^2)/4])/7
```

Maple [C] time = 0.273, size = 43, normalized size = 0.1

$$-\frac{(-7i+3x)\left(27x^2-4\right)}{21}\frac{1}{\sqrt[3]{-27x^2+4}} + \frac{12\sqrt[3]{2}x}{7}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{27x^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*I*x)^2/(-27*x^2+4)^(1/3),x)
```

```
[Out] -1/21*(-7*I+3*x)*(27*x^2-4)/(-27*x^2+4)^(1/3)+12/7*2^(1/3)*x*hypergeom([1/3
,1/2],[3/2],27/4*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3ix+2)^2}{\left(-27x^2+4\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*I*x)^2/(-27*x^2+4)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((3*I*x + 2)^2/(-27*x^2 + 4)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{21\, x \mathrm{integral}\left(\frac{32\left(-27\, x^2+4\right)^{\frac{2}{3}}}{21\left(27\, x^4-4\, x^2\right)}, x\right) + \left(3\, x^2 - 7i\, x - 8\right)\left(-27\, x^2 + 4\right)^{\frac{2}{3}}}{21\, x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*I*x)^2/(-27*x^2+4)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/21*(21*x*integral(32/21*(-27*x^2 + 4)^(2/3)/(27*x^4 - 4*x^2), x) + (3*x^2
 - 7*I*x - 8)*(-27*x^2 + 4)^(2/3))/x
```

Sympy [A] time = 3.47145, size = 73, normalized size = 0.13

$$-\frac{3\sqrt[3]{2}x^3{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\frac{27x^2e^{2i\pi}}{4}\right)}{2} + 2\sqrt[3]{2}x{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{3}{2}\end{matrix}\middle|\frac{27x^2e^{2i\pi}}{4}\right) - \frac{i\left(4-27x^2\right)^{\frac{2}{3}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*I*x)**2/(-27*x**2+4)**(1/3),x)
```

```
[Out] -3*2**(1/3)*x**3*hyper((1/3, 3/2), (5/2,), 27*x**2*exp_polar(2*I*pi)/4)/2 +
 2*2**(1/3)*x*hyper((1/3, 1/2), (3/2,), 27*x**2*exp_polar(2*I*pi)/4) - I*(4
 - 27*x**2)**(2/3)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3i\, x + 2)^2}{\left(-27\, x^2 + 4\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*I*x)^2/(-27*x^2+4)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((3*I*x + 2)^2/(-27*x^2 + 4)^(1/3), x)
```

3.708 $\int \frac{2+3ix}{\sqrt[3]{4-27x^2}}\,dx$

Optimal. Leaf size=531

$$\frac{2\,2^{5/6}\left(2^{2/3}-\sqrt[3]{4-27x^2}\right)\sqrt{\frac{\left(4-27x^2\right)^{2/3}+2^{2/3}\sqrt[3]{4-27x^2}+2\sqrt[3]{2}}{\left(2^{2/3}(1-\sqrt{3})-\sqrt[3]{4-27x^2}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{2^{2/3}(1+\sqrt{3})-\sqrt[3]{4-27x^2}}{2^{2/3}(1-\sqrt{3})-\sqrt[3]{4-27x^2}}\right),4\sqrt{3}-7\right)}{9\sqrt[4]{3}\sqrt{-\frac{2^{2/3}-\sqrt[3]{4-27x^2}}{\left(2^{2/3}(1-\sqrt{3})-\sqrt[3]{4-27x^2}\right)^2}}x}-\frac{6}{2^{2/3}\left(1-\sqrt{3}\right)}$$

```
[Out] (-I/12)*(4 - 27*x^2)^(2/3) - (6*x)/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1
/3)) - (2^(1/3)*Sqrt[2 + Sqrt[3]]*(2^(2/3) - (4 - 27*x^2)^(1/3))*Sqrt[(2*2^
(1/3) + 2^(2/3)*(4 - 27*x^2)^(1/3) + (4 - 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt
[3]) - (4 - 27*x^2)^(1/3))^2]*EllipticE[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4
- 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))], -7 + 4*Sqrt
[3]])/(3*3^(3/4)*x*Sqrt[-((2^(2/3) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt
[3]) - (4 - 27*x^2)^(1/3))^2)]) + (2*2^(5/6)*(2^(2/3) - (4 - 27*x^2)^(1/3))
*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 - 27*x^2)^(1/3) + (4 - 27*x^2)^(2/3))/(2^(2/3
)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))^2]*EllipticF[ArcSin[(2^(2/3)*(1 + Sqr
t[3]) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))],
-7 + 4*Sqrt[3]])/(9*3^(1/4)*x*Sqrt[-((2^(2/3) - (4 - 27*x^2)^(1/3))/(2^(2/3
)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.261419, antiderivative size = 531, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {641, 235, 304, 219, 1879}

$$-\frac{6x}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}}-\frac{1}{12}i\left(4-27x^2\right)^{2/3}+\frac{2\,2^{5/6}\left(2^{2/3}-\sqrt[3]{4-27x^2}\right)\sqrt{\frac{\left(4-27x^2\right)^{2/3}+2^{2/3}\sqrt[3]{4-27x^2}+2\sqrt[3]{2}}{\left(2^{2/3}(1-\sqrt{3})-\sqrt[3]{4-27x^2}\right)^2}}F\left(\sin^{-1}\left(\frac{2^{2/3}}{2^{2/3}}\right.\right.}{9\sqrt[4]{3}\sqrt{-\frac{2^{2/3}-\sqrt[3]{4-27x^2}}{\left(2^{2/3}(1-\sqrt{3})-\sqrt[3]{4-27x^2}\right)^2}}x}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + (3*I)*x)/(4 - 27*x^2)^(1/3),x]
```

```
[Out] (-I/12)*(4 - 27*x^2)^(2/3) - (6*x)/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1
/3)) - (2^(1/3)*Sqrt[2 + Sqrt[3]]*(2^(2/3) - (4 - 27*x^2)^(1/3))*Sqrt[(2*2^
(1/3) + 2^(2/3)*(4 - 27*x^2)^(1/3) + (4 - 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt
[3]) - (4 - 27*x^2)^(1/3))^2]*EllipticE[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4
- 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))], -7 + 4*Sqrt
[3]])/(3*3^(3/4)*x*Sqrt[-((2^(2/3) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt
[3]) - (4 - 27*x^2)^(1/3))^2)]) + (2*2^(5/6)*(2^(2/3) - (4 - 27*x^2)^(1/3))
*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 - 27*x^2)^(1/3) + (4 - 27*x^2)^(2/3))/(2^(2/3
)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))^2]*EllipticF[ArcSin[(2^(2/3)*(1 + Sqr
t[3]) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))],
-7 + 4*Sqrt[3]])/(9*3^(1/4)*x*Sqrt[-((2^(2/3) - (4 - 27*x^2)^(1/3))/(2^(2/3
)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))^2)])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3ix}{\sqrt[3]{4-27x^2}}\,dx &= -\frac{1}{12}i\left(4-27x^2\right)^{2/3}+2\int \frac{1}{\sqrt[3]{4-27x^2}}\,dx\\
&= -\frac{1}{12}i\left(4-27x^2\right)^{2/3}-\frac{\sqrt{-x^2}\,\text{Subst}\left(\int \frac{x}{\sqrt{-4+x^3}}\,dx,x,\sqrt[3]{4-27x^2}\right)}{\sqrt{3}x}\\
&= -\frac{1}{12}i\left(4-27x^2\right)^{2/3}+\frac{\sqrt{-x^2}\,\text{Subst}\left(\int \frac{2^{2/3}\left(1+\sqrt{3}\right)-x}{\sqrt{-4+x^3}}\,dx,x,\sqrt[3]{4-27x^2}\right)}{\sqrt{3}x}-\frac{\left(2\sqrt[6]{2}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-4+x^3}}\,d\right.}{\sqrt{3\left(2-\sqrt{3}\right)}x}\\
&= -\frac{1}{12}i\left(4-27x^2\right)^{2/3}-\frac{6x}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}}-\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(2^{2/3}-\sqrt[3]{4-27x^2}\right)\sqrt{\frac{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{4-27x^2}\ldots}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{\ldots}\right.}}}{3\,3^{3/4}x\sqrt{-\frac{\ldots}{\left(2^{2/3}\ldots\right.}}}
\end{aligned}$$

Mathematica [C] time = 0.019041, size = 42, normalized size = 0.08

$$\sqrt[3]{2}x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{27x^2}{4}\right)-\frac{1}{12}i\left(4-27x^2\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + (3*I)*x)/(4 - 27*x^2)^(1/3),x]
```

```
[Out] (-I/12)*(4 - 27*x^2)^(2/3) + 2^(1/3)*x*Hypergeometric2F1[1/3, 1/2, 3/2, (27
*x^2)/4]
```

Maple [C] time = 0.247, size = 37, normalized size = 0.1

$$\frac{i}{12}\left(27x^2-4\right)\frac{1}{\sqrt[3]{-27x^2+4}}+\sqrt[3]{2}x{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{27x^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*I*x)/(-27*x^2+4)^(1/3),x)
```

```
[Out] 1/12*I*(27*x^2-4)/(-27*x^2+4)^(1/3)+2^(1/3)*x*hypergeom([1/3,1/2],[3/2],27/
4*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{3ix+2}{\left(-27x^2+4\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*I*x)/(-27*x^2+4)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((3*I*x + 2)/(-27*x^2 + 4)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{36\,x\text{integral}\left(\frac{8\left(-27x^2+4\right)^{\frac{2}{3}}}{9\left(27x^4-4x^2\right)},x\right)+\left(-27x^2+4\right)^{\frac{2}{3}}(-3ix-8)}{36x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*I*x)/(-27*x^2+4)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/36*(36*x*integral(8/9*(-27*x^2 + 4)^(2/3)/(27*x^4 - 4*x^2), x) + (-27*x^2
 + 4)^(2/3)*(-3*I*x - 8))/x
```

Sympy [A] time = 1.95779, size = 39, normalized size = 0.07

$$\sqrt[3]{2}x{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{3}{2}\end{matrix}\middle|\frac{27x^2e^{2i\pi}}{4}\right)-\frac{i\left(4-27x^2\right)^{\frac{2}{3}}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*I*x)/(-27*x**2+4)**(1/3),x)
```

```
[Out] 2**(1/3)*x*hyper((1/3, 1/2), (3/2,), 27*x**2*exp_polar(2*I*pi)/4) - I*(4 -
27*x**2)**(2/3)/12
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3i\,x+2}{\left(-27\,x^2+4\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*I*x)/(-27*x^2+4)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((3*I*x + 2)/(-27*x^2 + 4)^(1/3), x)
```

3.709 $\int \frac{1}{(2+3ix)\sqrt[3]{4-27x^2}}\,dx$

Optimal. Leaf size=109

$$-\frac{i\log\left(27\ 2^{2/3}\sqrt[3]{4-27x^2}+81ix-54\right)}{12\sqrt[3]{2}}+\frac{i\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{\sqrt[3]{2}(2-3ix)}{\sqrt{3}\sqrt[3]{4-27x^2}}\right)}{6\sqrt[3]{2}\sqrt{3}}+\frac{i\log(2+3ix)}{12\sqrt[3]{2}}$$

```
[Out] ((I/6)*ArcTan[1/Sqrt[3] + (2^(1/3)*(2 - (3*I)*x))/(Sqrt[3]*(4 - 27*x^2)^(1/
3))])/(2^(1/3)*Sqrt[3]) + ((I/12)*Log[2 + (3*I)*x])/2^(1/3) - ((I/12)*Log[-
54 + (81*I)*x + 27*2^(2/3)*(4 - 27*x^2)^(1/3)])/2^(1/3)
```

Rubi [A] time = 0.0162252, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {751}

$$-\frac{i\log\left(27\ 2^{2/3}\sqrt[3]{4-27x^2}+81ix-54\right)}{12\sqrt[3]{2}}+\frac{i\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{\sqrt[3]{2}(2-3ix)}{\sqrt{3}\sqrt[3]{4-27x^2}}\right)}{6\sqrt[3]{2}\sqrt{3}}+\frac{i\log(2+3ix)}{12\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + (3*I)*x)*(4 - 27*x^2)^(1/3)),x]
```

```
[Out] ((I/6)*ArcTan[1/Sqrt[3] + (2^(1/3)*(2 - (3*I)*x))/(Sqrt[3]*(4 - 27*x^2)^(1/
3))])/(2^(1/3)*Sqrt[3]) + ((I/12)*Log[2 + (3*I)*x])/2^(1/3) - ((I/12)*Log[-
54 + (81*I)*x + 27*2^(2/3)*(4 - 27*x^2)^(1/3)])/2^(1/3)
```

Rule 751

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/3)), x_Symbol] :> With[
{q = Rt[(6*c^2*e^2)/d^2, 3]}, -Simp[(Sqrt[3]*c*e*ArcTan[1/Sqrt[3] + (2*c*(d
 - e*x))/(Sqrt[3]*d*q*(a + c*x^2)^(1/3))])/(d^2*q^2), x] + (-Simp[(3*c*e*Lo
g[d + e*x])/(2*d^2*q^2), x] + Simp[(3*c*e*Log[c*d - c*e*x - d*q*(a + c*x^2)
^(1/3)])/(2*d^2*q^2), x])] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - 3*a*e^2
, 0]
```

Rubi steps

$$\int \frac{1}{(2+3ix)\sqrt[3]{4-27x^2}}\,dx=\frac{i\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{\sqrt[3]{2}(2-3ix)}{\sqrt{3}\sqrt[3]{4-27x^2}}\right)}{6\sqrt[3]{2}\sqrt{3}}+\frac{i\log(2+3ix)}{12\sqrt[3]{2}}-\frac{i\log\left(-54+81ix+27\ 2^{2/3}\sqrt[3]{4-27x^2}\right)}{12\sqrt[3]{2}}$$

Mathematica [C] time = 0.0829706, size = 125, normalized size = 1.15

$$\frac{i\sqrt[3]{\frac{2\sqrt{3}-9x}{-3x+2i}}\sqrt[3]{\frac{9x+2\sqrt{3}}{3x-2i}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{2\left(3i+\sqrt{3}\right)}{6i-9x},\frac{2\left(-3i+\sqrt{3}\right)}{9x-6i}\right)}{2\ 3^{2/3}\sqrt[3]{4-27x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 + (3*I)*x)*(4 - 27*x^2)^(1/3)),x]
```

```
[Out] ((I/2)*((2*Sqrt[3] - 9*x)/(2*I - 3*x))^(1/3)*((2*Sqrt[3] + 9*x)/(-2*I + 3*x
))^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, (2*(3*I + Sqrt[3]))/(6*I - 9*x), (2*(
-3*I + Sqrt[3]))/(-6*I + 9*x)])/(3^(2/3)*(4 - 27*x^2)^(1/3))
```

Maple [F] time = 0.411, size = 0, normalized size = 0.

$$\int \frac{1}{2+3ix}\frac{1}{\sqrt[3]{-27x^{2}+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*I*x)/(-27*x^2+4)^(1/3),x)
```

```
[Out] int(1/(2+3*I*x)/(-27*x^2+4)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-27x^{2}+4\right)^{\frac{1}{3}}(3ix+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*I*x)/(-27*x^2+4)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((-27*x^2 + 4)^(1/3)*(3*I*x + 2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*I*x)/(-27*x^2+4)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{4-27x^{2}}\,(3ix+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*I*x)/(-27*x**2+4)**(1/3),x)
```

```
[Out] Integral(1/((4 - 27*x**2)**(1/3)*(3*I*x + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-27\,x^2+4\right)^{\frac{1}{3}}\left(3i\,x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*I*x)/(-27*x^2+4)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((-27*x^2 + 4)^(1/3)*(3*I*x + 2)), x)
```

3.710 $\int \frac{1}{(2+3ix)^2 \sqrt[3]{4-27x^2}} \, dx$

Optimal. Leaf size=650

$$\frac{\left(2^{2/3}-\sqrt[3]{4-27x^2}\right)\sqrt{\frac{\left(4-27x^2\right)^{2/3}+2^{2/3}\sqrt[3]{4-27x^2}+2\sqrt[3]{2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{4-27x^2}}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}}\right),4\sqrt{3}-7\right)}{72\sqrt[6]{2}\sqrt[4]{3}\sqrt{-\frac{2^{2/3}-\sqrt[3]{4-27x^2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}\,x}-\frac{3x}{16\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4\right.}$$

```
[Out] ((I/48)*(4 - 27*x^2)^(2/3))/(2 + (3*I)*x) - (3*x)/(16*(2^(2/3)*(1 - Sqrt[3]
) - (4 - 27*x^2)^(1/3))) + ((I/24)*ArcTan[1/Sqrt[3] + (2^(1/3)*(2 - (3*I)*x
))/(Sqrt[3]*(4 - 27*x^2)^(1/3))]])/(2^(1/3)*Sqrt[3]) - (Sqrt[2 + Sqrt[3]]*(2
^(2/3) - (4 - 27*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 - 27*x^2)^(1/3) +
 (4 - 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))^2]*Ellipt
icE[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[
3]) - (4 - 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(48*2^(2/3)*3^(3/4)*x*Sqrt[-((
2^(2/3) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))^
2)]) + ((2^(2/3) - (4 - 27*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 - 27*x^
2)^(1/3) + (4 - 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))
^2]*EllipticF[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 - 27*x^2)^(1/3))/(2^(2/3)*
(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(72*2^(1/6)*3^(1/4)*
x*Sqrt[-((2^(2/3) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^
2)^(1/3))^2)]) + ((I/48)*Log[2 + (3*I)*x])/2^(1/3) - ((I/48)*Log[-54 + (81*
I)*x + 27*2^(2/3)*(4 - 27*x^2)^(1/3)])/2^(1/3)
```

Rubi [A] time = 0.350024, antiderivative size = 650, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {745, 844, 235, 304, 219, 1879, 751}

$$-\frac{3x}{16\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)}+\frac{i\left(4-27x^2\right)^{2/3}}{48(2+3ix)}-\frac{i\log\left(27\,2^{2/3}\sqrt[3]{4-27x^2}+81ix-54\right)}{48\sqrt[3]{2}}+\frac{i\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{\sqrt[3]{2}(2-3ix)}{\sqrt{3}\sqrt[3]{4-27x^2}}\right)}{24\sqrt[3]{2}\sqrt{3}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + (3*I)*x)^2*(4 - 27*x^2)^(1/3)),x]
```

```
[Out] ((I/48)*(4 - 27*x^2)^(2/3))/(2 + (3*I)*x) - (3*x)/(16*(2^(2/3)*(1 - Sqrt[3]
) - (4 - 27*x^2)^(1/3))) + ((I/24)*ArcTan[1/Sqrt[3] + (2^(1/3)*(2 - (3*I)*x
))/(Sqrt[3]*(4 - 27*x^2)^(1/3))]])/(2^(1/3)*Sqrt[3]) - (Sqrt[2 + Sqrt[3]]*(2
^(2/3) - (4 - 27*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 - 27*x^2)^(1/3) +
 (4 - 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))^2]*Ellipt
icE[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[
3]) - (4 - 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(48*2^(2/3)*3^(3/4)*x*Sqrt[-((
2^(2/3) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))^
2)]) + ((2^(2/3) - (4 - 27*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 - 27*x^
2)^(1/3) + (4 - 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))
^2]*EllipticF[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 - 27*x^2)^(1/3))/(2^(2/3)*
(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(72*2^(1/6)*3^(1/4)*
x*Sqrt[-((2^(2/3) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^
2)^(1/3))^2)]) + ((I/48)*Log[2 + (3*I)*x])/2^(1/3) - ((I/48)*Log[-54 + (81*
```

```
I)*x + 27*2^(2/3)*(4 - 27*x^2)^(1/3)])/2^(1/3)
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 751

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/3)), x_Symbol] :> With[
{q = Rt[(6*c^2*e^2)/d^2, 3]}, -Simp[(Sqrt[3]*c*e*ArcTan[1/Sqrt[3] + (2*c*(d
 - e*x))/(Sqrt[3]*d*q*(a + c*x^2)^(1/3))])/(d^2*q^2), x] + (-Simp[(3*c*e*Lo
g[d + e*x])/(2*d^2*q^2), x] + Simp[(3*c*e*Log[c*d - c*e*x - d*q*(a + c*x^2)
^(1/3)])/(2*d^2*q^2), x])] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - 3*a*e^2
```

, 0]

Rubi steps

$$\int \frac{1}{(2+3ix)^2\sqrt[3]{4-27x^2}}\,dx = \frac{i\left(4-27x^2\right)^{2/3}}{48(2+3ix)} - \frac{3}{16}\int \frac{-2-ix}{(2+3ix)\sqrt[3]{4-27x^2}}\,dx$$

$$= \frac{i\left(4-27x^2\right)^{2/3}}{48(2+3ix)} + \frac{1}{16}\int \frac{1}{\sqrt[3]{4-27x^2}}\,dx + \frac{1}{4}\int \frac{1}{(2+3ix)\sqrt[3]{4-27x^2}}\,dx$$

$$= \frac{i\left(4-27x^2\right)^{2/3}}{48(2+3ix)} + \frac{i\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{\sqrt[3]{2}(2-3ix)}{\sqrt{3}\sqrt[3]{4-27x^2}}\right)}{24\sqrt[3]{2}\sqrt{3}} + \frac{i\log(2+3ix)}{48\sqrt[3]{2}} - \frac{i\log\left(-54+81ix+27\,2^{2/3}\sqrt[3]{\cdots}\right.}{48\sqrt[3]{2}}$$

$$= \frac{i\left(4-27x^2\right)^{2/3}}{48(2+3ix)} + \frac{i\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{\sqrt[3]{2}(2-3ix)}{\sqrt{3}\sqrt[3]{4-27x^2}}\right)}{24\sqrt[3]{2}\sqrt{3}} + \frac{i\log(2+3ix)}{48\sqrt[3]{2}} - \frac{i\log\left(-54+81ix+27\,2^{2/3}\sqrt[3]{\cdots}\right.}{48\sqrt[3]{2}}$$

$$= \frac{i\left(4-27x^2\right)^{2/3}}{48(2+3ix)} - \frac{3x}{16\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)} + \frac{i\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{\sqrt[3]{2}(2-3ix)}{\sqrt{3}\sqrt[3]{4-27x^2}}\right)}{24\sqrt[3]{2}\sqrt{3}} - \frac{\sqrt{2+\sqrt{3}}\left(\cdots\right.}{\cdots}$$

Mathematica [C] time = 0.108249, size = 132, normalized size = 0.2

$$\frac{\sqrt[3]{\frac{2\sqrt{3}-9x}{-3x+2i}}\sqrt[3]{\frac{9x+2\sqrt{3}}{3x-2i}}F_1\left(\frac{5}{3};\frac{1}{3},\frac{1}{3};\frac{8}{3};\frac{2\left(3i+\sqrt{3}\right)}{6i-9x},\frac{2\left(-3i+\sqrt{3}\right)}{9x-6i}\right)}{5\,3^{2/3}(3x-2i)\sqrt[3]{4-27x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 + (3*I)*x)^2*(4 - 27*x^2)^(1/3)),x]
```

```
[Out] (((2*Sqrt[3] - 9*x)/(2*I - 3*x))^(1/3)*((2*Sqrt[3] + 9*x)/(-2*I + 3*x))^(1/
3)*AppellF1[5/3, 1/3, 1/3, 8/3, (2*(3*I + Sqrt[3]))/(6*I - 9*x), (2*(-3*I +
 Sqrt[3]))/(-6*I + 9*x)])/(5*3^(2/3)*(-2*I + 3*x)*(4 - 27*x^2)^(1/3))
```

Maple [F] time = 0.436, size = 0, normalized size = 0.

$$\int \frac{1}{(2+3\,ix)^2}\frac{1}{\sqrt[3]{-27\,x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*I*x)^2/(-27*x^2+4)^(1/3),x)
```

```
[Out] int(1/(2+3*I*x)^2/(-27*x^2+4)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-27\,x^2+4\right)^{\frac{1}{3}}(3i\,x+2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*I*x)^2/(-27*x^2+4)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((-27*x^2 + 4)^(1/3)*(3*I*x + 2)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*I*x)^2/(-27*x^2+4)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{4-27x^2}\,(3ix+2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*I*x)**2/(-27*x**2+4)**(1/3),x)
```

```
[Out] Integral(1/((4 - 27*x**2)**(1/3)*(3*I*x + 2)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-27\,x^2+4\right)^{\frac{1}{3}}(3i\,x+2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*I*x)^2/(-27*x^2+4)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((-27*x^2 + 4)^(1/3)*(3*I*x + 2)^2), x)
```

3.711 $\int \frac{1}{(2+3ix)^3\sqrt[3]{4-27x^2}}\,dx$

Optimal. Leaf size=676

$$\frac{\left(2^{2/3}-\sqrt[3]{4-27x^2}\right)\sqrt{\frac{\left(4-27x^2\right)^{2/3}+2^{2/3}\sqrt[3]{4-27x^2}+2\sqrt[3]{2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{4-27x^2}}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}}\right),4\sqrt{3}-7\right)}{144\sqrt[6]{2}\sqrt[4]{3}\sqrt{-\frac{2^{2/3}-\sqrt[3]{4-27x^2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}x}-\frac{3x}{32\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4\right.}}$$

```
[Out] ((I/96)*(4 - 27*x^2)^(2/3))/(2 + (3*I)*x)^2 + ((I/96)*(4 - 27*x^2)^(2/3))/(
2 + (3*I)*x) - (3*x)/(32*(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))) + ((
I/96)*ArcTan[1/Sqrt[3] + (2^(1/3)*(2 - (3*I)*x))/(Sqrt[3]*(4 - 27*x^2)^(1/3
))])/(2^(1/3)*Sqrt[3]) - (Sqrt[2 + Sqrt[3]]*(2^(2/3) - (4 - 27*x^2)^(1/3))*
Sqrt[(2*2^(1/3) + 2^(2/3)*(4 - 27*x^2)^(1/3) + (4 - 27*x^2)^(2/3))/(2^(2/3)
*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))^2]*EllipticE[ArcSin[(2^(2/3)*(1 + Sqrt
[3]) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))], -
7 + 4*Sqrt[3]])/(96*2^(2/3)*3^(3/4)*x*Sqrt[-((2^(2/3) - (4 - 27*x^2)^(1/3))
/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))^2)]) + ((2^(2/3) - (4 - 27*x^
2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 - 27*x^2)^(1/3) + (4 - 27*x^2)^(2/3)
)/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))^2]*EllipticF[ArcSin[(2^(2/3)
*(1 + Sqrt[3]) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^
(1/3))], -7 + 4*Sqrt[3]])/(144*2^(1/6)*3^(1/4)*x*Sqrt[-((2^(2/3) - (4 - 27*
x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))^2)]) + ((I/192)*Lo
g[2 + (3*I)*x])/2^(1/3) - ((I/192)*Log[-54 + (81*I)*x + 27*2^(2/3)*(4 - 27*
x^2)^(1/3)])/2^(1/3)
```

Rubi [A] time = 0.412514, antiderivative size = 676, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {745, 835, 844, 235, 304, 219, 1879, 751}

$$-\frac{3x}{32\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)}+\frac{i\left(4-27x^2\right)^{2/3}}{96(2+3ix)}+\frac{i\left(4-27x^2\right)^{2/3}}{96(2+3ix)^2}-\frac{i\log\left(27\,2^{2/3}\sqrt[3]{4-27x^2}+81ix-54\right)}{192\sqrt[3]{2}}+\frac{i\tan^{-1}\left(\right.}{9}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + (3*I)*x)^3*(4 - 27*x^2)^(1/3)),x]
```

```
[Out] ((I/96)*(4 - 27*x^2)^(2/3))/(2 + (3*I)*x)^2 + ((I/96)*(4 - 27*x^2)^(2/3))/(
2 + (3*I)*x) - (3*x)/(32*(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))) + ((
I/96)*ArcTan[1/Sqrt[3] + (2^(1/3)*(2 - (3*I)*x))/(Sqrt[3]*(4 - 27*x^2)^(1/3
))])/(2^(1/3)*Sqrt[3]) - (Sqrt[2 + Sqrt[3]]*(2^(2/3) - (4 - 27*x^2)^(1/3))*
Sqrt[(2*2^(1/3) + 2^(2/3)*(4 - 27*x^2)^(1/3) + (4 - 27*x^2)^(2/3))/(2^(2/3)
*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))^2]*EllipticE[ArcSin[(2^(2/3)*(1 + Sqrt
[3]) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))], -
7 + 4*Sqrt[3]])/(96*2^(2/3)*3^(3/4)*x*Sqrt[-((2^(2/3) - (4 - 27*x^2)^(1/3))
/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))^2)]) + ((2^(2/3) - (4 - 27*x^
2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 - 27*x^2)^(1/3) + (4 - 27*x^2)^(2/3)
)/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))^2]*EllipticF[ArcSin[(2^(2/3)
*(1 + Sqrt[3]) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^
(1/3))], -7 + 4*Sqrt[3]])/(144*2^(1/6)*3^(1/4)*x*Sqrt[-((2^(2/3) - (4 - 27*
x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))^2)]) + ((I/192)*Lo
```

```
g[2 + (3*I)*x])/2^(1/3) - ((I/192)*Log[-54 + (81*I)*x + 27*2^(2/3)*(4 - 27*
x^2)^(1/3)])/2^(1/3)
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
```

```
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 751

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/3)), x_Symbol] :> With[
{q = Rt[(6*c^2*e^2)/d^2, 3]}, -Simp[(Sqrt[3]*c*e*ArcTan[1/Sqrt[3] + (2*c*(d
 - e*x))/(Sqrt[3]*d*q*(a + c*x^2)^(1/3))])/(d^2*q^2), x] + (-Simp[(3*c*e*Lo
g[d + e*x])/(2*d^2*q^2), x] + Simp[(3*c*e*Log[c*d - c*e*x - d*q*(a + c*x^2)
^(1/3)])/(2*d^2*q^2), x])] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - 3*a*e^2
, 0]
```

Rubi steps

$$\int \frac{1}{(2+3ix)^3\sqrt[3]{4-27x^2}}\,dx = \frac{i\left(4-27x^2\right)^{2/3}}{96(2+3ix)^2} - \frac{3}{32}\int \frac{-4+2ix}{(2+3ix)^2\sqrt[3]{4-27x^2}}\,dx$$

$$= \frac{i\left(4-27x^2\right)^{2/3}}{96(2+3ix)^2} + \frac{i\left(4-27x^2\right)^{2/3}}{96(2+3ix)} - \frac{\int \frac{-192-144ix}{(2+3ix)\sqrt[3]{4-27x^2}}\,dx}{1536}$$

$$= \frac{i\left(4-27x^2\right)^{2/3}}{96(2+3ix)^2} + \frac{i\left(4-27x^2\right)^{2/3}}{96(2+3ix)} + \frac{1}{32}\int \frac{1}{\sqrt[3]{4-27x^2}}\,dx + \frac{1}{16}\int \frac{1}{(2+3ix)\sqrt[3]{4-27x^2}}\,dx$$

$$= \frac{i\left(4-27x^2\right)^{2/3}}{96(2+3ix)^2} + \frac{i\left(4-27x^2\right)^{2/3}}{96(2+3ix)} + \frac{i\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{\sqrt[3]{2}(2-3ix)}{\sqrt{3}\sqrt[3]{4-27x^2}}\right)}{96\sqrt[3]{2}\sqrt{3}} + \frac{i\log(2+3ix)}{192\sqrt[3]{2}} - \frac{i\log\left(-54\right.}{}$$

$$= \frac{i\left(4-27x^2\right)^{2/3}}{96(2+3ix)^2} + \frac{i\left(4-27x^2\right)^{2/3}}{96(2+3ix)} + \frac{i\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{\sqrt[3]{2}(2-3ix)}{\sqrt{3}\sqrt[3]{4-27x^2}}\right)}{96\sqrt[3]{2}\sqrt{3}} + \frac{i\log(2+3ix)}{192\sqrt[3]{2}} - \frac{i\log\left(-54\right.}{}$$

$$= \frac{i\left(4-27x^2\right)^{2/3}}{96(2+3ix)^2} + \frac{i\left(4-27x^2\right)^{2/3}}{96(2+3ix)} - \frac{3x}{32\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)} + \frac{i\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{\sqrt[3]{2}(2-3i}{\sqrt{3}\sqrt[3]{4-2}}\right.}{96\sqrt[3]{2}\sqrt{3}}$$

Mathematica [C] time = 0.100555, size = 134, normalized size = 0.2

$$-\frac{i\sqrt[3]{\frac{2\sqrt{3}-9x}{-3x+2i}}\sqrt[3]{\frac{9x+2\sqrt{3}}{3x-2i}}F_1\left(\frac{8}{3};\frac{1}{3},\frac{1}{3};\frac{11}{3};\frac{2\left(3i+\sqrt{3}\right)}{6i-9x},\frac{2\left(-3i+\sqrt{3}\right)}{9x-6i}\right)}{8\ 3^{2/3}(-3x+2i)^2\sqrt[3]{4-27x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 + (3*I)*x)^3*(4 - 27*x^2)^(1/3)),x]
```

```
[Out] ((-I/8)*((2*Sqrt[3] - 9*x)/(2*I - 3*x))^(1/3)*((2*Sqrt[3] + 9*x)/(-2*I + 3*
x))^(1/3)*AppellF1[8/3, 1/3, 1/3, 11/3, (2*(3*I + Sqrt[3]))/(6*I - 9*x), (2
*(-3*I + Sqrt[3]))/(-6*I + 9*x)])/(3^(2/3)*(2*I - 3*x)^2*(4 - 27*x^2)^(1/3)
)
```

Maple [F] time = 0.463, size = 0, normalized size = 0.

$$\int \frac{1}{\left(2+3 i x\right)^{3}}\frac{1}{\sqrt[3]{-27 x^{2}+4}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*I*x)^3/(-27*x^2+4)^(1/3),x)
```

```
[Out] int(1/(2+3*I*x)^3/(-27*x^2+4)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-27 x^{2}+4\right)^{\frac{1}{3}} \left(3 i x+2\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*I*x)^3/(-27*x^2+4)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((-27*x^2 + 4)^(1/3)*(3*I*x + 2)^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*I*x)^3/(-27*x^2+4)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{4-27x^{2}} \left(3ix+2\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*I*x)**3/(-27*x**2+4)**(1/3),x)
```

```
[Out] Integral(1/((4 - 27*x**2)**(1/3)*(3*I*x + 2)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-27 x^{2}+4\right)^{\frac{1}{3}} \left(3 i x+2\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*I*x)^3/(-27*x^2+4)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((-27*x^2 + 4)^(1/3)*(3*I*x + 2)^3), x)
```

3.712 $\int \frac{1}{\left(\sqrt{3}+x\right)\sqrt[3]{1+x^2}}\,dx$

Optimal. Leaf size=104

$$\frac{\log\left(-\sqrt[3]{2}\sqrt{3}\sqrt[3]{x^2+1}-x+\sqrt{3}\right)}{2\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\left(\sqrt{3}-x\right)}{3\sqrt[3]{x^2+1}}+\frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log\left(x+\sqrt{3}\right)}{2\ 2^{2/3}}$$

```
[Out] -(ArcTan[1/Sqrt[3] + (2^(2/3)*(Sqrt[3] - x))/(3*(1 + x^2)^(1/3))]/(2^(2/3)*
Sqrt[3])) - Log[Sqrt[3] + x]/(2*2^(2/3)) + Log[Sqrt[3] - x - 2^(1/3)*Sqrt[3
]*(1 + x^2)^(1/3)]/(2*2^(2/3))
```

Rubi [A] time = 0.0129699, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {751}

$$\frac{\log\left(-\sqrt[3]{2}\sqrt{3}\sqrt[3]{x^2+1}-x+\sqrt{3}\right)}{2\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\left(\sqrt{3}-x\right)}{3\sqrt[3]{x^2+1}}+\frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log\left(x+\sqrt{3}\right)}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((Sqrt[3] + x)*(1 + x^2)^(1/3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] + (2^(2/3)*(Sqrt[3] - x))/(3*(1 + x^2)^(1/3))]/(2^(2/3)*
Sqrt[3])) - Log[Sqrt[3] + x]/(2*2^(2/3)) + Log[Sqrt[3] - x - 2^(1/3)*Sqrt[3
]*(1 + x^2)^(1/3)]/(2*2^(2/3))
```

Rule 751

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/3)), x_Symbol] :> With[
{q = Rt[(6*c^2*e^2)/d^2, 3]}, -Simp[(Sqrt[3]*c*e*ArcTan[1/Sqrt[3] + (2*c*(d
 - e*x))/(Sqrt[3]*d*q*(a + c*x^2)^(1/3))])/(d^2*q^2), x] + (-Simp[(3*c*e*Lo
g[d + e*x])/(2*d^2*q^2), x] + Simp[(3*c*e*Log[c*d - c*e*x - d*q*(a + c*x^2)
^(1/3)])/(2*d^2*q^2), x])] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - 3*a*e^2
, 0]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{3}+x\right)\sqrt[3]{1+x^2}}\,dx=-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2^{2/3}\left(\sqrt{3}-x\right)}{3\sqrt[3]{1+x^2}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log\left(\sqrt{3}+x\right)}{2\ 2^{2/3}}+\frac{\log\left(\sqrt{3}-x-\sqrt[3]{2}\sqrt{3}\sqrt[3]{1+x^2}\right)}{2\ 2^{2/3}}$$

Mathematica [C] time = 0.0800111, size = 102, normalized size = 0.98

$$-\frac{3\sqrt[3]{\frac{x-i}{x+\sqrt{3}}}\sqrt[3]{\frac{x+i}{x+\sqrt{3}}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{-i+\sqrt{3}}{x+\sqrt{3}},\frac{i+\sqrt{3}}{x+\sqrt{3}}\right)}{2\sqrt[3]{x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((Sqrt[3] + x)*(1 + x^2)^(1/3)),x]
```

```
[Out] (-3*((-I + x)/(Sqrt[3] + x))^(1/3)*((I + x)/(Sqrt[3] + x))^(1/3)*AppellF1[2
/3, 1/3, 1/3, 5/3, (-I + Sqrt[3])/(Sqrt[3] + x), (I + Sqrt[3])/(Sqrt[3] + x
)])/(2*(1 + x^2)^(1/3))
```

Maple [F] time = 0.377, size = 0, normalized size = 0.

$$\int \frac{1}{x+\sqrt{3}}\frac{1}{\sqrt[3]{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(1/3)/(x+3^(1/2)),x)
```

```
[Out] int(1/(x^2+1)^(1/3)/(x+3^(1/2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+1\right)^{\frac{1}{3}}\left(x+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/3)/(x+3^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 1)^(1/3)*(x + sqrt(3))), x)
```

Fricas [B] time = 43.6501, size = 799, normalized size = 7.68

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(6\cdot 4^{\frac{2}{3}}\left(x^4+8\sqrt{3}x^3-18\,x^2-27\right)\left(x^2+1\right)^{\frac{2}{3}}+4^{\frac{1}{3}}\left(x^6+99\,x^4+243\,x^2+12\sqrt{3}\left(x^5+10\,x^3+9\,x\right)\right.\right.}{6\left(x^6-225\,x^4-405\,x^2-243\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/3)/(x+3^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(6*4^(2/3)*(x^4 + 8*sqrt(3)
*x^3 - 18*x^2 - 27)*(x^2 + 1)^(2/3) + 4^(1/3)*(x^6 + 99*x^4 + 243*x^2 + 12*
sqrt(3)*(x^5 + 10*x^3 + 9*x) + 81) + 4*(21*x^4 + 54*x^2 + sqrt(3)*(x^5 - 42
*x^3 - 27*x) + 81)*(x^2 + 1)^(1/3))/(x^6 - 225*x^4 - 405*x^2 - 243)) - 1/24
*4^(2/3)*log((3*4^(2/3)*(x^2 - 2*sqrt(3)*x + 3)*(x^2 + 1)^(2/3) + 4^(1/3)*(
x^4 + 18*x^2 - 4*sqrt(3)*(x^3 + 3*x) + 9) + 2*(9*x^2 - sqrt(3)*(x^3 + 9*x)
+ 9)*(x^2 + 1)^(1/3))/(x^4 - 6*x^2 + 9)) + 1/12*4^(2/3)*log((4^(1/3)*(x^2 -
 2*sqrt(3)*x + 3) + 2*(x^2 + 1)^(1/3)*(sqrt(3)*x - 3))/(x^2 - 3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x+\sqrt{3}\right)\sqrt[3]{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(1/3)/(x+3**(1/2)),x)
```

```
[Out] Integral(1/((x + sqrt(3))*(x**2 + 1)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+1\right)^{\frac{1}{3}}\left(x+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/3)/(x+3^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 1)^(1/3)*(x + sqrt(3))), x)
```

3.713 $\int \frac{1}{\left(\sqrt{3}-x\right)\sqrt[3]{1+x^2}}\,dx$

Optimal. Leaf size=101

$$-\frac{\log\left(-\sqrt[3]{2}\sqrt{3}\sqrt[3]{x^2+1}+x+\sqrt{3}\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\left(x+\sqrt{3}\right)}{3\sqrt[3]{x^2+1}}+\frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(\sqrt{3}-x\right)}{2\ 2^{2/3}}$$

```
[Out] ArcTan[1/Sqrt[3] + (2^(2/3)*(Sqrt[3] + x))/(3*(1 + x^2)^(1/3))]/(2^(2/3)*Sq
rt[3]) + Log[Sqrt[3] - x]/(2*2^(2/3)) - Log[Sqrt[3] + x - 2^(1/3)*Sqrt[3]*(
1 + x^2)^(1/3)]/(2*2^(2/3))
```

Rubi [A] time = 0.0144118, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {751}

$$-\frac{\log\left(-\sqrt[3]{2}\sqrt{3}\sqrt[3]{x^2+1}+x+\sqrt{3}\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\left(x+\sqrt{3}\right)}{3\sqrt[3]{x^2+1}}+\frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(\sqrt{3}-x\right)}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((Sqrt[3] - x)*(1 + x^2)^(1/3)),x]
```

```
[Out] ArcTan[1/Sqrt[3] + (2^(2/3)*(Sqrt[3] + x))/(3*(1 + x^2)^(1/3))]/(2^(2/3)*Sq
rt[3]) + Log[Sqrt[3] - x]/(2*2^(2/3)) - Log[Sqrt[3] + x - 2^(1/3)*Sqrt[3]*(
1 + x^2)^(1/3)]/(2*2^(2/3))
```

Rule 751

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/3)), x_Symbol] :> With[
{q = Rt[(6*c^2*e^2)/d^2, 3]}, -Simp[(Sqrt[3]*c*e*ArcTan[1/Sqrt[3] + (2*c*(d
 - e*x))/(Sqrt[3]*d*q*(a + c*x^2)^(1/3))])/(d^2*q^2), x] + (-Simp[(3*c*e*Lo
g[d + e*x])/(2*d^2*q^2), x] + Simp[(3*c*e*Log[c*d - c*e*x - d*q*(a + c*x^2)
^(1/3)])/(2*d^2*q^2), x])] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - 3*a*e^2
, 0]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{3}-x\right)\sqrt[3]{1+x^2}}\,dx=\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2^{2/3}\left(\sqrt{3}+x\right)}{3\sqrt[3]{1+x^2}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(\sqrt{3}-x\right)}{2\ 2^{2/3}}-\frac{\log\left(\sqrt{3}+x-\sqrt[3]{2}\sqrt{3}\sqrt[3]{1+x^2}\right)}{2\ 2^{2/3}}$$

Mathematica [C] time = 0.0828518, size = 110, normalized size = 1.09

$$\frac{3\sqrt[3]{\frac{x-i}{x-\sqrt{3}}}\sqrt[3]{\frac{x+i}{x-\sqrt{3}}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{-i+\sqrt{3}}{\sqrt{3}-x},\frac{i+\sqrt{3}}{\sqrt{3}-x}\right)}{2\sqrt[3]{x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((Sqrt[3] - x)*(1 + x^2)^(1/3)),x]
```

```
[Out] (3*((-I + x)/(-Sqrt[3] + x))^(1/3)*((I + x)/(-Sqrt[3] + x))^(1/3)*AppellF1[
2/3, 1/3, 1/3, 5/3, (-I + Sqrt[3])/(Sqrt[3] - x), (I + Sqrt[3])/(Sqrt[3] -
x)])/(2*(1 + x^2)^(1/3))
```

Maple [F] time = 0.413, size = 0, normalized size = 0.

$$\int \frac{1}{-x+\sqrt{3}}\frac{1}{\sqrt[3]{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(1/3)/(-x+3^(1/2)),x)
```

```
[Out] int(1/(x^2+1)^(1/3)/(-x+3^(1/2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(x^2+1\right)^{\frac{1}{3}}\left(x-\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/3)/(-x+3^(1/2)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((x^2 + 1)^(1/3)*(x - sqrt(3))), x)
```

Fricas [B] time = 40.936, size = 1006, normalized size = 9.96

$$\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(6\cdot 4^{\frac{2}{3}}\left(8\sqrt{3}\,(-1)^{\frac{2}{3}}x^3-(-1)^{\frac{2}{3}}\left(x^4-18x^2-27\right)\right)\left(x^2+1\right)^{\frac{2}{3}}-4^{\frac{1}{3}}\left(x^6+99x^4+243x^2-1\right.\right.}{6\left(x^6-22\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/3)/(-x+3^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*sqrt(3)*(6*4^(2/3)*(8*sqr
t(3)*(-1)^(2/3)*x^3 - (-1)^(2/3)*(x^4 - 18*x^2 - 27))*(x^2 + 1)^(2/3) - 4^(
1/3)*(x^6 + 99*x^4 + 243*x^2 - 12*sqrt(3)*(x^5 + 10*x^3 + 9*x) + 81) - 4*(x
^2 + 1)^(1/3)*(sqrt(3)*(-1)^(1/3)*(x^5 - 42*x^3 - 27*x) - 3*(-1)^(1/3)*(7*x
^4 + 18*x^2 + 27)))/(x^6 - 225*x^4 - 405*x^2 - 243)) - 1/24*4^(2/3)*(-1)^(1
/3)*log(-(3*4^(2/3)*(x^2 + 1)^(2/3)*(2*sqrt(3)*(-1)^(1/3)*x + (-1)^(1/3)*(x
^2 + 3)) - 4^(1/3)*(4*sqrt(3)*(-1)^(2/3)*(x^3 + 3*x) + (-1)^(2/3)*(x^4 + 18
*x^2 + 9)) - 2*(9*x^2 + sqrt(3)*(x^3 + 9*x) + 9)*(x^2 + 1)^(1/3))/(x^4 - 6*
x^2 + 9)) + 1/12*4^(2/3)*(-1)^(1/3)*log(-(4^(1/3)*(x^2 + 2*sqrt(3)*x + 3) +
 2*(x^2 + 1)^(1/3)*(sqrt(3)*(-1)^(1/3)*x + 3*(-1)^(1/3)))/(x^2 - 3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{x\sqrt[3]{x^2+1} - \sqrt{3}\sqrt[3]{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(1/3)/(-x+3**(1/2)),x)
```

```
[Out] -Integral(1/(x*(x**2 + 1)**(1/3) - sqrt(3)*(x**2 + 1)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(x^2+1)^{\frac{1}{3}}\left(x-\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/3)/(-x+3^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(-1/((x^2 + 1)^(1/3)*(x - sqrt(3))), x)
```

3.714 $\int \frac{1}{(3-x)\sqrt[3]{1-x^2}}\,dx$

Optimal. Leaf size=78

$$-\frac{1}{4}\log(3-x)+\frac{3}{8}\log\left(-\frac{1}{2}(x+1)^{2/3}-\sqrt[3]{1-x}\right)-\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{(x+1)^{2/3}}{\sqrt{3}\sqrt[3]{1-x}}\right)$$

```
[Out] -(Sqrt[3]*ArcTan[1/Sqrt[3] - (1 + x)^(2/3)/(Sqrt[3]*(1 - x)^(1/3))])/4 - Lo
g[3 - x]/4 + (3*Log[-(1 - x)^(1/3) - (1 + x)^(2/3)/2])/8
```

Rubi [A] time = 0.0155426, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {753, 123}

$$-\frac{1}{4}\log(3-x)+\frac{3}{8}\log\left(-\frac{1}{2}(x+1)^{2/3}-\sqrt[3]{1-x}\right)-\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{(x+1)^{2/3}}{\sqrt{3}\sqrt[3]{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x)*(1 - x^2)^(1/3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[1/Sqrt[3] - (1 + x)^(2/3)/(Sqrt[3]*(1 - x)^(1/3))])/4 - Lo
g[3 - x]/4 + (3*Log[-(1 - x)^(1/3) - (1 + x)^(2/3)/2])/8
```

Rule 753

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/3)), x_Symbol] :> Dist[
a^(1/3), Int[1/((d + e*x)*(1 - (3*e*x)/d)^(1/3)*(1 + (3*e*x)/d)^(1/3)), x],
 x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + 9*a*e^2, 0] && GtQ[a, 0]
```

Rule 123

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)*((e_.) + (f_.)*(x_))
^(1/3)), x_Symbol] :> With[{q = Rt[(b*(b*e - a*f))/(b*c - a*d)^2, 3]}, -Sim
p[Log[a + b*x]/(2*q*(b*c - a*d)), x] + (-Simp[(Sqrt[3]*ArcTan[1/Sqrt[3] + (
2*q*(c + d*x)^(2/3))/(Sqrt[3]*(e + f*x)^(1/3))])/(2*q*(b*c - a*d)), x] + Si
mp[(3*Log[q*(c + d*x)^(2/3) - (e + f*x)^(1/3)])/(4*q*(b*c - a*d)), x])] /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - b*c*f - a*d*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(3-x)\sqrt[3]{1-x^2}}\,dx &= \int \frac{1}{\sqrt[3]{1-x}(3-x)\sqrt[3]{1+x}}\,dx\\ &= -\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{(1+x)^{2/3}}{\sqrt{3}\sqrt[3]{1-x}}\right)-\frac{1}{4}\log(3-x)+\frac{3}{8}\log\left(-\sqrt[3]{1-x}-\frac{1}{2}(1+x)^{2/3}\right)\end{aligned}$$

Mathematica [C] time = 0.0527986, size = 68, normalized size = 0.87

$$\frac{3\sqrt[3]{\frac{x-1}{x-3}}\sqrt[3]{\frac{x+1}{x-3}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};-\frac{4}{x-3},-\frac{2}{x-3}\right)}{2\sqrt[3]{1-x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3 - x)*(1 - x^2)^(1/3)),x]
```

```
[Out] (3*((-1 + x)/(-3 + x))^(1/3)*((1 + x)/(-3 + x))^(1/3)*AppellF1[2/3, 1/3, 1/
3, 5/3, -4/(-3 + x), -2/(-3 + x)])/(2*(1 - x^2)^(1/3))
```

Maple [F] time = 0.418, size = 0, normalized size = 0.

$$\int \frac{1}{3-x}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-x)/(-x^2+1)^(1/3),x)
```

```
[Out] int(1/(3-x)/(-x^2+1)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(-x^2+1\right)^{\frac{1}{3}}(x-3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)/(-x^2+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -integrate(1/((-x^2 + 1)^(1/3)*(x - 3)), x)
```

Fricas [A] time = 5.59644, size = 350, normalized size = 4.49

$$-\frac{1}{4}\sqrt{3}\arctan\left(\frac{18031\sqrt{3}\left(-x^2+1\right)^{\frac{1}{3}}(x+1)+\sqrt{3}\left(5054x^2-8497x+23659\right)+57889\sqrt{3}\left(-x^2+1\right)^{\frac{2}{3}}}{6859x^2+240699x-220122}\right)+\frac{1}{8}\log\left(\frac{x^2+6(\cdot}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)/(-x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(3)*arctan((18031*sqrt(3)*(-x^2 + 1)^(1/3)*(x + 1) + sqrt(3)*(5054
*x^2 - 8497*x + 23659) + 57889*sqrt(3)*(-x^2 + 1)^(2/3))/(6859*x^2 + 240699
*x - 220122)) + 1/8*log((x^2 + 6*(-x^2 + 1)^(1/3)*(x + 1) - 6*x + 12*(-x^2
+ 1)^(2/3) + 9)/(x^2 - 6*x + 9))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{x\sqrt[3]{1-x^2}-3\sqrt[3]{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)/(-x**2+1)**(1/3),x)
```

```
[Out] -Integral(1/(x*(1 - x**2)**(1/3) - 3*(1 - x**2)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(-x^2+1\right)^{\frac{1}{3}}(x-3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)/(-x^2+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-1/((-x^2 + 1)^(1/3)*(x - 3)), x)
```

3.715 $\int \frac{1}{(3+x)\sqrt[3]{1-x^2}}\,dx$

Optimal. Leaf size=76

$$\frac{1}{4}\log(x+3)-\frac{3}{8}\log\left(-\frac{1}{2}(1-x)^{2/3}-\sqrt[3]{x+1}\right)+\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{(1-x)^{2/3}}{\sqrt{3}\sqrt[3]{x+1}}\right)$$

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] - (1 - x)^(2/3)/(Sqrt[3]*(1 + x)^(1/3))])/4 + Log
[3 + x]/4 - (3*Log[-(1 - x)^(2/3)/2 - (1 + x)^(1/3)])/8
```

Rubi [A] time = 0.0144284, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {753, 123}

$$\frac{1}{4}\log(x+3)-\frac{3}{8}\log\left(-\frac{1}{2}(1-x)^{2/3}-\sqrt[3]{x+1}\right)+\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{(1-x)^{2/3}}{\sqrt{3}\sqrt[3]{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 + x)*(1 - x^2)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] - (1 - x)^(2/3)/(Sqrt[3]*(1 + x)^(1/3))])/4 + Log
[3 + x]/4 - (3*Log[-(1 - x)^(2/3)/2 - (1 + x)^(1/3)])/8
```

Rule 753

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/3)), x_Symbol] :> Dist[
a^(1/3), Int[1/((d + e*x)*(1 - (3*e*x)/d)^(1/3)*(1 + (3*e*x)/d)^(1/3)), x],
 x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + 9*a*e^2, 0] && GtQ[a, 0]
```

Rule 123

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)*((e_.) + (f_.)*(x_))
^(1/3)), x_Symbol] :> With[{q = Rt[(b*(b*e - a*f))/(b*c - a*d)^2, 3]}, -Sim
p[Log[a + b*x]/(2*q*(b*c - a*d)), x] + (-Simp[(Sqrt[3]*ArcTan[1/Sqrt[3] + (
2*q*(c + d*x)^(2/3))/(Sqrt[3]*(e + f*x)^(1/3))])/(2*q*(b*c - a*d)), x] + Si
mp[(3*Log[q*(c + d*x)^(2/3) - (e + f*x)^(1/3)])/(4*q*(b*c - a*d)), x])] /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - b*c*f - a*d*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(3+x)\sqrt[3]{1-x^2}}\,dx&=\int \frac{1}{\sqrt[3]{1-x}\sqrt[3]{1+x}(3+x)}\,dx\\&=\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{(1-x)^{2/3}}{\sqrt{3}\sqrt[3]{1+x}}\right)+\frac{1}{4}\log(3+x)-\frac{3}{8}\log\left(-\frac{1}{2}(1-x)^{2/3}-\sqrt[3]{1+x}\right)\end{aligned}$$

Mathematica [C] time = 0.039634, size = 68, normalized size = 0.89

$$-\frac{3\sqrt[3]{\frac{x-1}{x+3}}\sqrt[3]{\frac{x+1}{x+3}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{4}{x+3},\frac{2}{x+3}\right)}{2\sqrt[3]{1-x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3 + x)*(1 - x^2)^(1/3)),x]
```

```
[Out] (-3*((-1 + x)/(3 + x))^(1/3)*((1 + x)/(3 + x))^(1/3)*AppellF1[2/3, 1/3, 1/3
, 5/3, 4/(3 + x), 2/(3 + x)])/(2*(1 - x^2)^(1/3))
```

Maple [F] time = 0.392, size = 0, normalized size = 0.

$$\int \frac{1}{3+x}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+x)/(-x^2+1)^(1/3),x)
```

```
[Out] int(1/(3+x)/(-x^2+1)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^2+1\right)^{\frac{1}{3}}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(-x^2+1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x^2 + 1)^(1/3)*(x + 3)), x)
```

Fricas [B] time = 5.34306, size = 350, normalized size = 4.61

$$\frac{1}{4}\sqrt{3}\arctan\left(-\frac{18031\sqrt{3}\left(-x^2+1\right)^{\frac{1}{3}}(x-1)-\sqrt{3}\left(5054\,x^2+8497\,x+23659\right)-57889\sqrt{3}\left(-x^2+1\right)^{\frac{2}{3}}}{6859\,x^2-240699\,x-220122}\right)-\frac{1}{8}\log\left(\frac{x^2-6}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(-x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(3)*arctan(-(18031*sqrt(3)*(-x^2 + 1)^(1/3)*(x - 1) - sqrt(3)*(5054
*x^2 + 8497*x + 23659) - 57889*sqrt(3)*(-x^2 + 1)^(2/3))/(6859*x^2 - 240699
*x - 220122)) - 1/8*log((x^2 - 6*(-x^2 + 1)^(1/3)*(x - 1) + 6*x + 12*(-x^2
+ 1)^(2/3) + 9)/(x^2 + 6*x + 9))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{-(x-1)(x+1)}\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(-x**2+1)**(1/3),x)
```

```
[Out] Integral(1/((-(x - 1)*(x + 1))**(1/3)*(x + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^2+1\right)^{\frac{1}{3}}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(-x^2+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((-x^2 + 1)^(1/3)*(x + 3)), x)
```

3.716 $\int \frac{1}{(d+ex)\sqrt[3]{d^2-9e^2x^2}}\,dx$

Optimal. Leaf size=206

$$\frac{\sqrt[3]{1-\frac{9e^2x^2}{d^2}}\log(d+ex)}{4e\sqrt[3]{d^2-9e^2x^2}}-\frac{3\sqrt[3]{1-\frac{9e^2x^2}{d^2}}\log\left(-\frac{1}{2}\left(1-\frac{3ex}{d}\right)^{2/3}-\sqrt[3]{\frac{3ex}{d}+1}\right)}{8e\sqrt[3]{d^2-9e^2x^2}}+\frac{\sqrt{3}\sqrt[3]{1-\frac{9e^2x^2}{d^2}}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{\left(1-\frac{3ex}{d}\right)^{2/3}}{\sqrt{3}\sqrt[3]{\frac{3ex}{d}+1}}\right)}{4e\sqrt[3]{d^2-9e^2x^2}}$$

```
[Out] (Sqrt[3]*(1 - (9*e^2*x^2)/d^2)^(1/3)*ArcTan[1/Sqrt[3] - (1 - (3*e*x)/d)^(2/
3)/(Sqrt[3]*(1 + (3*e*x)/d)^(1/3))])/(4*e*(d^2 - 9*e^2*x^2)^(1/3)) + ((1 -
(9*e^2*x^2)/d^2)^(1/3)*Log[d + e*x])/(4*e*(d^2 - 9*e^2*x^2)^(1/3)) - (3*(1
- (9*e^2*x^2)/d^2)^(1/3)*Log[-(1 - (3*e*x)/d)^(2/3)/2 - (1 + (3*e*x)/d)^(1/
3)])/(8*e*(d^2 - 9*e^2*x^2)^(1/3))
```

Rubi [A] time = 0.0651082, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {754, 753, 123}

$$\frac{\sqrt[3]{1-\frac{9e^2x^2}{d^2}}\log(d+ex)}{4e\sqrt[3]{d^2-9e^2x^2}}-\frac{3\sqrt[3]{1-\frac{9e^2x^2}{d^2}}\log\left(-\frac{1}{2}\left(1-\frac{3ex}{d}\right)^{2/3}-\sqrt[3]{\frac{3ex}{d}+1}\right)}{8e\sqrt[3]{d^2-9e^2x^2}}+\frac{\sqrt{3}\sqrt[3]{1-\frac{9e^2x^2}{d^2}}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{\left(1-\frac{3ex}{d}\right)^{2/3}}{\sqrt{3}\sqrt[3]{\frac{3ex}{d}+1}}\right)}{4e\sqrt[3]{d^2-9e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(d^2 - 9*e^2*x^2)^(1/3)),x]
```


```
[Out] (Sqrt[3]*(1 - (9*e^2*x^2)/d^2)^(1/3)*ArcTan[1/Sqrt[3] - (1 - (3*e*x)/d)^(2/
3)/(Sqrt[3]*(1 + (3*e*x)/d)^(1/3))])/(4*e*(d^2 - 9*e^2*x^2)^(1/3)) + ((1 -
(9*e^2*x^2)/d^2)^(1/3)*Log[d + e*x])/(4*e*(d^2 - 9*e^2*x^2)^(1/3)) - (3*(1
- (9*e^2*x^2)/d^2)^(1/3)*Log[-(1 - (3*e*x)/d)^(2/3)/2 - (1 + (3*e*x)/d)^(1/
3)])/(8*e*(d^2 - 9*e^2*x^2)^(1/3))
```


Rule 754

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/3)), x_Symbol] :> Dist[
(1 + (c*x^2)/a)^(1/3)/(a + c*x^2)^(1/3), Int[1/((d + e*x)*(1 + (c*x^2)/a)^(
1/3)), x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + 9*a*e^2, 0] &&  !GtQ
[a, 0]
```

Rule 753

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/3)), x_Symbol] :> Dist[
a^(1/3), Int[1/((d + e*x)*(1 - (3*e*x)/d)^(1/3)*(1 + (3*e*x)/d)^(1/3)), x],
 x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + 9*a*e^2, 0] && GtQ[a, 0]
```

Rule 123

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)*((e_.) + (f_.)*(x_))
^(1/3)), x_Symbol] :> With[{q = Rt[(b*(b*e - a*f))/(b*c - a*d)^2, 3]}, -Sim
p[Log[a + b*x]/(2*q*(b*c - a*d)), x] + (-Simp[(Sqrt[3]*ArcTan[1/Sqrt[3] + (
2*q*(c + d*x)^(2/3))/(Sqrt[3]*(e + f*x)^(1/3))])/(2*q*(b*c - a*d)), x] + Si
mp[(3*Log[q*(c + d*x)^(2/3) - (e + f*x)^(1/3)])/(4*q*(b*c - a*d)), x])] /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - b*c*f - a*d*f, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\sqrt[3]{d^2-9e^2x^2}}\,dx = \frac{\sqrt[3]{1-\frac{9e^2x^2}{d^2}}\int \frac{1}{(d+ex)\sqrt[3]{1-\frac{9e^2x^2}{d^2}}}\,dx}{\sqrt[3]{d^2-9e^2x^2}}$$

$$= \frac{\sqrt[3]{1-\frac{9e^2x^2}{d^2}}\int \frac{1}{(d+ex)\sqrt[3]{1-\frac{3ex}{d}}\sqrt[3]{1+\frac{3ex}{d}}}\,dx}{\sqrt[3]{d^2-9e^2x^2}}$$

$$= \frac{\sqrt{3}\sqrt[3]{1-\frac{9e^2x^2}{d^2}}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{\left(1-\frac{3ex}{d}\right)^{2/3}}{\sqrt{3}\sqrt[3]{1+\frac{3ex}{d}}}\right)}{4e\sqrt[3]{d^2-9e^2x^2}}+\frac{\sqrt[3]{1-\frac{9e^2x^2}{d^2}}\log(d+ex)}{4e\sqrt[3]{d^2-9e^2x^2}}-\frac{3\sqrt[3]{1-\frac{9e^2x^2}{d^2}}\log\left(-\frac{1}{2}\left(1-\right.\right.}{8e\sqrt[3]{d^2-9\right.}}$$

Mathematica [C] time = 0.105073, size = 155, normalized size = 0.75

$$-\frac{\sqrt[3]{3}\sqrt[3]{-\frac{e\left(\sqrt{\frac{d^2}{e^2}}-3x\right)}{d+ex}}\sqrt[3]{\frac{e\left(\sqrt{\frac{d^2}{e^2}}+3x\right)}{d+ex}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{3d-\sqrt{\frac{d^2}{e^2}}e}{3d+3ex},\frac{3d+\sqrt{\frac{d^2}{e^2}}e}{3d+3ex}\right)}{2e\sqrt[3]{d^2-9e^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x)*(d^2 - 9*e^2*x^2)^(1/3)),x]
```

```
[Out] -(3^(1/3)*(-((e*(Sqrt[d^2/e^2] - 3*x))/(d + e*x)))^(1/3)*((e*(Sqrt[d^2/e^2]
 + 3*x))/(d + e*x))^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, (3*d - Sqrt[d^2/e^2]
*e)/(3*d + 3*e*x), (3*d + Sqrt[d^2/e^2]*e)/(3*d + 3*e*x)])/(2*e*(d^2 - 9*e^
2*x^2)^(1/3))
```

Maple [F] time = 0.521, size = 0, normalized size = 0.

$$\int \frac{1}{ex+d}\frac{1}{\sqrt[3]{-9\,e^2x^2+d^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(-9*e^2*x^2+d^2)^(1/3),x)
```

```
[Out] int(1/(e*x+d)/(-9*e^2*x^2+d^2)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-9\,e^2x^2+d^2\right)^{\frac{1}{3}}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-9*e^2*x^2+d^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((-9*e^2*x^2 + d^2)^(1/3)*(e*x + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-9*e^2*x^2+d^2)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{-(-d+3ex)(d+3ex)}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-9*e**2*x**2+d**2)**(1/3),x)
```

```
[Out] Integral(1/((-(-d + 3*e*x)*(d + 3*e*x))**(1/3)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-9\,e^2x^2+d^2\right)^{\frac{1}{3}}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-9*e^2*x^2+d^2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((-9*e^2*x^2 + d^2)^(1/3)*(e*x + d)), x)
```

3.717 $\int \frac{1}{(a+bx)\sqrt[4]{c+dx^2}}\,dx$

Optimal. Leaf size=278

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{a^2d+b^2c}}\right)}{\sqrt{b}\sqrt[4]{a^2d+b^2c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{a^2d+b^2c}}\right)}{\sqrt{b}\sqrt[4]{a^2d+b^2c}}-\frac{a\sqrt[4]{c}\sqrt{-\frac{dx^2}{c}}\Pi\left(-\frac{b\sqrt{c}}{\sqrt{da^2+b^2c}};\sin^{-1}\left(\frac{\sqrt[4]{dx^2+c}}{\sqrt[4]{c}}\right)\middle|-1\right)}{bx\sqrt{a^2d+b^2c}}+\frac{a\sqrt[4]{c}\sqrt{-\frac{dx^2}{c}}\Pi\left(\frac{b\sqrt{c}}{\sqrt{da^2+b^2c}};\sin^{-1}\right.}{bx\sqrt{a^2d+b^2c}}$$

```
[Out] ArcTan[(Sqrt[b]*(c + d*x^2)^(1/4))/(b^2*c + a^2*d)^(1/4)]/(Sqrt[b]*(b^2*c +
 a^2*d)^(1/4)) - ArcTanh[(Sqrt[b]*(c + d*x^2)^(1/4))/(b^2*c + a^2*d)^(1/4)]
/(Sqrt[b]*(b^2*c + a^2*d)^(1/4)) - (a*c^(1/4)*Sqrt[-((d*x^2)/c)]*EllipticPi
[-((b*Sqrt[c])/Sqrt[b^2*c + a^2*d]), ArcSin[(c + d*x^2)^(1/4)/c^(1/4)], -1]
)/(b*Sqrt[b^2*c + a^2*d]*x) + (a*c^(1/4)*Sqrt[-((d*x^2)/c)]*EllipticPi[(b*S
qrt[c])/Sqrt[b^2*c + a^2*d], ArcSin[(c + d*x^2)^(1/4)/c^(1/4)], -1])/(b*Sqr
t[b^2*c + a^2*d]*x)
```

Rubi [A] time = 0.282689, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {746, 399, 490, 1218, 444, 63, 298, 205, 208}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{a^2d+b^2c}}\right)}{\sqrt{b}\sqrt[4]{a^2d+b^2c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{a^2d+b^2c}}\right)}{\sqrt{b}\sqrt[4]{a^2d+b^2c}}-\frac{a\sqrt[4]{c}\sqrt{-\frac{dx^2}{c}}\Pi\left(-\frac{b\sqrt{c}}{\sqrt{da^2+b^2c}};\sin^{-1}\left(\frac{\sqrt[4]{dx^2+c}}{\sqrt[4]{c}}\right)\middle|-1\right)}{bx\sqrt{a^2d+b^2c}}+\frac{a\sqrt[4]{c}\sqrt{-\frac{dx^2}{c}}\Pi\left(\frac{b\sqrt{c}}{\sqrt{da^2+b^2c}};\sin^{-1}\right.}{bx\sqrt{a^2d+b^2c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x^2)^(1/4)),x]
```


```
[Out] ArcTan[(Sqrt[b]*(c + d*x^2)^(1/4))/(b^2*c + a^2*d)^(1/4)]/(Sqrt[b]*(b^2*c +
 a^2*d)^(1/4)) - ArcTanh[(Sqrt[b]*(c + d*x^2)^(1/4))/(b^2*c + a^2*d)^(1/4)]
/(Sqrt[b]*(b^2*c + a^2*d)^(1/4)) - (a*c^(1/4)*Sqrt[-((d*x^2)/c)]*EllipticPi
[-((b*Sqrt[c])/Sqrt[b^2*c + a^2*d]), ArcSin[(c + d*x^2)^(1/4)/c^(1/4)], -1]
)/(b*Sqrt[b^2*c + a^2*d]*x) + (a*c^(1/4)*Sqrt[-((d*x^2)/c)]*EllipticPi[(b*S
qrt[c])/Sqrt[b^2*c + a^2*d], ArcSin[(c + d*x^2)^(1/4)/c^(1/4)], -1])/(b*Sqr
t[b^2*c + a^2*d]*x)
```


Rule 746

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
```

```
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)\sqrt[4]{c+dx^2}}\,dx = a\int \frac{1}{(a^2-b^2x^2)\sqrt[4]{c+dx^2}}\,dx - b\int \frac{x}{(a^2-b^2x^2)\sqrt[4]{c+dx^2}}\,dx$$

$$= -\left(\frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{(a^2-b^2x)\sqrt[4]{c+dx}}\,dx, x, x^2\right)\right) + \frac{\left(2a\sqrt{-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{x^2}{(b^2c+a^2d-b^2x^4)\sqrt{1-\frac{x^4}{c}}}\,dx,\right.}{x}$$

$$= -\frac{(2b)\,\text{Subst}\left(\int \frac{x^2}{a^2+\frac{b^2c}{d}-\frac{b^2x^4}{d}}\,dx, x, \sqrt[4]{c+dx^2}\right)}{d} + \frac{\left(a\sqrt{-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{b^2c+a^2d}-bx^2\right)\sqrt{1-\frac{x^4}{c}}}\,dx, x,\right.}{bx}$$

$$= -\frac{a\sqrt[4]{c}\sqrt{-\frac{dx^2}{c}}\,\Pi\left(-\frac{b\sqrt{c}}{\sqrt{b^2c+a^2d}}; \sin^{-1}\left(\frac{\sqrt[4]{c+dx^2}}{\sqrt[4]{c}}\right)\middle|-1\right)}{b\sqrt{b^2c+a^2d}\,x} + \frac{a\sqrt[4]{c}\sqrt{-\frac{dx^2}{c}}\,\Pi\left(\frac{b\sqrt{c}}{\sqrt{b^2c+a^2d}}; \sin^{-1}\left(\frac{\sqrt[4]{c+dx^2}}{\sqrt[4]{c}}\right)\middle|-1\right.}{b\sqrt{b^2c+a^2d}\,x}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{b^2c+a^2d}}\right)}{\sqrt{b}\sqrt[4]{b^2c+a^2d}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{b^2c+a^2d}}\right)}{\sqrt{b}\sqrt[4]{b^2c+a^2d}} - \frac{a\sqrt[4]{c}\sqrt{-\frac{dx^2}{c}}\,\Pi\left(-\frac{b\sqrt{c}}{\sqrt{b^2c+a^2d}}; \sin^{-1}\left(\frac{\sqrt[4]{c+dx^2}}{\sqrt[4]{c}}\right)\middle|-1\right)}{b\sqrt{b^2c+a^2d}\,x} + \cdots$$

Mathematica [C] time = 0.102812, size = 126, normalized size = 0.45

$$-\frac{2\sqrt[4]{\frac{b\left(x-\sqrt{-\frac{c}{d}}\right)}{a+bx}}\sqrt[4]{\frac{b\left(\sqrt{-\frac{c}{d}}+x\right)}{a+bx}}F_1\left(\frac{1}{2};\frac{1}{4},\frac{1}{4};\frac{3}{2};\frac{a-b\sqrt{-\frac{c}{d}}}{a+bx},\frac{a+b\sqrt{-\frac{c}{d}}}{a+bx}\right)}{b\sqrt[4]{c+dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x)*(c + d*x^2)^(1/4)),x]
```

```
[Out] (-2*((b*(-Sqrt[-(c/d)] + x))/(a + b*x))^(1/4)*((b*(Sqrt[-(c/d)] + x))/(a +
b*x))^(1/4)*AppellF1[1/2, 1/4, 1/4, 3/2, (a - b*Sqrt[-(c/d)])/(a + b*x), (a
 + b*Sqrt[-(c/d)])/(a + b*x)])/(b*(c + d*x^2)^(1/4))
```

Maple [F] time = 0.567, size = 0, normalized size = 0.

$$\int \frac{1}{bx+a}\frac{1}{\sqrt[4]{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x^2+c)^(1/4),x)
```

```
[Out] int(1/(b*x+a)/(d*x^2+c)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx^2+c)^{\frac{1}{4}}(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x^2+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^2 + c)^(1/4)*(b*x + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x^2+c)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)\sqrt[4]{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x**2+c)**(1/4),x)
```

```
[Out] Integral(1/((a + b*x)*(c + d*x**2)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^2+c\right)^{\frac{1}{4}}(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x^2+c)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^2 + c)^(1/4)*(b*x + a)), x)
```

3.718 $\int \frac{1}{(a+bx)\left(c+dx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=268

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{a^2d+b^2c}}\right)}{\left(a^2d+b^2c\right)^{3/4}}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{a^2d+b^2c}}\right)}{\left(a^2d+b^2c\right)^{3/4}}+\frac{a\sqrt[4]{c}\sqrt{-\frac{dx^2}{c}}\,\Pi\left(-\frac{b\sqrt{c}}{\sqrt{da^2+b^2c}};\sin^{-1}\left(\frac{\sqrt[4]{dx^2+c}}{\sqrt[4]{c}}\right)\middle|-1\right)}{x\left(a^2d+b^2c\right)}+\frac{a\sqrt[4]{c}\sqrt{-\frac{dx^2}{c}}\,\Pi\left(\frac{b\sqrt{c}}{\sqrt{da^2+\cdots}}\right.}{x\left(a^2\cdots\right.}$$

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*(c + d*x^2)^(1/4))/(b^2*c + a^2*d)^(1/4)])/(b^2*
c + a^2*d)^(3/4)) - (Sqrt[b]*ArcTanh[(Sqrt[b]*(c + d*x^2)^(1/4))/(b^2*c + a
^2*d)^(1/4)])/(b^2*c + a^2*d)^(3/4) + (a*c^(1/4)*Sqrt[-((d*x^2)/c)]*Ellipti
cPi[-((b*Sqrt[c])/Sqrt[b^2*c + a^2*d]), ArcSin[(c + d*x^2)^(1/4)/c^(1/4)],
-1])/((b^2*c + a^2*d)*x) + (a*c^(1/4)*Sqrt[-((d*x^2)/c)]*EllipticPi[(b*Sqrt
[c])/Sqrt[b^2*c + a^2*d], ArcSin[(c + d*x^2)^(1/4)/c^(1/4)], -1])/((b^2*c +
 a^2*d)*x)
```

Rubi [A] time = 0.269515, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {747, 401, 108, 409, 1218, 444, 63, 212, 208, 205}

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{a^2d+b^2c}}\right)}{\left(a^2d+b^2c\right)^{3/4}}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{a^2d+b^2c}}\right)}{\left(a^2d+b^2c\right)^{3/4}}+\frac{a\sqrt[4]{c}\sqrt{-\frac{dx^2}{c}}\,\Pi\left(-\frac{b\sqrt{c}}{\sqrt{da^2+b^2c}};\sin^{-1}\left(\frac{\sqrt[4]{dx^2+c}}{\sqrt[4]{c}}\right)\middle|-1\right)}{x\left(a^2d+b^2c\right)}+\frac{a\sqrt[4]{c}\sqrt{-\frac{dx^2}{c}}\,\Pi\left(\frac{b\sqrt{c}}{\sqrt{da^2+\cdots}}\right.}{x\left(a^2\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x^2)^(3/4)),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*(c + d*x^2)^(1/4))/(b^2*c + a^2*d)^(1/4)])/(b^2*
c + a^2*d)^(3/4)) - (Sqrt[b]*ArcTanh[(Sqrt[b]*(c + d*x^2)^(1/4))/(b^2*c + a
^2*d)^(1/4)])/(b^2*c + a^2*d)^(3/4) + (a*c^(1/4)*Sqrt[-((d*x^2)/c)]*Ellipti
cPi[-((b*Sqrt[c])/Sqrt[b^2*c + a^2*d]), ArcSin[(c + d*x^2)^(1/4)/c^(1/4)],
-1])/((b^2*c + a^2*d)*x) + (a*c^(1/4)*Sqrt[-((d*x^2)/c)]*EllipticPi[(b*Sqrt
[c])/Sqrt[b^2*c + a^2*d], ArcSin[(c + d*x^2)^(1/4)/c^(1/4)], -1])/((b^2*c +
 a^2*d)*x)
```

Rule 747

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[-((b*x^2)/a)]/(2*x), Subst[Int[1/(Sqrt[-((b*x)/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 108

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - (d*e
)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
```

```
x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)\left(c+dx^2\right)^{3/4}}\,dx = a\int \frac{1}{\left(a^2-b^2x^2\right)\left(c+dx^2\right)^{3/4}}\,dx - b\int \frac{x}{\left(a^2-b^2x^2\right)\left(c+dx^2\right)^{3/4}}\,dx$$

$$= -\left(\frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{\left(a^2-b^2x\right)(c+dx)^{3/4}}\,dx,x,x^2\right)\right) + \frac{\left(a\sqrt{-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-\frac{dx}{c}}\left(a^2-b^2x\right)(c+dx)^{3/4}}\right.}{2x}$$

$$= -\frac{(2b)\,\text{Subst}\left(\int \frac{1}{a^2+\frac{b^2c}{d}-\frac{b^2x^4}{d}}\,dx,x,\sqrt[4]{c+dx^2}\right)}{d} - \frac{\left(2a\sqrt{-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\left(-b^2c-a^2d+b^2x^4\right)\sqrt{1-\frac{x^4}{c}}}\,dx\right.}{x}$$

$$= -\frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{b^2c+a^2d}-bx^2}\,dx,x,\sqrt[4]{c+dx^2}\right)}{\sqrt{b^2c+a^2d}} - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{b^2c+a^2d}+bx^2}\,dx,x,\sqrt[4]{c+dx^2}\right)}{\sqrt{b^2c+a^2d}} + \frac{(a\cdots}{\cdots}$$

$$= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{b^2c+a^2d}}\right)}{\left(b^2c+a^2d\right)^{3/4}} - \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{b^2c+a^2d}}\right)}{\left(b^2c+a^2d\right)^{3/4}} + \frac{a\sqrt[4]{c}\sqrt{-\frac{dx^2}{c}}\,\Pi\left(-\frac{b\sqrt{c}}{\sqrt{b^2c+a^2d}};\sin^{-1}\left(\frac{\sqrt[4]{c+dx^2}}{\sqrt[4]{c}}\right.\right.}{\left(b^2c+a^2d\right)x}$$

Mathematica [A] time = 0.201889, size = 237, normalized size = 0.88

$$-\frac{\sqrt{b}x\sqrt[4]{a^2d+b^2c}\left(\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{a^2d+b^2c}}\right)+\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{a^2d+b^2c}}\right)\right)+a\sqrt[4]{c}\sqrt{-\frac{dx^2}{c}}\,\Pi\left(-\frac{b\sqrt{c}}{\sqrt{da^2+b^2c}};-\sin^{-1}\left(\frac{\sqrt[4]{dx^2+c}}{\sqrt[4]{c}}\right)\middle|-1\right)+a\sqrt[4]{c}\sqrt{-\cdots}}{x\left(a^2d+b^2c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x^2)^(3/4)),x]
```

```
[Out] -((Sqrt[b]*(b^2*c + a^2*d)^(1/4)*x*(ArcTan[(Sqrt[b]*(c + d*x^2)^(1/4))/(b^2
*c + a^2*d)^(1/4)] + ArcTanh[(Sqrt[b]*(c + d*x^2)^(1/4))/(b^2*c + a^2*d)^(1
/4)]) + a*c^(1/4)*Sqrt[-((d*x^2)/c)]*EllipticPi[-((b*Sqrt[c])/Sqrt[b^2*c +
a^2*d]), -ArcSin[(c + d*x^2)^(1/4)/c^(1/4)], -1] + a*c^(1/4)*Sqrt[-((d*x^2)
/c)]*EllipticPi[(b*Sqrt[c])/Sqrt[b^2*c + a^2*d], -ArcSin[(c + d*x^2)^(1/4)/
c^(1/4)], -1])/((b^2*c + a^2*d)*x))
```

Maple [F] time = 0.615, size = 0, normalized size = 0.

$$\int \frac{1}{bx+a}\left(dx^2+c\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x^2+c)^(3/4),x)
```

```
[Out] int(1/(b*x+a)/(d*x^2+c)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^2+c\right)^{\frac{3}{4}}(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x^2+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^2 + c)^(3/4)*(b*x + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x^2+c)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)\left(c+dx^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x**2+c)**(3/4),x)
```

```
[Out] Integral(1/((a + b*x)*(c + d*x**2)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^2+c\right)^{\frac{3}{4}}(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x^2+c)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^2 + c)^(3/4)*(b*x + a)), x)
```

3.719 $\int \frac{1}{(d+ex)^{3/2}\sqrt[4]{a+cx^2}}\,dx$

Optimal. Leaf size=200

$$-\frac{2\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt[4]{-\frac{\left(\sqrt{-a}+\sqrt{c}x\right)\left(\sqrt{-a}e+\sqrt{c}d\right)}{\left(\sqrt{-a}-\sqrt{c}x\right)\left(\sqrt{c}d-\sqrt{-a}e\right)}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\frac{2\sqrt{-a}\sqrt{c}(d+ex)}{\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\sqrt{-a}-\sqrt{c}x\right)}\right)}{\sqrt[4]{a+cx^2}\sqrt{d+ex}\left(\sqrt{-a}e+\sqrt{c}d\right)}$$

```
[Out] (-2*(Sqrt[-a] - Sqrt[c]*x)*(-(((Sqrt[c]*d + Sqrt[-a]*e)*(Sqrt[-a] + Sqrt[c]
*x))/((Sqrt[c]*d - Sqrt[-a]*e)*(Sqrt[-a] - Sqrt[c]*x))))^(1/4)*Hypergeometr
ic2F1[-1/2, 1/4, 1/2, (2*Sqrt[-a]*Sqrt[c]*(d + e*x))/((Sqrt[c]*d - Sqrt[-a]
*e)*(Sqrt[-a] - Sqrt[c]*x))])/((Sqrt[c]*d + Sqrt[-a]*e)*Sqrt[d + e*x]*(a +
c*x^2)^(1/4))
```

Rubi [A] time = 0.0944947, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {727}

$$-\frac{2\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt[4]{-\frac{\left(\sqrt{-a}+\sqrt{c}x\right)\left(\sqrt{-a}e+\sqrt{c}d\right)}{\left(\sqrt{-a}-\sqrt{c}x\right)\left(\sqrt{c}d-\sqrt{-a}e\right)}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\frac{2\sqrt{-a}\sqrt{c}(d+ex)}{\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\sqrt{-a}-\sqrt{c}x\right)}\right)}{\sqrt[4]{a+cx^2}\sqrt{d+ex}\left(\sqrt{-a}e+\sqrt{c}d\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*(a + c*x^2)^(1/4)),x]
```

```
[Out] (-2*(Sqrt[-a] - Sqrt[c]*x)*(-(((Sqrt[c]*d + Sqrt[-a]*e)*(Sqrt[-a] + Sqrt[c]
*x))/((Sqrt[c]*d - Sqrt[-a]*e)*(Sqrt[-a] - Sqrt[c]*x))))^(1/4)*Hypergeometr
ic2F1[-1/2, 1/4, 1/2, (2*Sqrt[-a]*Sqrt[c]*(d + e*x))/((Sqrt[c]*d - Sqrt[-a]
*e)*(Sqrt[-a] - Sqrt[c]*x))])/((Sqrt[c]*d + Sqrt[-a]*e)*Sqrt[d + e*x]*(a +
c*x^2)^(1/4))
```

Rule 727

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((Rt[-(a*c), 2] - c*x)*(d + e*x)^(m + 1)*(a + c*x^2)^p*Hypergeometric2F1[m
+ 1, -p, m + 2, (2*c*Rt[-(a*c), 2]*(d + e*x))/((c*d - e*Rt[-(a*c), 2])*(Rt[
-(a*c), 2] - c*x))])/((m + 1)*(c*d + e*Rt[-(a*c), 2])*(((c*d + e*Rt[-(a*c),
 2])*(Rt[-(a*c), 2] + c*x))/((c*d - e*Rt[-(a*c), 2])*(-Rt[-(a*c), 2] + c*x)
))^p), x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !Int
egerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\sqrt[4]{a+cx^2}}\,dx=-\frac{2\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt[4]{-\frac{\left(\sqrt{c}d+\sqrt{-a}e\right)\left(\sqrt{-a}+\sqrt{c}x\right)}{\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\sqrt{-a}-\sqrt{c}x\right)}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\frac{2\sqrt{-a}\sqrt{c}(d+ex)}{\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\sqrt{-a}-\sqrt{c}x\right)}\right)}{\left(\sqrt{c}d+\sqrt{-a}e\right)\sqrt{d+ex}\sqrt[4]{a+cx^2}}$$

Mathematica [A] time = 0.517099, size = 108, normalized size = 0.54

$$\frac{\left(a+cx^2\right)^{3/4}(cdx-ae)\,_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{(ae-cdx)^2}{ac(d+ex)^2}\right)}{ac(d+ex)^{5/2}\left(\frac{\left(a+cx^2\right)\left(ae^2+cd^2\right)}{ac(d+ex)^2}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*(a + c*x^2)^(1/4)),x]
```

```
[Out] ((-(a*e) + c*d*x)*(a + c*x^2)^(3/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((a*e
 - c*d*x)^2/(a*c*(d + e*x)^2))])/(a*c*(d + e*x)^(5/2)*(((c*d^2 + a*e^2)*(a
+ c*x^2))/(a*c*(d + e*x)^2))^(3/4))
```

Maple [F] time = 0.668, size = 0, normalized size = 0.

$$\int (ex+d)^{-\frac{3}{2}}\frac{1}{\sqrt[4]{cx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(c*x^2+a)^(1/4),x)
```

```
[Out] int(1/(e*x+d)^(3/2)/(c*x^2+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+a\right)^{\frac{1}{4}}(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)^(1/4)*(e*x + d)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(cx^2+a\right)^{\frac{3}{4}}\sqrt{ex+d}}{ce^2x^4+2\,cdex^3+2\,adex+ad^2+\left(cd^2+ae^2\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + a)^(3/4)*sqrt(e*x + d)/(c*e^2*x^4 + 2*c*d*e*x^3 + 2*a*d*e
*x + a*d^2 + (c*d^2 + a*e^2)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{a+cx^2}\,(d+ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(c*x**2+a)**(1/4),x)
```

```
[Out] Integral(1/((a + c*x**2)**(1/4)*(d + e*x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+a\right)^{\frac{1}{4}}(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^2 + a)^(1/4)*(e*x + d)^(3/2)), x)
```

3.720 $\int \frac{1}{(1+x)\sqrt[6]{1+x^2}}\,dx$

Optimal. Leaf size=203

$$xF_1\left(\frac{1}{2};1,\frac{1}{6};\frac{3}{2};x^2,-x^2\right)+\frac{\log\left(\sqrt[3]{x^2+1}-\sqrt[6]{2}\sqrt[6]{x^2+1}+\sqrt[3]{2}\right)}{4\sqrt[6]{2}}-\frac{\log\left(\sqrt[3]{x^2+1}+\sqrt[6]{2}\sqrt[6]{x^2+1}+\sqrt[3]{2}\right)}{4\sqrt[6]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1-2^{5/6}}{\sqrt{}}\right.}{2\sqrt[6]{2}}$$

```
[Out] x*AppellF1[1/2, 1, 1/6, 3/2, x^2, -x^2] - (Sqrt[3]*ArcTan[(1 - 2^(5/6)*(1 +
 x^2)^(1/6))/Sqrt[3]])/(2*2^(1/6)) + (Sqrt[3]*ArcTan[(1 + 2^(5/6)*(1 + x^2)
^(1/6))/Sqrt[3]])/(2*2^(1/6)) - ArcTanh[(1 + x^2)^(1/6)/2^(1/6)]/2^(1/6) +
Log[2^(1/3) - 2^(1/6)*(1 + x^2)^(1/6) + (1 + x^2)^(1/3)]/(4*2^(1/6)) - Log[
2^(1/3) + 2^(1/6)*(1 + x^2)^(1/6) + (1 + x^2)^(1/3)]/(4*2^(1/6))
```

Rubi [A] time = 0.367675, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {757, 429, 444, 63, 296, 634, 618, 204, 628, 206}

$$xF_1\left(\frac{1}{2};1,\frac{1}{6};\frac{3}{2};x^2,-x^2\right)+\frac{\log\left(\sqrt[3]{x^2+1}-\sqrt[6]{2}\sqrt[6]{x^2+1}+\sqrt[3]{2}\right)}{4\sqrt[6]{2}}-\frac{\log\left(\sqrt[3]{x^2+1}+\sqrt[6]{2}\sqrt[6]{x^2+1}+\sqrt[3]{2}\right)}{4\sqrt[6]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1-2^{5/6}}{\sqrt{}}\right.}{2\sqrt[6]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(1 + x^2)^(1/6)),x]
```

```
[Out] x*AppellF1[1/2, 1, 1/6, 3/2, x^2, -x^2] - (Sqrt[3]*ArcTan[(1 - 2^(5/6)*(1 +
 x^2)^(1/6))/Sqrt[3]])/(2*2^(1/6)) + (Sqrt[3]*ArcTan[(1 + 2^(5/6)*(1 + x^2)
^(1/6))/Sqrt[3]])/(2*2^(1/6)) - ArcTanh[(1 + x^2)^(1/6)/2^(1/6)]/2^(1/6) +
Log[2^(1/3) - 2^(1/6)*(1 + x^2)^(1/6) + (1 + x^2)^(1/3)]/(4*2^(1/6)) - Log[
2^(1/3) + 2^(1/6)*(1 + x^2)^(1/6) + (1 + x^2)^(1/3)]/(4*2^(1/6))
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(1+x)\sqrt[6]{1+x^2}}\,dx = \int \left(\frac{1}{\left(1-x^2\right)\sqrt[6]{1+x^2}} + \frac{x}{\left(-1+x^2\right)\sqrt[6]{1+x^2}}\right)dx$$
$$= \int \frac{1}{\left(1-x^2\right)\sqrt[6]{1+x^2}}\,dx + \int \frac{x}{\left(-1+x^2\right)\sqrt[6]{1+x^2}}\,dx$$
$$= xF_1\left(\frac{1}{2};1,\frac{1}{6};\frac{3}{2};x^2,-x^2\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt[6]{1+x}}\,dx, x, x^2\right)$$
$$= xF_1\left(\frac{1}{2};1,\frac{1}{6};\frac{3}{2};x^2,-x^2\right) + 3\,\text{Subst}\left(\int \frac{x^4}{-2+x^6}\,dx, x, \sqrt[6]{1+x^2}\right)$$
$$= xF_1\left(\frac{1}{2};1,\frac{1}{6};\frac{3}{2};x^2,-x^2\right) - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2^{5/6}}-\frac{x}{2}}{\sqrt[3]{2}-\sqrt[6]{2}x+x^2}\,dx, x, \sqrt[6]{1+x^2}\right)}{\sqrt[6]{2}} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2^{5/6}}+\frac{x}{2}}{\sqrt[3]{2}+\sqrt[6]{2}x+x^2}\,dx, x, \right.}{\sqrt[6]{2}}$$
$$= xF_1\left(\frac{1}{2};1,\frac{1}{6};\frac{3}{2};x^2,-x^2\right) - \frac{\tanh^{-1}\left(\frac{\sqrt[6]{1+x^2}}{\sqrt[6]{2}}\right)}{\sqrt[6]{2}} + \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-\sqrt[6]{2}x+x^2}\,dx, x, \sqrt[6]{1+x^2}\right) +$$
$$= xF_1\left(\frac{1}{2};1,\frac{1}{6};\frac{3}{2};x^2,-x^2\right) - \frac{\tanh^{-1}\left(\frac{\sqrt[6]{1+x^2}}{\sqrt[6]{2}}\right)}{\sqrt[6]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[6]{2}\sqrt[6]{1+x^2}+\sqrt[3]{1+x^2}\right)}{4\sqrt[6]{2}} - \frac{\log\left(\sqrt[3]{2}+\right.}{}$$
$$= xF_1\left(\frac{1}{2};1,\frac{1}{6};\frac{3}{2};x^2,-x^2\right) - \frac{\sqrt{3}\tan^{-1}\left(\frac{1-2^{5/6}\sqrt[6]{1+x^2}}{\sqrt{3}}\right)}{2\sqrt[6]{2}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+2^{5/6}\sqrt[6]{1+x^2}}{\sqrt{3}}\right)}{2\sqrt[6]{2}} - \frac{\tanh^{-1}\left(\frac{\sqrt[6]{1+x^2}}{\sqrt[6]{2}}\right)}{\sqrt[6]{2}}$$

Mathematica [C] time = 0.0385877, size = 72, normalized size = 0.35

$$-\frac{3\sqrt[6]{\frac{x-i}{x+1}}\sqrt[6]{\frac{x+i}{x+1}}F_1\left(\frac{1}{3};\frac{1}{6},\frac{1}{6};\frac{4}{3};\frac{1-i}{x+1},\frac{1+i}{x+1}\right)}{\sqrt[6]{x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 + x)*(1 + x^2)^(1/6)),x]
```

```
[Out] (-3*((-I + x)/(1 + x))^(1/6)*((I + x)/(1 + x))^(1/6)*AppellF1[1/3, 1/6, 1/6
, 4/3, (1 - I)/(1 + x), (1 + I)/(1 + x)])/(1 + x^2)^(1/6)
```

Maple [F] time = 0.366, size = 0, normalized size = 0.

$$\int \frac{1}{1+x}\frac{1}{\sqrt[6]{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(x^2+1)^(1/6),x)
```

```
[Out] int(1/(1+x)/(x^2+1)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+1\right)^{\frac{1}{6}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+1)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 1)^(1/6)*(x + 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+1)^(1/6),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x+1)\sqrt[6]{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x**2+1)**(1/6),x)
```

```
[Out] Integral(1/((x + 1)*(x**2 + 1)**(1/6)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+1\right)^{\frac{1}{6}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+1)^(1/6),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 1)^(1/6)*(x + 1)), x)
```

3.721 $\int (d+ex)^m \left(a+cx^2\right)^3 dx$

Optimal. Leaf size=223

$$-\frac{4c^2d\left(3ae^2+5cd^2\right)(d+ex)^{m+4}}{e^7(m+4)}+\frac{3c^2\left(ae^2+5cd^2\right)(d+ex)^{m+5}}{e^7(m+5)}+\frac{\left(ae^2+cd^2\right)^3(d+ex)^{m+1}}{e^7(m+1)}-\frac{6cd\left(ae^2+cd^2\right)^2(d+ex}{e^7(m+2)}$$

```
[Out] ((c*d^2 + a*e^2)^3*(d + e*x)^(1 + m))/(e^7*(1 + m)) - (6*c*d*(c*d^2 + a*e^2
)^2*(d + e*x)^(2 + m))/(e^7*(2 + m)) + (3*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^
2)*(d + e*x)^(3 + m))/(e^7*(3 + m)) - (4*c^2*d*(5*c*d^2 + 3*a*e^2)*(d + e*x
)^(4 + m))/(e^7*(4 + m)) + (3*c^2*(5*c*d^2 + a*e^2)*(d + e*x)^(5 + m))/(e^7
*(5 + m)) - (6*c^3*d*(d + e*x)^(6 + m))/(e^7*(6 + m)) + (c^3*(d + e*x)^(7 +
 m))/(e^7*(7 + m))
```

Rubi [A] time = 0.133102, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$-\frac{4c^2d\left(3ae^2+5cd^2\right)(d+ex)^{m+4}}{e^7(m+4)}+\frac{3c^2\left(ae^2+5cd^2\right)(d+ex)^{m+5}}{e^7(m+5)}+\frac{\left(ae^2+cd^2\right)^3(d+ex)^{m+1}}{e^7(m+1)}-\frac{6cd\left(ae^2+cd^2\right)^2(d+ex}{e^7(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a + c*x^2)^3,x]
```

```
[Out] ((c*d^2 + a*e^2)^3*(d + e*x)^(1 + m))/(e^7*(1 + m)) - (6*c*d*(c*d^2 + a*e^2
)^2*(d + e*x)^(2 + m))/(e^7*(2 + m)) + (3*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^
2)*(d + e*x)^(3 + m))/(e^7*(3 + m)) - (4*c^2*d*(5*c*d^2 + 3*a*e^2)*(d + e*x
)^(4 + m))/(e^7*(4 + m)) + (3*c^2*(5*c*d^2 + a*e^2)*(d + e*x)^(5 + m))/(e^7
*(5 + m)) - (6*c^3*d*(d + e*x)^(6 + m))/(e^7*(6 + m)) + (c^3*(d + e*x)^(7 +
 m))/(e^7*(7 + m))
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (d+ex)^m \left(a+cx^2\right)^3 dx=\int \left(\frac{\left(cd^2+ae^2\right)^3(d+ex)^m}{e^6}-\frac{6cd\left(cd^2+ae^2\right)^2(d+ex)^{1+m}}{e^6}+\frac{3c\left(cd^2+ae^2\right)\left(5cd^2+ae^2\right)(}{e^6}\right.$$

$$=\frac{\left(cd^2+ae^2\right)^3(d+ex)^{1+m}}{e^7(1+m)}-\frac{6cd\left(cd^2+ae^2\right)^2(d+ex)^{2+m}}{e^7(2+m)}+\frac{3c\left(cd^2+ae^2\right)\left(5cd^2+ae^2\right)(d}{e^7(3+m)}$$

Mathematica [A] time = 0.699113, size = 379, normalized size = 1.7

$$\frac{(d+ex)^{m+1}\left(\frac{6\left((m+6)\left(ae^2+cd^2\right)\left(4(m+4)\left(ae^2+cd^2\right)\left(ae^2\left(m^2+5m+6\right)+c\left(2d^2-2de(m+1)x+e^2\left(m^2+3m+2\right)x^2\right)\right)-4cd(m+1)(d+ex)\left(ae^2\left(m^2+7m+12\right)+c\left(2d^2-2\right.\right.\right.\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(a + c*x^2)^3,x]
```

```
[Out] ((d + e*x)^(1 + m)*((a + c*x^2)^3 + (6*((c*d^2 + a*e^2)*(6 + m)*(e^4*(1 + m
)*(2 + m)*(3 + m)*(4 + m)*(a + c*x^2)^2 + 4*(c*d^2 + a*e^2)*(4 + m)*(a*e^2*
(6 + 5*m + m^2) + c*(2*d^2 - 2*d*e*(1 + m)*x + e^2*(2 + 3*m + m^2)*x^2)) -
4*c*d*(1 + m)*(d + e*x)*(a*e^2*(12 + 7*m + m^2) + c*(2*d^2 - 2*d*e*(2 + m)*
x + e^2*(6 + 5*m + m^2)*x^2))) - c*d*(1 + m)*(d + e*x)*(e^4*(2 + m)*(3 + m)
*(4 + m)*(5 + m)*(a + c*x^2)^2 + 4*(c*d^2 + a*e^2)*(5 + m)*(a*e^2*(12 + 7*m
 + m^2) + c*(2*d^2 - 2*d*e*(2 + m)*x + e^2*(6 + 5*m + m^2)*x^2)) - 4*c*d*(2
 + m)*(d + e*x)*(a*e^2*(20 + 9*m + m^2) + c*(2*d^2 - 2*d*e*(3 + m)*x + e^2*
(12 + 7*m + m^2)*x^2)))))/(e^6*(1 + m)*(2 + m)*(3 + m)*(4 + m)*(5 + m)*(6 +
 m))))/(e*(7 + m))
```

Maple [B] time = 0.049, size = 1140, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+a)^3,x)
```

```
[Out] (e*x+d)^(1+m)*(c^3*e^6*m^6*x^6+21*c^3*e^6*m^5*x^6+3*a*c^2*e^6*m^6*x^4-6*c^3
*d*e^5*m^5*x^5+175*c^3*e^6*m^4*x^6+69*a*c^2*e^6*m^5*x^4-90*c^3*d*e^5*m^4*x^
5+735*c^3*e^6*m^3*x^6+3*a^2*c*e^6*m^6*x^2-12*a*c^2*d*e^5*m^5*x^3+621*a*c^2*
e^6*m^4*x^4+30*c^3*d^2*e^4*m^4*x^4-510*c^3*d*e^5*m^3*x^5+1624*c^3*e^6*m^2*x
^6+75*a^2*c*e^6*m^5*x^2-228*a*c^2*d*e^5*m^4*x^3+2775*a*c^2*e^6*m^3*x^4+300*
c^3*d^2*e^4*m^3*x^4-1350*c^3*d*e^5*m^2*x^5+1764*c^3*e^6*m*x^6+a^3*e^6*m^6-6
*a^2*c*d*e^5*m^5*x+741*a^2*c*e^6*m^4*x^2+36*a*c^2*d^2*e^4*m^4*x^2-1572*a*c^
2*d*e^5*m^3*x^3+6432*a*c^2*e^6*m^2*x^4-120*c^3*d^3*e^3*m^3*x^3+1050*c^3*d^2
*e^4*m^2*x^4-1644*c^3*d*e^5*m*x^5+720*c^3*e^6*x^6+27*a^3*e^6*m^5-138*a^2*c*
d*e^5*m^4*x+3657*a^2*c*e^6*m^3*x^2+576*a*c^2*d^2*e^4*m^3*x^2-4812*a*c^2*d*e
^5*m^2*x^3+7236*a*c^2*e^6*m*x^4-720*c^3*d^3*e^3*m^2*x^3+1500*c^3*d^2*e^4*m*
x^4-720*c^3*d*e^5*x^5+295*a^3*e^6*m^4+6*a^2*c*d^2*e^4*m^4-1206*a^2*c*d*e^5*
m^3*x+9336*a^2*c*e^6*m^2*x^2-72*a*c^2*d^3*e^3*m^3*x+2988*a*c^2*d^2*e^4*m^2*
x^2-6480*a*c^2*d*e^5*m*x^3+3024*a*c^2*e^6*x^4+360*c^3*d^4*e^2*m^2*x^2-1320*
c^3*d^3*e^3*m*x^3+720*c^3*d^2*e^4*x^4+1665*a^3*e^6*m^3+132*a^2*c*d^2*e^4*m^
3-4902*a^2*c*d*e^5*m^2*x+11388*a^2*c*e^6*m*x^2-1008*a*c^2*d^3*e^3*m^2*x+547
2*a*c^2*d^2*e^4*m*x^2-3024*a*c^2*d*e^5*x^3+1080*c^3*d^4*e^2*m*x^2-720*c^3*d
^3*e^3*x^3+5104*a^3*e^6*m^2+1074*a^2*c*d^2*e^4*m^2-8868*a^2*c*d*e^5*m*x+504
0*a^2*c*e^6*x^2+72*a*c^2*d^4*e^2*m^2-3960*a*c^2*d^3*e^3*m*x+3024*a*c^2*d^2*
e^4*x^2-720*c^3*d^5*e*m*x+720*c^3*d^4*e^2*x^2+8028*a^3*e^6*m+3828*a^2*c*d^2
*e^4*m-5040*a^2*c*d*e^5*x+936*a*c^2*d^4*e^2*m-3024*a*c^2*d^3*e^3*x-720*c^3*
d^5*e*x+5040*a^3*e^6+5040*a^2*c*d^2*e^4+3024*a*c^2*d^4*e^2+720*c^3*d^6)/e^7
/(m^7+28*m^6+322*m^5+1960*m^4+6769*m^3+13132*m^2+13068*m+5040)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.2828, size = 2692, normalized size = 12.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] (a^3*d*e^6*m^6 + 27*a^3*d*e^6*m^5 + 720*c^3*d^7 + 3024*a*c^2*d^5*e^2 + 5040
*a^2*c*d^3*e^4 + 5040*a^3*d*e^6 + (c^3*e^7*m^6 + 21*c^3*e^7*m^5 + 175*c^3*e
^7*m^4 + 735*c^3*e^7*m^3 + 1624*c^3*e^7*m^2 + 1764*c^3*e^7*m + 720*c^3*e^7)
*x^7 + (c^3*d*e^6*m^6 + 15*c^3*d*e^6*m^5 + 85*c^3*d*e^6*m^4 + 225*c^3*d*e^6
*m^3 + 274*c^3*d*e^6*m^2 + 120*c^3*d*e^6*m)*x^6 + 3*(a*c^2*e^7*m^6 + 1008*a
*c^2*e^7 - (2*c^3*d^2*e^5 - 23*a*c^2*e^7)*m^5 - (20*c^3*d^2*e^5 - 207*a*c^2
*e^7)*m^4 - 5*(14*c^3*d^2*e^5 - 185*a*c^2*e^7)*m^3 - 4*(25*c^3*d^2*e^5 - 53
6*a*c^2*e^7)*m^2 - 12*(4*c^3*d^2*e^5 - 201*a*c^2*e^7)*m)*x^5 + (6*a^2*c*d^3
*e^4 + 295*a^3*d*e^6)*m^4 + 3*(a*c^2*d*e^6*m^6 + 19*a*c^2*d*e^6*m^5 + (10*c
^3*d^3*e^4 + 131*a*c^2*d*e^6)*m^4 + (60*c^3*d^3*e^4 + 401*a*c^2*d*e^6)*m^3
+ 10*(11*c^3*d^3*e^4 + 54*a*c^2*d*e^6)*m^2 + 12*(5*c^3*d^3*e^4 + 21*a*c^2*d
*e^6)*m)*x^4 + 3*(44*a^2*c*d^3*e^4 + 555*a^3*d*e^6)*m^3 + 3*(a^2*c*e^7*m^6
+ 1680*a^2*c*e^7 - (4*a*c^2*d^2*e^5 - 25*a^2*c*e^7)*m^5 - (64*a*c^2*d^2*e^5
 - 247*a^2*c*e^7)*m^4 - (40*c^3*d^4*e^3 + 332*a*c^2*d^2*e^5 - 1219*a^2*c*e^
7)*m^3 - 8*(15*c^3*d^4*e^3 + 76*a*c^2*d^2*e^5 - 389*a^2*c*e^7)*m^2 - 4*(20*
c^3*d^4*e^3 + 84*a*c^2*d^2*e^5 - 949*a^2*c*e^7)*m)*x^3 + 2*(36*a*c^2*d^5*e^
2 + 537*a^2*c*d^3*e^4 + 2552*a^3*d*e^6)*m^2 + 3*(a^2*c*d*e^6*m^6 + 23*a^2*c
*d*e^6*m^5 + 3*(4*a*c^2*d^3*e^4 + 67*a^2*c*d*e^6)*m^4 + (168*a*c^2*d^3*e^4
+ 817*a^2*c*d*e^6)*m^3 + 2*(60*c^3*d^5*e^2 + 330*a*c^2*d^3*e^4 + 739*a^2*c*
d*e^6)*m^2 + 24*(5*c^3*d^5*e^2 + 21*a*c^2*d^3*e^4 + 35*a^2*c*d*e^6)*m)*x^2
+ 12*(78*a*c^2*d^5*e^2 + 319*a^2*c*d^3*e^4 + 669*a^3*d*e^6)*m + (a^3*e^7*m^
6 + 5040*a^3*e^7 - 3*(2*a^2*c*d^2*e^5 - 9*a^3*e^7)*m^5 - (132*a^2*c*d^2*e^5
 - 295*a^3*e^7)*m^4 - 3*(24*a*c^2*d^4*e^3 + 358*a^2*c*d^2*e^5 - 555*a^3*e^7
)*m^3 - 4*(234*a*c^2*d^4*e^3 + 957*a^2*c*d^2*e^5 - 1276*a^3*e^7)*m^2 - 36*(
20*c^3*d^6*e + 84*a*c^2*d^4*e^3 + 140*a^2*c*d^2*e^5 - 223*a^3*e^7)*m)*x)*(e
*x + d)^m/(e^7*m^7 + 28*e^7*m^6 + 322*e^7*m^5 + 1960*e^7*m^4 + 6769*e^7*m^3
 + 13132*e^7*m^2 + 13068*e^7*m + 5040*e^7)
```

Sympy [A] time = 18.1348, size = 15781, normalized size = 70.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+a)**3,x)
```

```
[Out] Piecewise((d**m*(a**3*x + a**2*c*x**3 + 3*a*c**2*x**5/5 + c**3*x**7/7), Eq(
e, 0)), (-10*a**3*d**4*e**6/(60*d**10*e**7 + 360*d**9*e**8*x + 900*d**8*e**
9*x**2 + 1200*d**7*e**10*x**3 + 900*d**6*e**11*x**4 + 360*d**5*e**12*x**5 +
 60*d**4*e**13*x**6) + 60*a**2*c*d**3*e**7*x**3/(60*d**10*e**7 + 360*d**9*e
**8*x + 900*d**8*e**9*x**2 + 1200*d**7*e**10*x**3 + 900*d**6*e**11*x**4 + 3
60*d**5*e**12*x**5 + 60*d**4*e**13*x**6) + 45*a**2*c*d**2*e**8*x**4/(60*d**
10*e**7 + 360*d**9*e**8*x + 900*d**8*e**9*x**2 + 1200*d**7*e**10*x**3 + 900
*d**6*e**11*x**4 + 360*d**5*e**12*x**5 + 60*d**4*e**13*x**6) + 18*a**2*c*d*
```

```
e**9*x**5/(60*d**10*e**7 + 360*d**9*e**8*x + 900*d**8*e**9*x**2 + 1200*d**7
*e**10*x**3 + 900*d**6*e**11*x**4 + 360*d**5*e**12*x**5 + 60*d**4*e**13*x**
6) + 3*a**2*c*e**10*x**6/(60*d**10*e**7 + 360*d**9*e**8*x + 900*d**8*e**9*x
**2 + 1200*d**7*e**10*x**3 + 900*d**6*e**11*x**4 + 360*d**5*e**12*x**5 + 60
*d**4*e**13*x**6) + 36*a*c**2*d**3*e**7*x**5/(60*d**10*e**7 + 360*d**9*e**8
*x + 900*d**8*e**9*x**2 + 1200*d**7*e**10*x**3 + 900*d**6*e**11*x**4 + 360*
d**5*e**12*x**5 + 60*d**4*e**13*x**6) + 6*a*c**2*d**2*e**8*x**6/(60*d**10*e
**7 + 360*d**9*e**8*x + 900*d**8*e**9*x**2 + 1200*d**7*e**10*x**3 + 900*d**
6*e**11*x**4 + 360*d**5*e**12*x**5 + 60*d**4*e**13*x**6) + 60*c**3*d**10*lo
g(d/e + x)/(60*d**10*e**7 + 360*d**9*e**8*x + 900*d**8*e**9*x**2 + 1200*d**
7*e**10*x**3 + 900*d**6*e**11*x**4 + 360*d**5*e**12*x**5 + 60*d**4*e**13*x*
*6) + 22*c**3*d**10/(60*d**10*e**7 + 360*d**9*e**8*x + 900*d**8*e**9*x**2 +
 1200*d**7*e**10*x**3 + 900*d**6*e**11*x**4 + 360*d**5*e**12*x**5 + 60*d**4
*e**13*x**6) + 360*c**3*d**9*e*x*log(d/e + x)/(60*d**10*e**7 + 360*d**9*e**
8*x + 900*d**8*e**9*x**2 + 1200*d**7*e**10*x**3 + 900*d**6*e**11*x**4 + 360
*d**5*e**12*x**5 + 60*d**4*e**13*x**6) + 72*c**3*d**9*e*x/(60*d**10*e**7 +
360*d**9*e**8*x + 900*d**8*e**9*x**2 + 1200*d**7*e**10*x**3 + 900*d**6*e**1
1*x**4 + 360*d**5*e**12*x**5 + 60*d**4*e**13*x**6) + 900*c**3*d**8*e**2*x**
2*log(d/e + x)/(60*d**10*e**7 + 360*d**9*e**8*x + 900*d**8*e**9*x**2 + 1200
*d**7*e**10*x**3 + 900*d**6*e**11*x**4 + 360*d**5*e**12*x**5 + 60*d**4*e**1
3*x**6) + 1200*c**3*d**7*e**3*x**3*log(d/e + x)/(60*d**10*e**7 + 360*d**9*e
**8*x + 900*d**8*e**9*x**2 + 1200*d**7*e**10*x**3 + 900*d**6*e**11*x**4 + 3
60*d**5*e**12*x**5 + 60*d**4*e**13*x**6) - 300*c**3*d**7*e**3*x**3/(60*d**1
0*e**7 + 360*d**9*e**8*x + 900*d**8*e**9*x**2 + 1200*d**7*e**10*x**3 + 900*
d**6*e**11*x**4 + 360*d**5*e**12*x**5 + 60*d**4*e**13*x**6) + 900*c**3*d**6
*e**4*x**4*log(d/e + x)/(60*d**10*e**7 + 360*d**9*e**8*x + 900*d**8*e**9*x*
*2 + 1200*d**7*e**10*x**3 + 900*d**6*e**11*x**4 + 360*d**5*e**12*x**5 + 60*
d**4*e**13*x**6) - 525*c**3*d**6*e**4*x**4/(60*d**10*e**7 + 360*d**9*e**8*x
 + 900*d**8*e**9*x**2 + 1200*d**7*e**10*x**3 + 900*d**6*e**11*x**4 + 360*d*
*5*e**12*x**5 + 60*d**4*e**13*x**6) + 360*c**3*d**5*e**5*x**5*log(d/e + x)/
(60*d**10*e**7 + 360*d**9*e**8*x + 900*d**8*e**9*x**2 + 1200*d**7*e**10*x**
3 + 900*d**6*e**11*x**4 + 360*d**5*e**12*x**5 + 60*d**4*e**13*x**6) - 390*c
**3*d**5*e**5*x**5/(60*d**10*e**7 + 360*d**9*e**8*x + 900*d**8*e**9*x**2 +
1200*d**7*e**10*x**3 + 900*d**6*e**11*x**4 + 360*d**5*e**12*x**5 + 60*d**4*
e**13*x**6) + 60*c**3*d**4*e**6*x**6*log(d/e + x)/(60*d**10*e**7 + 360*d**9
*e**8*x + 900*d**8*e**9*x**2 + 1200*d**7*e**10*x**3 + 900*d**6*e**11*x**4 +
 360*d**5*e**12*x**5 + 60*d**4*e**13*x**6) - 125*c**3*d**4*e**6*x**6/(60*d*
*10*e**7 + 360*d**9*e**8*x + 900*d**8*e**9*x**2 + 1200*d**7*e**10*x**3 + 90
0*d**6*e**11*x**4 + 360*d**5*e**12*x**5 + 60*d**4*e**13*x**6), Eq(m, -7)),
(-2*a**3*d**3*e**6/(10*d**8*e**7 + 50*d**7*e**8*x + 100*d**6*e**9*x**2 + 10
0*d**5*e**10*x**3 + 50*d**4*e**11*x**4 + 10*d**3*e**12*x**5) + 10*a**2*c*d*
*2*e**7*x**3/(10*d**8*e**7 + 50*d**7*e**8*x + 100*d**6*e**9*x**2 + 100*d**5
*e**10*x**3 + 50*d**4*e**11*x**4 + 10*d**3*e**12*x**5) + 5*a**2*c*d*e**8*x*
*4/(10*d**8*e**7 + 50*d**7*e**8*x + 100*d**6*e**9*x**2 + 100*d**5*e**10*x**
3 + 50*d**4*e**11*x**4 + 10*d**3*e**12*x**5) + a**2*c*e**9*x**5/(10*d**8*e*
*7 + 50*d**7*e**8*x + 100*d**6*e**9*x**2 + 100*d**5*e**10*x**3 + 50*d**4*e*
*11*x**4 + 10*d**3*e**12*x**5) + 6*a*c**2*d**2*e**7*x**5/(10*d**8*e**7 + 50
*d**7*e**8*x + 100*d**6*e**9*x**2 + 100*d**5*e**10*x**3 + 50*d**4*e**11*x**
4 + 10*d**3*e**12*x**5) - 60*c**3*d**9*log(d/e + x)/(10*d**8*e**7 + 50*d**7
*e**8*x + 100*d**6*e**9*x**2 + 100*d**5*e**10*x**3 + 50*d**4*e**11*x**4 + 1
0*d**3*e**12*x**5) - 27*c**3*d**9/(10*d**8*e**7 + 50*d**7*e**8*x + 100*d**6
*e**9*x**2 + 100*d**5*e**10*x**3 + 50*d**4*e**11*x**4 + 10*d**3*e**12*x**5)
 - 300*c**3*d**8*e*x*log(d/e + x)/(10*d**8*e**7 + 50*d**7*e**8*x + 100*d**6
*e**9*x**2 + 100*d**5*e**10*x**3 + 50*d**4*e**11*x**4 + 10*d**3*e**12*x**5)
 - 75*c**3*d**8*e*x/(10*d**8*e**7 + 50*d**7*e**8*x + 100*d**6*e**9*x**2 + 1
00*d**5*e**10*x**3 + 50*d**4*e**11*x**4 + 10*d**3*e**12*x**5) - 600*c**3*d*
*7*e**2*x**2*log(d/e + x)/(10*d**8*e**7 + 50*d**7*e**8*x + 100*d**6*e**9*x*
*2 + 100*d**5*e**10*x**3 + 50*d**4*e**11*x**4 + 10*d**3*e**12*x**5) - 600*c
**3*d**6*e**3*x**3*log(d/e + x)/(10*d**8*e**7 + 50*d**7*e**8*x + 100*d**6*e
```

9*x2 + 100*d**5*e**10*x**3 + 50*d**4*e**11*x**4 + 10*d**3*e**12*x**5) +
200*c**3*d**6*e**3*x**3/(10*d**8*e**7 + 50*d**7*e**8*x + 100*d**6*e**9*x**
2 + 100*d**5*e**10*x**3 + 50*d**4*e**11*x**4 + 10*d**3*e**12*x**5) - 300*c*
*3*d**5*e**4*x**4*log(d/e + x)/(10*d**8*e**7 + 50*d**7*e**8*x + 100*d**6*e*
*9*x**2 + 100*d**5*e**10*x**3 + 50*d**4*e**11*x**4 + 10*d**3*e**12*x**5) +
250*c**3*d**5*e**4*x**4/(10*d**8*e**7 + 50*d**7*e**8*x + 100*d**6*e**9*x**2
+ 100*d**5*e**10*x**3 + 50*d**4*e**11*x**4 + 10*d**3*e**12*x**5) - 60*c**3
*d**4*e**5*x**5*log(d/e + x)/(10*d**8*e**7 + 50*d**7*e**8*x + 100*d**6*e**9
*x**2 + 100*d**5*e**10*x**3 + 50*d**4*e**11*x**4 + 10*d**3*e**12*x**5) + 11
0*c**3*d**4*e**5*x**5/(10*d**8*e**7 + 50*d**7*e**8*x + 100*d**6*e**9*x**2 +
100*d**5*e**10*x**3 + 50*d**4*e**11*x**4 + 10*d**3*e**12*x**5) + 10*c**3*d
3*e6*x**6/(10*d**8*e**7 + 50*d**7*e**8*x + 100*d**6*e**9*x**2 + 100*d**
5*e**10*x**3 + 50*d**4*e**11*x**4 + 10*d**3*e**12*x**5), Eq(m, -6)), (-a**3
*d**2*e**6/(4*d**6*e**7 + 16*d**5*e**8*x + 24*d**4*e**9*x**2 + 16*d**3*e**1
0*x**3 + 4*d**2*e**11*x**4) + 4*a**2*c*d*e**7*x**3/(4*d**6*e**7 + 16*d**5*e
8*x + 24*d4*e**9*x**2 + 16*d**3*e**10*x**3 + 4*d**2*e**11*x**4) + a**2*
c*e**8*x**4/(4*d**6*e**7 + 16*d**5*e**8*x + 24*d**4*e**9*x**2 + 16*d**3*e**
10*x**3 + 4*d**2*e**11*x**4) + 12*a*c**2*d**6*e**2*log(d/e + x)/(4*d**6*e**
7 + 16*d**5*e**8*x + 24*d**4*e**9*x**2 + 16*d**3*e**10*x**3 + 4*d**2*e**11*
x**4) + 7*a*c**2*d**6*e**2/(4*d**6*e**7 + 16*d**5*e**8*x + 24*d**4*e**9*x**
2 + 16*d**3*e**10*x**3 + 4*d**2*e**11*x**4) + 48*a*c**2*d**5*e**3*x*log(d/e
+ x)/(4*d**6*e**7 + 16*d**5*e**8*x + 24*d**4*e**9*x**2 + 16*d**3*e**10*x**
3 + 4*d**2*e**11*x**4) + 16*a*c**2*d**5*e**3*x/(4*d**6*e**7 + 16*d**5*e**8*
x + 24*d**4*e**9*x**2 + 16*d**3*e**10*x**3 + 4*d**2*e**11*x**4) + 72*a*c**2
*d**4*e**4*x**2*log(d/e + x)/(4*d**6*e**7 + 16*d**5*e**8*x + 24*d**4*e**9*x
2 + 16*d3*e**10*x**3 + 4*d**2*e**11*x**4) + 48*a*c**2*d**3*e**5*x**3*lo
g(d/e + x)/(4*d**6*e**7 + 16*d**5*e**8*x + 24*d**4*e**9*x**2 + 16*d**3*e**1
0*x**3 + 4*d**2*e**11*x**4) - 24*a*c**2*d**3*e**5*x**3/(4*d**6*e**7 + 16*d*
*5*e**8*x + 24*d**4*e**9*x**2 + 16*d**3*e**10*x**3 + 4*d**2*e**11*x**4) + 1
2*a*c**2*d**2*e**6*x**4*log(d/e + x)/(4*d**6*e**7 + 16*d**5*e**8*x + 24*d**
4*e**9*x**2 + 16*d**3*e**10*x**3 + 4*d**2*e**11*x**4) - 18*a*c**2*d**2*e**6
*x**4/(4*d**6*e**7 + 16*d**5*e**8*x + 24*d**4*e**9*x**2 + 16*d**3*e**10*x**
3 + 4*d**2*e**11*x**4) + 60*c**3*d**8*log(d/e + x)/(4*d**6*e**7 + 16*d**5*e
8*x + 24*d4*e**9*x**2 + 16*d**3*e**10*x**3 + 4*d**2*e**11*x**4) + 35*c*
*3*d**8/(4*d**6*e**7 + 16*d**5*e**8*x + 24*d**4*e**9*x**2 + 16*d**3*e**10*x
3 + 4*d2*e**11*x**4) + 240*c**3*d**7*e*x*log(d/e + x)/(4*d**6*e**7 + 16
*d**5*e**8*x + 24*d**4*e**9*x**2 + 16*d**3*e**10*x**3 + 4*d**2*e**11*x**4)
+ 80*c**3*d**7*e*x/(4*d**6*e**7 + 16*d**5*e**8*x + 24*d**4*e**9*x**2 + 16*d
3*e10*x**3 + 4*d**2*e**11*x**4) + 360*c**3*d**6*e**2*x**2*log(d/e + x)/
(4*d**6*e**7 + 16*d**5*e**8*x + 24*d**4*e**9*x**2 + 16*d**3*e**10*x**3 + 4*
d**2*e**11*x**4) + 240*c**3*d**5*e**3*x**3*log(d/e + x)/(4*d**6*e**7 + 16*d
5*e8*x + 24*d**4*e**9*x**2 + 16*d**3*e**10*x**3 + 4*d**2*e**11*x**4) -
120*c**3*d**5*e**3*x**3/(4*d**6*e**7 + 16*d**5*e**8*x + 24*d**4*e**9*x**2 +
16*d**3*e**10*x**3 + 4*d**2*e**11*x**4) + 60*c**3*d**4*e**4*x**4*log(d/e +
x)/(4*d**6*e**7 + 16*d**5*e**8*x + 24*d**4*e**9*x**2 + 16*d**3*e**10*x**3
+ 4*d**2*e**11*x**4) - 90*c**3*d**4*e**4*x**4/(4*d**6*e**7 + 16*d**5*e**8*x
+ 24*d**4*e**9*x**2 + 16*d**3*e**10*x**3 + 4*d**2*e**11*x**4) - 12*c**3*d*
*3*e**5*x**5/(4*d**6*e**7 + 16*d**5*e**8*x + 24*d**4*e**9*x**2 + 16*d**3*e*
*10*x**3 + 4*d**2*e**11*x**4) + 2*c**3*d**2*e**6*x**6/(4*d**6*e**7 + 16*d**
5*e**8*x + 24*d**4*e**9*x**2 + 16*d**3*e**10*x**3 + 4*d**2*e**11*x**4), Eq(
m, -5)), (-a**3*d*e**6/(3*d**4*e**7 + 9*d**3*e**8*x + 9*d**2*e**9*x**2 + 3*
d*e**10*x**3) + 3*a**2*c*e**7*x**3/(3*d**4*e**7 + 9*d**3*e**8*x + 9*d**2*e*
*9*x**2 + 3*d*e**10*x**3) - 36*a*c**2*d**5*e**2*log(d/e + x)/(3*d**4*e**7 +
9*d**3*e**8*x + 9*d**2*e**9*x**2 + 3*d*e**10*x**3) - 30*a*c**2*d**5*e**2/(
3*d**4*e**7 + 9*d**3*e**8*x + 9*d**2*e**9*x**2 + 3*d*e**10*x**3) - 108*a*c*
*2*d**4*e**3*x*log(d/e + x)/(3*d**4*e**7 + 9*d**3*e**8*x + 9*d**2*e**9*x**2
+ 3*d*e**10*x**3) - 54*a*c**2*d**4*e**3*x/(3*d**4*e**7 + 9*d**3*e**8*x + 9
*d**2*e**9*x**2 + 3*d*e**10*x**3) - 108*a*c**2*d**3*e**4*x**2*log(d/e + x)/
(3*d**4*e**7 + 9*d**3*e**8*x + 9*d**2*e**9*x**2 + 3*d*e**10*x**3) - 36*a*c*

*2*d**2*e**5*x**3*log(d/e + x)/(3*d**4*e**7 + 9*d**3*e**8*x + 9*d**2*e**9*x
2 + 3*d*e10*x**3) + 36*a*c**2*d**2*e**5*x**3/(3*d**4*e**7 + 9*d**3*e**8
*x + 9*d**2*e**9*x**2 + 3*d*e**10*x**3) + 9*a*c**2*d*e**6*x**4/(3*d**4*e**7
 + 9*d**3*e**8*x + 9*d**2*e**9*x**2 + 3*d*e**10*x**3) - 60*c**3*d**7*log(d/
e + x)/(3*d**4*e**7 + 9*d**3*e**8*x + 9*d**2*e**9*x**2 + 3*d*e**10*x**3) -
50*c**3*d**7/(3*d**4*e**7 + 9*d**3*e**8*x + 9*d**2*e**9*x**2 + 3*d*e**10*x*
*3) - 180*c**3*d**6*e*x*log(d/e + x)/(3*d**4*e**7 + 9*d**3*e**8*x + 9*d**2*
e**9*x**2 + 3*d*e**10*x**3) - 90*c**3*d**6*e*x/(3*d**4*e**7 + 9*d**3*e**8*x
 + 9*d**2*e**9*x**2 + 3*d*e**10*x**3) - 180*c**3*d**5*e**2*x**2*log(d/e + x
)/(3*d**4*e**7 + 9*d**3*e**8*x + 9*d**2*e**9*x**2 + 3*d*e**10*x**3) - 60*c*
*3*d**4*e**3*x**3*log(d/e + x)/(3*d**4*e**7 + 9*d**3*e**8*x + 9*d**2*e**9*x
2 + 3*d*e10*x**3) + 60*c**3*d**4*e**3*x**3/(3*d**4*e**7 + 9*d**3*e**8*x
 + 9*d**2*e**9*x**2 + 3*d*e**10*x**3) + 15*c**3*d**3*e**4*x**4/(3*d**4*e**7
 + 9*d**3*e**8*x + 9*d**2*e**9*x**2 + 3*d*e**10*x**3) - 3*c**3*d**2*e**5*x*
*5/(3*d**4*e**7 + 9*d**3*e**8*x + 9*d**2*e**9*x**2 + 3*d*e**10*x**3) + c**3
*d*e**6*x**6/(3*d**4*e**7 + 9*d**3*e**8*x + 9*d**2*e**9*x**2 + 3*d*e**10*x*
*3), Eq(m, -4)), (-2*a**3*e**6/(4*d**2*e**7 + 8*d*e**8*x + 4*e**9*x**2) + 1
2*a**2*c*d**2*e**4*log(d/e + x)/(4*d**2*e**7 + 8*d*e**8*x + 4*e**9*x**2) +
18*a**2*c*d**2*e**4/(4*d**2*e**7 + 8*d*e**8*x + 4*e**9*x**2) + 24*a**2*c*d*
e**5*x*log(d/e + x)/(4*d**2*e**7 + 8*d*e**8*x + 4*e**9*x**2) + 24*a**2*c*d*
e**5*x/(4*d**2*e**7 + 8*d*e**8*x + 4*e**9*x**2) + 12*a**2*c*e**6*x**2*log(d
/e + x)/(4*d**2*e**7 + 8*d*e**8*x + 4*e**9*x**2) + 72*a*c**2*d**4*e**2*log(
d/e + x)/(4*d**2*e**7 + 8*d*e**8*x + 4*e**9*x**2) + 108*a*c**2*d**4*e**2/(4
*d**2*e**7 + 8*d*e**8*x + 4*e**9*x**2) + 144*a*c**2*d**3*e**3*x*log(d/e + x
)/(4*d**2*e**7 + 8*d*e**8*x + 4*e**9*x**2) + 144*a*c**2*d**3*e**3*x/(4*d**2
*e**7 + 8*d*e**8*x + 4*e**9*x**2) + 72*a*c**2*d**2*e**4*x**2*log(d/e + x)/(
4*d**2*e**7 + 8*d*e**8*x + 4*e**9*x**2) - 24*a*c**2*d*e**5*x**3/(4*d**2*e**
7 + 8*d*e**8*x + 4*e**9*x**2) + 6*a*c**2*e**6*x**4/(4*d**2*e**7 + 8*d*e**8*
x + 4*e**9*x**2) + 60*c**3*d**6*log(d/e + x)/(4*d**2*e**7 + 8*d*e**8*x + 4*
e**9*x**2) + 90*c**3*d**6/(4*d**2*e**7 + 8*d*e**8*x + 4*e**9*x**2) + 120*c*
*3*d**5*e*x*log(d/e + x)/(4*d**2*e**7 + 8*d*e**8*x + 4*e**9*x**2) + 120*c**
3*d**5*e*x/(4*d**2*e**7 + 8*d*e**8*x + 4*e**9*x**2) + 60*c**3*d**4*e**2*x**
2*log(d/e + x)/(4*d**2*e**7 + 8*d*e**8*x + 4*e**9*x**2) - 20*c**3*d**3*e**3
*x**3/(4*d**2*e**7 + 8*d*e**8*x + 4*e**9*x**2) + 5*c**3*d**2*e**4*x**4/(4*d
2*e7 + 8*d*e**8*x + 4*e**9*x**2) - 2*c**3*d*e**5*x**5/(4*d**2*e**7 + 8*
d*e**8*x + 4*e**9*x**2) + c**3*e**6*x**6/(4*d**2*e**7 + 8*d*e**8*x + 4*e**9
*x**2), Eq(m, -3)), (-10*a**3*e**6/(10*d*e**7 + 10*e**8*x) - 60*a**2*c*d**2
*e**4*log(d/e + x)/(10*d*e**7 + 10*e**8*x) - 60*a**2*c*d**2*e**4/(10*d*e**7
 + 10*e**8*x) - 60*a**2*c*d*e**5*x*log(d/e + x)/(10*d*e**7 + 10*e**8*x) + 3
0*a**2*c*e**6*x**2/(10*d*e**7 + 10*e**8*x) - 120*a*c**2*d**4*e**2*log(d/e +
 x)/(10*d*e**7 + 10*e**8*x) - 120*a*c**2*d**4*e**2/(10*d*e**7 + 10*e**8*x)
- 120*a*c**2*d**3*e**3*x*log(d/e + x)/(10*d*e**7 + 10*e**8*x) + 60*a*c**2*d
2*e4*x**2/(10*d*e**7 + 10*e**8*x) - 20*a*c**2*d*e**5*x**3/(10*d*e**7 +
10*e**8*x) + 10*a*c**2*e**6*x**4/(10*d*e**7 + 10*e**8*x) - 60*c**3*d**6*log
(d/e + x)/(10*d*e**7 + 10*e**8*x) - 60*c**3*d**6/(10*d*e**7 + 10*e**8*x) -
60*c**3*d**5*e*x*log(d/e + x)/(10*d*e**7 + 10*e**8*x) + 30*c**3*d**4*e**2*x
2/(10*d*e7 + 10*e**8*x) - 10*c**3*d**3*e**3*x**3/(10*d*e**7 + 10*e**8*x
) + 5*c**3*d**2*e**4*x**4/(10*d*e**7 + 10*e**8*x) - 3*c**3*d*e**5*x**5/(10*
d*e**7 + 10*e**8*x) + 2*c**3*e**6*x**6/(10*d*e**7 + 10*e**8*x), Eq(m, -2)),
 (a**3*log(d/e + x)/e + 3*a**2*c*d**2*log(d/e + x)/e**3 - 3*a**2*c*d*x/e**2
 + 3*a**2*c*x**2/(2*e) + 3*a*c**2*d**4*log(d/e + x)/e**5 - 3*a*c**2*d**3*x/
e**4 + 3*a*c**2*d**2*x**2/(2*e**3) - a*c**2*d*x**3/e**2 + 3*a*c**2*x**4/(4*
e) + c**3*d**6*log(d/e + x)/e**7 - c**3*d**5*x/e**6 + c**3*d**4*x**2/(2*e**
5) - c**3*d**3*x**3/(3*e**4) + c**3*d**2*x**4/(4*e**3) - c**3*d*x**5/(5*e**
2) + c**3*x**6/(6*e), Eq(m, -1)), (a**3*d*e**6*m**6*(d + e*x)**m/(e**7*m**7
 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e
7*m2 + 13068*e**7*m + 5040*e**7) + 27*a**3*d*e**6*m**5*(d + e*x)**m/(e*
*7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 +
13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 295*a**3*d*e**6*m**4*(d + e*x

)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7
*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1665*a**3*d*e**6*m**3
*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 +
6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 5104*a**3*d*
e**6*m**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**
7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 802
8*a**3*d*e**6*m*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 19
60*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7)
 + 5040*a**3*d*e**6*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5
+ 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e
7) + a3*e**7*m**6*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m
5 + 1960*e7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 50
40*e**7) + 27*a**3*e**7*m**5*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322
*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7
*m + 5040*e**7) + 295*a**3*e**7*m**4*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m*
*6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13
068*e**7*m + 5040*e**7) + 1665*a**3*e**7*m**3*x*(d + e*x)**m/(e**7*m**7 + 2
8*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*
m**2 + 13068*e**7*m + 5040*e**7) + 5104*a**3*e**7*m**2*x*(d + e*x)**m/(e**7
*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13
132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 8028*a**3*e**7*m*x*(d + e*x)**m
/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**
3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 5040*a**3*e**7*x*(d + e*x
)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7
*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 6*a**2*c*d**3*e**4*m*
4(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4
+ 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 132*a**2*c
*d**3*e**4*m**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 19
60*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7)
 + 1074*a**2*c*d**3*e**4*m**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*
e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*
m + 5040*e**7) + 3828*a**2*c*d**3*e**4*m*(d + e*x)**m/(e**7*m**7 + 28*e**7*
m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 +
13068*e**7*m + 5040*e**7) + 5040*a**2*c*d**3*e**4*(d + e*x)**m/(e**7*m**7 +
 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**
7*m**2 + 13068*e**7*m + 5040*e**7) - 6*a**2*c*d**2*e**5*m**5*x*(d + e*x)**m
/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**
3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 132*a**2*c*d**2*e**5*m**4
x(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4
+ 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 1074*a**2*
c*d**2*e**5*m**3*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 +
 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e*
*7) - 3828*a**2*c*d**2*e**5*m**2*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 +
 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*
e**7*m + 5040*e**7) - 5040*a**2*c*d**2*e**5*m*x*(d + e*x)**m/(e**7*m**7 + 2
8*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*
m**2 + 13068*e**7*m + 5040*e**7) + 3*a**2*c*d*e**6*m**6*x**2*(d + e*x)**m/(
e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3
+ 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 69*a**2*c*d*e**6*m**5*x**2*
(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6
769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 603*a**2*c*d*
e**6*m**4*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 196
0*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7)
+ 2451*a**2*c*d*e**6*m**3*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322
*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7
*m + 5040*e**7) + 4434*a**2*c*d*e**6*m**2*x**2*(d + e*x)**m/(e**7*m**7 + 28
*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m
2 + 13068*e7*m + 5040*e**7) + 2520*a**2*c*d*e**6*m*x**2*(d + e*x)**m/(e
7*m7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 +

13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 3*a**2*c*e**7*m**6*x**3*(d +
e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*
e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 75*a**2*c*e**7*m*
*5*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*
m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 741*a
2*c*e7*m**4*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5
 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*
e**7) + 3657*a**2*c*e**7*m**3*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 +
 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*
e**7*m + 5040*e**7) + 9336*a**2*c*e**7*m**2*x**3*(d + e*x)**m/(e**7*m**7 +
28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7
*m**2 + 13068*e**7*m + 5040*e**7) + 11388*a**2*c*e**7*m*x**3*(d + e*x)**m/(
e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3
 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 5040*a**2*c*e**7*x**3*(d +
e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e
7*m3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 72*a*c**2*d**5*e**
2*m**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m
4 + 6769*e7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 936*a*
c**2*d**5*e**2*m*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1
960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7
) + 3024*a*c**2*d**5*e**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7
*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m +
5040*e**7) - 72*a*c**2*d**4*e**3*m**3*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m
6 + 322*e7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 1
3068*e**7*m + 5040*e**7) - 936*a*c**2*d**4*e**3*m**2*x*(d + e*x)**m/(e**7*m
7 + 28*e7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 1313
2*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 3024*a*c**2*d**4*e**3*m*x*(d + e*
x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**
7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 36*a*c**2*d**3*e**4*
m**4*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**
7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 504
*a*c**2*d**3*e**4*m**3*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e*
*7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m
 + 5040*e**7) + 1980*a*c**2*d**3*e**4*m**2*x**2*(d + e*x)**m/(e**7*m**7 + 28
*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m
2 + 13068*e7*m + 5040*e**7) + 1512*a*c**2*d**3*e**4*m*x**2*(d + e*x)**m
/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**
3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 12*a*c**2*d**2*e**5*m**5*
x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**
4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 192*a*c*
*2*d**2*e**5*m**4*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m*
*5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 504
0*e**7) - 996*a*c**2*d**2*e**5*m**3*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*
m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 +
13068*e**7*m + 5040*e**7) - 1824*a*c**2*d**2*e**5*m**2*x**3*(d + e*x)**m/(e
7*m7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 +
 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 1008*a*c**2*d**2*e**5*m*x**3
*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 +
6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 3*a*c**2*d*e
6*m6*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960
*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) +
 57*a*c**2*d*e**6*m**5*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e*
*7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m
 + 5040*e**7) + 393*a*c**2*d*e**6*m**4*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**
7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2
 + 13068*e**7*m + 5040*e**7) + 1203*a*c**2*d*e**6*m**3*x**4*(d + e*x)**m/(e*
*7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 +
13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1620*a*c**2*d*e**6*m**2*x**4*
(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6

769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 756*a*c**2*d*
e**6*m*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e
7*m4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 3
*a*c**2*e**7*m**6*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m*
*5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 504
0*e**7) + 69*a*c**2*e**7*m**5*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 +
 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*
e**7*m + 5040*e**7) + 621*a*c**2*e**7*m**4*x**5*(d + e*x)**m/(e**7*m**7 + 2
8*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*
m**2 + 13068*e**7*m + 5040*e**7) + 2775*a*c**2*e**7*m**3*x**5*(d + e*x)**m/
(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3
 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 6432*a*c**2*e**7*m**2*x**5
*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 +
6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 7236*a*c**2*
e**7*m*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e
7*m4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 3
024*a*c**2*e**7*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5
 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*
e**7) + 720*c**3*d**7*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**
5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040
*e**7) - 720*c**3*d**6*e*m*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e
7*m5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m
 + 5040*e**7) + 360*c**3*d**5*e**2*m**2*x**2*(d + e*x)**m/(e**7*m**7 + 28*e
7*m6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**
2 + 13068*e**7*m + 5040*e**7) + 360*c**3*d**5*e**2*m*x**2*(d + e*x)**m/(e**
7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 1
3132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 120*c**3*d**4*e**3*m**3*x**3*(
d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 67
69*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 360*c**3*d**4*
e**3*m**2*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 196
0*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7)
- 240*c**3*d**4*e**3*m*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e*
*7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m
+ 5040*e**7) + 30*c**3*d**3*e**4*m**4*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**
7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2
+ 13068*e**7*m + 5040*e**7) + 180*c**3*d**3*e**4*m**3*x**4*(d + e*x)**m/(e*
*7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 +
13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 330*c**3*d**3*e**4*m**2*x**4*
(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6
769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 180*c**3*d**3
*e**4*m*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*
e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) -
6*c**3*d**2*e**5*m**5*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**
7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m +
 5040*e**7) - 60*c**3*d**2*e**5*m**4*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7
*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 +
 13068*e**7*m + 5040*e**7) - 210*c**3*d**2*e**5*m**3*x**5*(d + e*x)**m/(e**
7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 1
3132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 300*c**3*d**2*e**5*m**2*x**5*(
d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 67
69*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 144*c**3*d**2*
e**5*m*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e
7*m4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + c
3*d*e6*m**6*x**6*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5
 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*
e**7) + 15*c**3*d*e**6*m**5*x**6*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 3
22*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e*
*7*m + 5040*e**7) + 85*c**3*d*e**6*m**4*x**6*(d + e*x)**m/(e**7*m**7 + 28*e
7*m6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**

```
2 + 13068*e**7*m + 5040*e**7) + 225*c**3*d*e**6*m**3*x**6*(d + e*x)**m/(e**
7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 1
3132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 274*c**3*d*e**6*m**2*x**6*(d +
 e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*
e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 120*c**3*d*e**6*m
*x**6*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m*
*4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + c**3*e*
*7*m**6*x**7*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*
e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) +
21*c**3*e**7*m**5*x**7*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m*
*5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 504
0*e**7) + 175*c**3*e**7*m**4*x**7*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 +
322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e
**7*m + 5040*e**7) + 735*c**3*e**7*m**3*x**7*(d + e*x)**m/(e**7*m**7 + 28*e
**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**
2 + 13068*e**7*m + 5040*e**7) + 1624*c**3*e**7*m**2*x**7*(d + e*x)**m/(e**7
*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13
132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1764*c**3*e**7*m*x**7*(d + e*x)
**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*
m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 720*c**3*e**7*x**7*(d
+ e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769
*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7), True))
```

Giac [B] time = 1.18065, size = 2808, normalized size = 12.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*c^3*m^6*x^7*e^7 + (x*e + d)^m*c^3*d*m^6*x^6*e^6 + 21*(x*e + d)
^m*c^3*m^5*x^7*e^7 + 15*(x*e + d)^m*c^3*d*m^5*x^6*e^6 - 6*(x*e + d)^m*c^3*d
^2*m^5*x^5*e^5 + 3*(x*e + d)^m*a*c^2*m^6*x^5*e^7 + 175*(x*e + d)^m*c^3*m^4*
x^7*e^7 + 3*(x*e + d)^m*a*c^2*d*m^6*x^4*e^6 + 85*(x*e + d)^m*c^3*d*m^4*x^6*
e^6 - 60*(x*e + d)^m*c^3*d^2*m^4*x^5*e^5 + 30*(x*e + d)^m*c^3*d^3*m^4*x^4*e
^4 + 69*(x*e + d)^m*a*c^2*m^5*x^5*e^7 + 735*(x*e + d)^m*c^3*m^3*x^7*e^7 + 5
7*(x*e + d)^m*a*c^2*d*m^5*x^4*e^6 + 225*(x*e + d)^m*c^3*d*m^3*x^6*e^6 - 12*
(x*e + d)^m*a*c^2*d^2*m^5*x^3*e^5 - 210*(x*e + d)^m*c^3*d^2*m^3*x^5*e^5 + 1
80*(x*e + d)^m*c^3*d^3*m^3*x^4*e^4 - 120*(x*e + d)^m*c^3*d^4*m^3*x^3*e^3 +
3*(x*e + d)^m*a^2*c*m^6*x^3*e^7 + 621*(x*e + d)^m*a*c^2*m^4*x^5*e^7 + 1624*
(x*e + d)^m*c^3*m^2*x^7*e^7 + 3*(x*e + d)^m*a^2*c*d*m^6*x^2*e^6 + 393*(x*e
+ d)^m*a*c^2*d*m^4*x^4*e^6 + 274*(x*e + d)^m*c^3*d*m^2*x^6*e^6 - 192*(x*e +
 d)^m*a*c^2*d^2*m^4*x^3*e^5 - 300*(x*e + d)^m*c^3*d^2*m^2*x^5*e^5 + 36*(x*e
 + d)^m*a*c^2*d^3*m^4*x^2*e^4 + 330*(x*e + d)^m*c^3*d^3*m^2*x^4*e^4 - 360*(
x*e + d)^m*c^3*d^4*m^2*x^3*e^3 + 360*(x*e + d)^m*c^3*d^5*m^2*x^2*e^2 + 75*(
x*e + d)^m*a^2*c*m^5*x^3*e^7 + 2775*(x*e + d)^m*a*c^2*m^3*x^5*e^7 + 1764*(x
*e + d)^m*c^3*m*x^7*e^7 + 69*(x*e + d)^m*a^2*c*d*m^5*x^2*e^6 + 1203*(x*e +
d)^m*a*c^2*d*m^3*x^4*e^6 + 120*(x*e + d)^m*c^3*d*m*x^6*e^6 - 6*(x*e + d)^m*
a^2*c*d^2*m^5*x*e^5 - 996*(x*e + d)^m*a*c^2*d^2*m^3*x^3*e^5 - 144*(x*e + d)
^m*c^3*d^2*m*x^5*e^5 + 504*(x*e + d)^m*a*c^2*d^3*m^3*x^2*e^4 + 180*(x*e + d
)^m*c^3*d^3*m*x^4*e^4 - 72*(x*e + d)^m*a*c^2*d^4*m^3*x*e^3 - 240*(x*e + d)^
m*c^3*d^4*m*x^3*e^3 + 360*(x*e + d)^m*c^3*d^5*m*x^2*e^2 - 720*(x*e + d)^m*c
^3*d^6*m*x*e + (x*e + d)^m*a^3*m^6*x*e^7 + 741*(x*e + d)^m*a^2*c*m^4*x^3*e^
7 + 6432*(x*e + d)^m*a*c^2*m^2*x^5*e^7 + 720*(x*e + d)^m*c^3*x^7*e^7 + (x*e
 + d)^m*a^3*d*m^6*e^6 + 603*(x*e + d)^m*a^2*c*d*m^4*x^2*e^6 + 1620*(x*e + d
)^m*a*c^2*d*m^2*x^4*e^6 - 132*(x*e + d)^m*a^2*c*d^2*m^4*x*e^5 - 1824*(x*e +
```

```
d)^m*a*c^2*d^2*m^2*x^3*e^5 + 6*(x*e + d)^m*a^2*c*d^3*m^4*e^4 + 1980*(x*e +
d)^m*a*c^2*d^3*m^2*x^2*e^4 - 936*(x*e + d)^m*a*c^2*d^4*m^2*x*e^3 + 72*(x*e
 + d)^m*a*c^2*d^5*m^2*e^2 + 720*(x*e + d)^m*c^3*d^7 + 27*(x*e + d)^m*a^3*m^
5*x*e^7 + 3657*(x*e + d)^m*a^2*c*m^3*x^3*e^7 + 7236*(x*e + d)^m*a*c^2*m*x^5
*e^7 + 27*(x*e + d)^m*a^3*d*m^5*e^6 + 2451*(x*e + d)^m*a^2*c*d*m^3*x^2*e^6
+ 756*(x*e + d)^m*a*c^2*d*m*x^4*e^6 - 1074*(x*e + d)^m*a^2*c*d^2*m^3*x*e^5
- 1008*(x*e + d)^m*a*c^2*d^2*m*x^3*e^5 + 132*(x*e + d)^m*a^2*c*d^3*m^3*e^4
+ 1512*(x*e + d)^m*a*c^2*d^3*m*x^2*e^4 - 3024*(x*e + d)^m*a*c^2*d^4*m*x*e^3
 + 936*(x*e + d)^m*a*c^2*d^5*m*e^2 + 295*(x*e + d)^m*a^3*m^4*x*e^7 + 9336*(
x*e + d)^m*a^2*c*m^2*x^3*e^7 + 3024*(x*e + d)^m*a*c^2*x^5*e^7 + 295*(x*e +
d)^m*a^3*d*m^4*e^6 + 4434*(x*e + d)^m*a^2*c*d*m^2*x^2*e^6 - 3828*(x*e + d)^
m*a^2*c*d^2*m^2*x*e^5 + 1074*(x*e + d)^m*a^2*c*d^3*m^2*e^4 + 3024*(x*e + d)
^m*a*c^2*d^5*e^2 + 1665*(x*e + d)^m*a^3*m^3*x*e^7 + 11388*(x*e + d)^m*a^2*c
*m*x^3*e^7 + 1665*(x*e + d)^m*a^3*d*m^3*e^6 + 2520*(x*e + d)^m*a^2*c*d*m*x^
2*e^6 - 5040*(x*e + d)^m*a^2*c*d^2*m*x*e^5 + 3828*(x*e + d)^m*a^2*c*d^3*m*e
^4 + 5104*(x*e + d)^m*a^3*m^2*x*e^7 + 5040*(x*e + d)^m*a^2*c*x^3*e^7 + 5104
*(x*e + d)^m*a^3*d*m^2*e^6 + 5040*(x*e + d)^m*a^2*c*d^3*e^4 + 8028*(x*e + d
)^m*a^3*m*x*e^7 + 8028*(x*e + d)^m*a^3*d*m*e^6 + 5040*(x*e + d)^m*a^3*x*e^7
 + 5040*(x*e + d)^m*a^3*d*e^6)/(m^7*e^7 + 28*m^6*e^7 + 322*m^5*e^7 + 1960*m
^4*e^7 + 6769*m^3*e^7 + 13132*m^2*e^7 + 13068*m*e^7 + 5040*e^7)
```

3.722 $\int (d+ex)^m \left(a+cx^2\right)^2 dx$

Optimal. Leaf size=140

$$\frac{\left(ae^2+cd^2\right)^2 (d+ex)^{m+1}}{e^5(m+1)}-\frac{4cd\left(ae^2+cd^2\right)(d+ex)^{m+2}}{e^5(m+2)}+\frac{2c\left(ae^2+3cd^2\right)(d+ex)^{m+3}}{e^5(m+3)}-\frac{4c^2d(d+ex)^{m+4}}{e^5(m+4)}+\frac{c^2(d+ex)^{m+5}}{e^5(m+5)}$$

```
[Out] ((c*d^2 + a*e^2)^2*(d + e*x)^(1 + m))/(e^5*(1 + m)) - (4*c*d*(c*d^2 + a*e^2
)*(d + e*x)^(2 + m))/(e^5*(2 + m)) + (2*c*(3*c*d^2 + a*e^2)*(d + e*x)^(3 +
m))/(e^5*(3 + m)) - (4*c^2*d*(d + e*x)^(4 + m))/(e^5*(4 + m)) + (c^2*(d + e
*x)^(5 + m))/(e^5*(5 + m))
```

Rubi [A] time = 0.0687169, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$\frac{\left(ae^2+cd^2\right)^2 (d+ex)^{m+1}}{e^5(m+1)}-\frac{4cd\left(ae^2+cd^2\right)(d+ex)^{m+2}}{e^5(m+2)}+\frac{2c\left(ae^2+3cd^2\right)(d+ex)^{m+3}}{e^5(m+3)}-\frac{4c^2d(d+ex)^{m+4}}{e^5(m+4)}+\frac{c^2(d+ex)^{m+5}}{e^5(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a + c*x^2)^2,x]
```


```
[Out] ((c*d^2 + a*e^2)^2*(d + e*x)^(1 + m))/(e^5*(1 + m)) - (4*c*d*(c*d^2 + a*e^2
)*(d + e*x)^(2 + m))/(e^5*(2 + m)) + (2*c*(3*c*d^2 + a*e^2)*(d + e*x)^(3 +
m))/(e^5*(3 + m)) - (4*c^2*d*(d + e*x)^(4 + m))/(e^5*(4 + m)) + (c^2*(d + e
*x)^(5 + m))/(e^5*(5 + m))
```


Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (d+ex)^m \left(a+cx^2\right)^2 dx = \int \left(\frac{\left(cd^2+ae^2\right)^2 (d+ex)^m}{e^4}-\frac{4cd\left(cd^2+ae^2\right)(d+ex)^{1+m}}{e^4}+\frac{2c\left(3cd^2+ae^2\right)(d+ex)^{2+m}}{e^4}-\frac{4c}{}\right.$$
$$=\frac{\left(cd^2+ae^2\right)^2 (d+ex)^{1+m}}{e^5(1+m)}-\frac{4cd\left(cd^2+ae^2\right)(d+ex)^{2+m}}{e^5(2+m)}+\frac{2c\left(3cd^2+ae^2\right)(d+ex)^{3+m}}{e^5(3+m)}-\frac{4c^2}{}$$

Mathematica [A] time = 0.188468, size = 176, normalized size = 1.26

$$\frac{(d+ex)^{m+1}\left(\frac{4\left(ae^2+cd^2\right)\left(ae^2\left(m^2+5m+6\right)+c\left(2d^2-2de(m+1)x+e^2\left(m^2+3m+2\right)x^2\right)\right)}{e^4(m+1)(m+2)(m+3)}-\frac{4cd(d+ex)\left(ae^2\left(m^2+7m+12\right)+c\left(2d^2-2de(m+2)x+e^2\left(m^2+5m+6\right)x^2\right)\right)}{e^4(m+2)(m+3)(m+4)}+\right.}{e(m+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(a + c*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + m)*((a + c*x^2)^2 + (4*(c*d^2 + a*e^2)*(a*e^2*(6 + 5*m + m^
2) + c*(2*d^2 - 2*d*e*(1 + m)*x + e^2*(2 + 3*m + m^2)*x^2)))/(e^4*(1 + m)*(
2 + m)*(3 + m)) - (4*c*d*(d + e*x)*(a*e^2*(12 + 7*m + m^2) + c*(2*d^2 - 2*d
*e*(2 + m)*x + e^2*(6 + 5*m + m^2)*x^2)))/(e^4*(2 + m)*(3 + m)*(4 + m))))/(
e*(5 + m))
```

Maple [B] time = 0.047, size = 420, normalized size = 3.

$$(ex+d)^{1+m}\left(c^2e^4m^4x^4+10\,c^2e^4m^3x^4+2\,ace^4m^4x^2-4\,c^2de^3m^3x^3+35\,c^2e^4m^2x^4+24\,ace^4m^3x^2-24\,c^2de^3m^2x^3+50\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+a)^2,x)
```

```
[Out] (e*x+d)^(1+m)*(c^2*e^4*m^4*x^4+10*c^2*e^4*m^3*x^4+2*a*c*e^4*m^4*x^2-4*c^2*d
*e^3*m^3*x^3+35*c^2*e^4*m^2*x^4+24*a*c*e^4*m^3*x^2-24*c^2*d*e^3*m^2*x^3+50*
c^2*e^4*m*x^4+a^2*e^4*m^4-4*a*c*d*e^3*m^3*x+98*a*c*e^4*m^2*x^2+12*c^2*d^2*e
^2*m^2*x^2-44*c^2*d*e^3*m*x^3+24*c^2*e^4*x^4+14*a^2*e^4*m^3-40*a*c*d*e^3*m^
2*x+156*a*c*e^4*m*x^2+36*c^2*d^2*e^2*m*x^2-24*c^2*d*e^3*x^3+71*a^2*e^4*m^2+
4*a*c*d^2*e^2*m^2-116*a*c*d*e^3*m*x+80*a*c*e^4*x^2-24*c^2*d^3*e*m*x+24*c^2*
d^2*e^2*x^2+154*a^2*e^4*m+36*a*c*d^2*e^2*m-80*a*c*d*e^3*x-24*c^2*d^3*e*x+12
0*a^2*e^4+80*a*c*d^2*e^2+24*c^2*d^4)/e^5/(m^5+15*m^4+85*m^3+225*m^2+274*m+1
20)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.22426, size = 1092, normalized size = 7.8

$$\left(a^2de^4m^4+14\,a^2de^4m^3+24\,c^2d^5+80\,acd^3e^2+120\,a^2de^4+\left(c^2e^5m^4+10\,c^2e^5m^3+35\,c^2e^5m^2+50\,c^2e^5m+24\,c^2e^5\right)x^5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] (a^2*d*e^4*m^4 + 14*a^2*d*e^4*m^3 + 24*c^2*d^5 + 80*a*c*d^3*e^2 + 120*a^2*d
*e^4 + (c^2*e^5*m^4 + 10*c^2*e^5*m^3 + 35*c^2*e^5*m^2 + 50*c^2*e^5*m + 24*c
^2*e^5)*x^5 + (c^2*d*e^4*m^4 + 6*c^2*d*e^4*m^3 + 11*c^2*d*e^4*m^2 + 6*c^2*d
*e^4*m)*x^4 + 2*(a*c*e^5*m^4 + 40*a*c*e^5 - 2*(c^2*d^2*e^3 - 6*a*c*e^5)*m^3
 - (6*c^2*d^2*e^3 - 49*a*c*e^5)*m^2 - 2*(2*c^2*d^2*e^3 - 39*a*c*e^5)*m)*x^3
 + (4*a*c*d^3*e^2 + 71*a^2*d*e^4)*m^2 + 2*(a*c*d*e^4*m^4 + 10*a*c*d*e^4*m^3
 + (6*c^2*d^3*e^2 + 29*a*c*d*e^4)*m^2 + 2*(3*c^2*d^3*e^2 + 10*a*c*d*e^4)*m)
```

```
*x^2 + 2*(18*a*c*d^3*e^2 + 77*a^2*d*e^4)*m + (a^2*e^5*m^4 + 120*a^2*e^5 - 2
*(2*a*c*d^2*e^3 - 7*a^2*e^5)*m^3 - (36*a*c*d^2*e^3 - 71*a^2*e^5)*m^2 - 2*(1
2*c^2*d^4*e + 40*a*c*d^2*e^3 - 77*a^2*e^5)*m)*x)*(e*x + d)^m/(e^5*m^5 + 15*
e^5*m^4 + 85*e^5*m^3 + 225*e^5*m^2 + 274*e^5*m + 120*e^5)
```

Sympy [A] time = 5.7532, size = 5049, normalized size = 36.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+a)**2,x)
```

```
[Out] Piecewise((d**m*(a**2*x + 2*a*c*x**3/3 + c**2*x**5/5), Eq(e, 0)), (-3*a**2*
d**2*e**4/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8
*x**3 + 12*d**2*e**9*x**4) + 8*a*c*d*e**5*x**3/(12*d**6*e**5 + 48*d**5*e**6
*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4) + 2*a*c*e**
6*x**4/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x*
*3 + 12*d**2*e**9*x**4) + 12*c**2*d**6*log(d/e + x)/(12*d**6*e**5 + 48*d**5
*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4) + 7*c*
*2*d**6/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x
**3 + 12*d**2*e**9*x**4) + 48*c**2*d**5*e*x*log(d/e + x)/(12*d**6*e**5 + 48
*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4) +
 16*c**2*d**5*e*x/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d
**3*e**8*x**3 + 12*d**2*e**9*x**4) + 72*c**2*d**4*e**2*x**2*log(d/e + x)/(1
2*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d
**2*e**9*x**4) + 48*c**2*d**3*e**3*x**3*log(d/e + x)/(12*d**6*e**5 + 48*d**
5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4) - 24*
c**2*d**3*e**3*x**3/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48
*d**3*e**8*x**3 + 12*d**2*e**9*x**4) + 12*c**2*d**2*e**4*x**4*log(d/e + x)/
(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12
*d**2*e**9*x**4) - 18*c**2*d**2*e**4*x**4/(12*d**6*e**5 + 48*d**5*e**6*x +
72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4), Eq(m, -5)), (-a
**2*d*e**4/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3)
 + 2*a*c*e**5*x**3/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e*
*8*x**3) - 12*c**2*d**5*log(d/e + x)/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*
e**7*x**2 + 3*d*e**8*x**3) - 10*c**2*d**5/(3*d**4*e**5 + 9*d**3*e**6*x + 9*
d**2*e**7*x**2 + 3*d*e**8*x**3) - 36*c**2*d**4*e*x*log(d/e + x)/(3*d**4*e**
5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3) - 18*c**2*d**4*e*x/(3
*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3) - 36*c**2*d*
*3*e**2*x**2*log(d/e + x)/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 +
 3*d*e**8*x**3) - 12*c**2*d**2*e**3*x**3*log(d/e + x)/(3*d**4*e**5 + 9*d**3
*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3) + 12*c**2*d**2*e**3*x**3/(3*d**
4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3) + 3*c**2*d*e**4*
x**4/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3), Eq(m
, -4)), (-a**2*e**4/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 4*a*c*d**2*e
**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 6*a*c*d**2*e**2
/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 8*a*c*d*e**3*x*log(d/e + x)/(2*
d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 8*a*c*d*e**3*x/(2*d**2*e**5 + 4*d*e
**6*x + 2*e**7*x**2) + 4*a*c*e**4*x**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6
*x + 2*e**7*x**2) + 12*c**2*d**4*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2
*e**7*x**2) + 18*c**2*d**4/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 24*c*
*2*d**3*e*x*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 24*c**2
*d**3*e*x/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 12*c**2*d**2*e**2*x**2
*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 4*c**2*d*e**3*x**3
/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + c**2*e**4*x**4/(2*d**2*e**5 + 4
*d*e**6*x + 2*e**7*x**2), Eq(m, -3)), (-3*a**2*e**4/(3*d*e**5 + 3*e**6*x) -
```

12*a*c*d**2*e**2*log(d/e + x)/(3*d*e**5 + 3*e**6*x) - 12*a*c*d**2*e**2/(3*
d*e**5 + 3*e**6*x) - 12*a*c*d*e**3*x*log(d/e + x)/(3*d*e**5 + 3*e**6*x) + 6
*a*c*e**4*x**2/(3*d*e**5 + 3*e**6*x) - 12*c**2*d**4*log(d/e + x)/(3*d*e**5
+ 3*e**6*x) - 12*c**2*d**4/(3*d*e**5 + 3*e**6*x) - 12*c**2*d**3*e*x*log(d/e
+ x)/(3*d*e**5 + 3*e**6*x) + 6*c**2*d**2*e**2*x**2/(3*d*e**5 + 3*e**6*x) -
2*c**2*d*e**3*x**3/(3*d*e**5 + 3*e**6*x) + c**2*e**4*x**4/(3*d*e**5 + 3*e*
*6*x), Eq(m, -2)), (a**2*log(d/e + x)/e + 2*a*c*d**2*log(d/e + x)/e**3 - 2*
a*c*d*x/e**2 + a*c*x**2/e + c**2*d**4*log(d/e + x)/e**5 - c**2*d**3*x/e**4
+ c**2*d**2*x**2/(2*e**3) - c**2*d*x**3/(3*e**2) + c**2*x**4/(4*e), Eq(m, -
1)), (a**2*d*e**4*m**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**
3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 14*a**2*d*e**4*m**3*(d + e*x)*
*m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m +
120*e**5) + 71*a**2*d*e**4*m**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85
*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 154*a**2*d*e**4*m*(d
+ e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e*
*5*m + 120*e**5) + 120*a**2*d*e**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 +
85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + a**2*e**5*m**4*x*(
d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*
e**5*m + 120*e**5) + 14*a**2*e**5*m**3*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*
m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 71*a**2*e**5
*m**2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m*
*2 + 274*e**5*m + 120*e**5) + 154*a**2*e**5*m*x*(d + e*x)**m/(e**5*m**5 + 1
5*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 120*a
2*e5*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5
*m**2 + 274*e**5*m + 120*e**5) + 4*a*c*d**3*e**2*m**2*(d + e*x)**m/(e**5*m*
*5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) +
36*a*c*d**3*e**2*m*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 +
225*e**5*m**2 + 274*e**5*m + 120*e**5) + 80*a*c*d**3*e**2*(d + e*x)**m/(e*
*5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e*
*5) - 4*a*c*d**2*e**3*m**3*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e*
*5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 36*a*c*d**2*e**3*m**2*x*
(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274
*e**5*m + 120*e**5) - 80*a*c*d**2*e**3*m*x*(d + e*x)**m/(e**5*m**5 + 15*e**
5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 2*a*c*d*e*
*4*m**4*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e*
*5*m**2 + 274*e**5*m + 120*e**5) + 20*a*c*d*e**4*m**3*x**2*(d + e*x)**m/(e*
*5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e*
*5) + 58*a*c*d*e**4*m**2*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e
5*m3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 40*a*c*d*e**4*m*x**2*(d
+ e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e
5*m + 120*e5) + 2*a*c*e**5*m**4*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*
m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 24*a*c*e**5*
m**3*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*
m**2 + 274*e**5*m + 120*e**5) + 98*a*c*e**5*m**2*x**3*(d + e*x)**m/(e**5*m*
*5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) +
156*a*c*e**5*m*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3
+ 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 80*a*c*e**5*x**3*(d + e*x)**m/(e
5*m5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e
5) + 24*c2*d**5*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 +
225*e**5*m**2 + 274*e**5*m + 120*e**5) - 24*c**2*d**4*e*m*x*(d + e*x)**m/(
e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*
e**5) + 12*c**2*d**3*e**2*m**2*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4
+ 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 12*c**2*d**3*e**2
*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m*
*2 + 274*e**5*m + 120*e**5) - 4*c**2*d**2*e**3*m**3*x**3*(d + e*x)**m/(e**5
*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5
) - 12*c**2*d**2*e**3*m**2*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85
*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 8*c**2*d**2*e**3*m*x*
3(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 +

```
274*e**5*m + 120*e**5) + c**2*d*e**4*m**4*x**4*(d + e*x)**m/(e**5*m**5 + 15
*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 6*c**2
*d*e**4*m**3*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 2
25*e**5*m**2 + 274*e**5*m + 120*e**5) + 11*c**2*d*e**4*m**2*x**4*(d + e*x)*
*m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m +
120*e**5) + 6*c**2*d*e**4*m*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 8
5*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + c**2*e**5*m**4*x**5*
(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274
*e**5*m + 120*e**5) + 10*c**2*e**5*m**3*x**5*(d + e*x)**m/(e**5*m**5 + 15*e
**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 35*c**2*
e**5*m**2*x**5*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*
e**5*m**2 + 274*e**5*m + 120*e**5) + 50*c**2*e**5*m*x**5*(d + e*x)**m/(e**5
*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5
) + 24*c**2*e**5*x**5*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3
 + 225*e**5*m**2 + 274*e**5*m + 120*e**5), True))
```

Giac [B] time = 1.15116, size = 1145, normalized size = 8.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*c^2*m^4*x^5*e^5 + (x*e + d)^m*c^2*d*m^4*x^4*e^4 + 10*(x*e + d)
^m*c^2*m^3*x^5*e^5 + 6*(x*e + d)^m*c^2*d*m^3*x^4*e^4 - 4*(x*e + d)^m*c^2*d^
2*m^3*x^3*e^3 + 2*(x*e + d)^m*a*c*m^4*x^3*e^5 + 35*(x*e + d)^m*c^2*m^2*x^5*
e^5 + 2*(x*e + d)^m*a*c*d*m^4*x^2*e^4 + 11*(x*e + d)^m*c^2*d*m^2*x^4*e^4 -
12*(x*e + d)^m*c^2*d^2*m^2*x^3*e^3 + 12*(x*e + d)^m*c^2*d^3*m^2*x^2*e^2 + 2
4*(x*e + d)^m*a*c*m^3*x^3*e^5 + 50*(x*e + d)^m*c^2*m*x^5*e^5 + 20*(x*e + d)
^m*a*c*d*m^3*x^2*e^4 + 6*(x*e + d)^m*c^2*d*m*x^4*e^4 - 4*(x*e + d)^m*a*c*d^
2*m^3*x*e^3 - 8*(x*e + d)^m*c^2*d^2*m*x^3*e^3 + 12*(x*e + d)^m*c^2*d^3*m*x^
2*e^2 - 24*(x*e + d)^m*c^2*d^4*m*x*e + (x*e + d)^m*a^2*m^4*x*e^5 + 98*(x*e
+ d)^m*a*c*m^2*x^3*e^5 + 24*(x*e + d)^m*c^2*x^5*e^5 + (x*e + d)^m*a^2*d*m^4
*e^4 + 58*(x*e + d)^m*a*c*d*m^2*x^2*e^4 - 36*(x*e + d)^m*a*c*d^2*m^2*x*e^3
+ 4*(x*e + d)^m*a*c*d^3*m^2*e^2 + 24*(x*e + d)^m*c^2*d^5 + 14*(x*e + d)^m*a
^2*m^3*x*e^5 + 156*(x*e + d)^m*a*c*m*x^3*e^5 + 14*(x*e + d)^m*a^2*d*m^3*e^4
 + 40*(x*e + d)^m*a*c*d*m*x^2*e^4 - 80*(x*e + d)^m*a*c*d^2*m*x*e^3 + 36*(x*
e + d)^m*a*c*d^3*m*e^2 + 71*(x*e + d)^m*a^2*m^2*x*e^5 + 80*(x*e + d)^m*a*c*
x^3*e^5 + 71*(x*e + d)^m*a^2*d*m^2*e^4 + 80*(x*e + d)^m*a*c*d^3*e^2 + 154*(
x*e + d)^m*a^2*m*x*e^5 + 154*(x*e + d)^m*a^2*d*m*e^4 + 120*(x*e + d)^m*a^2*
x*e^5 + 120*(x*e + d)^m*a^2*d*e^4)/(m^5*e^5 + 15*m^4*e^5 + 85*m^3*e^5 + 225
*m^2*e^5 + 274*m*e^5 + 120*e^5)
```

3.723 $\int (d+ex)^m \left(a+cx^2\right) dx$

Optimal. Leaf size=70

$$\frac{\left(ae^2+cd^2\right)(d+ex)^{m+1}}{e^3(m+1)}-\frac{2cd(d+ex)^{m+2}}{e^3(m+2)}+\frac{c(d+ex)^{m+3}}{e^3(m+3)}$$

```
[Out] ((c*d^2 + a*e^2)*(d + e*x)^(1 + m))/(e^3*(1 + m)) - (2*c*d*(d + e*x)^(2 + m
))/(e^3*(2 + m)) + (c*(d + e*x)^(3 + m))/(e^3*(3 + m))
```

Rubi [A] time = 0.0295774, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {697}

$$\frac{\left(ae^2+cd^2\right)(d+ex)^{m+1}}{e^3(m+1)}-\frac{2cd(d+ex)^{m+2}}{e^3(m+2)}+\frac{c(d+ex)^{m+3}}{e^3(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a + c*x^2),x]
```


```
[Out] ((c*d^2 + a*e^2)*(d + e*x)^(1 + m))/(e^3*(1 + m)) - (2*c*d*(d + e*x)^(2 + m
))/(e^3*(2 + m)) + (c*(d + e*x)^(3 + m))/(e^3*(3 + m))
```


Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^m \left(a+cx^2\right) dx &= \int \left(\frac{\left(cd^2+ae^2\right)(d+ex)^m}{e^2}-\frac{2cd(d+ex)^{1+m}}{e^2}+\frac{c(d+ex)^{2+m}}{e^2}\right) dx\\ &= \frac{\left(cd^2+ae^2\right)(d+ex)^{1+m}}{e^3(1+m)}-\frac{2cd(d+ex)^{2+m}}{e^3(2+m)}+\frac{c(d+ex)^{3+m}}{e^3(3+m)}\end{aligned}$$

Mathematica [A] time = 0.0402471, size = 59, normalized size = 0.84

$$\frac{(d+ex)^{m+1}\left(\frac{ae^2+cd^2}{m+1}+\frac{c(d+ex)^2}{m+3}-\frac{2cd(d+ex)}{m+2}\right)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(a + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + m)*((c*d^2 + a*e^2)/(1 + m) - (2*c*d*(d + e*x))/(2 + m) + (
c*(d + e*x)^2)/(3 + m)))/e^3
```

Maple [A] time = 0.043, size = 100, normalized size = 1.4

$$\frac{(ex+d)^{1+m}\left(ce^2m^2x^2+3\,ce^2mx^2+ae^2m^2-2\,cdemx+2\,ce^2x^2+5\,ae^2m-2\,cdex+6\,ae^2+2\,cd^2\right)}{e^3\left(m^3+6\,m^2+11\,m+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+a),x)
```

```
[Out] (e*x+d)^(1+m)*(c*e^2*m^2*x^2+3*c*e^2*m*x^2+a*e^2*m^2-2*c*d*e*m*x+2*c*e^2*x^
2+5*a*e^2*m-2*c*d*e*x+6*a*e^2+2*c*d^2)/e^3/(m^3+6*m^2+11*m+6)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.0741, size = 308, normalized size = 4.4

$$\frac{\left(ade^2m^2+5\,ade^2m+2\,cd^3+6\,ade^2+\left(ce^3m^2+3\,ce^3m+2\,ce^3\right)x^3+\left(cde^2m^2+cde^2m\right)x^2+\left(ae^3m^2+6\,ae^3-\left(2\,cd^2e-5\,a\right.\right.\right.}{e^3m^3+6\,e^3m^2+11\,e^3m+6\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+a),x, algorithm="fricas")
```

```
[Out] (a*d*e^2*m^2 + 5*a*d*e^2*m + 2*c*d^3 + 6*a*d*e^2 + (c*e^3*m^2 + 3*c*e^3*m +
 2*c*e^3)*x^3 + (c*d*e^2*m^2 + c*d*e^2*m)*x^2 + (a*e^3*m^2 + 6*a*e^3 - (2*c
*d^2*e - 5*a*e^3)*m)*x)*(e*x + d)^m/(e^3*m^3 + 6*e^3*m^2 + 11*e^3*m + 6*e^3
)
```

Sympy [A] time = 1.4262, size = 952, normalized size = 13.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+a),x)
```

```
[Out] Piecewise((d**m*(a*x + c*x**3/3), Eq(e, 0)), (-a*e**2/(2*d**2*e**3 + 4*d*e*
*4*x + 2*e**5*x**2) + 2*c*d**2*log(d/e + x)/(2*d**2*e**3 + 4*d*e**4*x + 2*e
**5*x**2) + 3*c*d**2/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2) + 4*c*d*e*x*l
og(d/e + x)/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2) + 4*c*d*e*x/(2*d**2*e*
*3 + 4*d*e**4*x + 2*e**5*x**2) + 2*c*e**2*x**2*log(d/e + x)/(2*d**2*e**3 +
4*d*e**4*x + 2*e**5*x**2), Eq(m, -3)), (-a*e**2/(d*e**3 + e**4*x) - 2*c*d**
2*log(d/e + x)/(d*e**3 + e**4*x) - 2*c*d**2/(d*e**3 + e**4*x) - 2*c*d*e*x*l
```

```
og(d/e + x)/(d*e**3 + e**4*x) + c*e**2*x**2/(d*e**3 + e**4*x), Eq(m, -2)),
(a*log(d/e + x)/e + c*d**2*log(d/e + x)/e**3 - c*d*x/e**2 + c*x**2/(2*e), E
q(m, -1)), (a*d*e**2*m**2*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m
 + 6*e**3) + 5*a*d*e**2*m*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m
 + 6*e**3) + 6*a*d*e**2*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m +
 6*e**3) + a*e**3*m**2*x*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m
+ 6*e**3) + 5*a*e**3*m*x*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m
+ 6*e**3) + 6*a*e**3*x*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m +
6*e**3) + 2*c*d**3*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e*
*3) - 2*c*d**2*e*m*x*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*
e**3) + c*d*e**2*m**2*x**2*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*
m + 6*e**3) + c*d*e**2*m*x**2*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e*
*3*m + 6*e**3) + c*e**3*m**2*x**3*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 1
1*e**3*m + 6*e**3) + 3*c*e**3*m*x**3*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2
+ 11*e**3*m + 6*e**3) + 2*c*e**3*x**3*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2
 + 11*e**3*m + 6*e**3), True))
```

<u>Giac</u> [B] time = 1.12598, size = 319, normalized size = 4.56

$$\frac{(xe+d)^m cm^2x^3e^3 + (xe+d)^m cdm^2x^2e^2 + 3\,(xe+d)^m cmx^3e^3 + (xe+d)^m cdmx^2e^2 - 2\,(xe+d)^m cd^2mxe + (xe+d)^m am}{m^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+a),x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*c*m^2*x^3*e^3 + (x*e + d)^m*c*d*m^2*x^2*e^2 + 3*(x*e + d)^m*c*
m*x^3*e^3 + (x*e + d)^m*c*d*m*x^2*e^2 - 2*(x*e + d)^m*c*d^2*m*x*e + (x*e +
d)^m*a*m^2*x*e^3 + 2*(x*e + d)^m*c*x^3*e^3 + (x*e + d)^m*a*d*m^2*e^2 + 2*(x
*e + d)^m*c*d^3 + 5*(x*e + d)^m*a*m*x*e^3 + 5*(x*e + d)^m*a*d*m*e^2 + 6*(x*
e + d)^m*a*x*e^3 + 6*(x*e + d)^m*a*d*e^2)/(m^3*e^3 + 6*m^2*e^3 + 11*m*e^3 +
 6*e^3)
```

3.724 $\int \frac{(d+ex)^m}{a+cx^2} dx$

Optimal. Leaf size=167

$$\frac{(d+ex)^{m+1} {}_2F_1\left(1,m+1;m+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{-a}(m+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}-\frac{(d+ex)^{m+1} {}_2F_1\left(1,m+1;m+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{-a}(m+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}$$

```
[Out] ((d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (Sqrt[c]*(d + e*x))/(
Sqrt[c]*d - Sqrt[-a]*e)])/(2*Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + m)) - (
(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (Sqrt[c]*(d + e*x))/(S
qrt[c]*d + Sqrt[-a]*e)])/(2*Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + m))
```

Rubi [A] time = 0.208284, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {712, 68}

$$\frac{(d+ex)^{m+1} {}_2F_1\left(1,m+1;m+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{-a}(m+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}-\frac{(d+ex)^{m+1} {}_2F_1\left(1,m+1;m+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{-a}(m+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(a + c*x^2),x]
```


```
[Out] ((d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (Sqrt[c]*(d + e*x))/(
Sqrt[c]*d - Sqrt[-a]*e)])/(2*Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + m)) - (
(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (Sqrt[c]*(d + e*x))/(S
qrt[c]*d + Sqrt[-a]*e)])/(2*Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + m))
```


Rule 712

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^m}{a+cx^2} dx &= \int \left(\frac{\sqrt{-a}(d+ex)^m}{2a\left(\sqrt{-a}-\sqrt{c}x\right)}+\frac{\sqrt{-a}(d+ex)^m}{2a\left(\sqrt{-a}+\sqrt{c}x\right)}\right) dx\\
&= -\frac{\int \frac{(d+ex)^m}{\sqrt{-a}-\sqrt{c}x} dx}{2\sqrt{-a}}-\frac{\int \frac{(d+ex)^m}{\sqrt{-a}+\sqrt{c}x} dx}{2\sqrt{-a}}\\
&= \frac{(d+ex)^{1+m} {}_2F_1\left(1,1+m;2+m;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(1+m)}-\frac{(d+ex)^{1+m} {}_2F_1\left(1,1+m;2+m;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(1+m)}
\end{aligned}$$

Mathematica [A] time = 0.157862, size = 145, normalized size = 0.87

$$\frac{(d+ex)^{m+1}\left(\frac{{}_2F_1\left(1,m+1;m+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\sqrt{c}d-\sqrt{-a}e}-\frac{{}_2F_1\left(1,m+1;m+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{\sqrt{-a}e+\sqrt{c}d}\right)}{2\sqrt{-a}(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(a + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + m)*(Hypergeometric2F1[1, 1 + m, 2 + m, (Sqrt[c]*(d + e*x))/
(Sqrt[c]*d - Sqrt[-a]*e)]/(Sqrt[c]*d - Sqrt[-a]*e) - Hypergeometric2F1[1, 1
 + m, 2 + m, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]/(Sqrt[c]*d + Sqr
t[-a]*e)))/(2*Sqrt[-a]*(1 + m))
```

Maple [F] time = 0.587, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(c*x^2+a),x)
```

```
[Out] int((e*x+d)^m/(c*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/(c*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^m}{cx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m/(c*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m}{a+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(c*x**2+a),x)
```

```
[Out] Integral((d + e*x)**m/(a + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(c*x^2 + a), x)
```

3.725 $\int \frac{(d+ex)^m}{(a+cx^2)^2} dx$

Optimal. Leaf size=304

$$-\frac{(d+ex)^{m+1}\left(\sqrt{-a}\sqrt{c}dem+ae^2(1-m)+cd^2\right){}_2F_1\left(1,m+1;m+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4(-a)^{3/2}(m+1)\left(\sqrt{c}d-\sqrt{-a}e\right)\left(ae^2+cd^2\right)}+\frac{(d+ex)^{m+1}\left(-\sqrt{-a}\sqrt{c}dem+ae^2(1-\right.}{4(-a)^{3/2}(m+1)\left(\sqrt{\right.}}$$

```
[Out] ((a*e + c*d*x)*(d + e*x)^(1 + m))/(2*a*(c*d^2 + a*e^2)*(a + c*x^2)) - ((c*d
^2 + a*e^2*(1 - m) + Sqrt[-a]*Sqrt[c]*d*e*m)*(d + e*x)^(1 + m)*Hypergeometr
ic2F1[1, 1 + m, 2 + m, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*(-
a)^(3/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + m)) + ((c*d^2 + a*e^
2*(1 - m) - Sqrt[-a]*Sqrt[c]*d*e*m)*(d + e*x)^(1 + m)*Hypergeometric2F1[1,
1 + m, 2 + m, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(4*(-a)^(3/2)*
(Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + m))
```

Rubi [A] time = 0.380677, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {741, 831, 68}

$$-\frac{(d+ex)^{m+1}\left(\sqrt{-a}\sqrt{c}dem+ae^2(1-m)+cd^2\right){}_2F_1\left(1,m+1;m+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4(-a)^{3/2}(m+1)\left(\sqrt{c}d-\sqrt{-a}e\right)\left(ae^2+cd^2\right)}+\frac{(d+ex)^{m+1}\left(-\sqrt{-a}\sqrt{c}dem+ae^2(1-\right.}{4(-a)^{3/2}(m+1)\left(\sqrt{\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(a + c*x^2)^2,x]
```

```
[Out] ((a*e + c*d*x)*(d + e*x)^(1 + m))/(2*a*(c*d^2 + a*e^2)*(a + c*x^2)) - ((c*d
^2 + a*e^2*(1 - m) + Sqrt[-a]*Sqrt[c]*d*e*m)*(d + e*x)^(1 + m)*Hypergeometr
ic2F1[1, 1 + m, 2 + m, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*(-
a)^(3/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + m)) + ((c*d^2 + a*e^
2*(1 - m) - Sqrt[-a]*Sqrt[c]*d*e*m)*(d + e*x)^(1 + m)*Hypergeometric2F1[1,
1 + m, 2 + m, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(4*(-a)^(3/2)*
(Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + m))
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
```

```
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(d+ex)^m}{\left(a+cx^2\right)^2}\,dx = \frac{(ae+cdx)(d+ex)^{1+m}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \frac{(d+ex)^m\left(-cd^2-ae^2(1-m)+cdemx\right)}{a+cx^2}\,dx}{2a\left(cd^2+ae^2\right)}$$

$$= \frac{(ae+cdx)(d+ex)^{1+m}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \left(\frac{\left(\sqrt{-a}\left(-cd^2-ae^2(1-m)\right)-a\sqrt{c}dem\right)(d+ex)^m}{2a\left(\sqrt{-a}-\sqrt{c}x\right)} + \frac{\left(\sqrt{-a}\left(-cd^2-ae^2(1-m)\right)+a\sqrt{c}dem\right)(d+ex)^m}{2a\left(\sqrt{-a}+\sqrt{c}x\right)}\right) dx}{2a\left(cd^2+ae^2\right)}$$

$$= \frac{(ae+cdx)(d+ex)^{1+m}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{\left(cd^2+ae^2(1-m)-\sqrt{-a}\sqrt{c}dem\right)\int \frac{(d+ex)^m}{\sqrt{-a}-\sqrt{c}x}\,dx}{4(-a)^{3/2}\left(cd^2+ae^2\right)} + \frac{\left(cd^2+ae^2(1-m)+\sqrt{-a}\right.}{4(-a)^{3/2}\left(c\right.}$$

$$= \frac{(ae+cdx)(d+ex)^{1+m}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\left(cd^2+ae^2(1-m)+\sqrt{-a}\sqrt{c}dem\right)(d+ex)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}}\right.}{4(-a)^{3/2}\left(\sqrt{c}d-\sqrt{-a}e\right)\left(cd^2+ae^2\right)(1+m)}$$

Mathematica [A] time = 0.361402, size = 253, normalized size = 0.83

$$\frac{(d+ex)^{m+1}\left(\frac{\left(\sqrt{-a}\sqrt{c}dem-ae^2(m-1)+cd^2\right){}_2F_1\left(1,m+1;m+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\sqrt{-a}(m+1)\left(\sqrt{c}d-\sqrt{-a}e\right)} + \frac{\left(\sqrt{-a}\sqrt{c}dem+ae^2(m-1)-cd^2\right){}_2F_1\left(1,m+1;m+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{\sqrt{-a}(m+1)\left(\sqrt{-a}e+\sqrt{c}d\right)} + \frac{2(ae+cdx)}{a+cx^2}\right)}{4a\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(a + c*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + m)*((2*(a*e + c*d*x))/(a + c*x^2) + ((c*d^2 - a*e^2*(-1 + m
) + Sqrt[-a]*Sqrt[c]*d*e*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (Sqrt[c]*(d
+ e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(1 +
m)) + ((-(c*d^2) + a*e^2*(-1 + m) + Sqrt[-a]*Sqrt[c]*d*e*m)*Hypergeometric2
F1[1, 1 + m, 2 + m, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[-a
]*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + m))))/(4*a*(c*d^2 + a*e^2))
```

Maple [F] time = 0.545, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(cx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(c*x^2+a)^2,x)
```

```
[Out] int((e*x+d)^m/(c*x^2+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(cx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/(c*x^2 + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^m}{c^2x^4+2\,acx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(cx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(c*x^2 + a)^2, x)
```

3.726 $\int \frac{(d+ex)^m}{\left(a+cx^2\right)^3} dx$

Optimal. Leaf size=472

$$\frac{(d+ex)^{m+1}\left(a\sqrt{c}dem\left(ae^2(5-2m)+3cd^2\right)-\sqrt{-a}\left(a^2e^4\left(m^2-4m+3\right)+acd^2e^2\left(-m^2-2m+6\right)+3c^2d^4\right)\right){}_2F_1\left(1,m+1\right.}{16a^3(m+1)\left(\sqrt{c}d-\sqrt{-a}e\right)\left(ae^2+cd^2\right)^2}$$

```
[Out] ((a*e + c*d*x)*(d + e*x)^(1 + m))/(4*a*(c*d^2 + a*e^2)*(a + c*x^2)^2) + ((d
 + e*x)^(1 + m)*(a*e*(a*e^2*(3 - m) + c*d^2*(1 + m)) + c*d*(3*c*d^2 + a*e^2
*(5 - 2*m))*x))/(8*a^2*(c*d^2 + a*e^2)^2*(a + c*x^2)) + ((a*Sqrt[c]*d*e*(3*
c*d^2 + a*e^2*(5 - 2*m))*m - Sqrt[-a]*(3*c^2*d^4 + a*c*d^2*e^2*(6 - 2*m - m
^2) + a^2*e^4*(3 - 4*m + m^2)))*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 +
m, 2 + m, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(16*a^3*(Sqrt[c]*d
 - Sqrt[-a]*e)*(c*d^2 + a*e^2)^2*(1 + m)) + ((a*Sqrt[c]*d*e*(3*c*d^2 + a*e^
2*(5 - 2*m))*m + Sqrt[-a]*(3*c^2*d^4 + a*c*d^2*e^2*(6 - 2*m - m^2) + a^2*e^
4*(3 - 4*m + m^2)))*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (S
qrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(16*a^3*(Sqrt[c]*d + Sqrt[-a]*
e)*(c*d^2 + a*e^2)^2*(1 + m))
```

Rubi [A] time = 0.81205, antiderivative size = 472, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {741, 823, 831, 68}

$$\frac{(d+ex)^{m+1}\left(a\sqrt{c}dem\left(ae^2(5-2m)+3cd^2\right)-\sqrt{-a}\left(a^2e^4\left(m^2-4m+3\right)+acd^2e^2\left(-m^2-2m+6\right)+3c^2d^4\right)\right){}_2F_1\left(1,m+1\right.}{16a^3(m+1)\left(\sqrt{c}d-\sqrt{-a}e\right)\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(a + c*x^2)^3,x]
```


```
[Out] ((a*e + c*d*x)*(d + e*x)^(1 + m))/(4*a*(c*d^2 + a*e^2)*(a + c*x^2)^2) + ((d
 + e*x)^(1 + m)*(a*e*(a*e^2*(3 - m) + c*d^2*(1 + m)) + c*d*(3*c*d^2 + a*e^2
*(5 - 2*m))*x))/(8*a^2*(c*d^2 + a*e^2)^2*(a + c*x^2)) + ((a*Sqrt[c]*d*e*(3*
c*d^2 + a*e^2*(5 - 2*m))*m - Sqrt[-a]*(3*c^2*d^4 + a*c*d^2*e^2*(6 - 2*m - m
^2) + a^2*e^4*(3 - 4*m + m^2)))*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 +
m, 2 + m, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(16*a^3*(Sqrt[c]*d
 - Sqrt[-a]*e)*(c*d^2 + a*e^2)^2*(1 + m)) + ((a*Sqrt[c]*d*e*(3*c*d^2 + a*e^
2*(5 - 2*m))*m + Sqrt[-a]*(3*c^2*d^4 + a*c*d^2*e^2*(6 - 2*m - m^2) + a^2*e^
4*(3 - 4*m + m^2)))*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (S
qrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(16*a^3*(Sqrt[c]*d + Sqrt[-a]*
e)*(c*d^2 + a*e^2)^2*(1 + m))
```


Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(d+ex)^m}{\left(a+cx^2\right)^3}\,dx = \frac{(ae+cdx)(d+ex)^{1+m}}{4a\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} - \frac{\int \frac{(d+ex)^m\left(-3cd^2-ae^2(3-m)-cde(2-m)x\right)}{\left(a+cx^2\right)^2}\,dx}{4a\left(cd^2+ae^2\right)}$$

$$= \frac{(ae+cdx)(d+ex)^{1+m}}{4a\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{(d+ex)^{1+m}\left(ae\left(ae^2(3-m)+cd^2(1+m)\right)+cd\left(3cd^2+ae^2(5-2m)\right)x\right.}{8a^2\left(cd^2+ae^2\right)^2\left(a+cx^2\right)}$$

$$= \frac{(ae+cdx)(d+ex)^{1+m}}{4a\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{(d+ex)^{1+m}\left(ae\left(ae^2(3-m)+cd^2(1+m)\right)+cd\left(3cd^2+ae^2(5-2m)\right)x\right.}{8a^2\left(cd^2+ae^2\right)^2\left(a+cx^2\right)}$$

$$= \frac{(ae+cdx)(d+ex)^{1+m}}{4a\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{(d+ex)^{1+m}\left(ae\left(ae^2(3-m)+cd^2(1+m)\right)+cd\left(3cd^2+ae^2(5-2m)\right)x\right.}{8a^2\left(cd^2+ae^2\right)^2\left(a+cx^2\right)}$$

$$= \frac{(ae+cdx)(d+ex)^{1+m}}{4a\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{(d+ex)^{1+m}\left(ae\left(ae^2(3-m)+cd^2(1+m)\right)+cd\left(3cd^2+ae^2(5-2m)\right)x\right.}{8a^2\left(cd^2+ae^2\right)^2\left(a+cx^2\right)}$$

Mathematica [A] time = 0.923583, size = 396, normalized size = 0.84

$$\frac{(d+ex)^{m+1}\left(\frac{\frac{\left(\sqrt{-a}\left(-a^2e^4\left(m^2-4m+3\right)+acd^2e^2\left(m^2+2m-6\right)-3c^2d^4\right)+a\sqrt{c}dem\left(ae^2(5-2m)+3cd^2\right)\right){}_2F_1\left(1,m+1;m+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\sqrt{c}d-\sqrt{-a}e}+\frac{\left(\sqrt{-a}\left(a^2e^4\left(m^2-4m+3\right)-acd^2e^2\left(m^2+2m-6\right)+3c^2\right.\right.}{}}{a(m+1)}\right.}{16a^2\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(a + c*x^2)^3,x]
```

```
[Out] ((d + e*x)^(1 + m)*((4*a*(c*d^2 + a*e^2)*(a*e + c*d*x))/(a + c*x^2)^2 + (2*
(-(a^2*e^3*(-3 + m)) + 3*c^2*d^3*x + a*c*d*e*(d*(1 + m) + e*(5 - 2*m)*x)))/
(a + c*x^2) + (((a*Sqrt[c]*d*e*(3*c*d^2 + a*e^2*(5 - 2*m))*m + Sqrt[-a]*(-3
*c^2*d^4 - a^2*e^4*(3 - 4*m + m^2) + a*c*d^2*e^2*(-6 + 2*m + m^2)))*Hyperge
ometric2F1[1, 1 + m, 2 + m, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/
(Sqrt[c]*d - Sqrt[-a]*e) + ((a*Sqrt[c]*d*e*(3*c*d^2 + a*e^2*(5 - 2*m))*m +
Sqrt[-a]*(3*c^2*d^4 + a^2*e^4*(3 - 4*m + m^2) - a*c*d^2*e^2*(-6 + 2*m + m^2
)))*Hypergeometric2F1[1, 1 + m, 2 + m, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqr
t[-a]*e)])/(Sqrt[c]*d + Sqrt[-a]*e))/(a*(1 + m))))/(16*a^2*(c*d^2 + a*e^2)^
2)
```

Maple [F] time = 0.604, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{(cx^2+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(c*x^2+a)^3,x)
```

```
[Out] int((e*x+d)^m/(c*x^2+a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{(cx^2+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/(c*x^2 + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(ex+d)^m}{c^3x^6+3\,ac^2x^4+3\,a^2cx^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m/(c^3*x^6 + 3*a*c^2*x^4 + 3*a^2*c*x^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(c*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(cx^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(c*x^2 + a)^3, x)
```

3.727 $\int (d+ex)^m \left(a+cx^2\right)^{3/2} dx$

Optimal. Leaf size=154

$$\frac{\left(a+cx^2\right)^{3/2} (d+ex)^{m+1} F_1\left(m+1;-\frac{3}{2},-\frac{3}{2};m+2;\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e(m+1)\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{3/2}\left(1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}\right)^{3/2}}$$

```
[Out] ((d + e*x)^(1 + m)*(a + c*x^2)^(3/2)*AppellF1[1 + m, -3/2, -3/2, 2 + m, (d
+ e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]), (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])])/(
e*(1 + m)*(1 - (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]))^(3/2)*(1 - (d + e*x)/(
d + (Sqrt[-a]*e)/Sqrt[c]))^(3/2))
```

Rubi [A] time = 0.135235, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {760, 133}

$$\frac{\left(a+cx^2\right)^{3/2} (d+ex)^{m+1} F_1\left(m+1;-\frac{3}{2},-\frac{3}{2};m+2;\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e(m+1)\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{3/2}\left(1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a + c*x^2)^(3/2),x]
```

```
[Out] ((d + e*x)^(1 + m)*(a + c*x^2)^(3/2)*AppellF1[1 + m, -3/2, -3/2, 2 + m, (d
+ e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]), (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])])/(
e*(1 + m)*(1 - (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]))^(3/2)*(1 - (d + e*x)/(
d + (Sqrt[-a]*e)/Sqrt[c]))^(3/2))
```

Rule 760

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + (e*q)/c))^p*
(1 - (d + e*x)/(d - (e*q)/c))^p), Subst[Int[x^m*Simp[1 - x/(d + (e*q)/c), x
]^p*Simp[1 - x/(d - (e*q)/c), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (d+ex)^m \left(a+cx^2\right)^{3/2} dx = \frac{\left(a+cx^2\right)^{3/2} \text{Subst}\left(\int x^m \left(1-\frac{x}{d-\frac{\sqrt{-ae}}{\sqrt{c}}}\right)^{3/2} \left(1-\frac{x}{d+\frac{\sqrt{-ae}}{\sqrt{c}}}\right)^{3/2} dx, x, d+ex\right)}{e\left(1-\frac{d+ex}{d-\frac{\sqrt{-ae}}{\sqrt{c}}}\right)^{3/2} \left(1-\frac{d+ex}{d+\frac{\sqrt{-ae}}{\sqrt{c}}}\right)^{3/2}}$$

$$= \frac{(d+ex)^{1+m} \left(a+cx^2\right)^{3/2} F_1\left(1+m;-\frac{3}{2},-\frac{3}{2};2+m;\frac{d+ex}{d-\frac{\sqrt{-ae}}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-ae}}{\sqrt{c}}}\right)}{e(1+m)\left(1-\frac{d+ex}{d-\frac{\sqrt{-ae}}{\sqrt{c}}}\right)^{3/2} \left(1-\frac{d+ex}{d+\frac{\sqrt{-ae}}{\sqrt{c}}}\right)^{3/2}}$$

Mathematica [F] time = 0.0809725, size = 0, normalized size = 0.

$$\int (d+ex)^m \left(a+cx^2\right)^{3/2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m*(a + c*x^2)^(3/2),x]
```

```
[Out] Integrate[(d + e*x)^m*(a + c*x^2)^(3/2), x]
```

Maple [F] time = 0.558, size = 0, normalized size = 0.

$$\int (ex+d)^m \left(cx^2+a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+a)^(3/2),x)
```

```
[Out] int((e*x+d)^m*(c*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+a\right)^{\frac{3}{2}} (ex+d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^(3/2)*(e*x + d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2+a\right)^{\frac{3}{2}} (ex+d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + a)^(3/2)*(e*x + d)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + cx^2\right)^{\frac{3}{2}} (d + ex)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+a)**(3/2),x)
```

```
[Out] Integral((a + c*x**2)**(3/2)*(d + e*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + a\right)^{\frac{3}{2}} (ex + d)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^(3/2)*(e*x + d)^m, x)
```

3.728 $\int (d+ex)^m\sqrt{a+cx^2}\,dx$

Optimal. Leaf size=154

$$\frac{\sqrt{a+cx^2}(d+ex)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e(m+1)\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}}}$$

```
[Out] ((d + e*x)^(1 + m)*Sqrt[a + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m, (d +
e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]), (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])])/(e*
(1 + m)*Sqrt[1 - (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c])]*Sqrt[1 - (d + e*x)/(
d + (Sqrt[-a]*e)/Sqrt[c])])
```

Rubi [A] time = 0.0659291, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {760, 133}

$$\frac{\sqrt{a+cx^2}(d+ex)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e(m+1)\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*Sqrt[a + c*x^2],x]
```

```
[Out] ((d + e*x)^(1 + m)*Sqrt[a + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m, (d +
e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]), (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])])/(e*
(1 + m)*Sqrt[1 - (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c])]*Sqrt[1 - (d + e*x)/(
d + (Sqrt[-a]*e)/Sqrt[c])])
```

Rule 760

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + (e*q)/c))^p*
(1 - (d + e*x)/(d - (e*q)/c))^p), Subst[Int[x^m*Simp[1 - x/(d + (e*q)/c), x
]^p*Simp[1 - x/(d - (e*q)/c), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (d+ex)^m\sqrt{a+cx^2}\,dx = \frac{\sqrt{a+cx^2}\,\mathrm{Subst}\left(\int x^m\sqrt{1-\frac{x}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{x}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}}\,dx,x,d+ex\right)}{e\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}}}$$

$$= \frac{(d+ex)^{1+m}\sqrt{a+cx^2}F_1\left(1+m;-\frac{1}{2},-\frac{1}{2};2+m;\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e(1+m)\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}}}$$

Mathematica [A] time = 0.109778, size = 159, normalized size = 1.03

$$\frac{\sqrt{a+cx^2}(d+ex)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{d+ex}{d-\sqrt{-\frac{a}{c}}e},\frac{d+ex}{d+\sqrt{-\frac{a}{c}}e}\right)}{e(m+1)\sqrt{\frac{e\left(\sqrt{-\frac{a}{c}}-x\right)}{e\sqrt{-\frac{a}{c}}+d}}\sqrt{\frac{e\left(\sqrt{-\frac{a}{c}}+x\right)}{e\sqrt{-\frac{a}{c}}-d}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^m*Sqrt[a + c*x^2],x]
```

```
[Out] ((d + e*x)^(1 + m)*Sqrt[a + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m, (d +
e*x)/(d - Sqrt[-(a/c)]*e), (d + e*x)/(d + Sqrt[-(a/c)]*e)])/(e*(1 + m)*Sqrt
[(e*(Sqrt[-(a/c)] - x))/(d + Sqrt[-(a/c)]*e)]*Sqrt[(e*(Sqrt[-(a/c)] + x))/(
-d + Sqrt[-(a/c)]*e)])
```

Maple [F] time = 0.555, size = 0, normalized size = 0.

$$\int (ex+d)^m\sqrt{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+a)^(1/2),x)
```

```
[Out] int((e*x+d)^m*(c*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2+a}(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^2+a}(ex+d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*(e*x + d)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+cx^2}\,(d+ex)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2+a}(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)^m, x)
```

3.729 $\int \frac{(d+ex)^m}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=154

$$\frac{(d+ex)^{m+1}\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}}F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e(m+1)\sqrt{a+cx^2}}$$

```
[Out] ((d + e*x)^(1 + m)*Sqrt[1 - (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c])]*Sqrt[1 -
(d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])]*AppellF1[1 + m, 1/2, 1/2, 2 + m, (d +
 e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]), (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])])/(e
*(1 + m)*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.0615328, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {760, 133}

$$\frac{(d+ex)^{m+1}\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}}F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e(m+1)\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/Sqrt[a + c*x^2],x]
```

```
[Out] ((d + e*x)^(1 + m)*Sqrt[1 - (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c])]*Sqrt[1 -
(d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])]*AppellF1[1 + m, 1/2, 1/2, 2 + m, (d +
 e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]), (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])])/(e
*(1 + m)*Sqrt[a + c*x^2])
```

Rule 760

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + (e*q)/c))^p*
(1 - (d + e*x)/(d - (e*q)/c))^p), Subst[Int[x^m*Simp[1 - x/(d + (e*q)/c), x
]^p*Simp[1 - x/(d - (e*q)/c), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{(d+ex)^m}{\sqrt{a+cx^2}}\,dx = \frac{\left(\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}}\right)\text{Subst}\left(\int \frac{x^m}{\sqrt{1-\frac{x}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{x}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}}}\,dx, x, d+ex\right)}{e\sqrt{a+cx^2}}$$

$$= \frac{(d+ex)^{1+m}\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}}\,F_1\left(1+m;\frac{1}{2},\frac{1}{2};2+m;\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e(1+m)\sqrt{a+cx^2}}$$

Mathematica [A] time = 0.112678, size = 159, normalized size = 1.03

$$\frac{(d+ex)^{m+1}\sqrt{\frac{e\left(\sqrt{-\frac{a}{c}}-x\right)}{e\sqrt{-\frac{a}{c}}+d}}\sqrt{\frac{e\left(\sqrt{-\frac{a}{c}}+x\right)}{e\sqrt{-\frac{a}{c}}-d}}\,F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{d+ex}{d-\sqrt{-\frac{a}{c}}e},\frac{d+ex}{d+\sqrt{-\frac{a}{c}}e}\right)}{e(m+1)\sqrt{a+cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^m/Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[(e*(Sqrt[-(a/c)] - x))/(d + Sqrt[-(a/c)]*e)]*Sqrt[(e*(Sqrt[-(a/c)] +
x))/(-d + Sqrt[-(a/c)]*e)]*(d + e*x)^(1 + m)*AppellF1[1 + m, 1/2, 1/2, 2 +
m, (d + e*x)/(d - Sqrt[-(a/c)]*e), (d + e*x)/(d + Sqrt[-(a/c)]*e)])/(e*(1 +
 m)*Sqrt[a + c*x^2])
```

Maple [F] time = 180., size = 0, normalized size = 0.

hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(c*x^2+a)^(1/2),x)
```

```
[Out] int((e*x+d)^m/(c*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\sqrt{cx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/sqrt(c*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^m}{\sqrt{cx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m/sqrt(c*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m}{\sqrt{a+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**m/sqrt(a + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\sqrt{cx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/sqrt(c*x^2 + a), x)
```

3.730 $\int \frac{(d+ex)^m}{\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=154

$$\frac{(d+ex)^{m+1}\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{3/2}\left(1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}\right)^{3/2}F_1\left(m+1;\frac{3}{2},\frac{3}{2};m+2;\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e(m+1)\left(a+cx^2\right)^{3/2}}$$

```
[Out] ((d + e*x)^(1 + m)*(1 - (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]))^(3/2)*(1 - (d
 + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c]))^(3/2)*AppellF1[1 + m, 3/2, 3/2, 2 + m,
(d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]), (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])]
)/(e*(1 + m)*(a + c*x^2)^(3/2))
```

Rubi [A] time = 0.0625726, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {760, 133}

$$\frac{(d+ex)^{m+1}\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{3/2}\left(1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}\right)^{3/2}F_1\left(m+1;\frac{3}{2},\frac{3}{2};m+2;\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e(m+1)\left(a+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(a + c*x^2)^(3/2),x]
```


```
[Out] ((d + e*x)^(1 + m)*(1 - (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]))^(3/2)*(1 - (d
 + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c]))^(3/2)*AppellF1[1 + m, 3/2, 3/2, 2 + m,
(d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]), (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])]
)/(e*(1 + m)*(a + c*x^2)^(3/2))
```


Rule 760

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + (e*q)/c))^p*
(1 - (d + e*x)/(d - (e*q)/c))^p), Subst[Int[x^m*Simp[1 - x/(d + (e*q)/c), x
]^p*Simp[1 - x/(d - (e*q)/c), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{(d+ex)^m}{\left(a+cx^2\right)^{3/2}}\,dx = \frac{\left(\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{3/2}\left(1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{3/2}\right)\text{Subst}\left(\int \frac{x^m}{\left(1-\frac{x}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{3/2}\left(1-\frac{x}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{3/2}}dx, x, d+ex\right)}{e\left(a+cx^2\right)^{3/2}}$$

$$= \frac{(d+ex)^{1+m}\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{3/2}\left(1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{3/2} F_1\left(1+m;\frac{3}{2},\frac{3}{2};2+m;\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e(1+m)\left(a+cx^2\right)^{3/2}}$$

Mathematica [A] time = 0.194306, size = 159, normalized size = 1.03

$$\frac{(d+ex)^{m+1}\left(\frac{e\left(\sqrt{-\frac{a}{c}}-x\right)}{e\sqrt{-\frac{a}{c}}+d}\right)^{3/2}\left(\frac{e\left(\sqrt{-\frac{a}{c}}+x\right)}{e\sqrt{-\frac{a}{c}}-d}\right)^{3/2} F_1\left(m+1;\frac{3}{2},\frac{3}{2};m+2;\frac{d+ex}{d-\sqrt{-\frac{a}{c}}e},\frac{d+ex}{d+\sqrt{-\frac{a}{c}}e}\right)}{e(m+1)\left(a+cx^2\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^m/(a + c*x^2)^(3/2),x]
```

```
[Out] (((e*(Sqrt[-(a/c)] - x))/(d + Sqrt[-(a/c)]*e))^(3/2)*((e*(Sqrt[-(a/c)] + x)
)/(-d + Sqrt[-(a/c)]*e))^(3/2)*(d + e*x)^(1 + m)*AppellF1[1 + m, 3/2, 3/2,
2 + m, (d + e*x)/(d - Sqrt[-(a/c)]*e), (d + e*x)/(d + Sqrt[-(a/c)]*e)])/(e*
(1 + m)*(a + c*x^2)^(3/2))
```

Maple [F] time = 0.531, size = 0, normalized size = 0.

$$\int (ex+d)^m \left(cx^2+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(c*x^2+a)^(3/2),x)
```

```
[Out] int((e*x+d)^m/(c*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(cx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/(c*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+a}(ex+d)^m}{c^2x^4+2\,acx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*(e*x + d)^m/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m}{\left(a+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral((d + e*x)**m/(a + c*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(cx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(c*x^2 + a)^(3/2), x)
```

3.731 $\int (d+ex)^m \left(a+cx^2\right)^p \, dx$

Optimal. Leaf size=152

$$\frac{\left(a+cx^2\right)^p (d+ex)^{m+1} \left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p} \left(1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}\right)^{-p} F_1\left(m+1;-p,-p;m+2;\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e(m+1)}$$

```
[Out] ((d + e*x)^(1 + m)*(a + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (d + e*x)/(
d - (Sqrt[-a]*e)/Sqrt[c]), (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])])/(e*(1 + m
)*(1 - (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]))^p*(1 - (d + e*x)/(d + (Sqrt[-a
]*e)/Sqrt[c]))^p)
```

Rubi [A] time = 0.064112, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {760, 133}

$$\frac{\left(a+cx^2\right)^p (d+ex)^{m+1} \left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p} \left(1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}\right)^{-p} F_1\left(m+1;-p,-p;m+2;\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a + c*x^2)^p,x]
```

```
[Out] ((d + e*x)^(1 + m)*(a + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (d + e*x)/(
d - (Sqrt[-a]*e)/Sqrt[c]), (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])])/(e*(1 + m
)*(1 - (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]))^p*(1 - (d + e*x)/(d + (Sqrt[-a
]*e)/Sqrt[c]))^p)
```

Rule 760

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + (e*q)/c))^p*
(1 - (d + e*x)/(d - (e*q)/c))^p), Subst[Int[x^m*Simp[1 - x/(d + (e*q)/c), x
]^p*Simp[1 - x/(d - (e*q)/c), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (d+ex)^m \left(a+cx^2\right)^p \, dx = \frac{\left(\left(a+cx^2\right)^p \left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p} \left(1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\right) \text{Subst}\left(\int x^m \left(1-\frac{x}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^p \left(1-\frac{x}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^p dx, x \right.}{e}$$

$$= \frac{(d+ex)^{1+m} \left(a+cx^2\right)^p \left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p} \left(1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p} F_1\left(1+m;-p,-p;2+m;\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}, \frac{d+}{d+} \right.}{e(1+m)}$$

Mathematica [A] time = 0.136545, size = 157, normalized size = 1.03

$$\frac{\left(a+cx^2\right)^p (d+ex)^{m+1} \left(\frac{e\left(\sqrt{-\frac{a}{c}}-x\right)}{e\sqrt{-\frac{a}{c}}+d}\right)^{-p} \left(\frac{e\left(\sqrt{-\frac{a}{c}}+x\right)}{e\sqrt{-\frac{a}{c}}-d}\right)^{-p} F_1\left(m+1;-p,-p;m+2;\frac{d+ex}{d-\sqrt{-\frac{a}{c}}e},\frac{d+ex}{d+\sqrt{-\frac{a}{c}}e}\right)}{e(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^m*(a + c*x^2)^p,x]
```

```
[Out] ((d + e*x)^(1 + m)*(a + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (d + e*x)/(
d - Sqrt[-(a/c)]*e), (d + e*x)/(d + Sqrt[-(a/c)]*e)])/(e*(1 + m)*((e*(Sqrt[
-(a/c)] - x))/(d + Sqrt[-(a/c)]*e))^p*((e*(Sqrt[-(a/c)] + x))/(-d + Sqrt[-(
a/c)]*e))^p)
```

Maple [F] time = 0.599, size = 0, normalized size = 0.

$$\int (ex+d)^m \left(cx^2+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+a)^p,x)
```

```
[Out] int((e*x+d)^m*(c*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+a\right)^p (ex+d)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^p*(e*x + d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2+a\right)^p (ex+d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + a)^p*(e*x + d)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + a\right)^p (ex + d)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^p*(e*x + d)^m, x)
```

3.732 $\int (d+ex)^3 \left(a+cx^2\right)^p \, dx$

Optimal. Leaf size=178

$$-\frac{e\left(a+cx^2\right)^{p+1}\left((2p+3)\left(ae^2-cd^2(2p+5)\right)-2cde(p+1)(p+3)x\right)}{2c^2(p+2)\left(2p^2+5p+3\right)}-\frac{dx\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(3ae^2-cd^2(2p+3)\right)}{c(2p+3)}$$

```
[Out] (e*(d + e*x)^2*(a + c*x^2)^(1 + p))/(2*c*(2 + p)) - (e*((3 + 2*p)*(a*e^2 -
c*d^2*(5 + 2*p)) - 2*c*d*e*(1 + p)*(3 + p)*x)*(a + c*x^2)^(1 + p))/(2*c^2*(
2 + p)*(3 + 5*p + 2*p^2)) - (d*(3*a*e^2 - c*d^2*(3 + 2*p))*x*(a + c*x^2)^p*
Hypergeometric2F1[1/2, -p, 3/2, -((c*x^2)/a)])/(c*(3 + 2*p)*(1 + (c*x^2)/a)
^p)
```

Rubi [A] time = 0.151959, antiderivative size = 169, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {743, 780, 246, 245}

$$-\frac{e\left(a+cx^2\right)^{p+1}\left((2p+3)\left(ae^2-cd^2(2p+5)\right)-2cde(p+1)(p+3)x\right)}{2c^2(p+2)\left(2p^2+5p+3\right)}+dx\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d^2-\frac{3ae^2}{2cp+3c}\right){}_2F_1$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + c*x^2)^p,x]
```

```
[Out] (e*(d + e*x)^2*(a + c*x^2)^(1 + p))/(2*c*(2 + p)) - (e*((3 + 2*p)*(a*e^2 -
c*d^2*(5 + 2*p)) - 2*c*d*e*(1 + p)*(3 + p)*x)*(a + c*x^2)^(1 + p))/(2*c^2*(
2 + p)*(3 + 5*p + 2*p^2)) + (d*(d^2 - (3*a*e^2)/(3*c + 2*c*p))*x*(a + c*x^2
)^p*Hypergeometric2F1[1/2, -p, 3/2, -((c*x^2)/a)])/(1 + (c*x^2)/a)^p
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^3\left(a+cx^2\right)^p\,dx &= \frac{e(d+ex)^2\left(a+cx^2\right)^{1+p}}{2c(2+p)}+\frac{\int (d+ex)\left(-2\left(ae^2-cd^2(2+p)\right)+2cde(3+p)x\right)\left(a+cx^2\right)^p\,dx}{2c(2+p)}\\ &= \frac{e(d+ex)^2\left(a+cx^2\right)^{1+p}}{2c(2+p)}-\frac{e\left((3+2p)\left(ae^2-cd^2(5+2p)\right)-2cde(1+p)(3+p)x\right)\left(a+cx^2\right)^{1+p}}{2c^2(2+p)\left(3+5p+2p^2\right)}\\ &= \frac{e(d+ex)^2\left(a+cx^2\right)^{1+p}}{2c(2+p)}-\frac{e\left((3+2p)\left(ae^2-cd^2(5+2p)\right)-2cde(1+p)(3+p)x\right)\left(a+cx^2\right)^{1+p}}{2c^2(2+p)\left(3+5p+2p^2\right)}\\ &= \frac{e(d+ex)^2\left(a+cx^2\right)^{1+p}}{2c(2+p)}-\frac{e\left((3+2p)\left(ae^2-cd^2(5+2p)\right)-2cde(1+p)(3+p)x\right)\left(a+cx^2\right)^{1+p}}{2c^2(2+p)\left(3+5p+2p^2\right)}\end{aligned}$$

Mathematica [A] time = 0.229372, size = 223, normalized size = 1.25

$$\frac{\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(e\left(-a^2e^2\left(\left(\frac{cx^2}{a}+1\right)^p-1\right)+c^2x^2\left(\frac{cx^2}{a}+1\right)^p\left(3d^2(p+2)+e^2(p+1)x^2\right)+2c^2de\left(p^2+3p+2\right)x^3\,{}_2F_1\right.\right.}{2c^2(p+1)(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + c*x^2)^p,x]
```

```
[Out] ((a + c*x^2)^p*(2*c^2*d^3*(2 + 3*p + p^2)*x*Hypergeometric2F1[1/2, -p, 3/2,
 -((c*x^2)/a)] + e*(c^2*x^2*(1 + (c*x^2)/a)^p*(3*d^2*(2 + p) + e^2*(1 + p)*
x^2) - a^2*e^2*(-1 + (1 + (c*x^2)/a)^p) + a*c*(e^2*p*x^2*(1 + (c*x^2)/a)^p
+ 3*d^2*(2 + p)*(-1 + (1 + (c*x^2)/a)^p)) + 2*c^2*d*e*(2 + 3*p + p^2)*x^3*H
ypergeometric2F1[3/2, -p, 5/2, -((c*x^2)/a)])))/(2*c^2*(1 + p)*(2 + p)*(1 +
 (c*x^2)/a)^p)
```

Maple [F] time = 0.452, size = 0, normalized size = 0.

$$\int (ex+d)^3\left(cx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+a)^p,x)
```

```
[Out] int((e*x+d)^3*(c*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3\left(cx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3*(c*x^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3\right)\left(cx^2 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(c*x^2 + a)^p, x)
```

Sympy [C] time = 17.2661, size = 468, normalized size = 2.63

$$a^p d^3 x {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{cx^2 e^{i\pi}}{a}\right) + a^p d e^2 x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{cx^2 e^{i\pi}}{a}\right) + 3d^2e\left(\begin{cases} \frac{a^p x^2}{2} & \text{for } c = 0 \\ \frac{\begin{cases} \frac{(a+cx^2)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log\left(a + cx^2\right) & \text{otherwise} \end{cases}}{2c} & \text{otherwise} \end{cases}\right) + e^3 \left(\ldots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a)**p,x)
```

```
[Out] a**p*d**3*x*hyper((1/2, -p), (3/2,), c*x**2*exp_polar(I*pi)/a) + a**p*d*e**
2*x**3*hyper((3/2, -p), (5/2,), c*x**2*exp_polar(I*pi)/a) + 3*d**2*e*Piecew
ise((a**p*x**2/2, Eq(c, 0)), (Piecewise(((a + c*x**2)**(p + 1)/(p + 1), Ne(
p, -1)), (log(a + c*x**2), True))/(2*c), True)) + e**3*Piecewise((a**p*x**4
/4, Eq(c, 0)), (a*log(-I*sqrt(a)*sqrt(1/c) + x)/(2*a*c**2 + 2*c**3*x**2) +
a*log(I*sqrt(a)*sqrt(1/c) + x)/(2*a*c**2 + 2*c**3*x**2) + a/(2*a*c**2 + 2*c
**3*x**2) + c*x**2*log(-I*sqrt(a)*sqrt(1/c) + x)/(2*a*c**2 + 2*c**3*x**2) +
 c*x**2*log(I*sqrt(a)*sqrt(1/c) + x)/(2*a*c**2 + 2*c**3*x**2), Eq(p, -2)),
(-a*log(-I*sqrt(a)*sqrt(1/c) + x)/(2*c**2) - a*log(I*sqrt(a)*sqrt(1/c) + x)
/(2*c**2) + x**2/(2*c), Eq(p, -1)), (-a**2*(a + c*x**2)**p/(2*c**2*p**2 + 6
*c**2*p + 4*c**2) + a*c*p*x**2*(a + c*x**2)**p/(2*c**2*p**2 + 6*c**2*p + 4*
c**2) + c**2*p*x**4*(a + c*x**2)**p/(2*c**2*p**2 + 6*c**2*p + 4*c**2) + c**
2*x**4*(a + c*x**2)**p/(2*c**2*p**2 + 6*c**2*p + 4*c**2), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^3 \left(cx^2 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(c*x^2 + a)^p, x)
```

3.733 $\int (d+ex)^2 \left(a+cx^2\right)^p \, dx$

Optimal. Leaf size=133

$$-\frac{x\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(ae^2-cd^2(2p+3)\right){}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{cx^2}{a}\right)}{c(2p+3)}+\frac{e(d+ex)\left(a+cx^2\right)^{p+1}}{c(2p+3)}+\frac{de(p+2)\left(a+cx^2\right)^{p+1}}{c(p+1)(2p+3)}$$

```
[Out] (d*e*(2 + p)*(a + c*x^2)^(1 + p))/(c*(1 + p)*(3 + 2*p)) + (e*(d + e*x)*(a +
 c*x^2)^(1 + p))/(c*(3 + 2*p)) - ((a*e^2 - c*d^2*(3 + 2*p))*x*(a + c*x^2)^p
*Hypergeometric2F1[1/2, -p, 3/2, -((c*x^2)/a)])/(c*(3 + 2*p)*(1 + (c*x^2)/a
)^p)
```

Rubi [A] time = 0.0730234, antiderivative size = 125, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {743, 641, 246, 245}

$$x\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d^2-\frac{ae^2}{2cp+3c}\right){}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{cx^2}{a}\right)+\frac{e(d+ex)\left(a+cx^2\right)^{p+1}}{c(2p+3)}+\frac{de(p+2)\left(a+cx^2\right)^{p+1}}{c(p+1)(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + c*x^2)^p,x]
```

```
[Out] (d*e*(2 + p)*(a + c*x^2)^(1 + p))/(c*(1 + p)*(3 + 2*p)) + (e*(d + e*x)*(a +
 c*x^2)^(1 + p))/(c*(3 + 2*p)) + ((d^2 - (a*e^2)/(3*c + 2*c*p))*x*(a + c*x^
2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((c*x^2)/a)])/(1 + (c*x^2)/a)^p
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int (d+ex)^2\left(a+cx^2\right)^p\,dx=\frac{e(d+ex)\left(a+cx^2\right)^{1+p}}{c(3+2p)}+\frac{\int\left(-ae^2+cd^2(3+2p)+2cde(2+p)x\right)\left(a+cx^2\right)^p\,dx}{c(3+2p)}$$
$$=\frac{de(2+p)\left(a+cx^2\right)^{1+p}}{c(1+p)(3+2p)}+\frac{e(d+ex)\left(a+cx^2\right)^{1+p}}{c(3+2p)}+\left(d^2-\frac{ae^2}{3c+2cp}\right)\int\left(a+cx^2\right)^p\,dx$$
$$=\frac{de(2+p)\left(a+cx^2\right)^{1+p}}{c(1+p)(3+2p)}+\frac{e(d+ex)\left(a+cx^2\right)^{1+p}}{c(3+2p)}+\left(\left(d^2-\frac{ae^2}{3c+2cp}\right)\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)$$
$$=\frac{de(2+p)\left(a+cx^2\right)^{1+p}}{c(1+p)(3+2p)}+\frac{e(d+ex)\left(a+cx^2\right)^{1+p}}{c(3+2p)}+\left(d^2-\frac{ae^2}{3c+2cp}\right)x\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}$$

Mathematica [A] time = 0.100026, size = 133, normalized size = 1.

$$\frac{\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(3cd^2(p+1)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{cx^2}{a}\right)+e\left(3d\left(cx^2\left(\frac{cx^2}{a}+1\right)^p+a\left(\left(\frac{cx^2}{a}+1\right)^p-1\right)\right)+ce(p+1)x^3\,{}_2F_1\left(\frac{3}{2}\right.\right.\right.}{3c(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + c*x^2)^p,x]
```

```
[Out] ((a + c*x^2)^p*(3*c*d^2*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((c*x^2)
/a)] + e*(3*d*(c*x^2*(1 + (c*x^2)/a)^p + a*(-1 + (1 + (c*x^2)/a)^p)) + c*e*
(1 + p)*x^3*Hypergeometric2F1[3/2, -p, 5/2, -((c*x^2)/a)])))/(3*c*(1 + p)*(
1 + (c*x^2)/a)^p)
```

Maple [F] time = 0.45, size = 0, normalized size = 0.

$$\int (ex+d)^2\left(cx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a)^p,x)
```

```
[Out] int((e*x+d)^2*(c*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^2\left(cx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2*(c*x^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^2 + 2\,dex + d^2\right)\left(cx^2 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(c*x^2 + a)^p, x)
```

Sympy [A] time = 12.7396, size = 97, normalized size = 0.73

$$a^p d^2 x {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{cx^2 e^{i\pi}}{a}\right) + \frac{a^p e^2 x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{cx^2 e^{i\pi}}{a}\right)}{3} + 2de\left(\begin{cases} \frac{a^p x^2}{2} & \text{for}\ c = 0 \\ \frac{\begin{cases} \frac{\left(a+cx^2\right)^{p+1}}{p+1} & \text{for}\ p \neq -1 \\ \log\left(a + cx^2\right) & \text{otherwise} \end{cases}}{2c} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+a)**p,x)
```

```
[Out] a**p*d**2*x*hyper((1/2, -p), (3/2,), c*x**2*exp_polar(I*pi)/a) + a**p*e**2*
x**3*hyper((3/2, -p), (5/2,), c*x**2*exp_polar(I*pi)/a)/3 + 2*d*e*Piecewise
((a**p*x**2/2, Eq(c, 0)), (Piecewise(((a + c*x**2)**(p + 1)/(p + 1), Ne(p,
-1)), (log(a + c*x**2), True))/(2*c), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^2\left(cx^2 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(c*x^2 + a)^p, x)
```

3.734 $\int (d+ex)\left(a+cx^2\right)^p\,dx$

Optimal. Leaf size=61

$$\frac{dx\left(a+cx^2\right)^{p+1}\,{}_2F_1\left(1,p+\frac{3}{2};\frac{3}{2};-\frac{cx^2}{a}\right)}{a}+\frac{e\left(a+cx^2\right)^{p+1}}{2c(p+1)}$$

```
[Out] (e*(a + c*x^2)^(1 + p))/(2*c*(1 + p)) + (d*x*(a + c*x^2)^(1 + p)*Hypergeome
tric2F1[1, 3/2 + p, 3/2, -((c*x^2)/a)])/a
```

Rubi [A] time = 0.0203105, antiderivative size = 70, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {641, 246, 245}

$$dx\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{cx^2}{a}\right)+\frac{e\left(a+cx^2\right)^{p+1}}{2c(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + c*x^2)^p,x]
```

```
[Out] (e*(a + c*x^2)^(1 + p))/(2*c*(1 + p)) + (d*x*(a + c*x^2)^p*Hypergeometric2F
1[1/2, -p, 3/2, -((c*x^2)/a)])/(1 + (c*x^2)/a)^p
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(a+cx^2\right)^p\,dx &= \frac{e\left(a+cx^2\right)^{1+p}}{2c(1+p)}+d\int \left(a+cx^2\right)^p\,dx\\ &= \frac{e\left(a+cx^2\right)^{1+p}}{2c(1+p)}+\left(d\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int \left(1+\frac{cx^2}{a}\right)^p\,dx\\ &= \frac{e\left(a+cx^2\right)^{1+p}}{2c(1+p)}+dx\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{cx^2}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.0502764, size = 98, normalized size = 1.61

$$\frac{\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(2cd(p+1)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{cx^2}{a}\right)+cex^2\left(\frac{cx^2}{a}+1\right)^p+ae\left(\left(\frac{cx^2}{a}+1\right)^p-1\right)\right)}{2c(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + c*x^2)^p,x]
```

```
[Out] ((a + c*x^2)^p*(c*e*x^2*(1 + (c*x^2)/a)^p + a*e*(-1 + (1 + (c*x^2)/a)^p) +
2*c*d*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((c*x^2)/a)]))/(2*c*(1 + p
)*(1 + (c*x^2)/a)^p)
```

Maple [F] time = 0.309, size = 0, normalized size = 0.

$$\int (ex+d)\left(cx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+a)^p,x)
```

```
[Out] int((e*x+d)*(c*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)\left(cx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(c*x^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex+d)\left(cx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)*(c*x^2 + a)^p, x)
```

Sympy [A] time = 6.73627, size = 61, normalized size = 1.

$$a^p dx\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{cx^2e^{i\pi}}{a}\right)+e\left(\begin{cases}\frac{a^px^2}{2} & \text{for } c=0\\ \frac{\begin{cases}\frac{\left(a+cx^2\right)^{p+1}}{p+1} & \text{for } p\neq-1\\ \log\left(a+cx^2\right) & \text{otherwise}\end{cases}}{2c} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+a)**p,x)
```

```
[Out] a**p*d*x*hyper((1/2, -p), (3/2,), c*x**2*exp_polar(I*pi)/a) + e*Piecewise((
a**p*x**2/2, Eq(c, 0)), (Piecewise(((a + c*x**2)**(p + 1)/(p + 1), Ne(p, -1
)), (log(a + c*x**2), True))/(2*c), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)\left(cx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(c*x^2 + a)^p, x)
```

3.735 $\int \left(a+cx^2\right)^p\,dx$

Optimal. Leaf size=35

$$\frac{x\left(a+cx^2\right)^{p+1}\,{}_2F_1\left(1,p+\frac{3}{2};\frac{3}{2};-\frac{cx^2}{a}\right)}{a}$$

```
[Out] (x*(a + c*x^2)^(1 + p)*Hypergeometric2F1[1, 3/2 + p, 3/2, -((c*x^2)/a)])/a
```

Rubi [A] time = 0.0091919, antiderivative size = 44, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {246, 245}

$$x\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{cx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^p,x]
```

```
[Out] (x*(a + c*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((c*x^2)/a)])/(1 + (c*x^2
)/a)^p
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left(a+cx^2\right)^p\,dx &= \left(\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int \left(1+\frac{cx^2}{a}\right)^p\,dx\\ &= x\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{cx^2}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.0034646, size = 44, normalized size = 1.26

$$x\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{cx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^p,x]
```

```
[Out] (x*(a + c*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((c*x^2)/a)])/(1 + (c*x^2
)/a)^p
```

Maple [F] time = 0.312, size = 0, normalized size = 0.

$$\int \left(cx^2 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^p,x)
```

```
[Out] int((c*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + a)^p, x)
```

Sympy [C] time = 2.59831, size = 22, normalized size = 0.63

$$a^p x {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{cx^2 e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**p,x)
```

```
[Out] a**p*x*hyper((1/2, -p), (3/2,), c*x**2*exp_polar(I*pi)/a)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^p, x)
```

3.736 $\int \frac{(a+cx^2)^p}{d+ex}\,dx$

Optimal. Leaf size=125

$$\frac{x\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-\frac{e\left(a+cx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{e^2\left(cx^2+a\right)}{cd^2+ae^2}\right)}{2(p+1)\left(ae^2+cd^2\right)}$$

```
[Out] (x*(a + c*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((c*x^2)/a), (e^2*x^2)/d^2])/(d
*(1 + (c*x^2)/a)^p) - (e*(a + c*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2
+ p, (e^2*(a + c*x^2))/(c*d^2 + a*e^2)])/(2*(c*d^2 + a*e^2)*(1 + p))
```

Rubi [A] time = 0.118191, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {757, 430, 429, 444, 68}

$$\frac{x\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-\frac{e\left(a+cx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{e^2\left(cx^2+a\right)}{cd^2+ae^2}\right)}{2(p+1)\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^p/(d + e*x),x]
```

```
[Out] (x*(a + c*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((c*x^2)/a), (e^2*x^2)/d^2])/(d
*(1 + (c*x^2)/a)^p) - (e*(a + c*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2
+ p, (e^2*(a + c*x^2))/(c*d^2 + a*e^2)])/(2*(c*d^2 + a*e^2)*(1 + p))
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
1, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+cx^2\right)^p}{d+ex}\,dx &= \int \left(\frac{d\left(a+cx^2\right)^p}{d^2-e^2x^2}+\frac{ex\left(a+cx^2\right)^p}{-d^2+e^2x^2}\right)dx\\
&= d\int \frac{\left(a+cx^2\right)^p}{d^2-e^2x^2}\,dx+e\int \frac{x\left(a+cx^2\right)^p}{-d^2+e^2x^2}\,dx\\
&= \frac{1}{2}e\,\mathrm{Subst}\left(\int \frac{(a+cx)^p}{-d^2+e^2x}\,dx,x,x^2\right)+\left(d\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{cx^2}{a}\right)^p}{d^2-e^2x^2}\,dx\\
&= \frac{x\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-\frac{e\left(a+cx^2\right)^{1+p}\,{}_2F_1\left(1,1+p;2+p;\frac{e^2\left(a+cx^2\right)}{cd^2+ae^2}\right)}{2\left(cd^2+ae^2\right)(1+p)}
\end{aligned}$$

Mathematica [A] time = 0.0890637, size = 131, normalized size = 1.05

$$\frac{\left(a+cx^2\right)^p\left(\frac{e\left(x-\sqrt{-\frac{a}{c}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{c}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{c}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{c}}e}{d+ex}\right)}{2ep}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + c*x^2)^p/(d + e*x),x]
```

```
[Out] ((a + c*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/c)]*e)/(d + e*
x), (d + Sqrt[-(a/c)]*e)/(d + e*x)])/(2*e*p*((e*(-Sqrt[-(a/c)] + x))/(d + e
*x))^p*((e*(Sqrt[-(a/c)] + x))/(d + e*x))^p)
```

Maple [F] time = 0.564, size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+a\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^p/(e*x+d),x)
```

```
[Out] int((c*x^2+a)^p/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+a\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^p/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+a\right)^p}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + a)^p/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+cx^2\right)^p}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**p/(e*x+d),x)
```

```
[Out] Integral((a + c*x**2)**p/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+a\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^p/(e*x + d), x)
```

3.737 $\int \frac{(a+cx^2)^p}{(d+ex)^2} dx$

Optimal. Leaf size=191

$$\frac{x\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2}+\frac{e^2x^3\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{3}{2};-p,2;\frac{5}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^4}-\frac{cde\left(a+c\right.}{}$$

```
[Out] (x*(a + c*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((c*x^2)/a), (e^2*x^2)/d^2])/(d
^2*(1 + (c*x^2)/a)^p) + (e^2*x^3*(a + c*x^2)^p*AppellF1[3/2, -p, 2, 5/2, -(
(c*x^2)/a), (e^2*x^2)/d^2])/(3*d^4*(1 + (c*x^2)/a)^p) - (c*d*e*(a + c*x^2)^
(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2*(a + c*x^2))/(c*d^2 + a*e^2
)])/((c*d^2 + a*e^2)^2*(1 + p))
```

Rubi [A] time = 0.178933, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {757, 430, 429, 444, 68, 511, 510}

$$\frac{x\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2}+\frac{e^2x^3\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{3}{2};-p,2;\frac{5}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^4}-\frac{cde\left(a+c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^p/(d + e*x)^2,x]
```


```
[Out] (x*(a + c*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((c*x^2)/a), (e^2*x^2)/d^2])/(d
^2*(1 + (c*x^2)/a)^p) + (e^2*x^3*(a + c*x^2)^p*AppellF1[3/2, -p, 2, 5/2, -(
(c*x^2)/a), (e^2*x^2)/d^2])/(3*d^4*(1 + (c*x^2)/a)^p) - (c*d*e*(a + c*x^2)^
(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2*(a + c*x^2))/(c*d^2 + a*e^2
)])/((c*d^2 + a*e^2)^2*(1 + p))
```


Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+cx^2\right)^p}{(d+ex)^2}\,dx &= \int \left(\frac{d^2\left(a+cx^2\right)^p}{\left(d^2-e^2x^2\right)^2}-\frac{2dex\left(a+cx^2\right)^p}{\left(d^2-e^2x^2\right)^2}+\frac{e^2x^2\left(a+cx^2\right)^p}{\left(-d^2+e^2x^2\right)^2}\right)dx\\
&= d^2\int \frac{\left(a+cx^2\right)^p}{\left(d^2-e^2x^2\right)^2}\,dx-(2de)\int \frac{x\left(a+cx^2\right)^p}{\left(d^2-e^2x^2\right)^2}\,dx+e^2\int \frac{x^2\left(a+cx^2\right)^p}{\left(-d^2+e^2x^2\right)^2}\,dx\\
&= -\left((de)\,\text{Subst}\left(\int \frac{(a+cx)^p}{\left(d^2-e^2x\right)^2}\,dx,x,x^2\right)\right)+\left(d^2\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{cx^2}{a}\right)^p}{\left(d^2-e^2x^2\right)^2}\,dx+\left(e^2\left(a+c\right.\right.\\
&= \frac{x\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2}+\frac{e^2x^3\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}F_1\left(\frac{3}{2};-p,2;\frac{5}{2};-\frac{cx^2}{a},\right.}{3d^4}\end{aligned}$$

Mathematica [A] time = 0.0840536, size = 141, normalized size = 0.74

$$\frac{\left(a+cx^2\right)^p\left(\frac{e\left(x-\sqrt{-\frac{a}{c}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{c}}+x\right)}{d+ex}\right)^{-p}F_1\left(1-2p;-p,-p;2-2p;\frac{d-\sqrt{-\frac{a}{c}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{c}}e}{d+ex}\right)}{e(2p-1)(d+ex)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + c*x^2)^p/(d + e*x)^2,x]
```

```
[Out] ((a + c*x^2)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/c)]*e)/(d +
 e*x), (d + Sqrt[-(a/c)]*e)/(d + e*x)])/(e*(-1 + 2*p)*((e*(-Sqrt[-(a/c)] +
```

```
x))/(d + e*x))^p*((e*(Sqrt[-(a/c)] + x))/(d + e*x))^p*(d + e*x))
```

Maple [F] time = 0.555, size = 0, normalized size = 0.

$$\int \frac{(cx^2 + a)^p}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^p/(e*x+d)^2,x)
```

```
[Out] int((c*x^2+a)^p/(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + a)^p}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^p/(e*x + d)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^2 + a)^p}{e^2x^2 + 2\,dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + a)^p/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**p/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+a\right)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^p/(e*x + d)^2, x)
```

3.738 $\int \frac{(a+cx^2)^p}{(d+ex)^3}\,dx$

Optimal. Leaf size=322

$$\frac{x\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3}+\frac{e^2x^3\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{3}{2};-p,3;\frac{5}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5}-\frac{3c^2d^2e\left(a\right.}{}$$

```
[Out] -(d^2*e*(a + c*x^2)^(1 + p))/(4*(c*d^2 + a*e^2)*(d^2 - e^2*x^2)^2) + (x*(a
+ c*x^2)^p*AppellF1[1/2, -p, 3, 3/2, -((c*x^2)/a), (e^2*x^2)/d^2])/(d^3*(1
+ (c*x^2)/a)^p) + (e^2*x^3*(a + c*x^2)^p*AppellF1[3/2, -p, 3, 5/2, -((c*x^2
)/a), (e^2*x^2)/d^2])/(d^5*(1 + (c*x^2)/a)^p) + (c*e*(2*a*e^2 + c*d^2*(1 +
p))*(a + c*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2*(a + c*x^2)
)/(c*d^2 + a*e^2)])/(4*(c*d^2 + a*e^2)^3*(1 + p)) - (3*c^2*d^2*e*(a + c*x^2
)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, (e^2*(a + c*x^2))/(c*d^2 + a*e
^2)])/(2*(c*d^2 + a*e^2)^3*(1 + p))
```

Rubi [A] time = 0.315816, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {757, 430, 429, 444, 68, 511, 510, 446, 78}

$$\frac{x\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3}+\frac{e^2x^3\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{3}{2};-p,3;\frac{5}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5}-\frac{3c^2d^2e\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^p/(d + e*x)^3,x]
```


```
[Out] -(d^2*e*(a + c*x^2)^(1 + p))/(4*(c*d^2 + a*e^2)*(d^2 - e^2*x^2)^2) + (x*(a
+ c*x^2)^p*AppellF1[1/2, -p, 3, 3/2, -((c*x^2)/a), (e^2*x^2)/d^2])/(d^3*(1
+ (c*x^2)/a)^p) + (e^2*x^3*(a + c*x^2)^p*AppellF1[3/2, -p, 3, 5/2, -((c*x^2
)/a), (e^2*x^2)/d^2])/(d^5*(1 + (c*x^2)/a)^p) + (c*e*(2*a*e^2 + c*d^2*(1 +
p))*(a + c*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2*(a + c*x^2)
)/(c*d^2 + a*e^2)])/(4*(c*d^2 + a*e^2)^3*(1 + p)) - (3*c^2*d^2*e*(a + c*x^2
)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, (e^2*(a + c*x^2))/(c*d^2 + a*e
^2)])/(2*(c*d^2 + a*e^2)^3*(1 + p))
```


Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rubi steps

$$\int \frac{(a+cx^2)^p}{(d+ex)^3}\,dx = \int \left(\frac{d^3(a+cx^2)^p}{(d^2-e^2x^2)^3} - \frac{3d^2ex(a+cx^2)^p}{(d^2-e^2x^2)^3} + \frac{3de^2x^2(a+cx^2)^p}{(d^2-e^2x^2)^3} + \frac{e^3x^3(a+cx^2)^p}{(-d^2+e^2x^2)^3}\right)dx$$

$$= d^3\int \frac{(a+cx^2)^p}{(d^2-e^2x^2)^3}\,dx - (3d^2e)\int \frac{x(a+cx^2)^p}{(d^2-e^2x^2)^3}\,dx + (3de^2)\int \frac{x^2(a+cx^2)^p}{(d^2-e^2x^2)^3}\,dx + e^3\int \frac{x^3(a+cx^2}{(-d^2+e^2x^2}$$

$$= -\left(\frac{1}{2}(3d^2e)\,\mathrm{Subst}\left(\int \frac{(a+cx)^p}{(d^2-e^2x)^3}\,dx, x, x^2\right)\right) + \frac{1}{2}e^3\,\mathrm{Subst}\left(\int \frac{x(a+cx)^p}{(-d^2+e^2x)^3}\,dx, x, x^2\right) + \left(d^3(a+c\right.$$

$$= -\frac{d^2e(a+cx^2)^{1+p}}{4(cd^2+ae^2)(d^2-e^2x^2)^2} + \frac{x(a+cx^2)^p\left(1+\frac{cx^2}{a}\right)^{-p} F_1\left(\frac{1}{2}; -p, 3; \frac{3}{2}; -\frac{cx^2}{a}, \frac{e^2x^2}{d^2}\right)}{d^3} + \frac{e^2x^3(a+cx^2)^p}{}$$

$$= -\frac{d^2e(a+cx^2)^{1+p}}{4(cd^2+ae^2)(d^2-e^2x^2)^2} + \frac{x(a+cx^2)^p\left(1+\frac{cx^2}{a}\right)^{-p} F_1\left(\frac{1}{2}; -p, 3; \frac{3}{2}; -\frac{cx^2}{a}, \frac{e^2x^2}{d^2}\right)}{d^3} + \frac{e^2x^3(a+cx^2)^p}{}$$

Mathematica [A] time = 0.121307, size = 142, normalized size = 0.44

$$\frac{(a+cx^2)^p \left(\frac{e\left(x-\sqrt{-\frac{a}{c}}\right)}{d+ex}\right)^{-p} \left(\frac{e\left(\sqrt{-\frac{a}{c}}+x\right)}{d+ex}\right)^{-p} F_1\left(2-2p; -p, -p; 3-2p; \frac{d-\sqrt{-\frac{a}{c}}e}{d+ex}, \frac{d+\sqrt{-\frac{a}{c}}e}{d+ex}\right)}{2e(p-1)(d+ex)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + c*x^2)^p/(d + e*x)^3,x]
```

```
[Out] ((a + c*x^2)^p*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (d - Sqrt[-(a/c)]*e)/(d +
 e*x), (d + Sqrt[-(a/c)]*e)/(d + e*x)])/(2*e*(-1 + p)*((e*(-Sqrt[-(a/c)] +
x))/(d + e*x))^p*((e*(Sqrt[-(a/c)] + x))/(d + e*x))^p*(d + e*x)^2)
```

Maple [F] time = 0.58, size = 0, normalized size = 0.

$$\int \frac{(cx^2+a)^p}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^p/(e*x+d)^3,x)
```

```
[Out] int((c*x^2+a)^p/(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2+a)^p}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^p/(e*x + d)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+a\right)^p}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + a)^p/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**p/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+a\right)^p}{\left(ex+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^p/(e*x + d)^3, x)
```

3.739 $\int (d+ex)^{-2p} \left(a+cx^2\right)^p \, dx$

Optimal. Leaf size=160

$$\frac{\left(a+cx^2\right)^p (d+ex)^{1-2p} \left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p} \left(1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}\right)^{-p} F_1\left(1-2p;-p,-p;2-2p;\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e(1-2p)}$$

```
[Out] ((d + e*x)^(1 - 2*p)*(a + c*x^2)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d +
e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]), (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])])/(e*
(1 - 2*p)*(1 - (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]))^p*(1 - (d + e*x)/(d +
(Sqrt[-a]*e)/Sqrt[c]))^p)
```

Rubi [A] time = 0.0852414, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {760, 133}

$$\frac{\left(a+cx^2\right)^p (d+ex)^{1-2p} \left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p} \left(1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}\right)^{-p} F_1\left(1-2p;-p,-p;2-2p;\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e(1-2p)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^p/(d + e*x)^(2*p),x]
```

```
[Out] ((d + e*x)^(1 - 2*p)*(a + c*x^2)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d +
e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]), (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])])/(e*
(1 - 2*p)*(1 - (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]))^p*(1 - (d + e*x)/(d +
(Sqrt[-a]*e)/Sqrt[c]))^p)
```

Rule 760

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + (e*q)/c))^p*
(1 - (d + e*x)/(d - (e*q)/c))^p), Subst[Int[x^m*Simp[1 - x/(d + (e*q)/c), x
]^p*Simp[1 - x/(d - (e*q)/c), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (d+ex)^{-2p}\left(a+cx^2\right)^p dx = \frac{\left(\left(a+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\right)\text{Subst}\left(\int x^{-2p}\left(1-\frac{x}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^p\left(1-\frac{x}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^p dx, x,}{e}$$

$$= \frac{(d+ex)^{1-2p}\left(a+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p} F_1\left(1-2p;-p,-p;2-2p;\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right.}{e(1-2p)}$$

Mathematica [A] time = 0.126451, size = 166, normalized size = 1.04

$$-\frac{\left(a+cx^2\right)^p (d+ex)^{1-2p}\left(\frac{e\left(\sqrt{-\frac{a}{c}}-x\right)}{e\sqrt{-\frac{a}{c}}+d}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{c}}+x\right)}{e\sqrt{-\frac{a}{c}}-d}\right)^{-p} F_1\left(1-2p;-p,-p;2-2p;\frac{d+ex}{d-\sqrt{-\frac{a}{c}}e},\frac{d+ex}{d+\sqrt{-\frac{a}{c}}e}\right)}{e(2p-1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + c*x^2)^p/(d + e*x)^(2*p),x]
```

```
[Out] -(((d + e*x)^(1 - 2*p)*(a + c*x^2)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d
+ e*x)/(d - Sqrt[-(a/c)]*e), (d + e*x)/(d + Sqrt[-(a/c)]*e)])/(e*(-1 + 2*p)
*((e*(Sqrt[-(a/c)] - x))/(d + Sqrt[-(a/c)]*e))^p*((e*(Sqrt[-(a/c)] + x))/(-
d + Sqrt[-(a/c)]*e))^p))
```

Maple [F] time = 0.583, size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+a\right)^p}{(ex+d)^{2p}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^p/((e*x+d)^(2*p)),x)
```

```
[Out] int((c*x^2+a)^p/((e*x+d)^(2*p)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+a\right)^p}{(ex+d)^{2p}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p/((e*x+d)^(2*p)),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^p/(e*x + d)^(2*p), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+a\right)^p}{(ex+d)^{2p}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p/((e*x+d)^(2*p)),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + a)^p/(e*x + d)^(2*p), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**p/((e*x+d)**(2*p)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+a\right)^p}{(ex+d)^{2p}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p/((e*x+d)^(2*p)),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^p/(e*x + d)^(2*p), x)
```

3.740 $\int (d+ex)^{-1-2p} \left(a+cx^2\right)^p \, dx$

Optimal. Leaf size=155

$$-\frac{\left(a+cx^2\right)^p (d+ex)^{-2p} \left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p} \left(1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}\right)^{-p} F_1\left(-2p;-p,-p;1-2p;\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{2ep}$$

```
[Out] -((a + c*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (d + e*x)/(d - (Sqrt[-a]*e)
/Sqrt[c]), (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])])/(2*e*p*(d + e*x)^(2*p)*(1
 - (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]))^p*(1 - (d + e*x)/(d + (Sqrt[-a]*e)
/Sqrt[c]))^p)
```

Rubi [A] time = 0.0650256, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {760, 133}

$$-\frac{\left(a+cx^2\right)^p (d+ex)^{-2p} \left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p} \left(1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}\right)^{-p} F_1\left(-2p;-p,-p;1-2p;\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{2ep}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(-1 - 2*p)*(a + c*x^2)^p,x]
```

```
[Out] -((a + c*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (d + e*x)/(d - (Sqrt[-a]*e)
/Sqrt[c]), (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])])/(2*e*p*(d + e*x)^(2*p)*(1
 - (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]))^p*(1 - (d + e*x)/(d + (Sqrt[-a]*e)
/Sqrt[c]))^p)
```

Rule 760

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + (e*q)/c))^p*
(1 - (d + e*x)/(d - (e*q)/c))^p), Subst[Int[x^m*Simp[1 - x/(d + (e*q)/c), x
]^p*Simp[1 - x/(d - (e*q)/c), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (d+ex)^{-1-2p}\left(a+cx^2\right)^p\,dx = \frac{\left(\left(a+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\sqrt{-ae}}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{d+\frac{\sqrt{-ae}}{\sqrt{c}}}\right)^{-p}\right)\text{Subst}\left(\int x^{-1-2p}\left(1-\frac{x}{d-\frac{\sqrt{-ae}}{\sqrt{c}}}\right)^p\left(1-\frac{x}{d+\frac{\sqrt{-ae}}{\sqrt{c}}}\right.\cdots\right.}{e}$$

$$= -\frac{(d+ex)^{-2p}\left(a+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\sqrt{-ae}}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{d+\frac{\sqrt{-ae}}{\sqrt{c}}}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d+ex}{d-\frac{\sqrt{-ae}}{\sqrt{c}}},\cdots\right.}{2ep}$$

Mathematica [A] time = 0.100414, size = 160, normalized size = 1.03

$$-\frac{\left(a+cx^2\right)^p(d+ex)^{-2p}\left(\frac{e\left(\sqrt{-\frac{a}{c}}-x\right)}{e\sqrt{-\frac{a}{c}}+d}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{c}}+x\right)}{e\sqrt{-\frac{a}{c}}-d}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d+ex}{d-\sqrt{-\frac{a}{c}}e},\frac{d+ex}{d+\sqrt{-\frac{a}{c}}e}\right)}{2ep}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^(-1 - 2*p)*(a + c*x^2)^p,x]
```

```
[Out] -((a + c*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (d + e*x)/(d - Sqrt[-(a/c)]
*e), (d + e*x)/(d + Sqrt[-(a/c)]*e)])/(2*e*p*((e*(Sqrt[-(a/c)] - x))/(d + S
qrt[-(a/c)]*e))^p*((e*(Sqrt[-(a/c)] + x))/(-d + Sqrt[-(a/c)]*e))^p*(d + e*x
)^(2*p))
```

Maple [F] time = 0.593, size = 0, normalized size = 0.

$$\int (ex+d)^{-1-2p}\left(cx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(-1-2*p)*(c*x^2+a)^p,x)
```

```
[Out] int((e*x+d)^(-1-2*p)*(c*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+a\right)^p(ex+d)^{-2p-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-1-2*p)*(c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^p*(e*x + d)^(-2*p - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2+a\right)^p(ex+d)^{-2p-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-1-2*p)*(c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + a)^p*(e*x + d)^(-2*p - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(-1-2*p)*(c*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + a\right)^p (ex + d)^{-2p-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-1-2*p)*(c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^p*(e*x + d)^(-2*p - 1), x)
```

3.741 $\int (d+ex)^{-2-2p}\left(a+cx^2\right)^p\,dx$

Optimal. Leaf size=209

$$-\frac{\left(\sqrt{-a}-\sqrt{c}x\right)\left(a+cx^2\right)^p(d+ex)^{-2p-1}\left(-\frac{\left(\sqrt{-a}+\sqrt{c}x\right)\left(\sqrt{-a}e+\sqrt{c}d\right)}{\left(\sqrt{-a}-\sqrt{c}x\right)\left(\sqrt{c}d-\sqrt{-a}e\right)}\right)^{-p}{}_2F_1\left(-2p-1,-p;-2p;\frac{2\sqrt{-a}\sqrt{c}(d+ex)}{\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\sqrt{-a}-\sqrt{c}x\right)}\right)}{(2p+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}$$

```
[Out] -(((Sqrt[-a] - Sqrt[c]*x)*(d + e*x)^(-1 - 2*p)*(a + c*x^2)^p*Hypergeometric
2F1[-1 - 2*p, -p, -2*p, (2*Sqrt[-a]*Sqrt[c]*(d + e*x))/((Sqrt[c]*d - Sqrt[-
a]*e)*(Sqrt[-a] - Sqrt[c]*x))])/((Sqrt[c]*d + Sqrt[-a]*e)*(1 + 2*p)*(-(((Sq
rt[c]*d + Sqrt[-a]*e)*(Sqrt[-a] + Sqrt[c]*x))/((Sqrt[c]*d - Sqrt[-a]*e)*(Sq
rt[-a] - Sqrt[c]*x))))^p))
```

Rubi [A] time = 0.0839888, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {727}

$$-\frac{\left(\sqrt{-a}-\sqrt{c}x\right)\left(a+cx^2\right)^p(d+ex)^{-2p-1}\left(-\frac{\left(\sqrt{-a}+\sqrt{c}x\right)\left(\sqrt{-a}e+\sqrt{c}d\right)}{\left(\sqrt{-a}-\sqrt{c}x\right)\left(\sqrt{c}d-\sqrt{-a}e\right)}\right)^{-p}{}_2F_1\left(-2p-1,-p;-2p;\frac{2\sqrt{-a}\sqrt{c}(d+ex)}{\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\sqrt{-a}-\sqrt{c}x\right)}\right)}{(2p+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(-2 - 2*p)*(a + c*x^2)^p,x]
```

```
[Out] -(((Sqrt[-a] - Sqrt[c]*x)*(d + e*x)^(-1 - 2*p)*(a + c*x^2)^p*Hypergeometric
2F1[-1 - 2*p, -p, -2*p, (2*Sqrt[-a]*Sqrt[c]*(d + e*x))/((Sqrt[c]*d - Sqrt[-
a]*e)*(Sqrt[-a] - Sqrt[c]*x))])/((Sqrt[c]*d + Sqrt[-a]*e)*(1 + 2*p)*(-(((Sq
rt[c]*d + Sqrt[-a]*e)*(Sqrt[-a] + Sqrt[c]*x))/((Sqrt[c]*d - Sqrt[-a]*e)*(Sq
rt[-a] - Sqrt[c]*x))))^p))
```

Rule 727

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((Rt[-(a*c), 2] - c*x)*(d + e*x)^(m + 1)*(a + c*x^2)^p*Hypergeometric2F1[m
+ 1, -p, m + 2, (2*c*Rt[-(a*c), 2]*(d + e*x))/((c*d - e*Rt[-(a*c), 2])*(Rt[
-(a*c), 2] - c*x))])/((m + 1)*(c*d + e*Rt[-(a*c), 2])*(((c*d + e*Rt[-(a*c),
 2])*(Rt[-(a*c), 2] + c*x))/((c*d - e*Rt[-(a*c), 2])*(-Rt[-(a*c), 2] + c*x)
))^p), x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Int
egerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int (d+ex)^{-2-2p}\left(a+cx^2\right)^p\,dx=-\frac{\left(\sqrt{-a}-\sqrt{c}x\right)\left(-\frac{\left(\sqrt{c}d+\sqrt{-a}e\right)\left(\sqrt{-a}+\sqrt{c}x\right)}{\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\sqrt{-a}-\sqrt{c}x\right)}\right)^{-p}(d+ex)^{-1-2p}\left(a+cx^2\right)^p{}_2F_1\left(-1-2p,-p;-2\ldots\right)}{\left(\sqrt{c}d+\sqrt{-a}e\right)(1+2p)}$$

Mathematica [A] time = 0.13412, size = 200, normalized size = 0.96

$$\frac{\left(\sqrt{c}x-\sqrt{-a}\right)\left(a+cx^2\right)^p(d+ex)^{-2p-1}\left(1-\frac{c(d+ex)}{cd-\sqrt{-a}\sqrt{c}e}\right)^{-p}\left(1-\frac{c(d+ex)}{\sqrt{-a}\sqrt{c}e+cd}\right)^p{}_2F_1\left(-2p-1,-p;-2p;\frac{2\sqrt{-a}\sqrt{c}(d+ex)}{\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\sqrt{-a}-\sqrt{c}x\right)}\right)}{(2p+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^(-2 - 2*p)*(a + c*x^2)^p,x]
```

```
[Out] ((-Sqrt[-a] + Sqrt[c]*x)*(d + e*x)^(-1 - 2*p)*(a + c*x^2)^p*(1 - (c*(d + e*
x))/(c*d + Sqrt[-a]*Sqrt[c]*e))^p*Hypergeometric2F1[-1 - 2*p, -p, -2*p, (2*
Sqrt[-a]*Sqrt[c]*(d + e*x))/((Sqrt[c]*d - Sqrt[-a]*e)*(Sqrt[-a] - Sqrt[c]*x
))])/((Sqrt[c]*d + Sqrt[-a]*e)*(1 + 2*p)*(1 - (c*(d + e*x))/(c*d - Sqrt[-a]
*Sqrt[c]*e))^p)
```

Maple [F] time = 0.583, size = 0, normalized size = 0.

$$\int (ex+d)^{-2-2p} \left(cx^2+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(-2-2*p)*(c*x^2+a)^p,x)
```

```
[Out] int((e*x+d)^(-2-2*p)*(c*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+a\right)^p (ex+d)^{-2p-2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-2-2*p)*(c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^p*(e*x + d)^(-2*p - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2+a\right)^p (ex+d)^{-2p-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-2-2*p)*(c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + a)^p*(e*x + d)^(-2*p - 2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(-2-2*p)*(c*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + a\right)^p (ex + d)^{-2p-2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-2-2*p)*(c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^p*(e*x + d)^(-2*p - 2), x)
```

3.742 $\int (d+ex)^{-3-2p}\left(a+cx^2\right)^p\,dx$

Optimal. Leaf size=270

$$-\frac{cd\left(\sqrt{-a}-\sqrt{c}x\right)\left(a+cx^2\right)^p(d+ex)^{-2p-1}\left(-\frac{\left(\sqrt{-a}+\sqrt{c}x\right)\left(\sqrt{-a}e+\sqrt{c}d\right)}{\left(\sqrt{-a}-\sqrt{c}x\right)\left(\sqrt{c}d-\sqrt{-a}e\right)}\right)^{-p}{}_2F_1\left(-2p-1,-p;-2p;\frac{2\sqrt{-a}\sqrt{c}(d+ex)}{\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\sqrt{-a}-\sqrt{c}x\right)}\right)}{(2p+1)\left(\sqrt{-a}e+\sqrt{c}d\right)\left(ae^2+cd^2\right)}-\frac{e\left(a+c\right.}{2(}$$

```
[Out] -(e*(a + c*x^2)^(1 + p))/(2*(c*d^2 + a*e^2)*(1 + p)*(d + e*x)^(2*(1 + p)))
- (c*d*(Sqrt[-a] - Sqrt[c]*x)*(d + e*x)^(-1 - 2*p)*(a + c*x^2)^p*Hypergeome
tric2F1[-1 - 2*p, -p, -2*p, (2*Sqrt[-a]*Sqrt[c]*(d + e*x))/((Sqrt[c]*d - Sq
rt[-a]*e)*(Sqrt[-a] - Sqrt[c]*x))])/((Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*e^
2)*(1 + 2*p)*(-(((Sqrt[c]*d + Sqrt[-a]*e)*(Sqrt[-a] + Sqrt[c]*x))/((Sqrt[c]
*d - Sqrt[-a]*e)*(Sqrt[-a] - Sqrt[c]*x))))^p)
```

Rubi [A] time = 0.0857186, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {731, 727}

$$-\frac{cd\left(\sqrt{-a}-\sqrt{c}x\right)\left(a+cx^2\right)^p(d+ex)^{-2p-1}\left(-\frac{\left(\sqrt{-a}+\sqrt{c}x\right)\left(\sqrt{-a}e+\sqrt{c}d\right)}{\left(\sqrt{-a}-\sqrt{c}x\right)\left(\sqrt{c}d-\sqrt{-a}e\right)}\right)^{-p}{}_2F_1\left(-2p-1,-p;-2p;\frac{2\sqrt{-a}\sqrt{c}(d+ex)}{\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\sqrt{-a}-\sqrt{c}x\right)}\right)}{(2p+1)\left(\sqrt{-a}e+\sqrt{c}d\right)\left(ae^2+cd^2\right)}-\frac{e\left(a+c\right.}{2(}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(-3 - 2*p)*(a + c*x^2)^p,x]
```

```
[Out] -(e*(a + c*x^2)^(1 + p))/(2*(c*d^2 + a*e^2)*(1 + p)*(d + e*x)^(2*(1 + p)))
- (c*d*(Sqrt[-a] - Sqrt[c]*x)*(d + e*x)^(-1 - 2*p)*(a + c*x^2)^p*Hypergeome
tric2F1[-1 - 2*p, -p, -2*p, (2*Sqrt[-a]*Sqrt[c]*(d + e*x))/((Sqrt[c]*d - Sq
rt[-a]*e)*(Sqrt[-a] - Sqrt[c]*x))])/((Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*e^
2)*(1 + 2*p)*(-(((Sqrt[c]*d + Sqrt[-a]*e)*(Sqrt[-a] + Sqrt[c]*x))/((Sqrt[c]
*d - Sqrt[-a]*e)*(Sqrt[-a] - Sqrt[c]*x))))^p)
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 727

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((Rt[-(a*c), 2] - c*x)*(d + e*x)^(m + 1)*(a + c*x^2)^p*Hypergeometric2F1[m
+ 1, -p, m + 2, (2*c*Rt[-(a*c), 2]*(d + e*x))/((c*d - e*Rt[-(a*c), 2])*(Rt[
-(a*c), 2] - c*x))])/((m + 1)*(c*d + e*Rt[-(a*c), 2])*(((c*d + e*Rt[-(a*c),
 2])*(Rt[-(a*c), 2] + c*x))/((c*d - e*Rt[-(a*c), 2])*(-Rt[-(a*c), 2] + c*x)
))^p), x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Int
egerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int (d+ex)^{-3-2p}\left(a+cx^2\right)^p\,dx = -\frac{e(d+ex)^{-2(1+p)}\left(a+cx^2\right)^{1+p}}{2\left(cd^2+ae^2\right)(1+p)} + \frac{(cd)\int (d+ex)^{-2-2p}\left(a+cx^2\right)^p\,dx}{cd^2+ae^2}$$

$$= -\frac{e(d+ex)^{-2(1+p)}\left(a+cx^2\right)^{1+p}}{2\left(cd^2+ae^2\right)(1+p)} - \frac{cd\left(\sqrt{-a}-\sqrt{c}x\right)\left(-\frac{\left(\sqrt{c}d+\sqrt{-a}e\right)\left(\sqrt{-a}+\sqrt{c}x\right)}{\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\sqrt{-a}-\sqrt{c}x\right)}\right)^{-p}(d+ex)^{-1\cdots}}{\left(\sqrt{c}d+\sqrt{-a}e\right)\cdots}$$

Mathematica [A] time = 49.557, size = 368, normalized size = 1.36

$$\frac{2^{-2p-3}\mathrm{Gamma}\left(-p-\frac{1}{2}\right)\left(a+cx^2\right)^p(d+ex)^{-2(p+1)}\left(\frac{e\left(\sqrt{-\frac{a}{c}}-x\right)}{e\sqrt{-\frac{a}{c}}+d}\right)^{-p}\left(1-\frac{d+ex}{e\sqrt{-\frac{a}{c}}+d}\right)^{p+1}\left(\mathrm{Gamma}(1-2p)\mathrm{Gamma}(-p)\left(e\sqrt{-\cdots}\right.\right.}{\sqrt{\pi}e(p+1)\mathrm{G}\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^(-3 - 2*p)*(a + c*x^2)^p,x]
```

```
[Out] (2^(-3 - 2*p)*(a + c*x^2)^p*(1 - (d + e*x)/(d + Sqrt[-(a/c)]*e))^(1 + p)*Ga
mma[-1/2 - p]*((d + Sqrt[-(a/c)]*e)*(2*d*(1 + p) + e*(Sqrt[-(a/c)] + 2*Sqrt
[-(a/c)]*p + x))*Gamma[1 - 2*p]*Gamma[-p]*Hypergeometric2F1[1, -p, -2*p, (2
*Sqrt[-(a/c)]*(d + e*x))/((d + Sqrt[-(a/c)]*e)*(Sqrt[-(a/c)] + x))] + (2*e*
(a + Sqrt[-(a/c)]*c*x)*(d + e*x)*Gamma[1 - p]*Gamma[-2*p]*Hypergeometric2F1
[2, 1 - p, 1 - 2*p, (2*Sqrt[-(a/c)]*(d + e*x))/((d + Sqrt[-(a/c)]*e)*(Sqrt[
-(a/c)] + x))])/(c*(Sqrt[-(a/c)] + x))))/(e*(d + Sqrt[-(a/c)]*e)^2*(1 + p)*
Sqrt[Pi]*((e*(Sqrt[-(a/c)] - x))/(d + Sqrt[-(a/c)]*e))^p*(d + e*x)^(2*(1 +
p))*Gamma[1 - 2*p]*Gamma[-2*p])
```

Maple [F] time = 0.614, size = 0, normalized size = 0.

$$\int (ex+d)^{-3-2p}\left(cx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(-3-2*p)*(c*x^2+a)^p,x)
```

```
[Out] int((e*x+d)^(-3-2*p)*(c*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+a\right)^p(ex+d)^{-2p-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-3-2*p)*(c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^p*(e*x + d)^(-2*p - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2+a\right)^p(ex+d)^{-2p-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-3-2*p)*(c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + a)^p*(e*x + d)^(-2*p - 3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(-3-2*p)*(c*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+a\right)^p(ex+d)^{-2p-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-3-2*p)*(c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^p*(e*x + d)^(-2*p - 3), x)
```

3.743 $\int (d+ex)^{-4-2p} \left(a+cx^2\right)^p \, dx$

Optimal. Leaf size=347

$$\frac{c\left(\sqrt{-a}-\sqrt{c}x\right)\left(a+cx^2\right)^p (d+ex)^{-2p-1}\left(ae^2-cd^2(2p+3)\right)\left(-\frac{\left(\sqrt{-a}+\sqrt{c}x\right)\left(\sqrt{-a}e+\sqrt{c}d\right)}{\left(\sqrt{-a}-\sqrt{c}x\right)\left(\sqrt{c}d-\sqrt{-a}e\right)}\right)^{-p} {}_2F_1\left(-2p-1,-p;-2p;\frac{2\sqrt{-a}\sqrt{c}}{\left(\sqrt{c}d-\sqrt{-a}e\right)}\right.}{(2p+1)(2p+3)\left(\sqrt{-a}e+\sqrt{c}d\right)\left(ae^2+cd^2\right)^2}$$

```
[Out] -((e*(d + e*x)^(-3 - 2*p)*(a + c*x^2)^(1 + p))/((c*d^2 + a*e^2)*(3 + 2*p)))
 - (c*d*e*(2 + p)*(a + c*x^2)^(1 + p))/((c*d^2 + a*e^2)^2*(1 + p)*(3 + 2*p)
*(d + e*x)^(2*(1 + p))) + (c*(a*e^2 - c*d^2*(3 + 2*p))*(Sqrt[-a] - Sqrt[c]*
x)*(d + e*x)^(-1 - 2*p)*(a + c*x^2)^p*Hypergeometric2F1[-1 - 2*p, -p, -2*p,
 (2*Sqrt[-a]*Sqrt[c]*(d + e*x))/((Sqrt[c]*d - Sqrt[-a]*e)*(Sqrt[-a] - Sqrt[
c]*x))])/((Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*e^2)^2*(1 + 2*p)*(3 + 2*p)*(-
(((Sqrt[c]*d + Sqrt[-a]*e)*(Sqrt[-a] + Sqrt[c]*x))/((Sqrt[c]*d - Sqrt[-a]*e
)*(Sqrt[-a] - Sqrt[c]*x))))^p)
```

Rubi [A] time = 0.182588, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {745, 807, 727}

$$\frac{c\left(\sqrt{-a}-\sqrt{c}x\right)\left(a+cx^2\right)^p (d+ex)^{-2p-1}\left(ae^2-cd^2(2p+3)\right)\left(-\frac{\left(\sqrt{-a}+\sqrt{c}x\right)\left(\sqrt{-a}e+\sqrt{c}d\right)}{\left(\sqrt{-a}-\sqrt{c}x\right)\left(\sqrt{c}d-\sqrt{-a}e\right)}\right)^{-p} {}_2F_1\left(-2p-1,-p;-2p;\frac{2\sqrt{-a}\sqrt{c}}{\left(\sqrt{c}d-\sqrt{-a}e\right)}\right.}{(2p+1)(2p+3)\left(\sqrt{-a}e+\sqrt{c}d\right)\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(-4 - 2*p)*(a + c*x^2)^p,x]
```

```
[Out] -((e*(d + e*x)^(-3 - 2*p)*(a + c*x^2)^(1 + p))/((c*d^2 + a*e^2)*(3 + 2*p)))
 - (c*d*e*(2 + p)*(a + c*x^2)^(1 + p))/((c*d^2 + a*e^2)^2*(1 + p)*(3 + 2*p)
*(d + e*x)^(2*(1 + p))) + (c*(a*e^2 - c*d^2*(3 + 2*p))*(Sqrt[-a] - Sqrt[c]*
x)*(d + e*x)^(-1 - 2*p)*(a + c*x^2)^p*Hypergeometric2F1[-1 - 2*p, -p, -2*p,
 (2*Sqrt[-a]*Sqrt[c]*(d + e*x))/((Sqrt[c]*d - Sqrt[-a]*e)*(Sqrt[-a] - Sqrt[
c]*x))])/((Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*e^2)^2*(1 + 2*p)*(3 + 2*p)*(-
(((Sqrt[c]*d + Sqrt[-a]*e)*(Sqrt[-a] + Sqrt[c]*x))/((Sqrt[c]*d - Sqrt[-a]*e
)*(Sqrt[-a] - Sqrt[c]*x))))^p)
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 727

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((Rt[-(a*c), 2] - c*x)*(d + e*x)^(m + 1)*(a + c*x^2)^p*Hypergeometric2F1[m
+ 1, -p, m + 2, (2*c*Rt[-(a*c), 2]*(d + e*x))/((c*d - e*Rt[-(a*c), 2])*(Rt[
-(a*c), 2] - c*x))])/((m + 1)*(c*d + e*Rt[-(a*c), 2])*(((c*d + e*Rt[-(a*c),
 2])*(Rt[-(a*c), 2] + c*x))/((c*d - e*Rt[-(a*c), 2])*(-Rt[-(a*c), 2] + c*x)
))^p), x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Int
egerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int (d+ex)^{-4-2p}\left(a+cx^2\right)^p\,dx = -\frac{e(d+ex)^{-3-2p}\left(a+cx^2\right)^{1+p}}{\left(cd^2+ae^2\right)(3+2p)} - \frac{c\int (d+ex)^{-3-2p}(-d(3+2p)+ex)\left(a+cx^2\right)^p\,dx}{\left(cd^2+ae^2\right)(3+2p)}$$
$$= -\frac{e(d+ex)^{-3-2p}\left(a+cx^2\right)^{1+p}}{\left(cd^2+ae^2\right)(3+2p)} - \frac{cde(2+p)(d+ex)^{-2(1+p)}\left(a+cx^2\right)^{1+p}}{\left(cd^2+ae^2\right)^2(1+p)(3+2p)} - \frac{\left(c\left(ae^2-cd^2(3+\right.\right.}{\left(\right.}$$
$$= -\frac{e(d+ex)^{-3-2p}\left(a+cx^2\right)^{1+p}}{\left(cd^2+ae^2\right)(3+2p)} - \frac{cde(2+p)(d+ex)^{-2(1+p)}\left(a+cx^2\right)^{1+p}}{\left(cd^2+ae^2\right)^2(1+p)(3+2p)} + \frac{c\left(ae^2-cd^2(3+\right.}{}$$

Mathematica [B] time = 25.074, size = 1439, normalized size = 4.15

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^(-4 - 2*p)*(a + c*x^2)^p,x]
```

```
[Out] (-2*(d + e*x)^(-3 - 2*p)*(a + c*x^2)^p*(1 - (d + e*x)/(d + Sqrt[-(a/c)]*e))
^(1 + p)*Gamma[-2*(1 + p)]*((d + Sqrt[-(a/c)]*e)^3*(Sqrt[-(a/c)] + x)*Gamma
[1 - 2*p]*Gamma[-p]*Hypergeometric2F1[1, -p, -2*p, (2*Sqrt[-(a/c)]*(d + e*x
))/((d + Sqrt[-(a/c)]*e)*(Sqrt[-(a/c)] + x))] + 3*(d + Sqrt[-(a/c)]*e)^3*p*
(Sqrt[-(a/c)] + x)*Gamma[1 - 2*p]*Gamma[-p]*Hypergeometric2F1[1, -p, -2*p,
(2*Sqrt[-(a/c)]*(d + e*x))/((d + Sqrt[-(a/c)]*e)*(Sqrt[-(a/c)] + x))] + 2*(
d + Sqrt[-(a/c)]*e)^3*p^2*(Sqrt[-(a/c)] + x)*Gamma[1 - 2*p]*Gamma[-p]*Hyper
geometric2F1[1, -p, -2*p, (2*Sqrt[-(a/c)]*(d + e*x))/((d + Sqrt[-(a/c)]*e)*
(Sqrt[-(a/c)] + x))] + (d + Sqrt[-(a/c)]*e)^2*(Sqrt[-(a/c)] + x)*(d + e*x)*
Gamma[1 - 2*p]*Gamma[-p]*Hypergeometric2F1[1, -p, -2*p, (2*Sqrt[-(a/c)]*(d
+ e*x))/((d + Sqrt[-(a/c)]*e)*(Sqrt[-(a/c)] + x))] + 2*(d + Sqrt[-(a/c)]*e)
^2*p*(Sqrt[-(a/c)] + x)*(d + e*x)*Gamma[1 - 2*p]*Gamma[-p]*Hypergeometric2F
1[1, -p, -2*p, (2*Sqrt[-(a/c)]*(d + e*x))/((d + Sqrt[-(a/c)]*e)*(Sqrt[-(a/c
)] + x))] + (d + Sqrt[-(a/c)]*e)*(Sqrt[-(a/c)] + x)*(d + e*x)^2*Gamma[1 - 2
*p]*Gamma[-p]*Hypergeometric2F1[1, -p, -2*p, (2*Sqrt[-(a/c)]*(d + e*x))/((d
 + Sqrt[-(a/c)]*e)*(Sqrt[-(a/c)] + x))] - 3*Sqrt[-(a/c)]*(d + Sqrt[-(a/c)]*
e)^2*(d + e*x)*Gamma[1 - p]*Gamma[-2*p]*Hypergeometric2F1[2, 1 - p, 1 - 2*p
, (2*Sqrt[-(a/c)]*(d + e*x))/((d + Sqrt[-(a/c)]*e)*(Sqrt[-(a/c)] + x))] - 4
*Sqrt[-(a/c)]*(d + Sqrt[-(a/c)]*e)^2*p*(d + e*x)*Gamma[1 - p]*Gamma[-2*p]*H
ypergeometric2F1[2, 1 - p, 1 - 2*p, (2*Sqrt[-(a/c)]*(d + e*x))/((d + Sqrt[-
(a/c)]*e)*(Sqrt[-(a/c)] + x))] + 4*Sqrt[-(a/c)]*(d + Sqrt[-(a/c)]*e)*p*(d +
 e*x)^2*Gamma[1 - p]*Gamma[-2*p]*Hypergeometric2F1[2, 1 - p, 1 - 2*p, (2*Sq
rt[-(a/c)]*(d + e*x))/((d + Sqrt[-(a/c)]*e)*(Sqrt[-(a/c)] + x))] + 3*Sqrt[-
(a/c)]*(d + e*x)^3*Gamma[1 - p]*Gamma[-2*p]*Hypergeometric2F1[2, 1 - p, 1 -
 2*p, (2*Sqrt[-(a/c)]*(d + e*x))/((d + Sqrt[-(a/c)]*e)*(Sqrt[-(a/c)] + x))]
 + Sqrt[-(a/c)]*(d + Sqrt[-(a/c)]*e)^2*(d + e*x)*Gamma[1 - p]*Gamma[-2*p]*H
```

```
ypergeometricPFQ[{2, 2, 1 - p}, {1, 1 - 2*p}, (2*Sqrt[-(a/c)]*(d + e*x))/((
d + Sqrt[-(a/c)]*e)*(Sqrt[-(a/c)] + x))] - 2*Sqrt[-(a/c)]*(d + Sqrt[-(a/c)]
*e)*(d + e*x)^2*Gamma[1 - p]*Gamma[-2*p]*HypergeometricPFQ[{2, 2, 1 - p}, {
1, 1 - 2*p}, (2*Sqrt[-(a/c)]*(d + e*x))/((d + Sqrt[-(a/c)]*e)*(Sqrt[-(a/c)]
 + x))] + Sqrt[-(a/c)]*(d + e*x)^3*Gamma[1 - p]*Gamma[-2*p]*HypergeometricP
FQ[{2, 2, 1 - p}, {1, 1 - 2*p}, (2*Sqrt[-(a/c)]*(d + e*x))/((d + Sqrt[-(a/c
)]*e)*(Sqrt[-(a/c)] + x))]))/(e*(d + Sqrt[-(a/c)]*e)^3*(3 + 2*p)*((e*(Sqrt[
-(a/c)] - x))/(d + Sqrt[-(a/c)]*e))^p*(Sqrt[-(a/c)] + x)*Gamma[1 - 2*p]*Gam
ma[-2*p]*Gamma[-p])
```

Maple [F] time = 0.594, size = 0, normalized size = 0.

$$\int (ex+d)^{-4-2p} \left(cx^2+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(-4-2*p)*(c*x^2+a)^p,x)

[Out] int((e*x+d)^(-4-2*p)*(c*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+a\right)^p (ex+d)^{-2p-4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-4-2*p)*(c*x^2+a)^p,x, algorithm="maxima")

[Out] integrate((c*x^2 + a)^p*(e*x + d)^(-2*p - 4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2+a\right)^p (ex+d)^{-2p-4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-4-2*p)*(c*x^2+a)^p,x, algorithm="fricas")

[Out] integral((c*x^2 + a)^p*(e*x + d)^(-2*p - 4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(-4-2*p)*(c*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + a\right)^p (ex + d)^{-2p-4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-4-2*p)*(c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^p*(e*x + d)^(-2*p - 4), x)
```

3.744 $\int (d+ex)^{-5-2p}\left(a+cx^2\right)^p\,dx$

Optimal. Leaf size=436

$$\frac{c^2d\left(\sqrt{-a}-\sqrt{c}x\right)\left(a+cx^2\right)^p(d+ex)^{-2p-1}\left(3ae^2-cd^2(2p+3)\right)\left(-\frac{\left(\sqrt{-a}+\sqrt{c}x\right)\left(\sqrt{-a}e+\sqrt{c}d\right)}{\left(\sqrt{-a}-\sqrt{c}x\right)\left(\sqrt{c}d-\sqrt{-a}e\right)}\right)^{-p}{}_2F_1\left(-2p-1,-p;-2p;\frac{2\sqrt{}}{\left(\sqrt{c}d-\sqrt{}\right.}\right.}{(2p+1)(2p+3)\left(\sqrt{-a}e+\sqrt{c}d\right)\left(ae^2+cd^2\right)^3}$$

```
[Out] -((c*d*e*(3 + p)*(d + e*x)^(-3 - 2*p)*(a + c*x^2)^(1 + p))/((c*d^2 + a*e^2)
^2*(2 + p)*(3 + 2*p))) + (c*e*(a*e^2*(3 + 2*p) - c*d^2*(9 + 8*p + 2*p^2))*(
a + c*x^2)^(1 + p))/(2*(c*d^2 + a*e^2)^3*(1 + p)*(2 + p)*(3 + 2*p)*(d + e*x
)^(2*(1 + p))) - (e*(a + c*x^2)^(1 + p))/(2*(c*d^2 + a*e^2)*(2 + p)*(d + e*
x)^(2*(2 + p))) + (c^2*d*(3*a*e^2 - c*d^2*(3 + 2*p))*(Sqrt[-a] - Sqrt[c]*x)
*(d + e*x)^(-1 - 2*p)*(a + c*x^2)^p*Hypergeometric2F1[-1 - 2*p, -p, -2*p, (
2*Sqrt[-a]*Sqrt[c]*(d + e*x))/((Sqrt[c]*d - Sqrt[-a]*e)*(Sqrt[-a] - Sqrt[c]
*x))])/((Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*e^2)^3*(1 + 2*p)*(3 + 2*p)*(-((
(Sqrt[c]*d + Sqrt[-a]*e)*(Sqrt[-a] + Sqrt[c]*x))/((Sqrt[c]*d - Sqrt[-a]*e)*
(Sqrt[-a] - Sqrt[c]*x))))^p)
```

Rubi [A] time = 0.454356, antiderivative size = 436, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {745, 837, 807, 727}

$$\frac{c^2d\left(\sqrt{-a}-\sqrt{c}x\right)\left(a+cx^2\right)^p(d+ex)^{-2p-1}\left(3ae^2-cd^2(2p+3)\right)\left(-\frac{\left(\sqrt{-a}+\sqrt{c}x\right)\left(\sqrt{-a}e+\sqrt{c}d\right)}{\left(\sqrt{-a}-\sqrt{c}x\right)\left(\sqrt{c}d-\sqrt{-a}e\right)}\right)^{-p}{}_2F_1\left(-2p-1,-p;-2p;\frac{2\sqrt{}}{\left(\sqrt{c}d-\sqrt{}\right.}\right.}{(2p+1)(2p+3)\left(\sqrt{-a}e+\sqrt{c}d\right)\left(ae^2+cd^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(-5 - 2*p)*(a + c*x^2)^p,x]
```

```
[Out] -((c*d*e*(3 + p)*(d + e*x)^(-3 - 2*p)*(a + c*x^2)^(1 + p))/((c*d^2 + a*e^2)
^2*(2 + p)*(3 + 2*p))) + (c*e*(a*e^2*(3 + 2*p) - c*d^2*(9 + 8*p + 2*p^2))*(
a + c*x^2)^(1 + p))/(2*(c*d^2 + a*e^2)^3*(1 + p)*(2 + p)*(3 + 2*p)*(d + e*x
)^(2*(1 + p))) - (e*(a + c*x^2)^(1 + p))/(2*(c*d^2 + a*e^2)*(2 + p)*(d + e*
x)^(2*(2 + p))) + (c^2*d*(3*a*e^2 - c*d^2*(3 + 2*p))*(Sqrt[-a] - Sqrt[c]*x)
*(d + e*x)^(-1 - 2*p)*(a + c*x^2)^p*Hypergeometric2F1[-1 - 2*p, -p, -2*p, (
2*Sqrt[-a]*Sqrt[c]*(d + e*x))/((Sqrt[c]*d - Sqrt[-a]*e)*(Sqrt[-a] - Sqrt[c]
*x))])/((Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*e^2)^3*(1 + 2*p)*(3 + 2*p)*(-((
(Sqrt[c]*d + Sqrt[-a]*e)*(Sqrt[-a] + Sqrt[c]*x))/((Sqrt[c]*d - Sqrt[-a]*e)*
(Sqrt[-a] - Sqrt[c]*x))))^p)
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
```

```
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2
 + a*e^2, 0] && ILtQ[Simplify[m + 2*p + 3], 0] && NeQ[m, -1]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 727

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((Rt[-(a*c), 2] - c*x)*(d + e*x)^(m + 1)*(a + c*x^2)^p*Hypergeometric2F1[m
+ 1, -p, m + 2, (2*c*Rt[-(a*c), 2]*(d + e*x))/((c*d - e*Rt[-(a*c), 2])*(Rt[
-(a*c), 2] - c*x))])/((m + 1)*(c*d + e*Rt[-(a*c), 2])*(((c*d + e*Rt[-(a*c),
 2])*(Rt[-(a*c), 2] + c*x))/((c*d - e*Rt[-(a*c), 2])*(-Rt[-(a*c), 2] + c*x)
))^p), x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Int
egerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int (d+ex)^{-5-2p}\left(a+cx^2\right)^p\,dx = -\frac{e(d+ex)^{-2(2+p)}\left(a+cx^2\right)^{1+p}}{2\left(cd^2+ae^2\right)(2+p)} - \frac{c\int (d+ex)^{-4-2p}(-2d(2+p)+2ex)\left(a+cx^2\right)^p\,dx}{2\left(cd^2+ae^2\right)(2+p)}$$

$$= -\frac{cde(3+p)(d+ex)^{-3-2p}\left(a+cx^2\right)^{1+p}}{\left(cd^2+ae^2\right)^2(2+p)(3+2p)} - \frac{e(d+ex)^{-2(2+p)}\left(a+cx^2\right)^{1+p}}{2\left(cd^2+ae^2\right)(2+p)} + \frac{c\int (d+ex)^{-3-2p}}{}$$

$$= -\frac{cde(3+p)(d+ex)^{-3-2p}\left(a+cx^2\right)^{1+p}}{\left(cd^2+ae^2\right)^2(2+p)(3+2p)} + \frac{ce\left(ae^2(3+2p)-cd^2\left(9+8p+2p^2\right)\right)(d+ex)^{-2(}}{2\left(cd^2+ae^2\right)^3(1+p)(2+p)(3+2}$$

$$= -\frac{cde(3+p)(d+ex)^{-3-2p}\left(a+cx^2\right)^{1+p}}{\left(cd^2+ae^2\right)^2(2+p)(3+2p)} + \frac{ce\left(ae^2(3+2p)-cd^2\left(9+8p+2p^2\right)\right)(d+ex)^{-2(}}{2\left(cd^2+ae^2\right)^3(1+p)(2+p)(3+2}$$

Mathematica [B] time = 98.1975, size = 2500, normalized size = 5.73

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^(-5 - 2*p)*(a + c*x^2)^p,x]
```

```
[Out] -((a + c*x^2)^p*(1 - (d + e*x)/(d + Sqrt[-(a/c)]*e))^(1 + p)*(6*(d + Sqrt[-
(a/c)]*e)^4*p*(Sqrt[-(a/c)] + x)*Gamma[-p]*Gamma[-2*(1 + p)]*Hypergeometric
2F1[1, -p, -3 - 2*p, (2*Sqrt[-(a/c)]*(d + e*x))/((d + Sqrt[-(a/c)]*e)*(Sqrt
[-(a/c)] + x))] + 22*(d + Sqrt[-(a/c)]*e)^4*p^2*(Sqrt[-(a/c)] + x)*Gamma[-p
]*Gamma[-2*(1 + p)]*Hypergeometric2F1[1, -p, -3 - 2*p, (2*Sqrt[-(a/c)]*(d +
 e*x))/((d + Sqrt[-(a/c)]*e)*(Sqrt[-(a/c)] + x))] + 24*(d + Sqrt[-(a/c)]*e)
^4*p^3*(Sqrt[-(a/c)] + x)*Gamma[-p]*Gamma[-2*(1 + p)]*Hypergeometric2F1[1,
-p, -3 - 2*p, (2*Sqrt[-(a/c)]*(d + e*x))/((d + Sqrt[-(a/c)]*e)*(Sqrt[-(a/c)
] + x))] + 8*(d + Sqrt[-(a/c)]*e)^4*p^4*(Sqrt[-(a/c)] + x)*Gamma[-p]*Gamma[
```

```
-2*(1 + p)]*Hypergeometric2F1[1, -p, -3 - 2*p, (2*Sqrt[-(a/c)]*(d + e*x))/(
(d + Sqrt[-(a/c)]*e)*(Sqrt[-(a/c)] + x))] + 6*(d + Sqrt[-(a/c)]*e)^2*p*(Sqr
t[-(a/c)] + x)*(d + e*x)^2*Gamma[-p]*Gamma[-2*(1 + p)]*Hypergeometric2F1[1,
 -p, -1 - 2*p, (2*Sqrt[-(a/c)]*(d + e*x))/((d + Sqrt[-(a/c)]*e)*(Sqrt[-(a/c
)] + x))] + 12*(d + Sqrt[-(a/c)]*e)^2*p^2*(Sqrt[-(a/c)] + x)*(d + e*x)^2*Ga
mma[-p]*Gamma[-2*(1 + p)]*Hypergeometric2F1[1, -p, -1 - 2*p, (2*Sqrt[-(a/c)
]*(d + e*x))/((d + Sqrt[-(a/c)]*e)*(Sqrt[-(a/c)] + x))] + 6*(d + Sqrt[-(a/c
)]*e)*p*(Sqrt[-(a/c)] + x)*(d + e*x)^3*Gamma[-p]*Gamma[-2*(1 + p)]*Hypergeo
metric2F1[1, -p, -2*p, (2*Sqrt[-(a/c)]*(d + e*x))/((d + Sqrt[-(a/c)]*e)*(Sq
rt[-(a/c)] + x))] + 6*(d + Sqrt[-(a/c)]*e)^3*p*(Sqrt[-(a/c)] + x)*(d + e*x)
*Gamma[-p]*Gamma[-2*(1 + p)]*Hypergeometric2F1[1, -p, -2*(1 + p), (2*Sqrt[-
(a/c)]*(d + e*x))/((d + Sqrt[-(a/c)]*e)*(Sqrt[-(a/c)] + x))] + 18*(d + Sqrt
[-(a/c)]*e)^3*p^2*(Sqrt[-(a/c)] + x)*(d + e*x)*Gamma[-p]*Gamma[-2*(1 + p)]*
Hypergeometric2F1[1, -p, -2*(1 + p), (2*Sqrt[-(a/c)]*(d + e*x))/((d + Sqrt[
-(a/c)]*e)*(Sqrt[-(a/c)] + x))] + 12*(d + Sqrt[-(a/c)]*e)^3*p^3*(Sqrt[-(a/c
)] + x)*(d + e*x)*Gamma[-p]*Gamma[-2*(1 + p)]*Hypergeometric2F1[1, -p, -2*(
1 + p), (2*Sqrt[-(a/c)]*(d + e*x))/((d + Sqrt[-(a/c)]*e)*(Sqrt[-(a/c)] + x)
)] + 18*Sqrt[-(a/c)]*(d + Sqrt[-(a/c)]*e)^2*p*(d + e*x)^2*Gamma[-3 - 2*p]*G
amma[1 - p]*Hypergeometric2F1[2, 1 - p, -1 - 2*p, (2*Sqrt[-(a/c)]*(d + e*x)
)/((d + Sqrt[-(a/c)]*e)*(Sqrt[-(a/c)] + x))] + 48*Sqrt[-(a/c)]*(d + Sqrt[-(
a/c)]*e)^2*p^2*(d + e*x)^2*Gamma[-3 - 2*p]*Gamma[1 - p]*Hypergeometric2F1[2
, 1 - p, -1 - 2*p, (2*Sqrt[-(a/c)]*(d + e*x))/((d + Sqrt[-(a/c)]*e)*(Sqrt[-
(a/c)] + x))] + 24*Sqrt[-(a/c)]*(d + Sqrt[-(a/c)]*e)^2*p^3*(d + e*x)^2*Gamm
a[-3 - 2*p]*Gamma[1 - p]*Hypergeometric2F1[2, 1 - p, -1 - 2*p, (2*Sqrt[-(a/
c)]*(d + e*x))/((d + Sqrt[-(a/c)]*e)*(Sqrt[-(a/c)] + x))] + 33*Sqrt[-(a/c)]
*(d + e*x)^4*Gamma[-3 - 2*p]*Gamma[1 - p]*Hypergeometric2F1[2, 1 - p, 1 - 2
*p, (2*Sqrt[-(a/c)]*(d + e*x))/((d + Sqrt[-(a/c)]*e)*(Sqrt[-(a/c)] + x))] +
 22*Sqrt[-(a/c)]*p*(d + e*x)^4*Gamma[-3 - 2*p]*Gamma[1 - p]*Hypergeometric2
F1[2, 1 - p, 1 - 2*p, (2*Sqrt[-(a/c)]*(d + e*x))/((d + Sqrt[-(a/c)]*e)*(Sqr
t[-(a/c)] + x))] + 54*Sqrt[-(a/c)]*(d + Sqrt[-(a/c)]*e)*p*(d + e*x)^3*Gamma
[-3 - 2*p]*Gamma[1 - p]*Hypergeometric2F1[2, 1 - p, -2*p, (2*Sqrt[-(a/c)]*(
d + e*x))/((d + Sqrt[-(a/c)]*e)*(Sqrt[-(a/c)] + x))] + 36*Sqrt[-(a/c)]*(d +
 Sqrt[-(a/c)]*e)*p^2*(d + e*x)^3*Gamma[-3 - 2*p]*Gamma[1 - p]*Hypergeometri
c2F1[2, 1 - p, -2*p, (2*Sqrt[-(a/c)]*(d + e*x))/((d + Sqrt[-(a/c)]*e)*(Sqrt
[-(a/c)] + x))] + 18*Sqrt[-(a/c)]*(d + e*x)^4*Gamma[-3 - 2*p]*Gamma[1 - p]*
HypergeometricPFQ[{2, 2, 1 - p}, {1, 1 - 2*p}, (2*Sqrt[-(a/c)]*(d + e*x))/(
(d + Sqrt[-(a/c)]*e)*(Sqrt[-(a/c)] + x))] + 12*Sqrt[-(a/c)]*p*(d + e*x)^4*G
amma[-3 - 2*p]*Gamma[1 - p]*HypergeometricPFQ[{2, 2, 1 - p}, {1, 1 - 2*p},
(2*Sqrt[-(a/c)]*(d + e*x))/((d + Sqrt[-(a/c)]*e)*(Sqrt[-(a/c)] + x))] + 18*
Sqrt[-(a/c)]*(d + Sqrt[-(a/c)]*e)*p*(d + e*x)^3*Gamma[-3 - 2*p]*Gamma[1 - p
]*HypergeometricPFQ[{2, 2, 1 - p}, {1, -2*p}, (2*Sqrt[-(a/c)]*(d + e*x))/((
d + Sqrt[-(a/c)]*e)*(Sqrt[-(a/c)] + x))] + 12*Sqrt[-(a/c)]*(d + Sqrt[-(a/c)
]*e)*p^2*(d + e*x)^3*Gamma[-3 - 2*p]*Gamma[1 - p]*HypergeometricPFQ[{2, 2,
1 - p}, {1, -2*p}, (2*Sqrt[-(a/c)]*(d + e*x))/((d + Sqrt[-(a/c)]*e)*(Sqrt[-
(a/c)] + x))] + 3*Sqrt[-(a/c)]*(d + e*x)^4*Gamma[-3 - 2*p]*Gamma[1 - p]*Hyp
ergeometricPFQ[{2, 2, 2, 1 - p}, {1, 1, 1 - 2*p}, (2*Sqrt[-(a/c)]*(d + e*x)
)/((d + Sqrt[-(a/c)]*e)*(Sqrt[-(a/c)] + x))] + 2*Sqrt[-(a/c)]*p*(d + e*x)^4
*Gamma[-3 - 2*p]*Gamma[1 - p]*HypergeometricPFQ[{2, 2, 2, 1 - p}, {1, 1, 1
- 2*p}, (2*Sqrt[-(a/c)]*(d + e*x))/((d + Sqrt[-(a/c)]*e)*(Sqrt[-(a/c)] + x)
)]))/(4*e*(d + Sqrt[-(a/c)]*e)^4*p*(1 + p)*(2 + p)*(1 + 2*p)*(3 + 2*p)*((e*
(Sqrt[-(a/c)] - x))/(d + Sqrt[-(a/c)]*e))^p*(Sqrt[-(a/c)] + x)*(d + e*x)^(2
*(2 + p))*Gamma[-p]*Gamma[-2*(1 + p)])
```

Maple [F] time = 0.597, size = 0, normalized size = 0.

$$\int (ex+d)^{-5-2p} \left(cx^2+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(-5-2*p)*(c*x^2+a)^p,x)
```

```
[Out] int((e*x+d)^(-5-2*p)*(c*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + a\right)^p (ex + d)^{-2p-5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-5-2*p)*(c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^p*(e*x + d)^(-2*p - 5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2 + a\right)^p (ex + d)^{-2p-5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-5-2*p)*(c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + a)^p*(e*x + d)^(-2*p - 5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(-5-2*p)*(c*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + a\right)^p (ex + d)^{-2p-5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-5-2*p)*(c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^p*(e*x + d)^(-2*p - 5), x)
```

3.745 $\int (d+ex)^{-6-2p}\left(a+cx^2\right)^p\,dx$

Optimal. Leaf size=559

$$-\frac{c^2\left(\sqrt{-a}-\sqrt{c}x\right)\left(a+cx^2\right)^p(d+ex)^{-2p-1}\left(3a^2e^4-6acd^2e^2(2p+5)+c^2d^4\left(4p^2+16p+15\right)\right)\left(-\frac{\left(\sqrt{-a}+\sqrt{c}x\right)\left(\sqrt{-a}e+\sqrt{c}d\right)}{\left(\sqrt{-a}-\sqrt{c}x\right)\left(\sqrt{c}d-\sqrt{-a}e\right)}\right)^{-p}}{(2p+1)(2p+3)(2p+5)\left(\sqrt{-a}e+\sqrt{c}d\right)\left(ae^2+cd^2\right)^4}$$

```
[Out] -((e*(d + e*x)^(-5 - 2*p)*(a + c*x^2)^(1 + p))/((c*d^2 + a*e^2)*(5 + 2*p)))
 + (c*e*(3*a*e^2*(2 + p) - c*d^2*(18 + 11*p + 2*p^2))*(d + e*x)^(-3 - 2*p)*
(a + c*x^2)^(1 + p))/((c*d^2 + a*e^2)^3*(2 + p)*(3 + 2*p)*(5 + 2*p)) + (c^2
*d*e*(3 + p)*(a*e^2*(8 + 5*p) - c*d^2*(8 + 7*p + 2*p^2))*(a + c*x^2)^(1 + p
))/((c*d^2 + a*e^2)^4*(1 + p)*(2 + p)*(3 + 2*p)*(5 + 2*p)*(d + e*x)^(2*(1 +
 p))) - (c*d*e*(4 + p)*(a + c*x^2)^(1 + p))/((c*d^2 + a*e^2)^2*(2 + p)*(5 +
 2*p)*(d + e*x)^(2*(2 + p))) - (c^2*(3*a^2*e^4 - 6*a*c*d^2*e^2*(5 + 2*p) +
c^2*d^4*(15 + 16*p + 4*p^2))*(Sqrt[-a] - Sqrt[c]*x)*(d + e*x)^(-1 - 2*p)*(a
 + c*x^2)^p*Hypergeometric2F1[-1 - 2*p, -p, -2*p, (2*Sqrt[-a]*Sqrt[c]*(d +
e*x))/((Sqrt[c]*d - Sqrt[-a]*e)*(Sqrt[-a] - Sqrt[c]*x))])/((Sqrt[c]*d + Sqr
t[-a]*e)*(c*d^2 + a*e^2)^4*(1 + 2*p)*(3 + 2*p)*(5 + 2*p)*(-(((Sqrt[c]*d + S
qrt[-a]*e)*(Sqrt[-a] + Sqrt[c]*x))/((Sqrt[c]*d - Sqrt[-a]*e)*(Sqrt[-a] - Sq
rt[c]*x))))^p)
```

Rubi [A] time = 0.731034, antiderivative size = 559, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {745, 837, 807, 727}

$$-\frac{c^2\left(\sqrt{-a}-\sqrt{c}x\right)\left(a+cx^2\right)^p(d+ex)^{-2p-1}\left(3a^2e^4-6acd^2e^2(2p+5)+c^2d^4\left(4p^2+16p+15\right)\right)\left(-\frac{\left(\sqrt{-a}+\sqrt{c}x\right)\left(\sqrt{-a}e+\sqrt{c}d\right)}{\left(\sqrt{-a}-\sqrt{c}x\right)\left(\sqrt{c}d-\sqrt{-a}e\right)}\right)^{-p}}{(2p+1)(2p+3)(2p+5)\left(\sqrt{-a}e+\sqrt{c}d\right)\left(ae^2+cd^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(-6 - 2*p)*(a + c*x^2)^p,x]
```


```
[Out] -((e*(d + e*x)^(-5 - 2*p)*(a + c*x^2)^(1 + p))/((c*d^2 + a*e^2)*(5 + 2*p)))
 + (c*e*(3*a*e^2*(2 + p) - c*d^2*(18 + 11*p + 2*p^2))*(d + e*x)^(-3 - 2*p)*
(a + c*x^2)^(1 + p))/((c*d^2 + a*e^2)^3*(2 + p)*(3 + 2*p)*(5 + 2*p)) + (c^2
*d*e*(3 + p)*(a*e^2*(8 + 5*p) - c*d^2*(8 + 7*p + 2*p^2))*(a + c*x^2)^(1 + p
))/((c*d^2 + a*e^2)^4*(1 + p)*(2 + p)*(3 + 2*p)*(5 + 2*p)*(d + e*x)^(2*(1 +
 p))) - (c*d*e*(4 + p)*(a + c*x^2)^(1 + p))/((c*d^2 + a*e^2)^2*(2 + p)*(5 +
 2*p)*(d + e*x)^(2*(2 + p))) - (c^2*(3*a^2*e^4 - 6*a*c*d^2*e^2*(5 + 2*p) +
c^2*d^4*(15 + 16*p + 4*p^2))*(Sqrt[-a] - Sqrt[c]*x)*(d + e*x)^(-1 - 2*p)*(a
 + c*x^2)^p*Hypergeometric2F1[-1 - 2*p, -p, -2*p, (2*Sqrt[-a]*Sqrt[c]*(d +
e*x))/((Sqrt[c]*d - Sqrt[-a]*e)*(Sqrt[-a] - Sqrt[c]*x))])/((Sqrt[c]*d + Sqr
t[-a]*e)*(c*d^2 + a*e^2)^4*(1 + 2*p)*(3 + 2*p)*(5 + 2*p)*(-(((Sqrt[c]*d + S
qrt[-a]*e)*(Sqrt[-a] + Sqrt[c]*x))/((Sqrt[c]*d - Sqrt[-a]*e)*(Sqrt[-a] - Sq
rt[c]*x))))^p)
```


Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
```

```
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2
 + a*e^2, 0] && ILtQ[Simplify[m + 2*p + 3], 0] && NeQ[m, -1]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 727

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((Rt[-(a*c), 2] - c*x)*(d + e*x)^(m + 1)*(a + c*x^2)^p*Hypergeometric2F1[m
+ 1, -p, m + 2, (2*c*Rt[-(a*c), 2]*(d + e*x))/((c*d - e*Rt[-(a*c), 2])*(Rt[
-(a*c), 2] - c*x))])/((m + 1)*(c*d + e*Rt[-(a*c), 2])*(((c*d + e*Rt[-(a*c),
 2])*(Rt[-(a*c), 2] + c*x))/((c*d - e*Rt[-(a*c), 2])*(-Rt[-(a*c), 2] + c*x)
))^p), x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Int
egerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^{-6-2p}\left(a+cx^2\right)^p\,dx &= -\frac{e(d+ex)^{-5-2p}\left(a+cx^2\right)^{1+p}}{\left(cd^2+ae^2\right)(5+2p)}-\frac{c\int (d+ex)^{-5-2p}(-d(5+2p)+3ex)\left(a+cx^2\right)^p\,dx}{\left(cd^2+ae^2\right)(5+2p)}\\
&= -\frac{e(d+ex)^{-5-2p}\left(a+cx^2\right)^{1+p}}{\left(cd^2+ae^2\right)(5+2p)}-\frac{cde(4+p)(d+ex)^{-2(2+p)}\left(a+cx^2\right)^{1+p}}{\left(cd^2+ae^2\right)^2(2+p)(5+2p)}+\frac{c\int (d+ex)^{-4-2p}\cdots}{\cdots}\\
&= -\frac{e(d+ex)^{-5-2p}\left(a+cx^2\right)^{1+p}}{\left(cd^2+ae^2\right)(5+2p)}+\frac{ce\left(3ae^2(2+p)-cd^2\left(18+11p+2p^2\right)\right)(d+ex)^{-3-2p}\left(a+\cdots\right.}{\left(cd^2+ae^2\right)^3(2+p)(3+2p)(5+2p)}\\
&= -\frac{e(d+ex)^{-5-2p}\left(a+cx^2\right)^{1+p}}{\left(cd^2+ae^2\right)(5+2p)}+\frac{ce\left(3ae^2(2+p)-cd^2\left(18+11p+2p^2\right)\right)(d+ex)^{-3-2p}\left(a+\cdots\right.}{\left(cd^2+ae^2\right)^3(2+p)(3+2p)(5+2p)}\\
&= -\frac{e(d+ex)^{-5-2p}\left(a+cx^2\right)^{1+p}}{\left(cd^2+ae^2\right)(5+2p)}+\frac{ce\left(3ae^2(2+p)-cd^2\left(18+11p+2p^2\right)\right)(d+ex)^{-3-2p}\left(a+\cdots\right.}{\left(cd^2+ae^2\right)^3(2+p)(3+2p)(5+2p)}
\end{aligned}$$

Mathematica [B] time = 53.7932, size = 5685, normalized size = 10.17

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^(-6 - 2*p)*(a + c*x^2)^p,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.599, size = 0, normalized size = 0.

$$\int (ex+d)^{-6-2p} \left(cx^2+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(-6-2*p)*(c*x^2+a)^p,x)
```

```
[Out] int((e*x+d)^(-6-2*p)*(c*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+a\right)^p (ex+d)^{-2p-6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-6-2*p)*(c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^p*(e*x + d)^(-2*p - 6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2+a\right)^p (ex+d)^{-2p-6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-6-2*p)*(c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + a)^p*(e*x + d)^(-2*p - 6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(-6-2*p)*(c*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+a\right)^p (ex+d)^{-2p-6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-6-2*p)*(c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^p*(e*x + d)^(-2*p - 6), x)
```

3.746 $\int \frac{(3-4x)^n}{\sqrt{1-x^2}} dx$

Optimal. Leaf size=43

$$\sqrt{2}7^n\sqrt{x+1}F_1\left(\frac{1}{2};-n,\frac{1}{2};\frac{3}{2};\frac{4(x+1)}{7},\frac{x+1}{2}\right)$$

```
[Out] Sqrt[2]*7^n*Sqrt[1 + x]*AppellF1[1/2, -n, 1/2, 3/2, (4*(1 + x))/7, (1 + x)/
2]
```

Rubi [A] time = 0.0209597, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {756, 138}

$$\sqrt{2}7^n\sqrt{x+1}F_1\left(\frac{1}{2};-n,\frac{1}{2};\frac{3}{2};\frac{4(x+1)}{7},\frac{x+1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 4*x)^n/Sqrt[1 - x^2],x]
```

```
[Out] Sqrt[2]*7^n*Sqrt[1 + x]*AppellF1[1/2, -n, 1/2, 3/2, (4*(1 + x))/7, (1 + x)/
2]
```

Rule 756

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[(
d + e*x)^m*(Rt[a, 2] + Rt[-c, 2]*x)^p*(Rt[a, 2] - Rt[-c, 2]*x)^p, x] /; Fre
eQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && GtQ[
a, 0] && LtQ[c, 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\begin{aligned} \int \frac{(3-4x)^n}{\sqrt{1-x^2}} dx &= \int \frac{(3-4x)^n}{\sqrt{1-x}\sqrt{1+x}} dx \\ &= \sqrt{2}7^n\sqrt{1+x}F_1\left(\frac{1}{2};-n,\frac{1}{2};\frac{3}{2};\frac{4(1+x)}{7},\frac{1+x}{2}\right) \end{aligned}$$

Mathematica [A] time = 0.0320823, size = 48, normalized size = 1.12

$$-\frac{(3-4x)^{n+1}F_1\left(n+1;\frac{1}{2},\frac{1}{2};n+2;\frac{1}{7}(3-4x),4x-3\right)}{\sqrt{7}(n+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - 4*x)^n/Sqrt[1 - x^2],x]
```

```
[Out] -(((3 - 4*x)^(1 + n)*AppellF1[1 + n, 1/2, 1/2, 2 + n, (3 - 4*x)/7, -3 + 4*x
])/(Sqrt[7]*(1 + n)))
```

Maple [F] time = 0.415, size = 0, normalized size = 0.

$$\int (3-4x)^n \frac{1}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-4*x)^n/(-x^2+1)^(1/2),x)
```

```
[Out] int((3-4*x)^n/(-x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-4x+3)^n}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*x)^n/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-4*x + 3)^n/sqrt(-x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^2+1}(-4x+3)^n}{x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*x)^n/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^2 + 1)*(-4*x + 3)^n/(x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3-4x)^n}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*x)**n/(-x**2+1)**(1/2),x)
```

```
[Out] Integral((3 - 4*x)**n/sqrt(-(x - 1)*(x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-4x+3)^n}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*x)^n/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-4*x + 3)^n/sqrt(-x^2 + 1), x)
```

3.747 $\int \frac{(a+bx)^6}{a^2-b^2x^2}\,dx$

Optimal. Leaf size=83

$$-\frac{4a^3(a+bx)^2}{b}-\frac{4a^2(a+bx)^3}{3b}-\frac{32a^5\log(a-bx)}{b}-16a^4x-\frac{a(a+bx)^4}{2b}-\frac{(a+bx)^5}{5b}$$

```
[Out] -16*a^4*x - (4*a^3*(a + b*x)^2)/b - (4*a^2*(a + b*x)^3)/(3*b) - (a*(a + b*x
)^4)/(2*b) - (a + b*x)^5/(5*b) - (32*a^5*Log[a - b*x])/b
```

Rubi [A] time = 0.0319296, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {627, 43}

$$-\frac{4a^3(a+bx)^2}{b}-\frac{4a^2(a+bx)^3}{3b}-\frac{32a^5\log(a-bx)}{b}-16a^4x-\frac{a(a+bx)^4}{2b}-\frac{(a+bx)^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^6/(a^2 - b^2*x^2),x]
```

```
[Out] -16*a^4*x - (4*a^3*(a + b*x)^2)/b - (4*a^2*(a + b*x)^3)/(3*b) - (a*(a + b*x
)^4)/(2*b) - (a + b*x)^5/(5*b) - (32*a^5*Log[a - b*x])/b
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^6}{a^2-b^2x^2}\,dx &= \int \frac{(a+bx)^5}{a-bx}\,dx\\ &= \int \left(-16a^4+\frac{32a^5}{a-bx}-8a^3(a+bx)-4a^2(a+bx)^2-2a(a+bx)^3-(a+bx)^4\right)dx\\ &= -16a^4x-\frac{4a^3(a+bx)^2}{b}-\frac{4a^2(a+bx)^3}{3b}-\frac{a(a+bx)^4}{2b}-\frac{(a+bx)^5}{5b}-\frac{32a^5\log(a-bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0074237, size = 65, normalized size = 0.78

$$-\frac{16}{3}a^2b^2x^3-13a^3bx^2-\frac{32a^5\log(a-bx)}{b}-31a^4x-\frac{3}{2}ab^3x^4-\frac{b^4x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^6/(a^2 - b^2*x^2),x]
```

```
[Out] -31*a^4*x - 13*a^3*b*x^2 - (16*a^2*b^2*x^3)/3 - (3*a*b^3*x^4)/2 - (b^4*x^5)
/5 - (32*a^5*Log[a - b*x])/b
```

Maple [A] time = 0.041, size = 61, normalized size = 0.7

$$-\frac{b^4x^5}{5}-\frac{3\,ab^3x^4}{2}-\frac{16\,b^2a^2x^3}{3}-13\,a^3bx^2-31\,a^4x-32\,\frac{a^5\ln(bx-a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^6/(-b^2*x^2+a^2),x)
```

```
[Out] -1/5*b^4*x^5-3/2*a*b^3*x^4-16/3*b^2*a^2*x^3-13*a^3*b*x^2-31*a^4*x-32*a^5/b*
ln(b*x-a)
```

Maxima [A] time = 1.2065, size = 81, normalized size = 0.98

$$-\frac{1}{5}\,b^4x^5-\frac{3}{2}\,ab^3x^4-\frac{16}{3}\,a^2b^2x^3-13\,a^3bx^2-31\,a^4x-\frac{32\,a^5\log(bx-a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(-b^2*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/5*b^4*x^5 - 3/2*a*b^3*x^4 - 16/3*a^2*b^2*x^3 - 13*a^3*b*x^2 - 31*a^4*x -
 32*a^5*log(b*x - a)/b
```

Fricas [A] time = 1.94812, size = 147, normalized size = 1.77

$$-\frac{6\,b^5x^5+45\,ab^4x^4+160\,a^2b^3x^3+390\,a^3b^2x^2+930\,a^4bx+960\,a^5\log(bx-a)}{30\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(-b^2*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/30*(6*b^5*x^5 + 45*a*b^4*x^4 + 160*a^2*b^3*x^3 + 390*a^3*b^2*x^2 + 930*a
^4*b*x + 960*a^5*log(b*x - a))/b
```

Sympy [A] time = 0.350551, size = 65, normalized size = 0.78

$$-\frac{32a^5\log(-a+bx)}{b}-31a^4x-13a^3bx^2-\frac{16a^2b^2x^3}{3}-\frac{3ab^3x^4}{2}-\frac{b^4x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**6/(-b**2*x**2+a**2),x)
```

```
[Out] -32*a**5*log(-a + b*x)/b - 31*a**4*x - 13*a**3*b*x**2 - 16*a**2*b**2*x**3/3
 - 3*a*b**3*x**4/2 - b**4*x**5/5
```

Giac [A] time = 1.17242, size = 97, normalized size = 1.17

$$-\frac{32\,a^5\log\left(|bx-a|\right)}{b}-\frac{6\,b^9x^5+45\,ab^8x^4+160\,a^2b^7x^3+390\,a^3b^6x^2+930\,a^4b^5x}{30\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(-b^2*x^2+a^2),x, algorithm="giac")
```

```
[Out] -32*a^5*log(abs(b*x - a))/b - 1/30*(6*b^9*x^5 + 45*a*b^8*x^4 + 160*a^2*b^7*
x^3 + 390*a^3*b^6*x^2 + 930*a^4*b^5*x)/b^5
```

3.748 $\int \frac{(a+bx)^5}{a^2-b^2x^2} dx$

Optimal. Leaf size=66

$$-\frac{2a^2(a+bx)^2}{b}-\frac{16a^4\log(a-bx)}{b}-8a^3x-\frac{2a(a+bx)^3}{3b}-\frac{(a+bx)^4}{4b}$$

```
[Out] -8*a^3*x - (2*a^2*(a + b*x)^2)/b - (2*a*(a + b*x)^3)/(3*b) - (a + b*x)^4/(4
*b) - (16*a^4*Log[a - b*x])/b
```

Rubi [A] time = 0.0253941, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {627, 43}

$$-\frac{2a^2(a+bx)^2}{b}-\frac{16a^4\log(a-bx)}{b}-8a^3x-\frac{2a(a+bx)^3}{3b}-\frac{(a+bx)^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a^2 - b^2*x^2),x]
```

```
[Out] -8*a^3*x - (2*a^2*(a + b*x)^2)/b - (2*a*(a + b*x)^3)/(3*b) - (a + b*x)^4/(4
*b) - (16*a^4*Log[a - b*x])/b
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^5}{a^2-b^2x^2}\,dx &= \int \frac{(a+bx)^4}{a-bx}\,dx \\
&= \int \left(-8a^3+\frac{16a^4}{a-bx}-4a^2(a+bx)-2a(a+bx)^2-(a+bx)^3\right)dx \\
&= -8a^3x-\frac{2a^2(a+bx)^2}{b}-\frac{2a(a+bx)^3}{3b}-\frac{(a+bx)^4}{4b}-\frac{16a^4\log(a-bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0059495, size = 54, normalized size = 0.82

$$-\frac{11}{2}a^2bx^2-\frac{16a^4\log(a-bx)}{b}-15a^3x-\frac{5}{3}ab^2x^3-\frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a^2 - b^2*x^2),x]
```

```
[Out] -15*a^3*x - (11*a^2*b*x^2)/2 - (5*a*b^2*x^3)/3 - (b^3*x^4)/4 - (16*a^4*Log[
a - b*x])/b
```

Maple [A] time = 0.041, size = 50, normalized size = 0.8

$$-\frac{b^3x^4}{4}-\frac{5\,ab^2x^3}{3}-\frac{11\,a^2bx^2}{2}-15\,xa^3-16\,\frac{a^4\ln(bx-a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(-b^2*x^2+a^2),x)
```

```
[Out] -1/4*b^3*x^4-5/3*a*b^2*x^3-11/2*a^2*b*x^2-15*x*a^3-16*a^4/b*ln(b*x-a)
```

Maxima [A] time = 1.20587, size = 66, normalized size = 1.

$$-\frac{1}{4}\,b^3x^4-\frac{5}{3}\,ab^2x^3-\frac{11}{2}\,a^2bx^2-15\,a^3x-\frac{16\,a^4\log(bx-a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(-b^2*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/4*b^3*x^4 - 5/3*a*b^2*x^3 - 11/2*a^2*b*x^2 - 15*a^3*x - 16*a^4*log(b*x -
 a)/b
```

Fricas [A] time = 1.82077, size = 122, normalized size = 1.85

$$-\frac{3\,b^4x^4+20\,ab^3x^3+66\,a^2b^2x^2+180\,a^3bx+192\,a^4\log(bx-a)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(-b^2*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*b^4*x^4 + 20*a*b^3*x^3 + 66*a^2*b^2*x^2 + 180*a^3*b*x + 192*a^4*lo
g(b*x - a))/b
```

Sympy [A] time = 0.327504, size = 53, normalized size = 0.8

$$-\frac{16a^4\log(-a+bx)}{b}-15a^3x-\frac{11a^2bx^2}{2}-\frac{5ab^2x^3}{3}-\frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(-b**2*x**2+a**2),x)
```

```
[Out] -16*a**4*log(-a + b*x)/b - 15*a**3*x - 11*a**2*b*x**2/2 - 5*a*b**2*x**3/3 -
 b**3*x**4/4
```

Giac [A] time = 1.17887, size = 82, normalized size = 1.24

$$-\frac{16\,a^4\log\left(|bx-a|\right)}{b}-\frac{3\,b^7x^4+20\,ab^6x^3+66\,a^2b^5x^2+180\,a^3b^4x}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(-b^2*x^2+a^2),x, algorithm="giac")
```

```
[Out] -16*a^4*log(abs(b*x - a))/b - 1/12*(3*b^7*x^4 + 20*a*b^6*x^3 + 66*a^2*b^5*x
^2 + 180*a^3*b^4*x)/b^4
```

3.749 $\int \frac{(a+bx)^4}{a^2-b^2x^2} dx$

Optimal. Leaf size=49

$$-\frac{8a^3 \log(a-bx)}{b} - 4a^2x - \frac{a(a+bx)^2}{b} - \frac{(a+bx)^3}{3b}$$

```
[Out] -4*a^2*x - (a*(a + b*x)^2)/b - (a + b*x)^3/(3*b) - (8*a^3*Log[a - b*x])/b
```

Rubi [A] time = 0.0205057, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {627, 43}

$$-\frac{8a^3 \log(a-bx)}{b} - 4a^2x - \frac{a(a+bx)^2}{b} - \frac{(a+bx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(a^2 - b^2*x^2),x]
```

```
[Out] -4*a^2*x - (a*(a + b*x)^2)/b - (a + b*x)^3/(3*b) - (8*a^3*Log[a - b*x])/b
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^4}{a^2-b^2x^2} \, dx &= \int \frac{(a+bx)^3}{a-bx} \, dx \\
&= \int \left(-4a^2 + \frac{8a^3}{a-bx} - 2a(a+bx) - (a+bx)^2\right) dx \\
&= -4a^2x - \frac{a(a+bx)^2}{b} - \frac{(a+bx)^3}{3b} - \frac{8a^3 \log(a-bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0054987, size = 39, normalized size = 0.8

$$-\frac{8a^3 \log(a-bx)}{b} - 7a^2x - 2abx^2 - \frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(a^2 - b^2*x^2),x]
```

```
[Out] -7*a^2*x - 2*a*b*x^2 - (b^2*x^3)/3 - (8*a^3*Log[a - b*x])/b
```

Maple [A] time = 0.041, size = 39, normalized size = 0.8

$$-\frac{b^2x^3}{3} - 2\,abx^2 - 7\,a^2x - 8\,\frac{a^3\ln(bx - a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(-b^2*x^2+a^2),x)
```

```
[Out] -1/3*b^2*x^3-2*a*b*x^2-7*a^2*x-8/b*a^3*ln(b*x-a)
```

Maxima [A] time = 1.23004, size = 51, normalized size = 1.04

$$-\frac{1}{3}\,b^2x^3 - 2\,abx^2 - 7\,a^2x - \frac{8\,a^3\log(bx - a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(-b^2*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/3*b^2*x^3 - 2*a*b*x^2 - 7*a^2*x - 8*a^3*log(b*x - a)/b
```

Fricas [A] time = 1.71158, size = 90, normalized size = 1.84

$$-\frac{b^3x^3 + 6\,ab^2x^2 + 21\,a^2bx + 24\,a^3\log(bx - a)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(-b^2*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/3*(b^3*x^3 + 6*a*b^2*x^2 + 21*a^2*b*x + 24*a^3*log(b*x - a))/b
```

Sympy [A] time = 0.315249, size = 37, normalized size = 0.76

$$-\frac{8a^3\log(-a + bx)}{b} - 7a^2x - 2abx^2 - \frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(-b**2*x**2+a**2),x)
```

```
[Out] -8*a**3*log(-a + b*x)/b - 7*a**2*x - 2*a*b*x**2 - b**2*x**3/3
```

Giac [A] time = 1.22229, size = 66, normalized size = 1.35

$$-\frac{8\,a^3\log(|bx - a|)}{b} - \frac{b^5x^3 + 6\,ab^4x^2 + 21\,a^2b^3x}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(-b^2*x^2+a^2),x, algorithm="giac")
```

```
[Out] -8*a^3*log(abs(b*x - a))/b - 1/3*(b^5*x^3 + 6*a*b^4*x^2 + 21*a^2*b^3*x)/b^3
```

3.750 $\int \frac{(a+bx)^3}{a^2-b^2x^2}\,dx$

Optimal. Leaf size=28

$$-\frac{4a^2\log(a-bx)}{b}-3ax-\frac{bx^2}{2}$$

```
[Out] -3*a*x - (b*x^2)/2 - (4*a^2*Log[a - b*x])/b
```

Rubi [A] time = 0.0146225, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {627, 43}

$$-\frac{4a^2\log(a-bx)}{b}-3ax-\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(a^2 - b^2*x^2),x]
```

```
[Out] -3*a*x - (b*x^2)/2 - (4*a^2*Log[a - b*x])/b
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{a^2-b^2x^2}\,dx &= \int \frac{(a+bx)^2}{a-bx}\,dx\\ &= \int \left(-3a-bx+\frac{4a^2}{a-bx}\right)dx\\ &= -3ax-\frac{bx^2}{2}-\frac{4a^2\log(a-bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0048804, size = 28, normalized size = 1.

$$-\frac{4a^2\log(a-bx)}{b}-3ax-\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(a^2 - b^2*x^2),x]
```

```
[Out] -3*a*x - (b*x^2)/2 - (4*a^2*Log[a - b*x])/b
```

Maple [A] time = 0.041, size = 28, normalized size = 1.

$$-\frac{bx^2}{2} - 3\,ax - 4\,\frac{a^2\ln(bx-a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(-b^2*x^2+a^2),x)
```

```
[Out] -1/2*b*x^2-3*a*x-4/b*a^2*ln(b*x-a)
```

Maxima [A] time = 1.23439, size = 36, normalized size = 1.29

$$-\frac{1}{2}\,bx^2 - 3\,ax - \frac{4\,a^2\log(bx-a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(-b^2*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/2*b*x^2 - 3*a*x - 4*a^2*log(b*x - a)/b
```

Fricas [A] time = 1.62271, size = 66, normalized size = 2.36

$$-\frac{b^2x^2 + 6\,abx + 8\,a^2\log(bx-a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(-b^2*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*x^2 + 6*a*b*x + 8*a^2*log(b*x - a))/b
```

Sympy [A] time = 0.291171, size = 26, normalized size = 0.93

$$-\frac{4a^2\log(-a+bx)}{b} - 3ax - \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(-b**2*x**2+a**2),x)
```

```
[Out] -4*a**2*log(-a + b*x)/b - 3*a*x - b*x**2/2
```

Giac [A] time = 1.23165, size = 51, normalized size = 1.82

$$-\frac{4\,a^2\log(|bx-a|)}{b} - \frac{b^3x^2 + 6\,ab^2x}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(-b^2*x^2+a^2),x, algorithm="giac")
```

```
[Out] -4*a^2*log(abs(b*x - a))/b - 1/2*(b^3*x^2 + 6*a*b^2*x)/b^2
```

3.751 $\int \frac{(a+bx)^2}{a^2-b^2x^2} dx$

Optimal. Leaf size=17

$$-\frac{2a\log(a-bx)}{b} - x$$

```
[Out] -x - (2*a*Log[a - b*x])/b
```

Rubi [A] time = 0.0142805, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {627, 43}

$$-\frac{2a\log(a-bx)}{b} - x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a^2 - b^2*x^2),x]

[Out] -x - (2*a*Log[a - b*x])/b
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2}{a^2-b^2x^2} dx &= \int \frac{a+bx}{a-bx} dx \\
&= \int \left(-1 + \frac{2a}{a-bx}\right) dx \\
&= -x - \frac{2a\log(a-bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0029428, size = 17, normalized size = 1.

$$-\frac{2a\log(a-bx)}{b} - x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a^2 - b^2*x^2),x]
```

```
[Out] -x - (2*a*Log[a - b*x])/b
```

Maple [A] time = 0.039, size = 19, normalized size = 1.1

$$-x-2\,{\frac {a\ln \left(bx-a \right) }{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b^2*x^2+a^2),x)
```

```
[Out] -x-2/b*a*ln(b*x-a)
```

Maxima [A] time = 1.03761, size = 24, normalized size = 1.41

$$-x - \frac{2 \, a \log\left(b x - a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b^2*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -x - 2*a*log(b*x - a)/b
```

Fricas [A] time = 1.83442, size = 39, normalized size = 2.29

$$-\frac{b x + 2 \, a \log\left(b x - a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b^2*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -(b*x + 2*a*log(b*x - a))/b
```

Sympy [A] time = 0.269772, size = 14, normalized size = 0.82

$$- \frac{2 a \log{\left (- a + b x \right)}}{b} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b**2*x**2+a**2),x)
```

```
[Out] -2*a*log(-a + b*x)/b - x
```

Giac [A] time = 1.18335, size = 26, normalized size = 1.53

$$-x - \frac{2 \, a \log\left({\left| b x - a \right|}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b^2*x^2+a^2),x, algorithm="giac")
```

```
[Out] -x - 2*a*log(abs(b*x - a))/b
```

3.752 $\int \frac{a+bx}{a^2-b^2x^2} \, dx$

Optimal. Leaf size=12

$$-\frac{\log(a-bx)}{b}$$

```
[Out] -(Log[a - b*x]/b)
```

Rubi [A] time = 0.0044518, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {627, 31}

$$-\frac{\log(a-bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a^2 - b^2*x^2),x]
```

```
[Out] -(Log[a - b*x]/b)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx}{a^2-b^2x^2} \, dx &= \int \frac{1}{a-bx} \, dx \\ &= -\frac{\log(a-bx)}{b} \end{aligned}$$

Mathematica [A] time = 0.0011062, size = 12, normalized size = 1.

$$-\frac{\log(a-bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a^2 - b^2*x^2),x]
```

```
[Out] -(Log[a - b*x]/b)
```

Maple [A] time = 0.038, size = 14, normalized size = 1.2

$$-\frac{\ln (bx-a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-b^2*x^2+a^2),x)
```

```
[Out] -1/b*ln(b*x-a)
```

Maxima [A] time = 1.40966, size = 18, normalized size = 1.5

$$-\frac{\log (bx-a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b^2*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -log(b*x - a)/b
```

Fricas [A] time = 1.73668, size = 23, normalized size = 1.92

$$-\frac{\log (bx-a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b^2*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -log(b*x - a)/b
```

Sympy [A] time = 0.067708, size = 8, normalized size = 0.67

$$-\frac{\log (-a+bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b**2*x**2+a**2),x)
```

```
[Out] -log(-a + b*x)/b
```

Giac [A] time = 1.20268, size = 19, normalized size = 1.58

$$-\frac{\log (|bx-a|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b^2*x^2+a^2),x, algorithm="giac")
```

```
[Out] -log(abs(b*x - a))/b
```

3.753 $\int \frac{1}{(a+bx)(a^2-b^2x^2)} dx$

Optimal. Leaf size=35

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2b} - \frac{1}{2ab(a+bx)}$$

```
[Out] -1/(2*a*b*(a + b*x)) + ArcTanh[(b*x)/a]/(2*a^2*b)
```

Rubi [A] time = 0.0308144, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {627, 44, 208}

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2b} - \frac{1}{2ab(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(a^2 - b^2*x^2)),x]
```

```
[Out] -1/(2*a*b*(a + b*x)) + ArcTanh[(b*x)/a]/(2*a^2*b)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)\left(a^2-b^2x^2\right)} dx &= \int \frac{1}{(a-bx)(a+bx)^2} dx \\
&= \int \left(\frac{1}{2a(a+bx)^2} + \frac{1}{2a\left(a^2-b^2x^2\right)}\right) dx \\
&= -\frac{1}{2ab(a+bx)} + \frac{\int \frac{1}{a^2-b^2x^2} dx}{2a} \\
&= -\frac{1}{2ab(a+bx)} + \frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2b}
\end{aligned}$$

Mathematica [A] time = 0.0114628, size = 47, normalized size = 1.34

$$\frac{-(a+bx)\log(a-bx)+(a+bx)\log(a+bx)-2a}{4a^2b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(a^2 - b^2*x^2)),x]
```

```
[Out] (-2*a - (a + b*x)*Log[a - b*x] + (a + b*x)*Log[a + b*x])/(4*a^2*b*(a + b*x)
)
```

Maple [A] time = 0.047, size = 47, normalized size = 1.3

$$\frac{\ln(bx+a)}{4\,ba^2}-\frac{1}{2\,ab\,(bx+a)}-\frac{\ln(bx-a)}{4\,ba^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(-b^2*x^2+a^2),x)
```

```
[Out] 1/4/b/a^2*ln(b*x+a)-1/2/a/b/(b*x+a)-1/4/b/a^2*ln(b*x-a)
```

Maxima [A] time = 1.34774, size = 63, normalized size = 1.8

$$-\frac{1}{2\left(ab^2x+a^2b\right)}+\frac{\log(bx+a)}{4\,a^2b}-\frac{\log(bx-a)}{4\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b^2*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/2/(a*b^2*x + a^2*b) + 1/4*log(b*x + a)/(a^2*b) - 1/4*log(b*x - a)/(a^2*b
)
```

Fricas [A] time = 1.73136, size = 109, normalized size = 3.11

$$\frac{(bx+a)\log(bx+a)-(bx+a)\log(bx-a)-2\,a}{4\left(a^2b^2x+a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b^2*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/4*((b*x + a)*log(b*x + a) - (b*x + a)*log(b*x - a) - 2*a)/(a^2*b^2*x + a^
3*b)
```

Sympy [A] time = 0.395931, size = 39, normalized size = 1.11

$$-\frac{1}{2a^2b+2ab^2x}-\frac{\frac{\log\left(-\frac{a}{b}+x\right)}{4}-\frac{\log\left(\frac{a}{b}+x\right)}{4}}{a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b**2*x**2+a**2),x)
```

```
[Out] -1/(2*a**2*b + 2*a*b**2*x) - (log(-a/b + x)/4 - log(a/b + x)/4)/(a**2*b)
```

Giac [A] time = 1.20726, size = 65, normalized size = 1.86

$$\frac{\log\left(|bx+a|\right)}{4\,a^2b}-\frac{\log\left(|bx-a|\right)}{4\,a^2b}-\frac{1}{2\,(bx+a)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b^2*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(b*x + a))/(a^2*b) - 1/4*log(abs(b*x - a))/(a^2*b) - 1/2/((b*x +
 a)*a*b)
```

3.754 $\int \frac{1}{(a+bx)^2(a^2-b^2x^2)}\,dx$

Optimal. Leaf size=52

$$-\frac{1}{4a^2b(a+bx)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{4a^3b}-\frac{1}{4ab(a+bx)^2}$$

```
[Out] -1/(4*a*b*(a + b*x)^2) - 1/(4*a^2*b*(a + b*x)) + ArcTanh[(b*x)/a]/(4*a^3*b)
```

Rubi [A] time = 0.0373936, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {627, 44, 208}

$$-\frac{1}{4a^2b(a+bx)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{4a^3b}-\frac{1}{4ab(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(a^2 - b^2*x^2)),x]
```

```
[Out] -1/(4*a*b*(a + b*x)^2) - 1/(4*a^2*b*(a + b*x)) + ArcTanh[(b*x)/a]/(4*a^3*b)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^2\left(a^2-b^2x^2\right)}\,dx &= \int \frac{1}{(a-bx)(a+bx)^3}\,dx\\
&= \int \left(\frac{1}{2a(a+bx)^3}+\frac{1}{4a^2(a+bx)^2}+\frac{1}{4a^2\left(a^2-b^2x^2\right)}\right)dx\\
&= -\frac{1}{4ab(a+bx)^2}-\frac{1}{4a^2b(a+bx)}+\frac{\int \frac{1}{a^2-b^2x^2}\,dx}{4a^2}\\
&= -\frac{1}{4ab(a+bx)^2}-\frac{1}{4a^2b(a+bx)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{4a^3b}
\end{aligned}$$

Mathematica [A] time = 0.0168035, size = 58, normalized size = 1.12

$$\frac{-2a(2a+bx)+(a+bx)^2(-\log(a-bx))+(a+bx)^2\log(a+bx)}{8a^3b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(a^2 - b^2*x^2)),x]
```

```
[Out] (-2*a*(2*a + b*x) - (a + b*x)^2*Log[a - b*x] + (a + b*x)^2*Log[a + b*x])/(8
*a^3*b*(a + b*x)^2)
```

Maple [A] time = 0.047, size = 62, normalized size = 1.2

$$\frac{\ln(bx+a)}{8\,ba^3}-\frac{1}{4\,ba^2\,(bx+a)}-\frac{1}{4\,ab\,(bx+a)^2}-\frac{\ln(bx-a)}{8\,ba^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(-b^2*x^2+a^2),x)
```

```
[Out] 1/8/b/a^3*ln(b*x+a)-1/4/a^2/b/(b*x+a)-1/4/a/b/(b*x+a)^2-1/8/b/a^3*ln(b*x-a)
```

Maxima [A] time = 1.14737, size = 90, normalized size = 1.73

$$-\frac{bx+2\,a}{4\left(a^2b^3x^2+2\,a^3b^2x+a^4b\right)}+\frac{\log(bx+a)}{8\,a^3b}-\frac{\log(bx-a)}{8\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b^2*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/4*(b*x + 2*a)/(a^2*b^3*x^2 + 2*a^3*b^2*x + a^4*b) + 1/8*log(b*x + a)/(a^
3*b) - 1/8*log(b*x - a)/(a^3*b)
```

Fricas [A] time = 1.85799, size = 192, normalized size = 3.69

$$-\frac{2\,abx+4\,a^2-\left(b^2x^2+2\,abx+a^2\right)\log(bx+a)+\left(b^2x^2+2\,abx+a^2\right)\log(bx-a)}{8\left(a^3b^3x^2+2\,a^4b^2x+a^5b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b^2*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/8*(2*a*b*x + 4*a^2 - (b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a) + (b^2*x^2 +
 2*a*b*x + a^2)*log(b*x - a))/(a^3*b^3*x^2 + 2*a^4*b^2*x + a^5*b)
```

Sympy [A] time = 0.492546, size = 58, normalized size = 1.12

$$-\frac{2a+bx}{4a^4b+8a^3b^2x+4a^2b^3x^2}-\frac{\frac{\log\left(-\frac{a}{b}+x\right)}{8}-\frac{\log\left(\frac{a}{b}+x\right)}{8}}{a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(-b**2*x**2+a**2),x)
```

```
[Out] -(2*a + b*x)/(4*a**4*b + 8*a**3*b**2*x + 4*a**2*b**3*x**2) - (log(-a/b + x)
/8 - log(a/b + x)/8)/(a**3*b)
```

Giac [A] time = 1.20897, size = 69, normalized size = 1.33

$$-\frac{\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}}{4\,a^2b^2}-\frac{\log\left(\left|-\frac{2\,a}{bx+a}+1\right|\right)}{8\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b^2*x^2+a^2),x, algorithm="giac")
```

```
[Out] -1/4*(b/(b*x + a) + a*b/(b*x + a)^2)/(a^2*b^2) - 1/8*log(abs(-2*a/(b*x + a)
 + 1))/(a^3*b)
```

3.755 $\int \frac{1}{(a+bx)^3\left(a^2-b^2x^2\right)}\,dx$

Optimal. Leaf size=69

$$-\frac{1}{8a^3b(a+bx)}-\frac{1}{8a^2b(a+bx)^2}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{8a^4b}-\frac{1}{6ab(a+bx)^3}$$

```
[Out] -1/(6*a*b*(a + b*x)^3) - 1/(8*a^2*b*(a + b*x)^2) - 1/(8*a^3*b*(a + b*x)) +
ArcTanh[(b*x)/a]/(8*a^4*b)
```

Rubi [A] time = 0.0457732, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {627, 44, 208}

$$-\frac{1}{8a^3b(a+bx)}-\frac{1}{8a^2b(a+bx)^2}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{8a^4b}-\frac{1}{6ab(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(a^2 - b^2*x^2)),x]
```

```
[Out] -1/(6*a*b*(a + b*x)^3) - 1/(8*a^2*b*(a + b*x)^2) - 1/(8*a^3*b*(a + b*x)) +
ArcTanh[(b*x)/a]/(8*a^4*b)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^3(a^2-b^2x^2)}\,dx = \int \frac{1}{(a-bx)(a+bx)^4}\,dx$$
$$= \int \left(\frac{1}{2a(a+bx)^4} + \frac{1}{4a^2(a+bx)^3} + \frac{1}{8a^3(a+bx)^2} + \frac{1}{8a^3(a^2-b^2x^2)}\right) dx$$
$$= -\frac{1}{6ab(a+bx)^3} - \frac{1}{8a^2b(a+bx)^2} - \frac{1}{8a^3b(a+bx)} + \frac{\int \frac{1}{a^2-b^2x^2}\,dx}{8a^3}$$
$$= -\frac{1}{6ab(a+bx)^3} - \frac{1}{8a^2b(a+bx)^2} - \frac{1}{8a^3b(a+bx)} + \frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{8a^4b}$$

Mathematica [A] time = 0.0215884, size = 71, normalized size = 1.03

$$\frac{-2a\left(10a^2+9abx+3b^2x^2\right)-3(a+bx)^3\log(a-bx)+3(a+bx)^3\log(a+bx)}{48a^4b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(a^2 - b^2*x^2)),x]
```

```
[Out] (-2*a*(10*a^2 + 9*a*b*x + 3*b^2*x^2) - 3*(a + b*x)^3*Log[a - b*x] + 3*(a +
b*x)^3*Log[a + b*x])/(48*a^4*b*(a + b*x)^3)
```

Maple [A] time = 0.049, size = 77, normalized size = 1.1

$$\frac{\ln(bx+a)}{16\,a^4b} - \frac{1}{8\,a^3b\,(bx+a)} - \frac{1}{8\,ba^2\,(bx+a)^2} - \frac{1}{6\,ab\,(bx+a)^3} - \frac{\ln(bx-a)}{16\,a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(-b^2*x^2+a^2),x)
```

```
[Out] 1/16/a^4/b*ln(b*x+a)-1/8/a^3/b/(b*x+a)-1/8/a^2/b/(b*x+a)^2-1/6/a/b/(b*x+a)^
3-1/16/a^4/b*ln(b*x-a)
```

Maxima [A] time = 1.02723, size = 122, normalized size = 1.77

$$-\frac{3\,b^2x^2+9\,abx+10\,a^2}{24\left(a^3b^4x^3+3\,a^4b^3x^2+3\,a^5b^2x+a^6b\right)} + \frac{\log(bx+a)}{16\,a^4b} - \frac{\log(bx-a)}{16\,a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(-b^2*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/24*(3*b^2*x^2 + 9*a*b*x + 10*a^2)/(a^3*b^4*x^3 + 3*a^4*b^3*x^2 + 3*a^5*b
^2*x + a^6*b) + 1/16*log(b*x + a)/(a^4*b) - 1/16*log(b*x - a)/(a^4*b)
```

Fricas [B] time = 1.78484, size = 288, normalized size = 4.17

$$-\frac{6\,ab^2x^2 + 18\,a^2bx + 20\,a^3 - 3\,\left(b^3x^3 + 3\,ab^2x^2 + 3\,a^2bx + a^3\right)\log\left(bx + a\right) + 3\,\left(b^3x^3 + 3\,ab^2x^2 + 3\,a^2bx + a^3\right)\log\left(bx - a\right)}{48\,\left(a^4b^4x^3 + 3\,a^5b^3x^2 + 3\,a^6b^2x + a^7b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(-b^2*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/48*(6*a*b^2*x^2 + 18*a^2*b*x + 20*a^3 - 3*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2
*b*x + a^3)*log(b*x + a) + 3*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*log(
b*x - a))/(a^4*b^4*x^3 + 3*a^5*b^3*x^2 + 3*a^6*b^2*x + a^7*b)
```

Sympy [A] time = 0.609055, size = 83, normalized size = 1.2

$$-\frac{10a^2 + 9abx + 3b^2x^2}{24a^6b + 72a^5b^2x + 72a^4b^3x^2 + 24a^3b^4x^3} - \frac{\frac{\log\left(-\frac{a}{b}+x\right)}{16} - \frac{\log\left(\frac{a}{b}+x\right)}{16}}{a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(-b**2*x**2+a**2),x)
```

```
[Out] -(10*a**2 + 9*a*b*x + 3*b**2*x**2)/(24*a**6*b + 72*a**5*b**2*x + 72*a**4*b*
*3*x**2 + 24*a**3*b**4*x**3) - (log(-a/b + x)/16 - log(a/b + x)/16)/(a**4*b
)
```

Giac [A] time = 1.24327, size = 95, normalized size = 1.38

$$\frac{\log\left(|bx + a|\right)}{16\,a^4b} - \frac{\log\left(|bx - a|\right)}{16\,a^4b} - \frac{3\,ab^2x^2 + 9\,a^2bx + 10\,a^3}{24\,(bx + a)^3a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(-b^2*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/16*log(abs(b*x + a))/(a^4*b) - 1/16*log(abs(b*x - a))/(a^4*b) - 1/24*(3*a
*b^2*x^2 + 9*a^2*b*x + 10*a^3)/((b*x + a)^3*a^4*b)
```

3.756 $\int \frac{1}{(a+bx)^4(a^2-b^2x^2)}\,dx$

Optimal. Leaf size=86

$$-\frac{1}{16a^4b(a+bx)}-\frac{1}{16a^3b(a+bx)^2}-\frac{1}{12a^2b(a+bx)^3}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{16a^5b}-\frac{1}{8ab(a+bx)^4}$$

```
[Out] -1/(8*a*b*(a + b*x)^4) - 1/(12*a^2*b*(a + b*x)^3) - 1/(16*a^3*b*(a + b*x)^2
) - 1/(16*a^4*b*(a + b*x)) + ArcTanh[(b*x)/a]/(16*a^5*b)
```

Rubi [A] time = 0.0530616, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {627, 44, 208}

$$-\frac{1}{16a^4b(a+bx)}-\frac{1}{16a^3b(a+bx)^2}-\frac{1}{12a^2b(a+bx)^3}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{16a^5b}-\frac{1}{8ab(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^4*(a^2 - b^2*x^2)),x]
```

```
[Out] -1/(8*a*b*(a + b*x)^4) - 1/(12*a^2*b*(a + b*x)^3) - 1/(16*a^3*b*(a + b*x)^2
) - 1/(16*a^4*b*(a + b*x)) + ArcTanh[(b*x)/a]/(16*a^5*b)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^4(a^2-b^2x^2)}\,dx = \int \frac{1}{(a-bx)(a+bx)^5}\,dx$$
$$= \int \left(\frac{1}{2a(a+bx)^5} + \frac{1}{4a^2(a+bx)^4} + \frac{1}{8a^3(a+bx)^3} + \frac{1}{16a^4(a+bx)^2} + \frac{1}{16a^4(a^2-b^2x^2)}\right) dx$$
$$= -\frac{1}{8ab(a+bx)^4} - \frac{1}{12a^2b(a+bx)^3} - \frac{1}{16a^3b(a+bx)^2} - \frac{1}{16a^4b(a+bx)} + \frac{\int \frac{1}{a^2-b^2x^2}\,dx}{16a^4}$$
$$= -\frac{1}{8ab(a+bx)^4} - \frac{1}{12a^2b(a+bx)^3} - \frac{1}{16a^3b(a+bx)^2} - \frac{1}{16a^4b(a+bx)} + \frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{16a^5b}$$

Mathematica [A] time = 0.0243488, size = 82, normalized size = 0.95

$$\frac{-2a\left(19a^2bx+16a^3+12ab^2x^2+3b^3x^3\right)-3(a+bx)^4\log(a-bx)+3(a+bx)^4\log(a+bx)}{96a^5b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^4*(a^2 - b^2*x^2)),x]
```

```
[Out] (-2*a*(16*a^3 + 19*a^2*b*x + 12*a*b^2*x^2 + 3*b^3*x^3) - 3*(a + b*x)^4*Log[
a - b*x] + 3*(a + b*x)^4*Log[a + b*x])/(96*a^5*b*(a + b*x)^4)
```

Maple [A] time = 0.049, size = 92, normalized size = 1.1

$$\frac{\ln(bx+a)}{32\,a^5b} - \frac{1}{16\,a^4b\,(bx+a)} - \frac{1}{16\,a^3b\,(bx+a)^2} - \frac{1}{12\,ba^2\,(bx+a)^3} - \frac{1}{8\,ab\,(bx+a)^4} - \frac{\ln(bx-a)}{32\,a^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^4/(-b^2*x^2+a^2),x)
```

```
[Out] 1/32/a^5/b*ln(b*x+a)-1/16/a^4/b/(b*x+a)-1/16/a^3/b/(b*x+a)^2-1/12/a^2/b/(b*
x+a)^3-1/8/a/b/(b*x+a)^4-1/32/a^5/b*ln(b*x-a)
```

Maxima [A] time = 1.03286, size = 151, normalized size = 1.76

$$-\frac{3\,b^3x^3+12\,ab^2x^2+19\,a^2bx+16\,a^3}{48\left(a^4b^5x^4+4\,a^5b^4x^3+6\,a^6b^3x^2+4\,a^7b^2x+a^8b\right)} + \frac{\log(bx+a)}{32\,a^5b} - \frac{\log(bx-a)}{32\,a^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(-b^2*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/48*(3*b^3*x^3 + 12*a*b^2*x^2 + 19*a^2*b*x + 16*a^3)/(a^4*b^5*x^4 + 4*a^5
*b^4*x^3 + 6*a^6*b^3*x^2 + 4*a^7*b^2*x + a^8*b) + 1/32*log(b*x + a)/(a^5*b)
 - 1/32*log(b*x - a)/(a^5*b)
```

Fricas [B] time = 1.7232, size = 375, normalized size = 4.36

$$-\frac{6\,ab^3x^3+24\,a^2b^2x^2+38\,a^3bx+32\,a^4-3\left(b^4x^4+4\,ab^3x^3+6\,a^2b^2x^2+4\,a^3bx+a^4\right)\log\left(bx+a\right)+3\left(b^4x^4+4\,ab^3x^3\right.}{96\left(a^5b^5x^4+4\,a^6b^4x^3+6\,a^7b^3x^2+4\,a^8b^2x+a^9b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(-b^2*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/96*(6*a*b^3*x^3 + 24*a^2*b^2*x^2 + 38*a^3*b*x + 32*a^4 - 3*(b^4*x^4 + 4*
a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*log(b*x + a) + 3*(b^4*x^4 + 4*
a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*log(b*x - a))/(a^5*b^5*x^4 + 4
*a^6*b^4*x^3 + 6*a^7*b^3*x^2 + 4*a^8*b^2*x + a^9*b)
```

Sympy [A] time = 0.708482, size = 107, normalized size = 1.24

$$-\frac{16a^3+19a^2bx+12ab^2x^2+3b^3x^3}{48a^8b+192a^7b^2x+288a^6b^3x^2+192a^5b^4x^3+48a^4b^5x^4}-\frac{\frac{\log\left(-\frac{a}{b}+x\right)}{32}-\frac{\log\left(\frac{a}{b}+x\right)}{32}}{a^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**4/(-b**2*x**2+a**2),x)
```

```
[Out] -(16*a**3 + 19*a**2*b*x + 12*a*b**2*x**2 + 3*b**3*x**3)/(48*a**8*b + 192*a*
*7*b**2*x + 288*a**6*b**3*x**2 + 192*a**5*b**4*x**3 + 48*a**4*b**5*x**4) -
(log(-a/b + x)/32 - log(a/b + x)/32)/(a**5*b)
```

Giac [A] time = 1.223, size = 109, normalized size = 1.27

$$\frac{\log\left(|bx+a|\right)}{32\,a^5b}-\frac{\log\left(|bx-a|\right)}{32\,a^5b}-\frac{3\,ab^3x^3+12\,a^2b^2x^2+19\,a^3bx+16\,a^4}{48\left(bx+a\right)^4a^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(-b^2*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/32*log(abs(b*x + a))/(a^5*b) - 1/32*log(abs(b*x - a))/(a^5*b) - 1/48*(3*a
*b^3*x^3 + 12*a^2*b^2*x^2 + 19*a^3*b*x + 16*a^4)/((b*x + a)^4*a^5*b)
```

3.757 $\int \frac{(a+bx)^7}{\left(a^2-b^2x^2\right)^2}\,dx$

Optimal. Leaf size=70

$$\frac{23}{2}a^2bx^2+\frac{32a^5}{b(a-bx)}+\frac{80a^4\log(a-bx)}{b}+49a^3x+\frac{7}{3}ab^2x^3+\frac{b^3x^4}{4}$$

```
[Out] 49*a^3*x + (23*a^2*b*x^2)/2 + (7*a*b^2*x^3)/3 + (b^3*x^4)/4 + (32*a^5)/(b*(
a - b*x)) + (80*a^4*Log[a - b*x])/b
```

Rubi [A] time = 0.0509393, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {627, 43}

$$\frac{23}{2}a^2bx^2+\frac{32a^5}{b(a-bx)}+\frac{80a^4\log(a-bx)}{b}+49a^3x+\frac{7}{3}ab^2x^3+\frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/(a^2 - b^2*x^2)^2,x]
```

```
[Out] 49*a^3*x + (23*a^2*b*x^2)/2 + (7*a*b^2*x^3)/3 + (b^3*x^4)/4 + (32*a^5)/(b*(
a - b*x)) + (80*a^4*Log[a - b*x])/b
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{\left(a^2-b^2x^2\right)^2}\,dx &= \int \frac{(a+bx)^5}{(a-bx)^2}\,dx\\ &= \int \left(49a^3+23a^2bx+7ab^2x^2+b^3x^3+\frac{32a^5}{(a-bx)^2}-\frac{80a^4}{a-bx}\right)dx\\ &= 49a^3x+\frac{23}{2}a^2bx^2+\frac{7}{3}ab^2x^3+\frac{b^3x^4}{4}+\frac{32a^5}{b(a-bx)}+\frac{80a^4\log(a-bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0269898, size = 71, normalized size = 1.01

$$\frac{23}{2}a^2bx^2-\frac{32a^5}{b(bx-a)}+\frac{80a^4\log(a-bx)}{b}+49a^3x+\frac{7}{3}ab^2x^3+\frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/(a^2 - b^2*x^2)^2,x]
```

```
[Out] 49*a^3*x + (23*a^2*b*x^2)/2 + (7*a*b^2*x^3)/3 + (b^3*x^4)/4 - (32*a^5)/(b*(
-a + b*x)) + (80*a^4*Log[a - b*x])/b
```

Maple [A] time = 0.044, size = 67, normalized size = 1.

$$\frac{b^3x^4}{4}+\frac{7\,ab^2x^3}{3}+\frac{23\,a^2bx^2}{2}+49\,xa^3-32\,\frac{a^5}{b\,(bx-a)}+80\,\frac{a^4\ln{(bx-a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/(-b^2*x^2+a^2)^2,x)
```

```
[Out] 1/4*b^3*x^4+7/3*a*b^2*x^3+23/2*a^2*b*x^2+49*x*a^3-32/b*a^5/(b*x-a)+80*a^4/b
*ln(b*x-a)
```

Maxima [A] time = 1.09474, size = 89, normalized size = 1.27

$$\frac{1}{4}\,b^3x^4+\frac{7}{3}\,ab^2x^3+\frac{23}{2}\,a^2bx^2-\frac{32\,a^5}{b^2x-ab}+49\,a^3x+\frac{80\,a^4\log{(bx-a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*x^4 + 7/3*a*b^2*x^3 + 23/2*a^2*b*x^2 - 32*a^5/(b^2*x - a*b) + 49*a^
3*x + 80*a^4*log(b*x - a)/b
```

Fricas [A] time = 1.70059, size = 192, normalized size = 2.74

$$\frac{3\,b^5x^5+25\,ab^4x^4+110\,a^2b^3x^3+450\,a^3b^2x^2-588\,a^4bx-384\,a^5+960\,\left(a^4bx-a^5\right)\log{(bx-a)}}{12\,\left(b^2x-ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^5*x^5 + 25*a*b^4*x^4 + 110*a^2*b^3*x^3 + 450*a^3*b^2*x^2 - 588*a^
4*b*x - 384*a^5 + 960*(a^4*b*x - a^5)*log(b*x - a))/(b^2*x - a*b)
```

Sympy [A] time = 0.442633, size = 65, normalized size = 0.93

$$-\frac{32a^5}{-ab+b^2x}+\frac{80a^4\log{(-a+bx)}}{b}+49a^3x+\frac{23a^2bx^2}{2}+\frac{7ab^2x^3}{3}+\frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/(-b**2*x**2+a**2)**2,x)
```

```
[Out] -32*a**5/(-a*b + b**2*x) + 80*a**4*log(-a + b*x)/b + 49*a**3*x + 23*a**2*b*
x**2/2 + 7*a*b**2*x**3/3 + b**3*x**4/4
```

Giac [A] time = 1.22894, size = 105, normalized size = 1.5

$$\frac{80\,a^4\log\left(|bx-a|\right)}{b}-\frac{32\,a^5}{(bx-a)b}+\frac{3\,b^{11}x^4+28\,ab^{10}x^3+138\,a^2b^9x^2+588\,a^3b^8x}{12\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 80*a^4*log(abs(b*x - a))/b - 32*a^5/((b*x - a)*b) + 1/12*(3*b^11*x^4 + 28*a
*b^10*x^3 + 138*a^2*b^9*x^2 + 588*a^3*b^8*x)/b^8
```

3.758 $\int \frac{(a+bx)^6}{\left(a^2-b^2x^2\right)^2} \, dx$

Optimal. Leaf size=55

$$\frac{16a^4}{b(a-bx)}+\frac{32a^3\log(a-bx)}{b}+17a^2x+3abx^2+\frac{b^2x^3}{3}$$

```
[Out] 17*a^2*x + 3*a*b*x^2 + (b^2*x^3)/3 + (16*a^4)/(b*(a - b*x)) + (32*a^3*Log[a
 - b*x])/b
```

Rubi [A] time = 0.0410961, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {627, 43}

$$\frac{16a^4}{b(a-bx)}+\frac{32a^3\log(a-bx)}{b}+17a^2x+3abx^2+\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^6/(a^2 - b^2*x^2)^2,x]
```

```
[Out] 17*a^2*x + 3*a*b*x^2 + (b^2*x^3)/3 + (16*a^4)/(b*(a - b*x)) + (32*a^3*Log[a
 - b*x])/b
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^6}{\left(a^2-b^2x^2\right)^2} \, dx &= \int \frac{(a+bx)^4}{(a-bx)^2} \, dx \\
&= \int \left(17a^2+6abx+b^2x^2+\frac{16a^4}{(a-bx)^2}-\frac{32a^3}{a-bx}\right) dx \\
&= 17a^2x+3abx^2+\frac{b^2x^3}{3}+\frac{16a^4}{b(a-bx)}+\frac{32a^3\log(a-bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0280465, size = 56, normalized size = 1.02

$$-\frac{16a^4}{b(bx-a)}+\frac{32a^3\log(a-bx)}{b}+17a^2x+3abx^2+\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^6/(a^2 - b^2*x^2)^2,x]
```

```
[Out] 17*a^2*x + 3*a*b*x^2 + (b^2*x^3)/3 - (16*a^4)/(b*(-a + b*x)) + (32*a^3*Log[
a - b*x])/b
```

Maple [A] time = 0.046, size = 56, normalized size = 1.

$$\frac{b^2x^3}{3}+3\,abx^2+17\,a^2x-16\,\frac{a^4}{b\,(bx-a)}+32\,\frac{a^3\ln(bx-a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^6/(-b^2*x^2+a^2)^2,x)
```

```
[Out] 1/3*b^2*x^3+3*a*b*x^2+17*a^2*x-16*a^4/b/(b*x-a)+32/b*a^3*ln(b*x-a)
```

Maxima [A] time = 1.11841, size = 74, normalized size = 1.35

$$\frac{1}{3}\,b^2x^3+3\,abx^2-\frac{16\,a^4}{b^2x-ab}+17\,a^2x+\frac{32\,a^3\log(bx-a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3 + 3*a*b*x^2 - 16*a^4/(b^2*x - a*b) + 17*a^2*x + 32*a^3*log(b*x
- a)/b
```

Fricas [A] time = 1.66251, size = 157, normalized size = 2.85

$$\frac{b^4x^4+8\,ab^3x^3+42\,a^2b^2x^2-51\,a^3bx-48\,a^4+96\,\left(a^3bx-a^4\right)\log(bx-a)}{3\,\left(b^2x-ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*x^4 + 8*a*b^3*x^3 + 42*a^2*b^2*x^2 - 51*a^3*b*x - 48*a^4 + 96*(a^3
*b*x - a^4)*log(b*x - a))/(b^2*x - a*b)
```

Sympy [A] time = 0.397923, size = 49, normalized size = 0.89

$$-\frac{16a^4}{-ab+b^2x}+\frac{32a^3\log(-a+bx)}{b}+17a^2x+3abx^2+\frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**6/(-b**2*x**2+a**2)**2,x)
```

```
[Out] -16*a**4/(-a*b + b**2*x) + 32*a**3*log(-a + b*x)/b + 17*a**2*x + 3*a*b*x**2
 + b**2*x**3/3
```

Giac [A] time = 1.21901, size = 89, normalized size = 1.62

$$\frac{32\,a^3\log\left(|bx-a|\right)}{b}-\frac{16\,a^4}{(bx-a)b}+\frac{b^8x^3+9\,ab^7x^2+51\,a^2b^6x}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 32*a^3*log(abs(b*x - a))/b - 16*a^4/((b*x - a)*b) + 1/3*(b^8*x^3 + 9*a*b^7*
x^2 + 51*a^2*b^6*x)/b^6
```

3.759 $\int \frac{(a+bx)^5}{\left(a^2-b^2x^2\right)^2}\,dx$

Optimal. Leaf size=44

$$\frac{8a^3}{b(a-bx)}+\frac{12a^2\log(a-bx)}{b}+5ax+\frac{bx^2}{2}$$

```
[Out] 5*a*x + (b*x^2)/2 + (8*a^3)/(b*(a - b*x)) + (12*a^2*Log[a - b*x])/b
```

Rubi [A] time = 0.0309178, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {627, 43}

$$\frac{8a^3}{b(a-bx)}+\frac{12a^2\log(a-bx)}{b}+5ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a^2 - b^2*x^2)^2,x]
```

```
[Out] 5*a*x + (b*x^2)/2 + (8*a^3)/(b*(a - b*x)) + (12*a^2*Log[a - b*x])/b
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{\left(a^2-b^2x^2\right)^2}\,dx &= \int \frac{(a+bx)^3}{(a-bx)^2}\,dx\\ &= \int \left(5a+bx+\frac{8a^3}{(a-bx)^2}-\frac{12a^2}{a-bx}\right)dx\\ &= 5ax+\frac{bx^2}{2}+\frac{8a^3}{b(a-bx)}+\frac{12a^2\log(a-bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0197676, size = 45, normalized size = 1.02

$$-\frac{8a^3}{b(bx-a)}+\frac{12a^2\log(a-bx)}{b}+5ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a^2 - b^2*x^2)^2,x]
```

```
[Out] 5*a*x + (b*x^2)/2 - (8*a^3)/(b*(-a + b*x)) + (12*a^2*Log[a - b*x])/b
```

Maple [A] time = 0.046, size = 45, normalized size = 1.

$$\frac{bx^2}{2}+5\,ax-8\,\frac{a^3}{b\,(bx-a)}+12\,\frac{a^2\ln(bx-a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(-b^2*x^2+a^2)^2,x)
```

```
[Out] 1/2*b*x^2+5*a*x-8/b*a^3/(b*x-a)+12/b*a^2*ln(b*x-a)
```

Maxima [A] time = 1.04831, size = 59, normalized size = 1.34

$$\frac{1}{2}\,bx^2-\frac{8\,a^3}{b^2x-ab}+5\,ax+\frac{12\,a^2\log(bx-a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 - 8*a^3/(b^2*x - a*b) + 5*a*x + 12*a^2*log(b*x - a)/b
```

Fricas [A] time = 1.73501, size = 134, normalized size = 3.05

$$\frac{b^3x^3+9\,ab^2x^2-10\,a^2bx-16\,a^3+24\left(a^2bx-a^3\right)\log(bx-a)}{2\left(b^2x-ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 + 9*a*b^2*x^2 - 10*a^2*b*x - 16*a^3 + 24*(a^2*b*x - a^3)*log(b
*x - a))/(b^2*x - a*b)
```

Sympy [A] time = 0.383747, size = 37, normalized size = 0.84

$$-\frac{8a^3}{-ab+b^2x}+\frac{12a^2\log(-a+bx)}{b}+5ax+\frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(-b**2*x**2+a**2)**2,x)
```

```
[Out] -8*a**3/(-a*b + b**2*x) + 12*a**2*log(-a + b*x)/b + 5*a*x + b*x**2/2
```

Giac [A] time = 1.22885, size = 74, normalized size = 1.68

$$\frac{12\,a^2\log\left(|bx-a|\right)}{b}-\frac{8\,a^3}{(bx-a)b}+\frac{b^5x^2+10\,ab^4x}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 12*a^2*log(abs(b*x - a))/b - 8*a^3/((b*x - a)*b) + 1/2*(b^5*x^2 + 10*a*b^4*
x)/b^4
```

3.760 $\int \frac{(a+bx)^4}{\left(a^2-b^2x^2\right)^2} dx$

Optimal. Leaf size=31

$$\frac{4a^2}{b(a-bx)}+\frac{4a\log(a-bx)}{b}+x$$

```
[Out] x + (4*a^2)/(b*(a - b*x)) + (4*a*Log[a - b*x])/b
```

Rubi [A] time = 0.0214676, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {627, 43}

$$\frac{4a^2}{b(a-bx)}+\frac{4a\log(a-bx)}{b}+x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(a^2 - b^2*x^2)^2,x]
```

```
[Out] x + (4*a^2)/(b*(a - b*x)) + (4*a*Log[a - b*x])/b
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{\left(a^2-b^2x^2\right)^2} dx &= \int \frac{(a+bx)^2}{(a-bx)^2} dx\\ &= \int \left(1+\frac{4a^2}{(a-bx)^2}-\frac{4a}{a-bx}\right) dx\\ &= x+\frac{4a^2}{b(a-bx)}+\frac{4a\log(a-bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0195169, size = 32, normalized size = 1.03

$$-\frac{4a^2}{b(bx-a)}+\frac{4a\log(a-bx)}{b}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(a^2 - b^2*x^2)^2,x]
```

```
[Out] x - (4*a^2)/(b*(-a + b*x)) + (4*a*Log[a - b*x])/b
```

Maple [A] time = 0.043, size = 34, normalized size = 1.1

$$x-4\,{\frac{{a}^{2}}{b\left(bx-a\right)}}+4\,{\frac{a\ln\left(bx-a\right)}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(-b^2*x^2+a^2)^2,x)
```

```
[Out] x-4/b*a^2/(b*x-a)+4/b*a*ln(b*x-a)
```

Maxima [A] time = 1.02119, size = 45, normalized size = 1.45

$$-\frac{4\,a^{2}}{b^{2}x-ab}+x+\frac{4\,a\log\left(bx-a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -4*a^2/(b^2*x - a*b) + x + 4*a*log(b*x - a)/b
```

Fricas [A] time = 1.75862, size = 97, normalized size = 3.13

$$\frac{b^{2}x^{2}-abx-4\,a^{2}+4\,\left(abx-a^{2}\right)\log\left(bx-a\right)}{b^{2}x-ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] (b^2*x^2 - a*b*x - 4*a^2 + 4*(a*b*x - a^2)*log(b*x - a))/(b^2*x - a*b)
```

Sympy [A] time = 0.341498, size = 26, normalized size = 0.84

$$-\frac{4a^{2}}{-ab+b^{2}x}+\frac{4a\log\left(-a+bx\right)}{b}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(-b**2*x**2+a**2)**2,x)
```

```
[Out] -4*a**2/(-a*b + b**2*x) + 4*a*log(-a + b*x)/b + x
```

Giac [A] time = 1.24342, size = 46, normalized size = 1.48

$$x + \frac{4\,a\log\left(|bx - a|\right)}{b} - \frac{4\,a^2}{(bx - a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] x + 4*a*log(abs(b*x - a))/b - 4*a^2/((b*x - a)*b)
```

3.761 $\int \frac{(a+bx)^3}{\left(a^2-b^2x^2\right)^2} \, dx$

Optimal. Leaf size=26

$$\frac{2a}{b(a-bx)}+\frac{\log(a-bx)}{b}$$

```
[Out] (2*a)/(b*(a - b*x)) + Log[a - b*x]/b
```

Rubi [A] time = 0.017393, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {627, 43}

$$\frac{2a}{b(a-bx)}+\frac{\log(a-bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(a^2 - b^2*x^2)^2,x]
```

```
[Out] (2*a)/(b*(a - b*x)) + Log[a - b*x]/b
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{\left(a^2-b^2x^2\right)^2}\,dx &= \int \frac{a+bx}{(a-bx)^2}\,dx\\ &= \int \left(\frac{2a}{(a-bx)^2}+\frac{1}{-a+bx}\right)dx\\ &= \frac{2a}{b(a-bx)}+\frac{\log(a-bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0077067, size = 23, normalized size = 0.88

$$\frac{\frac{2a}{a-bx}+\log(a-bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(a^2 - b^2*x^2)^2,x]
```

```
[Out] ((2*a)/(a - b*x) + Log[a - b*x])/b
```

Maple [A] time = 0.043, size = 29, normalized size = 1.1

$$-2\,{\frac{a}{b\left(bx-a\right)}}+{\frac{\ln\left(bx-a\right)}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(-b^2*x^2+a^2)^2,x)
```

```
[Out] -2/b*a/(b*x-a)+1/b*ln(b*x-a)
```

Maxima [A] time = 1.07968, size = 38, normalized size = 1.46

$$-\frac{2\,a}{b^{2}x-ab}+\frac{\log\left(bx-a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -2*a/(b^2*x - a*b) + log(b*x - a)/b
```

Fricas [A] time = 1.74987, size = 62, normalized size = 2.38

$$\frac{(bx-a)\log\left(bx-a\right)-2\,a}{b^{2}x-ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] ((b*x - a)*log(b*x - a) - 2*a)/(b^2*x - a*b)
```

Sympy [A] time = 0.320725, size = 19, normalized size = 0.73

$$-\frac{2a}{-ab+b^{2}x}+\frac{\log\left(-a+bx\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(-b**2*x**2+a**2)**2,x)
```

```
[Out] -2*a/(-a*b + b**2*x) + log(-a + b*x)/b
```

Giac [A] time = 1.25697, size = 39, normalized size = 1.5

$$\frac{\log\left(|bx - a|\right)}{b} - \frac{2\,a}{(bx - a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] log(abs(b*x - a))/b - 2*a/((b*x - a)*b)
```

3.762 $\int \frac{(a+bx)^2}{(a^2-b^2x^2)^2} dx$

Optimal. Leaf size=12

$$\frac{1}{b(a-bx)}$$

```
[Out] 1/(b*(a - b*x))
```

Rubi [A] time = 0.0061974, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {627, 32}

$$\frac{1}{b(a-bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a^2 - b^2*x^2)^2,x]
```

```
[Out] 1/(b*(a - b*x))
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{\left(a^2-b^2x^2\right)^2} dx &= \int \frac{1}{(a-bx)^2} dx\\ &= \frac{1}{b(a-bx)}\end{aligned}$$

Mathematica [A] time = 0.0020324, size = 12, normalized size = 1.

$$\frac{1}{b(a-bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a^2 - b^2*x^2)^2,x]
```

```
[Out] 1/(b*(a - b*x))
```

Maple [A] time = 0.037, size = 15, normalized size = 1.3

$$-\frac{1}{b\,(bx-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b^2*x^2+a^2)^2,x)
```

```
[Out] -1/b/(b*x-a)
```

Maxima [A] time = 1.32449, size = 19, normalized size = 1.58

$$-\frac{1}{b^2x-ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/(b^2*x - a*b)
```

Fricas [A] time = 1.70612, size = 24, normalized size = 2.

$$-\frac{1}{b^2x-ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/(b^2*x - a*b)
```

Sympy [A] time = 0.296404, size = 10, normalized size = 0.83

$$-\frac{1}{-ab+b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b**2*x**2+a**2)**2,x)
```

```
[Out] -1/(-a*b + b**2*x)
```

Giac [A] time = 1.19993, size = 19, normalized size = 1.58

$$-\frac{1}{(bx-a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -1/((b*x - a)*b)
```

3.763 $\int \frac{a+bx}{\left(a^2-b^2x^2\right)^2} dx$

Optimal. Leaf size=36

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2b}+\frac{1}{2ab(a-bx)}$$

```
[Out] 1/(2*a*b*(a - b*x)) + ArcTanh[(b*x)/a]/(2*a^2*b)
```

Rubi [A] time = 0.0261313, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {627, 44, 208}

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2b}+\frac{1}{2ab(a-bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a^2 - b^2*x^2)^2,x]
```

```
[Out] 1/(2*a*b*(a - b*x)) + ArcTanh[(b*x)/a]/(2*a^2*b)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+bx}{\left(a^2-b^2x^2\right)^2}\,dx = \int \frac{1}{(a-bx)^2(a+bx)}\,dx$$
$$= \int \left(\frac{1}{2a(a-bx)^2} + \frac{1}{2a\left(a^2-b^2x^2\right)}\right)dx$$
$$= \frac{1}{2ab(a-bx)} + \frac{\int \frac{1}{a^2-b^2x^2}\,dx}{2a}$$
$$= \frac{1}{2ab(a-bx)} + \frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2b}$$

Mathematica [A] time = 0.0099478, size = 50, normalized size = 1.39

$$\frac{(bx-a)\log(a-bx)+(a-bx)\log(a+bx)+2a}{4a^2b(a-bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a^2 - b^2*x^2)^2,x]
```

```
[Out] (2*a + (-a + b*x)*Log[a - b*x] + (a - b*x)*Log[a + b*x])/(4*a^2*b*(a - b*x)
)
```

Maple [A] time = 0.047, size = 49, normalized size = 1.4

$$\frac{\ln(bx+a)}{4\,ba^2} - \frac{\ln(bx-a)}{4\,ba^2} - \frac{1}{2\,ab\,(bx-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-b^2*x^2+a^2)^2,x)
```

```
[Out] 1/4/b/a^2*ln(b*x+a)-1/4/b/a^2*ln(b*x-a)-1/2/b/a/(b*x-a)
```

Maxima [A] time = 1.06567, size = 65, normalized size = 1.81

$$-\frac{1}{2\left(ab^2x-a^2b\right)} + \frac{\log(bx+a)}{4\,a^2b} - \frac{\log(bx-a)}{4\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2/(a*b^2*x - a^2*b) + 1/4*log(b*x + a)/(a^2*b) - 1/4*log(b*x - a)/(a^2*b
)
```

Fricas [A] time = 1.8425, size = 109, normalized size = 3.03

$$\frac{(bx-a)\log(bx+a)-(bx-a)\log(bx-a)-2\,a}{4\left(a^2b^2x-a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((b*x - a)*log(b*x + a) - (b*x - a)*log(b*x - a) - 2*a)/(a^2*b^2*x - a^
3*b)
```

Sympy [A] time = 0.391543, size = 37, normalized size = 1.03

$$-\frac{1}{-2a^2b+2ab^2x}+\frac{-\frac{\log\left(-\frac{a}{b}+x\right)}{4}+\frac{\log\left(\frac{a}{b}+x\right)}{4}}{a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b**2*x**2+a**2)**2,x)
```

```
[Out] -1/(-2*a**2*b + 2*a*b**2*x) + (-log(-a/b + x)/4 + log(a/b + x)/4)/(a**2*b)
```

Giac [A] time = 1.23356, size = 68, normalized size = 1.89

$$\frac{\log\left(|bx+a|\right)}{4\,a^2b}-\frac{\log\left(|bx-a|\right)}{4\,a^2b}-\frac{1}{2\,(bx-a)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*log(abs(b*x + a))/(a^2*b) - 1/4*log(abs(b*x - a))/(a^2*b) - 1/2/((b*x -
 a)*a*b)
```

3.764 $\int \frac{1}{(a+bx)(a^2-b^2x^2)^2} dx$

Optimal. Leaf size=70

$$\frac{1}{8a^3b(a-bx)}-\frac{1}{4a^3b(a+bx)}-\frac{1}{8a^2b(a+bx)^2}+\frac{3\tanh^{-1}\left(\frac{bx}{a}\right)}{8a^4b}$$

```
[Out] 1/(8*a^3*b*(a - b*x)) - 1/(8*a^2*b*(a + b*x)^2) - 1/(4*a^3*b*(a + b*x)) + (
3*ArcTanh[(b*x)/a])/(8*a^4*b)
```

Rubi [A] time = 0.0460265, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {627, 44, 208}

$$\frac{1}{8a^3b(a-bx)}-\frac{1}{4a^3b(a+bx)}-\frac{1}{8a^2b(a+bx)^2}+\frac{3\tanh^{-1}\left(\frac{bx}{a}\right)}{8a^4b}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(a^2 - b^2*x^2)^2),x]
```

```
[Out] 1/(8*a^3*b*(a - b*x)) - 1/(8*a^2*b*(a + b*x)^2) - 1/(4*a^3*b*(a + b*x)) + (
3*ArcTanh[(b*x)/a])/(8*a^4*b)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)\left(a^2-b^2x^2\right)^2}\,dx=\int \frac{1}{(a-bx)^2(a+bx)^3}\,dx$$
$$=\int \left(\frac{1}{8a^3(a-bx)^2}+\frac{1}{4a^2(a+bx)^3}+\frac{1}{4a^3(a+bx)^2}+\frac{3}{8a^3\left(a^2-b^2x^2\right)}\right)dx$$
$$=\frac{1}{8a^3b(a-bx)}-\frac{1}{8a^2b(a+bx)^2}-\frac{1}{4a^3b(a+bx)}+\frac{3\int \frac{1}{a^2-b^2x^2}\,dx}{8a^3}$$
$$=\frac{1}{8a^3b(a-bx)}-\frac{1}{8a^2b(a+bx)^2}-\frac{1}{4a^3b(a+bx)}+\frac{3\tanh^{-1}\left(\frac{bx}{a}\right)}{8a^4b}$$

Mathematica [A] time = 0.0203476, size = 87, normalized size = 1.24

$$-\frac{1}{8a^3b(bx-a)}-\frac{1}{4a^3b(a+bx)}-\frac{1}{8a^2b(a+bx)^2}-\frac{3\log(a-bx)}{16a^4b}+\frac{3\log(a+bx)}{16a^4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(a^2 - b^2*x^2)^2),x]
```

```
[Out] -1/(8*a^3*b*(-a + b*x)) - 1/(8*a^2*b*(a + b*x)^2) - 1/(4*a^3*b*(a + b*x)) -
 (3*Log[a - b*x])/(16*a^4*b) + (3*Log[a + b*x])/(16*a^4*b)
```

Maple [A] time = 0.051, size = 79, normalized size = 1.1

$$\frac{3\ln(bx+a)}{16a^4b}-\frac{1}{4a^3b(bx+a)}-\frac{1}{8ba^2(bx+a)^2}-\frac{3\ln(bx-a)}{16a^4b}-\frac{1}{8a^3b(bx-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(-b^2*x^2+a^2)^2,x)
```

```
[Out] 3/16/a^4/b*ln(b*x+a)-1/4/a^3/b/(b*x+a)-1/8/a^2/b/(b*x+a)^2-3/16/a^4/b*ln(b*
x-a)-1/8/b/a^3/(b*x-a)
```

Maxima [A] time = 1.04205, size = 122, normalized size = 1.74

$$-\frac{3b^2x^2+3abx-2a^2}{8\left(a^3b^4x^3+a^4b^3x^2-a^5b^2x-a^6b\right)}+\frac{3\log(bx+a)}{16a^4b}-\frac{3\log(bx-a)}{16a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(3*b^2*x^2 + 3*a*b*x - 2*a^2)/(a^3*b^4*x^3 + a^4*b^3*x^2 - a^5*b^2*x -
 a^6*b) + 3/16*log(b*x + a)/(a^4*b) - 3/16*log(b*x - a)/(a^4*b)
```

Fricas [B] time = 1.7458, size = 269, normalized size = 3.84

$$-\frac{6\,ab^2x^2+6\,a^2bx-4\,a^3-3\,\left(b^3x^3+ab^2x^2-a^2bx-a^3\right)\log\left(bx+a\right)+3\,\left(b^3x^3+ab^2x^2-a^2bx-a^3\right)\log\left(bx-a\right)}{16\,\left(a^4b^4x^3+a^5b^3x^2-a^6b^2x-a^7b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(6*a*b^2*x^2 + 6*a^2*b*x - 4*a^3 - 3*(b^3*x^3 + a*b^2*x^2 - a^2*b*x -
 a^3)*log(b*x + a) + 3*(b^3*x^3 + a*b^2*x^2 - a^2*b*x - a^3)*log(b*x - a))/
(a^4*b^4*x^3 + a^5*b^3*x^2 - a^6*b^2*x - a^7*b)
```

Sympy [A] time = 0.578813, size = 85, normalized size = 1.21

$$-\frac{-2a^2+3abx+3b^2x^2}{-8a^6b-8a^5b^2x+8a^4b^3x^2+8a^3b^4x^3}+\frac{-\frac{3\log\left(-\frac{a}{b}+x\right)}{16}+\frac{3\log\left(\frac{a}{b}+x\right)}{16}}{a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b**2*x**2+a**2)**2,x)
```

```
[Out] -(-2*a**2 + 3*a*b*x + 3*b**2*x**2)/(-8*a**6*b - 8*a**5*b**2*x + 8*a**4*b**3
*x**2 + 8*a**3*b**4*x**3) + (-3*log(-a/b + x)/16 + 3*log(a/b + x)/16)/(a**4
*b)
```

Giac [A] time = 1.24312, size = 107, normalized size = 1.53

$$\frac{3\,\log\left(|bx+a|\right)}{16\,a^4b}-\frac{3\,\log\left(|bx-a|\right)}{16\,a^4b}-\frac{3\,ab^2x^2+3\,a^2bx-2\,a^3}{8\,(bx+a)^2(bx-a)a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 3/16*log(abs(b*x + a))/(a^4*b) - 3/16*log(abs(b*x - a))/(a^4*b) - 1/8*(3*a*
b^2*x^2 + 3*a^2*b*x - 2*a^3)/((b*x + a)^2*(b*x - a)*a^4*b)
```

3.765 $\int \frac{1}{(a+bx)^2\left(a^2-b^2x^2\right)^2}\,dx$

Optimal. Leaf size=87

$$\frac{1}{16a^4b(a-bx)}-\frac{3}{16a^4b(a+bx)}-\frac{1}{8a^3b(a+bx)^2}-\frac{1}{12a^2b(a+bx)^3}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{4a^5b}$$

```
[Out] 1/(16*a^4*b*(a - b*x)) - 1/(12*a^2*b*(a + b*x)^3) - 1/(8*a^3*b*(a + b*x)^2)
 - 3/(16*a^4*b*(a + b*x)) + ArcTanh[(b*x)/a]/(4*a^5*b)
```

Rubi [A] time = 0.0527658, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {627, 44, 208}

$$\frac{1}{16a^4b(a-bx)}-\frac{3}{16a^4b(a+bx)}-\frac{1}{8a^3b(a+bx)^2}-\frac{1}{12a^2b(a+bx)^3}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{4a^5b}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(a^2 - b^2*x^2)^2),x]
```

```
[Out] 1/(16*a^4*b*(a - b*x)) - 1/(12*a^2*b*(a + b*x)^3) - 1/(8*a^3*b*(a + b*x)^2)
 - 3/(16*a^4*b*(a + b*x)) + ArcTanh[(b*x)/a]/(4*a^5*b)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^2\left(a^2-b^2x^2\right)^2}\,dx = \int \frac{1}{(a-bx)^2(a+bx)^4}\,dx$$
$$= \int \left(\frac{1}{16a^4(a-bx)^2} + \frac{1}{4a^2(a+bx)^4} + \frac{1}{4a^3(a+bx)^3} + \frac{3}{16a^4(a+bx)^2} + \frac{1}{4a^4\left(a^2-b^2x^2\right)}\right)$$
$$= \frac{1}{16a^4b(a-bx)} - \frac{1}{12a^2b(a+bx)^3} - \frac{1}{8a^3b(a+bx)^2} - \frac{3}{16a^4b(a+bx)} + \frac{\int \frac{1}{a^2-b^2x^2}\,dx}{4a^4}$$
$$= \frac{1}{16a^4b(a-bx)} - \frac{1}{12a^2b(a+bx)^3} - \frac{1}{8a^3b(a+bx)^2} - \frac{3}{16a^4b(a+bx)} + \frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{4a^5b}$$

Mathematica [A] time = 0.040318, size = 75, normalized size = 0.86

$$\frac{\frac{2a\left(a^2bx-4a^3+6ab^2x^2+3b^3x^3\right)}{(a-bx)(a+bx)^3} - 3\log(a-bx) + 3\log(a+bx)}{24a^5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(a^2 - b^2*x^2)^2),x]
```

```
[Out] ((2*a*(-4*a^3 + a^2*b*x + 6*a*b^2*x^2 + 3*b^3*x^3))/((a - b*x)*(a + b*x)^3)
 - 3*Log[a - b*x] + 3*Log[a + b*x])/(24*a^5*b)
```

Maple [A] time = 0.052, size = 94, normalized size = 1.1

$$\frac{\ln(bx+a)}{8a^5b} - \frac{3}{16a^4b(bx+a)} - \frac{1}{8a^3b(bx+a)^2} - \frac{1}{12ba^2(bx+a)^3} - \frac{\ln(bx-a)}{8a^5b} - \frac{1}{16a^4b(bx-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(-b^2*x^2+a^2)^2,x)
```

```
[Out] 1/8/a^5/b*ln(b*x+a)-3/16/a^4/b/(b*x+a)-1/8/a^3/b/(b*x+a)^2-1/12/a^2/b/(b*x+
a)^3-1/8/a^5/b*ln(b*x-a)-1/16/a^4/b/(b*x-a)
```

Maxima [A] time = 1.01817, size = 136, normalized size = 1.56

$$-\frac{3b^3x^3 + 6ab^2x^2 + a^2bx - 4a^3}{12\left(a^4b^5x^4 + 2a^5b^4x^3 - 2a^7b^2x - a^8b\right)} + \frac{\log(bx+a)}{8a^5b} - \frac{\log(bx-a)}{8a^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/12*(3*b^3*x^3 + 6*a*b^2*x^2 + a^2*b*x - 4*a^3)/(a^4*b^5*x^4 + 2*a^5*b^4*
x^3 - 2*a^7*b^2*x - a^8*b) + 1/8*log(b*x + a)/(a^5*b) - 1/8*log(b*x - a)/(a
^5*b)
```

Fricas [A] time = 1.79541, size = 308, normalized size = 3.54

$$-\frac{6\,ab^3x^3+12\,a^2b^2x^2+2\,a^3bx-8\,a^4-3\,\left(b^4x^4+2\,ab^3x^3-2\,a^3bx-a^4\right)\log\left(bx+a\right)+3\,\left(b^4x^4+2\,ab^3x^3-2\,a^3bx-a^4\right)\log}{24\,\left(a^5b^5x^4+2\,a^6b^4x^3-2\,a^8b^2x-a^9b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(6*a*b^3*x^3 + 12*a^2*b^2*x^2 + 2*a^3*b*x - 8*a^4 - 3*(b^4*x^4 + 2*a*
b^3*x^3 - 2*a^3*b*x - a^4)*log(b*x + a) + 3*(b^4*x^4 + 2*a*b^3*x^3 - 2*a^3*
b*x - a^4)*log(b*x - a))/(a^5*b^5*x^4 + 2*a^6*b^4*x^3 - 2*a^8*b^2*x - a^9*b
)
```

Sympy [A] time = 0.668692, size = 92, normalized size = 1.06

$$-\frac{-4a^3+a^2bx+6ab^2x^2+3b^3x^3}{-12a^8b-24a^7b^2x+24a^5b^4x^3+12a^4b^5x^4}+\frac{-\frac{\log\left(-\frac{a}{b}+x\right)}{8}+\frac{\log\left(\frac{a}{b}+x\right)}{8}}{a^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(-b**2*x**2+a**2)**2,x)
```

```
[Out] -(-4*a**3 + a**2*b*x + 6*a*b**2*x**2 + 3*b**3*x**3)/(-12*a**8*b - 24*a**7*b
**2*x + 24*a**5*b**4*x**3 + 12*a**4*b**5*x**4) + (-log(-a/b + x)/8 + log(a/
b + x)/8)/(a**5*b)
```

Giac [A] time = 1.19928, size = 134, normalized size = 1.54

$$-\frac{\log\left(\left|-\frac{2\,a}{bx+a}+1\right|\right)}{8\,a^5b}+\frac{1}{32\,a^5b\left(\frac{2\,a}{bx+a}-1\right)}-\frac{\frac{9\,a^2b^5}{bx+a}+\frac{6\,a^3b^5}{(bx+a)^2}+\frac{4\,a^4b^5}{(bx+a)^3}}{48\,a^6b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(-2*a/(b*x + a) + 1))/(a^5*b) + 1/32/(a^5*b*(2*a/(b*x + a) - 1)
) - 1/48*(9*a^2*b^5/(b*x + a) + 6*a^3*b^5/(b*x + a)^2 + 4*a^4*b^5/(b*x + a)
^3)/(a^6*b^6)
```

3.766 $\int \frac{1}{(a+bx)^3\left(a^2-b^2x^2\right)^2}\,dx$

Optimal. Leaf size=104

$$\frac{1}{32a^5b(a-bx)}-\frac{1}{8a^5b(a+bx)}-\frac{3}{32a^4b(a+bx)^2}-\frac{1}{12a^3b(a+bx)^3}-\frac{1}{16a^2b(a+bx)^4}+\frac{5\tanh^{-1}\left(\frac{bx}{a}\right)}{32a^6b}$$

```
[Out] 1/(32*a^5*b*(a - b*x)) - 1/(16*a^2*b*(a + b*x)^4) - 1/(12*a^3*b*(a + b*x)^3
) - 3/(32*a^4*b*(a + b*x)^2) - 1/(8*a^5*b*(a + b*x)) + (5*ArcTanh[(b*x)/a])
/(32*a^6*b)
```

Rubi [A] time = 0.0658568, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {627, 44, 208}

$$\frac{1}{32a^5b(a-bx)}-\frac{1}{8a^5b(a+bx)}-\frac{3}{32a^4b(a+bx)^2}-\frac{1}{12a^3b(a+bx)^3}-\frac{1}{16a^2b(a+bx)^4}+\frac{5\tanh^{-1}\left(\frac{bx}{a}\right)}{32a^6b}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(a^2 - b^2*x^2)^2),x]
```

```
[Out] 1/(32*a^5*b*(a - b*x)) - 1/(16*a^2*b*(a + b*x)^4) - 1/(12*a^3*b*(a + b*x)^3
) - 3/(32*a^4*b*(a + b*x)^2) - 1/(8*a^5*b*(a + b*x)) + (5*ArcTanh[(b*x)/a])
/(32*a^6*b)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^3\left(a^2-b^2x^2\right)^2}\,dx = \int \frac{1}{(a-bx)^2(a+bx)^5}\,dx$$
$$= \int \left(\frac{1}{32a^5(a-bx)^2} + \frac{1}{4a^2(a+bx)^5} + \frac{1}{4a^3(a+bx)^4} + \frac{3}{16a^4(a+bx)^3} + \frac{1}{8a^5(a+bx)^2} + \frac{}{32a^5}\right.$$
$$= \frac{1}{32a^5b(a-bx)} - \frac{1}{16a^2b(a+bx)^4} - \frac{1}{12a^3b(a+bx)^3} - \frac{3}{32a^4b(a+bx)^2} - \frac{1}{8a^5b(a+bx)} + \frac{5\int}{}$$
$$= \frac{1}{32a^5b(a-bx)} - \frac{1}{16a^2b(a+bx)^4} - \frac{1}{12a^3b(a+bx)^3} - \frac{3}{32a^4b(a+bx)^2} - \frac{1}{8a^5b(a+bx)} + \frac{5\,\mathrm{t}}{}$$

Mathematica [A] time = 0.028483, size = 112, normalized size = 1.08

$$\frac{70a^3b^2x^2 + 90a^2b^3x^3 - 30a^4bx - 64a^5 + 30ab^4x^4 - 15(a-bx)(a+bx)^4\log(a-bx) + 15(a-bx)(a+bx)^4\log(a+bx)}{192a^6b(a-bx)(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(a^2 - b^2*x^2)^2),x]
```

```
[Out] (-64*a^5 - 30*a^4*b*x + 70*a^3*b^2*x^2 + 90*a^2*b^3*x^3 + 30*a*b^4*x^4 - 15
*(a - b*x)*(a + b*x)^4*Log[a - b*x] + 15*(a - b*x)*(a + b*x)^4*Log[a + b*x]
)/(192*a^6*b*(a - b*x)*(a + b*x)^4)
```

Maple [A] time = 0.051, size = 109, normalized size = 1.1

$$\frac{5\ln(bx+a)}{64a^6b} - \frac{1}{8a^5b(bx+a)} - \frac{3}{32a^4b(bx+a)^2} - \frac{1}{12a^3b(bx+a)^3} - \frac{1}{16ba^2(bx+a)^4} - \frac{5\ln(bx-a)}{64a^6b} - \frac{1}{32a^5b(bx-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(-b^2*x^2+a^2)^2,x)
```

```
[Out] 5/64/a^6/b*ln(b*x+a)-1/8/a^5/b/(b*x+a)-3/32/a^4/b/(b*x+a)^2-1/12/a^3/b/(b*x
+a)^3-1/16/a^2/b/(b*x+a)^4-5/64/a^6/b*ln(b*x-a)-1/32/a^5/b/(b*x-a)
```

Maxima [A] time = 1.06657, size = 182, normalized size = 1.75

$$-\frac{15b^4x^4 + 45ab^3x^3 + 35a^2b^2x^2 - 15a^3bx - 32a^4}{96\left(a^5b^6x^5 + 3a^6b^5x^4 + 2a^7b^4x^3 - 2a^8b^3x^2 - 3a^9b^2x - a^{10}b\right)} + \frac{5\log(bx+a)}{64a^6b} - \frac{5\log(bx-a)}{64a^6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/96*(15*b^4*x^4 + 45*a*b^3*x^3 + 35*a^2*b^2*x^2 - 15*a^3*b*x - 32*a^4)/(a
^5*b^6*x^5 + 3*a^6*b^5*x^4 + 2*a^7*b^4*x^3 - 2*a^8*b^3*x^2 - 3*a^9*b^2*x -
a^10*b) + 5/64*log(b*x + a)/(a^6*b) - 5/64*log(b*x - a)/(a^6*b)
```

Fricas [B] time = 1.76579, size = 471, normalized size = 4.53

$$-\frac{30\,ab^4x^4+90\,a^2b^3x^3+70\,a^3b^2x^2-30\,a^4bx-64\,a^5-15\left(b^5x^5+3\,ab^4x^4+2\,a^2b^3x^3-2\,a^3b^2x^2-3\,a^4bx-a^5\right)\log\left(bx\right.}{192\left(a^6b^6x^5+3\,a^7b^5x^4+2\,a^8b^4x^3-2\,a^9b^3x^2-3\,a^{10}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/192*(30*a*b^4*x^4 + 90*a^2*b^3*x^3 + 70*a^3*b^2*x^2 - 30*a^4*b*x - 64*a^
5 - 15*(b^5*x^5 + 3*a*b^4*x^4 + 2*a^2*b^3*x^3 - 2*a^3*b^2*x^2 - 3*a^4*b*x -
 a^5)*log(b*x + a) + 15*(b^5*x^5 + 3*a*b^4*x^4 + 2*a^2*b^3*x^3 - 2*a^3*b^2*
x^2 - 3*a^4*b*x - a^5)*log(b*x - a))/(a^6*b^6*x^5 + 3*a^7*b^5*x^4 + 2*a^8*b
^4*x^3 - 2*a^9*b^3*x^2 - 3*a^10*b^2*x - a^11*b)
```

Sympy [A] time = 0.864606, size = 133, normalized size = 1.28

$$-\frac{-32a^4-15a^3bx+35a^2b^2x^2+45ab^3x^3+15b^4x^4}{-96a^{10}b-288a^9b^2x-192a^8b^3x^2+192a^7b^4x^3+288a^6b^5x^4+96a^5b^6x^5}+\frac{-\frac{5\log\left(-\frac{a}{b}+x\right)}{64}+\frac{5\log\left(\frac{a}{b}+x\right)}{64}}{a^6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(-b**2*x**2+a**2)**2,x)
```

```
[Out] -(-32*a**4 - 15*a**3*b*x + 35*a**2*b**2*x**2 + 45*a*b**3*x**3 + 15*b**4*x**
4)/(-96*a**10*b - 288*a**9*b**2*x - 192*a**8*b**3*x**2 + 192*a**7*b**4*x**3
 + 288*a**6*b**5*x**4 + 96*a**5*b**6*x**5) + (-5*log(-a/b + x)/64 + 5*log(a
/b + x)/64)/(a**6*b)
```

Giac [A] time = 1.18172, size = 136, normalized size = 1.31

$$\frac{5\,\log\left(|bx+a|\right)}{64\,a^6b}-\frac{5\,\log\left(|bx-a|\right)}{64\,a^6b}-\frac{15\,ab^4x^4+45\,a^2b^3x^3+35\,a^3b^2x^2-15\,a^4bx-32\,a^5}{96\,(bx+a)^4(bx-a)a^6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 5/64*log(abs(b*x + a))/(a^6*b) - 5/64*log(abs(b*x - a))/(a^6*b) - 1/96*(15*
a*b^4*x^4 + 45*a^2*b^3*x^3 + 35*a^3*b^2*x^2 - 15*a^4*b*x - 32*a^5)/((b*x +
a)^4*(b*x - a)*a^6*b)
```

3.767 $\int \frac{(a+bx)^8}{\left(a^2-b^2x^2\right)^3}\,dx$

Optimal. Leaf size=71

$$\frac{16a^5}{b(a-bx)^2}-\frac{80a^4}{b(a-bx)}-\frac{80a^3\log(a-bx)}{b}-31a^2x-4abx^2-\frac{b^2x^3}{3}$$

```
[Out] -31*a^2*x - 4*a*b*x^2 - (b^2*x^3)/3 + (16*a^5)/(b*(a - b*x)^2) - (80*a^4)/(
b*(a - b*x)) - (80*a^3*Log[a - b*x])/b
```

Rubi [A] time = 0.0514683, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {627, 43}

$$\frac{16a^5}{b(a-bx)^2}-\frac{80a^4}{b(a-bx)}-\frac{80a^3\log(a-bx)}{b}-31a^2x-4abx^2-\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^8/(a^2 - b^2*x^2)^3,x]
```

```
[Out] -31*a^2*x - 4*a*b*x^2 - (b^2*x^3)/3 + (16*a^5)/(b*(a - b*x)^2) - (80*a^4)/(
b*(a - b*x)) - (80*a^3*Log[a - b*x])/b
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^8}{\left(a^2-b^2x^2\right)^3}\,dx &= \int \frac{(a+bx)^5}{(a-bx)^3}\,dx\\ &= \int \left(-31a^2-8abx-b^2x^2+\frac{32a^5}{(a-bx)^3}-\frac{80a^4}{(a-bx)^2}+\frac{80a^3}{a-bx}\right)dx\\ &= -31a^2x-4abx^2-\frac{b^2x^3}{3}+\frac{16a^5}{b(a-bx)^2}-\frac{80a^4}{b(a-bx)}-\frac{80a^3\log(a-bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0330701, size = 73, normalized size = 1.03

$$\frac{16a^5}{b(bx-a)^2}+\frac{80a^4}{b(bx-a)}-\frac{80a^3\log(a-bx)}{b}-31a^2x-4abx^2-\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^8/(a^2 - b^2*x^2)^3,x]
```

```
[Out] -31*a^2*x - 4*a*b*x^2 - (b^2*x^3)/3 + (16*a^5)/(b*(-a + b*x)^2) + (80*a^4)/
(b*(-a + b*x)) - (80*a^3*Log[a - b*x])/b
```

Maple [A] time = 0.046, size = 73, normalized size = 1.

$$-\frac{b^2x^3}{3} - 4\,abx^2 - 31\,a^2x + 16\,\frac{a^5}{b\,(bx-a)^2} + 80\,\frac{a^4}{b\,(bx-a)} - 80\,\frac{a^3\ln(bx-a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^8/(-b^2*x^2+a^2)^3,x)
```

```
[Out] -1/3*b^2*x^3-4*a*b*x^2-31*a^2*x+16/b*a^5/(b*x-a)^2+80*a^4/b/(b*x-a)-80/b*a^
3*ln(b*x-a)
```

Maxima [A] time = 1.08801, size = 101, normalized size = 1.42

$$-\frac{1}{3}\,b^2x^3 - 4\,abx^2 - 31\,a^2x - \frac{80\,a^3\log(bx-a)}{b} + \frac{16\left(5\,a^4bx - 4\,a^5\right)}{b^3x^2 - 2\,ab^2x + a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/3*b^2*x^3 - 4*a*b*x^2 - 31*a^2*x - 80*a^3*log(b*x - a)/b + 16*(5*a^4*b*x
 - 4*a^5)/(b^3*x^2 - 2*a*b^2*x + a^2*b)
```

Fricas [A] time = 1.70623, size = 231, normalized size = 3.25

$$-\frac{b^5x^5 + 10\,ab^4x^4 + 70\,a^2b^3x^3 - 174\,a^3b^2x^2 - 147\,a^4bx + 192\,a^5 + 240\left(a^3b^2x^2 - 2\,a^4bx + a^5\right)\log(bx-a)}{3\left(b^3x^2 - 2\,ab^2x + a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(b^5*x^5 + 10*a*b^4*x^4 + 70*a^2*b^3*x^3 - 174*a^3*b^2*x^2 - 147*a^4*b
*x + 192*a^5 + 240*(a^3*b^2*x^2 - 2*a^4*b*x + a^5)*log(b*x - a))/(b^3*x^2 -
 2*a*b^2*x + a^2*b)
```

Sympy [A] time = 0.513292, size = 70, normalized size = 0.99

$$-\frac{80a^3\log(-a+bx)}{b} - 31a^2x - 4abx^2 - \frac{b^2x^3}{3} + \frac{-64a^5 + 80a^4bx}{a^2b - 2ab^2x + b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**8/(-b**2*x**2+a**2)**3,x)
```

```
[Out] -80*a**3*log(-a + b*x)/b - 31*a**2*x - 4*a*b*x**2 - b**2*x**3/3 + (-64*a**5
 + 80*a**4*b*x)/(a**2*b - 2*a*b**2*x + b**3*x**2)
```

Giac [A] time = 1.23523, size = 103, normalized size = 1.45

$$-\frac{80\,a^3\log\left(|bx-a|\right)}{b}+\frac{16\left(5\,a^4bx-4\,a^5\right)}{(bx-a)^2b}-\frac{b^{11}x^3+12\,ab^{10}x^2+93\,a^2b^9x}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -80*a^3*log(abs(b*x - a))/b + 16*(5*a^4*b*x - 4*a^5)/((b*x - a)^2*b) - 1/3*
(b^11*x^3 + 12*a*b^10*x^2 + 93*a^2*b^9*x)/b^9
```

3.768 $\int \frac{(a+bx)^7}{(a^2-b^2x^2)^3} \, dx$

Optimal. Leaf size=60

$$\frac{8a^4}{b(a-bx)^2} - \frac{32a^3}{b(a-bx)} - \frac{24a^2\log(a-bx)}{b} - 7ax - \frac{bx^2}{2}$$

```
[Out] -7*a*x - (b*x^2)/2 + (8*a^4)/(b*(a - b*x)^2) - (32*a^3)/(b*(a - b*x)) - (24
*a^2*Log[a - b*x])/b
```

Rubi [A] time = 0.0406074, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {627, 43}

$$\frac{8a^4}{b(a-bx)^2} - \frac{32a^3}{b(a-bx)} - \frac{24a^2\log(a-bx)}{b} - 7ax - \frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/(a^2 - b^2*x^2)^3,x]
```

```
[Out] -7*a*x - (b*x^2)/2 + (8*a^4)/(b*(a - b*x)^2) - (32*a^3)/(b*(a - b*x)) - (24
*a^2*Log[a - b*x])/b
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^7}{(a^2-b^2x^2)^3} \, dx &= \int \frac{(a+bx)^4}{(a-bx)^3} \, dx \\
&= \int \left(-7a - bx + \frac{16a^4}{(a-bx)^3} - \frac{32a^3}{(a-bx)^2} + \frac{24a^2}{a-bx}\right) dx \\
&= -7ax - \frac{bx^2}{2} + \frac{8a^4}{b(a-bx)^2} - \frac{32a^3}{b(a-bx)} - \frac{24a^2\log(a-bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0280393, size = 62, normalized size = 1.03

$$\frac{8a^4}{b(bx-a)^2} + \frac{32a^3}{b(bx-a)} - \frac{24a^2\log(a-bx)}{b} - 7ax - \frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/(a^2 - b^2*x^2)^3,x]
```

```
[Out] -7*a*x - (b*x^2)/2 + (8*a^4)/(b*(-a + b*x)^2) + (32*a^3)/(b*(-a + b*x)) - (
24*a^2*Log[a - b*x])/b
```

Maple [A] time = 0.047, size = 62, normalized size = 1.

$$-\frac{bx^2}{2} - 7\,ax + 8\,\frac{a^4}{b\,(bx-a)^2} + 32\,\frac{a^3}{b\,(bx-a)} - 24\,\frac{a^2\ln(bx-a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/(-b^2*x^2+a^2)^3,x)
```

```
[Out] -1/2*b*x^2-7*a*x+8*a^4/b/(b*x-a)^2+32/b*a^3/(b*x-a)-24/b*a^2*ln(b*x-a)
```

Maxima [A] time = 1.08461, size = 86, normalized size = 1.43

$$-\frac{1}{2}\,bx^2 - 7\,ax - \frac{24\,a^2\log(bx-a)}{b} + \frac{8\left(4\,a^3bx - 3\,a^4\right)}{b^3x^2 - 2\,ab^2x + a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/2*b*x^2 - 7*a*x - 24*a^2*log(b*x - a)/b + 8*(4*a^3*b*x - 3*a^4)/(b^3*x^2
 - 2*a*b^2*x + a^2*b)
```

Fricas [A] time = 1.62566, size = 203, normalized size = 3.38

$$-\frac{b^4x^4 + 12\,ab^3x^3 - 27\,a^2b^2x^2 - 50\,a^3bx + 48\,a^4 + 48\left(a^2b^2x^2 - 2\,a^3bx + a^4\right)\log(bx-a)}{2\left(b^3x^2 - 2\,ab^2x + a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b^4*x^4 + 12*a*b^3*x^3 - 27*a^2*b^2*x^2 - 50*a^3*b*x + 48*a^4 + 48*(a
^2*b^2*x^2 - 2*a^3*b*x + a^4)*log(b*x - a))/(b^3*x^2 - 2*a*b^2*x + a^2*b)
```

Sympy [A] time = 0.495016, size = 58, normalized size = 0.97

$$-\frac{24a^2\log(-a+bx)}{b} - 7ax - \frac{bx^2}{2} + \frac{-24a^4 + 32a^3bx}{a^2b - 2ab^2x + b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/(-b**2*x**2+a**2)**3,x)
```

```
[Out] -24*a**2*log(-a + b*x)/b - 7*a*x - b*x**2/2 + (-24*a**4 + 32*a**3*b*x)/(a**
2*b - 2*a*b**2*x + b**3*x**2)
```

Giac [A] time = 1.225, size = 88, normalized size = 1.47

$$-\frac{24\,a^2\log\left(|bx-a|\right)}{b}+\frac{8\left(4\,a^3bx-3\,a^4\right)}{(bx-a)^2b}-\frac{b^7x^2+14\,ab^6x}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -24*a^2*log(abs(b*x - a))/b + 8*(4*a^3*b*x - 3*a^4)/((b*x - a)^2*b) - 1/2*(
b^7*x^2 + 14*a*b^6*x)/b^6
```

3.769 $\int \frac{(a+bx)^6}{\left(a^2-b^2x^2\right)^3}\,dx$

Optimal. Leaf size=49

$$\frac{4a^3}{b(a-bx)^2}-\frac{12a^2}{b(a-bx)}-\frac{6a\log(a-bx)}{b}-x$$

```
[Out] -x + (4*a^3)/(b*(a - b*x)^2) - (12*a^2)/(b*(a - b*x)) - (6*a*Log[a - b*x])/
b
```

Rubi [A] time = 0.0295678, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {627, 43}

$$\frac{4a^3}{b(a-bx)^2}-\frac{12a^2}{b(a-bx)}-\frac{6a\log(a-bx)}{b}-x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^6/(a^2 - b^2*x^2)^3,x]
```

```
[Out] -x + (4*a^3)/(b*(a - b*x)^2) - (12*a^2)/(b*(a - b*x)) - (6*a*Log[a - b*x])/
b
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^6}{\left(a^2-b^2x^2\right)^3}\,dx &= \int \frac{(a+bx)^3}{(a-bx)^3}\,dx\\ &= \int \left(-1+\frac{8a^3}{(a-bx)^3}-\frac{12a^2}{(a-bx)^2}+\frac{6a}{a-bx}\right)dx\\ &= -x+\frac{4a^3}{b(a-bx)^2}-\frac{12a^2}{b(a-bx)}-\frac{6a\log(a-bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.044352, size = 41, normalized size = 0.84

$$\frac{4a^2(3bx-2a)}{b(a-bx)^2}-\frac{6a\log(a-bx)}{b}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^6/(a^2 - b^2*x^2)^3,x]
```

```
[Out] -x + (4*a^2*(-2*a + 3*b*x))/(b*(a - b*x)^2) - (6*a*Log[a - b*x])/b
```

Maple [A] time = 0.045, size = 53, normalized size = 1.1

$$-x+4\,{\frac {{a}^{3}}{b \left(bx-a \right) ^{2}}}+12\,{\frac {{a}^{2}}{b \left(bx-a \right) }}-6\,{\frac {a\ln \left(bx-a \right) }{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^6/(-b^2*x^2+a^2)^3,x)
```

```
[Out] -x+4/b*a^3/(b*x-a)^2+12/b*a^2/(b*x-a)-6/b*a*ln(b*x-a)
```

Maxima [A] time = 1.03819, size = 74, normalized size = 1.51

$$-x - \frac{6 \, a \log\left(b x - a\right)}{b} + \frac{4 \, {\left(3 \, a^{2} b x - 2 \, a^{3}\right)}}{b^{3} x^{2} - 2 \, a b^{2} x + a^{2} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -x - 6*a*log(b*x - a)/b + 4*(3*a^2*b*x - 2*a^3)/(b^3*x^2 - 2*a*b^2*x + a^2*
b)
```

Fricas [A] time = 1.74147, size = 167, normalized size = 3.41

$$-\frac{b^{3} x^{3} - 2 \, a b^{2} x^{2} - 11 \, a^{2} b x + 8 \, a^{3} + 6 \, {\left(a b^{2} x^{2} - 2 \, a^{2} b x + a^{3}\right)} \log\left(b x - a\right)}{b^{3} x^{2} - 2 \, a b^{2} x + a^{2} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -(b^3*x^3 - 2*a*b^2*x^2 - 11*a^2*b*x + 8*a^3 + 6*(a*b^2*x^2 - 2*a^2*b*x + a
^3)*log(b*x - a))/(b^3*x^2 - 2*a*b^2*x + a^2*b)
```

Sympy [A] time = 0.426304, size = 46, normalized size = 0.94

$$- \frac{6 a \log{\left (- a + b x \right)}}{b} - x + \frac{- 8 a^{3} + 12 a^{2} b x}{a^{2} b - 2 a b^{2} x + b^{3} x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**6/(-b**2*x**2+a**2)**3,x)
```

```
[Out] -6*a*log(-a + b*x)/b - x + (-8*a**3 + 12*a**2*b*x)/(a**2*b - 2*a*b**2*x + b
**3*x**2)
```

Giac [A] time = 1.17542, size = 62, normalized size = 1.27

$$-x-\frac{6\,a\log\left(|bx-a|\right)}{b}+\frac{4\left(3\,a^{2}bx-2\,a^{3}\right)}{(bx-a)^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -x - 6*a*log(abs(b*x - a))/b + 4*(3*a^2*b*x - 2*a^3)/((b*x - a)^2*b)
```

3.770 $\int \frac{(a+bx)^5}{\left(a^2-b^2x^2\right)^3} dx$

Optimal. Leaf size=43

$$\frac{2a^2}{b(a-bx)^2}-\frac{4a}{b(a-bx)}-\frac{\log(a-bx)}{b}$$

```
[Out] (2*a^2)/(b*(a - b*x)^2) - (4*a)/(b*(a - b*x)) - Log[a - b*x]/b
```

Rubi [A] time = 0.0246914, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {627, 43}

$$\frac{2a^2}{b(a-bx)^2}-\frac{4a}{b(a-bx)}-\frac{\log(a-bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a^2 - b^2*x^2)^3,x]

[Out] (2*a^2)/(b*(a - b*x)^2) - (4*a)/(b*(a - b*x)) - Log[a - b*x]/b
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^5}{\left(a^2-b^2x^2\right)^3} dx &= \int \frac{(a+bx)^2}{(a-bx)^3} dx \\
&= \int \left(\frac{4a^2}{(a-bx)^3}-\frac{4a}{(a-bx)^2}+\frac{1}{a-bx}\right) dx \\
&= \frac{2a^2}{b(a-bx)^2}-\frac{4a}{b(a-bx)}-\frac{\log(a-bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0201887, size = 30, normalized size = 0.7

$$-\frac{\frac{2a(a-2bx)}{(a-bx)^2}+\log(a-bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a^2 - b^2*x^2)^3,x]
```

```
[Out] -(((2*a*(a - 2*b*x))/(a - b*x)^2 + Log[a - b*x])/b)
```

Maple [A] time = 0.044, size = 47, normalized size = 1.1

$$2\,\frac{a^2}{b\,(bx-a)^2}+4\,\frac{a}{b\,(bx-a)}-\frac{\ln(bx-a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(-b^2*x^2+a^2)^3,x)
```

```
[Out] 2/b*a^2/(b*x-a)^2+4/b*a/(b*x-a)-1/b*ln(b*x-a)
```

Maxima [A] time = 1.00333, size = 66, normalized size = 1.53

$$\frac{2\left(2\,abx-a^2\right)}{b^3x^2-2\,ab^2x+a^2b}-\frac{\log(bx-a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 2*(2*a*b*x - a^2)/(b^3*x^2 - 2*a*b^2*x + a^2*b) - log(b*x - a)/b
```

Fricas [A] time = 1.7725, size = 122, normalized size = 2.84

$$\frac{4\,abx-2\,a^2-\left(b^2x^2-2\,abx+a^2\right)\log(bx-a)}{b^3x^2-2\,ab^2x+a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] (4*a*b*x - 2*a^2 - (b^2*x^2 - 2*a*b*x + a^2)*log(b*x - a))/(b^3*x^2 - 2*a*b
^2*x + a^2*b)
```

Sympy [A] time = 0.420473, size = 39, normalized size = 0.91

$$\frac{-2a^2+4abx}{a^2b-2ab^2x+b^3x^2}-\frac{\log(-a+bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(-b**2*x**2+a**2)**3,x)
```

```
[Out] (-2*a**2 + 4*a*b*x)/(a**2*b - 2*a*b**2*x + b**3*x**2) - log(-a + b*x)/b
```

Giac [A] time = 1.1591, size = 54, normalized size = 1.26

$$-\frac{\log\left(|bx-a|\right)}{b}+\frac{2\left(2\,abx-a^2\right)}{(bx-a)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -log(abs(b*x - a))/b + 2*(2*a*b*x - a^2)/((b*x - a)^2*b)
```

3.771 $\int \frac{(a+bx)^4}{\left(a^2-b^2x^2\right)^3} \, dx$

Optimal. Leaf size=10

$$\frac{x}{(a-bx)^2}$$

[Out] x/(a - b*x)^2

Rubi [A] time = 0.0070928, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {627, 34}

$$\frac{x}{(a-bx)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^4/(a^2 - b^2*x^2)^3,x]

[Out] x/(a - b*x)^2

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 34

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(d*x*(
a + b*x)^(m + 1))/(b*(m + 2)), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{\left(a^2-b^2x^2\right)^3} \, dx &= \int \frac{a+bx}{(a-bx)^3} \, dx\\ &= \frac{x}{(a-bx)^2}\end{aligned}$$

Mathematica [A] time = 0.0035456, size = 10, normalized size = 1.

$$\frac{x}{(a-bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^4/(a^2 - b^2*x^2)^3,x]

[Out] x/(a - b*x)^2

Maple [B] time = 0.044, size = 29, normalized size = 2.9

$$\frac{a}{b\left(bx-a\right)^{2}}+\frac{1}{b\left(bx-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(-b^2*x^2+a^2)^3,x)
```

```
[Out] 1/b*a/(b*x-a)^2+1/b/(b*x-a)
```

Maxima [A] time = 1.00771, size = 27, normalized size = 2.7

$$\frac{x}{b^{2}x^{2}-2\,abx+a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] x/(b^2*x^2 - 2*a*b*x + a^2)
```

Fricas [A] time = 1.65899, size = 39, normalized size = 3.9

$$\frac{x}{b^{2}x^{2}-2\,abx+a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] x/(b^2*x^2 - 2*a*b*x + a^2)
```

Sympy [B] time = 0.357719, size = 17, normalized size = 1.7

$$\frac{x}{a^{2}-2abx+b^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(-b**2*x**2+a**2)**3,x)
```

```
[Out] x/(a**2 - 2*a*b*x + b**2*x**2)
```

Giac [A] time = 1.19041, size = 15, normalized size = 1.5

$$\frac{x}{\left(bx-a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(-b^2*x^2+a^2)^3,x, algorithm="giac")

[Out] x/(b*x - a)^2
```

3.772 $\int \frac{(a+bx)^3}{\left(a^2-b^2x^2\right)^3}\,dx$

Optimal. Leaf size=15

$$\frac{1}{2b(a-bx)^2}$$

```
[Out] 1/(2*b*(a - b*x)^2)
```

Rubi [A] time = 0.0066662, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {627, 32}

$$\frac{1}{2b(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(a^2 - b^2*x^2)^3,x]
```

```
[Out] 1/(2*b*(a - b*x)^2)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^3}{\left(a^2-b^2x^2\right)^3}\,dx &= \int \frac{1}{(a-bx)^3}\,dx \\ &= \frac{1}{2b(a-bx)^2} \end{aligned}$$

Mathematica [A] time = 0.001539, size = 15, normalized size = 1.

$$\frac{1}{2b(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(a^2 - b^2*x^2)^3,x]
```

```
[Out] 1/(2*b*(a - b*x)^2)
```

Maple [A] time = 0.039, size = 15, normalized size = 1.

$$\frac{1}{2\,b\,(bx-a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(-b^2*x^2+a^2)^3,x)
```

```
[Out] 1/2/b/(b*x-a)^2
```

Maxima [A] time = 1.02489, size = 32, normalized size = 2.13

$$\frac{1}{2\left(b^3x^2-2\,ab^2x+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/2/(b^3*x^2 - 2*a*b^2*x + a^2*b)
```

Fricas [A] time = 1.6542, size = 47, normalized size = 3.13

$$\frac{1}{2\left(b^3x^2-2\,ab^2x+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/2/(b^3*x^2 - 2*a*b^2*x + a^2*b)
```

Sympy [B] time = 0.347631, size = 24, normalized size = 1.6

$$\frac{1}{2a^2b-4ab^2x+2b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(-b**2*x**2+a**2)**3,x)
```

```
[Out] 1/(2*a**2*b - 4*a*b**2*x + 2*b**3*x**2)
```

Giac [A] time = 1.19786, size = 19, normalized size = 1.27

$$\frac{1}{2\,(bx-a)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/2/((b*x - a)^2*b)
```

3.773 $\int \frac{(a+bx)^2}{(a^2-b^2x^2)^3} dx$

Optimal. Leaf size=54

$$\frac{1}{4a^2b(a-bx)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{4a^3b}+\frac{1}{4ab(a-bx)^2}$$

```
[Out] 1/(4*a*b*(a - b*x)^2) + 1/(4*a^2*b*(a - b*x)) + ArcTanh[(b*x)/a]/(4*a^3*b)
```

Rubi [A] time = 0.0348784, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {627, 44, 208}

$$\frac{1}{4a^2b(a-bx)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{4a^3b}+\frac{1}{4ab(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a^2 - b^2*x^2)^3,x]
```

```
[Out] 1/(4*a*b*(a - b*x)^2) + 1/(4*a^2*b*(a - b*x)) + ArcTanh[(b*x)/a]/(4*a^3*b)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^2}{\left(a^2-b^2x^2\right)^3}\,dx = \int \frac{1}{(a-bx)^3(a+bx)}\,dx$$
$$= \int \left(\frac{1}{2a(a-bx)^3} + \frac{1}{4a^2(a-bx)^2} + \frac{1}{4a^2\left(a^2-b^2x^2\right)}\right)dx$$
$$= \frac{1}{4ab(a-bx)^2} + \frac{1}{4a^2b(a-bx)} + \frac{\int \frac{1}{a^2-b^2x^2}\,dx}{4a^2}$$
$$= \frac{1}{4ab(a-bx)^2} + \frac{1}{4a^2b(a-bx)} + \frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{4a^3b}$$

Mathematica [A] time = 0.0153305, size = 62, normalized size = 1.15

$$\frac{2a(2a-bx)+(a-bx)^2(-\log(a-bx))+(a-bx)^2\log(a+bx)}{8a^3b(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a^2 - b^2*x^2)^3,x]
```

```
[Out] (2*a*(2*a - b*x) - (a - b*x)^2*Log[a - b*x] + (a - b*x)^2*Log[a + b*x])/(8*
a^3*b*(a - b*x)^2)
```

Maple [A] time = 0.046, size = 66, normalized size = 1.2

$$\frac{\ln(bx+a)}{8\,ba^3} - \frac{\ln(bx-a)}{8\,ba^3} - \frac{1}{4\,ba^2\,(bx-a)} + \frac{1}{4\,ab\,(bx-a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b^2*x^2+a^2)^3,x)
```

```
[Out] 1/8/b/a^3*ln(b*x+a)-1/8/b/a^3*ln(b*x-a)-1/4/b/a^2/(b*x-a)+1/4/b/a/(b*x-a)^2
```

Maxima [A] time = 1.07009, size = 90, normalized size = 1.67

$$-\frac{bx-2\,a}{4\left(a^2b^3x^2-2\,a^3b^2x+a^4b\right)} + \frac{\log(bx+a)}{8\,a^3b} - \frac{\log(bx-a)}{8\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b*x - 2*a)/(a^2*b^3*x^2 - 2*a^3*b^2*x + a^4*b) + 1/8*log(b*x + a)/(a^
3*b) - 1/8*log(b*x - a)/(a^3*b)
```

Fricas [A] time = 1.75228, size = 192, normalized size = 3.56

$$-\frac{2\,abx-4\,a^2-\left(b^2x^2-2\,abx+a^2\right)\log(bx+a)+\left(b^2x^2-2\,abx+a^2\right)\log(bx-a)}{8\left(a^3b^3x^2-2\,a^4b^2x+a^5b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*a*b*x - 4*a^2 - (b^2*x^2 - 2*a*b*x + a^2)*log(b*x + a) + (b^2*x^2 -
 2*a*b*x + a^2)*log(b*x - a))/(a^3*b^3*x^2 - 2*a^4*b^2*x + a^5*b)
```

Sympy [A] time = 0.496473, size = 58, normalized size = 1.07

$$-\frac{-2a+bx}{4a^4b-8a^3b^2x+4a^2b^3x^2}-\frac{\frac{\log\left(-\frac{a}{b}+x\right)}{8}-\frac{\log\left(\frac{a}{b}+x\right)}{8}}{a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b**2*x**2+a**2)**3,x)
```

```
[Out] -(-2*a + b*x)/(4*a**4*b - 8*a**3*b**2*x + 4*a**2*b**3*x**2) - (log(-a/b + x
)/8 - log(a/b + x)/8)/(a**3*b)
```

Giac [A] time = 1.17014, size = 81, normalized size = 1.5

$$\frac{\log\left(|bx+a|\right)}{8\,a^3b}-\frac{\log\left(|bx-a|\right)}{8\,a^3b}-\frac{abx-2\,a^2}{4\,(bx-a)^2a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*log(abs(b*x + a))/(a^3*b) - 1/8*log(abs(b*x - a))/(a^3*b) - 1/4*(a*b*x
- 2*a^2)/((b*x - a)^2*a^3*b)
```

3.774 $\int \frac{a+bx}{(a^2-b^2x^2)^3}\,dx$

Optimal. Leaf size=71

$$\frac{1}{4a^3b(a-bx)}-\frac{1}{8a^3b(a+bx)}+\frac{1}{8a^2b(a-bx)^2}+\frac{3\tanh^{-1}\left(\frac{bx}{a}\right)}{8a^4b}$$

```
[Out] 1/(8*a^2*b*(a - b*x)^2) + 1/(4*a^3*b*(a - b*x)) - 1/(8*a^3*b*(a + b*x)) + (
3*ArcTanh[(b*x)/a])/(8*a^4*b)
```

Rubi [A] time = 0.0418414, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {627, 44, 208}

$$\frac{1}{4a^3b(a-bx)}-\frac{1}{8a^3b(a+bx)}+\frac{1}{8a^2b(a-bx)^2}+\frac{3\tanh^{-1}\left(\frac{bx}{a}\right)}{8a^4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a^2 - b^2*x^2)^3,x]
```

```
[Out] 1/(8*a^2*b*(a - b*x)^2) + 1/(4*a^3*b*(a - b*x)) - 1/(8*a^3*b*(a + b*x)) + (
3*ArcTanh[(b*x)/a])/(8*a^4*b)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+bx}{\left(a^2-b^2x^2\right)^3}\,dx = \int \frac{1}{(a-bx)^3(a+bx)^2}\,dx$$
$$= \int \left(\frac{1}{4a^2(a-bx)^3} + \frac{1}{4a^3(a-bx)^2} + \frac{1}{8a^3(a+bx)^2} + \frac{3}{8a^3\left(a^2-b^2x^2\right)}\right) dx$$
$$= \frac{1}{8a^2b(a-bx)^2} + \frac{1}{4a^3b(a-bx)} - \frac{1}{8a^3b(a+bx)} + \frac{3\int \frac{1}{a^2-b^2x^2}\,dx}{8a^3}$$
$$= \frac{1}{8a^2b(a-bx)^2} + \frac{1}{4a^3b(a-bx)} - \frac{1}{8a^3b(a+bx)} + \frac{3\tanh^{-1}\left(\frac{bx}{a}\right)}{8a^4b}$$

Mathematica [A] time = 0.0320932, size = 65, normalized size = 0.92

$$\frac{\frac{2a\left(2a^2+3abx-3b^2x^2\right)}{(a-bx)^2(a+bx)} - 3\log(a-bx) + 3\log(a+bx)}{16a^4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a^2 - b^2*x^2)^3,x]
```

```
[Out] ((2*a*(2*a^2 + 3*a*b*x - 3*b^2*x^2))/((a - b*x)^2*(a + b*x)) - 3*Log[a - b*
x] + 3*Log[a + b*x])/(16*a^4*b)
```

Maple [A] time = 0.048, size = 81, normalized size = 1.1

$$\frac{3\ln(bx+a)}{16\,a^4b} - \frac{1}{8\,a^3b\,(bx+a)} - \frac{3\ln(bx-a)}{16\,a^4b} - \frac{1}{4\,a^3b\,(bx-a)} + \frac{1}{8\,ba^2\,(bx-a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-b^2*x^2+a^2)^3,x)
```

```
[Out] 3/16/a^4/b*ln(b*x+a)-1/8/a^3/b/(b*x+a)-3/16/a^4/b*ln(b*x-a)-1/4/b/a^3/(b*x-
a)+1/8/b/a^2/(b*x-a)^2
```

Maxima [A] time = 1.11657, size = 122, normalized size = 1.72

$$-\frac{3\,b^2x^2 - 3\,abx - 2\,a^2}{8\left(a^3b^4x^3 - a^4b^3x^2 - a^5b^2x + a^6b\right)} + \frac{3\log(bx+a)}{16\,a^4b} - \frac{3\log(bx-a)}{16\,a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*b^2*x^2 - 3*a*b*x - 2*a^2)/(a^3*b^4*x^3 - a^4*b^3*x^2 - a^5*b^2*x +
 a^6*b) + 3/16*log(b*x + a)/(a^4*b) - 3/16*log(b*x - a)/(a^4*b)
```

Fricas [B] time = 1.80226, size = 269, normalized size = 3.79

$$-\frac{6\,ab^2x^2 - 6\,a^2bx - 4\,a^3 - 3\left(b^3x^3 - ab^2x^2 - a^2bx + a^3\right)\log\left(bx + a\right) + 3\left(b^3x^3 - ab^2x^2 - a^2bx + a^3\right)\log\left(bx - a\right)}{16\left(a^4b^4x^3 - a^5b^3x^2 - a^6b^2x + a^7b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*a*b^2*x^2 - 6*a^2*b*x - 4*a^3 - 3*(b^3*x^3 - a*b^2*x^2 - a^2*b*x +
 a^3)*log(b*x + a) + 3*(b^3*x^3 - a*b^2*x^2 - a^2*b*x + a^3)*log(b*x - a))/
(a^4*b^4*x^3 - a^5*b^3*x^2 - a^6*b^2*x + a^7*b)
```

Sympy [A] time = 0.58414, size = 87, normalized size = 1.23

$$-\frac{-2a^2 - 3abx + 3b^2x^2}{8a^6b - 8a^5b^2x - 8a^4b^3x^2 + 8a^3b^4x^3} - \frac{\frac{3\log\left(-\frac{a}{b}+x\right)}{16} - \frac{3\log\left(\frac{a}{b}+x\right)}{16}}{a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b**2*x**2+a**2)**3,x)
```

```
[Out] -(-2*a**2 - 3*a*b*x + 3*b**2*x**2)/(8*a**6*b - 8*a**5*b**2*x - 8*a**4*b**3*
x**2 + 8*a**3*b**4*x**3) - (3*log(-a/b + x)/16 - 3*log(a/b + x)/16)/(a**4*b
)
```

Giac [A] time = 1.23955, size = 107, normalized size = 1.51

$$\frac{3\,\log\left(|bx + a|\right)}{16\,a^4b} - \frac{3\,\log\left(|bx - a|\right)}{16\,a^4b} - \frac{3\,ab^2x^2 - 3\,a^2bx - 2\,a^3}{8\,(bx + a)(bx - a)^2a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 3/16*log(abs(b*x + a))/(a^4*b) - 3/16*log(abs(b*x - a))/(a^4*b) - 1/8*(3*a*
b^2*x^2 - 3*a^2*b*x - 2*a^3)/((b*x + a)*(b*x - a)^2*a^4*b)
```

3.775 $\int \frac{1}{(a+bx)\left(a^2-b^2x^2\right)^3}\,dx$

Optimal. Leaf size=105

$$\frac{1}{8a^5b(a-bx)}-\frac{3}{16a^5b(a+bx)}+\frac{1}{32a^4b(a-bx)^2}-\frac{3}{32a^4b(a+bx)^2}-\frac{1}{24a^3b(a+bx)^3}+\frac{5\tanh^{-1}\left(\frac{bx}{a}\right)}{16a^6b}$$

```
[Out] 1/(32*a^4*b*(a - b*x)^2) + 1/(8*a^5*b*(a - b*x)) - 1/(24*a^3*b*(a + b*x)^3)
 - 3/(32*a^4*b*(a + b*x)^2) - 3/(16*a^5*b*(a + b*x)) + (5*ArcTanh[(b*x)/a])
/(16*a^6*b)
```

Rubi [A] time = 0.0655311, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {627, 44, 208}

$$\frac{1}{8a^5b(a-bx)}-\frac{3}{16a^5b(a+bx)}+\frac{1}{32a^4b(a-bx)^2}-\frac{3}{32a^4b(a+bx)^2}-\frac{1}{24a^3b(a+bx)^3}+\frac{5\tanh^{-1}\left(\frac{bx}{a}\right)}{16a^6b}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(a^2 - b^2*x^2)^3),x]
```

```
[Out] 1/(32*a^4*b*(a - b*x)^2) + 1/(8*a^5*b*(a - b*x)) - 1/(24*a^3*b*(a + b*x)^3)
 - 3/(32*a^4*b*(a + b*x)^2) - 3/(16*a^5*b*(a + b*x)) + (5*ArcTanh[(b*x)/a])
/(16*a^6*b)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)\left(a^2-b^2x^2\right)^3}\,dx = \int \frac{1}{(a-bx)^3(a+bx)^4}\,dx$$
$$= \int \left(\frac{1}{16a^4(a-bx)^3} + \frac{1}{8a^5(a-bx)^2} + \frac{1}{8a^3(a+bx)^4} + \frac{3}{16a^4(a+bx)^3} + \frac{3}{16a^5(a+bx)^2} + \frac{}{1}\right.$$
$$= \frac{1}{32a^4b(a-bx)^2} + \frac{1}{8a^5b(a-bx)} - \frac{1}{24a^3b(a+bx)^3} - \frac{3}{32a^4b(a+bx)^2} - \frac{3}{16a^5b(a+bx)} + \frac{5}{}$$
$$= \frac{1}{32a^4b(a-bx)^2} + \frac{1}{8a^5b(a-bx)} - \frac{1}{24a^3b(a+bx)^3} - \frac{3}{32a^4b(a+bx)^2} - \frac{3}{16a^5b(a+bx)} + \frac{5}{}$$

Mathematica [A] time = 0.0429631, size = 87, normalized size = 0.83

$$\frac{-\frac{2a\left(-25a^2b^2x^2-25a^3bx+8a^4+15ab^3x^3+15b^4x^4\right)}{(a-bx)^2(a+bx)^3}-15\log(a-bx)+15\log(a+bx)}{96a^6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(a^2 - b^2*x^2)^3),x]
```

```
[Out] ((-2*a*(8*a^4 - 25*a^3*b*x - 25*a^2*b^2*x^2 + 15*a*b^3*x^3 + 15*b^4*x^4))/(
(a - b*x)^2*(a + b*x)^3) - 15*Log[a - b*x] + 15*Log[a + b*x])/(96*a^6*b)
```

Maple [A] time = 0.051, size = 111, normalized size = 1.1

$$\frac{5\ln(bx+a)}{32a^6b} - \frac{3}{16a^5b(bx+a)} - \frac{3}{32a^4b(bx+a)^2} - \frac{1}{24a^3b(bx+a)^3} - \frac{5\ln(bx-a)}{32a^6b} - \frac{1}{8a^5b(bx-a)} + \frac{1}{32a^4b(bx-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(-b^2*x^2+a^2)^3,x)
```

```
[Out] 5/32/a^6/b*ln(b*x+a)-3/16/a^5/b/(b*x+a)-3/32/a^4/b/(b*x+a)^2-1/24/a^3/b/(b*
x+a)^3-5/32/a^6/b*ln(b*x-a)-1/8/a^5/b/(b*x-a)+1/32/a^4/b/(b*x-a)^2
```

Maxima [A] time = 1.29289, size = 178, normalized size = 1.7

$$-\frac{15\,b^4x^4+15\,ab^3x^3-25\,a^2b^2x^2-25\,a^3bx+8\,a^4}{48\left(a^5b^6x^5+a^6b^5x^4-2\,a^7b^4x^3-2\,a^8b^3x^2+a^9b^2x+a^{10}b\right)} + \frac{5\,\log(bx+a)}{32\,a^6b} - \frac{5\,\log(bx-a)}{32\,a^6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(15*b^4*x^4 + 15*a*b^3*x^3 - 25*a^2*b^2*x^2 - 25*a^3*b*x + 8*a^4)/(a^
5*b^6*x^5 + a^6*b^5*x^4 - 2*a^7*b^4*x^3 - 2*a^8*b^3*x^2 + a^9*b^2*x + a^10*
b) + 5/32*log(b*x + a)/(a^6*b) - 5/32*log(b*x - a)/(a^6*b)
```

Fricas [B] time = 1.81665, size = 454, normalized size = 4.32

$$-\frac{30\,ab^4x^4+30\,a^2b^3x^3-50\,a^3b^2x^2-50\,a^4bx+16\,a^5-15\left(b^5x^5+ab^4x^4-2\,a^2b^3x^3-2\,a^3b^2x^2+a^4bx+a^5\right)\log\left(bx+a\right)+}{96\left(a^6b^6x^5+a^7b^5x^4-2\,a^8b^4x^3-2\,a^9b^3x^2+a^{10}b^2x+a^{11}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/96*(30*a*b^4*x^4 + 30*a^2*b^3*x^3 - 50*a^3*b^2*x^2 - 50*a^4*b*x + 16*a^5
 - 15*(b^5*x^5 + a*b^4*x^4 - 2*a^2*b^3*x^3 - 2*a^3*b^2*x^2 + a^4*b*x + a^5)
*log(b*x + a) + 15*(b^5*x^5 + a*b^4*x^4 - 2*a^2*b^3*x^3 - 2*a^3*b^2*x^2 + a
^4*b*x + a^5)*log(b*x - a))/(a^6*b^6*x^5 + a^7*b^5*x^4 - 2*a^8*b^4*x^3 - 2*
a^9*b^3*x^2 + a^10*b^2*x + a^11*b)
```

Sympy [A] time = 0.858839, size = 134, normalized size = 1.28

$$-\frac{8a^4-25a^3bx-25a^2b^2x^2+15ab^3x^3+15b^4x^4}{48a^{10}b+48a^9b^2x-96a^8b^3x^2-96a^7b^4x^3+48a^6b^5x^4+48a^5b^6x^5}-\frac{\frac{5\log\left(-\frac{a}{b}+x\right)}{32}-\frac{5\log\left(\frac{a}{b}+x\right)}{32}}{a^6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b**2*x**2+a**2)**3,x)
```

```
[Out] -(8*a**4 - 25*a**3*b*x - 25*a**2*b**2*x**2 + 15*a*b**3*x**3 + 15*b**4*x**4)
/(48*a**10*b + 48*a**9*b**2*x - 96*a**8*b**3*x**2 - 96*a**7*b**4*x**3 + 48*
a**6*b**5*x**4 + 48*a**5*b**6*x**5) - (5*log(-a/b + x)/32 - 5*log(a/b + x)/
32)/(a**6*b)
```

Giac [A] time = 1.17042, size = 136, normalized size = 1.3

$$\frac{5\,\log\left(|bx+a|\right)}{32\,a^6b}-\frac{5\,\log\left(|bx-a|\right)}{32\,a^6b}-\frac{15\,ab^4x^4+15\,a^2b^3x^3-25\,a^3b^2x^2-25\,a^4bx+8\,a^5}{48\,(bx+a)^3(bx-a)^2a^6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 5/32*log(abs(b*x + a))/(a^6*b) - 5/32*log(abs(b*x - a))/(a^6*b) - 1/48*(15*
a*b^4*x^4 + 15*a^2*b^3*x^3 - 25*a^3*b^2*x^2 - 25*a^4*b*x + 8*a^5)/((b*x + a
)^3*(b*x - a)^2*a^6*b)
```

3.776 $\int \frac{1}{(a+bx)^2\left(a^2-b^2x^2\right)^3}\,dx$

Optimal. Leaf size=122

$$\frac{5}{64a^6b(a-bx)}-\frac{5}{32a^6b(a+bx)}+\frac{1}{64a^5b(a-bx)^2}-\frac{3}{32a^5b(a+bx)^2}-\frac{1}{16a^4b(a+bx)^3}-\frac{1}{32a^3b(a+bx)^4}+\frac{15\tanh^{-1}\left(\right.}{64a^7b}$$

```
[Out] 1/(64*a^5*b*(a - b*x)^2) + 5/(64*a^6*b*(a - b*x)) - 1/(32*a^3*b*(a + b*x)^4
) - 1/(16*a^4*b*(a + b*x)^3) - 3/(32*a^5*b*(a + b*x)^2) - 5/(32*a^6*b*(a +
b*x)) + (15*ArcTanh[(b*x)/a])/(64*a^7*b)
```

Rubi [A] time = 0.0788462, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {627, 44, 208}

$$\frac{5}{64a^6b(a-bx)}-\frac{5}{32a^6b(a+bx)}+\frac{1}{64a^5b(a-bx)^2}-\frac{3}{32a^5b(a+bx)^2}-\frac{1}{16a^4b(a+bx)^3}-\frac{1}{32a^3b(a+bx)^4}+\frac{15\tanh^{-1}\left(\right.}{64a^7b}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(a^2 - b^2*x^2)^3),x]
```


```
[Out] 1/(64*a^5*b*(a - b*x)^2) + 5/(64*a^6*b*(a - b*x)) - 1/(32*a^3*b*(a + b*x)^4
) - 1/(16*a^4*b*(a + b*x)^3) - 3/(32*a^5*b*(a + b*x)^2) - 5/(32*a^6*b*(a +
b*x)) + (15*ArcTanh[(b*x)/a])/(64*a^7*b)
```


Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^2\left(a^2-b^2x^2\right)^3}\,dx=\int \frac{1}{(a-bx)^3(a+bx)^5}\,dx$$
$$=\int \left(\frac{1}{32a^5(a-bx)^3}+\frac{5}{64a^6(a-bx)^2}+\frac{1}{8a^3(a+bx)^5}+\frac{3}{16a^4(a+bx)^4}+\frac{3}{16a^5(a+bx)^3}+\frac{}{32}\right.$$
$$=\frac{1}{64a^5b(a-bx)^2}+\frac{5}{64a^6b(a-bx)}-\frac{1}{32a^3b(a+bx)^4}-\frac{1}{16a^4b(a+bx)^3}-\frac{3}{32a^5b(a+bx)^2}-$$
$$=\frac{1}{64a^5b(a-bx)^2}+\frac{5}{64a^6b(a-bx)}-\frac{1}{32a^3b(a+bx)^4}-\frac{1}{16a^4b(a+bx)^3}-\frac{3}{32a^5b(a+bx)^2}-$$

Mathematica [A] time = 0.0531594, size = 98, normalized size = 0.8

$$\frac{\frac{2a\left(50a^3b^2x^2+10a^2b^3x^3+17a^4bx-16a^5-30ab^4x^4-15b^5x^5\right)}{(a-bx)^2(a+bx)^4}-15\log(a-bx)+15\log(a+bx)}{128a^7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(a^2 - b^2*x^2)^3),x]
```

```
[Out] ((2*a*(-16*a^5 + 17*a^4*b*x + 50*a^3*b^2*x^2 + 10*a^2*b^3*x^3 - 30*a*b^4*x^
4 - 15*b^5*x^5))/((a - b*x)^2*(a + b*x)^4) - 15*Log[a - b*x] + 15*Log[a + b
*x])/(128*a^7*b)
```

Maple [A] time = 0.049, size = 126, normalized size = 1.

$$\frac{15\ln(bx+a)}{128\,ba^7}-\frac{5}{32\,a^6b\,(bx+a)}-\frac{3}{32\,a^5b\,(bx+a)^2}-\frac{1}{16\,a^4b\,(bx+a)^3}-\frac{1}{32\,a^3b\,(bx+a)^4}-\frac{15\ln(bx-a)}{128\,ba^7}-\frac{5}{64\,a^6b\,(bx-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(-b^2*x^2+a^2)^3,x)
```

```
[Out] 15/128/a^7/b*ln(b*x+a)-5/32/a^6/b/(b*x+a)-3/32/a^5/b/(b*x+a)^2-1/16/a^4/b/(
b*x+a)^3-1/32/a^3/b/(b*x+a)^4-15/128/a^7/b*ln(b*x-a)-5/64/a^6/b/(b*x-a)+1/6
4/a^5/b/(b*x-a)^2
```

Maxima [A] time = 1.31346, size = 211, normalized size = 1.73

$$-\frac{15\,b^5x^5+30\,ab^4x^4-10\,a^2b^3x^3-50\,a^3b^2x^2-17\,a^4bx+16\,a^5}{64\left(a^6b^7x^6+2\,a^7b^6x^5-a^8b^5x^4-4\,a^9b^4x^3-a^{10}b^3x^2+2\,a^{11}b^2x+a^{12}b\right)}+\frac{15\log(bx+a)}{128\,a^7b}-\frac{15\log(bx-a)}{128\,a^7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/64*(15*b^5*x^5 + 30*a*b^4*x^4 - 10*a^2*b^3*x^3 - 50*a^3*b^2*x^2 - 17*a^4
*b*x + 16*a^5)/(a^6*b^7*x^6 + 2*a^7*b^6*x^5 - a^8*b^5*x^4 - 4*a^9*b^4*x^3 -
 a^10*b^3*x^2 + 2*a^11*b^2*x + a^12*b) + 15/128*log(b*x + a)/(a^7*b) - 15/1
```

```
28*log(b*x - a)/(a^7*b)
```

Fricas [B] time = 1.70633, size = 547, normalized size = 4.48

$$-\frac{30\,ab^5x^5+60\,a^2b^4x^4-20\,a^3b^3x^3-100\,a^4b^2x^2-34\,a^5bx+32\,a^6-15\left(b^6x^6+2\,ab^5x^5-a^2b^4x^4-4\,a^3b^3x^3-a^4b^2x^2+\right.}{128\left(a^7b^7x^6+2\,a^8b^6x^5-a^9b^5x^4-4\,a^{10}b^4x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(30*a*b^5*x^5 + 60*a^2*b^4*x^4 - 20*a^3*b^3*x^3 - 100*a^4*b^2*x^2 -
34*a^5*b*x + 32*a^6 - 15*(b^6*x^6 + 2*a*b^5*x^5 - a^2*b^4*x^4 - 4*a^3*b^3*x
^3 - a^4*b^2*x^2 + 2*a^5*b*x + a^6)*log(b*x + a) + 15*(b^6*x^6 + 2*a*b^5*x^
5 - a^2*b^4*x^4 - 4*a^3*b^3*x^3 - a^4*b^2*x^2 + 2*a^5*b*x + a^6)*log(b*x -
a))/(a^7*b^7*x^6 + 2*a^8*b^6*x^5 - a^9*b^5*x^4 - 4*a^10*b^4*x^3 - a^11*b^3*
x^2 + 2*a^12*b^2*x + a^13*b)
```

Sympy [A] time = 1.04247, size = 158, normalized size = 1.3

$$-\frac{16a^5-17a^4bx-50a^3b^2x^2-10a^2b^3x^3+30ab^4x^4+15b^5x^5}{64a^{12}b+128a^{11}b^2x-64a^{10}b^3x^2-256a^9b^4x^3-64a^8b^5x^4+128a^7b^6x^5+64a^6b^7x^6}-\frac{\frac{15\log\left(-\frac{a}{b}+x\right)}{128}-\frac{15\log\left(\frac{a}{b}+x\right)}{128}}{a^7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(-b**2*x**2+a**2)**3,x)
```

```
[Out] -(16*a**5 - 17*a**4*b*x - 50*a**3*b**2*x**2 - 10*a**2*b**3*x**3 + 30*a*b**4
*x**4 + 15*b**5*x**5)/(64*a**12*b + 128*a**11*b**2*x - 64*a**10*b**3*x**2 -
 256*a**9*b**4*x**3 - 64*a**8*b**5*x**4 + 128*a**7*b**6*x**5 + 64*a**6*b**7
*x**6) - (15*log(-a/b + x)/128 - 15*log(a/b + x)/128)/(a**7*b)
```

Giac [A] time = 1.20814, size = 169, normalized size = 1.39

$$-\frac{15\,\log\left(\left|-\frac{2\,a}{bx+a}+1\right|\right)}{128\,a^7b}+\frac{\frac{24\,a}{bx+a}-11}{256\,a^7b\left(\frac{2\,a}{bx+a}-1\right)^2}-\frac{\frac{5\,a^6b^{11}}{bx+a}+\frac{3\,a^7b^{11}}{(bx+a)^2}+\frac{2\,a^8b^{11}}{(bx+a)^3}+\frac{a^9b^{11}}{(bx+a)^4}}{32\,a^{12}b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -15/128*log(abs(-2*a/(b*x + a) + 1))/(a^7*b) + 1/256*(24*a/(b*x + a) - 11)/
(a^7*b*(2*a/(b*x + a) - 1)^2) - 1/32*(5*a^6*b^11/(b*x + a) + 3*a^7*b^11/(b*
x + a)^2 + 2*a^8*b^11/(b*x + a)^3 + a^9*b^11/(b*x + a)^4)/(a^12*b^12)
```

3.777 $\int (a+bx)^4\sqrt{a^2-b^2x^2}\,dx$

Optimal. Leaf size=173

$$\frac{21}{16}a^4x\sqrt{a^2-b^2x^2}-\frac{7a^3\left(a^2-b^2x^2\right)^{3/2}}{8b}-\frac{21a^2(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{40b}-\frac{3a(a+bx)^2\left(a^2-b^2x^2\right)^{3/2}}{10b}-\frac{(a+bx)^3\left(a^2-b^2x^2\right)^{3/2}}{6b}$$

```
[Out] (21*a^4*x*Sqrt[a^2 - b^2*x^2])/16 - (7*a^3*(a^2 - b^2*x^2)^(3/2))/(8*b) - (
21*a^2*(a + b*x)*(a^2 - b^2*x^2)^(3/2))/(40*b) - (3*a*(a + b*x)^2*(a^2 - b^
2*x^2)^(3/2))/(10*b) - ((a + b*x)^3*(a^2 - b^2*x^2)^(3/2))/(6*b) + (21*a^6*
ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]])/(16*b)
```

Rubi [A] time = 0.0757016, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {671, 641, 195, 217, 203}

$$\frac{21}{16}a^4x\sqrt{a^2-b^2x^2}-\frac{7a^3\left(a^2-b^2x^2\right)^{3/2}}{8b}-\frac{21a^2(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{40b}-\frac{3a(a+bx)^2\left(a^2-b^2x^2\right)^{3/2}}{10b}-\frac{(a+bx)^3\left(a^2-b^2x^2\right)^{3/2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*Sqrt[a^2 - b^2*x^2],x]
```

```
[Out] (21*a^4*x*Sqrt[a^2 - b^2*x^2])/16 - (7*a^3*(a^2 - b^2*x^2)^(3/2))/(8*b) - (
21*a^2*(a + b*x)*(a^2 - b^2*x^2)^(3/2))/(40*b) - (3*a*(a + b*x)^2*(a^2 - b^
2*x^2)^(3/2))/(10*b) - ((a + b*x)^3*(a^2 - b^2*x^2)^(3/2))/(6*b) + (21*a^6*
ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]])/(16*b)
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+bx)^4\sqrt{a^2-b^2x^2}\,dx = -\frac{(a+bx)^3\left(a^2-b^2x^2\right)^{3/2}}{6b}+\frac{1}{2}(3a)\int (a+bx)^3\sqrt{a^2-b^2x^2}\,dx$$
$$= -\frac{3a(a+bx)^2\left(a^2-b^2x^2\right)^{3/2}}{10b}-\frac{(a+bx)^3\left(a^2-b^2x^2\right)^{3/2}}{6b}+\frac{1}{10}\left(21a^2\right)\int (a+bx)^2\sqrt{a^2-b^2x^2}$$
$$= -\frac{21a^2(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{40b}-\frac{3a(a+bx)^2\left(a^2-b^2x^2\right)^{3/2}}{10b}-\frac{(a+bx)^3\left(a^2-b^2x^2\right)^{3/2}}{6b}+\frac{1}{8}$$
$$= -\frac{7a^3\left(a^2-b^2x^2\right)^{3/2}}{8b}-\frac{21a^2(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{40b}-\frac{3a(a+bx)^2\left(a^2-b^2x^2\right)^{3/2}}{10b}-(a+bx)$$
$$= \frac{21}{16}a^4x\sqrt{a^2-b^2x^2}-\frac{7a^3\left(a^2-b^2x^2\right)^{3/2}}{8b}-\frac{21a^2(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{40b}-\frac{3a(a+bx)^2\left(a^2-b\right.}{10b}$$
$$= \frac{21}{16}a^4x\sqrt{a^2-b^2x^2}-\frac{7a^3\left(a^2-b^2x^2\right)^{3/2}}{8b}-\frac{21a^2(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{40b}-\frac{3a(a+bx)^2\left(a^2-b\right.}{10b}$$
$$= \frac{21}{16}a^4x\sqrt{a^2-b^2x^2}-\frac{7a^3\left(a^2-b^2x^2\right)^{3/2}}{8b}-\frac{21a^2(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{40b}-\frac{3a(a+bx)^2\left(a^2-b\right.}{10b}$$

Mathematica [A] time = 0.21049, size = 123, normalized size = 0.71

$$\frac{\sqrt{a^2-b^2x^2}\left(\sqrt{1-\frac{b^2x^2}{a^2}}\left(256a^3b^2x^2+350a^2b^3x^3-75a^4bx-448a^5+192ab^4x^4+40b^5x^5\right)+315a^5\sin^{-1}\left(\frac{bx}{a}\right)\right)}{240b\sqrt{1-\frac{b^2x^2}{a^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*Sqrt[a^2 - b^2*x^2],x]
```

```
[Out] (Sqrt[a^2 - b^2*x^2]*(Sqrt[1 - (b^2*x^2)/a^2]*(-448*a^5 - 75*a^4*b*x + 256*
a^3*b^2*x^2 + 350*a^2*b^3*x^3 + 192*a*b^4*x^4 + 40*b^5*x^5) + 315*a^5*ArcSi
n[(b*x)/a]))/(240*b*Sqrt[1 - (b^2*x^2)/a^2])
```

Maple [A] time = 0.058, size = 139, normalized size = 0.8

$$-\frac{b^2x^3}{6}\left(-b^2x^2+a^2\right)^{\frac{3}{2}}-\frac{13\,a^2x}{8}\left(-b^2x^2+a^2\right)^{\frac{3}{2}}+\frac{21\,a^4x}{16}\sqrt{-b^2x^2+a^2}+\frac{21\,a^6}{16}\arctan\left(x\sqrt{b^2}\frac{1}{\sqrt{-b^2x^2+a^2}}\right)\frac{1}{\sqrt{b^2}}-\frac{4\,abx}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(-b^2*x^2+a^2)^(1/2),x)
```

```
[Out] -1/6*b^2*x^3*(-b^2*x^2+a^2)^(3/2)-13/8*a^2*x*(-b^2*x^2+a^2)^(3/2)+21/16*a^4
*x*(-b^2*x^2+a^2)^(1/2)+21/16*a^6/(b^2)^(1/2)*arctan((b^2)^(1/2)*x/(-b^2*x^
2+a^2)^(1/2))-4/5*a*b*x^2*(-b^2*x^2+a^2)^(3/2)-28/15*a^3*(-b^2*x^2+a^2)^(3/
```

```
2)/b
```

Maxima [A] time = 1.60884, size = 177, normalized size = 1.02

$$-\frac{1}{6}\left(-b^2x^2+a^2\right)^{\frac{3}{2}}b^2x^3+\frac{21\,a^6\arcsin\left(\frac{b^2x}{\sqrt{a^2b^2}}\right)}{16\,\sqrt{b^2}}+\frac{21}{16}\,\sqrt{-b^2x^2+a^2}a^4x-\frac{4}{5}\left(-b^2x^2+a^2\right)^{\frac{3}{2}}abx^2-\frac{13}{8}\left(-b^2x^2+a^2\right)^{\frac{3}{2}}a^2x-\frac{28}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(-b^2*x^2+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*(-b^2*x^2 + a^2)^(3/2)*b^2*x^3 + 21/16*a^6*arcsin(b^2*x/sqrt(a^2*b^2))
/sqrt(b^2) + 21/16*sqrt(-b^2*x^2 + a^2)*a^4*x - 4/5*(-b^2*x^2 + a^2)^(3/2)*
a*b*x^2 - 13/8*(-b^2*x^2 + a^2)^(3/2)*a^2*x - 28/15*(-b^2*x^2 + a^2)^(3/2)*
a^3/b
```

Fricas [A] time = 1.83845, size = 234, normalized size = 1.35

$$-\frac{630\,a^6\arctan\left(-\frac{a-\sqrt{-b^2x^2+a^2}}{bx}\right)-\left(40\,b^5x^5+192\,ab^4x^4+350\,a^2b^3x^3+256\,a^3b^2x^2-75\,a^4bx-448\,a^5\right)\sqrt{-b^2x^2+a^2}}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(-b^2*x^2+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/240*(630*a^6*arctan(-(a - sqrt(-b^2*x^2 + a^2))/(b*x)) - (40*b^5*x^5 + 1
92*a*b^4*x^4 + 350*a^2*b^3*x^3 + 256*a^3*b^2*x^2 - 75*a^4*b*x - 448*a^5)*sq
rt(-b^2*x^2 + a^2))/b
```

Sympy [C] time = 12.3879, size = 706, normalized size = 4.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(-b**2*x**2+a**2)**(1/2),x)
```

```
[Out] a**4*Piecewise((-I*a**2*acosh(b*x/a)/(2*b) - I*a*x/(2*sqrt(-1 + b**2*x**2/a
**2)) + I*b**2*x**3/(2*a*sqrt(-1 + b**2*x**2/a**2)), Abs(b**2*x**2)/Abs(a**
2) > 1), (a**2*asin(b*x/a)/(2*b) + a*x*sqrt(1 - b**2*x**2/a**2)/2, True)) +
 4*a**3*b*Piecewise((x**2*sqrt(a**2)/2, Eq(b**2, 0)), (-(a**2 - b**2*x**2)*
*(3/2)/(3*b**2), True)) + 6*a**2*b**2*Piecewise((-I*a**4*acosh(b*x/a)/(8*b*
*3) + I*a**3*x/(8*b**2*sqrt(-1 + b**2*x**2/a**2)) - 3*I*a*x**3/(8*sqrt(-1 +
 b**2*x**2/a**2)) + I*b**2*x**5/(4*a*sqrt(-1 + b**2*x**2/a**2)), Abs(b**2*x
**2)/Abs(a**2) > 1), (a**4*asin(b*x/a)/(8*b**3) - a**3*x/(8*b**2*sqrt(1 - b
**2*x**2/a**2)) + 3*a*x**3/(8*sqrt(1 - b**2*x**2/a**2)) - b**2*x**5/(4*a*sq
rt(1 - b**2*x**2/a**2)), True)) + 4*a*b**3*Piecewise((-2*a**4*sqrt(a**2 - b
**2*x**2)/(15*b**4) - a**2*x**2*sqrt(a**2 - b**2*x**2)/(15*b**2) + x**4*sqr
t(a**2 - b**2*x**2)/5, Ne(b, 0)), (x**4*sqrt(a**2)/4, True)) + b**4*Piecewi
se((-I*a**6*acosh(b*x/a)/(16*b**5) + I*a**5*x/(16*b**4*sqrt(-1 + b**2*x**2/
```

```
a**2)) - I*a**3*x**3/(48*b**2*sqrt(-1 + b**2*x**2/a**2)) - 5*I*a*x**5/(24*s
qrt(-1 + b**2*x**2/a**2)) + I*b**2*x**7/(6*a*sqrt(-1 + b**2*x**2/a**2)), Ab
s(b**2*x**2)/Abs(a**2) > 1), (a**6*asin(b*x/a)/(16*b**5) - a**5*x/(16*b**4*
sqrt(1 - b**2*x**2/a**2)) + a**3*x**3/(48*b**2*sqrt(1 - b**2*x**2/a**2)) +
5*a*x**5/(24*sqrt(1 - b**2*x**2/a**2)) - b**2*x**7/(6*a*sqrt(1 - b**2*x**2/
a**2)), True))
```

Giac [A] time = 1.16998, size = 123, normalized size = 0.71

$$\frac{21\,a^6\arcsin\left(\frac{bx}{a}\right)\operatorname{sgn}(a)\operatorname{sgn}(b)}{16\,|b|}-\frac{1}{240}\left(\frac{448\,a^5}{b}+\left(75\,a^4-2\left(128\,a^3b+\left(175\,a^2b^2+4\left(5\,b^4x+24\,ab^3\right)x\right)x\right)x\right)x\right)\sqrt{-b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(-b^2*x^2+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] 21/16*a^6*arcsin(b*x/a)*sgn(a)*sgn(b)/abs(b) - 1/240*(448*a^5/b + (75*a^4 -
 2*(128*a^3*b + (175*a^2*b^2 + 4*(5*b^4*x + 24*a*b^3)*x)*x)*x)*x)*sqrt(-b^2
*x^2 + a^2)
```

3.778 $\int (a+bx)^3\sqrt{a^2-b^2x^2}\,dx$

Optimal. Leaf size=140

$$\frac{7}{8}a^3x\sqrt{a^2-b^2x^2}-\frac{7a^2\left(a^2-b^2x^2\right)^{3/2}}{12b}-\frac{7a(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{20b}-\frac{(a+bx)^2\left(a^2-b^2x^2\right)^{3/2}}{5b}+\frac{7a^5\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{8b}$$

```
[Out] (7*a^3*x*Sqrt[a^2 - b^2*x^2])/8 - (7*a^2*(a^2 - b^2*x^2)^(3/2))/(12*b) - (7
*a*(a + b*x)*(a^2 - b^2*x^2)^(3/2))/(20*b) - ((a + b*x)^2*(a^2 - b^2*x^2)^(
3/2))/(5*b) + (7*a^5*ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]])/(8*b)
```

Rubi [A] time = 0.0501128, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {671, 641, 195, 217, 203}

$$\frac{7}{8}a^3x\sqrt{a^2-b^2x^2}-\frac{7a^2\left(a^2-b^2x^2\right)^{3/2}}{12b}-\frac{7a(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{20b}-\frac{(a+bx)^2\left(a^2-b^2x^2\right)^{3/2}}{5b}+\frac{7a^5\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*Sqrt[a^2 - b^2*x^2],x]
```

```
[Out] (7*a^3*x*Sqrt[a^2 - b^2*x^2])/8 - (7*a^2*(a^2 - b^2*x^2)^(3/2))/(12*b) - (7
*a*(a + b*x)*(a^2 - b^2*x^2)^(3/2))/(20*b) - ((a + b*x)^2*(a^2 - b^2*x^2)^(
3/2))/(5*b) + (7*a^5*ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]])/(8*b)
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+bx)^3\sqrt{a^2-b^2x^2}\,dx = -\frac{(a+bx)^2\left(a^2-b^2x^2\right)^{3/2}}{5b} + \frac{1}{5}(7a)\int (a+bx)^2\sqrt{a^2-b^2x^2}\,dx$$
$$= -\frac{7a(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{20b} - \frac{(a+bx)^2\left(a^2-b^2x^2\right)^{3/2}}{5b} + \frac{1}{4}\left(7a^2\right)\int (a+bx)\sqrt{a^2-b^2x^2}\,dx$$
$$= -\frac{7a^2\left(a^2-b^2x^2\right)^{3/2}}{12b} - \frac{7a(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{20b} - \frac{(a+bx)^2\left(a^2-b^2x^2\right)^{3/2}}{5b} + \frac{1}{4}\left(7a^3\right)\int \sqrt{}$$
$$= \frac{7}{8}a^3x\sqrt{a^2-b^2x^2} - \frac{7a^2\left(a^2-b^2x^2\right)^{3/2}}{12b} - \frac{7a(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{20b} - \frac{(a+bx)^2\left(a^2-b^2x^2\right)^{3}}{5b}$$
$$= \frac{7}{8}a^3x\sqrt{a^2-b^2x^2} - \frac{7a^2\left(a^2-b^2x^2\right)^{3/2}}{12b} - \frac{7a(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{20b} - \frac{(a+bx)^2\left(a^2-b^2x^2\right)^{3}}{5b}$$
$$= \frac{7}{8}a^3x\sqrt{a^2-b^2x^2} - \frac{7a^2\left(a^2-b^2x^2\right)^{3/2}}{12b} - \frac{7a(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{20b} - \frac{(a+bx)^2\left(a^2-b^2x^2\right)^{3}}{5b}$$

Mathematica [A] time = 0.218372, size = 112, normalized size = 0.8

$$\frac{\sqrt{a^2-b^2x^2}\left(\sqrt{1-\frac{b^2x^2}{a^2}}\left(112a^2b^2x^2+15a^3bx-136a^4+90ab^3x^3+24b^4x^4\right)+105a^4\sin^{-1}\left(\frac{bx}{a}\right)\right)}{120b\sqrt{1-\frac{b^2x^2}{a^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*Sqrt[a^2 - b^2*x^2],x]
```

```
[Out] (Sqrt[a^2 - b^2*x^2]*(Sqrt[1 - (b^2*x^2)/a^2]*(-136*a^4 + 15*a^3*b*x + 112*
a^2*b^2*x^2 + 90*a*b^3*x^3 + 24*b^4*x^4) + 105*a^4*ArcSin[(b*x)/a]))/(120*b
*Sqrt[1 - (b^2*x^2)/a^2])
```

Maple [A] time = 0.05, size = 114, normalized size = 0.8

$$-\frac{bx^2}{5}\left(-b^2x^2+a^2\right)^{\frac{3}{2}} - \frac{17a^2}{15b}\left(-b^2x^2+a^2\right)^{\frac{3}{2}} - \frac{3ax}{4}\left(-b^2x^2+a^2\right)^{\frac{3}{2}} + \frac{7xa^3}{8}\sqrt{-b^2x^2+a^2} + \frac{7a^5}{8}\arctan\left(x\sqrt{b^2}\frac{1}{\sqrt{-b^2x^2+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(-b^2*x^2+a^2)^(1/2),x)
```

```
[Out] -1/5*b*x^2*(-b^2*x^2+a^2)^(3/2)-17/15*a^2*(-b^2*x^2+a^2)^(3/2)/b-3/4*a*x*(-
b^2*x^2+a^2)^(3/2)+7/8*a^3*x*(-b^2*x^2+a^2)^(1/2)+7/8*a^5/(b^2)^(1/2)*arcta
n((b^2)^(1/2)*x/(-b^2*x^2+a^2)^(1/2))
```

Maxima [A] time = 1.5549, size = 143, normalized size = 1.02

$$\frac{7\,a^{5}\arcsin\left(\frac{b^{2}x}{\sqrt{a^{2}b^{2}}}\right)}{8\,\sqrt{b^{2}}}+\frac{7}{8}\,\sqrt{-b^{2}x^{2}+a^{2}}a^{3}x-\frac{1}{5}\left(-b^{2}x^{2}+a^{2}\right)^{\frac{3}{2}}bx^{2}-\frac{3}{4}\left(-b^{2}x^{2}+a^{2}\right)^{\frac{3}{2}}ax-\frac{17\left(-b^{2}x^{2}+a^{2}\right)^{\frac{3}{2}}a^{2}}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(-b^2*x^2+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 7/8*a^5*arcsin(b^2*x/sqrt(a^2*b^2))/sqrt(b^2) + 7/8*sqrt(-b^2*x^2 + a^2)*a^
3*x - 1/5*(-b^2*x^2 + a^2)^(3/2)*b*x^2 - 3/4*(-b^2*x^2 + a^2)^(3/2)*a*x - 1
7/15*(-b^2*x^2 + a^2)^(3/2)*a^2/b
```

Fricas [A] time = 1.81893, size = 208, normalized size = 1.49

$$-\frac{210\,a^{5}\arctan\left(-\frac{a-\sqrt{-b^{2}x^{2}+a^{2}}}{bx}\right)-\left(24\,b^{4}x^{4}+90\,ab^{3}x^{3}+112\,a^{2}b^{2}x^{2}+15\,a^{3}bx-136\,a^{4}\right)\sqrt{-b^{2}x^{2}+a^{2}}}{120\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(-b^2*x^2+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/120*(210*a^5*arctan(-(a - sqrt(-b^2*x^2 + a^2))/(b*x)) - (24*b^4*x^4 + 9
0*a*b^3*x^3 + 112*a^2*b^2*x^2 + 15*a^3*b*x - 136*a^4)*sqrt(-b^2*x^2 + a^2))
/b
```

Sympy [C] time = 6.80029, size = 442, normalized size = 3.16

$$a^{3}\left(\begin{cases}-\frac{ia^{2}\operatorname{acosh}\left(\frac{bx}{a}\right)}{2b}-\frac{iax}{2\sqrt{-1+\frac{b^{2}x^{2}}{a^{2}}}}+\frac{ib^{2}x^{3}}{2a\sqrt{-1+\frac{b^{2}x^{2}}{a^{2}}}} & \text{for}\ \frac{|b^{2}x^{2}|}{|a^{2}|}>1\\ \frac{a^{2}\operatorname{asin}\left(\frac{bx}{a}\right)}{2b}+\frac{ax\sqrt{1-\frac{b^{2}x^{2}}{a^{2}}}}{2} & \text{otherwise}\end{cases}\right)+3a^{2}b\left(\begin{cases}\frac{x^{2}\sqrt{a^{2}}}{2} & \text{for}\ b^{2}=0\\ -\frac{\left(a^{2}-b^{2}x^{2}\right)^{\frac{3}{2}}}{3b^{2}} & \text{otherwise}\end{cases}\right)+3ab^{2}\left(\begin{cases}-\frac{ia^{4}\operatorname{acosh}\left(\frac{bx}{a}\right)}{8b^{3}} \cdots\\ \frac{a^{4}\operatorname{asin}\left(\frac{bx}{a}\right)}{8b^{3}}- \cdots\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(-b**2*x**2+a**2)**(1/2),x)
```

```
[Out] a**3*Piecewise((-I*a**2*acosh(b*x/a)/(2*b) - I*a*x/(2*sqrt(-1 + b**2*x**2/a
**2)) + I*b**2*x**3/(2*a*sqrt(-1 + b**2*x**2/a**2)), Abs(b**2*x**2)/Abs(a**
2) > 1), (a**2*asin(b*x/a)/(2*b) + a*x*sqrt(1 - b**2*x**2/a**2)/2, True)) +
 3*a**2*b*Piecewise((x**2*sqrt(a**2)/2, Eq(b**2, 0)), (-(a**2 - b**2*x**2)*
*(3/2)/(3*b**2), True)) + 3*a*b**2*Piecewise((-I*a**4*acosh(b*x/a)/(8*b**3)
 + I*a**3*x/(8*b**2*sqrt(-1 + b**2*x**2/a**2)) - 3*I*a*x**3/(8*sqrt(-1 + b*
*2*x**2/a**2)) + I*b**2*x**5/(4*a*sqrt(-1 + b**2*x**2/a**2)), Abs(b**2*x**2
)/Abs(a**2) > 1), (a**4*asin(b*x/a)/(8*b**3) - a**3*x/(8*b**2*sqrt(1 - b**2
*x**2/a**2)) + 3*a*x**3/(8*sqrt(1 - b**2*x**2/a**2)) - b**2*x**5/(4*a*sqrt(
1 - b**2*x**2/a**2)), True)) + b**3*Piecewise((-2*a**4*sqrt(a**2 - b**2*x**
2)/(15*b**4) - a**2*x**2*sqrt(a**2 - b**2*x**2)/(15*b**2) + x**4*sqrt(a**2
- b**2*x**2)/5, Ne(b, 0)), (x**4*sqrt(a**2)/4, True))
```

Giac [A] time = 1.1991, size = 109, normalized size = 0.78

$$\frac{7\,a^5\arcsin\left(\frac{bx}{a}\right)\mathrm{sgn}\left(a\right)\mathrm{sgn}\left(b\right)}{8\,|b|}-\frac{1}{120}\,\sqrt{-b^2x^2+a^2}\left(\frac{136\,a^4}{b}-\left(15\,a^3+2\left(56\,a^2b+3\left(4\,b^3x+15\,ab^2\right)x\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(-b^2*x^2+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] 7/8*a^5*arcsin(b*x/a)*sgn(a)*sgn(b)/abs(b) - 1/120*sqrt(-b^2*x^2 + a^2)*(13
6*a^4/b - (15*a^3 + 2*(56*a^2*b + 3*(4*b^3*x + 15*a*b^2)*x)*x)*x)
```

3.779 $\int (a+bx)^2\sqrt{a^2-b^2x^2}\,dx$

Optimal. Leaf size=107

$$\frac{5}{8}a^2x\sqrt{a^2-b^2x^2}-\frac{5a\left(a^2-b^2x^2\right)^{3/2}}{12b}-\frac{(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{4b}+\frac{5a^4\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{8b}$$

```
[Out] (5*a^2*x*Sqrt[a^2 - b^2*x^2])/8 - (5*a*(a^2 - b^2*x^2)^(3/2))/(12*b) - ((a
+ b*x)*(a^2 - b^2*x^2)^(3/2))/(4*b) + (5*a^4*ArcTan[(b*x)/Sqrt[a^2 - b^2*x^
2]])/(8*b)
```

Rubi [A] time = 0.0339475, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {671, 641, 195, 217, 203}

$$\frac{5}{8}a^2x\sqrt{a^2-b^2x^2}-\frac{5a\left(a^2-b^2x^2\right)^{3/2}}{12b}-\frac{(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{4b}+\frac{5a^4\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*Sqrt[a^2 - b^2*x^2],x]
```

```
[Out] (5*a^2*x*Sqrt[a^2 - b^2*x^2])/8 - (5*a*(a^2 - b^2*x^2)^(3/2))/(12*b) - ((a
+ b*x)*(a^2 - b^2*x^2)^(3/2))/(4*b) + (5*a^4*ArcTan[(b*x)/Sqrt[a^2 - b^2*x^
2]])/(8*b)
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int (a+bx)^2\sqrt{a^2-b^2x^2}\,dx &= -\frac{(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{4b}+\frac{1}{4}(5a)\int (a+bx)\sqrt{a^2-b^2x^2}\,dx\\
&= -\frac{5a\left(a^2-b^2x^2\right)^{3/2}}{12b}-\frac{(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{4b}+\frac{1}{4}\left(5a^2\right)\int \sqrt{a^2-b^2x^2}\,dx\\
&= \frac{5}{8}a^2x\sqrt{a^2-b^2x^2}-\frac{5a\left(a^2-b^2x^2\right)^{3/2}}{12b}-\frac{(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{4b}+\frac{1}{8}\left(5a^4\right)\int \frac{1}{\sqrt{a^2-b^2x^2}}\,dx\\
&= \frac{5}{8}a^2x\sqrt{a^2-b^2x^2}-\frac{5a\left(a^2-b^2x^2\right)^{3/2}}{12b}-\frac{(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{4b}+\frac{1}{8}\left(5a^4\right)\text{Subst}\left(\int \frac{1}{1+b^2\cdots}\right.\\
&= \frac{5}{8}a^2x\sqrt{a^2-b^2x^2}-\frac{5a\left(a^2-b^2x^2\right)^{3/2}}{12b}-\frac{(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{4b}+\frac{5a^4\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{8b}
\end{aligned}$$

Mathematica [A] time = 0.179711, size = 101, normalized size = 0.94

$$\frac{\sqrt{a^2-b^2x^2}\left(\sqrt{1-\frac{b^2x^2}{a^2}}\left(9a^2bx-16a^3+16ab^2x^2+6b^3x^3\right)+15a^3\sin^{-1}\left(\frac{bx}{a}\right)\right)}{24b\sqrt{1-\frac{b^2x^2}{a^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*Sqrt[a^2 - b^2*x^2],x]
```

```
[Out] (Sqrt[a^2 - b^2*x^2]*(Sqrt[1 - (b^2*x^2)/a^2]*(-16*a^3 + 9*a^2*b*x + 16*a*b
^2*x^2 + 6*b^3*x^3) + 15*a^3*ArcSin[(b*x)/a]))/(24*b*Sqrt[1 - (b^2*x^2)/a^2
])
```

Maple [A] time = 0.048, size = 91, normalized size = 0.9

$$-\frac{x}{4}\left(-b^2x^2+a^2\right)^{\frac{3}{2}}+\frac{5a^2x}{8}\sqrt{-b^2x^2+a^2}+\frac{5a^4}{8}\arctan\left(x\sqrt{b^2}\frac{1}{\sqrt{-b^2x^2+a^2}}\right)\frac{1}{\sqrt{b^2}}-\frac{2a}{3b}\left(-b^2x^2+a^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(-b^2*x^2+a^2)^(1/2),x)
```

```
[Out] -1/4*x*(-b^2*x^2+a^2)^(3/2)+5/8*a^2*x*(-b^2*x^2+a^2)^(1/2)+5/8*a^4/(b^2)^(1
/2)*arctan((b^2)^(1/2)*x/(-b^2*x^2+a^2)^(1/2))-2/3*a*(-b^2*x^2+a^2)^(3/2)/b
```

Maxima [A] time = 1.67389, size = 112, normalized size = 1.05

$$\frac{5a^4\arcsin\left(\frac{b^2x}{\sqrt{a^2b^2}}\right)}{8\sqrt{b^2}}+\frac{5}{8}\sqrt{-b^2x^2+a^2}a^2x-\frac{1}{4}\left(-b^2x^2+a^2\right)^{\frac{3}{2}}x-\frac{2\left(-b^2x^2+a^2\right)^{\frac{3}{2}}a}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b^2*x^2+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/8*a^4*arcsin(b^2*x/sqrt(a^2*b^2))/sqrt(b^2) + 5/8*sqrt(-b^2*x^2 + a^2)*a^
2*x - 1/4*(-b^2*x^2 + a^2)^(3/2)*x - 2/3*(-b^2*x^2 + a^2)^(3/2)*a/b
```

Fricas [A] time = 1.82214, size = 177, normalized size = 1.65

$$-\frac{30\,a^4\arctan\left(-\frac{a-\sqrt{-b^2x^2+a^2}}{bx}\right)-\left(6\,b^3x^3+16\,ab^2x^2+9\,a^2bx-16\,a^3\right)\sqrt{-b^2x^2+a^2}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b^2*x^2+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(30*a^4*arctan(-(a - sqrt(-b^2*x^2 + a^2))/(b*x)) - (6*b^3*x^3 + 16*a
*b^2*x^2 + 9*a^2*b*x - 16*a^3)*sqrt(-b^2*x^2 + a^2))/b
```

Sympy [C] time = 6.28458, size = 354, normalized size = 3.31

$$a^2\left(\begin{cases}-\frac{ia^2\operatorname{acosh}\left(\frac{bx}{a}\right)}{2b}-\frac{iax}{2\sqrt{-1+\frac{b^2x^2}{a^2}}}+\frac{ib^2x^3}{2a\sqrt{-1+\frac{b^2x^2}{a^2}}} & \text{for}\ \frac{|b^2x^2|}{|a^2|}>1\\ \frac{a^2\operatorname{asin}\left(\frac{bx}{a}\right)}{2b}+\frac{ax\sqrt{1-\frac{b^2x^2}{a^2}}}{2} & \text{otherwise}\end{cases}\right)+2ab\left(\begin{cases}\frac{x^2\sqrt{a^2}}{2} & \text{for}\ b^2=0\\ -\frac{\left(a^2-b^2x^2\right)^{\frac{3}{2}}}{3b^2} & \text{otherwise}\end{cases}\right)+b^2\left(\begin{cases}-\frac{ia^4\operatorname{acosh}\left(\frac{bx}{a}\right)}{8b^3}+\dots\\ \frac{a^4\operatorname{asin}\left(\frac{bx}{a}\right)}{8b^3}-\frac{\dots}{8b^2\dots}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(-b**2*x**2+a**2)**(1/2),x)
```

```
[Out] a**2*Piecewise((-I*a**2*acosh(b*x/a)/(2*b) - I*a*x/(2*sqrt(-1 + b**2*x**2/a
**2)) + I*b**2*x**3/(2*a*sqrt(-1 + b**2*x**2/a**2)), Abs(b**2*x**2)/Abs(a**
2) > 1), (a**2*asin(b*x/a)/(2*b) + a*x*sqrt(1 - b**2*x**2/a**2)/2, True)) +
 2*a*b*Piecewise((x**2*sqrt(a**2)/2, Eq(b**2, 0)), (-(a**2 - b**2*x**2)**(3
/2)/(3*b**2), True)) + b**2*Piecewise((-I*a**4*acosh(b*x/a)/(8*b**3) + I*a*
*3*x/(8*b**2*sqrt(-1 + b**2*x**2/a**2)) - 3*I*a*x**3/(8*sqrt(-1 + b**2*x**2
/a**2)) + I*b**2*x**5/(4*a*sqrt(-1 + b**2*x**2/a**2)), Abs(b**2*x**2)/Abs(a
**2) > 1), (a**4*asin(b*x/a)/(8*b**3) - a**3*x/(8*b**2*sqrt(1 - b**2*x**2/a
**2)) + 3*a*x**3/(8*sqrt(1 - b**2*x**2/a**2)) - b**2*x**5/(4*a*sqrt(1 - b**
2*x**2/a**2)), True))
```

Giac [A] time = 1.24354, size = 93, normalized size = 0.87

$$\frac{5\,a^4\arcsin\left(\frac{bx}{a}\right)\operatorname{sgn}\left(a\right)\operatorname{sgn}\left(b\right)}{8\,|b|}-\frac{1}{24}\,\sqrt{-b^2x^2+a^2}\left(\frac{16\,a^3}{b}-\left(9\,a^2+2\left(3\,b^2x+8\,ab\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b^2*x^2+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] 5/8*a^4*arcsin(b*x/a)*sgn(a)*sgn(b)/abs(b) - 1/24*sqrt(-b^2*x^2 + a^2)*(16*
a^3/b - (9*a^2 + 2*(3*b^2*x + 8*a*b)*x)*x)
```

3.780 $\int (a+bx)\sqrt{a^2-b^2x^2}\,dx$

Optimal. Leaf size=76

$$\frac{1}{2}ax\sqrt{a^2-b^2x^2}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{3b}+\frac{a^3\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{2b}$$

```
[Out] (a*x*Sqrt[a^2 - b^2*x^2])/2 - (a^2 - b^2*x^2)^(3/2)/(3*b) + (a^3*ArcTan[(b*
x)/Sqrt[a^2 - b^2*x^2]])/(2*b)
```

Rubi [A] time = 0.0201091, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {641, 195, 217, 203}

$$\frac{1}{2}ax\sqrt{a^2-b^2x^2}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{3b}+\frac{a^3\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*Sqrt[a^2 - b^2*x^2],x]
```

```
[Out] (a*x*Sqrt[a^2 - b^2*x^2])/2 - (a^2 - b^2*x^2)^(3/2)/(3*b) + (a^3*ArcTan[(b*
x)/Sqrt[a^2 - b^2*x^2]])/(2*b)
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+bx)\sqrt{a^2-b^2x^2}\,dx = -\frac{\left(a^2-b^2x^2\right)^{3/2}}{3b} + a\int \sqrt{a^2-b^2x^2}\,dx$$
$$= \frac{1}{2}ax\sqrt{a^2-b^2x^2} - \frac{\left(a^2-b^2x^2\right)^{3/2}}{3b} + \frac{1}{2}a^3\int \frac{1}{\sqrt{a^2-b^2x^2}}\,dx$$
$$= \frac{1}{2}ax\sqrt{a^2-b^2x^2} - \frac{\left(a^2-b^2x^2\right)^{3/2}}{3b} + \frac{1}{2}a^3\,\mathrm{Subst}\left(\int \frac{1}{1+b^2x^2}\,dx, x, \frac{x}{\sqrt{a^2-b^2x^2}}\right)$$
$$= \frac{1}{2}ax\sqrt{a^2-b^2x^2} - \frac{\left(a^2-b^2x^2\right)^{3/2}}{3b} + \frac{a^3\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{2b}$$

Mathematica [A] time = 0.0765633, size = 90, normalized size = 1.18

$$\frac{\sqrt{a^2-b^2x^2}\left(\left(-2a^2+3abx+2b^2x^2\right)\sqrt{1-\frac{b^2x^2}{a^2}}+3a^2\sin^{-1}\left(\frac{bx}{a}\right)\right)}{6b\sqrt{1-\frac{b^2x^2}{a^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*Sqrt[a^2 - b^2*x^2],x]
```

```
[Out] (Sqrt[a^2 - b^2*x^2]*((-2*a^2 + 3*a*b*x + 2*b^2*x^2)*Sqrt[1 - (b^2*x^2)/a^2
] + 3*a^2*ArcSin[(b*x)/a]))/(6*b*Sqrt[1 - (b^2*x^2)/a^2])
```

Maple [A] time = 0.048, size = 71, normalized size = 0.9

$$-\frac{1}{3\,b}\left(-b^2x^2+a^2\right)^{\frac{3}{2}}+\frac{ax}{2}\sqrt{-b^2x^2+a^2}+\frac{a^3}{2}\arctan\left(x\sqrt{b^2}\frac{1}{\sqrt{-b^2x^2+a^2}}\right)\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(-b^2*x^2+a^2)^(1/2),x)
```

```
[Out] -1/3*(-b^2*x^2+a^2)^(3/2)/b+1/2*a*x*(-b^2*x^2+a^2)^(1/2)+1/2*a^3/(b^2)^(1/2
)*arctan((b^2)^(1/2)*x/(-b^2*x^2+a^2)^(1/2))
```

Maxima [A] time = 1.7378, size = 85, normalized size = 1.12

$$\frac{a^3\arcsin\left(\frac{b^2x}{\sqrt{a^2b^2}}\right)}{2\sqrt{b^2}}+\frac{1}{2}\sqrt{-b^2x^2+a^2}ax-\frac{\left(-b^2x^2+a^2\right)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b^2*x^2+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*a^3*arcsin(b^2*x/sqrt(a^2*b^2))/sqrt(b^2) + 1/2*sqrt(-b^2*x^2 + a^2)*a*
x - 1/3*(-b^2*x^2 + a^2)^(3/2)/b
```

Fricas [A] time = 1.81042, size = 150, normalized size = 1.97

$$-\frac{6\,a^3 \arctan\left(-\frac{a-\sqrt{-b^2x^2+a^2}}{bx}\right) - \left(2\,b^2x^2 + 3\,abx - 2\,a^2\right)\sqrt{-b^2x^2 + a^2}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b^2*x^2+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(6*a^3*arctan(-(a - sqrt(-b^2*x^2 + a^2))/(b*x)) - (2*b^2*x^2 + 3*a*b*
x - 2*a^2)*sqrt(-b^2*x^2 + a^2))/b
```

Sympy [C] time = 3.34282, size = 146, normalized size = 1.92

$$a\left(\begin{cases} -\frac{ia^2 \operatorname{acosh}\left(\frac{bx}{a}\right)}{2b} - \frac{iax}{2\sqrt{-1+\frac{b^2x^2}{a^2}}} + \frac{ib^2x^3}{2a\sqrt{-1+\frac{b^2x^2}{a^2}}} & \text{for}\ \frac{|b^2x^2|}{|a^2|} > 1 \\ \frac{a^2 \operatorname{asin}\left(\frac{bx}{a}\right)}{2b} + \frac{ax\sqrt{1-\frac{b^2x^2}{a^2}}}{2} & \text{otherwise} \end{cases}\right) + b\left(\begin{cases} \frac{x^2\sqrt{a^2}}{2} & \text{for}\ b^2 = 0 \\ -\frac{\left(a^2-b^2x^2\right)^{\frac{3}{2}}}{3b^2} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b**2*x**2+a**2)**(1/2),x)
```

```
[Out] a*Piecewise((-I*a**2*acosh(b*x/a)/(2*b) - I*a*x/(2*sqrt(-1 + b**2*x**2/a**2
)) + I*b**2*x**3/(2*a*sqrt(-1 + b**2*x**2/a**2)), Abs(b**2*x**2)/Abs(a**2)
> 1), (a**2*asin(b*x/a)/(2*b) + a*x*sqrt(1 - b**2*x**2/a**2)/2, True)) + b*
Piecewise((x**2*sqrt(a**2)/2, Eq(b**2, 0)), (-(a**2 - b**2*x**2)**(3/2)/(3*
b**2), True))
```

Giac [A] time = 1.20121, size = 76, normalized size = 1.

$$\frac{a^3 \arcsin\left(\frac{bx}{a}\right) \operatorname{sgn}(a) \operatorname{sgn}(b)}{2\,|b|} + \frac{1}{6}\sqrt{-b^2x^2 + a^2}\left((2\,bx + 3\,a)x - \frac{2\,a^2}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b^2*x^2+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*a^3*arcsin(b*x/a)*sgn(a)*sgn(b)/abs(b) + 1/6*sqrt(-b^2*x^2 + a^2)*((2*b
*x + 3*a)*x - 2*a^2/b)
```

3.781 $\int \frac{\sqrt{a^2-b^2x^2}}{a+bx}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{a^2-b^2x^2}}{b}+\frac{a\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{b}$$

```
[Out] Sqrt[a^2 - b^2*x^2]/b + (a*ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]])/b
```

Rubi [A] time = 0.0161951, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {665, 217, 203}

$$\frac{\sqrt{a^2-b^2x^2}}{b}+\frac{a\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 - b^2*x^2]/(a + b*x),x]
```

```
[Out] Sqrt[a^2 - b^2*x^2]/b + (a*ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]])/b
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2-b^2x^2}}{a+bx}\,dx &= \frac{\sqrt{a^2-b^2x^2}}{b}+a\int \frac{1}{\sqrt{a^2-b^2x^2}}\,dx\\ &= \frac{\sqrt{a^2-b^2x^2}}{b}+a\,\mathrm{Subst}\left(\int \frac{1}{1+b^2x^2}\,dx,x,\frac{x}{\sqrt{a^2-b^2x^2}}\right)\\ &= \frac{\sqrt{a^2-b^2x^2}}{b}+\frac{a\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0358361, size = 43, normalized size = 0.93

$$\frac{\sqrt{a^2-b^2x^2}+a\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 - b^2*x^2]/(a + b*x),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^2] + a*ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]])/b
```

Maple [A] time = 0.044, size = 77, normalized size = 1.7

$$\frac{1}{b}\sqrt{-\left(x+\frac{a}{b}\right)^2b^2+2\left(x+\frac{a}{b}\right)ab}+a\arctan\left(x\sqrt{b^2}\frac{1}{\sqrt{-\left(x+\frac{a}{b}\right)^2b^2+2\left(x+\frac{a}{b}\right)ab}}\right)\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^2*x^2+a^2)^(1/2)/(b*x+a),x)
```

```
[Out] 1/b*(-(x+1/b*a)^2*b^2+2*(x+1/b*a)*a*b)^(1/2)+a/(b^2)^(1/2)*arctan((b^2)^(1/
2)*x/(-(x+1/b*a)^2*b^2+2*(x+1/b*a)*a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77752, size = 101, normalized size = 2.2

$$-\frac{2\,a\arctan\left(-\frac{a-\sqrt{-b^2x^2+a^2}}{bx}\right)-\sqrt{-b^2x^2+a^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -(2*a*arctan(-(a - sqrt(-b^2*x^2 + a^2))/(b*x)) - sqrt(-b^2*x^2 + a^2))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(-a+bx)(a+bx)}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**2*x**2+a**2)**(1/2)/(b*x+a),x)
```

```
[Out] Integral(sqrt(-(-a + b*x)*(a + b*x))/(a + b*x), x)
```

Giac [A] time = 1.28768, size = 49, normalized size = 1.07

$$\frac{a\arcsin\left(\frac{bx}{a}\right)\operatorname{sgn}(a)\operatorname{sgn}(b)}{|b|}+\frac{\sqrt{-b^2x^2+a^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] a*arcsin(b*x/a)*sgn(a)*sgn(b)/abs(b) + sqrt(-b^2*x^2 + a^2)/b
```

3.782 $\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^2} dx$

Optimal. Leaf size=54

$$-\frac{2\sqrt{a^2-b^2x^2}}{b(a+bx)}-\frac{\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{b}$$

```
[Out] (-2*Sqrt[a^2 - b^2*x^2])/(b*(a + b*x)) - ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]]/
b
```

Rubi [A] time = 0.0140286, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {663, 217, 203}

$$-\frac{2\sqrt{a^2-b^2x^2}}{b(a+bx)}-\frac{\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 - b^2*x^2]/(a + b*x)^2,x]
```

```
[Out] (-2*Sqrt[a^2 - b^2*x^2])/(b*(a + b*x)) - ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]]/
b
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^2} dx &= -\frac{2\sqrt{a^2-b^2x^2}}{b(a+bx)}-\int \frac{1}{\sqrt{a^2-b^2x^2}} dx\\
&= -\frac{2\sqrt{a^2-b^2x^2}}{b(a+bx)}-\text{Subst}\left(\int \frac{1}{1+b^2x^2} dx, x, \frac{x}{\sqrt{a^2-b^2x^2}}\right)\\
&= -\frac{2\sqrt{a^2-b^2x^2}}{b(a+bx)}-\frac{\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0551268, size = 51, normalized size = 0.94

$$-\frac{\frac{2\sqrt{a^2-b^2x^2}}{a+bx}+\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 - b^2*x^2]/(a + b*x)^2,x]
```

```
[Out] -(((2*Sqrt[a^2 - b^2*x^2])/(a + b*x) + ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]])/b
)
```

Maple [B] time = 0.049, size = 126, normalized size = 2.3

$$-\frac{1}{ab^3}\left(-\left(x+\frac{a}{b}\right)^2 b^2+2\left(x+\frac{a}{b}\right)ab\right)^{\frac{3}{2}}\left(x+\frac{a}{b}\right)^{-2}-\frac{1}{ab}\sqrt{-\left(x+\frac{a}{b}\right)^2 b^2+2\left(x+\frac{a}{b}\right)ab}-\arctan\Bigg(x\sqrt{b^2}\frac{1}{\sqrt{-\left(x+\frac{a}{b}\right)^2 b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^2*x^2+a^2)^(1/2)/(b*x+a)^2,x)
```

```
[Out] -1/b^3/a/(x+1/b*a)^2*(-(x+1/b*a)^2*b^2+2*(x+1/b*a)*a*b)^(3/2)-1/b/a*(-(x+1/
b*a)^2*b^2+2*(x+1/b*a)*a*b)^(1/2)-1/(b^2)^(1/2)*arctan((b^2)^(1/2)*x/(-(x+1
/b*a)^2*b^2+2*(x+1/b*a)*a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.93435, size = 142, normalized size = 2.63

$$-\frac{2\left(bx-(bx+a)\arctan\left(-\frac{a-\sqrt{-b^2x^2+a^2}}{bx}\right)+a+\sqrt{-b^2x^2+a^2}\right)}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -2*(b*x - (b*x + a)*arctan(-(a - sqrt(-b^2*x^2 + a^2))/(b*x)) + a + sqrt(-b
^2*x^2 + a^2))/(b^2*x + a*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(-a+bx)(a+bx)}}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**2*x**2+a**2)**(1/2)/(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(-(-a + b*x)*(a + b*x))/(a + b*x)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.783 $\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^3} dx$

Optimal. Leaf size=33

$$-\frac{\left(a^2-b^2x^2\right)^{3/2}}{3ab(a+bx)^3}$$

```
[Out] -(a^2 - b^2*x^2)^(3/2)/(3*a*b*(a + b*x)^3)
```

Rubi [A] time = 0.0094301, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {651}

$$-\frac{\left(a^2-b^2x^2\right)^{3/2}}{3ab(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 - b^2*x^2]/(a + b*x)^3,x]
```

```
[Out] -(a^2 - b^2*x^2)^(3/2)/(3*a*b*(a + b*x)^3)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^3} dx = -\frac{\left(a^2-b^2x^2\right)^{3/2}}{3ab(a+bx)^3}$$

Mathematica [A] time = 0.0325069, size = 39, normalized size = 1.18

$$-\frac{(a-bx)\sqrt{a^2-b^2x^2}}{3ab(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 - b^2*x^2]/(a + b*x)^3,x]
```

```
[Out] -((a - b*x)*Sqrt[a^2 - b^2*x^2])/(3*a*b*(a + b*x)^2)
```

Maple [A] time = 0.043, size = 36, normalized size = 1.1

$$-\frac{-bx+a}{3\,(bx+a)^2\,ba}\sqrt{-b^2x^2+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^2*x^2+a^2)^(1/2)/(b*x+a)^3,x)
```

```
[Out] -1/3/(b*x+a)^2*(-b*x+a)/b/a*(-b^2*x^2+a^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.7758, size = 134, normalized size = 4.06

$$-\frac{b^2x^2 + 2\,abx + a^2 - \sqrt{-b^2x^2 + a^2}(bx - a)}{3\left(ab^3x^2 + 2\,a^2b^2x + a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(b^2*x^2 + 2*a*b*x + a^2 - sqrt(-b^2*x^2 + a^2)*(b*x - a))/(a*b^3*x^2
+ 2*a^2*b^2*x + a^3*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(-a + bx)(a + bx)}}{(a + bx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**2*x**2+a**2)**(1/2)/(b*x+a)**3,x)
```

```
[Out] Integral(sqrt(-(-a + b*x)*(a + b*x))/(a + b*x)**3, x)
```

Giac [B] time = 1.28214, size = 100, normalized size = 3.03

$$\frac{2\left(\frac{3\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^2}{b^4x^2}+1\right)}{3\,a\left(\frac{ab+\sqrt{-b^2x^2+a^2}|b|}{b^2x}+1\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 2/3*(3*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^2/(b^4*x^2) + 1)/(a*((a*b + sqrt
(-b^2*x^2 + a^2)*abs(b))/(b^2*x) + 1)^3*abs(b))
```

3.784 $\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^4} dx$

Optimal. Leaf size=67

$$-\frac{\left(a^2-b^2x^2\right)^{3/2}}{15a^2b(a+bx)^3}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{5ab(a+bx)^4}$$

```
[Out] -(a^2 - b^2*x^2)^(3/2)/(5*a*b*(a + b*x)^4) - (a^2 - b^2*x^2)^(3/2)/(15*a^2*
b*(a + b*x)^3)
```

Rubi [A] time = 0.0208856, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 651}

$$-\frac{\left(a^2-b^2x^2\right)^{3/2}}{15a^2b(a+bx)^3}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{5ab(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 - b^2*x^2]/(a + b*x)^4,x]
```

```
[Out] -(a^2 - b^2*x^2)^(3/2)/(5*a*b*(a + b*x)^4) - (a^2 - b^2*x^2)^(3/2)/(15*a^2*
b*(a + b*x)^3)
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^4} dx &= -\frac{\left(a^2-b^2x^2\right)^{3/2}}{5ab(a+bx)^4}+\frac{\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^3} dx}{5a}\\ &= -\frac{\left(a^2-b^2x^2\right)^{3/2}}{5ab(a+bx)^4}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{15a^2b(a+bx)^3}\end{aligned}$$

Mathematica [A] time = 0.0374707, size = 51, normalized size = 0.76

$$\frac{\sqrt{a^2-b^2x^2}\left(-4a^2+3abx+b^2x^2\right)}{15a^2b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 - b^2*x^2]/(a + b*x)^4,x]
```

```
[Out] (Sqrt[a^2 - b^2*x^2]*(-4*a^2 + 3*a*b*x + b^2*x^2))/(15*a^2*b*(a + b*x)^3)
```

Maple [A] time = 0.044, size = 43, normalized size = 0.6

$$-\frac{(bx+4a)(-bx+a)}{15(bx+a)^3ba^2}\sqrt{-b^2x^2+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^2*x^2+a^2)^(1/2)/(b*x+a)^4,x)
```

```
[Out] -1/15*(-b*x+a)*(b*x+4*a)*(-b^2*x^2+a^2)^(1/2)/(b*x+a)^3/b/a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.87622, size = 213, normalized size = 3.18

$$-\frac{4b^3x^3+12ab^2x^2+12a^2bx+4a^3-\left(b^2x^2+3abx-4a^2\right)\sqrt{-b^2x^2+a^2}}{15\left(a^2b^4x^3+3a^3b^3x^2+3a^4b^2x+a^5b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/15*(4*b^3*x^3 + 12*a*b^2*x^2 + 12*a^2*b*x + 4*a^3 - (b^2*x^2 + 3*a*b*x -
 4*a^2)*sqrt(-b^2*x^2 + a^2))/(a^2*b^4*x^3 + 3*a^3*b^3*x^2 + 3*a^4*b^2*x +
a^5*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-(-a+bx)(a+bx)}}{(a+bx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**2*x**2+a**2)**(1/2)/(b*x+a)**4,x)
```

[Out] Integral(sqrt(-(-a + b*x)*(a + b*x))/(a + b*x)**4, x)

Giac [B] time = 1.2234, size = 223, normalized size = 3.33

$$\frac{2\left(\frac{5\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)}{b^2x}+\frac{25\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^2}{b^4x^2}+\frac{15\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^3}{b^6x^3}+\frac{15\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^4}{b^8x^4}+4\right)}{15\,a^2\left(\frac{ab+\sqrt{-b^2x^2+a^2}|b|}{b^2x}+1\right)^5|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^4,x, algorithm="giac")

[Out] 2/15*(5*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))/(b^2*x) + 25*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^2/(b^4*x^2) + 15*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^3/(b^6*x^3) + 15*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^4/(b^8*x^4) + 4)/(a^2*((a*b + sqrt(-b^2*x^2 + a^2)*abs(b))/(b^2*x) + 1)^5*abs(b))

3.785 $\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^5} \, dx$

Optimal. Leaf size=100

$$-\frac{2\left(a^2-b^2x^2\right)^{3/2}}{105a^3b(a+bx)^3}-\frac{2\left(a^2-b^2x^2\right)^{3/2}}{35a^2b(a+bx)^4}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{7ab(a+bx)^5}$$

```
[Out] -(a^2 - b^2*x^2)^(3/2)/(7*a*b*(a + b*x)^5) - (2*(a^2 - b^2*x^2)^(3/2))/(35*
a^2*b*(a + b*x)^4) - (2*(a^2 - b^2*x^2)^(3/2))/(105*a^3*b*(a + b*x)^3)
```

Rubi [A] time = 0.0357099, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 651}

$$-\frac{2\left(a^2-b^2x^2\right)^{3/2}}{105a^3b(a+bx)^3}-\frac{2\left(a^2-b^2x^2\right)^{3/2}}{35a^2b(a+bx)^4}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{7ab(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 - b^2*x^2]/(a + b*x)^5,x]
```

```
[Out] -(a^2 - b^2*x^2)^(3/2)/(7*a*b*(a + b*x)^5) - (2*(a^2 - b^2*x^2)^(3/2))/(35*
a^2*b*(a + b*x)^4) - (2*(a^2 - b^2*x^2)^(3/2))/(105*a^3*b*(a + b*x)^3)
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^5} \, dx &= -\frac{\left(a^2-b^2x^2\right)^{3/2}}{7ab(a+bx)^5}+\frac{2\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^4} \, dx}{7a}\\
&= -\frac{\left(a^2-b^2x^2\right)^{3/2}}{7ab(a+bx)^5}-\frac{2\left(a^2-b^2x^2\right)^{3/2}}{35a^2b(a+bx)^4}+\frac{2\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^3} \, dx}{35a^2}\\
&= -\frac{\left(a^2-b^2x^2\right)^{3/2}}{7ab(a+bx)^5}-\frac{2\left(a^2-b^2x^2\right)^{3/2}}{35a^2b(a+bx)^4}-\frac{2\left(a^2-b^2x^2\right)^{3/2}}{105a^3b(a+bx)^3}
\end{aligned}$$

Mathematica [A] time = 0.0409247, size = 63, normalized size = 0.63

$$\frac{\sqrt{a^2-b^2x^2}\left(13a^2bx-23a^3+8ab^2x^2+2b^3x^3\right)}{105a^3b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 - b^2*x^2]/(a + b*x)^5,x]
```

```
[Out] (Sqrt[a^2 - b^2*x^2]*(-23*a^3 + 13*a^2*b*x + 8*a*b^2*x^2 + 2*b^3*x^3))/(105
*a^3*b*(a + b*x)^4)
```

Maple [A] time = 0.043, size = 55, normalized size = 0.6

$$-\frac{\left(2\,b^{2}x^{2}+10\,abx+23\,a^{2}\right)(-bx+a)}{105\,(bx+a)^{4}\,a^{3}b}\sqrt{-b^{2}x^{2}+a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^2*x^2+a^2)^(1/2)/(b*x+a)^5,x)
```

```
[Out] -1/105*(-b*x+a)*(2*b^2*x^2+10*a*b*x+23*a^2)*(-b^2*x^2+a^2)^(1/2)/(b*x+a)^4/
a^3/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.91067, size = 290, normalized size = 2.9

$$-\frac{23\,b^{4}x^{4}+92\,ab^{3}x^{3}+138\,a^{2}b^{2}x^{2}+92\,a^{3}bx+23\,a^{4}-\left(2\,b^{3}x^{3}+8\,ab^{2}x^{2}+13\,a^{2}bx-23\,a^{3}\right)\sqrt{-b^{2}x^{2}+a^{2}}}{105\left(a^{3}b^{5}x^{4}+4\,a^{4}b^{4}x^{3}+6\,a^{5}b^{3}x^{2}+4\,a^{6}b^{2}x+a^{7}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/105*(23*b^4*x^4 + 92*a*b^3*x^3 + 138*a^2*b^2*x^2 + 92*a^3*b*x + 23*a^4 -
 (2*b^3*x^3 + 8*a*b^2*x^2 + 13*a^2*b*x - 23*a^3)*sqrt(-b^2*x^2 + a^2))/(a^3
*b^5*x^4 + 4*a^4*b^4*x^3 + 6*a^5*b^3*x^2 + 4*a^6*b^2*x + a^7*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-(-a+bx)(a+bx)}}{(a+bx)^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**2*x**2+a**2)**(1/2)/(b*x+a)**5,x)
```

```
[Out] Integral(sqrt(-(-a + b*x)*(a + b*x))/(a + b*x)**5, x)
```

Giac [C] time = 1.21159, size = 120, normalized size = 1.2

$$-\frac{1}{420}\left(\frac{\left(15\left(\frac{2a}{bx+a}-1\right)^{\frac{7}{2}}+42\left(\frac{2a}{bx+a}-1\right)^{\frac{5}{2}}+35\left(\frac{2a}{bx+a}-1\right)^{\frac{3}{2}}\right)\operatorname{sgn}\left(\frac{1}{bx+a}\right)\operatorname{sgn}(b)}{a^3b^2}+\frac{8i\operatorname{sgn}\left(\frac{1}{bx+a}\right)\operatorname{sgn}(b)}{a^3b^2}\right)|b|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/420*((15*(2*a/(b*x + a) - 1)^(7/2) + 42*(2*a/(b*x + a) - 1)^(5/2) + 35*(
2*a/(b*x + a) - 1)^(3/2))*sgn(1/(b*x + a))*sgn(b)/(a^3*b^2) + 8*I*sgn(1/(b*
x + a))*sgn(b)/(a^3*b^2))*abs(b)
```

3.786 $\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^6}\,dx$

Optimal. Leaf size=133

$$-\frac{2\left(a^2-b^2x^2\right)^{3/2}}{315a^4b(a+bx)^3}-\frac{2\left(a^2-b^2x^2\right)^{3/2}}{105a^3b(a+bx)^4}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{21a^2b(a+bx)^5}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{9ab(a+bx)^6}$$

```
[Out] -(a^2 - b^2*x^2)^(3/2)/(9*a*b*(a + b*x)^6) - (a^2 - b^2*x^2)^(3/2)/(21*a^2*
b*(a + b*x)^5) - (2*(a^2 - b^2*x^2)^(3/2))/(105*a^3*b*(a + b*x)^4) - (2*(a^
2 - b^2*x^2)^(3/2))/(315*a^4*b*(a + b*x)^3)
```

Rubi [A] time = 0.0533064, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 651}

$$-\frac{2\left(a^2-b^2x^2\right)^{3/2}}{315a^4b(a+bx)^3}-\frac{2\left(a^2-b^2x^2\right)^{3/2}}{105a^3b(a+bx)^4}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{21a^2b(a+bx)^5}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{9ab(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 - b^2*x^2]/(a + b*x)^6,x]
```

```
[Out] -(a^2 - b^2*x^2)^(3/2)/(9*a*b*(a + b*x)^6) - (a^2 - b^2*x^2)^(3/2)/(21*a^2*
b*(a + b*x)^5) - (2*(a^2 - b^2*x^2)^(3/2))/(105*a^3*b*(a + b*x)^4) - (2*(a^
2 - b^2*x^2)^(3/2))/(315*a^4*b*(a + b*x)^3)
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^6}\,dx &= -\frac{\left(a^2-b^2x^2\right)^{3/2}}{9ab(a+bx)^6}+\frac{\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^5}\,dx}{3a}\\
&= -\frac{\left(a^2-b^2x^2\right)^{3/2}}{9ab(a+bx)^6}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{21a^2b(a+bx)^5}+\frac{2\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^4}\,dx}{21a^2}\\
&= -\frac{\left(a^2-b^2x^2\right)^{3/2}}{9ab(a+bx)^6}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{21a^2b(a+bx)^5}-\frac{2\left(a^2-b^2x^2\right)^{3/2}}{105a^3b(a+bx)^4}+\frac{2\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^3}\,dx}{105a^3}\\
&= -\frac{\left(a^2-b^2x^2\right)^{3/2}}{9ab(a+bx)^6}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{21a^2b(a+bx)^5}-\frac{2\left(a^2-b^2x^2\right)^{3/2}}{105a^3b(a+bx)^4}-\frac{2\left(a^2-b^2x^2\right)^{3/2}}{315a^4b(a+bx)^3}
\end{aligned}$$

Mathematica [A] time = 0.0449516, size = 74, normalized size = 0.56

$$\frac{\sqrt{a^2-b^2x^2}\left(21a^2b^2x^2+25a^3bx-58a^4+10ab^3x^3+2b^4x^4\right)}{315a^4b(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 - b^2*x^2]/(a + b*x)^6,x]
```

```
[Out] (Sqrt[a^2 - b^2*x^2]*(-58*a^4 + 25*a^3*b*x + 21*a^2*b^2*x^2 + 10*a*b^3*x^3
+ 2*b^4*x^4))/(315*a^4*b*(a + b*x)^5)
```

Maple [A] time = 0.043, size = 66, normalized size = 0.5

$$-\frac{\left(2\,b^3x^3+12\,ab^2x^2+33\,xa^2b+58\,a^3\right)(-bx+a)}{315\,(bx+a)^5\,a^4b}\sqrt{-b^2x^2+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^2*x^2+a^2)^(1/2)/(b*x+a)^6,x)
```

```
[Out] -1/315*(-b*x+a)*(2*b^3*x^3+12*a*b^2*x^2+33*a^2*b*x+58*a^3)*(-b^2*x^2+a^2)^(
1/2)/(b*x+a)^5/a^4/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.07224, size = 366, normalized size = 2.75

$$-\frac{58\,b^5x^5+290\,ab^4x^4+580\,a^2b^3x^3+580\,a^3b^2x^2+290\,a^4bx+58\,a^5-\left(2\,b^4x^4+10\,ab^3x^3+21\,a^2b^2x^2+25\,a^3bx-58\,a}{315\left(a^4b^6x^5+5\,a^5b^5x^4+10\,a^6b^4x^3+10\,a^7b^3x^2+5\,a^8b^2x+a^9b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] -1/315*(58*b^5*x^5 + 290*a*b^4*x^4 + 580*a^2*b^3*x^3 + 580*a^3*b^2*x^2 + 29
0*a^4*b*x + 58*a^5 - (2*b^4*x^4 + 10*a*b^3*x^3 + 21*a^2*b^2*x^2 + 25*a^3*b*
x - 58*a^4)*sqrt(-b^2*x^2 + a^2))/(a^4*b^6*x^5 + 5*a^5*b^5*x^4 + 10*a^6*b^4
*x^3 + 10*a^7*b^3*x^2 + 5*a^8*b^2*x + a^9*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(-a+bx)(a+bx)}}{(a+bx)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**2*x**2+a**2)**(1/2)/(b*x+a)**6,x)
```

```
[Out] Integral(sqrt(-(-a + b*x)*(a + b*x))/(a + b*x)**6, x)
```

Giac [B] time = 1.2051, size = 390, normalized size = 2.93

$$\frac{2\left(\frac{207\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)}{b^2x}+\frac{1143\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^2}{b^4x^2}+\frac{2247\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^3}{b^6x^3}+\frac{3843\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^4}{b^8x^4}+\frac{3465\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^5}{b^{10}x^5}+2625\left(\cdots\right.\right.}{315\,a^4\left(\frac{ab+\sqrt{-b^2x^2+a^2}|b|}{b^2x}+1\right)^9|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 2/315*(207*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))/(b^2*x) + 1143*(a*b + sqrt(-
b^2*x^2 + a^2)*abs(b))^2/(b^4*x^2) + 2247*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b
))^3/(b^6*x^3) + 3843*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^4/(b^8*x^4) + 346
5*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^5/(b^10*x^5) + 2625*(a*b + sqrt(-b^2*
x^2 + a^2)*abs(b))^6/(b^12*x^6) + 945*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^7
/(b^14*x^7) + 315*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^8/(b^16*x^8) + 58)/(a
^4*((a*b + sqrt(-b^2*x^2 + a^2)*abs(b))/(b^2*x) + 1)^9*abs(b))
```

3.787 $\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^7} dx$

Optimal. Leaf size=166

$$-\frac{8\left(a^2-b^2x^2\right)^{3/2}}{3465a^5b(a+bx)^3}-\frac{8\left(a^2-b^2x^2\right)^{3/2}}{1155a^4b(a+bx)^4}-\frac{4\left(a^2-b^2x^2\right)^{3/2}}{231a^3b(a+bx)^5}-\frac{4\left(a^2-b^2x^2\right)^{3/2}}{99a^2b(a+bx)^6}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{11ab(a+bx)^7}$$

```
[Out] -(a^2 - b^2*x^2)^(3/2)/(11*a*b*(a + b*x)^7) - (4*(a^2 - b^2*x^2)^(3/2))/(99
*a^2*b*(a + b*x)^6) - (4*(a^2 - b^2*x^2)^(3/2))/(231*a^3*b*(a + b*x)^5) - (
8*(a^2 - b^2*x^2)^(3/2))/(1155*a^4*b*(a + b*x)^4) - (8*(a^2 - b^2*x^2)^(3/2
))/(3465*a^5*b*(a + b*x)^3)
```

Rubi [A] time = 0.0708873, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 651}

$$-\frac{8\left(a^2-b^2x^2\right)^{3/2}}{3465a^5b(a+bx)^3}-\frac{8\left(a^2-b^2x^2\right)^{3/2}}{1155a^4b(a+bx)^4}-\frac{4\left(a^2-b^2x^2\right)^{3/2}}{231a^3b(a+bx)^5}-\frac{4\left(a^2-b^2x^2\right)^{3/2}}{99a^2b(a+bx)^6}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{11ab(a+bx)^7}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 - b^2*x^2]/(a + b*x)^7,x]
```

```
[Out] -(a^2 - b^2*x^2)^(3/2)/(11*a*b*(a + b*x)^7) - (4*(a^2 - b^2*x^2)^(3/2))/(99
*a^2*b*(a + b*x)^6) - (4*(a^2 - b^2*x^2)^(3/2))/(231*a^3*b*(a + b*x)^5) - (
8*(a^2 - b^2*x^2)^(3/2))/(1155*a^4*b*(a + b*x)^4) - (8*(a^2 - b^2*x^2)^(3/2
))/(3465*a^5*b*(a + b*x)^3)
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^7}\,dx = -\frac{\left(a^2-b^2x^2\right)^{3/2}}{11ab(a+bx)^7} + \frac{4\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^6}\,dx}{11a}$$
$$= -\frac{\left(a^2-b^2x^2\right)^{3/2}}{11ab(a+bx)^7} - \frac{4\left(a^2-b^2x^2\right)^{3/2}}{99a^2b(a+bx)^6} + \frac{4\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^5}\,dx}{33a^2}$$
$$= -\frac{\left(a^2-b^2x^2\right)^{3/2}}{11ab(a+bx)^7} - \frac{4\left(a^2-b^2x^2\right)^{3/2}}{99a^2b(a+bx)^6} - \frac{4\left(a^2-b^2x^2\right)^{3/2}}{231a^3b(a+bx)^5} + \frac{8\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^4}\,dx}{231a^3}$$
$$= -\frac{\left(a^2-b^2x^2\right)^{3/2}}{11ab(a+bx)^7} - \frac{4\left(a^2-b^2x^2\right)^{3/2}}{99a^2b(a+bx)^6} - \frac{4\left(a^2-b^2x^2\right)^{3/2}}{231a^3b(a+bx)^5} - \frac{8\left(a^2-b^2x^2\right)^{3/2}}{1155a^4b(a+bx)^4} + \frac{8\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^3}\,dx}{1155a^4}$$
$$= -\frac{\left(a^2-b^2x^2\right)^{3/2}}{11ab(a+bx)^7} - \frac{4\left(a^2-b^2x^2\right)^{3/2}}{99a^2b(a+bx)^6} - \frac{4\left(a^2-b^2x^2\right)^{3/2}}{231a^3b(a+bx)^5} - \frac{8\left(a^2-b^2x^2\right)^{3/2}}{1155a^4b(a+bx)^4} - \frac{8\left(a^2-b^2x^2\right)^{3/2}}{3465a^5b(a+bx)^3}$$

Mathematica [A] time = 0.0453151, size = 85, normalized size = 0.51

$$\frac{\sqrt{a^2-b^2x^2}\left(184a^3b^2x^2+124a^2b^3x^3+183a^4bx-547a^5+48ab^4x^4+8b^5x^5\right)}{3465a^5b(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 - b^2*x^2]/(a + b*x)^7,x]
```

```
[Out] (Sqrt[a^2 - b^2*x^2]*(-547*a^5 + 183*a^4*b*x + 184*a^3*b^2*x^2 + 124*a^2*b^
3*x^3 + 48*a*b^4*x^4 + 8*b^5*x^5))/(3465*a^5*b*(a + b*x)^6)
```

Maple [A] time = 0.043, size = 77, normalized size = 0.5

$$-\frac{\left(8\,b^4x^4+56\,ab^3x^3+180\,b^2x^2a^2+364\,xa^3b+547\,a^4\right)(-bx+a)}{3465\,(bx+a)^6\,a^5b}\sqrt{-b^2x^2+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^2*x^2+a^2)^(1/2)/(b*x+a)^7,x)
```

```
[Out] -1/3465*(-b*x+a)*(8*b^4*x^4+56*a*b^3*x^3+180*a^2*b^2*x^2+364*a^3*b*x+547*a^
4)*(-b^2*x^2+a^2)^(1/2)/(b*x+a)^6/a^5/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.5877, size = 456, normalized size = 2.75

$$-\frac{547\,b^6x^6+3282\,ab^5x^5+8205\,a^2b^4x^4+10940\,a^3b^3x^3+8205\,a^4b^2x^2+3282\,a^5bx+547\,a^6-\left(8\,b^5x^5+48\,ab^4x^4+12\right.}{3465\left(a^5b^7x^6+6\,a^6b^6x^5+15\,a^7b^5x^4+20\,a^8b^4x^3+15\,a^9b^3x^2+6\,a^{10}b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/3465*(547*b^6*x^6 + 3282*a*b^5*x^5 + 8205*a^2*b^4*x^4 + 10940*a^3*b^3*x^
3 + 8205*a^4*b^2*x^2 + 3282*a^5*b*x + 547*a^6 - (8*b^5*x^5 + 48*a*b^4*x^4 +
 124*a^2*b^3*x^3 + 184*a^3*b^2*x^2 + 183*a^4*b*x - 547*a^5)*sqrt(-b^2*x^2 +
 a^2))/(a^5*b^7*x^6 + 6*a^6*b^6*x^5 + 15*a^7*b^5*x^4 + 20*a^8*b^4*x^3 + 15*
a^9*b^3*x^2 + 6*a^10*b^2*x + a^11*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(-a+bx)(a+bx)}}{(a+bx)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**2*x**2+a**2)**(1/2)/(b*x+a)**7,x)
```

```
[Out] Integral(sqrt(-(-a + b*x)*(a + b*x))/(a + b*x)**7, x)
```

Giac [B] time = 1.24197, size = 474, normalized size = 2.86

$$\frac{2\left(\frac{2552\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)}{b^2x}+\frac{16225\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^2}{b^4x^2}+\frac{42900\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^3}{b^6x^3}+\frac{92730\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^4}{b^8x^4}+\frac{122892\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)}{b^{10}x^5}\right.}{3465\,a^5\left(\frac{ab}{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 2/3465*(2552*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))/(b^2*x) + 16225*(a*b + sqr
t(-b^2*x^2 + a^2)*abs(b))^2/(b^4*x^2) + 42900*(a*b + sqrt(-b^2*x^2 + a^2)*a
bs(b))^3/(b^6*x^3) + 92730*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^4/(b^8*x^4)
+ 122892*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^5/(b^10*x^5) + 129822*(a*b + s
qrt(-b^2*x^2 + a^2)*abs(b))^6/(b^12*x^6) + 87780*(a*b + sqrt(-b^2*x^2 + a^2
)*abs(b))^7/(b^14*x^7) + 47355*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^8/(b^16*
x^8) + 13860*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^9/(b^18*x^9) + 3465*(a*b +
 sqrt(-b^2*x^2 + a^2)*abs(b))^10/(b^20*x^10) + 547)/(a^5*((a*b + sqrt(-b^2*
x^2 + a^2)*abs(b))/(b^2*x) + 1)^11*abs(b))
```

3.788 $\int (a+bx)^3 \left(a^2-b^2x^2\right)^{3/2} dx$

Optimal. Leaf size=164

$$\frac{9}{16}a^5x\sqrt{a^2-b^2x^2}+\frac{3}{8}a^3x\left(a^2-b^2x^2\right)^{3/2}-\frac{3a^2\left(a^2-b^2x^2\right)^{5/2}}{10b}-\frac{3a(a+bx)\left(a^2-b^2x^2\right)^{5/2}}{14b}-\frac{(a+bx)^2\left(a^2-b^2x^2\right)^{5/2}}{7b}+\frac{9a^7}{}$$

```
[Out] (9*a^5*x*Sqrt[a^2 - b^2*x^2])/16 + (3*a^3*x*(a^2 - b^2*x^2)^(3/2))/8 - (3*a
^2*(a^2 - b^2*x^2)^(5/2))/(10*b) - (3*a*(a + b*x)*(a^2 - b^2*x^2)^(5/2))/(1
4*b) - ((a + b*x)^2*(a^2 - b^2*x^2)^(5/2))/(7*b) + (9*a^7*ArcTan[(b*x)/Sqrt
[a^2 - b^2*x^2]])/(16*b)
```

Rubi [A] time = 0.0629918, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {671, 641, 195, 217, 203}

$$\frac{9}{16}a^5x\sqrt{a^2-b^2x^2}+\frac{3}{8}a^3x\left(a^2-b^2x^2\right)^{3/2}-\frac{3a^2\left(a^2-b^2x^2\right)^{5/2}}{10b}-\frac{3a(a+bx)\left(a^2-b^2x^2\right)^{5/2}}{14b}-\frac{(a+bx)^2\left(a^2-b^2x^2\right)^{5/2}}{7b}+\frac{9a^7}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(a^2 - b^2*x^2)^(3/2),x]
```

```
[Out] (9*a^5*x*Sqrt[a^2 - b^2*x^2])/16 + (3*a^3*x*(a^2 - b^2*x^2)^(3/2))/8 - (3*a
^2*(a^2 - b^2*x^2)^(5/2))/(10*b) - (3*a*(a + b*x)*(a^2 - b^2*x^2)^(5/2))/(1
4*b) - ((a + b*x)^2*(a^2 - b^2*x^2)^(5/2))/(7*b) + (9*a^7*ArcTan[(b*x)/Sqrt
[a^2 - b^2*x^2]])/(16*b)
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+bx)^3 \left(a^2-b^2x^2\right)^{3/2} dx = -\frac{(a+bx)^2 \left(a^2-b^2x^2\right)^{5/2}}{7b} + \frac{1}{7}(9a) \int (a+bx)^2 \left(a^2-b^2x^2\right)^{3/2} dx$$
$$= -\frac{3a(a+bx)\left(a^2-b^2x^2\right)^{5/2}}{14b} - \frac{(a+bx)^2\left(a^2-b^2x^2\right)^{5/2}}{7b} + \frac{1}{2}\left(3a^2\right) \int (a+bx)\left(a^2-b^2x^2\right)$$
$$= -\frac{3a^2\left(a^2-b^2x^2\right)^{5/2}}{10b} - \frac{3a(a+bx)\left(a^2-b^2x^2\right)^{5/2}}{14b} - \frac{(a+bx)^2\left(a^2-b^2x^2\right)^{5/2}}{7b} + \frac{1}{2}\left(3a^3\right) \int$$
$$= \frac{3}{8}a^3x\left(a^2-b^2x^2\right)^{3/2} - \frac{3a^2\left(a^2-b^2x^2\right)^{5/2}}{10b} - \frac{3a(a+bx)\left(a^2-b^2x^2\right)^{5/2}}{14b} - \frac{(a+bx)^2\left(a^2-\right.}{7b}$$
$$= \frac{9}{16}a^5x\sqrt{a^2-b^2x^2} + \frac{3}{8}a^3x\left(a^2-b^2x^2\right)^{3/2} - \frac{3a^2\left(a^2-b^2x^2\right)^{5/2}}{10b} - \frac{3a(a+bx)\left(a^2-b^2x^2\right)^{5}}{14b}$$
$$= \frac{9}{16}a^5x\sqrt{a^2-b^2x^2} + \frac{3}{8}a^3x\left(a^2-b^2x^2\right)^{3/2} - \frac{3a^2\left(a^2-b^2x^2\right)^{5/2}}{10b} - \frac{3a(a+bx)\left(a^2-b^2x^2\right)^{5}}{14b}$$
$$= \frac{9}{16}a^5x\sqrt{a^2-b^2x^2} + \frac{3}{8}a^3x\left(a^2-b^2x^2\right)^{3/2} - \frac{3a^2\left(a^2-b^2x^2\right)^{5/2}}{10b} - \frac{3a(a+bx)\left(a^2-b^2x^2\right)^{5}}{14b}$$

Mathematica [A] time = 0.224929, size = 134, normalized size = 0.82

$$\frac{\sqrt{a^2-b^2x^2}\left(\sqrt{1-\frac{b^2x^2}{a^2}}\left(656a^4b^2x^2+350a^3b^3x^3-208a^2b^4x^4+245a^5bx-368a^6-280ab^5x^5-80b^6x^6\right)+315a^6\sin^{-1}\right.}{560b\sqrt{1-\frac{b^2x^2}{a^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(a^2 - b^2*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^2]*(Sqrt[1 - (b^2*x^2)/a^2]*(-368*a^6 + 245*a^5*b*x + 656
*a^4*b^2*x^2 + 350*a^3*b^3*x^3 - 208*a^2*b^4*x^4 - 280*a*b^5*x^5 - 80*b^6*x
^6) + 315*a^6*ArcSin[(b*x)/a]))/(560*b*Sqrt[1 - (b^2*x^2)/a^2])
```

Maple [A] time = 0.055, size = 134, normalized size = 0.8

$$-\frac{bx^2}{7}\left(-b^2x^2+a^2\right)^{\frac{5}{2}} - \frac{23\,a^2}{35\,b}\left(-b^2x^2+a^2\right)^{\frac{5}{2}} - \frac{ax}{2}\left(-b^2x^2+a^2\right)^{\frac{5}{2}} + \frac{3\,xa^3}{8}\left(-b^2x^2+a^2\right)^{\frac{3}{2}} + \frac{9\,a^5x}{16}\sqrt{-b^2x^2+a^2} + \frac{9\,a^7}{16}\,\text{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(-b^2*x^2+a^2)^(3/2),x)
```

```
[Out] -1/7*b*x^2*(-b^2*x^2+a^2)^(5/2)-23/35*a^2*(-b^2*x^2+a^2)^(5/2)/b-1/2*a*x*(-
b^2*x^2+a^2)^(5/2)+3/8*a^3*x*(-b^2*x^2+a^2)^(3/2)+9/16*a^5*x*(-b^2*x^2+a^2)
```

```
^(1/2)+9/16*a^7/(b^2)^(1/2)*arctan((b^2)^(1/2)*x/(-b^2*x^2+a^2)^(1/2))
```

Maxima [A] time = 1.56827, size = 170, normalized size = 1.04

$$\frac{9\,a^7\arcsin\left(\frac{b^2x}{\sqrt{a^2b^2}}\right)}{16\,\sqrt{b^2}}+\frac{9}{16}\,\sqrt{-b^2x^2+a^2}a^5x+\frac{3}{8}\left(-b^2x^2+a^2\right)^{\frac{3}{2}}a^3x-\frac{1}{7}\left(-b^2x^2+a^2\right)^{\frac{5}{2}}bx^2-\frac{1}{2}\left(-b^2x^2+a^2\right)^{\frac{5}{2}}ax-\frac{23\left(-b^2x^2\right.}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(-b^2*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 9/16*a^7*arcsin(b^2*x/sqrt(a^2*b^2))/sqrt(b^2) + 9/16*sqrt(-b^2*x^2 + a^2)*
a^5*x + 3/8*(-b^2*x^2 + a^2)^(3/2)*a^3*x - 1/7*(-b^2*x^2 + a^2)^(5/2)*b*x^2
 - 1/2*(-b^2*x^2 + a^2)^(5/2)*a*x - 23/35*(-b^2*x^2 + a^2)^(5/2)*a^2/b
```

Fricas [A] time = 2.05022, size = 259, normalized size = 1.58

$$-\frac{630\,a^7\arctan\left(-\frac{a-\sqrt{-b^2x^2+a^2}}{bx}\right)+\left(80\,b^6x^6+280\,ab^5x^5+208\,a^2b^4x^4-350\,a^3b^3x^3-656\,a^4b^2x^2-245\,a^5bx+368\,a^6\right)\sqrt{-b}}{560\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(-b^2*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/560*(630*a^7*arctan(-(a - sqrt(-b^2*x^2 + a^2))/(b*x)) + (80*b^6*x^6 + 2
80*a*b^5*x^5 + 208*a^2*b^4*x^4 - 350*a^3*b^3*x^3 - 656*a^4*b^2*x^2 - 245*a^
5*b*x + 368*a^6)*sqrt(-b^2*x^2 + a^2))/b
```

Sympy [C] time = 13.6904, size = 821, normalized size = 5.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(-b**2*x**2+a**2)**(3/2),x)
```

```
[Out] a**5*Piecewise((-I*a**2*acosh(b*x/a)/(2*b) - I*a*x/(2*sqrt(-1 + b**2*x**2/a
**2)) + I*b**2*x**3/(2*a*sqrt(-1 + b**2*x**2/a**2)), Abs(b**2*x**2)/Abs(a**
2) > 1), (a**2*asin(b*x/a)/(2*b) + a*x*sqrt(1 - b**2*x**2/a**2)/2, True)) +
 3*a**4*b*Piecewise((x**2*sqrt(a**2)/2, Eq(b**2, 0)), (-(a**2 - b**2*x**2)*
*(3/2)/(3*b**2), True)) + 2*a**3*b**2*Piecewise((-I*a**4*acosh(b*x/a)/(8*b*
*3) + I*a**3*x/(8*b**2*sqrt(-1 + b**2*x**2/a**2)) - 3*I*a*x**3/(8*sqrt(-1 +
 b**2*x**2/a**2)) + I*b**2*x**5/(4*a*sqrt(-1 + b**2*x**2/a**2)), Abs(b**2*x
**2)/Abs(a**2) > 1), (a**4*asin(b*x/a)/(8*b**3) - a**3*x/(8*b**2*sqrt(1 - b
**2*x**2/a**2)) + 3*a*x**3/(8*sqrt(1 - b**2*x**2/a**2)) - b**2*x**5/(4*a*sq
rt(1 - b**2*x**2/a**2)), True)) - 2*a**2*b**3*Piecewise((-2*a**4*sqrt(a**2
- b**2*x**2)/(15*b**4) - a**2*x**2*sqrt(a**2 - b**2*x**2)/(15*b**2) + x**4*
sqrt(a**2 - b**2*x**2)/5, Ne(b, 0)), (x**4*sqrt(a**2)/4, True)) - 3*a*b**4*
Piecewise((-I*a**6*acosh(b*x/a)/(16*b**5) + I*a**5*x/(16*b**4*sqrt(-1 + b**
2*x**2/a**2)) - I*a**3*x**3/(48*b**2*sqrt(-1 + b**2*x**2/a**2)) - 5*I*a*x**
```

```
5/(24*sqrt(-1 + b**2*x**2/a**2)) + I*b**2*x**7/(6*a*sqrt(-1 + b**2*x**2/a**
2)), Abs(b**2*x**2)/Abs(a**2) > 1), (a**6*asin(b*x/a)/(16*b**5) - a**5*x/(1
6*b**4*sqrt(1 - b**2*x**2/a**2)) + a**3*x**3/(48*b**2*sqrt(1 - b**2*x**2/a*
*2)) + 5*a*x**5/(24*sqrt(1 - b**2*x**2/a**2)) - b**2*x**7/(6*a*sqrt(1 - b**
2*x**2/a**2)), True)) - b**5*Piecewise((-8*a**6*sqrt(a**2 - b**2*x**2)/(105
*b**6) - 4*a**4*x**2*sqrt(a**2 - b**2*x**2)/(105*b**4) - a**2*x**4*sqrt(a**
2 - b**2*x**2)/(35*b**2) + x**6*sqrt(a**2 - b**2*x**2)/7, Ne(b, 0)), (x**6*
sqrt(a**2)/6, True))
```

Giac [A] time = 1.26588, size = 140, normalized size = 0.85

$$\frac{9\,a^7\arcsin\left(\frac{bx}{a}\right)\operatorname{sgn}(a)\operatorname{sgn}(b)}{16\,|b|}-\frac{1}{560}\left(\frac{368\,a^6}{b}-\left(245\,a^5+2\left(328\,a^4b+\left(175\,a^3b^2-4\left(26\,a^2b^3+5\left(2\,b^5x+7\,ab^4\right)x\right)x\right)x\right)x\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(-b^2*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 9/16*a^7*arcsin(b*x/a)*sgn(a)*sgn(b)/abs(b) - 1/560*(368*a^6/b - (245*a^5 +
 2*(328*a^4*b + (175*a^3*b^2 - 4*(26*a^2*b^3 + 5*(2*b^5*x + 7*a*b^4)*x)*x)*
x)*x)*x)*sqrt(-b^2*x^2 + a^2)
```

3.789 $\int (a+bx)^2 \left(a^2-b^2x^2\right)^{3/2} \, dx$

Optimal. Leaf size=131

$$\frac{7}{16}a^4x\sqrt{a^2-b^2x^2}+\frac{7}{24}a^2x\left(a^2-b^2x^2\right)^{3/2}-\frac{7a\left(a^2-b^2x^2\right)^{5/2}}{30b}-\frac{(a+bx)\left(a^2-b^2x^2\right)^{5/2}}{6b}+\frac{7a^6\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{16b}$$

```
[Out] (7*a^4*x*Sqrt[a^2 - b^2*x^2])/16 + (7*a^2*x*(a^2 - b^2*x^2)^(3/2))/24 - (7*
a*(a^2 - b^2*x^2)^(5/2))/(30*b) - ((a + b*x)*(a^2 - b^2*x^2)^(5/2))/(6*b) +
 (7*a^6*ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]])/(16*b)
```

Rubi [A] time = 0.0437812, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {671, 641, 195, 217, 203}

$$\frac{7}{16}a^4x\sqrt{a^2-b^2x^2}+\frac{7}{24}a^2x\left(a^2-b^2x^2\right)^{3/2}-\frac{7a\left(a^2-b^2x^2\right)^{5/2}}{30b}-\frac{(a+bx)\left(a^2-b^2x^2\right)^{5/2}}{6b}+\frac{7a^6\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(a^2 - b^2*x^2)^(3/2),x]
```

```
[Out] (7*a^4*x*Sqrt[a^2 - b^2*x^2])/16 + (7*a^2*x*(a^2 - b^2*x^2)^(3/2))/24 - (7*
a*(a^2 - b^2*x^2)^(5/2))/(30*b) - ((a + b*x)*(a^2 - b^2*x^2)^(5/2))/(6*b) +
 (7*a^6*ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]])/(16*b)
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+bx)^2 \left(a^2-b^2x^2\right)^{3/2} dx = -\frac{(a+bx)\left(a^2-b^2x^2\right)^{5/2}}{6b} + \frac{1}{6}(7a)\int (a+bx)\left(a^2-b^2x^2\right)^{3/2} dx$$
$$= -\frac{7a\left(a^2-b^2x^2\right)^{5/2}}{30b} - \frac{(a+bx)\left(a^2-b^2x^2\right)^{5/2}}{6b} + \frac{1}{6}\left(7a^2\right)\int \left(a^2-b^2x^2\right)^{3/2} dx$$
$$= \frac{7}{24}a^2x\left(a^2-b^2x^2\right)^{3/2} - \frac{7a\left(a^2-b^2x^2\right)^{5/2}}{30b} - \frac{(a+bx)\left(a^2-b^2x^2\right)^{5/2}}{6b} + \frac{1}{8}\left(7a^4\right)\int \sqrt{a^2-}$$
$$= \frac{7}{16}a^4x\sqrt{a^2-b^2x^2} + \frac{7}{24}a^2x\left(a^2-b^2x^2\right)^{3/2} - \frac{7a\left(a^2-b^2x^2\right)^{5/2}}{30b} - \frac{(a+bx)\left(a^2-b^2x^2\right)^{5/2}}{6b}$$
$$= \frac{7}{16}a^4x\sqrt{a^2-b^2x^2} + \frac{7}{24}a^2x\left(a^2-b^2x^2\right)^{3/2} - \frac{7a\left(a^2-b^2x^2\right)^{5/2}}{30b} - \frac{(a+bx)\left(a^2-b^2x^2\right)^{5/2}}{6b}$$
$$= \frac{7}{16}a^4x\sqrt{a^2-b^2x^2} + \frac{7}{24}a^2x\left(a^2-b^2x^2\right)^{3/2} - \frac{7a\left(a^2-b^2x^2\right)^{5/2}}{30b} - \frac{(a+bx)\left(a^2-b^2x^2\right)^{5/2}}{6b}$$

Mathematica [A] time = 0.183362, size = 123, normalized size = 0.94

$$\frac{\sqrt{a^2-b^2x^2}\left(\sqrt{1-\frac{b^2x^2}{a^2}}\left(192a^3b^2x^2+10a^2b^3x^3+135a^4bx-96a^5-96ab^4x^4-40b^5x^5\right)+105a^5\sin^{-1}\left(\frac{bx}{a}\right)\right)}{240b\sqrt{1-\frac{b^2x^2}{a^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(a^2 - b^2*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^2]*(Sqrt[1 - (b^2*x^2)/a^2]*(-96*a^5 + 135*a^4*b*x + 192*
a^3*b^2*x^2 + 10*a^2*b^3*x^3 - 96*a*b^4*x^4 - 40*b^5*x^5) + 105*a^5*ArcSin[
(b*x)/a]))/(240*b*Sqrt[1 - (b^2*x^2)/a^2])
```

Maple [A] time = 0.051, size = 111, normalized size = 0.9

$$-\frac{x}{6}\left(-b^2x^2+a^2\right)^{\frac{5}{2}} + \frac{7a^2x}{24}\left(-b^2x^2+a^2\right)^{\frac{3}{2}} + \frac{7a^4x}{16}\sqrt{-b^2x^2+a^2} + \frac{7a^6}{16}\arctan\left(x\sqrt{b^2}\frac{1}{\sqrt{-b^2x^2+a^2}}\right)\frac{1}{\sqrt{b^2}} - \frac{2a}{5b}\left(-b^2x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(-b^2*x^2+a^2)^(3/2),x)
```

```
[Out] -1/6*x*(-b^2*x^2+a^2)^(5/2)+7/24*a^2*x*(-b^2*x^2+a^2)^(3/2)+7/16*a^4*x*(-b^
2*x^2+a^2)^(1/2)+7/16*a^6/(b^2)^(1/2)*arctan((b^2)^(1/2)*x/(-b^2*x^2+a^2)^(
1/2))-2/5*a*(-b^2*x^2+a^2)^(5/2)/b
```

Maxima [A] time = 1.6062, size = 139, normalized size = 1.06

$$\frac{7\,a^6\arcsin\left(\frac{b^2x}{\sqrt{a^2b^2}}\right)}{16\,\sqrt{b^2}}+\frac{7}{16}\,\sqrt{-b^2x^2+a^2}a^4x+\frac{7}{24}\left(-b^2x^2+a^2\right)^{\frac{3}{2}}a^2x-\frac{1}{6}\left(-b^2x^2+a^2\right)^{\frac{5}{2}}x-\frac{2\left(-b^2x^2+a^2\right)^{\frac{5}{2}}a}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b^2*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 7/16*a^6*arcsin(b^2*x/sqrt(a^2*b^2))/sqrt(b^2) + 7/16*sqrt(-b^2*x^2 + a^2)*
a^4*x + 7/24*(-b^2*x^2 + a^2)^(3/2)*a^2*x - 1/6*(-b^2*x^2 + a^2)^(5/2)*x -
2/5*(-b^2*x^2 + a^2)^(5/2)*a/b
```

Fricas [A] time = 2.08246, size = 231, normalized size = 1.76

$$-\frac{210\,a^6\arctan\left(-\frac{a-\sqrt{-b^2x^2+a^2}}{bx}\right)+\left(40\,b^5x^5+96\,ab^4x^4-10\,a^2b^3x^3-192\,a^3b^2x^2-135\,a^4bx+96\,a^5\right)\sqrt{-b^2x^2+a^2}}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b^2*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/240*(210*a^6*arctan(-(a - sqrt(-b^2*x^2 + a^2))/(b*x)) + (40*b^5*x^5 + 9
6*a*b^4*x^4 - 10*a^2*b^3*x^3 - 192*a^3*b^2*x^2 - 135*a^4*b*x + 96*a^5)*sqrt
(-b^2*x^2 + a^2))/b
```

Sympy [C] time = 9.46096, size = 498, normalized size = 3.8

$$a^4\left(\begin{cases}-\frac{ia^2\operatorname{acosh}\left(\frac{bx}{a}\right)}{2b}-\frac{iax}{2\sqrt{-1+\frac{b^2x^2}{a^2}}}+\frac{ib^2x^3}{2a\sqrt{-1+\frac{b^2x^2}{a^2}}} & \text{for}\ \frac{|b^2x^2|}{|a^2|}>1\\ \frac{a^2\operatorname{asin}\left(\frac{bx}{a}\right)}{2b}+\frac{ax\sqrt{1-\frac{b^2x^2}{a^2}}}{2} & \text{otherwise}\end{cases}\right)+2a^3b\left(\begin{cases}\frac{x^2\sqrt{a^2}}{2} & \text{for}\ b^2=0\\ -\frac{\left(a^2-b^2x^2\right)^{\frac{3}{2}}}{3b^2} & \text{otherwise}\end{cases}\right)-2ab^3\left(\begin{cases}-\frac{2a^4\sqrt{a^2-b^2x^2}}{15b^4} & \\ \frac{x^4\sqrt{a^2}}{4} & \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(-b**2*x**2+a**2)**(3/2),x)
```

```
[Out] a**4*Piecewise((-I*a**2*acosh(b*x/a)/(2*b) - I*a*x/(2*sqrt(-1 + b**2*x**2/a
**2)) + I*b**2*x**3/(2*a*sqrt(-1 + b**2*x**2/a**2)), Abs(b**2*x**2)/Abs(a**
2) > 1), (a**2*asin(b*x/a)/(2*b) + a*x*sqrt(1 - b**2*x**2/a**2)/2, True)) +
 2*a**3*b*Piecewise((x**2*sqrt(a**2)/2, Eq(b**2, 0)), (-(a**2 - b**2*x**2)*
*(3/2)/(3*b**2), True)) - 2*a*b**3*Piecewise((-2*a**4*sqrt(a**2 - b**2*x**2
)/(15*b**4) - a**2*x**2*sqrt(a**2 - b**2*x**2)/(15*b**2) + x**4*sqrt(a**2 -
 b**2*x**2)/5, Ne(b, 0)), (x**4*sqrt(a**2)/4, True)) - b**4*Piecewise((-I*a
**6*acosh(b*x/a)/(16*b**5) + I*a**5*x/(16*b**4*sqrt(-1 + b**2*x**2/a**2)) -
 I*a**3*x**3/(48*b**2*sqrt(-1 + b**2*x**2/a**2)) - 5*I*a*x**5/(24*sqrt(-1 +
 b**2*x**2/a**2)) + I*b**2*x**7/(6*a*sqrt(-1 + b**2*x**2/a**2)), Abs(b**2*x
**2)/Abs(a**2) > 1), (a**6*asin(b*x/a)/(16*b**5) - a**5*x/(16*b**4*sqrt(1 -
 b**2*x**2/a**2)) + a**3*x**3/(48*b**2*sqrt(1 - b**2*x**2/a**2)) + 5*a*x**5
/(24*sqrt(1 - b**2*x**2/a**2)) - b**2*x**7/(6*a*sqrt(1 - b**2*x**2/a**2)),
```

```
True))
```

Giac [A] time = 1.24505, size = 124, normalized size = 0.95

$$\frac{7\,a^{6}\arcsin\left(\frac{bx}{a}\right)\operatorname{sgn}\left(a\right)\operatorname{sgn}\left(b\right)}{16\,|b|}-\frac{1}{240}\left(\frac{96\,a^{5}}{b}-\left(135\,a^{4}+2\left(96\,a^{3}b+\left(5\,a^{2}b^{2}-4\left(5\,b^{4}x+12\,ab^{3}\right)x\right)x\right)x\right)x\right)\sqrt{-b^{2}x^{2}+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b^2*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 7/16*a^6*arcsin(b*x/a)*sgn(a)*sgn(b)/abs(b) - 1/240*(96*a^5/b - (135*a^4 +
2*(96*a^3*b + (5*a^2*b^2 - 4*(5*b^4*x + 12*a*b^3)*x)*x)*x)*x)*sqrt(-b^2*x^2
 + a^2)
```

3.790 $\int (a+bx)\left(a^2-b^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=100

$$\frac{3}{8}a^3x\sqrt{a^2-b^2x^2}+\frac{1}{4}ax\left(a^2-b^2x^2\right)^{3/2}-\frac{\left(a^2-b^2x^2\right)^{5/2}}{5b}+\frac{3a^5\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{8b}$$

```
[Out] (3*a^3*x*Sqrt[a^2 - b^2*x^2])/8 + (a*x*(a^2 - b^2*x^2)^(3/2))/4 - (a^2 - b^
2*x^2)^(5/2)/(5*b) + (3*a^5*ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]])/(8*b)
```

Rubi [A] time = 0.0262414, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {641, 195, 217, 203}

$$\frac{3}{8}a^3x\sqrt{a^2-b^2x^2}+\frac{1}{4}ax\left(a^2-b^2x^2\right)^{3/2}-\frac{\left(a^2-b^2x^2\right)^{5/2}}{5b}+\frac{3a^5\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(a^2 - b^2*x^2)^(3/2),x]
```

```
[Out] (3*a^3*x*Sqrt[a^2 - b^2*x^2])/8 + (a*x*(a^2 - b^2*x^2)^(3/2))/4 - (a^2 - b^
2*x^2)^(5/2)/(5*b) + (3*a^5*ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]])/(8*b)
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+bx)\left(a^2-b^2x^2\right)^{3/2} dx = -\frac{\left(a^2-b^2x^2\right)^{5/2}}{5b} + a\int \left(a^2-b^2x^2\right)^{3/2} dx$$
$$= \frac{1}{4}ax\left(a^2-b^2x^2\right)^{3/2} - \frac{\left(a^2-b^2x^2\right)^{5/2}}{5b} + \frac{1}{4}\left(3a^3\right)\int \sqrt{a^2-b^2x^2}\,dx$$
$$= \frac{3}{8}a^3x\sqrt{a^2-b^2x^2} + \frac{1}{4}ax\left(a^2-b^2x^2\right)^{3/2} - \frac{\left(a^2-b^2x^2\right)^{5/2}}{5b} + \frac{1}{8}\left(3a^5\right)\int \frac{1}{\sqrt{a^2-b^2x^2}}dx$$
$$= \frac{3}{8}a^3x\sqrt{a^2-b^2x^2} + \frac{1}{4}ax\left(a^2-b^2x^2\right)^{3/2} - \frac{\left(a^2-b^2x^2\right)^{5/2}}{5b} + \frac{1}{8}\left(3a^5\right)\text{Subst}\left(\int \frac{1}{1+b^2x^2}d\right.$$
$$= \frac{3}{8}a^3x\sqrt{a^2-b^2x^2} + \frac{1}{4}ax\left(a^2-b^2x^2\right)^{3/2} - \frac{\left(a^2-b^2x^2\right)^{5/2}}{5b} + \frac{3a^5\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{8b}$$

Mathematica [A] time = 0.145797, size = 112, normalized size = 1.12

$$\frac{\sqrt{a^2-b^2x^2}\left(\sqrt{1-\frac{b^2x^2}{a^2}}\left(16a^2b^2x^2+25a^3bx-8a^4-10ab^3x^3-8b^4x^4\right)+15a^4\sin^{-1}\left(\frac{bx}{a}\right)\right)}{40b\sqrt{1-\frac{b^2x^2}{a^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(a^2 - b^2*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^2]*(Sqrt[1 - (b^2*x^2)/a^2]*(-8*a^4 + 25*a^3*b*x + 16*a^2
*b^2*x^2 - 10*a*b^3*x^3 - 8*b^4*x^4) + 15*a^4*ArcSin[(b*x)/a]))/(40*b*Sqrt[
1 - (b^2*x^2)/a^2])
```

Maple [A] time = 0.047, size = 91, normalized size = 0.9

$$-\frac{1}{5\,b}\left(-b^2x^2+a^2\right)^{\frac{5}{2}} + \frac{ax}{4}\left(-b^2x^2+a^2\right)^{\frac{3}{2}} + \frac{3\,xa^3}{8}\sqrt{-b^2x^2+a^2} + \frac{3\,a^5}{8}\arctan\left(x\sqrt{b^2}\frac{1}{\sqrt{-b^2x^2+a^2}}\right)\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(-b^2*x^2+a^2)^(3/2),x)
```

```
[Out] -1/5*(-b^2*x^2+a^2)^(5/2)/b+1/4*a*x*(-b^2*x^2+a^2)^(3/2)+3/8*a^3*x*(-b^2*x^
2+a^2)^(1/2)+3/8*a^5/(b^2)^(1/2)*arctan((b^2)^(1/2)*x/(-b^2*x^2+a^2)^(1/2))
```

Maxima [A] time = 1.68208, size = 112, normalized size = 1.12

$$\frac{3\,a^5\arcsin\left(\frac{b^2x}{\sqrt{a^2b^2}}\right)}{8\,\sqrt{b^2}} + \frac{3}{8}\,\sqrt{-b^2x^2+a^2}a^3x + \frac{1}{4}\left(-b^2x^2+a^2\right)^{\frac{3}{2}}ax - \frac{\left(-b^2x^2+a^2\right)^{\frac{5}{2}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b^2*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/8*a^5*arcsin(b^2*x/sqrt(a^2*b^2))/sqrt(b^2) + 3/8*sqrt(-b^2*x^2 + a^2)*a^
3*x + 1/4*(-b^2*x^2 + a^2)^(3/2)*a*x - 1/5*(-b^2*x^2 + a^2)^(5/2)/b
```

Fricas [A] time = 2.11073, size = 200, normalized size = 2.

$$-\frac{30\,a^5\arctan\left(-\frac{a-\sqrt{-b^2x^2+a^2}}{bx}\right)+\left(8\,b^4x^4+10\,ab^3x^3-16\,a^2b^2x^2-25\,a^3bx+8\,a^4\right)\sqrt{-b^2x^2+a^2}}{40\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b^2*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/40*(30*a^5*arctan(-(a - sqrt(-b^2*x^2 + a^2))/(b*x)) + (8*b^4*x^4 + 10*a
*b^3*x^3 - 16*a^2*b^2*x^2 - 25*a^3*b*x + 8*a^4)*sqrt(-b^2*x^2 + a^2))/b
```

Sympy [C] time = 7.83493, size = 439, normalized size = 4.39

$$a^3\left(\begin{cases}-\frac{ia^2\operatorname{acosh}\left(\frac{bx}{a}\right)}{2b}-\frac{iax}{2\sqrt{-1+\frac{b^2x^2}{a^2}}}+\frac{ib^2x^3}{2a\sqrt{-1+\frac{b^2x^2}{a^2}}} & \text{for}\ \frac{|b^2x^2|}{|a^2|}>1\\ \frac{a^2\operatorname{asin}\left(\frac{bx}{a}\right)}{2b}+\frac{ax\sqrt{1-\frac{b^2x^2}{a^2}}}{2} & \text{otherwise}\end{cases}\right)+a^2b\left(\begin{cases}\frac{x^2\sqrt{a^2}}{2} & \text{for}\ b^2=0\\ -\frac{\left(a^2-b^2x^2\right)^{\frac{3}{2}}}{3b^2} & \text{otherwise}\end{cases}\right)-ab^2\left(\begin{cases}-\frac{ia^4\operatorname{acosh}\left(\frac{bx}{a}\right)}{8b^3}+ \\ \frac{a^4\operatorname{asin}\left(\frac{bx}{a}\right)}{8b^3}-\frac{}{8b^2}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b**2*x**2+a**2)**(3/2),x)
```

```
[Out] a**3*Piecewise((-I*a**2*acosh(b*x/a)/(2*b) - I*a*x/(2*sqrt(-1 + b**2*x**2/a
**2)) + I*b**2*x**3/(2*a*sqrt(-1 + b**2*x**2/a**2)), Abs(b**2*x**2)/Abs(a**
2) > 1), (a**2*asin(b*x/a)/(2*b) + a*x*sqrt(1 - b**2*x**2/a**2)/2, True)) +
 a**2*b*Piecewise((x**2*sqrt(a**2)/2, Eq(b**2, 0)), (-(a**2 - b**2*x**2)**(
3/2)/(3*b**2), True)) - a*b**2*Piecewise((-I*a**4*acosh(b*x/a)/(8*b**3) + I
*a**3*x/(8*b**2*sqrt(-1 + b**2*x**2/a**2)) - 3*I*a*x**3/(8*sqrt(-1 + b**2*x
**2/a**2)) + I*b**2*x**5/(4*a*sqrt(-1 + b**2*x**2/a**2)), Abs(b**2*x**2)/Ab
s(a**2) > 1), (a**4*asin(b*x/a)/(8*b**3) - a**3*x/(8*b**2*sqrt(1 - b**2*x**
2/a**2)) + 3*a*x**3/(8*sqrt(1 - b**2*x**2/a**2)) - b**2*x**5/(4*a*sqrt(1 -
b**2*x**2/a**2)), True)) - b**3*Piecewise((-2*a**4*sqrt(a**2 - b**2*x**2)/(
15*b**4) - a**2*x**2*sqrt(a**2 - b**2*x**2)/(15*b**2) + x**4*sqrt(a**2 - b*
*2*x**2)/5, Ne(b, 0)), (x**4*sqrt(a**2)/4, True))
```

Giac [A] time = 1.2365, size = 109, normalized size = 1.09

$$\frac{3\,a^5\arcsin\left(\frac{bx}{a}\right)\operatorname{sgn}(a)\operatorname{sgn}(b)}{8\,|b|}-\frac{1}{40}\,\sqrt{-b^2x^2+a^2}\left(\frac{8\,a^4}{b}-\left(25\,a^3+2\left(8\,a^2b-\left(4\,b^3x+5\,ab^2\right)x\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b^2*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 3/8*a^5*arcsin(b*x/a)*sgn(a)*sgn(b)/abs(b) - 1/40*sqrt(-b^2*x^2 + a^2)*(8*a
^4/b - (25*a^3 + 2*(8*a^2*b - (4*b^3*x + 5*a*b^2)*x)*x)*x)
```

3.791 $\int \frac{\left(a^2-b^2x^2\right)^{3/2}}{a+bx} dx$

Optimal. Leaf size=76

$$\frac{1}{2}ax\sqrt{a^2-b^2x^2}+\frac{\left(a^2-b^2x^2\right)^{3/2}}{3b}+\frac{a^3\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{2b}$$

```
[Out] (a*x*Sqrt[a^2 - b^2*x^2])/2 + (a^2 - b^2*x^2)^(3/2)/(3*b) + (a^3*ArcTan[(b*
x)/Sqrt[a^2 - b^2*x^2]])/(2*b)
```

Rubi [A] time = 0.0221436, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {665, 195, 217, 203}

$$\frac{1}{2}ax\sqrt{a^2-b^2x^2}+\frac{\left(a^2-b^2x^2\right)^{3/2}}{3b}+\frac{a^3\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*x^2)^(3/2)/(a + b*x),x]
```

```
[Out] (a*x*Sqrt[a^2 - b^2*x^2])/2 + (a^2 - b^2*x^2)^(3/2)/(3*b) + (a^3*ArcTan[(b*
x)/Sqrt[a^2 - b^2*x^2]])/(2*b)
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a^2-b^2x^2\right)^{3/2}}{a+bx}\,dx = \frac{\left(a^2-b^2x^2\right)^{3/2}}{3b}+a\int \sqrt{a^2-b^2x^2}\,dx$$
$$= \frac{1}{2}ax\sqrt{a^2-b^2x^2}+\frac{\left(a^2-b^2x^2\right)^{3/2}}{3b}+\frac{1}{2}a^3\int \frac{1}{\sqrt{a^2-b^2x^2}}\,dx$$
$$= \frac{1}{2}ax\sqrt{a^2-b^2x^2}+\frac{\left(a^2-b^2x^2\right)^{3/2}}{3b}+\frac{1}{2}a^3\,\text{Subst}\left(\int \frac{1}{1+b^2x^2}\,dx,x,\frac{x}{\sqrt{a^2-b^2x^2}}\right)$$
$$= \frac{1}{2}ax\sqrt{a^2-b^2x^2}+\frac{\left(a^2-b^2x^2\right)^{3/2}}{3b}+\frac{a^3\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{2b}$$

Mathematica [A] time = 0.0784287, size = 90, normalized size = 1.18

$$\frac{\sqrt{a^2-b^2x^2}\left(\left(2a^2+3abx-2b^2x^2\right)\sqrt{1-\frac{b^2x^2}{a^2}}+3a^2\sin^{-1}\left(\frac{bx}{a}\right)\right)}{6b\sqrt{1-\frac{b^2x^2}{a^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*x^2)^(3/2)/(a + b*x),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^2]*((2*a^2 + 3*a*b*x - 2*b^2*x^2)*Sqrt[1 - (b^2*x^2)/a^2]
 + 3*a^2*ArcSin[(b*x)/a]))/(6*b*Sqrt[1 - (b^2*x^2)/a^2])
```

Maple [A] time = 0.047, size = 113, normalized size = 1.5

$$\frac{1}{3b}\left(-\left(x+\frac{a}{b}\right)^2b^2+2\left(x+\frac{a}{b}\right)ab\right)^{\frac{3}{2}}+\frac{ax}{2}\sqrt{-\left(x+\frac{a}{b}\right)^2b^2+2\left(x+\frac{a}{b}\right)ab}+\frac{a^3}{2}\arctan\left(x\sqrt{b^2}\frac{1}{\sqrt{-\left(x+\frac{a}{b}\right)^2b^2+2\left(x+\frac{a}{b}\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^2*x^2+a^2)^(3/2)/(b*x+a),x)
```

```
[Out] 1/3/b*(-(x+1/b*a)^2*b^2+2*(x+1/b*a)*a*b)^(3/2)+1/2*a*(-(x+1/b*a)^2*b^2+2*(x
+1/b*a)*a*b)^(1/2)*x+1/2*a^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*x/(-(x+1/b*a)^2
*b^2+2*(x+1/b*a)*a*b)^(1/2))
```

Maxima [C] time = 1.73481, size = 119, normalized size = 1.57

$$-\frac{i\,a^3\arcsin\left(\frac{bx}{a}+2\right)}{2\,b}+\frac{1}{2}\sqrt{b^2x^2+4\,abx+3\,a^2}ax+\frac{\sqrt{b^2x^2+4\,abx+3\,a^2}a^2}{b}+\frac{\left(-b^2x^2+a^2\right)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*I*a^3*arcsin(b*x/a + 2)/b + 1/2*sqrt(b^2*x^2 + 4*a*b*x + 3*a^2)*a*x +
sqrt(b^2*x^2 + 4*a*b*x + 3*a^2)*a^2/b + 1/3*(-b^2*x^2 + a^2)^(3/2)/b
```

Fricas [A] time = 2.01181, size = 150, normalized size = 1.97

$$-\frac{6\,a^3\arctan\left(-\frac{a-\sqrt{-b^2x^2+a^2}}{bx}\right)+\left(2\,b^2x^2-3\,abx-2\,a^2\right)\sqrt{-b^2x^2+a^2}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/6*(6*a^3*arctan(-(a - sqrt(-b^2*x^2 + a^2))/(b*x)) + (2*b^2*x^2 - 3*a*b*
x - 2*a^2)*sqrt(-b^2*x^2 + a^2))/b
```

Sympy [C] time = 4.00543, size = 146, normalized size = 1.92

$$a\left(\begin{cases} -\frac{ia^2\operatorname{acosh}\left(\frac{bx}{a}\right)}{2b}-\frac{iax}{2\sqrt{-1+\frac{b^2x^2}{a^2}}}+\frac{ib^2x^3}{2a\sqrt{-1+\frac{b^2x^2}{a^2}}} & \text{for}\ \frac{|b^2x^2|}{|a^2|}>1 \\ \frac{a^2\operatorname{asin}\left(\frac{bx}{a}\right)}{2b}+\frac{ax\sqrt{1-\frac{b^2x^2}{a^2}}}{2} & \text{otherwise} \end{cases}\right)-b\left(\begin{cases} \frac{x^2\sqrt{a^2}}{2} & \text{for}\ b^2=0 \\ -\frac{\left(a^2-b^2x^2\right)^{\frac{3}{2}}}{3b^2} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**2*x**2+a**2)**(3/2)/(b*x+a),x)
```

```
[Out] a*Piecewise((-I*a**2*acosh(b*x/a)/(2*b) - I*a*x/(2*sqrt(-1 + b**2*x**2/a**2
)) + I*b**2*x**3/(2*a*sqrt(-1 + b**2*x**2/a**2)), Abs(b**2*x**2)/Abs(a**2)
> 1), (a**2*asin(b*x/a)/(2*b) + a*x*sqrt(1 - b**2*x**2/a**2)/2, True)) - b*
Piecewise((x**2*sqrt(a**2)/2, Eq(b**2, 0)), (-(a**2 - b**2*x**2)**(3/2)/(3*
b**2), True))
```

Giac [A] time = 1.23443, size = 76, normalized size = 1.

$$\frac{a^3\arcsin\left(\frac{bx}{a}\right)\operatorname{sgn}\left(a\right)\operatorname{sgn}\left(b\right)}{2\,|b|}-\frac{1}{6}\,\sqrt{-b^2x^2+a^2}\left((2\,bx-3\,a)x-\frac{2\,a^2}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*a^3*arcsin(b*x/a)*sgn(a)*sgn(b)/abs(b) - 1/6*sqrt(-b^2*x^2 + a^2)*((2*b
*x - 3*a)*x - 2*a^2/b)
```

3.792 $\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^2} dx$

Optimal. Leaf size=85

$$\frac{3a\sqrt{a^2-b^2x^2}}{2b}+\frac{(a^2-b^2x^2)^{3/2}}{2b(a+bx)}+\frac{3a^2\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{2b}$$

```
[Out] (3*a*Sqrt[a^2 - b^2*x^2])/(2*b) + (a^2 - b^2*x^2)^(3/2)/(2*b*(a + b*x)) + (
3*a^2*ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]])/(2*b)
```

Rubi [A] time = 0.0293329, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {665, 217, 203}

$$\frac{3a\sqrt{a^2-b^2x^2}}{2b}+\frac{(a^2-b^2x^2)^{3/2}}{2b(a+bx)}+\frac{3a^2\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*x^2)^(3/2)/(a + b*x)^2,x]
```

```
[Out] (3*a*Sqrt[a^2 - b^2*x^2])/(2*b) + (a^2 - b^2*x^2)^(3/2)/(2*b*(a + b*x)) + (
3*a^2*ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]])/(2*b)
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2-b^2x^2\right)^{3/2}}{(a+bx)^2}\,dx &= \frac{\left(a^2-b^2x^2\right)^{3/2}}{2b(a+bx)}+\frac{1}{2}(3a)\int \frac{\sqrt{a^2-b^2x^2}}{a+bx}\,dx\\ &= \frac{3a\sqrt{a^2-b^2x^2}}{2b}+\frac{\left(a^2-b^2x^2\right)^{3/2}}{2b(a+bx)}+\frac{1}{2}\left(3a^2\right)\int \frac{1}{\sqrt{a^2-b^2x^2}}\,dx\\ &= \frac{3a\sqrt{a^2-b^2x^2}}{2b}+\frac{\left(a^2-b^2x^2\right)^{3/2}}{2b(a+bx)}+\frac{1}{2}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1+b^2x^2}\,dx,x,\frac{x}{\sqrt{a^2-b^2x^2}}\right)\\ &= \frac{3a\sqrt{a^2-b^2x^2}}{2b}+\frac{\left(a^2-b^2x^2\right)^{3/2}}{2b(a+bx)}+\frac{3a^2\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0561479, size = 60, normalized size = 0.71

$$\left(\frac{2a}{b}-\frac{x}{2}\right)\sqrt{a^2-b^2x^2}+\frac{3a^2\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*x^2)^(3/2)/(a + b*x)^2,x]
```

```
[Out] ((2*a)/b - x/2)*Sqrt[a^2 - b^2*x^2] + (3*a^2*ArcTan[(b*x)/Sqrt[a^2 - b^2*x^
2]])/(2*b)
```

Maple [B] time = 0.05, size = 158, normalized size = 1.9

$$\frac{1}{ab^3}\left(-\left(x+\frac{a}{b}\right)^2b^2+2\left(x+\frac{a}{b}\right)ab\right)^{\frac{5}{2}}\left(x+\frac{a}{b}\right)^{-2}+\frac{1}{ab}\left(-\left(x+\frac{a}{b}\right)^2b^2+2\left(x+\frac{a}{b}\right)ab\right)^{\frac{3}{2}}+\frac{3x}{2}\sqrt{-\left(x+\frac{a}{b}\right)^2b^2+2\left(x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^2*x^2+a^2)^(3/2)/(b*x+a)^2,x)
```

```
[Out] 1/b^3/a/(x+1/b*a)^2*(-(x+1/b*a)^2*b^2+2*(x+1/b*a)*a*b)^(5/2)+1/b/a*(-(x+1/b
*a)^2*b^2+2*(x+1/b*a)*a*b)^(3/2)+3/2*(-(x+1/b*a)^2*b^2+2*(x+1/b*a)*a*b)^(1/
2)*x+3/2*a^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*x/(-(x+1/b*a)^2*b^2+2*(x+1/b*a)
*a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.08218, size = 126, normalized size = 1.48

$$-\frac{6\,a^2\arctan\left(-\frac{a-\sqrt{-b^2x^2+a^2}}{bx}\right)+\sqrt{-b^2x^2+a^2}(bx-4\,a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(6*a^2*arctan(-(a - sqrt(-b^2*x^2 + a^2))/(b*x)) + sqrt(-b^2*x^2 + a^2
)*(b*x - 4*a))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-\left(-a+bx\right)\left(a+bx\right)\right)^{\frac{3}{2}}}{\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**2*x**2+a**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Integral((-(-a + b*x)*(a + b*x))**(3/2)/(a + b*x)**2, x)
```

Giac [A] time = 1.26223, size = 163, normalized size = 1.92

$$-\frac{\left(12\,a^3b^3\arctan\left(\sqrt{\frac{2\,a}{bx+a}-1}\right)\mathrm{sgn}\left(\frac{1}{bx+a}\right)\mathrm{sgn}\left(b\right)-\frac{\left(5\,a^3b^3\left(\frac{2\,a}{bx+a}-1\right)^{\frac{3}{2}}\mathrm{sgn}\left(\frac{1}{bx+a}\right)\mathrm{sgn}(b)+3\,a^3b^3\sqrt{\frac{2\,a}{bx+a}-1}\mathrm{sgn}\left(\frac{1}{bx+a}\right)\mathrm{sgn}(b)\right)(bx+a)^2}{a^2}\right)|b|}{4\,ab^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*(12*a^3*b^3*arctan(sqrt(2*a/(b*x + a) - 1))*sgn(1/(b*x + a))*sgn(b) -
(5*a^3*b^3*(2*a/(b*x + a) - 1)^(3/2)*sgn(1/(b*x + a))*sgn(b) + 3*a^3*b^3*sq
rt(2*a/(b*x + a) - 1)*sgn(1/(b*x + a))*sgn(b))*(b*x + a)^2/a^2)*abs(b)/(a*b
^5)
```

3.793 $\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^3}\,dx$

Optimal. Leaf size=76

$$-\frac{2\left(a^2-b^2x^2\right)^{3/2}}{b(a+bx)^2}-\frac{3\sqrt{a^2-b^2x^2}}{b}-\frac{3a\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{b}$$

```
[Out] (-3*Sqrt[a^2 - b^2*x^2])/b - (2*(a^2 - b^2*x^2)^(3/2))/(b*(a + b*x)^2) - (3
*a*ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]])/b
```

Rubi [A] time = 0.0277218, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {663, 665, 217, 203}

$$-\frac{2\left(a^2-b^2x^2\right)^{3/2}}{b(a+bx)^2}-\frac{3\sqrt{a^2-b^2x^2}}{b}-\frac{3a\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*x^2)^(3/2)/(a + b*x)^3,x]
```

```
[Out] (-3*Sqrt[a^2 - b^2*x^2])/b - (2*(a^2 - b^2*x^2)^(3/2))/(b*(a + b*x)^2) - (3
*a*ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]])/b
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a^2-b^2x^2\right)^{3/2}}{(a+bx)^3}\,dx = -\frac{2\left(a^2-b^2x^2\right)^{3/2}}{b(a+bx)^2} - 3\int \frac{\sqrt{a^2-b^2x^2}}{a+bx}\,dx$$
$$= -\frac{3\sqrt{a^2-b^2x^2}}{b} - \frac{2\left(a^2-b^2x^2\right)^{3/2}}{b(a+bx)^2} - (3a)\int \frac{1}{\sqrt{a^2-b^2x^2}}\,dx$$
$$= -\frac{3\sqrt{a^2-b^2x^2}}{b} - \frac{2\left(a^2-b^2x^2\right)^{3/2}}{b(a+bx)^2} - (3a)\,\mathrm{Subst}\left(\int \frac{1}{1+b^2x^2}\,dx, x, \frac{x}{\sqrt{a^2-b^2x^2}}\right)$$
$$= -\frac{3\sqrt{a^2-b^2x^2}}{b} - \frac{2\left(a^2-b^2x^2\right)^{3/2}}{b(a+bx)^2} - \frac{3a\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{b}$$

Mathematica [A] time = 0.0853772, size = 60, normalized size = 0.79

$$-\frac{\frac{\sqrt{a^2-b^2x^2}(5a+bx)}{a+bx} + 3a\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*x^2)^(3/2)/(a + b*x)^3,x]
```

```
[Out] -((((5*a + b*x)*Sqrt[a^2 - b^2*x^2])/(a + b*x) + 3*a*ArcTan[(b*x)/Sqrt[a^2
 - b^2*x^2]])/b)
```

Maple [B] time = 0.047, size = 206, normalized size = 2.7

$$-\frac{1}{b^4a}\left(-\left(x+\frac{a}{b}\right)^2b^2+2\left(x+\frac{a}{b}\right)ab\right)^{\frac{5}{2}}\left(x+\frac{a}{b}\right)^{-3}-2\frac{1}{b^3a^2}\left(-\left(x+\frac{a}{b}\right)^2b^2+2\left(x+\frac{a}{b}\right)ab\right)^{5/2}\left(x+\frac{a}{b}\right)^{-2}-2\frac{1}{ba^2}\left(-\left(x+\frac{a}{b}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^2*x^2+a^2)^(3/2)/(b*x+a)^3,x)
```

```
[Out] -1/b^4/a/(x+1/b*a)^3*(-(x+1/b*a)^2*b^2+2*(x+1/b*a)*a*b)^(5/2)-2/b^3/a^2/(x+
1/b*a)^2*(-(x+1/b*a)^2*b^2+2*(x+1/b*a)*a*b)^(5/2)-2/b/a^2*(-(x+1/b*a)^2*b^2
+2*(x+1/b*a)*a*b)^(3/2)-3/a*(-(x+1/b*a)^2*b^2+2*(x+1/b*a)*a*b)^(1/2)*x-3*a/
(b^2)^(1/2)*arctan((b^2)^(1/2)*x/(-(x+1/b*a)^2*b^2+2*(x+1/b*a)*a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.15592, size = 174, normalized size = 2.29

$$-\frac{5\,abx+5\,a^2-6\left(abx+a^2\right)\arctan\left(-\frac{a-\sqrt{-b^2x^2+a^2}}{bx}\right)+\sqrt{-b^2x^2+a^2}(bx+5\,a)}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(5*a*b*x + 5*a^2 - 6*(a*b*x + a^2)*arctan(-(a - sqrt(-b^2*x^2 + a^2))/(b*x
)) + sqrt(-b^2*x^2 + a^2)*(b*x + 5*a))/(b^2*x + a*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-\left(-a+bx\right)\left(a+bx\right)\right)^{\frac{3}{2}}}{\left(a+bx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**2*x**2+a**2)**(3/2)/(b*x+a)**3,x)
```

```
[Out] Integral((-(-a + b*x)*(a + b*x))**(3/2)/(a + b*x)**3, x)
```

Giac [A] time = 1.2186, size = 104, normalized size = 1.37

$$-\frac{3\,a\arcsin\left(\frac{bx}{a}\right)\operatorname{sgn}\left(a\right)\operatorname{sgn}\left(b\right)}{|b|}-\frac{\sqrt{-b^2x^2+a^2}}{b}+\frac{8\,a}{\left(\frac{ab+\sqrt{-b^2x^2+a^2}|b|}{b^2x}+1\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -3*a*arcsin(b*x/a)*sgn(a)*sgn(b)/abs(b) - sqrt(-b^2*x^2 + a^2)/b + 8*a/(((a
*b + sqrt(-b^2*x^2 + a^2)*abs(b))/(b^2*x) + 1)*abs(b))
```

3.794 $\int \frac{\left(a^2-b^2x^2\right)^{3/2}}{(a+bx)^4}\,dx$

Optimal. Leaf size=83

$$-\frac{2\left(a^2-b^2x^2\right)^{3/2}}{3b(a+bx)^3}+\frac{2\sqrt{a^2-b^2x^2}}{b(a+bx)}+\frac{\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{b}$$

```
[Out] (2*Sqrt[a^2 - b^2*x^2])/(b*(a + b*x)) - (2*(a^2 - b^2*x^2)^(3/2))/(3*b*(a +
 b*x)^3) + ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]]/b
```

Rubi [A] time = 0.0229014, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {663, 217, 203}

$$-\frac{2\left(a^2-b^2x^2\right)^{3/2}}{3b(a+bx)^3}+\frac{2\sqrt{a^2-b^2x^2}}{b(a+bx)}+\frac{\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*x^2)^(3/2)/(a + b*x)^4,x]
```

```
[Out] (2*Sqrt[a^2 - b^2*x^2])/(b*(a + b*x)) - (2*(a^2 - b^2*x^2)^(3/2))/(3*b*(a +
 b*x)^3) + ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]]/b
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a^2-b^2x^2\right)^{3/2}}{(a+bx)^4}\,dx &= -\frac{2\left(a^2-b^2x^2\right)^{3/2}}{3b(a+bx)^3}-\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^2}\,dx\\
&= \frac{2\sqrt{a^2-b^2x^2}}{b(a+bx)}-\frac{2\left(a^2-b^2x^2\right)^{3/2}}{3b(a+bx)^3}+\int \frac{1}{\sqrt{a^2-b^2x^2}}\,dx\\
&= \frac{2\sqrt{a^2-b^2x^2}}{b(a+bx)}-\frac{2\left(a^2-b^2x^2\right)^{3/2}}{3b(a+bx)^3}+\text{Subst}\left(\int \frac{1}{1+b^2x^2}\,dx,x,\frac{x}{\sqrt{a^2-b^2x^2}}\right)\\
&= \frac{2\sqrt{a^2-b^2x^2}}{b(a+bx)}-\frac{2\left(a^2-b^2x^2\right)^{3/2}}{3b(a+bx)^3}+\frac{\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.0695871, size = 61, normalized size = 0.73

$$
\frac{\frac{4\sqrt{a^2-b^2x^2}(a+2bx)}{(a+bx)^2}+3\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{3b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*x^2)^(3/2)/(a + b*x)^4,x]
```

```
[Out] ((4*(a + 2*b*x)*Sqrt[a^2 - b^2*x^2])/(a + b*x)^2 + 3*ArcTan[(b*x)/Sqrt[a^2
- b^2*x^2]])/(3*b)
```

Maple [B] time = 0.053, size = 248, normalized size = 3.

$$
-\frac{1}{3\,b^5a}\left(-\left(x+\frac{a}{b}\right)^2b^2+2\left(x+\frac{a}{b}\right)ab\right)^{\frac{5}{2}}\left(x+\frac{a}{b}\right)^{-4}+\frac{1}{3\,a^2b^4}\left(-\left(x+\frac{a}{b}\right)^2b^2+2\left(x+\frac{a}{b}\right)ab\right)^{\frac{5}{2}}\left(x+\frac{a}{b}\right)^{-3}+\frac{2}{3\,b^3a^3}\left(-(x
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^2*x^2+a^2)^(3/2)/(b*x+a)^4,x)
```

```
[Out] -1/3/b^5/a/(x+1/b*a)^4*(-(x+1/b*a)^2*b^2+2*(x+1/b*a)*a*b)^(5/2)+1/3/b^4/a^2
/(x+1/b*a)^3*(-(x+1/b*a)^2*b^2+2*(x+1/b*a)*a*b)^(5/2)+2/3/b^3/a^3/(x+1/b*a)
^2*(-(x+1/b*a)^2*b^2+2*(x+1/b*a)*a*b)^(5/2)+2/3/b/a^3*(-(x+1/b*a)^2*b^2+2*(
x+1/b*a)*a*b)^(3/2)+1/a^2*(-(x+1/b*a)^2*b^2+2*(x+1/b*a)*a*b)^(1/2)*x+1/(b^2
)^(1/2)*arctan((b^2)^(1/2)*x/(-(x+1/b*a)^2*b^2+2*(x+1/b*a)*a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.0793, size = 235, normalized size = 2.83

$$\frac{2\left(2\,b^2x^2+4\,abx+2\,a^2-3\left(b^2x^2+2\,abx+a^2\right)\arctan\left(-\frac{a-\sqrt{-b^2x^2+a^2}}{bx}\right)+2\,\sqrt{-b^2x^2+a^2}(2\,bx+a)\right)}{3\left(b^3x^2+2\,ab^2x+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 2/3*(2*b^2*x^2 + 4*a*b*x + 2*a^2 - 3*(b^2*x^2 + 2*a*b*x + a^2)*arctan(-(a -
 sqrt(-b^2*x^2 + a^2))/(b*x)) + 2*sqrt(-b^2*x^2 + a^2)*(2*b*x + a))/(b^3*x^
2 + 2*a*b^2*x + a^2*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(-a+bx\right)\left(a+bx\right)\right)^{\frac{3}{2}}}{\left(a+bx\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**2*x**2+a**2)**(3/2)/(b*x+a)**4,x)
```

```
[Out] Integral((-(-a + b*x)*(a + b*x))**(3/2)/(a + b*x)**4, x)
```

Giac [A] time = 1.25805, size = 116, normalized size = 1.4

$$\frac{\arcsin\left(\frac{bx}{a}\right)\operatorname{sgn}\left(a\right)\operatorname{sgn}\left(b\right)}{|b|}-\frac{8\left(\frac{3\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)}{b^2x}+1\right)}{3\left(\frac{ab+\sqrt{-b^2x^2+a^2}|b|}{b^2x}+1\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] arcsin(b*x/a)*sgn(a)*sgn(b)/abs(b) - 8/3*(3*(a*b + sqrt(-b^2*x^2 + a^2)*abs
(b))/(b^2*x) + 1)/(((a*b + sqrt(-b^2*x^2 + a^2)*abs(b))/(b^2*x) + 1)^3*abs(
b))
```

3.795 $\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^5}\,dx$

Optimal. Leaf size=33

$$-\frac{(a^2-b^2x^2)^{5/2}}{5ab(a+bx)^5}$$

```
[Out] -(a^2 - b^2*x^2)^(5/2)/(5*a*b*(a + b*x)^5)
```

Rubi [A] time = 0.0090839, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {651}

$$-\frac{(a^2-b^2x^2)^{5/2}}{5ab(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*x^2)^(3/2)/(a + b*x)^5,x]
```

```
[Out] -(a^2 - b^2*x^2)^(5/2)/(5*a*b*(a + b*x)^5)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^5}\,dx = -\frac{(a^2-b^2x^2)^{5/2}}{5ab(a+bx)^5}$$

Mathematica [A] time = 0.0455292, size = 41, normalized size = 1.24

$$-\frac{(a-bx)^2\sqrt{a^2-b^2x^2}}{5ab(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*x^2)^(3/2)/(a + b*x)^5,x]
```

```
[Out] -((a - b*x)^2*Sqrt[a^2 - b^2*x^2])/(5*a*b*(a + b*x)^3)
```

Maple [A] time = 0.043, size = 36, normalized size = 1.1

$$-\frac{-bx+a}{5\,(bx+a)^4\,ba}\left(-b^2x^2+a^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^2*x^2+a^2)^(3/2)/(b*x+a)^5,x)
```

```
[Out] -1/5/(b*x+a)^4*(-b*x+a)/b/a*(-b^2*x^2+a^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.13154, size = 198, normalized size = 6.

$$-\frac{b^3x^3 + 3\,ab^2x^2 + 3\,a^2bx + a^3 + \left(b^2x^2 - 2\,abx + a^2\right)\sqrt{-b^2x^2 + a^2}}{5\left(ab^4x^3 + 3\,a^2b^3x^2 + 3\,a^3b^2x + a^4b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/5*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3 + (b^2*x^2 - 2*a*b*x + a^2)*s
qrt(-b^2*x^2 + a^2))/(a*b^4*x^3 + 3*a^2*b^3*x^2 + 3*a^3*b^2*x + a^4*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(-a + bx\right)\left(a + bx\right)\right)^{\frac{3}{2}}}{\left(a + bx\right)^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**2*x**2+a**2)**(3/2)/(b*x+a)**5,x)
```

```
[Out] Integral((-(-a + b*x)*(a + b*x))**(3/2)/(a + b*x)**5, x)
```

Giac [C] time = 1.29834, size = 139, normalized size = 4.21

$$-\frac{1}{15}\left(-\frac{3i\,\mathrm{sgn}\left(\frac{1}{bx+a}\right)\mathrm{sgn}\left(b\right)}{ab^2} - \frac{5\left(\frac{2a}{bx+a} - 1\right)^{\frac{3}{2}}\mathrm{sgn}\left(\frac{1}{bx+a}\right)\mathrm{sgn}\left(b\right) - \left(3\left(\frac{2a}{bx+a} - 1\right)^{\frac{5}{2}} + 5\left(\frac{2a}{bx+a} - 1\right)^{\frac{3}{2}}\right)\mathrm{sgn}\left(\frac{1}{bx+a}\right)\mathrm{sgn}\left(b\right)}{ab^2}\right)|b|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/15*(-3*I*sgn(1/(b*x + a))*sgn(b)/(a*b^2) - (5*(2*a/(b*x + a) - 1)^(3/2)*
sgn(1/(b*x + a))*sgn(b) - (3*(2*a/(b*x + a) - 1)^(5/2) + 5*(2*a/(b*x + a) -
 1)^(3/2))*sgn(1/(b*x + a))*sgn(b))/(a*b^2))*abs(b)
```

3.796 $\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^6} dx$

Optimal. Leaf size=67

$$-\frac{(a^2-b^2x^2)^{5/2}}{35a^2b(a+bx)^5}-\frac{(a^2-b^2x^2)^{5/2}}{7ab(a+bx)^6}$$

```
[Out] -(a^2 - b^2*x^2)^(5/2)/(7*a*b*(a + b*x)^6) - (a^2 - b^2*x^2)^(5/2)/(35*a^2*
b*(a + b*x)^5)
```

Rubi [A] time = 0.0211747, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 651}

$$-\frac{(a^2-b^2x^2)^{5/2}}{35a^2b(a+bx)^5}-\frac{(a^2-b^2x^2)^{5/2}}{7ab(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*x^2)^(3/2)/(a + b*x)^6,x]
```

```
[Out] -(a^2 - b^2*x^2)^(5/2)/(7*a*b*(a + b*x)^6) - (a^2 - b^2*x^2)^(5/2)/(35*a^2*
b*(a + b*x)^5)
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^6} dx &= -\frac{(a^2-b^2x^2)^{5/2}}{7ab(a+bx)^6}+\frac{\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^5} dx}{7a}\\ &= -\frac{(a^2-b^2x^2)^{5/2}}{7ab(a+bx)^6}-\frac{(a^2-b^2x^2)^{5/2}}{35a^2b(a+bx)^5}\end{aligned}$$

Mathematica [A] time = 0.0505912, size = 48, normalized size = 0.72

$$-\frac{(a-bx)^2(6a+bx)\sqrt{a^2-b^2x^2}}{35a^2b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*x^2)^(3/2)/(a + b*x)^6,x]
```

```
[Out] -((a - b*x)^2*(6*a + b*x)*Sqrt[a^2 - b^2*x^2])/(35*a^2*b*(a + b*x)^4)
```

Maple [A] time = 0.045, size = 43, normalized size = 0.6

$$-\frac{(bx+6a)(-bx+a)}{35(bx+a)^5 ba^2}\left(-b^2x^2+a^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^2*x^2+a^2)^(3/2)/(b*x+a)^6,x)
```

```
[Out] -1/35*(-b*x+a)*(b*x+6*a)*(-b^2*x^2+a^2)^(3/2)/(b*x+a)^5/b/a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.23308, size = 281, normalized size = 4.19

$$-\frac{6b^4x^4+24ab^3x^3+36a^2b^2x^2+24a^3bx+6a^4+\left(b^3x^3+4ab^2x^2-11a^2bx+6a^3\right)\sqrt{-b^2x^2+a^2}}{35\left(a^2b^5x^4+4a^3b^4x^3+6a^4b^3x^2+4a^5b^2x+a^6b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] -1/35*(6*b^4*x^4 + 24*a*b^3*x^3 + 36*a^2*b^2*x^2 + 24*a^3*b*x + 6*a^4 + (b^
3*x^3 + 4*a*b^2*x^2 - 11*a^2*b*x + 6*a^3)*sqrt(-b^2*x^2 + a^2))/(a^2*b^5*x^
4 + 4*a^3*b^4*x^3 + 6*a^4*b^3*x^2 + 4*a^5*b^2*x + a^6*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(-a+bx\right)\left(a+bx\right)\right)^{\frac{3}{2}}}{\left(a+bx\right)^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**2*x**2+a**2)**(3/2)/(b*x+a)**6,x)
```

```
[Out] Integral((-(-a + b*x)*(a + b*x))**(3/2)/(a + b*x)**6, x)
```

Giac [B] time = 1.28945, size = 306, normalized size = 4.57

$$\frac{2\left(\frac{7\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)}{b^2x}+\frac{91\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^2}{b^4x^2}+\frac{70\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^3}{b^6x^3}+\frac{140\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^4}{b^8x^4}+\frac{35\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^5}{b^{10}x^5}+\frac{35\left(ab+\sqrt{-b^2x^2+\cdots\right.}{b^{12}x^6}\right.}{35\,a^2\left(\frac{ab+\sqrt{-b^2x^2+a^2}|b|}{b^2x}+1\right)^7|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 2/35*(7*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))/(b^2*x) + 91*(a*b + sqrt(-b^2*x
^2 + a^2)*abs(b))^2/(b^4*x^2) + 70*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^3/(b
^6*x^3) + 140*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^4/(b^8*x^4) + 35*(a*b + s
qrt(-b^2*x^2 + a^2)*abs(b))^5/(b^10*x^5) + 35*(a*b + sqrt(-b^2*x^2 + a^2)*a
bs(b))^6/(b^12*x^6) + 6)/(a^2*((a*b + sqrt(-b^2*x^2 + a^2)*abs(b))/(b^2*x)
+ 1)^7*abs(b))
```

3.797 $\int \frac{\left(a^2-b^2x^2\right)^{3/2}}{(a+bx)^7} dx$

Optimal. Leaf size=100

$$-\frac{2\left(a^2-b^2x^2\right)^{5/2}}{315a^3b(a+bx)^5}-\frac{2\left(a^2-b^2x^2\right)^{5/2}}{63a^2b(a+bx)^6}-\frac{\left(a^2-b^2x^2\right)^{5/2}}{9ab(a+bx)^7}$$

```
[Out] -(a^2 - b^2*x^2)^(5/2)/(9*a*b*(a + b*x)^7) - (2*(a^2 - b^2*x^2)^(5/2))/(63*
a^2*b*(a + b*x)^6) - (2*(a^2 - b^2*x^2)^(5/2))/(315*a^3*b*(a + b*x)^5)
```

Rubi [A] time = 0.0367267, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 651}

$$-\frac{2\left(a^2-b^2x^2\right)^{5/2}}{315a^3b(a+bx)^5}-\frac{2\left(a^2-b^2x^2\right)^{5/2}}{63a^2b(a+bx)^6}-\frac{\left(a^2-b^2x^2\right)^{5/2}}{9ab(a+bx)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*x^2)^(3/2)/(a + b*x)^7,x]
```

```
[Out] -(a^2 - b^2*x^2)^(5/2)/(9*a*b*(a + b*x)^7) - (2*(a^2 - b^2*x^2)^(5/2))/(63*
a^2*b*(a + b*x)^6) - (2*(a^2 - b^2*x^2)^(5/2))/(315*a^3*b*(a + b*x)^5)
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2-b^2x^2\right)^{3/2}}{(a+bx)^7} dx &= -\frac{\left(a^2-b^2x^2\right)^{5/2}}{9ab(a+bx)^7}+\frac{2\int \frac{\left(a^2-b^2x^2\right)^{3/2}}{(a+bx)^6} dx}{9a}\\
&= -\frac{\left(a^2-b^2x^2\right)^{5/2}}{9ab(a+bx)^7}-\frac{2\left(a^2-b^2x^2\right)^{5/2}}{63a^2b(a+bx)^6}+\frac{2\int \frac{\left(a^2-b^2x^2\right)^{3/2}}{(a+bx)^5} dx}{63a^2}\\
&= -\frac{\left(a^2-b^2x^2\right)^{5/2}}{9ab(a+bx)^7}-\frac{2\left(a^2-b^2x^2\right)^{5/2}}{63a^2b(a+bx)^6}-\frac{2\left(a^2-b^2x^2\right)^{5/2}}{315a^3b(a+bx)^5}
\end{aligned}$$

Mathematica [A] time = 0.0519771, size = 60, normalized size = 0.6

$$-\frac{(a-bx)^2\sqrt{a^2-b^2x^2}\left(47a^2+14abx+2b^2x^2\right)}{315a^3b(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*x^2)^(3/2)/(a + b*x)^7,x]
```

```
[Out] -((a - b*x)^2*Sqrt[a^2 - b^2*x^2]*(47*a^2 + 14*a*b*x + 2*b^2*x^2))/(315*a^3
*b*(a + b*x)^5)
```

Maple [A] time = 0.045, size = 55, normalized size = 0.6

$$-\frac{\left(2\,b^2x^2+14\,abx+47\,a^2\right)(-bx+a)}{315\,(bx+a)^6\,a^3b}\left(-b^2x^2+a^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^2*x^2+a^2)^(3/2)/(b*x+a)^7,x)
```

```
[Out] -1/315*(-b*x+a)*(2*b^2*x^2+14*a*b*x+47*a^2)*(-b^2*x^2+a^2)^(3/2)/(b*x+a)^6/
a^3/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.52184, size = 366, normalized size = 3.66

$$-\frac{47\,b^5x^5+235\,ab^4x^4+470\,a^2b^3x^3+470\,a^3b^2x^2+235\,a^4bx+47\,a^5+\left(2\,b^4x^4+10\,ab^3x^3+21\,a^2b^2x^2-80\,a^3bx+47\,a^4\right)}{315\left(a^3b^6x^5+5\,a^4b^5x^4+10\,a^5b^4x^3+10\,a^6b^3x^2+5\,a^7b^2x+a^8b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/315*(47*b^5*x^5 + 235*a*b^4*x^4 + 470*a^2*b^3*x^3 + 470*a^3*b^2*x^2 + 23
5*a^4*b*x + 47*a^5 + (2*b^4*x^4 + 10*a*b^3*x^3 + 21*a^2*b^2*x^2 - 80*a^3*b*
x + 47*a^4)*sqrt(-b^2*x^2 + a^2))/(a^3*b^6*x^5 + 5*a^4*b^5*x^4 + 10*a^5*b^4
*x^3 + 10*a^6*b^3*x^2 + 5*a^7*b^2*x + a^8*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(-a+bx\right)\left(a+bx\right)\right)^{\frac{3}{2}}}{\left(a+bx\right)^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**2*x**2+a**2)**(3/2)/(b*x+a)**7,x)
```

```
[Out] Integral((-(-a + b*x)*(a + b*x))**(3/2)/(a + b*x)**7, x)
```

Giac [B] time = 1.24946, size = 390, normalized size = 3.9

$$\frac{2\left(\frac{108\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)}{b^2x}+\frac{1062\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^2}{b^4x^2}+\frac{1638\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^3}{b^6x^3}+\frac{3402\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^4}{b^8x^4}+\frac{2520\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^5}{b^{10}x^5}+\cdots\right.}{315\,a^3\left(\frac{ab+\sqrt{-b^2x^2+a^2}|b|}{b^2x}+1\right)^9|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 2/315*(108*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))/(b^2*x) + 1062*(a*b + sqrt(-
b^2*x^2 + a^2)*abs(b))^2/(b^4*x^2) + 1638*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b
))^3/(b^6*x^3) + 3402*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^4/(b^8*x^4) + 252
0*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^5/(b^10*x^5) + 2310*(a*b + sqrt(-b^2*
x^2 + a^2)*abs(b))^6/(b^12*x^6) + 630*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^7
/(b^14*x^7) + 315*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^8/(b^16*x^8) + 47)/(a
^3*((a*b + sqrt(-b^2*x^2 + a^2)*abs(b))/(b^2*x) + 1)^9*abs(b))
```

3.798 $\int \frac{\left(a^2-b^2x^2\right)^{3/2}}{(a+bx)^8}\,dx$

Optimal. Leaf size=133

$$-\frac{2\left(a^2-b^2x^2\right)^{5/2}}{1155a^4b(a+bx)^5}-\frac{2\left(a^2-b^2x^2\right)^{5/2}}{231a^3b(a+bx)^6}-\frac{\left(a^2-b^2x^2\right)^{5/2}}{33a^2b(a+bx)^7}-\frac{\left(a^2-b^2x^2\right)^{5/2}}{11ab(a+bx)^8}$$

```
[Out] -(a^2 - b^2*x^2)^(5/2)/(11*a*b*(a + b*x)^8) - (a^2 - b^2*x^2)^(5/2)/(33*a^2
*b*(a + b*x)^7) - (2*(a^2 - b^2*x^2)^(5/2))/(231*a^3*b*(a + b*x)^6) - (2*(a
^2 - b^2*x^2)^(5/2))/(1155*a^4*b*(a + b*x)^5)
```

Rubi [A] time = 0.0554768, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 651}

$$-\frac{2\left(a^2-b^2x^2\right)^{5/2}}{1155a^4b(a+bx)^5}-\frac{2\left(a^2-b^2x^2\right)^{5/2}}{231a^3b(a+bx)^6}-\frac{\left(a^2-b^2x^2\right)^{5/2}}{33a^2b(a+bx)^7}-\frac{\left(a^2-b^2x^2\right)^{5/2}}{11ab(a+bx)^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*x^2)^(3/2)/(a + b*x)^8,x]
```

```
[Out] -(a^2 - b^2*x^2)^(5/2)/(11*a*b*(a + b*x)^8) - (a^2 - b^2*x^2)^(5/2)/(33*a^2
*b*(a + b*x)^7) - (2*(a^2 - b^2*x^2)^(5/2))/(231*a^3*b*(a + b*x)^6) - (2*(a
^2 - b^2*x^2)^(5/2))/(1155*a^4*b*(a + b*x)^5)
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^8}\,dx = -\frac{(a^2-b^2x^2)^{5/2}}{11ab(a+bx)^8} + \frac{3\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^7}dx}{11a}$$
$$= -\frac{(a^2-b^2x^2)^{5/2}}{11ab(a+bx)^8} - \frac{(a^2-b^2x^2)^{5/2}}{33a^2b(a+bx)^7} + \frac{2\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^6}dx}{33a^2}$$
$$= -\frac{(a^2-b^2x^2)^{5/2}}{11ab(a+bx)^8} - \frac{(a^2-b^2x^2)^{5/2}}{33a^2b(a+bx)^7} - \frac{2(a^2-b^2x^2)^{5/2}}{231a^3b(a+bx)^6} + \frac{2\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^5}dx}{231a^3}$$
$$= -\frac{(a^2-b^2x^2)^{5/2}}{11ab(a+bx)^8} - \frac{(a^2-b^2x^2)^{5/2}}{33a^2b(a+bx)^7} - \frac{2(a^2-b^2x^2)^{5/2}}{231a^3b(a+bx)^6} - \frac{2(a^2-b^2x^2)^{5/2}}{1155a^4b(a+bx)^5}$$

Mathematica [A] time = 0.0575, size = 71, normalized size = 0.53

$$-\frac{(a-bx)^2\sqrt{a^2-b^2x^2}\left(61a^2bx+152a^3+16ab^2x^2+2b^3x^3\right)}{1155a^4b(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*x^2)^(3/2)/(a + b*x)^8,x]
```

```
[Out] -((a - b*x)^2*Sqrt[a^2 - b^2*x^2]*(152*a^3 + 61*a^2*b*x + 16*a*b^2*x^2 + 2*
b^3*x^3))/(1155*a^4*b*(a + b*x)^6)
```

Maple [A] time = 0.045, size = 66, normalized size = 0.5

$$-\frac{\left(2b^3x^3+16ab^2x^2+61xa^2b+152a^3\right)(-bx+a)}{1155(bx+a)^7a^4b}\left(-b^2x^2+a^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^2*x^2+a^2)^(3/2)/(b*x+a)^8,x)
```

```
[Out] -1/1155*(-b*x+a)*(2*b^3*x^3+16*a*b^2*x^2+61*a^2*b*x+152*a^3)*(-b^2*x^2+a^2)
^(3/2)/(b*x+a)^7/a^4/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.14769, size = 448, normalized size = 3.37

$$-\frac{152\,b^6x^6+912\,ab^5x^5+2280\,a^2b^4x^4+3040\,a^3b^3x^3+2280\,a^4b^2x^2+912\,a^5bx+152\,a^6+\left(2\,b^5x^5+12\,ab^4x^4+31\,a^2b^3x^3\right.}{1155\left(a^4b^7x^6+6\,a^5b^6x^5+15\,a^6b^5x^4+20\,a^7b^4x^3+15\,a^8b^3x^2+6\,a^9b^2x+a^{10}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^8,x, algorithm="fricas")
```

```
[Out] -1/1155*(152*b^6*x^6 + 912*a*b^5*x^5 + 2280*a^2*b^4*x^4 + 3040*a^3*b^3*x^3
+ 2280*a^4*b^2*x^2 + 912*a^5*b*x + 152*a^6 + (2*b^5*x^5 + 12*a*b^4*x^4 + 31
*a^2*b^3*x^3 + 46*a^3*b^2*x^2 - 243*a^4*b*x + 152*a^5)*sqrt(-b^2*x^2 + a^2)
)/(a^4*b^7*x^6 + 6*a^5*b^6*x^5 + 15*a^6*b^5*x^4 + 20*a^7*b^4*x^3 + 15*a^8*b
^3*x^2 + 6*a^9*b^2*x + a^10*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**2*x**2+a**2)**(3/2)/(b*x+a)**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24842, size = 474, normalized size = 3.56

$$\frac{2\left(\frac{517\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)}{b^2x}+\frac{4895\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^2}{b^4x^2}+\frac{11220\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^3}{b^6x^3}+\frac{27060\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^4}{b^8x^4}+\frac{32802\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^5}{b^{10}x^5}+\frac{37}{}\right.}{1155\,a^4\left(\frac{ab+\sqrt{-b^2}}{b^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^8,x, algorithm="giac")
```

```
[Out] 2/1155*(517*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))/(b^2*x) + 4895*(a*b + sqrt(
-b^2*x^2 + a^2)*abs(b))^2/(b^4*x^2) + 11220*(a*b + sqrt(-b^2*x^2 + a^2)*abs
(b))^3/(b^6*x^3) + 27060*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^4/(b^8*x^4) +
32802*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^5/(b^10*x^5) + 37422*(a*b + sqrt(
-b^2*x^2 + a^2)*abs(b))^6/(b^12*x^6) + 23100*(a*b + sqrt(-b^2*x^2 + a^2)*ab
s(b))^7/(b^14*x^7) + 13860*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^8/(b^16*x^8)
 + 3465*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^9/(b^18*x^9) + 1155*(a*b + sqrt
(-b^2*x^2 + a^2)*abs(b))^10/(b^20*x^10) + 152)/(a^4*((a*b + sqrt(-b^2*x^2 +
 a^2)*abs(b))/(b^2*x) + 1)^11*abs(b))
```

3.799 $\int \frac{\left(a^2-b^2x^2\right)^{3/2}}{(a+bx)^9} dx$

Optimal. Leaf size=166

$$-\frac{8\left(a^2-b^2x^2\right)^{5/2}}{15015a^5b(a+bx)^5}-\frac{8\left(a^2-b^2x^2\right)^{5/2}}{3003a^4b(a+bx)^6}-\frac{4\left(a^2-b^2x^2\right)^{5/2}}{429a^3b(a+bx)^7}-\frac{4\left(a^2-b^2x^2\right)^{5/2}}{143a^2b(a+bx)^8}-\frac{\left(a^2-b^2x^2\right)^{5/2}}{13ab(a+bx)^9}$$

```
[Out] -(a^2 - b^2*x^2)^(5/2)/(13*a*b*(a + b*x)^9) - (4*(a^2 - b^2*x^2)^(5/2))/(14
3*a^2*b*(a + b*x)^8) - (4*(a^2 - b^2*x^2)^(5/2))/(429*a^3*b*(a + b*x)^7) -
(8*(a^2 - b^2*x^2)^(5/2))/(3003*a^4*b*(a + b*x)^6) - (8*(a^2 - b^2*x^2)^(5/
2))/(15015*a^5*b*(a + b*x)^5)
```

Rubi [A] time = 0.0750511, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 651}

$$-\frac{8\left(a^2-b^2x^2\right)^{5/2}}{15015a^5b(a+bx)^5}-\frac{8\left(a^2-b^2x^2\right)^{5/2}}{3003a^4b(a+bx)^6}-\frac{4\left(a^2-b^2x^2\right)^{5/2}}{429a^3b(a+bx)^7}-\frac{4\left(a^2-b^2x^2\right)^{5/2}}{143a^2b(a+bx)^8}-\frac{\left(a^2-b^2x^2\right)^{5/2}}{13ab(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*x^2)^(3/2)/(a + b*x)^9,x]
```


```
[Out] -(a^2 - b^2*x^2)^(5/2)/(13*a*b*(a + b*x)^9) - (4*(a^2 - b^2*x^2)^(5/2))/(14
3*a^2*b*(a + b*x)^8) - (4*(a^2 - b^2*x^2)^(5/2))/(429*a^3*b*(a + b*x)^7) -
(8*(a^2 - b^2*x^2)^(5/2))/(3003*a^4*b*(a + b*x)^6) - (8*(a^2 - b^2*x^2)^(5/
2))/(15015*a^5*b*(a + b*x)^5)
```


Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^9}\,dx = -\frac{(a^2-b^2x^2)^{5/2}}{13ab(a+bx)^9} + \frac{4\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^8}\,dx}{13a}$$

$$= -\frac{(a^2-b^2x^2)^{5/2}}{13ab(a+bx)^9} - \frac{4(a^2-b^2x^2)^{5/2}}{143a^2b(a+bx)^8} + \frac{12\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^7}\,dx}{143a^2}$$

$$= -\frac{(a^2-b^2x^2)^{5/2}}{13ab(a+bx)^9} - \frac{4(a^2-b^2x^2)^{5/2}}{143a^2b(a+bx)^8} - \frac{4(a^2-b^2x^2)^{5/2}}{429a^3b(a+bx)^7} + \frac{8\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^6}\,dx}{429a^3}$$

$$= -\frac{(a^2-b^2x^2)^{5/2}}{13ab(a+bx)^9} - \frac{4(a^2-b^2x^2)^{5/2}}{143a^2b(a+bx)^8} - \frac{4(a^2-b^2x^2)^{5/2}}{429a^3b(a+bx)^7} - \frac{8(a^2-b^2x^2)^{5/2}}{3003a^4b(a+bx)^6} + \frac{8\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^5}\,dx}{3003a^4}$$

$$= -\frac{(a^2-b^2x^2)^{5/2}}{13ab(a+bx)^9} - \frac{4(a^2-b^2x^2)^{5/2}}{143a^2b(a+bx)^8} - \frac{4(a^2-b^2x^2)^{5/2}}{429a^3b(a+bx)^7} - \frac{8(a^2-b^2x^2)^{5/2}}{3003a^4b(a+bx)^6} - \frac{8(a^2-b^2x^2)^{5/2}}{15015a^5b(a+bx)^5}$$

Mathematica [A] time = 0.0591876, size = 82, normalized size = 0.49

$$-\frac{(a-bx)^2\sqrt{a^2-b^2x^2}\left(308a^2b^2x^2+852a^3bx+1763a^4+72ab^3x^3+8b^4x^4\right)}{15015a^5b(a+bx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*x^2)^(3/2)/(a + b*x)^9,x]
```

```
[Out] -((a - b*x)^2*Sqrt[a^2 - b^2*x^2]*(1763*a^4 + 852*a^3*b*x + 308*a^2*b^2*x^2
 + 72*a*b^3*x^3 + 8*b^4*x^4))/(15015*a^5*b*(a + b*x)^7)
```

Maple [A] time = 0.046, size = 77, normalized size = 0.5

$$-\frac{\left(8\,b^4x^4+72\,ab^3x^3+308\,b^2x^2a^2+852\,xa^3b+1763\,a^4\right)(-bx+a)}{15015\,(bx+a)^8\,a^5b}\left(-b^2x^2+a^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^2*x^2+a^2)^(3/2)/(b*x+a)^9,x)
```

```
[Out] -1/15015*(-b*x+a)*(8*b^4*x^4+72*a*b^3*x^3+308*a^2*b^2*x^2+852*a^3*b*x+1763*
a^4)*(-b^2*x^2+a^2)^(3/2)/(b*x+a)^8/a^5/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.91788, size = 544, normalized size = 3.28

$$-\frac{1763\,b^7x^7+12341\,ab^6x^6+37023\,a^2b^5x^5+61705\,a^3b^4x^4+61705\,a^4b^3x^3+37023\,a^5b^2x^2+12341\,a^6bx+1763\,a^7+\left(\right.}{15015\left(a^5b^8x^7+7\,a^6b^7x^6+21\,a^7b^6x^5+35\,a^8b^5x^4+35\,a^9\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^9,x, algorithm="fricas")
```

```
[Out] -1/15015*(1763*b^7*x^7 + 12341*a*b^6*x^6 + 37023*a^2*b^5*x^5 + 61705*a^3*b^
4*x^4 + 61705*a^4*b^3*x^3 + 37023*a^5*b^2*x^2 + 12341*a^6*b*x + 1763*a^7 +
(8*b^6*x^6 + 56*a*b^5*x^5 + 172*a^2*b^4*x^4 + 308*a^3*b^3*x^3 + 367*a^4*b^2
*x^2 - 2674*a^5*b*x + 1763*a^6)*sqrt(-b^2*x^2 + a^2))/(a^5*b^8*x^7 + 7*a^6*
b^7*x^6 + 21*a^7*b^6*x^5 + 35*a^8*b^5*x^4 + 35*a^9*b^4*x^3 + 21*a^10*b^3*x^
2 + 7*a^11*b^2*x + a^12*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**2*x**2+a**2)**(3/2)/(b*x+a)**9,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25809, size = 558, normalized size = 3.36

$$2\left(\frac{7904\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)}{b^2x}+\frac{77454\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^2}{b^4x^2}+\frac{233948\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^3}{b^6x^3}+\frac{659945\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^4}{b^8x^4}+\frac{1094808\left(ab+\sqrt{-b^2x^2+a^2}\right.}{b^{10}x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^9,x, algorithm="giac")
```

```
[Out] 2/15015*(7904*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))/(b^2*x) + 77454*(a*b + sq
rt(-b^2*x^2 + a^2)*abs(b))^2/(b^4*x^2) + 233948*(a*b + sqrt(-b^2*x^2 + a^2)
*abs(b))^3/(b^6*x^3) + 659945*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^4/(b^8*x^
4) + 1094808*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^5/(b^10*x^5) + 1559844*(a*
b + sqrt(-b^2*x^2 + a^2)*abs(b))^6/(b^12*x^6) + 1465464*(a*b + sqrt(-b^2*x^
2 + a^2)*abs(b))^7/(b^14*x^7) + 1174173*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))
^8/(b^16*x^8) + 600600*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^9/(b^18*x^9) + 2
70270*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^10/(b^20*x^10) + 60060*(a*b + sqr
t(-b^2*x^2 + a^2)*abs(b))^11/(b^22*x^11) + 15015*(a*b + sqrt(-b^2*x^2 + a^2
)*abs(b))^12/(b^24*x^12) + 1763)/(a^5*((a*b + sqrt(-b^2*x^2 + a^2)*abs(b))/
(b^2*x) + 1)^13*abs(b))
```

3.800 $\int (d+ex)^3 \left(d^2-e^2x^2\right)^{7/2} dx$

Optimal. Leaf size=212

$$\frac{91}{256}d^9x\sqrt{d^2-e^2x^2}+\frac{91}{384}d^7x\left(d^2-e^2x^2\right)^{3/2}+\frac{91}{480}d^5x\left(d^2-e^2x^2\right)^{5/2}+\frac{13}{80}d^3x\left(d^2-e^2x^2\right)^{7/2}-\frac{13d^2\left(d^2-e^2x^2\right)^{9/2}}{90e}-\frac{13d(\cdots}{}$$

```
[Out] (91*d^9*x*Sqrt[d^2 - e^2*x^2])/256 + (91*d^7*x*(d^2 - e^2*x^2)^(3/2))/384 +
 (91*d^5*x*(d^2 - e^2*x^2)^(5/2))/480 + (13*d^3*x*(d^2 - e^2*x^2)^(7/2))/80
 - (13*d^2*(d^2 - e^2*x^2)^(9/2))/(90*e) - (13*d*(d + e*x)*(d^2 - e^2*x^2)^
(9/2))/(110*e) - ((d + e*x)^2*(d^2 - e^2*x^2)^(9/2))/(11*e) + (91*d^11*ArcT
an[(e*x)/Sqrt[d^2 - e^2*x^2]])/(256*e)
```

Rubi [A] time = 0.0900898, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {671, 641, 195, 217, 203}

$$\frac{91}{256}d^9x\sqrt{d^2-e^2x^2}+\frac{91}{384}d^7x\left(d^2-e^2x^2\right)^{3/2}+\frac{91}{480}d^5x\left(d^2-e^2x^2\right)^{5/2}+\frac{13}{80}d^3x\left(d^2-e^2x^2\right)^{7/2}-\frac{13d^2\left(d^2-e^2x^2\right)^{9/2}}{90e}-\frac{13d(\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (91*d^9*x*Sqrt[d^2 - e^2*x^2])/256 + (91*d^7*x*(d^2 - e^2*x^2)^(3/2))/384 +
 (91*d^5*x*(d^2 - e^2*x^2)^(5/2))/480 + (13*d^3*x*(d^2 - e^2*x^2)^(7/2))/80
 - (13*d^2*(d^2 - e^2*x^2)^(9/2))/(90*e) - (13*d*(d + e*x)*(d^2 - e^2*x^2)^
(9/2))/(110*e) - ((d + e*x)^2*(d^2 - e^2*x^2)^(9/2))/(11*e) + (91*d^11*ArcT
an[(e*x)/Sqrt[d^2 - e^2*x^2]])/(256*e)
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^3 \left(d^2-e^2x^2\right)^{7/2} dx = -\frac{(d+ex)^2\left(d^2-e^2x^2\right)^{9/2}}{11e} + \frac{1}{11}(13d)\int (d+ex)^2\left(d^2-e^2x^2\right)^{7/2} dx$$

$$= -\frac{13d(d+ex)\left(d^2-e^2x^2\right)^{9/2}}{110e} - \frac{(d+ex)^2\left(d^2-e^2x^2\right)^{9/2}}{11e} + \frac{1}{10}\left(13d^2\right)\int (d+ex)\left(d^2-e^2x\right.$$

$$= -\frac{13d^2\left(d^2-e^2x^2\right)^{9/2}}{90e} - \frac{13d(d+ex)\left(d^2-e^2x^2\right)^{9/2}}{110e} - \frac{(d+ex)^2\left(d^2-e^2x^2\right)^{9/2}}{11e} + \frac{1}{10}\left(13d\right.$$

$$= \frac{13}{80}d^3x\left(d^2-e^2x^2\right)^{7/2} - \frac{13d^2\left(d^2-e^2x^2\right)^{9/2}}{90e} - \frac{13d(d+ex)\left(d^2-e^2x^2\right)^{9/2}}{110e} - \frac{(d+ex)^2\left(d^2\right.}{11}$$

$$= \frac{91}{480}d^5x\left(d^2-e^2x^2\right)^{5/2} + \frac{13}{80}d^3x\left(d^2-e^2x^2\right)^{7/2} - \frac{13d^2\left(d^2-e^2x^2\right)^{9/2}}{90e} - \frac{13d(d+ex)\left(d^2-\right.}{110e}$$

$$= \frac{91}{384}d^7x\left(d^2-e^2x^2\right)^{3/2} + \frac{91}{480}d^5x\left(d^2-e^2x^2\right)^{5/2} + \frac{13}{80}d^3x\left(d^2-e^2x^2\right)^{7/2} - \frac{13d^2\left(d^2-e^2x\right.}{90e}$$

$$= \frac{91}{256}d^9x\sqrt{d^2-e^2x^2} + \frac{91}{384}d^7x\left(d^2-e^2x^2\right)^{3/2} + \frac{91}{480}d^5x\left(d^2-e^2x^2\right)^{5/2} + \frac{13}{80}d^3x\left(d^2-e^2x\right.$$

$$= \frac{91}{256}d^9x\sqrt{d^2-e^2x^2} + \frac{91}{384}d^7x\left(d^2-e^2x^2\right)^{3/2} + \frac{91}{480}d^5x\left(d^2-e^2x^2\right)^{5/2} + \frac{13}{80}d^3x\left(d^2-e^2x\right.$$

$$= \frac{91}{256}d^9x\sqrt{d^2-e^2x^2} + \frac{91}{384}d^7x\left(d^2-e^2x^2\right)^{3/2} + \frac{91}{480}d^5x\left(d^2-e^2x^2\right)^{5/2} + \frac{13}{80}d^3x\left(d^2-e^2x\right.$$

Mathematica [A] time = 0.31498, size = 178, normalized size = 0.84

$$\frac{\sqrt{d^2-e^2x^2}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\left(167680d^8e^2x^2+12210d^7e^3x^3-222720d^6e^4x^4-142296d^5e^5x^5+110080d^4e^6x^6+131472d^3e^7x\right.\right.}{126720e\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(Sqrt[1 - (e^2*x^2)/d^2]*(-44800*d^10 + 81675*d^9*e*x
+ 167680*d^8*e^2*x^2 + 12210*d^7*e^3*x^3 - 222720*d^6*e^4*x^4 - 142296*d^5*
e^5*x^5 + 110080*d^4*e^6*x^6 + 131472*d^3*e^7*x^7 + 1280*d^2*e^8*x^8 - 3801
6*d*e^9*x^9 - 11520*e^10*x^10) + 45045*d^10*ArcSin[(e*x)/d]))/(126720*e*Sqr
t[1 - (e^2*x^2)/d^2])
```

Maple [A] time = 0.079, size = 174, normalized size = 0.8

$$-\frac{ex^2}{11}\left(-e^2x^2+d^2\right)^{\frac{9}{2}} - \frac{35d^2}{99e}\left(-e^2x^2+d^2\right)^{\frac{9}{2}} - \frac{3dx}{10}\left(-e^2x^2+d^2\right)^{\frac{9}{2}} + \frac{13d^3x}{80}\left(-e^2x^2+d^2\right)^{\frac{7}{2}} + \frac{91d^5x}{480}\left(-e^2x^2+d^2\right)^{\frac{5}{2}} + \frac{9}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/11*e*x^2*(-e^2*x^2+d^2)^(9/2)-35/99*d^2*(-e^2*x^2+d^2)^(9/2)/e-3/10*d*x*
(-e^2*x^2+d^2)^(9/2)+13/80*d^3*x*(-e^2*x^2+d^2)^(7/2)+91/480*d^5*x*(-e^2*x^
2+d^2)^(5/2)+91/384*d^7*x*(-e^2*x^2+d^2)^(3/2)+91/256*d^9*x*(-e^2*x^2+d^2)^
(1/2)+91/256*d^11/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Maxima [A] time = 1.5995, size = 224, normalized size = 1.06

$$\frac{91\,d^{11}\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{256\,\sqrt{e^2}}+\frac{91}{256}\,\sqrt{-e^2x^2+d^2}d^9x+\frac{91}{384}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^7x+\frac{91}{480}\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^5x+\frac{13}{80}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^3x-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 91/256*d^11*arcsin(e^2*x/sqrt(d^2*e^2))/sqrt(e^2) + 91/256*sqrt(-e^2*x^2 +
d^2)*d^9*x + 91/384*(-e^2*x^2 + d^2)^(3/2)*d^7*x + 91/480*(-e^2*x^2 + d^2)^
(5/2)*d^5*x + 13/80*(-e^2*x^2 + d^2)^(7/2)*d^3*x - 1/11*(-e^2*x^2 + d^2)^(9
/2)*e*x^2 - 3/10*(-e^2*x^2 + d^2)^(9/2)*d*x - 35/99*(-e^2*x^2 + d^2)^(9/2)*
d^2/e
```

Fricas [A] time = 2.28852, size = 405, normalized size = 1.91

$$-\frac{90090\,d^{11}\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(11520\,e^{10}x^{10}+38016\,de^9x^9-1280\,d^2e^8x^8-131472\,d^3e^7x^7-110080\,d^4e^6x^6+1422\right.}{126720\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/126720*(90090*d^11*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (11520*e^
10*x^10 + 38016*d*e^9*x^9 - 1280*d^2*e^8*x^8 - 131472*d^3*e^7*x^7 - 110080*
d^4*e^6*x^6 + 142296*d^5*e^5*x^5 + 222720*d^6*e^4*x^4 - 12210*d^7*e^3*x^3 -
 167680*d^8*e^2*x^2 - 81675*d^9*e*x + 44800*d^10)*sqrt(-e^2*x^2 + d^2))/e
```

Sympy [C] time = 34.6203, size = 1503, normalized size = 7.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d**9*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d
**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**
2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) +
 3*d**8*e*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)*
```

```
*(3/2)/(3*e**2), True)) - 8*d**6*e**3*Piecewise((-2*d**4*sqrt(d**2 - e**2*x
**2)/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**
2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) - 6*d**5*e**4*Piece
wise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**
2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24
*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)),
Abs(e**2*x**2)/Abs(d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e**
4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2))
+ 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**
2/d**2)), True)) + 6*d**4*e**5*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(1
05*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt(d
**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**
6*sqrt(d**2)/6, True)) + 8*d**3*e**6*Piecewise((-5*I*d**8*acosh(e*x/d)/(128
*e**7) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(3
84*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x
**2/d**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*s
qrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (5*d**8*asin(e*x/
d)/(128*e**7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/
(384*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**
2/d**2)) + 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 -
 e**2*x**2/d**2)), True)) - 3*d*e**8*Piecewise((-7*I*d**10*acosh(e*x/d)/(25
6*e**9) + 7*I*d**9*x/(256*e**8*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**7*x**3/(
768*e**6*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**5*x**5/(1920*e**4*sqrt(-1 + e*
*2*x**2/d**2)) - I*d**3*x**7/(480*e**2*sqrt(-1 + e**2*x**2/d**2)) - 9*I*d*x
**9/(80*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**11/(10*d*sqrt(-1 + e**2*x**2
/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (7*d**10*asin(e*x/d)/(256*e**9) - 7
*d**9*x/(256*e**8*sqrt(1 - e**2*x**2/d**2)) + 7*d**7*x**3/(768*e**6*sqrt(1
- e**2*x**2/d**2)) + 7*d**5*x**5/(1920*e**4*sqrt(1 - e**2*x**2/d**2)) + d**
3*x**7/(480*e**2*sqrt(1 - e**2*x**2/d**2)) + 9*d*x**9/(80*sqrt(1 - e**2*x**
2/d**2)) - e**2*x**11/(10*d*sqrt(1 - e**2*x**2/d**2)), True)) - e**9*Piecew
ise((-128*d**10*sqrt(d**2 - e**2*x**2)/(3465*e**10) - 64*d**8*x**2*sqrt(d**
2 - e**2*x**2)/(3465*e**8) - 16*d**6*x**4*sqrt(d**2 - e**2*x**2)/(1155*e**6
) - 8*d**4*x**6*sqrt(d**2 - e**2*x**2)/(693*e**4) - d**2*x**8*sqrt(d**2 - e
**2*x**2)/(99*e**2) + x**10*sqrt(d**2 - e**2*x**2)/11, Ne(e, 0)), (x**10*sq
rt(d**2)/10, True))
```

Giac [A] time = 1.26169, size = 186, normalized size = 0.88

$$\frac{91}{256}\,d^{11}\arcsin\left(\frac{xe}{d}\right)e^{(-1)}\mathrm{sgn}\left(d\right) - \frac{1}{126720}\left(44800\,d^{10}e^{(-1)} - \left(81675\,d^{9} + 2\left(83840\,d^{8}e + \left(6105\,d^{7}e^{2} - 4\left(27840\,d^{6}e^{3} + \right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] 91/256*d^11*arcsin(x*e/d)*e^(-1)*sgn(d) - 1/126720*(44800*d^10*e^(-1) - (81
675*d^9 + 2*(83840*d^8*e + (6105*d^7*e^2 - 4*(27840*d^6*e^3 + (17787*d^5*e^
4 - 2*(6880*d^4*e^5 + (8217*d^3*e^6 + 8*(10*d^2*e^7 - 9*(10*x*e^9 + 33*d*e^
8)*x)*x)*x)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

3.801 $\int (d+ex)^2 \left(d^2-e^2x^2\right)^{7/2} dx$

Optimal. Leaf size=179

$$\frac{77}{256}d^8x\sqrt{d^2-e^2x^2}+\frac{77}{384}d^6x\left(d^2-e^2x^2\right)^{3/2}+\frac{77}{480}d^4x\left(d^2-e^2x^2\right)^{5/2}+\frac{11}{80}d^2x\left(d^2-e^2x^2\right)^{7/2}-\frac{11d\left(d^2-e^2x^2\right)^{9/2}}{90e}-\frac{(d+ex)}{}$$

```
[Out] (77*d^8*x*Sqrt[d^2 - e^2*x^2])/256 + (77*d^6*x*(d^2 - e^2*x^2)^(3/2))/384 +
 (77*d^4*x*(d^2 - e^2*x^2)^(5/2))/480 + (11*d^2*x*(d^2 - e^2*x^2)^(7/2))/80
 - (11*d*(d^2 - e^2*x^2)^(9/2))/(90*e) - ((d + e*x)*(d^2 - e^2*x^2)^(9/2))/
(10*e) + (77*d^10*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(256*e)
```

Rubi [A] time = 0.0669763, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {671, 641, 195, 217, 203}

$$\frac{77}{256}d^8x\sqrt{d^2-e^2x^2}+\frac{77}{384}d^6x\left(d^2-e^2x^2\right)^{3/2}+\frac{77}{480}d^4x\left(d^2-e^2x^2\right)^{5/2}+\frac{11}{80}d^2x\left(d^2-e^2x^2\right)^{7/2}-\frac{11d\left(d^2-e^2x^2\right)^{9/2}}{90e}-\frac{(d+ex)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (77*d^8*x*Sqrt[d^2 - e^2*x^2])/256 + (77*d^6*x*(d^2 - e^2*x^2)^(3/2))/384 +
 (77*d^4*x*(d^2 - e^2*x^2)^(5/2))/480 + (11*d^2*x*(d^2 - e^2*x^2)^(7/2))/80
 - (11*d*(d^2 - e^2*x^2)^(9/2))/(90*e) - ((d + e*x)*(d^2 - e^2*x^2)^(9/2))/
(10*e) + (77*d^10*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(256*e)
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^2\left(d^2-e^2x^2\right)^{7/2}dx = -\frac{(d+ex)\left(d^2-e^2x^2\right)^{9/2}}{10e}+\frac{1}{10}(11d)\int (d+ex)\left(d^2-e^2x^2\right)^{7/2}dx$$

$$= -\frac{11d\left(d^2-e^2x^2\right)^{9/2}}{90e}-\frac{(d+ex)\left(d^2-e^2x^2\right)^{9/2}}{10e}+\frac{1}{10}\left(11d^2\right)\int\left(d^2-e^2x^2\right)^{7/2}dx$$

$$= \frac{11}{80}d^2x\left(d^2-e^2x^2\right)^{7/2}-\frac{11d\left(d^2-e^2x^2\right)^{9/2}}{90e}-\frac{(d+ex)\left(d^2-e^2x^2\right)^{9/2}}{10e}+\frac{1}{80}\left(77d^4\right)\int\left(d^2\right.$$

$$= \frac{77}{480}d^4x\left(d^2-e^2x^2\right)^{5/2}+\frac{11}{80}d^2x\left(d^2-e^2x^2\right)^{7/2}-\frac{11d\left(d^2-e^2x^2\right)^{9/2}}{90e}-\frac{(d+ex)\left(d^2-e^2x\right.}{10e}$$

$$= \frac{77}{384}d^6x\left(d^2-e^2x^2\right)^{3/2}+\frac{77}{480}d^4x\left(d^2-e^2x^2\right)^{5/2}+\frac{11}{80}d^2x\left(d^2-e^2x^2\right)^{7/2}-\frac{11d\left(d^2-e^2x\right.}{90e}$$

$$= \frac{77}{256}d^8x\sqrt{d^2-e^2x^2}+\frac{77}{384}d^6x\left(d^2-e^2x^2\right)^{3/2}+\frac{77}{480}d^4x\left(d^2-e^2x^2\right)^{5/2}+\frac{11}{80}d^2x\left(d^2-e^2\right.$$

$$= \frac{77}{256}d^8x\sqrt{d^2-e^2x^2}+\frac{77}{384}d^6x\left(d^2-e^2x^2\right)^{3/2}+\frac{77}{480}d^4x\left(d^2-e^2x^2\right)^{5/2}+\frac{11}{80}d^2x\left(d^2-e^2\right.$$

$$= \frac{77}{256}d^8x\sqrt{d^2-e^2x^2}+\frac{77}{384}d^6x\left(d^2-e^2x^2\right)^{3/2}+\frac{77}{480}d^4x\left(d^2-e^2x^2\right)^{5/2}+\frac{11}{80}d^2x\left(d^2-e^2\right.$$

Mathematica [A] time = 0.355226, size = 118, normalized size = 0.66

$$\frac{\left(d^2-e^2x^2\right)^{9/2}\left(\frac{33d^2\left(-326d^4e^2x^3+200d^2e^4x^5+\frac{105d^7\sin^{-1}\left(\frac{ex}{d}\right)}{e\sqrt{1-\frac{e^2x^2}{d^2}}}+279d^6x-48e^6x^7\right)}{\left(d^2-e^2x^2\right)^4}-\frac{2560d}{e}-1152x\right)}{11520}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((d^2 - e^2*x^2)^(9/2)*((-2560*d)/e - 1152*x + (33*d^2*(279*d^6*x - 326*d^4
*e^2*x^3 + 200*d^2*e^4*x^5 - 48*e^6*x^7 + (105*d^7*ArcSin[(e*x)/d])/(e*Sqrt
[1 - (e^2*x^2)/d^2])))/(d^2 - e^2*x^2)^4))/11520
```

Maple [A] time = 0.05, size = 151, normalized size = 0.8

$$-\frac{x}{10}\left(-e^2x^2+d^2\right)^{\frac{9}{2}}+\frac{11\,d^2x}{80}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}+\frac{77\,d^4x}{480}\left(-e^2x^2+d^2\right)^{\frac{5}{2}}+\frac{77\,d^6x}{384}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}+\frac{77\,d^8x}{256}\sqrt{-e^2x^2+d^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/10*x*(-e^2*x^2+d^2)^(9/2)+11/80*d^2*x*(-e^2*x^2+d^2)^(7/2)+77/480*d^4*x*
(-e^2*x^2+d^2)^(5/2)+77/384*d^6*x*(-e^2*x^2+d^2)^(3/2)+77/256*d^8*x*(-e^2*x
^2+d^2)^(1/2)+77/256*d^10/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(
1/2))-2/9*d*(-e^2*x^2+d^2)^(9/2)/e
```

Maxima [A] time = 1.71957, size = 193, normalized size = 1.08

$$\frac{77\,d^{10}\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{256\,\sqrt{e^2}}+\frac{77}{256}\,\sqrt{-e^2x^2+d^2}d^8x+\frac{77}{384}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^6x+\frac{77}{480}\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^4x+\frac{11}{80}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2x -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 77/256*d^10*arcsin(e^2*x/sqrt(d^2*e^2))/sqrt(e^2) + 77/256*sqrt(-e^2*x^2 +
d^2)*d^8*x + 77/384*(-e^2*x^2 + d^2)^(3/2)*d^6*x + 77/480*(-e^2*x^2 + d^2)^
(5/2)*d^4*x + 11/80*(-e^2*x^2 + d^2)^(7/2)*d^2*x - 1/10*(-e^2*x^2 + d^2)^(9
/2)*x - 2/9*(-e^2*x^2 + d^2)^(9/2)*d/e
```

Fricas [A] time = 2.1594, size = 355, normalized size = 1.98

$$-\frac{6930\,d^{10}\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(1152\,e^9x^9+2560\,de^8x^8-3024\,d^2e^7x^7-10240\,d^3e^6x^6+312\,d^4e^5x^5+15360\,d^5e^4x^4+\right.}{11520\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/11520*(6930*d^10*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (1152*e^9*x
^9 + 2560*d*e^8*x^8 - 3024*d^2*e^7*x^7 - 10240*d^3*e^6*x^6 + 312*d^4*e^5*x^
5 + 15360*d^5*e^4*x^4 + 6150*d^6*e^3*x^3 - 10240*d^7*e^2*x^2 - 8055*d^8*e*x
 + 2560*d^9)*sqrt(-e^2*x^2 + d^2))/e
```

Sympy [C] time = 31.3627, size = 1420, normalized size = 7.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d**8*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d
**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**
2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) +
 2*d**7*e*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)*
*(3/2)/(3*e**2), True)) - 2*d**6*e**2*Piecewise((-I*d**4*acosh(e*x/d)/(8*e*
*3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 +
 e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x
```

```
**2)/Abs(d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e
**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sq
rt(1 - e**2*x**2/d**2)), True)) - 6*d**5*e**3*Piecewise((-2*d**4*sqrt(d**2
- e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*
sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) + 6*d**3*e*
*5*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(
d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) +
 x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True)) + 2*d
**2*e**6*Piecewise((-5*I*d**8*acosh(e*x/d)/(128*e**7) + 5*I*d**7*x/(128*e**
6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*e**4*sqrt(-1 + e**2*x**2/
d**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d*x**7/(48*
sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-1 + e**2*x**2/d**2)), A
bs(e**2*x**2)/Abs(d**2) > 1), (5*d**8*asin(e*x/d)/(128*e**7) - 5*d**7*x/(12
8*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4*sqrt(1 - e**2*x**2
/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**2/d**2)) + 7*d*x**7/(48*sqrt
(1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2/d**2)), True)) -
2*d*e**7*Piecewise((-16*d**8*sqrt(d**2 - e**2*x**2)/(315*e**8) - 8*d**6*x**
2*sqrt(d**2 - e**2*x**2)/(315*e**6) - 2*d**4*x**4*sqrt(d**2 - e**2*x**2)/(1
05*e**4) - d**2*x**6*sqrt(d**2 - e**2*x**2)/(63*e**2) + x**8*sqrt(d**2 - e*
*2*x**2)/9, Ne(e, 0)), (x**8*sqrt(d**2)/8, True)) - e**8*Piecewise((-7*I*d*
*10*acosh(e*x/d)/(256*e**9) + 7*I*d**9*x/(256*e**8*sqrt(-1 + e**2*x**2/d**2
)) - 7*I*d**7*x**3/(768*e**6*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**5*x**5/(19
20*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**7/(480*e**2*sqrt(-1 + e**2*x
**2/d**2)) - 9*I*d*x**9/(80*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**11/(10*d
*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (7*d**10*asin(e
*x/d)/(256*e**9) - 7*d**9*x/(256*e**8*sqrt(1 - e**2*x**2/d**2)) + 7*d**7*x*
*3/(768*e**6*sqrt(1 - e**2*x**2/d**2)) + 7*d**5*x**5/(1920*e**4*sqrt(1 - e*
*2*x**2/d**2)) + d**3*x**7/(480*e**2*sqrt(1 - e**2*x**2/d**2)) + 9*d*x**9/(
80*sqrt(1 - e**2*x**2/d**2)) - e**2*x**11/(10*d*sqrt(1 - e**2*x**2/d**2)),
True))
```

Giac [A] time = 1.26565, size = 173, normalized size = 0.97

$$\frac{77}{256}\,d^{10}\arcsin\left(\frac{xe}{d}\right)e^{(-1)}\mathrm{sgn}\left(d\right)-\frac{1}{11520}\left(2560\,d^{9}e^{(-1)}-\left(8055\,d^{8}+2\left(5120\,d^{7}e-\left(3075\,d^{6}e^{2}+4\left(1920\,d^{5}e^{3}+\left(39\,d^{4}e\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] 77/256*d^10*arcsin(x*e/d)*e^(-1)*sgn(d) - 1/11520*(2560*d^9*e^(-1) - (8055*
d^8 + 2*(5120*d^7*e - (3075*d^6*e^2 + 4*(1920*d^5*e^3 + (39*d^4*e^4 - 2*(64
0*d^3*e^5 + (189*d^2*e^6 - 8*(9*x*e^8 + 20*d*e^7)*x)*x)*x)*x)*x)*x)*x)*x)*s
qrt(-x^2*e^2 + d^2)
```

3.802 $\int (d+ex)\left(d^2-e^2x^2\right)^{7/2}\,dx$

Optimal. Leaf size=148

$$\frac{35}{128}d^7x\sqrt{d^2-e^2x^2}+\frac{35}{192}d^5x\left(d^2-e^2x^2\right)^{3/2}+\frac{7}{48}d^3x\left(d^2-e^2x^2\right)^{5/2}+\frac{1}{8}dx\left(d^2-e^2x^2\right)^{7/2}-\frac{\left(d^2-e^2x^2\right)^{9/2}}{9e}+\frac{35d^9\tan^{-1}\left(\right.}{128e}$$

```
[Out] (35*d^7*x*Sqrt[d^2 - e^2*x^2])/128 + (35*d^5*x*(d^2 - e^2*x^2)^(3/2))/192 +
 (7*d^3*x*(d^2 - e^2*x^2)^(5/2))/48 + (d*x*(d^2 - e^2*x^2)^(7/2))/8 - (d^2
- e^2*x^2)^(9/2)/(9*e) + (35*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(128*e)
```

Rubi [A] time = 0.0466568, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {641, 195, 217, 203}

$$\frac{35}{128}d^7x\sqrt{d^2-e^2x^2}+\frac{35}{192}d^5x\left(d^2-e^2x^2\right)^{3/2}+\frac{7}{48}d^3x\left(d^2-e^2x^2\right)^{5/2}+\frac{1}{8}dx\left(d^2-e^2x^2\right)^{7/2}-\frac{\left(d^2-e^2x^2\right)^{9/2}}{9e}+\frac{35d^9\tan^{-1}\left(\right.}{128e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (35*d^7*x*Sqrt[d^2 - e^2*x^2])/128 + (35*d^5*x*(d^2 - e^2*x^2)^(3/2))/192 +
 (7*d^3*x*(d^2 - e^2*x^2)^(5/2))/48 + (d*x*(d^2 - e^2*x^2)^(7/2))/8 - (d^2
- e^2*x^2)^(9/2)/(9*e) + (35*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(128*e)
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (d+ex)\left(d^2-e^2x^2\right)^{7/2}\,dx = -\frac{\left(d^2-e^2x^2\right)^{9/2}}{9e} + d\int \left(d^2-e^2x^2\right)^{7/2}\,dx$$
$$= \frac{1}{8}dx\left(d^2-e^2x^2\right)^{7/2} - \frac{\left(d^2-e^2x^2\right)^{9/2}}{9e} + \frac{1}{8}\left(7d^3\right)\int \left(d^2-e^2x^2\right)^{5/2}\,dx$$
$$= \frac{7}{48}d^3x\left(d^2-e^2x^2\right)^{5/2} + \frac{1}{8}dx\left(d^2-e^2x^2\right)^{7/2} - \frac{\left(d^2-e^2x^2\right)^{9/2}}{9e} + \frac{1}{48}\left(35d^5\right)\int \left(d^2-e^2x^2\right)^{3/}$$
$$= \frac{35}{192}d^5x\left(d^2-e^2x^2\right)^{3/2} + \frac{7}{48}d^3x\left(d^2-e^2x^2\right)^{5/2} + \frac{1}{8}dx\left(d^2-e^2x^2\right)^{7/2} - \frac{\left(d^2-e^2x^2\right)^{9/2}}{9e} + \frac{1}{6}$$
$$= \frac{35}{128}d^7x\sqrt{d^2-e^2x^2} + \frac{35}{192}d^5x\left(d^2-e^2x^2\right)^{3/2} + \frac{7}{48}d^3x\left(d^2-e^2x^2\right)^{5/2} + \frac{1}{8}dx\left(d^2-e^2x^2\right)^{7/2}$$
$$= \frac{35}{128}d^7x\sqrt{d^2-e^2x^2} + \frac{35}{192}d^5x\left(d^2-e^2x^2\right)^{3/2} + \frac{7}{48}d^3x\left(d^2-e^2x^2\right)^{5/2} + \frac{1}{8}dx\left(d^2-e^2x^2\right)^{7/2}$$
$$= \frac{35}{128}d^7x\sqrt{d^2-e^2x^2} + \frac{35}{192}d^5x\left(d^2-e^2x^2\right)^{3/2} + \frac{7}{48}d^3x\left(d^2-e^2x^2\right)^{5/2} + \frac{1}{8}dx\left(d^2-e^2x^2\right)^{7/2}$$

Mathematica [A] time = 0.263427, size = 114, normalized size = 0.77

$$\frac{1}{384}d\sqrt{d^2-e^2x^2}\left(-326d^4e^2x^3+200d^2e^4x^5+\frac{105d^7\sin^{-1}\left(\frac{ex}{d}\right)}{e\sqrt{1-\frac{e^2x^2}{d^2}}}+279d^6x-48e^6x^7\right)-\frac{\left(d^2-e^2x^2\right)^{9/2}}{9e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] -(d^2 - e^2*x^2)^(9/2)/(9*e) + (d*Sqrt[d^2 - e^2*x^2]*(279*d^6*x - 326*d^4*
e^2*x^3 + 200*d^2*e^4*x^5 - 48*e^6*x^7 + (105*d^7*ArcSin[(e*x)/d])/(e*Sqrt[
1 - (e^2*x^2)/d^2])))/384
```

Maple [A] time = 0.046, size = 131, normalized size = 0.9

$$-\frac{1}{9e}\left(-e^2x^2+d^2\right)^{\frac{9}{2}}+\frac{dx}{8}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}+\frac{7d^3x}{48}\left(-e^2x^2+d^2\right)^{\frac{5}{2}}+\frac{35d^5x}{192}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}+\frac{35d^7x}{128}\sqrt{-e^2x^2+d^2}+\frac{35d^9}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/9*(-e^2*x^2+d^2)^(9/2)/e+1/8*d*x*(-e^2*x^2+d^2)^(7/2)+7/48*d^3*x*(-e^2*x
^2+d^2)^(5/2)+35/192*d^5*x*(-e^2*x^2+d^2)^(3/2)+35/128*d^7*x*(-e^2*x^2+d^2)
^(1/2)+35/128*d^9/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Maxima [A] time = 1.52541, size = 166, normalized size = 1.12

$$\frac{35\,d^9\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{128\sqrt{e^2}}+\frac{35}{128}\sqrt{-e^2x^2+d^2}d^7x+\frac{35}{192}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5x+\frac{7}{48}\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3x+\frac{1}{8}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}dx-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 35/128*d^9*arcsin(e^2*x/sqrt(d^2*e^2))/sqrt(e^2) + 35/128*sqrt(-e^2*x^2 + d
^2)*d^7*x + 35/192*(-e^2*x^2 + d^2)^(3/2)*d^5*x + 7/48*(-e^2*x^2 + d^2)^(5/
2)*d^3*x + 1/8*(-e^2*x^2 + d^2)^(7/2)*d*x - 1/9*(-e^2*x^2 + d^2)^(9/2)/e
```

Fricas [A] time = 2.18489, size = 311, normalized size = 2.1

$$-\frac{630\,d^9\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(128\,e^8x^8+144\,de^7x^7-512\,d^2e^6x^6-600\,d^3e^5x^5+768\,d^4e^4x^4+978\,d^5e^3x^3-512\,d^6e^2x}{1152\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/1152*(630*d^9*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (128*e^8*x^8 +
 144*d*e^7*x^7 - 512*d^2*e^6*x^6 - 600*d^3*e^5*x^5 + 768*d^4*e^4*x^4 + 978*
d^5*e^3*x^3 - 512*d^6*e^2*x^2 - 837*d^7*e*x + 128*d^8)*sqrt(-e^2*x^2 + d^2)
)/e
```

Sympy [C] time = 24.1894, size = 1290, normalized size = 8.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d**7*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d
**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**
2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) +
 d**6*e*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(
3/2)/(3*e**2), True)) - 3*d**5*e**2*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3
) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e
**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**
2)/Abs(d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**
2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt
(1 - e**2*x**2/d**2)), True)) - 3*d**4*e**3*Piecewise((-2*d**4*sqrt(d**2 -
e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sq
rt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) + 3*d**3*e**4
*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e*
*2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x*
*5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d*
*2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(
16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d
**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e*
*2*x**2/d**2)), True)) + 3*d**2*e**5*Piecewise((-8*d**6*sqrt(d**2 - e**2*x*
*2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*
sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0))
, (x**6*sqrt(d**2)/6, True)) - d*e**6*Piecewise((-5*I*d**8*acosh(e*x/d)/(12
8*e**7) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(
```

```
384*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*
x**2/d**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*
sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (5*d**8*asin(e*x
/d)/(128*e**7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3
/(384*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x*
*2/d**2)) + 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1
- e**2*x**2/d**2)), True)) - e**7*Piecewise((-16*d**8*sqrt(d**2 - e**2*x**2
)/(315*e**8) - 8*d**6*x**2*sqrt(d**2 - e**2*x**2)/(315*e**6) - 2*d**4*x**4*
sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**6*sqrt(d**2 - e**2*x**2)/(63*e*
*2) + x**8*sqrt(d**2 - e**2*x**2)/9, Ne(e, 0)), (x**8*sqrt(d**2)/8, True))
```

Giac [A] time = 1.22357, size = 161, normalized size = 1.09

$$\frac{35}{128}d^9\arcsin\left(\frac{xe}{d}\right)e^{(-1)}\mathrm{sgn}\left(d\right)-\frac{1}{1152}\left(128\,d^8e^{(-1)}-\left(837\,d^7+2\left(256\,d^6e-\left(489\,d^5e^2+4\left(96\,d^4e^3-\left(75\,d^3e^4+2\left(32\right.\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] 35/128*d^9*arcsin(x*e/d)*e^(-1)*sgn(d) - 1/1152*(128*d^8*e^(-1) - (837*d^7
+ 2*(256*d^6*e - (489*d^5*e^2 + 4*(96*d^4*e^3 - (75*d^3*e^4 + 2*(32*d^2*e^5
 - (8*x*e^7 + 9*d*e^6)*x)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

3.803 $\int \frac{(d^2-e^2x^2)^{7/2}}{d+ex}\,dx$

Optimal. Leaf size=124

$$\frac{5}{16}d^5x\sqrt{d^2-e^2x^2}+\frac{5}{24}d^3x\left(d^2-e^2x^2\right)^{3/2}+\frac{1}{6}dx\left(d^2-e^2x^2\right)^{5/2}+\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e}+\frac{5d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e}$$

```
[Out] (5*d^5*x*Sqrt[d^2 - e^2*x^2])/16 + (5*d^3*x*(d^2 - e^2*x^2)^(3/2))/24 + (d*
x*(d^2 - e^2*x^2)^(5/2))/6 + (d^2 - e^2*x^2)^(7/2)/(7*e) + (5*d^7*ArcTan[(e
*x)/Sqrt[d^2 - e^2*x^2]])/(16*e)
```

Rubi [A] time = 0.0385643, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {665, 195, 217, 203}

$$\frac{5}{16}d^5x\sqrt{d^2-e^2x^2}+\frac{5}{24}d^3x\left(d^2-e^2x^2\right)^{3/2}+\frac{1}{6}dx\left(d^2-e^2x^2\right)^{5/2}+\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e}+\frac{5d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(7/2)/(d + e*x),x]
```

```
[Out] (5*d^5*x*Sqrt[d^2 - e^2*x^2])/16 + (5*d^3*x*(d^2 - e^2*x^2)^(3/2))/24 + (d*
x*(d^2 - e^2*x^2)^(5/2))/6 + (d^2 - e^2*x^2)^(7/2)/(7*e) + (5*d^7*ArcTan[(e
*x)/Sqrt[d^2 - e^2*x^2]])/(16*e)
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{7/2}}{d+ex}\,dx = \frac{(d^2-e^2x^2)^{7/2}}{7e} + d\int (d^2-e^2x^2)^{5/2}\,dx$$
$$= \frac{1}{6}dx\left(d^2-e^2x^2\right)^{5/2} + \frac{(d^2-e^2x^2)^{7/2}}{7e} + \frac{1}{6}\left(5d^3\right)\int \left(d^2-e^2x^2\right)^{3/2}\,dx$$
$$= \frac{5}{24}d^3x\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{6}dx\left(d^2-e^2x^2\right)^{5/2} + \frac{(d^2-e^2x^2)^{7/2}}{7e} + \frac{1}{8}\left(5d^5\right)\int \sqrt{d^2-e^2x^2}\,dx$$
$$= \frac{5}{16}d^5x\sqrt{d^2-e^2x^2} + \frac{5}{24}d^3x\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{6}dx\left(d^2-e^2x^2\right)^{5/2} + \frac{(d^2-e^2x^2)^{7/2}}{7e} + \frac{1}{16}\left(5d^7\right)\int -$$
$$= \frac{5}{16}d^5x\sqrt{d^2-e^2x^2} + \frac{5}{24}d^3x\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{6}dx\left(d^2-e^2x^2\right)^{5/2} + \frac{(d^2-e^2x^2)^{7/2}}{7e} + \frac{1}{16}\left(5d^7\right)\text{Sub}$$
$$= \frac{5}{16}d^5x\sqrt{d^2-e^2x^2} + \frac{5}{24}d^3x\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{6}dx\left(d^2-e^2x^2\right)^{5/2} + \frac{(d^2-e^2x^2)^{7/2}}{7e} + \frac{5d^7\tan^{-1}\left(\frac{}{\sqrt{}}\right.}{16e}$$

Mathematica [A] time = 0.0969637, size = 113, normalized size = 0.91

$$\frac{\sqrt{d^2-e^2x^2}\left(-144d^4e^2x^2-182d^3e^3x^3+144d^2e^4x^4+231d^5ex+48d^6+56de^5x^5-48e^6x^6\right)+105d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{336e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(7/2)/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(48*d^6 + 231*d^5*e*x - 144*d^4*e^2*x^2 - 182*d^3*e^3*
x^3 + 144*d^2*e^4*x^4 + 56*d*e^5*x^5 - 48*e^6*x^6) + 105*d^7*ArcTan[(e*x)/S
qrt[d^2 - e^2*x^2]])/(336*e)
```

Maple [A] time = 0.047, size = 181, normalized size = 1.5

$$\frac{1}{7e}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{\frac{7}{2}} + \frac{dx}{6}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{\frac{5}{2}} + \frac{5d^3x}{24}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{\frac{3}{2}} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(7/2)/(e*x+d),x)
```

```
[Out] 1/7/e*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(7/2)+1/6*d*(-(d/e+x)^2*e^2+2*d*e*(d/e
+x))^(5/2)*x+5/24*d^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x+5/16*d^5*(-(d/
e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x+5/16*d^7/(e^2)^(1/2)*arctan((e^2)^(1/2)*x
/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))
```

Maxima [C] time = 1.59661, size = 174, normalized size = 1.4

$$-\frac{5i\,d^7\arcsin\left(\frac{ex}{d}+2\right)}{16\,e} + \frac{5}{16}\sqrt{e^2x^2+4\,dex+3\,d^2}d^5x + \frac{5\sqrt{e^2x^2+4\,dex+3\,d^2}d^6}{8\,e} + \frac{5}{24}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3x + \frac{1}{6}\left(-e^2x^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -5/16*I*d^7*arcsin(e*x/d + 2)/e + 5/16*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^5*
x + 5/8*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^6/e + 5/24*(-e^2*x^2 + d^2)^(3/2)
*d^3*x + 1/6*(-e^2*x^2 + d^2)^(5/2)*d*x + 1/7*(-e^2*x^2 + d^2)^(7/2)/e
```

Fricas [A] time = 2.15538, size = 257, normalized size = 2.07

$$-\frac{210\,d^7\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(48\,e^6x^6-56\,de^5x^5-144\,d^2e^4x^4+182\,d^3e^3x^3+144\,d^4e^2x^2-231\,d^5ex-48\,d^6\right)\sqrt{-e^2x^2}}{336\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/336*(210*d^7*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (48*e^6*x^6 - 5
6*d*e^5*x^5 - 144*d^2*e^4*x^4 + 182*d^3*e^3*x^3 + 144*d^4*e^2*x^2 - 231*d^5
*e*x - 48*d^6)*sqrt(-e^2*x^2 + d^2))/e
```

Sympy [C] time = 14.1586, size = 818, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(7/2)/(e*x+d),x)
```

```
[Out] d**5*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d
**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**
2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) -
 d**4*e*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(
3/2)/(3*e**2), True)) - 2*d**3*e**2*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3
) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e
**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**
2)/Abs(d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**
2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt
(1 - e**2*x**2/d**2)), True)) + 2*d**2*e**3*Piecewise((-2*d**4*sqrt(d**2 -
e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sq
rt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) + d*e**4*Piec
ewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x*
*2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(2
4*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)),
 Abs(e**2*x**2)/Abs(d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e*
*4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2))
 + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x*
*2/d**2)), True)) - e**5*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**
6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt(d**2 -
e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt
(d**2)/6, True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.804 $\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=132

$$\frac{7}{16}d^4x\sqrt{d^2-e^2x^2}+\frac{7}{24}d^2x\left(d^2-e^2x^2\right)^{3/2}+\frac{7d\left(d^2-e^2x^2\right)^{5/2}}{30e}+\frac{(d-ex)\left(d^2-e^2x^2\right)^{5/2}}{6e}+\frac{7d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e}$$

```
[Out] (7*d^4*x*Sqrt[d^2 - e^2*x^2])/16 + (7*d^2*x*(d^2 - e^2*x^2)^(3/2))/24 + (7*
d*(d^2 - e^2*x^2)^(5/2))/(30*e) + ((d - e*x)*(d^2 - e^2*x^2)^(5/2))/(6*e) +
 (7*d^6*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e)
```

Rubi [A] time = 0.0508933, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {655, 671, 641, 195, 217, 203}

$$\frac{7}{16}d^4x\sqrt{d^2-e^2x^2}+\frac{7}{24}d^2x\left(d^2-e^2x^2\right)^{3/2}+\frac{7d\left(d^2-e^2x^2\right)^{5/2}}{30e}+\frac{(d-ex)\left(d^2-e^2x^2\right)^{5/2}}{6e}+\frac{7d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^2,x]
```

```
[Out] (7*d^4*x*Sqrt[d^2 - e^2*x^2])/16 + (7*d^2*x*(d^2 - e^2*x^2)^(3/2))/24 + (7*
d*(d^2 - e^2*x^2)^(5/2))/(30*e) + ((d - e*x)*(d^2 - e^2*x^2)^(5/2))/(6*e) +
 (7*d^6*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e)
```

Rule 655

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(2*m)/a^m, Int[(a + c*x^2)^(m + p)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d,
 e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IntegerQ[m] && R
ationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^2}\,dx = \int (d-ex)^2\left(d^2-e^2x^2\right)^{3/2}\,dx$$

$$= \frac{(d-ex)\left(d^2-e^2x^2\right)^{5/2}}{6e} + \frac{1}{6}(7d)\int (d-ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$$

$$= \frac{7d\left(d^2-e^2x^2\right)^{5/2}}{30e} + \frac{(d-ex)\left(d^2-e^2x^2\right)^{5/2}}{6e} + \frac{1}{6}\left(7d^2\right)\int \left(d^2-e^2x^2\right)^{3/2}\,dx$$

$$= \frac{7}{24}d^2x\left(d^2-e^2x^2\right)^{3/2} + \frac{7d\left(d^2-e^2x^2\right)^{5/2}}{30e} + \frac{(d-ex)\left(d^2-e^2x^2\right)^{5/2}}{6e} + \frac{1}{8}\left(7d^4\right)\int \sqrt{d^2-e^2x^2}\,dx$$

$$= \frac{7}{16}d^4x\sqrt{d^2-e^2x^2} + \frac{7}{24}d^2x\left(d^2-e^2x^2\right)^{3/2} + \frac{7d\left(d^2-e^2x^2\right)^{5/2}}{30e} + \frac{(d-ex)\left(d^2-e^2x^2\right)^{5/2}}{6e} + \frac{1}{16}\left(7d\right.$$

$$= \frac{7}{16}d^4x\sqrt{d^2-e^2x^2} + \frac{7}{24}d^2x\left(d^2-e^2x^2\right)^{3/2} + \frac{7d\left(d^2-e^2x^2\right)^{5/2}}{30e} + \frac{(d-ex)\left(d^2-e^2x^2\right)^{5/2}}{6e} + \frac{1}{16}\left(7d\right.$$

$$= \frac{7}{16}d^4x\sqrt{d^2-e^2x^2} + \frac{7}{24}d^2x\left(d^2-e^2x^2\right)^{3/2} + \frac{7d\left(d^2-e^2x^2\right)^{5/2}}{30e} + \frac{(d-ex)\left(d^2-e^2x^2\right)^{5/2}}{6e} + \frac{7d^6\,\mathrm{ta}}{}$$

Mathematica [A] time = 0.0778846, size = 102, normalized size = 0.77

$$\frac{\sqrt{d^2-e^2x^2}\left(-192d^3e^2x^2+10d^2e^3x^3+135d^4ex+96d^5+96de^4x^4-40e^5x^5\right)+105d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{240e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^2,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(96*d^5 + 135*d^4*e*x - 192*d^3*e^2*x^2 + 10*d^2*e^3*x
^3 + 96*d*e^4*x^4 - 40*e^5*x^5) + 105*d^6*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]
)/(240*e)
```

Maple [B] time = 0.05, size = 228, normalized size = 1.7

$$\frac{1}{5e^3d}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{\frac{9}{2}}\left(\frac{d}{e}+x\right)^{-2}+\frac{1}{5de}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{\frac{7}{2}}+\frac{7x}{30}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^2,x)
```

```
[Out] 1/5/e^3/d/(d/e+x)^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(9/2)+1/5/e/d*(-(d/e+x)^
2*e^2+2*d*e*(d/e+x))^(7/2)+7/30*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)*x+7/24
*d^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x+7/16*d^4*(-(d/e+x)^2*e^2+2*d*e*
(d/e+x))^(1/2)*x+7/16*d^6/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+
2*d*e*(d/e+x))^(1/2))
```

Maxima [C] time = 1.58455, size = 188, normalized size = 1.42

$$-\frac{7i\,d^6\arcsin\left(\frac{ex}{d}+2\right)}{16\,e}+\frac{7}{16}\sqrt{e^2x^2+4\,dex+3\,d^2}d^4x+\frac{7\sqrt{e^2x^2+4\,dex+3\,d^2}d^5}{8\,e}+\frac{7}{24}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2x+\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{6\left(e^2x+de\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -7/16*I*d^6*arcsin(e*x/d + 2)/e + 7/16*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^4*
x + 7/8*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^5/e + 7/24*(-e^2*x^2 + d^2)^(3/2)
*d^2*x + 1/6*(-e^2*x^2 + d^2)^(7/2)/(e^2*x + d*e) + 7/30*(-e^2*x^2 + d^2)^(
5/2)*d/e
```

Fricas [A] time = 2.17988, size = 231, normalized size = 1.75

$$-\frac{210\,d^6\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(40\,e^5x^5-96\,de^4x^4-10\,d^2e^3x^3+192\,d^3e^2x^2-135\,d^4ex-96\,d^5\right)\sqrt{-e^2x^2+d^2}}{240\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/240*(210*d^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (40*e^5*x^5 - 9
6*d*e^4*x^4 - 10*d^2*e^3*x^3 + 192*d^3*e^2*x^2 - 135*d^4*e*x - 96*d^5)*sqrt
(-e^2*x^2 + d^2))/e
```

Sympy [C] time = 15.2106, size = 498, normalized size = 3.77

$$d^4\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e}-\frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \frac{|e^2x^2|}{|d^2|}>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e}+\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise}\end{cases}\right)-2d^3e\left(\begin{cases}\frac{x^2\sqrt{d^2}}{2} & \text{for}\ e^2=0\\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise}\end{cases}\right)+2de^3\left(\begin{cases}-\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4}- \\ \frac{x^4\sqrt{d^2}}{4}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(7/2)/(e*x+d)**2,x)
```

```
[Out] d**4*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d
**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**
```

```
2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) -
 2*d**3*e*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)*
*(3/2)/(3*e**2), True)) + 2*d*e**3*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2
)/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 -
 e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) - e**4*Piecewise((-I*d
**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) -
 I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 +
 e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x
**2)/Abs(d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 -
 e**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5
/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)),
True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.805 $\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^3} dx$

Optimal. Leaf size=142

$$\frac{7}{8}d^3x\sqrt{d^2-e^2x^2}+\frac{7d^2\left(d^2-e^2x^2\right)^{3/2}}{12e}+\frac{7d(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{20e}+\frac{(d-ex)^2\left(d^2-e^2x^2\right)^{3/2}}{5e}+\frac{7d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}$$

```
[Out] (7*d^3*x*Sqrt[d^2 - e^2*x^2])/8 + (7*d^2*(d^2 - e^2*x^2)^(3/2))/(12*e) + (7
*d*(d - e*x)*(d^2 - e^2*x^2)^(3/2))/(20*e) + ((d - e*x)^2*(d^2 - e^2*x^2)^(
3/2))/(5*e) + (7*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e)
```

Rubi [A] time = 0.0594916, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {655, 671, 641, 195, 217, 203}

$$\frac{7}{8}d^3x\sqrt{d^2-e^2x^2}+\frac{7d^2\left(d^2-e^2x^2\right)^{3/2}}{12e}+\frac{7d(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{20e}+\frac{(d-ex)^2\left(d^2-e^2x^2\right)^{3/2}}{5e}+\frac{7d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^3,x]
```

```
[Out] (7*d^3*x*Sqrt[d^2 - e^2*x^2])/8 + (7*d^2*(d^2 - e^2*x^2)^(3/2))/(12*e) + (7
*d*(d - e*x)*(d^2 - e^2*x^2)^(3/2))/(20*e) + ((d - e*x)^2*(d^2 - e^2*x^2)^(
3/2))/(5*e) + (7*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e)
```

Rule 655

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(2*m)/a^m, Int[(a + c*x^2)^(m + p)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d,
 e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IntegerQ[m] && R
ationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^3}\,dx = \int (d-ex)^3\sqrt{d^2-e^2x^2}\,dx$$

$$= \frac{(d-ex)^2\left(d^2-e^2x^2\right)^{3/2}}{5e} + \frac{1}{5}(7d)\int (d-ex)^2\sqrt{d^2-e^2x^2}\,dx$$

$$= \frac{7d(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{20e} + \frac{(d-ex)^2\left(d^2-e^2x^2\right)^{3/2}}{5e} + \frac{1}{4}\left(7d^2\right)\int (d-ex)\sqrt{d^2-e^2x^2}\,dx$$

$$= \frac{7d^2\left(d^2-e^2x^2\right)^{3/2}}{12e} + \frac{7d(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{20e} + \frac{(d-ex)^2\left(d^2-e^2x^2\right)^{3/2}}{5e} + \frac{1}{4}\left(7d^3\right)\int \sqrt{d^2-e^2x}$$

$$= \frac{7}{8}d^3x\sqrt{d^2-e^2x^2} + \frac{7d^2\left(d^2-e^2x^2\right)^{3/2}}{12e} + \frac{7d(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{20e} + \frac{(d-ex)^2\left(d^2-e^2x^2\right)^{3/2}}{5e} + \frac{1}{8}$$

$$= \frac{7}{8}d^3x\sqrt{d^2-e^2x^2} + \frac{7d^2\left(d^2-e^2x^2\right)^{3/2}}{12e} + \frac{7d(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{20e} + \frac{(d-ex)^2\left(d^2-e^2x^2\right)^{3/2}}{5e} + \frac{1}{8}$$

$$= \frac{7}{8}d^3x\sqrt{d^2-e^2x^2} + \frac{7d^2\left(d^2-e^2x^2\right)^{3/2}}{12e} + \frac{7d(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{20e} + \frac{(d-ex)^2\left(d^2-e^2x^2\right)^{3/2}}{5e} + \frac{7}{}$$

Mathematica [A] time = 0.074503, size = 91, normalized size = 0.64

$$\frac{\sqrt{d^2-e^2x^2}\left(-112d^2e^2x^2+15d^3ex+136d^4+90de^3x^3-24e^4x^4\right)+105d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{120e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^3,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(136*d^4 + 15*d^3*e*x - 112*d^2*e^2*x^2 + 90*d*e^3*x^3
 - 24*e^4*x^4) + 105*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(120*e)
```

Maple [B] time = 0.051, size = 274, normalized size = 1.9

$$\frac{1}{3e^4d}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{\frac{9}{2}}\left(\frac{d}{e}+x\right)^{-3} + \frac{2}{5e^3d^2}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{\frac{9}{2}}\left(\frac{d}{e}+x\right)^{-2} + \frac{2}{5ed^2}\left(-\left(\frac{d}{e}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^3,x)
```

```
[Out] 1/3/e^4/d/(d/e+x)^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(9/2)+2/5/e^3/d^2/(d/e+x
)^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(9/2)+2/5/e/d^2*(-(d/e+x)^2*e^2+2*d*e*(d
/e+x))^(7/2)+7/15/d*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)*x+7/12*d*(-(d/e+x)
^2*e^2+2*d*e*(d/e+x))^(3/2)*x+7/8*d^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*
x+7/8*d^5/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(
1/2))
```

Maxima [C] time = 1.58163, size = 216, normalized size = 1.52

$$-\frac{7 i d^5 \arcsin\left(\frac{ex}{d}+2\right)}{8e} + \frac{7}{8}\sqrt{e^2x^2+4dex+3d^2}d^3x + \frac{7\sqrt{e^2x^2+4dex+3d^2}d^4}{4e} + \frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{5\left(e^3x^2+2de^2x+d^2e\right)} + \frac{7\left(-e^2x^2+d\right.}{20\left(e^2x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -7/8*I*d^5*arcsin(e*x/d + 2)/e + 7/8*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^3*x
+ 7/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^4/e + 1/5*(-e^2*x^2 + d^2)^(7/2)/(e
^3*x^2 + 2*d*e^2*x + d^2*e) + 7/20*(-e^2*x^2 + d^2)^(5/2)*d/(e^2*x + d*e) +
 7/12*(-e^2*x^2 + d^2)^(3/2)*d^2/e
```

Fricas [A] time = 2.28994, size = 208, normalized size = 1.46

$$-\frac{210\,d^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + \left(24\,e^4x^4 - 90\,de^3x^3 + 112\,d^2e^2x^2 - 15\,d^3ex - 136\,d^4\right)\sqrt{-e^2x^2+d^2}}{120\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/120*(210*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (24*e^4*x^4 - 9
0*d*e^3*x^3 + 112*d^2*e^2*x^2 - 15*d^3*e*x - 136*d^4)*sqrt(-e^2*x^2 + d^2))
/e
```

Sympy [C] time = 21.5633, size = 442, normalized size = 3.11

$$d^3\left(\begin{cases} -\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e} - \frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \frac{|e^2x^2|}{|d^2|} > 1 \\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e} + \frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise} \end{cases}\right) - 3d^2e\left(\begin{cases} \frac{x^2\sqrt{d^2}}{2} & \text{for } e^2 = 0 \\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise} \end{cases}\right) + 3de^2\left(\left(\begin{cases} -\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3} \\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3} - \end{cases}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(7/2)/(e*x+d)**3,x)
```

```
[Out] d**3*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d
**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**
```

```
2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) -
 3*d**2*e*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)*
*(3/2)/(3*e**2), True)) + 3*d*e**2*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3)
 + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e*
*2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2
)/Abs(d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2
*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(
1 - e**2*x**2/d**2)), True)) - e**3*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**
2)/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2
- e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.806 $\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^4} dx$

Optimal. Leaf size=136

$$\frac{35}{8}d^2x\sqrt{d^2-e^2x^2}+\frac{35d\left(d^2-e^2x^2\right)^{3/2}}{12e}+\frac{2\left(d^2-e^2x^2\right)^{7/2}}{e(d+ex)^3}+\frac{7(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{35d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}$$

```
[Out] (35*d^2*x*Sqrt[d^2 - e^2*x^2])/8 + (35*d*(d^2 - e^2*x^2)^(3/2))/(12*e) + (7
*(d - e*x)*(d^2 - e^2*x^2)^(3/2))/(4*e) + (2*(d^2 - e^2*x^2)^(7/2))/(e*(d +
 e*x)^3) + (35*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e)
```

Rubi [A] time = 0.0566787, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {663, 655, 671, 641, 195, 217, 203}

$$\frac{35}{8}d^2x\sqrt{d^2-e^2x^2}+\frac{35d\left(d^2-e^2x^2\right)^{3/2}}{12e}+\frac{2\left(d^2-e^2x^2\right)^{7/2}}{e(d+ex)^3}+\frac{7(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{35d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^4,x]
```

```
[Out] (35*d^2*x*Sqrt[d^2 - e^2*x^2])/8 + (35*d*(d^2 - e^2*x^2)^(3/2))/(12*e) + (7
*(d - e*x)*(d^2 - e^2*x^2)^(3/2))/(4*e) + (2*(d^2 - e^2*x^2)^(7/2))/(e*(d +
 e*x)^3) + (35*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e)
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(2*m)/a^m, Int[(a + c*x^2)^(m + p)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d,
 e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IntegerQ[m] && R
ationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^4}\,dx &= \frac{2\left(d^2-e^2x^2\right)^{7/2}}{e(d+ex)^3}+7\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx\\
&= \frac{2\left(d^2-e^2x^2\right)^{7/2}}{e(d+ex)^3}+7\int (d-ex)^2\sqrt{d^2-e^2x^2}\,dx\\
&= \frac{7(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{2\left(d^2-e^2x^2\right)^{7/2}}{e(d+ex)^3}+\frac{1}{4}(35d)\int (d-ex)\sqrt{d^2-e^2x^2}\,dx\\
&= \frac{35d\left(d^2-e^2x^2\right)^{3/2}}{12e}+\frac{7(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{2\left(d^2-e^2x^2\right)^{7/2}}{e(d+ex)^3}+\frac{1}{4}\left(35d^2\right)\int \sqrt{d^2-e^2x^2}\,dx\\
&= \frac{35}{8}d^2x\sqrt{d^2-e^2x^2}+\frac{35d\left(d^2-e^2x^2\right)^{3/2}}{12e}+\frac{7(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{2\left(d^2-e^2x^2\right)^{7/2}}{e(d+ex)^3}+\frac{1}{8}\left(35d^4\right)\\
&= \frac{35}{8}d^2x\sqrt{d^2-e^2x^2}+\frac{35d\left(d^2-e^2x^2\right)^{3/2}}{12e}+\frac{7(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{2\left(d^2-e^2x^2\right)^{7/2}}{e(d+ex)^3}+\frac{1}{8}\left(35d^4\right)\\
&= \frac{35}{8}d^2x\sqrt{d^2-e^2x^2}+\frac{35d\left(d^2-e^2x^2\right)^{3/2}}{12e}+\frac{7(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{2\left(d^2-e^2x^2\right)^{7/2}}{e(d+ex)^3}+\frac{35d^4\tan}{}
\end{aligned}$$

Mathematica [A] time = 0.0784545, size = 80, normalized size = 0.59

$$\frac{\sqrt{d^2-e^2x^2}\left(-81d^2ex+160d^3+32de^2x^2-6e^3x^3\right)+105d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{24e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(160*d^3 - 81*d^2*e*x + 32*d*e^2*x^2 - 6*e^3*x^3) + 10
5*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(24*e)
```

Maple [B] time = 0.05, size = 317, normalized size = 2.3

$$\frac{1}{e^5 d}\left(-\left(\frac{d}{e}+x\right)^2 e^2+2de\left(\frac{d}{e}+x\right)\right)^{\frac{9}{2}}\left(\frac{d}{e}+x\right)^{-4}+\frac{5}{3e^4d^2}\left(-\left(\frac{d}{e}+x\right)^2 e^2+2de\left(\frac{d}{e}+x\right)\right)^{\frac{9}{2}}\left(\frac{d}{e}+x\right)^{-3}+2\frac{1}{e^3d^3}\left(-\left(\frac{d}{e}+x\right)^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^4,x)
```

```
[Out] 1/e^5/d/(d/e+x)^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(9/2)+5/3/e^4/d^2/(d/e+x)^
3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(9/2)+2/e^3/d^3/(d/e+x)^2*(-(d/e+x)^2*e^2+
2*d*e*(d/e+x))^(9/2)+2/e/d^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(7/2)+7/3/d^2*(
-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)*x+35/12*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^
(3/2)*x+35/8*d^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x+35/8*d^4/(e^2)^(1/2
)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.19882, size = 181, normalized size = 1.33

$$-\frac{210\,d^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(6\,e^3x^3-32\,de^2x^2+81\,d^2ex-160\,d^3\right)\sqrt{-e^2x^2+d^2}}{24\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/24*(210*d^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (6*e^3*x^3 - 32*
d*e^2*x^2 + 81*d^2*e*x - 160*d^3)*sqrt(-e^2*x^2 + d^2))/e
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(7/2)/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**(7/2)/(d + e*x)**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.807 $\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^5}\,dx$

Optimal. Leaf size=132

$$-\frac{2\left(d^2-e^2x^2\right)^{7/2}}{e(d+ex)^4}-\frac{14\left(d^2-e^2x^2\right)^{5/2}}{e(d+ex)^2}-\frac{35\left(d^2-e^2x^2\right)^{3/2}}{3e}-\frac{35}{2}dx\sqrt{d^2-e^2x^2}-\frac{35d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

```
[Out] (-35*d*x*Sqrt[d^2 - e^2*x^2])/2 - (35*(d^2 - e^2*x^2)^(3/2))/(3*e) - (14*(d
^2 - e^2*x^2)^(5/2))/(e*(d + e*x)^2) - (2*(d^2 - e^2*x^2)^(7/2))/(e*(d + e*
x)^4) - (35*d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)
```

Rubi [A] time = 0.0508519, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {663, 665, 195, 217, 203}

$$-\frac{2\left(d^2-e^2x^2\right)^{7/2}}{e(d+ex)^4}-\frac{14\left(d^2-e^2x^2\right)^{5/2}}{e(d+ex)^2}-\frac{35\left(d^2-e^2x^2\right)^{3/2}}{3e}-\frac{35}{2}dx\sqrt{d^2-e^2x^2}-\frac{35d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^5,x]
```


```
[Out] (-35*d*x*Sqrt[d^2 - e^2*x^2])/2 - (35*(d^2 - e^2*x^2)^(3/2))/(3*e) - (14*(d
^2 - e^2*x^2)^(5/2))/(e*(d + e*x)^2) - (2*(d^2 - e^2*x^2)^(7/2))/(e*(d + e*
x)^4) - (35*d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)
```


Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^5}\,dx = -\frac{2\left(d^2-e^2x^2\right)^{7/2}}{e(d+ex)^4} - 7\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^3}\,dx$$
$$= -\frac{14\left(d^2-e^2x^2\right)^{5/2}}{e(d+ex)^2} - \frac{2\left(d^2-e^2x^2\right)^{7/2}}{e(d+ex)^4} - 35\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{d+ex}\,dx$$
$$= -\frac{35\left(d^2-e^2x^2\right)^{3/2}}{3e} - \frac{14\left(d^2-e^2x^2\right)^{5/2}}{e(d+ex)^2} - \frac{2\left(d^2-e^2x^2\right)^{7/2}}{e(d+ex)^4} - (35d)\int \sqrt{d^2-e^2x^2}\,dx$$
$$= -\frac{35}{2}dx\sqrt{d^2-e^2x^2} - \frac{35\left(d^2-e^2x^2\right)^{3/2}}{3e} - \frac{14\left(d^2-e^2x^2\right)^{5/2}}{e(d+ex)^2} - \frac{2\left(d^2-e^2x^2\right)^{7/2}}{e(d+ex)^4} - \frac{1}{2}\left(35d^3\right)\int \frac{}{\sqrt{d^2}}$$
$$= -\frac{35}{2}dx\sqrt{d^2-e^2x^2} - \frac{35\left(d^2-e^2x^2\right)^{3/2}}{3e} - \frac{14\left(d^2-e^2x^2\right)^{5/2}}{e(d+ex)^2} - \frac{2\left(d^2-e^2x^2\right)^{7/2}}{e(d+ex)^4} - \frac{1}{2}\left(35d^3\right)\text{Subst}$$
$$= -\frac{35}{2}dx\sqrt{d^2-e^2x^2} - \frac{35\left(d^2-e^2x^2\right)^{3/2}}{3e} - \frac{14\left(d^2-e^2x^2\right)^{5/2}}{e(d+ex)^2} - \frac{2\left(d^2-e^2x^2\right)^{7/2}}{e(d+ex)^4} - \frac{35d^3\tan^{-1}\left(\frac{e}{\sqrt{d^2-}}\right)}{2e}$$

Mathematica [A] time = 0.0993727, size = 85, normalized size = 0.64

$$\frac{1}{6}\sqrt{d^2-e^2x^2}\left(-\frac{96d^3}{e(d+ex)} - \frac{70d^2}{e} + 15dx - 2ex^2\right) - \frac{35d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^5,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*((-70*d^2)/e + 15*d*x - 2*e*x^2 - (96*d^3)/(e*(d + e*x
))))/6 - (35*d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)
```

Maple [B] time = 0.051, size = 364, normalized size = 2.8

$$-\frac{1}{e^6 d}\left(-\left(\frac{d}{e}+x\right)^2 e^2 + 2de\left(\frac{d}{e}+x\right)\right)^{\frac{9}{2}}\left(\frac{d}{e}+x\right)^{-5} - 4\frac{1}{e^5 d^2}\left(-\left(\frac{d}{e}+x\right)^2 e^2 + 2de\left(\frac{d}{e}+x\right)\right)^{9/2}\left(\frac{d}{e}+x\right)^{-4} - \frac{20}{3e^4 d^3}\left(-\left(\frac{d}{e}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^5,x)
```

```
[Out] -1/e^6/d/(d/e+x)^5*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(9/2)-4/e^5/d^2/(d/e+x)^4
*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(9/2)-20/3/e^4/d^3/(d/e+x)^3*(-(d/e+x)^2*e^
2+2*d*e*(d/e+x))^(9/2)-8/e^3/d^4/(d/e+x)^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(
9/2)-8/e/d^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(7/2)-28/3/d^3*(-(d/e+x)^2*e^2+
2*d*e*(d/e+x))^(5/2)*x-35/3/d*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x-35/2*d
```

```
*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x-35/2*d^3/(e^2)^(1/2)*arctan((e^2)^(
1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.23631, size = 244, normalized size = 1.85

$$-\frac{166\,d^3ex+166\,d^4-210\left(d^3ex+d^4\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(2\,e^3x^3-13\,de^2x^2+55\,d^2ex+166\,d^3\right)\sqrt{-e^2x^2+d^2}}{6\left(e^2x+de\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^5,x, algorithm="fricas")
```

```
[Out] -1/6*(166*d^3*e*x + 166*d^4 - 210*(d^3*e*x + d^4)*arctan(-(d - sqrt(-e^2*x^
2 + d^2))/(e*x)) + (2*e^3*x^3 - 13*d*e^2*x^2 + 55*d^2*e*x + 166*d^3)*sqrt(-
e^2*x^2 + d^2))/(e^2*x + d*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}{\left(d+ex\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(7/2)/(e*x+d)**5,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**(7/2)/(d + e*x)**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.808 $\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^6} dx$

Optimal. Leaf size=145

$$-\frac{2(d^2-e^2x^2)^{7/2}}{3e(d+ex)^5}+\frac{14(d^2-e^2x^2)^{5/2}}{3e(d+ex)^3}+\frac{35(d^2-e^2x^2)^{3/2}}{6e(d+ex)}+\frac{35d\sqrt{d^2-e^2x^2}}{2e}+\frac{35d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

```
[Out] (35*d*Sqrt[d^2 - e^2*x^2])/(2*e) + (35*(d^2 - e^2*x^2)^(3/2))/(6*e*(d + e*x
)) + (14*(d^2 - e^2*x^2)^(5/2))/(3*e*(d + e*x)^3) - (2*(d^2 - e^2*x^2)^(7/2
))/(3*e*(d + e*x)^5) + (35*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)
```

Rubi [A] time = 0.0587709, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {663, 665, 217, 203}

$$-\frac{2(d^2-e^2x^2)^{7/2}}{3e(d+ex)^5}+\frac{14(d^2-e^2x^2)^{5/2}}{3e(d+ex)^3}+\frac{35(d^2-e^2x^2)^{3/2}}{6e(d+ex)}+\frac{35d\sqrt{d^2-e^2x^2}}{2e}+\frac{35d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^6,x]
```

```
[Out] (35*d*Sqrt[d^2 - e^2*x^2])/(2*e) + (35*(d^2 - e^2*x^2)^(3/2))/(6*e*(d + e*x
)) + (14*(d^2 - e^2*x^2)^(5/2))/(3*e*(d + e*x)^3) - (2*(d^2 - e^2*x^2)^(7/2
))/(3*e*(d + e*x)^5) + (35*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^6}\,dx = -\frac{2\left(d^2-e^2x^2\right)^{7/2}}{3e(d+ex)^5}-\frac{7}{3}\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}\,dx$$
$$=\frac{14\left(d^2-e^2x^2\right)^{5/2}}{3e(d+ex)^3}-\frac{2\left(d^2-e^2x^2\right)^{7/2}}{3e(d+ex)^5}+\frac{35}{3}\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{(d+ex)^2}\,dx$$
$$=\frac{35\left(d^2-e^2x^2\right)^{3/2}}{6e(d+ex)}+\frac{14\left(d^2-e^2x^2\right)^{5/2}}{3e(d+ex)^3}-\frac{2\left(d^2-e^2x^2\right)^{7/2}}{3e(d+ex)^5}+\frac{1}{2}(35d)\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}\,dx$$
$$=\frac{35d\sqrt{d^2-e^2x^2}}{2e}+\frac{35\left(d^2-e^2x^2\right)^{3/2}}{6e(d+ex)}+\frac{14\left(d^2-e^2x^2\right)^{5/2}}{3e(d+ex)^3}-\frac{2\left(d^2-e^2x^2\right)^{7/2}}{3e(d+ex)^5}+\frac{1}{2}\left(35d^2\right)\int \frac{1}{\sqrt{d^2-e^2x}}$$
$$=\frac{35d\sqrt{d^2-e^2x^2}}{2e}+\frac{35\left(d^2-e^2x^2\right)^{3/2}}{6e(d+ex)}+\frac{14\left(d^2-e^2x^2\right)^{5/2}}{3e(d+ex)^3}-\frac{2\left(d^2-e^2x^2\right)^{7/2}}{3e(d+ex)^5}+\frac{1}{2}\left(35d^2\right)\text{Subst}\left(\int \frac{}{1}\right.$$
$$=\frac{35d\sqrt{d^2-e^2x^2}}{2e}+\frac{35\left(d^2-e^2x^2\right)^{3/2}}{6e(d+ex)}+\frac{14\left(d^2-e^2x^2\right)^{5/2}}{3e(d+ex)^3}-\frac{2\left(d^2-e^2x^2\right)^{7/2}}{3e(d+ex)^5}+\frac{35d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

Mathematica [A] time = 0.0972618, size = 87, normalized size = 0.6

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(229d^2ex+164d^3+30de^2x^2-3e^3x^3\right)}{(d+ex)^2}+105d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{6e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^6,x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(164*d^3 + 229*d^2*e*x + 30*d*e^2*x^2 - 3*e^3*x^3))/(
d + e*x)^2 + 105*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(6*e)
```

Maple [B] time = 0.051, size = 407, normalized size = 2.8

$$-\frac{1}{3e^7d}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{\frac{9}{2}}\left(\frac{d}{e}+x\right)^{-6}+\frac{1}{e^6d^2}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{\frac{9}{2}}\left(\frac{d}{e}+x\right)^{-5}+4\frac{1}{e^5d^3}\left(-\left(\frac{d}{e}+x\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^6,x)
```

```
[Out] -1/3/e^7/d/(d/e+x)^6*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(9/2)+1/e^6/d^2/(d/e+x)
^5*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(9/2)+4/e^5/d^3/(d/e+x)^4*(-(d/e+x)^2*e^2
+2*d*e*(d/e+x))^(9/2)+20/3/e^4/d^4/(d/e+x)^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))
^(9/2)+8/e^3/d^5/(d/e+x)^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(9/2)+8/e/d^5*(-(
d/e+x)^2*e^2+2*d*e*(d/e+x))^(7/2)+28/3/d^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(
5/2)*x+35/3/d^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x+35/2*(-(d/e+x)^2*e^2
+2*d*e*(d/e+x))^(1/2)*x+35/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)
^2*e^2+2*d*e*(d/e+x))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.30443, size = 312, normalized size = 2.15

$$\frac{164\,d^2e^2x^2 + 328\,d^3ex + 164\,d^4 - 210\left(d^2e^2x^2 + 2\,d^3ex + d^4\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) - \left(3\,e^3x^3 - 30\,de^2x^2 - 229\,d^2ex - \right.}{6\left(e^3x^2 + 2\,de^2x + d^2e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^6,x, algorithm="fricas")
```

```
[Out] 1/6*(164*d^2*e^2*x^2 + 328*d^3*e*x + 164*d^4 - 210*(d^2*e^2*x^2 + 2*d^3*e*x
 + d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (3*e^3*x^3 - 30*d*e^2*x
^2 - 229*d^2*e*x - 164*d^3)*sqrt(-e^2*x^2 + d^2))/(e^3*x^2 + 2*d*e^2*x + d^
2*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(-d + ex\right)\left(d + ex\right)\right)^{\frac{7}{2}}}{\left(d + ex\right)^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(7/2)/(e*x+d)**6,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**(7/2)/(d + e*x)**6, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^6,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.809 $\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^7} dx$

Optimal. Leaf size=138

$$-\frac{2(d^2-e^2x^2)^{7/2}}{5e(d+ex)^6}+\frac{14(d^2-e^2x^2)^{5/2}}{15e(d+ex)^4}-\frac{14(d^2-e^2x^2)^{3/2}}{3e(d+ex)^2}-\frac{7\sqrt{d^2-e^2x^2}}{e}-\frac{7d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

```
[Out] (-7*Sqrt[d^2 - e^2*x^2])/e - (14*(d^2 - e^2*x^2)^(3/2))/(3*e*(d + e*x)^2) +
 (14*(d^2 - e^2*x^2)^(5/2))/(15*e*(d + e*x)^4) - (2*(d^2 - e^2*x^2)^(7/2))/
(5*e*(d + e*x)^6) - (7*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e
```

Rubi [A] time = 0.0588687, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {663, 665, 217, 203}

$$-\frac{2(d^2-e^2x^2)^{7/2}}{5e(d+ex)^6}+\frac{14(d^2-e^2x^2)^{5/2}}{15e(d+ex)^4}-\frac{14(d^2-e^2x^2)^{3/2}}{3e(d+ex)^2}-\frac{7\sqrt{d^2-e^2x^2}}{e}-\frac{7d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^7,x]
```

```
[Out] (-7*Sqrt[d^2 - e^2*x^2])/e - (14*(d^2 - e^2*x^2)^(3/2))/(3*e*(d + e*x)^2) +
 (14*(d^2 - e^2*x^2)^(5/2))/(15*e*(d + e*x)^4) - (2*(d^2 - e^2*x^2)^(7/2))/
(5*e*(d + e*x)^6) - (7*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^7}\,dx = -\frac{2\left(d^2-e^2x^2\right)^{7/2}}{5e(d+ex)^6} - \frac{7}{5}\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^5}\,dx$$

$$= \frac{14\left(d^2-e^2x^2\right)^{5/2}}{15e(d+ex)^4} - \frac{2\left(d^2-e^2x^2\right)^{7/2}}{5e(d+ex)^6} + \frac{7}{3}\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{(d+ex)^3}\,dx$$

$$= -\frac{14\left(d^2-e^2x^2\right)^{3/2}}{3e(d+ex)^2} + \frac{14\left(d^2-e^2x^2\right)^{5/2}}{15e(d+ex)^4} - \frac{2\left(d^2-e^2x^2\right)^{7/2}}{5e(d+ex)^6} - 7\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}\,dx$$

$$= -\frac{7\sqrt{d^2-e^2x^2}}{e} - \frac{14\left(d^2-e^2x^2\right)^{3/2}}{3e(d+ex)^2} + \frac{14\left(d^2-e^2x^2\right)^{5/2}}{15e(d+ex)^4} - \frac{2\left(d^2-e^2x^2\right)^{7/2}}{5e(d+ex)^6} - (7d)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,d$$

$$= -\frac{7\sqrt{d^2-e^2x^2}}{e} - \frac{14\left(d^2-e^2x^2\right)^{3/2}}{3e(d+ex)^2} + \frac{14\left(d^2-e^2x^2\right)^{5/2}}{15e(d+ex)^4} - \frac{2\left(d^2-e^2x^2\right)^{7/2}}{5e(d+ex)^6} - (7d)\,\text{Subst}\left(\int \frac{1}{1+e}\right.$$

$$= -\frac{7\sqrt{d^2-e^2x^2}}{e} - \frac{14\left(d^2-e^2x^2\right)^{3/2}}{3e(d+ex)^2} + \frac{14\left(d^2-e^2x^2\right)^{5/2}}{15e(d+ex)^4} - \frac{2\left(d^2-e^2x^2\right)^{7/2}}{5e(d+ex)^6} - \frac{7d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

Mathematica [A] time = 0.0871775, size = 87, normalized size = 0.63

$$-\frac{\sqrt{d^2-e^2x^2}\left(381d^2ex+167d^3+277de^2x^2+15e^3x^3\right)}{15e(d+ex)^3} - \frac{7d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^7,x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]*(167*d^3 + 381*d^2*e*x + 277*d*e^2*x^2 + 15*e^3*x^3))
/(15*e*(d + e*x)^3) - (7*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e
```

Maple [B] time = 0.051, size = 454, normalized size = 3.3

$$-\frac{1}{5e^8d}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{\frac{9}{2}}\left(\frac{d}{e}+x\right)^{-7} + \frac{2}{15e^7d^2}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{\frac{9}{2}}\left(\frac{d}{e}+x\right)^{-6} - \frac{2}{5e^6d^3}\left(-\left(\frac{d}{e}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^7,x)
```

```
[Out] -1/5/e^8/d/(d/e+x)^7*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(9/2)+2/15/e^7/d^2/(d/e
+x)^6*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(9/2)-2/5/e^6/d^3/(d/e+x)^5*(-(d/e+x)^
2*e^2+2*d*e*(d/e+x))^(9/2)-8/5/e^5/d^4/(d/e+x)^4*(-(d/e+x)^2*e^2+2*d*e*(d/e
+x))^(9/2)-8/3/e^4/d^5/(d/e+x)^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(9/2)-16/5/
e^3/d^6/(d/e+x)^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(9/2)-16/5/e/d^6*(-(d/e+x)
^2*e^2+2*d*e*(d/e+x))^(7/2)-56/15/d^5*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)*
x-14/3/d^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x-7/d*(-(d/e+x)^2*e^2+2*d*e
*(d/e+x))^(1/2)*x-7*d/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*
e*(d/e+x))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.56656, size = 379, normalized size = 2.75

$$-\frac{167\,de^3x^3 + 501\,d^2e^2x^2 + 501\,d^3ex + 167\,d^4 - 210\left(de^3x^3 + 3\,d^2e^2x^2 + 3\,d^3ex + d^4\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + \left(15\,e^3x^3 +\right.}{15\left(e^4x^3 + 3\,de^3x^2 + 3\,d^2e^2x + d^3e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^7,x, algorithm="fricas")
```

```
[Out] -1/15*(167*d*e^3*x^3 + 501*d^2*e^2*x^2 + 501*d^3*e*x + 167*d^4 - 210*(d*e^3
*x^3 + 3*d^2*e^2*x^2 + 3*d^3*e*x + d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/
(e*x)) + (15*e^3*x^3 + 277*d*e^2*x^2 + 381*d^2*e*x + 167*d^3)*sqrt(-e^2*x^2
 + d^2))/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(7/2)/(e*x+d)**7,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^7,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.810 $\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^8}\,dx$

Optimal. Leaf size=143

$$-\frac{2\left(d^2-e^2x^2\right)^{7/2}}{7e(d+ex)^7}+\frac{2\left(d^2-e^2x^2\right)^{5/2}}{5e(d+ex)^5}-\frac{2\left(d^2-e^2x^2\right)^{3/2}}{3e(d+ex)^3}+\frac{2\sqrt{d^2-e^2x^2}}{e(d+ex)}+\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

```
[Out] (2*Sqrt[d^2 - e^2*x^2])/(e*(d + e*x)) - (2*(d^2 - e^2*x^2)^(3/2))/(3*e*(d +
 e*x)^3) + (2*(d^2 - e^2*x^2)^(5/2))/(5*e*(d + e*x)^5) - (2*(d^2 - e^2*x^2)
^(7/2))/(7*e*(d + e*x)^7) + ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e
```

Rubi [A] time = 0.041401, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {663, 217, 203}

$$-\frac{2\left(d^2-e^2x^2\right)^{7/2}}{7e(d+ex)^7}+\frac{2\left(d^2-e^2x^2\right)^{5/2}}{5e(d+ex)^5}-\frac{2\left(d^2-e^2x^2\right)^{3/2}}{3e(d+ex)^3}+\frac{2\sqrt{d^2-e^2x^2}}{e(d+ex)}+\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^8,x]
```

```
[Out] (2*Sqrt[d^2 - e^2*x^2])/(e*(d + e*x)) - (2*(d^2 - e^2*x^2)^(3/2))/(3*e*(d +
 e*x)^3) + (2*(d^2 - e^2*x^2)^(5/2))/(5*e*(d + e*x)^5) - (2*(d^2 - e^2*x^2)
^(7/2))/(7*e*(d + e*x)^7) + ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^8}\,dx = -\frac{2\left(d^2-e^2x^2\right)^{7/2}}{7e(d+ex)^7} - \int \frac{\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^6}\,dx$$
$$= \frac{2\left(d^2-e^2x^2\right)^{5/2}}{5e(d+ex)^5} - \frac{2\left(d^2-e^2x^2\right)^{7/2}}{7e(d+ex)^7} + \int \frac{\left(d^2-e^2x^2\right)^{3/2}}{(d+ex)^4}\,dx$$
$$= -\frac{2\left(d^2-e^2x^2\right)^{3/2}}{3e(d+ex)^3} + \frac{2\left(d^2-e^2x^2\right)^{5/2}}{5e(d+ex)^5} - \frac{2\left(d^2-e^2x^2\right)^{7/2}}{7e(d+ex)^7} - \int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx$$
$$= \frac{2\sqrt{d^2-e^2x^2}}{e(d+ex)} - \frac{2\left(d^2-e^2x^2\right)^{3/2}}{3e(d+ex)^3} + \frac{2\left(d^2-e^2x^2\right)^{5/2}}{5e(d+ex)^5} - \frac{2\left(d^2-e^2x^2\right)^{7/2}}{7e(d+ex)^7} + \int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$
$$= \frac{2\sqrt{d^2-e^2x^2}}{e(d+ex)} - \frac{2\left(d^2-e^2x^2\right)^{3/2}}{3e(d+ex)^3} + \frac{2\left(d^2-e^2x^2\right)^{5/2}}{5e(d+ex)^5} - \frac{2\left(d^2-e^2x^2\right)^{7/2}}{7e(d+ex)^7} + \text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, -\right.$$
$$= \frac{2\sqrt{d^2-e^2x^2}}{e(d+ex)} - \frac{2\left(d^2-e^2x^2\right)^{3/2}}{3e(d+ex)^3} + \frac{2\left(d^2-e^2x^2\right)^{5/2}}{5e(d+ex)^5} - \frac{2\left(d^2-e^2x^2\right)^{7/2}}{7e(d+ex)^7} + \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

Mathematica [A] time = 0.0950117, size = 85, normalized size = 0.59

$$\frac{8\sqrt{d^2-e^2x^2}\left(76d^2ex+19d^3+71de^2x^2+44e^3x^3\right)}{105e(d+ex)^4} + \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^8,x]
```

```
[Out] (8*Sqrt[d^2 - e^2*x^2]*(19*d^3 + 76*d^2*e*x + 71*d*e^2*x^2 + 44*e^3*x^3))/(
105*e*(d + e*x)^4) + ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e
```

Maple [B] time = 0.051, size = 496, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^8,x)
```

```
[Out] -1/7/e^9/d/(d/e+x)^8*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(9/2)+1/35/e^8/d^2/(d/e
+x)^7*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(9/2)-2/105/e^7/d^3/(d/e+x)^6*(-(d/e+x
)^2*e^2+2*d*e*(d/e+x))^(9/2)+2/35/e^6/d^4/(d/e+x)^5*(-(d/e+x)^2*e^2+2*d*e*(
d/e+x))^(9/2)+8/35/e^5/d^5/(d/e+x)^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(9/2)+8
/21/e^4/d^6/(d/e+x)^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(9/2)+16/35/e^3/d^7/(d
/e+x)^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(9/2)+16/35/e/d^7*(-(d/e+x)^2*e^2+2*
d*e*(d/e+x))^(7/2)+8/15/d^6*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)*x+2/3/d^4*
(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x+1/d^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))
^(1/2)*x+1/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^
(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.5588, size = 432, normalized size = 3.02

$$\frac{2\left(76\,e^{4}x^{4}+304\,de^{3}x^{3}+456\,d^{2}e^{2}x^{2}+304\,d^{3}ex+76\,d^{4}-105\left(e^{4}x^{4}+4\,de^{3}x^{3}+6\,d^{2}e^{2}x^{2}+4\,d^{3}ex+d^{4}\right)\arctan\left(-\frac{d-\sqrt{}}{}\right.\right.}{105\left(e^{5}x^{4}+4\,de^{4}x^{3}+6\,d^{2}e^{3}x^{2}+4\,d^{3}e^{2}x+d^{4}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^8,x, algorithm="fricas")
```

```
[Out] 2/105*(76*e^4*x^4 + 304*d*e^3*x^3 + 456*d^2*e^2*x^2 + 304*d^3*e*x + 76*d^4
- 105*(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4)*arctan(-(d
- sqrt(-e^2*x^2 + d^2))/(e*x)) + 4*(44*e^3*x^3 + 71*d*e^2*x^2 + 76*d^2*e*x
+ 19*d^3)*sqrt(-e^2*x^2 + d^2))/(e^5*x^4 + 4*d*e^4*x^3 + 6*d^2*e^3*x^2 + 4*
d^3*e^2*x + d^4*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(7/2)/(e*x+d)**8,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^8,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.811 $\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^9} dx$

Optimal. Leaf size=33

$$-\frac{(d^2-e^2x^2)^{9/2}}{9de(d+ex)^9}$$

```
[Out] -(d^2 - e^2*x^2)^(9/2)/(9*d*e*(d + e*x)^9)
```

Rubi [A] time = 0.0090743, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {651}

$$-\frac{(d^2-e^2x^2)^{9/2}}{9de(d+ex)^9}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^9,x]
```

```
[Out] -(d^2 - e^2*x^2)^(9/2)/(9*d*e*(d + e*x)^9)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^9} dx = -\frac{(d^2-e^2x^2)^{9/2}}{9de(d+ex)^9}$$

Mathematica [A] time = 0.0626643, size = 41, normalized size = 1.24

$$-\frac{(d-ex)^4\sqrt{d^2-e^2x^2}}{9de(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^9,x]
```

```
[Out] -((d - e*x)^4*Sqrt[d^2 - e^2*x^2])/(9*d*e*(d + e*x)^5)
```

Maple [A] time = 0.042, size = 36, normalized size = 1.1

$$-\frac{-ex+d}{9\,(ex+d)^8\,de}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^9,x)
```

```
[Out] -1/9/(e*x+d)^8*(-e*x+d)/d/e*(-e^2*x^2+d^2)^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.37059, size = 333, normalized size = 10.09

$$-\frac{e^5x^5 + 5\,de^4x^4 + 10\,d^2e^3x^3 + 10\,d^3e^2x^2 + 5\,d^4ex + d^5 + \left(e^4x^4 - 4\,de^3x^3 + 6\,d^2e^2x^2 - 4\,d^3ex + d^4\right)\sqrt{-e^2x^2 + d^2}}{9\left(de^6x^5 + 5\,d^2e^5x^4 + 10\,d^3e^4x^3 + 10\,d^4e^3x^2 + 5\,d^5e^2x + d^6e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^9,x, algorithm="fricas")
```

```
[Out] -1/9*(e^5*x^5 + 5*d*e^4*x^4 + 10*d^2*e^3*x^3 + 10*d^3*e^2*x^2 + 5*d^4*e*x +
 d^5 + (e^4*x^4 - 4*d*e^3*x^3 + 6*d^2*e^2*x^2 - 4*d^3*e*x + d^4)*sqrt(-e^2*
x^2 + d^2))/(d*e^6*x^5 + 5*d^2*e^5*x^4 + 10*d^3*e^4*x^3 + 10*d^4*e^3*x^2 +
5*d^5*e^2*x + d^6*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(7/2)/(e*x+d)**9,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^9,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.812 $\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^{10}}\,dx$

Optimal. Leaf size=67

$$-\frac{(d^2-e^2x^2)^{9/2}}{99d^2e(d+ex)^9}-\frac{(d^2-e^2x^2)^{9/2}}{11de(d+ex)^{10}}$$

```
[Out] -(d^2 - e^2*x^2)^(9/2)/(11*d*e*(d + e*x)^10) - (d^2 - e^2*x^2)^(9/2)/(99*d^
2*e*(d + e*x)^9)
```

Rubi [A] time = 0.021408, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 651}

$$-\frac{(d^2-e^2x^2)^{9/2}}{99d^2e(d+ex)^9}-\frac{(d^2-e^2x^2)^{9/2}}{11de(d+ex)^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^10,x]
```

```
[Out] -(d^2 - e^2*x^2)^(9/2)/(11*d*e*(d + e*x)^10) - (d^2 - e^2*x^2)^(9/2)/(99*d^
2*e*(d + e*x)^9)
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^{10}}\,dx &= -\frac{(d^2-e^2x^2)^{9/2}}{11de(d+ex)^{10}}+\frac{\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^9}\,dx}{11d}\\ &= -\frac{(d^2-e^2x^2)^{9/2}}{11de(d+ex)^{10}}-\frac{(d^2-e^2x^2)^{9/2}}{99d^2e(d+ex)^9}\end{aligned}$$

Mathematica [A] time = 0.0671445, size = 48, normalized size = 0.72

$$-\frac{(d-ex)^4(10d+ex)\sqrt{d^2-e^2x^2}}{99d^2e(d+ex)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^10,x]
```

```
[Out] -((d - e*x)^4*(10*d + e*x)*Sqrt[d^2 - e^2*x^2])/(99*d^2*e*(d + e*x)^6)
```

Maple [A] time = 0.043, size = 43, normalized size = 0.6

$$-\frac{(ex+10\,d)\,(-ex+d)}{99\,(ex+d)^{9}\,d^{2}e}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^10,x)
```

```
[Out] -1/99*(-e*x+d)*(e*x+10*d)*(-e^2*x^2+d^2)^(7/2)/(e*x+d)^9/d^2/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.80534, size = 428, normalized size = 6.39

$$-\frac{10\,e^{6}x^{6}+60\,de^{5}x^{5}+150\,d^{2}e^{4}x^{4}+200\,d^{3}e^{3}x^{3}+150\,d^{4}e^{2}x^{2}+60\,d^{5}ex+10\,d^{6}+\left(e^{5}x^{5}+6\,de^{4}x^{4}-34\,d^{2}e^{3}x^{3}+56\,d^{3}e^{2}\right.}{99\left(d^{2}e^{7}x^{6}+6\,d^{3}e^{6}x^{5}+15\,d^{4}e^{5}x^{4}+20\,d^{5}e^{4}x^{3}+15\,d^{6}e^{3}x^{2}+6\,d^{7}e^{2}x+d^{8}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^10,x, algorithm="fricas")
```

```
[Out] -1/99*(10*e^6*x^6 + 60*d*e^5*x^5 + 150*d^2*e^4*x^4 + 200*d^3*e^3*x^3 + 150*
d^4*e^2*x^2 + 60*d^5*e*x + 10*d^6 + (e^5*x^5 + 6*d*e^4*x^4 - 34*d^2*e^3*x^3
 + 56*d^3*e^2*x^2 - 39*d^4*e*x + 10*d^5)*sqrt(-e^2*x^2 + d^2))/(d^2*e^7*x^6
 + 6*d^3*e^6*x^5 + 15*d^4*e^5*x^4 + 20*d^5*e^4*x^3 + 15*d^6*e^3*x^2 + 6*d^7
*e^2*x + d^8*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(7/2)/(e*x+d)**10,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^10,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.813 $\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^{11}}\,dx$

Optimal. Leaf size=100

$$-\frac{2\left(d^2-e^2x^2\right)^{9/2}}{1287d^3e(d+ex)^9}-\frac{2\left(d^2-e^2x^2\right)^{9/2}}{143d^2e(d+ex)^{10}}-\frac{\left(d^2-e^2x^2\right)^{9/2}}{13de(d+ex)^{11}}$$

```
[Out] -(d^2 - e^2*x^2)^(9/2)/(13*d*e*(d + e*x)^11) - (2*(d^2 - e^2*x^2)^(9/2))/(1
43*d^2*e*(d + e*x)^10) - (2*(d^2 - e^2*x^2)^(9/2))/(1287*d^3*e*(d + e*x)^9)
```

Rubi [A] time = 0.0359951, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 651}

$$-\frac{2\left(d^2-e^2x^2\right)^{9/2}}{1287d^3e(d+ex)^9}-\frac{2\left(d^2-e^2x^2\right)^{9/2}}{143d^2e(d+ex)^{10}}-\frac{\left(d^2-e^2x^2\right)^{9/2}}{13de(d+ex)^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^11,x]
```

```
[Out] -(d^2 - e^2*x^2)^(9/2)/(13*d*e*(d + e*x)^11) - (2*(d^2 - e^2*x^2)^(9/2))/(1
43*d^2*e*(d + e*x)^10) - (2*(d^2 - e^2*x^2)^(9/2))/(1287*d^3*e*(d + e*x)^9)
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^{11}}\,dx &= -\frac{\left(d^2-e^2x^2\right)^{9/2}}{13de(d+ex)^{11}}+\frac{2\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^{10}}\,dx}{13d}\\
&= -\frac{\left(d^2-e^2x^2\right)^{9/2}}{13de(d+ex)^{11}}-\frac{2\left(d^2-e^2x^2\right)^{9/2}}{143d^2e(d+ex)^{10}}+\frac{2\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^9}\,dx}{143d^2}\\
&= -\frac{\left(d^2-e^2x^2\right)^{9/2}}{13de(d+ex)^{11}}-\frac{2\left(d^2-e^2x^2\right)^{9/2}}{143d^2e(d+ex)^{10}}-\frac{2\left(d^2-e^2x^2\right)^{9/2}}{1287d^3e(d+ex)^9}
\end{aligned}$$

Mathematica [A] time = 0.0715973, size = 60, normalized size = 0.6

$$-\frac{(d-ex)^4\sqrt{d^2-e^2x^2}\left(119d^2+22dex+2e^2x^2\right)}{1287d^3e(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^11,x]
```

```
[Out] -((d - e*x)^4*Sqrt[d^2 - e^2*x^2]*(119*d^2 + 22*d*e*x + 2*e^2*x^2))/(1287*d
^3*e*(d + e*x)^7)
```

Maple [A] time = 0.044, size = 55, normalized size = 0.6

$$-\frac{\left(2\,e^2x^2+22\,dex+119\,d^2\right)(-ex+d)}{1287\,(ex+d)^{10}\,d^3e}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^11,x)
```

```
[Out] -1/1287*(-e*x+d)*(2*e^2*x^2+22*d*e*x+119*d^2)*(-e^2*x^2+d^2)^(7/2)/(e*x+d)^
10/d^3/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.66569, size = 522, normalized size = 5.22

$$-\frac{119\,e^7x^7+833\,de^6x^6+2499\,d^2e^5x^5+4165\,d^3e^4x^4+4165\,d^4e^3x^3+2499\,d^5e^2x^2+833\,d^6ex+119\,d^7+\left(2\,e^6x^6+14\,de^5x\right.}{1287\left(d^3e^8x^7+7\,d^4e^7x^6+21\,d^5e^6x^5+35\,d^6e^5x^4+35\,d^7e^4x^3+21\,d^8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^11,x, algorithm="fricas")
```

```
[Out] -1/1287*(119*e^7*x^7 + 833*d*e^6*x^6 + 2499*d^2*e^5*x^5 + 4165*d^3*e^4*x^4
+ 4165*d^4*e^3*x^3 + 2499*d^5*e^2*x^2 + 833*d^6*e*x + 119*d^7 + (2*e^6*x^6
+ 14*d*e^5*x^5 + 43*d^2*e^4*x^4 - 352*d^3*e^3*x^3 + 628*d^4*e^2*x^2 - 454*d
^5*e*x + 119*d^6)*sqrt(-e^2*x^2 + d^2))/(d^3*e^8*x^7 + 7*d^4*e^7*x^6 + 21*d
^5*e^6*x^5 + 35*d^6*e^5*x^4 + 35*d^7*e^4*x^3 + 21*d^8*e^3*x^2 + 7*d^9*e^2*x
 + d^10*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(7/2)/(e*x+d)**11,x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^11,x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.814 $\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^{12}} dx$

Optimal. Leaf size=133

$$-\frac{2(d^2-e^2x^2)^{9/2}}{6435d^4e(d+ex)^9}-\frac{2(d^2-e^2x^2)^{9/2}}{715d^3e(d+ex)^{10}}-\frac{(d^2-e^2x^2)^{9/2}}{65d^2e(d+ex)^{11}}-\frac{(d^2-e^2x^2)^{9/2}}{15de(d+ex)^{12}}$$

```
[Out] -(d^2 - e^2*x^2)^(9/2)/(15*d*e*(d + e*x)^12) - (d^2 - e^2*x^2)^(9/2)/(65*d^
2*e*(d + e*x)^11) - (2*(d^2 - e^2*x^2)^(9/2))/(715*d^3*e*(d + e*x)^10) - (2
*(d^2 - e^2*x^2)^(9/2))/(6435*d^4*e*(d + e*x)^9)
```

Rubi [A] time = 0.0559151, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 651}

$$-\frac{2(d^2-e^2x^2)^{9/2}}{6435d^4e(d+ex)^9}-\frac{2(d^2-e^2x^2)^{9/2}}{715d^3e(d+ex)^{10}}-\frac{(d^2-e^2x^2)^{9/2}}{65d^2e(d+ex)^{11}}-\frac{(d^2-e^2x^2)^{9/2}}{15de(d+ex)^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^12,x]
```

```
[Out] -(d^2 - e^2*x^2)^(9/2)/(15*d*e*(d + e*x)^12) - (d^2 - e^2*x^2)^(9/2)/(65*d^
2*e*(d + e*x)^11) - (2*(d^2 - e^2*x^2)^(9/2))/(715*d^3*e*(d + e*x)^10) - (2
*(d^2 - e^2*x^2)^(9/2))/(6435*d^4*e*(d + e*x)^9)
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^{12}}\,dx &= -\frac{\left(d^2-e^2x^2\right)^{9/2}}{15de(d+ex)^{12}}+\frac{\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^{11}}\,dx}{5d} \\
&= -\frac{\left(d^2-e^2x^2\right)^{9/2}}{15de(d+ex)^{12}}-\frac{\left(d^2-e^2x^2\right)^{9/2}}{65d^2e(d+ex)^{11}}+\frac{2\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^{10}}\,dx}{65d^2} \\
&= -\frac{\left(d^2-e^2x^2\right)^{9/2}}{15de(d+ex)^{12}}-\frac{\left(d^2-e^2x^2\right)^{9/2}}{65d^2e(d+ex)^{11}}-\frac{2\left(d^2-e^2x^2\right)^{9/2}}{715d^3e(d+ex)^{10}}+\frac{2\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^{9}}\,dx}{715d^3} \\
&= -\frac{\left(d^2-e^2x^2\right)^{9/2}}{15de(d+ex)^{12}}-\frac{\left(d^2-e^2x^2\right)^{9/2}}{65d^2e(d+ex)^{11}}-\frac{2\left(d^2-e^2x^2\right)^{9/2}}{715d^3e(d+ex)^{10}}-\frac{2\left(d^2-e^2x^2\right)^{9/2}}{6435d^4e(d+ex)^{9}}
\end{aligned}
$$

Mathematica [A] time = 0.0742261, size = 71, normalized size = 0.53

$$
-\frac{(d-ex)^4\sqrt{d^2-e^2x^2}\left(141d^2ex+548d^3+24de^2x^2+2e^3x^3\right)}{6435d^4e(d+ex)^8}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^12,x]
```

```
[Out] -((d - e*x)^4*Sqrt[d^2 - e^2*x^2]*(548*d^3 + 141*d^2*e*x + 24*d*e^2*x^2 + 2
*e^3*x^3))/(6435*d^4*e*(d + e*x)^8)
```

Maple [A] time = 0.045, size = 66, normalized size = 0.5

$$
-\frac{\left(2\,e^3x^3+24\,e^2x^2d+141\,xd^2e+548\,d^3\right)(-ex+d)}{6435\,(ex+d)^{11}\,d^4e}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^12,x)
```

```
[Out] -1/6435*(-e*x+d)*(2*e^3*x^3+24*d*e^2*x^2+141*d^2*e*x+548*d^3)*(-e^2*x^2+d^2
)^(7/2)/(e*x+d)^11/d^4/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^12,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.07142, size = 612, normalized size = 4.6

$$-\frac{548\,e^8x^8+4384\,de^7x^7+15344\,d^2e^6x^6+30688\,d^3e^5x^5+38360\,d^4e^4x^4+30688\,d^5e^3x^3+15344\,d^6e^2x^2+4384\,d^7ex+54}{6435\,\left(d^4e^9x^8+8\,d^5e^8x^7+28\,d^6e^7x^6+56\,d^7e^6x^5+70\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^12,x, algorithm="fricas")
```

```
[Out] -1/6435*(548*e^8*x^8 + 4384*d*e^7*x^7 + 15344*d^2*e^6*x^6 + 30688*d^3*e^5*x
^5 + 38360*d^4*e^4*x^4 + 30688*d^5*e^3*x^3 + 15344*d^6*e^2*x^2 + 4384*d^7*e
*x + 548*d^8 + (2*e^7*x^7 + 16*d*e^6*x^6 + 57*d^2*e^5*x^5 + 120*d^3*e^4*x^4
 - 1440*d^4*e^3*x^3 + 2748*d^5*e^2*x^2 - 2051*d^6*e*x + 548*d^7)*sqrt(-e^2*
x^2 + d^2))/(d^4*e^9*x^8 + 8*d^5*e^8*x^7 + 28*d^6*e^7*x^6 + 56*d^7*e^6*x^5
+ 70*d^8*e^5*x^4 + 56*d^9*e^4*x^3 + 28*d^10*e^3*x^2 + 8*d^11*e^2*x + d^12*e
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(7/2)/(e*x+d)**12,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^12,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.815 $\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^{13}}\,dx$

Optimal. Leaf size=166

$$-\frac{8\left(d^2-e^2x^2\right)^{9/2}}{109395d^5e(d+ex)^9}-\frac{8\left(d^2-e^2x^2\right)^{9/2}}{12155d^4e(d+ex)^{10}}-\frac{4\left(d^2-e^2x^2\right)^{9/2}}{1105d^3e(d+ex)^{11}}-\frac{4\left(d^2-e^2x^2\right)^{9/2}}{255d^2e(d+ex)^{12}}-\frac{\left(d^2-e^2x^2\right)^{9/2}}{17de(d+ex)^{13}}$$

```
[Out] -(d^2 - e^2*x^2)^(9/2)/(17*d*e*(d + e*x)^13) - (4*(d^2 - e^2*x^2)^(9/2))/(2
55*d^2*e*(d + e*x)^12) - (4*(d^2 - e^2*x^2)^(9/2))/(1105*d^3*e*(d + e*x)^11
) - (8*(d^2 - e^2*x^2)^(9/2))/(12155*d^4*e*(d + e*x)^10) - (8*(d^2 - e^2*x^
2)^(9/2))/(109395*d^5*e*(d + e*x)^9)
```

Rubi [A] time = 0.0758125, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 651}

$$-\frac{8\left(d^2-e^2x^2\right)^{9/2}}{109395d^5e(d+ex)^9}-\frac{8\left(d^2-e^2x^2\right)^{9/2}}{12155d^4e(d+ex)^{10}}-\frac{4\left(d^2-e^2x^2\right)^{9/2}}{1105d^3e(d+ex)^{11}}-\frac{4\left(d^2-e^2x^2\right)^{9/2}}{255d^2e(d+ex)^{12}}-\frac{\left(d^2-e^2x^2\right)^{9/2}}{17de(d+ex)^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^13,x]
```

```
[Out] -(d^2 - e^2*x^2)^(9/2)/(17*d*e*(d + e*x)^13) - (4*(d^2 - e^2*x^2)^(9/2))/(2
55*d^2*e*(d + e*x)^12) - (4*(d^2 - e^2*x^2)^(9/2))/(1105*d^3*e*(d + e*x)^11
) - (8*(d^2 - e^2*x^2)^(9/2))/(12155*d^4*e*(d + e*x)^10) - (8*(d^2 - e^2*x^
2)^(9/2))/(109395*d^5*e*(d + e*x)^9)
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^{13}} dx = -\frac{(d^2-e^2x^2)^{9/2}}{17de(d+ex)^{13}} + \frac{4\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^{12}} dx}{17d}$$

$$= -\frac{(d^2-e^2x^2)^{9/2}}{17de(d+ex)^{13}} - \frac{4(d^2-e^2x^2)^{9/2}}{255d^2e(d+ex)^{12}} + \frac{4\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^{11}} dx}{85d^2}$$

$$= -\frac{(d^2-e^2x^2)^{9/2}}{17de(d+ex)^{13}} - \frac{4(d^2-e^2x^2)^{9/2}}{255d^2e(d+ex)^{12}} - \frac{4(d^2-e^2x^2)^{9/2}}{1105d^3e(d+ex)^{11}} + \frac{8\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^{10}} dx}{1105d^3}$$

$$= -\frac{(d^2-e^2x^2)^{9/2}}{17de(d+ex)^{13}} - \frac{4(d^2-e^2x^2)^{9/2}}{255d^2e(d+ex)^{12}} - \frac{4(d^2-e^2x^2)^{9/2}}{1105d^3e(d+ex)^{11}} - \frac{8(d^2-e^2x^2)^{9/2}}{12155d^4e(d+ex)^{10}} + \frac{8\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^{9}} d}{12155d^4}$$

$$= -\frac{(d^2-e^2x^2)^{9/2}}{17de(d+ex)^{13}} - \frac{4(d^2-e^2x^2)^{9/2}}{255d^2e(d+ex)^{12}} - \frac{4(d^2-e^2x^2)^{9/2}}{1105d^3e(d+ex)^{11}} - \frac{8(d^2-e^2x^2)^{9/2}}{12155d^4e(d+ex)^{10}} - \frac{8(d^2-e^2x^2)^{9}}{109395d^5e(d+}$$

Mathematica [A] time = 0.0816001, size = 82, normalized size = 0.49

$$-\frac{(d-ex)^4\sqrt{d^2-e^2x^2}\left(660d^2e^2x^2+2756d^3ex+8627d^4+104de^3x^3+8e^4x^4\right)}{109395d^5e(d+ex)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^13,x]
```

```
[Out] -((d - e*x)^4*Sqrt[d^2 - e^2*x^2]*(8627*d^4 + 2756*d^3*e*x + 660*d^2*e^2*x^
2 + 104*d*e^3*x^3 + 8*e^4*x^4))/(109395*d^5*e*(d + e*x)^9)
```

Maple [A] time = 0.045, size = 77, normalized size = 0.5

$$-\frac{\left(8e^4x^4+104e^3x^3d+660e^2x^2d^2+2756xd^3e+8627d^4\right)(-ex+d)}{109395(ex+d)^{12}d^5e}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^13,x)
```

```
[Out] -1/109395*(-e*x+d)*(8*e^4*x^4+104*d*e^3*x^3+660*d^2*e^2*x^2+2756*d^3*e*x+86
27*d^4)*(-e^2*x^2+d^2)^(7/2)/(e*x+d)^12/d^5/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 7.09905, size = 720, normalized size = 4.34

$$-\frac{8627\,e^9x^9+77643\,de^8x^8+310572\,d^2e^7x^7+724668\,d^3e^6x^6+1087002\,d^4e^5x^5+1087002\,d^5e^4x^4+724668\,d^6e^3x^3+3}{109395\left(d^5e^{10}x^9+9\,d^6e^9x^8+36\,d^7\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^13,x, algorithm="fricas")
```

```
[Out] -1/109395*(8627*e^9*x^9 + 77643*d*e^8*x^8 + 310572*d^2*e^7*x^7 + 724668*d^3
*e^6*x^6 + 1087002*d^4*e^5*x^5 + 1087002*d^5*e^4*x^4 + 724668*d^6*e^3*x^3 +
 310572*d^7*e^2*x^2 + 77643*d^8*e*x + 8627*d^9 + (8*e^8*x^8 + 72*d*e^7*x^7
+ 292*d^2*e^6*x^6 + 708*d^3*e^5*x^5 + 1155*d^4*e^4*x^4 - 20508*d^5*e^3*x^3
+ 41398*d^6*e^2*x^2 - 31752*d^7*e*x + 8627*d^8)*sqrt(-e^2*x^2 + d^2))/(d^5*
e^10*x^9 + 9*d^6*e^9*x^8 + 36*d^7*e^8*x^7 + 84*d^8*e^7*x^6 + 126*d^9*e^6*x^
5 + 126*d^10*e^5*x^4 + 84*d^11*e^4*x^3 + 36*d^12*e^3*x^2 + 9*d^13*e^2*x + d
^14*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(7/2)/(e*x+d)**13,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^13,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.816 $\int \frac{\sqrt{a^2-b^2x^2}}{a-bx}\,dx$

Optimal. Leaf size=47

$$\frac{a\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{b}-\frac{\sqrt{a^2-b^2x^2}}{b}$$

```
[Out] -(Sqrt[a^2 - b^2*x^2]/b) + (a*ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]])/b
```

Rubi [A] time = 0.015241, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {665, 217, 203}

$$\frac{a\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{b}-\frac{\sqrt{a^2-b^2x^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 - b^2*x^2]/(a - b*x),x]
```

```
[Out] -(Sqrt[a^2 - b^2*x^2]/b) + (a*ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]])/b
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2-b^2x^2}}{a-bx}\,dx &= -\frac{\sqrt{a^2-b^2x^2}}{b}+a\int \frac{1}{\sqrt{a^2-b^2x^2}}\,dx\\
&= -\frac{\sqrt{a^2-b^2x^2}}{b}+a\,\text{Subst}\left(\int \frac{1}{1+b^2x^2}\,dx,x,\frac{x}{\sqrt{a^2-b^2x^2}}\right)\\
&= -\frac{\sqrt{a^2-b^2x^2}}{b}+\frac{a\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0452098, size = 47, normalized size = 1.

$$\frac{a\tan^{-1}\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{b}-\frac{\sqrt{a^2-b^2x^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 - b^2*x^2]/(a - b*x),x]
```

```
[Out] -(Sqrt[a^2 - b^2*x^2]/b) + (a*ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]])/b
```

Maple [A] time = 0.044, size = 82, normalized size = 1.7

$$-\frac{1}{b}\sqrt{-\left(x-\frac{a}{b}\right)^2b^2-2\left(x-\frac{a}{b}\right)ab}+a\arctan\left(x\sqrt{b^2}\frac{1}{\sqrt{-\left(x-\frac{a}{b}\right)^2b^2-2\left(x-\frac{a}{b}\right)ab}}\right)\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^2*x^2+a^2)^(1/2)/(-b*x+a),x)
```

```
[Out] -1/b*(-(x-1/b*a)^2*b^2-2*(x-1/b*a)*a*b)^(1/2)+a/(b^2)^(1/2)*arctan((b^2)^(1
/2)*x/(-(x-1/b*a)^2*b^2-2*(x-1/b*a)*a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(1/2)/(-b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.05957, size = 101, normalized size = 2.15

$$-\frac{2\,a\arctan\left(-\frac{a-\sqrt{-b^2x^2+a^2}}{bx}\right)+\sqrt{-b^2x^2+a^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(1/2)/(-b*x+a),x, algorithm="fricas")
```

```
[Out] -(2*a*arctan(-(a - sqrt(-b^2*x^2 + a^2))/(b*x)) + sqrt(-b^2*x^2 + a^2))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{a^2 - b^2x^2}}{-a + bx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**2*x**2+a**2)**(1/2)/(-b*x+a),x)
```

```
[Out] -Integral(sqrt(a**2 - b**2*x**2)/(-a + b*x), x)
```

Giac [A] time = 1.41494, size = 50, normalized size = 1.06

$$\frac{a \arcsin\left(\frac{bx}{a}\right) \operatorname{sgn}(a) \operatorname{sgn}(b)}{|b|} - \frac{\sqrt{-b^2x^2 + a^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^(1/2)/(-b*x+a),x, algorithm="giac")
```

```
[Out] a*arcsin(b*x/a)*sgn(a)*sgn(b)/abs(b) - sqrt(-b^2*x^2 + a^2)/b
```

3.817 $\int (a+bx)^2\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx$

Optimal. Leaf size=130

$$\frac{5}{8}a^2x\sqrt{cx^2-\frac{a^2c}{b^2}}+\frac{5ab\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{12c}+\frac{b(a+bx)\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{4c}-\frac{5a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{cx^2-\frac{a^2c}{b^2}}}\right)}{8b^2}$$

```
[Out] (5*a^2*x*Sqrt[-((a^2*c)/b^2) + c*x^2])/8 + (5*a*b*(-((a^2*c)/b^2) + c*x^2)^
(3/2))/(12*c) + (b*(a + b*x)*(-((a^2*c)/b^2) + c*x^2)^(3/2))/(4*c) - (5*a^4
*Sqrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[-((a^2*c)/b^2) + c*x^2]])/(8*b^2)
```

Rubi [A] time = 0.0531535, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {671, 641, 195, 217, 206}

$$\frac{5}{8}a^2x\sqrt{cx^2-\frac{a^2c}{b^2}}+\frac{5ab\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{12c}+\frac{b(a+bx)\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{4c}-\frac{5a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{cx^2-\frac{a^2c}{b^2}}}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*Sqrt[-((a^2*c)/b^2) + c*x^2],x]
```

```
[Out] (5*a^2*x*Sqrt[-((a^2*c)/b^2) + c*x^2])/8 + (5*a*b*(-((a^2*c)/b^2) + c*x^2)^
(3/2))/(12*c) + (b*(a + b*x)*(-((a^2*c)/b^2) + c*x^2)^(3/2))/(4*c) - (5*a^4
*Sqrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[-((a^2*c)/b^2) + c*x^2]])/(8*b^2)
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+bx)^2\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx = \frac{b(a+bx)\left(-\frac{a^2c}{b^2}+cx^2\right)^{3/2}}{4c}+\frac{1}{4}(5a)\int (a+bx)\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx$$

$$= \frac{5ab\left(-\frac{a^2c}{b^2}+cx^2\right)^{3/2}}{12c}+\frac{b(a+bx)\left(-\frac{a^2c}{b^2}+cx^2\right)^{3/2}}{4c}+\frac{1}{4}\left(5a^2\right)\int \sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx$$

$$= \frac{5}{8}a^2x\sqrt{-\frac{a^2c}{b^2}+cx^2}+\frac{5ab\left(-\frac{a^2c}{b^2}+cx^2\right)^{3/2}}{12c}+\frac{b(a+bx)\left(-\frac{a^2c}{b^2}+cx^2\right)^{3/2}}{4c}-\frac{\left(5a^4c\right)\int \frac{1}{\sqrt{-\frac{a^2c}{b^2}+cx^2}}}{8b^2}$$

$$= \frac{5}{8}a^2x\sqrt{-\frac{a^2c}{b^2}+cx^2}+\frac{5ab\left(-\frac{a^2c}{b^2}+cx^2\right)^{3/2}}{12c}+\frac{b(a+bx)\left(-\frac{a^2c}{b^2}+cx^2\right)^{3/2}}{4c}-\frac{\left(5a^4c\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{5}{8}a^2x\sqrt{-\frac{a^2c}{b^2}+cx^2}+\frac{5ab\left(-\frac{a^2c}{b^2}+cx^2\right)^{3/2}}{12c}+\frac{b(a+bx)\left(-\frac{a^2c}{b^2}+cx^2\right)^{3/2}}{4c}-\frac{5a^4\sqrt{c}\tanh^{-1}\left(\right.}{8b^2}$$

Mathematica [A] time = 0.150186, size = 103, normalized size = 0.79

$$\frac{\sqrt{c\left(x^2-\frac{a^2}{b^2}\right)}\left(\sqrt{1-\frac{b^2x^2}{a^2}}\left(9a^2bx-16a^3+16ab^2x^2+6b^3x^3\right)+15a^3\sin^{-1}\left(\frac{bx}{a}\right)\right)}{24b\sqrt{1-\frac{b^2x^2}{a^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*Sqrt[-((a^2*c)/b^2) + c*x^2],x]
```

```
[Out] (Sqrt[c*(-(a^2/b^2) + x^2)]*(Sqrt[1 - (b^2*x^2)/a^2]*(-16*a^3 + 9*a^2*b*x +
 16*a*b^2*x^2 + 6*b^3*x^3) + 15*a^3*ArcSin[(b*x)/a]))/(24*b*Sqrt[1 - (b^2*x
^2)/a^2])
```

Maple [A] time = 0.264, size = 113, normalized size = 0.9

$$\frac{b^2x}{4c}\left(-\frac{a^2c}{b^2}+cx^2\right)^{\frac{3}{2}}+\frac{5a^2x}{8}\sqrt{-\frac{a^2c}{b^2}+cx^2}-\frac{5a^4}{8b^2}\sqrt{c}\ln\left(x\sqrt{c}+\sqrt{-\frac{a^2c}{b^2}+cx^2}\right)+\frac{2ab}{3c}\left(\frac{c\left(b^2x^2-a^2\right)}{b^2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(-a^2*c/b^2+c*x^2)^(1/2),x)
```

```
[Out] 1/4*b^2*x*(-a^2*c/b^2+c*x^2)^(3/2)/c+5/8*a^2*x*(-a^2*c/b^2+c*x^2)^(1/2)-5/8
/b^2*a^4*c^(1/2)*ln(x*c^(1/2)+(-a^2*c/b^2+c*x^2)^(1/2))+2/3*a*b/c*(c*(b^2*x
```

```
^2-a^2)/b^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-a^2*c/b^2+c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.2209, size = 506, normalized size = 3.89

$$\left[\frac{15\,a^4\sqrt{c}\log\left(2\,b^2cx^2-2\,b^2\sqrt{c}x\sqrt{\frac{b^2cx^2-a^2c}{b^2}}-a^2c\right)+2\left(6\,b^4x^3+16\,ab^3x^2+9\,a^2b^2x-16\,a^3b\right)\sqrt{\frac{b^2cx^2-a^2c}{b^2}}}{48\,b^2},\ \frac{15\,a^4\sqrt{-c}\,\mathrm{arct}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-a^2*c/b^2+c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*a^4*sqrt(c)*log(2*b^2*c*x^2 - 2*b^2*sqrt(c)*x*sqrt((b^2*c*x^2 - a
^2*c)/b^2) - a^2*c) + 2*(6*b^4*x^3 + 16*a*b^3*x^2 + 9*a^2*b^2*x - 16*a^3*b)
*sqrt((b^2*c*x^2 - a^2*c)/b^2))/b^2, 1/24*(15*a^4*sqrt(-c)*arctan(b^2*sqrt(
-c)*x*sqrt((b^2*c*x^2 - a^2*c)/b^2)/(b^2*c*x^2 - a^2*c)) + (6*b^4*x^3 + 16*
a*b^3*x^2 + 9*a^2*b^2*x - 16*a^3*b)*sqrt((b^2*c*x^2 - a^2*c)/b^2))/b^2]
```

Sympy [C] time = 6.30035, size = 411, normalized size = 3.16

$$a^2\left(\begin{cases}-\frac{a^2\sqrt{c}\,\mathrm{acosh}\left(\frac{bx}{a}\right)}{2b^2}-\frac{a\sqrt{c}x}{2b\sqrt{-1+\frac{b^2x^2}{a^2}}}+\frac{b\sqrt{c}x^3}{2a\sqrt{-1+\frac{b^2x^2}{a^2}}} & \text{for}\ \frac{|b^2x^2|}{|a^2|}>1\\ \frac{ia^2\sqrt{c}\,\mathrm{asin}\left(\frac{bx}{a}\right)}{2b^2}+\frac{ia\sqrt{c}x\sqrt{1-\frac{b^2x^2}{a^2}}}{2b} & \text{otherwise}\end{cases}\right)+2ab\left(\begin{cases}0 & \text{for}\ c=0\\ \frac{\left(-\frac{a^2c}{b^2}+cx^2\right)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)+b^2\left(\begin{cases}-\frac{a^4\sqrt{c}\,\mathrm{acosh}}{8b^4}\\ \frac{ia^4\sqrt{c}\,\mathrm{asin}\left(\frac{b}{a}\right.}{8b^4}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(-a**2*c/b**2+c*x**2)**(1/2),x)
```

```
[Out] a**2*Piecewise((-a**2*sqrt(c)*acosh(b*x/a)/(2*b**2) - a*sqrt(c)*x/(2*b*sqrt
(-1 + b**2*x**2/a**2)) + b*sqrt(c)*x**3/(2*a*sqrt(-1 + b**2*x**2/a**2)), Ab
s(b**2*x**2)/Abs(a**2) > 1), (I*a**2*sqrt(c)*asin(b*x/a)/(2*b**2) + I*a*sqr
t(c)*x*sqrt(1 - b**2*x**2/a**2)/(2*b), True)) + 2*a*b*Piecewise((0, Eq(c, 0
)), ((-a**2*c/b**2 + c*x**2)**(3/2)/(3*c), True)) + b**2*Piecewise((-a**4*s
qrt(c)*acosh(b*x/a)/(8*b**4) + a**3*sqrt(c)*x/(8*b**3*sqrt(-1 + b**2*x**2/a
**2)) - 3*a*sqrt(c)*x**3/(8*b*sqrt(-1 + b**2*x**2/a**2)) + b*sqrt(c)*x**5/(
4*a*sqrt(-1 + b**2*x**2/a**2)), Abs(b**2*x**2)/Abs(a**2) > 1), (I*a**4*sqrt
```

```
(c)*asin(b*x/a)/(8*b**4) - I*a**3*sqrt(c)*x/(8*b**3*sqrt(1 - b**2*x**2/a**2
)) + 3*I*a*sqrt(c)*x**3/(8*b*sqrt(1 - b**2*x**2/a**2)) - I*b*sqrt(c)*x**5/(
4*a*sqrt(1 - b**2*x**2/a**2)), True))
```

Giac [A] time = 1.307, size = 136, normalized size = 1.05

$$\frac{\left(\frac{15\,a^4\sqrt{c}\log\left(\left|-\sqrt{b^2c}x+\sqrt{b^2cx^2-a^2c}\right|\right)}{|b|}-\sqrt{b^2cx^2-a^2c}\left(\frac{16\,a^3}{b}-\left(9\,a^2+2\left(3\,b^2x+8\,ab\right)x\right)x\right)\right)|b|}{24\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-a^2*c/b^2+c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(15*a^4*sqrt(c)*log(abs(-sqrt(b^2*c)*x + sqrt(b^2*c*x^2 - a^2*c)))/abs
(b) - sqrt(b^2*c*x^2 - a^2*c)*(16*a^3/b - (9*a^2 + 2*(3*b^2*x + 8*a*b)*x)*x
))*abs(b)/b^2
```

3.818 $\int (a+bx)^3\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx$

Optimal. Leaf size=167

$$\frac{7}{8}a^3x\sqrt{cx^2-\frac{a^2c}{b^2}}+\frac{7a^2b\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{12c}+\frac{7ab(a+bx)\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{20c}+\frac{b(a+bx)^2\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{5c}-\frac{7a^5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{cx^2-\frac{a^2c}{b^2}}}\right)}{8b^2}$$

```
[Out] (7*a^3*x*Sqrt[-((a^2*c)/b^2) + c*x^2])/8 + (7*a^2*b*(-((a^2*c)/b^2) + c*x^2
)^(3/2))/(12*c) + (7*a*b*(a + b*x)*(-((a^2*c)/b^2) + c*x^2)^(3/2))/(20*c) +
 (b*(a + b*x)^2*(-((a^2*c)/b^2) + c*x^2)^(3/2))/(5*c) - (7*a^5*Sqrt[c]*ArcT
anh[(Sqrt[c]*x)/Sqrt[-((a^2*c)/b^2) + c*x^2]])/(8*b^2)
```

Rubi [A] time = 0.0783444, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {671, 641, 195, 217, 206}

$$\frac{7}{8}a^3x\sqrt{cx^2-\frac{a^2c}{b^2}}+\frac{7a^2b\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{12c}+\frac{7ab(a+bx)\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{20c}+\frac{b(a+bx)^2\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{5c}-\frac{7a^5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{cx^2-\frac{a^2c}{b^2}}}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*Sqrt[-((a^2*c)/b^2) + c*x^2],x]
```

```
[Out] (7*a^3*x*Sqrt[-((a^2*c)/b^2) + c*x^2])/8 + (7*a^2*b*(-((a^2*c)/b^2) + c*x^2
)^(3/2))/(12*c) + (7*a*b*(a + b*x)*(-((a^2*c)/b^2) + c*x^2)^(3/2))/(20*c) +
 (b*(a + b*x)^2*(-((a^2*c)/b^2) + c*x^2)^(3/2))/(5*c) - (7*a^5*Sqrt[c]*ArcT
anh[(Sqrt[c]*x)/Sqrt[-((a^2*c)/b^2) + c*x^2]])/(8*b^2)
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+bx)^3\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx = \frac{b(a+bx)^2\left(-\frac{a^2c}{b^2}+cx^2\right)^{3/2}}{5c}+\frac{1}{5}(7a)\int (a+bx)^2\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx$$

$$= \frac{7ab(a+bx)\left(-\frac{a^2c}{b^2}+cx^2\right)^{3/2}}{20c}+\frac{b(a+bx)^2\left(-\frac{a^2c}{b^2}+cx^2\right)^{3/2}}{5c}+\frac{1}{4}\left(7a^2\right)\int (a+bx)\sqrt{-\frac{a^2c}{b^2}+c$$

$$= \frac{7a^2b\left(-\frac{a^2c}{b^2}+cx^2\right)^{3/2}}{12c}+\frac{7ab(a+bx)\left(-\frac{a^2c}{b^2}+cx^2\right)^{3/2}}{20c}+\frac{b(a+bx)^2\left(-\frac{a^2c}{b^2}+cx^2\right)^{3/2}}{5c}+\frac{1}{4}\left(7a^3$$

$$= \frac{7}{8}a^3x\sqrt{-\frac{a^2c}{b^2}+cx^2}+\frac{7a^2b\left(-\frac{a^2c}{b^2}+cx^2\right)^{3/2}}{12c}+\frac{7ab(a+bx)\left(-\frac{a^2c}{b^2}+cx^2\right)^{3/2}}{20c}+\frac{b(a+bx)^2\left(-\right.}{5c}$$

$$= \frac{7}{8}a^3x\sqrt{-\frac{a^2c}{b^2}+cx^2}+\frac{7a^2b\left(-\frac{a^2c}{b^2}+cx^2\right)^{3/2}}{12c}+\frac{7ab(a+bx)\left(-\frac{a^2c}{b^2}+cx^2\right)^{3/2}}{20c}+\frac{b(a+bx)^2\left(-\right.}{5c}$$

$$= \frac{7}{8}a^3x\sqrt{-\frac{a^2c}{b^2}+cx^2}+\frac{7a^2b\left(-\frac{a^2c}{b^2}+cx^2\right)^{3/2}}{12c}+\frac{7ab(a+bx)\left(-\frac{a^2c}{b^2}+cx^2\right)^{3/2}}{20c}+\frac{b(a+bx)^2\left(-\right.}{5c}$$

Mathematica [A] time = 0.177469, size = 114, normalized size = 0.68

$$\frac{\sqrt{c\left(x^2-\frac{a^2}{b^2}\right)}\left(\sqrt{1-\frac{b^2x^2}{a^2}}\left(112a^2b^2x^2+15a^3bx-136a^4+90ab^3x^3+24b^4x^4\right)+105a^4\sin^{-1}\left(\frac{bx}{a}\right)\right)}{120b\sqrt{1-\frac{b^2x^2}{a^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*Sqrt[-((a^2*c)/b^2) + c*x^2],x]
```

```
[Out] (Sqrt[c*(-(a^2/b^2) + x^2)]*(Sqrt[1 - (b^2*x^2)/a^2]*(-136*a^4 + 15*a^3*b*x
 + 112*a^2*b^2*x^2 + 90*a*b^3*x^3 + 24*b^4*x^4) + 105*a^4*ArcSin[(b*x)/a]))
/(120*b*Sqrt[1 - (b^2*x^2)/a^2])
```

Maple [A] time = 0.154, size = 169, normalized size = 1.

$$\frac{b^3x^2}{5\,c}\left(-\frac{a^2c}{b^2}+cx^2\right)^{\frac{3}{2}}+\frac{2\,ba^2}{15\,c}\left(-\frac{a^2c}{b^2}+cx^2\right)^{\frac{3}{2}}+\frac{3\,b^2ax}{4\,c}\left(-\frac{a^2c}{b^2}+cx^2\right)^{\frac{3}{2}}+\frac{7\,xa^3}{8}\sqrt{-\frac{a^2c}{b^2}+cx^2}-\frac{7\,a^5}{8\,b^2}\sqrt{c}\ln\left(x\sqrt{c}+\sqrt{-\frac{a^2c}{b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(-a^2*c/b^2+c*x^2)^(1/2),x)
```

```
[Out] 1/5*b^3*x^2*(-a^2*c/b^2+c*x^2)^(3/2)/c+2/15*a^2*b*(-a^2*c/b^2+c*x^2)^(3/2)/
c+3/4*b^2*a*x*(-a^2*c/b^2+c*x^2)^(3/2)/c+7/8*a^3*x*(-a^2*c/b^2+c*x^2)^(1/2)
-7/8/b^2*a^5*c^(1/2)*ln(x*c^(1/2)+(-a^2*c/b^2+c*x^2)^(1/2))+b*a^2/c*(c*(b^2
*x^2-a^2)/b^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(-a^2*c/b^2+c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.31945, size = 568, normalized size = 3.4

$$\left[\frac{105\,a^5\sqrt{c}\log\left(2\,b^2cx^2-2\,b^2\sqrt{c}x\sqrt{\frac{b^2cx^2-a^2c}{b^2}}-a^2c\right)+2\left(24\,b^5x^4+90\,ab^4x^3+112\,a^2b^3x^2+15\,a^3b^2x-136\,a^4b\right)\sqrt{\frac{b^2cx^2-}{b^2}}}{240\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(-a^2*c/b^2+c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/240*(105*a^5*sqrt(c)*log(2*b^2*c*x^2 - 2*b^2*sqrt(c)*x*sqrt((b^2*c*x^2 -
 a^2*c)/b^2) - a^2*c) + 2*(24*b^5*x^4 + 90*a*b^4*x^3 + 112*a^2*b^3*x^2 + 15
*a^3*b^2*x - 136*a^4*b)*sqrt((b^2*c*x^2 - a^2*c)/b^2))/b^2, 1/120*(105*a^5*
sqrt(-c)*arctan(b^2*sqrt(-c)*x*sqrt((b^2*c*x^2 - a^2*c)/b^2)/(b^2*c*x^2 - a
^2*c)) + (24*b^5*x^4 + 90*a*b^4*x^3 + 112*a^2*b^3*x^2 + 15*a^3*b^2*x - 136*
a^4*b)*sqrt((b^2*c*x^2 - a^2*c)/b^2))/b^2]
```

Sympy [C] time = 6.71576, size = 495, normalized size = 2.96

$$-\frac{2a^4\sqrt{-\frac{a^2c}{b^2}+cx^2}}{15b}+a^3\left(\begin{cases}-\frac{a^2\sqrt{c}\operatorname{acosh}\left(\frac{bx}{a}\right)}{2b^2}-\frac{a\sqrt{c}x}{2b\sqrt{-1+\frac{b^2x^2}{a^2}}}+\frac{b\sqrt{c}x^3}{2a\sqrt{-1+\frac{b^2x^2}{a^2}}} & \text{for } \frac{|b^2x^2|}{|a^2|}>1\\ \frac{ia^2\sqrt{c}\operatorname{asin}\left(\frac{bx}{a}\right)}{2b^2}+\frac{ia\sqrt{c}x\sqrt{1-\frac{b^2x^2}{a^2}}}{2b} & \text{otherwise}\end{cases}\right)-\frac{a^2bx^2\sqrt{-\frac{a^2c}{b^2}+cx^2}}{15}+3a^2b\left(\left\{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(-a**2*c/b**2+c*x**2)**(1/2),x)
```

```
[Out] -2*a**4*sqrt(-a**2*c/b**2 + c*x**2)/(15*b) + a**3*Piecewise((-a**2*sqrt(c)*
acosh(b*x/a)/(2*b**2) - a*sqrt(c)*x/(2*b*sqrt(-1 + b**2*x**2/a**2)) + b*sqr
```

```
t(c)*x**3/(2*a*sqrt(-1 + b**2*x**2/a**2)), Abs(b**2*x**2)/Abs(a**2) > 1), (
I*a**2*sqrt(c)*asin(b*x/a)/(2*b**2) + I*a*sqrt(c)*x*sqrt(1 - b**2*x**2/a**2
)/(2*b), True)) - a**2*b*x**2*sqrt(-a**2*c/b**2 + c*x**2)/15 + 3*a**2*b*Pie
cewise((0, Eq(c, 0)), ((-a**2*c/b**2 + c*x**2)**(3/2)/(3*c), True)) + 3*a*b
**2*Piecewise((-a**4*sqrt(c)*acosh(b*x/a)/(8*b**4) + a**3*sqrt(c)*x/(8*b**3
*sqrt(-1 + b**2*x**2/a**2)) - 3*a*sqrt(c)*x**3/(8*b*sqrt(-1 + b**2*x**2/a**
2)) + b*sqrt(c)*x**5/(4*a*sqrt(-1 + b**2*x**2/a**2)), Abs(b**2*x**2)/Abs(a*
*2) > 1), (I*a**4*sqrt(c)*asin(b*x/a)/(8*b**4) - I*a**3*sqrt(c)*x/(8*b**3*s
qrt(1 - b**2*x**2/a**2)) + 3*I*a*sqrt(c)*x**3/(8*b*sqrt(1 - b**2*x**2/a**2)
) - I*b*sqrt(c)*x**5/(4*a*sqrt(1 - b**2*x**2/a**2)), True)) + b**3*x**4*sqr
t(-a**2*c/b**2 + c*x**2)/5
```

Giac [A] time = 1.31103, size = 153, normalized size = 0.92

$$\frac{\left(\frac{105\,a^5\sqrt{c}\log\left(\left|-\sqrt{b^2c}x+\sqrt{b^2cx^2-a^2c}\right|\right)}{|b|}-\sqrt{b^2cx^2-a^2c}\left(\frac{136\,a^4}{b}-\left(15\,a^3+2\left(56\,a^2b+3\left(4\,b^3x+15\,ab^2\right)x\right)x\right)x\right)\right)|b|}{120\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(-a^2*c/b^2+c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/120*(105*a^5*sqrt(c)*log(abs(-sqrt(b^2*c)*x + sqrt(b^2*c*x^2 - a^2*c)))/a
bs(b) - sqrt(b^2*c*x^2 - a^2*c)*(136*a^4/b - (15*a^3 + 2*(56*a^2*b + 3*(4*b
^3*x + 15*a*b^2)*x)*x)*x))*abs(b)/b^2
```

3.819 $\int (1+x)\sqrt{-1+x^2}\,dx$

Optimal. Leaf size=44

$$\frac{1}{3}\left(x^2-1\right)^{3/2}+\frac{1}{2}x\sqrt{x^2-1}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

```
[Out] (x*Sqrt[-1 + x^2])/2 + (-1 + x^2)^(3/2)/3 - ArcTanh[x/Sqrt[-1 + x^2]]/2
```

Rubi [A] time = 0.0069667, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {641, 195, 217, 206}

$$\frac{1}{3}\left(x^2-1\right)^{3/2}+\frac{1}{2}x\sqrt{x^2-1}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)*Sqrt[-1 + x^2],x]
```

```
[Out] (x*Sqrt[-1 + x^2])/2 + (-1 + x^2)^(3/2)/3 - ArcTanh[x/Sqrt[-1 + x^2]]/2
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int(1+x)\sqrt{-1+x^2}\,dx &= \frac{1}{3}\left(-1+x^2\right)^{3/2}+\int\sqrt{-1+x^2}\,dx\\ &= \frac{1}{2}x\sqrt{-1+x^2}+\frac{1}{3}\left(-1+x^2\right)^{3/2}-\frac{1}{2}\int\frac{1}{\sqrt{-1+x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{-1+x^2}+\frac{1}{3}\left(-1+x^2\right)^{3/2}-\frac{1}{2}\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{-1+x^2}}\right)\\ &= \frac{1}{2}x\sqrt{-1+x^2}+\frac{1}{3}\left(-1+x^2\right)^{3/2}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0313316, size = 49, normalized size = 1.11

$$\frac{\left(x^2-1\right)\left(\sqrt{1-x^2}\left(2x^2+3x-2\right)+3\sin^{-1}(x)\right)}{6\sqrt{-\left(x^2-1\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)*Sqrt[-1 + x^2],x]
```

```
[Out] ((-1 + x^2)*(Sqrt[1 - x^2]*(-2 + 3*x + 2*x^2) + 3*ArcSin[x]))/(6*Sqrt[-(-1
+ x^2)^2])
```

Maple [A] time = 0.04, size = 33, normalized size = 0.8

$$\frac{x}{2}\sqrt{x^2-1}-\frac{1}{2}\ln\left(x+\sqrt{x^2-1}\right)+\frac{1}{3}\left(x^2-1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)*(x^2-1)^(1/2),x)
```

```
[Out] 1/2*x*(x^2-1)^(1/2)-1/2*ln(x+(x^2-1)^(1/2))+1/3*(x^2-1)^(3/2)
```

Maxima [A] time = 1.02171, size = 49, normalized size = 1.11

$$\frac{1}{3}\left(x^2-1\right)^{\frac{3}{2}}+\frac{1}{2}\sqrt{x^2-1}x-\frac{1}{2}\log\left(2x+2\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(x^2 - 1)^(3/2) + 1/2*sqrt(x^2 - 1)*x - 1/2*log(2*x + 2*sqrt(x^2 - 1))
```

Fricas [A] time = 2.13397, size = 90, normalized size = 2.05

$$\frac{1}{6}\left(2x^2+3x-2\right)\sqrt{x^2-1}+\frac{1}{2}\log\left(-x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^2 + 3*x - 2)*sqrt(x^2 - 1) + 1/2*log(-x + sqrt(x^2 - 1))
```

Sympy [A] time = 0.202426, size = 39, normalized size = 0.89

$$\frac{x^2\sqrt{x^2-1}}{3}+\frac{x\sqrt{x^2-1}}{2}-\frac{\sqrt{x^2-1}}{3}-\frac{\operatorname{acosh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*(x**2-1)**(1/2),x)
```

```
[Out] x**2*sqrt(x**2 - 1)/3 + x*sqrt(x**2 - 1)/2 - sqrt(x**2 - 1)/3 - acosh(x)/2
```

Giac [A] time = 1.33293, size = 46, normalized size = 1.05

$$\frac{1}{6}\left((2x+3)x-2\right)\sqrt{x^2-1}+\frac{1}{2}\log\left(\left|-x+\sqrt{x^2-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*((2*x + 3)*x - 2)*sqrt(x^2 - 1) + 1/2*log(abs(-x + sqrt(x^2 - 1)))
```

3.820 $\int (1+x)\sqrt{1-x^2}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}(x)$$

```
[Out] (x*Sqrt[1 - x^2])/2 - (1 - x^2)^(3/2)/3 + ArcSin[x]/2
```

Rubi [A] time = 0.0062876, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {641, 195, 216}

$$-\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)*Sqrt[1 - x^2],x]
```

```
[Out] (x*Sqrt[1 - x^2])/2 - (1 - x^2)^(3/2)/3 + ArcSin[x]/2
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int (1+x)\sqrt{1-x^2}\,dx &= -\frac{1}{3}\left(1-x^2\right)^{3/2}+\int \sqrt{1-x^2}\,dx\\ &= \frac{1}{2}x\sqrt{1-x^2}-\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{1-x^2}-\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0094504, size = 31, normalized size = 0.82

$$\frac{1}{6}\left(\sqrt{1-x^2}\left(2x^2+3x-2\right)+3\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)*Sqrt[1 - x^2],x]
```

```
[Out] (Sqrt[1 - x^2]*(-2 + 3*x + 2*x^2) + 3*ArcSin[x])/6
```

Maple [A] time = 0.04, size = 29, normalized size = 0.8

$$-\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}+\frac{\arcsin(x)}{2}+\frac{x}{2}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)*(-x^2+1)^(1/2),x)
```

```
[Out] -1/3*(-x^2+1)^(3/2)+1/2*arcsin(x)+1/2*x*(-x^2+1)^(1/2)
```

Maxima [A] time = 1.51298, size = 38, normalized size = 1.

$$-\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}+\frac{1}{2}\sqrt{-x^2+1}x+\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-x^2 + 1)^(3/2) + 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Fricas [A] time = 2.3934, size = 96, normalized size = 2.53

$$\frac{1}{6}\left(2x^2+3x-2\right)\sqrt{-x^2+1}-\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^2 + 3*x - 2)*sqrt(-x^2 + 1) - arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] time = 0.194033, size = 39, normalized size = 1.03

$$\frac{x^2\sqrt{1-x^2}}{3}+\frac{x\sqrt{1-x^2}}{2}-\frac{\sqrt{1-x^2}}{3}+\frac{\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*(-x**2+1)**(1/2),x)
```

```
[Out] x**2*sqrt(1 - x**2)/3 + x*sqrt(1 - x**2)/2 - sqrt(1 - x**2)/3 + asin(x)/2
```

Giac [A] time = 1.19394, size = 34, normalized size = 0.89

$$\frac{1}{6}\left((2x+3)x-2\right)\sqrt{-x^2+1}+\frac{1}{2}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*((2*x + 3)*x - 2)*sqrt(-x^2 + 1) + 1/2*arcsin(x)
```

3.821 $\int \frac{\sqrt{1-x^2}}{1+x} \, dx$

Optimal. Leaf size=14

$$\sqrt{1-x^2}+\sin^{-1}(x)$$

```
[Out] Sqrt[1 - x^2] + ArcSin[x]
```

Rubi [A] time = 0.0058539, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {665, 216}

$$\sqrt{1-x^2}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/(1 + x),x]
```

```
[Out] Sqrt[1 - x^2] + ArcSin[x]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{1-x^2}}{1+x} \, dx &= \sqrt{1-x^2}+\int \frac{1}{\sqrt{1-x^2}} \, dx \\ &= \sqrt{1-x^2}+\sin^{-1}(x) \end{aligned}$$

Mathematica [B] time = 0.0486737, size = 30, normalized size = 2.14

$$\sqrt{1-x^2}-2 \sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/(1 + x),x]
```

```
[Out] Sqrt[1 - x^2] - 2*ArcSin[Sqrt[1 - x]/Sqrt[2]]
```

Maple [A] time = 0.04, size = 18, normalized size = 1.3

$$\sqrt{-\left(1+x\right)^{2}+2+2x}+\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(1+x),x)
```

```
[Out] (-(1+x)^2+2+2*x)^(1/2)+arcsin(x)
```

Maxima [A] time = 1.51349, size = 16, normalized size = 1.14

$$\sqrt{-x^{2}+1}+\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1+x),x, algorithm="maxima")
```

```
[Out] sqrt(-x^2 + 1) + arcsin(x)
```

Fricas [B] time = 2.31635, size = 69, normalized size = 4.93

$$\sqrt{-x^{2}+1}-2\arctan\left(\frac{\sqrt{-x^{2}+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1+x),x, algorithm="fricas")
```

```
[Out] sqrt(-x^2 + 1) - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] time = 1.68256, size = 15, normalized size = 1.07

$$\begin{cases}\sqrt{1-x^{2}}+\operatorname{asin}\left(x\right) & \text{for}\: x>-1\wedge x<1\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(1+x),x)
```

```
[Out] Piecewise((sqrt(1 - x**2) + asin(x), (x > -1) & (x < 1)))
```

Giac [A] time = 1.20619, size = 16, normalized size = 1.14

$$\sqrt{-x^{2}+1}+\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1+x),x, algorithm="giac")
```

```
[Out] sqrt(-x^2 + 1) + arcsin(x)
```

3.822 $\int (1-x)\sqrt{1-x^2}\,dx$

Optimal. Leaf size=38

$$\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}(x)$$

```
[Out] (x*Sqrt[1 - x^2])/2 + (1 - x^2)^(3/2)/3 + ArcSin[x]/2
```

Rubi [A] time = 0.0061408, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {641, 195, 216}

$$\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)*Sqrt[1 - x^2],x]
```

```
[Out] (x*Sqrt[1 - x^2])/2 + (1 - x^2)^(3/2)/3 + ArcSin[x]/2
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int (1-x)\sqrt{1-x^2}\,dx &= \frac{1}{3}\left(1-x^2\right)^{3/2}+\int \sqrt{1-x^2}\,dx\\ &= \frac{1}{2}x\sqrt{1-x^2}+\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{1-x^2}+\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0149647, size = 32, normalized size = 0.84

$$\frac{1}{6}\left(3\sin^{-1}(x)-\sqrt{1-x^2}\left(2x^2-3x-2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)*Sqrt[1 - x^2],x]
```

```
[Out] (-(Sqrt[1 - x^2]*(-2 - 3*x + 2*x^2)) + 3*ArcSin[x])/6
```

Maple [A] time = 0.043, size = 29, normalized size = 0.8

$$\frac{1}{3}\left(-x^{2}+1\right)^{\frac{3}{2}}+\frac{\arcsin\left(x\right)}{2}+\frac{x}{2}\sqrt{-x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)*(-x^2+1)^(1/2),x)
```

```
[Out] 1/3*(-x^2+1)^(3/2)+1/2*arcsin(x)+1/2*x*(-x^2+1)^(1/2)
```

Maxima [A] time = 1.56012, size = 38, normalized size = 1.

$$\frac{1}{3}\left(-x^{2}+1\right)^{\frac{3}{2}}+\frac{1}{2}\sqrt{-x^{2}+1}x+\frac{1}{2}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(-x^2 + 1)^(3/2) + 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Fricas [A] time = 2.32611, size = 97, normalized size = 2.55

$$-\frac{1}{6}\left(2x^{2}-3x-2\right)\sqrt{-x^{2}+1}-\arctan\left(\frac{\sqrt{-x^{2}+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(2*x^2 - 3*x - 2)*sqrt(-x^2 + 1) - arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] time = 0.195379, size = 39, normalized size = 1.03

$$-\frac{x^{2}\sqrt{1-x^{2}}}{3}+\frac{x\sqrt{1-x^{2}}}{2}+\frac{\sqrt{1-x^{2}}}{3}+\frac{\operatorname{asin}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*(-x**2+1)**(1/2),x)
```

```
[Out] -x**2*sqrt(1 - x**2)/3 + x*sqrt(1 - x**2)/2 + sqrt(1 - x**2)/3 + asin(x)/2
```

Giac [A] time = 1.20527, size = 34, normalized size = 0.89

$$-\frac{1}{6}\left((2x-3)x-2\right)\sqrt{-x^2+1}+\frac{1}{2}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*((2*x - 3)*x - 2)*sqrt(-x^2 + 1) + 1/2*arcsin(x)
```

3.823 $\int \frac{\sqrt{1-x^2}}{1-x} dx$

Optimal. Leaf size=16

$$\sin^{-1}(x) - \sqrt{1-x^2}$$

```
[Out] -Sqrt[1 - x^2] + ArcSin[x]
```

Rubi [A] time = 0.0064882, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {665, 216}

$$\sin^{-1}(x) - \sqrt{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/(1 - x),x]

[Out] -Sqrt[1 - x^2] + ArcSin[x]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{1-x^2}}{1-x} dx &= -\sqrt{1-x^2} + \int \frac{1}{\sqrt{1-x^2}} dx \\ &= -\sqrt{1-x^2} + \sin^{-1}(x) \end{aligned}$$

Mathematica [B] time = 0.0495471, size = 44, normalized size = 2.75

$$\frac{x^2 + 2\sqrt{1-x^2}\sin^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right) - 1}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/(1 - x),x]

[Out] (-1 + x^2 + 2*Sqrt[1 - x^2]*ArcSin[Sqrt[1 + x]/Sqrt[2]])/Sqrt[1 - x^2]
```

Maple [A] time = 0.04, size = 20, normalized size = 1.3

$$-\sqrt{-\left(-1+x\right)^{2}+2-2x}+\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(1-x),x)
```

```
[Out] -(-(-1+x)^2+2-2*x)^(1/2)+arcsin(x)
```

Maxima [A] time = 1.70816, size = 19, normalized size = 1.19

$$-\sqrt{-x^{2}+1}+\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1-x),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1) + arcsin(x)
```

Fricas [B] time = 2.21357, size = 70, normalized size = 4.38

$$-\sqrt{-x^{2}+1}-2\arctan\left(\frac{\sqrt{-x^{2}+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1-x),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 + 1) - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] time = 1.82402, size = 17, normalized size = 1.06

$$-\begin{cases}\sqrt{1-x^{2}}-\operatorname{asin}\left(x\right) & \text{for}\: x>-1\wedge x<1\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(1-x),x)
```

```
[Out] -Piecewise((sqrt(1 - x**2) - asin(x), (x > -1) & (x < 1)))
```

Giac [A] time = 1.21201, size = 19, normalized size = 1.19

$$-\sqrt{-x^{2}+1}+\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1-x),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 1) + arcsin(x)
```

3.824 $\int \frac{\sqrt{1-x^2}}{(1-x)^2} \, dx$

Optimal. Leaf size=25

$$\frac{2\sqrt{1-x^2}}{1-x} - \sin^{-1}(x)$$

```
[Out] (2*Sqrt[1 - x^2])/(1 - x) - ArcSin[x]
```

Rubi [A] time = 0.0068515, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {663, 216}

$$\frac{2\sqrt{1-x^2}}{1-x} - \sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/(1 - x)^2,x]

[Out] (2*Sqrt[1 - x^2])/(1 - x) - ArcSin[x]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-x^2}}{(1-x)^2} \, dx &= \frac{2\sqrt{1-x^2}}{1-x} - \int \frac{1}{\sqrt{1-x^2}} \, dx\\ &= \frac{2\sqrt{1-x^2}}{1-x} - \sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0548329, size = 50, normalized size = 2.

$$2\sqrt{1-x^2}\left(\frac{1}{1-x} + \frac{\sinh^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)}{\sqrt{x-1}\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/(1 - x)^2,x]
```

```
[Out] 2*Sqrt[1 - x^2]*((1 - x)^(-1) + ArcSinh[Sqrt[-1 + x]/Sqrt[2]]/(Sqrt[-1 + x]
*Sqrt[1 + x]))
```

Maple [A] time = 0.045, size = 40, normalized size = 1.6

$$\frac{1}{(-1+x)^2}\left(-(-1+x)^2+2-2x\right)^{\frac{3}{2}}+\sqrt{-(-1+x)^2+2-2x}-\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(1-x)^2,x)
```

```
[Out] 1/(-1+x)^2*(-(-1+x)^2+2-2*x)^(3/2)+(-(-1+x)^2+2-2*x)^(1/2)-arcsin(x)
```

Maxima [A] time = 1.53006, size = 28, normalized size = 1.12

$$-\frac{2\sqrt{-x^2+1}}{x-1}-\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1-x)^2,x, algorithm="maxima")
```

```
[Out] -2*sqrt(-x^2 + 1)/(x - 1) - arcsin(x)
```

Fricas [A] time = 2.14285, size = 104, normalized size = 4.16

$$\frac{2\left((x-1)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+x-\sqrt{-x^2+1}-1\right)}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1-x)^2,x, algorithm="fricas")
```

```
[Out] 2*((x - 1)*arctan((sqrt(-x^2 + 1) - 1)/x) + x - sqrt(-x^2 + 1) - 1)/(x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-(x-1)(x+1)}}{(x-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(1-x)**2,x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/(x - 1)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1-x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.825 $\int \frac{\sqrt{1-x^2}}{(1-x)^3} dx$

Optimal. Leaf size=22

$$\frac{(1-x^2)^{3/2}}{3(1-x)^3}$$

```
[Out] (1 - x^2)^(3/2)/(3*(1 - x)^3)
```

Rubi [A] time = 0.0057352, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {651}

$$\frac{(1-x^2)^{3/2}}{3(1-x)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/(1 - x)^3,x]

[Out] (1 - x^2)^(3/2)/(3*(1 - x)^3)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{\sqrt{1-x^2}}{(1-x)^3} dx = \frac{(1-x^2)^{3/2}}{3(1-x)^3}$$

Mathematica [A] time = 0.0167045, size = 23, normalized size = 1.05

$$\frac{(x+1)\sqrt{1-x^2}}{3(x-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/(1 - x)^3,x]

[Out] ((1 + x)*Sqrt[1 - x^2])/(3*(-1 + x)^2)
```

Maple [A] time = 0.041, size = 20, normalized size = 0.9

$$\frac{1+x}{3(-1+x)^2}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(1-x)^3,x)
```

```
[Out] 1/3*(1+x)*(-x^2+1)^(1/2)/(-1+x)^2
```

Maxima [B] time = 0.994955, size = 51, normalized size = 2.32

$$\frac{2\sqrt{-x^2+1}}{3\left(x^2-2x+1\right)}+\frac{\sqrt{-x^2+1}}{3(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1-x)^3,x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 1/3*sqrt(-x^2 + 1)/(x - 1)
```

Fricas [B] time = 2.11562, size = 84, normalized size = 3.82

$$\frac{x^2+\sqrt{-x^2+1}(x+1)-2x+1}{3\left(x^2-2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1-x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(x^2 + sqrt(-x^2 + 1)*(x + 1) - 2*x + 1)/(x^2 - 2*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{1-x^2}}{x^3-3x^2+3x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(1-x)**3,x)
```

```
[Out] -Integral(sqrt(1 - x**2)/(x**3 - 3*x**2 + 3*x - 1), x)
```

Giac [B] time = 1.2547, size = 55, normalized size = 2.5

$$\frac{2\left(\frac{3\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+1\right)}{3\left(\frac{\sqrt{-x^2+1}-1}{x}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1-x)^3,x, algorithm="giac")
```

```
[Out] 2/3*(3*(sqrt(-x^2 + 1) - 1)^2/x^2 + 1)/((sqrt(-x^2 + 1) - 1)/x + 1)^3
```

3.826 $\int \frac{(d+ex)^5}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=182

$$-\frac{63d^4\sqrt{d^2-e^2x^2}}{8e}-\frac{21d^3(d+ex)\sqrt{d^2-e^2x^2}}{8e}-\frac{21d^2(d+ex)^2\sqrt{d^2-e^2x^2}}{20e}-\frac{9d(d+ex)^3\sqrt{d^2-e^2x^2}}{20e}-\frac{(d+ex)^4\sqrt{d^2-e^2}}{5e}$$

```
[Out] (-63*d^4*Sqrt[d^2 - e^2*x^2])/(8*e) - (21*d^3*(d + e*x)*Sqrt[d^2 - e^2*x^2]
)/(8*e) - (21*d^2*(d + e*x)^2*Sqrt[d^2 - e^2*x^2])/(20*e) - (9*d*(d + e*x)^
3*Sqrt[d^2 - e^2*x^2])/(20*e) - ((d + e*x)^4*Sqrt[d^2 - e^2*x^2])/(5*e) + (
63*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e)
```

Rubi [A] time = 0.078631, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {671, 641, 217, 203}

$$-\frac{63d^4\sqrt{d^2-e^2x^2}}{8e}-\frac{21d^3(d+ex)\sqrt{d^2-e^2x^2}}{8e}-\frac{21d^2(d+ex)^2\sqrt{d^2-e^2x^2}}{20e}-\frac{9d(d+ex)^3\sqrt{d^2-e^2x^2}}{20e}-\frac{(d+ex)^4\sqrt{d^2-e^2}}{5e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-63*d^4*Sqrt[d^2 - e^2*x^2])/(8*e) - (21*d^3*(d + e*x)*Sqrt[d^2 - e^2*x^2]
)/(8*e) - (21*d^2*(d + e*x)^2*Sqrt[d^2 - e^2*x^2])/(20*e) - (9*d*(d + e*x)^
3*Sqrt[d^2 - e^2*x^2])/(20*e) - ((d + e*x)^4*Sqrt[d^2 - e^2*x^2])/(5*e) + (
63*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e)
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^5}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{(d+ex)^4\sqrt{d^2-e^2x^2}}{5e} + \frac{1}{5}(9d)\int \frac{(d+ex)^4}{\sqrt{d^2-e^2x^2}}\,dx$$
$$= -\frac{9d(d+ex)^3\sqrt{d^2-e^2x^2}}{20e} - \frac{(d+ex)^4\sqrt{d^2-e^2x^2}}{5e} + \frac{1}{20}\left(63d^2\right)\int \frac{(d+ex)^3}{\sqrt{d^2-e^2x^2}}\,dx$$
$$= -\frac{21d^2(d+ex)^2\sqrt{d^2-e^2x^2}}{20e} - \frac{9d(d+ex)^3\sqrt{d^2-e^2x^2}}{20e} - \frac{(d+ex)^4\sqrt{d^2-e^2x^2}}{5e} + \frac{1}{4}\left(21d^3\right)\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x}}$$
$$= -\frac{21d^3(d+ex)\sqrt{d^2-e^2x^2}}{8e} - \frac{21d^2(d+ex)^2\sqrt{d^2-e^2x^2}}{20e} - \frac{9d(d+ex)^3\sqrt{d^2-e^2x^2}}{20e} - \frac{(d+ex)^4\sqrt{d^2-e^2x}}{5e}$$
$$= -\frac{63d^4\sqrt{d^2-e^2x^2}}{8e} - \frac{21d^3(d+ex)\sqrt{d^2-e^2x^2}}{8e} - \frac{21d^2(d+ex)^2\sqrt{d^2-e^2x^2}}{20e} - \frac{9d(d+ex)^3\sqrt{d^2-e^2x^2}}{20e} -$$
$$= -\frac{63d^4\sqrt{d^2-e^2x^2}}{8e} - \frac{21d^3(d+ex)\sqrt{d^2-e^2x^2}}{8e} - \frac{21d^2(d+ex)^2\sqrt{d^2-e^2x^2}}{20e} - \frac{9d(d+ex)^3\sqrt{d^2-e^2x^2}}{20e} -$$
$$= -\frac{63d^4\sqrt{d^2-e^2x^2}}{8e} - \frac{21d^3(d+ex)\sqrt{d^2-e^2x^2}}{8e} - \frac{21d^2(d+ex)^2\sqrt{d^2-e^2x^2}}{20e} - \frac{9d(d+ex)^3\sqrt{d^2-e^2x^2}}{20e} -$$

Mathematica [A] time = 0.121002, size = 92, normalized size = 0.51

$$\frac{315d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \sqrt{d^2-e^2x^2}\left(144d^2e^2x^2+275d^3ex+488d^4+50de^3x^3+8e^4x^4\right)}{40e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(Sqrt[d^2 - e^2*x^2]*(488*d^4 + 275*d^3*e*x + 144*d^2*e^2*x^2 + 50*d*e^3*
x^3 + 8*e^4*x^4)) + 315*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(40*e)
```

Maple [A] time = 0.061, size = 144, normalized size = 0.8

$$-\frac{e^3x^4}{5}\sqrt{-e^2x^2+d^2} - \frac{18\,ed^2x^2}{5}\sqrt{-e^2x^2+d^2} - \frac{61\,d^4}{5\,e}\sqrt{-e^2x^2+d^2} - \frac{5\,de^2x^3}{4}\sqrt{-e^2x^2+d^2} - \frac{55\,d^3x}{8}\sqrt{-e^2x^2+d^2} + \frac{63\,d^5}{8}\,\mathrm{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/5*e^3*x^4*(-e^2*x^2+d^2)^(1/2)-18/5*e*d^2*x^2*(-e^2*x^2+d^2)^(1/2)-61/5*
d^4*(-e^2*x^2+d^2)^(1/2)/e-5/4*d*e^2*x^3*(-e^2*x^2+d^2)^(1/2)-55/8*d^3*x*(-
e^2*x^2+d^2)^(1/2)+63/8*d^5/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)
^(1/2))
```

Maxima [A] time = 1.68987, size = 184, normalized size = 1.01

$$-\frac{1}{5}\sqrt{-e^2x^2+d^2}e^3x^4 - \frac{5}{4}\sqrt{-e^2x^2+d^2}de^2x^3 - \frac{18}{5}\sqrt{-e^2x^2+d^2}d^2ex^2 + \frac{63\,d^5\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{8\sqrt{e^2}} - \frac{55}{8}\sqrt{-e^2x^2+d^2}d^3x - \frac{61\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(-e^2*x^2 + d^2)*e^3*x^4 - 5/4*sqrt(-e^2*x^2 + d^2)*d*e^2*x^3 - 18
/5*sqrt(-e^2*x^2 + d^2)*d^2*e*x^2 + 63/8*d^5*arcsin(e^2*x/sqrt(d^2*e^2))/sq
rt(e^2) - 55/8*sqrt(-e^2*x^2 + d^2)*d^3*x - 61/5*sqrt(-e^2*x^2 + d^2)*d^4/e
```

Fricas [A] time = 2.1493, size = 207, normalized size = 1.14

$$-\frac{630\,d^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(8\,e^4x^4+50\,de^3x^3+144\,d^2e^2x^2+275\,d^3ex+488\,d^4\right)\sqrt{-e^2x^2+d^2}}{40\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/40*(630*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (8*e^4*x^4 + 50*
d*e^3*x^3 + 144*d^2*e^2*x^2 + 275*d^3*e*x + 488*d^4)*sqrt(-e^2*x^2 + d^2))/
e
```

Sympy [A] time = 9.07527, size = 644, normalized size = 3.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**5*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/sqrt(d**2), (d**2 >
 0) & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d**2))/sqrt(d**2),
(d**2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e**2/d**2))/sqrt(-d
**2), (d**2 < 0) & (e**2 < 0))) + 5*d**4*e*Piecewise((x**2/(2*sqrt(d**2)),
Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True)) + 10*d**3*e**2*Piecewis
e((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d**2)/(2*e**2)
, Abs(e**2*x**2)/Abs(d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x/(2*e**2*s
qrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2)), True)) + 10
*d**2*e**3*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) - x**2*sqrt(d
**2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)), True)) + 5*d*e*
*4*Piecewise((-3*I*d**4*acosh(e*x/d)/(8*e**5) + 3*I*d**3*x/(8*e**4*sqrt(-1
+ e**2*x**2/d**2)) - I*d*x**3/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - I*x**5/(
4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (3*d**4*asin
(e*x/d)/(8*e**5) - 3*d**3*x/(8*e**4*sqrt(1 - e**2*x**2/d**2)) + d*x**3/(8*e
**2*sqrt(1 - e**2*x**2/d**2)) + x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True))
 + e**5*Piecewise((-8*d**4*sqrt(d**2 - e**2*x**2)/(15*e**6) - 4*d**2*x**2*s
qrt(d**2 - e**2*x**2)/(15*e**4) - x**4*sqrt(d**2 - e**2*x**2)/(5*e**2), Ne(
e, 0)), (x**6/(6*sqrt(d**2)), True))
```

Giac [A] time = 1.27637, size = 99, normalized size = 0.54

$$\frac{63}{8}\,d^5\arcsin\left(\frac{xe}{d}\right)e^{(-1)}\mathrm{sgn}\left(d\right)-\frac{1}{40}\left(488\,d^4e^{(-1)}+\left(275\,d^3+2\left(72\,d^2e+\left(4\,xe^3+25\,de^2\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 63/8*d^5*arcsin(x*e/d)*e^(-1)*sgn(d) - 1/40*(488*d^4*e^(-1) + (275*d^3 + 2*
(72*d^2*e + (4*x*e^3 + 25*d*e^2)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

3.827 $\int \frac{(d+ex)^4}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=149

$$-\frac{35d^3\sqrt{d^2-e^2x^2}}{8e}-\frac{35d^2(d+ex)\sqrt{d^2-e^2x^2}}{24e}-\frac{7d(d+ex)^2\sqrt{d^2-e^2x^2}}{12e}-\frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{4e}+\frac{35d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}$$

```
[Out] (-35*d^3*Sqrt[d^2 - e^2*x^2])/(8*e) - (35*d^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]
)/(24*e) - (7*d*(d + e*x)^2*Sqrt[d^2 - e^2*x^2])/(12*e) - ((d + e*x)^3*Sqrt
[d^2 - e^2*x^2])/(4*e) + (35*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e)
```

Rubi [A] time = 0.0618129, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {671, 641, 217, 203}

$$-\frac{35d^3\sqrt{d^2-e^2x^2}}{8e}-\frac{35d^2(d+ex)\sqrt{d^2-e^2x^2}}{24e}-\frac{7d(d+ex)^2\sqrt{d^2-e^2x^2}}{12e}-\frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{4e}+\frac{35d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-35*d^3*Sqrt[d^2 - e^2*x^2])/(8*e) - (35*d^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]
)/(24*e) - (7*d*(d + e*x)^2*Sqrt[d^2 - e^2*x^2])/(12*e) - ((d + e*x)^3*Sqrt
[d^2 - e^2*x^2])/(4*e) + (35*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e)
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^4}{\sqrt{d^2-e^2x^2}}dx = -\frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{4e} + \frac{1}{4}(7d)\int \frac{(d+ex)^3}{\sqrt{d^2-e^2x^2}}dx$$
$$= -\frac{7d(d+ex)^2\sqrt{d^2-e^2x^2}}{12e} - \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{4e} + \frac{1}{12}\left(35d^2\right)\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}dx$$
$$= -\frac{35d^2(d+ex)\sqrt{d^2-e^2x^2}}{24e} - \frac{7d(d+ex)^2\sqrt{d^2-e^2x^2}}{12e} - \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{4e} + \frac{1}{8}\left(35d^3\right)\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}$$
$$= -\frac{35d^3\sqrt{d^2-e^2x^2}}{8e} - \frac{35d^2(d+ex)\sqrt{d^2-e^2x^2}}{24e} - \frac{7d(d+ex)^2\sqrt{d^2-e^2x^2}}{12e} - \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{4e} + \frac{1}{8}(3$$
$$= -\frac{35d^3\sqrt{d^2-e^2x^2}}{8e} - \frac{35d^2(d+ex)\sqrt{d^2-e^2x^2}}{24e} - \frac{7d(d+ex)^2\sqrt{d^2-e^2x^2}}{12e} - \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{4e} + \frac{1}{8}(3$$
$$= -\frac{35d^3\sqrt{d^2-e^2x^2}}{8e} - \frac{35d^2(d+ex)\sqrt{d^2-e^2x^2}}{24e} - \frac{7d(d+ex)^2\sqrt{d^2-e^2x^2}}{12e} - \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{4e} + \frac{35d^4}{}$$

Mathematica [A] time = 0.0835836, size = 81, normalized size = 0.54

$$\frac{105d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \sqrt{d^2-e^2x^2}\left(81d^2ex + 160d^3 + 32de^2x^2 + 6e^3x^3\right)}{24e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(Sqrt[d^2 - e^2*x^2]*(160*d^3 + 81*d^2*e*x + 32*d*e^2*x^2 + 6*e^3*x^3)) +
 105*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(24*e)
```

Maple [A] time = 0.056, size = 119, normalized size = 0.8

$$-\frac{e^2x^3}{4}\sqrt{-e^2x^2+d^2} - \frac{27\,d^2x}{8}\sqrt{-e^2x^2+d^2} + \frac{35\,d^4}{8}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right)\frac{1}{\sqrt{e^2}} - \frac{4\,dex^2}{3}\sqrt{-e^2x^2+d^2} - \frac{20\,d^3}{3\,e}\sqrt{-e^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/4*e^2*x^3*(-e^2*x^2+d^2)^(1/2)-27/8*d^2*x*(-e^2*x^2+d^2)^(1/2)+35/8*d^4/
(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-4/3*e*d*x^2*(-e^2*x^
2+d^2)^(1/2)-20/3*d^3*(-e^2*x^2+d^2)^(1/2)/e
```

Maxima [A] time = 1.6912, size = 150, normalized size = 1.01

$$-\frac{1}{4}\sqrt{-e^2x^2+d^2}e^2x^3 - \frac{4}{3}\sqrt{-e^2x^2+d^2}dex^2 + \frac{35\,d^4\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{8\sqrt{e^2}} - \frac{27}{8}\sqrt{-e^2x^2+d^2}d^2x - \frac{20\sqrt{-e^2x^2+d^2}d^3}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-e^2*x^2 + d^2)*e^2*x^3 - 4/3*sqrt(-e^2*x^2 + d^2)*d*e*x^2 + 35/8
*d^4*arcsin(e^2*x/sqrt(d^2*e^2))/sqrt(e^2) - 27/8*sqrt(-e^2*x^2 + d^2)*d^2*
x - 20/3*sqrt(-e^2*x^2 + d^2)*d^3/e
```

Fricas [A] time = 2.19204, size = 181, normalized size = 1.21

$$-\frac{210\,d^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(6\,e^3x^3+32\,de^2x^2+81\,d^2ex+160\,d^3\right)\sqrt{-e^2x^2+d^2}}{24\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(210*d^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (6*e^3*x^3 + 32*
d*e^2*x^2 + 81*d^2*e*x + 160*d^3)*sqrt(-e^2*x^2 + d^2))/e
```

Sympy [A] time = 8.19789, size = 549, normalized size = 3.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**4*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/sqrt(d**2), (d**2 >
 0) & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d**2))/sqrt(d**2),
(d**2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e**2/d**2))/sqrt(-d
**2), (d**2 < 0) & (e**2 < 0))) + 4*d**3*e*Piecewise((x**2/(2*sqrt(d**2)),
Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True)) + 6*d**2*e**2*Piecewise
((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d**2)/(2*e**2),
 Abs(e**2*x**2)/Abs(d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x/(2*e**2*sq
rt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2)), True)) + 4*d
*e**3*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) - x**2*sqrt(d**2 -
 e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)), True)) + e**4*Piecew
ise((-3*I*d**4*acosh(e*x/d)/(8*e**5) + 3*I*d**3*x/(8*e**4*sqrt(-1 + e**2*x*
*2/d**2)) - I*d*x**3/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - I*x**5/(4*d*sqrt(
-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (3*d**4*asin(e*x/d)/(
8*e**5) - 3*d**3*x/(8*e**4*sqrt(1 - e**2*x**2/d**2)) + d*x**3/(8*e**2*sqrt(
1 - e**2*x**2/d**2)) + x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [A] time = 1.29989, size = 85, normalized size = 0.57

$$\frac{35}{8}\,d^4\arcsin\left(\frac{xe}{d}\right)e^{(-1)}\mathrm{sgn}\left(d\right)-\frac{1}{24}\left(160\,d^3e^{(-1)}+\left(81\,d^2+2\left(3\,xe^2+16\,de\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 35/8*d^4*arcsin(x*e/d)*e^(-1)*sgn(d) - 1/24*(160*d^3*e^(-1) + (81*d^2 + 2*(
3*x*e^2 + 16*d*e)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

3.828 $\int \frac{(d+ex)^3}{\sqrt{d^2-e^2x^2}} dx$

Optimal. Leaf size=116

$$-\frac{5d^2\sqrt{d^2-e^2x^2}}{2e}-\frac{5d(d+ex)\sqrt{d^2-e^2x^2}}{6e}-\frac{(d+ex)^2\sqrt{d^2-e^2x^2}}{3e}+\frac{5d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

```
[Out] (-5*d^2*Sqrt[d^2 - e^2*x^2])/(2*e) - (5*d*(d + e*x)*Sqrt[d^2 - e^2*x^2])/(6
*e) - ((d + e*x)^2*Sqrt[d^2 - e^2*x^2])/(3*e) + (5*d^3*ArcTan[(e*x)/Sqrt[d^
2 - e^2*x^2]])/(2*e)
```

Rubi [A] time = 0.0427732, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {671, 641, 217, 203}

$$-\frac{5d^2\sqrt{d^2-e^2x^2}}{2e}-\frac{5d(d+ex)\sqrt{d^2-e^2x^2}}{6e}-\frac{(d+ex)^2\sqrt{d^2-e^2x^2}}{3e}+\frac{5d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-5*d^2*Sqrt[d^2 - e^2*x^2])/(2*e) - (5*d*(d + e*x)*Sqrt[d^2 - e^2*x^2])/(6
*e) - ((d + e*x)^2*Sqrt[d^2 - e^2*x^2])/(3*e) + (5*d^3*ArcTan[(e*x)/Sqrt[d^
2 - e^2*x^2]])/(2*e)
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^3}{\sqrt{d^2-e^2x^2}}dx = -\frac{(d+ex)^2\sqrt{d^2-e^2x^2}}{3e} + \frac{1}{3}(5d)\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}dx$$
$$= -\frac{5d(d+ex)\sqrt{d^2-e^2x^2}}{6e} - \frac{(d+ex)^2\sqrt{d^2-e^2x^2}}{3e} + \frac{1}{2}\left(5d^2\right)\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}dx$$
$$= -\frac{5d^2\sqrt{d^2-e^2x^2}}{2e} - \frac{5d(d+ex)\sqrt{d^2-e^2x^2}}{6e} - \frac{(d+ex)^2\sqrt{d^2-e^2x^2}}{3e} + \frac{1}{2}\left(5d^3\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}dx$$
$$= -\frac{5d^2\sqrt{d^2-e^2x^2}}{2e} - \frac{5d(d+ex)\sqrt{d^2-e^2x^2}}{6e} - \frac{(d+ex)^2\sqrt{d^2-e^2x^2}}{3e} + \frac{1}{2}\left(5d^3\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}dx\right.$$
$$= -\frac{5d^2\sqrt{d^2-e^2x^2}}{2e} - \frac{5d(d+ex)\sqrt{d^2-e^2x^2}}{6e} - \frac{(d+ex)^2\sqrt{d^2-e^2x^2}}{3e} + \frac{5d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

Mathematica [A] time = 0.0600884, size = 70, normalized size = 0.6

$$\frac{15d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \sqrt{d^2-e^2x^2}\left(22d^2+9dex+2e^2x^2\right)}{6e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(Sqrt[d^2 - e^2*x^2]*(22*d^2 + 9*d*e*x + 2*e^2*x^2)) + 15*d^3*ArcTan[(e*x
)/Sqrt[d^2 - e^2*x^2]])/(6*e)
```

Maple [A] time = 0.05, size = 94, normalized size = 0.8

$$-\frac{ex^2}{3}\sqrt{-e^2x^2+d^2} - \frac{11\,d^2}{3\,e}\sqrt{-e^2x^2+d^2} - \frac{3\,dx}{2}\sqrt{-e^2x^2+d^2} + \frac{5\,d^3}{2}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right)\frac{1}{\sqrt{e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/3*e*x^2*(-e^2*x^2+d^2)^(1/2)-11/3*d^2*(-e^2*x^2+d^2)^(1/2)/e-3/2*d*x*(-e
^2*x^2+d^2)^(1/2)+5/2*d^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(
1/2))
```

Maxima [A] time = 1.7962, size = 116, normalized size = 1.

$$-\frac{1}{3}\sqrt{-e^2x^2+d^2}ex^2 + \frac{5\,d^3\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{2\sqrt{e^2}} - \frac{3}{2}\sqrt{-e^2x^2+d^2}dx - \frac{11\sqrt{-e^2x^2+d^2}d^2}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-e^2*x^2 + d^2)*e*x^2 + 5/2*d^3*arcsin(e^2*x/sqrt(d^2*e^2))/sqrt(
e^2) - 3/2*sqrt(-e^2*x^2 + d^2)*d*x - 11/3*sqrt(-e^2*x^2 + d^2)*d^2/e
```

Fricas [A] time = 2.16217, size = 153, normalized size = 1.32

$$-\frac{30\,d^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(2\,e^2x^2+9\,dex+22\,d^2\right)\sqrt{-e^2x^2+d^2}}{6\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(30*d^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (2*e^2*x^2 + 9*d*e
*x + 22*d^2)*sqrt(-e^2*x^2 + d^2))/e
```

Sympy [A] time = 4.36979, size = 338, normalized size = 2.91

$$d^3\left(\begin{cases}\frac{\sqrt{\frac{d^2}{e^2}}\operatorname{asin}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for}\ d^2>0\wedge e^2>0\\ \frac{\sqrt{-\frac{d^2}{e^2}}\operatorname{asinh}\left(x\sqrt{-\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for}\ d^2>0\wedge e^2<0\\ \frac{\sqrt{\frac{d^2}{e^2}}\operatorname{acosh}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{-d^2}} & \text{for}\ d^2<0\wedge e^2<0\end{cases}\right)+3d^2e\left(\begin{cases}\frac{x^2}{2\sqrt{d^2}} & \text{for}\ e^2=0\\ -\frac{\sqrt{d^2-e^2x^2}}{e^2} & \text{otherwise}\end{cases}\right)+3de^2\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^3}-\frac{idx\sqrt{-1+\frac{e^2x^2}{d^2}}}{2e^2} \\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e^3}-\frac{dx}{2e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{\ }{2d\sqrt{\ }}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**3*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/sqrt(d**2), (d**2 >
 0) & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d**2))/sqrt(d**2),
(d**2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e**2/d**2))/sqrt(-d
**2), (d**2 < 0) & (e**2 < 0))) + 3*d**2*e*Piecewise((x**2/(2*sqrt(d**2)),
Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True)) + 3*d*e**2*Piecewise((-
I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d**2)/(2*e**2), Ab
s(e**2*x**2)/Abs(d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x/(2*e**2*sqrt(
1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2)), True)) + e**3*P
iecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) - x**2*sqrt(d**2 - e**2*x
**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)), True))
```

Giac [A] time = 1.36994, size = 70, normalized size = 0.6

$$\frac{5}{2}\,d^3\arcsin\left(\frac{xe}{d}\right)e^{(-1)}\mathrm{sgn}\left(d\right)-\frac{1}{6}\,\sqrt{-x^2e^2+d^2}\left(22\,d^2e^{(-1)}+\left(2\,xe+9\,d\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 5/2*d^3*arcsin(x*e/d)*e^(-1)*sgn(d) - 1/6*sqrt(-x^2*e^2 + d^2)*(22*d^2*e^(-
1) + (2*x*e + 9*d)*x)
```

3.829 $\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=74

$$-\frac{2d\sqrt{d^2-e^2x^2}}{e}-\frac{1}{2}x\sqrt{d^2-e^2x^2}+\frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

```
[Out] (-2*d*Sqrt[d^2 - e^2*x^2])/e - (x*Sqrt[d^2 - e^2*x^2])/2 + (3*d^2*ArcTan[(e
*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)
```

Rubi [A] time = 0.0267756, antiderivative size = 83, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {671, 641, 217, 203}

$$-\frac{3d\sqrt{d^2-e^2x^2}}{2e}-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}+\frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-3*d*Sqrt[d^2 - e^2*x^2])/(2*e) - ((d + e*x)*Sqrt[d^2 - e^2*x^2])/(2*e) +
(3*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e} + \frac{1}{2}(3d)\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}\,dx$$
$$= -\frac{3d\sqrt{d^2-e^2x^2}}{2e} - \frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e} + \frac{1}{2}\left(3d^2\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$
$$= -\frac{3d\sqrt{d^2-e^2x^2}}{2e} - \frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e} + \frac{1}{2}\left(3d^2\right)\mathrm{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)$$
$$= -\frac{3d\sqrt{d^2-e^2x^2}}{2e} - \frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e} + \frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

Mathematica [A] time = 0.0366859, size = 58, normalized size = 0.78

$$\frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - (4d+ex)\sqrt{d^2-e^2x^2}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-((4*d + e*x)*Sqrt[d^2 - e^2*x^2]) + 3*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2
]])/(2*e)
```

Maple [A] time = 0.049, size = 71, normalized size = 1.

$$-\frac{x}{2}\sqrt{-e^2x^2+d^2} + \frac{3\,d^2}{2}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right)\frac{1}{\sqrt{e^2}} - 2\,\frac{d\sqrt{-e^2x^2+d^2}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/2*x*(-e^2*x^2+d^2)^(1/2)+3/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*
x^2+d^2)^(1/2))-2*d*(-e^2*x^2+d^2)^(1/2)/e
```

Maxima [A] time = 1.73513, size = 85, normalized size = 1.15

$$\frac{3\,d^2\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{2\sqrt{e^2}} - \frac{1}{2}\sqrt{-e^2x^2+d^2}x - \frac{2\sqrt{-e^2x^2+d^2}d}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/2*d^2*arcsin(e^2*x/sqrt(d^2*e^2))/sqrt(e^2) - 1/2*sqrt(-e^2*x^2 + d^2)*x
- 2*sqrt(-e^2*x^2 + d^2)*d/e
```

Fricas [A] time = 2.15827, size = 126, normalized size = 1.7

$$-\frac{6\,d^2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\sqrt{-e^2x^2+d^2}(ex+4\,d)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(6*d^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + sqrt(-e^2*x^2 + d^2
)*(e*x + 4*d))/e
```

Sympy [A] time = 3.7291, size = 270, normalized size = 3.65

$$d^2\left(\begin{cases}\frac{\sqrt{\frac{d^2}{e^2}}\operatorname{asin}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for } d^2>0\wedge e^2>0\\ \frac{\sqrt{-\frac{d^2}{e^2}}\operatorname{asinh}\left(x\sqrt{-\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for } d^2>0\wedge e^2<0\\ \frac{\sqrt{\frac{d^2}{e^2}}\operatorname{acosh}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{-d^2}} & \text{for } d^2<0\wedge e^2<0\end{cases}\right)+2de\left(\begin{cases}\frac{x^2}{2\sqrt{d^2}} & \text{for } e^2=0\\ -\frac{\sqrt{d^2-e^2x^2}}{e^2} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^3}-\frac{idx\sqrt{-1+\frac{e^2x^2}{d^2}}}{2e^2} \\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e^3}-\frac{dx}{2e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{}{2d\sqrt{}}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/sqrt(d**2), (d**2 >
 0) & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d**2))/sqrt(d**2),
(d**2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e**2/d**2))/sqrt(-d
**2), (d**2 < 0) & (e**2 < 0))) + 2*d*e*Piecewise((x**2/(2*sqrt(d**2)), Eq(
e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True)) + e**2*Piecewise((-I*d**2*
acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d**2)/(2*e**2), Abs(e**2*
x**2)/Abs(d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x/(2*e**2*sqrt(1 - e**
2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [A] time = 1.33922, size = 54, normalized size = 0.73

$$\frac{3}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^{(-1)}\operatorname{sgn}(d)-\frac{1}{2}\,\sqrt{-x^2e^2+d^2}\left(4\,de^{(-1)}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 3/2*d^2*arcsin(x*e/d)*e^(-1)*sgn(d) - 1/2*sqrt(-x^2*e^2 + d^2)*(4*d*e^(-1)
+ x)
```

3.830 $\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=47

$$\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}-\frac{\sqrt{d^2-e^2x^2}}{e}$$

```
[Out] -(Sqrt[d^2 - e^2*x^2]/e) + (d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e
```

Rubi [A] time = 0.0123493, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {641, 217, 203}

$$\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}-\frac{\sqrt{d^2-e^2x^2}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/e) + (d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}\,dx &= -\frac{\sqrt{d^2-e^2x^2}}{e}+d\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx\\
&= -\frac{\sqrt{d^2-e^2x^2}}{e}+d\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)\\
&= -\frac{\sqrt{d^2-e^2x^2}}{e}+\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}
\end{aligned}$$

Mathematica [A] time = 0.0152537, size = 47, normalized size = 1.

$$\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}-\frac{\sqrt{d^2-e^2x^2}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/e) + (d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e
```

Maple [A] time = 0.044, size = 50, normalized size = 1.1

$$-\frac{1}{e}\sqrt{-e^2x^2+d^2}+d\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right)\frac{1}{\sqrt{e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -(-e^2*x^2+d^2)^(1/2)/e+d/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(
1/2))
```

Maxima [A] time = 1.96742, size = 57, normalized size = 1.21

$$\frac{d\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{\sqrt{e^2}}-\frac{\sqrt{-e^2x^2+d^2}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] d*arcsin(e^2*x/sqrt(d^2*e^2))/sqrt(e^2) - sqrt(-e^2*x^2 + d^2)/e
```

Fricas [A] time = 2.13811, size = 101, normalized size = 2.15

$$-\frac{2\,d\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\sqrt{-e^2x^2+d^2}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*d*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + sqrt(-e^2*x^2 + d^2))/e
```

Sympy [A] time = 1.52875, size = 42, normalized size = 0.89

$$\begin{cases}\frac{d\left(\begin{cases}\operatorname{asin}\left(ex\sqrt{\frac{1}{d^2}}\right) & \text{for}\ d^2>0\end{cases}\right)-\sqrt{d^2-e^2x^2}}{e} & \text{for}\ e\neq 0\\ \frac{dx}{\sqrt{d^2}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Piecewise(((d*Piecewise((asin(e*x*sqrt(d**(-2))), d**2 > 0)) - sqrt(d**2 -
e**2*x**2))/e, Ne(e, 0)), (d*x/sqrt(d**2), True))
```

Giac [A] time = 1.26415, size = 43, normalized size = 0.91

$$d \arcsin\left(\frac{xe}{d}\right)e^{(-1)}\mathrm{sgn}\left(d\right) - \sqrt{-x^2e^2 + d^2}e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] d*arcsin(x*e/d)*e^(-1)*sgn(d) - sqrt(-x^2*e^2 + d^2)*e^(-1)
```

3.831 $\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=31

$$-\frac{\sqrt{d^2-e^2x^2}}{de(d+ex)}$$

```
[Out] -(Sqrt[d^2 - e^2*x^2]/(d*e*(d + e*x)))
```

Rubi [A] time = 0.0094422, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {651}

$$-\frac{\sqrt{d^2-e^2x^2}}{de(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/(d*e*(d + e*x)))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{de(d+ex)}$$

Mathematica [A] time = 0.0057505, size = 32, normalized size = 1.03

$$-\frac{\sqrt{d^2-e^2x^2}}{d^2e+de^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/(d^2*e + d*e^2*x))
```

Maple [A] time = 0.045, size = 29, normalized size = 0.9

$$-\frac{-ex+d}{de}\frac{1}{\sqrt{-e^2x^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -(-e*x+d)/d/e/(-e^2*x^2+d^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.01641, size = 72, normalized size = 2.32

$$-\frac{ex + d + \sqrt{-e^2x^2 + d^2}}{de^2x + d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(e*x + d + sqrt(-e^2*x^2 + d^2))/(d*e^2*x + d^2*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\left(-d + ex\right)\left(d + ex\right)}\left(d + ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.832 $\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=67

$$-\frac{\sqrt{d^2-e^2x^2}}{3d^2e(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{3de(d+ex)^2}$$

```
[Out] -Sqrt[d^2 - e^2*x^2]/(3*d*e*(d + e*x)^2) - Sqrt[d^2 - e^2*x^2]/(3*d^2*e*(d
+ e*x))
```

Rubi [A] time = 0.02218, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 651}

$$-\frac{\sqrt{d^2-e^2x^2}}{3d^2e(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{3de(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2]/(3*d*e*(d + e*x)^2) - Sqrt[d^2 - e^2*x^2]/(3*d^2*e*(d
+ e*x))
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx &= -\frac{\sqrt{d^2-e^2x^2}}{3de(d+ex)^2}+\frac{\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{3d}\\ &= -\frac{\sqrt{d^2-e^2x^2}}{3de(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}}{3d^2e(d+ex)}\end{aligned}$$

Mathematica [A] time = 0.0415059, size = 40, normalized size = 0.6

$$-\frac{(2d+ex)\sqrt{d^2-e^2x^2}}{3d^2e(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -((2*d + e*x)*Sqrt[d^2 - e^2*x^2])/(3*d^2*e*(d + e*x)^2)
```

Maple [A] time = 0.044, size = 43, normalized size = 0.6

$$-\frac{(-ex+d)(ex+2d)}{(3ex+3d)d^2e}\frac{1}{\sqrt{-e^2x^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/3*(-e*x+d)*(e*x+2*d)/(e*x+d)/d^2/e/(-e^2*x^2+d^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.12626, size = 144, normalized size = 2.15

$$-\frac{2e^2x^2+4dex+2d^2+\sqrt{-e^2x^2+d^2}(ex+2d)}{3\left(d^2e^3x^2+2d^3e^2x+d^4e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*e^2*x^2 + 4*d*e*x + 2*d^2 + sqrt(-e^2*x^2 + d^2)*(e*x + 2*d))/(d^2*
e^3*x^2 + 2*d^3*e^2*x + d^4*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(-d+ex)(d+ex)}(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.833 $\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=100

$$-\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d+ex)}-\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}$$

```
[Out] -Sqrt[d^2 - e^2*x^2]/(5*d*e*(d + e*x)^3) - (2*Sqrt[d^2 - e^2*x^2])/(15*d^2*
e*(d + e*x)^2) - (2*Sqrt[d^2 - e^2*x^2])/(15*d^3*e*(d + e*x))
```

Rubi [A] time = 0.0376555, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 651}

$$-\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d+ex)}-\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2]/(5*d*e*(d + e*x)^3) - (2*Sqrt[d^2 - e^2*x^2])/(15*d^2*
e*(d + e*x)^2) - (2*Sqrt[d^2 - e^2*x^2])/(15*d^3*e*(d + e*x))
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx &= -\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}+\frac{2\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx}{5d}\\ &= -\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}-\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d+ex)^2}+\frac{2\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{15d^2}\\ &= -\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}-\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d+ex)^2}-\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d+ex)}\end{aligned}$$

Mathematica [A] time = 0.039272, size = 52, normalized size = 0.52

$$-\frac{\sqrt{d^2-e^2x^2}\left(7d^2+6dex+2e^2x^2\right)}{15d^3e(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]*(7*d^2 + 6*d*e*x + 2*e^2*x^2))/(15*d^3*e*(d + e*x)^3)
```

Maple [A] time = 0.043, size = 55, normalized size = 0.6

$$-\frac{(-ex+d)\left(2\,e^2x^2+6\,dex+7\,d^2\right)}{15\,ed^3\,(ex+d)^2}\frac{1}{\sqrt{-e^2x^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/15*(-e*x+d)*(2*e^2*x^2+6*d*e*x+7*d^2)/(e*x+d)^2/d^3/e/(-e^2*x^2+d^2)^(1/
2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.11496, size = 216, normalized size = 2.16

$$-\frac{7\,e^3x^3+21\,de^2x^2+21\,d^2ex+7\,d^3+\left(2\,e^2x^2+6\,dex+7\,d^2\right)\sqrt{-e^2x^2+d^2}}{15\,\left(d^3e^4x^3+3\,d^4e^3x^2+3\,d^5e^2x+d^6e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(7*e^3*x^3 + 21*d*e^2*x^2 + 21*d^2*e*x + 7*d^3 + (2*e^2*x^2 + 6*d*e*x
 + 7*d^2)*sqrt(-e^2*x^2 + d^2))/(d^3*e^4*x^3 + 3*d^4*e^3*x^2 + 3*d^5*e^2*x
+ d^6*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-(-d+ex)\,(d+ex)}\,(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.834 $\int \frac{1}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=133

$$-\frac{2\sqrt{d^2-e^2x^2}}{35d^4e(d+ex)}-\frac{2\sqrt{d^2-e^2x^2}}{35d^3e(d+ex)^2}-\frac{3\sqrt{d^2-e^2x^2}}{35d^2e(d+ex)^3}-\frac{\sqrt{d^2-e^2x^2}}{7de(d+ex)^4}$$

```
[Out] -Sqrt[d^2 - e^2*x^2]/(7*d*e*(d + e*x)^4) - (3*Sqrt[d^2 - e^2*x^2])/(35*d^2*
e*(d + e*x)^3) - (2*Sqrt[d^2 - e^2*x^2])/(35*d^3*e*(d + e*x)^2) - (2*Sqrt[d
^2 - e^2*x^2])/(35*d^4*e*(d + e*x))
```

Rubi [A] time = 0.0552447, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 651}

$$-\frac{2\sqrt{d^2-e^2x^2}}{35d^4e(d+ex)}-\frac{2\sqrt{d^2-e^2x^2}}{35d^3e(d+ex)^2}-\frac{3\sqrt{d^2-e^2x^2}}{35d^2e(d+ex)^3}-\frac{\sqrt{d^2-e^2x^2}}{7de(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^4*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2]/(7*d*e*(d + e*x)^4) - (3*Sqrt[d^2 - e^2*x^2])/(35*d^2*
e*(d + e*x)^3) - (2*Sqrt[d^2 - e^2*x^2])/(35*d^3*e*(d + e*x)^2) - (2*Sqrt[d
^2 - e^2*x^2])/(35*d^4*e*(d + e*x))
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx &= -\frac{\sqrt{d^2-e^2x^2}}{7de(d+ex)^4}+\frac{3\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx}{7d}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{7de(d+ex)^4}-\frac{3\sqrt{d^2-e^2x^2}}{35d^2e(d+ex)^3}+\frac{6\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx}{35d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{7de(d+ex)^4}-\frac{3\sqrt{d^2-e^2x^2}}{35d^2e(d+ex)^3}-\frac{2\sqrt{d^2-e^2x^2}}{35d^3e(d+ex)^2}+\frac{2\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{35d^3}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{7de(d+ex)^4}-\frac{3\sqrt{d^2-e^2x^2}}{35d^2e(d+ex)^3}-\frac{2\sqrt{d^2-e^2x^2}}{35d^3e(d+ex)^2}-\frac{2\sqrt{d^2-e^2x^2}}{35d^4e(d+ex)}
\end{aligned}$$

Mathematica [A] time = 0.0423093, size = 63, normalized size = 0.47

$$-\frac{\sqrt{d^2-e^2x^2}\left(13d^2ex+12d^3+8de^2x^2+2e^3x^3\right)}{35d^4e(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^4*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]*(12*d^3 + 13*d^2*e*x + 8*d*e^2*x^2 + 2*e^3*x^3))/(35*
d^4*e*(d + e*x)^4)
```

Maple [A] time = 0.043, size = 66, normalized size = 0.5

$$-\frac{(-ex+d)\left(2e^3x^3+8e^2x^2d+13xd^2e+12d^3\right)}{35ed^4(ex+d)^3}\frac{1}{\sqrt{-e^2x^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/35*(-e*x+d)*(2*e^3*x^3+8*d*e^2*x^2+13*d^2*e*x+12*d^3)/(e*x+d)^3/d^4/e/(-
e^2*x^2+d^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.28272, size = 288, normalized size = 2.17

$$-\frac{12e^4x^4+48de^3x^3+72d^2e^2x^2+48d^3ex+12d^4+\left(2e^3x^3+8de^2x^2+13d^2ex+12d^3\right)\sqrt{-e^2x^2+d^2}}{35\left(d^4e^5x^4+4d^5e^4x^3+6d^6e^3x^2+4d^7e^2x+d^8e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/35*(12*e^4*x^4 + 48*d*e^3*x^3 + 72*d^2*e^2*x^2 + 48*d^3*e*x + 12*d^4 + (
2*e^3*x^3 + 8*d*e^2*x^2 + 13*d^2*e*x + 12*d^3)*sqrt(-e^2*x^2 + d^2))/(d^4*e
^5*x^4 + 4*d^5*e^4*x^3 + 6*d^6*e^3*x^2 + 4*d^7*e^2*x + d^8*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(-d+ex)(d+ex)}(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**4/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.835 $\int \frac{1}{(d+ex)^5\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=166

$$-\frac{8\sqrt{d^2-e^2x^2}}{315d^5e(d+ex)}-\frac{8\sqrt{d^2-e^2x^2}}{315d^4e(d+ex)^2}-\frac{4\sqrt{d^2-e^2x^2}}{105d^3e(d+ex)^3}-\frac{4\sqrt{d^2-e^2x^2}}{63d^2e(d+ex)^4}-\frac{\sqrt{d^2-e^2x^2}}{9de(d+ex)^5}$$

```
[Out] -Sqrt[d^2 - e^2*x^2]/(9*d*e*(d + e*x)^5) - (4*Sqrt[d^2 - e^2*x^2])/(63*d^2*
e*(d + e*x)^4) - (4*Sqrt[d^2 - e^2*x^2])/(105*d^3*e*(d + e*x)^3) - (8*Sqrt[
d^2 - e^2*x^2])/(315*d^4*e*(d + e*x)^2) - (8*Sqrt[d^2 - e^2*x^2])/(315*d^5*
e*(d + e*x))
```

Rubi [A] time = 0.0721729, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 651}

$$-\frac{8\sqrt{d^2-e^2x^2}}{315d^5e(d+ex)}-\frac{8\sqrt{d^2-e^2x^2}}{315d^4e(d+ex)^2}-\frac{4\sqrt{d^2-e^2x^2}}{105d^3e(d+ex)^3}-\frac{4\sqrt{d^2-e^2x^2}}{63d^2e(d+ex)^4}-\frac{\sqrt{d^2-e^2x^2}}{9de(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^5*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2]/(9*d*e*(d + e*x)^5) - (4*Sqrt[d^2 - e^2*x^2])/(63*d^2*
e*(d + e*x)^4) - (4*Sqrt[d^2 - e^2*x^2])/(105*d^3*e*(d + e*x)^3) - (8*Sqrt[
d^2 - e^2*x^2])/(315*d^4*e*(d + e*x)^2) - (8*Sqrt[d^2 - e^2*x^2])/(315*d^5*
e*(d + e*x))
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^5\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{9de(d+ex)^5} + \frac{4\int \frac{1}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx}{9d}$$
$$= -\frac{\sqrt{d^2-e^2x^2}}{9de(d+ex)^5} - \frac{4\sqrt{d^2-e^2x^2}}{63d^2e(d+ex)^4} + \frac{4\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx}{21d^2}$$
$$= -\frac{\sqrt{d^2-e^2x^2}}{9de(d+ex)^5} - \frac{4\sqrt{d^2-e^2x^2}}{63d^2e(d+ex)^4} - \frac{4\sqrt{d^2-e^2x^2}}{105d^3e(d+ex)^3} + \frac{8\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx}{105d^3}$$
$$= -\frac{\sqrt{d^2-e^2x^2}}{9de(d+ex)^5} - \frac{4\sqrt{d^2-e^2x^2}}{63d^2e(d+ex)^4} - \frac{4\sqrt{d^2-e^2x^2}}{105d^3e(d+ex)^3} - \frac{8\sqrt{d^2-e^2x^2}}{315d^4e(d+ex)^2} + \frac{8\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}}{315d^4}$$
$$= -\frac{\sqrt{d^2-e^2x^2}}{9de(d+ex)^5} - \frac{4\sqrt{d^2-e^2x^2}}{63d^2e(d+ex)^4} - \frac{4\sqrt{d^2-e^2x^2}}{105d^3e(d+ex)^3} - \frac{8\sqrt{d^2-e^2x^2}}{315d^4e(d+ex)^2} - \frac{8\sqrt{d^2-e^2x^2}}{315d^5e(d+ex)}$$

Mathematica [A] time = 0.0482122, size = 74, normalized size = 0.45

$$-\frac{\sqrt{d^2-e^2x^2}\left(84d^2e^2x^2+100d^3ex+83d^4+40de^3x^3+8e^4x^4\right)}{315d^5e(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^5*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]*(83*d^4 + 100*d^3*e*x + 84*d^2*e^2*x^2 + 40*d*e^3*x^3
 + 8*e^4*x^4))/(315*d^5*e*(d + e*x)^5)
```

Maple [A] time = 0.045, size = 77, normalized size = 0.5

$$-\frac{(-ex+d)\left(8e^4x^4+40e^3x^3d+84e^2x^2d^2+100xd^3e+83d^4\right)}{315ed^5(ex+d)^4}\frac{1}{\sqrt{-e^2x^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^5/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/315*(-e*x+d)*(8*e^4*x^4+40*d*e^3*x^3+84*d^2*e^2*x^2+100*d^3*e*x+83*d^4)/
(e*x+d)^4/d^5/e/(-e^2*x^2+d^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^5/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.40017, size = 369, normalized size = 2.22

$$-\frac{83\,e^5x^5+415\,de^4x^4+830\,d^2e^3x^3+830\,d^3e^2x^2+415\,d^4ex+83\,d^5+\left(8\,e^4x^4+40\,de^3x^3+84\,d^2e^2x^2+100\,d^3ex+83\,d^4\right)}{315\left(d^5e^6x^5+5\,d^6e^5x^4+10\,d^7e^4x^3+10\,d^8e^3x^2+5\,d^9e^2x+d^{10}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^5/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/315*(83*e^5*x^5 + 415*d*e^4*x^4 + 830*d^2*e^3*x^3 + 830*d^3*e^2*x^2 + 41
5*d^4*e*x + 83*d^5 + (8*e^4*x^4 + 40*d*e^3*x^3 + 84*d^2*e^2*x^2 + 100*d^3*e
*x + 83*d^4)*sqrt(-e^2*x^2 + d^2))/(d^5*e^6*x^5 + 5*d^6*e^5*x^4 + 10*d^7*e^
4*x^3 + 10*d^8*e^3*x^2 + 5*d^9*e^2*x + d^10*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**5/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**5), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^5/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.836 $\int \frac{(d+ex)^6}{\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=143

$$\frac{2(d+ex)^5}{3e\left(d^2-e^2x^2\right)^{3/2}}-\frac{14(d+ex)^3}{3e\sqrt{d^2-e^2x^2}}-\frac{35\sqrt{d^2-e^2x^2}(d+ex)}{6e}-\frac{35d\sqrt{d^2-e^2x^2}}{2e}+\frac{35d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

```
[Out] (2*(d + e*x)^5)/(3*e*(d^2 - e^2*x^2)^(3/2)) - (14*(d + e*x)^3)/(3*e*Sqrt[d^
2 - e^2*x^2]) - (35*d*Sqrt[d^2 - e^2*x^2])/(2*e) - (35*(d + e*x)*Sqrt[d^2 -
 e^2*x^2])/(6*e) + (35*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)
```

Rubi [A] time = 0.0584483, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {669, 671, 641, 217, 203}

$$\frac{2(d+ex)^5}{3e\left(d^2-e^2x^2\right)^{3/2}}-\frac{14(d+ex)^3}{3e\sqrt{d^2-e^2x^2}}-\frac{35\sqrt{d^2-e^2x^2}(d+ex)}{6e}-\frac{35d\sqrt{d^2-e^2x^2}}{2e}+\frac{35d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^6/(d^2 - e^2*x^2)^(5/2),x]
```


```
[Out] (2*(d + e*x)^5)/(3*e*(d^2 - e^2*x^2)^(3/2)) - (14*(d + e*x)^3)/(3*e*Sqrt[d^
2 - e^2*x^2]) - (35*d*Sqrt[d^2 - e^2*x^2])/(2*e) - (35*(d + e*x)*Sqrt[d^2 -
 e^2*x^2])/(6*e) + (35*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)
```


Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^6}{\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{2(d+ex)^5}{3e\left(d^2-e^2x^2\right)^{3/2}} - \frac{7}{3}\int \frac{(d+ex)^4}{\left(d^2-e^2x^2\right)^{3/2}}\,dx$$
$$= \frac{2(d+ex)^5}{3e\left(d^2-e^2x^2\right)^{3/2}} - \frac{14(d+ex)^3}{3e\sqrt{d^2-e^2x^2}} + \frac{35}{3}\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$$
$$= \frac{2(d+ex)^5}{3e\left(d^2-e^2x^2\right)^{3/2}} - \frac{14(d+ex)^3}{3e\sqrt{d^2-e^2x^2}} - \frac{35(d+ex)\sqrt{d^2-e^2x^2}}{6e} + \frac{1}{2}(35d)\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}\,dx$$
$$= \frac{2(d+ex)^5}{3e\left(d^2-e^2x^2\right)^{3/2}} - \frac{14(d+ex)^3}{3e\sqrt{d^2-e^2x^2}} - \frac{35d\sqrt{d^2-e^2x^2}}{2e} - \frac{35(d+ex)\sqrt{d^2-e^2x^2}}{6e} + \frac{1}{2}\left(35d^2\right)\int \frac{1}{\sqrt{d^2-\ldots}}$$
$$= \frac{2(d+ex)^5}{3e\left(d^2-e^2x^2\right)^{3/2}} - \frac{14(d+ex)^3}{3e\sqrt{d^2-e^2x^2}} - \frac{35d\sqrt{d^2-e^2x^2}}{2e} - \frac{35(d+ex)\sqrt{d^2-e^2x^2}}{6e} + \frac{1}{2}\left(35d^2\right)\mathrm{Subst}\left(\int \ldots\right.$$
$$= \frac{2(d+ex)^5}{3e\left(d^2-e^2x^2\right)^{3/2}} - \frac{14(d+ex)^3}{3e\sqrt{d^2-e^2x^2}} - \frac{35d\sqrt{d^2-e^2x^2}}{2e} - \frac{35(d+ex)\sqrt{d^2-e^2x^2}}{6e} + \frac{35d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x\ldots}}\right.}{2e}$$

Mathematica [A] time = 0.21352, size = 121, normalized size = 0.85

$$\frac{(d+ex)\left(\sqrt{1-\frac{e^2x^2}{d^2}}\left(-229d^2ex+164d^3+30de^2x^2+3e^3x^3\right)-105d(d-ex)^2\sin^{-1}\left(\frac{ex}{d}\right)\right)}{6e(ex-d)\sqrt{d^2-e^2x^2}\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^6/(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] ((d + e*x)*(Sqrt[1 - (e^2*x^2)/d^2]*(164*d^3 - 229*d^2*e*x + 30*d*e^2*x^2 +
 3*e^3*x^3) - 105*d*(d - e*x)^2*ArcSin[(e*x)/d]))/(6*e*(-d + e*x)*Sqrt[d^2
- e^2*x^2]*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [A] time = 0.08, size = 189, normalized size = 1.3

$$-\frac{e^4x^5}{2}\left(-e^2x^2+d^2\right)^{-\frac{3}{2}} + \frac{35\,d^2e^2x^3}{6}\left(-e^2x^2+d^2\right)^{-\frac{3}{2}} - \frac{131\,d^2x}{6}\frac{1}{\sqrt{-e^2x^2+d^2}} + \frac{35\,d^2}{2}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right)\frac{1}{\sqrt{e^2}} - 6\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^6/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/2*e^4*x^5/(-e^2*x^2+d^2)^(3/2)+35/6*e^2*d^2*x^3/(-e^2*x^2+d^2)^(3/2)-131
/6*d^2*x/(-e^2*x^2+d^2)^(1/2)+35/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e
^2*x^2+d^2)^(1/2))-6*d*e^3*x^4/(-e^2*x^2+d^2)^(3/2)+44*d^3*e*x^2/(-e^2*x^2+
```

```
d^2)^(3/2)-82/3*d^5/e/(-e^2*x^2+d^2)^(3/2)+16/3*d^4*x/(-e^2*x^2+d^2)^(3/2)
```

Maxima [A] time = 1.65743, size = 284, normalized size = 1.99

$$\frac{35}{6}d^2e^4x\left(\frac{3x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}-\frac{2d^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}\right)-\frac{e^4x^5}{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{6de^3x^4}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{44d^3ex^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{16d^4x}{3\left(-e^2x^2+d^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 35/6*d^2*e^4*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2
)^(3/2)*e^4)) - 1/2*e^4*x^5/(-e^2*x^2 + d^2)^(3/2) - 6*d*e^3*x^4/(-e^2*x^2
+ d^2)^(3/2) + 44*d^3*e*x^2/(-e^2*x^2 + d^2)^(3/2) + 16/3*d^4*x/(-e^2*x^2 +
 d^2)^(3/2) - 82/3*d^5/((-e^2*x^2 + d^2)^(3/2)*e) - 61/6*d^2*x/sqrt(-e^2*x^
2 + d^2) + 35/2*d^2*arcsin(e^2*x/sqrt(d^2*e^2))/sqrt(e^2)
```

Fricas [A] time = 2.22356, size = 313, normalized size = 2.19

$$-\frac{164d^2e^2x^2-328d^3ex+164d^4+210\left(d^2e^2x^2-2d^3ex+d^4\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(3e^3x^3+30de^2x^2-229d^2ex\right.}{6\left(e^3x^2-2de^2x+d^2e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*(164*d^2*e^2*x^2 - 328*d^3*e*x + 164*d^4 + 210*(d^2*e^2*x^2 - 2*d^3*e*
x + d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (3*e^3*x^3 + 30*d*e^2*
x^2 - 229*d^2*e*x + 164*d^3)*sqrt(-e^2*x^2 + d^2))/(e^3*x^2 - 2*d*e^2*x + d
^2*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^6}{(-(-d+ex)(d+ex))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**6/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral((d + e*x)**6/(-(-d + e*x)*(d + e*x))**(5/2), x)
```

Giac [A] time = 1.2638, size = 131, normalized size = 0.92

$$\frac{35}{2}d^2\arcsin\left(\frac{xe}{d}\right)e^{(-1)}\mathrm{sgn}(d)-\frac{\left(164d^5e^{(-1)}+\left(99d^4-\left(264d^3e+\left(166d^2e^2-3\left(xe^4+12de^3\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}}{6\left(x^2e^2-d^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 35/2*d^2*arcsin(x*e/d)*e^(-1)*sgn(d) - 1/6*(164*d^5*e^(-1) + (99*d^4 - (264
*d^3*e + (166*d^2*e^2 - 3*(x*e^4 + 12*d*e^3)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^
2)/(x^2*e^2 - d^2)^2
```

3.837 $\int \frac{(d+ex)^5}{\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=108

$$\frac{2(d+ex)^4}{3e\left(d^2-e^2x^2\right)^{3/2}}-\frac{10(d+ex)^2}{3e\sqrt{d^2-e^2x^2}}-\frac{5\sqrt{d^2-e^2x^2}}{e}+\frac{5d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

```
[Out] (2*(d + e*x)^4)/(3*e*(d^2 - e^2*x^2)^(3/2)) - (10*(d + e*x)^2)/(3*e*Sqrt[d^
2 - e^2*x^2]) - (5*Sqrt[d^2 - e^2*x^2])/e + (5*d*ArcTan[(e*x)/Sqrt[d^2 - e^
2*x^2]])/e
```

Rubi [A] time = 0.0398871, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {669, 641, 217, 203}

$$\frac{2(d+ex)^4}{3e\left(d^2-e^2x^2\right)^{3/2}}-\frac{10(d+ex)^2}{3e\sqrt{d^2-e^2x^2}}-\frac{5\sqrt{d^2-e^2x^2}}{e}+\frac{5d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5/(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (2*(d + e*x)^4)/(3*e*(d^2 - e^2*x^2)^(3/2)) - (10*(d + e*x)^2)/(3*e*Sqrt[d^
2 - e^2*x^2]) - (5*Sqrt[d^2 - e^2*x^2])/e + (5*d*ArcTan[(e*x)/Sqrt[d^2 - e^
2*x^2]])/e
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^5}{\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{2(d+ex)^4}{3e\left(d^2-e^2x^2\right)^{3/2}} - \frac{5}{3}\int \frac{(d+ex)^3}{\left(d^2-e^2x^2\right)^{3/2}}\,dx$$
$$= \frac{2(d+ex)^4}{3e\left(d^2-e^2x^2\right)^{3/2}} - \frac{10(d+ex)^2}{3e\sqrt{d^2-e^2x^2}} + 5\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}\,dx$$
$$= \frac{2(d+ex)^4}{3e\left(d^2-e^2x^2\right)^{3/2}} - \frac{10(d+ex)^2}{3e\sqrt{d^2-e^2x^2}} - \frac{5\sqrt{d^2-e^2x^2}}{e} + (5d)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$
$$= \frac{2(d+ex)^4}{3e\left(d^2-e^2x^2\right)^{3/2}} - \frac{10(d+ex)^2}{3e\sqrt{d^2-e^2x^2}} - \frac{5\sqrt{d^2-e^2x^2}}{e} + (5d)\,\mathrm{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)$$
$$= \frac{2(d+ex)^4}{3e\left(d^2-e^2x^2\right)^{3/2}} - \frac{10(d+ex)^2}{3e\sqrt{d^2-e^2x^2}} - \frac{5\sqrt{d^2-e^2x^2}}{e} + \frac{5d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

Mathematica [A] time = 0.157079, size = 109, normalized size = 1.01

$$\frac{(d+ex)\left(\left(23d^2-34dex+3e^2x^2\right)\sqrt{1-\frac{e^2x^2}{d^2}}-15(d-ex)^2\sin^{-1}\left(\frac{ex}{d}\right)\right)}{3e(ex-d)\sqrt{d^2-e^2x^2}\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5/(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] ((d + e*x)*((23*d^2 - 34*d*e*x + 3*e^2*x^2)*Sqrt[1 - (e^2*x^2)/d^2] - 15*(d
 - e*x)^2*ArcSin[(e*x)/d]))/(3*e*(-d + e*x)*Sqrt[d^2 - e^2*x^2]*Sqrt[1 - (e
^2*x^2)/d^2])
```

Maple [A] time = 0.062, size = 160, normalized size = 1.5

$$-e^3x^4\left(-e^2x^2+d^2\right)^{-\frac{3}{2}}+14\frac{ed^2x^2}{\left(-e^2x^2+d^2\right)^{3/2}}-\frac{23\,d^4}{3\,e}\left(-e^2x^2+d^2\right)^{-\frac{3}{2}}+\frac{5\,de^2x^3}{3}\left(-e^2x^2+d^2\right)^{-\frac{3}{2}}-\frac{23\,dx}{3}\frac{1}{\sqrt{-e^2x^2+d^2}}+5\,\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -e^3*x^4/(-e^2*x^2+d^2)^(3/2)+14*e*d^2*x^2/(-e^2*x^2+d^2)^(3/2)-23/3*d^4/e/
(-e^2*x^2+d^2)^(3/2)+5/3*d*e^2*x^3/(-e^2*x^2+d^2)^(3/2)-23/3*d*x/(-e^2*x^2+
d^2)^(1/2)+5*d/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+11/3*
d^3*x/(-e^2*x^2+d^2)^(3/2)
```

Maxima [A] time = 1.87467, size = 244, normalized size = 2.26

$$\frac{5}{3}de^4x\left(\frac{3\,x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}-\frac{2\,d^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}\right)-\frac{e^3x^4}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{14\,d^2ex^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{11\,d^3x}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{23\,d^4}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e}-\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/3*d*e^4*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(
3/2)*e^4)) - e^3*x^4/(-e^2*x^2 + d^2)^(3/2) + 14*d^2*e*x^2/(-e^2*x^2 + d^2)
^(3/2) + 11/3*d^3*x/(-e^2*x^2 + d^2)^(3/2) - 23/3*d^4/((-e^2*x^2 + d^2)^(3/
2)*e) - 13/3*d*x/sqrt(-e^2*x^2 + d^2) + 5*d*arcsin(e^2*x/sqrt(d^2*e^2))/sqr
t(e^2)
```

Fricas [A] time = 2.1472, size = 277, normalized size = 2.56

$$-\frac{23\,de^2x^2-46\,d^2ex+23\,d^3+30\left(de^2x^2-2\,d^2ex+d^3\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(3\,e^2x^2-34\,dex+23\,d^2\right)\sqrt{-e^2x^2+d^2}}{3\left(e^3x^2-2\,de^2x+d^2e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(23*d*e^2*x^2 - 46*d^2*e*x + 23*d^3 + 30*(d*e^2*x^2 - 2*d^2*e*x + d^3)
*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (3*e^2*x^2 - 34*d*e*x + 23*d^2
)*sqrt(-e^2*x^2 + d^2))/(e^3*x^2 - 2*d*e^2*x + d^2*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(d+ex)^5}{(-(-d+ex)(d+ex))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral((d + e*x)**5/(-(-d + e*x)*(d + e*x))**(5/2), x)
```

Giac [A] time = 1.34395, size = 116, normalized size = 1.07

$$5\,d\arcsin\left(\frac{xe}{d}\right)e^{(-1)}\mathrm{sgn}\left(d\right)-\frac{\left(23\,d^4e^{(-1)}+\left(12\,d^3-\left(42\,d^2e-\left(3\,xe^3-28\,de^2\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}}{3\left(x^2e^2-d^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 5*d*arcsin(x*e/d)*e^(-1)*sgn(d) - 1/3*(23*d^4*e^(-1) + (12*d^3 - (42*d^2*e
- (3*x*e^3 - 28*d*e^2)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^2
```

3.838 $\int \frac{(d+ex)^4}{\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=81

$$\frac{2(d+ex)^3}{3e\left(d^2-e^2x^2\right)^{3/2}}-\frac{2(d+ex)}{e\sqrt{d^2-e^2x^2}}+\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

```
[Out] (2*(d + e*x)^3)/(3*e*(d^2 - e^2*x^2)^(3/2)) - (2*(d + e*x))/(e*Sqrt[d^2 - e
^2*x^2]) + ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e
```

Rubi [A] time = 0.0231172, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {669, 653, 217, 203}

$$\frac{2(d+ex)^3}{3e\left(d^2-e^2x^2\right)^{3/2}}-\frac{2(d+ex)}{e\sqrt{d^2-e^2x^2}}+\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (2*(d + e*x)^3)/(3*e*(d^2 - e^2*x^2)^(3/2)) - (2*(d + e*x))/(e*Sqrt[d^2 - e
^2*x^2]) + ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^4}{\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{2(d+ex)^3}{3e\left(d^2-e^2x^2\right)^{3/2}} - \int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{3/2}}\,dx$$
$$= \frac{2(d+ex)^3}{3e\left(d^2-e^2x^2\right)^{3/2}} - \frac{2(d+ex)}{e\sqrt{d^2-e^2x^2}} + \int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$
$$= \frac{2(d+ex)^3}{3e\left(d^2-e^2x^2\right)^{3/2}} - \frac{2(d+ex)}{e\sqrt{d^2-e^2x^2}} + \text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)$$
$$= \frac{2(d+ex)^3}{3e\left(d^2-e^2x^2\right)^{3/2}} - \frac{2(d+ex)}{e\sqrt{d^2-e^2x^2}} + \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

Mathematica [A] time = 0.120354, size = 100, normalized size = 1.23

$$-\frac{(d+ex)\left(4d(d-2ex)\sqrt{1-\frac{e^2x^2}{d^2}} - 3(d-ex)^2\sin^{-1}\left(\frac{ex}{d}\right)\right)}{3de(d-ex)\sqrt{d^2-e^2x^2}\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] -((d + e*x)*(4*d*(d - 2*e*x)*Sqrt[1 - (e^2*x^2)/d^2] - 3*(d - e*x)^2*ArcSin
[(e*x)/d]))/(3*d*e*(d - e*x)*Sqrt[d^2 - e^2*x^2]*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [A] time = 0.059, size = 132, normalized size = 1.6

$$\frac{e^2x^3}{3}\left(-e^2x^2+d^2\right)^{-\frac{3}{2}} - \frac{7x}{3}\frac{1}{\sqrt{-e^2x^2+d^2}} + \arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right)\frac{1}{\sqrt{e^2}} + 4\frac{dex^2}{\left(-e^2x^2+d^2\right)^{3/2}} - \frac{4d^3}{3e}\left(-e^2x^2+d^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] 1/3*e^2*x^3/(-e^2*x^2+d^2)^(3/2)-7/3*x/(-e^2*x^2+d^2)^(1/2)+1/(e^2)^(1/2)*a
rctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+4*e*d*x^2/(-e^2*x^2+d^2)^(3/2)-4/
3*d^3/e/(-e^2*x^2+d^2)^(3/2)+7/3*d^2*x/(-e^2*x^2+d^2)^(3/2)
```

Maxima [B] time = 1.68638, size = 207, normalized size = 2.56

$$\frac{1}{3}e^4x\left(\frac{3x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2} - \frac{2d^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}\right) + \frac{4dex^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}} + \frac{7d^2x}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}} - \frac{4d^3}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e} - \frac{5x}{3\sqrt{-e^2x^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*e^4*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/
2)*e^4)) + 4*d*e*x^2/(-e^2*x^2 + d^2)^(3/2) + 7/3*d^2*x/(-e^2*x^2 + d^2)^(3
/2) - 4/3*d^3/((-e^2*x^2 + d^2)^(3/2)*e) - 5/3*x/sqrt(-e^2*x^2 + d^2) + arc
sin(e^2*x/sqrt(d^2*e^2))/sqrt(e^2)
```

Fricas [A] time = 2.03055, size = 236, normalized size = 2.91

$$-\frac{2\left(2\,e^{2}x^{2}-4\,dex+2\,d^{2}+3\left(e^{2}x^{2}-2\,dex+d^{2}\right)\arctan\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{ex}\right)-2\,\sqrt{-e^{2}x^{2}+d^{2}}(2\,ex-d)\right)}{3\left(e^{3}x^{2}-2\,de^{2}x+d^{2}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*e^2*x^2 - 4*d*e*x + 2*d^2 + 3*(e^2*x^2 - 2*d*e*x + d^2)*arctan(-(d
- sqrt(-e^2*x^2 + d^2))/(e*x)) - 2*sqrt(-e^2*x^2 + d^2)*(2*e*x - d))/(e^3*x
^2 - 2*d*e^2*x + d^2*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(d+ex)^{4}}{(-(-d+ex)(d+ex))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral((d + e*x)**4/(-(-d + e*x)*(d + e*x))**(5/2), x)
```

Giac [A] time = 1.41552, size = 89, normalized size = 1.1

$$\arcsin\left(\frac{xe}{d}\right)e^{(-1)}\mathrm{sgn}\,(d)-\frac{4\left(d^{3}e^{(-1)}-\left(2\,xe^{2}+3\,de\right)x^{2}\right)\sqrt{-x^{2}e^{2}+d^{2}}}{3\left(x^{2}e^{2}-d^{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] arcsin(x*e/d)*e^(-1)*sgn(d) - 4/3*(d^3*e^(-1) - (2*x*e^2 + 3*d*e)*x^2)*sqrt
(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^2
```

3.839 $\int \frac{(d+ex)^3}{\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=33

$$\frac{(d+ex)^3}{3de\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] (d + e*x)^3/(3*d*e*(d^2 - e^2*x^2)^(3/2))
```

Rubi [A] time = 0.0099128, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {651}

$$\frac{(d+ex)^3}{3de\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (d + e*x)^3/(3*d*e*(d^2 - e^2*x^2)^(3/2))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(d^2-e^2x^2\right)^{5/2}}\,dx=\frac{(d+ex)^3}{3de\left(d^2-e^2x^2\right)^{3/2}}$$

Mathematica [A] time = 0.0445357, size = 41, normalized size = 1.24

$$\frac{(d+ex)^2}{3de(d-ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (d + e*x)^2/(3*d*e*(d - e*x)*Sqrt[d^2 - e^2*x^2])
```

Maple [A] time = 0.042, size = 36, normalized size = 1.1

$$\frac{(ex+d)^4(-ex+d)}{3\,de}\left(-e^2x^2+d^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] 1/3*(e*x+d)^4*(-e*x+d)/d/e/(-e^2*x^2+d^2)^(5/2)
```

Maxima [B] time = 1.19949, size = 108, normalized size = 3.27

$$\frac{ex^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{4\,dx}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{d^2}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e}-\frac{x}{3\sqrt{-e^2x^2+d^2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] e*x^2/(-e^2*x^2 + d^2)^(3/2) + 4/3*d*x/(-e^2*x^2 + d^2)^(3/2) + 1/3*d^2/((-
e^2*x^2 + d^2)^(3/2)*e) - 1/3*x/(sqrt(-e^2*x^2 + d^2)*d)
```

Fricas [B] time = 2.06081, size = 132, normalized size = 4.

$$\frac{e^2x^2-2\,dex+d^2+\sqrt{-e^2x^2+d^2}(ex+d)}{3\left(de^3x^2-2\,d^2e^2x+d^3e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(e^2*x^2 - 2*d*e*x + d^2 + sqrt(-e^2*x^2 + d^2)*(e*x + d))/(d*e^3*x^2 -
 2*d^2*e^2*x + d^3*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(d+ex)^3}{(-(-d+ex)(d+ex))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral((d + e*x)**3/(-(-d + e*x)*(d + e*x))**(5/2), x)
```

Giac [A] time = 1.30452, size = 76, normalized size = 2.3

$$\frac{\sqrt{-x^2e^2+d^2}\left(d^2e^{(-1)}+\left(x\left(\frac{xe^2}{d}+3\,e\right)+3\,d\right)x\right)}{3\left(x^2e^2-d^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(-x^2*e^2 + d^2)*(d^2*e^(-1) + (x*(x*e^2/d + 3*e) + 3*d)*x)/(x^2*e^
2 - d^2)^2
```

3.840 $\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=53

$$\frac{x}{3d^2\sqrt{d^2-e^2x^2}}+\frac{2(d+ex)}{3e(d^2-e^2x^2)^{3/2}}$$

```
[Out] (2*(d + e*x))/(3*e*(d^2 - e^2*x^2)^(3/2)) + x/(3*d^2*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.0133077, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {653, 191}

$$\frac{x}{3d^2\sqrt{d^2-e^2x^2}}+\frac{2(d+ex)}{3e(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (2*(d + e*x))/(3*e*(d^2 - e^2*x^2)^(3/2)) + x/(3*d^2*Sqrt[d^2 - e^2*x^2])
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{5/2}}\,dx &= \frac{2(d+ex)}{3e(d^2-e^2x^2)^{3/2}}+\frac{1}{3}\int \frac{1}{(d^2-e^2x^2)^{3/2}}\,dx\\ &= \frac{2(d+ex)}{3e(d^2-e^2x^2)^{3/2}}+\frac{x}{3d^2\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0281359, size = 47, normalized size = 0.89

$$\frac{(2d-ex)(d+ex)}{3d^2e(d-ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] ((2*d - e*x)*(d + e*x))/(3*d^2*e*(d - e*x)*Sqrt[d^2 - e^2*x^2])
```

Maple [A] time = 0.044, size = 44, normalized size = 0.8

$$\frac{(ex+d)^3(-ex+d)(-ex+2d)}{3d^2e}\left(-e^2x^2+d^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] 1/3*(e*x+d)^3*(-e*x+d)*(-e*x+2*d)/d^2/e/(-e^2*x^2+d^2)^(5/2)
```

Maxima [A] time = 1.14806, size = 78, normalized size = 1.47

$$\frac{2x}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{2d}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e}+\frac{x}{3\sqrt{-e^2x^2+d^2}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x/(-e^2*x^2 + d^2)^(3/2) + 2/3*d/((-e^2*x^2 + d^2)^(3/2)*e) + 1/3*x/(sq
rt(-e^2*x^2 + d^2)*d^2)
```

Fricas [A] time = 2.03883, size = 143, normalized size = 2.7

$$\frac{2e^2x^2-4dex+2d^2-\sqrt{-e^2x^2+d^2}(ex-2d)}{3\left(d^2e^3x^2-2d^3e^2x+d^4e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*e^2*x^2 - 4*d*e*x + 2*d^2 - sqrt(-e^2*x^2 + d^2)*(e*x - 2*d))/(d^2*e
^3*x^2 - 2*d^3*e^2*x + d^4*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(d+ex)^2}{(-(-d+ex)(d+ex))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral((d + e*x)**2/(-(-d + e*x)*(d + e*x))**(5/2), x)
```

Giac [A] time = 1.45495, size = 65, normalized size = 1.23

$$-\frac{\sqrt{-x^2e^2+d^2}\left(x\left(\frac{x^2e^2}{d^2}-3\right)-2\,de^{(-1)}\right)}{3\left(x^2e^2-d^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(-x^2*e^2 + d^2)*(x*(x^2*e^2/d^2 - 3) - 2*d*e^(-1))/(x^2*e^2 - d^2
)^2
```

3.841 $\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=56

$$\frac{2x}{3d^3\sqrt{d^2-e^2x^2}}+\frac{d+ex}{3de\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] (d + e*x)/(3*d*e*(d^2 - e^2*x^2)^(3/2)) + (2*x)/(3*d^3*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.0123644, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {639, 191}

$$\frac{2x}{3d^3\sqrt{d^2-e^2x^2}}+\frac{d+ex}{3de\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (d + e*x)/(3*d*e*(d^2 - e^2*x^2)^(3/2)) + (2*x)/(3*d^3*Sqrt[d^2 - e^2*x^2])
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{5/2}}\,dx &= \frac{d+ex}{3de\left(d^2-e^2x^2\right)^{3/2}}+\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3d}\\ &= \frac{d+ex}{3de\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{3d^3\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0237811, size = 51, normalized size = 0.91

$$\frac{d^2+2dex-2e^2x^2}{3d^3e(d-ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (d^2 + 2*d*e*x - 2*e^2*x^2)/(3*d^3*e*(d - e*x)*Sqrt[d^2 - e^2*x^2])
```

Maple [A] time = 0.044, size = 53, normalized size = 1.

$$\frac{(ex+d)^2(-ex+d)\left(-2e^2x^2+2dex+d^2\right)}{3d^3e}\left(-e^2x^2+d^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] 1/3*(e*x+d)^2*(-e*x+d)*(-2*e^2*x^2+2*d*e*x+d^2)/d^3/e/(-e^2*x^2+d^2)^(5/2)
```

Maxima [A] time = 1.20168, size = 81, normalized size = 1.45

$$\frac{x}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d}+\frac{1}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e}+\frac{2x}{3\sqrt{-e^2x^2+d^2}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x/((-e^2*x^2 + d^2)^(3/2)*d) + 1/3/((-e^2*x^2 + d^2)^(3/2)*e) + 2/3*x/(
sqrt(-e^2*x^2 + d^2)*d^3)
```

Fricas [B] time = 2.08311, size = 192, normalized size = 3.43

$$\frac{e^3x^3-de^2x^2-d^2ex+d^3-\left(2e^2x^2-2dex-d^2\right)\sqrt{-e^2x^2+d^2}}{3\left(d^3e^4x^3-d^4e^3x^2-d^5e^2x+d^6e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(e^3*x^3 - d*e^2*x^2 - d^2*e*x + d^3 - (2*e^2*x^2 - 2*d*e*x - d^2)*sqrt
(-e^2*x^2 + d^2))/(d^3*e^4*x^3 - d^4*e^3*x^2 - d^5*e^2*x + d^6*e)
```

Sympy [C] time = 4.95444, size = 298, normalized size = 5.32

$$d\left(\begin{cases}\frac{3id^2x}{-3d^7\sqrt{-1+\frac{e^2x^2}{d^2}}+3d^5e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{2ie^2x^3}{-3d^7\sqrt{-1+\frac{e^2x^2}{d^2}}+3d^5e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \frac{|e^2x^2|}{|d^2|}>1\\ -\frac{3d^2x}{-3d^7\sqrt{1-\frac{e^2x^2}{d^2}}+3d^5e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{2e^2x^3}{-3d^7\sqrt{1-\frac{e^2x^2}{d^2}}+3d^5e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+e\left(\begin{cases}-\frac{1}{-3d^2e^2\sqrt{d^2-e^2x^2}+3e^4x^2\sqrt{d^2-e^2x^2}} & \text{for}\\ \frac{x^2}{2\left(d^2\right)^{\frac{5}{2}}} & \text{oth}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d*Piecewise((3*I*d**2*x/(-3*d**7*sqrt(-1 + e**2*x**2/d**2) + 3*d**5*e**2*x*
*2*sqrt(-1 + e**2*x**2/d**2)) - 2*I*e**2*x**3/(-3*d**7*sqrt(-1 + e**2*x**2/
d**2) + 3*d**5*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**
2) > 1), (-3*d**2*x/(-3*d**7*sqrt(1 - e**2*x**2/d**2) + 3*d**5*e**2*x**2*sq
rt(1 - e**2*x**2/d**2)) + 2*e**2*x**3/(-3*d**7*sqrt(1 - e**2*x**2/d**2) + 3
*d**5*e**2*x**2*sqrt(1 - e**2*x**2/d**2)), True)) + e*Piecewise((-1/(-3*d**
2*e**2*sqrt(d**2 - e**2*x**2) + 3*e**4*x**2*sqrt(d**2 - e**2*x**2)), Ne(e,
0)), (x**2/(2*(d**2)**(5/2)), True))
```

Giac [A] time = 1.42761, size = 70, normalized size = 1.25

$$-\frac{\sqrt{-x^2e^2 + d^2}\left(x\left(\frac{2x^2e^2}{d^3} - \frac{3}{d}\right) - e^{(-1)}\right)}{3\left(x^2e^2 - d^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(-x^2*e^2 + d^2)*(x*(2*x^2*e^2/d^3 - 3/d) - e^(-1))/(x^2*e^2 - d^2
)^2
```

3.842 $\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=82

$$\frac{8x}{15d^5\sqrt{d^2-e^2x^2}}+\frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] (4*x)/(15*d^3*(d^2 - e^2*x^2)^(3/2)) - 1/(5*d*e*(d + e*x)*(d^2 - e^2*x^2)^(
3/2)) + (8*x)/(15*d^5*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.0206295, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {659, 192, 191}

$$\frac{8x}{15d^5\sqrt{d^2-e^2x^2}}+\frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (4*x)/(15*d^3*(d^2 - e^2*x^2)^(3/2)) - 1/(5*d*e*(d + e*x)*(d^2 - e^2*x^2)^(
3/2)) + (8*x)/(15*d^5*Sqrt[d^2 - e^2*x^2])
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = -\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$
$$= \frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{8\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^3}$$
$$= \frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{8x}{15d^5\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.0649542, size = 82, normalized size = 1.

$$-\frac{\sqrt{d^2-e^2x^2}\left(-12d^2e^2x^2-12d^3ex+3d^4+8de^3x^3+8e^4x^4\right)}{15d^5e(d-ex)^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]*(3*d^4 - 12*d^3*e*x - 12*d^2*e^2*x^2 + 8*d*e^3*x^3 +
8*e^4*x^4))/(15*d^5*e*(d - e*x)^2*(d + e*x)^3)
```

Maple [A] time = 0.045, size = 70, normalized size = 0.9

$$-\frac{(-ex+d)\left(8\,e^4x^4+8\,e^3x^3d-12\,e^2x^2d^2-12\,xd^3e+3\,d^4\right)}{15\,d^5e}\left(-e^2x^2+d^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/15*(-e*x+d)*(8*e^4*x^4+8*d*e^3*x^3-12*d^2*e^2*x^2-12*d^3*e*x+3*d^4)/d^5/
e/(-e^2*x^2+d^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.17436, size = 342, normalized size = 4.17

$$-\frac{3\,e^5x^5+3\,de^4x^4-6\,d^2e^3x^3-6\,d^3e^2x^2+3\,d^4ex+3\,d^5+\left(8\,e^4x^4+8\,de^3x^3-12\,d^2e^2x^2-12\,d^3ex+3\,d^4\right)\sqrt{-e^2x^2+d^2}}{15\left(d^5e^6x^5+d^6e^5x^4-2\,d^7e^4x^3-2\,d^8e^3x^2+d^9e^2x+d^{10}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*e^5*x^5 + 3*d*e^4*x^4 - 6*d^2*e^3*x^3 - 6*d^3*e^2*x^2 + 3*d^4*e*x
+ 3*d^5 + (8*e^4*x^4 + 8*d*e^3*x^3 - 12*d^2*e^2*x^2 - 12*d^3*e*x + 3*d^4)*s
qrt(-e^2*x^2 + d^2))/(d^5*e^6*x^5 + d^6*e^5*x^4 - 2*d^7*e^4*x^3 - 2*d^8*e^3
*x^2 + d^9*e^2*x + d^10*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(1/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\mathrm{undef}, \mathrm{undef}, \mathrm{undef}, \mathrm{undef}, \mathrm{undef}, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, 1]
```

3.843 $\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=115

$$\frac{8x}{21d^6\sqrt{d^2-e^2x^2}}+\frac{4x}{21d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{1}{7d^2e(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{1}{7de(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] (4*x)/(21*d^4*(d^2 - e^2*x^2)^(3/2)) - 1/(7*d*e*(d + e*x)^2*(d^2 - e^2*x^2)
^(3/2)) - 1/(7*d^2*e*(d + e*x)*(d^2 - e^2*x^2)^(3/2)) + (8*x)/(21*d^6*Sqrt[
d^2 - e^2*x^2])
```

Rubi [A] time = 0.0353934, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {659, 192, 191}

$$\frac{8x}{21d^6\sqrt{d^2-e^2x^2}}+\frac{4x}{21d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{1}{7d^2e(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{1}{7de(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (4*x)/(21*d^4*(d^2 - e^2*x^2)^(3/2)) - 1/(7*d*e*(d + e*x)^2*(d^2 - e^2*x^2)
^(3/2)) - 1/(7*d^2*e*(d + e*x)*(d^2 - e^2*x^2)^(3/2)) + (8*x)/(21*d^6*Sqrt[
d^2 - e^2*x^2])
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}\,dx = -\frac{1}{7de(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{5\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx}{7d}$$

$$= -\frac{1}{7de(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{1}{7d^2e(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{7d^2}$$

$$= \frac{4x}{21d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{1}{7de(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{1}{7d^2e(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{8\int \frac{1}{(d^2-}}{2}$$

$$= \frac{4x}{21d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{1}{7de(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{1}{7d^2e(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{8}{21d^6\sqrt{d}}$$

Mathematica [A] time = 0.0612647, size = 93, normalized size = 0.81

$$\frac{\sqrt{d^2-e^2x^2}\left(24d^3e^2x^2+4d^2e^3x^3+9d^4ex-6d^5-16de^4x^4-8e^5x^5\right)}{21d^6e(d-ex)^2(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-6*d^5 + 9*d^4*e*x + 24*d^3*e^2*x^2 + 4*d^2*e^3*x^3 -
 16*d*e^4*x^4 - 8*e^5*x^5))/(21*d^6*e*(d - e*x)^2*(d + e*x)^4)
```

Maple [A] time = 0.046, size = 88, normalized size = 0.8

$$-\frac{(-ex+d)\left(8e^5x^5+16e^4x^4d-4e^3x^3d^2-24e^2x^2d^3-9xd^4e+6d^5\right)}{(21ex+21d)d^6e}\left(-e^2x^2+d^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/21*(-e*x+d)*(8*e^5*x^5+16*d*e^4*x^4-4*d^2*e^3*x^3-24*d^3*e^2*x^2-9*d^4*e
*x+6*d^5)/(e*x+d)/d^6/e/(-e^2*x^2+d^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.35041, size = 413, normalized size = 3.59

$$-\frac{6\,e^{6}x^{6}+12\,de^{5}x^{5}-6\,d^{2}e^{4}x^{4}-24\,d^{3}e^{3}x^{3}-6\,d^{4}e^{2}x^{2}+12\,d^{5}ex+6\,d^{6}+\left(8\,e^{5}x^{5}+16\,de^{4}x^{4}-4\,d^{2}e^{3}x^{3}-24\,d^{3}e^{2}x^{2}-9\,d}{21\,\left(d^{6}e^{7}x^{6}+2\,d^{7}e^{6}x^{5}-d^{8}e^{5}x^{4}-4\,d^{9}e^{4}x^{3}-d^{10}e^{3}x^{2}+2\,d^{11}e^{2}x+d^{12}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/21*(6*e^6*x^6 + 12*d*e^5*x^5 - 6*d^2*e^4*x^4 - 24*d^3*e^3*x^3 - 6*d^4*e^
2*x^2 + 12*d^5*e*x + 6*d^6 + (8*e^5*x^5 + 16*d*e^4*x^4 - 4*d^2*e^3*x^3 - 24
*d^3*e^2*x^2 - 9*d^4*e*x + 6*d^5)*sqrt(-e^2*x^2 + d^2))/(d^6*e^7*x^6 + 2*d^
7*e^6*x^5 - d^8*e^5*x^4 - 4*d^9*e^4*x^3 - d^10*e^3*x^2 + 2*d^11*e^2*x + d^1
2*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(1/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.844 $\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{5/2}} dx$

Optimal. Leaf size=148

$$\frac{16x}{63d^7\sqrt{d^2-e^2x^2}}+\frac{8x}{63d^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{2}{21d^3e(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{2}{21d^2e(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{1}{9de(d+ex)^3\left(d^2-\right.}$$

```
[Out] (8*x)/(63*d^5*(d^2 - e^2*x^2)^(3/2)) - 1/(9*d*e*(d + e*x)^3*(d^2 - e^2*x^2)
^(3/2)) - 2/(21*d^2*e*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)) - 2/(21*d^3*e*(d +
 e*x)*(d^2 - e^2*x^2)^(3/2)) + (16*x)/(63*d^7*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.053541, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {659, 192, 191}

$$\frac{16x}{63d^7\sqrt{d^2-e^2x^2}}+\frac{8x}{63d^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{2}{21d^3e(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{2}{21d^2e(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{1}{9de(d+ex)^3\left(d^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (8*x)/(63*d^5*(d^2 - e^2*x^2)^(3/2)) - 1/(9*d*e*(d + e*x)^3*(d^2 - e^2*x^2)
^(3/2)) - 2/(21*d^2*e*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)) - 2/(21*d^3*e*(d +
 e*x)*(d^2 - e^2*x^2)^(3/2)) + (16*x)/(63*d^7*Sqrt[d^2 - e^2*x^2])
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}\,dx = -\frac{1}{9de(d+ex)^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{2\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{3d}$$

$$= -\frac{1}{9de(d+ex)^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{2}{21d^2e(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{10\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx}{21d^2}$$

$$= -\frac{1}{9de(d+ex)^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{2}{21d^2e(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{2}{21d^3e(d+ex)\left(d^2-e^2x\right.}$$

$$= \frac{8x}{63d^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{1}{9de(d+ex)^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{2}{21d^2e(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{}{21}$$

$$= \frac{8x}{63d^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{1}{9de(d+ex)^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{2}{21d^2e(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{}{21}$$

Mathematica [A] time = 0.0679166, size = 104, normalized size = 0.7

$$-\frac{\sqrt{d^2-e^2x^2}\left(-66d^4e^2x^2-56d^3e^3x^3+24d^2e^4x^4-6d^5ex+19d^6+48de^5x^5+16e^6x^6\right)}{63d^7e(d-ex)^2(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]*(19*d^6 - 6*d^5*e*x - 66*d^4*e^2*x^2 - 56*d^3*e^3*x^3
 + 24*d^2*e^4*x^4 + 48*d*e^5*x^5 + 16*e^6*x^6))/(63*d^7*e*(d - e*x)^2*(d +
e*x)^5)
```

Maple [A] time = 0.045, size = 99, normalized size = 0.7

$$-\frac{(-ex+d)\left(16e^6x^6+48e^5x^5d+24e^4x^4d^2-56e^3x^3d^3-66e^2x^2d^4-6xd^5e+19d^6\right)}{63ed^7(ex+d)^2}\left(-e^2x^2+d^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/63*(-e*x+d)*(16*e^6*x^6+48*d*e^5*x^5+24*d^2*e^4*x^4-56*d^3*e^3*x^3-66*d^
4*e^2*x^2-6*d^5*e*x+19*d^6)/(e*x+d)^2/d^7/e/(-e^2*x^2+d^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.9897, size = 493, normalized size = 3.33

$$-\frac{19\,e^{7}x^{7}+57\,de^{6}x^{6}+19\,d^{2}e^{5}x^{5}-95\,d^{3}e^{4}x^{4}-95\,d^{4}e^{3}x^{3}+19\,d^{5}e^{2}x^{2}+57\,d^{6}ex+19\,d^{7}+\left(16\,e^{6}x^{6}+48\,de^{5}x^{5}+24\,d^{2}e^{4}x^{4}\right.}{63\left(d^{7}e^{8}x^{7}+3\,d^{8}e^{7}x^{6}+d^{9}e^{6}x^{5}-5\,d^{10}e^{5}x^{4}-5\,d^{11}e^{4}x^{3}+d^{12}e^{3}x^{2}+3\,d^{13}e^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/63*(19*e^7*x^7 + 57*d*e^6*x^6 + 19*d^2*e^5*x^5 - 95*d^3*e^4*x^4 - 95*d^4
*e^3*x^3 + 19*d^5*e^2*x^2 + 57*d^6*e*x + 19*d^7 + (16*e^6*x^6 + 48*d*e^5*x^
5 + 24*d^2*e^4*x^4 - 56*d^3*e^3*x^3 - 66*d^4*e^2*x^2 - 6*d^5*e*x + 19*d^6)*
sqrt(-e^2*x^2 + d^2))/(d^7*e^8*x^7 + 3*d^8*e^7*x^6 + d^9*e^6*x^5 - 5*d^10*e
^5*x^4 - 5*d^11*e^4*x^3 + d^12*e^3*x^2 + 3*d^13*e^2*x + d^14*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(1/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, 1]
```

3.845 $\int \frac{1}{(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=181

$$\frac{16x}{99d^8\sqrt{d^2-e^2x^2}}+\frac{8x}{99d^6\left(d^2-e^2x^2\right)^{3/2}}-\frac{2}{33d^4e(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{2}{33d^3e(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{7}{99d^2e(d+ex)}$$

```
[Out] (8*x)/(99*d^6*(d^2 - e^2*x^2)^(3/2)) - 1/(11*d*e*(d + e*x)^4*(d^2 - e^2*x^2
)^(3/2)) - 7/(99*d^2*e*(d + e*x)^3*(d^2 - e^2*x^2)^(3/2)) - 2/(33*d^3*e*(d
+ e*x)^2*(d^2 - e^2*x^2)^(3/2)) - 2/(33*d^4*e*(d + e*x)*(d^2 - e^2*x^2)^(3/
2)) + (16*x)/(99*d^8*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.0747313, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {659, 192, 191}

$$\frac{16x}{99d^8\sqrt{d^2-e^2x^2}}+\frac{8x}{99d^6\left(d^2-e^2x^2\right)^{3/2}}-\frac{2}{33d^4e(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{2}{33d^3e(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{7}{99d^2e(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^4*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (8*x)/(99*d^6*(d^2 - e^2*x^2)^(3/2)) - 1/(11*d*e*(d + e*x)^4*(d^2 - e^2*x^2
)^(3/2)) - 7/(99*d^2*e*(d + e*x)^3*(d^2 - e^2*x^2)^(3/2)) - 2/(33*d^3*e*(d
+ e*x)^2*(d^2 - e^2*x^2)^(3/2)) - 2/(33*d^4*e*(d + e*x)*(d^2 - e^2*x^2)^(3/
2)) + (16*x)/(99*d^8*Sqrt[d^2 - e^2*x^2])
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}}\,dx = -\frac{1}{11de(d+ex)^4 \left(d^2-e^2x^2\right)^{3/2}} + \frac{7\int \frac{1}{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}\,dx}{11d}$$

$$= -\frac{1}{11de(d+ex)^4 \left(d^2-e^2x^2\right)^{3/2}} - \frac{7}{99d^2e(d+ex)^3 \left(d^2-e^2x^2\right)^{3/2}} + \frac{14\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{33d^2}$$

$$= -\frac{1}{11de(d+ex)^4 \left(d^2-e^2x^2\right)^{3/2}} - \frac{7}{99d^2e(d+ex)^3 \left(d^2-e^2x^2\right)^{3/2}} - \frac{2}{33d^3e(d+ex)^2 \left(d^2-e^2x^2\right.}$$

$$= -\frac{1}{11de(d+ex)^4 \left(d^2-e^2x^2\right)^{3/2}} - \frac{7}{99d^2e(d+ex)^3 \left(d^2-e^2x^2\right)^{3/2}} - \frac{2}{33d^3e(d+ex)^2 \left(d^2-e^2x^2\right.}$$

$$= \frac{8x}{99d^6 \left(d^2-e^2x^2\right)^{3/2}} - \frac{1}{11de(d+ex)^4 \left(d^2-e^2x^2\right)^{3/2}} - \frac{7}{99d^2e(d+ex)^3 \left(d^2-e^2x^2\right)^{3/2}} - \frac{}{33d^3}$$

$$= \frac{8x}{99d^6 \left(d^2-e^2x^2\right)^{3/2}} - \frac{1}{11de(d+ex)^4 \left(d^2-e^2x^2\right)^{3/2}} - \frac{7}{99d^2e(d+ex)^3 \left(d^2-e^2x^2\right)^{3/2}} - \frac{}{33d^3}$$

Mathematica [A] time = 0.0675843, size = 115, normalized size = 0.64

$$-\frac{\sqrt{d^2-e^2x^2}\left(-72d^5e^2x^2-122d^4e^3x^3-32d^3e^4x^4+72d^2e^5x^5+13d^6ex+28d^7+64de^6x^6+16e^7x^7\right)}{99d^8e(d-ex)^2(d+ex)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^4*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]*(28*d^7 + 13*d^6*e*x - 72*d^5*e^2*x^2 - 122*d^4*e^3*x
^3 - 32*d^3*e^4*x^4 + 72*d^2*e^5*x^5 + 64*d*e^6*x^6 + 16*e^7*x^7))/(99*d^8*
e*(d - e*x)^2*(d + e*x)^6)
```

Maple [A] time = 0.047, size = 110, normalized size = 0.6

$$-\frac{(-ex+d)\left(16\,e^7x^7+64\,e^6x^6d+72\,e^5x^5d^2-32\,e^4x^4d^3-122\,e^3x^3d^4-72\,e^2x^2d^5+13\,xd^6e+28\,d^7\right)}{99\,ed^8\,(ex+d)^3}\left(-e^2x^2+d^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^4/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/99*(-e*x+d)*(16*e^7*x^7+64*d*e^6*x^6+72*d^2*e^5*x^5-32*d^3*e^4*x^4-122*d
^4*e^3*x^3-72*d^5*e^2*x^2+13*d^6*e*x+28*d^7)/(e*x+d)^3/d^8/e/(-e^2*x^2+d^2)
^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.69455, size = 582, normalized size = 3.22

$$-\frac{28\,e^8x^8+112\,de^7x^7+112\,d^2e^6x^6-112\,d^3e^5x^5-280\,d^4e^4x^4-112\,d^5e^3x^3+112\,d^6e^2x^2+112\,d^7ex+28\,d^8+\left(16\,e^7x^7\right.}{99\left(d^8e^9x^8+4\,d^9e^8x^7+4\,d^{10}e^7x^6-4\,d^{11}e^6x^5-10\,d^{12}e^5x^4-4\,d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/99*(28*e^8*x^8 + 112*d*e^7*x^7 + 112*d^2*e^6*x^6 - 112*d^3*e^5*x^5 - 280
*d^4*e^4*x^4 - 112*d^5*e^3*x^3 + 112*d^6*e^2*x^2 + 112*d^7*e*x + 28*d^8 + (
16*e^7*x^7 + 64*d*e^6*x^6 + 72*d^2*e^5*x^5 - 32*d^3*e^4*x^4 - 122*d^4*e^3*x
^3 - 72*d^5*e^2*x^2 + 13*d^6*e*x + 28*d^7)*sqrt(-e^2*x^2 + d^2))/(d^8*e^9*x
^8 + 4*d^9*e^8*x^7 + 4*d^10*e^7*x^6 - 4*d^11*e^6*x^5 - 10*d^12*e^5*x^4 - 4*
d^13*e^4*x^3 + 4*d^14*e^3*x^2 + 4*d^15*e^2*x + d^16*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**4/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(1/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\mathrm{undef},\mathrm{undef},\mathrm{undef},\mathrm{undef},\mathrm{undef},1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, 1]
```

3.846 $\int \frac{(d+ex)^9}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=206

$$\frac{2(d+ex)^8}{5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{22(d+ex)^6}{15e\left(d^2-e^2x^2\right)^{3/2}}+\frac{66(d+ex)^4}{5e\sqrt{d^2-e^2x^2}}+\frac{77\sqrt{d^2-e^2x^2}(d+ex)^2}{5e}+\frac{77d\sqrt{d^2-e^2x^2}(d+ex)}{2e}+\frac{231d^2\sqrt{d^2-}}{2e}$$

```
[Out] (2*(d + e*x)^8)/(5*e*(d^2 - e^2*x^2)^(5/2)) - (22*(d + e*x)^6)/(15*e*(d^2 -
 e^2*x^2)^(3/2)) + (66*(d + e*x)^4)/(5*e*Sqrt[d^2 - e^2*x^2]) + (231*d^2*Sq
rt[d^2 - e^2*x^2])/(2*e) + (77*d*(d + e*x)*Sqrt[d^2 - e^2*x^2])/(2*e) + (77
*(d + e*x)^2*Sqrt[d^2 - e^2*x^2])/(5*e) - (231*d^3*ArcTan[(e*x)/Sqrt[d^2 -
e^2*x^2]])/(2*e)
```

Rubi [A] time = 0.101482, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {669, 671, 641, 217, 203}

$$\frac{2(d+ex)^8}{5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{22(d+ex)^6}{15e\left(d^2-e^2x^2\right)^{3/2}}+\frac{66(d+ex)^4}{5e\sqrt{d^2-e^2x^2}}+\frac{77\sqrt{d^2-e^2x^2}(d+ex)^2}{5e}+\frac{77d\sqrt{d^2-e^2x^2}(d+ex)}{2e}+\frac{231d^2\sqrt{d^2-}}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^9/(d^2 - e^2*x^2)^(7/2),x]
```


```
[Out] (2*(d + e*x)^8)/(5*e*(d^2 - e^2*x^2)^(5/2)) - (22*(d + e*x)^6)/(15*e*(d^2 -
 e^2*x^2)^(3/2)) + (66*(d + e*x)^4)/(5*e*Sqrt[d^2 - e^2*x^2]) + (231*d^2*Sq
rt[d^2 - e^2*x^2])/(2*e) + (77*d*(d + e*x)*Sqrt[d^2 - e^2*x^2])/(2*e) + (77
*(d + e*x)^2*Sqrt[d^2 - e^2*x^2])/(5*e) - (231*d^3*ArcTan[(e*x)/Sqrt[d^2 -
e^2*x^2]])/(2*e)
```


Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^9}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{2(d+ex)^8}{5e\left(d^2-e^2x^2\right)^{5/2}} - \frac{11}{5}\int \frac{(d+ex)^7}{\left(d^2-e^2x^2\right)^{5/2}}\,dx$$
$$= \frac{2(d+ex)^8}{5e\left(d^2-e^2x^2\right)^{5/2}} - \frac{22(d+ex)^6}{15e\left(d^2-e^2x^2\right)^{3/2}} + \frac{33}{5}\int \frac{(d+ex)^5}{\left(d^2-e^2x^2\right)^{3/2}}\,dx$$
$$= \frac{2(d+ex)^8}{5e\left(d^2-e^2x^2\right)^{5/2}} - \frac{22(d+ex)^6}{15e\left(d^2-e^2x^2\right)^{3/2}} + \frac{66(d+ex)^4}{5e\sqrt{d^2-e^2x^2}} - \frac{231}{5}\int \frac{(d+ex)^3}{\sqrt{d^2-e^2x^2}}\,dx$$
$$= \frac{2(d+ex)^8}{5e\left(d^2-e^2x^2\right)^{5/2}} - \frac{22(d+ex)^6}{15e\left(d^2-e^2x^2\right)^{3/2}} + \frac{66(d+ex)^4}{5e\sqrt{d^2-e^2x^2}} + \frac{77(d+ex)^2\sqrt{d^2-e^2x^2}}{5e} - (77d)\int \frac{(d}{\sqrt{d^2}}$$
$$= \frac{2(d+ex)^8}{5e\left(d^2-e^2x^2\right)^{5/2}} - \frac{22(d+ex)^6}{15e\left(d^2-e^2x^2\right)^{3/2}} + \frac{66(d+ex)^4}{5e\sqrt{d^2-e^2x^2}} + \frac{77d(d+ex)\sqrt{d^2-e^2x^2}}{2e} + \frac{77(d+ex)^2}{5e}$$
$$= \frac{2(d+ex)^8}{5e\left(d^2-e^2x^2\right)^{5/2}} - \frac{22(d+ex)^6}{15e\left(d^2-e^2x^2\right)^{3/2}} + \frac{66(d+ex)^4}{5e\sqrt{d^2-e^2x^2}} + \frac{231d^2\sqrt{d^2-e^2x^2}}{2e} + \frac{77d(d+ex)\sqrt{d^2-}}{2e}$$
$$= \frac{2(d+ex)^8}{5e\left(d^2-e^2x^2\right)^{5/2}} - \frac{22(d+ex)^6}{15e\left(d^2-e^2x^2\right)^{3/2}} + \frac{66(d+ex)^4}{5e\sqrt{d^2-e^2x^2}} + \frac{231d^2\sqrt{d^2-e^2x^2}}{2e} + \frac{77d(d+ex)\sqrt{d^2-}}{2e}$$
$$= \frac{2(d+ex)^8}{5e\left(d^2-e^2x^2\right)^{5/2}} - \frac{22(d+ex)^6}{15e\left(d^2-e^2x^2\right)^{3/2}} + \frac{66(d+ex)^4}{5e\sqrt{d^2-e^2x^2}} + \frac{231d^2\sqrt{d^2-e^2x^2}}{2e} + \frac{77d(d+ex)\sqrt{d^2-}}{2e}$$

Mathematica [A] time = 0.342552, size = 144, normalized size = 0.7

$$\frac{(d+ex)\left(\sqrt{1-\frac{e^2x^2}{d^2}}\left(8711d^3e^2x^2-815d^2e^3x^3-12843d^4ex+5446d^5-105de^4x^4-10e^5x^5\right)-3465d^2(d-ex)^3\sin^{-1}\left(\frac{ex}{d}\right.\right.}{30e(d-ex)^2\sqrt{d^2-e^2x^2}\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^9/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((d + e*x)*(Sqrt[1 - (e^2*x^2)/d^2]*(5446*d^5 - 12843*d^4*e*x + 8711*d^3*e^
2*x^2 - 815*d^2*e^3*x^3 - 105*d*e^4*x^4 - 10*e^5*x^5) - 3465*d^2*(d - e*x)^
3*ArcSin[(e*x)/d]))/(30*e*(d - e*x)^2*Sqrt[d^2 - e^2*x^2]*Sqrt[1 - (e^2*x^2
)/d^2])
```

Maple [A] time = 0.209, size = 309, normalized size = 1.5

$$-\frac{152\,d^7x}{5}\left(-e^2x^2+d^2\right)^{-\frac{5}{2}} + \frac{157\,d^5x}{15}\left(-e^2x^2+d^2\right)^{-\frac{3}{2}} + 63\,\frac{d^5e^2x^3}{\left(-e^2x^2+d^2\right)^{5/2}} - \frac{231\,d^3}{2}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right)\frac{1}{\sqrt{e^2}} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^9/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -152/5*d^7*x/(-e^2*x^2+d^2)^(5/2)+157/15*d^5*x/(-e^2*x^2+d^2)^(3/2)+63*d^5*
e^2*x^3/(-e^2*x^2+d^2)^(5/2)-231/2*d^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e
^2*x^2+d^2)^(1/2))+4093/30*d^3*x/(-e^2*x^2+d^2)^(1/2)-9/2*d*e^6*x^7/(-e^2*x
^2+d^2)^(5/2)+231/10*d^3*e^4*x^5/(-e^2*x^2+d^2)^(5/2)-77/2*d^3*e^2*x^3/(-e^
2*x^2+d^2)^(3/2)-116/3*e^5*d^2*x^6/(-e^2*x^2+d^2)^(5/2)+358*e^3*d^4*x^4/(-e
^2*x^2+d^2)^(5/2)-1348/3*e*d^6*x^2/(-e^2*x^2+d^2)^(5/2)-1/3*e^7*x^8/(-e^2*x
^2+d^2)^(5/2)+2723/15*d^8/e/(-e^2*x^2+d^2)^(5/2)
```

Maxima [B] time = 1.87807, size = 518, normalized size = 2.51

$$-\frac{e^7x^8}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{9\,de^6x^7}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{77}{10}d^3e^6x\left(\frac{15\,x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{20\,d^2x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{8\,d^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}\right)-\frac{116\,d^2e^5x}{3\left(-e^2x^2+d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^9/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/3*e^7*x^8/(-e^2*x^2 + d^2)^(5/2) - 9/2*d*e^6*x^7/(-e^2*x^2 + d^2)^(5/2)
+ 77/10*d^3*e^6*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*
x^2 + d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - 116/3*d^2*e^5
*x^6/(-e^2*x^2 + d^2)^(5/2) - 77/2*d^3*e^4*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)
*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4)) + 358*d^4*e^3*x^4/(-e^2*x^2 + d
^2)^(5/2) + 63*d^5*e^2*x^3/(-e^2*x^2 + d^2)^(5/2) - 1348/3*d^6*e*x^2/(-e^2*
x^2 + d^2)^(5/2) - 152/5*d^7*x/(-e^2*x^2 + d^2)^(5/2) + 2723/15*d^8/((-e^2*
x^2 + d^2)^(5/2)*e) + 619/15*d^5*x/(-e^2*x^2 + d^2)^(3/2) - 989/30*d^3*x/sq
rt(-e^2*x^2 + d^2) - 231/2*d^3*arcsin(e^2*x/sqrt(d^2*e^2))/sqrt(e^2)
```

Fricas [A] time = 2.54416, size = 447, normalized size = 2.17

$$\frac{5446\,d^3e^3x^3-16338\,d^4e^2x^2+16338\,d^5ex-5446\,d^6+6930\left(d^3e^3x^3-3\,d^4e^2x^2+3\,d^5ex-d^6\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(1}{30\left(e^4x^3-3\,de^3x^2+3\,d^2e^2x-d^3e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^9/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/30*(5446*d^3*e^3*x^3 - 16338*d^4*e^2*x^2 + 16338*d^5*e*x - 5446*d^6 + 693
0*(d^3*e^3*x^3 - 3*d^4*e^2*x^2 + 3*d^5*e*x - d^6)*arctan(-(d - sqrt(-e^2*x^
2 + d^2))/(e*x)) + (10*e^5*x^5 + 105*d*e^4*x^4 + 815*d^2*e^3*x^3 - 8711*d^3
*e^2*x^2 + 12843*d^4*e*x - 5446*d^5)*sqrt(-e^2*x^2 + d^2))/(e^4*x^3 - 3*d*e
^3*x^2 + 3*d^2*e^2*x - d^3*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(d+ex)^9}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**9/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**9/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [A] time = 1.32293, size = 174, normalized size = 0.84

$$-\frac{231}{2}\,d^3\arcsin\left(\frac{xe}{d}\right)e^{(-1)}\mathrm{sgn}\,(d)-\frac{\left(5446\,d^8e^{(-1)}+\left(3495\,d^7-\left(13480\,d^6e+\left(7765\,d^5e^2-\left(10740\,d^4e^3+\left(5941\,d^3e^4-5\right.\right.\right.\right.\right.\right.}{30\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^9/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -231/2*d^3*arcsin(x*e/d)*e^(-1)*sgn(d) - 1/30*(5446*d^8*e^(-1) + (3495*d^7
- (13480*d^6*e + (7765*d^5*e^2 - (10740*d^4*e^3 + (5941*d^3*e^4 - 5*(232*d^
2*e^5 + (2*x*e^7 + 27*d*e^6)*x)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)/(x^2
*e^2 - d^2)^3
```

3.847 $\int \frac{(d+ex)^8}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=173

$$\frac{2(d+ex)^7}{5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{6(d+ex)^5}{5e\left(d^2-e^2x^2\right)^{3/2}}+\frac{42(d+ex)^3}{5e\sqrt{d^2-e^2x^2}}+\frac{21\sqrt{d^2-e^2x^2}(d+ex)}{2e}+\frac{63d\sqrt{d^2-e^2x^2}}{2e}-\frac{63d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

```
[Out] (2*(d + e*x)^7)/(5*e*(d^2 - e^2*x^2)^(5/2)) - (6*(d + e*x)^5)/(5*e*(d^2 - e
^2*x^2)^(3/2)) + (42*(d + e*x)^3)/(5*e*Sqrt[d^2 - e^2*x^2]) + (63*d*Sqrt[d^
2 - e^2*x^2])/(2*e) + (21*(d + e*x)*Sqrt[d^2 - e^2*x^2])/(2*e) - (63*d^2*Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)
```

Rubi [A] time = 0.0768588, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {669, 671, 641, 217, 203}

$$\frac{2(d+ex)^7}{5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{6(d+ex)^5}{5e\left(d^2-e^2x^2\right)^{3/2}}+\frac{42(d+ex)^3}{5e\sqrt{d^2-e^2x^2}}+\frac{21\sqrt{d^2-e^2x^2}(d+ex)}{2e}+\frac{63d\sqrt{d^2-e^2x^2}}{2e}-\frac{63d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^8/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (2*(d + e*x)^7)/(5*e*(d^2 - e^2*x^2)^(5/2)) - (6*(d + e*x)^5)/(5*e*(d^2 - e
^2*x^2)^(3/2)) + (42*(d + e*x)^3)/(5*e*Sqrt[d^2 - e^2*x^2]) + (63*d*Sqrt[d^
2 - e^2*x^2])/(2*e) + (21*(d + e*x)*Sqrt[d^2 - e^2*x^2])/(2*e) - (63*d^2*Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^8}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{2(d+ex)^7}{5e(d^2-e^2x^2)^{5/2}} - \frac{9}{5}\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{5/2}}\,dx$$

$$= \frac{2(d+ex)^7}{5e(d^2-e^2x^2)^{5/2}} - \frac{6(d+ex)^5}{5e(d^2-e^2x^2)^{3/2}} + \frac{21}{5}\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{3/2}}\,dx$$

$$= \frac{2(d+ex)^7}{5e(d^2-e^2x^2)^{5/2}} - \frac{6(d+ex)^5}{5e(d^2-e^2x^2)^{3/2}} + \frac{42(d+ex)^3}{5e\sqrt{d^2-e^2x^2}} - 21\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$$

$$= \frac{2(d+ex)^7}{5e(d^2-e^2x^2)^{5/2}} - \frac{6(d+ex)^5}{5e(d^2-e^2x^2)^{3/2}} + \frac{42(d+ex)^3}{5e\sqrt{d^2-e^2x^2}} + \frac{21(d+ex)\sqrt{d^2-e^2x^2}}{2e} - \frac{1}{2}(63d)\int \frac{d}{\sqrt{d^2}}$$

$$= \frac{2(d+ex)^7}{5e(d^2-e^2x^2)^{5/2}} - \frac{6(d+ex)^5}{5e(d^2-e^2x^2)^{3/2}} + \frac{42(d+ex)^3}{5e\sqrt{d^2-e^2x^2}} + \frac{63d\sqrt{d^2-e^2x^2}}{2e} + \frac{21(d+ex)\sqrt{d^2-e^2x^2}}{2e}$$

$$= \frac{2(d+ex)^7}{5e(d^2-e^2x^2)^{5/2}} - \frac{6(d+ex)^5}{5e(d^2-e^2x^2)^{3/2}} + \frac{42(d+ex)^3}{5e\sqrt{d^2-e^2x^2}} + \frac{63d\sqrt{d^2-e^2x^2}}{2e} + \frac{21(d+ex)\sqrt{d^2-e^2x^2}}{2e}$$

$$= \frac{2(d+ex)^7}{5e(d^2-e^2x^2)^{5/2}} - \frac{6(d+ex)^5}{5e(d^2-e^2x^2)^{3/2}} + \frac{42(d+ex)^3}{5e\sqrt{d^2-e^2x^2}} + \frac{63d\sqrt{d^2-e^2x^2}}{2e} + \frac{21(d+ex)\sqrt{d^2-e^2x^2}}{2e}$$

Mathematica [A] time = 0.282841, size = 131, normalized size = 0.76

$$\frac{(d+ex)\left(\sqrt{1-\frac{e^2x^2}{d^2}}\left(801d^2e^2x^2-1163d^3ex+496d^4-65de^3x^3-5e^4x^4\right)-315d(d-ex)^3\sin^{-1}\left(\frac{ex}{d}\right)\right)}{10e(d-ex)^2\sqrt{d^2-e^2x^2}\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^8/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((d + e*x)*(Sqrt[1 - (e^2*x^2)/d^2]*(496*d^4 - 1163*d^3*e*x + 801*d^2*e^2*x^2 - 65*d*e^3*x^3 - 5*e^4*x^4) - 315*d*(d - e*x)^3*ArcSin[(e*x)/d]))/(10*e*(d - e*x)^2*Sqrt[d^2 - e^2*x^2]*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [A] time = 0.148, size = 284, normalized size = 1.6

$$-\frac{76\,d^6x}{5}\left(-e^2x^2+d^2\right)^{-\frac{5}{2}} - \frac{e^6x^7}{2}\left(-e^2x^2+d^2\right)^{-\frac{5}{2}} + \frac{248\,d^7}{5\,e}\left(-e^2x^2+d^2\right)^{-\frac{5}{2}} - \frac{21\,d^2e^2x^3}{2}\left(-e^2x^2+d^2\right)^{-\frac{3}{2}} + 35\,\frac{d^4e^2x^3}{\left(-e^2x^2+d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^8/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -76/5*d^6*x/(-e^2*x^2+d^2)^(5/2)-1/2*e^6*x^7/(-e^2*x^2+d^2)^(5/2)+248/5*d^7
/e/(-e^2*x^2+d^2)^(5/2)-21/2*e^2*d^2*x^3/(-e^2*x^2+d^2)^(3/2)+35*d^4*e^2*x^
3/(-e^2*x^2+d^2)^(5/2)+63/10*e^4*d^2*x^5/(-e^2*x^2+d^2)^(5/2)-8*e^5*d*x^6/(
-e^2*x^2+d^2)^(5/2)+104*e^3*d^3*x^4/(-e^2*x^2+d^2)^(5/2)-120*e*d^5*x^2/(-e^
2*x^2+d^2)^(5/2)-63/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(
1/2))+27/5*d^4*x/(-e^2*x^2+d^2)^(3/2)+423/10*d^2*x/(-e^2*x^2+d^2)^(1/2)
```

Maxima [B] time = 1.81844, size = 485, normalized size = 2.8

$$-\frac{e^6x^7}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{21}{10}d^2e^6x\left(\frac{15\,x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{20\,d^2x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{8\,d^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}\right)-\frac{8\,de^5x^6}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{21}{2}d^2e^4x\left(\frac{}{\left(-e^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^8/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/2*e^6*x^7/(-e^2*x^2 + d^2)^(5/2) + 21/10*d^2*e^6*x*(15*x^4/((-e^2*x^2 +
d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^
2 + d^2)^(5/2)*e^6)) - 8*d*e^5*x^6/(-e^2*x^2 + d^2)^(5/2) - 21/2*d^2*e^4*x*
(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4)) +
 104*d^3*e^3*x^4/(-e^2*x^2 + d^2)^(5/2) + 35*d^4*e^2*x^3/(-e^2*x^2 + d^2)^(
5/2) - 120*d^5*e*x^2/(-e^2*x^2 + d^2)^(5/2) - 76/5*d^6*x/(-e^2*x^2 + d^2)^(
5/2) + 248/5*d^7/((-e^2*x^2 + d^2)^(5/2)*e) + 69/5*d^4*x/(-e^2*x^2 + d^2)^(
3/2) - 39/10*d^2*x/sqrt(-e^2*x^2 + d^2) - 63/2*d^2*arcsin(e^2*x/sqrt(d^2*e^
2))/sqrt(e^2)
```

Fricas [A] time = 2.3162, size = 409, normalized size = 2.36

$$\frac{496\,d^2e^3x^3-1488\,d^3e^2x^2+1488\,d^4ex-496\,d^5+630\left(d^2e^3x^3-3\,d^3e^2x^2+3\,d^4ex-d^5\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(5\,e^4x^4\right.}{10\left(e^4x^3-3\,de^3x^2+3\,d^2e^2x-d^3e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^8/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/10*(496*d^2*e^3*x^3 - 1488*d^3*e^2*x^2 + 1488*d^4*e*x - 496*d^5 + 630*(d^
2*e^3*x^3 - 3*d^3*e^2*x^2 + 3*d^4*e*x - d^5)*arctan(-(d - sqrt(-e^2*x^2 + d
^2))/(e*x)) + (5*e^4*x^4 + 65*d*e^3*x^3 - 801*d^2*e^2*x^2 + 1163*d^3*e*x -
496*d^4)*sqrt(-e^2*x^2 + d^2))/(e^4*x^3 - 3*d*e^3*x^2 + 3*d^2*e^2*x - d^3*e
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^8}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**8/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**8/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [A] time = 1.34107, size = 159, normalized size = 0.92

$$-\frac{63}{2} d^2 \arcsin\left(\frac{xe}{d}\right) e^{(-1)}\mathrm{sgn}\left(d\right) - \frac{\left(496 d^7 e^{(-1)} + \left(325 d^6 - \left(1200 d^5 e + \left(655 d^4 e^2 - \left(1040 d^3 e^3 + \left(591 d^2 e^4 - 5 \left(xe^6 + 1\right.\right.\right.\right.\right.\right.\right.}{10 \left(x^2 e^2 - d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^8/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -63/2*d^2*arcsin(x*e/d)*e^(-1)*sgn(d) - 1/10*(496*d^7*e^(-1) + (325*d^6 - (
1200*d^5*e + (655*d^4*e^2 - (1040*d^3*e^3 + (591*d^2*e^4 - 5*(x*e^6 + 16*d*
e^5)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^3
```

3.848 $\int \frac{(d+ex)^7}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=138

$$\frac{2(d+ex)^6}{5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{14(d+ex)^4}{15e\left(d^2-e^2x^2\right)^{3/2}}+\frac{14(d+ex)^2}{3e\sqrt{d^2-e^2x^2}}+\frac{7\sqrt{d^2-e^2x^2}}{e}-\frac{7d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

```
[Out] (2*(d + e*x)^6)/(5*e*(d^2 - e^2*x^2)^(5/2)) - (14*(d + e*x)^4)/(15*e*(d^2 -
 e^2*x^2)^(3/2)) + (14*(d + e*x)^2)/(3*e*Sqrt[d^2 - e^2*x^2]) + (7*Sqrt[d^2
 - e^2*x^2])/e - (7*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e
```

Rubi [A] time = 0.0572427, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {669, 641, 217, 203}

$$\frac{2(d+ex)^6}{5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{14(d+ex)^4}{15e\left(d^2-e^2x^2\right)^{3/2}}+\frac{14(d+ex)^2}{3e\sqrt{d^2-e^2x^2}}+\frac{7\sqrt{d^2-e^2x^2}}{e}-\frac{7d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^7/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (2*(d + e*x)^6)/(5*e*(d^2 - e^2*x^2)^(5/2)) - (14*(d + e*x)^4)/(15*e*(d^2 -
 e^2*x^2)^(3/2)) + (14*(d + e*x)^2)/(3*e*Sqrt[d^2 - e^2*x^2]) + (7*Sqrt[d^2
 - e^2*x^2])/e - (7*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^7}{(d^2-e^2x^2)^{7/2}}\,dx &= \frac{2(d+ex)^6}{5e(d^2-e^2x^2)^{5/2}} - \frac{7}{5}\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{5/2}}\,dx \\ &= \frac{2(d+ex)^6}{5e(d^2-e^2x^2)^{5/2}} - \frac{14(d+ex)^4}{15e(d^2-e^2x^2)^{3/2}} + \frac{7}{3}\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{3/2}}\,dx \\ &= \frac{2(d+ex)^6}{5e(d^2-e^2x^2)^{5/2}} - \frac{14(d+ex)^4}{15e(d^2-e^2x^2)^{3/2}} + \frac{14(d+ex)^2}{3e\sqrt{d^2-e^2x^2}} - 7\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}\,dx \\ &= \frac{2(d+ex)^6}{5e(d^2-e^2x^2)^{5/2}} - \frac{14(d+ex)^4}{15e(d^2-e^2x^2)^{3/2}} + \frac{14(d+ex)^2}{3e\sqrt{d^2-e^2x^2}} + \frac{7\sqrt{d^2-e^2x^2}}{e} - (7d)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx \\ &= \frac{2(d+ex)^6}{5e(d^2-e^2x^2)^{5/2}} - \frac{14(d+ex)^4}{15e(d^2-e^2x^2)^{3/2}} + \frac{14(d+ex)^2}{3e\sqrt{d^2-e^2x^2}} + \frac{7\sqrt{d^2-e^2x^2}}{e} - (7d)\,\text{Subst}\left(\int \frac{1}{1+e^2x\ldots}\right. \\ &= \frac{2(d+ex)^6}{5e(d^2-e^2x^2)^{5/2}} - \frac{14(d+ex)^4}{15e(d^2-e^2x^2)^{3/2}} + \frac{14(d+ex)^2}{3e\sqrt{d^2-e^2x^2}} + \frac{7\sqrt{d^2-e^2x^2}}{e} - \frac{7d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}\end{aligned}$$

Mathematica [A] time = 0.240212, size = 119, normalized size = 0.86

$$\frac{(d+ex)\left(\sqrt{1-\frac{e^2x^2}{d^2}}\left(-381d^2ex+167d^3+277de^2x^2-15e^3x^3\right)-105(d-ex)^3\sin^{-1}\left(\frac{ex}{d}\right)\right)}{15e(d-ex)^2\sqrt{d^2-e^2x^2}\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^7/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((d + e*x)*(Sqrt[1 - (e^2*x^2)/d^2]*(167*d^3 - 381*d^2*e*x + 277*d*e^2*x^2
- 15*e^3*x^3) - 105*(d - e*x)^3*ArcSin[(e*x)/d]))/(15*e*(d - e*x)^2*Sqrt[d^
2 - e^2*x^2]*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [B] time = 0.106, size = 253, normalized size = 1.8

$$-e^5x^6\left(-e^2x^2+d^2\right)^{-\frac{5}{2}}+27\frac{e^3d^2x^4}{\left(-e^2x^2+d^2\right)^{5/2}}-\frac{73\,ed^4x^2}{3}\left(-e^2x^2+d^2\right)^{-\frac{5}{2}}+\frac{167\,d^6}{15\,e}\left(-e^2x^2+d^2\right)^{-\frac{5}{2}}+\frac{7\,de^4x^5}{5}\left(-e^2x^2+d^2\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^7/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -e^5*x^6/(-e^2*x^2+d^2)^(5/2)+27*e^3*d^2*x^4/(-e^2*x^2+d^2)^(5/2)-73/3*e*d^
4*x^2/(-e^2*x^2+d^2)^(5/2)+167/15*d^6/e/(-e^2*x^2+d^2)^(5/2)+7/5*d*e^4*x^5/
(-e^2*x^2+d^2)^(5/2)-7/3*d*e^2*x^3/(-e^2*x^2+d^2)^(3/2)+176/15*d*x/(-e^2*x^
2+d^2)^(1/2)-7*d/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+35/
2*d^3*e^2*x^3/(-e^2*x^2+d^2)^(5/2)-61/10*d^5*x/(-e^2*x^2+d^2)^(5/2)+71/30*d
^3*x/(-e^2*x^2+d^2)^(3/2)
```

Maxima [B] time = 2.28444, size = 443, normalized size = 3.21

$$\frac{7}{15}de^6x\left(\frac{15x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{20d^2x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{8d^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}\right)-\frac{e^5x^6}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{7}{3}de^4x\left(\frac{3x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}-\frac{2d}{\left(-e^2x^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 7/15*d*e^6*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 +
 d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - e^5*x^6/(-e^2*x^2
+ d^2)^(5/2) - 7/3*d*e^4*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e
^2*x^2 + d^2)^(3/2)*e^4)) + 27*d^2*e^3*x^4/(-e^2*x^2 + d^2)^(5/2) + 35/2*d^
3*e^2*x^3/(-e^2*x^2 + d^2)^(5/2) - 73/3*d^4*e*x^2/(-e^2*x^2 + d^2)^(5/2) -
61/10*d^5*x/(-e^2*x^2 + d^2)^(5/2) + 167/15*d^6/((-e^2*x^2 + d^2)^(5/2)*e)
+ 127/30*d^3*x/(-e^2*x^2 + d^2)^(3/2) + 22/15*d*x/sqrt(-e^2*x^2 + d^2) - 7*
d*arcsin(e^2*x/sqrt(d^2*e^2))/sqrt(e^2)
```

Fricas [A] time = 2.28796, size = 378, normalized size = 2.74

$$\frac{167\,de^3x^3-501\,d^2e^2x^2+501\,d^3ex-167\,d^4+210\left(de^3x^3-3\,d^2e^2x^2+3\,d^3ex-d^4\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(15\,e^3x^3-27\right.}{15\left(e^4x^3-3\,de^3x^2+3\,d^2e^2x-d^3e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(167*d*e^3*x^3 - 501*d^2*e^2*x^2 + 501*d^3*e*x - 167*d^4 + 210*(d*e^3*
x^3 - 3*d^2*e^2*x^2 + 3*d^3*e*x - d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(
e*x)) + (15*e^3*x^3 - 277*d*e^2*x^2 + 381*d^2*e*x - 167*d^3)*sqrt(-e^2*x^2
+ d^2))/(e^4*x^3 - 3*d*e^3*x^2 + 3*d^2*e^2*x - d^3*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(d+ex)^7}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**7/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**7/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [A] time = 1.34006, size = 144, normalized size = 1.04

$$-7\,d\arcsin\left(\frac{xe}{d}\right)e^{(-1)}\mathrm{sgn}\left(d\right)-\frac{\left(167\,d^6e^{(-1)}+\left(120\,d^5-\left(365\,d^4e+\left(160\,d^3e^2-\left(405\,d^2e^3-\left(15\,xe^5-232\,de^4\right)x\right)x\right)x\right)x\right)x\right)\sqrt{}}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -7*d*arcsin(x*e/d)*e^(-1)*sgn(d) - 1/15*(167*d^6*e^(-1) + (120*d^5 - (365*d
^4*e + (160*d^3*e^2 - (405*d^2*e^3 - (15*x*e^5 - 232*d*e^4)*x)*x)*x)*x)*x)*
sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^3
```

3.849 $\int \frac{(d+ex)^6}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=112

$$\frac{2(d+ex)^5}{5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)^3}{3e\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d+ex)}{e\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

```
[Out] (2*(d + e*x)^5)/(5*e*(d^2 - e^2*x^2)^(5/2)) - (2*(d + e*x)^3)/(3*e*(d^2 - e
^2*x^2)^(3/2)) + (2*(d + e*x))/(e*Sqrt[d^2 - e^2*x^2]) - ArcTan[(e*x)/Sqrt[
d^2 - e^2*x^2]]/e
```

Rubi [A] time = 0.0331972, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {669, 653, 217, 203}

$$\frac{2(d+ex)^5}{5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)^3}{3e\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d+ex)}{e\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^6/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (2*(d + e*x)^5)/(5*e*(d^2 - e^2*x^2)^(5/2)) - (2*(d + e*x)^3)/(3*e*(d^2 - e
^2*x^2)^(3/2)) + (2*(d + e*x))/(e*Sqrt[d^2 - e^2*x^2]) - ArcTan[(e*x)/Sqrt[
d^2 - e^2*x^2]]/e
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^6}{\left(d^2-e^2x^2\right)^{7/2}}dx = \frac{2(d+ex)^5}{5e\left(d^2-e^2x^2\right)^{5/2}} - \int \frac{(d+ex)^4}{\left(d^2-e^2x^2\right)^{5/2}}dx$$
$$= \frac{2(d+ex)^5}{5e\left(d^2-e^2x^2\right)^{5/2}} - \frac{2(d+ex)^3}{3e\left(d^2-e^2x^2\right)^{3/2}} + \int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{3/2}}dx$$
$$= \frac{2(d+ex)^5}{5e\left(d^2-e^2x^2\right)^{5/2}} - \frac{2(d+ex)^3}{3e\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(d+ex)}{e\sqrt{d^2-e^2x^2}} - \int \frac{1}{\sqrt{d^2-e^2x^2}}dx$$
$$= \frac{2(d+ex)^5}{5e\left(d^2-e^2x^2\right)^{5/2}} - \frac{2(d+ex)^3}{3e\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(d+ex)}{e\sqrt{d^2-e^2x^2}} - \text{Subst}\left(\int \frac{1}{1+e^2x^2}dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)$$
$$= \frac{2(d+ex)^5}{5e\left(d^2-e^2x^2\right)^{5/2}} - \frac{2(d+ex)^3}{3e\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(d+ex)}{e\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

Mathematica [A] time = 0.193265, size = 113, normalized size = 1.01

$$\frac{(d+ex)\left(2d\left(13d^2-24dex+23e^2x^2\right)\sqrt{1-\frac{e^2x^2}{d^2}}-15(d-ex)^3\sin^{-1}\left(\frac{ex}{d}\right)\right)}{15de(d-ex)^2\sqrt{d^2-e^2x^2}\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^6/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((d + e*x)*(2*d*(13*d^2 - 24*d*e*x + 23*e^2*x^2)*Sqrt[1 - (e^2*x^2)/d^2] -
15*(d - e*x)^3*ArcSin[(e*x)/d]))/(15*d*e*(d - e*x)^2*Sqrt[d^2 - e^2*x^2]*Sq
rt[1 - (e^2*x^2)/d^2])
```

Maple [B] time = 0.084, size = 225, normalized size = 2.

$$\frac{e^4x^5}{5}\left(-e^2x^2+d^2\right)^{-\frac{5}{2}} - \frac{e^2x^3}{3}\left(-e^2x^2+d^2\right)^{-\frac{3}{2}} + \frac{38\,x}{15}\frac{1}{\sqrt{-e^2x^2+d^2}} - \arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right)\frac{1}{\sqrt{e^2}} + 6\frac{e^3dx^4}{\left(-e^2x^2+d^2\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^6/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/5*e^4*x^5/(-e^2*x^2+d^2)^(5/2)-1/3*e^2*x^3/(-e^2*x^2+d^2)^(3/2)+38/15*x/(
-e^2*x^2+d^2)^(1/2)-1/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)
)+6*d*e^3*x^4/(-e^2*x^2+d^2)^(5/2)-4/3*d^3*e*x^2/(-e^2*x^2+d^2)^(5/2)+26/15
*d^5/e/(-e^2*x^2+d^2)^(5/2)+15/2*e^2*d^2*x^3/(-e^2*x^2+d^2)^(5/2)-13/10*d^4
*x/(-e^2*x^2+d^2)^(5/2)+23/30*d^2*x/(-e^2*x^2+d^2)^(3/2)
```

Maxima [B] time = 1.98363, size = 405, normalized size = 3.62

$$\frac{1}{15}e^6x\left(\frac{15\,x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2} - \frac{20\,d^2x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4} + \frac{8\,d^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}\right) - \frac{1}{3}e^4x\left(\frac{3\,x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2} - \frac{2\,d^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}\right) + \frac{6}{(-e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*e^6*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d
^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - 1/3*e^4*x*(3*x^2/((-
e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4)) + 6*d*e^3*x
^4/(-e^2*x^2 + d^2)^(5/2) + 15/2*d^2*e^2*x^3/(-e^2*x^2 + d^2)^(5/2) - 4/3*d
^3*e*x^2/(-e^2*x^2 + d^2)^(5/2) - 13/10*d^4*x/(-e^2*x^2 + d^2)^(5/2) + 26/1
5*d^5/((-e^2*x^2 + d^2)^(5/2)*e) + 31/30*d^2*x/(-e^2*x^2 + d^2)^(3/2) + 16/
15*x/sqrt(-e^2*x^2 + d^2) - arcsin(e^2*x/sqrt(d^2*e^2))/sqrt(e^2)
```

Fricas [A] time = 2.20407, size = 333, normalized size = 2.97

$$\frac{2\left(13\,e^3x^3 - 39\,de^2x^2 + 39\,d^2ex - 13\,d^3 + 15\left(e^3x^3 - 3\,de^2x^2 + 3\,d^2ex - d^3\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) - \left(23\,e^2x^2 - 24\,dex + 1\right.}{15\left(e^4x^3 - 3\,de^3x^2 + 3\,d^2e^2x - d^3e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(13*e^3*x^3 - 39*d*e^2*x^2 + 39*d^2*e*x - 13*d^3 + 15*(e^3*x^3 - 3*d*e
^2*x^2 + 3*d^2*e*x - d^3)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (23*e
^2*x^2 - 24*d*e*x + 13*d^2)*sqrt(-e^2*x^2 + d^2))/(e^4*x^3 - 3*d*e^3*x^2 +
3*d^2*e^2*x - d^3*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^6}{\left(-(-d+ex)(d+ex)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**6/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**6/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [A] time = 1.37681, size = 128, normalized size = 1.14

$$-\arcsin\left(\frac{xe}{d}\right)e^{(-1)}\mathrm{sgn}\left(d\right) - \frac{2\left(13\,d^5e^{(-1)} + \left(15\,d^4 - \left(10\,d^3e - \left(10\,d^2e^2 + \left(23\,xe^4 + 45\,de^3\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2 + d^2}}{15\left(x^2e^2 - d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -arcsin(x*e/d)*e^(-1)*sgn(d) - 2/15*(13*d^5*e^(-1) + (15*d^4 - (10*d^3*e -
(10*d^2*e^2 + (23*x*e^4 + 45*d*e^3)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)/(x^2*e
^2 - d^2)^3
```

3.850 $\int \frac{(d+ex)^5}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=33

$$\frac{(d+ex)^5}{5de\left(d^2-e^2x^2\right)^{5/2}}$$

```
[Out] (d + e*x)^5/(5*d*e*(d^2 - e^2*x^2)^(5/2))
```

Rubi [A] time = 0.0094206, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {651}

$$\frac{(d+ex)^5}{5de\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d + e*x)^5/(5*d*e*(d^2 - e^2*x^2)^(5/2))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{(d+ex)^5}{\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{(d+ex)^5}{5de\left(d^2-e^2x^2\right)^{5/2}}$$

Mathematica [A] time = 0.0944502, size = 41, normalized size = 1.24

$$\frac{(d+ex)^3}{5de(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d + e*x)^3/(5*d*e*(d - e*x)^2*Sqrt[d^2 - e^2*x^2])
```

Maple [A] time = 0.043, size = 36, normalized size = 1.1

$$\frac{(ex+d)^6(-ex+d)}{5\,de}\left(-e^2x^2+d^2\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/5*(e*x+d)^6*(-e*x+d)/d/e/(-e^2*x^2+d^2)^(7/2)
```

Maxima [B] time = 1.17891, size = 200, normalized size = 6.06

$$\frac{e^3x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{5\,de^2x^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{2\,d^2ex^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{7\,d^3x}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{d^4}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{dx}{10\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{x}{5\sqrt{-e^2x^2+d^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] e^3*x^4/(-e^2*x^2 + d^2)^(5/2) + 5/2*d*e^2*x^3/(-e^2*x^2 + d^2)^(5/2) + 2*d
^2*e*x^2/(-e^2*x^2 + d^2)^(5/2) + 7/10*d^3*x/(-e^2*x^2 + d^2)^(5/2) + 1/5*d
^4/((-e^2*x^2 + d^2)^(5/2)*e) + 1/10*d*x/(-e^2*x^2 + d^2)^(3/2) + 1/5*x/(sq
rt(-e^2*x^2 + d^2)*d)
```

Fricas [B] time = 1.965, size = 197, normalized size = 5.97

$$\frac{e^3x^3-3\,de^2x^2+3\,d^2ex-d^3-\left(e^2x^2+2\,dex+d^2\right)\sqrt{-e^2x^2+d^2}}{5\left(de^4x^3-3\,d^2e^3x^2+3\,d^3e^2x-d^4e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(e^3*x^3 - 3*d*e^2*x^2 + 3*d^2*e*x - d^3 - (e^2*x^2 + 2*d*e*x + d^2)*sq
rt(-e^2*x^2 + d^2))/(d*e^4*x^3 - 3*d^2*e^3*x^2 + 3*d^3*e^2*x - d^4*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(d+ex)^5}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**5/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [B] time = 1.30536, size = 103, normalized size = 3.12

$$-\frac{\left(d^4e^{(-1)}+\left(5\,d^3+\left(10\,d^2e+\left(x\left(\frac{xe^4}{d}+5\,e^3\right)+10\,de^2\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}}{5\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/5*(d^4*e^(-1) + (5*d^3 + (10*d^2*e + (x*(x*e^4/d + 5*e^3) + 10*d*e^2)*x)
*x)*x)*sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^3
```

3.851 $\int \frac{(d+ex)^4}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=67

$$\frac{(d+ex)^4}{3de\left(d^2-e^2x^2\right)^{5/2}}-\frac{(d+ex)^5}{15d^2e\left(d^2-e^2x^2\right)^{5/2}}$$

```
[Out] (d + e*x)^4/(3*d*e*(d^2 - e^2*x^2)^(5/2)) - (d + e*x)^5/(15*d^2*e*(d^2 - e^
2*x^2)^(5/2))
```

Rubi [A] time = 0.0227085, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 651}

$$\frac{(d+ex)^4}{3de\left(d^2-e^2x^2\right)^{5/2}}-\frac{(d+ex)^5}{15d^2e\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d + e*x)^4/(3*d*e*(d^2 - e^2*x^2)^(5/2)) - (d + e*x)^5/(15*d^2*e*(d^2 - e^
2*x^2)^(5/2))
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^4}{\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{(d+ex)^4}{3de\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{(d+ex)^5}{\left(d^2-e^2x^2\right)^{7/2}}\,dx}{3d}\\ &= \frac{(d+ex)^4}{3de\left(d^2-e^2x^2\right)^{5/2}}-\frac{(d+ex)^5}{15d^2e\left(d^2-e^2x^2\right)^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.0687201, size = 49, normalized size = 0.73

$$\frac{(4d-ex)(d+ex)^2}{15d^2e(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((4*d - e*x)*(d + e*x)^2)/(15*d^2*e*(d - e*x)^2*Sqrt[d^2 - e^2*x^2])
```

Maple [A] time = 0.043, size = 44, normalized size = 0.7

$$\frac{(ex+d)^5(-ex+d)(-ex+4d)}{15d^2e}\left(-e^2x^2+d^2\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/15*(e*x+d)^5*(-e*x+d)*(-e*x+4*d)/d^2/e/(-e^2*x^2+d^2)^(7/2)
```

Maxima [B] time = 1.17327, size = 166, normalized size = 2.48

$$\frac{e^2x^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{4\,dex^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{11\,d^2x}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{4\,d^3}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}-\frac{x}{30\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{x}{15\sqrt{-e^2x^2+d^2}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/2*e^2*x^3/(-e^2*x^2 + d^2)^(5/2) + 4/3*d*e*x^2/(-e^2*x^2 + d^2)^(5/2) + 1
1/10*d^2*x/(-e^2*x^2 + d^2)^(5/2) + 4/15*d^3/((-e^2*x^2 + d^2)^(5/2)*e) - 1
/30*x/(-e^2*x^2 + d^2)^(3/2) - 1/15*x/(sqrt(-e^2*x^2 + d^2)*d^2)
```

Fricas [A] time = 2.08619, size = 212, normalized size = 3.16

$$\frac{4\,e^3x^3-12\,de^2x^2+12\,d^2ex-4\,d^3+\left(e^2x^2-3\,dex-4\,d^2\right)\sqrt{-e^2x^2+d^2}}{15\left(d^2e^4x^3-3\,d^3e^3x^2+3\,d^4e^2x-d^5e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(4*e^3*x^3 - 12*d*e^2*x^2 + 12*d^2*e*x - 4*d^3 + (e^2*x^2 - 3*d*e*x -
4*d^2)*sqrt(-e^2*x^2 + d^2))/(d^2*e^4*x^3 - 3*d^3*e^3*x^2 + 3*d^4*e^2*x - d
^5*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(d+ex)^4}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**4/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [A] time = 1.34625, size = 95, normalized size = 1.42

$$-\frac{\left(4\,d^3e^{(-1)}+\left(15\,d^2-\left(x\left(\frac{x^2e^4}{d^2}-10\,e^2\right)-20\,de\right)x\right)x\right)\sqrt{-x^2e^2+d^2}}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*(4*d^3*e^(-1) + (15*d^2 - (x*(x^2*e^4/d^2 - 10*e^2) - 20*d*e)*x)*x)*s
qrt(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^3
```

3.852 $\int \frac{(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=103

$$\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}+\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2}+\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}$$

```
[Out] Sqrt[d^2 - e^2*x^2]/(5*d*e*(d - e*x)^3) + (2*Sqrt[d^2 - e^2*x^2])/(15*d^2*e
*(d - e*x)^2) + (2*Sqrt[d^2 - e^2*x^2])/(15*d^3*e*(d - e*x))
```

Rubi [A] time = 0.0436404, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {655, 659, 651}

$$\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}+\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2}+\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/(5*d*e*(d - e*x)^3) + (2*Sqrt[d^2 - e^2*x^2])/(15*d^2*e
*(d - e*x)^2) + (2*Sqrt[d^2 - e^2*x^2])/(15*d^3*e*(d - e*x))
```

Rule 655

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(2*m)/a^m, Int[(a + c*x^2)^(m + p)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d,
 e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IntegerQ[m] && R
ationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{1}{(d-ex)^3\sqrt{d^2-e^2x^2}}\,dx$$
$$= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3} + \frac{2\int \frac{1}{(d-ex)^2\sqrt{d^2-e^2x^2}}\,dx}{5d}$$
$$= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3} + \frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2} + \frac{2\int \frac{1}{(d-ex)\sqrt{d^2-e^2x^2}}\,dx}{15d^2}$$
$$= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3} + \frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2} + \frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}$$

Mathematica [A] time = 0.0625678, size = 58, normalized size = 0.56

$$\frac{(d+ex)\left(7d^2-6dex+2e^2x^2\right)}{15d^3e(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((d + e*x)*(7*d^2 - 6*d*e*x + 2*e^2*x^2))/(15*d^3*e*(d - e*x)^2*Sqrt[d^2 - 
e^2*x^2])
```

Maple [A] time = 0.043, size = 55, normalized size = 0.5

$$\frac{(ex+d)^4(-ex+d)\left(2e^2x^2-6dex+7d^2\right)}{15d^3e}\left(-e^2x^2+d^2\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/15*(e*x+d)^4*(-e*x+d)*(2*e^2*x^2-6*d*e*x+7*d^2)/d^3/e/(-e^2*x^2+d^2)^(7/2
)
```

Maxima [A] time = 1.30242, size = 136, normalized size = 1.32

$$\frac{ex^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{4\,dx}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{7\,d^2}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} + \frac{x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d} + \frac{2\,x}{15\sqrt{-e^2x^2+d^2}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/3*e*x^2/(-e^2*x^2 + d^2)^(5/2) + 4/5*d*x/(-e^2*x^2 + d^2)^(5/2) + 7/15*d^
2/((-e^2*x^2 + d^2)^(5/2)*e) + 1/15*x/((-e^2*x^2 + d^2)^(3/2)*d) + 2/15*x/(
sqrt(-e^2*x^2 + d^2)*d^3)
```

Fricas [A] time = 2.13638, size = 215, normalized size = 2.09

$$\frac{7\,e^3x^3-21\,de^2x^2+21\,d^2ex-7\,d^3-\left(2\,e^2x^2-6\,dex+7\,d^2\right)\sqrt{-e^2x^2+d^2}}{15\,\left(d^3e^4x^3-3\,d^4e^3x^2+3\,d^5e^2x-d^6e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(7*e^3*x^3 - 21*d*e^2*x^2 + 21*d^2*e*x - 7*d^3 - (2*e^2*x^2 - 6*d*e*x
+ 7*d^2)*sqrt(-e^2*x^2 + d^2))/(d^3*e^4*x^3 - 3*d^4*e^3*x^2 + 3*d^5*e^2*x -
 d^6*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^3}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [A] time = 1.27872, size = 95, normalized size = 0.92

$$-\frac{\sqrt{-x^2e^2+d^2}\left(7\,d^2e^{(-1)}+\left(\left(x\left(\frac{2\,x^2e^4}{d^3}-\frac{5\,e^2}{d}\right)+5\,e\right)x+15\,d\right)x\right)}{15\,\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*(7*d^2*e^(-1) + ((x*(2*x^2*e^4/d^3 - 5*e^2/d) +
5*e)*x + 15*d)*x)/(x^2*e^2 - d^2)^3
```

3.853 $\int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=77

$$\frac{2x}{5d^4\sqrt{d^2-e^2x^2}}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d+ex)}{5e\left(d^2-e^2x^2\right)^{5/2}}$$

```
[Out] (2*(d + e*x))/(5*e*(d^2 - e^2*x^2)^(5/2)) + x/(5*d^2*(d^2 - e^2*x^2)^(3/2))
 + (2*x)/(5*d^4*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.018428, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {653, 192, 191}

$$\frac{2x}{5d^4\sqrt{d^2-e^2x^2}}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d+ex)}{5e\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (2*(d + e*x))/(5*e*(d^2 - e^2*x^2)^(5/2)) + x/(5*d^2*(d^2 - e^2*x^2)^(3/2))
 + (2*x)/(5*d^4*Sqrt[d^2 - e^2*x^2])
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{2(d+ex)}{5e\left(d^2-e^2x^2\right)^{5/2}}+\frac{3}{5}\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx$$

$$= \frac{2(d+ex)}{5e\left(d^2-e^2x^2\right)^{5/2}}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{5d^2}$$

$$= \frac{2(d+ex)}{5e\left(d^2-e^2x^2\right)^{5/2}}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{5d^4\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.044081, size = 63, normalized size = 0.82

$$\frac{d^2ex+2d^3-4de^2x^2+2e^3x^3}{5d^4e(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (2*d^3 + d^2*e*x - 4*d*e^2*x^2 + 2*e^3*x^3)/(5*d^4*e*(d - e*x)^2*Sqrt[d^2 -
 e^2*x^2])
```

Maple [A] time = 0.045, size = 65, normalized size = 0.8

$$\frac{(ex+d)^3(-ex+d)\left(2e^3x^3-4e^2x^2d+xd^2e+2d^3\right)}{5d^4e}\left(-e^2x^2+d^2\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/5*(e*x+d)^3*(-e*x+d)*(2*e^3*x^3-4*d*e^2*x^2+d^2*e*x+2*d^3)/d^4/e/(-e^2*x^
2+d^2)^(7/2)
```

Maxima [A] time = 1.19045, size = 105, normalized size = 1.36

$$\frac{2x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{2d}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{x}{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2}+\frac{2x}{5\sqrt{-e^2x^2+d^2}d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/5*x/(-e^2*x^2 + d^2)^(5/2) + 2/5*d/((-e^2*x^2 + d^2)^(5/2)*e) + 1/5*x/((-
e^2*x^2 + d^2)^(3/2)*d^2) + 2/5*x/(sqrt(-e^2*x^2 + d^2)*d^4)
```

Fricas [A] time = 2.11157, size = 230, normalized size = 2.99

$$\frac{2e^4x^4-4de^3x^3+4d^3ex-2d^4-\left(2e^3x^3-4de^2x^2+d^2ex+2d^3\right)\sqrt{-e^2x^2+d^2}}{5\left(d^4e^5x^4-2d^5e^4x^3+2d^7e^2x-d^8e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*e^4*x^4 - 4*d*e^3*x^3 + 4*d^3*e*x - 2*d^4 - (2*e^3*x^3 - 4*d*e^2*x^2
 + d^2*e*x + 2*d^3)*sqrt(-e^2*x^2 + d^2))/(d^4*e^5*x^4 - 2*d^5*e^4*x^3 + 2*
d^7*e^2*x - d^8*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^2}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [A] time = 1.42641, size = 82, normalized size = 1.06

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(x^2\left(\frac{2x^2e^4}{d^4}-\frac{5e^2}{d^2}\right)+5\right)x+2de^{(-1)}\right)}{5\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/5*sqrt(-x^2*e^2 + d^2)*((x^2*(2*x^2*e^4/d^4 - 5*e^2/d^2) + 5)*x + 2*d*e^
(-1))/(x^2*e^2 - d^2)^3
```

3.854 $\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=80

$$\frac{8x}{15d^5\sqrt{d^2-e^2x^2}}+\frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{d+ex}{5de\left(d^2-e^2x^2\right)^{5/2}}$$

```
[Out] (d + e*x)/(5*d*e*(d^2 - e^2*x^2)^(5/2)) + (4*x)/(15*d^3*(d^2 - e^2*x^2)^(3/
2)) + (8*x)/(15*d^5*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.0193956, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {639, 192, 191}

$$\frac{8x}{15d^5\sqrt{d^2-e^2x^2}}+\frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{d+ex}{5de\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d + e*x)/(5*d*e*(d^2 - e^2*x^2)^(5/2)) + (4*x)/(15*d^3*(d^2 - e^2*x^2)^(3/
2)) + (8*x)/(15*d^5*Sqrt[d^2 - e^2*x^2])
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d+ex}{5de\left(d^2-e^2x^2\right)^{5/2}} + \frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= \frac{d+ex}{5de\left(d^2-e^2x^2\right)^{5/2}} + \frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{8\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^3}$$

$$= \frac{d+ex}{5de\left(d^2-e^2x^2\right)^{5/2}} + \frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{8x}{15d^5\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.0317078, size = 82, normalized size = 1.02

$$\frac{-12d^2e^2x^2+12d^3ex+3d^4-8de^3x^3+8e^4x^4}{15d^5e(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (3*d^4 + 12*d^3*e*x - 12*d^2*e^2*x^2 - 8*d*e^3*x^3 + 8*e^4*x^4)/(15*d^5*e*(
d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Maple [A] time = 0.045, size = 77, normalized size = 1.

$$\frac{(ex+d)^2(-ex+d)\left(8e^4x^4-8e^3x^3d-12e^2x^2d^2+12xd^3e+3d^4\right)}{15d^5e}\left(-e^2x^2+d^2\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/15*(e*x+d)^2*(-e*x+d)*(8*e^4*x^4-8*d*e^3*x^3-12*d^2*e^2*x^2+12*d^3*e*x+3*
d^4)/d^5/e/(-e^2*x^2+d^2)^(7/2)
```

Maxima [A] time = 1.27569, size = 108, normalized size = 1.35

$$\frac{x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d} + \frac{1}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} + \frac{4x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3} + \frac{8x}{15\sqrt{-e^2x^2+d^2}d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*x/((-e^2*x^2 + d^2)^(5/2)*d) + 1/5/((-e^2*x^2 + d^2)^(5/2)*e) + 4/15*x/
((-e^2*x^2 + d^2)^(3/2)*d^3) + 8/15*x/(sqrt(-e^2*x^2 + d^2)*d^5)
```

Fricas [B] time = 2.28307, size = 340, normalized size = 4.25

$$\frac{3\,e^{5}x^{5}-3\,de^{4}x^{4}-6\,d^{2}e^{3}x^{3}+6\,d^{3}e^{2}x^{2}+3\,d^{4}ex-3\,d^{5}-\left(8\,e^{4}x^{4}-8\,de^{3}x^{3}-12\,d^{2}e^{2}x^{2}+12\,d^{3}ex+3\,d^{4}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{15\left(d^{5}e^{6}x^{5}-d^{6}e^{5}x^{4}-2\,d^{7}e^{4}x^{3}+2\,d^{8}e^{3}x^{2}+d^{9}e^{2}x-d^{10}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*e^5*x^5 - 3*d*e^4*x^4 - 6*d^2*e^3*x^3 + 6*d^3*e^2*x^2 + 3*d^4*e*x -
 3*d^5 - (8*e^4*x^4 - 8*d*e^3*x^3 - 12*d^2*e^2*x^2 + 12*d^3*e*x + 3*d^4)*sq
rt(-e^2*x^2 + d^2))/(d^5*e^6*x^5 - d^6*e^5*x^4 - 2*d^7*e^4*x^3 + 2*d^8*e^3*
x^2 + d^9*e^2*x - d^10*e)
```

Sympy [C] time = 10.657, size = 605, normalized size = 7.56

$$d\left(\begin{cases}-\frac{15id^{4}x}{15d^{11}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}-30d^{9}e^{2}x^{2}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}+15d^{7}e^{4}x^{4}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}+\frac{20id^{2}e^{2}x^{3}}{15d^{11}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}-30d^{9}e^{2}x^{2}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}+15d^{7}e^{4}x^{4}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}-\frac{}{15d^{11}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}\cdots} \\ \frac{15d^{4}x}{15d^{11}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}-30d^{9}e^{2}x^{2}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}+15d^{7}e^{4}x^{4}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}-\frac{20d^{2}e^{2}x^{3}}{15d^{11}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}-30d^{9}e^{2}x^{2}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}+15d^{7}e^{4}x^{4}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}+\frac{8e^{4}x}{15d^{11}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}-30d^{9}e^{2}x^{2}\sqrt{1\cdots}}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((-15*I*d**4*x/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*d**9*e**
2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2*x**2/d*
*2)) + 20*I*d**2*e**2*x**3/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*d**9*e*
*2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2*x**2/d
**2)) - 8*I*e**4*x**5/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*d**9*e**2*x*
*2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2*x**2/d**2))
, Abs(e**2*x**2)/Abs(d**2) > 1), (15*d**4*x/(15*d**11*sqrt(1 - e**2*x**2/d*
*2) - 30*d**9*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(1
 - e**2*x**2/d**2)) - 20*d**2*e**2*x**3/(15*d**11*sqrt(1 - e**2*x**2/d**2)
- 30*d**9*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(1 - e
**2*x**2/d**2)) + 8*e**4*x**5/(15*d**11*sqrt(1 - e**2*x**2/d**2) - 30*d**9*
e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(1 - e**2*x**2/d
**2)), True)) + e*Piecewise((1/(5*d**4*e**2*sqrt(d**2 - e**2*x**2) - 10*d**
2*e**4*x**2*sqrt(d**2 - e**2*x**2) + 5*e**6*x**4*sqrt(d**2 - e**2*x**2)), N
e(e, 0)), (x**2/(2*(d**2)**(7/2)), True))
```

Giac [A] time = 1.33083, size = 88, normalized size = 1.1

$$-\frac{\sqrt{-x^{2}e^{2}+d^{2}}\left(\left(4\,x^{2}\left(\frac{2\,x^{2}e^{4}}{d^{5}}-\frac{5\,e^{2}}{d^{3}}\right)+\frac{15}{d}\right)x+3\,e^{(-1)}\right)}{15\left(x^{2}e^{2}-d^{2}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*((4*x^2*(2*x^2*e^4/d^5 - 5*e^2/d^3) + 15/d)*x +
3*e^(-1))/(x^2*e^2 - d^2)^3
```

3.855 $\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=106

$$\frac{16x}{35d^7\sqrt{d^2-e^2x^2}}+\frac{8x}{35d^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{6x}{35d^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{1}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}$$

```
[Out] (6*x)/(35*d^3*(d^2 - e^2*x^2)^(5/2)) - 1/(7*d*e*(d + e*x)*(d^2 - e^2*x^2)^(
5/2)) + (8*x)/(35*d^5*(d^2 - e^2*x^2)^(3/2)) + (16*x)/(35*d^7*Sqrt[d^2 - e^
2*x^2])
```

Rubi [A] time = 0.0277526, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {659, 192, 191}

$$\frac{16x}{35d^7\sqrt{d^2-e^2x^2}}+\frac{8x}{35d^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{6x}{35d^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{1}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (6*x)/(35*d^3*(d^2 - e^2*x^2)^(5/2)) - 1/(7*d*e*(d + e*x)*(d^2 - e^2*x^2)^(
5/2)) + (8*x)/(35*d^5*(d^2 - e^2*x^2)^(3/2)) + (16*x)/(35*d^7*Sqrt[d^2 - e^
2*x^2])
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}dx = -\frac{1}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}} + \frac{6\int \frac{1}{\left(d^2-e^2x^2\right)^{7/2}}dx}{7d}$$

$$= \frac{6x}{35d^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}} + \frac{24\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}dx}{35d^3}$$

$$= \frac{6x}{35d^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}} + \frac{8x}{35d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{16\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{35d^5}$$

$$= \frac{6x}{35d^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}} + \frac{8x}{35d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{16x}{35d^7\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.0801594, size = 104, normalized size = 0.98

$$\frac{\sqrt{d^2-e^2x^2}\left(30d^4e^2x^2-40d^3e^3x^3-40d^2e^4x^4+30d^5ex-5d^6+16de^5x^5+16e^6x^6\right)}{35d^7e(d-ex)^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-5*d^6 + 30*d^5*e*x + 30*d^4*e^2*x^2 - 40*d^3*e^3*x^3
 - 40*d^2*e^4*x^4 + 16*d*e^5*x^5 + 16*e^6*x^6))/(35*d^7*e*(d - e*x)^3*(d +
e*x)^4)
```

Maple [A] time = 0.045, size = 92, normalized size = 0.9

$$-\frac{(-ex+d)\left(-16\,e^6x^6-16\,e^5x^5d+40\,e^4x^4d^2+40\,e^3x^3d^3-30\,e^2x^2d^4-30\,xd^5e+5\,d^6\right)}{35\,d^7e}\left(-e^2x^2+d^2\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/35*(-e*x+d)*(-16*e^6*x^6-16*d*e^5*x^5+40*d^2*e^4*x^4+40*d^3*e^3*x^3-30*d
^4*e^2*x^2-30*d^5*e*x+5*d^6)/d^7/e/(-e^2*x^2+d^2)^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.05304, size = 487, normalized size = 4.59

$$-\frac{5\,e^{7}x^{7}+5\,de^{6}x^{6}-15\,d^{2}e^{5}x^{5}-15\,d^{3}e^{4}x^{4}+15\,d^{4}e^{3}x^{3}+15\,d^{5}e^{2}x^{2}-5\,d^{6}ex-5\,d^{7}+\left(16\,e^{6}x^{6}+16\,de^{5}x^{5}-40\,d^{2}e^{4}x^{4}-40\,d}{35\left(d^{7}e^{8}x^{7}+d^{8}e^{7}x^{6}-3\,d^{9}e^{6}x^{5}-3\,d^{10}e^{5}x^{4}+3\,d^{11}e^{4}x^{3}+3\,d^{12}e^{3}x^{2}-d^{13}e^{2}x-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/35*(5*e^7*x^7 + 5*d*e^6*x^6 - 15*d^2*e^5*x^5 - 15*d^3*e^4*x^4 + 15*d^4*e
^3*x^3 + 15*d^5*e^2*x^2 - 5*d^6*e*x - 5*d^7 + (16*e^6*x^6 + 16*d*e^5*x^5 -
40*d^2*e^4*x^4 - 40*d^3*e^3*x^3 + 30*d^4*e^2*x^2 + 30*d^5*e*x - 5*d^6)*sqrt
(-e^2*x^2 + d^2))/(d^7*e^8*x^7 + d^8*e^7*x^6 - 3*d^9*e^6*x^5 - 3*d^10*e^5*x
^4 + 3*d^11*e^4*x^3 + 3*d^12*e^3*x^2 - d^13*e^2*x - d^14*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(1/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, undef, undef, 1]
```

3.856 $\int \frac{1}{(d+ex)^2(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=139

$$\frac{16x}{45d^8\sqrt{d^2-e^2x^2}}+\frac{8x}{45d^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{15d^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{1}{9d^2e(d+ex)\left(d^2-e^2x^2\right)^{5/2}}-\frac{1}{9de(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}$$

```
[Out] (2*x)/(15*d^4*(d^2 - e^2*x^2)^(5/2)) - 1/(9*d*e*(d + e*x)^2*(d^2 - e^2*x^2)
^(5/2)) - 1/(9*d^2*e*(d + e*x)*(d^2 - e^2*x^2)^(5/2)) + (8*x)/(45*d^6*(d^2
- e^2*x^2)^(3/2)) + (16*x)/(45*d^8*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.0466924, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {659, 192, 191}

$$\frac{16x}{45d^8\sqrt{d^2-e^2x^2}}+\frac{8x}{45d^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{15d^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{1}{9d^2e(d+ex)\left(d^2-e^2x^2\right)^{5/2}}-\frac{1}{9de(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (2*x)/(15*d^4*(d^2 - e^2*x^2)^(5/2)) - 1/(9*d*e*(d + e*x)^2*(d^2 - e^2*x^2)
^(5/2)) - 1/(9*d^2*e*(d + e*x)*(d^2 - e^2*x^2)^(5/2)) + (8*x)/(45*d^6*(d^2
- e^2*x^2)^(3/2)) + (16*x)/(45*d^8*Sqrt[d^2 - e^2*x^2])
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = -\frac{1}{9de(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{7\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx}{9d}$$

$$= -\frac{1}{9de(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{9d^2e(d+ex)\left(d^2-e^2x^2\right)^{5/2}} + \frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{7/2}}\,dx}{3d^2}$$

$$= \frac{2x}{15d^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{9de(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{9d^2e(d+ex)\left(d^2-e^2x^2\right)^{5/2}} + \frac{8\int \frac{}{(d^2-}}{15}$$

$$= \frac{2x}{15d^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{9de(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{9d^2e(d+ex)\left(d^2-e^2x^2\right)^{5/2}} + \frac{}{45d^6\left(d\right.}$$

$$= \frac{2x}{15d^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{9de(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{9d^2e(d+ex)\left(d^2-e^2x^2\right)^{5/2}} + \frac{}{45d^6\left(d\right.}$$

Mathematica [A] time = 0.0757517, size = 115, normalized size = 0.83

$$\frac{\sqrt{d^2-e^2x^2}\left(60d^5e^2x^2-10d^4e^3x^3-80d^3e^4x^4-24d^2e^5x^5+25d^6ex-10d^7+32de^6x^6+16e^7x^7\right)}{45d^8e(d-ex)^3(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-10*d^7 + 25*d^6*e*x + 60*d^5*e^2*x^2 - 10*d^4*e^3*x^
3 - 80*d^3*e^4*x^4 - 24*d^2*e^5*x^5 + 32*d*e^6*x^6 + 16*e^7*x^7))/(45*d^8*e
*(d - e*x)^3*(d + e*x)^5)
```

Maple [A] time = 0.048, size = 110, normalized size = 0.8

$$-\frac{(-ex+d)\left(-16\,e^7x^7-32\,e^6x^6d+24\,e^5x^5d^2+80\,e^4x^4d^3+10\,e^3x^3d^4-60\,e^2x^2d^5-25\,xd^6e+10\,d^7\right)}{(45\,ex+45\,d)\,d^8e}\left(-e^2x^2+d^2\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/45*(-e*x+d)*(-16*e^7*x^7-32*d*e^6*x^6+24*d^2*e^5*x^5+80*d^3*e^4*x^4+10*d
^4*e^3*x^3-60*d^5*e^2*x^2-25*d^6*e*x+10*d^7)/(e*x+d)/d^8/e/(-e^2*x^2+d^2)^(
7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [B] time = 3.84771, size = 524, normalized size = 3.77

$$-\frac{10\,e^{8}x^{8}+20\,de^{7}x^{7}-20\,d^{2}e^{6}x^{6}-60\,d^{3}e^{5}x^{5}+60\,d^{5}e^{3}x^{3}+20\,d^{6}e^{2}x^{2}-20\,d^{7}ex-10\,d^{8}+\left(16\,e^{7}x^{7}+32\,de^{6}x^{6}-24\,d^{2}e^{5}\right.}{45\left(d^{8}e^{9}x^{8}+2\,d^{9}e^{8}x^{7}-2\,d^{10}e^{7}x^{6}-6\,d^{11}e^{6}x^{5}+6\,d^{13}e^{4}x^{3}+2\,d^{14}e^{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/45*(10*e^8*x^8 + 20*d*e^7*x^7 - 20*d^2*e^6*x^6 - 60*d^3*e^5*x^5 + 60*d^5
*e^3*x^3 + 20*d^6*e^2*x^2 - 20*d^7*e*x - 10*d^8 + (16*e^7*x^7 + 32*d*e^6*x^
6 - 24*d^2*e^5*x^5 - 80*d^3*e^4*x^4 - 10*d^4*e^3*x^3 + 60*d^5*e^2*x^2 + 25*
d^6*e*x - 10*d^7)*sqrt(-e^2*x^2 + d^2))/(d^8*e^9*x^8 + 2*d^9*e^8*x^7 - 2*d^
10*e^7*x^6 - 6*d^11*e^6*x^5 + 6*d^13*e^4*x^3 + 2*d^14*e^3*x^2 - 2*d^15*e^2*
x - d^16*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(1/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.857 $\int \frac{1}{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=172

$$\frac{128x}{495d^9\sqrt{d^2-e^2x^2}}+\frac{64x}{495d^7\left(d^2-e^2x^2\right)^{3/2}}+\frac{16x}{165d^5\left(d^2-e^2x^2\right)^{5/2}}-\frac{8}{99d^3e(d+ex)\left(d^2-e^2x^2\right)^{5/2}}-\frac{8}{99d^2e(d+ex)^2\left(d^2-e^2\right.}$$

```
[Out] (16*x)/(165*d^5*(d^2 - e^2*x^2)^(5/2)) - 1/(11*d*e*(d + e*x)^3*(d^2 - e^2*x
^2)^(5/2)) - 8/(99*d^2*e*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2)) - 8/(99*d^3*e*(
d + e*x)*(d^2 - e^2*x^2)^(5/2)) + (64*x)/(495*d^7*(d^2 - e^2*x^2)^(3/2)) +
(128*x)/(495*d^9*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.0639118, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {659, 192, 191}

$$\frac{128x}{495d^9\sqrt{d^2-e^2x^2}}+\frac{64x}{495d^7\left(d^2-e^2x^2\right)^{3/2}}+\frac{16x}{165d^5\left(d^2-e^2x^2\right)^{5/2}}-\frac{8}{99d^3e(d+ex)\left(d^2-e^2x^2\right)^{5/2}}-\frac{8}{99d^2e(d+ex)^2\left(d^2-e^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (16*x)/(165*d^5*(d^2 - e^2*x^2)^(5/2)) - 1/(11*d*e*(d + e*x)^3*(d^2 - e^2*x
^2)^(5/2)) - 8/(99*d^2*e*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2)) - 8/(99*d^3*e*(
d + e*x)*(d^2 - e^2*x^2)^(5/2)) + (64*x)/(495*d^7*(d^2 - e^2*x^2)^(3/2)) +
(128*x)/(495*d^9*Sqrt[d^2 - e^2*x^2])
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = -\frac{1}{11de(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{8\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}\,dx}{11d}$$

$$= -\frac{1}{11de(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{8}{99d^2e(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{56\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,c}{99d^2}$$

$$= -\frac{1}{11de(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{8}{99d^2e(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{8}{99d^3e(d+ex)\left(d^2-e^2\right.}$$

$$= \frac{16x}{165d^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{11de(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{8}{99d^2e(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}} -$$

$$= \frac{16x}{165d^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{11de(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{8}{99d^2e(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}} -$$

$$= \frac{16x}{165d^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{11de(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{8}{99d^2e(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}} -$$

Mathematica [A] time = 0.0825094, size = 126, normalized size = 0.73

$$\frac{\sqrt{d^2-e^2x^2}\left(680d^6e^2x^2+400d^5e^3x^3-720d^4e^4x^4-832d^3e^5x^5+64d^2e^6x^6+120d^7ex-125d^8+384de^7x^7+128e^8x^8\right)}{495d^9e(d-ex)^3(d+ex)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-125*d^8 + 120*d^7*e*x + 680*d^6*e^2*x^2 + 400*d^5*e^
3*x^3 - 720*d^4*e^4*x^4 - 832*d^3*e^5*x^5 + 64*d^2*e^6*x^6 + 384*d*e^7*x^7
+ 128*e^8*x^8))/(495*d^9*e*(d - e*x)^3*(d + e*x)^6)
```

Maple [A] time = 0.046, size = 121, normalized size = 0.7

$$-\frac{(-ex+d)\left(-128\,e^8x^8-384\,e^7x^7d-64\,e^6x^6d^2+832\,e^5x^5d^3+720\,e^4x^4d^4-400\,e^3x^3d^5-680\,e^2x^2d^6-120\,xd^7e+125\right.}{495\,ed^9\,(ex+d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/495*(-e*x+d)*(-128*e^8*x^8-384*d*e^7*x^7-64*d^2*e^6*x^6+832*d^3*e^5*x^5+
720*d^4*e^4*x^4-400*d^5*e^3*x^3-680*d^6*e^2*x^2-120*d^7*e*x+125*d^8)/(e*x+d
)^2/d^9/e/(-e^2*x^2+d^2)^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.7211, size = 575, normalized size = 3.34

$$-\frac{125\,e^{9}x^{9}+375\,de^{8}x^{8}-1000\,d^{3}e^{6}x^{6}-750\,d^{4}e^{5}x^{5}+750\,d^{5}e^{4}x^{4}+1000\,d^{6}e^{3}x^{3}-375\,d^{8}ex-125\,d^{9}+\left(128\,e^{8}x^{8}+384\,de^{7}x\right.}{495\left(d^{9}e^{10}x^{9}+3\,d^{10}e^{9}x^{8}-8\,d^{12}e^{7}x^{6}-6\,d^{13}e^{6}x^{5}+6\,d^{1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/495*(125*e^9*x^9 + 375*d*e^8*x^8 - 1000*d^3*e^6*x^6 - 750*d^4*e^5*x^5 +
750*d^5*e^4*x^4 + 1000*d^6*e^3*x^3 - 375*d^8*e*x - 125*d^9 + (128*e^8*x^8 +
 384*d*e^7*x^7 + 64*d^2*e^6*x^6 - 832*d^3*e^5*x^5 - 720*d^4*e^4*x^4 + 400*d
^5*e^3*x^3 + 680*d^6*e^2*x^2 + 120*d^7*e*x - 125*d^8)*sqrt(-e^2*x^2 + d^2))
/(d^9*e^10*x^9 + 3*d^10*e^9*x^8 - 8*d^12*e^7*x^6 - 6*d^13*e^6*x^5 + 6*d^14*
e^5*x^4 + 8*d^15*e^4*x^3 - 3*d^17*e^2*x - d^18*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(1/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\mathrm{undef},\mathrm{undef},\mathrm{undef},\mathrm{undef},\mathrm{undef},\mathrm{undef},\mathrm{undef},1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, undef, undef, 1]
```

3.858 $\int \frac{1}{(d+ex)^4 \left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=205

$$\frac{128x}{715d^{10}\sqrt{d^2-e^2x^2}}+\frac{64x}{715d^8\left(d^2-e^2x^2\right)^{3/2}}+\frac{48x}{715d^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{8}{143d^4e(d+ex)\left(d^2-e^2x^2\right)^{5/2}}-\frac{8}{143d^3e(d+ex)^2\left(d\right.}$$

```
[Out] (48*x)/(715*d^6*(d^2 - e^2*x^2)^(5/2)) - 1/(13*d*e*(d + e*x)^4*(d^2 - e^2*x
^2)^(5/2)) - 9/(143*d^2*e*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2)) - 8/(143*d^3*e
*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2)) - 8/(143*d^4*e*(d + e*x)*(d^2 - e^2*x^2
)^(5/2)) + (64*x)/(715*d^8*(d^2 - e^2*x^2)^(3/2)) + (128*x)/(715*d^10*Sqrt[
d^2 - e^2*x^2])
```

Rubi [A] time = 0.0889444, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {659, 192, 191}

$$\frac{128x}{715d^{10}\sqrt{d^2-e^2x^2}}+\frac{64x}{715d^8\left(d^2-e^2x^2\right)^{3/2}}+\frac{48x}{715d^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{8}{143d^4e(d+ex)\left(d^2-e^2x^2\right)^{5/2}}-\frac{8}{143d^3e(d+ex)^2\left(d\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```


```
[Out] (48*x)/(715*d^6*(d^2 - e^2*x^2)^(5/2)) - 1/(13*d*e*(d + e*x)^4*(d^2 - e^2*x
^2)^(5/2)) - 9/(143*d^2*e*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2)) - 8/(143*d^3*e
*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2)) - 8/(143*d^4*e*(d + e*x)*(d^2 - e^2*x^2
)^(5/2)) + (64*x)/(715*d^8*(d^2 - e^2*x^2)^(3/2)) + (128*x)/(715*d^10*Sqrt[
d^2 - e^2*x^2])
```


Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^4(d^2-e^2x^2)^{7/2}}dx = -\frac{1}{13de(d+ex)^4(d^2-e^2x^2)^{5/2}} + \frac{9\int \frac{1}{(d+ex)^3(d^2-e^2x^2)^{7/2}}dx}{13d}$$

$$= -\frac{1}{13de(d+ex)^4(d^2-e^2x^2)^{5/2}} - \frac{9}{143d^2e(d+ex)^3(d^2-e^2x^2)^{5/2}} + \frac{72\int \frac{1}{(d+ex)^2(d^2-e^2x^2)^{7/2}}dx}{143d^2}$$

$$= -\frac{1}{13de(d+ex)^4(d^2-e^2x^2)^{5/2}} - \frac{9}{143d^2e(d+ex)^3(d^2-e^2x^2)^{5/2}} - \frac{8}{143d^3e(d+ex)^2(d^2-e^2x^2)^{5/2}}$$

$$= -\frac{1}{13de(d+ex)^4(d^2-e^2x^2)^{5/2}} - \frac{9}{143d^2e(d+ex)^3(d^2-e^2x^2)^{5/2}} - \frac{8}{143d^3e(d+ex)^2(d^2-e^2x^2)^{5/2}}$$

$$= \frac{48x}{715d^6(d^2-e^2x^2)^{5/2}} - \frac{1}{13de(d+ex)^4(d^2-e^2x^2)^{5/2}} - \frac{9}{143d^2e(d+ex)^3(d^2-e^2x^2)^{5/2}} - \frac{}{14}$$

$$= \frac{48x}{715d^6(d^2-e^2x^2)^{5/2}} - \frac{1}{13de(d+ex)^4(d^2-e^2x^2)^{5/2}} - \frac{9}{143d^2e(d+ex)^3(d^2-e^2x^2)^{5/2}} - \frac{}{14}$$

$$= \frac{48x}{715d^6(d^2-e^2x^2)^{5/2}} - \frac{1}{13de(d+ex)^4(d^2-e^2x^2)^{5/2}} - \frac{9}{143d^2e(d+ex)^3(d^2-e^2x^2)^{5/2}} - \frac{}{14}$$

Mathematica [A] time = 0.0886175, size = 137, normalized size = 0.67

$$\frac{\sqrt{d^2-e^2x^2}\left(800d^7e^2x^2+1080d^6e^3x^3-320d^5e^4x^4-1552d^4e^5x^5-768d^3e^6x^6+448d^2e^7x^7-5d^8ex-180d^9+512de^8x^8+\right.}{715d^{10}e(d-ex)^3(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-180*d^9 - 5*d^8*e*x + 800*d^7*e^2*x^2 + 1080*d^6*e^3
*x^3 - 320*d^5*e^4*x^4 - 1552*d^4*e^5*x^5 - 768*d^3*e^6*x^6 + 448*d^2*e^7*x
^7 + 512*d*e^8*x^8 + 128*e^9*x^9))/(715*d^10*e*(d - e*x)^3*(d + e*x)^7)
```

Maple [A] time = 0.047, size = 132, normalized size = 0.6

$$-\frac{(-ex+d)\left(-128e^9x^9-512e^8x^8d-448e^7x^7d^2+768e^6x^6d^3+1552e^5x^5d^4+320e^4x^4d^5-1080e^3x^3d^6-800e^2x^2d^7+5\right.}{715ed^{10}(ex+d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/715*(-e*x+d)*(-128*e^9*x^9-512*d*e^8*x^8-448*d^2*e^7*x^7+768*d^3*e^6*x^6
+1552*d^4*e^5*x^5+320*d^5*e^4*x^4-1080*d^6*e^3*x^3-800*d^7*e^2*x^2+5*d^8*e*
x+180*d^9)/(e*x+d)^3/d^10/e/(-e^2*x^2+d^2)^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 7.73555, size = 709, normalized size = 3.46

$$-\frac{180\,e^{10}x^{10}+720\,de^{9}x^{9}+540\,d^{2}e^{8}x^{8}-1440\,d^{3}e^{7}x^{7}-2520\,d^{4}e^{6}x^{6}+2520\,d^{6}e^{4}x^{4}+1440\,d^{7}e^{3}x^{3}-540\,d^{8}e^{2}x^{2}-720\,d^{9}}{715\left(d^{10}e^{11}x^{10}+4\,d^{11}e^{10}x^{9}+3\,d^{12}e^{9}x^{8}-8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/715*(180*e^10*x^10 + 720*d*e^9*x^9 + 540*d^2*e^8*x^8 - 1440*d^3*e^7*x^7
- 2520*d^4*e^6*x^6 + 2520*d^6*e^4*x^4 + 1440*d^7*e^3*x^3 - 540*d^8*e^2*x^2
- 720*d^9*e*x - 180*d^10 + (128*e^9*x^9 + 512*d*e^8*x^8 + 448*d^2*e^7*x^7 -
 768*d^3*e^6*x^6 - 1552*d^4*e^5*x^5 - 320*d^5*e^4*x^4 + 1080*d^6*e^3*x^3 +
800*d^7*e^2*x^2 - 5*d^8*e*x - 180*d^9)*sqrt(-e^2*x^2 + d^2))/(d^10*e^11*x^1
0 + 4*d^11*e^10*x^9 + 3*d^12*e^9*x^8 - 8*d^13*e^8*x^7 - 14*d^14*e^7*x^6 + 1
4*d^16*e^5*x^4 + 8*d^17*e^4*x^3 - 3*d^18*e^3*x^2 - 4*d^19*e^2*x - d^20*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, undef, undef, 1]
```

3.859 $\int \frac{1}{(d+ex)^5\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=238

$$\frac{256x}{2145d^{11}\sqrt{d^2-e^2x^2}}+\frac{128x}{2145d^9\left(d^2-e^2x^2\right)^{3/2}}+\frac{32x}{715d^7\left(d^2-e^2x^2\right)^{5/2}}-\frac{16}{429d^5e(d+ex)\left(d^2-e^2x^2\right)^{5/2}}-\frac{16}{429d^4e(d+ex)^2\left(d^2\right.}$$

```
[Out] (32*x)/(715*d^7*(d^2 - e^2*x^2)^(5/2)) - 1/(15*d*e*(d + e*x)^5*(d^2 - e^2*x
^2)^(5/2)) - 2/(39*d^2*e*(d + e*x)^4*(d^2 - e^2*x^2)^(5/2)) - 6/(143*d^3*e*
(d + e*x)^3*(d^2 - e^2*x^2)^(5/2)) - 16/(429*d^4*e*(d + e*x)^2*(d^2 - e^2*x
^2)^(5/2)) - 16/(429*d^5*e*(d + e*x)*(d^2 - e^2*x^2)^(5/2)) + (128*x)/(2145
*d^9*(d^2 - e^2*x^2)^(3/2)) + (256*x)/(2145*d^11*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.112964, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {659, 192, 191}

$$\frac{256x}{2145d^{11}\sqrt{d^2-e^2x^2}}+\frac{128x}{2145d^9\left(d^2-e^2x^2\right)^{3/2}}+\frac{32x}{715d^7\left(d^2-e^2x^2\right)^{5/2}}-\frac{16}{429d^5e(d+ex)\left(d^2-e^2x^2\right)^{5/2}}-\frac{16}{429d^4e(d+ex)^2\left(d^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^5*(d^2 - e^2*x^2)^(7/2)),x]
```


```
[Out] (32*x)/(715*d^7*(d^2 - e^2*x^2)^(5/2)) - 1/(15*d*e*(d + e*x)^5*(d^2 - e^2*x
^2)^(5/2)) - 2/(39*d^2*e*(d + e*x)^4*(d^2 - e^2*x^2)^(5/2)) - 6/(143*d^3*e*
(d + e*x)^3*(d^2 - e^2*x^2)^(5/2)) - 16/(429*d^4*e*(d + e*x)^2*(d^2 - e^2*x
^2)^(5/2)) - 16/(429*d^5*e*(d + e*x)*(d^2 - e^2*x^2)^(5/2)) + (128*x)/(2145
*d^9*(d^2 - e^2*x^2)^(3/2)) + (256*x)/(2145*d^11*Sqrt[d^2 - e^2*x^2])
```


Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^5 \left(d^2-e^2x^2\right)^{7/2}}\,dx = -\frac{1}{15de(d+ex)^5 \left(d^2-e^2x^2\right)^{5/2}} + \frac{2\int \frac{1}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx}{3d}$$

$$= -\frac{1}{15de(d+ex)^5 \left(d^2-e^2x^2\right)^{5/2}} - \frac{2}{39d^2e(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} + \frac{6\int \frac{1}{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}\,d}{13d^2}$$

$$= -\frac{1}{15de(d+ex)^5 \left(d^2-e^2x^2\right)^{5/2}} - \frac{2}{39d^2e(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} - \frac{6}{143d^3e(d+ex)^3 \left(d^2-\right.}$$

$$= -\frac{1}{15de(d+ex)^5 \left(d^2-e^2x^2\right)^{5/2}} - \frac{2}{39d^2e(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} - \frac{6}{143d^3e(d+ex)^3 \left(d^2-\right.}$$

$$= -\frac{1}{15de(d+ex)^5 \left(d^2-e^2x^2\right)^{5/2}} - \frac{2}{39d^2e(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} - \frac{6}{143d^3e(d+ex)^3 \left(d^2-\right.}$$

$$= \frac{32x}{715d^7 \left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{15de(d+ex)^5 \left(d^2-e^2x^2\right)^{5/2}} - \frac{2}{39d^2e(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} -$$

$$= \frac{32x}{715d^7 \left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{15de(d+ex)^5 \left(d^2-e^2x^2\right)^{5/2}} - \frac{2}{39d^2e(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} -$$

$$= \frac{32x}{715d^7 \left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{15de(d+ex)^5 \left(d^2-e^2x^2\right)^{5/2}} - \frac{2}{39d^2e(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} -$$

Mathematica [A] time = 0.0965126, size = 148, normalized size = 0.62

$$\frac{\sqrt{d^2-e^2x^2}\left(1590d^8e^2x^2+3760d^7e^3x^3+1520d^6e^4x^4-3744d^5e^5x^5-4640d^4e^6x^6-640d^3e^7x^7+1920d^2e^8x^8-370d^9e\right.}{2145d^{11}e(d-ex)^3(d+ex)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^5*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-503*d^10 - 370*d^9*e*x + 1590*d^8*e^2*x^2 + 3760*d^7
*e^3*x^3 + 1520*d^6*e^4*x^4 - 3744*d^5*e^5*x^5 - 4640*d^4*e^6*x^6 - 640*d^3
*e^7*x^7 + 1920*d^2*e^8*x^8 + 1280*d*e^9*x^9 + 256*e^10*x^10))/(2145*d^11*e
*(d - e*x)^3*(d + e*x)^8)
```

Maple [A] time = 0.048, size = 143, normalized size = 0.6

$$-\frac{(-ex+d)\left(-256e^{10}x^{10}-1280e^9x^9d-1920e^8x^8d^2+640e^7x^7d^3+4640e^6x^6d^4+3744e^5x^5d^5-1520e^4x^4d^6-3760e\right.}{2145ed^{11}(ex+d)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^5/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/2145*(-e*x+d)*(-256*e^10*x^10-1280*d*e^9*x^9-1920*d^2*e^8*x^8+640*d^3*e^
7*x^7+4640*d^4*e^6*x^6+3744*d^5*e^5*x^5-1520*d^6*e^4*x^4-3760*d^7*e^3*x^3-1
```

```
590*d^8*e^2*x^2+370*d^9*e*x+503*d^10)/(e*x+d)^4/d^11/e/(-e^2*x^2+d^2)^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^5/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 11.6702, size = 863, normalized size = 3.63

$$-\frac{503\,e^{11}x^{11} + 2515\,de^{10}x^{10} + 3521\,d^2e^9x^9 - 2515\,d^3e^8x^8 - 11066\,d^4e^7x^7 - 7042\,d^5e^6x^6 + 7042\,d^6e^5x^5 + 11066\,d^7e^4x^4 + 2}{2145\left(d^{11}e^{12}x^{11} + 5\,d^{12}e^{11}x^{1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^5/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/2145*(503*e^11*x^11 + 2515*d*e^10*x^10 + 3521*d^2*e^9*x^9 - 2515*d^3*e^8
*x^8 - 11066*d^4*e^7*x^7 - 7042*d^5*e^6*x^6 + 7042*d^6*e^5*x^5 + 11066*d^7*
e^4*x^4 + 2515*d^8*e^3*x^3 - 3521*d^9*e^2*x^2 - 2515*d^10*e*x - 503*d^11 +
(256*e^10*x^10 + 1280*d*e^9*x^9 + 1920*d^2*e^8*x^8 - 640*d^3*e^7*x^7 - 4640
*d^4*e^6*x^6 - 3744*d^5*e^5*x^5 + 1520*d^6*e^4*x^4 + 3760*d^7*e^3*x^3 + 159
0*d^8*e^2*x^2 - 370*d^9*e*x - 503*d^10)*sqrt(-e^2*x^2 + d^2))/(d^11*e^12*x^
11 + 5*d^12*e^11*x^10 + 7*d^13*e^10*x^9 - 5*d^14*e^9*x^8 - 22*d^15*e^8*x^7
- 14*d^16*e^7*x^6 + 14*d^17*e^6*x^5 + 22*d^18*e^5*x^4 + 5*d^19*e^4*x^3 - 7*
d^20*e^3*x^2 - 5*d^21*e^2*x - d^22*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**5/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^5/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.860 $\int \frac{1+x}{\sqrt{1-x^2}} \, dx$

Optimal. Leaf size=16

$$\sin^{-1}(x) - \sqrt{1-x^2}$$

```
[Out] -Sqrt[1 - x^2] + ArcSin[x]
```

Rubi [A] time = 0.0043927, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {641, 216}

$$\sin^{-1}(x) - \sqrt{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/Sqrt[1 - x^2],x]

[Out] -Sqrt[1 - x^2] + ArcSin[x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x}{\sqrt{1-x^2}} \, dx &= -\sqrt{1-x^2} + \int \frac{1}{\sqrt{1-x^2}} \, dx \\ &= -\sqrt{1-x^2} + \sin^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.0057832, size = 16, normalized size = 1.

$$\sin^{-1}(x) - \sqrt{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/Sqrt[1 - x^2],x]

[Out] -Sqrt[1 - x^2] + ArcSin[x]
```

Maple [A] time = 0.042, size = 15, normalized size = 0.9

$$\arcsin(x) - \sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-x^2+1)^(1/2),x)
```

```
[Out] arcsin(x)-(-x^2+1)^(1/2)
```

Maxima [A] time = 1.64176, size = 19, normalized size = 1.19

$$-\sqrt{-x^2+1}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1) + arcsin(x)
```

Fricas [B] time = 2.03078, size = 70, normalized size = 4.38

$$-\sqrt{-x^2+1}-2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 + 1) - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] time = 0.137741, size = 10, normalized size = 0.62

$$-\sqrt{1-x^2}+\operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**2+1)**(1/2),x)
```

```
[Out] -sqrt(1 - x**2) + asin(x)
```

Giac [A] time = 1.27079, size = 19, normalized size = 1.19

$$-\sqrt{-x^2+1}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 1) + arcsin(x)
```

3.861 $\int \frac{1-x}{\sqrt{1-x^2}} \, dx$

Optimal. Leaf size=14

$$\sqrt{1-x^2}+\sin^{-1}(x)$$

```
[Out] Sqrt[1 - x^2] + ArcSin[x]
```

Rubi [A] time = 0.0042074, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {641, 216}

$$\sqrt{1-x^2}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/Sqrt[1 - x^2],x]
```

```
[Out] Sqrt[1 - x^2] + ArcSin[x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned} \int \frac{1-x}{\sqrt{1-x^2}} \, dx &= \sqrt{1-x^2}+\int \frac{1}{\sqrt{1-x^2}} \, dx \\ &= \sqrt{1-x^2}+\sin^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.0056058, size = 14, normalized size = 1.

$$\sqrt{1-x^2}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/Sqrt[1 - x^2],x]
```

```
[Out] Sqrt[1 - x^2] + ArcSin[x]
```

Maple [A] time = 0.041, size = 13, normalized size = 0.9

$$\arcsin(x)+\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/(-x^2+1)^(1/2),x)
```

```
[Out] arcsin(x)+(-x^2+1)^(1/2)
```

Maxima [A] time = 1.52499, size = 16, normalized size = 1.14

$$\sqrt{-x^2+1}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-x^2 + 1) + arcsin(x)
```

Fricas [B] time = 2.04453, size = 69, normalized size = 4.93

$$\sqrt{-x^2+1}-2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(-x^2 + 1) - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] time = 0.131368, size = 10, normalized size = 0.71

$$\sqrt{1-x^2}+\operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(-x**2+1)**(1/2),x)
```

```
[Out] sqrt(1 - x**2) + asin(x)
```

Giac [A] time = 1.19809, size = 16, normalized size = 1.14

$$\sqrt{-x^2+1}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(-x^2 + 1) + arcsin(x)
```

3.862 $\int (d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}\,dx$

Optimal. Leaf size=160

$$-\frac{256d^3\left(cd^2-ce^2x^2\right)^{3/2}}{315ce(d+ex)^{3/2}}-\frac{64d^2\left(cd^2-ce^2x^2\right)^{3/2}}{105ce\sqrt{d+ex}}-\frac{8d\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}{21ce}-\frac{2(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{3/2}}{9ce}$$

```
[Out] (-256*d^3*(c*d^2 - c*e^2*x^2)^(3/2))/(315*c*e*(d + e*x)^(3/2)) - (64*d^2*(c
*d^2 - c*e^2*x^2)^(3/2))/(105*c*e*Sqrt[d + e*x]) - (8*d*Sqrt[d + e*x]*(c*d^
2 - c*e^2*x^2)^(3/2))/(21*c*e) - (2*(d + e*x)^(3/2)*(c*d^2 - c*e^2*x^2)^(3/
2))/(9*c*e)
```

Rubi [A] time = 0.0729391, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {657, 649}

$$-\frac{256d^3\left(cd^2-ce^2x^2\right)^{3/2}}{315ce(d+ex)^{3/2}}-\frac{64d^2\left(cd^2-ce^2x^2\right)^{3/2}}{105ce\sqrt{d+ex}}-\frac{8d\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}{21ce}-\frac{2(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{3/2}}{9ce}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)*Sqrt[c*d^2 - c*e^2*x^2],x]
```

```
[Out] (-256*d^3*(c*d^2 - c*e^2*x^2)^(3/2))/(315*c*e*(d + e*x)^(3/2)) - (64*d^2*(c
*d^2 - c*e^2*x^2)^(3/2))/(105*c*e*Sqrt[d + e*x]) - (8*d*Sqrt[d + e*x]*(c*d^
2 - c*e^2*x^2)^(3/2))/(21*c*e) - (2*(d + e*x)^(3/2)*(c*d^2 - c*e^2*x^2)^(3/
2))/(9*c*e)
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int (d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}\,dx=-\frac{2(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{3/2}}{9ce}+\frac{1}{3}(4d)\int (d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}\,dx$$
$$=-\frac{8d\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}{21ce}-\frac{2(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{3/2}}{9ce}+\frac{1}{21}\left(32d^2\right)\int \sqrt{d+ex}\sqrt{\cdot}$$
$$=-\frac{64d^2\left(cd^2-ce^2x^2\right)^{3/2}}{105ce\sqrt{d+ex}}-\frac{8d\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}{21ce}-\frac{2(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{3/2}}{9ce}+$$
$$=-\frac{256d^3\left(cd^2-ce^2x^2\right)^{3/2}}{315ce(d+ex)^{3/2}}-\frac{64d^2\left(cd^2-ce^2x^2\right)^{3/2}}{105ce\sqrt{d+ex}}-\frac{8d\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}{21ce}-\frac{2(d+$$

Mathematica [A] time = 0.0629891, size = 75, normalized size = 0.47

$$-\frac{2\left(-156d^2e^2x^2+2d^3ex+319d^4-130de^3x^3-35e^4x^4\right)\sqrt{c\left(d^2-e^2x^2\right)}}{315e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)*Sqrt[c*d^2 - c*e^2*x^2],x]
```

```
[Out] (-2*Sqrt[c*(d^2 - e^2*x^2)]*(319*d^4 + 2*d^3*e*x - 156*d^2*e^2*x^2 - 130*d*
e^3*x^3 - 35*e^4*x^4))/(315*e*Sqrt[d + e*x])
```

Maple [A] time = 0.044, size = 66, normalized size = 0.4

$$-\frac{(-2\,ex+2\,d)\left(35\,e^3x^3+165\,de^2x^2+321\,d^2xe+319\,d^3\right)}{315\,e}\sqrt{-ce^2x^2+cd^2}\frac{1}{\sqrt{ex+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(-c*e^2*x^2+c*d^2)^(1/2),x)
```

```
[Out] -2/315*(-e*x+d)*(35*e^3*x^3+165*d*e^2*x^2+321*d^2*e*x+319*d^3)*(-c*e^2*x^2+
c*d^2)^(1/2)/e/(e*x+d)^(1/2)
```

Maxima [A] time = 1.10864, size = 111, normalized size = 0.69

$$\frac{2\left(35\,\sqrt{c}e^4x^4+130\,\sqrt{c}de^3x^3+156\,\sqrt{c}d^2e^2x^2-2\,\sqrt{c}d^3ex-319\,\sqrt{c}d^4\right)(ex+d)\sqrt{-ex+d}}{315\left(e^2x+de\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*sqrt(c)*e^4*x^4 + 130*sqrt(c)*d*e^3*x^3 + 156*sqrt(c)*d^2*e^2*x^2
 - 2*sqrt(c)*d^3*e*x - 319*sqrt(c)*d^4)*(e*x + d)*sqrt(-e*x + d)/(e^2*x + d
*e)
```

Fricas [A] time = 2.08586, size = 174, normalized size = 1.09

$$\frac{2\left(35\,e^4x^4+130\,de^3x^3+156\,d^2e^2x^2-2\,d^3ex-319\,d^4\right)\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}}{315\left(e^2x+de\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*e^4*x^4 + 130*d*e^3*x^3 + 156*d^2*e^2*x^2 - 2*d^3*e*x - 319*d^4)*
sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)/(e^2*x + d*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-c\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(-c*e**2*x**2+c*d**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-d + e*x)*(d + e*x))*(d + e*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-ce^2x^2+cd^2}(ex+d)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c*e^2*x^2 + c*d^2)*(e*x + d)^(5/2), x)
```

3.863 $\int (d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}\,dx$

Optimal. Leaf size=119

$$-\frac{64d^2\left(cd^2-ce^2x^2\right)^{3/2}}{105ce(d+ex)^{3/2}}-\frac{16d\left(cd^2-ce^2x^2\right)^{3/2}}{35ce\sqrt{d+ex}}-\frac{2\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}{7ce}$$

```
[Out] (-64*d^2*(c*d^2 - c*e^2*x^2)^(3/2))/(105*c*e*(d + e*x)^(3/2)) - (16*d*(c*d^
2 - c*e^2*x^2)^(3/2))/(35*c*e*Sqrt[d + e*x]) - (2*Sqrt[d + e*x]*(c*d^2 - c*
e^2*x^2)^(3/2))/(7*c*e)
```

Rubi [A] time = 0.0476059, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {657, 649}

$$-\frac{64d^2\left(cd^2-ce^2x^2\right)^{3/2}}{105ce(d+ex)^{3/2}}-\frac{16d\left(cd^2-ce^2x^2\right)^{3/2}}{35ce\sqrt{d+ex}}-\frac{2\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}{7ce}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*Sqrt[c*d^2 - c*e^2*x^2],x]
```

```
[Out] (-64*d^2*(c*d^2 - c*e^2*x^2)^(3/2))/(105*c*e*(d + e*x)^(3/2)) - (16*d*(c*d^
2 - c*e^2*x^2)^(3/2))/(35*c*e*Sqrt[d + e*x]) - (2*Sqrt[d + e*x]*(c*d^2 - c*
e^2*x^2)^(3/2))/(7*c*e)
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}\,dx &= -\frac{2\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}{7ce}+\frac{1}{7}(8d)\int \sqrt{d+ex}\sqrt{cd^2-ce^2x^2}\,dx\\
&= -\frac{16d\left(cd^2-ce^2x^2\right)^{3/2}}{35ce\sqrt{d+ex}}-\frac{2\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}{7ce}+\frac{1}{35}\left(32d^2\right)\int \frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}\,dx\\
&= -\frac{64d^2\left(cd^2-ce^2x^2\right)^{3/2}}{105ce(d+ex)^{3/2}}-\frac{16d\left(cd^2-ce^2x^2\right)^{3/2}}{35ce\sqrt{d+ex}}-\frac{2\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}{7ce}
\end{aligned}$$

Mathematica [A] time = 0.0532616, size = 64, normalized size = 0.54

$$\frac{2\left(17d^2ex-71d^3+39de^2x^2+15e^3x^3\right)\sqrt{c\left(d^2-e^2x^2\right)}}{105e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)*Sqrt[c*d^2 - c*e^2*x^2],x]
```

```
[Out] (2*Sqrt[c*(d^2 - e^2*x^2)]*(-71*d^3 + 17*d^2*e*x + 39*d*e^2*x^2 + 15*e^3*x^
3))/(105*e*Sqrt[d + e*x])
```

Maple [A] time = 0.043, size = 55, normalized size = 0.5

$$-\frac{(-2ex+2d)\left(15e^{2}x^{2}+54dxe+71d^{2}\right)}{105e}\sqrt{-ce^{2}x^{2}+cd^{2}}\frac{1}{\sqrt{ex+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(-c*e^2*x^2+c*d^2)^(1/2),x)
```

```
[Out] -2/105*(-e*x+d)*(15*e^2*x^2+54*d*e*x+71*d^2)*(-c*e^2*x^2+c*d^2)^(1/2)/(e*x+
d)^(1/2)/e
```

Maxima [A] time = 1.18132, size = 92, normalized size = 0.77

$$\frac{2\left(15\sqrt{c}e^{3}x^{3}+39\sqrt{c}de^{2}x^{2}+17\sqrt{c}d^{2}ex-71\sqrt{c}d^{3}\right)(ex+d)\sqrt{-ex+d}}{105\left(e^{2}x+de\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*sqrt(c)*e^3*x^3 + 39*sqrt(c)*d*e^2*x^2 + 17*sqrt(c)*d^2*e*x - 71*
sqrt(c)*d^3)*(e*x + d)*sqrt(-e*x + d)/(e^2*x + d*e)
```

Fricas [A] time = 2.05922, size = 149, normalized size = 1.25

$$\frac{2\left(15e^{3}x^{3}+39de^{2}x^{2}+17d^{2}ex-71d^{3}\right)\sqrt{-ce^{2}x^{2}+cd^{2}}\sqrt{ex+d}}{105\left(e^{2}x+de\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*e^3*x^3 + 39*d*e^2*x^2 + 17*d^2*e*x - 71*d^3)*sqrt(-c*e^2*x^2 + c
*d^2)*sqrt(e*x + d)/(e^2*x + d*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-c\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(-c*e**2*x**2+c*d**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-d + e*x)*(d + e*x))*(d + e*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-ce^2x^2 + cd^2}(ex + d)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c*e^2*x^2 + c*d^2)*(e*x + d)^(3/2), x)
```

3.864 $\int \sqrt{d+ex}\sqrt{cd^2-ce^2x^2}\,dx$

Optimal. Leaf size=78

$$-\frac{2\left(cd^2-ce^2x^2\right)^{3/2}}{5ce\sqrt{d+ex}}-\frac{8d\left(cd^2-ce^2x^2\right)^{3/2}}{15ce(d+ex)^{3/2}}$$

```
[Out] (-8*d*(c*d^2 - c*e^2*x^2)^(3/2))/(15*c*e*(d + e*x)^(3/2)) - (2*(c*d^2 - c*e
^2*x^2)^(3/2))/(5*c*e*Sqrt[d + e*x])
```

Rubi [A] time = 0.0279207, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {657, 649}

$$-\frac{2\left(cd^2-ce^2x^2\right)^{3/2}}{5ce\sqrt{d+ex}}-\frac{8d\left(cd^2-ce^2x^2\right)^{3/2}}{15ce(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*Sqrt[c*d^2 - c*e^2*x^2],x]
```


```
[Out] (-8*d*(c*d^2 - c*e^2*x^2)^(3/2))/(15*c*e*(d + e*x)^(3/2)) - (2*(c*d^2 - c*e
^2*x^2)^(3/2))/(5*c*e*Sqrt[d + e*x])
```


Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{d+ex}\sqrt{cd^2-ce^2x^2}\,dx &= -\frac{2\left(cd^2-ce^2x^2\right)^{3/2}}{5ce\sqrt{d+ex}}+\frac{1}{5}(4d)\int \frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}\,dx\\ &= -\frac{8d\left(cd^2-ce^2x^2\right)^{3/2}}{15ce(d+ex)^{3/2}}-\frac{2\left(cd^2-ce^2x^2\right)^{3/2}}{5ce\sqrt{d+ex}}\end{aligned}$$

Mathematica [A] time = 0.0432887, size = 53, normalized size = 0.68

$$-\frac{2\left(7d^2-4dex-3e^2x^2\right)\sqrt{c\left(d^2-e^2x^2\right)}}{15e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*Sqrt[c*d^2 - c*e^2*x^2],x]
```

```
[Out] (-2*(7*d^2 - 4*d*e*x - 3*e^2*x^2)*Sqrt[c*(d^2 - e^2*x^2)])/(15*e*Sqrt[d + e
*x])
```

Maple [A] time = 0.045, size = 44, normalized size = 0.6

$$-\frac{(-2\,ex+2\,d)\,(3\,ex+7\,d)}{15\,e}\sqrt{-ce^2x^2+cd^2}\frac{1}{\sqrt{ex+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(-c*e^2*x^2+c*d^2)^(1/2),x)
```

```
[Out] -2/15*(-e*x+d)*(3*e*x+7*d)*(-c*e^2*x^2+c*d^2)^(1/2)/e/(e*x+d)^(1/2)
```

Maxima [A] time = 1.05922, size = 73, normalized size = 0.94

$$\frac{2\left(3\,\sqrt{c}e^2x^2+4\,\sqrt{c}dex-7\,\sqrt{c}d^2\right)(ex+d)\sqrt{-ex+d}}{15\left(e^2x+de\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*sqrt(c)*e^2*x^2 + 4*sqrt(c)*d*e*x - 7*sqrt(c)*d^2)*(e*x + d)*sqrt(-
e*x + d)/(e^2*x + d*e)
```

Fricas [A] time = 2.07991, size = 120, normalized size = 1.54

$$\frac{2\,\sqrt{-ce^2x^2+cd^2}\left(3\,e^2x^2+4\,dex-7\,d^2\right)\sqrt{ex+d}}{15\left(e^2x+de\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*sqrt(-c*e^2*x^2 + c*d^2)*(3*e^2*x^2 + 4*d*e*x - 7*d^2)*sqrt(e*x + d)/(
e^2*x + d*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-c\,(-d+ex)\,(d+ex)}\sqrt{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(-c*e**2*x**2+c*d**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-d + e*x)*(d + e*x))*sqrt(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-ce^2x^2 + cd^2}\sqrt{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d), x)
```

3.865 $\int \frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=38

$$-\frac{2\left(cd^2-ce^2x^2\right)^{3/2}}{3ce(d+ex)^{3/2}}$$

```
[Out] (-2*(c*d^2 - c*e^2*x^2)^(3/2))/(3*c*e*(d + e*x)^(3/2))
```

Rubi [A] time = 0.012885, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {649}

$$-\frac{2\left(cd^2-ce^2x^2\right)^{3/2}}{3ce(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*d^2 - c*e^2*x^2]/Sqrt[d + e*x],x]
```

```
[Out] (-2*(c*d^2 - c*e^2*x^2)^(3/2))/(3*c*e*(d + e*x)^(3/2))
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}\,dx=-\frac{2\left(cd^2-ce^2x^2\right)^{3/2}}{3ce(d+ex)^{3/2}}$$

Mathematica [A] time = 0.0374206, size = 40, normalized size = 1.05

$$-\frac{2(d-ex)\sqrt{c\left(d^2-e^2x^2\right)}}{3e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*d^2 - c*e^2*x^2]/Sqrt[d + e*x],x]
```

```
[Out] (-2*(d - e*x)*Sqrt[c*(d^2 - e^2*x^2)])/(3*e*Sqrt[d + e*x])
```

Maple [A] time = 0.04, size = 36, normalized size = 1.

$$-\frac{-2\,ex+2\,d}{3\,e}\sqrt{-ce^2x^2+cd^2}\frac{1}{\sqrt{ex+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(1/2),x)

[Out] -2/3*(-e*x+d)*(-c*e^2*x^2+c*d^2)^(1/2)/e/(e*x+d)^(1/2)

Maxima [A] time = 1.06111, size = 35, normalized size = 0.92

$$\frac{2\left(\sqrt{c}ex-\sqrt{c}d\right)\sqrt{-ex+d}}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima")

[Out] 2/3*(sqrt(c)*e*x - sqrt(c)*d)*sqrt(-e*x + d)/e

Fricas [A] time = 2.01604, size = 92, normalized size = 2.42

$$\frac{2\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}(ex-d)}{3\left(e^2x+de\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")

[Out] 2/3*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)*(e*x - d)/(e^2*x + d*e)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-c\left(-d+ex\right)\left(d+ex\right)}}{\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c*e**2*x**2+c*d**2)**(1/2)/(e*x+d)**(1/2),x)

[Out] Integral(sqrt(-c*(-d + e*x)*(d + e*x))/sqrt(d + e*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-ce^2x^2+cd^2}}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-c*e^2*x^2 + c*d^2)/sqrt(e*x + d), x)

3.866 $\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=99

$$\frac{2\sqrt{cd^2-ce^2x^2}}{e\sqrt{d+ex}}-\frac{2\sqrt{2}\sqrt{c}\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{e}$$

```
[Out] (2*Sqrt[c*d^2 - c*e^2*x^2])/(e*Sqrt[d + e*x]) - (2*Sqrt[2]*Sqrt[c]*Sqrt[d]*
ArcTanh[Sqrt[c*d^2 - c*e^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/e
```

Rubi [A] time = 0.0553575, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {665, 661, 208}

$$\frac{2\sqrt{cd^2-ce^2x^2}}{e\sqrt{d+ex}}-\frac{2\sqrt{2}\sqrt{c}\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*d^2 - c*e^2*x^2]/(d + e*x)^(3/2),x]
```

```
[Out] (2*Sqrt[c*d^2 - c*e^2*x^2])/(e*Sqrt[d + e*x]) - (2*Sqrt[2]*Sqrt[c]*Sqrt[d]*
ArcTanh[Sqrt[c*d^2 - c*e^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/e
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{3/2}}\,dx = \frac{2\sqrt{cd^2-ce^2x^2}}{e\sqrt{d+ex}} + (2cd)\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\,dx$$
$$= \frac{2\sqrt{cd^2-ce^2x^2}}{e\sqrt{d+ex}} + (4cde)\,\text{Subst}\left(\int \frac{1}{-2cde^2+e^2x^2}\,dx, x, \frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}\right)$$
$$= \frac{2\sqrt{cd^2-ce^2x^2}}{e\sqrt{d+ex}} - \frac{2\sqrt{2}\sqrt{c}\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{e}$$

Mathematica [A] time = 0.0925152, size = 98, normalized size = 0.99

$$\frac{2\sqrt{c\left(d^2-e^2x^2\right)}\left(\frac{1}{\sqrt{d+ex}} - \frac{\sqrt{2}\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{\sqrt{2}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*d^2 - c*e^2*x^2]/(d + e*x)^(3/2),x]
```

```
[Out] (2*Sqrt[c*(d^2 - e^2*x^2)]*(1/Sqrt[d + e*x] - (Sqrt[2]*Sqrt[d]*ArcTanh[Sqrt
[d^2 - e^2*x^2]/(Sqrt[2]*Sqrt[d]*Sqrt[d + e*x])])/Sqrt[d^2 - e^2*x^2]))/e
```

Maple [A] time = 0.243, size = 97, normalized size = 1.

$$-2\,\frac{\sqrt{-c\left(e^2x^2-d^2\right)}}{\sqrt{ex+d}\sqrt{-(ex-d)\,ce}\sqrt{cd}}\left(cd\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{-(ex-d)\,c}\sqrt{2}}{\sqrt{cd}}\right)-\sqrt{-(ex-d)\,c}\sqrt{cd}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(3/2),x)
```

```
[Out] -2*(-c*(e^2*x^2-d^2))^(1/2)*(c*d*2^(1/2)*arctanh(1/2*(-(e*x-d)*c)^(1/2)*2^(
1/2)/(c*d)^(1/2))-(-(e*x-d)*c)^(1/2)*(c*d)^(1/2))/(e*x+d)^(1/2)/(-(e*x-d)*c
)^(1/2)/e/(c*d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-ce^2x^2+cd^2}}{(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-c*e^2*x^2 + c*d^2)/(e*x + d)^(3/2), x)
```

Fricas [A] time = 2.30145, size = 533, normalized size = 5.38

$$\left[\frac{\sqrt{2}\sqrt{cd}(ex+d)\log\left(-\frac{ce^2x^2-2\,cdex-3\,cd^2+2\,\sqrt{2}\sqrt{-ce^2x^2+cd^2}\sqrt{cd}\sqrt{ex+d}}{e^2x^2+2\,dex+d^2}\right)+2\,\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}}{e^2x+de},-\frac{2\left(\sqrt{2}\sqrt{-cd}(ex+d)\arctan\left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(2)*sqrt(c*d)*(e*x + d)*log(-(c*e^2*x^2 - 2*c*d*e*x - 3*c*d^2 + 2*sqr
t(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(c*d)*sqrt(e*x + d))/(e^2*x^2 + 2*d*e*x +
 d^2)) + 2*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d))/(e^2*x + d*e), -2*(sqrt(
2)*sqrt(-c*d)*(e*x + d)*arctan(sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(-c*d)*
sqrt(e*x + d)/(c*e^2*x^2 - c*d^2)) - sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)
)/(e^2*x + d*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-c\,(-d+ex)\,(d+ex)}}{(d+ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e**2*x**2+c*d**2)**(1/2)/(e*x+d)**(3/2),x)
```

```
[Out] Integral(sqrt(-c*(-d + e*x)*(d + e*x))/(d + e*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-ce^2x^2+cd^2}}{(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c*e^2*x^2 + c*d^2)/(e*x + d)^(3/2), x)
```

3.867 $\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=98

$$\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{2}\sqrt{d}e}-\frac{\sqrt{cd^2-ce^2x^2}}{e(d+ex)^{3/2}}$$

```
[Out] -(Sqrt[c*d^2 - c*e^2*x^2]/(e*(d + e*x)^(3/2))) + (Sqrt[c]*ArcTanh[Sqrt[c*d^
2 - c*e^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/(Sqrt[2]*Sqrt[d]*e
)
```

Rubi [A] time = 0.0482074, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {663, 661, 208}

$$\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{2}\sqrt{d}e}-\frac{\sqrt{cd^2-ce^2x^2}}{e(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*d^2 - c*e^2*x^2]/(d + e*x)^(5/2),x]
```

```
[Out] -(Sqrt[c*d^2 - c*e^2*x^2]/(e*(d + e*x)^(3/2))) + (Sqrt[c]*ArcTanh[Sqrt[c*d^
2 - c*e^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/(Sqrt[2]*Sqrt[d]*e
)
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{5/2}}\,dx = -\frac{\sqrt{cd^2-ce^2x^2}}{e(d+ex)^{3/2}} - \frac{1}{2}c\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\,dx$$
$$= -\frac{\sqrt{cd^2-ce^2x^2}}{e(d+ex)^{3/2}} - (ce)\,\mathrm{Subst}\left(\int \frac{1}{-2cde^2+e^2x^2}\,dx, x, \frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}\right)$$
$$= -\frac{\sqrt{cd^2-ce^2x^2}}{e(d+ex)^{3/2}} + \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{2}\sqrt{d}e}$$

Mathematica [A] time = 0.112271, size = 101, normalized size = 1.03

$$\frac{\sqrt{c\left(d^2-e^2x^2\right)}\left(\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{\sqrt{2}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{d}\sqrt{d^2-e^2x^2}} - \frac{2}{(d+ex)^{3/2}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*d^2 - c*e^2*x^2]/(d + e*x)^(5/2),x]
```

```
[Out] (Sqrt[c*(d^2 - e^2*x^2)]*(-2/(d + e*x)^(3/2) + (Sqrt[2]*ArcTanh[Sqrt[d^2 -
e^2*x^2]/(Sqrt[2]*Sqrt[d]*Sqrt[d + e*x])])/(Sqrt[d]*Sqrt[d^2 - e^2*x^2])))/
(2*e)
```

Maple [A] time = 0.171, size = 127, normalized size = 1.3

$$\frac{1}{2e}\sqrt{-c\left(e^2x^2-d^2\right)}\left(\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{-(ex-d)c}\frac{1}{\sqrt{cd}}\right)xce + cd\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{-(ex-d)c}\frac{1}{\sqrt{cd}}\right) - 2\sqrt{-(ex-d)c}\sqrt{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(5/2),x)
```

```
[Out] 1/2*(-c*(e^2*x^2-d^2))^(1/2)*(2^(1/2)*arctanh(1/2*(-(e*x-d)*c)^(1/2)*2^(1/2
)/(c*d)^(1/2))*x*c*e+c*d*2^(1/2)*arctanh(1/2*(-(e*x-d)*c)^(1/2)*2^(1/2)/(c*
d)^(1/2))-2*(-(e*x-d)*c)^(1/2)*(c*d)^(1/2))/(e*x+d)^(3/2)/(-(e*x-d)*c)^(1/2
)/e/(c*d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-ce^2x^2+cd^2}}{(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-c*e^2*x^2 + c*d^2)/(e*x + d)^(5/2), x)
```

Fricas [A] time = 2.3749, size = 640, normalized size = 6.53

$$\left[\frac{\sqrt{\frac{1}{2}}\left(e^2x^2+2\,dex+d^2\right)\sqrt{\frac{c}{d}}\log\left(-\frac{ce^2x^2-2\,cdex-3\,cd^2-4\,\sqrt{\frac{1}{2}}\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}d\sqrt{\frac{c}{d}}}{e^2x^2+2\,dex+d^2}\right)-2\,\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}}{2\left(e^3x^2+2\,de^2x+d^2e\right)},\ \frac{\sqrt{\frac{1}{2}}\left(e^2x^2+2\,d\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(1/2)*(e^2*x^2 + 2*d*e*x + d^2)*sqrt(c/d)*log(-(c*e^2*x^2 - 2*c*d
*e*x - 3*c*d^2 - 4*sqrt(1/2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)*d*sqrt(
c/d))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)
)/(e^3*x^2 + 2*d*e^2*x + d^2*e), (sqrt(1/2)*(e^2*x^2 + 2*d*e*x + d^2)*sqrt(
-c/d)*arctan(2*sqrt(1/2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)*d*sqrt(-c/d
)/(c*e^2*x^2 - c*d^2)) - sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d))/(e^3*x^2 +
 2*d*e^2*x + d^2*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-d+ex\right)\left(d+ex\right)}}{\left(d+ex\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e**2*x**2+c*d**2)**(1/2)/(e*x+d)**(5/2),x)
```

```
[Out] Integral(sqrt(-c*(-d + e*x)*(d + e*x))/(d + e*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-ce^2x^2+cd^2}}{\left(ex+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c*e^2*x^2 + c*d^2)/(e*x + d)^(5/2), x)
```

3.868 $\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=141

$$\frac{\sqrt{cd^2-ce^2x^2}}{8de(d+ex)^{3/2}}-\frac{\sqrt{cd^2-ce^2x^2}}{2e(d+ex)^{5/2}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{8\sqrt{2}d^{3/2}e}$$

```
[Out] -Sqrt[c*d^2 - c*e^2*x^2]/(2*e*(d + e*x)^(5/2)) + Sqrt[c*d^2 - c*e^2*x^2]/(8
*d*e*(d + e*x)^(3/2)) + (Sqrt[c]*ArcTanh[Sqrt[c*d^2 - c*e^2*x^2]/(Sqrt[2]*S
qrt[c]*Sqrt[d]*Sqrt[d + e*x])])/(8*Sqrt[2]*d^(3/2)*e)
```

Rubi [A] time = 0.0776742, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {663, 673, 661, 208}

$$\frac{\sqrt{cd^2-ce^2x^2}}{8de(d+ex)^{3/2}}-\frac{\sqrt{cd^2-ce^2x^2}}{2e(d+ex)^{5/2}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{8\sqrt{2}d^{3/2}e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*d^2 - c*e^2*x^2]/(d + e*x)^(7/2),x]
```

```
[Out] -Sqrt[c*d^2 - c*e^2*x^2]/(2*e*(d + e*x)^(5/2)) + Sqrt[c*d^2 - c*e^2*x^2]/(8
*d*e*(d + e*x)^(3/2)) + (Sqrt[c]*ArcTanh[Sqrt[c*d^2 - c*e^2*x^2]/(Sqrt[2]*S
qrt[c]*Sqrt[d]*Sqrt[d + e*x])])/(8*Sqrt[2]*d^(3/2)*e)
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[(m + 2*
p + 2)/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; Fr
eeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p
+ 1, 0] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{7/2}}\,dx &= -\frac{\sqrt{cd^2-ce^2x^2}}{2e(d+ex)^{5/2}}-\frac{1}{4}c\int \frac{1}{(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}\,dx\\
&= -\frac{\sqrt{cd^2-ce^2x^2}}{2e(d+ex)^{5/2}}+\frac{\sqrt{cd^2-ce^2x^2}}{8de(d+ex)^{3/2}}-\frac{c\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\,dx}{16d}\\
&= -\frac{\sqrt{cd^2-ce^2x^2}}{2e(d+ex)^{5/2}}+\frac{\sqrt{cd^2-ce^2x^2}}{8de(d+ex)^{3/2}}-\frac{(ce)\,\mathrm{Subst}\left(\int \frac{1}{-2cde^2+e^2x^2}\,dx,x,\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}\right)}{8d}\\
&= -\frac{\sqrt{cd^2-ce^2x^2}}{2e(d+ex)^{5/2}}+\frac{\sqrt{cd^2-ce^2x^2}}{8de(d+ex)^{3/2}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{8\sqrt{2}d^{3/2}e}
\end{aligned}
$$

Mathematica [A] time = 0.144759, size = 111, normalized size = 0.79

$$
\frac{\sqrt{c\left(d^2-e^2x^2\right)}\left(\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{\sqrt{2}\sqrt{d}\sqrt{d+ex}}\right)}{d^{3/2}\sqrt{d^2-e^2x^2}}+\frac{2ex-6d}{d(d+ex)^{5/2}}\right)}{16e}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*d^2 - c*e^2*x^2]/(d + e*x)^(7/2),x]
```

```
[Out] (Sqrt[c*(d^2 - e^2*x^2)]*((-6*d + 2*e*x)/(d*(d + e*x)^(5/2)) + (Sqrt[2]*Arc
Tanh[Sqrt[d^2 - e^2*x^2]/(Sqrt[2]*Sqrt[d]*Sqrt[d + e*x])])/(d^(3/2)*Sqrt[d^
2 - e^2*x^2])))/(16*e)
```

Maple [A] time = 0.169, size = 190, normalized size = 1.4

$$
\frac{1}{16\,de}\sqrt{-c\left(e^2x^2-d^2\right)}\left(\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{-(ex-d)\,c}\frac{1}{\sqrt{cd}}\right)x^2ce^2+2\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{-(ex-d)\,c}\sqrt{2}}{\sqrt{cd}}\right)xcde+\sqrt{2}\mathrm{Art}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(7/2),x)
```

```
[Out] 1/16*(-c*(e^2*x^2-d^2))^(1/2)*(2^(1/2)*arctanh(1/2*(-(e*x-d)*c)^(1/2)*2^(1/
2)/(c*d)^(1/2))*x^2*c*e^2+2*2^(1/2)*arctanh(1/2*(-(e*x-d)*c)^(1/2)*2^(1/2)/
(c*d)^(1/2))*x*c*d*e+2^(1/2)*arctanh(1/2*(-(e*x-d)*c)^(1/2)*2^(1/2)/(c*d)^(
1/2))*c*d^2+2*x*e*(-(e*x-d)*c)^(1/2)*(c*d)^(1/2)-6*(-(e*x-d)*c)^(1/2)*(c*d)
^(1/2)*d)/(e*x+d)^(5/2)/(-(e*x-d)*c)^(1/2)/e/d/(c*d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\sqrt{-ce^2x^2+cd^2}}{(ex+d)^{\frac{7}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-c*e^2*x^2 + c*d^2)/(e*x + d)^(7/2), x)
```

Fricas [A] time = 2.38676, size = 776, normalized size = 5.5

$$\left[\frac{\sqrt{\frac{1}{2}}\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\sqrt{\frac{c}{d}}\log\left(-\frac{ce^2x^2-2\,cdex-3\,cd^2-4\,\sqrt{\frac{1}{2}}\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}d\sqrt{\frac{c}{d}}}{e^2x^2+2\,dex+d^2}\right)+2\,\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}(ex-3\,d}{16\left(de^4x^3+3\,d^2e^3x^2+3\,d^3e^2x+d^4e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/16*(sqrt(1/2)*(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(c/d)*log(-(
c*e^2*x^2 - 2*c*d*e*x - 3*c*d^2 - 4*sqrt(1/2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt
(e*x + d)*d*sqrt(c/d))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*sqrt(-c*e^2*x^2 + c*d
^2)*sqrt(e*x + d)*(e*x - 3*d))/(d*e^4*x^3 + 3*d^2*e^3*x^2 + 3*d^3*e^2*x + d
^4*e), 1/8*(sqrt(1/2)*(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(-c/d)*
arctan(2*sqrt(1/2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)*d*sqrt(-c/d)/(c*e
^2*x^2 - c*d^2)) + sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)*(e*x - 3*d))/(d*e
^4*x^3 + 3*d^2*e^3*x^2 + 3*d^3*e^2*x + d^4*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-d+ex\right)\left(d+ex\right)}}{\left(d+ex\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e**2*x**2+c*d**2)**(1/2)/(e*x+d)**(7/2),x)
```

```
[Out] Integral(sqrt(-c*(-d + e*x)*(d + e*x))/(d + e*x)**(7/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-ce^2x^2+cd^2}}{\left(ex+d\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c*e^2*x^2 + c*d^2)/(e*x + d)^(7/2), x)
```

3.869 $\int (d+ex)^{5/2} \left(cd^2-ce^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=201

$$-\frac{4096d^4\left(cd^2-ce^2x^2\right)^{5/2}}{15015ce(d+ex)^{5/2}}-\frac{1024d^3\left(cd^2-ce^2x^2\right)^{5/2}}{3003ce(d+ex)^{3/2}}-\frac{128d^2\left(cd^2-ce^2x^2\right)^{5/2}}{429ce\sqrt{d+ex}}-\frac{32d\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{5/2}}{143ce}-2(d+e$$

```
[Out] (-4096*d^4*(c*d^2 - c*e^2*x^2)^(5/2))/(15015*c*e*(d + e*x)^(5/2)) - (1024*d
^3*(c*d^2 - c*e^2*x^2)^(5/2))/(3003*c*e*(d + e*x)^(3/2)) - (128*d^2*(c*d^2
- c*e^2*x^2)^(5/2))/(429*c*e*Sqrt[d + e*x]) - (32*d*Sqrt[d + e*x]*(c*d^2 -
c*e^2*x^2)^(5/2))/(143*c*e) - (2*(d + e*x)^(3/2)*(c*d^2 - c*e^2*x^2)^(5/2))
/(13*c*e)
```

Rubi [A] time = 0.0996525, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {657, 649}

$$-\frac{4096d^4\left(cd^2-ce^2x^2\right)^{5/2}}{15015ce(d+ex)^{5/2}}-\frac{1024d^3\left(cd^2-ce^2x^2\right)^{5/2}}{3003ce(d+ex)^{3/2}}-\frac{128d^2\left(cd^2-ce^2x^2\right)^{5/2}}{429ce\sqrt{d+ex}}-\frac{32d\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{5/2}}{143ce}-2(d+e$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)*(c*d^2 - c*e^2*x^2)^(3/2),x]
```

```
[Out] (-4096*d^4*(c*d^2 - c*e^2*x^2)^(5/2))/(15015*c*e*(d + e*x)^(5/2)) - (1024*d
^3*(c*d^2 - c*e^2*x^2)^(5/2))/(3003*c*e*(d + e*x)^(3/2)) - (128*d^2*(c*d^2
- c*e^2*x^2)^(5/2))/(429*c*e*Sqrt[d + e*x]) - (32*d*Sqrt[d + e*x]*(c*d^2 -
c*e^2*x^2)^(5/2))/(143*c*e) - (2*(d + e*x)^(3/2)*(c*d^2 - c*e^2*x^2)^(5/2))
/(13*c*e)
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int (d+ex)^{5/2}\left(cd^2-ce^2x^2\right)^{3/2}dx = -\frac{2(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{5/2}}{13ce}+\frac{1}{13}(16d)\int (d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{3/2}dx$$
$$= -\frac{32d\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{5/2}}{143ce}-\frac{2(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{5/2}}{13ce}+\frac{1}{143}\left(192d^2\right)\int \sqrt{d+e}$$
$$= -\frac{128d^2\left(cd^2-ce^2x^2\right)^{5/2}}{429ce\sqrt{d+ex}}-\frac{32d\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{5/2}}{143ce}-\frac{2(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{5/2}}{13ce}$$
$$= -\frac{1024d^3\left(cd^2-ce^2x^2\right)^{5/2}}{3003ce(d+ex)^{3/2}}-\frac{128d^2\left(cd^2-ce^2x^2\right)^{5/2}}{429ce\sqrt{d+ex}}-\frac{32d\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{5/2}}{143ce}-2$$
$$= -\frac{4096d^4\left(cd^2-ce^2x^2\right)^{5/2}}{15015ce(d+ex)^{5/2}}-\frac{1024d^3\left(cd^2-ce^2x^2\right)^{5/2}}{3003ce(d+ex)^{3/2}}-\frac{128d^2\left(cd^2-ce^2x^2\right)^{5/2}}{429ce\sqrt{d+ex}}-32d\sqrt{}$$

Mathematica [A] time = 0.0673561, size = 84, normalized size = 0.42

$$-\frac{2c(d-ex)^2\left(14210d^2e^2x^2+16700d^3ex+9683d^4+6300de^3x^3+1155e^4x^4\right)\sqrt{c\left(d^2-e^2x^2\right)}}{15015e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)*(c*d^2 - c*e^2*x^2)^(3/2),x]
```

```
[Out] (-2*c*(d - e*x)^2*Sqrt[c*(d^2 - e^2*x^2)]*(9683*d^4 + 16700*d^3*e*x + 14210
*d^2*e^2*x^2 + 6300*d*e^3*x^3 + 1155*e^4*x^4))/(15015*e*Sqrt[d + e*x])
```

Maple [A] time = 0.043, size = 77, normalized size = 0.4

$$-\frac{(-2\,ex+2\,d)\left(1155\,e^4x^4+6300\,de^3x^3+14210\,d^2e^2x^2+16700\,d^3xe+9683\,d^4\right)}{15015\,e}\left(-ce^2x^2+cd^2\right)^{\frac{3}{2}}(ex+d)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(-c*e^2*x^2+c*d^2)^(3/2),x)
```

```
[Out] -2/15015*(-e*x+d)*(1155*e^4*x^4+6300*d*e^3*x^3+14210*d^2*e^2*x^2+16700*d^3*
e*x+9683*d^4)*(-c*e^2*x^2+c*d^2)^(3/2)/e/(e*x+d)^(3/2)
```

Maxima [A] time = 1.27707, size = 149, normalized size = 0.74

$$-\frac{2\left(1155\,c^{\frac{3}{2}}e^6x^6+3990\,c^{\frac{3}{2}}de^5x^5+2765\,c^{\frac{3}{2}}d^2e^4x^4-5420\,c^{\frac{3}{2}}d^3e^3x^3-9507\,c^{\frac{3}{2}}d^4e^2x^2-2666\,c^{\frac{3}{2}}d^5ex+9683\,c^{\frac{3}{2}}d^6\right)(ex+d)\sqrt{}}{15015\left(e^2x+de\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/15015*(1155*c^(3/2)*e^6*x^6 + 3990*c^(3/2)*d*e^5*x^5 + 2765*c^(3/2)*d^2*
e^4*x^4 - 5420*c^(3/2)*d^3*e^3*x^3 - 9507*c^(3/2)*d^4*e^2*x^2 - 2666*c^(3/2
```

```
)*d^5*e*x + 9683*c^(3/2)*d^6)*(e*x + d)*sqrt(-e*x + d)/(e^2*x + d*e)
```

Fricas [A] time = 2.34063, size = 259, normalized size = 1.29

$$-\frac{2\left(1155\,ce^6x^6+3990\,cde^5x^5+2765\,cd^2e^4x^4-5420\,cd^3e^3x^3-9507\,cd^4e^2x^2-2666\,cd^5ex+9683\,cd^6\right)\sqrt{-ce^2x^2+cd^2}}{15015\left(e^2x+de\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/15015*(1155*c*e^6*x^6 + 3990*c*d*e^5*x^5 + 2765*c*d^2*e^4*x^4 - 5420*c*d
^3*e^3*x^3 - 9507*c*d^4*e^2*x^2 - 2666*c*d^5*e*x + 9683*c*d^6)*sqrt(-c*e^2*
x^2 + c*d^2)*sqrt(e*x + d)/(e^2*x + d*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-c\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(-c*e**2*x**2+c*d**2)**(3/2),x)
```

```
[Out] Integral((-c*(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-ce^2x^2+cd^2\right)^{\frac{3}{2}}\left(ex+d\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-c*e^2*x^2 + c*d^2)^(3/2)*(e*x + d)^(5/2), x)
```

3.870 $\int (d+ex)^{3/2} \left(cd^2-ce^2x^2\right)^{3/2} \, dx$

Optimal. Leaf size=160

$$-\frac{256d^3\left(cd^2-ce^2x^2\right)^{5/2}}{1155ce(d+ex)^{5/2}}-\frac{64d^2\left(cd^2-ce^2x^2\right)^{5/2}}{231ce(d+ex)^{3/2}}-\frac{8d\left(cd^2-ce^2x^2\right)^{5/2}}{33ce\sqrt{d+ex}}-\frac{2\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{5/2}}{11ce}$$

```
[Out] (-256*d^3*(c*d^2 - c*e^2*x^2)^(5/2))/(1155*c*e*(d + e*x)^(5/2)) - (64*d^2*(
c*d^2 - c*e^2*x^2)^(5/2))/(231*c*e*(d + e*x)^(3/2)) - (8*d*(c*d^2 - c*e^2*x
^2)^(5/2))/(33*c*e*Sqrt[d + e*x]) - (2*Sqrt[d + e*x]*(c*d^2 - c*e^2*x^2)^(5
/2))/(11*c*e)
```

Rubi [A] time = 0.0700215, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {657, 649}

$$-\frac{256d^3\left(cd^2-ce^2x^2\right)^{5/2}}{1155ce(d+ex)^{5/2}}-\frac{64d^2\left(cd^2-ce^2x^2\right)^{5/2}}{231ce(d+ex)^{3/2}}-\frac{8d\left(cd^2-ce^2x^2\right)^{5/2}}{33ce\sqrt{d+ex}}-\frac{2\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{5/2}}{11ce}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*(c*d^2 - c*e^2*x^2)^(3/2),x]
```

```
[Out] (-256*d^3*(c*d^2 - c*e^2*x^2)^(5/2))/(1155*c*e*(d + e*x)^(5/2)) - (64*d^2*(
c*d^2 - c*e^2*x^2)^(5/2))/(231*c*e*(d + e*x)^(3/2)) - (8*d*(c*d^2 - c*e^2*x
^2)^(5/2))/(33*c*e*Sqrt[d + e*x]) - (2*Sqrt[d + e*x]*(c*d^2 - c*e^2*x^2)^(5
/2))/(11*c*e)
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{3/2}\,dx &= -\frac{2\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{5/2}}{11ce}+\frac{1}{11}(12d)\int \sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}\,dx\\
&= -\frac{8d\left(cd^2-ce^2x^2\right)^{5/2}}{33ce\sqrt{d+ex}}-\frac{2\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{5/2}}{11ce}+\frac{1}{33}\left(32d^2\right)\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{\sqrt{d+ex}}\,dx\\
&= -\frac{64d^2\left(cd^2-ce^2x^2\right)^{5/2}}{231ce(d+ex)^{3/2}}-\frac{8d\left(cd^2-ce^2x^2\right)^{5/2}}{33ce\sqrt{d+ex}}-\frac{2\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{5/2}}{11ce}+\frac{1}{231}\left(128d\right.\\
&= -\frac{256d^3\left(cd^2-ce^2x^2\right)^{5/2}}{1155ce(d+ex)^{5/2}}-\frac{64d^2\left(cd^2-ce^2x^2\right)^{5/2}}{231ce(d+ex)^{3/2}}-\frac{8d\left(cd^2-ce^2x^2\right)^{5/2}}{33ce\sqrt{d+ex}}-\frac{2\sqrt{d+ex}\left(cd\right.}{11c}
\end{aligned}$$

Mathematica [A] time = 0.0567404, size = 73, normalized size = 0.46

$$-\frac{2c(d-ex)^2\left(755d^2ex+533d^3+455de^2x^2+105e^3x^3\right)\sqrt{c\left(d^2-e^2x^2\right)}}{1155e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)*(c*d^2 - c*e^2*x^2)^(3/2),x]
```

```
[Out] (-2*c*(d - e*x)^2*Sqrt[c*(d^2 - e^2*x^2)]*(533*d^3 + 755*d^2*e*x + 455*d*e^
2*x^2 + 105*e^3*x^3))/(1155*e*Sqrt[d + e*x])
```

Maple [A] time = 0.049, size = 66, normalized size = 0.4

$$-\frac{(-2\,ex+2\,d)\left(105\,e^3x^3+455\,de^2x^2+755\,d^2xe+533\,d^3\right)}{1155\,e}\left(-ce^2x^2+cd^2\right)^{\frac{3}{2}}(ex+d)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(-c*e^2*x^2+c*d^2)^(3/2),x)
```

```
[Out] -2/1155*(-e*x+d)*(105*e^3*x^3+455*d*e^2*x^2+755*d^2*e*x+533*d^3)*(-c*e^2*x^
2+c*d^2)^(3/2)/e/(e*x+d)^(3/2)
```

Maxima [A] time = 1.28505, size = 130, normalized size = 0.81

$$-\frac{2\left(105\,c^{\frac{3}{2}}e^5x^5+245\,c^{\frac{3}{2}}de^4x^4-50\,c^{\frac{3}{2}}d^2e^3x^3-522\,c^{\frac{3}{2}}d^3e^2x^2-311\,c^{\frac{3}{2}}d^4ex+533\,c^{\frac{3}{2}}d^5\right)(ex+d)\sqrt{-ex+d}}{1155\left(e^2x+de\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/1155*(105*c^(3/2)*e^5*x^5 + 245*c^(3/2)*d*e^4*x^4 - 50*c^(3/2)*d^2*e^3*x
^3 - 522*c^(3/2)*d^3*e^2*x^2 - 311*c^(3/2)*d^4*e*x + 533*c^(3/2)*d^5)*(e*x
+ d)*sqrt(-e*x + d)/(e^2*x + d*e)
```

Fricas [A] time = 2.46514, size = 220, normalized size = 1.38

$$-\frac{2\left(105\,ce^5x^5+245\,cde^4x^4-50\,cd^2e^3x^3-522\,cd^3e^2x^2-311\,cd^4ex+533\,cd^5\right)\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}}{1155\left(e^2x+de\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/1155*(105*c*e^5*x^5 + 245*c*d*e^4*x^4 - 50*c*d^2*e^3*x^3 - 522*c*d^3*e^2
*x^2 - 311*c*d^4*e*x + 533*c*d^5)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)/(e
```

```
^2*x + d*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (-c(-d+ex)(d+ex))^{\frac{3}{2}}(d+ex)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(-c*e**2*x**2+c*d**2)**(3/2),x)
```

```
[Out] Integral((-c*(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-ce^2x^2+cd^2\right)^{\frac{3}{2}}(ex+d)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-c*e^2*x^2 + c*d^2)^(3/2)*(e*x + d)^(3/2), x)
```

3.871 $\int \sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2} dx$

Optimal. Leaf size=119

$$-\frac{2\left(cd^2-ce^2x^2\right)^{5/2}}{9ce\sqrt{d+ex}}-\frac{16d\left(cd^2-ce^2x^2\right)^{5/2}}{63ce(d+ex)^{3/2}}-\frac{64d^2\left(cd^2-ce^2x^2\right)^{5/2}}{315ce(d+ex)^{5/2}}$$

```
[Out] (-64*d^2*(c*d^2 - c*e^2*x^2)^(5/2))/(315*c*e*(d + e*x)^(5/2)) - (16*d*(c*d^
2 - c*e^2*x^2)^(5/2))/(63*c*e*(d + e*x)^(3/2)) - (2*(c*d^2 - c*e^2*x^2)^(5/
2))/(9*c*e*Sqrt[d + e*x])
```

Rubi [A] time = 0.0498016, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {657, 649}

$$-\frac{2\left(cd^2-ce^2x^2\right)^{5/2}}{9ce\sqrt{d+ex}}-\frac{16d\left(cd^2-ce^2x^2\right)^{5/2}}{63ce(d+ex)^{3/2}}-\frac{64d^2\left(cd^2-ce^2x^2\right)^{5/2}}{315ce(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(c*d^2 - c*e^2*x^2)^(3/2),x]
```

```
[Out] (-64*d^2*(c*d^2 - c*e^2*x^2)^(5/2))/(315*c*e*(d + e*x)^(5/2)) - (16*d*(c*d^
2 - c*e^2*x^2)^(5/2))/(63*c*e*(d + e*x)^(3/2)) - (2*(c*d^2 - c*e^2*x^2)^(5/
2))/(9*c*e*Sqrt[d + e*x])
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2} dx &= -\frac{2\left(cd^2-ce^2x^2\right)^{5/2}}{9ce\sqrt{d+ex}}+\frac{1}{9}(8d)\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{\sqrt{d+ex}} dx\\
&= -\frac{16d\left(cd^2-ce^2x^2\right)^{5/2}}{63ce(d+ex)^{3/2}}-\frac{2\left(cd^2-ce^2x^2\right)^{5/2}}{9ce\sqrt{d+ex}}+\frac{1}{63}\left(32d^2\right)\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{3/2}} dx\\
&= -\frac{64d^2\left(cd^2-ce^2x^2\right)^{5/2}}{315ce(d+ex)^{5/2}}-\frac{16d\left(cd^2-ce^2x^2\right)^{5/2}}{63ce(d+ex)^{3/2}}-\frac{2\left(cd^2-ce^2x^2\right)^{5/2}}{9ce\sqrt{d+ex}}
\end{aligned}$$

Mathematica [A] time = 0.0475711, size = 62, normalized size = 0.52

$$-\frac{2c(d-ex)^2\left(107d^2+110dex+35e^2x^2\right)\sqrt{c\left(d^2-e^2x^2\right)}}{315e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*(c*d^2 - c*e^2*x^2)^(3/2),x]
```

```
[Out] (-2*c*(d - e*x)^2*Sqrt[c*(d^2 - e^2*x^2)]*(107*d^2 + 110*d*e*x + 35*e^2*x^2
))/(315*e*Sqrt[d + e*x])
```

Maple [A] time = 0.046, size = 55, normalized size = 0.5

$$-\frac{(-2\,ex+2\,d)\left(35\,e^2x^2+110\,dex+107\,d^2\right)}{315\,e}\left(-ce^2x^2+cd^2\right)^{\frac{3}{2}}(ex+d)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(-c*e^2*x^2+c*d^2)^(3/2),x)
```

```
[Out] -2/315*(-e*x+d)*(35*e^2*x^2+110*d*e*x+107*d^2)*(-c*e^2*x^2+c*d^2)^(3/2)/e/(
e*x+d)^(3/2)
```

Maxima [A] time = 1.21692, size = 111, normalized size = 0.93

$$-\frac{2\left(35\,c^{\frac{3}{2}}e^4x^4+40\,c^{\frac{3}{2}}de^3x^3-78\,c^{\frac{3}{2}}d^2e^2x^2-104\,c^{\frac{3}{2}}d^3ex+107\,c^{\frac{3}{2}}d^4\right)(ex+d)\sqrt{-ex+d}}{315\left(e^2x+de\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/315*(35*c^(3/2)*e^4*x^4 + 40*c^(3/2)*d*e^3*x^3 - 78*c^(3/2)*d^2*e^2*x^2
- 104*c^(3/2)*d^3*e*x + 107*c^(3/2)*d^4)*(e*x + d)*sqrt(-e*x + d)/(e^2*x +
d*e)
```

Fricas [A] time = 2.25751, size = 189, normalized size = 1.59

$$-\frac{2\left(35\,ce^4x^4+40\,cde^3x^3-78\,cd^2e^2x^2-104\,cd^3ex+107\,cd^4\right)\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}}{315\left(e^2x+de\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/315*(35*c*e^4*x^4 + 40*c*d*e^3*x^3 - 78*c*d^2*e^2*x^2 - 104*c*d^3*e*x +
107*c*d^4)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)/(e^2*x + d*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (-c(-d+ex)(d+ex))^{\frac{3}{2}}\sqrt{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(-c*e**2*x**2+c*d**2)**(3/2),x)
```

```
[Out] Integral((-c*(-d + e*x)*(d + e*x))**(3/2)*sqrt(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-ce^2x^2+cd^2\right)^{\frac{3}{2}}\sqrt{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-c*e^2*x^2 + c*d^2)^(3/2)*sqrt(e*x + d), x)
```

3.872 $\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=78

$$-\frac{2\left(cd^2-ce^2x^2\right)^{5/2}}{7ce(d+ex)^{3/2}}-\frac{8d\left(cd^2-ce^2x^2\right)^{5/2}}{35ce(d+ex)^{5/2}}$$

```
[Out] (-8*d*(c*d^2 - c*e^2*x^2)^(5/2))/(35*c*e*(d + e*x)^(5/2)) - (2*(c*d^2 - c*e
^2*x^2)^(5/2))/(7*c*e*(d + e*x)^(3/2))
```

Rubi [A] time = 0.0306673, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {657, 649}

$$-\frac{2\left(cd^2-ce^2x^2\right)^{5/2}}{7ce(d+ex)^{3/2}}-\frac{8d\left(cd^2-ce^2x^2\right)^{5/2}}{35ce(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 - c*e^2*x^2)^(3/2)/Sqrt[d + e*x],x]
```

```
[Out] (-8*d*(c*d^2 - c*e^2*x^2)^(5/2))/(35*c*e*(d + e*x)^(5/2)) - (2*(c*d^2 - c*e
^2*x^2)^(5/2))/(7*c*e*(d + e*x)^(3/2))
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{\sqrt{d+ex}}\,dx &= -\frac{2\left(cd^2-ce^2x^2\right)^{5/2}}{7ce(d+ex)^{3/2}}+\frac{1}{7}(4d)\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx\\ &= -\frac{8d\left(cd^2-ce^2x^2\right)^{5/2}}{35ce(d+ex)^{5/2}}-\frac{2\left(cd^2-ce^2x^2\right)^{5/2}}{7ce(d+ex)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0487574, size = 51, normalized size = 0.65

$$-\frac{2c(d-ex)^2(9d+5ex)\sqrt{c\left(d^2-e^2x^2\right)}}{35e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 - c*e^2*x^2)^(3/2)/Sqrt[d + e*x],x]
```

```
[Out] (-2*c*(d - e*x)^2*(9*d + 5*e*x)*Sqrt[c*(d^2 - e^2*x^2)])/(35*e*Sqrt[d + e*x
])
```

Maple [A] time = 0.041, size = 44, normalized size = 0.6

$$-\frac{(-2\,ex+2\,d)\,(5\,ex+9\,d)}{35\,e}\left(-ce^2x^2+cd^2\right)^{\frac{3}{2}}(ex+d)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(1/2),x)
```

```
[Out] -2/35*(-e*x+d)*(5*e*x+9*d)*(-c*e^2*x^2+c*d^2)^(3/2)/e/(e*x+d)^(3/2)
```

Maxima [A] time = 1.08443, size = 74, normalized size = 0.95

$$-\frac{2\left(5\,c^{\frac{3}{2}}e^3x^3-c^{\frac{3}{2}}de^2x^2-13\,c^{\frac{3}{2}}d^2ex+9\,c^{\frac{3}{2}}d^3\right)\sqrt{-ex+d}}{35\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/35*(5*c^(3/2)*e^3*x^3 - c^(3/2)*d*e^2*x^2 - 13*c^(3/2)*d^2*e*x + 9*c^(3/
2)*d^3)*sqrt(-e*x + d)/e
```

Fricas [A] time = 1.99887, size = 153, normalized size = 1.96

$$-\frac{2\left(5\,ce^3x^3-cde^2x^2-13\,cd^2ex+9\,cd^3\right)\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}}{35\left(e^2x+de\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/35*(5*c*e^3*x^3 - c*d*e^2*x^2 - 13*c*d^2*e*x + 9*c*d^3)*sqrt(-c*e^2*x^2
+ c*d^2)*sqrt(e*x + d)/(e^2*x + d*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-c\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}}{\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e**2*x**2+c*d**2)**(3/2)/(e*x+d)**(1/2),x)
```

```
[Out] Integral((-c*(-d + e*x)*(d + e*x))**(3/2)/sqrt(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-ce^2x^2 + cd^2\right)^{\frac{3}{2}}}{\sqrt{ex + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-c*e^2*x^2 + c*d^2)^(3/2)/sqrt(e*x + d), x)
```

3.873 $\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{2\left(cd^2-ce^2x^2\right)^{5/2}}{5ce(d+ex)^{5/2}}$$

```
[Out] (-2*(c*d^2 - c*e^2*x^2)^(5/2))/(5*c*e*(d + e*x)^(5/2))
```

Rubi [A] time = 0.0134863, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {649}

$$-\frac{2\left(cd^2-ce^2x^2\right)^{5/2}}{5ce(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(3/2),x]

[Out] (-2*(c*d^2 - c*e^2*x^2)^(5/2))/(5*c*e*(d + e*x)^(5/2))
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=-\frac{2\left(cd^2-ce^2x^2\right)^{5/2}}{5ce(d+ex)^{5/2}}$$

Mathematica [A] time = 0.0504065, size = 43, normalized size = 1.13

$$-\frac{2c(d-ex)^2\sqrt{c\left(d^2-e^2x^2\right)}}{5e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(3/2),x]

[Out] (-2*c*(d - e*x)^2*Sqrt[c*(d^2 - e^2*x^2)])/(5*e*Sqrt[d + e*x])
```

Maple [A] time = 0.042, size = 36, normalized size = 1.

$$-\frac{-2\,ex+2\,d}{5\,e}\left(-ce^2x^2+cd^2\right)^{\frac{3}{2}}(ex+d)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(3/2),x)
```

```
[Out] -2/5*(-e*x+d)*(-c*e^2*x^2+c*d^2)^(3/2)/e/(e*x+d)^(3/2)
```

Maxima [A] time = 1.05618, size = 53, normalized size = 1.39

$$-\frac{2\left(c^{\frac{3}{2}}e^2x^2 - 2\,c^{\frac{3}{2}}dex + c^{\frac{3}{2}}d^2\right)\sqrt{-ex+d}}{5\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/5*(c^(3/2)*e^2*x^2 - 2*c^(3/2)*d*e*x + c^(3/2)*d^2)*sqrt(-e*x + d)/e
```

Fricas [A] time = 2.07715, size = 123, normalized size = 3.24

$$-\frac{2\left(ce^2x^2 - 2\,cdex + cd^2\right)\sqrt{-ce^2x^2 + cd^2}\sqrt{ex+d}}{5\left(e^2x + de\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(c*e^2*x^2 - 2*c*d*e*x + c*d^2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)
/(e^2*x + d*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}}{\left(d+ex\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e**2*x**2+c*d**2)**(3/2)/(e*x+d)**(3/2),x)
```

```
[Out] Integral((-c*(-d + e*x)*(d + e*x))**(3/2)/(d + e*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-ce^2x^2 + cd^2\right)^{\frac{3}{2}}}{\left(ex+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(3/2),x, algorithm="giac")

[Out] integrate((-c*e^2*x^2 + c*d^2)^(3/2)/(e*x + d)^(3/2), x)
```

3.874 $\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=136

$$-\frac{4\sqrt{2}c^{3/2}d^{3/2}\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{e}+\frac{4cd\sqrt{cd^2-ce^2x^2}}{e\sqrt{d+ex}}+\frac{2\left(cd^2-ce^2x^2\right)^{3/2}}{3e(d+ex)^{3/2}}$$

```
[Out] (4*c*d*Sqrt[c*d^2 - c*e^2*x^2])/(e*Sqrt[d + e*x]) + (2*(c*d^2 - c*e^2*x^2)^
(3/2))/(3*e*(d + e*x)^(3/2)) - (4*Sqrt[2]*c^(3/2)*d^(3/2)*ArcTanh[Sqrt[c*d^
2 - c*e^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/e
```

Rubi [A] time = 0.0719828, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {665, 661, 208}

$$-\frac{4\sqrt{2}c^{3/2}d^{3/2}\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{e}+\frac{4cd\sqrt{cd^2-ce^2x^2}}{e\sqrt{d+ex}}+\frac{2\left(cd^2-ce^2x^2\right)^{3/2}}{3e(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(5/2),x]
```

```
[Out] (4*c*d*Sqrt[c*d^2 - c*e^2*x^2])/(e*Sqrt[d + e*x]) + (2*(c*d^2 - c*e^2*x^2)^
(3/2))/(3*e*(d + e*x)^(3/2)) - (4*Sqrt[2]*c^(3/2)*d^(3/2)*ArcTanh[Sqrt[c*d^
2 - c*e^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/e
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{5/2}}\,dx = \frac{2\left(cd^2-ce^2x^2\right)^{3/2}}{3e(d+ex)^{3/2}} + (2cd)\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{3/2}}\,dx$$
$$= \frac{4cd\sqrt{cd^2-ce^2x^2}}{e\sqrt{d+ex}} + \frac{2\left(cd^2-ce^2x^2\right)^{3/2}}{3e(d+ex)^{3/2}} + \left(4c^2d^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\,dx$$
$$= \frac{4cd\sqrt{cd^2-ce^2x^2}}{e\sqrt{d+ex}} + \frac{2\left(cd^2-ce^2x^2\right)^{3/2}}{3e(d+ex)^{3/2}} + \left(8c^2d^2e\right)\text{Subst}\left(\int \frac{1}{-2cde^2+e^2x^2}\,dx, x, \frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}\right)$$
$$= \frac{4cd\sqrt{cd^2-ce^2x^2}}{e\sqrt{d+ex}} + \frac{2\left(cd^2-ce^2x^2\right)^{3/2}}{3e(d+ex)^{3/2}} - \frac{4\sqrt{2}c^{3/2}d^{3/2}\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{e}$$

Mathematica [A] time = 0.210561, size = 110, normalized size = 0.81

$$\frac{2c\sqrt{c\left(d^2-e^2x^2\right)}\left(\frac{7d-ex}{\sqrt{d+ex}} - \frac{6\sqrt{2}d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{\sqrt{2}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{d^2-e^2x^2}}\right)}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(5/2),x]
```

```
[Out] (2*c*Sqrt[c*(d^2 - e^2*x^2)]*((7*d - e*x)/Sqrt[d + e*x] - (6*Sqrt[2]*d^(3/2
)*ArcTanh[Sqrt[d^2 - e^2*x^2]/(Sqrt[2]*Sqrt[d]*Sqrt[d + e*x])])/Sqrt[d^2 -
e^2*x^2]))/(3*e)
```

Maple [A] time = 0.167, size = 122, normalized size = 0.9

$$-\frac{2c}{3e}\sqrt{-c\left(e^2x^2-d^2\right)}\left(6\sqrt{2}\text{Artanh}\left(1/2\frac{\sqrt{-(ex-d)c}\sqrt{2}}{\sqrt{cd}}\right)cd^2 + xe\sqrt{-(ex-d)c}\sqrt{cd} - 7\sqrt{-(ex-d)c}\sqrt{cd}d\right)\frac{1}{\sqrt{ex+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(5/2),x)
```

```
[Out] -2/3*(-c*(e^2*x^2-d^2))^(1/2)*c*(6*2^(1/2)*arctanh(1/2*(-(e*x-d)*c)^(1/2)*2
^(1/2)/(c*d)^(1/2))*c*d^2+x*e*(-(e*x-d)*c)^(1/2)*(c*d)^(1/2)-7*(-(e*x-d)*c)
^(1/2)*(c*d)^(1/2)*d)/(e*x+d)^(1/2)/(-(e*x-d)*c)^(1/2)/e/(c*d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-ce^2x^2+cd^2\right)^{\frac{3}{2}}}{(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-c*e^2*x^2 + c*d^2)^(3/2)/(e*x + d)^(5/2), x)
```

Fricas [A] time = 2.22102, size = 609, normalized size = 4.48

$$\left[\frac{2\left(3\sqrt{2}\left(cdex+cd^2\right)\sqrt{cd}\log\left(-\frac{ce^2x^2-2\,cdex-3\,cd^2+2\,\sqrt{2}\sqrt{-ce^2x^2+cd^2}\sqrt{cd}\sqrt{ex+d}}{e^2x^2+2\,dex+d^2}\right)-\sqrt{-ce^2x^2+cd^2}(cex-7\,cd)\sqrt{ex+d}\right)}{3\left(e^2x+de\right)},-\frac{2\left(6\sqrt{2}\left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [2/3*(3*sqrt(2)*(c*d*e*x + c*d^2)*sqrt(c*d)*log(-(c*e^2*x^2 - 2*c*d*e*x - 3
*c*d^2 + 2*sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(c*d)*sqrt(e*x + d))/(e^2*x
^2 + 2*d*e*x + d^2)) - sqrt(-c*e^2*x^2 + c*d^2)*(c*e*x - 7*c*d)*sqrt(e*x +
d))/(e^2*x + d*e), -2/3*(6*sqrt(2)*(c*d*e*x + c*d^2)*sqrt(-c*d)*arctan(sqrt
(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(-c*d)*sqrt(e*x + d)/(c*e^2*x^2 - c*d^2))
+ sqrt(-c*e^2*x^2 + c*d^2)*(c*e*x - 7*c*d)*sqrt(e*x + d))/(e^2*x + d*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}}{\left(d+ex\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e**2*x**2+c*d**2)**(3/2)/(e*x+d)**(5/2),x)
```

```
[Out] Integral((-c*(-d + e*x)*(d + e*x))**(3/2)/(d + e*x)**(5/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.875 $\int \frac{(cd^2-ce^2x^2)^{3/2}}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=133

$$\frac{3\sqrt{2}c^{3/2}\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{e}-\frac{3c\sqrt{cd^2-ce^2x^2}}{e\sqrt{d+ex}}-\frac{(cd^2-ce^2x^2)^{3/2}}{e(d+ex)^{5/2}}$$

```
[Out] (-3*c*Sqrt[c*d^2 - c*e^2*x^2])/(e*Sqrt[d + e*x]) - (c*d^2 - c*e^2*x^2)^(3/2
)/(e*(d + e*x)^(5/2)) + (3*Sqrt[2]*c^(3/2)*Sqrt[d]*ArcTanh[Sqrt[c*d^2 - c*e
^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/e
```

Rubi [A] time = 0.0789978, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {663, 665, 661, 208}

$$\frac{3\sqrt{2}c^{3/2}\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{e}-\frac{3c\sqrt{cd^2-ce^2x^2}}{e\sqrt{d+ex}}-\frac{(cd^2-ce^2x^2)^{3/2}}{e(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(7/2),x]
```


```
[Out] (-3*c*Sqrt[c*d^2 - c*e^2*x^2])/(e*Sqrt[d + e*x]) - (c*d^2 - c*e^2*x^2)^(3/2
)/(e*(d + e*x)^(5/2)) + (3*Sqrt[2]*c^(3/2)*Sqrt[d]*ArcTanh[Sqrt[c*d^2 - c*e
^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/e
```


Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx &= -\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{e(d+ex)^{5/2}}-\frac{1}{2}(3c)\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{3/2}}\,dx\\
&= -\frac{3c\sqrt{cd^2-ce^2x^2}}{e\sqrt{d+ex}}-\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{e(d+ex)^{5/2}}-\left(3c^2d\right)\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\,dx\\
&= -\frac{3c\sqrt{cd^2-ce^2x^2}}{e\sqrt{d+ex}}-\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{e(d+ex)^{5/2}}-\left(6c^2de\right)\text{Subst}\left(\int \frac{1}{-2cde^2+e^2x^2}\,dx,x,\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}\right)\\
&= -\frac{3c\sqrt{cd^2-ce^2x^2}}{e\sqrt{d+ex}}-\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{e(d+ex)^{5/2}}+\frac{3\sqrt{2}c^{3/2}\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{e}
\end{aligned}
$$

Mathematica [A] time = 0.203967, size = 107, normalized size = 0.8

$$
\frac{c\sqrt{c\left(d^2-e^2x^2\right)}\left(\frac{3\sqrt{2}\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{\sqrt{2}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{d^2-e^2x^2}}-\frac{2(2d+ex)}{(d+ex)^{3/2}}\right)}{e}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(7/2),x]
```

```
[Out] (c*Sqrt[c*(d^2 - e^2*x^2)]*((-2*(2*d + e*x))/(d + e*x)^(3/2) + (3*Sqrt[2]*S
qrt[d]*ArcTanh[Sqrt[d^2 - e^2*x^2]/(Sqrt[2]*Sqrt[d]*Sqrt[d + e*x])])/Sqrt[d
^2 - e^2*x^2]))/e
```

Maple [A] time = 0.171, size = 154, normalized size = 1.2

$$
\frac{c}{e}\sqrt{-c\left(e^2x^2-d^2\right)}\left(3\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{-(ex-d)c}\sqrt{2}}{\sqrt{cd}}\right)xcde+3\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{-(ex-d)c}\sqrt{2}}{\sqrt{cd}}\right)cd^2-2xe\sqrt{-(ex-d)c}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(7/2),x)
```

```
[Out] (-c*(e^2*x^2-d^2))^(1/2)*c*(3*2^(1/2)*arctanh(1/2*(-(e*x-d)*c)^(1/2)*2^(1/2
)/(c*d)^(1/2))*x*c*d*e+3*2^(1/2)*arctanh(1/2*(-(e*x-d)*c)^(1/2)*2^(1/2)/(c*
d)^(1/2))*c*d^2-2*x*e*(-(e*x-d)*c)^(1/2)*(c*d)^(1/2)-4*(-(e*x-d)*c)^(1/2)*(
c*d)^(1/2)*d)/(e*x+d)^(3/2)/(-(e*x-d)*c)^(1/2)/e/(c*d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\left(-ce^2x^2+cd^2\right)^{\frac{3}{2}}}{(ex+d)^{\frac{7}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((-c*e^2*x^2 + c*d^2)^(3/2)/(e*x + d)^(7/2), x)
```

Fricas [A] time = 2.21309, size = 689, normalized size = 5.18

$$\left[\frac{3\sqrt{2}\left(ce^2x^2+2\,cdex+cd^2\right)\sqrt{cd}\log\left(-\frac{ce^2x^2-2\,cdex-3\,cd^2-2\,\sqrt{2}\sqrt{-ce^2x^2+cd^2}\sqrt{cd}\sqrt{ex+d}}{e^2x^2+2\,dex+d^2}\right)-4\,\sqrt{-ce^2x^2+cd^2}(cex+2\,cd)\sqrt{ex+d}}{2\left(e^3x^2+2\,de^2x+d^2e\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*sqrt(2)*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*sqrt(c*d)*log(-(c*e^2*x^2 -
 2*c*d*e*x - 3*c*d^2 - 2*sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(c*d)*sqrt(e*
x + d))/(e^2*x^2 + 2*d*e*x + d^2)) - 4*sqrt(-c*e^2*x^2 + c*d^2)*(c*e*x + 2*
c*d)*sqrt(e*x + d))/(e^3*x^2 + 2*d*e^2*x + d^2*e), (3*sqrt(2)*(c*e^2*x^2 +
2*c*d*e*x + c*d^2)*sqrt(-c*d)*arctan(sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(
-c*d)*sqrt(e*x + d)/(c*e^2*x^2 - c*d^2)) - 2*sqrt(-c*e^2*x^2 + c*d^2)*(c*e*
x + 2*c*d)*sqrt(e*x + d))/(e^3*x^2 + 2*d*e^2*x + d^2*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}}{\left(d+ex\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e**2*x**2+c*d**2)**(3/2)/(e*x+d)**(7/2),x)
```

```
[Out] Integral((-c*(-d + e*x)*(d + e*x))**(3/2)/(d + e*x)**(7/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-ce^2x^2+cd^2\right)^{\frac{3}{2}}}{\left(ex+d\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((-c*e^2*x^2 + c*d^2)^(3/2)/(e*x + d)^(7/2), x)
```

3.876 $\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{9/2}}\,dx$

Optimal. Leaf size=139

$$-\frac{3c^{3/2}\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{4\sqrt{2}\sqrt{d}e}+\frac{3c\sqrt{cd^2-ce^2x^2}}{4e(d+ex)^{3/2}}-\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{2e(d+ex)^{7/2}}$$

```
[Out] (3*c*Sqrt[c*d^2 - c*e^2*x^2])/(4*e*(d + e*x)^(3/2)) - (c*d^2 - c*e^2*x^2)^(
3/2)/(2*e*(d + e*x)^(7/2)) - (3*c^(3/2)*ArcTanh[Sqrt[c*d^2 - c*e^2*x^2]/(Sq
rt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/(4*Sqrt[2]*Sqrt[d]*e)
```

Rubi [A] time = 0.0706147, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {663, 661, 208}

$$-\frac{3c^{3/2}\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{4\sqrt{2}\sqrt{d}e}+\frac{3c\sqrt{cd^2-ce^2x^2}}{4e(d+ex)^{3/2}}-\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{2e(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(9/2),x]
```

```
[Out] (3*c*Sqrt[c*d^2 - c*e^2*x^2])/(4*e*(d + e*x)^(3/2)) - (c*d^2 - c*e^2*x^2)^(
3/2)/(2*e*(d + e*x)^(7/2)) - (3*c^(3/2)*ArcTanh[Sqrt[c*d^2 - c*e^2*x^2]/(Sq
rt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/(4*Sqrt[2]*Sqrt[d]*e)
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{9/2}}\,dx = -\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{2e(d+ex)^{7/2}} - \frac{1}{4}(3c)\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{5/2}}\,dx$$
$$= \frac{3c\sqrt{cd^2-ce^2x^2}}{4e(d+ex)^{3/2}} - \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{2e(d+ex)^{7/2}} + \frac{1}{8}\left(3c^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\,dx$$
$$= \frac{3c\sqrt{cd^2-ce^2x^2}}{4e(d+ex)^{3/2}} - \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{2e(d+ex)^{7/2}} + \frac{1}{4}\left(3c^2e\right)\text{Subst}\left(\int \frac{1}{-2cde^2+e^2x^2}\,dx, x, \frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}\right)$$
$$= \frac{3c\sqrt{cd^2-ce^2x^2}}{4e(d+ex)^{3/2}} - \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{2e(d+ex)^{7/2}} - \frac{3c^{3/2}\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{4\sqrt{2}\sqrt{de}}$$

Mathematica [A] time = 0.173428, size = 109, normalized size = 0.78

$$\frac{c\sqrt{c\left(d^2-e^2x^2\right)}\left(\frac{2(d+5ex)}{(d+ex)^{5/2}} - \frac{3\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{\sqrt{2}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{d}\sqrt{d^2-e^2x^2}}\right)}{8e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(9/2),x]
```

```
[Out] (c*Sqrt[c*(d^2 - e^2*x^2)]*((2*(d + 5*e*x))/(d + e*x)^(5/2) - (3*Sqrt[2]*Ar
cTanh[Sqrt[d^2 - e^2*x^2]/(Sqrt[2]*Sqrt[d]*Sqrt[d + e*x])])/(Sqrt[d]*Sqrt[d
^2 - e^2*x^2])))/(8*e)
```

Maple [A] time = 0.175, size = 190, normalized size = 1.4

$$-\frac{c}{8e}\sqrt{-c\left(e^2x^2-d^2\right)}\left(3\sqrt{2}\text{Artanh}\left(1/2\frac{\sqrt{-(ex-d)c}\sqrt{2}}{\sqrt{cd}}\right)x^2ce^2 + 6\sqrt{2}\text{Artanh}\left(1/2\frac{\sqrt{-(ex-d)c}\sqrt{2}}{\sqrt{cd}}\right)xcde + 3\sqrt{2}\text{Ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(9/2),x)
```

```
[Out] -1/8*(-c*(e^2*x^2-d^2))^(1/2)*c*(3*2^(1/2)*arctanh(1/2*(-(e*x-d)*c)^(1/2)*2
^(1/2)/(c*d)^(1/2))*x^2*c*e^2+6*2^(1/2)*arctanh(1/2*(-(e*x-d)*c)^(1/2)*2^(1
/2)/(c*d)^(1/2))*x*c*d*e+3*2^(1/2)*arctanh(1/2*(-(e*x-d)*c)^(1/2)*2^(1/2)/(
c*d)^(1/2))*c*d^2-10*x*e*(-(e*x-d)*c)^(1/2)*(c*d)^(1/2)-2*(-(e*x-d)*c)^(1/2
)*(c*d)^(1/2)*d)/(e*x+d)^(5/2)/(-(e*x-d)*c)^(1/2)/e/(c*d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-ce^2x^2+cd^2\right)^{\frac{3}{2}}}{(ex+d)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((-c*e^2*x^2 + c*d^2)^(3/2)/(e*x + d)^(9/2), x)
```

Fricas [A] time = 2.16347, size = 803, normalized size = 5.78

$$\left[\frac{3\sqrt{\frac{1}{2}}\left(ce^3x^3+3\,cde^2x^2+3\,cd^2ex+cd^3\right)\sqrt{\frac{c}{d}}\log\left(-\frac{ce^2x^2-2\,cdex-3\,cd^2+4\,\sqrt{\frac{1}{2}}\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}d\sqrt{\frac{c}{d}}}{e^2x^2+2\,dex+d^2}\right)+2\,\sqrt{-ce^2x^2+cd^2}(5\,cex+cd}{8\left(e^4x^3+3\,de^3x^2+3\,d^2e^2x+d^3e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(1/2)*(c*e^3*x^3 + 3*c*d*e^2*x^2 + 3*c*d^2*e*x + c*d^3)*sqrt(c/
d)*log(-(c*e^2*x^2 - 2*c*d*e*x - 3*c*d^2 + 4*sqrt(1/2)*sqrt(-c*e^2*x^2 + c*
d^2)*sqrt(e*x + d)*d*sqrt(c/d))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*sqrt(-c*e^2*
x^2 + c*d^2)*(5*c*e*x + c*d)*sqrt(e*x + d))/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*
e^2*x + d^3*e), -1/4*(3*sqrt(1/2)*(c*e^3*x^3 + 3*c*d*e^2*x^2 + 3*c*d^2*e*x
+ c*d^3)*sqrt(-c/d)*arctan(2*sqrt(1/2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x +
d)*d*sqrt(-c/d)/(c*e^2*x^2 - c*d^2)) - sqrt(-c*e^2*x^2 + c*d^2)*(5*c*e*x +
c*d)*sqrt(e*x + d))/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}}{\left(d+ex\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e**2*x**2+c*d**2)**(3/2)/(e*x+d)**(9/2),x)
```

```
[Out] Integral((-c*(-d + e*x)*(d + e*x))**(3/2)/(d + e*x)**(9/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-ce^2x^2+cd^2\right)^{\frac{3}{2}}}{\left(ex+d\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((-c*e^2*x^2 + c*d^2)^(3/2)/(e*x + d)^(9/2), x)
```

3.877 $\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{11/2}}\,dx$

Optimal. Leaf size=178

$$-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{16\sqrt{2}d^{3/2}e}-\frac{c\sqrt{cd^2-ce^2x^2}}{16de(d+ex)^{3/2}}+\frac{c\sqrt{cd^2-ce^2x^2}}{4e(d+ex)^{5/2}}-\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{3e(d+ex)^{9/2}}$$

```
[Out] (c*Sqrt[c*d^2 - c*e^2*x^2])/(4*e*(d + e*x)^(5/2)) - (c*Sqrt[c*d^2 - c*e^2*x
^2])/(16*d*e*(d + e*x)^(3/2)) - (c*d^2 - c*e^2*x^2)^(3/2)/(3*e*(d + e*x)^(9
/2)) - (c^(3/2)*ArcTanh[Sqrt[c*d^2 - c*e^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d]*Sq
rt[d + e*x])])/(16*Sqrt[2]*d^(3/2)*e)
```

Rubi [A] time = 0.0967206, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {663, 673, 661, 208}

$$-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{16\sqrt{2}d^{3/2}e}-\frac{c\sqrt{cd^2-ce^2x^2}}{16de(d+ex)^{3/2}}+\frac{c\sqrt{cd^2-ce^2x^2}}{4e(d+ex)^{5/2}}-\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{3e(d+ex)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(11/2),x]
```

```
[Out] (c*Sqrt[c*d^2 - c*e^2*x^2])/(4*e*(d + e*x)^(5/2)) - (c*Sqrt[c*d^2 - c*e^2*x
^2])/(16*d*e*(d + e*x)^(3/2)) - (c*d^2 - c*e^2*x^2)^(3/2)/(3*e*(d + e*x)^(9
/2)) - (c^(3/2)*ArcTanh[Sqrt[c*d^2 - c*e^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d]*Sq
rt[d + e*x])])/(16*Sqrt[2]*d^(3/2)*e)
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[(m + 2*
p + 2)/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; Fr
eeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p
+ 1, 0] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{11/2}}\,dx &= -\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{3e(d+ex)^{9/2}}-\frac{1}{2}c\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{7/2}}\,dx\\
&= \frac{c\sqrt{cd^2-ce^2x^2}}{4e(d+ex)^{5/2}}-\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{3e(d+ex)^{9/2}}+\frac{1}{8}c^2\int \frac{1}{(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}\,dx\\
&= \frac{c\sqrt{cd^2-ce^2x^2}}{4e(d+ex)^{5/2}}-\frac{c\sqrt{cd^2-ce^2x^2}}{16de(d+ex)^{3/2}}-\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{3e(d+ex)^{9/2}}+\frac{c^2\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\,dx}{32d}\\
&= \frac{c\sqrt{cd^2-ce^2x^2}}{4e(d+ex)^{5/2}}-\frac{c\sqrt{cd^2-ce^2x^2}}{16de(d+ex)^{3/2}}-\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{3e(d+ex)^{9/2}}+\frac{\left(c^2e\right)\text{Subst}\left(\int \frac{1}{-2cde^2+e^2x^2}\,dx,x,\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}\right)}{16d}\\
&= \frac{c\sqrt{cd^2-ce^2x^2}}{4e(d+ex)^{5/2}}-\frac{c\sqrt{cd^2-ce^2x^2}}{16de(d+ex)^{3/2}}-\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{3e(d+ex)^{9/2}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{16\sqrt{2}d^{3/2}e}
\end{aligned}$$

Mathematica [A] time = 0.247028, size = 134, normalized size = 0.75

$$\frac{\left(c\left(d^2-e^2x^2\right)\right)^{3/2}\left(-\frac{2\sqrt{d}\left(7d^2-22dex+3e^2x^2\right)}{(d-ex)(d+ex)^{9/2}}-\frac{3\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{\sqrt{2}\sqrt{d}\sqrt{d+ex}}\right)}{\left(d^2-e^2x^2\right)^{3/2}}\right)}{96d^{3/2}e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(11/2),x]
```

```
[Out] ((c*(d^2 - e^2*x^2))^(3/2)*((-2*Sqrt[d]*(7*d^2 - 22*d*e*x + 3*e^2*x^2))/((d
 - e*x)*(d + e*x)^(9/2)) - (3*Sqrt[2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/(Sqrt[2]*
Sqrt[d]*Sqrt[d + e*x])])/(d^2 - e^2*x^2)^(3/2)))/(96*d^(3/2)*e)
```

Maple [A] time = 0.174, size = 259, normalized size = 1.5

$$-\frac{c}{96\,de}\sqrt{-c\left(e^2x^2-d^2\right)}\left(3\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{-(ex-d)c}\sqrt{2}}{\sqrt{cd}}\right)x^3ce^3+9\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{-(ex-d)c}\sqrt{2}}{\sqrt{cd}}\right)x^2cde^2+9\sqrt{2}\text{A}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(11/2),x)
```

```
[Out] -1/96*(-c*(e^2*x^2-d^2))^(1/2)*c*(3*2^(1/2)*arctanh(1/2*(-(e*x-d)*c)^(1/2)*
2^(1/2)/(c*d)^(1/2))*x^3*c*e^3+9*2^(1/2)*arctanh(1/2*(-(e*x-d)*c)^(1/2)*2^(
1/2)/(c*d)^(1/2))*x^2*c*d*e^2+9*2^(1/2)*arctanh(1/2*(-(e*x-d)*c)^(1/2)*2^(1
/2)/(c*d)^(1/2))*x*c*d^2*e+3*2^(1/2)*arctanh(1/2*(-(e*x-d)*c)^(1/2)*2^(1/2)
/(c*d)^(1/2))*c*d^3+6*x^2*e^2*(-(e*x-d)*c)^(1/2)*(c*d)^(1/2)-44*x*d*e*(-(e*
x-d)*c)^(1/2)*(c*d)^(1/2)+14*(-(e*x-d)*c)^(1/2)*(c*d)^(1/2)*d^2)/(e*x+d)^(7
/2)/(-(e*x-d)*c)^(1/2)/e/d/(c*d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-ce^2x^2 + cd^2\right)^{\frac{3}{2}}}{(ex + d)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((-c*e^2*x^2 + c*d^2)^(3/2)/(e*x + d)^(11/2), x)
```

Fricas [A] time = 2.25512, size = 965, normalized size = 5.42

$$\left[\frac{3\sqrt{\frac{1}{2}}\left(ce^4x^4 + 4\,cde^3x^3 + 6\,cd^2e^2x^2 + 4\,cd^3ex + cd^4\right)\sqrt{\frac{c}{d}}\log\left(-\frac{ce^2x^2 - 2\,cdex - 3\,cd^2 + 4\,\sqrt{\frac{1}{2}}\sqrt{-ce^2x^2 + cd^2}\sqrt{ex + d}d\sqrt{\frac{c}{d}}}{e^2x^2 + 2\,dex + d^2}\right) - 2\left(3\,ce^2x^2 - \right.}{96\left(de^5x^4 + 4\,d^2e^4x^3 + 6\,d^3e^3x^2 + 4\,d^4e^2x + d^5e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(11/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*sqrt(1/2)*(c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e
*x + c*d^4)*sqrt(c/d)*log(-(c*e^2*x^2 - 2*c*d*e*x - 3*c*d^2 + 4*sqrt(1/2)*s
qrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)*d*sqrt(c/d))/(e^2*x^2 + 2*d*e*x + d^2
)) - 2*(3*c*e^2*x^2 - 22*c*d*e*x + 7*c*d^2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e
*x + d))/(d*e^5*x^4 + 4*d^2*e^4*x^3 + 6*d^3*e^3*x^2 + 4*d^4*e^2*x + d^5*e),
 -1/48*(3*sqrt(1/2)*(c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*
e*x + c*d^4)*sqrt(-c/d)*arctan(2*sqrt(1/2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*
x + d)*d*sqrt(-c/d)/(c*e^2*x^2 - c*d^2)) + (3*c*e^2*x^2 - 22*c*d*e*x + 7*c*
d^2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d))/(d*e^5*x^4 + 4*d^2*e^4*x^3 + 6
*d^3*e^3*x^2 + 4*d^4*e^2*x + d^5*e)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e**2*x**2+c*d**2)**(3/2)/(e*x+d)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-ce^2x^2 + cd^2\right)^{\frac{3}{2}}}{(ex + d)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((-c*e^2*x^2 + c*d^2)^(3/2)/(e*x + d)^(11/2), x)
```

3.878 $\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{13/2}}\,dx$

Optimal. Leaf size=217

$$-\frac{3c^{3/2}\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{256\sqrt{2}d^{5/2}e}-\frac{3c\sqrt{cd^2-ce^2x^2}}{256d^2e(d+ex)^{3/2}}-\frac{c\sqrt{cd^2-ce^2x^2}}{64de(d+ex)^{5/2}}+\frac{c\sqrt{cd^2-ce^2x^2}}{8e(d+ex)^{7/2}}-\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{4e(d+ex)^{11/2}}$$

```
[Out] (c*Sqrt[c*d^2 - c*e^2*x^2])/(8*e*(d + e*x)^(7/2)) - (c*Sqrt[c*d^2 - c*e^2*x
^2])/(64*d*e*(d + e*x)^(5/2)) - (3*c*Sqrt[c*d^2 - c*e^2*x^2])/(256*d^2*e*(d
 + e*x)^(3/2)) - (c*d^2 - c*e^2*x^2)^(3/2)/(4*e*(d + e*x)^(11/2)) - (3*c^(3
/2)*ArcTanh[Sqrt[c*d^2 - c*e^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])
])/(256*Sqrt[2]*d^(5/2)*e)
```

Rubi [A] time = 0.134371, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {663, 673, 661, 208}

$$-\frac{3c^{3/2}\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{256\sqrt{2}d^{5/2}e}-\frac{3c\sqrt{cd^2-ce^2x^2}}{256d^2e(d+ex)^{3/2}}-\frac{c\sqrt{cd^2-ce^2x^2}}{64de(d+ex)^{5/2}}+\frac{c\sqrt{cd^2-ce^2x^2}}{8e(d+ex)^{7/2}}-\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{4e(d+ex)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(13/2),x]
```

```
[Out] (c*Sqrt[c*d^2 - c*e^2*x^2])/(8*e*(d + e*x)^(7/2)) - (c*Sqrt[c*d^2 - c*e^2*x
^2])/(64*d*e*(d + e*x)^(5/2)) - (3*c*Sqrt[c*d^2 - c*e^2*x^2])/(256*d^2*e*(d
 + e*x)^(3/2)) - (c*d^2 - c*e^2*x^2)^(3/2)/(4*e*(d + e*x)^(11/2)) - (3*c^(3
/2)*ArcTanh[Sqrt[c*d^2 - c*e^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])
])/(256*Sqrt[2]*d^(5/2)*e)
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[(m + 2*
p + 2)/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; Fr
eeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p
+ 1, 0] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{13/2}}\,dx = -\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{4e(d+ex)^{11/2}} - \frac{1}{8}(3c)\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{9/2}}\,dx$$

$$= \frac{c\sqrt{cd^2-ce^2x^2}}{8e(d+ex)^{7/2}} - \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{4e(d+ex)^{11/2}} + \frac{1}{16}c^2\int \frac{1}{(d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}}\,dx$$

$$= \frac{c\sqrt{cd^2-ce^2x^2}}{8e(d+ex)^{7/2}} - \frac{c\sqrt{cd^2-ce^2x^2}}{64de(d+ex)^{5/2}} - \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{4e(d+ex)^{11/2}} + \frac{\left(3c^2\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}\,dx}{128d}$$

$$= \frac{c\sqrt{cd^2-ce^2x^2}}{8e(d+ex)^{7/2}} - \frac{c\sqrt{cd^2-ce^2x^2}}{64de(d+ex)^{5/2}} - \frac{3c\sqrt{cd^2-ce^2x^2}}{256d^2e(d+ex)^{3/2}} - \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{4e(d+ex)^{11/2}} + \frac{\left(3c^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2}}}{512d^2}$$

$$= \frac{c\sqrt{cd^2-ce^2x^2}}{8e(d+ex)^{7/2}} - \frac{c\sqrt{cd^2-ce^2x^2}}{64de(d+ex)^{5/2}} - \frac{3c\sqrt{cd^2-ce^2x^2}}{256d^2e(d+ex)^{3/2}} - \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{4e(d+ex)^{11/2}} + \frac{\left(3c^2e\right)\text{Subst}\left(\int \frac{}{-2cd}\right.}{25}$$

$$= \frac{c\sqrt{cd^2-ce^2x^2}}{8e(d+ex)^{7/2}} - \frac{c\sqrt{cd^2-ce^2x^2}}{64de(d+ex)^{5/2}} - \frac{3c\sqrt{cd^2-ce^2x^2}}{256d^2e(d+ex)^{3/2}} - \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{4e(d+ex)^{11/2}} - \frac{3c^{3/2}\tanh^{-1}\left(\frac{\sqrt{cd^2-c}}{\sqrt{2}\sqrt{c}\sqrt{d}}\right.}{256\sqrt{2}d^{5/2}e}$$

Mathematica [A] time = 0.272458, size = 145, normalized size = 0.67

$$\frac{\left(c\left(d^2-e^2x^2\right)\right)^{3/2}\left(-\frac{2\sqrt{d}\left(-79d^2ex+39d^3+13de^2x^2+3e^3x^3\right)}{(d-ex)(d+ex)^{11/2}} - \frac{3\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{\sqrt{2}\sqrt{d}\sqrt{d+ex}}\right)}{\left(d^2-e^2x^2\right)^{3/2}}\right)}{512d^{5/2}e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(13/2),x]
```

```
[Out] ((c*(d^2 - e^2*x^2))^(3/2)*((-2*Sqrt[d]*(39*d^3 - 79*d^2*e*x + 13*d*e^2*x^2
 + 3*e^3*x^3))/((d - e*x)*(d + e*x)^(11/2)) - (3*Sqrt[2]*ArcTanh[Sqrt[d^2 -
 e^2*x^2]/(Sqrt[2]*Sqrt[d]*Sqrt[d + e*x])])/(d^2 - e^2*x^2)^(3/2)))/(512*d^
(5/2)*e)
```

Maple [A] time = 0.176, size = 325, normalized size = 1.5

$$-\frac{c}{512\,ed^2}\sqrt{-c\left(e^2x^2-d^2\right)}\left(3\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{-(ex-d)\,c}\sqrt{2}}{\sqrt{cd}}\right)x^4ce^4 + 12\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{-(ex-d)\,c}\sqrt{2}}{\sqrt{cd}}\right)x^3cde^3 + 18\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(13/2),x)
```

```
[Out] -1/512*(-c*(e^2*x^2-d^2))^(1/2)*c*(3*2^(1/2)*arctanh(1/2*(-(e*x-d)*c)^(1/2)
*2^(1/2)/(c*d)^(1/2))*x^4*c*e^4+12*2^(1/2)*arctanh(1/2*(-(e*x-d)*c)^(1/2)*2
^(1/2)/(c*d)^(1/2))*x^3*c*d*e^3+18*2^(1/2)*arctanh(1/2*(-(e*x-d)*c)^(1/2)*2
^(1/2)/(c*d)^(1/2))*x^2*c*d^2*e^2+12*2^(1/2)*arctanh(1/2*(-(e*x-d)*c)^(1/2)
```

```
*2^(1/2)/(c*d)^(1/2))*x*c*d^3*e+6*x^3*e^3*(c*d)^(1/2)*(-(e*x-d)*c)^(1/2)+3*
2^(1/2)*arctanh(1/2*(-(e*x-d)*c)^(1/2)*2^(1/2)/(c*d)^(1/2))*c*d^4+26*x^2*d*
e^2*(c*d)^(1/2)*(-(e*x-d)*c)^(1/2)-158*x*d^2*e*(c*d)^(1/2)*(-(e*x-d)*c)^(1/
2)+78*(-(e*x-d)*c)^(1/2)*(c*d)^(1/2)*d^3)/(e*x+d)^(9/2)/(-(e*x-d)*c)^(1/2)/
e/d^2/(c*d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-ce^2x^2+cd^2\right)^{\frac{3}{2}}}{(ex+d)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(13/2),x, algorithm="maxima")
```

```
[Out] integrate((-c*e^2*x^2 + c*d^2)^(3/2)/(e*x + d)^(13/2), x)
```

Fricas [A] time = 2.28808, size = 1130, normalized size = 5.21

$$\left[\frac{3\sqrt{\frac{1}{2}}\left(ce^5x^5+5\,cde^4x^4+10\,cd^2e^3x^3+10\,cd^3e^2x^2+5\,cd^4ex+cd^5\right)\sqrt{\frac{c}{d}}\log\left(-\frac{ce^2x^2-2\,cdex-3\,cd^2+4\sqrt{\frac{1}{2}}\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}d\sqrt{\frac{c}{d}}}{e^2x^2+2\,dex+d^2}\right)}{512\left(d^2e^6x^5+5\,d^3e^5x^4+10\,d^4e^4x^3+10\,d^5e^3x^2+5\,d^6e^2x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(13/2),x, algorithm="fricas")
```

```
[Out] [1/512*(3*sqrt(1/2)*(c*e^5*x^5 + 5*c*d*e^4*x^4 + 10*c*d^2*e^3*x^3 + 10*c*d^
3*e^2*x^2 + 5*c*d^4*e*x + c*d^5)*sqrt(c/d)*log(-(c*e^2*x^2 - 2*c*d*e*x - 3*
c*d^2 + 4*sqrt(1/2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)*d*sqrt(c/d))/(e^
2*x^2 + 2*d*e*x + d^2)) - 2*(3*c*e^3*x^3 + 13*c*d*e^2*x^2 - 79*c*d^2*e*x +
39*c*d^3)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d))/(d^2*e^6*x^5 + 5*d^3*e^5*
x^4 + 10*d^4*e^4*x^3 + 10*d^5*e^3*x^2 + 5*d^6*e^2*x + d^7*e), -1/256*(3*sqr
t(1/2)*(c*e^5*x^5 + 5*c*d*e^4*x^4 + 10*c*d^2*e^3*x^3 + 10*c*d^3*e^2*x^2 + 5
*c*d^4*e*x + c*d^5)*sqrt(-c/d)*arctan(2*sqrt(1/2)*sqrt(-c*e^2*x^2 + c*d^2)*
sqrt(e*x + d)*d*sqrt(-c/d)/(c*e^2*x^2 - c*d^2)) + (3*c*e^3*x^3 + 13*c*d*e^2
*x^2 - 79*c*d^2*e*x + 39*c*d^3)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d))/(d^
2*e^6*x^5 + 5*d^3*e^5*x^4 + 10*d^4*e^4*x^3 + 10*d^5*e^3*x^2 + 5*d^6*e^2*x +
 d^7*e)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e**2*x**2+c*d**2)**(3/2)/(e*x+d)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c e^2 x^2 + c d^2\right)^{\frac{3}{2}}}{(ex+d)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(13/2),x, algorithm="giac")
```

```
[Out] integrate((-c*e^2*x^2 + c*d^2)^(3/2)/(e*x + d)^(13/2), x)
```

3.879 $\int \frac{(d+ex)^{7/2}}{\sqrt{cd^2-ce^2x^2}}\,dx$

Optimal. Leaf size=160

$$-\frac{256d^3\sqrt{cd^2-ce^2x^2}}{35ce\sqrt{d+ex}}-\frac{64d^2\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}{35ce}-\frac{24d(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}{35ce}-\frac{2(d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}}{7ce}$$

```
[Out] (-256*d^3*Sqrt[c*d^2 - c*e^2*x^2])/(35*c*e*Sqrt[d + e*x]) - (64*d^2*Sqrt[d
+ e*x]*Sqrt[c*d^2 - c*e^2*x^2])/(35*c*e) - (24*d*(d + e*x)^(3/2)*Sqrt[c*d^2
 - c*e^2*x^2])/(35*c*e) - (2*(d + e*x)^(5/2)*Sqrt[c*d^2 - c*e^2*x^2])/(7*c*
e)
```

Rubi [A] time = 0.0800169, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {657, 649}

$$-\frac{256d^3\sqrt{cd^2-ce^2x^2}}{35ce\sqrt{d+ex}}-\frac{64d^2\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}{35ce}-\frac{24d(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}{35ce}-\frac{2(d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}}{7ce}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/Sqrt[c*d^2 - c*e^2*x^2],x]
```

```
[Out] (-256*d^3*Sqrt[c*d^2 - c*e^2*x^2])/(35*c*e*Sqrt[d + e*x]) - (64*d^2*Sqrt[d
+ e*x]*Sqrt[c*d^2 - c*e^2*x^2])/(35*c*e) - (24*d*(d + e*x)^(3/2)*Sqrt[c*d^2
 - c*e^2*x^2])/(35*c*e) - (2*(d + e*x)^(5/2)*Sqrt[c*d^2 - c*e^2*x^2])/(7*c*
e)
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^{7/2}}{\sqrt{cd^2-ce^2x^2}}\,dx &= -\frac{2(d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}}{7ce}+\frac{1}{7}(12d)\int \frac{(d+ex)^{5/2}}{\sqrt{cd^2-ce^2x^2}}\,dx\\
&= -\frac{24d(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}{35ce}-\frac{2(d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}}{7ce}+\frac{1}{35}\left(96d^2\right)\int \frac{(d+ex)^{3/2}}{\sqrt{cd^2-ce^2x^2}}\,dx\\
&= -\frac{64d^2\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}{35ce}-\frac{24d(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}{35ce}-\frac{2(d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}}{7ce}+\frac{1}{35}(12\\
&= -\frac{256d^3\sqrt{cd^2-ce^2x^2}}{35ce\sqrt{d+ex}}-\frac{64d^2\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}{35ce}-\frac{24d(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}{35ce}-\frac{2(d+ex)^{5/2}}{7}
\end{aligned}$$

Mathematica [A] time = 0.0753387, size = 70, normalized size = 0.44

$$-\frac{2(d-ex)\sqrt{d+ex}\left(71d^2ex+177d^3+27de^2x^2+5e^3x^3\right)}{35e\sqrt{c\left(d^2-e^2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)/Sqrt[c*d^2 - c*e^2*x^2],x]
```

```
[Out] (-2*(d - e*x)*Sqrt[d + e*x]*(177*d^3 + 71*d^2*e*x + 27*d*e^2*x^2 + 5*e^3*x^
3))/(35*e*Sqrt[c*(d^2 - e^2*x^2)])
```

Maple [A] time = 0.043, size = 66, normalized size = 0.4

$$-\frac{(-2\,ex+2\,d)\left(5\,e^3x^3+27\,de^2x^2+71\,d^2xe+177\,d^3\right)}{35\,e}\sqrt{ex+d}\frac{1}{\sqrt{-ce^2x^2+cd^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/(-c*e^2*x^2+c*d^2)^(1/2),x)
```

```
[Out] -2/35*(-e*x+d)*(5*e^3*x^3+27*d*e^2*x^2+71*d^2*e*x+177*d^3)*(e*x+d)^(1/2)/e/
(-c*e^2*x^2+c*d^2)^(1/2)
```

Maxima [A] time = 1.06837, size = 77, normalized size = 0.48

$$\frac{2\left(5\,e^4x^4+22\,de^3x^3+44\,d^2e^2x^2+106\,d^3ex-177\,d^4\right)}{35\,\sqrt{-ex+d}\sqrt{ce}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*e^4*x^4 + 22*d*e^3*x^3 + 44*d^2*e^2*x^2 + 106*d^3*e*x - 177*d^4)/(s
qrt(-e*x + d)*sqrt(c)*e)
```

Fricas [A] time = 2.11261, size = 154, normalized size = 0.96

$$-\frac{2\left(5\,e^3x^3+27\,de^2x^2+71\,d^2ex+177\,d^3\right)\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}}{35\left(ce^2x+cde\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/35*(5*e^3*x^3 + 27*d*e^2*x^2 + 71*d^2*e*x + 177*d^3)*sqrt(-c*e^2*x^2 + c
*d^2)*sqrt(e*x + d)/(c*e^2*x + c*d*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^{\frac{7}{2}}}{\sqrt{-c(-d+ex)(d+ex)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/(-c*e**2*x**2+c*d**2)**(1/2),x)
```

```
[Out] Integral((d + e*x)**(7/2)/sqrt(-c*(-d + e*x)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{7}{2}}}{\sqrt{-ce^2x^2+cd^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(7/2)/sqrt(-c*e^2*x^2 + c*d^2), x)
```

3.880 $\int \frac{(d+ex)^{5/2}}{\sqrt{cd^2-ce^2x^2}}\,dx$

Optimal. Leaf size=119

$$-\frac{64d^2\sqrt{cd^2-ce^2x^2}}{15ce\sqrt{d+ex}}-\frac{16d\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}{15ce}-\frac{2(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}{5ce}$$

```
[Out] (-64*d^2*Sqrt[c*d^2 - c*e^2*x^2])/(15*c*e*Sqrt[d + e*x]) - (16*d*Sqrt[d + e
*x]*Sqrt[c*d^2 - c*e^2*x^2])/(15*c*e) - (2*(d + e*x)^(3/2)*Sqrt[c*d^2 - c*e
^2*x^2])/(5*c*e)
```

Rubi [A] time = 0.0490256, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {657, 649}

$$-\frac{64d^2\sqrt{cd^2-ce^2x^2}}{15ce\sqrt{d+ex}}-\frac{16d\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}{15ce}-\frac{2(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}{5ce}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/Sqrt[c*d^2 - c*e^2*x^2],x]
```

```
[Out] (-64*d^2*Sqrt[c*d^2 - c*e^2*x^2])/(15*c*e*Sqrt[d + e*x]) - (16*d*Sqrt[d + e
*x]*Sqrt[c*d^2 - c*e^2*x^2])/(15*c*e) - (2*(d + e*x)^(3/2)*Sqrt[c*d^2 - c*e
^2*x^2])/(5*c*e)
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^{5/2}}{\sqrt{cd^2-ce^2x^2}}\,dx &= -\frac{2(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}{5ce}+\frac{1}{5}(8d)\int \frac{(d+ex)^{3/2}}{\sqrt{cd^2-ce^2x^2}}\,dx\\ &= -\frac{16d\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}{15ce}-\frac{2(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}{5ce}+\frac{1}{15}\left(32d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}\,dx\\ &= -\frac{64d^2\sqrt{cd^2-ce^2x^2}}{15ce\sqrt{d+ex}}-\frac{16d\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}{15ce}-\frac{2(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}{5ce}\end{aligned}$$

Mathematica [A] time = 0.0576308, size = 59, normalized size = 0.5

$$-\frac{2(d-ex)\sqrt{d+ex}\left(43d^2+14dex+3e^2x^2\right)}{15e\sqrt{c\left(d^2-e^2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/Sqrt[c*d^2 - c*e^2*x^2],x]
```

```
[Out] (-2*(d - e*x)*Sqrt[d + e*x]*(43*d^2 + 14*d*e*x + 3*e^2*x^2))/(15*e*Sqrt[c*(
d^2 - e^2*x^2)])
```

Maple [A] time = 0.042, size = 55, normalized size = 0.5

$$-\frac{(-2\,ex+2\,d)\left(3\,e^2x^2+14\,dxe+43\,d^2\right)}{15\,e}\sqrt{ex+d}\frac{1}{\sqrt{-ce^2x^2+cd^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(1/2),x)
```

```
[Out] -2/15*(-e*x+d)*(3*e^2*x^2+14*d*e*x+43*d^2)*(e*x+d)^(1/2)/e/(-c*e^2*x^2+c*d^
2)^(1/2)
```

Maxima [A] time = 1.08892, size = 78, normalized size = 0.66

$$\frac{2\left(3\,\sqrt{c}e^3x^3+11\,\sqrt{c}de^2x^2+29\,\sqrt{c}d^2ex-43\,\sqrt{c}d^3\right)}{15\,\sqrt{-ex+d}ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*sqrt(c)*e^3*x^3 + 11*sqrt(c)*d*e^2*x^2 + 29*sqrt(c)*d^2*e*x - 43*sq
rt(c)*d^3)/(sqrt(-e*x + d)*c*e)
```

Fricas [A] time = 2.04872, size = 130, normalized size = 1.09

$$-\frac{2\,\sqrt{-ce^2x^2+cd^2}\left(3\,e^2x^2+14\,dex+43\,d^2\right)\sqrt{ex+d}}{15\,\left(ce^2x+cde\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*sqrt(-c*e^2*x^2 + c*d^2)*(3*e^2*x^2 + 14*d*e*x + 43*d^2)*sqrt(e*x + d
)/(c*e^2*x + c*d*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(d+ex)^{\frac{5}{2}}}{\sqrt{-c\,(-d+ex)\,(d+ex)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(-c*e**2*x**2+c*d**2)**(1/2),x)
```

```
[Out] Integral((d + e*x)**(5/2)/sqrt(-c*(-d + e*x)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\sqrt{-ce^2x^2+cd^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(5/2)/sqrt(-c*e^2*x^2 + c*d^2), x)
```

3.881 $\int \frac{(d+ex)^{3/2}}{\sqrt{cd^2-ce^2x^2}}\,dx$

Optimal. Leaf size=78

$$-\frac{8d\sqrt{cd^2-ce^2x^2}}{3ce\sqrt{d+ex}}-\frac{2\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}{3ce}$$

```
[Out] (-8*d*Sqrt[c*d^2 - c*e^2*x^2])/(3*c*e*Sqrt[d + e*x]) - (2*Sqrt[d + e*x]*Sqr
t[c*d^2 - c*e^2*x^2])/(3*c*e)
```

Rubi [A] time = 0.0301264, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {657, 649}

$$-\frac{8d\sqrt{cd^2-ce^2x^2}}{3ce\sqrt{d+ex}}-\frac{2\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}{3ce}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/Sqrt[c*d^2 - c*e^2*x^2],x]
```

```
[Out] (-8*d*Sqrt[c*d^2 - c*e^2*x^2])/(3*c*e*Sqrt[d + e*x]) - (2*Sqrt[d + e*x]*Sqr
t[c*d^2 - c*e^2*x^2])/(3*c*e)
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^{3/2}}{\sqrt{cd^2-ce^2x^2}}\,dx &= -\frac{2\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}{3ce}+\frac{1}{3}(4d)\int \frac{\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}\,dx\\ &= -\frac{8d\sqrt{cd^2-ce^2x^2}}{3ce\sqrt{d+ex}}-\frac{2\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}{3ce}\end{aligned}$$

Mathematica [A] time = 0.0481572, size = 47, normalized size = 0.6

$$-\frac{2(d-ex)\sqrt{d+ex}(5d+ex)}{3e\sqrt{c\left(d^2-e^2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/Sqrt[c*d^2 - c*e^2*x^2],x]
```

```
[Out] (-2*(d - e*x)*Sqrt[d + e*x]*(5*d + e*x))/(3*e*Sqrt[c*(d^2 - e^2*x^2)])
```

Maple [A] time = 0.042, size = 43, normalized size = 0.6

$$-\frac{(-2\,ex+2\,d)\,(ex+5\,d)}{3\,e}\sqrt{ex+d}\frac{1}{\sqrt{-ce^2x^2+cd^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(1/2),x)
```

```
[Out] -2/3*(-e*x+d)*(e*x+5*d)*(e*x+d)^(1/2)/e/(-c*e^2*x^2+c*d^2)^(1/2)
```

Maxima [A] time = 1.04609, size = 46, normalized size = 0.59

$$\frac{2\left(e^2x^2+4\,dex-5\,d^2\right)}{3\,\sqrt{-ex+d}\sqrt{c}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(e^2*x^2 + 4*d*e*x - 5*d^2)/(sqrt(-e*x + d)*sqrt(c)*e)
```

Fricas [A] time = 2.13827, size = 101, normalized size = 1.29

$$-\frac{2\,\sqrt{-ce^2x^2+cd^2}(ex+5\,d)\sqrt{ex+d}}{3\left(ce^2x+cde\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(-c*e^2*x^2 + c*d^2)*(e*x + 5*d)*sqrt(e*x + d)/(c*e^2*x + c*d*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(d+ex)^{\frac{3}{2}}}{\sqrt{-c\,(-d+ex)\,(d+ex)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(-c*e**2*x**2+c*d**2)**(1/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)/sqrt(-c*(-d + e*x)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{-ce^2x^2+cd^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)/sqrt(-c*e^2*x^2 + c*d^2), x)
```

3.882 $\int \frac{\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}} dx$

Optimal. Leaf size=36

$$-\frac{2\sqrt{cd^2 - ce^2x^2}}{ce\sqrt{d + ex}}$$

```
[Out] (-2*Sqrt[c*d^2 - c*e^2*x^2])/(c*e*Sqrt[d + e*x])
```

Rubi [A] time = 0.0127333, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {649}

$$-\frac{2\sqrt{cd^2 - ce^2x^2}}{ce\sqrt{d + ex}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/Sqrt[c*d^2 - c*e^2*x^2],x]
```

```
[Out] (-2*Sqrt[c*d^2 - c*e^2*x^2])/(c*e*Sqrt[d + e*x])
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{\sqrt{d + ex}}{\sqrt{cd^2 - ce^2x^2}} dx = -\frac{2\sqrt{cd^2 - ce^2x^2}}{ce\sqrt{d + ex}}$$

Mathematica [A] time = 0.0439942, size = 35, normalized size = 0.97

$$-\frac{2\sqrt{c\left(d^2 - e^2x^2\right)}}{ce\sqrt{d + ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/Sqrt[c*d^2 - c*e^2*x^2],x]
```

```
[Out] (-2*Sqrt[c*(d^2 - e^2*x^2)])/(c*e*Sqrt[d + e*x])
```

Maple [A] time = 0.042, size = 36, normalized size = 1.

$$-2\,\frac{(-ex + d)\sqrt{ex + d}}{e\sqrt{-ce^2x^2 + cd^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2),x)
```

```
[Out] -2*(-e*x+d)*(e*x+d)^(1/2)/e/(-c*e^2*x^2+c*d^2)^(1/2)
```

Maxima [A] time = 1.13023, size = 39, normalized size = 1.08

$$\frac{2\left(\sqrt{c}ex - \sqrt{c}d\right)}{\sqrt{-ex + d}ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(c)*e*x - sqrt(c)*d)/(sqrt(-e*x + d)*c*e)
```

Fricas [A] time = 2.11583, size = 82, normalized size = 2.28

$$-\frac{2\sqrt{-ce^2x^2 + cd^2}\sqrt{ex + d}}{ce^2x + cde}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)/(c*e^2*x + c*d*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d + ex}}{\sqrt{-c(-d + ex)(d + ex)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(-c*e**2*x**2+c*d**2)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/sqrt(-c*(-d + e*x)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex + d}}{\sqrt{-ce^2x^2 + cd^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)/sqrt(-c*e^2*x^2 + c*d^2), x)
```

3.883 $\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\,dx$

Optimal. Leaf size=65

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{c}\sqrt{d}e}$$

```
[Out] -((Sqrt[2]*ArcTanh[Sqrt[c*d^2 - c*e^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d
+ e*x])])/(Sqrt[c]*Sqrt[d]*e))
```

Rubi [A] time = 0.0287519, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {661, 208}

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{c}\sqrt{d}e}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*Sqrt[c*d^2 - c*e^2*x^2]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[Sqrt[c*d^2 - c*e^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d
+ e*x])])/(Sqrt[c]*Sqrt[d]*e))
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\,dx &= (2e)\,\text{Subst}\left(\int \frac{1}{-2cde^2+e^2x^2}\,dx, x, \frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}\right)\\ &= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{c}\sqrt{d}e}\end{aligned}$$

Mathematica [A] time = 0.0484258, size = 86, normalized size = 1.32

$$-\frac{\sqrt{2}\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{\sqrt{2}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{d}e\sqrt{c\left(d^2-e^2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*Sqrt[c*d^2 - c*e^2*x^2]),x]
```

```
[Out] -((Sqrt[2]*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/(Sqrt[2]*Sqrt[d]
*Sqrt[d + e*x])])/(Sqrt[d]*e*Sqrt[c*(d^2 - e^2*x^2)]))
```

Maple [A] time = 0.162, size = 74, normalized size = 1.1

$$-\frac{\sqrt{2}}{e}\sqrt{-c\left(e^2x^2-d^2\right)}\text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{-(ex-d)c}\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{ex+d}}\frac{1}{\sqrt{-(ex-d)c}}\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2),x)
```

```
[Out] -1/(e*x+d)^(1/2)*(-c*(e^2*x^2-d^2))^(1/2)/(-(e*x-d)*c)^(1/2)/e*2^(1/2)/(c*d
)^(1/2)*arctanh(1/2*(-(e*x-d)*c)^(1/2)*2^(1/2)/(c*d)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)), x)
```

Fricas [A] time = 2.11131, size = 371, normalized size = 5.71

$$\left[\frac{\sqrt{2}\sqrt{\frac{1}{cd}}\log\left(-\frac{e^2x^2-2\,dex+2\,\sqrt{2}\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}d\sqrt{\frac{1}{cd}}-3\,d^2}{e^2x^2+2\,dex+d^2}\right)}{2\,e},-\frac{\sqrt{2}\sqrt{-\frac{1}{cd}}\arctan\left(\frac{\sqrt{2}\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}d\sqrt{-\frac{1}{cd}}}{e^2x^2-d^2}\right)}{e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*sqrt(1/(c*d))*log(-(e^2*x^2 - 2*d*e*x + 2*sqrt(2)*sqrt(-c*e^2*
x^2 + c*d^2)*sqrt(e*x + d)*d*sqrt(1/(c*d)) - 3*d^2)/(e^2*x^2 + 2*d*e*x + d^
2))/e, -sqrt(2)*sqrt(-1/(c*d))*arctan(sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt
(e*x + d)*d*sqrt(-1/(c*d))/(e^2*x^2 - d^2))/e]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-c\,(-d+ex)\,(d+ex)}\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(-c*e**2*x**2+c*d**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-c*(-d + e*x)*(d + e*x))*sqrt(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-ce^2x^2 + cd^2}\sqrt{ex + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)), x)
```

3.884 $\int \frac{1}{(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}\,dx$

Optimal. Leaf size=109

$$-\frac{\sqrt{cd^2-ce^2x^2}}{2cde(d+ex)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{2\sqrt{2}\sqrt{c}d^{3/2}e}$$

```
[Out] -Sqrt[c*d^2 - c*e^2*x^2]/(2*c*d*e*(d + e*x)^(3/2)) - ArcTanh[Sqrt[c*d^2 - c
*e^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])]/(2*Sqrt[2]*Sqrt[c]*d^(3/
2)*e)
```

Rubi [A] time = 0.0510786, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {673, 661, 208}

$$-\frac{\sqrt{cd^2-ce^2x^2}}{2cde(d+ex)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{2\sqrt{2}\sqrt{c}d^{3/2}e}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*Sqrt[c*d^2 - c*e^2*x^2]),x]
```

```
[Out] -Sqrt[c*d^2 - c*e^2*x^2]/(2*c*d*e*(d + e*x)^(3/2)) - ArcTanh[Sqrt[c*d^2 - c
*e^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])]/(2*Sqrt[2]*Sqrt[c]*d^(3/
2)*e)
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[(m + 2*
p + 2)/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; Fr
eeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p
+ 1, 0] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}\,dx = -\frac{\sqrt{cd^2-ce^2x^2}}{2cde(d+ex)^{3/2}} + \frac{\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\,dx}{4d}$$
$$= -\frac{\sqrt{cd^2-ce^2x^2}}{2cde(d+ex)^{3/2}} + \frac{e\,\text{Subst}\left(\int \frac{1}{-2cde^2+e^2x^2}\,dx, x, \frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}\right)}{2d}$$
$$= -\frac{\sqrt{cd^2-ce^2x^2}}{2cde(d+ex)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{2\sqrt{2}\sqrt{c}d^{3/2}e}$$

Mathematica [A] time = 0.0811625, size = 122, normalized size = 1.12

$$\frac{-\sqrt{2}\sqrt{d+ex}\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{\sqrt{2}\sqrt{d}\sqrt{d+ex}}\right)-2\sqrt{d}(d-ex)}{4d^{3/2}e\sqrt{d+ex}\sqrt{c\left(d^2-e^2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*Sqrt[c*d^2 - c*e^2*x^2]),x]
```

```
[Out] (-2*Sqrt[d]*(d - e*x) - Sqrt[2]*Sqrt[d + e*x]*Sqrt[d^2 - e^2*x^2]*ArcTanh[S
qrt[d^2 - e^2*x^2]/(Sqrt[2]*Sqrt[d]*Sqrt[d + e*x])])/(4*d^(3/2)*e*Sqrt[d +
e*x]*Sqrt[c*(d^2 - e^2*x^2)])
```

Maple [A] time = 0.169, size = 133, normalized size = 1.2

$$-\frac{1}{4\,ced}\sqrt{-c\left(e^2x^2-d^2\right)}\left(\sqrt{2}\text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{-(ex-d)\,c}\frac{1}{\sqrt{cd}}\right)xce+cd\sqrt{2}\text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{-(ex-d)\,c}\frac{1}{\sqrt{cd}}\right)+2\sqrt{-(ex-d)\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(1/2),x)
```

```
[Out] -1/4/(e*x+d)^(3/2)*(-c*(e^2*x^2-d^2))^(1/2)/c*(2^(1/2)*arctanh(1/2*(-(e*x-d
)*c)^(1/2)*2^(1/2)/(c*d)^(1/2))*x*c*e+c*d*2^(1/2)*arctanh(1/2*(-(e*x-d)*c)^
(1/2)*2^(1/2)/(c*d)^(1/2))+2*(-(e*x-d)*c)^(1/2)*(c*d)^(1/2))/(-(e*x-d)*c)^(
1/2)/e/d/(c*d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-ce^2x^2+cd^2}(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*e^2*x^2 + c*d^2)*(e*x + d)^(3/2)), x)
```

Fricas [A] time = 2.29677, size = 668, normalized size = 6.13

$$\left[\frac{\sqrt{2}\left(e^2x^2+2\,dex+d^2\right)\sqrt{cd}\log\left(-\frac{ce^2x^2-2\,cdex-3\,cd^2+2\,\sqrt{2}\sqrt{-ce^2x^2+cd^2}\sqrt{cd}\sqrt{ex+d}}{e^2x^2+2\,dex+d^2}\right)-4\,\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}d}{8\left(cd^2e^3x^2+2\,cd^3e^2x+cd^4e\right)},-\frac{\sqrt{2}\left(e^2x^2+2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*(e^2*x^2 + 2*d*e*x + d^2)*sqrt(c*d)*log(-(c*e^2*x^2 - 2*c*d*e
*x - 3*c*d^2 + 2*sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(c*d)*sqrt(e*x + d))/
(e^2*x^2 + 2*d*e*x + d^2)) - 4*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)*d)/(c
*d^2*e^3*x^2 + 2*c*d^3*e^2*x + c*d^4*e), -1/4*(sqrt(2)*(e^2*x^2 + 2*d*e*x +
 d^2)*sqrt(-c*d)*arctan(sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(-c*d)*sqrt(e*
x + d)/(c*e^2*x^2 - c*d^2)) + 2*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)*d)/(
c*d^2*e^3*x^2 + 2*c*d^3*e^2*x + c*d^4*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-c\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(-c*e**2*x**2+c*d**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-c*(-d + e*x)*(d + e*x))*(d + e*x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-ce^2x^2+cd^2}(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c*e^2*x^2 + c*d^2)*(e*x + d)^(3/2)), x)
```

3.885 $\int \frac{1}{(d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}}\,dx$

Optimal. Leaf size=150

$$-\frac{3\sqrt{cd^2-ce^2x^2}}{16cd^2e(d+ex)^{3/2}}-\frac{\sqrt{cd^2-ce^2x^2}}{4cde(d+ex)^{5/2}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{16\sqrt{2}\sqrt{c}d^{5/2}e}$$

```
[Out] -Sqrt[c*d^2 - c*e^2*x^2]/(4*c*d*e*(d + e*x)^(5/2)) - (3*Sqrt[c*d^2 - c*e^2*
x^2])/(16*c*d^2*e*(d + e*x)^(3/2)) - (3*ArcTanh[Sqrt[c*d^2 - c*e^2*x^2]/(Sq
rt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/(16*Sqrt[2]*Sqrt[c]*d^(5/2)*e)
```

Rubi [A] time = 0.0720232, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {673, 661, 208}

$$-\frac{3\sqrt{cd^2-ce^2x^2}}{16cd^2e(d+ex)^{3/2}}-\frac{\sqrt{cd^2-ce^2x^2}}{4cde(d+ex)^{5/2}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{16\sqrt{2}\sqrt{c}d^{5/2}e}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(5/2)*Sqrt[c*d^2 - c*e^2*x^2]),x]
```

```
[Out] -Sqrt[c*d^2 - c*e^2*x^2]/(4*c*d*e*(d + e*x)^(5/2)) - (3*Sqrt[c*d^2 - c*e^2*
x^2])/(16*c*d^2*e*(d + e*x)^(3/2)) - (3*ArcTanh[Sqrt[c*d^2 - c*e^2*x^2]/(Sq
rt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/(16*Sqrt[2]*Sqrt[c]*d^(5/2)*e)
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[(m + 2*
p + 2)/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; Fr
eeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p
+ 1, 0] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}}\,dx &= -\frac{\sqrt{cd^2-ce^2x^2}}{4cde(d+ex)^{5/2}}+\frac{3\int \frac{1}{(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}\,dx}{8d}\\
&= -\frac{\sqrt{cd^2-ce^2x^2}}{4cde(d+ex)^{5/2}}-\frac{3\sqrt{cd^2-ce^2x^2}}{16cd^2e(d+ex)^{3/2}}+\frac{3\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\,dx}{32d^2}\\
&= -\frac{\sqrt{cd^2-ce^2x^2}}{4cde(d+ex)^{5/2}}-\frac{3\sqrt{cd^2-ce^2x^2}}{16cd^2e(d+ex)^{3/2}}+\frac{(3e)\,\mathrm{Subst}\left(\int \frac{1}{-2cde^2+e^2x^2}\,dx,x,\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}\right)}{16d^2}\\
&= -\frac{\sqrt{cd^2-ce^2x^2}}{4cde(d+ex)^{5/2}}-\frac{3\sqrt{cd^2-ce^2x^2}}{16cd^2e(d+ex)^{3/2}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{16\sqrt{2}\sqrt{c}d^{5/2}e}
\end{aligned}$$

Mathematica [A] time = 0.110124, size = 140, normalized size = 0.93

$$\frac{2\sqrt{d}\sqrt{d+ex}\left(-7d^2+4dex+3e^2x^2\right)-3\sqrt{2}(d+ex)^2\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{\sqrt{2}\sqrt{d}\sqrt{d+ex}}\right)}{32d^{5/2}e(d+ex)^2\sqrt{c\left(d^2-e^2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(5/2)*Sqrt[c*d^2 - c*e^2*x^2]),x]
```

```
[Out] (2*Sqrt[d]*Sqrt[d + e*x]*(-7*d^2 + 4*d*e*x + 3*e^2*x^2) - 3*Sqrt[2]*(d + e*
x)^2*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/(Sqrt[2]*Sqrt[d]*Sqrt[
d + e*x]])])/(32*d^(5/2)*e*(d + e*x)^2*Sqrt[c*(d^2 - e^2*x^2)])
```

Maple [A] time = 0.165, size = 195, normalized size = 1.3

$$-\frac{1}{32\,d^2ec}\sqrt{-c\left(e^2x^2-d^2\right)}\left(3\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{-(ex-d)c}\sqrt{2}}{\sqrt{cd}}\right)x^2ce^2+6\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{-(ex-d)c}\sqrt{2}}{\sqrt{cd}}\right)xcde+3\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(1/2),x)
```

```
[Out] -1/32/(e*x+d)^(5/2)*(-c*(e^2*x^2-d^2))^(1/2)/c*(3*2^(1/2)*arctanh(1/2*(-(e*
x-d)*c)^(1/2)*2^(1/2)/(c*d)^(1/2))*x^2*c*e^2+6*2^(1/2)*arctanh(1/2*(-(e*x-d
)*c)^(1/2)*2^(1/2)/(c*d)^(1/2))*x*c*d*e+3*2^(1/2)*arctanh(1/2*(-(e*x-d)*c)^
(1/2)*2^(1/2)/(c*d)^(1/2))*c*d^2+6*x*e*(-(e*x-d)*c)^(1/2)*(c*d)^(1/2)+14*(-
(e*x-d)*c)^(1/2)*(c*d)^(1/2)*d)/(-(e*x-d)*c)^(1/2)/e/d^2/(c*d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-ce^2x^2+cd^2}(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*e^2*x^2 + c*d^2)*(e*x + d)^(5/2)), x)
```

Fricas [A] time = 2.10015, size = 811, normalized size = 5.41

$$\left[\frac{3\sqrt{2}\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\sqrt{cd}\log\left(-\frac{ce^2x^2-2\,cdex-3\,cd^2+2\,\sqrt{2}\sqrt{-ce^2x^2+cd^2}\sqrt{cd}\sqrt{ex+d}}{e^2x^2+2\,dex+d^2}\right)-4\,\sqrt{-ce^2x^2+cd^2}\left(3\,dex+7\,d^2\right)\sqrt{}}{64\left(cd^3e^4x^3+3\,cd^4e^3x^2+3\,cd^5e^2x+cd^6e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/64*(3*sqrt(2)*(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(c*d)*log(-(
c*e^2*x^2 - 2*c*d*e*x - 3*c*d^2 + 2*sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(c
*d)*sqrt(e*x + d))/(e^2*x^2 + 2*d*e*x + d^2)) - 4*sqrt(-c*e^2*x^2 + c*d^2)*
(3*d*e*x + 7*d^2)*sqrt(e*x + d))/(c*d^3*e^4*x^3 + 3*c*d^4*e^3*x^2 + 3*c*d^5
*e^2*x + c*d^6*e), -1/32*(3*sqrt(2)*(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^
3)*sqrt(-c*d)*arctan(sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(-c*d)*sqrt(e*x +
 d)/(c*e^2*x^2 - c*d^2)) + 2*sqrt(-c*e^2*x^2 + c*d^2)*(3*d*e*x + 7*d^2)*sqr
t(e*x + d))/(c*d^3*e^4*x^3 + 3*c*d^4*e^3*x^2 + 3*c*d^5*e^2*x + c*d^6*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-c\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(5/2)/(-c*e**2*x**2+c*d**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-c*(-d + e*x)*(d + e*x))*(d + e*x)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.886 $\int \frac{(d+ex)^{9/2}}{\left(cd^2-ce^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=160

$$-\frac{2(d+ex)^{7/2}}{5ce\sqrt{cd^2-ce^2x^2}}-\frac{8d(d+ex)^{5/2}}{5ce\sqrt{cd^2-ce^2x^2}}-\frac{64d^2(d+ex)^{3/2}}{5ce\sqrt{cd^2-ce^2x^2}}+\frac{256d^3\sqrt{d+ex}}{5ce\sqrt{cd^2-ce^2x^2}}$$

```
[Out] (256*d^3*Sqrt[d + e*x])/(5*c*e*Sqrt[c*d^2 - c*e^2*x^2]) - (64*d^2*(d + e*x)
^(3/2))/(5*c*e*Sqrt[c*d^2 - c*e^2*x^2]) - (8*d*(d + e*x)^(5/2))/(5*c*e*Sqrt
[c*d^2 - c*e^2*x^2]) - (2*(d + e*x)^(7/2))/(5*c*e*Sqrt[c*d^2 - c*e^2*x^2])
```

Rubi [A] time = 0.0726321, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {657, 649}

$$-\frac{2(d+ex)^{7/2}}{5ce\sqrt{cd^2-ce^2x^2}}-\frac{8d(d+ex)^{5/2}}{5ce\sqrt{cd^2-ce^2x^2}}-\frac{64d^2(d+ex)^{3/2}}{5ce\sqrt{cd^2-ce^2x^2}}+\frac{256d^3\sqrt{d+ex}}{5ce\sqrt{cd^2-ce^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(9/2)/(c*d^2 - c*e^2*x^2)^(3/2),x]
```

```
[Out] (256*d^3*Sqrt[d + e*x])/(5*c*e*Sqrt[c*d^2 - c*e^2*x^2]) - (64*d^2*(d + e*x)
^(3/2))/(5*c*e*Sqrt[c*d^2 - c*e^2*x^2]) - (8*d*(d + e*x)^(5/2))/(5*c*e*Sqrt
[c*d^2 - c*e^2*x^2]) - (2*(d + e*x)^(7/2))/(5*c*e*Sqrt[c*d^2 - c*e^2*x^2])
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^{9/2}}{\left(cd^2-ce^2x^2\right)^{3/2}}\,dx &= -\frac{2(d+ex)^{7/2}}{5ce\sqrt{cd^2-ce^2x^2}}+\frac{1}{5}(12d)\int \frac{(d+ex)^{7/2}}{\left(cd^2-ce^2x^2\right)^{3/2}}\,dx\\
&= -\frac{8d(d+ex)^{5/2}}{5ce\sqrt{cd^2-ce^2x^2}}-\frac{2(d+ex)^{7/2}}{5ce\sqrt{cd^2-ce^2x^2}}+\frac{1}{5}\left(32d^2\right)\int \frac{(d+ex)^{5/2}}{\left(cd^2-ce^2x^2\right)^{3/2}}\,dx\\
&= -\frac{64d^2(d+ex)^{3/2}}{5ce\sqrt{cd^2-ce^2x^2}}-\frac{8d(d+ex)^{5/2}}{5ce\sqrt{cd^2-ce^2x^2}}-\frac{2(d+ex)^{7/2}}{5ce\sqrt{cd^2-ce^2x^2}}+\frac{1}{5}\left(128d^3\right)\int \frac{(d+ex)^{3/2}}{\left(cd^2-ce^2x^2\right)^{3/2}}\,dx\\
&= \frac{256d^3\sqrt{d+ex}}{5ce\sqrt{cd^2-ce^2x^2}}-\frac{64d^2(d+ex)^{3/2}}{5ce\sqrt{cd^2-ce^2x^2}}-\frac{8d(d+ex)^{5/2}}{5ce\sqrt{cd^2-ce^2x^2}}-\frac{2(d+ex)^{7/2}}{5ce\sqrt{cd^2-ce^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.0834965, size = 66, normalized size = 0.41

$$-\frac{2\sqrt{d+ex}\left(43d^2ex-91d^3+7de^2x^2+e^3x^3\right)}{5ce\sqrt{c\left(d^2-e^2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(9/2)/(c*d^2 - c*e^2*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(-91*d^3 + 43*d^2*e*x + 7*d*e^2*x^2 + e^3*x^3))/(5*c*e*Sq
rt[c*(d^2 - e^2*x^2)])
```

Maple [A] time = 0.043, size = 66, normalized size = 0.4

$$\frac{(-2\,ex+2\,d)\left(-e^3x^3-7\,de^2x^2-43\,d^2xe+91\,d^3\right)}{5\,e}(ex+d)^{\frac{3}{2}}\left(-ce^2x^2+cd^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(9/2)/(-c*e^2*x^2+c*d^2)^(3/2),x)
```

```
[Out] 2/5*(-e*x+d)*(-e^3*x^3-7*d*e^2*x^2-43*d^2*e*x+91*d^3)*(e*x+d)^(3/2)/e/(-c*e
^2*x^2+c*d^2)^(3/2)
```

Maxima [A] time = 1.11729, size = 61, normalized size = 0.38

$$-\frac{2\left(e^3x^3+7\,de^2x^2+43\,d^2ex-91\,d^3\right)}{5\sqrt{-ex+d}c^{\frac{3}{2}}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/5*(e^3*x^3 + 7*d*e^2*x^2 + 43*d^2*e*x - 91*d^3)/(sqrt(-e*x + d)*c^(3/2)*
e)
```

Fricas [A] time = 2.18276, size = 157, normalized size = 0.98

$$\frac{2\left(e^3x^3+7\,de^2x^2+43\,d^2ex-91\,d^3\right)\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}}{5\left(c^2e^3x^2-c^2d^2e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(e^3*x^3 + 7*d*e^2*x^2 + 43*d^2*e*x - 91*d^3)*sqrt(-c*e^2*x^2 + c*d^2)*
sqrt(e*x + d)/(c^2*e^3*x^2 - c^2*d^2*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(9/2)/(-c*e**2*x**2+c*d**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.887 $\int \frac{(d+ex)^{7/2}}{\left(cd^2-ce^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=119

$$-\frac{2(d+ex)^{5/2}}{3ce\sqrt{cd^2-ce^2x^2}}-\frac{16d(d+ex)^{3/2}}{3ce\sqrt{cd^2-ce^2x^2}}+\frac{64d^2\sqrt{d+ex}}{3ce\sqrt{cd^2-ce^2x^2}}$$

```
[Out] (64*d^2*Sqrt[d + e*x])/(3*c*e*Sqrt[c*d^2 - c*e^2*x^2]) - (16*d*(d + e*x)^(3
/2))/(3*c*e*Sqrt[c*d^2 - c*e^2*x^2]) - (2*(d + e*x)^(5/2))/(3*c*e*Sqrt[c*d^
2 - c*e^2*x^2])
```

Rubi [A] time = 0.0486483, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {657, 649}

$$-\frac{2(d+ex)^{5/2}}{3ce\sqrt{cd^2-ce^2x^2}}-\frac{16d(d+ex)^{3/2}}{3ce\sqrt{cd^2-ce^2x^2}}+\frac{64d^2\sqrt{d+ex}}{3ce\sqrt{cd^2-ce^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/(c*d^2 - c*e^2*x^2)^(3/2),x]
```

```
[Out] (64*d^2*Sqrt[d + e*x])/(3*c*e*Sqrt[c*d^2 - c*e^2*x^2]) - (16*d*(d + e*x)^(3
/2))/(3*c*e*Sqrt[c*d^2 - c*e^2*x^2]) - (2*(d + e*x)^(5/2))/(3*c*e*Sqrt[c*d^
2 - c*e^2*x^2])
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^{7/2}}{\left(cd^2-ce^2x^2\right)^{3/2}}\,dx &= -\frac{2(d+ex)^{5/2}}{3ce\sqrt{cd^2-ce^2x^2}}+\frac{1}{3}(8d)\int \frac{(d+ex)^{5/2}}{\left(cd^2-ce^2x^2\right)^{3/2}}\,dx\\
&= -\frac{16d(d+ex)^{3/2}}{3ce\sqrt{cd^2-ce^2x^2}}-\frac{2(d+ex)^{5/2}}{3ce\sqrt{cd^2-ce^2x^2}}+\frac{1}{3}\left(32d^2\right)\int \frac{(d+ex)^{3/2}}{\left(cd^2-ce^2x^2\right)^{3/2}}\,dx\\
&= \frac{64d^2\sqrt{d+ex}}{3ce\sqrt{cd^2-ce^2x^2}}-\frac{16d(d+ex)^{3/2}}{3ce\sqrt{cd^2-ce^2x^2}}-\frac{2(d+ex)^{5/2}}{3ce\sqrt{cd^2-ce^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.0589753, size = 55, normalized size = 0.46

$$-\frac{2\sqrt{d+ex}\left(-23d^2+10dex+e^2x^2\right)}{3ce\sqrt{c\left(d^2-e^2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)/(c*d^2 - c*e^2*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(-23*d^2 + 10*d*e*x + e^2*x^2))/(3*c*e*Sqrt[c*(d^2 - e^2*
x^2)])
```

Maple [A] time = 0.042, size = 55, normalized size = 0.5

$$\frac{(-2\,ex+2\,d)\left(-e^2x^2-10\,dxe+23\,d^2\right)}{3\,e}(ex+d)^{\frac{3}{2}}\left(-ce^2x^2+cd^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/(-c*e^2*x^2+c*d^2)^(3/2),x)
```

```
[Out] 2/3*(-e*x+d)*(-e^2*x^2-10*d*e*x+23*d^2)*(e*x+d)^(3/2)/e/(-c*e^2*x^2+c*d^2)^
(3/2)
```

Maxima [A] time = 1.13826, size = 58, normalized size = 0.49

$$-\frac{2\left(\sqrt{c}e^2x^2+10\,\sqrt{c}dex-23\,\sqrt{c}d^2\right)}{3\,\sqrt{-ex+d}c^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/3*(sqrt(c)*e^2*x^2 + 10*sqrt(c)*d*e*x - 23*sqrt(c)*d^2)/(sqrt(-e*x + d)*
c^2*e)
```

Fricas [A] time = 2.11554, size = 135, normalized size = 1.13

$$\frac{2\,\sqrt{-ce^2x^2+cd^2}\left(e^2x^2+10\,dex-23\,d^2\right)\sqrt{ex+d}}{3\left(c^2e^3x^2-c^2d^2e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(-c*e^2*x^2 + c*d^2)*(e^2*x^2 + 10*d*e*x - 23*d^2)*sqrt(e*x + d)/(c
^2*e^3*x^2 - c^2*d^2*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^{\frac{7}{2}}}{(-c(-d+ex)(d+ex))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/(-c*e**2*x**2+c*d**2)**(3/2),x)
```

```
[Out] Integral((d + e*x)**(7/2)/(-c*(-d + e*x)*(d + e*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.888 $\int \frac{(d+ex)^{5/2}}{\left(cd^2-ce^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{8d\sqrt{d+ex}}{ce\sqrt{cd^2-ce^2x^2}}-\frac{2(d+ex)^{3/2}}{ce\sqrt{cd^2-ce^2x^2}}$$

```
[Out] (8*d*Sqrt[d + e*x])/(c*e*Sqrt[c*d^2 - c*e^2*x^2]) - (2*(d + e*x)^(3/2))/(c*
e*Sqrt[c*d^2 - c*e^2*x^2])
```

Rubi [A] time = 0.0288803, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {657, 649}

$$\frac{8d\sqrt{d+ex}}{ce\sqrt{cd^2-ce^2x^2}}-\frac{2(d+ex)^{3/2}}{ce\sqrt{cd^2-ce^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(c*d^2 - c*e^2*x^2)^(3/2),x]
```

```
[Out] (8*d*Sqrt[d + e*x])/(c*e*Sqrt[c*d^2 - c*e^2*x^2]) - (2*(d + e*x)^(3/2))/(c*
e*Sqrt[c*d^2 - c*e^2*x^2])
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^{5/2}}{\left(cd^2-ce^2x^2\right)^{3/2}}\,dx &= -\frac{2(d+ex)^{3/2}}{ce\sqrt{cd^2-ce^2x^2}}+(4d)\int \frac{(d+ex)^{3/2}}{\left(cd^2-ce^2x^2\right)^{3/2}}\,dx\\ &= \frac{8d\sqrt{d+ex}}{ce\sqrt{cd^2-ce^2x^2}}-\frac{2(d+ex)^{3/2}}{ce\sqrt{cd^2-ce^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0556384, size = 43, normalized size = 0.58

$$\frac{2(3d-ex)\sqrt{d+ex}}{ce\sqrt{c\left(d^2-e^2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(c*d^2 - c*e^2*x^2)^(3/2),x]
```

```
[Out] (2*(3*d - e*x)*Sqrt[d + e*x])/(c*e*Sqrt[c*(d^2 - e^2*x^2)])
```

Maple [A] time = 0.039, size = 44, normalized size = 0.6

$$2\,\frac{(-ex+d)\,(-ex+3\,d)\,(ex+d)^{3/2}}{e\left(-ce^{2}x^{2}+cd^{2}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(3/2),x)
```

```
[Out] 2*(-e*x+d)*(-e*x+3*d)*(e*x+d)^(3/2)/e/(-c*e^2*x^2+c*d^2)^(3/2)
```

Maxima [A] time = 1.14239, size = 31, normalized size = 0.42

$$-\frac{2\,(ex-3\,d)}{\sqrt{-ex+d}c^{\frac{3}{2}}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*(e*x - 3*d)/(sqrt(-e*x + d)*c^(3/2)*e)
```

Fricas [A] time = 2.04419, size = 108, normalized size = 1.46

$$\frac{2\,\sqrt{-ce^{2}x^{2}+cd^{2}}\sqrt{ex+d}(ex-3\,d)}{c^{2}e^{3}x^{2}-c^{2}d^{2}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)*(e*x - 3*d)/(c^2*e^3*x^2 - c^2*d^2
*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(d+ex)^{\frac{5}{2}}}{(-c\,(-d+ex)\,(d+ex))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(-c*e**2*x**2+c*d**2)**(3/2),x)
```

```
[Out] Integral((d + e*x)**(5/2)/(-c*(-d + e*x)*(d + e*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.889 $\int \frac{(d+ex)^{3/2}}{\left(cd^2-ce^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=36

$$\frac{2\sqrt{d+ex}}{ce\sqrt{cd^2-ce^2x^2}}$$

```
[Out] (2*Sqrt[d + e*x])/(c*e*Sqrt[c*d^2 - c*e^2*x^2])
```

Rubi [A] time = 0.0130347, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {649}

$$\frac{2\sqrt{d+ex}}{ce\sqrt{cd^2-ce^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(c*d^2 - c*e^2*x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[d + e*x])/(c*e*Sqrt[c*d^2 - c*e^2*x^2])
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\left(cd^2-ce^2x^2\right)^{3/2}}\,dx = \frac{2\sqrt{d+ex}}{ce\sqrt{cd^2-ce^2x^2}}$$

Mathematica [A] time = 0.0451778, size = 35, normalized size = 0.97

$$\frac{2\sqrt{d+ex}}{ce\sqrt{c\left(d^2-e^2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(c*d^2 - c*e^2*x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[d + e*x])/(c*e*Sqrt[c*(d^2 - e^2*x^2)])
```

Maple [A] time = 0.04, size = 36, normalized size = 1.

$$2\,\frac{(-ex+d)\,(ex+d)^{3/2}}{e\left(-ce^2x^2+cd^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(3/2),x)
```

```
[Out] 2*(-e*x+d)*(e*x+d)^(3/2)/e/(-c*e^2*x^2+c*d^2)^(3/2)
```

Maxima [A] time = 1.1656, size = 22, normalized size = 0.61

$$\frac{2}{\sqrt{-ex+d}c^{\frac{3}{2}}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/(sqrt(-e*x + d)*c^(3/2)*e)
```

Fricas [A] time = 2.22249, size = 93, normalized size = 2.58

$$-\frac{2\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}}{c^2e^3x^2-c^2d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)/(c^2*e^3*x^2 - c^2*d^2*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^{\frac{3}{2}}}{(-c(-d+ex)(d+ex))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(-c*e**2*x**2+c*d**2)**(3/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)/(-c*(-d + e*x)*(d + e*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.890 $\int \frac{\sqrt{d+ex}}{\left(cd^2-ce^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=104

$$\frac{\sqrt{d+ex}}{cde\sqrt{cd^2-ce^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{2}c^{3/2}d^{3/2}e}$$

```
[Out] Sqrt[d + e*x]/(c*d*e*Sqrt[c*d^2 - c*e^2*x^2]) - ArcTanh[Sqrt[c*d^2 - c*e^2*
x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])]/(Sqrt[2]*c^(3/2)*d^(3/2)*e)
```

Rubi [A] time = 0.0506586, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {667, 661, 208}

$$\frac{\sqrt{d+ex}}{cde\sqrt{cd^2-ce^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{2}c^{3/2}d^{3/2}e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(c*d^2 - c*e^2*x^2)^(3/2),x]
```

```
[Out] Sqrt[d + e*x]/(c*d*e*Sqrt[c*d^2 - c*e^2*x^2]) - ArcTanh[Sqrt[c*d^2 - c*e^2*
x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])]/(Sqrt[2]*c^(3/2)*d^(3/2)*e)
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(d*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(p + 1)), x] + Dist[(d*(m + 2*p
 + 2))/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && LtQ[0, m,
1] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\left(cd^2-ce^2x^2\right)^{3/2}}\,dx = \frac{\sqrt{d+ex}}{cde\sqrt{cd^2-ce^2x^2}} + \frac{\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\,dx}{2cd}$$

$$= \frac{\sqrt{d+ex}}{cde\sqrt{cd^2-ce^2x^2}} + \frac{e\,\text{Subst}\left(\int \frac{1}{-2cde^2+e^2x^2}\,dx, x, \frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}\right)}{cd}$$

$$= \frac{\sqrt{d+ex}}{cde\sqrt{cd^2-ce^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{2}c^{3/2}d^{3/2}e}$$

Mathematica [A] time = 0.0535869, size = 110, normalized size = 1.06

$$\frac{2\sqrt{d}\sqrt{d+ex} - \sqrt{2}\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{\sqrt{2}\sqrt{d}\sqrt{d+ex}}\right)}{2cd^{3/2}e\sqrt{c\left(d^2-e^2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(c*d^2 - c*e^2*x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[d]*Sqrt[d + e*x] - Sqrt[2]*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^2 - e
^2*x^2]/(Sqrt[2]*Sqrt[d]*Sqrt[d + e*x])])/(2*c*d^(3/2)*e*Sqrt[c*(d^2 - e^2*
x^2)])
```

Maple [A] time = 0.169, size = 98, normalized size = 0.9

$$\frac{1}{2\,c^2\,(ex-d)\,ed}\sqrt{-c\left(e^2x^2-d^2\right)}\left(\sqrt{2}\text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{-(ex-d)\,c}\frac{1}{\sqrt{cd}}\right)\sqrt{-(ex-d)\,c}-2\sqrt{cd}\right)\frac{1}{\sqrt{ex+d}}\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(3/2),x)
```

```
[Out] 1/2/(e*x+d)^(1/2)*(-c*(e^2*x^2-d^2))^(1/2)*(2^(1/2)*arctanh(1/2*(-(e*x-d)*c
)^(1/2)*2^(1/2)/(c*d)^(1/2))*(-(e*x-d)*c)^(1/2)-2*(c*d)^(1/2))/c^2/(e*x-d)/
e/d/(c*d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\left(-ce^2x^2+cd^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(-c*e^2*x^2 + c*d^2)^(3/2), x)
```

Fricas [A] time = 2.2346, size = 609, normalized size = 5.86

$$\left[\frac{\sqrt{2}\left(e^2x^2-d^2\right)\sqrt{cd}\log\left(-\frac{ce^2x^2-2\,cdex-3\,cd^2+2\,\sqrt{2}\sqrt{-ce^2x^2+cd^2}\sqrt{cd}\sqrt{ex+d}}{e^2x^2+2\,dex+d^2}\right)-4\,\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}d}{4\left(c^2d^2e^3x^2-c^2d^4e\right)},-\frac{\sqrt{2}\left(e^2x^2-d^2\right)\sqrt{-cd}\,\mathrm{arc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*(e^2*x^2 - d^2)*sqrt(c*d)*log(-(c*e^2*x^2 - 2*c*d*e*x - 3*c*d
^2 + 2*sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(c*d)*sqrt(e*x + d))/(e^2*x^2 +
 2*d*e*x + d^2)) - 4*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)*d)/(c^2*d^2*e^3
*x^2 - c^2*d^4*e), -1/2*(sqrt(2)*(e^2*x^2 - d^2)*sqrt(-c*d)*arctan(sqrt(2)*
sqrt(-c*e^2*x^2 + c*d^2)*sqrt(-c*d)*sqrt(e*x + d)/(c*e^2*x^2 - c*d^2)) + 2*
sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)*d)/(c^2*d^2*e^3*x^2 - c^2*d^4*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{d+ex}}{\left(-c\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(-c*e**2*x**2+c*d**2)**(3/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/(-c*(-d + e*x)*(d + e*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.891 $\int \frac{1}{\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=150

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{4\sqrt{2}c^{3/2}d^{5/2}e}+\frac{3\sqrt{d+ex}}{4cd^2e\sqrt{cd^2-ce^2x^2}}-\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}$$

```
[Out] -1/(2*c*d*e*Sqrt[d + e*x]*Sqrt[c*d^2 - c*e^2*x^2]) + (3*Sqrt[d + e*x])/(4*c
*d^2*e*Sqrt[c*d^2 - c*e^2*x^2]) - (3*ArcTanh[Sqrt[c*d^2 - c*e^2*x^2]/(Sqrt[
2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/(4*Sqrt[2]*c^(3/2)*d^(5/2)*e)
```

Rubi [A] time = 0.0727255, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {673, 667, 661, 208}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{4\sqrt{2}c^{3/2}d^{5/2}e}+\frac{3\sqrt{d+ex}}{4cd^2e\sqrt{cd^2-ce^2x^2}}-\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(c*d^2 - c*e^2*x^2)^(3/2)),x]
```


```
[Out] -1/(2*c*d*e*Sqrt[d + e*x]*Sqrt[c*d^2 - c*e^2*x^2]) + (3*Sqrt[d + e*x])/(4*c
*d^2*e*Sqrt[c*d^2 - c*e^2*x^2]) - (3*ArcTanh[Sqrt[c*d^2 - c*e^2*x^2]/(Sqrt[
2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/(4*Sqrt[2]*c^(3/2)*d^(5/2)*e)
```


Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[(m + 2*
p + 2)/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; Fr
eeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p
+ 1, 0] && IntegerQ[2*p]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(d*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(p + 1)), x] + Dist[(d*(m + 2*p
 + 2))/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && LtQ[0, m,
1] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}\,dx = -\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}+\frac{3\int \frac{\sqrt{d+ex}}{\left(cd^2-ce^2x^2\right)^{3/2}}\,dx}{4d}$$
$$= -\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}+\frac{3\sqrt{d+ex}}{4cd^2e\sqrt{cd^2-ce^2x^2}}+\frac{3\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\,dx}{8cd^2}$$
$$= -\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}+\frac{3\sqrt{d+ex}}{4cd^2e\sqrt{cd^2-ce^2x^2}}+\frac{(3e)\,\mathrm{Subst}\left(\int \frac{1}{-2cde^2+e^2x^2}\,dx, x, \frac{\sqrt{cd^2}}{\sqrt{c}}\right)}{4cd^2}$$
$$= -\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}+\frac{3\sqrt{d+ex}}{4cd^2e\sqrt{cd^2-ce^2x^2}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{4\sqrt{2}c^{3/2}d^{5/2}e}$$

Mathematica [A] time = 0.0873345, size = 128, normalized size = 0.85

$$\frac{2\sqrt{d}\sqrt{d+ex}(d+3ex)-3\sqrt{2}(d+ex)\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{\sqrt{2}\sqrt{d}\sqrt{d+ex}}\right)}{8cd^{5/2}e(d+ex)\sqrt{c\left(d^2-e^2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(c*d^2 - c*e^2*x^2)^(3/2)),x]
```

```
[Out] (2*Sqrt[d]*Sqrt[d + e*x]*(d + 3*e*x) - 3*Sqrt[2]*(d + e*x)*Sqrt[d^2 - e^2*x
^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/(Sqrt[2]*Sqrt[d]*Sqrt[d + e*x])])/(8*c*d^(5
/2)*e*(d + e*x)*Sqrt[c*(d^2 - e^2*x^2)])
```

Maple [A] time = 0.194, size = 152, normalized size = 1.

$$\frac{1}{8c^2(ex-d)ed^2}\sqrt{-c\left(e^2x^2-d^2\right)}\left(3\sqrt{-(ex-d)c}\sqrt{2}\mathrm{Artanh}\left(1/2\frac{\sqrt{-(ex-d)c}\sqrt{2}}{\sqrt{cd}}\right)xe+3\sqrt{-(ex-d)c}\sqrt{2}\mathrm{Artanh}\left(1/2\frac{\sqrt{}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(3/2),x)
```

```
[Out] 1/8/(e*x+d)^(3/2)*(-c*(e^2*x^2-d^2))^(1/2)/c^2*(3*(-(e*x-d)*c)^(1/2)*2^(1/2
)*arctanh(1/2*(-(e*x-d)*c)^(1/2)*2^(1/2)/(c*d)^(1/2))*x*e+3*(-(e*x-d)*c)^(1
/2)*2^(1/2)*arctanh(1/2*(-(e*x-d)*c)^(1/2)*2^(1/2)/(c*d)^(1/2))*d-6*(c*d)^(
1/2)*x*e-2*(c*d)^(1/2)*d)/(e*x-d)/e/d^2/(c*d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-ce^2x^2+cd^2\right)^{\frac{3}{2}}\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-c*e^2*x^2 + c*d^2)^(3/2)*sqrt(e*x + d)), x)
```

Fricas [A] time = 2.2437, size = 805, normalized size = 5.37

$$\left[\frac{3\,\sqrt{2}\left(e^3x^3+de^2x^2-d^2ex-d^3\right)\sqrt{cd}\log\left(-\frac{ce^2x^2-2\,cdex-3\,cd^2+2\,\sqrt{2}\sqrt{-ce^2x^2+cd^2}\sqrt{cd}\sqrt{ex+d}}{e^2x^2+2\,dex+d^2}\right)-4\,\sqrt{-ce^2x^2+cd^2}\left(3\,dex+d^2\right)\sqrt{ex}}{16\left(c^2d^3e^4x^3+c^2d^4e^3x^2-c^2d^5e^2x-c^2d^6e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(2)*(e^3*x^3 + d*e^2*x^2 - d^2*e*x - d^3)*sqrt(c*d)*log(-(c*e^
2*x^2 - 2*c*d*e*x - 3*c*d^2 + 2*sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(c*d)*
sqrt(e*x + d))/(e^2*x^2 + 2*d*e*x + d^2)) - 4*sqrt(-c*e^2*x^2 + c*d^2)*(3*d
*e*x + d^2)*sqrt(e*x + d))/(c^2*d^3*e^4*x^3 + c^2*d^4*e^3*x^2 - c^2*d^5*e^2
*x - c^2*d^6*e), -1/8*(3*sqrt(2)*(e^3*x^3 + d*e^2*x^2 - d^2*e*x - d^3)*sqrt
(-c*d)*arctan(sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(-c*d)*sqrt(e*x + d)/(c*
e^2*x^2 - c*d^2)) + 2*sqrt(-c*e^2*x^2 + c*d^2)*(3*d*e*x + d^2)*sqrt(e*x + d
))/(c^2*d^3*e^4*x^3 + c^2*d^4*e^3*x^2 - c^2*d^5*e^2*x - c^2*d^6*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-c\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(-c*e**2*x**2+c*d**2)**(3/2),x)
```

```
[Out] Integral(1/((-c*(-d + e*x)*(d + e*x))**(3/2)*sqrt(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.892 $\int \frac{1}{(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=191

$$-\frac{15\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{32\sqrt{2}c^{3/2}d^{7/2}e}+\frac{15\sqrt{d+ex}}{32cd^3e\sqrt{cd^2-ce^2x^2}}-\frac{5}{16cd^2e\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}-\frac{1}{4cde(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}$$

```
[Out] -1/(4*c*d*e*(d + e*x)^(3/2)*Sqrt[c*d^2 - c*e^2*x^2]) - 5/(16*c*d^2*e*Sqrt[d
 + e*x]*Sqrt[c*d^2 - c*e^2*x^2]) + (15*Sqrt[d + e*x])/(32*c*d^3*e*Sqrt[c*d^
2 - c*e^2*x^2]) - (15*ArcTanh[Sqrt[c*d^2 - c*e^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt
[d]*Sqrt[d + e*x])])/(32*Sqrt[2]*c^(3/2)*d^(7/2)*e)
```

Rubi [A] time = 0.106512, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {673, 667, 661, 208}

$$-\frac{15\tanh^{-1}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{32\sqrt{2}c^{3/2}d^{7/2}e}+\frac{15\sqrt{d+ex}}{32cd^3e\sqrt{cd^2-ce^2x^2}}-\frac{5}{16cd^2e\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}-\frac{1}{4cde(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*(c*d^2 - c*e^2*x^2)^(3/2)),x]
```

```
[Out] -1/(4*c*d*e*(d + e*x)^(3/2)*Sqrt[c*d^2 - c*e^2*x^2]) - 5/(16*c*d^2*e*Sqrt[d
 + e*x]*Sqrt[c*d^2 - c*e^2*x^2]) + (15*Sqrt[d + e*x])/(32*c*d^3*e*Sqrt[c*d^
2 - c*e^2*x^2]) - (15*ArcTanh[Sqrt[c*d^2 - c*e^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt
[d]*Sqrt[d + e*x])])/(32*Sqrt[2]*c^(3/2)*d^(7/2)*e)
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[(m + 2*
p + 2)/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; Fr
eeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p
+ 1, 0] && IntegerQ[2*p]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(d*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(p + 1)), x] + Dist[(d*(m + 2*p
 + 2))/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && LtQ[0, m,
1] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{3/2}}\,dx = -\frac{1}{4cde(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}} + \frac{5\int \frac{1}{\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}\,dx}{8d}$$

$$= -\frac{1}{4cde(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}} - \frac{5}{16cd^2e\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}} + \frac{15\int \frac{\sqrt{d+ex}}{\left(cd^2-ce^2x^2\right)^{3/2}}\,d}{32d^2}$$

$$= -\frac{1}{4cde(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}} - \frac{5}{16cd^2e\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}} + \frac{15\sqrt{d+ex}}{32cd^3e\sqrt{cd^2-ce^2}}$$

$$= -\frac{1}{4cde(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}} - \frac{5}{16cd^2e\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}} + \frac{15\sqrt{d+ex}}{32cd^3e\sqrt{cd^2-ce^2}}$$

$$= -\frac{1}{4cde(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}} - \frac{5}{16cd^2e\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}} + \frac{15\sqrt{d+ex}}{32cd^3e\sqrt{cd^2-ce^2}}$$

Mathematica [A] time = 0.116679, size = 143, normalized size = 0.75

$$\frac{2\sqrt{d}\sqrt{d+ex}\left(-3d^2+20dex+15e^2x^2\right)-15\sqrt{2}(d+ex)^2\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{\sqrt{2}\sqrt{d}\sqrt{d+ex}}\right)}{64cd^{7/2}e(d+ex)^2\sqrt{c\left(d^2-e^2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*(c*d^2 - c*e^2*x^2)^(3/2)),x]
```

```
[Out] (2*Sqrt[d]*Sqrt[d + e*x]*(-3*d^2 + 20*d*e*x + 15*e^2*x^2) - 15*Sqrt[2]*(d +
 e*x)^2*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/(Sqrt[2]*Sqrt[d]*Sq
rt[d + e*x]])])/(64*c*d^(7/2)*e*(d + e*x)^2*Sqrt[c*(d^2 - e^2*x^2)])
```

Maple [A] time = 0.175, size = 217, normalized size = 1.1

$$\frac{1}{64\,c^2\,(ex-d)\,ed^3}\sqrt{-c\left(e^2x^2-d^2\right)}\left(15\sqrt{-(ex-d)\,c}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{-(ex-d)\,c}\sqrt{2}}{\sqrt{cd}}\right)x^2e^2+30\sqrt{-(ex-d)\,c}\sqrt{2}\mathrm{Artan}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(3/2),x)
```

```
[Out] 1/64/(e*x+d)^(5/2)*(-c*(e^2*x^2-d^2))^(1/2)/c^2*(15*(-(e*x-d)*c)^(1/2)*2^(1
/2)*arctanh(1/2*(-(e*x-d)*c)^(1/2)*2^(1/2)/(c*d)^(1/2))*x^2*e^2+30*(-(e*x-d
)*c)^(1/2)*2^(1/2)*arctanh(1/2*(-(e*x-d)*c)^(1/2)*2^(1/2)/(c*d)^(1/2))*x*d*
e+15*(-(e*x-d)*c)^(1/2)*2^(1/2)*arctanh(1/2*(-(e*x-d)*c)^(1/2)*2^(1/2)/(c*d
)^(1/2))*d^2-30*(c*d)^(1/2)*x^2*e^2-40*(c*d)^(1/2)*x*d*e+6*(c*d)^(1/2)*d^2)
/(e*x-d)/e/d^3/(c*d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-ce^2x^2 + cd^2\right)^{\frac{3}{2}}(ex + d)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-c*e^2*x^2 + c*d^2)^(3/2)*(e*x + d)^(3/2)), x)
```

Fricas [A] time = 2.26468, size = 886, normalized size = 4.64

$$\left[\frac{15\,\sqrt{2}\left(e^4x^4 + 2\,de^3x^3 - 2\,d^3ex - d^4\right)\sqrt{cd}\log\left(-\frac{ce^2x^2 - 2\,cdex - 3\,cd^2 + 2\,\sqrt{2}\sqrt{-ce^2x^2 + cd^2}\sqrt{cd}\sqrt{ex + d}}{e^2x^2 + 2\,dex + d^2}\right) - 4\,\sqrt{-ce^2x^2 + cd^2}\left(15\,de^2x^2 + 20\right.}{128\left(c^2d^4e^5x^4 + 2\,c^2d^5e^4x^3 - 2\,c^2d^7e^2x - c^2d^8e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/128*(15*sqrt(2)*(e^4*x^4 + 2*d*e^3*x^3 - 2*d^3*e*x - d^4)*sqrt(c*d)*log(
-(c*e^2*x^2 - 2*c*d*e*x - 3*c*d^2 + 2*sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt
(c*d)*sqrt(e*x + d))/(e^2*x^2 + 2*d*e*x + d^2)) - 4*sqrt(-c*e^2*x^2 + c*d^2
)*(15*d*e^2*x^2 + 20*d^2*e*x - 3*d^3)*sqrt(e*x + d))/(c^2*d^4*e^5*x^4 + 2*c
^2*d^5*e^4*x^3 - 2*c^2*d^7*e^2*x - c^2*d^8*e), -1/64*(15*sqrt(2)*(e^4*x^4 +
 2*d*e^3*x^3 - 2*d^3*e*x - d^4)*sqrt(-c*d)*arctan(sqrt(2)*sqrt(-c*e^2*x^2 +
 c*d^2)*sqrt(-c*d)*sqrt(e*x + d)/(c*e^2*x^2 - c*d^2)) + 2*sqrt(-c*e^2*x^2 +
 c*d^2)*(15*d*e^2*x^2 + 20*d^2*e*x - 3*d^3)*sqrt(e*x + d))/(c^2*d^4*e^5*x^4
 + 2*c^2*d^5*e^4*x^3 - 2*c^2*d^7*e^2*x - c^2*d^8*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-c\left(-d + ex\right)\left(d + ex\right)\right)^{\frac{3}{2}}\left(d + ex\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(-c*e**2*x**2+c*d**2)**(3/2),x)
```

```
[Out] Integral(1/((-c*(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, 2]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, undef, undef, 2]
```

3.893 $\int \frac{1}{\sqrt{-1+x}\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=31

$$\sqrt{2}\tan^{-1}\left(\frac{\sqrt{1-x^2}}{\sqrt{2}\sqrt{x-1}}\right)$$

```
[Out] Sqrt[2]*ArcTan[Sqrt[1 - x^2]/(Sqrt[2]*Sqrt[-1 + x])]
```

Rubi [A] time = 0.0103923, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {661, 203}

$$\sqrt{2}\tan^{-1}\left(\frac{\sqrt{1-x^2}}{\sqrt{2}\sqrt{x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x]*Sqrt[1 - x^2]),x]
```

```
[Out] Sqrt[2]*ArcTan[Sqrt[1 - x^2]/(Sqrt[2]*Sqrt[-1 + x])]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+x}\sqrt{1-x^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \frac{\sqrt{1-x^2}}{\sqrt{-1+x}}\right)\\ &= \sqrt{2}\tan^{-1}\left(\frac{\sqrt{1-x^2}}{\sqrt{2}\sqrt{-1+x}}\right)\end{aligned}$$

Mathematica [A] time = 0.027545, size = 46, normalized size = 1.48

$$-\frac{\sqrt{2}\sqrt{x-1}\sqrt{x+1}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + x]*Sqrt[1 - x^2]),x]
```

```
[Out] -((Sqrt[2]*Sqrt[-1 + x]*Sqrt[1 + x]*ArcTanh[Sqrt[1 + x]/Sqrt[2]])/Sqrt[1 -
x^2])
```

Maple [A] time = 0.085, size = 39, normalized size = 1.3

$$\sqrt{2}\sqrt{-x^2+1}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-1-x}\right)\frac{1}{\sqrt{-1+x}}\frac{1}{\sqrt{-1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+x)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] 1/(-1+x)^(1/2)*(-x^2+1)^(1/2)/(-1-x)^(1/2)*2^(1/2)*arctan(1/2*(-1-x)^(1/2)*
2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+1}\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(x - 1)), x)
```

Fricas [A] time = 2.1309, size = 84, normalized size = 2.71

$$\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+1}\sqrt{x-1}}{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*arctan(sqrt(2)*sqrt(-x^2 + 1)*sqrt(x - 1)/(x^2 - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x-1)(x+1)}\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x - 1)*(x + 1))*sqrt(x - 1)), x)
```

Giac [C] time = 1.24628, size = 49, normalized size = 1.58

$$\frac{1}{2}i\left(\sqrt{2}\log\left(\sqrt{2}+\sqrt{x+1}\right)-\sqrt{2}\log\left(-\sqrt{2}+\sqrt{x+1}\right)\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*I*(sqrt(2)*log(sqrt(2) + sqrt(x + 1)) - sqrt(2)*log(-sqrt(2) + sqrt(x +
 1)))*sgn(x)
```

3.894 $\int (2+ex)^{5/2}\sqrt{12-3e^2x^2}\,dx$

Optimal. Leaf size=87

$$\frac{2(2-ex)^{9/2}}{3\sqrt{3}e}-\frac{24\sqrt{3}(2-ex)^{7/2}}{7e}+\frac{96\sqrt{3}(2-ex)^{5/2}}{5e}-\frac{128(2-ex)^{3/2}}{\sqrt{3}e}$$

```
[Out] (-128*(2 - e*x)^(3/2))/(Sqrt[3]*e) + (96*Sqrt[3]*(2 - e*x)^(5/2))/(5*e) - (
24*Sqrt[3]*(2 - e*x)^(7/2))/(7*e) + (2*(2 - e*x)^(9/2))/(3*Sqrt[3]*e)
```

Rubi [A] time = 0.0267335, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {627, 43}

$$\frac{2(2-ex)^{9/2}}{3\sqrt{3}e}-\frac{24\sqrt{3}(2-ex)^{7/2}}{7e}+\frac{96\sqrt{3}(2-ex)^{5/2}}{5e}-\frac{128(2-ex)^{3/2}}{\sqrt{3}e}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + e*x)^(5/2)*Sqrt[12 - 3*e^2*x^2],x]
```

```
[Out] (-128*(2 - e*x)^(3/2))/(Sqrt[3]*e) + (96*Sqrt[3]*(2 - e*x)^(5/2))/(5*e) - (
24*Sqrt[3]*(2 - e*x)^(7/2))/(7*e) + (2*(2 - e*x)^(9/2))/(3*Sqrt[3]*e)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (2+ex)^{5/2}\sqrt{12-3e^2x^2}\,dx &= \int \sqrt{6-3ex}(2+ex)^3\,dx\\ &= \int \left(64\sqrt{6-3ex}-16(6-3ex)^{3/2}+\frac{4}{3}(6-3ex)^{5/2}-\frac{1}{27}(6-3ex)^{7/2}\right)dx\\ &= -\frac{128(2-ex)^{3/2}}{\sqrt{3}e}+\frac{96\sqrt{3}(2-ex)^{5/2}}{5e}-\frac{24\sqrt{3}(2-ex)^{7/2}}{7e}+\frac{2(2-ex)^{9/2}}{3\sqrt{3}e}\end{aligned}$$

Mathematica [A] time = 0.0752209, size = 58, normalized size = 0.67

$$\frac{2(ex-2)\sqrt{4-e^2x^2}\left(35e^3x^3+330e^2x^2+1284ex+2552\right)}{105e\sqrt{3ex+6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + e*x)^(5/2)*Sqrt[12 - 3*e^2*x^2],x]
```

```
[Out] (2*(-2 + e*x)*Sqrt[4 - e^2*x^2]*(2552 + 1284*e*x + 330*e^2*x^2 + 35*e^3*x^3
))/(105*e*Sqrt[6 + 3*e*x])
```

Maple [A] time = 0.043, size = 52, normalized size = 0.6

$$\frac{(2\,ex-4)\left(35\,e^{3}x^{3}+330\,e^{2}x^{2}+1284\,ex+2552\right)}{315\,e}\sqrt{-3\,e^{2}x^{2}+12}\frac{1}{\sqrt{ex+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+2)^(5/2)*(-3*e^2*x^2+12)^(1/2),x)
```

```
[Out] 2/315*(e*x-2)*(35*e^3*x^3+330*e^2*x^2+1284*e*x+2552)*(-3*e^2*x^2+12)^(1/2)/
e/(e*x+2)^(1/2)
```

Maxima [C] time = 1.71542, size = 96, normalized size = 1.1

$$\frac{\left(70i\,\sqrt{3}e^{4}x^{4}+520i\,\sqrt{3}e^{3}x^{3}+1248i\,\sqrt{3}e^{2}x^{2}-32i\,\sqrt{3}ex-10208i\,\sqrt{3}\right)(ex+2)\sqrt{ex-2}}{315\left(e^{2}x+2\,e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(5/2)*(-3*e^2*x^2+12)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/315*(70*I*sqrt(3)*e^4*x^4 + 520*I*sqrt(3)*e^3*x^3 + 1248*I*sqrt(3)*e^2*x^
2 - 32*I*sqrt(3)*e*x - 10208*I*sqrt(3))*(e*x + 2)*sqrt(e*x - 2)/(e^2*x + 2*
e)
```

Fricas [A] time = 2.11747, size = 154, normalized size = 1.77

$$\frac{2\left(35\,e^{4}x^{4}+260\,e^{3}x^{3}+624\,e^{2}x^{2}-16\,ex-5104\right)\sqrt{-3\,e^{2}x^{2}+12}\sqrt{ex+2}}{315\left(e^{2}x+2\,e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(5/2)*(-3*e^2*x^2+12)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*e^4*x^4 + 260*e^3*x^3 + 624*e^2*x^2 - 16*e*x - 5104)*sqrt(-3*e^2*
x^2 + 12)*sqrt(e*x + 2)/(e^2*x + 2*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)**(5/2)*(-3*e**2*x**2+12)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-3\,e^2x^2+12}(ex+2)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(5/2)*(-3*e^2*x^2+12)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-3*e^2*x^2 + 12)*(e*x + 2)^(5/2), x)
```

3.895 $\int (2+ex)^{3/2}\sqrt{12-3e^2x^2}\,dx$

Optimal. Leaf size=65

$$-\frac{2\sqrt{3}(2-ex)^{7/2}}{7e}+\frac{16\sqrt{3}(2-ex)^{5/2}}{5e}-\frac{32(2-ex)^{3/2}}{\sqrt{3}e}$$

```
[Out] (-32*(2 - e*x)^(3/2))/(Sqrt[3]*e) + (16*Sqrt[3]*(2 - e*x)^(5/2))/(5*e) - (2
*Sqrt[3]*(2 - e*x)^(7/2))/(7*e)
```

Rubi [A] time = 0.0202403, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {627, 43}

$$-\frac{2\sqrt{3}(2-ex)^{7/2}}{7e}+\frac{16\sqrt{3}(2-ex)^{5/2}}{5e}-\frac{32(2-ex)^{3/2}}{\sqrt{3}e}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + e*x)^(3/2)*Sqrt[12 - 3*e^2*x^2],x]
```

```
[Out] (-32*(2 - e*x)^(3/2))/(Sqrt[3]*e) + (16*Sqrt[3]*(2 - e*x)^(5/2))/(5*e) - (2
*Sqrt[3]*(2 - e*x)^(7/2))/(7*e)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (2+ex)^{3/2}\sqrt{12-3e^2x^2}\,dx &= \int \sqrt{6-3ex}(2+ex)^2\,dx\\
&= \int \left(16\sqrt{6-3ex}-\frac{8}{3}(6-3ex)^{3/2}+\frac{1}{9}(6-3ex)^{5/2}\right)dx\\
&= -\frac{32(2-ex)^{3/2}}{\sqrt{3}e}+\frac{16\sqrt{3}(2-ex)^{5/2}}{5e}-\frac{2\sqrt{3}(2-ex)^{7/2}}{7e}
\end{aligned}$$

Mathematica [A] time = 0.0554336, size = 50, normalized size = 0.77

$$\frac{2(ex-2)\sqrt{4-e^2x^2}\left(15e^2x^2+108ex+284\right)}{35e\sqrt{3ex+6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + e*x)^(3/2)*Sqrt[12 - 3*e^2*x^2],x]
```

```
[Out] (2*(-2 + e*x)*Sqrt[4 - e^2*x^2]*(284 + 108*e*x + 15*e^2*x^2))/(35*e*Sqrt[6
+ 3*e*x])
```

Maple [A] time = 0.042, size = 44, normalized size = 0.7

$$\frac{(2\,ex-4)\left(15\,e^2x^2+108\,ex+284\right)}{105\,e}\sqrt{-3\,e^2x^2+12}\frac{1}{\sqrt{ex+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+2)^(3/2)*(-3*e^2*x^2+12)^(1/2),x)
```

```
[Out] 2/105*(e*x-2)*(15*e^2*x^2+108*e*x+284)*(-3*e^2*x^2+12)^(1/2)/(e*x+2)^(1/2)/
e
```

Maxima [C] time = 1.71175, size = 81, normalized size = 1.25

$$\frac{\left(30i\sqrt{3}e^3x^3+156i\sqrt{3}e^2x^2+136i\sqrt{3}ex-1136i\sqrt{3}\right)(ex+2)\sqrt{ex-2}}{105\left(e^2x+2\,e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(3/2)*(-3*e^2*x^2+12)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/105*(30*I*sqrt(3)*e^3*x^3 + 156*I*sqrt(3)*e^2*x^2 + 136*I*sqrt(3)*e*x - 1
136*I*sqrt(3))*(e*x + 2)*sqrt(e*x - 2)/(e^2*x + 2*e)
```

Fricas [A] time = 2.09546, size = 132, normalized size = 2.03

$$\frac{2\left(15\,e^3x^3+78\,e^2x^2+68\,ex-568\right)\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}}{105\left(e^2x+2\,e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(3/2)*(-3*e^2*x^2+12)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*e^3*x^3 + 78*e^2*x^2 + 68*e*x - 568)*sqrt(-3*e^2*x^2 + 12)*sqrt(e
*x + 2)/(e^2*x + 2*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\sqrt{3}\left(\int 2\sqrt{ex+2}\sqrt{-e^2x^2+4}\,dx+\int ex\sqrt{ex+2}\sqrt{-e^2x^2+4}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)**(3/2)*(-3*e**2*x**2+12)**(1/2),x)
```

```
[Out] sqrt(3)*(Integral(2*sqrt(e*x + 2)*sqrt(-e**2*x**2 + 4), x) + Integral(e*x*s
qrt(e*x + 2)*sqrt(-e**2*x**2 + 4), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-3\,e^2x^2+12}(ex+2)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(3/2)*(-3*e^2*x^2+12)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-3*e^2*x^2 + 12)*(e*x + 2)^(3/2), x)
```

3.896 $\int \sqrt{2+ex}\sqrt{12-3e^2x^2}\,dx$

Optimal. Leaf size=43

$$\frac{2\sqrt{3}(2-ex)^{5/2}}{5e}-\frac{8(2-ex)^{3/2}}{\sqrt{3}e}$$

```
[Out] (-8*(2 - e*x)^(3/2))/(Sqrt[3]*e) + (2*Sqrt[3]*(2 - e*x)^(5/2))/(5*e)
```

Rubi [A] time = 0.0153062, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {627, 43}

$$\frac{2\sqrt{3}(2-ex)^{5/2}}{5e}-\frac{8(2-ex)^{3/2}}{\sqrt{3}e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + e*x]*Sqrt[12 - 3*e^2*x^2],x]
```

```
[Out] (-8*(2 - e*x)^(3/2))/(Sqrt[3]*e) + (2*Sqrt[3]*(2 - e*x)^(5/2))/(5*e)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{2+ex}\sqrt{12-3e^2x^2}\,dx &= \int \sqrt{6-3ex}(2+ex)\,dx\\ &= \int \left(4\sqrt{6-3ex}-\frac{1}{3}(6-3ex)^{3/2}\right)dx\\ &= -\frac{8(2-ex)^{3/2}}{\sqrt{3}e}+\frac{2\sqrt{3}(2-ex)^{5/2}}{5e}\end{aligned}$$

Mathematica [A] time = 0.0439205, size = 42, normalized size = 0.98

$$\frac{2(ex-2)(3ex+14)\sqrt{4-e^2x^2}}{5e\sqrt{3ex+6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + e*x]*Sqrt[12 - 3*e^2*x^2],x]
```

```
[Out] (2*(-2 + e*x)*(14 + 3*e*x)*Sqrt[4 - e^2*x^2])/(5*e*Sqrt[6 + 3*e*x])
```

Maple [A] time = 0.039, size = 36, normalized size = 0.8

$$\frac{(2ex-4)(3ex+14)}{15e}\sqrt{-3e^2x^2+12}\frac{1}{\sqrt{ex+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+2)^(1/2)*(-3*e^2*x^2+12)^(1/2),x)
```

```
[Out] 2/15*(e*x-2)*(3*e*x+14)*(-3*e^2*x^2+12)^(1/2)/e/(e*x+2)^(1/2)
```

Maxima [C] time = 2.1625, size = 66, normalized size = 1.53

$$\frac{\left(6i\sqrt{3}e^2x^2+16i\sqrt{3}ex-56i\sqrt{3}\right)(ex+2)\sqrt{ex-2}}{15\left(e^2x+2e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(1/2)*(-3*e^2*x^2+12)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(6*I*sqrt(3)*e^2*x^2 + 16*I*sqrt(3)*e*x - 56*I*sqrt(3))*(e*x + 2)*sqrt
(e*x - 2)/(e^2*x + 2*e)
```

Fricas [A] time = 2.11975, size = 109, normalized size = 2.53

$$\frac{2\left(3e^2x^2+8ex-28\right)\sqrt{-3e^2x^2+12}\sqrt{ex+2}}{15\left(e^2x+2e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(1/2)*(-3*e^2*x^2+12)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*e^2*x^2 + 8*e*x - 28)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2)/(e^2*x +
2*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\sqrt{3}\int\sqrt{ex+2}\sqrt{-e^2x^2+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)**(1/2)*(-3*e**2*x**2+12)**(1/2),x)
```

```
[Out] sqrt(3)*Integral(sqrt(e*x + 2)*sqrt(-e**2*x**2 + 4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-3e^2x^2+12}\sqrt{ex+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(1/2)*(-3*e^2*x^2+12)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2), x)
```

3.897 $\int \frac{\sqrt{12-3e^2x^2}}{\sqrt{2+ex}}\,dx$

Optimal. Leaf size=20

$$-\frac{2(2-ex)^{3/2}}{\sqrt{3}e}$$

```
[Out] (-2*(2 - e*x)^(3/2))/(Sqrt[3]*e)
```

Rubi [A] time = 0.0090526, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {627, 32}

$$-\frac{2(2-ex)^{3/2}}{\sqrt{3}e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[12 - 3*e^2*x^2]/Sqrt[2 + e*x],x]
```

```
[Out] (-2*(2 - e*x)^(3/2))/(Sqrt[3]*e)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{12-3e^2x^2}}{\sqrt{2+ex}}\,dx &= \int \sqrt{6-3ex}\,dx\\ &= -\frac{2(2-ex)^{3/2}}{\sqrt{3}e}\end{aligned}$$

Mathematica [A] time = 0.0380533, size = 34, normalized size = 1.7

$$\frac{2(ex-2)\sqrt{4-e^2x^2}}{e\sqrt{3ex+6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[12 - 3*e^2*x^2]/Sqrt[2 + e*x],x]
```

```
[Out] (2*(-2 + e*x)*Sqrt[4 - e^2*x^2])/(e*Sqrt[6 + 3*e*x])
```

Maple [A] time = 0.04, size = 30, normalized size = 1.5

$$\frac{2ex-4}{3e}\sqrt{-3e^2x^2+12}\frac{1}{\sqrt{ex+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(1/2),x)
```

```
[Out] 2/3*(e*x-2)*(-3*e^2*x^2+12)^(1/2)/e/(e*x+2)^(1/2)
```

Maxima [C] time = 1.87714, size = 34, normalized size = 1.7

$$\frac{\left(2i\sqrt{3}ex-4i\sqrt{3}\right)\sqrt{ex-2}}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*I*sqrt(3)*e*x - 4*I*sqrt(3))*sqrt(e*x - 2)/e
```

Fricas [B] time = 2.04526, size = 88, normalized size = 4.4

$$\frac{2\sqrt{-3e^2x^2+12}\sqrt{ex+2}(ex-2)}{3\left(e^2x+2e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2)*(e*x - 2)/(e^2*x + 2*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\sqrt{3}\int\frac{\sqrt{-e^2x^2+4}}{\sqrt{ex+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e**2*x**2+12)**(1/2)/(e*x+2)**(1/2),x)
```

```
[Out] sqrt(3)*Integral(sqrt(-e**2*x**2 + 4)/sqrt(e*x + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-3e^2x^2+12}}{\sqrt{ex+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-3*e^2*x^2 + 12)/sqrt(e*x + 2), x)
```

3.898 $\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{3/2}}\,dx$

Optimal. Leaf size=46

$$\frac{2\sqrt{3}\sqrt{2-ex}}{e}-\frac{4\sqrt{3}\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{e}$$

```
[Out] (2*Sqrt[3]*Sqrt[2 - e*x])/e - (4*Sqrt[3]*ArcTanh[Sqrt[2 - e*x]/2])/e
```

Rubi [A] time = 0.0202946, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {627, 50, 63, 206}

$$\frac{2\sqrt{3}\sqrt{2-ex}}{e}-\frac{4\sqrt{3}\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[12 - 3*e^2*x^2]/(2 + e*x)^(3/2),x]
```

```
[Out] (2*Sqrt[3]*Sqrt[2 - e*x])/e - (4*Sqrt[3]*ArcTanh[Sqrt[2 - e*x]/2])/e
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{3/2}}\,dx = \int \frac{\sqrt{6-3ex}}{2+ex}\,dx$$
$$= \frac{2\sqrt{3}\sqrt{2-ex}}{e} + 12\int \frac{1}{\sqrt{6-3ex}(2+ex)}\,dx$$
$$= \frac{2\sqrt{3}\sqrt{2-ex}}{e} - \frac{8\,\text{Subst}\left(\int \frac{1}{4-\frac{x^2}{3}}\,dx, x, \sqrt{6-3ex}\right)}{e}$$
$$= \frac{2\sqrt{3}\sqrt{2-ex}}{e} - \frac{4\sqrt{3}\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{e}$$

Mathematica [A] time = 0.0602468, size = 63, normalized size = 1.37

$$\frac{2\sqrt{12-3e^2x^2}\left(\sqrt{ex-2}-2\tan^{-1}\left(\frac{1}{2}\sqrt{ex-2}\right)\right)}{e\sqrt{ex-2}\sqrt{ex+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[12 - 3*e^2*x^2]/(2 + e*x)^(3/2),x]
```

```
[Out] (2*Sqrt[12 - 3*e^2*x^2]*(Sqrt[-2 + e*x] - 2*ArcTan[Sqrt[-2 + e*x]/2]))/(e*S
qrt[-2 + e*x]*Sqrt[2 + e*x])
```

Maple [A] time = 0.143, size = 66, normalized size = 1.4

$$-2\,\frac{\sqrt{-e^2x^2+4}\left(2\sqrt{3}\text{Artanh}\left(1/6\sqrt{3}\sqrt{-3\,ex+6}\right)-\sqrt{-3\,ex+6}\right)\sqrt{3}}{\sqrt{ex+2}\sqrt{-3\,ex+6}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(3/2),x)
```

```
[Out] -2*(-e^2*x^2+4)^(1/2)*(2*3^(1/2)*arctanh(1/6*3^(1/2)*(-3*e*x+6)^(1/2))-(-3*
e*x+6)^(1/2))/(e*x+2)^(1/2)*3^(1/2)/(-3*e*x+6)^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-3\,e^2x^2+12}}{(ex+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-3*e^2*x^2 + 12)/(e*x + 2)^(3/2), x)
```

Fricas [B] time = 2.13564, size = 239, normalized size = 5.2

$$\frac{2\left(\sqrt{3}(ex+2)\log\left(-\frac{3\,e^2x^2-12\,ex+4\,\sqrt{3}\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}-36}{e^2x^2+4\,ex+4}\right)+\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}\right)}{e^2x+2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(3)*(e*x + 2)*log(-(3*e^2*x^2 - 12*e*x + 4*sqrt(3)*sqrt(-3*e^2*x^2 +
 12)*sqrt(e*x + 2) - 36)/(e^2*x^2 + 4*e*x + 4)) + sqrt(-3*e^2*x^2 + 12)*sqr
t(e*x + 2))/(e^2*x + 2*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\sqrt{3}\int\frac{\sqrt{-e^2x^2+4}}{ex\sqrt{ex+2}+2\sqrt{ex+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e**2*x**2+12)**(1/2)/(e*x+2)**(3/2),x)
```

```
[Out] sqrt(3)*Integral(sqrt(-e**2*x**2 + 4)/(e*x*sqrt(e*x + 2) + 2*sqrt(e*x + 2))
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-3\,e^2x^2+12}}{(ex+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-3*e^2*x^2 + 12)/(e*x + 2)^(3/2), x)
```

3.899 $\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{5/2}}\,dx$

Optimal. Leaf size=55

$$\frac{\sqrt{3}\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{2e}-\frac{\sqrt{3}\sqrt{2-ex}}{e(ex+2)}$$

```
[Out] -((Sqrt[3]*Sqrt[2 - e*x])/(e*(2 + e*x))) + (Sqrt[3]*ArcTanh[Sqrt[2 - e*x]/2
])/(2*e)
```

Rubi [A] time = 0.020318, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {627, 47, 63, 206}

$$\frac{\sqrt{3}\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{2e}-\frac{\sqrt{3}\sqrt{2-ex}}{e(ex+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[12 - 3*e^2*x^2]/(2 + e*x)^(5/2),x]
```

```
[Out] -((Sqrt[3]*Sqrt[2 - e*x])/(e*(2 + e*x))) + (Sqrt[3]*ArcTanh[Sqrt[2 - e*x]/2
])/(2*e)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{5/2}}\,dx &= \int \frac{\sqrt{6-3ex}}{(2+ex)^2}\,dx\\ &= -\frac{\sqrt{3}\sqrt{2-ex}}{e(2+ex)}-\frac{3}{2}\int \frac{1}{\sqrt{6-3ex}(2+ex)}\,dx\\ &= -\frac{\sqrt{3}\sqrt{2-ex}}{e(2+ex)}+\frac{\text{Subst}\left(\int \frac{1}{4-\frac{x^2}{3}}\,dx,x,\sqrt{6-3ex}\right)}{e}\\ &= -\frac{\sqrt{3}\sqrt{2-ex}}{e(2+ex)}+\frac{\sqrt{3}\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{2e}\end{aligned}$$

Mathematica [A] time = 0.0887889, size = 74, normalized size = 1.35

$$-\frac{\sqrt{12-3e^2x^2}\left(2ex+\sqrt{2-ex}(ex+2)\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)-4\right)}{2e(ex-2)(ex+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[12 - 3*e^2*x^2]/(2 + e*x)^(5/2),x]
```

```
[Out] -(Sqrt[12 - 3*e^2*x^2]*(-4 + 2*e*x + Sqrt[2 - e*x]*(2 + e*x)*ArcTanh[Sqrt[2
 - e*x]/2]))/(2*e*(-2 + e*x)*(2 + e*x)^(3/2))
```

Maple [B] time = 0.149, size = 88, normalized size = 1.6

$$\frac{\sqrt{3}}{2e}\sqrt{-e^2x^2+4}\left(\sqrt{3}\text{Artanh}\left(\frac{\sqrt{3}}{6}\sqrt{-3ex+6}\right)xe+2\sqrt{3}\text{Artanh}\left(1/6\sqrt{3}\sqrt{-3ex+6}\right)-2\sqrt{-3ex+6}\right)\frac{1}{\sqrt{(ex+2)^3}}\frac{1}{\sqrt{-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(5/2),x)
```

```
[Out] 1/2*(-e^2*x^2+4)^(1/2)*(3^(1/2)*arctanh(1/6*3^(1/2)*(-3*e*x+6)^(1/2))*x*e+2
*3^(1/2)*arctanh(1/6*3^(1/2)*(-3*e*x+6)^(1/2))-2*(-3*e*x+6)^(1/2))/((e*x+2)
^3)^(1/2)*3^(1/2)/(-3*e*x+6)^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-3e^2x^2+12}}{(ex+2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-3*e^2*x^2 + 12)/(e*x + 2)^(5/2), x)
```

Fricas [B] time = 1.88265, size = 277, normalized size = 5.04

$$\frac{\sqrt{3}\left(e^2x^2+4\,ex+4\right)\log\left(-\frac{3\,e^2x^2-12\,ex-4\,\sqrt{3}\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}-36}{e^2x^2+4\,ex+4}\right)-4\,\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}}{4\left(e^3x^2+4\,e^2x+4\,e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(3)*(e^2*x^2 + 4*e*x + 4)*log(-(3*e^2*x^2 - 12*e*x - 4*sqrt(3)*sqr
t(-3*e^2*x^2 + 12)*sqrt(e*x + 2) - 36)/(e^2*x^2 + 4*e*x + 4)) - 4*sqrt(-3*e
^2*x^2 + 12)*sqrt(e*x + 2))/(e^3*x^2 + 4*e^2*x + 4*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\sqrt{3}\int\frac{\sqrt{-e^2x^2+4}}{e^2x^2\sqrt{ex+2}+4ex\sqrt{ex+2}+4\sqrt{ex+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e**2*x**2+12)**(1/2)/(e*x+2)**(5/2),x)
```

```
[Out] sqrt(3)*Integral(sqrt(-e**2*x**2 + 4)/(e**2*x**2*sqrt(e*x + 2) + 4*e*x*sqrt
(e*x + 2) + 4*sqrt(e*x + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-3\,e^2x^2+12}}{(ex+2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-3*e^2*x^2 + 12)/(e*x + 2)^(5/2), x)
```

3.900 $\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{7/2}}\,dx$

Optimal. Leaf size=86

$$\frac{\sqrt{3}\sqrt{2-ex}}{16e(ex+2)}-\frac{\sqrt{3}\sqrt{2-ex}}{2e(ex+2)^2}+\frac{\sqrt{3}\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{32e}$$

```
[Out] -(Sqrt[3]*Sqrt[2 - e*x])/(2*e*(2 + e*x)^2) + (Sqrt[3]*Sqrt[2 - e*x])/(16*e*
(2 + e*x)) + (Sqrt[3]*ArcTanh[Sqrt[2 - e*x]/2])/(32*e)
```

Rubi [A] time = 0.0290316, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {627, 47, 51, 63, 206}

$$\frac{\sqrt{3}\sqrt{2-ex}}{16e(ex+2)}-\frac{\sqrt{3}\sqrt{2-ex}}{2e(ex+2)^2}+\frac{\sqrt{3}\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{32e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[12 - 3*e^2*x^2]/(2 + e*x)^(7/2),x]
```

```
[Out] -(Sqrt[3]*Sqrt[2 - e*x])/(2*e*(2 + e*x)^2) + (Sqrt[3]*Sqrt[2 - e*x])/(16*e*
(2 + e*x)) + (Sqrt[3]*ArcTanh[Sqrt[2 - e*x]/2])/(32*e)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{7/2}}\,dx &= \int \frac{\sqrt{6-3ex}}{(2+ex)^3}\,dx \\
&= -\frac{\sqrt{3}\sqrt{2-ex}}{2e(2+ex)^2} - \frac{3}{4}\int \frac{1}{\sqrt{6-3ex}(2+ex)^2}\,dx \\
&= -\frac{\sqrt{3}\sqrt{2-ex}}{2e(2+ex)^2} + \frac{\sqrt{3}\sqrt{2-ex}}{16e(2+ex)} - \frac{3}{32}\int \frac{1}{\sqrt{6-3ex}(2+ex)}\,dx \\
&= -\frac{\sqrt{3}\sqrt{2-ex}}{2e(2+ex)^2} + \frac{\sqrt{3}\sqrt{2-ex}}{16e(2+ex)} + \frac{\text{Subst}\left(\int \frac{1}{4-\frac{x^2}{3}}\,dx, x, \sqrt{6-3ex}\right)}{16e} \\
&= -\frac{\sqrt{3}\sqrt{2-ex}}{2e(2+ex)^2} + \frac{\sqrt{3}\sqrt{2-ex}}{16e(2+ex)} + \frac{\sqrt{3}\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{32e}
\end{aligned}$$

Mathematica [C] time = 0.0679962, size = 54, normalized size = 0.63

$$\frac{(ex-2)\sqrt{4-e^2x^2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{1}{2}-\frac{ex}{4}\right)}{32e\sqrt{3ex+6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[12 - 3*e^2*x^2]/(2 + e*x)^(7/2),x]
```

```
[Out] ((-2 + e*x)*Sqrt[4 - e^2*x^2]*Hypergeometric2F1[3/2, 3, 5/2, 1/2 - (e*x)/4]
)/(32*e*Sqrt[6 + 3*e*x])
```

Maple [A] time = 0.15, size = 125, normalized size = 1.5

$$\frac{\sqrt{3}}{32\,e}\sqrt{-e^2x^2+4}\left(\sqrt{3}\text{Artanh}\left(\frac{\sqrt{3}}{6}\sqrt{-3\,ex+6}\right)x^2e^2+4\,\sqrt{3}\text{Artanh}\left(1/6\,\sqrt{3}\sqrt{-3\,ex+6}\right)xe+2\,xe\sqrt{-3\,ex+6}+4\,\sqrt{3}\text{Arta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(7/2),x)
```

```
[Out] 1/32*(-e^2*x^2+4)^(1/2)*(3^(1/2)*arctanh(1/6*3^(1/2)*(-3*e*x+6)^(1/2))*x^2*
e^2+4*3^(1/2)*arctanh(1/6*3^(1/2)*(-3*e*x+6)^(1/2))*x*e+2*x*e*(-3*e*x+6)^(1
/2)+4*3^(1/2)*arctanh(1/6*3^(1/2)*(-3*e*x+6)^(1/2))-12*(-3*e*x+6)^(1/2))*3^
(1/2)/((e*x+2)^5)^(1/2)/(-3*e*x+6)^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-3\,e^2x^2+12}}{(ex+2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-3*e^2*x^2 + 12)/(e*x + 2)^(7/2), x)
```

Fricas [B] time = 1.87538, size = 327, normalized size = 3.8

$$\frac{\sqrt{3}\left(e^3x^3+6\,e^2x^2+12\,ex+8\right)\log\left(-\frac{3\,e^2x^2-12\,ex-4\,\sqrt{3}\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}-36}{e^2x^2+4\,ex+4}\right)+4\,\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}(ex-6)}{64\left(e^4x^3+6\,e^3x^2+12\,e^2x+8\,e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/64*(sqrt(3)*(e^3*x^3 + 6*e^2*x^2 + 12*e*x + 8)*log(-(3*e^2*x^2 - 12*e*x -
 4*sqrt(3)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2) - 36)/(e^2*x^2 + 4*e*x + 4))
 + 4*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2)*(e*x - 6))/(e^4*x^3 + 6*e^3*x^2 +
12*e^2*x + 8*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e**2*x**2+12)**(1/2)/(e*x+2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-3\,e^2x^2+12}}{(ex+2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-3*e^2*x^2 + 12)/(e*x + 2)^(7/2), x)
```

3.901 $\int (2+ex)^{5/2}\left(12-3e^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=109

$$-\frac{6\sqrt{3}(2-ex)^{13/2}}{13e}+\frac{96\sqrt{3}(2-ex)^{11/2}}{11e}-\frac{64\sqrt{3}(2-ex)^{9/2}}{e}+\frac{1536\sqrt{3}(2-ex)^{7/2}}{7e}-\frac{1536\sqrt{3}(2-ex)^{5/2}}{5e}$$

```
[Out] (-1536*Sqrt[3]*(2 - e*x)^(5/2))/(5*e) + (1536*Sqrt[3]*(2 - e*x)^(7/2))/(7*e
) - (64*Sqrt[3]*(2 - e*x)^(9/2))/e + (96*Sqrt[3]*(2 - e*x)^(11/2))/(11*e) -
 (6*Sqrt[3]*(2 - e*x)^(13/2))/(13*e)
```

Rubi [A] time = 0.0274405, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {627, 43}

$$-\frac{6\sqrt{3}(2-ex)^{13/2}}{13e}+\frac{96\sqrt{3}(2-ex)^{11/2}}{11e}-\frac{64\sqrt{3}(2-ex)^{9/2}}{e}+\frac{1536\sqrt{3}(2-ex)^{7/2}}{7e}-\frac{1536\sqrt{3}(2-ex)^{5/2}}{5e}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + e*x)^(5/2)*(12 - 3*e^2*x^2)^(3/2),x]
```

```
[Out] (-1536*Sqrt[3]*(2 - e*x)^(5/2))/(5*e) + (1536*Sqrt[3]*(2 - e*x)^(7/2))/(7*e
) - (64*Sqrt[3]*(2 - e*x)^(9/2))/e + (96*Sqrt[3]*(2 - e*x)^(11/2))/(11*e) -
 (6*Sqrt[3]*(2 - e*x)^(13/2))/(13*e)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (2+ex)^{5/2}\left(12-3e^2x^2\right)^{3/2}\,dx &= \int (6-3ex)^{3/2}(2+ex)^4\,dx\\ &= \int \left(256(6-3ex)^{3/2}-\frac{256}{3}(6-3ex)^{5/2}+\frac{32}{3}(6-3ex)^{7/2}-\frac{16}{27}(6-3ex)^{9/2}+\frac{1}{81}(6-3ex)\right.\\ &= -\frac{1536\sqrt{3}(2-ex)^{5/2}}{5e}+\frac{1536\sqrt{3}(2-ex)^{7/2}}{7e}-\frac{64\sqrt{3}(2-ex)^{9/2}}{e}+\frac{96\sqrt{3}(2-ex)^{11/2}}{11e}-6\sqrt{}\end{aligned}$$

Mathematica [A] time = 0.0682521, size = 67, normalized size = 0.61

$$-\frac{2(ex-2)^2\sqrt{12-3e^2x^2}\left(1155e^4x^4+12600e^3x^3+56840e^2x^2+133600ex+154928\right)}{5005e\sqrt{ex+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + e*x)^(5/2)*(12 - 3*e^2*x^2)^(3/2),x]
```

```
[Out] (-2*(-2 + e*x)^2*Sqrt[12 - 3*e^2*x^2]*(154928 + 133600*e*x + 56840*e^2*x^2
+ 12600*e^3*x^3 + 1155*e^4*x^4))/(5005*e*Sqrt[2 + e*x])
```

Maple [A] time = 0.043, size = 60, normalized size = 0.6

$$\frac{(2\,ex-4)\left(1155\,e^4x^4+12600\,e^3x^3+56840\,e^2x^2+133600\,ex+154928\right)}{15015\,e}\left(-3\,e^2x^2+12\right)^{\frac{3}{2}}(ex+2)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+2)^(5/2)*(-3*e^2*x^2+12)^(3/2),x)
```

```
[Out] 2/15015*(e*x-2)*(1155*e^4*x^4+12600*e^3*x^3+56840*e^2*x^2+133600*e*x+154928
)*(-3*e^2*x^2+12)^(3/2)/e/(e*x+2)^(3/2)
```

Maxima [C] time = 1.77502, size = 126, normalized size = 1.16

$$-\frac{\left(2310i\,\sqrt{3}e^6x^6+15960i\,\sqrt{3}e^5x^5+22120i\,\sqrt{3}e^4x^4-86720i\,\sqrt{3}e^3x^3-304224i\,\sqrt{3}e^2x^2-170624i\,\sqrt{3}ex+1239424i\,\sqrt{\cdots}\right.}{5005\left(e^2x+2\,e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(5/2)*(-3*e^2*x^2+12)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/5005*(2310*I*sqrt(3)*e^6*x^6 + 15960*I*sqrt(3)*e^5*x^5 + 22120*I*sqrt(3)
*e^4*x^4 - 86720*I*sqrt(3)*e^3*x^3 - 304224*I*sqrt(3)*e^2*x^2 - 170624*I*sq
rt(3)*e*x + 1239424*I*sqrt(3))*(e*x + 2)*sqrt(e*x - 2)/(e^2*x + 2*e)
```

Fricas [A] time = 1.84497, size = 215, normalized size = 1.97

$$-\frac{2\left(1155\,e^6x^6+7980\,e^5x^5+11060\,e^4x^4-43360\,e^3x^3-152112\,e^2x^2-85312\,ex+619712\right)\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}}{5005\left(e^2x+2\,e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(5/2)*(-3*e^2*x^2+12)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5005*(1155*e^6*x^6 + 7980*e^5*x^5 + 11060*e^4*x^4 - 43360*e^3*x^3 - 1521
12*e^2*x^2 - 85312*e*x + 619712)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2)/(e^2*x
 + 2*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)**(5/2)*(-3*e**2*x**2+12)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-3 e^2 x^2 + 12\right)^{\frac{3}{2}} (ex + 2)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(5/2)*(-3*e^2*x^2+12)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-3*e^2*x^2 + 12)^(3/2)*(e*x + 2)^(5/2), x)
```

3.902 $\int (2+ex)^{3/2} \left(12-3e^2x^2\right)^{3/2} \, dx$

Optimal. Leaf size=87

$$\frac{6\sqrt{3}(2-ex)^{11/2}}{11e}-\frac{8\sqrt{3}(2-ex)^{9/2}}{e}+\frac{288\sqrt{3}(2-ex)^{7/2}}{7e}-\frac{384\sqrt{3}(2-ex)^{5/2}}{5e}$$

```
[Out] (-384*Sqrt[3]*(2 - e*x)^(5/2))/(5*e) + (288*Sqrt[3]*(2 - e*x)^(7/2))/(7*e)
- (8*Sqrt[3]*(2 - e*x)^(9/2))/e + (6*Sqrt[3]*(2 - e*x)^(11/2))/(11*e)
```

Rubi [A] time = 0.0239898, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {627, 43}

$$\frac{6\sqrt{3}(2-ex)^{11/2}}{11e}-\frac{8\sqrt{3}(2-ex)^{9/2}}{e}+\frac{288\sqrt{3}(2-ex)^{7/2}}{7e}-\frac{384\sqrt{3}(2-ex)^{5/2}}{5e}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + e*x)^(3/2)*(12 - 3*e^2*x^2)^(3/2),x]
```

```
[Out] (-384*Sqrt[3]*(2 - e*x)^(5/2))/(5*e) + (288*Sqrt[3]*(2 - e*x)^(7/2))/(7*e)
- (8*Sqrt[3]*(2 - e*x)^(9/2))/e + (6*Sqrt[3]*(2 - e*x)^(11/2))/(11*e)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (2+ex)^{3/2}\left(12-3e^2x^2\right)^{3/2}\,dx &= \int (6-3ex)^{3/2}(2+ex)^3\,dx\\
&= \int \left(64(6-3ex)^{3/2}-16(6-3ex)^{5/2}+\frac{4}{3}(6-3ex)^{7/2}-\frac{1}{27}(6-3ex)^{9/2}\right)dx\\
&= -\frac{384\sqrt{3}(2-ex)^{5/2}}{5e}+\frac{288\sqrt{3}(2-ex)^{7/2}}{7e}-\frac{8\sqrt{3}(2-ex)^{9/2}}{e}+\frac{6\sqrt{3}(2-ex)^{11/2}}{11e}
\end{aligned}$$

Mathematica [A] time = 0.0599333, size = 59, normalized size = 0.68

$$-\frac{2(ex-2)^2\sqrt{12-3e^2x^2}\left(105e^3x^3+910e^2x^2+3020ex+4264\right)}{385e\sqrt{ex+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + e*x)^(3/2)*(12 - 3*e^2*x^2)^(3/2),x]
```

```
[Out] (-2*(-2 + e*x)^2*Sqrt[12 - 3*e^2*x^2]*(4264 + 3020*e*x + 910*e^2*x^2 + 105*
e^3*x^3))/(385*e*Sqrt[2 + e*x])
```

Maple [A] time = 0.043, size = 52, normalized size = 0.6

$$\frac{(2\,ex-4)\left(105\,e^{3}x^{3}+910\,e^{2}x^{2}+3020\,ex+4264\right)}{1155\,e}\left(-3\,e^{2}x^{2}+12\right)^{\frac{3}{2}}(ex+2)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+2)^(3/2)*(-3*e^2*x^2+12)^(3/2),x)
```

```
[Out] 2/1155*(e*x-2)*(105*e^3*x^3+910*e^2*x^2+3020*e*x+4264)*(-3*e^2*x^2+12)^(3/2
)/e/(e*x+2)^(3/2)
```

Maxima [C] time = 1.95948, size = 111, normalized size = 1.28

$$-\frac{\left(210i\,\sqrt{3}e^{5}x^{5}+980i\,\sqrt{3}e^{4}x^{4}-400i\,\sqrt{3}e^{3}x^{3}-8352i\,\sqrt{3}e^{2}x^{2}-9952i\,\sqrt{3}ex+34112i\,\sqrt{3}\right)(ex+2)\sqrt{ex-2}}{385\left(e^{2}x+2\,e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(3/2)*(-3*e^2*x^2+12)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/385*(210*I*sqrt(3)*e^5*x^5 + 980*I*sqrt(3)*e^4*x^4 - 400*I*sqrt(3)*e^3*x
^3 - 8352*I*sqrt(3)*e^2*x^2 - 9952*I*sqrt(3)*e*x + 34112*I*sqrt(3))*(e*x +
2)*sqrt(e*x - 2)/(e^2*x + 2*e)
```

Fricas [A] time = 1.84532, size = 181, normalized size = 2.08

$$-\frac{2\left(105\,e^{5}x^{5}+490\,e^{4}x^{4}-200\,e^{3}x^{3}-4176\,e^{2}x^{2}-4976\,ex+17056\right)\sqrt{-3\,e^{2}x^{2}+12}\sqrt{ex+2}}{385\left(e^{2}x+2\,e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(3/2)*(-3*e^2*x^2+12)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/385*(105*e^5*x^5 + 490*e^4*x^4 - 200*e^3*x^3 - 4176*e^2*x^2 - 4976*e*x +
 17056)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2)/(e^2*x + 2*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)**(3/2)*(-3*e**2*x**2+12)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-3\,e^2x^2+12\right)^{\frac{3}{2}}(ex+2)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(3/2)*(-3*e^2*x^2+12)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-3*e^2*x^2 + 12)^(3/2)*(e*x + 2)^(3/2), x)
```

3.903 $\int \sqrt{2+ex}\left(12-3e^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=65

$$-\frac{2(2-ex)^{9/2}}{\sqrt{3}e}+\frac{48\sqrt{3}(2-ex)^{7/2}}{7e}-\frac{96\sqrt{3}(2-ex)^{5/2}}{5e}$$

```
[Out] (-96*Sqrt[3]*(2 - e*x)^(5/2))/(5*e) + (48*Sqrt[3]*(2 - e*x)^(7/2))/(7*e) -
(2*(2 - e*x)^(9/2))/(Sqrt[3]*e)
```

Rubi [A] time = 0.0196335, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {627, 43}

$$-\frac{2(2-ex)^{9/2}}{\sqrt{3}e}+\frac{48\sqrt{3}(2-ex)^{7/2}}{7e}-\frac{96\sqrt{3}(2-ex)^{5/2}}{5e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + e*x]*(12 - 3*e^2*x^2)^(3/2),x]
```

```
[Out] (-96*Sqrt[3]*(2 - e*x)^(5/2))/(5*e) + (48*Sqrt[3]*(2 - e*x)^(7/2))/(7*e) -
(2*(2 - e*x)^(9/2))/(Sqrt[3]*e)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{2+ex}\left(12-3e^2x^2\right)^{3/2}\,dx &= \int (6-3ex)^{3/2}(2+ex)^2\,dx\\ &= \int \left(16(6-3ex)^{3/2}-\frac{8}{3}(6-3ex)^{5/2}+\frac{1}{9}(6-3ex)^{7/2}\right)dx\\ &= -\frac{96\sqrt{3}(2-ex)^{5/2}}{5e}+\frac{48\sqrt{3}(2-ex)^{7/2}}{7e}-\frac{2(2-ex)^{9/2}}{\sqrt{3}e}\end{aligned}$$

Mathematica [A] time = 0.0447866, size = 52, normalized size = 0.8

$$-\frac{2(ex-2)^2\sqrt{4-e^2x^2}\left(35e^2x^2+220ex+428\right)}{35e\sqrt{3ex+6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + e*x]*(12 - 3*e^2*x^2)^(3/2),x]
```

```
[Out] (-2*(-2 + e*x)^2*Sqrt[4 - e^2*x^2]*(428 + 220*e*x + 35*e^2*x^2))/(35*e*Sqrt
[6 + 3*e*x])
```

Maple [A] time = 0.043, size = 44, normalized size = 0.7

$$\frac{(2\,ex-4)\left(35\,e^2x^2+220\,ex+428\right)}{315\,e}\left(-3\,e^2x^2+12\right)^{\frac{3}{2}}(ex+2)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+2)^(1/2)*(-3*e^2*x^2+12)^(3/2),x)
```

```
[Out] 2/315*(e*x-2)*(35*e^2*x^2+220*e*x+428)*(-3*e^2*x^2+12)^(3/2)/e/(e*x+2)^(3/2
)
```

Maxima [C] time = 1.99547, size = 96, normalized size = 1.48

$$-\frac{\left(70i\,\sqrt{3}e^4x^4+160i\,\sqrt{3}e^3x^3-624i\,\sqrt{3}e^2x^2-1664i\,\sqrt{3}ex+3424i\,\sqrt{3}\right)(ex+2)\sqrt{ex-2}}{105\left(e^2x+2\,e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(1/2)*(-3*e^2*x^2+12)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/105*(70*I*sqrt(3)*e^4*x^4 + 160*I*sqrt(3)*e^3*x^3 - 624*I*sqrt(3)*e^2*x^
2 - 1664*I*sqrt(3)*e*x + 3424*I*sqrt(3))*(e*x + 2)*sqrt(e*x - 2)/(e^2*x + 2
*e)
```

Fricas [A] time = 1.86763, size = 155, normalized size = 2.38

$$-\frac{2\left(35\,e^4x^4+80\,e^3x^3-312\,e^2x^2-832\,ex+1712\right)\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}}{105\left(e^2x+2\,e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(1/2)*(-3*e^2*x^2+12)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/105*(35*e^4*x^4 + 80*e^3*x^3 - 312*e^2*x^2 - 832*e*x + 1712)*sqrt(-3*e^2
*x^2 + 12)*sqrt(e*x + 2)/(e^2*x + 2*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$3\sqrt{3}\left(\int 4\sqrt{ex+2}\sqrt{-e^2x^2+4}\,dx+\int -e^2x^2\sqrt{ex+2}\sqrt{-e^2x^2+4}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)**(1/2)*(-3*e**2*x**2+12)**(3/2),x)
```

```
[Out] 3*sqrt(3)*(Integral(4*sqrt(e*x + 2)*sqrt(-e**2*x**2 + 4), x) + Integral(-e*
*2*x**2*sqrt(e*x + 2)*sqrt(-e**2*x**2 + 4), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-3 e^2 x^2 + 12\right)^{\frac{3}{2}} \sqrt{ex + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(1/2)*(-3*e^2*x^2+12)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-3*e^2*x^2 + 12)^(3/2)*sqrt(e*x + 2), x)
```

3.904 $\int \frac{(12-3e^2x^2)^{3/2}}{\sqrt{2+ex}}\,dx$

Optimal. Leaf size=45

$$\frac{6\sqrt{3}(2-ex)^{7/2}}{7e}-\frac{24\sqrt{3}(2-ex)^{5/2}}{5e}$$

```
[Out] (-24*Sqrt[3]*(2 - e*x)^(5/2))/(5*e) + (6*Sqrt[3]*(2 - e*x)^(7/2))/(7*e)
```

Rubi [A] time = 0.0156231, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {627, 43}

$$\frac{6\sqrt{3}(2-ex)^{7/2}}{7e}-\frac{24\sqrt{3}(2-ex)^{5/2}}{5e}$$

Antiderivative was successfully verified.

```
[In] Int[(12 - 3*e^2*x^2)^(3/2)/Sqrt[2 + e*x],x]
```

```
[Out] (-24*Sqrt[3]*(2 - e*x)^(5/2))/(5*e) + (6*Sqrt[3]*(2 - e*x)^(7/2))/(7*e)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(12-3e^2x^2)^{3/2}}{\sqrt{2+ex}}\,dx &= \int (6-3ex)^{3/2}(2+ex)\,dx\\
&= \int \left(4(6-3ex)^{3/2}-\frac{1}{3}(6-3ex)^{5/2}\right)dx\\
&= -\frac{24\sqrt{3}(2-ex)^{5/2}}{5e}+\frac{6\sqrt{3}(2-ex)^{7/2}}{7e}
\end{aligned}$$

Mathematica [A] time = 0.0545767, size = 43, normalized size = 0.96

$$-\frac{6(ex-2)^2(5ex+18)\sqrt{12-3e^2x^2}}{35e\sqrt{ex+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(12 - 3*e^2*x^2)^(3/2)/Sqrt[2 + e*x],x]
```

```
[Out] (-6*(-2 + e*x)^2*(18 + 5*e*x)*Sqrt[12 - 3*e^2*x^2])/(35*e*Sqrt[2 + e*x])
```

Maple [A] time = 0.04, size = 36, normalized size = 0.8

$$\frac{(2\,ex-4)\,(5\,ex+18)}{35\,e}\left(-3\,e^{2}x^{2}+12\right)^{\frac{3}{2}}(ex+2)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(1/2),x)
```

```
[Out] 2/35*(e*x-2)*(5*e*x+18)*(-3*e^2*x^2+12)^(3/2)/e/(e*x+2)^(3/2)
```

Maxima [C] time = 1.78863, size = 63, normalized size = 1.4

$$-\frac{\left(30i\,\sqrt{3}e^{3}x^{3}-12i\,\sqrt{3}e^{2}x^{2}-312i\,\sqrt{3}ex+432i\,\sqrt{3}\right)\sqrt{ex-2}}{35\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/35*(30*I*sqrt(3)*e^3*x^3 - 12*I*sqrt(3)*e^2*x^2 - 312*I*sqrt(3)*e*x + 43
2*I*sqrt(3))*sqrt(e*x - 2)/e
```

Fricas [A] time = 1.77618, size = 128, normalized size = 2.84

$$-\frac{6\left(5\,e^{3}x^{3}-2\,e^{2}x^{2}-52\,ex+72\right)\sqrt{-3\,e^{2}x^{2}+12}\sqrt{ex+2}}{35\left(e^{2}x+2\,e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -6/35*(5*e^3*x^3 - 2*e^2*x^2 - 52*e*x + 72)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x
+ 2)/(e^2*x + 2*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$3\sqrt{3}\left(\int\frac{4\sqrt{-e^{2}x^{2}+4}}{\sqrt{ex+2}}\,dx+\int-\frac{e^{2}x^{2}\sqrt{-e^{2}x^{2}+4}}{\sqrt{ex+2}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e**2*x**2+12)**(3/2)/(e*x+2)**(1/2),x)
```

```
[Out] 3*sqrt(3)*(Integral(4*sqrt(-e**2*x**2 + 4)/sqrt(e*x + 2), x) + Integral(-e*
*2*x**2*sqrt(-e**2*x**2 + 4)/sqrt(e*x + 2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-3\,e^2x^2+12\right)^{\frac{3}{2}}}{\sqrt{ex+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-3*e^2*x^2 + 12)^(3/2)/sqrt(e*x + 2), x)
```

3.905 $\int \frac{\left(12-3e^2x^2\right)^{3/2}}{(2+ex)^{3/2}}\,dx$

Optimal. Leaf size=22

$$-\frac{6\sqrt{3}(2-ex)^{5/2}}{5e}$$

```
[Out] (-6*Sqrt[3]*(2 - e*x)^(5/2))/(5*e)
```

Rubi [A] time = 0.0094595, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {627, 32}

$$-\frac{6\sqrt{3}(2-ex)^{5/2}}{5e}$$

Antiderivative was successfully verified.

```
[In] Int[(12 - 3*e^2*x^2)^(3/2)/(2 + e*x)^(3/2),x]
```

```
[Out] (-6*Sqrt[3]*(2 - e*x)^(5/2))/(5*e)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(12-3e^2x^2\right)^{3/2}}{(2+ex)^{3/2}}\,dx &= \int (6-3ex)^{3/2}\,dx\\ &= -\frac{6\sqrt{3}(2-ex)^{5/2}}{5e}\end{aligned}$$

Mathematica [A] time = 0.0447272, size = 37, normalized size = 1.68

$$-\frac{6(ex-2)^2\sqrt{12-3e^2x^2}}{5e\sqrt{ex+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(12 - 3*e^2*x^2)^(3/2)/(2 + e*x)^(3/2),x]
```

```
[Out] (-6*(-2 + e*x)^2*Sqrt[12 - 3*e^2*x^2])/(5*e*Sqrt[2 + e*x])
```

Maple [A] time = 0.042, size = 30, normalized size = 1.4

$$\frac{2ex-4}{5e}\left(-3e^{2}x^{2}+12\right)^{\frac{3}{2}}(ex+2)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(3/2),x)
```

```
[Out] 2/5*(e*x-2)*(-3*e^2*x^2+12)^(3/2)/e/(e*x+2)^(3/2)
```

Maxima [C] time = 1.78105, size = 49, normalized size = 2.23

$$-\frac{\left(6i\sqrt{3}e^{2}x^{2}-24i\sqrt{3}ex+24i\sqrt{3}\right)\sqrt{ex-2}}{5e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/5*(6*I*sqrt(3)*e^2*x^2 - 24*I*sqrt(3)*e*x + 24*I*sqrt(3))*sqrt(e*x - 2)/
e
```

Fricas [B] time = 1.7918, size = 105, normalized size = 4.77

$$-\frac{6\left(e^{2}x^{2}-4ex+4\right)\sqrt{-3e^{2}x^{2}+12}\sqrt{ex+2}}{5\left(e^{2}x+2e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -6/5*(e^2*x^2 - 4*e*x + 4)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2)/(e^2*x + 2*e
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$3\sqrt{3}\left(\int\frac{4\sqrt{-e^{2}x^{2}+4}}{ex\sqrt{ex+2}+2\sqrt{ex+2}}\,dx+\int-\frac{e^{2}x^{2}\sqrt{-e^{2}x^{2}+4}}{ex\sqrt{ex+2}+2\sqrt{ex+2}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e**2*x**2+12)**(3/2)/(e*x+2)**(3/2),x)
```

```
[Out] 3*sqrt(3)*(Integral(4*sqrt(-e**2*x**2 + 4)/(e*x*sqrt(e*x + 2) + 2*sqrt(e*x
+ 2)), x) + Integral(-e**2*x**2*sqrt(-e**2*x**2 + 4)/(e*x*sqrt(e*x + 2) + 2
*sqrt(e*x + 2)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-3\,e^{2}x^{2}+12\right)^{\frac{3}{2}}}{\left(ex+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-3*e^2*x^2 + 12)^(3/2)/(e*x + 2)^(3/2), x)
```

3.906 $\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{5/2}}\,dx$

Optimal. Leaf size=66

$$\frac{2\sqrt{3}(2-ex)^{3/2}}{e}+\frac{24\sqrt{3}\sqrt{2-ex}}{e}-\frac{48\sqrt{3}\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{e}$$

```
[Out] (24*Sqrt[3]*Sqrt[2 - e*x])/e + (2*Sqrt[3]*(2 - e*x)^(3/2))/e - (48*Sqrt[3]*
ArcTanh[Sqrt[2 - e*x]/2])/e
```

Rubi [A] time = 0.028687, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {627, 50, 63, 206}

$$\frac{2\sqrt{3}(2-ex)^{3/2}}{e}+\frac{24\sqrt{3}\sqrt{2-ex}}{e}-\frac{48\sqrt{3}\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(12 - 3*e^2*x^2)^(3/2)/(2 + e*x)^(5/2),x]
```

```
[Out] (24*Sqrt[3]*Sqrt[2 - e*x])/e + (2*Sqrt[3]*(2 - e*x)^(3/2))/e - (48*Sqrt[3]*
ArcTanh[Sqrt[2 - e*x]/2])/e
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(12-3e^2x^2\right)^{3/2}}{(2+ex)^{5/2}}\,dx &= \int \frac{(6-3ex)^{3/2}}{2+ex}\,dx\\
&= \frac{2\sqrt{3}(2-ex)^{3/2}}{e}+12\int \frac{\sqrt{6-3ex}}{2+ex}\,dx\\
&= \frac{24\sqrt{3}\sqrt{2-ex}}{e}+\frac{2\sqrt{3}(2-ex)^{3/2}}{e}+144\int \frac{1}{\sqrt{6-3ex}(2+ex)}\,dx\\
&= \frac{24\sqrt{3}\sqrt{2-ex}}{e}+\frac{2\sqrt{3}(2-ex)^{3/2}}{e}-\frac{96\,\text{Subst}\left(\int \frac{1}{4-\frac{x^2}{3}}\,dx,x,\sqrt{6-3ex}\right)}{e}\\
&= \frac{24\sqrt{3}\sqrt{2-ex}}{e}+\frac{2\sqrt{3}(2-ex)^{3/2}}{e}-\frac{48\sqrt{3}\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{e}
\end{aligned}
$$

Mathematica [A] time = 0.0691722, size = 69, normalized size = 1.05

$$
-\frac{2\sqrt{12-3e^2x^2}\left(\sqrt{ex-2}(ex-14)+24\tan^{-1}\left(\frac{1}{2}\sqrt{ex-2}\right)\right)}{e\sqrt{ex-2}\sqrt{ex+2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(12 - 3*e^2*x^2)^(3/2)/(2 + e*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[12 - 3*e^2*x^2]*((-14 + e*x)*Sqrt[-2 + e*x] + 24*ArcTan[Sqrt[-2 +
e*x]/2]))/(e*Sqrt[-2 + e*x]*Sqrt[2 + e*x])
```

Maple [A] time = 0.124, size = 77, normalized size = 1.2

$$
-2\,\frac{\sqrt{-e^2x^2+4}\left(xe\sqrt{-3\,ex+6}+24\,\sqrt{3}\text{Artanh}\left(1/6\,\sqrt{3}\sqrt{-3\,ex+6}\right)-14\,\sqrt{-3\,ex+6}\right)\sqrt{3}}{\sqrt{ex+2}\sqrt{-3\,ex+6}e}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(5/2),x)
```

```
[Out] -2*(-e^2*x^2+4)^(1/2)*(x*e*(-3*e*x+6)^(1/2)+24*3^(1/2)*arctanh(1/6*3^(1/2)*
(-3*e*x+6)^(1/2))-14*(-3*e*x+6)^(1/2))*3^(1/2)/(e*x+2)^(1/2)/(-3*e*x+6)^(1/
2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\left(-3\,e^2x^2+12\right)^{\frac{3}{2}}}{(ex+2)^{\frac{5}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-3*e^2*x^2 + 12)^(3/2)/(e*x + 2)^(5/2), x)
```

Fricas [B] time = 1.89295, size = 258, normalized size = 3.91

$$\frac{2\left(12\sqrt{3}(ex+2)\log\left(-\frac{3e^2x^2-12ex+4\sqrt{3}\sqrt{-3e^2x^2+12}\sqrt{ex+2}-36}{e^2x^2+4ex+4}\right)-\sqrt{-3e^2x^2+12}\sqrt{ex+2}(ex-14)\right)}{e^2x+2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2*(12*sqrt(3)*(e*x + 2)*log(-(3*e^2*x^2 - 12*e*x + 4*sqrt(3)*sqrt(-3*e^2*x^
2 + 12)*sqrt(e*x + 2) - 36)/(e^2*x^2 + 4*e*x + 4)) - sqrt(-3*e^2*x^2 + 12)*
sqrt(e*x + 2)*(e*x - 14))/(e^2*x + 2*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e**2*x**2+12)**(3/2)/(e*x+2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.907 $\int \frac{\left(12-3e^2x^2\right)^{3/2}}{(2+ex)^{7/2}}\,dx$

Optimal. Leaf size=73

$$-\frac{3\sqrt{3}(2-ex)^{3/2}}{e(ex+2)}-\frac{9\sqrt{3}\sqrt{2-ex}}{e}+\frac{18\sqrt{3}\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{e}$$

```
[Out] (-9*Sqrt[3]*Sqrt[2 - e*x])/e - (3*Sqrt[3]*(2 - e*x)^(3/2))/(e*(2 + e*x)) +
(18*Sqrt[3]*ArcTanh[Sqrt[2 - e*x]/2])/e
```

Rubi [A] time = 0.0276186, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {627, 47, 50, 63, 206}

$$-\frac{3\sqrt{3}(2-ex)^{3/2}}{e(ex+2)}-\frac{9\sqrt{3}\sqrt{2-ex}}{e}+\frac{18\sqrt{3}\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(12 - 3*e^2*x^2)^(3/2)/(2 + e*x)^(7/2),x]
```

```
[Out] (-9*Sqrt[3]*Sqrt[2 - e*x])/e - (3*Sqrt[3]*(2 - e*x)^(3/2))/(e*(2 + e*x)) +
(18*Sqrt[3]*ArcTanh[Sqrt[2 - e*x]/2])/e
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{7/2}}\,dx &= \int \frac{(6-3ex)^{3/2}}{(2+ex)^2}\,dx\\ &= -\frac{3\sqrt{3}(2-ex)^{3/2}}{e(2+ex)}-\frac{9}{2}\int \frac{\sqrt{6-3ex}}{2+ex}\,dx\\ &= -\frac{9\sqrt{3}\sqrt{2-ex}}{e}-\frac{3\sqrt{3}(2-ex)^{3/2}}{e(2+ex)}-54\int \frac{1}{\sqrt{6-3ex}(2+ex)}\,dx\\ &= -\frac{9\sqrt{3}\sqrt{2-ex}}{e}-\frac{3\sqrt{3}(2-ex)^{3/2}}{e(2+ex)}+\frac{36\,\mathrm{Subst}\left(\int \frac{1}{4-\frac{x^2}{3}}\,dx,x,\sqrt{6-3ex}\right)}{e}\\ &= -\frac{9\sqrt{3}\sqrt{2-ex}}{e}-\frac{3\sqrt{3}(2-ex)^{3/2}}{e(2+ex)}+\frac{18\sqrt{3}\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{e}\end{aligned}$$

Mathematica [C] time = 0.0632678, size = 55, normalized size = 0.75

$$-\frac{3(ex-2)^2\sqrt{12-3e^2x^2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{1}{2}-\frac{ex}{4}\right)}{40e\sqrt{ex+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(12 - 3*e^2*x^2)^(3/2)/(2 + e*x)^(7/2),x]
```

```
[Out] (-3*(-2 + e*x)^2*Sqrt[12 - 3*e^2*x^2]*Hypergeometric2F1[2, 5/2, 7/2, 1/2 -
(e*x)/4])/(40*e*Sqrt[2 + e*x])
```

Maple [A] time = 0.131, size = 101, normalized size = 1.4

$$6\,\frac{\sqrt{-e^2x^2+4}\left(3\sqrt{3}\mathrm{Artanh}\left(1/6\sqrt{3}\sqrt{-3ex+6}\right)xe-xe\sqrt{-3ex+6}+6\sqrt{3}\mathrm{Artanh}\left(1/6\sqrt{3}\sqrt{-3ex+6}\right)-4\sqrt{-3ex+}\right.}{\sqrt{(ex+2)^3}\sqrt{-3ex+6}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(7/2),x)
```

```
[Out] 6*(-e^2*x^2+4)^(1/2)*(3*3^(1/2)*arctanh(1/6*3^(1/2)*(-3*e*x+6)^(1/2))*x*e-x
*e*(-3*e*x+6)^(1/2)+6*3^(1/2)*arctanh(1/6*3^(1/2)*(-3*e*x+6)^(1/2))-4*(-3*e
*x+6)^(1/2))/((e*x+2)^3)^(1/2)*3^(1/2)/(-3*e*x+6)^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-3\,e^2x^2+12\right)^{\frac{3}{2}}}{(ex+2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((-3*e^2*x^2 + 12)^(3/2)/(e*x + 2)^(7/2), x)
```

Fricas [B] time = 1.81619, size = 290, normalized size = 3.97

$$\frac{3\left(3\,\sqrt{3}\left(e^2x^2+4\,ex+4\right)\log\left(-\frac{3\,e^2x^2-12\,ex-4\,\sqrt{3}\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}-36}{e^2x^2+4\,ex+4}\right)-2\,\sqrt{-3\,e^2x^2+12}(ex+4)\sqrt{ex+2}\right)}{e^3x^2+4\,e^2x+4\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(7/2),x, algorithm="fricas")
```

```
[Out] 3*(3*sqrt(3)*(e^2*x^2 + 4*e*x + 4)*log(-(3*e^2*x^2 - 12*e*x - 4*sqrt(3)*sqr
t(-3*e^2*x^2 + 12)*sqrt(e*x + 2) - 36)/(e^2*x^2 + 4*e*x + 4)) - 2*sqrt(-3*e
^2*x^2 + 12)*(e*x + 4)*sqrt(e*x + 2))/(e^3*x^2 + 4*e^2*x + 4*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e**2*x**2+12)**(3/2)/(e*x+2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-3\,e^2x^2+12\right)^{\frac{3}{2}}}{(ex+2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((-3*e^2*x^2 + 12)^(3/2)/(e*x + 2)^(7/2), x)
```

3.908 $\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{9/2}}\,dx$

Optimal. Leaf size=86

$$-\frac{3\sqrt{3}(2-ex)^{3/2}}{2e(ex+2)^2}+\frac{9\sqrt{3}\sqrt{2-ex}}{4e(ex+2)}-\frac{9\sqrt{3}\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{8e}$$

```
[Out] (-3*Sqrt[3]*(2 - e*x)^(3/2))/(2*e*(2 + e*x)^2) + (9*Sqrt[3]*Sqrt[2 - e*x])/
(4*e*(2 + e*x)) - (9*Sqrt[3]*ArcTanh[Sqrt[2 - e*x]/2])/(8*e)
```

Rubi [A] time = 0.0290324, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {627, 47, 63, 206}

$$-\frac{3\sqrt{3}(2-ex)^{3/2}}{2e(ex+2)^2}+\frac{9\sqrt{3}\sqrt{2-ex}}{4e(ex+2)}-\frac{9\sqrt{3}\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{8e}$$

Antiderivative was successfully verified.

```
[In] Int[(12 - 3*e^2*x^2)^(3/2)/(2 + e*x)^(9/2),x]
```

```
[Out] (-3*Sqrt[3]*(2 - e*x)^(3/2))/(2*e*(2 + e*x)^2) + (9*Sqrt[3]*Sqrt[2 - e*x])/
(4*e*(2 + e*x)) - (9*Sqrt[3]*ArcTanh[Sqrt[2 - e*x]/2])/(8*e)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(12-3e^2x^2\right)^{3/2}}{(2+ex)^{9/2}}\,dx &= \int \frac{(6-3ex)^{3/2}}{(2+ex)^3}\,dx\\ &= -\frac{3\sqrt{3}(2-ex)^{3/2}}{2e(2+ex)^2}-\frac{9}{4}\int \frac{\sqrt{6-3ex}}{(2+ex)^2}\,dx\\ &= -\frac{3\sqrt{3}(2-ex)^{3/2}}{2e(2+ex)^2}+\frac{9\sqrt{3}\sqrt{2-ex}}{4e(2+ex)}+\frac{27}{8}\int \frac{1}{\sqrt{6-3ex}(2+ex)}\,dx\\ &= -\frac{3\sqrt{3}(2-ex)^{3/2}}{2e(2+ex)^2}+\frac{9\sqrt{3}\sqrt{2-ex}}{4e(2+ex)}-\frac{9\,\text{Subst}\left(\int \frac{1}{4-\frac{x^2}{3}}\,dx,x,\sqrt{6-3ex}\right)}{4e}\\ &= -\frac{3\sqrt{3}(2-ex)^{3/2}}{2e(2+ex)^2}+\frac{9\sqrt{3}\sqrt{2-ex}}{4e(2+ex)}-\frac{9\sqrt{3}\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{8e}\end{aligned}$$

Mathematica [A] time = 0.131689, size = 88, normalized size = 1.02

$$\frac{3\sqrt{12-3e^2x^2}\left(2\left(5e^2x^2-8ex-4\right)+3\sqrt{2-ex}(ex+2)^2\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)\right)}{8e(ex-2)(ex+2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(12 - 3*e^2*x^2)^(3/2)/(2 + e*x)^(9/2),x]
```

```
[Out] (3*Sqrt[12 - 3*e^2*x^2]*(2*(-4 - 8*e*x + 5*e^2*x^2) + 3*Sqrt[2 - e*x]*(2 +
e*x)^2*ArcTanh[Sqrt[2 - e*x]/2]))/(8*e*(-2 + e*x)*(2 + e*x)^(5/2))
```

Maple [A] time = 0.132, size = 126, normalized size = 1.5

$$-\frac{3\sqrt{3}}{8e}\sqrt{-e^2x^2+4}\left(3\sqrt{3}\text{Artanh}\left(1/6\sqrt{3}\sqrt{-3ex+6}\right)x^2e^2+12\sqrt{3}\text{Artanh}\left(1/6\sqrt{3}\sqrt{-3ex+6}\right)xe-10xe\sqrt{-3ex+6}+1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(9/2),x)
```

```
[Out] -3/8*(-e^2*x^2+4)^(1/2)*(3*3^(1/2)*arctanh(1/6*3^(1/2)*(-3*e*x+6)^(1/2))*x^
2*e^2+12*3^(1/2)*arctanh(1/6*3^(1/2)*(-3*e*x+6)^(1/2))*x*e-10*x*e*(-3*e*x+6
)^(1/2)+12*3^(1/2)*arctanh(1/6*3^(1/2)*(-3*e*x+6)^(1/2))-4*(-3*e*x+6)^(1/2)
)*3^(1/2)/((e*x+2)^5)^(1/2)/(-3*e*x+6)^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-3e^2x^2+12\right)^{\frac{3}{2}}}{(ex+2)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((-3*e^2*x^2 + 12)^(3/2)/(e*x + 2)^(9/2), x)
```

Fricas [B] time = 1.87931, size = 332, normalized size = 3.86

$$\frac{3\left(3\sqrt{3}\left(e^{3}x^{3}+6e^{2}x^{2}+12ex+8\right)\log\left(-\frac{3e^{2}x^{2}-12ex+4\sqrt{3}\sqrt{-3e^{2}x^{2}+12}\sqrt{ex+2}-36}{e^{2}x^{2}+4ex+4}\right)+4\sqrt{-3e^{2}x^{2}+12}(5ex+2)\sqrt{ex+2}\right)}{16\left(e^{4}x^{3}+6e^{3}x^{2}+12e^{2}x+8e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(9/2),x, algorithm="fricas")
```

```
[Out] 3/16*(3*sqrt(3)*(e^3*x^3 + 6*e^2*x^2 + 12*e*x + 8)*log(-(3*e^2*x^2 - 12*e*x
 + 4*sqrt(3)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2) - 36)/(e^2*x^2 + 4*e*x + 4
)) + 4*sqrt(-3*e^2*x^2 + 12)*(5*e*x + 2)*sqrt(e*x + 2))/(e^4*x^3 + 6*e^3*x^
2 + 12*e^2*x + 8*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e**2*x**2+12)**(3/2)/(e*x+2)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-3e^{2}x^{2}+12\right)^{\frac{3}{2}}}{(ex+2)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((-3*e^2*x^2 + 12)^(3/2)/(e*x + 2)^(9/2), x)
```

3.909 $\int \frac{\left(12-3e^2x^2\right)^{3/2}}{(2+ex)^{11/2}}\,dx$

Optimal. Leaf size=113

$$-\frac{\sqrt{3}(2-ex)^{3/2}}{e(ex+2)^3}-\frac{3\sqrt{3}\sqrt{2-ex}}{32e(ex+2)}+\frac{3\sqrt{3}\sqrt{2-ex}}{4e(ex+2)^2}-\frac{3\sqrt{3}\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{64e}$$

```
[Out] -((Sqrt[3]*(2 - e*x)^(3/2))/(e*(2 + e*x)^3)) + (3*Sqrt[3]*Sqrt[2 - e*x])/(4
*e*(2 + e*x)^2) - (3*Sqrt[3]*Sqrt[2 - e*x])/(32*e*(2 + e*x)) - (3*Sqrt[3]*A
rcTanh[Sqrt[2 - e*x]/2])/(64*e)
```

Rubi [A] time = 0.0389677, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {627, 47, 51, 63, 206}

$$-\frac{\sqrt{3}(2-ex)^{3/2}}{e(ex+2)^3}-\frac{3\sqrt{3}\sqrt{2-ex}}{32e(ex+2)}+\frac{3\sqrt{3}\sqrt{2-ex}}{4e(ex+2)^2}-\frac{3\sqrt{3}\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{64e}$$

Antiderivative was successfully verified.

```
[In] Int[(12 - 3*e^2*x^2)^(3/2)/(2 + e*x)^(11/2),x]
```

```
[Out] -((Sqrt[3]*(2 - e*x)^(3/2))/(e*(2 + e*x)^3)) + (3*Sqrt[3]*Sqrt[2 - e*x])/(4
*e*(2 + e*x)^2) - (3*Sqrt[3]*Sqrt[2 - e*x])/(32*e*(2 + e*x)) - (3*Sqrt[3]*A
rcTanh[Sqrt[2 - e*x]/2])/(64*e)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{11/2}}\,dx &= \int \frac{(6-3ex)^{3/2}}{(2+ex)^4}\,dx \\
&= -\frac{\sqrt{3}(2-ex)^{3/2}}{e(2+ex)^3} - \frac{3}{2}\int \frac{\sqrt{6-3ex}}{(2+ex)^3}\,dx \\
&= -\frac{\sqrt{3}(2-ex)^{3/2}}{e(2+ex)^3} + \frac{3\sqrt{3}\sqrt{2-ex}}{4e(2+ex)^2} + \frac{9}{8}\int \frac{1}{\sqrt{6-3ex}(2+ex)^2}\,dx \\
&= -\frac{\sqrt{3}(2-ex)^{3/2}}{e(2+ex)^3} + \frac{3\sqrt{3}\sqrt{2-ex}}{4e(2+ex)^2} - \frac{3\sqrt{3}\sqrt{2-ex}}{32e(2+ex)} + \frac{9}{64}\int \frac{1}{\sqrt{6-3ex}(2+ex)}\,dx \\
&= -\frac{\sqrt{3}(2-ex)^{3/2}}{e(2+ex)^3} + \frac{3\sqrt{3}\sqrt{2-ex}}{4e(2+ex)^2} - \frac{3\sqrt{3}\sqrt{2-ex}}{32e(2+ex)} - \frac{3\,\text{Subst}\left(\int \frac{1}{4-\frac{x^2}{3}}\,dx, x, \sqrt{6-3ex}\right)}{32e} \\
&= -\frac{\sqrt{3}(2-ex)^{3/2}}{e(2+ex)^3} + \frac{3\sqrt{3}\sqrt{2-ex}}{4e(2+ex)^2} - \frac{3\sqrt{3}\sqrt{2-ex}}{32e(2+ex)} - \frac{3\sqrt{3}\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{64e}
\end{aligned}$$

Mathematica [C] time = 0.0846051, size = 55, normalized size = 0.49

$$-\frac{3(ex-2)^2\sqrt{12-3e^2x^2}\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{1}{2}-\frac{ex}{4}\right)}{640e\sqrt{ex+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(12 - 3*e^2*x^2)^(3/2)/(2 + e*x)^(11/2),x]
```

```
[Out] (-3*(-2 + e*x)^2*Sqrt[12 - 3*e^2*x^2]*Hypergeometric2F1[5/2, 4, 7/2, 1/2 -
(e*x)/4])/(640*e*Sqrt[2 + e*x])
```

Maple [A] time = 0.154, size = 167, normalized size = 1.5

$$-\frac{\sqrt{3}}{64\,e}\sqrt{-e^2x^2+4}\left(3\sqrt{3}\text{Artanh}\left(1/6\sqrt{3}\sqrt{-3\,ex+6}\right)x^3e^3+18\sqrt{3}\text{Artanh}\left(1/6\sqrt{3}\sqrt{-3\,ex+6}\right)x^2e^2+6\,x^2e^2\sqrt{-3\,ex+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(11/2),x)
```

```
[Out] -1/64*(-e^2*x^2+4)^(1/2)*(3*3^(1/2)*arctanh(1/6*3^(1/2)*(-3*e*x+6)^(1/2))*x
^3*e^3+18*3^(1/2)*arctanh(1/6*3^(1/2)*(-3*e*x+6)^(1/2))*x^2*e^2+6*x^2*e^2*(
```

```
-3*e*x+6)^(1/2)+36*3^(1/2)*arctanh(1/6*3^(1/2)*(-3*e*x+6)^(1/2))*x*e-88*x*e
*(-3*e*x+6)^(1/2)+24*3^(1/2)*arctanh(1/6*3^(1/2)*(-3*e*x+6)^(1/2))+56*(-3*e
*x+6)^(1/2))*3^(1/2)/((e*x+2)^7)^(1/2)/(-3*e*x+6)^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-3\,e^2x^2+12\right)^{\frac{3}{2}}}{(ex+2)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((-3*e^2*x^2 + 12)^(3/2)/(e*x + 2)^(11/2), x)
```

Fricas [A] time = 1.85877, size = 390, normalized size = 3.45

$$\frac{3\sqrt{3}\left(e^4x^4+8\,e^3x^3+24\,e^2x^2+32\,ex+16\right)\log\left(-\frac{3\,e^2x^2-12\,ex+4\,\sqrt{3}\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}-36}{e^2x^2+4\,ex+4}\right)-4\left(3\,e^2x^2-44\,ex+28\right)\sqrt{-3\,e^2x^2+}}{128\left(e^5x^4+8\,e^4x^3+24\,e^3x^2+32\,e^2x+16\,e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(11/2),x, algorithm="fricas")
```

```
[Out] 1/128*(3*sqrt(3)*(e^4*x^4 + 8*e^3*x^3 + 24*e^2*x^2 + 32*e*x + 16)*log(-(3*e
^2*x^2 - 12*e*x + 4*sqrt(3)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2) - 36)/(e^2*
x^2 + 4*e*x + 4)) - 4*(3*e^2*x^2 - 44*e*x + 28)*sqrt(-3*e^2*x^2 + 12)*sqrt(
e*x + 2))/(e^5*x^4 + 8*e^4*x^3 + 24*e^3*x^2 + 32*e^2*x + 16*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e**2*x**2+12)**(3/2)/(e*x+2)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-3\,e^2x^2+12\right)^{\frac{3}{2}}}{(ex+2)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(11/2),x, algorithm="giac")

[Out] integrate((-3*e^2*x^2 + 12)^(3/2)/(e*x + 2)^(11/2), x)
```

3.910 $\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{13/2}}\,dx$

Optimal. Leaf size=144

$$-\frac{3\sqrt{3}(2-ex)^{3/2}}{4e(ex+2)^4}-\frac{9\sqrt{3}\sqrt{2-ex}}{1024e(ex+2)}-\frac{3\sqrt{3}\sqrt{2-ex}}{128e(ex+2)^2}+\frac{3\sqrt{3}\sqrt{2-ex}}{8e(ex+2)^3}-\frac{9\sqrt{3}\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{2048e}$$

```
[Out] (-3*Sqrt[3]*(2 - e*x)^(3/2))/(4*e*(2 + e*x)^4) + (3*Sqrt[3]*Sqrt[2 - e*x])/
(8*e*(2 + e*x)^3) - (3*Sqrt[3]*Sqrt[2 - e*x])/(128*e*(2 + e*x)^2) - (9*Sqrt
[3]*Sqrt[2 - e*x])/(1024*e*(2 + e*x)) - (9*Sqrt[3]*ArcTanh[Sqrt[2 - e*x]/2]
)/(2048*e)
```

Rubi [A] time = 0.0516731, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {627, 47, 51, 63, 206}

$$-\frac{3\sqrt{3}(2-ex)^{3/2}}{4e(ex+2)^4}-\frac{9\sqrt{3}\sqrt{2-ex}}{1024e(ex+2)}-\frac{3\sqrt{3}\sqrt{2-ex}}{128e(ex+2)^2}+\frac{3\sqrt{3}\sqrt{2-ex}}{8e(ex+2)^3}-\frac{9\sqrt{3}\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{2048e}$$

Antiderivative was successfully verified.

```
[In] Int[(12 - 3*e^2*x^2)^(3/2)/(2 + e*x)^(13/2),x]
```

```
[Out] (-3*Sqrt[3]*(2 - e*x)^(3/2))/(4*e*(2 + e*x)^4) + (3*Sqrt[3]*Sqrt[2 - e*x])/
(8*e*(2 + e*x)^3) - (3*Sqrt[3]*Sqrt[2 - e*x])/(128*e*(2 + e*x)^2) - (9*Sqrt
[3]*Sqrt[2 - e*x])/(1024*e*(2 + e*x)) - (9*Sqrt[3]*ArcTanh[Sqrt[2 - e*x]/2]
)/(2048*e)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(12-3e^2x^2\right)^{3/2}}{(2+ex)^{13/2}}\,dx &= \int \frac{(6-3ex)^{3/2}}{(2+ex)^5}\,dx\\
&= -\frac{3\sqrt{3}(2-ex)^{3/2}}{4e(2+ex)^4}-\frac{9}{8}\int \frac{\sqrt{6-3ex}}{(2+ex)^4}\,dx\\
&= -\frac{3\sqrt{3}(2-ex)^{3/2}}{4e(2+ex)^4}+\frac{3\sqrt{3}\sqrt{2-ex}}{8e(2+ex)^3}+\frac{9}{16}\int \frac{1}{\sqrt{6-3ex}(2+ex)^3}\,dx\\
&= -\frac{3\sqrt{3}(2-ex)^{3/2}}{4e(2+ex)^4}+\frac{3\sqrt{3}\sqrt{2-ex}}{8e(2+ex)^3}-\frac{3\sqrt{3}\sqrt{2-ex}}{128e(2+ex)^2}+\frac{27}{256}\int \frac{1}{\sqrt{6-3ex}(2+ex)^2}\,dx\\
&= -\frac{3\sqrt{3}(2-ex)^{3/2}}{4e(2+ex)^4}+\frac{3\sqrt{3}\sqrt{2-ex}}{8e(2+ex)^3}-\frac{3\sqrt{3}\sqrt{2-ex}}{128e(2+ex)^2}-\frac{9\sqrt{3}\sqrt{2-ex}}{1024e(2+ex)}+\frac{27\int \frac{1}{\sqrt{6-3ex}(2+ex)}\,dx}{2048}\\
&= -\frac{3\sqrt{3}(2-ex)^{3/2}}{4e(2+ex)^4}+\frac{3\sqrt{3}\sqrt{2-ex}}{8e(2+ex)^3}-\frac{3\sqrt{3}\sqrt{2-ex}}{128e(2+ex)^2}-\frac{9\sqrt{3}\sqrt{2-ex}}{1024e(2+ex)}-\frac{9\,\mathrm{Subst}\left(\int \frac{1}{4-\frac{x^2}{3}}\,dx,x,\sqrt{\right.}{1024e}\\
&= -\frac{3\sqrt{3}(2-ex)^{3/2}}{4e(2+ex)^4}+\frac{3\sqrt{3}\sqrt{2-ex}}{8e(2+ex)^3}-\frac{3\sqrt{3}\sqrt{2-ex}}{128e(2+ex)^2}-\frac{9\sqrt{3}\sqrt{2-ex}}{1024e(2+ex)}-\frac{9\sqrt{3}\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{2048e}
\end{aligned}$$

Mathematica [C] time = 0.0846702, size = 55, normalized size = 0.38

$$-\frac{3(ex-2)^2\sqrt{12-3e^2x^2}\,{}_2F_1\left(\frac{5}{2},5;\frac{7}{2};\frac{1}{2}-\frac{ex}{4}\right)}{2560e\sqrt{ex+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(12 - 3*e^2*x^2)^(3/2)/(2 + e*x)^(13/2),x]
```

```
[Out] (-3*(-2 + e*x)^2*Sqrt[12 - 3*e^2*x^2]*Hypergeometric2F1[5/2, 5, 7/2, 1/2 -
(e*x)/4])/(2560*e*Sqrt[2 + e*x])
```

Maple [A] time = 0.149, size = 208, normalized size = 1.4

$$-\frac{3\sqrt{3}}{2048\,e}\sqrt{-e^2x^2+4}\left(3\sqrt{3}\mathrm{Artanh}\left(1/6\sqrt{3}\sqrt{-3ex+6}\right)x^4e^4+24\sqrt{3}\mathrm{Artanh}\left(1/6\sqrt{3}\sqrt{-3ex+6}\right)x^3e^3+6x^3e^3\sqrt{-3ex\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(13/2),x)
```

```
[Out] -3/2048*(-e^2*x^2+4)^(1/2)*(3*3^(1/2)*arctanh(1/6*3^(1/2)*(-3*e*x+6)^(1/2))
*x^4*e^4+24*3^(1/2)*arctanh(1/6*3^(1/2)*(-3*e*x+6)^(1/2))*x^3*e^3+6*x^3*e^3
*(-3*e*x+6)^(1/2)+72*3^(1/2)*arctanh(1/6*3^(1/2)*(-3*e*x+6)^(1/2))*x^2*e^2+
52*x^2*e^2*(-3*e*x+6)^(1/2)+96*3^(1/2)*arctanh(1/6*3^(1/2)*(-3*e*x+6)^(1/2)
)*x*e-632*x*e*(-3*e*x+6)^(1/2)+48*3^(1/2)*arctanh(1/6*3^(1/2)*(-3*e*x+6)^(1
/2))+624*(-3*e*x+6)^(1/2))*3^(1/2)/((e*x+2)^9)^(1/2)/(-3*e*x+6)^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-3\,e^2x^2+12\right)^{\frac{3}{2}}}{(ex+2)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(13/2),x, algorithm="maxima")
```

```
[Out] integrate((-3*e^2*x^2 + 12)^(3/2)/(e*x + 2)^(13/2), x)
```

Fricas [A] time = 1.77994, size = 450, normalized size = 3.12

$$\frac{3\left(3\,\sqrt{3}\left(e^5x^5+10\,e^4x^4+40\,e^3x^3+80\,e^2x^2+80\,ex+32\right)\log\left(-\frac{3\,e^2x^2-12\,ex+4\,\sqrt{3}\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}-36}{e^2x^2+4\,ex+4}\right)-4\left(3\,e^3x^3+26\,e^2x^2-\right.}{4096\left(e^6x^5+10\,e^5x^4+40\,e^4x^3+80\,e^3x^2+80\,e^2x+32\,e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(13/2),x, algorithm="fricas")
```

```
[Out] 3/4096*(3*sqrt(3)*(e^5*x^5 + 10*e^4*x^4 + 40*e^3*x^3 + 80*e^2*x^2 + 80*e*x
+ 32)*log(-(3*e^2*x^2 - 12*e*x + 4*sqrt(3)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x +
 2) - 36)/(e^2*x^2 + 4*e*x + 4)) - 4*(3*e^3*x^3 + 26*e^2*x^2 - 316*e*x + 31
2)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2))/(e^6*x^5 + 10*e^5*x^4 + 40*e^4*x^3
+ 80*e^3*x^2 + 80*e^2*x + 32*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e**2*x**2+12)**(3/2)/(e*x+2)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-3\,e^2x^2+12\right)^{\frac{3}{2}}}{\left(ex+2\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(13/2),x, algorithm="giac")
```

```
[Out] integrate((-3*e^2*x^2 + 12)^(3/2)/(e*x + 2)^(13/2), x)
```

3.911 $\int \frac{(2+ex)^{7/2}}{\sqrt{12-3e^2x^2}}\,dx$

Optimal. Leaf size=85

$$\frac{2(2-ex)^{7/2}}{7\sqrt{3}e}-\frac{8\sqrt{3}(2-ex)^{5/2}}{5e}+\frac{32(2-ex)^{3/2}}{\sqrt{3}e}-\frac{128\sqrt{2-ex}}{\sqrt{3}e}$$

```
[Out] (-128*Sqrt[2 - e*x])/(Sqrt[3]*e) + (32*(2 - e*x)^(3/2))/(Sqrt[3]*e) - (8*Sq
rt[3]*(2 - e*x)^(5/2))/(5*e) + (2*(2 - e*x)^(7/2))/(7*Sqrt[3]*e)
```

Rubi [A] time = 0.0233863, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {627, 43}

$$\frac{2(2-ex)^{7/2}}{7\sqrt{3}e}-\frac{8\sqrt{3}(2-ex)^{5/2}}{5e}+\frac{32(2-ex)^{3/2}}{\sqrt{3}e}-\frac{128\sqrt{2-ex}}{\sqrt{3}e}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + e*x)^(7/2)/Sqrt[12 - 3*e^2*x^2],x]
```

```
[Out] (-128*Sqrt[2 - e*x])/(Sqrt[3]*e) + (32*(2 - e*x)^(3/2))/(Sqrt[3]*e) - (8*Sq
rt[3]*(2 - e*x)^(5/2))/(5*e) + (2*(2 - e*x)^(7/2))/(7*Sqrt[3]*e)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(2+ex)^{7/2}}{\sqrt{12-3e^2x^2}}\,dx&=\int \frac{(2+ex)^3}{\sqrt{6-3ex}}\,dx\\&=\int \left(\frac{64}{\sqrt{6-3ex}}-16\sqrt{6-3ex}+\frac{4}{3}(6-3ex)^{3/2}-\frac{1}{27}(6-3ex)^{5/2}\right)dx\\&=-\frac{128\sqrt{2-ex}}{\sqrt{3}e}+\frac{32(2-ex)^{3/2}}{\sqrt{3}e}-\frac{8\sqrt{3}(2-ex)^{5/2}}{5e}+\frac{2(2-ex)^{7/2}}{7\sqrt{3}e}\end{aligned}$$

Mathematica [A] time = 0.0827437, size = 57, normalized size = 0.67

$$\frac{2(ex-2)\sqrt{ex+2}\left(5e^3x^3+54e^2x^2+284ex+1416\right)}{35e\sqrt{12-3e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + e*x)^(7/2)/Sqrt[12 - 3*e^2*x^2],x]
```

```
[Out] (2*(-2 + e*x)*Sqrt[2 + e*x]*(1416 + 284*e*x + 54*e^2*x^2 + 5*e^3*x^3))/(35*
e*Sqrt[12 - 3*e^2*x^2])
```

Maple [A] time = 0.043, size = 52, normalized size = 0.6

$$\frac{(2ex-4)\left(5e^3x^3+54e^2x^2+284ex+1416\right)}{35e}\sqrt{ex+2}\frac{1}{\sqrt{-3e^2x^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+2)^(7/2)/(-3*e^2*x^2+12)^(1/2),x)
```

```
[Out] 2/35*(e*x-2)*(5*e^3*x^3+54*e^2*x^2+284*e*x+1416)*(e*x+2)^(1/2)/e/(-3*e^2*x^
2+12)^(1/2)
```

Maxima [C] time = 1.90065, size = 61, normalized size = 0.72

$$-\frac{2i\sqrt{3}\left(5e^4x^4+44e^3x^3+176e^2x^2+848ex-2832\right)}{105\sqrt{ex-2}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(7/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/105*I*sqrt(3)*(5*e^4*x^4 + 44*e^3*x^3 + 176*e^2*x^2 + 848*e*x - 2832)/(s
qrt(e*x - 2)*e)
```

Fricas [A] time = 1.81925, size = 135, normalized size = 1.59

$$-\frac{2\left(5e^3x^3+54e^2x^2+284ex+1416\right)\sqrt{-3e^2x^2+12}\sqrt{ex+2}}{105\left(e^2x+2e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(7/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/105*(5*e^3*x^3 + 54*e^2*x^2 + 284*e*x + 1416)*sqrt(-3*e^2*x^2 + 12)*sqrt
(e*x + 2)/(e^2*x + 2*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)**(7/2)/(-3*e**2*x**2+12)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+2)^{\frac{7}{2}}}{\sqrt{-3\,e^2x^2+12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(7/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + 2)^(7/2)/sqrt(-3*e^2*x^2 + 12), x)
```

3.912 $\int \frac{(2+ex)^{5/2}}{\sqrt{12-3e^2x^2}}\,dx$

Optimal. Leaf size=65

$$-\frac{2(2-ex)^{5/2}}{5\sqrt{3}e}+\frac{16(2-ex)^{3/2}}{3\sqrt{3}e}-\frac{32\sqrt{2-ex}}{\sqrt{3}e}$$

```
[Out] (-32*Sqrt[2 - e*x])/(Sqrt[3]*e) + (16*(2 - e*x)^(3/2))/(3*Sqrt[3]*e) - (2*(
2 - e*x)^(5/2))/(5*Sqrt[3]*e)
```

Rubi [A] time = 0.0199924, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {627, 43}

$$-\frac{2(2-ex)^{5/2}}{5\sqrt{3}e}+\frac{16(2-ex)^{3/2}}{3\sqrt{3}e}-\frac{32\sqrt{2-ex}}{\sqrt{3}e}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + e*x)^(5/2)/Sqrt[12 - 3*e^2*x^2],x]
```

```
[Out] (-32*Sqrt[2 - e*x])/(Sqrt[3]*e) + (16*(2 - e*x)^(3/2))/(3*Sqrt[3]*e) - (2*(
2 - e*x)^(5/2))/(5*Sqrt[3]*e)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+ex)^{5/2}}{\sqrt{12-3e^2x^2}}\,dx &= \int \frac{(2+ex)^2}{\sqrt{6-3ex}}\,dx\\
&= \int \left(\frac{16}{\sqrt{6-3ex}}-\frac{8}{3}\sqrt{6-3ex}+\frac{1}{9}(6-3ex)^{3/2}\right)dx\\
&= -\frac{32\sqrt{2-ex}}{\sqrt{3}e}+\frac{16(2-ex)^{3/2}}{3\sqrt{3}e}-\frac{2(2-ex)^{5/2}}{5\sqrt{3}e}
\end{aligned}$$

Mathematica [A] time = 0.0591318, size = 49, normalized size = 0.75

$$\frac{2(ex-2)\sqrt{ex+2}\left(3e^2x^2+28ex+172\right)}{15e\sqrt{12-3e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + e*x)^(5/2)/Sqrt[12 - 3*e^2*x^2],x]
```

```
[Out] (2*(-2 + e*x)*Sqrt[2 + e*x]*(172 + 28*e*x + 3*e^2*x^2))/(15*e*Sqrt[12 - 3*e
^2*x^2])
```

Maple [A] time = 0.043, size = 44, normalized size = 0.7

$$\frac{(2\,ex-4)\left(3\,e^2x^2+28\,ex+172\right)}{15\,e}\sqrt{ex+2}\frac{1}{\sqrt{-3\,e^2x^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/2),x)
```

```
[Out] 2/15*(e*x-2)*(3*e^2*x^2+28*e*x+172)*(e*x+2)^(1/2)/e/(-3*e^2*x^2+12)^(1/2)
```

Maxima [C] time = 1.88108, size = 63, normalized size = 0.97

$$-\frac{6i\,\sqrt{3}e^3x^3+44i\,\sqrt{3}e^2x^2+232i\,\sqrt{3}ex-688i\,\sqrt{3}}{45\,\sqrt{ex-2}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/45*(6*I*sqrt(3)*e^3*x^3 + 44*I*sqrt(3)*e^2*x^2 + 232*I*sqrt(3)*e*x - 688
*I*sqrt(3))/(sqrt(e*x - 2)*e)
```

Fricas [A] time = 1.82014, size = 113, normalized size = 1.74

$$-\frac{2\left(3\,e^2x^2+28\,ex+172\right)\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}}{45\left(e^2x+2\,e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/45*(3*e^2*x^2 + 28*e*x + 172)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2)/(e^2*x
 + 2*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)**(5/2)/(-3*e**2*x**2+12)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+2)^{\frac{5}{2}}}{\sqrt{-3\,e^2x^2+12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + 2)^(5/2)/sqrt(-3*e^2*x^2 + 12), x)
```

3.913 $\int \frac{(2+ex)^{3/2}}{\sqrt{12-3e^2x^2}} \, dx$

Optimal. Leaf size=43

$$\frac{2(2-ex)^{3/2}}{3\sqrt{3}e}-\frac{8\sqrt{2-ex}}{\sqrt{3}e}$$

```
[Out] (-8*Sqrt[2 - e*x])/(Sqrt[3]*e) + (2*(2 - e*x)^(3/2))/(3*Sqrt[3]*e)
```

Rubi [A] time = 0.0167946, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {627, 43}

$$\frac{2(2-ex)^{3/2}}{3\sqrt{3}e}-\frac{8\sqrt{2-ex}}{\sqrt{3}e}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + e*x)^(3/2)/Sqrt[12 - 3*e^2*x^2],x]
```

```
[Out] (-8*Sqrt[2 - e*x])/(Sqrt[3]*e) + (2*(2 - e*x)^(3/2))/(3*Sqrt[3]*e)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(2+ex)^{3/2}}{\sqrt{12-3e^2x^2}} \, dx &= \int \frac{2+ex}{\sqrt{6-3ex}} \, dx\\ &= \int \left(\frac{4}{\sqrt{6-3ex}}-\frac{1}{3}\sqrt{6-3ex}\right) dx\\ &= -\frac{8\sqrt{2-ex}}{\sqrt{3}e}+\frac{2(2-ex)^{3/2}}{3\sqrt{3}e}\end{aligned}$$

Mathematica [A] time = 0.0506008, size = 40, normalized size = 0.93

$$\frac{2(ex-2)\sqrt{ex+2}(ex+10)}{3e\sqrt{12-3e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + e*x)^(3/2)/Sqrt[12 - 3*e^2*x^2],x]
```

```
[Out] (2*(-2 + e*x)*Sqrt[2 + e*x]*(10 + e*x))/(3*e*Sqrt[12 - 3*e^2*x^2])
```

Maple [A] time = 0.04, size = 35, normalized size = 0.8

$$\frac{(2ex-4)(ex+10)}{3e}\sqrt{ex+2}\frac{1}{\sqrt{-3e^2x^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/2),x)
```

```
[Out] 2/3*(e*x-2)*(e*x+10)*(e*x+2)^(1/2)/e/(-3*e^2*x^2+12)^(1/2)
```

Maxima [C] time = 1.96245, size = 38, normalized size = 0.88

$$-\frac{2i\sqrt{3}\left(e^2x^2+8ex-20\right)}{9\sqrt{ex-2}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/9*I*sqrt(3)*(e^2*x^2 + 8*e*x - 20)/(sqrt(e*x - 2)*e)
```

Fricas [A] time = 1.72781, size = 90, normalized size = 2.09

$$-\frac{2\sqrt{-3e^2x^2+12}(ex+10)\sqrt{ex+2}}{9\left(e^2x+2e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/9*sqrt(-3*e^2*x^2 + 12)*(e*x + 10)*sqrt(e*x + 2)/(e^2*x + 2*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{3}\left(\int\frac{2\sqrt{ex+2}}{\sqrt{-e^2x^2+4}}\,dx+\int\frac{ex\sqrt{ex+2}}{\sqrt{-e^2x^2+4}}\,dx\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)**(3/2)/(-3*e**2*x**2+12)**(1/2),x)
```

```
[Out] sqrt(3)*(Integral(2*sqrt(e*x + 2)/sqrt(-e**2*x**2 + 4), x) + Integral(e*x*s
qrt(e*x + 2)/sqrt(-e**2*x**2 + 4), x))/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+2)^{\frac{3}{2}}}{\sqrt{-3\,e^2x^2+12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + 2)^(3/2)/sqrt(-3*e^2*x^2 + 12), x)
```

3.914 $\int \frac{\sqrt{2+ex}}{\sqrt{12-3e^2x^2}}\,dx$

Optimal. Leaf size=20

$$-\frac{2\sqrt{2-ex}}{\sqrt{3}e}$$

```
[Out] (-2*Sqrt[2 - e*x])/(Sqrt[3]*e)
```

Rubi [A] time = 0.0085675, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {627, 32}

$$-\frac{2\sqrt{2-ex}}{\sqrt{3}e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + e*x]/Sqrt[12 - 3*e^2*x^2],x]
```

```
[Out] (-2*Sqrt[2 - e*x])/(Sqrt[3]*e)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2+ex}}{\sqrt{12-3e^2x^2}}\,dx &= \int \frac{1}{\sqrt{6-3ex}}\,dx\\ &= -\frac{2\sqrt{2-ex}}{\sqrt{3}e}\end{aligned}$$

Mathematica [A] time = 0.0347937, size = 33, normalized size = 1.65

$$\frac{2(ex-2)\sqrt{ex+2}}{e\sqrt{12-3e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + e*x]/Sqrt[12 - 3*e^2*x^2],x]
```

```
[Out] (2*(-2 + e*x)*Sqrt[2 + e*x])/(e*Sqrt[12 - 3*e^2*x^2])
```

Maple [A] time = 0.039, size = 30, normalized size = 1.5

$$2\,\frac{(ex-2)\sqrt{ex+2}}{e\sqrt{-3\,e^{2}x^{2}+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/2),x)
```

```
[Out] 2*(e*x-2)*(e*x+2)^(1/2)/e/(-3*e^2*x^2+12)^(1/2)
```

Maxima [C] time = 1.73152, size = 34, normalized size = 1.7

$$-\frac{2i\sqrt{3}ex-4i\sqrt{3}}{3\sqrt{ex-2}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(2*I*sqrt(3)*e*x - 4*I*sqrt(3))/(sqrt(e*x - 2)*e)
```

Fricas [A] time = 1.75109, size = 76, normalized size = 3.8

$$-\frac{2\sqrt{-3\,e^{2}x^{2}+12}\sqrt{ex+2}}{3\left(e^{2}x+2\,e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2)/(e^2*x + 2*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{3}\int\frac{\sqrt{ex+2}}{\sqrt{-e^{2}x^{2}+4}}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)**(1/2)/(-3*e**2*x**2+12)**(1/2),x)
```

```
[Out] sqrt(3)*Integral(sqrt(e*x + 2)/sqrt(-e**2*x**2 + 4), x)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{ex+2}}{\sqrt{-3\,e^{2}x^{2}+12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + 2)/sqrt(-3*e^2*x^2 + 12), x)
```

3.915 $\int \frac{1}{\sqrt{2+ex}\sqrt{12-3e^2x^2}}\,dx$

Optimal. Leaf size=25

$$-\frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{\sqrt{3}e}$$

[Out] -(ArcTanh[Sqrt[2 - e*x]/2]/(Sqrt[3]*e))

Rubi [A] time = 0.0153917, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {627, 63, 206}

$$-\frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{\sqrt{3}e}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[2 + e*x]*Sqrt[12 - 3*e^2*x^2]),x]

[Out] -(ArcTanh[Sqrt[2 - e*x]/2]/(Sqrt[3]*e))

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+ex}\sqrt{12-3e^2x^2}}\,dx &= \int \frac{1}{\sqrt{6-3ex}(2+ex)}\,dx\\ &= -\frac{2\,\text{Subst}\left(\int \frac{1}{4-\frac{x^2}{3}}\,dx, x, \sqrt{6-3ex}\right)}{3e}\\ &= -\frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{\sqrt{3}e}\end{aligned}$$

Mathematica [A] time = 0.042496, size = 50, normalized size = 2.

$$\frac{\sqrt{ex-2}\sqrt{ex+2}\tan^{-1}\left(\frac{1}{2}\sqrt{ex-2}\right)}{e\sqrt{12-3e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 + e*x]*Sqrt[12 - 3*e^2*x^2]),x]
```

```
[Out] (Sqrt[-2 + e*x]*Sqrt[2 + e*x]*ArcTan[Sqrt[-2 + e*x]/2])/(e*Sqrt[12 - 3*e^2*
x^2])
```

Maple [B] time = 0.125, size = 50, normalized size = 2.

$$-\frac{\sqrt{3}}{3e}\sqrt{-e^2x^2+4}\operatorname{Artanh}\left(\frac{\sqrt{3}}{6}\sqrt{-3\,ex+6}\right)\frac{1}{\sqrt{ex+2}}\frac{1}{\sqrt{-ex+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/2),x)
```

```
[Out] -1/3*(-e^2*x^2+4)^(1/2)*3^(1/2)*arctanh(1/6*3^(1/2)*(-3*e*x+6)^(1/2))/(e*x+
2)^(1/2)/(-e*x+2)^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2)), x)
```

Fricas [B] time = 1.89868, size = 158, normalized size = 6.32

$$\frac{\sqrt{3}\log\left(-\frac{3\,e^2x^2-12\,ex+4\,\sqrt{3}\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}-36}{e^2x^2+4\,ex+4}\right)}{6\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-(3*e^2*x^2 - 12*e*x + 4*sqrt(3)*sqrt(-3*e^2*x^2 + 12)*sqrt
(e*x + 2) - 36)/(e^2*x^2 + 4*e*x + 4))/e
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{3}\int\frac{1}{\sqrt{ex+2}\sqrt{-e^2x^2+4}}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)**(1/2)/(-3*e**2*x**2+12)**(1/2),x)
```

```
[Out] sqrt(3)*Integral(1/(sqrt(e*x + 2)*sqrt(-e**2*x**2 + 4)), x)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2)), x)
```

3.916 $\int \frac{1}{(2+ex)^{3/2}\sqrt{12-3e^2x^2}}\,dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{2-ex}}{4\sqrt{3}e(ex+2)}-\frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{8\sqrt{3}e}$$

```
[Out] -Sqrt[2 - e*x]/(4*Sqrt[3]*e*(2 + e*x)) - ArcTanh[Sqrt[2 - e*x]/2]/(8*Sqrt[3
]*e)
```

Rubi [A] time = 0.0207186, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {627, 51, 63, 206}

$$-\frac{\sqrt{2-ex}}{4\sqrt{3}e(ex+2)}-\frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{8\sqrt{3}e}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + e*x)^(3/2)*Sqrt[12 - 3*e^2*x^2]),x]
```

```
[Out] -Sqrt[2 - e*x]/(4*Sqrt[3]*e*(2 + e*x)) - ArcTanh[Sqrt[2 - e*x]/2]/(8*Sqrt[3
]*e)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(2+ex)^{3/2}\sqrt{12-3e^2x^2}}\,dx &= \int \frac{1}{\sqrt{6-3ex}(2+ex)^2}\,dx\\ &= -\frac{\sqrt{2-ex}}{4\sqrt{3}e(2+ex)}+\frac{1}{8}\int \frac{1}{\sqrt{6-3ex}(2+ex)}\,dx\\ &= -\frac{\sqrt{2-ex}}{4\sqrt{3}e(2+ex)}-\frac{\text{Subst}\left(\int \frac{1}{4-\frac{x^2}{3}}\,dx,x,\sqrt{6-3ex}\right)}{12e}\\ &= -\frac{\sqrt{2-ex}}{4\sqrt{3}e(2+ex)}-\frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{8\sqrt{3}e}\end{aligned}$$

Mathematica [A] time = 0.0614101, size = 54, normalized size = 0.95

$$\frac{-2\sqrt{2-ex}-(ex+2)\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{8\sqrt{3}e(ex+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2 + e*x)^(3/2)*Sqrt[12 - 3*e^2*x^2]),x]
```

```
[Out] (-2*Sqrt[2 - e*x] - (2 + e*x)*ArcTanh[Sqrt[2 - e*x]/2])/(8*Sqrt[3]*e*(2 + e
*x))
```

Maple [B] time = 0.127, size = 88, normalized size = 1.5

$$-\frac{\sqrt{3}}{24\,e}\sqrt{-e^2x^2+4}\left(\sqrt{3}\text{Artanh}\left(\frac{\sqrt{3}}{6}\sqrt{-3\,ex+6}\right)xe+2\,\sqrt{3}\text{Artanh}\left(1/6\,\sqrt{3}\sqrt{-3\,ex+6}\right)+2\,\sqrt{-3\,ex+6}\right)\frac{1}{\sqrt{(ex+2)^3}}\frac{1}{\sqrt{-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/2),x)
```

```
[Out] -1/24*(-e^2*x^2+4)^(1/2)*(3^(1/2)*arctanh(1/6*3^(1/2)*(-3*e*x+6)^(1/2))*x*e
+2*3^(1/2)*arctanh(1/6*3^(1/2)*(-3*e*x+6)^(1/2))+2*(-3*e*x+6)^(1/2))/((e*x+
2)^3)^(1/2)*3^(1/2)/(-3*e*x+6)^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,e^2x^2+12}(ex+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-3*e^2*x^2 + 12)*(e*x + 2)^(3/2)), x)
```

Fricas [B] time = 1.78991, size = 278, normalized size = 4.88

$$\frac{\sqrt{3}\left(e^{2}x^{2}+4\,ex+4\right)\log\left(-\frac{3\,e^{2}x^{2}-12\,ex+4\,\sqrt{3}\sqrt{-3\,e^{2}x^{2}+12}\sqrt{ex+2}-36}{e^{2}x^{2}+4\,ex+4}\right)-4\,\sqrt{-3\,e^{2}x^{2}+12}\sqrt{ex+2}}{48\,\left(e^{3}x^{2}+4\,e^{2}x+4\,e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*(sqrt(3)*(e^2*x^2 + 4*e*x + 4)*log(-(3*e^2*x^2 - 12*e*x + 4*sqrt(3)*sq
rt(-3*e^2*x^2 + 12)*sqrt(e*x + 2) - 36)/(e^2*x^2 + 4*e*x + 4)) - 4*sqrt(-3*
e^2*x^2 + 12)*sqrt(e*x + 2))/(e^3*x^2 + 4*e^2*x + 4*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{3}\int\frac{1}{ex\sqrt{ex+2}\sqrt{-e^{2}x^{2}+4}+2\sqrt{ex+2}\sqrt{-e^{2}x^{2}+4}}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)**(3/2)/(-3*e**2*x**2+12)**(1/2),x)
```

```
[Out] sqrt(3)*Integral(1/(e*x*sqrt(e*x + 2)*sqrt(-e**2*x**2 + 4) + 2*sqrt(e*x + 2
)*sqrt(-e**2*x**2 + 4)), x)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-3\,e^{2}x^{2}+12}(ex+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-3*e^2*x^2 + 12)*(e*x + 2)^(3/2)), x)
```

3.917 $\int \frac{1}{(2+ex)^{5/2}\sqrt{12-3e^2x^2}}\,dx$

Optimal. Leaf size=86

$$-\frac{\sqrt{3}\sqrt{2-ex}}{64e(ex+2)}-\frac{\sqrt{2-ex}}{8\sqrt{3}e(ex+2)^2}-\frac{\sqrt{3}\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{128e}$$

```
[Out] -Sqrt[2 - e*x]/(8*Sqrt[3]*e*(2 + e*x)^2) - (Sqrt[3]*Sqrt[2 - e*x])/(64*e*(2
 + e*x)) - (Sqrt[3]*ArcTanh[Sqrt[2 - e*x]/2])/(128*e)
```

Rubi [A] time = 0.0278343, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {627, 51, 63, 206}

$$-\frac{\sqrt{3}\sqrt{2-ex}}{64e(ex+2)}-\frac{\sqrt{2-ex}}{8\sqrt{3}e(ex+2)^2}-\frac{\sqrt{3}\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{128e}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + e*x)^(5/2)*Sqrt[12 - 3*e^2*x^2]),x]
```

```
[Out] -Sqrt[2 - e*x]/(8*Sqrt[3]*e*(2 + e*x)^2) - (Sqrt[3]*Sqrt[2 - e*x])/(64*e*(2
 + e*x)) - (Sqrt[3]*ArcTanh[Sqrt[2 - e*x]/2])/(128*e)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(2+ex)^{5/2}\sqrt{12-3e^2x^2}}\,dx = \int \frac{1}{\sqrt{6-3ex}(2+ex)^3}\,dx$$
$$= -\frac{\sqrt{2-ex}}{8\sqrt{3}e(2+ex)^2} + \frac{3}{16}\int \frac{1}{\sqrt{6-3ex}(2+ex)^2}\,dx$$
$$= -\frac{\sqrt{2-ex}}{8\sqrt{3}e(2+ex)^2} - \frac{\sqrt{3}\sqrt{2-ex}}{64e(2+ex)} + \frac{3}{128}\int \frac{1}{\sqrt{6-3ex}(2+ex)}\,dx$$
$$= -\frac{\sqrt{2-ex}}{8\sqrt{3}e(2+ex)^2} - \frac{\sqrt{3}\sqrt{2-ex}}{64e(2+ex)} - \frac{\text{Subst}\left(\int \frac{1}{4-\frac{x^2}{3}}\,dx, x, \sqrt{6-3ex}\right)}{64e}$$
$$= -\frac{\sqrt{2-ex}}{8\sqrt{3}e(2+ex)^2} - \frac{\sqrt{3}\sqrt{2-ex}}{64e(2+ex)} - \frac{\sqrt{3}\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{128e}$$

Mathematica [C] time = 0.0608295, size = 53, normalized size = 0.62

$$\frac{(ex-2)\sqrt{ex+2}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{1}{2}-\frac{ex}{4}\right)}{32e\sqrt{12-3e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2 + e*x)^(5/2)*Sqrt[12 - 3*e^2*x^2]),x]
```

```
[Out] ((-2 + e*x)*Sqrt[2 + e*x]*Hypergeometric2F1[1/2, 3, 3/2, 1/2 - (e*x)/4])/(3
2*e*Sqrt[12 - 3*e^2*x^2])
```

Maple [A] time = 0.128, size = 126, normalized size = 1.5

$$-\frac{\sqrt{3}}{384\,e}\sqrt{-e^2x^2+4}\left(3\sqrt{3}\text{Artanh}\left(1/6\sqrt{3}\sqrt{-3\,ex+6}\right)x^2e^2+12\sqrt{3}\text{Artanh}\left(1/6\sqrt{3}\sqrt{-3\,ex+6}\right)xe+6\,xe\sqrt{-3\,ex+6}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/2),x)
```

```
[Out] -1/384*(-e^2*x^2+4)^(1/2)*(3*3^(1/2)*arctanh(1/6*3^(1/2)*(-3*e*x+6)^(1/2))*
x^2*e^2+12*3^(1/2)*arctanh(1/6*3^(1/2)*(-3*e*x+6)^(1/2))*x*e+6*x*e*(-3*e*x+
6)^(1/2)+12*3^(1/2)*arctanh(1/6*3^(1/2)*(-3*e*x+6)^(1/2))+28*(-3*e*x+6)^(1/
2))/((e*x+2)^5)^(1/2)*3^(1/2)/(-3*e*x+6)^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,e^2x^2+12}(ex+2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-3*e^2*x^2 + 12)*(e*x + 2)^(5/2)), x)
```

Fricas [B] time = 1.90118, size = 335, normalized size = 3.9

$$\frac{3\sqrt{3}\left(e^{3}x^{3}+6\,e^{2}x^{2}+12\,ex+8\right)\log\left(-\frac{3\,e^{2}x^{2}-12\,ex+4\,\sqrt{3}\sqrt{-3\,e^{2}x^{2}+12}\sqrt{ex+2}-36}{e^{2}x^{2}+4\,ex+4}\right)-4\,\sqrt{-3\,e^{2}x^{2}+12}(3\,ex+14)\sqrt{ex+2}}{768\left(e^{4}x^{3}+6\,e^{3}x^{2}+12\,e^{2}x+8\,e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(3*sqrt(3)*(e^3*x^3 + 6*e^2*x^2 + 12*e*x + 8)*log(-(3*e^2*x^2 - 12*e*
x + 4*sqrt(3)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2) - 36)/(e^2*x^2 + 4*e*x +
4)) - 4*sqrt(-3*e^2*x^2 + 12)*(3*e*x + 14)*sqrt(e*x + 2))/(e^4*x^3 + 6*e^3*
x^2 + 12*e^2*x + 8*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)**(5/2)/(-3*e**2*x**2+12)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.918 $\int \frac{(2+ex)^{11/2}}{(12-3e^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=111

$$-\frac{2(2-ex)^{7/2}}{21\sqrt{3}e}+\frac{32(2-ex)^{5/2}}{15\sqrt{3}e}-\frac{64(2-ex)^{3/2}}{3\sqrt{3}e}+\frac{512\sqrt{2-ex}}{3\sqrt{3}e}+\frac{512}{3\sqrt{3}e\sqrt{2-ex}}$$

```
[Out] 512/(3*Sqrt[3]*e*Sqrt[2 - e*x]) + (512*Sqrt[2 - e*x])/(3*Sqrt[3]*e) - (64*(
2 - e*x)^(3/2))/(3*Sqrt[3]*e) + (32*(2 - e*x)^(5/2))/(15*Sqrt[3]*e) - (2*(2
 - e*x)^(7/2))/(21*Sqrt[3]*e)
```

Rubi [A] time = 0.0285124, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {627, 43}

$$-\frac{2(2-ex)^{7/2}}{21\sqrt{3}e}+\frac{32(2-ex)^{5/2}}{15\sqrt{3}e}-\frac{64(2-ex)^{3/2}}{3\sqrt{3}e}+\frac{512\sqrt{2-ex}}{3\sqrt{3}e}+\frac{512}{3\sqrt{3}e\sqrt{2-ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + e*x)^(11/2)/(12 - 3*e^2*x^2)^(3/2),x]
```

```
[Out] 512/(3*Sqrt[3]*e*Sqrt[2 - e*x]) + (512*Sqrt[2 - e*x])/(3*Sqrt[3]*e) - (64*(
2 - e*x)^(3/2))/(3*Sqrt[3]*e) + (32*(2 - e*x)^(5/2))/(15*Sqrt[3]*e) - (2*(2
 - e*x)^(7/2))/(21*Sqrt[3]*e)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(2+ex)^{11/2}}{(12-3e^2x^2)^{3/2}}\,dx &= \int \frac{(2+ex)^4}{(6-3ex)^{3/2}}\,dx\\ &= \int \left(\frac{256}{(6-3ex)^{3/2}}-\frac{256}{3\sqrt{6-3ex}}+\frac{32}{3}\sqrt{6-3ex}-\frac{16}{27}(6-3ex)^{3/2}+\frac{1}{81}(6-3ex)^{5/2}\right)dx\\ &= \frac{512}{3\sqrt{3}e\sqrt{2-ex}}+\frac{512\sqrt{2-ex}}{3\sqrt{3}e}-\frac{64(2-ex)^{3/2}}{3\sqrt{3}e}+\frac{32(2-ex)^{5/2}}{15\sqrt{3}e}-\frac{2(2-ex)^{7/2}}{21\sqrt{3}e}\end{aligned}$$

Mathematica [A] time = 0.101524, size = 60, normalized size = 0.54

$$-\frac{2\sqrt{ex+2}\left(5e^4x^4+72e^3x^3+568e^2x^2+5664ex-23216\right)}{105e\sqrt{12-3e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + e*x)^(11/2)/(12 - 3*e^2*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[2 + e*x]*(-23216 + 5664*e*x + 568*e^2*x^2 + 72*e^3*x^3 + 5*e^4*x^4
))/(105*e*Sqrt[12 - 3*e^2*x^2])
```

Maple [A] time = 0.043, size = 60, normalized size = 0.5

$$\frac{(2\,ex-4)\left(5\,e^{4}x^{4}+72\,e^{3}x^{3}+568\,e^{2}x^{2}+5664\,ex-23216\right)}{35\,e}(ex+2)^{\frac{3}{2}}\left(-3\,e^{2}x^{2}+12\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+2)^(11/2)/(-3*e^2*x^2+12)^(3/2),x)
```

```
[Out] 2/35*(e*x-2)*(5*e^4*x^4+72*e^3*x^3+568*e^2*x^2+5664*e*x-23216)*(e*x+2)^(3/2
)/e/(-3*e^2*x^2+12)^(3/2)
```

Maxima [C] time = 1.67088, size = 78, normalized size = 0.7

$$\frac{10i\,\sqrt{3}e^{4}x^{4}+144i\,\sqrt{3}e^{3}x^{3}+1136i\,\sqrt{3}e^{2}x^{2}+11328i\,\sqrt{3}ex-46432i\,\sqrt{3}}{315\,\sqrt{ex-2}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(11/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/315*(10*I*sqrt(3)*e^4*x^4 + 144*I*sqrt(3)*e^3*x^3 + 1136*I*sqrt(3)*e^2*x^
2 + 11328*I*sqrt(3)*e*x - 46432*I*sqrt(3))/(sqrt(e*x - 2)*e)
```

Fricas [A] time = 1.84925, size = 158, normalized size = 1.42

$$\frac{2\left(5\,e^{4}x^{4}+72\,e^{3}x^{3}+568\,e^{2}x^{2}+5664\,ex-23216\right)\sqrt{-3\,e^{2}x^{2}+12}\sqrt{ex+2}}{315\left(e^{3}x^{2}-4\,e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(11/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/315*(5*e^4*x^4 + 72*e^3*x^3 + 568*e^2*x^2 + 5664*e*x - 23216)*sqrt(-3*e^2
*x^2 + 12)*sqrt(e*x + 2)/(e^3*x^2 - 4*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)**(11/2)/(-3*e**2*x**2+12)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(11/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.919 $\int \frac{(2+ex)^{9/2}}{\left(12-3e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{2(2-ex)^{5/2}}{15\sqrt{3}e}-\frac{8(2-ex)^{3/2}}{3\sqrt{3}e}+\frac{32\sqrt{2-ex}}{\sqrt{3}e}+\frac{128}{3\sqrt{3}e\sqrt{2-ex}}$$

```
[Out] 128/(3*Sqrt[3]*e*Sqrt[2 - e*x]) + (32*Sqrt[2 - e*x])/(Sqrt[3]*e) - (8*(2 -
e*x)^(3/2))/(3*Sqrt[3]*e) + (2*(2 - e*x)^(5/2))/(15*Sqrt[3]*e)
```

Rubi [A] time = 0.0245445, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {627, 43}

$$\frac{2(2-ex)^{5/2}}{15\sqrt{3}e}-\frac{8(2-ex)^{3/2}}{3\sqrt{3}e}+\frac{32\sqrt{2-ex}}{\sqrt{3}e}+\frac{128}{3\sqrt{3}e\sqrt{2-ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + e*x)^(9/2)/(12 - 3*e^2*x^2)^(3/2),x]
```

```
[Out] 128/(3*Sqrt[3]*e*Sqrt[2 - e*x]) + (32*Sqrt[2 - e*x])/(Sqrt[3]*e) - (8*(2 -
e*x)^(3/2))/(3*Sqrt[3]*e) + (2*(2 - e*x)^(5/2))/(15*Sqrt[3]*e)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(2+ex)^{9/2}}{\left(12-3e^2x^2\right)^{3/2}}\,dx &= \int \frac{(2+ex)^3}{(6-3ex)^{3/2}}\,dx\\ &= \int \left(\frac{64}{(6-3ex)^{3/2}}-\frac{16}{\sqrt{6-3ex}}+\frac{4}{3}\sqrt{6-3ex}-\frac{1}{27}(6-3ex)^{3/2}\right)dx\\ &= \frac{128}{3\sqrt{3}e\sqrt{2-ex}}+\frac{32\sqrt{2-ex}}{\sqrt{3}e}-\frac{8(2-ex)^{3/2}}{3\sqrt{3}e}+\frac{2(2-ex)^{5/2}}{15\sqrt{3}e}\end{aligned}$$

Mathematica [A] time = 0.0805123, size = 51, normalized size = 0.59

$$-\frac{2\sqrt{ex+2}\left(e^3x^3+14e^2x^2+172ex-728\right)}{15e\sqrt{12-3e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + e*x)^(9/2)/(12 - 3*e^2*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[2 + e*x]*(-728 + 172*e*x + 14*e^2*x^2 + e^3*x^3))/(15*e*Sqrt[12 -
3*e^2*x^2])
```

Maple [A] time = 0.044, size = 51, normalized size = 0.6

$$\frac{(2ex-4)\left(e^3x^3+14e^2x^2+172ex-728\right)}{5e}(ex+2)^{\frac{3}{2}}\left(-3e^2x^2+12\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+2)^(9/2)/(-3*e^2*x^2+12)^(3/2),x)
```

```
[Out] 2/5*(e*x-2)*(e^3*x^3+14*e^2*x^2+172*e*x-728)*(e*x+2)^(3/2)/e/(-3*e^2*x^2+12
)^(3/2)
```

Maxima [C] time = 1.57932, size = 49, normalized size = 0.56

$$\frac{2i\sqrt{3}\left(e^3x^3+14e^2x^2+172ex-728\right)}{45\sqrt{ex-2}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(9/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/45*I*sqrt(3)*(e^3*x^3 + 14*e^2*x^2 + 172*e*x - 728)/(sqrt(e*x - 2)*e)
```

Fricas [A] time = 1.75109, size = 131, normalized size = 1.51

$$\frac{2\left(e^3x^3+14e^2x^2+172ex-728\right)\sqrt{-3e^2x^2+12}\sqrt{ex+2}}{45\left(e^3x^2-4e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(9/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/45*(e^3*x^3 + 14*e^2*x^2 + 172*e*x - 728)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x
+ 2)/(e^3*x^2 - 4*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)**(9/2)/(-3*e**2*x**2+12)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(9/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.920 $\int \frac{(2+ex)^{7/2}}{\left(12-3e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{2(2-ex)^{3/2}}{9\sqrt{3}e}+\frac{16\sqrt{2-ex}}{3\sqrt{3}e}+\frac{32}{3\sqrt{3}e\sqrt{2-ex}}$$

```
[Out] 32/(3*Sqrt[3]*e*Sqrt[2 - e*x]) + (16*Sqrt[2 - e*x])/(3*Sqrt[3]*e) - (2*(2 -
 e*x)^(3/2))/(9*Sqrt[3]*e)
```

Rubi [A] time = 0.021156, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {627, 43}

$$-\frac{2(2-ex)^{3/2}}{9\sqrt{3}e}+\frac{16\sqrt{2-ex}}{3\sqrt{3}e}+\frac{32}{3\sqrt{3}e\sqrt{2-ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + e*x)^(7/2)/(12 - 3*e^2*x^2)^(3/2),x]
```

```
[Out] 32/(3*Sqrt[3]*e*Sqrt[2 - e*x]) + (16*Sqrt[2 - e*x])/(3*Sqrt[3]*e) - (2*(2 -
 e*x)^(3/2))/(9*Sqrt[3]*e)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(2+ex)^{7/2}}{\left(12-3e^2x^2\right)^{3/2}}\,dx &= \int \frac{(2+ex)^2}{(6-3ex)^{3/2}}\,dx\\ &= \int \left(\frac{16}{(6-3ex)^{3/2}}-\frac{8}{3\sqrt{6-3ex}}+\frac{1}{9}\sqrt{6-3ex}\right)dx\\ &= \frac{32}{3\sqrt{3}e\sqrt{2-ex}}+\frac{16\sqrt{2-ex}}{3\sqrt{3}e}-\frac{2(2-ex)^{3/2}}{9\sqrt{3}e}\end{aligned}$$

Mathematica [A] time = 0.0643718, size = 43, normalized size = 0.64

$$-\frac{2\sqrt{ex+2}\left(e^2x^2+20ex-92\right)}{9e\sqrt{12-3e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + e*x)^(7/2)/(12 - 3*e^2*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[2 + e*x]*(-92 + 20*e*x + e^2*x^2))/(9*e*Sqrt[12 - 3*e^2*x^2])
```

Maple [A] time = 0.042, size = 43, normalized size = 0.6

$$\frac{(2ex-4)\left(e^2x^2+20ex-92\right)}{3e}(ex+2)^{\frac{3}{2}}\left(-3e^2x^2+12\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+2)^(7/2)/(-3*e^2*x^2+12)^(3/2),x)
```

```
[Out] 2/3*(e*x-2)*(e^2*x^2+20*e*x-92)*(e*x+2)^(3/2)/e/(-3*e^2*x^2+12)^(3/2)
```

Maxima [C] time = 1.68838, size = 49, normalized size = 0.73

$$\frac{2i\sqrt{3}e^2x^2+40i\sqrt{3}ex-184i\sqrt{3}}{27\sqrt{ex-2}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(7/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/27*(2*I*sqrt(3)*e^2*x^2 + 40*I*sqrt(3)*e*x - 184*I*sqrt(3))/(sqrt(e*x - 2
)*e)
```

Fricas [A] time = 1.85742, size = 111, normalized size = 1.66

$$\frac{2\left(e^2x^2+20ex-92\right)\sqrt{-3e^2x^2+12}\sqrt{ex+2}}{27\left(e^3x^2-4e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(7/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/27*(e^2*x^2 + 20*e*x - 92)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2)/(e^3*x^2 -
 4*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)**(7/2)/(-3*e**2*x**2+12)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(7/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.921 $\int \frac{(2+ex)^{5/2}}{\left(12-3e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=45

$$\frac{2\sqrt{2-ex}}{3\sqrt{3}e}+\frac{8}{3\sqrt{3}e\sqrt{2-ex}}$$

```
[Out] 8/(3*Sqrt[3]*e*Sqrt[2 - e*x]) + (2*Sqrt[2 - e*x])/(3*Sqrt[3]*e)
```

Rubi [A] time = 0.0171074, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {627, 43}

$$\frac{2\sqrt{2-ex}}{3\sqrt{3}e}+\frac{8}{3\sqrt{3}e\sqrt{2-ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + e*x)^(5/2)/(12 - 3*e^2*x^2)^(3/2),x]
```

```
[Out] 8/(3*Sqrt[3]*e*Sqrt[2 - e*x]) + (2*Sqrt[2 - e*x])/(3*Sqrt[3]*e)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(2+ex)^{5/2}}{\left(12-3e^2x^2\right)^{3/2}}\,dx &= \int \frac{2+ex}{(6-3ex)^{3/2}}\,dx\\ &= \int \left(\frac{4}{(6-3ex)^{3/2}}-\frac{1}{3\sqrt{6-3ex}}\right)dx\\ &= \frac{8}{3\sqrt{3}e\sqrt{2-ex}}+\frac{2\sqrt{2-ex}}{3\sqrt{3}e}\end{aligned}$$

Mathematica [A] time = 0.0566121, size = 35, normalized size = 0.78

$$-\frac{2(ex-6)\sqrt{ex+2}}{3e\sqrt{12-3e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + e*x)^(5/2)/(12 - 3*e^2*x^2)^(3/2),x]
```

```
[Out] (-2*(-6 + e*x)*Sqrt[2 + e*x])/(3*e*Sqrt[12 - 3*e^2*x^2])
```

Maple [A] time = 0.04, size = 35, normalized size = 0.8

$$2\,{\frac {(ex-2)\,(ex-6)\,(ex+2)^{3/2}}{e\left(-3\,e^{2}x^{2}+12\right)^{3/2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+2)^(5/2)/(-3*e^2*x^2+12)^(3/2),x)
```

```
[Out] 2*(e*x-2)*(e*x-6)*(e*x+2)^(3/2)/e/(-3*e^2*x^2+12)^(3/2)
```

Maxima [C] time = 1.55537, size = 27, normalized size = 0.6

$$\frac{2i\,\sqrt{3}(ex-6)}{9\,\sqrt{ex-2}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(5/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/9*I*sqrt(3)*(e*x - 6)/(sqrt(e*x - 2)*e)
```

Fricas [A] time = 1.82831, size = 90, normalized size = 2.

$$\frac{2\,\sqrt{-3\,e^{2}x^{2}+12}\sqrt{ex+2}(ex-6)}{9\left(e^{3}x^{2}-4\,e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(5/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/9*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2)*(e*x - 6)/(e^3*x^2 - 4*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)**(5/2)/(-3*e**2*x**2+12)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(5/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.922 $\int \frac{(2+ex)^{3/2}}{\left(12-3e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=22

$$\frac{2}{3\sqrt{3}e\sqrt{2-ex}}$$

```
[Out] 2/(3*Sqrt[3]*e*Sqrt[2 - e*x])
```

Rubi [A] time = 0.0096814, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {627, 32}

$$\frac{2}{3\sqrt{3}e\sqrt{2-ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + e*x)^(3/2)/(12 - 3*e^2*x^2)^(3/2),x]
```

```
[Out] 2/(3*Sqrt[3]*e*Sqrt[2 - e*x])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(2+ex)^{3/2}}{\left(12-3e^2x^2\right)^{3/2}}\,dx &= \int \frac{1}{(6-3ex)^{3/2}}\,dx\\ &= \frac{2}{3\sqrt{3}e\sqrt{2-ex}}\end{aligned}$$

Mathematica [A] time = 0.0455999, size = 30, normalized size = 1.36

$$\frac{2\sqrt{ex+2}}{3e\sqrt{12-3e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + e*x)^(3/2)/(12 - 3*e^2*x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[2 + e*x])/(3*e*Sqrt[12 - 3*e^2*x^2])
```

Maple [A] time = 0.04, size = 30, normalized size = 1.4

$$-2\,\frac{(ex-2)\,(ex+2)^{3/2}}{e\left(-3\,e^{2}x^{2}+12\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+2)^(3/2)/(-3*e^2*x^2+12)^(3/2),x)
```

```
[Out] -2*(e*x-2)*(e*x+2)^(3/2)/e/(-3*e^2*x^2+12)^(3/2)
```

Maxima [C] time = 1.71926, size = 20, normalized size = 0.91

$$-\frac{2i\,\sqrt{3}}{9\,\sqrt{ex-2e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(3/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/9*I*sqrt(3)/(sqrt(e*x - 2)*e)
```

Fricas [B] time = 1.67795, size = 78, normalized size = 3.55

$$-\frac{2\,\sqrt{-3\,e^{2}x^{2}+12}\sqrt{ex+2}}{9\left(e^{3}x^{2}-4\,e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(3/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/9*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2)/(e^3*x^2 - 4*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{3}\left(\int\frac{2\sqrt{ex+2}}{-e^{2}x^{2}\sqrt{-e^{2}x^{2}+4}+4\sqrt{-e^{2}x^{2}+4}}\,dx+\int\frac{ex\sqrt{ex+2}}{-e^{2}x^{2}\sqrt{-e^{2}x^{2}+4}+4\sqrt{-e^{2}x^{2}+4}}\,dx\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)**(3/2)/(-3*e**2*x**2+12)**(3/2),x)
```

```
[Out] sqrt(3)*(Integral(2*sqrt(e*x + 2)/(-e**2*x**2*sqrt(-e**2*x**2 + 4) + 4*sqrt
(-e**2*x**2 + 4)), x) + Integral(e*x*sqrt(e*x + 2)/(-e**2*x**2*sqrt(-e**2*x
**2 + 4) + 4*sqrt(-e**2*x**2 + 4)), x))/9
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(3/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.923 $\int \frac{\sqrt{2+ex}}{(12-3e^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=50

$$\frac{1}{6\sqrt{3}e\sqrt{2-ex}}-\frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{12\sqrt{3}e}$$

```
[Out] 1/(6*Sqrt[3]*e*Sqrt[2 - e*x]) - ArcTanh[Sqrt[2 - e*x]/2]/(12*Sqrt[3]*e)
```

Rubi [A] time = 0.0209086, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {627, 51, 63, 206}

$$\frac{1}{6\sqrt{3}e\sqrt{2-ex}}-\frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{12\sqrt{3}e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + e*x]/(12 - 3*e^2*x^2)^(3/2),x]

[Out] 1/(6*Sqrt[3]*e*Sqrt[2 - e*x]) - ArcTanh[Sqrt[2 - e*x]/2]/(12*Sqrt[3]*e)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{2+ex}}{\left(12-3e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{(6-3ex)^{3/2}(2+ex)}\,dx$$
$$= \frac{1}{6\sqrt{3}e\sqrt{2-ex}} + \frac{1}{12}\int \frac{1}{\sqrt{6-3ex}(2+ex)}\,dx$$
$$= \frac{1}{6\sqrt{3}e\sqrt{2-ex}} - \frac{\text{Subst}\left(\int \frac{1}{4-\frac{x^2}{3}}\,dx, x, \sqrt{6-3ex}\right)}{18e}$$
$$= \frac{1}{6\sqrt{3}e\sqrt{2-ex}} - \frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{12\sqrt{3}e}$$

Mathematica [C] time = 0.0423811, size = 48, normalized size = 0.96

$$\frac{\sqrt{ex+2}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{1}{2}-\frac{ex}{4}\right)}{6e\sqrt{12-3e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + e*x]/(12 - 3*e^2*x^2)^(3/2),x]
```

```
[Out] (Sqrt[2 + e*x]*Hypergeometric2F1[-1/2, 1, 1/2, 1/2 - (e*x)/4])/(6*e*Sqrt[12
 - 3*e^2*x^2])
```

Maple [A] time = 0.054, size = 60, normalized size = 1.2

$$\frac{1}{(108\,ex-216)\,e}\sqrt{-3\,e^2x^2+12}\left(\sqrt{3}\text{Artanh}\left(\frac{\sqrt{3}}{6}\sqrt{-3\,ex+6}\right)\sqrt{-3\,ex+6}-6\right)\frac{1}{\sqrt{ex+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(3/2),x)
```

```
[Out] 1/108/(e*x+2)^(1/2)*(-3*e^2*x^2+12)^(1/2)*(3^(1/2)*arctanh(1/6*3^(1/2)*(-3*
e*x+6)^(1/2))*(-3*e*x+6)^(1/2)-6)/(e*x-2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+2}}{\left(-3\,e^2x^2+12\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + 2)/(-3*e^2*x^2 + 12)^(3/2), x)
```

Fricas [B] time = 1.84519, size = 254, normalized size = 5.08

$$\frac{\sqrt{3}\left(e^{2}x^{2}-4\right)\log\left(-\frac{3\,e^{2}x^{2}-12\,ex+4\,\sqrt{3}\sqrt{-3\,e^{2}x^{2}+12}\sqrt{ex+2}-36}{e^{2}x^{2}+4\,ex+4}\right)-4\,\sqrt{-3\,e^{2}x^{2}+12}\sqrt{ex+2}}{72\,\left(e^{3}x^{2}-4\,e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/72*(sqrt(3)*(e^2*x^2 - 4)*log(-(3*e^2*x^2 - 12*e*x + 4*sqrt(3)*sqrt(-3*e^
2*x^2 + 12)*sqrt(e*x + 2) - 36)/(e^2*x^2 + 4*e*x + 4)) - 4*sqrt(-3*e^2*x^2
+ 12)*sqrt(e*x + 2))/(e^3*x^2 - 4*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{3}\int\frac{\sqrt{ex+2}}{-e^{2}x^{2}\sqrt{-e^{2}x^{2}+4}+4\sqrt{-e^{2}x^{2}+4}}\,dx}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)**(1/2)/(-3*e**2*x**2+12)**(3/2),x)
```

```
[Out] sqrt(3)*Integral(sqrt(e*x + 2)/(-e**2*x**2*sqrt(-e**2*x**2 + 4) + 4*sqrt(-e
**2*x**2 + 4)), x)/9
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_{0}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.924 $\int \frac{1}{\sqrt{2+ex}\left(12-3e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{1}{16\sqrt{3}e\sqrt{2-ex}}-\frac{1}{12\sqrt{3}e\sqrt{2-ex}(ex+2)}-\frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{32\sqrt{3}e}$$

```
[Out] 1/(16*Sqrt[3]*e*Sqrt[2 - e*x]) - 1/(12*Sqrt[3]*e*Sqrt[2 - e*x]*(2 + e*x)) -
 ArcTanh[Sqrt[2 - e*x]/2]/(32*Sqrt[3]*e)
```

Rubi [A] time = 0.0303714, antiderivative size = 86, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {627, 51, 63, 206}

$$-\frac{\sqrt{2-ex}}{16\sqrt{3}e(ex+2)}+\frac{1}{6\sqrt{3}e\sqrt{2-ex}(ex+2)}-\frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{32\sqrt{3}e}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 + e*x]*(12 - 3*e^2*x^2)^(3/2)),x]
```

```
[Out] 1/(6*Sqrt[3]*e*Sqrt[2 - e*x]*(2 + e*x)) - Sqrt[2 - e*x]/(16*Sqrt[3]*e*(2 +
e*x)) - ArcTanh[Sqrt[2 - e*x]/2]/(32*Sqrt[3]*e)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+ex}\left(12-3e^2x^2\right)^{3/2}}\,dx &= \int \frac{1}{(6-3ex)^{3/2}(2+ex)^2}\,dx\\ &= \frac{1}{6\sqrt{3}e\sqrt{2-ex}(2+ex)}+\frac{1}{4}\int \frac{1}{\sqrt{6-3ex}(2+ex)^2}\,dx\\ &= \frac{1}{6\sqrt{3}e\sqrt{2-ex}(2+ex)}-\frac{\sqrt{2-ex}}{16\sqrt{3}e(2+ex)}+\frac{1}{32}\int \frac{1}{\sqrt{6-3ex}(2+ex)}\,dx\\ &= \frac{1}{6\sqrt{3}e\sqrt{2-ex}(2+ex)}-\frac{\sqrt{2-ex}}{16\sqrt{3}e(2+ex)}-\frac{\text{Subst}\left(\int \frac{1}{4-\frac{x^2}{3}}\,dx,x,\sqrt{6-3ex}\right)}{48e}\\ &= \frac{1}{6\sqrt{3}e\sqrt{2-ex}(2+ex)}-\frac{\sqrt{2-ex}}{16\sqrt{3}e(2+ex)}-\frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{32\sqrt{3}e}\end{aligned}$$

Mathematica [C] time = 0.0496762, size = 48, normalized size = 0.61

$$\frac{\sqrt{ex+2}\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{1}{2}-\frac{ex}{4}\right)}{24e\sqrt{12-3e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 + e*x]*(12 - 3*e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[2 + e*x]*Hypergeometric2F1[-1/2, 2, 1/2, 1/2 - (e*x)/4])/(24*e*Sqrt[1
2 - 3*e^2*x^2])
```

Maple [A] time = 0.058, size = 93, normalized size = 1.2

$$\frac{1}{(288\,ex-576)\,e}\sqrt{-3\,e^2x^2+12}\left(\sqrt{3}\text{Artanh}\left(\frac{\sqrt{3}}{6}\sqrt{-3\,ex+6}\right)\sqrt{-3\,ex+6}xe+2\,\sqrt{3}\text{Artanh}\left(1/6\,\sqrt{3}\sqrt{-3\,ex+6}\right)\sqrt{-3\,ex+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(3/2),x)
```

```
[Out] 1/288/(e*x+2)^(3/2)*(-3*e^2*x^2+12)^(1/2)*(3^(1/2)*arctanh(1/6*3^(1/2)*(-3*
e*x+6)^(1/2))*(-3*e*x+6)^(1/2)*x*e+2*3^(1/2)*arctanh(1/6*3^(1/2)*(-3*e*x+6)
^(1/2))*(-3*e*x+6)^(1/2)-6*e*x-4)/(e*x-2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3\,e^2x^2+12\right)^{\frac{3}{2}}\sqrt{ex+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*e^2*x^2 + 12)^(3/2)*sqrt(e*x + 2)), x)
```

Fricas [B] time = 1.85329, size = 331, normalized size = 4.19

$$\frac{3\sqrt{3}\left(e^3x^3+2e^2x^2-4ex-8\right)\log\left(-\frac{3e^2x^2-12ex+4\sqrt{3}\sqrt{-3e^2x^2+12}\sqrt{ex+2}-36}{e^2x^2+4ex+4}\right)-4\sqrt{-3e^2x^2+12}(3ex+2)\sqrt{ex+2}}{576\left(e^4x^3+2e^3x^2-4e^2x-8e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/576*(3*sqrt(3)*(e^3*x^3 + 2*e^2*x^2 - 4*e*x - 8)*log(-(3*e^2*x^2 - 12*e*x
 + 4*sqrt(3)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2) - 36)/(e^2*x^2 + 4*e*x + 4
)) - 4*sqrt(-3*e^2*x^2 + 12)*(3*e*x + 2)*sqrt(e*x + 2))/(e^4*x^3 + 2*e^3*x^
2 - 4*e^2*x - 8*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{3}\int\frac{1}{-e^2x^2\sqrt{ex+2}\sqrt{-e^2x^2+4}+4\sqrt{ex+2}\sqrt{-e^2x^2+4}}\,dx}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)**(1/2)/(-3*e**2*x**2+12)**(3/2),x)
```

```
[Out] sqrt(3)*Integral(1/(-e**2*x**2*sqrt(e*x + 2)*sqrt(-e**2*x**2 + 4) + 4*sqrt(
e*x + 2)*sqrt(-e**2*x**2 + 4)), x)/9
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.925 $\int \frac{1}{(2+ex)^{3/2}\left(12-3e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=108

$$\frac{5}{256\sqrt{3}e\sqrt{2-ex}}-\frac{5}{192\sqrt{3}e\sqrt{2-ex}(ex+2)}-\frac{1}{24\sqrt{3}e\sqrt{2-ex}(ex+2)^2}-\frac{5\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{512\sqrt{3}e}$$

```
[Out] 5/(256*Sqrt[3]*e*Sqrt[2 - e*x]) - 1/(24*Sqrt[3]*e*Sqrt[2 - e*x]*(2 + e*x)^2
) - 5/(192*Sqrt[3]*e*Sqrt[2 - e*x]*(2 + e*x)) - (5*ArcTanh[Sqrt[2 - e*x]/2]
)/(512*Sqrt[3]*e)
```

Rubi [A] time = 0.0414894, antiderivative size = 115, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {627, 51, 63, 206}

$$-\frac{5\sqrt{2-ex}}{256\sqrt{3}e(ex+2)}-\frac{5\sqrt{2-ex}}{96\sqrt{3}e(ex+2)^2}+\frac{1}{6\sqrt{3}e\sqrt{2-ex}(ex+2)^2}-\frac{5\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right)}{512\sqrt{3}e}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + e*x)^(3/2)*(12 - 3*e^2*x^2)^(3/2)),x]
```

```
[Out] 1/(6*Sqrt[3]*e*Sqrt[2 - e*x]*(2 + e*x)^2) - (5*Sqrt[2 - e*x])/(96*Sqrt[3]*e
*(2 + e*x)^2) - (5*Sqrt[2 - e*x])/(256*Sqrt[3]*e*(2 + e*x)) - (5*ArcTanh[Sq
rt[2 - e*x]/2])/(512*Sqrt[3]*e)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(2+ex)^{3/2}\left(12-3e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{(6-3ex)^{3/2}(2+ex)^3}\,dx$$
$$= \frac{1}{6\sqrt{3}e\sqrt{2-ex}(2+ex)^2} + \frac{5}{12}\int \frac{1}{\sqrt{6-3ex}(2+ex)^3}\,dx$$
$$= \frac{1}{6\sqrt{3}e\sqrt{2-ex}(2+ex)^2} - \frac{5\sqrt{2-ex}}{96\sqrt{3}e(2+ex)^2} + \frac{5}{64}\int \frac{1}{\sqrt{6-3ex}(2+ex)^2}\,dx$$
$$= \frac{1}{6\sqrt{3}e\sqrt{2-ex}(2+ex)^2} - \frac{5\sqrt{2-ex}}{96\sqrt{3}e(2+ex)^2} - \frac{5\sqrt{2-ex}}{256\sqrt{3}e(2+ex)} + \frac{5}{512}\int \frac{1}{\sqrt{6-3ex}(2$$
$$= \frac{1}{6\sqrt{3}e\sqrt{2-ex}(2+ex)^2} - \frac{5\sqrt{2-ex}}{96\sqrt{3}e(2+ex)^2} - \frac{5\sqrt{2-ex}}{256\sqrt{3}e(2+ex)} - \frac{5\,\mathrm{Subst}\left(\int \frac{1}{4-\frac{x^2}{3}}\,dx\right.}{768e}$$
$$= \frac{1}{6\sqrt{3}e\sqrt{2-ex}(2+ex)^2} - \frac{5\sqrt{2-ex}}{96\sqrt{3}e(2+ex)^2} - \frac{5\sqrt{2-ex}}{256\sqrt{3}e(2+ex)} - \frac{5\tanh^{-1}\left(\frac{1}{2}\sqrt{2-ex}\right.}{512\sqrt{3}e}$$

Mathematica [C] time = 0.0501724, size = 48, normalized size = 0.44

$$\frac{\sqrt{ex+2}\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\frac{1}{2}-\frac{ex}{4}\right)}{96e\sqrt{12-3e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2 + e*x)^(3/2)*(12 - 3*e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[2 + e*x]*Hypergeometric2F1[-1/2, 3, 1/2, 1/2 - (e*x)/4])/(96*e*Sqrt[1
2 - 3*e^2*x^2])
```

Maple [A] time = 0.059, size = 135, normalized size = 1.3

$$\frac{1}{(4608\,ex-9216)\,e}\sqrt{-3\,e^2x^2+12}\left(5\,\sqrt{3}\mathrm{Artanh}\left(1/6\,\sqrt{3}\sqrt{-3\,ex+6}\right)\sqrt{-3\,ex+6}x^2e^2+20\,\sqrt{3}\mathrm{Artanh}\left(1/6\,\sqrt{3}\sqrt{-3\,ex+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+2)^(3/2)/(-3*e^2*x^2+12)^(3/2),x)
```

```
[Out] 1/4608/(e*x+2)^(5/2)*(-3*e^2*x^2+12)^(1/2)*(5*3^(1/2)*arctanh(1/6*3^(1/2)*(
-3*e*x+6)^(1/2))*(-3*e*x+6)^(1/2)*x^2*e^2+20*3^(1/2)*arctanh(1/6*3^(1/2)*(-
3*e*x+6)^(1/2))*(-3*e*x+6)^(1/2)*x*e-30*e^2*x^2+20*3^(1/2)*arctanh(1/6*3^(1
/2)*(-3*e*x+6)^(1/2))*(-3*e*x+6)^(1/2)-80*e*x+24)/(e*x-2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3\,e^2x^2+12\right)^{\frac{3}{2}}(ex+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)^(3/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*e^2*x^2 + 12)^(3/2)*(e*x + 2)^(3/2)), x)
```

Fricas [A] time = 1.83962, size = 359, normalized size = 3.32

$$\frac{15\sqrt{3}\left(e^4x^4+4e^3x^3-16ex-16\right)\log\left(-\frac{3e^2x^2-12ex+4\sqrt{3}\sqrt{-3e^2x^2+12}\sqrt{ex+2}-36}{e^2x^2+4ex+4}\right)-4\left(15e^2x^2+40ex-12\right)\sqrt{-3e^2x^2+12}\sqrt{ex+2}}{9216\left(e^5x^4+4e^4x^3-16e^2x-16e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)^(3/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/9216*(15*sqrt(3)*(e^4*x^4 + 4*e^3*x^3 - 16*e*x - 16)*log(-(3*e^2*x^2 - 12
*e*x + 4*sqrt(3)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2) - 36)/(e^2*x^2 + 4*e*x
 + 4)) - 4*(15*e^2*x^2 + 40*e*x - 12)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2))/
(e^5*x^4 + 4*e^4*x^3 - 16*e^2*x - 16*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)**(3/2)/(-3*e**2*x**2+12)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, undef, undef, −2]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)^(3/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, undef, undef, -2]
```

3.926 $\int \frac{1}{\sqrt{1-x}(1+x)}\,dx$

Optimal. Leaf size=23

$$-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

```
[Out] -(Sqrt[2]*ArcTanh[Sqrt[1 - x]/Sqrt[2]])
```

Rubi [A] time = 0.0061987, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {63, 206}

$$-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x]*(1 + x)),x]

[Out] -(Sqrt[2]*ArcTanh[Sqrt[1 - x]/Sqrt[2]])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-x}(1+x)}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1-x}\right)\right)\\ &= -\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0044313, size = 23, normalized size = 1.

$$-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x]*(1 + x)),x]
```

```
[Out] -(Sqrt[2]*ArcTanh[Sqrt[1 - x]/Sqrt[2]])
```

Maple [A] time = 0.041, size = 19, normalized size = 0.8

$$-\operatorname{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{1-x}\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(1-x)^(1/2),x)
```

```
[Out] -arctanh(1/2*(1-x)^(1/2)*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.74477, size = 46, normalized size = 2.

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{2}+\sqrt{-x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*log(-(sqrt(2) - sqrt(-x + 1))/(sqrt(2) + sqrt(-x + 1)))
```

Fricas [A] time = 1.8582, size = 80, normalized size = 3.48

$$\frac{1}{2}\sqrt{2}\log\left(\frac{x+2\sqrt{2}\sqrt{-x+1}-3}{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log((x + 2*sqrt(2)*sqrt(-x + 1) - 3)/(x + 1))
```

Sympy [A] time = 1.51749, size = 44, normalized size = 1.91

$$\begin{cases} -\sqrt{2}\operatorname{acosh}\left(\frac{\sqrt{2}}{\sqrt{x+1}}\right) & \text{for}\ \frac{2}{|x+1|}>1 \\ \sqrt{2}i\operatorname{asin}\left(\frac{\sqrt{2}}{\sqrt{x+1}}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(1-x)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(2)*acosh(sqrt(2)/sqrt(x + 1)), 2/Abs(x + 1) > 1), (sqrt(2)
*I*asin(sqrt(2)/sqrt(x + 1)), True))
```

Giac [B] time = 1.20572, size = 51, normalized size = 2.22

$$-\frac{1}{2}\sqrt{2}\log\left(\sqrt{2}+\sqrt{-x+1}\right)+\frac{1}{2}\sqrt{2}\log\left(\left|-\sqrt{2}+\sqrt{-x+1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(sqrt(2) + sqrt(-x + 1)) + 1/2*sqrt(2)*log(abs(-sqrt(2) + s
qrt(-x + 1)))
```

3.927 $\int \frac{1}{\sqrt{1+x}\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=23

$$-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

```
[Out] -(Sqrt[2]*ArcTanh[Sqrt[1 - x]/Sqrt[2]])
```

Rubi [A] time = 0.0103184, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {627, 63, 206}

$$-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + x]*Sqrt[1 - x^2]),x]

[Out] -(Sqrt[2]*ArcTanh[Sqrt[1 - x]/Sqrt[2]])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+x}\sqrt{1-x^2}}\,dx &= \int \frac{1}{\sqrt{1-x}(1+x)}\,dx\\ &= -\left(2\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1-x}\right)\right)\\ &= -\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0260191, size = 45, normalized size = 1.96

$$\frac{\sqrt{2}\sqrt{x-1}\sqrt{x+1}\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + x]*Sqrt[1 - x^2]),x]
```

```
[Out] (Sqrt[2]*Sqrt[-1 + x]*Sqrt[1 + x]*ArcTan[Sqrt[-1 + x]/Sqrt[2]])/Sqrt[1 - x^
2]
```

Maple [B] time = 0.084, size = 40, normalized size = 1.7

$$-\sqrt{2}\sqrt{-x^2+1}\text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{1-x}\right)\frac{1}{\sqrt{1+x}}\frac{1}{\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] -1/(1+x)^(1/2)*(-x^2+1)^(1/2)/(1-x)^(1/2)*2^(1/2)*arctanh(1/2*(1-x)^(1/2)*2
^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+1}\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(x + 1)), x)
```

Fricas [B] time = 1.74303, size = 122, normalized size = 5.3

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{x^2+2\sqrt{2}\sqrt{-x^2+1}\sqrt{x+1}-2x-3}{x^2+2x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-(x^2 + 2*sqrt(2)*sqrt(-x^2 + 1)*sqrt(x + 1) - 2*x - 3)/(x^
2 + 2*x + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x-1)(x+1)}\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x - 1)*(x + 1))*sqrt(x + 1)), x)
```

Giac [B] time = 1.25514, size = 50, normalized size = 2.17

$$-\frac{1}{2}\sqrt{2}\log\left(\sqrt{2}+\sqrt{-x+1}\right)+\frac{1}{2}\sqrt{2}\log\left(\sqrt{2}-\sqrt{-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(sqrt(2) + sqrt(-x + 1)) + 1/2*sqrt(2)*log(sqrt(2) - sqrt(-
x + 1))
```

3.928 $\int \frac{1}{\sqrt{1-ax}(1+ax)}\,dx$

Optimal. Leaf size=27

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{a}$$

```
[Out] -((Sqrt[2]*ArcTanh[Sqrt[1 - a*x]/Sqrt[2]])/a)
```

Rubi [A] time = 0.0081858, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {63, 206}

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - a*x]*(1 + a*x)),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[Sqrt[1 - a*x]/Sqrt[2]])/a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-ax}(1+ax)}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1-ax}\right)}{a}\\ &= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.0069099, size = 27, normalized size = 1.

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - a*x]*(1 + a*x)),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[Sqrt[1 - a*x]/Sqrt[2]])/a)
```

Maple [A] time = 0.043, size = 23, normalized size = 0.9

$$-\frac{\sqrt{2}}{a}\operatorname{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{-ax+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)/(-a*x+1)^(1/2),x)
```

```
[Out] -arctanh(1/2*(-a*x+1)^(1/2)*2^(1/2))*2^(1/2)/a
```

Maxima [A] time = 1.67271, size = 53, normalized size = 1.96

$$\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{-ax+1}}{\sqrt{2}+\sqrt{-ax+1}}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*log(-(sqrt(2) - sqrt(-a*x + 1))/(sqrt(2) + sqrt(-a*x + 1)))/a
```

Fricas [A] time = 1.78686, size = 90, normalized size = 3.33

$$\frac{\sqrt{2}\log\left(\frac{ax+2\,\sqrt{2}\sqrt{-ax+1}-3}{ax+1}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log((a*x + 2*sqrt(2)*sqrt(-a*x + 1) - 3)/(a*x + 1))/a
```

Sympy [A] time = 5.7172, size = 65, normalized size = 2.41

$$\begin{cases} \frac{2\left(\begin{cases} -\frac{\sqrt{2}\operatorname{acoth}\left(\frac{\sqrt{2}}{\sqrt{-ax+1}}\right)}{2} & \text{for}\ \frac{1}{-ax+1} > \frac{1}{2} \\ -\frac{\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}}{\sqrt{-ax+1}}\right)}{2} & \text{for}\ \frac{1}{-ax+1} < \frac{1}{2} \end{cases}\right)}{a} & \text{for}\ a \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)/(-a*x+1)**(1/2),x)
```

```
[Out] Piecewise((2*Piecewise((-sqrt(2)*acoth(sqrt(2)/sqrt(-a*x + 1))/2, 1/(-a*x +
 1) > 1/2), (-sqrt(2)*atanh(sqrt(2)/sqrt(-a*x + 1))/2, 1/(-a*x + 1) < 1/2))
/a, Ne(a, 0)), (x, True))
```

Giac [A] time = 1.14819, size = 57, normalized size = 2.11

$$\frac{\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\sqrt{-ax+1}\right|}{2\left(\sqrt{2}+\sqrt{-ax+1}\right)}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*log(1/2*abs(-2*sqrt(2) + 2*sqrt(-a*x + 1))/(sqrt(2) + sqrt(-a*x
 + 1)))/a
```

3.929 $\int \frac{1}{\sqrt{1+ax}\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=27

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{a}$$

```
[Out] -((Sqrt[2]*ArcTanh[Sqrt[1 - a*x]/Sqrt[2]])/a)
```

Rubi [A] time = 0.0140314, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {627, 63, 206}

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + a*x]*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[Sqrt[1 - a*x]/Sqrt[2]])/a)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+ax}\sqrt{1-a^2x^2}}\,dx &= \int \frac{1}{\sqrt{1-ax}(1+ax)}\,dx\\ &= -\frac{2\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1-ax}\right)}{a}\\ &= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.0467998, size = 53, normalized size = 1.96

$$\frac{\sqrt{ax+1}\sqrt{2ax-2}\tan^{-1}\left(\frac{\sqrt{ax-1}}{\sqrt{2}}\right)}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + a*x]*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (Sqrt[1 + a*x]*Sqrt[-2 + 2*a*x]*ArcTan[Sqrt[-1 + a*x]/Sqrt[2]])/(a*Sqrt[1 -
 a^2*x^2])
```

Maple [B] time = 0.122, size = 50, normalized size = 1.9

$$-\frac{\sqrt{2}}{a}\sqrt{-a^2x^2+1}\,\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{-ax+1}\right)\frac{1}{\sqrt{ax+1}}\frac{1}{\sqrt{-ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^(1/2)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/(a*x+1)^(1/2)*(-a^2*x^2+1)^(1/2)/(-a*x+1)^(1/2)/a*2^(1/2)*arctanh(1/2*(-
a*x+1)^(1/2)*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a^2x^2+1}\sqrt{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^(1/2)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*sqrt(a*x + 1)), x)
```

Fricas [B] time = 1.77288, size = 149, normalized size = 5.52

$$\frac{\sqrt{2}\log\left(-\frac{a^2x^2-2\,ax+2\,\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{ax+1}-3}{a^2x^2+2\,ax+1}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^(1/2)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-(a^2*x^2 - 2*a*x + 2*sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(a*x +
 1) - 3)/(a^2*x^2 + 2*a*x + 1))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(ax-1)(ax+1)}\sqrt{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**(1/2)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(a*x - 1)*(a*x + 1))*sqrt(a*x + 1)), x)
```

Giac [A] time = 1.24203, size = 58, normalized size = 2.15

$$-\frac{\sqrt{2}\log\left(\sqrt{2}+\sqrt{-ax+1}\right)-\sqrt{2}\log\left(\sqrt{2}-\sqrt{-ax+1}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^(1/2)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(2)*log(sqrt(2) + sqrt(-a*x + 1)) - sqrt(2)*log(sqrt(2) - sqrt(-a
*x + 1)))/a
```

3.930 $\int \sqrt{2+ex}\sqrt[4]{12-3e^2x^2}\,dx$

Optimal. Leaf size=309

$$-\frac{\sqrt[4]{3}(ex+2)^{3/4}(2-ex)^{5/4}}{2e}+\frac{3\sqrt[4]{3}(ex+2)^{3/4}\sqrt[4]{2-ex}}{2e}+\frac{3\sqrt[4]{3}\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}-\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{2\sqrt{2}e}-\frac{3\sqrt[4]{3}\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\dots}{\dots}\right)}{2\dots}$$

```
[Out] (3*3^(1/4)*(2 - e*x)^(1/4)*(2 + e*x)^(3/4))/(2*e) - (3^(1/4)*(2 - e*x)^(5/4
)*(2 + e*x)^(3/4))/(2*e) + (3*3^(1/4)*ArcTan[1 - (Sqrt[2]*(2 - e*x)^(1/4))/
(2 + e*x)^(1/4)])/(Sqrt[2]*e) - (3*3^(1/4)*ArcTan[1 + (Sqrt[2]*(2 - e*x)^(1
/4))/(2 + e*x)^(1/4)])/(Sqrt[2]*e) + (3*3^(1/4)*Log[(Sqrt[6 - 3*e*x] - Sqrt
[6]*(2 - e*x)^(1/4)*(2 + e*x)^(1/4) + Sqrt[3]*Sqrt[2 + e*x])/Sqrt[2 + e*x]]
)/(2*Sqrt[2]*e) - (3*3^(1/4)*Log[(Sqrt[6 - 3*e*x] + Sqrt[6]*(2 - e*x)^(1/4)
*(2 + e*x)^(1/4) + Sqrt[3]*Sqrt[2 + e*x])/Sqrt[2 + e*x]])/(2*Sqrt[2]*e)
```

Rubi [A] time = 0.335062, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {675, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\sqrt[4]{3}(ex+2)^{3/4}(2-ex)^{5/4}}{2e}+\frac{3\sqrt[4]{3}(ex+2)^{3/4}\sqrt[4]{2-ex}}{2e}+\frac{3\sqrt[4]{3}\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}-\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{2\sqrt{2}e}-\frac{3\sqrt[4]{3}\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\dots}{\dots}\right)}{2\dots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + e*x]*(12 - 3*e^2*x^2)^(1/4),x]
```


```
[Out] (3*3^(1/4)*(2 - e*x)^(1/4)*(2 + e*x)^(3/4))/(2*e) - (3^(1/4)*(2 - e*x)^(5/4
)*(2 + e*x)^(3/4))/(2*e) + (3*3^(1/4)*ArcTan[1 - (Sqrt[2]*(2 - e*x)^(1/4))/
(2 + e*x)^(1/4)])/(Sqrt[2]*e) - (3*3^(1/4)*ArcTan[1 + (Sqrt[2]*(2 - e*x)^(1
/4))/(2 + e*x)^(1/4)])/(Sqrt[2]*e) + (3*3^(1/4)*Log[(Sqrt[6 - 3*e*x] - Sqrt
[6]*(2 - e*x)^(1/4)*(2 + e*x)^(1/4) + Sqrt[3]*Sqrt[2 + e*x])/Sqrt[2 + e*x]]
)/(2*Sqrt[2]*e) - (3*3^(1/4)*Log[(Sqrt[6 - 3*e*x] + Sqrt[6]*(2 - e*x)^(1/4)
*(2 + e*x)^(1/4) + Sqrt[3]*Sqrt[2 + e*x])/Sqrt[2 + e*x]])/(2*Sqrt[2]*e)
```


Rule 675

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[(
d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] && E
qQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && GtQ[a, 0] && GtQ[d, 0] &&  !IGtQ[m
, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{2+ex}\sqrt[4]{12-3e^2x^2}\,dx = \int \sqrt[4]{6-3ex}(2+ex)^{3/4}\,dx$$
$$= -\frac{\sqrt[4]{3}(2-ex)^{5/4}(2+ex)^{3/4}}{2e} + \frac{3}{2}\int \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\,dx$$
$$= \frac{3\sqrt[4]{3}\sqrt[4]{2-ex}(2+ex)^{3/4}}{2e} - \frac{\sqrt[4]{3}(2-ex)^{5/4}(2+ex)^{3/4}}{2e} + \frac{9}{2}\int \frac{1}{(6-3ex)^{3/4}\sqrt[4]{2+ex}}\,dx$$
$$= \frac{3\sqrt[4]{3}\sqrt[4]{2-ex}(2+ex)^{3/4}}{2e} - \frac{\sqrt[4]{3}(2-ex)^{5/4}(2+ex)^{3/4}}{2e} - \frac{6\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{4-\frac{x^4}{3}}}dx, x, \sqrt[4]{6-3ex}\right)}{e}$$
$$= \frac{3\sqrt[4]{3}\sqrt[4]{2-ex}(2+ex)^{3/4}}{2e} - \frac{\sqrt[4]{3}(2-ex)^{5/4}(2+ex)^{3/4}}{2e} - \frac{6\,\text{Subst}\left(\int \frac{1}{1+\frac{x^4}{3}}dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{e}$$
$$= \frac{3\sqrt[4]{3}\sqrt[4]{2-ex}(2+ex)^{3/4}}{2e} - \frac{\sqrt[4]{3}(2-ex)^{5/4}(2+ex)^{3/4}}{2e} - \frac{\sqrt{3}\,\text{Subst}\left(\int \frac{\sqrt{3}-x^2}{1+\frac{x^4}{3}}dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{e} -$$
$$= \frac{3\sqrt[4]{3}\sqrt[4]{2-ex}(2+ex)^{3/4}}{2e} - \frac{\sqrt[4]{3}(2-ex)^{5/4}(2+ex)^{3/4}}{2e} + \frac{\left(3\sqrt[4]{3}\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{3}+2x}{-\sqrt{3}-\sqrt{2}\sqrt[4]{3}x-x^2}dx,\right.}{2\sqrt{2}e}$$
$$= \frac{3\sqrt[4]{3}\sqrt[4]{2-ex}(2+ex)^{3/4}}{2e} - \frac{\sqrt[4]{3}(2-ex)^{5/4}(2+ex)^{3/4}}{2e} + \frac{3\sqrt[4]{3}\log\left(\frac{\sqrt{2-ex}-\sqrt{2}\sqrt[4]{2-ex}\sqrt[4]{2+ex}+\sqrt{2+ex}}{\sqrt{2+ex}}\right.}{2\sqrt{2}e}$$
$$= \frac{3\sqrt[4]{3}\sqrt[4]{2-ex}(2+ex)^{3/4}}{2e} - \frac{\sqrt[4]{3}(2-ex)^{5/4}(2+ex)^{3/4}}{2e} + \frac{3\sqrt[4]{3}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{2+ex}}\right)}{\sqrt{2}e} - \frac{3\sqrt[4]{3}\tan}{}$$

Mathematica [C] time = 0.0493846, size = 60, normalized size = 0.19

$$\frac{8\sqrt{2}(ex-2)\sqrt[4]{12-3e^2x^2}\,{}_2F_1\left(-\frac{3}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}-\frac{ex}{4}\right)}{5e\sqrt[4]{ex+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + e*x]*(12 - 3*e^2*x^2)^(1/4),x]
```

```
[Out] (8*Sqrt[2]*(-2 + e*x)*(12 - 3*e^2*x^2)^(1/4)*Hypergeometric2F1[-3/4, 5/4, 9
/4, 1/2 - (e*x)/4])/(5*e*(2 + e*x)^(1/4))
```

Maple [F] time = 0.171, size = 0, normalized size = 0.

$$\int \sqrt{ex+2}\sqrt[4]{-3\,e^2x^2+12}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+2)^(1/2)*(-3*e^2*x^2+12)^(1/4),x)
```

```
[Out] int((e*x+2)^(1/2)*(-3*e^2*x^2+12)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-3\,e^{2}x^{2}+12\right)^{\frac{1}{4}}\sqrt{ex+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(1/2)*(-3*e^2*x^2+12)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x + 2), x)
```

Fricas [B] time = 2.06396, size = 1532, normalized size = 4.96

$$12\cdot 3^{\frac{1}{4}}\sqrt{2}e\frac{1}{e^{4}}^{\frac{1}{4}}\arctan\left(-\frac{3^{\frac{3}{4}}\sqrt{2}\left(-3\,e^{2}x^{2}+12\right)^{\frac{1}{4}}\sqrt{ex+2}e^{3}\frac{1}{e^{4}}^{\frac{3}{4}}-3^{\frac{3}{4}}\sqrt{2}\left(e^{4}x+2\,e^{3}\right)\sqrt{\frac{3^{\frac{1}{4}}\sqrt{2}\left(-3\,e^{2}x^{2}+12\right)^{\frac{1}{4}}\sqrt{ex+2}e\frac{1}{e^{4}}^{\frac{1}{4}}+\sqrt{3}\left(e^{3}x+2\,e^{2}\right)\sqrt{\frac{1}{e^{4}}}+\sqrt{-3\,e^{2}x^{2}+12}}{ex+2}}\frac{1}{e^{4}}^{\frac{3}{4}}+3\,ex+6}{3\,(ex+2)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(1/2)*(-3*e^2*x^2+12)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/4*(12*3^(1/4)*sqrt(2)*e*(e^(-4))^(1/4)*arctan(-1/3*(3^(3/4)*sqrt(2)*(-3*e
^2*x^2 + 12)^(1/4)*sqrt(e*x + 2)*e^3*(e^(-4))^(3/4) - 3^(3/4)*sqrt(2)*(e^4*
x + 2*e^3)*sqrt((3^(1/4)*sqrt(2)*(-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x + 2)*e*(e
^(-4))^(1/4) + sqrt(3)*(e^3*x + 2*e^2)*sqrt(e^(-4)) + sqrt(-3*e^2*x^2 + 12)
)/(e*x + 2))*(e^(-4))^(3/4) + 3*e*x + 6)/(e*x + 2)) + 12*3^(1/4)*sqrt(2)*e*
(e^(-4))^(1/4)*arctan(-1/3*(3^(3/4)*sqrt(2)*(-3*e^2*x^2 + 12)^(1/4)*sqrt(e*
x + 2)*e^3*(e^(-4))^(3/4) - 3^(3/4)*sqrt(2)*(e^4*x + 2*e^3)*sqrt(-(3^(1/4)*
sqrt(2)*(-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x + 2)*e*(e^(-4))^(1/4) - sqrt(3)*(e
^3*x + 2*e^2)*sqrt(e^(-4)) - sqrt(-3*e^2*x^2 + 12))/(e*x + 2))*(e^(-4))^(3/
4) - 3*e*x - 6)/(e*x + 2)) - 3*3^(1/4)*sqrt(2)*e*(e^(-4))^(1/4)*log((3^(1/4
)*sqrt(2)*(-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x + 2)*e*(e^(-4))^(1/4) + sqrt(3)*
(e^3*x + 2*e^2)*sqrt(e^(-4)) + sqrt(-3*e^2*x^2 + 12))/(e*x + 2)) + 3*3^(1/4
)*sqrt(2)*e*(e^(-4))^(1/4)*log(-(3^(1/4)*sqrt(2)*(-3*e^2*x^2 + 12)^(1/4)*sq
rt(e*x + 2)*e*(e^(-4))^(1/4) - sqrt(3)*(e^3*x + 2*e^2)*sqrt(e^(-4)) - sqrt(
-3*e^2*x^2 + 12))/(e*x + 2)) + 2*(-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x + 2)*(e*x
 + 1))/e
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\sqrt[4]{3}\int\sqrt{ex+2}\sqrt[4]{-e^{2}x^{2}+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)**(1/2)*(-3*e**2*x**2+12)**(1/4),x)
```

```
[Out] 3**(1/4)*Integral(sqrt(e*x + 2)*(-e**2*x**2 + 4)**(1/4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(1/2)*(-3*e^2*x^2+12)^(1/4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.931 $\int \frac{\sqrt[4]{12-3e^2x^2}}{\sqrt{2+ex}}\,dx$

Optimal. Leaf size=269

$$\frac{\sqrt[4]{3}\sqrt[4]{2-ex}(ex+2)^{3/4}}{e}+\frac{\sqrt[4]{3}\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}-\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}e}-\frac{\sqrt[4]{3}\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}+\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}e}+\frac{\sqrt{2}\sqrt[4]{3}\tan^{-}}{}$$

```
[Out] (3^(1/4)*(2 - e*x)^(1/4)*(2 + e*x)^(3/4))/e + (Sqrt[2]*3^(1/4)*ArcTan[1 - (
Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/e - (Sqrt[2]*3^(1/4)*ArcTan[1 +
(Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/e + (3^(1/4)*Log[(Sqrt[6 - 3*e*
x] - Sqrt[6]*(2 - e*x)^(1/4)*(2 + e*x)^(1/4) + Sqrt[3]*Sqrt[2 + e*x])/Sqrt[
2 + e*x]])/(Sqrt[2]*e) - (3^(1/4)*Log[(Sqrt[6 - 3*e*x] + Sqrt[6]*(2 - e*x)^
(1/4)*(2 + e*x)^(1/4) + Sqrt[3]*Sqrt[2 + e*x])/Sqrt[2 + e*x]])/(Sqrt[2]*e)
```

Rubi [A] time = 0.26134, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {675, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{3}\sqrt[4]{2-ex}(ex+2)^{3/4}}{e}+\frac{\sqrt[4]{3}\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}-\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}e}-\frac{\sqrt[4]{3}\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}+\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}e}+\frac{\sqrt{2}\sqrt[4]{3}\tan^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(12 - 3*e^2*x^2)^(1/4)/Sqrt[2 + e*x],x]
```

```
[Out] (3^(1/4)*(2 - e*x)^(1/4)*(2 + e*x)^(3/4))/e + (Sqrt[2]*3^(1/4)*ArcTan[1 - (
Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/e - (Sqrt[2]*3^(1/4)*ArcTan[1 +
(Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/e + (3^(1/4)*Log[(Sqrt[6 - 3*e*
x] - Sqrt[6]*(2 - e*x)^(1/4)*(2 + e*x)^(1/4) + Sqrt[3]*Sqrt[2 + e*x])/Sqrt[
2 + e*x]])/(Sqrt[2]*e) - (3^(1/4)*Log[(Sqrt[6 - 3*e*x] + Sqrt[6]*(2 - e*x)^
(1/4)*(2 + e*x)^(1/4) + Sqrt[3]*Sqrt[2 + e*x])/Sqrt[2 + e*x]])/(Sqrt[2]*e)
```

Rule 675

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[(
d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] && E
qQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && GtQ[a, 0] && GtQ[d, 0] &&  !IGtQ[m
, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{\sqrt{2+ex}}\,dx = \int \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\,dx$$
$$= \frac{\sqrt[4]{3}\sqrt[4]{2-ex}(2+ex)^{3/4}}{e} + 3\int \frac{1}{(6-3ex)^{3/4}\sqrt[4]{2+ex}}\,dx$$
$$= \frac{\sqrt[4]{3}\sqrt[4]{2-ex}(2+ex)^{3/4}}{e} - \frac{4\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{4-\frac{x^4}{3}}}\,dx, x, \sqrt[4]{6-3ex}\right)}{e}$$
$$= \frac{\sqrt[4]{3}\sqrt[4]{2-ex}(2+ex)^{3/4}}{e} - \frac{4\,\text{Subst}\left(\int \frac{1}{1+\frac{x^4}{3}}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{e}$$
$$= \frac{\sqrt[4]{3}\sqrt[4]{2-ex}(2+ex)^{3/4}}{e} - \frac{2\,\text{Subst}\left(\int \frac{\sqrt{3}-x^2}{1+\frac{x^4}{3}}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{\sqrt{3}e} - \frac{2\,\text{Subst}\left(\int \frac{\sqrt{3}+x^2}{1+\frac{x^4}{3}}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{\sqrt{3}e}$$
$$= \frac{\sqrt[4]{3}\sqrt[4]{2-ex}(2+ex)^{3/4}}{e} + \frac{\sqrt[4]{3}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{3}+2x}{-\sqrt{3}-\sqrt{2}\sqrt[4]{3}x-x^2}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{\sqrt{2}e} + \frac{\sqrt[4]{3}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{3}-2x}{-\sqrt{3}+\sqrt{2}\sqrt[4]{3}x-x^2}\,d\right.}{\sqrt{2}e}$$
$$= \frac{\sqrt[4]{3}\sqrt[4]{2-ex}(2+ex)^{3/4}}{e} + \frac{\sqrt[4]{3}\log\left(\frac{\sqrt{2-ex}-\sqrt{2}\sqrt[4]{2-ex}\sqrt[4]{2+ex}+\sqrt{2+ex}}{\sqrt{2+ex}}\right)}{\sqrt{2}e} - \frac{\sqrt[4]{3}\log\left(\frac{\sqrt{2-ex}+\sqrt{2}\sqrt[4]{2-ex}\sqrt[4]{2+ex}+\sqrt{2+ex}}{\sqrt{2+ex}}\right.}{\sqrt{2}e}$$
$$= \frac{\sqrt[4]{3}\sqrt[4]{2-ex}(2+ex)^{3/4}}{e} + \frac{\sqrt{2}\sqrt[4]{3}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{2+ex}}\right)}{e} - \frac{\sqrt{2}\sqrt[4]{3}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{2+ex}}\right)}{e} + \frac{\sqrt[4]{3}\log\left(\sqrt{2-e}\right.}{}$$

Mathematica [C] time = 0.0476005, size = 60, normalized size = 0.22

$$\frac{2\sqrt{2}(ex-2)\sqrt[4]{12-3e^2x^2}\,{}_2F_1\left(\frac{1}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}-\frac{ex}{4}\right)}{5e\sqrt[4]{ex+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(12 - 3*e^2*x^2)^(1/4)/Sqrt[2 + e*x],x]
```

```
[Out] (2*Sqrt[2]*(-2 + e*x)*(12 - 3*e^2*x^2)^(1/4)*Hypergeometric2F1[1/4, 5/4, 9/
4, 1/2 - (e*x)/4])/(5*e*(2 + e*x)^(1/4))
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \sqrt[4]{-3e^2x^2+12}\frac{1}{\sqrt{ex+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(1/2),x)
```

```
[Out] int((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-3\,e^2x^2+12\right)^{\frac{1}{4}}}{\sqrt{ex+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-3*e^2*x^2 + 12)^(1/4)/sqrt(e*x + 2), x)
```

Fricas [B] time = 2.18322, size = 1511, normalized size = 5.62

$$4\cdot 3^{\frac{1}{4}}\sqrt{2}e\frac{1}{e^4}^{\frac{1}{4}}\arctan\left(-\frac{3^{\frac{3}{4}}\sqrt{2}\left(-3\,e^2x^2+12\right)^{\frac{1}{4}}\sqrt{ex+2}e^3\frac{1}{e^4}^{\frac{3}{4}}-3^{\frac{3}{4}}\sqrt{2}\left(e^4x+2\,e^3\right)\sqrt{\frac{3^{\frac{1}{4}}\sqrt{2}\left(-3\,e^2x^2+12\right)^{\frac{1}{4}}\sqrt{ex+2}e\frac{1}{e^4}^{\frac{1}{4}}+\sqrt{3}\left(e^3x+2\,e^2\right)\sqrt{\frac{1}{e^4}}+\sqrt{-3\,e^2x^2+12}}{ex+2}}\frac{1}{e^4}^{\frac{3}{4}}+3\,ex}{3\,(ex+2)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*3^(1/4)*sqrt(2)*e*(e^(-4))^(1/4)*arctan(-1/3*(3^(3/4)*sqrt(2)*(-3*e^
2*x^2 + 12)^(1/4)*sqrt(e*x + 2)*e^3*(e^(-4))^(3/4) - 3^(3/4)*sqrt(2)*(e^4*x
 + 2*e^3)*sqrt((3^(1/4)*sqrt(2)*(-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x + 2)*e*(e^
(-4))^(1/4) + sqrt(3)*(e^3*x + 2*e^2)*sqrt(e^(-4)) + sqrt(-3*e^2*x^2 + 12))
/(e*x + 2))*(e^(-4))^(3/4) + 3*e*x + 6)/(e*x + 2)) + 4*3^(1/4)*sqrt(2)*e*(e
^(-4))^(1/4)*arctan(-1/3*(3^(3/4)*sqrt(2)*(-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x
+ 2)*e^3*(e^(-4))^(3/4) - 3^(3/4)*sqrt(2)*(e^4*x + 2*e^3)*sqrt(-(3^(1/4)*sq
rt(2)*(-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x + 2)*e*(e^(-4))^(1/4) - sqrt(3)*(e^3
*x + 2*e^2)*sqrt(e^(-4)) - sqrt(-3*e^2*x^2 + 12))/(e*x + 2))*(e^(-4))^(3/4)
 - 3*e*x - 6)/(e*x + 2)) - 3^(1/4)*sqrt(2)*e*(e^(-4))^(1/4)*log((3^(1/4)*sq
rt(2)*(-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x + 2)*e*(e^(-4))^(1/4) + sqrt(3)*(e^3
*x + 2*e^2)*sqrt(e^(-4)) + sqrt(-3*e^2*x^2 + 12))/(e*x + 2)) + 3^(1/4)*sqrt
(2)*e*(e^(-4))^(1/4)*log(-(3^(1/4)*sqrt(2)*(-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x
 + 2)*e*(e^(-4))^(1/4) - sqrt(3)*(e^3*x + 2*e^2)*sqrt(e^(-4)) - sqrt(-3*e^2
*x^2 + 12))/(e*x + 2)) + 2*(-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x + 2))/e
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\sqrt[4]{3}\int \frac{\sqrt[4]{-e^2x^2+4}}{\sqrt{ex+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e**2*x**2+12)**(1/4)/(e*x+2)**(1/2),x)
```

```
[Out] 3**(1/4)*Integral((-e**2*x**2 + 4)**(1/4)/sqrt(e*x + 2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.932 $\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{3/2}}\,dx$

Optimal. Leaf size=270

$$-\frac{4\sqrt[4]{3}\sqrt[4]{2-ex}}{e\sqrt[4]{ex+2}}-\frac{\sqrt[4]{3}\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}-\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}e}+\frac{\sqrt[4]{3}\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}+\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}e}-\frac{\sqrt{2}\sqrt[4]{3}\tan^{-1}\left(1\cdot\right.}{e}$$

```
[Out] (-4*3^(1/4)*(2 - e*x)^(1/4))/(e*(2 + e*x)^(1/4)) - (Sqrt[2]*3^(1/4)*ArcTan[
1 - (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/e + (Sqrt[2]*3^(1/4)*ArcTan
[1 + (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/e - (3^(1/4)*Log[(Sqrt[6 -
 3*e*x] - Sqrt[6]*(2 - e*x)^(1/4)*(2 + e*x)^(1/4) + Sqrt[3]*Sqrt[2 + e*x])/
Sqrt[2 + e*x]])/(Sqrt[2]*e) + (3^(1/4)*Log[(Sqrt[6 - 3*e*x] + Sqrt[6]*(2 -
e*x)^(1/4)*(2 + e*x)^(1/4) + Sqrt[3]*Sqrt[2 + e*x])/Sqrt[2 + e*x]])/(Sqrt[2
]*e)
```

Rubi [A] time = 0.262523, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {675, 47, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{4\sqrt[4]{3}\sqrt[4]{2-ex}}{e\sqrt[4]{ex+2}}-\frac{\sqrt[4]{3}\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}-\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}e}+\frac{\sqrt[4]{3}\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}+\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}e}-\frac{\sqrt{2}\sqrt[4]{3}\tan^{-1}\left(1\cdot\right.}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(12 - 3*e^2*x^2)^(1/4)/(2 + e*x)^(3/2),x]
```


```
[Out] (-4*3^(1/4)*(2 - e*x)^(1/4))/(e*(2 + e*x)^(1/4)) - (Sqrt[2]*3^(1/4)*ArcTan[
1 - (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/e + (Sqrt[2]*3^(1/4)*ArcTan
[1 + (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/e - (3^(1/4)*Log[(Sqrt[6 -
 3*e*x] - Sqrt[6]*(2 - e*x)^(1/4)*(2 + e*x)^(1/4) + Sqrt[3]*Sqrt[2 + e*x])/
Sqrt[2 + e*x]])/(Sqrt[2]*e) + (3^(1/4)*Log[(Sqrt[6 - 3*e*x] + Sqrt[6]*(2 -
e*x)^(1/4)*(2 + e*x)^(1/4) + Sqrt[3]*Sqrt[2 + e*x])/Sqrt[2 + e*x]])/(Sqrt[2
]*e)
```


Rule 675

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[(
d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] && E
qQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && GtQ[a, 0] && GtQ[d, 0] &&  !IGtQ[m
, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{3/2}}\,dx = \int \frac{\sqrt[4]{6-3ex}}{(2+ex)^{5/4}}\,dx$$
$$= -\frac{4\sqrt[4]{3}\sqrt[4]{2-ex}}{e\sqrt[4]{2+ex}} - 3\int \frac{1}{(6-3ex)^{3/4}\sqrt[4]{2+ex}}\,dx$$
$$= -\frac{4\sqrt[4]{3}\sqrt[4]{2-ex}}{e\sqrt[4]{2+ex}} + \frac{4\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{4-\frac{x^4}{3}}}\,dx, x, \sqrt[4]{6-3ex}\right)}{e}$$
$$= -\frac{4\sqrt[4]{3}\sqrt[4]{2-ex}}{e\sqrt[4]{2+ex}} + \frac{4\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{x^4}{3}}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{e}$$
$$= -\frac{4\sqrt[4]{3}\sqrt[4]{2-ex}}{e\sqrt[4]{2+ex}} + \frac{2\,\mathrm{Subst}\left(\int \frac{\sqrt{3}-x^2}{1+\frac{x^4}{3}}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{\sqrt{3}e} + \frac{2\,\mathrm{Subst}\left(\int \frac{\sqrt{3}+x^2}{1+\frac{x^4}{3}}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{\sqrt{3}e}$$
$$= -\frac{4\sqrt[4]{3}\sqrt[4]{2-ex}}{e\sqrt[4]{2+ex}} - \frac{\sqrt[4]{3}\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{3}+2x}{-\sqrt{3}-\sqrt{2}\sqrt[4]{3}x-x^2}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{\sqrt{2}e} - \frac{\sqrt[4]{3}\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{3}-2x}{-\sqrt{3}+\sqrt{2}\sqrt[4]{3}x-x^2}\,dx, x,\right.}{\sqrt{2}e}$$
$$= -\frac{4\sqrt[4]{3}\sqrt[4]{2-ex}}{e\sqrt[4]{2+ex}} - \frac{\sqrt[4]{3}\log\left(\frac{\sqrt{2-ex}-\sqrt{2}\sqrt[4]{2-ex}\sqrt[4]{2+ex}+\sqrt{2+ex}}{\sqrt{2+ex}}\right)}{\sqrt{2}e} + \frac{\sqrt[4]{3}\log\left(\frac{\sqrt{2-ex}+\sqrt{2}\sqrt[4]{2-ex}\sqrt[4]{2+ex}+\sqrt{2+ex}}{\sqrt{2+ex}}\right)}{\sqrt{2}e} + \cdots$$
$$= -\frac{4\sqrt[4]{3}\sqrt[4]{2-ex}}{e\sqrt[4]{2+ex}} - \frac{\sqrt{2}\sqrt[4]{3}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{2+ex}}\right)}{e} + \frac{\sqrt{2}\sqrt[4]{3}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{2+ex}}\right)}{e} - \frac{\sqrt[4]{3}\log\left(\frac{\sqrt{2-ex}-\sqrt{2}\cdots}{}\right.}{\sqrt{\cdots}}$$

Mathematica [C] time = 0.056021, size = 60, normalized size = 0.22

$$\frac{(ex-2)\sqrt[4]{12-3e^2x^2}\,{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}-\frac{ex}{4}\right)}{5\sqrt{2}e\sqrt[4]{ex+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(12 - 3*e^2*x^2)^(1/4)/(2 + e*x)^(3/2),x]
```

```
[Out] ((-2 + e*x)*(12 - 3*e^2*x^2)^(1/4)*Hypergeometric2F1[5/4, 5/4, 9/4, 1/2 - (
e*x)/4])/(5*Sqrt[2]*e*(2 + e*x)^(1/4))
```

Maple [F] time = 0.485, size = 0, normalized size = 0.

$$\int \sqrt[4]{-3\,e^2x^2+12}\,(ex+2)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(3/2),x)
```

```
[Out] int((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-3\,e^2x^2+12\right)^{\frac{1}{4}}}{\left(ex+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-3*e^2*x^2 + 12)^(1/4)/(e*x + 2)^(3/2), x)
```

Fricas [B] time = 2.05574, size = 1593, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(4*3^(1/4)*sqrt(2)*(e^2*x + 2*e)*(e^(-4))^(1/4)*arctan(-1/3*(3^(3/4)*s
qrt(2)*(-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x + 2)*e^3*(e^(-4))^(3/4) - 3^(3/4)*s
qrt(2)*(e^4*x + 2*e^3)*sqrt((3^(1/4)*sqrt(2)*(-3*e^2*x^2 + 12)^(1/4)*sqrt(e
*x + 2)*e*(e^(-4))^(1/4) + sqrt(3)*(e^3*x + 2*e^2)*sqrt(e^(-4)) + sqrt(-3*e
^2*x^2 + 12))/(e*x + 2))*(e^(-4))^(3/4) + 3*e*x + 6)/(e*x + 2)) + 4*3^(1/4)
*sqrt(2)*(e^2*x + 2*e)*(e^(-4))^(1/4)*arctan(-1/3*(3^(3/4)*sqrt(2)*(-3*e^2*
x^2 + 12)^(1/4)*sqrt(e*x + 2)*e^3*(e^(-4))^(3/4) - 3^(3/4)*sqrt(2)*(e^4*x +
 2*e^3)*sqrt(-(3^(1/4)*sqrt(2)*(-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x + 2)*e*(e^(
-4))^(1/4) - sqrt(3)*(e^3*x + 2*e^2)*sqrt(e^(-4)) - sqrt(-3*e^2*x^2 + 12))/
(e*x + 2))*(e^(-4))^(3/4) - 3*e*x - 6)/(e*x + 2)) - 3^(1/4)*sqrt(2)*(e^2*x
+ 2*e)*(e^(-4))^(1/4)*log((3^(1/4)*sqrt(2)*(-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x
 + 2)*e*(e^(-4))^(1/4) + sqrt(3)*(e^3*x + 2*e^2)*sqrt(e^(-4)) + sqrt(-3*e^2
*x^2 + 12))/(e*x + 2)) + 3^(1/4)*sqrt(2)*(e^2*x + 2*e)*(e^(-4))^(1/4)*log(-
(3^(1/4)*sqrt(2)*(-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x + 2)*e*(e^(-4))^(1/4) - s
qrt(3)*(e^3*x + 2*e^2)*sqrt(e^(-4)) - sqrt(-3*e^2*x^2 + 12))/(e*x + 2)) + 8
*(-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x + 2))/(e^2*x + 2*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\sqrt[4]{3}\int \frac{\sqrt[4]{-e^2x^2+4}}{ex\sqrt{ex+2}+2\sqrt{ex+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e**2*x**2+12)**(1/4)/(e*x+2)**(3/2),x)
```

```
[Out] 3**(1/4)*Integral((-e**2*x**2 + 4)**(1/4)/(e*x*sqrt(e*x + 2) + 2*sqrt(e*x +
 2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.933 $\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{5/2}}\,dx$

Optimal. Leaf size=35

$$-\frac{\sqrt[4]{3}\left(4-e^2x^2\right)^{5/4}}{5e(ex+2)^{5/2}}$$

```
[Out] -(3^(1/4)*(4 - e^2*x^2)^(5/4))/(5*e*(2 + e*x)^(5/2))
```

Rubi [A] time = 0.0100162, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {651}

$$-\frac{\sqrt[4]{3}\left(4-e^2x^2\right)^{5/4}}{5e(ex+2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(12 - 3*e^2*x^2)^(1/4)/(2 + e*x)^(5/2),x]
```

```
[Out] -(3^(1/4)*(4 - e^2*x^2)^(5/4))/(5*e*(2 + e*x)^(5/2))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{5/2}}\,dx = -\frac{\sqrt[4]{3}\left(4-e^2x^2\right)^{5/4}}{5e(2+ex)^{5/2}}$$

Mathematica [A] time = 0.0533383, size = 35, normalized size = 1.

$$\frac{(ex-2)\sqrt[4]{12-3e^2x^2}}{5e(ex+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(12 - 3*e^2*x^2)^(1/4)/(2 + e*x)^(5/2),x]
```

```
[Out] ((-2 + e*x)*(12 - 3*e^2*x^2)^(1/4))/(5*e*(2 + e*x)^(3/2))
```

Maple [A] time = 0.049, size = 30, normalized size = 0.9

$$\frac{ex-2}{5e}\sqrt[4]{-3e^2x^2+12}\,(ex+2)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(5/2),x)
```

```
[Out] 1/5*(e*x-2)/(e*x+2)^(3/2)/e*(-3*e^2*x^2+12)^(1/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-3\,e^2x^2+12\right)^{\frac{1}{4}}}{(ex+2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-3*e^2*x^2 + 12)^(1/4)/(e*x + 2)^(5/2), x)
```

Fricas [A] time = 1.7725, size = 107, normalized size = 3.06

$$\frac{\left(-3\,e^2x^2+12\right)^{\frac{1}{4}}\sqrt{ex+2}(ex-2)}{5\left(e^3x^2+4\,e^2x+4\,e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/5*(-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x + 2)*(e*x - 2)/(e^3*x^2 + 4*e^2*x + 4*
e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e**2*x**2+12)**(1/4)/(e*x+2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25996, size = 62, normalized size = 1.77

$$-\frac{3^{\frac{1}{4}}\left(-(xe+2)^2+4\,xe+8\right)^{\frac{1}{4}}\left(\frac{4}{xe+2}-1\right)e^{(-1)}}{5\sqrt{xe+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/5*3^(1/4)*(-(x*e + 2)^2 + 4*x*e + 8)^(1/4)*(4/(x*e + 2) - 1)*e^(-1)/sqrt
(x*e + 2)
```

3.934 $\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{7/2}} dx$

Optimal. Leaf size=71

$$-\frac{(4-e^2x^2)^{5/4}}{15\ 3^{3/4}e(ex+2)^{5/2}}-\frac{(4-e^2x^2)^{5/4}}{3\ 3^{3/4}e(ex+2)^{7/2}}$$

```
[Out] -(4 - e^2*x^2)^(5/4)/(3*3^(3/4)*e*(2 + e*x)^(7/2)) - (4 - e^2*x^2)^(5/4)/(1
5*3^(3/4)*e*(2 + e*x)^(5/2))
```

Rubi [A] time = 0.0260016, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 651}

$$-\frac{(4-e^2x^2)^{5/4}}{15\ 3^{3/4}e(ex+2)^{5/2}}-\frac{(4-e^2x^2)^{5/4}}{3\ 3^{3/4}e(ex+2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(12 - 3*e^2*x^2)^(1/4)/(2 + e*x)^(7/2),x]
```

```
[Out] -(4 - e^2*x^2)^(5/4)/(3*3^(3/4)*e*(2 + e*x)^(7/2)) - (4 - e^2*x^2)^(5/4)/(1
5*3^(3/4)*e*(2 + e*x)^(5/2))
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{7/2}}\,dx &= -\frac{(4-e^2x^2)^{5/4}}{3\ 3^{3/4}e(2+ex)^{7/2}}+\frac{1}{9}\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{5/2}}\,dx\\ &= -\frac{(4-e^2x^2)^{5/4}}{3\ 3^{3/4}e(2+ex)^{7/2}}-\frac{(4-e^2x^2)^{5/4}}{15\ 3^{3/4}e(2+ex)^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.0637389, size = 45, normalized size = 0.63

$$\frac{(ex-2)(ex+7)\sqrt[4]{4-e^2x^2}}{15\ 3^{3/4}e(ex+2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(12 - 3*e^2*x^2)^(1/4)/(2 + e*x)^(7/2),x]
```

```
[Out] ((-2 + e*x)*(7 + e*x)*(4 - e^2*x^2)^(1/4))/(15*3^(3/4)*e*(2 + e*x)^(5/2))
```

Maple [A] time = 0.042, size = 35, normalized size = 0.5

$$\frac{(ex-2)(ex+7)}{45\,e}\sqrt[4]{-3\,e^2x^2+12}\,(ex+2)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(7/2),x)
```

```
[Out] 1/45*(e*x-2)*(e*x+7)*(-3*e^2*x^2+12)^(1/4)/(e*x+2)^(5/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-3\,e^2x^2+12\right)^{\frac{1}{4}}}{(ex+2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((-3*e^2*x^2 + 12)^(1/4)/(e*x + 2)^(7/2), x)
```

Fricas [A] time = 1.75313, size = 143, normalized size = 2.01

$$\frac{\left(e^2x^2+5\,ex-14\right)\left(-3\,e^2x^2+12\right)^{\frac{1}{4}}\sqrt{ex+2}}{45\left(e^4x^3+6\,e^3x^2+12\,e^2x+8\,e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/45*(e^2*x^2 + 5*e*x - 14)*(-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x + 2)/(e^4*x^3
+ 6*e^3*x^2 + 12*e^2*x + 8*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e**2*x**2+12)**(1/4)/(e*x+2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26122, size = 124, normalized size = 1.75

$$-\frac{1}{180}\cdot 3^{\frac{1}{4}}\left(\frac{9\left(-(xe+2)^2+4\,xe+8\right)^{\frac{1}{4}}\left(\frac{4}{xe+2}-1\right)}{\sqrt{xe+2}}+\frac{5\left((xe+2)^2-8\,xe\right)\left(-(xe+2)^2+4\,xe+8\right)^{\frac{1}{4}}}{(xe+2)^{\frac{5}{2}}}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/180*3^(1/4)*(9*(-(x*e + 2)^2 + 4*x*e + 8)^(1/4)*(4/(x*e + 2) - 1)/sqrt(x
*e + 2) + 5*((x*e + 2)^2 - 8*x*e)*(-(x*e + 2)^2 + 4*x*e + 8)^(1/4)/(x*e + 2
)^(5/2))*e^(-1)
```

3.935 $\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{9/2}}\,dx$

Optimal. Leaf size=106

$$-\frac{2\left(4-e^2x^2\right)^{5/4}}{195\ 3^{3/4}e(ex+2)^{5/2}}-\frac{2\left(4-e^2x^2\right)^{5/4}}{39\ 3^{3/4}e(ex+2)^{7/2}}-\frac{\sqrt[4]{3}\left(4-e^2x^2\right)^{5/4}}{13e(ex+2)^{9/2}}$$

```
[Out] -(3^(1/4)*(4 - e^2*x^2)^(5/4))/(13*e*(2 + e*x)^(9/2)) - (2*(4 - e^2*x^2)^(5
/4))/(39*3^(3/4)*e*(2 + e*x)^(7/2)) - (2*(4 - e^2*x^2)^(5/4))/(195*3^(3/4)*
e*(2 + e*x)^(5/2))
```

Rubi [A] time = 0.040833, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 651}

$$-\frac{2\left(4-e^2x^2\right)^{5/4}}{195\ 3^{3/4}e(ex+2)^{5/2}}-\frac{2\left(4-e^2x^2\right)^{5/4}}{39\ 3^{3/4}e(ex+2)^{7/2}}-\frac{\sqrt[4]{3}\left(4-e^2x^2\right)^{5/4}}{13e(ex+2)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(12 - 3*e^2*x^2)^(1/4)/(2 + e*x)^(9/2),x]
```

```
[Out] -(3^(1/4)*(4 - e^2*x^2)^(5/4))/(13*e*(2 + e*x)^(9/2)) - (2*(4 - e^2*x^2)^(5
/4))/(39*3^(3/4)*e*(2 + e*x)^(7/2)) - (2*(4 - e^2*x^2)^(5/4))/(195*3^(3/4)*
e*(2 + e*x)^(5/2))
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{9/2}}\,dx &= -\frac{\sqrt[4]{3}\left(4-e^2x^2\right)^{5/4}}{13e(2+ex)^{9/2}}+\frac{2}{13}\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{7/2}}\,dx\\
&= -\frac{\sqrt[4]{3}\left(4-e^2x^2\right)^{5/4}}{13e(2+ex)^{9/2}}-\frac{2\left(4-e^2x^2\right)^{5/4}}{39\ 3^{3/4}e(2+ex)^{7/2}}+\frac{2}{117}\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{5/2}}\,dx\\
&= -\frac{\sqrt[4]{3}\left(4-e^2x^2\right)^{5/4}}{13e(2+ex)^{9/2}}-\frac{2\left(4-e^2x^2\right)^{5/4}}{39\ 3^{3/4}e(2+ex)^{7/2}}-\frac{2\left(4-e^2x^2\right)^{5/4}}{195\ 3^{3/4}e(2+ex)^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0762167, size = 57, normalized size = 0.54

$$\frac{\sqrt[4]{4-e^2x^2}\left(2e^3x^3+14e^2x^2+37ex-146\right)}{195\ 3^{3/4}e(ex+2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(12 - 3*e^2*x^2)^(1/4)/(2 + e*x)^(9/2),x]
```

```
[Out] ((4 - e^2*x^2)^(1/4)*(-146 + 37*e*x + 14*e^2*x^2 + 2*e^3*x^3))/(195*3^(3/4)
*e*(2 + e*x)^(7/2))
```

Maple [A] time = 0.042, size = 44, normalized size = 0.4

$$\frac{(ex-2)\left(2e^2x^2+18ex+73\right)}{585e}\sqrt[4]{-3e^2x^2+12}\,(ex+2)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(9/2),x)
```

```
[Out] 1/585*(e*x-2)*(2*e^2*x^2+18*e*x+73)*(-3*e^2*x^2+12)^(1/4)/(e*x+2)^(7/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-3e^2x^2+12\right)^{\frac{1}{4}}}{(ex+2)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((-3*e^2*x^2 + 12)^(1/4)/(e*x + 2)^(9/2), x)
```

Fricas [A] time = 1.84171, size = 186, normalized size = 1.75

$$\frac{\left(2e^3x^3+14e^2x^2+37ex-146\right)\left(-3e^2x^2+12\right)^{\frac{1}{4}}\sqrt{ex+2}}{585\left(e^5x^4+8e^4x^3+24e^3x^2+32e^2x+16e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/585*(2*e^3*x^3 + 14*e^2*x^2 + 37*e*x - 146)*(-3*e^2*x^2 + 12)^(1/4)*sqrt(
e*x + 2)/(e^5*x^4 + 8*e^4*x^3 + 24*e^3*x^2 + 32*e^2*x + 16*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e**2*x**2+12)**(1/4)/(e*x+2)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32091, size = 197, normalized size = 1.86

$$-\frac{1}{9360}\cdot 3^{\frac{1}{4}}\left(\frac{117\left(-(xe+2)^2+4\,xe+8\right)^{\frac{1}{4}}\left(\frac{4}{xe+2}-1\right)}{\sqrt{xe+2}}+\frac{130\left((xe+2)^2-8\,xe\right)\left(-(xe+2)^2+4\,xe+8\right)^{\frac{1}{4}}}{(xe+2)^{\frac{5}{2}}}-\frac{45\left((xe+2)^3-12\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(9/2),x, algorithm="giac")
```

```
[Out] -1/9360*3^(1/4)*(117*(-(x*e + 2)^2 + 4*x*e + 8)^(1/4)*(4/(x*e + 2) - 1)/sqr
t(x*e + 2) + 130*((x*e + 2)^2 - 8*x*e)*(-(x*e + 2)^2 + 4*x*e + 8)^(1/4)/(x*
e + 2)^(5/2) - 45*((x*e + 2)^3 - 12*(x*e + 2)^2 + 48*x*e + 32)*(-(x*e + 2)^
2 + 4*x*e + 8)^(1/4)/(x*e + 2)^(7/2))*e^(-1)
```

3.936 $\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{11/2}}\,dx$

Optimal. Leaf size=141

$$-\frac{2\left(4-e^2x^2\right)^{5/4}}{1105\ 3^{3/4}e(ex+2)^{5/2}}-\frac{2\left(4-e^2x^2\right)^{5/4}}{221\ 3^{3/4}e(ex+2)^{7/2}}-\frac{3\sqrt[4]{3}\left(4-e^2x^2\right)^{5/4}}{221e(ex+2)^{9/2}}-\frac{\sqrt[4]{3}\left(4-e^2x^2\right)^{5/4}}{17e(ex+2)^{11/2}}$$

```
[Out] -(3^(1/4)*(4 - e^2*x^2)^(5/4))/(17*e*(2 + e*x)^(11/2)) - (3*3^(1/4)*(4 - e^
2*x^2)^(5/4))/(221*e*(2 + e*x)^(9/2)) - (2*(4 - e^2*x^2)^(5/4))/(221*3^(3/4
)*e*(2 + e*x)^(7/2)) - (2*(4 - e^2*x^2)^(5/4))/(1105*3^(3/4)*e*(2 + e*x)^(5
/2))
```

Rubi [A] time = 0.0577388, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 651}

$$-\frac{2\left(4-e^2x^2\right)^{5/4}}{1105\ 3^{3/4}e(ex+2)^{5/2}}-\frac{2\left(4-e^2x^2\right)^{5/4}}{221\ 3^{3/4}e(ex+2)^{7/2}}-\frac{3\sqrt[4]{3}\left(4-e^2x^2\right)^{5/4}}{221e(ex+2)^{9/2}}-\frac{\sqrt[4]{3}\left(4-e^2x^2\right)^{5/4}}{17e(ex+2)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(12 - 3*e^2*x^2)^(1/4)/(2 + e*x)^(11/2),x]
```

```
[Out] -(3^(1/4)*(4 - e^2*x^2)^(5/4))/(17*e*(2 + e*x)^(11/2)) - (3*3^(1/4)*(4 - e^
2*x^2)^(5/4))/(221*e*(2 + e*x)^(9/2)) - (2*(4 - e^2*x^2)^(5/4))/(221*3^(3/4
)*e*(2 + e*x)^(7/2)) - (2*(4 - e^2*x^2)^(5/4))/(1105*3^(3/4)*e*(2 + e*x)^(5
/2))
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{11/2}}\,dx = -\frac{\sqrt[4]{3}\left(4-e^2x^2\right)^{5/4}}{17e(2+ex)^{11/2}} + \frac{3}{17}\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{9/2}}\,dx$$
$$= -\frac{\sqrt[4]{3}\left(4-e^2x^2\right)^{5/4}}{17e(2+ex)^{11/2}} - \frac{3\sqrt[4]{3}\left(4-e^2x^2\right)^{5/4}}{221e(2+ex)^{9/2}} + \frac{6}{221}\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{7/2}}\,dx$$
$$= -\frac{\sqrt[4]{3}\left(4-e^2x^2\right)^{5/4}}{17e(2+ex)^{11/2}} - \frac{3\sqrt[4]{3}\left(4-e^2x^2\right)^{5/4}}{221e(2+ex)^{9/2}} - \frac{2\left(4-e^2x^2\right)^{5/4}}{221\ 3^{3/4}e(2+ex)^{7/2}} + \frac{2}{663}\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{5/2}}\,dx$$
$$= -\frac{\sqrt[4]{3}\left(4-e^2x^2\right)^{5/4}}{17e(2+ex)^{11/2}} - \frac{3\sqrt[4]{3}\left(4-e^2x^2\right)^{5/4}}{221e(2+ex)^{9/2}} - \frac{2\left(4-e^2x^2\right)^{5/4}}{221\ 3^{3/4}e(2+ex)^{7/2}} - \frac{2\left(4-e^2x^2\right)^{5/4}}{1105\ 3^{3/4}e(2+ex)^{5/2}}$$

Mathematica [A] time = 0.0773567, size = 65, normalized size = 0.46

$$\frac{\sqrt[4]{4-e^2x^2}\left(2e^4x^4+18e^3x^3+65e^2x^2+123ex-682\right)}{1105\ 3^{3/4}e(ex+2)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(12 - 3*e^2*x^2)^(1/4)/(2 + e*x)^(11/2),x]
```

```
[Out] ((4 - e^2*x^2)^(1/4)*(-682 + 123*e*x + 65*e^2*x^2 + 18*e^3*x^3 + 2*e^4*x^4)
)/(1105*3^(3/4)*e*(2 + e*x)^(9/2))
```

Maple [A] time = 0.041, size = 52, normalized size = 0.4

$$\frac{(ex-2)\left(2\,e^3x^3+22\,e^2x^2+109\,ex+341\right)}{3315\,e}\sqrt[4]{-3\,e^2x^2+12}\,(ex+2)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(11/2),x)
```

```
[Out] 1/3315*(e*x-2)*(2*e^3*x^3+22*e^2*x^2+109*e*x+341)*(-3*e^2*x^2+12)^(1/4)/(e*
x+2)^(9/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-3\,e^2x^2+12\right)^{\frac{1}{4}}}{(ex+2)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((-3*e^2*x^2 + 12)^(1/4)/(e*x + 2)^(11/2), x)
```

Fricas [A] time = 1.82339, size = 225, normalized size = 1.6

$$\frac{\left(2 e^4 x^4 + 18 e^3 x^3 + 65 e^2 x^2 + 123 ex - 682\right)\left(-3 e^2 x^2 + 12\right)^{\frac{1}{4}} \sqrt{ex + 2}}{3315 \left(e^6 x^5 + 10 e^5 x^4 + 40 e^4 x^3 + 80 e^3 x^2 + 80 e^2 x + 32 e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(11/2),x, algorithm="fricas")
```

```
[Out] 1/3315*(2*e^4*x^4 + 18*e^3*x^3 + 65*e^2*x^2 + 123*e*x - 682)*(-3*e^2*x^2 +
12)^(1/4)*sqrt(e*x + 2)/(e^6*x^5 + 10*e^5*x^4 + 40*e^4*x^3 + 80*e^3*x^2 + 8
0*e^2*x + 32*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e**2*x**2+12)**(1/4)/(e*x+2)**(11/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3661, size = 284, normalized size = 2.01

$$-\frac{1}{212160} \cdot 3^{\frac{1}{4}} \left(\frac{663 \left(-(xe+2)^2 + 4 xe + 8\right)^{\frac{1}{4}} \left(\frac{4}{xe+2} - 1\right)}{\sqrt{xe+2}} + \frac{1105 \left((xe+2)^2 - 8 xe\right)\left(-(xe+2)^2 + 4 xe + 8\right)^{\frac{1}{4}}}{(xe+2)^{\frac{5}{2}}} - \frac{765 \left((xe + \right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(11/2),x, algorithm="giac")
```

```
[Out] -1/212160*3^(1/4)*(663*(-(x*e + 2)^2 + 4*x*e + 8)^(1/4)*(4/(x*e + 2) - 1)/s
qrt(x*e + 2) + 1105*((x*e + 2)^2 - 8*x*e)*(-(x*e + 2)^2 + 4*x*e + 8)^(1/4)/
(x*e + 2)^(5/2) - 765*((x*e + 2)^3 - 12*(x*e + 2)^2 + 48*x*e + 32)*(-(x*e +
 2)^2 + 4*x*e + 8)^(1/4)/(x*e + 2)^(7/2) + 195*((x*e + 2)^4 - 16*(x*e + 2)^
3 + 96*(x*e + 2)^2 - 256*x*e - 256)*(-(x*e + 2)^2 + 4*x*e + 8)^(1/4)/(x*e +
 2)^(9/2))*e^(-1)
```

3.937 $\int \frac{(2+ex)^{5/2}}{\sqrt[4]{12-3e^2x^2}}\,dx$

Optimal. Leaf size=340

$$-\frac{(2-ex)^{3/4}(ex+2)^{9/4}}{3\sqrt[4]{3}e}-\frac{3^{3/4}(2-ex)^{3/4}(ex+2)^{5/4}}{2e}-\frac{5\,3^{3/4}(2-ex)^{3/4}\sqrt[4]{ex+2}}{2e}-\frac{5\,3^{3/4}\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}-\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{2\sqrt{2}e}$$

```
[Out] (-5*3^(3/4)*(2 - e*x)^(3/4)*(2 + e*x)^(1/4))/(2*e) - (3^(3/4)*(2 - e*x)^(3/
4)*(2 + e*x)^(5/4))/(2*e) - ((2 - e*x)^(3/4)*(2 + e*x)^(9/4))/(3*3^(1/4)*e)
 + (5*3^(3/4)*ArcTan[1 - (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/(Sqrt[
2]*e) - (5*3^(3/4)*ArcTan[1 + (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/(
Sqrt[2]*e) - (5*3^(3/4)*Log[(Sqrt[6 - 3*e*x] - Sqrt[6]*(2 - e*x)^(1/4)*(2 +
 e*x)^(1/4) + Sqrt[3]*Sqrt[2 + e*x])/Sqrt[2 + e*x]])/(2*Sqrt[2]*e) + (5*3^(
3/4)*Log[(Sqrt[6 - 3*e*x] + Sqrt[6]*(2 - e*x)^(1/4)*(2 + e*x)^(1/4) + Sqrt[
3]*Sqrt[2 + e*x])/Sqrt[2 + e*x]])/(2*Sqrt[2]*e)
```

Rubi [A] time = 0.298083, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {675, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{(2-ex)^{3/4}(ex+2)^{9/4}}{3\sqrt[4]{3}e}-\frac{3^{3/4}(2-ex)^{3/4}(ex+2)^{5/4}}{2e}-\frac{5\,3^{3/4}(2-ex)^{3/4}\sqrt[4]{ex+2}}{2e}-\frac{5\,3^{3/4}\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}-\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{2\sqrt{2}e}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + e*x)^(5/2)/(12 - 3*e^2*x^2)^(1/4),x]
```

```
[Out] (-5*3^(3/4)*(2 - e*x)^(3/4)*(2 + e*x)^(1/4))/(2*e) - (3^(3/4)*(2 - e*x)^(3/
4)*(2 + e*x)^(5/4))/(2*e) - ((2 - e*x)^(3/4)*(2 + e*x)^(9/4))/(3*3^(1/4)*e)
 + (5*3^(3/4)*ArcTan[1 - (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/(Sqrt[
2]*e) - (5*3^(3/4)*ArcTan[1 + (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/(
Sqrt[2]*e) - (5*3^(3/4)*Log[(Sqrt[6 - 3*e*x] - Sqrt[6]*(2 - e*x)^(1/4)*(2 +
 e*x)^(1/4) + Sqrt[3]*Sqrt[2 + e*x])/Sqrt[2 + e*x]])/(2*Sqrt[2]*e) + (5*3^(
3/4)*Log[(Sqrt[6 - 3*e*x] + Sqrt[6]*(2 - e*x)^(1/4)*(2 + e*x)^(1/4) + Sqrt[
3]*Sqrt[2 + e*x])/Sqrt[2 + e*x]])/(2*Sqrt[2]*e)
```

Rule 675

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[(
d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] && E
qQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && GtQ[a, 0] && GtQ[d, 0] &&  !IGtQ[m
, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(2+ex)^{5/2}}{\sqrt[4]{12-3e^2x^2}}\,dx = \int \frac{(2+ex)^{9/4}}{\sqrt[4]{6-3ex}}\,dx$$
$$= -\frac{(2-ex)^{3/4}(2+ex)^{9/4}}{3\sqrt[4]{3}e} + 3\int \frac{(2+ex)^{5/4}}{\sqrt[4]{6-3ex}}\,dx$$
$$= -\frac{3^{3/4}(2-ex)^{3/4}(2+ex)^{5/4}}{2e} - \frac{(2-ex)^{3/4}(2+ex)^{9/4}}{3\sqrt[4]{3}e} + \frac{15}{2}\int \frac{\sqrt[4]{2+ex}}{\sqrt[4]{6-3ex}}\,dx$$
$$= -\frac{5\,3^{3/4}(2-ex)^{3/4}\sqrt[4]{2+ex}}{2e} - \frac{3^{3/4}(2-ex)^{3/4}(2+ex)^{5/4}}{2e} - \frac{(2-ex)^{3/4}(2+ex)^{9/4}}{3\sqrt[4]{3}e} + \frac{15}{2}\int \frac{1}{\sqrt[4]{6-3ex}(2-\cdots}$$
$$= -\frac{5\,3^{3/4}(2-ex)^{3/4}\sqrt[4]{2+ex}}{2e} - \frac{3^{3/4}(2-ex)^{3/4}(2+ex)^{5/4}}{2e} - \frac{(2-ex)^{3/4}(2+ex)^{9/4}}{3\sqrt[4]{3}e} - \frac{10\,\mathrm{Subst}\left(\int \frac{x^2}{\left(4-\frac{x^4}{3}\right)\cdots}\right.}{e\cdots}$$
$$= -\frac{5\,3^{3/4}(2-ex)^{3/4}\sqrt[4]{2+ex}}{2e} - \frac{3^{3/4}(2-ex)^{3/4}(2+ex)^{5/4}}{2e} - \frac{(2-ex)^{3/4}(2+ex)^{9/4}}{3\sqrt[4]{3}e} - \frac{10\,\mathrm{Subst}\left(\int \frac{x^2}{1+\frac{x^4}{3}}d\cdots\right.}{e}$$
$$= -\frac{5\,3^{3/4}(2-ex)^{3/4}\sqrt[4]{2+ex}}{2e} - \frac{3^{3/4}(2-ex)^{3/4}(2+ex)^{5/4}}{2e} - \frac{(2-ex)^{3/4}(2+ex)^{9/4}}{3\sqrt[4]{3}e} + \frac{5\,\mathrm{Subst}\left(\int \frac{\sqrt{3}-x^2}{1+\frac{x^4}{3}}d\cdots\right.}{e}$$
$$= -\frac{5\,3^{3/4}(2-ex)^{3/4}\sqrt[4]{2+ex}}{2e} - \frac{3^{3/4}(2-ex)^{3/4}(2+ex)^{5/4}}{2e} - \frac{(2-ex)^{3/4}(2+ex)^{9/4}}{3\sqrt[4]{3}e} - \frac{15\,\mathrm{Subst}\left(\int \frac{\cdots}{\sqrt{3}-\sqrt{2}\cdots}\right.}{2\cdots}$$
$$= -\frac{5\,3^{3/4}(2-ex)^{3/4}\sqrt[4]{2+ex}}{2e} - \frac{3^{3/4}(2-ex)^{3/4}(2+ex)^{5/4}}{2e} - \frac{(2-ex)^{3/4}(2+ex)^{9/4}}{3\sqrt[4]{3}e} - \frac{5\,3^{3/4}\log\left(\frac{\sqrt{2-ex}-\sqrt{\cdots}}{\cdots}\right.}{2\sqrt{\cdots}}$$
$$= -\frac{5\,3^{3/4}(2-ex)^{3/4}\sqrt[4]{2+ex}}{2e} - \frac{3^{3/4}(2-ex)^{3/4}(2+ex)^{5/4}}{2e} - \frac{(2-ex)^{3/4}(2+ex)^{9/4}}{3\sqrt[4]{3}e} + \frac{5\,3^{3/4}\tan^{-1}\left(1-\frac{\sqrt{\cdots}}{\cdots}\right.}{\sqrt{2}e}$$

Mathematica [C] time = 0.0698542, size = 60, normalized size = 0.18

$$\frac{64\sqrt{2}(ex-2)\sqrt[4]{ex+2}\,{}_2F_1\left(-\frac{9}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}-\frac{ex}{4}\right)}{3e\sqrt[4]{12-3e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + e*x)^(5/2)/(12 - 3*e^2*x^2)^(1/4),x]
```

```
[Out] (64*Sqrt[2]*(-2 + e*x)*(2 + e*x)^(1/4)*Hypergeometric2F1[-9/4, 3/4, 7/4, 1/
2 - (e*x)/4])/(3*e*(12 - 3*e^2*x^2)^(1/4))
```

Maple [F] time = 0.159, size = 0, normalized size = 0.

$$\int (ex+2)^{\frac{5}{2}}\frac{1}{\sqrt[4]{-3\,e^2x^2+12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/4),x)
```

```
[Out] int((e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+2)^{\frac{5}{2}}}{\left(-3\,e^2x^2+12\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((e*x + 2)^(5/2)/(-3*e^2*x^2 + 12)^(1/4), x)
```

Fricas [B] time = 2.12063, size = 1804, normalized size = 5.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/36*(180*27^(1/4)*sqrt(2)*(e^2*x + 2*e)*(e^(-4))^(1/4)*arctan(-1/27*(27^(3
/4)*sqrt(2)*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e*x + 2)*e^3*(e^(-4))^(3/4) + 27*e
^2*x^2 - 27^(3/4)*sqrt(2)*(e^5*x^2 - 4*e^3)*sqrt((27^(1/4)*sqrt(2)*(-3*e^2*
x^2 + 12)^(3/4)*sqrt(e*x + 2)*e*(e^(-4))^(1/4) + 3*sqrt(3)*(e^4*x^2 - 4*e^2
)*sqrt(e^(-4)) - 3*sqrt(-3*e^2*x^2 + 12)*(e*x + 2))/(e^2*x^2 - 4))*(e^(-4))
^(3/4) - 108)/(e^2*x^2 - 4)) + 180*27^(1/4)*sqrt(2)*(e^2*x + 2*e)*(e^(-4))^
(1/4)*arctan(-1/27*(27^(3/4)*sqrt(2)*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e*x + 2)*
e^3*(e^(-4))^(3/4) - 27*e^2*x^2 - 27^(3/4)*sqrt(2)*(e^5*x^2 - 4*e^3)*sqrt(-
(27^(1/4)*sqrt(2)*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e*x + 2)*e*(e^(-4))^(1/4) -
3*sqrt(3)*(e^4*x^2 - 4*e^2)*sqrt(e^(-4)) + 3*sqrt(-3*e^2*x^2 + 12)*(e*x + 2
))/(e^2*x^2 - 4))*(e^(-4))^(3/4) + 108)/(e^2*x^2 - 4)) - 45*27^(1/4)*sqrt(2
)*(e^2*x + 2*e)*(e^(-4))^(1/4)*log((27^(1/4)*sqrt(2)*(-3*e^2*x^2 + 12)^(3/4
)*sqrt(e*x + 2)*e*(e^(-4))^(1/4) + 3*sqrt(3)*(e^4*x^2 - 4*e^2)*sqrt(e^(-4))
 - 3*sqrt(-3*e^2*x^2 + 12)*(e*x + 2))/(e^2*x^2 - 4)) + 45*27^(1/4)*sqrt(2)*
(e^2*x + 2*e)*(e^(-4))^(1/4)*log(-(27^(1/4)*sqrt(2)*(-3*e^2*x^2 + 12)^(3/4)
*sqrt(e*x + 2)*e*(e^(-4))^(1/4) - 3*sqrt(3)*(e^4*x^2 - 4*e^2)*sqrt(e^(-4))
+ 3*sqrt(-3*e^2*x^2 + 12)*(e*x + 2))/(e^2*x^2 - 4)) - 2*(2*e^2*x^2 + 17*e*x
 + 71)*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e*x + 2))/(e^2*x + 2*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)**(5/2)/(-3*e**2*x**2+12)**(1/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+2)^{\frac{5}{2}}}{\left(-3\,e^2x^2+12\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((e*x + 2)^(5/2)/(-3*e^2*x^2 + 12)^(1/4), x)
```

3.938 $\int \frac{(2+ex)^{3/2}}{\sqrt[4]{12-3e^2x^2}}\,dx$

Optimal. Leaf size=309

$$-\frac{(2-ex)^{3/4}(ex+2)^{5/4}}{2\sqrt[4]{3}e}-\frac{5(2-ex)^{3/4}\sqrt[4]{ex+2}}{2\sqrt[4]{3}e}-\frac{5\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}-\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{2\sqrt{2}\sqrt[4]{3}e}+\frac{5\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}+\sqrt{6}\sqrt[4]{2-e\ldots}}{\sqrt{ex+2}}\right)}{2\sqrt{2}\sqrt[4]{3}e}$$

```
[Out] (-5*(2 - e*x)^(3/4)*(2 + e*x)^(1/4))/(2*3^(1/4)*e) - ((2 - e*x)^(3/4)*(2 +
e*x)^(5/4))/(2*3^(1/4)*e) + (5*ArcTan[1 - (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*
x)^(1/4)])/(Sqrt[2]*3^(1/4)*e) - (5*ArcTan[1 + (Sqrt[2]*(2 - e*x)^(1/4))/(2
 + e*x)^(1/4)])/(Sqrt[2]*3^(1/4)*e) - (5*Log[(Sqrt[6 - 3*e*x] - Sqrt[6]*(2
- e*x)^(1/4)*(2 + e*x)^(1/4) + Sqrt[3]*Sqrt[2 + e*x])/Sqrt[2 + e*x]])/(2*Sq
rt[2]*3^(1/4)*e) + (5*Log[(Sqrt[6 - 3*e*x] + Sqrt[6]*(2 - e*x)^(1/4)*(2 + e
*x)^(1/4) + Sqrt[3]*Sqrt[2 + e*x])/Sqrt[2 + e*x]])/(2*Sqrt[2]*3^(1/4)*e)
```

Rubi [A] time = 0.269418, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {675, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{(2-ex)^{3/4}(ex+2)^{5/4}}{2\sqrt[4]{3}e}-\frac{5(2-ex)^{3/4}\sqrt[4]{ex+2}}{2\sqrt[4]{3}e}-\frac{5\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}-\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{2\sqrt{2}\sqrt[4]{3}e}+\frac{5\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}+\sqrt{6}\sqrt[4]{2-e\ldots}}{\sqrt{ex+2}}\right)}{2\sqrt{2}\sqrt[4]{3}e}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + e*x)^(3/2)/(12 - 3*e^2*x^2)^(1/4),x]
```


```
[Out] (-5*(2 - e*x)^(3/4)*(2 + e*x)^(1/4))/(2*3^(1/4)*e) - ((2 - e*x)^(3/4)*(2 +
e*x)^(5/4))/(2*3^(1/4)*e) + (5*ArcTan[1 - (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*
x)^(1/4)])/(Sqrt[2]*3^(1/4)*e) - (5*ArcTan[1 + (Sqrt[2]*(2 - e*x)^(1/4))/(2
 + e*x)^(1/4)])/(Sqrt[2]*3^(1/4)*e) - (5*Log[(Sqrt[6 - 3*e*x] - Sqrt[6]*(2
- e*x)^(1/4)*(2 + e*x)^(1/4) + Sqrt[3]*Sqrt[2 + e*x])/Sqrt[2 + e*x]])/(2*Sq
rt[2]*3^(1/4)*e) + (5*Log[(Sqrt[6 - 3*e*x] + Sqrt[6]*(2 - e*x)^(1/4)*(2 + e
*x)^(1/4) + Sqrt[3]*Sqrt[2 + e*x])/Sqrt[2 + e*x]])/(2*Sqrt[2]*3^(1/4)*e)
```


Rule 675

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[(
d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] && E
qQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && GtQ[a, 0] && GtQ[d, 0] &&  !IGtQ[m
, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(2+ex)^{3/2}}{\sqrt[4]{12-3e^2x^2}}\,dx = \int \frac{(2+ex)^{5/4}}{\sqrt[4]{6-3ex}}\,dx$$
$$= -\frac{(2-ex)^{3/4}(2+ex)^{5/4}}{2\sqrt[4]{3}e} + \frac{5}{2}\int \frac{\sqrt[4]{2+ex}}{\sqrt[4]{6-3ex}}\,dx$$
$$= -\frac{5(2-ex)^{3/4}\sqrt[4]{2+ex}}{2\sqrt[4]{3}e} - \frac{(2-ex)^{3/4}(2+ex)^{5/4}}{2\sqrt[4]{3}e} + \frac{5}{2}\int \frac{1}{\sqrt[4]{6-3ex}(2+ex)^{3/4}}\,dx$$
$$= -\frac{5(2-ex)^{3/4}\sqrt[4]{2+ex}}{2\sqrt[4]{3}e} - \frac{(2-ex)^{3/4}(2+ex)^{5/4}}{2\sqrt[4]{3}e} - \frac{10\,\mathrm{Subst}\left(\int \frac{x^2}{\left(4-\frac{x^4}{3}\right)^{3/4}}dx, x, \sqrt[4]{6-3ex}\right)}{3e}$$
$$= -\frac{5(2-ex)^{3/4}\sqrt[4]{2+ex}}{2\sqrt[4]{3}e} - \frac{(2-ex)^{3/4}(2+ex)^{5/4}}{2\sqrt[4]{3}e} - \frac{10\,\mathrm{Subst}\left(\int \frac{x^2}{1+\frac{x^4}{3}}dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{3e}$$
$$= -\frac{5(2-ex)^{3/4}\sqrt[4]{2+ex}}{2\sqrt[4]{3}e} - \frac{(2-ex)^{3/4}(2+ex)^{5/4}}{2\sqrt[4]{3}e} + \frac{5\,\mathrm{Subst}\left(\int \frac{\sqrt{3}-x^2}{1+\frac{x^4}{3}}dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{3e} - \frac{5\,\mathrm{Subst}\left(\int \frac{\sqrt{}}{1}\right.}{}$$
$$= -\frac{5(2-ex)^{3/4}\sqrt[4]{2+ex}}{2\sqrt[4]{3}e} - \frac{(2-ex)^{3/4}(2+ex)^{5/4}}{2\sqrt[4]{3}e} - \frac{5\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{3}-\sqrt{2}\sqrt[4]{3}x+x^2}dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{2e} - \frac{5\,\mathrm{Sub}}{}$$
$$= -\frac{5(2-ex)^{3/4}\sqrt[4]{2+ex}}{2\sqrt[4]{3}e} - \frac{(2-ex)^{3/4}(2+ex)^{5/4}}{2\sqrt[4]{3}e} - \frac{5\log\left(\frac{\sqrt{2-ex}-\sqrt{2}\sqrt[4]{2-ex}\sqrt[4]{2+ex}+\sqrt{2+ex}}{\sqrt{2+ex}}\right)}{2\sqrt{2}\sqrt[4]{3}e} + \frac{5\log\left(\frac{\sqrt{2-e}}{}\right.}{}$$
$$= -\frac{5(2-ex)^{3/4}\sqrt[4]{2+ex}}{2\sqrt[4]{3}e} - \frac{(2-ex)^{3/4}(2+ex)^{5/4}}{2\sqrt[4]{3}e} + \frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{2+ex}}\right)}{\sqrt{2}\sqrt[4]{3}e} - \frac{5\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{2+ex}}\right)}{\sqrt{2}\sqrt[4]{3}e}$$

Mathematica [C] time = 0.054909, size = 60, normalized size = 0.19

$$\frac{16\sqrt{2}(ex-2)\sqrt[4]{ex+2}\,{}_2F_1\left(-\frac{5}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}-\frac{ex}{4}\right)}{3e\sqrt[4]{12-3e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + e*x)^(3/2)/(12 - 3*e^2*x^2)^(1/4),x]
```

```
[Out] (16*Sqrt[2]*(-2 + e*x)*(2 + e*x)^(1/4)*Hypergeometric2F1[-5/4, 3/4, 7/4, 1/
2 - (e*x)/4])/(3*e*(12 - 3*e^2*x^2)^(1/4))
```

Maple [F] time = 0.067, size = 0, normalized size = 0.

$$\int (ex+2)^{\frac{3}{2}} \frac{1}{\sqrt[4]{-3\,e^2x^2+12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/4),x)
```

```
[Out] int((e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+2)^{\frac{3}{2}}}{\left(-3\,e^2x^2+12\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((e*x + 2)^(3/2)/(-3*e^2*x^2 + 12)^(1/4), x)
```

Fricas [B] time = 2.22365, size = 1828, normalized size = 5.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/12*(60*sqrt(2)*(1/3)^(1/4)*(e^2*x + 2*e)*(e^(-4))^(1/4)*arctan(-(sqrt(2)*
(1/3)^(3/4)*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e*x + 2)*e^3*(e^(-4))^(3/4) + e^2*
x^2 - sqrt(3)*sqrt(2)*(1/3)^(3/4)*(e^5*x^2 - 4*e^3)*sqrt((sqrt(2)*(1/3)^(1/
4)*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e*x + 2)*e*(e^(-4))^(1/4) + 3*sqrt(1/3)*(e^
4*x^2 - 4*e^2)*sqrt(e^(-4)) - sqrt(-3*e^2*x^2 + 12)*(e*x + 2))/(e^2*x^2 - 4
))*(e^(-4))^(3/4) - 4)/(e^2*x^2 - 4)) + 60*sqrt(2)*(1/3)^(1/4)*(e^2*x + 2*e
)*(e^(-4))^(1/4)*arctan(-(sqrt(2)*(1/3)^(3/4)*(-3*e^2*x^2 + 12)^(3/4)*sqrt(
e*x + 2)*e^3*(e^(-4))^(3/4) - e^2*x^2 - sqrt(3)*sqrt(2)*(1/3)^(3/4)*(e^5*x^
2 - 4*e^3)*sqrt(-(sqrt(2)*(1/3)^(1/4)*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e*x + 2)
*e*(e^(-4))^(1/4) - 3*sqrt(1/3)*(e^4*x^2 - 4*e^2)*sqrt(e^(-4)) + sqrt(-3*e^
2*x^2 + 12)*(e*x + 2))/(e^2*x^2 - 4))*(e^(-4))^(3/4) + 4)/(e^2*x^2 - 4)) -
15*sqrt(2)*(1/3)^(1/4)*(e^2*x + 2*e)*(e^(-4))^(1/4)*log(3*(sqrt(2)*(1/3)^(1
/4)*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e*x + 2)*e*(e^(-4))^(1/4) + 3*sqrt(1/3)*(e
^4*x^2 - 4*e^2)*sqrt(e^(-4)) - sqrt(-3*e^2*x^2 + 12)*(e*x + 2))/(e^2*x^2 -
4)) + 15*sqrt(2)*(1/3)^(1/4)*(e^2*x + 2*e)*(e^(-4))^(1/4)*log(-3*(sqrt(2)*(
1/3)^(1/4)*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e*x + 2)*e*(e^(-4))^(1/4) - 3*sqrt(
1/3)*(e^4*x^2 - 4*e^2)*sqrt(e^(-4)) + sqrt(-3*e^2*x^2 + 12)*(e*x + 2))/(e^2
*x^2 - 4)) - 2*(-3*e^2*x^2 + 12)^(3/4)*(e*x + 7)*sqrt(e*x + 2))/(e^2*x + 2*
e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{3^{\frac{3}{4}}\left(\int \frac{2\sqrt{ex+2}}{\sqrt[4]{-e^2x^2+4}}\,dx + \int \frac{ex\sqrt{ex+2}}{\sqrt[4]{-e^2x^2+4}}\,dx\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)**(3/2)/(-3*e**2*x**2+12)**(1/4),x)
```

```
[Out] 3**(3/4)*(Integral(2*sqrt(e*x + 2)/(-e**2*x**2 + 4)**(1/4), x) + Integral(e
*x*sqrt(e*x + 2)/(-e**2*x**2 + 4)**(1/4), x))/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+2)^{\frac{3}{2}}}{\left(-3\,e^2x^2+12\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((e*x + 2)^(3/2)/(-3*e^2*x^2 + 12)^(1/4), x)
```

3.939 $\int \frac{\sqrt{2+ex}}{\sqrt[4]{12-3e^2x^2}}\,dx$

Optimal. Leaf size=270

$$-\frac{(2-ex)^{3/4}\sqrt[4]{ex+2}}{\sqrt[4]{3}e}-\frac{\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}-\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}\sqrt[4]{3}e}+\frac{\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}+\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}\sqrt[4]{3}e}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{2-\cdots}}{\sqrt[4]{ex+\cdots}}\right.}{\sqrt[4]{3}e}$$

```
[Out] -(((2 - e*x)^(3/4)*(2 + e*x)^(1/4))/(3^(1/4)*e)) + (Sqrt[2]*ArcTan[1 - (Sqr
t[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/(3^(1/4)*e) - (Sqrt[2]*ArcTan[1 + (
Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/(3^(1/4)*e) - Log[(Sqrt[6 - 3*e*
x] - Sqrt[6]*(2 - e*x)^(1/4)*(2 + e*x)^(1/4) + Sqrt[3]*Sqrt[2 + e*x])/Sqrt[
2 + e*x]]/(Sqrt[2]*3^(1/4)*e) + Log[(Sqrt[6 - 3*e*x] + Sqrt[6]*(2 - e*x)^(1
/4)*(2 + e*x)^(1/4) + Sqrt[3]*Sqrt[2 + e*x])/Sqrt[2 + e*x]]/(Sqrt[2]*3^(1/4
)*e)
```

Rubi [A] time = 0.23944, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {675, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{(2-ex)^{3/4}\sqrt[4]{ex+2}}{\sqrt[4]{3}e}-\frac{\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}-\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}\sqrt[4]{3}e}+\frac{\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}+\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}\sqrt[4]{3}e}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{2-\cdots}}{\sqrt[4]{ex+\cdots}}\right.}{\sqrt[4]{3}e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + e*x]/(12 - 3*e^2*x^2)^(1/4),x]
```

```
[Out] -(((2 - e*x)^(3/4)*(2 + e*x)^(1/4))/(3^(1/4)*e)) + (Sqrt[2]*ArcTan[1 - (Sqr
t[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/(3^(1/4)*e) - (Sqrt[2]*ArcTan[1 + (
Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/(3^(1/4)*e) - Log[(Sqrt[6 - 3*e*
x] - Sqrt[6]*(2 - e*x)^(1/4)*(2 + e*x)^(1/4) + Sqrt[3]*Sqrt[2 + e*x])/Sqrt[
2 + e*x]]/(Sqrt[2]*3^(1/4)*e) + Log[(Sqrt[6 - 3*e*x] + Sqrt[6]*(2 - e*x)^(1
/4)*(2 + e*x)^(1/4) + Sqrt[3]*Sqrt[2 + e*x])/Sqrt[2 + e*x]]/(Sqrt[2]*3^(1/4
)*e)
```

Rule 675

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[(
d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] && E
qQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && GtQ[a, 0] && GtQ[d, 0] &&  !IGtQ[m
, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{2+ex}}{\sqrt[4]{12-3e^2x^2}}\,dx = \int \frac{\sqrt[4]{2+ex}}{\sqrt[4]{6-3ex}}\,dx$$
$$= -\frac{(2-ex)^{3/4}\sqrt[4]{2+ex}}{\sqrt[4]{3}e} + \int \frac{1}{\sqrt[4]{6-3ex}(2+ex)^{3/4}}\,dx$$
$$= -\frac{(2-ex)^{3/4}\sqrt[4]{2+ex}}{\sqrt[4]{3}e} - \frac{4\,\mathrm{Subst}\left(\int \frac{x^2}{\left(4-\frac{x^4}{3}\right)^{3/4}}\,dx, x, \sqrt[4]{6-3ex}\right)}{3e}$$
$$= -\frac{(2-ex)^{3/4}\sqrt[4]{2+ex}}{\sqrt[4]{3}e} - \frac{4\,\mathrm{Subst}\left(\int \frac{x^2}{1+\frac{x^4}{3}}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{3e}$$
$$= -\frac{(2-ex)^{3/4}\sqrt[4]{2+ex}}{\sqrt[4]{3}e} + \frac{2\,\mathrm{Subst}\left(\int \frac{\sqrt{3}-x^2}{1+\frac{x^4}{3}}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{3e} - \frac{2\,\mathrm{Subst}\left(\int \frac{\sqrt{3}+x^2}{1+\frac{x^4}{3}}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{3e}$$
$$= -\frac{(2-ex)^{3/4}\sqrt[4]{2+ex}}{\sqrt[4]{3}e} - \frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{3}-\sqrt{2}\sqrt[4]{3}x+x^2}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{e} - \frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{3}+\sqrt{2}\sqrt[4]{3}x+x^2}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{e}$$
$$= -\frac{(2-ex)^{3/4}\sqrt[4]{2+ex}}{\sqrt[4]{3}e} - \frac{\log\left(\frac{\sqrt{2-ex}-\sqrt{2}\sqrt[4]{2-ex}\sqrt[4]{2+ex}+\sqrt{2+ex}}{\sqrt{2+ex}}\right)}{\sqrt{2}\sqrt[4]{3}e} + \frac{\log\left(\frac{\sqrt{2-ex}+\sqrt{2}\sqrt[4]{2-ex}\sqrt[4]{2+ex}+\sqrt{2+ex}}{\sqrt{2+ex}}\right)}{\sqrt{2}\sqrt[4]{3}e} - \sqrt{2}\,\mathrm{Su}$$
$$= -\frac{(2-ex)^{3/4}\sqrt[4]{2+ex}}{\sqrt[4]{3}e} + \frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{2+ex}}\right)}{\sqrt[4]{3}e} - \frac{\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{2+ex}}\right)}{\sqrt[4]{3}e} - \frac{\log\left(\frac{\sqrt{2-ex}-\sqrt{2}\sqrt[4]{2-ex}\sqrt[4]{2}}{\sqrt{2+ex}}\right.}{\sqrt{2}\sqrt[4]{3}e}$$

Mathematica [C] time = 0.0461471, size = 60, normalized size = 0.22

$$\frac{4\sqrt{2}(ex-2)\sqrt[4]{ex+2}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}-\frac{ex}{4}\right)}{3e\sqrt[4]{12-3e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + e*x]/(12 - 3*e^2*x^2)^(1/4),x]
```

```
[Out] (4*Sqrt[2]*(-2 + e*x)*(2 + e*x)^(1/4)*Hypergeometric2F1[-1/4, 3/4, 7/4, 1/2
 - (e*x)/4])/(3*e*(12 - 3*e^2*x^2)^(1/4))
```

Maple [F] time = 0.075, size = 0, normalized size = 0.

$$\int \sqrt{ex+2}\frac{1}{\sqrt[4]{-3\,e^2x^2+12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/4),x)
```

```
[Out] int((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+2}}{\left(-3\,e^2x^2+12\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + 2)/(-3*e^2*x^2 + 12)^(1/4), x)
```

Fricas [B] time = 2.07716, size = 1810, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/6*(12*sqrt(2)*(1/3)^(1/4)*(e^2*x + 2*e)*(e^(-4))^(1/4)*arctan(-(sqrt(2)*(
1/3)^(3/4)*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e*x + 2)*e^3*(e^(-4))^(3/4) + e^2*x
^2 - sqrt(3)*sqrt(2)*(1/3)^(3/4)*(e^5*x^2 - 4*e^3)*sqrt((sqrt(2)*(1/3)^(1/4
)*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e*x + 2)*e*(e^(-4))^(1/4) + 3*sqrt(1/3)*(e^4
*x^2 - 4*e^2)*sqrt(e^(-4)) - sqrt(-3*e^2*x^2 + 12)*(e*x + 2))/(e^2*x^2 - 4)
)*(e^(-4))^(3/4) - 4)/(e^2*x^2 - 4)) + 12*sqrt(2)*(1/3)^(1/4)*(e^2*x + 2*e)
*(e^(-4))^(1/4)*arctan(-(sqrt(2)*(1/3)^(3/4)*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e
*x + 2)*e^3*(e^(-4))^(3/4) - e^2*x^2 - sqrt(3)*sqrt(2)*(1/3)^(3/4)*(e^5*x^2
 - 4*e^3)*sqrt(-(sqrt(2)*(1/3)^(1/4)*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e*x + 2)*
e*(e^(-4))^(1/4) - 3*sqrt(1/3)*(e^4*x^2 - 4*e^2)*sqrt(e^(-4)) + sqrt(-3*e^2
*x^2 + 12)*(e*x + 2))/(e^2*x^2 - 4))*(e^(-4))^(3/4) + 4)/(e^2*x^2 - 4)) - 3
*sqrt(2)*(1/3)^(1/4)*(e^2*x + 2*e)*(e^(-4))^(1/4)*log(3*(sqrt(2)*(1/3)^(1/4
)*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e*x + 2)*e*(e^(-4))^(1/4) + 3*sqrt(1/3)*(e^4
*x^2 - 4*e^2)*sqrt(e^(-4)) - sqrt(-3*e^2*x^2 + 12)*(e*x + 2))/(e^2*x^2 - 4)
) + 3*sqrt(2)*(1/3)^(1/4)*(e^2*x + 2*e)*(e^(-4))^(1/4)*log(-3*(sqrt(2)*(1/3
)^(1/4)*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e*x + 2)*e*(e^(-4))^(1/4) - 3*sqrt(1/3
)*(e^4*x^2 - 4*e^2)*sqrt(e^(-4)) + sqrt(-3*e^2*x^2 + 12)*(e*x + 2))/(e^2*x^
2 - 4)) - 2*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e*x + 2))/(e^2*x + 2*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{3^{\frac{3}{4}}\int \frac{\sqrt{ex+2}}{\sqrt[4]{-e^2x^2+4}}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)**(1/2)/(-3*e**2*x**2+12)**(1/4),x)
```

```
[Out] 3**(3/4)*Integral(sqrt(e*x + 2)/(-e**2*x**2 + 4)**(1/4), x)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+2}}{\left(-3\,e^2x^2+12\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + 2)/(-3*e^2*x^2 + 12)^(1/4), x)
```

3.940 $\int \frac{1}{\sqrt{2+ex}\sqrt[4]{12-3e^2x^2}}\,dx$

Optimal. Leaf size=241

$$-\frac{\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}-\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}\sqrt[4]{3}e}+\frac{\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}+\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}\sqrt[4]{3}e}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt[4]{3}e}-\frac{\sqrt{2}\tan^{-1}\left(\right.}{\sqrt[4]{}}$$

```
[Out] (Sqrt[2]*ArcTan[1 - (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/(3^(1/4)*e)
 - (Sqrt[2]*ArcTan[1 + (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/(3^(1/4)
*e) - Log[(Sqrt[6 - 3*e*x] - Sqrt[6]*(2 - e*x)^(1/4)*(2 + e*x)^(1/4) + Sqrt
[3]*Sqrt[2 + e*x])/Sqrt[2 + e*x]]/(Sqrt[2]*3^(1/4)*e) + Log[(Sqrt[6 - 3*e*x
] + Sqrt[6]*(2 - e*x)^(1/4)*(2 + e*x)^(1/4) + Sqrt[3]*Sqrt[2 + e*x])/Sqrt[2
 + e*x]]/(Sqrt[2]*3^(1/4)*e)
```

Rubi [A] time = 0.244801, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {675, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}-\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}\sqrt[4]{3}e}+\frac{\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}+\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}\sqrt[4]{3}e}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt[4]{3}e}-\frac{\sqrt{2}\tan^{-1}\left(\right.}{\sqrt[4]{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 + e*x]*(12 - 3*e^2*x^2)^(1/4)),x]
```


```
[Out] (Sqrt[2]*ArcTan[1 - (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/(3^(1/4)*e)
 - (Sqrt[2]*ArcTan[1 + (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/(3^(1/4)
*e) - Log[(Sqrt[6 - 3*e*x] - Sqrt[6]*(2 - e*x)^(1/4)*(2 + e*x)^(1/4) + Sqrt
[3]*Sqrt[2 + e*x])/Sqrt[2 + e*x]]/(Sqrt[2]*3^(1/4)*e) + Log[(Sqrt[6 - 3*e*x
] + Sqrt[6]*(2 - e*x)^(1/4)*(2 + e*x)^(1/4) + Sqrt[3]*Sqrt[2 + e*x])/Sqrt[2
 + e*x]]/(Sqrt[2]*3^(1/4)*e)
```


Rule 675

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[(
d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] && E
qQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && GtQ[a, 0] && GtQ[d, 0] &&  !IGtQ[m
, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+ex}\sqrt[4]{12-3e^2x^2}}\,dx = \int \frac{1}{\sqrt[4]{6-3ex}(2+ex)^{3/4}}\,dx$$
$$= -\frac{4\,\text{Subst}\left(\int \frac{x^2}{\left(4-\frac{x^4}{3}\right)^{3/4}}\,dx, x, \sqrt[4]{6-3ex}\right)}{3e}$$
$$= -\frac{4\,\text{Subst}\left(\int \frac{x^2}{1+\frac{x^4}{3}}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{3e}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{\sqrt{3}-x^2}{1+\frac{x^4}{3}}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{3e} - \frac{2\,\text{Subst}\left(\int \frac{\sqrt{3}+x^2}{1+\frac{x^4}{3}}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{3e}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{3}-\sqrt{2}\sqrt[4]{3}x+x^2}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{e} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{3}+\sqrt{2}\sqrt[4]{3}x+x^2}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{e} - \text{Subst}$$
$$= -\frac{\log\left(\frac{\sqrt{2-ex}-\sqrt{2}\sqrt[4]{2-ex}\sqrt[4]{2+ex}+\sqrt{2+ex}}{\sqrt{2+ex}}\right)}{\sqrt{2}\sqrt[4]{3}e} + \frac{\log\left(\frac{\sqrt{2-ex}+\sqrt{2}\sqrt[4]{2-ex}\sqrt[4]{2+ex}+\sqrt{2+ex}}{\sqrt{2+ex}}\right)}{\sqrt{2}\sqrt[4]{3}e} - \sqrt{2}\,\text{Subst}\left(\int \frac{}{-1}\right.$$
$$= \frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{2+ex}}\right)}{\sqrt[4]{3}e} - \frac{\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{2+ex}}\right)}{\sqrt[4]{3}e} - \frac{\log\left(\frac{\sqrt{2-ex}-\sqrt{2}\sqrt[4]{2-ex}\sqrt[4]{2+ex}+\sqrt{2+ex}}{\sqrt{2+ex}}\right)}{\sqrt{2}\sqrt[4]{3}e} -$$

Mathematica [C] time = 0.050139, size = 60, normalized size = 0.25

$$\frac{\sqrt{2}(ex-2)\sqrt[4]{ex+2}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}-\frac{ex}{4}\right)}{3e\sqrt[4]{12-3e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 + e*x]*(12 - 3*e^2*x^2)^(1/4)),x]
```

```
[Out] (Sqrt[2]*(-2 + e*x)*(2 + e*x)^(1/4)*Hypergeometric2F1[3/4, 3/4, 7/4, 1/2 -
(e*x)/4])/(3*e*(12 - 3*e^2*x^2)^(1/4))
```

Maple [F] time = 0.465, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ex+2}}\frac{1}{\sqrt[4]{-3\,e^2x^2+12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/4),x)
```

```
[Out] int(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3\,e^2x^2+12\right)^{\frac{1}{4}}\sqrt{ex+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x + 2)), x)
```

Fricas [B] time = 2.15836, size = 1654, normalized size = 6.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*(1/3)^(1/4)*(e^(-4))^(1/4)*arctan(-(sqrt(2)*(1/3)^(3/4)*(-3*e^2*x
^2 + 12)^(3/4)*sqrt(e*x + 2)*e^3*(e^(-4))^(3/4) + e^2*x^2 - sqrt(3)*sqrt(2)
*(1/3)^(3/4)*(e^5*x^2 - 4*e^3)*sqrt((sqrt(2)*(1/3)^(1/4)*(-3*e^2*x^2 + 12)^
(3/4)*sqrt(e*x + 2)*e*(e^(-4))^(1/4) + 3*sqrt(1/3)*(e^4*x^2 - 4*e^2)*sqrt(e
^(-4)) - sqrt(-3*e^2*x^2 + 12)*(e*x + 2))/(e^2*x^2 - 4))*(e^(-4))^(3/4) - 4
)/(e^2*x^2 - 4)) + 2*sqrt(2)*(1/3)^(1/4)*(e^(-4))^(1/4)*arctan(-(sqrt(2)*(1
/3)^(3/4)*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e*x + 2)*e^3*(e^(-4))^(3/4) - e^2*x^
2 - sqrt(3)*sqrt(2)*(1/3)^(3/4)*(e^5*x^2 - 4*e^3)*sqrt(-(sqrt(2)*(1/3)^(1/4
)*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e*x + 2)*e*(e^(-4))^(1/4) - 3*sqrt(1/3)*(e^4
*x^2 - 4*e^2)*sqrt(e^(-4)) + sqrt(-3*e^2*x^2 + 12)*(e*x + 2))/(e^2*x^2 - 4)
)*(e^(-4))^(3/4) + 4)/(e^2*x^2 - 4)) - 1/2*sqrt(2)*(1/3)^(1/4)*(e^(-4))^(1/
4)*log(3*(sqrt(2)*(1/3)^(1/4)*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e*x + 2)*e*(e^(-
4))^(1/4) + 3*sqrt(1/3)*(e^4*x^2 - 4*e^2)*sqrt(e^(-4)) - sqrt(-3*e^2*x^2 +
12)*(e*x + 2))/(e^2*x^2 - 4)) + 1/2*sqrt(2)*(1/3)^(1/4)*(e^(-4))^(1/4)*log(
-3*(sqrt(2)*(1/3)^(1/4)*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e*x + 2)*e*(e^(-4))^(1
/4) - 3*sqrt(1/3)*(e^4*x^2 - 4*e^2)*sqrt(e^(-4)) + sqrt(-3*e^2*x^2 + 12)*(e
*x + 2))/(e^2*x^2 - 4))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{3^{\frac{3}{4}} \int \frac{1}{\sqrt{ex+2}\,\sqrt[4]{-e^2x^2+4}}\, dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)**(1/2)/(-3*e**2*x**2+12)**(1/4),x)
```

```
[Out] 3**(3/4)*Integral(1/(sqrt(e*x + 2)*(-e**2*x**2 + 4)**(1/4)), x)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3\,e^2x^2+12\right)^{\frac{1}{4}}\sqrt{ex+2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x + 2)), x)
```

3.941 $\int \frac{1}{(2+ex)^{3/2}\sqrt[4]{12-3e^2x^2}}\,dx$

Optimal. Leaf size=35

$$-\frac{\left(4-e^2x^2\right)^{3/4}}{3\sqrt[4]{3}e(ex+2)^{3/2}}$$

```
[Out] -(4 - e^2*x^2)^(3/4)/(3*3^(1/4)*e*(2 + e*x)^(3/2))
```

Rubi [A] time = 0.0141082, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {651}

$$-\frac{\left(4-e^2x^2\right)^{3/4}}{3\sqrt[4]{3}e(ex+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + e*x)^(3/2)*(12 - 3*e^2*x^2)^(1/4)),x]
```

```
[Out] -(4 - e^2*x^2)^(3/4)/(3*3^(1/4)*e*(2 + e*x)^(3/2))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{1}{(2+ex)^{3/2}\sqrt[4]{12-3e^2x^2}}\,dx=-\frac{\left(4-e^2x^2\right)^{3/4}}{3\sqrt[4]{3}e(2+ex)^{3/2}}$$

Mathematica [A] time = 0.0460585, size = 35, normalized size = 1.

$$\frac{ex-2}{3e\sqrt{ex+2}\sqrt[4]{12-3e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2 + e*x)^(3/2)*(12 - 3*e^2*x^2)^(1/4)),x]
```

```
[Out] (-2 + e*x)/(3*e*Sqrt[2 + e*x]*(12 - 3*e^2*x^2)^(1/4))
```

Maple [A] time = 0.043, size = 30, normalized size = 0.9

$$\frac{ex-2}{3e}\frac{1}{\sqrt{ex+2}}\frac{1}{\sqrt[4]{-3e^2x^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/4),x)
```

```
[Out] 1/3*(e*x-2)/(e*x+2)^(1/2)/e/(-3*e^2*x^2+12)^(1/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3\,e^2x^2+12\right)^{\frac{1}{4}}(ex+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*e^2*x^2 + 12)^(1/4)*(e*x + 2)^(3/2)), x)
```

Fricas [A] time = 1.85474, size = 95, normalized size = 2.71

$$-\frac{\left(-3\,e^2x^2+12\right)^{\frac{3}{4}}\sqrt{ex+2}}{9\left(e^3x^2+4\,e^2x+4\,e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/9*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e*x + 2)/(e^3*x^2 + 4*e^2*x + 4*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{3^{\frac{3}{4}}\int \frac{1}{ex\sqrt{ex+2}\,\sqrt[4]{-e^2x^2+4}+2\sqrt{ex+2}\,\sqrt[4]{-e^2x^2+4}}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)**(3/2)/(-3*e**2*x**2+12)**(1/4),x)
```

```
[Out] 3**(3/4)*Integral(1/(e*x*sqrt(e*x + 2)*(-e**2*x**2 + 4)**(1/4) + 2*sqrt(e*x
 + 2)*(-e**2*x**2 + 4)**(1/4)), x)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3\,e^2x^2+12\right)^{\frac{1}{4}}(ex+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-3*e^2*x^2 + 12)^(1/4)*(e*x + 2)^(3/2)), x)
```

3.942 $\int \frac{1}{(2+ex)^{5/2} \sqrt[4]{12-3e^2x^2}} \, dx$

Optimal. Leaf size=71

$$-\frac{(4-e^2x^2)^{3/4}}{21\sqrt[4]{3}e(ex+2)^{3/2}}-\frac{(4-e^2x^2)^{3/4}}{7\sqrt[4]{3}e(ex+2)^{5/2}}$$

```
[Out] -(4 - e^2*x^2)^(3/4)/(7*3^(1/4)*e*(2 + e*x)^(5/2)) - (4 - e^2*x^2)^(3/4)/(2
1*3^(1/4)*e*(2 + e*x)^(3/2))
```

Rubi [A] time = 0.0264299, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 651}

$$-\frac{(4-e^2x^2)^{3/4}}{21\sqrt[4]{3}e(ex+2)^{3/2}}-\frac{(4-e^2x^2)^{3/4}}{7\sqrt[4]{3}e(ex+2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + e*x)^(5/2)*(12 - 3*e^2*x^2)^(1/4)),x]
```

```
[Out] -(4 - e^2*x^2)^(3/4)/(7*3^(1/4)*e*(2 + e*x)^(5/2)) - (4 - e^2*x^2)^(3/4)/(2
1*3^(1/4)*e*(2 + e*x)^(3/2))
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(2+ex)^{5/2}\sqrt[4]{12-3e^2x^2}}\,dx &= -\frac{(4-e^2x^2)^{3/4}}{7\sqrt[4]{3}e(2+ex)^{5/2}}+\frac{1}{7}\int \frac{1}{(2+ex)^{3/2}\sqrt[4]{12-3e^2x^2}}\,dx\\ &= -\frac{(4-e^2x^2)^{3/4}}{7\sqrt[4]{3}e(2+ex)^{5/2}}-\frac{(4-e^2x^2)^{3/4}}{21\sqrt[4]{3}e(2+ex)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0572088, size = 40, normalized size = 0.56

$$\frac{(ex-2)(ex+5)}{21e(ex+2)^{3/2}\sqrt[4]{12-3e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2 + e*x)^(5/2)*(12 - 3*e^2*x^2)^(1/4)),x]
```

```
[Out] ((-2 + e*x)*(5 + e*x))/(21*e*(2 + e*x)^(3/2)*(12 - 3*e^2*x^2)^(1/4))
```

Maple [A] time = 0.043, size = 35, normalized size = 0.5

$$\frac{(ex-2)(ex+5)}{21e}(ex+2)^{-\frac{3}{2}}\frac{1}{\sqrt[4]{-3e^2x^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/4),x)
```

```
[Out] 1/21*(e*x-2)*(e*x+5)/(e*x+2)^(3/2)/e/(-3*e^2*x^2+12)^(1/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3e^2x^2+12\right)^{\frac{1}{4}}(ex+2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*e^2*x^2 + 12)^(1/4)*(e*x + 2)^(5/2)), x)
```

Fricas [A] time = 1.81542, size = 127, normalized size = 1.79

$$-\frac{\left(-3e^2x^2+12\right)^{\frac{3}{4}}(ex+5)\sqrt{ex+2}}{63\left(e^4x^3+6e^3x^2+12e^2x+8e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/63*(-3*e^2*x^2 + 12)^(3/4)*(e*x + 5)*sqrt(e*x + 2)/(e^4*x^3 + 6*e^3*x^2
+ 12*e^2*x + 8*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)**(5/2)/(-3*e**2*x**2+12)**(1/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3\,e^2x^2+12\right)^{\frac{1}{4}}(ex+2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-3*e^2*x^2 + 12)^(1/4)*(e*x + 2)^(5/2)), x)
```

3.943 $\int \frac{1}{(2+ex)^{7/2}\sqrt[4]{12-3e^2x^2}}\,dx$

Optimal. Leaf size=106

$$-\frac{2\left(4-e^2x^2\right)^{3/4}}{231\sqrt[4]{3}e(ex+2)^{3/2}}-\frac{2\left(4-e^2x^2\right)^{3/4}}{77\sqrt[4]{3}e(ex+2)^{5/2}}-\frac{\left(4-e^2x^2\right)^{3/4}}{11\sqrt[4]{3}e(ex+2)^{7/2}}$$

```
[Out] -(4 - e^2*x^2)^(3/4)/(11*3^(1/4)*e*(2 + e*x)^(7/2)) - (2*(4 - e^2*x^2)^(3/4
))/(77*3^(1/4)*e*(2 + e*x)^(5/2)) - (2*(4 - e^2*x^2)^(3/4))/(231*3^(1/4)*e*
(2 + e*x)^(3/2))
```

Rubi [A] time = 0.0510352, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 651}

$$-\frac{2\left(4-e^2x^2\right)^{3/4}}{231\sqrt[4]{3}e(ex+2)^{3/2}}-\frac{2\left(4-e^2x^2\right)^{3/4}}{77\sqrt[4]{3}e(ex+2)^{5/2}}-\frac{\left(4-e^2x^2\right)^{3/4}}{11\sqrt[4]{3}e(ex+2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + e*x)^(7/2)*(12 - 3*e^2*x^2)^(1/4)),x]
```

```
[Out] -(4 - e^2*x^2)^(3/4)/(11*3^(1/4)*e*(2 + e*x)^(7/2)) - (2*(4 - e^2*x^2)^(3/4
))/(77*3^(1/4)*e*(2 + e*x)^(5/2)) - (2*(4 - e^2*x^2)^(3/4))/(231*3^(1/4)*e*
(2 + e*x)^(3/2))
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(2+ex)^{7/2}\sqrt[4]{12-3e^2x^2}}\,dx &= -\frac{\left(4-e^2x^2\right)^{3/4}}{11\sqrt[4]{3}e(2+ex)^{7/2}}+\frac{2}{11}\int \frac{1}{(2+ex)^{5/2}\sqrt[4]{12-3e^2x^2}}\,dx\\
&= -\frac{\left(4-e^2x^2\right)^{3/4}}{11\sqrt[4]{3}e(2+ex)^{7/2}}-\frac{2\left(4-e^2x^2\right)^{3/4}}{77\sqrt[4]{3}e(2+ex)^{5/2}}+\frac{2}{77}\int \frac{1}{(2+ex)^{3/2}\sqrt[4]{12-3e^2x^2}}\,dx\\
&= -\frac{\left(4-e^2x^2\right)^{3/4}}{11\sqrt[4]{3}e(2+ex)^{7/2}}-\frac{2\left(4-e^2x^2\right)^{3/4}}{77\sqrt[4]{3}e(2+ex)^{5/2}}-\frac{2\left(4-e^2x^2\right)^{3/4}}{231\sqrt[4]{3}e(2+ex)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0692178, size = 49, normalized size = 0.46

$$\frac{(ex-2)\left(2e^2x^2+14ex+41\right)}{231e(ex+2)^{5/2}\sqrt[4]{12-3e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2 + e*x)^(7/2)*(12 - 3*e^2*x^2)^(1/4)),x]
```

```
[Out] ((-2 + e*x)*(41 + 14*e*x + 2*e^2*x^2))/(231*e*(2 + e*x)^(5/2)*(12 - 3*e^2*x
^2)^(1/4))
```

Maple [A] time = 0.042, size = 44, normalized size = 0.4

$$\frac{(ex-2)\left(2e^2x^2+14ex+41\right)}{231e}(ex+2)^{-\frac{5}{2}}\frac{1}{\sqrt[4]{-3e^2x^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+2)^(7/2)/(-3*e^2*x^2+12)^(1/4),x)
```

```
[Out] 1/231*(e*x-2)*(2*e^2*x^2+14*e*x+41)/(e*x+2)^(5/2)/e/(-3*e^2*x^2+12)^(1/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(-3e^2x^2+12\right)^{\frac{1}{4}}(ex+2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)^(7/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*e^2*x^2 + 12)^(1/4)*(e*x + 2)^(7/2)), x)
```

Fricas [A] time = 1.8617, size = 169, normalized size = 1.59

$$-\frac{\left(2e^2x^2+14ex+41\right)\left(-3e^2x^2+12\right)^{\frac{3}{4}}\sqrt{ex+2}}{693\left(e^5x^4+8e^4x^3+24e^3x^2+32e^2x+16e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)^(7/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/693*(2*e^2*x^2 + 14*e*x + 41)*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e*x + 2)/(e^5
*x^4 + 8*e^4*x^3 + 24*e^3*x^2 + 32*e^2*x + 16*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)**(7/2)/(-3*e**2*x**2+12)**(1/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3\,e^2x^2+12\right)^{\frac{1}{4}}(ex+2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)^(7/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-3*e^2*x^2 + 12)^(1/4)*(e*x + 2)^(7/2)), x)
```

3.944 $\int \frac{1}{(2+ex)^{9/2} \sqrt[4]{12-3e^2x^2}} \, dx$

Optimal. Leaf size=141

$$-\frac{2\left(4-e^2x^2\right)^{3/4}}{1155\sqrt[4]{3}e(ex+2)^{3/2}}-\frac{2\left(4-e^2x^2\right)^{3/4}}{385\sqrt[4]{3}e(ex+2)^{5/2}}-\frac{\left(4-e^2x^2\right)^{3/4}}{55\sqrt[4]{3}e(ex+2)^{7/2}}-\frac{\left(4-e^2x^2\right)^{3/4}}{15\sqrt[4]{3}e(ex+2)^{9/2}}$$

```
[Out] -(4 - e^2*x^2)^(3/4)/(15*3^(1/4)*e*(2 + e*x)^(9/2)) - (4 - e^2*x^2)^(3/4)/(
55*3^(1/4)*e*(2 + e*x)^(7/2)) - (2*(4 - e^2*x^2)^(3/4))/(385*3^(1/4)*e*(2 +
 e*x)^(5/2)) - (2*(4 - e^2*x^2)^(3/4))/(1155*3^(1/4)*e*(2 + e*x)^(3/2))
```

Rubi [A] time = 0.0729962, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 651}

$$-\frac{2\left(4-e^2x^2\right)^{3/4}}{1155\sqrt[4]{3}e(ex+2)^{3/2}}-\frac{2\left(4-e^2x^2\right)^{3/4}}{385\sqrt[4]{3}e(ex+2)^{5/2}}-\frac{\left(4-e^2x^2\right)^{3/4}}{55\sqrt[4]{3}e(ex+2)^{7/2}}-\frac{\left(4-e^2x^2\right)^{3/4}}{15\sqrt[4]{3}e(ex+2)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + e*x)^(9/2)*(12 - 3*e^2*x^2)^(1/4)),x]
```

```
[Out] -(4 - e^2*x^2)^(3/4)/(15*3^(1/4)*e*(2 + e*x)^(9/2)) - (4 - e^2*x^2)^(3/4)/(
55*3^(1/4)*e*(2 + e*x)^(7/2)) - (2*(4 - e^2*x^2)^(3/4))/(385*3^(1/4)*e*(2 +
 e*x)^(5/2)) - (2*(4 - e^2*x^2)^(3/4))/(1155*3^(1/4)*e*(2 + e*x)^(3/2))
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(2+ex)^{9/2}\sqrt[4]{12-3e^2x^2}}\,dx &= -\frac{\left(4-e^2x^2\right)^{3/4}}{15\sqrt[4]{3}e(2+ex)^{9/2}}+\frac{1}{5}\int \frac{1}{(2+ex)^{7/2}\sqrt[4]{12-3e^2x^2}}\,dx\\
&= -\frac{\left(4-e^2x^2\right)^{3/4}}{15\sqrt[4]{3}e(2+ex)^{9/2}}-\frac{\left(4-e^2x^2\right)^{3/4}}{55\sqrt[4]{3}e(2+ex)^{7/2}}+\frac{2}{55}\int \frac{1}{(2+ex)^{5/2}\sqrt[4]{12-3e^2x^2}}\,dx\\
&= -\frac{\left(4-e^2x^2\right)^{3/4}}{15\sqrt[4]{3}e(2+ex)^{9/2}}-\frac{\left(4-e^2x^2\right)^{3/4}}{55\sqrt[4]{3}e(2+ex)^{7/2}}-\frac{2\left(4-e^2x^2\right)^{3/4}}{385\sqrt[4]{3}e(2+ex)^{5/2}}+\frac{2}{385}\int \frac{1}{(2+ex)^{3/2}\sqrt[4]{12}}\\
&= -\frac{\left(4-e^2x^2\right)^{3/4}}{15\sqrt[4]{3}e(2+ex)^{9/2}}-\frac{\left(4-e^2x^2\right)^{3/4}}{55\sqrt[4]{3}e(2+ex)^{7/2}}-\frac{2\left(4-e^2x^2\right)^{3/4}}{385\sqrt[4]{3}e(2+ex)^{5/2}}-\frac{2\left(4-e^2x^2\right)^{3/4}}{1155\sqrt[4]{3}e(2+ex)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0704015, size = 57, normalized size = 0.4

$$\frac{(ex-2)\left(2e^3x^3+18e^2x^2+69ex+159\right)}{1155e(ex+2)^{7/2}\sqrt[4]{12-3e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2 + e*x)^(9/2)*(12 - 3*e^2*x^2)^(1/4)),x]
```

```
[Out] ((-2 + e*x)*(159 + 69*e*x + 18*e^2*x^2 + 2*e^3*x^3))/(1155*e*(2 + e*x)^(7/2
)*(12 - 3*e^2*x^2)^(1/4))
```

Maple [A] time = 0.04, size = 52, normalized size = 0.4

$$\frac{(ex-2)\left(2\,e^3x^3+18\,e^2x^2+69\,ex+159\right)}{1155\,e}(ex+2)^{-\frac{7}{2}}\frac{1}{\sqrt[4]{-3\,e^2x^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+2)^(9/2)/(-3*e^2*x^2+12)^(1/4),x)
```

```
[Out] 1/1155*(e*x-2)*(2*e^3*x^3+18*e^2*x^2+69*e*x+159)/(e*x+2)^(7/2)/e/(-3*e^2*x^
2+12)^(1/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3\,e^2x^2+12\right)^{\frac{1}{4}}(ex+2)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)^(9/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*e^2*x^2 + 12)^(1/4)*(e*x + 2)^(9/2)), x)
```

Fricas [A] time = 1.93047, size = 208, normalized size = 1.48

$$-\frac{\left(2\,e^3x^3+18\,e^2x^2+69\,ex+159\right)\left(-3\,e^2x^2+12\right)^{\frac{3}{4}}\sqrt{ex+2}}{3465\left(e^6x^5+10\,e^5x^4+40\,e^4x^3+80\,e^3x^2+80\,e^2x+32\,e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)^(9/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/3465*(2*e^3*x^3 + 18*e^2*x^2 + 69*e*x + 159)*(-3*e^2*x^2 + 12)^(3/4)*sqr
t(e*x + 2)/(e^6*x^5 + 10*e^5*x^4 + 40*e^4*x^3 + 80*e^3*x^2 + 80*e^2*x + 32*
e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)**(9/2)/(-3*e**2*x**2+12)**(1/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-3\,e^2x^2+12\right)^{\frac{1}{4}}(ex+2)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+2)^(9/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-3*e^2*x^2 + 12)^(1/4)*(e*x + 2)^(9/2)), x)
```

3.945 $\int (a+bx)^m \left(a^2-b^2x^2\right)^3 \, dx$

Optimal. Leaf size=84

$$\frac{8a^3(a+bx)^{m+4}}{b(m+4)}-\frac{12a^2(a+bx)^{m+5}}{b(m+5)}+\frac{6a(a+bx)^{m+6}}{b(m+6)}-\frac{(a+bx)^{m+7}}{b(m+7)}$$

```
[Out] (8*a^3*(a + b*x)^(4 + m))/(b*(4 + m)) - (12*a^2*(a + b*x)^(5 + m))/(b*(5 +
m)) + (6*a*(a + b*x)^(6 + m))/(b*(6 + m)) - (a + b*x)^(7 + m)/(b*(7 + m))
```

Rubi [A] time = 0.0418968, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {627, 43}

$$\frac{8a^3(a+bx)^{m+4}}{b(m+4)}-\frac{12a^2(a+bx)^{m+5}}{b(m+5)}+\frac{6a(a+bx)^{m+6}}{b(m+6)}-\frac{(a+bx)^{m+7}}{b(m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(a^2 - b^2*x^2)^3,x]
```

```
[Out] (8*a^3*(a + b*x)^(4 + m))/(b*(4 + m)) - (12*a^2*(a + b*x)^(5 + m))/(b*(5 +
m)) + (6*a*(a + b*x)^(6 + m))/(b*(6 + m)) - (a + b*x)^(7 + m)/(b*(7 + m))
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (a+bx)^m \left(a^2-b^2x^2\right)^3 \, dx &= \int (a-bx)^3(a+bx)^{3+m} \, dx\\
&= \int \left(8a^3(a+bx)^{3+m}-12a^2(a+bx)^{4+m}+6a(a+bx)^{5+m}-(a+bx)^{6+m}\right) dx\\
&= \frac{8a^3(a+bx)^{4+m}}{b(4+m)}-\frac{12a^2(a+bx)^{5+m}}{b(5+m)}+\frac{6a(a+bx)^{6+m}}{b(6+m)}-\frac{(a+bx)^{7+m}}{b(7+m)}
\end{aligned}$$

Mathematica [A] time = 0.0645084, size = 68, normalized size = 0.81

$$\frac{(a+bx)^{m+4}\left(-\frac{12a^2(a+bx)}{m+5}+\frac{8a^3}{m+4}+\frac{6a(a+bx)^2}{m+6}-\frac{(a+bx)^3}{m+7}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(a^2 - b^2*x^2)^3,x]
```

```
[Out] ((a + b*x)^(4 + m)*((8*a^3)/(4 + m) - (12*a^2*(a + b*x))/(5 + m) + (6*a*(a
+ b*x)^2)/(6 + m) - (a + b*x)^3/(7 + m)))/b
```

Maple [B] time = 0.046, size = 178, normalized size = 2.1

$$\frac{(bx+a)^{4+m}\left(-b^3m^3x^3+3\,ab^2m^3x^2-15\,b^3m^2x^3-3\,a^2bm^3x+51\,ab^2m^2x^2-74\,b^3mx^3+a^3m^3-57\,a^2bm^2x+276\,ab^2\cdots\right.}{b\left(m^4+22\,m^3+179\,m^2+638\,m+8\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(-b^2*x^2+a^2)^3,x)
```

```
[Out] (b*x+a)^(4+m)*(-b^3*m^3*x^3+3*a*b^2*m^3*x^2-15*b^3*m^2*x^3-3*a^2*b*m^3*x+51
*a*b^2*m^2*x^2-74*b^3*m*x^3+a^3*m^3-57*a^2*b*m^2*x+276*a*b^2*m*x^2-120*b^3*
x^3+21*a^3*m^2-354*a^2*b*m*x+480*a*b^2*x^2+152*a^3*m-696*a^2*b*x+384*a^3)/b
/(m^4+22*m^3+179*m^2+638*m+840)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.99885, size = 682, normalized size = 8.12

$$\frac{\left(a^7m^3+21\,a^7m^2+152\,a^7m-\left(b^7m^3+15\,b^7m^2+74\,b^7m+120\,b^7\right)x^7+384\,a^7-\left(ab^6m^3+9\,ab^6m^2+20\,ab^6m\right)x^6+3\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] (a^7*m^3 + 21*a^7*m^2 + 152*a^7*m - (b^7*m^3 + 15*b^7*m^2 + 74*b^7*m + 120*
b^7)*x^7 + 384*a^7 - (a*b^6*m^3 + 9*a*b^6*m^2 + 20*a*b^6*m)*x^6 + 3*(a^2*b^
5*m^3 + 19*a^2*b^5*m^2 + 102*a^2*b^5*m + 168*a^2*b^5)*x^5 + 3*(a^3*b^4*m^3
+ 13*a^3*b^4*m^2 + 32*a^3*b^4*m)*x^4 - 3*(a^4*b^3*m^3 + 23*a^4*b^3*m^2 + 16
2*a^4*b^3*m + 280*a^4*b^3)*x^3 - 3*(a^5*b^2*m^3 + 17*a^5*b^2*m^2 + 76*a^5*b
^2*m)*x^2 + (a^6*b*m^3 + 27*a^6*b*m^2 + 254*a^6*b*m + 840*a^6*b)*x)*(b*x +
a)^m/(b*m^4 + 22*b*m^3 + 179*b*m^2 + 638*b*m + 840*b)
```

Sympy [A] time = 5.32617, size = 2096, normalized size = 24.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(-b**2*x**2+a**2)**3,x)
```

```
[Out] Piecewise((a**6*a**m*x, Eq(b, 0)), (-15*a**3*log(a/b + x)/(15*a**3*b + 45*a
**2*b**2*x + 45*a*b**3*x**2 + 15*b**4*x**3) - 8*a**3/(15*a**3*b + 45*a**2*b
**2*x + 45*a*b**3*x**2 + 15*b**4*x**3) - 45*a**2*b*x*log(a/b + x)/(15*a**3*
b + 45*a**2*b**2*x + 45*a*b**3*x**2 + 15*b**4*x**3) + 6*a**2*b*x/(15*a**3*b
 + 45*a**2*b**2*x + 45*a*b**3*x**2 + 15*b**4*x**3) - 45*a*b**2*x**2*log(a/b
 + x)/(15*a**3*b + 45*a**2*b**2*x + 45*a*b**3*x**2 + 15*b**4*x**3) + 6*a*b*
*2*x**2/(15*a**3*b + 45*a**2*b**2*x + 45*a*b**3*x**2 + 15*b**4*x**3) - 15*b
**3*x**3*log(a/b + x)/(15*a**3*b + 45*a**2*b**2*x + 45*a*b**3*x**2 + 15*b**
4*x**3) + 32*b**3*x**3/(15*a**3*b + 45*a**2*b**2*x + 45*a*b**3*x**2 + 15*b*
*4*x**3), Eq(m, -7)), (12*a**3*log(a/b + x)/(2*a**2*b + 4*a*b**2*x + 2*b**3
*x**2) + 5*a**3/(2*a**2*b + 4*a*b**2*x + 2*b**3*x**2) + 24*a**2*b*x*log(a/b
 + x)/(2*a**2*b + 4*a*b**2*x + 2*b**3*x**2) + 12*a*b**2*x**2*log(a/b + x)/(
2*a**2*b + 4*a*b**2*x + 2*b**3*x**2) - 15*a*b**2*x**2/(2*a**2*b + 4*a*b**2*
x + 2*b**3*x**2) - 2*b**3*x**3/(2*a**2*b + 4*a*b**2*x + 2*b**3*x**2), Eq(m,
 -6)), (-48*a**3*log(a/b + x)/(4*a*b + 4*b**2*x) - 29*a**3/(4*a*b + 4*b**2*
x) - 48*a**2*b*x*log(a/b + x)/(4*a*b + 4*b**2*x) + 23*a**2*b*x/(4*a*b + 4*b
**2*x) + 18*a*b**2*x**2/(4*a*b + 4*b**2*x) - 2*b**3*x**3/(4*a*b + 4*b**2*x)
, Eq(m, -5)), (8*a**3*log(a/b + x)/b - 7*a**2*x + 2*a*b*x**2 - b**2*x**3/3,
 Eq(m, -4)), (a**7*m**3*(a + b*x)**m/(b*m**4 + 22*b*m**3 + 179*b*m**2 + 638
*b*m + 840*b) + 21*a**7*m**2*(a + b*x)**m/(b*m**4 + 22*b*m**3 + 179*b*m**2
+ 638*b*m + 840*b) + 152*a**7*m*(a + b*x)**m/(b*m**4 + 22*b*m**3 + 179*b*m*
*2 + 638*b*m + 840*b) + 384*a**7*(a + b*x)**m/(b*m**4 + 22*b*m**3 + 179*b*m
**2 + 638*b*m + 840*b) + a**6*b*m**3*x*(a + b*x)**m/(b*m**4 + 22*b*m**3 + 1
79*b*m**2 + 638*b*m + 840*b) + 27*a**6*b*m**2*x*(a + b*x)**m/(b*m**4 + 22*b
*m**3 + 179*b*m**2 + 638*b*m + 840*b) + 254*a**6*b*m*x*(a + b*x)**m/(b*m**4
 + 22*b*m**3 + 179*b*m**2 + 638*b*m + 840*b) + 840*a**6*b*x*(a + b*x)**m/(b
*m**4 + 22*b*m**3 + 179*b*m**2 + 638*b*m + 840*b) - 3*a**5*b**2*m**3*x**2*(
a + b*x)**m/(b*m**4 + 22*b*m**3 + 179*b*m**2 + 638*b*m + 840*b) - 51*a**5*b
**2*m**2*x**2*(a + b*x)**m/(b*m**4 + 22*b*m**3 + 179*b*m**2 + 638*b*m + 840
*b) - 228*a**5*b**2*m*x**2*(a + b*x)**m/(b*m**4 + 22*b*m**3 + 179*b*m**2 +
638*b*m + 840*b) - 3*a**4*b**3*m**3*x**3*(a + b*x)**m/(b*m**4 + 22*b*m**3 +
 179*b*m**2 + 638*b*m + 840*b) - 69*a**4*b**3*m**2*x**3*(a + b*x)**m/(b*m**
4 + 22*b*m**3 + 179*b*m**2 + 638*b*m + 840*b) - 486*a**4*b**3*m*x**3*(a + b
*x)**m/(b*m**4 + 22*b*m**3 + 179*b*m**2 + 638*b*m + 840*b) - 840*a**4*b**3*
x**3*(a + b*x)**m/(b*m**4 + 22*b*m**3 + 179*b*m**2 + 638*b*m + 840*b) + 3*a
**3*b**4*m**3*x**4*(a + b*x)**m/(b*m**4 + 22*b*m**3 + 179*b*m**2 + 638*b*m
+ 840*b) + 39*a**3*b**4*m**2*x**4*(a + b*x)**m/(b*m**4 + 22*b*m**3 + 179*b*
m**2 + 638*b*m + 840*b) + 96*a**3*b**4*m*x**4*(a + b*x)**m/(b*m**4 + 22*b*m
**3 + 179*b*m**2 + 638*b*m + 840*b) + 3*a**2*b**5*m**3*x**5*(a + b*x)**m/(b
*m**4 + 22*b*m**3 + 179*b*m**2 + 638*b*m + 840*b) + 57*a**2*b**5*m**2*x**5*
(a + b*x)**m/(b*m**4 + 22*b*m**3 + 179*b*m**2 + 638*b*m + 840*b) + 306*a**2
*b**5*m*x**5*(a + b*x)**m/(b*m**4 + 22*b*m**3 + 179*b*m**2 + 638*b*m + 840*
b) + 504*a**2*b**5*x**5*(a + b*x)**m/(b*m**4 + 22*b*m**3 + 179*b*m**2 + 638
*b*m + 840*b) - a*b**6*m**3*x**6*(a + b*x)**m/(b*m**4 + 22*b*m**3 + 179*b*m
**2 + 638*b*m + 840*b) - 9*a*b**6*m**2*x**6*(a + b*x)**m/(b*m**4 + 22*b*m**
3 + 179*b*m**2 + 638*b*m + 840*b) - 20*a*b**6*m*x**6*(a + b*x)**m/(b*m**4 +
 22*b*m**3 + 179*b*m**2 + 638*b*m + 840*b) - b**7*m**3*x**7*(a + b*x)**m/(b
*m**4 + 22*b*m**3 + 179*b*m**2 + 638*b*m + 840*b) - 15*b**7*m**2*x**7*(a +
b*x)**m/(b*m**4 + 22*b*m**3 + 179*b*m**2 + 638*b*m + 840*b) - 74*b**7*m*x**
7*(a + b*x)**m/(b*m**4 + 22*b*m**3 + 179*b*m**2 + 638*b*m + 840*b) - 120*b*
*7*x**7*(a + b*x)**m/(b*m**4 + 22*b*m**3 + 179*b*m**2 + 638*b*m + 840*b), T
rue))
```

Giac [B] time = 1.15659, size = 740, normalized size = 8.81

$$-\frac{(bx+a)^m b^7 m^3 x^7 + (bx+a)^m a b^6 m^3 x^6 + 15\,(bx+a)^m b^7 m^2 x^7 - 3\,(bx+a)^m a^2 b^5 m^3 x^5 + 9\,(bx+a)^m a b^6 m^2 x^6 + 74\,(bx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -((b*x + a)^m*b^7*m^3*x^7 + (b*x + a)^m*a*b^6*m^3*x^6 + 15*(b*x + a)^m*b^7*
m^2*x^7 - 3*(b*x + a)^m*a^2*b^5*m^3*x^5 + 9*(b*x + a)^m*a*b^6*m^2*x^6 + 74*
(b*x + a)^m*b^7*m*x^7 - 3*(b*x + a)^m*a^3*b^4*m^3*x^4 - 57*(b*x + a)^m*a^2*
b^5*m^2*x^5 + 20*(b*x + a)^m*a*b^6*m*x^6 + 120*(b*x + a)^m*b^7*x^7 + 3*(b*x
 + a)^m*a^4*b^3*m^3*x^3 - 39*(b*x + a)^m*a^3*b^4*m^2*x^4 - 306*(b*x + a)^m*
a^2*b^5*m*x^5 + 3*(b*x + a)^m*a^5*b^2*m^3*x^2 + 69*(b*x + a)^m*a^4*b^3*m^2*
x^3 - 96*(b*x + a)^m*a^3*b^4*m*x^4 - 504*(b*x + a)^m*a^2*b^5*x^5 - (b*x + a
)^m*a^6*b*m^3*x + 51*(b*x + a)^m*a^5*b^2*m^2*x^2 + 486*(b*x + a)^m*a^4*b^3*
m*x^3 - (b*x + a)^m*a^7*m^3 - 27*(b*x + a)^m*a^6*b*m^2*x + 228*(b*x + a)^m*
a^5*b^2*m*x^2 + 840*(b*x + a)^m*a^4*b^3*x^3 - 21*(b*x + a)^m*a^7*m^2 - 254*
(b*x + a)^m*a^6*b*m*x - 152*(b*x + a)^m*a^7*m - 840*(b*x + a)^m*a^6*b*x - 3
84*(b*x + a)^m*a^7)/(b*m^4 + 22*b*m^3 + 179*b*m^2 + 638*b*m + 840*b)
```

3.946 $\int (a+bx)^m \left(a^2-b^2x^2\right)^2 \, dx$

Optimal. Leaf size=61

$$\frac{4a^2(a+bx)^{m+3}}{b(m+3)}-\frac{4a(a+bx)^{m+4}}{b(m+4)}+\frac{(a+bx)^{m+5}}{b(m+5)}$$

```
[Out] (4*a^2*(a + b*x)^(3 + m))/(b*(3 + m)) - (4*a*(a + b*x)^(4 + m))/(b*(4 + m))
 + (a + b*x)^(5 + m)/(b*(5 + m))
```

Rubi [A] time = 0.0279287, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {627, 43}

$$\frac{4a^2(a+bx)^{m+3}}{b(m+3)}-\frac{4a(a+bx)^{m+4}}{b(m+4)}+\frac{(a+bx)^{m+5}}{b(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(a^2 - b^2*x^2)^2,x]
```

```
[Out] (4*a^2*(a + b*x)^(3 + m))/(b*(3 + m)) - (4*a*(a + b*x)^(4 + m))/(b*(4 + m))
 + (a + b*x)^(5 + m)/(b*(5 + m))
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^m \left(a^2-b^2x^2\right)^2 \, dx &= \int (a-bx)^2(a+bx)^{2+m} \, dx\\ &= \int \left(4a^2(a+bx)^{2+m}-4a(a+bx)^{3+m}+(a+bx)^{4+m}\right) dx\\ &= \frac{4a^2(a+bx)^{3+m}}{b(3+m)}-\frac{4a(a+bx)^{4+m}}{b(4+m)}+\frac{(a+bx)^{5+m}}{b(5+m)}\end{aligned}$$

Mathematica [A] time = 0.0530235, size = 50, normalized size = 0.82

$$\frac{(a+bx)^{m+3}\left(\frac{4a^2}{m+3}-\frac{4a(a+bx)}{m+4}+\frac{(a+bx)^2}{m+5}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(a^2 - b^2*x^2)^2,x]
```

```
[Out] ((a + b*x)^(3 + m)*((4*a^2)/(3 + m) - (4*a*(a + b*x))/(4 + m) + (a + b*x)^2
/(5 + m)))/b
```

Maple [A] time = 0.046, size = 94, normalized size = 1.5

$$\frac{(bx+a)^{3+m}\left(b^2m^2x^2-2\,abm^2x+7\,b^2mx^2+a^2m^2-18\,abmx+12\,b^2x^2+11\,ma^2-36\,abx+32\,a^2\right)}{b\left(m^3+12\,m^2+47\,m+60\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(-b^2*x^2+a^2)^2,x)
```

```
[Out] (b*x+a)^(3+m)*(b^2*m^2*x^2-2*a*b*m^2*x+7*b^2*m*x^2+a^2*m^2-18*a*b*m*x+12*b^
2*x^2+11*a^2*m-36*a*b*x+32*a^2)/b/(m^3+12*m^2+47*m+60)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.96081, size = 363, normalized size = 5.95

$$\frac{\left(a^5m^2+11\,a^5m+\left(b^5m^2+7\,b^5m+12\,b^5\right)x^5+32\,a^5+\left(ab^4m^2+3\,ab^4m\right)x^4-2\left(a^2b^3m^2+11\,a^2b^3m+20\,a^2b^3\right)x^3-2\,(\cdots}{bm^3+12\,bm^2+47\,bm+60\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] (a^5*m^2 + 11*a^5*m + (b^5*m^2 + 7*b^5*m + 12*b^5)*x^5 + 32*a^5 + (a*b^4*m^
2 + 3*a*b^4*m)*x^4 - 2*(a^2*b^3*m^2 + 11*a^2*b^3*m + 20*a^2*b^3)*x^3 - 2*(a
^3*b^2*m^2 + 7*a^3*b^2*m)*x^2 + (a^4*b*m^2 + 15*a^4*b*m + 60*a^4*b)*x)*(b*x
 + a)^m/(b*m^3 + 12*b*m^2 + 47*b*m + 60*b)
```

Sympy [A] time = 2.48281, size = 945, normalized size = 15.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(-b**2*x**2+a**2)**2,x)
```

```
[Out] Piecewise((a**4*a**m*x, Eq(b, 0)), (3*a**2*log(a/b + x)/(3*a**2*b + 6*a*b**
2*x + 3*b**3*x**2) + a**2/(3*a**2*b + 6*a*b**2*x + 3*b**3*x**2) + 6*a*b*x*l
og(a/b + x)/(3*a**2*b + 6*a*b**2*x + 3*b**3*x**2) + 2*a*b*x/(3*a**2*b + 6*a
*b**2*x + 3*b**3*x**2) + 3*b**2*x**2*log(a/b + x)/(3*a**2*b + 6*a*b**2*x +
3*b**3*x**2) - 5*b**2*x**2/(3*a**2*b + 6*a*b**2*x + 3*b**3*x**2), Eq(m, -5)
), (-12*a**2*log(a/b + x)/(3*a*b + 3*b**2*x) - 11*a**2/(3*a*b + 3*b**2*x) -
 12*a*b*x*log(a/b + x)/(3*a*b + 3*b**2*x) + 4*a*b*x/(3*a*b + 3*b**2*x) + 3*
b**2*x**2/(3*a*b + 3*b**2*x), Eq(m, -4)), (4*a**2*log(a/b + x)/b - 3*a*x +
b*x**2/2, Eq(m, -3)), (a**5*m**2*(a + b*x)**m/(b*m**3 + 12*b*m**2 + 47*b*m
+ 60*b) + 11*a**5*m*(a + b*x)**m/(b*m**3 + 12*b*m**2 + 47*b*m + 60*b) + 32*
a**5*(a + b*x)**m/(b*m**3 + 12*b*m**2 + 47*b*m + 60*b) + a**4*b*m**2*x*(a +
 b*x)**m/(b*m**3 + 12*b*m**2 + 47*b*m + 60*b) + 15*a**4*b*m*x*(a + b*x)**m/
(b*m**3 + 12*b*m**2 + 47*b*m + 60*b) + 60*a**4*b*x*(a + b*x)**m/(b*m**3 + 1
2*b*m**2 + 47*b*m + 60*b) - 2*a**3*b**2*m**2*x**2*(a + b*x)**m/(b*m**3 + 12
*b*m**2 + 47*b*m + 60*b) - 14*a**3*b**2*m*x**2*(a + b*x)**m/(b*m**3 + 12*b*
m**2 + 47*b*m + 60*b) - 2*a**2*b**3*m**2*x**3*(a + b*x)**m/(b*m**3 + 12*b*m
**2 + 47*b*m + 60*b) - 22*a**2*b**3*m*x**3*(a + b*x)**m/(b*m**3 + 12*b*m**2
 + 47*b*m + 60*b) - 40*a**2*b**3*x**3*(a + b*x)**m/(b*m**3 + 12*b*m**2 + 47
*b*m + 60*b) + a*b**4*m**2*x**4*(a + b*x)**m/(b*m**3 + 12*b*m**2 + 47*b*m +
 60*b) + 3*a*b**4*m*x**4*(a + b*x)**m/(b*m**3 + 12*b*m**2 + 47*b*m + 60*b)
+ b**5*m**2*x**5*(a + b*x)**m/(b*m**3 + 12*b*m**2 + 47*b*m + 60*b) + 7*b**5
*m*x**5*(a + b*x)**m/(b*m**3 + 12*b*m**2 + 47*b*m + 60*b) + 12*b**5*x**5*(a
 + b*x)**m/(b*m**3 + 12*b*m**2 + 47*b*m + 60*b), True))
```

Giac [B] time = 1.28296, size = 389, normalized size = 6.38

$$\frac{(bx+a)^m b^5 m^2 x^5 + (bx+a)^m a b^4 m^2 x^4 + 7\,(bx+a)^m b^5 m x^5 - 2\,(bx+a)^m a^2 b^3 m^2 x^3 + 3\,(bx+a)^m a b^4 m x^4 + 12\,(bx+a)^m b^5 \cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^m*b^5*m^2*x^5 + (b*x + a)^m*a*b^4*m^2*x^4 + 7*(b*x + a)^m*b^5*m*
x^5 - 2*(b*x + a)^m*a^2*b^3*m^2*x^3 + 3*(b*x + a)^m*a*b^4*m*x^4 + 12*(b*x +
 a)^m*b^5*x^5 - 2*(b*x + a)^m*a^3*b^2*m^2*x^2 - 22*(b*x + a)^m*a^2*b^3*m*x^
3 + (b*x + a)^m*a^4*b*m^2*x - 14*(b*x + a)^m*a^3*b^2*m*x^2 - 40*(b*x + a)^m
*a^2*b^3*x^3 + (b*x + a)^m*a^5*m^2 + 15*(b*x + a)^m*a^4*b*m*x + 11*(b*x + a
)^m*a^5*m + 60*(b*x + a)^m*a^4*b*x + 32*(b*x + a)^m*a^5)/(b*m^3 + 12*b*m^2
+ 47*b*m + 60*b)
```

3.947 $\int (a+bx)^m \left(a^2-b^2x^2\right) dx$

Optimal. Leaf size=40

$$\frac{2a(a+bx)^{m+2}}{b(m+2)}-\frac{(a+bx)^{m+3}}{b(m+3)}$$

```
[Out] (2*a*(a + b*x)^(2 + m))/(b*(2 + m)) - (a + b*x)^(3 + m)/(b*(3 + m))
```

Rubi [A] time = 0.0165849, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {627, 43}

$$\frac{2a(a+bx)^{m+2}}{b(m+2)}-\frac{(a+bx)^{m+3}}{b(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(a^2 - b^2*x^2),x]
```

```
[Out] (2*a*(a + b*x)^(2 + m))/(b*(2 + m)) - (a + b*x)^(3 + m)/(b*(3 + m))
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^m \left(a^2-b^2x^2\right) dx &= \int (a-bx)(a+bx)^{1+m}\, dx\\ &= \int \left(2a(a+bx)^{1+m}-(a+bx)^{2+m}\right) dx\\ &= \frac{2a(a+bx)^{2+m}}{b(2+m)}-\frac{(a+bx)^{3+m}}{b(3+m)}\end{aligned}$$

Mathematica [A] time = 0.0284567, size = 36, normalized size = 0.9

$$\frac{(a+bx)^{m+2}(a(m+4)-b(m+2)x)}{b(m+2)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(a^2 - b^2*x^2),x]
```

```
[Out] ((a + b*x)^(2 + m)*(a*(4 + m) - b*(2 + m)*x))/(b*(2 + m)*(3 + m))
```

Maple [A] time = 0.04, size = 40, normalized size = 1.

$$\frac{(bx+a)^{2+m}(-bmx+am-2\,bx+4\,a)}{b\left(m^2+5\,m+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(-b^2*x^2+a^2),x)
```

```
[Out] (b*x+a)^(2+m)*(-b*m*x+a*m-2*b*x+4*a)/b/(m^2+5*m+6)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(-b^2*x^2+a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.81246, size = 151, normalized size = 3.78

$$-\frac{\left(ab^2mx^2-a^3m+\left(b^3m+2\,b^3\right)x^3-4\,a^3-\left(a^2bm+6\,a^2b\right)x\right)(bx+a)^m}{bm^2+5\,bm+6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(-b^2*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -(a*b^2*m*x^2 - a^3*m + (b^3*m + 2*b^3)*x^3 - 4*a^3 - (a^2*b*m + 6*a^2*b)*x
)*(b*x + a)^m/(b*m^2 + 5*b*m + 6*b)
```

Sympy [A] time = 1.05682, size = 267, normalized size = 6.68

$$\begin{cases} a^2a^mx & \text{for}\ b=0 \\ -\frac{a\log\left(\frac{a}{b}+x\right)}{ab+b^2x}-\frac{2a}{ab+b^2x}-\frac{bx\log\left(\frac{a}{b}+x\right)}{ab+b^2x} & \text{for}\ m=-3 \\ \frac{2a\log\left(\frac{a}{b}+x\right)}{b}-x & \text{for}\ m=-2 \\ \frac{a^3m(a+bx)^m}{bm^2+5bm+6b}+\frac{4a^3(a+bx)^m}{bm^2+5bm+6b}+\frac{a^2bmx(a+bx)^m}{bm^2+5bm+6b}+\frac{6a^2bx(a+bx)^m}{bm^2+5bm+6b}-\frac{ab^2mx^2(a+bx)^m}{bm^2+5bm+6b}-\frac{b^3mx^3(a+bx)^m}{bm^2+5bm+6b}-\frac{2b^3x^3(a+bx)^m}{bm^2+5bm+6b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(-b**2*x**2+a**2),x)
```

```
[Out] Piecewise((a**2*a**m*x, Eq(b, 0)), (-a*log(a/b + x)/(a*b + b**2*x) - 2*a/(a
*b + b**2*x) - b*x*log(a/b + x)/(a*b + b**2*x), Eq(m, -3)), (2*a*log(a/b +
```

```
x)/b - x, Eq(m, -2)), (a**3*m*(a + b*x)**m/(b*m**2 + 5*b*m + 6*b) + 4*a**3*
(a + b*x)**m/(b*m**2 + 5*b*m + 6*b) + a**2*b*m*x*(a + b*x)**m/(b*m**2 + 5*b
*m + 6*b) + 6*a**2*b*x*(a + b*x)**m/(b*m**2 + 5*b*m + 6*b) - a*b**2*m*x**2*
(a + b*x)**m/(b*m**2 + 5*b*m + 6*b) - b**3*m*x**3*(a + b*x)**m/(b*m**2 + 5*
b*m + 6*b) - 2*b**3*x**3*(a + b*x)**m/(b*m**2 + 5*b*m + 6*b), True))
```

Giac [B] time = 1.2403, size = 159, normalized size = 3.98

$$-\frac{(bx+a)^m b^3 m x^3 + (bx+a)^m a b^2 m x^2 + 2\,(bx+a)^m b^3 x^3 - (bx+a)^m a^2 b m x - (bx+a)^m a^3 m - 6\,(bx+a)^m a^2 b x - 4\,(bx}{bm^2 + 5\,bm + 6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(-b^2*x^2+a^2),x, algorithm="giac")
```

```
[Out] -((b*x + a)^m*b^3*m*x^3 + (b*x + a)^m*a*b^2*m*x^2 + 2*(b*x + a)^m*b^3*x^3 -
 (b*x + a)^m*a^2*b*m*x - (b*x + a)^m*a^3*m - 6*(b*x + a)^m*a^2*b*x - 4*(b*x
 + a)^m*a^3)/(b*m^2 + 5*b*m + 6*b)
```

3.948 $\int \frac{(a+bx)^m}{a^2-b^2x^2} \, dx$

Optimal. Leaf size=38

$$\frac{(a+bx)^m {}_2F_1\left(1,m;m+1;\frac{a+bx}{2a}\right)}{2abm}$$

```
[Out] ((a + b*x)^m*Hypergeometric2F1[1, m, 1 + m, (a + b*x)/(2*a)])/(2*a*b*m)
```

Rubi [A] time = 0.0133872, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {627, 68}

$$\frac{(a+bx)^m {}_2F_1\left(1,m;m+1;\frac{a+bx}{2a}\right)}{2abm}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m/(a^2 - b^2*x^2),x]
```

```
[Out] ((a + b*x)^m*Hypergeometric2F1[1, m, 1 + m, (a + b*x)/(2*a)])/(2*a*b*m)
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^m}{a^2-b^2x^2} \, dx &= \int \frac{(a+bx)^{-1+m}}{a-bx} \, dx\\ &= \frac{(a+bx)^m {}_2F_1\left(1,m;1+m;\frac{a+bx}{2a}\right)}{2abm}\end{aligned}$$

Mathematica [A] time = 0.0421651, size = 59, normalized size = 1.55

$$\frac{(a+bx)^m \left(m(a+bx) {}_2F_1\left(1,m+1;m+2;\frac{a+bx}{2a}\right)+2a(m+1)\right)}{4a^2bm(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m/(a^2 - b^2*x^2),x]
```

```
[Out] ((a + b*x)^m*(2*a*(1 + m) + m*(a + b*x)*Hypergeometric2F1[1, 1 + m, 2 + m,
(a + b*x)/(2*a)]))/(4*a^2*b*m*(1 + m))
```

Maple [F] time = 0.522, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^m}{-b^2x^2+a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m/(-b^2*x^2+a^2),x)
```

```
[Out] int((b*x+a)^m/(-b^2*x^2+a^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(bx+a)^m}{b^2x^2-a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(-b^2*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -integrate((b*x + a)^m/(b^2*x^2 - a^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(bx+a)^m}{b^2x^2-a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(-b^2*x^2+a^2),x, algorithm="fricas")
```

```
[Out] integral(-(b*x + a)^m/(b^2*x^2 - a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(a+bx)^m}{-a^2+b^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m/(-b**2*x**2+a**2),x)
```

```
[Out] -Integral((a + b*x)**m/(-a**2 + b**2*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(bx+a)^m}{b^2x^2-a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(-b^2*x^2+a^2),x, algorithm="giac")
```

```
[Out] integrate(-(b*x + a)^m/(b^2*x^2 - a^2), x)
```

3.949 $\int \frac{(a+bx)^m}{\left(a^2-b^2x^2\right)^2} dx$

Optimal. Leaf size=44

$$-\frac{(a+bx)^{m-1} {}_2F_1\left(2,m-1;m;\frac{a+bx}{2a}\right)}{4a^2b(1-m)}$$

```
[Out] -((a + b*x)^(-1 + m)*Hypergeometric2F1[2, -1 + m, m, (a + b*x)/(2*a)])/(4*a
^2*b*(1 - m))
```

Rubi [A] time = 0.0154593, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {627, 68}

$$-\frac{(a+bx)^{m-1} {}_2F_1\left(2,m-1;m;\frac{a+bx}{2a}\right)}{4a^2b(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m/(a^2 - b^2*x^2)^2,x]
```

```
[Out] -((a + b*x)^(-1 + m)*Hypergeometric2F1[2, -1 + m, m, (a + b*x)/(2*a)])/(4*a
^2*b*(1 - m))
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^m}{\left(a^2-b^2x^2\right)^2} dx &= \int \frac{(a+bx)^{-2+m}}{(a-bx)^2} dx \\ &= -\frac{(a+bx)^{-1+m} {}_2F_1\left(2,-1+m;m;\frac{a+bx}{2a}\right)}{4a^2b(1-m)} \end{aligned}$$

Mathematica [B] time = 0.150645, size = 102, normalized size = 2.32

$$\frac{(a+bx)^m\left(\frac{2(a+bx) {}_2F_1\left(1,m+1;m+2;\frac{a+bx}{2a}\right)}{m+1}+\frac{(a+bx) {}_2F_1\left(2,m+1;m+2;\frac{a+bx}{2a}\right)}{m+1}+4a\left(\frac{a}{(m-1)(a+bx)}+\frac{1}{m}\right)\right)}{16a^4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m/(a^2 - b^2*x^2)^2,x]
```

```
[Out] ((a + b*x)^m*(4*a*(m^(-1) + a/((-1 + m)*(a + b*x))) + (2*(a + b*x)*Hypergeo
metric2F1[1, 1 + m, 2 + m, (a + b*x)/(2*a)])/(1 + m) + ((a + b*x)*Hypergeom
etric2F1[2, 1 + m, 2 + m, (a + b*x)/(2*a)])/(1 + m)))/(16*a^4*b)
```

Maple [F] time = 0.505, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^m}{\left(-b^2x^2+a^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m/(-b^2*x^2+a^2)^2,x)
```

```
[Out] int((b*x+a)^m/(-b^2*x^2+a^2)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^m}{\left(b^2x^2-a^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^m/(b^2*x^2 - a^2)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx+a)^m}{b^4x^4-2\,a^2b^2x^2+a^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^m/(b^4*x^4 - 2*a^2*b^2*x^2 + a^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^m}{(-a+bx)^2\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m/(-b**2*x**2+a**2)**2,x)
```

```
[Out] Integral((a + b*x)**m/((-a + b*x)**2*(a + b*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^m}{\left(b^2x^2-a^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^m/(b^2*x^2 - a^2)^2, x)
```

3.950 $\int \frac{(a+bx)^m}{\left(a^2-b^2x^2\right)^3} \, dx$

Optimal. Leaf size=46

$$-\frac{(a+bx)^{m-2} \, _2F_1\left(3,m-2;m-1;\frac{a+bx}{2a}\right)}{8a^3b(2-m)}$$

```
[Out] -((a + b*x)^(-2 + m)*Hypergeometric2F1[3, -2 + m, -1 + m, (a + b*x)/(2*a)])
/(8*a^3*b*(2 - m))
```

Rubi [A] time = 0.0151704, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {627, 68}

$$-\frac{(a+bx)^{m-2} \, _2F_1\left(3,m-2;m-1;\frac{a+bx}{2a}\right)}{8a^3b(2-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m/(a^2 - b^2*x^2)^3,x]
```

```
[Out] -((a + b*x)^(-2 + m)*Hypergeometric2F1[3, -2 + m, -1 + m, (a + b*x)/(2*a)])
/(8*a^3*b*(2 - m))
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^m}{\left(a^2-b^2x^2\right)^3} \, dx &= \int \frac{(a+bx)^{-3+m}}{(a-bx)^3} \, dx\\ &= -\frac{(a+bx)^{-2+m} \, _2F_1\left(3,-2+m;-1+m;\frac{a+bx}{2a}\right)}{8a^3b(2-m)}\end{aligned}$$

Mathematica [B] time = 0.185065, size = 153, normalized size = 3.33

$$\frac{(a+bx)^m\left(\frac{8a^3}{(m-2)(a+bx)^2}+\frac{12a^2}{(m-1)(a+bx)}+\frac{6(a+bx)\,_2F_1\left(1,m+1;m+2;\frac{a+bx}{2a}\right)}{m+1}+\frac{3(a+bx)\,_2F_1\left(2,m+1;m+2;\frac{a+bx}{2a}\right)}{m+1}+\frac{(a+bx)\,_2F_1\left(3,m+1;m+2;\frac{a+bx}{2a}\right)}{m+1}+\frac{12a}{m}\right.}{64a^6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m/(a^2 - b^2*x^2)^3,x]
```

```
[Out] ((a + b*x)^m*((12*a)/m + (8*a^3)/((-2 + m)*(a + b*x)^2) + (12*a^2)/((-1 + m
)*(a + b*x)) + (6*(a + b*x)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*x)/(2
*a)])/(1 + m) + (3*(a + b*x)*Hypergeometric2F1[2, 1 + m, 2 + m, (a + b*x)/(
2*a)])/(1 + m) + ((a + b*x)*Hypergeometric2F1[3, 1 + m, 2 + m, (a + b*x)/(2
*a)])/(1 + m)))/(64*a^6*b)
```

Maple [F] time = 0.536, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^m}{\left(-b^2x^2+a^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m/(-b^2*x^2+a^2)^3,x)
```

```
[Out] int((b*x+a)^m/(-b^2*x^2+a^2)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(bx+a)^m}{\left(b^2x^2-a^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -integrate((b*x + a)^m/(b^2*x^2 - a^2)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(bx+a)^m}{b^6x^6-3\,a^2b^4x^4+3\,a^4b^2x^2-a^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] integral(-(b*x + a)^m/(b^6*x^6 - 3*a^2*b^4*x^4 + 3*a^4*b^2*x^2 - a^6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(a+bx)^m}{-a^6+3a^4b^2x^2-3a^2b^4x^4+b^6x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m/(-b**2*x**2+a**2)**3,x)
```

```
[Out] -Integral((a + b*x)**m/(-a**6 + 3*a**4*b**2*x**2 - 3*a**2*b**4*x**4 + b**6*
x**6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(bx+a)^m}{(b^2x^2-a^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] integrate(-(b*x + a)^m/(b^2*x^2 - a^2)^3, x)
```

3.951 $\int (d+ex)^m \left(d^2-e^2x^2\right)^{7/2} dx$

Optimal. Leaf size=59

$$\frac{\left(d^2-e^2x^2\right)^{9/2}(d+ex)^m {}_2F_1\left(1,m+9;m+\frac{11}{2};\frac{d+ex}{2d}\right)}{de(2m+9)}$$

```
[Out] ((d + e*x)^m*(d^2 - e^2*x^2)^(9/2)*Hypergeometric2F1[1, 9 + m, 11/2 + m, (d
 + e*x)/(2*d)])/(d*e*(9 + 2*m))
```

Rubi [A] time = 0.0490376, antiderivative size = 83, normalized size of antiderivative = 1.41, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {680, 678, 69}

$$-\frac{2^{m+\frac{9}{2}}\left(d^2-e^2x^2\right)^{9/2}(d+ex)^m\left(\frac{ex}{d}+1\right)^{-m-\frac{9}{2}} {}_2F_1\left(\frac{9}{2},-m-\frac{7}{2};\frac{11}{2};\frac{d-ex}{2d}\right)}{9de}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] -(2^(9/2 + m)*(d + e*x)^m*(1 + (e*x)/d)^(-9/2 - m)*(d^2 - e^2*x^2)^(9/2)*Hy
pergeometric2F1[9/2, -7/2 - m, 11/2, (d - e*x)/(2*d)])/(9*d*e)
```

Rule 680

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^IntPart[m]*(d + e*x)^FracPart[m])/(1 + (e*x)/d)^FracPart[m], Int[(1 + (e
*x)/d)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && EqQ[c*d^2 +
a*e^2, 0] && !IntegerQ[p] && !(IntegerQ[m] || GtQ[d, 0])
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) && !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (d+ex)^m \left(d^2-e^2x^2\right)^{7/2} dx = \left((d+ex)^m \left(1+\frac{ex}{d}\right)^{-m}\right) \int \left(1+\frac{ex}{d}\right)^m \left(d^2-e^2x^2\right)^{7/2} dx$$
$$= \frac{\left((d+ex)^m \left(1+\frac{ex}{d}\right)^{-\frac{9}{2}-m} \left(d^2-e^2x^2\right)^{9/2}\right) \int \left(1+\frac{ex}{d}\right)^{\frac{7}{2}+m} \left(d^2-dex\right)^{7/2} dx}{\left(d^2-dex\right)^{9/2}}$$
$$= -\frac{2^{\frac{9}{2}+m}(d+ex)^m \left(1+\frac{ex}{d}\right)^{-\frac{9}{2}-m} \left(d^2-e^2x^2\right)^{9/2} {}_2F_1\left(\frac{9}{2},-\frac{7}{2}-m;\frac{11}{2};\frac{d-ex}{2d}\right)}{9de}$$

Mathematica [C] time = 0.516095, size = 347, normalized size = 5.88

$$\frac{(d+ex)^m \left(\frac{ex}{d}+1\right)^{-m-\frac{1}{2}} \left(-105d^4e^3x^3\sqrt{d-ex}\sqrt{d+ex}F_1\left(3;-\frac{1}{2},-m-\frac{1}{2};4;\frac{ex}{d},-\frac{ex}{d}\right)+63d^2e^5x^5\sqrt{d-ex}\sqrt{d+ex}F_1\left(5;-\frac{1}{2},-\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^m*(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((d + e*x)^m*(1 + (e*x)/d)^(-1/2 - m)*(-105*d^4*e^3*x^3*Sqrt[d - e*x]*Sqrt[
d + e*x]*AppellF1[3, -1/2, -1/2 - m, 4, (e*x)/d, -((e*x)/d)] + 63*d^2*e^5*x
^5*Sqrt[d - e*x]*Sqrt[d + e*x]*AppellF1[5, -1/2, -1/2 - m, 6, (e*x)/d, -((e
*x)/d)] - 15*e^7*x^7*Sqrt[d - e*x]*Sqrt[d + e*x]*AppellF1[7, -1/2, -1/2 - m
, 8, (e*x)/d, -((e*x)/d)] - 35*2^(3/2 + m)*d^7*Sqrt[1 - (e*x)/d]*Sqrt[d^2 -
 e^2*x^2]*Hypergeometric2F1[3/2, -1/2 - m, 5/2, (d - e*x)/(2*d)] + 35*2^(3/
2 + m)*d^6*e*x*Sqrt[1 - (e*x)/d]*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[3/2,
 -1/2 - m, 5/2, (d - e*x)/(2*d)]))/(105*e*Sqrt[1 - (e*x)/d])
```

Maple [F] time = 0.506, size = 0, normalized size = 0.

$$\int (ex+d)^m \left(-e^2x^2+d^2\right)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] int((e*x+d)^m*(-e^2*x^2+d^2)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-e^2x^2+d^2\right)^{\frac{7}{2}} (ex+d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(e^6x^6 - 3\,d^2e^4x^4 + 3\,d^4e^2x^2 - d^6\right)\sqrt{-e^2x^2 + d^2}(ex + d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(e^6*x^6 - 3*d^2*e^4*x^4 + 3*d^4*e^2*x^2 - d^6)*sqrt(-e^2*x^2 + d
^2)*(e*x + d)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-e^2x^2 + d^2\right)^{\frac{7}{2}}(ex + d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^m, x)
```

3.952 $\int (d+ex)^m \left(d^2-e^2x^2\right)^{5/2} dx$

Optimal. Leaf size=83

$$-\frac{2^{m+\frac{7}{2}}\left(d^2-e^2x^2\right)^{7/2}(d+ex)^m\left(\frac{ex}{d}+1\right)^{-m-\frac{7}{2}}{}_2F_1\left(\frac{7}{2},-m-\frac{5}{2};\frac{9}{2};\frac{d-ex}{2d}\right)}{7de}$$

```
[Out] -(2^(7/2 + m)*(d + e*x)^m*(1 + (e*x)/d)^(-7/2 - m)*(d^2 - e^2*x^2)^(7/2)*Hy
pergeometric2F1[7/2, -5/2 - m, 9/2, (d - e*x)/(2*d)])/(7*d*e)
```

Rubi [A] time = 0.0475875, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {680, 678, 69}

$$-\frac{2^{m+\frac{7}{2}}\left(d^2-e^2x^2\right)^{7/2}(d+ex)^m\left(\frac{ex}{d}+1\right)^{-m-\frac{7}{2}}{}_2F_1\left(\frac{7}{2},-m-\frac{5}{2};\frac{9}{2};\frac{d-ex}{2d}\right)}{7de}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] -(2^(7/2 + m)*(d + e*x)^m*(1 + (e*x)/d)^(-7/2 - m)*(d^2 - e^2*x^2)^(7/2)*Hy
pergeometric2F1[7/2, -5/2 - m, 9/2, (d - e*x)/(2*d)])/(7*d*e)
```

Rule 680

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^IntPart[m]*(d + e*x)^FracPart[m])/(1 + (e*x)/d)^FracPart[m], Int[(1 + (e
*x)/d)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && EqQ[c*d^2 +
a*e^2, 0] && !IntegerQ[p] && !(IntegerQ[m] || GtQ[d, 0])
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) && !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (d+ex)^m \left(d^2-e^2x^2\right)^{5/2} dx = \left((d+ex)^m \left(1+\frac{ex}{d}\right)^{-m}\right) \int \left(1+\frac{ex}{d}\right)^m \left(d^2-e^2x^2\right)^{5/2} dx$$
$$= \frac{\left((d+ex)^m \left(1+\frac{ex}{d}\right)^{-\frac{7}{2}-m} \left(d^2-e^2x^2\right)^{7/2}\right) \int \left(1+\frac{ex}{d}\right)^{\frac{5}{2}+m} \left(d^2-dex\right)^{5/2} dx}{\left(d^2-dex\right)^{7/2}}$$
$$= -\frac{2^{\frac{7}{2}+m}(d+ex)^m \left(1+\frac{ex}{d}\right)^{-\frac{7}{2}-m} \left(d^2-e^2x^2\right)^{7/2} {}_2F_1\left(\frac{7}{2},-\frac{5}{2}-m;\frac{9}{2};\frac{d-ex}{2d}\right)}{7de}$$

Mathematica [C] time = 0.39913, size = 227, normalized size = 2.73

$$\frac{(d+ex)^m \left(\frac{ex}{d}+1\right)^{-m-\frac{1}{2}} \left(-10d^2e^3x^3\sqrt{d-ex}\sqrt{d+ex}F_1\left(3;-\frac{1}{2},-m-\frac{1}{2};4;\frac{ex}{d},-\frac{ex}{d}\right)+3e^5x^5\sqrt{d-ex}\sqrt{d+ex}F_1\left(5;-\frac{1}{2},-m\right.\right.}{15e\sqrt{1-\frac{ex}{d}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^m*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] ((d + e*x)^m*(1 + (e*x)/d)^(-1/2 - m)*(-10*d^2*e^3*x^3*Sqrt[d - e*x]*Sqrt[d
 + e*x]*AppellF1[3, -1/2, -1/2 - m, 4, (e*x)/d, -((e*x)/d)] + 3*e^5*x^5*Sqr
t[d - e*x]*Sqrt[d + e*x]*AppellF1[5, -1/2, -1/2 - m, 6, (e*x)/d, -((e*x)/d)
] - 5*2^(3/2 + m)*d^4*(d - e*x)*Sqrt[1 - (e*x)/d]*Sqrt[d^2 - e^2*x^2]*Hyper
geometric2F1[3/2, -1/2 - m, 5/2, (d - e*x)/(2*d)]))/(15*e*Sqrt[1 - (e*x)/d]
)
```

Maple [F] time = 0.484, size = 0, normalized size = 0.

$$\int (ex+d)^m \left(-e^2x^2+d^2\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] int((e*x+d)^m*(-e^2*x^2+d^2)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^4x^4 - 2\,d^2e^2x^2 + d^4\right)\sqrt{-e^2x^2 + d^2}(ex + d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((e^4*x^4 - 2*d^2*e^2*x^2 + d^4)*sqrt(-e^2*x^2 + d^2)*(e*x + d)^m,
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-e^2x^2 + d^2\right)^{\frac{5}{2}}(ex + d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^m, x)
```

3.953 $\int (d+ex)^m \left(d^2-e^2x^2\right)^{3/2} \, dx$

Optimal. Leaf size=59

$$\frac{\left(d^2-e^2x^2\right)^{5/2}(d+ex)^m \,_2F_1\left(1,m+5;m+\frac{7}{2};\frac{d+ex}{2d}\right)}{de(2m+5)}$$

```
[Out] ((d + e*x)^m*(d^2 - e^2*x^2)^(5/2)*Hypergeometric2F1[1, 5 + m, 7/2 + m, (d
+ e*x)/(2*d)])/(d*e*(5 + 2*m))
```

Rubi [A] time = 0.0473627, antiderivative size = 83, normalized size of antiderivative = 1.41, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {680, 678, 69}

$$-\frac{2^{m+\frac{5}{2}}\left(d^2-e^2x^2\right)^{5/2}(d+ex)^m\left(\frac{ex}{d}+1\right)^{-m-\frac{5}{2}} \,_2F_1\left(\frac{5}{2},-m-\frac{3}{2};\frac{7}{2};\frac{d-ex}{2d}\right)}{5de}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] -(2^(5/2 + m)*(d + e*x)^m*(1 + (e*x)/d)^(-5/2 - m)*(d^2 - e^2*x^2)^(5/2)*Hy
pergeometric2F1[5/2, -3/2 - m, 7/2, (d - e*x)/(2*d)])/(5*d*e)
```

Rule 680

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^IntPart[m]*(d + e*x)^FracPart[m])/(1 + (e*x)/d)^FracPart[m], Int[(1 + (e
*x)/d)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && EqQ[c*d^2 +
a*e^2, 0] && !IntegerQ[p] && !(IntegerQ[m] || GtQ[d, 0])
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) && !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (d+ex)^m \left(d^2-e^2x^2\right)^{3/2} dx = \left((d+ex)^m \left(1+\frac{ex}{d}\right)^{-m}\right) \int \left(1+\frac{ex}{d}\right)^m \left(d^2-e^2x^2\right)^{3/2} dx$$
$$= \frac{\left((d+ex)^m \left(1+\frac{ex}{d}\right)^{-\frac{5}{2}-m} \left(d^2-e^2x^2\right)^{5/2}\right) \int \left(1+\frac{ex}{d}\right)^{\frac{3}{2}+m} \left(d^2-dex\right)^{3/2} dx}{\left(d^2-dex\right)^{5/2}}$$
$$= -\frac{2^{\frac{5}{2}+m}(d+ex)^m \left(1+\frac{ex}{d}\right)^{-\frac{5}{2}-m} \left(d^2-e^2x^2\right)^{5/2} {}_2F_1\left(\frac{5}{2},-\frac{3}{2}-m;\frac{7}{2};\frac{d-ex}{2d}\right)}{5de}$$

Mathematica [C] time = 0.237686, size = 191, normalized size = 3.24

$$-\frac{2^m(d+ex)^m \left(\frac{ex}{d}+1\right)^{-2m-\frac{1}{2}} \left(e^3x^3\sqrt{d-ex}\sqrt{d+ex}\left(\frac{ex}{2d}+\frac{1}{2}\right)^m F_1\left(3;-\frac{1}{2},-m-\frac{1}{2};4;\frac{ex}{d},-\frac{ex}{d}\right)+2d^2(d-ex)\sqrt{2-\frac{2ex}{d}}\sqrt{d^2-e}\right.}{3e\sqrt{1-\frac{ex}{d}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^m*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] -(2^m*(d + e*x)^m*(1 + (e*x)/d)^(-1/2 - 2*m)*(e^3*x^3*Sqrt[d - e*x]*Sqrt[d
+ e*x]*(1/2 + (e*x)/(2*d))^m*AppellF1[3, -1/2, -1/2 - m, 4, (e*x)/d, -((e*x
)/d)] + 2*d^2*(d - e*x)*Sqrt[2 - (2*e*x)/d]*(1 + (e*x)/d)^m*Sqrt[d^2 - e^2*
x^2]*Hypergeometric2F1[3/2, -1/2 - m, 5/2, (d - e*x)/(2*d)]))/(3*e*Sqrt[1 -
 (e*x)/d])
```

Maple [F] time = 0.479, size = 0, normalized size = 0.

$$\int (ex+d)^m \left(-e^2x^2+d^2\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] int((e*x+d)^m*(-e^2*x^2+d^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-e^2x^2+d^2\right)^{\frac{3}{2}} (ex+d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(e^2x^2-d^2\right)\sqrt{-e^2x^2+d^2}(ex+d)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(e^2*x^2 - d^2)*sqrt(-e^2*x^2 + d^2)*(e*x + d)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^m, x)
```

3.954 $\int (d+ex)^m \sqrt{d^2-e^2x^2}\,dx$

Optimal. Leaf size=67

$$\frac{(d-ex)\sqrt{d^2-e^2x^2}(d+ex)^{m+1}\,{}_2F_1\left(1,m+3;m+\frac{5}{2};\frac{d+ex}{2d}\right)}{de(2m+3)}$$

```
[Out] ((d - e*x)*(d + e*x)^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[1, 3 + m
, 5/2 + m, (d + e*x)/(2*d)])/(d*e*(3 + 2*m))
```

Rubi [A] time = 0.0469089, antiderivative size = 83, normalized size of antiderivative = 1.24, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {680, 678, 69}

$$-\frac{2^{m+\frac{3}{2}}\left(d^2-e^2x^2\right)^{3/2}(d+ex)^m\left(\frac{ex}{d}+1\right)^{-m-\frac{3}{2}}\,{}_2F_1\left(\frac{3}{2},-m-\frac{1}{2};\frac{5}{2};\frac{d-ex}{2d}\right)}{3de}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -(2^(3/2 + m)*(d + e*x)^m*(1 + (e*x)/d)^(-3/2 - m)*(d^2 - e^2*x^2)^(3/2)*Hy
pergeometric2F1[3/2, -1/2 - m, 5/2, (d - e*x)/(2*d)])/(3*d*e)
```

Rule 680

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^IntPart[m]*(d + e*x)^FracPart[m])/(1 + (e*x)/d)^FracPart[m], Int[(1 + (e
*x)/d)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && EqQ[c*d^2 +
a*e^2, 0] && !IntegerQ[p] && !(IntegerQ[m] || GtQ[d, 0])
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) && !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (d+ex)^m\sqrt{d^2-e^2x^2}\,dx = \left((d+ex)^m\left(1+\frac{ex}{d}\right)^{-m}\right)\int \left(1+\frac{ex}{d}\right)^m\sqrt{d^2-e^2x^2}\,dx$$
$$= \frac{\left((d+ex)^m\left(1+\frac{ex}{d}\right)^{-\frac{3}{2}-m}\left(d^2-e^2x^2\right)^{3/2}\right)\int \left(1+\frac{ex}{d}\right)^{\frac{1}{2}+m}\sqrt{d^2-dex}\,dx}{\left(d^2-dex\right)^{3/2}}$$
$$= -\frac{2^{\frac{3}{2}+m}(d+ex)^m\left(1+\frac{ex}{d}\right)^{-\frac{3}{2}-m}\left(d^2-e^2x^2\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},-\frac{1}{2}-m;\frac{5}{2};\frac{d-ex}{2d}\right)}{3de}$$

Mathematica [A] time = 0.0639009, size = 86, normalized size = 1.28

$$-\frac{2^{m+\frac{3}{2}}(d-ex)\sqrt{d^2-e^2x^2}(d+ex)^m\left(\frac{ex}{d}+1\right)^{-m-\frac{1}{2}}\,{}_2F_1\left(\frac{3}{2},-m-\frac{1}{2};\frac{5}{2};\frac{d-ex}{2d}\right)}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -(2^(3/2 + m)*(d - e*x)*(d + e*x)^m*(1 + (e*x)/d)^(-1/2 - m)*Sqrt[d^2 - e^2
*x^2]*Hypergeometric2F1[3/2, -1/2 - m, 5/2, (d - e*x)/(2*d)])/(3*e)
```

Maple [F] time = 0.481, size = 0, normalized size = 0.

$$\int (ex+d)^m\sqrt{-e^2x^2+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] int((e*x+d)^m*(-e^2*x^2+d^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-e^2x^2+d^2}(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)*(e*x + d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{-e^2x^2+d^2}(ex+d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-e^2*x^2 + d^2)*(e*x + d)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-(-d+ex)(d+ex)}(d+ex)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-e^2x^2+d^2}(ex+d)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)*(e*x + d)^m, x)
```

3.955 $\int \frac{(d+ex)^m}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=67

$$\frac{(d-ex)(d+ex)^{m+1}\,{}_2F_1\left(1,m+1;m+\frac{3}{2};\frac{d+ex}{2d}\right)}{de(2m+1)\sqrt{d^2-e^2x^2}}$$

```
[Out] ((d - e*x)*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 3/2 + m, (d + e*x)
/(2*d)])/(d*e*(1 + 2*m)*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.0451431, antiderivative size = 81, normalized size of antiderivative = 1.21, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {680, 678, 69}

$$-\frac{2^{m+\frac{1}{2}}\sqrt{d^2-e^2x^2}(d+ex)^m\left(\frac{ex}{d}+1\right)^{-m-\frac{1}{2}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{d-ex}{2d}\right)}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -((2^(1/2 + m)*(d + e*x)^m*(1 + (e*x)/d)^(-1/2 - m)*Sqrt[d^2 - e^2*x^2]*Hyp
ergeometric2F1[1/2, 1/2 - m, 3/2, (d - e*x)/(2*d)])/(d*e))
```

Rule 680

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^IntPart[m]*(d + e*x)^FracPart[m])/(1 + (e*x)/d)^FracPart[m], Int[(1 + (e
*x)/d)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && EqQ[c*d^2 +
a*e^2, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] || GtQ[d, 0])
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(d+ex)^m}{\sqrt{d^2-e^2x^2}}\,dx = \left((d+ex)^m\left(1+\frac{ex}{d}\right)^{-m}\right)\int \frac{\left(1+\frac{ex}{d}\right)^m}{\sqrt{d^2-e^2x^2}}\,dx$$
$$= \frac{\left((d+ex)^m\left(1+\frac{ex}{d}\right)^{-\frac{1}{2}-m}\sqrt{d^2-e^2x^2}\right)\int \frac{\left(1+\frac{ex}{d}\right)^{-\frac{1}{2}+m}}{\sqrt{d^2-dex}}\,dx}{\sqrt{d^2-dex}}$$
$$= -\frac{2^{\frac{1}{2}+m}(d+ex)^m\left(1+\frac{ex}{d}\right)^{-\frac{1}{2}-m}\sqrt{d^2-e^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{d-ex}{2d}\right)}{de}$$

Mathematica [A] time = 0.0816501, size = 84, normalized size = 1.25

$$-\frac{2^{m+\frac{1}{2}}(d-ex)(d+ex)^m\left(\frac{ex}{d}+1\right)^{\frac{1}{2}-m}{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{d-ex}{2d}\right)}{e\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -((2^(1/2 + m)*(d - e*x)*(d + e*x)^m*(1 + (e*x)/d)^(1/2 - m)*Hypergeometric
2F1[1/2, 1/2 - m, 3/2, (d - e*x)/(2*d)])/(e*Sqrt[d^2 - e^2*x^2]))
```

Maple [F] time = 0.484, size = 0, normalized size = 0.

$$\int (ex+d)^m \frac{1}{\sqrt{-e^2x^2+d^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] int((e*x+d)^m/(-e^2*x^2+d^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\sqrt{-e^2x^2+d^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/sqrt(-e^2*x^2 + d^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-e^2x^2+d^2}(ex+d)^m}{e^2x^2-d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-e^2*x^2 + d^2)*(e*x + d)^m/(e^2*x^2 - d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m}{\sqrt{-(-d+ex)(d+ex)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral((d + e*x)**m/sqrt(-(-d + e*x)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\sqrt{-e^2x^2+d^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/sqrt(-e^2*x^2 + d^2), x)
```

3.956 $\int \frac{(d+ex)^m}{\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=80

$$\frac{2^{m-\frac{1}{2}}(d+ex)^m\left(\frac{ex}{d}+1\right)^{\frac{1}{2}-m}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{1}{2};\frac{d-ex}{2d}\right)}{de\sqrt{d^2-e^2x^2}}$$

```
[Out] (2^(-1/2 + m)*(d + e*x)^m*(1 + (e*x)/d)^(1/2 - m)*Hypergeometric2F1[-1/2, 3
/2 - m, 1/2, (d - e*x)/(2*d)])/(d*e*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.0539989, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {680, 678, 69}

$$\frac{2^{m-\frac{1}{2}}(d+ex)^m\left(\frac{ex}{d}+1\right)^{\frac{1}{2}-m}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{1}{2};\frac{d-ex}{2d}\right)}{de\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (2^(-1/2 + m)*(d + e*x)^m*(1 + (e*x)/d)^(1/2 - m)*Hypergeometric2F1[-1/2, 3
/2 - m, 1/2, (d - e*x)/(2*d)])/(d*e*Sqrt[d^2 - e^2*x^2])
```

Rule 680

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^IntPart[m]*(d + e*x)^FracPart[m])/(1 + (e*x)/d)^FracPart[m], Int[(1 + (e
*x)/d)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && EqQ[c*d^2 +
a*e^2, 0] && !IntegerQ[p] && !(IntegerQ[m] || GtQ[d, 0])
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) && !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(d+ex)^m}{\left(d^2-e^2x^2\right)^{3/2}}\,dx = \left((d+ex)^m\left(1+\frac{ex}{d}\right)^{-m}\right)\int \frac{\left(1+\frac{ex}{d}\right)^m}{\left(d^2-e^2x^2\right)^{3/2}}\,dx$$

$$= \frac{\left((d+ex)^m\left(1+\frac{ex}{d}\right)^{\frac{1}{2}-m}\sqrt{d^2-dex}\right)\int \frac{\left(1+\frac{ex}{d}\right)^{-\frac{3}{2}+m}}{\left(d^2-dex\right)^{3/2}}\,dx}{\sqrt{d^2-e^2x^2}}$$

$$= \frac{2^{-\frac{1}{2}+m}(d+ex)^m\left(1+\frac{ex}{d}\right)^{\frac{1}{2}-m}{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{1}{2};\frac{d-ex}{2d}\right)}{de\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.0700174, size = 80, normalized size = 1.

$$\frac{2^{m-\frac{1}{2}}(d+ex)^m\left(\frac{ex}{d}+1\right)^{\frac{1}{2}-m}{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{1}{2};\frac{d-ex}{2d}\right)}{de\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (2^(-1/2 + m)*(d + e*x)^m*(1 + (e*x)/d)^(1/2 - m)*Hypergeometric2F1[-1/2, 3
/2 - m, 1/2, (d - e*x)/(2*d)])/(d*e*Sqrt[d^2 - e^2*x^2])
```

Maple [F] time = 0.483, size = 0, normalized size = 0.

$$\int (ex+d)^m\left(-e^2x^2+d^2\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] int((e*x+d)^m/(-e^2*x^2+d^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/(-e^2*x^2 + d^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-e^2x^2+d^2}(ex+d)^m}{e^4x^4-2\,d^2e^2x^2+d^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-e^2*x^2 + d^2)*(e*x + d)^m/(e^4*x^4 - 2*d^2*e^2*x^2 + d^4),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m}{(-(-d+ex)(d+ex))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral((d + e*x)**m/(-(-d + e*x)*(d + e*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(-e^2*x^2 + d^2)^(3/2), x)
```

3.957 $\int \frac{(d+ex)^m}{\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=60

$$-\frac{(d+ex)^m\,{}_2F_1\left(1,m-3;m-\frac{1}{2};\frac{d+ex}{2d}\right)}{de(3-2m)\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] -(((d + e*x)^m*Hypergeometric2F1[1, -3 + m, -1/2 + m, (d + e*x)/(2*d)])/(d*
e*(3 - 2*m)*(d^2 - e^2*x^2)^(3/2)))
```

Rubi [A] time = 0.052316, antiderivative size = 83, normalized size of antiderivative = 1.38, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {680, 678, 69}

$$\frac{2^{m-\frac{3}{2}}(d+ex)^m\left(\frac{ex}{d}+1\right)^{\frac{3}{2}-m}\,{}_2F_1\left(-\frac{3}{2},\frac{5}{2}-m;-\frac{1}{2};\frac{d-ex}{2d}\right)}{3de\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (2^(-3/2 + m)*(d + e*x)^m*(1 + (e*x)/d)^(3/2 - m)*Hypergeometric2F1[-3/2, 5
/2 - m, -1/2, (d - e*x)/(2*d)])/(3*d*e*(d^2 - e^2*x^2)^(3/2))
```

Rule 680

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^IntPart[m]*(d + e*x)^FracPart[m])/(1 + (e*x)/d)^FracPart[m], Int[(1 + (e
*x)/d)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && EqQ[c*d^2 +
a*e^2, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] || GtQ[d, 0])
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(d+ex)^m}{(d^2-e^2x^2)^{5/2}}\,dx = \left((d+ex)^m\left(1+\frac{ex}{d}\right)^{-m}\right)\int \frac{\left(1+\frac{ex}{d}\right)^m}{(d^2-e^2x^2)^{5/2}}\,dx$$
$$= \frac{\left((d+ex)^m\left(1+\frac{ex}{d}\right)^{\frac{3}{2}-m}(d^2-dex)^{3/2}\right)\int \frac{\left(1+\frac{ex}{d}\right)^{-\frac{5}{2}+m}}{(d^2-dex)^{5/2}}\,dx}{(d^2-e^2x^2)^{3/2}}$$
$$= \frac{2^{-\frac{3}{2}+m}(d+ex)^m\left(1+\frac{ex}{d}\right)^{\frac{3}{2}-m}\,{}_2F_1\left(-\frac{3}{2},\frac{5}{2}-m;-\frac{1}{2};\frac{d-ex}{2d}\right)}{3de\left(d^2-e^2x^2\right)^{3/2}}$$

Mathematica [A] time = 0.111728, size = 92, normalized size = 1.53

$$\frac{2^{m-\frac{3}{2}}(d+ex)^m\left(\frac{ex}{d}+1\right)^{\frac{1}{2}-m}\,{}_2F_1\left(-\frac{3}{2},\frac{5}{2}-m;-\frac{1}{2};\frac{d-ex}{2d}\right)}{\left(3d^3e-3d^2e^2x\right)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (2^(-3/2 + m)*(d + e*x)^m*(1 + (e*x)/d)^(1/2 - m)*Hypergeometric2F1[-3/2, 5
/2 - m, -1/2, (d - e*x)/(2*d)])/((3*d^3*e - 3*d^2*e^2*x)*Sqrt[d^2 - e^2*x^2
])
```

Maple [F] time = 0.484, size = 0, normalized size = 0.

$$\int (ex+d)^m\left(-e^2x^2+d^2\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] int((e*x+d)^m/(-e^2*x^2+d^2)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/(-e^2*x^2 + d^2)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-e^2x^2+d^2}(ex+d)^m}{e^6x^6-3\,d^2e^4x^4+3\,d^4e^2x^2-d^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-e^2*x^2 + d^2)*(e*x + d)^m/(e^6*x^6 - 3*d^2*e^4*x^4 + 3*d^4
*e^2*x^2 - d^6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(d+ex)^m}{(-(-d+ex)(d+ex))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral((d + e*x)**m/(-(-d + e*x)*(d + e*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ex+d)^m}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(-e^2*x^2 + d^2)^(5/2), x)
```

3.958 $\int \frac{(d+ex)^m}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{(d-ex)(d+ex)^{m+1}\,{}_2F_1\left(1,m-5;m-\frac{3}{2};\frac{d+ex}{2d}\right)}{de(5-2m)\left(d^2-e^2x^2\right)^{7/2}}$$

```
[Out] -(((d - e*x)*(d + e*x)^(1 + m)*Hypergeometric2F1[1, -5 + m, -3/2 + m, (d +
e*x)/(2*d)])/(d*e*(5 - 2*m)*(d^2 - e^2*x^2)^(7/2)))
```

Rubi [A] time = 0.0566717, antiderivative size = 83, normalized size of antiderivative = 1.22, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {680, 678, 69}

$$\frac{2^{m-\frac{5}{2}}(d+ex)^m\left(\frac{ex}{d}+1\right)^{\frac{5}{2}-m}\,{}_2F_1\left(-\frac{5}{2},\frac{7}{2}-m;-\frac{3}{2};\frac{d-ex}{2d}\right)}{5de\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (2^(-5/2 + m)*(d + e*x)^m*(1 + (e*x)/d)^(5/2 - m)*Hypergeometric2F1[-5/2, 7
/2 - m, -3/2, (d - e*x)/(2*d)])/(5*d*e*(d^2 - e^2*x^2)^(5/2))
```

Rule 680

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^IntPart[m]*(d + e*x)^FracPart[m])/(1 + (e*x)/d)^FracPart[m], Int[(1 + (e
*x)/d)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && EqQ[c*d^2 +
a*e^2, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] || GtQ[d, 0])
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(d+ex)^m}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \left((d+ex)^m\left(1+\frac{ex}{d}\right)^{-m}\right)\int \frac{\left(1+\frac{ex}{d}\right)^m}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{\left((d+ex)^m\left(1+\frac{ex}{d}\right)^{\frac{5}{2}-m}\left(d^2-dex\right)^{5/2}\right)\int \frac{\left(1+\frac{ex}{d}\right)^{-\frac{7}{2}+m}}{\left(d^2-dex\right)^{7/2}}\,dx}{\left(d^2-e^2x^2\right)^{5/2}}$$

$$= \frac{2^{-\frac{5}{2}+m}(d+ex)^m\left(1+\frac{ex}{d}\right)^{\frac{5}{2}-m}\,{}_2F_1\left(-\frac{5}{2},\frac{7}{2}-m;-\frac{3}{2};\frac{d-ex}{2d}\right)}{5de\left(d^2-e^2x^2\right)^{5/2}}$$

Mathematica [A] time = 0.103493, size = 91, normalized size = 1.34

$$\frac{2^{m-\frac{5}{2}}(d+ex)^m\left(\frac{ex}{d}+1\right)^{\frac{1}{2}-m}\,{}_2F_1\left(-\frac{5}{2},\frac{7}{2}-m;-\frac{3}{2};\frac{d-ex}{2d}\right)}{5d^3e(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (2^(-5/2 + m)*(d + e*x)^m*(1 + (e*x)/d)^(1/2 - m)*Hypergeometric2F1[-5/2, 7
/2 - m, -3/2, (d - e*x)/(2*d)])/(5*d^3*e*(d - e*x)^2*Sqrt[d^2 - e^2*x^2])
```

Maple [F] time = 0.479, size = 0, normalized size = 0.

$$\int (ex+d)^m\left(-e^2x^2+d^2\right)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] int((e*x+d)^m/(-e^2*x^2+d^2)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-e^2x^2+d^2}(ex+d)^m}{e^8x^8-4\,d^2e^6x^6+6\,d^4e^4x^4-4\,d^6e^2x^2+d^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-e^2*x^2 + d^2)*(e*x + d)^m/(e^8*x^8 - 4*d^2*e^6*x^6 + 6*d^4*
e^4*x^4 - 4*d^6*e^2*x^2 + d^8), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

3.959 $\int (a+bx)^m \left(a^2-b^2x^2\right)^p \, dx$

Optimal. Leaf size=63

$$\frac{(a+bx)^m \left(a^2-b^2x^2\right)^{p+1} {}_2F_1\left(1,m+2p+2;m+p+2;\frac{a+bx}{2a}\right)}{2ab(m+p+1)}$$

```
[Out] ((a + b*x)^m*(a^2 - b^2*x^2)^(1 + p)*Hypergeometric2F1[1, 2 + m + 2*p, 2 +
m + p, (a + b*x)/(2*a)])/(2*a*b*(1 + m + p))
```

Rubi [A] time = 0.055875, antiderivative size = 85, normalized size of antiderivative = 1.35, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {680, 678, 69}

$$-\frac{2^{m+p}(a+bx)^m \left(a^2-b^2x^2\right)^{p+1} \left(\frac{bx}{a}+1\right)^{-m-p-1} {}_2F_1\left(-m-p,p+1;p+2;\frac{a-bx}{2a}\right)}{ab(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(a^2 - b^2*x^2)^p,x]
```

```
[Out] -((2^(m + p)*(a + b*x)^m*(1 + (b*x)/a)^(-1 - m - p)*(a^2 - b^2*x^2)^(1 + p)
*Hypergeometric2F1[-m - p, 1 + p, 2 + p, (a - b*x)/(2*a)])/(a*b*(1 + p)))
```

Rule 680

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^IntPart[m]*(d + e*x)^FracPart[m])/(1 + (e*x)/d)^FracPart[m], Int[(1 + (e
*x)/d)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && EqQ[c*d^2 +
a*e^2, 0] && !IntegerQ[p] && !(IntegerQ[m] || GtQ[d, 0])
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) && !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (a+bx)^m \left(a^2-b^2x^2\right)^p dx = \left((a+bx)^m \left(1+\frac{bx}{a}\right)^{-m}\right) \int \left(1+\frac{bx}{a}\right)^m \left(a^2-b^2x^2\right)^p dx$$
$$= \left((a+bx)^m \left(1+\frac{bx}{a}\right)^{-1-m-p} \left(a^2-abx\right)^{-1-p} \left(a^2-b^2x^2\right)^{1+p}\right) \int \left(1+\frac{bx}{a}\right)^{m+p} \left(a^2-abx\right)^p dx$$
$$= -\frac{2^{m+p}(a+bx)^m \left(1+\frac{bx}{a}\right)^{-1-m-p} \left(a^2-b^2x^2\right)^{1+p} {}_2F_1\left(-m-p,1+p;2+p;\frac{a-bx}{2a}\right)}{ab(1+p)}$$

Mathematica [A] time = 0.0899089, size = 85, normalized size = 1.35

$$\frac{2^{m+p}(bx-a)(a+bx)^m \left(a^2-b^2x^2\right)^p \left(\frac{bx}{a}+1\right)^{-m-p} {}_2F_1\left(-m-p,p+1;p+2;\frac{a-bx}{2a}\right)}{b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(a^2 - b^2*x^2)^p,x]
```

```
[Out] (2^(m + p)*(-a + b*x)*(a + b*x)^m*(1 + (b*x)/a)^(-m - p)*(a^2 - b^2*x^2)^p*
Hypergeometric2F1[-m - p, 1 + p, 2 + p, (a - b*x)/(2*a)])/(b*(1 + p))
```

Maple [F] time = 0.625, size = 0, normalized size = 0.

$$\int (bx+a)^m \left(-b^2x^2+a^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(-b^2*x^2+a^2)^p,x)
```

```
[Out] int((b*x+a)^m*(-b^2*x^2+a^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-b^2x^2+a^2\right)^p (bx+a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(-b^2*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((-b^2*x^2 + a^2)^p*(b*x + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-b^2x^2+a^2\right)^p (bx+a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(-b^2*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b^2*x^2 + a^2)^p*(b*x + a)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\left(-a+bx\right)\left(a+bx\right)\right)^{p}\left(a+bx\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(-b**2*x**2+a**2)**p,x)
```

```
[Out] Integral((-(-a + b*x)*(a + b*x))**p*(a + b*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-b^{2}x^{2}+a^{2}\right)^{p}\left(bx+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(-b^2*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((-b^2*x^2 + a^2)^p*(b*x + a)^m, x)
```

3.960 $\int (d+ex)^3 \left(1-\frac{e^2x^2}{d^2}\right)^p \, dx$

Optimal. Leaf size=57

$$-\frac{d^4 2^{p+3} \left(\frac{d-ex}{d}\right)^{p+1} {}_2F_1\left(-p-3,p+1;p+2;\frac{d-ex}{2d}\right)}{e(p+1)}$$

```
[Out] -((2^(3 + p)*d^4*((d - e*x)/d)^(1 + p)*Hypergeometric2F1[-3 - p, 1 + p, 2 +
 p, (d - e*x)/(2*d)])/(e*(1 + p)))
```

Rubi [A] time = 0.0386394, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {676, 69}

$$-\frac{d^4 2^{p+3} \left(\frac{d-ex}{d}\right)^{p+1} {}_2F_1\left(-p-3,p+1;p+2;\frac{d-ex}{2d}\right)}{e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(1 - (e^2*x^2)/d^2)^p,x]
```

```
[Out] -((2^(3 + p)*d^4*((d - e*x)/d)^(1 + p)*Hypergeometric2F1[-3 - p, 1 + p, 2 +
 p, (d - e*x)/(2*d)])/(e*(1 + p)))
```

Rule 676

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(a^(p + 1)*d^(m - 1)*((d - e*x)/d)^(p + 1))/(a/d + (c*x)/e)^(p + 1), Int[(1
 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d, e, m}, x]
&& EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || GtQ[d, 0]) &&
GtQ[a, 0] &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^3 \left(1-\frac{e^2x^2}{d^2}\right)^p \, dx &= \left(d^2 \left(\frac{d-ex}{d}\right)^{1+p} \left(\frac{1}{d}-\frac{ex}{d^2}\right)^{-1-p}\right) \int \left(\frac{1}{d}-\frac{ex}{d^2}\right)^p \left(1+\frac{ex}{d}\right)^{3+p} \, dx \\
&= -\frac{2^{3+p} d^4 \left(\frac{d-ex}{d}\right)^{1+p} {}_2F_1\left(-3-p,1+p;2+p;\frac{d-ex}{2d}\right)}{e(1+p)}
\end{aligned}$$

Mathematica [B] time = 0.112035, size = 116, normalized size = 2.04

$$d^3 x \, {}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)+de^2x^3 \, {}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)-\frac{2d^4\left(1-\frac{e^2x^2}{d^2}\right)^{p+1}}{e(p+1)}+\frac{d^4\left(1-\frac{e^2x^2}{d^2}\right)^{p+2}}{2e(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(1 - (e^2*x^2)/d^2)^p,x]
```

```
[Out] (-2*d^4*(1 - (e^2*x^2)/d^2)^(1 + p))/(e*(1 + p)) + (d^4*(1 - (e^2*x^2)/d^2)
^(2 + p))/(2*e*(2 + p)) + d^3*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d
^2] + d*e^2*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2]
```

Maple [A] time = 0.616, size = 104, normalized size = 1.8

$$\frac{e^3x^4}{4}{}_2F_1(2,-p;\,3;\,\frac{e^2x^2}{d^2})+de^2x^3{}_2F_1(\frac{3}{2},-p;\,\frac{5}{2};\,\frac{e^2x^2}{d^2})+\frac{3\,d^2ex^2}{2}{}_2F_1(1,-p;\,2;\,\frac{e^2x^2}{d^2})+d^3x{}_2F_1(\frac{1}{2},-p;\,\frac{3}{2};\,\frac{e^2x^2}{d^2})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(1-e^2*x^2/d^2)^p,x)
```

```
[Out] 1/4*e^3*x^4*hypergeom([2,-p],[3],e^2*x^2/d^2)+d*e^2*x^3*hypergeom([3/2,-p],
[5/2],e^2*x^2/d^2)+3/2*e*d^2*x^2*hypergeom([1,-p],[2],e^2*x^2/d^2)+d^3*x*hy
pergeom([1/2,-p],[3/2],e^2*x^2/d^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3\left(-\frac{e^2x^2}{d^2}+1\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(1-e^2*x^2/d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2/d^2 + 1)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\left(-\frac{e^2x^2-d^2}{d^2}\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(1-e^2*x^2/d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(-(e^2*x^2 - d^2)/d^2)^p
, x)
```

Sympy [B] time = 6.26666, size = 479, normalized size = 8.4

$$d^3x{}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right) + 3d^2e\left(\begin{cases} \frac{x^2}{2} & \text{for } e^2 = 0 \\ -\frac{d^2\left(\begin{cases} \frac{\left(1-\frac{e^2x^2}{d^2}\right)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log\left(1 - \frac{e^2x^2}{d^2}\right) & \text{otherwise} \end{cases}\right)}{2e^2} & \text{otherwise} \end{cases}\right) + de^2x^3{}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right) + e^3\left(\begin{cases} \frac{x^4}{4} \\ - \\ - \\ - \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(1-e**2*x**2/d**2)**p,x)
```

```
[Out] d**3*x*hyper((1/2, -p), (3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2) + 3*d**2*
e*Piecewise((x**2/2, Eq(e**2, 0)), (-d**2*Piecewise(((1 - e**2*x**2/d**2)**
(p + 1)/(p + 1), Ne(p, -1)), (log(1 - e**2*x**2/d**2), True))/(2*e**2), Tru
e)) + d*e**2*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar(2*I*pi)/d**2
) + e**3*Piecewise((x**4/4, Eq(e, 0)), (-d**6*log(-d/e + x)/(-2*d**2*e**4 +
 2*e**6*x**2) - d**6*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**6/(-2*d
**2*e**4 + 2*e**6*x**2) + d**4*e**2*x**2*log(-d/e + x)/(-2*d**2*e**4 + 2*e*
*6*x**2) + d**4*e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2), Eq(p,
-2)), (-d**4*log(-d/e + x)/(2*e**4) - d**4*log(d/e + x)/(2*e**4) - d**2*x**
2/(2*e**2), Eq(p, -1)), (-d**4*(1 - e**2*x**2/d**2)**p/(2*e**4*p**2 + 6*e**
4*p + 4*e**4) - d**2*e**2*p*x**2*(1 - e**2*x**2/d**2)**p/(2*e**4*p**2 + 6*e
**4*p + 4*e**4) + e**4*p*x**4*(1 - e**2*x**2/d**2)**p/(2*e**4*p**2 + 6*e**4
*p + 4*e**4) + e**4*x**4*(1 - e**2*x**2/d**2)**p/(2*e**4*p**2 + 6*e**4*p +
4*e**4), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3\left(-\frac{e^2x^2}{d^2}+1\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(1-e^2*x^2/d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2/d^2 + 1)^p, x)
```

3.961 $\int (d+ex)^2 \left(1-\frac{e^2x^2}{d^2}\right)^p \, dx$

Optimal. Leaf size=57

$$-\frac{d^3 2^{p+2} \left(\frac{d-ex}{d}\right)^{p+1} {}_2F_1\left(-p-2,p+1;p+2;\frac{d-ex}{2d}\right)}{e(p+1)}$$

```
[Out] -((2^(2 + p)*d^3*((d - e*x)/d)^(1 + p)*Hypergeometric2F1[-2 - p, 1 + p, 2 +
 p, (d - e*x)/(2*d)])/(e*(1 + p)))
```

Rubi [A] time = 0.0345681, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {676, 69}

$$-\frac{d^3 2^{p+2} \left(\frac{d-ex}{d}\right)^{p+1} {}_2F_1\left(-p-2,p+1;p+2;\frac{d-ex}{2d}\right)}{e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(1 - (e^2*x^2)/d^2)^p,x]
```

```
[Out] -((2^(2 + p)*d^3*((d - e*x)/d)^(1 + p)*Hypergeometric2F1[-2 - p, 1 + p, 2 +
 p, (d - e*x)/(2*d)])/(e*(1 + p)))
```

Rule 676

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(a^(p + 1)*d^(m - 1)*((d - e*x)/d)^(p + 1))/(a/d + (c*x)/e)^(p + 1), Int[(1
 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d, e, m}, x]
&& EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || GtQ[d, 0]) &&
GtQ[a, 0] &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^2 \left(1-\frac{e^2x^2}{d^2}\right)^p \, dx &= \left(d\left(\frac{d-ex}{d}\right)^{1+p}\left(\frac{1}{d}-\frac{ex}{d^2}\right)^{-1-p}\right)\int \left(\frac{1}{d}-\frac{ex}{d^2}\right)^p \left(1+\frac{ex}{d}\right)^{2+p} \, dx\\
&= -\frac{2^{2+p} d^3 \left(\frac{d-ex}{d}\right)^{1+p} {}_2F_1\left(-2-p,1+p;2+p;\frac{d-ex}{2d}\right)}{e(1+p)}
\end{aligned}$$

Mathematica [A] time = 0.0656816, size = 86, normalized size = 1.51

$$d^2 x \, {}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)+\frac{1}{3}e^2x^3 \, {}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)-\frac{d^3\left(1-\frac{e^2x^2}{d^2}\right)^{p+1}}{e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(1 - (e^2*x^2)/d^2)^p,x]
```

```
[Out] -((d^3*(1 - (e^2*x^2)/d^2)^(1 + p))/(e*(1 + p))) + d^2*x*Hypergeometric2F1[
1/2, -p, 3/2, (e^2*x^2)/d^2] + (e^2*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^
2*x^2)/d^2])/3
```

Maple [A] time = 0.495, size = 75, normalized size = 1.3

$$\frac{e^2x^3}{3}{}_2\mathrm{F}_1(\frac{3}{2},-p;\ \frac{5}{2};\ \frac{e^2x^2}{d^2})+edx^2{}_2\mathrm{F}_1(1,-p;\ 2;\ \frac{e^2x^2}{d^2})+d^2x{}_2\mathrm{F}_1(\frac{1}{2},-p;\ \frac{3}{2};\ \frac{e^2x^2}{d^2})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(1-e^2*x^2/d^2)^p,x)
```

```
[Out] 1/3*e^2*x^3*hypergeom([3/2,-p],[5/2],e^2*x^2/d^2)+e*d*x^2*hypergeom([1,-p],
[2],e^2*x^2/d^2)+d^2*x*hypergeom([1/2,-p],[3/2],e^2*x^2/d^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^2\left(-\frac{e^2x^2}{d^2}+1\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(1-e^2*x^2/d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2/d^2 + 1)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(e^2x^2+2\,dex+d^2\right)\left(-\frac{e^2x^2-d^2}{d^2}\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(1-e^2*x^2/d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(-(e^2*x^2 - d^2)/d^2)^p, x)
```

Sympy [C] time = 3.89185, size = 116, normalized size = 2.04

$$d^2x{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)+2de\left(\begin{cases}\frac{x^2}{2} & \text{for } e^2=0\\ -\frac{d^2\left(\begin{cases}\frac{\left(1-\frac{e^2x^2}{d^2}\right)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(1-\frac{e^2x^2}{d^2}\right) & \text{otherwise}\end{cases}\right)}{2e^2} & \text{otherwise}\end{cases}\right)+\frac{e^2x^3{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(1-e**2*x**2/d**2)**p,x)
```

```
[Out] d**2*x*hyper((1/2, -p), (3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2) + 2*d*e*P
iecewise((x**2/2, Eq(e**2, 0)), (-d**2*Piecewise(((1 - e**2*x**2/d**2)**(p
+ 1)/(p + 1), Ne(p, -1)), (log(1 - e**2*x**2/d**2), True))/(2*e**2), True))
 + e**2*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^2\left(-\frac{e^2x^2}{d^2}+1\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(1-e^2*x^2/d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2/d^2 + 1)^p, x)
```

3.962 $\int (d+ex)\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx$

Optimal. Leaf size=57

$$-\frac{d^2 2^{p+1}\left(\frac{d-ex}{d}\right)^{p+1} {}_2F_1\left(-p-1,p+1;p+2;\frac{d-ex}{2d}\right)}{e(p+1)}$$

```
[Out] -((2^(1 + p)*d^2*((d - e*x)/d)^(1 + p)*Hypergeometric2F1[-1 - p, 1 + p, 2 +
 p, (d - e*x)/(2*d)])/(e*(1 + p)))
```

Rubi [A] time = 0.0156348, antiderivative size = 56, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {641, 245}

$$dx\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{d^2\left(1-\frac{e^2x^2}{d^2}\right)^{p+1}}{2e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(1 - (e^2*x^2)/d^2)^p,x]
```

```
[Out] -(d^2*(1 - (e^2*x^2)/d^2)^(1 + p))/(2*e*(1 + p)) + d*x*Hypergeometric2F1[1/
2, -p, 3/2, (e^2*x^2)/d^2]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx &= -\frac{d^2\left(1-\frac{e^2x^2}{d^2}\right)^{1+p}}{2e(1+p)}+d\int\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx\\ &= -\frac{d^2\left(1-\frac{e^2x^2}{d^2}\right)^{1+p}}{2e(1+p)}+dx\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0282315, size = 56, normalized size = 0.98

$$dx\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{d^2\left(1-\frac{e^2x^2}{d^2}\right)^{p+1}}{2e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(1 - (e^2*x^2)/d^2)^p,x]
```

```
[Out] -(d^2*(1 - (e^2*x^2)/d^2)^(1 + p))/(2*e*(1 + p)) + d*x*Hypergeometric2F1[1/
2, -p, 3/2, (e^2*x^2)/d^2]
```

Maple [A] time = 0.354, size = 47, normalized size = 0.8

$$\frac{ex^2}{2}{}_2\mathrm{F}_1(1,-p;\,2;\,\frac{e^2x^2}{d^2})+dx{}_2\mathrm{F}_1(\frac{1}{2},-p;\,\frac{3}{2};\,\frac{e^2x^2}{d^2})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(1-e^2*x^2/d^2)^p,x)
```

```
[Out] 1/2*e*x^2*hypergeom([1,-p],[2],e^2*x^2/d^2)+d*x*hypergeom([1/2,-p],[3/2],e^
2*x^2/d^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)\left(-\frac{e^2x^2}{d^2}+1\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(1-e^2*x^2/d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2/d^2 + 1)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((ex+d)\left(-\frac{e^2x^2-d^2}{d^2}\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(1-e^2*x^2/d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)*(-(e^2*x^2 - d^2)/d^2)^p, x)
```

Sympy [A] time = 3.25252, size = 78, normalized size = 1.37

$$dx{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)+e\left(\begin{cases}\frac{x^2}{2} & \text{for } e^2=0\\ -\frac{d^2\left(\begin{cases}\frac{\left(1-\frac{e^2x^2}{d^2}\right)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(1-\frac{e^2x^2}{d^2}\right) & \text{otherwise}\end{cases}\right)}{2e^2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(1-e**2*x**2/d**2)**p,x)
```

```
[Out] d*x*hyper((1/2, -p), (3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2) + e*Piecewis
e((x**2/2, Eq(e**2, 0)), (-d**2*Piecewise(((1 - e**2*x**2/d**2)**(p + 1)/(p
 + 1), Ne(p, -1)), (log(1 - e**2*x**2/d**2), True))/(2*e**2), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)\left(-\frac{e^2x^2}{d^2}+1\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(1-e^2*x^2/d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2/d^2 + 1)^p, x)
```

3.963 $\int \left(1-\frac{e^2x^2}{d^2}\right)^p dx$

Optimal. Leaf size=22

$$x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)$$

```
[Out] x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2]
```

Rubi [A] time = 0.0049424, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {245}

$$x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - (e^2*x^2)/d^2)^p,x]
```

```
[Out] x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \left(1-\frac{e^2x^2}{d^2}\right)^p dx = x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)$$

Mathematica [A] time = 0.0020593, size = 22, normalized size = 1.

$$x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - (e^2*x^2)/d^2)^p,x]
```

```
[Out] x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2]
```

Maple [A] time = 0.343, size = 21, normalized size = 1.

$$x{}_2\mathrm{F}_1(\frac{1}{2},-p;\,\frac{3}{2};\,\frac{e^2x^2}{d^2})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-e^2*x^2/d^2)^p,x)
```

```
[Out] x*hypergeom([1/2,-p],[3/2],e^2*x^2/d^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\frac{e^2x^2}{d^2}+1\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-e^2*x^2/d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2/d^2 + 1)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-\frac{e^2x^2-d^2}{d^2}\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-e^2*x^2/d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-(e^2*x^2 - d^2)/d^2)^p, x)
```

Sympy [C] time = 1.0558, size = 24, normalized size = 1.09

$$x_2F_1\left(\begin{matrix}\frac{1}{2}, -p\\ \frac{3}{2}\end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-e**2*x**2/d**2)**p,x)
```

```
[Out] x*hyper((1/2, -p), (3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\frac{e^2x^2}{d^2}+1\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-e^2*x^2/d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2/d^2 + 1)^p, x)
```

3.964 $\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^p}{d+ex} \, dx$

Optimal. Leaf size=41

$$\frac{2^p \left(\frac{d+ex}{d}\right)^p {}_2F_1\left(-p,p;p+1;\frac{d+ex}{2d}\right)}{ep}$$

```
[Out] (2^p*((d + e*x)/d)^p*Hypergeometric2F1[-p, p, 1 + p, (d + e*x)/(2*d)])/(e*p
)
```

Rubi [A] time = 0.03575, antiderivative size = 54, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {676, 69}

$$-\frac{2^{p-1} \left(\frac{d-ex}{d}\right)^{p+1} {}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)}{e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - (e^2*x^2)/d^2)^p/(d + e*x),x]
```

```
[Out] -((2^(-1 + p)*((d - e*x)/d)^(1 + p)*Hypergeometric2F1[1 - p, 1 + p, 2 + p,
(d - e*x)/(2*d)])/(e*(1 + p)))
```

Rule 676

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(a^(p + 1)*d^(m - 1)*((d - e*x)/d)^(p + 1))/(a/d + (c*x)/e)^(p + 1), Int[(1
 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d, e, m}, x]
&& EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || GtQ[d, 0]) &&
GtQ[a, 0] &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^p}{d+ex} \, dx &= \frac{\left(\left(\frac{d-ex}{d}\right)^{1+p} \left(\frac{1}{d}-\frac{ex}{d^2}\right)^{-1-p}\right) \int \left(\frac{1}{d}-\frac{ex}{d^2}\right)^p \left(1+\frac{ex}{d}\right)^{-1+p} \, dx}{d^2}\\
&= -\frac{2^{-1+p} \left(\frac{d-ex}{d}\right)^{1+p} {}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)}{e(1+p)}
\end{aligned}$$

Mathematica [A] time = 0.0859912, size = 76, normalized size = 1.85

$$-\frac{2^{p-1}(d-ex)\left(\frac{ex}{d}+1\right)^{-p}\left(1-\frac{e^2x^2}{d^2}\right)^p {}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)}{de(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - (e^2*x^2)/d^2)^p/(d + e*x),x]
```

```
[Out] -((2^(-1 + p)*(d - e*x)*(1 - (e^2*x^2)/d^2)^p*Hypergeometric2F1[1 - p, 1 +
p, 2 + p, (d - e*x)/(2*d)])/(d*e*(1 + p)*(1 + (e*x)/d)^p))
```

Maple [F] time = 0.529, size = 0, normalized size = 0.

$$\int \frac{1}{ex+d}\left(1-\frac{e^2x^2}{d^2}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-e^2*x^2/d^2)^p/(e*x+d),x)
```

```
[Out] int((1-e^2*x^2/d^2)^p/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\frac{e^2x^2}{d^2}+1\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-e^2*x^2/d^2)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2/d^2 + 1)^p/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-\frac{e^2x^2-d^2}{d^2}\right)^p}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-e^2*x^2/d^2)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((-(e^2*x^2 - d^2)/d^2)^p/(e*x + d), x)
```

Sympy [C] time = 5.18054, size = 323, normalized size = 7.88

$$\begin{cases} \frac{0^p \log\left(-1+\frac{e^2x^2}{d^2}\right)}{2e} + \frac{0^p \operatorname{acoth}\left(\frac{ex}{d}\right)}{e} + \frac{dd^{-2p}e^{2p}px^{2p}e^{i\pi p}\Gamma(p)\Gamma\left(\frac{1}{2}-p\right){}_2F_1\left(\begin{matrix} 1-p, \frac{1}{2}-p \\ \frac{3}{2}-p \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{2e^2x\Gamma\left(\frac{3}{2}-p\right)\Gamma(p+1)} + \frac{ex^2\Gamma(p)\Gamma(1-p){}_3F_2\left(\begin{matrix} 2,1,1-p \\ 2,2 \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2d^2\Gamma(-p)\Gamma(p+1)} & \text{for } \frac{|e^2 \\ \frac{0^p \log\left(1-\frac{e^2x^2}{d^2}\right)}{2e} + \frac{0^p \operatorname{atanh}\left(\frac{ex}{d}\right)}{e} + \frac{dd^{-2p}e^{2p}px^{2p}e^{i\pi p}\Gamma(p)\Gamma\left(\frac{1}{2}-p\right){}_2F_1\left(\begin{matrix} 1-p, \frac{1}{2}-p \\ \frac{3}{2}-p \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{2e^2x\Gamma\left(\frac{3}{2}-p\right)\Gamma(p+1)} + \frac{ex^2\Gamma(p)\Gamma(1-p){}_3F_2\left(\begin{matrix} 2,1,1-p \\ 2,2 \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2d^2\Gamma(-p)\Gamma(p+1)} & \text{other} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-e**2*x**2/d**2)**p/(e*x+d),x)
```

```
[Out] Piecewise((0**p*log(-1 + e**2*x**2/d**2)/(2*e) + 0**p*acoth(e*x/d)/e + d*d*
*(-2*p)*e**(2*p)*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((1 -
p, 1/2 - p), (3/2 - p,), d**2/(e**2*x**2))/(2*e**2*x*gamma(3/2 - p)*gamma(p
 + 1)) + e*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), e**2*x**
2*exp_polar(2*I*pi)/d**2)/(2*d**2*gamma(-p)*gamma(p + 1)), Abs(e**2*x**2)/A
bs(d**2) > 1), (0**p*log(1 - e**2*x**2/d**2)/(2*e) + 0**p*atanh(e*x/d)/e +
d*d**(-2*p)*e**(2*p)*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((
1 - p, 1/2 - p), (3/2 - p,), d**2/(e**2*x**2))/(2*e**2*x*gamma(3/2 - p)*gam
ma(p + 1)) + e*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), e**2
*x**2*exp_polar(2*I*pi)/d**2)/(2*d**2*gamma(-p)*gamma(p + 1)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\frac{e^2x^2}{d^2}+1\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-e^2*x^2/d^2)^p/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2/d^2 + 1)^p/(e*x + d), x)
```

3.965 $\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=57

$$-\frac{2^{p-2}\left(\frac{d-ex}{d}\right)^{p+1}\,{}_2F_1\left(2-p,p+1;p+2;\frac{d-ex}{2d}\right)}{de(p+1)}$$

```
[Out] -((2^(-2 + p)*((d - e*x)/d)^(1 + p)*Hypergeometric2F1[2 - p, 1 + p, 2 + p,
(d - e*x)/(2*d)])/(d*e*(1 + p)))
```

Rubi [A] time = 0.0396949, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {676, 69}

$$-\frac{2^{p-2}\left(\frac{d-ex}{d}\right)^{p+1}\,{}_2F_1\left(2-p,p+1;p+2;\frac{d-ex}{2d}\right)}{de(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - (e^2*x^2)/d^2)^p/(d + e*x)^2,x]
```

```
[Out] -((2^(-2 + p)*((d - e*x)/d)^(1 + p)*Hypergeometric2F1[2 - p, 1 + p, 2 + p,
(d - e*x)/(2*d)])/(d*e*(1 + p)))
```

Rule 676

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(a^(p + 1)*d^(m - 1)*((d - e*x)/d)^(p + 1))/(a/d + (c*x)/e)^(p + 1), Int[(1
 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d, e, m}, x]
&& EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || GtQ[d, 0]) &&
GtQ[a, 0] &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^p}{(d+ex)^2}\,dx &= \frac{\left(\left(\frac{d-ex}{d}\right)^{1+p}\left(\frac{1}{d}-\frac{ex}{d^2}\right)^{-1-p}\right)\int \left(\frac{1}{d}-\frac{ex}{d^2}\right)^p\left(1+\frac{ex}{d}\right)^{-2+p}\,dx}{d^3}\\ &= -\frac{2^{-2+p}\left(\frac{d-ex}{d}\right)^{1+p}\,{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)}{de(1+p)}\end{aligned}$$

Mathematica [A] time = 0.0784986, size = 76, normalized size = 1.33

$$-\frac{2^{p-2}(d-ex)\left(\frac{ex}{d}+1\right)^{-p}\left(1-\frac{e^2x^2}{d^2}\right)^p {}_2F_1\left(2-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^2e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - (e^2*x^2)/d^2)^p/(d + e*x)^2,x]
```

```
[Out] -((2^(-2 + p)*(d - e*x)*(1 - (e^2*x^2)/d^2)^p*Hypergeometric2F1[2 - p, 1 +
p, 2 + p, (d - e*x)/(2*d)])/(d^2*e*(1 + p)*(1 + (e*x)/d)^p))
```

Maple [F] time = 0.546, size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)^2}\left(1-\frac{e^2x^2}{d^2}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-e^2*x^2/d^2)^p/(e*x+d)^2,x)
```

```
[Out] int((1-e^2*x^2/d^2)^p/(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\frac{e^2x^2}{d^2}+1\right)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-e^2*x^2/d^2)^p/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2/d^2 + 1)^p/(e*x + d)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-\frac{e^2x^2-d^2}{d^2}\right)^p}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-e^2*x^2/d^2)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((-(e^2*x^2 - d^2)/d^2)^p/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(-1+\frac{ex}{d}\right)\left(1+\frac{ex}{d}\right)\right)^p}{(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-e**2*x**2/d**2)**p/(e*x+d)**2,x)
```

```
[Out] Integral((-(-1 + e*x/d)*(1 + e*x/d))**p/(d + e*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\frac{e^2x^2}{d^2}+1\right)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-e^2*x^2/d^2)^p/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2/d^2 + 1)^p/(e*x + d)^2, x)
```

3.966 $\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^p}{(d+ex)^3}\,dx$

Optimal. Leaf size=57

$$-\frac{2^{p-3}\left(\frac{d-ex}{d}\right)^{p+1}{}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^2e(p+1)}$$

```
[Out] -((2^(-3 + p)*((d - e*x)/d)^(1 + p)*Hypergeometric2F1[3 - p, 1 + p, 2 + p,
(d - e*x)/(2*d)])/(d^2*e*(1 + p)))
```

Rubi [A] time = 0.0365362, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {676, 69}

$$-\frac{2^{p-3}\left(\frac{d-ex}{d}\right)^{p+1}{}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^2e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - (e^2*x^2)/d^2)^p/(d + e*x)^3,x]
```

```
[Out] -((2^(-3 + p)*((d - e*x)/d)^(1 + p)*Hypergeometric2F1[3 - p, 1 + p, 2 + p,
(d - e*x)/(2*d)])/(d^2*e*(1 + p)))
```

Rule 676

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(a^(p + 1)*d^(m - 1)*((d - e*x)/d)^(p + 1))/(a/d + (c*x)/e)^(p + 1), Int[(1
 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d, e, m}, x]
&& EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || GtQ[d, 0]) &&
GtQ[a, 0] &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^p}{(d+ex)^3}\,dx &= \frac{\left(\left(\frac{d-ex}{d}\right)^{1+p}\left(\frac{1}{d}-\frac{ex}{d^2}\right)^{-1-p}\right)\int \left(\frac{1}{d}-\frac{ex}{d^2}\right)^p\left(1+\frac{ex}{d}\right)^{-3+p}\,dx}{d^4}\\ &= -\frac{2^{-3+p}\left(\frac{d-ex}{d}\right)^{1+p}{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^2e(1+p)}\end{aligned}$$

Mathematica [A] time = 0.0768739, size = 76, normalized size = 1.33

$$-\frac{2^{p-3}(d-ex)\left(\frac{ex}{d}+1\right)^{-p}\left(1-\frac{e^2x^2}{d^2}\right)^p {}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^3e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - (e^2*x^2)/d^2)^p/(d + e*x)^3,x]
```

```
[Out] -((2^(-3 + p)*(d - e*x)*(1 - (e^2*x^2)/d^2)^p*Hypergeometric2F1[3 - p, 1 +
p, 2 + p, (d - e*x)/(2*d)])/(d^3*e*(1 + p)*(1 + (e*x)/d)^p))
```

Maple [F] time = 0.61, size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)^3}\left(1-\frac{e^2x^2}{d^2}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-e^2*x^2/d^2)^p/(e*x+d)^3,x)
```

```
[Out] int((1-e^2*x^2/d^2)^p/(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\frac{e^2x^2}{d^2}+1\right)^p}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-e^2*x^2/d^2)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2/d^2 + 1)^p/(e*x + d)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-\frac{e^2x^2-d^2}{d^2}\right)^p}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-e^2*x^2/d^2)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((-(e^2*x^2 - d^2)/d^2)^p/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(-1+\frac{ex}{d}\right)\left(1+\frac{ex}{d}\right)\right)^{p}}{\left(d+ex\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-e**2*x**2/d**2)**p/(e*x+d)**3,x)
```

```
[Out] Integral((-(-1 + e*x/d)*(1 + e*x/d))**p/(d + e*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\frac{e^{2}x^{2}}{d^{2}}+1\right)^{p}}{\left(ex+d\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-e^2*x^2/d^2)^p/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2/d^2 + 1)^p/(e*x + d)^3, x)
```

3.967 $\int (a+bx)^3 \left(a^2-b^2x^2\right)^p \, dx$

Optimal. Leaf size=60

$$\frac{(a+bx)^3 \left(a^2-b^2x^2\right)^{p+1} {}_2F_1\left(1,2p+5;p+5;\frac{a+bx}{2a}\right)}{2ab(p+4)}$$

```
[Out] ((a + b*x)^3*(a^2 - b^2*x^2)^(1 + p)*Hypergeometric2F1[1, 5 + 2*p, 5 + p, (
a + b*x)/(2*a)])/(2*a*b*(4 + p))
```

Rubi [A] time = 0.0305929, antiderivative size = 73, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {678, 69}

$$-\frac{a^2 2^{p+3} \left(\frac{bx}{a}+1\right)^{-p-1} \left(a^2-b^2x^2\right)^{p+1} {}_2F_1\left(-p-3,p+1;p+2;\frac{a-bx}{2a}\right)}{b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(a^2 - b^2*x^2)^p,x]
```

```
[Out] -((2^(3 + p)*a^2*(1 + (b*x)/a)^(-1 - p)*(a^2 - b^2*x^2)^(1 + p)*Hypergeomet
ric2F1[-3 - p, 1 + p, 2 + p, (a - b*x)/(2*a)])/(b*(1 + p)))
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3 \left(a^2-b^2x^2\right)^p \, dx &= \left(a^2(a-bx)^{-1-p} \left(1+\frac{bx}{a}\right)^{-1-p} \left(a^2-b^2x^2\right)^{1+p}\right) \int (a-bx)^p \left(1+\frac{bx}{a}\right)^{3+p} \, dx\\ &= -\frac{2^{3+p} a^2 \left(1+\frac{bx}{a}\right)^{-1-p} \left(a^2-b^2x^2\right)^{1+p} {}_2F_1\left(-3-p,1+p;2+p;\frac{a-bx}{2a}\right)}{b(1+p)}\end{aligned}$$

Mathematica [B] time = 0.178725, size = 155, normalized size = 2.58

$$\frac{1}{2} \left(a^2-b^2x^2\right)^p \left(2a^3x \left(1-\frac{b^2x^2}{a^2}\right)^{-p} {}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{b^2x^2}{a^2}\right)+2ab^2x^3 \left(1-\frac{b^2x^2}{a^2}\right)^{-p} {}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{b^2x^2}{a^2}\right)+\frac{\left(b^2x^2-a^2\right) \left(a^2(3p\right.}{b(p+1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(a^2 - b^2*x^2)^p,x]
```

```
[Out] ((a^2 - b^2*x^2)^p*(((-a^2 + b^2*x^2)*(a^2*(7 + 3*p) + b^2*(1 + p)*x^2))/(b
*(1 + p)*(2 + p)) + (2*a^3*x*Hypergeometric2F1[1/2, -p, 3/2, (b^2*x^2)/a^2]
)/(1 - (b^2*x^2)/a^2)^p + (2*a*b^2*x^3*Hypergeometric2F1[3/2, -p, 5/2, (b^2
*x^2)/a^2])/(1 - (b^2*x^2)/a^2)^p))/2
```

Maple [F] time = 0.51, size = 0, normalized size = 0.

$$\int (bx+a)^3 \left(-b^2x^2+a^2\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(-b^2*x^2+a^2)^p,x)
```

```
[Out] int((b*x+a)^3*(-b^2*x^2+a^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^3 \left(-b^2x^2+a^2\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(-b^2*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^3*(-b^2*x^2 + a^2)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)\left(-b^2x^2+a^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(-b^2*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*(-b^2*x^2 + a^2)^p, x)
```

Sympy [B] time = 5.8479, size = 476, normalized size = 7.93

$$a^3a^{2p}x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{b^2x^2e^{2i\pi}}{a^2}\right)+3a^2b\left(\begin{cases}\frac{x^2\left(a^2\right)^p}{2} & \text{for } b^2=0\\ -\frac{\begin{cases}\frac{\left(a^2-b^2x^2\right)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(a^2-b^2x^2\right) & \text{otherwise}\end{cases}}{2b^2} & \text{otherwise}\end{cases}\right)+aa^{2p}b^2x^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{b^2x^2e^{2i\pi}}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(-b**2*x**2+a**2)**p,x)
```

```
[Out] a**3*a**(2*p)*x*hyper((1/2, -p), (3/2,), b**2*x**2*exp_polar(2*I*pi)/a**2)
+ 3*a**2*b*Piecewise((x**2*(a**2)**p/2, Eq(b**2, 0)), (-Piecewise(((a**2 -
b**2*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(a**2 - b**2*x**2), True))/(2*
b**2), True)) + a*a**(2*p)*b**2*x**3*hyper((3/2, -p), (5/2,), b**2*x**2*exp
_polar(2*I*pi)/a**2) + b**3*Piecewise((x**4*(a**2)**p/4, Eq(b, 0)), (-a**2*
log(-a/b + x)/(-2*a**2*b**4 + 2*b**6*x**2) - a**2*log(a/b + x)/(-2*a**2*b**
4 + 2*b**6*x**2) - a**2/(-2*a**2*b**4 + 2*b**6*x**2) + b**2*x**2*log(-a/b +
 x)/(-2*a**2*b**4 + 2*b**6*x**2) + b**2*x**2*log(a/b + x)/(-2*a**2*b**4 + 2
*b**6*x**2), Eq(p, -2)), (-a**2*log(-a/b + x)/(2*b**4) - a**2*log(a/b + x)/
(2*b**4) - x**2/(2*b**2), Eq(p, -1)), (-a**4*(a**2 - b**2*x**2)**p/(2*b**4*
p**2 + 6*b**4*p + 4*b**4) - a**2*b**2*p*x**2*(a**2 - b**2*x**2)**p/(2*b**4*
p**2 + 6*b**4*p + 4*b**4) + b**4*p*x**4*(a**2 - b**2*x**2)**p/(2*b**4*p**2
+ 6*b**4*p + 4*b**4) + b**4*x**4*(a**2 - b**2*x**2)**p/(2*b**4*p**2 + 6*b**
4*p + 4*b**4), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^3\left(-b^2x^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(-b^2*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^3*(-b^2*x^2 + a^2)^p, x)
```

3.968 $\int (a+bx)^2 \left(a^2-b^2x^2\right)^p \, dx$

Optimal. Leaf size=60

$$\frac{(a+bx)^2 \left(a^2-b^2x^2\right)^{p+1} {}_2F_1\left(1,2(p+2);p+4;\frac{a+bx}{2a}\right)}{2ab(p+3)}$$

```
[Out] ((a + b*x)^2*(a^2 - b^2*x^2)^(1 + p)*Hypergeometric2F1[1, 2*(2 + p), 4 + p,
 (a + b*x)/(2*a)])/(2*a*b*(3 + p))
```

Rubi [A] time = 0.0289164, antiderivative size = 71, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {678, 69}

$$-\frac{a2^{p+2}\left(\frac{bx}{a}+1\right)^{-p-1}\left(a^2-b^2x^2\right)^{p+1} {}_2F_1\left(-p-2,p+1;p+2;\frac{a-bx}{2a}\right)}{b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(a^2 - b^2*x^2)^p,x]
```

```
[Out] -((2^(2 + p)*a*(1 + (b*x)/a)^(-1 - p)*(a^2 - b^2*x^2)^(1 + p)*Hypergeometri
c2F1[-2 - p, 1 + p, 2 + p, (a - b*x)/(2*a)])/(b*(1 + p)))
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2 \left(a^2-b^2x^2\right)^p \, dx &= \left(a(a-bx)^{-1-p}\left(1+\frac{bx}{a}\right)^{-1-p}\left(a^2-b^2x^2\right)^{1+p}\right)\int (a-bx)^p\left(1+\frac{bx}{a}\right)^{2+p} dx\\ &= -\frac{2^{2+p}a\left(1+\frac{bx}{a}\right)^{-1-p}\left(a^2-b^2x^2\right)^{1+p} {}_2F_1\left(-2-p,1+p;2+p;\frac{a-bx}{2a}\right)}{b(1+p)}\end{aligned}$$

Mathematica [B] time = 0.0607434, size = 134, normalized size = 2.23

$$\frac{\left(a^2-b^2x^2\right)^p\left(1-\frac{b^2x^2}{a^2}\right)^{-p}\left(3a^2b(p+1)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{b^2x^2}{a^2}\right)+b^3(p+1)x^3\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{b^2x^2}{a^2}\right)-3a\left(a^2-b^2x^2\right)\left(1-\frac{b^2x^2}{a^2}\right.\right.}{3b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(a^2 - b^2*x^2)^p,x]
```

```
[Out] ((a^2 - b^2*x^2)^p*(-3*a*(a^2 - b^2*x^2)*(1 - (b^2*x^2)/a^2)^p + 3*a^2*b*(1
 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, (b^2*x^2)/a^2] + b^3*(1 + p)*x^3*Hy
pergeometric2F1[3/2, -p, 5/2, (b^2*x^2)/a^2]))/(3*b*(1 + p)*(1 - (b^2*x^2)/
a^2)^p)
```

Maple [F] time = 0.488, size = 0, normalized size = 0.

$$\int (bx+a)^2 \left(-b^2x^2+a^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(-b^2*x^2+a^2)^p,x)
```

```
[Out] int((b*x+a)^2*(-b^2*x^2+a^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^2 \left(-b^2x^2+a^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b^2*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^2*(-b^2*x^2 + a^2)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2x^2+2\,abx+a^2\right)\left(-b^2x^2+a^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b^2*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*(-b^2*x^2 + a^2)^p, x)
```

Sympy [A] time = 3.71884, size = 124, normalized size = 2.07

$$a^2a^{2p}x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{b^2x^2e^{2i\pi}}{a^2}\right)+2ab\left(\begin{cases}\frac{x^2\left(a^2\right)^p}{2} & \text{for}\ b^2=0\\ -\frac{\begin{cases}\frac{\left(a^2-b^2x^2\right)^{p+1}}{p+1} & \text{for}\ p\neq -1\\ \log\left(a^2-b^2x^2\right) & \text{otherwise}\end{cases}}{2b^2} & \text{otherwise}\end{cases}\right)+\frac{a^{2p}b^2x^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{b^2x^2e^{2i\pi}}{a^2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(-b**2*x**2+a**2)**p,x)
```

```
[Out] a**2*a**(2*p)*x*hyper((1/2, -p), (3/2,), b**2*x**2*exp_polar(2*I*pi)/a**2)
+ 2*a*b*Piecewise((x**2*(a**2)**p/2, Eq(b**2, 0)), (-Piecewise(((a**2 - b**
2*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(a**2 - b**2*x**2), True))/(2*b**
2), True)) + a**(2*p)*b**2*x**3*hyper((3/2, -p), (5/2,), b**2*x**2*exp_pola
r(2*I*pi)/a**2)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^2 \left(-b^2x^2+a^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b^2*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^2*(-b^2*x^2 + a^2)^p, x)
```

3.969 $\int (a+bx)\left(a^2-b^2x^2\right)^p\,dx$

Optimal. Leaf size=83

$$ax\left(a^2-b^2x^2\right)^p\left(1-\frac{b^2x^2}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{b^2x^2}{a^2}\right)-\frac{\left(a^2-b^2x^2\right)^{p+1}}{2b(p+1)}$$

```
[Out] -(a^2 - b^2*x^2)^(1 + p)/(2*b*(1 + p)) + (a*x*(a^2 - b^2*x^2)^p*Hypergeomet
ric2F1[1/2, -p, 3/2, (b^2*x^2)/a^2])/(1 - (b^2*x^2)/a^2)^p
```

Rubi [A] time = 0.022037, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {641, 246, 245}

$$ax\left(a^2-b^2x^2\right)^p\left(1-\frac{b^2x^2}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{b^2x^2}{a^2}\right)-\frac{\left(a^2-b^2x^2\right)^{p+1}}{2b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(a^2 - b^2*x^2)^p,x]
```

```
[Out] -(a^2 - b^2*x^2)^(1 + p)/(2*b*(1 + p)) + (a*x*(a^2 - b^2*x^2)^p*Hypergeomet
ric2F1[1/2, -p, 3/2, (b^2*x^2)/a^2])/(1 - (b^2*x^2)/a^2)^p
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int (a+bx)\left(a^2-b^2x^2\right)^p\,dx &= -\frac{\left(a^2-b^2x^2\right)^{1+p}}{2b(1+p)}+a\int\left(a^2-b^2x^2\right)^p\,dx\\
&= -\frac{\left(a^2-b^2x^2\right)^{1+p}}{2b(1+p)}+\left(a\left(a^2-b^2x^2\right)^p\left(1-\frac{b^2x^2}{a^2}\right)^{-p}\right)\int\left(1-\frac{b^2x^2}{a^2}\right)^p\,dx\\
&= -\frac{\left(a^2-b^2x^2\right)^{1+p}}{2b(1+p)}+ax\left(a^2-b^2x^2\right)^p\left(1-\frac{b^2x^2}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{b^2x^2}{a^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0463599, size = 83, normalized size = 1.

$$ax\left(a^2-b^2x^2\right)^p\left(1-\frac{b^2x^2}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{b^2x^2}{a^2}\right)-\frac{\left(a^2-b^2x^2\right)^{p+1}}{2b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(a^2 - b^2*x^2)^p,x]
```

```
[Out] -(a^2 - b^2*x^2)^(1 + p)/(2*b*(1 + p)) + (a*x*(a^2 - b^2*x^2)^p*Hypergeomet
ric2F1[1/2, -p, 3/2, (b^2*x^2)/a^2])/(1 - (b^2*x^2)/a^2)^p
```

Maple [F] time = 0.381, size = 0, normalized size = 0.

$$\int (bx+a)\left(-b^2x^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(-b^2*x^2+a^2)^p,x)
```

```
[Out] int((b*x+a)*(-b^2*x^2+a^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)\left(-b^2x^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b^2*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)*(-b^2*x^2 + a^2)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx+a)\left(-b^2x^2+a^2\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b^2*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)*(-b^2*x^2 + a^2)^p, x)
```

Sympy [A] time = 2.94537, size = 82, normalized size = 0.99

$$aa^{2p}x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{b^2x^2e^{2i\pi}}{a^2}\right)+b\left(\begin{cases}\frac{x^2\left(a^2\right)^p}{2} & \text{for } b^2=0\\ -\frac{\begin{cases}\frac{\left(a^2-b^2x^2\right)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(a^2-b^2x^2\right) & \text{otherwise}\end{cases}}{2b^2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b**2*x**2+a**2)**p,x)
```

```
[Out] a*a**(2*p)*x*hyper((1/2, -p), (3/2,), b**2*x**2*exp_polar(2*I*pi)/a**2) + b
*Piecewise((x**2*(a**2)**p/2, Eq(b**2, 0)), (-Piecewise(((a**2 - b**2*x**2)
**(p + 1)/(p + 1), Ne(p, -1)), (log(a**2 - b**2*x**2), True))/(2*b**2), Tru
e))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)\left(-b^2x^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b^2*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*(-b^2*x^2 + a^2)^p, x)
```

3.970 $\int \frac{(a^2-b^2x^2)^p}{a+bx} \, dx$

Optimal. Leaf size=55

$$\frac{(a-bx)\left(a^2-b^2x^2\right)^p {}_2F_1\left(1,2p+1;p+1;\frac{a+bx}{2a}\right)}{2abp}$$

```
[Out] ((a - b*x)*(a^2 - b^2*x^2)^p*Hypergeometric2F1[1, 1 + 2*p, 1 + p, (a + b*x)
/(2*a)])/(2*a*b*p)
```

Rubi [A] time = 0.0299029, antiderivative size = 73, normalized size of antiderivative = 1.33, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {678, 69}

$$-\frac{2^{p-1}\left(\frac{bx}{a}+1\right)^{-p-1}\left(a^2-b^2x^2\right)^{p+1} {}_2F_1\left(1-p,p+1;p+2;\frac{a-bx}{2a}\right)}{a^2b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*x^2)^p/(a + b*x),x]
```

```
[Out] -((2^(-1 + p)*(1 + (b*x)/a)^(-1 - p)*(a^2 - b^2*x^2)^(1 + p)*Hypergeometric
2F1[1 - p, 1 + p, 2 + p, (a - b*x)/(2*a)])/(a^2*b*(1 + p)))
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2-b^2x^2\right)^p}{a+bx}\,dx &= \frac{\left((a-bx)^{-1-p}\left(1+\frac{bx}{a}\right)^{-1-p}\left(a^2-b^2x^2\right)^{1+p}\right)\int (a-bx)^p\left(1+\frac{bx}{a}\right)^{-1+p}\,dx}{a^2}\\ &= -\frac{2^{-1+p}\left(1+\frac{bx}{a}\right)^{-1-p}\left(a^2-b^2x^2\right)^{1+p} {}_2F_1\left(1-p,1+p;2+p;\frac{a-bx}{2a}\right)}{a^2b(1+p)}\end{aligned}$$

Mathematica [A] time = 0.0516805, size = 75, normalized size = 1.36

$$-\frac{2^{p-1}(a-bx)\left(\frac{bx}{a}+1\right)^{-p}\left(a^2-b^2x^2\right)^p {}_2F_1\left(1-p,p+1;p+2;\frac{a-bx}{2a}\right)}{ab(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*x^2)^p/(a + b*x),x]
```

```
[Out] -((2^(-1 + p)*(a - b*x)*(a^2 - b^2*x^2)^p*Hypergeometric2F1[1 - p, 1 + p, 2
 + p, (a - b*x)/(2*a)])/(a*b*(1 + p)*(1 + (b*x)/a)^p))
```

Maple [F] time = 0.532, size = 0, normalized size = 0.

$$\int \frac{\left(-b^{2} x^{2}+a^{2}\right)^{p}}{b x+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^2*x^2+a^2)^p/(b*x+a),x)
```

```
[Out] int((-b^2*x^2+a^2)^p/(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-b^{2} x^{2}+a^{2}\right)^{p}}{b x+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^p/(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((-b^2*x^2 + a^2)^p/(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-b^{2} x^{2}+a^{2}\right)^{p}}{b x+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^p/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((-b^2*x^2 + a^2)^p/(b*x + a), x)
```

Sympy [C] time = 5.05242, size = 323, normalized size = 5.87

$$\begin{cases} \frac{0^{p} \log{\left(-1+\frac{b^{2} x^{2}}{a^{2}} \right)}}{2 b} + \frac{0^{p} \operatorname{acoth}{\left(\frac{b x}{a} \right)}}{b} + \frac{a b^{2 p} p x^{2 p} e^{i \pi p} \Gamma\left(p\right) \Gamma\left(\frac{1}{2}-p\right) {{}_{2}F_{1}\left(\begin{matrix} 1 - p, \frac{1}{2} - p \\ \frac{3}{2} - p \end{matrix}\middle| {\frac{a^{2}}{b^{2} x^{2}}} \right)}}{2 b^{2} x \Gamma\left(\frac{3}{2}-p\right) \Gamma\left(p+1\right)} + \frac{a^{2 p} b x^{2} \Gamma\left(p\right) \Gamma\left(1-p\right) {{}_{3}F_{2}\left(\begin{matrix} 2, 1, 1 - p \\ 2, 2 \end{matrix}\middle| {\frac{b^{2} x^{2} e^{2 i \pi}}{a^{2}}} \right)}}{2 a^{2} \Gamma\left(-p\right) \Gamma\left(p+1\right)} & \text{for}\: \frac{\left|{b^{2} x^{2}}\right|}{\left|{a^{2}}\right|} > 1 \\ \frac{0^{p} \log{\left(1-\frac{b^{2} x^{2}}{a^{2}} \right)}}{2 b} + \frac{0^{p} \operatorname{atanh}{\left(\frac{b x}{a} \right)}}{b} + \frac{a b^{2 p} p x^{2 p} e^{i \pi p} \Gamma\left(p\right) \Gamma\left(\frac{1}{2}-p\right) {{}_{2}F_{1}\left(\begin{matrix} 1 - p, \frac{1}{2} - p \\ \frac{3}{2} - p \end{matrix}\middle| {\frac{a^{2}}{b^{2} x^{2}}} \right)}}{2 b^{2} x \Gamma\left(\frac{3}{2}-p\right) \Gamma\left(p+1\right)} + \frac{a^{2 p} b x^{2} \Gamma\left(p\right) \Gamma\left(1-p\right) {{}_{3}F_{2}\left(\begin{matrix} 2, 1, 1 - p \\ 2, 2 \end{matrix}\middle| {\frac{b^{2} x^{2} e^{2 i \pi}}{a^{2}}} \right)}}{2 a^{2} \Gamma\left(-p\right) \Gamma\left(p+1\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**2*x**2+a**2)**p/(b*x+a),x)
```

```
[Out] Piecewise((0**p*log(-1 + b**2*x**2/a**2)/(2*b) + 0**p*acoth(b*x/a)/b + a*b*
*(2*p)*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((1 - p, 1/2 - p
), (3/2 - p,), a**2/(b**2*x**2))/(2*b**2*x*gamma(3/2 - p)*gamma(p + 1)) + a
**(2*p)*b*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), b**2*x**2
*exp_polar(2*I*pi)/a**2)/(2*a**2*gamma(-p)*gamma(p + 1)), Abs(b**2*x**2)/Ab
s(a**2) > 1), (0**p*log(1 - b**2*x**2/a**2)/(2*b) + 0**p*atanh(b*x/a)/b + a
*b**(2*p)*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((1 - p, 1/2
- p), (3/2 - p,), a**2/(b**2*x**2))/(2*b**2*x*gamma(3/2 - p)*gamma(p + 1))
+ a**(2*p)*b*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), b**2*x
**2*exp_polar(2*I*pi)/a**2)/(2*a**2*gamma(-p)*gamma(p + 1)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-b^2x^2+a^2\right)^p}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^p/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((-b^2*x^2 + a^2)^p/(b*x + a), x)
```

3.971 $\int \frac{(a^2-b^2x^2)^p}{(a+bx)^2}\,dx$

Optimal. Leaf size=58

$$-\frac{(a^2-b^2x^2)^{p+1}\,{}_2F_1\left(1,2p;p;\frac{a+bx}{2a}\right)}{2ab(1-p)(a+bx)^2}$$

```
[Out] -((a^2 - b^2*x^2)^(1 + p)*Hypergeometric2F1[1, 2*p, p, (a + b*x)/(2*a)])/(2
*a*b*(1 - p)*(a + b*x)^2)
```

Rubi [A] time = 0.0309562, antiderivative size = 73, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {678, 69}

$$-\frac{2^{p-2}\left(\frac{bx}{a}+1\right)^{-p-1}(a^2-b^2x^2)^{p+1}\,{}_2F_1\left(2-p,p+1;p+2;\frac{a-bx}{2a}\right)}{a^3b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*x^2)^p/(a + b*x)^2,x]
```

```
[Out] -((2^(-2 + p)*(1 + (b*x)/a)^(-1 - p)*(a^2 - b^2*x^2)^(1 + p)*Hypergeometric
2F1[2 - p, 1 + p, 2 + p, (a - b*x)/(2*a)])/(a^3*b*(1 + p)))
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2-b^2x^2)^p}{(a+bx)^2}\,dx &= \frac{\left((a-bx)^{-1-p}\left(1+\frac{bx}{a}\right)^{-1-p}(a^2-b^2x^2)^{1+p}\right)\int (a-bx)^p\left(1+\frac{bx}{a}\right)^{-2+p}\,dx}{a^3}\\ &= -\frac{2^{-2+p}\left(1+\frac{bx}{a}\right)^{-1-p}(a^2-b^2x^2)^{1+p}\,{}_2F_1\left(2-p,1+p;2+p;\frac{a-bx}{2a}\right)}{a^3b(1+p)}\end{aligned}$$

Mathematica [A] time = 0.0531989, size = 75, normalized size = 1.29

$$-\frac{2^{p-2}(a-bx)\left(\frac{bx}{a}+1\right)^{-p}\left(a^2-b^2x^2\right)^p {}_2F_1\left(2-p,p+1;p+2;\frac{a-bx}{2a}\right)}{a^2b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*x^2)^p/(a + b*x)^2,x]
```

```
[Out] -((2^(-2 + p)*(a - b*x)*(a^2 - b^2*x^2)^p*Hypergeometric2F1[2 - p, 1 + p, 2
 + p, (a - b*x)/(2*a)])/(a^2*b*(1 + p)*(1 + (b*x)/a)^p))
```

Maple [F] time = 0.541, size = 0, normalized size = 0.

$$\int \frac{\left(-b^2x^2+a^2\right)^p}{\left(bx+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^2*x^2+a^2)^p/(b*x+a)^2,x)
```

```
[Out] int((-b^2*x^2+a^2)^p/(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-b^2x^2+a^2\right)^p}{\left(bx+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^p/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((-b^2*x^2 + a^2)^p/(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-b^2x^2+a^2\right)^p}{b^2x^2+2\,abx+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^p/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((-b^2*x^2 + a^2)^p/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(-a+bx\right)\left(a+bx\right)\right)^p}{\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**2*x**2+a**2)**p/(b*x+a)**2,x)
```

```
[Out] Integral((-(-a + b*x)*(a + b*x))**p/(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-b^2x^2 + a^2\right)^p}{(bx + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^p/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((-b^2*x^2 + a^2)^p/(b*x + a)^2, x)
```

3.972 $\int \frac{(a^2-b^2x^2)^p}{(a+bx)^3}\,dx$

Optimal. Leaf size=62

$$-\frac{(a^2-b^2x^2)^{p+1}\,{}_2F_1\left(1,2p-1;p-1;\frac{a+bx}{2a}\right)}{2ab(2-p)(a+bx)^3}$$

```
[Out] -((a^2 - b^2*x^2)^(1 + p)*Hypergeometric2F1[1, -1 + 2*p, -1 + p, (a + b*x)/
(2*a)])/(2*a*b*(2 - p)*(a + b*x)^3)
```

Rubi [A] time = 0.0305619, antiderivative size = 73, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {678, 69}

$$-\frac{2^{p-3}\left(\frac{bx}{a}+1\right)^{-p-1}(a^2-b^2x^2)^{p+1}\,{}_2F_1\left(3-p,p+1;p+2;\frac{a-bx}{2a}\right)}{a^4b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*x^2)^p/(a + b*x)^3,x]
```

```
[Out] -((2^(-3 + p)*(1 + (b*x)/a)^(-1 - p)*(a^2 - b^2*x^2)^(1 + p)*Hypergeometric
2F1[3 - p, 1 + p, 2 + p, (a - b*x)/(2*a)])/(a^4*b*(1 + p)))
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2-b^2x^2)^p}{(a+bx)^3}\,dx &= \frac{\left((a-bx)^{-1-p}\left(1+\frac{bx}{a}\right)^{-1-p}(a^2-b^2x^2)^{1+p}\right)\int (a-bx)^p\left(1+\frac{bx}{a}\right)^{-3+p}\,dx}{a^4}\\ &= -\frac{2^{-3+p}\left(1+\frac{bx}{a}\right)^{-1-p}(a^2-b^2x^2)^{1+p}\,{}_2F_1\left(3-p,1+p;2+p;\frac{a-bx}{2a}\right)}{a^4b(1+p)}\end{aligned}$$

Mathematica [A] time = 0.0556562, size = 75, normalized size = 1.21

$$-\frac{2^{p-3}(a-bx)\left(\frac{bx}{a}+1\right)^{-p}\left(a^2-b^2x^2\right)^p {}_2F_1\left(3-p,p+1;p+2;\frac{a-bx}{2a}\right)}{a^3b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*x^2)^p/(a + b*x)^3,x]
```

```
[Out] -((2^(-3 + p)*(a - b*x)*(a^2 - b^2*x^2)^p*Hypergeometric2F1[3 - p, 1 + p, 2
 + p, (a - b*x)/(2*a)])/(a^3*b*(1 + p)*(1 + (b*x)/a)^p))
```

Maple [F] time = 0.564, size = 0, normalized size = 0.

$$\int \frac{\left(-b^2x^2+a^2\right)^p}{(bx+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^2*x^2+a^2)^p/(b*x+a)^3,x)
```

```
[Out] int((-b^2*x^2+a^2)^p/(b*x+a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-b^2x^2+a^2\right)^p}{(bx+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^p/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((-b^2*x^2 + a^2)^p/(b*x + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-b^2x^2+a^2\right)^p}{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^p/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral((-b^2*x^2 + a^2)^p/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(-a+bx)(a+bx))^p}{(a+bx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**2*x**2+a**2)**p/(b*x+a)**3,x)
```

```
[Out] Integral((-(-a + b*x)*(a + b*x))**p/(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-b^2x^2+a^2\right)^p}{(bx+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^p/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((-b^2*x^2 + a^2)^p/(b*x + a)^3, x)
```

3.973 $\int (a+bx)^{3/2} \left(a^2-b^2x^2\right)^p \, dx$

Optimal. Leaf size=85

$$-\frac{2^{p+\frac{3}{2}} \sqrt{a+bx} \left(\frac{bx}{a}+1\right)^{-p-\frac{3}{2}} \left(a^2-b^2x^2\right)^{p+1} {}_2F_1\left(-p-\frac{3}{2},p+1;p+2;\frac{a-bx}{2a}\right)}{b(p+1)}$$

```
[Out] -((2^(3/2 + p)*Sqrt[a + b*x]*(1 + (b*x)/a)^(-3/2 - p)*(a^2 - b^2*x^2)^(1 +
p)*Hypergeometric2F1[-3/2 - p, 1 + p, 2 + p, (a - b*x)/(2*a)])/(b*(1 + p)))
```

Rubi [A] time = 0.0638669, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {680, 678, 69}

$$-\frac{2^{p+\frac{3}{2}} \sqrt{a+bx} \left(\frac{bx}{a}+1\right)^{-p-\frac{3}{2}} \left(a^2-b^2x^2\right)^{p+1} {}_2F_1\left(-p-\frac{3}{2},p+1;p+2;\frac{a-bx}{2a}\right)}{b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*(a^2 - b^2*x^2)^p,x]
```

```
[Out] -((2^(3/2 + p)*Sqrt[a + b*x]*(1 + (b*x)/a)^(-3/2 - p)*(a^2 - b^2*x^2)^(1 +
p)*Hypergeometric2F1[-3/2 - p, 1 + p, 2 + p, (a - b*x)/(2*a)])/(b*(1 + p)))
```

Rule 680

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^IntPart[m]*(d + e*x)^FracPart[m])/(1 + (e*x)/d)^FracPart[m], Int[(1 + (e
*x)/d)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && EqQ[c*d^2 +
a*e^2, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] || GtQ[d, 0])
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (a+bx)^{3/2}\left(a^2-b^2x^2\right)^p\,dx = \frac{\left(a\sqrt{a+bx}\right)\int \left(1+\frac{bx}{a}\right)^{3/2}\left(a^2-b^2x^2\right)^p\,dx}{\sqrt{1+\frac{bx}{a}}}$$

$$= \left(a\sqrt{a+bx}\left(1+\frac{bx}{a}\right)^{-\frac{3}{2}-p}\left(a^2-abx\right)^{-1-p}\left(a^2-b^2x^2\right)^{1+p}\right)\int \left(1+\frac{bx}{a}\right)^{\frac{3}{2}+p}\left(a^2-abx\right)^p\,dx$$

$$= -\frac{2^{\frac{3}{2}+p}\sqrt{a+bx}\left(1+\frac{bx}{a}\right)^{-\frac{3}{2}-p}\left(a^2-b^2x^2\right)^{1+p}\,{}_2F_1\left(-\frac{3}{2}-p,1+p;2+p;\frac{a-bx}{2a}\right)}{b(1+p)}$$

Mathematica [C] time = 0.256433, size = 189, normalized size = 2.22

$$\frac{2^{p-1}\sqrt{a+bx}\left(1-\frac{bx}{a}\right)^{-p}\left(\frac{bx}{a}+1\right)^{-2p-\frac{1}{2}}\left(b^2(p+1)x^2(a-bx)^p(a+bx)^p\left(\frac{bx}{2a}+\frac{1}{2}\right)^p F_1\left(2;-p,-p-\frac{1}{2};3;\frac{bx}{a},-\frac{bx}{a}\right)-2\sqrt{2}a(a\right.}{b(p+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)^(3/2)*(a^2 - b^2*x^2)^p,x]
```

```
[Out] (2^(-1 + p)*Sqrt[a + b*x]*(1 + (b*x)/a)^(-1/2 - 2*p)*(b^2*(1 + p)*x^2*(a -
b*x)^p*(a + b*x)^p*(1/2 + (b*x)/(2*a))^p*AppellF1[2, -p, -1/2 - p, 3, (b*x)
/a, -((b*x)/a)] - 2*Sqrt[2]*a*(a - b*x)*(a^2 - b^2*x^2)^p*(1 - (b^2*x^2)/a^
2)^p*Hypergeometric2F1[-1/2 - p, 1 + p, 2 + p, (a - b*x)/(2*a)]))/(b*(1 + p
)*(1 - (b*x)/a)^p)
```

Maple [F] time = 0.514, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}}\left(-b^2x^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(-b^2*x^2+a^2)^p,x)
```

```
[Out] int((b*x+a)^(3/2)*(-b^2*x^2+a^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}}\left(-b^2x^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(-b^2*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)*(-b^2*x^2 + a^2)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx+a)^{\frac{3}{2}}\left(-b^2x^2+a^2\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(-b^2*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)*(-b^2*x^2 + a^2)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(-b**2*x**2+a**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}}\left(-b^2x^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(-b^2*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/2)*(-b^2*x^2 + a^2)^p, x)
```

3.974 $\int \sqrt{a+bx}\left(a^2-b^2x^2\right)^p\,dx$

Optimal. Leaf size=88

$$-\frac{2^{p+\frac{1}{2}}\sqrt{a+bx}\left(\frac{bx}{a}+1\right)^{-p-\frac{3}{2}}\left(a^2-b^2x^2\right)^{p+1}{}_2F_1\left(-p-\frac{1}{2},p+1;p+2;\frac{a-bx}{2a}\right)}{ab(p+1)}$$

```
[Out] -((2^(1/2 + p)*Sqrt[a + b*x]*(1 + (b*x)/a)^(-3/2 - p)*(a^2 - b^2*x^2)^(1 +
p)*Hypergeometric2F1[-1/2 - p, 1 + p, 2 + p, (a - b*x)/(2*a)])/(a*b*(1 + p)
))
```

Rubi [A] time = 0.0587131, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {680, 678, 69}

$$-\frac{2^{p+\frac{1}{2}}\sqrt{a+bx}\left(\frac{bx}{a}+1\right)^{-p-\frac{3}{2}}\left(a^2-b^2x^2\right)^{p+1}{}_2F_1\left(-p-\frac{1}{2},p+1;p+2;\frac{a-bx}{2a}\right)}{ab(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*(a^2 - b^2*x^2)^p,x]
```

```
[Out] -((2^(1/2 + p)*Sqrt[a + b*x]*(1 + (b*x)/a)^(-3/2 - p)*(a^2 - b^2*x^2)^(1 +
p)*Hypergeometric2F1[-1/2 - p, 1 + p, 2 + p, (a - b*x)/(2*a)])/(a*b*(1 + p)
))
```

Rule 680

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^IntPart[m]*(d + e*x)^FracPart[m])/(1 + (e*x)/d)^FracPart[m], Int[(1 + (e
*x)/d)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && EqQ[c*d^2 +
a*e^2, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] || GtQ[d, 0])
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \sqrt{a+bx}\left(a^2-b^2x^2\right)^p\,dx = \frac{\sqrt{a+bx}\int \sqrt{1+\frac{bx}{a}}\left(a^2-b^2x^2\right)^p\,dx}{\sqrt{1+\frac{bx}{a}}}$$
$$= \left(\sqrt{a+bx}\left(1+\frac{bx}{a}\right)^{-\frac{3}{2}-p}\left(a^2-abx\right)^{-1-p}\left(a^2-b^2x^2\right)^{1+p}\right)\int \left(1+\frac{bx}{a}\right)^{\frac{1}{2}+p}\left(a^2-abx\right)^p\,dx$$
$$= -\frac{2^{\frac{1}{2}+p}\sqrt{a+bx}\left(1+\frac{bx}{a}\right)^{-\frac{3}{2}-p}\left(a^2-b^2x^2\right)^{1+p}\,{}_2F_1\left(-\frac{1}{2}-p,1+p;2+p;\frac{a-bx}{2a}\right)}{ab(1+p)}$$

Mathematica [A] time = 0.0936701, size = 89, normalized size = 1.01

$$\frac{2^{p+\frac{1}{2}}(bx-a)\sqrt{a+bx}\left(\frac{bx}{a}+1\right)^{-p-\frac{1}{2}}\left(a^2-b^2x^2\right)^p\,{}_2F_1\left(-p-\frac{1}{2},p+1;p+2;\frac{a-bx}{2a}\right)}{b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*(a^2 - b^2*x^2)^p,x]
```

```
[Out] (2^(1/2 + p)*(-a + b*x)*Sqrt[a + b*x]*(1 + (b*x)/a)^(-1/2 - p)*(a^2 - b^2*x
^2)^p*Hypergeometric2F1[-1/2 - p, 1 + p, 2 + p, (a - b*x)/(2*a)])/(b*(1 + p
))
```

Maple [F] time = 0.516, size = 0, normalized size = 0.

$$\int \sqrt{bx+a}\left(-b^2x^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(-b^2*x^2+a^2)^p,x)
```

```
[Out] int((b*x+a)^(1/2)*(-b^2*x^2+a^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx+a}\left(-b^2x^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(-b^2*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)*(-b^2*x^2 + a^2)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{bx+a}\left(-b^2x^2+a^2\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(-b^2*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(-b^2*x^2 + a^2)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (-(-a+bx)(a+bx))^p\sqrt{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(-b**2*x**2+a**2)**p,x)
```

```
[Out] Integral((-(-a + b*x)*(a + b*x))**p*sqrt(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx+a}\left(-b^2x^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(-b^2*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x + a)*(-b^2*x^2 + a^2)^p, x)
```

3.975 $\int \frac{(a^2-b^2x^2)^p}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=88

$$-\frac{2^{p-\frac{1}{2}}\left(\frac{bx}{a}+1\right)^{-p-\frac{1}{2}}\left(a^2-b^2x^2\right)^{p+1}{}_2F_1\left(\frac{1}{2}-p,p+1;p+2;\frac{a-bx}{2a}\right)}{ab(p+1)\sqrt{a+bx}}$$

```
[Out] -((2^(-1/2 + p)*(1 + (b*x)/a)^(-1/2 - p)*(a^2 - b^2*x^2)^(1 + p)*Hypergeome
tric2F1[1/2 - p, 1 + p, 2 + p, (a - b*x)/(2*a)])/(a*b*(1 + p)*Sqrt[a + b*x]
))
```

Rubi [A] time = 0.0666792, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {680, 678, 69}

$$-\frac{2^{p-\frac{1}{2}}\left(\frac{bx}{a}+1\right)^{-p-\frac{1}{2}}\left(a^2-b^2x^2\right)^{p+1}{}_2F_1\left(\frac{1}{2}-p,p+1;p+2;\frac{a-bx}{2a}\right)}{ab(p+1)\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*x^2)^p/Sqrt[a + b*x],x]
```

```
[Out] -((2^(-1/2 + p)*(1 + (b*x)/a)^(-1/2 - p)*(a^2 - b^2*x^2)^(1 + p)*Hypergeome
tric2F1[1/2 - p, 1 + p, 2 + p, (a - b*x)/(2*a)])/(a*b*(1 + p)*Sqrt[a + b*x]
))
```

Rule 680

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^IntPart[m]*(d + e*x)^FracPart[m])/(1 + (e*x)/d)^FracPart[m], Int[(1 + (e
*x)/d)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && EqQ[c*d^2 +
a*e^2, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] || GtQ[d, 0])
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{\left(a^2-b^2x^2\right)^p}{\sqrt{a+bx}}\,dx = \frac{\sqrt{1+\frac{bx}{a}}\int \frac{\left(a^2-b^2x^2\right)^p}{\sqrt{1+\frac{bx}{a}}}\,dx}{\sqrt{a+bx}}$$

$$= \frac{\left(\left(1+\frac{bx}{a}\right)^{-\frac{1}{2}-p}\left(a^2-abx\right)^{-1-p}\left(a^2-b^2x^2\right)^{1+p}\right)\int \left(1+\frac{bx}{a}\right)^{-\frac{1}{2}+p}\left(a^2-abx\right)^p\,dx}{\sqrt{a+bx}}$$

$$= -\frac{2^{-\frac{1}{2}+p}\left(1+\frac{bx}{a}\right)^{-\frac{1}{2}-p}\left(a^2-b^2x^2\right)^{1+p}\,{}_2F_1\left(\frac{1}{2}-p,1+p;2+p;\frac{a-bx}{2a}\right)}{ab(1+p)\sqrt{a+bx}}$$

Mathematica [A] time = 0.0946472, size = 89, normalized size = 1.01

$$\frac{2^{p-\frac{1}{2}}(bx-a)\left(\frac{bx}{a}+1\right)^{\frac{1}{2}-p}\left(a^2-b^2x^2\right)^p\,{}_2F_1\left(\frac{1}{2}-p,p+1;p+2;\frac{a-bx}{2a}\right)}{b(p+1)\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*x^2)^p/Sqrt[a + b*x],x]
```

```
[Out] (2^(-1/2 + p)*(-a + b*x)*(1 + (b*x)/a)^(1/2 - p)*(a^2 - b^2*x^2)^p*Hypergeo
metric2F1[1/2 - p, 1 + p, 2 + p, (a - b*x)/(2*a)])/(b*(1 + p)*Sqrt[a + b*x]
)
```

Maple [F] time = 0.52, size = 0, normalized size = 0.

$$\int \left(-b^2x^2+a^2\right)^p\frac{1}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^2*x^2+a^2)^p/(b*x+a)^(1/2),x)
```

```
[Out] int((-b^2*x^2+a^2)^p/(b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-b^2x^2+a^2\right)^p}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^p/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-b^2*x^2 + a^2)^p/sqrt(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-b^2x^2+a^2\right)^p}{\sqrt{bx+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^p/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((-b^2*x^2 + a^2)^p/sqrt(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-\left(-a+bx\right)\left(a+bx\right)\right)^p}{\sqrt{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**2*x**2+a**2)**p/(b*x+a)**(1/2),x)
```

```
[Out] Integral((-(-a + b*x)*(a + b*x))**p/sqrt(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-b^2x^2+a^2\right)^p}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^p/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-b^2*x^2 + a^2)^p/sqrt(b*x + a), x)
```

3.976 $\int \frac{\left(a^2-b^2x^2\right)^p}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{2^{p-\frac{3}{2}}\left(\frac{bx}{a}+1\right)^{-p-\frac{1}{2}}\left(a^2-b^2x^2\right)^{p+1}\,{}_2F_1\left(\frac{3}{2}-p,p+1;p+2;\frac{a-bx}{2a}\right)}{a^2b(p+1)\sqrt{a+bx}}$$

```
[Out] -((2^(-3/2 + p)*(1 + (b*x)/a)^(-1/2 - p)*(a^2 - b^2*x^2)^(1 + p)*Hypergeome
tric2F1[3/2 - p, 1 + p, 2 + p, (a - b*x)/(2*a)])/(a^2*b*(1 + p)*Sqrt[a + b*
x]))
```

Rubi [A] time = 0.0717756, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {680, 678, 69}

$$-\frac{2^{p-\frac{3}{2}}\left(\frac{bx}{a}+1\right)^{-p-\frac{1}{2}}\left(a^2-b^2x^2\right)^{p+1}\,{}_2F_1\left(\frac{3}{2}-p,p+1;p+2;\frac{a-bx}{2a}\right)}{a^2b(p+1)\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*x^2)^p/(a + b*x)^(3/2),x]
```

```
[Out] -((2^(-3/2 + p)*(1 + (b*x)/a)^(-1/2 - p)*(a^2 - b^2*x^2)^(1 + p)*Hypergeome
tric2F1[3/2 - p, 1 + p, 2 + p, (a - b*x)/(2*a)])/(a^2*b*(1 + p)*Sqrt[a + b*
x]))
```

Rule 680

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^IntPart[m]*(d + e*x)^FracPart[m])/(1 + (e*x)/d)^FracPart[m], Int[(1 + (e
*x)/d)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && EqQ[c*d^2 +
a*e^2, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] || GtQ[d, 0])
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(a^2-b^2x^2)^p}{(a+bx)^{3/2}}\,dx = \frac{\sqrt{1+\frac{bx}{a}}\int \frac{(a^2-b^2x^2)^p}{\left(1+\frac{bx}{a}\right)^{3/2}}\,dx}{a\sqrt{a+bx}}$$

$$= \frac{\left(\left(1+\frac{bx}{a}\right)^{-\frac{1}{2}-p}(a^2-abx)^{-1-p}(a^2-b^2x^2)^{1+p}\right)\int \left(1+\frac{bx}{a}\right)^{-\frac{3}{2}+p}(a^2-abx)^p\,dx}{a\sqrt{a+bx}}$$

$$= -\frac{2^{-\frac{3}{2}+p}\left(1+\frac{bx}{a}\right)^{-\frac{1}{2}-p}(a^2-b^2x^2)^{1+p}\,{}_2F_1\left(\frac{3}{2}-p,1+p;2+p;\frac{a-bx}{2a}\right)}{a^2b(1+p)\sqrt{a+bx}}$$

Mathematica [A] time = 0.111484, size = 92, normalized size = 1.05

$$-\frac{2^{p-\frac{3}{2}}(a-bx)\left(\frac{bx}{a}+1\right)^{\frac{1}{2}-p}(a^2-b^2x^2)^p\,{}_2F_1\left(\frac{3}{2}-p,p+1;p+2;\frac{a-bx}{2a}\right)}{ab(p+1)\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*x^2)^p/(a + b*x)^(3/2),x]
```

```
[Out] -((2^(-3/2 + p)*(a - b*x)*(1 + (b*x)/a)^(1/2 - p)*(a^2 - b^2*x^2)^p*Hyperge
ometric2F1[3/2 - p, 1 + p, 2 + p, (a - b*x)/(2*a)])/(a*b*(1 + p)*Sqrt[a + b
*x]))
```

Maple [F] time = 0.516, size = 0, normalized size = 0.

$$\int (-b^2x^2+a^2)^p (bx+a)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^2*x^2+a^2)^p/(b*x+a)^(3/2),x)
```

```
[Out] int((-b^2*x^2+a^2)^p/(b*x+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-b^2x^2+a^2)^p}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^p/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-b^2*x^2 + a^2)^p/(b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}\left(-b^2x^2+a^2\right)^p}{b^2x^2+2\,abx+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^p/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(-b^2*x^2 + a^2)^p/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(-a+bx)(a+bx))^p}{(a+bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**2*x**2+a**2)**p/(b*x+a)**(3/2),x)
```

```
[Out] Integral((-(-a + b*x)*(a + b*x))**p/(a + b*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-b^2x^2+a^2\right)^p}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*x^2+a^2)^p/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-b^2*x^2 + a^2)^p/(b*x + a)^(3/2), x)
```

3.977 $\int \left(-a\left(a^2-b^2x^2\right)^p+(a+bx)\left(a^2-b^2x^2\right)^p\right)\,dx$

Optimal. Leaf size=28

$$-\frac{\left(a^2-b^2x^2\right)^{p+1}}{2b(p+1)}$$

```
[Out] -(a^2 - b^2*x^2)^(1 + p)/(2*b*(1 + p))
```

Rubi [A] time = 0.0413844, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {246, 245, 641}

$$-\frac{\left(a^2-b^2x^2\right)^{p+1}}{2b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[-(a*(a^2 - b^2*x^2)^p) + (a + b*x)*(a^2 - b^2*x^2)^p,x]
```

```
[Out] -(a^2 - b^2*x^2)^(1 + p)/(2*b*(1 + p))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(-a\left(a^2-b^2x^2\right)^p+(a+bx)\left(a^2-b^2x^2\right)^p\right)dx &= -\left(a\int \left(a^2-b^2x^2\right)^p\,dx\right)+\int (a+bx)\left(a^2-b^2x^2\right)^p\,dx\\
&= -\frac{\left(a^2-b^2x^2\right)^{1+p}}{2b(1+p)}+a\int \left(a^2-b^2x^2\right)^p\,dx-\left(a\left(a^2-b^2x^2\right)^p\left(1-\frac{b^2x^2}{a^2}\right)^{-p}\right.\\
&= -\frac{\left(a^2-b^2x^2\right)^{1+p}}{2b(1+p)}-ax\left(a^2-b^2x^2\right)^p\left(1-\frac{b^2x^2}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{b^2x^2}{a^2}\right)+\\
&= -\frac{\left(a^2-b^2x^2\right)^{1+p}}{2b(1+p)}
\end{aligned}$$

Mathematica [A] time = 0.0051315, size = 28, normalized size = 1.

$$-\frac{\left(a^2-b^2x^2\right)^{p+1}}{2b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[-(a*(a^2 - b^2*x^2)^p) + (a + b*x)*(a^2 - b^2*x^2)^p,x]
```

```
[Out] -(a^2 - b^2*x^2)^(1 + p)/(2*b*(1 + p))
```

Maple [A] time = 0.039, size = 36, normalized size = 1.3

$$-\frac{(bx+a)\,(-bx+a)\left(-b^2x^2+a^2\right)^p}{2\,b\left(1+p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-a*(-b^2*x^2+a^2)^p+(b*x+a)*(-b^2*x^2+a^2)^p,x)
```

```
[Out] -1/2*(b*x+a)*(-b*x+a)*(-b^2*x^2+a^2)^p/b/(1+p)
```

Maxima [A] time = 1.39302, size = 57, normalized size = 2.04

$$\frac{\left(b^2x^2-a^2\right)e^{\left(p\log(bx+a)+p\log(-bx+a)\right)}}{2\,b\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-a*(-b^2*x^2+a^2)^p+(b*x+a)*(-b^2*x^2+a^2)^p,x, algorithm="maxima
")
```

```
[Out] 1/2*(b^2*x^2 - a^2)*e^(p*log(b*x + a) + p*log(-b*x + a))/(b*(p + 1))
```

Fricas [A] time = 1.97114, size = 68, normalized size = 2.43

$$\frac{\left(b^2x^2-a^2\right)\left(-b^2x^2+a^2\right)^p}{2\left(bp+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-a*(-b^2*x^2+a^2)^p+(b*x+a)*(-b^2*x^2+a^2)^p,x, algorithm="fricas
")
```

```
[Out] 1/2*(b^2*x^2 - a^2)*(-b^2*x^2 + a^2)^p/(b*p + b)
```

Sympy [A] time = 3.82176, size = 49, normalized size = 1.75

$$b\left(\begin{cases} \frac{x^2(a^2)^p}{2} & \text{for } b^2 = 0 \\ -\frac{\begin{cases} \frac{(a^2-b^2x^2)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log\left(a^2 - b^2x^2\right) & \text{otherwise} \end{cases}}{2b^2} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-a*(-b**2*x**2+a**2)**p+(b*x+a)*(-b**2*x**2+a**2)**p,x)
```

```
[Out] b*Piecewise((x**2*(a**2)**p/2, Eq(b**2, 0)), (-Piecewise(((a**2 - b**2*x**2
)**(p + 1)/(p + 1), Ne(p, -1)), (log(a**2 - b**2*x**2), True))/(2*b**2), Tr
ue))
```

Giac [A] time = 1.2646, size = 35, normalized size = 1.25

$$-\frac{\left(-b^2x^2 + a^2\right)^{p+1}}{2\,b\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-a*(-b^2*x^2+a^2)^p+(b*x+a)*(-b^2*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] -1/2*(-b^2*x^2 + a^2)^(p + 1)/(b*(p + 1))
```

3.978 $\int (d+ex)^2 \left(cd^2+2cdex+ce^2x^2\right) dx$

Optimal. Leaf size=15

$$\frac{c(d+ex)^5}{5e}$$

```
[Out] (c*(d + e*x)^5)/(5*e)
```

Rubi [A] time = 0.0044967, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {27, 12, 32}

$$\frac{c(d+ex)^5}{5e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2),x]
```

```
[Out] (c*(d + e*x)^5)/(5*e)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int (d+ex)^2 \left(cd^2+2cdex+ce^2x^2\right) dx &= \int c(d+ex)^4 \, dx \\ &= c \int (d+ex)^4 \, dx \\ &= \frac{c(d+ex)^5}{5e} \end{aligned}$$

Mathematica [A] time = 0.0029526, size = 15, normalized size = 1.

$$\frac{c(d+ex)^5}{5e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2),x]
```

```
[Out] (c*(d + e*x)^5)/(5*e)
```

Maple [B] time = 0.04, size = 48, normalized size = 3.2

$$\frac{e^4cx^5}{5} + de^3cx^4 + 2\,d^2ce^2x^3 + 2\,cd^3ex^2 + cd^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2),x)
```

```
[Out] 1/5*e^4*c*x^5+d*e^3*c*x^4+2*d^2*c*e^2*x^3+2*c*d^3*e*x^2+c*d^4*x
```

Maxima [B] time = 1.09967, size = 63, normalized size = 4.2

$$\frac{1}{5}\,ce^4x^5 + cde^3x^4 + 2\,cd^2e^2x^3 + 2\,cd^3ex^2 + cd^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="maxima")
```

```
[Out] 1/5*c*e^4*x^5 + c*d*e^3*x^4 + 2*c*d^2*e^2*x^3 + 2*c*d^3*e*x^2 + c*d^4*x
```

Fricas [B] time = 1.61673, size = 99, normalized size = 6.6

$$\frac{1}{5}x^5e^4c + x^4e^3dc + 2x^3e^2d^2c + 2x^2ed^3c + xd^4c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="fricas")
```

```
[Out] 1/5*x^5*e^4*c + x^4*e^3*d*c + 2*x^3*e^2*d^2*c + 2*x^2*e*d^3*c + x*d^4*c
```

Sympy [B] time = 0.072436, size = 51, normalized size = 3.4

$$cd^4x + 2cd^3ex^2 + 2cd^2e^2x^3 + cde^3x^4 + \frac{ce^4x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*e**2*x**2+2*c*d*e*x+c*d**2),x)
```

```
[Out] c*d**4*x + 2*c*d**3*e*x**2 + 2*c*d**2*e**2*x**3 + c*d*e**3*x**4 + c*e**4*x*
*5/5
```

Giac [B] time = 1.1605, size = 61, normalized size = 4.07

$$\frac{1}{5}\,cx^5e^4 + cdx^4e^3 + 2\,cd^2x^3e^2 + 2\,cd^3x^2e + cd^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")
```

```
[Out] 1/5*c*x^5*e^4 + c*d*x^4*e^3 + 2*c*d^2*x^3*e^2 + 2*c*d^3*x^2*e + c*d^4*x
```

3.979 $\int (d+ex)\left(cd^2+2cdex+ce^2x^2\right)\,dx$

Optimal. Leaf size=15

$$\frac{c(d+ex)^4}{4e}$$

```
[Out] (c*(d + e*x)^4)/(4*e)
```

Rubi [A] time = 0.0040956, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {27, 12, 32}

$$\frac{c(d+ex)^4}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2),x]
```

```
[Out] (c*(d + e*x)^4)/(4*e)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(cd^2+2cdex+ce^2x^2\right)\,dx &= \int c(d+ex)^3\,dx\\ &= c\int (d+ex)^3\,dx\\ &= \frac{c(d+ex)^4}{4e}\end{aligned}$$

Mathematica [A] time = 0.0016316, size = 15, normalized size = 1.

$$\frac{c(d+ex)^4}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2),x]
```

```
[Out] (c*(d + e*x)^4)/(4*e)
```

Maple [B] time = 0.039, size = 36, normalized size = 2.4

$$\frac{ce^3x^4}{4} + dce^2x^3 + \frac{3\,d^2ecx^2}{2} + cd^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2),x)
```

```
[Out] 1/4*c*e^3*x^4+d*c*e^2*x^3+3/2*d^2*e*c*x^2+c*d^3*x
```

Maxima [B] time = 1.1008, size = 41, normalized size = 2.73

$$\frac{\left(ce^2x^2 + 2\,cdex + cd^2\right)^2}{4\,ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="maxima")
```

```
[Out] 1/4*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^2/(c*e)
```

Fricas [B] time = 1.56161, size = 77, normalized size = 5.13

$$\frac{1}{4}x^4e^3c + x^3e^2dc + \frac{3}{2}x^2ed^2c + xd^3c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*e^3*c + x^3*e^2*d*c + 3/2*x^2*e*d^2*c + x*d^3*c
```

Sympy [B] time = 0.075352, size = 39, normalized size = 2.6

$$cd^3x + \frac{3cd^2ex^2}{2} + cde^2x^3 + \frac{ce^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*e**2*x**2+2*c*d*e*x+c*d**2),x)
```

```
[Out] c*d**3*x + 3*c*d**2*e*x**2/2 + c*d*e**2*x**3 + c*e**3*x**4/4
```

Giac [B] time = 1.18386, size = 46, normalized size = 3.07

$$\frac{1}{4}\,cx^4e^3 + cdx^3e^2 + \frac{3}{2}\,cd^2x^2e + cd^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")
```

```
[Out] 1/4*c*x^4*e^3 + c*d*x^3*e^2 + 3/2*c*d^2*x^2*e + c*d^3*x
```

3.980 $\int \left(cd^2+2cdex+ce^2x^2\right)\,dx$

Optimal. Leaf size=25

$$cd^2x+cdex^2+\frac{1}{3}ce^2x^3$$

```
[Out] c*d^2*x + c*d*e*x^2 + (c*e^2*x^3)/3
```

Rubi [A] time = 0.0058733, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$cd^2x+cdex^2+\frac{1}{3}ce^2x^3$$

Antiderivative was successfully verified.

```
[In] Int[c*d^2 + 2*c*d*e*x + c*e^2*x^2,x]
```

```
[Out] c*d^2*x + c*d*e*x^2 + (c*e^2*x^3)/3
```

Rubi steps

$$\int \left(cd^2+2cdex+ce^2x^2\right)\,dx=cd^2x+cdex^2+\frac{1}{3}ce^2x^3$$

Mathematica [A] time = 0.0000363, size = 25, normalized size = 1.

$$cd^2x+cdex^2+\frac{1}{3}ce^2x^3$$

Antiderivative was successfully verified.

```
[In] Integrate[c*d^2 + 2*c*d*e*x + c*e^2*x^2,x]
```

```
[Out] c*d^2*x + c*d*e*x^2 + (c*e^2*x^3)/3
```

Maple [A] time = 0.038, size = 24, normalized size = 1.

$$cd^2x+cdex^2+\frac{ce^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*e^2*x^2+2*c*d*e*x+c*d^2,x)
```

```
[Out] c*d^2*x+c*d*e*x^2+1/3*c*e^2*x^3
```

Maxima [A] time = 1.01181, size = 31, normalized size = 1.24

$$\frac{1}{3}ce^2x^3 + cdex^2 + cd^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*e^2*x^2+2*c*d*e*x+c*d^2,x, algorithm="maxima")
```

```
[Out] 1/3*c*e^2*x^3 + c*d*e*x^2 + c*d^2*x
```

Fricas [A] time = 1.52701, size = 50, normalized size = 2.

$$\frac{1}{3}x^3e^2c + x^2edc + xd^2c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*e^2*x^2+2*c*d*e*x+c*d^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3*e^2*c + x^2*e*d*c + x*d^2*c
```

Sympy [A] time = 0.068721, size = 24, normalized size = 0.96

$$cd^2x + cdex^2 + \frac{ce^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*e**2*x**2+2*c*d*e*x+c*d**2,x)
```

```
[Out] c*d**2*x + c*d*e*x**2 + c*e**2*x**3/3
```

Giac [A] time = 1.13855, size = 31, normalized size = 1.24

$$\frac{1}{3}cx^3e^2 + cdx^2e + cd^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*e^2*x^2+2*c*d*e*x+c*d^2,x, algorithm="giac")
```

```
[Out] 1/3*c*x^3*e^2 + c*d*x^2*e + c*d^2*x
```

3.981 $\int \frac{cd^2+2cdex+ce^2x^2}{d+ex} \, dx$

Optimal. Leaf size=14

$$cdx + \frac{1}{2}cex^2$$

```
[Out] c*d*x + (c*e*x^2)/2
```

Rubi [A] time = 0.0083603, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {24}

$$cdx + \frac{1}{2}cex^2$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)/(d + e*x),x]
```

```
[Out] c*d*x + (c*e*x^2)/2
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{cd^2 + 2cdex + ce^2x^2}{d + ex} \, dx &= \frac{\int \left(cde^2 + ce^3x\right) dx}{e^2} \\ &= cdx + \frac{1}{2}cex^2 \end{aligned}$$

Mathematica [A] time = 0.0005952, size = 14, normalized size = 1.

$$c\left(dx + \frac{ex^2}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)/(d + e*x),x]
```

```
[Out] c*(d*x + (e*x^2)/2)
```

Maple [A] time = 0.038, size = 13, normalized size = 0.9

$$c\left(\frac{ex^2}{2} + dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d),x)
```

```
[Out] c*(1/2*e*x^2+d*x)
```

Maxima [A] time = 1.11703, size = 16, normalized size = 1.14

$$\frac{1}{2}cex^2 + cdx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*c*e*x^2 + c*d*x
```

Fricas [A] time = 1.93557, size = 28, normalized size = 2.

$$\frac{1}{2}cex^2 + cdx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/2*c*e*x^2 + c*d*x
```

Sympy [A] time = 0.085971, size = 12, normalized size = 0.86

$$cdx + \frac{cex^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)/(e*x+d),x)
```

```
[Out] c*d*x + c*e*x**2/2
```

Giac [A] time = 1.13159, size = 26, normalized size = 1.86

$$\frac{1}{2}\left(cx^2e^3 + 2\,cdxe^2\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d),x, algorithm="giac")
```

```
[Out] 1/2*(c*x^2*e^3 + 2*c*d*x*e^2)*e^(-2)
```

3.982 $\int \frac{cd^2+2cdex+ce^2x^2}{(d+ex)^2}\,dx$

Optimal. Leaf size=3

$$cx$$

[Out] c*x

Rubi [A] time = 0.0037241, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {24, 21, 8}

$$cx$$

Antiderivative was successfully verified.

[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)/(d + e*x)^2,x]

[Out] c*x

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{cd^2+2cdex+ce^2x^2}{(d+ex)^2}\,dx &= \frac{\int \frac{cde^2+ce^3x}{d+ex}\,dx}{e^2}\\ &= c\int 1\,dx\\ &= cx\end{aligned}$$

Mathematica [A] time = 0.0003464, size = 3, normalized size = 1.

$$cx$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)/(d + e*x)^2,x]
```

```
[Out] c*x
```

Maple [A] time = 0.038, size = 4, normalized size = 1.3

$$cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^2,x)
```

```
[Out] c*x
```

Maxima [A] time = 1.12504, size = 4, normalized size = 1.33

$$cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] c*x
```

Fricas [A] time = 1.9908, size = 7, normalized size = 2.33

$$cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] c*x
```

Sympy [A] time = 0.081061, size = 2, normalized size = 0.67

$$cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)/(e*x+d)**2,x)
```

```
[Out] c*x
```

Giac [C] time = 1.16001, size = 149, normalized size = 49.67

$$-2\left(e^{(-1)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^2}\right)-\frac{de^{(-1)}}{xe+d}\right)cd+\left(2\,de^{(-3)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^2}\right)+(xe+d)e^{(-3)}-\frac{d^2e^{(-3)}}{xe+d}\right)ce^2-\frac{cd^2e^{(-1)}}{xe+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -2*(e^(-1)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2) - d*e^(-1)/(x*e + d))*c*d +
 (2*d*e^(-3)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2) + (x*e + d)*e^(-3) - d^2*
e^(-3)/(x*e + d))*c*e^2 - c*d^2*e^(-1)/(x*e + d)
```

3.983 $\int \frac{cd^2+2cdex+ce^2x^2}{(d+ex)^3} dx$

Optimal. Leaf size=11

$$\frac{c\log(d+ex)}{e}$$

```
[Out] (c*Log[d + e*x])/e
```

Rubi [A] time = 0.0061493, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {24, 21, 31}

$$\frac{c\log(d+ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)/(d + e*x)^3,x]
```

```
[Out] (c*Log[d + e*x])/e
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{cd^2+2cdex+ce^2x^2}{(d+ex)^3} dx &= \frac{\int \frac{cde^2+ce^3x}{(d+ex)^2} dx}{e^2}\\ &= c\int \frac{1}{d+ex} dx\\ &= \frac{c\log(d+ex)}{e}\end{aligned}$$

Mathematica [A] time = 0.0013314, size = 11, normalized size = 1.

$$\frac{c\log(d+ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)/(d + e*x)^3,x]
```

```
[Out] (c*Log[d + e*x])/e
```

Maple [A] time = 0.038, size = 12, normalized size = 1.1

$$\frac{c \ln (ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^3,x)
```

```
[Out] c*ln(e*x+d)/e
```

Maxima [A] time = 1.15981, size = 15, normalized size = 1.36

$$\frac{c \log (ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] c*log(e*x + d)/e
```

Fricas [A] time = 2.05157, size = 24, normalized size = 2.18

$$\frac{c \log (ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] c*log(e*x + d)/e
```

Sympy [A] time = 0.083459, size = 8, normalized size = 0.73

$$\frac{c \log (d + ex)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)/(e*x+d)**3,x)
```

```
[Out] c*log(d + e*x)/e
```

Giac [A] time = 1.20382, size = 16, normalized size = 1.45

$$ce^{(-1)}\log(|xe+d|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] c*e^(-1)*log(abs(x*e + d))
```

3.984 $\int \frac{cd^2+2cdex+ce^2x^2}{(d+ex)^4} dx$

Optimal. Leaf size=13

$$-\frac{c}{e(d+ex)}$$

```
[Out] -(c/(e*(d + e*x)))
```

Rubi [A] time = 0.0064377, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {24, 21, 32}

$$-\frac{c}{e(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)/(d + e*x)^4,x]
```

```
[Out] -(c/(e*(d + e*x)))
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{cd^2+2cdex+ce^2x^2}{(d+ex)^4}\,dx &= \frac{\int \frac{cde^2+ce^3x}{(d+ex)^3}\,dx}{e^2}\\ &= c\int \frac{1}{(d+ex)^2}\,dx\\ &= -\frac{c}{e(d+ex)}\end{aligned}$$

Mathematica [A] time = 0.0033505, size = 13, normalized size = 1.

$$-\frac{c}{e(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)/(d + e*x)^4,x]
```

```
[Out] -(c/(e*(d + e*x)))
```

Maple [A] time = 0.038, size = 14, normalized size = 1.1

$$-\frac{c}{e\,(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^4,x)
```

```
[Out] -c/e/(e*x+d)
```

Maxima [A] time = 1.16147, size = 19, normalized size = 1.46

$$-\frac{c}{e^2x+de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -c/(e^2*x + d*e)
```

Fricas [A] time = 1.98543, size = 24, normalized size = 1.85

$$-\frac{c}{e^2x+de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -c/(e^2*x + d*e)
```

Sympy [A] time = 0.314067, size = 10, normalized size = 0.77

$$-\frac{c}{de+e^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)/(e*x+d)**4,x)
```

```
[Out] -c/(d*e + e**2*x)
```

Giac [B] time = 1.22292, size = 46, normalized size = 3.54

$$-\frac{\left(cx^2e^4 + 2\,cdxe^3 + cd^2e^2\right)e^{(-3)}}{(xe + d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -(c*x^2*e^4 + 2*c*d*x*e^3 + c*d^2*e^2)*e^(-3)/(x*e + d)^3
```

3.985 $\int \frac{cd^2+2cdex+ce^2x^2}{(d+ex)^5} dx$

Optimal. Leaf size=15

$$-\frac{c}{2e(d+ex)^2}$$

[Out] -c/(2*e*(d + e*x)^2)

Rubi [A] time = 0.0063687, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {24, 21, 32}

$$-\frac{c}{2e(d+ex)^2}$$

Antiderivative was successfully verified.

[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)/(d + e*x)^5,x]

[Out] -c/(2*e*(d + e*x)^2)

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{cd^2+2cdex+ce^2x^2}{(d+ex)^5} dx &= \frac{\int \frac{cde^2+ce^3x}{(d+ex)^4} dx}{e^2}\\ &= c\int \frac{1}{(d+ex)^3} dx\\ &= -\frac{c}{2e(d+ex)^2}\end{aligned}$$

Mathematica [A] time = 0.0033624, size = 15, normalized size = 1.

$$-\frac{c}{2e(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)/(d + e*x)^5,x]
```

```
[Out] -c/(2*e*(d + e*x)^2)
```

Maple [A] time = 0.037, size = 14, normalized size = 0.9

$$-\frac{c}{2\,e\,(ex+d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^5,x)
```

```
[Out] -1/2*c/e/(e*x+d)^2
```

Maxima [A] time = 1.18599, size = 34, normalized size = 2.27

$$-\frac{c}{2\left(e^3x^2+2\,de^2x+d^2e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^5,x, algorithm="maxima")
```

```
[Out] -1/2*c/(e^3*x^2 + 2*d*e^2*x + d^2*e)
```

Fricas [A] time = 1.93154, size = 51, normalized size = 3.4

$$-\frac{c}{2\left(e^3x^2+2\,de^2x+d^2e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^5,x, algorithm="fricas")
```

```
[Out] -1/2*c/(e^3*x^2 + 2*d*e^2*x + d^2*e)
```

Sympy [B] time = 0.41448, size = 26, normalized size = 1.73

$$-\frac{c}{2d^2e+4de^2x+2e^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)/(e*x+d)**5,x)
```

```
[Out] -c/(2*d**2*e + 4*d*e**2*x + 2*e**3*x**2)
```

Giac [A] time = 1.16165, size = 18, normalized size = 1.2

$$-\frac{ce^{(-1)}}{2\,(xe+d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] -1/2*c*e^(-1)/(x*e + d)^2
```

3.986 $\int \frac{cd^2+2cdex+ce^2x^2}{(d+ex)^6} dx$

Optimal. Leaf size=15

$$-\frac{c}{3e(d+ex)^3}$$

[Out] -c/(3*e*(d + e*x)^3)

Rubi [A] time = 0.0063929, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {24, 21, 32}

$$-\frac{c}{3e(d+ex)^3}$$

Antiderivative was successfully verified.

[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)/(d + e*x)^6,x]

[Out] -c/(3*e*(d + e*x)^3)

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_Symbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[m, -1]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{cd^2+2cdex+ce^2x^2}{(d+ex)^6} dx &= \frac{\int \frac{cde^2+ce^3x}{(d+ex)^5} dx}{e^2} \\
&= c\int \frac{1}{(d+ex)^4} dx \\
&= -\frac{c}{3e(d+ex)^3}
\end{aligned}$$

Mathematica [A] time = 0.0036726, size = 15, normalized size = 1.

$$-\frac{c}{3e(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)/(d + e*x)^6,x]
```

```
[Out] -c/(3*e*(d + e*x)^3)
```

Maple [A] time = 0.04, size = 14, normalized size = 0.9

$$-\frac{c}{3\,e\,(ex+d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^6,x)
```

```
[Out] -1/3*c/e/(e*x+d)^3
```

Maxima [B] time = 1.14637, size = 49, normalized size = 3.27

$$-\frac{c}{3\,\left(e^4x^3+3\,de^3x^2+3\,d^2e^2x+d^3e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^6,x, algorithm="maxima")
```

```
[Out] -1/3*c/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e)
```

Fricas [B] time = 1.97777, size = 73, normalized size = 4.87

$$-\frac{c}{3\,\left(e^4x^3+3\,de^3x^2+3\,d^2e^2x+d^3e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^6,x, algorithm="fricas")
```

```
[Out] -1/3*c/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e)
```

Sympy [B] time = 0.488633, size = 37, normalized size = 2.47

$$-\frac{c}{3d^3e+9d^2e^2x+9de^3x^2+3e^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)/(e*x+d)**6,x)
```

```
[Out] -c/(3*d**3*e + 9*d**2*e**2*x + 9*d*e**3*x**2 + 3*e**4*x**3)
```

Giac [B] time = 1.2113, size = 46, normalized size = 3.07

$$-\frac{\left(cx^2e^4 + 2\,cdxe^3 + cd^2e^2\right)e^{(-3)}}{3\,(xe + d)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^6,x, algorithm="giac")
```

```
[Out] -1/3*(c*x^2*e^4 + 2*c*d*x*e^3 + c*d^2*e^2)*e^(-3)/(x*e + d)^5
```

3.987 $\int (d+ex)^2 \left(cd^2+2cdex+ce^2x^2\right)^2 \, dx$

Optimal. Leaf size=17

$$\frac{c^2(d+ex)^7}{7e}$$

```
[Out] (c^2*(d + e*x)^7)/(7*e)
```

Rubi [A] time = 0.0048802, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {27, 12, 32}

$$\frac{c^2(d+ex)^7}{7e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]
```

```
[Out] (c^2*(d + e*x)^7)/(7*e)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^2 \left(cd^2+2cdex+ce^2x^2\right)^2 \, dx &= \int c^2(d+ex)^6 \, dx \\
&= c^2 \int (d+ex)^6 \, dx \\
&= \frac{c^2(d+ex)^7}{7e}
\end{aligned}$$

Mathematica [A] time = 0.0026172, size = 17, normalized size = 1.

$$\frac{c^2(d+ex)^7}{7e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]
```

```
[Out] (c^2*(d + e*x)^7)/(7*e)
```

Maple [B] time = 0.039, size = 86, normalized size = 5.1

$$\frac{e^6c^2x^7}{7} + de^5c^2x^6 + 3\,d^2c^2e^4x^5 + 5\,d^3c^2e^3x^4 + 5\,d^4c^2e^2x^3 + 3\,d^5c^2ex^2 + d^6c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x)
```

```
[Out] 1/7*e^6*c^2*x^7+d*e^5*c^2*x^6+3*d^2*c^2*e^4*x^5+5*d^3*c^2*e^3*x^4+5*d^4*c^2
*e^2*x^3+3*d^5*c^2*e*x^2+d^6*c^2*x
```

Maxima [B] time = 1.14724, size = 115, normalized size = 6.76

$$\frac{1}{7}\,c^2e^6x^7 + c^2de^5x^6 + 3\,c^2d^2e^4x^5 + 5\,c^2d^3e^3x^4 + 5\,c^2d^4e^2x^3 + 3\,c^2d^5ex^2 + c^2d^6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="maxima")
```

```
[Out] 1/7*c^2*e^6*x^7 + c^2*d*e^5*x^6 + 3*c^2*d^2*e^4*x^5 + 5*c^2*d^3*e^3*x^4 + 5
*c^2*d^4*e^2*x^3 + 3*c^2*d^5*e*x^2 + c^2*d^6*x
```

Fricas [B] time = 1.7823, size = 166, normalized size = 9.76

$$\frac{1}{7}x^7e^6c^2 + x^6e^5dc^2 + 3x^5e^4d^2c^2 + 5x^4e^3d^3c^2 + 5x^3e^2d^4c^2 + 3x^2ed^5c^2 + xd^6c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*e^6*c^2 + x^6*e^5*d*c^2 + 3*x^5*e^4*d^2*c^2 + 5*x^4*e^3*d^3*c^2 + 5
*x^3*e^2*d^4*c^2 + 3*x^2*e*d^5*c^2 + x*d^6*c^2
```

Sympy [B] time = 0.089141, size = 90, normalized size = 5.29

$$c^2d^6x + 3c^2d^5ex^2 + 5c^2d^4e^2x^3 + 5c^2d^3e^3x^4 + 3c^2d^2e^4x^5 + c^2de^5x^6 + \frac{c^2e^6x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*e**2*x**2+2*c*d*e*x+c*d**2)**2,x)
```

```
[Out] c**2*d**6*x + 3*c**2*d**5*e*x**2 + 5*c**2*d**4*e**2*x**3 + 5*c**2*d**3*e**3
*x**4 + 3*c**2*d**2*e**4*x**5 + c**2*d*e**5*x**6 + c**2*e**6*x**7/7
```

Giac [B] time = 1.21732, size = 109, normalized size = 6.41

$$\frac{1}{7}c^2x^7e^6 + c^2dx^6e^5 + 3c^2d^2x^5e^4 + 5c^2d^3x^4e^3 + 5c^2d^4x^3e^2 + 3c^2d^5x^2e + c^2d^6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")
```

```
[Out] 1/7*c^2*x^7*e^6 + c^2*d*x^6*e^5 + 3*c^2*d^2*x^5*e^4 + 5*c^2*d^3*x^4*e^3 + 5
*c^2*d^4*x^3*e^2 + 3*c^2*d^5*x^2*e + c^2*d^6*x
```

3.988 $\int (d+ex)\left(cd^2+2cdex+ce^2x^2\right)^2 dx$

Optimal. Leaf size=17

$$\frac{c^2(d+ex)^6}{6e}$$

```
[Out] (c^2*(d + e*x)^6)/(6*e)
```

Rubi [A] time = 0.0043832, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {27, 12, 32}

$$\frac{c^2(d+ex)^6}{6e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]
```

```
[Out] (c^2*(d + e*x)^6)/(6*e)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(cd^2+2cdex+ce^2x^2\right)^2 dx &= \int c^2(d+ex)^5 dx\\ &= c^2\int (d+ex)^5 dx\\ &= \frac{c^2(d+ex)^6}{6e}\end{aligned}$$

Mathematica [A] time = 0.0020731, size = 17, normalized size = 1.

$$\frac{c^2(d+ex)^6}{6e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]
```

```
[Out] (c^2*(d + e*x)^6)/(6*e)
```

Maple [B] time = 0.038, size = 72, normalized size = 4.2

$$\frac{c^2e^5x^6}{6} + dc^2e^4x^5 + \frac{5\,d^2c^2e^3x^4}{2} + \frac{10\,d^3c^2e^2x^3}{3} + \frac{5\,c^2ed^4x^2}{2} + c^2d^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x)
```

```
[Out] 1/6*c^2*e^5*x^6+d*c^2*e^4*x^5+5/2*d^2*c^2*e^3*x^4+10/3*d^3*c^2*e^2*x^3+5/2*
c^2*e*d^4*x^2+c^2*d^5*x
```

Maxima [B] time = 1.22369, size = 41, normalized size = 2.41

$$\frac{\left(ce^2x^2 + 2\,cdex + cd^2\right)^3}{6\,ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^3/(c*e)
```

Fricas [B] time = 1.72845, size = 149, normalized size = 8.76

$$\frac{1}{6}x^6e^5c^2 + x^5e^4dc^2 + \frac{5}{2}x^4e^3d^2c^2 + \frac{10}{3}x^3e^2d^3c^2 + \frac{5}{2}x^2ed^4c^2 + xd^5c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*x^6*e^5*c^2 + x^5*e^4*d*c^2 + 5/2*x^4*e^3*d^2*c^2 + 10/3*x^3*e^2*d^3*c^
2 + 5/2*x^2*e*d^4*c^2 + x*d^5*c^2
```

Sympy [B] time = 0.084157, size = 80, normalized size = 4.71

$$c^2d^5x + \frac{5c^2d^4ex^2}{2} + \frac{10c^2d^3e^2x^3}{3} + \frac{5c^2d^2e^3x^4}{2} + c^2de^4x^5 + \frac{c^2e^5x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*e**2*x**2+2*c*d*e*x+c*d**2)**2,x)
```

```
[Out] c**2*d**5*x + 5*c**2*d**4*e*x**2/2 + 10*c**2*d**3*e**2*x**3/3 + 5*c**2*d**2
*e**3*x**4/2 + c**2*d*e**4*x**5 + c**2*e**5*x**6/6
```

Giac [B] time = 1.16845, size = 92, normalized size = 5.41

$$\frac{1}{6}\,c^2x^6e^5 + c^2dx^5e^4 + \frac{5}{2}\,c^2d^2x^4e^3 + \frac{10}{3}\,c^2d^3x^3e^2 + \frac{5}{2}\,c^2d^4x^2e + c^2d^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*c^2*x^6*e^5 + c^2*d*x^5*e^4 + 5/2*c^2*d^2*x^4*e^3 + 10/3*c^2*d^3*x^3*e^
2 + 5/2*c^2*d^4*x^2*e + c^2*d^5*x
```

3.989 $\int \left(cd^2 + 2cdex + ce^2x^2\right)^2 \, dx$

Optimal. Leaf size=17

$$\frac{c^2(d+ex)^5}{5e}$$

```
[Out] (c^2*(d + e*x)^5)/(5*e)
```

Rubi [A] time = 0.003937, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {27, 12, 32}

$$\frac{c^2(d+ex)^5}{5e}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]
```

```
[Out] (c^2*(d + e*x)^5)/(5*e)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(cd^2 + 2cdex + ce^2x^2\right)^2 \, dx &= \int c^2(d+ex)^4 \, dx \\ &= c^2 \int (d+ex)^4 \, dx \\ &= \frac{c^2(d+ex)^5}{5e}\end{aligned}$$

Mathematica [A] time = 0.001093, size = 17, normalized size = 1.

$$\frac{c^2(d+ex)^5}{5e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]
```

```
[Out] (c^2*(d + e*x)^5)/(5*e)
```

Maple [B] time = 0.039, size = 58, normalized size = 3.4

$$\frac{c^2e^4x^5}{5} + c^2de^3x^4 + 2\,c^2d^2e^2x^3 + 2\,c^2d^3ex^2 + c^2d^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x)
```

```
[Out] 1/5*c^2*e^4*x^5+c^2*d*e^3*x^4+2*c^2*d^2*e^2*x^3+2*c^2*d^3*e*x^2+c^2*d^4*x
```

Maxima [B] time = 1.22374, size = 92, normalized size = 5.41

$$\frac{1}{5}\,c^2e^4x^5 + c^2de^3x^4 + \frac{4}{3}\,c^2d^2e^2x^3 + c^2d^4x + \frac{2}{3}\left(ce^2x^3 + 3\,cdex^2\right)cd^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="maxima")
```

```
[Out] 1/5*c^2*e^4*x^5 + c^2*d*e^3*x^4 + 4/3*c^2*d^2*e^2*x^3 + c^2*d^4*x + 2/3*(c*
e^2*x^3 + 3*c*d*e*x^2)*c*d^2
```

Fricas [B] time = 1.85975, size = 112, normalized size = 6.59

$$\frac{1}{5}x^5e^4c^2 + x^4e^3dc^2 + 2x^3e^2d^2c^2 + 2x^2ed^3c^2 + xd^4c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*e^4*c^2 + x^4*e^3*d*c^2 + 2*x^3*e^2*d^2*c^2 + 2*x^2*e*d^3*c^2 + x*d
^4*c^2
```

Sympy [B] time = 0.074471, size = 60, normalized size = 3.53

$$c^2d^4x + 2c^2d^3ex^2 + 2c^2d^2e^2x^3 + c^2de^3x^4 + \frac{c^2e^4x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**2,x)
```

```
[Out] c**2*d**4*x + 2*c**2*d**3*e*x**2 + 2*c**2*d**2*e**2*x**3 + c**2*d*e**3*x**4
 + c**2*e**4*x**5/5
```

Giac [B] time = 1.19167, size = 74, normalized size = 4.35

$$\frac{1}{5}\,c^2x^5e^4 + c^2dx^4e^3 + 2\,c^2d^2x^3e^2 + 2\,c^2d^3x^2e + c^2d^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")
```

```
[Out] 1/5*c^2*x^5*e^4 + c^2*d*x^4*e^3 + 2*c^2*d^2*x^3*e^2 + 2*c^2*d^3*x^2*e + c^2
*d^4*x
```

3.990 $\int \frac{\left(cd^2+2cdex+ce^2x^2\right)^2}{d+ex} dx$

Optimal. Leaf size=17

$$\frac{c^2(d+ex)^4}{4e}$$

```
[Out] (c^2*(d + e*x)^4)/(4*e)
```

Rubi [A] time = 0.0047706, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {27, 12, 32}

$$\frac{c^2(d+ex)^4}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x),x]
```

```
[Out] (c^2*(d + e*x)^4)/(4*e)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(cd^2+2cdex+ce^2x^2\right)^2}{d+ex} dx &= \int c^2(d+ex)^3 dx\\ &= c^2 \int (d+ex)^3 dx\\ &= \frac{c^2(d+ex)^4}{4e}\end{aligned}$$

Mathematica [A] time = 0.0014003, size = 17, normalized size = 1.

$$\frac{c^2(d+ex)^4}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x),x]
```

```
[Out] (c^2*(d + e*x)^4)/(4*e)
```

Maple [B] time = 0.039, size = 36, normalized size = 2.1

$$c^{2}\left(\frac{e^{3}x^{4}}{4}+de^{2}x^{3}+\frac{3\,d^{2}ex^{2}}{2}+d^{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d),x)
```

```
[Out] c^2*(1/4*e^3*x^4+d*e^2*x^3+3/2*d^2*e*x^2+d^3*x)
```

Maxima [B] time = 1.20283, size = 58, normalized size = 3.41

$$\frac{1}{4}\,c^{2}e^{3}x^{4}+c^{2}de^{2}x^{3}+\frac{3}{2}\,c^{2}d^{2}ex^{2}+c^{2}d^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*c^2*e^3*x^4 + c^2*d*e^2*x^3 + 3/2*c^2*d^2*e*x^2 + c^2*d^3*x
```

Fricas [B] time = 1.9784, size = 88, normalized size = 5.18

$$\frac{1}{4}\,c^{2}e^{3}x^{4}+c^{2}de^{2}x^{3}+\frac{3}{2}\,c^{2}d^{2}ex^{2}+c^{2}d^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*c^2*e^3*x^4 + c^2*d*e^2*x^3 + 3/2*c^2*d^2*e*x^2 + c^2*d^3*x
```

Sympy [B] time = 0.104091, size = 46, normalized size = 2.71

$$c^{2}d^{3}x+\frac{3c^{2}d^{2}ex^{2}}{2}+c^{2}de^{2}x^{3}+\frac{c^{2}e^{3}x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**2/(e*x+d),x)
```

```
[Out] c**2*d**3*x + 3*c**2*d**2*e*x**2/2 + c**2*d*e**2*x**3 + c**2*e**3*x**4/4
```

Giac [B] time = 1.17281, size = 66, normalized size = 3.88

$$\frac{1}{4}\left(c^{2}x^{4}e^{7} + 4\,c^{2}dx^{3}e^{6} + 6\,c^{2}d^{2}x^{2}e^{5} + 4\,c^{2}d^{3}xe^{4}\right)e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d),x, algorithm="giac")
```

```
[Out] 1/4*(c^2*x^4*e^7 + 4*c^2*d*x^3*e^6 + 6*c^2*d^2*x^2*e^5 + 4*c^2*d^3*x*e^4)*e
^(-4)
```

3.991 $\int \frac{\left(cd^2+2cdex+ce^2x^2\right)^2}{(d+ex)^2}\,dx$

Optimal. Leaf size=17

$$\frac{c^2(d+ex)^3}{3e}$$

```
[Out] (c^2*(d + e*x)^3)/(3*e)
```

Rubi [A] time = 0.0051889, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {27, 12, 32}

$$\frac{c^2(d+ex)^3}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x)^2,x]
```

```
[Out] (c^2*(d + e*x)^3)/(3*e)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(cd^2+2cdex+ce^2x^2\right)^2}{(d+ex)^2}\,dx &= \int c^2(d+ex)^2\,dx\\ &= c^2\int (d+ex)^2\,dx\\ &= \frac{c^2(d+ex)^3}{3e}\end{aligned}$$

Mathematica [A] time = 0.0014096, size = 17, normalized size = 1.

$$\frac{c^2(d+ex)^3}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x)^2,x]
```

```
[Out] (c^2*(d + e*x)^3)/(3*e)
```

Maple [A] time = 0.038, size = 16, normalized size = 0.9

$$\frac{c^2 (ex+d)^3}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^2,x)
```

```
[Out] 1/3*c^2*(e*x+d)^3/e
```

Maxima [A] time = 1.22575, size = 39, normalized size = 2.29

$$\frac{1}{3}c^2e^2x^3 + c^2dex^2 + c^2d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/3*c^2*e^2*x^3 + c^2*d*e*x^2 + c^2*d^2*x
```

Fricas [A] time = 1.95184, size = 58, normalized size = 3.41

$$\frac{1}{3}c^2e^2x^3 + c^2dex^2 + c^2d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/3*c^2*e^2*x^3 + c^2*d*e*x^2 + c^2*d^2*x
```

Sympy [B] time = 0.115574, size = 29, normalized size = 1.71

$$c^2d^2x + c^2dex^2 + \frac{c^2e^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**2/(e*x+d)**2,x)
```

```
[Out] c**2*d**2*x + c**2*d*e*x**2 + c**2*e**2*x**3/3
```

Giac [A] time = 1.1677, size = 20, normalized size = 1.18

$$\frac{1}{3}(xe+d)^3c^2e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/3*(x*e + d)^3*c^2*e^(-1)
```

3.992 $\int \frac{(cd^2+2cdex+ce^2x^2)^2}{(d+ex)^3} dx$

Optimal. Leaf size=17

$$\frac{c^2(d+ex)^2}{2e}$$

```
[Out] (c^2*(d + e*x)^2)/(2*e)
```

Rubi [A] time = 0.0035307, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {27, 9}

$$\frac{c^2(d+ex)^2}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x)^3,x]
```

```
[Out] (c^2*(d + e*x)^2)/(2*e)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 9

```
Int[(a_)*((b_) + (c_.)*(x_)), x_Symbol] :> Simp[(a*(b + c*x)^2)/(2*c), x] /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{(cd^2+2cdex+ce^2x^2)^2}{(d+ex)^3} dx &= \int c^2(d+ex)\, dx \\ &= \frac{c^2(d+ex)^2}{2e} \end{aligned}$$

Mathematica [A] time = 0.000673, size = 16, normalized size = 0.94

$$c^2\left(dx+\frac{ex^2}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x)^3,x]
```

```
[Out] c^2*(d*x + (e*x^2)/2)
```

Maple [A] time = 0.038, size = 15, normalized size = 0.9

$$c^2\left(\frac{ex^2}{2}+dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^3,x)
```

```
[Out] c^2*(1/2*e*x^2+d*x)
```

Maxima [A] time = 1.16155, size = 22, normalized size = 1.29

$$\frac{1}{2}\,c^2ex^2+c^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*c^2*e*x^2 + c^2*d*x
```

Fricas [A] time = 2.05921, size = 34, normalized size = 2.

$$\frac{1}{2}\,c^2ex^2+c^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/2*c^2*e*x^2 + c^2*d*x
```

Sympy [A] time = 0.116893, size = 15, normalized size = 0.88

$$c^2dx+\frac{c^2ex^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**2/(e*x+d)**3,x)
```

```
[Out] c**2*d*x + c**2*e*x**2/2
```

Giac [A] time = 1.14977, size = 31, normalized size = 1.82

$$\frac{1}{2}\left(c^2x^2e^7+2\,c^2dxe^6\right)e^{(-6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/2*(c^2*x^2*e^7 + 2*c^2*d*x*e^6)*e^(-6)
```

3.993 $\int \frac{(cd^2+2cdex+ce^2x^2)^2}{(d+ex)^4} dx$

Optimal. Leaf size=5

$$c^2x$$

[Out] c^2*x

Rubi [A] time = 0.0022651, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {27, 8}

$$c^2x$$

Antiderivative was successfully verified.

[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x)^4,x]

[Out] c^2*x

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(cd^2+2cdex+ce^2x^2)^2}{(d+ex)^4} dx = \int c^2 dx$$
$$= c^2x$$

Mathematica [A] time = 0.0002489, size = 5, normalized size = 1.

$$c^2x$$

Antiderivative was successfully verified.

[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x)^4,x]

[Out] c^2*x

Maple [A] time = 0.04, size = 6, normalized size = 1.2

$$c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^4,x)
```

```
[Out] c^2*x
```

Maxima [A] time = 1.14836, size = 7, normalized size = 1.4

$$c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] c^2*x
```

Fricas [A] time = 2.01003, size = 9, normalized size = 1.8

$$c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] c^2*x
```

Sympy [A] time = 0.095872, size = 3, normalized size = 0.6

$$c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**2/(e*x+d)**4,x)
```

```
[Out] c**2*x
```

Giac [A] time = 1.16299, size = 7, normalized size = 1.4

$$c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] c^2*x
```

3.994 $\int \frac{(cd^2+2cdex+ce^2x^2)^2}{(d+ex)^5} dx$

Optimal. Leaf size=13

$$\frac{c^2 \log(d+ex)}{e}$$

```
[Out] (c^2*Log[d + e*x])/e
```

Rubi [A] time = 0.0044983, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {27, 12, 31}

$$\frac{c^2 \log(d+ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x)^5,x]
```

```
[Out] (c^2*Log[d + e*x])/e
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(cd^2+2cdex+ce^2x^2)^2}{(d+ex)^5} dx &= \int \frac{c^2}{d+ex} dx\\ &= c^2 \int \frac{1}{d+ex} dx\\ &= \frac{c^2 \log(d+ex)}{e}\end{aligned}$$

Mathematica [A] time = 0.0013471, size = 13, normalized size = 1.

$$\frac{c^2 \log(d+ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x)^5,x]
```

```
[Out] (c^2*Log[d + e*x])/e
```

Maple [A] time = 0.039, size = 14, normalized size = 1.1

$$\frac{c^2 \ln (ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^5,x)
```

```
[Out] c^2*ln(e*x+d)/e
```

Maxima [A] time = 1.17349, size = 18, normalized size = 1.38

$$\frac{c^2 \log (ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^5,x, algorithm="maxima")
```

```
[Out] c^2*log(e*x + d)/e
```

Fricas [A] time = 1.98157, size = 27, normalized size = 2.08

$$\frac{c^2 \log (ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^5,x, algorithm="fricas")
```

```
[Out] c^2*log(e*x + d)/e
```

Sympy [A] time = 0.138259, size = 10, normalized size = 0.77

$$\frac{c^2 \log (d + ex)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**2/(e*x+d)**5,x)
```

```
[Out] c**2*log(d + e*x)/e
```

Giac [A] time = 1.19822, size = 35, normalized size = 2.69

$$-c^2 e^{(-1)} \log\left(\frac{|xe + d|e^{(-1)}}{(xe + d)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] -c^2*e^(-1)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2)
```

3.995 $\int \frac{(cd^2+2cdex+ce^2x^2)^2}{(d+ex)^6} \, dx$

Optimal. Leaf size=15

$$-\frac{c^2}{e(d+ex)}$$

```
[Out] -(c^2/(e*(d + e*x)))
```

Rubi [A] time = 0.0046743, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {27, 12, 32}

$$-\frac{c^2}{e(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x)^6,x]

[Out] -(c^2/(e*(d + e*x)))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(cd^2+2cdex+ce^2x^2)^2}{(d+ex)^6} \, dx &= \int \frac{c^2}{(d+ex)^2} \, dx \\
&= c^2 \int \frac{1}{(d+ex)^2} \, dx \\
&= -\frac{c^2}{e(d+ex)}
\end{aligned}$$

Mathematica [A] time = 0.0017567, size = 15, normalized size = 1.

$$-\frac{c^2}{e(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x)^6,x]
```

```
[Out] -(c^2/(e*(d + e*x)))
```

Maple [A] time = 0.039, size = 16, normalized size = 1.1

$$-\frac{c^2}{e\,(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^6,x)
```

```
[Out] -c^2/e/(e*x+d)
```

Maxima [A] time = 1.17471, size = 22, normalized size = 1.47

$$-\frac{c^2}{e^2x+de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^6,x, algorithm="maxima")
```

```
[Out] -c^2/(e^2*x + d*e)
```

Fricas [A] time = 2.03494, size = 27, normalized size = 1.8

$$-\frac{c^2}{e^2x+de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^6,x, algorithm="fricas")
```

```
[Out] -c^2/(e^2*x + d*e)
```

Sympy [A] time = 0.314408, size = 12, normalized size = 0.8

$$-\frac{c^2}{de+e^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**2/(e*x+d)**6,x)
```

```
[Out] -c**2/(d*e + e**2*x)
```

Giac [B] time = 1.20877, size = 89, normalized size = 5.93

$$-\frac{\left(c^{2}x^{4}e^{8}+4\,c^{2}dx^{3}e^{7}+6\,c^{2}d^{2}x^{2}e^{6}+4\,c^{2}d^{3}xe^{5}+c^{2}d^{4}e^{4}\right)e^{(-5)}}{(xe+d)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^6,x, algorithm="giac")
```

```
[Out] -(c^2*x^4*e^8 + 4*c^2*d*x^3*e^7 + 6*c^2*d^2*x^2*e^6 + 4*c^2*d^3*x*e^5 + c^2
*d^4*e^4)*e^(-5)/(x*e + d)^5
```

3.996 $\int \frac{\left(cd^2+2cdex+ce^2x^2\right)^2}{(d+ex)^7} dx$

Optimal. Leaf size=17

$$-\frac{c^2}{2e(d+ex)^2}$$

```
[Out] -c^2/(2*e*(d + e*x)^2)
```

Rubi [A] time = 0.0051904, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {27, 12, 32}

$$-\frac{c^2}{2e(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x)^7,x]
```

```
[Out] -c^2/(2*e*(d + e*x)^2)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(cd^2+2cdex+ce^2x^2\right)^2}{(d+ex)^7} dx &= \int \frac{c^2}{(d+ex)^3} dx\\ &= c^2 \int \frac{1}{(d+ex)^3} dx\\ &= -\frac{c^2}{2e(d+ex)^2}\end{aligned}$$

Mathematica [A] time = 0.001941, size = 17, normalized size = 1.

$$-\frac{c^2}{2e(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x)^7,x]
```

```
[Out] -c^2/(2*e*(d + e*x)^2)
```

Maple [A] time = 0.04, size = 16, normalized size = 0.9

$$-\frac{c^2}{2\,e\,(ex+d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^7,x)
```

```
[Out] -1/2*c^2/e/(e*x+d)^2
```

Maxima [A] time = 1.21949, size = 36, normalized size = 2.12

$$-\frac{c^2}{2\left(e^3x^2+2\,de^2x+d^2e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^7,x, algorithm="maxima")
```

```
[Out] -1/2*c^2/(e^3*x^2 + 2*d*e^2*x + d^2*e)
```

Fricas [A] time = 1.9613, size = 54, normalized size = 3.18

$$-\frac{c^2}{2\left(e^3x^2+2\,de^2x+d^2e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^7,x, algorithm="fricas")
```

```
[Out] -1/2*c^2/(e^3*x^2 + 2*d*e^2*x + d^2*e)
```

Sympy [A] time = 0.690977, size = 27, normalized size = 1.59

$$-\frac{c^2}{2d^2e+4de^2x+2e^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**2/(e*x+d)**7,x)
```

```
[Out] -c**2/(2*d**2*e + 4*d*e**2*x + 2*e**3*x**2)
```

Giac [B] time = 1.22104, size = 89, normalized size = 5.24

$$-\frac{\left(c^2x^4e^8+4\,c^2dx^3e^7+6\,c^2d^2x^2e^6+4\,c^2d^3xe^5+c^2d^4e^4\right)e^{(-5)}}{2\,(xe+d)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^7,x, algorithm="giac")
```

```
[Out] -1/2*(c^2*x^4*e^8 + 4*c^2*d*x^3*e^7 + 6*c^2*d^2*x^2*e^6 + 4*c^2*d^3*x*e^5 +
 c^2*d^4*e^4)*e^(-5)/(x*e + d)^6
```

3.997 $\int \frac{\left(cd^2+2cdex+ce^2x^2\right)^2}{(d+ex)^8} dx$

Optimal. Leaf size=17

$$-\frac{c^2}{3e(d+ex)^3}$$

```
[Out] -c^2/(3*e*(d + e*x)^3)
```

Rubi [A] time = 0.0047521, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {27, 12, 32}

$$-\frac{c^2}{3e(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x)^8,x]
```

```
[Out] -c^2/(3*e*(d + e*x)^3)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(cd^2+2cdex+ce^2x^2\right)^2}{(d+ex)^8} \, dx &= \int \frac{c^2}{(d+ex)^4} \, dx \\
&= c^2 \int \frac{1}{(d+ex)^4} \, dx \\
&= -\frac{c^2}{3e(d+ex)^3}
\end{aligned}$$

Mathematica [A] time = 0.0019206, size = 17, normalized size = 1.

$$-\frac{c^2}{3e(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x)^8,x]
```

```
[Out] -c^2/(3*e*(d + e*x)^3)
```

Maple [A] time = 0.039, size = 16, normalized size = 0.9

$$-\frac{c^2}{3\,e\,(ex+d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^8,x)
```

```
[Out] -1/3*c^2/e/(e*x+d)^3
```

Maxima [B] time = 1.19668, size = 51, normalized size = 3.

$$-\frac{c^2}{3\left(e^4x^3+3\,de^3x^2+3\,d^2e^2x+d^3e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^8,x, algorithm="maxima")
```

```
[Out] -1/3*c^2/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e)
```

Fricas [B] time = 1.93112, size = 76, normalized size = 4.47

$$-\frac{c^2}{3\left(e^4x^3+3\,de^3x^2+3\,d^2e^2x+d^3e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^8,x, algorithm="fricas")
```

```
[Out] -1/3*c^2/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e)
```

Sympy [B] time = 0.66286, size = 39, normalized size = 2.29

$$-\frac{c^2}{3d^3e+9d^2e^2x+9de^3x^2+3e^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**2/(e*x+d)**8,x)
```

```
[Out] -c**2/(3*d**3*e + 9*d**2*e**2*x + 9*d*e**3*x**2 + 3*e**4*x**3)
```

Giac [B] time = 1.26211, size = 89, normalized size = 5.24

$$-\frac{\left(c^{2}x^{4}e^{8}+4\,c^{2}dx^{3}e^{7}+6\,c^{2}d^{2}x^{2}e^{6}+4\,c^{2}d^{3}xe^{5}+c^{2}d^{4}e^{4}\right)e^{(-5)}}{3\,(xe+d)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^8,x, algorithm="giac")
```

```
[Out] -1/3*(c^2*x^4*e^8 + 4*c^2*d*x^3*e^7 + 6*c^2*d^2*x^2*e^6 + 4*c^2*d^3*x*e^5 +
 c^2*d^4*e^4)*e^(-5)/(x*e + d)^7
```

3.998 $\int \frac{(d+ex)^5}{cd^2+2cdex+ce^2x^2} \, dx$

Optimal. Leaf size=17

$$\frac{(d+ex)^4}{4ce}$$

```
[Out] (d + e*x)^4/(4*c*e)
```

Rubi [A] time = 0.0047771, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {27, 12, 32}

$$\frac{(d+ex)^4}{4ce}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5/(c*d^2 + 2*c*d*e*x + c*e^2*x^2),x]
```

```
[Out] (d + e*x)^4/(4*c*e)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(d+ex)^5}{cd^2+2cdex+ce^2x^2} \, dx &= \int \frac{(d+ex)^3}{c} \, dx \\ &= \frac{\int (d+ex)^3 \, dx}{c} \\ &= \frac{(d+ex)^4}{4ce} \end{aligned}$$

Mathematica [A] time = 0.0009105, size = 17, normalized size = 1.

$$\frac{(d+ex)^4}{4ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5/(c*d^2 + 2*c*d*e*x + c*e^2*x^2),x]
```

```
[Out] (d + e*x)^4/(4*c*e)
```

Maple [A] time = 0.039, size = 16, normalized size = 0.9

$$\frac{(ex+d)^4}{4\,ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2),x)
```

```
[Out] 1/4*(e*x+d)^4/c/e
```

Maxima [B] time = 1.24114, size = 50, normalized size = 2.94

$$\frac{e^3x^4 + 4\,de^2x^3 + 6\,d^2ex^2 + 4\,d^3x}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="maxima")
```

```
[Out] 1/4*(e^3*x^4 + 4*d*e^2*x^3 + 6*d^2*e*x^2 + 4*d^3*x)/c
```

Fricas [B] time = 1.96849, size = 74, normalized size = 4.35

$$\frac{e^3x^4 + 4\,de^2x^3 + 6\,d^2ex^2 + 4\,d^3x}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="fricas")
```

```
[Out] 1/4*(e^3*x^4 + 4*d*e^2*x^3 + 6*d^2*e*x^2 + 4*d^3*x)/c
```

Sympy [B] time = 0.114562, size = 39, normalized size = 2.29

$$\frac{d^3x}{c} + \frac{3d^2ex^2}{2c} + \frac{de^2x^3}{c} + \frac{e^3x^4}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5/(c*e**2*x**2+2*c*d*e*x+c*d**2),x)
```

```
[Out] d**3*x/c + 3*d**2*e*x**2/(2*c) + d*e**2*x**3/c + e**3*x**4/(4*c)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.999 $\int \frac{(d+ex)^4}{cd^2+2cdex+ce^2x^2}\,dx$

Optimal. Leaf size=17

$$\frac{(d+ex)^3}{3ce}$$

```
[Out] (d + e*x)^3/(3*c*e)
```

Rubi [A] time = 0.0047593, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {27, 12, 32}

$$\frac{(d+ex)^3}{3ce}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(c*d^2 + 2*c*d*e*x + c*e^2*x^2),x]
```

```
[Out] (d + e*x)^3/(3*c*e)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^4}{cd^2+2cdex+ce^2x^2}\,dx &= \int \frac{(d+ex)^2}{c}\,dx\\ &= \frac{\int (d+ex)^2\,dx}{c}\\ &= \frac{(d+ex)^3}{3ce}\end{aligned}$$

Mathematica [A] time = 0.0008047, size = 17, normalized size = 1.

$$\frac{(d+ex)^3}{3ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(c*d^2 + 2*c*d*e*x + c*e^2*x^2),x]
```

```
[Out] (d + e*x)^3/(3*c*e)
```

Maple [A] time = 0.039, size = 16, normalized size = 0.9

$$\frac{(ex+d)^3}{3\,ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2),x)
```

```
[Out] 1/3*(e*x+d)^3/c/e
```

Maxima [A] time = 1.10039, size = 35, normalized size = 2.06

$$\frac{e^2x^3+3\,dex^2+3\,d^2x}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="maxima")
```

```
[Out] 1/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)/c
```

Fricas [A] time = 1.92547, size = 53, normalized size = 3.12

$$\frac{e^2x^3+3\,dex^2+3\,d^2x}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="fricas")
```

```
[Out] 1/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)/c
```

Sympy [B] time = 0.117196, size = 24, normalized size = 1.41

$$\frac{d^2x}{c}+\frac{dex^2}{c}+\frac{e^2x^3}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(c*e**2*x**2+2*c*d*e*x+c*d**2),x)
```

```
[Out] d**2*x/c + d*e*x**2/c + e**2*x**3/(3*c)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1000 $\int \frac{(d+ex)^3}{cd^2+2cdex+ce^2x^2}\,dx$

Optimal. Leaf size=17

$$\frac{(d+ex)^2}{2ce}$$

```
[Out] (d + e*x)^2/(2*c*e)
```

Rubi [A] time = 0.0034212, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {27, 9}

$$\frac{(d+ex)^2}{2ce}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(c*d^2 + 2*c*d*e*x + c*e^2*x^2),x]
```

```
[Out] (d + e*x)^2/(2*c*e)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 9

```
Int[(a_)*((b_) + (c_.)*(x_)), x_Symbol] :> Simp[(a*(b + c*x)^2)/(2*c), x] /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^3}{cd^2+2cdex+ce^2x^2}\,dx &= \int \frac{d+ex}{c}\,dx\\ &= \frac{(d+ex)^2}{2ce}\end{aligned}$$

Mathematica [A] time = 0.0005302, size = 16, normalized size = 0.94

$$\frac{dx+\frac{ex^2}{2}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(c*d^2 + 2*c*d*e*x + c*e^2*x^2),x]
```

```
[Out] (d*x + (e*x^2)/2)/c
```

Maple [A] time = 0.04, size = 15, normalized size = 0.9

$$\frac{1}{c}\left(\frac{ex^2}{2}+dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2),x)
```

```
[Out] 1/c*(1/2*e*x^2+d*x)
```

Maxima [A] time = 1.20135, size = 20, normalized size = 1.18

$$\frac{ex^2+2\,dx}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="maxima")
```

```
[Out] 1/2*(e*x^2 + 2*d*x)/c
```

Fricas [A] time = 1.93851, size = 31, normalized size = 1.82

$$\frac{ex^2+2\,dx}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="fricas")
```

```
[Out] 1/2*(e*x^2 + 2*d*x)/c
```

Sympy [A] time = 0.16671, size = 12, normalized size = 0.71

$$\frac{dx}{c}+\frac{ex^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*e**2*x**2+2*c*d*e*x+c*d**2),x)
```

```
[Out] d*x/c + e*x**2/(2*c)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1001 $\int \frac{(d+ex)^2}{cd^2+2cdex+ce^2x^2} dx$

Optimal. Leaf size=5

$$\frac{x}{c}$$

[Out] x/c

Rubi [A] time = 0.0023776, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {27, 8}

$$\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(c*d^2 + 2*c*d*e*x + c*e^2*x^2),x]
```

```
[Out] x/c
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \frac{(d+ex)^2}{cd^2+2cdex+ce^2x^2} \, dx &= \int \frac{1}{c} \, dx \\ &= \frac{x}{c} \end{aligned}$$

Mathematica [A] time = 0.0002426, size = 5, normalized size = 1.

$$\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(c*d^2 + 2*c*d*e*x + c*e^2*x^2),x]
```

```
[Out] x/c
```

Maple [A] time = 0.037, size = 6, normalized size = 1.2

$$\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2),x)
```

```
[Out] x/c
```

Maxima [A] time = 1.25048, size = 7, normalized size = 1.4

$$\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="maxima")
```

```
[Out] x/c
```

Fricas [A] time = 2.0046, size = 7, normalized size = 1.4

$$\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="fricas")
```

```
[Out] x/c
```

Sympy [A] time = 0.096697, size = 2, normalized size = 0.4

$$\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*e**2*x**2+2*c*d*e*x+c*d**2),x)
```

```
[Out] x/c
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1002 $\int \frac{d+ex}{cd^2+2cdex+ce^2x^2} dx$

Optimal. Leaf size=13

$$\frac{\log(d+ex)}{ce}$$

```
[Out] Log[d + e*x]/(c*e)
```

Rubi [A] time = 0.0040553, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {27, 12, 31}

$$\frac{\log(d+ex)}{ce}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(c*d^2 + 2*c*d*e*x + c*e^2*x^2),x]
```

```
[Out] Log[d + e*x]/(c*e)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex}{cd^2+2cdex+ce^2x^2} dx &= \int \frac{1}{c(d+ex)} dx\\ &= \frac{\int \frac{1}{d+ex} dx}{c}\\ &= \frac{\log(d+ex)}{ce}\end{aligned}$$

Mathematica [A] time = 0.0017076, size = 16, normalized size = 1.23

$$\frac{\log(cd+cex)}{ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(c*d^2 + 2*c*d*e*x + c*e^2*x^2),x]
```

```
[Out] Log[c*d + c*e*x]/(c*e)
```

Maple [A] time = 0.039, size = 14, normalized size = 1.1

$$\frac{\ln (ex + d)}{ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2),x)
```

```
[Out] ln(e*x+d)/c/e
```

Maxima [B] time = 1.17815, size = 39, normalized size = 3.

$$\frac{\log \left(ce^2x^2 + 2\,cdex + cd^2\right)}{2\,ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(c*e)
```

Fricas [A] time = 1.91153, size = 27, normalized size = 2.08

$$\frac{\log (ex + d)}{ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="fricas")
```

```
[Out] log(e*x + d)/(c*e)
```

Sympy [A] time = 0.137631, size = 12, normalized size = 0.92

$$\frac{\log (cd + cex)}{ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*e**2*x**2+2*c*d*e*x+c*d**2),x)
```

```
[Out] log(c*d + c*e*x)/(c*e)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1003 $\int \frac{1}{cd^2+2cdex+ce^2x^2} \, dx$

Optimal. Leaf size=15

$$-\frac{1}{ce(d+ex)}$$

```
[Out] -(1/(c*e*(d + e*x)))
```

Rubi [A] time = 0.0040105, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {27, 12, 32}

$$-\frac{1}{ce(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(-1),x]
```

```
[Out] -(1/(c*e*(d + e*x)))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{cd^2+2cdex+ce^2x^2} \, dx &= \int \frac{1}{c(d+ex)^2} \, dx\\ &= \frac{\int \frac{1}{(d+ex)^2} \, dx}{c}\\ &= -\frac{1}{ce(d+ex)}\end{aligned}$$

Mathematica [A] time = 0.0018185, size = 15, normalized size = 1.

$$-\frac{1}{ce(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(-1),x]
```

```
[Out] -(1/(c*e*(d + e*x)))
```

Maple [A] time = 0.042, size = 16, normalized size = 1.1

$$-\frac{1}{ce\left(ex+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*e^2*x^2+2*c*d*e*x+c*d^2),x)
```

```
[Out] -1/c/e/(e*x+d)
```

Maxima [A] time = 1.19525, size = 20, normalized size = 1.33

$$-\frac{1}{ce^2x+cde}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="maxima")
```

```
[Out] -1/(c*e^2*x + c*d*e)
```

Fricas [A] time = 1.9736, size = 30, normalized size = 2.

$$-\frac{1}{ce^2x+cde}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="fricas")
```

```
[Out] -1/(c*e^2*x + c*d*e)
```

Sympy [A] time = 0.327937, size = 14, normalized size = 0.93

$$-\frac{1}{cde+ce^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*e**2*x**2+2*c*d*e*x+c*d**2),x)
```

```
[Out] -1/(c*d*e + c*e**2*x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.1004 $\int \frac{1}{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)}\,dx$

Optimal. Leaf size=17

$$-\frac{1}{2ce(d+ex)^2}$$

```
[Out] -1/(2*c*e*(d + e*x)^2)
```

Rubi [A] time = 0.0047097, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {27, 12, 32}

$$-\frac{1}{2ce(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)),x]
```

```
[Out] -1/(2*c*e*(d + e*x)^2)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)}\,dx &= \int \frac{1}{c(d+ex)^3}\,dx\\ &= \frac{\int \frac{1}{(d+ex)^3}\,dx}{c}\\ &= -\frac{1}{2ce(d+ex)^2}\end{aligned}$$

Mathematica [A] time = 0.0020022, size = 17, normalized size = 1.

$$-\frac{1}{2ce(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)),x]
```

```
[Out] -1/(2*c*e*(d + e*x)^2)
```

Maple [A] time = 0.041, size = 16, normalized size = 0.9

$$-\frac{1}{2\,ce\,(ex+d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2),x)
```

```
[Out] -1/2/c/e/(e*x+d)^2
```

Maxima [A] time = 1.08266, size = 36, normalized size = 2.12

$$-\frac{1}{2\left(ce^3x^2+2\,cde^2x+cd^2e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="maxima")
```

```
[Out] -1/2/(c*e^3*x^2 + 2*c*d*e^2*x + c*d^2*e)
```

Fricas [A] time = 2.09091, size = 57, normalized size = 3.35

$$-\frac{1}{2\left(ce^3x^2+2\,cde^2x+cd^2e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="fricas")
```

```
[Out] -1/2/(c*e^3*x^2 + 2*c*d*e^2*x + c*d^2*e)
```

Sympy [B] time = 0.506623, size = 31, normalized size = 1.82

$$-\frac{1}{2cd^2e+4cde^2x+2ce^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*e**2*x**2+2*c*d*e*x+c*d**2),x)
```

```
[Out] -1/(2*c*d**2*e + 4*c*d*e**2*x + 2*c*e**3*x**2)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1005 $\int \frac{1}{(d+ex)^2(cd^2+2cdex+ce^2x^2)} dx$

Optimal. Leaf size=17

$$-\frac{1}{3ce(d+ex)^3}$$

```
[Out] -1/(3*c*e*(d + e*x)^3)
```

Rubi [A] time = 0.0046357, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {27, 12, 32}

$$-\frac{1}{3ce(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)),x]
```

```
[Out] -1/(3*c*e*(d + e*x)^3)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)^2\left(cd^2+2cdex+ce^2x^2\right)} dx &= \int \frac{1}{c(d+ex)^4} dx\\ &= \frac{\int \frac{1}{(d+ex)^4} dx}{c}\\ &= -\frac{1}{3ce(d+ex)^3}\end{aligned}$$

Mathematica [A] time = 0.002144, size = 17, normalized size = 1.

$$-\frac{1}{3ce(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)),x]
```

```
[Out] -1/(3*c*e*(d + e*x)^3)
```

Maple [A] time = 0.043, size = 16, normalized size = 0.9

$$-\frac{1}{3\,ce\,(ex+d)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2),x)
```

```
[Out] -1/3/c/e/(e*x+d)^3
```

Maxima [B] time = 1.27748, size = 53, normalized size = 3.12

$$-\frac{1}{3\left(ce^{4}x^{3}+3\,cde^{3}x^{2}+3\,cd^{2}e^{2}x+cd^{3}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="maxima")
```

```
[Out] -1/3/(c*e^4*x^3 + 3*c*d*e^3*x^2 + 3*c*d^2*e^2*x + c*d^3*e)
```

Fricas [B] time = 2.01887, size = 81, normalized size = 4.76

$$-\frac{1}{3\left(ce^{4}x^{3}+3\,cde^{3}x^{2}+3\,cd^{2}e^{2}x+cd^{3}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="fricas")
```

```
[Out] -1/3/(c*e^4*x^3 + 3*c*d*e^3*x^2 + 3*c*d^2*e^2*x + c*d^3*e)
```

Sympy [B] time = 0.486444, size = 44, normalized size = 2.59

$$-\frac{1}{3cd^{3}e+9cd^{2}e^{2}x+9cde^{3}x^{2}+3ce^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*e**2*x**2+2*c*d*e*x+c*d**2),x)
```

```
[Out] -1/(3*c*d**3*e + 9*c*d**2*e**2*x + 9*c*d*e**3*x**2 + 3*c*e**4*x**3)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1006 $\int \frac{1}{(d+ex)^3(cd^2+2cdex+ce^2x^2)}\,dx$

Optimal. Leaf size=17

$$-\frac{1}{4ce(d+ex)^4}$$

```
[Out] -1/(4*c*e*(d + e*x)^4)
```

Rubi [A] time = 0.004754, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {27, 12, 32}

$$-\frac{1}{4ce(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)),x]
```

```
[Out] -1/(4*c*e*(d + e*x)^4)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)^3\left(cd^2+2cdex+ce^2x^2\right)}\,dx &= \int \frac{1}{c(d+ex)^5}\,dx\\
&= \frac{\int \frac{1}{(d+ex)^5}\,dx}{c}\\
&= -\frac{1}{4ce(d+ex)^4}
\end{aligned}$$

Mathematica [A] time = 0.004563, size = 17, normalized size = 1.

$$-\frac{1}{4ce(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)),x]
```

```
[Out] -1/(4*c*e*(d + e*x)^4)
```

Maple [A] time = 0.043, size = 16, normalized size = 0.9

$$-\frac{1}{4\,ce\left(ex+d\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2),x)
```

```
[Out] -1/4/c/e/(e*x+d)^4
```

Maxima [B] time = 1.00505, size = 69, normalized size = 4.06

$$-\frac{1}{4\left(ce^{5}x^{4}+4\,cde^{4}x^{3}+6\,cd^{2}e^{3}x^{2}+4\,cd^{3}e^{2}x+cd^{4}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="maxima")
```

```
[Out] -1/4/(c*e^5*x^4 + 4*c*d*e^4*x^3 + 6*c*d^2*e^3*x^2 + 4*c*d^3*e^2*x + c*d^4*e
)
```

Fricas [B] time = 1.85085, size = 105, normalized size = 6.18

$$-\frac{1}{4\left(ce^{5}x^{4}+4\,cde^{4}x^{3}+6\,cd^{2}e^{3}x^{2}+4\,cd^{3}e^{2}x+cd^{4}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="fricas")
```

```
[Out] -1/4/(c*e^5*x^4 + 4*c*d*e^4*x^3 + 6*c*d^2*e^3*x^2 + 4*c*d^3*e^2*x + c*d^4*e
)
```

Sympy [B] time = 0.65278, size = 58, normalized size = 3.41

$$-\frac{1}{4cd^{4}e+16cd^{3}e^{2}x+24cd^{2}e^{3}x^{2}+16cde^{4}x^{3}+4ce^{5}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*e**2*x**2+2*c*d*e*x+c*d**2),x)
```

```
[Out] -1/(4*c*d**4*e + 16*c*d**3*e**2*x + 24*c*d**2*e**3*x**2 + 16*c*d*e**4*x**3
+ 4*c*e**5*x**4)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1007 $\int \frac{(d+ex)^7}{\left(cd^2+2cdex+ce^2x^2\right)^2}\,dx$

Optimal. Leaf size=17

$$\frac{(d+ex)^4}{4c^2e}$$

[Out] (d + e*x)^4/(4*c^2*e)

Rubi [A] time = 0.0049706, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {27, 12, 32}

$$\frac{(d+ex)^4}{4c^2e}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^7/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]

[Out] (d + e*x)^4/(4*c^2*e)

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^7}{\left(cd^2+2cdex+ce^2x^2\right)^2}\,dx &= \int \frac{(d+ex)^3}{c^2}\,dx\\ &= \frac{\int (d+ex)^3\,dx}{c^2}\\ &= \frac{(d+ex)^4}{4c^2e}\end{aligned}$$

Mathematica [A] time = 0.0010034, size = 17, normalized size = 1.

$$\frac{(d+ex)^4}{4c^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^7/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]
```

```
[Out] (d + e*x)^4/(4*c^2*e)
```

Maple [A] time = 0.038, size = 16, normalized size = 0.9

$$\frac{(ex+d)^4}{4\,c^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^7/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x)
```

```
[Out] 1/4*(e*x+d)^4/c^2/e
```

Maxima [B] time = 1.02417, size = 50, normalized size = 2.94

$$\frac{e^3x^4+4\,de^2x^3+6\,d^2ex^2+4\,d^3x}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(e^3*x^4 + 4*d*e^2*x^3 + 6*d^2*e*x^2 + 4*d^3*x)/c^2
```

Fricas [B] time = 2.06592, size = 77, normalized size = 4.53

$$\frac{e^3x^4+4\,de^2x^3+6\,d^2ex^2+4\,d^3x}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(e^3*x^4 + 4*d*e^2*x^3 + 6*d^2*e*x^2 + 4*d^3*x)/c^2
```

Sympy [B] time = 0.162961, size = 46, normalized size = 2.71

$$\frac{d^3x}{c^2}+\frac{3d^2ex^2}{2c^2}+\frac{de^2x^3}{c^2}+\frac{e^3x^4}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**7/(c*e**2*x**2+2*c*d*e*x+c*d**2)**2,x)
```

```
[Out] d**3*x/c**2 + 3*d**2*e*x**2/(2*c**2) + d*e**2*x**3/c**2 + e**3*x**4/(4*c**2
)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1008 $\int \frac{(d+ex)^6}{\left(cd^2+2cdex+ce^2x^2\right)^2}\,dx$

Optimal. Leaf size=17

$$\frac{(d+ex)^3}{3c^2e}$$

```
[Out] (d + e*x)^3/(3*c^2*e)
```

Rubi [A] time = 0.0048266, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {27, 12, 32}

$$\frac{(d+ex)^3}{3c^2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^6/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]
```

```
[Out] (d + e*x)^3/(3*c^2*e)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^6}{\left(cd^2+2cdex+ce^2x^2\right)^2}\,dx &= \int \frac{(d+ex)^2}{c^2}\,dx\\ &= \frac{\int (d+ex)^2\,dx}{c^2}\\ &= \frac{(d+ex)^3}{3c^2e}\end{aligned}$$

Mathematica [A] time = 0.0008689, size = 17, normalized size = 1.

$$\frac{(d+ex)^3}{3c^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^6/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]
```

```
[Out] (d + e*x)^3/(3*c^2*e)
```

Maple [A] time = 0.039, size = 16, normalized size = 0.9

$$\frac{(ex+d)^3}{3c^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^6/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x)
```

```
[Out] 1/3*(e*x+d)^3/c^2/e
```

Maxima [A] time = 1.0044, size = 35, normalized size = 2.06

$$\frac{e^2x^3+3\,dex^2+3\,d^2x}{3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)/c^2
```

Fricas [A] time = 1.87924, size = 55, normalized size = 3.24

$$\frac{e^2x^3+3\,dex^2+3\,d^2x}{3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)/c^2
```

Sympy [B] time = 0.131206, size = 29, normalized size = 1.71

$$\frac{d^2x}{c^2}+\frac{dex^2}{c^2}+\frac{e^2x^3}{3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**6/(c*e**2*x**2+2*c*d*e*x+c*d**2)**2,x)
```

```
[Out] d**2*x/c**2 + d*e*x**2/c**2 + e**2*x**3/(3*c**2)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1009 $\int \frac{(d+ex)^5}{\left(cd^2+2cdex+ce^2x^2\right)^2} dx$

Optimal. Leaf size=17

$$\frac{(d+ex)^2}{2c^2e}$$

```
[Out] (d + e*x)^2/(2*c^2*e)
```

Rubi [A] time = 0.0035537, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {27, 9}

$$\frac{(d+ex)^2}{2c^2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]
```

```
[Out] (d + e*x)^2/(2*c^2*e)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 9

```
Int[(a_)*((b_) + (c_.)*(x_)), x_Symbol] :> Simp[(a*(b + c*x)^2)/(2*c), x] /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{(d+ex)^5}{\left(cd^2+2cdex+ce^2x^2\right)^2} dx = \int \frac{d+ex}{c^2} dx$$
$$= \frac{(d+ex)^2}{2c^2e}$$

Mathematica [A] time = 0.0006573, size = 16, normalized size = 0.94

$$\frac{dx+\frac{ex^2}{2}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]
```

```
[Out] (d*x + (e*x^2)/2)/c^2
```

Maple [A] time = 0.04, size = 15, normalized size = 0.9

$$\frac{1}{c^2}\left(\frac{ex^2}{2}+dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x)
```

```
[Out] 1/c^2*(1/2*e*x^2+d*x)
```

Maxima [A] time = 1.11812, size = 20, normalized size = 1.18

$$\frac{ex^2+2\,dx}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(e*x^2 + 2*d*x)/c^2
```

Fricas [A] time = 2.01251, size = 34, normalized size = 2.

$$\frac{ex^2+2\,dx}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(e*x^2 + 2*d*x)/c^2
```

Sympy [A] time = 0.138297, size = 15, normalized size = 0.88

$$\frac{dx}{c^2}+\frac{ex^2}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5/(c*e**2*x**2+2*c*d*e*x+c*d**2)**2,x)
```

```
[Out] d*x/c**2 + e*x**2/(2*c**2)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1010 $\int \frac{(d+ex)^4}{\left(cd^2+2cdex+ce^2x^2\right)^2}\,dx$

Optimal. Leaf size=5

$$\frac{x}{c^2}$$

```
[Out] x/c^2
```

Rubi [A] time = 0.0027614, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {27, 8}

$$\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]
```

```
[Out] x/c^2
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \frac{(d+ex)^4}{\left(cd^2+2cdex+ce^2x^2\right)^2}\,dx &= \int \frac{1}{c^2}\,dx \\ &= \frac{x}{c^2} \end{aligned}$$

Mathematica [A] time = 0.0003174, size = 5, normalized size = 1.

$$\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]
```

```
[Out] x/c^2
```

Maple [A] time = 0.04, size = 6, normalized size = 1.2

$$\frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x)
```

```
[Out] x/c^2
```

Maxima [A] time = 1.08951, size = 7, normalized size = 1.4

$$\frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="maxima")
```

```
[Out] x/c^2
```

Fricas [A] time = 2.04359, size = 9, normalized size = 1.8

$$\frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="fricas")
```

```
[Out] x/c^2
```

Sympy [A] time = 0.12804, size = 3, normalized size = 0.6

$$\frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(c*e**2*x**2+2*c*d*e*x+c*d**2)**2,x)
```

```
[Out] x/c**2
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1011 $\int \frac{(d+ex)^3}{\left(cd^2+2cdex+ce^2x^2\right)^2} dx$

Optimal. Leaf size=13

$$\frac{\log(d+ex)}{c^2e}$$

```
[Out] Log[d + e*x]/(c^2*e)
```

Rubi [A] time = 0.0046287, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {27, 12, 31}

$$\frac{\log(d+ex)}{c^2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]
```

```
[Out] Log[d + e*x]/(c^2*e)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^3}{\left(cd^2+2cdex+ce^2x^2\right)^2} dx &= \int \frac{1}{c^2(d+ex)} dx\\
&= \frac{\int \frac{1}{d+ex} dx}{c^2}\\
&= \frac{\log(d+ex)}{c^2e}
\end{aligned}$$

Mathematica [A] time = 0.0014898, size = 13, normalized size = 1.

$$\frac{\log(d+ex)}{c^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]
```

```
[Out] Log[d + e*x]/(c^2*e)
```

Maple [A] time = 0.041, size = 14, normalized size = 1.1

$$\frac{\ln(ex+d)}{c^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x)
```

```
[Out] ln(e*x+d)/c^2/e
```

Maxima [A] time = 1.16106, size = 18, normalized size = 1.38

$$\frac{\log(ex+d)}{c^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="maxima")
```

```
[Out] log(e*x + d)/(c^2*e)
```

Fricas [A] time = 1.828, size = 30, normalized size = 2.31

$$\frac{\log(ex+d)}{c^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="fricas")
```

```
[Out] log(e*x + d)/(c^2*e)
```

Sympy [A] time = 0.162771, size = 17, normalized size = 1.31

$$\frac{\log\left(c^2d+c^2ex\right)}{c^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*e**2*x**2+2*c*d*e*x+c*d**2)**2,x)
```

```
[Out] log(c**2*d + c**2*e*x)/(c**2*e)
```

<u>Giac</u> [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.1012 $\int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^2} dx$

Optimal. Leaf size=15

$$-\frac{1}{c^2e(d+ex)}$$

```
[Out] -(1/(c^2*e*(d + e*x)))
```

Rubi [A] time = 0.0048716, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {27, 12, 32}

$$-\frac{1}{c^2e(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]
```

```
[Out] -(1/(c^2*e*(d + e*x)))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^2}\,dx &= \int \frac{1}{c^2(d+ex)^2}\,dx\\ &= \frac{\int \frac{1}{(d+ex)^2}\,dx}{c^2}\\ &= -\frac{1}{c^2e(d+ex)}\end{aligned}$$

Mathematica [A] time = 0.0021272, size = 15, normalized size = 1.

$$-\frac{1}{c^2e(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]
```

```
[Out] -(1/(c^2*e*(d + e*x)))
```

Maple [A] time = 0.039, size = 16, normalized size = 1.1

$$-\frac{1}{c^2e\,(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x)
```

```
[Out] -1/c^2/e/(e*x+d)
```

Maxima [A] time = 1.08718, size = 26, normalized size = 1.73

$$-\frac{1}{c^2e^2x+c^2de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="maxima")
```

```
[Out] -1/(c^2*e^2*x + c^2*d*e)
```

Fricas [A] time = 1.92588, size = 35, normalized size = 2.33

$$-\frac{1}{c^2e^2x+c^2de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="fricas")
```

```
[Out] -1/(c^2*e^2*x + c^2*d*e)
```

Sympy [A] time = 0.52727, size = 17, normalized size = 1.13

$$-\frac{1}{c^2de+c^2e^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*e**2*x**2+2*c*d*e*x+c*d**2)**2,x)
```

```
[Out] -1/(c**2*d*e + c**2*e**2*x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1013 $\int \frac{d+ex}{(cd^2+2cdex+ce^2x^2)^2}\,dx$

Optimal. Leaf size=17

$$-\frac{1}{2c^2e(d+ex)^2}$$

```
[Out] -1/(2*c^2*e*(d + e*x)^2)
```

Rubi [A] time = 0.0043401, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {27, 12, 32}

$$-\frac{1}{2c^2e(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]
```

```
[Out] -1/(2*c^2*e*(d + e*x)^2)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex}{(cd^2+2cdex+ce^2x^2)^2}\,dx &= \int \frac{1}{c^2(d+ex)^3}\,dx\\ &= \frac{\int \frac{1}{(d+ex)^3}\,dx}{c^2}\\ &= -\frac{1}{2c^2e(d+ex)^2}\end{aligned}$$

Mathematica [A] time = 0.0020644, size = 17, normalized size = 1.

$$-\frac{1}{2c^2e(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]
```

```
[Out] -1/(2*c^2*e*(d + e*x)^2)
```

Maple [A] time = 0.041, size = 16, normalized size = 0.9

$$-\frac{1}{2\,c^2e\,(ex+d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x)
```

```
[Out] -1/2/c^2/e/(e*x+d)^2
```

Maxima [B] time = 1.08769, size = 41, normalized size = 2.41

$$-\frac{1}{2\left(ce^2x^2+2\,cdex+cd^2\right)ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2/((c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c*e)
```

Fricas [B] time = 2.03139, size = 65, normalized size = 3.82

$$-\frac{1}{2\left(c^2e^3x^2+2\,c^2de^2x+c^2d^2e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2/(c^2*e^3*x^2 + 2*c^2*d*e^2*x + c^2*d^2*e)
```

Sympy [B] time = 0.621539, size = 36, normalized size = 2.12

$$-\frac{1}{2c^2d^2e+4c^2de^2x+2c^2e^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*e**2*x**2+2*c*d*e*x+c*d**2)**2,x)
```

```
[Out] -1/(2*c**2*d**2*e + 4*c**2*d*e**2*x + 2*c**2*e**3*x**2)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1014 $\int \frac{1}{\left(cd^2+2cdex+ce^2x^2\right)^2} dx$

Optimal. Leaf size=17

$$-\frac{1}{3c^2e(d+ex)^3}$$

```
[Out] -1/(3*c^2*e*(d + e*x)^3)
```

Rubi [A] time = 0.004028, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {27, 12, 32}

$$-\frac{1}{3c^2e(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(-2),x]

[Out] -1/(3*c^2*e*(d + e*x)^3)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(cd^2+2cdex+ce^2x^2\right)^2} dx &= \int \frac{1}{c^2(d+ex)^4} dx\\ &= \frac{\int \frac{1}{(d+ex)^4} dx}{c^2}\\ &= -\frac{1}{3c^2e(d+ex)^3}\end{aligned}$$

Mathematica [A] time = 0.0020688, size = 17, normalized size = 1.

$$-\frac{1}{3c^2e(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(-2),x]
```

```
[Out] -1/(3*c^2*e*(d + e*x)^3)
```

Maple [A] time = 0.041, size = 16, normalized size = 0.9

$$-\frac{1}{3\,c^2e\,(ex+d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x)
```

```
[Out] -1/3/c^2/e/(e*x+d)^3
```

Maxima [B] time = 1.00626, size = 63, normalized size = 3.71

$$-\frac{1}{3\left(c^2e^4x^3+3\,c^2de^3x^2+3\,c^2d^2e^2x+c^2d^3e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="maxima")
```

```
[Out] -1/3/(c^2*e^4*x^3 + 3*c^2*d*e^3*x^2 + 3*c^2*d^2*e^2*x + c^2*d^3*e)
```

Fricas [B] time = 1.95349, size = 92, normalized size = 5.41

$$-\frac{1}{3\left(c^2e^4x^3+3\,c^2de^3x^2+3\,c^2d^2e^2x+c^2d^3e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3/(c^2*e^4*x^3 + 3*c^2*d*e^3*x^2 + 3*c^2*d^2*e^2*x + c^2*d^3*e)
```

Sympy [B] time = 0.692472, size = 51, normalized size = 3.

$$-\frac{1}{3c^2d^3e+9c^2d^2e^2x+9c^2de^3x^2+3c^2e^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*e**2*x**2+2*c*d*e*x+c*d**2)**2,x)
```

```
[Out] -1/(3*c**2*d**3*e + 9*c**2*d**2*e**2*x + 9*c**2*d*e**3*x**2 + 3*c**2*e**4*x
**3)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1015 $\int \frac{1}{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^2} dx$

Optimal. Leaf size=17

$$-\frac{1}{4c^2e(d+ex)^4}$$

```
[Out] -1/(4*c^2*e*(d + e*x)^4)
```

Rubi [A] time = 0.0047134, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {27, 12, 32}

$$-\frac{1}{4c^2e(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2),x]
```

```
[Out] -1/(4*c^2*e*(d + e*x)^4)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^2} dx &= \int \frac{1}{c^2(d+ex)^5} dx \\
&= \frac{\int \frac{1}{(d+ex)^5} dx}{c^2} \\
&= -\frac{1}{4c^2e(d+ex)^4}
\end{aligned}$$

Mathematica [A] time = 0.0021628, size = 17, normalized size = 1.

$$-\frac{1}{4c^2e(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2),x]
```

```
[Out] -1/(4*c^2*e*(d + e*x)^4)
```

Maple [A] time = 0.042, size = 16, normalized size = 0.9

$$-\frac{1}{4\,c^{2}e\left(ex+d\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x)
```

```
[Out] -1/4/c^2/e/(e*x+d)^4
```

Maxima [B] time = 1.03265, size = 82, normalized size = 4.82

$$-\frac{1}{4\left(c^{2}e^{5}x^{4}+4\,c^{2}de^{4}x^{3}+6\,c^{2}d^{2}e^{3}x^{2}+4\,c^{2}d^{3}e^{2}x+c^{2}d^{4}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4/(c^2*e^5*x^4 + 4*c^2*d*e^4*x^3 + 6*c^2*d^2*e^3*x^2 + 4*c^2*d^3*e^2*x +
 c^2*d^4*e)
```

Fricas [B] time = 1.97832, size = 119, normalized size = 7.

$$-\frac{1}{4\left(c^{2}e^{5}x^{4}+4\,c^{2}de^{4}x^{3}+6\,c^{2}d^{2}e^{3}x^{2}+4\,c^{2}d^{3}e^{2}x+c^{2}d^{4}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="fricas")
```

```
[Out] -1/4/(c^2*e^5*x^4 + 4*c^2*d*e^4*x^3 + 6*c^2*d^2*e^3*x^2 + 4*c^2*d^3*e^2*x +
 c^2*d^4*e)
```

Sympy [B] time = 0.601638, size = 66, normalized size = 3.88

$$-\frac{1}{4c^{2}d^{4}e+16c^{2}d^{3}e^{2}x+24c^{2}d^{2}e^{3}x^{2}+16c^{2}de^{4}x^{3}+4c^{2}e^{5}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*e**2*x**2+2*c*d*e*x+c*d**2)**2,x)
```

```
[Out] -1/(4*c**2*d**4*e + 16*c**2*d**3*e**2*x + 24*c**2*d**2*e**3*x**2 + 16*c**2*
d*e**4*x**3 + 4*c**2*e**5*x**4)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1016 $\int \frac{1}{(d+ex)^2\left(cd^2+2cdex+ce^2x^2\right)^2}\,dx$

Optimal. Leaf size=17

$$-\frac{1}{5c^2e(d+ex)^5}$$

```
[Out] -1/(5*c^2*e*(d + e*x)^5)
```

Rubi [A] time = 0.0047017, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {27, 12, 32}

$$-\frac{1}{5c^2e(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2),x]

[Out] -1/(5*c^2*e*(d + e*x)^5)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)^2\left(cd^2+2cdex+ce^2x^2\right)^2}\,dx &= \int \frac{1}{c^2(d+ex)^6}\,dx\\ &= \frac{\int \frac{1}{(d+ex)^6}\,dx}{c^2}\\ &= -\frac{1}{5c^2e(d+ex)^5}\end{aligned}$$

Mathematica [A] time = 0.0045773, size = 17, normalized size = 1.

$$-\frac{1}{5c^2e(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2),x]
```

```
[Out] -1/(5*c^2*e*(d + e*x)^5)
```

Maple [A] time = 0.043, size = 16, normalized size = 0.9

$$-\frac{1}{5\,c^2e\,(ex+d)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x)
```

```
[Out] -1/5/c^2/e/(e*x+d)^5
```

Maxima [B] time = 1.0261, size = 101, normalized size = 5.94

$$-\frac{1}{5\,\left(c^2e^6x^5+5\,c^2de^5x^4+10\,c^2d^2e^4x^3+10\,c^2d^3e^3x^2+5\,c^2d^4e^2x+c^2d^5e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="maxima")
```

```
[Out] -1/5/(c^2*e^6*x^5 + 5*c^2*d*e^5*x^4 + 10*c^2*d^2*e^4*x^3 + 10*c^2*d^3*e^3*x
^2 + 5*c^2*d^4*e^2*x + c^2*d^5*e)
```

Fricas [B] time = 2.10266, size = 149, normalized size = 8.76

$$-\frac{1}{5\,\left(c^2e^6x^5+5\,c^2de^5x^4+10\,c^2d^2e^4x^3+10\,c^2d^3e^3x^2+5\,c^2d^4e^2x+c^2d^5e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="fricas")
```

```
[Out] -1/5/(c^2*e^6*x^5 + 5*c^2*d*e^5*x^4 + 10*c^2*d^2*e^4*x^3 + 10*c^2*d^3*e^3*x
^2 + 5*c^2*d^4*e^2*x + c^2*d^5*e)
```

Sympy [B] time = 0.822059, size = 82, normalized size = 4.82

$$-\frac{1}{5c^2d^5e+25c^2d^4e^2x+50c^2d^3e^3x^2+50c^2d^2e^4x^3+25c^2de^5x^4+5c^2e^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*e**2*x**2+2*c*d*e*x+c*d**2)**2,x)
```

```
[Out] -1/(5*c**2*d**5*e + 25*c**2*d**4*e**2*x + 50*c**2*d**3*e**3*x**2 + 50*c**2*
d**2*e**4*x**3 + 25*c**2*d*e**5*x**4 + 5*c**2*e**6*x**5)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1017 $\int \frac{(d+ex)^9}{\left(cd^2+2cdex+ce^2x^2\right)^3}\,dx$

Optimal. Leaf size=17

$$\frac{(d+ex)^4}{4c^3e}$$

```
[Out] (d + e*x)^4/(4*c^3*e)
```

Rubi [A] time = 0.0050063, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {27, 12, 32}

$$\frac{(d+ex)^4}{4c^3e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^9/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]
```

```
[Out] (d + e*x)^4/(4*c^3*e)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^9}{\left(cd^2+2cdex+ce^2x^2\right)^3}\,dx &= \int \frac{(d+ex)^3}{c^3}\,dx\\ &= \frac{\int (d+ex)^3\,dx}{c^3}\\ &= \frac{(d+ex)^4}{4c^3e}\end{aligned}$$

Mathematica [A] time = 0.0014609, size = 17, normalized size = 1.

$$\frac{(d+ex)^4}{4c^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^9/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]
```

```
[Out] (d + e*x)^4/(4*c^3*e)
```

Maple [A] time = 0.04, size = 16, normalized size = 0.9

$$\frac{(ex+d)^4}{4c^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^9/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x)
```

```
[Out] 1/4*(e*x+d)^4/c^3/e
```

Maxima [B] time = 1.03142, size = 50, normalized size = 2.94

$$\frac{e^3x^4 + 4\,de^2x^3 + 6\,d^2ex^2 + 4\,d^3x}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^9/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(e^3*x^4 + 4*d*e^2*x^3 + 6*d^2*e*x^2 + 4*d^3*x)/c^3
```

Fricas [B] time = 1.94569, size = 77, normalized size = 4.53

$$\frac{e^3x^4 + 4\,de^2x^3 + 6\,d^2ex^2 + 4\,d^3x}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^9/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(e^3*x^4 + 4*d*e^2*x^3 + 6*d^2*e*x^2 + 4*d^3*x)/c^3
```

Sympy [B] time = 0.163009, size = 46, normalized size = 2.71

$$\frac{d^3x}{c^3} + \frac{3d^2ex^2}{2c^3} + \frac{de^2x^3}{c^3} + \frac{e^3x^4}{4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**9/(c*e**2*x**2+2*c*d*e*x+c*d**2)**3,x)
```

```
[Out] d**3*x/c**3 + 3*d**2*e*x**2/(2*c**3) + d*e**2*x**3/c**3 + e**3*x**4/(4*c**3
)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^9/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1018 $\int \frac{(d+ex)^8}{\left(cd^2+2cdex+ce^2x^2\right)^3} \, dx$

Optimal. Leaf size=17

$$\frac{(d+ex)^3}{3c^3e}$$

[Out] (d + e*x)^3/(3*c^3*e)

Rubi [A] time = 0.0048614, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {27, 12, 32}

$$\frac{(d+ex)^3}{3c^3e}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^8/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]

[Out] (d + e*x)^3/(3*c^3*e)

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^8}{\left(cd^2+2cdex+ce^2x^2\right)^3} \, dx &= \int \frac{(d+ex)^2}{c^3} \, dx\\ &= \frac{\int (d+ex)^2 \, dx}{c^3}\\ &= \frac{(d+ex)^3}{3c^3e}\end{aligned}$$

Mathematica [A] time = 0.0009025, size = 17, normalized size = 1.

$$\frac{(d+ex)^3}{3c^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^8/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]
```

```
[Out] (d + e*x)^3/(3*c^3*e)
```

Maple [A] time = 0.04, size = 16, normalized size = 0.9

$$\frac{(ex+d)^3}{3\,c^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^8/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x)
```

```
[Out] 1/3*(e*x+d)^3/c^3/e
```

Maxima [A] time = 1.07916, size = 35, normalized size = 2.06

$$\frac{e^2x^3+3\,dex^2+3\,d^2x}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^8/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)/c^3
```

Fricas [A] time = 1.98393, size = 55, normalized size = 3.24

$$\frac{e^2x^3+3\,dex^2+3\,d^2x}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^8/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)/c^3
```

Sympy [B] time = 0.145524, size = 29, normalized size = 1.71

$$\frac{d^2x}{c^3}+\frac{dex^2}{c^3}+\frac{e^2x^3}{3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**8/(c*e**2*x**2+2*c*d*e*x+c*d**2)**3,x)
```

```
[Out] d**2*x/c**3 + d*e*x**2/c**3 + e**2*x**3/(3*c**3)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^8/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1019 $\int \frac{(d+ex)^7}{\left(cd^2+2cdex+ce^2x^2\right)^3} dx$

Optimal. Leaf size=17

$$\frac{(d+ex)^2}{2c^3e}$$

```
[Out] (d + e*x)^2/(2*c^3*e)
```

Rubi [A] time = 0.0035288, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {27, 9}

$$\frac{(d+ex)^2}{2c^3e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^7/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]
```

```
[Out] (d + e*x)^2/(2*c^3*e)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Cancel[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 9

```
Int[(a_)*((b_) + (c_.)*(x_)), x_Symbol] :> Simp[(a*(b + c*x)^2)/(2*c), x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^7}{\left(cd^2+2cdex+ce^2x^2\right)^3} dx &= \int \frac{d+ex}{c^3} dx\\ &= \frac{(d+ex)^2}{2c^3e}\end{aligned}$$

Mathematica [A] time = 0.0006766, size = 16, normalized size = 0.94

$$\frac{dx+\frac{ex^2}{2}}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^7/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]
```

```
[Out] (d*x + (e*x^2)/2)/c^3
```

Maple [A] time = 0.039, size = 15, normalized size = 0.9

$$\frac{1}{c^3}\left(\frac{ex^2}{2}+dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^7/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x)
```

```
[Out] 1/c^3*(1/2*e*x^2+d*x)
```

Maxima [A] time = 1.06581, size = 20, normalized size = 1.18

$$\frac{ex^2+2\,dx}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(e*x^2 + 2*d*x)/c^3
```

Fricas [A] time = 1.97146, size = 34, normalized size = 2.

$$\frac{ex^2+2\,dx}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(e*x^2 + 2*d*x)/c^3
```

Sympy [A] time = 0.148425, size = 15, normalized size = 0.88

$$\frac{dx}{c^3}+\frac{ex^2}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**7/(c*e**2*x**2+2*c*d*e*x+c*d**2)**3,x)
```

```
[Out] d*x/c**3 + e*x**2/(2*c**3)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1020 $\int \frac{(d+ex)^6}{(cd^2+2cdex+ce^2x^2)^3} dx$

Optimal. Leaf size=5

$$\frac{x}{c^3}$$

[Out] x/c^3

Rubi [A] time = 0.002476, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {27, 8}

$$\frac{x}{c^3}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^6/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]

[Out] x/c^3

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \frac{(d+ex)^6}{\left(cd^2+2cdex+ce^2x^2\right)^3} dx &= \int \frac{1}{c^3} dx \\ &= \frac{x}{c^3} \end{aligned}$$

Mathematica [A] time = 0.0003703, size = 5, normalized size = 1.

$$\frac{x}{c^3}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^6/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]

[Out] x/c^3

Maple [A] time = 0.038, size = 6, normalized size = 1.2

$$\frac{x}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^6/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x)
```

```
[Out] x/c^3
```

Maxima [A] time = 0.987178, size = 7, normalized size = 1.4

$$\frac{x}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="maxima")
```

```
[Out] x/c^3
```

Fricas [A] time = 2.01358, size = 9, normalized size = 1.8

$$\frac{x}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="fricas")
```

```
[Out] x/c^3
```

Sympy [A] time = 0.134455, size = 3, normalized size = 0.6

$$\frac{x}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**6/(c*e**2*x**2+2*c*d*e*x+c*d**2)**3,x)
```

```
[Out] x/c**3
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1021 $\int \frac{(d+ex)^5}{\left(cd^2+2cdex+ce^2x^2\right)^3} \, dx$

Optimal. Leaf size=13

$$\frac{\log(d+ex)}{c^3e}$$

```
[Out] Log[d + e*x]/(c^3*e)
```

Rubi [A] time = 0.0046667, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {27, 12, 31}

$$\frac{\log(d+ex)}{c^3e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]
```

```
[Out] Log[d + e*x]/(c^3*e)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^5}{\left(cd^2+2cdex+ce^2x^2\right)^3} \, dx &= \int \frac{1}{c^3(d+ex)} \, dx \\
&= \frac{\int \frac{1}{d+ex} \, dx}{c^3} \\
&= \frac{\log(d+ex)}{c^3e}
\end{aligned}$$

Mathematica [A] time = 0.0014406, size = 13, normalized size = 1.

$$\frac{\log(d+ex)}{c^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]
```

```
[Out] Log[d + e*x]/(c^3*e)
```

Maple [A] time = 0.039, size = 14, normalized size = 1.1

$$\frac{\ln\left(ex+d\right)}{c^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x)
```

```
[Out] ln(e*x+d)/c^3/e
```

Maxima [A] time = 0.976387, size = 18, normalized size = 1.38

$$\frac{\log\left(ex+d\right)}{c^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="maxima")
```

```
[Out] log(e*x + d)/(c^3*e)
```

Fricas [A] time = 2.06933, size = 30, normalized size = 2.31

$$\frac{\log\left(ex+d\right)}{c^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="fricas")
```

```
[Out] log(e*x + d)/(c^3*e)
```

Sympy [A] time = 0.162682, size = 17, normalized size = 1.31

$$\frac{\log\left(c^3d+c^3ex\right)}{c^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5/(c*e**2*x**2+2*c*d*e*x+c*d**2)**3,x)
```

```
[Out] log(c**3*d + c**3*e*x)/(c**3*e)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1022 $\int \frac{(d+ex)^4}{(cd^2+2cdex+ce^2x^2)^3} dx$

Optimal. Leaf size=15

$$-\frac{1}{c^3e(d+ex)}$$

```
[Out] -(1/(c^3*e*(d + e*x)))
```

Rubi [A] time = 0.0048168, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {27, 12, 32}

$$-\frac{1}{c^3e(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]
```

```
[Out] -(1/(c^3*e*(d + e*x)))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^4}{(cd^2+2cdex+ce^2x^2)^3} dx &= \int \frac{1}{c^3(d+ex)^2} dx \\ &= \frac{\int \frac{1}{(d+ex)^2} dx}{c^3} \\ &= -\frac{1}{c^3e(d+ex)}\end{aligned}$$

Mathematica [A] time = 0.0020717, size = 15, normalized size = 1.

$$-\frac{1}{c^3e(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]
```

```
[Out] -(1/(c^3*e*(d + e*x)))
```

Maple [A] time = 0.038, size = 16, normalized size = 1.1

$$-\frac{1}{c^3e\,(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x)
```

```
[Out] -1/c^3/e/(e*x+d)
```

Maxima [A] time = 0.991647, size = 26, normalized size = 1.73

$$-\frac{1}{c^3e^2x+c^3de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="maxima")
```

```
[Out] -1/(c^3*e^2*x + c^3*d*e)
```

Fricas [A] time = 2.00171, size = 35, normalized size = 2.33

$$-\frac{1}{c^3e^2x+c^3de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="fricas")
```

```
[Out] -1/(c^3*e^2*x + c^3*d*e)
```

Sympy [A] time = 0.40185, size = 17, normalized size = 1.13

$$-\frac{1}{c^3de+c^3e^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(c*e**2*x**2+2*c*d*e*x+c*d**2)**3,x)
```

```
[Out] -1/(c**3*d*e + c**3*e**2*x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.1023 $\int \frac{(d+ex)^3}{\left(cd^2+2cdex+ce^2x^2\right)^3}\,dx$

Optimal. Leaf size=17

$$-\frac{1}{2c^3e(d+ex)^2}$$

[Out] -1/(2*c^3*e*(d + e*x)^2)

Rubi [A] time = 0.004698, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {27, 12, 32}

$$-\frac{1}{2c^3e(d+ex)^2}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^3/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]

[Out] -1/(2*c^3*e*(d + e*x)^2)

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^3}{\left(cd^2+2cdex+ce^2x^2\right)^3}\,dx &= \int \frac{1}{c^3(d+ex)^3}\,dx\\
&= \frac{\int \frac{1}{(d+ex)^3}\,dx}{c^3}\\
&= -\frac{1}{2c^3e(d+ex)^2}
\end{aligned}$$

Mathematica [A] time = 0.0023602, size = 17, normalized size = 1.

$$-\frac{1}{2c^3e(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]
```

```
[Out] -1/(2*c^3*e*(d + e*x)^2)
```

Maple [A] time = 0.059, size = 16, normalized size = 0.9

$$-\frac{1}{2\,c^{3}e\,(ex+d)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x)
```

```
[Out] -1/2/c^3/e/(e*x+d)^2
```

Maxima [B] time = 0.97591, size = 45, normalized size = 2.65

$$-\frac{1}{2\left(c^{3}e^{3}x^{2}+2\,c^{3}de^{2}x+c^{3}d^{2}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="maxima")
```

```
[Out] -1/2/(c^3*e^3*x^2 + 2*c^3*d*e^2*x + c^3*d^2*e)
```

Fricas [B] time = 1.92044, size = 65, normalized size = 3.82

$$-\frac{1}{2\left(c^{3}e^{3}x^{2}+2\,c^{3}de^{2}x+c^{3}d^{2}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2/(c^3*e^3*x^2 + 2*c^3*d*e^2*x + c^3*d^2*e)
```

Sympy [B] time = 0.474627, size = 36, normalized size = 2.12

$$-\frac{1}{2c^{3}d^{2}e+4c^{3}de^{2}x+2c^{3}e^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*e**2*x**2+2*c*d*e*x+c*d**2)**3,x)
```

```
[Out] -1/(2*c**3*d**2*e + 4*c**3*d*e**2*x + 2*c**3*e**3*x**2)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1024 $\int \frac{(d+ex)^2}{\left(cd^2+2cdex+ce^2x^2\right)^3}\,dx$

Optimal. Leaf size=17

$$-\frac{1}{3c^3e(d+ex)^3}$$

```
[Out] -1/(3*c^3*e*(d + e*x)^3)
```

Rubi [A] time = 0.0055453, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {27, 12, 32}

$$-\frac{1}{3c^3e(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]
```

```
[Out] -1/(3*c^3*e*(d + e*x)^3)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2}{\left(cd^2+2cdex+ce^2x^2\right)^3}\,dx &= \int \frac{1}{c^3(d+ex)^4}\,dx\\ &= \frac{\int \frac{1}{(d+ex)^4}\,dx}{c^3}\\ &= -\frac{1}{3c^3e(d+ex)^3}\end{aligned}$$

Mathematica [A] time = 0.00257, size = 17, normalized size = 1.

$$-\frac{1}{3c^3e(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]
```

```
[Out] -1/(3*c^3*e*(d + e*x)^3)
```

Maple [A] time = 0.042, size = 16, normalized size = 0.9

$$-\frac{1}{3\,c^3e\,(ex+d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x)
```

```
[Out] -1/3/c^3/e/(e*x+d)^3
```

Maxima [B] time = 0.991338, size = 63, normalized size = 3.71

$$-\frac{1}{3\left(c^3e^4x^3+3\,c^3de^3x^2+3\,c^3d^2e^2x+c^3d^3e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="maxima")
```

```
[Out] -1/3/(c^3*e^4*x^3 + 3*c^3*d*e^3*x^2 + 3*c^3*d^2*e^2*x + c^3*d^3*e)
```

Fricas [B] time = 1.90593, size = 92, normalized size = 5.41

$$-\frac{1}{3\left(c^3e^4x^3+3\,c^3de^3x^2+3\,c^3d^2e^2x+c^3d^3e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="fricas")
```

```
[Out] -1/3/(c^3*e^4*x^3 + 3*c^3*d*e^3*x^2 + 3*c^3*d^2*e^2*x + c^3*d^3*e)
```

Sympy [B] time = 0.591096, size = 51, normalized size = 3.

$$-\frac{1}{3c^3d^3e+9c^3d^2e^2x+9c^3de^3x^2+3c^3e^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*e**2*x**2+2*c*d*e*x+c*d**2)**3,x)
```

```
[Out] -1/(3*c**3*d**3*e + 9*c**3*d**2*e**2*x + 9*c**3*d*e**3*x**2 + 3*c**3*e**4*x
**3)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1025 $\int \frac{d+ex}{\left(cd^2+2cdex+ce^2x^2\right)^3}\,dx$

Optimal. Leaf size=17

$$-\frac{1}{4c^3e(d+ex)^4}$$

```
[Out] -1/(4*c^3*e*(d + e*x)^4)
```

Rubi [A] time = 0.0044241, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {27, 12, 32}

$$-\frac{1}{4c^3e(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]
```

```
[Out] -1/(4*c^3*e*(d + e*x)^4)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex}{\left(cd^2+2cdex+ce^2x^2\right)^3}\,dx &= \int \frac{1}{c^3(d+ex)^5}\,dx\\ &= \frac{\int \frac{1}{(d+ex)^5}\,dx}{c^3}\\ &= -\frac{1}{4c^3e(d+ex)^4}\end{aligned}$$

Mathematica [A] time = 0.0026144, size = 17, normalized size = 1.

$$-\frac{1}{4c^3e(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]
```

```
[Out] -1/(4*c^3*e*(d + e*x)^4)
```

Maple [A] time = 0.042, size = 16, normalized size = 0.9

$$-\frac{1}{4\,c^{3}e\left(ex+d\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x)
```

```
[Out] -1/4/c^3/e/(e*x+d)^4
```

Maxima [B] time = 0.97872, size = 41, normalized size = 2.41

$$-\frac{1}{4\left(ce^{2}x^{2}+2\,cdex+cd^{2}\right)^{2}ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4/((c*e^2*x^2 + 2*c*d*e*x + c*d^2)^2*c*e)
```

Fricas [B] time = 2.02453, size = 119, normalized size = 7.

$$-\frac{1}{4\left(c^{3}e^{5}x^{4}+4\,c^{3}de^{4}x^{3}+6\,c^{3}d^{2}e^{3}x^{2}+4\,c^{3}d^{3}e^{2}x+c^{3}d^{4}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4/(c^3*e^5*x^4 + 4*c^3*d*e^4*x^3 + 6*c^3*d^2*e^3*x^2 + 4*c^3*d^3*e^2*x +
 c^3*d^4*e)
```

Sympy [B] time = 0.772306, size = 66, normalized size = 3.88

$$-\frac{1}{4c^{3}d^{4}e+16c^{3}d^{3}e^{2}x+24c^{3}d^{2}e^{3}x^{2}+16c^{3}de^{4}x^{3}+4c^{3}e^{5}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*e**2*x**2+2*c*d*e*x+c*d**2)**3,x)
```

```
[Out] -1/(4*c**3*d**4*e + 16*c**3*d**3*e**2*x + 24*c**3*d**2*e**3*x**2 + 16*c**3*
d*e**4*x**3 + 4*c**3*e**5*x**4)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1026 $\int \frac{1}{(cd^2+2cdex+ce^2x^2)^3}\,dx$

Optimal. Leaf size=17

$$-\frac{1}{5c^3e(d+ex)^5}$$

```
[Out] -1/(5*c^3*e*(d + e*x)^5)
```

Rubi [A] time = 0.0040724, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {27, 12, 32}

$$-\frac{1}{5c^3e(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(-3),x]
```

```
[Out] -1/(5*c^3*e*(d + e*x)^5)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Cancel[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(cd^2+2cdex+ce^2x^2\right)^3}\,dx &= \int \frac{1}{c^3(d+ex)^6}\,dx\\ &= \frac{\int \frac{1}{(d+ex)^6}\,dx}{c^3}\\ &= -\frac{1}{5c^3e(d+ex)^5}\end{aligned}$$

Mathematica [A] time = 0.0027721, size = 17, normalized size = 1.

$$-\frac{1}{5c^3e(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(-3),x]
```

```
[Out] -1/(5*c^3*e*(d + e*x)^5)
```

Maple [A] time = 0.04, size = 16, normalized size = 0.9

$$-\frac{1}{5\,c^{3}e\left(ex+d\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x)
```

```
[Out] -1/5/c^3/e/(e*x+d)^5
```

Maxima [B] time = 0.978386, size = 101, normalized size = 5.94

$$-\frac{1}{5\left(c^{3}e^{6}x^{5}+5\,c^{3}de^{5}x^{4}+10\,c^{3}d^{2}e^{4}x^{3}+10\,c^{3}d^{3}e^{3}x^{2}+5\,c^{3}d^{4}e^{2}x+c^{3}d^{5}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="maxima")
```

```
[Out] -1/5/(c^3*e^6*x^5 + 5*c^3*d*e^5*x^4 + 10*c^3*d^2*e^4*x^3 + 10*c^3*d^3*e^3*x
^2 + 5*c^3*d^4*e^2*x + c^3*d^5*e)
```

Fricas [B] time = 1.98565, size = 149, normalized size = 8.76

$$-\frac{1}{5\left(c^{3}e^{6}x^{5}+5\,c^{3}de^{5}x^{4}+10\,c^{3}d^{2}e^{4}x^{3}+10\,c^{3}d^{3}e^{3}x^{2}+5\,c^{3}d^{4}e^{2}x+c^{3}d^{5}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="fricas")
```

```
[Out] -1/5/(c^3*e^6*x^5 + 5*c^3*d*e^5*x^4 + 10*c^3*d^2*e^4*x^3 + 10*c^3*d^3*e^3*x
^2 + 5*c^3*d^4*e^2*x + c^3*d^5*e)
```

Sympy [B] time = 0.745794, size = 82, normalized size = 4.82

$$-\frac{1}{5c^{3}d^{5}e+25c^{3}d^{4}e^{2}x+50c^{3}d^{3}e^{3}x^{2}+50c^{3}d^{2}e^{4}x^{3}+25c^{3}de^{5}x^{4}+5c^{3}e^{6}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*e**2*x**2+2*c*d*e*x+c*d**2)**3,x)
```

```
[Out] -1/(5*c**3*d**5*e + 25*c**3*d**4*e**2*x + 50*c**3*d**3*e**3*x**2 + 50*c**3*
d**2*e**4*x**3 + 25*c**3*d*e**5*x**4 + 5*c**3*e**6*x**5)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1027 $\int \frac{1}{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^3}\,dx$

Optimal. Leaf size=17

$$-\frac{1}{6c^3e(d+ex)^6}$$

```
[Out] -1/(6*c^3*e*(d + e*x)^6)
```

Rubi [A] time = 0.0046868, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {27, 12, 32}

$$-\frac{1}{6c^3e(d+ex)^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3),x]

[Out] -1/(6*c^3*e*(d + e*x)^6)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^3}\,dx &= \int \frac{1}{c^3(d+ex)^7}\,dx\\ &= \frac{\int \frac{1}{(d+ex)^7}\,dx}{c^3}\\ &= -\frac{1}{6c^3e(d+ex)^6}\end{aligned}$$

Mathematica [A] time = 0.0048614, size = 17, normalized size = 1.

$$-\frac{1}{6c^3e(d+ex)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3),x]
```

```
[Out] -1/(6*c^3*e*(d + e*x)^6)
```

Maple [A] time = 0.041, size = 16, normalized size = 0.9

$$-\frac{1}{6\,c^{3}e\,(ex+d)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x)
```

```
[Out] -1/6/c^3/e/(e*x+d)^6
```

Maxima [B] time = 0.991984, size = 120, normalized size = 7.06

$$-\frac{1}{6\left(c^{3}e^{7}x^{6}+6\,c^{3}de^{6}x^{5}+15\,c^{3}d^{2}e^{5}x^{4}+20\,c^{3}d^{3}e^{4}x^{3}+15\,c^{3}d^{4}e^{3}x^{2}+6\,c^{3}d^{5}e^{2}x+c^{3}d^{6}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="maxima")
```

```
[Out] -1/6/(c^3*e^7*x^6 + 6*c^3*d*e^6*x^5 + 15*c^3*d^2*e^5*x^4 + 20*c^3*d^3*e^4*x
^3 + 15*c^3*d^4*e^3*x^2 + 6*c^3*d^5*e^2*x + c^3*d^6*e)
```

Fricas [B] time = 1.99252, size = 177, normalized size = 10.41

$$-\frac{1}{6\left(c^{3}e^{7}x^{6}+6\,c^{3}de^{6}x^{5}+15\,c^{3}d^{2}e^{5}x^{4}+20\,c^{3}d^{3}e^{4}x^{3}+15\,c^{3}d^{4}e^{3}x^{2}+6\,c^{3}d^{5}e^{2}x+c^{3}d^{6}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="fricas")
```

```
[Out] -1/6/(c^3*e^7*x^6 + 6*c^3*d*e^6*x^5 + 15*c^3*d^2*e^5*x^4 + 20*c^3*d^3*e^4*x
^3 + 15*c^3*d^4*e^3*x^2 + 6*c^3*d^5*e^2*x + c^3*d^6*e)
```

Sympy [B] time = 0.744703, size = 97, normalized size = 5.71

$$-\frac{1}{6c^{3}d^{6}e+36c^{3}d^{5}e^{2}x+90c^{3}d^{4}e^{3}x^{2}+120c^{3}d^{3}e^{4}x^{3}+90c^{3}d^{2}e^{5}x^{4}+36c^{3}de^{6}x^{5}+6c^{3}e^{7}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*e**2*x**2+2*c*d*e*x+c*d**2)**3,x)
```

```
[Out] -1/(6*c**3*d**6*e + 36*c**3*d**5*e**2*x + 90*c**3*d**4*e**3*x**2 + 120*c**3
*d**3*e**4*x**3 + 90*c**3*d**2*e**5*x**4 + 36*c**3*d*e**6*x**5 + 6*c**3*e**
7*x**6)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1028 $\int \frac{1}{(d+ex)^2\left(cd^2+2cdex+ce^2x^2\right)^3} dx$

Optimal. Leaf size=17

$$-\frac{1}{7c^3e(d+ex)^7}$$

```
[Out] -1/(7*c^3*e*(d + e*x)^7)
```

Rubi [A] time = 0.0047815, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {27, 12, 32}

$$-\frac{1}{7c^3e(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3),x]
```

```
[Out] -1/(7*c^3*e*(d + e*x)^7)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)^2\left(cd^2+2cdex+ce^2x^2\right)^3} dx &= \int \frac{1}{c^3(d+ex)^8} dx \\
&= \frac{\int \frac{1}{(d+ex)^8} dx}{c^3} \\
&= -\frac{1}{7c^3e(d+ex)^7}
\end{aligned}$$

Mathematica [A] time = 0.0045791, size = 17, normalized size = 1.

$$-\frac{1}{7c^3e(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3),x]
```

```
[Out] -1/(7*c^3*e*(d + e*x)^7)
```

Maple [A] time = 0.042, size = 16, normalized size = 0.9

$$-\frac{1}{7\,c^{3}e\,(ex+d)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x)
```

```
[Out] -1/7/c^3/e/(e*x+d)^7
```

Maxima [B] time = 1.02177, size = 139, normalized size = 8.18

$$-\frac{1}{7\left(c^{3}e^{8}x^{7}+7\,c^{3}de^{7}x^{6}+21\,c^{3}d^{2}e^{6}x^{5}+35\,c^{3}d^{3}e^{5}x^{4}+35\,c^{3}d^{4}e^{4}x^{3}+21\,c^{3}d^{5}e^{3}x^{2}+7\,c^{3}d^{6}e^{2}x+c^{3}d^{7}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="maxima")
```

```
[Out] -1/7/(c^3*e^8*x^7 + 7*c^3*d*e^7*x^6 + 21*c^3*d^2*e^6*x^5 + 35*c^3*d^3*e^5*x
^4 + 35*c^3*d^4*e^4*x^3 + 21*c^3*d^5*e^3*x^2 + 7*c^3*d^6*e^2*x + c^3*d^7*e)
```

Fricas [B] time = 2.01563, size = 205, normalized size = 12.06

$$-\frac{1}{7\left(c^{3}e^{8}x^{7}+7\,c^{3}de^{7}x^{6}+21\,c^{3}d^{2}e^{6}x^{5}+35\,c^{3}d^{3}e^{5}x^{4}+35\,c^{3}d^{4}e^{4}x^{3}+21\,c^{3}d^{5}e^{3}x^{2}+7\,c^{3}d^{6}e^{2}x+c^{3}d^{7}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="fricas")
```

```
[Out] -1/7/(c^3*e^8*x^7 + 7*c^3*d*e^7*x^6 + 21*c^3*d^2*e^6*x^5 + 35*c^3*d^3*e^5*x
^4 + 35*c^3*d^4*e^4*x^3 + 21*c^3*d^5*e^3*x^2 + 7*c^3*d^6*e^2*x + c^3*d^7*e)
```

Sympy [B] time = 1.03015, size = 112, normalized size = 6.59

$$-\frac{1}{7c^{3}d^{7}e+49c^{3}d^{6}e^{2}x+147c^{3}d^{5}e^{3}x^{2}+245c^{3}d^{4}e^{4}x^{3}+245c^{3}d^{3}e^{5}x^{4}+147c^{3}d^{2}e^{6}x^{5}+49c^{3}de^{7}x^{6}+7c^{3}e^{8}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*e**2*x**2+2*c*d*e*x+c*d**2)**3,x)
```

```
[Out] -1/(7*c**3*d**7*e + 49*c**3*d**6*e**2*x + 147*c**3*d**5*e**3*x**2 + 245*c**
3*d**4*e**4*x**3 + 245*c**3*d**3*e**5*x**4 + 147*c**3*d**2*e**6*x**5 + 49*c
**3*d*e**7*x**6 + 7*c**3*e**8*x**7)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1029 $\int (d+ex)^3\sqrt{cd^2+2cdex+ce^2x^2}\,dx$

Optimal. Leaf size=34

$$\frac{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}{5c^2e}$$

```
[Out] (c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(5*c^2*e)
```

Rubi [A] time = 0.0235951, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {643, 629}

$$\frac{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}{5c^2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]
```

```
[Out] (c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(5*c^2*e)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^3\sqrt{cd^2+2cdex+ce^2x^2}\,dx &= \frac{\int (d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}\,dx}{c}\\ &= \frac{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}{5c^2e}\end{aligned}$$

Mathematica [A] time = 0.0111601, size = 27, normalized size = 0.79

$$\frac{(d+ex)^4\sqrt{c(d+ex)^2}}{5e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]
```

```
[Out] ((d + e*x)^4*Sqrt[c*(d + e*x)^2])/(5*e)
```

Maple [B] time = 0.042, size = 73, normalized size = 2.2

$$\frac{x\left(e^4x^4+5\,de^3x^3+10\,d^2e^2x^2+10\,d^3ex+5\,d^4\right)}{5\,ex+5\,d}\sqrt{ce^2x^2+2\,cdex+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x)
```

```
[Out] 1/5*x*(e^4*x^4+5*d*e^3*x^3+10*d^2*e^2*x^2+10*d^3*e*x+5*d^4)*(c*e^2*x^2+2*c*
d*e*x+c*d^2)^(1/2)/(e*x+d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.15252, size = 158, normalized size = 4.65

$$\frac{\left(e^4x^5+5\,de^3x^4+10\,d^2e^2x^3+10\,d^3ex^2+5\,d^4x\right)\sqrt{ce^2x^2+2\,cdex+cd^2}}{5\,(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/5*(e^4*x^5 + 5*d*e^3*x^4 + 10*d^2*e^2*x^3 + 10*d^3*e*x^2 + 5*d^4*x)*sqrt(
c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(e*x + d)
```

Sympy [A] time = 0.823304, size = 187, normalized size = 5.5

$$\begin{cases} \frac{d^4\sqrt{cd^2+2cdex+ce^2x^2}}{5e}+\frac{4d^3x\sqrt{cd^2+2cdex+ce^2x^2}}{5}+\frac{6d^2ex^2\sqrt{cd^2+2cdex+ce^2x^2}}{5}+\frac{4de^2x^3\sqrt{cd^2+2cdex+ce^2x^2}}{5}+\frac{e^3x^4\sqrt{cd^2+2cdex+ce^2x^2}}{5} & \text{for } e \neq 0 \\ d^3x\sqrt{cd^2} & \text{otherwi} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2),x)
```

```
[Out] Piecewise((d**4*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(5*e) + 4*d**3*x*sqr
t(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/5 + 6*d**2*e*x**2*sqrt(c*d**2 + 2*c*d*e
```

```
*x + c*e**2*x**2)/5 + 4*d*e**2*x**3*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/
5 + e**3*x**4*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/5, Ne(e, 0)), (d**3*x*
sqrt(c*d**2), True))
```

Giac [A] time = 1.20402, size = 82, normalized size = 2.41

$$\frac{1}{5}\left(d^4e^{(-1)} + \left(4\,d^3 + \left(6\,d^2e + \left(xe^3 + 4\,de^2\right)x\right)x\right)x\right)\sqrt{cx^2e^2 + 2\,cdxe + cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*(d^4*e^(-1) + (4*d^3 + (6*d^2*e + (x*e^3 + 4*d*e^2)*x)*x)*x)*sqrt(c*x^2
*e^2 + 2*c*d*x*e + c*d^2)
```

3.1030 $\int (d+ex)^2\sqrt{cd^2+2cdex+ce^2x^2}\,dx$

Optimal. Leaf size=39

$$\frac{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}{4ce}$$

```
[Out] ((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))/(4*c*e)
```

Rubi [A] time = 0.0215713, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {642, 609}

$$\frac{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}{4ce}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]
```

```
[Out] ((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))/(4*c*e)
```

Rule 642

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^m/c^(m/2), Int[(a + b*x + c*x^2)^(p + m/2), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && EqQ[2*c*d
 - b*e, 0] && IntegerQ[m/2]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^2\sqrt{cd^2+2cdex+ce^2x^2}\,dx &= \frac{\int \left(cd^2+2cdex+ce^2x^2\right)^{3/2}\,dx}{c}\\ &= \frac{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}{4ce}\end{aligned}$$

Mathematica [A] time = 0.0140682, size = 28, normalized size = 0.72

$$\frac{(d+ex)\left(c(d+ex)^2\right)^{3/2}}{4ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]
```

```
[Out] ((d + e*x)*(c*(d + e*x)^2)^(3/2))/(4*c*e)
```

Maple [A] time = 0.042, size = 62, normalized size = 1.6

$$\frac{x\left(e^{3}x^{3}+4\,de^{2}x^{2}+6\,d^{2}ex+4\,d^{3}\right)}{4\,ex+4\,d}\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x)
```

```
[Out] 1/4*x*(e^3*x^3+4*d*e^2*x^2+6*d^2*e*x+4*d^3)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/
2)/(e*x+d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.05364, size = 134, normalized size = 3.44

$$\frac{\left(e^{3}x^{4}+4\,de^{2}x^{3}+6\,d^{2}ex^{2}+4\,d^{3}x\right)\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}}{4\,(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/4*(e^3*x^4 + 4*d*e^2*x^3 + 6*d^2*e*x^2 + 4*d^3*x)*sqrt(c*e^2*x^2 + 2*c*d*
e*x + c*d^2)/(e*x + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{c\,(d+ex)^{2}}\,(d+ex)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2),x)
```

```
[Out] Integral(sqrt(c*(d + e*x)**2)*(d + e*x)**2, x)
```

Giac [A] time = 1.20574, size = 69, normalized size = 1.77

$$\frac{1}{4}\sqrt{cx^2e^2+2\,cdxe+cd^2}\left(d^3e^{(-1)}+\left(3\,d^2+\left(xe^2+3\,de\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^2*e^2 + 2*c*d*x*e + c*d^2)*(d^3*e^(-1) + (3*d^2 + (x*e^2 + 3*d
*e)*x)*x)
```

3.1031 $\int (d+ex)\sqrt{cd^2+2cdex+ce^2x^2}\,dx$

Optimal. Leaf size=34

$$\frac{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}{3ce}$$

```
[Out] (c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(3*c*e)
```

Rubi [A] time = 0.0087348, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {629}

$$\frac{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}{3ce}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]
```

```
[Out] (c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(3*c*e)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (d+ex)\sqrt{cd^2+2cdex+ce^2x^2}\,dx=\frac{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}{3ce}$$

Mathematica [A] time = 0.0086244, size = 23, normalized size = 0.68

$$\frac{\left(c(d+ex)^2\right)^{3/2}}{3ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]
```

```
[Out] (c*(d + e*x)^2)^(3/2)/(3*c*e)
```

Maple [A] time = 0.04, size = 51, normalized size = 1.5

$$\frac{x\left(e^2x^2+3\,dex+3\,d^2\right)}{3\,ex+3\,d}\sqrt{ce^2x^2+2\,cdex+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x)
```

```
[Out] 1/3*x*(e^2*x^2+3*d*e*x+3*d^2)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)
```

Maxima [A] time = 0.985503, size = 41, normalized size = 1.21

$$\frac{\left(ce^2x^2 + 2\,cdex + cd^2\right)^{\frac{3}{2}}}{3\,ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^(3/2)/(c*e)
```

Fricas [A] time = 2.04189, size = 112, normalized size = 3.29

$$\frac{\sqrt{ce^2x^2 + 2\,cdex + cd^2}\left(e^2x^3 + 3\,dex^2 + 3\,d^2x\right)}{3\,(ex + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)/(e*
x + d)
```

Sympy [A] time = 0.292331, size = 107, normalized size = 3.15

$$\begin{cases} \frac{d^2\sqrt{cd^2+2cdex+ce^2x^2}}{3e} + \frac{2dx\sqrt{cd^2+2cdex+ce^2x^2}}{3} + \frac{ex^2\sqrt{cd^2+2cdex+ce^2x^2}}{3} & \text{for}\ e \neq 0 \\ dx\sqrt{cd^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2),x)
```

```
[Out] Piecewise((d**2*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(3*e) + 2*d*x*sqrt(c
*d**2 + 2*c*d*e*x + c*e**2*x**2)/3 + e*x**2*sqrt(c*d**2 + 2*c*d*e*x + c*e**
2*x**2)/3, Ne(e, 0)), (d*x*sqrt(c*d**2), True))
```

Giac [A] time = 1.16425, size = 55, normalized size = 1.62

$$\frac{1}{3}\,\sqrt{cx^2e^2 + 2\,cdxe + cd^2}\left(d^2e^{(-1)} + (xe + 2\,d)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(c*x^2*e^2 + 2*c*d*x*e + c*d^2)*(d^2*e^(-1) + (x*e + 2*d)*x)
```

3.1032 $\int \sqrt{cd^2 + 2cdex + ce^2x^2}\,dx$

Optimal. Leaf size=36

$$\frac{(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}{2e}$$

```
[Out] ((d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/(2*e)
```

Rubi [A] time = 0.0061876, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {609}

$$\frac{(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]

[Out] ((d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/(2*e)
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \sqrt{cd^2+2cdex+ce^2x^2}\,dx = \frac{(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}{2e}$$

Mathematica [A] time = 0.0133653, size = 31, normalized size = 0.86

$$\frac{cx(d+ex)(2d+ex)}{2\sqrt{c(d+ex)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]

[Out] (c*x*(d + e*x)*(2*d + e*x))/(2*Sqrt[c*(d + e*x)^2])
```

Maple [A] time = 0.04, size = 40, normalized size = 1.1

$$\frac{x\,(ex+2\,d)}{2\,ex+2\,d}\sqrt{ce^2x^2+2\,cdex+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x)
```

```
[Out] 1/2*x*(e*x+2*d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.02898, size = 90, normalized size = 2.5

$$\frac{\sqrt{ce^2x^2 + 2\,cdex + cd^2}\left(ex^2 + 2\,dx\right)}{2\,(ex + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*(e*x^2 + 2*d*x)/(e*x + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cd^2 + 2cdex + ce^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2),x)
```

```
[Out] Integral(sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2), x)
```

Giac [A] time = 1.26372, size = 41, normalized size = 1.14

$$\frac{1}{2}\sqrt{cx^2e^2 + 2\,cdxe + cd^2}\left(de^{(-1)} + x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2*e^2 + 2*c*d*x*e + c*d^2)*(d*e^(-1) + x)
```

3.1033 $\int \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{d+ex}\,dx$

Optimal. Leaf size=28

$$\frac{\sqrt{cd^2+2cdex+ce^2x^2}}{e}$$

```
[Out] Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/e
```

Rubi [A] time = 0.0237031, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {643, 629}

$$\frac{\sqrt{cd^2+2cdex+ce^2x^2}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(d + e*x),x]
```

```
[Out] Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/e
```

Rule 643

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{d+ex}\,dx &= c\int \frac{d+ex}{\sqrt{cd^2+2cdex+ce^2x^2}}\,dx\\ &= \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{e}\end{aligned}$$

Mathematica [A] time = 0.0060449, size = 21, normalized size = 0.75

$$\frac{cx(d+ex)}{\sqrt{c(d+ex)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(d + e*x),x]
```

```
[Out] (c*x*(d + e*x))/Sqrt[c*(d + e*x)^2]
```

Maple [A] time = 0.041, size = 32, normalized size = 1.1

$$\frac{x}{ex+d}\sqrt{ce^2x^2+2\,cdex+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d),x)
```

```
[Out] (c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.21814, size = 66, normalized size = 2.36

$$\frac{\sqrt{ce^2x^2+2\,cdex+cd^2}x}{ex+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*x/(e*x + d)
```

Sympy [A] time = 1.65946, size = 37, normalized size = 1.32

$$\begin{cases} \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{e} & \text{for } e \neq 0 \\ \frac{x\sqrt{cd^2}}{d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2)/(e*x+d),x)
```

```
[Out] Piecewise((sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/e, Ne(e, 0)), (x*sqrt(c*d
**2)/d, True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1034 $\int \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{(d+ex)^2} dx$

Optimal. Leaf size=40

$$\frac{c(d+ex)\log(d+ex)}{e\sqrt{cd^2+2cdex+ce^2x^2}}$$

```
[Out] (c*(d + e*x)*Log[d + e*x])/(e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])
```

Rubi [A] time = 0.0253335, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {642, 608, 31}

$$\frac{c(d+ex)\log(d+ex)}{e\sqrt{cd^2+2cdex+ce^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(d + e*x)^2,x]
```

```
[Out] (c*(d + e*x)*Log[d + e*x])/(e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^m/c^(m/2), Int[(a + b*x + c*x^2)^(p + m/2), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && EqQ[2*c*d
 - b*e, 0] && IntegerQ[m/2]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{(d+ex)^2} dx &= c\int \frac{1}{\sqrt{cd^2+2cdex+ce^2x^2}} dx \\
&= \frac{\left(c\left(cde+ce^2x\right)\right)\int \frac{1}{cde+ce^2x} dx}{\sqrt{cd^2+2cdex+ce^2x^2}} \\
&= \frac{c(d+ex)\log(d+ex)}{e\sqrt{cd^2+2cdex+ce^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.0077057, size = 29, normalized size = 0.72

$$\frac{c(d+ex)\log(d+ex)}{e\sqrt{c(d+ex)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(d + e*x)^2,x]
```

```
[Out] (c*(d + e*x)*Log[d + e*x])/(e*Sqrt[c*(d + e*x)^2])
```

Maple [A] time = 0.042, size = 40, normalized size = 1.

$$\frac{\ln(ex+d)}{(ex+d)e}\sqrt{ce^2x^2+2cdex+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^2,x)
```

```
[Out] (c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)*ln(e*x+d)/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.3914, size = 86, normalized size = 2.15

$$\frac{\sqrt{ce^2x^2+2cdex+cd^2}\log(ex+d)}{e^2x+de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^2,x, algorithm="fricas"
)
```

```
[Out] sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*log(e*x + d)/(e^2*x + d*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c(d+ex)^2}}{(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2)/(e*x+d)**2,x)
```

```
[Out] Integral(sqrt(c*(d + e*x)**2)/(d + e*x)**2, x)
```

Giac [A] time = 1.35305, size = 11, normalized size = 0.28

$$2 C_0 \sqrt{c} e^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 2*C_0*sqrt(c)*e^3
```

3.1035 $\int \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{(d+ex)^3} dx$

Optimal. Leaf size=30

$$-\frac{c}{e\sqrt{cd^2 + 2cdex + ce^2x^2}}$$

```
[Out] -(c/(e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]))
```

Rubi [A] time = 0.0233458, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {643, 629}

$$-\frac{c}{e\sqrt{cd^2 + 2cdex + ce^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(d + e*x)^3,x]

[Out] -(c/(e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]))
```

Rule 643

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^3} dx &= c^2 \int \frac{d + ex}{\left(cd^2 + 2cdex + ce^2x^2\right)^{3/2}} dx \\ &= -\frac{c}{e\sqrt{cd^2 + 2cdex + ce^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0074524, size = 19, normalized size = 0.63

$$-\frac{c}{e\sqrt{c(d + ex)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(d + e*x)^3,x]

[Out] -(c/(e*Sqrt[c*(d + e*x)^2]))
```

Maple [A] time = 0.04, size = 35, normalized size = 1.2

$$-\frac{1}{\left(ex+d\right)^{2}e}\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^3,x)
```

```
[Out] -1/(e*x+d)^2/e*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.40205, size = 92, normalized size = 3.07

$$-\frac{\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}}{e^{3}x^{2}+2\,de^{2}x+d^{2}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^3,x, algorithm="fricas"
)
```

```
[Out] -sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(e^3*x^2 + 2*d*e^2*x + d^2*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\left(d+ex\right)^{2}}}{\left(d+ex\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2)/(e*x+d)**3,x)
```

```
[Out] Integral(sqrt(c*(d + e*x)**2)/(d + e*x)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1036 $\int \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=39

$$-\frac{c}{2e(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}$$

```
[Out] -c/(2*e*(d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])
```

Rubi [A] time = 0.0193965, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {642, 607}

$$-\frac{c}{2e(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(d + e*x)^4,x]
```

```
[Out] -c/(2*e*(d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^m/c^(m/2), Int[(a + b*x + c*x^2)^(p + m/2), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && EqQ[2*c*d
 - b*e, 0] && IntegerQ[m/2]
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{(d+ex)^4}\,dx &= c^2\int \frac{1}{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}\,dx\\ &= -\frac{c}{2e(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.005925, size = 27, normalized size = 0.69

$$-\frac{\sqrt{c(d+ex)^2}}{2e(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(d + e*x)^4,x]
```

```
[Out] -Sqrt[c*(d + e*x)^2]/(2*e*(d + e*x)^3)
```

Maple [A] time = 0.04, size = 35, normalized size = 0.9

$$-\frac{1}{2\,(ex+d)^3\,e}\sqrt{ce^2x^2+2\,cdex+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^4,x)
```

```
[Out] -1/2/(e*x+d)^3/e*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.3728, size = 119, normalized size = 3.05

$$-\frac{\sqrt{ce^2x^2+2\,cdex+cd^2}}{2\left(e^4x^3+3\,de^3x^2+3\,d^2e^2x+d^3e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas"
)
```

```
[Out] -1/2*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2
*x + d^3*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c\,(d+ex)^2}}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(sqrt(c*(d + e*x)**2)/(d + e*x)**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1037 $\int \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{(d+ex)^5}\,dx$

Optimal. Leaf size=34

$$-\frac{c^2}{3e\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}$$

```
[Out] -c^2/(3*e*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))
```

Rubi [A] time = 0.0229168, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {643, 629}

$$-\frac{c^2}{3e\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(d + e*x)^5,x]
```

```
[Out] -c^2/(3*e*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))
```

Rule 643

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{(d+ex)^5}\,dx &= c^3\int \frac{d+ex}{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}\,dx\\ &= -\frac{c^2}{3e\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0073914, size = 27, normalized size = 0.79

$$-\frac{\sqrt{c(d+ex)^2}}{3e(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(d + e*x)^5,x]
```

```
[Out] -Sqrt[c*(d + e*x)^2]/(3*e*(d + e*x)^4)
```

Maple [A] time = 0.041, size = 35, normalized size = 1.

$$-\frac{1}{3\left(ex+d\right)^{4}e}\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^5,x)
```

```
[Out] -1/3/(e*x+d)^4/e*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^5,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 2.31593, size = 140, normalized size = 4.12

$$-\frac{\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}}{3\left(e^{5}x^{4}+4\,de^{4}x^{3}+6\,d^{2}e^{3}x^{2}+4\,d^{3}e^{2}x+d^{4}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^5,x, algorithm="fricas"
)
```

```
[Out] -1/3*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(e^5*x^4 + 4*d*e^4*x^3 + 6*d^2*e^3
*x^2 + 4*d^3*e^2*x + d^4*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\left(d+ex\right)^{2}}}{\left(d+ex\right)^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2)/(e*x+d)**5,x)
```

```
[Out] Integral(sqrt(c*(d + e*x)**2)/(d + e*x)**5, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1038 $\int \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{(d+ex)^6}\,dx$

Optimal. Leaf size=41

$$-\frac{c^2}{4e(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}$$

```
[Out] -c^2/(4*e*(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))
```

Rubi [A] time = 0.0192795, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {642, 607}

$$-\frac{c^2}{4e(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(d + e*x)^6,x]
```

```
[Out] -c^2/(4*e*(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))
```

Rule 642

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^m/c^(m/2), Int[(a + b*x + c*x^2)^(p + m/2), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && EqQ[2*c*d
 - b*e, 0] && IntegerQ[m/2]
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{(d+ex)^6}\,dx &= c^3\int \frac{1}{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}\,dx\\ &= -\frac{c^2}{4e(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0065646, size = 27, normalized size = 0.66

$$-\frac{\sqrt{c(d+ex)^2}}{4e(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(d + e*x)^6,x]
```

```
[Out] -Sqrt[c*(d + e*x)^2]/(4*e*(d + e*x)^5)
```

Maple [A] time = 0.041, size = 35, normalized size = 0.9

$$-\frac{1}{4\,(ex+d)^5\,e}\sqrt{ce^2x^2+2\,cdex+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^6,x)
```

```
[Out] -1/4/(e*x+d)^5/e*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^6,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 2.34392, size = 165, normalized size = 4.02

$$-\frac{\sqrt{ce^2x^2+2\,cdex+cd^2}}{4\left(e^6x^5+5\,de^5x^4+10\,d^2e^4x^3+10\,d^3e^3x^2+5\,d^4e^2x+d^5e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^6,x, algorithm="fricas"
)
```

```
[Out] -1/4*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(e^6*x^5 + 5*d*e^5*x^4 + 10*d^2*e^
4*x^3 + 10*d^3*e^3*x^2 + 5*d^4*e^2*x + d^5*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c\,(d+ex)^2}}{(d+ex)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2)/(e*x+d)**6,x)
```

```
[Out] Integral(sqrt(c*(d + e*x)**2)/(d + e*x)**6, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^6,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1039 $\int (d+ex)^3 \left(cd^2+2cdex+ce^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=34

$$\frac{\left(cd^2+2cdex+ce^2x^2\right)^{7/2}}{7c^2e}$$

```
[Out] (c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(7/2)/(7*c^2*e)
```

Rubi [A] time = 0.0244234, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {643, 629}

$$\frac{\left(cd^2+2cdex+ce^2x^2\right)^{7/2}}{7c^2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2),x]
```

```
[Out] (c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(7/2)/(7*c^2*e)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^3 \left(cd^2+2cdex+ce^2x^2\right)^{3/2}\,dx &= \frac{\int (d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{5/2}\,dx}{c}\\ &= \frac{\left(cd^2+2cdex+ce^2x^2\right)^{7/2}}{7c^2e}\end{aligned}$$

Mathematica [A] time = 0.0168272, size = 27, normalized size = 0.79

$$\frac{(d+ex)^4\left(c(d+ex)^2\right)^{3/2}}{7e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2),x]
```

```
[Out] ((d + e*x)^4*(c*(d + e*x)^2)^(3/2))/(7*e)
```

Maple [B] time = 0.042, size = 95, normalized size = 2.8

$$\frac{x\left(e^{6}x^{6}+7\,d e^{5}x^{5}+21\,d^{2}e^{4}x^{4}+35\,d^{3}e^{3}x^{3}+35\,d^{4}e^{2}x^{2}+21\,d^{5}ex+7\,d^{6}\right)}{7\,\left(ex+d\right)^{3}}\left(c e^{2}x^{2}+2\,cdex+c d^{2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x)
```

```
[Out] 1/7*x*(e^6*x^6+7*d*e^5*x^5+21*d^2*e^4*x^4+35*d^3*e^3*x^3+35*d^4*e^2*x^2+21*
d^5*e*x+7*d^6)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.21906, size = 223, normalized size = 6.56

$$\frac{\left(c e^{6}x^{7}+7\,c d e^{5}x^{6}+21\,c d^{2}e^{4}x^{5}+35\,c d^{3}e^{3}x^{4}+35\,c d^{4}e^{2}x^{3}+21\,c d^{5}ex^{2}+7\,c d^{6}x\right)\sqrt{c e^{2}x^{2}+2\,cdex+c d^{2}}}{7\,\left(ex+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/7*(c*e^6*x^7 + 7*c*d*e^5*x^6 + 21*c*d^2*e^4*x^5 + 35*c*d^3*e^3*x^4 + 35*c
*d^4*e^2*x^3 + 21*c*d^5*e*x^2 + 7*c*d^6*x)*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d
^2)/(e*x + d)
```

Sympy [A] time = 4.61971, size = 277, normalized size = 8.15

$$\begin{cases}\frac{cd^{6}\sqrt{cd^{2}+2cdex+ce^{2}x^{2}}}{7e} + \frac{6cd^{5}x\sqrt{cd^{2}+2cdex+ce^{2}x^{2}}}{7} + \frac{15cd^{4}ex^{2}\sqrt{cd^{2}+2cdex+ce^{2}x^{2}}}{7} + \frac{20cd^{3}e^{2}x^{3}\sqrt{cd^{2}+2cdex+ce^{2}x^{2}}}{7} + \frac{15cd^{2}e^{3}x^{4}\sqrt{cd^{2}+2cdex+ce^{2}x^{2}}}{7} + \\ d^{3}x\left(cd^{2}\right)^{\frac{3}{2}}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2),x)
```

```
[Out] Piecewise((c*d**6*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(7*e) + 6*c*d**5*x
*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/7 + 15*c*d**4*e*x**2*sqrt(c*d**2 +
2*c*d*e*x + c*e**2*x**2)/7 + 20*c*d**3*e**2*x**3*sqrt(c*d**2 + 2*c*d*e*x +
c*e**2*x**2)/7 + 15*c*d**2*e**3*x**4*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)
/7 + 6*c*d*e**4*x**5*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/7 + c*e**5*x**6
*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/7, Ne(e, 0)), (d**3*x*(c*d**2)**(3/
2), True))
```

Giac [B] time = 1.29958, size = 119, normalized size = 3.5

$$\frac{1}{7}\left(cd^6e^{(-1)}+\left(6\,cd^5+\left(15\,cd^4e+\left(20\,cd^3e^2+\left(15\,cd^2e^3+\left(cxe^5+6\,cde^4\right)x\right)x\right)x\right)x\right)x\right)\sqrt{cx^2e^2+2\,cdxe+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/7*(c*d^6*e^(-1) + (6*c*d^5 + (15*c*d^4*e + (20*c*d^3*e^2 + (15*c*d^2*e^3
+ (c*x*e^5 + 6*c*d*e^4)*x)*x)*x)*x)*x)*sqrt(c*x^2*e^2 + 2*c*d*x*e + c*d^2)
```

3.1040 $\int (d+ex)^2 \left(cd^2+2cdex+ce^2x^2\right)^{3/2} dx$

Optimal. Leaf size=39

$$\frac{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}{6ce}$$

```
[Out] ((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2))/(6*c*e)
```

Rubi [A] time = 0.0210601, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {642, 609}

$$\frac{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}{6ce}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2),x]
```

```
[Out] ((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2))/(6*c*e)
```

Rule 642

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^m/c^(m/2), Int[(a + b*x + c*x^2)^(p + m/2), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && EqQ[2*c*d
 - b*e, 0] && IntegerQ[m/2]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned} \int (d+ex)^2 \left(cd^2+2cdex+ce^2x^2\right)^{3/2} dx &= \frac{\int \left(cd^2+2cdex+ce^2x^2\right)^{5/2} dx}{c} \\ &= \frac{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}{6ce} \end{aligned}$$

Mathematica [A] time = 0.0242349, size = 28, normalized size = 0.72

$$\frac{(d+ex)\left(c(d+ex)^2\right)^{5/2}}{6ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2),x]
```

```
[Out] ((d + e*x)*(c*(d + e*x)^2)^(5/2))/(6*c*e)
```

Maple [B] time = 0.04, size = 84, normalized size = 2.2

$$\frac{x\left(e^{5}x^{5}+6\,de^{4}x^{4}+15\,d^{2}e^{3}x^{3}+20\,d^{3}e^{2}x^{2}+15\,d^{4}ex+6\,d^{5}\right)}{6\,\left(ex+d\right)^{3}}\left(ce^{2}x^{2}+2\,cdex+cd^{2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x)
```

```
[Out] 1/6*x*(e^5*x^5+6*d*e^4*x^4+15*d^2*e^3*x^3+20*d^3*e^2*x^2+15*d^4*e*x+6*d^5)*
(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.27949, size = 197, normalized size = 5.05

$$\frac{\left(ce^{5}x^{6}+6\,cde^{4}x^{5}+15\,cd^{2}e^{3}x^{4}+20\,cd^{3}e^{2}x^{3}+15\,cd^{4}ex^{2}+6\,cd^{5}x\right)\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}}{6\,(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/6*(c*e^5*x^6 + 6*c*d*e^4*x^5 + 15*c*d^2*e^3*x^4 + 20*c*d^3*e^2*x^3 + 15*c
*d^4*e*x^2 + 6*c*d^5*x)*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(e*x + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c\,(d+ex)^{2}\right)^{\frac{3}{2}}(d+ex)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2),x)
```

```
[Out] Integral((c*(d + e*x)**2)**(3/2)*(d + e*x)**2, x)
```

Giac [B] time = 1.17377, size = 104, normalized size = 2.67

$$\frac{1}{6}\left(cd^5e^{(-1)} + \left(5\,cd^4 + \left(10\,cd^3e + \left(10\,cd^2e^2 + \left(cxe^4 + 5\,cde^3\right)x\right)x\right)x\right)x\right)\sqrt{cx^2e^2 + 2\,cdxe + cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/6*(c*d^5*e^(-1) + (5*c*d^4 + (10*c*d^3*e + (10*c*d^2*e^2 + (c*x*e^4 + 5*c
*d*e^3)*x)*x)*x)*x)*sqrt(c*x^2*e^2 + 2*c*d*x*e + c*d^2)
```

3.1041 $\int (d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=34

$$\frac{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}{5ce}$$

```
[Out] (c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(5*c*e)
```

Rubi [A] time = 0.0089651, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {629}

$$\frac{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}{5ce}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2),x]
```

```
[Out] (c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(5*c*e)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}\,dx=\frac{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}{5ce}$$

Mathematica [A] time = 0.0129433, size = 23, normalized size = 0.68

$$\frac{\left(c(d+ex)^2\right)^{5/2}}{5ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2),x]
```

```
[Out] (c*(d + e*x)^2)^(5/2)/(5*c*e)
```

Maple [B] time = 0.042, size = 73, normalized size = 2.2

$$\frac{x\left(e^4x^4+5\,de^3x^3+10\,d^2e^2x^2+10\,d^3ex+5\,d^4\right)}{5\,(ex+d)^3}\left(ce^2x^2+2\,cdex+cd^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x)
```

```
[Out] 1/5*x*(e^4*x^4+5*d*e^3*x^3+10*d^2*e^2*x^2+10*d^3*e*x+5*d^4)*(c*e^2*x^2+2*c*
d*e*x+c*d^2)^(3/2)/(e*x+d)^3
```

Maxima [A] time = 1.0186, size = 41, normalized size = 1.21

$$\frac{\left(ce^2x^2 + 2\,cdex + cd^2\right)^{\frac{5}{2}}}{5\,ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^(5/2)/(c*e)
```

Fricas [B] time = 2.30631, size = 171, normalized size = 5.03

$$\frac{\left(ce^4x^5 + 5\,cde^3x^4 + 10\,cd^2e^2x^3 + 10\,cd^3ex^2 + 5\,cd^4x\right)\sqrt{ce^2x^2 + 2\,cdex + cd^2}}{5\,(ex + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*(c*e^4*x^5 + 5*c*d*e^3*x^4 + 10*c*d^2*e^2*x^3 + 10*c*d^3*e*x^2 + 5*c*d^
4*x)*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(e*x + d)
```

Sympy [A] time = 2.24087, size = 194, normalized size = 5.71

$$\begin{cases} \frac{cd^4\sqrt{cd^2+2cdex+ce^2x^2}}{5e} + \frac{4cd^3x\sqrt{cd^2+2cdex+ce^2x^2}}{5} + \frac{6cd^2ex^2\sqrt{cd^2+2cdex+ce^2x^2}}{5} + \frac{4cde^2x^3\sqrt{cd^2+2cdex+ce^2x^2}}{5} + \frac{ce^3x^4\sqrt{cd^2+2cdex+ce^2x^2}}{5} & \text{for } e \\ dx\left(cd^2\right)^{\frac{3}{2}} & \text{othe} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2),x)
```

```
[Out] Piecewise((c*d**4*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(5*e) + 4*c*d**3*x
*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/5 + 6*c*d**2*e*x**2*sqrt(c*d**2 + 2
*c*d*e*x + c*e**2*x**2)/5 + 4*c*d*e**2*x**3*sqrt(c*d**2 + 2*c*d*e*x + c*e**
2*x**2)/5 + c*e**3*x**4*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/5, Ne(e, 0))
, (d*x*(c*d**2)**(3/2), True))
```

Giac [B] time = 1.2066, size = 89, normalized size = 2.62

$$\frac{1}{5}\left(cd^4e^{(-1)} + \left(4\,cd^3 + \left(6\,cd^2e + \left(cxe^3 + 4\,cde^2\right)x\right)x\right)x\right)\sqrt{cx^2e^2 + 2\,cdxe + cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/5*(c*d^4*e^(-1) + (4*c*d^3 + (6*c*d^2*e + (c*x*e^3 + 4*c*d*e^2)*x)*x)*x)*
sqrt(c*x^2*e^2 + 2*c*d*x*e + c*d^2)
```

3.1042 $\int \left(cd^2+2cdex+ce^2x^2\right)^{3/2} dx$

Optimal. Leaf size=36

$$\frac{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}{4e}$$

```
[Out] ((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))/(4*e)
```

Rubi [A] time = 0.0060748, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {609}

$$\frac{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2),x]
```

```
[Out] ((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))/(4*e)
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \left(cd^2+2cdex+ce^2x^2\right)^{3/2} dx=\frac{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}{4e}$$

Mathematica [A] time = 0.0018106, size = 25, normalized size = 0.69

$$\frac{(d+ex)\left(c(d+ex)^2\right)^{3/2}}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2),x]
```

```
[Out] ((d + e*x)*(c*(d + e*x)^2)^(3/2))/(4*e)
```

Maple [A] time = 0.04, size = 62, normalized size = 1.7

$$\frac{x\left(e^3x^3+4\,de^2x^2+6\,d^2ex+4\,d^3\right)}{4\,(ex+d)^3}\left(ce^2x^2+2\,cdex+cd^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x)
```

```
[Out] 1/4*x*(e^3*x^3+4*d*e^2*x^2+6*d^2*e*x+4*d^3)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/
2)/(e*x+d)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.18211, size = 144, normalized size = 4.

$$\frac{\left(ce^3x^4 + 4\,cde^2x^3 + 6\,cd^2ex^2 + 4\,cd^3x\right)\sqrt{ce^2x^2 + 2\,cdex + cd^2}}{4\,(ex + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(c*e^3*x^4 + 4*c*d*e^2*x^3 + 6*c*d^2*e*x^2 + 4*c*d^3*x)*sqrt(c*e^2*x^2
+ 2*c*d*e*x + c*d^2)/(e*x + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cd^2 + 2cdex + ce^2x^2\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2),x)
```

```
[Out] Integral((c*d**2 + 2*c*d*e*x + c*e**2*x**2)**(3/2), x)
```

Giac [A] time = 1.29678, size = 74, normalized size = 2.06

$$\frac{1}{4}\left(cd^3e^{(-1)} + \left(3\,cd^2 + \left(cxe^2 + 3\,cde\right)x\right)x\right)\sqrt{cx^2e^2 + 2\,cdxe + cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(c*d^3*e^(-1) + (3*c*d^2 + (c*x*e^2 + 3*c*d*e)*x)*x)*sqrt(c*x^2*e^2 + 2
*c*d*x*e + c*d^2)
```

3.1043 $\int \frac{(cd^2+2cdex+ce^2x^2)^{3/2}}{d+ex} dx$

Optimal. Leaf size=31

$$\frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{3e}$$

```
[Out] (c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(3*e)
```

Rubi [A] time = 0.0232013, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {643, 629}

$$\frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] (c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(3*e)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{d + ex} dx &= c \int (d + ex)\sqrt{cd^2 + 2cdex + ce^2x^2} dx \\
&= \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{3e}
\end{aligned}$$

Mathematica [A] time = 0.002872, size = 20, normalized size = 0.65

$$\frac{(c(d + ex)^2)^{3/2}}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] (c*(d + e*x)^2)^(3/2)/(3*e)
```

Maple [A] time = 0.041, size = 51, normalized size = 1.7

$$\frac{x\left(e^2x^2+3\,dex+3\,d^2\right)}{3\,\left(ex+d\right)^3}\left(ce^2x^2+2\,cdex+cd^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d),x)
```

```
[Out] 1/3*x*(e^2*x^2+3*d*e*x+3*d^2)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.14175, size = 120, normalized size = 3.87

$$\frac{\left(ce^2x^3+3\,cdex^2+3\,cd^2x\right)\sqrt{ce^2x^2+2\,cdex+cd^2}}{3\,(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/3*(c*e^2*x^3 + 3*c*d*e*x^2 + 3*c*d^2*x)*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^
2)/(e*x + d)
```

Sympy [A] time = 3.00671, size = 39, normalized size = 1.26

$$\begin{cases} \frac{\left(cd^2+2cdex+ce^2x^2\right)^{\frac{3}{2}}}{3e} & \text{for } e \neq 0 \\ \frac{x\left(cd^2\right)^{\frac{3}{2}}}{d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2)/(e*x+d),x)
```

```
[Out] Piecewise(((c*d**2 + 2*c*d*e*x + c*e**2*x**2)**(3/2)/(3*e), Ne(e, 0)), (x*(
c*d**2)**(3/2)/d, True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1044 $\int \frac{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}{(d+ex)^2} dx$

Optimal. Leaf size=37

$$\frac{c(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}{2e}$$

```
[Out] (c*(d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/(2*e)
```

Rubi [A] time = 0.0209133, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {642, 609}

$$\frac{c(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(d + e*x)^2,x]
```

```
[Out] (c*(d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/(2*e)
```

Rule 642

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^m/c^(m/2), Int[(a + b*x + c*x^2)^(p + m/2), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && EqQ[2*c*d
 - b*e, 0] && IntegerQ[m/2]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned} \int \frac{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}{(d+ex)^2} dx &= c \int \sqrt{cd^2+2cdex+ce^2x^2} \, dx \\ &= \frac{c(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}{2e} \end{aligned}$$

Mathematica [A] time = 0.0023317, size = 33, normalized size = 0.89

$$\frac{c^2x(d+ex)(2d+ex)}{2\sqrt{c(d+ex)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(d + e*x)^2,x]
```

```
[Out] (c^2*x*(d + e*x)*(2*d + e*x))/(2*Sqrt[c*(d + e*x)^2])
```

Maple [A] time = 0.039, size = 40, normalized size = 1.1

$$\frac{x\left(ex+2\,d\right)}{2\,\left(ex+d\right)^{3}}\left(ce^{2}x^{2}+2\,cdex+cd^{2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^2,x)
```

```
[Out] 1/2*x*(e*x+2*d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.27804, size = 96, normalized size = 2.59

$$\frac{\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}\left(cex^{2}+2\,cdx\right)}{2\,\left(ex+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^2,x, algorithm="fricas"
)
```

```
[Out] 1/2*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*(c*e*x^2 + 2*c*d*x)/(e*x + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(d+ex\right)^{2}\right)^{\frac{3}{2}}}{\left(d+ex\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2)/(e*x+d)**2,x)
```

```
[Out] Integral((c*(d + e*x)**2)**(3/2)/(d + e*x)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1045 $\int \frac{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}{(d+ex)^3} dx$

Optimal. Leaf size=29

$$\frac{c\sqrt{cd^2+2cdex+ce^2x^2}}{e}$$

```
[Out] (c*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/e
```

Rubi [A] time = 0.0224896, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {643, 629}

$$\frac{c\sqrt{cd^2+2cdex+ce^2x^2}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(d + e*x)^3,x]
```

```
[Out] (c*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/e
```

Rule 643

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}{(d+ex)^3} dx &= c^2 \int \frac{d+ex}{\sqrt{cd^2+2cdex+ce^2x^2}} dx\\ &= \frac{c\sqrt{cd^2+2cdex+ce^2x^2}}{e}\end{aligned}$$

Mathematica [A] time = 0.0090703, size = 22, normalized size = 0.76

$$\frac{x\left(c(d+ex)^2\right)^{3/2}}{(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(d + e*x)^3,x]
```

```
[Out] (x*(c*(d + e*x)^2)^(3/2))/(d + e*x)^3
```

Maple [A] time = 0.04, size = 32, normalized size = 1.1

$$\frac{x}{(ex+d)^{3}}\left(ce^{2}x^{2}+2\,cdex+cd^{2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^3,x)
```

```
[Out] (c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^3*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.34877, size = 69, normalized size = 2.38

$$\frac{\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}cx}{ex+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^3,x, algorithm="fricas"
)
```

```
[Out] sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c*x/(e*x + d)
```

Sympy [A] time = 3.65716, size = 39, normalized size = 1.34

$$c\left(\begin{cases}\frac{x\sqrt{cd^{2}}}{d} & \text{for}\ e=0\\ \frac{\sqrt{cd^{2}+2cdex+ce^{2}x^{2}}}{e} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2)/(e*x+d)**3,x)
```

```
[Out] c*Piecewise((x*sqrt(c*d**2)/d, Eq(e, 0)), (sqrt(c*d**2 + 2*c*d*e*x + c*e**2
*x**2)/e, True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1046 $\int \frac{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=42

$$\frac{c^2(d+ex)\log(d+ex)}{e\sqrt{cd^2+2cdex+ce^2x^2}}$$

```
[Out] (c^2*(d + e*x)*Log[d + e*x])/(e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])
```

Rubi [A] time = 0.0246343, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {642, 608, 31}

$$\frac{c^2(d+ex)\log(d+ex)}{e\sqrt{cd^2+2cdex+ce^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(d + e*x)^4,x]

[Out] (c^2*(d + e*x)*Log[d + e*x])/(e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^m/c^(m/2), Int[(a + b*x + c*x^2)^(p + m/2), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && EqQ[2*c*d
 - b*e, 0] && IntegerQ[m/2]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}{(d+ex)^4}\,dx &= c^2\int \frac{1}{\sqrt{cd^2+2cdex+ce^2x^2}}\,dx\\
&= \frac{\left(c^2\left(cde+ce^2x\right)\right)\int \frac{1}{cde+ce^2x}\,dx}{\sqrt{cd^2+2cdex+ce^2x^2}}\\
&= \frac{c^2(d+ex)\log(d+ex)}{e\sqrt{cd^2+2cdex+ce^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.0038717, size = 31, normalized size = 0.74

$$\frac{c^2(d+ex)\log(d+ex)}{e\sqrt{c(d+ex)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(d + e*x)^4,x]
```

```
[Out] (c^2*(d + e*x)*Log[d + e*x])/(e*Sqrt[c*(d + e*x)^2])
```

Maple [A] time = 0.043, size = 40, normalized size = 1.

$$\frac{\ln(ex+d)}{(ex+d)^3 e}\left(ce^2x^2+2\,cdex+cd^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^4,x)
```

```
[Out] (c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^3*ln(e*x+d)/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^4,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.2917, size = 89, normalized size = 2.12

$$\frac{\sqrt{ce^2x^2+2\,cdex+cd^2}c\log(ex+d)}{e^2x+de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^4,x, algorithm="fricas"
)
```

```
[Out] sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c*log(e*x + d)/(e^2*x + d*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c\,(d+ex)^2\right)^{\frac{3}{2}}}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2)/(e*x+d)**4,x)
```

```
[Out] Integral((c*(d + e*x)**2)**(3/2)/(d + e*x)**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1047 $\int \frac{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}{(d+ex)^5}\,dx$

Optimal. Leaf size=32

$$-\frac{c^2}{e\sqrt{cd^2+2cdex+ce^2x^2}}$$

```
[Out] -(c^2/(e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]))
```

Rubi [A] time = 0.0233844, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {643, 629}

$$-\frac{c^2}{e\sqrt{cd^2+2cdex+ce^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(d + e*x)^5,x]
```

```
[Out] -(c^2/(e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]))
```

Rule 643

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}{(d+ex)^5}\,dx &= c^3\int \frac{d+ex}{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}\,dx\\ &= -\frac{c^2}{e\sqrt{cd^2+2cdex+ce^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0109843, size = 25, normalized size = 0.78

$$-\frac{\left(c(d+ex)^2\right)^{3/2}}{e(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(d + e*x)^5,x]
```

```
[Out] -((c*(d + e*x)^2)^(3/2)/(e*(d + e*x)^4))
```

Maple [A] time = 0.04, size = 35, normalized size = 1.1

$$-\frac{1}{(ex+d)^4 e}\left(ce^2x^2+2\,cdex+cd^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^5,x)
```

```
[Out] -1/(e*x+d)^4/e*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^5,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.52517, size = 95, normalized size = 2.97

$$-\frac{\sqrt{ce^2x^2+2\,cdex+cd^2}c}{e^3x^2+2\,de^2x+d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^5,x, algorithm="fricas"
)
```

```
[Out] -sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c/(e^3*x^2 + 2*d*e^2*x + d^2*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c\left(d+ex\right)^2\right)^{\frac{3}{2}}}{\left(d+ex\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2)/(e*x+d)**5,x)
```

```
[Out] Integral((c*(d + e*x)**2)**(3/2)/(d + e*x)**5, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1048 $\int \frac{(cd^2+2cdex+ce^2x^2)^{3/2}}{(d+ex)^6} dx$

Optimal. Leaf size=41

$$-\frac{c^2}{2e(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}$$

```
[Out] -c^2/(2*e*(d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])
```

Rubi [A] time = 0.0197819, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {642, 607}

$$-\frac{c^2}{2e(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(d + e*x)^6,x]
```

```
[Out] -c^2/(2*e*(d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^m/c^(m/2), Int[(a + b*x + c*x^2)^(p + m/2), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && EqQ[2*c*d
 - b*e, 0] && IntegerQ[m/2]
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cd^2+2cdex+ce^2x^2)^{3/2}}{(d+ex)^6} dx &= c^3 \int \frac{1}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx\\ &= -\frac{c^2}{2e(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0087903, size = 27, normalized size = 0.66

$$-\frac{(c(d+ex)^2)^{3/2}}{2e(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(d + e*x)^6,x]
```

```
[Out] -(c*(d + e*x)^2)^(3/2)/(2*e*(d + e*x)^5)
```

Maple [A] time = 0.041, size = 35, normalized size = 0.9

$$-\frac{1}{2\left(ex+d\right)^{5}e}\left(ce^{2}x^{2}+2\,cdex+cd^{2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^6,x)
```

```
[Out] -1/2/(e*x+d)^5/e*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^6,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.41897, size = 122, normalized size = 2.98

$$-\frac{\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}c}{2\left(e^{4}x^{3}+3\,de^{3}x^{2}+3\,d^{2}e^{2}x+d^{3}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^6,x, algorithm="fricas"
)
```

```
[Out] -1/2*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e
^2*x + d^3*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c\left(d+ex\right)^{2}\right)^{\frac{3}{2}}}{\left(d+ex\right)^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2)/(e*x+d)**6,x)
```

```
[Out] Integral((c*(d + e*x)**2)**(3/2)/(d + e*x)**6, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^6,x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.1049 $\int \frac{(cd^2+2cdex+ce^2x^2)^{3/2}}{(d+ex)^7} dx$

Optimal. Leaf size=34

$$-\frac{c^3}{3e\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}$$

```
[Out] -c^3/(3*e*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))
```

Rubi [A] time = 0.0236374, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {643, 629}

$$-\frac{c^3}{3e\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(d + e*x)^7,x]
```

```
[Out] -c^3/(3*e*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))
```

Rule 643

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}{(d+ex)^7}\,dx &= c^4 \int \frac{d+ex}{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}\,dx\\ &= -\frac{c^3}{3e\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0089959, size = 27, normalized size = 0.79

$$-\frac{\left(c(d+ex)^2\right)^{3/2}}{3e(d+ex)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(d + e*x)^7,x]
```

```
[Out] -(c*(d + e*x)^2)^(3/2)/(3*e*(d + e*x)^6)
```

Maple [A] time = 0.04, size = 35, normalized size = 1.

$$-\frac{1}{3\left(ex+d\right)^{6}e}\left(ce^{2}x^{2}+2\,cdex+cd^{2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^7,x)
```

```
[Out] -1/3/(e*x+d)^6/e*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^7,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 2.44491, size = 143, normalized size = 4.21

$$-\frac{\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}c}{3\left(e^{5}x^{4}+4\,de^{4}x^{3}+6\,d^{2}e^{3}x^{2}+4\,d^{3}e^{2}x+d^{4}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^7,x, algorithm="fricas"
)
```

```
[Out] -1/3*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c/(e^5*x^4 + 4*d*e^4*x^3 + 6*d^2*e
^3*x^2 + 4*d^3*e^2*x + d^4*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c\left(d+ex\right)^{2}\right)^{\frac{3}{2}}}{\left(d+ex\right)^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2)/(e*x+d)**7,x)
```

```
[Out] Integral((c*(d + e*x)**2)**(3/2)/(d + e*x)**7, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^7,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1050 $\int (d+ex)^3 \left(cd^2+2cdex+ce^2x^2\right)^{5/2} dx$

Optimal. Leaf size=34

$$\frac{\left(cd^2+2cdex+ce^2x^2\right)^{9/2}}{9c^2e}$$

```
[Out] (c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(9/2)/(9*c^2*e)
```

Rubi [A] time = 0.0238558, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {643, 629}

$$\frac{\left(cd^2+2cdex+ce^2x^2\right)^{9/2}}{9c^2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2),x]
```

```
[Out] (c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(9/2)/(9*c^2*e)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^3 \left(cd^2+2cdex+ce^2x^2\right)^{5/2} dx &= \frac{\int (d+ex) \left(cd^2+2cdex+ce^2x^2\right)^{7/2} dx}{c}\\ &= \frac{\left(cd^2+2cdex+ce^2x^2\right)^{9/2}}{9c^2e}\end{aligned}$$

Mathematica [A] time = 0.0264728, size = 27, normalized size = 0.79

$$\frac{(d+ex)^4 \left(c(d+ex)^2\right)^{5/2}}{9e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2),x]
```

```
[Out] ((d + e*x)^4*(c*(d + e*x)^2)^(5/2))/(9*e)
```

Maple [B] time = 0.04, size = 117, normalized size = 3.4

$$\frac{x\left(e^8x^8+9\,de^7x^7+36\,d^2e^6x^6+84\,d^3e^5x^5+126\,d^4e^4x^4+126\,d^5e^3x^3+84\,d^6e^2x^2+36\,d^7ex+9\,d^8\right)}{9\,(ex+d)^5}\left(ce^2x^2+2\,cdex+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x)
```

```
[Out] 1/9*x*(e^8*x^8+9*d*e^7*x^7+36*d^2*e^6*x^6+84*d^3*e^5*x^5+126*d^4*e^4*x^4+12
6*d^5*e^3*x^3+84*d^6*e^2*x^2+36*d^7*e*x+9*d^8)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^
(5/2)/(e*x+d)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.37562, size = 301, normalized size = 8.85

$$\frac{\left(c^2e^8x^9+9\,c^2de^7x^8+36\,c^2d^2e^6x^7+84\,c^2d^3e^5x^6+126\,c^2d^4e^4x^5+126\,c^2d^5e^3x^4+84\,c^2d^6e^2x^3+36\,c^2d^7ex^2+9\,c^2d^8x\right)}{9\,(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/9*(c^2*e^8*x^9 + 9*c^2*d*e^7*x^8 + 36*c^2*d^2*e^6*x^7 + 84*c^2*d^3*e^5*x^
6 + 126*c^2*d^4*e^4*x^5 + 126*c^2*d^5*e^3*x^4 + 84*c^2*d^6*e^2*x^3 + 36*c^2
*d^7*e*x^2 + 9*c^2*d^8*x)*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(e*x + d)
```

Sympy [A] time = 19.2363, size = 374, normalized size = 11.

$$\begin{cases} \frac{c^2d^8\sqrt{cd^2+2cdex+ce^2x^2}}{9e} + \frac{8c^2d^7x\sqrt{cd^2+2cdex+ce^2x^2}}{9} + \frac{28c^2d^6ex^2\sqrt{cd^2+2cdex+ce^2x^2}}{9} + \frac{56c^2d^5e^2x^3\sqrt{cd^2+2cdex+ce^2x^2}}{9} + \frac{70c^2d^4e^3x^4\sqrt{cd^2+2cdex+ce}}{9} \\ d^3x\left(cd^2\right)^{\frac{5}{2}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2),x)
```

```
[Out] Piecewise((c**2*d**8*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(9*e) + 8*c**2*
d**7*x*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/9 + 28*c**2*d**6*e*x**2*sqrt(
c*d**2 + 2*c*d*e*x + c*e**2*x**2)/9 + 56*c**2*d**5*e**2*x**3*sqrt(c*d**2 +
2*c*d*e*x + c*e**2*x**2)/9 + 70*c**2*d**4*e**3*x**4*sqrt(c*d**2 + 2*c*d*e*x
 + c*e**2*x**2)/9 + 56*c**2*d**3*e**4*x**5*sqrt(c*d**2 + 2*c*d*e*x + c*e**2
*x**2)/9 + 28*c**2*d**2*e**5*x**6*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/9
+ 8*c**2*d*e**6*x**7*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/9 + c**2*e**7*x
**8*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/9, Ne(e, 0)), (d**3*x*(c*d**2)**
(5/2), True))
```

Giac [B] time = 1.20719, size = 173, normalized size = 5.09

$$\frac{1}{9}\left(c^2d^8e^{(-1)} + \left(8\,c^2d^7 + \left(28\,c^2d^6e + \left(56\,c^2d^5e^2 + \left(70\,c^2d^4e^3 + \left(56\,c^2d^3e^4 + \left(28\,c^2d^2e^5 + \left(c^2xe^7 + 8\,c^2de^6\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/9*(c^2*d^8*e^(-1) + (8*c^2*d^7 + (28*c^2*d^6*e + (56*c^2*d^5*e^2 + (70*c^
2*d^4*e^3 + (56*c^2*d^3*e^4 + (28*c^2*d^2*e^5 + (c^2*x*e^7 + 8*c^2*d*e^6)*x
)*x)*x)*x)*x)*x)*x)*sqrt(c*x^2*e^2 + 2*c*d*x*e + c*d^2)
```

3.1051 $\int (d+ex)^2 \left(cd^2+2cdex+ce^2x^2\right)^{5/2} dx$

Optimal. Leaf size=39

$$\frac{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{7/2}}{8ce}$$

```
[Out] ((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(7/2))/(8*c*e)
```

Rubi [A] time = 0.0207566, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {642, 609}

$$\frac{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{7/2}}{8ce}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2),x]
```

```
[Out] ((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(7/2))/(8*c*e)
```

Rule 642

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^m/c^(m/2), Int[(a + b*x + c*x^2)^(p + m/2), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && EqQ[2*c*d
 - b*e, 0] && IntegerQ[m/2]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^2 \left(cd^2+2cdex+ce^2x^2\right)^{5/2} dx &= \frac{\int \left(cd^2+2cdex+ce^2x^2\right)^{7/2} dx}{c}\\ &= \frac{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{7/2}}{8ce}\end{aligned}$$

Mathematica [A] time = 0.0200629, size = 28, normalized size = 0.72

$$\frac{(d+ex)\left(c(d+ex)^2\right)^{7/2}}{8ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2),x]
```

```
[Out] ((d + e*x)*(c*(d + e*x)^2)^(7/2))/(8*c*e)
```

Maple [B] time = 0.04, size = 106, normalized size = 2.7

$$\frac{x\left(e^{7}x^{7}+8\,d e^{6}x^{6}+28\,d^{2}e^{5}x^{5}+56\,d^{3}e^{4}x^{4}+70\,d^{4}e^{3}x^{3}+56\,d^{5}e^{2}x^{2}+28\,d^{6}ex+8\,d^{7}\right)}{8\,\left(ex+d\right)^{5}}\left(ce^{2}x^{2}+2\,cdex+cd^{2}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x)
```

```
[Out] 1/8*x*(e^7*x^7+8*d*e^6*x^6+28*d^2*e^5*x^5+56*d^3*e^4*x^4+70*d^4*e^3*x^3+56*
d^5*e^2*x^2+28*d^6*e*x+8*d^7)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.50481, size = 270, normalized size = 6.92

$$\frac{\left(c^{2}e^{7}x^{8}+8\,c^{2}de^{6}x^{7}+28\,c^{2}d^{2}e^{5}x^{6}+56\,c^{2}d^{3}e^{4}x^{5}+70\,c^{2}d^{4}e^{3}x^{4}+56\,c^{2}d^{5}e^{2}x^{3}+28\,c^{2}d^{6}ex^{2}+8\,c^{2}d^{7}x\right)\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}}{8\,(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/8*(c^2*e^7*x^8 + 8*c^2*d*e^6*x^7 + 28*c^2*d^2*e^5*x^6 + 56*c^2*d^3*e^4*x^
5 + 70*c^2*d^4*e^3*x^4 + 56*c^2*d^5*e^2*x^3 + 28*c^2*d^6*e*x^2 + 8*c^2*d^7*
x)*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(e*x + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c\left(d+ex\right)^{2}\right)^{\frac{5}{2}}\left(d+ex\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2),x)
```

```
[Out] Integral((c*(d + e*x)**2)**(5/2)*(d + e*x)**2, x)
```

Giac [B] time = 1.20787, size = 155, normalized size = 3.97

$$\frac{1}{8}\left(c^2d^7e^{(-1)}+\left(7\,c^2d^6+\left(21\,c^2d^5e+\left(35\,c^2d^4e^2+\left(35\,c^2d^3e^3+\left(21\,c^2d^2e^4+\left(c^2xe^6+7\,c^2de^5\right)x\right)x\right)x\right)x\right)x\right)x\right)\sqrt{cx^2e^2+2\,c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/8*(c^2*d^7*e^(-1) + (7*c^2*d^6 + (21*c^2*d^5*e + (35*c^2*d^4*e^2 + (35*c^
2*d^3*e^3 + (21*c^2*d^2*e^4 + (c^2*x*e^6 + 7*c^2*d*e^5)*x)*x)*x)*x)*x)*x)*s
qrt(c*x^2*e^2 + 2*c*d*x*e + c*d^2)
```

3.1052 $\int (d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=34

$$\frac{\left(cd^2+2cdex+ce^2x^2\right)^{7/2}}{7ce}$$

```
[Out] (c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(7/2)/(7*c*e)
```

Rubi [A] time = 0.0090822, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {629}

$$\frac{\left(cd^2+2cdex+ce^2x^2\right)^{7/2}}{7ce}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2),x]
```

```
[Out] (c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(7/2)/(7*c*e)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{5/2}\,dx=\frac{\left(cd^2+2cdex+ce^2x^2\right)^{7/2}}{7ce}$$

Mathematica [A] time = 0.0186751, size = 23, normalized size = 0.68

$$\frac{\left(c(d+ex)^2\right)^{7/2}}{7ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2),x]
```

```
[Out] (c*(d + e*x)^2)^(7/2)/(7*c*e)
```

Maple [B] time = 0.04, size = 95, normalized size = 2.8

$$\frac{x\left(e^6x^6+7\,de^5x^5+21\,d^2e^4x^4+35\,d^3e^3x^3+35\,d^4e^2x^2+21\,d^5ex+7\,d^6\right)}{7\,(ex+d)^5}\left(ce^2x^2+2\,cdex+cd^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x)
```

```
[Out] 1/7*x*(e^6*x^6+7*d*e^5*x^5+21*d^2*e^4*x^4+35*d^3*e^3*x^3+35*d^4*e^2*x^2+21*
d^5*e*x+7*d^6)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^5
```

Maxima [A] time = 1.01183, size = 41, normalized size = 1.21

$$\frac{\left(ce^2x^2 + 2\,cdex + cd^2\right)^{\frac{7}{2}}}{7\,ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/7*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^(7/2)/(c*e)
```

Fricas [B] time = 2.34068, size = 242, normalized size = 7.12

$$\frac{\left(c^2e^6x^7 + 7\,c^2de^5x^6 + 21\,c^2d^2e^4x^5 + 35\,c^2d^3e^3x^4 + 35\,c^2d^4e^2x^3 + 21\,c^2d^5ex^2 + 7\,c^2d^6x\right)\sqrt{ce^2x^2 + 2\,cdex + cd^2}}{7\,(ex + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/7*(c^2*e^6*x^7 + 7*c^2*d*e^5*x^6 + 21*c^2*d^2*e^4*x^5 + 35*c^2*d^3*e^3*x^
4 + 35*c^2*d^4*e^2*x^3 + 21*c^2*d^5*e*x^2 + 7*c^2*d^6*x)*sqrt(c*e^2*x^2 + 2
*c*d*e*x + c*d^2)/(e*x + d)
```

Sympy [A] time = 8.19646, size = 287, normalized size = 8.44

$$\begin{cases} \frac{c^2d^6\sqrt{cd^2+2cdex+ce^2x^2}}{7e} + \frac{6c^2d^5x\sqrt{cd^2+2cdex+ce^2x^2}}{7} + \frac{15c^2d^4ex^2\sqrt{cd^2+2cdex+ce^2x^2}}{7} + \frac{20c^2d^3e^2x^3\sqrt{cd^2+2cdex+ce^2x^2}}{7} + \frac{15c^2d^2e^3x^4\sqrt{cd^2+2cdex+ce^2}}{7} \\ dx\left(cd^2\right)^{\frac{5}{2}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2),x)
```

```
[Out] Piecewise((c**2*d**6*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(7*e) + 6*c**2*
d**5*x*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/7 + 15*c**2*d**4*e*x**2*sqrt(
c*d**2 + 2*c*d*e*x + c*e**2*x**2)/7 + 20*c**2*d**3*e**2*x**3*sqrt(c*d**2 +
2*c*d*e*x + c*e**2*x**2)/7 + 15*c**2*d**2*e**3*x**4*sqrt(c*d**2 + 2*c*d*e*x
 + c*e**2*x**2)/7 + 6*c**2*d*e**4*x**5*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**
2)/7 + c**2*e**5*x**6*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/7, Ne(e, 0)),
(d*x*(c*d**2)**(5/2), True))
```

Giac [B] time = 1.21988, size = 138, normalized size = 4.06

$$\frac{1}{7}\left(c^2d^6e^{(-1)}+\left(6\,c^2d^5+\left(15\,c^2d^4e+\left(20\,c^2d^3e^2+\left(15\,c^2d^2e^3+\left(c^2xe^5+6\,c^2de^4\right)x\right)x\right)x\right)x\right)x\right)\sqrt{cx^2e^2+2\,cdxe+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/7*(c^2*d^6*e^(-1) + (6*c^2*d^5 + (15*c^2*d^4*e + (20*c^2*d^3*e^2 + (15*c^
2*d^2*e^3 + (c^2*x*e^5 + 6*c^2*d*e^4)*x)*x)*x)*x)*x)*sqrt(c*x^2*e^2 + 2*c*d
*x*e + c*d^2)
```

3.1053 $\int \left(cd^2+2cdex+ce^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=36

$$\frac{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}{6e}$$

```
[Out] ((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2))/(6*e)
```

Rubi [A] time = 0.0067982, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {609}

$$\frac{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}{6e}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2),x]
```

```
[Out] ((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2))/(6*e)
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \left(cd^2+2cdex+ce^2x^2\right)^{5/2}\,dx=\frac{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}{6e}$$

Mathematica [A] time = 0.0017505, size = 25, normalized size = 0.69

$$\frac{(d+ex)\left(c(d+ex)^2\right)^{5/2}}{6e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2),x]
```

```
[Out] ((d + e*x)*(c*(d + e*x)^2)^(5/2))/(6*e)
```

Maple [B] time = 0.041, size = 84, normalized size = 2.3

$$\frac{x\left(e^5x^5+6\,de^4x^4+15\,d^2e^3x^3+20\,d^3e^2x^2+15\,d^4ex+6\,d^5\right)}{6\,(ex+d)^5}\left(ce^2x^2+2\,cdex+cd^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x)
```

```
[Out] 1/6*x*(e^5*x^5+6*d*e^4*x^4+15*d^2*e^3*x^3+20*d^3*e^2*x^2+15*d^4*e*x+6*d^5)*
(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.35627, size = 213, normalized size = 5.92

$$\frac{\left(c^{2}e^{5}x^{6} + 6\,c^{2}de^{4}x^{5} + 15\,c^{2}d^{2}e^{3}x^{4} + 20\,c^{2}d^{3}e^{2}x^{3} + 15\,c^{2}d^{4}ex^{2} + 6\,c^{2}d^{5}x\right)\sqrt{ce^{2}x^{2} + 2\,cdex + cd^{2}}}{6\,(ex + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(c^2*e^5*x^6 + 6*c^2*d*e^4*x^5 + 15*c^2*d^2*e^3*x^4 + 20*c^2*d^3*e^2*x^
3 + 15*c^2*d^4*e*x^2 + 6*c^2*d^5*x)*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(e*
x + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cd^{2} + 2cdex + ce^{2}x^{2}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2),x)
```

```
[Out] Integral((c*d**2 + 2*c*d*e*x + c*e**2*x**2)**(5/2), x)
```

Giac [B] time = 1.31023, size = 120, normalized size = 3.33

$$\frac{1}{6}\left(c^{2}d^{5}e^{(-1)} + \left(5\,c^{2}d^{4} + \left(10\,c^{2}d^{3}e + \left(10\,c^{2}d^{2}e^{2} + \left(c^{2}xe^{4} + 5\,c^{2}de^{3}\right)x\right)x\right)x\right)x\right)\sqrt{cx^{2}e^{2} + 2\,cdxe + cd^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(c^2*d^5*e^(-1) + (5*c^2*d^4 + (10*c^2*d^3*e + (10*c^2*d^2*e^2 + (c^2*x
*e^4 + 5*c^2*d*e^3)*x)*x)*x)*x)*sqrt(c*x^2*e^2 + 2*c*d*x*e + c*d^2)
```

3.1054 $\int \frac{(cd^2+2cdex+ce^2x^2)^{5/2}}{d+ex} dx$

Optimal. Leaf size=31

$$\frac{(cd^2+2cdex+ce^2x^2)^{5/2}}{5e}$$

```
[Out] (c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(5*e)
```

Rubi [A] time = 0.0237312, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {643, 629}

$$\frac{(cd^2+2cdex+ce^2x^2)^{5/2}}{5e}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x),x]
```

```
[Out] (c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(5*e)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cd^2+2cdex+ce^2x^2)^{5/2}}{d+ex} dx &= c\int (d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{3/2} dx\\ &= \frac{(cd^2+2cdex+ce^2x^2)^{5/2}}{5e}\end{aligned}$$

Mathematica [A] time = 0.0054284, size = 20, normalized size = 0.65

$$\frac{\left(c(d+ex)^2\right)^{5/2}}{5e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x),x]
```

```
[Out] (c*(d + e*x)^2)^(5/2)/(5*e)
```

Maple [B] time = 0.04, size = 73, normalized size = 2.4

$$\frac{x\left(e^4x^4+5\,de^3x^3+10\,d^2e^2x^2+10\,d^3ex+5\,d^4\right)}{5\,\left(ex+d\right)^5}\left(ce^2x^2+2\,cdex+cd^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d),x)
```

```
[Out] 1/5*x*(e^4*x^4+5*d*e^3*x^3+10*d^2*e^2*x^2+10*d^3*e*x+5*d^4)*(c*e^2*x^2+2*c*
d*e*x+c*d^2)^(5/2)/(e*x+d)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.37133, size = 185, normalized size = 5.97

$$\frac{\left(c^2e^4x^5+5\,c^2de^3x^4+10\,c^2d^2e^2x^3+10\,c^2d^3ex^2+5\,c^2d^4x\right)\sqrt{ce^2x^2+2\,cdex+cd^2}}{5\,(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/5*(c^2*e^4*x^5 + 5*c^2*d*e^3*x^4 + 10*c^2*d^2*e^2*x^3 + 10*c^2*d^3*e*x^2
+ 5*c^2*d^4*x)*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(e*x + d)
```

Sympy [A] time = 5.57784, size = 39, normalized size = 1.26

$$\begin{cases} \frac{\left(cd^2+2cdex+ce^2x^2\right)^{\frac{5}{2}}}{5e} & \text{for}\: e \neq 0 \\ \frac{x\left(cd^2\right)^{\frac{5}{2}}}{d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2)/(e*x+d),x)
```

```
[Out] Piecewise(((c*d**2 + 2*c*d*e*x + c*e**2*x**2)**(5/2)/(5*e), Ne(e, 0)), (x*(
c*d**2)**(5/2)/d, True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1055 $\int \frac{(cd^2+2cdex+ce^2x^2)^{5/2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=37

$$\frac{c(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}{4e}$$

```
[Out] (c*(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))/(4*e)
```

Rubi [A] time = 0.0209725, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {642, 609}

$$\frac{c(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x)^2,x]
```

```
[Out] (c*(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))/(4*e)
```

Rule 642

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^m/c^(m/2), Int[(a + b*x + c*x^2)^(p + m/2), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && EqQ[2*c*d
 - b*e, 0] && IntegerQ[m/2]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}{(d+ex)^2}\,dx &= c\int \left(cd^2+2cdex+ce^2x^2\right)^{3/2}\,dx\\ &= \frac{c(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}{4e}\end{aligned}$$

Mathematica [A] time = 0.0048314, size = 26, normalized size = 0.7

$$\frac{c(d+ex)\left(c(d+ex)^2\right)^{3/2}}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x)^2,x]
```

```
[Out] (c*(d + e*x)*(c*(d + e*x)^2)^(3/2))/(4*e)
```

Maple [A] time = 0.042, size = 62, normalized size = 1.7

$$\frac{x\left(e^{3}x^{3}+4\,d e^{2}x^{2}+6\,d^{2}e x+4\,d^{3}\right)}{4\,\left(ex+d\right)^{5}}\left(c e^{2}x^{2}+2\,c d e x+c d^{2}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^2,x)
```

```
[Out] 1/4*x*(e^3*x^3+4*d*e^2*x^2+6*d^2*e*x+4*d^3)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/
2)/(e*x+d)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.25465, size = 155, normalized size = 4.19

$$\frac{\left(c^{2}e^{3}x^{4}+4\,c^{2}d e^{2}x^{3}+6\,c^{2}d^{2}e x^{2}+4\,c^{2}d^{3}x\right)\sqrt{c e^{2}x^{2}+2\,c d e x+c d^{2}}}{4\,(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas"
)
```

```
[Out] 1/4*(c^2*e^3*x^4 + 4*c^2*d*e^2*x^3 + 6*c^2*d^2*e*x^2 + 4*c^2*d^3*x)*sqrt(c*
e^2*x^2 + 2*c*d*e*x + c*d^2)/(e*x + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\,(d+ex)^{2}\right)^{\frac{5}{2}}}{(d+ex)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] Integral((c*(d + e*x)**2)**(5/2)/(d + e*x)**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.1056 $\int \frac{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}{(d+ex)^3} dx$

Optimal. Leaf size=32

$$\frac{c\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}{3e}$$

```
[Out] (c*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))/(3*e)
```

Rubi [A] time = 0.0227479, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {643, 629}

$$\frac{c\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x)^3,x]

[Out] (c*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))/(3*e)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}{(d+ex)^3} dx &= c^2 \int (d+ex)\sqrt{cd^2+2cdex+ce^2x^2}\, dx\\ &= \frac{c\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}{3e}\end{aligned}$$

Mathematica [A] time = 0.0060843, size = 21, normalized size = 0.66

$$\frac{c\left(c(d+ex)^2\right)^{3/2}}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x)^3,x]
```

```
[Out] (c*(c*(d + e*x)^2)^(3/2))/(3*e)
```

Maple [A] time = 0.04, size = 51, normalized size = 1.6

$$\frac{x\left(e^{2}x^{2}+3\,dex+3\,d^{2}\right)}{3\,\left(ex+d\right)^{5}}\left(ce^{2}x^{2}+2\,cdex+cd^{2}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^3,x)
```

```
[Out] 1/3*x*(e^2*x^2+3*d*e*x+3*d^2)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 2.33436, size = 128, normalized size = 4.

$$\frac{\left(c^{2}e^{2}x^{3}+3\,c^{2}dex^{2}+3\,c^{2}d^{2}x\right)\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}}{3\,(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^3,x, algorithm="fricas"
)
```

```
[Out] 1/3*(c^2*e^2*x^3 + 3*c^2*d*e*x^2 + 3*c^2*d^2*x)*sqrt(c*e^2*x^2 + 2*c*d*e*x
+ c*d^2)/(e*x + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c\,(d+ex)^{2}\right)^{\frac{5}{2}}}{(d+ex)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2)/(e*x+d)**3,x)
```

```
[Out] Integral((c*(d + e*x)**2)**(5/2)/(d + e*x)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1057 $\int \frac{(cd^2+2cdex+ce^2x^2)^{5/2}}{(d+ex)^4} dx$

Optimal. Leaf size=39

$$\frac{c^2(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}{2e}$$

```
[Out] (c^2*(d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/(2*e)
```

Rubi [A] time = 0.0208637, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {642, 609}

$$\frac{c^2(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x)^4,x]
```

```
[Out] (c^2*(d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/(2*e)
```

Rule 642

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^m/c^(m/2), Int[(a + b*x + c*x^2)^(p + m/2), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && EqQ[2*c*d
 - b*e, 0] && IntegerQ[m/2]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{(cd^2+2cdex+ce^2x^2)^{5/2}}{(d+ex)^4} dx &= c^2 \int \sqrt{cd^2+2cdex+ce^2x^2} dx \\ &= \frac{c^2(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}{2e}\end{aligned}$$

Mathematica [A] time = 0.0022836, size = 33, normalized size = 0.85

$$\frac{c^3x(d+ex)(2d+ex)}{2\sqrt{c(d+ex)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x)^4,x]
```

```
[Out] (c^3*x*(d + e*x)*(2*d + e*x))/(2*Sqrt[c*(d + e*x)^2])
```

Maple [A] time = 0.039, size = 40, normalized size = 1.

$$\frac{x\,(ex+2\,d)}{2\,(ex+d)^{5}}\left(ce^{2}x^{2}+2\,cdex+cd^{2}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^4,x)
```

```
[Out] 1/2*x*(e*x+2*d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.21686, size = 101, normalized size = 2.59

$$\frac{\left(c^{2}ex^{2}+2\,c^{2}dx\right)\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}}{2\,(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas"
)
```

```
[Out] 1/2*(c^2*e*x^2 + 2*c^2*d*x)*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(e*x + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c\,(d+ex)^{2}\right)^{\frac{5}{2}}}{(d+ex)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2)/(e*x+d)**4,x)
```

```
[Out] Integral((c*(d + e*x)**2)**(5/2)/(d + e*x)**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1058 $\int \frac{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}{(d+ex)^5}\,dx$

Optimal. Leaf size=31

$$\frac{c^2\sqrt{cd^2+2cdex+ce^2x^2}}{e}$$

```
[Out] (c^2*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/e
```

Rubi [A] time = 0.0227217, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {643, 629}

$$\frac{c^2\sqrt{cd^2+2cdex+ce^2x^2}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x)^5,x]
```

```
[Out] (c^2*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/e
```

Rule 643

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}{(d+ex)^5}\,dx &= c^3\int \frac{d+ex}{\sqrt{cd^2+2cdex+ce^2x^2}}\,dx\\ &= \frac{c^2\sqrt{cd^2+2cdex+ce^2x^2}}{e}\end{aligned}$$

Mathematica [A] time = 0.0063779, size = 23, normalized size = 0.74

$$\frac{c^3x(d+ex)}{\sqrt{c(d+ex)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x)^5,x]
```

```
[Out] (c^3*x*(d + e*x))/Sqrt[c*(d + e*x)^2]
```

Maple [A] time = 0.041, size = 32, normalized size = 1.

$$\frac{x}{(ex+d)^5}\left(ce^2x^2+2\,cdex+cd^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^5,x)
```

```
[Out] (c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^5*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^5,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.26425, size = 72, normalized size = 2.32

$$\frac{\sqrt{ce^2x^2+2\,cdex+cd^2}c^2x}{ex+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^5,x, algorithm="fricas"
)
```

```
[Out] sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c^2*x/(e*x + d)
```

Sympy [A] time = 8.02297, size = 41, normalized size = 1.32

$$c^2\left(\begin{cases}\frac{x\sqrt{cd^2}}{d} & \text{for } e=0\\ \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{e} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2)/(e*x+d)**5,x)
```

```
[Out] c**2*Piecewise((x*sqrt(c*d**2)/d, Eq(e, 0)), (sqrt(c*d**2 + 2*c*d*e*x + c*e
**2*x**2)/e, True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1059 $\int \frac{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}{(d+ex)^6}\,dx$

Optimal. Leaf size=42

$$\frac{c^3(d+ex)\log(d+ex)}{e\sqrt{cd^2+2cdex+ce^2x^2}}$$

```
[Out] (c^3*(d + e*x)*Log[d + e*x])/(e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])
```

Rubi [A] time = 0.0249265, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {642, 608, 31}

$$\frac{c^3(d+ex)\log(d+ex)}{e\sqrt{cd^2+2cdex+ce^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x)^6,x]
```

```
[Out] (c^3*(d + e*x)*Log[d + e*x])/(e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^m/c^(m/2), Int[(a + b*x + c*x^2)^(p + m/2), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && EqQ[2*c*d
 - b*e, 0] && IntegerQ[m/2]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}{(d+ex)^6}\,dx &= c^3\int \frac{1}{\sqrt{cd^2+2cdex+ce^2x^2}}\,dx\\ &= \frac{\left(c^3\left(cde+ce^2x\right)\right)\int \frac{1}{cde+ce^2x}\,dx}{\sqrt{cd^2+2cdex+ce^2x^2}}\\ &= \frac{c^3(d+ex)\log(d+ex)}{e\sqrt{cd^2+2cdex+ce^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0037168, size = 31, normalized size = 0.74

$$\frac{c^3(d+ex)\log(d+ex)}{e\sqrt{c(d+ex)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x)^6,x]
```

```
[Out] (c^3*(d + e*x)*Log[d + e*x])/(e*Sqrt[c*(d + e*x)^2])
```

Maple [A] time = 0.041, size = 40, normalized size = 1.

$$\frac{\ln(ex+d)}{(ex+d)^5 e}\left(ce^2x^2+2\,cdex+cd^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^6,x)
```

```
[Out] (c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^5*ln(e*x+d)/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^6,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.29899, size = 92, normalized size = 2.19

$$\frac{\sqrt{ce^2x^2+2\,cdex+cd^2}c^2\log(ex+d)}{e^2x+de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^6,x, algorithm="fricas"
)
```

```
[Out] sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c^2*log(e*x + d)/(e^2*x + d*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c\,(d+ex)^2\right)^{\frac{5}{2}}}{(d+ex)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2)/(e*x+d)**6,x)
```

```
[Out] Integral((c*(d + e*x)**2)**(5/2)/(d + e*x)**6, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^6,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1060 $\int \frac{(cd^2+2cdex+ce^2x^2)^{5/2}}{(d+ex)^7}\,dx$

Optimal. Leaf size=32

$$-\frac{c^3}{e\sqrt{cd^2+2cdex+ce^2x^2}}$$

```
[Out] -(c^3/(e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]))
```

Rubi [A] time = 0.0238005, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {643, 629}

$$-\frac{c^3}{e\sqrt{cd^2+2cdex+ce^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x)^7,x]
```

```
[Out] -(c^3/(e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]))
```

Rule 643

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}{(d+ex)^7}\,dx &= c^4\int \frac{d+ex}{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}\,dx\\ &= -\frac{c^3}{e\sqrt{cd^2+2cdex+ce^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0129414, size = 25, normalized size = 0.78

$$-\frac{\left(c(d+ex)^2\right)^{5/2}}{e(d+ex)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x)^7,x]
```

```
[Out] -((c*(d + e*x)^2)^(5/2)/(e*(d + e*x)^6))
```

Maple [A] time = 0.039, size = 35, normalized size = 1.1

$$-\frac{1}{\left(ex+d\right)^{6}e}\left(ce^{2}x^{2}+2\,cdex+cd^{2}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^7,x)
```

```
[Out] -1/(e*x+d)^6/e*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^7,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.25277, size = 97, normalized size = 3.03

$$-\frac{\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}c^{2}}{e^{3}x^{2}+2\,de^{2}x+d^{2}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^7,x, algorithm="fricas"
)
```

```
[Out] -sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c^2/(e^3*x^2 + 2*d*e^2*x + d^2*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c\left(d+ex\right)^{2}\right)^{\frac{5}{2}}}{\left(d+ex\right)^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2)/(e*x+d)**7,x)
```

```
[Out] Integral((c*(d + e*x)**2)**(5/2)/(d + e*x)**7, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^7,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1061 $\int \frac{(cd^2+2cdex+ce^2x^2)^{5/2}}{(d+ex)^8} dx$

Optimal. Leaf size=41

$$-\frac{c^3}{2e(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}$$

```
[Out] -c^3/(2*e*(d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])
```

Rubi [A] time = 0.0215237, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {642, 607}

$$-\frac{c^3}{2e(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x)^8,x]
```

```
[Out] -c^3/(2*e*(d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^m/c^(m/2), Int[(a + b*x + c*x^2)^(p + m/2), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && EqQ[2*c*d
 - b*e, 0] && IntegerQ[m/2]
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cd^2+2cdex+ce^2x^2)^{5/2}}{(d+ex)^8} dx &= c^4 \int \frac{1}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx\\ &= -\frac{c^3}{2e(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0109381, size = 27, normalized size = 0.66

$$-\frac{(c(d+ex)^2)^{5/2}}{2e(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x)^8,x]
```

```
[Out] -(c*(d + e*x)^2)^(5/2)/(2*e*(d + e*x)^7)
```

Maple [A] time = 0.04, size = 35, normalized size = 0.9

$$-\frac{1}{2\,(ex+d)^7\,e}\left(ce^2x^2+2\,cdex+cd^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^8,x)
```

```
[Out] -1/2/(e*x+d)^7/e*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^8,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.3915, size = 124, normalized size = 3.02

$$-\frac{\sqrt{ce^2x^2+2\,cdex+cd^2}c^2}{2\left(e^4x^3+3\,de^3x^2+3\,d^2e^2x+d^3e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^8,x, algorithm="fricas"
)
```

```
[Out] -1/2*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c^2/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2
*e^2*x + d^3*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2)/(e*x+d)**8,x)
```

```
[Out] Timed out
```

<u>**Giac**</u> **[F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^8,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1062 $\int \frac{(d+ex)^4}{\sqrt{cd^2+2cdex+ce^2x^2}}\,dx$

Optimal. Leaf size=39

$$\frac{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}{4c^2e}$$

```
[Out] ((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))/(4*c^2*e)
```

Rubi [A] time = 0.0215258, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {642, 609}

$$\frac{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}{4c^2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]
```

```
[Out] ((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))/(4*c^2*e)
```

Rule 642

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^m/c^(m/2), Int[(a + b*x + c*x^2)^(p + m/2), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && EqQ[2*c*d
 - b*e, 0] && IntegerQ[m/2]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^4}{\sqrt{cd^2+2cdex+ce^2x^2}}\,dx &= \frac{\int \left(cd^2+2cdex+ce^2x^2\right)^{3/2}\,dx}{c^2}\\ &= \frac{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}{4c^2e}\end{aligned}$$

Mathematica [A] time = 0.008763, size = 27, normalized size = 0.69

$$\frac{(d+ex)^5}{4e\sqrt{c(d+ex)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]
```

```
[Out] (d + e*x)^5/(4*e*Sqrt[c*(d + e*x)^2])
```

Maple [A] time = 0.042, size = 60, normalized size = 1.5

$$\frac{x\left(e^{3}x^{3}+4\,de^{2}x^{2}+6\,d^{2}ex+4\,d^{3}\right)(ex+d)}{4}\frac{1}{\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x)
```

```
[Out] 1/4*x*(e^3*x^3+4*d*e^2*x^2+6*d^2*e*x+4*d^3)*(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*
d^2)^(1/2)
```

Maxima [B] time = 1.17514, size = 246, normalized size = 6.31

$$\frac{3\,c^{2}d^{4}e^{4}\log\left(x+\frac{d}{e}\right)}{2\,\left(ce^{2}\right)^{\frac{5}{2}}}-\frac{3\,cd^{3}e^{3}x}{2\,\left(ce^{2}\right)^{\frac{3}{2}}}+\frac{3\,d^{2}e^{2}x^{2}}{4\,\sqrt{ce^{2}}}-\frac{3}{2}\,d^{4}\sqrt{\frac{1}{ce^{2}}}\log\left(x+\frac{d}{e}\right)+\frac{\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}e^{2}x^{3}}{4\,c}+\frac{3\,\sqrt{ce^{2}x^{2}+2\,cde}}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 3/2*c^2*d^4*e^4*log(x + d/e)/(c*e^2)^(5/2) - 3/2*c*d^3*e^3*x/(c*e^2)^(3/2)
+ 3/4*d^2*e^2*x^2/sqrt(c*e^2) - 3/2*d^4*sqrt(1/(c*e^2))*log(x + d/e) + 1/4*
sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*e^2*x^3/c + 3/4*sqrt(c*e^2*x^2 + 2*c*d*
e*x + c*d^2)*d*e*x^2/c + 5/2*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*d^3/(c*e)
```

Fricas [A] time = 2.44388, size = 139, normalized size = 3.56

$$\frac{\left(e^{3}x^{4}+4\,de^{2}x^{3}+6\,d^{2}ex^{2}+4\,d^{3}x\right)\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}}{4\,(cex+cd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/4*(e^3*x^4 + 4*d*e^2*x^3 + 6*d^2*e*x^2 + 4*d^3*x)*sqrt(c*e^2*x^2 + 2*c*d*
e*x + c*d^2)/(c*e*x + c*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(d+ex)^{4}}{\sqrt{c\,(d+ex)^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2),x)
```

```
[Out] Integral((d + e*x)**4/sqrt(c*(d + e*x)**2), x)
```

Giac [A] time = 1.38227, size = 85, normalized size = 2.18

$$\frac{1}{4}\sqrt{cx^2e^2 + 2\,cdxe + cd^2}\left(\frac{d^3e^{(-1)}}{c} + \left(x\left(\frac{xe^2}{c} + \frac{3\,de}{c}\right) + \frac{3\,d^2}{c}\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^2*e^2 + 2*c*d*x*e + c*d^2)*(d^3*e^(-1)/c + (x*(x*e^2/c + 3*d*e
/c) + 3*d^2/c)*x)
```

3.1063 $\int \frac{(d+ex)^3}{\sqrt{cd^2+2cdex+ce^2x^2}}\,dx$

Optimal. Leaf size=34

$$\frac{(cd^2+2cdex+ce^2x^2)^{3/2}}{3c^2e}$$

```
[Out] (c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(3*c^2*e)
```

Rubi [A] time = 0.0232877, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {643, 629}

$$\frac{(cd^2+2cdex+ce^2x^2)^{3/2}}{3c^2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]
```

```
[Out] (c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(3*c^2*e)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^3}{\sqrt{cd^2+2cdex+ce^2x^2}}\,dx &= \frac{\int (d+ex)\sqrt{cd^2+2cdex+ce^2x^2}\,dx}{c}\\ &= \frac{(cd^2+2cdex+ce^2x^2)^{3/2}}{3c^2e}\end{aligned}$$

Mathematica [A] time = 0.0059435, size = 27, normalized size = 0.79

$$\frac{(d+ex)^4}{3e\sqrt{c(d+ex)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]
```

```
[Out] (d + e*x)^4/(3*e*Sqrt[c*(d + e*x)^2])
```

Maple [A] time = 0.041, size = 49, normalized size = 1.4

$$\frac{x\left(e^{2}x^{2}+3\,dex+3\,d^{2}\right)(ex+d)}{3}\frac{1}{\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x)
```

```
[Out] 1/3*x*(e^2*x^2+3*d*e*x+3*d^2)*(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)
```

Maxima [B] time = 1.21266, size = 197, normalized size = 5.79

$$\frac{4\,c^{2}d^{3}e^{4}\log\left(x+\frac{d}{e}\right)}{3\left(ce^{2}\right)^{\frac{5}{2}}}-\frac{4\,cd^{2}e^{3}x}{3\left(ce^{2}\right)^{\frac{3}{2}}}+\frac{2\,de^{2}x^{2}}{3\sqrt{ce^{2}}}-\frac{4}{3}\,d^{3}\sqrt{\frac{1}{ce^{2}}}\log\left(x+\frac{d}{e}\right)+\frac{\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}ex^{2}}{3\,c}+\frac{7\sqrt{ce^{2}x^{2}+2\,cdex+c}}{3\,ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 4/3*c^2*d^3*e^4*log(x + d/e)/(c*e^2)^(5/2) - 4/3*c*d^2*e^3*x/(c*e^2)^(3/2)
+ 2/3*d*e^2*x^2/sqrt(c*e^2) - 4/3*d^3*sqrt(1/(c*e^2))*log(x + d/e) + 1/3*sq
rt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*e*x^2/c + 7/3*sqrt(c*e^2*x^2 + 2*c*d*e*x
+ c*d^2)*d^2/(c*e)
```

Fricas [A] time = 2.33509, size = 117, normalized size = 3.44

$$\frac{\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}\left(e^{2}x^{3}+3\,dex^{2}+3\,d^{2}x\right)}{3\,(cex+cd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/3*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)/(c*
e*x + c*d)
```

Sympy [A] time = 1.0791, size = 114, normalized size = 3.35

$$\begin{cases}\frac{d^{2}\sqrt{cd^{2}+2cdex+ce^{2}x^{2}}}{3ce}+\frac{2dx\sqrt{cd^{2}+2cdex+ce^{2}x^{2}}}{3c}+\frac{ex^{2}\sqrt{cd^{2}+2cdex+ce^{2}x^{2}}}{3c} & \text{for}\ e\neq 0\\ \frac{d^{3}x}{\sqrt{cd^{2}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2),x)
```

```
[Out] Piecewise((d**2*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(3*c*e) + 2*d*x*sqrt
(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(3*c) + e*x**2*sqrt(c*d**2 + 2*c*d*e*x +
 c*e**2*x**2)/(3*c), Ne(e, 0)), (d**3*x/sqrt(c*d**2), True))
```

Giac [A] time = 1.3571, size = 68, normalized size = 2.

$$\frac{1}{3}\sqrt{cx^2e^2 + 2\,cdxe + cd^2}\left(x\left(\frac{xe}{c} + \frac{2\,d}{c}\right) + \frac{d^2e^{(-1)}}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(c*x^2*e^2 + 2*c*d*x*e + c*d^2)*(x*(x*e/c + 2*d/c) + d^2*e^(-1)/c)
```

3.1064 $\int \frac{(d+ex)^2}{\sqrt{cd^2+2cdex+ce^2x^2}}\,dx$

Optimal. Leaf size=39

$$\frac{(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}{2ce}$$

```
[Out] ((d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/(2*c*e)
```

Rubi [A] time = 0.0215158, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {642, 609}

$$\frac{(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}{2ce}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]
```

```
[Out] ((d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/(2*c*e)
```

Rule 642

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^m/c^(m/2), Int[(a + b*x + c*x^2)^(p + m/2), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && EqQ[2*c*d
 - b*e, 0] && IntegerQ[m/2]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2}{\sqrt{cd^2+2cdex+ce^2x^2}}\,dx &= \frac{\int \sqrt{cd^2+2cdex+ce^2x^2}\,dx}{c}\\ &= \frac{(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}{2ce}\end{aligned}$$

Mathematica [A] time = 0.003141, size = 30, normalized size = 0.77

$$\frac{x(d+ex)(2d+ex)}{2\sqrt{c(d+ex)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]
```

```
[Out] (x*(d + e*x)*(2*d + e*x))/(2*Sqrt[c*(d + e*x)^2])
```

Maple [A] time = 0.04, size = 38, normalized size = 1.

$$\frac{x\,(ex+2\,d)\,(ex+d)}{2}\frac{1}{\sqrt{ce^2x^2+2\,cdex+cd^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x)
```

```
[Out] 1/2*x*(e*x+2*d)*(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)
```

Maxima [B] time = 1.16027, size = 147, normalized size = 3.77

$$\frac{c^2d^2e^4\log\left(x+\frac{d}{e}\right)}{\left(ce^2\right)^{\frac{5}{2}}}-\frac{cde^3x}{\left(ce^2\right)^{\frac{3}{2}}}+\frac{e^2x^2}{2\,\sqrt{ce^2}}-d^2\sqrt{\frac{1}{ce^2}}\log\left(x+\frac{d}{e}\right)+\frac{2\,\sqrt{ce^2x^2+2\,cdex+cd^2}d}{ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="maxima"
)
```

```
[Out] c^2*d^2*e^4*log(x + d/e)/(c*e^2)^(5/2) - c*d*e^3*x/(c*e^2)^(3/2) + 1/2*e^2*
x^2/sqrt(c*e^2) - d^2*sqrt(1/(c*e^2))*log(x + d/e) + 2*sqrt(c*e^2*x^2 + 2*c
*d*e*x + c*d^2)*d/(c*e)
```

Fricas [A] time = 2.38543, size = 96, normalized size = 2.46

$$\frac{\sqrt{ce^2x^2+2\,cdex+cd^2}\left(ex^2+2\,dx\right)}{2\,(cex+cd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/2*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*(e*x^2 + 2*d*x)/(c*e*x + c*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(d+ex)^2}{\sqrt{c\,(d+ex)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2),x)
```

```
[Out] Integral((d + e*x)**2/sqrt(c*(d + e*x)**2), x)
```

Giac [A] time = 1.39398, size = 50, normalized size = 1.28

$$\frac{1}{2}\sqrt{cx^2e^2 + 2\,cdxe + cd^2}\left(\frac{de^{(-1)}}{c} + \frac{x}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2*e^2 + 2*c*d*x*e + c*d^2)*(d*e^(-1)/c + x/c)
```

3.1065 $\int \frac{d+ex}{\sqrt{cd^2+2cdex+ce^2x^2}}\,dx$

Optimal. Leaf size=31

$$\frac{\sqrt{cd^2+2cdex+ce^2x^2}}{ce}$$

```
[Out] Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(c*e)
```

Rubi [A] time = 0.0087882, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {629}

$$\frac{\sqrt{cd^2+2cdex+ce^2x^2}}{ce}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]
```

```
[Out] Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(c*e)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex}{\sqrt{cd^2+2cdex+ce^2x^2}}\,dx=\frac{\sqrt{cd^2+2cdex+ce^2x^2}}{ce}$$

Mathematica [A] time = 0.0025526, size = 20, normalized size = 0.65

$$\frac{x(d+ex)}{\sqrt{c(d+ex)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]
```

```
[Out] (x*(d + e*x))/Sqrt[c*(d + e*x)^2]
```

Maple [A] time = 0.04, size = 30, normalized size = 1.

$$(ex+d)\,x\frac{1}{\sqrt{ce^2x^2+2\,cdex+cd^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x)
```

```
[Out] 1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)*(e*x+d)*x
```

Maxima [A] time = 1.20373, size = 39, normalized size = 1.26

$$\frac{\sqrt{ce^2x^2 + 2\,cdex + cd^2}}{ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(c*e)
```

Fricas [A] time = 2.27197, size = 72, normalized size = 2.32

$$\frac{\sqrt{ce^2x^2 + 2\,cdex + cd^2}x}{cex + cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*x/(c*e*x + c*d)
```

Sympy [A] time = 0.827076, size = 39, normalized size = 1.26

$$\begin{cases} \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{ce} & \text{for}\: e \neq 0 \\ \frac{dx}{\sqrt{cd^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2),x)
```

```
[Out] Piecewise((sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(c*e), Ne(e, 0)), (d*x/sq
rt(c*d**2), True))
```

Giac [A] time = 1.34488, size = 38, normalized size = 1.23

$$\frac{\sqrt{cx^2e^2 + 2\,cdxe + cd^2}e^{(-1)}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(c*x^2*e^2 + 2*c*d*x*e + c*d^2)*e^(-1)/c
```

3.1066 $\int \frac{1}{\sqrt{cd^2+2cdex+ce^2x^2}}\,dx$

Optimal. Leaf size=39

$$\frac{(d+ex)\log(d+ex)}{e\sqrt{cd^2+2cdex+ce^2x^2}}$$

```
[Out] ((d + e*x)*Log[d + e*x])/(e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])
```

Rubi [A] time = 0.0093111, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {608, 31}

$$\frac{(d+ex)\log(d+ex)}{e\sqrt{cd^2+2cdex+ce^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]
```

```
[Out] ((d + e*x)*Log[d + e*x])/(e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{cd^2+2cdex+ce^2x^2}}\,dx &= \frac{\left(cde+ce^2x\right)\int \frac{1}{cde+ce^2x}\,dx}{\sqrt{cd^2+2cdex+ce^2x^2}}\\ &= \frac{(d+ex)\log(d+ex)}{e\sqrt{cd^2+2cdex+ce^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.006548, size = 28, normalized size = 0.72

$$\frac{(d+ex)\log(d+ex)}{e\sqrt{c(d+ex)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]
```

```
[Out] ((d + e*x)*Log[d + e*x])/(e*Sqrt[c*(d + e*x)^2])
```

Maple [A] time = 0.04, size = 38, normalized size = 1.

$$\frac{(ex+d)\ln(ex+d)}{e}\frac{1}{\sqrt{ce^2x^2+2\,cdex+cd^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x)
```

```
[Out] (e*x+d)*ln(e*x+d)/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)
```

Maxima [A] time = 1.14099, size = 24, normalized size = 0.62

$$\sqrt{\frac{1}{ce^2}}\log\left(x+\frac{d}{e}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(1/(c*e^2))*log(x + d/e)
```

Fricas [A] time = 2.33178, size = 92, normalized size = 2.36

$$\frac{\sqrt{ce^2x^2+2\,cdex+cd^2}\log(ex+d)}{ce^2x+cde}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*log(e*x + d)/(c*e^2*x + c*d*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{cd^2+2cdex+ce^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2), x)
```

Giac [A] time = 1.3566, size = 73, normalized size = 1.87

$$-\frac{e^{(-1)}\log\left(\left|-\sqrt{c}de^2-\left(\sqrt{c}xe-\sqrt{cx^2e^2+2\,cdxe+cd^2}\right)e^2\right|\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac")
```

```
[Out] -e^(-1)*log(abs(-sqrt(c)*d*e^2 - (sqrt(c)*x*e - sqrt(c*x^2*e^2 + 2*c*d*x*e
+ c*d^2))*e^2))/sqrt(c)
```

3.1067 $\int \frac{1}{(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}\,dx$

Optimal. Leaf size=29

$$-\frac{1}{e\sqrt{cd^2+2cdex+ce^2x^2}}$$

[Out] -(1/(e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]))

Rubi [A] time = 0.0244909, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {643, 629}

$$-\frac{1}{e\sqrt{cd^2+2cdex+ce^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]),x]

[Out] -(1/(e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]))

Rule 643

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}\,dx &= c\int \frac{d+ex}{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}\,dx\\ &= -\frac{1}{e\sqrt{cd^2+2cdex+ce^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0106617, size = 18, normalized size = 0.62

$$-\frac{1}{e\sqrt{c(d+ex)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]),x]

```
[Out] -(1/(e*Sqrt[c*(d + e*x)^2]))
```

Maple [A] time = 0.039, size = 28, normalized size = 1.

$$-\frac{1}{e}\frac{1}{\sqrt{ce^2x^2+2\,cdex+cd^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x)
```

```
[Out] -1/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)
```

Maxima [A] time = 1.11979, size = 26, normalized size = 0.9

$$-\frac{1}{\sqrt{ce^2}x+\sqrt{c}de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -1/(sqrt(c)*e^2*x + sqrt(c)*d*e)
```

Fricas [A] time = 2.4122, size = 100, normalized size = 3.45

$$-\frac{\sqrt{ce^2x^2+2\,cdex+cd^2}}{ce^3x^2+2\,cde^2x+cd^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(c*e^3*x^2 + 2*c*d*e^2*x + c*d^2*e)
```

Sympy [A] time = 1.90278, size = 41, normalized size = 1.41

$$\begin{cases} -\frac{1}{e\sqrt{cd^2+2cdex+ce^2x^2}} & \text{for}\ e\neq 0\\ \frac{x}{d\sqrt{cd^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2),x)
```

```
[Out] Piecewise((-1/(e*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)), Ne(e, 0)), (x/(d*
sqrt(c*d**2)), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1068 $\int \frac{1}{(d+ex)^2\sqrt{cd^2+2cdex+ce^2x^2}}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{2e(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}$$

```
[Out] -1/(2*e*(d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])
```

Rubi [A] time = 0.0193157, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {642, 607}

$$-\frac{1}{2e(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]),x]
```

```
[Out] -1/(2*e*(d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^m/c^(m/2), Int[(a + b*x + c*x^2)^(p + m/2), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && EqQ[2*c*d
 - b*e, 0] && IntegerQ[m/2]
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)^2\sqrt{cd^2+2cdex+ce^2x^2}}\,dx &= c\int \frac{1}{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}\,dx\\ &= -\frac{1}{2e(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0098509, size = 26, normalized size = 0.68

$$-\frac{c(d+ex)}{2e\left(c(d+ex)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]),x]
```

```
[Out] -(c*(d + e*x))/(2*e*(c*(d + e*x)^2)^(3/2))
```

Maple [A] time = 0.041, size = 35, normalized size = 0.9

$$-\frac{1}{2e\left(ex+d\right)}\frac{1}{\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x)
```

```
[Out] -1/2/e/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)
```

Maxima [A] time = 1.20221, size = 45, normalized size = 1.18

$$-\frac{1}{2\left(\sqrt{c}e^{3}x^{2}+2\sqrt{c}de^{2}x+\sqrt{c}d^{2}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/2/(sqrt(c)*e^3*x^2 + 2*sqrt(c)*d*e^2*x + sqrt(c)*d^2*e)
```

Fricas [A] time = 2.38721, size = 130, normalized size = 3.42

$$-\frac{\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}}{2\left(ce^{4}x^{3}+3\,cde^{3}x^{2}+3\,cd^{2}e^{2}x+cd^{3}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="frica
s")
```

```
[Out] -1/2*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(c*e^4*x^3 + 3*c*d*e^3*x^2 + 3*c*d
^2*e^2*x + c*d^3*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{c\left(d+ex\right)^{2}}\left(d+ex\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*(d + e*x)**2)*(d + e*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

3.1069 $\int \frac{1}{(d+ex)^3\sqrt{cd^2+2cdex+ce^2x^2}}\,dx$

Optimal. Leaf size=32

$$-\frac{c}{3e\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}$$

```
[Out] -c/(3*e*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))
```

Rubi [A] time = 0.0235819, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {643, 629}

$$-\frac{c}{3e\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]),x]
```

```
[Out] -c/(3*e*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))
```

Rule 643

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)^3\sqrt{cd^2+2cdex+ce^2x^2}}\,dx &= c^2\int \frac{d+ex}{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}\,dx\\ &= -\frac{c}{3e\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0147183, size = 21, normalized size = 0.66

$$-\frac{c}{3e\left(c(d+ex)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]),x]
```

```
[Out] -c/(3*e*(c*(d + e*x)^2)^(3/2))
```

Maple [A] time = 0.041, size = 35, normalized size = 1.1

$$-\frac{1}{3\,\left(ex+d\right)^{2}e}\frac{1}{\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x)
```

```
[Out] -1/3/(e*x+d)^2/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)
```

Maxima [A] time = 1.06041, size = 63, normalized size = 1.97

$$-\frac{1}{3\left(\sqrt{c}e^{4}x^{3}+3\,\sqrt{c}de^{3}x^{2}+3\,\sqrt{c}d^{2}e^{2}x+\sqrt{c}d^{3}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/3/(sqrt(c)*e^4*x^3 + 3*sqrt(c)*d*e^3*x^2 + 3*sqrt(c)*d^2*e^2*x + sqrt(c)
*d^3*e)
```

Fricas [B] time = 2.38018, size = 154, normalized size = 4.81

$$-\frac{\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}}{3\left(ce^{5}x^{4}+4\,cde^{4}x^{3}+6\,cd^{2}e^{3}x^{2}+4\,cd^{3}e^{2}x+cd^{4}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="frica
s")
```

```
[Out] -1/3*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(c*e^5*x^4 + 4*c*d*e^4*x^3 + 6*c*d
^2*e^3*x^2 + 4*c*d^3*e^2*x + c*d^4*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{c\left(d+ex\right)^{2}}\left(d+ex\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*(d + e*x)**2)*(d + e*x)**3), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError
```

3.1070 $\int \frac{1}{(d+ex)^4\sqrt{cd^2+2cdex+ce^2x^2}}\,dx$

Optimal. Leaf size=39

$$-\frac{c}{4e(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}$$

```
[Out] -c/(4*e*(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))
```

Rubi [A] time = 0.0193221, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {642, 607}

$$-\frac{c}{4e(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^4*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]),x]
```

```
[Out] -c/(4*e*(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))
```

Rule 642

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^m/c^(m/2), Int[(a + b*x + c*x^2)^(p + m/2), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && EqQ[2*c*d
 - b*e, 0] && IntegerQ[m/2]
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)^4\sqrt{cd^2+2cdex+ce^2x^2}}\,dx &= c^2\int \frac{1}{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}\,dx\\
&= -\frac{c}{4e(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0107383, size = 27, normalized size = 0.69

$$-\frac{1}{4e(d+ex)^3\sqrt{c(d+ex)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^4*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]),x]
```

```
[Out] -1/(4*e*(d + e*x)^3*Sqrt[c*(d + e*x)^2])
```

Maple [A] time = 0.043, size = 35, normalized size = 0.9

$$-\frac{1}{4\left(ex+d\right)^{3}e\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x)
```

```
[Out] -1/4/(e*x+d)^3/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)
```

Maxima [A] time = 1.15207, size = 82, normalized size = 2.1

$$-\frac{1}{4\left(\sqrt{c}e^{5}x^{4}+4\sqrt{c}de^{4}x^{3}+6\sqrt{c}d^{2}e^{3}x^{2}+4\sqrt{c}d^{3}e^{2}x+\sqrt{c}d^{4}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/4/(sqrt(c)*e^5*x^4 + 4*sqrt(c)*d*e^4*x^3 + 6*sqrt(c)*d^2*e^3*x^2 + 4*sqr
t(c)*d^3*e^2*x + sqrt(c)*d^4*e)
```

Fricas [B] time = 2.27123, size = 181, normalized size = 4.64

$$-\frac{\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}}{4\left(ce^{6}x^{5}+5\,cde^{5}x^{4}+10\,cd^{2}e^{4}x^{3}+10\,cd^{3}e^{3}x^{2}+5\,cd^{4}e^{2}x+cd^{5}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="frica
s")
```

```
[Out] -1/4*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(c*e^6*x^5 + 5*c*d*e^5*x^4 + 10*c*
d^2*e^4*x^3 + 10*c*d^3*e^3*x^2 + 5*c*d^4*e^2*x + c*d^5*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c\left(d+ex\right)^{2}}\left(d+ex\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**4/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*(d + e*x)**2)*(d + e*x)**4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError
```

3.1071 $\int \frac{(d+ex)^4}{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=39

$$\frac{(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}{2c^2e}$$

```
[Out] ((d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/(2*c^2*e)
```

Rubi [A] time = 0.0223517, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {642, 609}

$$\frac{(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}{2c^2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2),x]
```

```
[Out] ((d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/(2*c^2*e)
```

Rule 642

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^m/c^(m/2), Int[(a + b*x + c*x^2)^(p + m/2), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && EqQ[2*c*d
 - b*e, 0] && IntegerQ[m/2]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^4}{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}\,dx &= \frac{\int \sqrt{cd^2+2cdex+ce^2x^2}\,dx}{c^2}\\ &= \frac{(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}{2c^2e}\end{aligned}$$

Mathematica [A] time = 0.0050569, size = 33, normalized size = 0.85

$$\frac{x(d+ex)(2d+ex)}{2c\sqrt{c(d+ex)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2),x]
```

```
[Out] (x*(d + e*x)*(2*d + e*x))/(2*c*Sqrt[c*(d + e*x)^2])
```

Maple [A] time = 0.039, size = 40, normalized size = 1.

$$\frac{x\,(ex+2\,d)\,(ex+d)^{3}}{2}\left(ce^{2}x^{2}+2\,cdex+cd^{2}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x)
```

```
[Out] 1/2*x*(e*x+2*d)*(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)
```

Maxima [B] time = 1.32177, size = 134, normalized size = 3.44

$$\frac{e^{2}x^{3}}{2\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}c}+\frac{3\,dex^{2}}{2\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}c}-\frac{d^{3}}{\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] 1/2*e^2*x^3/(sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c) + 3/2*d*e*x^2/(sqrt(c*e
^2*x^2 + 2*c*d*e*x + c*d^2)*c) - d^3/(sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c
*e)
```

Fricas [A] time = 2.23836, size = 101, normalized size = 2.59

$$\frac{\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}\left(ex^{2}+2\,dx\right)}{2\left(c^{2}ex+c^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/2*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*(e*x^2 + 2*d*x)/(c^2*e*x + c^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(d+ex)^{4}}{\left(c\,(d+ex)^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2),x)
```

```
[Out] Integral((d + e*x)**4/(c*(d + e*x)**2)**(3/2), x)
```

Giac [A] time = 1.33488, size = 88, normalized size = 2.26

$$\frac{4\,C_0 d e^{(-1)} - \frac{2\,d^3 e^{(-1)}}{c} + \left(x\left(\frac{xe^2}{c} + \frac{3\,de}{c}\right) + 4\,C_0\right)x}{2\,\sqrt{cx^2e^2 + 2\,cdxe + cd^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(4*C_0*d*e^(-1) - 2*d^3*e^(-1)/c + (x*(x*e^2/c + 3*d*e/c) + 4*C_0)*x)/s
qrt(c*x^2*e^2 + 2*c*d*x*e + c*d^2)
```

3.1072 $\int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=31

$$\frac{\sqrt{cd^2+2cdex+ce^2x^2}}{c^2e}$$

```
[Out] Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(c^2*e)
```

Rubi [A] time = 0.0238644, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {643, 629}

$$\frac{\sqrt{cd^2+2cdex+ce^2x^2}}{c^2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2),x]
```

```
[Out] Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(c^2*e)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^3}{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}\,dx &= \frac{\int \frac{d+ex}{\sqrt{cd^2+2cdex+ce^2x^2}}\,dx}{c}\\ &= \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{c^2e}\end{aligned}$$

Mathematica [A] time = 0.0073226, size = 22, normalized size = 0.71

$$\frac{x(d+ex)^3}{\left(c(d+ex)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2),x]
```

```
[Out] (x*(d + e*x)^3)/(c*(d + e*x)^2)^(3/2)
```

Maple [A] time = 0.04, size = 32, normalized size = 1.

$$(ex+d)^3 x \left(ce^2x^2+2\,cdex+cd^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x)
```

```
[Out] 1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)*(e*x+d)^3*x
```

Maxima [B] time = 1.28817, size = 86, normalized size = 2.77

$$\frac{ex^2}{\sqrt{ce^2x^2+2\,cdex+cd^2}c}-\frac{d^2}{\sqrt{ce^2x^2+2\,cdex+cd^2}ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] e*x^2/(sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c) - d^2/(sqrt(c*e^2*x^2 + 2*c*d
*e*x + c*d^2)*c*e)
```

Fricas [A] time = 2.42828, size = 77, normalized size = 2.48

$$\frac{\sqrt{ce^2x^2+2\,cdex+cd^2}x}{c^2ex+c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*x/(c^2*e*x + c^2*d)
```

Sympy [A] time = 0.91692, size = 42, normalized size = 1.35

$$\begin{cases} \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{c^2e} & \text{for}\ e\neq 0 \\ \frac{d^3x}{\left(cd^2\right)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2),x)
```

```
[Out] Piecewise((sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(c**2*e), Ne(e, 0)), (d**
3*x/(c*d**2)**(3/2), True))
```

Giac [A] time = 1.33103, size = 72, normalized size = 2.32

$$\frac{2\,C_0de^{(-1)} + \left(2\,C_0 + \frac{xe}{c}\right)x - \frac{d^2e^{(-1)}}{c}}{\sqrt{cx^2e^2 + 2\,cdxe + cd^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac")
```

```
[Out] (2*C_0*d*e^(-1) + (2*C_0 + x*e/c)*x - d^2*e^(-1)/c)/sqrt(c*x^2*e^2 + 2*c*d*
x*e + c*d^2)
```

3.1073 $\int \frac{(d+ex)^2}{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=42

$$\frac{(d+ex)\log(d+ex)}{ce\sqrt{cd^2+2cdex+ce^2x^2}}$$

```
[Out] ((d + e*x)*Log[d + e*x])/(c*e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])
```

Rubi [A] time = 0.0252252, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {642, 608, 31}

$$\frac{(d+ex)\log(d+ex)}{ce\sqrt{cd^2+2cdex+ce^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2),x]
```

```
[Out] ((d + e*x)*Log[d + e*x])/(c*e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^m/c^(m/2), Int[(a + b*x + c*x^2)^(p + m/2), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && EqQ[2*c*d
 - b*e, 0] && IntegerQ[m/2]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2}{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}\,dx &= \frac{\int \frac{1}{\sqrt{cd^2+2cdex+ce^2x^2}}\,dx}{c}\\
&= \frac{\left(cde+ce^2x\right)\int \frac{1}{cde+ce^2x}\,dx}{c\sqrt{cd^2+2cdex+ce^2x^2}}\\
&= \frac{(d+ex)\log(d+ex)}{ce\sqrt{cd^2+2cdex+ce^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.0028975, size = 31, normalized size = 0.74

$$\frac{(d+ex)\log(d+ex)}{ce\sqrt{c(d+ex)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2),x]
```

```
[Out] ((d + e*x)*Log[d + e*x])/(c*e*Sqrt[c*(d + e*x)^2])
```

Maple [A] time = 0.042, size = 40, normalized size = 1.

$$\frac{(ex+d)^3\ln(ex+d)}{e}\left(ce^2x^2+2\,cdex+cd^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x)
```

```
[Out] 1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)*(e*x+d)^3*ln(e*x+d)/e
```

Maxima [B] time = 1.14424, size = 166, normalized size = 3.95

$$\frac{3\,c^2d^2e^4}{2\left(ce^2\right)^{\frac{7}{2}}\left(x+\frac{d}{e}\right)^2}+\frac{2\,cde^3x}{\left(ce^2\right)^{\frac{5}{2}}\left(x+\frac{d}{e}\right)^2}+\frac{e^2\log\left(x+\frac{d}{e}\right)}{\left(ce^2\right)^{\frac{3}{2}}}-\frac{2\,d}{\sqrt{ce^2x^2+2\,cdex+cd^2}ce}+\frac{d^2}{2\left(ce^2\right)^{\frac{3}{2}}\left(x+\frac{d}{e}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] 3/2*c^2*d^2*e^4/((c*e^2)^(7/2)*(x + d/e)^2) + 2*c*d*e^3*x/((c*e^2)^(5/2)*(x
 + d/e)^2) + e^2*log(x + d/e)/(c*e^2)^(3/2) - 2*d/(sqrt(c*e^2*x^2 + 2*c*d*e
*x + c*d^2)*c*e) + 1/2*d^2/((c*e^2)^(3/2)*(x + d/e)^2)
```

Fricas [A] time = 2.35377, size = 97, normalized size = 2.31

$$\frac{\sqrt{ce^2x^2+2\,cdex+cd^2}\log(ex+d)}{c^2e^2x+c^2de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*log(e*x + d)/(c^2*e^2*x + c^2*d*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^2}{\left(c(d+ex)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2),x)
```

```
[Out] Integral((d + e*x)**2/(c*(d + e*x)**2)**(3/2), x)
```

Giac [B] time = 1.41241, size = 119, normalized size = 2.83

$$\frac{2\left(C_0de^{(-1)}+C_0x\right)}{\sqrt{cx^2e^2+2\,cdxe+cd^2}}-\frac{e^{(-1)}\log\left(\left|-\sqrt{c}de^2-\left(\sqrt{c}xe-\sqrt{cx^2e^2+2\,cdxe+cd^2}\right)e^2\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2*(C_0*d*e^(-1) + C_0*x)/sqrt(c*x^2*e^2 + 2*c*d*x*e + c*d^2) - e^(-1)*log(a
bs(-sqrt(c)*d*e^2 - (sqrt(c)*x*e - sqrt(c*x^2*e^2 + 2*c*d*x*e + c*d^2))*e^2
))/c^(3/2)
```

3.1074 $\int \frac{d+ex}{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=32

$$-\frac{1}{ce\sqrt{cd^2+2cdex+ce^2x^2}}$$

```
[Out] -(1/(c*e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]))
```

Rubi [A] time = 0.0092454, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {629}

$$-\frac{1}{ce\sqrt{cd^2+2cdex+ce^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2),x]
```

```
[Out] -(1/(c*e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]))
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex}{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}\,dx = -\frac{1}{ce\sqrt{cd^2+2cdex+ce^2x^2}}$$

Mathematica [A] time = 0.0064515, size = 21, normalized size = 0.66

$$-\frac{1}{ce\sqrt{c(d+ex)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2),x]
```

```
[Out] -(1/(c*e*Sqrt[c*(d + e*x)^2]))
```

Maple [A] time = 0.04, size = 35, normalized size = 1.1

$$-\frac{(ex+d)^2}{e}\left(ce^2x^2+2\,cdex+cd^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x)
```

```
[Out] -(e*x+d)^2/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)
```

Maxima [A] time = 1.19602, size = 41, normalized size = 1.28

$$-\frac{1}{\sqrt{ce^2x^2 + 2\,cdex + cd^2}ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/(sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c*e)
```

Fricas [A] time = 2.44006, size = 108, normalized size = 3.38

$$-\frac{\sqrt{ce^2x^2 + 2\,cdex + cd^2}}{c^2e^3x^2 + 2\,c^2de^2x + c^2d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(c^2*e^3*x^2 + 2*c^2*d*e^2*x + c^2*d^2
*e)
```

Sympy [A] time = 1.19376, size = 42, normalized size = 1.31

$$\begin{cases} -\frac{1}{ce\sqrt{cd^2+2cdex+ce^2x^2}} & \text{for}\ e \neq 0 \\ \frac{dx}{\left(cd^2\right)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2),x)
```

```
[Out] Piecewise((-1/(c*e*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)), Ne(e, 0)), (d*x
/(c*d**2)**(3/2), True))
```

Giac [A] time = 1.36931, size = 55, normalized size = 1.72

$$\frac{2\,C_0de^{(-1)} + 2\,C_0x - \frac{e^{(-1)}}{c}}{\sqrt{cx^2e^2 + 2\,cdxe + cd^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac")
```

```
[Out] (2*C_0*d*e^(-1) + 2*C_0*x - e^(-1)/c)/sqrt(c*x^2*e^2 + 2*c*d*x*e + c*d^2)
```

3.1075 $\int \frac{1}{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}} dx$

Optimal. Leaf size=41

$$-\frac{1}{2ce(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}$$

```
[Out] -1/(2*c*e*(d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])
```

Rubi [A] time = 0.0051423, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {607}

$$-\frac{1}{2ce(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(-3/2),x]
```

```
[Out] -1/(2*c*e*(d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}} dx = -\frac{1}{2ce(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}$$

Mathematica [A] time = 0.0026425, size = 25, normalized size = 0.61

$$-\frac{d+ex}{2e\left(c(d+ex)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(-3/2),x]
```

```
[Out] -(d + e*x)/(2*e*(c*(d + e*x)^2)^(3/2))
```

Maple [A] time = 0.042, size = 33, normalized size = 0.8

$$-\frac{ex+d}{2e}\left(ce^2x^2+2cdex+cd^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x)
```

```
[Out] -1/2*(e*x+d)/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)
```

Maxima [A] time = 1.25243, size = 24, normalized size = 0.59

$$-\frac{1}{2\left(ce^2\right)^{\frac{3}{2}}\left(x+\frac{d}{e}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2/((c*e^2)^(3/2)*(x + d/e)^2)
```

Fricas [A] time = 2.30546, size = 140, normalized size = 3.41

$$-\frac{\sqrt{ce^2x^2+2\,cdex+cd^2}}{2\left(c^2e^4x^3+3\,c^2de^3x^2+3\,c^2d^2e^2x+c^2d^3e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(c^2*e^4*x^3 + 3*c^2*d*e^3*x^2 + 3
*c^2*d^2*e^2*x + c^2*d^3*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cd^2+2cdex+ce^2x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2),x)
```

```
[Out] Integral((c*d**2 + 2*c*d*e*x + c*e**2*x**2)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.1076 $\int \frac{1}{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=31

$$-\frac{1}{3e\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}$$

```
[Out] -1/(3*e*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))
```

Rubi [A] time = 0.0251365, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {643, 629}

$$-\frac{1}{3e\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)),x]
```

```
[Out] -1/(3*e*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))
```

Rule 643

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}\,dx &= c\int \frac{d+ex}{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}\,dx\\ &= -\frac{1}{3e\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0198879, size = 20, normalized size = 0.65

$$-\frac{1}{3e\left(c(d+ex)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)),x]
```

```
[Out] -1/(3*e*(c*(d + e*x)^2)^(3/2))
```

Maple [A] time = 0.046, size = 28, normalized size = 0.9

$$-\frac{1}{3e}\left(ce^2x^2+2\,cdex+cd^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x)
```

```
[Out] -1/3/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)
```

Maxima [A] time = 1.13796, size = 63, normalized size = 2.03

$$-\frac{1}{3\left(c^{\frac{3}{2}}e^4x^3+3\,c^{\frac{3}{2}}de^3x^2+3\,c^{\frac{3}{2}}d^2e^2x+c^{\frac{3}{2}}d^3e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] -1/3/(c^(3/2)*e^4*x^3 + 3*c^(3/2)*d*e^3*x^2 + 3*c^(3/2)*d^2*e^2*x + c^(3/2)
*d^3*e)
```

Fricas [B] time = 2.40767, size = 167, normalized size = 5.39

$$-\frac{\sqrt{ce^2x^2+2\,cdex+cd^2}}{3\left(c^2e^5x^4+4\,c^2de^4x^3+6\,c^2d^2e^3x^2+4\,c^2d^3e^2x+c^2d^4e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/3*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(c^2*e^5*x^4 + 4*c^2*d*e^4*x^3 + 6
*c^2*d^2*e^3*x^2 + 4*c^2*d^3*e^2*x + c^2*d^4*e)
```

Sympy [A] time = 2.66401, size = 42, normalized size = 1.35

$$\begin{cases} -\frac{1}{3e\left(cd^2+2cdex+ce^2x^2\right)^{\frac{3}{2}}} & \text{for}\ e\neq 0 \\ \frac{x}{d\left(cd^2\right)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2),x)
```

```
[Out] Piecewise((-1/(3*e*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**(3/2)), Ne(e, 0)), (
x/(d*(c*d**2)**(3/2)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.1077 $\int \frac{1}{(d+ex)^2(cd^2+2cdex+ce^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{4e(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}$$

```
[Out] -1/(4*e*(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))
```

Rubi [A] time = 0.0202065, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {642, 607}

$$-\frac{1}{4e(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)),x]
```

```
[Out] -1/(4*e*(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))
```

Rule 642

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^m/c^(m/2), Int[(a + b*x + c*x^2)^(p + m/2), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && EqQ[2*c*d
 - b*e, 0] && IntegerQ[m/2]
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)^2\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}\,dx &= c\int \frac{1}{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}\,dx\\ &= -\frac{1}{4e(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0177119, size = 26, normalized size = 0.68

$$-\frac{c(d+ex)}{4e\left(c(d+ex)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)),x]
```

```
[Out] -(c*(d + e*x))/(4*e*(c*(d + e*x)^2)^(5/2))
```

Maple [A] time = 0.041, size = 35, normalized size = 0.9

$$-\frac{1}{4\,e\,(ex+d)}\left(ce^2x^2+2\,cdex+cd^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x)
```

```
[Out] -1/4/e/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)
```

Maxima [A] time = 1.20297, size = 82, normalized size = 2.16

$$-\frac{1}{4\left(c^{\frac{3}{2}}e^5x^4+4\,c^{\frac{3}{2}}de^4x^3+6\,c^{\frac{3}{2}}d^2e^3x^2+4\,c^{\frac{3}{2}}d^3e^2x+c^{\frac{3}{2}}d^4e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="maxim
a")
```

```
[Out] -1/4/(c^(3/2)*e^5*x^4 + 4*c^(3/2)*d*e^4*x^3 + 6*c^(3/2)*d^2*e^3*x^2 + 4*c^(
3/2)*d^3*e^2*x + c^(3/2)*d^4*e)
```

Fricas [B] time = 2.42569, size = 197, normalized size = 5.18

$$-\frac{\sqrt{ce^2x^2+2\,cdex+cd^2}}{4\left(c^2e^6x^5+5\,c^2de^5x^4+10\,c^2d^2e^4x^3+10\,c^2d^3e^3x^2+5\,c^2d^4e^2x+c^2d^5e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="frica
s")
```

```
[Out] -1/4*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(c^2*e^6*x^5 + 5*c^2*d*e^5*x^4 + 1
0*c^2*d^2*e^4*x^3 + 10*c^2*d^3*e^3*x^2 + 5*c^2*d^4*e^2*x + c^2*d^5*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c\,(d+ex)^2\right)^{\frac{3}{2}}(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2),x)
```

```
[Out] Integral(1/((c*(d + e*x)**2)**(3/2)*(d + e*x)**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.1078 $\int \frac{1}{(d+ex)^3\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=32

$$-\frac{c}{5e\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}$$

```
[Out] -c/(5*e*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2))
```

Rubi [A] time = 0.023793, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {643, 629}

$$-\frac{c}{5e\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)),x]
```

```
[Out] -c/(5*e*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2))
```

Rule 643

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)^3\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}\,dx &= c^2\int \frac{d+ex}{\left(cd^2+2cdex+ce^2x^2\right)^{7/2}}\,dx\\ &= -\frac{c}{5e\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.0262628, size = 21, normalized size = 0.66

$$-\frac{c}{5e\left(c(d+ex)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)),x]
```

```
[Out] -c/(5*e*(c*(d + e*x)^2)^(5/2))
```

Maple [A] time = 0.041, size = 35, normalized size = 1.1

$$-\frac{1}{5\,(ex+d)^2\,e}\left(ce^2x^2+2\,cdex+cd^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x)
```

```
[Out] -1/5/(e*x+d)^2/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)
```

Maxima [B] time = 1.20147, size = 101, normalized size = 3.16

$$-\frac{1}{5\left(c^{\frac{3}{2}}e^6x^5+5\,c^{\frac{3}{2}}de^5x^4+10\,c^{\frac{3}{2}}d^2e^4x^3+10\,c^{\frac{3}{2}}d^3e^3x^2+5\,c^{\frac{3}{2}}d^4e^2x+c^{\frac{3}{2}}d^5e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="maxim
a")
```

```
[Out] -1/5/(c^(3/2)*e^6*x^5 + 5*c^(3/2)*d*e^5*x^4 + 10*c^(3/2)*d^2*e^4*x^3 + 10*c
^(3/2)*d^3*e^3*x^2 + 5*c^(3/2)*d^4*e^2*x + c^(3/2)*d^5*e)
```

Fricas [B] time = 2.35955, size = 225, normalized size = 7.03

$$-\frac{\sqrt{ce^2x^2+2\,cdex+cd^2}}{5\left(c^2e^7x^6+6\,c^2de^6x^5+15\,c^2d^2e^5x^4+20\,c^2d^3e^4x^3+15\,c^2d^4e^3x^2+6\,c^2d^5e^2x+c^2d^6e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="frica
s")
```

```
[Out] -1/5*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(c^2*e^7*x^6 + 6*c^2*d*e^6*x^5 + 1
5*c^2*d^2*e^5*x^4 + 20*c^2*d^3*e^4*x^3 + 15*c^2*d^4*e^3*x^2 + 6*c^2*d^5*e^2
*x + c^2*d^6*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(c\,(d+ex)^2\right)^{\frac{3}{2}}(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2),x)
```

```
[Out] Integral(1/((c*(d + e*x)**2)**(3/2)*(d + e*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\text{undef}, \text{undef}, \text{undef}, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac"
)
```

```
[Out] [undef, undef, undef, 1]
```

3.1079 $\int \frac{(d+ex)^6}{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=39

$$\frac{(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}{2c^3e}$$

```
[Out] ((d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/(2*c^3*e)
```

Rubi [A] time = 0.0225454, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {642, 609}

$$\frac{(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}{2c^3e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^6/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2),x]
```

```
[Out] ((d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/(2*c^3*e)
```

Rule 642

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^m/c^(m/2), Int[(a + b*x + c*x^2)^(p + m/2), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && EqQ[2*c*d
 - b*e, 0] && IntegerQ[m/2]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^6}{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}\,dx &= \frac{\int \sqrt{cd^2+2cdex+ce^2x^2}\,dx}{c^3}\\ &= \frac{(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}{2c^3e}\end{aligned}$$

Mathematica [A] time = 0.0032829, size = 33, normalized size = 0.85

$$\frac{x(d+ex)(2d+ex)}{2c^2\sqrt{c(d+ex)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^6/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2),x]
```

```
[Out] (x*(d + e*x)*(2*d + e*x))/(2*c^2*Sqrt[c*(d + e*x)^2])
```

Maple [A] time = 0.04, size = 40, normalized size = 1.

$$\frac{x\left(ex+2d\right)\left(ex+d\right)^{5}}{2}\left(ce^{2}x^{2}+2cdex+cd^{2}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^6/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x)
```

```
[Out] 1/2*x*(e*x+2*d)*(e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)
```

Maxima [B] time = 1.27596, size = 313, normalized size = 8.03

$$\frac{e^{4}x^{5}}{2\left(ce^{2}x^{2}+2cdex+cd^{2}\right)^{\frac{3}{2}}c}+\frac{5de^{3}x^{4}}{2\left(ce^{2}x^{2}+2cdex+cd^{2}\right)^{\frac{3}{2}}c}-\frac{25c^{2}d^{6}e^{4}}{4\left(ce^{2}\right)^{\frac{9}{2}}\left(x+\frac{d}{e}\right)^{4}}-\frac{10d^{3}ex^{2}}{\left(ce^{2}x^{2}+2cdex+cd^{2}\right)^{\frac{3}{2}}c}+\frac{50cd}{3\left(ce^{2}\right)^{\frac{7}{2}}\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="maxima"
)
```

```
[Out] 1/2*e^4*x^5/((c*e^2*x^2 + 2*c*d*e*x + c*d^2)^(3/2)*c) + 5/2*d*e^3*x^4/((c*e
^2*x^2 + 2*c*d*e*x + c*d^2)^(3/2)*c) - 25/4*c^2*d^6*e^4/((c*e^2)^(9/2)*(x +
 d/e)^4) - 10*d^3*e*x^2/((c*e^2*x^2 + 2*c*d*e*x + c*d^2)^(3/2)*c) + 50/3*c*
d^5*e^3/((c*e^2)^(7/2)*(x + d/e)^3) - 26/3*d^5/((c*e^2*x^2 + 2*c*d*e*x + c*
d^2)^(3/2)*c*e) - 25/2*d^4*e^2/((c*e^2)^(5/2)*(x + d/e)^2) + 25/4*d^6/((c*e
^2)^(5/2)*(x + d/e)^4)
```

Fricas [A] time = 2.30854, size = 101, normalized size = 2.59

$$\frac{\sqrt{ce^{2}x^{2}+2cdex+cd^{2}}\left(ex^{2}+2dx\right)}{2\left(c^{3}ex+c^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/2*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*(e*x^2 + 2*d*x)/(c^3*e*x + c^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(d+ex\right)^{6}}{\left(c\left(d+ex\right)^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**6/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2),x)
```

```
[Out] Integral((d + e*x)**6/(c*(d + e*x)**2)**(5/2), x)
```

Giac [B] time = 1.48774, size = 146, normalized size = 3.74

$$-\frac{\frac{9\,d^5e^{(-1)}}{c} - 4\,C_0d^3e^{(-3)} - \left(12\,C_0d^2e^{(-2)} - \frac{25\,d^4}{c} - \left(\frac{20\,d^3e}{c} - 12\,C_0de^{(-1)} - \left(x\left(\frac{xe^4}{c} + \frac{5\,de^3}{c}\right) + 4\,C_0\right)x\right)x\right)x}{2\left(cx^2e^2 + 2\,cdxe + cd^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*(9*d^5*e^(-1)/c - 4*C_0*d^3*e^(-3) - (12*C_0*d^2*e^(-2) - 25*d^4/c - (
20*d^3*e/c - 12*C_0*d*e^(-1) - (x*(x*e^4/c + 5*d*e^3/c) + 4*C_0)*x)*x)*x)/(
c*x^2*e^2 + 2*c*d*x*e + c*d^2)^(3/2)
```

3.1080 $\int \frac{(d+ex)^5}{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=31

$$\frac{\sqrt{cd^2+2cdex+ce^2x^2}}{c^3e}$$

```
[Out] Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(c^3*e)
```

Rubi [A] time = 0.0237814, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {643, 629}

$$\frac{\sqrt{cd^2+2cdex+ce^2x^2}}{c^3e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2),x]
```

```
[Out] Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(c^3*e)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^5}{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}\,dx &= \frac{\int \frac{d+ex}{\sqrt{cd^2+2cdex+ce^2x^2}}\,dx}{c^2}\\ &= \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{c^3e}\end{aligned}$$

Mathematica [A] time = 0.0050517, size = 23, normalized size = 0.74

$$\frac{x(d+ex)}{c^2\sqrt{c(d+ex)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2),x]
```

```
[Out] (x*(d + e*x))/(c^2*Sqrt[c*(d + e*x)^2])
```

Maple [A] time = 0.039, size = 32, normalized size = 1.

$$(ex+d)^5 x\left(ce^2x^2+2cdex+cd^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x)
```

```
[Out] 1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)*(e*x+d)^5*x
```

Maxima [B] time = 1.25439, size = 266, normalized size = 8.58

$$\frac{e^3x^4}{\left(ce^2x^2+2cdex+cd^2\right)^{\frac{3}{2}}c}-\frac{4c^2d^5e^4}{\left(ce^2\right)^{\frac{9}{2}}\left(x+\frac{d}{e}\right)^4}-\frac{6d^2ex^2}{\left(ce^2x^2+2cdex+cd^2\right)^{\frac{3}{2}}c}+\frac{32cd^4e^3}{3\left(ce^2\right)^{\frac{7}{2}}\left(x+\frac{d}{e}\right)^3}-\frac{17d^4}{3\left(ce^2x^2+2cdex+cd^2\right)^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="maxima"
)
```

```
[Out] e^3*x^4/((c*e^2*x^2 + 2*c*d*e*x + c*d^2)^(3/2)*c) - 4*c^2*d^5*e^4/((c*e^2)^
(9/2)*(x + d/e)^4) - 6*d^2*e*x^2/((c*e^2*x^2 + 2*c*d*e*x + c*d^2)^(3/2)*c)
+ 32/3*c*d^4*e^3/((c*e^2)^(7/2)*(x + d/e)^3) - 17/3*d^4/((c*e^2*x^2 + 2*c*d
*e*x + c*d^2)^(3/2)*c*e) - 8*d^3*e^2/((c*e^2)^(5/2)*(x + d/e)^2) + 4*d^5/((
c*e^2)^(5/2)*(x + d/e)^4)
```

Fricas [A] time = 2.38364, size = 77, normalized size = 2.48

$$\frac{\sqrt{ce^2x^2+2cdex+cd^2}\,x}{c^3ex+c^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="fricas"
)
```

```
[Out] sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*x/(c^3*e*x + c^3*d)
```

Sympy [A] time = 1.70706, size = 42, normalized size = 1.35

$$\begin{cases}\frac{\sqrt{cd^2+2cdex+ce^2x^2}}{c^3e} & \text{for } e\neq 0\\ \frac{d^5x}{\left(cd^2\right)^{\frac{5}{2}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2),x)
```

```
[Out] Piecewise((sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(c**3*e), Ne(e, 0)), (d**
5*x/(c*d**2)**(5/2), True))
```

Giac [B] time = 1.45984, size = 126, normalized size = 4.06

$$\frac{2\,C_0 d^3 e^{(-3)} - \frac{3\,d^4 e^{(-1)}}{c} + \left(6\,C_0 d^2 e^{(-2)} - \frac{8\,d^3}{c} + \left(6\,C_0 d e^{(-1)} + \left(2\,C_0 + \frac{x e^3}{c}\right)x - \frac{6\,d^2 e}{c}\right)x\right)x}{\left(c x^2 e^2 + 2\,c d x e + c d^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="giac")
```

```
[Out] (2*C_0*d^3*e^(-3) - 3*d^4*e^(-1)/c + (6*C_0*d^2*e^(-2) - 8*d^3/c + (6*C_0*d
*e^(-1) + (2*C_0 + x*e^3/c)*x - 6*d^2*e/c)*x)*x)/(c*x^2*e^2 + 2*c*d*x*e + c
*d^2)^(3/2)
```

3.1081 $\int \frac{(d+ex)^4}{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=42

$$\frac{(d+ex)\log(d+ex)}{c^2e\sqrt{cd^2+2cdex+ce^2x^2}}$$

```
[Out] ((d + e*x)*Log[d + e*x])/(c^2*e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])
```

Rubi [A] time = 0.0258827, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {642, 608, 31}

$$\frac{(d+ex)\log(d+ex)}{c^2e\sqrt{cd^2+2cdex+ce^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2),x]
```

```
[Out] ((d + e*x)*Log[d + e*x])/(c^2*e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^m/c^(m/2), Int[(a + b*x + c*x^2)^(p + m/2), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && EqQ[2*c*d
 - b*e, 0] && IntegerQ[m/2]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^4}{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}\,dx &= \frac{\int \frac{1}{\sqrt{cd^2+2cdex+ce^2x^2}}\,dx}{c^2}\\
&= \frac{\left(cde+ce^2x\right)\int \frac{1}{cde+ce^2x}\,dx}{c^2\sqrt{cd^2+2cdex+ce^2x^2}}\\
&= \frac{(d+ex)\log(d+ex)}{c^2e\sqrt{cd^2+2cdex+ce^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.0070995, size = 31, normalized size = 0.74

$$\frac{(d+ex)\log(d+ex)}{c^2e\sqrt{c(d+ex)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2),x]
```

```
[Out] ((d + e*x)*Log[d + e*x])/(c^2*e*Sqrt[c*(d + e*x)^2])
```

Maple [A] time = 0.042, size = 40, normalized size = 1.

$$\frac{(ex+d)^5\ln(ex+d)}{e}\left(ce^2x^2+2\,cdex+cd^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x)
```

```
[Out] 1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)*(e*x+d)^5*ln(e*x+d)/e
```

Maxima [B] time = 1.57926, size = 606, normalized size = 14.43

$$\frac{1}{12}e^4\left(\frac{48\sqrt{c}de^3x^3+108\sqrt{c}d^2e^2x^2+88\sqrt{c}d^3ex+25\sqrt{c}d^4}{c^3e^9x^4+4\,c^3de^8x^3+6\,c^3d^2e^7x^2+4\,c^3d^3e^6x+c^3d^4e^5}+\frac{12\log(ex+d)}{c^{\frac{5}{2}}e^5}\right)-\frac{1}{3}de^3\left(\frac{3\,c^2d^3e}{\left(ce^2\right)^{\frac{9}{2}}\left(x+\frac{d}{e}\right)^4}+\frac{1}{\left(ce^2x^2+2\,c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="maxima"
)
```

```
[Out] 1/12*e^4*((48*sqrt(c)*d*e^3*x^3 + 108*sqrt(c)*d^2*e^2*x^2 + 88*sqrt(c)*d^3*
e*x + 25*sqrt(c)*d^4)/(c^3*e^9*x^4 + 4*c^3*d*e^8*x^3 + 6*c^3*d^2*e^7*x^2 +
4*c^3*d^3*e^6*x + c^3*d^4*e^5) + 12*log(e*x + d)/(c^(5/2)*e^5)) - 1/3*d*e^3
*(3*c^2*d^3*e/((c*e^2)^(9/2)*(x + d/e)^4) + 12*x^2/((c*e^2*x^2 + 2*c*d*e*x
+ c*d^2)^(3/2)*c*e^2) - 8*c*d^2/((c*e^2)^(7/2)*(x + d/e)^3) + 8*d^2/((c*e^2
*x^2 + 2*c*d*e*x + c*d^2)^(3/2)*c*e^4) + 6*d/((c*e^2)^(5/2)*e*(x + d/e)^2)
- 6*d^3/((c*e^2)^(5/2)*e^3*(x + d/e)^4)) - 1/2*d^2*e^2*(3*c^2*d^2*e^2/((c*e
^2)^(9/2)*(x + d/e)^4) - 8*c*d*e/((c*e^2)^(7/2)*(x + d/e)^3) + 6/((c*e^2)^(
5/2)*(x + d/e)^2)) - 1/3*d^3*e*(4/((c*e^2*x^2 + 2*c*d*e*x + c*d^2)^(3/2)*c*
e^2) - 3*d/((c*e^2)^(5/2)*e*(x + d/e)^4)) - 1/4*d^4/((c*e^2)^(5/2)*(x + d/e
)^4)
```

Fricas [A] time = 2.32264, size = 97, normalized size = 2.31

$$\frac{\sqrt{ce^2x^2+2\,cdex+cd^2}\log(ex+d)}{c^3e^2x+c^3de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="fricas"
)
```

```
[Out] sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*log(e*x + d)/(c^3*e^2*x + c^3*d*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^4}{\left(c\,(d+ex)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2),x)
```

```
[Out] Integral((d + e*x)**4/(c*(d + e*x)**2)**(5/2), x)
```

Giac [B] time = 1.55926, size = 149, normalized size = 3.55

$$\frac{2\left(C_0d^3e^{(-3)}+\left(3\,C_0d^2e^{(-2)}+\left(3\,C_0de^{(-1)}+C_0x\right)x\right)x\right)}{\left(cx^2e^2+2\,cdxe+cd^2\right)^{\frac{3}{2}}}-\frac{e^{(-1)}\log\left(\left|-\sqrt{c}de^2-\left(\sqrt{c}xe-\sqrt{cx^2e^2+2\,cdxe+cd^2}\right)e^2\right|\right)}{c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 2*(C_0*d^3*e^(-3) + (3*C_0*d^2*e^(-2) + (3*C_0*d*e^(-1) + C_0*x)*x)*x)/(c*x
^2*e^2 + 2*c*d*x*e + c*d^2)^(3/2) - e^(-1)*log(abs(-sqrt(c)*d*e^2 - (sqrt(c
)*x*e - sqrt(c*x^2*e^2 + 2*c*d*x*e + c*d^2))*e^2))/c^(5/2)
```

3.1082 $\int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx$

Optimal. Leaf size=32

$$-\frac{1}{c^2 e\sqrt{cd^2+2cdex+ce^2x^2}}$$

```
[Out] -(1/(c^2*e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]))
```

Rubi [A] time = 0.0246531, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {643, 629}

$$-\frac{1}{c^2 e\sqrt{cd^2+2cdex+ce^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2),x]

[Out] -(1/(c^2*e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]))
```

Rule 643

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^{5/2}}\,dx &= \frac{\int \frac{d+ex}{(cd^2+2cdex+ce^2x^2)^{3/2}}\,dx}{c}\\ &= -\frac{1}{c^2 e\sqrt{cd^2+2cdex+ce^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0132627, size = 21, normalized size = 0.66

$$-\frac{1}{c^2 e\sqrt{c(d+ex)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2),x]
```

```
[Out] -(1/(c^2*e*Sqrt[c*(d + e*x)^2]))
```

Maple [A] time = 0.04, size = 35, normalized size = 1.1

$$-\frac{(ex+d)^4}{e}\left(ce^2x^2+2\,cdex+cd^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x)
```

```
[Out] -(e*x+d)^4/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)
```

Maxima [B] time = 1.57334, size = 215, normalized size = 6.72

$$-\frac{c^2d^3e^4}{\left(ce^2\right)^{\frac{9}{2}}\left(x+\frac{d}{e}\right)^4}-\frac{ex^2}{\left(ce^2x^2+2\,cdex+cd^2\right)^{\frac{3}{2}}c}+\frac{8\,cd^2e^3}{3\,\left(ce^2\right)^{\frac{7}{2}}\left(x+\frac{d}{e}\right)^3}-\frac{5\,d^2}{3\left(ce^2x^2+2\,cdex+cd^2\right)^{\frac{3}{2}}ce}-\frac{2\,de^2}{\left(ce^2\right)^{\frac{5}{2}}\left(x+\frac{d}{e}\right)^2}+\frac{}{\left(ce\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="maxima"
)
```

```
[Out] -c^2*d^3*e^4/((c*e^2)^(9/2)*(x + d/e)^4) - e*x^2/((c*e^2*x^2 + 2*c*d*e*x +
c*d^2)^(3/2)*c) + 8/3*c*d^2*e^3/((c*e^2)^(7/2)*(x + d/e)^3) - 5/3*d^2/((c*e
^2*x^2 + 2*c*d*e*x + c*d^2)^(3/2)*c*e) - 2*d*e^2/((c*e^2)^(5/2)*(x + d/e)^2
) + d^3/((c*e^2)^(5/2)*(x + d/e)^4)
```

Fricas [A] time = 2.40682, size = 108, normalized size = 3.38

$$-\frac{\sqrt{ce^2x^2+2\,cdex+cd^2}}{c^3e^3x^2+2\,c^3de^2x+c^3d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="fricas"
)
```

```
[Out] -sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(c^3*e^3*x^2 + 2*c^3*d*e^2*x + c^3*d^2
*e)
```

Sympy [A] time = 1.52782, size = 70, normalized size = 2.19

$$\begin{cases} -\frac{\sqrt{cd^2+2cdex+ce^2x^2}}{c^3d^2e+2c^3de^2x+c^3e^3x^2} & \text{for}\: e \neq 0 \\ \frac{d^3x}{\left(cd^2\right)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2),x)
```

```
[Out] Piecewise((-sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(c**3*d**2*e + 2*c**3*d*
e**2*x + c**3*e**3*x**2), Ne(e, 0)), (d**3*x/(c*d**2)**(5/2), True))
```

Giac [B] time = 1.40174, size = 107, normalized size = 3.34

$$\frac{2\,C_0d^3e^{(-3)}+\left(6\,C_0d^2e^{(-2)}+\left(6\,C_0de^{(-1)}+2\,C_0x-\frac{e}{c}\right)x-\frac{2\,d}{c}\right)x-\frac{d^2e^{(-1)}}{c}}{\left(cx^2e^2+2\,cdxe+cd^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="giac")
```

```
[Out] (2*C_0*d^3*e^(-3) + (6*C_0*d^2*e^(-2) + (6*C_0*d*e^(-1) + 2*C_0*x - e/c)*x
- 2*d/c)*x - d^2*e^(-1)/c)/(c*x^2*e^2 + 2*c*d*x*e + c*d^2)^(3/2)
```

3.1083 $\int \frac{(d+ex)^2}{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{2c^2e(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}$$

```
[Out] -1/(2*c^2*e*(d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])
```

Rubi [A] time = 0.0215117, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {642, 607}

$$-\frac{1}{2c^2e(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2),x]
```

```
[Out] -1/(2*c^2*e*(d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^m/c^(m/2), Int[(a + b*x + c*x^2)^(p + m/2), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && EqQ[2*c*d
 - b*e, 0] && IntegerQ[m/2]
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2}{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}\,dx &= \frac{\int \frac{1}{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}\,dx}{c}\\ &= -\frac{1}{2c^2e(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0096567, size = 28, normalized size = 0.68

$$-\frac{d+ex}{2ce\left(c(d+ex)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2),x]
```

```
[Out] -(d + e*x)/(2*c*e*(c*(d + e*x)^2)^(3/2))
```

Maple [A] time = 0.041, size = 35, normalized size = 0.9

$$-\frac{(ex+d)^3}{2e}\left(ce^2x^2+2\,cdex+cd^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x)
```

```
[Out] -1/2*(e*x+d)^3/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)
```

Maxima [B] time = 1.27339, size = 167, normalized size = 4.07

$$-\frac{c^2d^2e^4}{4\left(ce^2\right)^{\frac{9}{2}}\left(x+\frac{d}{e}\right)^4}+\frac{2\,cde^3}{3\left(ce^2\right)^{\frac{7}{2}}\left(x+\frac{d}{e}\right)^3}-\frac{2\,d}{3\left(ce^2x^2+2\,cdex+cd^2\right)^{\frac{3}{2}}ce}-\frac{e^2}{2\left(ce^2\right)^{\frac{5}{2}}\left(x+\frac{d}{e}\right)^2}+\frac{d^2}{4\left(ce^2\right)^{\frac{5}{2}}\left(x+\frac{d}{e}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="maxima"
)
```

```
[Out] -1/4*c^2*d^2*e^4/((c*e^2)^(9/2)*(x + d/e)^4) + 2/3*c*d*e^3/((c*e^2)^(7/2)*(
x + d/e)^3) - 2/3*d/((c*e^2*x^2 + 2*c*d*e*x + c*d^2)^(3/2)*c*e) - 1/2*e^2/(
(c*e^2)^(5/2)*(x + d/e)^2) + 1/4*d^2/((c*e^2)^(5/2)*(x + d/e)^4)
```

Fricas [A] time = 2.38008, size = 140, normalized size = 3.41

$$-\frac{\sqrt{ce^2x^2+2\,cdex+cd^2}}{2\left(c^3e^4x^3+3\,c^3de^3x^2+3\,c^3d^2e^2x+c^3d^3e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="fricas"
)
```

```
[Out] -1/2*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(c^3*e^4*x^3 + 3*c^3*d*e^3*x^2 + 3
*c^3*d^2*e^2*x + c^3*d^3*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(d+ex)^2}{\left(c\,(d+ex)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2),x)
```

```
[Out] Integral((d + e*x)**2/(c*(d + e*x)**2)**(5/2), x)
```

Giac [A] time = 1.53967, size = 95, normalized size = 2.32

$$\frac{4\,C_0d^3e^{(-3)} + \left(12\,C_0d^2e^{(-2)} + 4\left(3\,C_0de^{(-1)} + C_0x\right)x - \frac{1}{c}\right)x - \frac{de^{(-1)}}{c}}{2\left(cx^2e^2 + 2\,cdxe + cd^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2*(4*C_0*d^3*e^(-3) + (12*C_0*d^2*e^(-2) + 4*(3*C_0*d*e^(-1) + C_0*x)*x -
 1/c)*x - d*e^(-1)/c)/(c*x^2*e^2 + 2*c*d*x*e + c*d^2)^(3/2)
```

3.1084 $\int \frac{d+ex}{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}} dx$

Optimal. Leaf size=34

$$-\frac{1}{3ce\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}$$

```
[Out] -1/(3*c*e*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))
```

Rubi [A] time = 0.0093726, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {629}

$$-\frac{1}{3ce\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2),x]
```

```
[Out] -1/(3*c*e*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex}{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}\,dx=-\frac{1}{3ce\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}$$

Mathematica [A] time = 0.0112833, size = 30, normalized size = 0.88

$$-\frac{\sqrt{c(d+ex)^2}}{3c^3e(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2),x]
```

```
[Out] -Sqrt[c*(d + e*x)^2]/(3*c^3*e*(d + e*x)^4)
```

Maple [A] time = 0.04, size = 35, normalized size = 1.

$$-\frac{(ex+d)^2}{3e}\left(ce^2x^2+2cdex+cd^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x)
```

```
[Out] -1/3*(e*x+d)^2/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)
```

Maxima [A] time = 1.57778, size = 41, normalized size = 1.21

$$-\frac{1}{3\left(ce^2x^2+2\,cdex+cd^2\right)^{\frac{3}{2}}ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3/((c*e^2*x^2 + 2*c*d*e*x + c*d^2)^(3/2)*c*e)
```

Fricas [B] time = 2.18579, size = 167, normalized size = 4.91

$$-\frac{\sqrt{ce^2x^2+2\,cdex+cd^2}}{3\left(c^3e^5x^4+4\,c^3de^4x^3+6\,c^3d^2e^3x^2+4\,c^3d^3e^2x+c^3d^4e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(c^3*e^5*x^4 + 4*c^3*d*e^4*x^3 + 6
*c^3*d^2*e^3*x^2 + 4*c^3*d^3*e^2*x + c^3*d^4*e)
```

Sympy [A] time = 1.64703, size = 124, normalized size = 3.65

$$\begin{cases} -\frac{1}{3c^2d^2e\sqrt{cd^2+2cdex+ce^2x^2}+6c^2de^2x\sqrt{cd^2+2cdex+ce^2x^2}+3c^2e^3x^2\sqrt{cd^2+2cdex+ce^2x^2}} & \text{for}\ e\neq 0 \\ \frac{dx}{\left(cd^2\right)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2),x)
```

```
[Out] Piecewise((-1/(3*c**2*d**2*e*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2) + 6*c**
2*d*e**2*x*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2) + 3*c**2*e**3*x**2*sqrt(c
*d**2 + 2*c*d*e*x + c*e**2*x**2)), Ne(e, 0)), (d*x/(c*d**2)**(5/2), True))
```

Giac [B] time = 1.33218, size = 86, normalized size = 2.53

$$\frac{6\,C_0d^3e^{(-3)}+6\left(3\,C_0d^2e^{(-2)}+\left(3\,C_0de^{(-1)}+C_0x\right)x\right)x-\frac{e^{(-1)}}{c}}{3\left(cx^2e^2+2\,cdxe+cd^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(6*C_0*d^3*e^(-3) + 6*(3*C_0*d^2*e^(-2) + (3*C_0*d*e^(-1) + C_0*x)*x)*x
 - e^(-1)/c)/(c*x^2*e^2 + 2*c*d*x*e + c*d^2)^(3/2)
```

3.1085 $\int \frac{1}{(cd^2+2cdex+ce^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{4ce(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}$$

```
[Out] -1/(4*c*e*(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))
```

Rubi [A] time = 0.0050822, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {607}

$$-\frac{1}{4ce(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(-5/2),x]
```

```
[Out] -1/(4*c*e*(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}\,dx=-\frac{1}{4ce(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}$$

Mathematica [A] time = 0.0028933, size = 25, normalized size = 0.61

$$-\frac{d+ex}{4e\left(c(d+ex)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(-5/2),x]
```

```
[Out] -(d + e*x)/(4*e*(c*(d + e*x)^2)^(5/2))
```

Maple [A] time = 0.039, size = 33, normalized size = 0.8

$$-\frac{ex+d}{4\,e}\left(ce^2x^2+2\,cdex+cd^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x)
```

```
[Out] -1/4*(e*x+d)/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)
```

Maxima [A] time = 1.14462, size = 24, normalized size = 0.59

$$-\frac{1}{4\left(ce^2\right)^{\frac{5}{2}}\left(x+\frac{d}{e}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4/((c*e^2)^(5/2)*(x + d/e)^4)
```

Fricas [B] time = 2.28296, size = 197, normalized size = 4.8

$$-\frac{\sqrt{ce^2x^2+2\,cdex+cd^2}}{4\left(c^3e^6x^5+5\,c^3de^5x^4+10\,c^3d^2e^4x^3+10\,c^3d^3e^3x^2+5\,c^3d^4e^2x+c^3d^5e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(c^3*e^6*x^5 + 5*c^3*d*e^5*x^4 + 1
0*c^3*d^2*e^4*x^3 + 10*c^3*d^3*e^3*x^2 + 5*c^3*d^4*e^2*x + c^3*d^5*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(cd^2+2cdex+ce^2x^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2),x)
```

```
[Out] Integral((c*d**2 + 2*c*d*e*x + c*e**2*x**2)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.1086 $\int \frac{1}{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=31

$$-\frac{1}{5e\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}$$

```
[Out] -1/(5*e*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2))
```

Rubi [A] time = 0.0242698, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {643, 629}

$$-\frac{1}{5e\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)),x]
```

```
[Out] -1/(5*e*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2))
```

Rule 643

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}\,dx &= c\int \frac{d+ex}{\left(cd^2+2cdex+ce^2x^2\right)^{7/2}}\,dx\\
&= -\frac{1}{5e\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0266593, size = 20, normalized size = 0.65

$$-\frac{1}{5e\left(c(d+ex)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)),x]
```

```
[Out] -1/(5*e*(c*(d + e*x)^2)^(5/2))
```

Maple [A] time = 0.042, size = 28, normalized size = 0.9

$$-\frac{1}{5e}\left(ce^{2}x^{2}+2\,cdex+cd^{2}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x)
```

```
[Out] -1/5/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)
```

Maxima [B] time = 1.21331, size = 101, normalized size = 3.26

$$-\frac{1}{5\left(c^{\frac{5}{2}}e^{6}x^{5}+5\,c^{\frac{5}{2}}de^{5}x^{4}+10\,c^{\frac{5}{2}}d^{2}e^{4}x^{3}+10\,c^{\frac{5}{2}}d^{3}e^{3}x^{2}+5\,c^{\frac{5}{2}}d^{4}e^{2}x+c^{\frac{5}{2}}d^{5}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="maxima"
)
```

```
[Out] -1/5/(c^(5/2)*e^6*x^5 + 5*c^(5/2)*d*e^5*x^4 + 10*c^(5/2)*d^2*e^4*x^3 + 10*c
^(5/2)*d^3*e^3*x^2 + 5*c^(5/2)*d^4*e^2*x + c^(5/2)*d^5*e)
```

Fricas [B] time = 2.32166, size = 225, normalized size = 7.26

$$-\frac{\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}}{5\left(c^{3}e^{7}x^{6}+6\,c^{3}de^{6}x^{5}+15\,c^{3}d^{2}e^{5}x^{4}+20\,c^{3}d^{3}e^{4}x^{3}+15\,c^{3}d^{4}e^{3}x^{2}+6\,c^{3}d^{5}e^{2}x+c^{3}d^{6}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="fricas"
)
```

```
[Out] -1/5*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(c^3*e^7*x^6 + 6*c^3*d*e^6*x^5 + 1
5*c^3*d^2*e^5*x^4 + 20*c^3*d^3*e^4*x^3 + 15*c^3*d^4*e^3*x^2 + 6*c^3*d^5*e^2
*x + c^3*d^6*e)
```

Sympy [A] time = 3.66332, size = 42, normalized size = 1.35

$$\begin{cases} -\frac{1}{5e\left(cd^{2}+2cdex+ce^{2}x^{2}\right)^{\frac{5}{2}}} & \text{for}\: e \neq 0 \\ \frac{x}{d\left(cd^{2}\right)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2),x)
```

```
[Out] Piecewise((-1/(5*e*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**(5/2)), Ne(e, 0)), (
x/(d*(c*d**2)**(5/2)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\mathrm{undef}, \mathrm{undef}, \mathrm{undef}, \mathrm{undef}, \mathrm{undef}, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, 1]
```

3.1087 $\int \frac{1}{(d+ex)^2\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{6e(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}$$

```
[Out] -1/(6*e*(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2))
```

Rubi [A] time = 0.0206601, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {642, 607}

$$-\frac{1}{6e(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)),x]
```

```
[Out] -1/(6*e*(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2))
```

Rule 642

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^m/c^(m/2), Int[(a + b*x + c*x^2)^(p + m/2), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && EqQ[2*c*d
 - b*e, 0] && IntegerQ[m/2]
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)^2\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}\,dx &= c\int \frac{1}{\left(cd^2+2cdex+ce^2x^2\right)^{7/2}}\,dx\\ &= -\frac{1}{6e(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.0177792, size = 26, normalized size = 0.68

$$-\frac{c(d+ex)}{6e\left(c(d+ex)^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)),x]
```

```
[Out] -(c*(d + e*x))/(6*e*(c*(d + e*x)^2)^(7/2))
```

Maple [A] time = 0.042, size = 35, normalized size = 0.9

$$-\frac{1}{6e(ex+d)}\left(ce^2x^2+2cdex+cd^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x)
```

```
[Out] -1/6/e/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)
```

Maxima [B] time = 1.16159, size = 120, normalized size = 3.16

$$-\frac{1}{6\left(c^{\frac{5}{2}}e^7x^6+6c^{\frac{5}{2}}de^6x^5+15c^{\frac{5}{2}}d^2e^5x^4+20c^{\frac{5}{2}}d^3e^4x^3+15c^{\frac{5}{2}}d^4e^3x^2+6c^{\frac{5}{2}}d^5e^2x+c^{\frac{5}{2}}d^6e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="maxim
a")
```

```
[Out] -1/6/(c^(5/2)*e^7*x^6 + 6*c^(5/2)*d*e^6*x^5 + 15*c^(5/2)*d^2*e^5*x^4 + 20*c
^(5/2)*d^3*e^4*x^3 + 15*c^(5/2)*d^4*e^3*x^2 + 6*c^(5/2)*d^5*e^2*x + c^(5/2)
*d^6*e)
```

Fricas [B] time = 2.26976, size = 254, normalized size = 6.68

$$-\frac{\sqrt{ce^2x^2+2cdex+cd^2}}{6\left(c^3e^8x^7+7c^3de^7x^6+21c^3d^2e^6x^5+35c^3d^3e^5x^4+35c^3d^4e^4x^3+21c^3d^5e^3x^2+7c^3d^6e^2x+c^3d^7e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="frica
s")
```

```
[Out] -1/6*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(c^3*e^8*x^7 + 7*c^3*d*e^7*x^6 + 2
1*c^3*d^2*e^6*x^5 + 35*c^3*d^3*e^5*x^4 + 35*c^3*d^4*e^4*x^3 + 21*c^3*d^5*e^
3*x^2 + 7*c^3*d^6*e^2*x + c^3*d^7*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c\left(d+ex\right)^2\right)^{\frac{5}{2}}\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2),x)
```

```
[Out] Integral(1/((c*(d + e*x)**2)**(5/2)*(d + e*x)**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.1088 $\int \frac{1}{(d+ex)^3(cd^2+2cdex+ce^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=32

$$-\frac{c}{7e\left(cd^2+2cdex+ce^2x^2\right)^{7/2}}$$

```
[Out] -c/(7*e*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(7/2))
```

Rubi [A] time = 0.0231345, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {643, 629}

$$-\frac{c}{7e\left(cd^2+2cdex+ce^2x^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)),x]
```

```
[Out] -c/(7*e*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(7/2))
```

Rule 643

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)^3\left(cd^2+2cdex+ce^2x^2\right)^{5/2}}\,dx &= c^2\int \frac{d+ex}{\left(cd^2+2cdex+ce^2x^2\right)^{9/2}}\,dx\\ &= -\frac{c}{7e\left(cd^2+2cdex+ce^2x^2\right)^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.0272238, size = 21, normalized size = 0.66

$$-\frac{c}{7e\left(c(d+ex)^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)),x]
```

```
[Out] -c/(7*e*(c*(d + e*x)^2)^(7/2))
```

Maple [A] time = 0.042, size = 35, normalized size = 1.1

$$-\frac{1}{7\,(ex+d)^{2}\,e}\left(ce^{2}x^{2}+2\,cdex+cd^{2}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x)
```

```
[Out] -1/7/(e*x+d)^2/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)
```

Maxima [B] time = 1.14404, size = 139, normalized size = 4.34

$$-\frac{1}{7\left(c^{\frac{5}{2}}e^{8}x^{7}+7\,c^{\frac{5}{2}}de^{7}x^{6}+21\,c^{\frac{5}{2}}d^{2}e^{6}x^{5}+35\,c^{\frac{5}{2}}d^{3}e^{5}x^{4}+35\,c^{\frac{5}{2}}d^{4}e^{4}x^{3}+21\,c^{\frac{5}{2}}d^{5}e^{3}x^{2}+7\,c^{\frac{5}{2}}d^{6}e^{2}x+c^{\frac{5}{2}}d^{7}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="maxim
a")
```

```
[Out] -1/7/(c^(5/2)*e^8*x^7 + 7*c^(5/2)*d*e^7*x^6 + 21*c^(5/2)*d^2*e^6*x^5 + 35*c
^(5/2)*d^3*e^5*x^4 + 35*c^(5/2)*d^4*e^4*x^3 + 21*c^(5/2)*d^5*e^3*x^2 + 7*c^
(5/2)*d^6*e^2*x + c^(5/2)*d^7*e)
```

Fricas [B] time = 2.3652, size = 282, normalized size = 8.81

$$-\frac{\sqrt{ce^{2}x^{2}+2\,cdex+cd^{2}}}{7\left(c^{3}e^{9}x^{8}+8\,c^{3}de^{8}x^{7}+28\,c^{3}d^{2}e^{7}x^{6}+56\,c^{3}d^{3}e^{6}x^{5}+70\,c^{3}d^{4}e^{5}x^{4}+56\,c^{3}d^{5}e^{4}x^{3}+28\,c^{3}d^{6}e^{3}x^{2}+8\,c^{3}d^{7}e^{2}x+c^{3}d^{8}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="frica
s")
```

```
[Out] -1/7*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(c^3*e^9*x^8 + 8*c^3*d*e^8*x^7 + 2
8*c^3*d^2*e^7*x^6 + 56*c^3*d^3*e^6*x^5 + 70*c^3*d^4*e^5*x^4 + 56*c^3*d^5*e^
4*x^3 + 28*c^3*d^6*e^3*x^2 + 8*c^3*d^7*e^2*x + c^3*d^8*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c\,(d+ex)^{2}\right)^{\frac{5}{2}}(d+ex)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2),x)
```

```
[Out] Integral(1/((c*(d + e*x)**2)**(5/2)*(d + e*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="giac"
)
```

```
[Out] [undef, undef, undef, undef, undef, 1]
```

3.1089 $\int (d+ex)^m \left(cd^2+2cdex+ce^2x^2\right)^2 \, dx$

Optimal. Leaf size=21

$$\frac{c^2(d+ex)^{m+5}}{e(m+5)}$$

```
[Out] (c^2*(d + e*x)^(5 + m))/(e*(5 + m))
```

Rubi [A] time = 0.0093606, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {27, 12, 32}

$$\frac{c^2(d+ex)^{m+5}}{e(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]
```

```
[Out] (c^2*(d + e*x)^(5 + m))/(e*(5 + m))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^m \left(cd^2+2cdex+ce^2x^2\right)^2 \, dx &= \int c^2(d+ex)^{4+m} \, dx \\
&= c^2 \int (d+ex)^{4+m} \, dx \\
&= \frac{c^2(d+ex)^{5+m}}{e(5+m)}
\end{aligned}$$

Mathematica [A] time = 0.0168074, size = 22, normalized size = 1.05

$$\frac{c^2(d+ex)^{m+5}}{em+5e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]
```

```
[Out] (c^2*(d + e*x)^(5 + m))/(5*e + e*m)
```

Maple [A] time = 0.04, size = 40, normalized size = 1.9

$$\frac{(ex+d)^{1+m}\,c^2\left(e^2x^2+2\,dex+d^2\right)^2}{e\,(5+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x)
```

```
[Out] (e*x+d)^(1+m)*c^2*(e^2*x^2+2*d*e*x+d^2)^2/e/(5+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.44608, size = 169, normalized size = 8.05

$$\frac{\left(c^2e^5x^5+5\,c^2de^4x^4+10\,c^2d^2e^3x^3+10\,c^2d^3e^2x^2+5\,c^2d^4ex+c^2d^5\right)(ex+d)^m}{em+5\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="fricas")
```

```
[Out] (c^2*e^5*x^5 + 5*c^2*d*e^4*x^4 + 10*c^2*d^2*e^3*x^3 + 10*c^2*d^3*e^2*x^2 +
5*c^2*d^4*e*x + c^2*d^5)*(e*x + d)^m/(e*m + 5*e)
```

Sympy [A] time = 1.45693, size = 185, normalized size = 8.81

$$\begin{cases} \frac{c^2x}{d} & \text{for}\ e=0 \wedge m=-5 \\ c^2d^4d^mx & \text{for}\ e=0 \\ \frac{c^2\log\left(\frac{d}{e}+x\right)}{e} & \text{for}\ m=-5 \\ \frac{c^2d^5(d+ex)^m}{em+5e}+\frac{5c^2d^4ex(d+ex)^m}{em+5e}+\frac{10c^2d^3e^2x^2(d+ex)^m}{em+5e}+\frac{10c^2d^2e^3x^3(d+ex)^m}{em+5e}+\frac{5c^2de^4x^4(d+ex)^m}{em+5e}+\frac{c^2e^5x^5(d+ex)^m}{em+5e} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*e**2*x**2+2*c*d*e*x+c*d**2)**2,x)
```

```
[Out] Piecewise((c**2*x/d, Eq(e, 0) & Eq(m, -5)), (c**2*d**4*d**m*x, Eq(e, 0)), (
c**2*log(d/e + x)/e, Eq(m, -5)), (c**2*d**5*(d + e*x)**m/(e*m + 5*e) + 5*c*
*2*d**4*e*x*(d + e*x)**m/(e*m + 5*e) + 10*c**2*d**3*e**2*x**2*(d + e*x)**m/
(e*m + 5*e) + 10*c**2*d**2*e**3*x**3*(d + e*x)**m/(e*m + 5*e) + 5*c**2*d*e*
*4*x**4*(d + e*x)**m/(e*m + 5*e) + c**2*e**5*x**5*(d + e*x)**m/(e*m + 5*e),
 True))
```

Giac [B] time = 1.33514, size = 169, normalized size = 8.05

$$\frac{(xe+d)^m c^2x^5e^5 + 5\,(xe+d)^m c^2dx^4e^4 + 10\,(xe+d)^m c^2d^2x^3e^3 + 10\,(xe+d)^m c^2d^3x^2e^2 + 5\,(xe+d)^m c^2d^4xe + (xe+d)^m c}{me+5\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*c^2*x^5*e^5 + 5*(x*e + d)^m*c^2*d*x^4*e^4 + 10*(x*e + d)^m*c^2
*d^2*x^3*e^3 + 10*(x*e + d)^m*c^2*d^3*x^2*e^2 + 5*(x*e + d)^m*c^2*d^4*x*e +
 (x*e + d)^m*c^2*d^5)/(m*e + 5*e)
```

3.1090 $\int (d+ex)^m \left(cd^2+2cdex+ce^2x^2\right)\,dx$

Optimal. Leaf size=19

$$\frac{c(d+ex)^{m+3}}{e(m+3)}$$

```
[Out] (c*(d + e*x)^(3 + m))/(e*(3 + m))
```

Rubi [A] time = 0.0077997, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {27, 12, 32}

$$\frac{c(d+ex)^{m+3}}{e(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(c*d^2 + 2*c*d*e*x + c*e^2*x^2),x]
```

```
[Out] (c*(d + e*x)^(3 + m))/(e*(3 + m))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^m \left(cd^2+2cdex+ce^2x^2\right)\,dx &= \int c(d+ex)^{2+m}\,dx\\ &= c\int (d+ex)^{2+m}\,dx\\ &= \frac{c(d+ex)^{3+m}}{e(3+m)}\end{aligned}$$

Mathematica [A] time = 0.0140483, size = 20, normalized size = 1.05

$$\frac{c(d+ex)^{m+3}}{em+3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(c*d^2 + 2*c*d*e*x + c*e^2*x^2),x]
```

```
[Out] (c*(d + e*x)^(3 + m))/(3*e + e*m)
```

Maple [A] time = 0.043, size = 36, normalized size = 1.9

$$\frac{(ex+d)^{1+m} c \left(e^2x^2+2\,dex+d^2\right)}{e\,(3+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*e^2*x^2+2*c*d*e*x+c*d^2),x)
```

```
[Out] (e*x+d)^(1+m)*c*(e^2*x^2+2*d*e*x+d^2)/e/(3+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.48514, size = 101, normalized size = 5.32

$$\frac{\left(ce^3x^3+3\,cde^2x^2+3\,cd^2ex+cd^3\right)(ex+d)^m}{em+3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="fricas")
```

```
[Out] (c*e^3*x^3 + 3*c*d*e^2*x^2 + 3*c*d^2*e*x + c*d^3)*(e*x + d)^m/(e*m + 3*e)
```

Sympy [A] time = 0.70723, size = 116, normalized size = 6.11

$$\begin{cases} \frac{cx}{d} & \text{for}\: e = 0 \wedge m = -3 \\ cd^2d^mx & \text{for}\: e = 0 \\ \frac{c\log\left(\frac{d}{e}+x\right)}{e} & \text{for}\: m = -3 \\ \frac{cd^3(d+ex)^m}{em+3e} + \frac{3cd^2ex(d+ex)^m}{em+3e} + \frac{3cde^2x^2(d+ex)^m}{em+3e} + \frac{ce^3x^3(d+ex)^m}{em+3e} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*e**2*x**2+2*c*d*e*x+c*d**2),x)
```

```
[Out] Piecewise((c*x/d, Eq(e, 0) & Eq(m, -3)), (c*d**2*d**m*x, Eq(e, 0)), (c*log(
d/e + x)/e, Eq(m, -3)), (c*d**3*(d + e*x)**m/(e*m + 3*e) + 3*c*d**2*e*x*(d
+ e*x)**m/(e*m + 3*e) + 3*c*d*e**2*x**2*(d + e*x)**m/(e*m + 3*e) + c*e**3*x
**3*(d + e*x)**m/(e*m + 3*e), True))
```

Giac [B] time = 1.27151, size = 101, normalized size = 5.32

$$\frac{(xe+d)^m cx^3e^3 + 3\,(xe+d)^m cdx^2e^2 + 3\,(xe+d)^m cd^2xe + (xe+d)^m cd^3}{me+3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*c*x^3*e^3 + 3*(x*e + d)^m*c*d*x^2*e^2 + 3*(x*e + d)^m*c*d^2*x*
e + (x*e + d)^m*c*d^3)/(m*e + 3*e)
```

3.1091 $\int \frac{(d+ex)^m}{cd^2+2cdex+ce^2x^2}\,dx$

Optimal. Leaf size=24

$$-\frac{(d+ex)^{m-1}}{ce(1-m)}$$

```
[Out] -((d + e*x)^(-1 + m)/(c*e*(1 - m)))
```

Rubi [A] time = 0.0115782, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {27, 12, 32}

$$-\frac{(d+ex)^{m-1}}{ce(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(c*d^2 + 2*c*d*e*x + c*e^2*x^2),x]
```

```
[Out] -((d + e*x)^(-1 + m)/(c*e*(1 - m)))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^m}{cd^2+2cdex+ce^2x^2}\,dx &= \int \frac{(d+ex)^{-2+m}}{c}\,dx\\ &= \frac{\int (d+ex)^{-2+m}\,dx}{c}\\ &= -\frac{(d+ex)^{-1+m}}{ce(1-m)}\end{aligned}$$

Mathematica [A] time = 0.0140982, size = 21, normalized size = 0.88

$$\frac{(d+ex)^{m-1}}{ce(m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(c*d^2 + 2*c*d*e*x + c*e^2*x^2),x]
```

```
[Out] (d + e*x)^(-1 + m)/(c*e*(-1 + m))
```

Maple [A] time = 0.041, size = 22, normalized size = 0.9

$$\frac{(ex+d)^{-1+m}}{ce\,(-1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(c*e^2*x^2+2*c*d*e*x+c*d^2),x)
```

```
[Out] (e*x+d)^(-1+m)/c/e/(-1+m)
```

Maxima [A] time = 1.18196, size = 36, normalized size = 1.5

$$\frac{(ex+d)^{m}}{ce^{2}(m-1)x+cde(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="maxima")
```

```
[Out] (e*x + d)^m/(c*e^2*(m - 1)*x + c*d*e*(m - 1))
```

Fricas [A] time = 2.33927, size = 72, normalized size = 3.

$$\frac{(ex+d)^{m}}{cdem-cde+\left(ce^{2}m-ce^{2}\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="fricas")
```

```
[Out] (e*x + d)^m/(c*d*e*m - c*d*e + (c*e^2*m - c*e^2)*x)
```

Sympy [A] time = 1.18284, size = 63, normalized size = 2.62

$$\begin{cases} \text{NaN} & \text{for}\: d=0 \wedge e=0 \wedge m=1 \\ 0^{m}\tilde{\infty}x & \text{for}\: d=-ex \\ \frac{d^{m}x}{cd^{2}} & \text{for}\: e=0 \\ \frac{\log\left(\frac{d}{e}+x\right)}{ce} & \text{for}\: m=1 \\ \frac{(d+ex)^{m}}{cdem-cde+ce^{2}mx-ce^{2}x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(c*e**2*x**2+2*c*d*e*x+c*d**2),x)
```

```
[Out] Piecewise((nan, Eq(d, 0) & Eq(e, 0) & Eq(m, 1)), (0**m*zoo*x, Eq(d, -e*x)),
 (d**m*x/(c*d**2), Eq(e, 0)), (log(d/e + x)/(c*e), Eq(m, 1)), ((d + e*x)**m
/(c*d*e*m - c*d*e + c*e**2*m*x - c*e**2*x), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{ce^2x^2 + 2\,cdex + cd^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(c*e^2*x^2 + 2*c*d*e*x + c*d^2), x)
```

3.1092 $\int \frac{(d+ex)^m}{\left(cd^2+2cdex+ce^2x^2\right)^2} dx$

Optimal. Leaf size=24

$$-\frac{(d+ex)^{m-3}}{c^2e(3-m)}$$

```
[Out] -((d + e*x)^(-3 + m)/(c^2*e*(3 - m)))
```

Rubi [A] time = 0.0101768, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {27, 12, 32}

$$-\frac{(d+ex)^{m-3}}{c^2e(3-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]
```

```
[Out] -((d + e*x)^(-3 + m)/(c^2*e*(3 - m)))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^m}{\left(cd^2+2cdex+ce^2x^2\right)^2} dx &= \int \frac{(d+ex)^{-4+m}}{c^2} dx\\ &= \frac{\int (d+ex)^{-4+m} dx}{c^2}\\ &= -\frac{(d+ex)^{-3+m}}{c^2e(3-m)}\end{aligned}$$

Mathematica [A] time = 0.0148665, size = 21, normalized size = 0.88

$$\frac{(d+ex)^{m-3}}{c^2e(m-3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]
```

```
[Out] (d + e*x)^(-3 + m)/(c^2*e*(-3 + m))
```

Maple [A] time = 0.04, size = 40, normalized size = 1.7

$$\frac{(ex+d)^{-1+m}}{\left(e^2x^2+2\,dex+d^2\right)c^2e\,(-3+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x)
```

```
[Out] (e*x+d)^(-1+m)/(e^2*x^2+2*d*e*x+d^2)/c^2/e/(-3+m)
```

Maxima [B] time = 1.37361, size = 88, normalized size = 3.67

$$\frac{(ex+d)^m}{c^2e^4(m-3)x^3+3\,c^2de^3(m-3)x^2+3\,c^2d^2e^2(m-3)x+c^2d^3e(m-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="maxima")
```

```
[Out] (e*x + d)^m/(c^2*e^4*(m - 3)*x^3 + 3*c^2*d*e^3*(m - 3)*x^2 + 3*c^2*d^2*e^2*
(m - 3)*x + c^2*d^3*e*(m - 3))
```

Fricas [B] time = 2.47713, size = 196, normalized size = 8.17

$$\frac{(ex+d)^m}{c^2d^3em-3\,c^2d^3e+\left(c^2e^4m-3\,c^2e^4\right)x^3+3\left(c^2de^3m-3\,c^2de^3\right)x^2+3\left(c^2d^2e^2m-3\,c^2d^2e^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="fricas")
```

```
[Out] (e*x + d)^m/(c^2*d^3*e*m - 3*c^2*d^3*e + (c^2*e^4*m - 3*c^2*e^4)*x^3 + 3*(c
^2*d*e^3*m - 3*c^2*d*e^3)*x^2 + 3*(c^2*d^2*e^2*m - 3*c^2*d^2*e^2)*x)
```

Sympy [A] time = 2.14732, size = 136, normalized size = 5.67

$$\begin{cases} \frac{x}{c^2d} & \text{for } e=0 \wedge m=3 \\ \frac{d^mx}{c^2d^4} & \text{for } e=0 \\ \frac{\log\left(\frac{d}{e}+x\right)}{c^2e} & \text{for } m=3 \\ \frac{(d+ex)^m}{c^2d^3em-3c^2d^3e+3c^2d^2e^2mx-9c^2d^2e^2x+3c^2de^3mx^2-9c^2de^3x^2+c^2e^4mx^3-3c^2e^4x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(c*e**2*x**2+2*c*d*e*x+c*d**2)**2,x)
```

```
[Out] Piecewise((x/(c**2*d), Eq(e, 0) & Eq(m, 3)), (d**m*x/(c**2*d**4), Eq(e, 0))
, (log(d/e + x)/(c**2*e), Eq(m, 3)), ((d + e*x)**m/(c**2*d**3*e*m - 3*c**2*
d**3*e + 3*c**2*d**2*e**2*m*x - 9*c**2*d**2*e**2*x + 3*c**2*d*e**3*m*x**2 -
 9*c**2*d*e**3*x**2 + c**2*e**4*m*x**3 - 3*c**2*e**4*x**3), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(ce^2x^2+2\,cdex+cd^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^2, x)
```

3.1093 $\int \frac{(d+ex)^m}{\left(cd^2+2cdex+ce^2x^2\right)^3} dx$

Optimal. Leaf size=24

$$-\frac{(d+ex)^{m-5}}{c^3e(5-m)}$$

```
[Out] -((d + e*x)^(-5 + m)/(c^3*e*(5 - m)))
```

Rubi [A] time = 0.0102396, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {27, 12, 32}

$$-\frac{(d+ex)^{m-5}}{c^3e(5-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]
```

```
[Out] -((d + e*x)^(-5 + m)/(c^3*e*(5 - m)))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^m}{\left(cd^2+2cdex+ce^2x^2\right)^3} dx &= \int \frac{(d+ex)^{-6+m}}{c^3} dx \\
&= \frac{\int (d+ex)^{-6+m} dx}{c^3} \\
&= -\frac{(d+ex)^{-5+m}}{c^3e(5-m)}
\end{aligned}$$

Mathematica [A] time = 0.0144844, size = 21, normalized size = 0.88

$$\frac{(d+ex)^{m-5}}{c^3e(m-5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]
```

```
[Out] (d + e*x)^(-5 + m)/(c^3*e*(-5 + m))
```

Maple [A] time = 0.042, size = 40, normalized size = 1.7

$$\frac{(ex+d)^{-1+m}}{\left(e^2x^2+2\,dex+d^2\right)^2 c^3e\,(-5+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x)
```

```
[Out] (e*x+d)^(-1+m)/(e^2*x^2+2*d*e*x+d^2)^2/c^3/e/(-5+m)
```

Maxima [B] time = 1.20799, size = 134, normalized size = 5.58

$$\frac{(ex+d)^m}{c^3e^6(m-5)x^5+5\,c^3de^5(m-5)x^4+10\,c^3d^2e^4(m-5)x^3+10\,c^3d^3e^3(m-5)x^2+5\,c^3d^4e^2(m-5)x+c^3d^5e(m-5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="maxima")
```

```
[Out] (e*x + d)^m/(c^3*e^6*(m - 5)*x^5 + 5*c^3*d*e^5*(m - 5)*x^4 + 10*c^3*d^2*e^4
*(m - 5)*x^3 + 10*c^3*d^3*e^3*(m - 5)*x^2 + 5*c^3*d^4*e^2*(m - 5)*x + c^3*d
^5*e*(m - 5))
```

Fricas [B] time = 2.49311, size = 306, normalized size = 12.75

$$\frac{(ex+d)^m}{c^3d^5em-5\,c^3d^5e+\left(c^3e^6m-5\,c^3e^6\right)x^5+5\left(c^3de^5m-5\,c^3de^5\right)x^4+10\left(c^3d^2e^4m-5\,c^3d^2e^4\right)x^3+10\left(c^3d^3e^3m-5\,c^3d^3e^3\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="fricas")
```

```
[Out] (e*x + d)^m/(c^3*d^5*e*m - 5*c^3*d^5*e + (c^3*e^6*m - 5*c^3*e^6)*x^5 + 5*(c
^3*d*e^5*m - 5*c^3*d*e^5)*x^4 + 10*(c^3*d^2*e^4*m - 5*c^3*d^2*e^4)*x^3 + 10
*(c^3*d^3*e^3*m - 5*c^3*d^3*e^3)*x^2 + 5*(c^3*d^4*e^2*m - 5*c^3*d^4*e^2)*x)
```

Sympy [A] time = 3.70306, size = 201, normalized size = 8.38

$$\begin{cases} \frac{x}{c^3d} & \text{for}\: e=0 \wedge m \\ \frac{d^m x}{c^3d^6} & \text{for}\: e=0 \\ \frac{\log\left(\frac{d}{e}+x\right)}{c^3e} & \text{for}\: m=5 \\ \frac{(d+ex)^m}{c^3d^5em-5c^3d^5e+5c^3d^4e^2mx-25c^3d^4e^2x+10c^3d^3e^3mx^2-50c^3d^3e^3x^2+10c^3d^2e^4mx^3-50c^3d^2e^4x^3+5c^3de^5mx^4-25c^3de^5x^4+c^3e^6mx^5-5c^3e^6x^5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(c*e**2*x**2+2*c*d*e*x+c*d**2)**3,x)
```

```
[Out] Piecewise((x/(c**3*d), Eq(e, 0) & Eq(m, 5)), (d**m*x/(c**3*d**6), Eq(e, 0))
, (log(d/e + x)/(c**3*e), Eq(m, 5)), ((d + e*x)**m/(c**3*d**5*e*m - 5*c**3*
d**5*e + 5*c**3*d**4*e**2*m*x - 25*c**3*d**4*e**2*x + 10*c**3*d**3*e**3*m*x
**2 - 50*c**3*d**3*e**3*x**2 + 10*c**3*d**2*e**4*m*x**3 - 50*c**3*d**2*e**4
*x**3 + 5*c**3*d*e**5*m*x**4 - 25*c**3*d*e**5*x**4 + c**3*e**6*m*x**5 - 5*c
**3*e**6*x**5), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{(ce^2x^2+2\,cdex+cd^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^3, x)
```

3.1094 $\int (d+ex)^m \left(cd^2+2cdex+ce^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=42

$$\frac{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}(d+ex)^{m+1}}{e(m+4)}$$

```
[Out] ((d + e*x)^(1 + m)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))/(e*(4 + m))
```

Rubi [A] time = 0.0189694, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {644, 32}

$$\frac{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}(d+ex)^{m+1}}{e(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2),x]
```

```
[Out] ((d + e*x)^(1 + m)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))/(e*(4 + m))
```

Rule 644

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[(a + b*x + c*x^2)^p/(d + e*x)^(2*p), Int[(d + e*x)^(m + 2*p),
x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !Integ
erQ[p] && EqQ[2*c*d - b*e, 0] &&  !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^m \left(cd^2+2cdex+ce^2x^2\right)^{3/2}\,dx &= \frac{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}\int (d+ex)^{3+m}\,dx}{(d+ex)^3}\\ &= \frac{(d+ex)^{1+m}\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}{e(4+m)}\end{aligned}$$

Mathematica [A] time = 0.0295078, size = 31, normalized size = 0.74

$$\frac{\left(c(d+ex)^2\right)^{3/2}(d+ex)^{m+1}}{e(m+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2),x]
```

```
[Out] ((d + e*x)^(1 + m)*(c*(d + e*x)^2)^(3/2))/(e*(4 + m))
```

Maple [A] time = 0.038, size = 41, normalized size = 1.

$$\frac{(ex+d)^{1+m}}{e(4+m)}\left(ce^2x^2+2\,cdex+cd^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x)
```

```
[Out] (e*x+d)^(1+m)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/e/(4+m)
```

Maxima [A] time = 1.18199, size = 95, normalized size = 2.26

$$\frac{\left(c^{\frac{3}{2}}e^4x^4+4\,c^{\frac{3}{2}}de^3x^3+6\,c^{\frac{3}{2}}d^2e^2x^2+4\,c^{\frac{3}{2}}d^3ex+c^{\frac{3}{2}}d^4\right)(ex+d)^m}{e(m+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] (c^(3/2)*e^4*x^4 + 4*c^(3/2)*d*e^3*x^3 + 6*c^(3/2)*d^2*e^2*x^2 + 4*c^(3/2)*
d^3*e*x + c^(3/2)*d^4)*(e*x + d)^m/(e*(m + 4))
```

Fricas [A] time = 2.55927, size = 150, normalized size = 3.57

$$\frac{\left(ce^3x^3+3\,cde^2x^2+3\,cd^2ex+cd^3\right)\sqrt{ce^2x^2+2\,cdex+cd^2}(ex+d)^m}{em+4\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] (c*e^3*x^3 + 3*c*d*e^2*x^2 + 3*c*d^2*e*x + c*d^3)*sqrt(c*e^2*x^2 + 2*c*d*e*
x + c*d^2)*(e*x + d)^m/(e*m + 4*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\left(d+ex\right)^2\right)^{\frac{3}{2}}\left(d+ex\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2),x)
```

```
[Out] Integral((c*(d + e*x)**2)**(3/2)*(d + e*x)**m, x)
```

Giac [B] time = 1.27467, size = 140, normalized size = 3.33

$$\frac{(xe+d)^m c^{\frac{3}{2}} x^4 e^4 + 4\,(xe+d)^m c^{\frac{3}{2}} d x^3 e^3 + 6\,(xe+d)^m c^{\frac{3}{2}} d^2 x^2 e^2 + 4\,(xe+d)^m c^{\frac{3}{2}} d^3 x e + (xe+d)^m c^{\frac{3}{2}} d^4}{me+4\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*c^(3/2)*x^4*e^4 + 4*(x*e + d)^m*c^(3/2)*d*x^3*e^3 + 6*(x*e + d
)^m*c^(3/2)*d^2*x^2*e^2 + 4*(x*e + d)^m*c^(3/2)*d^3*x*e + (x*e + d)^m*c^(3/
2)*d^4)/(m*e + 4*e)
```

3.1095 $\int (d+ex)^m \sqrt{cd^2+2cdex+ce^2x^2}\,dx$

Optimal. Leaf size=42

$$\frac{\sqrt{cd^2+2cdex+ce^2x^2}(d+ex)^{m+1}}{e(m+2)}$$

```
[Out] ((d + e*x)^(1 + m)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/(e*(2 + m))
```

Rubi [A] time = 0.0181988, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {644, 32}

$$\frac{\sqrt{cd^2+2cdex+ce^2x^2}(d+ex)^{m+1}}{e(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]
```

```
[Out] ((d + e*x)^(1 + m)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/(e*(2 + m))
```

Rule 644

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[(a + b*x + c*x^2)^p/(d + e*x)^(2*p), Int[(d + e*x)^(m + 2*p),
x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !Integ
erQ[p] && EqQ[2*c*d - b*e, 0] && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^m\sqrt{cd^2+2cdex+ce^2x^2}\,dx &= \frac{\sqrt{cd^2+2cdex+ce^2x^2}\int (d+ex)^{1+m}\,dx}{d+ex}\\ &= \frac{(d+ex)^{1+m}\sqrt{cd^2+2cdex+ce^2x^2}}{e(2+m)}\end{aligned}$$

Mathematica [A] time = 0.0175014, size = 31, normalized size = 0.74

$$\frac{\sqrt{c(d+ex)^2}(d+ex)^{m+1}}{e(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]
```

```
[Out] ((d + e*x)^(1 + m)*Sqrt[c*(d + e*x)^2])/(e*(2 + m))
```

Maple [A] time = 0.044, size = 41, normalized size = 1.

$$\frac{(ex+d)^{1+m}}{e(2+m)}\sqrt{ce^2x^2+2\,cdex+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x)
```

```
[Out] (e*x+d)^(1+m)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/e/(2+m)
```

Maxima [A] time = 1.20951, size = 57, normalized size = 1.36

$$\frac{\left(\sqrt{c}e^2x^2+2\,\sqrt{c}dex+\sqrt{c}d^2\right)(ex+d)^m}{e(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="maxima"
)
```

```
[Out] (sqrt(c)*e^2*x^2 + 2*sqrt(c)*d*e*x + sqrt(c)*d^2)*(e*x + d)^m/(e*(m + 2))
```

Fricas [A] time = 2.4213, size = 96, normalized size = 2.29

$$\frac{\sqrt{ce^2x^2+2\,cdex+cd^2}(ex+d)(ex+d)^m}{em+2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="fricas"
)
```

```
[Out] sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*(e*x + d)*(e*x + d)^m/(e*m + 2*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{c\,(d+ex)^2}\,(d+ex)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2),x)
```

```
[Out] Integral(sqrt(c*(d + e*x)**2)*(d + e*x)**m, x)
```

Giac [A] time = 1.19182, size = 84, normalized size = 2.

$$\frac{(xe+d)^m\sqrt{c}x^2e^2+2\,(xe+d)^m\sqrt{c}dxe+(xe+d)^m\sqrt{c}d^2}{me+2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*sqrt(c)*x^2*e^2 + 2*(x*e + d)^m*sqrt(c)*d*x*e + (x*e + d)^m*sq
rt(c)*d^2)/(m*e + 2*e)
```

3.1096 $\int \frac{(d+ex)^m}{\sqrt{cd^2+2cdex+ce^2x^2}} dx$

Optimal. Leaf size=40

$$\frac{(d+ex)^{m+1}}{em\sqrt{cd^2+2cdex+ce^2x^2}}$$

```
[Out] (d + e*x)^(1 + m)/(e*m*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])
```

Rubi [A] time = 0.018051, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {644, 32}

$$\frac{(d+ex)^{m+1}}{em\sqrt{cd^2+2cdex+ce^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]
```

```
[Out] (d + e*x)^(1 + m)/(e*m*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])
```

Rule 644

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[(a + b*x + c*x^2)^p/(d + e*x)^(2*p), Int[(d + e*x)^(m + 2*p),
x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !Integ
erQ[p] && EqQ[2*c*d - b*e, 0] &&  !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(d+ex)^m}{\sqrt{cd^2+2cdex+ce^2x^2}} dx &= \frac{(d+ex)\int (d+ex)^{-1+m} dx}{\sqrt{cd^2+2cdex+ce^2x^2}} \\ &= \frac{(d+ex)^{1+m}}{em\sqrt{cd^2+2cdex+ce^2x^2}} \end{aligned}$$

Mathematica [A] time = 0.0121871, size = 29, normalized size = 0.72

$$\frac{(d+ex)^{m+1}}{em\sqrt{c(d+ex)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]
```

```
[Out] (d + e*x)^(1 + m)/(e*m*Sqrt[c*(d + e*x)^2])
```

Maple [A] time = 0.04, size = 39, normalized size = 1.

$$\frac{(ex+d)^{1+m}}{em}\frac{1}{\sqrt{ce^2x^2+2\,cdex+cd^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x)
```

```
[Out] (e*x+d)^(1+m)/e/m/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)
```

Maxima [A] time = 1.22615, size = 23, normalized size = 0.57

$$\frac{(ex+d)^m}{\sqrt{c}em}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="maxima"
)
```

```
[Out] (e*x + d)^m/(sqrt(c)*e*m)
```

Fricas [A] time = 2.36558, size = 96, normalized size = 2.4

$$\frac{\sqrt{ce^2x^2+2\,cdex+cd^2}(ex+d)^m}{ce^2mx+cdem}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="fricas"
)
```

```
[Out] sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*(e*x + d)^m/(c*e^2*m*x + c*d*e*m)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m}{\sqrt{c\,(d+ex)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2),x)
```

```
[Out] Integral((d + e*x)**m/sqrt(c*(d + e*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\sqrt{ce^2x^2+2\,cdex+cd^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2), x)
```

3.1097 $\int \frac{(d+ex)^m}{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=45

$$-\frac{(d+ex)^{m+1}}{e(2-m)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}$$

```
[Out] -((d + e*x)^(1 + m)/(e*(2 - m)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)))
```

Rubi [A] time = 0.0202965, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {644, 32}

$$-\frac{(d+ex)^{m+1}}{e(2-m)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2),x]
```

```
[Out] -((d + e*x)^(1 + m)/(e*(2 - m)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)))
```

Rule 644

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[(a + b*x + c*x^2)^p/(d + e*x)^(2*p), Int[(d + e*x)^(m + 2*p),
x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !Integ
erQ[p] && EqQ[2*c*d - b*e, 0] &&  !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^m}{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}\,dx &= \frac{(d+ex)^3\int (d+ex)^{-3+m}\,dx}{\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}\\ &= -\frac{(d+ex)^{1+m}}{e(2-m)\left(cd^2+2cdex+ce^2x^2\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.019931, size = 31, normalized size = 0.69

$$\frac{(d+ex)^{m+1}}{e(m-2)\left(c(d+ex)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2),x]
```

```
[Out] (d + e*x)^(1 + m)/(e*(-2 + m)*(c*(d + e*x)^2)^(3/2))
```

Maple [A] time = 0.039, size = 41, normalized size = 0.9

$$\frac{(ex+d)^{1+m}}{e(-2+m)}\left(ce^2x^2+2\,cdex+cd^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x)
```

```
[Out] (e*x+d)^(1+m)/e/(-2+m)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)
```

Maxima [A] time = 1.07959, size = 69, normalized size = 1.53

$$\frac{(ex+d)^m\sqrt{c}}{c^2e^3(m-2)x^2+2\,c^2de^2(m-2)x+c^2d^2e(m-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] (e*x + d)^m*sqrt(c)/(c^2*e^3*(m - 2)*x^2 + 2*c^2*d*e^2*(m - 2)*x + c^2*d^2*
e*(m - 2))
```

Fricas [B] time = 2.47161, size = 244, normalized size = 5.42

$$\frac{\sqrt{ce^2x^2+2\,cdex+cd^2}(ex+d)^m}{c^2d^3em-2\,c^2d^3e+\left(c^2e^4m-2\,c^2e^4\right)x^3+3\left(c^2de^3m-2\,c^2de^3\right)x^2+3\left(c^2d^2e^2m-2\,c^2d^2e^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*(e*x + d)^m/(c^2*d^3*e*m - 2*c^2*d^3*e
+ (c^2*e^4*m - 2*c^2*e^4)*x^3 + 3*(c^2*d*e^3*m - 2*c^2*d*e^3)*x^2 + 3*(c^2*
d^2*e^2*m - 2*c^2*d^2*e^2)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(d+ex)^m}{\left(c\,(d+ex)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2),x)
```

```
[Out] Integral((d + e*x)**m/(c*(d + e*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(ce^2x^2+2\,cdex+cd^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^(3/2), x)
```

3.1098 $\int (d+ex)^m \left(cd^2+2cdex+ce^2x^2\right)^p \, dx$

Optimal. Leaf size=43

$$\frac{(d+ex)^{m+1}\left(cd^2+2cdex+ce^2x^2\right)^p}{e(m+2p+1)}$$

```
[Out] ((d + e*x)^(1 + m)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p)/(e*(1 + m + 2*p))
```

Rubi [A] time = 0.0177609, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {644, 32}

$$\frac{(d+ex)^{m+1}\left(cd^2+2cdex+ce^2x^2\right)^p}{e(m+2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]
```

```
[Out] ((d + e*x)^(1 + m)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p)/(e*(1 + m + 2*p))
```

Rule 644

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[(a + b*x + c*x^2)^p/(d + e*x)^(2*p), Int[(d + e*x)^(m + 2*p),
x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !Integ
erQ[p] && EqQ[2*c*d - b*e, 0] &&  !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^m \left(cd^2+2cdex+ce^2x^2\right)^p \, dx &= \left((d+ex)^{-2p}\left(cd^2+2cdex+ce^2x^2\right)^p\right)\int (d+ex)^{m+2p} \, dx\\ &= \frac{(d+ex)^{1+m}\left(cd^2+2cdex+ce^2x^2\right)^p}{e(1+m+2p)}\end{aligned}$$

Mathematica [A] time = 0.0169445, size = 32, normalized size = 0.74

$$\frac{(d+ex)^{m+1}\left(c(d+ex)^2\right)^p}{em+2ep+e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]
```

```
[Out] ((d + e*x)^(1 + m)*(c*(d + e*x)^2)^p)/(e + e*m + 2*e*p)
```

Maple [A] time = 0.041, size = 44, normalized size = 1.

$$\frac{(ex+d)^{1+m}\left(ce^2x^2+2\,cdex+cd^2\right)^p}{e\left(1+m+2\,p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x)
```

```
[Out] (e*x+d)^(1+m)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p/e/(1+m+2*p)
```

Maxima [A] time = 1.09992, size = 58, normalized size = 1.35

$$\frac{(c^pex+c^pd)e^{\left(m\log(ex+d)+2\,p\log(ex+d)\right)}}{e\left(m+2\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="maxima")
```

```
[Out] (c^p*e*x + c^p*d)*e^(m*log(e*x + d) + 2*p*log(e*x + d))/(e*(m + 2*p + 1))
```

Fricas [A] time = 2.43167, size = 99, normalized size = 2.3

$$\frac{(ex+d)(ex+d)^me^{\left(2\,p\log(ex+d)+p\log(c)\right)}}{em+2\,ep+e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="fricas")
```

```
[Out] (e*x + d)*(e*x + d)^m*e^(2*p*log(e*x + d) + p*log(c))/(e*m + 2*e*p + e)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*e**2*x**2+2*c*d*e*x+c*d**2)**p,x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.24254, size = 93, normalized size = 2.16

$$\frac{(xe+d)^mxe^{\left(2\,p\log(xe+d)+p\log(c)+1\right)}+(xe+d)^mde^{\left(2\,p\log(xe+d)+p\log(c)\right)}}{me+2\,pe+e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*x*e^(2*p*log(x*e + d) + p*log(c) + 1) + (x*e + d)^m*d*e^(2*p*l
og(x*e + d) + p*log(c)))/(m*e + 2*p*e + e)
```

3.1099 $\int (d+ex)^p \left(cd^2+2cdex+ce^2x^2\right)^{-p} dx$

Optimal. Leaf size=44

$$\frac{(d+ex)^{p+1}\left(cd^2+2cdex+ce^2x^2\right)^{-p}}{e(1-p)}$$

```
[Out] (d + e*x)^(1 + p)/(e*(1 - p)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p)
```

Rubi [A] time = 0.0173963, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {644, 32}

$$\frac{(d+ex)^{p+1}\left(cd^2+2cdex+ce^2x^2\right)^{-p}}{e(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^p/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]
```

```
[Out] (d + e*x)^(1 + p)/(e*(1 - p)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p)
```

Rule 644

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[(a + b*x + c*x^2)^p/(d + e*x)^(2*p), Int[(d + e*x)^(m + 2*p),
x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !Integ
erQ[p] && EqQ[2*c*d - b*e, 0] && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^p \left(cd^2+2cdex+ce^2x^2\right)^{-p} dx &= \left((d+ex)^{2p}\left(cd^2+2cdex+ce^2x^2\right)^{-p}\right)\int (d+ex)^{-p} dx\\ &= \frac{(d+ex)^{1+p}\left(cd^2+2cdex+ce^2x^2\right)^{-p}}{e(1-p)}\end{aligned}$$

Mathematica [A] time = 0.0160117, size = 31, normalized size = 0.7

$$\frac{(d+ex)^{p+1}\left(c(d+ex)^2\right)^{-p}}{e-ep}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^p/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]
```

```
[Out] (d + e*x)^(1 + p)/((e - e*p)*(c*(d + e*x)^2)^p)
```

Maple [A] time = 0.04, size = 44, normalized size = 1.

$$-\frac{(ex+d)^{1+p}}{e\left(p-1\right)\left(ce^{2}x^{2}+2\,cdex+cd^{2}\right)^{p}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^p/((c*e^2*x^2+2*c*d*e*x+c*d^2)^p),x)
```

```
[Out] -(e*x+d)^(1+p)/e/(p-1)/((c*e^2*x^2+2*c*d*e*x+c*d^2)^p)
```

Maxima [A] time = 1.15625, size = 39, normalized size = 0.89

$$-\frac{ex+d}{(ex+d)^{p}c^{p}e\left(p-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^p/((c*e^2*x^2+2*c*d*e*x+c*d^2)^p),x, algorithm="maxima")
```

```
[Out] -(e*x + d)/((e*x + d)^p*c^p*e*(p - 1))
```

Fricas [A] time = 2.44774, size = 54, normalized size = 1.23

$$-\frac{ex+d}{\left(ep-e\right)(ex+d)^{p}c^{p}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^p/((c*e^2*x^2+2*c*d*e*x+c*d^2)^p),x, algorithm="fricas")
```

```
[Out] -(e*x + d)/((e*p - e)*(e*x + d)^p*c^p)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**p/((c*e**2*x**2+2*c*d*e*x+c*d**2)**p),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.2373, size = 93, normalized size = 2.11

$$-\frac{(xe+d)^{p}xe^{\left(-2\,p\log(xe+d)-p\log(c)+1\right)}+(xe+d)^{p}de^{\left(-2\,p\log(xe+d)-p\log(c)\right)}}{pe-e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^p/((c*e^2*x^2+2*c*d*e*x+c*d^2)^p),x, algorithm="giac")
```

```
[Out] -((x*e + d)^p*x*e^(-2*p*log(x*e + d) - p*log(c) + 1) + (x*e + d)^p*d*e^(-2*
p*log(x*e + d) - p*log(c)))/(p*e - e)
```

3.1100 $\int (d+ex)^3 \left(cd^2+2cdex+ce^2x^2\right)^p \, dx$

Optimal. Leaf size=39

$$\frac{\left(cd^2+2cdex+ce^2x^2\right)^{p+2}}{2c^2e(p+2)}$$

```
[Out] (c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(2 + p)/(2*c^2*e*(2 + p))
```

Rubi [A] time = 0.0256012, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {643, 629}

$$\frac{\left(cd^2+2cdex+ce^2x^2\right)^{p+2}}{2c^2e(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]
```

```
[Out] (c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(2 + p)/(2*c^2*e*(2 + p))
```

Rule 643

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^3 \left(cd^2+2cdex+ce^2x^2\right)^p \, dx &= \frac{\int (d+ex) \left(cd^2+2cdex+ce^2x^2\right)^{1+p} \, dx}{c}\\ &= \frac{\left(cd^2+2cdex+ce^2x^2\right)^{2+p}}{2c^2e(2+p)}\end{aligned}$$

Mathematica [A] time = 0.0166515, size = 30, normalized size = 0.77

$$\frac{(d+ex)^4 \left(c(d+ex)^2\right)^p}{2e(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]
```

```
[Out] ((d + e*x)^4*(c*(d + e*x)^2)^p)/(2*e*(2 + p))
```

Maple [A] time = 0.04, size = 40, normalized size = 1.

$$\frac{(ex+d)^4\left(ce^2x^2+2\,cdex+cd^2\right)^p}{2\,e\left(2+p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x)
```

```
[Out] 1/2*(e*x+d)^4/e/(2+p)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p
```

Maxima [B] time = 1.97006, size = 423, normalized size = 10.85

$$\frac{(c^pex+c^pd)(ex+d)^{2\,p}d^3}{e\left(2\,p+1\right)}+\frac{3\left(c^pe^2\left(2\,p+1\right)x^2+2\,c^pdepx-c^pd^2\right)(ex+d)^{2\,p}d^2}{2\left(2\,p^2+3\,p+1\right)e}+\frac{3\left(\left(2\,p^2+3\,p+1\right)c^pe^3x^3+\left(2\,p^2+p\right)\cdots}{\left(4\,p^3+12\,p^2\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="maxima")
```

```
[Out] (c^p*e*x + c^p*d)*(e*x + d)^(2*p)*d^3/(e*(2*p + 1)) + 3/2*(c^p*e^2*(2*p + 1
)*x^2 + 2*c^p*d*e*p*x - c^p*d^2)*(e*x + d)^(2*p)*d^2/((2*p^2 + 3*p + 1)*e)
+ 3*((2*p^2 + 3*p + 1)*c^p*e^3*x^3 + (2*p^2 + p)*c^p*d*e^2*x^2 - 2*c^p*d^2*
e*p*x + c^p*d^3)*(e*x + d)^(2*p)*d/((4*p^3 + 12*p^2 + 11*p + 3)*e) + 1/2*((
4*p^3 + 12*p^2 + 11*p + 3)*c^p*e^4*x^4 + 2*(2*p^3 + 3*p^2 + p)*c^p*d*e^3*x^
3 - 3*(2*p^2 + p)*c^p*d^2*e^2*x^2 + 6*c^p*d^3*e*p*x - 3*c^p*d^4)*(e*x + d)^
(2*p)/((4*p^4 + 20*p^3 + 35*p^2 + 25*p + 6)*e)
```

Fricas [A] time = 2.29006, size = 147, normalized size = 3.77

$$\frac{\left(e^4x^4+4\,de^3x^3+6\,d^2e^2x^2+4\,d^3ex+d^4\right)\left(ce^2x^2+2\,cdex+cd^2\right)^p}{2\left(ep+2\,e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="fricas")
```

```
[Out] 1/2*(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4)*(c*e^2*x^2 +
2*c*d*e*x + c*d^2)^p/(e*p + 2*e)
```

Sympy [A] time = 1.06861, size = 233, normalized size = 5.97

$$\begin{cases}\frac{x}{c^2d} & \text{for} \cdots\\ d^3x\left(cd^2\right)^p & \text{for} \cdots\\ \frac{\log\left(\frac{d}{e}+x\right)}{c^2e} & \text{for} \cdots\\ \frac{d^4\left(cd^2+2cdex+ce^2x^2\right)^p}{2ep+4e}+\frac{4d^3ex\left(cd^2+2cdex+ce^2x^2\right)^p}{2ep+4e}+\frac{6d^2e^2x^2\left(cd^2+2cdex+ce^2x^2\right)^p}{2ep+4e}+\frac{4de^3x^3\left(cd^2+2cdex+ce^2x^2\right)^p}{2ep+4e}+\frac{e^4x^4\left(cd^2+2cdex+ce^2x^2\right)^p}{2ep+4e} & \text{oth}\cdots\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*e**2*x**2+2*c*d*e*x+c*d**2)**p,x)
```

```
[Out] Piecewise((x/(c**2*d), Eq(e, 0) & Eq(p, -2)), (d**3*x*(c*d**2)**p, Eq(e, 0)
), (log(d/e + x)/(c**2*e), Eq(p, -2)), (d**4*(c*d**2 + 2*c*d*e*x + c*e**2*x
**2)**p/(2*e*p + 4*e) + 4*d**3*e*x*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2
*e*p + 4*e) + 6*d**2*e**2*x**2*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*e*p
 + 4*e) + 4*d*e**3*x**3*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*e*p + 4*e)
 + e**4*x**4*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*e*p + 4*e), True))
```

Giac [B] time = 1.28653, size = 216, normalized size = 5.54

$$\frac{\left(cx^2e^2 + 2\,cdxe + cd^2\right)^p x^4e^4 + 4\left(cx^2e^2 + 2\,cdxe + cd^2\right)^p dx^3e^3 + 6\left(cx^2e^2 + 2\,cdxe + cd^2\right)^p d^2x^2e^2 + 4\left(cx^2e^2 + 2\,cdxe + cd^2\right.}{2\left(pe + 2\,e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="giac")
```

```
[Out] 1/2*((c*x^2*e^2 + 2*c*d*x*e + c*d^2)^p*x^4*e^4 + 4*(c*x^2*e^2 + 2*c*d*x*e +
 c*d^2)^p*d*x^3*e^3 + 6*(c*x^2*e^2 + 2*c*d*x*e + c*d^2)^p*d^2*x^2*e^2 + 4*(
c*x^2*e^2 + 2*c*d*x*e + c*d^2)^p*d^3*x*e + (c*x^2*e^2 + 2*c*d*x*e + c*d^2)^
p*d^4)/(p*e + 2*e)
```

3.1101 $\int (d+ex)^2 \left(cd^2+2cdex+ce^2x^2\right)^p \, dx$

Optimal. Leaf size=43

$$\frac{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{p+1}}{ce(2p+3)}$$

```
[Out] ((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(1 + p))/(c*e*(3 + 2*p))
```

Rubi [A] time = 0.0259138, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {642, 609}

$$\frac{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{p+1}}{ce(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]
```

```
[Out] ((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(1 + p))/(c*e*(3 + 2*p))
```

Rule 642

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^m/c^(m/2), Int[(a + b*x + c*x^2)^(p + m/2), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && EqQ[2*c*d
 - b*e, 0] && IntegerQ[m/2]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^2 \left(cd^2+2cdex+ce^2x^2\right)^p \, dx &= \frac{\int \left(cd^2+2cdex+ce^2x^2\right)^{1+p} \, dx}{c}\\ &= \frac{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{1+p}}{ce(3+2p)}\end{aligned}$$

Mathematica [A] time = 0.0185436, size = 32, normalized size = 0.74

$$\frac{(d+ex)\left(c(d+ex)^2\right)^{p+1}}{ce(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]
```

```
[Out] ((d + e*x)*(c*(d + e*x)^2)^(1 + p))/(c*e*(3 + 2*p))
```

Maple [A] time = 0.039, size = 41, normalized size = 1.

$$\frac{(ex+d)^3\left(ce^2x^2+2\,cdex+cd^2\right)^p}{e\left(3+2\,p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x)
```

```
[Out] (e*x+d)^3/e/(3+2*p)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p
```

Maxima [B] time = 1.29265, size = 246, normalized size = 5.72

$$\frac{(c^pex+c^pd)(ex+d)^{2\,p}d^2}{e\left(2\,p+1\right)}+\frac{\left(c^pe^2\left(2\,p+1\right)x^2+2\,c^pdepx-c^pd^2\right)(ex+d)^{2\,p}d}{\left(2\,p^2+3\,p+1\right)e}+\frac{\left(\left(2\,p^2+3\,p+1\right)c^pe^3x^3+\left(2\,p^2+p\right)c^pde^2x^2\right.}{\left(4\,p^3+12\,p^2+11\,p\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="maxima")
```

```
[Out] (c^p*e*x + c^p*d)*(e*x + d)^(2*p)*d^2/(e*(2*p + 1)) + (c^p*e^2*(2*p + 1)*x^
2 + 2*c^p*d*e*p*x - c^p*d^2)*(e*x + d)^(2*p)*d/((2*p^2 + 3*p + 1)*e) + ((2*
p^2 + 3*p + 1)*c^p*e^3*x^3 + (2*p^2 + p)*c^p*d*e^2*x^2 - 2*c^p*d^2*e*p*x +
c^p*d^3)*(e*x + d)^(2*p)/((4*p^3 + 12*p^2 + 11*p + 3)*e)
```

Fricas [A] time = 2.61892, size = 123, normalized size = 2.86

$$\frac{\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\left(ce^2x^2+2\,cdex+cd^2\right)^p}{2\,ep+3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="fricas")
```

```
[Out] (e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p
/(2*e*p + 3*e)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*e**2*x**2+2*c*d*e*x+c*d**2)**p,x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.21143, size = 173, normalized size = 4.02

$$\frac{\left(cx^2e^2 + 2\,cdxe + cd^2\right)^p x^3e^3 + 3\left(cx^2e^2 + 2\,cdxe + cd^2\right)^p dx^2e^2 + 3\left(cx^2e^2 + 2\,cdxe + cd^2\right)^p d^2xe + \left(cx^2e^2 + 2\,cdxe + cd^2\right)}{2\,pe + 3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="giac")
```

```
[Out] ((c*x^2*e^2 + 2*c*d*x*e + c*d^2)^p*x^3*e^3 + 3*(c*x^2*e^2 + 2*c*d*x*e + c*d
^2)^p*d*x^2*e^2 + 3*(c*x^2*e^2 + 2*c*d*x*e + c*d^2)^p*d^2*x*e + (c*x^2*e^2
+ 2*c*d*x*e + c*d^2)^p*d^3)/(2*p*e + 3*e)
```

3.1102 $\int (d+ex)\left(cd^2+2cdex+ce^2x^2\right)^p\,dx$

Optimal. Leaf size=39

$$\frac{\left(cd^2+2cdex+ce^2x^2\right)^{p+1}}{2ce(p+1)}$$

```
[Out] (c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(1 + p)/(2*c*e*(1 + p))
```

Rubi [A] time = 0.0103326, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {629}

$$\frac{\left(cd^2+2cdex+ce^2x^2\right)^{p+1}}{2ce(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]
```

```
[Out] (c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(1 + p)/(2*c*e*(1 + p))
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (d+ex)\left(cd^2+2cdex+ce^2x^2\right)^p\,dx=\frac{\left(cd^2+2cdex+ce^2x^2\right)^{1+p}}{2ce(1+p)}$$

Mathematica [A] time = 0.0143138, size = 28, normalized size = 0.72

$$\frac{\left(c(d+ex)^2\right)^{p+1}}{2ce(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]
```

```
[Out] (c*(d + e*x)^2)^(1 + p)/(2*c*e*(1 + p))
```

Maple [A] time = 0.042, size = 40, normalized size = 1.

$$\frac{(ex+d)^2\left(ce^2x^2+2\,cdex+cd^2\right)^p}{2\,e\left(1+p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x)
```

```
[Out] 1/2*(e*x+d)^2/e/(1+p)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.622, size = 101, normalized size = 2.59

$$\frac{\left(e^{2} x^{2} + 2\, d e x + d^{2}\right) \left(c e^{2} x^{2} + 2\, c d e x + c d^{2}\right)^{p}}{2 \left(e p + e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="fricas")
```

```
[Out] 1/2*(e^2*x^2 + 2*d*e*x + d^2)*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p/(e*p + e)
```

Sympy [A] time = 0.472735, size = 139, normalized size = 3.56

$$\begin{cases} \frac{x}{c d} & \text{for}\: e = 0 \wedge p = -1 \\ d x \left(c d^{2}\right)^{p} & \text{for}\: e = 0 \\ \frac{\log{\left(\frac{d}{e} + x \right)}}{c e} & \text{for}\: p = -1 \\ \frac{d^{2} \left(c d^{2} + 2 c d e x + c e^{2} x^{2}\right)^{p}}{2 e p + 2 e} + \frac{2 d e x \left(c d^{2} + 2 c d e x + c e^{2} x^{2}\right)^{p}}{2 e p + 2 e} + \frac{e^{2} x^{2} \left(c d^{2} + 2 c d e x + c e^{2} x^{2}\right)^{p}}{2 e p + 2 e} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*e**2*x**2+2*c*d*e*x+c*d**2)**p,x)
```

```
[Out] Piecewise((x/(c*d), Eq(e, 0) & Eq(p, -1)), (d*x*(c*d**2)**p, Eq(e, 0)), (lo
g(d/e + x)/(c*e), Eq(p, -1)), (d**2*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(
2*e*p + 2*e) + 2*d*e*x*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*e*p + 2*e)
+ e**2*x**2*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*e*p + 2*e), True))
```

Giac [B] time = 1.19729, size = 127, normalized size = 3.26

$$\frac{\left(c x^{2} e^{2} + 2\, c d x e + c d^{2}\right)^{p} x^{2} e^{2} + 2 \left(c x^{2} e^{2} + 2\, c d x e + c d^{2}\right)^{p} d x e + \left(c x^{2} e^{2} + 2\, c d x e + c d^{2}\right)^{p} d^{2}}{2 \left(p e + e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="giac")
```

```
[Out] 1/2*((c*x^2*e^2 + 2*c*d*x*e + c*d^2)^p*x^2*e^2 + 2*(c*x^2*e^2 + 2*c*d*x*e +
 c*d^2)^p*d*x*e + (c*x^2*e^2 + 2*c*d*x*e + c*d^2)^p*d^2)/(p*e + e)
```

3.1103 $\int \left(cd^2+2cdex+ce^2x^2\right)^p\,dx$

Optimal. Leaf size=38

$$\frac{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^p}{e(2p+1)}$$

```
[Out] ((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p)/(e*(1 + 2*p))
```

Rubi [A] time = 0.0097498, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {609}

$$\frac{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^p}{e(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]
```

```
[Out] ((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p)/(e*(1 + 2*p))
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \left(cd^2+2cdex+ce^2x^2\right)^p\,dx=\frac{(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^p}{e(1+2p)}$$

Mathematica [A] time = 0.0101832, size = 25, normalized size = 0.66

$$\frac{(d+ex)\left(c(d+ex)^2\right)^p}{2ep+e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]
```

```
[Out] ((d + e*x)*(c*(d + e*x)^2)^p)/(e + 2*e*p)
```

Maple [A] time = 0.039, size = 39, normalized size = 1.

$$\frac{(ex+d)\left(ce^2x^2+2\,cdex+cd^2\right)^p}{e\left(1+2\,p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x)
```

```
[Out] (e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p/e/(1+2*p)
```

Maxima [A] time = 1.24028, size = 43, normalized size = 1.13

$$\frac{(c^p ex + c^p d)(ex + d)^{2p}}{e(2p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="maxima")
```

```
[Out] (c^p*e*x + c^p*d)*(e*x + d)^(2*p)/(e*(2*p + 1))
```

Fricas [A] time = 2.52531, size = 77, normalized size = 2.03

$$\frac{(ex + d)\left(ce^2x^2 + 2\,cdex + cd^2\right)^p}{2\,ep + e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="fricas")
```

```
[Out] (e*x + d)*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p/(2*e*p + e)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**p,x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.32154, size = 84, normalized size = 2.21

$$\frac{\left(cx^2e^2 + 2\,cdxe + cd^2\right)^p xe + \left(cx^2e^2 + 2\,cdxe + cd^2\right)^p d}{2\,pe + e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="giac")
```

```
[Out] ((c*x^2*e^2 + 2*c*d*x*e + c*d^2)^p*x*e + (c*x^2*e^2 + 2*c*d*x*e + c*d^2)^p*
d)/(2*p*e + e)
```

3.1104 $\int \frac{(cd^2+2cdex+ce^2x^2)^p}{d+ex} dx$

Optimal. Leaf size=32

$$\frac{(cd^2 + 2cdex + ce^2x^2)^p}{2ep}$$

```
[Out] (c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p/(2*e*p)
```

Rubi [A] time = 0.0212296, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {643, 629}

$$\frac{(cd^2 + 2cdex + ce^2x^2)^p}{2ep}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p/(d + e*x),x]
```

```
[Out] (c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p/(2*e*p)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{d + ex} dx &= c \int (d + ex)(cd^2 + 2cdex + ce^2x^2)^{-1+p} dx \\ &= \frac{(cd^2 + 2cdex + ce^2x^2)^p}{2ep} \end{aligned}$$

Mathematica [A] time = 0.0064535, size = 21, normalized size = 0.66

$$\frac{(c(d + ex)^2)^p}{2ep}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p/(d + e*x),x]
```

```
[Out] (c*(d + e*x)^2)^p/(2*e*p)
```

Maple [A] time = 0.039, size = 31, normalized size = 1.

$$\frac{\left(ce^{2}x^{2}+2\,cdex+cd^{2}\right)^{p}}{2\,ep}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d),x)
```

```
[Out] 1/2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p/e/p
```

Maxima [A] time = 1.15541, size = 27, normalized size = 0.84

$$\frac{(ex+d)^{2\,p}c^{p}}{2\,ep}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*(e*x + d)^(2*p)*c^p/(e*p)
```

Fricas [A] time = 2.51845, size = 61, normalized size = 1.91

$$\frac{\left(ce^{2}x^{2}+2\,cdex+cd^{2}\right)^{p}}{2\,ep}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/2*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p/(e*p)
```

Sympy [A] time = 0.348116, size = 48, normalized size = 1.5

$$\begin{cases} \frac{x}{d} & \text{for}\: e = 0 \wedge p = 0 \\ \frac{\log{\left(\frac{d}{e}+x \right)}}{e} & \text{for}\: p = 0 \\ \frac{x\left(cd^{2}\right)^{p}}{d} & \text{for}\: e = 0 \\ \frac{\left(cd^{2}+2cdex+ce^{2}x^{2}\right)^{p}}{2ep} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**p/(e*x+d),x)
```

```
[Out] Piecewise((x/d, Eq(e, 0) & Eq(p, 0)), (log(d/e + x)/e, Eq(p, 0)), (x*(c*d**
2)**p/d, Eq(e, 0)), ((c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*e*p), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ce^2x^2 + 2\,cdex + cd^2\right)^p}{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p/(e*x + d), x)
```

3.1105 $\int \frac{\left(cd^2+2cdex+ce^2x^2\right)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=42

$$-\frac{c(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{p-1}}{e(1-2p)}$$

```
[Out] -((c*(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(-1 + p))/(e*(1 - 2*p)))
```

Rubi [A] time = 0.0256373, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {642, 609}

$$-\frac{c(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{p-1}}{e(1-2p)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p/(d + e*x)^2,x]
```

```
[Out] -((c*(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(-1 + p))/(e*(1 - 2*p)))
```

Rule 642

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^m/c^(m/2), Int[(a + b*x + c*x^2)^(p + m/2), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && EqQ[2*c*d
 - b*e, 0] && IntegerQ[m/2]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(cd^2+2cdex+ce^2x^2\right)^p}{(d+ex)^2}\,dx &= c\int \left(cd^2+2cdex+ce^2x^2\right)^{-1+p}\,dx\\ &= -\frac{c(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{-1+p}}{e(1-2p)}\end{aligned}$$

Mathematica [A] time = 0.0166086, size = 30, normalized size = 0.71

$$\frac{c(d+ex)\left(c(d+ex)^2\right)^{p-1}}{e(2p-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p/(d + e*x)^2,x]
```

```
[Out] (c*(d + e*x)*(c*(d + e*x)^2)^(-1 + p))/(e*(-1 + 2*p))
```

Maple [A] time = 0.039, size = 41, normalized size = 1.

$$\frac{\left(ce^2x^2 + 2\,cdex + cd^2\right)^p}{(ex + d)\left(-1 + 2\,p\right)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d)^2,x)
```

```
[Out] 1/(e*x+d)/(-1+2*p)/e*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p
```

Maxima [A] time = 1.31708, size = 46, normalized size = 1.1

$$\frac{(ex + d)^{2\,p}c^p}{e^2\left(2\,p - 1\right)x + de\left(2\,p - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] (e*x + d)^(2*p)*c^p/(e^2*(2*p - 1)*x + d*e*(2*p - 1))
```

Fricas [A] time = 2.44252, size = 96, normalized size = 2.29

$$\frac{\left(ce^2x^2 + 2\,cdex + cd^2\right)^p}{2\,dep - de + \left(2\,e^2p - e^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] (c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p/(2*d*e*p - d*e + (2*e^2*p - e^2)*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**p/(e*x+d)**2,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ce^2x^2 + 2\,cdex + cd^2\right)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p/(e*x + d)^2, x)
```

3.1106 $\int \frac{\left(cd^2+2cdex+ce^2x^2\right)^p}{(d+ex)^3}\,dx$

Optimal. Leaf size=39

$$-\frac{c\left(cd^2+2cdex+ce^2x^2\right)^{p-1}}{2e(1-p)}$$

```
[Out] -(c*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(-1 + p))/(2*e*(1 - p))
```

Rubi [A] time = 0.0256884, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {643, 629}

$$-\frac{c\left(cd^2+2cdex+ce^2x^2\right)^{p-1}}{2e(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p/(d + e*x)^3,x]
```

```
[Out] -(c*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(-1 + p))/(2*e*(1 - p))
```

Rule 643

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(cd^2+2cdex+ce^2x^2\right)^p}{(d+ex)^3}\,dx &= c^2\int (d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{-2+p}\,dx\\ &= -\frac{c\left(cd^2+2cdex+ce^2x^2\right)^{-1+p}}{2e(1-p)}\end{aligned}$$

Mathematica [A] time = 0.0124948, size = 26, normalized size = 0.67

$$\frac{c\left(c(d+ex)^2\right)^{p-1}}{2e(p-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p/(d + e*x)^3,x]
```

```
[Out] (c*(c*(d + e*x)^2)^(-1 + p))/(2*e*(-1 + p))
```

Maple [A] time = 0.039, size = 40, normalized size = 1.

$$\frac{\left(ce^{2}x^{2}+2\,cdex+cd^{2}\right)^{p}}{2\,\left(ex+d\right)^{2}\left(p-1\right)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d)^3,x)
```

```
[Out] 1/2/(e*x+d)^2/(p-1)/e*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p
```

Maxima [A] time = 1.27352, size = 61, normalized size = 1.56

$$\frac{\left(ex+d\right)^{2\,p}c^{p}}{2\left(e^{3}\left(p-1\right)x^{2}+2\,de^{2}\left(p-1\right)x+d^{2}e\left(p-1\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(e*x + d)^(2*p)*c^p/(e^3*(p - 1)*x^2 + 2*d*e^2*(p - 1)*x + d^2*e*(p - 1
))
```

Fricas [A] time = 2.48723, size = 136, normalized size = 3.49

$$\frac{\left(ce^{2}x^{2}+2\,cdex+cd^{2}\right)^{p}}{2\left(d^{2}ep-d^{2}e+\left(e^{3}p-e^{3}\right)x^{2}+2\left(de^{2}p-de^{2}\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p/(d^2*e*p - d^2*e + (e^3*p - e^3)*x^2
+ 2*(d*e^2*p - d*e^2)*x)
```

Sympy [A] time = 1.16808, size = 100, normalized size = 2.56

$$\begin{cases}\frac{cx}{d} & \text{for}\: e=0 \wedge p=1\\ \frac{x\left(cd^{2}\right)^{p}}{d^{3}} & \text{for}\: e=0\\ \frac{c\log\left(\frac{d}{e}+x\right)}{e} & \text{for}\: p=1\\ \frac{\left(cd^{2}+2cdex+ce^{2}x^{2}\right)^{p}}{2d^{2}ep-2d^{2}e+4de^{2}px-4de^{2}x+2e^{3}px^{2}-2e^{3}x^{2}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**p/(e*x+d)**3,x)
```

```
[Out] Piecewise((c*x/d, Eq(e, 0) & Eq(p, 1)), (x*(c*d**2)**p/d**3, Eq(e, 0)), (c*
log(d/e + x)/e, Eq(p, 1)), ((c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*d**2*e
*p - 2*d**2*e + 4*d*e**2*p*x - 4*d*e**2*x + 2*e**3*p*x**2 - 2*e**3*x**2), T
rue))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ce^2x^2 + 2\,cdex + cd^2\right)^p}{(ex + d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p/(e*x + d)^3, x)
```

3.1107 $\int (d+ex)^{-1-2p}\left(cd^2+2cdex+ce^2x^2\right)^p\,dx$

Optimal. Leaf size=41

$$\frac{(d+ex)^{-2p}\log(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^p}{e}$$

```
[Out] ((c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p*Log[d + e*x])/(e*(d + e*x)^(2*p))
```

Rubi [A] time = 0.0147421, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {644, 31}

$$\frac{(d+ex)^{-2p}\log(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^p}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(-1 - 2*p)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]
```

```
[Out] ((c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p*Log[d + e*x])/(e*(d + e*x)^(2*p))
```

Rule 644

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[(a + b*x + c*x^2)^p/(d + e*x)^(2*p), Int[(d + e*x)^(m + 2*p),
x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !Integ
erQ[p] && EqQ[2*c*d - b*e, 0] &&  !IntegerQ[m]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^{-1-2p}\left(cd^2+2cdex+ce^2x^2\right)^p\,dx &= \left((d+ex)^{-2p}\left(cd^2+2cdex+ce^2x^2\right)^p\right)\int \frac{1}{d+ex}\,dx\\ &= \frac{(d+ex)^{-2p}\left(cd^2+2cdex+ce^2x^2\right)^p\log(d+ex)}{e}\end{aligned}$$

Mathematica [A] time = 0.0070629, size = 30, normalized size = 0.73

$$\frac{(d+ex)^{-2p}\log(d+ex)\left(c(d+ex)^2\right)^p}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(-1 - 2*p)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]
```

```
[Out] ((c*(d + e*x)^2)^p*Log[d + e*x])/(e*(d + e*x)^(2*p))
```

Maple [B] time = 0.079, size = 95, normalized size = 2.3

$$x\ln\left(ex+d\right){\mathrm e}^{\left(-1-2p\right)\ln\left(ex+d\right)}{\mathrm e}^{p\ln\left(c e^{2} x^{2}+2 c d e x+c d^{2}\right)}+\frac{d\ln\left(ex+d\right){\mathrm e}^{\left(-1-2p\right)\ln\left(ex+d\right)}{\mathrm e}^{p\ln\left(c e^{2} x^{2}+2 c d e x+c d^{2}\right)}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(-1-2*p)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x)
```

```
[Out] x*ln(e*x+d)*exp((-1-2*p)*ln(e*x+d))*exp(p*ln(c*e^2*x^2+2*c*d*e*x+c*d^2))+d/
e*ln(e*x+d)*exp((-1-2*p)*ln(e*x+d))*exp(p*ln(c*e^2*x^2+2*c*d*e*x+c*d^2))
```

Maxima [A] time = 1.34949, size = 18, normalized size = 0.44

$$\frac{c^p\log\left(ex+d\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-1-2*p)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="maxi
ma")
```

```
[Out] c^p*log(e*x + d)/e
```

Fricas [A] time = 2.40496, size = 27, normalized size = 0.66

$$\frac{c^p\log\left(ex+d\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-1-2*p)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="fric
as")
```

```
[Out] c^p*log(e*x + d)/e
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c\left(d+ex\right)^{2}\right)^{p}\left(d+ex\right)^{-2p-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(-1-2*p)*(c*e**2*x**2+2*c*d*e*x+c*d**2)**p,x)
```

```
[Out] Integral((c*(d + e*x)**2)**p*(d + e*x)**(-2*p - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(ce^{2}x^{2}+2\,cdex+cd^{2}\right)^{p}\left(ex+d\right)^{-2p-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-1-2*p)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="giac
")
```

```
[Out] integrate((c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p*(e*x + d)^(-2*p - 1), x)
```

3.1108 $\int (d+ex)^{-1+2p}\left(cd^2+2cdex+ce^2x^2\right)^{-p}\,dx$

Optimal. Leaf size=43

$$\frac{(d+ex)^{2p}\log(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{-p}}{e}$$

```
[Out] ((d + e*x)^(2*p)*Log[d + e*x])/(e*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p)
```

Rubi [A] time = 0.0146628, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {644, 31}

$$\frac{(d+ex)^{2p}\log(d+ex)\left(cd^2+2cdex+ce^2x^2\right)^{-p}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(-1 + 2*p)/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]
```

```
[Out] ((d + e*x)^(2*p)*Log[d + e*x])/(e*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p)
```

Rule 644

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[(a + b*x + c*x^2)^p/(d + e*x)^(2*p), Int[(d + e*x)^(m + 2*p),
x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !Integ
erQ[p] && EqQ[2*c*d - b*e, 0] && !IntegerQ[m]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^{-1+2p}\left(cd^2+2cdex+ce^2x^2\right)^{-p}\,dx &= \left((d+ex)^{2p}\left(cd^2+2cdex+ce^2x^2\right)^{-p}\right)\int \frac{1}{d+ex}\,dx\\ &= \frac{(d+ex)^{2p}\left(cd^2+2cdex+ce^2x^2\right)^{-p}\log(d+ex)}{e}\end{aligned}$$

Mathematica [A] time = 0.0060687, size = 32, normalized size = 0.74

$$\frac{(d+ex)^{2p}\log(d+ex)\left(c(d+ex)^2\right)^{-p}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(-1 + 2*p)/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]
```

```
[Out] ((d + e*x)^(2*p)*Log[d + e*x])/(e*(c*(d + e*x)^2)^p)
```

Maple [A] time = 0.077, size = 74, normalized size = 1.7

$$\frac{1}{\mathrm{e}^{p\ln\left(ce^{2}x^{2}+2\,cdex+cd^{2}\right)}}\left(x\ln\left(ex+d\right)\mathrm{e}^{\left(-1+2\,p\right)\ln\left(ex+d\right)}+\frac{d\ln\left(ex+d\right)\mathrm{e}^{\left(-1+2\,p\right)\ln\left(ex+d\right)}}{e}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(-1+2*p)/((c*e^2*x^2+2*c*d*e*x+c*d^2)^p),x)
```

```
[Out] (x*ln(e*x+d)*exp((-1+2*p)*ln(e*x+d))+d/e*ln(e*x+d)*exp((-1+2*p)*ln(e*x+d)))
/exp(p*ln(c*e^2*x^2+2*c*d*e*x+c*d^2))
```

Maxima [A] time = 1.16358, size = 20, normalized size = 0.47

$$\frac{\log\left(ex+d\right)}{c^{p}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-1+2*p)/((c*e^2*x^2+2*c*d*e*x+c*d^2)^p),x, algorithm="ma
xima")
```

```
[Out] log(e*x + d)/(c^p*e)
```

Fricas [A] time = 2.36749, size = 30, normalized size = 0.7

$$\frac{\log\left(ex+d\right)}{c^{p}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-1+2*p)/((c*e^2*x^2+2*c*d*e*x+c*d^2)^p),x, algorithm="fr
icas")
```

```
[Out] log(e*x + d)/(c^p*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(-1+2*p)/((c*e**2*x**2+2*c*d*e*x+c*d**2)**p),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{2p-1}}{\left(ce^2x^2+2\,cdex+cd^2\right)^p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-1+2*p)/((c*e^2*x^2+2*c*d*e*x+c*d^2)^p),x, algorithm="gi
ac")
```

```
[Out] integrate((e*x + d)^(2*p - 1)/(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p, x)
```

3.1109 $\int (bd+2cdx)^4 \left(a+bx+cx^2\right) \, dx$

Optimal. Leaf size=45

$$\frac{d^4(b+2cx)^7}{56c^2}-\frac{d^4\left(b^2-4ac\right)(b+2cx)^5}{40c^2}$$

```
[Out] -((b^2 - 4*a*c)*d^4*(b + 2*c*x)^5)/(40*c^2) + (d^4*(b + 2*c*x)^7)/(56*c^2)
```

Rubi [A] time = 0.0751143, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {683}

$$\frac{d^4(b+2cx)^7}{56c^2}-\frac{d^4\left(b^2-4ac\right)(b+2cx)^5}{40c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^4*(a + b*x + c*x^2),x]
```

```
[Out] -((b^2 - 4*a*c)*d^4*(b + 2*c*x)^5)/(40*c^2) + (d^4*(b + 2*c*x)^7)/(56*c^2)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int (bd+2cdx)^4 \left(a+bx+cx^2\right) \, dx &= \int \left(\frac{\left(-b^2+4ac\right)(bd+2cdx)^4}{4c}+\frac{(bd+2cdx)^6}{4cd^2}\right) \, dx\\ &= -\frac{\left(b^2-4ac\right)d^4(b+2cx)^5}{40c^2}+\frac{d^4(b+2cx)^7}{56c^2}\end{aligned}$$

Mathematica [B] time = 0.0171741, size = 102, normalized size = 2.27

$$d^4\left(\frac{8}{5}c^3x^5\left(2ac+7b^2\right)+8bc^2x^4\left(ac+b^2\right)+b^2cx^3\left(8ac+3b^2\right)+\frac{1}{2}b^3x^2\left(8ac+b^2\right)+ab^4x+8bc^4x^6+\frac{16c^5x^7}{7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^4*(a + b*x + c*x^2),x]
```

```
[Out] d^4*(a*b^4*x + (b^3*(b^2 + 8*a*c)*x^2)/2 + b^2*c*(3*b^2 + 8*a*c)*x^3 + 8*b*
c^2*(b^2 + a*c)*x^4 + (8*c^3*(7*b^2 + 2*a*c)*x^5)/5 + 8*b*c^4*x^6 + (16*c^5
*x^7)/7)
```

Maple [B] time = 0.04, size = 137, normalized size = 3.

$$\frac{16 c^5 d^4 x^7}{7} + 8 b d^4 c^4 x^6 + \frac{\left(16 c^4 d^4 a + 56 b^2 d^4 c^3\right) x^5}{5} + \frac{\left(32 b d^4 c^3 a + 32 b^3 d^4 c^2\right) x^4}{4} + \frac{\left(24 b^2 d^4 c^2 a + 9 b^4 d^4 c\right) x^3}{3} + \frac{\left(8 b^3 d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^4*(c*x^2+b*x+a),x)
```

```
[Out] 16/7*c^5*d^4*x^7+8*b*d^4*c^4*x^6+1/5*(16*a*c^4*d^4+56*b^2*c^3*d^4)*x^5+1/4*
(32*a*b*c^3*d^4+32*b^3*c^2*d^4)*x^4+1/3*(24*a*b^2*c^2*d^4+9*b^4*c*d^4)*x^3+
1/2*(8*a*b^3*c*d^4+b^5*d^4)*x^2+b^4*d^4*a*x
```

Maxima [B] time = 1.22842, size = 162, normalized size = 3.6

$$\frac{16}{7} c^5 d^4 x^7 + 8 b c^4 d^4 x^6 + a b^4 d^4 x + \frac{8}{5} \left(7 b^2 c^3 + 2 a c^4\right) d^4 x^5 + 8 \left(b^3 c^2 + a b c^3\right) d^4 x^4 + \left(3 b^4 c + 8 a b^2 c^2\right) d^4 x^3 + \frac{1}{2} \left(b^5 + 8 a b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4*(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 16/7*c^5*d^4*x^7 + 8*b*c^4*d^4*x^6 + a*b^4*d^4*x + 8/5*(7*b^2*c^3 + 2*a*c^4
)*d^4*x^5 + 8*(b^3*c^2 + a*b*c^3)*d^4*x^4 + (3*b^4*c + 8*a*b^2*c^2)*d^4*x^3
 + 1/2*(b^5 + 8*a*b^3*c)*d^4*x^2
```

Fricas [B] time = 2.02305, size = 286, normalized size = 6.36

$$\frac{16}{7} x^7 d^4 c^5 + 8 x^6 d^4 c^4 b + \frac{56}{5} x^5 d^4 c^3 b^2 + \frac{16}{5} x^5 d^4 c^4 a + 8 x^4 d^4 c^2 b^3 + 8 x^4 d^4 c^3 b a + 3 x^3 d^4 c b^4 + 8 x^3 d^4 c^2 b^2 a + \frac{1}{2} x^2 d^4 b^5 + 4 x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4*(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 16/7*x^7*d^4*c^5 + 8*x^6*d^4*c^4*b + 56/5*x^5*d^4*c^3*b^2 + 16/5*x^5*d^4*c^
4*a + 8*x^4*d^4*c^2*b^3 + 8*x^4*d^4*c^3*b*a + 3*x^3*d^4*c*b^4 + 8*x^3*d^4*c
^2*b^2*a + 1/2*x^2*d^4*b^5 + 4*x^2*d^4*c*b^3*a + x*d^4*b^4*a
```

Sympy [B] time = 0.123086, size = 143, normalized size = 3.18

$$a b^4 d^4 x + 8 b c^4 d^4 x^6 + \frac{16 c^5 d^4 x^7}{7} + x^5 \left(\frac{16 a c^4 d^4}{5} + \frac{56 b^2 c^3 d^4}{5}\right) + x^4 \left(8 a b c^3 d^4 + 8 b^3 c^2 d^4\right) + x^3 \left(8 a b^2 c^2 d^4 + 3 b^4 c d^4\right) + x^2 \left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**4*(c*x**2+b*x+a),x)
```

```
[Out] a*b**4*d**4*x + 8*b*c**4*d**4*x**6 + 16*c**5*d**4*x**7/7 + x**5*(16*a*c**4*
d**4/5 + 56*b**2*c**3*d**4/5) + x**4*(8*a*b*c**3*d**4 + 8*b**3*c**2*d**4) +
 x**3*(8*a*b**2*c**2*d**4 + 3*b**4*c*d**4) + x**2*(4*a*b**3*c*d**4 + b**5*d
```

```
**4/2)
```

Giac [B] time = 1.21602, size = 185, normalized size = 4.11

$$\frac{16}{7}\,c^5d^4x^7 + 8\,bc^4d^4x^6 + \frac{56}{5}\,b^2c^3d^4x^5 + \frac{16}{5}\,ac^4d^4x^5 + 8\,b^3c^2d^4x^4 + 8\,abc^3d^4x^4 + 3\,b^4cd^4x^3 + 8\,ab^2c^2d^4x^3 + \frac{1}{2}\,b^5d^4x^2 + 4\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4*(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 16/7*c^5*d^4*x^7 + 8*b*c^4*d^4*x^6 + 56/5*b^2*c^3*d^4*x^5 + 16/5*a*c^4*d^4*
x^5 + 8*b^3*c^2*d^4*x^4 + 8*a*b*c^3*d^4*x^4 + 3*b^4*c*d^4*x^3 + 8*a*b^2*c^2
*d^4*x^3 + 1/2*b^5*d^4*x^2 + 4*a*b^3*c*d^4*x^2 + a*b^4*d^4*x
```

3.1110 $\int (bd+2cdx)^3 \left(a+bx+cx^2\right) dx$

Optimal. Leaf size=45

$$\frac{d^3(b+2cx)^6}{48c^2}-\frac{d^3\left(b^2-4ac\right)(b+2cx)^4}{32c^2}$$

```
[Out] -((b^2 - 4*a*c)*d^3*(b + 2*c*x)^4)/(32*c^2) + (d^3*(b + 2*c*x)^6)/(48*c^2)
```

Rubi [A] time = 0.0535636, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {683}

$$\frac{d^3(b+2cx)^6}{48c^2}-\frac{d^3\left(b^2-4ac\right)(b+2cx)^4}{32c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^3*(a + b*x + c*x^2),x]
```

```
[Out] -((b^2 - 4*a*c)*d^3*(b + 2*c*x)^4)/(32*c^2) + (d^3*(b + 2*c*x)^6)/(48*c^2)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int (bd+2cdx)^3\left(a+bx+cx^2\right)dx &= \int \left(\frac{\left(-b^2+4ac\right)(bd+2cdx)^3}{4c}+\frac{(bd+2cdx)^5}{4cd^2}\right)dx\\ &= -\frac{\left(b^2-4ac\right)d^3(b+2cx)^4}{32c^2}+\frac{d^3(b+2cx)^6}{48c^2}\end{aligned}$$

Mathematica [A] time = 0.0145711, size = 66, normalized size = 1.47

$$\frac{1}{6}d^3x(b+cx)\left(6a\left(b^2+2bcx+2c^2x^2\right)+x\left(11b^2cx+3b^3+16bc^2x^2+8c^3x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^3*(a + b*x + c*x^2),x]
```

```
[Out] (d^3*x*(b + c*x)*(6*a*(b^2 + 2*b*c*x + 2*c^2*x^2) + x*(3*b^3 + 11*b^2*c*x +
 16*b*c^2*x^2 + 8*c^3*x^3)))/6
```

Maple [B] time = 0.039, size = 108, normalized size = 2.4

$$\frac{4\,c^4d^3x^6}{3}+4\,bd^3c^3x^5+\frac{\left(8\,c^3d^3a+18\,b^2d^3c^2\right)x^4}{4}+\frac{\left(12\,bd^3c^2a+7\,b^3d^3c\right)x^3}{3}+\frac{\left(6\,b^2d^3ca+b^4d^3\right)x^2}{2}+b^3d^3ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^3*(c*x^2+b*x+a),x)
```

```
[Out] 4/3*c^4*d^3*x^6+4*b*d^3*c^3*x^5+1/4*(8*a*c^3*d^3+18*b^2*c^2*d^3)*x^4+1/3*(1
2*a*b*c^2*d^3+7*b^3*c*d^3)*x^3+1/2*(6*a*b^2*c*d^3+b^4*d^3)*x^2+b^3*d^3*a*x
```

Maxima [B] time = 1.23227, size = 131, normalized size = 2.91

$$\frac{4}{3}c^4d^3x^6 + 4\,bc^3d^3x^5 + ab^3d^3x + \frac{1}{2}\left(9\,b^2c^2 + 4\,ac^3\right)d^3x^4 + \frac{1}{3}\left(7\,b^3c + 12\,abc^2\right)d^3x^3 + \frac{1}{2}\left(b^4 + 6\,ab^2c\right)d^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3*(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 4/3*c^4*d^3*x^6 + 4*b*c^3*d^3*x^5 + a*b^3*d^3*x + 1/2*(9*b^2*c^2 + 4*a*c^3)
*d^3*x^4 + 1/3*(7*b^3*c + 12*a*b*c^2)*d^3*x^3 + 1/2*(b^4 + 6*a*b^2*c)*d^3*x
^2
```

Fricas [B] time = 2.02252, size = 225, normalized size = 5.

$$\frac{4}{3}x^6d^3c^4 + 4x^5d^3c^3b + \frac{9}{2}x^4d^3c^2b^2 + 2x^4d^3c^3a + \frac{7}{3}x^3d^3cb^3 + 4x^3d^3c^2ba + \frac{1}{2}x^2d^3b^4 + 3x^2d^3cb^2a + xd^3b^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3*(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 4/3*x^6*d^3*c^4 + 4*x^5*d^3*c^3*b + 9/2*x^4*d^3*c^2*b^2 + 2*x^4*d^3*c^3*a +
 7/3*x^3*d^3*c*b^3 + 4*x^3*d^3*c^2*b*a + 1/2*x^2*d^3*b^4 + 3*x^2*d^3*c*b^2*
a + x*d^3*b^3*a
```

Sympy [B] time = 0.120087, size = 114, normalized size = 2.53

$$ab^3d^3x + 4bc^3d^3x^5 + \frac{4c^4d^3x^6}{3} + x^4\left(2ac^3d^3 + \frac{9b^2c^2d^3}{2}\right) + x^3\left(4abc^2d^3 + \frac{7b^3cd^3}{3}\right) + x^2\left(3ab^2cd^3 + \frac{b^4d^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**3*(c*x**2+b*x+a),x)
```

```
[Out] a*b**3*d**3*x + 4*b*c**3*d**3*x**5 + 4*c**4*d**3*x**6/3 + x**4*(2*a*c**3*d*
*3 + 9*b**2*c**2*d**3/2) + x**3*(4*a*b*c**2*d**3 + 7*b**3*c*d**3/3) + x**2*
(3*a*b**2*c*d**3 + b**4*d**3/2)
```

Giac [B] time = 1.22207, size = 146, normalized size = 3.24

$$\frac{4}{3}c^4d^3x^6 + 4\,bc^3d^3x^5 + \frac{9}{2}\,b^2c^2d^3x^4 + 2\,ac^3d^3x^4 + \frac{7}{3}\,b^3cd^3x^3 + 4\,abc^2d^3x^3 + \frac{1}{2}\,b^4d^3x^2 + 3\,ab^2cd^3x^2 + ab^3d^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3*(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 4/3*c^4*d^3*x^6 + 4*b*c^3*d^3*x^5 + 9/2*b^2*c^2*d^3*x^4 + 2*a*c^3*d^3*x^4 +
 7/3*b^3*c*d^3*x^3 + 4*a*b*c^2*d^3*x^3 + 1/2*b^4*d^3*x^2 + 3*a*b^2*c*d^3*x^
2 + a*b^3*d^3*x
```

3.1111 $\int (bd+2cdx)^2 \left(a+bx+cx^2\right) \, dx$

Optimal. Leaf size=45

$$\frac{d^2(b+2cx)^5}{40c^2}-\frac{d^2\left(b^2-4ac\right)(b+2cx)^3}{24c^2}$$

```
[Out] -((b^2 - 4*a*c)*d^2*(b + 2*c*x)^3)/(24*c^2) + (d^2*(b + 2*c*x)^5)/(40*c^2)
```

Rubi [A] time = 0.0397384, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {683}

$$\frac{d^2(b+2cx)^5}{40c^2}-\frac{d^2\left(b^2-4ac\right)(b+2cx)^3}{24c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^2*(a + b*x + c*x^2),x]
```

```
[Out] -((b^2 - 4*a*c)*d^2*(b + 2*c*x)^3)/(24*c^2) + (d^2*(b + 2*c*x)^5)/(40*c^2)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int (bd+2cdx)^2\left(a+bx+cx^2\right)\,dx&=\int \left(\frac{\left(-b^2+4ac\right)(bd+2cdx)^2}{4c}+\frac{(bd+2cdx)^4}{4cd^2}\right)dx\\&=-\frac{\left(b^2-4ac\right)d^2(b+2cx)^3}{24c^2}+\frac{d^2(b+2cx)^5}{40c^2}\end{aligned}$$

Mathematica [A] time = 0.0099655, size = 64, normalized size = 1.42

$$d^2\left(\frac{1}{3}cx^3\left(4ac+5b^2\right)+\frac{1}{2}bx^2\left(4ac+b^2\right)+ab^2x+2bc^2x^4+\frac{4c^3x^5}{5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^2*(a + b*x + c*x^2),x]
```

```
[Out] d^2*(a*b^2*x + (b*(b^2 + 4*a*c)*x^2)/2 + (c*(5*b^2 + 4*a*c)*x^3)/3 + 2*b*c^
2*x^4 + (4*c^3*x^5)/5)
```

Maple [A] time = 0.039, size = 79, normalized size = 1.8

$$\frac{4\,c^3d^2x^5}{5}+2\,bd^2c^2x^4+\frac{\left(4\,c^2d^2a+5\,b^2d^2c\right)x^3}{3}+\frac{\left(4\,bd^2ca+b^3d^2\right)x^2}{2}+b^2d^2ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^2*(c*x^2+b*x+a),x)
```

```
[Out] 4/5*c^3*d^2*x^5+2*b*d^2*c^2*x^4+1/3*(4*a*c^2*d^2+5*b^2*c*d^2)*x^3+1/2*(4*a*
b*c*d^2+b^3*d^2)*x^2+b^2*d^2*a*x
```

Maxima [A] time = 1.26757, size = 96, normalized size = 2.13

$$\frac{4}{5}c^3d^2x^5 + 2\,bc^2d^2x^4 + ab^2d^2x + \frac{1}{3}\left(5\,b^2c + 4\,ac^2\right)d^2x^3 + \frac{1}{2}\left(b^3 + 4\,abc\right)d^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2*(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 4/5*c^3*d^2*x^5 + 2*b*c^2*d^2*x^4 + a*b^2*d^2*x + 1/3*(5*b^2*c + 4*a*c^2)*d
^2*x^3 + 1/2*(b^3 + 4*a*b*c)*d^2*x^2
```

Fricas [A] time = 1.968, size = 169, normalized size = 3.76

$$\frac{4}{5}x^5d^2c^3 + 2x^4d^2c^2b + \frac{5}{3}x^3d^2cb^2 + \frac{4}{3}x^3d^2c^2a + \frac{1}{2}x^2d^2b^3 + 2x^2d^2cba + xd^2b^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2*(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 4/5*x^5*d^2*c^3 + 2*x^4*d^2*c^2*b + 5/3*x^3*d^2*c*b^2 + 4/3*x^3*d^2*c^2*a +
 1/2*x^2*d^2*b^3 + 2*x^2*d^2*c*b*a + x*d^2*b^2*a
```

Sympy [B] time = 0.084387, size = 85, normalized size = 1.89

$$ab^2d^2x + 2bc^2d^2x^4 + \frac{4c^3d^2x^5}{5} + x^3\left(\frac{4ac^2d^2}{3} + \frac{5b^2cd^2}{3}\right) + x^2\left(2abcd^2 + \frac{b^3d^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**2*(c*x**2+b*x+a),x)
```

```
[Out] a*b**2*d**2*x + 2*b*c**2*d**2*x**4 + 4*c**3*d**2*x**5/5 + x**3*(4*a*c**2*d*
*2/3 + 5*b**2*c*d**2/3) + x**2*(2*a*b*c*d**2 + b**3*d**2/2)
```

Giac [A] time = 1.19415, size = 107, normalized size = 2.38

$$\frac{4}{5}c^3d^2x^5 + 2\,bc^2d^2x^4 + \frac{5}{3}\,b^2cd^2x^3 + \frac{4}{3}\,ac^2d^2x^3 + \frac{1}{2}\,b^3d^2x^2 + 2\,abcd^2x^2 + ab^2d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2*(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 4/5*c^3*d^2*x^5 + 2*b*c^2*d^2*x^4 + 5/3*b^2*c*d^2*x^3 + 4/3*a*c^2*d^2*x^3 +
 1/2*b^3*d^2*x^2 + 2*a*b*c*d^2*x^2 + a*b^2*d^2*x
```

3.1112 $\int (bd+2cdx)\left(a+bx+cx^2\right)\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}d\left(a+bx+cx^2\right)^2$$

```
[Out] (d*(a + b*x + c*x^2)^2)/2
```

Rubi [A] time = 0.004063, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {629}

$$\frac{1}{2}d\left(a+bx+cx^2\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)*(a + b*x + c*x^2),x]

[Out] (d*(a + b*x + c*x^2)^2)/2
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (bd+2cdx)\left(a+bx+cx^2\right)\,dx=\frac{1}{2}d\left(a+bx+cx^2\right)^2$$

Mathematica [A] time = 0.0047767, size = 22, normalized size = 1.29

$$\frac{1}{2}dx(b+cx)(2a+x(b+cx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)*(a + b*x + c*x^2),x]

[Out] (d*x*(b + c*x)*(2*a + x*(b + c*x)))/2
```

Maple [B] time = 0.038, size = 39, normalized size = 2.3

$$\frac{c^2dx^4}{2}+bcdx^3+\frac{\left(2\,acd+b^2d\right)x^2}{2}+bdax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)*(c*x^2+b*x+a),x)
```

```
[Out] 1/2*c^2*d*x^4+b*c*d*x^3+1/2*(2*a*c*d+b^2*d)*x^2+b*d*a*x
```

Maxima [A] time = 1.17061, size = 20, normalized size = 1.18

$$\frac{1}{2}\left(cx^2 + bx + a\right)^2 d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)*(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(c*x^2 + b*x + a)^2*d
```

Fricas [B] time = 1.942, size = 88, normalized size = 5.18

$$\frac{1}{2}x^4dc^2 + x^3dcb + \frac{1}{2}x^2db^2 + x^2dca + xdba$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)*(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*x^4*d*c^2 + x^3*d*c*b + 1/2*x^2*d*b^2 + x^2*d*c*a + x*d*b*a
```

Sympy [B] time = 0.097042, size = 39, normalized size = 2.29

$$abdx + bcdx^3 + \frac{c^2dx^4}{2} + x^2\left(acd + \frac{b^2d}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)*(c*x**2+b*x+a),x)
```

```
[Out] a*b*d*x + b*c*d*x**3 + c**2*d*x**4/2 + x**2*(a*c*d + b**2*d/2)
```

Giac [B] time = 1.24304, size = 51, normalized size = 3.

$$\frac{1}{2}c^2dx^4 + bcdx^3 + \frac{1}{2}b^2dx^2 + acdx^2 + abdx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)*(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*c^2*d*x^4 + b*c*d*x^3 + 1/2*b^2*d*x^2 + a*c*d*x^2 + a*b*d*x
```

3.1113 $\int \frac{a+bx+cx^2}{bd+2cdx}\,dx$

Optimal. Leaf size=48

$$-\frac{(b^2-4ac)\log(b+2cx)}{8c^2d}+\frac{bx}{4cd}+\frac{x^2}{4d}$$

```
[Out] (b*x)/(4*c*d) + x^2/(4*d) - ((b^2 - 4*a*c)*Log[b + 2*c*x])/(8*c^2*d)
```

Rubi [A] time = 0.0350725, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {683}

$$-\frac{(b^2-4ac)\log(b+2cx)}{8c^2d}+\frac{bx}{4cd}+\frac{x^2}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(b*d + 2*c*d*x),x]
```

```
[Out] (b*x)/(4*c*d) + x^2/(4*d) - ((b^2 - 4*a*c)*Log[b + 2*c*x])/(8*c^2*d)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{bd+2cdx}\,dx &= \int \left(\frac{b}{4cd}+\frac{x}{2d}+\frac{-b^2+4ac}{4cd(b+2cx)}\right)dx\\ &= \frac{bx}{4cd}+\frac{x^2}{4d}-\frac{(b^2-4ac)\log(b+2cx)}{8c^2d}\end{aligned}$$

Mathematica [A] time = 0.0133888, size = 37, normalized size = 0.77

$$\frac{2cx(b+cx)-(b^2-4ac)\log(b+2cx)}{8c^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(b*d + 2*c*d*x),x]
```

```
[Out] (2*c*x*(b + c*x) - (b^2 - 4*a*c)*Log[b + 2*c*x])/(8*c^2*d)
```

Maple [A] time = 0.041, size = 54, normalized size = 1.1

$$\frac{x^2}{4d}+\frac{bx}{4cd}+\frac{\ln(2cx+b)a}{2cd}-\frac{\ln(2cx+b)b^2}{8c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(2*c*d*x+b*d),x)
```

```
[Out] 1/4*x^2/d+1/4*b*x/c/d+1/2/d/c*ln(2*c*x+b)*a-1/8/d/c^2*ln(2*c*x+b)*b^2
```

Maxima [A] time = 1.1732, size = 55, normalized size = 1.15

$$\frac{cx^2 + bx}{4\,cd} - \frac{\left(b^2 - 4\,ac\right)\log\left(2\,cx + b\right)}{8\,c^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d),x, algorithm="maxima")
```

```
[Out] 1/4*(c*x^2 + b*x)/(c*d) - 1/8*(b^2 - 4*a*c)*log(2*c*x + b)/(c^2*d)
```

Fricas [A] time = 2.22093, size = 89, normalized size = 1.85

$$\frac{2\,c^2x^2 + 2\,bcx - \left(b^2 - 4\,ac\right)\log\left(2\,cx + b\right)}{8\,c^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d),x, algorithm="fricas")
```

```
[Out] 1/8*(2*c^2*x^2 + 2*b*c*x - (b^2 - 4*a*c)*log(2*c*x + b))/(c^2*d)
```

Sympy [A] time = 0.434418, size = 37, normalized size = 0.77

$$\frac{bx}{4cd} + \frac{x^2}{4d} + \frac{\left(4ac - b^2\right)\log\left(b + 2cx\right)}{8c^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(2*c*d*x+b*d),x)
```

```
[Out] b*x/(4*c*d) + x**2/(4*d) + (4*a*c - b**2)*log(b + 2*c*x)/(8*c**2*d)
```

Giac [A] time = 1.17559, size = 63, normalized size = 1.31

$$-\frac{\left(b^2 - 4\,ac\right)\log\left(|2\,cx + b|\right)}{8\,c^2 d} + \frac{c^2dx^2 + bcdx}{4\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d),x, algorithm="giac")
```

```
[Out] -1/8*(b^2 - 4*a*c)*log(abs(2*c*x + b))/(c^2*d) + 1/4*(c^2*d*x^2 + b*c*d*x)/
(c^2*d^2)
```

3.1114 $\int \frac{a+bx+cx^2}{(bd+2cdx)^2} dx$

Optimal. Leaf size=38

$$\frac{b^2-4ac}{8c^2d^2(b+2cx)}+\frac{x}{4cd^2}$$

```
[Out] x/(4*c*d^2) + (b^2 - 4*a*c)/(8*c^2*d^2*(b + 2*c*x))
```

Rubi [A] time = 0.0276715, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {683}

$$\frac{b^2-4ac}{8c^2d^2(b+2cx)}+\frac{x}{4cd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(b*d + 2*c*d*x)^2,x]
```

```
[Out] x/(4*c*d^2) + (b^2 - 4*a*c)/(8*c^2*d^2*(b + 2*c*x))
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{(bd+2cdx)^2}\,dx &= \int \left(\frac{1}{4cd^2}+\frac{-b^2+4ac}{4cd^2(b+2cx)^2}\right)dx\\ &= \frac{x}{4cd^2}+\frac{b^2-4ac}{8c^2d^2(b+2cx)}\end{aligned}$$

Mathematica [A] time = 0.0106645, size = 41, normalized size = 1.08

$$\frac{\frac{b^2-4ac}{8c^2(b+2cx)}+\frac{b+2cx}{8c^2}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(b*d + 2*c*d*x)^2,x]
```

```
[Out] ((b^2 - 4*a*c)/(8*c^2*(b + 2*c*x)) + (b + 2*c*x)/(8*c^2))/d^2
```

Maple [A] time = 0.043, size = 35, normalized size = 0.9

$$\frac{1}{d^2}\left(\frac{x}{4\,c}-\frac{4\,ac-b^2}{8\,c^2\,(2\,cx+b)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(2*c*d*x+b*d)^2,x)
```

```
[Out] 1/d^2*(1/4*x/c-1/8*(4*a*c-b^2)/c^2/(2*c*x+b))
```

Maxima [A] time = 1.34769, size = 54, normalized size = 1.42

$$\frac{b^2 - 4\,ac}{8\left(2\,c^3d^2x + bc^2d^2\right)} + \frac{x}{4\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(b^2 - 4*a*c)/(2*c^3*d^2*x + b*c^2*d^2) + 1/4*x/(c*d^2)
```

Fricas [A] time = 2.20489, size = 90, normalized size = 2.37

$$\frac{4\,c^2x^2 + 2\,bcx + b^2 - 4\,ac}{8\left(2\,c^3d^2x + bc^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*c^2*x^2 + 2*b*c*x + b^2 - 4*a*c)/(2*c^3*d^2*x + b*c^2*d^2)
```

Sympy [A] time = 0.470093, size = 36, normalized size = 0.95

$$-\frac{4ac - b^2}{8bc^2d^2 + 16c^3d^2x} + \frac{x}{4cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(2*c*d*x+b*d)**2,x)
```

```
[Out] -(4*a*c - b**2)/(8*b*c**2*d**2 + 16*c**3*d**2*x) + x/(4*c*d**2)
```

Giac [B] time = 1.14319, size = 230, normalized size = 6.05

$$-\frac{1}{8}c\left(\frac{b^2}{(2\,cdx + bd)c^3d} - \frac{2\,b\log\left(\frac{|2\,cdx+bd|}{2\,(2\,cdx+bd)^2|c||d|}\right)}{c^3d^2} - \frac{2\,cdx + bd}{c^3d^3}\right) + \frac{b\left(\frac{b}{(2\,cdx+bd)c} - \frac{\log\left(\frac{|2\,cdx+bd|}{2\,(2\,cdx+bd)^2|c||d|}\right)}{cd}\right)}{4\,cd} - \frac{a}{2\,(2\,cdx + bd)cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^2,x, algorithm="giac")
```

```
[Out] -1/8*c*(b^2/((2*c*d*x + b*d)*c^3*d) - 2*b*log(1/2*abs(2*c*d*x + b*d)/((2*c*
d*x + b*d)^2*abs(c)*abs(d)))/(c^3*d^2) - (2*c*d*x + b*d)/(c^3*d^3)) + 1/4*b
*(b/((2*c*d*x + b*d)*c) - log(1/2*abs(2*c*d*x + b*d)/((2*c*d*x + b*d)^2*abs
(c)*abs(d)))/(c*d))/(c*d) - 1/2*a/((2*c*d*x + b*d)*c*d)
```

3.1115 $\int \frac{a+bx+cx^2}{(bd+2cdx)^3} dx$

Optimal. Leaf size=44

$$\frac{b^2-4ac}{16c^2d^3(b+2cx)^2}+\frac{\log(b+2cx)}{8c^2d^3}$$

```
[Out] (b^2 - 4*a*c)/(16*c^2*d^3*(b + 2*c*x)^2) + Log[b + 2*c*x]/(8*c^2*d^3)
```

Rubi [A] time = 0.0326059, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {683}

$$\frac{b^2-4ac}{16c^2d^3(b+2cx)^2}+\frac{\log(b+2cx)}{8c^2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(b*d + 2*c*d*x)^3,x]
```

```
[Out] (b^2 - 4*a*c)/(16*c^2*d^3*(b + 2*c*x)^2) + Log[b + 2*c*x]/(8*c^2*d^3)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{(bd+2cdx)^3}\,dx &= \int \left(\frac{-b^2+4ac}{4cd^3(b+2cx)^3}+\frac{1}{4cd^3(b+2cx)}\right)dx\\ &= \frac{b^2-4ac}{16c^2d^3(b+2cx)^2}+\frac{\log(b+2cx)}{8c^2d^3}\end{aligned}$$

Mathematica [A] time = 0.0154983, size = 37, normalized size = 0.84

$$\frac{\frac{b^2-4ac}{(b+2cx)^2}+2\log(b+2cx)}{16c^2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(b*d + 2*c*d*x)^3,x]
```

```
[Out] ((b^2 - 4*a*c)/(b + 2*c*x)^2 + 2*Log[b + 2*c*x])/(16*c^2*d^3)
```

Maple [A] time = 0.044, size = 53, normalized size = 1.2

$$-\frac{a}{4\,cd^3\,(2\,cx+b)^2}+\frac{b^2}{16\,c^2d^3\,(2\,cx+b)^2}+\frac{\ln(2\,cx+b)}{8\,c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(2*c*d*x+b*d)^3,x)
```

```
[Out] -1/4/d^3/c/(2*c*x+b)^2*a+1/16/d^3/c^2/(2*c*x+b)^2*b^2+1/8*ln(2*c*x+b)/c^2/d
^3
```

Maxima [A] time = 1.18744, size = 81, normalized size = 1.84

$$\frac{b^2 - 4\,ac}{16\left(4\,c^4d^3x^2 + 4\,bc^3d^3x + b^2c^2d^3\right)} + \frac{\log\left(2\,cx + b\right)}{8\,c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(b^2 - 4*a*c)/(4*c^4*d^3*x^2 + 4*b*c^3*d^3*x + b^2*c^2*d^3) + 1/8*log(
2*c*x + b)/(c^2*d^3)
```

Fricas [A] time = 2.33354, size = 153, normalized size = 3.48

$$\frac{b^2 - 4\,ac + 2\left(4\,c^2x^2 + 4\,bcx + b^2\right)\log\left(2\,cx + b\right)}{16\left(4\,c^4d^3x^2 + 4\,bc^3d^3x + b^2c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(b^2 - 4*a*c + 2*(4*c^2*x^2 + 4*b*c*x + b^2)*log(2*c*x + b))/(4*c^4*d^
3*x^2 + 4*b*c^3*d^3*x + b^2*c^2*d^3)
```

Sympy [A] time = 0.621888, size = 60, normalized size = 1.36

$$-\frac{4ac - b^2}{16b^2c^2d^3 + 64bc^3d^3x + 64c^4d^3x^2} + \frac{\log\left(b + 2cx\right)}{8c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(2*c*d*x+b*d)**3,x)
```

```
[Out] -(4*a*c - b**2)/(16*b**2*c**2*d**3 + 64*b*c**3*d**3*x + 64*c**4*d**3*x**2)
+ log(b + 2*c*x)/(8*c**2*d**3)
```

Giac [A] time = 1.22163, size = 55, normalized size = 1.25

$$\frac{\log\left(\left|2\,cx + b\right|\right)}{8\,c^2d^3} + \frac{b^2 - 4\,ac}{16\left(2\,cx + b\right)^2c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^3,x, algorithm="giac")
```

```
[Out] 1/8*log(abs(2*c*x + b))/(c^2*d^3) + 1/16*(b^2 - 4*a*c)/((2*c*x + b)^2*c^2*d
^3)
```

3.1116 $\int \frac{a+bx+cx^2}{(bd+2cdx)^4}\,dx$

Optimal. Leaf size=45

$$\frac{b^2-4ac}{24c^2d^4(b+2cx)^3}-\frac{1}{8c^2d^4(b+2cx)}$$

```
[Out] (b^2 - 4*a*c)/(24*c^2*d^4*(b + 2*c*x)^3) - 1/(8*c^2*d^4*(b + 2*c*x))
```

Rubi [A] time = 0.0317319, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {683}

$$\frac{b^2-4ac}{24c^2d^4(b+2cx)^3}-\frac{1}{8c^2d^4(b+2cx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(b*d + 2*c*d*x)^4,x]
```

```
[Out] (b^2 - 4*a*c)/(24*c^2*d^4*(b + 2*c*x)^3) - 1/(8*c^2*d^4*(b + 2*c*x))
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] && !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{(bd+2cdx)^4}\,dx &= \int \left(\frac{-b^2+4ac}{4cd^4(b+2cx)^4}+\frac{1}{4cd^4(b+2cx)^2}\right)dx\\ &= \frac{b^2-4ac}{24c^2d^4(b+2cx)^3}-\frac{1}{8c^2d^4(b+2cx)}\end{aligned}$$

Mathematica [A] time = 0.0156746, size = 38, normalized size = 0.84

$$-\frac{2c\left(a+3cx^2\right)+b^2+6bcx}{12c^2d^4(b+2cx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(b*d + 2*c*d*x)^4,x]
```

```
[Out] -(b^2 + 6*b*c*x + 2*c*(a + 3*c*x^2))/(12*c^2*d^4*(b + 2*c*x)^3)
```

Maple [A] time = 0.044, size = 42, normalized size = 0.9

$$\frac{1}{d^4}\left(-\frac{4\,ac-b^2}{24\,c^2\,(2\,cx+b)^3}-\frac{1}{8\,c^2\,(2\,cx+b)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(2*c*d*x+b*d)^4,x)
```

```
[Out] 1/d^4*(-1/24*(4*a*c-b^2)/c^2/(2*c*x+b)^3-1/8/c^2/(2*c*x+b))
```

Maxima [A] time = 1.16272, size = 96, normalized size = 2.13

$$-\frac{6\,c^2x^2+6\,bcx+b^2+2\,ac}{12\,\left(8\,c^5d^4x^3+12\,bc^4d^4x^2+6\,b^2c^3d^4x+b^3c^2d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^4,x, algorithm="maxima")
```

```
[Out] -1/12*(6*c^2*x^2 + 6*b*c*x + b^2 + 2*a*c)/(8*c^5*d^4*x^3 + 12*b*c^4*d^4*x^2
 + 6*b^2*c^3*d^4*x + b^3*c^2*d^4)
```

Fricas [A] time = 2.35558, size = 149, normalized size = 3.31

$$-\frac{6\,c^2x^2+6\,bcx+b^2+2\,ac}{12\,\left(8\,c^5d^4x^3+12\,bc^4d^4x^2+6\,b^2c^3d^4x+b^3c^2d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^4,x, algorithm="fricas")
```

```
[Out] -1/12*(6*c^2*x^2 + 6*b*c*x + b^2 + 2*a*c)/(8*c^5*d^4*x^3 + 12*b*c^4*d^4*x^2
 + 6*b^2*c^3*d^4*x + b^3*c^2*d^4)
```

Sympy [A] time = 0.651829, size = 75, normalized size = 1.67

$$-\frac{2ac+b^2+6bcx+6c^2x^2}{12b^3c^2d^4+72b^2c^3d^4x+144bc^4d^4x^2+96c^5d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(2*c*d*x+b*d)**4,x)
```

```
[Out] -(2*a*c + b**2 + 6*b*c*x + 6*c**2*x**2)/(12*b**3*c**2*d**4 + 72*b**2*c**3*d
**4*x + 144*b*c**4*d**4*x**2 + 96*c**5*d**4*x**3)
```

Giac [A] time = 1.25477, size = 50, normalized size = 1.11

$$-\frac{6\,c^2x^2+6\,bcx+b^2+2\,ac}{12\,(2\,cx+b)^3c^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^4,x, algorithm="giac")
```

```
[Out] -1/12*(6*c^2*x^2 + 6*b*c*x + b^2 + 2*a*c)/((2*c*x + b)^3*c^2*d^4)
```

3.1117 $\int \frac{a+bx+cx^2}{(bd+2cdx)^5}\,dx$

Optimal. Leaf size=37

$$\frac{(a+bx+cx^2)^2}{2d^5(b^2-4ac)(b+2cx)^4}$$

```
[Out] (a + b*x + c*x^2)^2/(2*(b^2 - 4*a*c)*d^5*(b + 2*c*x)^4)
```

Rubi [A] time = 0.010164, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {682}

$$\frac{(a+bx+cx^2)^2}{2d^5(b^2-4ac)(b+2cx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(b*d + 2*c*d*x)^5,x]
```


```
[Out] (a + b*x + c*x^2)^2/(2*(b^2 - 4*a*c)*d^5*(b + 2*c*x)^4)
```


Rule 682

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c,
0] && EqQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(bd+2cdx)^5}\,dx=\frac{(a+bx+cx^2)^2}{2(b^2-4ac)d^5(b+2cx)^4}$$

Mathematica [A] time = 0.0148447, size = 38, normalized size = 1.03

$$-\frac{4c(a+2cx^2)+b^2+8bcx}{32c^2d^5(b+2cx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(b*d + 2*c*d*x)^5,x]
```

```
[Out] -(b^2 + 8*b*c*x + 4*c*(a + 2*c*x^2))/(32*c^2*d^5*(b + 2*c*x)^4)
```

Maple [A] time = 0.046, size = 42, normalized size = 1.1

$$\frac{1}{d^5}\left(-\frac{1}{16c^2(2cx+b)^2}-\frac{4ac-b^2}{32c^2(2cx+b)^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(2*c*d*x+b*d)^5,x)
```

```
[Out] 1/d^5*(-1/16/c^2/(2*c*x+b)^2-1/32*(4*a*c-b^2)/c^2/(2*c*x+b)^4)
```

Maxima [B] time = 1.22513, size = 115, normalized size = 3.11

$$-\frac{8\,c^2x^2 + 8\,bcx + b^2 + 4\,ac}{32\left(16\,c^6d^5x^4 + 32\,bc^5d^5x^3 + 24\,b^2c^4d^5x^2 + 8\,b^3c^3d^5x + b^4c^2d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^5,x, algorithm="maxima")
```

```
[Out] -1/32*(8*c^2*x^2 + 8*b*c*x + b^2 + 4*a*c)/(16*c^6*d^5*x^4 + 32*b*c^5*d^5*x^
3 + 24*b^2*c^4*d^5*x^2 + 8*b^3*c^3*d^5*x + b^4*c^2*d^5)
```

Fricas [B] time = 2.4989, size = 178, normalized size = 4.81

$$-\frac{8\,c^2x^2 + 8\,bcx + b^2 + 4\,ac}{32\left(16\,c^6d^5x^4 + 32\,bc^5d^5x^3 + 24\,b^2c^4d^5x^2 + 8\,b^3c^3d^5x + b^4c^2d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^5,x, algorithm="fricas")
```

```
[Out] -1/32*(8*c^2*x^2 + 8*b*c*x + b^2 + 4*a*c)/(16*c^6*d^5*x^4 + 32*b*c^5*d^5*x^
3 + 24*b^2*c^4*d^5*x^2 + 8*b^3*c^3*d^5*x + b^4*c^2*d^5)
```

Sympy [B] time = 0.869247, size = 90, normalized size = 2.43

$$-\frac{4ac + b^2 + 8bcx + 8c^2x^2}{32b^4c^2d^5 + 256b^3c^3d^5x + 768b^2c^4d^5x^2 + 1024bc^5d^5x^3 + 512c^6d^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(2*c*d*x+b*d)**5,x)
```

```
[Out] -(4*a*c + b**2 + 8*b*c*x + 8*c**2*x**2)/(32*b**4*c**2*d**5 + 256*b**3*c**3*
d**5*x + 768*b**2*c**4*d**5*x**2 + 1024*b*c**5*d**5*x**3 + 512*c**6*d**5*x*
*4)
```

Giac [A] time = 1.18428, size = 81, normalized size = 2.19

$$\frac{\frac{b^2}{(2\,cdx+bd)^4c^2} - \frac{4\,a}{(2\,cdx+bd)^4c} - \frac{2}{(2\,cdx+bd)^2c^2d^2}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^5,x, algorithm="giac")
```

```
[Out] 1/32*(b^2/((2*c*d*x + b*d)^4*c^2) - 4*a/((2*c*d*x + b*d)^4*c) - 2/((2*c*d*x
 + b*d)^2*c^2*d^2))/d
```

3.1118 $\int \frac{a+bx+cx^2}{(bd+2cdx)^6} dx$

Optimal. Leaf size=45

$$\frac{b^2-4ac}{40c^2d^6(b+2cx)^5}-\frac{1}{24c^2d^6(b+2cx)^3}$$

```
[Out] (b^2 - 4*a*c)/(40*c^2*d^6*(b + 2*c*x)^5) - 1/(24*c^2*d^6*(b + 2*c*x)^3)
```

Rubi [A] time = 0.0328772, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {683}

$$\frac{b^2-4ac}{40c^2d^6(b+2cx)^5}-\frac{1}{24c^2d^6(b+2cx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(b*d + 2*c*d*x)^6,x]
```

```
[Out] (b^2 - 4*a*c)/(40*c^2*d^6*(b + 2*c*x)^5) - 1/(24*c^2*d^6*(b + 2*c*x)^3)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] && !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{(bd+2cdx)^6}\,dx&=\int \left(\frac{-b^2+4ac}{4cd^6(b+2cx)^6}+\frac{1}{4cd^6(b+2cx)^4}\right)dx\\&=\frac{b^2-4ac}{40c^2d^6(b+2cx)^5}-\frac{1}{24c^2d^6(b+2cx)^3}\end{aligned}$$

Mathematica [A] time = 0.0167315, size = 43, normalized size = 0.96

$$\frac{\frac{b^2-4ac}{40c^2(b+2cx)^5}-\frac{1}{24c^2(b+2cx)^3}}{d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(b*d + 2*c*d*x)^6,x]
```

```
[Out] ((b^2 - 4*a*c)/(40*c^2*(b + 2*c*x)^5) - 1/(24*c^2*(b + 2*c*x)^3))/d^6
```

Maple [A] time = 0.044, size = 42, normalized size = 0.9

$$\frac{1}{d^6}\left(-\frac{1}{24\,c^2\,(2\,cx+b)^3}-\frac{4\,ac-b^2}{40\,c^2\,(2\,cx+b)^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(2*c*d*x+b*d)^6,x)
```

```
[Out] 1/d^6*(-1/24/c^2/(2*c*x+b)^3-1/40*(4*a*c-b^2)/c^2/(2*c*x+b)^5)
```

Maxima [B] time = 1.45589, size = 134, normalized size = 2.98

$$-\frac{10\,c^2x^2+10\,bcx+b^2+6\,ac}{60\,\left(32\,c^7d^6x^5+80\,bc^6d^6x^4+80\,b^2c^5d^6x^3+40\,b^3c^4d^6x^2+10\,b^4c^3d^6x+b^5c^2d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^6,x, algorithm="maxima")
```

```
[Out] -1/60*(10*c^2*x^2 + 10*b*c*x + b^2 + 6*a*c)/(32*c^7*d^6*x^5 + 80*b*c^6*d^6*
x^4 + 80*b^2*c^5*d^6*x^3 + 40*b^3*c^4*d^6*x^2 + 10*b^4*c^3*d^6*x + b^5*c^2*
d^6)
```

Fricas [B] time = 2.47657, size = 211, normalized size = 4.69

$$-\frac{10\,c^2x^2+10\,bcx+b^2+6\,ac}{60\,\left(32\,c^7d^6x^5+80\,bc^6d^6x^4+80\,b^2c^5d^6x^3+40\,b^3c^4d^6x^2+10\,b^4c^3d^6x+b^5c^2d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^6,x, algorithm="fricas")
```

```
[Out] -1/60*(10*c^2*x^2 + 10*b*c*x + b^2 + 6*a*c)/(32*c^7*d^6*x^5 + 80*b*c^6*d^6*
x^4 + 80*b^2*c^5*d^6*x^3 + 40*b^3*c^4*d^6*x^2 + 10*b^4*c^3*d^6*x + b^5*c^2*
d^6)
```

Sympy [B] time = 1.14445, size = 105, normalized size = 2.33

$$-\frac{6ac+b^2+10bcx+10c^2x^2}{60b^5c^2d^6+600b^4c^3d^6x+2400b^3c^4d^6x^2+4800b^2c^5d^6x^3+4800bc^6d^6x^4+1920c^7d^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(2*c*d*x+b*d)**6,x)
```

```
[Out] -(6*a*c + b**2 + 10*b*c*x + 10*c**2*x**2)/(60*b**5*c**2*d**6 + 600*b**4*c**
3*d**6*x + 2400*b**3*c**4*d**6*x**2 + 4800*b**2*c**5*d**6*x**3 + 4800*b*c**
6*d**6*x**4 + 1920*c**7*d**6*x**5)
```

Giac [A] time = 1.1946, size = 50, normalized size = 1.11

$$-\frac{10\,c^2x^2+10\,bcx+b^2+6\,ac}{60\,(2\,cx+b)^5c^2d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^6,x, algorithm="giac")
```

```
[Out] -1/60*(10*c^2*x^2 + 10*b*c*x + b^2 + 6*a*c)/((2*c*x + b)^5*c^2*d^6)
```

3.1119 $\int \frac{a+bx+cx^2}{(bd+2cdx)^7} dx$

Optimal. Leaf size=45

$$\frac{b^2-4ac}{48c^2d^7(b+2cx)^6}-\frac{1}{32c^2d^7(b+2cx)^4}$$

```
[Out] (b^2 - 4*a*c)/(48*c^2*d^7*(b + 2*c*x)^6) - 1/(32*c^2*d^7*(b + 2*c*x)^4)
```

Rubi [A] time = 0.0325614, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {683}

$$\frac{b^2-4ac}{48c^2d^7(b+2cx)^6}-\frac{1}{32c^2d^7(b+2cx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(b*d + 2*c*d*x)^7,x]
```

```
[Out] (b^2 - 4*a*c)/(48*c^2*d^7*(b + 2*c*x)^6) - 1/(32*c^2*d^7*(b + 2*c*x)^4)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{(bd+2cdx)^7}\,dx &= \int \left(\frac{-b^2+4ac}{4cd^7(b+2cx)^7}+\frac{1}{4cd^7(b+2cx)^5}\right)dx\\ &= \frac{b^2-4ac}{48c^2d^7(b+2cx)^6}-\frac{1}{32c^2d^7(b+2cx)^4}\end{aligned}$$

Mathematica [A] time = 0.0138337, size = 43, normalized size = 0.96

$$\frac{\frac{b^2-4ac}{48c^2(b+2cx)^6}-\frac{1}{32c^2(b+2cx)^4}}{d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(b*d + 2*c*d*x)^7,x]
```

```
[Out] ((b^2 - 4*a*c)/(48*c^2*(b + 2*c*x)^6) - 1/(32*c^2*(b + 2*c*x)^4))/d^7
```

Maple [A] time = 0.045, size = 42, normalized size = 0.9

$$\frac{1}{d^7}\left(-\frac{4\,ac-b^2}{48\,c^2\,(2\,cx+b)^6}-\frac{1}{32\,c^2\,(2\,cx+b)^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(2*c*d*x+b*d)^7,x)
```

```
[Out] 1/d^7*(-1/48*(4*a*c-b^2)/c^2/(2*c*x+b)^6-1/32/c^2/(2*c*x+b)^4)
```

Maxima [B] time = 1.14208, size = 153, normalized size = 3.4

$$-\frac{12\,c^2x^2+12\,bcx+b^2+8\,ac}{96\left(64\,c^8d^7x^6+192\,bc^7d^7x^5+240\,b^2c^6d^7x^4+160\,b^3c^5d^7x^3+60\,b^4c^4d^7x^2+12\,b^5c^3d^7x+b^6c^2d^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^7,x, algorithm="maxima")
```

```
[Out] -1/96*(12*c^2*x^2 + 12*b*c*x + b^2 + 8*a*c)/(64*c^8*d^7*x^6 + 192*b*c^7*d^7
*x^5 + 240*b^2*c^6*d^7*x^4 + 160*b^3*c^5*d^7*x^3 + 60*b^4*c^4*d^7*x^2 + 12*
b^5*c^3*d^7*x + b^6*c^2*d^7)
```

Fricas [B] time = 2.36044, size = 243, normalized size = 5.4

$$-\frac{12\,c^2x^2+12\,bcx+b^2+8\,ac}{96\left(64\,c^8d^7x^6+192\,bc^7d^7x^5+240\,b^2c^6d^7x^4+160\,b^3c^5d^7x^3+60\,b^4c^4d^7x^2+12\,b^5c^3d^7x+b^6c^2d^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^7,x, algorithm="fricas")
```

```
[Out] -1/96*(12*c^2*x^2 + 12*b*c*x + b^2 + 8*a*c)/(64*c^8*d^7*x^6 + 192*b*c^7*d^7
*x^5 + 240*b^2*c^6*d^7*x^4 + 160*b^3*c^5*d^7*x^3 + 60*b^4*c^4*d^7*x^2 + 12*
b^5*c^3*d^7*x + b^6*c^2*d^7)
```

Sympy [B] time = 1.48938, size = 121, normalized size = 2.69

$$-\frac{8ac+b^2+12bcx+12c^2x^2}{96b^6c^2d^7+1152b^5c^3d^7x+5760b^4c^4d^7x^2+15360b^3c^5d^7x^3+23040b^2c^6d^7x^4+18432bc^7d^7x^5+6144c^8d^7x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(2*c*d*x+b*d)**7,x)
```

```
[Out] -(8*a*c + b**2 + 12*b*c*x + 12*c**2*x**2)/(96*b**6*c**2*d**7 + 1152*b**5*c*
*3*d**7*x + 5760*b**4*c**4*d**7*x**2 + 15360*b**3*c**5*d**7*x**3 + 23040*b*
*2*c**6*d**7*x**4 + 18432*b*c**7*d**7*x**5 + 6144*c**8*d**7*x**6)
```

Giac [A] time = 1.20721, size = 50, normalized size = 1.11

$$-\frac{12\,c^2x^2+12\,bcx+b^2+8\,ac}{96\,(2\,cx+b)^6c^2d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^7,x, algorithm="giac")
```

```
[Out] -1/96*(12*c^2*x^2 + 12*b*c*x + b^2 + 8*a*c)/((2*c*x + b)^6*c^2*d^7)
```

3.1120 $\int \frac{a+bx+cx^2}{(bd+2cdx)^8} dx$

Optimal. Leaf size=45

$$\frac{b^2-4ac}{56c^2d^8(b+2cx)^7}-\frac{1}{40c^2d^8(b+2cx)^5}$$

```
[Out] (b^2 - 4*a*c)/(56*c^2*d^8*(b + 2*c*x)^7) - 1/(40*c^2*d^8*(b + 2*c*x)^5)
```

Rubi [A] time = 0.0322843, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {683}

$$\frac{b^2-4ac}{56c^2d^8(b+2cx)^7}-\frac{1}{40c^2d^8(b+2cx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(b*d + 2*c*d*x)^8,x]
```

```
[Out] (b^2 - 4*a*c)/(56*c^2*d^8*(b + 2*c*x)^7) - 1/(40*c^2*d^8*(b + 2*c*x)^5)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] && !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{(bd+2cdx)^8}\,dx &= \int \left(\frac{-b^2+4ac}{4cd^8(b+2cx)^8}+\frac{1}{4cd^8(b+2cx)^6}\right)dx\\ &= \frac{b^2-4ac}{56c^2d^8(b+2cx)^7}-\frac{1}{40c^2d^8(b+2cx)^5}\end{aligned}$$

Mathematica [A] time = 0.0149663, size = 39, normalized size = 0.87

$$\frac{5\left(b^2-4ac\right)-7(b+2cx)^2}{280c^2d^8(b+2cx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(b*d + 2*c*d*x)^8,x]
```

```
[Out] (5*(b^2 - 4*a*c) - 7*(b + 2*c*x)^2)/(280*c^2*d^8*(b + 2*c*x)^7)
```

Maple [A] time = 0.043, size = 42, normalized size = 0.9

$$\frac{1}{d^8}\left(-\frac{4\,ac-b^2}{56\,c^2\,(2\,cx+b)^7}-\frac{1}{40\,c^2\,(2\,cx+b)^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(2*c*d*x+b*d)^8,x)
```

```
[Out] 1/d^8*(-1/56*(4*a*c-b^2)/c^2/(2*c*x+b)^7-1/40/c^2/(2*c*x+b)^5)
```

Maxima [B] time = 1.15251, size = 171, normalized size = 3.8

$$-\frac{14\,c^2x^2 + 14\,bcx + b^2 + 10\,ac}{140\left(128\,c^9d^8x^7 + 448\,bc^8d^8x^6 + 672\,b^2c^7d^8x^5 + 560\,b^3c^6d^8x^4 + 280\,b^4c^5d^8x^3 + 84\,b^5c^4d^8x^2 + 14\,b^6c^3d^8x + b^7c^2d^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^8,x, algorithm="maxima")
```

```
[Out] -1/140*(14*c^2*x^2 + 14*b*c*x + b^2 + 10*a*c)/(128*c^9*d^8*x^7 + 448*b*c^8*
d^8*x^6 + 672*b^2*c^7*d^8*x^5 + 560*b^3*c^6*d^8*x^4 + 280*b^4*c^5*d^8*x^3 +
 84*b^5*c^4*d^8*x^2 + 14*b^6*c^3*d^8*x + b^7*c^2*d^8)
```

Fricas [B] time = 2.09167, size = 277, normalized size = 6.16

$$-\frac{14\,c^2x^2 + 14\,bcx + b^2 + 10\,ac}{140\left(128\,c^9d^8x^7 + 448\,bc^8d^8x^6 + 672\,b^2c^7d^8x^5 + 560\,b^3c^6d^8x^4 + 280\,b^4c^5d^8x^3 + 84\,b^5c^4d^8x^2 + 14\,b^6c^3d^8x + b^7c^2d^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^8,x, algorithm="fricas")
```

```
[Out] -1/140*(14*c^2*x^2 + 14*b*c*x + b^2 + 10*a*c)/(128*c^9*d^8*x^7 + 448*b*c^8*
d^8*x^6 + 672*b^2*c^7*d^8*x^5 + 560*b^3*c^6*d^8*x^4 + 280*b^4*c^5*d^8*x^3 +
 84*b^5*c^4*d^8*x^2 + 14*b^6*c^3*d^8*x + b^7*c^2*d^8)
```

Sympy [B] time = 1.40456, size = 136, normalized size = 3.02

$$-\frac{10ac + b^2 + 14bcx + 14c^2x^2}{140b^7c^2d^8 + 1960b^6c^3d^8x + 11760b^5c^4d^8x^2 + 39200b^4c^5d^8x^3 + 78400b^3c^6d^8x^4 + 94080b^2c^7d^8x^5 + 62720bc^8d^8x^6 + 179}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(2*c*d*x+b*d)**8,x)
```

```
[Out] -(10*a*c + b**2 + 14*b*c*x + 14*c**2*x**2)/(140*b**7*c**2*d**8 + 1960*b**6*
c**3*d**8*x + 11760*b**5*c**4*d**8*x**2 + 39200*b**4*c**5*d**8*x**3 + 78400
*b**3*c**6*d**8*x**4 + 94080*b**2*c**7*d**8*x**5 + 62720*b*c**8*d**8*x**6 +
 17920*c**9*d**8*x**7)
```

Giac [A] time = 1.20759, size = 50, normalized size = 1.11

$$-\frac{14\,c^2x^2 + 14\,bcx + b^2 + 10\,ac}{140\,(2\,cx + b)^7\,c^2d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^8,x, algorithm="giac")
```

```
[Out] -1/140*(14*c^2*x^2 + 14*b*c*x + b^2 + 10*a*c)/((2*c*x + b)^7*c^2*d^8)
```

3.1121 $\int (bd+2cdx)^5 \left(a+bx+cx^2\right)^2 \, dx$

Optimal. Leaf size=73

$$-\frac{d^5 \left(b^2-4 a c\right) (b+2 c x)^8}{128 c^3}+\frac{d^5 \left(b^2-4 a c\right)^2 (b+2 c x)^6}{192 c^3}+\frac{d^5 (b+2 c x)^{10}}{320 c^3}$$

```
[Out] ((b^2 - 4*a*c)^2*d^5*(b + 2*c*x)^6)/(192*c^3) - ((b^2 - 4*a*c)*d^5*(b + 2*c
*x)^8)/(128*c^3) + (d^5*(b + 2*c*x)^10)/(320*c^3)
```

Rubi [A] time = 0.169635, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$-\frac{d^5 \left(b^2-4 a c\right) (b+2 c x)^8}{128 c^3}+\frac{d^5 \left(b^2-4 a c\right)^2 (b+2 c x)^6}{192 c^3}+\frac{d^5 (b+2 c x)^{10}}{320 c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^5*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((b^2 - 4*a*c)^2*d^5*(b + 2*c*x)^6)/(192*c^3) - ((b^2 - 4*a*c)*d^5*(b + 2*c
*x)^8)/(128*c^3) + (d^5*(b + 2*c*x)^10)/(320*c^3)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int (bd+2cdx)^5 \left(a+bx+cx^2\right)^2 \, dx &= \int \left(\frac{\left(-b^2+4ac\right)^2 (bd+2cdx)^5}{16c^2}+\frac{\left(-b^2+4ac\right) (bd+2cdx)^7}{8c^2d^2}+\frac{(bd+2cdx)^9}{16c^2d^4}\right) dx\\ &= \frac{\left(b^2-4ac\right)^2 d^5 (b+2cx)^6}{192c^3}-\frac{\left(b^2-4ac\right) d^5 (b+2cx)^8}{128c^3}+\frac{d^5 (b+2cx)^{10}}{320c^3}\end{aligned}$$

Mathematica [B] time = 0.0598018, size = 168, normalized size = 2.3

$$\frac{1}{15} d^5 x (b+cx) \left(5a^2 \left(28b^2c^2x^2+12b^3cx+3b^4+32bc^3x^3+16c^4x^4\right)+5ax \left(56b^3c^2x^2+88b^2c^3x^3+19b^4cx+3b^5+72bc^4x^4\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^5*(a + b*x + c*x^2)^2,x]
```

```
[Out] (d^5*x*(b + c*x)*(5*a^2*(3*b^4 + 12*b^3*c*x + 28*b^2*c^2*x^2 + 32*b*c^3*x^3
 + 16*c^4*x^4) + x^2*(b + c*x)^2*(5*b^4 + 30*b^3*c*x + 78*b^2*c^2*x^2 + 96*
b*c^3*x^3 + 48*c^4*x^4) + 5*a*x*(3*b^5 + 19*b^4*c*x + 56*b^3*c^2*x^2 + 88*b
```

```
^2*c^3*x^3 + 72*b*c^4*x^4 + 24*c^5*x^5)))/15
```

Maple [B] time = 0.059, size = 362, normalized size = 5.

$$\frac{16 c^7 d^5 x^{10}}{5} + 16 b d^5 c^6 x^9 + \frac{\left(240 b^2 d^5 c^5 + 32 c^5 d^5 \left(2 ac + b^2\right)\right) x^8}{8} + \frac{\left(200 b^3 d^5 c^4 + 80 b d^5 c^4 \left(2 ac + b^2\right) + 64 c^5 d^5 ab\right) x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^5*(c*x^2+b*x+a)^2,x)
```

```
[Out] 16/5*c^7*d^5*x^10+16*b*d^5*c^6*x^9+1/8*(240*b^2*d^5*c^5+32*c^5*d^5*(2*a*c+b
^2))*x^8+1/7*(200*b^3*d^5*c^4+80*b*d^5*c^4*(2*a*c+b^2)+64*c^5*d^5*a*b)*x^7+
1/6*(90*b^4*d^5*c^3+80*b^2*d^5*c^3*(2*a*c+b^2)+160*b^2*d^5*c^4*a+32*c^5*d^5
*a^2)*x^6+1/5*(21*b^5*d^5*c^2+40*b^3*d^5*c^2*(2*a*c+b^2)+160*b^3*d^5*c^3*a+
80*b*d^5*c^4*a^2)*x^5+1/4*(2*b^6*d^5*c+10*b^4*d^5*c*(2*a*c+b^2)+80*b^4*d^5*
c^2*a+80*b^2*d^5*c^3*a^2)*x^4+1/3*(b^5*d^5*(2*a*c+b^2)+20*b^5*d^5*c*a+40*b^
3*d^5*c^2*a^2)*x^3+1/2*(10*a^2*b^4*c*d^5+2*a*b^6*d^5)*x^2+b^5*d^5*a^2*x
```

Maxima [B] time = 1.20255, size = 320, normalized size = 4.38

$$\frac{16}{5} c^7 d^5 x^{10} + 16 bc^6 d^5 x^9 + 2 \left(17 b^2 c^5 + 4 ac^6\right) d^5 x^8 + a^2 b^5 d^5 x + 8 \left(5 b^3 c^4 + 4 abc^5\right) d^5 x^7 + \frac{1}{3} \left(85 b^4 c^3 + 160 ab^2 c^4 + 16 a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^5*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 16/5*c^7*d^5*x^10 + 16*b*c^6*d^5*x^9 + 2*(17*b^2*c^5 + 4*a*c^6)*d^5*x^8 + a
^2*b^5*d^5*x + 8*(5*b^3*c^4 + 4*a*b*c^5)*d^5*x^7 + 1/3*(85*b^4*c^3 + 160*a*
b^2*c^4 + 16*a^2*c^5)*d^5*x^6 + 1/5*(61*b^5*c^2 + 240*a*b^3*c^3 + 80*a^2*b*
c^4)*d^5*x^5 + (3*b^6*c + 25*a*b^4*c^2 + 20*a^2*b^2*c^3)*d^5*x^4 + 1/3*(b^7
 + 22*a*b^5*c + 40*a^2*b^3*c^2)*d^5*x^3 + (a*b^6 + 5*a^2*b^4*c)*d^5*x^2
```

Fricas [B] time = 2.08502, size = 605, normalized size = 8.29

$$\frac{16}{5} x^{10} d^5 c^7 + 16 x^9 d^5 c^6 b + 34 x^8 d^5 c^5 b^2 + 8 x^8 d^5 c^6 a + 40 x^7 d^5 c^4 b^3 + 32 x^7 d^5 c^5 ba + \frac{85}{3} x^6 d^5 c^3 b^4 + \frac{160}{3} x^6 d^5 c^4 b^2 a + \frac{16}{3} x^6 d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^5*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 16/5*x^10*d^5*c^7 + 16*x^9*d^5*c^6*b + 34*x^8*d^5*c^5*b^2 + 8*x^8*d^5*c^6*a
 + 40*x^7*d^5*c^4*b^3 + 32*x^7*d^5*c^5*b*a + 85/3*x^6*d^5*c^3*b^4 + 160/3*x
^6*d^5*c^4*b^2*a + 16/3*x^6*d^5*c^5*a^2 + 61/5*x^5*d^5*c^2*b^5 + 48*x^5*d^5
*c^3*b^3*a + 16*x^5*d^5*c^4*b*a^2 + 3*x^4*d^5*c*b^6 + 25*x^4*d^5*c^2*b^4*a
+ 20*x^4*d^5*c^3*b^2*a^2 + 1/3*x^3*d^5*b^7 + 22/3*x^3*d^5*c*b^5*a + 40/3*x^
3*d^5*c^2*b^3*a^2 + x^2*d^5*b^6*a + 5*x^2*d^5*c*b^4*a^2 + x*d^5*b^5*a^2
```

Sympy [B] time = 0.136747, size = 291, normalized size = 3.99

$$a^2b^5d^5x + 16bc^6d^5x^9 + \frac{16c^7d^5x^{10}}{5} + x^8\left(8ac^6d^5 + 34b^2c^5d^5\right) + x^7\left(32abc^5d^5 + 40b^3c^4d^5\right) + x^6\left(\frac{16a^2c^5d^5}{3} + \frac{160ab^2c^4d^5}{3} +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**5*(c*x**2+b*x+a)**2,x)
```

```
[Out] a**2*b**5*d**5*x + 16*b*c**6*d**5*x**9 + 16*c**7*d**5*x**10/5 + x**8*(8*a*c
**6*d**5 + 34*b**2*c**5*d**5) + x**7*(32*a*b*c**5*d**5 + 40*b**3*c**4*d**5)
 + x**6*(16*a**2*c**5*d**5/3 + 160*a*b**2*c**4*d**5/3 + 85*b**4*c**3*d**5/3
) + x**5*(16*a**2*b*c**4*d**5 + 48*a*b**3*c**3*d**5 + 61*b**5*c**2*d**5/5)
+ x**4*(20*a**2*b**2*c**3*d**5 + 25*a*b**4*c**2*d**5 + 3*b**6*c*d**5) + x**
3*(40*a**2*b**3*c**2*d**5/3 + 22*a*b**5*c*d**5/3 + b**7*d**5/3) + x**2*(5*a
**2*b**4*c*d**5 + a*b**6*d**5)
```

Giac [B] time = 1.18512, size = 386, normalized size = 5.29

$$\frac{16}{5}c^7d^5x^{10} + 16\,bc^6d^5x^9 + 34\,b^2c^5d^5x^8 + 8\,ac^6d^5x^8 + 40\,b^3c^4d^5x^7 + 32\,abc^5d^5x^7 + \frac{85}{3}\,b^4c^3d^5x^6 + \frac{160}{3}\,ab^2c^4d^5x^6 + \frac{16}{3}\,a^2c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^5*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 16/5*c^7*d^5*x^10 + 16*b*c^6*d^5*x^9 + 34*b^2*c^5*d^5*x^8 + 8*a*c^6*d^5*x^8
 + 40*b^3*c^4*d^5*x^7 + 32*a*b*c^5*d^5*x^7 + 85/3*b^4*c^3*d^5*x^6 + 160/3*a
*b^2*c^4*d^5*x^6 + 16/3*a^2*c^5*d^5*x^6 + 61/5*b^5*c^2*d^5*x^5 + 48*a*b^3*c
^3*d^5*x^5 + 16*a^2*b*c^4*d^5*x^5 + 3*b^6*c*d^5*x^4 + 25*a*b^4*c^2*d^5*x^4
+ 20*a^2*b^2*c^3*d^5*x^4 + 1/3*b^7*d^5*x^3 + 22/3*a*b^5*c*d^5*x^3 + 40/3*a^
2*b^3*c^2*d^5*x^3 + a*b^6*d^5*x^2 + 5*a^2*b^4*c*d^5*x^2 + a^2*b^5*d^5*x
```

3.1122 $\int (bd+2cdx)^4 \left(a+bx+cx^2\right)^2 \, dx$

Optimal. Leaf size=73

$$-\frac{d^4 \left(b^2-4ac\right) (b+2cx)^7}{112c^3}+\frac{d^4 \left(b^2-4ac\right)^2 (b+2cx)^5}{160c^3}+\frac{d^4(b+2cx)^9}{288c^3}$$

```
[Out] ((b^2 - 4*a*c)^2*d^4*(b + 2*c*x)^5)/(160*c^3) - ((b^2 - 4*a*c)*d^4*(b + 2*c
*x)^7)/(112*c^3) + (d^4*(b + 2*c*x)^9)/(288*c^3)
```

Rubi [A] time = 0.124237, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$-\frac{d^4 \left(b^2-4ac\right) (b+2cx)^7}{112c^3}+\frac{d^4 \left(b^2-4ac\right)^2 (b+2cx)^5}{160c^3}+\frac{d^4(b+2cx)^9}{288c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^4*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((b^2 - 4*a*c)^2*d^4*(b + 2*c*x)^5)/(160*c^3) - ((b^2 - 4*a*c)*d^4*(b + 2*c
*x)^7)/(112*c^3) + (d^4*(b + 2*c*x)^9)/(288*c^3)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] && !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int (bd+2cdx)^4 \left(a+bx+cx^2\right)^2 \, dx &= \int \left(\frac{\left(-b^2+4ac\right)^2 (bd+2cdx)^4}{16c^2}+\frac{\left(-b^2+4ac\right) (bd+2cdx)^6}{8c^2d^2}+\frac{(bd+2cdx)^8}{16c^2d^4}\right) dx\\ &= \frac{\left(b^2-4ac\right)^2 d^4(b+2cx)^5}{160c^3}-\frac{\left(b^2-4ac\right) d^4(b+2cx)^7}{112c^3}+\frac{d^4(b+2cx)^9}{288c^3}\end{aligned}$$

Mathematica [B] time = 0.026301, size = 179, normalized size = 2.45

$$d^4 \left(\frac{1}{5}c^2x^5 \left(16a^2c^2+112ab^2c+41b^4\right)+\frac{1}{2}bcx^4 \left(16a^2c^2+32ab^2c+5b^4\right)+\frac{1}{3}b^2x^3 \left(24a^2c^2+18ab^2c+b^4\right)+a^2b^4x+\frac{8}{7}c^4$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^4*(a + b*x + c*x^2)^2,x]
```

```
[Out] d^4*(a^2*b^4*x + a*b^3*(b^2 + 4*a*c)*x^2 + (b^2*(b^4 + 18*a*b^2*c + 24*a^2*
c^2)*x^3)/3 + (b*c*(5*b^4 + 32*a*b^2*c + 16*a^2*c^2)*x^4)/2 + (c^2*(41*b^4
+ 112*a*b^2*c + 16*a^2*c^2)*x^5)/5 + (4*b*c^3*(11*b^2 + 12*a*c)*x^6)/3 + (8
```

```
*c^4*(13*b^2 + 4*a*c)*x^7)/7 + 8*b*c^5*x^8 + (16*c^6*x^9)/9)
```

Maple [B] time = 0.039, size = 300, normalized size = 4.1

$$\frac{16 c^6 d^4 x^9}{9} + 8 b d^4 c^5 x^8 + \frac{\left(88 b^2 d^4 c^4 + 16 c^4 d^4 \left(2 ac + b^2\right)\right) x^7}{7} + \frac{\left(56 b^3 d^4 c^3 + 32 b d^4 c^3 \left(2 ac + b^2\right) + 32 c^4 d^4 ab\right) x^6}{6} + \frac{(17 b^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^4*(c*x^2+b*x+a)^2,x)
```

```
[Out] 16/9*c^6*d^4*x^9+8*b*d^4*c^5*x^8+1/7*(88*b^2*d^4*c^4+16*c^4*d^4*(2*a*c+b^2)
)*x^7+1/6*(56*b^3*d^4*c^3+32*b*d^4*c^3*(2*a*c+b^2)+32*c^4*d^4*a*b)*x^6+1/5*
(17*b^4*d^4*c^2+24*b^2*d^4*c^2*(2*a*c+b^2)+64*b^2*d^4*c^3*a+16*c^4*d^4*a^2)
*x^5+1/4*(2*b^5*d^4*c+8*b^3*d^4*c*(2*a*c+b^2)+48*b^3*d^4*c^2*a+32*b*d^4*c^3
*a^2)*x^4+1/3*(b^4*d^4*(2*a*c+b^2)+16*b^4*d^4*c*a+24*b^2*d^4*c^2*a^2)*x^3+1
/2*(8*a^2*b^3*c*d^4+2*a*b^5*d^4)*x^2+b^4*d^4*a^2*x
```

Maxima [B] time = 1.20309, size = 271, normalized size = 3.71

$$\frac{16}{9} c^6 d^4 x^9 + 8 b c^5 d^4 x^8 + \frac{8}{7} \left(13 b^2 c^4 + 4 ac^5\right) d^4 x^7 + a^2 b^4 d^4 x + \frac{4}{3} \left(11 b^3 c^3 + 12 abc^4\right) d^4 x^6 + \frac{1}{5} \left(41 b^4 c^2 + 112 ab^2 c^3 + 16 a^2 c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 16/9*c^6*d^4*x^9 + 8*b*c^5*d^4*x^8 + 8/7*(13*b^2*c^4 + 4*a*c^5)*d^4*x^7 + a
^2*b^4*d^4*x + 4/3*(11*b^3*c^3 + 12*a*b*c^4)*d^4*x^6 + 1/5*(41*b^4*c^2 + 11
2*a*b^2*c^3 + 16*a^2*c^4)*d^4*x^5 + 1/2*(5*b^5*c + 32*a*b^3*c^2 + 16*a^2*b*
c^3)*d^4*x^4 + 1/3*(b^6 + 18*a*b^4*c + 24*a^2*b^2*c^2)*d^4*x^3 + (a*b^5 + 4
*a^2*b^3*c)*d^4*x^2
```

Fricas [B] time = 1.93283, size = 510, normalized size = 6.99

$$\frac{16}{9} x^9 d^4 c^6 + 8 x^8 d^4 c^5 b + \frac{104}{7} x^7 d^4 c^4 b^2 + \frac{32}{7} x^7 d^4 c^5 a + \frac{44}{3} x^6 d^4 c^3 b^3 + 16 x^6 d^4 c^4 ba + \frac{41}{5} x^5 d^4 c^2 b^4 + \frac{112}{5} x^5 d^4 c^3 b^2 a + \frac{16}{5} x^5 d^4 c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 16/9*x^9*d^4*c^6 + 8*x^8*d^4*c^5*b + 104/7*x^7*d^4*c^4*b^2 + 32/7*x^7*d^4*c
^5*a + 44/3*x^6*d^4*c^3*b^3 + 16*x^6*d^4*c^4*b*a + 41/5*x^5*d^4*c^2*b^4 + 1
12/5*x^5*d^4*c^3*b^2*a + 16/5*x^5*d^4*c^4*a^2 + 5/2*x^4*d^4*c*b^5 + 16*x^4*
d^4*c^2*b^3*a + 8*x^4*d^4*c^3*b*a^2 + 1/3*x^3*d^4*b^6 + 6*x^3*d^4*c*b^4*a +
 8*x^3*d^4*c^2*b^2*a^2 + x^2*d^4*b^5*a + 4*x^2*d^4*c*b^3*a^2 + x*d^4*b^4*a^
2
```

Sympy [B] time = 0.105083, size = 248, normalized size = 3.4

$$a^2b^4d^4x + 8bc^5d^4x^8 + \frac{16c^6d^4x^9}{9} + x^7\left(\frac{32ac^5d^4}{7} + \frac{104b^2c^4d^4}{7}\right) + x^6\left(16abc^4d^4 + \frac{44b^3c^3d^4}{3}\right) + x^5\left(\frac{16a^2c^4d^4}{5} + \frac{112ab^2c^3}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**4*(c*x**2+b*x+a)**2,x)
```

```
[Out] a**2*b**4*d**4*x + 8*b*c**5*d**4*x**8 + 16*c**6*d**4*x**9/9 + x**7*(32*a*c*
*5*d**4/7 + 104*b**2*c**4*d**4/7) + x**6*(16*a*b*c**4*d**4 + 44*b**3*c**3*d
**4/3) + x**5*(16*a**2*c**4*d**4/5 + 112*a*b**2*c**3*d**4/5 + 41*b**4*c**2*
d**4/5) + x**4*(8*a**2*b*c**3*d**4 + 16*a*b**3*c**2*d**4 + 5*b**5*c*d**4/2)
 + x**3*(8*a**2*b**2*c**2*d**4 + 6*a*b**4*c*d**4 + b**6*d**4/3) + x**2*(4*a
**2*b**3*c*d**4 + a*b**5*d**4)
```

Giac [B] time = 1.27151, size = 324, normalized size = 4.44

$$\frac{16}{9}c^6d^4x^9 + 8\,bc^5d^4x^8 + \frac{104}{7}b^2c^4d^4x^7 + \frac{32}{7}ac^5d^4x^7 + \frac{44}{3}b^3c^3d^4x^6 + 16\,abc^4d^4x^6 + \frac{41}{5}b^4c^2d^4x^5 + \frac{112}{5}ab^2c^3d^4x^5 + \frac{1}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 16/9*c^6*d^4*x^9 + 8*b*c^5*d^4*x^8 + 104/7*b^2*c^4*d^4*x^7 + 32/7*a*c^5*d^4
*x^7 + 44/3*b^3*c^3*d^4*x^6 + 16*a*b*c^4*d^4*x^6 + 41/5*b^4*c^2*d^4*x^5 + 1
12/5*a*b^2*c^3*d^4*x^5 + 16/5*a^2*c^4*d^4*x^5 + 5/2*b^5*c*d^4*x^4 + 16*a*b^
3*c^2*d^4*x^4 + 8*a^2*b*c^3*d^4*x^4 + 1/3*b^6*d^4*x^3 + 6*a*b^4*c*d^4*x^3 +
 8*a^2*b^2*c^2*d^4*x^3 + a*b^5*d^4*x^2 + 4*a^2*b^3*c*d^4*x^2 + a^2*b^4*d^4*
x
```

3.1123 $\int (bd+2cdx)^3 \left(a+bx+cx^2\right)^2 \, dx$

Optimal. Leaf size=55

$$\frac{1}{12}d^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3+\frac{1}{4}d^3(b+2cx)^2\left(a+bx+cx^2\right)^3$$

```
[Out] ((b^2 - 4*a*c)*d^3*(a + b*x + c*x^2)^3)/12 + (d^3*(b + 2*c*x)^2*(a + b*x +
c*x^2)^3)/4
```

Rubi [A] time = 0.0244556, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {692, 629}

$$\frac{1}{12}d^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3+\frac{1}{4}d^3(b+2cx)^2\left(a+bx+cx^2\right)^3$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^3*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((b^2 - 4*a*c)*d^3*(a + b*x + c*x^2)^3)/12 + (d^3*(b + 2*c*x)^2*(a + b*x +
c*x^2)^3)/4
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (bd+2cdx)^3\left(a+bx+cx^2\right)^2\,dx &= \frac{1}{4}d^3(b+2cx)^2\left(a+bx+cx^2\right)^3+\frac{1}{4}\left(\left(b^2-4ac\right)d^2\right)\int (bd+2cdx)\left(a+bx+cx^2\right)^2\,dx\\ &= \frac{1}{12}\left(b^2-4ac\right)d^3\left(a+bx+cx^2\right)^3+\frac{1}{4}d^3(b+2cx)^2\left(a+bx+cx^2\right)^3\end{aligned}$$

Mathematica [A] time = 0.0198803, size = 97, normalized size = 1.76

$$\frac{1}{3}d^3x(b+cx)\left(3a^2\left(b^2+2bcx+2c^2x^2\right)+ax\left(11b^2cx+3b^3+16bc^2x^2+8c^3x^3\right)+x^2(b+cx)^2\left(b^2+3bcx+3c^2x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^3*(a + b*x + c*x^2)^2,x]
```

```
[Out] (d^3*x*(b + c*x)*(3*a^2*(b^2 + 2*b*c*x + 2*c^2*x^2) + x^2*(b + c*x)^2*(b^2
+ 3*b*c*x + 3*c^2*x^2) + a*x*(3*b^3 + 11*b^2*c*x + 16*b*c^2*x^2 + 8*c^3*x^3
)))/3
```

Maple [B] time = 0.041, size = 237, normalized size = 4.3

$$c^5d^3x^8 + 4\,bd^3c^4x^7 + \frac{\left(30\,b^2d^3c^3 + 8\,c^3d^3\left(2\,ac + b^2\right)\right)x^6}{6} + \frac{\left(13\,b^3d^3c^2 + 12\,bd^3c^2\left(2\,ac + b^2\right) + 16\,c^3d^3ab\right)x^5}{5} + \frac{\left(2\,b^4d^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^3*(c*x^2+b*x+a)^2,x)
```

```
[Out] c^5*d^3*x^8+4*b*d^3*c^4*x^7+1/6*(30*b^2*d^3*c^3+8*c^3*d^3*(2*a*c+b^2))*x^6+
1/5*(13*b^3*d^3*c^2+12*b*d^3*c^2*(2*a*c+b^2)+16*c^3*d^3*a*b)*x^5+1/4*(2*b^4
*d^3*c+6*b^2*d^3*c*(2*a*c+b^2)+24*b^2*d^3*c^2*a+8*c^3*d^3*a^2)*x^4+1/3*(b^3
*d^3*(2*a*c+b^2)+12*b^3*d^3*c*a+12*b*d^3*c^2*a^2)*x^3+1/2*(6*a^2*b^2*c*d^3+
2*a*b^4*d^3)*x^2+b^3*d^3*a^2*x
```

Maxima [B] time = 1.20458, size = 217, normalized size = 3.95

$$c^5d^3x^8 + 4\,bc^4d^3x^7 + \frac{1}{3}\left(19\,b^2c^3 + 8\,ac^4\right)d^3x^6 + a^2b^3d^3x + \left(5\,b^3c^2 + 8\,abc^3\right)d^3x^5 + \left(2\,b^4c + 9\,ab^2c^2 + 2\,a^2c^3\right)d^3x^4 + \frac{1}{3}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] c^5*d^3*x^8 + 4*b*c^4*d^3*x^7 + 1/3*(19*b^2*c^3 + 8*a*c^4)*d^3*x^6 + a^2*b^
3*d^3*x + (5*b^3*c^2 + 8*a*b*c^3)*d^3*x^5 + (2*b^4*c + 9*a*b^2*c^2 + 2*a^2*
c^3)*d^3*x^4 + 1/3*(b^5 + 14*a*b^3*c + 12*a^2*b*c^2)*d^3*x^3 + (a*b^4 + 3*a
^2*b^2*c)*d^3*x^2
```

Fricas [B] time = 1.72548, size = 393, normalized size = 7.15

$$x^8d^3c^5 + 4x^7d^3c^4b + \frac{19}{3}x^6d^3c^3b^2 + \frac{8}{3}x^6d^3c^4a + 5x^5d^3c^2b^3 + 8x^5d^3c^3ba + 2x^4d^3cb^4 + 9x^4d^3c^2b^2a + 2x^4d^3c^3a^2 + \frac{1}{3}x^3d^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] x^8*d^3*c^5 + 4*x^7*d^3*c^4*b + 19/3*x^6*d^3*c^3*b^2 + 8/3*x^6*d^3*c^4*a +
5*x^5*d^3*c^2*b^3 + 8*x^5*d^3*c^3*b*a + 2*x^4*d^3*c*b^4 + 9*x^4*d^3*c^2*b^2
*a + 2*x^4*d^3*c^3*a^2 + 1/3*x^3*d^3*b^5 + 14/3*x^3*d^3*c*b^3*a + 4*x^3*d^3
*c^2*b*a^2 + x^2*d^3*b^4*a + 3*x^2*d^3*c*b^2*a^2 + x*d^3*b^3*a^2
```

Sympy [B] time = 0.100154, size = 194, normalized size = 3.53

$$a^2b^3d^3x + 4bc^4d^3x^7 + c^5d^3x^8 + x^6\left(\frac{8ac^4d^3}{3} + \frac{19b^2c^3d^3}{3}\right) + x^5\left(8abc^3d^3 + 5b^3c^2d^3\right) + x^4\left(2a^2c^3d^3 + 9ab^2c^2d^3 + 2b^4cd^3\right) + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**3*(c*x**2+b*x+a)**2,x)
```

```
[Out] a**2*b**3*d**3*x + 4*b*c**4*d**3*x**7 + c**5*d**3*x**8 + x**6*(8*a*c**4*d**
3/3 + 19*b**2*c**3*d**3/3) + x**5*(8*a*b*c**3*d**3 + 5*b**3*c**2*d**3) + x*
*4*(2*a**2*c**3*d**3 + 9*a*b**2*c**2*d**3 + 2*b**4*c*d**3) + x**3*(4*a**2*b
*c**2*d**3 + 14*a*b**3*c*d**3/3 + b**5*d**3/3) + x**2*(3*a**2*b**2*c*d**3 +
 a*b**4*d**3)
```

Giac [B] time = 1.21721, size = 261, normalized size = 4.75

$$c^5d^3x^8 + 4\,bc^4d^3x^7 + \frac{19}{3}\,b^2c^3d^3x^6 + \frac{8}{3}\,ac^4d^3x^6 + 5\,b^3c^2d^3x^5 + 8\,abc^3d^3x^5 + 2\,b^4cd^3x^4 + 9\,ab^2c^2d^3x^4 + 2\,a^2c^3d^3x^4 + \frac{1}{3}\,b^5d^3 \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] c^5*d^3*x^8 + 4*b*c^4*d^3*x^7 + 19/3*b^2*c^3*d^3*x^6 + 8/3*a*c^4*d^3*x^6 +
5*b^3*c^2*d^3*x^5 + 8*a*b*c^3*d^3*x^5 + 2*b^4*c*d^3*x^4 + 9*a*b^2*c^2*d^3*x
^4 + 2*a^2*c^3*d^3*x^4 + 1/3*b^5*d^3*x^3 + 14/3*a*b^3*c*d^3*x^3 + 4*a^2*b*c
^2*d^3*x^3 + a*b^4*d^3*x^2 + 3*a^2*b^2*c*d^3*x^2 + a^2*b^3*d^3*x
```

3.1124 $\int (bd+2cdx)^2 \left(a+bx+cx^2\right)^2 \, dx$

Optimal. Leaf size=73

$$-\frac{d^2 \left(b^2-4ac\right) (b+2cx)^5}{80c^3}+\frac{d^2 \left(b^2-4ac\right)^2 (b+2cx)^3}{96c^3}+\frac{d^2(b+2cx)^7}{224c^3}$$

```
[Out] ((b^2 - 4*a*c)^2*d^2*(b + 2*c*x)^3)/(96*c^3) - ((b^2 - 4*a*c)*d^2*(b + 2*c*
x)^5)/(80*c^3) + (d^2*(b + 2*c*x)^7)/(224*c^3)
```

Rubi [A] time = 0.0843479, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$-\frac{d^2 \left(b^2-4ac\right) (b+2cx)^5}{80c^3}+\frac{d^2 \left(b^2-4ac\right)^2 (b+2cx)^3}{96c^3}+\frac{d^2(b+2cx)^7}{224c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^2*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((b^2 - 4*a*c)^2*d^2*(b + 2*c*x)^3)/(96*c^3) - ((b^2 - 4*a*c)*d^2*(b + 2*c*
x)^5)/(80*c^3) + (d^2*(b + 2*c*x)^7)/(224*c^3)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int (bd+2cdx)^2 \left(a+bx+cx^2\right)^2 \, dx &= \int \left(\frac{\left(-b^2+4ac\right)^2 (bd+2cdx)^2}{16c^2}+\frac{\left(-b^2+4ac\right) (bd+2cdx)^4}{8c^2d^2}+\frac{(bd+2cdx)^6}{16c^2d^4}\right) dx\\ &= \frac{\left(b^2-4ac\right)^2 d^2(b+2cx)^3}{96c^3}-\frac{\left(b^2-4ac\right) d^2(b+2cx)^5}{80c^3}+\frac{d^2(b+2cx)^7}{224c^3}\end{aligned}$$

Mathematica [A] time = 0.0183955, size = 111, normalized size = 1.52

$$d^2 \left(\frac{1}{3}x^3 \left(4a^2c^2+10ab^2c+b^4\right)+a^2b^2x+\frac{1}{5}c^2x^5 \left(8ac+13b^2\right)+\frac{1}{2}bcx^4 \left(8ac+3b^2\right)+abx^2 \left(2ac+b^2\right)+2bc^3x^6+\frac{4c^4x^7}{7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^2*(a + b*x + c*x^2)^2,x]
```

```
[Out] d^2*(a^2*b^2*x + a*b*(b^2 + 2*a*c)*x^2 + ((b^4 + 10*a*b^2*c + 4*a^2*c^2)*x^
3)/3 + (b*c*(3*b^2 + 8*a*c)*x^4)/2 + (c^2*(13*b^2 + 8*a*c)*x^5)/5 + 2*b*c^3
*x^6 + (4*c^4*x^7)/7)
```

Maple [B] time = 0.039, size = 176, normalized size = 2.4

$$\frac{4\,c^4d^2x^7}{7}+2\,bd^2c^3x^6+\frac{\left(9\,b^2d^2c^2+4\,c^2d^2\left(2\,ac+b^2\right)\right)x^5}{5}+\frac{\left(2\,b^3d^2c+4\,bd^2c\left(2\,ac+b^2\right)+8\,c^2d^2ab\right)x^4}{4}+\frac{\left(b^2d^2\left(2\,ac+b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^2*(c*x^2+b*x+a)^2,x)
```

```
[Out] 4/7*c^4*d^2*x^7+2*b*d^2*c^3*x^6+1/5*(9*b^2*d^2*c^2+4*c^2*d^2*(2*a*c+b^2))*x
^5+1/4*(2*b^3*d^2*c+4*b*d^2*c*(2*a*c+b^2)+8*c^2*d^2*a*b)*x^4+1/3*(b^2*d^2*(
2*a*c+b^2)+8*b^2*d^2*c*a+4*c^2*d^2*a^2)*x^3+1/2*(4*a^2*b*c*d^2+2*a*b^3*d^2)
*x^2+b^2*d^2*a^2*x
```

Maxima [A] time = 1.14253, size = 171, normalized size = 2.34

$$\frac{4}{7}\,c^4d^2x^7+2\,bc^3d^2x^6+\frac{1}{5}\left(13\,b^2c^2+8\,ac^3\right)d^2x^5+a^2b^2d^2x+\frac{1}{2}\left(3\,b^3c+8\,abc^2\right)d^2x^4+\frac{1}{3}\left(b^4+10\,ab^2c+4\,a^2c^2\right)d^2x^3+\left(ab\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 4/7*c^4*d^2*x^7 + 2*b*c^3*d^2*x^6 + 1/5*(13*b^2*c^2 + 8*a*c^3)*d^2*x^5 + a^
2*b^2*d^2*x + 1/2*(3*b^3*c + 8*a*b*c^2)*d^2*x^4 + 1/3*(b^4 + 10*a*b^2*c + 4
*a^2*c^2)*d^2*x^3 + (a*b^3 + 2*a^2*b*c)*d^2*x^2
```

Fricas [B] time = 1.67429, size = 315, normalized size = 4.32

$$\frac{4}{7}x^7d^2c^4+2x^6d^2c^3b+\frac{13}{5}x^5d^2c^2b^2+\frac{8}{5}x^5d^2c^3a+\frac{3}{2}x^4d^2cb^3+4x^4d^2c^2ba+\frac{1}{3}x^3d^2b^4+\frac{10}{3}x^3d^2cb^2a+\frac{4}{3}x^3d^2c^2a^2+x^2d^2b^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 4/7*x^7*d^2*c^4 + 2*x^6*d^2*c^3*b + 13/5*x^5*d^2*c^2*b^2 + 8/5*x^5*d^2*c^3*
a + 3/2*x^4*d^2*c*b^3 + 4*x^4*d^2*c^2*b*a + 1/3*x^3*d^2*b^4 + 10/3*x^3*d^2*
c*b^2*a + 4/3*x^3*d^2*c^2*a^2 + x^2*d^2*b^3*a + 2*x^2*d^2*c*b*a^2 + x*d^2*b
^2*a^2
```

Sympy [B] time = 0.099276, size = 156, normalized size = 2.14

$$a^2b^2d^2x+2bc^3d^2x^6+\frac{4c^4d^2x^7}{7}+x^5\left(\frac{8ac^3d^2}{5}+\frac{13b^2c^2d^2}{5}\right)+x^4\left(4abc^2d^2+\frac{3b^3cd^2}{2}\right)+x^3\left(\frac{4a^2c^2d^2}{3}+\frac{10ab^2cd^2}{3}+\frac{b^4d^2}{3}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**2*(c*x**2+b*x+a)**2,x)
```

```
[Out] a**2*b**2*d**2*x + 2*b*c**3*d**2*x**6 + 4*c**4*d**2*x**7/7 + x**5*(8*a*c**3
*d**2/5 + 13*b**2*c**2*d**2/5) + x**4*(4*a*b*c**2*d**2 + 3*b**3*c*d**2/2) +
 x**3*(4*a**2*c**2*d**2/3 + 10*a*b**2*c*d**2/3 + b**4*d**2/3) + x**2*(2*a**
2*b*c*d**2 + a*b**3*d**2)
```

Giac [B] time = 1.18166, size = 200, normalized size = 2.74

$$\frac{4}{7}c^4d^2x^7 + 2\,bc^3d^2x^6 + \frac{13}{5}b^2c^2d^2x^5 + \frac{8}{5}ac^3d^2x^5 + \frac{3}{2}b^3cd^2x^4 + 4\,abc^2d^2x^4 + \frac{1}{3}b^4d^2x^3 + \frac{10}{3}ab^2cd^2x^3 + \frac{4}{3}a^2c^2d^2x^3 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 4/7*c^4*d^2*x^7 + 2*b*c^3*d^2*x^6 + 13/5*b^2*c^2*d^2*x^5 + 8/5*a*c^3*d^2*x^
5 + 3/2*b^3*c*d^2*x^4 + 4*a*b*c^2*d^2*x^4 + 1/3*b^4*d^2*x^3 + 10/3*a*b^2*c*
d^2*x^3 + 4/3*a^2*c^2*d^2*x^3 + a*b^3*d^2*x^2 + 2*a^2*b*c*d^2*x^2 + a^2*b^2
*d^2*x
```

3.1125 $\int (bd+2cdx)\left(a+bx+cx^2\right)^2\,dx$

Optimal. Leaf size=17

$$\frac{1}{3}d\left(a+bx+cx^2\right)^3$$

[Out] (d*(a + b*x + c*x^2)^3)/3

Rubi [A] time = 0.0055592, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {629}

$$\frac{1}{3}d\left(a+bx+cx^2\right)^3$$

Antiderivative was successfully verified.

[In] Int[(b*d + 2*c*d*x)*(a + b*x + c*x^2)^2,x]

[Out] (d*(a + b*x + c*x^2)^3)/3

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (bd+2cdx)\left(a+bx+cx^2\right)^2\,dx=\frac{1}{3}d\left(a+bx+cx^2\right)^3$$

Mathematica [B] time = 0.0099525, size = 37, normalized size = 2.18

$$\frac{1}{3}dx(b+cx)\left(3a^2+3ax(b+cx)+x^2(b+cx)^2\right)$$

Antiderivative was successfully verified.

[In] Integrate[(b*d + 2*c*d*x)*(a + b*x + c*x^2)^2,x]

[Out] (d*x*(b + c*x)*(3*a^2 + 3*a*x*(b + c*x) + x^2*(b + c*x)^2))/3

Maple [B] time = 0.039, size = 95, normalized size = 5.6

$$\frac{c^3dx^6}{3}+bdc^2x^5+\frac{\left(2\,b^2dc+2\,cd\left(2\,ac+b^2\right)\right)x^4}{4}+\frac{\left(bd\left(2\,ac+b^2\right)+4\,cabd\right)x^3}{3}+\frac{\left(2\,cda^2+2\,ab^2d\right)x^2}{2}+bda^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)*(c*x^2+b*x+a)^2,x)
```

```
[Out] 1/3*c^3*d*x^6+b*d*c^2*x^5+1/4*(2*b^2*d*c+2*c*d*(2*a*c+b^2))*x^4+1/3*(b*d*(2
*a*c+b^2)+4*c*a*b*d)*x^3+1/2*(2*a^2*c*d+2*a*b^2*d)*x^2+b*d*a^2*x
```

Maxima [A] time = 1.16972, size = 20, normalized size = 1.18

$$\frac{1}{3}\left(cx^2+bx+a\right)^3 d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(c*x^2 + b*x + a)^3*d
```

Fricas [B] time = 1.84776, size = 174, normalized size = 10.24

$$\frac{1}{3}x^6dc^3+x^5dc^2b+x^4dcb^2+x^4dc^2a+\frac{1}{3}x^3db^3+2x^3dcba+x^2db^2a+x^2dca^2+xdba^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^6*d*c^3 + x^5*d*c^2*b + x^4*d*c*b^2 + x^4*d*c^2*a + 1/3*x^3*d*b^3 + 2
*x^3*d*c*b*a + x^2*d*b^2*a + x^2*d*c*a^2 + x*d*b*a^2
```

Sympy [B] time = 0.085105, size = 80, normalized size = 4.71

$$a^2bdx+bc^2dx^5+\frac{c^3dx^6}{3}+x^4\left(ac^2d+b^2cd\right)+x^3\left(2abcd+\frac{b^3d}{3}\right)+x^2\left(a^2cd+ab^2d\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)*(c*x**2+b*x+a)**2,x)
```

```
[Out] a**2*b*d*x + b*c**2*d*x**5 + c**3*d*x**6/3 + x**4*(a*c**2*d + b**2*c*d) + x
**3*(2*a*b*c*d + b**3*d/3) + x**2*(a**2*c*d + a*b**2*d)
```

Giac [B] time = 1.23593, size = 108, normalized size = 6.35

$$\frac{1}{3}c^3dx^6+bc^2dx^5+b^2cdx^4+ac^2dx^4+\frac{1}{3}b^3dx^3+2\,abcdx^3+ab^2dx^2+a^2cdx^2+a^2bdx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*c^3*d*x^6 + b*c^2*d*x^5 + b^2*c*d*x^4 + a*c^2*d*x^4 + 1/3*b^3*d*x^3 + 2
*a*b*c*d*x^3 + a*b^2*d*x^2 + a^2*c*d*x^2 + a^2*b*d*x
```

3.1126 $\int \frac{\left(a+bx+cx^2\right)^2}{bd+2cdx}\,dx$

Optimal. Leaf size=72

$$-\frac{\left(b^2-4ac\right)(b+2cx)^2}{32c^3d}+\frac{\left(b^2-4ac\right)^2\log(b+2cx)}{32c^3d}+\frac{(b+2cx)^4}{128c^3d}$$

```
[Out] -((b^2 - 4*a*c)*(b + 2*c*x)^2)/(32*c^3*d) + (b + 2*c*x)^4/(128*c^3*d) + ((b
^2 - 4*a*c)^2*Log[b + 2*c*x])/(32*c^3*d)
```

Rubi [A] time = 0.0722829, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$-\frac{\left(b^2-4ac\right)(b+2cx)^2}{32c^3d}+\frac{\left(b^2-4ac\right)^2\log(b+2cx)}{32c^3d}+\frac{(b+2cx)^4}{128c^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x),x]
```

```
[Out] -((b^2 - 4*a*c)*(b + 2*c*x)^2)/(32*c^3*d) + (b + 2*c*x)^4/(128*c^3*d) + ((b
^2 - 4*a*c)^2*Log[b + 2*c*x])/(32*c^3*d)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx+cx^2\right)^2}{bd+2cdx}\,dx &= \int \left(\frac{\left(-b^2+4ac\right)^2}{16c^2(bd+2cdx)}+\frac{\left(-b^2+4ac\right)(bd+2cdx)}{8c^2d^2}+\frac{(bd+2cdx)^3}{16c^2d^4}\right)dx\\ &= -\frac{\left(b^2-4ac\right)(b+2cx)^2}{32c^3d}+\frac{(b+2cx)^4}{128c^3d}+\frac{\left(b^2-4ac\right)^2\log(b+2cx)}{32c^3d}\end{aligned}$$

Mathematica [A] time = 0.0270897, size = 61, normalized size = 0.85

$$\frac{2cx(b+cx)\left(2c\left(4a+cx^2\right)-b^2+2bcx\right)+\left(b^2-4ac\right)^2\log(b+2cx)}{32c^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x),x]
```

```
[Out] (2*c*x*(b + c*x)*(-b^2 + 2*b*c*x + 2*c*(4*a + c*x^2)) + (b^2 - 4*a*c)^2*Log
[b + 2*c*x])/(32*c^3*d)
```

Maple [A] time = 0.043, size = 121, normalized size = 1.7

$$\frac{cx^4}{8d}+\frac{bx^3}{4d}+\frac{ax^2}{2d}+\frac{b^2x^2}{16cd}+\frac{abx}{2cd}-\frac{b^3x}{16c^2d}+\frac{\ln(2cx+b)a^2}{2cd}-\frac{\ln(2cx+b)ab^2}{4c^2d}+\frac{\ln(2cx+b)b^4}{32dc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(2*c*d*x+b*d),x)
```

```
[Out] 1/8*c/d*x^4+1/4/d*b*x^3+1/2/d*a*x^2+1/16/d/c*x^2*b^2+1/2/d/c*a*b*x-1/16/d/c
^2*b^3*x+1/2/d/c*ln(2*c*x+b)*a^2-1/4/d/c^2*ln(2*c*x+b)*a*b^2+1/32/d/c^3*ln(
2*c*x+b)*b^4
```

Maxima [A] time = 1.22093, size = 120, normalized size = 1.67

$$\frac{2\,c^3x^4+4\,bc^2x^3+\left(b^2c+8\,ac^2\right)x^2-\left(b^3-8\,abc\right)x}{16\,c^2d}+\frac{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\log\left(2\,cx+b\right)}{32\,c^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d),x, algorithm="maxima")
```

```
[Out] 1/16*(2*c^3*x^4 + 4*b*c^2*x^3 + (b^2*c + 8*a*c^2)*x^2 - (b^3 - 8*a*b*c)*x)/
(c^2*d) + 1/32*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*log(2*c*x + b)/(c^3*d)
```

Fricas [A] time = 1.93636, size = 192, normalized size = 2.67

$$\frac{4\,c^4x^4+8\,bc^3x^3+2\left(b^2c^2+8\,ac^3\right)x^2-2\left(b^3c-8\,abc^2\right)x+\left(b^4-8\,ab^2c+16\,a^2c^2\right)\log\left(2\,cx+b\right)}{32\,c^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d),x, algorithm="fricas")
```

```
[Out] 1/32*(4*c^4*x^4 + 8*b*c^3*x^3 + 2*(b^2*c^2 + 8*a*c^3)*x^2 - 2*(b^3*c - 8*a*
b*c^2)*x + (b^4 - 8*a*b^2*c + 16*a^2*c^2)*log(2*c*x + b))/(c^3*d)
```

Sympy [A] time = 0.538411, size = 76, normalized size = 1.06

$$\frac{bx^3}{4d}+\frac{cx^4}{8d}+\frac{x^2\left(8ac+b^2\right)}{16cd}+\frac{x\left(8abc-b^3\right)}{16c^2d}+\frac{\left(4ac-b^2\right)^2\log\left(b+2cx\right)}{32c^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(2*c*d*x+b*d),x)
```

```
[Out] b*x**3/(4*d) + c*x**4/(8*d) + x**2*(8*a*c + b**2)/(16*c*d) + x*(8*a*b*c - b
**3)/(16*c**2*d) + (4*a*c - b**2)**2*log(b + 2*c*x)/(32*c**3*d)
```

Giac [A] time = 1.22185, size = 157, normalized size = 2.18

$$\frac{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\log\left(|2\,cx+b|\right)}{32\,c^3d}+\frac{2\,c^5d^3x^4+4\,bc^4d^3x^3+b^2c^3d^3x^2+8\,ac^4d^3x^2-b^3c^2d^3x+8\,abc^3d^3x}{16\,c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d),x, algorithm="giac")
```

```
[Out] 1/32*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*log(abs(2*c*x + b))/(c^3*d) + 1/16*(2*c
^5*d^3*x^4 + 4*b*c^4*d^3*x^3 + b^2*c^3*d^3*x^2 + 8*a*c^4*d^3*x^2 - b^3*c^2*
d^3*x + 8*a*b*c^3*d^3*x)/(c^4*d^4)
```

3.1127 $\int \frac{(a+bx+cx^2)^2}{(bd+2cdx)^2} dx$

Optimal. Leaf size=72

$$-\frac{x\left(b^2-8ac\right)}{16c^2d^2}-\frac{\left(b^2-4ac\right)^2}{32c^3d^2(b+2cx)}+\frac{bx^2}{8cd^2}+\frac{x^3}{12d^2}$$

```
[Out] -((b^2 - 8*a*c)*x)/(16*c^2*d^2) + (b*x^2)/(8*c*d^2) + x^3/(12*d^2) - (b^2 -
 4*a*c)^2/(32*c^3*d^2*(b + 2*c*x))
```

Rubi [A] time = 0.0598793, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$-\frac{x\left(b^2-8ac\right)}{16c^2d^2}-\frac{\left(b^2-4ac\right)^2}{32c^3d^2(b+2cx)}+\frac{bx^2}{8cd^2}+\frac{x^3}{12d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x)^2,x]
```

```
[Out] -((b^2 - 8*a*c)*x)/(16*c^2*d^2) + (b*x^2)/(8*c*d^2) + x^3/(12*d^2) - (b^2 -
 4*a*c)^2/(32*c^3*d^2*(b + 2*c*x))
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx+cx^2\right)^2}{(bd+2cdx)^2}\,dx &= \int \left(\frac{-b^2+8ac}{16c^2d^2}+\frac{bx}{4cd^2}+\frac{x^2}{4d^2}+\frac{\left(-b^2+4ac\right)^2}{16c^2d^2(b+2cx)^2}\right)dx\\ &= -\frac{\left(b^2-8ac\right)x}{16c^2d^2}+\frac{bx^2}{8cd^2}+\frac{x^3}{12d^2}-\frac{\left(b^2-4ac\right)^2}{32c^3d^2(b+2cx)}\end{aligned}$$

Mathematica [A] time = 0.0519079, size = 59, normalized size = 0.82

$$\frac{-\frac{6x\left(b^2-8ac\right)}{c^2}-\frac{3\left(b^2-4ac\right)^2}{c^3(b+2cx)}+\frac{12bx^2}{c}+8x^3}{96d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x)^2,x]
```

```
[Out] ((-6*(b^2 - 8*a*c)*x)/c^2 + (12*b*x^2)/c + 8*x^3 - (3*(b^2 - 4*a*c)^2)/(c^3
*(b + 2*c*x)))/(96*d^2)
```

Maple [A] time = 0.042, size = 70, normalized size = 1.

$$\frac{1}{d^2}\left(\frac{1}{16\,c^2}\left(\frac{4\,x^3c^2}{3}+2\,bcx^2+8\,acx-b^2x\right)-\frac{16\,a^2c^2-8\,acb^2+b^4}{32\,c^3\,(2\,cx+b)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^2,x)
```

```
[Out] 1/d^2*(1/16/c^2*(4/3*x^3*c^2+2*b*c*x^2+8*a*c*x-b^2*x)-1/32*(16*a^2*c^2-8*a*
b^2*c+b^4)/c^3/(2*c*x+b))
```

Maxima [A] time = 1.92837, size = 104, normalized size = 1.44

$$-\frac{b^4-8\,ab^2c+16\,a^2c^2}{32\left(2\,c^4d^2x+bc^3d^2\right)}+\frac{4\,c^2x^3+6\,bcx^2-3\left(b^2-8\,ac\right)x}{48\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^2,x, algorithm="maxima")
```

```
[Out] -1/32*(b^4 - 8*a*b^2*c + 16*a^2*c^2)/(2*c^4*d^2*x + b*c^3*d^2) + 1/48*(4*c^
2*x^3 + 6*b*c*x^2 - 3*(b^2 - 8*a*c)*x)/(c^2*d^2)
```

Fricas [A] time = 1.8754, size = 182, normalized size = 2.53

$$\frac{16\,c^4x^4+32\,bc^3x^3+96\,ac^3x^2-3\,b^4+24\,ab^2c-48\,a^2c^2-6\left(b^3c-8\,abc^2\right)x}{96\left(2\,c^4d^2x+bc^3d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^2,x, algorithm="fricas")
```

```
[Out] 1/96*(16*c^4*x^4 + 32*b*c^3*x^3 + 96*a*c^3*x^2 - 3*b^4 + 24*a*b^2*c - 48*a^
2*c^2 - 6*(b^3*c - 8*a*b*c^2)*x)/(2*c^4*d^2*x + b*c^3*d^2)
```

Sympy [A] time = 0.6431, size = 78, normalized size = 1.08

$$\frac{bx^2}{8cd^2}-\frac{16a^2c^2-8ab^2c+b^4}{32bc^3d^2+64c^4d^2x}+\frac{x^3}{12d^2}+\frac{x\left(8ac-b^2\right)}{16c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(2*c*d*x+b*d)**2,x)
```

```
[Out] b*x**2/(8*c*d**2) - (16*a**2*c**2 - 8*a*b**2*c + b**4)/(32*b*c**3*d**2 + 64
*c**4*d**2*x) + x**3/(12*d**2) + x*(8*a*c - b**2)/(16*c**2*d**2)
```

Giac [B] time = 1.2094, size = 181, normalized size = 2.51

$$-\frac{(2\,cdx+bd)^3\left(\frac{6\,b^2d^2}{(2\,cdx+bd)^2}-\frac{24\,acd^2}{(2\,cdx+bd)^2}-1\right)}{96\,c^3d^5}-\frac{\frac{b^4c^3d^7}{2\,cdx+bd}-\frac{8\,ab^2c^4d^7}{2\,cdx+bd}+\frac{16\,a^2c^5d^7}{2\,cdx+bd}}{32\,c^6d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^2,x, algorithm="giac")
```

```
[Out] -1/96*(2*c*d*x + b*d)^3*(6*b^2*d^2/(2*c*d*x + b*d)^2 - 24*a*c*d^2/(2*c*d*x
+ b*d)^2 - 1)/(c^3*d^5) - 1/32*(b^4*c^3*d^7/(2*c*d*x + b*d) - 8*a*b^2*c^4*d
^7/(2*c*d*x + b*d) + 16*a^2*c^5*d^7/(2*c*d*x + b*d))/(c^6*d^8)
```

3.1128 $\int \frac{(a+bx+cx^2)^2}{(bd+2cdx)^3} dx$

Optimal. Leaf size=79

$$-\frac{(b^2-4ac)^2}{64c^3d^3(b+2cx)^2}-\frac{(b^2-4ac)\log(b+2cx)}{16c^3d^3}+\frac{bx}{16c^2d^3}+\frac{x^2}{16cd^3}$$

```
[Out] (b*x)/(16*c^2*d^3) + x^2/(16*c*d^3) - (b^2 - 4*a*c)^2/(64*c^3*d^3*(b + 2*c*
x)^2) - ((b^2 - 4*a*c)*Log[b + 2*c*x])/(16*c^3*d^3)
```

Rubi [A] time = 0.0635056, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$-\frac{(b^2-4ac)^2}{64c^3d^3(b+2cx)^2}-\frac{(b^2-4ac)\log(b+2cx)}{16c^3d^3}+\frac{bx}{16c^2d^3}+\frac{x^2}{16cd^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x)^3,x]
```

```
[Out] (b*x)/(16*c^2*d^3) + x^2/(16*c*d^3) - (b^2 - 4*a*c)^2/(64*c^3*d^3*(b + 2*c*
x)^2) - ((b^2 - 4*a*c)*Log[b + 2*c*x])/(16*c^3*d^3)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx+cx^2)^2}{(bd+2cdx)^3} dx &= \int \left(\frac{b}{16c^2d^3}+\frac{x}{8cd^3}+\frac{(-b^2+4ac)^2}{16c^2d^3(b+2cx)^3}+\frac{-b^2+4ac}{8c^2d^3(b+2cx)}\right) dx\\ &= \frac{bx}{16c^2d^3}+\frac{x^2}{16cd^3}-\frac{(b^2-4ac)^2}{64c^3d^3(b+2cx)^2}-\frac{(b^2-4ac)\log(b+2cx)}{16c^3d^3}\end{aligned}$$

Mathematica [A] time = 0.0406992, size = 61, normalized size = 0.77

$$\frac{-\frac{(b^2-4ac)^2}{(b+2cx)^2}-4(b^2-4ac)\log(b+2cx)+4bcx+4c^2x^2}{64c^3d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x)^3,x]
```

```
[Out] (4*b*c*x + 4*c^2*x^2 - (b^2 - 4*a*c)^2/(b + 2*c*x)^2 - 4*(b^2 - 4*a*c)*Log[
b + 2*c*x])/(64*c^3*d^3)
```

Maple [A] time = 0.046, size = 115, normalized size = 1.5

$$\frac{x^2}{16\,cd^3}+\frac{bx}{16\,c^2d^3}-\frac{a^2}{4\,cd^3\,(2\,cx+b)^2}+\frac{b^2a}{8\,c^2d^3\,(2\,cx+b)^2}-\frac{b^4}{64\,c^3d^3\,(2\,cx+b)^2}+\frac{\ln(2\,cx+b)\,a}{4\,c^2d^3}-\frac{\ln(2\,cx+b)\,b^2}{16\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^3,x)
```

```
[Out] 1/16*x^2/c/d^3+1/16*b*x/c^2/d^3-1/4/d^3/c/(2*c*x+b)^2*a^2+1/8/d^3/c^2/(2*c*
x+b)^2*a*b^2-1/64/d^3/c^3/(2*c*x+b)^2*b^4+1/4/d^3/c^2*ln(2*c*x+b)*a-1/16/d^
3/c^3*ln(2*c*x+b)*b^2
```

Maxima [A] time = 1.13658, size = 130, normalized size = 1.65

$$-\frac{b^4-8\,ab^2c+16\,a^2c^2}{64\left(4\,c^5d^3x^2+4\,bc^4d^3x+b^2c^3d^3\right)}+\frac{cx^2+bx}{16\,c^2d^3}-\frac{\left(b^2-4\,ac\right)\log(2\,cx+b)}{16\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^3,x, algorithm="maxima")
```

```
[Out] -1/64*(b^4 - 8*a*b^2*c + 16*a^2*c^2)/(4*c^5*d^3*x^2 + 4*b*c^4*d^3*x + b^2*c
^3*d^3) + 1/16*(c*x^2 + b*x)/(c^2*d^3) - 1/16*(b^2 - 4*a*c)*log(2*c*x + b)/
(c^3*d^3)
```

Fricas [B] time = 2.02038, size = 312, normalized size = 3.95

$$\frac{16\,c^4x^4+32\,bc^3x^3+20\,b^2c^2x^2+4\,b^3cx-b^4+8\,ab^2c-16\,a^2c^2-4\left(b^4-4\,ab^2c+4\left(b^2c^2-4\,ac^3\right)x^2+4\left(b^3c-4\,abc^2\right)\right.}{64\left(4\,c^5d^3x^2+4\,bc^4d^3x+b^2c^3d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(16*c^4*x^4 + 32*b*c^3*x^3 + 20*b^2*c^2*x^2 + 4*b^3*c*x - b^4 + 8*a*b^
2*c - 16*a^2*c^2 - 4*(b^4 - 4*a*b^2*c + 4*(b^2*c^2 - 4*a*c^3)*x^2 + 4*(b^3*
c - 4*a*b*c^2)*x)*log(2*c*x + b))/(4*c^5*d^3*x^2 + 4*b*c^4*d^3*x + b^2*c^3*
d^3)
```

Sympy [A] time = 1.08025, size = 102, normalized size = 1.29

$$\frac{bx}{16c^2d^3}-\frac{16a^2c^2-8ab^2c+b^4}{64b^2c^3d^3+256bc^4d^3x+256c^5d^3x^2}+\frac{x^2}{16cd^3}+\frac{\left(4ac-b^2\right)\log(b+2cx)}{16c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(2*c*d*x+b*d)**3,x)
```

```
[Out] b*x/(16*c**2*d**3) - (16*a**2*c**2 - 8*a*b**2*c + b**4)/(64*b**2*c**3*d**3
+ 256*b*c**4*d**3*x + 256*c**5*d**3*x**2) + x**2/(16*c*d**3) + (4*a*c - b**
2)*log(b + 2*c*x)/(16*c**3*d**3)
```

Giac [A] time = 1.13045, size = 119, normalized size = 1.51

$$-\frac{\left(b^2-4\,ac\right)\log\left(|2\,cx+b|\right)}{16\,c^3d^3}-\frac{b^4-8\,ab^2c+16\,a^2c^2}{64\,(2\,cx+b)^2c^3d^3}+\frac{c^5d^3x^2+bc^4d^3x}{16\,c^6d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^3,x, algorithm="giac")
```

```
[Out] -1/16*(b^2 - 4*a*c)*log(abs(2*c*x + b))/(c^3*d^3) - 1/64*(b^4 - 8*a*b^2*c +
 16*a^2*c^2)/((2*c*x + b)^2*c^3*d^3) + 1/16*(c^5*d^3*x^2 + b*c^4*d^3*x)/(c^
6*d^6)
```

3.1129 $\int \frac{(a+bx+cx^2)^2}{(bd+2cdx)^4} dx$

Optimal. Leaf size=66

$$-\frac{(b^2-4ac)^2}{96c^3d^4(b+2cx)^3}+\frac{b^2-4ac}{16c^3d^4(b+2cx)}+\frac{x}{16c^2d^4}$$

```
[Out] x/(16*c^2*d^4) - (b^2 - 4*a*c)^2/(96*c^3*d^4*(b + 2*c*x)^3) + (b^2 - 4*a*c)
/(16*c^3*d^4*(b + 2*c*x))
```

Rubi [A] time = 0.0541672, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$-\frac{(b^2-4ac)^2}{96c^3d^4(b+2cx)^3}+\frac{b^2-4ac}{16c^3d^4(b+2cx)}+\frac{x}{16c^2d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x)^4,x]
```

```
[Out] x/(16*c^2*d^4) - (b^2 - 4*a*c)^2/(96*c^3*d^4*(b + 2*c*x)^3) + (b^2 - 4*a*c)
/(16*c^3*d^4*(b + 2*c*x))
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx+cx^2)^2}{(bd+2cdx)^4}\,dx &= \int \left(\frac{1}{16c^2d^4}+\frac{(-b^2+4ac)^2}{16c^2d^4(b+2cx)^4}+\frac{-b^2+4ac}{8c^2d^4(b+2cx)^2}\right)dx\\ &= \frac{x}{16c^2d^4}-\frac{(b^2-4ac)^2}{96c^3d^4(b+2cx)^3}+\frac{b^2-4ac}{16c^3d^4(b+2cx)}\end{aligned}$$

Mathematica [A] time = 0.0567088, size = 53, normalized size = 0.8

$$\frac{-\frac{(b^2-4ac)^2}{(b+2cx)^3}+\frac{6(b^2-4ac)}{b+2cx}+6cx}{96c^3d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x)^4,x]
```

```
[Out] (6*c*x - (b^2 - 4*a*c)^2/(b + 2*c*x)^3 + (6*(b^2 - 4*a*c))/(b + 2*c*x))/(96
*c^3*d^4)
```

Maple [A] time = 0.044, size = 67, normalized size = 1.

$$\frac{1}{d^4}\left(\frac{x}{16\,c^2}-\frac{16\,a^2c^2-8\,acb^2+b^4}{96\,c^3\,(2\,cx+b)^3}-\frac{4\,ac-b^2}{16\,c^3\,(2\,cx+b)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^4,x)
```

```
[Out] 1/d^4*(1/16*x/c^2-1/96/c^3*(16*a^2*c^2-8*a*b^2*c+b^4)/(2*c*x+b)^3-1/16/c^3*
(4*a*c-b^2)/(2*c*x+b))
```

Maxima [A] time = 1.23916, size = 157, normalized size = 2.38

$$\frac{5\,b^4-16\,ab^2c-16\,a^2c^2+24\,\left(b^2c^2-4\,ac^3\right)x^2+24\,\left(b^3c-4\,abc^2\right)x}{96\,\left(8\,c^6d^4x^3+12\,bc^5d^4x^2+6\,b^2c^4d^4x+b^3c^3d^4\right)}+\frac{x}{16\,c^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^4,x, algorithm="maxima")
```

```
[Out] 1/96*(5*b^4 - 16*a*b^2*c - 16*a^2*c^2 + 24*(b^2*c^2 - 4*a*c^3)*x^2 + 24*(b^
3*c - 4*a*b*c^2)*x)/(8*c^6*d^4*x^3 + 12*b*c^5*d^4*x^2 + 6*b^2*c^4*d^4*x + b
^3*c^3*d^4) + 1/16*x/(c^2*d^4)
```

Fricas [B] time = 2.02515, size = 263, normalized size = 3.98

$$\frac{48\,c^4x^4+72\,bc^3x^3+5\,b^4-16\,ab^2c-16\,a^2c^2+12\,\left(5\,b^2c^2-8\,ac^3\right)x^2+6\,\left(5\,b^3c-16\,abc^2\right)x}{96\,\left(8\,c^6d^4x^3+12\,bc^5d^4x^2+6\,b^2c^4d^4x+b^3c^3d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^4,x, algorithm="fricas")
```

```
[Out] 1/96*(48*c^4*x^4 + 72*b*c^3*x^3 + 5*b^4 - 16*a*b^2*c - 16*a^2*c^2 + 12*(5*b
^2*c^2 - 8*a*c^3)*x^2 + 6*(5*b^3*c - 16*a*b*c^2)*x)/(8*c^6*d^4*x^3 + 12*b*c
^5*d^4*x^2 + 6*b^2*c^4*d^4*x + b^3*c^3*d^4)
```

Sympy [A] time = 1.35265, size = 117, normalized size = 1.77

$$-\frac{16a^2c^2+16ab^2c-5b^4+x^2\left(96ac^3-24b^2c^2\right)+x\left(96abc^2-24b^3c\right)}{96b^3c^3d^4+576b^2c^4d^4x+1152bc^5d^4x^2+768c^6d^4x^3}+\frac{x}{16c^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(2*c*d*x+b*d)**4,x)
```

```
[Out] -(16*a**2*c**2 + 16*a*b**2*c - 5*b**4 + x**2*(96*a*c**3 - 24*b**2*c**2) + x
*(96*a*b*c**2 - 24*b**3*c))/(96*b**3*c**3*d**4 + 576*b**2*c**4*d**4*x + 115
```

```
2*b*c**5*d**4*x**2 + 768*c**6*d**4*x**3) + x/(16*c**2*d**4)
```

Giac [A] time = 1.1374, size = 111, normalized size = 1.68

$$\frac{x}{16\,c^2d^4}+\frac{24\,b^2c^2x^2-96\,ac^3x^2+24\,b^3cx-96\,abc^2x+5\,b^4-16\,ab^2c-16\,a^2c^2}{96\,(2\,cx+b)^3c^3d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^4,x, algorithm="giac")
```

```
[Out] 1/16*x/(c^2*d^4) + 1/96*(24*b^2*c^2*x^2 - 96*a*c^3*x^2 + 24*b^3*c*x - 96*a*
b*c^2*x + 5*b^4 - 16*a*b^2*c - 16*a^2*c^2)/((2*c*x + b)^3*c^3*d^4)
```

3.1130 $\int \frac{\left(a+bx+cx^2\right)^2}{(bd+2cdx)^5}\,dx$

Optimal. Leaf size=72

$$-\frac{\left(b^2-4ac\right)^2}{128c^3d^5(b+2cx)^4}+\frac{b^2-4ac}{32c^3d^5(b+2cx)^2}+\frac{\log(b+2cx)}{32c^3d^5}$$

```
[Out] -(b^2 - 4*a*c)^2/(128*c^3*d^5*(b + 2*c*x)^4) + (b^2 - 4*a*c)/(32*c^3*d^5*(b
 + 2*c*x)^2) + Log[b + 2*c*x]/(32*c^3*d^5)
```

Rubi [A] time = 0.0576435, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$-\frac{\left(b^2-4ac\right)^2}{128c^3d^5(b+2cx)^4}+\frac{b^2-4ac}{32c^3d^5(b+2cx)^2}+\frac{\log(b+2cx)}{32c^3d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x)^5,x]
```

```
[Out] -(b^2 - 4*a*c)^2/(128*c^3*d^5*(b + 2*c*x)^4) + (b^2 - 4*a*c)/(32*c^3*d^5*(b
 + 2*c*x)^2) + Log[b + 2*c*x]/(32*c^3*d^5)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx+cx^2\right)^2}{(bd+2cdx)^5}\,dx &= \int \left(\frac{\left(-b^2+4ac\right)^2}{16c^2d^5(b+2cx)^5}+\frac{-b^2+4ac}{8c^2d^5(b+2cx)^3}+\frac{1}{16c^2d^5(b+2cx)}\right)dx\\ &= -\frac{\left(b^2-4ac\right)^2}{128c^3d^5(b+2cx)^4}+\frac{b^2-4ac}{32c^3d^5(b+2cx)^2}+\frac{\log(b+2cx)}{32c^3d^5}\end{aligned}$$

Mathematica [A] time = 0.0348702, size = 59, normalized size = 0.82

$$\frac{\frac{\left(b^2-4ac\right)\left(4c\left(a+4cx^2\right)+3b^2+16bcx\right)}{(b+2cx)^4}+4\log(b+2cx)}{128c^3d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x)^5,x]
```

```
[Out] (((b^2 - 4*a*c)*(3*b^2 + 16*b*c*x + 4*c*(a + 4*c*x^2)))/(b + 2*c*x)^4 + 4*L
og[b + 2*c*x])/(128*c^3*d^5)
```

Maple [A] time = 0.045, size = 111, normalized size = 1.5

$$-\frac{a}{8\,c^{2}d^{5}\left(2\,cx+b\right)^{2}}+\frac{b^{2}}{32\,c^{3}d^{5}\left(2\,cx+b\right)^{2}}-\frac{a^{2}}{8\,d^{5}c\left(2\,cx+b\right)^{4}}+\frac{b^{2}a}{16\,c^{2}d^{5}\left(2\,cx+b\right)^{4}}-\frac{b^{4}}{128\,c^{3}d^{5}\left(2\,cx+b\right)^{4}}+\frac{\ln\left(2\,cx+b\right)}{32\,c^{3}d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^5,x)
```

```
[Out] -1/8/d^5/c^2/(2*c*x+b)^2*a+1/32/d^5/c^3/(2*c*x+b)^2*b^2-1/8/d^5/c/(2*c*x+b)
^4*a^2+1/16/d^5/c^2/(2*c*x+b)^4*a*b^2-1/128/d^5/c^3/(2*c*x+b)^4*b^4+1/32*ln
(2*c*x+b)/c^3/d^5
```

Maxima [B] time = 1.20123, size = 184, normalized size = 2.56

$$\frac{3\,b^{4}-8\,ab^{2}c-16\,a^{2}c^{2}+16\,\left(b^{2}c^{2}-4\,ac^{3}\right)x^{2}+16\,\left(b^{3}c-4\,abc^{2}\right)x}{128\,\left(16\,c^{7}d^{5}x^{4}+32\,bc^{6}d^{5}x^{3}+24\,b^{2}c^{5}d^{5}x^{2}+8\,b^{3}c^{4}d^{5}x+b^{4}c^{3}d^{5}\right)}+\frac{\log\left(2\,cx+b\right)}{32\,c^{3}d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^5,x, algorithm="maxima")
```

```
[Out] 1/128*(3*b^4 - 8*a*b^2*c - 16*a^2*c^2 + 16*(b^2*c^2 - 4*a*c^3)*x^2 + 16*(b^
3*c - 4*a*b*c^2)*x)/(16*c^7*d^5*x^4 + 32*b*c^6*d^5*x^3 + 24*b^2*c^5*d^5*x^2
 + 8*b^3*c^4*d^5*x + b^4*c^3*d^5) + 1/32*log(2*c*x + b)/(c^3*d^5)
```

Fricas [B] time = 2.05401, size = 360, normalized size = 5.

$$\frac{3\,b^{4}-8\,ab^{2}c-16\,a^{2}c^{2}+16\,\left(b^{2}c^{2}-4\,ac^{3}\right)x^{2}+16\,\left(b^{3}c-4\,abc^{2}\right)x+4\,\left(16\,c^{4}x^{4}+32\,bc^{3}x^{3}+24\,b^{2}c^{2}x^{2}+8\,b^{3}cx+b^{4}\right)\log\left(2\,cx+b\right)}{128\,\left(16\,c^{7}d^{5}x^{4}+32\,bc^{6}d^{5}x^{3}+24\,b^{2}c^{5}d^{5}x^{2}+8\,b^{3}c^{4}d^{5}x+b^{4}c^{3}d^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^5,x, algorithm="fricas")
```

```
[Out] 1/128*(3*b^4 - 8*a*b^2*c - 16*a^2*c^2 + 16*(b^2*c^2 - 4*a*c^3)*x^2 + 16*(b^
3*c - 4*a*b*c^2)*x + 4*(16*c^4*x^4 + 32*b*c^3*x^3 + 24*b^2*c^2*x^2 + 8*b^3*
c*x + b^4)*log(2*c*x + b))/(16*c^7*d^5*x^4 + 32*b*c^6*d^5*x^3 + 24*b^2*c^5*
d^5*x^2 + 8*b^3*c^4*d^5*x + b^4*c^3*d^5)
```

Sympy [B] time = 2.07894, size = 139, normalized size = 1.93

$$-\frac{16a^{2}c^{2}+8ab^{2}c-3b^{4}+x^{2}\left(64ac^{3}-16b^{2}c^{2}\right)+x\left(64abc^{2}-16b^{3}c\right)}{128b^{4}c^{3}d^{5}+1024b^{3}c^{4}d^{5}x+3072b^{2}c^{5}d^{5}x^{2}+4096bc^{6}d^{5}x^{3}+2048c^{7}d^{5}x^{4}}+\frac{\log\left(b+2cx\right)}{32c^{3}d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(2*c*d*x+b*d)**5,x)
```

```
[Out] -(16*a**2*c**2 + 8*a*b**2*c - 3*b**4 + x**2*(64*a*c**3 - 16*b**2*c**2) + x*
(64*a*b*c**2 - 16*b**3*c))/(128*b**4*c**3*d**5 + 1024*b**3*c**4*d**5*x + 30
72*b**2*c**5*d**5*x**2 + 4096*b*c**6*d**5*x**3 + 2048*c**7*d**5*x**4) + log
(b + 2*c*x)/(32*c**3*d**5)
```

Giac [B] time = 1.17544, size = 197, normalized size = 2.74

$$-\frac{\log\left(\frac{1}{4\,(2\,cdx+bd)^2c^2d^2}\right)}{64\,c^3d^5}-\frac{\frac{b^4c^3d^9}{(2\,cdx+bd)^4}-\frac{8\,ab^2c^4d^9}{(2\,cdx+bd)^4}+\frac{16\,a^2c^5d^9}{(2\,cdx+bd)^4}-\frac{4\,b^2c^3d^7}{(2\,cdx+bd)^2}+\frac{16\,ac^4d^7}{(2\,cdx+bd)^2}}{128\,c^6d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^5,x, algorithm="giac")
```

```
[Out] -1/64*log(1/4/((2*c*d*x + b*d)^2*c^2*d^2))/(c^3*d^5) - 1/128*(b^4*c^3*d^9/(
2*c*d*x + b*d)^4 - 8*a*b^2*c^4*d^9/(2*c*d*x + b*d)^4 + 16*a^2*c^5*d^9/(2*c*
d*x + b*d)^4 - 4*b^2*c^3*d^7/(2*c*d*x + b*d)^2 + 16*a*c^4*d^7/(2*c*d*x + b*
d)^2)/(c^6*d^10)
```

3.1131 $\int \frac{(a+bx+cx^2)^2}{(bd+2cdx)^6} dx$

Optimal. Leaf size=73

$$-\frac{(b^2-4ac)^2}{160c^3d^6(b+2cx)^5}+\frac{b^2-4ac}{48c^3d^6(b+2cx)^3}-\frac{1}{32c^3d^6(b+2cx)}$$

```
[Out] -(b^2 - 4*a*c)^2/(160*c^3*d^6*(b + 2*c*x)^5) + (b^2 - 4*a*c)/(48*c^3*d^6*(b
 + 2*c*x)^3) - 1/(32*c^3*d^6*(b + 2*c*x))
```

Rubi [A] time = 0.0569732, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$-\frac{(b^2-4ac)^2}{160c^3d^6(b+2cx)^5}+\frac{b^2-4ac}{48c^3d^6(b+2cx)^3}-\frac{1}{32c^3d^6(b+2cx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x)^6,x]
```

```
[Out] -(b^2 - 4*a*c)^2/(160*c^3*d^6*(b + 2*c*x)^5) + (b^2 - 4*a*c)/(48*c^3*d^6*(b
 + 2*c*x)^3) - 1/(32*c^3*d^6*(b + 2*c*x))
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] && !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx+cx^2)^2}{(bd+2cdx)^6} dx &= \int \left(\frac{(-b^2+4ac)^2}{16c^2d^6(b+2cx)^6}+\frac{-b^2+4ac}{8c^2d^6(b+2cx)^4}+\frac{1}{16c^2d^6(b+2cx)^2}\right) dx\\ &= -\frac{(b^2-4ac)^2}{160c^3d^6(b+2cx)^5}+\frac{b^2-4ac}{48c^3d^6(b+2cx)^3}-\frac{1}{32c^3d^6(b+2cx)}\end{aligned}$$

Mathematica [A] time = 0.034111, size = 59, normalized size = 0.81

$$\frac{10(b^2-4ac)(b+2cx)^2-3(b^2-4ac)^2-15(b+2cx)^4}{480c^3d^6(b+2cx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x)^6,x]
```

```
[Out] (-3*(b^2 - 4*a*c)^2 + 10*(b^2 - 4*a*c)*(b + 2*c*x)^2 - 15*(b + 2*c*x)^4)/(4
80*c^3*d^6*(b + 2*c*x)^5)
```

Maple [A] time = 0.045, size = 74, normalized size = 1.

$$\frac{1}{d^6}\left(-\frac{4\,ac-b^2}{48\,c^3\,(2\,cx+b)^3}-\frac{1}{32\,c^3\,(2\,cx+b)}-\frac{16\,a^2c^2-8\,acb^2+b^4}{160\,c^3\,(2\,cx+b)^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^6,x)
```

```
[Out] 1/d^6*(-1/48*(4*a*c-b^2)/c^3/(2*c*x+b)^3-1/32/c^3/(2*c*x+b)-1/160*(16*a^2*c
^2-8*a*b^2*c+b^4)/c^3/(2*c*x+b)^5)
```

Maxima [B] time = 1.28222, size = 201, normalized size = 2.75

$$-\frac{30\,c^4x^4+60\,bc^3x^3+b^4+2\,ab^2c+6\,a^2c^2+20\left(2\,b^2c^2+ac^3\right)x^2+10\left(b^3c+2\,abc^2\right)x}{60\left(32\,c^8d^6x^5+80\,bc^7d^6x^4+80\,b^2c^6d^6x^3+40\,b^3c^5d^6x^2+10\,b^4c^4d^6x+b^5c^3d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^6,x, algorithm="maxima")
```

```
[Out] -1/60*(30*c^4*x^4 + 60*b*c^3*x^3 + b^4 + 2*a*b^2*c + 6*a^2*c^2 + 20*(2*b^2*
c^2 + a*c^3)*x^2 + 10*(b^3*c + 2*a*b*c^2)*x)/(32*c^8*d^6*x^5 + 80*b*c^7*d^6
*x^4 + 80*b^2*c^6*d^6*x^3 + 40*b^3*c^5*d^6*x^2 + 10*b^4*c^4*d^6*x + b^5*c^3
*d^6)
```

Fricas [B] time = 1.98415, size = 313, normalized size = 4.29

$$-\frac{30\,c^4x^4+60\,bc^3x^3+b^4+2\,ab^2c+6\,a^2c^2+20\left(2\,b^2c^2+ac^3\right)x^2+10\left(b^3c+2\,abc^2\right)x}{60\left(32\,c^8d^6x^5+80\,bc^7d^6x^4+80\,b^2c^6d^6x^3+40\,b^3c^5d^6x^2+10\,b^4c^4d^6x+b^5c^3d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^6,x, algorithm="fricas")
```

```
[Out] -1/60*(30*c^4*x^4 + 60*b*c^3*x^3 + b^4 + 2*a*b^2*c + 6*a^2*c^2 + 20*(2*b^2*
c^2 + a*c^3)*x^2 + 10*(b^3*c + 2*a*b*c^2)*x)/(32*c^8*d^6*x^5 + 80*b*c^7*d^6
*x^4 + 80*b^2*c^6*d^6*x^3 + 40*b^3*c^5*d^6*x^2 + 10*b^4*c^4*d^6*x + b^5*c^3
*d^6)
```

Sympy [B] time = 1.89372, size = 156, normalized size = 2.14

$$-\frac{6a^2c^2+2ab^2c+b^4+60bc^3x^3+30c^4x^4+x^2\left(20ac^3+40b^2c^2\right)+x\left(20abc^2+10b^3c\right)}{60b^5c^3d^6+600b^4c^4d^6x+2400b^3c^5d^6x^2+4800b^2c^6d^6x^3+4800bc^7d^6x^4+1920c^8d^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(2*c*d*x+b*d)**6,x)
```

```
[Out] -(6*a**2*c**2 + 2*a*b**2*c + b**4 + 60*b*c**3*x**3 + 30*c**4*x**4 + x**2*(2
0*a*c**3 + 40*b**2*c**2) + x*(20*a*b*c**2 + 10*b**3*c))/(60*b**5*c**3*d**6
+ 600*b**4*c**4*d**6*x + 2400*b**3*c**5*d**6*x**2 + 4800*b**2*c**6*d**6*x**
3 + 4800*b*c**7*d**6*x**4 + 1920*c**8*d**6*x**5)
```

Giac [A] time = 1.19, size = 117, normalized size = 1.6

$$-\frac{30\,c^4x^4+60\,bc^3x^3+40\,b^2c^2x^2+20\,ac^3x^2+10\,b^3cx+20\,abc^2x+b^4+2\,ab^2c+6\,a^2c^2}{60\,(2\,cx+b)^5c^3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^6,x, algorithm="giac")
```

```
[Out] -1/60*(30*c^4*x^4 + 60*b*c^3*x^3 + 40*b^2*c^2*x^2 + 20*a*c^3*x^2 + 10*b^3*c
*x + 20*a*b*c^2*x + b^4 + 2*a*b^2*c + 6*a^2*c^2)/((2*c*x + b)^5*c^3*d^6)
```

3.1132 $\int \frac{(a+bx+cx^2)^2}{(bd+2cdx)^7} dx$

Optimal. Leaf size=37

$$\frac{(a+bx+cx^2)^3}{3d^7(b^2-4ac)(b+2cx)^6}$$

```
[Out] (a + b*x + c*x^2)^3/(3*(b^2 - 4*a*c)*d^7*(b + 2*c*x)^6)
```

Rubi [A] time = 0.0143671, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {682}

$$\frac{(a+bx+cx^2)^3}{3d^7(b^2-4ac)(b+2cx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x)^7,x]
```

```
[Out] (a + b*x + c*x^2)^3/(3*(b^2 - 4*a*c)*d^7*(b + 2*c*x)^6)
```

Rule 682

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c,
0] && EqQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^2}{(bd+2cdx)^7} dx = \frac{(a+bx+cx^2)^3}{3(b^2-4ac)d^7(b+2cx)^6}$$

Mathematica [A] time = 0.0290149, size = 65, normalized size = 1.76

$$-\frac{16a^2c^2-3(b^2-4ac)(b+2cx)^2-8ab^2c+b^4+3(b+2cx)^4}{192c^3d^7(b+2cx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x)^7,x]
```

```
[Out] -(b^4 - 8*a*b^2*c + 16*a^2*c^2 - 3*(b^2 - 4*a*c)*(b + 2*c*x)^2 + 3*(b + 2*c
*x)^4)/(192*c^3*d^7*(b + 2*c*x)^6)
```

Maple [B] time = 0.044, size = 74, normalized size = 2.

$$\frac{1}{d^{7}}\left(-\frac{16\,a^{2}c^{2}-8\,acb^{2}+b^{4}}{192\,c^{3}\left(2\,cx+b\right)^{6}}-\frac{1}{64\,c^{3}\left(2\,cx+b\right)^{2}}-\frac{4\,ac-b^{2}}{64\,c^{3}\left(2\,cx+b\right)^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^7,x)
```

```
[Out] 1/d^7*(-1/192*(16*a^2*c^2-8*a*b^2*c+b^4)/c^3/(2*c*x+b)^6-1/64/c^3/(2*c*x+b)
^2-1/64*(4*a*c-b^2)/c^3/(2*c*x+b)^4)
```

Maxima [B] time = 1.18392, size = 221, normalized size = 5.97

$$-\frac{48\,c^{4}x^{4}+96\,bc^{3}x^{3}+b^{4}+4\,ab^{2}c+16\,a^{2}c^{2}+12\left(5\,b^{2}c^{2}+4\,ac^{3}\right)x^{2}+12\left(b^{3}c+4\,abc^{2}\right)x}{192\left(64\,c^{9}d^{7}x^{6}+192\,bc^{8}d^{7}x^{5}+240\,b^{2}c^{7}d^{7}x^{4}+160\,b^{3}c^{6}d^{7}x^{3}+60\,b^{4}c^{5}d^{7}x^{2}+12\,b^{5}c^{4}d^{7}x+b^{6}c^{3}d^{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^7,x, algorithm="maxima")
```

```
[Out] -1/192*(48*c^4*x^4 + 96*b*c^3*x^3 + b^4 + 4*a*b^2*c + 16*a^2*c^2 + 12*(5*b^
2*c^2 + 4*a*c^3)*x^2 + 12*(b^3*c + 4*a*b*c^2)*x)/(64*c^9*d^7*x^6 + 192*b*c^
8*d^7*x^5 + 240*b^2*c^7*d^7*x^4 + 160*b^3*c^6*d^7*x^3 + 60*b^4*c^5*d^7*x^2
+ 12*b^5*c^4*d^7*x + b^6*c^3*d^7)
```

Fricas [B] time = 1.88119, size = 351, normalized size = 9.49

$$-\frac{48\,c^{4}x^{4}+96\,bc^{3}x^{3}+b^{4}+4\,ab^{2}c+16\,a^{2}c^{2}+12\left(5\,b^{2}c^{2}+4\,ac^{3}\right)x^{2}+12\left(b^{3}c+4\,abc^{2}\right)x}{192\left(64\,c^{9}d^{7}x^{6}+192\,bc^{8}d^{7}x^{5}+240\,b^{2}c^{7}d^{7}x^{4}+160\,b^{3}c^{6}d^{7}x^{3}+60\,b^{4}c^{5}d^{7}x^{2}+12\,b^{5}c^{4}d^{7}x+b^{6}c^{3}d^{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^7,x, algorithm="fricas")
```

```
[Out] -1/192*(48*c^4*x^4 + 96*b*c^3*x^3 + b^4 + 4*a*b^2*c + 16*a^2*c^2 + 12*(5*b^
2*c^2 + 4*a*c^3)*x^2 + 12*(b^3*c + 4*a*b*c^2)*x)/(64*c^9*d^7*x^6 + 192*b*c^
8*d^7*x^5 + 240*b^2*c^7*d^7*x^4 + 160*b^3*c^6*d^7*x^3 + 60*b^4*c^5*d^7*x^2
+ 12*b^5*c^4*d^7*x + b^6*c^3*d^7)
```

Sympy [B] time = 3.89161, size = 172, normalized size = 4.65

$$-\frac{16a^{2}c^{2}+4ab^{2}c+b^{4}+96bc^{3}x^{3}+48c^{4}x^{4}+x^{2}\left(48ac^{3}+60b^{2}c^{2}\right)+x\left(48abc^{2}+12b^{3}c\right)}{192b^{6}c^{3}d^{7}+2304b^{5}c^{4}d^{7}x+11520b^{4}c^{5}d^{7}x^{2}+30720b^{3}c^{6}d^{7}x^{3}+46080b^{2}c^{7}d^{7}x^{4}+36864bc^{8}d^{7}x^{5}+12288c^{9}d^{7}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(2*c*d*x+b*d)**7,x)
```

```
[Out] -(16*a**2*c**2 + 4*a*b**2*c + b**4 + 96*b*c**3*x**3 + 48*c**4*x**4 + x**2*(
48*a*c**3 + 60*b**2*c**2) + x*(48*a*b*c**2 + 12*b**3*c))/(192*b**6*c**3*d**
```

```
7 + 2304*b**5*c**4*d**7*x + 11520*b**4*c**5*d**7*x**2 + 30720*b**3*c**6*d**
7*x**3 + 46080*b**2*c**7*d**7*x**4 + 36864*b*c**8*d**7*x**5 + 12288*c**9*d*
*7*x**6)
```

Giac [B] time = 1.24655, size = 117, normalized size = 3.16

$$-\frac{48\,c^4x^4+96\,bc^3x^3+60\,b^2c^2x^2+48\,ac^3x^2+12\,b^3cx+48\,abc^2x+b^4+4\,ab^2c+16\,a^2c^2}{192\,(2\,cx+b)^6c^3d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^7,x, algorithm="giac")
```

```
[Out] -1/192*(48*c^4*x^4 + 96*b*c^3*x^3 + 60*b^2*c^2*x^2 + 48*a*c^3*x^2 + 12*b^3*
c*x + 48*a*b*c^2*x + b^4 + 4*a*b^2*c + 16*a^2*c^2)/((2*c*x + b)^6*c^3*d^7)
```

3.1133 $\int \frac{(a+bx+cx^2)^2}{(bd+2cdx)^8} dx$

Optimal. Leaf size=73

$$-\frac{(b^2-4ac)^2}{224c^3d^8(b+2cx)^7}+\frac{b^2-4ac}{80c^3d^8(b+2cx)^5}-\frac{1}{96c^3d^8(b+2cx)^3}$$

```
[Out] -(b^2 - 4*a*c)^2/(224*c^3*d^8*(b + 2*c*x)^7) + (b^2 - 4*a*c)/(80*c^3*d^8*(b
 + 2*c*x)^5) - 1/(96*c^3*d^8*(b + 2*c*x)^3)
```

Rubi [A] time = 0.0566795, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$-\frac{(b^2-4ac)^2}{224c^3d^8(b+2cx)^7}+\frac{b^2-4ac}{80c^3d^8(b+2cx)^5}-\frac{1}{96c^3d^8(b+2cx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x)^8,x]
```

```
[Out] -(b^2 - 4*a*c)^2/(224*c^3*d^8*(b + 2*c*x)^7) + (b^2 - 4*a*c)/(80*c^3*d^8*(b
 + 2*c*x)^5) - 1/(96*c^3*d^8*(b + 2*c*x)^3)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx+cx^2)^2}{(bd+2cdx)^8} dx &= \int \left(\frac{(-b^2+4ac)^2}{16c^2d^8(b+2cx)^8}+\frac{-b^2+4ac}{8c^2d^8(b+2cx)^6}+\frac{1}{16c^2d^8(b+2cx)^4}\right) dx\\ &= -\frac{(b^2-4ac)^2}{224c^3d^8(b+2cx)^7}+\frac{b^2-4ac}{80c^3d^8(b+2cx)^5}-\frac{1}{96c^3d^8(b+2cx)^3}\end{aligned}$$

Mathematica [A] time = 0.0319239, size = 59, normalized size = 0.81

$$\frac{42(b^2-4ac)(b+2cx)^2-15(b^2-4ac)^2-35(b+2cx)^4}{3360c^3d^8(b+2cx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x)^8,x]
```

```
[Out] (-15*(b^2 - 4*a*c)^2 + 42*(b^2 - 4*a*c)*(b + 2*c*x)^2 - 35*(b + 2*c*x)^4)/(
3360*c^3*d^8*(b + 2*c*x)^7)
```

Maple [A] time = 0.044, size = 74, normalized size = 1.

$$\frac{1}{d^8}\left(-\frac{1}{96\,c^3\,(2\,cx+b)^3}-\frac{16\,a^2c^2-8\,acb^2+b^4}{224\,c^3\,(2\,cx+b)^7}-\frac{4\,ac-b^2}{80\,c^3\,(2\,cx+b)^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^8,x)
```

```
[Out] 1/d^8*(-1/96/c^3/(2*c*x+b)^3-1/224*(16*a^2*c^2-8*a*b^2*c+b^4)/c^3/(2*c*x+b)
^7-1/80*(4*a*c-b^2)/c^3/(2*c*x+b)^5)
```

Maxima [B] time = 1.13351, size = 238, normalized size = 3.26

$$-\frac{70\,c^4x^4+140\,bc^3x^3+b^4+6\,ab^2c+30\,a^2c^2+84\,\left(b^2c^2+ac^3\right)x^2+14\,\left(b^3c+6\,abc^2\right)x}{420\,\left(128\,c^{10}d^8x^7+448\,bc^9d^8x^6+672\,b^2c^8d^8x^5+560\,b^3c^7d^8x^4+280\,b^4c^6d^8x^3+84\,b^5c^5d^8x^2+14\,b^6c^4d^8x+b^7c^3d^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^8,x, algorithm="maxima")
```

```
[Out] -1/420*(70*c^4*x^4 + 140*b*c^3*x^3 + b^4 + 6*a*b^2*c + 30*a^2*c^2 + 84*(b^2
*c^2 + a*c^3)*x^2 + 14*(b^3*c + 6*a*b*c^2)*x)/(128*c^10*d^8*x^7 + 448*b*c^9
*d^8*x^6 + 672*b^2*c^8*d^8*x^5 + 560*b^3*c^7*d^8*x^4 + 280*b^4*c^6*d^8*x^3
+ 84*b^5*c^5*d^8*x^2 + 14*b^6*c^4*d^8*x + b^7*c^3*d^8)
```

Fricas [B] time = 2.00608, size = 379, normalized size = 5.19

$$-\frac{70\,c^4x^4+140\,bc^3x^3+b^4+6\,ab^2c+30\,a^2c^2+84\,\left(b^2c^2+ac^3\right)x^2+14\,\left(b^3c+6\,abc^2\right)x}{420\,\left(128\,c^{10}d^8x^7+448\,bc^9d^8x^6+672\,b^2c^8d^8x^5+560\,b^3c^7d^8x^4+280\,b^4c^6d^8x^3+84\,b^5c^5d^8x^2+14\,b^6c^4d^8x+b^7c^3d^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^8,x, algorithm="fricas")
```

```
[Out] -1/420*(70*c^4*x^4 + 140*b*c^3*x^3 + b^4 + 6*a*b^2*c + 30*a^2*c^2 + 84*(b^2
*c^2 + a*c^3)*x^2 + 14*(b^3*c + 6*a*b*c^2)*x)/(128*c^10*d^8*x^7 + 448*b*c^9
*d^8*x^6 + 672*b^2*c^8*d^8*x^5 + 560*b^3*c^7*d^8*x^4 + 280*b^4*c^6*d^8*x^3
+ 84*b^5*c^5*d^8*x^2 + 14*b^6*c^4*d^8*x + b^7*c^3*d^8)
```

Sympy [B] time = 2.75623, size = 187, normalized size = 2.56

$$-\frac{30a^2c^2+6ab^2c+b^4+140bc^3x^3+70c^4x^4+x^2\left(84ac^3+84b^2c^2\right)+x\left(84abc^2+14b^3c\right)}{420b^7c^3d^8+5880b^6c^4d^8x+35280b^5c^5d^8x^2+117600b^4c^6d^8x^3+235200b^3c^7d^8x^4+282240b^2c^8d^8x^5+188160bc^9d^8x^6+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(2*c*d*x+b*d)**8,x)
```

```
[Out] -(30*a**2*c**2 + 6*a*b**2*c + b**4 + 140*b*c**3*x**3 + 70*c**4*x**4 + x**2*
(84*a*c**3 + 84*b**2*c**2) + x*(84*a*b*c**2 + 14*b**3*c))/(420*b**7*c**3*d*
*8 + 5880*b**6*c**4*d**8*x + 35280*b**5*c**5*d**8*x**2 + 117600*b**4*c**6*d
**8*x**3 + 235200*b**3*c**7*d**8*x**4 + 282240*b**2*c**8*d**8*x**5 + 188160
*b*c**9*d**8*x**6 + 53760*c**10*d**8*x**7)
```

Giac [A] time = 1.21734, size = 117, normalized size = 1.6

$$-\frac{70\,c^4x^4+140\,bc^3x^3+84\,b^2c^2x^2+84\,ac^3x^2+14\,b^3cx+84\,abc^2x+b^4+6\,ab^2c+30\,a^2c^2}{420\,(2\,cx+b)^7c^3d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^8,x, algorithm="giac")
```

```
[Out] -1/420*(70*c^4*x^4 + 140*b*c^3*x^3 + 84*b^2*c^2*x^2 + 84*a*c^3*x^2 + 14*b^3
*c*x + 84*a*b*c^2*x + b^4 + 6*a*b^2*c + 30*a^2*c^2)/((2*c*x + b)^7*c^3*d^8)
```

3.1134 $\int \frac{(a+bx+cx^2)^2}{(bd+2cdx)^9} dx$

Optimal. Leaf size=73

$$-\frac{\left(b^2-4ac\right)^2}{256c^3d^9(b+2cx)^8}+\frac{b^2-4ac}{96c^3d^9(b+2cx)^6}-\frac{1}{128c^3d^9(b+2cx)^4}$$

```
[Out] -(b^2 - 4*a*c)^2/(256*c^3*d^9*(b + 2*c*x)^8) + (b^2 - 4*a*c)/(96*c^3*d^9*(b
 + 2*c*x)^6) - 1/(128*c^3*d^9*(b + 2*c*x)^4)
```

Rubi [A] time = 0.0548289, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$-\frac{\left(b^2-4ac\right)^2}{256c^3d^9(b+2cx)^8}+\frac{b^2-4ac}{96c^3d^9(b+2cx)^6}-\frac{1}{128c^3d^9(b+2cx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x)^9,x]
```

```
[Out] -(b^2 - 4*a*c)^2/(256*c^3*d^9*(b + 2*c*x)^8) + (b^2 - 4*a*c)/(96*c^3*d^9*(b
 + 2*c*x)^6) - 1/(128*c^3*d^9*(b + 2*c*x)^4)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx+cx^2\right)^2}{(bd+2cdx)^9}\,dx &= \int \left(\frac{\left(-b^2+4ac\right)^2}{16c^2d^9(b+2cx)^9}+\frac{-b^2+4ac}{8c^2d^9(b+2cx)^7}+\frac{1}{16c^2d^9(b+2cx)^5}\right)dx\\ &= -\frac{\left(b^2-4ac\right)^2}{256c^3d^9(b+2cx)^8}+\frac{b^2-4ac}{96c^3d^9(b+2cx)^6}-\frac{1}{128c^3d^9(b+2cx)^4}\end{aligned}$$

Mathematica [A] time = 0.0260712, size = 59, normalized size = 0.81

$$\frac{8\left(b^2-4ac\right)(b+2cx)^2-3\left(b^2-4ac\right)^2-6(b+2cx)^4}{768c^3d^9(b+2cx)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x)^9,x]
```

```
[Out] (-3*(b^2 - 4*a*c)^2 + 8*(b^2 - 4*a*c)*(b + 2*c*x)^2 - 6*(b + 2*c*x)^4)/(768
*c^3*d^9*(b + 2*c*x)^8)
```

Maple [A] time = 0.043, size = 74, normalized size = 1.

$$\frac{1}{d^9}\left(-\frac{4ac-b^2}{96c^3(2cx+b)^6}-\frac{16a^2c^2-8acb^2+b^4}{256c^3(2cx+b)^8}-\frac{1}{128c^3(2cx+b)^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^9,x)
```

```
[Out] 1/d^9*(-1/96*(4*a*c-b^2)/c^3/(2*c*x+b)^6-1/256*(16*a^2*c^2-8*a*b^2*c+b^4)/c
^3/(2*c*x+b)^8-1/128/c^3/(2*c*x+b)^4)
```

Maxima [B] time = 1.06347, size = 259, normalized size = 3.55

$$-\frac{96c^4x^4+192bc^3x^3+b^4+8ab^2c+48a^2c^2+16\left(7b^2c^2+8ac^3\right)x^2+16\left(b^3c+8abc^2\right)x}{768\left(256c^{11}d^9x^8+1024bc^{10}d^9x^7+1792b^2c^9d^9x^6+1792b^3c^8d^9x^5+1120b^4c^7d^9x^4+448b^5c^6d^9x^3+112b^6c^5d^9x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^9,x, algorithm="maxima")
```

```
[Out] -1/768*(96*c^4*x^4 + 192*b*c^3*x^3 + b^4 + 8*a*b^2*c + 48*a^2*c^2 + 16*(7*b
^2*c^2 + 8*a*c^3)*x^2 + 16*(b^3*c + 8*a*b*c^2)*x)/(256*c^11*d^9*x^8 + 1024*
b*c^10*d^9*x^7 + 1792*b^2*c^9*d^9*x^6 + 1792*b^3*c^8*d^9*x^5 + 1120*b^4*c^7
*d^9*x^4 + 448*b^5*c^6*d^9*x^3 + 112*b^6*c^5*d^9*x^2 + 16*b^7*c^4*d^9*x + b
^8*c^3*d^9)
```

Fricas [B] time = 1.97067, size = 423, normalized size = 5.79

$$-\frac{96c^4x^4+192bc^3x^3+b^4+8ab^2c+48a^2c^2+16\left(7b^2c^2+8ac^3\right)x^2+16\left(b^3c+8abc^2\right)x}{768\left(256c^{11}d^9x^8+1024bc^{10}d^9x^7+1792b^2c^9d^9x^6+1792b^3c^8d^9x^5+1120b^4c^7d^9x^4+448b^5c^6d^9x^3+112b^6c^5d^9x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^9,x, algorithm="fricas")
```

```
[Out] -1/768*(96*c^4*x^4 + 192*b*c^3*x^3 + b^4 + 8*a*b^2*c + 48*a^2*c^2 + 16*(7*b
^2*c^2 + 8*a*c^3)*x^2 + 16*(b^3*c + 8*a*b*c^2)*x)/(256*c^11*d^9*x^8 + 1024*
b*c^10*d^9*x^7 + 1792*b^2*c^9*d^9*x^6 + 1792*b^3*c^8*d^9*x^5 + 1120*b^4*c^7
*d^9*x^4 + 448*b^5*c^6*d^9*x^3 + 112*b^6*c^5*d^9*x^2 + 16*b^7*c^4*d^9*x + b
^8*c^3*d^9)
```

Sympy [B] time = 5.11816, size = 202, normalized size = 2.77

$$-\frac{48a^2c^2+8ab^2c+b^4+192bc^3x^3+96c^4x^4+x^2\left(128ac^3+112b^2c^2\right)+x\left(128abc^2+\right.}{768b^8c^3d^9+12288b^7c^4d^9x+86016b^6c^5d^9x^2+344064b^5c^6d^9x^3+860160b^4c^7d^9x^4+1376256b^3c^8d^9x^5+1376256b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(2*c*d*x+b*d)**9,x)
```

```
[Out] -(48*a**2*c**2 + 8*a*b**2*c + b**4 + 192*b*c**3*x**3 + 96*c**4*x**4 + x**2*
(128*a*c**3 + 112*b**2*c**2) + x*(128*a*b*c**2 + 16*b**3*c))/(768*b**8*c**3
*d**9 + 12288*b**7*c**4*d**9*x + 86016*b**6*c**5*d**9*x**2 + 344064*b**5*c*
*6*d**9*x**3 + 860160*b**4*c**7*d**9*x**4 + 1376256*b**3*c**8*d**9*x**5 + 1
376256*b**2*c**9*d**9*x**6 + 786432*b*c**10*d**9*x**7 + 196608*c**11*d**9*x
**8)
```

Giac [A] time = 1.1501, size = 117, normalized size = 1.6

$$-\frac{96\,c^4x^4 + 192\,bc^3x^3 + 112\,b^2c^2x^2 + 128\,ac^3x^2 + 16\,b^3cx + 128\,abc^2x + b^4 + 8\,ab^2c + 48\,a^2c^2}{768\,(2\,cx + b)^8c^3d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^9,x, algorithm="giac")
```

```
[Out] -1/768*(96*c^4*x^4 + 192*b*c^3*x^3 + 112*b^2*c^2*x^2 + 128*a*c^3*x^2 + 16*b
^3*c*x + 128*a*b*c^2*x + b^4 + 8*a*b^2*c + 48*a^2*c^2)/((2*c*x + b)^8*c^3*d
^9)
```

3.1135 $\int \frac{(a+bx+cx^2)^2}{(bd+2cdx)^{10}} dx$

Optimal. Leaf size=73

$$-\frac{(b^2-4ac)^2}{288c^3d^{10}(b+2cx)^9}+\frac{b^2-4ac}{112c^3d^{10}(b+2cx)^7}-\frac{1}{160c^3d^{10}(b+2cx)^5}$$

```
[Out] -(b^2 - 4*a*c)^2/(288*c^3*d^10*(b + 2*c*x)^9) + (b^2 - 4*a*c)/(112*c^3*d^10
*(b + 2*c*x)^7) - 1/(160*c^3*d^10*(b + 2*c*x)^5)
```

Rubi [A] time = 0.0575813, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$-\frac{(b^2-4ac)^2}{288c^3d^{10}(b+2cx)^9}+\frac{b^2-4ac}{112c^3d^{10}(b+2cx)^7}-\frac{1}{160c^3d^{10}(b+2cx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x)^10,x]
```

```
[Out] -(b^2 - 4*a*c)^2/(288*c^3*d^10*(b + 2*c*x)^9) + (b^2 - 4*a*c)/(112*c^3*d^10
*(b + 2*c*x)^7) - 1/(160*c^3*d^10*(b + 2*c*x)^5)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx+cx^2)^2}{(bd+2cdx)^{10}}\,dx &= \int \left(\frac{(-b^2+4ac)^2}{16c^2d^{10}(b+2cx)^{10}}+\frac{-b^2+4ac}{8c^2d^{10}(b+2cx)^8}+\frac{1}{16c^2d^{10}(b+2cx)^6}\right)dx\\ &= -\frac{(b^2-4ac)^2}{288c^3d^{10}(b+2cx)^9}+\frac{b^2-4ac}{112c^3d^{10}(b+2cx)^7}-\frac{1}{160c^3d^{10}(b+2cx)^5}\end{aligned}$$

Mathematica [A] time = 0.0404149, size = 59, normalized size = 0.81

$$\frac{90(b^2-4ac)(b+2cx)^2-35(b^2-4ac)^2-63(b+2cx)^4}{10080c^3d^{10}(b+2cx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x)^10,x]
```

```
[Out] (-35*(b^2 - 4*a*c)^2 + 90*(b^2 - 4*a*c)*(b + 2*c*x)^2 - 63*(b + 2*c*x)^4)/(
10080*c^3*d^10*(b + 2*c*x)^9)
```

Maple [A] time = 0.046, size = 74, normalized size = 1.

$$\frac{1}{d^{10}}\left(-\frac{4ac-b^{2}}{112\,c^{3}\left(2cx+b\right)^{7}}-\frac{1}{160\,c^{3}\left(2cx+b\right)^{5}}-\frac{16\,a^{2}c^{2}-8\,acb^{2}+b^{4}}{288\,c^{3}\left(2cx+b\right)^{9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^10,x)
```

```
[Out] 1/d^10*(-1/112*(4*a*c-b^2)/c^3/(2*c*x+b)^7-1/160/c^3/(2*c*x+b)^5-1/288*(16*
a^2*c^2-8*a*b^2*c+b^4)/c^3/(2*c*x+b)^9)
```

Maxima [B] time = 1.48932, size = 278, normalized size = 3.81

$$-\frac{126\,c^{4}x^{4}+252\,bc^{3}x^{3}+b^{4}+10\,ab^{2}c+70\,a^{2}c^{2}+36\left(4\,b^{2}c^{2}+5\,ac^{3}\right)x^{2}+18\left(b^{3}c+\right.}{1260\left(512\,c^{12}d^{10}x^{9}+2304\,bc^{11}d^{10}x^{8}+4608\,b^{2}c^{10}d^{10}x^{7}+5376\,b^{3}c^{9}d^{10}x^{6}+4032\,b^{4}c^{8}d^{10}x^{5}+2016\,b^{5}c^{7}d^{10}x^{4}+672\,b^{6}c^{6}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^10,x, algorithm="maxima")
```

```
[Out] -1/1260*(126*c^4*x^4 + 252*b*c^3*x^3 + b^4 + 10*a*b^2*c + 70*a^2*c^2 + 36*(
4*b^2*c^2 + 5*a*c^3)*x^2 + 18*(b^3*c + 10*a*b*c^2)*x)/(512*c^12*d^10*x^9 +
2304*b*c^11*d^10*x^8 + 4608*b^2*c^10*d^10*x^7 + 5376*b^3*c^9*d^10*x^6 + 403
2*b^4*c^8*d^10*x^5 + 2016*b^5*c^7*d^10*x^4 + 672*b^6*c^6*d^10*x^3 + 144*b^7
*c^5*d^10*x^2 + 18*b^8*c^4*d^10*x + b^9*c^3*d^10)
```

Fricas [B] time = 2.05418, size = 474, normalized size = 6.49

$$-\frac{126\,c^{4}x^{4}+252\,bc^{3}x^{3}+b^{4}+10\,ab^{2}c+70\,a^{2}c^{2}+36\left(4\,b^{2}c^{2}+5\,ac^{3}\right)x^{2}+18\left(b^{3}c+\right.}{1260\left(512\,c^{12}d^{10}x^{9}+2304\,bc^{11}d^{10}x^{8}+4608\,b^{2}c^{10}d^{10}x^{7}+5376\,b^{3}c^{9}d^{10}x^{6}+4032\,b^{4}c^{8}d^{10}x^{5}+2016\,b^{5}c^{7}d^{10}x^{4}+672\,b^{6}c^{6}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^10,x, algorithm="fricas")
```

```
[Out] -1/1260*(126*c^4*x^4 + 252*b*c^3*x^3 + b^4 + 10*a*b^2*c + 70*a^2*c^2 + 36*(
4*b^2*c^2 + 5*a*c^3)*x^2 + 18*(b^3*c + 10*a*b*c^2)*x)/(512*c^12*d^10*x^9 +
2304*b*c^11*d^10*x^8 + 4608*b^2*c^10*d^10*x^7 + 5376*b^3*c^9*d^10*x^6 + 403
2*b^4*c^8*d^10*x^5 + 2016*b^5*c^7*d^10*x^4 + 672*b^6*c^6*d^10*x^3 + 144*b^7
*c^5*d^10*x^2 + 18*b^8*c^4*d^10*x + b^9*c^3*d^10)
```

Sympy [B] time = 4.73929, size = 218, normalized size = 2.99

$$-\frac{70a^{2}c^{2}+10ab^{2}c+b^{4}+252bc^{3}x^{3}+126c^{4}x^{4}+x^{2}\left(180ac^{3}+144b^{2}c^{2}\right.}{1260b^{9}c^{3}d^{10}+22680b^{8}c^{4}d^{10}x+181440b^{7}c^{5}d^{10}x^{2}+846720b^{6}c^{6}d^{10}x^{3}+2540160b^{5}c^{7}d^{10}x^{4}+5080320b^{4}c^{8}d^{10}x^{5}+6773}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(2*c*d*x+b*d)**10,x)
```

```
[Out] -(70*a**2*c**2 + 10*a*b**2*c + b**4 + 252*b*c**3*x**3 + 126*c**4*x**4 + x**
2*(180*a*c**3 + 144*b**2*c**2) + x*(180*a*b*c**2 + 18*b**3*c))/(1260*b**9*c
**3*d**10 + 22680*b**8*c**4*d**10*x + 181440*b**7*c**5*d**10*x**2 + 846720*
b**6*c**6*d**10*x**3 + 2540160*b**5*c**7*d**10*x**4 + 5080320*b**4*c**8*d**
10*x**5 + 6773760*b**3*c**9*d**10*x**6 + 5806080*b**2*c**10*d**10*x**7 + 29
03040*b*c**11*d**10*x**8 + 645120*c**12*d**10*x**9)
```

Giac [A] time = 1.13713, size = 117, normalized size = 1.6

$$-\frac{126\,c^4x^4+252\,bc^3x^3+144\,b^2c^2x^2+180\,ac^3x^2+18\,b^3cx+180\,abc^2x+b^4+10\,ab^2c+70\,a^2c^2}{1260\,(2\,cx+b)^9c^3d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^10,x, algorithm="giac")
```

```
[Out] -1/1260*(126*c^4*x^4 + 252*b*c^3*x^3 + 144*b^2*c^2*x^2 + 180*a*c^3*x^2 + 18
*b^3*c*x + 180*a*b*c^2*x + b^4 + 10*a*b^2*c + 70*a^2*c^2)/((2*c*x + b)^9*c^
3*d^10)
```

3.1136 $\int \frac{(a+bx+cx^2)^2}{(bd+2cdx)^{11}}\,dx$

Optimal. Leaf size=73

$$-\frac{(b^2-4ac)^2}{320c^3d^{11}(b+2cx)^{10}}+\frac{b^2-4ac}{128c^3d^{11}(b+2cx)^8}-\frac{1}{192c^3d^{11}(b+2cx)^6}$$

```
[Out] -(b^2 - 4*a*c)^2/(320*c^3*d^11*(b + 2*c*x)^10) + (b^2 - 4*a*c)/(128*c^3*d^1
1*(b + 2*c*x)^8) - 1/(192*c^3*d^11*(b + 2*c*x)^6)
```

Rubi [A] time = 0.0542056, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$-\frac{(b^2-4ac)^2}{320c^3d^{11}(b+2cx)^{10}}+\frac{b^2-4ac}{128c^3d^{11}(b+2cx)^8}-\frac{1}{192c^3d^{11}(b+2cx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x)^11,x]
```

```
[Out] -(b^2 - 4*a*c)^2/(320*c^3*d^11*(b + 2*c*x)^10) + (b^2 - 4*a*c)/(128*c^3*d^1
1*(b + 2*c*x)^8) - 1/(192*c^3*d^11*(b + 2*c*x)^6)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx+cx^2)^2}{(bd+2cdx)^{11}}\,dx &= \int \left(\frac{(-b^2+4ac)^2}{16c^2d^{11}(b+2cx)^{11}}+\frac{-b^2+4ac}{8c^2d^{11}(b+2cx)^9}+\frac{1}{16c^2d^{11}(b+2cx)^7}\right)dx\\ &= -\frac{(b^2-4ac)^2}{320c^3d^{11}(b+2cx)^{10}}+\frac{b^2-4ac}{128c^3d^{11}(b+2cx)^8}-\frac{1}{192c^3d^{11}(b+2cx)^6}\end{aligned}$$

Mathematica [A] time = 0.0324032, size = 59, normalized size = 0.81

$$\frac{15(b^2-4ac)(b+2cx)^2-6(b^2-4ac)^2-10(b+2cx)^4}{1920c^3d^{11}(b+2cx)^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x)^11,x]
```

```
[Out] (-6*(b^2 - 4*a*c)^2 + 15*(b^2 - 4*a*c)*(b + 2*c*x)^2 - 10*(b + 2*c*x)^4)/(1
920*c^3*d^11*(b + 2*c*x)^10)
```

Maple [A] time = 0.046, size = 74, normalized size = 1.

$$\frac{1}{d^{11}}\left(-\frac{1}{192\,c^3\,(2\,cx+b)^6}-\frac{16\,a^2c^2-8\,acb^2+b^4}{320\,c^3\,(2\,cx+b)^{10}}-\frac{4\,ac-b^2}{128\,c^3\,(2\,cx+b)^8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^11,x)
```

```
[Out] 1/d^11*(-1/192/c^3/(2*c*x+b)^6-1/320*(16*a^2*c^2-8*a*b^2*c+b^4)/c^3/(2*c*x+
b)^10-1/128*(4*a*c-b^2)/c^3/(2*c*x+b)^8)
```

Maxima [B] time = 1.21255, size = 297, normalized size = 4.07

$$-\frac{160\,c^4x^4+320\,bc^3x^3+b^4+12\,ab^2c+96\,a^2c^2+60\,(3\,b^2c^2+4\,ac^3}{1920\,(1024\,c^{13}d^{11}x^{10}+5120\,bc^{12}d^{11}x^9+11520\,b^2c^{11}d^{11}x^8+15360\,b^3c^{10}d^{11}x^7+13440\,b^4c^9d^{11}x^6+8064\,b^5c^8d^{11}x^5+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^11,x, algorithm="maxima")
```

```
[Out] -1/1920*(160*c^4*x^4 + 320*b*c^3*x^3 + b^4 + 12*a*b^2*c + 96*a^2*c^2 + 60*(
3*b^2*c^2 + 4*a*c^3)*x^2 + 20*(b^3*c + 12*a*b*c^2)*x)/(1024*c^13*d^11*x^10
+ 5120*b*c^12*d^11*x^9 + 11520*b^2*c^11*d^11*x^8 + 15360*b^3*c^10*d^11*x^7
+ 13440*b^4*c^9*d^11*x^6 + 8064*b^5*c^8*d^11*x^5 + 3360*b^6*c^7*d^11*x^4 +
960*b^7*c^6*d^11*x^3 + 180*b^8*c^5*d^11*x^2 + 20*b^9*c^4*d^11*x + b^10*c^3*
d^11)
```

Fricas [B] time = 1.97898, size = 516, normalized size = 7.07

$$-\frac{160\,c^4x^4+320\,bc^3x^3+b^4+12\,ab^2c+96\,a^2c^2+60\,(3\,b^2c^2+4\,ac^3}{1920\,(1024\,c^{13}d^{11}x^{10}+5120\,bc^{12}d^{11}x^9+11520\,b^2c^{11}d^{11}x^8+15360\,b^3c^{10}d^{11}x^7+13440\,b^4c^9d^{11}x^6+8064\,b^5c^8d^{11}x^5+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^11,x, algorithm="fricas")
```

```
[Out] -1/1920*(160*c^4*x^4 + 320*b*c^3*x^3 + b^4 + 12*a*b^2*c + 96*a^2*c^2 + 60*(
3*b^2*c^2 + 4*a*c^3)*x^2 + 20*(b^3*c + 12*a*b*c^2)*x)/(1024*c^13*d^11*x^10
+ 5120*b*c^12*d^11*x^9 + 11520*b^2*c^11*d^11*x^8 + 15360*b^3*c^10*d^11*x^7
+ 13440*b^4*c^9*d^11*x^6 + 8064*b^5*c^8*d^11*x^5 + 3360*b^6*c^7*d^11*x^4 +
960*b^7*c^6*d^11*x^3 + 180*b^8*c^5*d^11*x^2 + 20*b^9*c^4*d^11*x + b^10*c^3*
d^11)
```

Sympy [B] time = 9.00974, size = 233, normalized size = 3.19

$$-\frac{96a^2c^2+12ab^2c+b^4+320bc^3x^3+160c^4x^4+x^2}{1920b^{10}c^3d^{11}+38400b^9c^4d^{11}x+345600b^8c^5d^{11}x^2+1843200b^7c^6d^{11}x^3+6451200b^6c^7d^{11}x^4+15482880b^5c^8d^{11}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(2*c*d*x+b*d)**11,x)
```

```
[Out] -(96*a**2*c**2 + 12*a*b**2*c + b**4 + 320*b*c**3*x**3 + 160*c**4*x**4 + x**
2*(240*a*c**3 + 180*b**2*c**2) + x*(240*a*b*c**2 + 20*b**3*c))/(1920*b**10*
c**3*d**11 + 38400*b**9*c**4*d**11*x + 345600*b**8*c**5*d**11*x**2 + 184320
0*b**7*c**6*d**11*x**3 + 6451200*b**6*c**7*d**11*x**4 + 15482880*b**5*c**8*
d**11*x**5 + 25804800*b**4*c**9*d**11*x**6 + 29491200*b**3*c**10*d**11*x**7
 + 22118400*b**2*c**11*d**11*x**8 + 9830400*b*c**12*d**11*x**9 + 1966080*c*
*13*d**11*x**10)
```

Giac [A] time = 1.17609, size = 117, normalized size = 1.6

$$-\frac{160\,c^4x^4 + 320\,bc^3x^3 + 180\,b^2c^2x^2 + 240\,ac^3x^2 + 20\,b^3cx + 240\,abc^2x + b^4 + 12\,ab^2c + 96\,a^2c^2}{1920\,(2\,cx + b)^{10}c^3d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^11,x, algorithm="giac")
```

```
[Out] -1/1920*(160*c^4*x^4 + 320*b*c^3*x^3 + 180*b^2*c^2*x^2 + 240*a*c^3*x^2 + 20
*b^3*c*x + 240*a*b*c^2*x + b^4 + 12*a*b^2*c + 96*a^2*c^2)/((2*c*x + b)^10*c
^3*d^11)
```

3.1137 $\int (bd+2cdx)^5\left(a+bx+cx^2\right)^3\,dx$

Optimal. Leaf size=101

$$-\frac{3d^5\left(b^2-4ac\right)(b+2cx)^{10}}{1280c^4}+\frac{3d^5\left(b^2-4ac\right)^2(b+2cx)^8}{1024c^4}-\frac{d^5\left(b^2-4ac\right)^3(b+2cx)^6}{768c^4}+\frac{d^5(b+2cx)^{12}}{1536c^4}$$

```
[Out] -((b^2 - 4*a*c)^3*d^5*(b + 2*c*x)^6)/(768*c^4) + (3*(b^2 - 4*a*c)^2*d^5*(b
+ 2*c*x)^8)/(1024*c^4) - (3*(b^2 - 4*a*c)*d^5*(b + 2*c*x)^10)/(1280*c^4) +
(d^5*(b + 2*c*x)^12)/(1536*c^4)
```

Rubi [A] time = 0.244876, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$-\frac{3d^5\left(b^2-4ac\right)(b+2cx)^{10}}{1280c^4}+\frac{3d^5\left(b^2-4ac\right)^2(b+2cx)^8}{1024c^4}-\frac{d^5\left(b^2-4ac\right)^3(b+2cx)^6}{768c^4}+\frac{d^5(b+2cx)^{12}}{1536c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^5*(a + b*x + c*x^2)^3,x]
```

```
[Out] -((b^2 - 4*a*c)^3*d^5*(b + 2*c*x)^6)/(768*c^4) + (3*(b^2 - 4*a*c)^2*d^5*(b
+ 2*c*x)^8)/(1024*c^4) - (3*(b^2 - 4*a*c)*d^5*(b + 2*c*x)^10)/(1280*c^4) +
(d^5*(b + 2*c*x)^12)/(1536*c^4)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] && !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\int (bd+2cdx)^5\left(a+bx+cx^2\right)^3\,dx=\int\left(\frac{\left(-b^2+4ac\right)^3(bd+2cdx)^5}{64c^3}+\frac{3\left(-b^2+4ac\right)^2(bd+2cdx)^7}{64c^3d^2}+\frac{3\left(-b^2+4ac\right)(b}{64c^3d^{}}\right.$$

$$=-\frac{\left(b^2-4ac\right)^3d^5(b+2cx)^6}{768c^4}+\frac{3\left(b^2-4ac\right)^2d^5(b+2cx)^8}{1024c^4}-\frac{3\left(b^2-4ac\right)d^5(b+2cx}{1280c^4}$$

Mathematica [B] time = 0.084249, size = 224, normalized size = 2.22

$$\frac{1}{60}d^5x(b+cx)\left(20a^3\left(28b^2c^2x^2+12b^3cx+3b^4+32bc^3x^3+16c^4x^4\right)+30a^2x\left(56b^3c^2x^2+88b^2c^3x^3+19b^4cx+3b^5+7\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^5*(a + b*x + c*x^2)^3,x]
```

```
[Out] (d^5*x*(b + c*x)*(20*a^3*(3*b^4 + 12*b^3*c*x + 28*b^2*c^2*x^2 + 32*b*c^3*x^
3 + 16*c^4*x^4) + 12*a*x^2*(b + c*x)^2*(5*b^4 + 30*b^3*c*x + 78*b^2*c^2*x^2
```

```
 + 96*b*c^3*x^3 + 48*c^4*x^4) + x^3*(b + c*x)^3*(15*b^4 + 96*b^3*c*x + 256*
b^2*c^2*x^2 + 320*b*c^3*x^3 + 160*c^4*x^4) + 30*a^2*x*(3*b^5 + 19*b^4*c*x +
 56*b^3*c^2*x^2 + 88*b^2*c^3*x^3 + 72*b*c^4*x^4 + 24*c^5*x^5)))/60
```

Maple [B] time = 0.041, size = 810, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^5*(c*x^2+b*x+a)^3,x)
```

```
[Out] 8/3*c^8*d^5*x^12+16*b*d^5*c^7*x^11+1/10*(320*b^2*d^5*c^6+32*c^5*d^5*(a*c^2+
2*b^2*c+c*(2*a*c+b^2)))*x^10+1/9*(280*b^3*d^5*c^5+80*b*d^5*c^4*(a*c^2+2*b^2
*c+c*(2*a*c+b^2))+32*c^5*d^5*(4*a*b*c+b*(2*a*c+b^2)))*x^9+1/8*(130*b^4*d^5*
c^4+80*b^2*d^5*c^3*(a*c^2+2*b^2*c+c*(2*a*c+b^2))+80*b*d^5*c^4*(4*a*b*c+b*(2
*a*c+b^2))+32*c^5*d^5*(a*(2*a*c+b^2)+2*b^2*a+a^2*c))*x^8+1/7*(31*b^5*d^5*c^
3+40*b^3*d^5*c^2*(a*c^2+2*b^2*c+c*(2*a*c+b^2))+80*b^2*d^5*c^3*(4*a*b*c+b*(2
*a*c+b^2))+80*b*d^5*c^4*(a*(2*a*c+b^2)+2*b^2*a+a^2*c)+96*c^5*d^5*b*a^2)*x^7
+1/6*(3*b^6*d^5*c^2+10*b^4*d^5*c*(a*c^2+2*b^2*c+c*(2*a*c+b^2))+40*b^3*d^5*c
^2*(4*a*b*c+b*(2*a*c+b^2))+80*b^2*d^5*c^3*(a*(2*a*c+b^2)+2*b^2*a+a^2*c)+240
*b^2*d^5*c^4*a^2+32*c^5*d^5*a^3)*x^6+1/5*(b^5*d^5*(a*c^2+2*b^2*c+c*(2*a*c+b
^2))+10*b^4*d^5*c*(4*a*b*c+b*(2*a*c+b^2))+40*b^3*d^5*c^2*(a*(2*a*c+b^2)+2*b
^2*a+a^2*c)+240*b^3*d^5*c^3*a^2+80*b*d^5*c^4*a^3)*x^5+1/4*(b^5*d^5*(4*a*b*c
+b*(2*a*c+b^2))+10*b^4*d^5*c*(a*(2*a*c+b^2)+2*b^2*a+a^2*c)+120*b^4*d^5*c^2*
a^2+80*b^2*d^5*c^3*a^3)*x^4+1/3*(b^5*d^5*(a*(2*a*c+b^2)+2*b^2*a+a^2*c)+30*b
^5*d^5*c*a^2+40*b^3*d^5*c^2*a^3)*x^3+1/2*(10*a^3*b^4*c*d^5+3*a^2*b^6*d^5)*x
^2+b^5*d^5*a^3*x
```

Maxima [B] time = 1.1377, size = 462, normalized size = 4.57

$$\frac{8}{3}c^8d^5x^{12} + 16\,bc^7d^5x^{11} + \frac{16}{5}\left(13\,b^2c^6 + 3\,ac^7\right)d^5x^{10} + \frac{8}{3}\left(23\,b^3c^5 + 18\,abc^6\right)d^5x^9 + a^3b^5d^5x + \frac{3}{4}\left(75\,b^4c^4 + 136\,ab^2c^5 + 16\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^5*(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 8/3*c^8*d^5*x^12 + 16*b*c^7*d^5*x^11 + 16/5*(13*b^2*c^6 + 3*a*c^7)*d^5*x^10
 + 8/3*(23*b^3*c^5 + 18*a*b*c^6)*d^5*x^9 + a^3*b^5*d^5*x + 3/4*(75*b^4*c^4
+ 136*a*b^2*c^5 + 16*a^2*c^6)*d^5*x^8 + 3*(11*b^5*c^3 + 40*a*b^3*c^4 + 16*a
^2*b*c^5)*d^5*x^7 + 1/6*(73*b^6*c^2 + 510*a*b^4*c^3 + 480*a^2*b^2*c^4 + 32*
a^3*c^5)*d^5*x^6 + 1/5*(13*b^7*c + 183*a*b^5*c^2 + 360*a^2*b^3*c^3 + 80*a^3
*b*c^4)*d^5*x^5 + 1/4*(b^8 + 36*a*b^6*c + 150*a^2*b^4*c^2 + 80*a^3*b^2*c^3)
*d^5*x^4 + 1/3*(3*a*b^7 + 33*a^2*b^5*c + 40*a^3*b^3*c^2)*d^5*x^3 + 1/2*(3*a
^2*b^6 + 10*a^3*b^4*c)*d^5*x^2
```

Fricas [B] time = 1.72415, size = 907, normalized size = 8.98

$$\frac{8}{3}x^{12}d^5c^8 + 16x^{11}d^5c^7b + \frac{208}{5}x^{10}d^5c^6b^2 + \frac{48}{5}x^{10}d^5c^7a + \frac{184}{3}x^9d^5c^5b^3 + 48x^9d^5c^6ba + \frac{225}{4}x^8d^5c^4b^4 + 102x^8d^5c^5b^2a + 12$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^5*(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 8/3*x^12*d^5*c^8 + 16*x^11*d^5*c^7*b + 208/5*x^10*d^5*c^6*b^2 + 48/5*x^10*d
^5*c^7*a + 184/3*x^9*d^5*c^5*b^3 + 48*x^9*d^5*c^6*b*a + 225/4*x^8*d^5*c^4*b
^4 + 102*x^8*d^5*c^5*b^2*a + 12*x^8*d^5*c^6*a^2 + 33*x^7*d^5*c^3*b^5 + 120*
x^7*d^5*c^4*b^3*a + 48*x^7*d^5*c^5*b*a^2 + 73/6*x^6*d^5*c^2*b^6 + 85*x^6*d^
5*c^3*b^4*a + 80*x^6*d^5*c^4*b^2*a^2 + 16/3*x^6*d^5*c^5*a^3 + 13/5*x^5*d^5*
c*b^7 + 183/5*x^5*d^5*c^2*b^5*a + 72*x^5*d^5*c^3*b^3*a^2 + 16*x^5*d^5*c^4*b
*a^3 + 1/4*x^4*d^5*b^8 + 9*x^4*d^5*c*b^6*a + 75/2*x^4*d^5*c^2*b^4*a^2 + 20*
x^4*d^5*c^3*b^2*a^3 + x^3*d^5*b^7*a + 11*x^3*d^5*c*b^5*a^2 + 40/3*x^3*d^5*c
^2*b^3*a^3 + 3/2*x^2*d^5*b^6*a^2 + 5*x^2*d^5*c*b^4*a^3 + x*d^5*b^5*a^3
```

Sympy [B] time = 0.127715, size = 428, normalized size = 4.24

$$a^3b^5d^5x + 16bc^7d^5x^{11} + \frac{8c^8d^5x^{12}}{3} + x^{10}\left(\frac{48ac^7d^5}{5} + \frac{208b^2c^6d^5}{5}\right) + x^9\left(48abc^6d^5 + \frac{184b^3c^5d^5}{3}\right) + x^8\left(12a^2c^6d^5 + 102a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**5*(c*x**2+b*x+a)**3,x)
```

```
[Out] a**3*b**5*d**5*x + 16*b*c**7*d**5*x**11 + 8*c**8*d**5*x**12/3 + x**10*(48*a
*c**7*d**5/5 + 208*b**2*c**6*d**5/5) + x**9*(48*a*b*c**6*d**5 + 184*b**3*c*
*5*d**5/3) + x**8*(12*a**2*c**6*d**5 + 102*a*b**2*c**5*d**5 + 225*b**4*c**4
*d**5/4) + x**7*(48*a**2*b*c**5*d**5 + 120*a*b**3*c**4*d**5 + 33*b**5*c**3*
d**5) + x**6*(16*a**3*c**5*d**5/3 + 80*a**2*b**2*c**4*d**5 + 85*a*b**4*c**3
*d**5 + 73*b**6*c**2*d**5/6) + x**5*(16*a**3*b*c**4*d**5 + 72*a**2*b**3*c**
3*d**5 + 183*a*b**5*c**2*d**5/5 + 13*b**7*c*d**5/5) + x**4*(20*a**3*b**2*c*
*3*d**5 + 75*a**2*b**4*c**2*d**5/2 + 9*a*b**6*c*d**5 + b**8*d**5/4) + x**3*
(40*a**3*b**3*c**2*d**5/3 + 11*a**2*b**5*c*d**5 + a*b**7*d**5) + x**2*(5*a*
*3*b**4*c*d**5 + 3*a**2*b**6*d**5/2)
```

Giac [B] time = 1.23557, size = 572, normalized size = 5.66

$$\frac{8}{3}c^8d^5x^{12} + 16\,bc^7d^5x^{11} + \frac{208}{5}b^2c^6d^5x^{10} + \frac{48}{5}ac^7d^5x^{10} + \frac{184}{3}b^3c^5d^5x^9 + 48\,abc^6d^5x^9 + \frac{225}{4}b^4c^4d^5x^8 + 102\,ab^2c^5d^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^5*(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 8/3*c^8*d^5*x^12 + 16*b*c^7*d^5*x^11 + 208/5*b^2*c^6*d^5*x^10 + 48/5*a*c^7*
d^5*x^10 + 184/3*b^3*c^5*d^5*x^9 + 48*a*b*c^6*d^5*x^9 + 225/4*b^4*c^4*d^5*x
^8 + 102*a*b^2*c^5*d^5*x^8 + 12*a^2*c^6*d^5*x^8 + 33*b^5*c^3*d^5*x^7 + 120*
a*b^3*c^4*d^5*x^7 + 48*a^2*b*c^5*d^5*x^7 + 73/6*b^6*c^2*d^5*x^6 + 85*a*b^4*
c^3*d^5*x^6 + 80*a^2*b^2*c^4*d^5*x^6 + 16/3*a^3*c^5*d^5*x^6 + 13/5*b^7*c*d^
5*x^5 + 183/5*a*b^5*c^2*d^5*x^5 + 72*a^2*b^3*c^3*d^5*x^5 + 16*a^3*b*c^4*d^5
*x^5 + 1/4*b^8*d^5*x^4 + 9*a*b^6*c*d^5*x^4 + 75/2*a^2*b^4*c^2*d^5*x^4 + 20*
a^3*b^2*c^3*d^5*x^4 + a*b^7*d^5*x^3 + 11*a^2*b^5*c*d^5*x^3 + 40/3*a^3*b^3*c
^2*d^5*x^3 + 3/2*a^2*b^6*d^5*x^2 + 5*a^3*b^4*c*d^5*x^2 + a^3*b^5*d^5*x
```

3.1138 $\int (bd+2cdx)^4 \left(a+bx+cx^2\right)^3 \, dx$

Optimal. Leaf size=101

$$-\frac{d^4 \left(b^2-4ac\right) (b+2cx)^9}{384c^4}+\frac{3d^4 \left(b^2-4ac\right)^2 (b+2cx)^7}{896c^4}-\frac{d^4 \left(b^2-4ac\right)^3 (b+2cx)^5}{640c^4}+\frac{d^4(b+2cx)^{11}}{1408c^4}$$

```
[Out] -((b^2 - 4*a*c)^3*d^4*(b + 2*c*x)^5)/(640*c^4) + (3*(b^2 - 4*a*c)^2*d^4*(b
+ 2*c*x)^7)/(896*c^4) - ((b^2 - 4*a*c)*d^4*(b + 2*c*x)^9)/(384*c^4) + (d^4*
(b + 2*c*x)^11)/(1408*c^4)
```

Rubi [A] time = 0.191634, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$-\frac{d^4 \left(b^2-4ac\right) (b+2cx)^9}{384c^4}+\frac{3d^4 \left(b^2-4ac\right)^2 (b+2cx)^7}{896c^4}-\frac{d^4 \left(b^2-4ac\right)^3 (b+2cx)^5}{640c^4}+\frac{d^4(b+2cx)^{11}}{1408c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^4*(a + b*x + c*x^2)^3,x]
```

```
[Out] -((b^2 - 4*a*c)^3*d^4*(b + 2*c*x)^5)/(640*c^4) + (3*(b^2 - 4*a*c)^2*d^4*(b
+ 2*c*x)^7)/(896*c^4) - ((b^2 - 4*a*c)*d^4*(b + 2*c*x)^9)/(384*c^4) + (d^4*
(b + 2*c*x)^11)/(1408*c^4)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] && !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\int (bd+2cdx)^4 \left(a+bx+cx^2\right)^3 \, dx = \int \left(\frac{\left(-b^2+4ac\right)^3 (bd+2cdx)^4}{64c^3}+\frac{3\left(-b^2+4ac\right)^2 (bd+2cdx)^6}{64c^3d^2}+\frac{3\left(-b^2+4ac\right) (bd+\right.$$

$$= -\frac{\left(b^2-4ac\right)^3 d^4(b+2cx)^5}{640c^4}+\frac{3\left(b^2-4ac\right)^2 d^4(b+2cx)^7}{896c^4}-\frac{\left(b^2-4ac\right) d^4(b+2cx)^9}{384c^4}+$$

Mathematica [B] time = 0.042036, size = 259, normalized size = 2.56

$$d^4 \left(\frac{3}{7}c^3x^7 \left(16a^2c^2+104ab^2c+43b^4\right)+\frac{1}{2}bc^2x^6 \left(48a^2c^2+88ab^2c+17b^4\right)+\frac{1}{5}cx^5 \left(168a^2b^2c^2+16a^3c^3+123ab^4c+11b^6\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^4*(a + b*x + c*x^2)^3,x]
```

```
[Out] d^4*(a^3*b^4*x + (a^2*b^3*(3*b^2 + 8*a*c)*x^2)/2 + a*b^2*(b^4 + 9*a*b^2*c +
 8*a^2*c^2)*x^3 + (b*(b^6 + 30*a*b^4*c + 96*a^2*b^2*c^2 + 32*a^3*c^3)*x^4)/
```

```
4 + (c*(11*b^6 + 123*a*b^4*c + 168*a^2*b^2*c^2 + 16*a^3*c^3)*x^5)/5 + (b*c^
2*(17*b^4 + 88*a*b^2*c + 48*a^2*c^2)*x^6)/2 + (3*c^3*(43*b^4 + 104*a*b^2*c
+ 16*a^2*c^2)*x^7)/7 + 24*b*c^4*(b^2 + a*c)*x^8 + (8*c^5*(7*b^2 + 2*a*c)*x^
9)/3 + 8*b*c^6*x^10 + (16*c^7*x^11)/11)
```

Maple [B] time = 0.038, size = 672, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^4*(c*x^2+b*x+a)^3,x)
```

```
[Out] 16/11*c^7*d^4*x^11+8*b*d^4*c^6*x^10+1/9*(120*b^2*d^4*c^5+16*c^4*d^4*(a*c^2+
2*b^2*c+c*(2*a*c+b^2)))*x^9+1/8*(80*b^3*d^4*c^4+32*b*d^4*c^3*(a*c^2+2*b^2*c
+c*(2*a*c+b^2))+16*c^4*d^4*(4*a*b*c+b*(2*a*c+b^2)))*x^8+1/7*(25*b^4*d^4*c^3
+24*b^2*d^4*c^2*(a*c^2+2*b^2*c+c*(2*a*c+b^2))+32*b*d^4*c^3*(4*a*b*c+b*(2*a*
c+b^2))+16*c^4*d^4*(a*(2*a*c+b^2)+2*b^2*a+a^2*c))*x^7+1/6*(3*b^5*d^4*c^2+8*
b^3*d^4*c*(a*c^2+2*b^2*c+c*(2*a*c+b^2))+24*b^2*d^4*c^2*(4*a*b*c+b*(2*a*c+b^
2))+32*b*d^4*c^3*(a*(2*a*c+b^2)+2*b^2*a+a^2*c)+48*c^4*d^4*b*a^2)*x^6+1/5*(b
^4*d^4*(a*c^2+2*b^2*c+c*(2*a*c+b^2))+8*b^3*d^4*c*(4*a*b*c+b*(2*a*c+b^2))+24
*b^2*d^4*c^2*(a*(2*a*c+b^2)+2*b^2*a+a^2*c)+96*b^2*d^4*c^3*a^2+16*c^4*d^4*a^
3)*x^5+1/4*(b^4*d^4*(4*a*b*c+b*(2*a*c+b^2))+8*b^3*d^4*c*(a*(2*a*c+b^2)+2*b^
2*a+a^2*c)+72*b^3*d^4*c^2*a^2+32*b*d^4*c^3*a^3)*x^4+1/3*(b^4*d^4*(a*(2*a*c+
b^2)+2*b^2*a+a^2*c)+24*b^4*d^4*c*a^2+24*b^2*d^4*c^2*a^3)*x^3+1/2*(8*a^3*b^3
*c*d^4+3*a^2*b^5*d^4)*x^2+b^4*d^4*a^3*x
```

Maxima [B] time = 1.16763, size = 392, normalized size = 3.88

$$\frac{16}{11}\,c^7d^4x^{11} + 8\,bc^6d^4x^{10} + \frac{8}{3}\left(7\,b^2c^5 + 2\,ac^6\right)d^4x^9 + 24\left(b^3c^4 + abc^5\right)d^4x^8 + a^3b^4d^4x + \frac{3}{7}\left(43\,b^4c^3 + 104\,ab^2c^4 + 16\,a^2c^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4*(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 16/11*c^7*d^4*x^11 + 8*b*c^6*d^4*x^10 + 8/3*(7*b^2*c^5 + 2*a*c^6)*d^4*x^9 +
 24*(b^3*c^4 + a*b*c^5)*d^4*x^8 + a^3*b^4*d^4*x + 3/7*(43*b^4*c^3 + 104*a*b
^2*c^4 + 16*a^2*c^5)*d^4*x^7 + 1/2*(17*b^5*c^2 + 88*a*b^3*c^3 + 48*a^2*b*c^
4)*d^4*x^6 + 1/5*(11*b^6*c + 123*a*b^4*c^2 + 168*a^2*b^2*c^3 + 16*a^3*c^4)*
d^4*x^5 + 1/4*(b^7 + 30*a*b^5*c + 96*a^2*b^3*c^2 + 32*a^3*b*c^3)*d^4*x^4 +
(a*b^6 + 9*a^2*b^4*c + 8*a^3*b^2*c^2)*d^4*x^3 + 1/2*(3*a^2*b^5 + 8*a^3*b^3*
c)*d^4*x^2
```

Fricas [B] time = 1.70301, size = 776, normalized size = 7.68

$$\frac{16}{11}x^{11}d^4c^7 + 8x^{10}d^4c^6b + \frac{56}{3}x^9d^4c^5b^2 + \frac{16}{3}x^9d^4c^6a + 24x^8d^4c^4b^3 + 24x^8d^4c^5ba + \frac{129}{7}x^7d^4c^3b^4 + \frac{312}{7}x^7d^4c^4b^2a + \frac{48}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4*(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 16/11*x^11*d^4*c^7 + 8*x^10*d^4*c^6*b + 56/3*x^9*d^4*c^5*b^2 + 16/3*x^9*d^4
*c^6*a + 24*x^8*d^4*c^4*b^3 + 24*x^8*d^4*c^5*b*a + 129/7*x^7*d^4*c^3*b^4 +
312/7*x^7*d^4*c^4*b^2*a + 48/7*x^7*d^4*c^5*a^2 + 17/2*x^6*d^4*c^2*b^5 + 44*
x^6*d^4*c^3*b^3*a + 24*x^6*d^4*c^4*b*a^2 + 11/5*x^5*d^4*c*b^6 + 123/5*x^5*d
^4*c^2*b^4*a + 168/5*x^5*d^4*c^3*b^2*a^2 + 16/5*x^5*d^4*c^4*a^3 + 1/4*x^4*d
^4*b^7 + 15/2*x^4*d^4*c*b^5*a + 24*x^4*d^4*c^2*b^3*a^2 + 8*x^4*d^4*c^3*b*a^
3 + x^3*d^4*b^6*a + 9*x^3*d^4*c*b^4*a^2 + 8*x^3*d^4*c^2*b^2*a^3 + 3/2*x^2*d
^4*b^5*a^2 + 4*x^2*d^4*c*b^3*a^3 + x*d^4*b^4*a^3
```

Sympy [B] time = 0.126409, size = 371, normalized size = 3.67

$$a^3b^4d^4x + 8bc^6d^4x^{10} + \frac{16c^7d^4x^{11}}{11} + x^9\left(\frac{16ac^6d^4}{3} + \frac{56b^2c^5d^4}{3}\right) + x^8\left(24abc^5d^4 + 24b^3c^4d^4\right) + x^7\left(\frac{48a^2c^5d^4}{7} + \frac{312ab^2c^4d^4}{7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**4*(c*x**2+b*x+a)**3,x)
```

```
[Out] a**3*b**4*d**4*x + 8*b*c**6*d**4*x**10 + 16*c**7*d**4*x**11/11 + x**9*(16*a
*c**6*d**4/3 + 56*b**2*c**5*d**4/3) + x**8*(24*a*b*c**5*d**4 + 24*b**3*c**4
*d**4) + x**7*(48*a**2*c**5*d**4/7 + 312*a*b**2*c**4*d**4/7 + 129*b**4*c**3
*d**4/7) + x**6*(24*a**2*b*c**4*d**4 + 44*a*b**3*c**3*d**4 + 17*b**5*c**2*d
**4/2) + x**5*(16*a**3*c**4*d**4/5 + 168*a**2*b**2*c**3*d**4/5 + 123*a*b**4
*c**2*d**4/5 + 11*b**6*c*d**4/5) + x**4*(8*a**3*b*c**3*d**4 + 24*a**2*b**3*
c**2*d**4 + 15*a*b**5*c*d**4/2 + b**7*d**4/4) + x**3*(8*a**3*b**2*c**2*d**4
 + 9*a**2*b**4*c*d**4 + a*b**6*d**4) + x**2*(4*a**3*b**3*c*d**4 + 3*a**2*b*
*5*d**4/2)
```

Giac [B] time = 1.28277, size = 487, normalized size = 4.82

$$\frac{16}{11}c^7d^4x^{11} + 8\,bc^6d^4x^{10} + \frac{56}{3}b^2c^5d^4x^9 + \frac{16}{3}ac^6d^4x^9 + 24\,b^3c^4d^4x^8 + 24\,abc^5d^4x^8 + \frac{129}{7}b^4c^3d^4x^7 + \frac{312}{7}ab^2c^4d^4x^7 + \frac{48}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4*(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 16/11*c^7*d^4*x^11 + 8*b*c^6*d^4*x^10 + 56/3*b^2*c^5*d^4*x^9 + 16/3*a*c^6*d
^4*x^9 + 24*b^3*c^4*d^4*x^8 + 24*a*b*c^5*d^4*x^8 + 129/7*b^4*c^3*d^4*x^7 +
312/7*a*b^2*c^4*d^4*x^7 + 48/7*a^2*c^5*d^4*x^7 + 17/2*b^5*c^2*d^4*x^6 + 44*
a*b^3*c^3*d^4*x^6 + 24*a^2*b*c^4*d^4*x^6 + 11/5*b^6*c*d^4*x^5 + 123/5*a*b^4
*c^2*d^4*x^5 + 168/5*a^2*b^2*c^3*d^4*x^5 + 16/5*a^3*c^4*d^4*x^5 + 1/4*b^7*d
^4*x^4 + 15/2*a*b^5*c*d^4*x^4 + 24*a^2*b^3*c^2*d^4*x^4 + 8*a^3*b*c^3*d^4*x^
4 + a*b^6*d^4*x^3 + 9*a^2*b^4*c*d^4*x^3 + 8*a^3*b^2*c^2*d^4*x^3 + 3/2*a^2*b
^5*d^4*x^2 + 4*a^3*b^3*c*d^4*x^2 + a^3*b^4*d^4*x
```

3.1139 $\int (bd+2cdx)^3 \left(a+bx+cx^2\right)^3 \, dx$

Optimal. Leaf size=55

$$\frac{1}{20}d^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4+\frac{1}{5}d^3(b+2cx)^2\left(a+bx+cx^2\right)^4$$

```
[Out] ((b^2 - 4*a*c)*d^3*(a + b*x + c*x^2)^4)/20 + (d^3*(b + 2*c*x)^2*(a + b*x +
c*x^2)^4)/5
```

Rubi [A] time = 0.0250731, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {692, 629}

$$\frac{1}{20}d^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4+\frac{1}{5}d^3(b+2cx)^2\left(a+bx+cx^2\right)^4$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^3*(a + b*x + c*x^2)^3,x]
```

```
[Out] ((b^2 - 4*a*c)*d^3*(a + b*x + c*x^2)^4)/20 + (d^3*(b + 2*c*x)^2*(a + b*x +
c*x^2)^4)/5
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (bd+2cdx)^3\left(a+bx+cx^2\right)^3\,dx &= \frac{1}{5}d^3(b+2cx)^2\left(a+bx+cx^2\right)^4+\frac{1}{5}\left(\left(b^2-4ac\right)d^2\right)\int (bd+2cdx)\left(a+bx+cx^2\right)^3 \\ &= \frac{1}{20}\left(b^2-4ac\right)d^3\left(a+bx+cx^2\right)^4+\frac{1}{5}d^3(b+2cx)^2\left(a+bx+cx^2\right)^4\end{aligned}$$

Mathematica [B] time = 0.0331185, size = 132, normalized size = 2.4

$$\frac{1}{20}d^3x(b+cx)\left(20a^3\left(b^2+2bcx+2c^2x^2\right)+10a^2x\left(11b^2cx+3b^3+16bc^2x^2+8c^3x^3\right)+20ax^2(b+cx)^2\left(b^2+3bcx+3c^2\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^3*(a + b*x + c*x^2)^3,x]
```

```
[Out] (d^3*x*(b + c*x)*(20*a^3*(b^2 + 2*b*c*x + 2*c^2*x^2) + 20*a*x^2*(b + c*x)^2
*(b^2 + 3*b*c*x + 3*c^2*x^2) + x^3*(b + c*x)^3*(5*b^2 + 16*b*c*x + 16*c^2*x
^2) + 10*a^2*x*(3*b^3 + 11*b^2*c*x + 16*b*c^2*x^2 + 8*c^3*x^3)))/20
```

Maple [B] time = 0.041, size = 534, normalized size = 9.7

$$\frac{4c^6d^3x^{10}}{5}+4bd^3c^5x^9+\frac{\left(42b^2d^3c^4+8c^3d^3\left(ac^2+2b^2c+c\left(2ac+b^2\right)\right)\right)x^8}{8}+\frac{\left(19b^3d^3c^3+12bd^3c^2\left(ac^2+2b^2c+c\left(2a\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^3*(c*x^2+b*x+a)^3,x)
```

```
[Out] 4/5*c^6*d^3*x^10+4*b*d^3*c^5*x^9+1/8*(42*b^2*d^3*c^4+8*c^3*d^3*(a*c^2+2*b^2
*c+c*(2*a*c+b^2)))*x^8+1/7*(19*b^3*d^3*c^3+12*b*d^3*c^2*(a*c^2+2*b^2*c+c*(2
*a*c+b^2))+8*c^3*d^3*(4*a*b*c+b*(2*a*c+b^2)))*x^7+1/6*(3*b^4*d^3*c^2+6*b^2*
d^3*c*(a*c^2+2*b^2*c+c*(2*a*c+b^2))+12*b*d^3*c^2*(4*a*b*c+b*(2*a*c+b^2))+8*
c^3*d^3*(a*(2*a*c+b^2)+2*b^2*a+a^2*c))*x^6+1/5*(b^3*d^3*(a*c^2+2*b^2*c+c*(2
*a*c+b^2))+6*b^2*d^3*c*(4*a*b*c+b*(2*a*c+b^2))+12*b*d^3*c^2*(a*(2*a*c+b^2)+
2*b^2*a+a^2*c)+24*c^3*d^3*b*a^2)*x^5+1/4*(b^3*d^3*(4*a*b*c+b*(2*a*c+b^2))+6
*b^2*d^3*c*(a*(2*a*c+b^2)+2*b^2*a+a^2*c)+36*b^2*d^3*c^2*a^2+8*c^3*d^3*a^3)*
x^4+1/3*(b^3*d^3*(a*(2*a*c+b^2)+2*b^2*a+a^2*c)+18*b^3*d^3*c*a^2+12*b*d^3*c^
2*a^3)*x^3+1/2*(6*a^3*b^2*c*d^3+3*a^2*b^4*d^3)*x^2+b^3*d^3*a^3*x
```

Maxima [B] time = 1.10006, size = 328, normalized size = 5.96

$$\frac{4}{5}c^6d^3x^{10}+4bc^5d^3x^9+\frac{3}{4}\left(11b^2c^4+4ac^5\right)d^3x^8+3\left(3b^3c^3+4abc^4\right)d^3x^7+a^3b^3d^3x+\frac{1}{2}\left(11b^4c^2+38ab^2c^3+8a^2c^4\right)d^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3*(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 4/5*c^6*d^3*x^10 + 4*b*c^5*d^3*x^9 + 3/4*(11*b^2*c^4 + 4*a*c^5)*d^3*x^8 + 3
*(3*b^3*c^3 + 4*a*b*c^4)*d^3*x^7 + a^3*b^3*d^3*x + 1/2*(11*b^4*c^2 + 38*a*b
^2*c^3 + 8*a^2*c^4)*d^3*x^6 + 3/5*(3*b^5*c + 25*a*b^3*c^2 + 20*a^2*b*c^3)*d
^3*x^5 + 1/4*(b^6 + 24*a*b^4*c + 54*a^2*b^2*c^2 + 8*a^3*c^3)*d^3*x^4 + (a*b
^5 + 7*a^2*b^3*c + 4*a^3*b*c^2)*d^3*x^3 + 3/2*(a^2*b^4 + 2*a^3*b^2*c)*d^3*x
^2
```

Fricas [B] time = 1.81675, size = 614, normalized size = 11.16

$$\frac{4}{5}x^{10}d^3c^6+4x^9d^3c^5b+\frac{33}{4}x^8d^3c^4b^2+3x^8d^3c^5a+9x^7d^3c^3b^3+12x^7d^3c^4ba+\frac{11}{2}x^6d^3c^2b^4+19x^6d^3c^3b^2a+4x^6d^3c^4a^2+\frac{9}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3*(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 4/5*x^10*d^3*c^6 + 4*x^9*d^3*c^5*b + 33/4*x^8*d^3*c^4*b^2 + 3*x^8*d^3*c^5*a
 + 9*x^7*d^3*c^3*b^3 + 12*x^7*d^3*c^4*b*a + 11/2*x^6*d^3*c^2*b^4 + 19*x^6*d
```

```
^3*c^3*b^2*a + 4*x^6*d^3*c^4*a^2 + 9/5*x^5*d^3*c*b^5 + 15*x^5*d^3*c^2*b^3*a
 + 12*x^5*d^3*c^3*b*a^2 + 1/4*x^4*d^3*b^6 + 6*x^4*d^3*c*b^4*a + 27/2*x^4*d^
3*c^2*b^2*a^2 + 2*x^4*d^3*c^3*a^3 + x^3*d^3*b^5*a + 7*x^3*d^3*c*b^3*a^2 + 4
*x^3*d^3*c^2*b*a^3 + 3/2*x^2*d^3*b^4*a^2 + 3*x^2*d^3*c*b^2*a^3 + x*d^3*b^3*
a^3
```

Sympy [B] time = 0.185013, size = 299, normalized size = 5.44

$$a^3b^3d^3x + 4bc^5d^3x^9 + \frac{4c^6d^3x^{10}}{5} + x^8\left(3ac^5d^3 + \frac{33b^2c^4d^3}{4}\right) + x^7\left(12abc^4d^3 + 9b^3c^3d^3\right) + x^6\left(4a^2c^4d^3 + 19ab^2c^3d^3 + \frac{11}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**3*(c*x**2+b*x+a)**3,x)
```

```
[Out] a**3*b**3*d**3*x + 4*b*c**5*d**3*x**9 + 4*c**6*d**3*x**10/5 + x**8*(3*a*c**
5*d**3 + 33*b**2*c**4*d**3/4) + x**7*(12*a*b*c**4*d**3 + 9*b**3*c**3*d**3)
+ x**6*(4*a**2*c**4*d**3 + 19*a*b**2*c**3*d**3 + 11*b**4*c**2*d**3/2) + x**
5*(12*a**2*b*c**3*d**3 + 15*a*b**3*c**2*d**3 + 9*b**5*c*d**3/5) + x**4*(2*a
**3*c**3*d**3 + 27*a**2*b**2*c**2*d**3/2 + 6*a*b**4*c*d**3 + b**6*d**3/4) +
 x**3*(4*a**3*b*c**2*d**3 + 7*a**2*b**3*c*d**3 + a*b**5*d**3) + x**2*(3*a**
3*b**2*c*d**3 + 3*a**2*b**4*d**3/2)
```

Giac [B] time = 1.1588, size = 402, normalized size = 7.31

$$\frac{4}{5}\,c^6d^3x^{10} + 4\,bc^5d^3x^9 + \frac{33}{4}\,b^2c^4d^3x^8 + 3\,ac^5d^3x^8 + 9\,b^3c^3d^3x^7 + 12\,abc^4d^3x^7 + \frac{11}{2}\,b^4c^2d^3x^6 + 19\,ab^2c^3d^3x^6 + 4\,a^2c^4d^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3*(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 4/5*c^6*d^3*x^10 + 4*b*c^5*d^3*x^9 + 33/4*b^2*c^4*d^3*x^8 + 3*a*c^5*d^3*x^8
 + 9*b^3*c^3*d^3*x^7 + 12*a*b*c^4*d^3*x^7 + 11/2*b^4*c^2*d^3*x^6 + 19*a*b^2
*c^3*d^3*x^6 + 4*a^2*c^4*d^3*x^6 + 9/5*b^5*c*d^3*x^5 + 15*a*b^3*c^2*d^3*x^5
 + 12*a^2*b*c^3*d^3*x^5 + 1/4*b^6*d^3*x^4 + 6*a*b^4*c*d^3*x^4 + 27/2*a^2*b^
2*c^2*d^3*x^4 + 2*a^3*c^3*d^3*x^4 + a*b^5*d^3*x^3 + 7*a^2*b^3*c*d^3*x^3 + 4
*a^3*b*c^2*d^3*x^3 + 3/2*a^2*b^4*d^3*x^2 + 3*a^3*b^2*c*d^3*x^2 + a^3*b^3*d^
3*x
```

3.1140 $\int (bd+2cdx)^2 \left(a+bx+cx^2\right)^3 \, dx$

Optimal. Leaf size=101

$$-\frac{3d^2\left(b^2-4ac\right)(b+2cx)^7}{896c^4}+\frac{3d^2\left(b^2-4ac\right)^2(b+2cx)^5}{640c^4}-\frac{d^2\left(b^2-4ac\right)^3(b+2cx)^3}{384c^4}+\frac{d^2(b+2cx)^9}{1152c^4}$$

```
[Out] -((b^2 - 4*a*c)^3*d^2*(b + 2*c*x)^3)/(384*c^4) + (3*(b^2 - 4*a*c)^2*d^2*(b
+ 2*c*x)^5)/(640*c^4) - (3*(b^2 - 4*a*c)*d^2*(b + 2*c*x)^7)/(896*c^4) + (d^
2*(b + 2*c*x)^9)/(1152*c^4)
```

Rubi [A] time = 0.12897, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$-\frac{3d^2\left(b^2-4ac\right)(b+2cx)^7}{896c^4}+\frac{3d^2\left(b^2-4ac\right)^2(b+2cx)^5}{640c^4}-\frac{d^2\left(b^2-4ac\right)^3(b+2cx)^3}{384c^4}+\frac{d^2(b+2cx)^9}{1152c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^2*(a + b*x + c*x^2)^3,x]
```

```
[Out] -((b^2 - 4*a*c)^3*d^2*(b + 2*c*x)^3)/(384*c^4) + (3*(b^2 - 4*a*c)^2*d^2*(b
+ 2*c*x)^5)/(640*c^4) - (3*(b^2 - 4*a*c)*d^2*(b + 2*c*x)^7)/(896*c^4) + (d^
2*(b + 2*c*x)^9)/(1152*c^4)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] && !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\int (bd+2cdx)^2\left(a+bx+cx^2\right)^3\,dx=\int\left(\frac{\left(-b^2+4ac\right)^3(bd+2cdx)^2}{64c^3}+\frac{3\left(-b^2+4ac\right)^2(bd+2cdx)^4}{64c^3d^2}+\frac{3\left(-b^2+4ac\right)(bd+\right.$$

$$=-\frac{\left(b^2-4ac\right)^3d^2(b+2cx)^3}{384c^4}+\frac{3\left(b^2-4ac\right)^2d^2(b+2cx)^5}{640c^4}-\frac{3\left(b^2-4ac\right)d^2(b+2cx)^7}{896c^4}$$

Mathematica [A] time = 0.0302483, size = 179, normalized size = 1.77

$$d^2\left(\frac{1}{5}cx^5\left(12a^2c^2+39ab^2c+7b^4\right)+\frac{1}{4}bx^4\left(24a^2c^2+18ab^2c+b^4\right)+\frac{1}{3}ax^3\left(4a^2c^2+15ab^2c+3b^4\right)+\frac{1}{2}a^2bx^2\left(4ac+3b^2\right)+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^2*(a + b*x + c*x^2)^3,x]
```

```
[Out] d^2*(a^3*b^2*x + (a^2*b*(3*b^2 + 4*a*c)*x^2)/2 + (a*(3*b^4 + 15*a*b^2*c + 4
*a^2*c^2)*x^3)/3 + (b*(b^4 + 18*a*b^2*c + 24*a^2*c^2)*x^4)/4 + (c*(7*b^4 +
```

```
39*a*b^2*c + 12*a^2*c^2)*x^5)/5 + (b*c^2*(19*b^2 + 36*a*c)*x^6)/6 + (c^3*(2
5*b^2 + 12*a*c)*x^7)/7 + 2*b*c^4*x^8 + (4*c^5*x^9)/9)
```

Maple [B] time = 0.04, size = 396, normalized size = 3.9

$$\frac{4\,c^5d^2x^9}{9}+2\,bd^2c^4x^8+\frac{\left(13\,b^2d^2c^3+4\,c^2d^2\left(ac^2+2\,b^2c+c\left(2\,ac+b^2\right)\right)\right)x^7}{7}+\frac{\left(3\,b^3d^2c^2+4\,bd^2c\left(ac^2+2\,b^2c+c\left(2\,ac\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^2*(c*x^2+b*x+a)^3,x)
```

```
[Out] 4/9*c^5*d^2*x^9+2*b*d^2*c^4*x^8+1/7*(13*b^2*d^2*c^3+4*c^2*d^2*(a*c^2+2*b^2*
c+c*(2*a*c+b^2)))*x^7+1/6*(3*b^3*d^2*c^2+4*b*d^2*c*(a*c^2+2*b^2*c+c*(2*a*c+
b^2))+4*c^2*d^2*(4*a*b*c+b*(2*a*c+b^2)))*x^6+1/5*(b^2*d^2*(a*c^2+2*b^2*c+c*
(2*a*c+b^2))+4*b*d^2*c*(4*a*b*c+b*(2*a*c+b^2))+4*c^2*d^2*(a*(2*a*c+b^2)+2*b
^2*a+a^2*c))*x^5+1/4*(b^2*d^2*(4*a*b*c+b*(2*a*c+b^2))+4*b*d^2*c*(a*(2*a*c+b
^2)+2*b^2*a+a^2*c)+12*a^2*b*c^2*d^2)*x^4+1/3*(b^2*d^2*(a*(2*a*c+b^2)+2*b^2*
a+a^2*c)+12*b^2*d^2*c*a^2+4*c^2*d^2*a^3)*x^3+1/2*(4*a^3*b*c*d^2+3*a^2*b^3*d
^2)*x^2+b^2*d^2*a^3*x
```

Maxima [B] time = 1.14134, size = 267, normalized size = 2.64

$$\frac{4}{9}\,c^5d^2x^9+2\,bc^4d^2x^8+\frac{1}{7}\left(25\,b^2c^3+12\,ac^4\right)d^2x^7+\frac{1}{6}\left(19\,b^3c^2+36\,abc^3\right)d^2x^6+a^3b^2d^2x+\frac{1}{5}\left(7\,b^4c+39\,ab^2c^2+12\,a^2c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2*(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 4/9*c^5*d^2*x^9 + 2*b*c^4*d^2*x^8 + 1/7*(25*b^2*c^3 + 12*a*c^4)*d^2*x^7 + 1
/6*(19*b^3*c^2 + 36*a*b*c^3)*d^2*x^6 + a^3*b^2*d^2*x + 1/5*(7*b^4*c + 39*a*
b^2*c^2 + 12*a^2*c^3)*d^2*x^5 + 1/4*(b^5 + 18*a*b^3*c + 24*a^2*b*c^2)*d^2*x
^4 + 1/3*(3*a*b^4 + 15*a^2*b^2*c + 4*a^3*c^2)*d^2*x^3 + 1/2*(3*a^2*b^3 + 4*
a^3*b*c)*d^2*x^2
```

Fricas [B] time = 1.70073, size = 500, normalized size = 4.95

$$\frac{4}{9}x^9d^2c^5+2x^8d^2c^4b+\frac{25}{7}x^7d^2c^3b^2+\frac{12}{7}x^7d^2c^4a+\frac{19}{6}x^6d^2c^2b^3+6x^6d^2c^3ba+\frac{7}{5}x^5d^2cb^4+\frac{39}{5}x^5d^2c^2b^2a+\frac{12}{5}x^5d^2c^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2*(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 4/9*x^9*d^2*c^5 + 2*x^8*d^2*c^4*b + 25/7*x^7*d^2*c^3*b^2 + 12/7*x^7*d^2*c^4
*a + 19/6*x^6*d^2*c^2*b^3 + 6*x^6*d^2*c^3*b*a + 7/5*x^5*d^2*c*b^4 + 39/5*x^
5*d^2*c^2*b^2*a + 12/5*x^5*d^2*c^3*a^2 + 1/4*x^4*d^2*b^5 + 9/2*x^4*d^2*c*b^
3*a + 6*x^4*d^2*c^2*b*a^2 + x^3*d^2*b^4*a + 5*x^3*d^2*c*b^2*a^2 + 4/3*x^3*d
^2*c^2*a^3 + 3/2*x^2*d^2*b^3*a^2 + 2*x^2*d^2*c*b*a^3 + x*d^2*b^2*a^3
```

Sympy [B] time = 0.108754, size = 246, normalized size = 2.44

$$a^3b^2d^2x + 2bc^4d^2x^8 + \frac{4c^5d^2x^9}{9} + x^7\left(\frac{12ac^4d^2}{7} + \frac{25b^2c^3d^2}{7}\right) + x^6\left(6abc^3d^2 + \frac{19b^3c^2d^2}{6}\right) + x^5\left(\frac{12a^2c^3d^2}{5} + \frac{39ab^2c^2d^2}{5} + \frac{7b}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**2*(c*x**2+b*x+a)**3,x)
```

```
[Out] a**3*b**2*d**2*x + 2*b*c**4*d**2*x**8 + 4*c**5*d**2*x**9/9 + x**7*(12*a*c**
4*d**2/7 + 25*b**2*c**3*d**2/7) + x**6*(6*a*b*c**3*d**2 + 19*b**3*c**2*d**2
/6) + x**5*(12*a**2*c**3*d**2/5 + 39*a*b**2*c**2*d**2/5 + 7*b**4*c*d**2/5)
+ x**4*(6*a**2*b*c**2*d**2 + 9*a*b**3*c*d**2/2 + b**5*d**2/4) + x**3*(4*a**
3*c**2*d**2/3 + 5*a**2*b**2*c*d**2 + a*b**4*d**2) + x**2*(2*a**3*b*c*d**2 +
 3*a**2*b**3*d**2/2)
```

Giac [B] time = 1.17085, size = 317, normalized size = 3.14

$$\frac{4}{9}c^5d^2x^9 + 2\,bc^4d^2x^8 + \frac{25}{7}\,b^2c^3d^2x^7 + \frac{12}{7}\,ac^4d^2x^7 + \frac{19}{6}\,b^3c^2d^2x^6 + 6\,abc^3d^2x^6 + \frac{7}{5}\,b^4cd^2x^5 + \frac{39}{5}\,ab^2c^2d^2x^5 + \frac{12}{5}\,a^2c^3d^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2*(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 4/9*c^5*d^2*x^9 + 2*b*c^4*d^2*x^8 + 25/7*b^2*c^3*d^2*x^7 + 12/7*a*c^4*d^2*x
^7 + 19/6*b^3*c^2*d^2*x^6 + 6*a*b*c^3*d^2*x^6 + 7/5*b^4*c*d^2*x^5 + 39/5*a*
b^2*c^2*d^2*x^5 + 12/5*a^2*c^3*d^2*x^5 + 1/4*b^5*d^2*x^4 + 9/2*a*b^3*c*d^2*
x^4 + 6*a^2*b*c^2*d^2*x^4 + a*b^4*d^2*x^3 + 5*a^2*b^2*c*d^2*x^3 + 4/3*a^3*c
^2*d^2*x^3 + 3/2*a^2*b^3*d^2*x^2 + 2*a^3*b*c*d^2*x^2 + a^3*b^2*d^2*x
```

3.1141 $\int (bd + 2cdx) \left(a + bx + cx^2\right)^3 \, dx$

Optimal. Leaf size=17

$$\frac{1}{4} d \left(a + bx + cx^2\right)^4$$

```
[Out] (d*(a + b*x + c*x^2)^4)/4
```

Rubi [A] time = 0.0054493, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {629}

$$\frac{1}{4} d \left(a + bx + cx^2\right)^4$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)*(a + b*x + c*x^2)^3,x]
```

```
[Out] (d*(a + b*x + c*x^2)^4)/4
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (bd + 2cdx) \left(a + bx + cx^2\right)^3 \, dx = \frac{1}{4} d \left(a + bx + cx^2\right)^4$$

Mathematica [B] time = 0.0176819, size = 52, normalized size = 3.06

$$\frac{1}{4} dx(b + cx) \left(6a^2 x(b + cx) + 4a^3 + 4ax^2(b + cx)^2 + x^3(b + cx)^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)*(a + b*x + c*x^2)^3,x]
```

```
[Out] (d*x*(b + c*x)*(4*a^3 + 6*a^2*x*(b + c*x) + 4*a*x^2*(b + c*x)^2 + x^3*(b +
c*x)^3))/4
```

Maple [B] time = 0.039, size = 231, normalized size = 13.6

$$\frac{c^4 d x^8}{4} + bdc^3 x^7 + \frac{\left(3\, b^2 d c^2 + 2\, cd \left(a c^2 + 2\, b^2 c + c \left(2\, ac + b^2\right)\right)\right) x^6}{6} + \frac{\left(bd \left(a c^2 + 2\, b^2 c + c \left(2\, ac + b^2\right)\right) + 2\, cd \left(4\, abc + b\right.\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)*(c*x^2+b*x+a)^3,x)
```

```
[Out] 1/4*c^4*d*x^8+b*d*c^3*x^7+1/6*(3*b^2*d*c^2+2*c*d*(a*c^2+2*b^2*c+c*(2*a*c+b^
2)))*x^6+1/5*(b*d*(a*c^2+2*b^2*c+c*(2*a*c+b^2))+2*c*d*(4*a*b*c+b*(2*a*c+b^2
)))*x^5+1/4*(b*d*(4*a*b*c+b*(2*a*c+b^2))+2*c*d*(a*(2*a*c+b^2)+2*b^2*a+a^2*c
))*x^4+1/3*(b*d*(a*(2*a*c+b^2)+2*b^2*a+a^2*c)+6*c*d*b*a^2)*x^3+1/2*(2*a^3*c
*d+3*a^2*b^2*d)*x^2+b*d*a^3*x
```

Maxima [A] time = 1.23692, size = 20, normalized size = 1.18

$$\frac{1}{4}\left(cx^2+bx+a\right)^4 d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)*(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(c*x^2 + b*x + a)^4*d
```

Fricas [B] time = 1.65394, size = 306, normalized size = 18.

$$\frac{1}{4}x^8dc^4+x^7dc^3b+\frac{3}{2}x^6dc^2b^2+x^6dc^3a+x^5dcb^3+3x^5dc^2ba+\frac{1}{4}x^4db^4+3x^4dcb^2a+\frac{3}{2}x^4dc^2a^2+x^3db^3a+3x^3dcba^2+\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)*(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*x^8*d*c^4 + x^7*d*c^3*b + 3/2*x^6*d*c^2*b^2 + x^6*d*c^3*a + x^5*d*c*b^3
 + 3*x^5*d*c^2*b*a + 1/4*x^4*d*b^4 + 3*x^4*d*c*b^2*a + 3/2*x^4*d*c^2*a^2 +
x^3*d*b^3*a + 3*x^3*d*c*b*a^2 + 3/2*x^2*d*b^2*a^2 + x^2*d*c*a^3 + x*d*b*a^3
```

Sympy [B] time = 0.128285, size = 144, normalized size = 8.47

$$a^3bdx+bc^3dx^7+\frac{c^4dx^8}{4}+x^6\left(ac^3d+\frac{3b^2c^2d}{2}\right)+x^5\left(3abc^2d+b^3cd\right)+x^4\left(\frac{3a^2c^2d}{2}+3ab^2cd+\frac{b^4d}{4}\right)+x^3\left(3a^2bcd+ab^3d\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)*(c*x**2+b*x+a)**3,x)
```

```
[Out] a**3*b*d*x + b*c**3*d*x**7 + c**4*d*x**8/4 + x**6*(a*c**3*d + 3*b**2*c**2*d
/2) + x**5*(3*a*b*c**2*d + b**3*c*d) + x**4*(3*a**2*c**2*d/2 + 3*a*b**2*c*d
 + b**4*d/4) + x**3*(3*a**2*b*c*d + a*b**3*d) + x**2*(a**3*c*d + 3*a**2*b**
2*d/2)
```

Giac [B] time = 1.22418, size = 189, normalized size = 11.12

$$\frac{1}{4}c^4dx^8+bc^3dx^7+\frac{3}{2}b^2c^2dx^6+ac^3dx^6+b^3cdx^5+3\,abc^2dx^5+\frac{1}{4}b^4dx^4+3\,ab^2cdx^4+\frac{3}{2}a^2c^2dx^4+ab^3dx^3+3\,a^2bcdx^3+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)*(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*c^4*d*x^8 + b*c^3*d*x^7 + 3/2*b^2*c^2*d*x^6 + a*c^3*d*x^6 + b^3*c*d*x^5
 + 3*a*b*c^2*d*x^5 + 1/4*b^4*d*x^4 + 3*a*b^2*c*d*x^4 + 3/2*a^2*c^2*d*x^4 +
a*b^3*d*x^3 + 3*a^2*b*c*d*x^3 + 3/2*a^2*b^2*d*x^2 + a^3*c*d*x^2 + a^3*b*d*x
```

3.1142 $\int \frac{\left(a+bx+cx^2\right)^3}{bd+2cdx} dx$

Optimal. Leaf size=100

$$-\frac{3\left(b^2-4ac\right)(b+2cx)^4}{512c^4d}+\frac{3\left(b^2-4ac\right)^2(b+2cx)^2}{256c^4d}-\frac{\left(b^2-4ac\right)^3\log(b+2cx)}{128c^4d}+\frac{(b+2cx)^6}{768c^4d}$$

```
[Out] (3*(b^2 - 4*a*c)^2*(b + 2*c*x)^2)/(256*c^4*d) - (3*(b^2 - 4*a*c)*(b + 2*c*x
)^4)/(512*c^4*d) + (b + 2*c*x)^6/(768*c^4*d) - ((b^2 - 4*a*c)^3*Log[b + 2*c
*x])/(128*c^4*d)
```

Rubi [A] time = 0.117778, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$-\frac{3\left(b^2-4ac\right)(b+2cx)^4}{512c^4d}+\frac{3\left(b^2-4ac\right)^2(b+2cx)^2}{256c^4d}-\frac{\left(b^2-4ac\right)^3\log(b+2cx)}{128c^4d}+\frac{(b+2cx)^6}{768c^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x),x]
```

```
[Out] (3*(b^2 - 4*a*c)^2*(b + 2*c*x)^2)/(256*c^4*d) - (3*(b^2 - 4*a*c)*(b + 2*c*x
)^4)/(512*c^4*d) + (b + 2*c*x)^6/(768*c^4*d) - ((b^2 - 4*a*c)^3*Log[b + 2*c
*x])/(128*c^4*d)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^3}{bd+2cdx} dx = \int \left(\frac{\left(-b^2+4ac\right)^3}{64c^3(bd+2cdx)}+\frac{3\left(-b^2+4ac\right)^2(bd+2cdx)}{64c^3d^2}+\frac{3\left(-b^2+4ac\right)(bd+2cdx)^3}{64c^3d^4}+\frac{(bd+2cdx)^5}{64c^3d^6}\right.$$

$$=\frac{3\left(b^2-4ac\right)^2(b+2cx)^2}{256c^4d}-\frac{3\left(b^2-4ac\right)(b+2cx)^4}{512c^4d}+\frac{(b+2cx)^6}{768c^4d}-\frac{\left(b^2-4ac\right)^3\log(b+2cx)}{128c^4d}$$

Mathematica [A] time = 0.0451798, size = 111, normalized size = 1.11

$$\frac{2cx(b+cx)\left(8c^2\left(18a^2+9acx^2+2c^2x^4\right)+2b^2c\left(5cx^2-18a\right)+8bc^2x\left(9a+4cx^2\right)-6b^3cx+3b^4\right)-3\left(b^2-4ac\right)^3\log(b+2}{384c^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x),x]
```

```
[Out] (2*c*x*(b + c*x)*(3*b^4 - 6*b^3*c*x + 8*b*c^2*x*(9*a + 4*c*x^2) + 2*b^2*c*(
-18*a + 5*c*x^2) + 8*c^2*(18*a^2 + 9*a*c*x^2 + 2*c^2*x^4)) - 3*(b^2 - 4*a*c
```

```
)^3*Log[b + 2*c*x])/(384*c^4*d)
```

Maple [B] time = 0.041, size = 222, normalized size = 2.2

$$\frac{c^2x^6}{12\,d}+\frac{bc x^5}{4\,d}+\frac{3\,a x^4c}{8\,d}+\frac{7\,x^4b^2}{32\,d}+\frac{3\,a x^3b}{4\,d}+\frac{x^3b^3}{48\,cd}+\frac{3\,a^2x^2}{4\,d}+\frac{3\,ab^2x^2}{16\,cd}-\frac{x^2b^4}{64\,c^2d}+\frac{3\,ba^2x}{4\,cd}-\frac{3\,ab^3x}{16\,c^2d}+\frac{b^5x}{64\,dc^3}+\frac{\ln(2\,cx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(2*c*d*x+b*d),x)
```

```
[Out] 1/12/d*c^2*x^6+1/4/d*c*b*x^5+3/8/d*x^4*a*c+7/32/d*x^4*b^2+3/4/d*x^3*a*b+1/4
8/d/c*x^3*b^3+3/4/d*a^2*x^2+3/16/d/c*x^2*a*b^2-1/64/d/c^2*x^2*b^4+3/4/d/c*a
^2*b*x-3/16/d/c^2*a*b^3*x+1/64/d/c^3*b^5*x+1/2/d/c*ln(2*c*x+b)*a^3-3/8/d/c^
2*ln(2*c*x+b)*a^2*b^2+3/32/d/c^3*ln(2*c*x+b)*a*b^4-1/128/d/c^4*ln(2*c*x+b)*
b^6
```

Maxima [A] time = 1.14785, size = 220, normalized size = 2.2

$$\frac{16\,c^5x^6+48\,bc^4x^5+6\left(7\,b^2c^3+12\,ac^4\right)x^4+4\left(b^3c^2+36\,abc^3\right)x^3-3\left(b^4c-12\,ab^2c^2-48\,a^2c^3\right)x^2+3\left(b^5-12\,ab^3c+4\right.}{192\,c^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d),x, algorithm="maxima")
```

```
[Out] 1/192*(16*c^5*x^6 + 48*b*c^4*x^5 + 6*(7*b^2*c^3 + 12*a*c^4)*x^4 + 4*(b^3*c^
2 + 36*a*b*c^3)*x^3 - 3*(b^4*c - 12*a*b^2*c^2 - 48*a^2*c^3)*x^2 + 3*(b^5 -
12*a*b^3*c + 48*a^2*b*c^2)*x)/(c^3*d) - 1/128*(b^6 - 12*a*b^4*c + 48*a^2*b^
2*c^2 - 64*a^3*c^3)*log(2*c*x + b)/(c^4*d)
```

Fricas [A] time = 1.97344, size = 356, normalized size = 3.56

$$\frac{32\,c^6x^6+96\,bc^5x^5+12\left(7\,b^2c^4+12\,ac^5\right)x^4+8\left(b^3c^3+36\,abc^4\right)x^3-6\left(b^4c^2-12\,ab^2c^3-48\,a^2c^4\right)x^2+6\left(b^5c-12\,ab^3c\right.}{384\,c^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d),x, algorithm="fricas")
```

```
[Out] 1/384*(32*c^6*x^6 + 96*b*c^5*x^5 + 12*(7*b^2*c^4 + 12*a*c^5)*x^4 + 8*(b^3*c
^3 + 36*a*b*c^4)*x^3 - 6*(b^4*c^2 - 12*a*b^2*c^3 - 48*a^2*c^4)*x^2 + 6*(b^5
*c - 12*a*b^3*c^2 + 48*a^2*b*c^3)*x - 3*(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2
- 64*a^3*c^3)*log(2*c*x + b))/(c^4*d)
```

Sympy [A] time = 1.08956, size = 141, normalized size = 1.41

$$\frac{bcx^5}{4d}+\frac{c^2x^6}{12d}+\frac{x^4\left(12ac+7b^2\right)}{32d}+\frac{x^3\left(36abc+b^3\right)}{48cd}+\frac{x^2\left(48a^2c^2+12ab^2c-b^4\right)}{64c^2d}+\frac{x\left(48a^2bc^2-12ab^3c+b^5\right)}{64c^3d}+\frac{\left(4ac-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(2*c*d*x+b*d),x)
```

```
[Out] b*c*x**5/(4*d) + c**2*x**6/(12*d) + x**4*(12*a*c + 7*b**2)/(32*d) + x**3*(3
6*a*b*c + b**3)/(48*c*d) + x**2*(48*a**2*c**2 + 12*a*b**2*c - b**4)/(64*c**
2*d) + x*(48*a**2*b*c**2 - 12*a*b**3*c + b**5)/(64*c**3*d) + (4*a*c - b**2)
**3*log(b + 2*c*x)/(128*c**4*d)
```

Giac [B] time = 1.16337, size = 288, normalized size = 2.88

$$-\frac{\left(b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3\right)\log\left(|2\,cx+b|\right)}{128\,c^4d}+\frac{16\,c^8d^5x^6+48\,bc^7d^5x^5+42\,b^2c^6d^5x^4+72\,ac^7d^5x^4+4\,b^3c^5d^5x^3+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d),x, algorithm="giac")
```

```
[Out] -1/128*(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*log(abs(2*c*x + b))
/(c^4*d) + 1/192*(16*c^8*d^5*x^6 + 48*b*c^7*d^5*x^5 + 42*b^2*c^6*d^5*x^4 +
72*a*c^7*d^5*x^4 + 4*b^3*c^5*d^5*x^3 + 144*a*b*c^6*d^5*x^3 - 3*b^4*c^4*d^5*
x^2 + 36*a*b^2*c^5*d^5*x^2 + 144*a^2*c^6*d^5*x^2 + 3*b^5*c^3*d^5*x - 36*a*b
^3*c^4*d^5*x + 144*a^2*b*c^5*d^5*x)/(c^6*d^6)
```

3.1143 $\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^2} dx$

Optimal. Leaf size=94

$$-\frac{(b^2-4ac)(b+2cx)^3}{128c^4d^2}+\frac{3x(b^2-4ac)^2}{64c^3d^2}+\frac{(b^2-4ac)^3}{128c^4d^2(b+2cx)}+\frac{(b+2cx)^5}{640c^4d^2}$$

```
[Out] (3*(b^2 - 4*a*c)^2*x)/(64*c^3*d^2) + (b^2 - 4*a*c)^3/(128*c^4*d^2*(b + 2*c*
x)) - ((b^2 - 4*a*c)*(b + 2*c*x)^3)/(128*c^4*d^2) + (b + 2*c*x)^5/(640*c^4*
d^2)
```

Rubi [A] time = 0.0945815, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$-\frac{(b^2-4ac)(b+2cx)^3}{128c^4d^2}+\frac{3x(b^2-4ac)^2}{64c^3d^2}+\frac{(b^2-4ac)^3}{128c^4d^2(b+2cx)}+\frac{(b+2cx)^5}{640c^4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^2,x]
```

```
[Out] (3*(b^2 - 4*a*c)^2*x)/(64*c^3*d^2) + (b^2 - 4*a*c)^3/(128*c^4*d^2*(b + 2*c*
x)) - ((b^2 - 4*a*c)*(b + 2*c*x)^3)/(128*c^4*d^2) + (b + 2*c*x)^5/(640*c^4*
d^2)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^2} dx &= \int \left(\frac{3(-b^2+4ac)^2}{64c^3d^2}+\frac{(-b^2+4ac)^3}{64c^3(bd+2cdx)^2}+\frac{3(-b^2+4ac)(bd+2cdx)^2}{64c^3d^4}+\frac{(bd+2cdx)^4}{64c^3d^6}\right) dx\\ &= \frac{3(b^2-4ac)^2 x}{64c^3d^2}+\frac{(b^2-4ac)^3}{128c^4d^2(b+2cx)}-\frac{(b^2-4ac)(b+2cx)^3}{128c^4d^2}+\frac{(b+2cx)^5}{640c^4d^2}\end{aligned}$$

Mathematica [A] time = 0.0791392, size = 101, normalized size = 1.07

$$\frac{\frac{10x(48a^2c^2-12ab^2c+b^4)}{c^3}-\frac{20bx^2(b^2-12ac)}{c^2}+\frac{5(b^2-4ac)^3}{c^4(b+2cx)}+\frac{40x^3(4ac+b^2)}{c}+80bx^4+32cx^5}{640d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^2,x]
```

```
[Out] ((10*(b^4 - 12*a*b^2*c + 48*a^2*c^2)*x)/c^3 - (20*b*(b^2 - 12*a*c)*x^2)/c^2
 + (40*(b^2 + 4*a*c)*x^3)/c + 80*b*x^4 + 32*c*x^5 + (5*(b^2 - 4*a*c)^3)/(c^
4*(b + 2*c*x)))/(640*d^2)
```

Maple [A] time = 0.044, size = 135, normalized size = 1.4

$$\frac{1}{d^2}\left(\frac{1}{64\,c^3}\left(\frac{16\,x^5c^4}{5}+8\,bx^4c^3+16\,x^3ac^3+4\,b^2c^2x^3+24\,x^2abc^2-2\,x^2b^3c+48\,a^2c^2x-12\,acb^2x+b^4x\right)-\frac{64\,a^3c^3-48\,a^2b}{128\,c^4\,(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^2,x)
```

```
[Out] 1/d^2*(1/64/c^3*(16/5*x^5*c^4+8*b*x^4*c^3+16*x^3*a*c^3+4*b^2*c^2*x^3+24*x^2
*a*b*c^2-2*x^2*b^3*c+48*a^2*c^2*x-12*a*c*b^2*x+b^4*x)-1/128*(64*a^3*c^3-48*
a^2*b^2*c^2+12*a*b^4*c-b^6)/c^4/(2*c*x+b))
```

Maxima [A] time = 1.16678, size = 186, normalized size = 1.98

$$\frac{b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3}{128\left(2\,c^5d^2x+bc^4d^2\right)}+\frac{16\,c^4x^5+40\,bc^3x^4+20\left(b^2c^2+4\,ac^3\right)x^3-10\left(b^3c-12\,abc^2\right)x^2+5\left(b^4-12\,ab^2c+4\right.}{320\,c^3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^2,x, algorithm="maxima")
```

```
[Out] 1/128*(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)/(2*c^5*d^2*x + b*c^4
*d^2) + 1/320*(16*c^4*x^5 + 40*b*c^3*x^4 + 20*(b^2*c^2 + 4*a*c^3)*x^3 - 10*
(b^3*c - 12*a*b*c^2)*x^2 + 5*(b^4 - 12*a*b^2*c + 48*a^2*c^2)*x)/(c^3*d^2)
```

Fricas [A] time = 1.90991, size = 305, normalized size = 3.24

$$\frac{64\,c^6x^6+192\,bc^5x^5+640\,abc^4x^3+960\,a^2c^4x^2+5\,b^6-60\,ab^4c+240\,a^2b^2c^2-320\,a^3c^3+160\left(b^2c^4+2\,ac^5\right)x^4+10\left(b^5c\right.}{640\left(2\,c^5d^2x+bc^4d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^2,x, algorithm="fricas")
```

```
[Out] 1/640*(64*c^6*x^6 + 192*b*c^5*x^5 + 640*a*b*c^4*x^3 + 960*a^2*c^4*x^2 + 5*b
^6 - 60*a*b^4*c + 240*a^2*b^2*c^2 - 320*a^3*c^3 + 160*(b^2*c^4 + 2*a*c^5)*x
^4 + 10*(b^5*c - 12*a*b^3*c^2 + 48*a^2*b*c^3)*x)/(2*c^5*d^2*x + b*c^4*d^2)
```

Sympy [A] time = 0.973318, size = 143, normalized size = 1.52

$$\frac{bx^4}{8d^2}+\frac{cx^5}{20d^2}-\frac{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}{128bc^4d^2+256c^5d^2x}+\frac{x^3\left(4ac+b^2\right)}{16cd^2}+\frac{x^2\left(12abc-b^3\right)}{32c^2d^2}+\frac{x\left(48a^2c^2-12ab^2c+b^4\right)}{64c^3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(2*c*d*x+b*d)**2,x)
```

```
[Out] b*x**4/(8*d**2) + c*x**5/(20*d**2) - (64*a**3*c**3 - 48*a**2*b**2*c**2 + 12
*a*b**4*c - b**6)/(128*b*c**4*d**2 + 256*c**5*d**2*x) + x**3*(4*a*c + b**2)
/(16*c*d**2) + x**2*(12*a*b*c - b**3)/(32*c**2*d**2) + x*(48*a**2*c**2 - 12
*a*b**2*c + b**4)/(64*c**3*d**2)
```

Giac [B] time = 1.22794, size = 298, normalized size = 3.17

$$\frac{\left(\frac{15\,b^4d^4}{(2\,cdx+bd)^4}-\frac{120\,ab^2cd^4}{(2\,cdx+bd)^4}+\frac{240\,a^2c^2d^4}{(2\,cdx+bd)^4}-\frac{5\,b^2d^2}{(2\,cdx+bd)^2}+\frac{20\,acd^2}{(2\,cdx+bd)^2}+1\right)(2\,cdx+bd)^5}{640\,c^4d^7}+\frac{\frac{b^6c^5d^{11}}{2\,cdx+bd}-\frac{12\,ab^4c^6d^{11}}{2\,cdx+bd}+\frac{48\,a^2b^2c^7d^{11}}{2\,cdx+bd}-\frac{64\,a}{2\,c}}{128\,c^9d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^2,x, algorithm="giac")
```

```
[Out] 1/640*(15*b^4*d^4/(2*c*d*x + b*d)^4 - 120*a*b^2*c*d^4/(2*c*d*x + b*d)^4 + 2
40*a^2*c^2*d^4/(2*c*d*x + b*d)^4 - 5*b^2*d^2/(2*c*d*x + b*d)^2 + 20*a*c*d^2
/(2*c*d*x + b*d)^2 + 1)*(2*c*d*x + b*d)^5/(c^4*d^7) + 1/128*(b^6*c^5*d^11/(
2*c*d*x + b*d) - 12*a*b^4*c^6*d^11/(2*c*d*x + b*d) + 48*a^2*b^2*c^7*d^11/(2
*c*d*x + b*d) - 64*a^3*c^8*d^11/(2*c*d*x + b*d))/(c^9*d^12)
```

3.1144 $\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^3}\,dx$

Optimal. Leaf size=100

$$-\frac{3(b^2-4ac)(b+2cx)^2}{256c^4d^3}+\frac{(b^2-4ac)^3}{256c^4d^3(b+2cx)^2}+\frac{3(b^2-4ac)^2\log(b+2cx)}{128c^4d^3}+\frac{(b+2cx)^4}{512c^4d^3}$$

```
[Out] (b^2 - 4*a*c)^3/(256*c^4*d^3*(b + 2*c*x)^2) - (3*(b^2 - 4*a*c)*(b + 2*c*x)^
2)/(256*c^4*d^3) + (b + 2*c*x)^4/(512*c^4*d^3) + (3*(b^2 - 4*a*c)^2*Log[b +
 2*c*x])/(128*c^4*d^3)
```

Rubi [A] time = 0.0982906, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$-\frac{3(b^2-4ac)(b+2cx)^2}{256c^4d^3}+\frac{(b^2-4ac)^3}{256c^4d^3(b+2cx)^2}+\frac{3(b^2-4ac)^2\log(b+2cx)}{128c^4d^3}+\frac{(b+2cx)^4}{512c^4d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^3,x]
```

```
[Out] (b^2 - 4*a*c)^3/(256*c^4*d^3*(b + 2*c*x)^2) - (3*(b^2 - 4*a*c)*(b + 2*c*x)^
2)/(256*c^4*d^3) + (b + 2*c*x)^4/(512*c^4*d^3) + (3*(b^2 - 4*a*c)^2*Log[b +
 2*c*x])/(128*c^4*d^3)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^3}\,dx &= \int \left(\frac{(-b^2+4ac)^3}{64c^3(bd+2cdx)^3}+\frac{3(-b^2+4ac)^2}{64c^3d^2(bd+2cdx)}+\frac{3(-b^2+4ac)(bd+2cdx)}{64c^3d^4}+\frac{(bd+2cdx)^3}{64c^3d^6}\right)dx\\ &= \frac{(b^2-4ac)^3}{256c^4d^3(b+2cx)^2}-\frac{3(b^2-4ac)(b+2cx)^2}{256c^4d^3}+\frac{(b+2cx)^4}{512c^4d^3}+\frac{3(b^2-4ac)^2\log(b+2cx)}{128c^4d^3}\end{aligned}$$

Mathematica [A] time = 0.0946953, size = 90, normalized size = 0.9

$$\frac{-\frac{8bx(b^2-6ac)}{c^3}+\frac{(b^2-4ac)^3}{c^4(b+2cx)^2}+\frac{6(b^2-4ac)^2\log(b+2cx)}{c^4}+\frac{48ax^2}{c}+\frac{16bx^3}{c}+8x^4}{256d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^3,x]
```

```
[Out] ((-8*b*(b^2 - 6*a*c)*x)/c^3 + (48*a*x^2)/c + (16*b*x^3)/c + 8*x^4 + (b^2 -
4*a*c)^3/(c^4*(b + 2*c*x)^2) + (6*(b^2 - 4*a*c)^2*Log[b + 2*c*x])/c^4)/(256
*d^3)
```

Maple [B] time = 0.047, size = 192, normalized size = 1.9

$$\frac{x^4}{32\,d^3}+\frac{bx^3}{16\,cd^3}+\frac{3\,ax^2}{16\,cd^3}+\frac{3\,abx}{16\,c^2d^3}-\frac{b^3x}{32\,c^3d^3}-\frac{a^3}{4\,cd^3\,(2\,cx+b)^2}+\frac{3\,b^2a^2}{16\,c^2d^3\,(2\,cx+b)^2}-\frac{3\,ab^4}{64\,c^3d^3\,(2\,cx+b)^2}+\frac{}{256\,d^3c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^3,x)
```

```
[Out] 1/32/d^3*x^4+1/16/d^3/c*x^3*b+3/16/d^3/c*a*x^2+3/16/d^3/c^2*a*b*x-1/32/d^3/
c^3*b^3*x-1/4/d^3/c/(2*c*x+b)^2*a^3+3/16/d^3/c^2/(2*c*x+b)^2*a^2*b^2-3/64/d
^3/c^3/(2*c*x+b)^2*a*b^4+1/256/d^3/c^4/(2*c*x+b)^2*b^6+3/8/d^3/c^2*ln(2*c*x
+b)*a^2-3/16/d^3/c^3*ln(2*c*x+b)*a*b^2+3/128/d^3/c^4*ln(2*c*x+b)*b^4
```

Maxima [A] time = 1.16199, size = 198, normalized size = 1.98

$$\frac{b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3}{256\left(4\,c^6d^3x^2+4\,bc^5d^3x+b^2c^4d^3\right)}+\frac{c^3x^4+2\,bc^2x^3+6\,ac^2x^2-\left(b^3-6\,abc\right)x}{32\,c^3d^3}+\frac{3\left(b^4-8\,ab^2c+16\,a^2c^2\right)\log\left(2\,cx+\right.}{128\,c^4d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^3,x, algorithm="maxima")
```

```
[Out] 1/256*(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)/(4*c^6*d^3*x^2 + 4*b
*c^5*d^3*x + b^2*c^4*d^3) + 1/32*(c^3*x^4 + 2*b*c^2*x^3 + 6*a*c^2*x^2 - (b^
3 - 6*a*b*c)*x)/(c^3*d^3) + 3/128*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*log(2*c*x
+ b)/(c^4*d^3)
```

Fricas [B] time = 1.99976, size = 539, normalized size = 5.39

$$\frac{32\,c^6x^6+96\,bc^5x^5+b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3+24\left(3\,b^2c^4+8\,ac^5\right)x^4-16\left(b^3c^3-24\,abc^4\right)x^3-16\left(2\,b^4c^2-\right.}{256\left(4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^3,x, algorithm="fricas")
```

```
[Out] 1/256*(32*c^6*x^6 + 96*b*c^5*x^5 + b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a
^3*c^3 + 24*(3*b^2*c^4 + 8*a*c^5)*x^4 - 16*(b^3*c^3 - 24*a*b*c^4)*x^3 - 16*
(2*b^4*c^2 - 15*a*b^2*c^3)*x^2 - 8*(b^5*c - 6*a*b^3*c^2)*x + 6*(b^6 - 8*a*b
^4*c + 16*a^2*b^2*c^2 + 4*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + 4*(b^5
*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x)*log(2*c*x + b))/(4*c^6*d^3*x^2 + 4*b*c^
5*d^3*x + b^2*c^4*d^3)
```

Sympy [A] time = 2.45082, size = 150, normalized size = 1.5

$$\frac{3ax^2}{16cd^3} + \frac{bx^3}{16cd^3} - \frac{64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6}{256b^2c^4d^3 + 1024bc^5d^3x + 1024c^6d^3x^2} + \frac{x^4}{32d^3} + \frac{x\left(6abc - b^3\right)}{32c^3d^3} + \frac{3\left(4ac - b^2\right)^2\log\left(b + 2cx\right)}{128c^4d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(2*c*d*x+b*d)**3,x)
```

```
[Out] 3*a*x**2/(16*c*d**3) + b*x**3/(16*c*d**3) - (64*a**3*c**3 - 48*a**2*b**2*c*
*2 + 12*a*b**4*c - b**6)/(256*b**2*c**4*d**3 + 1024*b*c**5*d**3*x + 1024*c*
*6*d**3*x**2) + x**4/(32*d**3) + x*(6*a*b*c - b**3)/(32*c**3*d**3) + 3*(4*a
*c - b**2)**2*log(b + 2*c*x)/(128*c**4*d**3)
```

Giac [A] time = 1.16787, size = 200, normalized size = 2.

$$\frac{3\left(b^4 - 8\,ab^2c + 16\,a^2c^2\right)\log\left(|2\,cx + b|\right)}{128\,c^4d^3} + \frac{b^6 - 12\,ab^4c + 48\,a^2b^2c^2 - 64\,a^3c^3}{256\,(2\,cx + b)^2c^4d^3} + \frac{c^{12}d^9x^4 + 2\,bc^{11}d^9x^3 + 6\,ac^{11}d^9x^2 - b^3c^9d}{32\,c^{12}d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^3,x, algorithm="giac")
```

```
[Out] 3/128*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*log(abs(2*c*x + b))/(c^4*d^3) + 1/256*
(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)/((2*c*x + b)^2*c^4*d^3) +
1/32*(c^12*d^9*x^4 + 2*b*c^11*d^9*x^3 + 6*a*c^11*d^9*x^2 - b^3*c^9*d^9*x +
6*a*b*c^10*d^9*x)/(c^12*d^12)
```

3.1145 $\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^4} dx$

Optimal. Leaf size=103

$$\frac{(b^2-4ac)^3}{384c^4d^4(b+2cx)^3}-\frac{3(b^2-4ac)^2}{128c^4d^4(b+2cx)}-\frac{x(b^2-6ac)}{32c^3d^4}+\frac{bx^2}{32c^2d^4}+\frac{x^3}{48cd^4}$$

```
[Out] -((b^2 - 6*a*c)*x)/(32*c^3*d^4) + (b*x^2)/(32*c^2*d^4) + x^3/(48*c*d^4) + (
b^2 - 4*a*c)^3/(384*c^4*d^4*(b + 2*c*x)^3) - (3*(b^2 - 4*a*c)^2)/(128*c^4*d
^4*(b + 2*c*x))
```

Rubi [A] time = 0.0941266, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$\frac{(b^2-4ac)^3}{384c^4d^4(b+2cx)^3}-\frac{3(b^2-4ac)^2}{128c^4d^4(b+2cx)}-\frac{x(b^2-6ac)}{32c^3d^4}+\frac{bx^2}{32c^2d^4}+\frac{x^3}{48cd^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^4,x]
```

```
[Out] -((b^2 - 6*a*c)*x)/(32*c^3*d^4) + (b*x^2)/(32*c^2*d^4) + x^3/(48*c*d^4) + (
b^2 - 4*a*c)^3/(384*c^4*d^4*(b + 2*c*x)^3) - (3*(b^2 - 4*a*c)^2)/(128*c^4*d
^4*(b + 2*c*x))
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^4} dx &= \int \left(\frac{-b^2+6ac}{32c^3d^4}+\frac{bx}{16c^2d^4}+\frac{x^2}{16cd^4}+\frac{(-b^2+4ac)^3}{64c^3d^4(b+2cx)^4}+\frac{3(-b^2+4ac)^2}{64c^3d^4(b+2cx)^2}\right) dx\\ &= -\frac{(b^2-6ac)x}{32c^3d^4}+\frac{bx^2}{32c^2d^4}+\frac{x^3}{48cd^4}+\frac{(b^2-4ac)^3}{384c^4d^4(b+2cx)^3}-\frac{3(b^2-4ac)^2}{128c^4d^4(b+2cx)}\end{aligned}$$

Mathematica [A] time = 0.064124, size = 81, normalized size = 0.79

$$\frac{\frac{(b^2-4ac)^3}{(b+2cx)^3}-\frac{9(b^2-4ac)^2}{b+2cx}+12cx(6ac-b^2)+12bc^2x^2+8c^3x^3}{384c^4d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^4,x]
```

```
[Out] (12*c*(-b^2 + 6*a*c)*x + 12*b*c^2*x^2 + 8*c^3*x^3 + (b^2 - 4*a*c)^3/(b + 2*
c*x)^3 - (9*(b^2 - 4*a*c)^2)/(b + 2*c*x))/(384*c^4*d^4)
```

Maple [A] time = 0.046, size = 116, normalized size = 1.1

$$\frac{1}{d^4}\left(\frac{1}{32\,c^3}\left(\frac{2\,x^3c^2}{3}+bcx^2+6\,acx-b^2x\right)-\frac{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}{384\,c^4\left(2\,cx+b\right)^3}-\frac{48\,a^2c^2-24\,acb^2+3\,b^4}{128\,c^4\left(2\,cx+b\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^4,x)
```

```
[Out] 1/d^4*(1/32/c^3*(2/3*x^3*c^2+b*c*x^2+6*a*c*x-b^2*x)-1/384*(64*a^3*c^3-48*a^
2*b^2*c^2+12*a*b^4*c-b^6)/c^4/(2*c*x+b)^3-1/128*(48*a^2*c^2-24*a*b^2*c+3*b^
4)/c^4/(2*c*x+b))
```

Maxima [A] time = 1.1684, size = 236, normalized size = 2.29

$$-\frac{2\,b^6-15\,ab^4c+24\,a^2b^2c^2+16\,a^3c^3+9\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)x^2+9\left(b^5c-8\,ab^3c^2+16\,a^2bc^3\right)x}{96\left(8\,c^7d^4x^3+12\,bc^6d^4x^2+6\,b^2c^5d^4x+b^3c^4d^4\right)}+\frac{2\,c^2x^3+3\,bcx^2-}{96\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^4,x, algorithm="maxima")
```

```
[Out] -1/96*(2*b^6 - 15*a*b^4*c + 24*a^2*b^2*c^2 + 16*a^3*c^3 + 9*(b^4*c^2 - 8*a*
b^2*c^3 + 16*a^2*c^4)*x^2 + 9*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x)/(8*c^
7*d^4*x^3 + 12*b*c^6*d^4*x^2 + 6*b^2*c^5*d^4*x + b^3*c^4*d^4) + 1/96*(2*c^2
*x^3 + 3*b*c*x^2 - 3*(b^2 - 6*a*c)*x)/(c^3*d^4)
```

Fricas [B] time = 1.92167, size = 417, normalized size = 4.05

$$\frac{16\,c^6x^6+48\,bc^5x^5-2\,b^6+15\,ab^4c-24\,a^2b^2c^2-16\,a^3c^3+24\left(b^2c^4+6\,ac^5\right)x^4-8\left(2\,b^3c^3-27\,abc^4\right)x^3-12\left(2\,b^4c^2-15\,a\right.}{96\left(8\,c^7d^4x^3+12\,bc^6d^4x^2+6\,b^2c^5d^4x+b^3c^4d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^4,x, algorithm="fricas")
```

```
[Out] 1/96*(16*c^6*x^6 + 48*b*c^5*x^5 - 2*b^6 + 15*a*b^4*c - 24*a^2*b^2*c^2 - 16*
a^3*c^3 + 24*(b^2*c^4 + 6*a*c^5)*x^4 - 8*(2*b^3*c^3 - 27*a*b*c^4)*x^3 - 12*
(2*b^4*c^2 - 15*a*b^2*c^3 + 12*a^2*c^4)*x^2 - 6*(2*b^5*c - 15*a*b^3*c^2 + 2
4*a^2*b*c^3)*x)/(8*c^7*d^4*x^3 + 12*b*c^6*d^4*x^2 + 6*b^2*c^5*d^4*x + b^3*c
^4*d^4)
```

Sympy [A] time = 2.40521, size = 185, normalized size = 1.8

$$\frac{bx^2}{32c^2d^4}-\frac{16a^3c^3+24a^2b^2c^2-15ab^4c+2b^6+x^2\left(144a^2c^4-72ab^2c^3+9b^4c^2\right)+x\left(144a^2bc^3-72ab^3c^2+9b^5c\right)}{96b^3c^4d^4+576b^2c^5d^4x+1152bc^6d^4x^2+768c^7d^4x^3}+\frac{x^3}{48cd^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(2*c*d*x+b*d)**4,x)
```

```
[Out] b*x**2/(32*c**2*d**4) - (16*a**3*c**3 + 24*a**2*b**2*c**2 - 15*a*b**4*c + 2
*b**6 + x**2*(144*a**2*c**4 - 72*a*b**2*c**3 + 9*b**4*c**2) + x*(144*a**2*b
*c**3 - 72*a*b**3*c**2 + 9*b**5*c))/(96*b**3*c**4*d**4 + 576*b**2*c**5*d**4
*x + 1152*b*c**6*d**4*x**2 + 768*c**7*d**4*x**3) + x**3/(48*c*d**4) + x*(6*
a*c - b**2)/(32*c**3*d**4)
```

Giac [A] time = 1.201, size = 221, normalized size = 2.15

$$-\frac{9\,b^4c^2x^2 - 72\,ab^2c^3x^2 + 144\,a^2c^4x^2 + 9\,b^5cx - 72\,ab^3c^2x + 144\,a^2bc^3x + 2\,b^6 - 15\,ab^4c + 24\,a^2b^2c^2 + 16\,a^3c^3}{96\,(2\,cx + b)^3c^4d^4} + \frac{2\,c^{11}a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^4,x, algorithm="giac")
```

```
[Out] -1/96*(9*b^4*c^2*x^2 - 72*a*b^2*c^3*x^2 + 144*a^2*c^4*x^2 + 9*b^5*c*x - 72*
a*b^3*c^2*x + 144*a^2*b*c^3*x + 2*b^6 - 15*a*b^4*c + 24*a^2*b^2*c^2 + 16*a^
3*c^3)/((2*c*x + b)^3*c^4*d^4) + 1/96*(2*c^11*d^8*x^3 + 3*b*c^10*d^8*x^2 -
3*b^2*c^9*d^8*x + 18*a*c^10*d^8*x)/(c^12*d^12)
```

3.1146 $\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^5}\,dx$

Optimal. Leaf size=107

$$\frac{(b^2-4ac)^3}{512c^4d^5(b+2cx)^4}-\frac{3(b^2-4ac)^2}{256c^4d^5(b+2cx)^2}-\frac{3(b^2-4ac)\log(b+2cx)}{128c^4d^5}+\frac{bx}{64c^3d^5}+\frac{x^2}{64c^2d^5}$$

```
[Out] (b*x)/(64*c^3*d^5) + x^2/(64*c^2*d^5) + (b^2 - 4*a*c)^3/(512*c^4*d^5*(b + 2
*c*x)^4) - (3*(b^2 - 4*a*c)^2)/(256*c^4*d^5*(b + 2*c*x)^2) - (3*(b^2 - 4*a*
c)*Log[b + 2*c*x])/(128*c^4*d^5)
```

Rubi [A] time = 0.0982241, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$\frac{(b^2-4ac)^3}{512c^4d^5(b+2cx)^4}-\frac{3(b^2-4ac)^2}{256c^4d^5(b+2cx)^2}-\frac{3(b^2-4ac)\log(b+2cx)}{128c^4d^5}+\frac{bx}{64c^3d^5}+\frac{x^2}{64c^2d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^5,x]
```

```
[Out] (b*x)/(64*c^3*d^5) + x^2/(64*c^2*d^5) + (b^2 - 4*a*c)^3/(512*c^4*d^5*(b + 2
*c*x)^4) - (3*(b^2 - 4*a*c)^2)/(256*c^4*d^5*(b + 2*c*x)^2) - (3*(b^2 - 4*a*
c)*Log[b + 2*c*x])/(128*c^4*d^5)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^5}\,dx &= \int \left(\frac{b}{64c^3d^5}+\frac{x}{32c^2d^5}+\frac{(-b^2+4ac)^3}{64c^3d^5(b+2cx)^5}+\frac{3(-b^2+4ac)^2}{64c^3d^5(b+2cx)^3}+\frac{3(-b^2+4ac)}{64c^3d^5(b+2cx)}\right)dx\\ &= \frac{bx}{64c^3d^5}+\frac{x^2}{64c^2d^5}+\frac{(b^2-4ac)^3}{512c^4d^5(b+2cx)^4}-\frac{3(b^2-4ac)^2}{256c^4d^5(b+2cx)^2}-\frac{3(b^2-4ac)\log(b+2cx)}{128c^4d^5}\end{aligned}$$

Mathematica [A] time = 0.0427481, size = 80, normalized size = 0.75

$$\frac{\frac{(b^2-4ac)^3}{(b+2cx)^4}-\frac{6(b^2-4ac)^2}{(b+2cx)^2}-12(b^2-4ac)\log(b+2cx)+8bcx+8c^2x^2}{512c^4d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^5,x]
```

```
[Out] (8*b*c*x + 8*c^2*x^2 + (b^2 - 4*a*c)^3/(b + 2*c*x)^4 - (6*(b^2 - 4*a*c)^2)/
(b + 2*c*x)^2 - 12*(b^2 - 4*a*c)*Log[b + 2*c*x])/(512*c^4*d^5)
```

Maple [A] time = 0.047, size = 195, normalized size = 1.8

$$\frac{x^2}{64c^2d^5}+\frac{bx}{64c^3d^5}-\frac{3a^2}{16c^2d^5(2cx+b)^2}+\frac{3b^2a}{32c^3d^5(2cx+b)^2}-\frac{3b^4}{256c^4d^5(2cx+b)^2}-\frac{a^3}{8d^5c(2cx+b)^4}+\frac{3b^2a^2}{32c^2d^5(2cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^5,x)
```

```
[Out] 1/64*x^2/c^2/d^5+1/64*b*x/c^3/d^5-3/16/d^5/c^2/(2*c*x+b)^2*a^2+3/32/d^5/c^3
/(2*c*x+b)^2*a*b^2-3/256/d^5/c^4/(2*c*x+b)^2*b^4-1/8/d^5/c/(2*c*x+b)^4*a^3+
3/32/d^5/c^2/(2*c*x+b)^4*a^2*b^2-3/128/d^5/c^3/(2*c*x+b)^4*a*b^4+1/512/d^5/
c^4/(2*c*x+b)^4*b^6+3/32/d^5/c^3*ln(2*c*x+b)*a-3/128/d^5/c^4*ln(2*c*x+b)*b^
2
```

Maxima [A] time = 1.13375, size = 262, normalized size = 2.45

$$-\frac{5b^6-36ab^4c+48a^2b^2c^2+64a^3c^3+24\left(b^4c^2-8ab^2c^3+16a^2c^4\right)x^2+24\left(b^5c-8ab^3c^2+16a^2bc^3\right)x}{512\left(16c^8d^5x^4+32bc^7d^5x^3+24b^2c^6d^5x^2+8b^3c^5d^5x+b^4c^4d^5\right)}+\frac{cx^2+bx}{64c^3d^5}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^5,x, algorithm="maxima")
```

```
[Out] -1/512*(5*b^6 - 36*a*b^4*c + 48*a^2*b^2*c^2 + 64*a^3*c^3 + 24*(b^4*c^2 - 8*
a*b^2*c^3 + 16*a^2*c^4)*x^2 + 24*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x)/(1
6*c^8*d^5*x^4 + 32*b*c^7*d^5*x^3 + 24*b^2*c^6*d^5*x^2 + 8*b^3*c^5*d^5*x + b
^4*c^4*d^5) + 1/64*(c*x^2 + b*x)/(c^3*d^5) - 3/128*(b^2 - 4*a*c)*log(2*c*x
+ b)/(c^4*d^5)
```

Fricas [B] time = 2.07979, size = 624, normalized size = 5.83

$$\frac{128c^6x^6+384bc^5x^5+448b^2c^4x^4+256b^3c^3x^3-5b^6+36ab^4c-48a^2b^2c^2-64a^3c^3+48\left(b^4c^2+4ab^2c^3-8a^2c^4\right)x^2}{512\left(16c^8d^5x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^5,x, algorithm="fricas")
```

```
[Out] 1/512*(128*c^6*x^6 + 384*b*c^5*x^5 + 448*b^2*c^4*x^4 + 256*b^3*c^3*x^3 - 5*
b^6 + 36*a*b^4*c - 48*a^2*b^2*c^2 - 64*a^3*c^3 + 48*(b^4*c^2 + 4*a*b^2*c^3
- 8*a^2*c^4)*x^2 - 16*(b^5*c - 12*a*b^3*c^2 + 24*a^2*b*c^3)*x - 12*(b^6 - 4
*a*b^4*c + 16*(b^2*c^4 - 4*a*c^5)*x^4 + 32*(b^3*c^3 - 4*a*b*c^4)*x^3 + 24*(
b^4*c^2 - 4*a*b^2*c^3)*x^2 + 8*(b^5*c - 4*a*b^3*c^2)*x)*log(2*c*x + b))/(16
*c^8*d^5*x^4 + 32*b*c^7*d^5*x^3 + 24*b^2*c^6*d^5*x^2 + 8*b^3*c^5*d^5*x + b^
4*c^4*d^5)
```

Sympy [A] time = 5.38392, size = 209, normalized size = 1.95

$$\frac{bx}{64c^3d^5} - \frac{64a^3c^3 + 48a^2b^2c^2 - 36ab^4c + 5b^6 + x^2\left(384a^2c^4 - 192ab^2c^3 + 24b^4c^2\right) + x\left(384a^2bc^3 - 192ab^3c^2 + 24b^5c\right)}{512b^4c^4d^5 + 4096b^3c^5d^5x + 12288b^2c^6d^5x^2 + 16384bc^7d^5x^3 + 8192c^8d^5x^4} + \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(2*c*d*x+b*d)**5,x)
```

```
[Out] b*x/(64*c**3*d**5) - (64*a**3*c**3 + 48*a**2*b**2*c**2 - 36*a*b**4*c + 5*b*
*6 + x**2*(384*a**2*c**4 - 192*a*b**2*c**3 + 24*b**4*c**2) + x*(384*a**2*b*
c**3 - 192*a*b**3*c**2 + 24*b**5*c))/(512*b**4*c**4*d**5 + 4096*b**3*c**5*d
**5*x + 12288*b**2*c**6*d**5*x**2 + 16384*b*c**7*d**5*x**3 + 8192*c**8*d**5
*x**4) + x**2/(64*c**2*d**5) + 3*(4*a*c - b**2)*log(b + 2*c*x)/(128*c**4*d*
*5)
```

Giac [B] time = 1.21066, size = 354, normalized size = 3.31

$$\frac{3\left(b^2 - 4\,ac\right)\log\left(\frac{1}{4\,(2\,cdx+bd)^2c^2d^2}\right)}{256\,c^4d^5} - \frac{(2\,cdx + bd)^2\left(\frac{3\,b^2d^2}{(2\,cdx+bd)^2} - \frac{12\,acd^2}{(2\,cdx+bd)^2} - 1\right)}{256\,c^4d^7} + \frac{\frac{b^6c^8d^{17}}{(2\,cdx+bd)^4} - \frac{12\,ab^4c^9d^{17}}{(2\,cdx+bd)^4} + \frac{48\,a^2b^2c^{10}d^{17}}{(2\,cdx+bd)^4} - \frac{64\,a^3 \ldots}{(2\,cd \ldots}}{512\,c \ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^5,x, algorithm="giac")
```

```
[Out] 3/256*(b^2 - 4*a*c)*log(1/4/((2*c*d*x + b*d)^2*c^2*d^2))/(c^4*d^5) - 1/256*
(2*c*d*x + b*d)^2*(3*b^2*d^2/(2*c*d*x + b*d)^2 - 12*a*c*d^2/(2*c*d*x + b*d)
^2 - 1)/(c^4*d^7) + 1/512*(b^6*c^8*d^17/(2*c*d*x + b*d)^4 - 12*a*b^4*c^9*d^
17/(2*c*d*x + b*d)^4 + 48*a^2*b^2*c^10*d^17/(2*c*d*x + b*d)^4 - 64*a^3*c^11
*d^17/(2*c*d*x + b*d)^4 - 6*b^4*c^8*d^15/(2*c*d*x + b*d)^2 + 48*a*b^2*c^9*d
^15/(2*c*d*x + b*d)^2 - 96*a^2*c^10*d^15/(2*c*d*x + b*d)^2)/(c^12*d^18)
```

3.1147 $\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^6} dx$

Optimal. Leaf size=94

$$\frac{(b^2-4ac)^3}{640c^4d^6(b+2cx)^5}-\frac{(b^2-4ac)^2}{128c^4d^6(b+2cx)^3}+\frac{3(b^2-4ac)}{128c^4d^6(b+2cx)}+\frac{x}{64c^3d^6}$$

```
[Out] x/(64*c^3*d^6) + (b^2 - 4*a*c)^3/(640*c^4*d^6*(b + 2*c*x)^5) - (b^2 - 4*a*c
)^2/(128*c^4*d^6*(b + 2*c*x)^3) + (3*(b^2 - 4*a*c))/(128*c^4*d^6*(b + 2*c*x
))
```

Rubi [A] time = 0.0854048, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$\frac{(b^2-4ac)^3}{640c^4d^6(b+2cx)^5}-\frac{(b^2-4ac)^2}{128c^4d^6(b+2cx)^3}+\frac{3(b^2-4ac)}{128c^4d^6(b+2cx)}+\frac{x}{64c^3d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^6,x]
```

```
[Out] x/(64*c^3*d^6) + (b^2 - 4*a*c)^3/(640*c^4*d^6*(b + 2*c*x)^5) - (b^2 - 4*a*c
)^2/(128*c^4*d^6*(b + 2*c*x)^3) + (3*(b^2 - 4*a*c))/(128*c^4*d^6*(b + 2*c*x
))
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^6} dx &= \int \left(\frac{1}{64c^3d^6}+\frac{(-b^2+4ac)^3}{64c^3d^6(b+2cx)^6}+\frac{3(-b^2+4ac)^2}{64c^3d^6(b+2cx)^4}+\frac{3(-b^2+4ac)}{64c^3d^6(b+2cx)^2}\right) dx\\ &= \frac{x}{64c^3d^6}+\frac{(b^2-4ac)^3}{640c^4d^6(b+2cx)^5}-\frac{(b^2-4ac)^2}{128c^4d^6(b+2cx)^3}+\frac{3(b^2-4ac)}{128c^4d^6(b+2cx)}\end{aligned}$$

Mathematica [A] time = 0.0629908, size = 72, normalized size = 0.77

$$\frac{\frac{(b^2-4ac)^3}{(b+2cx)^5}-\frac{5(b^2-4ac)^2}{(b+2cx)^3}+\frac{15(b^2-4ac)}{b+2cx}+10cx}{640c^4d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^6,x]
```

```
[Out] (10*c*x + (b^2 - 4*a*c)^3/(b + 2*c*x)^5 - (5*(b^2 - 4*a*c)^2)/(b + 2*c*x)^3
 + (15*(b^2 - 4*a*c))/(b + 2*c*x))/(640*c^4*d^6)
```

Maple [A] time = 0.047, size = 114, normalized size = 1.2

$$\frac{1}{d^6}\left(\frac{x}{64\,c^3}-\frac{48\,a^2c^2-24\,acb^2+3\,b^4}{384\,c^4\,(2\,cx+b)^3}-\frac{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}{640\,c^4\,(2\,cx+b)^5}-\frac{12\,ac-3\,b^2}{128\,c^4\,(2\,cx+b)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^6,x)
```

```
[Out] 1/d^6*(1/64*x/c^3-1/384*(48*a^2*c^2-24*a*b^2*c+3*b^4)/c^4/(2*c*x+b)^3-1/640
/c^4*(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(2*c*x+b)^5-1/128*(12*a*c-3
*b^2)/c^4/(2*c*x+b))
```

Maxima [B] time = 1.23413, size = 294, normalized size = 3.13

$$\frac{11\,b^6-32\,ab^4c-32\,a^2b^2c^2-64\,a^3c^3+240\left(b^2c^4-4\,ac^5\right)x^4+480\left(b^3c^3-4\,abc^4\right)x^3+20\left(17\,b^4c^2-64\,ab^2c^3-16\,a^2c^4\right)x^2}{640\left(32\,c^9d^6x^5+80\,bc^8d^6x^4+80\,b^2c^7d^6x^3+40\,b^3c^6d^6x^2+10\,b^4c^5d^6x+b^5c^4d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^6,x, algorithm="maxima")
```

```
[Out] 1/640*(11*b^6 - 32*a*b^4*c - 32*a^2*b^2*c^2 - 64*a^3*c^3 + 240*(b^2*c^4 - 4
*a*c^5)*x^4 + 480*(b^3*c^3 - 4*a*b*c^4)*x^3 + 20*(17*b^4*c^2 - 64*a*b^2*c^3
 - 16*a^2*c^4)*x^2 + 20*(5*b^5*c - 16*a*b^3*c^2 - 16*a^2*b*c^3)*x)/(32*c^9*
d^6*x^5 + 80*b*c^8*d^6*x^4 + 80*b^2*c^7*d^6*x^3 + 40*b^3*c^6*d^6*x^2 + 10*b
^4*c^5*d^6*x + b^5*c^4*d^6) + 1/64*x/(c^3*d^6)
```

Fricas [B] time = 1.93359, size = 493, normalized size = 5.24

$$\frac{320\,c^6x^6+800\,bc^5x^5+11\,b^6-32\,ab^4c-32\,a^2b^2c^2-64\,a^3c^3+80\left(13\,b^2c^4-12\,ac^5\right)x^4+80\left(11\,b^3c^3-24\,abc^4\right)x^3+40\,(11}{640\left(32\,c^9d^6x^5+80\,bc^8d^6x^4+80\,b^2c^7d^6x^3+40\,b^3c^6d^6x^2+10\,b^4c^5d^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^6,x, algorithm="fricas")
```

```
[Out] 1/640*(320*c^6*x^6 + 800*b*c^5*x^5 + 11*b^6 - 32*a*b^4*c - 32*a^2*b^2*c^2 -
 64*a^3*c^3 + 80*(13*b^2*c^4 - 12*a*c^5)*x^4 + 80*(11*b^3*c^3 - 24*a*b*c^4)
*x^3 + 40*(11*b^4*c^2 - 32*a*b^2*c^3 - 8*a^2*c^4)*x^2 + 10*(11*b^5*c - 32*a
*b^3*c^2 - 32*a^2*b*c^3)*x)/(32*c^9*d^6*x^5 + 80*b*c^8*d^6*x^4 + 80*b^2*c^7
*d^6*x^3 + 40*b^3*c^6*d^6*x^2 + 10*b^4*c^5*d^6*x + b^5*c^4*d^6)
```

Sympy [B] time = 5.23528, size = 223, normalized size = 2.37

$$-\frac{64a^3c^3+32a^2b^2c^2+32ab^4c-11b^6+x^4\left(960ac^5-240b^2c^4\right)+x^3\left(1920abc^4-480b^3c^3\right)+x^2\left(320a^2c^4+1280ab^2c^3-34\right.}{640b^5c^4d^6+6400b^4c^5d^6x+25600b^3c^6d^6x^2+51200b^2c^7d^6x^3+51200bc^8d^6x^4+204}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(2*c*d*x+b*d)**6,x)
```

```
[Out] -(64*a**3*c**3 + 32*a**2*b**2*c**2 + 32*a*b**4*c - 11*b**6 + x**4*(960*a*c*
*5 - 240*b**2*c**4) + x**3*(1920*a*b*c**4 - 480*b**3*c**3) + x**2*(320*a**2
*c**4 + 1280*a*b**2*c**3 - 340*b**4*c**2) + x*(320*a**2*b*c**3 + 320*a*b**3
*c**2 - 100*b**5*c))/(640*b**5*c**4*d**6 + 6400*b**4*c**5*d**6*x + 25600*b*
*3*c**6*d**6*x**2 + 51200*b**2*c**7*d**6*x**3 + 51200*b*c**8*d**6*x**4 + 20
480*c**9*d**6*x**5) + x/(64*c**3*d**6)
```

Giac [A] time = 1.18641, size = 216, normalized size = 2.3

$$\frac{x}{64\,c^3d^6}+\frac{240\,b^2c^4x^4-960\,ac^5x^4+480\,b^3c^3x^3-1920\,abc^4x^3+340\,b^4c^2x^2-1280\,ab^2c^3x^2-320\,a^2c^4x^2+100\,b^5cx-}{640\,(2\,cx+b)^5c^4d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^6,x, algorithm="giac")
```

```
[Out] 1/64*x/(c^3*d^6) + 1/640*(240*b^2*c^4*x^4 - 960*a*c^5*x^4 + 480*b^3*c^3*x^3
 - 1920*a*b*c^4*x^3 + 340*b^4*c^2*x^2 - 1280*a*b^2*c^3*x^2 - 320*a^2*c^4*x^
2 + 100*b^5*c*x - 320*a*b^3*c^2*x - 320*a^2*b*c^3*x + 11*b^6 - 32*a*b^4*c -
 32*a^2*b^2*c^2 - 64*a^3*c^3)/((2*c*x + b)^5*c^4*d^6)
```

3.1148 $\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^7} dx$

Optimal. Leaf size=100

$$\frac{(b^2-4ac)^3}{768c^4d^7(b+2cx)^6}-\frac{3(b^2-4ac)^2}{512c^4d^7(b+2cx)^4}+\frac{3(b^2-4ac)}{256c^4d^7(b+2cx)^2}+\frac{\log(b+2cx)}{128c^4d^7}$$

```
[Out] (b^2 - 4*a*c)^3/(768*c^4*d^7*(b + 2*c*x)^6) - (3*(b^2 - 4*a*c)^2)/(512*c^4*
d^7*(b + 2*c*x)^4) + (3*(b^2 - 4*a*c))/(256*c^4*d^7*(b + 2*c*x)^2) + Log[b
+ 2*c*x]/(128*c^4*d^7)
```

Rubi [A] time = 0.0828874, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$\frac{(b^2-4ac)^3}{768c^4d^7(b+2cx)^6}-\frac{3(b^2-4ac)^2}{512c^4d^7(b+2cx)^4}+\frac{3(b^2-4ac)}{256c^4d^7(b+2cx)^2}+\frac{\log(b+2cx)}{128c^4d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^7,x]
```

```
[Out] (b^2 - 4*a*c)^3/(768*c^4*d^7*(b + 2*c*x)^6) - (3*(b^2 - 4*a*c)^2)/(512*c^4*
d^7*(b + 2*c*x)^4) + (3*(b^2 - 4*a*c))/(256*c^4*d^7*(b + 2*c*x)^2) + Log[b
+ 2*c*x]/(128*c^4*d^7)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] && !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^7}dx &= \int \left(\frac{(-b^2+4ac)^3}{64c^3d^7(b+2cx)^7}+\frac{3(-b^2+4ac)^2}{64c^3d^7(b+2cx)^5}+\frac{3(-b^2+4ac)}{64c^3d^7(b+2cx)^3}+\frac{1}{64c^3d^7(b+2cx)}\right)dx\\ &= \frac{(b^2-4ac)^3}{768c^4d^7(b+2cx)^6}-\frac{3(b^2-4ac)^2}{512c^4d^7(b+2cx)^4}+\frac{3(b^2-4ac)}{256c^4d^7(b+2cx)^2}+\frac{\log(b+2cx)}{128c^4d^7}\end{aligned}$$

Mathematica [A] time = 0.0422631, size = 78, normalized size = 0.78

$$\frac{\frac{2(b^2-4ac)^3}{(b+2cx)^6}-\frac{9(b^2-4ac)^2}{(b+2cx)^4}+\frac{18(b^2-4ac)}{(b+2cx)^2}+12\log(b+2cx)}{1536c^4d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^7,x]
```

```
[Out] ((2*(b^2 - 4*a*c)^3)/(b + 2*c*x)^6 - (9*(b^2 - 4*a*c)^2)/(b + 2*c*x)^4 + (1
8*(b^2 - 4*a*c))/(b + 2*c*x)^2 + 12*Log[b + 2*c*x])/(1536*c^4*d^7)
```

Maple [B] time = 0.069, size = 191, normalized size = 1.9

$$-\frac{a^{3}}{12\,d^{7}c\,(2\,cx+b)^{6}}+\frac{b^{2}a^{2}}{16\,d^{7}c^{2}\,(2\,cx+b)^{6}}-\frac{b^{4}a}{64\,d^{7}c^{3}\,(2\,cx+b)^{6}}+\frac{b^{6}}{768\,c^{4}d^{7}\,(2\,cx+b)^{6}}-\frac{3\,a}{64\,d^{7}c^{3}\,(2\,cx+b)^{2}}+\frac{3}{256\,c^{4}d^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^7,x)
```

```
[Out] -1/12/d^7/c/(2*c*x+b)^6*a^3+1/16/d^7/c^2/(2*c*x+b)^6*a^2*b^2-1/64/d^7/c^3/(
2*c*x+b)^6*a*b^4+1/768/d^7/c^4/(2*c*x+b)^6*b^6-3/64/d^7/c^3/(2*c*x+b)^2*a+3
/256/d^7/c^4/(2*c*x+b)^2*b^2-3/32/d^7/c^2/(2*c*x+b)^4*a^2+3/64/d^7/c^3/(2*c
*x+b)^4*a*b^2-3/512/d^7/c^4/(2*c*x+b)^4*b^4+1/128*ln(2*c*x+b)/c^4/d^7
```

Maxima [B] time = 1.14426, size = 321, normalized size = 3.21

$$\frac{11\,b^{6}-24\,ab^{4}c-48\,a^{2}b^{2}c^{2}-128\,a^{3}c^{3}+288\,\left(b^{2}c^{4}-4\,ac^{5}\right)x^{4}+576\,\left(b^{3}c^{3}-4\,abc^{4}\right)x^{3}+36\,\left(11\,b^{4}c^{2}-40\,ab^{2}c^{3}-16\,a^{2}c\right.}{1536\,\left(64\,c^{10}d^{7}x^{6}+192\,bc^{9}d^{7}x^{5}+240\,b^{2}c^{8}d^{7}x^{4}+160\,b^{3}c^{7}d^{7}x^{3}+60\,b^{4}c^{6}d^{7}x^{2}+12\,b^{5}c^{5}d^{7}x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^7,x, algorithm="maxima")
```

```
[Out] 1/1536*(11*b^6 - 24*a*b^4*c - 48*a^2*b^2*c^2 - 128*a^3*c^3 + 288*(b^2*c^4 -
 4*a*c^5)*x^4 + 576*(b^3*c^3 - 4*a*b*c^4)*x^3 + 36*(11*b^4*c^2 - 40*a*b^2*c
^3 - 16*a^2*c^4)*x^2 + 36*(3*b^5*c - 8*a*b^3*c^2 - 16*a^2*b*c^3)*x)/(64*c^1
0*d^7*x^6 + 192*b*c^9*d^7*x^5 + 240*b^2*c^8*d^7*x^4 + 160*b^3*c^7*d^7*x^3 +
 60*b^4*c^6*d^7*x^2 + 12*b^5*c^5*d^7*x + b^6*c^4*d^7) + 1/128*log(2*c*x + b
)/(c^4*d^7)
```

Fricas [B] time = 2.04045, size = 643, normalized size = 6.43

$$\frac{11\,b^{6}-24\,ab^{4}c-48\,a^{2}b^{2}c^{2}-128\,a^{3}c^{3}+288\,\left(b^{2}c^{4}-4\,ac^{5}\right)x^{4}+576\,\left(b^{3}c^{3}-4\,abc^{4}\right)x^{3}+36\,\left(11\,b^{4}c^{2}-40\,ab^{2}c^{3}-16\,a^{2}c\right.}{1536\,\left(64\,c^{10}d^{7}x^{6}+192\,bc^{9}d^{7}x^{5}+240\,b^{2}c^{8}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^7,x, algorithm="fricas")
```

```
[Out] 1/1536*(11*b^6 - 24*a*b^4*c - 48*a^2*b^2*c^2 - 128*a^3*c^3 + 288*(b^2*c^4 -
 4*a*c^5)*x^4 + 576*(b^3*c^3 - 4*a*b*c^4)*x^3 + 36*(11*b^4*c^2 - 40*a*b^2*c
^3 - 16*a^2*c^4)*x^2 + 36*(3*b^5*c - 8*a*b^3*c^2 - 16*a^2*b*c^3)*x + 12*(64
*c^6*x^6 + 192*b*c^5*x^5 + 240*b^2*c^4*x^4 + 160*b^3*c^3*x^3 + 60*b^4*c^2*x
^2 + 12*b^5*c*x + b^6)*log(2*c*x + b))/(64*c^10*d^7*x^6 + 192*b*c^9*d^7*x^5
 + 240*b^2*c^8*d^7*x^4 + 160*b^3*c^7*d^7*x^3 + 60*b^4*c^6*d^7*x^2 + 12*b^5*
c^5*d^7*x + b^6*c^4*d^7)
```

Sympy [B] time = 14.7638, size = 245, normalized size = 2.45

$$-\frac{128a^3c^3 + 48a^2b^2c^2 + 24ab^4c - 11b^6 + x^4\left(1152ac^5 - 288b^2c^4\right) + x^3\left(2304abc^4 - 576b^3c^3\right) + x^2\left(576a^2c^4 + 1440ab^2c^3 - \right.}{1536b^6c^4d^7 + 18432b^5c^5d^7x + 92160b^4c^6d^7x^2 + 245760b^3c^7d^7x^3 + 368640b^2c^8d^7x^4 + 294912bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(2*c*d*x+b*d)**7,x)
```

```
[Out] -(128*a**3*c**3 + 48*a**2*b**2*c**2 + 24*a*b**4*c - 11*b**6 + x**4*(1152*a*
c**5 - 288*b**2*c**4) + x**3*(2304*a*b*c**4 - 576*b**3*c**3) + x**2*(576*a*
*2*c**4 + 1440*a*b**2*c**3 - 396*b**4*c**2) + x*(576*a**2*b*c**3 + 288*a*b*
*3*c**2 - 108*b**5*c))/(1536*b**6*c**4*d**7 + 18432*b**5*c**5*d**7*x + 9216
0*b**4*c**6*d**7*x**2 + 245760*b**3*c**7*d**7*x**3 + 368640*b**2*c**8*d**7*
x**4 + 294912*b*c**9*d**7*x**5 + 98304*c**10*d**7*x**6) + log(b + 2*c*x)/(1
28*c**4*d**7)
```

Giac [A] time = 1.18739, size = 220, normalized size = 2.2

$$\frac{\log\left(|2\,cx + b|\right)}{128\,c^4d^7} + \frac{11\,b^6 - 24\,ab^4c - 48\,a^2b^2c^2 - 128\,a^3c^3 + 288\left(b^2c^4 - 4\,ac^5\right)x^4 + 576\left(b^3c^3 - 4\,abc^4\right)x^3 + 36\left(11\,b^4c^2 - 40\right.}{1536\,(2\,cx + b)^6c^4d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^7,x, algorithm="giac")
```

```
[Out] 1/128*log(abs(2*c*x + b))/(c^4*d^7) + 1/1536*(11*b^6 - 24*a*b^4*c - 48*a^2*
b^2*c^2 - 128*a^3*c^3 + 288*(b^2*c^4 - 4*a*c^5)*x^4 + 576*(b^3*c^3 - 4*a*b*
c^4)*x^3 + 36*(11*b^4*c^2 - 40*a*b^2*c^3 - 16*a^2*c^4)*x^2 + 36*(3*b^5*c -
8*a*b^3*c^2 - 16*a^2*b*c^3)*x)/((2*c*x + b)^6*c^4*d^7)
```

3.1149 $\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^8} dx$

Optimal. Leaf size=101

$$\frac{(b^2-4ac)^3}{896c^4d^8(b+2cx)^7}-\frac{3(b^2-4ac)^2}{640c^4d^8(b+2cx)^5}+\frac{b^2-4ac}{128c^4d^8(b+2cx)^3}-\frac{1}{128c^4d^8(b+2cx)}$$

```
[Out] (b^2 - 4*a*c)^3/(896*c^4*d^8*(b + 2*c*x)^7) - (3*(b^2 - 4*a*c)^2)/(640*c^4*
d^8*(b + 2*c*x)^5) + (b^2 - 4*a*c)/(128*c^4*d^8*(b + 2*c*x)^3) - 1/(128*c^4
*d^8*(b + 2*c*x))
```

Rubi [A] time = 0.080555, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$\frac{(b^2-4ac)^3}{896c^4d^8(b+2cx)^7}-\frac{3(b^2-4ac)^2}{640c^4d^8(b+2cx)^5}+\frac{b^2-4ac}{128c^4d^8(b+2cx)^3}-\frac{1}{128c^4d^8(b+2cx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^8,x]
```

```
[Out] (b^2 - 4*a*c)^3/(896*c^4*d^8*(b + 2*c*x)^7) - (3*(b^2 - 4*a*c)^2)/(640*c^4*
d^8*(b + 2*c*x)^5) + (b^2 - 4*a*c)/(128*c^4*d^8*(b + 2*c*x)^3) - 1/(128*c^4
*d^8*(b + 2*c*x))
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^8}dx &= \int \left(\frac{(-b^2+4ac)^3}{64c^3d^8(b+2cx)^8}+\frac{3(-b^2+4ac)^2}{64c^3d^8(b+2cx)^6}+\frac{3(-b^2+4ac)}{64c^3d^8(b+2cx)^4}+\frac{1}{64c^3d^8(b+2cx)^2}\right)dx\\ &= \frac{(b^2-4ac)^3}{896c^4d^8(b+2cx)^7}-\frac{3(b^2-4ac)^2}{640c^4d^8(b+2cx)^5}+\frac{b^2-4ac}{128c^4d^8(b+2cx)^3}-\frac{1}{128c^4d^8(b+2cx)}\end{aligned}$$

Mathematica [A] time = 0.0485275, size = 79, normalized size = 0.78

$$\frac{35(b^2-4ac)(b+2cx)^4-21(b^2-4ac)^2(b+2cx)^2+5(b^2-4ac)^3-35(b+2cx)^6}{4480c^4d^8(b+2cx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^8,x]
```

```
[Out] (5*(b^2 - 4*a*c)^3 - 21*(b^2 - 4*a*c)^2*(b + 2*c*x)^2 + 35*(b^2 - 4*a*c)*(b
 + 2*c*x)^4 - 35*(b + 2*c*x)^6)/(4480*c^4*d^8*(b + 2*c*x)^7)
```

Maple [A] time = 0.046, size = 121, normalized size = 1.2

$$\frac{1}{d^8}\left(-\frac{12\,ac-3\,b^2}{384\,c^4\,(2\,cx+b)^3}-\frac{48\,a^2c^2-24\,acb^2+3\,b^4}{640\,c^4\,(2\,cx+b)^5}-\frac{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}{896\,c^4\,(2\,cx+b)^7}-\frac{1}{128\,c^4\,(2\,cx+b)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^8,x)
```

```
[Out] 1/d^8*(-1/384*(12*a*c-3*b^2)/c^4/(2*c*x+b)^3-1/640*(48*a^2*c^2-24*a*b^2*c+3
*b^4)/c^4/(2*c*x+b)^5-1/896*(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/c^4/
(2*c*x+b)^7-1/128/c^4/(2*c*x+b))
```

Maxima [B] time = 1.23609, size = 335, normalized size = 3.32

$$-\frac{140\,c^6x^6+420\,bc^5x^5+b^6+2\,ab^4c+6\,a^2b^2c^2+20\,a^3c^3+70\left(7\,b^2c^4+2\,ac^5\right)x^4+280\left(b^3c^3+abc^4\right)x^3+84\left(b^4c^2+2\,ab\right.}{280\left(128\,c^{11}d^8x^7+448\,bc^{10}d^8x^6+672\,b^2c^9d^8x^5+560\,b^3c^8d^8x^4+280\,b^4c^7d^8x^3+84\,b^5c^6d^8x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^8,x, algorithm="maxima")
```

```
[Out] -1/280*(140*c^6*x^6 + 420*b*c^5*x^5 + b^6 + 2*a*b^4*c + 6*a^2*b^2*c^2 + 20*
a^3*c^3 + 70*(7*b^2*c^4 + 2*a*c^5)*x^4 + 280*(b^3*c^3 + a*b*c^4)*x^3 + 84*(
b^4*c^2 + 2*a*b^2*c^3 + a^2*c^4)*x^2 + 14*(b^5*c + 2*a*b^3*c^2 + 6*a^2*b*c^
3)*x)/(128*c^11*d^8*x^7 + 448*b*c^10*d^8*x^6 + 672*b^2*c^9*d^8*x^5 + 560*b^
3*c^8*d^8*x^4 + 280*b^4*c^7*d^8*x^3 + 84*b^5*c^6*d^8*x^2 + 14*b^6*c^5*d^8*x
 + b^7*c^4*d^8)
```

Fricas [B] time = 1.95219, size = 529, normalized size = 5.24

$$-\frac{140\,c^6x^6+420\,bc^5x^5+b^6+2\,ab^4c+6\,a^2b^2c^2+20\,a^3c^3+70\left(7\,b^2c^4+2\,ac^5\right)x^4+280\left(b^3c^3+abc^4\right)x^3+84\left(b^4c^2+2\,ab\right.}{280\left(128\,c^{11}d^8x^7+448\,bc^{10}d^8x^6+672\,b^2c^9d^8x^5+560\,b^3c^8d^8x^4+280\,b^4c^7d^8x^3+84\,b^5c^6d^8x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^8,x, algorithm="fricas")
```

```
[Out] -1/280*(140*c^6*x^6 + 420*b*c^5*x^5 + b^6 + 2*a*b^4*c + 6*a^2*b^2*c^2 + 20*
a^3*c^3 + 70*(7*b^2*c^4 + 2*a*c^5)*x^4 + 280*(b^3*c^3 + a*b*c^4)*x^3 + 84*(
b^4*c^2 + 2*a*b^2*c^3 + a^2*c^4)*x^2 + 14*(b^5*c + 2*a*b^3*c^2 + 6*a^2*b*c^
3)*x)/(128*c^11*d^8*x^7 + 448*b*c^10*d^8*x^6 + 672*b^2*c^9*d^8*x^5 + 560*b^
3*c^8*d^8*x^4 + 280*b^4*c^7*d^8*x^3 + 84*b^5*c^6*d^8*x^2 + 14*b^6*c^5*d^8*x
 + b^7*c^4*d^8)
```

Sympy [B] time = 9.30292, size = 262, normalized size = 2.59

$$-\frac{20a^3c^3 + 6a^2b^2c^2 + 2ab^4c + b^6 + 420bc^5x^5 + 140c^6x^6 + x^4\left(140ac^5 + 490b^2c^4\right) + x^3\left(280abc^4 + 280b^3c^3\right) + x^2\left(84a^2c\right.}{280b^7c^4d^8 + 3920b^6c^5d^8x + 23520b^5c^6d^8x^2 + 78400b^4c^7d^8x^3 + 156800b^3c^8d^8x^4 + 188160b^2c^9d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(2*c*d*x+b*d)**8,x)
```

```
[Out] -(20*a**3*c**3 + 6*a**2*b**2*c**2 + 2*a*b**4*c + b**6 + 420*b*c**5*x**5 + 1
40*c**6*x**6 + x**4*(140*a*c**5 + 490*b**2*c**4) + x**3*(280*a*b*c**4 + 280
*b**3*c**3) + x**2*(84*a**2*c**4 + 168*a*b**2*c**3 + 84*b**4*c**2) + x*(84*
a**2*b*c**3 + 28*a*b**3*c**2 + 14*b**5*c))/(280*b**7*c**4*d**8 + 3920*b**6*
c**5*d**8*x + 23520*b**5*c**6*d**8*x**2 + 78400*b**4*c**7*d**8*x**3 + 15680
0*b**3*c**8*d**8*x**4 + 188160*b**2*c**9*d**8*x**5 + 125440*b*c**10*d**8*x*
*6 + 35840*c**11*d**8*x**7)
```

Giac [A] time = 1.18763, size = 223, normalized size = 2.21

$$-\frac{140\,c^6x^6 + 420\,bc^5x^5 + 490\,b^2c^4x^4 + 140\,ac^5x^4 + 280\,b^3c^3x^3 + 280\,abc^4x^3 + 84\,b^4c^2x^2 + 168\,ab^2c^3x^2 + 84\,a^2c^4x^2 + 1}{280\,(2\,cx + b)^7\,c^4d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^8,x, algorithm="giac")
```

```
[Out] -1/280*(140*c^6*x^6 + 420*b*c^5*x^5 + 490*b^2*c^4*x^4 + 140*a*c^5*x^4 + 280
*b^3*c^3*x^3 + 280*a*b*c^4*x^3 + 84*b^4*c^2*x^2 + 168*a*b^2*c^3*x^2 + 84*a^
2*c^4*x^2 + 14*b^5*c*x + 28*a*b^3*c^2*x + 84*a^2*b*c^3*x + b^6 + 2*a*b^4*c
+ 6*a^2*b^2*c^2 + 20*a^3*c^3)/((2*c*x + b)^7*c^4*d^8)
```

3.1150 $\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^9}\,dx$

Optimal. Leaf size=37

$$\frac{(a+bx+cx^2)^4}{4d^9(b^2-4ac)(b+2cx)^8}$$

```
[Out] (a + b*x + c*x^2)^4/(4*(b^2 - 4*a*c)*d^9*(b + 2*c*x)^8)
```

Rubi [A] time = 0.0133028, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {682}

$$\frac{(a+bx+cx^2)^4}{4d^9(b^2-4ac)(b+2cx)^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^9,x]
```

```
[Out] (a + b*x + c*x^2)^4/(4*(b^2 - 4*a*c)*d^9*(b + 2*c*x)^8)
```

Rule 682

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c,
0] && EqQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^9}\,dx = \frac{(a+bx+cx^2)^4}{4(b^2-4ac)d^9(b+2cx)^8}$$

Mathematica [B] time = 0.0524216, size = 96, normalized size = 2.59

$$\frac{48a^2b^2c^2-64a^3c^3+6(b^2-4ac)(b+2cx)^4-4(b^2-4ac)^2(b+2cx)^2-12ab^4c+b^6-4(b+2cx)^6}{1024c^4d^9(b+2cx)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^9,x]
```

```
[Out] (b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3 - 4*(b^2 - 4*a*c)^2*(b + 2*
c*x)^2 + 6*(b^2 - 4*a*c)*(b + 2*c*x)^4 - 4*(b + 2*c*x)^6)/(1024*c^4*d^9*(b
+ 2*c*x)^8)
```

Maple [B] time = 0.046, size = 121, normalized size = 3.3

$$\frac{1}{d^9}\left(-\frac{48\,a^2c^2-24\,acb^2+3\,b^4}{768\,c^4\,(2\,cx+b)^6}-\frac{1}{256\,c^4\,(2\,cx+b)^2}-\frac{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}{1024\,c^4\,(2\,cx+b)^8}-\frac{12\,ac-3\,b^2}{512\,c^4\,(2\,cx+b)^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^9,x)
```

```
[Out] 1/d^9*(-1/768*(48*a^2*c^2-24*a*b^2*c+3*b^4)/c^4/(2*c*x+b)^6-1/256/c^4/(2*c*
x+b)^2-1/1024*(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/c^4/(2*c*x+b)^8-1/
512*(12*a*c-3*b^2)/c^4/(2*c*x+b)^4)
```

Maxima [B] time = 1.43157, size = 359, normalized size = 9.7

$$-\frac{256\,c^6x^6+768\,bc^5x^5+b^6+4\,ab^4c+16\,a^2b^2c^2+64\,a^3c^3+96\left(9\,b^2c^4+4\,ac^5\right)x^4+64\left(7\,b^3c^3+12\,abc^4\right)x^3+16\left(7\,b^4\right.}{1024\left(256\,c^{12}d^9x^8+1024\,bc^{11}d^9x^7+1792\,b^2c^{10}d^9x^6+1792\,b^3c^9d^9x^5+1120\,b^4c^8d^9x^4+448\,b^5c^7d^9\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^9,x, algorithm="maxima")
```

```
[Out] -1/1024*(256*c^6*x^6 + 768*b*c^5*x^5 + b^6 + 4*a*b^4*c + 16*a^2*b^2*c^2 + 6
4*a^3*c^3 + 96*(9*b^2*c^4 + 4*a*c^5)*x^4 + 64*(7*b^3*c^3 + 12*a*b*c^4)*x^3
+ 16*(7*b^4*c^2 + 28*a*b^2*c^3 + 16*a^2*c^4)*x^2 + 16*(b^5*c + 4*a*b^3*c^2
+ 16*a^2*b*c^3)*x)/(256*c^12*d^9*x^8 + 1024*b*c^11*d^9*x^7 + 1792*b^2*c^10*
d^9*x^6 + 1792*b^3*c^9*d^9*x^5 + 1120*b^4*c^8*d^9*x^4 + 448*b^5*c^7*d^9*x^3
 + 112*b^6*c^6*d^9*x^2 + 16*b^7*c^5*d^9*x + b^8*c^4*d^9)
```

Fricas [B] time = 1.98935, size = 585, normalized size = 15.81

$$-\frac{256\,c^6x^6+768\,bc^5x^5+b^6+4\,ab^4c+16\,a^2b^2c^2+64\,a^3c^3+96\left(9\,b^2c^4+4\,ac^5\right)x^4+64\left(7\,b^3c^3+12\,abc^4\right)x^3+16\left(7\,b^4\right.}{1024\left(256\,c^{12}d^9x^8+1024\,bc^{11}d^9x^7+1792\,b^2c^{10}d^9x^6+1792\,b^3c^9d^9x^5+1120\,b^4c^8d^9x^4+448\,b^5c^7d^9\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^9,x, algorithm="fricas")
```

```
[Out] -1/1024*(256*c^6*x^6 + 768*b*c^5*x^5 + b^6 + 4*a*b^4*c + 16*a^2*b^2*c^2 + 6
4*a^3*c^3 + 96*(9*b^2*c^4 + 4*a*c^5)*x^4 + 64*(7*b^3*c^3 + 12*a*b*c^4)*x^3
+ 16*(7*b^4*c^2 + 28*a*b^2*c^3 + 16*a^2*c^4)*x^2 + 16*(b^5*c + 4*a*b^3*c^2
+ 16*a^2*b*c^3)*x)/(256*c^12*d^9*x^8 + 1024*b*c^11*d^9*x^7 + 1792*b^2*c^10*
d^9*x^6 + 1792*b^3*c^9*d^9*x^5 + 1120*b^4*c^8*d^9*x^4 + 448*b^5*c^7*d^9*x^3
 + 112*b^6*c^6*d^9*x^2 + 16*b^7*c^5*d^9*x + b^8*c^4*d^9)
```

Sympy [B] time = 22.5226, size = 277, normalized size = 7.49

$$-\frac{64a^3c^3+16a^2b^2c^2+4ab^4c+b^6+768bc^5x^5+256c^6x^6+x^4\left(384ac^5+864b^2c^4\right)+x^3\left(768abc^4+448b^3c^3\right)+x^2\left(256\right.}{1024b^8c^4d^9+16384b^7c^5d^9x+114688b^6c^6d^9x^2+458752b^5c^7d^9x^3+1146880b^4c^8d^9x^4+1835008b^3c^9d^9x^5+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(2*c*d*x+b*d)**9,x)
```

```
[Out] -(64*a**3*c**3 + 16*a**2*b**2*c**2 + 4*a*b**4*c + b**6 + 768*b*c**5*x**5 +
256*c**6*x**6 + x**4*(384*a*c**5 + 864*b**2*c**4) + x**3*(768*a*b*c**4 + 44
8*b**3*c**3) + x**2*(256*a**2*c**4 + 448*a*b**2*c**3 + 112*b**4*c**2) + x*(
256*a**2*b*c**3 + 64*a*b**3*c**2 + 16*b**5*c))/(1024*b**8*c**4*d**9 + 16384
*b**7*c**5*d**9*x + 114688*b**6*c**6*d**9*x**2 + 458752*b**5*c**7*d**9*x**3
 + 1146880*b**4*c**8*d**9*x**4 + 1835008*b**3*c**9*d**9*x**5 + 1835008*b**2
*c**10*d**9*x**6 + 1048576*b*c**11*d**9*x**7 + 262144*c**12*d**9*x**8)
```

Giac [B] time = 1.17934, size = 223, normalized size = 6.03

$$-\frac{256\,c^6x^6 + 768\,bc^5x^5 + 864\,b^2c^4x^4 + 384\,ac^5x^4 + 448\,b^3c^3x^3 + 768\,abc^4x^3 + 112\,b^4c^2x^2 + 448\,ab^2c^3x^2 + 256\,a^2c^4x^2 + 16}{1024\,(2\,cx + b)^8c^4d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^9,x, algorithm="giac")
```

```
[Out] -1/1024*(256*c^6*x^6 + 768*b*c^5*x^5 + 864*b^2*c^4*x^4 + 384*a*c^5*x^4 + 44
8*b^3*c^3*x^3 + 768*a*b*c^4*x^3 + 112*b^4*c^2*x^2 + 448*a*b^2*c^3*x^2 + 256
*a^2*c^4*x^2 + 16*b^5*c*x + 64*a*b^3*c^2*x + 256*a^2*b*c^3*x + b^6 + 4*a*b^
4*c + 16*a^2*b^2*c^2 + 64*a^3*c^3)/((2*c*x + b)^8*c^4*d^9)
```

3.1151 $\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^{10}}\,dx$

Optimal. Leaf size=101

$$\frac{(b^2-4ac)^3}{1152c^4d^{10}(b+2cx)^9}-\frac{3(b^2-4ac)^2}{896c^4d^{10}(b+2cx)^7}+\frac{3(b^2-4ac)}{640c^4d^{10}(b+2cx)^5}-\frac{1}{384c^4d^{10}(b+2cx)^3}$$

```
[Out] (b^2 - 4*a*c)^3/(1152*c^4*d^10*(b + 2*c*x)^9) - (3*(b^2 - 4*a*c)^2)/(896*c^
4*d^10*(b + 2*c*x)^7) + (3*(b^2 - 4*a*c))/(640*c^4*d^10*(b + 2*c*x)^5) - 1/
(384*c^4*d^10*(b + 2*c*x)^3)
```

Rubi [A] time = 0.0816032, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$\frac{(b^2-4ac)^3}{1152c^4d^{10}(b+2cx)^9}-\frac{3(b^2-4ac)^2}{896c^4d^{10}(b+2cx)^7}+\frac{3(b^2-4ac)}{640c^4d^{10}(b+2cx)^5}-\frac{1}{384c^4d^{10}(b+2cx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^10,x]
```

```
[Out] (b^2 - 4*a*c)^3/(1152*c^4*d^10*(b + 2*c*x)^9) - (3*(b^2 - 4*a*c)^2)/(896*c^
4*d^10*(b + 2*c*x)^7) + (3*(b^2 - 4*a*c))/(640*c^4*d^10*(b + 2*c*x)^5) - 1/
(384*c^4*d^10*(b + 2*c*x)^3)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^{10}}\,dx &= \int \left(\frac{(-b^2+4ac)^3}{64c^3d^{10}(b+2cx)^{10}}+\frac{3(-b^2+4ac)^2}{64c^3d^{10}(b+2cx)^8}+\frac{3(-b^2+4ac)}{64c^3d^{10}(b+2cx)^6}+\frac{1}{64c^3d^{10}(b+2cx)^4}\right)dx\\ &= \frac{(b^2-4ac)^3}{1152c^4d^{10}(b+2cx)^9}-\frac{3(b^2-4ac)^2}{896c^4d^{10}(b+2cx)^7}+\frac{3(b^2-4ac)}{640c^4d^{10}(b+2cx)^5}-\frac{1}{384c^4d^{10}(b+2cx)^3}\end{aligned}$$

Mathematica [A] time = 0.0577125, size = 79, normalized size = 0.78

$$\frac{189(b^2-4ac)(b+2cx)^4-135(b^2-4ac)^2(b+2cx)^2+35(b^2-4ac)^3-105(b+2cx)^6}{40320c^4d^{10}(b+2cx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^10,x]
```

```
[Out] (35*(b^2 - 4*a*c)^3 - 135*(b^2 - 4*a*c)^2*(b + 2*c*x)^2 + 189*(b^2 - 4*a*c)
*(b + 2*c*x)^4 - 105*(b + 2*c*x)^6)/(40320*c^4*d^10*(b + 2*c*x)^9)
```

Maple [A] time = 0.048, size = 121, normalized size = 1.2

$$\frac{1}{d^{10}}\left(-\frac{1}{384\,c^4\left(2\,cx+b\right)^3}-\frac{12\,ac-3\,b^2}{640\,c^4\left(2\,cx+b\right)^5}-\frac{48\,a^2c^2-24\,acb^2+3\,b^4}{896\,c^4\left(2\,cx+b\right)^7}-\frac{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}{1152\,c^4\left(2\,cx+b\right)^9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^10,x)
```

```
[Out] 1/d^10*(-1/384/c^4/(2*c*x+b)^3-1/640*(12*a*c-3*b^2)/c^4/(2*c*x+b)^5-1/896*(
48*a^2*c^2-24*a*b^2*c+3*b^4)/c^4/(2*c*x+b)^7-1/1152*(64*a^3*c^3-48*a^2*b^2*
c^2+12*a*b^4*c-b^6)/c^4/(2*c*x+b)^9)
```

Maxima [B] time = 1.89643, size = 378, normalized size = 3.74

$$-\frac{420\,c^6x^6+1260\,bc^5x^5+b^6+6\,ab^4c+30\,a^2b^2c^2+140\,a^3c^3+126\left(11\,b^2c^4+6\,ac^5\right)x^4+168\left(4\,b^3c^3+9\,abc^4\right)x^3+36\,(4\,b}{2520\,(512\,c^{13}d^{10}x^9+2304\,bc^{12}d^{10}x^8+4608\,b^2c^{11}d^{10}x^7+5376\,b^3c^{10}d^{10}x^6+4032\,b^4c^9d^{10}x^5+2016\,b^5c^8d^{10}x^4+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^10,x, algorithm="maxima")
```

```
[Out] -1/2520*(420*c^6*x^6 + 1260*b*c^5*x^5 + b^6 + 6*a*b^4*c + 30*a^2*b^2*c^2 +
140*a^3*c^3 + 126*(11*b^2*c^4 + 6*a*c^5)*x^4 + 168*(4*b^3*c^3 + 9*a*b*c^4)*
x^3 + 36*(4*b^4*c^2 + 24*a*b^2*c^3 + 15*a^2*c^4)*x^2 + 18*(b^5*c + 6*a*b^3*
c^2 + 30*a^2*b*c^3)*x)/(512*c^13*d^10*x^9 + 2304*b*c^12*d^10*x^8 + 4608*b^2
*c^11*d^10*x^7 + 5376*b^3*c^10*d^10*x^6 + 4032*b^4*c^9*d^10*x^5 + 2016*b^5*
c^8*d^10*x^4 + 672*b^6*c^7*d^10*x^3 + 144*b^7*c^6*d^10*x^2 + 18*b^8*c^5*d^1
0*x + b^9*c^4*d^10)
```

Fricas [B] time = 1.9383, size = 636, normalized size = 6.3

$$-\frac{420\,c^6x^6+1260\,bc^5x^5+b^6+6\,ab^4c+30\,a^2b^2c^2+140\,a^3c^3+126\left(11\,b^2c^4+6\,ac^5\right)x^4+168\left(4\,b^3c^3+9\,abc^4\right)x^3+36\,(4\,b}{2520\,(512\,c^{13}d^{10}x^9+2304\,bc^{12}d^{10}x^8+4608\,b^2c^{11}d^{10}x^7+5376\,b^3c^{10}d^{10}x^6+4032\,b^4c^9d^{10}x^5+2016\,b^5c^8d^{10}x^4+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^10,x, algorithm="fricas")
```

```
[Out] -1/2520*(420*c^6*x^6 + 1260*b*c^5*x^5 + b^6 + 6*a*b^4*c + 30*a^2*b^2*c^2 +
140*a^3*c^3 + 126*(11*b^2*c^4 + 6*a*c^5)*x^4 + 168*(4*b^3*c^3 + 9*a*b*c^4)*
x^3 + 36*(4*b^4*c^2 + 24*a*b^2*c^3 + 15*a^2*c^4)*x^2 + 18*(b^5*c + 6*a*b^3*
c^2 + 30*a^2*b*c^3)*x)/(512*c^13*d^10*x^9 + 2304*b*c^12*d^10*x^8 + 4608*b^2
*c^11*d^10*x^7 + 5376*b^3*c^10*d^10*x^6 + 4032*b^4*c^9*d^10*x^5 + 2016*b^5*
c^8*d^10*x^4 + 672*b^6*c^7*d^10*x^3 + 144*b^7*c^6*d^10*x^2 + 18*b^8*c^5*d^1
0*x + b^9*c^4*d^10)
```

Sympy [B] time = 12.05, size = 292, normalized size = 2.89

$$-\frac{140a^3c^3+30a^2b^2c^2+6ab^4c+b^6+1260bc^5x^5+420c^6x^6+x^4\left(756ac^5+1386b^2c^4\right)+x^3\left(1512abc^4+672b^3c\right.}{2520b^9c^4d^{10}+45360b^8c^5d^{10}x+362880b^7c^6d^{10}x^2+1693440b^6c^7d^{10}x^3+5080320b^5c^8d^{10}x^4+10160640b^4c^9d^{10}x^5+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(2*c*d*x+b*d)**10,x)
```

```
[Out] -(140*a**3*c**3 + 30*a**2*b**2*c**2 + 6*a*b**4*c + b**6 + 1260*b*c**5*x**5
+ 420*c**6*x**6 + x**4*(756*a*c**5 + 1386*b**2*c**4) + x**3*(1512*a*b*c**4
+ 672*b**3*c**3) + x**2*(540*a**2*c**4 + 864*a*b**2*c**3 + 144*b**4*c**2) +
 x*(540*a**2*b*c**3 + 108*a*b**3*c**2 + 18*b**5*c))/(2520*b**9*c**4*d**10 +
 45360*b**8*c**5*d**10*x + 362880*b**7*c**6*d**10*x**2 + 1693440*b**6*c**7*
d**10*x**3 + 5080320*b**5*c**8*d**10*x**4 + 10160640*b**4*c**9*d**10*x**5 +
 13547520*b**3*c**10*d**10*x**6 + 11612160*b**2*c**11*d**10*x**7 + 5806080*
b*c**12*d**10*x**8 + 1290240*c**13*d**10*x**9)
```

Giac [A] time = 1.1778, size = 223, normalized size = 2.21

$$-\frac{420\,c^6x^6+1260\,bc^5x^5+1386\,b^2c^4x^4+756\,ac^5x^4+672\,b^3c^3x^3+1512\,abc^4x^3+144\,b^4c^2x^2+864\,ab^2c^3x^2+540\,a^2c^4}{2520\,(2\,cx+b)^9c^4d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^10,x, algorithm="giac")
```

```
[Out] -1/2520*(420*c^6*x^6 + 1260*b*c^5*x^5 + 1386*b^2*c^4*x^4 + 756*a*c^5*x^4 +
672*b^3*c^3*x^3 + 1512*a*b*c^4*x^3 + 144*b^4*c^2*x^2 + 864*a*b^2*c^3*x^2 +
540*a^2*c^4*x^2 + 18*b^5*c*x + 108*a*b^3*c^2*x + 540*a^2*b*c^3*x + b^6 + 6*
a*b^4*c + 30*a^2*b^2*c^2 + 140*a^3*c^3)/((2*c*x + b)^9*c^4*d^10)
```

3.1152 $\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^{11}}\,dx$

Optimal. Leaf size=101

$$\frac{(b^2-4ac)^3}{1280c^4d^{11}(b+2cx)^{10}}-\frac{3(b^2-4ac)^2}{1024c^4d^{11}(b+2cx)^8}+\frac{b^2-4ac}{256c^4d^{11}(b+2cx)^6}-\frac{1}{512c^4d^{11}(b+2cx)^4}$$

```
[Out] (b^2 - 4*a*c)^3/(1280*c^4*d^11*(b + 2*c*x)^10) - (3*(b^2 - 4*a*c)^2)/(1024*
c^4*d^11*(b + 2*c*x)^8) + (b^2 - 4*a*c)/(256*c^4*d^11*(b + 2*c*x)^6) - 1/(5
12*c^4*d^11*(b + 2*c*x)^4)
```

Rubi [A] time = 0.0814115, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$\frac{(b^2-4ac)^3}{1280c^4d^{11}(b+2cx)^{10}}-\frac{3(b^2-4ac)^2}{1024c^4d^{11}(b+2cx)^8}+\frac{b^2-4ac}{256c^4d^{11}(b+2cx)^6}-\frac{1}{512c^4d^{11}(b+2cx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^11,x]
```

```
[Out] (b^2 - 4*a*c)^3/(1280*c^4*d^11*(b + 2*c*x)^10) - (3*(b^2 - 4*a*c)^2)/(1024*
c^4*d^11*(b + 2*c*x)^8) + (b^2 - 4*a*c)/(256*c^4*d^11*(b + 2*c*x)^6) - 1/(5
12*c^4*d^11*(b + 2*c*x)^4)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^{11}}\,dx&=\int \left(\frac{(-b^2+4ac)^3}{64c^3d^{11}(b+2cx)^{11}}+\frac{3(-b^2+4ac)^2}{64c^3d^{11}(b+2cx)^9}+\frac{3(-b^2+4ac)}{64c^3d^{11}(b+2cx)^7}+\frac{1}{64c^3d^{11}(b+2cx)^5}\right)dx\\&=\frac{(b^2-4ac)^3}{1280c^4d^{11}(b+2cx)^{10}}-\frac{3(b^2-4ac)^2}{1024c^4d^{11}(b+2cx)^8}+\frac{b^2-4ac}{256c^4d^{11}(b+2cx)^6}-\frac{1}{512c^4d^{11}(b+2cx)^4}\end{aligned}$$

Mathematica [A] time = 0.0567742, size = 79, normalized size = 0.78

$$\frac{20(b^2-4ac)(b+2cx)^4-15(b^2-4ac)^2(b+2cx)^2+4(b^2-4ac)^3-10(b+2cx)^6}{5120c^4d^{11}(b+2cx)^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^11,x]
```

```
[Out] (4*(b^2 - 4*a*c)^3 - 15*(b^2 - 4*a*c)^2*(b + 2*c*x)^2 + 20*(b^2 - 4*a*c)*(b
 + 2*c*x)^4 - 10*(b + 2*c*x)^6)/(5120*c^4*d^11*(b + 2*c*x)^10)
```

Maple [A] time = 0.046, size = 121, normalized size = 1.2

$$\frac{1}{d^{11}}\left(-\frac{12\,ac-3\,b^{2}}{768\,c^{4}\left(2\,cx+b\right)^{6}}-\frac{64\,a^{3}c^{3}-48\,a^{2}b^{2}c^{2}+12\,ab^{4}c-b^{6}}{1280\,c^{4}\left(2\,cx+b\right)^{10}}-\frac{48\,a^{2}c^{2}-24\,acb^{2}+3\,b^{4}}{1024\,c^{4}\left(2\,cx+b\right)^{8}}-\frac{1}{512\,c^{4}\left(2\,cx+b\right)^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^11,x)
```

```
[Out] 1/d^11*(-1/768*(12*a*c-3*b^2)/c^4/(2*c*x+b)^6-1/1280*(64*a^3*c^3-48*a^2*b^2
*c^2+12*a*b^4*c-b^6)/c^4/(2*c*x+b)^10-1/1024*(48*a^2*c^2-24*a*b^2*c+3*b^4)/
c^4/(2*c*x+b)^8-1/512/c^4/(2*c*x+b)^4)
```

Maxima [B] time = 2.35701, size = 397, normalized size = 3.93

$$-\frac{640\,c^{6}x^{6}+1920\,bc^{5}x^{5}+b^{6}+8\,ab^{4}c+48\,a^{2}b^{2}c^{2}+256\,a^{3}c^{3}+160\left(13\,b^{2}c^{4}+8\,ac^{5}\right)x^{4}+320\left(3\,b^{3}c^{3}+8\,abc^{4}\right)x^{3}}{5120\left(1024\,c^{14}d^{11}x^{10}+5120\,bc^{13}d^{11}x^{9}+11520\,b^{2}c^{12}d^{11}x^{8}+15360\,b^{3}c^{11}d^{11}x^{7}+13440\,b^{4}c^{10}d^{11}x^{6}+8064\,b^{5}c^{9}d^{11}x^{5}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^11,x, algorithm="maxima")
```

```
[Out] -1/5120*(640*c^6*x^6 + 1920*b*c^5*x^5 + b^6 + 8*a*b^4*c + 48*a^2*b^2*c^2 +
256*a^3*c^3 + 160*(13*b^2*c^4 + 8*a*c^5)*x^4 + 320*(3*b^3*c^3 + 8*a*b*c^4)*
x^3 + 60*(3*b^4*c^2 + 24*a*b^2*c^3 + 16*a^2*c^4)*x^2 + 20*(b^5*c + 8*a*b^3*
c^2 + 48*a^2*b*c^3)*x)/(1024*c^14*d^11*x^10 + 5120*b*c^13*d^11*x^9 + 11520*
b^2*c^12*d^11*x^8 + 15360*b^3*c^11*d^11*x^7 + 13440*b^4*c^10*d^11*x^6 + 806
4*b^5*c^9*d^11*x^5 + 3360*b^6*c^8*d^11*x^4 + 960*b^7*c^7*d^11*x^3 + 180*b^8
*c^6*d^11*x^2 + 20*b^9*c^5*d^11*x + b^10*c^4*d^11)
```

Fricas [B] time = 2.05677, size = 678, normalized size = 6.71

$$-\frac{640\,c^{6}x^{6}+1920\,bc^{5}x^{5}+b^{6}+8\,ab^{4}c+48\,a^{2}b^{2}c^{2}+256\,a^{3}c^{3}+160\left(13\,b^{2}c^{4}+8\,ac^{5}\right)x^{4}+320\left(3\,b^{3}c^{3}+8\,abc^{4}\right)x^{3}}{5120\left(1024\,c^{14}d^{11}x^{10}+5120\,bc^{13}d^{11}x^{9}+11520\,b^{2}c^{12}d^{11}x^{8}+15360\,b^{3}c^{11}d^{11}x^{7}+13440\,b^{4}c^{10}d^{11}x^{6}+8064\,b^{5}c^{9}d^{11}x^{5}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^11,x, algorithm="fricas")
```

```
[Out] -1/5120*(640*c^6*x^6 + 1920*b*c^5*x^5 + b^6 + 8*a*b^4*c + 48*a^2*b^2*c^2 +
256*a^3*c^3 + 160*(13*b^2*c^4 + 8*a*c^5)*x^4 + 320*(3*b^3*c^3 + 8*a*b*c^4)*
x^3 + 60*(3*b^4*c^2 + 24*a*b^2*c^3 + 16*a^2*c^4)*x^2 + 20*(b^5*c + 8*a*b^3*
c^2 + 48*a^2*b*c^3)*x)/(1024*c^14*d^11*x^10 + 5120*b*c^13*d^11*x^9 + 11520*
b^2*c^12*d^11*x^8 + 15360*b^3*c^11*d^11*x^7 + 13440*b^4*c^10*d^11*x^6 + 806
4*b^5*c^9*d^11*x^5 + 3360*b^6*c^8*d^11*x^4 + 960*b^7*c^7*d^11*x^3 + 180*b^8
*c^6*d^11*x^2 + 20*b^9*c^5*d^11*x + b^10*c^4*d^11)
```

Sympy [B] time = 42.6112, size = 308, normalized size = 3.05

$$-\frac{256a^3c^3 + 48a^2b^2c^2 + 8ab^4c + b^6 + 1920bc^5x^5 + 640c^6x^6 + x^4\left(1280ac^5 + 2080b^2c^4\right) + x^3\left(2560ab\right.}{5120b^{10}c^4d^{11} + 102400b^9c^5d^{11}x + 921600b^8c^6d^{11}x^2 + 4915200b^7c^7d^{11}x^3 + 17203200b^6c^8d^{11}x^4 + 41287680b^5c^9d^{11}x^5 +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(2*c*d*x+b*d)**11,x)
```

```
[Out] -(256*a**3*c**3 + 48*a**2*b**2*c**2 + 8*a*b**4*c + b**6 + 1920*b*c**5*x**5
+ 640*c**6*x**6 + x**4*(1280*a*c**5 + 2080*b**2*c**4) + x**3*(2560*a*b*c**4
 + 960*b**3*c**3) + x**2*(960*a**2*c**4 + 1440*a*b**2*c**3 + 180*b**4*c**2)
 + x*(960*a**2*b*c**3 + 160*a*b**3*c**2 + 20*b**5*c))/(5120*b**10*c**4*d**1
1 + 102400*b**9*c**5*d**11*x + 921600*b**8*c**6*d**11*x**2 + 4915200*b**7*c
**7*d**11*x**3 + 17203200*b**6*c**8*d**11*x**4 + 41287680*b**5*c**9*d**11*x
**5 + 68812800*b**4*c**10*d**11*x**6 + 78643200*b**3*c**11*d**11*x**7 + 589
82400*b**2*c**12*d**11*x**8 + 26214400*b*c**13*d**11*x**9 + 5242880*c**14*d
**11*x**10)
```

Giac [A] time = 1.17969, size = 223, normalized size = 2.21

$$-\frac{640\,c^6x^6 + 1920\,bc^5x^5 + 2080\,b^2c^4x^4 + 1280\,ac^5x^4 + 960\,b^3c^3x^3 + 2560\,abc^4x^3 + 180\,b^4c^2x^2 + 1440\,ab^2c^3x^2 + 960\,a^2c^4x}{5120\,(2\,cx + b)^{10}c^4d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^11,x, algorithm="giac")
```

```
[Out] -1/5120*(640*c^6*x^6 + 1920*b*c^5*x^5 + 2080*b^2*c^4*x^4 + 1280*a*c^5*x^4 +
 960*b^3*c^3*x^3 + 2560*a*b*c^4*x^3 + 180*b^4*c^2*x^2 + 1440*a*b^2*c^3*x^2
+ 960*a^2*c^4*x^2 + 20*b^5*c*x + 160*a*b^3*c^2*x + 960*a^2*b*c^3*x + b^6 +
8*a*b^4*c + 48*a^2*b^2*c^2 + 256*a^3*c^3)/((2*c*x + b)^10*c^4*d^11)
```

3.1153 $\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^{12}}\,dx$

Optimal. Leaf size=101

$$\frac{(b^2-4ac)^3}{1408c^4d^{12}(b+2cx)^{11}}-\frac{(b^2-4ac)^2}{384c^4d^{12}(b+2cx)^9}+\frac{3(b^2-4ac)}{896c^4d^{12}(b+2cx)^7}-\frac{1}{640c^4d^{12}(b+2cx)^5}$$

```
[Out] (b^2 - 4*a*c)^3/(1408*c^4*d^12*(b + 2*c*x)^11) - (b^2 - 4*a*c)^2/(384*c^4*d
^12*(b + 2*c*x)^9) + (3*(b^2 - 4*a*c))/(896*c^4*d^12*(b + 2*c*x)^7) - 1/(64
0*c^4*d^12*(b + 2*c*x)^5)
```

Rubi [A] time = 0.0846991, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$\frac{(b^2-4ac)^3}{1408c^4d^{12}(b+2cx)^{11}}-\frac{(b^2-4ac)^2}{384c^4d^{12}(b+2cx)^9}+\frac{3(b^2-4ac)}{896c^4d^{12}(b+2cx)^7}-\frac{1}{640c^4d^{12}(b+2cx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^12,x]
```

```
[Out] (b^2 - 4*a*c)^3/(1408*c^4*d^12*(b + 2*c*x)^11) - (b^2 - 4*a*c)^2/(384*c^4*d
^12*(b + 2*c*x)^9) + (3*(b^2 - 4*a*c))/(896*c^4*d^12*(b + 2*c*x)^7) - 1/(64
0*c^4*d^12*(b + 2*c*x)^5)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^{12}}\,dx&=\int \left(\frac{(-b^2+4ac)^3}{64c^3d^{12}(b+2cx)^{12}}+\frac{3(-b^2+4ac)^2}{64c^3d^{12}(b+2cx)^{10}}+\frac{3(-b^2+4ac)}{64c^3d^{12}(b+2cx)^8}+\frac{1}{64c^3d^{12}(b+2cx)^6}\right)dx\\&=\frac{(b^2-4ac)^3}{1408c^4d^{12}(b+2cx)^{11}}-\frac{(b^2-4ac)^2}{384c^4d^{12}(b+2cx)^9}+\frac{3(b^2-4ac)}{896c^4d^{12}(b+2cx)^7}-\frac{1}{640c^4d^{12}(b+2cx)^5}\end{aligned}$$

Mathematica [A] time = 0.0626427, size = 79, normalized size = 0.78

$$\frac{495(b^2-4ac)(b+2cx)^4-385(b^2-4ac)^2(b+2cx)^2+105(b^2-4ac)^3-231(b+2cx)^6}{147840c^4d^{12}(b+2cx)^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^12,x]
```

```
[Out] (105*(b^2 - 4*a*c)^3 - 385*(b^2 - 4*a*c)^2*(b + 2*c*x)^2 + 495*(b^2 - 4*a*c
)*(b + 2*c*x)^4 - 231*(b + 2*c*x)^6)/(147840*c^4*d^12*(b + 2*c*x)^11)
```

Maple [A] time = 0.047, size = 121, normalized size = 1.2

$$\frac{1}{d^{12}}\left(-\frac{1}{640\,c^4\,(2\,cx+b)^5}-\frac{12\,ac-3\,b^2}{896\,c^4\,(2\,cx+b)^7}-\frac{48\,a^2c^2-24\,acb^2+3\,b^4}{1152\,c^4\,(2\,cx+b)^9}-\frac{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}{1408\,c^4\,(2\,cx+b)^{11}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^12,x)
```

```
[Out] 1/d^12*(-1/640/c^4/(2*c*x+b)^5-1/896*(12*a*c-3*b^2)/c^4/(2*c*x+b)^7-1/1152*
(48*a^2*c^2-24*a*b^2*c+3*b^4)/c^4/(2*c*x+b)^9-1/1408*(64*a^3*c^3-48*a^2*b^2
*c^2+12*a*b^4*c-b^6)/c^4/(2*c*x+b)^11)
```

Maxima [B] time = 1.36038, size = 413, normalized size = 4.09

$$-\frac{924\,c^6x^6+2772\,bc^5x^5+b^6+10\,ab^4c+70\,a^2b^2c^2+420\,a^3c^3+990\left(3\,b^2c^4+2\,ac^5\right)x^4+1320\left(b^3c^3+\right.}{9240\left(2048\,c^{15}d^{12}x^{11}+11264\,bc^{14}d^{12}x^{10}+28160\,b^2c^{13}d^{12}x^9+42240\,b^3c^{12}d^{12}x^8+42240\,b^4c^{11}d^{12}x^7+29568\,b^5c^{10}d^{12}x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^12,x, algorithm="maxima")
```

```
[Out] -1/9240*(924*c^6*x^6 + 2772*b*c^5*x^5 + b^6 + 10*a*b^4*c + 70*a^2*b^2*c^2 +
 420*a^3*c^3 + 990*(3*b^2*c^4 + 2*a*c^5)*x^4 + 1320*(b^3*c^3 + 3*a*b*c^4)*x
^3 + 220*(b^4*c^2 + 10*a*b^2*c^3 + 7*a^2*c^4)*x^2 + 22*(b^5*c + 10*a*b^3*c^
2 + 70*a^2*b*c^3)*x)/(2048*c^15*d^12*x^11 + 11264*b*c^14*d^12*x^10 + 28160*
b^2*c^13*d^12*x^9 + 42240*b^3*c^12*d^12*x^8 + 42240*b^4*c^11*d^12*x^7 + 295
68*b^5*c^10*d^12*x^6 + 14784*b^6*c^9*d^12*x^5 + 5280*b^7*c^8*d^12*x^4 + 132
0*b^8*c^7*d^12*x^3 + 220*b^9*c^6*d^12*x^2 + 22*b^10*c^5*d^12*x + b^11*c^4*d
^12)
```

Fricas [B] time = 2.07111, size = 717, normalized size = 7.1

$$-\frac{924\,c^6x^6+2772\,bc^5x^5+b^6+10\,ab^4c+70\,a^2b^2c^2+420\,a^3c^3+990\left(3\,b^2c^4+2\,ac^5\right)x^4+1320\left(b^3c^3+\right.}{9240\left(2048\,c^{15}d^{12}x^{11}+11264\,bc^{14}d^{12}x^{10}+28160\,b^2c^{13}d^{12}x^9+42240\,b^3c^{12}d^{12}x^8+42240\,b^4c^{11}d^{12}x^7+29568\,b^5c^{10}d^{12}x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^12,x, algorithm="fricas")
```

```
[Out] -1/9240*(924*c^6*x^6 + 2772*b*c^5*x^5 + b^6 + 10*a*b^4*c + 70*a^2*b^2*c^2 +
 420*a^3*c^3 + 990*(3*b^2*c^4 + 2*a*c^5)*x^4 + 1320*(b^3*c^3 + 3*a*b*c^4)*x
^3 + 220*(b^4*c^2 + 10*a*b^2*c^3 + 7*a^2*c^4)*x^2 + 22*(b^5*c + 10*a*b^3*c^
2 + 70*a^2*b*c^3)*x)/(2048*c^15*d^12*x^11 + 11264*b*c^14*d^12*x^10 + 28160*
b^2*c^13*d^12*x^9 + 42240*b^3*c^12*d^12*x^8 + 42240*b^4*c^11*d^12*x^7 + 295
68*b^5*c^10*d^12*x^6 + 14784*b^6*c^9*d^12*x^5 + 5280*b^7*c^8*d^12*x^4 + 132
0*b^8*c^7*d^12*x^3 + 220*b^9*c^6*d^12*x^2 + 22*b^10*c^5*d^12*x + b^11*c^4*d
```

```
^12)
```

Sympy [B] time = 20.5477, size = 323, normalized size = 3.2

$$-\frac{420a^3c^3 + 70a^2b^2c^2 + 10ab^4c + b^6 + 2772bc^5x^5 + 924c^6x^6 + x^4\left(1980ac^5 + 297\right.}{9240b^{11}c^4d^{12} + 203280b^{10}c^5d^{12}x + 2032800b^9c^6d^{12}x^2 + 12196800b^8c^7d^{12}x^3 + 48787200b^7c^8d^{12}x^4 + 136604160b^6c^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(2*c*d*x+b*d)**12,x)
```

```
[Out] -(420*a**3*c**3 + 70*a**2*b**2*c**2 + 10*a*b**4*c + b**6 + 2772*b*c**5*x**5
 + 924*c**6*x**6 + x**4*(1980*a*c**5 + 2970*b**2*c**4) + x**3*(3960*a*b*c**
4 + 1320*b**3*c**3) + x**2*(1540*a**2*c**4 + 2200*a*b**2*c**3 + 220*b**4*c*
*2) + x*(1540*a**2*b*c**3 + 220*a*b**3*c**2 + 22*b**5*c))/(9240*b**11*c**4*
d**12 + 203280*b**10*c**5*d**12*x + 2032800*b**9*c**6*d**12*x**2 + 12196800
*b**8*c**7*d**12*x**3 + 48787200*b**7*c**8*d**12*x**4 + 136604160*b**6*c**9
*d**12*x**5 + 273208320*b**5*c**10*d**12*x**6 + 390297600*b**4*c**11*d**12*
x**7 + 390297600*b**3*c**12*d**12*x**8 + 260198400*b**2*c**13*d**12*x**9 +
104079360*b*c**14*d**12*x**10 + 18923520*c**15*d**12*x**11)
```

Giac [A] time = 1.17052, size = 223, normalized size = 2.21

$$-\frac{924\,c^6x^6 + 2772\,bc^5x^5 + 2970\,b^2c^4x^4 + 1980\,ac^5x^4 + 1320\,b^3c^3x^3 + 3960\,abc^4x^3 + 220\,b^4c^2x^2 + 2200\,ab^2c^3x^2 + 1540}{9240\,(2\,cx + b)^{11}c^4d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^12,x, algorithm="giac")
```

```
[Out] -1/9240*(924*c^6*x^6 + 2772*b*c^5*x^5 + 2970*b^2*c^4*x^4 + 1980*a*c^5*x^4 +
 1320*b^3*c^3*x^3 + 3960*a*b*c^4*x^3 + 220*b^4*c^2*x^2 + 2200*a*b^2*c^3*x^2
 + 1540*a^2*c^4*x^2 + 22*b^5*c*x + 220*a*b^3*c^2*x + 1540*a^2*b*c^3*x + b^6
 + 10*a*b^4*c + 70*a^2*b^2*c^2 + 420*a^3*c^3)/((2*c*x + b)^11*c^4*d^12)
```

3.1154 $\int \frac{(bd+2cdx)^8}{a+bx+cx^2} dx$

Optimal. Leaf size=122

$$\frac{2}{5}d^8\left(b^2-4ac\right)(b+2cx)^5+\frac{2}{3}d^8\left(b^2-4ac\right)^2(b+2cx)^3+2d^8\left(b^2-4ac\right)^3(b+2cx)-2d^8\left(b^2-4ac\right)^{7/2}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)$$

```
[Out] 2*(b^2 - 4*a*c)^3*d^8*(b + 2*c*x) + (2*(b^2 - 4*a*c)^2*d^8*(b + 2*c*x)^3)/3
 + (2*(b^2 - 4*a*c)*d^8*(b + 2*c*x)^5)/5 + (2*d^8*(b + 2*c*x)^7)/7 - 2*(b^2
 - 4*a*c)^(7/2)*d^8*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]
```

Rubi [A] time = 0.150405, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {692, 618, 206}

$$\frac{2}{5}d^8\left(b^2-4ac\right)(b+2cx)^5+\frac{2}{3}d^8\left(b^2-4ac\right)^2(b+2cx)^3+2d^8\left(b^2-4ac\right)^3(b+2cx)-2d^8\left(b^2-4ac\right)^{7/2}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^8/(a + b*x + c*x^2),x]
```

```
[Out] 2*(b^2 - 4*a*c)^3*d^8*(b + 2*c*x) + (2*(b^2 - 4*a*c)^2*d^8*(b + 2*c*x)^3)/3
 + (2*(b^2 - 4*a*c)*d^8*(b + 2*c*x)^5)/5 + (2*d^8*(b + 2*c*x)^7)/7 - 2*(b^2
 - 4*a*c)^(7/2)*d^8*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(bd+2cdx)^8}{a+bx+cx^2}\,dx = \frac{2}{7}d^8(b+2cx)^7 + \left(\left(b^2-4ac\right)d^2\right)\int \frac{(bd+2cdx)^6}{a+bx+cx^2}\,dx$$
$$= \frac{2}{5}\left(b^2-4ac\right)d^8(b+2cx)^5 + \frac{2}{7}d^8(b+2cx)^7 + \left(\left(b^2-4ac\right)^2 d^4\right)\int \frac{(bd+2cdx)^4}{a+bx+cx^2}\,dx$$
$$= \frac{2}{3}\left(b^2-4ac\right)^2 d^8(b+2cx)^3 + \frac{2}{5}\left(b^2-4ac\right)d^8(b+2cx)^5 + \frac{2}{7}d^8(b+2cx)^7 + \left(\left(b^2-4ac\right)^3 d^6\right)\int \frac{(b}{a}$$
$$= 2\left(b^2-4ac\right)^3 d^8(b+2cx) + \frac{2}{3}\left(b^2-4ac\right)^2 d^8(b+2cx)^3 + \frac{2}{5}\left(b^2-4ac\right)d^8(b+2cx)^5 + \frac{2}{7}d^8(b+2c$$
$$= 2\left(b^2-4ac\right)^3 d^8(b+2cx) + \frac{2}{3}\left(b^2-4ac\right)^2 d^8(b+2cx)^3 + \frac{2}{5}\left(b^2-4ac\right)d^8(b+2cx)^5 + \frac{2}{7}d^8(b+2c$$
$$= 2\left(b^2-4ac\right)^3 d^8(b+2cx) + \frac{2}{3}\left(b^2-4ac\right)^2 d^8(b+2cx)^3 + \frac{2}{5}\left(b^2-4ac\right)d^8(b+2cx)^5 + \frac{2}{7}d^8(b+2c$$

Mathematica [A] time = 0.113214, size = 188, normalized size = 1.54

$$d^8\left(\frac{16}{105}cx\left(112b^2c^2\left(15a^2-10acx^2+12c^2x^4\right)+840bc^3x\left(a^2-acx^2+c^2x^4\right)+16c^3\left(35a^2cx^2-105a^3-21ac^2x^4+15c^3\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^8/(a + b*x + c*x^2),x]
```

```
[Out] d^8*((16*c*x*(105*b^6 + 315*b^5*c*x + 420*b^3*c^2*x*(-2*a + 3*c*x^2) + 70*b
^4*c*(-9*a + 11*c*x^2) + 840*b*c^3*x*(a^2 - a*c*x^2 + c^2*x^4) + 112*b^2*c^
2*(15*a^2 - 10*a*c*x^2 + 12*c^2*x^4) + 16*c^3*(-105*a^3 + 35*a^2*c*x^2 - 21
*a*c^2*x^4 + 15*c^3*x^6)))/105 + 2*(-b^2 + 4*a*c)^(7/2)*ArcTan[(b + 2*c*x)/
Sqrt[-b^2 + 4*a*c]])
```

Maple [B] time = 0.151, size = 432, normalized size = 3.5

$$\frac{256\,d^8c^7x^7}{7} + 128\,d^8bc^6x^6 - \frac{256\,d^8x^5ac^6}{5} + \frac{1024\,d^8x^5b^2c^5}{5} - 128\,d^8x^4abc^5 + 192\,d^8x^4b^3c^4 + \frac{256\,d^8x^3a^2c^5}{3} - \frac{512\,d^8x^3a}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^8/(c*x^2+b*x+a),x)
```

```
[Out] 256/7*d^8*c^7*x^7+128*d^8*b*c^6*x^6-256/5*d^8*x^5*a*c^6+1024/5*d^8*x^5*b^2*
c^5-128*d^8*x^4*a*b*c^5+192*d^8*x^4*b^3*c^4+256/3*d^8*x^3*a^2*c^5-512/3*d^8
*x^3*a*b^2*c^4+352/3*d^8*x^3*b^4*c^3+128*d^8*x^2*a^2*b*c^4-128*d^8*x^2*a*b^
3*c^3+48*d^8*x^2*b^5*c^2-256*d^8*a^3*c^4*x+256*d^8*b^2*a^2*c^3*x-96*d^8*a*b
^4*c^2*x+16*d^8*b^6*c*x+512*d^8/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b
^2)^(1/2))*a^4*c^4-512*d^8/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2))*a^3*b^2*c^3+192*d^8/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1
/2))*a^2*b^4*c^2-32*d^8/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2
))*a*b^6*c+2*d^8/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^8
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^8/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.94672, size = 1166, normalized size = 9.56

$$\left[\frac{256}{7}\,c^7d^8x^7+128\,bc^6d^8x^6+\frac{256}{5}\left(4\,b^2c^5-ac^6\right)d^8x^5+64\left(3\,b^3c^4-2\,abc^5\right)d^8x^4+\frac{32}{3}\left(11\,b^4c^3-16\,ab^2c^4+8\,a^2c^5\right)d^8x^3+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^8/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [256/7*c^7*d^8*x^7 + 128*b*c^6*d^8*x^6 + 256/5*(4*b^2*c^5 - a*c^6)*d^8*x^5
+ 64*(3*b^3*c^4 - 2*a*b*c^5)*d^8*x^4 + 32/3*(11*b^4*c^3 - 16*a*b^2*c^4 + 8*
a^2*c^5)*d^8*x^3 + 16*(3*b^5*c^2 - 8*a*b^3*c^3 + 8*a^2*b*c^4)*d^8*x^2 - (b^
6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(b^2 - 4*a*c)*d^8*log((2*
c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b
*x + a)) + 16*(b^6*c - 6*a*b^4*c^2 + 16*a^2*b^2*c^3 - 16*a^3*c^4)*d^8*x, 25
6/7*c^7*d^8*x^7 + 128*b*c^6*d^8*x^6 + 256/5*(4*b^2*c^5 - a*c^6)*d^8*x^5 + 6
4*(3*b^3*c^4 - 2*a*b*c^5)*d^8*x^4 + 32/3*(11*b^4*c^3 - 16*a*b^2*c^4 + 8*a^2
*c^5)*d^8*x^3 + 16*(3*b^5*c^2 - 8*a*b^3*c^3 + 8*a^2*b*c^4)*d^8*x^2 - 2*(b^6
 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-b^2 + 4*a*c)*d^8*arctan(
-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 16*(b^6*c - 6*a*b^4*c^2 +
16*a^2*b^2*c^3 - 16*a^3*c^4)*d^8*x]
```

Sympy [B] time = 1.38732, size = 502, normalized size = 4.11

$$128bc^6d^8x^6+\frac{256c^7d^8x^7}{7}-d^8\sqrt{-\left(4ac-b^2\right)^7}\log\left(x+\frac{64a^3bc^3d^8-48a^2b^3c^2d^8+12ab^5cd^8-b^7d^8-d^8\sqrt{-\left(4ac-b^2\right)^7}}{128a^3c^4d^8-96a^2b^2c^3d^8+24ab^4c^2d^8-2b^6cd^8}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**8/(c*x**2+b*x+a),x)
```

```
[Out] 128*b*c**6*d**8*x**6 + 256*c**7*d**8*x**7/7 - d**8*sqrt(-(4*a*c - b**2)**7)
*log(x + (64*a**3*b*c**3*d**8 - 48*a**2*b**3*c**2*d**8 + 12*a*b**5*c*d**8 -
 b**7*d**8 - d**8*sqrt(-(4*a*c - b**2)**7))/(128*a**3*c**4*d**8 - 96*a**2*b
**2*c**3*d**8 + 24*a*b**4*c**2*d**8 - 2*b**6*c*d**8)) + d**8*sqrt(-(4*a*c -
 b**2)**7)*log(x + (64*a**3*b*c**3*d**8 - 48*a**2*b**3*c**2*d**8 + 12*a*b**
5*c*d**8 - b**7*d**8 + d**8*sqrt(-(4*a*c - b**2)**7))/(128*a**3*c**4*d**8 -
 96*a**2*b**2*c**3*d**8 + 24*a*b**4*c**2*d**8 - 2*b**6*c*d**8)) + x**5*(-25
6*a*c**6*d**8/5 + 1024*b**2*c**5*d**8/5) + x**4*(-128*a*b*c**5*d**8 + 192*b
**3*c**4*d**8) + x**3*(256*a**2*c**5*d**8/3 - 512*a*b**2*c**4*d**8/3 + 352*
b**4*c**3*d**8/3) + x**2*(128*a**2*b*c**4*d**8 - 128*a*b**3*c**3*d**8 + 48*
b**5*c**2*d**8) + x*(-256*a**3*c**4*d**8 + 256*a**2*b**2*c**3*d**8 - 96*a*b
**4*c**2*d**8 + 16*b**6*c*d**8)
```

Giac [B] time = 1.16147, size = 423, normalized size = 3.47

$$\frac{2\left(b^8d^8-16\,ab^6cd^8+96\,a^2b^4c^2d^8-256\,a^3b^2c^3d^8+256\,a^4c^4d^8\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}+\frac{16\left(240\,c^{14}d^8x^7+840\,bc^{13}d^8x^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^8/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 2*(b^8*d^8 - 16*a*b^6*c*d^8 + 96*a^2*b^4*c^2*d^8 - 256*a^3*b^2*c^3*d^8 + 25
6*a^4*c^4*d^8)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c) +
16/105*(240*c^14*d^8*x^7 + 840*b*c^13*d^8*x^6 + 1344*b^2*c^12*d^8*x^5 - 336
*a*c^13*d^8*x^5 + 1260*b^3*c^11*d^8*x^4 - 840*a*b*c^12*d^8*x^4 + 770*b^4*c^
10*d^8*x^3 - 1120*a*b^2*c^11*d^8*x^3 + 560*a^2*c^12*d^8*x^3 + 315*b^5*c^9*d
^8*x^2 - 840*a*b^3*c^10*d^8*x^2 + 840*a^2*b*c^11*d^8*x^2 + 105*b^6*c^8*d^8*
x - 630*a*b^4*c^9*d^8*x + 1680*a^2*b^2*c^10*d^8*x - 1680*a^3*c^11*d^8*x)/c^
7
```

3.1155 $\int \frac{(bd+2cdx)^7}{a+bx+cx^2} dx$

Optimal. Leaf size=86

$$d^7 \left(b^2-4ac\right)^3 \log \left(a+bx+cx^2\right)+\frac{1}{2}d^7 \left(b^2-4ac\right)(b+2cx)^4+d^7 \left(b^2-4ac\right)^2 (b+2cx)^2+\frac{1}{3}d^7(b+2cx)^6$$

```
[Out] (b^2 - 4*a*c)^2*d^7*(b + 2*c*x)^2 + ((b^2 - 4*a*c)*d^7*(b + 2*c*x)^4)/2 + (
d^7*(b + 2*c*x)^6)/3 + (b^2 - 4*a*c)^3*d^7*Log[a + b*x + c*x^2]
```

Rubi [A] time = 0.0701957, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {692, 628}

$$d^7 \left(b^2-4ac\right)^3 \log \left(a+bx+cx^2\right)+\frac{1}{2}d^7 \left(b^2-4ac\right)(b+2cx)^4+d^7 \left(b^2-4ac\right)^2 (b+2cx)^2+\frac{1}{3}d^7(b+2cx)^6$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^7/(a + b*x + c*x^2),x]
```

```
[Out] (b^2 - 4*a*c)^2*d^7*(b + 2*c*x)^2 + ((b^2 - 4*a*c)*d^7*(b + 2*c*x)^4)/2 + (
d^7*(b + 2*c*x)^6)/3 + (b^2 - 4*a*c)^3*d^7*Log[a + b*x + c*x^2]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(bd+2cdx)^7}{a+bx+cx^2}\,dx &= \frac{1}{3}d^7(b+2cx)^6+\left(\left(b^2-4ac\right)d^2\right)\int \frac{(bd+2cdx)^5}{a+bx+cx^2}\,dx\\ &= \frac{1}{2}\left(b^2-4ac\right)d^7(b+2cx)^4+\frac{1}{3}d^7(b+2cx)^6+\left(\left(b^2-4ac\right)^2 d^4\right)\int \frac{(bd+2cdx)^3}{a+bx+cx^2}\,dx\\ &= \left(b^2-4ac\right)^2 d^7(b+2cx)^2+\frac{1}{2}\left(b^2-4ac\right)d^7(b+2cx)^4+\frac{1}{3}d^7(b+2cx)^6+\left(\left(b^2-4ac\right)^3 d^6\right)\int \frac{bd+2}{a+bx}\\ &= \left(b^2-4ac\right)^2 d^7(b+2cx)^2+\frac{1}{2}\left(b^2-4ac\right)d^7(b+2cx)^4+\frac{1}{3}d^7(b+2cx)^6+\left(b^2-4ac\right)^3 d^7\log\left(a+bx+\right.\end{aligned}$$

Mathematica [A] time = 0.0552827, size = 110, normalized size = 1.28

$$d^7\left(\frac{4}{3}cx(b+cx)\left(8c^2\left(6a^2-3acx^2+2c^2x^4\right)+b^2\left(34c^2x^2-36ac\right)+8bc^2x\left(4cx^2-3a\right)+18b^3cx+9b^4\right)+\left(b^2-4ac\right)^3\log(a\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^7/(a + b*x + c*x^2),x]
```

```
[Out] d^7*((4*c*x*(b + c*x)*(9*b^4 + 18*b^3*c*x + 8*b*c^2*x*(-3*a + 4*c*x^2) + b^
2*(-36*a*c + 34*c^2*x^2) + 8*c^2*(6*a^2 - 3*a*c*x^2 + 2*c^2*x^4)))/3 + (b^2
 - 4*a*c)^3*Log[a + x*(b + c*x)])
```

Maple [B] time = 0.042, size = 243, normalized size = 2.8

$$\frac{64\,d^7c^6x^6}{3}+64\,d^7bc^5x^5-32\,d^7x^4ac^5+88\,d^7x^4b^2c^4-64\,d^7x^3abc^4+\frac{208\,d^7x^3b^3c^3}{3}+64\,d^7x^2a^2c^4-80\,d^7x^2ab^2c^3+36\,$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^7/(c*x^2+b*x+a),x)
```

```
[Out] 64/3*d^7*c^6*x^6+64*d^7*b*c^5*x^5-32*d^7*x^4*a*c^5+88*d^7*x^4*b^2*c^4-64*d^
7*x^3*a*b*c^4+208/3*d^7*x^3*b^3*c^3+64*d^7*x^2*a^2*c^4-80*d^7*x^2*a*b^2*c^3
+36*d^7*x^2*b^4*c^2+64*d^7*b*a^2*c^3*x-48*d^7*a*b^3*c^2*x+12*d^7*b^5*c*x-64
*d^7*ln(c*x^2+b*x+a)*a^3*c^3+48*d^7*ln(c*x^2+b*x+a)*a^2*b^2*c^2-12*d^7*ln(c
*x^2+b*x+a)*a*b^4*c+d^7*ln(c*x^2+b*x+a)*b^6
```

Maxima [B] time = 2.38226, size = 244, normalized size = 2.84

$$\frac{64}{3}\,c^6d^7x^6+64\,bc^5d^7x^5+8\left(11\,b^2c^4-4\,ac^5\right)d^7x^4+\frac{16}{3}\left(13\,b^3c^3-12\,abc^4\right)d^7x^3+4\left(9\,b^4c^2-20\,ab^2c^3+16\,a^2c^4\right)d^7x^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^7/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 64/3*c^6*d^7*x^6 + 64*b*c^5*d^7*x^5 + 8*(11*b^2*c^4 - 4*a*c^5)*d^7*x^4 + 16
/3*(13*b^3*c^3 - 12*a*b*c^4)*d^7*x^3 + 4*(9*b^4*c^2 - 20*a*b^2*c^3 + 16*a^2
*c^4)*d^7*x^2 + 4*(3*b^5*c - 12*a*b^3*c^2 + 16*a^2*b*c^3)*d^7*x + (b^6 - 12
*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*d^7*log(c*x^2 + b*x + a)
```

Fricas [B] time = 1.63438, size = 393, normalized size = 4.57

$$\frac{64}{3}\,c^6d^7x^6+64\,bc^5d^7x^5+8\left(11\,b^2c^4-4\,ac^5\right)d^7x^4+\frac{16}{3}\left(13\,b^3c^3-12\,abc^4\right)d^7x^3+4\left(9\,b^4c^2-20\,ab^2c^3+16\,a^2c^4\right)d^7x^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^7/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 64/3*c^6*d^7*x^6 + 64*b*c^5*d^7*x^5 + 8*(11*b^2*c^4 - 4*a*c^5)*d^7*x^4 + 16
/3*(13*b^3*c^3 - 12*a*b*c^4)*d^7*x^3 + 4*(9*b^4*c^2 - 20*a*b^2*c^3 + 16*a^2
*c^4)*d^7*x^2 + 4*(3*b^5*c - 12*a*b^3*c^2 + 16*a^2*b*c^3)*d^7*x + (b^6 - 12
*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*d^7*log(c*x^2 + b*x + a)
```

Sympy [B] time = 1.27699, size = 185, normalized size = 2.15

$$64bc^5d^7x^5 + \frac{64c^6d^7x^6}{3} - d^7\left(4ac - b^2\right)^3\log\left(a + bx + cx^2\right) + x^4\left(-32ac^5d^7 + 88b^2c^4d^7\right) + x^3\left(-64abc^4d^7 + \frac{208b^3c^3d^7}{3}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**7/(c*x**2+b*x+a),x)
```

```
[Out] 64*b*c**5*d**7*x**5 + 64*c**6*d**7*x**6/3 - d**7*(4*a*c - b**2)**3*log(a +
b*x + c*x**2) + x**4*(-32*a*c**5*d**7 + 88*b**2*c**4*d**7) + x**3*(-64*a*b*
c**4*d**7 + 208*b**3*c**3*d**7/3) + x**2*(64*a**2*c**4*d**7 - 80*a*b**2*c**
3*d**7 + 36*b**4*c**2*d**7) + x*(64*a**2*b*c**3*d**7 - 48*a*b**3*c**2*d**7
+ 12*b**5*c*d**7)
```

Giac [B] time = 1.18118, size = 296, normalized size = 3.44

$$\left(b^6d^7 - 12\,ab^4cd^7 + 48\,a^2b^2c^2d^7 - 64\,a^3c^3d^7\right)\log\left(cx^2 + bx + a\right) + \frac{4\left(16\,c^{12}d^7x^6 + 48\,bc^{11}d^7x^5 + 66\,b^2c^{10}d^7x^4 - 24\,ac^{11}d^7\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^7/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] (b^6*d^7 - 12*a*b^4*c*d^7 + 48*a^2*b^2*c^2*d^7 - 64*a^3*c^3*d^7)*log(c*x^2
+ b*x + a) + 4/3*(16*c^12*d^7*x^6 + 48*b*c^11*d^7*x^5 + 66*b^2*c^10*d^7*x^4
 - 24*a*c^11*d^7*x^4 + 52*b^3*c^9*d^7*x^3 - 48*a*b*c^10*d^7*x^3 + 27*b^4*c^
8*d^7*x^2 - 60*a*b^2*c^9*d^7*x^2 + 48*a^2*c^10*d^7*x^2 + 9*b^5*c^7*d^7*x -
36*a*b^3*c^8*d^7*x + 48*a^2*b*c^9*d^7*x)/c^6
```

3.1156 $\int \frac{(bd+2cdx)^6}{a+bx+cx^2}\,dx$

Optimal. Leaf size=97

$$\frac{2}{3}d^6\left(b^2-4ac\right)(b+2cx)^3+2d^6\left(b^2-4ac\right)^2(b+2cx)-2d^6\left(b^2-4ac\right)^{5/2}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+\frac{2}{5}d^6(b+2cx)^5$$

```
[Out] 2*(b^2 - 4*a*c)^2*d^6*(b + 2*c*x) + (2*(b^2 - 4*a*c)*d^6*(b + 2*c*x)^3)/3 +
 (2*d^6*(b + 2*c*x)^5)/5 - 2*(b^2 - 4*a*c)^(5/2)*d^6*ArcTanh[(b + 2*c*x)/Sq
rt[b^2 - 4*a*c]]
```

Rubi [A] time = 0.0804153, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {692, 618, 206}

$$\frac{2}{3}d^6\left(b^2-4ac\right)(b+2cx)^3+2d^6\left(b^2-4ac\right)^2(b+2cx)-2d^6\left(b^2-4ac\right)^{5/2}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+\frac{2}{5}d^6(b+2cx)^5$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^6/(a + b*x + c*x^2),x]
```

```
[Out] 2*(b^2 - 4*a*c)^2*d^6*(b + 2*c*x) + (2*(b^2 - 4*a*c)*d^6*(b + 2*c*x)^3)/3 +
 (2*d^6*(b + 2*c*x)^5)/5 - 2*(b^2 - 4*a*c)^(5/2)*d^6*ArcTanh[(b + 2*c*x)/Sq
rt[b^2 - 4*a*c]]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(bd+2cdx)^6}{a+bx+cx^2}\,dx = \frac{2}{5}d^6(b+2cx)^5 + \left(\left(b^2-4ac\right)d^2\right)\int \frac{(bd+2cdx)^4}{a+bx+cx^2}\,dx$$
$$= \frac{2}{3}\left(b^2-4ac\right)d^6(b+2cx)^3 + \frac{2}{5}d^6(b+2cx)^5 + \left(\left(b^2-4ac\right)^2 d^4\right)\int \frac{(bd+2cdx)^2}{a+bx+cx^2}\,dx$$
$$= 2\left(b^2-4ac\right)^2 d^6(b+2cx) + \frac{2}{3}\left(b^2-4ac\right)d^6(b+2cx)^3 + \frac{2}{5}d^6(b+2cx)^5 + \left(\left(b^2-4ac\right)^3 d^6\right)\int \frac{1}{a+bx}$$
$$= 2\left(b^2-4ac\right)^2 d^6(b+2cx) + \frac{2}{3}\left(b^2-4ac\right)d^6(b+2cx)^3 + \frac{2}{5}d^6(b+2cx)^5 - \left(2\left(b^2-4ac\right)^3 d^6\right)\text{Subst}\Big($$
$$= 2\left(b^2-4ac\right)^2 d^6(b+2cx) + \frac{2}{3}\left(b^2-4ac\right)d^6(b+2cx)^3 + \frac{2}{5}d^6(b+2cx)^5 - 2\left(b^2-4ac\right)^{5/2} d^6\tanh^{-1}\Big($$

Mathematica [A] time = 0.0757721, size = 120, normalized size = 1.24

$$d^6\left(\frac{4}{15}cx\left(16c^2\left(15a^2-5acx^2+3c^2x^4\right)+20b^2c\left(7cx^2-9a\right)+120bc^2x\left(cx^2-a\right)+90b^3cx+45b^4\right)-2\left(4ac-b^2\right)^{5/2}\tan^{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^6/(a + b*x + c*x^2),x]
```

```
[Out] d^6*((4*c*x*(45*b^4 + 90*b^3*c*x + 120*b*c^2*x*(-a + c*x^2) + 20*b^2*c*(-9*
a + 7*c*x^2) + 16*c^2*(15*a^2 - 5*a*c*x^2 + 3*c^2*x^4)))/15 - 2*(-b^2 + 4*a
*c)^(5/2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])
```

Maple [B] time = 0.149, size = 284, normalized size = 2.9

$$\frac{64\,d^6c^5x^5}{5}+32\,d^6bc^4x^4-\frac{64\,d^6x^3ac^4}{3}+\frac{112\,d^6x^3b^2c^3}{3}-32\,d^6x^2abc^3+24\,d^6x^2b^3c^2+64\,d^6a^2c^3x-48\,d^6ab^2c^2x+12\,d^6cb^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^6/(c*x^2+b*x+a),x)
```

```
[Out] 64/5*d^6*c^5*x^5+32*d^6*b*c^4*x^4-64/3*d^6*x^3*a*c^4+112/3*d^6*x^3*b^2*c^3-
32*d^6*x^2*a*b*c^3+24*d^6*x^2*b^3*c^2+64*d^6*a^2*c^3*x-48*d^6*a*b^2*c^2*x+1
2*d^6*c*b^4*x-128*d^6/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
*a^3*c^3+96*d^6/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b
^2*c^2-24*d^6/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^4*c
+2*d^6/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^6/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.75762, size = 780, normalized size = 8.04

$$\left[\frac{64}{5}c^5d^6x^5 + 32\,bc^4d^6x^4 + \frac{16}{3}\left(7\,b^2c^3 - 4\,ac^4\right)d^6x^3 + 8\left(3\,b^3c^2 - 4\,abc^3\right)d^6x^2 + \left(b^4 - 8\,ab^2c + 16\,a^2c^2\right)\sqrt{b^2 - 4\,ac}d^6\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^6/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [64/5*c^5*d^6*x^5 + 32*b*c^4*d^6*x^4 + 16/3*(7*b^2*c^3 - 4*a*c^4)*d^6*x^3 +
 8*(3*b^3*c^2 - 4*a*b*c^3)*d^6*x^2 + (b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(b^
2 - 4*a*c)*d^6*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(
2*c*x + b))/(c*x^2 + b*x + a)) + 4*(3*b^4*c - 12*a*b^2*c^2 + 16*a^2*c^3)*d^
6*x, 64/5*c^5*d^6*x^5 + 32*b*c^4*d^6*x^4 + 16/3*(7*b^2*c^3 - 4*a*c^4)*d^6*x
^3 + 8*(3*b^3*c^2 - 4*a*b*c^3)*d^6*x^2 - 2*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*s
qrt(-b^2 + 4*a*c)*d^6*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
 + 4*(3*b^4*c - 12*a*b^2*c^2 + 16*a^2*c^3)*d^6*x]
```

Sympy [B] time = 0.948615, size = 337, normalized size = 3.47

$$32bc^4d^6x^4 + \frac{64c^5d^6x^5}{5} + d^6\sqrt{-\left(4ac - b^2\right)^5}\log\left(x + \frac{16a^2bc^2d^6 - 8ab^3cd^6 + b^5d^6 - d^6\sqrt{-\left(4ac - b^2\right)^5}}{32a^2c^3d^6 - 16ab^2c^2d^6 + 2b^4cd^6}\right) - d^6\sqrt{-\left(4ac - \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**6/(c*x**2+b*x+a),x)
```

```
[Out] 32*b*c**4*d**6*x**4 + 64*c**5*d**6*x**5/5 + d**6*sqrt(-(4*a*c - b**2)**5)*l
og(x + (16*a**2*b*c**2*d**6 - 8*a*b**3*c*d**6 + b**5*d**6 - d**6*sqrt(-(4*a
*c - b**2)**5))/(32*a**2*c**3*d**6 - 16*a*b**2*c**2*d**6 + 2*b**4*c*d**6))
- d**6*sqrt(-(4*a*c - b**2)**5)*log(x + (16*a**2*b*c**2*d**6 - 8*a*b**3*c*d
**6 + b**5*d**6 + d**6*sqrt(-(4*a*c - b**2)**5))/(32*a**2*c**3*d**6 - 16*a*
b**2*c**2*d**6 + 2*b**4*c*d**6)) + x**3*(-64*a*c**4*d**6/3 + 112*b**2*c**3*
d**6/3) + x**2*(-32*a*b*c**3*d**6 + 24*b**3*c**2*d**6) + x*(64*a**2*c**3*d*
*6 - 48*a*b**2*c**2*d**6 + 12*b**4*c*d**6)
```

Giac [B] time = 1.2476, size = 266, normalized size = 2.74

$$\frac{2\left(b^6d^6 - 12\,ab^4cd^6 + 48\,a^2b^2c^2d^6 - 64\,a^3c^3d^6\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2 + 4\,ac}} + \frac{4\left(48\,c^{10}d^6x^5 + 120\,bc^9d^6x^4 + 140\,b^2c^8d^6x^3 - 80\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^6/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 2*(b^6*d^6 - 12*a*b^4*c*d^6 + 48*a^2*b^2*c^2*d^6 - 64*a^3*c^3*d^6)*arctan((
2*c*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c) + 4/15*(48*c^10*d^6*x^5 +
 120*b*c^9*d^6*x^4 + 140*b^2*c^8*d^6*x^3 - 80*a*c^9*d^6*x^3 + 90*b^3*c^7*d^
6*x^2 - 120*a*b*c^8*d^6*x^2 + 45*b^4*c^6*d^6*x - 180*a*b^2*c^7*d^6*x + 240*
a^2*c^8*d^6*x)/c^5
```

3.1157 $\int \frac{(bd+2cdx)^5}{a+bx+cx^2} dx$

Optimal. Leaf size=61

$$d^5\left(b^2-4ac\right)^2\log\left(a+bx+cx^2\right)+d^5\left(b^2-4ac\right)(b+2cx)^2+\frac{1}{2}d^5(b+2cx)^4$$

```
[Out] (b^2 - 4*a*c)*d^5*(b + 2*c*x)^2 + (d^5*(b + 2*c*x)^4)/2 + (b^2 - 4*a*c)^2*d
^5*Log[a + b*x + c*x^2]
```

Rubi [A] time = 0.0426205, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {692, 628}

$$d^5\left(b^2-4ac\right)^2\log\left(a+bx+cx^2\right)+d^5\left(b^2-4ac\right)(b+2cx)^2+\frac{1}{2}d^5(b+2cx)^4$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^5/(a + b*x + c*x^2),x]
```

```
[Out] (b^2 - 4*a*c)*d^5*(b + 2*c*x)^2 + (d^5*(b + 2*c*x)^4)/2 + (b^2 - 4*a*c)^2*d
^5*Log[a + b*x + c*x^2]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(bd+2cdx)^5}{a+bx+cx^2}\,dx &= \frac{1}{2}d^5(b+2cx)^4+\left(\left(b^2-4ac\right)d^2\right)\int \frac{(bd+2cdx)^3}{a+bx+cx^2}\,dx\\ &= \left(b^2-4ac\right)d^5(b+2cx)^2+\frac{1}{2}d^5(b+2cx)^4+\left(\left(b^2-4ac\right)^2d^4\right)\int \frac{bd+2cdx}{a+bx+cx^2}\,dx\\ &= \left(b^2-4ac\right)d^5(b+2cx)^2+\frac{1}{2}d^5(b+2cx)^4+\left(b^2-4ac\right)^2d^5\log\left(a+bx+cx^2\right)\end{aligned}$$

Mathematica [A] time = 0.0232375, size = 54, normalized size = 0.89

$$d^5\left(8cx(b+cx)\left(c\left(cx^2-2a\right)+b^2+bcx\right)+\left(b^2-4ac\right)^2\log(a+x(b+cx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^5/(a + b*x + c*x^2),x]
```

```
[Out] d^5*(8*c*x*(b + c*x)*(b^2 + b*c*x + c*(-2*a + c*x^2)) + (b^2 - 4*a*c)^2*Log
[a + x*(b + c*x)])
```

Maple [B] time = 0.041, size = 133, normalized size = 2.2

$$8\,x^4c^4d^5+16\,x^3bc^3d^5-16\,x^2ac^3d^5+16\,x^2b^2c^2d^5+16\,\ln\left(cx^2+bx+a\right)a^2c^2d^5-8\,\ln\left(cx^2+bx+a\right)ab^2cd^5+\ln\left(cx^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^5/(c*x^2+b*x+a),x)
```

```
[Out] 8*x^4*c^4*d^5+16*x^3*b*c^3*d^5-16*x^2*a*c^3*d^5+16*x^2*b^2*c^2*d^5+16*ln(c*
x^2+b*x+a)*a^2*c^2*d^5-8*ln(c*x^2+b*x+a)*a*b^2*c*d^5+ln(c*x^2+b*x+a)*b^4*d^
5-16*x*a*b*c^2*d^5+8*x*b^3*c*d^5
```

Maxima [A] time = 1.58318, size = 134, normalized size = 2.2

$$8\,c^4d^5x^4+16\,bc^3d^5x^3+16\left(b^2c^2-ac^3\right)d^5x^2+8\left(b^3c-2\,abc^2\right)d^5x+\left(b^4-8\,ab^2c+16\,a^2c^2\right)d^5\log\left(cx^2+bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^5/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 8*c^4*d^5*x^4 + 16*b*c^3*d^5*x^3 + 16*(b^2*c^2 - a*c^3)*d^5*x^2 + 8*(b^3*c
- 2*a*b*c^2)*d^5*x + (b^4 - 8*a*b^2*c + 16*a^2*c^2)*d^5*log(c*x^2 + b*x + a
)
```

Fricas [A] time = 1.69205, size = 207, normalized size = 3.39

$$8\,c^4d^5x^4+16\,bc^3d^5x^3+16\left(b^2c^2-ac^3\right)d^5x^2+8\left(b^3c-2\,abc^2\right)d^5x+\left(b^4-8\,ab^2c+16\,a^2c^2\right)d^5\log\left(cx^2+bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^5/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 8*c^4*d^5*x^4 + 16*b*c^3*d^5*x^3 + 16*(b^2*c^2 - a*c^3)*d^5*x^2 + 8*(b^3*c
- 2*a*b*c^2)*d^5*x + (b^4 - 8*a*b^2*c + 16*a^2*c^2)*d^5*log(c*x^2 + b*x + a
)
```

Sympy [A] time = 0.874252, size = 99, normalized size = 1.62

$$16bc^3d^5x^3+8c^4d^5x^4+d^5\left(4ac-b^2\right)^2\log\left(a+bx+cx^2\right)+x^2\left(-16ac^3d^5+16b^2c^2d^5\right)+x\left(-16abc^2d^5+8b^3cd^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**5/(c*x**2+b*x+a),x)
```

```
[Out] 16*b*c**3*d**5*x**3 + 8*c**4*d**5*x**4 + d**5*(4*a*c - b**2)**2*log(a + b*x
 + c*x**2) + x**2*(-16*a*c**3*d**5 + 16*b**2*c**2*d**5) + x*(-16*a*b*c**2*d
**5 + 8*b**3*c*d**5)
```

Giac [A] time = 1.20101, size = 159, normalized size = 2.61

$$\left(b^4d^5-8\,ab^2cd^5+16\,a^2c^2d^5\right)\log\left(cx^2+bx+a\right)+\frac{8\left(c^8d^5x^4+2\,bc^7d^5x^3+2\,b^2c^6d^5x^2-2\,ac^7d^5x^2+b^3c^5d^5x-2\,abc^6d^5x\right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^5/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] (b^4*d^5 - 8*a*b^2*c*d^5 + 16*a^2*c^2*d^5)*log(c*x^2 + b*x + a) + 8*(c^8*d^
5*x^4 + 2*b*c^7*d^5*x^3 + 2*b^2*c^6*d^5*x^2 - 2*a*c^7*d^5*x^2 + b^3*c^5*d^5
*x - 2*a*b*c^6*d^5*x)/c^4
```

3.1158 $\int \frac{(bd+2cdx)^4}{a+bx+cx^2} dx$

Optimal. Leaf size=72

$$2d^4\left(b^2-4ac\right)(b+2cx)-2d^4\left(b^2-4ac\right)^{3/2}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+\frac{2}{3}d^4(b+2cx)^3$$

```
[Out] 2*(b^2 - 4*a*c)*d^4*(b + 2*c*x) + (2*d^4*(b + 2*c*x)^3)/3 - 2*(b^2 - 4*a*c)
^(3/2)*d^4*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]
```

Rubi [A] time = 0.070038, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {692, 618, 206}

$$2d^4\left(b^2-4ac\right)(b+2cx)-2d^4\left(b^2-4ac\right)^{3/2}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+\frac{2}{3}d^4(b+2cx)^3$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^4/(a + b*x + c*x^2),x]
```

```
[Out] 2*(b^2 - 4*a*c)*d^4*(b + 2*c*x) + (2*d^4*(b + 2*c*x)^3)/3 - 2*(b^2 - 4*a*c)
^(3/2)*d^4*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(bd+2cdx)^4}{a+bx+cx^2}\,dx = \frac{2}{3}d^4(b+2cx)^3+\left(\left(b^2-4ac\right)d^2\right)\int \frac{(bd+2cdx)^2}{a+bx+cx^2}\,dx$$
$$= 2\left(b^2-4ac\right)d^4(b+2cx)+\frac{2}{3}d^4(b+2cx)^3+\left(\left(b^2-4ac\right)^2d^4\right)\int \frac{1}{a+bx+cx^2}\,dx$$
$$= 2\left(b^2-4ac\right)d^4(b+2cx)+\frac{2}{3}d^4(b+2cx)^3-\left(2\left(b^2-4ac\right)^2d^4\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)$$
$$= 2\left(b^2-4ac\right)d^4(b+2cx)+\frac{2}{3}d^4(b+2cx)^3-2\left(b^2-4ac\right)^{3/2}d^4\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)$$

Mathematica [A] time = 0.0418401, size = 72, normalized size = 1.

$$d^4\left(\frac{8}{3}cx\left(2c\left(cx^2-3a\right)+3b^2+3bcx\right)+2\left(4ac-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^4/(a + b*x + c*x^2),x]
```

```
[Out] d^4*((8*c*x*(3*b^2 + 3*b*c*x + 2*c*(-3*a + c*x^2)))/3 + 2*(-b^2 + 4*a*c)^(3
/2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])
```

Maple [B] time = 0.147, size = 170, normalized size = 2.4

$$\frac{16\,d^4c^3x^3}{3}+8\,d^4bc^2x^2-16\,d^4ac^2x+8\,d^4b^2cx+32\,\frac{d^4a^2c^2}{\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)-16\,\frac{d^4acb^2}{\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^4/(c*x^2+b*x+a),x)
```

```
[Out] 16/3*d^4*c^3*x^3+8*d^4*b*c^2*x^2-16*d^4*a*c^2*x+8*d^4*b^2*c*x+32*d^4/(4*a*c
-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*c^2-16*d^4/(4*a*c-b^2)^
(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c*b^2+2*d^4/(4*a*c-b^2)^(1/2)*a
rctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72199, size = 464, normalized size = 6.44

$$\left[\frac{16}{3}c^3d^4x^3+8\,bc^2d^4x^2-\left(b^2-4\,ac\right)^{\frac{3}{2}}d^4\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+8\left(b^2c-2\,ac^2\right)d^4x,\,\frac{16}{3}c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [16/3*c^3*d^4*x^3 + 8*b*c^2*d^4*x^2 - (b^2 - 4*a*c)^(3/2)*d^4*log((2*c^2*x^
2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a
)) + 8*(b^2*c - 2*a*c^2)*d^4*x, 16/3*c^3*d^4*x^3 + 8*b*c^2*d^4*x^2 - 2*(b^2
 - 4*a*c)*sqrt(-b^2 + 4*a*c)*d^4*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^
2 - 4*a*c)) + 8*(b^2*c - 2*a*c^2)*d^4*x]
```

Sympy [B] time = 0.745157, size = 204, normalized size = 2.83

$$8bc^2d^4x^2 + \frac{16c^3d^4x^3}{3} - d^4\sqrt{-\left(4ac-b^2\right)^3}\log\left(x + \frac{4abcd^4 - b^3d^4 - d^4\sqrt{-\left(4ac-b^2\right)^3}}{8ac^2d^4 - 2b^2cd^4}\right) + d^4\sqrt{-\left(4ac-b^2\right)^3}\log\left(x + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**4/(c*x**2+b*x+a),x)
```

```
[Out] 8*b*c**2*d**4*x**2 + 16*c**3*d**4*x**3/3 - d**4*sqrt(-(4*a*c - b**2)**3)*lo
g(x + (4*a*b*c*d**4 - b**3*d**4 - d**4*sqrt(-(4*a*c - b**2)**3))/(8*a*c**2*
d**4 - 2*b**2*c*d**4)) + d**4*sqrt(-(4*a*c - b**2)**3)*log(x + (4*a*b*c*d**
4 - b**3*d**4 + d**4*sqrt(-(4*a*c - b**2)**3))/(8*a*c**2*d**4 - 2*b**2*c*d*
*4)) + x*(-16*a*c**2*d**4 + 8*b**2*c*d**4)
```

Giac [A] time = 1.11598, size = 155, normalized size = 2.15

$$\frac{2\left(b^4d^4 - 8\,ab^2cd^4 + 16\,a^2c^2d^4\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}} + \frac{8\left(2\,c^6d^4x^3 + 3\,bc^5d^4x^2 + 3\,b^2c^4d^4x - 6\,ac^5d^4x\right)}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 2*(b^4*d^4 - 8*a*b^2*c*d^4 + 16*a^2*c^2*d^4)*arctan((2*c*x + b)/sqrt(-b^2 +
 4*a*c))/sqrt(-b^2 + 4*a*c) + 8/3*(2*c^6*d^4*x^3 + 3*b*c^5*d^4*x^2 + 3*b^2*
c^4*d^4*x - 6*a*c^5*d^4*x)/c^3
```

3.1159 $\int \frac{(bd+2cdx)^3}{a+bx+cx^2} dx$

Optimal. Leaf size=36

$$d^3 \left(b^2-4ac\right) \log \left(a+bx+cx^2\right)+d^3(b+2cx)^2$$

```
[Out] d^3*(b + 2*c*x)^2 + (b^2 - 4*a*c)*d^3*Log[a + b*x + c*x^2]
```

Rubi [A] time = 0.0228052, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {692, 628}

$$d^3 \left(b^2-4ac\right) \log \left(a+bx+cx^2\right)+d^3(b+2cx)^2$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^3/(a + b*x + c*x^2),x]
```

```
[Out] d^3*(b + 2*c*x)^2 + (b^2 - 4*a*c)*d^3*Log[a + b*x + c*x^2]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(bd+2cdx)^3}{a+bx+cx^2} dx &= d^3(b+2cx)^2+\left(\left(b^2-4ac\right)d^2\right)\int \frac{bd+2cdx}{a+bx+cx^2} dx \\ &= d^3(b+2cx)^2+\left(b^2-4ac\right)d^3 \log \left(a+bx+cx^2\right) \end{aligned}$$

Mathematica [A] time = 0.0123265, size = 33, normalized size = 0.92

$$d^3 \left(\left(b^2-4ac\right) \log (a+x(b+cx))+4cx(b+cx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^3/(a + b*x + c*x^2),x]
```

```
[Out] d^3*(4*c*x*(b + c*x) + (b^2 - 4*a*c)*Log[a + x*(b + c*x)])
```

Maple [A] time = 0.042, size = 57, normalized size = 1.6

$$4x^2c^2d^3 - 4\ln\left(cx^2 + bx + a\right)acd^3 + \ln\left(cx^2 + bx + a\right)b^2d^3 + 4xbcd^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^3/(c*x^2+b*x+a),x)
```

```
[Out] 4*x^2*c^2*d^3-4*ln(c*x^2+b*x+a)*a*c*d^3+ln(c*x^2+b*x+a)*b^2*d^3+4*x*b*c*d^3
```

Maxima [A] time = 1.14233, size = 58, normalized size = 1.61

$$4c^2d^3x^2 + 4bcd^3x + \left(b^2 - 4ac\right)d^3\log\left(cx^2 + bx + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 4*c^2*d^3*x^2 + 4*b*c*d^3*x + (b^2 - 4*a*c)*d^3*log(c*x^2 + b*x + a)
```

Fricas [A] time = 1.67158, size = 95, normalized size = 2.64

$$4c^2d^3x^2 + 4bcd^3x + \left(b^2 - 4ac\right)d^3\log\left(cx^2 + bx + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 4*c^2*d^3*x^2 + 4*b*c*d^3*x + (b^2 - 4*a*c)*d^3*log(c*x^2 + b*x + a)
```

Sympy [A] time = 0.624678, size = 44, normalized size = 1.22

$$4bcd^3x + 4c^2d^3x^2 - d^3\left(4ac - b^2\right)\log\left(a + bx + cx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**3/(c*x**2+b*x+a),x)
```

```
[Out] 4*b*c*d**3*x + 4*c**2*d**3*x**2 - d**3*(4*a*c - b**2)*log(a + b*x + c*x**2)
```

Giac [A] time = 1.12149, size = 72, normalized size = 2.

$$\left(b^2d^3 - 4acd^3\right)\log\left(cx^2 + bx + a\right) + \frac{4\left(c^4d^3x^2 + bc^3d^3x\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] (b^2*d^3 - 4*a*c*d^3)*log(c*x^2 + b*x + a) + 4*(c^4*d^3*x^2 + b*c^3*d^3*x)/
c^2
```

3.1160 $\int \frac{(bd+2cdx)^2}{a+bx+cx^2} dx$

Optimal. Leaf size=49

$$2d^2(b+2cx)-2d^2\sqrt{b^2-4ac}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)$$

```
[Out] 2*d^2*(b + 2*c*x) - 2*Sqrt[b^2 - 4*a*c]*d^2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]]
```

Rubi [A] time = 0.0308715, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {692, 618, 206}

$$2d^2(b+2cx)-2d^2\sqrt{b^2-4ac}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^2/(a + b*x + c*x^2),x]
```

```
[Out] 2*d^2*(b + 2*c*x) - 2*Sqrt[b^2 - 4*a*c]*d^2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(bd+2cdx)^2}{a+bx+cx^2}\,dx &= 2d^2(b+2cx)+\left(\left(b^2-4ac\right)d^2\right)\int \frac{1}{a+bx+cx^2}\,dx\\ &= 2d^2(b+2cx)-\left(2\left(b^2-4ac\right)d^2\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)\\ &= 2d^2(b+2cx)-2\sqrt{b^2-4ac}d^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\end{aligned}$$

Mathematica [A] time = 0.0243931, size = 47, normalized size = 0.96

$$d^2\left(4cx-2\sqrt{4ac-b^2}\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^2/(a + b*x + c*x^2),x]
```

```
[Out] d^2*(4*c*x - 2*Sqrt[-b^2 + 4*a*c]*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])
```

Maple [A] time = 0.148, size = 88, normalized size = 1.8

$$4\,c{d}^{2}x-8\,{\frac{a{d}^{2}c}{\sqrt{4\,ac-{b}^{2}}}\arctan \left({\frac{2\,cx+b}{\sqrt{4\,ac-{b}^{2}}}} \right) }+2\,{\frac{{b}^{2}{d}^{2}}{\sqrt{4\,ac-{b}^{2}}}\arctan \left({\frac{2\,cx+b}{\sqrt{4\,ac-{b}^{2}}}} \right) }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^2/(c*x^2+b*x+a),x)
```

```
[Out] 4*c*d^2*x-8*d^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c+2
*d^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71929, size = 300, normalized size = 6.12

$$\left[4\,cd^{2}x+\sqrt{b^{2}-4\,ac}d^{2}\log\left(\frac{2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac-\sqrt{b^{2}-4\,ac}(2\,cx+b)}{cx^{2}+bx+a}\right),4\,cd^{2}x-2\,\sqrt{-b^{2}+4\,ac}d^{2}\arctan\left(-\frac{\sqrt{-b^{2}}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [4*c*d^2*x + sqrt(b^2 - 4*a*c)*d^2*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c -
 sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)), 4*c*d^2*x - 2*sqrt(-b^2
 + 4*a*c)*d^2*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))]
```

Sympy [B] time = 1.0984, size = 99, normalized size = 2.02

$$4cd^{2}x+d^{2}\sqrt{-4ac+b^{2}}\log\left(x+\frac{bd^{2}-d^{2}\sqrt{-4ac+b^{2}}}{2cd^{2}}\right)-d^{2}\sqrt{-4ac+b^{2}}\log\left(x+\frac{bd^{2}+d^{2}\sqrt{-4ac+b^{2}}}{2cd^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**2/(c*x**2+b*x+a),x)
```

```
[Out] 4*c*d**2*x + d**2*sqrt(-4*a*c + b**2)*log(x + (b*d**2 - d**2*sqrt(-4*a*c +
b**2))/(2*c*d**2)) - d**2*sqrt(-4*a*c + b**2)*log(x + (b*d**2 + d**2*sqrt(-
4*a*c + b**2))/(2*c*d**2))
```

Giac [A] time = 1.13945, size = 77, normalized size = 1.57

$$4\,cd^2x+\frac{2\left(b^2d^2-4\,acd^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 4*c*d^2*x + 2*(b^2*d^2 - 4*a*c*d^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/
sqrt(-b^2 + 4*a*c)
```

3.1161 $\int \frac{bd+2cdx}{a+bx+cx^2} dx$

Optimal. Leaf size=13

$$d\log\left(a+bx+cx^2\right)$$

```
[Out] d*Log[a + b*x + c*x^2]
```

Rubi [A] time = 0.0048115, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {628}

$$d\log\left(a+bx+cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)/(a + b*x + c*x^2),x]
```

```
[Out] d*Log[a + b*x + c*x^2]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{bd+2cdx}{a+bx+cx^2}\,dx = d\log\left(a+bx+cx^2\right)$$

Mathematica [A] time = 0.002052, size = 12, normalized size = 0.92

$$d\log(a+x(b+cx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)/(a + b*x + c*x^2),x]
```

```
[Out] d*Log[a + x*(b + c*x)]
```

Maple [A] time = 0.04, size = 14, normalized size = 1.1

$$d\ln\left(cx^2+bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)/(c*x^2+b*x+a),x)
```

```
[Out] d*ln(c*x^2+b*x+a)
```

Maxima [A] time = 1.29871, size = 18, normalized size = 1.38

$$d\log\left(cx^2+bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] d*log(c*x^2 + b*x + a)
```

Fricas [A] time = 1.7289, size = 32, normalized size = 2.46

$$d\log\left(cx^2+bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] d*log(c*x^2 + b*x + a)
```

Sympy [A] time = 0.333279, size = 12, normalized size = 0.92

$$d\log\left(a+bx+cx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)/(c*x**2+b*x+a),x)
```

```
[Out] d*log(a + b*x + c*x**2)
```

Giac [A] time = 1.22793, size = 18, normalized size = 1.38

$$d\log\left(cx^2+bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] d*log(c*x^2 + b*x + a)
```

3.1162 $\int \frac{1}{(bd+2cdx)\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=48

$$\frac{\log\left(a+bx+cx^2\right)}{d\left(b^2-4ac\right)}-\frac{2\log(b+2cx)}{d\left(b^2-4ac\right)}$$

```
[Out] (-2*Log[b + 2*c*x])/((b^2 - 4*a*c)*d) + Log[a + b*x + c*x^2]/((b^2 - 4*a*c)
*d)
```

Rubi [A] time = 0.0233232, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {681, 31, 628}

$$\frac{\log\left(a+bx+cx^2\right)}{d\left(b^2-4ac\right)}-\frac{2\log(b+2cx)}{d\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*Log[b + 2*c*x])/((b^2 - 4*a*c)*d) + Log[a + b*x + c*x^2]/((b^2 - 4*a*c)
*d)
```

Rule 681

```
Int[1/(((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
:> Dist[(-4*b*c)/(d*(b^2 - 4*a*c)), Int[1/(b + 2*c*x), x], x] + Dist[b^2/(d
^2*(b^2 - 4*a*c)), Int[(d + e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(bd+2cdx)\left(a+bx+cx^2\right)}\,dx&=\frac{\int \frac{bd+2cdx}{a+bx+cx^2}\,dx}{\left(b^2-4ac\right)d^2}-\frac{(4c)\int \frac{1}{b+2cx}\,dx}{\left(b^2-4ac\right)d}\\&=-\frac{2\log(b+2cx)}{\left(b^2-4ac\right)d}+\frac{\log\left(a+bx+cx^2\right)}{\left(b^2-4ac\right)d}\end{aligned}$$

Mathematica [A] time = 0.0203116, size = 34, normalized size = 0.71

$$\frac{\log(a+x(b+cx))-2\log(b+2cx)}{d\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*Log[b + 2*c*x] + Log[a + x*(b + c*x)])/((b^2 - 4*a*c)*d)
```

Maple [A] time = 0.044, size = 54, normalized size = 1.1

$$-\frac{\ln\left(cx^2+bx+a\right)}{d\left(4\,ac-b^2\right)}+2\,\frac{\ln\left(2\,cx+b\right)}{d\left(4\,ac-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)/(c*x^2+b*x+a),x)
```

```
[Out] -1/d/(4*a*c-b^2)*ln(c*x^2+b*x+a)+2/d/(4*a*c-b^2)*ln(2*c*x+b)
```

Maxima [A] time = 1.20289, size = 65, normalized size = 1.35

$$\frac{\log\left(cx^2+bx+a\right)}{\left(b^2-4\,ac\right)d}-\frac{2\,\log\left(2\,cx+b\right)}{\left(b^2-4\,ac\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] log(c*x^2 + b*x + a)/((b^2 - 4*a*c)*d) - 2*log(2*c*x + b)/((b^2 - 4*a*c)*d)
```

Fricas [A] time = 1.64533, size = 82, normalized size = 1.71

$$\frac{\log\left(cx^2+bx+a\right)-2\,\log\left(2\,cx+b\right)}{\left(b^2-4\,ac\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] (log(c*x^2 + b*x + a) - 2*log(2*c*x + b))/((b^2 - 4*a*c)*d)
```

Sympy [A] time = 0.92411, size = 42, normalized size = 0.88

$$\frac{2\log\left(\frac{b}{2c}+x\right)}{d\left(4ac-b^2\right)}-\frac{\log\left(\frac{a}{c}+\frac{bx}{c}+x^2\right)}{d\left(4ac-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)/(c*x**2+b*x+a),x)
```

```
[Out] 2*log(b/(2*c) + x)/(d*(4*a*c - b**2)) - log(a/c + b*x/c + x**2)/(d*(4*a*c -
 b**2))
```

Giac [A] time = 1.19323, size = 77, normalized size = 1.6

$$-\frac{2\,c^2\log\left(|2\,cx+b|\right)}{b^2c^2d-4\,ac^3d}+\frac{\log\left(cx^2+bx+a\right)}{b^2d-4\,acd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -2*c^2*log(abs(2*c*x + b))/(b^2*c^2*d - 4*a*c^3*d) + log(c*x^2 + b*x + a)/(
b^2*d - 4*a*c*d)
```

3.1163 $\int \frac{1}{(bd+2cdx)^2(a+bx+cx^2)}\,dx$

Optimal. Leaf size=61

$$\frac{2}{d^2(b^2-4ac)(b+2cx)}-\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d^2(b^2-4ac)^{3/2}}$$

```
[Out] 2/((b^2 - 4*a*c)*d^2*(b + 2*c*x)) - (2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c
]])/((b^2 - 4*a*c)^(3/2)*d^2)
```

Rubi [A] time = 0.040675, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {693, 618, 206}

$$\frac{2}{d^2(b^2-4ac)(b+2cx)}-\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d^2(b^2-4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^2*(a + b*x + c*x^2)),x]
```

```
[Out] 2/((b^2 - 4*a*c)*d^2*(b + 2*c*x)) - (2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c
]])/((b^2 - 4*a*c)^(3/2)*d^2)
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)^2\left(a+bx+cx^2\right)}\,dx=\frac{2}{\left(b^2-4ac\right)d^2(b+2cx)}+\frac{\int \frac{1}{a+bx+cx^2}dx}{\left(b^2-4ac\right)d^2}$$
$$=\frac{2}{\left(b^2-4ac\right)d^2(b+2cx)}-\frac{2\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}dx,x,b+2cx\right)}{\left(b^2-4ac\right)d^2}$$
$$=\frac{2}{\left(b^2-4ac\right)d^2(b+2cx)}-\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}d^2}$$

Mathematica [A] time = 0.0507395, size = 63, normalized size = 1.03

$$\frac{\frac{2}{\left(b^2-4ac\right)(b+2cx)}-\frac{2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^2*(a + b*x + c*x^2)),x]
```

```
[Out] (2/((b^2 - 4*a*c)*(b + 2*c*x)) - (2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])
/(-b^2 + 4*a*c)^(3/2))/d^2
```

Maple [A] time = 0.152, size = 64, normalized size = 1.1

$$-2\,\frac{1}{d^2\left(4\,ac-b^2\right)^{3/2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)-2\,\frac{1}{d^2\left(4\,ac-b^2\right)(2\,cx+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^2/(c*x^2+b*x+a),x)
```

```
[Out] -2/d^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))-2/d^2/(4*a*c-b
^2)/(2*c*x+b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^2/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.11649, size = 564, normalized size = 9.25

$$\left[-\frac{\sqrt{b^2-4\,ac}(2\,cx+b)\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-2\,b^2+8\,ac}{2\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)d^2x+\left(b^5-8\,ab^3c+16\,a^2bc^2\right)d^2},-\frac{2\left(\sqrt{-b^2+4\,ac}(2\,cx+b)\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(}{b^2-4\,a}\right.\right.}{2\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)d^2x+\left(b^5-8\,ab\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [-(sqrt(b^2 - 4*a*c)*(2*c*x + b)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + s
qrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*b^2 + 8*a*c)/(2*(b^4*c
 - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2*x + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2),
 -2*(sqrt(-b^2 + 4*a*c)*(2*c*x + b)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/
(b^2 - 4*a*c)) - b^2 + 4*a*c)/(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2*x +
 (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)]
```

Sympy [B] time = 1.21653, size = 240, normalized size = 3.93

$$-\frac{2}{4abcd^2 - b^3d^2 + x\left(8ac^2d^2 - 2b^2cd^2\right)} + \frac{\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x + \frac{-16a^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}} + 8ab^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}} - b^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}} + b}{2c}\right)}{d^2} - \sqrt{-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**2/(c*x**2+b*x+a),x)
```

```
[Out] -2/(4*a*b*c*d**2 - b**3*d**2 + x*(8*a*c**2*d**2 - 2*b**2*c*d**2)) + sqrt(-1
/(4*a*c - b**2)**3)*log(x + (-16*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3) + 8*a
*b**2*c*sqrt(-1/(4*a*c - b**2)**3) - b**4*sqrt(-1/(4*a*c - b**2)**3) + b)/(
2*c))/d**2 - sqrt(-1/(4*a*c - b**2)**3)*log(x + (16*a**2*c**2*sqrt(-1/(4*a*
c - b**2)**3) - 8*a*b**2*c*sqrt(-1/(4*a*c - b**2)**3) + b**4*sqrt(-1/(4*a*c
 - b**2)**3) + b)/(2*c))/d**2
```

Giac [B] time = 1.17961, size = 158, normalized size = 2.59

$$\frac{2\,c^2d^3}{\left(b^2c^2d^4 - 4\,ac^3d^4\right)(2\,cdx + bd)} - \frac{2\,\arctan\left(-\frac{\frac{b^2d}{2\,cdx+bd} - \frac{4\,acd}{2\,cdx+bd}}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2 - 4\,ac\right)\sqrt{-b^2 + 4\,ac}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^2/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 2*c^2*d^3/((b^2*c^2*d^4 - 4*a*c^3*d^4)*(2*c*d*x + b*d)) - 2*arctan(-(b^2*d/
(2*c*d*x + b*d) - 4*a*c*d/(2*c*d*x + b*d))/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*
c)*sqrt(-b^2 + 4*a*c)*d^2)
```

3.1164 $\int \frac{1}{(bd+2cdx)^3(a+bx+cx^2)}\,dx$

Optimal. Leaf size=70

$$\frac{\log\left(a+bx+cx^2\right)}{d^3\left(b^2-4ac\right)^2}+\frac{1}{d^3\left(b^2-4ac\right)(b+2cx)^2}-\frac{2\log(b+2cx)}{d^3\left(b^2-4ac\right)^2}$$

```
[Out] 1/((b^2 - 4*a*c)*d^3*(b + 2*c*x)^2) - (2*Log[b + 2*c*x])/((b^2 - 4*a*c)^2*d
^3) + Log[a + b*x + c*x^2]/((b^2 - 4*a*c)^2*d^3)
```

Rubi [A] time = 0.0403733, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {693, 681, 31, 628}

$$\frac{\log\left(a+bx+cx^2\right)}{d^3\left(b^2-4ac\right)^2}+\frac{1}{d^3\left(b^2-4ac\right)(b+2cx)^2}-\frac{2\log(b+2cx)}{d^3\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^3*(a + b*x + c*x^2)),x]
```

```
[Out] 1/((b^2 - 4*a*c)*d^3*(b + 2*c*x)^2) - (2*Log[b + 2*c*x])/((b^2 - 4*a*c)^2*d
^3) + Log[a + b*x + c*x^2]/((b^2 - 4*a*c)^2*d^3)
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 681

```
Int[1/(((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
:> Dist[(-4*b*c)/(d*(b^2 - 4*a*c)), Int[1/(b + 2*c*x), x], x] + Dist[b^2/(d
^2*(b^2 - 4*a*c)), Int[(d + e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)^3(a+bx+cx^2)}\,dx = \frac{1}{(b^2-4ac)\,d^3(b+2cx)^2} + \frac{\int \frac{1}{(bd+2cdx)(a+bx+cx^2)}\,dx}{(b^2-4ac)\,d^2}$$
$$= \frac{1}{(b^2-4ac)\,d^3(b+2cx)^2} + \frac{\int \frac{bd+2cdx}{a+bx+cx^2}\,dx}{(b^2-4ac)^2\,d^4} - \frac{(4c)\int \frac{1}{b+2cx}\,dx}{(b^2-4ac)^2\,d^3}$$
$$= \frac{1}{(b^2-4ac)\,d^3(b+2cx)^2} - \frac{2\log(b+2cx)}{(b^2-4ac)^2\,d^3} + \frac{\log(a+bx+cx^2)}{(b^2-4ac)^2\,d^3}$$

Mathematica [A] time = 0.0391309, size = 65, normalized size = 0.93

$$\frac{\frac{\log(a+bx+cx^2)}{(b^2-4ac)^2} + \frac{1}{(b^2-4ac)(b+2cx)^2} - \frac{2\log(b+2cx)}{(b^2-4ac)^2}}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^3*(a + b*x + c*x^2)),x]
```

```
[Out] (1/((b^2 - 4*a*c)*(b + 2*c*x)^2) - (2*Log[b + 2*c*x])/(b^2 - 4*a*c)^2 + Log
[a + b*x + c*x^2]/(b^2 - 4*a*c)^2)/d^3
```

Maple [A] time = 0.046, size = 78, normalized size = 1.1

$$\frac{\ln(cx^2+bx+a)}{d^3(4ac-b^2)^2} - 2\frac{\ln(2cx+b)}{d^3(4ac-b^2)^2} - \frac{1}{d^3(4ac-b^2)(2cx+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^3/(c*x^2+b*x+a),x)
```

```
[Out] 1/d^3/(4*a*c-b^2)^2*ln(c*x^2+b*x+a)-2/d^3/(4*a*c-b^2)^2*ln(2*c*x+b)-1/d^3/(
4*a*c-b^2)/(2*c*x+b)^2
```

Maxima [A] time = 1.20086, size = 174, normalized size = 2.49

$$\frac{1}{4(b^2c^2-4ac^3)d^3x^2+4(b^3c-4abc^2)d^3x+(b^4-4ab^2c)d^3} + \frac{\log(cx^2+bx+a)}{(b^4-8ab^2c+16a^2c^2)d^3} - \frac{2\log(2cx+b)}{(b^4-8ab^2c+16a^2c^2)d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^3/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 1/(4*(b^2*c^2 - 4*a*c^3)*d^3*x^2 + 4*(b^3*c - 4*a*b*c^2)*d^3*x + (b^4 - 4*a
*b^2*c)*d^3) + log(c*x^2 + b*x + a)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*d^3) -
2*log(2*c*x + b)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*d^3)
```

Fricas [B] time = 1.99394, size = 338, normalized size = 4.83

$$\frac{b^2-4\,ac+\left(4\,c^2x^2+4\,bcx+b^2\right)\log\left(cx^2+bx+a\right)-2\left(4\,c^2x^2+4\,bcx+b^2\right)\log\left(2\,cx+b\right)}{4\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)d^3x^2+4\left(b^5c-8\,ab^3c^2+16\,a^2bc^3\right)d^3x+\left(b^6-8\,ab^4c+16\,a^2b^2c^2\right)d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^3/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] (b^2 - 4*a*c + (4*c^2*x^2 + 4*b*c*x + b^2)*log(c*x^2 + b*x + a) - 2*(4*c^2*
x^2 + 4*b*c*x + b^2)*log(2*c*x + b))/(4*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4
)*d^3*x^2 + 4*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3*x + (b^6 - 8*a*b^4*c
 + 16*a^2*b^2*c^2)*d^3)
```

Sympy [A] time = 2.34491, size = 119, normalized size = 1.7

$$-\frac{1}{4ab^2cd^3-b^4d^3+x^2\left(16ac^3d^3-4b^2c^2d^3\right)+x\left(16abc^2d^3-4b^3cd^3\right)}-\frac{2\log\left(\frac{b}{2c}+x\right)}{d^3\left(4ac-b^2\right)^2}+\frac{\log\left(\frac{a}{c}+\frac{bx}{c}+x^2\right)}{d^3\left(4ac-b^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**3/(c*x**2+b*x+a),x)
```

```
[Out] -1/(4*a*b**2*c*d**3 - b**4*d**3 + x**2*(16*a*c**3*d**3 - 4*b**2*c**2*d**3)
+ x*(16*a*b*c**2*d**3 - 4*b**3*c*d**3)) - 2*log(b/(2*c) + x)/(d**3*(4*a*c -
 b**2)**2) + log(a/c + b*x/c + x**2)/(d**3*(4*a*c - b**2)**2)
```

Giac [A] time = 1.16075, size = 150, normalized size = 2.14

$$-\frac{2\,c\log\left(|2\,cx+b|\right)}{b^4cd^3-8\,ab^2c^2d^3+16\,a^2c^3d^3}+\frac{\log\left(cx^2+bx+a\right)}{b^4d^3-8\,ab^2cd^3+16\,a^2c^2d^3}+\frac{1}{\left(b^2-4\,ac\right)(2\,cx+b)^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^3/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -2*c*log(abs(2*c*x + b))/(b^4*c*d^3 - 8*a*b^2*c^2*d^3 + 16*a^2*c^3*d^3) + l
og(c*x^2 + b*x + a)/(b^4*d^3 - 8*a*b^2*c*d^3 + 16*a^2*c^2*d^3) + 1/((b^2 -
4*a*c)*(2*c*x + b)^2*d^3)
```

3.1165 $\int \frac{1}{(bd+2cdx)^4(a+bx+cx^2)}\,dx$

Optimal. Leaf size=86

$$\frac{2}{d^4(b^2-4ac)^2(b+2cx)}+\frac{2}{3d^4(b^2-4ac)(b+2cx)^3}-\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d^4(b^2-4ac)^{5/2}}$$

```
[Out] 2/(3*(b^2 - 4*a*c)*d^4*(b + 2*c*x)^3) + 2/((b^2 - 4*a*c)^2*d^4*(b + 2*c*x))
 - (2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2)*d^4)
```

Rubi [A] time = 0.0670583, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {693, 618, 206}

$$\frac{2}{d^4(b^2-4ac)^2(b+2cx)}+\frac{2}{3d^4(b^2-4ac)(b+2cx)^3}-\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d^4(b^2-4ac)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^4*(a + b*x + c*x^2)),x]
```

```
[Out] 2/(3*(b^2 - 4*a*c)*d^4*(b + 2*c*x)^3) + 2/((b^2 - 4*a*c)^2*d^4*(b + 2*c*x))
 - (2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2)*d^4)
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)^4\left(a+bx+cx^2\right)}\,dx = \frac{2}{3\left(b^2-4ac\right)d^4(b+2cx)^3}+\frac{\int \frac{1}{(bd+2cdx)^2\left(a+bx+cx^2\right)}\,dx}{\left(b^2-4ac\right)d^2}$$
$$= \frac{2}{3\left(b^2-4ac\right)d^4(b+2cx)^3}+\frac{2}{\left(b^2-4ac\right)^2d^4(b+2cx)}+\frac{\int \frac{1}{a+bx+cx^2}\,dx}{\left(b^2-4ac\right)^2d^4}$$
$$= \frac{2}{3\left(b^2-4ac\right)d^4(b+2cx)^3}+\frac{2}{\left(b^2-4ac\right)^2d^4(b+2cx)}-\frac{2\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+\right.}{\left(b^2-4ac\right)^2d^4}$$
$$= \frac{2}{3\left(b^2-4ac\right)d^4(b+2cx)^3}+\frac{2}{\left(b^2-4ac\right)^2d^4(b+2cx)}-\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}d^4}$$

Mathematica [A] time = 0.0517138, size = 83, normalized size = 0.97

$$\frac{2\left(\frac{b^2-4ac}{(b+2cx)^3}+\frac{3\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{3}{b+2cx}\right)}{3d^4\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^4*(a + b*x + c*x^2)),x]
```

```
[Out] (2*((b^2 - 4*a*c)/(b + 2*c*x)^3 + 3/(b + 2*c*x) + (3*ArcTan[(b + 2*c*x)/Sqr
t[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]))/(3*(b^2 - 4*a*c)^2*d^4)
```

Maple [A] time = 0.153, size = 89, normalized size = 1.

$$2\,\frac{1}{d^4\left(4\,ac-b^2\right)^{5/2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)+2\,\frac{1}{d^4\left(4\,ac-b^2\right)^2(2\,cx+b)}-\frac{2}{3\,d^4\left(4\,ac-b^2\right)(2\,cx+b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^4/(c*x^2+b*x+a),x)
```

```
[Out] 2/d^4/(4*a*c-b^2)^(5/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))+2/d^4/(4*a*c-b^
2)^2/(2*c*x+b)-2/3/d^4/(4*a*c-b^2)/(2*c*x+b)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^4/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.02832, size = 1350, normalized size = 15.7

$$\left[\frac{8\,b^4-40\,ab^2c+32\,a^2c^2+24\,\left(b^2c^2-4\,ac^3\right)x^2+3\,\left(8\,c^3x^3+12\,bc^2x^2+6\,b^2cx+b^3\right)\sqrt{b^2-4\,ac}\log}{3\,\left(8\,\left(b^6c^3-12\,ab^4c^4+48\,a^2b^2c^5-64\,a^3c^6\right)d^4x^3+12\,\left(b^7c^2-12\,ab^5c^3+48\,a^2b^3c^4-64\,a^3bc^5\right)d^4x^2+6\,\left(b^8c-12\,ab^6c^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^4/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/3*(8*b^4 - 40*a*b^2*c + 32*a^2*c^2 + 24*(b^2*c^2 - 4*a*c^3)*x^2 + 3*(8*c
^3*x^3 + 12*b*c^2*x^2 + 6*b^2*c*x + b^3)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 +
 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a))
+ 24*(b^3*c - 4*a*b*c^2)*x)/(8*(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 6
4*a^3*c^6)*d^4*x^3 + 12*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b
*c^5)*d^4*x^2 + 6*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*
d^4*x + (b^9 - 12*a*b^7*c + 48*a^2*b^5*c^2 - 64*a^3*b^3*c^3)*d^4), 2/3*(4*b
^4 - 20*a*b^2*c + 16*a^2*c^2 + 12*(b^2*c^2 - 4*a*c^3)*x^2 - 3*(8*c^3*x^3 +
12*b*c^2*x^2 + 6*b^2*c*x + b^3)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*
c)*(2*c*x + b)/(b^2 - 4*a*c)) + 12*(b^3*c - 4*a*b*c^2)*x)/(8*(b^6*c^3 - 12*
a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*d^4*x^3 + 12*(b^7*c^2 - 12*a*b^5*c
^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^4*x^2 + 6*(b^8*c - 12*a*b^6*c^2 + 48*
a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^4*x + (b^9 - 12*a*b^7*c + 48*a^2*b^5*c^2 -
64*a^3*b^3*c^3)*d^4)]
```

Sympy [B] time = 2.57254, size = 442, normalized size = 5.14

$$\frac{-8ac+8b^2+24bcx+24c^2x^2}{48a^2b^3c^2d^4-24ab^5cd^4+3b^7d^4+x^3\left(384a^2c^5d^4-192ab^2c^4d^4+24b^4c^3d^4\right)+x^2\left(576a^2bc^4d^4-288ab^3c^3d^4+36b^5c^2d^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**4/(c*x**2+b*x+a),x)
```

```
[Out] (-8*a*c + 8*b**2 + 24*b*c*x + 24*c**2*x**2)/(48*a**2*b**3*c**2*d**4 - 24*a*
b**5*c*d**4 + 3*b**7*d**4 + x**3*(384*a**2*c**5*d**4 - 192*a*b**2*c**4*d**4
 + 24*b**4*c**3*d**4) + x**2*(576*a**2*b*c**4*d**4 - 288*a*b**3*c**3*d**4 +
 36*b**5*c**2*d**4) + x*(288*a**2*b**2*c**3*d**4 - 144*a*b**4*c**2*d**4 + 1
8*b**6*c*d**4)) - sqrt(-1/(4*a*c - b**2)**5)*log(x + (-64*a**3*c**3*sqrt(-1
/(4*a*c - b**2)**5) + 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5) - 12*a*b
**4*c*sqrt(-1/(4*a*c - b**2)**5) + b**6*sqrt(-1/(4*a*c - b**2)**5) + b)/(2*
c))/d**4 + sqrt(-1/(4*a*c - b**2)**5)*log(x + (64*a**3*c**3*sqrt(-1/(4*a*c
- b**2)**5) - 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5) + 12*a*b**4*c*sq
rt(-1/(4*a*c - b**2)**5) - b**6*sqrt(-1/(4*a*c - b**2)**5) + b)/(2*c))/d**4
```

Giac [A] time = 1.21531, size = 173, normalized size = 2.01

$$\frac{2\,\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4d^4-8\,ab^2cd^4+16\,a^2c^2d^4\right)\sqrt{-b^2+4\,ac}}+\frac{8\,\left(3\,c^2x^2+3\,bcx+b^2-ac\right)}{3\,\left(b^4d^4-8\,ab^2cd^4+16\,a^2c^2d^4\right)(2\,cx+b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^4/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4*d^4 - 8*a*b^2*c*d^4 + 16*a^2
*c^2*d^4)*sqrt(-b^2 + 4*a*c)) + 8/3*(3*c^2*x^2 + 3*b*c*x + b^2 - a*c)/((b^4
*d^4 - 8*a*b^2*c*d^4 + 16*a^2*c^2*d^4)*(2*c*x + b)^3)
```

3.1166 $\int \frac{(bd+2cdx)^8}{\left(a+bx+cx^2\right)^2} dx$

Optimal. Leaf size=126

$$\frac{28}{3}cd^8\left(b^2-4ac\right)(b+2cx)^3+28cd^8\left(b^2-4ac\right)^2(b+2cx)-28cd^8\left(b^2-4ac\right)^{5/2}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)-\frac{d^8(b+2cx)^7}{a+bx+cx^2}+$$

```
[Out] 28*c*(b^2 - 4*a*c)^2*d^8*(b + 2*c*x) + (28*c*(b^2 - 4*a*c)*d^8*(b + 2*c*x)^
3)/3 + (28*c*d^8*(b + 2*c*x)^5)/5 - (d^8*(b + 2*c*x)^7)/(a + b*x + c*x^2) -
 28*c*(b^2 - 4*a*c)^(5/2)*d^8*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]
```

Rubi [A] time = 0.104732, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {686, 692, 618, 206}

$$\frac{28}{3}cd^8\left(b^2-4ac\right)(b+2cx)^3+28cd^8\left(b^2-4ac\right)^2(b+2cx)-28cd^8\left(b^2-4ac\right)^{5/2}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)-\frac{d^8(b+2cx)^7}{a+bx+cx^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^8/(a + b*x + c*x^2)^2,x]
```

```
[Out] 28*c*(b^2 - 4*a*c)^2*d^8*(b + 2*c*x) + (28*c*(b^2 - 4*a*c)*d^8*(b + 2*c*x)^
3)/3 + (28*c*d^8*(b + 2*c*x)^5)/5 - (d^8*(b + 2*c*x)^7)/(a + b*x + c*x^2) -
 28*c*(b^2 - 4*a*c)^(5/2)*d^8*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\int \frac{(bd+2cdx)^8}{\left(a+bx+cx^2\right)^2}\,dx = -\frac{d^8(b+2cx)^7}{a+bx+cx^2}+\left(14cd^2\right)\int \frac{(bd+2cdx)^6}{a+bx+cx^2}\,dx$$
$$= \frac{28}{5}cd^8(b+2cx)^5-\frac{d^8(b+2cx)^7}{a+bx+cx^2}+\left(14c\left(b^2-4ac\right)d^4\right)\int \frac{(bd+2cdx)^4}{a+bx+cx^2}\,dx$$
$$= \frac{28}{3}c\left(b^2-4ac\right)d^8(b+2cx)^3+\frac{28}{5}cd^8(b+2cx)^5-\frac{d^8(b+2cx)^7}{a+bx+cx^2}+\left(14c\left(b^2-4ac\right)^2d^6\right)\int \frac{(bd+2c}{a+bx}$$
$$= 28c\left(b^2-4ac\right)^2d^8(b+2cx)+\frac{28}{3}c\left(b^2-4ac\right)d^8(b+2cx)^3+\frac{28}{5}cd^8(b+2cx)^5-\frac{d^8(b+2cx)^7}{a+bx+cx^2}+($$
$$= 28c\left(b^2-4ac\right)^2d^8(b+2cx)+\frac{28}{3}c\left(b^2-4ac\right)d^8(b+2cx)^3+\frac{28}{5}cd^8(b+2cx)^5-\frac{d^8(b+2cx)^7}{a+bx+cx^2}-($$
$$= 28c\left(b^2-4ac\right)^2d^8(b+2cx)+\frac{28}{3}c\left(b^2-4ac\right)d^8(b+2cx)^3+\frac{28}{5}cd^8(b+2cx)^5-\frac{d^8(b+2cx)^7}{a+bx+cx^2}-2$$

Mathematica [A] time = 0.0819658, size = 155, normalized size = 1.23

$$d^8\left(32c^2x\left(24a^2c^2-16ab^2c+3b^4\right)-\frac{512}{3}c^4x^3\left(ac-b^2\right)+128bc^3x^2\left(b^2-2ac\right)-\frac{\left(b^2-4ac\right)^3(b+2cx)}{a+x(b+cx)}-28c\left(4ac-b^2\right)^{5/2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^8/(a + b*x + c*x^2)^2,x]
```

```
[Out] d^8*(32*c^2*(3*b^4 - 16*a*b^2*c + 24*a^2*c^2)*x + 128*b*c^3*(b^2 - 2*a*c)*x
^2 - (512*c^4*(-b^2 + a*c)*x^3)/3 + 128*b*c^5*x^4 + (256*c^6*x^5)/5 - ((b^2
 - 4*a*c)^3*(b + 2*c*x))/(a + x*(b + c*x)) - 28*c*(-b^2 + 4*a*c)^(5/2)*ArcT
an[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])
```

Maple [B] time = 0.155, size = 479, normalized size = 3.8

$$\frac{256\,d^8c^6x^5}{5}+128\,d^8bc^5x^4-\frac{512\,d^8x^3ac^5}{3}+\frac{512\,d^8x^3b^2c^4}{3}-256\,d^8x^2abc^4+128\,d^8x^2b^3c^3+768\,d^8a^2c^4x-512\,d^8b^2ac^3x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^8/(c*x^2+b*x+a)^2,x)
```

```
[Out] 256/5*d^8*c^6*x^5+128*d^8*b*c^5*x^4-512/3*d^8*x^3*a*c^5+512/3*d^8*x^3*b^2*c
^4-256*d^8*x^2*a*b*c^4+128*d^8*x^2*b^3*c^3+768*d^8*a^2*c^4*x-512*d^8*b^2*a*
c^3*x+96*d^8*b^4*c^2*x+128*d^8/(c*x^2+b*x+a)*a^3*c^4*x-96*d^8/(c*x^2+b*x+a)
*b^2*a^2*c^3*x+24*d^8/(c*x^2+b*x+a)*a*b^4*c^2*x-2*d^8/(c*x^2+b*x+a)*b^6*c*x
+64*d^8/(c*x^2+b*x+a)*a^3*b*c^3-48*d^8/(c*x^2+b*x+a)*a^2*b^3*c^2+12*d^8/(c*
x^2+b*x+a)*a*b^5*c-d^8/(c*x^2+b*x+a)*b^7-1792*d^8*c^4/(4*a*c-b^2)^(1/2)*arc
tan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^3+1344*d^8*c^3/(4*a*c-b^2)^(1/2)*arctan(
(2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b^2-336*d^8*c^2/(4*a*c-b^2)^(1/2)*arctan((
2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^4+28*d^8*c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))*b^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^8/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.08082, size = 1634, normalized size = 12.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^8/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/15*(768*c^7*d^8*x^7 + 2688*b*c^6*d^8*x^6 + 896*(5*b^2*c^5 - 2*a*c^6)*d^8
*x^5 + 4480*(b^3*c^4 - a*b*c^5)*d^8*x^4 + 1120*(3*b^4*c^3 - 8*a*b^2*c^4 + 8
*a^2*c^5)*d^8*x^3 + 480*(3*b^5*c^2 - 12*a*b^3*c^3 + 16*a^2*b*c^4)*d^8*x^2 -
 30*(b^6*c - 60*a*b^4*c^2 + 304*a^2*b^2*c^3 - 448*a^3*c^4)*d^8*x - 15*(b^7
- 12*a*b^5*c + 48*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^8 + 210*((b^4*c^2 - 8*a*b^2
*c^3 + 16*a^2*c^4)*d^8*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^8*x + (
a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*d^8)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2
 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)
))/(c*x^2 + b*x + a), 1/15*(768*c^7*d^8*x^7 + 2688*b*c^6*d^8*x^6 + 896*(5*b
^2*c^5 - 2*a*c^6)*d^8*x^5 + 4480*(b^3*c^4 - a*b*c^5)*d^8*x^4 + 1120*(3*b^4*
c^3 - 8*a*b^2*c^4 + 8*a^2*c^5)*d^8*x^3 + 480*(3*b^5*c^2 - 12*a*b^3*c^3 + 16
*a^2*b*c^4)*d^8*x^2 - 30*(b^6*c - 60*a*b^4*c^2 + 304*a^2*b^2*c^3 - 448*a^3*
c^4)*d^8*x - 15*(b^7 - 12*a*b^5*c + 48*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^8 - 42
0*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8*x^2 + (b^5*c - 8*a*b^3*c^2 + 16
*a^2*b*c^3)*d^8*x + (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*d^8)*sqrt(-b^2 +
 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)))/(c*x^2 + b*x
 + a)]
```

Sympy [B] time = 3.36178, size = 476, normalized size = 3.78

$$128bc^5d^8x^4 + \frac{256c^6d^8x^5}{5} + 14cd^8\sqrt{-\left(4ac-b^2\right)^5}\log\left(x + \frac{224a^2bc^3d^8 - 112ab^3c^2d^8 + 14b^5cd^8 - 14cd^8\sqrt{-\left(4ac-b^2\right)^5}}{448a^2c^4d^8 - 224ab^2c^3d^8 + 28b^4c^2d^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**8/(c*x**2+b*x+a)**2,x)
```

```
[Out] 128*b*c**5*d**8*x**4 + 256*c**6*d**8*x**5/5 + 14*c*d**8*sqrt(-(4*a*c - b**2
)**5)*log(x + (224*a**2*b*c**3*d**8 - 112*a*b**3*c**2*d**8 + 14*b**5*c*d**8
 - 14*c*d**8*sqrt(-(4*a*c - b**2)**5))/(448*a**2*c**4*d**8 - 224*a*b**2*c**
3*d**8 + 28*b**4*c**2*d**8)) - 14*c*d**8*sqrt(-(4*a*c - b**2)**5)*log(x + (
224*a**2*b*c**3*d**8 - 112*a*b**3*c**2*d**8 + 14*b**5*c*d**8 + 14*c*d**8*sq
rt(-(4*a*c - b**2)**5))/(448*a**2*c**4*d**8 - 224*a*b**2*c**3*d**8 + 28*b**
```

```
4*c**2*d**8)) + x**3*(-512*a*c**5*d**8/3 + 512*b**2*c**4*d**8/3) + x**2*(-2
56*a*b*c**4*d**8 + 128*b**3*c**3*d**8) + x*(768*a**2*c**4*d**8 - 512*a*b**2
*c**3*d**8 + 96*b**4*c**2*d**8) + (64*a**3*b*c**3*d**8 - 48*a**2*b**3*c**2*
d**8 + 12*a*b**5*c*d**8 - b**7*d**8 + x*(128*a**3*c**4*d**8 - 96*a**2*b**2*
c**3*d**8 + 24*a*b**4*c**2*d**8 - 2*b**6*c*d**8))/(a + b*x + c*x**2)
```

Giac [B] time = 1.1537, size = 416, normalized size = 3.3

$$\frac{28\left(b^6cd^8 - 12\,ab^4c^2d^8 + 48\,a^2b^2c^3d^8 - 64\,a^3c^4d^8\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}} - \frac{2\,b^6cd^8x - 24\,ab^4c^2d^8x + 96\,a^2b^2c^3d^8x - 128\,a^3c^4}{cx^2 +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^8/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 28*(b^6*c*d^8 - 12*a*b^4*c^2*d^8 + 48*a^2*b^2*c^3*d^8 - 64*a^3*c^4*d^8)*arc
tan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c) - (2*b^6*c*d^8*x - 2
4*a*b^4*c^2*d^8*x + 96*a^2*b^2*c^3*d^8*x - 128*a^3*c^4*d^8*x + b^7*d^8 - 12
*a*b^5*c*d^8 + 48*a^2*b^3*c^2*d^8 - 64*a^3*b*c^3*d^8)/(c*x^2 + b*x + a) + 3
2/15*(24*c^16*d^8*x^5 + 60*b*c^15*d^8*x^4 + 80*b^2*c^14*d^8*x^3 - 80*a*c^15
*d^8*x^3 + 60*b^3*c^13*d^8*x^2 - 120*a*b*c^14*d^8*x^2 + 45*b^4*c^12*d^8*x -
 240*a*b^2*c^13*d^8*x + 360*a^2*c^14*d^8*x)/c^10
```

3.1167 $\int \frac{(bd+2cdx)^7}{\left(a+bx+cx^2\right)^2} dx$

Optimal. Leaf size=89

$$12cd^7\left(b^2-4ac\right)^2\log\left(a+bx+cx^2\right)+12cd^7\left(b^2-4ac\right)(b+2cx)^2-\frac{d^7(b+2cx)^6}{a+bx+cx^2}+6cd^7(b+2cx)^4$$

```
[Out] 12*c*(b^2 - 4*a*c)*d^7*(b + 2*c*x)^2 + 6*c*d^7*(b + 2*c*x)^4 - (d^7*(b + 2*
c*x)^6)/(a + b*x + c*x^2) + 12*c*(b^2 - 4*a*c)^2*d^7*Log[a + b*x + c*x^2]
```

Rubi [A] time = 0.06212, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {686, 692, 628}

$$12cd^7\left(b^2-4ac\right)^2\log\left(a+bx+cx^2\right)+12cd^7\left(b^2-4ac\right)(b+2cx)^2-\frac{d^7(b+2cx)^6}{a+bx+cx^2}+6cd^7(b+2cx)^4$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^7/(a + b*x + c*x^2)^2,x]
```

```
[Out] 12*c*(b^2 - 4*a*c)*d^7*(b + 2*c*x)^2 + 6*c*d^7*(b + 2*c*x)^4 - (d^7*(b + 2*
c*x)^6)/(a + b*x + c*x^2) + 12*c*(b^2 - 4*a*c)^2*d^7*Log[a + b*x + c*x^2]
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(bd+2cdx)^7}{\left(a+bx+cx^2\right)^2}\,dx = -\frac{d^7(b+2cx)^6}{a+bx+cx^2} + \left(12cd^2\right)\int \frac{(bd+2cdx)^5}{a+bx+cx^2}\,dx$$
$$= 6cd^7(b+2cx)^4 - \frac{d^7(b+2cx)^6}{a+bx+cx^2} + \left(12c\left(b^2-4ac\right)d^4\right)\int \frac{(bd+2cdx)^3}{a+bx+cx^2}\,dx$$
$$= 12c\left(b^2-4ac\right)d^7(b+2cx)^2 + 6cd^7(b+2cx)^4 - \frac{d^7(b+2cx)^6}{a+bx+cx^2} + \left(12c\left(b^2-4ac\right)^2d^6\right)\int \frac{bd+2cd}{a+bx+c}$$
$$= 12c\left(b^2-4ac\right)d^7(b+2cx)^2 + 6cd^7(b+2cx)^4 - \frac{d^7(b+2cx)^6}{a+bx+cx^2} + 12c\left(b^2-4ac\right)^2d^7\log\left(a+bx+c\right.$$

Mathematica [A] time = 0.0880245, size = 103, normalized size = 1.16

$$d^7\left(-16c^3x^2\left(8ac-5b^2\right)+16bc^2x\left(3b^2-8ac\right)-\frac{\left(b^2-4ac\right)^3}{a+x(b+cx)}+12c\left(b^2-4ac\right)^2\log(a+x(b+cx))+64bc^4x^3+32c^5x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^7/(a + b*x + c*x^2)^2,x]
```

```
[Out] d^7*(16*b*c^2*(3*b^2 - 8*a*c)*x - 16*c^3*(-5*b^2 + 8*a*c)*x^2 + 64*b*c^4*x^
3 + 32*c^5*x^4 - (b^2 - 4*a*c)^3/(a + x*(b + c*x)) + 12*c*(b^2 - 4*a*c)^2*L
og[a + x*(b + c*x)])
```

Maple [B] time = 0.048, size = 230, normalized size = 2.6

$$32\,d^7c^5x^4+64\,d^7bc^4x^3-128\,d^7x^2ac^4+80\,d^7x^2b^2c^3-128\,d^7abc^3x+48\,d^7b^3c^2x+64\,\frac{d^7a^3c^3}{cx^2+bx+a}-48\,\frac{d^7a^2b^2c^2}{cx^2+bx+a}+1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^7/(c*x^2+b*x+a)^2,x)
```

```
[Out] 32*d^7*c^5*x^4+64*d^7*b*c^4*x^3-128*d^7*x^2*a*c^4+80*d^7*x^2*b^2*c^3-128*d^
7*a*b*c^3*x+48*d^7*b^3*c^2*x+64*d^7/(c*x^2+b*x+a)*a^3*c^3-48*d^7/(c*x^2+b*x
+a)*a^2*b^2*c^2+12*d^7/(c*x^2+b*x+a)*a*b^4*c-d^7/(c*x^2+b*x+a)*b^6+192*d^7*
ln(c*x^2+b*x+a)*a^2*c^3-96*d^7*ln(c*x^2+b*x+a)*a*b^2*c^2+12*d^7*ln(c*x^2+b*
x+a)*b^4*c
```

Maxima [A] time = 1.11781, size = 209, normalized size = 2.35

$$32\,c^5d^7x^4+64\,bc^4d^7x^3+16\left(5\,b^2c^3-8\,ac^4\right)d^7x^2+16\left(3\,b^3c^2-8\,abc^3\right)d^7x+12\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)d^7\log\left(cx^2+bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^7/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 32*c^5*d^7*x^4 + 64*b*c^4*d^7*x^3 + 16*(5*b^2*c^3 - 8*a*c^4)*d^7*x^2 + 16*(
3*b^3*c^2 - 8*a*b*c^3)*d^7*x + 12*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^7*lo
g(c*x^2 + b*x + a) - (b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*d^7/(
```

```
c*x^2 + b*x + a)
```

Fricas [B] time = 1.9632, size = 587, normalized size = 6.6

$$\frac{32\,c^6d^7x^6 + 96\,bc^5d^7x^5 + 48\,\left(3\,b^2c^4 - 2\,ac^5\right)d^7x^4 + 64\,\left(2\,b^3c^3 - 3\,abc^4\right)d^7x^3 + 16\,\left(3\,b^4c^2 - 3\,ab^2c^3 - 8\,a^2c^4\right)d^7x^2 + 16\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^7/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (32*c^6*d^7*x^6 + 96*b*c^5*d^7*x^5 + 48*(3*b^2*c^4 - 2*a*c^5)*d^7*x^4 + 64*
(2*b^3*c^3 - 3*a*b*c^4)*d^7*x^3 + 16*(3*b^4*c^2 - 3*a*b^2*c^3 - 8*a^2*c^4)*
d^7*x^2 + 16*(3*a*b^3*c^2 - 8*a^2*b*c^3)*d^7*x - (b^6 - 12*a*b^4*c + 48*a^2
*b^2*c^2 - 64*a^3*c^3)*d^7 + 12*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7*x
^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^7*x + (a*b^4*c - 8*a^2*b^2*c^2
+ 16*a^3*c^3)*d^7)*log(c*x^2 + b*x + a))/(c*x^2 + b*x + a)
```

Sympy [A] time = 4.28406, size = 160, normalized size = 1.8

$$64bc^4d^7x^3 + 32c^5d^7x^4 + 12cd^7\left(4ac - b^2\right)^2\log\left(a + bx + cx^2\right) + x^2\left(-128ac^4d^7 + 80b^2c^3d^7\right) + x\left(-128abc^3d^7 + 48b^3c^2d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**7/(c*x**2+b*x+a)**2,x)
```

```
[Out] 64*b*c**4*d**7*x**3 + 32*c**5*d**7*x**4 + 12*c*d**7*(4*a*c - b**2)**2*log(a
 + b*x + c*x**2) + x**2*(-128*a*c**4*d**7 + 80*b**2*c**3*d**7) + x*(-128*a*
b*c**3*d**7 + 48*b**3*c**2*d**7) + (64*a**3*c**3*d**7 - 48*a**2*b**2*c**2*d
**7 + 12*a*b**4*c*d**7 - b**6*d**7)/(a + b*x + c*x**2)
```

Giac [B] time = 1.15258, size = 244, normalized size = 2.74

$$12\left(b^4cd^7 - 8\,ab^2c^2d^7 + 16\,a^2c^3d^7\right)\log\left(cx^2 + bx + a\right) - \frac{b^6d^7 - 12\,ab^4cd^7 + 48\,a^2b^2c^2d^7 - 64\,a^3c^3d^7}{cx^2 + bx + a} + \frac{16\left(2\,c^{13}d^7x^4 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^7/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 12*(b^4*c*d^7 - 8*a*b^2*c^2*d^7 + 16*a^2*c^3*d^7)*log(c*x^2 + b*x + a) - (b
^6*d^7 - 12*a*b^4*c*d^7 + 48*a^2*b^2*c^2*d^7 - 64*a^3*c^3*d^7)/(c*x^2 + b*x
 + a) + 16*(2*c^13*d^7*x^4 + 4*b*c^12*d^7*x^3 + 5*b^2*c^11*d^7*x^2 - 8*a*c^
12*d^7*x^2 + 3*b^3*c^10*d^7*x - 8*a*b*c^11*d^7*x)/c^8
```

3.1168 $\int \frac{(bd+2cdx)^6}{(a+bx+cx^2)^2}\,dx$

Optimal. Leaf size=100

$$20cd^6\left(b^2-4ac\right)(b+2cx)-20cd^6\left(b^2-4ac\right)^{3/2}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)-\frac{d^6(b+2cx)^5}{a+bx+cx^2}+\frac{20}{3}cd^6(b+2cx)^3$$

```
[Out] 20*c*(b^2 - 4*a*c)*d^6*(b + 2*c*x) + (20*c*d^6*(b + 2*c*x)^3)/3 - (d^6*(b +
 2*c*x)^5)/(a + b*x + c*x^2) - 20*c*(b^2 - 4*a*c)^(3/2)*d^6*ArcTanh[(b + 2*
c*x)/Sqrt[b^2 - 4*a*c]]
```

Rubi [A] time = 0.0742676, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {686, 692, 618, 206}

$$20cd^6\left(b^2-4ac\right)(b+2cx)-20cd^6\left(b^2-4ac\right)^{3/2}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)-\frac{d^6(b+2cx)^5}{a+bx+cx^2}+\frac{20}{3}cd^6(b+2cx)^3$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^6/(a + b*x + c*x^2)^2,x]
```

```
[Out] 20*c*(b^2 - 4*a*c)*d^6*(b + 2*c*x) + (20*c*d^6*(b + 2*c*x)^3)/3 - (d^6*(b +
 2*c*x)^5)/(a + b*x + c*x^2) - 20*c*(b^2 - 4*a*c)^(3/2)*d^6*ArcTanh[(b + 2*
c*x)/Sqrt[b^2 - 4*a*c]]
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(bd+2cdx)^6}{\left(a+bx+cx^2\right)^2}\,dx = -\frac{d^6(b+2cx)^5}{a+bx+cx^2}+\left(10cd^2\right)\int \frac{(bd+2cdx)^4}{a+bx+cx^2}\,dx$$
$$= \frac{20}{3}cd^6(b+2cx)^3-\frac{d^6(b+2cx)^5}{a+bx+cx^2}+\left(10c\left(b^2-4ac\right)d^4\right)\int \frac{(bd+2cdx)^2}{a+bx+cx^2}\,dx$$
$$= 20c\left(b^2-4ac\right)d^6(b+2cx)+\frac{20}{3}cd^6(b+2cx)^3-\frac{d^6(b+2cx)^5}{a+bx+cx^2}+\left(10c\left(b^2-4ac\right)^2d^6\right)\int \frac{1}{a+bx}$$
$$= 20c\left(b^2-4ac\right)d^6(b+2cx)+\frac{20}{3}cd^6(b+2cx)^3-\frac{d^6(b+2cx)^5}{a+bx+cx^2}-\left(20c\left(b^2-4ac\right)^2d^6\right)\text{Subst}\left(\int\right.$$
$$= 20c\left(b^2-4ac\right)d^6(b+2cx)+\frac{20}{3}cd^6(b+2cx)^3-\frac{d^6(b+2cx)^5}{a+bx+cx^2}-20c\left(b^2-4ac\right)^{3/2}d^6\tanh^{-1}\left(\right.$$

Mathematica [A] time = 0.0654078, size = 108, normalized size = 1.08

$$d^6\left(-16c^2x\left(8ac-3b^2\right)-\frac{\left(b^2-4ac\right)^2(b+2cx)}{a+x(b+cx)}+20c\left(4ac-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)+32bc^3x^2+\frac{64c^4x^3}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^6/(a + b*x + c*x^2)^2,x]
```

```
[Out] d^6*(-16*c^2*(-3*b^2 + 8*a*c)*x + 32*b*c^3*x^2 + (64*c^4*x^3)/3 - ((b^2 - 4
*a*c)^2*(b + 2*c*x))/(a + x*(b + c*x)) + 20*c*(-b^2 + 4*a*c)^(3/2)*ArcTan[(
b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])
```

Maple [B] time = 0.157, size = 312, normalized size = 3.1

$$\frac{64\,d^6c^4x^3}{3}+32\,d^6bc^3x^2-128\,d^6ac^3x+48\,d^6b^2c^2x-32\,\frac{d^6a^2c^3x}{cx^2+bx+a}+16\,\frac{d^6ab^2c^2x}{cx^2+bx+a}-2\,\frac{d^6cb^4x}{cx^2+bx+a}-16\,\frac{d^6a^2b}{cx^2+bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^6/(c*x^2+b*x+a)^2,x)
```

```
[Out] 64/3*d^6*c^4*x^3+32*d^6*b*c^3*x^2-128*d^6*a*c^3*x+48*d^6*b^2*c^2*x-32*d^6/(
c*x^2+b*x+a)*a^2*c^3*x+16*d^6/(c*x^2+b*x+a)*a*b^2*c^2*x-2*d^6/(c*x^2+b*x+a)
*c*b^4*x-16*d^6/(c*x^2+b*x+a)*a^2*b*c^2+8*d^6/(c*x^2+b*x+a)*a*b^3*c-d^6/(c*
x^2+b*x+a)*b^5+320*d^6*c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2))*a^2-160*d^6*c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
*a*b^2+20*d^6*c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^6/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.02972, size = 1085, normalized size = 10.85

$$\left[\frac{64\,c^5d^6x^5 + 160\,bc^4d^6x^4 + 80\left(3\,b^2c^3 - 4\,ac^4\right)d^6x^3 + 144\left(b^3c^2 - 2\,abc^3\right)d^6x^2 - 6\left(b^4c - 32\,ab^2c^2 + 80\,a^2c^3\right)d^6x - 3\left(b^5 - \cdots\right.}{3\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^6/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/3*(64*c^5*d^6*x^5 + 160*b*c^4*d^6*x^4 + 80*(3*b^2*c^3 - 4*a*c^4)*d^6*x^3
 + 144*(b^3*c^2 - 2*a*b*c^3)*d^6*x^2 - 6*(b^4*c - 32*a*b^2*c^2 + 80*a^2*c^3
)*d^6*x - 3*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^6 - 30*((b^2*c^2 - 4*a*c^3)*
d^6*x^2 + (b^3*c - 4*a*b*c^2)*d^6*x + (a*b^2*c - 4*a^2*c^2)*d^6)*sqrt(b^2 -
 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x +
 b))/(c*x^2 + b*x + a)))/(c*x^2 + b*x + a), 1/3*(64*c^5*d^6*x^5 + 160*b*c^4
*d^6*x^4 + 80*(3*b^2*c^3 - 4*a*c^4)*d^6*x^3 + 144*(b^3*c^2 - 2*a*b*c^3)*d^6
*x^2 - 6*(b^4*c - 32*a*b^2*c^2 + 80*a^2*c^3)*d^6*x - 3*(b^5 - 8*a*b^3*c + 1
6*a^2*b*c^2)*d^6 - 60*((b^2*c^2 - 4*a*c^3)*d^6*x^2 + (b^3*c - 4*a*b*c^2)*d^
6*x + (a*b^2*c - 4*a^2*c^2)*d^6)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a
*c)*(2*c*x + b)/(b^2 - 4*a*c)))/(c*x^2 + b*x + a)]
```

Sympy [B] time = 2.20969, size = 313, normalized size = 3.13

$$32bc^3d^6x^2 + \frac{64c^4d^6x^3}{3} - 10cd^6\sqrt{-\left(4ac-b^2\right)^3}\log\left(x + \frac{40abc^2d^6 - 10b^3cd^6 - 10cd^6\sqrt{-\left(4ac-b^2\right)^3}}{80ac^3d^6 - 20b^2c^2d^6}\right) + 10cd^6\sqrt{-\left(4ac - \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**6/(c*x**2+b*x+a)**2,x)
```

```
[Out] 32*b*c**3*d**6*x**2 + 64*c**4*d**6*x**3/3 - 10*c*d**6*sqrt(-(4*a*c - b**2)*
*3)*log(x + (40*a*b*c**2*d**6 - 10*b**3*c*d**6 - 10*c*d**6*sqrt(-(4*a*c - b
**2)**3))/(80*a*c**3*d**6 - 20*b**2*c**2*d**6)) + 10*c*d**6*sqrt(-(4*a*c -
b**2)**3)*log(x + (40*a*b*c**2*d**6 - 10*b**3*c*d**6 + 10*c*d**6*sqrt(-(4*a
*c - b**2)**3))/(80*a*c**3*d**6 - 20*b**2*c**2*d**6)) + x*(-128*a*c**3*d**6
 + 48*b**2*c**2*d**6) - (16*a**2*b*c**2*d**6 - 8*a*b**3*c*d**6 + b**5*d**6
+ x*(32*a**2*c**3*d**6 - 16*a*b**2*c**2*d**6 + 2*b**4*c*d**6))/(a + b*x + c
*x**2)
```

Giac [B] time = 1.18774, size = 266, normalized size = 2.66

$$\frac{20\left(b^4cd^6 - 8\,ab^2c^2d^6 + 16\,a^2c^3d^6\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2 + 4\,ac}} - \frac{2\,b^4cd^6x - 16\,ab^2c^2d^6x + 32\,a^2c^3d^6x + b^5d^6 - 8\,ab^3cd^6 + 16\,a^2bc\cdots}{cx^2 + bx + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^6/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 20*(b^4*c*d^6 - 8*a*b^2*c^2*d^6 + 16*a^2*c^3*d^6)*arctan((2*c*x + b)/sqrt(-
b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c) - (2*b^4*c*d^6*x - 16*a*b^2*c^2*d^6*x + 32
*a^2*c^3*d^6*x + b^5*d^6 - 8*a*b^3*c*d^6 + 16*a^2*b*c^2*d^6)/(c*x^2 + b*x +
 a) + 16/3*(4*c^10*d^6*x^3 + 6*b*c^9*d^6*x^2 + 9*b^2*c^8*d^6*x - 24*a*c^9*d
^6*x)/c^6
```

3.1169 $\int \frac{(bd+2cdx)^5}{(a+bx+cx^2)^2}\,dx$

Optimal. Leaf size=65

$$8cd^5\left(b^2-4ac\right)\log\left(a+bx+cx^2\right)-\frac{d^5(b+2cx)^4}{a+bx+cx^2}+8cd^5(b+2cx)^2$$

```
[Out] 8*c*d^5*(b + 2*c*x)^2 - (d^5*(b + 2*c*x)^4)/(a + b*x + c*x^2) + 8*c*(b^2 -
4*a*c)*d^5*Log[a + b*x + c*x^2]
```

Rubi [A] time = 0.039496, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {686, 692, 628}

$$8cd^5\left(b^2-4ac\right)\log\left(a+bx+cx^2\right)-\frac{d^5(b+2cx)^4}{a+bx+cx^2}+8cd^5(b+2cx)^2$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^5/(a + b*x + c*x^2)^2,x]
```

```
[Out] 8*c*d^5*(b + 2*c*x)^2 - (d^5*(b + 2*c*x)^4)/(a + b*x + c*x^2) + 8*c*(b^2 -
4*a*c)*d^5*Log[a + b*x + c*x^2]
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(bd+2cdx)^5}{\left(a+bx+cx^2\right)^2}\,dx = -\frac{d^5(b+2cx)^4}{a+bx+cx^2} + \left(8cd^2\right)\int \frac{(bd+2cdx)^3}{a+bx+cx^2}\,dx$$
$$= 8cd^5(b+2cx)^2 - \frac{d^5(b+2cx)^4}{a+bx+cx^2} + \left(8c\left(b^2-4ac\right)d^4\right)\int \frac{bd+2cdx}{a+bx+cx^2}\,dx$$
$$= 8cd^5(b+2cx)^2 - \frac{d^5(b+2cx)^4}{a+bx+cx^2} + 8c\left(b^2-4ac\right)d^5\log\left(a+bx+cx^2\right)$$

Mathematica [A] time = 0.0342878, size = 64, normalized size = 0.98

$$d^5\left(-\frac{\left(b^2-4ac\right)^2}{a+x(b+cx)} + 8c\left(b^2-4ac\right)\log(a+x(b+cx)) + 16bc^2x + 16c^3x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^5/(a + b*x + c*x^2)^2,x]
```

```
[Out] d^5*(16*b*c^2*x + 16*c^3*x^2 - (b^2 - 4*a*c)^2/(a + x*(b + c*x)) + 8*c*(b^2
 - 4*a*c)*Log[a + x*(b + c*x)])
```

Maple [A] time = 0.047, size = 128, normalized size = 2.

$$16\,d^5c^3x^2 + 16\,d^5bc^2x - 16\,\frac{d^5a^2c^2}{cx^2+bx+a} + 8\,\frac{d^5acb^2}{cx^2+bx+a} - \frac{d^5b^4}{cx^2+bx+a} - 32\,d^5\ln\left(cx^2+bx+a\right)ac^2 + 8\,d^5\ln\left(cx^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^5/(c*x^2+b*x+a)^2,x)
```

```
[Out] 16*d^5*c^3*x^2+16*d^5*b*c^2*x-16*d^5/(c*x^2+b*x+a)*a^2*c^2+8*d^5/(c*x^2+b*x
+a)*a*c*b^2-d^5/(c*x^2+b*x+a)*b^4-32*d^5*ln(c*x^2+b*x+a)*a*c^2+8*d^5*ln(c*x
^2+b*x+a)*b^2*c
```

Maxima [A] time = 1.1619, size = 116, normalized size = 1.78

$$16\,c^3d^5x^2 + 16\,bc^2d^5x + 8\left(b^2c - 4\,ac^2\right)d^5\log\left(cx^2+bx+a\right) - \frac{\left(b^4 - 8\,ab^2c + 16\,a^2c^2\right)d^5}{cx^2+bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^5/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 16*c^3*d^5*x^2 + 16*b*c^2*d^5*x + 8*(b^2*c - 4*a*c^2)*d^5*log(c*x^2 + b*x +
 a) - (b^4 - 8*a*b^2*c + 16*a^2*c^2)*d^5/(c*x^2 + b*x + a)
```

Fricas [B] time = 1.96973, size = 342, normalized size = 5.26

$$\frac{16\,c^4d^5x^4 + 32\,bc^3d^5x^3 + 16\,abc^2d^5x + 16\left(b^2c^2 + ac^3\right)d^5x^2 - \left(b^4 - 8\,ab^2c + 16\,a^2c^2\right)d^5 + 8\left(\left(b^2c^2 - 4\,ac^3\right)d^5x^2 + \left(b^3c \right.\right.}{cx^2+bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^5/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (16*c^4*d^5*x^4 + 32*b*c^3*d^5*x^3 + 16*a*b*c^2*d^5*x + 16*(b^2*c^2 + a*c^3
)*d^5*x^2 - (b^4 - 8*a*b^2*c + 16*a^2*c^2)*d^5 + 8*((b^2*c^2 - 4*a*c^3)*d^5
*x^2 + (b^3*c - 4*a*b*c^2)*d^5*x + (a*b^2*c - 4*a^2*c^2)*d^5)*log(c*x^2 + b
*x + a))/(c*x^2 + b*x + a)
```

Sympy [A] time = 2.30193, size = 90, normalized size = 1.38

$$16bc^2d^5x + 16c^3d^5x^2 - 8cd^5\left(4ac - b^2\right)\log\left(a + bx + cx^2\right) - \frac{16a^2c^2d^5 - 8ab^2cd^5 + b^4d^5}{a + bx + cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**5/(c*x**2+b*x+a)**2,x)
```

```
[Out] 16*b*c**2*d**5*x + 16*c**3*d**5*x**2 - 8*c*d**5*(4*a*c - b**2)*log(a + b*x
+ c*x**2) - (16*a**2*c**2*d**5 - 8*a*b**2*c*d**5 + b**4*d**5)/(a + b*x + c*
x**2)
```

Giac [A] time = 1.16529, size = 135, normalized size = 2.08

$$8\left(b^2cd^5 - 4\,ac^2d^5\right)\log\left(cx^2 + bx + a\right) - \frac{b^4d^5 - 8\,ab^2cd^5 + 16\,a^2c^2d^5}{cx^2 + bx + a} + \frac{16\left(c^7d^5x^2 + bc^6d^5x\right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^5/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 8*(b^2*c*d^5 - 4*a*c^2*d^5)*log(c*x^2 + b*x + a) - (b^4*d^5 - 8*a*b^2*c*d^5
 + 16*a^2*c^2*d^5)/(c*x^2 + b*x + a) + 16*(c^7*d^5*x^2 + b*c^6*d^5*x)/c^4
```

3.1170 $\int \frac{(bd+2cdx)^4}{\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=76

$$-12cd^4\sqrt{b^2-4ac}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)-\frac{d^4(b+2cx)^3}{a+bx+cx^2}+12cd^4(b+2cx)$$

```
[Out] 12*c*d^4*(b + 2*c*x) - (d^4*(b + 2*c*x)^3)/(a + b*x + c*x^2) - 12*c*Sqrt[b^
2 - 4*a*c]*d^4*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]
```

Rubi [A] time = 0.0507377, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {686, 692, 618, 206}

$$-12cd^4\sqrt{b^2-4ac}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)-\frac{d^4(b+2cx)^3}{a+bx+cx^2}+12cd^4(b+2cx)$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^4/(a + b*x + c*x^2)^2,x]
```

```
[Out] 12*c*d^4*(b + 2*c*x) - (d^4*(b + 2*c*x)^3)/(a + b*x + c*x^2) - 12*c*Sqrt[b^
2 - 4*a*c]*d^4*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(bd+2cdx)^4}{\left(a+bx+cx^2\right)^2}\,dx &= -\frac{d^4(b+2cx)^3}{a+bx+cx^2}+\left(6cd^2\right)\int \frac{(bd+2cdx)^2}{a+bx+cx^2}\,dx\\
&= 12cd^4(b+2cx)-\frac{d^4(b+2cx)^3}{a+bx+cx^2}+\left(6c\left(b^2-4ac\right)d^4\right)\int \frac{1}{a+bx+cx^2}\,dx\\
&= 12cd^4(b+2cx)-\frac{d^4(b+2cx)^3}{a+bx+cx^2}-\left(12c\left(b^2-4ac\right)d^4\right)\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)\\
&= 12cd^4(b+2cx)-\frac{d^4(b+2cx)^3}{a+bx+cx^2}-12c\sqrt{b^2-4ac}d^4\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0517713, size = 77, normalized size = 1.01

$$
d^4\left(-\frac{\left(b^2-4ac\right)(b+2cx)}{a+x(b+cx)}-12c\sqrt{4ac-b^2}\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)+16c^2x\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^4/(a + b*x + c*x^2)^2,x]
```

```
[Out] d^4*(16*c^2*x - ((b^2 - 4*a*c)*(b + 2*c*x))/(a + x*(b + c*x)) - 12*c*Sqrt[-
b^2 + 4*a*c]*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])
```

Maple [A] time = 0.157, size = 133, normalized size = 1.8

$$
16\,d^4c^2x+8\,\frac{d^4ac^2x}{cx^2+bx+a}-2\,\frac{d^4b^2cx}{cx^2+bx+a}+4\,\frac{d^4abc}{cx^2+bx+a}-\frac{d^4b^3}{cx^2+bx+a}-12\,d^4c\sqrt{4\,ac-b^2}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^4/(c*x^2+b*x+a)^2,x)
```

```
[Out] 16*d^4*c^2*x+8*d^4/(c*x^2+b*x+a)*a*c^2*x-2*d^4/(c*x^2+b*x+a)*b^2*c*x+4*d^4/
(c*x^2+b*x+a)*a*b*c-d^4/(c*x^2+b*x+a)*b^3-12*d^4*c*(4*a*c-b^2)^(1/2)*arctan
((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.08437, size = 647, normalized size = 8.51

$$\left[\frac{16\,c^3d^4x^3 + 16\,bc^2d^4x^2 - 2\left(b^2c - 12\,ac^2\right)d^4x - \left(b^3 - 4\,abc\right)d^4 + 6\left(c^2d^4x^2 + bcd^4x + acd^4\right)\sqrt{b^2 - 4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+\cdots}{\cdots}\right.}{cx^2 + bx + a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [(16*c^3*d^4*x^3 + 16*b*c^2*d^4*x^2 - 2*(b^2*c - 12*a*c^2)*d^4*x - (b^3 - 4
*a*b*c)*d^4 + 6*(c^2*d^4*x^2 + b*c*d^4*x + a*c*d^4)*sqrt(b^2 - 4*a*c)*log((
2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 +
 b*x + a)))/(c*x^2 + b*x + a), (16*c^3*d^4*x^3 + 16*b*c^2*d^4*x^2 - 2*(b^2*
c - 12*a*c^2)*d^4*x - (b^3 - 4*a*b*c)*d^4 - 12*(c^2*d^4*x^2 + b*c*d^4*x + a
*c*d^4)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*
a*c)))/(c*x^2 + b*x + a)]
```

Sympy [B] time = 1.65756, size = 173, normalized size = 2.28

$$16c^2d^4x + cd^4\sqrt{-144ac + 36b^2}\log\left(x + \frac{6bcd^4 - cd^4\sqrt{-144ac + 36b^2}}{12c^2d^4}\right) - cd^4\sqrt{-144ac + 36b^2}\log\left(x + \frac{6bcd^4 + cd^4\sqrt{-1\cdots}}{12c^2\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**4/(c*x**2+b*x+a)**2,x)
```

```
[Out] 16*c**2*d**4*x + c*d**4*sqrt(-144*a*c + 36*b**2)*log(x + (6*b*c*d**4 - c*d*
*4*sqrt(-144*a*c + 36*b**2))/(12*c**2*d**4)) - c*d**4*sqrt(-144*a*c + 36*b*
*2)*log(x + (6*b*c*d**4 + c*d**4*sqrt(-144*a*c + 36*b**2))/(12*c**2*d**4))
+ (4*a*b*c*d**4 - b**3*d**4 + x*(8*a*c**2*d**4 - 2*b**2*c*d**4))/(a + b*x +
 c*x**2)
```

Giac [A] time = 1.16134, size = 151, normalized size = 1.99

$$16\,c^2d^4x + \frac{12\left(b^2cd^4 - 4\,ac^2d^4\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2 + 4\,ac}} - \frac{2\,b^2cd^4x - 8\,ac^2d^4x + b^3d^4 - 4\,abcd^4}{cx^2 + bx + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 16*c^2*d^4*x + 12*(b^2*c*d^4 - 4*a*c^2*d^4)*arctan((2*c*x + b)/sqrt(-b^2 +
4*a*c))/sqrt(-b^2 + 4*a*c) - (2*b^2*c*d^4*x - 8*a*c^2*d^4*x + b^3*d^4 - 4*a
*b*c*d^4)/(c*x^2 + b*x + a)
```

3.1171 $\int \frac{(bd+2cdx)^3}{\left(a+bx+cx^2\right)^2} \, dx$

Optimal. Leaf size=43

$$4cd^3 \log \left(a + bx + cx^2\right) - \frac{d^3(b + 2cx)^2}{a + bx + cx^2}$$

```
[Out] -((d^3*(b + 2*c*x)^2)/(a + b*x + c*x^2)) + 4*c*d^3*Log[a + b*x + c*x^2]
```

Rubi [A] time = 0.0202037, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {686, 628}

$$4cd^3 \log \left(a + bx + cx^2\right) - \frac{d^3(b + 2cx)^2}{a + bx + cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^3/(a + b*x + c*x^2)^2,x]
```

```
[Out] -((d^3*(b + 2*c*x)^2)/(a + b*x + c*x^2)) + 4*c*d^3*Log[a + b*x + c*x^2]
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(bd + 2cdx)^3}{\left(a + bx + cx^2\right)^2} \, dx &= -\frac{d^3(b + 2cx)^2}{a + bx + cx^2} + \left(4cd^2\right) \int \frac{bd + 2cdx}{a + bx + cx^2} \, dx \\ &= -\frac{d^3(b + 2cx)^2}{a + bx + cx^2} + 4cd^3 \log \left(a + bx + cx^2\right) \end{aligned}$$

Mathematica [A] time = 0.020575, size = 42, normalized size = 0.98

$$d^3 \left(\frac{4ac - b^2}{a + bx + cx^2} + 4c \log \left(a + bx + cx^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^3/(a + b*x + c*x^2)^2,x]
```

```
[Out] d^3*((-b^2 + 4*a*c)/(a + b*x + c*x^2) + 4*c*Log[a + b*x + c*x^2])
```

Maple [A] time = 0.045, size = 58, normalized size = 1.4

$$4\,{\frac {{d}^{3}ac}{c{x}^{2}+bx+a}}-{\frac {{d}^{3}{b}^{2}}{c{x}^{2}+bx+a}}+4\,c{d}^{3}\ln \left(c{x}^{2}+bx+a \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^3/(c*x^2+b*x+a)^2,x)
```

```
[Out] 4*d^3/(c*x^2+b*x+a)*a*c-d^3/(c*x^2+b*x+a)*b^2+4*c*d^3*ln(c*x^2+b*x+a)
```

Maxima [A] time = 1.01148, size = 58, normalized size = 1.35

$$4\,c d^{3} \log\left(c x^{2} + b x + a\right) - \frac{{\left(b^{2} - 4\,a c\right)} d^{3}}{c x^{2} + b x + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 4*c*d^3*log(c*x^2 + b*x + a) - (b^2 - 4*a*c)*d^3/(c*x^2 + b*x + a)
```

Fricas [A] time = 2.01536, size = 136, normalized size = 3.16

$$-\frac{{\left(b^{2} - 4\,a c\right)} d^{3} - 4\,{\left(c^{2} d^{3} x^{2} + b c d^{3} x + a c d^{3}\right)} \log\left(c x^{2} + b x + a\right)}{c x^{2} + b x + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -((b^2 - 4*a*c)*d^3 - 4*(c^2*d^3*x^2 + b*c*d^3*x + a*c*d^3)*log(c*x^2 + b*x
 + a))/(c*x^2 + b*x + a)
```

Sympy [A] time = 1.21328, size = 42, normalized size = 0.98

$$4 c d^{3} \log{\left (a + b x + c x^{2} \right)} + \frac{4 a c d^{3} - b^{2} d^{3}}{a + b x + c x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**3/(c*x**2+b*x+a)**2,x)
```

```
[Out] 4*c*d**3*log(a + b*x + c*x**2) + (4*a*c*d**3 - b**2*d**3)/(a + b*x + c*x**2
)
```

Giac [A] time = 1.14961, size = 63, normalized size = 1.47

$$4\,cd^3\log\left(cx^2+bx+a\right)-\frac{b^2d^3-4\,acd^3}{cx^2+bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 4*c*d^3*log(c*x^2 + b*x + a) - (b^2*d^3 - 4*a*c*d^3)/(c*x^2 + b*x + a)
```

3.1172 $\int \frac{(bd+2cdx)^2}{\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=62

$$-\frac{4cd^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{d^2(b+2cx)}{a+bx+cx^2}$$

```
[Out] -((d^2*(b + 2*c*x))/(a + b*x + c*x^2)) - (4*c*d^2*ArcTanh[(b + 2*c*x)/Sqrt[
b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rubi [A] time = 0.0359027, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {686, 618, 206}

$$-\frac{4cd^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{d^2(b+2cx)}{a+bx+cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^2/(a + b*x + c*x^2)^2,x]
```

```
[Out] -((d^2*(b + 2*c*x))/(a + b*x + c*x^2)) - (4*c*d^2*ArcTanh[(b + 2*c*x)/Sqrt[
b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(bd+2cdx)^2}{\left(a+bx+cx^2\right)^2}\,dx = -\frac{d^2(b+2cx)}{a+bx+cx^2}+\left(2cd^2\right)\int \frac{1}{a+bx+cx^2}\,dx$$
$$= -\frac{d^2(b+2cx)}{a+bx+cx^2}-\left(4cd^2\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)$$
$$= -\frac{d^2(b+2cx)}{a+bx+cx^2}-\frac{4cd^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.0362394, size = 65, normalized size = 1.05

$$d^2\left(\frac{4c\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{-b-2cx}{a+bx+cx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^2/(a + b*x + c*x^2)^2,x]
```

```
[Out] d^2*((-b - 2*c*x)/(a + b*x + c*x^2) + (4*c*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4
*a*c]])/Sqrt[-b^2 + 4*a*c])
```

Maple [A] time = 0.152, size = 77, normalized size = 1.2

$$-2\,{\frac{c{d}^{2}x}{c{x}^{2}+bx+a}}-{\frac{{d}^{2}b}{c{x}^{2}+bx+a}}+4\,{\frac{c{d}^{2}}{\sqrt{4\,ac-{b}^{2}}}\arctan \left({\frac{2\,cx+b}{\sqrt{4\,ac-{b}^{2}}}} \right) }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^2/(c*x^2+b*x+a)^2,x)
```

```
[Out] -2*d^2/(c*x^2+b*x+a)*x*c-d^2/(c*x^2+b*x+a)*b+4*d^2*c/(4*a*c-b^2)^(1/2)*arct
an((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.07454, size = 667, normalized size = 10.76

$$\left[-\frac{2\left(b^2c-4\,ac^2\right)d^2x+\left(b^3-4\,abc\right)d^2-2\left(c^2d^2x^2+bcd^2x+acd^2\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)}{ab^2-4\,a^2c+\left(b^2c-4\,ac^2\right)x^2+\left(b^3-4\,abc\right)x},-\frac{2\,(b}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-(2*(b^2*c - 4*a*c^2)*d^2*x + (b^3 - 4*a*b*c)*d^2 - 2*(c^2*d^2*x^2 + b*c*d
^2*x + a*c*d^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c -
sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)))/(a*b^2 - 4*a^2*c + (b^2*
c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x), -(2*(b^2*c - 4*a*c^2)*d^2*x + (b^3 -
 4*a*b*c)*d^2 + 4*(c^2*d^2*x^2 + b*c*d^2*x + a*c*d^2)*sqrt(-b^2 + 4*a*c)*ar
ctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)))/(a*b^2 - 4*a^2*c + (b^
2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x)]
```

Sympy [B] time = 1.0763, size = 209, normalized size = 3.37

$$-2cd^2\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{-8ac^2d^2\sqrt{-\frac{1}{4ac-b^2}}+2b^2cd^2\sqrt{-\frac{1}{4ac-b^2}}+2bcd^2}{4c^2d^2}\right)+2cd^2\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{8ac^2d^2\sqrt{-\frac{1}{4ac}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**2/(c*x**2+b*x+a)**2,x)
```

```
[Out] -2*c*d**2*sqrt(-1/(4*a*c - b**2))*log(x + (-8*a*c**2*d**2*sqrt(-1/(4*a*c -
b**2)) + 2*b**2*c*d**2*sqrt(-1/(4*a*c - b**2)) + 2*b*c*d**2)/(4*c**2*d**2))
 + 2*c*d**2*sqrt(-1/(4*a*c - b**2))*log(x + (8*a*c**2*d**2*sqrt(-1/(4*a*c -
 b**2)) - 2*b**2*c*d**2*sqrt(-1/(4*a*c - b**2)) + 2*b*c*d**2)/(4*c**2*d**2)
) - (b*d**2 + 2*c*d**2*x)/(a + b*x + c*x**2)
```

Giac [A] time = 1.17002, size = 89, normalized size = 1.44

$$\frac{4\,cd^2\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}-\frac{2\,cd^2x+bd^2}{cx^2+bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 4*c*d^2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c) - (2*c*d^
2*x + b*d^2)/(c*x^2 + b*x + a)
```

3.1173 $\int \frac{bd+2cdx}{(a+bx+cx^2)^2} dx$

Optimal. Leaf size=15

$$-\frac{d}{a+bx+cx^2}$$

```
[Out] -(d/(a + b*x + c*x^2))
```

Rubi [A] time = 0.0051263, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {629}

$$-\frac{d}{a+bx+cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)/(a + b*x + c*x^2)^2,x]

[Out] -(d/(a + b*x + c*x^2))
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{bd+2cdx}{(a+bx+cx^2)^2} dx = -\frac{d}{a+bx+cx^2}$$

Mathematica [A] time = 0.0039089, size = 14, normalized size = 0.93

$$-\frac{d}{a+x(b+cx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)/(a + b*x + c*x^2)^2,x]

[Out] -(d/(a + x*(b + c*x)))
```

Maple [A] time = 0.037, size = 16, normalized size = 1.1

$$-\frac{d}{cx^2+bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)/(c*x^2+b*x+a)^2,x)
```

```
[Out] -d/(c*x^2+b*x+a)
```

Maxima [A] time = 1.05552, size = 20, normalized size = 1.33

$$-\frac{d}{cx^2+bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -d/(c*x^2 + b*x + a)
```

Fricas [A] time = 1.93995, size = 30, normalized size = 2.

$$-\frac{d}{cx^2+bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -d/(c*x^2 + b*x + a)
```

Sympy [A] time = 0.594069, size = 12, normalized size = 0.8

$$-\frac{d}{a+bx+cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)/(c*x**2+b*x+a)**2,x)
```

```
[Out] -d/(a + b*x + c*x**2)
```

Giac [A] time = 1.18969, size = 20, normalized size = 1.33

$$-\frac{d}{cx^2+bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -d/(c*x^2 + b*x + a)
```

3.1174 $\int \frac{1}{(bd+2cdx)\left(a+bx+cx^2\right)^2} dx$

Optimal. Leaf size=78

$$-\frac{1}{d\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{4c\log\left(a+bx+cx^2\right)}{d\left(b^2-4ac\right)^2}+\frac{8c\log(b+2cx)}{d\left(b^2-4ac\right)^2}$$

```
[Out] -(1/((b^2 - 4*a*c)*d*(a + b*x + c*x^2))) + (8*c*Log[b + 2*c*x])/((b^2 - 4*a
*c)^2*d) - (4*c*Log[a + b*x + c*x^2])/((b^2 - 4*a*c)^2*d)
```

Rubi [A] time = 0.0382858, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {687, 681, 31, 628}

$$-\frac{1}{d\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{4c\log\left(a+bx+cx^2\right)}{d\left(b^2-4ac\right)^2}+\frac{8c\log(b+2cx)}{d\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)*(a + b*x + c*x^2)^2),x]
```

```
[Out] -(1/((b^2 - 4*a*c)*d*(a + b*x + c*x^2))) + (8*c*Log[b + 2*c*x])/((b^2 - 4*a
*c)^2*d) - (4*c*Log[a + b*x + c*x^2])/((b^2 - 4*a*c)^2*d)
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 681

```
Int[1/(((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
:> Dist[(-4*b*c)/(d*(b^2 - 4*a*c)), Int[1/(b + 2*c*x), x], x] + Dist[b^2/(d
^2*(b^2 - 4*a*c)), Int[(d + e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)\left(a+bx+cx^2\right)^2}\,dx = -\frac{1}{\left(b^2-4ac\right)d\left(a+bx+cx^2\right)} - \frac{(4c)\int \frac{1}{(bd+2cdx)\left(a+bx+cx^2\right)}\,dx}{b^2-4ac}$$

$$= -\frac{1}{\left(b^2-4ac\right)d\left(a+bx+cx^2\right)} - \frac{(4c)\int \frac{bd+2cdx}{a+bx+cx^2}\,dx}{\left(b^2-4ac\right)^2 d^2} + \frac{\left(16c^2\right)\int \frac{1}{b+2cx}\,dx}{\left(b^2-4ac\right)^2 d}$$

$$= -\frac{1}{\left(b^2-4ac\right)d\left(a+bx+cx^2\right)} + \frac{8c\log(b+2cx)}{\left(b^2-4ac\right)^2 d} - \frac{4c\log\left(a+bx+cx^2\right)}{\left(b^2-4ac\right)^2 d}$$

Mathematica [A] time = 0.0556991, size = 59, normalized size = 0.76

$$\frac{-\frac{b^2-4ac}{a+x(b+cx)} - 4c\log(a+x(b+cx)) + 8c\log(b+2cx)}{d\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)*(a + b*x + c*x^2)^2),x]
```

```
[Out] (-((b^2 - 4*a*c)/(a + x*(b + c*x))) + 8*c*Log[b + 2*c*x] - 4*c*Log[a + x*(b
 + c*x)])/((b^2 - 4*a*c)^2*d)
```

Maple [A] time = 0.075, size = 119, normalized size = 1.5

$$4\,\frac{ac}{d\left(4\,ac-b^2\right)^2\left(cx^2+bx+a\right)} - \frac{b^2}{d\left(4\,ac-b^2\right)^2\left(cx^2+bx+a\right)} - 4\,\frac{c\ln\left(cx^2+bx+a\right)}{d\left(4\,ac-b^2\right)^2} + 8\,\frac{c\ln(2\,cx+b)}{d\left(4\,ac-b^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)/(c*x^2+b*x+a)^2,x)
```

```
[Out] 4/d/(4*a*c-b^2)^2/(c*x^2+b*x+a)*a*c-1/d/(4*a*c-b^2)^2/(c*x^2+b*x+a)*b^2-4/d
/(4*a*c-b^2)^2*c*ln(c*x^2+b*x+a)+8/d*c/(4*a*c-b^2)^2*ln(2*c*x+b)
```

Maxima [A] time = 1.17233, size = 163, normalized size = 2.09

$$-\frac{4\,c\log\left(cx^2+bx+a\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)d} + \frac{8\,c\log(2\,cx+b)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)d} - \frac{1}{\left(b^2c-4\,ac^2\right)dx^2+\left(b^3-4\,abc\right)dx+\left(ab^2-4\,a^2c\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -4*c*log(c*x^2 + b*x + a)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*d) + 8*c*log(2*c*
x + b)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*d) - 1/((b^2*c - 4*a*c^2)*d*x^2 + (b
^3 - 4*a*b*c)*d*x + (a*b^2 - 4*a^2*c)*d)
```

Fricas [A] time = 1.94166, size = 309, normalized size = 3.96

$$-\frac{b^2-4\,ac+4\left(c^2x^2+bcx+ac\right)\log\left(cx^2+bx+a\right)-8\left(c^2x^2+bcx+ac\right)\log\left(2\,cx+b\right)}{\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)dx^2+\left(b^5-8\,ab^3c+16\,a^2bc^2\right)dx+\left(ab^4-8\,a^2b^2c+16\,a^3c^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(b^2 - 4*a*c + 4*(c^2*x^2 + b*c*x + a*c)*log(c*x^2 + b*x + a) - 8*(c^2*x^2
 + b*c*x + a*c)*log(2*c*x + b))/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^2 +
 (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*
d)
```

Sympy [A] time = 2.2666, size = 102, normalized size = 1.31

$$\frac{8c\log\left(\frac{b}{2c}+x\right)}{d\left(4ac-b^2\right)^2}-\frac{4c\log\left(\frac{a}{c}+\frac{bx}{c}+x^2\right)}{d\left(4ac-b^2\right)^2}+\frac{1}{4a^2cd-ab^2d+x^2\left(4ac^2d-b^2cd\right)+x\left(4abcd-b^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)/(c*x**2+b*x+a)**2,x)
```

```
[Out] 8*c*log(b/(2*c) + x)/(d*(4*a*c - b**2)**2) - 4*c*log(a/c + b*x/c + x**2)/(d
*(4*a*c - b**2)**2) + 1/(4*a**2*c*d - a*b**2*d + x**2*(4*a*c**2*d - b**2*c*
d) + x*(4*a*b*c*d - b**3*d))
```

Giac [A] time = 1.15679, size = 146, normalized size = 1.87

$$\frac{8\,c^2\log\left(|2\,cx+b|\right)}{b^4cd-8\,ab^2c^2d+16\,a^2c^3d}-\frac{4\,c\log\left(cx^2+bx+a\right)}{b^4d-8\,ab^2cd+16\,a^2c^2d}-\frac{1}{\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 8*c^2*log(abs(2*c*x + b))/(b^4*c*d - 8*a*b^2*c^2*d + 16*a^2*c^3*d) - 4*c*lo
g(c*x^2 + b*x + a)/(b^4*d - 8*a*b^2*c*d + 16*a^2*c^2*d) - 1/((c*x^2 + b*x +
 a)*(b^2 - 4*a*c)*d)
```

3.1175 $\int \frac{1}{(bd+2cdx)^2 (a+bx+cx^2)^2} dx$

Optimal. Leaf size=98

$$-\frac{1}{d^2 (b^2-4ac) (b+2cx) (a+bx+cx^2)}-\frac{12c}{d^2 (b^2-4ac)^2 (b+2cx)}+\frac{12c \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d^2 (b^2-4ac)^{5/2}}$$

```
[Out] (-12*c)/((b^2 - 4*a*c)^2*d^2*(b + 2*c*x)) - 1/((b^2 - 4*a*c)*d^2*(b + 2*c*x
)*(a + b*x + c*x^2)) + (12*c*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((b^2
- 4*a*c)^(5/2)*d^2)
```

Rubi [A] time = 0.0575797, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {687, 693, 618, 206}

$$-\frac{1}{d^2 (b^2-4ac) (b+2cx) (a+bx+cx^2)}-\frac{12c}{d^2 (b^2-4ac)^2 (b+2cx)}+\frac{12c \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d^2 (b^2-4ac)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^2*(a + b*x + c*x^2)^2),x]
```

```
[Out] (-12*c)/((b^2 - 4*a*c)^2*d^2*(b + 2*c*x)) - 1/((b^2 - 4*a*c)*d^2*(b + 2*c*x
)*(a + b*x + c*x^2)) + (12*c*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((b^2
- 4*a*c)^(5/2)*d^2)
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)^2\left(a+bx+cx^2\right)^2}\,dx = -\frac{1}{\left(b^2-4ac\right)d^2(b+2cx)\left(a+bx+cx^2\right)}-\frac{(6c)\int \frac{1}{(bd+2cdx)^2\left(a+bx+cx^2\right)}\,dx}{b^2-4ac}$$

$$= -\frac{12c}{\left(b^2-4ac\right)^2 d^2(b+2cx)}-\frac{1}{\left(b^2-4ac\right)d^2(b+2cx)\left(a+bx+cx^2\right)}-\frac{(6c)\int \frac{1}{a+bx+cx^2}}{\left(b^2-4ac\right)^2 d^2}$$

$$= -\frac{12c}{\left(b^2-4ac\right)^2 d^2(b+2cx)}-\frac{1}{\left(b^2-4ac\right)d^2(b+2cx)\left(a+bx+cx^2\right)}+\frac{(12c)\,\mathrm{Subst}\left(\int\right.}{(b}$$

$$= -\frac{12c}{\left(b^2-4ac\right)^2 d^2(b+2cx)}-\frac{1}{\left(b^2-4ac\right)d^2(b+2cx)\left(a+bx+cx^2\right)}+\frac{12c\tanh^{-1}\left(\frac{b}{\sqrt{b}}\right.}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.121909, size = 84, normalized size = 0.86

$$-\frac{\frac{12c\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{b+2cx}{a+x(b+cx)}+\frac{8c}{b+2cx}}{d^2\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^2*(a + b*x + c*x^2)^2),x]
```

```
[Out] -(((8*c)/(b + 2*c*x) + (b + 2*c*x)/(a + x*(b + c*x)) + (12*c*ArcTan[(b + 2*
c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/((b^2 - 4*a*c)^2*d^2))
```

Maple [A] time = 0.158, size = 127, normalized size = 1.3

$$-2\,\frac{cx}{d^2\left(4\,ac-b^2\right)^2\left(cx^2+bx+a\right)}-\frac{b}{d^2\left(4\,ac-b^2\right)^2\left(cx^2+bx+a\right)}-12\,\frac{c}{d^2\left(4\,ac-b^2\right)^{5/2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)-8\,\frac{}{d^2\left(4\,ac\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^2/(c*x^2+b*x+a)^2,x)
```

```
[Out] -2/d^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)*x*c-1/d^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)*b-
12/d^2/(4*a*c-b^2)^(5/2)*c*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))-8/d^2/(4*a*c
-b^2)^2*c/(2*c*x+b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^2/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.15535, size = 1384, normalized size = 14.12

$$\left[-\frac{b^4 + 4\,ab^2c - 32\,a^2c^2 + 12\left(b^2c^2 - 4\,ac^3\right)x^2 - 6\left(2\,c^3x^3 + 3\,bc^2x^2 + abc + \left(b^2c + 2\,ac^2\right)x\right)\sqrt{b^2 - 4\,a}}{2\left(b^6c^2 - 12\,ab^4c^3 + 48\,a^2b^2c^4 - 64\,a^3c^5\right)d^2x^3 + 3\left(b^7c - 12\,ab^5c^2 + 48\,a^2b^3c^3 - 64\,a^3bc^4\right)d^2x^2 + \left(b^8 - 10\,ab^6c + 24\,a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^2/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-(b^4 + 4*a*b^2*c - 32*a^2*c^2 + 12*(b^2*c^2 - 4*a*c^3)*x^2 - 6*(2*c^3*x^3
 + 3*b*c^2*x^2 + a*b*c + (b^2*c + 2*a*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x +
 a)) + 12*(b^3*c - 4*a*b*c^2)*x)/(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^
4 - 64*a^3*c^5)*d^2*x^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3
*b*c^4)*d^2*x^2 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128
*a^4*c^4)*d^2*x + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^
2), -(b^4 + 4*a*b^2*c - 32*a^2*c^2 + 12*(b^2*c^2 - 4*a*c^3)*x^2 - 12*(2*c^3
*x^3 + 3*b*c^2*x^2 + a*b*c + (b^2*c + 2*a*c^2)*x)*sqrt(-b^2 + 4*a*c)*arctan
(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 12*(b^3*c - 4*a*b*c^2)*x)
/(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2*x^3 + 3*(b^7
*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2*x^2 + (b^8 - 10*a*b^
6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2*x + (a*b^7 - 12*a^
2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2)]
```

Sympy [B] time = 2.676, size = 457, normalized size = 4.66

$$\frac{6c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x + \frac{-384a^3c^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}} + 288a^2b^2c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}} - 72ab^4c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}} + 6b^6c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}} + 6bc}{12c^2}\right)}{d^2} - \frac{6c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**2/(c*x**2+b*x+a)**2,x)
```

```
[Out] 6*c*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-384*a**3*c**4*sqrt(-1/(4*a*c - b**
2)**5) + 288*a**2*b**2*c**3*sqrt(-1/(4*a*c - b**2)**5) - 72*a*b**4*c**2*sqr
t(-1/(4*a*c - b**2)**5) + 6*b**6*c*sqrt(-1/(4*a*c - b**2)**5) + 6*b*c)/(12*
c**2))/d**2 - 6*c*sqrt(-1/(4*a*c - b**2)**5)*log(x + (384*a**3*c**4*sqrt(-1
/(4*a*c - b**2)**5) - 288*a**2*b**2*c**3*sqrt(-1/(4*a*c - b**2)**5) + 72*a*
b**4*c**2*sqrt(-1/(4*a*c - b**2)**5) - 6*b**6*c*sqrt(-1/(4*a*c - b**2)**5)
+ 6*b*c)/(12*c**2))/d**2 - (8*a*c + b**2 + 12*b*c*x + 12*c**2*x**2)/(16*a**
3*b*c**2*d**2 - 8*a**2*b**3*c*d**2 + a*b**5*d**2 + x**3*(32*a**2*c**4*d**2
- 16*a*b**2*c**3*d**2 + 2*b**4*c**2*d**2) + x**2*(48*a**2*b*c**3*d**2 - 24*
a*b**3*c**2*d**2 + 3*b**5*c*d**2) + x*(32*a**3*c**3*d**2 - 6*a*b**4*c*d**2
```

```
+ b**6*d**2))
```

Giac [B] time = 1.23668, size = 297, normalized size = 3.03

$$-\frac{8\,c^5d^7}{\left(b^4c^4d^8-8\,ab^2c^5d^8+16\,a^2c^6d^8\right)(2\,cdx+bd)}-\frac{12\,c\arctan\left(\frac{\frac{b^2d}{2\,cdx+bd}-\frac{4\,acd}{2\,cdx+bd}}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}d^2}+\frac{4}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)(2\,cdx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^2/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -8*c^5*d^7/((b^4*c^4*d^8 - 8*a*b^2*c^5*d^8 + 16*a^2*c^6*d^8)*(2*c*d*x + b*d
)) - 12*c*arctan((b^2*d/(2*c*d*x + b*d) - 4*a*c*d/(2*c*d*x + b*d))/sqrt(-b^
2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)*d^2) + 4*c/(
(b^4 - 8*a*b^2*c + 16*a^2*c^2)*(2*c*d*x + b*d)*(b^2*d^2/(2*c*d*x + b*d)^2 -
 4*a*c*d^2/(2*c*d*x + b*d)^2 - 1)*d)
```

3.1176 $\int \frac{1}{(bd+2cdx)^3\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=110

$$-\frac{1}{d^3\left(b^2-4ac\right)(b+2cx)^2\left(a+bx+cx^2\right)}-\frac{8c\log\left(a+bx+cx^2\right)}{d^3\left(b^2-4ac\right)^3}-\frac{8c}{d^3\left(b^2-4ac\right)^2(b+2cx)^2}+\frac{16c\log(b+2cx)}{d^3\left(b^2-4ac\right)^3}$$

```
[Out] (-8*c)/((b^2 - 4*a*c)^2*d^3*(b + 2*c*x)^2) - 1/((b^2 - 4*a*c)*d^3*(b + 2*c*
x)^2*(a + b*x + c*x^2)) + (16*c*Log[b + 2*c*x])/((b^2 - 4*a*c)^3*d^3) - (8*
c*Log[a + b*x + c*x^2])/((b^2 - 4*a*c)^3*d^3)
```

Rubi [A] time = 0.0621794, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {687, 693, 681, 31, 628}

$$-\frac{1}{d^3\left(b^2-4ac\right)(b+2cx)^2\left(a+bx+cx^2\right)}-\frac{8c\log\left(a+bx+cx^2\right)}{d^3\left(b^2-4ac\right)^3}-\frac{8c}{d^3\left(b^2-4ac\right)^2(b+2cx)^2}+\frac{16c\log(b+2cx)}{d^3\left(b^2-4ac\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^3*(a + b*x + c*x^2)^2),x]
```

```
[Out] (-8*c)/((b^2 - 4*a*c)^2*d^3*(b + 2*c*x)^2) - 1/((b^2 - 4*a*c)*d^3*(b + 2*c*
x)^2*(a + b*x + c*x^2)) + (16*c*Log[b + 2*c*x])/((b^2 - 4*a*c)^3*d^3) - (8*
c*Log[a + b*x + c*x^2])/((b^2 - 4*a*c)^3*d^3)
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 681

```
Int[1/(((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
:> Dist[(-4*b*c)/(d*(b^2 - 4*a*c)), Int[1/(b + 2*c*x), x], x] + Dist[b^2/(d
^2*(b^2 - 4*a*c)), Int[(d + e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)^3\left(a+bx+cx^2\right)^2}\,dx = -\frac{1}{\left(b^2-4ac\right)d^3(b+2cx)^2\left(a+bx+cx^2\right)} - \frac{(8c)\int \frac{1}{(bd+2cdx)^3\left(a+bx+cx^2\right)}\,dx}{b^2-4ac}$$

$$= -\frac{8c}{\left(b^2-4ac\right)^2 d^3(b+2cx)^2} - \frac{1}{\left(b^2-4ac\right)d^3(b+2cx)^2\left(a+bx+cx^2\right)} - \frac{(8c)\int \frac{1}{(bd+2cd}$$

$$= -\frac{8c}{\left(b^2-4ac\right)^2 d^3(b+2cx)^2} - \frac{1}{\left(b^2-4ac\right)d^3(b+2cx)^2\left(a+bx+cx^2\right)} - \frac{(8c)\int \frac{bd+2cd}{a+bx+c}}{\left(b^2-4ac\right)^3}$$

$$= -\frac{8c}{\left(b^2-4ac\right)^2 d^3(b+2cx)^2} - \frac{1}{\left(b^2-4ac\right)d^3(b+2cx)^2\left(a+bx+cx^2\right)} + \frac{16c\log(b+2}{\left(b^2-4ac\right)^3}$$

Mathematica [A] time = 0.0902598, size = 79, normalized size = 0.72

$$\frac{-\frac{4c\left(b^2-4ac\right)}{(b+2cx)^2} + \frac{4ac-b^2}{a+x(b+cx)} - 8c\log(a+x(b+cx)) + 16c\log(b+2cx)}{d^3\left(b^2-4ac\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^3*(a + b*x + c*x^2)^2),x]
```

```
[Out] ((-4*c*(b^2 - 4*a*c))/(b + 2*c*x)^2 + (-b^2 + 4*a*c)/(a + x*(b + c*x)) + 16
*c*Log[b + 2*c*x] - 8*c*Log[a + x*(b + c*x)])/((b^2 - 4*a*c)^3*d^3)
```

Maple [A] time = 0.056, size = 144, normalized size = 1.3

$$-4\,\frac{ac}{d^3\left(4\,ac-b^2\right)^3\left(cx^2+bx+a\right)} + \frac{b^2}{d^3\left(4\,ac-b^2\right)^3\left(cx^2+bx+a\right)} + 8\,\frac{c\ln\left(cx^2+bx+a\right)}{d^3\left(4\,ac-b^2\right)^3} - 16\,\frac{c\ln\left(2\,cx+b\right)}{d^3\left(4\,ac-b^2\right)^3} - 4\,\frac{}{d^3\left(4\,ac\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^3/(c*x^2+b*x+a)^2,x)
```

```
[Out] -4/d^3/(4*a*c-b^2)^3/(c*x^2+b*x+a)*a*c+1/d^3/(4*a*c-b^2)^3/(c*x^2+b*x+a)*b^
2+8/d^3/(4*a*c-b^2)^3*c*ln(c*x^2+b*x+a)-16/d^3*c/(4*a*c-b^2)^3*ln(2*c*x+b)-
4/d^3*c/(4*a*c-b^2)^2/(2*c*x+b)^2
```

Maxima [B] time = 1.12979, size = 400, normalized size = 3.64

$$-\frac{8\,c^2x^2+8\,bcx+b^2+4\,ac}{4\,\left(b^4c^3-8\,ab^2c^4+16\,a^2c^5\right)d^3x^4+8\,\left(b^5c^2-8\,ab^3c^3+16\,a^2bc^4\right)d^3x^3+\left(5\,b^6c-36\,ab^4c^2+48\,a^2b^2c^3+64\,a^3c^4\right)d^3x^2+\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^3/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(8*c^2*x^2 + 8*b*c*x + b^2 + 4*a*c)/(4*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5
)*d^3*x^4 + 8*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^3*x^3 + (5*b^6*c - 3
6*a*b^4*c^2 + 48*a^2*b^2*c^3 + 64*a^3*c^4)*d^3*x^2 + (b^7 - 4*a*b^5*c - 16*
a^2*b^3*c^2 + 64*a^3*b*c^3)*d^3*x + (a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*
d^3) - 8*c*log(c*x^2 + b*x + a)/((b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^
3*c^3)*d^3) + 16*c*log(2*c*x + b)/((b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*
a^3*c^3)*d^3)
```

Fricas [B] time = 2.05137, size = 864, normalized size = 7.85

$$-\frac{b^4-16\,a^2c^2+8\,\left(b^2c^2-4\,ac^3\right)x^2+8\,\left(b^3c-4\,abc^2\right)x+8\,\left(4\,c^4x^4+8\,bc^3x^3+ab^2c+\left(5\,b^2c^2+4\,ac^3\right)x^2+\cdots\right.}{4\,\left(b^6c^3-12\,ab^4c^4+48\,a^2b^2c^5-64\,a^3c^6\right)d^3x^4+8\,\left(b^7c^2-12\,ab^5c^3+48\,a^2b^3c^4-64\,a^3bc^5\right)d^3x^3+\left(5\,b^8c-56\,ab^6c^2+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^3/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(b^4 - 16*a^2*c^2 + 8*(b^2*c^2 - 4*a*c^3)*x^2 + 8*(b^3*c - 4*a*b*c^2)*x +
8*(4*c^4*x^4 + 8*b*c^3*x^3 + a*b^2*c + (5*b^2*c^2 + 4*a*c^3)*x^2 + (b^3*c +
 4*a*b*c^2)*x)*log(c*x^2 + b*x + a) - 16*(4*c^4*x^4 + 8*b*c^3*x^3 + a*b^2*c
 + (5*b^2*c^2 + 4*a*c^3)*x^2 + (b^3*c + 4*a*b*c^2)*x)*log(2*c*x + b))/(4*(b
^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*d^3*x^4 + 8*(b^7*c^2 -
 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^3*x^3 + (5*b^8*c - 56*a*b^
6*c^2 + 192*a^2*b^4*c^3 - 128*a^3*b^2*c^4 - 256*a^4*c^5)*d^3*x^2 + (b^9 - 8
*a*b^7*c + 128*a^3*b^3*c^3 - 256*a^4*b*c^4)*d^3*x + (a*b^8 - 12*a^2*b^6*c +
 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^3)
```

Sympy [B] time = 6.44781, size = 303, normalized size = 2.75

$$-\frac{16c\log\left(\frac{b}{2c}+x\right)}{d^3\left(4ac-b^2\right)^3}+\frac{8c\log\left(\frac{a}{c}+\frac{bx}{c}+x^2\right)}{d^3\left(4ac-b^2\right)^3}-\frac{\cdots}{16a^3b^2c^2d^3-8a^2b^4cd^3+ab^6d^3+x^4\left(64a^2c^5d^3-32ab^2c^4d^3+4b^4c^3d^3\right)+\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**3/(c*x**2+b*x+a)**2,x)
```

```
[Out] -16*c*log(b/(2*c) + x)/(d**3*(4*a*c - b**2)**3) + 8*c*log(a/c + b*x/c + x**
2)/(d**3*(4*a*c - b**2)**3) - (4*a*c + b**2 + 8*b*c*x + 8*c**2*x**2)/(16*a*
*3*b**2*c**2*d**3 - 8*a**2*b**4*c*d**3 + a*b**6*d**3 + x**4*(64*a**2*c**5*d
**3 - 32*a*b**2*c**4*d**3 + 4*b**4*c**3*d**3) + x**3*(128*a**2*b*c**4*d**3
- 64*a*b**3*c**3*d**3 + 8*b**5*c**2*d**3) + x**2*(64*a**3*c**4*d**3 + 48*a*
*2*b**2*c**3*d**3 - 36*a*b**4*c**2*d**3 + 5*b**6*c*d**3) + x*(64*a**3*b*c**
```

```
3*d**3 - 16*a**2*b**3*c**2*d**3 - 4*a*b**5*c*d**3 + b**7*d**3))
```

Giac [A] time = 1.17857, size = 274, normalized size = 2.49

$$\frac{16\,c^2\log\left(|2\,cx+b|\right)}{b^6cd^3-12\,ab^4c^2d^3+48\,a^2b^2c^3d^3-64\,a^3c^4d^3}-\frac{8\,c\log\left(cx^2+bx+a\right)}{b^6d^3-12\,ab^4cd^3+48\,a^2b^2c^2d^3-64\,a^3c^3d^3}-\frac{b^4-16\,a^2c^2+8\left(b^2c^2-4\,a\right.}{\left(cx^2+bx+a\right)\left(b^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^3/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 16*c^2*log(abs(2*c*x + b))/(b^6*c*d^3 - 12*a*b^4*c^2*d^3 + 48*a^2*b^2*c^3*d
^3 - 64*a^3*c^4*d^3) - 8*c*log(c*x^2 + b*x + a)/(b^6*d^3 - 12*a*b^4*c*d^3 +
 48*a^2*b^2*c^2*d^3 - 64*a^3*c^3*d^3) - (b^4 - 16*a^2*c^2 + 8*(b^2*c^2 - 4*
a*c^3)*x^2 + 8*(b^3*c - 4*a*b*c^2)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)^3*(2
*c*x + b)^2*d^3)
```

3.1177 $\int \frac{(bd+2cdx)^{10}}{(a+bx+cx^2)^3} dx$

Optimal. Leaf size=160

$$84c^2d^{10}\left(b^2-4ac\right)(b+2cx)^3+252c^2d^{10}\left(b^2-4ac\right)^2(b+2cx)-252c^2d^{10}\left(b^2-4ac\right)^{5/2}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)-\frac{9cd^{10}(b}{a+bx}$$

```
[Out] 252*c^2*(b^2 - 4*a*c)^2*d^10*(b + 2*c*x) + 84*c^2*(b^2 - 4*a*c)*d^10*(b + 2
*c*x)^3 + (252*c^2*d^10*(b + 2*c*x)^5)/5 - (d^10*(b + 2*c*x)^9)/(2*(a + b*x
 + c*x^2)^2) - (9*c*d^10*(b + 2*c*x)^7)/(a + b*x + c*x^2) - 252*c^2*(b^2 -
4*a*c)^(5/2)*d^10*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]
```

Rubi [A] time = 0.13716, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {686, 692, 618, 206}

$$84c^2d^{10}\left(b^2-4ac\right)(b+2cx)^3+252c^2d^{10}\left(b^2-4ac\right)^2(b+2cx)-252c^2d^{10}\left(b^2-4ac\right)^{5/2}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)-\frac{9cd^{10}(b}{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^10/(a + b*x + c*x^2)^3,x]
```


```
[Out] 252*c^2*(b^2 - 4*a*c)^2*d^10*(b + 2*c*x) + 84*c^2*(b^2 - 4*a*c)*d^10*(b + 2
*c*x)^3 + (252*c^2*d^10*(b + 2*c*x)^5)/5 - (d^10*(b + 2*c*x)^9)/(2*(a + b*x
 + c*x^2)^2) - (9*c*d^10*(b + 2*c*x)^7)/(a + b*x + c*x^2) - 252*c^2*(b^2 -
4*a*c)^(5/2)*d^10*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]
```


Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(bd+2cdx)^{10}}{\left(a+bx+cx^2\right)^3}\,dx = -\frac{d^{10}(b+2cx)^9}{2\left(a+bx+cx^2\right)^2} + \left(9cd^2\right)\int \frac{(bd+2cdx)^8}{\left(a+bx+cx^2\right)^2}\,dx$$

$$= -\frac{d^{10}(b+2cx)^9}{2\left(a+bx+cx^2\right)^2} - \frac{9cd^{10}(b+2cx)^7}{a+bx+cx^2} + \left(126c^2d^4\right)\int \frac{(bd+2cdx)^6}{a+bx+cx^2}\,dx$$

$$= \frac{252}{5}c^2d^{10}(b+2cx)^5 - \frac{d^{10}(b+2cx)^9}{2\left(a+bx+cx^2\right)^2} - \frac{9cd^{10}(b+2cx)^7}{a+bx+cx^2} + \left(126c^2\left(b^2-4ac\right)d^6\right)\int \frac{(bd+2cdx}{a+bx+c}$$

$$= 84c^2\left(b^2-4ac\right)d^{10}(b+2cx)^3 + \frac{252}{5}c^2d^{10}(b+2cx)^5 - \frac{d^{10}(b+2cx)^9}{2\left(a+bx+cx^2\right)^2} - \frac{9cd^{10}(b+2cx)^7}{a+bx+cx^2} + (12$$

$$= 252c^2\left(b^2-4ac\right)^2d^{10}(b+2cx) + 84c^2\left(b^2-4ac\right)d^{10}(b+2cx)^3 + \frac{252}{5}c^2d^{10}(b+2cx)^5 - \frac{d^{10}(b+}{2\left(a+bx\right.}$$

$$= 252c^2\left(b^2-4ac\right)^2d^{10}(b+2cx) + 84c^2\left(b^2-4ac\right)d^{10}(b+2cx)^3 + \frac{252}{5}c^2d^{10}(b+2cx)^5 - \frac{d^{10}(b+}{2\left(a+bx\right.}$$

$$= 252c^2\left(b^2-4ac\right)^2d^{10}(b+2cx) + 84c^2\left(b^2-4ac\right)d^{10}(b+2cx)^3 + \frac{252}{5}c^2d^{10}(b+2cx)^5 - \frac{d^{10}(b+}{2\left(a+bx\right.}$$

Mathematica [A] time = 0.115103, size = 192, normalized size = 1.2

$$d^{10}\left(128c^3x\left(48a^2c^2-30ab^2c+5b^4\right)-256c^5x^3\left(4ac-3b^2\right)+128bc^4x^2\left(5b^2-12ac\right)-252c^2\left(4ac-b^2\right)^{5/2}\tan^{-1}\left(\frac{b+2c}{\sqrt{4ac-}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^10/(a + b*x + c*x^2)^3,x]
```

```
[Out] d^10*(128*c^3*(5*b^4 - 30*a*b^2*c + 48*a^2*c^2)*x + 128*b*c^4*(5*b^2 - 12*a
*c)*x^2 - 256*c^5*(-3*b^2 + 4*a*c)*x^3 + 512*b*c^6*x^4 + (1024*c^7*x^5)/5 -
 ((b^2 - 4*a*c)^4*(b + 2*c*x))/(2*(a + x*(b + c*x))^2) + (17*c*(-b^2 + 4*a*
c)^3*(b + 2*c*x))/(a + x*(b + c*x)) - 252*c^2*(-b^2 + 4*a*c)^(5/2)*ArcTan[(
b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])
```

Maple [B] time = 0.163, size = 751, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^10/(c*x^2+b*x+a)^3,x)
```

```
[Out] 6144*d^10*a^2*c^5*x+640*d^10*b^4*c^3*x+512*d^10*b*c^6*x^4-1024*d^10*x^3*a*c
^6+768*d^10*x^3*b^2*c^5+640*d^10*x^2*b^3*c^4+408*d^10/(c*x^2+b*x+a)^2*x^3*a
*b^4*c^4+3264*d^10/(c*x^2+b*x+a)^2*x^2*a^3*b*c^5-1632*d^10/(c*x^2+b*x+a)^2*
```

```
x^3*a^2*b^2*c^5-1536*d^10*x^2*a*b*c^5-3840*d^10*b^2*a*c^4*x-16128*d^10*c^5/
(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^3+252*d^10*c^2/(4*a
*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^6+2176*d^10/(c*x^2+b*x+
a)^2*x^3*a^3*c^6-34*d^10/(c*x^2+b*x+a)^2*x^3*b^6*c^3-51*d^10/(c*x^2+b*x+a)^
2*x^2*b^7*c^2+1920*d^10/(c*x^2+b*x+a)^2*x*a^4*c^5-18*d^10/(c*x^2+b*x+a)^2*x
*b^8*c+960*d^10/(c*x^2+b*x+a)^2*a^4*b*c^4-688*d^10/(c*x^2+b*x+a)^2*a^3*b^3*
c^3+156*d^10/(c*x^2+b*x+a)^2*a^2*b^5*c^2-9*d^10/(c*x^2+b*x+a)^2*a*b^7*c-302
4*d^10*c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^4-2448
*d^10/(c*x^2+b*x+a)^2*x^2*a^2*b^3*c^4+612*d^10/(c*x^2+b*x+a)^2*x^2*a*b^5*c^
3-288*d^10/(c*x^2+b*x+a)^2*x*a^3*b^2*c^4-504*d^10/(c*x^2+b*x+a)^2*x*a^2*b^4
*c^3+186*d^10/(c*x^2+b*x+a)^2*x*a*b^6*c^2+12096*d^10*c^4/(4*a*c-b^2)^(1/2)*
arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b^2+1024/5*d^10*c^7*x^5-1/2*d^10/(c
*x^2+b*x+a)^2*b^9
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^10/(c*x^2+b*x+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 2.16704, size = 2643, normalized size = 16.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^10/(c*x^2+b*x+a)^3,x, algorithm="fricas")

[Out] [1/10*(2048*c^9*d^10*x^9 + 9216*b*c^8*d^10*x^8 + 1536*(13*b^2*c^7 - 4*a*c^8
)*d^10*x^7 + 5376*(5*b^3*c^6 - 4*a*b*c^7)*d^10*x^6 + 5376*(5*b^4*c^5 - 10*a
*b^2*c^6 + 8*a^2*c^7)*d^10*x^5 + 6400*(3*b^5*c^4 - 10*a*b^3*c^5 + 12*a^2*b*
c^6)*d^10*x^4 + 20*(303*b^6*c^3 - 436*a*b^4*c^4 - 2736*a^2*b^2*c^5 + 6720*a
^3*c^6)*d^10*x^3 - 10*(51*b^7*c^2 - 1892*a*b^5*c^3 + 9488*a^2*b^3*c^4 - 140
16*a^3*b*c^5)*d^10*x^2 - 20*(9*b^8*c - 93*a*b^6*c^2 - 68*a^2*b^4*c^3 + 2064
*a^3*b^2*c^4 - 4032*a^4*c^5)*d^10*x - 5*(b^9 + 18*a*b^7*c - 312*a^2*b^5*c^2
 + 1376*a^3*b^3*c^3 - 1920*a^4*b*c^4)*d^10 + 1260*((b^4*c^4 - 8*a*b^2*c^5 +
 16*a^2*c^6)*d^10*x^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*d^10*x^3 +
 (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*d^10*x^2 + 2*(a*b^5*c^2 - 8*a^2*b^3*c
^3 + 16*a^3*b*c^4)*d^10*x + (a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^10
)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a
*c)*(2*c*x + b))/(c*x^2 + b*x + a)))/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2
+ 2*a*c)*x^2 + a^2), 1/10*(2048*c^9*d^10*x^9 + 9216*b*c^8*d^10*x^8 + 1536*(
13*b^2*c^7 - 4*a*c^8)*d^10*x^7 + 5376*(5*b^3*c^6 - 4*a*b*c^7)*d^10*x^6 + 53
76*(5*b^4*c^5 - 10*a*b^2*c^6 + 8*a^2*c^7)*d^10*x^5 + 6400*(3*b^5*c^4 - 10*a
*b^3*c^5 + 12*a^2*b*c^6)*d^10*x^4 + 20*(303*b^6*c^3 - 436*a*b^4*c^4 - 2736*
a^2*b^2*c^5 + 6720*a^3*c^6)*d^10*x^3 - 10*(51*b^7*c^2 - 1892*a*b^5*c^3 + 94
88*a^2*b^3*c^4 - 14016*a^3*b*c^5)*d^10*x^2 - 20*(9*b^8*c - 93*a*b^6*c^2 - 6
8*a^2*b^4*c^3 + 2064*a^3*b^2*c^4 - 4032*a^4*c^5)*d^10*x - 5*(b^9 + 18*a*b^7
*c - 312*a^2*b^5*c^2 + 1376*a^3*b^3*c^3 - 1920*a^4*b*c^4)*d^10 - 2520*((b^4
*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*d^10*x^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a
^2*b*c^5)*d^10*x^3 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*d^10*x^2 + 2*(a*b
```

```
^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*d^10*x + (a^2*b^4*c^2 - 8*a^3*b^2*c^
3 + 16*a^4*c^4)*d^10)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x
+ b)/(b^2 - 4*a*c)))/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a
^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**10/(c*x**2+b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.1988, size = 621, normalized size = 3.88

$$\frac{252\left(b^{6}c^{2}d^{10}-12\,ab^{4}c^{3}d^{10}+48\,a^{2}b^{2}c^{4}d^{10}-64\,a^{3}c^{5}d^{10}\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\sqrt{-b^{2}+4\,ac}}-\frac{68\,b^{6}c^{3}d^{10}x^{3}-816\,ab^{4}c^{4}d^{10}x^{3}+3264\,a^{2}b^{2}c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^10/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 252*(b^6*c^2*d^10 - 12*a*b^4*c^3*d^10 + 48*a^2*b^2*c^4*d^10 - 64*a^3*c^5*d^
10)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c) - 1/2*(68*b^6
*c^3*d^10*x^3 - 816*a*b^4*c^4*d^10*x^3 + 3264*a^2*b^2*c^5*d^10*x^3 - 4352*a
^3*c^6*d^10*x^3 + 102*b^7*c^2*d^10*x^2 - 1224*a*b^5*c^3*d^10*x^2 + 4896*a^2
*b^3*c^4*d^10*x^2 - 6528*a^3*b*c^5*d^10*x^2 + 36*b^8*c*d^10*x - 372*a*b^6*c
^2*d^10*x + 1008*a^2*b^4*c^3*d^10*x + 576*a^3*b^2*c^4*d^10*x - 3840*a^4*c^5
*d^10*x + b^9*d^10 + 18*a*b^7*c*d^10 - 312*a^2*b^5*c^2*d^10 + 1376*a^3*b^3*
c^3*d^10 - 1920*a^4*b*c^4*d^10)/(c*x^2 + b*x + a)^2 + 128/5*(8*c^22*d^10*x^
5 + 20*b*c^21*d^10*x^4 + 30*b^2*c^20*d^10*x^3 - 40*a*c^21*d^10*x^3 + 25*b^3
*c^19*d^10*x^2 - 60*a*b*c^20*d^10*x^2 + 25*b^4*c^18*d^10*x - 150*a*b^2*c^19
*d^10*x + 240*a^2*c^20*d^10*x)/c^15
```

3.1178 $\int \frac{(bd+2cdx)^9}{\left(a+bx+cx^2\right)^3}\,dx$

Optimal. Leaf size=123

$$96c^2d^9\left(b^2-4ac\right)^2\log\left(a+bx+cx^2\right)+96c^2d^9\left(b^2-4ac\right)(b+2cx)^2-\frac{8cd^9(b+2cx)^6}{a+bx+cx^2}-\frac{d^9(b+2cx)^8}{2\left(a+bx+cx^2\right)^2}+48c^2d^9(b$$

```
[Out] 96*c^2*(b^2 - 4*a*c)*d^9*(b + 2*c*x)^2 + 48*c^2*d^9*(b + 2*c*x)^4 - (d^9*(b
 + 2*c*x)^8)/(2*(a + b*x + c*x^2)^2) - (8*c*d^9*(b + 2*c*x)^6)/(a + b*x + c
*x^2) + 96*c^2*(b^2 - 4*a*c)^2*d^9*Log[a + b*x + c*x^2]
```

Rubi [A] time = 0.0848583, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {686, 692, 628}

$$96c^2d^9\left(b^2-4ac\right)^2\log\left(a+bx+cx^2\right)+96c^2d^9\left(b^2-4ac\right)(b+2cx)^2-\frac{8cd^9(b+2cx)^6}{a+bx+cx^2}-\frac{d^9(b+2cx)^8}{2\left(a+bx+cx^2\right)^2}+48c^2d^9(b$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^9/(a + b*x + c*x^2)^3,x]
```

```
[Out] 96*c^2*(b^2 - 4*a*c)*d^9*(b + 2*c*x)^2 + 48*c^2*d^9*(b + 2*c*x)^4 - (d^9*(b
 + 2*c*x)^8)/(2*(a + b*x + c*x^2)^2) - (8*c*d^9*(b + 2*c*x)^6)/(a + b*x + c
*x^2) + 96*c^2*(b^2 - 4*a*c)^2*d^9*Log[a + b*x + c*x^2]
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(bd+2cdx)^9}{(a+bx+cx^2)^3}\,dx = -\frac{d^9(b+2cx)^8}{2(a+bx+cx^2)^2} + (8cd^2)\int \frac{(bd+2cdx)^7}{(a+bx+cx^2)^2}\,dx$$
$$= -\frac{d^9(b+2cx)^8}{2(a+bx+cx^2)^2} - \frac{8cd^9(b+2cx)^6}{a+bx+cx^2} + (96c^2d^4)\int \frac{(bd+2cdx)^5}{a+bx+cx^2}\,dx$$
$$= 48c^2d^9(b+2cx)^4 - \frac{d^9(b+2cx)^8}{2(a+bx+cx^2)^2} - \frac{8cd^9(b+2cx)^6}{a+bx+cx^2} + \left(96c^2\left(b^2-4ac\right)d^6\right)\int \frac{(bd+2cdx)^3}{a+bx+cx^2}\,dx$$
$$= 96c^2\left(b^2-4ac\right)d^9(b+2cx)^2 + 48c^2d^9(b+2cx)^4 - \frac{d^9(b+2cx)^8}{2(a+bx+cx^2)^2} - \frac{8cd^9(b+2cx)^6}{a+bx+cx^2} + \left(96c^2\left(b\right.\right.$$
$$= 96c^2\left(b^2-4ac\right)d^9(b+2cx)^2 + 48c^2d^9(b+2cx)^4 - \frac{d^9(b+2cx)^8}{2(a+bx+cx^2)^2} - \frac{8cd^9(b+2cx)^6}{a+bx+cx^2} + 96c^2\left(b^2\right.$$

Mathematica [A] time = 0.0610094, size = 131, normalized size = 1.07

$$d^9\left(-384c^4x^2\left(2ac-b^2\right) + 256bc^3x\left(b^2-3ac\right) + 96c^2\left(b^2-4ac\right)^2\log(a+x(b+cx)) + \frac{16c\left(4ac-b^2\right)^3}{a+x(b+cx)} - \frac{\left(b^2-4ac\right)^4}{2(a+x(b+cx))^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^9/(a + b*x + c*x^2)^3,x]
```

```
[Out] d^9*(256*b*c^3*(b^2 - 3*a*c)*x - 384*c^4*(-b^2 + 2*a*c)*x^2 + 256*b*c^5*x^3
 + 128*c^6*x^4 - (b^2 - 4*a*c)^4/(2*(a + x*(b + c*x))^2) + (16*c*(-b^2 + 4*
a*c)^3)/(a + x*(b + c*x)) + 96*c^2*(b^2 - 4*a*c)^2*Log[a + x*(b + c*x)])
```

Maple [B] time = 0.052, size = 465, normalized size = 3.8

$$128\,d^9c^6x^4 + 256\,d^9bc^5x^3 - 768\,d^9x^2ac^5 + 384\,d^9x^2b^2c^4 - 768\,d^9abc^4x + 256\,d^9b^3c^3x + 1024\,\frac{d^9x^2a^3c^5}{\left(cx^2+bx+a\right)^2} - 768\,\frac{d^9x}{(cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^9/(c*x^2+b*x+a)^3,x)
```

```
[Out] 128*d^9*c^6*x^4+256*d^9*b*c^5*x^3-768*d^9*x^2*a*c^5+384*d^9*x^2*b^2*c^4-768
*d^9*a*b*c^4*x+256*d^9*b^3*c^3*x+1024*d^9/(c*x^2+b*x+a)^2*x^2*a^3*c^5-768*d
^9/(c*x^2+b*x+a)^2*x^2*a^2*b^2*c^4+192*d^9/(c*x^2+b*x+a)^2*x^2*a*b^4*c^3-16
*d^9/(c*x^2+b*x+a)^2*x^2*b^6*c^2+1024*d^9/(c*x^2+b*x+a)^2*x*a^3*b*c^4-768*d
^9/(c*x^2+b*x+a)^2*x*a^2*b^3*c^3+192*d^9/(c*x^2+b*x+a)^2*x*a*b^5*c^2-16*d^9
/(c*x^2+b*x+a)^2*x*b^7*c+896*d^9/(c*x^2+b*x+a)^2*a^4*c^4-640*d^9/(c*x^2+b*x
+a)^2*a^3*b^2*c^3+144*d^9/(c*x^2+b*x+a)^2*a^2*b^4*c^2-8*d^9/(c*x^2+b*x+a)^2
*a*b^6*c-1/2*d^9/(c*x^2+b*x+a)^2*b^8+1536*d^9*ln(c*x^2+b*x+a)*a^2*c^4-768*d
^9*ln(c*x^2+b*x+a)*a*b^2*c^3+96*d^9*ln(c*x^2+b*x+a)*b^4*c^2
```

Maxima [B] time = 1.17008, size = 375, normalized size = 3.05

$$128\,c^6d^9x^4 + 256\,bc^5d^9x^3 + 384\left(b^2c^4-2\,ac^5\right)d^9x^2 + 256\left(b^3c^3-3\,abc^4\right)d^9x + 96\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)d^9\log\left(cx^2+b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^9/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 128*c^6*d^9*x^4 + 256*b*c^5*d^9*x^3 + 384*(b^2*c^4 - 2*a*c^5)*d^9*x^2 + 256
*(b^3*c^3 - 3*a*b*c^4)*d^9*x + 96*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^9*
log(c*x^2 + b*x + a) - 1/2*(32*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 6
4*a^3*c^5)*d^9*x^2 + 32*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c
^4)*d^9*x + (b^8 + 16*a*b^6*c - 288*a^2*b^4*c^2 + 1280*a^3*b^2*c^3 - 1792*a
^4*c^4)*d^9)/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)
```

Fricas [B] time = 1.98246, size = 1035, normalized size = 8.41

$$\frac{256\,c^8d^9x^8 + 1024\,bc^7d^9x^7 + 1024\left(2\,b^2c^6 - ac^7\right)d^9x^6 + 512\left(5\,b^3c^5 - 6\,abc^6\right)d^9x^5 + 256\left(7\,b^4c^4 - 8\,ab^2c^5 - 11\,a^2c^6\right)d^9x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^9/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(256*c^8*d^9*x^8 + 1024*b*c^7*d^9*x^7 + 1024*(2*b^2*c^6 - a*c^7)*d^9*x^
6 + 512*(5*b^3*c^5 - 6*a*b*c^6)*d^9*x^5 + 256*(7*b^4*c^4 - 8*a*b^2*c^5 - 11
*a^2*c^6)*d^9*x^4 + 512*(b^5*c^3 + 2*a*b^3*c^4 - 11*a^2*b*c^5)*d^9*x^3 - 32
*(b^6*c^2 - 44*a*b^4*c^3 + 120*a^2*b^2*c^4 - 16*a^3*c^5)*d^9*x^2 - 32*(b^7*
c - 12*a*b^5*c^2 + 32*a^2*b^3*c^3 - 16*a^3*b*c^4)*d^9*x - (b^8 + 16*a*b^6*c
 - 288*a^2*b^4*c^2 + 1280*a^3*b^2*c^3 - 1792*a^4*c^4)*d^9 + 192*((b^4*c^4 -
 8*a*b^2*c^5 + 16*a^2*c^6)*d^9*x^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^
5)*d^9*x^3 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*d^9*x^2 + 2*(a*b^5*c^2 -
8*a^2*b^3*c^3 + 16*a^3*b*c^4)*d^9*x + (a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4
*c^4)*d^9)*log(c*x^2 + b*x + a))/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*
a*c)*x^2 + a^2)
```

Sympy [B] time = 31.3321, size = 320, normalized size = 2.6

$$256bc^5d^9x^3 + 128c^6d^9x^4 + 96c^2d^9\left(4ac - b^2\right)^2\log\left(a + bx + cx^2\right) + x^2\left(-768ac^5d^9 + 384b^2c^4d^9\right) + x\left(-768abc^4d^9 + 25\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**9/(c*x**2+b*x+a)**3,x)
```

```
[Out] 256*b*c**5*d**9*x**3 + 128*c**6*d**9*x**4 + 96*c**2*d**9*(4*a*c - b**2)**2*
log(a + b*x + c*x**2) + x**2*(-768*a*c**5*d**9 + 384*b**2*c**4*d**9) + x*(-
768*a*b*c**4*d**9 + 256*b**3*c**3*d**9) + (1792*a**4*c**4*d**9 - 1280*a**3*
b**2*c**3*d**9 + 288*a**2*b**4*c**2*d**9 - 16*a*b**6*c*d**9 - b**8*d**9 + x
**2*(2048*a**3*c**5*d**9 - 1536*a**2*b**2*c**4*d**9 + 384*a*b**4*c**3*d**9
- 32*b**6*c**2*d**9) + x*(2048*a**3*b*c**4*d**9 - 1536*a**2*b**3*c**3*d**9
+ 384*a*b**5*c**2*d**9 - 32*b**7*c*d**9))/(2*a**2 + 4*a*b*x + 4*b*c*x**3 +
2*c**2*x**4 + x**2*(4*a*c + 2*b**2))
```

Giac [B] time = 1.27181, size = 404, normalized size = 3.28

$$96\left(b^4c^2d^9 - 8\,ab^2c^3d^9 + 16\,a^2c^4d^9\right)\log\left(cx^2 + bx + a\right) - \frac{b^8d^9 + 16\,ab^6cd^9 - 288\,a^2b^4c^2d^9 + 1280\,a^3b^2c^3d^9 - 1792\,a^4c^4d^9}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^9/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 96*(b^4*c^2*d^9 - 8*a*b^2*c^3*d^9 + 16*a^2*c^4*d^9)*log(c*x^2 + b*x + a) -
1/2*(b^8*d^9 + 16*a*b^6*c*d^9 - 288*a^2*b^4*c^2*d^9 + 1280*a^3*b^2*c^3*d^9
- 1792*a^4*c^4*d^9 + 32*(b^6*c^2*d^9 - 12*a*b^4*c^3*d^9 + 48*a^2*b^2*c^4*d^
9 - 64*a^3*c^5*d^9)*x^2 + 32*(b^7*c*d^9 - 12*a*b^5*c^2*d^9 + 48*a^2*b^3*c^3
*d^9 - 64*a^3*b*c^4*d^9)*x)/(c*x^2 + b*x + a)^2 + 128*(c^18*d^9*x^4 + 2*b*c
^17*d^9*x^3 + 3*b^2*c^16*d^9*x^2 - 6*a*c^17*d^9*x^2 + 2*b^3*c^15*d^9*x - 6*
a*b*c^16*d^9*x)/c^12
```

3.1179 $\int \frac{(bd+2cdx)^8}{\left(a+bx+cx^2\right)^3} dx$

Optimal. Leaf size=134

$$140c^2d^8\left(b^2-4ac\right)(b+2cx)-140c^2d^8\left(b^2-4ac\right)^{3/2}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)-\frac{7cd^8(b+2cx)^5}{a+bx+cx^2}-\frac{d^8(b+2cx)^7}{2\left(a+bx+cx^2\right)^2}+\frac{140}{3}$$

```
[Out] 140*c^2*(b^2 - 4*a*c)*d^8*(b + 2*c*x) + (140*c^2*d^8*(b + 2*c*x)^3)/3 - (d^
8*(b + 2*c*x)^7)/(2*(a + b*x + c*x^2)^2) - (7*c*d^8*(b + 2*c*x)^5)/(a + b*x
 + c*x^2) - 140*c^2*(b^2 - 4*a*c)^(3/2)*d^8*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]]
```

Rubi [A] time = 0.101095, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {686, 692, 618, 206}

$$140c^2d^8\left(b^2-4ac\right)(b+2cx)-140c^2d^8\left(b^2-4ac\right)^{3/2}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)-\frac{7cd^8(b+2cx)^5}{a+bx+cx^2}-\frac{d^8(b+2cx)^7}{2\left(a+bx+cx^2\right)^2}+\frac{140}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^8/(a + b*x + c*x^2)^3,x]
```


```
[Out] 140*c^2*(b^2 - 4*a*c)*d^8*(b + 2*c*x) + (140*c^2*d^8*(b + 2*c*x)^3)/3 - (d^
8*(b + 2*c*x)^7)/(2*(a + b*x + c*x^2)^2) - (7*c*d^8*(b + 2*c*x)^5)/(a + b*x
 + c*x^2) - 140*c^2*(b^2 - 4*a*c)^(3/2)*d^8*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]]
```


Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(bd+2cdx)^8}{\left(a+bx+cx^2\right)^3}dx = -\frac{d^8(b+2cx)^7}{2\left(a+bx+cx^2\right)^2} + \left(7cd^2\right)\int \frac{(bd+2cdx)^6}{\left(a+bx+cx^2\right)^2}dx$$
$$= -\frac{d^8(b+2cx)^7}{2\left(a+bx+cx^2\right)^2} - \frac{7cd^8(b+2cx)^5}{a+bx+cx^2} + \left(70c^2d^4\right)\int \frac{(bd+2cdx)^4}{a+bx+cx^2}dx$$
$$= \frac{140}{3}c^2d^8(b+2cx)^3 - \frac{d^8(b+2cx)^7}{2\left(a+bx+cx^2\right)^2} - \frac{7cd^8(b+2cx)^5}{a+bx+cx^2} + \left(70c^2\left(b^2-4ac\right)d^6\right)\int \frac{(bd+2cdx)^2}{a+bx+cx^2}$$
$$= 140c^2\left(b^2-4ac\right)d^8(b+2cx) + \frac{140}{3}c^2d^8(b+2cx)^3 - \frac{d^8(b+2cx)^7}{2\left(a+bx+cx^2\right)^2} - \frac{7cd^8(b+2cx)^5}{a+bx+cx^2} + \Big(70c^2$$
$$= 140c^2\left(b^2-4ac\right)d^8(b+2cx) + \frac{140}{3}c^2d^8(b+2cx)^3 - \frac{d^8(b+2cx)^7}{2\left(a+bx+cx^2\right)^2} - \frac{7cd^8(b+2cx)^5}{a+bx+cx^2} - \Big(140c^2$$
$$= 140c^2\left(b^2-4ac\right)d^8(b+2cx) + \frac{140}{3}c^2d^8(b+2cx)^3 - \frac{d^8(b+2cx)^7}{2\left(a+bx+cx^2\right)^2} - \frac{7cd^8(b+2cx)^5}{a+bx+cx^2} - 140c^2$$

Mathematica [A] time = 0.0861327, size = 142, normalized size = 1.06

$$d^8\left(-256c^3x\left(3ac-b^2\right)+140c^2\left(4ac-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)-\frac{13c\left(b^2-4ac\right)^2(b+2cx)}{a+x(b+cx)}-\frac{\left(b^2-4ac\right)^3(b+2cx)}{2(a+x(b+cx))^2}+12\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^8/(a + b*x + c*x^2)^3,x]
```

```
[Out] d^8*(-256*c^3*(-b^2 + 3*a*c)*x + 128*b*c^4*x^2 + (256*c^5*x^3)/3 - ((b^2 -
4*a*c)^3*(b + 2*c*x))/(2*(a + x*(b + c*x))^2) - (13*c*(b^2 - 4*a*c)^2*(b +
2*c*x))/(a + x*(b + c*x)) + 140*c^2*(-b^2 + 4*a*c)^(3/2)*ArcTan[(b + 2*c*x)
/Sqrt[-b^2 + 4*a*c]])
```

Maple [B] time = 0.162, size = 526, normalized size = 3.9

$$\frac{256\,d^8c^5x^3}{3}+128\,d^8bc^4x^2-768\,d^8ac^4x+256\,d^8b^2c^3x-416\,\frac{d^8x^3a^2c^5}{\left(cx^2+bx+a\right)^2}+208\,\frac{d^8x^3ab^2c^4}{\left(cx^2+bx+a\right)^2}-26\,\frac{d^8x^3b^4c^3}{\left(cx^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^8/(c*x^2+b*x+a)^3,x)
```

```
[Out] 256/3*d^8*c^5*x^3+128*d^8*b*c^4*x^2-768*d^8*a*c^4*x+256*d^8*b^2*c^3*x-416*d
^8/(c*x^2+b*x+a)^2*x^3*a^2*c^5+208*d^8/(c*x^2+b*x+a)^2*x^3*a*b^2*c^4-26*d^8
/(c*x^2+b*x+a)^2*x^3*b^4*c^3-624*d^8/(c*x^2+b*x+a)^2*x^2*a^2*b*c^4+312*d^8/
(c*x^2+b*x+a)^2*x^2*a*b^3*c^3-39*d^8/(c*x^2+b*x+a)^2*x^2*b^5*c^2-352*d^8/(c
*x^2+b*x+a)^2*a^3*c^4*x-48*d^8/(c*x^2+b*x+a)^2*b^2*a^2*c^3*x+90*d^8/(c*x^2+
b*x+a)^2*a*b^4*c^2*x-14*d^8/(c*x^2+b*x+a)^2*b^6*c*x-176*d^8/(c*x^2+b*x+a)^2
```

```
*a^3*b*c^3+80*d^8/(c*x^2+b*x+a)^2*a^2*b^3*c^2-7*d^8/(c*x^2+b*x+a)^2*a*b^5*c
-1/2*d^8/(c*x^2+b*x+a)^2*b^7+2240*d^8*c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b
)/(4*a*c-b^2)^(1/2))*a^2-1120*d^8*c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4
*a*c-b^2)^(1/2))*a*b^2+140*d^8*c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*
c-b^2)^(1/2))*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^8/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.15014, size = 1879, normalized size = 14.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^8/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/6*(512*c^7*d^8*x^7 + 1792*b*c^6*d^8*x^6 + 3584*(b^2*c^5 - a*c^6)*d^8*x^5
 + 256*(15*b^3*c^4 - 26*a*b*c^5)*d^8*x^4 + 4*(345*b^4*c^3 + 312*a*b^2*c^4 -
 2800*a^2*c^5)*d^8*x^3 - 6*(39*b^5*c^2 - 824*a*b^3*c^3 + 2032*a^2*b*c^4)*d^
8*x^2 - 12*(7*b^6*c - 45*a*b^4*c^2 - 104*a^2*b^2*c^3 + 560*a^3*c^4)*d^8*x -
 3*(b^7 + 14*a*b^5*c - 160*a^2*b^3*c^2 + 352*a^3*b*c^3)*d^8 - 420*((b^2*c^4
 - 4*a*c^5)*d^8*x^4 + 2*(b^3*c^3 - 4*a*b*c^4)*d^8*x^3 + (b^4*c^2 - 2*a*b^2*
c^3 - 8*a^2*c^4)*d^8*x^2 + 2*(a*b^3*c^2 - 4*a^2*b*c^3)*d^8*x + (a^2*b^2*c^2
 - 4*a^3*c^3)*d^8)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c
 + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)))/(c^2*x^4 + 2*b*c*x^3
+ 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2), 1/6*(512*c^7*d^8*x^7 + 1792*b*c^6*d^8
*x^6 + 3584*(b^2*c^5 - a*c^6)*d^8*x^5 + 256*(15*b^3*c^4 - 26*a*b*c^5)*d^8*x
^4 + 4*(345*b^4*c^3 + 312*a*b^2*c^4 - 2800*a^2*c^5)*d^8*x^3 - 6*(39*b^5*c^2
 - 824*a*b^3*c^3 + 2032*a^2*b*c^4)*d^8*x^2 - 12*(7*b^6*c - 45*a*b^4*c^2 - 1
04*a^2*b^2*c^3 + 560*a^3*c^4)*d^8*x - 3*(b^7 + 14*a*b^5*c - 160*a^2*b^3*c^2
 + 352*a^3*b*c^3)*d^8 - 840*((b^2*c^4 - 4*a*c^5)*d^8*x^4 + 2*(b^3*c^3 - 4*a
*b*c^4)*d^8*x^3 + (b^4*c^2 - 2*a*b^2*c^3 - 8*a^2*c^4)*d^8*x^2 + 2*(a*b^3*c^
2 - 4*a^2*b*c^3)*d^8*x + (a^2*b^2*c^2 - 4*a^3*c^3)*d^8)*sqrt(-b^2 + 4*a*c)*
arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)))/(c^2*x^4 + 2*b*c*x^3
 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)]
```

Sympy [B] time = 12.3837, size = 469, normalized size = 3.5

$$128bc^4d^8x^2 + \frac{256c^5d^8x^3}{3} - 70c^2d^8\sqrt{-\left(4ac-b^2\right)^3}\log\left(x + \frac{280abc^3d^8 - 70b^3c^2d^8 - 70c^2d^8\sqrt{-\left(4ac-b^2\right)^3}}{560ac^4d^8 - 140b^2c^3d^8}\right) + 70c^2d^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**8/(c*x**2+b*x+a)**3,x)
```

```
[Out] 128*b*c**4*d**8*x**2 + 256*c**5*d**8*x**3/3 - 70*c**2*d**8*sqrt(-(4*a*c - b
**2)**3)*log(x + (280*a*b*c**3*d**8 - 70*b**3*c**2*d**8 - 70*c**2*d**8*sqrt
(-(4*a*c - b**2)**3))/(560*a*c**4*d**8 - 140*b**2*c**3*d**8)) + 70*c**2*d**
8*sqrt(-(4*a*c - b**2)**3)*log(x + (280*a*b*c**3*d**8 - 70*b**3*c**2*d**8 +
 70*c**2*d**8*sqrt(-(4*a*c - b**2)**3))/(560*a*c**4*d**8 - 140*b**2*c**3*d*
*8)) + x*(-768*a*c**4*d**8 + 256*b**2*c**3*d**8) - (352*a**3*b*c**3*d**8 -
160*a**2*b**3*c**2*d**8 + 14*a*b**5*c*d**8 + b**7*d**8 + x**3*(832*a**2*c**
5*d**8 - 416*a*b**2*c**4*d**8 + 52*b**4*c**3*d**8) + x**2*(1248*a**2*b*c**4
*d**8 - 624*a*b**3*c**3*d**8 + 78*b**5*c**2*d**8) + x*(704*a**3*c**4*d**8 +
 96*a**2*b**2*c**3*d**8 - 180*a*b**4*c**2*d**8 + 28*b**6*c*d**8))/(2*a**2 +
 4*a*b*x + 4*b*c*x**3 + 2*c**2*x**4 + x**2*(4*a*c + 2*b**2))
```

Giac [B] time = 1.19335, size = 425, normalized size = 3.17

$$\frac{140\left(b^4c^2d^8 - 8\,ab^2c^3d^8 + 16\,a^2c^4d^8\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}} - \frac{52\,b^4c^3d^8x^3 - 416\,ab^2c^4d^8x^3 + 832\,a^2c^5d^8x^3 + 78\,b^5c^2d^8x^2 - 6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^8/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 140*(b^4*c^2*d^8 - 8*a*b^2*c^3*d^8 + 16*a^2*c^4*d^8)*arctan((2*c*x + b)/sqr
t(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c) - 1/2*(52*b^4*c^3*d^8*x^3 - 416*a*b^2*c
^4*d^8*x^3 + 832*a^2*c^5*d^8*x^3 + 78*b^5*c^2*d^8*x^2 - 624*a*b^3*c^3*d^8*x
^2 + 1248*a^2*b*c^4*d^8*x^2 + 28*b^6*c*d^8*x - 180*a*b^4*c^2*d^8*x + 96*a^2
*b^2*c^3*d^8*x + 704*a^3*c^4*d^8*x + b^7*d^8 + 14*a*b^5*c*d^8 - 160*a^2*b^3
*c^2*d^8 + 352*a^3*b*c^3*d^8)/(c*x^2 + b*x + a)^2 + 128/3*(2*c^14*d^8*x^3 +
 3*b*c^13*d^8*x^2 + 6*b^2*c^12*d^8*x - 18*a*c^13*d^8*x)/c^9
```

3.1180 $\int \frac{(bd+2cdx)^7}{\left(a+bx+cx^2\right)^3} dx$

Optimal. Leaf size=97

$$48c^2d^7\left(b^2-4ac\right)\log\left(a+bx+cx^2\right)-\frac{6cd^7(b+2cx)^4}{a+bx+cx^2}-\frac{d^7(b+2cx)^6}{2\left(a+bx+cx^2\right)^2}+48c^2d^7(b+2cx)^2$$

```
[Out] 48*c^2*d^7*(b + 2*c*x)^2 - (d^7*(b + 2*c*x)^6)/(2*(a + b*x + c*x^2)^2) - (6
*c*d^7*(b + 2*c*x)^4)/(a + b*x + c*x^2) + 48*c^2*(b^2 - 4*a*c)*d^7*Log[a +
b*x + c*x^2]
```

Rubi [A] time = 0.0604783, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {686, 692, 628}

$$48c^2d^7\left(b^2-4ac\right)\log\left(a+bx+cx^2\right)-\frac{6cd^7(b+2cx)^4}{a+bx+cx^2}-\frac{d^7(b+2cx)^6}{2\left(a+bx+cx^2\right)^2}+48c^2d^7(b+2cx)^2$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^7/(a + b*x + c*x^2)^3,x]
```

```
[Out] 48*c^2*d^7*(b + 2*c*x)^2 - (d^7*(b + 2*c*x)^6)/(2*(a + b*x + c*x^2)^2) - (6
*c*d^7*(b + 2*c*x)^4)/(a + b*x + c*x^2) + 48*c^2*(b^2 - 4*a*c)*d^7*Log[a +
b*x + c*x^2]
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(bd+2cdx)^7}{\left(a+bx+cx^2\right)^3}\,dx = -\frac{d^7(b+2cx)^6}{2\left(a+bx+cx^2\right)^2} + \left(6cd^2\right)\int \frac{(bd+2cdx)^5}{\left(a+bx+cx^2\right)^2}\,dx$$
$$= -\frac{d^7(b+2cx)^6}{2\left(a+bx+cx^2\right)^2} - \frac{6cd^7(b+2cx)^4}{a+bx+cx^2} + \left(48c^2d^4\right)\int \frac{(bd+2cdx)^3}{a+bx+cx^2}\,dx$$
$$= 48c^2d^7(b+2cx)^2 - \frac{d^7(b+2cx)^6}{2\left(a+bx+cx^2\right)^2} - \frac{6cd^7(b+2cx)^4}{a+bx+cx^2} + \left(48c^2\left(b^2-4ac\right)d^6\right)\int \frac{bd+2cdx}{a+bx+cx^2}\,dx$$
$$= 48c^2d^7(b+2cx)^2 - \frac{d^7(b+2cx)^6}{2\left(a+bx+cx^2\right)^2} - \frac{6cd^7(b+2cx)^4}{a+bx+cx^2} + 48c^2\left(b^2-4ac\right)d^7\log\left(a+bx+cx^2\right)$$

Mathematica [A] time = 0.0376434, size = 92, normalized size = 0.95

$$d^7\left(48c^2\left(b^2-4ac\right)\log(a+x(b+cx)) - \frac{12c\left(b^2-4ac\right)^2}{a+x(b+cx)} - \frac{\left(b^2-4ac\right)^3}{2(a+x(b+cx))^2} + 64bc^3x + 64c^4x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^7/(a + b*x + c*x^2)^3,x]
```

```
[Out] d^7*(64*b*c^3*x + 64*c^4*x^2 - (b^2 - 4*a*c)^3/(2*(a + x*(b + c*x))^2) - (1
2*c*(b^2 - 4*a*c)^2)/(a + x*(b + c*x)) + 48*c^2*(b^2 - 4*a*c)*Log[a + x*(b
+ c*x)])
```

Maple [B] time = 0.051, size = 307, normalized size = 3.2

$$64\,d^7c^4x^2 + 64\,d^7bc^3x - 192\,\frac{d^7x^2a^2c^4}{\left(cx^2+bx+a\right)^2} + 96\,\frac{d^7x^2ab^2c^3}{\left(cx^2+bx+a\right)^2} - 12\,\frac{d^7x^2b^4c^2}{\left(cx^2+bx+a\right)^2} - 192\,\frac{d^7ba^2c^3x}{\left(cx^2+bx+a\right)^2} + 96\,\frac{d^7}{\left(cx^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^7/(c*x^2+b*x+a)^3,x)
```

```
[Out] 64*d^7*c^4*x^2+64*d^7*b*c^3*x-192*d^7/(c*x^2+b*x+a)^2*x^2*a^2*c^4+96*d^7/(c
*x^2+b*x+a)^2*x^2*a*b^2*c^3-12*d^7/(c*x^2+b*x+a)^2*x^2*b^4*c^2-192*d^7/(c*x
^2+b*x+a)^2*x*a^2*b*c^3+96*d^7/(c*x^2+b*x+a)^2*x*a*b^3*c^2-12*d^7/(c*x^2+b*
x+a)^2*x*b^5*c-160*d^7/(c*x^2+b*x+a)^2*a^3*c^3+72*d^7/(c*x^2+b*x+a)^2*a^2*b
^2*c^2-6*d^7/(c*x^2+b*x+a)^2*a*b^4*c-1/2*d^7/(c*x^2+b*x+a)^2*b^6-192*d^7*ln
(c*x^2+b*x+a)*a*c^3+48*d^7*ln(c*x^2+b*x+a)*b^2*c^2
```

Maxima [A] time = 1.12292, size = 255, normalized size = 2.63

$$64\,c^4d^7x^2 + 64\,bc^3d^7x + 48\left(b^2c^2-4\,ac^3\right)d^7\log\left(cx^2+bx+a\right) - \frac{24\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)d^7x^2 + 24\left(b^5c-8\,ab^3c^2+1\right.}{2\left(c^2x^4+2\,bcx^3+2\,ab\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^7/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 64*c^4*d^7*x^2 + 64*b*c^3*d^7*x + 48*(b^2*c^2 - 4*a*c^3)*d^7*log(c*x^2 + b*
x + a) - 1/2*(24*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7*x^2 + 24*(b^5*c -
 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^7*x + (b^6 + 12*a*b^4*c - 144*a^2*b^2*c^2 +
320*a^3*c^3)*d^7)/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)
```

Fricas [B] time = 2.11628, size = 724, normalized size = 7.46

$$\frac{128\,c^6d^7x^6 + 384\,bc^5d^7x^5 + 128\left(3\,b^2c^4 + 2\,ac^5\right)d^7x^4 + 128\left(b^3c^3 + 4\,abc^4\right)d^7x^3 - 8\left(3\,b^4c^2 - 56\,ab^2c^3 + 32\,a^2c^4\right)d^7x^2 - \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^7/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(128*c^6*d^7*x^6 + 384*b*c^5*d^7*x^5 + 128*(3*b^2*c^4 + 2*a*c^5)*d^7*x^
4 + 128*(b^3*c^3 + 4*a*b*c^4)*d^7*x^3 - 8*(3*b^4*c^2 - 56*a*b^2*c^3 + 32*a^
2*c^4)*d^7*x^2 - 8*(3*b^5*c - 24*a*b^3*c^2 + 32*a^2*b*c^3)*d^7*x - (b^6 + 1
2*a*b^4*c - 144*a^2*b^2*c^2 + 320*a^3*c^3)*d^7 + 96*((b^2*c^4 - 4*a*c^5)*d^
7*x^4 + 2*(b^3*c^3 - 4*a*b*c^4)*d^7*x^3 + (b^4*c^2 - 2*a*b^2*c^3 - 8*a^2*c^
4)*d^7*x^2 + 2*(a*b^3*c^2 - 4*a^2*b*c^3)*d^7*x + (a^2*b^2*c^2 - 4*a^3*c^3)*
d^7)*log(c*x^2 + b*x + a))/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x
^2 + a^2)
```

Sympy [B] time = 13.4142, size = 219, normalized size = 2.26

$$64bc^3d^7x + 64c^4d^7x^2 - 48c^2d^7\left(4ac - b^2\right)\log\left(a + bx + cx^2\right) - \frac{320a^3c^3d^7 - 144a^2b^2c^2d^7 + 12ab^4cd^7 + b^6d^7 + x^2\left(384a \cdots\right.}{2a^2 + 4abx + \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**7/(c*x**2+b*x+a)**3,x)
```

```
[Out] 64*b*c**3*d**7*x + 64*c**4*d**7*x**2 - 48*c**2*d**7*(4*a*c - b**2)*log(a +
b*x + c*x**2) - (320*a**3*c**3*d**7 - 144*a**2*b**2*c**2*d**7 + 12*a*b**4*c
*d**7 + b**6*d**7 + x**2*(384*a**2*c**4*d**7 - 192*a*b**2*c**3*d**7 + 24*b*
*4*c**2*d**7) + x*(384*a**2*b*c**3*d**7 - 192*a*b**3*c**2*d**7 + 24*b**5*c*
d**7))/(2*a**2 + 4*a*b*x + 4*b*c*x**3 + 2*c**2*x**4 + x**2*(4*a*c + 2*b**2)
)
```

Giac [B] time = 1.1791, size = 258, normalized size = 2.66

$$48\left(b^2c^2d^7 - 4\,ac^3d^7\right)\log\left(cx^2 + bx + a\right) - \frac{b^6d^7 + 12\,ab^4cd^7 - 144\,a^2b^2c^2d^7 + 320\,a^3c^3d^7 + 24\left(b^4c^2d^7 - 8\,ab^2c^3d^7 + 16 \cdots\right.}{2\left(cx^2 + bx + a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^7/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 48*(b^2*c^2*d^7 - 4*a*c^3*d^7)*log(c*x^2 + b*x + a) - 1/2*(b^6*d^7 + 12*a*b
^4*c*d^7 - 144*a^2*b^2*c^2*d^7 + 320*a^3*c^3*d^7 + 24*(b^4*c^2*d^7 - 8*a*b^
2*c^3*d^7 + 16*a^2*c^4*d^7)*x^2 + 24*(b^5*c*d^7 - 8*a*b^3*c^2*d^7 + 16*a^2*
b*c^3*d^7)*x)/(c*x^2 + b*x + a)^2 + 64*(c^10*d^7*x^2 + b*c^9*d^7*x)/c^6
```

3.1181 $\int \frac{(bd+2cdx)^6}{(a+bx+cx^2)^3} dx$

Optimal. Leaf size=108

$$-60c^2d^6\sqrt{b^2-4ac}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)-\frac{5cd^6(b+2cx)^3}{a+bx+cx^2}-\frac{d^6(b+2cx)^5}{2(a+bx+cx^2)^2}+60c^2d^6(b+2cx)$$

```
[Out] 60*c^2*d^6*(b + 2*c*x) - (d^6*(b + 2*c*x)^5)/(2*(a + b*x + c*x^2)^2) - (5*c
*d^6*(b + 2*c*x)^3)/(a + b*x + c*x^2) - 60*c^2*Sqrt[b^2 - 4*a*c]*d^6*ArcTan
h[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]
```

Rubi [A] time = 0.0726404, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {686, 692, 618, 206}

$$-60c^2d^6\sqrt{b^2-4ac}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)-\frac{5cd^6(b+2cx)^3}{a+bx+cx^2}-\frac{d^6(b+2cx)^5}{2(a+bx+cx^2)^2}+60c^2d^6(b+2cx)$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^6/(a + b*x + c*x^2)^3,x]
```

```
[Out] 60*c^2*d^6*(b + 2*c*x) - (d^6*(b + 2*c*x)^5)/(2*(a + b*x + c*x^2)^2) - (5*c
*d^6*(b + 2*c*x)^3)/(a + b*x + c*x^2) - 60*c^2*Sqrt[b^2 - 4*a*c]*d^6*ArcTan
h[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(bd+2cdx)^6}{\left(a+bx+cx^2\right)^3}\,dx = -\frac{d^6(b+2cx)^5}{2\left(a+bx+cx^2\right)^2}+\left(5cd^2\right)\int \frac{(bd+2cdx)^4}{\left(a+bx+cx^2\right)^2}\,dx$$
$$= -\frac{d^6(b+2cx)^5}{2\left(a+bx+cx^2\right)^2}-\frac{5cd^6(b+2cx)^3}{a+bx+cx^2}+\left(30c^2d^4\right)\int \frac{(bd+2cdx)^2}{a+bx+cx^2}\,dx$$
$$= 60c^2d^6(b+2cx)-\frac{d^6(b+2cx)^5}{2\left(a+bx+cx^2\right)^2}-\frac{5cd^6(b+2cx)^3}{a+bx+cx^2}+\left(30c^2\left(b^2-4ac\right)d^6\right)\int \frac{1}{a+bx+cx^2}$$
$$= 60c^2d^6(b+2cx)-\frac{d^6(b+2cx)^5}{2\left(a+bx+cx^2\right)^2}-\frac{5cd^6(b+2cx)^3}{a+bx+cx^2}-\left(60c^2\left(b^2-4ac\right)d^6\right)\text{Subst}\left(\int \frac{}{b^2-}\right.$$
$$= 60c^2d^6(b+2cx)-\frac{d^6(b+2cx)^5}{2\left(a+bx+cx^2\right)^2}-\frac{5cd^6(b+2cx)^3}{a+bx+cx^2}-60c^2\sqrt{b^2-4ac}\,d^6\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right.$$

Mathematica [A] time = 0.0650986, size = 113, normalized size = 1.05

$$d^6\left(-60c^2\sqrt{4ac-b^2}\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)+\frac{9c\left(4ac-b^2\right)(b+2cx)}{a+x(b+cx)}-\frac{\left(b^2-4ac\right)^2(b+2cx)}{2(a+x(b+cx))^2}+64c^3x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^6/(a + b*x + c*x^2)^3,x]
```

```
[Out] d^6*(64*c^3*x - ((b^2 - 4*a*c)^2*(b + 2*c*x))/(2*(a + x*(b + c*x))^2) + (9*
c*(-b^2 + 4*a*c)*(b + 2*c*x))/(a + x*(b + c*x)) - 60*c^2*Sqrt[-b^2 + 4*a*c]
*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])
```

Maple [B] time = 0.157, size = 289, normalized size = 2.7

$$64\,d^6c^3x+72\,\frac{d^6x^3ac^4}{\left(cx^2+bx+a\right)^2}-18\,\frac{d^6x^3b^2c^3}{\left(cx^2+bx+a\right)^2}+108\,\frac{d^6x^2abc^3}{\left(cx^2+bx+a\right)^2}-27\,\frac{d^6x^2b^3c^2}{\left(cx^2+bx+a\right)^2}+56\,\frac{d^6a^2c^3x}{\left(cx^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^6/(c*x^2+b*x+a)^3,x)
```

```
[Out] 64*d^6*c^3*x+72*d^6/(c*x^2+b*x+a)^2*x^3*a*c^4-18*d^6/(c*x^2+b*x+a)^2*x^3*b^
2*c^3+108*d^6/(c*x^2+b*x+a)^2*x^2*a*b*c^3-27*d^6/(c*x^2+b*x+a)^2*x^2*b^3*c^
2+56*d^6/(c*x^2+b*x+a)^2*a^2*c^3*x+26*d^6/(c*x^2+b*x+a)^2*a*b^2*c^2*x-10*d^
6/(c*x^2+b*x+a)^2*c*b^4*x+28*d^6/(c*x^2+b*x+a)^2*a^2*b*c^2-5*d^6/(c*x^2+b*x
+a)^2*a*b^3*c-1/2*d^6/(c*x^2+b*x+a)^2*b^5-60*d^6*c^2*(4*a*c-b^2)^(1/2)*arct
an((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^6/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.3224, size = 1235, normalized size = 11.44

$$\left[\frac{128\,c^5d^6x^5+256\,bc^4d^6x^4+4\left(23\,b^2c^3+100\,ac^4\right)d^6x^3-2\left(27\,b^3c^2-236\,abc^3\right)d^6x^2-4\left(5\,b^4c-13\,ab^2c^2-60\,a^2c^3\right)d^6x-}{2\left(c^2x^4+2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^6/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/2*(128*c^5*d^6*x^5 + 256*b*c^4*d^6*x^4 + 4*(23*b^2*c^3 + 100*a*c^4)*d^6*
x^3 - 2*(27*b^3*c^2 - 236*a*b*c^3)*d^6*x^2 - 4*(5*b^4*c - 13*a*b^2*c^2 - 60
*a^2*c^3)*d^6*x - (b^5 + 10*a*b^3*c - 56*a^2*b*c^2)*d^6 + 60*(c^4*d^6*x^4 +
 2*b*c^3*d^6*x^3 + 2*a*b*c^2*d^6*x + a^2*c^2*d^6 + (b^2*c^2 + 2*a*c^3)*d^6*
x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 -
4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)))/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b
^2 + 2*a*c)*x^2 + a^2), 1/2*(128*c^5*d^6*x^5 + 256*b*c^4*d^6*x^4 + 4*(23*b^
2*c^3 + 100*a*c^4)*d^6*x^3 - 2*(27*b^3*c^2 - 236*a*b*c^3)*d^6*x^2 - 4*(5*b^
4*c - 13*a*b^2*c^2 - 60*a^2*c^3)*d^6*x - (b^5 + 10*a*b^3*c - 56*a^2*b*c^2)*
d^6 - 120*(c^4*d^6*x^4 + 2*b*c^3*d^6*x^3 + 2*a*b*c^2*d^6*x + a^2*c^2*d^6 +
(b^2*c^2 + 2*a*c^3)*d^6*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*
(2*c*x + b)/(b^2 - 4*a*c)))/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*
x^2 + a^2)]
```

Sympy [B] time = 7.11762, size = 299, normalized size = 2.77

$$64c^3d^6x+c^2d^6\sqrt{-3600ac+900b^2}\log\left(x+\frac{30bc^2d^6-c^2d^6\sqrt{-3600ac+900b^2}}{60c^3d^6}\right)-c^2d^6\sqrt{-3600ac+900b^2}\log\left(x+\frac{30bc^2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**6/(c*x**2+b*x+a)**3,x)
```

```
[Out] 64*c**3*d**6*x + c**2*d**6*sqrt(-3600*a*c + 900*b**2)*log(x + (30*b*c**2*d*
*6 - c**2*d**6*sqrt(-3600*a*c + 900*b**2))/(60*c**3*d**6)) - c**2*d**6*sqrt
(-3600*a*c + 900*b**2)*log(x + (30*b*c**2*d**6 + c**2*d**6*sqrt(-3600*a*c +
 900*b**2))/(60*c**3*d**6)) + (56*a**2*b*c**2*d**6 - 10*a*b**3*c*d**6 - b**
5*d**6 + x**3*(144*a*c**4*d**6 - 36*b**2*c**3*d**6) + x**2*(216*a*b*c**3*d*
*6 - 54*b**3*c**2*d**6) + x*(112*a**2*c**3*d**6 + 52*a*b**2*c**2*d**6 - 20*
b**4*c*d**6))/(2*a**2 + 4*a*b*x + 4*b*c*x**3 + 2*c**2*x**4 + x**2*(4*a*c +
2*b**2))
```

Giac [A] time = 1.17917, size = 265, normalized size = 2.45

$$64\,c^3d^6x + \frac{60\left(b^2c^2d^6 - 4\,ac^3d^6\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2 + 4\,ac}} - \frac{36\,b^2c^3d^6x^3 - 144\,ac^4d^6x^3 + 54\,b^3c^2d^6x^2 - 216\,abc^3d^6x^2 + 20\,b}{2\left(cx^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^6/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 64*c^3*d^6*x + 60*(b^2*c^2*d^6 - 4*a*c^3*d^6)*arctan((2*c*x + b)/sqrt(-b^2
+ 4*a*c))/sqrt(-b^2 + 4*a*c) - 1/2*(36*b^2*c^3*d^6*x^3 - 144*a*c^4*d^6*x^3
+ 54*b^3*c^2*d^6*x^2 - 216*a*b*c^3*d^6*x^2 + 20*b^4*c*d^6*x - 52*a*b^2*c^2*
d^6*x - 112*a^2*c^3*d^6*x + b^5*d^6 + 10*a*b^3*c*d^6 - 56*a^2*b*c^2*d^6)/(c
*x^2 + b*x + a)^2
```

3.1182 $\int \frac{(bd+2cdx)^5}{\left(a+bx+cx^2\right)^3} \, dx$

Optimal. Leaf size=73

$$16c^2d^5\log\left(a+bx+cx^2\right)-\frac{4cd^5(b+2cx)^2}{a+bx+cx^2}-\frac{d^5(b+2cx)^4}{2\left(a+bx+cx^2\right)^2}$$

```
[Out] -(d^5*(b + 2*c*x)^4)/(2*(a + b*x + c*x^2)^2) - (4*c*d^5*(b + 2*c*x)^2)/(a +
 b*x + c*x^2) + 16*c^2*d^5*Log[a + b*x + c*x^2]
```

Rubi [A] time = 0.0379107, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {686, 628}

$$16c^2d^5\log\left(a+bx+cx^2\right)-\frac{4cd^5(b+2cx)^2}{a+bx+cx^2}-\frac{d^5(b+2cx)^4}{2\left(a+bx+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^5/(a + b*x + c*x^2)^3,x]
```

```
[Out] -(d^5*(b + 2*c*x)^4)/(2*(a + b*x + c*x^2)^2) - (4*c*d^5*(b + 2*c*x)^2)/(a +
 b*x + c*x^2) + 16*c^2*d^5*Log[a + b*x + c*x^2]
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bd+2cdx)^5}{\left(a+bx+cx^2\right)^3} \, dx &= -\frac{d^5(b+2cx)^4}{2\left(a+bx+cx^2\right)^2}+\left(4cd^2\right)\int \frac{(bd+2cdx)^3}{\left(a+bx+cx^2\right)^2} \, dx\\
&= -\frac{d^5(b+2cx)^4}{2\left(a+bx+cx^2\right)^2}-\frac{4cd^5(b+2cx)^2}{a+bx+cx^2}+\left(16c^2d^4\right)\int \frac{bd+2cdx}{a+bx+cx^2} \, dx\\
&= -\frac{d^5(b+2cx)^4}{2\left(a+bx+cx^2\right)^2}-\frac{4cd^5(b+2cx)^2}{a+bx+cx^2}+16c^2d^5\log\left(a+bx+cx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0418596, size = 65, normalized size = 0.89

$$d^5\left(16c^2\log(a+x(b+cx))-\frac{\left(b^2-4ac\right)\left(4c\left(3a+4cx^2\right)+b^2+16bcx\right)}{2(a+x(b+cx))^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^5/(a + b*x + c*x^2)^3,x]
```

```
[Out] d^5*(-((b^2 - 4*a*c)*(b^2 + 16*b*c*x + 4*c*(3*a + 4*c*x^2)))/(2*(a + x*(b +
 c*x))^2) + 16*c^2*Log[a + x*(b + c*x)])
```

Maple [B] time = 0.048, size = 181, normalized size = 2.5

$$32\frac{x^2ac^3d^5}{\left(cx^2+bx+a\right)^2}-8\frac{x^2b^2c^2d^5}{\left(cx^2+bx+a\right)^2}+32\frac{xabc^2d^5}{\left(cx^2+bx+a\right)^2}-8\frac{xb^3cd^5}{\left(cx^2+bx+a\right)^2}+24\frac{d^5a^2c^2}{\left(cx^2+bx+a\right)^2}-4\frac{d^5acb^2}{\left(cx^2+bx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^5/(c*x^2+b*x+a)^3,x)
```

```
[Out] 32*d^5/(c*x^2+b*x+a)^2*x^2*a*c^3-8*d^5/(c*x^2+b*x+a)^2*x^2*b^2*c^2+32*d^5/(
c*x^2+b*x+a)^2*b*a*c^2*x-8*d^5/(c*x^2+b*x+a)^2*b^3*c*x+24*d^5/(c*x^2+b*x+a)
^2*a^2*c^2-4*d^5/(c*x^2+b*x+a)^2*a*c*b^2-1/2*d^5/(c*x^2+b*x+a)^2*b^4+16*c^2
*d^5*ln(c*x^2+b*x+a)
```

Maxima [A] time = 1.2268, size = 167, normalized size = 2.29

$$16\,c^2d^5\log\left(cx^2+bx+a\right)-\frac{16\left(b^2c^2-4\,ac^3\right)d^5x^2+16\left(b^3c-4\,abc^2\right)d^5x+\left(b^4+8\,ab^2c-48\,a^2c^2\right)d^5}{2\left(c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\,ac\right)x^2+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^5/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 16*c^2*d^5*log(c*x^2 + b*x + a) - 1/2*(16*(b^2*c^2 - 4*a*c^3)*d^5*x^2 + 16*
(b^3*c - 4*a*b*c^2)*d^5*x + (b^4 + 8*a*b^2*c - 48*a^2*c^2)*d^5)/(c^2*x^4 +
2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)
```

Fricas [B] time = 2.35211, size = 385, normalized size = 5.27

$$-\frac{16\left(b^2c^2-4\,ac^3\right)d^5x^2+16\left(b^3c-4\,abc^2\right)d^5x+\left(b^4+8\,ab^2c-48\,a^2c^2\right)d^5-32\left(c^4d^5x^4+2\,bc^3d^5x^3+2\,abc^2d^5x+a^2c^2\right.}{2\left(c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\,ac\right)x^2+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^5/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(16*(b^2*c^2 - 4*a*c^3)*d^5*x^2 + 16*(b^3*c - 4*a*b*c^2)*d^5*x + (b^4
+ 8*a*b^2*c - 48*a^2*c^2)*d^5 - 32*(c^4*d^5*x^4 + 2*b*c^3*d^5*x^3 + 2*a*b*c
```

```
^2*d^5*x + a^2*c^2*d^5 + (b^2*c^2 + 2*a*c^3)*d^5*x^2)*log(c*x^2 + b*x + a))
/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)
```

Sympy [B] time = 6.81381, size = 141, normalized size = 1.93

$$16c^2d^5\log\left(a+bx+cx^2\right)+\frac{48a^2c^2d^5-8ab^2cd^5-b^4d^5+x^2\left(64ac^3d^5-16b^2c^2d^5\right)+x\left(64abc^2d^5-16b^3cd^5\right)}{2a^2+4abx+4bcx^3+2c^2x^4+x^2\left(4ac+2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**5/(c*x**2+b*x+a)**3,x)
```

```
[Out] 16*c**2*d**5*log(a + b*x + c*x**2) + (48*a**2*c**2*d**5 - 8*a*b**2*c*d**5 -
 b**4*d**5 + x**2*(64*a*c**3*d**5 - 16*b**2*c**2*d**5) + x*(64*a*b*c**2*d**
5 - 16*b**3*c*d**5))/(2*a**2 + 4*a*b*x + 4*b*c*x**3 + 2*c**2*x**4 + x**2*(4
*a*c + 2*b**2))
```

Giac [A] time = 1.1679, size = 149, normalized size = 2.04

$$16\,c^2d^5\log\left(cx^2+bx+a\right)-\frac{b^4d^5+8\,ab^2cd^5-48\,a^2c^2d^5+16\left(b^2c^2d^5-4\,ac^3d^5\right)x^2+16\left(b^3cd^5-4\,abc^2d^5\right)x}{2\left(cx^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^5/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 16*c^2*d^5*log(c*x^2 + b*x + a) - 1/2*(b^4*d^5 + 8*a*b^2*c*d^5 - 48*a^2*c^2
*d^5 + 16*(b^2*c^2*d^5 - 4*a*c^3*d^5)*x^2 + 16*(b^3*c*d^5 - 4*a*b*c^2*d^5)*
x)/(c*x^2 + b*x + a)^2
```

3.1183 $\int \frac{(bd+2cdx)^4}{(a+bx+cx^2)^3}\,dx$

Optimal. Leaf size=92

$$-\frac{12c^2d^4\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{3cd^4(b+2cx)}{a+bx+cx^2}-\frac{d^4(b+2cx)^3}{2\left(a+bx+cx^2\right)^2}$$

```
[Out] -(d^4*(b + 2*c*x)^3)/(2*(a + b*x + c*x^2)^2) - (3*c*d^4*(b + 2*c*x))/(a + b
*x + c*x^2) - (12*c^2*d^4*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2
- 4*a*c]
```

Rubi [A] time = 0.0566576, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {686, 618, 206}

$$-\frac{12c^2d^4\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{3cd^4(b+2cx)}{a+bx+cx^2}-\frac{d^4(b+2cx)^3}{2\left(a+bx+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^4/(a + b*x + c*x^2)^3,x]
```

```
[Out] -(d^4*(b + 2*c*x)^3)/(2*(a + b*x + c*x^2)^2) - (3*c*d^4*(b + 2*c*x))/(a + b
*x + c*x^2) - (12*c^2*d^4*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2
- 4*a*c]
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(bd+2cdx)^4}{\left(a+bx+cx^2\right)^3}\,dx = -\frac{d^4(b+2cx)^3}{2\left(a+bx+cx^2\right)^2}+\left(3cd^2\right)\int \frac{(bd+2cdx)^2}{\left(a+bx+cx^2\right)^2}\,dx$$
$$= -\frac{d^4(b+2cx)^3}{2\left(a+bx+cx^2\right)^2}-\frac{3cd^4(b+2cx)}{a+bx+cx^2}+\left(6c^2d^4\right)\int \frac{1}{a+bx+cx^2}\,dx$$
$$= -\frac{d^4(b+2cx)^3}{2\left(a+bx+cx^2\right)^2}-\frac{3cd^4(b+2cx)}{a+bx+cx^2}-\left(12c^2d^4\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)$$
$$= -\frac{d^4(b+2cx)^3}{2\left(a+bx+cx^2\right)^2}-\frac{3cd^4(b+2cx)}{a+bx+cx^2}-\frac{12c^2d^4\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.0647679, size = 89, normalized size = 0.97

$$d^4\left(\frac{12c^2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-\frac{(b+2cx)\left(2c\left(3a+5cx^2\right)+b^2+10bcx\right)}{2(a+x(b+cx))^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^4/(a + b*x + c*x^2)^3,x]
```

```
[Out] d^4*(-((b + 2*c*x)*(b^2 + 10*b*c*x + 2*c*(3*a + 5*c*x^2)))/(2*(a + x*(b + c
*x))^2) + (12*c^2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c
])
```

Maple [A] time = 0.157, size = 173, normalized size = 1.9

$$-10\frac{d^4c^3x^3}{\left(cx^2+bx+a\right)^2}-15\frac{d^4bc^2x^2}{\left(cx^2+bx+a\right)^2}-6\frac{d^4ac^2x}{\left(cx^2+bx+a\right)^2}-6\frac{d^4b^2cx}{\left(cx^2+bx+a\right)^2}-3\frac{d^4abc}{\left(cx^2+bx+a\right)^2}-\frac{d^4b^3}{2\left(cx^2+bx+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^4/(c*x^2+b*x+a)^3,x)
```

```
[Out] -10*d^4/(c*x^2+b*x+a)^2*c^3*x^3-15*d^4/(c*x^2+b*x+a)^2*b*c^2*x^2-6*d^4/(c*x
^2+b*x+a)^2*a*c^2*x-6*d^4/(c*x^2+b*x+a)^2*b^2*c*x-3*d^4/(c*x^2+b*x+a)^2*a*b
*c-1/2*d^4/(c*x^2+b*x+a)^2*b^3+12*d^4*c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b
)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.26966, size = 1312, normalized size = 14.26

$$\left[-\frac{20\left(b^2c^3-4\,ac^4\right)d^4x^3+30\left(b^3c^2-4\,abc^3\right)d^4x^2+12\left(b^4c-3\,ab^2c^2-4\,a^2c^3\right)d^4x+\left(b^5+2\,ab^3c-24\,a^2bc^2\right)d^4-12\left(c\right.}{2\left(\left(b^2c^2-4\,ac^3\right)x^4+a^2b^2-4\,a^3c+2\left(b^3c-4\,abc^2\right)x^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(20*(b^2*c^3 - 4*a*c^4)*d^4*x^3 + 30*(b^3*c^2 - 4*a*b*c^3)*d^4*x^2 +
12*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*x + (b^5 + 2*a*b^3*c - 24*a^2*b*c^
2)*d^4 - 12*(c^4*d^4*x^4 + 2*b*c^3*d^4*x^3 + 2*a*b*c^2*d^4*x + a^2*c^2*d^4
+ (b^2*c^2 + 2*a*c^3)*d^4*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x +
 b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)))/((b^2*c^2
 - 4*a*c^3)*x^4 + a^2*b^2 - 4*a^3*c + 2*(b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 2*
a*b^2*c - 8*a^2*c^2)*x^2 + 2*(a*b^3 - 4*a^2*b*c)*x), -1/2*(20*(b^2*c^3 - 4*
a*c^4)*d^4*x^3 + 30*(b^3*c^2 - 4*a*b*c^3)*d^4*x^2 + 12*(b^4*c - 3*a*b^2*c^2
 - 4*a^2*c^3)*d^4*x + (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^4 + 24*(c^4*d^4*x^
4 + 2*b*c^3*d^4*x^3 + 2*a*b*c^2*d^4*x + a^2*c^2*d^4 + (b^2*c^2 + 2*a*c^3)*d
^4*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*
a*c)))/((b^2*c^2 - 4*a*c^3)*x^4 + a^2*b^2 - 4*a^3*c + 2*(b^3*c - 4*a*b*c^2)
*x^3 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*x^2 + 2*(a*b^3 - 4*a^2*b*c)*x)]
```

Sympy [B] time = 3.96293, size = 303, normalized size = 3.29

$$-6c^2d^4\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{-24ac^3d^4\sqrt{-\frac{1}{4ac-b^2}}+6b^2c^2d^4\sqrt{-\frac{1}{4ac-b^2}}+6bc^2d^4}{12c^3d^4}\right)+6c^2d^4\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{24ac^3d^4}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**4/(c*x**2+b*x+a)**3,x)
```

```
[Out] -6*c**2*d**4*sqrt(-1/(4*a*c - b**2))*log(x + (-24*a*c**3*d**4*sqrt(-1/(4*a*
c - b**2)) + 6*b**2*c**2*d**4*sqrt(-1/(4*a*c - b**2)) + 6*b*c**2*d**4)/(12*
c**3*d**4)) + 6*c**2*d**4*sqrt(-1/(4*a*c - b**2))*log(x + (24*a*c**3*d**4*s
qrt(-1/(4*a*c - b**2)) - 6*b**2*c**2*d**4*sqrt(-1/(4*a*c - b**2)) + 6*b*c**
2*d**4)/(12*c**3*d**4)) - (6*a*b*c*d**4 + b**3*d**4 + 30*b*c**2*d**4*x**2 +
 20*c**3*d**4*x**3 + x*(12*a*c**2*d**4 + 12*b**2*c*d**4))/(2*a**2 + 4*a*b*x
 + 4*b*c*x**3 + 2*c**2*x**4 + x**2*(4*a*c + 2*b**2))
```

Giac [A] time = 1.26437, size = 154, normalized size = 1.67

$$\frac{12\,c^2d^4\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}-\frac{20\,c^3d^4x^3+30\,bc^2d^4x^2+12\,b^2cd^4x+12\,ac^2d^4x+b^3d^4+6\,abcd^4}{2\left(cx^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 12*c^2*d^4*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c) - 1/2*
(20*c^3*d^4*x^3 + 30*b*c^2*d^4*x^2 + 12*b^2*c*d^4*x + 12*a*c^2*d^4*x + b^3*
d^4 + 6*a*b*c*d^4)/(c*x^2 + b*x + a)^2
```

3.1184 $\int \frac{(bd+2cdx)^3}{\left(a+bx+cx^2\right)^3}\,dx$

Optimal. Leaf size=37

$$-\frac{d^3(b+2cx)^4}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

```
[Out] -(d^3*(b + 2*c*x)^4)/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2)
```

Rubi [A] time = 0.0145966, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {682}

$$-\frac{d^3(b+2cx)^4}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^3/(a + b*x + c*x^2)^3,x]
```

```
[Out] -(d^3*(b + 2*c*x)^4)/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2)
```

Rule 682

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c,
0] && EqQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(bd+2cdx)^3}{\left(a+bx+cx^2\right)^3}\,dx=-\frac{d^3(b+2cx)^4}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

Mathematica [A] time = 0.0243607, size = 38, normalized size = 1.03

$$-\frac{d^3\left(4c\left(a+2cx^2\right)+b^2+8bcx\right)}{2(a+x(b+cx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^3/(a + b*x + c*x^2)^3,x]
```

```
[Out] -(d^3*(b^2 + 8*b*c*x + 4*c*(a + 2*c*x^2)))/(2*(a + x*(b + c*x))^2)
```

Maple [A] time = 0.046, size = 40, normalized size = 1.1

$$\frac{d^3}{\left(cx^2+bx+a\right)^2}\left(-4\,c^2x^2-4\,bcx-2\,ac-\frac{b^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^3/(c*x^2+b*x+a)^3,x)
```

```
[Out] d^3*(-4*c^2*x^2-4*b*c*x-2*a*c-1/2*b^2)/(c*x^2+b*x+a)^2
```

Maxima [B] time = 1.13308, size = 96, normalized size = 2.59

$$-\frac{8\,c^{2}d^{3}x^{2}+8\,bcd^{3}x+\left(b^{2}+4\,ac\right)d^{3}}{2\left(c^{2}x^{4}+2\,bcx^{3}+2\,abx+\left(b^{2}+2\,ac\right)x^{2}+a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(8*c^2*d^3*x^2 + 8*b*c*d^3*x + (b^2 + 4*a*c)*d^3)/(c^2*x^4 + 2*b*c*x^3
 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)
```

Fricas [B] time = 2.03639, size = 154, normalized size = 4.16

$$-\frac{8\,c^{2}d^{3}x^{2}+8\,bcd^{3}x+\left(b^{2}+4\,ac\right)d^{3}}{2\left(c^{2}x^{4}+2\,bcx^{3}+2\,abx+\left(b^{2}+2\,ac\right)x^{2}+a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(8*c^2*d^3*x^2 + 8*b*c*d^3*x + (b^2 + 4*a*c)*d^3)/(c^2*x^4 + 2*b*c*x^3
 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)
```

Sympy [B] time = 2.43457, size = 80, normalized size = 2.16

$$-\frac{4acd^{3}+b^{2}d^{3}+8bcd^{3}x+8c^{2}d^{3}x^{2}}{2a^{2}+4abx+4bcx^{3}+2c^{2}x^{4}+x^{2}\left(4ac+2b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**3/(c*x**2+b*x+a)**3,x)
```

```
[Out] -(4*a*c*d**3 + b**2*d**3 + 8*b*c*d**3*x + 8*c**2*d**3*x**2)/(2*a**2 + 4*a*b
*x + 4*b*c*x**3 + 2*c**2*x**4 + x**2*(4*a*c + 2*b**2))
```

Giac [A] time = 1.18235, size = 65, normalized size = 1.76

$$-\frac{8\,c^{2}d^{3}x^{2}+8\,bcd^{3}x+b^{2}d^{3}+4\,acd^{3}}{2\left(cx^{2}+bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(8*c^2*d^3*x^2 + 8*b*c*d^3*x + b^2*d^3 + 4*a*c*d^3)/(c*x^2 + b*x + a)^
2
```

3.1185 $\int \frac{(bd+2cdx)^2}{(a+bx+cx^2)^3} dx$

Optimal. Leaf size=100

$$\frac{4c^2d^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{cd^2(b+2cx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{d^2(b+2cx)}{2(a+bx+cx^2)^2}$$

```
[Out] -(d^2*(b + 2*c*x))/(2*(a + b*x + c*x^2)^2) - (c*d^2*(b + 2*c*x))/((b^2 - 4*
a*c)*(a + b*x + c*x^2)) + (4*c^2*d^2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]
)/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.0528953, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {686, 614, 618, 206}

$$\frac{4c^2d^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{cd^2(b+2cx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{d^2(b+2cx)}{2(a+bx+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^2/(a + b*x + c*x^2)^3,x]
```

```
[Out] -(d^2*(b + 2*c*x))/(2*(a + b*x + c*x^2)^2) - (c*d^2*(b + 2*c*x))/((b^2 - 4*
a*c)*(a + b*x + c*x^2)) + (4*c^2*d^2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]
)/(b^2 - 4*a*c)^(3/2)
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(bd+2cdx)^2}{\left(a+bx+cx^2\right)^3}\,dx &= -\frac{d^2(b+2cx)}{2\left(a+bx+cx^2\right)^2}+\left(cd^2\right)\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx\\ &= -\frac{d^2(b+2cx)}{2\left(a+bx+cx^2\right)^2}-\frac{cd^2(b+2cx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\left(2c^2d^2\right)\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac}\\ &= -\frac{d^2(b+2cx)}{2\left(a+bx+cx^2\right)^2}-\frac{cd^2(b+2cx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\left(4c^2d^2\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{b^2-4ac}\\ &= -\frac{d^2(b+2cx)}{2\left(a+bx+cx^2\right)^2}-\frac{cd^2(b+2cx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{4c^2d^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0910937, size = 98, normalized size = 0.98

$$d^2\left(\frac{4c^2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}-\frac{(b+2cx)\left(2c\left(cx^2-a\right)+b^2+2bcx\right)}{2\left(b^2-4ac\right)(a+x(b+cx))^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^2/(a + b*x + c*x^2)^3,x]
```

```
[Out] d^2*(-((b + 2*c*x)*(b^2 + 2*b*c*x + 2*c*(-a + c*x^2)))/(2*(b^2 - 4*a*c)*(a
+ x*(b + c*x))^2) + (4*c^2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 +
4*a*c)^(3/2))
```

Maple [B] time = 0.155, size = 245, normalized size = 2.5

$$2\,\frac{d^2c^3x^3}{\left(cx^2+bx+a\right)^2\left(4\,ac-b^2\right)}+3\,\frac{d^2bc^2x^2}{\left(cx^2+bx+a\right)^2\left(4\,ac-b^2\right)}-2\,\frac{c^2d^2xa}{\left(cx^2+bx+a\right)^2\left(4\,ac-b^2\right)}+2\,\frac{cd^2xb^2}{\left(cx^2+bx+a\right)^2\left(4\,ac\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^2/(c*x^2+b*x+a)^3,x)
```

```
[Out] 2*d^2/(c*x^2+b*x+a)^2*c^3/(4*a*c-b^2)*x^3+3*d^2/(c*x^2+b*x+a)^2*b*c^2/(4*a*
c-b^2)*x^2-2*d^2/(c*x^2+b*x+a)^2*c^2/(4*a*c-b^2)*x*a+2*d^2/(c*x^2+b*x+a)^2*
c/(4*a*c-b^2)*x*b^2-d^2/(c*x^2+b*x+a)^2*b/(4*a*c-b^2)*a*c+1/2*d^2/(c*x^2+b*
x+a)^2*b^3/(4*a*c-b^2)+4*d^2*c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-
b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.15957, size = 1485, normalized size = 14.85

$$\left[-\frac{4\left(b^2c^3-4\,ac^4\right)d^2x^3+6\left(b^3c^2-4\,abc^3\right)d^2x^2+4\left(b^4c-5\,ab^2c^2+4\,a^2c^3\right)d^2x+\left(b^5-6\,ab^3c+8\,a^2bc^2\right)d^2+4\left(c^4d^2x^4+2\right.}{2\left(a^2b^4-8\,a^3b^2c+16\,a^4c^2+\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)x^4+2\left(b^5c-8\,ab^3c^2+16\,a^2b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(4*(b^2*c^3 - 4*a*c^4)*d^2*x^3 + 6*(b^3*c^2 - 4*a*b*c^3)*d^2*x^2 + 4*
(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d^2*x + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d
^2 + 4*(c^4*d^2*x^4 + 2*b*c^3*d^2*x^3 + 2*a*b*c^2*d^2*x + a^2*c^2*d^2 + (b^
2*c^2 + 2*a*c^3)*d^2*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2
- 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)))/(a^2*b^4 - 8*a
^3*b^2*c + 16*a^4*c^2 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c
 - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2
*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x), -1/2*(4*(b^2*c^3 - 4*a*c^4)*d^2*x
^3 + 6*(b^3*c^2 - 4*a*b*c^3)*d^2*x^2 + 4*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*
d^2*x + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d^2 - 8*(c^4*d^2*x^4 + 2*b*c^3*d^2*
x^3 + 2*a*b*c^2*d^2*x + a^2*c^2*d^2 + (b^2*c^2 + 2*a*c^3)*d^2*x^2)*sqrt(-b^
2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)))/(a^2*b^4
- 8*a^3*b^2*c + 16*a^4*c^2 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(
b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^
2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)]
```

Sympy [B] time = 2.7708, size = 430, normalized size = 4.3

$$-2c^2d^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x+\frac{-32a^2c^4d^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+16ab^2c^3d^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-2b^4c^2d^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+2bc^2d^2}{4c^3d^2}\right)+2c^2d^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**2/(c*x**2+b*x+a)**3,x)
```

```
[Out] -2*c**2*d**2*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**2*c**4*d**2*sqrt(-1
/(4*a*c - b**2)**3) + 16*a*b**2*c**3*d**2*sqrt(-1/(4*a*c - b**2)**3) - 2*b*
*4*c**2*d**2*sqrt(-1/(4*a*c - b**2)**3) + 2*b*c**2*d**2)/(4*c**3*d**2)) + 2
*c**2*d**2*sqrt(-1/(4*a*c - b**2)**3)*log(x + (32*a**2*c**4*d**2*sqrt(-1/(4
*a*c - b**2)**3) - 16*a*b**2*c**3*d**2*sqrt(-1/(4*a*c - b**2)**3) + 2*b**4*
c**2*d**2*sqrt(-1/(4*a*c - b**2)**3) + 2*b*c**2*d**2)/(4*c**3*d**2)) + (-2*
a*b*c*d**2 + b**3*d**2 + 6*b*c**2*d**2*x**2 + 4*c**3*d**2*x**3 + x*(-4*a*c*
*2*d**2 + 4*b**2*c*d**2))/(8*a**3*c - 2*a**2*b**2 + x**4*(8*a*c**3 - 2*b**2
*c**2) + x**3*(16*a*b*c**2 - 4*b**3*c) + x**2*(16*a**2*c**2 + 4*a*b**2*c -
```

```
2*b**4) + x*(16*a**2*b*c - 4*a*b**3))
```

Giac [A] time = 1.25046, size = 181, normalized size = 1.81

$$-\frac{4\,c^2d^2\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}-\frac{4\,c^3d^2x^3+6\,bc^2d^2x^2+4\,b^2cd^2x-4\,ac^2d^2x+b^3d^2-2\,abcd^2}{2\left(cx^2+bx+a\right)^2\left(b^2-4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] -4*c^2*d^2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2
+ 4*a*c)) - 1/2*(4*c^3*d^2*x^3 + 6*b*c^2*d^2*x^2 + 4*b^2*c*d^2*x - 4*a*c^2*
d^2*x + b^3*d^2 - 2*a*b*c*d^2)/((c*x^2 + b*x + a)^2*(b^2 - 4*a*c))
```

3.1186 $\int \frac{bd+2cdx}{(a+bx+cx^2)^3}\,dx$

Optimal. Leaf size=17

$$-\frac{d}{2\left(a+bx+cx^2\right)^2}$$

```
[Out] -d/(2*(a + b*x + c*x^2)^2)
```

Rubi [A] time = 0.0048227, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {629}

$$-\frac{d}{2\left(a+bx+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)/(a + b*x + c*x^2)^3,x]
```

```
[Out] -d/(2*(a + b*x + c*x^2)^2)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{bd+2cdx}{\left(a+bx+cx^2\right)^3}\,dx=-\frac{d}{2\left(a+bx+cx^2\right)^2}$$

Mathematica [A] time = 0.0064625, size = 16, normalized size = 0.94

$$-\frac{d}{2(a+x(b+cx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)/(a + b*x + c*x^2)^3,x]
```

```
[Out] -d/(2*(a + x*(b + c*x))^2)
```

Maple [A] time = 0.038, size = 16, normalized size = 0.9

$$-\frac{d}{2\left(cx^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)/(c*x^2+b*x+a)^3,x)
```

```
[Out] -1/2*d/(c*x^2+b*x+a)^2
```

Maxima [A] time = 1.33547, size = 20, normalized size = 1.18

$$-\frac{d}{2\left(cx^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*d/(c*x^2 + b*x + a)^2
```

Fricas [B] time = 2.02136, size = 89, normalized size = 5.24

$$-\frac{d}{2\left(c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\,ac\right)x^2+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*d/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)
```

Sympy [B] time = 1.81857, size = 44, normalized size = 2.59

$$-\frac{d}{2a^2+4abx+4bcx^3+2c^2x^4+x^2\left(4ac+2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)/(c*x**2+b*x+a)**3,x)
```

```
[Out] -d/(2*a**2 + 4*a*b*x + 4*b*c*x**3 + 2*c**2*x**4 + x**2*(4*a*c + 2*b**2))
```

Giac [A] time = 1.22106, size = 20, normalized size = 1.18

$$-\frac{d}{2\left(cx^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*d/(c*x^2 + b*x + a)^2
```

3.1187 $\int \frac{1}{(bd+2cdx)\left(a+bx+cx^2\right)^3}\,dx$

Optimal. Leaf size=112

$$\frac{16c^2\log\left(a+bx+cx^2\right)}{d\left(b^2-4ac\right)^3}-\frac{32c^2\log(b+2cx)}{d\left(b^2-4ac\right)^3}+\frac{4c}{d\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{1}{2d\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

```
[Out] -1/(2*(b^2 - 4*a*c)*d*(a + b*x + c*x^2)^2) + (4*c)/((b^2 - 4*a*c)^2*d*(a +
b*x + c*x^2)) - (32*c^2*Log[b + 2*c*x])/((b^2 - 4*a*c)^3*d) + (16*c^2*Log[a
 + b*x + c*x^2])/((b^2 - 4*a*c)^3*d)
```

Rubi [A] time = 0.0596157, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {687, 681, 31, 628}

$$\frac{16c^2\log\left(a+bx+cx^2\right)}{d\left(b^2-4ac\right)^3}-\frac{32c^2\log(b+2cx)}{d\left(b^2-4ac\right)^3}+\frac{4c}{d\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{1}{2d\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)*(a + b*x + c*x^2)^3),x]
```

```
[Out] -1/(2*(b^2 - 4*a*c)*d*(a + b*x + c*x^2)^2) + (4*c)/((b^2 - 4*a*c)^2*d*(a +
b*x + c*x^2)) - (32*c^2*Log[b + 2*c*x])/((b^2 - 4*a*c)^3*d) + (16*c^2*Log[a
 + b*x + c*x^2])/((b^2 - 4*a*c)^3*d)
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 681

```
Int[1/(((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
:> Dist[(-4*b*c)/(d*(b^2 - 4*a*c)), Int[1/(b + 2*c*x), x], x] + Dist[b^2/(d
^2*(b^2 - 4*a*c)), Int[(d + e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)\left(a+bx+cx^2\right)^3}\,dx = -\frac{1}{2\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^2} - \frac{(4c)\int \frac{1}{(bd+2cdx)\left(a+bx+cx^2\right)^2}\,dx}{b^2-4ac}$$

$$= -\frac{1}{2\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^2} + \frac{4c}{\left(b^2-4ac\right)^2 d\left(a+bx+cx^2\right)} + \frac{\left(16c^2\right)\int \frac{}{(bd+2cd}}{\left(b^2-4\right.}$$

$$= -\frac{1}{2\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^2} + \frac{4c}{\left(b^2-4ac\right)^2 d\left(a+bx+cx^2\right)} + \frac{\left(16c^2\right)\int \frac{bd+2cd}{a+bx+c}}{\left(b^2-4ac\right)^3 d}$$

$$= -\frac{1}{2\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^2} + \frac{4c}{\left(b^2-4ac\right)^2 d\left(a+bx+cx^2\right)} - \frac{32c^2\log(b+2c}{\left(b^2-4ac\right)^3 d}$$

Mathematica [A] time = 0.0747044, size = 90, normalized size = 0.8

$$\frac{\frac{8c\left(b^2-4ac\right)}{a+x(b+cx)} - \frac{\left(b^2-4ac\right)^2}{(a+x(b+cx))^2} + 32c^2\log(a+x(b+cx)) - 64c^2\log(b+2cx)}{2d\left(b^2-4ac\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)*(a + b*x + c*x^2)^3),x]
```

```
[Out] (-((b^2 - 4*a*c)^2/(a + x*(b + c*x))^2) + (8*c*(b^2 - 4*a*c))/(a + x*(b + c
*x)) - 64*c^2*Log[b + 2*c*x] + 32*c^2*Log[a + x*(b + c*x)])/(2*(b^2 - 4*a*c
)^3*d)
```

Maple [B] time = 0.057, size = 304, normalized size = 2.7

$$16\,\frac{ax^2c^3}{d\left(4\,ac-b^2\right)^3\left(cx^2+bx+a\right)^2} - 4\,\frac{b^2x^2c^2}{d\left(4\,ac-b^2\right)^3\left(cx^2+bx+a\right)^2} + 16\,\frac{bac^2x}{d\left(4\,ac-b^2\right)^3\left(cx^2+bx+a\right)^2} - 4\,\frac{}{d\left(4\,ac-b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)/(c*x^2+b*x+a)^3,x)
```

```
[Out] 16/d/(4*a*c-b^2)^3/(c*x^2+b*x+a)^2*x^2*a*c^3-4/d/(4*a*c-b^2)^3/(c*x^2+b*x+a
)^2*x^2*b^2*c^2+16/d/(4*a*c-b^2)^3/(c*x^2+b*x+a)^2*b*a*c^2*x-4/d/(4*a*c-b^2
)^3/(c*x^2+b*x+a)^2*b^3*c*x+24/d/(4*a*c-b^2)^3/(c*x^2+b*x+a)^2*a^2*c^2-8/d/
(4*a*c-b^2)^3/(c*x^2+b*x+a)^2*a*c*b^2+1/2/d/(4*a*c-b^2)^3/(c*x^2+b*x+a)^2*b
^4-16/d/(4*a*c-b^2)^3*c^2*ln(c*x^2+b*x+a)+32/d*c^2/(4*a*c-b^2)^3*ln(2*c*x+b
)
```

Maxima [B] time = 1.26214, size = 359, normalized size = 3.21

$$\frac{16\,c^2\log\left(cx^2+bx+a\right)}{\left(b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3\right)d} - \frac{32\,c^2\log\left(2\,cx+b\right)}{\left(b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3\right)d} + \frac{}{2\left(\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)dx^4+2\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 16*c^2*log(c*x^2 + b*x + a)/((b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^
3)*d) - 32*c^2*log(2*c*x + b)/((b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*
c^3)*d) + 1/2*(8*c^2*x^2 + 8*b*c*x - b^2 + 12*a*c)/((b^4*c^2 - 8*a*b^2*c^3
+ 16*a^2*c^4)*d*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*x^3 + (b^6 -
 6*a*b^4*c + 32*a^3*c^3)*d*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d*x
 + (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*d)
```

Fricas [B] time = 2.16355, size = 825, normalized size = 7.37

$$-\frac{b^4 - 16\,ab^2c + 48\,a^2c^2 - 8\left(b^2c^2 - 4\,ac^3\right)x^2 - 8\left(b^3c - 4\,abc^2\right)x - 32\left(c^4x^4 + 2\,bc^3x^3 + 2\,abc^2x + a^2c^2 + \left(b^2 \cdots\right.\right.}{2\left(\left(b^6c^2 - 12\,ab^4c^3 + 48\,a^2b^2c^4 - 64\,a^3c^5\right)dx^4 + 2\left(b^7c - 12\,ab^5c^2 + 48\,a^2b^3c^3 - 64\,a^3bc^4\right)dx^3 + \left(b^8 - 10\,ab^6c + 24\,a^2b^4c^2 \cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b^4 - 16*a*b^2*c + 48*a^2*c^2 - 8*(b^2*c^2 - 4*a*c^3)*x^2 - 8*(b^3*c
- 4*a*b*c^2)*x - 32*(c^4*x^4 + 2*b*c^3*x^3 + 2*a*b*c^2*x + a^2*c^2 + (b^2*c
^2 + 2*a*c^3)*x^2)*log(c*x^2 + b*x + a) + 64*(c^4*x^4 + 2*b*c^3*x^3 + 2*a*b
*c^2*x + a^2*c^2 + (b^2*c^2 + 2*a*c^3)*x^2)*log(2*c*x + b))/((b^6*c^2 - 12*
a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*x^4 + 2*(b^7*c - 12*a*b^5*c^2 +
48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 +
 32*a^3*b^2*c^3 - 128*a^4*c^4)*d*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3
*c^2 - 64*a^4*b*c^3)*d*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
5*c^3)*d)
```

Sympy [B] time = 6.37589, size = 246, normalized size = 2.2

$$\frac{32c^2\log\left(\frac{b}{2c} + x\right)}{d\left(4ac - b^2\right)^3} - \frac{16c^2\log\left(\frac{a}{c} + \frac{bx}{c} + x^2\right)}{d\left(4ac - b^2\right)^3} + \frac{12 \cdots}{32a^4c^2d - 16a^3b^2cd + 2a^2b^4d + x^4\left(32a^2c^4d - 16ab^2c^3d + 2b^4c^2d\right) + x^3\left(64 \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)/(c*x**2+b*x+a)**3,x)
```

```
[Out] 32*c**2*log(b/(2*c) + x)/(d*(4*a*c - b**2)**3) - 16*c**2*log(a/c + b*x/c +
x**2)/(d*(4*a*c - b**2)**3) + (12*a*c - b**2 + 8*b*c*x + 8*c**2*x**2)/(32*a
**4*c**2*d - 16*a**3*b**2*c*d + 2*a**2*b**4*d + x**4*(32*a**2*c**4*d - 16*a
*b**2*c**3*d + 2*b**4*c**2*d) + x**3*(64*a**2*b*c**3*d - 32*a*b**3*c**2*d +
 4*b**5*c*d) + x**2*(64*a**3*c**3*d - 12*a*b**4*c*d + 2*b**6*d) + x*(64*a**
3*b*c**2*d - 32*a**2*b**3*c*d + 4*a*b**5*d))
```

Giac [A] time = 1.18831, size = 254, normalized size = 2.27

$$-\frac{32\,c^3\log\left(|2\,cx + b|\right)}{b^6cd - 12\,ab^4c^2d + 48\,a^2b^2c^3d - 64\,a^3c^4d} + \frac{16\,c^2\log\left(cx^2 + bx + a\right)}{b^6d - 12\,ab^4cd + 48\,a^2b^2c^2d - 64\,a^3c^3d} - \frac{b^4 - 16\,ab^2c + 48\,a^2c^2 - 8\left(b^2c^2 - \cdots\right.}{2\left(cx^2 + bx + a\right)^2\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] -32*c^3*log(abs(2*c*x + b))/(b^6*c*d - 12*a*b^4*c^2*d + 48*a^2*b^2*c^3*d -
64*a^3*c^4*d) + 16*c^2*log(c*x^2 + b*x + a)/(b^6*d - 12*a*b^4*c*d + 48*a^2*
b^2*c^2*d - 64*a^3*c^3*d) - 1/2*(b^4 - 16*a*b^2*c + 48*a^2*c^2 - 8*(b^2*c^2
 - 4*a*c^3)*x^2 - 8*(b^3*c - 4*a*b*c^2)*x)/((c*x^2 + b*x + a)^2*(b^2 - 4*a*
c)^3*d)
```

3.1188 $\int \frac{1}{(bd+2cdx)^2\left(a+bx+cx^2\right)^3}\,dx$

Optimal. Leaf size=140

$$\frac{60c^2}{d^2\left(b^2-4ac\right)^3(b+2cx)}-\frac{60c^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d^2\left(b^2-4ac\right)^{7/2}}+\frac{5c}{d^2\left(b^2-4ac\right)^2(b+2cx)\left(a+bx+cx^2\right)}-\frac{1}{2d^2\left(b^2-4ac\right)(b+2cx)\left(a\right.}$$

```
[Out] (60*c^2)/((b^2 - 4*a*c)^3*d^2*(b + 2*c*x)) - 1/(2*(b^2 - 4*a*c)*d^2*(b + 2*
c*x)*(a + b*x + c*x^2)^2) + (5*c)/((b^2 - 4*a*c)^2*d^2*(b + 2*c*x)*(a + b*x
 + c*x^2)) - (60*c^2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)
^(7/2)*d^2)
```

Rubi [A] time = 0.0993551, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {687, 693, 618, 206}

$$\frac{60c^2}{d^2\left(b^2-4ac\right)^3(b+2cx)}-\frac{60c^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d^2\left(b^2-4ac\right)^{7/2}}+\frac{5c}{d^2\left(b^2-4ac\right)^2(b+2cx)\left(a+bx+cx^2\right)}-\frac{1}{2d^2\left(b^2-4ac\right)(b+2cx)\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^2*(a + b*x + c*x^2)^3),x]
```

```
[Out] (60*c^2)/((b^2 - 4*a*c)^3*d^2*(b + 2*c*x)) - 1/(2*(b^2 - 4*a*c)*d^2*(b + 2*
c*x)*(a + b*x + c*x^2)^2) + (5*c)/((b^2 - 4*a*c)^2*d^2*(b + 2*c*x)*(a + b*x
 + c*x^2)) - (60*c^2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)
^(7/2)*d^2)
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)^2\left(a+bx+cx^2\right)^3}\,dx = -\frac{1}{2\left(b^2-4ac\right)d^2(b+2cx)\left(a+bx+cx^2\right)^2} - \frac{(5c)\int \frac{1}{(bd+2cdx)^2\left(a+bx+cx^2\right)^2}\,dx}{b^2-4ac}$$

$$= -\frac{1}{2\left(b^2-4ac\right)d^2(b+2cx)\left(a+bx+cx^2\right)^2} + \frac{5c}{\left(b^2-4ac\right)^2 d^2(b+2cx)\left(a+bx+cx\right.}$$

$$= \frac{60c^2}{\left(b^2-4ac\right)^3 d^2(b+2cx)} - \frac{1}{2\left(b^2-4ac\right)d^2(b+2cx)\left(a+bx+cx^2\right)^2} + \frac{}{\left(b^2-4ac\right)^2}$$

$$= \frac{60c^2}{\left(b^2-4ac\right)^3 d^2(b+2cx)} - \frac{1}{2\left(b^2-4ac\right)d^2(b+2cx)\left(a+bx+cx^2\right)^2} + \frac{}{\left(b^2-4ac\right)^2}$$

$$= \frac{60c^2}{\left(b^2-4ac\right)^3 d^2(b+2cx)} - \frac{1}{2\left(b^2-4ac\right)d^2(b+2cx)\left(a+bx+cx^2\right)^2} + \frac{}{\left(b^2-4ac\right)^2}$$

Mathematica [A] time = 0.134535, size = 119, normalized size = 0.85

$$\frac{\frac{120c^2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - \frac{\left(b^2-4ac\right)(b+2cx)}{(a+x(b+cx))^2} + \frac{14c(b+2cx)}{a+x(b+cx)} + \frac{64c^2}{b+2cx}}{2d^2\left(b^2-4ac\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^2*(a + b*x + c*x^2)^3),x]
```

```
[Out] ((64*c^2)/(b + 2*c*x) - ((b^2 - 4*a*c)*(b + 2*c*x))/(a + x*(b + c*x))^2 + (
14*c*(b + 2*c*x))/(a + x*(b + c*x)) + (120*c^2*ArcTan[(b + 2*c*x)/Sqrt[-b^2
 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(b^2 - 4*a*c)^3*d^2)
```

Maple [B] time = 0.162, size = 273, normalized size = 2.

$$-14\frac{c^3x^3}{d^2\left(4\,ac-b^2\right)^3\left(cx^2+bx+a\right)^2} - 21\frac{bc^2x^2}{d^2\left(4\,ac-b^2\right)^3\left(cx^2+bx+a\right)^2} - 18\frac{ac^2x}{d^2\left(4\,ac-b^2\right)^3\left(cx^2+bx+a\right)^2} - 6\frac{}{d^2\left(4\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^2/(c*x^2+b*x+a)^3,x)
```

```
[Out] -14/d^2/(4*a*c-b^2)^3/(c*x^2+b*x+a)^2*c^3*x^3-21/d^2/(4*a*c-b^2)^3/(c*x^2+b
*x+a)^2*b*c^2*x^2-18/d^2/(4*a*c-b^2)^3/(c*x^2+b*x+a)^2*a*c^2*x-6/d^2/(4*a*c
-b^2)^3/(c*x^2+b*x+a)^2*b^2*c*x-9/d^2/(4*a*c-b^2)^3/(c*x^2+b*x+a)^2*a*b*c+1
/2/d^2/(4*a*c-b^2)^3/(c*x^2+b*x+a)^2*b^3-60/d^2/(4*a*c-b^2)^(7/2)*c^2*arcta
```

```
n((2*c*x+b)/(4*a*c-b^2)^(1/2))-32/d^2/(4*a*c-b^2)^3*c^2/(2*c*x+b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^2/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.36787, size = 2534, normalized size = 18.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^2/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(b^6 - 22*a*b^4*c + 8*a^2*b^2*c^2 + 256*a^3*c^3 - 120*(b^2*c^4 - 4*a*
c^5)*x^4 - 240*(b^3*c^3 - 4*a*b*c^4)*x^3 - 10*(13*b^4*c^2 - 32*a*b^2*c^3 -
80*a^2*c^4)*x^2 + 60*(2*c^5*x^5 + 5*b*c^4*x^4 + a^2*b*c^2 + 4*(b^2*c^3 + a*
c^4)*x^3 + (b^3*c^2 + 6*a*b*c^3)*x^2 + 2*(a*b^2*c^2 + a^2*c^3)*x)*sqrt(b^2
- 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x
+ b))/(c*x^2 + b*x + a)) - 10*(b^5*c + 16*a*b^3*c^2 - 80*a^2*b*c^3)*x)/(2*(
b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6 + 256*a^4*c^7)*d^
2*x^5 + 5*(b^9*c^2 - 16*a*b^7*c^3 + 96*a^2*b^5*c^4 - 256*a^3*b^3*c^5 + 256*
a^4*b*c^6)*d^2*x^4 + 4*(b^10*c - 15*a*b^8*c^2 + 80*a^2*b^6*c^3 - 160*a^3*b^
4*c^4 + 256*a^5*c^6)*d^2*x^3 + (b^11 - 10*a*b^9*c + 320*a^3*b^5*c^3 - 1280*
a^4*b^3*c^4 + 1536*a^5*b*c^5)*d^2*x^2 + 2*(a*b^10 - 15*a^2*b^8*c + 80*a^3*b
^6*c^2 - 160*a^4*b^4*c^3 + 256*a^6*c^5)*d^2*x + (a^2*b^9 - 16*a^3*b^7*c + 9
6*a^4*b^5*c^2 - 256*a^5*b^3*c^3 + 256*a^6*b*c^4)*d^2), -1/2*(b^6 - 22*a*b^4
*c + 8*a^2*b^2*c^2 + 256*a^3*c^3 - 120*(b^2*c^4 - 4*a*c^5)*x^4 - 240*(b^3*c
^3 - 4*a*b*c^4)*x^3 - 10*(13*b^4*c^2 - 32*a*b^2*c^3 - 80*a^2*c^4)*x^2 + 120
*(2*c^5*x^5 + 5*b*c^4*x^4 + a^2*b*c^2 + 4*(b^2*c^3 + a*c^4)*x^3 + (b^3*c^2
+ 6*a*b*c^3)*x^2 + 2*(a*b^2*c^2 + a^2*c^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sq
rt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 10*(b^5*c + 16*a*b^3*c^2 - 80
*a^2*b*c^3)*x)/(2*(b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^
6 + 256*a^4*c^7)*d^2*x^5 + 5*(b^9*c^2 - 16*a*b^7*c^3 + 96*a^2*b^5*c^4 - 256
*a^3*b^3*c^5 + 256*a^4*b*c^6)*d^2*x^4 + 4*(b^10*c - 15*a*b^8*c^2 + 80*a^2*b
^6*c^3 - 160*a^3*b^4*c^4 + 256*a^5*c^6)*d^2*x^3 + (b^11 - 10*a*b^9*c + 320*
a^3*b^5*c^3 - 1280*a^4*b^3*c^4 + 1536*a^5*b*c^5)*d^2*x^2 + 2*(a*b^10 - 15*a
^2*b^8*c + 80*a^3*b^6*c^2 - 160*a^4*b^4*c^3 + 256*a^6*c^5)*d^2*x + (a^2*b^9
 - 16*a^3*b^7*c + 96*a^4*b^5*c^2 - 256*a^5*b^3*c^3 + 256*a^6*b*c^4)*d^2)]
```

Sympy [B] time = 15.2761, size = 801, normalized size = 5.72

$$\frac{30c^2\sqrt{-\frac{1}{(4ac-b^2)^7}}\log\left(x+\frac{-7680a^4c^6\sqrt{-\frac{1}{(4ac-b^2)^7}}+7680a^3b^2c^5\sqrt{-\frac{1}{(4ac-b^2)^7}}-2880a^2b^4c^4\sqrt{-\frac{1}{(4ac-b^2)^7}}+480ab^6c^3\sqrt{-\frac{1}{(4ac-b^2)^7}}-30b^8c^2\sqrt{-\frac{1}{(4ac-b^2)^7}}+30b}{60c^3}\right.}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**2/(c*x**2+b*x+a)**3,x)
```

```
[Out] 30*c**2*sqrt(-1/(4*a*c - b**2)**7)*log(x + (-7680*a**4*c**6*sqrt(-1/(4*a*c
- b**2)**7) + 7680*a**3*b**2*c**5*sqrt(-1/(4*a*c - b**2)**7) - 2880*a**2*b*
*4*c**4*sqrt(-1/(4*a*c - b**2)**7) + 480*a*b**6*c**3*sqrt(-1/(4*a*c - b**2)
**7) - 30*b**8*c**2*sqrt(-1/(4*a*c - b**2)**7) + 30*b*c**2)/(60*c**3))/d**2
 - 30*c**2*sqrt(-1/(4*a*c - b**2)**7)*log(x + (7680*a**4*c**6*sqrt(-1/(4*a*
c - b**2)**7) - 7680*a**3*b**2*c**5*sqrt(-1/(4*a*c - b**2)**7) + 2880*a**2*
b**4*c**4*sqrt(-1/(4*a*c - b**2)**7) - 480*a*b**6*c**3*sqrt(-1/(4*a*c - b**
2)**7) + 30*b**8*c**2*sqrt(-1/(4*a*c - b**2)**7) + 30*b*c**2)/(60*c**3))/d*
*2 - (64*a**2*c**2 + 18*a*b**2*c - b**4 + 240*b*c**3*x**3 + 120*c**4*x**4 +
 x**2*(200*a*c**3 + 130*b**2*c**2) + x*(200*a*b*c**2 + 10*b**3*c))/(128*a**
5*b*c**3*d**2 - 96*a**4*b**3*c**2*d**2 + 24*a**3*b**5*c*d**2 - 2*a**2*b**7*
d**2 + x**5*(256*a**3*c**6*d**2 - 192*a**2*b**2*c**5*d**2 + 48*a*b**4*c**4*
d**2 - 4*b**6*c**3*d**2) + x**4*(640*a**3*b*c**5*d**2 - 480*a**2*b**3*c**4*
d**2 + 120*a*b**5*c**3*d**2 - 10*b**7*c**2*d**2) + x**3*(512*a**4*c**5*d**2
 + 128*a**3*b**2*c**4*d**2 - 288*a**2*b**4*c**3*d**2 + 88*a*b**6*c**2*d**2
- 8*b**8*c*d**2) + x**2*(768*a**4*b*c**4*d**2 - 448*a**3*b**3*c**3*d**2 + 4
8*a**2*b**5*c**2*d**2 + 12*a*b**7*c*d**2 - 2*b**9*d**2) + x*(256*a**5*c**4*
d**2 + 64*a**4*b**2*c**3*d**2 - 144*a**3*b**4*c**2*d**2 + 44*a**2*b**6*c*d*
*2 - 4*a*b**8*d**2))
```

Giac [B] time = 1.18273, size = 408, normalized size = 2.91

$$\frac{32\,c^8 d^{11}}{\left(b^6c^6d^{12} - 12\,ab^4c^7d^{12} + 48\,a^2b^2c^8d^{12} - 64\,a^3c^9d^{12}\right)(2\,cdx + bd)} - \frac{60\,c^2 \arctan\left(-\frac{\frac{b^2d}{2\,cdx+bd} - \frac{4\,acd}{2\,cdx+bd}}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^6 - 12\,ab^4c + 48\,a^2b^2c^2 - 64\,a^3c^3\right)\sqrt{-b^2 + 4\,ac}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^2/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 32*c^8*d^11/((b^6*c^6*d^12 - 12*a*b^4*c^7*d^12 + 48*a^2*b^2*c^8*d^12 - 64*a
^3*c^9*d^12)*(2*c*d*x + b*d)) - 60*c^2*arctan(-(b^2*d/(2*c*d*x + b*d) - 4*a
*c*d/(2*c*d*x + b*d))/sqrt(-b^2 + 4*a*c))/((b^6 - 12*a*b^4*c + 48*a^2*b^2*c
^2 - 64*a^3*c^3)*sqrt(-b^2 + 4*a*c)*d^2) - 4*(9*b^2*c^2*d/(2*c*d*x + b*d)^3
 - 36*a*c^3*d/(2*c*d*x + b*d)^3 - 7*c^2/((2*c*d*x + b*d)*d))/((b^6 - 12*a*b
^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*(b^2*d^2/(2*c*d*x + b*d)^2 - 4*a*c*d^2/
(2*c*d*x + b*d)^2 - 1)^2)
```

3.1189 $\int \frac{1}{(bd+2cdx)^3(a+bx+cx^2)^3}\,dx$

Optimal. Leaf size=154

$$\frac{48c^2\log\left(a+bx+cx^2\right)}{d^3\left(b^2-4ac\right)^4}+\frac{48c^2}{d^3\left(b^2-4ac\right)^3(b+2cx)^2}-\frac{96c^2\log(b+2cx)}{d^3\left(b^2-4ac\right)^4}+\frac{6c}{d^3\left(b^2-4ac\right)^2(b+2cx)^2\left(a+bx+cx^2\right)}-\frac{\quad}{2d^3\cdots}$$

```
[Out] (48*c^2)/((b^2 - 4*a*c)^3*d^3*(b + 2*c*x)^2) - 1/(2*(b^2 - 4*a*c)*d^3*(b +
2*c*x)^2*(a + b*x + c*x^2)^2) + (6*c)/((b^2 - 4*a*c)^2*d^3*(b + 2*c*x)^2*(a
 + b*x + c*x^2)) - (96*c^2*Log[b + 2*c*x])/((b^2 - 4*a*c)^4*d^3) + (48*c^2*
Log[a + b*x + c*x^2])/((b^2 - 4*a*c)^4*d^3)
```

Rubi [A] time = 0.0881354, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {687, 693, 681, 31, 628}

$$\frac{48c^2\log\left(a+bx+cx^2\right)}{d^3\left(b^2-4ac\right)^4}+\frac{48c^2}{d^3\left(b^2-4ac\right)^3(b+2cx)^2}-\frac{96c^2\log(b+2cx)}{d^3\left(b^2-4ac\right)^4}+\frac{6c}{d^3\left(b^2-4ac\right)^2(b+2cx)^2\left(a+bx+cx^2\right)}-\frac{\quad}{2d^3\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^3*(a + b*x + c*x^2)^3),x]
```

```
[Out] (48*c^2)/((b^2 - 4*a*c)^3*d^3*(b + 2*c*x)^2) - 1/(2*(b^2 - 4*a*c)*d^3*(b +
2*c*x)^2*(a + b*x + c*x^2)^2) + (6*c)/((b^2 - 4*a*c)^2*d^3*(b + 2*c*x)^2*(a
 + b*x + c*x^2)) - (96*c^2*Log[b + 2*c*x])/((b^2 - 4*a*c)^4*d^3) + (48*c^2*
Log[a + b*x + c*x^2])/((b^2 - 4*a*c)^4*d^3)
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 681

```
Int[1/(((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
:> Dist[(-4*b*c)/(d*(b^2 - 4*a*c)), Int[1/(b + 2*c*x), x], x] + Dist[b^2/(d
^2*(b^2 - 4*a*c)), Int[(d + e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)^3\left(a+bx+cx^2\right)^3}\,dx = -\frac{1}{2\left(b^2-4ac\right)d^3(b+2cx)^2\left(a+bx+cx^2\right)^2} - \frac{(6c)\int \frac{1}{(bd+2cdx)^3\left(a+bx+cx^2\right)^2}\,dx}{b^2-4ac}$$

$$= -\frac{1}{2\left(b^2-4ac\right)d^3(b+2cx)^2\left(a+bx+cx^2\right)^2} + \frac{6c}{\left(b^2-4ac\right)^2 d^3(b+2cx)^2\left(a+bx+\right.}$$

$$= \frac{48c^2}{\left(b^2-4ac\right)^3 d^3(b+2cx)^2} - \frac{1}{2\left(b^2-4ac\right)d^3(b+2cx)^2\left(a+bx+cx^2\right)^2} + \frac{}{\left(b^2-4a\right.}$$

$$= \frac{48c^2}{\left(b^2-4ac\right)^3 d^3(b+2cx)^2} - \frac{1}{2\left(b^2-4ac\right)d^3(b+2cx)^2\left(a+bx+cx^2\right)^2} + \frac{}{\left(b^2-4a\right.}$$

$$= \frac{48c^2}{\left(b^2-4ac\right)^3 d^3(b+2cx)^2} - \frac{1}{2\left(b^2-4ac\right)d^3(b+2cx)^2\left(a+bx+cx^2\right)^2} + \frac{}{\left(b^2-4a\right.}$$

Mathematica [A] time = 0.144913, size = 111, normalized size = 0.72

$$\frac{\frac{32c^2\left(b^2-4ac\right)}{(b+2cx)^2} + \frac{16c\left(b^2-4ac\right)}{a+x(b+cx)} - \frac{\left(b^2-4ac\right)^2}{(a+x(b+cx))^2} + 96c^2\log(a+x(b+cx)) - 192c^2\log(b+2cx)}{2d^3\left(b^2-4ac\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^3*(a + b*x + c*x^2)^3),x]
```

```
[Out] ((32*c^2*(b^2 - 4*a*c))/(b + 2*c*x)^2 - (b^2 - 4*a*c)^2/(a + x*(b + c*x))^2
 + (16*c*(b^2 - 4*a*c))/(a + x*(b + c*x)) - 192*c^2*Log[b + 2*c*x] + 96*c^2
*Log[a + x*(b + c*x)])/(2*(b^2 - 4*a*c)^4*d^3)
```

Maple [B] time = 0.057, size = 332, normalized size = 2.2

$$-32\,\frac{ax^2c^3}{d^3\left(4\,ac-b^2\right)^4\left(cx^2+bx+a\right)^2} + 8\,\frac{b^2x^2c^2}{d^3\left(4\,ac-b^2\right)^4\left(cx^2+bx+a\right)^2} - 32\,\frac{bac^2x}{d^3\left(4\,ac-b^2\right)^4\left(cx^2+bx+a\right)^2} + 8\,\frac{}{d^3\left(4\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^3/(c*x^2+b*x+a)^3,x)
```

[Out] -32/d^3/(4*a*c-b^2)^4/(c*x^2+b*x+a)^2*x^2*a*c^3+8/d^3/(4*a*c-b^2)^4/(c*x^2+b*x+a)^2*x^2*b^2*c^2-32/d^3/(4*a*c-b^2)^4/(c*x^2+b*x+a)^2*b*a*c^2*x+8/d^3/(4*a*c-b^2)^4/(c*x^2+b*x+a)^2*b^3*c*x-40/d^3/(4*a*c-b^2)^4/(c*x^2+b*x+a)^2*a^2*c^2+12/d^3/(4*a*c-b^2)^4/(c*x^2+b*x+a)^2*a*c*b^2-1/2/d^3/(4*a*c-b^2)^4/(c*x^2+b*x+a)^2*b^4+48/d^3/(4*a*c-b^2)^4*c^2*ln(c*x^2+b*x+a)-96/d^3*c^2/(4*a*c-b^2)^4*ln(2*c*x+b)-16/d^3*c^2/(4*a*c-b^2)^3/(2*c*x+b)^2

Maxima [B] time = 1.26, size = 747, normalized size = 4.85

$$\frac{}{2\left(4\left(b^6c^4-12\,ab^4c^5+48\,a^2b^2c^6-64\,a^3c^7\right)d^3x^6+12\left(b^7c^3-12\,ab^5c^4+48\,a^2b^3c^5-64\,a^3bc^6\right)d^3x^5+\left(13\,b^8c^2-148\,ab^6c^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*c*d*x+b*d)^3/(c*x^2+b*x+a)^3,x, algorithm="maxima")

[Out] 1/2*(96*c^4*x^4 + 192*b*c^3*x^3 - b^4 + 20*a*b^2*c + 32*a^2*c^2 + 36*(3*b^2*c^2 + 4*a*c^3)*x^2 + 12*(b^3*c + 12*a*b*c^2)*x)/(4*(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*d^3*x^6 + 12*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*d^3*x^5 + (13*b^8*c^2 - 148*a*b^6*c^3 + 528*a^2*b^4*c^4 - 448*a^3*b^2*c^5 - 512*a^4*c^6)*d^3*x^4 + 2*(3*b^9*c - 28*a*b^7*c^2 + 48*a^2*b^5*c^3 + 192*a^3*b^3*c^4 - 512*a^4*b*c^5)*d^3*x^3 + (b^10 - 2*a*b^8*c - 68*a^2*b^6*c^2 + 368*a^3*b^4*c^3 - 448*a^4*b^2*c^4 - 256*a^5*c^5)*d^3*x^2 + 2*(a*b^9 - 10*a^2*b^7*c + 24*a^3*b^5*c^2 + 32*a^4*b^3*c^3 - 128*a^5*b*c^4)*d^3*x + (a^2*b^8 - 12*a^3*b^6*c + 48*a^4*b^4*c^2 - 64*a^5*b^2*c^3)*d^3) + 48*c^2*log(c*x^2 + b*x + a)/((b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4)*d^3) - 96*c^2*log(2*c*x + b)/((b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4)*d^3)

Fricas [B] time = 2.33862, size = 1747, normalized size = 11.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*c*d*x+b*d)^3/(c*x^2+b*x+a)^3,x, algorithm="fricas")

[Out] -1/2*(b^6 - 24*a*b^4*c + 48*a^2*b^2*c^2 + 128*a^3*c^3 - 96*(b^2*c^4 - 4*a*c^5)*x^4 - 192*(b^3*c^3 - 4*a*b*c^4)*x^3 - 36*(3*b^4*c^2 - 8*a*b^2*c^3 - 16*a^2*c^4)*x^2 - 12*(b^5*c + 8*a*b^3*c^2 - 48*a^2*b*c^3)*x - 96*(4*c^6*x^6 + 12*b*c^5*x^5 + a^2*b^2*c^2 + (13*b^2*c^4 + 8*a*c^5)*x^4 + 2*(3*b^3*c^3 + 8*a*b*c^4)*x^3 + (b^4*c^2 + 10*a*b^2*c^3 + 4*a^2*c^4)*x^2 + 2*(a*b^3*c^2 + 2*a^2*b*c^3)*x)*log(c*x^2 + b*x + a) + 192*(4*c^6*x^6 + 12*b*c^5*x^5 + a^2*b^2*c^2 + (13*b^2*c^4 + 8*a*c^5)*x^4 + 2*(3*b^3*c^3 + 8*a*b*c^4)*x^3 + (b^4*c^2 + 10*a*b^2*c^3 + 4*a^2*c^4)*x^2 + 2*(a*b^3*c^2 + 2*a^2*b*c^3)*x)*log(2*c*x + b))/(4*(b^8*c^4 - 16*a*b^6*c^5 + 96*a^2*b^4*c^6 - 256*a^3*b^2*c^7 + 256*a^4*c^8)*d^3*x^6 + 12*(b^9*c^3 - 16*a*b^7*c^4 + 96*a^2*b^5*c^5 - 256*a^3*b^3*c^6 + 256*a^4*b*c^7)*d^3*x^5 + (13*b^10*c^2 - 200*a*b^8*c^3 + 1120*a^2*b^6*c^4 - 2560*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 + 2048*a^5*c^7)*d^3*x^4 + 2*(3*b^11*c - 40*a*b^9*c^2 + 160*a^2*b^7*c^3 - 1280*a^4*b^3*c^5 + 2048*a^5*b*c^6)*d^3*x^3 + (b^12 - 6*a*b^10*c - 60*a^2*b^8*c^2 + 640*a^3*b^6*c^3 - 1920*a^4*b^4*c^4 + 1536*a^5*b^2*c^5 + 1024*a^6*c^6)*d^3*x^2 + 2*(a*b^11 - 14*a^2*b^9*c + 64*a^3*b^7*c^2 - 64*a^4*b^5*c^3 - 256*a^5*b^3*c^4 + 512*a^6*b*c^5)*d^3*x + (a^2*b^10 - 16*a^3*b^8*c + 96*a^4*b^6*c^2 - 256*a^5*b^4*c^3 + 256*

```
a^6*b^2*c^4)*d^3)
```

Sympy [B] time = 27.5422, size = 597, normalized size = 3.88

$$-\frac{96c^2\log\left(\frac{b}{2c}+x\right)}{d^3\left(4ac-b^2\right)^4}+\frac{48c^2\log\left(\frac{a}{c}+\frac{bx}{c}+x^2\right)}{d^3\left(4ac-b^2\right)^4}-\frac{\cdots}{128a^5b^2c^3d^3-96a^4b^4c^2d^3+24a^3b^6cd^3-2a^2b^8d^3+x^6\left(512a^3c^7d^3-3\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**3/(c*x**2+b*x+a)**3,x)
```

```
[Out] -96*c**2*log(b/(2*c) + x)/(d**3*(4*a*c - b**2)**4) + 48*c**2*log(a/c + b*x/
c + x**2)/(d**3*(4*a*c - b**2)**4) - (32*a**2*c**2 + 20*a*b**2*c - b**4 + 1
92*b*c**3*x**3 + 96*c**4*x**4 + x**2*(144*a*c**3 + 108*b**2*c**2) + x*(144*
a*b*c**2 + 12*b**3*c))/(128*a**5*b**2*c**3*d**3 - 96*a**4*b**4*c**2*d**3 +
24*a**3*b**6*c*d**3 - 2*a**2*b**8*d**3 + x**6*(512*a**3*c**7*d**3 - 384*a**
2*b**2*c**6*d**3 + 96*a*b**4*c**5*d**3 - 8*b**6*c**4*d**3) + x**5*(1536*a**
3*b*c**6*d**3 - 1152*a**2*b**3*c**5*d**3 + 288*a*b**5*c**4*d**3 - 24*b**7*c
**3*d**3) + x**4*(1024*a**4*c**6*d**3 + 896*a**3*b**2*c**5*d**3 - 1056*a**2
*b**4*c**4*d**3 + 296*a*b**6*c**3*d**3 - 26*b**8*c**2*d**3) + x**3*(2048*a*
*4*b*c**5*d**3 - 768*a**3*b**3*c**4*d**3 - 192*a**2*b**5*c**3*d**3 + 112*a*
b**7*c**2*d**3 - 12*b**9*c*d**3) + x**2*(512*a**5*c**5*d**3 + 896*a**4*b**2
*c**4*d**3 - 736*a**3*b**4*c**3*d**3 + 136*a**2*b**6*c**2*d**3 + 4*a*b**8*c
*d**3 - 2*b**10*d**3) + x*(512*a**5*b*c**4*d**3 - 128*a**4*b**3*c**3*d**3 -
 96*a**3*b**5*c**2*d**3 + 40*a**2*b**7*c*d**3 - 4*a*b**9*d**3))
```

Giac [A] time = 1.2142, size = 408, normalized size = 2.65

$$-\frac{96\,c^3\log\left(|2\,cx+b|\right)}{b^8cd^3-16\,ab^6c^2d^3+96\,a^2b^4c^3d^3-256\,a^3b^2c^4d^3+256\,a^4c^5d^3}+\frac{48\,c^2\log\left(cx^2+bx+a\right)}{b^8d^3-16\,ab^6cd^3+96\,a^2b^4c^2d^3-256\,a^3b^2c^3d^3+25\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^3/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] -96*c^3*log(abs(2*c*x + b))/(b^8*c*d^3 - 16*a*b^6*c^2*d^3 + 96*a^2*b^4*c^3*
d^3 - 256*a^3*b^2*c^4*d^3 + 256*a^4*c^5*d^3) + 48*c^2*log(c*x^2 + b*x + a)/
(b^8*d^3 - 16*a*b^6*c*d^3 + 96*a^2*b^4*c^2*d^3 - 256*a^3*b^2*c^3*d^3 + 256*
a^4*c^4*d^3) + 1/2*(96*c^4*x^4 + 192*b*c^3*x^3 + 108*b^2*c^2*x^2 + 144*a*c^
3*x^2 + 12*b^3*c*x + 144*a*b*c^2*x - b^4 + 20*a*b^2*c + 32*a^2*c^2)/((b^6*d
^3 - 12*a*b^4*c*d^3 + 48*a^2*b^2*c^2*d^3 - 64*a^3*c^3*d^3)*(2*c^2*x^3 + 3*b
*c*x^2 + b^2*x + 2*a*c*x + a*b)^2)
```

3.1190 $\int \frac{1}{(bd+2cdx)^4\left(a+bx+cx^2\right)^3}\,dx$

Optimal. Leaf size=168

$$\frac{140c^2}{d^4\left(b^2-4ac\right)^4(b+2cx)}+\frac{140c^2}{3d^4\left(b^2-4ac\right)^3(b+2cx)^3}-\frac{140c^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d^4\left(b^2-4ac\right)^{9/2}}+\frac{7c}{d^4\left(b^2-4ac\right)^2(b+2cx)^3\left(a+bx+cx^2\right)}$$

```
[Out] (140*c^2)/(3*(b^2 - 4*a*c)^3*d^4*(b + 2*c*x)^3) + (140*c^2)/((b^2 - 4*a*c)^
4*d^4*(b + 2*c*x)) - 1/(2*(b^2 - 4*a*c)*d^4*(b + 2*c*x)^3*(a + b*x + c*x^2)
^2) + (7*c)/((b^2 - 4*a*c)^2*d^4*(b + 2*c*x)^3*(a + b*x + c*x^2)) - (140*c^
2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(9/2)*d^4)
```

Rubi [A] time = 0.146113, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {687, 693, 618, 206}

$$\frac{140c^2}{d^4\left(b^2-4ac\right)^4(b+2cx)}+\frac{140c^2}{3d^4\left(b^2-4ac\right)^3(b+2cx)^3}-\frac{140c^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d^4\left(b^2-4ac\right)^{9/2}}+\frac{7c}{d^4\left(b^2-4ac\right)^2(b+2cx)^3\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^4*(a + b*x + c*x^2)^3),x]
```

```
[Out] (140*c^2)/(3*(b^2 - 4*a*c)^3*d^4*(b + 2*c*x)^3) + (140*c^2)/((b^2 - 4*a*c)^
4*d^4*(b + 2*c*x)) - 1/(2*(b^2 - 4*a*c)*d^4*(b + 2*c*x)^3*(a + b*x + c*x^2)
^2) + (7*c)/((b^2 - 4*a*c)^2*d^4*(b + 2*c*x)^3*(a + b*x + c*x^2)) - (140*c^
2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(9/2)*d^4)
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)^4\left(a+bx+cx^2\right)^3}\,dx = -\frac{1}{2\left(b^2-4ac\right)d^4(b+2cx)^3\left(a+bx+cx^2\right)^2} - \frac{(7c)\int \frac{1}{(bd+2cdx)^4\left(a+bx+cx^2\right)^2}\,dx}{b^2-4ac}$$

$$= -\frac{1}{2\left(b^2-4ac\right)d^4(b+2cx)^3\left(a+bx+cx^2\right)^2} + \frac{7c}{\left(b^2-4ac\right)^2 d^4(b+2cx)^3\left(a+bx+\right.}$$

$$= \frac{140c^2}{3\left(b^2-4ac\right)^3 d^4(b+2cx)^3} - \frac{1}{2\left(b^2-4ac\right)d^4(b+2cx)^3\left(a+bx+cx^2\right)^2} + \frac{}{\left(b^2-4\right.}$$

$$= \frac{140c^2}{3\left(b^2-4ac\right)^3 d^4(b+2cx)^3} + \frac{140c^2}{\left(b^2-4ac\right)^4 d^4(b+2cx)} - \frac{1}{2\left(b^2-4ac\right)d^4(b+2cx)^3}$$

$$= \frac{140c^2}{3\left(b^2-4ac\right)^3 d^4(b+2cx)^3} + \frac{140c^2}{\left(b^2-4ac\right)^4 d^4(b+2cx)} - \frac{1}{2\left(b^2-4ac\right)d^4(b+2cx)^3}$$

$$= \frac{140c^2}{3\left(b^2-4ac\right)^3 d^4(b+2cx)^3} + \frac{140c^2}{\left(b^2-4ac\right)^4 d^4(b+2cx)} - \frac{1}{2\left(b^2-4ac\right)d^4(b+2cx)^3}$$

Mathematica [A] time = 0.338748, size = 140, normalized size = 0.83

$$\frac{\frac{64c^2\left(b^2-4ac\right)}{(b+2cx)^3} + \frac{840c^2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - \frac{3\left(b^2-4ac\right)(b+2cx)}{(a+x(b+cx))^2} + \frac{66c(b+2cx)}{a+x(b+cx)} + \frac{576c^2}{b+2cx}}{6d^4\left(b^2-4ac\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^4*(a + b*x + c*x^2)^3),x]
```

```
[Out] ((64*c^2*(b^2 - 4*a*c))/(b + 2*c*x)^3 + (576*c^2)/(b + 2*c*x) - (3*(b^2 - 4
*a*c)*(b + 2*c*x))/(a + x*(b + c*x))^2 + (66*c*(b + 2*c*x))/(a + x*(b + c*x
)) + (840*c^2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(
6*(b^2 - 4*a*c)^4*d^4)
```

Maple [A] time = 0.164, size = 301, normalized size = 1.8

$$22\,\frac{c^3x^3}{d^4\left(4\,ac-b^2\right)^4\left(cx^2+bx+a\right)^2} + 33\,\frac{bc^2x^2}{d^4\left(4\,ac-b^2\right)^4\left(cx^2+bx+a\right)^2} + 26\,\frac{ac^2x}{d^4\left(4\,ac-b^2\right)^4\left(cx^2+bx+a\right)^2} + 10\,\frac{}{d^4\left(4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^4/(c*x^2+b*x+a)^3,x)
```

```
[Out] 22/d^4/(4*a*c-b^2)^4/(c*x^2+b*x+a)^2*c^3*x^3+33/d^4/(4*a*c-b^2)^4/(c*x^2+b*
x+a)^2*b*c^2*x^2+26/d^4/(4*a*c-b^2)^4/(c*x^2+b*x+a)^2*a*c^2*x+10/d^4/(4*a*c
-b^2)^4/(c*x^2+b*x+a)^2*b^2*c*x+13/d^4/(4*a*c-b^2)^4/(c*x^2+b*x+a)^2*a*b*c-
1/2/d^4/(4*a*c-b^2)^4/(c*x^2+b*x+a)^2*b^3+140/d^4/(4*a*c-b^2)^(9/2)*c^2*arc
tan((2*c*x+b)/(4*a*c-b^2)^(1/2))+96/d^4/(4*a*c-b^2)^4*c^2/(2*c*x+b)-32/3/d^
4*c^2/(4*a*c-b^2)^3/(2*c*x+b)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^4/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.46057, size = 4655, normalized size = 27.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^4/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/6*(3*b^8 - 90*a*b^6*c - 328*a^2*b^4*c^2 + 2816*a^3*b^2*c^3 - 1024*a^4*c
^4 - 3360*(b^2*c^6 - 4*a*c^7)*x^6 - 10080*(b^3*c^5 - 4*a*b*c^6)*x^5 - 5600*
(2*b^4*c^4 - 7*a*b^2*c^5 - 4*a^2*c^6)*x^4 - 5600*(b^5*c^3 - 2*a*b^3*c^4 - 8
*a^2*b*c^5)*x^3 - 14*(83*b^6*c^2 + 204*a*b^4*c^3 - 2016*a^2*b^2*c^4 - 512*a
^3*c^5)*x^2 - 420*(8*c^7*x^7 + 28*b*c^6*x^6 + a^2*b^3*c^2 + 2*(19*b^2*c^5 +
 8*a*c^6)*x^5 + 5*(5*b^3*c^4 + 8*a*b*c^5)*x^4 + 4*(2*b^4*c^3 + 9*a*b^2*c^4
+ 2*a^2*c^5)*x^3 + (b^5*c^2 + 14*a*b^3*c^3 + 12*a^2*b*c^4)*x^2 + 2*(a*b^4*c
^2 + 3*a^2*b^2*c^3)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2
*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 14*(3*b^7*c + 12
4*a*b^5*c^2 - 416*a^2*b^3*c^3 - 512*a^3*b*c^4)*x)/(8*(b^10*c^5 - 20*a*b^8*c
^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*
d^4*x^7 + 28*(b^11*c^4 - 20*a*b^9*c^5 + 160*a^2*b^7*c^6 - 640*a^3*b^5*c^7 +
 1280*a^4*b^3*c^8 - 1024*a^5*b*c^9)*d^4*x^6 + 2*(19*b^12*c^3 - 372*a*b^10*c
^4 + 2880*a^2*b^8*c^5 - 10880*a^3*b^6*c^6 + 19200*a^4*b^4*c^7 - 9216*a^5*b^
2*c^8 - 8192*a^6*c^9)*d^4*x^5 + 5*(5*b^13*c^2 - 92*a*b^11*c^3 + 640*a^2*b^9
*c^4 - 1920*a^3*b^7*c^5 + 1280*a^4*b^5*c^6 + 5120*a^5*b^3*c^7 - 8192*a^6*b*
c^8)*d^4*x^4 + 4*(2*b^14*c - 31*a*b^12*c^2 + 142*a^2*b^10*c^3 + 120*a^3*b^8
*c^4 - 2880*a^4*b^6*c^5 + 8192*a^5*b^4*c^6 - 6656*a^6*b^2*c^7 - 2048*a^7*c^
8)*d^4*x^3 + (b^15 - 6*a*b^13*c - 108*a^2*b^11*c^2 + 1360*a^3*b^9*c^3 - 576
0*a^4*b^7*c^4 + 9216*a^5*b^5*c^5 + 1024*a^6*b^3*c^6 - 12288*a^7*b*c^7)*d^4*
x^2 + 2*(a*b^14 - 17*a^2*b^12*c + 100*a^3*b^10*c^2 - 160*a^4*b^8*c^3 - 640*
a^5*b^6*c^4 + 2816*a^6*b^4*c^5 - 3072*a^7*b^2*c^6)*d^4*x + (a^2*b^13 - 20*a
^3*b^11*c + 160*a^4*b^9*c^2 - 640*a^5*b^7*c^3 + 1280*a^6*b^5*c^4 - 1024*a^7
*b^3*c^5)*d^4), -1/6*(3*b^8 - 90*a*b^6*c - 328*a^2*b^4*c^2 + 2816*a^3*b^2*c
^3 - 1024*a^4*c^4 - 3360*(b^2*c^6 - 4*a*c^7)*x^6 - 10080*(b^3*c^5 - 4*a*b*c
^6)*x^5 - 5600*(2*b^4*c^4 - 7*a*b^2*c^5 - 4*a^2*c^6)*x^4 - 5600*(b^5*c^3 -
2*a*b^3*c^4 - 8*a^2*b*c^5)*x^3 - 14*(83*b^6*c^2 + 204*a*b^4*c^3 - 2016*a^2*
b^2*c^4 - 512*a^3*c^5)*x^2 + 840*(8*c^7*x^7 + 28*b*c^6*x^6 + a^2*b^3*c^2 +
2*(19*b^2*c^5 + 8*a*c^6)*x^5 + 5*(5*b^3*c^4 + 8*a*b*c^5)*x^4 + 4*(2*b^4*c^3
 + 9*a*b^2*c^4 + 2*a^2*c^5)*x^3 + (b^5*c^2 + 14*a*b^3*c^3 + 12*a^2*b*c^4)*x
```

```
^2 + 2*(a*b^4*c^2 + 3*a^2*b^2*c^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2
+ 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 14*(3*b^7*c + 124*a*b^5*c^2 - 416*a^2
*b^3*c^3 - 512*a^3*b*c^4)*x)/(8*(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7
- 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*d^4*x^7 + 28*(b^11*c^
4 - 20*a*b^9*c^5 + 160*a^2*b^7*c^6 - 640*a^3*b^5*c^7 + 1280*a^4*b^3*c^8 - 1
024*a^5*b*c^9)*d^4*x^6 + 2*(19*b^12*c^3 - 372*a*b^10*c^4 + 2880*a^2*b^8*c^5
 - 10880*a^3*b^6*c^6 + 19200*a^4*b^4*c^7 - 9216*a^5*b^2*c^8 - 8192*a^6*c^9)
*d^4*x^5 + 5*(5*b^13*c^2 - 92*a*b^11*c^3 + 640*a^2*b^9*c^4 - 1920*a^3*b^7*c
^5 + 1280*a^4*b^5*c^6 + 5120*a^5*b^3*c^7 - 8192*a^6*b*c^8)*d^4*x^4 + 4*(2*b
^14*c - 31*a*b^12*c^2 + 142*a^2*b^10*c^3 + 120*a^3*b^8*c^4 - 2880*a^4*b^6*c
^5 + 8192*a^5*b^4*c^6 - 6656*a^6*b^2*c^7 - 2048*a^7*c^8)*d^4*x^3 + (b^15 -
6*a*b^13*c - 108*a^2*b^11*c^2 + 1360*a^3*b^9*c^3 - 5760*a^4*b^7*c^4 + 9216*
a^5*b^5*c^5 + 1024*a^6*b^3*c^6 - 12288*a^7*b*c^7)*d^4*x^2 + 2*(a*b^14 - 17*
a^2*b^12*c + 100*a^3*b^10*c^2 - 160*a^4*b^8*c^3 - 640*a^5*b^6*c^4 + 2816*a^
6*b^4*c^5 - 3072*a^7*b^2*c^6)*d^4*x + (a^2*b^13 - 20*a^3*b^11*c + 160*a^4*b
^9*c^2 - 640*a^5*b^7*c^3 + 1280*a^6*b^5*c^4 - 1024*a^7*b^3*c^5)*d^4)]
```

Sympy [B] time = 97.5109, size = 1238, normalized size = 7.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**4/(c*x**2+b*x+a)**3,x)
```

```
[Out] -70*c**2*sqrt(-1/(4*a*c - b**2)**9)*log(x + (-71680*a**5*c**7*sqrt(-1/(4*a*
c - b**2)**9) + 89600*a**4*b**2*c**6*sqrt(-1/(4*a*c - b**2)**9) - 44800*a**
3*b**4*c**5*sqrt(-1/(4*a*c - b**2)**9) + 11200*a**2*b**6*c**4*sqrt(-1/(4*a*
c - b**2)**9) - 1400*a*b**8*c**3*sqrt(-1/(4*a*c - b**2)**9) + 70*b**10*c**2
*sqrt(-1/(4*a*c - b**2)**9) + 70*b*c**2)/(140*c**3))/d**4 + 70*c**2*sqrt(-1
/(4*a*c - b**2)**9)*log(x + (71680*a**5*c**7*sqrt(-1/(4*a*c - b**2)**9) - 8
9600*a**4*b**2*c**6*sqrt(-1/(4*a*c - b**2)**9) + 44800*a**3*b**4*c**5*sqrt(
-1/(4*a*c - b**2)**9) - 11200*a**2*b**6*c**4*sqrt(-1/(4*a*c - b**2)**9) + 1
400*a*b**8*c**3*sqrt(-1/(4*a*c - b**2)**9) - 70*b**10*c**2*sqrt(-1/(4*a*c -
 b**2)**9) + 70*b*c**2)/(140*c**3))/d**4 + (-256*a**3*c**3 + 640*a**2*b**2*
c**2 + 78*a*b**4*c - 3*b**6 + 10080*b*c**5*x**5 + 3360*c**6*x**6 + x**4*(56
00*a*c**5 + 11200*b**2*c**4) + x**3*(11200*a*b*c**4 + 5600*b**3*c**3) + x**
2*(1792*a**2*c**4 + 7504*a*b**2*c**3 + 1162*b**4*c**2) + x*(1792*a**2*b*c**
3 + 1904*a*b**3*c**2 + 42*b**5*c))/(1536*a**6*b**3*c**4*d**4 - 1536*a**5*b*
*5*c**3*d**4 + 576*a**4*b**7*c**2*d**4 - 96*a**3*b**9*c*d**4 + 6*a**2*b**11
*d**4 + x**7*(12288*a**4*c**9*d**4 - 12288*a**3*b**2*c**8*d**4 + 4608*a**2*
b**4*c**7*d**4 - 768*a*b**6*c**6*d**4 + 48*b**8*c**5*d**4) + x**6*(43008*a*
*4*b*c**8*d**4 - 43008*a**3*b**3*c**7*d**4 + 16128*a**2*b**5*c**6*d**4 - 26
88*a*b**7*c**5*d**4 + 168*b**9*c**4*d**4) + x**5*(24576*a**5*c**8*d**4 + 33
792*a**4*b**2*c**7*d**4 - 49152*a**3*b**4*c**6*d**4 + 20352*a**2*b**6*c**5*
d**4 - 3552*a*b**8*c**4*d**4 + 228*b**10*c**3*d**4) + x**4*(61440*a**5*b*c*
*7*d**4 - 23040*a**4*b**3*c**6*d**4 - 15360*a**3*b**5*c**5*d**4 + 10560*a**
2*b**7*c**4*d**4 - 2160*a*b**9*c**3*d**4 + 150*b**11*c**2*d**4) + x**3*(122
88*a**6*c**7*d**4 + 43008*a**5*b**2*c**6*d**4 - 38400*a**4*b**4*c**5*d**4 +
 7680*a**3*b**6*c**4*d**4 + 1200*a**2*b**8*c**3*d**4 - 552*a*b**10*c**2*d**
4 + 48*b**12*c*d**4) + x**2*(18432*a**6*b*c**6*d**4 + 3072*a**5*b**3*c**5*d
**4 - 13056*a**4*b**5*c**4*d**4 + 5376*a**3*b**7*c**3*d**4 - 696*a**2*b**9*
c**2*d**4 - 12*a*b**11*c*d**4 + 6*b**13*d**4) + x*(9216*a**6*b**2*c**5*d**4
 - 6144*a**5*b**4*c**4*d**4 + 384*a**4*b**6*c**3*d**4 + 576*a**3*b**8*c**2*
d**4 - 156*a**2*b**10*c*d**4 + 12*a*b**12*d**4))
```

Giac [A] time = 1.25099, size = 420, normalized size = 2.5

$$\frac{140\, c^2 \arctan\left(\frac{2\, cx+b}{\sqrt{-b^2+4\, ac}}\right)}{\left(b^8 d^4 - 16\, ab^6 cd^4 + 96\, a^2 b^4 c^2 d^4 - 256\, a^3 b^2 c^3 d^4 + 256\, a^4 c^4 d^4\right)\sqrt{-b^2 + 4\, ac}} + \frac{44\, c^3 x^3 + 66\, bc^2 x^2 + 20\, b^2 cx +}{2\left(b^8 d^4 - 16\, ab^6 cd^4 + 96\, a^2 b^4 c^2 d^4 - 256\, a^3 b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^4/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 140*c^2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^8*d^4 - 16*a*b^6*c*d^4 +
 96*a^2*b^4*c^2*d^4 - 256*a^3*b^2*c^3*d^4 + 256*a^4*c^4*d^4)*sqrt(-b^2 + 4*
a*c)) + 1/2*(44*c^3*x^3 + 66*b*c^2*x^2 + 20*b^2*c*x + 52*a*c^2*x - b^3 + 26
*a*b*c)/((b^8*d^4 - 16*a*b^6*c*d^4 + 96*a^2*b^4*c^2*d^4 - 256*a^3*b^2*c^3*d
^4 + 256*a^4*c^4*d^4)*(c*x^2 + b*x + a)^2) + 64/3*(18*c^4*x^2 + 18*b*c^3*x
+ 5*b^2*c^2 - 2*a*c^3)/((b^8*d^4 - 16*a*b^6*c*d^4 + 96*a^2*b^4*c^2*d^4 - 25
6*a^3*b^2*c^3*d^4 + 256*a^4*c^4*d^4)*(2*c*x + b)^3)
```

3.1191 $\int (bd+2cdx)^4\sqrt{a+bx+cx^2}\,dx$

Optimal. Leaf size=165

$$-\frac{d^4\left(b^2-4ac\right)^3\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{64c^{3/2}}-\frac{d^4\left(b^2-4ac\right)(b+2cx)^3\sqrt{a+bx+cx^2}}{48c}-\frac{d^4\left(b^2-4ac\right)^2(b+2cx)\sqrt{a+bx+c}}{32c}$$

```
[Out] -((b^2 - 4*a*c)^2*d^4*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(32*c) - ((b^2 - 4
*a*c)*d^4*(b + 2*c*x)^3*Sqrt[a + b*x + c*x^2])/(48*c) + (d^4*(b + 2*c*x)^5*
Sqrt[a + b*x + c*x^2])/(12*c) - ((b^2 - 4*a*c)^3*d^4*ArcTanh[(b + 2*c*x)/(2
*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(64*c^(3/2))
```

Rubi [A] time = 0.0976735, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {685, 692, 621, 206}

$$-\frac{d^4\left(b^2-4ac\right)^3\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{64c^{3/2}}-\frac{d^4\left(b^2-4ac\right)(b+2cx)^3\sqrt{a+bx+cx^2}}{48c}-\frac{d^4\left(b^2-4ac\right)^2(b+2cx)\sqrt{a+bx+c}}{32c}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^4*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] -((b^2 - 4*a*c)^2*d^4*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(32*c) - ((b^2 - 4
*a*c)*d^4*(b + 2*c*x)^3*Sqrt[a + b*x + c*x^2])/(48*c) + (d^4*(b + 2*c*x)^5*
Sqrt[a + b*x + c*x^2])/(12*c) - ((b^2 - 4*a*c)^3*d^4*ArcTanh[(b + 2*c*x)/(2
*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(64*c^(3/2))
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (bd+2cdx)^4\sqrt{a+bx+cx^2}\,dx = \frac{d^4(b+2cx)^5\sqrt{a+bx+cx^2}}{12c} - \frac{\left(b^2-4ac\right)\int \frac{(bd+2cdx)^4}{\sqrt{a+bx+cx^2}}\,dx}{24c}$$

$$= -\frac{\left(b^2-4ac\right)d^4(b+2cx)^3\sqrt{a+bx+cx^2}}{48c} + \frac{d^4(b+2cx)^5\sqrt{a+bx+cx^2}}{12c} - \frac{\left(\left(b^2-4ac\right)^2 \right.}{}$$

$$= -\frac{\left(b^2-4ac\right)^2 d^4(b+2cx)\sqrt{a+bx+cx^2}}{32c} - \frac{\left(b^2-4ac\right)d^4(b+2cx)^3\sqrt{a+bx+cx^2}}{48c} + \frac{d^4}{}$$

$$= -\frac{\left(b^2-4ac\right)^2 d^4(b+2cx)\sqrt{a+bx+cx^2}}{32c} - \frac{\left(b^2-4ac\right)d^4(b+2cx)^3\sqrt{a+bx+cx^2}}{48c} + \frac{d^4}{}$$

$$= -\frac{\left(b^2-4ac\right)^2 d^4(b+2cx)\sqrt{a+bx+cx^2}}{32c} - \frac{\left(b^2-4ac\right)d^4(b+2cx)^3\sqrt{a+bx+cx^2}}{48c} + \frac{d^4}{}$$

Mathematica [A] time = 0.888537, size = 203, normalized size = 1.23

$$d^4\left(\frac{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(2(b+2cx)\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\left(4c\left(a+2cx^2\right)+b^2+8bcx\right)-\sqrt{c}\sqrt{4a-\frac{b^2}{c}}\left(4ac-b^2\right)\sinh^{-1}\left(\frac{b+}{\sqrt{c}\sqrt{}}\right.\right.}{64c\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^4*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] d^4*(((b + 2*c*x)^3*(a + x*(b + c*x))^(3/2))/3 + ((b^2 - 4*a*c)*Sqrt[a + x*
(b + c*x)]*(2*(b + 2*c*x)*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*(b^2 +
 8*b*c*x + 4*c*(a + 2*c*x^2)) - Sqrt[4*a - b^2/c]*Sqrt[c]*(-b^2 + 4*a*c)*Ar
cSinh[(b + 2*c*x)/(Sqrt[4*a - b^2/c]*Sqrt[c])]))/(64*c*Sqrt[(c*(a + x*(b +
c*x)))/(-b^2 + 4*a*c)]))
```

Maple [B] time = 0.055, size = 413, normalized size = 2.5

$$\frac{7\,d^4b^3}{12}\left(cx^2+bx+a\right)^{\frac{3}{2}} + 4\,d^4c^2bx^2\left(cx^2+bx+a\right)^{3/2} + \frac{5\,d^4b^2cx}{2}\left(cx^2+bx+a\right)^{\frac{3}{2}} - \frac{d^4b^6}{64}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}} + \sqrt{cx^2+bx+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^4*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 7/12*d^4*b^3*(c*x^2+b*x+a)^(3/2)+4*d^4*c^2*b*x^2*(c*x^2+b*x+a)^(3/2)+5/2*d^
4*c*b^2*x*(c*x^2+b*x+a)^(3/2)-1/64*d^4*b^6/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(
c*x^2+b*x+a)^(1/2))-d^4*c*b*a*(c*x^2+b*x+a)^(3/2)-2*d^4*c^2*a*x*(c*x^2+b*x+
```

```
a)^(3/2)+d^4*c^2*a^2*x*(c*x^2+b*x+a)^(1/2)+1/2*d^4*c*a^2*(c*x^2+b*x+a)^(1/2
)*b+d^4*c^(3/2)*a^3*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+8/3*d^4*c^3
*x^3*(c*x^2+b*x+a)^(3/2)+1/16*d^4*b^4*x*(c*x^2+b*x+a)^(1/2)+1/32*d^4/c*b^5*
(c*x^2+b*x+a)^(1/2)-1/4*d^4*b^3*a*(c*x^2+b*x+a)^(1/2)+3/16*d^4*b^4/c^(1/2)*
ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/2*d^4*c*b^2*a*x*(c*x^2+b*x+
a)^(1/2)-3/4*d^4*c^(1/2)*b^2*a^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.27941, size = 1071, normalized size = 6.49

$$\left[-\frac{3\left(b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3\right)\sqrt{c}d^4\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right)-4\,(256\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(3*(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(c)*d^4*log
(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) -
 4*a*c) - 4*(256*c^6*d^4*x^5 + 640*b*c^5*d^4*x^4 + 16*(39*b^2*c^4 + 4*a*c^5
)*d^4*x^3 + 8*(37*b^3*c^3 + 12*a*b*c^4)*d^4*x^2 + 2*(31*b^4*c^2 + 48*a*b^2*
c^3 - 48*a^2*c^4)*d^4*x + (3*b^5*c + 32*a*b^3*c^2 - 48*a^2*b*c^3)*d^4)*sqrt
(c*x^2 + b*x + a))/c^2, 1/192*(3*(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^
3*c^3)*sqrt(-c)*d^4*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(
c^2*x^2 + b*c*x + a*c)) + 2*(256*c^6*d^4*x^5 + 640*b*c^5*d^4*x^4 + 16*(39*b
^2*c^4 + 4*a*c^5)*d^4*x^3 + 8*(37*b^3*c^3 + 12*a*b*c^4)*d^4*x^2 + 2*(31*b^4
*c^2 + 48*a*b^2*c^3 - 48*a^2*c^4)*d^4*x + (3*b^5*c + 32*a*b^3*c^2 - 48*a^2*
b*c^3)*d^4)*sqrt(c*x^2 + b*x + a))/c^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^4\left(\int b^4\sqrt{a+bx+cx^2}\,dx+\int 16c^4x^4\sqrt{a+bx+cx^2}\,dx+\int 32bc^3x^3\sqrt{a+bx+cx^2}\,dx+\int 24b^2c^2x^2\sqrt{a+bx+cx^2}\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**4*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] d**4*(Integral(b**4*sqrt(a + b*x + c*x**2), x) + Integral(16*c**4*x**4*sqrt
(a + b*x + c*x**2), x) + Integral(32*b*c**3*x**3*sqrt(a + b*x + c*x**2), x)
```

```
 + Integral(24*b**2*c**2*x**2*sqrt(a + b*x + c*x**2), x) + Integral(8*b**3*
c*x*sqrt(a + b*x + c*x**2), x))
```

Giac [A] time = 1.28851, size = 350, normalized size = 2.12

$$\frac{1}{96}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(2c^4d^4x+5bc^3d^4\right)x+\frac{39b^2c^7d^4+4ac^8d^4}{c^5}\right)x+\frac{37b^3c^6d^4+12abc^7d^4}{c^5}\right)x+\frac{31b^4c^5d^4+48ab\ldots}{c\ldots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(2*c^4*d^4*x + 5*b*c^3*d^4)*x + (39*
b^2*c^7*d^4 + 4*a*c^8*d^4)/c^5)*x + (37*b^3*c^6*d^4 + 12*a*b*c^7*d^4)/c^5)*
x + (31*b^4*c^5*d^4 + 48*a*b^2*c^6*d^4 - 48*a^2*c^7*d^4)/c^5)*x + (3*b^5*c^
4*d^4 + 32*a*b^3*c^5*d^4 - 48*a^2*b*c^6*d^4)/c^5) + 1/64*(b^6*d^4 - 12*a*b^
4*c*d^4 + 48*a^2*b^2*c^2*d^4 - 64*a^3*c^3*d^4)*log(abs(-2*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))*sqrt(c) - b))/c^(3/2)
```

3.1192 $\int (bd+2cdx)^3 \sqrt{a+bx+cx^2}\, dx$

Optimal. Leaf size=59

$$\frac{4}{15}d^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}+\frac{2}{5}d^3(b+2cx)^2\left(a+bx+cx^2\right)^{3/2}$$

```
[Out] (4*(b^2 - 4*a*c)*d^3*(a + b*x + c*x^2)^(3/2))/15 + (2*d^3*(b + 2*c*x)^2*(a
+ b*x + c*x^2)^(3/2))/5
```

Rubi [A] time = 0.0262283, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {692, 629}

$$\frac{4}{15}d^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}+\frac{2}{5}d^3(b+2cx)^2\left(a+bx+cx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^3*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (4*(b^2 - 4*a*c)*d^3*(a + b*x + c*x^2)^(3/2))/15 + (2*d^3*(b + 2*c*x)^2*(a
+ b*x + c*x^2)^(3/2))/5
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (bd+2cdx)^3\sqrt{a+bx+cx^2}\,dx &= \frac{2}{5}d^3(b+2cx)^2\left(a+bx+cx^2\right)^{3/2}+\frac{1}{5}\left(2\left(b^2-4ac\right)d^2\right)\int (bd+2cdx)\sqrt{a+bx+cx^2}\\ &= \frac{4}{15}\left(b^2-4ac\right)d^3\left(a+bx+cx^2\right)^{3/2}+\frac{2}{5}d^3(b+2cx)^2\left(a+bx+cx^2\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.0378078, size = 44, normalized size = 0.75

$$\frac{2}{15}d^3(a+x(b+cx))^{3/2}\left(4c\left(3cx^2-2a\right)+5b^2+12bcx\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^3*Sqrt[a + b*x + c*x^2],x]
```

[Out] (2*d^3*(a + x*(b + c*x))^(3/2)*(5*b^2 + 12*b*c*x + 4*c*(-2*a + 3*c*x^2)))/1
5

Maple [A] time = 0.045, size = 41, normalized size = 0.7

$$-\frac{\left(-24\,c^2x^2-24\,bcx+16\,ac-10\,b^2\right)d^3}{15}\left(cx^2+bx+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*d*x+b*d)^3*(c*x^2+b*x+a)^(1/2),x)

[Out] -2/15*(c*x^2+b*x+a)^(3/2)*(-12*c^2*x^2-12*b*c*x+8*a*c-5*b^2)*d^3

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*d*x+b*d)^3*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 2.25002, size = 198, normalized size = 3.36

$$\frac{2}{15}\left(12\,c^3d^3x^4+24\,bc^2d^3x^3+\left(17\,b^2c+4\,ac^2\right)d^3x^2+\left(5\,b^3+4\,abc\right)d^3x+\left(5\,ab^2-8\,a^2c\right)d^3\right)\sqrt{cx^2+bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*d*x+b*d)^3*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")

[Out] 2/15*(12*c^3*d^3*x^4 + 24*b*c^2*d^3*x^3 + (17*b^2*c + 4*a*c^2)*d^3*x^2 + (5
*b^3 + 4*a*b*c)*d^3*x + (5*a*b^2 - 8*a^2*c)*d^3)*sqrt(c*x^2 + b*x + a)

Sympy [B] time = 0.497091, size = 216, normalized size = 3.66

$$-\frac{16a^2cd^3\sqrt{a+bx+cx^2}}{15}+\frac{2ab^2d^3\sqrt{a+bx+cx^2}}{3}+\frac{8abcd^3x\sqrt{a+bx+cx^2}}{15}+\frac{8ac^2d^3x^2\sqrt{a+bx+cx^2}}{15}+\frac{2b^3d^3x\sqrt{a+bx+}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*d*x+b*d)**3*(c*x**2+b*x+a)**(1/2),x)

[Out] -16*a**2*c*d**3*sqrt(a + b*x + c*x**2)/15 + 2*a*b**2*d**3*sqrt(a + b*x + c*
x**2)/3 + 8*a*b*c*d**3*x*sqrt(a + b*x + c*x**2)/15 + 8*a*c**2*d**3*x**2*sqr
t(a + b*x + c*x**2)/15 + 2*b**3*d**3*x*sqrt(a + b*x + c*x**2)/3 + 34*b**2*c
*d**3*x**2*sqrt(a + b*x + c*x**2)/15 + 16*b*c**2*d**3*x**3*sqrt(a + b*x + c

```
*x**2)/5 + 8*c**3*d**3*x**4*sqrt(a + b*x + c*x**2)/5
```

Giac [B] time = 1.19079, size = 163, normalized size = 2.76

$$\frac{2}{15}\sqrt{cx^2+bx+a}\left(\left(\left(12\left(c^3d^3x+2\,bc^2d^3\right)x+\frac{17\,b^2c^5d^3+4\,ac^6d^3}{c^4}\right)x+\frac{5\,b^3c^4d^3+4\,abc^5d^3}{c^4}\right)x+\frac{5\,ab^2c^4d^3-8\,a^2c^5d^3}{c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*sqrt(c*x^2 + b*x + a)*(((12*(c^3*d^3*x + 2*b*c^2*d^3)*x + (17*b^2*c^5*
d^3 + 4*a*c^6*d^3)/c^4)*x + (5*b^3*c^4*d^3 + 4*a*b*c^5*d^3)/c^4)*x + (5*a*b
^2*c^4*d^3 - 8*a^2*c^5*d^3)/c^4)
```

3.1193 $\int (bd+2cdx)^2\sqrt{a+bx+cx^2}\,dx$

Optimal. Leaf size=123

$$-\frac{d^2\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{32c^{3/2}}-\frac{d^2\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}}{16c}+\frac{d^2(b+2cx)^3\sqrt{a+bx+cx^2}}{8c}$$

```
[Out] -((b^2 - 4*a*c)*d^2*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(16*c) + (d^2*(b + 2
*c*x)^3*Sqrt[a + b*x + c*x^2])/(8*c) - ((b^2 - 4*a*c)^2*d^2*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(32*c^(3/2))
```

Rubi [A] time = 0.0559653, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {685, 692, 621, 206}

$$-\frac{d^2\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{32c^{3/2}}-\frac{d^2\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}}{16c}+\frac{d^2(b+2cx)^3\sqrt{a+bx+cx^2}}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^2*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] -((b^2 - 4*a*c)*d^2*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(16*c) + (d^2*(b + 2
*c*x)^3*Sqrt[a + b*x + c*x^2])/(8*c) - ((b^2 - 4*a*c)^2*d^2*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(32*c^(3/2))
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (bd+2cdx)^2\sqrt{a+bx+cx^2}\,dx = \frac{d^2(b+2cx)^3\sqrt{a+bx+cx^2}}{8c} - \frac{\left(b^2-4ac\right)\int \frac{(bd+2cdx)^2}{\sqrt{a+bx+cx^2}}\,dx}{16c}$$
$$= -\frac{\left(b^2-4ac\right)d^2(b+2cx)\sqrt{a+bx+cx^2}}{16c} + \frac{d^2(b+2cx)^3\sqrt{a+bx+cx^2}}{8c} - \frac{\left(\left(b^2-4ac\right)\right.}{}$$
$$= -\frac{\left(b^2-4ac\right)d^2(b+2cx)\sqrt{a+bx+cx^2}}{16c} + \frac{d^2(b+2cx)^3\sqrt{a+bx+cx^2}}{8c} - \frac{\left(\left(b^2-4ac\right)\right.}{}$$
$$= -\frac{\left(b^2-4ac\right)d^2(b+2cx)\sqrt{a+bx+cx^2}}{16c} + \frac{d^2(b+2cx)^3\sqrt{a+bx+cx^2}}{8c} - \frac{\left(b^2-4ac\right)^2}{}$$

Mathematica [A] time = 0.371687, size = 140, normalized size = 1.14

$$\frac{d^2\sqrt{a+x(b+cx)}\left(2(b+2cx)\left(4c\left(a+2cx^2\right)+b^2+8bcx\right)-\frac{c^{3/2}\sqrt{4a-\frac{b^2}{c}}(a+x(b+cx))\sinh^{-1}\left(\frac{b+2cx}{\sqrt{c}\sqrt{4a-\frac{b^2}{c}}}\right)}{\left(\frac{c(a+x(b+cx))}{4ac-b^2}\right)^{3/2}}\right)}{32c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^2*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (d^2*Sqrt[a + x*(b + c*x)]*(2*(b + 2*c*x)*(b^2 + 8*b*c*x + 4*c*(a + 2*c*x^2
)) - (Sqrt[4*a - b^2/c]*c^(3/2)*(a + x*(b + c*x))*ArcSinh[(b + 2*c*x)/(Sqrt
[4*a - b^2/c]*Sqrt[c])])/((c*(a + x*(b + c*x)))/(-b^2 + 4*a*c))^(3/2)))/(32
*c)
```

Maple [B] time = 0.048, size = 230, normalized size = 1.9

$$d^2cx\left(cx^2+bx+a\right)^{\frac{3}{2}} + \frac{d^2b}{2}\left(cx^2+bx+a\right)^{\frac{3}{2}} + \frac{b^2d^2x}{8}\sqrt{cx^2+bx+a} + \frac{d^2b^3}{16\,c}\sqrt{cx^2+bx+a} + \frac{b^2d^2a}{4}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^2*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] d^2*c*x*(c*x^2+b*x+a)^(3/2)+1/2*d^2*b*(c*x^2+b*x+a)^(3/2)+1/8*d^2*b^2*x*(c*
x^2+b*x+a)^(1/2)+1/16*d^2/c*b^3*(c*x^2+b*x+a)^(1/2)+1/4*d^2*b^2/c^(1/2)*ln(
(1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/32*d^2*b^4/c^(3/2)*ln((1/2*b+c
*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2*d^2*c*a*x*(c*x^2+b*x+a)^(1/2)-1/4*d^2*
a*(c*x^2+b*x+a)^(1/2)*b-1/2*d^2*c^(1/2)*a^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b
*x+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.29423, size = 702, normalized size = 5.71

$$\left[\frac{\left(b^4 - 8\,ab^2c + 16\,a^2c^2\right)\sqrt{c}d^2\log\left(-8\,c^2x^2 - 8\,bcx - b^2 + 4\,\sqrt{cx^2 + bx + a}(2\,cx + b)\sqrt{c} - 4\,ac\right) + 4\left(16\,c^4d^2x^3 + 24\,bc^3d^2x^2\right.}{64\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/64*((b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c)*d^2*log(-8*c^2*x^2 - 8*b*c*x
- b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(16*c^4*d^
2*x^3 + 24*b*c^3*d^2*x^2 + 2*(5*b^2*c^2 + 4*a*c^3)*d^2*x + (b^3*c + 4*a*b*c
^2)*d^2)*sqrt(c*x^2 + b*x + a))/c^2, 1/32*((b^4 - 8*a*b^2*c + 16*a^2*c^2)*s
qrt(-c)*d^2*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2
+ b*c*x + a*c)) + 2*(16*c^4*d^2*x^3 + 24*b*c^3*d^2*x^2 + 2*(5*b^2*c^2 + 4*a
*c^3)*d^2*x + (b^3*c + 4*a*b*c^2)*d^2)*sqrt(c*x^2 + b*x + a))/c^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int b^2\sqrt{a + bx + cx^2}\,dx + \int 4c^2x^2\sqrt{a + bx + cx^2}\,dx + \int 4bcx\sqrt{a + bx + cx^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**2*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] d**2*(Integral(b**2*sqrt(a + b*x + c*x**2), x) + Integral(4*c**2*x**2*sqrt(
a + b*x + c*x**2), x) + Integral(4*b*c*x*sqrt(a + b*x + c*x**2), x))
```

Giac [A] time = 1.18944, size = 209, normalized size = 1.7

$$\frac{1}{16}\sqrt{cx^2 + bx + a}\left(2\left(4\left(2\,c^2d^2x + 3\,bcd^2\right)x + \frac{5\,b^2c^3d^2 + 4\,ac^4d^2}{c^3}\right)x + \frac{b^3c^2d^2 + 4\,abc^3d^2}{c^3}\right) + \frac{\left(b^4d^2 - 8\,ab^2cd^2 + 16\,a^2c^2d^2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(c*x^2 + b*x + a)*(2*(4*(2*c^2*d^2*x + 3*b*c*d^2)*x + (5*b^2*c^3*d
^2 + 4*a*c^4*d^2)/c^3)*x + (b^3*c^2*d^2 + 4*a*b*c^3*d^2)/c^3) + 1/32*(b^4*d
^2 - 8*a*b^2*c*d^2 + 16*a^2*c^2*d^2)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*sqrt(c) - b))/c^(3/2)
```

3.1194 $\int (bd + 2cdx)\sqrt{a + bx + cx^2}\,dx$

Optimal. Leaf size=19

$$\frac{2}{3}d\left(a+bx+cx^2\right)^{3/2}$$

```
[Out] (2*d*(a + b*x + c*x^2)^(3/2))/3
```

Rubi [A] time = 0.0063804, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {629}

$$\frac{2}{3}d\left(a+bx+cx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*d*(a + b*x + c*x^2)^(3/2))/3
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (bd+2cdx)\sqrt{a+bx+cx^2}\,dx = \frac{2}{3}d\left(a+bx+cx^2\right)^{3/2}$$

Mathematica [A] time = 0.0090106, size = 18, normalized size = 0.95

$$\frac{2}{3}d(a+x(b+cx))^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*d*(a + x*(b + c*x))^(3/2))/3
```

Maple [A] time = 0.043, size = 16, normalized size = 0.8

$$\frac{2d}{3}\left(cx^2+bx+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)*(c*x^2+b*x+a)^(1/2),x)
```

[Out] 2/3*d*(c*x^2+b*x+a)^(3/2)

Maxima [A] time = 1.17951, size = 20, normalized size = 1.05

$$\frac{2}{3}\left(cx^2+bx+a\right)^{\frac{3}{2}}d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*d*x+b*d)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")

[Out] 2/3*(c*x^2 + b*x + a)^(3/2)*d

Fricas [A] time = 2.18912, size = 69, normalized size = 3.63

$$\frac{2}{3}\left(cdx^2+bdx+ad\right)\sqrt{cx^2+bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*d*x+b*d)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")

[Out] 2/3*(c*d*x^2 + b*d*x + a*d)*sqrt(c*x^2 + b*x + a)

Sympy [B] time = 0.255468, size = 65, normalized size = 3.42

$$\frac{2ad\sqrt{a+bx+cx^2}}{3}+\frac{2bdx\sqrt{a+bx+cx^2}}{3}+\frac{2cdx^2\sqrt{a+bx+cx^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*d*x+b*d)*(c*x**2+b*x+a)**(1/2),x)

[Out] 2*a*d*sqrt(a + b*x + c*x**2)/3 + 2*b*d*x*sqrt(a + b*x + c*x**2)/3 + 2*c*d*x
2*sqrt(a + b*x + c*x2)/3

Giac [A] time = 1.1255, size = 20, normalized size = 1.05

$$\frac{2}{3}\left(cx^2+bx+a\right)^{\frac{3}{2}}d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*d*x+b*d)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")

[Out] 2/3*(c*x^2 + b*x + a)^(3/2)*d

3.1195 $\int \frac{\sqrt{a+bx+cx^2}}{bd+2cdx}\,dx$

Optimal. Leaf size=83

$$\frac{\sqrt{a+bx+cx^2}}{2cd}-\frac{\sqrt{b^2-4ac}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{4c^{3/2}d}$$

```
[Out] Sqrt[a + b*x + c*x^2]/(2*c*d) - (Sqrt[b^2 - 4*a*c]*ArcTan[(2*Sqrt[c]*Sqrt[a
 + b*x + c*x^2])/Sqrt[b^2 - 4*a*c]])/(4*c^(3/2)*d)
```

Rubi [A] time = 0.063362, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {685, 688, 205}

$$\frac{\sqrt{a+bx+cx^2}}{2cd}-\frac{\sqrt{b^2-4ac}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{4c^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(b*d + 2*c*d*x),x]
```

```
[Out] Sqrt[a + b*x + c*x^2]/(2*c*d) - (Sqrt[b^2 - 4*a*c]*ArcTan[(2*Sqrt[c]*Sqrt[a
 + b*x + c*x^2])/Sqrt[b^2 - 4*a*c]])/(4*c^(3/2)*d)
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{bd+2cdx}\,dx = \frac{\sqrt{a+bx+cx^2}}{2cd} - \frac{(b^2-4ac)\int \frac{1}{(bd+2cdx)\sqrt{a+bx+cx^2}}\,dx}{4c}$$
$$= \frac{\sqrt{a+bx+cx^2}}{2cd} - (b^2-4ac)\,\mathrm{Subst}\left(\int \frac{1}{2b^2cd-8ac^2d+8c^2dx^2}\,dx, x, \sqrt{a+bx+cx^2}\right)$$
$$= \frac{\sqrt{a+bx+cx^2}}{2cd} - \frac{\sqrt{b^2-4ac}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{4c^{3/2}d}$$

Mathematica [A] time = 0.0540887, size = 80, normalized size = 0.96

$$\frac{2\sqrt{c}\sqrt{a+x(b+cx)} - \sqrt{b^2-4ac}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+x(b+cx)}}{\sqrt{b^2-4ac}}\right)}{4c^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(b*d + 2*c*d*x),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)] - Sqrt[b^2 - 4*a*c]*ArcTan[(2*Sqrt[c]*Sqrt
[a + x*(b + c*x)])/Sqrt[b^2 - 4*a*c]])/(4*c^(3/2)*d)
```

Maple [B] time = 0.212, size = 244, normalized size = 2.9

$$\frac{1}{4cd}\sqrt{4\left(x+1/2\,\frac{b}{c}\right)^2 c+\frac{4ac-b^2}{c}} - \frac{a}{cd}\ln\left(\left(\frac{4ac-b^2}{2c}+\frac{1}{2}\sqrt{\frac{4ac-b^2}{c}}\sqrt{4\left(x+1/2\,\frac{b}{c}\right)^2 c+\frac{4ac-b^2}{c}}\right)\left(x+\frac{b}{2c}\right)^{-1}\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d),x)
```

```
[Out] 1/4/d/c*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2)-1/d/c/((4*a*c-b^2)/c)^(1/2)
*ln((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-
b^2)/c)^(1/2))/(x+1/2*b/c))*a+1/4/d/c^2/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*
c-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))
/(x+1/2*b/c))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.61645, size = 410, normalized size = 4.94

$$\left[\frac{\sqrt{-\frac{b^2-4\,ac}{c}}\log\left(-\frac{4\,c^2x^2+4\,bcx-b^2+8\,ac-4\,\sqrt{cx^2+bx+a}c\sqrt{-\frac{b^2-4\,ac}{c}}}{4\,c^2x^2+4\,bcx+b^2}\right)+4\,\sqrt{cx^2+bx+a}}{8\,cd}, \frac{\sqrt{\frac{b^2-4\,ac}{c}}\arctan\left(\frac{\sqrt{\frac{b^2-4\,ac}{c}}}{2\,\sqrt{cx^2+bx+a}}\right)+2\,\sqrt{cx^2+bx+a}}{4\,cd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-(b^2 - 4*a*c)/c)*log(-(4*c^2*x^2 + 4*b*c*x - b^2 + 8*a*c - 4*sq
rt(c*x^2 + b*x + a)*c*sqrt(-(b^2 - 4*a*c)/c))/(4*c^2*x^2 + 4*b*c*x + b^2))
+ 4*sqrt(c*x^2 + b*x + a))/(c*d), 1/4*(sqrt((b^2 - 4*a*c)/c)*arctan(1/2*sqr
t((b^2 - 4*a*c)/c)/sqrt(c*x^2 + b*x + a)) + 2*sqrt(c*x^2 + b*x + a))/(c*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{a+bx+cx^2}}{b+2cx}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(2*c*d*x+b*d),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(b + 2*c*x), x)/d
```

Giac [A] time = 1.16911, size = 131, normalized size = 1.58

$$-\frac{\left(b^2-4\,ac\right)\arctan\left(-\frac{2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)c+b\sqrt{c}}{\sqrt{b^2c-4\,ac^2}}\right)}{2\,\sqrt{b^2c-4\,ac^2}cd}+\frac{\sqrt{cx^2+bx+a}}{2\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d),x, algorithm="giac")
```

```
[Out] -1/2*(b^2 - 4*a*c)*arctan(-(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*c + b*sqr
t(c))/sqrt(b^2*c - 4*a*c^2))/(sqrt(b^2*c - 4*a*c^2)*c*d) + 1/2*sqrt(c*x^2 +
 b*x + a)/(c*d)
```

3.1196 $\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^2}\,dx$

Optimal. Leaf size=75

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{4c^{3/2}d^2}-\frac{\sqrt{a+bx+cx^2}}{2cd^2(b+2cx)}$$

```
[Out] -Sqrt[a + b*x + c*x^2]/(2*c*d^2*(b + 2*c*x)) + ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + b*x + c*x^2])]/(4*c^(3/2)*d^2)
```

Rubi [A] time = 0.0287825, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {684, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{4c^{3/2}d^2}-\frac{\sqrt{a+bx+cx^2}}{2cd^2(b+2cx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(b*d + 2*c*d*x)^2,x]
```

```
[Out] -Sqrt[a + b*x + c*x^2]/(2*c*d^2*(b + 2*c*x)) + ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + b*x + c*x^2])]/(4*c^(3/2)*d^2)
```

Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] && !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^2}\,dx = -\frac{\sqrt{a+bx+cx^2}}{2cd^2(b+2cx)} + \frac{\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{4cd^2}$$
$$= -\frac{\sqrt{a+bx+cx^2}}{2cd^2(b+2cx)} + \frac{\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{2cd^2}$$
$$= -\frac{\sqrt{a+bx+cx^2}}{2cd^2(b+2cx)} + \frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{4c^{3/2}d^2}$$

Mathematica [A] time = 0.299273, size = 114, normalized size = 1.52

$$\frac{\sqrt{a+x(b+cx)}\left(\frac{\sinh^{-1}\left(\frac{b+2cx}{\sqrt{c}\sqrt{4a-\frac{b^2}{c}}}\right)}{\sqrt{4a-\frac{b^2}{c}}\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}-\frac{2\sqrt{c}}{b+2cx}\right)}{4c^{3/2}d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(b*d + 2*c*d*x)^2,x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*((-2*Sqrt[c])/(b + 2*c*x) + ArcSinh[(b + 2*c*x)/(Sqr
t[4*a - b^2/c]*Sqrt[c])]/(Sqrt[4*a - b^2/c]*Sqrt[(c*(a + x*(b + c*x)))/(-b^
2 + 4*a*c)])))/(4*c^(3/2)*d^2)
```

Maple [B] time = 0.195, size = 291, normalized size = 3.9

$$-\frac{1}{cd^2\left(4\,ac-b^2\right)}\left(\left(x+\frac{b}{2\,c}\right)^2c+\frac{4\,ac-b^2}{4\,c}\right)^{\frac{3}{2}}\left(x+\frac{b}{2\,c}\right)^{-1}+\frac{x}{d^2\left(4\,ac-b^2\right)}\sqrt{\left(x+\frac{b}{2\,c}\right)^2c+\frac{4\,ac-b^2}{4\,c}}+\frac{b}{2\,cd^2\left(4\,ac-b^2\right)}\sqrt{\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^2,x)
```

```
[Out] -1/c/d^2/(4*a*c-b^2)/(x+1/2*b/c)*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(3/2)+
1/d^2/(4*a*c-b^2)*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2)*x+1/2/c/d^2/(4*
a*c-b^2)*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2)*b+1/c^(1/2)/d^2/(4*a*c-b
^2)*ln((x+1/2*b/c)*c^(1/2)+((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2))*a-1/4
/c^(3/2)/d^2/(4*a*c-b^2)*ln((x+1/2*b/c)*c^(1/2)+((x+1/2*b/c)^2*c+1/4*(4*a*c
-b^2)/c)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.71201, size = 450, normalized size = 6.

$$\left[\frac{(2\,cx+b)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right)-4\,\sqrt{cx^2+bx+a}\,c}{8\left(2\,c^3d^2x+bc^2d^2\right)},-\frac{(2\,cx+b)\sqrt{-c}\,\mathrm{arc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^2,x, algorithm="fricas")
```

```
[Out] [1/8*((2*c*x + b)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b
*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^2 + b*x + a)*c)/(2*c^3*d^
2*x + b*c^2*d^2), -1/4*((2*c*x + b)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x +
a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*sqrt(c*x^2 + b*x + a)*
c)/(2*c^3*d^2*x + b*c^2*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{a+bx+cx^2}}{b^2+4bcx+4c^2x^2}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(2*c*d*x+b*d)**2,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(b**2 + 4*b*c*x + 4*c**2*x**2), x)/d**2
```

Giac [B] time = 1.19959, size = 261, normalized size = 3.48

$$-\frac{1}{4}\,d^2\left(\frac{\left(\frac{c\arctan\left(\frac{\sqrt{-\frac{b^2cd^2}{(2\,cdx+bd)^2}+\frac{4\,ac^2d^2}{(2\,cdx+bd)^2}+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\sqrt{-\frac{b^2cd^2}{(2\,cdx+bd)^2}+\frac{4\,ac^2d^2}{(2\,cdx+bd)^2}+c}\right)\mathrm{sgn}\left(\frac{1}{2\,cdx+bd}\right)\mathrm{sgn}\left(c\right)\mathrm{sgn}\left(d\right)}{c^2d^4|c|}-\frac{\left(c\arctan\left(\frac{\sqrt{c}}{\sqrt{-c}}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^2,x, algorithm="giac")
```

```
[Out] -1/4*d^2*((c*arctan(sqrt(-b^2*c*d^2/(2*c*d*x + b*d)^2 + 4*a*c^2*d^2/(2*c*d*
x + b*d)^2 + c)/sqrt(-c))/sqrt(-c) + sqrt(-b^2*c*d^2/(2*c*d*x + b*d)^2 + 4*
```

```
a*c^2*d^2/(2*c*d*x + b*d)^2 + c))*sgn(1/(2*c*d*x + b*d))*sgn(c)*sgn(d)/(c^2
*d^4*abs(c)) - (c*arctan(sqrt(c)/sqrt(-c)) + sqrt(-c)*sqrt(c))*sgn(1/(2*c*d
*x + b*d))*sgn(c)*sgn(d)/(sqrt(-c)*c^2*d^4*abs(c)))*abs(c)
```

3.1197 $\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^3} dx$

Optimal. Leaf size=91

$$\frac{\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{8c^{3/2}d^3\sqrt{b^2-4ac}}-\frac{\sqrt{a+bx+cx^2}}{4cd^3(b+2cx)^2}$$

```
[Out] -Sqrt[a + b*x + c*x^2]/(4*c*d^3*(b + 2*c*x)^2) + ArcTan[(2*Sqrt[c]*Sqrt[a +
 b*x + c*x^2])/Sqrt[b^2 - 4*a*c]]/(8*c^(3/2)*Sqrt[b^2 - 4*a*c]*d^3)
```

Rubi [A] time = 0.051964, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {684, 688, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{8c^{3/2}d^3\sqrt{b^2-4ac}}-\frac{\sqrt{a+bx+cx^2}}{4cd^3(b+2cx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(b*d + 2*c*d*x)^3,x]
```

```
[Out] -Sqrt[a + b*x + c*x^2]/(4*c*d^3*(b + 2*c*x)^2) + ArcTan[(2*Sqrt[c]*Sqrt[a +
 b*x + c*x^2])/Sqrt[b^2 - 4*a*c]]/(8*c^(3/2)*Sqrt[b^2 - 4*a*c]*d^3)
```

Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^3}\,dx = -\frac{\sqrt{a+bx+cx^2}}{4cd^3(b+2cx)^2} + \frac{\int \frac{1}{(bd+2cdx)\sqrt{a+bx+cx^2}}\,dx}{8cd^2}$$
$$= -\frac{\sqrt{a+bx+cx^2}}{4cd^3(b+2cx)^2} + \frac{\text{Subst}\left(\int \frac{1}{2b^2cd-8ac^2d+8c^2dx^2}\,dx, x, \sqrt{a+bx+cx^2}\right)}{2d^2}$$
$$= -\frac{\sqrt{a+bx+cx^2}}{4cd^3(b+2cx)^2} + \frac{\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{8c^{3/2}\sqrt{b^2-4ac}\,d^3}$$

Mathematica [A] time = 0.201273, size = 103, normalized size = 1.13

$$\frac{-\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\tanh^{-1}\left(2\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\right)-\frac{2c(a+x(b+cx))}{(b+2cx)^2}}{8c^2d^3\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(b*d + 2*c*d*x)^3,x]
```

```
[Out] ((-2*c*(a + x*(b + c*x)))/(b + 2*c*x)^2 - Sqrt[(c*(a + x*(b + c*x)))/(-b^2
+ 4*a*c)]*ArcTanh[2*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]])/(8*c^2*d^3
*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.196, size = 340, normalized size = 3.7

$$-\frac{1}{4c^2d^3\left(4ac-b^2\right)}\left(\left(x+\frac{b}{2c}\right)^2c+\frac{4ac-b^2}{4c}\right)^{\frac{3}{2}}\left(x+\frac{b}{2c}\right)^{-2}+\frac{1}{8cd^3\left(4ac-b^2\right)}\sqrt{4\left(x+1/2\frac{b}{c}\right)^2c+\frac{4ac-b^2}{c}}-\frac{a}{2cd^3\left(4ac\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^3,x)
```

```
[Out] -1/4/d^3/c^2/(4*a*c-b^2)/(x+1/2*b/c)^2*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^
(3/2)+1/8/d^3/c/(4*a*c-b^2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2)-1/2/d^3
/c/(4*a*c-b^2)/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-b^2)
/c)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))*a+1/8/d^3/c
^2/(4*a*c-b^2)/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-b^2)
/c)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.07557, size = 806, normalized size = 8.86

$$\left[-\frac{\left(4\,c^2x^2+4\,bcx+b^2\right)\sqrt{-b^2c+4\,ac^2}\log\left(-\frac{4\,c^2x^2+4\,bcx-b^2+8\,ac-4\,\sqrt{-b^2c+4\,ac^2}\sqrt{cx^2+bx+a}}{4\,c^2x^2+4\,bcx+b^2}\right)+4\left(b^2c-4\,ac^2\right)\sqrt{cx^2+bx+a}}{16\left(4\left(b^2c^4-4\,ac^5\right)d^3x^2+4\left(b^3c^3-4\,abc^4\right)d^3x+\left(b^4c^2-4\,ab^2c^3\right)d^3\right)},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*((4*c^2*x^2 + 4*b*c*x + b^2)*sqrt(-b^2*c + 4*a*c^2)*log(-(4*c^2*x^2
+ 4*b*c*x - b^2 + 8*a*c - 4*sqrt(-b^2*c + 4*a*c^2)*sqrt(c*x^2 + b*x + a))/(
4*c^2*x^2 + 4*b*c*x + b^2)) + 4*(b^2*c - 4*a*c^2)*sqrt(c*x^2 + b*x + a))/(4
*(b^2*c^4 - 4*a*c^5)*d^3*x^2 + 4*(b^3*c^3 - 4*a*b*c^4)*d^3*x + (b^4*c^2 - 4
*a*b^2*c^3)*d^3), -1/8*((4*c^2*x^2 + 4*b*c*x + b^2)*sqrt(b^2*c - 4*a*c^2)*a
rctan(1/2*sqrt(b^2*c - 4*a*c^2)*sqrt(c*x^2 + b*x + a)/(c^2*x^2 + b*c*x + a*
c)) + 2*(b^2*c - 4*a*c^2)*sqrt(c*x^2 + b*x + a))/(4*(b^2*c^4 - 4*a*c^5)*d^3
*x^2 + 4*(b^3*c^3 - 4*a*b*c^4)*d^3*x + (b^4*c^2 - 4*a*b^2*c^3)*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{a+bx+cx^2}}{b^3+6b^2cx+12bc^2x^2+8c^3x^3}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(2*c*d*x+b*d)**3,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(b**3 + 6*b**2*c*x + 12*b*c**2*x**2 + 8*c**
3*x**3), x)/d**3
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1198 $\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^4} dx$

Optimal. Leaf size=39

$$\frac{2\left(a+bx+cx^2\right)^{3/2}}{3d^4\left(b^2-4ac\right)(b+2cx)^3}$$

```
[Out] (2*(a + b*x + c*x^2)^(3/2))/(3*(b^2 - 4*a*c)*d^4*(b + 2*c*x)^3)
```

Rubi [A] time = 0.0140489, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {682}

$$\frac{2\left(a+bx+cx^2\right)^{3/2}}{3d^4\left(b^2-4ac\right)(b+2cx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(b*d + 2*c*d*x)^4,x]
```

```
[Out] (2*(a + b*x + c*x^2)^(3/2))/(3*(b^2 - 4*a*c)*d^4*(b + 2*c*x)^3)
```

Rule 682

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c,
0] && EqQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^4} dx = \frac{2\left(a+bx+cx^2\right)^{3/2}}{3\left(b^2-4ac\right)d^4(b+2cx)^3}$$

Mathematica [A] time = 0.0184316, size = 38, normalized size = 0.97

$$\frac{2(a+x(b+cx))^{3/2}}{3d^4\left(b^2-4ac\right)(b+2cx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(b*d + 2*c*d*x)^4,x]
```

```
[Out] (2*(a + x*(b + c*x))^(3/2))/(3*(b^2 - 4*a*c)*d^4*(b + 2*c*x)^3)
```

Maple [A] time = 0.042, size = 38, normalized size = 1.

$$-\frac{2}{3\,(2\,cx+b)^3 d^4\left(4\,ac-b^2\right)}\left(cx^2+bx+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^4,x)
```

```
[Out] -2/3*(c*x^2+b*x+a)^(3/2)/(2*c*x+b)^3/d^4/(4*a*c-b^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.27809, size = 205, normalized size = 5.26

$$\frac{2\left(cx^{2}+bx+a\right)^{\frac{3}{2}}}{3\left(8\left(b^{2}c^{3}-4\,ac^{4}\right)d^{4}x^{3}+12\left(b^{3}c^{2}-4\,abc^{3}\right)d^{4}x^{2}+6\left(b^{4}c-4\,ab^{2}c^{2}\right)d^{4}x+\left(b^{5}-4\,ab^{3}c\right)d^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^4,x, algorithm="fricas")
```

```
[Out] 2/3*(c*x^2 + b*x + a)^(3/2)/(8*(b^2*c^3 - 4*a*c^4)*d^4*x^3 + 12*(b^3*c^2 -
4*a*b*c^3)*d^4*x^2 + 6*(b^4*c - 4*a*b^2*c^2)*d^4*x + (b^5 - 4*a*b^3*c)*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{a+bx+cx^{2}}}{b^{4}+8b^{3}cx+24b^{2}c^{2}x^{2}+32bc^{3}x^{3}+16c^{4}x^{4}}\,dx}{d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(2*c*d*x+b*d)**4,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(b**4 + 8*b**3*c*x + 24*b**2*c**2*x**2 + 32
*b*c**3*x**3 + 16*c**4*x**4), x)/d**4
```

Giac [B] time = 1.28705, size = 277, normalized size = 7.1

$$\frac{12\left(\sqrt{c}x-\sqrt{cx^{2}+bx+a}\right)^{4}c^{\frac{5}{2}}+24\left(\sqrt{c}x-\sqrt{cx^{2}+bx+a}\right)^{3}bc^{2}+18\left(\sqrt{c}x-\sqrt{cx^{2}+bx+a}\right)^{2}b^{2}c^{\frac{3}{2}}+6\left(\sqrt{c}x-\sqrt{cx^{2}+}\right.}{12\left(2\left(\sqrt{c}x-\sqrt{cx^{2}+bx+a}\right)^{2}c+2\left(\sqrt{c}x-\sqrt{cx^{2}+bx+a}\right)b\sqrt{c}+b^{2}-2\,ac\right)^{3}c^{2}d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^4,x, algorithm="giac")
```

```
[Out] 1/12*(12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*c^(5/2) + 24*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^3*b*c^2 + 18*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2
*c^(3/2) + 6*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*c + b^4*sqrt(c) - 2*a*
b^2*c^(3/2) + 4*a^2*c^(5/2))/((2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c +
2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*sqrt(c) + b^2 - 2*a*c)^3*c^2*d^4)
```

3.1199 $\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^5} dx$

Optimal. Leaf size=133

$$\frac{\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{32c^{3/2}d^5\left(b^2-4ac\right)^{3/2}}+\frac{\sqrt{a+bx+cx^2}}{16cd^5\left(b^2-4ac\right)(b+2cx)^2}-\frac{\sqrt{a+bx+cx^2}}{8cd^5(b+2cx)^4}$$

```
[Out] -Sqrt[a + b*x + c*x^2]/(8*c*d^5*(b + 2*c*x)^4) + Sqrt[a + b*x + c*x^2]/(16*
c*(b^2 - 4*a*c)*d^5*(b + 2*c*x)^2) + ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])/Sqrt[b^2 - 4*a*c]]/(32*c^(3/2)*(b^2 - 4*a*c)^(3/2)*d^5)
```

Rubi [A] time = 0.0892683, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {684, 693, 688, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{32c^{3/2}d^5\left(b^2-4ac\right)^{3/2}}+\frac{\sqrt{a+bx+cx^2}}{16cd^5\left(b^2-4ac\right)(b+2cx)^2}-\frac{\sqrt{a+bx+cx^2}}{8cd^5(b+2cx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(b*d + 2*c*d*x)^5,x]
```

```
[Out] -Sqrt[a + b*x + c*x^2]/(8*c*d^5*(b + 2*c*x)^4) + Sqrt[a + b*x + c*x^2]/(16*
c*(b^2 - 4*a*c)*d^5*(b + 2*c*x)^2) + ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])/Sqrt[b^2 - 4*a*c]]/(32*c^(3/2)*(b^2 - 4*a*c)^(3/2)*d^5)
```

Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^5}\,dx = -\frac{\sqrt{a+bx+cx^2}}{8cd^5(b+2cx)^4}+\frac{\int \frac{1}{(bd+2cdx)^3\sqrt{a+bx+cx^2}}\,dx}{16cd^2}$$
$$= -\frac{\sqrt{a+bx+cx^2}}{8cd^5(b+2cx)^4}+\frac{\sqrt{a+bx+cx^2}}{16c\left(b^2-4ac\right)d^5(b+2cx)^2}+\frac{\int \frac{1}{(bd+2cdx)\sqrt{a+bx+cx^2}}\,dx}{32c\left(b^2-4ac\right)d^4}$$
$$= -\frac{\sqrt{a+bx+cx^2}}{8cd^5(b+2cx)^4}+\frac{\sqrt{a+bx+cx^2}}{16c\left(b^2-4ac\right)d^5(b+2cx)^2}+\frac{\text{Subst}\left(\int \frac{1}{2b^2cd-8ac^2d+8c^2dx^2}\,dx,x,\sqrt{a+bx+cx^2}\right)}{8\left(b^2-4ac\right)d^4}$$
$$= -\frac{\sqrt{a+bx+cx^2}}{8cd^5(b+2cx)^4}+\frac{\sqrt{a+bx+cx^2}}{16c\left(b^2-4ac\right)d^5(b+2cx)^2}+\frac{\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{32c^{3/2}\left(b^2-4ac\right)^{3/2}d^5}$$

Mathematica [C] time = 0.0289667, size = 62, normalized size = 0.47

$$\frac{2(a+x(b+cx))^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{4c(a+x(b+cx))}{4ac-b^2}\right)}{3d^5\left(b^2-4ac\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(b*d + 2*c*d*x)^5,x]
```

```
[Out] (2*(a + x*(b + c*x))^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, (4*c*(a + x*(b +
c*x)))/(-b^2 + 4*a*c)])/(3*(b^2 - 4*a*c)^3*d^5)
```

Maple [B] time = 0.197, size = 400, normalized size = 3.

$$-\frac{1}{32\,c^4d^5\left(4\,ac-b^2\right)}\left(\left(x+\frac{b}{2\,c}\right)^2c+\frac{4\,ac-b^2}{4\,c}\right)^{\frac{3}{2}}\left(x+\frac{b}{2\,c}\right)^{-4}+\frac{1}{16\,c^2d^5\left(4\,ac-b^2\right)^2}\left(\left(x+\frac{b}{2\,c}\right)^2c+\frac{4\,ac-b^2}{4\,c}\right)^{\frac{3}{2}}\left(x+\frac{b}{2\,c}\right)^{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^5,x)
```

```
[Out] -1/32/d^5/c^4/(4*a*c-b^2)/(x+1/2*b/c)^4*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)
^(3/2)+1/16/d^5/c^2/(4*a*c-b^2)^2/(x+1/2*b/c)^2*((x+1/2*b/c)^2*c+1/4*(4*a*c
-b^2)/c)^(3/2)-1/32/d^5/c/(4*a*c-b^2)^2*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(
1/2)+1/8/d^5/c/(4*a*c-b^2)^2/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/
2*((4*a*c-b^2)/c)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c
))*a-1/32/d^5/c^2/(4*a*c-b^2)^2/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c
+1/2*((4*a*c-b^2)/c)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*
b/c))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 10.4778, size = 1532, normalized size = 11.52

$$\left[\frac{\left(16\,c^4x^4+32\,bc^3x^3+24\,b^2c^2x^2+8\,b^3cx+b^4\right)\sqrt{-b^2c+4\,ac^2}\log\left(-\frac{4\,c^2x^2+4\,bcx-b^2+8\,ac+4\,\sqrt{-b^2c+4\,ac^2}\sqrt{cx^2+bx+a}}{4\,c^2x^2+4\,bcx+b^2}\right)-4\left(b^4c-\right.}{64\left(16\left(b^4c^6-8\,ab^2c^7+16\,a^2c^8\right)d^5x^4+32\left(b^5c^5-8\,ab^3c^6+16\,a^2bc^7\right)d^5x^3+24\left(b^6c^4-8\,ab^4c^5+16\,a^2b^2c^6\right)d^5x^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^5,x, algorithm="fricas")
```

```
[Out] [1/64*((16*c^4*x^4 + 32*b*c^3*x^3 + 24*b^2*c^2*x^2 + 8*b^3*c*x + b^4)*sqrt(
-b^2*c + 4*a*c^2)*log(-(4*c^2*x^2 + 4*b*c*x - b^2 + 8*a*c + 4*sqrt(-b^2*c +
 4*a*c^2)*sqrt(c*x^2 + b*x + a))/(4*c^2*x^2 + 4*b*c*x + b^2)) - 4*(b^4*c -
12*a*b^2*c^2 + 32*a^2*c^3 - 4*(b^2*c^3 - 4*a*c^4)*x^2 - 4*(b^3*c^2 - 4*a*b*
c^3)*x)*sqrt(c*x^2 + b*x + a))/(16*(b^4*c^6 - 8*a*b^2*c^7 + 16*a^2*c^8)*d^5
*x^4 + 32*(b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*d^5*x^3 + 24*(b^6*c^4 - 8*
a*b^4*c^5 + 16*a^2*b^2*c^6)*d^5*x^2 + 8*(b^7*c^3 - 8*a*b^5*c^4 + 16*a^2*b^3
*c^5)*d^5*x + (b^8*c^2 - 8*a*b^6*c^3 + 16*a^2*b^4*c^4)*d^5), -1/32*((16*c^4
*x^4 + 32*b*c^3*x^3 + 24*b^2*c^2*x^2 + 8*b^3*c*x + b^4)*sqrt(b^2*c - 4*a*c^
2)*arctan(1/2*sqrt(b^2*c - 4*a*c^2)*sqrt(c*x^2 + b*x + a)/(c^2*x^2 + b*c*x
+ a*c)) + 2*(b^4*c - 12*a*b^2*c^2 + 32*a^2*c^3 - 4*(b^2*c^3 - 4*a*c^4)*x^2
- 4*(b^3*c^2 - 4*a*b*c^3)*x)*sqrt(c*x^2 + b*x + a))/(16*(b^4*c^6 - 8*a*b^2*
c^7 + 16*a^2*c^8)*d^5*x^4 + 32*(b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*d^5*x
^3 + 24*(b^6*c^4 - 8*a*b^4*c^5 + 16*a^2*b^2*c^6)*d^5*x^2 + 8*(b^7*c^3 - 8*a
*b^5*c^4 + 16*a^2*b^3*c^5)*d^5*x + (b^8*c^2 - 8*a*b^6*c^3 + 16*a^2*b^4*c^4)
*d^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{a+bx+cx^2}}{b^5+10b^4cx+40b^3c^2x^2+80b^2c^3x^3+80bc^4x^4+32c^5x^5}\,dx}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(2*c*d*x+b*d)**5,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(b**5 + 10*b**4*c*x + 40*b**3*c**2*x**2 + 8
0*b**2*c**3*x**3 + 80*b*c**4*x**4 + 32*c**5*x**5), x)/d**5
```

Giac [B] time = 1.69006, size = 859, normalized size = 6.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^5,x, algorithm="giac")
```

```
[Out] -1/12288*(sqrt(-b^2*c + 4*a*c^2)*log(abs(c))*sgn(1/(2*c*d*x + b*d))*sgn(c)*
sgn(d)/(b^8*c^9*d^13 - 16*a*b^6*c^10*d^13 + 96*a^2*b^4*c^11*d^13 - 256*a^3*
b^2*c^12*d^13 + 256*a^4*c^13*d^13) - 2*sqrt(-b^2*c*d^2/(2*c*d*x + b*d)^2 +
4*a*c^2*d^2/(2*c*d*x + b*d)^2 + c)*((b^2*c^3*d^5*abs(c)*sgn(1/(2*c*d*x + b*
d))*sgn(c)*sgn(d) - 4*a*c^4*d^5*abs(c)*sgn(1/(2*c*d*x + b*d))*sgn(c)*sgn(d)
)/(b^8*c^12*d^16 - 16*a*b^6*c^13*d^16 + 96*a^2*b^4*c^14*d^16 - 256*a^3*b^2*
c^15*d^16 + 256*a^4*c^16*d^16) - 2*(b^4*c^5*d^9*abs(c)*sgn(1/(2*c*d*x + b*d
))*sgn(c)*sgn(d) - 8*a*b^2*c^6*d^9*abs(c)*sgn(1/(2*c*d*x + b*d))*sgn(c)*sgn
(d) + 16*a^2*c^7*d^9*abs(c)*sgn(1/(2*c*d*x + b*d))*sgn(c)*sgn(d))/((b^8*c^1
2*d^16 - 16*a*b^6*c^13*d^16 + 96*a^2*b^4*c^14*d^16 - 256*a^3*b^2*c^15*d^16
+ 256*a^4*c^16*d^16)*(2*c*d*x + b*d)^2*c^2*d^2))/((2*c*d*x + b*d)*c*d) - 2*
sqrt(-b^2*c + 4*a*c^2)*log(abs(sqrt(-b^2*c*d^2/(2*c*d*x + b*d)^2 + 4*a*c^2*
d^2/(2*c*d*x + b*d)^2 + c) + sqrt(-b^2*c^3*d^4 + 4*a*c^4*d^4)/((2*c*d*x + b
*d)*c*d)))*sgn(1/(2*c*d*x + b*d))*sgn(c)*sgn(d)/((b^8*c^9 - 16*a*b^6*c^10 +
 96*a^2*b^4*c^11 - 256*a^3*b^2*c^12 + 256*a^4*c^13)*d^13))*d^2*abs(c)
```

3.1200 $\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^6}\,dx$

Optimal. Leaf size=79

$$\frac{4\left(a+bx+cx^2\right)^{3/2}}{15d^6\left(b^2-4ac\right)^2(b+2cx)^3}+\frac{2\left(a+bx+cx^2\right)^{3/2}}{5d^6\left(b^2-4ac\right)(b+2cx)^5}$$

```
[Out] (2*(a + b*x + c*x^2)^(3/2))/(5*(b^2 - 4*a*c)*d^6*(b + 2*c*x)^5) + (4*(a + b
*x + c*x^2)^(3/2))/(15*(b^2 - 4*a*c)^2*d^6*(b + 2*c*x)^3)
```

Rubi [A] time = 0.0348507, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {693, 682}

$$\frac{4\left(a+bx+cx^2\right)^{3/2}}{15d^6\left(b^2-4ac\right)^2(b+2cx)^3}+\frac{2\left(a+bx+cx^2\right)^{3/2}}{5d^6\left(b^2-4ac\right)(b+2cx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(b*d + 2*c*d*x)^6,x]
```

```
[Out] (2*(a + b*x + c*x^2)^(3/2))/(5*(b^2 - 4*a*c)*d^6*(b + 2*c*x)^5) + (4*(a + b
*x + c*x^2)^(3/2))/(15*(b^2 - 4*a*c)^2*d^6*(b + 2*c*x)^3)
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 682

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c,
0] && EqQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^6}\,dx &= \frac{2\left(a+bx+cx^2\right)^{3/2}}{5\left(b^2-4ac\right)d^6(b+2cx)^5}+\frac{2\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^4}\,dx}{5\left(b^2-4ac\right)d^2}\\ &= \frac{2\left(a+bx+cx^2\right)^{3/2}}{5\left(b^2-4ac\right)d^6(b+2cx)^5}+\frac{4\left(a+bx+cx^2\right)^{3/2}}{15\left(b^2-4ac\right)^2d^6(b+2cx)^3}\end{aligned}$$

Mathematica [A] time = 0.0293914, size = 62, normalized size = 0.78

$$\frac{2(a+x(b+cx))^{3/2}\left(4c\left(2cx^2-3a\right)+5b^2+8bcx\right)}{15d^6\left(b^2-4ac\right)^2(b+2cx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(b*d + 2*c*d*x)^6,x]
```

```
[Out] (2*(a + x*(b + c*x))^(3/2)*(5*b^2 + 8*b*c*x + 4*c*(-3*a + 2*c*x^2)))/(15*(b
^2 - 4*a*c)^2*d^6*(b + 2*c*x)^5)
```

Maple [A] time = 0.043, size = 70, normalized size = 0.9

$$-\frac{-16\,c^2x^2-16\,bcx+24\,ac-10\,b^2}{15\,(2\,cx+b)^5\,d^6\left(16\,a^2c^2-8\,acb^2+b^4\right)}\left(cx^2+bx+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^6,x)
```

```
[Out] -2/15*(-8*c^2*x^2-8*b*c*x+12*a*c-5*b^2)*(c*x^2+b*x+a)^(3/2)/(2*c*x+b)^5/d^6
/(16*a^2*c^2-8*a*b^2*c+b^4)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 21.128, size = 575, normalized size = 7.28

$$\frac{2\left(8\,c^3x^4+16\,bc^2x^3+5\,ab^2-12\,a^2c+\left(13\,b^2c-4\,ac^2\right)x^2\right.}{15\left(32\left(b^4c^5-8\,ab^2c^6+16\,a^2c^7\right)d^6x^5+80\left(b^5c^4-8\,ab^3c^5+16\,a^2bc^6\right)d^6x^4+80\left(b^6c^3-8\,ab^4c^4+16\,a^2b^2c^5\right)d^6x^3+40\left(b^7\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^6,x, algorithm="fricas")
```

```
[Out] 2/15*(8*c^3*x^4 + 16*b*c^2*x^3 + 5*a*b^2 - 12*a^2*c + (13*b^2*c - 4*a*c^2)*
x^2 + (5*b^3 - 4*a*b*c)*x)*sqrt(c*x^2 + b*x + a)/(32*(b^4*c^5 - 8*a*b^2*c^6
 + 16*a^2*c^7)*d^6*x^5 + 80*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*d^6*x^4
+ 80*(b^6*c^3 - 8*a*b^4*c^4 + 16*a^2*b^2*c^5)*d^6*x^3 + 40*(b^7*c^2 - 8*a*b
^5*c^3 + 16*a^2*b^3*c^4)*d^6*x^2 + 10*(b^8*c - 8*a*b^6*c^2 + 16*a^2*b^4*c^3
)*d^6*x + (b^9 - 8*a*b^7*c + 16*a^2*b^5*c^2)*d^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{a+bx+cx^2}}{b^6+12b^5cx+60b^4c^2x^2+160b^3c^3x^3+240b^2c^4x^4+192bc^5x^5+64c^6x^6}\,dx}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(2*c*d*x+b*d)**6,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(b**6 + 12*b**5*c*x + 60*b**4*c**2*x**2 + 1
60*b**3*c**3*x**3 + 240*b**2*c**4*x**4 + 192*b*c**5*x**5 + 64*c**6*x**6), x
)/d**6
```

Giac [B] time = 1.48839, size = 571, normalized size = 7.23

$$60\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^6c^{\frac{7}{2}}+180\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^5bc^3+220\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^4b^2c^{\frac{5}{2}}+20\left(\sqrt{c}x-\sqrt{cx^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^6,x, algorithm="giac")
```

```
[Out] 1/30*(60*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*c^(7/2) + 180*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^5*b*c^3 + 220*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b
^2*c^(5/2) + 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*c^(7/2) + 140*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^3*b^3*c^2 + 40*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^3*a*b*c^3 + 50*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^4*c^(3/2) + 2
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^2*c^(5/2) + 20*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^2*a^2*c^(7/2) + 10*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
b^5*c + 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*c^3 + b^6*sqrt(c) - 2*
a*b^4*c^(3/2) + 8*a^2*b^2*c^(5/2) - 4*a^3*c^(7/2))/((2*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^2*c + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*sqrt(c) + b^2
 - 2*a*c)^5*c^2*d^6)
```

3.1201 $\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^7}\,dx$

Optimal. Leaf size=175

$$\frac{\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{64c^{3/2}d^7\left(b^2-4ac\right)^{5/2}}+\frac{\sqrt{a+bx+cx^2}}{32cd^7\left(b^2-4ac\right)^2(b+2cx)^2}+\frac{\sqrt{a+bx+cx^2}}{48cd^7\left(b^2-4ac\right)(b+2cx)^4}-\frac{\sqrt{a+bx+cx^2}}{12cd^7(b+2cx)^6}$$

```
[Out] -Sqrt[a + b*x + c*x^2]/(12*c*d^7*(b + 2*c*x)^6) + Sqrt[a + b*x + c*x^2]/(48
*c*(b^2 - 4*a*c)*d^7*(b + 2*c*x)^4) + Sqrt[a + b*x + c*x^2]/(32*c*(b^2 - 4*
a*c)^2*d^7*(b + 2*c*x)^2) + ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b
^2 - 4*a*c]]/(64*c^(3/2)*(b^2 - 4*a*c)^(5/2)*d^7)
```

Rubi [A] time = 0.125092, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {684, 693, 688, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{64c^{3/2}d^7\left(b^2-4ac\right)^{5/2}}+\frac{\sqrt{a+bx+cx^2}}{32cd^7\left(b^2-4ac\right)^2(b+2cx)^2}+\frac{\sqrt{a+bx+cx^2}}{48cd^7\left(b^2-4ac\right)(b+2cx)^4}-\frac{\sqrt{a+bx+cx^2}}{12cd^7(b+2cx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(b*d + 2*c*d*x)^7,x]
```

```
[Out] -Sqrt[a + b*x + c*x^2]/(12*c*d^7*(b + 2*c*x)^6) + Sqrt[a + b*x + c*x^2]/(48
*c*(b^2 - 4*a*c)*d^7*(b + 2*c*x)^4) + Sqrt[a + b*x + c*x^2]/(32*c*(b^2 - 4*
a*c)^2*d^7*(b + 2*c*x)^2) + ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b
^2 - 4*a*c]]/(64*c^(3/2)*(b^2 - 4*a*c)^(5/2)*d^7)
```

Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
```

```
qQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^7}\,dx = -\frac{\sqrt{a+bx+cx^2}}{12cd^7(b+2cx)^6} + \frac{\int \frac{1}{(bd+2cdx)^5\sqrt{a+bx+cx^2}}\,dx}{24cd^2}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{12cd^7(b+2cx)^6} + \frac{\sqrt{a+bx+cx^2}}{48c\left(b^2-4ac\right)d^7(b+2cx)^4} + \frac{\int \frac{1}{(bd+2cdx)^3\sqrt{a+bx+cx^2}}\,dx}{32c\left(b^2-4ac\right)d^4}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{12cd^7(b+2cx)^6} + \frac{\sqrt{a+bx+cx^2}}{48c\left(b^2-4ac\right)d^7(b+2cx)^4} + \frac{\sqrt{a+bx+cx^2}}{32c\left(b^2-4ac\right)^2d^7(b+2cx)^2} + \frac{\int \frac{1}{(bd+2cdx)\sqrt{\ldots}}}{64c\left(b^2-\ldots\right.}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{12cd^7(b+2cx)^6} + \frac{\sqrt{a+bx+cx^2}}{48c\left(b^2-4ac\right)d^7(b+2cx)^4} + \frac{\sqrt{a+bx+cx^2}}{32c\left(b^2-4ac\right)^2d^7(b+2cx)^2} + \frac{\text{Subst}\left(\int \frac{\ldots}{2b\ldots}\right.}{\ldots}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{12cd^7(b+2cx)^6} + \frac{\sqrt{a+bx+cx^2}}{48c\left(b^2-4ac\right)d^7(b+2cx)^4} + \frac{\sqrt{a+bx+cx^2}}{32c\left(b^2-4ac\right)^2d^7(b+2cx)^2} + \frac{\tan^{-1}\left(\frac{2\sqrt{c}\ldots}{\ldots}\right.}{64c^{3/2}\left(b^2-\ldots\right.}$$

Mathematica [C] time = 0.0246535, size = 62, normalized size = 0.35

$$\frac{2(a+x(b+cx))^{3/2}\,{}_2F_1\left(\frac{3}{2},4;\frac{5}{2};\frac{4c(a+x(b+cx))}{4ac-b^2}\right)}{3d^7\left(b^2-4ac\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(b*d + 2*c*d*x)^7,x]
```

```
[Out] (2*(a + x*(b + c*x))^(3/2)*Hypergeometric2F1[3/2, 4, 5/2, (4*c*(a + x*(b +
c*x)))/(-b^2 + 4*a*c)])/(3*(b^2 - 4*a*c)^4*d^7)
```

Maple [B] time = 0.202, size = 460, normalized size = 2.6

$$-\frac{1}{192\,d^7c^6\left(4\,ac-b^2\right)}\left(\left(x+\frac{b}{2\,c}\right)^2c+\frac{4\,ac-b^2}{4\,c}\right)^{\frac{3}{2}}\left(x+\frac{b}{2\,c}\right)^{-6}+\frac{1}{64\,c^4d^7\left(4\,ac-b^2\right)^2}\left(\left(x+\frac{b}{2\,c}\right)^2c+\frac{4\,ac-b^2}{4\,c}\right)^{\frac{3}{2}}\left(x+\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^7,x)
```

```
[Out] -1/192/d^7/c^6/(4*a*c-b^2)/(x+1/2*b/c)^6*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c
)^(3/2)+1/64/d^7/c^4/(4*a*c-b^2)^2/(x+1/2*b/c)^4*((x+1/2*b/c)^2*c+1/4*(4*a*
c-b^2)/c)^(3/2)-1/32/d^7/c^2/(4*a*c-b^2)^3/(x+1/2*b/c)^2*((x+1/2*b/c)^2*c+1
```

```
/4*(4*a*c-b^2)/c)^(3/2)+1/64/d^7/c/(4*a*c-b^2)^3*(4*(x+1/2*b/c)^2*c+(4*a*c-
b^2)/c)^(1/2)-1/16/d^7/c/(4*a*c-b^2)^3/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c
-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/
(x+1/2*b/c))*a+1/64/d^7/c^2/(4*a*c-b^2)^3/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*
a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2
))/(x+1/2*b/c))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 43.5559, size = 2624, normalized size = 14.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^7,x, algorithm="fricas")
```

```
[Out] [-1/384*(3*(64*c^6*x^6 + 192*b*c^5*x^5 + 240*b^2*c^4*x^4 + 160*b^3*c^3*x^3
+ 60*b^4*c^2*x^2 + 12*b^5*c*x + b^6)*sqrt(-b^2*c + 4*a*c^2)*log(-(4*c^2*x^2
 + 4*b*c*x - b^2 + 8*a*c - 4*sqrt(-b^2*c + 4*a*c^2)*sqrt(c*x^2 + b*x + a))/
(4*c^2*x^2 + 4*b*c*x + b^2)) + 4*(3*b^6*c - 68*a*b^4*c^2 + 352*a^2*b^2*c^3
- 512*a^3*c^4 - 48*(b^2*c^5 - 4*a*c^6)*x^4 - 96*(b^3*c^4 - 4*a*b*c^5)*x^3 -
 16*(5*b^4*c^3 - 22*a*b^2*c^4 + 8*a^2*c^5)*x^2 - 32*(b^5*c^2 - 5*a*b^3*c^3
+ 4*a^2*b*c^4)*x)*sqrt(c*x^2 + b*x + a))/(64*(b^6*c^8 - 12*a*b^4*c^9 + 48*a
^2*b^2*c^10 - 64*a^3*c^11)*d^7*x^6 + 192*(b^7*c^7 - 12*a*b^5*c^8 + 48*a^2*b
^3*c^9 - 64*a^3*b*c^10)*d^7*x^5 + 240*(b^8*c^6 - 12*a*b^6*c^7 + 48*a^2*b^4*
c^8 - 64*a^3*b^2*c^9)*d^7*x^4 + 160*(b^9*c^5 - 12*a*b^7*c^6 + 48*a^2*b^5*c^
7 - 64*a^3*b^3*c^8)*d^7*x^3 + 60*(b^10*c^4 - 12*a*b^8*c^5 + 48*a^2*b^6*c^6
- 64*a^3*b^4*c^7)*d^7*x^2 + 12*(b^11*c^3 - 12*a*b^9*c^4 + 48*a^2*b^7*c^5 -
64*a^3*b^5*c^6)*d^7*x + (b^12*c^2 - 12*a*b^10*c^3 + 48*a^2*b^8*c^4 - 64*a^3
*b^6*c^5)*d^7), -1/192*(3*(64*c^6*x^6 + 192*b*c^5*x^5 + 240*b^2*c^4*x^4 + 1
60*b^3*c^3*x^3 + 60*b^4*c^2*x^2 + 12*b^5*c*x + b^6)*sqrt(b^2*c - 4*a*c^2)*a
rctan(1/2*sqrt(b^2*c - 4*a*c^2)*sqrt(c*x^2 + b*x + a)/(c^2*x^2 + b*c*x + a*
c)) + 2*(3*b^6*c - 68*a*b^4*c^2 + 352*a^2*b^2*c^3 - 512*a^3*c^4 - 48*(b^2*c
^5 - 4*a*c^6)*x^4 - 96*(b^3*c^4 - 4*a*b*c^5)*x^3 - 16*(5*b^4*c^3 - 22*a*b^2
*c^4 + 8*a^2*c^5)*x^2 - 32*(b^5*c^2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*x)*sqrt(c*
x^2 + b*x + a))/(64*(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11
)*d^7*x^6 + 192*(b^7*c^7 - 12*a*b^5*c^8 + 48*a^2*b^3*c^9 - 64*a^3*b*c^10)*d
^7*x^5 + 240*(b^8*c^6 - 12*a*b^6*c^7 + 48*a^2*b^4*c^8 - 64*a^3*b^2*c^9)*d^7
*x^4 + 160*(b^9*c^5 - 12*a*b^7*c^6 + 48*a^2*b^5*c^7 - 64*a^3*b^3*c^8)*d^7*x
^3 + 60*(b^10*c^4 - 12*a*b^8*c^5 + 48*a^2*b^6*c^6 - 64*a^3*b^4*c^7)*d^7*x^2
 + 12*(b^11*c^3 - 12*a*b^9*c^4 + 48*a^2*b^7*c^5 - 64*a^3*b^5*c^6)*d^7*x + (
b^12*c^2 - 12*a*b^10*c^3 + 48*a^2*b^8*c^4 - 64*a^3*b^6*c^5)*d^7)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{a+bx+cx^2}}{b^7+14b^6cx+84b^5c^2x^2+280b^4c^3x^3+560b^3c^4x^4+672b^2c^5x^5+448bc^6x^6+128c^7x^7}\,dx}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(2*c*d*x+b*d)**7,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(b**7 + 14*b**6*c*x + 84*b**5*c**2*x**2 + 2
80*b**4*c**3*x**3 + 560*b**3*c**4*x**4 + 672*b**2*c**5*x**5 + 448*b*c**6*x*
*6 + 128*c**7*x**7), x)/d**7
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^7,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1202 $\int (bd+2cdx)^5 \left(a+bx+cx^2\right)^{3/2} \, dx$

Optimal. Leaf size=98

$$\frac{16}{315} d^5 \left(b^2-4ac\right)^2 \left(a+bx+cx^2\right)^{5/2}+\frac{8}{63} d^5 \left(b^2-4ac\right) (b+2cx)^2 \left(a+bx+cx^2\right)^{5/2}+\frac{2}{9} d^5 (b+2cx)^4 \left(a+bx+cx^2\right)^{5/2}$$

```
[Out] (16*(b^2 - 4*a*c)^2*d^5*(a + b*x + c*x^2)^(5/2))/315 + (8*(b^2 - 4*a*c)*d^5
*(b + 2*c*x)^2*(a + b*x + c*x^2)^(5/2))/63 + (2*d^5*(b + 2*c*x)^4*(a + b*x
+ c*x^2)^(5/2))/9
```

Rubi [A] time = 0.0512991, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {692, 629}

$$\frac{16}{315} d^5 \left(b^2-4ac\right)^2 \left(a+bx+cx^2\right)^{5/2}+\frac{8}{63} d^5 \left(b^2-4ac\right) (b+2cx)^2 \left(a+bx+cx^2\right)^{5/2}+\frac{2}{9} d^5 (b+2cx)^4 \left(a+bx+cx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^5*(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (16*(b^2 - 4*a*c)^2*d^5*(a + b*x + c*x^2)^(5/2))/315 + (8*(b^2 - 4*a*c)*d^5
*(b + 2*c*x)^2*(a + b*x + c*x^2)^(5/2))/63 + (2*d^5*(b + 2*c*x)^4*(a + b*x
+ c*x^2)^(5/2))/9
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (bd+2cdx)^5 \left(a+bx+cx^2\right)^{3/2} \, dx &= \frac{2}{9} d^5 (b+2cx)^4 \left(a+bx+cx^2\right)^{5/2}+\frac{1}{9} \left(4 \left(b^2-4ac\right) d^2\right) \int (bd+2cdx)^3 \left(a+bx+cx^2\right. \\ &= \frac{8}{63} \left(b^2-4ac\right) d^5 (b+2cx)^2 \left(a+bx+cx^2\right)^{5/2}+\frac{2}{9} d^5 (b+2cx)^4 \left(a+bx+cx^2\right)^{5/2}+\frac{1}{6} \\ &= \frac{16}{315} \left(b^2-4ac\right)^2 d^5 \left(a+bx+cx^2\right)^{5/2}+\frac{8}{63} \left(b^2-4ac\right) d^5 (b+2cx)^2 \left(a+bx+cx^2\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.0773778, size = 92, normalized size = 0.94

$$\frac{2}{315} d^5 (a+x(b+cx))^{5/2} \left(16c^2 \left(8a^2-20acx^2+35c^2x^4\right)+8b^2c \left(115cx^2-18a\right)+160bc^2x \left(7cx^2-2a\right)+360b^3cx+63b^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^5*(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*d^5*(a + x*(b + c*x))^(5/2)*(63*b^4 + 360*b^3*c*x + 160*b*c^2*x*(-2*a +
7*c*x^2) + 8*b^2*c*(-18*a + 115*c*x^2) + 16*c^2*(8*a^2 - 20*a*c*x^2 + 35*c^
2*x^4)))/315
```

Maple [A] time = 0.045, size = 91, normalized size = 0.9

$$\frac{\left(1120\,c^4x^4+2240\,bc^3x^3-640\,x^2ac^3+1840\,x^2b^2c^2-640\,xbac^2+720\,xb^3c+256\,a^2c^2-288\,acb^2+126\,b^4\right)d^5}{315}\left(cx^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^5*(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 2/315*(c*x^2+b*x+a)^(5/2)*(560*c^4*x^4+1120*b*c^3*x^3-320*a*c^3*x^2+920*b^2
*c^2*x^2-320*a*b*c^2*x+360*b^3*c*x+128*a^2*c^2-144*a*b^2*c+63*b^4)*d^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^5*(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.93849, size = 564, normalized size = 5.76

$$\frac{2}{315}\left(560\,c^6d^5x^8+2240\,bc^5d^5x^7+40\left(93\,b^2c^4+20\,ac^5\right)d^5x^6+40\left(83\,b^3c^3+60\,abc^4\right)d^5x^5+\left(1703\,b^4c^2+2976\,ab^2c^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^5*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/315*(560*c^6*d^5*x^8 + 2240*b*c^5*d^5*x^7 + 40*(93*b^2*c^4 + 20*a*c^5)*d^
5*x^6 + 40*(83*b^3*c^3 + 60*a*b*c^4)*d^5*x^5 + (1703*b^4*c^2 + 2976*a*b^2*c
^3 + 48*a^2*c^4)*d^5*x^4 + 2*(243*b^5*c + 976*a*b^3*c^2 + 48*a^2*b*c^3)*d^5
*x^3 + (63*b^6 + 702*a*b^4*c + 120*a^2*b^2*c^2 - 64*a^3*c^3)*d^5*x^2 + 2*(6
3*a*b^5 + 36*a^2*b^3*c - 32*a^3*b*c^2)*d^5*x + (63*a^2*b^4 - 144*a^3*b^2*c
+ 128*a^4*c^2)*d^5)*sqrt(c*x^2 + b*x + a)
```

Sympy [B] time = 4.43183, size = 656, normalized size = 6.69

$$\frac{256a^4c^2d^5\sqrt{a+bx+cx^2}}{315}-\frac{32a^3b^2cd^5\sqrt{a+bx+cx^2}}{35}-\frac{128a^3bc^2d^5x\sqrt{a+bx+cx^2}}{315}-\frac{128a^3c^3d^5x^2\sqrt{a+bx+cx^2}}{315}+\frac{2a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**5*(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] 256*a**4*c**2*d**5*sqrt(a + b*x + c*x**2)/315 - 32*a**3*b**2*c*d**5*sqrt(a
+ b*x + c*x**2)/35 - 128*a**3*b*c**2*d**5*x*sqrt(a + b*x + c*x**2)/315 - 12
8*a**3*c**3*d**5*x**2*sqrt(a + b*x + c*x**2)/315 + 2*a**2*b**4*d**5*sqrt(a
+ b*x + c*x**2)/5 + 16*a**2*b**3*c*d**5*x*sqrt(a + b*x + c*x**2)/35 + 16*a*
*2*b**2*c**2*d**5*x**2*sqrt(a + b*x + c*x**2)/21 + 64*a**2*b*c**3*d**5*x**3
*sqrt(a + b*x + c*x**2)/105 + 32*a**2*c**4*d**5*x**4*sqrt(a + b*x + c*x**2)
/105 + 4*a*b**5*d**5*x*sqrt(a + b*x + c*x**2)/5 + 156*a*b**4*c*d**5*x**2*sq
rt(a + b*x + c*x**2)/35 + 3904*a*b**3*c**2*d**5*x**3*sqrt(a + b*x + c*x**2)
/315 + 1984*a*b**2*c**3*d**5*x**4*sqrt(a + b*x + c*x**2)/105 + 320*a*b*c**4
*d**5*x**5*sqrt(a + b*x + c*x**2)/21 + 320*a*c**5*d**5*x**6*sqrt(a + b*x +
c*x**2)/63 + 2*b**6*d**5*x**2*sqrt(a + b*x + c*x**2)/5 + 108*b**5*c*d**5*x*
*3*sqrt(a + b*x + c*x**2)/35 + 3406*b**4*c**2*d**5*x**4*sqrt(a + b*x + c*x*
*2)/315 + 1328*b**3*c**3*d**5*x**5*sqrt(a + b*x + c*x**2)/63 + 496*b**2*c**
4*d**5*x**6*sqrt(a + b*x + c*x**2)/21 + 128*b*c**5*d**5*x**7*sqrt(a + b*x +
 c*x**2)/9 + 32*c**6*d**5*x**8*sqrt(a + b*x + c*x**2)/9
```

Giac [B] time = 1.19533, size = 441, normalized size = 4.5

$$\frac{2}{315}\sqrt{cx^2+bx+a}\left(\left(\left(\left(\left(40\left(\left(14\left(c^6d^5x+4\,bc^5d^5\right)x+\frac{93\,b^2c^{12}d^5+20\,ac^{13}d^5}{c^8}\right)x+\frac{83\,b^3c^{11}d^5+60\,abc^{12}d^5}{c^8}\right)x+\frac{1703\,b^4c^{10}}{}\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^5*(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/315*sqrt(c*x^2 + b*x + a)*(((((40*((14*(c^6*d^5*x + 4*b*c^5*d^5)*x + (93*
b^2*c^12*d^5 + 20*a*c^13*d^5)/c^8)*x + (83*b^3*c^11*d^5 + 60*a*b*c^12*d^5)/
c^8)*x + (1703*b^4*c^10*d^5 + 2976*a*b^2*c^11*d^5 + 48*a^2*c^12*d^5)/c^8)*x
 + 2*(243*b^5*c^9*d^5 + 976*a*b^3*c^10*d^5 + 48*a^2*b*c^11*d^5)/c^8)*x + (6
3*b^6*c^8*d^5 + 702*a*b^4*c^9*d^5 + 120*a^2*b^2*c^10*d^5 - 64*a^3*c^11*d^5)
/c^8)*x + 2*(63*a*b^5*c^8*d^5 + 36*a^2*b^3*c^9*d^5 - 32*a^3*b*c^10*d^5)/c^8
)*x + (63*a^2*b^4*c^8*d^5 - 144*a^3*b^2*c^9*d^5 + 128*a^4*c^10*d^5)/c^8)
```

3.1203 $\int (bd+2cdx)^4 \left(a+bx+cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=207

$$-\frac{d^4\left(b^2-4ac\right)(b+2cx)^5\sqrt{a+bx+cx^2}}{128c^2}+\frac{d^4\left(b^2-4ac\right)^2(b+2cx)^3\sqrt{a+bx+cx^2}}{512c^2}+\frac{3d^4\left(b^2-4ac\right)^3(b+2cx)\sqrt{a+bx}}{1024c^2}$$

```
[Out] (3*(b^2 - 4*a*c)^3*d^4*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(1024*c^2) + ((b^
2 - 4*a*c)^2*d^4*(b + 2*c*x)^3*Sqrt[a + b*x + c*x^2])/(512*c^2) - ((b^2 - 4
*a*c)*d^4*(b + 2*c*x)^5*Sqrt[a + b*x + c*x^2])/(128*c^2) + (d^4*(b + 2*c*x)
^5*(a + b*x + c*x^2)^(3/2))/(16*c) + (3*(b^2 - 4*a*c)^4*d^4*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2048*c^(5/2))
```

Rubi [A] time = 0.128642, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {685, 692, 621, 206}

$$-\frac{d^4\left(b^2-4ac\right)(b+2cx)^5\sqrt{a+bx+cx^2}}{128c^2}+\frac{d^4\left(b^2-4ac\right)^2(b+2cx)^3\sqrt{a+bx+cx^2}}{512c^2}+\frac{3d^4\left(b^2-4ac\right)^3(b+2cx)\sqrt{a+bx}}{1024c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^4*(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (3*(b^2 - 4*a*c)^3*d^4*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(1024*c^2) + ((b^
2 - 4*a*c)^2*d^4*(b + 2*c*x)^3*Sqrt[a + b*x + c*x^2])/(512*c^2) - ((b^2 - 4
*a*c)*d^4*(b + 2*c*x)^5*Sqrt[a + b*x + c*x^2])/(128*c^2) + (d^4*(b + 2*c*x)
^5*(a + b*x + c*x^2)^(3/2))/(16*c) + (3*(b^2 - 4*a*c)^4*d^4*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2048*c^(5/2))
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (bd+2cdx)^4\left(a+bx+cx^2\right)^{3/2}\,dx = \frac{d^4(b+2cx)^5\left(a+bx+cx^2\right)^{3/2}}{16c} - \frac{\left(3\left(b^2-4ac\right)\right)\int (bd+2cdx)^4\sqrt{a+bx+cx^2}\,dx}{32c}$$

$$= -\frac{\left(b^2-4ac\right)d^4(b+2cx)^5\sqrt{a+bx+cx^2}}{128c^2} + \frac{d^4(b+2cx)^5\left(a+bx+cx^2\right)^{3/2}}{16c} + \frac{\left(b^2-4\right.}{}$$

$$= \frac{\left(b^2-4ac\right)^2 d^4(b+2cx)^3\sqrt{a+bx+cx^2}}{512c^2} - \frac{\left(b^2-4ac\right)d^4(b+2cx)^5\sqrt{a+bx+cx^2}}{128c^2} +$$

$$= \frac{3\left(b^2-4ac\right)^3 d^4(b+2cx)\sqrt{a+bx+cx^2}}{1024c^2} + \frac{\left(b^2-4ac\right)^2 d^4(b+2cx)^3\sqrt{a+bx+cx^2}}{512c^2}$$

$$= \frac{3\left(b^2-4ac\right)^3 d^4(b+2cx)\sqrt{a+bx+cx^2}}{1024c^2} + \frac{\left(b^2-4ac\right)^2 d^4(b+2cx)^3\sqrt{a+bx+cx^2}}{512c^2}$$

$$= \frac{3\left(b^2-4ac\right)^3 d^4(b+2cx)\sqrt{a+bx+cx^2}}{1024c^2} + \frac{\left(b^2-4ac\right)^2 d^4(b+2cx)^3\sqrt{a+bx+cx^2}}{512c^2}$$

Mathematica [A] time = 2.10874, size = 249, normalized size = 1.2

$$\frac{1}{4}d^4\left((b+2cx)^3(a+x(b+cx))^{5/2} - 2c\left(a-\frac{b^2}{4c}\right)(b+2cx)\sqrt{a+x(b+cx)}\left((a+x(b+cx))^2 - \frac{(a+x(b+cx))\left(2(b+2cx)\sqrt{\right.}\right.}{}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^4*(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (d^4*((b + 2*c*x)^3*(a + x*(b + c*x))^(5/2) - 2*(a - b^2/(4*c))*c*(b + 2*c*
x)*Sqrt[a + x*(b + c*x)]*((a + x*(b + c*x))^2 - ((a + x*(b + c*x))*(2*(b +
2*c*x)*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*(-3*b^2 + 8*b*c*x + 4*c*(
5*a + 2*c*x^2)) + 3*Sqrt[4*a - b^2/c]*Sqrt[c]*(-b^2 + 4*a*c)*ArcSinh[(b + 2
*c*x)/(Sqrt[4*a - b^2/c]*Sqrt[c])]))/(256*c*(b + 2*c*x)*((c*(a + x*(b + c*x
)))/(-b^2 + 4*a*c))^(3/2)))))/4
```

Maple [B] time = 0.056, size = 641, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^4*(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 3*d^4*c^2*b*x^2*(c*x^2+b*x+a)^(5/2)-1/2*d^4*c*b*a*(c*x^2+b*x+a)^(5/2)+7/4*d
^4*c*b^2*x*(c*x^2+b*x+a)^(5/2)-3/512*d^4/c*b^6*(c*x^2+b*x+a)^(1/2)*x+9/128*
d^4*b^4*(c*x^2+b*x+a)^(1/2)*x*a+9/64*d^4*b^4/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))*a^2-3/128*d^4*b^6/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))*a-3/8*d^4*c^(1/2)*b^2*a^3*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*
x+a)^(1/2))+1/4*d^4*c^2*a^2*x*(c*x^2+b*x+a)^(3/2)+1/8*d^4*c*a^2*(c*x^2+b*x+
a)^(3/2)*b+3/8*d^4*c^2*a^3*(c*x^2+b*x+a)^(1/2)*x+3/16*d^4*c*a^3*(c*x^2+b*x+
a)^(1/2)*b-d^4*c^2*a*x*(c*x^2+b*x+a)^(5/2)+2*d^4*c^3*x^3*(c*x^2+b*x+a)^(5/2
)+1/128*d^4/c*b^5*(c*x^2+b*x+a)^(3/2)-3/1024*d^4/c^2*b^7*(c*x^2+b*x+a)^(1/2
)+3/8*d^4*c^(3/2)*a^4*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/16*d^4*
b^3*a*(c*x^2+b*x+a)^(3/2)-9/64*d^4*b^3*a^2*(c*x^2+b*x+a)^(1/2)+1/64*d^4*b^4
*x*(c*x^2+b*x+a)^(3/2)+3/2048*d^4*b^8/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))+9/256*d^4/c*b^5*(c*x^2+b*x+a)^(1/2)*a+3/8*d^4*b^3*(c*x^2+b*x
+a)^(5/2)-9/32*d^4*c*b^2*a^2*(c*x^2+b*x+a)^(1/2)*x-1/8*d^4*c*b^2*a*x*(c*x^2
+b*x+a)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4*(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.80686, size = 1540, normalized size = 7.44

$$\left[\frac{3\left(b^8 - 16\,ab^6c + 96\,a^2b^4c^2 - 256\,a^3b^2c^3 + 256\,a^4c^4\right)\sqrt{c}d^4\log\left(-8\,c^2x^2 - 8\,bcx - b^2 - 4\,\sqrt{cx^2 + bx + a}(2\,cx + b)\sqrt{c} - \right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4096*(3*(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c
^4)*sqrt(c)*d^4*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2
*c*x + b)*sqrt(c) - 4*a*c) + 4*(2048*c^8*d^4*x^7 + 7168*b*c^7*d^4*x^6 + 768
*(13*b^2*c^6 + 4*a*c^7)*d^4*x^5 + 640*(11*b^3*c^5 + 12*a*b*c^6)*d^4*x^4 + 1
6*(161*b^4*c^4 + 472*a*b^2*c^5 + 16*a^2*c^6)*d^4*x^3 + 24*(17*b^5*c^3 + 152
*a*b^3*c^4 + 16*a^2*b*c^5)*d^4*x^2 + 2*(b^6*c^2 + 396*a*b^4*c^3 + 240*a^2*b
^2*c^4 - 192*a^3*c^5)*d^4*x - (3*b^7*c - 44*a*b^5*c^2 - 176*a^2*b^3*c^3 + 1
92*a^3*b*c^4)*d^4)*sqrt(c*x^2 + b*x + a))/c^3, -1/2048*(3*(b^8 - 16*a*b^6*c
 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4)*sqrt(-c)*d^4*arctan(1/2*
sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(20
48*c^8*d^4*x^7 + 7168*b*c^7*d^4*x^6 + 768*(13*b^2*c^6 + 4*a*c^7)*d^4*x^5 +
640*(11*b^3*c^5 + 12*a*b*c^6)*d^4*x^4 + 16*(161*b^4*c^4 + 472*a*b^2*c^5 + 1
6*a^2*c^6)*d^4*x^3 + 24*(17*b^5*c^3 + 152*a*b^3*c^4 + 16*a^2*b*c^5)*d^4*x^2
 + 2*(b^6*c^2 + 396*a*b^4*c^3 + 240*a^2*b^2*c^4 - 192*a^3*c^5)*d^4*x - (3*b
^7*c - 44*a*b^5*c^2 - 176*a^2*b^3*c^3 + 192*a^3*b*c^4)*d^4)*sqrt(c*x^2 + b*
```

```
x + a))/c^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^4\left(\int ab^4\sqrt{a+bx+cx^2}\,dx+\int b^5x\sqrt{a+bx+cx^2}\,dx+\int 16c^5x^6\sqrt{a+bx+cx^2}\,dx+\int 16ac^4x^4\sqrt{a+bx+cx^2}\,dx+\int 4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**4*(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] d**4*(Integral(a*b**4*sqrt(a + b*x + c*x**2), x) + Integral(b**5*x*sqrt(a +
 b*x + c*x**2), x) + Integral(16*c**5*x**6*sqrt(a + b*x + c*x**2), x) + Int
egral(16*a*c**4*x**4*sqrt(a + b*x + c*x**2), x) + Integral(48*b*c**4*x**5*s
qrt(a + b*x + c*x**2), x) + Integral(56*b**2*c**3*x**4*sqrt(a + b*x + c*x**
2), x) + Integral(32*b**3*c**2*x**3*sqrt(a + b*x + c*x**2), x) + Integral(9
*b**4*c*x**2*sqrt(a + b*x + c*x**2), x) + Integral(32*a*b*c**3*x**3*sqrt(a
+ b*x + c*x**2), x) + Integral(24*a*b**2*c**2*x**2*sqrt(a + b*x + c*x**2),
x) + Integral(8*a*b**3*c*x*sqrt(a + b*x + c*x**2), x))
```

Giac [B] time = 1.19455, size = 528, normalized size = 2.55

$$\frac{1}{1024}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(2\left(4\left(2c^5d^4x+7bc^4d^4\right)x+\frac{3\left(13b^2c^{10}d^4+4ac^{11}d^4\right)}{c^7}\right)x+\frac{5\left(11b^3c^9d^4+12abc^{10}d^4\right)}{c^7}\right)x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4*(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1024*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(2*(4*(2*c^5*d^4*x + 7*b*c^4*d^4)*
x + 3*(13*b^2*c^10*d^4 + 4*a*c^11*d^4)/c^7)*x + 5*(11*b^3*c^9*d^4 + 12*a*b*
c^10*d^4)/c^7)*x + (161*b^4*c^8*d^4 + 472*a*b^2*c^9*d^4 + 16*a^2*c^10*d^4)/
c^7)*x + 3*(17*b^5*c^7*d^4 + 152*a*b^3*c^8*d^4 + 16*a^2*b*c^9*d^4)/c^7)*x +
 (b^6*c^6*d^4 + 396*a*b^4*c^7*d^4 + 240*a^2*b^2*c^8*d^4 - 192*a^3*c^9*d^4)/
c^7)*x - (3*b^7*c^5*d^4 - 44*a*b^5*c^6*d^4 - 176*a^2*b^3*c^7*d^4 + 192*a^3*
b*c^8*d^4)/c^7) - 3/2048*(b^8*d^4 - 16*a*b^6*c*d^4 + 96*a^2*b^4*c^2*d^4 - 2
56*a^3*b^2*c^3*d^4 + 256*a^4*c^4*d^4)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))*sqrt(c) - b))/c^(5/2)
```

3.1204 $\int (bd+2cdx)^3 \left(a+bx+cx^2\right)^{3/2} \, dx$

Optimal. Leaf size=59

$$\frac{4}{35}d^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{5/2}+\frac{2}{7}d^3(b+2cx)^2\left(a+bx+cx^2\right)^{5/2}$$

```
[Out] (4*(b^2 - 4*a*c)*d^3*(a + b*x + c*x^2)^(5/2))/35 + (2*d^3*(b + 2*c*x)^2*(a
+ b*x + c*x^2)^(5/2))/7
```

Rubi [A] time = 0.0269461, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {692, 629}

$$\frac{4}{35}d^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{5/2}+\frac{2}{7}d^3(b+2cx)^2\left(a+bx+cx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^3*(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (4*(b^2 - 4*a*c)*d^3*(a + b*x + c*x^2)^(5/2))/35 + (2*d^3*(b + 2*c*x)^2*(a
+ b*x + c*x^2)^(5/2))/7
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (bd+2cdx)^3\left(a+bx+cx^2\right)^{3/2}\,dx &= \frac{2}{7}d^3(b+2cx)^2\left(a+bx+cx^2\right)^{5/2}+\frac{1}{7}\left(2\left(b^2-4ac\right)d^2\right)\int (bd+2cdx)\left(a+bx+c\right.\\ &= \frac{4}{35}\left(b^2-4ac\right)d^3\left(a+bx+cx^2\right)^{5/2}+\frac{2}{7}d^3(b+2cx)^2\left(a+bx+cx^2\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.0466772, size = 44, normalized size = 0.75

$$\frac{2}{35}d^3(a+x(b+cx))^{5/2}\left(4c\left(5cx^2-2a\right)+7b^2+20bcx\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^3*(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*d^3*(a + x*(b + c*x))^(5/2)*(7*b^2 + 20*b*c*x + 4*c*(-2*a + 5*c*x^2)))/3
5
```

Maple [A] time = 0.045, size = 41, normalized size = 0.7

$$-\frac{\left(-40\,c^2x^2-40\,bcx+16\,ac-14\,b^2\right)d^3}{35}\left(cx^2+bx+a\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^3*(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -2/35*(c*x^2+b*x+a)^(5/2)*(-20*c^2*x^2-20*b*c*x+8*a*c-7*b^2)*d^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3*(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.66994, size = 320, normalized size = 5.42

$$\frac{2}{35}\left(20\,c^4d^3x^6+60\,bc^3d^3x^5+\left(67\,b^2c^2+32\,ac^3\right)d^3x^4+2\left(17\,b^3c+32\,abc^2\right)d^3x^3+\left(7\,b^4+46\,ab^2c+4\,a^2c^2\right)d^3x^2+2\left(7\,ab\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(20*c^4*d^3*x^6 + 60*b*c^3*d^3*x^5 + (67*b^2*c^2 + 32*a*c^3)*d^3*x^4 +
 2*(17*b^3*c + 32*a*b*c^2)*d^3*x^3 + (7*b^4 + 46*a*b^2*c + 4*a^2*c^2)*d^3*x
^2 + 2*(7*a*b^3 + 2*a^2*b*c)*d^3*x + (7*a^2*b^2 - 8*a^3*c)*d^3)*sqrt(c*x^2
+ b*x + a)
```

Sympy [B] time = 1.83522, size = 371, normalized size = 6.29

$$-\frac{16a^3cd^3\sqrt{a+bx+cx^2}}{35}+\frac{2a^2b^2d^3\sqrt{a+bx+cx^2}}{5}+\frac{8a^2bcd^3x\sqrt{a+bx+cx^2}}{35}+\frac{8a^2c^2d^3x^2\sqrt{a+bx+cx^2}}{35}+\frac{4ab^3d^3x\sqrt{a+}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**3*(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] -16*a**3*c*d**3*sqrt(a + b*x + c*x**2)/35 + 2*a**2*b**2*d**3*sqrt(a + b*x +
 c*x**2)/5 + 8*a**2*b*c*d**3*x*sqrt(a + b*x + c*x**2)/35 + 8*a**2*c**2*d**3
*x**2*sqrt(a + b*x + c*x**2)/35 + 4*a*b**3*d**3*x*sqrt(a + b*x + c*x**2)/5
+ 92*a*b**2*c*d**3*x**2*sqrt(a + b*x + c*x**2)/35 + 128*a*b*c**2*d**3*x**3*
sqrt(a + b*x + c*x**2)/35 + 64*a*c**3*d**3*x**4*sqrt(a + b*x + c*x**2)/35 +
 2*b**4*d**3*x**2*sqrt(a + b*x + c*x**2)/5 + 68*b**3*c*d**3*x**3*sqrt(a + b
*x + c*x**2)/35 + 134*b**2*c**2*d**3*x**4*sqrt(a + b*x + c*x**2)/35 + 24*b*
c**3*d**3*x**5*sqrt(a + b*x + c*x**2)/7 + 8*c**4*d**3*x**6*sqrt(a + b*x + c
*x**2)/7
```

Giac [B] time = 1.13857, size = 269, normalized size = 4.56

$$\frac{2}{35}\sqrt{cx^2+bx+a}\left(\left(\left(\left(\left(20\left(c^4d^3x+3\,bc^3d^3\right)x+\frac{67\,b^2c^8d^3+32\,ac^9d^3}{c^6}\right)x+\frac{2\left(17\,b^3c^7d^3+32\,abc^8d^3\right)}{c^6}\right)x+\frac{7\,b^4c^6d^3+46\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3*(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/35*sqrt(c*x^2 + b*x + a)*(((((20*(c^4*d^3*x + 3*b*c^3*d^3)*x + (67*b^2*c^
8*d^3 + 32*a*c^9*d^3)/c^6)*x + 2*(17*b^3*c^7*d^3 + 32*a*b*c^8*d^3)/c^6)*x +
 (7*b^4*c^6*d^3 + 46*a*b^2*c^7*d^3 + 4*a^2*c^8*d^3)/c^6)*x + 2*(7*a*b^3*c^6
*d^3 + 2*a^2*b*c^7*d^3)/c^6)*x + (7*a^2*b^2*c^6*d^3 - 8*a^3*c^7*d^3)/c^6)
```

3.1205 $\int (bd+2cdx)^2 \left(a+bx+cx^2\right)^{3/2} dx$

Optimal. Leaf size=165

$$\frac{d^2\left(b^2-4ac\right)^2(b+2cx)\sqrt{a+bx+cx^2}}{128c^2}-\frac{d^2\left(b^2-4ac\right)(b+2cx)^3\sqrt{a+bx+cx^2}}{64c^2}+\frac{d^2\left(b^2-4ac\right)^3\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{5/2}}+$$

```
[Out] ((b^2 - 4*a*c)^2*d^2*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(128*c^2) - ((b^2 -
 4*a*c)*d^2*(b + 2*c*x)^3*Sqrt[a + b*x + c*x^2])/(64*c^2) + (d^2*(b + 2*c*x
)^3*(a + b*x + c*x^2)^(3/2))/(12*c) + ((b^2 - 4*a*c)^3*d^2*ArcTanh[(b + 2*c
*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(5/2))
```

Rubi [A] time = 0.0887438, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {685, 692, 621, 206}

$$\frac{d^2\left(b^2-4ac\right)^2(b+2cx)\sqrt{a+bx+cx^2}}{128c^2}-\frac{d^2\left(b^2-4ac\right)(b+2cx)^3\sqrt{a+bx+cx^2}}{64c^2}+\frac{d^2\left(b^2-4ac\right)^3\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{5/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^2*(a + b*x + c*x^2)^(3/2),x]
```


```
[Out] ((b^2 - 4*a*c)^2*d^2*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(128*c^2) - ((b^2 -
 4*a*c)*d^2*(b + 2*c*x)^3*Sqrt[a + b*x + c*x^2])/(64*c^2) + (d^2*(b + 2*c*x
)^3*(a + b*x + c*x^2)^(3/2))/(12*c) + ((b^2 - 4*a*c)^3*d^2*ArcTanh[(b + 2*c
*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(5/2))
```


Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (bd+2cdx)^2\left(a+bx+cx^2\right)^{3/2}dx = \frac{d^2(b+2cx)^3\left(a+bx+cx^2\right)^{3/2}}{12c} - \frac{\left(b^2-4ac\right)\int (bd+2cdx)^2\sqrt{a+bx+cx^2}\,dx}{8c}$$

$$= -\frac{\left(b^2-4ac\right)d^2(b+2cx)^3\sqrt{a+bx+cx^2}}{64c^2} + \frac{d^2(b+2cx)^3\left(a+bx+cx^2\right)^{3/2}}{12c} + \frac{\left(b^2\right.}{}$$

$$= \frac{\left(b^2-4ac\right)^2 d^2(b+2cx)\sqrt{a+bx+cx^2}}{128c^2} - \frac{\left(b^2-4ac\right)d^2(b+2cx)^3\sqrt{a+bx+cx^2}}{64c^2}$$

$$= \frac{\left(b^2-4ac\right)^2 d^2(b+2cx)\sqrt{a+bx+cx^2}}{128c^2} - \frac{\left(b^2-4ac\right)d^2(b+2cx)^3\sqrt{a+bx+cx^2}}{64c^2}$$

$$= \frac{\left(b^2-4ac\right)^2 d^2(b+2cx)\sqrt{a+bx+cx^2}}{128c^2} - \frac{\left(b^2-4ac\right)d^2(b+2cx)^3\sqrt{a+bx+cx^2}}{64c^2}$$

Mathematica [A] time = 0.456914, size = 211, normalized size = 1.28

$$\frac{1}{3}d^2(b+2cx)\sqrt{a+x(b+cx)}\left((a+x(b+cx))^2 - \frac{(a+x(b+cx))\left(2(b+2cx)\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\left(4c\left(5a+2cx^2\right)-3b^2+8bcx\right)\right.}{256c(b+2cx)\left(\frac{c(a+x(b+cx))}{4ac-b^2}\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^2*(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (d^2*(b + 2*c*x)*Sqrt[a + x*(b + c*x)]*((a + x*(b + c*x))^2 - ((a + x*(b +
c*x))*(2*(b + 2*c*x)*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*(-3*b^2 + 8
*b*c*x + 4*c*(5*a + 2*c*x^2)) + 3*Sqrt[4*a - b^2/c]*Sqrt[c]*(-b^2 + 4*a*c)*
ArcSinh[(b + 2*c*x)/(Sqrt[4*a - b^2/c]*Sqrt[c])]))/(256*c*(b + 2*c*x)*((c*(
a + x*(b + c*x)))/(-b^2 + 4*a*c))^(3/2))))/3
```

Maple [B] time = 0.049, size = 406, normalized size = 2.5

$$-\frac{a^2d^2b}{8}\sqrt{cx^2+bx+a} + \frac{b^2d^2x}{24}\left(cx^2+bx+a\right)^{\frac{3}{2}} + \frac{d^2b^3}{48\,c}\left(cx^2+bx+a\right)^{\frac{3}{2}} - \frac{d^2b^5}{128\,c^2}\sqrt{cx^2+bx+a} - \frac{ad^2b}{12}\left(cx^2+bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^2*(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -1/8*d^2*a^2*(c*x^2+b*x+a)^(1/2)*b+1/24*d^2*b^2*x*(c*x^2+b*x+a)^(3/2)+1/48*
d^2/c*b^3*(c*x^2+b*x+a)^(3/2)-1/128*d^2/c^2*b^5*(c*x^2+b*x+a)^(1/2)-1/12*d^
2*a*(c*x^2+b*x+a)^(3/2)*b+2/3*d^2*c*x*(c*x^2+b*x+a)^(5/2)+1/3*d^2*b*(c*x^2+
```

```
b*x+a)^(5/2)+3/16*d^2*b^2/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2
))*a^2-3/64*d^2*b^4/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+1
/8*d^2*b^2*(c*x^2+b*x+a)^(1/2)*x*a-1/64*d^2/c*b^4*(c*x^2+b*x+a)^(1/2)*x+1/1
6*d^2/c*b^3*(c*x^2+b*x+a)^(1/2)*a+1/256*d^2*b^6/c^(5/2)*ln((1/2*b+c*x)/c^(1
/2)+(c*x^2+b*x+a)^(1/2))-1/6*d^2*c*a*x*(c*x^2+b*x+a)^(3/2)-1/4*d^2*c*a^2*(c
*x^2+b*x+a)^(1/2)*x-1/4*d^2*c^(1/2)*a^3*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2*(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.86534, size = 1071, normalized size = 6.49

$$\left[-\frac{3\left(b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3\right)\sqrt{c}d^2\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right)-4\left(256\,c^6\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/1536*(3*(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(c)*d^2*lo
g(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c)
- 4*a*c) - 4*(256*c^6*d^2*x^5 + 640*b*c^5*d^2*x^4 + 16*(33*b^2*c^4 + 28*a*c
^5)*d^2*x^3 + 8*(19*b^3*c^3 + 84*a*b*c^4)*d^2*x^2 + 2*(b^4*c^2 + 144*a*b^2*
c^3 + 48*a^2*c^4)*d^2*x - (3*b^5*c - 32*a*b^3*c^2 - 48*a^2*b*c^3)*d^2)*sqrt
(c*x^2 + b*x + a))/c^3, -1/768*(3*(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a
^3*c^3)*sqrt(-c)*d^2*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/
(c^2*x^2 + b*c*x + a*c)) - 2*(256*c^6*d^2*x^5 + 640*b*c^5*d^2*x^4 + 16*(33*
b^2*c^4 + 28*a*c^5)*d^2*x^3 + 8*(19*b^3*c^3 + 84*a*b*c^4)*d^2*x^2 + 2*(b^4*
c^2 + 144*a*b^2*c^3 + 48*a^2*c^4)*d^2*x - (3*b^5*c - 32*a*b^3*c^2 - 48*a^2*
b*c^3)*d^2)*sqrt(c*x^2 + b*x + a))/c^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int ab^2\sqrt{a+bx+cx^2}\,dx+\int b^3x\sqrt{a+bx+cx^2}\,dx+\int 4c^3x^4\sqrt{a+bx+cx^2}\,dx+\int 4ac^2x^2\sqrt{a+bx+cx^2}\,dx+\int 8bc\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**2*(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] d**2*(Integral(a*b**2*sqrt(a + b*x + c*x**2), x) + Integral(b**3*x*sqrt(a +
 b*x + c*x**2), x) + Integral(4*c**3*x**4*sqrt(a + b*x + c*x**2), x) + Inte
```

```
gral(4*a*c**2*x**2*sqrt(a + b*x + c*x**2), x) + Integral(8*b*c**2*x**3*sqrt
(a + b*x + c*x**2), x) + Integral(5*b**2*c*x**2*sqrt(a + b*x + c*x**2), x)
+ Integral(4*a*b*c*x*sqrt(a + b*x + c*x**2), x))
```

Giac [A] time = 1.19731, size = 350, normalized size = 2.12

$$\frac{1}{384}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(2c^3d^2x+5bc^2d^2\right)x+\frac{33b^2c^6d^2+28ac^7d^2}{c^5}\right)x+\frac{19b^3c^5d^2+84abc^6d^2}{c^5}\right)x+\frac{b^4c^4d^2+14$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2*(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/384*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(2*c^3*d^2*x + 5*b*c^2*d^2)*x + (33
*b^2*c^6*d^2 + 28*a*c^7*d^2)/c^5)*x + (19*b^3*c^5*d^2 + 84*a*b*c^6*d^2)/c^5
)*x + (b^4*c^4*d^2 + 144*a*b^2*c^5*d^2 + 48*a^2*c^6*d^2)/c^5)*x - (3*b^5*c^
3*d^2 - 32*a*b^3*c^4*d^2 - 48*a^2*b*c^5*d^2)/c^5) - 1/256*(b^6*d^2 - 12*a*b
^4*c*d^2 + 48*a^2*b^2*c^2*d^2 - 64*a^3*c^3*d^2)*log(abs(-2*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))*sqrt(c) - b))/c^(5/2)
```

3.1206 $\int (bd + 2cdx) \left(a + bx + cx^2\right)^{3/2} dx$

Optimal. Leaf size=19

$$\frac{2}{5}d\left(a + bx + cx^2\right)^{5/2}$$

```
[Out] (2*d*(a + b*x + c*x^2)^(5/2))/5
```

Rubi [A] time = 0.0066193, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {629}

$$\frac{2}{5}d\left(a + bx + cx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)*(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*d*(a + b*x + c*x^2)^(5/2))/5
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (bd + 2cdx) \left(a + bx + cx^2\right)^{3/2} dx = \frac{2}{5}d\left(a + bx + cx^2\right)^{5/2}$$

Mathematica [A] time = 0.0144772, size = 18, normalized size = 0.95

$$\frac{2}{5}d(a + x(b + cx))^{5/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)*(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*d*(a + x*(b + c*x))^(5/2))/5
```

Maple [A] time = 0.042, size = 16, normalized size = 0.8

$$\frac{2d}{5}\left(cx^2 + bx + a\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)*(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 2/5*d*(c*x^2+b*x+a)^(5/2)
```

Maxima [A] time = 1.12318, size = 20, normalized size = 1.05

$$\frac{2}{5}\left(cx^2+bx+a\right)^{\frac{5}{2}}d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)*(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(c*x^2 + b*x + a)^(5/2)*d
```

Fricas [B] time = 2.63561, size = 128, normalized size = 6.74

$$\frac{2}{5}\left(c^2dx^4+2\,bcdx^3+2\,abdx+\left(b^2+2\,ac\right)dx^2+a^2d\right)\sqrt{cx^2+bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(c^2*d*x^4 + 2*b*c*d*x^3 + 2*a*b*d*x + (b^2 + 2*a*c)*d*x^2 + a^2*d)*sqr
t(c*x^2 + b*x + a)
```

Sympy [B] time = 0.939766, size = 146, normalized size = 7.68

$$\frac{2a^2d\sqrt{a+bx+cx^2}}{5}+\frac{4abdx\sqrt{a+bx+cx^2}}{5}+\frac{4acdx^2\sqrt{a+bx+cx^2}}{5}+\frac{2b^2dx^2\sqrt{a+bx+cx^2}}{5}+\frac{4bcdx^3\sqrt{a+bx+cx^2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)*(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] 2*a**2*d*sqrt(a + b*x + c*x**2)/5 + 4*a*b*d*x*sqrt(a + b*x + c*x**2)/5 + 4*
a*c*d*x**2*sqrt(a + b*x + c*x**2)/5 + 2*b**2*d*x**2*sqrt(a + b*x + c*x**2)/
5 + 4*b*c*d*x**3*sqrt(a + b*x + c*x**2)/5 + 2*c**2*d*x**4*sqrt(a + b*x + c*
x**2)/5
```

Giac [B] time = 1.22153, size = 88, normalized size = 4.63

$$\frac{2}{5}\left(a^2d+\left(2\,abd+\left(\left(c^2dx+2\,bcd\right)x+\frac{b^2c^4d+2\,ac^5d}{c^4}\right)x\right)x\right)\sqrt{cx^2+bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)*(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/5*(a^2*d + (2*a*b*d + ((c^2*d*x + 2*b*c*d)*x + (b^2*c^4*d + 2*a*c^5*d)/c^
4)*x)*x)*sqrt(c*x^2 + b*x + a)
```

3.1207 $\int \frac{(a+bx+cx^2)^{3/2}}{bd+2cdx}\,dx$

Optimal. Leaf size=115

$$-\frac{(b^2-4ac)\sqrt{a+bx+cx^2}}{8c^2d}+\frac{(b^2-4ac)^{3/2}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{16c^{5/2}d}+\frac{(a+bx+cx^2)^{3/2}}{6cd}$$

```
[Out] -((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])/(8*c^2*d) + (a + b*x + c*x^2)^(3/2)/
(6*c*d) + ((b^2 - 4*a*c)^(3/2)*ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqr
t[b^2 - 4*a*c]])/(16*c^(5/2)*d)
```

Rubi [A] time = 0.0827866, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {685, 688, 205}

$$-\frac{(b^2-4ac)\sqrt{a+bx+cx^2}}{8c^2d}+\frac{(b^2-4ac)^{3/2}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{16c^{5/2}d}+\frac{(a+bx+cx^2)^{3/2}}{6cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x),x]
```

```
[Out] -((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])/(8*c^2*d) + (a + b*x + c*x^2)^(3/2)/
(6*c*d) + ((b^2 - 4*a*c)^(3/2)*ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqr
t[b^2 - 4*a*c]])/(16*c^(5/2)*d)
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{3/2}}{bd+2cdx}\,dx = \frac{(a+bx+cx^2)^{3/2}}{6cd} - \frac{(b^2-4ac)\int \frac{\sqrt{a+bx+cx^2}}{bd+2cdx}\,dx}{4c}$$
$$= -\frac{(b^2-4ac)\sqrt{a+bx+cx^2}}{8c^2d} + \frac{(a+bx+cx^2)^{3/2}}{6cd} + \frac{(b^2-4ac)^2\int \frac{1}{(bd+2cdx)\sqrt{a+bx+cx^2}}\,dx}{16c^2}$$
$$= -\frac{(b^2-4ac)\sqrt{a+bx+cx^2}}{8c^2d} + \frac{(a+bx+cx^2)^{3/2}}{6cd} + \frac{(b^2-4ac)^2\,\text{Subst}\left(\int \frac{1}{2b^2cd-8ac^2d+8c^2dx^2}\,dx\right.}{4c}$$
$$= -\frac{(b^2-4ac)\sqrt{a+bx+cx^2}}{8c^2d} + \frac{(a+bx+cx^2)^{3/2}}{6cd} + \frac{(b^2-4ac)^{3/2}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{16c^{5/2}d}$$

Mathematica [A] time = 0.105074, size = 103, normalized size = 0.9

$$\frac{2\sqrt{c}\sqrt{a+x(b+cx)}\left(4c\left(4a+cx^2\right)-3b^2+4bcx\right)+3\left(b^2-4ac\right)^{3/2}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+x(b+cx)}}{\sqrt{b^2-4ac}}\right)}{48c^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-3*b^2 + 4*b*c*x + 4*c*(4*a + c*x^2)) + 3
*(b^2 - 4*a*c)^(3/2)*ArcTan[(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])/Sqrt[b^2 - 4*
a*c]])/(48*c^(5/2)*d)
```

Maple [B] time = 0.19, size = 430, normalized size = 3.7

$$\frac{1}{6\,cd}\left(\left(x+\frac{b}{2\,c}\right)^2 c+\frac{4\,ac-b^2}{4\,c}\right)^{\frac{3}{2}}+\frac{a}{4\,cd}\sqrt{4\left(x+1/2\,\frac{b}{c}\right)^2 c+\frac{4\,ac-b^2}{c}}-\frac{b^2}{16\,c^2d}\sqrt{4\left(x+1/2\,\frac{b}{c}\right)^2 c+\frac{4\,ac-b^2}{c}}-\frac{a^2}{cd}\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d),x)
```

```
[Out] 1/6/d/c*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(3/2)+1/4/d/c*(4*(x+1/2*b/c)^2*
c+(4*a*c-b^2)/c)^(1/2)*a-1/16/d/c^2*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2)
*b^2-1/d/c/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^
(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))*a^2+1/2/d/c^2/(
(4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/2)*(4*(x+
1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))*a*b^2-1/16/d/c^3/((4*a*c-b^
2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/2)*(4*(x+1/2*b/c)^
2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.69747, size = 570, normalized size = 4.96

$$\left[-\frac{3\left(b^{2}-4\,ac\right)\sqrt{-\frac{b^{2}-4\,ac}{c}}\log\left(-\frac{4\,c^{2}x^{2}+4\,bcx-b^{2}+8\,ac-4\,\sqrt{cx^{2}+bx+a}c\sqrt{-\frac{b^{2}-4\,ac}{c}}}{4\,c^{2}x^{2}+4\,bcx+b^{2}}\right)-4\left(4\,c^{2}x^{2}+4\,bcx-3\,b^{2}+16\,ac\right)\sqrt{cx^{2}+bx+a}}{96\,c^{2}d},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b^2 - 4*a*c)*sqrt(-(b^2 - 4*a*c)/c)*log(-(4*c^2*x^2 + 4*b*c*x -
b^2 + 8*a*c - 4*sqrt(c*x^2 + b*x + a)*c*sqrt(-(b^2 - 4*a*c)/c))/(4*c^2*x^2
+ 4*b*c*x + b^2)) - 4*(4*c^2*x^2 + 4*b*c*x - 3*b^2 + 16*a*c)*sqrt(c*x^2 + b
*x + a))/(c^2*d), -1/48*(3*(b^2 - 4*a*c)*sqrt((b^2 - 4*a*c)/c)*arctan(1/2*s
qrt((b^2 - 4*a*c)/c)/sqrt(c*x^2 + b*x + a)) - 2*(4*c^2*x^2 + 4*b*c*x - 3*b^
2 + 16*a*c)*sqrt(c*x^2 + b*x + a))/(c^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a\sqrt{a+bx+cx^{2}}}{b+2cx}\,dx+\int \frac{bx\sqrt{a+bx+cx^{2}}}{b+2cx}\,dx+\int \frac{cx^{2}\sqrt{a+bx+cx^{2}}}{b+2cx}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(2*c*d*x+b*d),x)
```

```
[Out] (Integral(a*sqrt(a + b*x + c*x**2)/(b + 2*c*x), x) + Integral(b*x*sqrt(a +
b*x + c*x**2)/(b + 2*c*x), x) + Integral(c*x**2*sqrt(a + b*x + c*x**2)/(b +
 2*c*x), x))/d
```

Giac [A] time = 1.15905, size = 201, normalized size = 1.75

$$\frac{1}{24}\sqrt{cx^{2}+bx+a}\left(4\,x\left(\frac{x}{d}+\frac{b}{cd}\right)-\frac{3\,b^{2}c^{3}d^{3}-16\,ac^{4}d^{3}}{c^{5}d^{4}}\right)+\frac{\left(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2}\right)\arctan\left(-\frac{2\left(\sqrt{c}x-\sqrt{cx^{2}+bx+a}\right)c+b\sqrt{c}}{\sqrt{b^{2}c-4\,ac^{2}}}\right)}{8\,\sqrt{b^{2}c-4\,ac^{2}}c^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + b*x + a)*(4*x*(x/d + b/(c*d)) - (3*b^2*c^3*d^3 - 16*a*c^4
*d^3)/(c^5*d^4)) + 1/8*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*arctan(-(2*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))*c + b*sqrt(c))/sqrt(b^2*c - 4*a*c^2))/(sqrt(b^2*c
 - 4*a*c^2)*c^2*d)
```

3.1208 $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(bd+2cdx)^2}\,dx$

Optimal. Leaf size=113

$$-\frac{3\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{32c^{5/2}d^2}+\frac{3(b+2cx)\sqrt{a+bx+cx^2}}{16c^2d^2}-\frac{\left(a+bx+cx^2\right)^{3/2}}{2cd^2(b+2cx)}$$

```
[Out] (3*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(16*c^2*d^2) - (a + b*x + c*x^2)^(3/2
)/(2*c*d^2*(b + 2*c*x)) - (3*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + b*x + c*x^2])])/(32*c^(5/2)*d^2)
```

Rubi [A] time = 0.0457616, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {684, 612, 621, 206}

$$-\frac{3\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{32c^{5/2}d^2}+\frac{3(b+2cx)\sqrt{a+bx+cx^2}}{16c^2d^2}-\frac{\left(a+bx+cx^2\right)^{3/2}}{2cd^2(b+2cx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^2,x]
```


```
[Out] (3*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(16*c^2*d^2) - (a + b*x + c*x^2)^(3/2
)/(2*c*d^2*(b + 2*c*x)) - (3*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + b*x + c*x^2])])/(32*c^(5/2)*d^2)
```


Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(bd+2cdx)^2}\,dx &= -\frac{\left(a+bx+cx^2\right)^{3/2}}{2cd^2(b+2cx)}+\frac{3\int \sqrt{a+bx+cx^2}\,dx}{4cd^2}\\ &= \frac{3(b+2cx)\sqrt{a+bx+cx^2}}{16c^2d^2}-\frac{\left(a+bx+cx^2\right)^{3/2}}{2cd^2(b+2cx)}-\frac{\left(3\left(b^2-4ac\right)\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{32c^2d^2}\\ &= \frac{3(b+2cx)\sqrt{a+bx+cx^2}}{16c^2d^2}-\frac{\left(a+bx+cx^2\right)^{3/2}}{2cd^2(b+2cx)}-\frac{\left(3\left(b^2-4ac\right)\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{16c^2d^2}\\ &= \frac{3(b+2cx)\sqrt{a+bx+cx^2}}{16c^2d^2}-\frac{\left(a+bx+cx^2\right)^{3/2}}{2cd^2(b+2cx)}-\frac{3\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{32c^{5/2}d^2}\end{aligned}$$

Mathematica [C] time = 0.0448098, size = 95, normalized size = 0.84

$$\frac{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{16c^2d^2(b+2cx)\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^2,x]
```

```
[Out] ((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]*Hypergeometric2F1[-3/2, -1/2, 1/2, (b
+ 2*c*x)^2/(b^2 - 4*a*c)])/(16*c^2*d^2*(b + 2*c*x)*Sqrt[(c*(a + x*(b + c*x)
))/(-b^2 + 4*a*c)])
```

Maple [B] time = 0.193, size = 570, normalized size = 5.

$$-\frac{1}{cd^2\left(4ac-b^2\right)}\left(\left(x+\frac{b}{2c}\right)^2c+\frac{4ac-b^2}{4c}\right)^{\frac{5}{2}}\left(x+\frac{b}{2c}\right)^{-1}+\frac{x}{d^2\left(4ac-b^2\right)}\left(\left(x+\frac{b}{2c}\right)^2c+\frac{4ac-b^2}{4c}\right)^{\frac{3}{2}}+\frac{b}{2cd^2\left(4ac-b^2\right)}\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^2,x)
```

```
[Out] -1/c/d^2/(4*a*c-b^2)/(x+1/2*b/c)*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(5/2)+
1/d^2/(4*a*c-b^2)*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(3/2)*x+1/2/c/d^2/(4*
a*c-b^2)*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(3/2)*b+3/2/d^2/(4*a*c-b^2)*((
x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2)*x*a-3/8/c/d^2/(4*a*c-b^2)*((x+1/2*b
/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2)*x*b^2+3/4/c/d^2/(4*a*c-b^2)*((x+1/2*b/c)^2
*c+1/4*(4*a*c-b^2)/c)^(1/2)*b*a-3/16/c^2/d^2/(4*a*c-b^2)*((x+1/2*b/c)^2*c+1
/4*(4*a*c-b^2)/c)^(1/2)*b^3+3/2/c^(1/2)/d^2/(4*a*c-b^2)*ln((x+1/2*b/c)*c^(1
/2)+((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2))*a^2-3/4/c^(3/2)/d^2/(4*a*c-b
^2)*ln((x+1/2*b/c)*c^(1/2)+((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2))*b^2*a
+3/32/c^(5/2)/d^2/(4*a*c-b^2)*ln((x+1/2*b/c)*c^(1/2)+((x+1/2*b/c)^2*c+1/4*(
4*a*c-b^2)/c)^(1/2))*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.72907, size = 647, normalized size = 5.73

$$\left[-\frac{3\left(b^3-4\,abc+2\left(b^2c-4\,ac^2\right)x\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right)-4\left(4\,c^3x^2+4\,b\right.}{64\left(2\,c^4d^2x+bc^3d^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^2,x, algorithm="fricas")
```

```
[Out] [-1/64*(3*(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x)*sqrt(c)*log(-8*c^2*x^2 -
8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(4
*c^3*x^2 + 4*b*c^2*x + 3*b^2*c - 8*a*c^2)*sqrt(c*x^2 + b*x + a))/(2*c^4*d^2
*x + b*c^3*d^2), 1/32*(3*(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x)*sqrt(-c)*a
rctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c
)) + 2*(4*c^3*x^2 + 4*b*c^2*x + 3*b^2*c - 8*a*c^2)*sqrt(c*x^2 + b*x + a))/(
2*c^4*d^2*x + b*c^3*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a\sqrt{a+bx+cx^2}}{b^2+4bcx+4c^2x^2}\,dx+\int \frac{bx\sqrt{a+bx+cx^2}}{b^2+4bcx+4c^2x^2}\,dx+\int \frac{cx^2\sqrt{a+bx+cx^2}}{b^2+4bcx+4c^2x^2}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(2*c*d*x+b*d)**2,x)
```

```
[Out] (Integral(a*sqrt(a + b*x + c*x**2)/(b**2 + 4*b*c*x + 4*c**2*x**2), x) + Int
egral(b*x*sqrt(a + b*x + c*x**2)/(b**2 + 4*b*c*x + 4*c**2*x**2), x) + Integ
ral(c*x**2*sqrt(a + b*x + c*x**2)/(b**2 + 4*b*c*x + 4*c**2*x**2), x))/d**2
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.1209 $\int \frac{(a+bx+cx^2)^{3/2}}{(bd+2cdx)^3} dx$

Optimal. Leaf size=115

$$-\frac{3\sqrt{b^2-4ac}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{32c^{5/2}d^3}+\frac{3\sqrt{a+bx+cx^2}}{16c^2d^3}-\frac{(a+bx+cx^2)^{3/2}}{4cd^3(b+2cx)^2}$$

```
[Out] (3*Sqrt[a + b*x + c*x^2])/(16*c^2*d^3) - (a + b*x + c*x^2)^(3/2)/(4*c*d^3*(
b + 2*c*x)^2) - (3*Sqrt[b^2 - 4*a*c]*ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])/Sqrt[b^2 - 4*a*c]])/(32*c^(5/2)*d^3)
```

Rubi [A] time = 0.0711629, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {684, 685, 688, 205}

$$-\frac{3\sqrt{b^2-4ac}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{32c^{5/2}d^3}+\frac{3\sqrt{a+bx+cx^2}}{16c^2d^3}-\frac{(a+bx+cx^2)^{3/2}}{4cd^3(b+2cx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^3,x]
```


```
[Out] (3*Sqrt[a + b*x + c*x^2])/(16*c^2*d^3) - (a + b*x + c*x^2)^(3/2)/(4*c*d^3*(
b + 2*c*x)^2) - (3*Sqrt[b^2 - 4*a*c]*ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])/Sqrt[b^2 - 4*a*c]])/(32*c^(5/2)*d^3)
```


Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(bd+2cdx)^3}\,dx = -\frac{\left(a+bx+cx^2\right)^{3/2}}{4cd^3(b+2cx)^2}+\frac{3\int \frac{\sqrt{a+bx+cx^2}}{bd+2cdx}\,dx}{8cd^2}$$
$$= \frac{3\sqrt{a+bx+cx^2}}{16c^2d^3}-\frac{\left(a+bx+cx^2\right)^{3/2}}{4cd^3(b+2cx)^2}-\frac{\left(3\left(b^2-4ac\right)\right)\int \frac{1}{(bd+2cdx)\sqrt{a+bx+cx^2}}\,dx}{32c^2d^2}$$
$$= \frac{3\sqrt{a+bx+cx^2}}{16c^2d^3}-\frac{\left(a+bx+cx^2\right)^{3/2}}{4cd^3(b+2cx)^2}-\frac{\left(3\left(b^2-4ac\right)\right)\text{Subst}\left(\int \frac{1}{2b^2cd-8ac^2d+8c^2dx^2}\,dx,x,\sqrt{a+}\right.}{8cd^2}$$
$$= \frac{3\sqrt{a+bx+cx^2}}{16c^2d^3}-\frac{\left(a+bx+cx^2\right)^{3/2}}{4cd^3(b+2cx)^2}-\frac{3\sqrt{b^2-4ac}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{32c^{5/2}d^3}$$

Mathematica [C] time = 0.032334, size = 62, normalized size = 0.54

$$\frac{2(a+x(b+cx))^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{4c(a+x(b+cx))}{4ac-b^2}\right)}{5d^3\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^3,x]
```

```
[Out] (2*(a + x*(b + c*x))^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, (4*c*(a + x*(b +
c*x)))/(-b^2 + 4*a*c)])/(5*(b^2 - 4*a*c)^2*d^3)
```

Maple [B] time = 0.191, size = 562, normalized size = 4.9

$$-\frac{1}{4c^2d^3\left(4ac-b^2\right)}\left(\left(x+\frac{b}{2c}\right)^2c+\frac{4ac-b^2}{4c}\right)^{\frac{5}{2}}\left(x+\frac{b}{2c}\right)^{-2}+\frac{1}{4cd^3\left(4ac-b^2\right)}\left(\left(x+\frac{b}{2c}\right)^2c+\frac{4ac-b^2}{4c}\right)^{\frac{3}{2}}+\frac{3}{8cd^3\left(4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^3,x)
```

```
[Out] -1/4/d^3/c^2/(4*a*c-b^2)/(x+1/2*b/c)^2*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^
(5/2)+1/4/d^3/c/(4*a*c-b^2)*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(3/2)+3/8/d
^3/c/(4*a*c-b^2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2)*a-3/32/d^3/c^2/(4*
a*c-b^2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2)*b^2-3/2/d^3/c/(4*a*c-b^2)/
((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/2)*(4*(x
+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))*a^2+3/4/d^3/c^2/(4*a*c-b^2
)/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/2)*(4*
(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))*a*b^2-3/32/d^3/c^3/(4*a*
c-b^2)/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/2
)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.63833, size = 705, normalized size = 6.13

$$\left[\frac{3\left(4c^2x^2+4bcx+b^2\right)\sqrt{-\frac{b^2-4ac}{c}}\log\left(-\frac{4c^2x^2+4bcx-b^2+8ac-4\sqrt{cx^2+bx+ac}\sqrt{-\frac{b^2-4ac}{c}}}{4c^2x^2+4bcx+b^2}\right)+4\left(8c^2x^2+8bcx+3b^2-4ac\right)\sqrt{cx^2+b}}{64\left(4c^4d^3x^2+4bc^3d^3x+b^2c^2d^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^3,x, algorithm="fricas")
```

```
[Out] [1/64*(3*(4*c^2*x^2 + 4*b*c*x + b^2)*sqrt(-(b^2 - 4*a*c)/c)*log(-(4*c^2*x^2
 + 4*b*c*x - b^2 + 8*a*c - 4*sqrt(c*x^2 + b*x + a)*c*sqrt(-(b^2 - 4*a*c)/c)
)/(4*c^2*x^2 + 4*b*c*x + b^2)) + 4*(8*c^2*x^2 + 8*b*c*x + 3*b^2 - 4*a*c)*sq
rt(c*x^2 + b*x + a))/(4*c^4*d^3*x^2 + 4*b*c^3*d^3*x + b^2*c^2*d^3), 1/32*(3
*(4*c^2*x^2 + 4*b*c*x + b^2)*sqrt((b^2 - 4*a*c)/c)*arctan(1/2*sqrt((b^2 - 4
*a*c)/c)/sqrt(c*x^2 + b*x + a)) + 2*(8*c^2*x^2 + 8*b*c*x + 3*b^2 - 4*a*c)*s
qrt(c*x^2 + b*x + a))/(4*c^4*d^3*x^2 + 4*b*c^3*d^3*x + b^2*c^2*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a\sqrt{a+bx+cx^2}}{b^3+6b^2cx+12bc^2x^2+8c^3x^3}\,dx+\int \frac{bx\sqrt{a+bx+cx^2}}{b^3+6b^2cx+12bc^2x^2+8c^3x^3}\,dx+\int \frac{cx^2\sqrt{a+bx+cx^2}}{b^3+6b^2cx+12bc^2x^2+8c^3x^3}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(2*c*d*x+b*d)**3,x)
```

```
[Out] (Integral(a*sqrt(a + b*x + c*x**2)/(b**3 + 6*b**2*c*x + 12*b*c**2*x**2 + 8*
c**3*x**3), x) + Integral(b*x*sqrt(a + b*x + c*x**2)/(b**3 + 6*b**2*c*x + 1
2*b*c**2*x**2 + 8*c**3*x**3), x) + Integral(c*x**2*sqrt(a + b*x + c*x**2)/(
b**3 + 6*b**2*c*x + 12*b*c**2*x**2 + 8*c**3*x**3), x))/d**3
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1210 $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(bd+2cdx)^4}\,dx$

Optimal. Leaf size=107

$$-\frac{\sqrt{a+bx+cx^2}}{8c^2d^4(b+2cx)}+\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}d^4}-\frac{\left(a+bx+cx^2\right)^{3/2}}{6cd^4(b+2cx)^3}$$

```
[Out] -Sqrt[a + b*x + c*x^2]/(8*c^2*d^4*(b + 2*c*x)) - (a + b*x + c*x^2)^(3/2)/(6
*c*d^4*(b + 2*c*x)^3) + ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])]/(16*c^(5/2)*d^4)
```

Rubi [A] time = 0.0501314, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {684, 621, 206}

$$-\frac{\sqrt{a+bx+cx^2}}{8c^2d^4(b+2cx)}+\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}d^4}-\frac{\left(a+bx+cx^2\right)^{3/2}}{6cd^4(b+2cx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^4,x]
```

```
[Out] -Sqrt[a + b*x + c*x^2]/(8*c^2*d^4*(b + 2*c*x)) - (a + b*x + c*x^2)^(3/2)/(6
*c*d^4*(b + 2*c*x)^3) + ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])]/(16*c^(5/2)*d^4)
```

Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(bd+2cdx)^4}\,dx = -\frac{\left(a+bx+cx^2\right)^{3/2}}{6cd^4(b+2cx)^3} + \frac{\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^2}\,dx}{4cd^2}$$
$$= -\frac{\sqrt{a+bx+cx^2}}{8c^2d^4(b+2cx)} - \frac{\left(a+bx+cx^2\right)^{3/2}}{6cd^4(b+2cx)^3} + \frac{\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{16c^2d^4}$$
$$= -\frac{\sqrt{a+bx+cx^2}}{8c^2d^4(b+2cx)} - \frac{\left(a+bx+cx^2\right)^{3/2}}{6cd^4(b+2cx)^3} + \frac{\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{8c^2d^4}$$
$$= -\frac{\sqrt{a+bx+cx^2}}{8c^2d^4(b+2cx)} - \frac{\left(a+bx+cx^2\right)^{3/2}}{6cd^4(b+2cx)^3} + \frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}d^4}$$

Mathematica [C] time = 0.0467636, size = 95, normalized size = 0.89

$$\frac{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{48c^2d^4(b+2cx)^3\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^4,x]
```

```
[Out] ((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]*Hypergeometric2F1[-3/2, -3/2, -1/2, (b
 + 2*c*x)^2/(b^2 - 4*a*c)])/(48*c^2*d^4*(b + 2*c*x)^3*Sqrt[(c*(a + x*(b + c
*x)))/(-b^2 + 4*a*c)])
```

Maple [B] time = 0.192, size = 629, normalized size = 5.9

$$-\frac{1}{12\,c^3d^4\left(4\,ac-b^2\right)}\left(\left(x+\frac{b}{2\,c}\right)^2c+\frac{4\,ac-b^2}{4\,c}\right)^{\frac{5}{2}}\left(x+\frac{b}{2\,c}\right)^{-3}-\frac{2}{3\,cd^4\left(4\,ac-b^2\right)^2}\left(\left(x+\frac{b}{2\,c}\right)^2c+\frac{4\,ac-b^2}{4\,c}\right)^{\frac{5}{2}}\left(x+\frac{b}{2\,c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^4,x)
```

```
[Out] -1/12/d^4/c^3/(4*a*c-b^2)/(x+1/2*b/c)^3*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)
^(5/2)-2/3/d^4/c/(4*a*c-b^2)^2/(x+1/2*b/c)*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)
/c)^(5/2)+2/3/d^4/(4*a*c-b^2)^2*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(3/2)*x
+1/3/d^4/c/(4*a*c-b^2)^2*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(3/2)*b+1/d^4/
(4*a*c-b^2)^2*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2)*x*a-1/4/d^4/c/(4*a*
c-b^2)^2*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2)*x*b^2+1/2/d^4/c/(4*a*c-b
^2)^2*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2)*b*a-1/8/d^4/c^2/(4*a*c-b^2)
^2*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2)*b^3+1/d^4/c^(1/2)/(4*a*c-b^2)^
2*ln((x+1/2*b/c)*c^(1/2)+((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2))*a^2-1/2
/d^4/c^(3/2)/(4*a*c-b^2)^2*ln((x+1/2*b/c)*c^(1/2)+((x+1/2*b/c)^2*c+1/4*(4*a
*c-b^2)/c)^(1/2))*b^2*a+1/16/d^4/c^(5/2)/(4*a*c-b^2)^2*ln((x+1/2*b/c)*c^(1/
2)+((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2))*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 7.30189, size = 776, normalized size = 7.25

$$\left[\frac{3\left(8c^3x^3+12bc^2x^2+6b^2cx+b^3\right)\sqrt{c}\log\left(-8c^2x^2-8bcx-b^2-4\sqrt{cx^2+bx+a}(2cx+b)\sqrt{c}-4ac\right)-4\left(16c^3x^2+16b\right.}{96\left(8c^6d^4x^3+12bc^5d^4x^2+6b^2c^4d^4x+b^3c^3d^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(3*(8*c^3*x^3 + 12*b*c^2*x^2 + 6*b^2*c*x + b^3)*sqrt(c)*log(-8*c^2*x^
2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) -
4*(16*c^3*x^2 + 16*b*c^2*x + 3*b^2*c + 4*a*c^2)*sqrt(c*x^2 + b*x + a))/(8*c
^6*d^4*x^3 + 12*b*c^5*d^4*x^2 + 6*b^2*c^4*d^4*x + b^3*c^3*d^4), -1/48*(3*(8
*c^3*x^3 + 12*b*c^2*x^2 + 6*b^2*c*x + b^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 +
 b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(16*c^3*x^2 + 1
6*b*c^2*x + 3*b^2*c + 4*a*c^2)*sqrt(c*x^2 + b*x + a))/(8*c^6*d^4*x^3 + 12*b
*c^5*d^4*x^2 + 6*b^2*c^4*d^4*x + b^3*c^3*d^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a\sqrt{a+bx+cx^2}}{b^4+8b^3cx+24b^2c^2x^2+32bc^3x^3+16c^4x^4}\,dx+\int\frac{bx\sqrt{a+bx+cx^2}}{b^4+8b^3cx+24b^2c^2x^2+32bc^3x^3+16c^4x^4}\,dx+\int\frac{cx^2\sqrt{a+bx+cx^2}}{b^4+8b^3cx+24b^2c^2x^2+32bc^3x^3+16c^4x^4}\,dx}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(2*c*d*x+b*d)**4,x)
```

```
[Out] (Integral(a*sqrt(a + b*x + c*x**2)/(b**4 + 8*b**3*c*x + 24*b**2*c**2*x**2 +
 32*b*c**3*x**3 + 16*c**4*x**4), x) + Integral(b*x*sqrt(a + b*x + c*x**2)/(
b**4 + 8*b**3*c*x + 24*b**2*c**2*x**2 + 32*b*c**3*x**3 + 16*c**4*x**4), x)
+ Integral(c*x**2*sqrt(a + b*x + c*x**2)/(b**4 + 8*b**3*c*x + 24*b**2*c**2*
x**2 + 32*b*c**3*x**3 + 16*c**4*x**4), x))/d**4
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1211 $\int \frac{(a+bx+cx^2)^{3/2}}{(bd+2cdx)^5} dx$

Optimal. Leaf size=123

$$\frac{3\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{128c^{5/2}d^5\sqrt{b^2-4ac}}-\frac{3\sqrt{a+bx+cx^2}}{64c^2d^5(b+2cx)^2}-\frac{(a+bx+cx^2)^{3/2}}{8cd^5(b+2cx)^4}$$

```
[Out] (-3*Sqrt[a + b*x + c*x^2])/(64*c^2*d^5*(b + 2*c*x)^2) - (a + b*x + c*x^2)^(
3/2)/(8*c*d^5*(b + 2*c*x)^4) + (3*ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/
Sqrt[b^2 - 4*a*c]])/(128*c^(5/2)*Sqrt[b^2 - 4*a*c]*d^5)
```

Rubi [A] time = 0.0731791, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {684, 688, 205}

$$\frac{3\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{128c^{5/2}d^5\sqrt{b^2-4ac}}-\frac{3\sqrt{a+bx+cx^2}}{64c^2d^5(b+2cx)^2}-\frac{(a+bx+cx^2)^{3/2}}{8cd^5(b+2cx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^5,x]
```


```
[Out] (-3*Sqrt[a + b*x + c*x^2])/(64*c^2*d^5*(b + 2*c*x)^2) - (a + b*x + c*x^2)^(
3/2)/(8*c*d^5*(b + 2*c*x)^4) + (3*ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/
Sqrt[b^2 - 4*a*c]])/(128*c^(5/2)*Sqrt[b^2 - 4*a*c]*d^5)
```


Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] && !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{3/2}}{(bd+2cdx)^5}dx = -\frac{(a+bx+cx^2)^{3/2}}{8cd^5(b+2cx)^4} + \frac{3\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^3}dx}{16cd^2}$$
$$= -\frac{3\sqrt{a+bx+cx^2}}{64c^2d^5(b+2cx)^2} - \frac{(a+bx+cx^2)^{3/2}}{8cd^5(b+2cx)^4} + \frac{3\int \frac{1}{(bd+2cdx)\sqrt{a+bx+cx^2}}dx}{128c^2d^4}$$
$$= -\frac{3\sqrt{a+bx+cx^2}}{64c^2d^5(b+2cx)^2} - \frac{(a+bx+cx^2)^{3/2}}{8cd^5(b+2cx)^4} + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{2b^2cd-8ac^2d+8c^2dx^2}dx, x, \sqrt{a+bx+cx^2}\right)}{32cd^4}$$
$$= -\frac{3\sqrt{a+bx+cx^2}}{64c^2d^5(b+2cx)^2} - \frac{(a+bx+cx^2)^{3/2}}{8cd^5(b+2cx)^4} + \frac{3\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{128c^{5/2}\sqrt{b^2-4ac}d^5}$$

Mathematica [A] time = 0.245584, size = 162, normalized size = 1.32

$$\frac{-2c\left(8a^2c+a\left(3b^2+28bcx+28c^2x^2\right)+x\left(23b^2cx+3b^3+40bc^2x^2+20c^3x^3\right)\right)-3(b+2cx)^4\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\tanh^{-1}\left(2\sqrt{\frac{c(a\cdots}{4\cdots}}\right.}{128c^3d^5(b+2cx)^4\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^5,x]
```

```
[Out] (-2*c*(8*a^2*c + a*(3*b^2 + 28*b*c*x + 28*c^2*x^2) + x*(3*b^3 + 23*b^2*c*x
+ 40*b*c^2*x^2 + 20*c^3*x^3)) - 3*(b + 2*c*x)^4*Sqrt[(c*(a + x*(b + c*x)))/
(-b^2 + 4*a*c)]*ArcTanh[2*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]])/(128
*c^3*d^5*(b + 2*c*x)^4*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.194, size = 622, normalized size = 5.1

$$-\frac{1}{32c^4d^5\left(4ac-b^2\right)}\left(\left(x+\frac{b}{2c}\right)^2c+\frac{4ac-b^2}{4c}\right)^{\frac{5}{2}}\left(x+\frac{b}{2c}\right)^{-4}-\frac{1}{16c^2d^5\left(4ac-b^2\right)^2}\left(\left(x+\frac{b}{2c}\right)^2c+\frac{4ac-b^2}{4c}\right)^{\frac{5}{2}}\left(x+\frac{b}{2c}\right)^{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^5,x)
```

```
[Out] -1/32/d^5/c^4/(4*a*c-b^2)/(x+1/2*b/c)^4*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)
^(5/2)-1/16/d^5/c^2/(4*a*c-b^2)^2/(x+1/2*b/c)^2*((x+1/2*b/c)^2*c+1/4*(4*a*c
-b^2)/c)^(5/2)+1/16/d^5/c/(4*a*c-b^2)^2*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)
^(3/2)+3/32/d^5/c/(4*a*c-b^2)^2*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2)*a-3
/128/d^5/c^2/(4*a*c-b^2)^2*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2)*b^2-3/8/
d^5/c/(4*a*c-b^2)^2/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*((4*a*c
-b^2)/c)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))*a^2+3/
16/d^5/c^2/(4*a*c-b^2)^2/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*((
4*a*c-b^2)/c)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))*a
*b^2-3/128/d^5/c^3/(4*a*c-b^2)^2/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/
c+1/2*((4*a*c-b^2)/c)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2
*b/c))*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 12.4489, size = 1320, normalized size = 10.73

$$\left[-\frac{3\left(16\,c^4x^4+32\,bc^3x^3+24\,b^2c^2x^2+8\,b^3cx+b^4\right)\sqrt{-b^2c+4\,ac^2}\log\left(-\frac{4\,c^2x^2+4\,bcx-b^2+8\,ac-4\,\sqrt{-b^2c+4\,ac^2}\sqrt{cx^2+bx+a}}{4\,c^2x^2+4\,bcx+b^2}\right)+4\,(3}{256\left(16\left(b^2c^7-4\,ac^8\right)d^5x^4+32\left(b^3c^6-4\,abc^7\right)d^5x^3+24\left(b^4c^5-4\,ab^2c^6\right)d^5x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^5,x, algorithm="fricas")
```

```
[Out] [-1/256*(3*(16*c^4*x^4 + 32*b*c^3*x^3 + 24*b^2*c^2*x^2 + 8*b^3*c*x + b^4)*s
qrt(-b^2*c + 4*a*c^2)*log(-(4*c^2*x^2 + 4*b*c*x - b^2 + 8*a*c - 4*sqrt(-b^2
*c + 4*a*c^2)*sqrt(c*x^2 + b*x + a))/(4*c^2*x^2 + 4*b*c*x + b^2)) + 4*(3*b^
4*c - 4*a*b^2*c^2 - 32*a^2*c^3 + 20*(b^2*c^3 - 4*a*c^4)*x^2 + 20*(b^3*c^2 -
 4*a*b*c^3)*x)*sqrt(c*x^2 + b*x + a))/(16*(b^2*c^7 - 4*a*c^8)*d^5*x^4 + 32*
(b^3*c^6 - 4*a*b*c^7)*d^5*x^3 + 24*(b^4*c^5 - 4*a*b^2*c^6)*d^5*x^2 + 8*(b^5
*c^4 - 4*a*b^3*c^5)*d^5*x + (b^6*c^3 - 4*a*b^4*c^4)*d^5), -1/128*(3*(16*c^4
*x^4 + 32*b*c^3*x^3 + 24*b^2*c^2*x^2 + 8*b^3*c*x + b^4)*sqrt(b^2*c - 4*a*c^
2)*arctan(1/2*sqrt(b^2*c - 4*a*c^2)*sqrt(c*x^2 + b*x + a)/(c^2*x^2 + b*c*x
+ a*c)) + 2*(3*b^4*c - 4*a*b^2*c^2 - 32*a^2*c^3 + 20*(b^2*c^3 - 4*a*c^4)*x^
2 + 20*(b^3*c^2 - 4*a*b*c^3)*x)*sqrt(c*x^2 + b*x + a))/(16*(b^2*c^7 - 4*a*c
^8)*d^5*x^4 + 32*(b^3*c^6 - 4*a*b*c^7)*d^5*x^3 + 24*(b^4*c^5 - 4*a*b^2*c^6)
*d^5*x^2 + 8*(b^5*c^4 - 4*a*b^3*c^5)*d^5*x + (b^6*c^3 - 4*a*b^4*c^4)*d^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a\sqrt{a+bx+cx^2}}{b^5+10b^4cx+40b^3c^2x^2+80b^2c^3x^3+80bc^4x^4+32c^5x^5}\,dx+\int\frac{bx\sqrt{a+bx+cx^2}}{b^5+10b^4cx+40b^3c^2x^2+80b^2c^3x^3+80bc^4x^4+32c^5x^5}\,dx+\int\frac{cx^2\sqrt{a+bx+c}}{b^5+10b^4cx+40b^3c^2x^2+80b^2c^3x}}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(2*c*d*x+b*d)**5,x)
```

```
[Out] (Integral(a*sqrt(a + b*x + c*x**2)/(b**5 + 10*b**4*c*x + 40*b**3*c**2*x**2
+ 80*b**2*c**3*x**3 + 80*b*c**4*x**4 + 32*c**5*x**5), x) + Integral(b*x*sqr
t(a + b*x + c*x**2)/(b**5 + 10*b**4*c*x + 40*b**3*c**2*x**2 + 80*b**2*c**3*
x**3 + 80*b*c**4*x**4 + 32*c**5*x**5), x) + Integral(c*x**2*sqrt(a + b*x +
c*x**2)/(b**5 + 10*b**4*c*x + 40*b**3*c**2*x**2 + 80*b**2*c**3*x**3 + 80*b*
c**4*x**4 + 32*c**5*x**5), x))/d**5
```

Giac [B] time = 1.79571, size = 911, normalized size = 7.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^5,x, algorithm="giac")
```

```
[Out] 1/96*(3*sqrt(-b^2*c + 4*a*c^2)*log(abs(c))*sgn(1/(2*c*d*x + b*d))*sgn(c)*sg
n(d)/(b^6*c^7*d^8*abs(c) - 12*a*b^4*c^8*d^8*abs(c) + 48*a^2*b^2*c^9*d^8*abs
(c) - 64*a^3*c^10*d^8*abs(c)) + 2*sqrt(-b^2*c*d^2/(2*c*d*x + b*d)^2 + 4*a*c
^2*d^2/(2*c*d*x + b*d)^2 + c)*(5*(b^4*c^5*d^10*sgn(1/(2*c*d*x + b*d))*sgn(c
)*sgn(d) - 8*a*b^2*c^6*d^10*sgn(1/(2*c*d*x + b*d))*sgn(c)*sgn(d) + 16*a^2*c
^7*d^10*sgn(1/(2*c*d*x + b*d))*sgn(c)*sgn(d))/(b^8*c^12*d^16 - 16*a*b^6*c^1
3*d^16 + 96*a^2*b^4*c^14*d^16 - 256*a^3*b^2*c^15*d^16 + 256*a^4*c^16*d^16)
- 2*(b^6*c^7*d^14*sgn(1/(2*c*d*x + b*d))*sgn(c)*sgn(d) - 12*a*b^4*c^8*d^14*
sgn(1/(2*c*d*x + b*d))*sgn(c)*sgn(d) + 48*a^2*b^2*c^9*d^14*sgn(1/(2*c*d*x +
 b*d))*sgn(c)*sgn(d) - 64*a^3*c^10*d^14*sgn(1/(2*c*d*x + b*d))*sgn(c)*sgn(d
))/((b^8*c^12*d^16 - 16*a*b^6*c^13*d^16 + 96*a^2*b^4*c^14*d^16 - 256*a^3*b^
2*c^15*d^16 + 256*a^4*c^16*d^16)*(2*c*d*x + b*d)^2*c^2*d^2))/((2*c*d*x + b*
d)*c*d) - 6*sqrt(-b^2*c + 4*a*c^2)*log(abs(sqrt(-b^2*c*d^2/(2*c*d*x + b*d)^
2 + 4*a*c^2*d^2/(2*c*d*x + b*d)^2 + c) + sqrt(-b^2*c^3*d^4 + 4*a*c^4*d^4)/(
(2*c*d*x + b*d)*c*d)))*sgn(1/(2*c*d*x + b*d))*sgn(c)*sgn(d)/((b^6*c^7 - 12*
a*b^4*c^8 + 48*a^2*b^2*c^9 - 64*a^3*c^10)*d^8*abs(c)))*d^2*abs(c)
```

3.1212 $\int \frac{(a+bx+cx^2)^{3/2}}{(bd+2cdx)^6} dx$

Optimal. Leaf size=39

$$\frac{2(a+bx+cx^2)^{5/2}}{5d^6(b^2-4ac)(b+2cx)^5}$$

```
[Out] (2*(a + b*x + c*x^2)^(5/2))/(5*(b^2 - 4*a*c)*d^6*(b + 2*c*x)^5)
```

Rubi [A] time = 0.0164574, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {682}

$$\frac{2(a+bx+cx^2)^{5/2}}{5d^6(b^2-4ac)(b+2cx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^6,x]

[Out] (2*(a + b*x + c*x^2)^(5/2))/(5*(b^2 - 4*a*c)*d^6*(b + 2*c*x)^5)
```

Rule 682

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c,
0] && EqQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{3/2}}{(bd+2cdx)^6} dx = \frac{2(a+bx+cx^2)^{5/2}}{5(b^2-4ac)d^6(b+2cx)^5}$$

Mathematica [A] time = 0.025451, size = 38, normalized size = 0.97

$$\frac{2(a+x(b+cx))^{5/2}}{5d^6(b^2-4ac)(b+2cx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^6,x]

[Out] (2*(a + x*(b + c*x))^(5/2))/(5*(b^2 - 4*a*c)*d^6*(b + 2*c*x)^5)
```

Maple [A] time = 0.043, size = 38, normalized size = 1.

$$-\frac{2}{5(2cx+b)^5 d^6(4ac-b^2)}(cx^2+bx+a)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^6,x)
```

```
[Out] -2/5*(c*x^2+b*x+a)^(5/2)/(2*c*x+b)^5/d^6/(4*a*c-b^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 24.2278, size = 383, normalized size = 9.82

$$\frac{2\left(c^2x^4 + 2\,bcx^3 + 2\,abx + \left(b^2 + 2\,ac\right)x^2 + a^2\right)\sqrt{cx^2 + bx + a}}{5\left(32\left(b^2c^5 - 4\,ac^6\right)d^6x^5 + 80\left(b^3c^4 - 4\,abc^5\right)d^6x^4 + 80\left(b^4c^3 - 4\,ab^2c^4\right)d^6x^3 + 40\left(b^5c^2 - 4\,ab^3c^3\right)d^6x^2 + 10\left(b^6c - 4\,ab^4c^2\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^6,x, algorithm="fricas")
```

```
[Out] 2/5*(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*sqrt(c*x^2 +
b*x + a)/(32*(b^2*c^5 - 4*a*c^6)*d^6*x^5 + 80*(b^3*c^4 - 4*a*b*c^5)*d^6*x^4
 + 80*(b^4*c^3 - 4*a*b^2*c^4)*d^6*x^3 + 40*(b^5*c^2 - 4*a*b^3*c^3)*d^6*x^2
+ 10*(b^6*c - 4*a*b^4*c^2)*d^6*x + (b^7 - 4*a*b^5*c)*d^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a\sqrt{a+bx+cx^2}}{b^6+12b^5cx+60b^4c^2x^2+160b^3c^3x^3+240b^2c^4x^4+192bc^5x^5+64c^6x^6}\,dx + \int \frac{bx\sqrt{a+bx+cx^2}}{b^6+12b^5cx+60b^4c^2x^2+160b^3c^3x^3+240b^2c^4x^4+192bc^5x^5+64c^6x^6}\,dx + \int \frac{}{b^6+12b}}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(2*c*d*x+b*d)**6,x)
```

```
[Out] (Integral(a*sqrt(a + b*x + c*x**2)/(b**6 + 12*b**5*c*x + 60*b**4*c**2*x**2
+ 160*b**3*c**3*x**3 + 240*b**2*c**4*x**4 + 192*b*c**5*x**5 + 64*c**6*x**6)
, x) + Integral(b*x*sqrt(a + b*x + c*x**2)/(b**6 + 12*b**5*c*x + 60*b**4*c*
*2*x**2 + 160*b**3*c**3*x**3 + 240*b**2*c**4*x**4 + 192*b*c**5*x**5 + 64*c*
*6*x**6), x) + Integral(c*x**2*sqrt(a + b*x + c*x**2)/(b**6 + 12*b**5*c*x +
 60*b**4*c**2*x**2 + 160*b**3*c**3*x**3 + 240*b**2*c**4*x**4 + 192*b*c**5*x
**5 + 64*c**6*x**6), x))/d**6
```

Giac [B] time = 1.84344, size = 802, normalized size = 20.56

$$80\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^8c^{\frac{9}{2}}+320\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^7bc^4+560\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^6b^2c^{\frac{7}{2}}+560\left(\sqrt{c}x-\sqrt{c\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^6,x, algorithm="giac")
```

```
[Out] 1/80*(80*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*c^(9/2) + 320*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^7*b*c^4 + 560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b
^2*c^(7/2) + 560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*c^3 + 360*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^4*b^4*c^(5/2) - 80*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^4*a*b^2*c^(7/2) + 160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c
^(9/2) + 160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^5*c^2 - 160*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^3*a*b^3*c^3 + 320*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^3*a^2*b*c^4 + 50*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^6*c^(3/2) - 1
20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^4*c^(5/2) + 240*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^2*a^2*b^2*c^(7/2) + 10*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*b^7*c - 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^5*c^2 + 80*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*a^2*b^3*c^3 + b^8*sqrt(c) - 6*a*b^6*c^(3/2) +
16*a^2*b^4*c^(5/2) - 16*a^3*b^2*c^(7/2) + 16*a^4*c^(9/2))/((2*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^2*c + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*sqrt(c
) + b^2 - 2*a*c)^5*c^3*d^6)
```

3.1213 $\int \frac{(a+bx+cx^2)^{3/2}}{(bd+2cdx)^7}\,dx$

Optimal. Leaf size=165

$$\frac{\sqrt{a+bx+cx^2}}{128c^2d^7\left(b^2-4ac\right)(b+2cx)^2}+\frac{\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{256c^{5/2}d^7\left(b^2-4ac\right)^{3/2}}-\frac{\sqrt{a+bx+cx^2}}{64c^2d^7(b+2cx)^4}-\frac{\left(a+bx+cx^2\right)^{3/2}}{12cd^7(b+2cx)^6}$$

```
[Out] -Sqrt[a + b*x + c*x^2]/(64*c^2*d^7*(b + 2*c*x)^4) + Sqrt[a + b*x + c*x^2]/(
128*c^2*(b^2 - 4*a*c)*d^7*(b + 2*c*x)^2) - (a + b*x + c*x^2)^(3/2)/(12*c*d^
7*(b + 2*c*x)^6) + ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*
c]]/(256*c^(5/2)*(b^2 - 4*a*c)^(3/2)*d^7)
```

Rubi [A] time = 0.113958, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {684, 693, 688, 205}

$$\frac{\sqrt{a+bx+cx^2}}{128c^2d^7\left(b^2-4ac\right)(b+2cx)^2}+\frac{\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{256c^{5/2}d^7\left(b^2-4ac\right)^{3/2}}-\frac{\sqrt{a+bx+cx^2}}{64c^2d^7(b+2cx)^4}-\frac{\left(a+bx+cx^2\right)^{3/2}}{12cd^7(b+2cx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^7,x]
```

```
[Out] -Sqrt[a + b*x + c*x^2]/(64*c^2*d^7*(b + 2*c*x)^4) + Sqrt[a + b*x + c*x^2]/(
128*c^2*(b^2 - 4*a*c)*d^7*(b + 2*c*x)^2) - (a + b*x + c*x^2)^(3/2)/(12*c*d^
7*(b + 2*c*x)^6) + ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*
c]]/(256*c^(5/2)*(b^2 - 4*a*c)^(3/2)*d^7)
```

Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
```

```
qQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(bd+2cdx)^7}\,dx = -\frac{\left(a+bx+cx^2\right)^{3/2}}{12cd^7(b+2cx)^6}+\frac{\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^5}\,dx}{8cd^2}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{64c^2d^7(b+2cx)^4}-\frac{\left(a+bx+cx^2\right)^{3/2}}{12cd^7(b+2cx)^6}+\frac{\int \frac{1}{(bd+2cdx)^3\sqrt{a+bx+cx^2}}\,dx}{128c^2d^4}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{64c^2d^7(b+2cx)^4}+\frac{\sqrt{a+bx+cx^2}}{128c^2\left(b^2-4ac\right)d^7(b+2cx)^2}-\frac{\left(a+bx+cx^2\right)^{3/2}}{12cd^7(b+2cx)^6}+\frac{\int \frac{1}{(bd+2cdx)\sqrt{a+bx+c}}}{256c^2\left(b^2-4ac\right.}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{64c^2d^7(b+2cx)^4}+\frac{\sqrt{a+bx+cx^2}}{128c^2\left(b^2-4ac\right)d^7(b+2cx)^2}-\frac{\left(a+bx+cx^2\right)^{3/2}}{12cd^7(b+2cx)^6}+\frac{\text{Subst}\left(\int \frac{}{2b^2cd-8c}\right.}{6}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{64c^2d^7(b+2cx)^4}+\frac{\sqrt{a+bx+cx^2}}{128c^2\left(b^2-4ac\right)d^7(b+2cx)^2}-\frac{\left(a+bx+cx^2\right)^{3/2}}{12cd^7(b+2cx)^6}+\frac{\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx}}{\sqrt{b^2-4a}}\right.}{256c^{5/2}\left(b^2-4ac\right.}$$

Mathematica [C] time = 0.0319288, size = 62, normalized size = 0.38

$$\frac{2(a+x(b+cx))^{5/2}\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{4c(a+x(b+cx))}{4ac-b^2}\right)}{5d^7\left(b^2-4ac\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^7,x]
```

```
[Out] (2*(a + x*(b + c*x))^(5/2)*Hypergeometric2F1[5/2, 4, 7/2, (4*c*(a + x*(b +
c*x)))/(-b^2 + 4*a*c)])/(5*(b^2 - 4*a*c)^4*d^7)
```

Maple [B] time = 0.23, size = 682, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^7,x)
```

```
[Out] -1/192/d^7/c^6/(4*a*c-b^2)/(x+1/2*b/c)^6*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c
)^(5/2)+1/192/d^7/c^4/(4*a*c-b^2)^2/(x+1/2*b/c)^4*((x+1/2*b/c)^2*c+1/4*(4*a
*c-b^2)/c)^(5/2)+1/96/d^7/c^2/(4*a*c-b^2)^3/(x+1/2*b/c)^2*((x+1/2*b/c)^2*c+
1/4*(4*a*c-b^2)/c)^(5/2)-1/96/d^7/c/(4*a*c-b^2)^3*((x+1/2*b/c)^2*c+1/4*(4*a
*c-b^2)/c)^(3/2)-1/64/d^7/c/(4*a*c-b^2)^3*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)
^(1/2)*a+1/256/d^7/c^2/(4*a*c-b^2)^3*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2
```

```
)*b^2+1/16/d^7/c/(4*a*c-b^2)^3/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+
1/2*((4*a*c-b^2)/c)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b
/c))*a^2-1/32/d^7/c^2/(4*a*c-b^2)^3/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^
2)/c+1/2*((4*a*c-b^2)/c)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+
1/2*b/c))*a*b^2+1/256/d^7/c^3/(4*a*c-b^2)^3/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(
4*a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1
/2))/(x+1/2*b/c))*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 48.1422, size = 2276, normalized size = 13.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^7,x, algorithm="fricas")
```

```
[Out] [1/1536*(3*(64*c^6*x^6 + 192*b*c^5*x^5 + 240*b^2*c^4*x^4 + 160*b^3*c^3*x^3
+ 60*b^4*c^2*x^2 + 12*b^5*c*x + b^6)*sqrt(-b^2*c + 4*a*c^2)*log(-(4*c^2*x^2
 + 4*b*c*x - b^2 + 8*a*c + 4*sqrt(-b^2*c + 4*a*c^2)*sqrt(c*x^2 + b*x + a))/
(4*c^2*x^2 + 4*b*c*x + b^2)) - 4*(3*b^6*c - 4*a*b^4*c^2 - 160*a^2*b^2*c^3 +
 512*a^3*c^4 - 48*(b^2*c^5 - 4*a*c^6)*x^4 - 96*(b^3*c^4 - 4*a*b*c^5)*x^3 -
16*(b^4*c^3 + 10*a*b^2*c^4 - 56*a^2*c^5)*x^2 + 32*(b^5*c^2 - 11*a*b^3*c^3 +
 28*a^2*b*c^4)*x)*sqrt(c*x^2 + b*x + a))/(64*(b^4*c^9 - 8*a*b^2*c^10 + 16*a
^2*c^11)*d^7*x^6 + 192*(b^5*c^8 - 8*a*b^3*c^9 + 16*a^2*b*c^10)*d^7*x^5 + 24
0*(b^6*c^7 - 8*a*b^4*c^8 + 16*a^2*b^2*c^9)*d^7*x^4 + 160*(b^7*c^6 - 8*a*b^5
*c^7 + 16*a^2*b^3*c^8)*d^7*x^3 + 60*(b^8*c^5 - 8*a*b^6*c^6 + 16*a^2*b^4*c^7
)*d^7*x^2 + 12*(b^9*c^4 - 8*a*b^7*c^5 + 16*a^2*b^5*c^6)*d^7*x + (b^10*c^3 -
 8*a*b^8*c^4 + 16*a^2*b^6*c^5)*d^7), -1/768*(3*(64*c^6*x^6 + 192*b*c^5*x^5
+ 240*b^2*c^4*x^4 + 160*b^3*c^3*x^3 + 60*b^4*c^2*x^2 + 12*b^5*c*x + b^6)*sq
rt(b^2*c - 4*a*c^2)*arctan(1/2*sqrt(b^2*c - 4*a*c^2)*sqrt(c*x^2 + b*x + a)/
(c^2*x^2 + b*c*x + a*c)) + 2*(3*b^6*c - 4*a*b^4*c^2 - 160*a^2*b^2*c^3 + 512
*a^3*c^4 - 48*(b^2*c^5 - 4*a*c^6)*x^4 - 96*(b^3*c^4 - 4*a*b*c^5)*x^3 - 16*(
b^4*c^3 + 10*a*b^2*c^4 - 56*a^2*c^5)*x^2 + 32*(b^5*c^2 - 11*a*b^3*c^3 + 28*
a^2*b*c^4)*x)*sqrt(c*x^2 + b*x + a))/(64*(b^4*c^9 - 8*a*b^2*c^10 + 16*a^2*c
^11)*d^7*x^6 + 192*(b^5*c^8 - 8*a*b^3*c^9 + 16*a^2*b*c^10)*d^7*x^5 + 240*(b
^6*c^7 - 8*a*b^4*c^8 + 16*a^2*b^2*c^9)*d^7*x^4 + 160*(b^7*c^6 - 8*a*b^5*c^7
 + 16*a^2*b^3*c^8)*d^7*x^3 + 60*(b^8*c^5 - 8*a*b^6*c^6 + 16*a^2*b^4*c^7)*d^
7*x^2 + 12*(b^9*c^4 - 8*a*b^7*c^5 + 16*a^2*b^5*c^6)*d^7*x + (b^10*c^3 - 8*a
*b^8*c^4 + 16*a^2*b^6*c^5)*d^7)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a\sqrt{a+bx+cx^2}}{b^7+14b^6cx+84b^5c^2x^2+280b^4c^3x^3+560b^3c^4x^4+672b^2c^5x^5+448bc^6x^6+128c^7x^7}\,dx + \int \frac{bx\sqrt{a+bx+cx^2}}{b^7+14b^6cx+84b^5c^2x^2+280b^4c^3x^3+560b^3c^4x^4+672b^2c^5x^5+448bc^6x^6+}}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(2*c*d*x+b*d)**7,x)
```

```
[Out] (Integral(a*sqrt(a + b*x + c*x**2)/(b**7 + 14*b**6*c*x + 84*b**5*c**2*x**2
+ 280*b**4*c**3*x**3 + 560*b**3*c**4*x**4 + 672*b**2*c**5*x**5 + 448*b*c**6
*x**6 + 128*c**7*x**7), x) + Integral(b*x*sqrt(a + b*x + c*x**2)/(b**7 + 14
*b**6*c*x + 84*b**5*c**2*x**2 + 280*b**4*c**3*x**3 + 560*b**3*c**4*x**4 + 6
72*b**2*c**5*x**5 + 448*b*c**6*x**6 + 128*c**7*x**7), x) + Integral(c*x**2*
sqrt(a + b*x + c*x**2)/(b**7 + 14*b**6*c*x + 84*b**5*c**2*x**2 + 280*b**4*c
**3*x**3 + 560*b**3*c**4*x**4 + 672*b**2*c**5*x**5 + 448*b*c**6*x**6 + 128*
c**7*x**7), x))/d**7
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^7,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1214 $\int \frac{(a+bx+cx^2)^{3/2}}{(bd+2cdx)^8} dx$

Optimal. Leaf size=79

$$\frac{4(a+bx+cx^2)^{5/2}}{35d^8(b^2-4ac)^2(b+2cx)^5}+\frac{2(a+bx+cx^2)^{5/2}}{7d^8(b^2-4ac)(b+2cx)^7}$$

```
[Out] (2*(a + b*x + c*x^2)^(5/2))/(7*(b^2 - 4*a*c)*d^8*(b + 2*c*x)^7) + (4*(a + b
*x + c*x^2)^(5/2))/(35*(b^2 - 4*a*c)^2*d^8*(b + 2*c*x)^5)
```

Rubi [A] time = 0.0348012, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {693, 682}

$$\frac{4(a+bx+cx^2)^{5/2}}{35d^8(b^2-4ac)^2(b+2cx)^5}+\frac{2(a+bx+cx^2)^{5/2}}{7d^8(b^2-4ac)(b+2cx)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^8,x]
```

```
[Out] (2*(a + b*x + c*x^2)^(5/2))/(7*(b^2 - 4*a*c)*d^8*(b + 2*c*x)^7) + (4*(a + b
*x + c*x^2)^(5/2))/(35*(b^2 - 4*a*c)^2*d^8*(b + 2*c*x)^5)
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 682

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c,
0] && EqQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx+cx^2)^{3/2}}{(bd+2cdx)^8} dx &= \frac{2(a+bx+cx^2)^{5/2}}{7(b^2-4ac)d^8(b+2cx)^7}+\frac{2\int \frac{(a+bx+cx^2)^{3/2}}{(bd+2cdx)^6} dx}{7(b^2-4ac)d^2}\\ &= \frac{2(a+bx+cx^2)^{5/2}}{7(b^2-4ac)d^8(b+2cx)^7}+\frac{4(a+bx+cx^2)^{5/2}}{35(b^2-4ac)^2d^8(b+2cx)^5}\end{aligned}$$

Mathematica [A] time = 0.0367531, size = 62, normalized size = 0.78

$$\frac{2(a+x(b+cx))^{5/2}\left(4c\left(2cx^2-5a\right)+7b^2+8bcx\right)}{35d^8\left(b^2-4ac\right)^2(b+2cx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^8,x]
```

```
[Out] (2*(a + x*(b + c*x))^(5/2)*(7*b^2 + 8*b*c*x + 4*c*(-5*a + 2*c*x^2)))/(35*(b
^2 - 4*a*c)^2*d^8*(b + 2*c*x)^7)
```

Maple [A] time = 0.044, size = 70, normalized size = 0.9

$$-\frac{-16\,c^2x^2-16\,bcx+40\,ac-14\,b^2}{35\,(2\,cx+b)^7\,d^8\left(16\,a^2c^2-8\,acb^2+b^4\right)}\left(cx^2+bx+a\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^8,x)
```

```
[Out] -2/35*(-8*c^2*x^2-8*b*c*x+20*a*c-7*b^2)*(c*x^2+b*x+a)^(5/2)/(2*c*x+b)^7/d^8
/(16*a^2*c^2-8*a*b^2*c+b^4)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 119.981, size = 841, normalized size = 10.65

$$\frac{2\left(8\,c^4x^6+24\,bc^3x^5+\left(31\,b^2c^2-4\,ac^3\right)x^4 \cdots\right.}{35\left(128\left(b^4c^7-8\,ab^2c^8+16\,a^2c^9\right)d^8x^7+448\left(b^5c^6-8\,ab^3c^7+16\,a^2bc^8\right)d^8x^6+672\left(b^6c^5-8\,ab^4c^6+16\,a^2b^2c^7\right)d^8x^5+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^8,x, algorithm="fricas")
```

```
[Out] 2/35*(8*c^4*x^6 + 24*b*c^3*x^5 + (31*b^2*c^2 - 4*a*c^3)*x^4 + 7*a^2*b^2 - 2
0*a^3*c + 2*(11*b^3*c - 4*a*b*c^2)*x^3 + (7*b^4 + 10*a*b^2*c - 32*a^2*c^2)*
x^2 + 2*(7*a*b^3 - 16*a^2*b*c)*x)*sqrt(c*x^2 + b*x + a)/(128*(b^4*c^7 - 8*a
*b^2*c^8 + 16*a^2*c^9)*d^8*x^7 + 448*(b^5*c^6 - 8*a*b^3*c^7 + 16*a^2*b*c^8)
*d^8*x^6 + 672*(b^6*c^5 - 8*a*b^4*c^6 + 16*a^2*b^2*c^7)*d^8*x^5 + 560*(b^7*
c^4 - 8*a*b^5*c^5 + 16*a^2*b^3*c^6)*d^8*x^4 + 280*(b^8*c^3 - 8*a*b^6*c^4 +
16*a^2*b^4*c^5)*d^8*x^3 + 84*(b^9*c^2 - 8*a*b^7*c^3 + 16*a^2*b^5*c^4)*d^8*x
```

```
^2 + 14*(b^10*c - 8*a*b^8*c^2 + 16*a^2*b^6*c^3)*d^8*x + (b^11 - 8*a*b^9*c +
 16*a^2*b^7*c^2)*d^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(2*c*d*x+b*d)**8,x)

[Out] Timed out
```

Giac [B] time = 2.00361, size = 1354, normalized size = 17.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^8,x, algorithm="giac")

[Out] 1/280*(560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*c^(11/2) + 2800*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^9*b*c^5 + 6160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^8*b^2*c^(9/2) + 560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*c^(11/2) + 7
840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^3*c^4 + 2240*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^7*a*b*c^5 + 6440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^
4*c^(7/2) + 3360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^2*c^(9/2) + 1120
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*c^(11/2) + 3640*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^5*b^5*c^3 + 2240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*
a*b^3*c^4 + 3360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b*c^5 + 1484*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^6*c^(5/2) + 392*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^4*a*b^4*c^(7/2) + 4032*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*
a^2*b^2*c^(9/2) + 224*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*c^(11/2) +
448*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^7*c^2 - 336*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^3*a*b^5*c^3 + 2464*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a
^2*b^3*c^4 + 448*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b*c^5 + 98*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^2*b^8*c^(3/2) - 224*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^2*a*b^6*c^(5/2) + 840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2
*b^4*c^(7/2) + 224*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^2*c^(9/2) +
112*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*c^(11/2) + 14*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))*b^9*c - 56*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^7*c
^2 + 168*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^5*c^3 + 112*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))*a^4*b*c^5 + b^10*sqrt(c) - 6*a*b^8*c^(3/2) + 20*a^2
*b^6*c^(5/2) - 24*a^3*b^4*c^(7/2) + 48*a^4*b^2*c^(9/2) - 16*a^5*c^(11/2))/(
(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c + 2*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))*b*sqrt(c) + b^2 - 2*a*c)^7*c^3*d^8)
```

3.1215 $\int \frac{(a+bx+cx^2)^{3/2}}{(bd+2cdx)^9}\,dx$

Optimal. Leaf size=207

$$\frac{3\sqrt{a+bx+cx^2}}{1024c^2d^9\left(b^2-4ac\right)^2(b+2cx)^2}+\frac{\sqrt{a+bx+cx^2}}{512c^2d^9\left(b^2-4ac\right)(b+2cx)^4}+\frac{3\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{2048c^{5/2}d^9\left(b^2-4ac\right)^{5/2}}-\frac{\sqrt{a+bx+cx^2}}{128c^2d^9(b+2cx)^6}-\frac{(a}{16}$$

```
[Out] -Sqrt[a + b*x + c*x^2]/(128*c^2*d^9*(b + 2*c*x)^6) + Sqrt[a + b*x + c*x^2]/
(512*c^2*(b^2 - 4*a*c)*d^9*(b + 2*c*x)^4) + (3*Sqrt[a + b*x + c*x^2])/(1024
*c^2*(b^2 - 4*a*c)^2*d^9*(b + 2*c*x)^2) - (a + b*x + c*x^2)^(3/2)/(16*c*d^9
*(b + 2*c*x)^8) + (3*ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*
a*c]])/(2048*c^(5/2)*(b^2 - 4*a*c)^(5/2)*d^9)
```

Rubi [A] time = 0.146302, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {684, 693, 688, 205}

$$\frac{3\sqrt{a+bx+cx^2}}{1024c^2d^9\left(b^2-4ac\right)^2(b+2cx)^2}+\frac{\sqrt{a+bx+cx^2}}{512c^2d^9\left(b^2-4ac\right)(b+2cx)^4}+\frac{3\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{2048c^{5/2}d^9\left(b^2-4ac\right)^{5/2}}-\frac{\sqrt{a+bx+cx^2}}{128c^2d^9(b+2cx)^6}-\frac{(a}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^9,x]
```

```
[Out] -Sqrt[a + b*x + c*x^2]/(128*c^2*d^9*(b + 2*c*x)^6) + Sqrt[a + b*x + c*x^2]/
(512*c^2*(b^2 - 4*a*c)*d^9*(b + 2*c*x)^4) + (3*Sqrt[a + b*x + c*x^2])/(1024
*c^2*(b^2 - 4*a*c)^2*d^9*(b + 2*c*x)^2) - (a + b*x + c*x^2)^(3/2)/(16*c*d^9
*(b + 2*c*x)^8) + (3*ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*
a*c]])/(2048*c^(5/2)*(b^2 - 4*a*c)^(5/2)*d^9)
```

Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] && !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
```

```
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{3/2}}{(bd+2cdx)^9}\,dx = -\frac{(a+bx+cx^2)^{3/2}}{16cd^9(b+2cx)^8} + \frac{3\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^7}\,dx}{32cd^2}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{128c^2d^9(b+2cx)^6} - \frac{(a+bx+cx^2)^{3/2}}{16cd^9(b+2cx)^8} + \frac{\int \frac{1}{(bd+2cdx)^5\sqrt{a+bx+cx^2}}\,dx}{256c^2d^4}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{128c^2d^9(b+2cx)^6} + \frac{\sqrt{a+bx+cx^2}}{512c^2(b^2-4ac)d^9(b+2cx)^4} - \frac{(a+bx+cx^2)^{3/2}}{16cd^9(b+2cx)^8} + \frac{3\int \frac{1}{(bd+2cdx)^3\sqrt{a+bx+c}}}{1024c^2(b^2-4ac}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{128c^2d^9(b+2cx)^6} + \frac{\sqrt{a+bx+cx^2}}{512c^2(b^2-4ac)d^9(b+2cx)^4} + \frac{3\sqrt{a+bx+cx^2}}{1024c^2(b^2-4ac)^2d^9(b+2cx)^2} - \frac{(a+}{16cd}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{128c^2d^9(b+2cx)^6} + \frac{\sqrt{a+bx+cx^2}}{512c^2(b^2-4ac)d^9(b+2cx)^4} + \frac{3\sqrt{a+bx+cx^2}}{1024c^2(b^2-4ac)^2d^9(b+2cx)^2} - \frac{(a+}{16cd}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{128c^2d^9(b+2cx)^6} + \frac{\sqrt{a+bx+cx^2}}{512c^2(b^2-4ac)d^9(b+2cx)^4} + \frac{3\sqrt{a+bx+cx^2}}{1024c^2(b^2-4ac)^2d^9(b+2cx)^2} - \frac{(a+}{16cd}$$

Mathematica [C] time = 0.0297943, size = 62, normalized size = 0.3

$$\frac{2(a+x(b+cx))^{5/2}\,{}_2F_1\left(\frac{5}{2},5;\frac{7}{2};\frac{4c(a+x(b+cx))}{4ac-b^2}\right)}{5d^9\left(b^2-4ac\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^9,x]
```

```
[Out] (2*(a + x*(b + c*x))^(5/2)*Hypergeometric2F1[5/2, 5, 7/2, (4*c*(a + x*(b +
c*x)))/(-b^2 + 4*a*c)])/(5*(b^2 - 4*a*c)^5*d^9)
```

Maple [B] time = 0.238, size = 742, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^9,x)
```

```
[Out] -1/1024/d^9/c^8/(4*a*c-b^2)/(x+1/2*b/c)^8*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/
c)^(5/2)+1/512/d^9/c^6/(4*a*c-b^2)^2/(x+1/2*b/c)^6*((x+1/2*b/c)^2*c+1/4*(4*
```

```
a*c-b^2)/c)^(5/2)-1/512/d^9/c^4/(4*a*c-b^2)^3/(x+1/2*b/c)^4*((x+1/2*b/c)^2*
c+1/4*(4*a*c-b^2)/c)^(5/2)-1/256/d^9/c^2/(4*a*c-b^2)^4/(x+1/2*b/c)^2*((x+1/
2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(5/2)+1/256/d^9/c/(4*a*c-b^2)^4*((x+1/2*b/c)^
2*c+1/4*(4*a*c-b^2)/c)^(3/2)+3/512/d^9/c/(4*a*c-b^2)^4*(4*(x+1/2*b/c)^2*c+(
4*a*c-b^2)/c)^(1/2)*a-3/2048/d^9/c^2/(4*a*c-b^2)^4*(4*(x+1/2*b/c)^2*c+(4*a*
c-b^2)/c)^(1/2)*b^2-3/128/d^9/c/(4*a*c-b^2)^4/((4*a*c-b^2)/c)^(1/2)*ln((1/2
*(4*a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^
(1/2))/(x+1/2*b/c))*a^2+3/256/d^9/c^2/(4*a*c-b^2)^4/((4*a*c-b^2)/c)^(1/2)*l
n((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^
2)/c)^(1/2))/(x+1/2*b/c))*a*b^2-3/2048/d^9/c^3/(4*a*c-b^2)^4/((4*a*c-b^2)/c
)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/2)*(4*(x+1/2*b/c)^2*c+
(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^9,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(2*c*d*x+b*d)**9,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^9,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1216 $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(bd+2cdx)^{10}}\,dx$

Optimal. Leaf size=118

$$\frac{16\left(a+bx+cx^2\right)^{5/2}}{315d^{10}\left(b^2-4ac\right)^3(b+2cx)^5}+\frac{8\left(a+bx+cx^2\right)^{5/2}}{63d^{10}\left(b^2-4ac\right)^2(b+2cx)^7}+\frac{2\left(a+bx+cx^2\right)^{5/2}}{9d^{10}\left(b^2-4ac\right)(b+2cx)^9}$$

```
[Out] (2*(a + b*x + c*x^2)^(5/2))/(9*(b^2 - 4*a*c)*d^10*(b + 2*c*x)^9) + (8*(a +
b*x + c*x^2)^(5/2))/(63*(b^2 - 4*a*c)^2*d^10*(b + 2*c*x)^7) + (16*(a + b*x
+ c*x^2)^(5/2))/(315*(b^2 - 4*a*c)^3*d^10*(b + 2*c*x)^5)
```

Rubi [A] time = 0.0572946, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {693, 682}

$$\frac{16\left(a+bx+cx^2\right)^{5/2}}{315d^{10}\left(b^2-4ac\right)^3(b+2cx)^5}+\frac{8\left(a+bx+cx^2\right)^{5/2}}{63d^{10}\left(b^2-4ac\right)^2(b+2cx)^7}+\frac{2\left(a+bx+cx^2\right)^{5/2}}{9d^{10}\left(b^2-4ac\right)(b+2cx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^10,x]
```


```
[Out] (2*(a + b*x + c*x^2)^(5/2))/(9*(b^2 - 4*a*c)*d^10*(b + 2*c*x)^9) + (8*(a +
b*x + c*x^2)^(5/2))/(63*(b^2 - 4*a*c)^2*d^10*(b + 2*c*x)^7) + (16*(a + b*x
+ c*x^2)^(5/2))/(315*(b^2 - 4*a*c)^3*d^10*(b + 2*c*x)^5)
```


Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 682

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c,
0] && EqQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{3/2}}{(bd+2cdx)^{10}}\,dx = \frac{2(a+bx+cx^2)^{5/2}}{9(b^2-4ac)d^{10}(b+2cx)^9} + \frac{4\int \frac{(a+bx+cx^2)^{3/2}}{(bd+2cdx)^8}dx}{9(b^2-4ac)d^2}$$

$$= \frac{2(a+bx+cx^2)^{5/2}}{9(b^2-4ac)d^{10}(b+2cx)^9} + \frac{8(a+bx+cx^2)^{5/2}}{63(b^2-4ac)^2d^{10}(b+2cx)^7} + \frac{8\int \frac{(a+bx+cx^2)^{3/2}}{(bd+2cdx)^6}dx}{63(b^2-4ac)^2d^4}$$

$$= \frac{2(a+bx+cx^2)^{5/2}}{9(b^2-4ac)d^{10}(b+2cx)^9} + \frac{8(a+bx+cx^2)^{5/2}}{63(b^2-4ac)^2d^{10}(b+2cx)^7} + \frac{16(a+bx+cx^2)^{5/2}}{315(b^2-4ac)^3d^{10}(b+2cx)^5}$$

Mathematica [A] time = 0.062807, size = 110, normalized size = 0.93

$$\frac{2(a+x(b+cx))^{5/2}\left(16c^2(35a^2-20acx^2+8c^2x^4)+8b^2c(34cx^2-45a)+64bc^2x(4cx^2-5a)+144b^3cx+63b^4\right)}{315d^{10}(b^2-4ac)^3(b+2cx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^10,x]
```

```
[Out] (2*(a + x*(b + c*x))^(5/2)*(63*b^4 + 144*b^3*c*x + 64*b*c^2*x*(-5*a + 4*c*x
^2) + 8*b^2*c*(-45*a + 34*c*x^2) + 16*c^2*(35*a^2 - 20*a*c*x^2 + 8*c^2*x^4)
))/(315*(b^2 - 4*a*c)^3*d^10*(b + 2*c*x)^9)
```

Maple [A] time = 0.046, size = 133, normalized size = 1.1

$$-\frac{256c^4x^4+512bc^3x^3-640ac^3x^2+544b^2c^2x^2-640abc^2x+288b^3cx+1120a^2c^2-720acb^2+126b^4}{315(2cx+b)^9d^{10}(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}(cx^2+bx+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^10,x)
```

```
[Out] -2/315*(128*c^4*x^4+256*b*c^3*x^3-320*a*c^3*x^2+272*b^2*c^2*x^2-320*a*b*c^2
*x+144*b^3*c*x+560*a^2*c^2-360*a*b^2*c+63*b^4)*(c*x^2+b*x+a)^(5/2)/(2*c*x+b
)^9/d^10/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^10,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(2*c*d*x+b*d)**10,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.09737, size = 1881, normalized size = 15.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^10,x, algorithm="giac")
```

```
[Out] 1/630*(3360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*c^(13/2) + 20160*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^11*b*c^6 + 54180*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^10*b^2*c^(11/2) + 5040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a*c^(1
3/2) + 86100*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^3*c^5 + 25200*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^9*a*b*c^6 + 90216*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^8*b^4*c^(9/2) + 53172*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^2*c^
(11/2) + 7056*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*c^(13/2) + 66024*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^5*c^4 + 61488*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^7*a*b^3*c^5 + 28224*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*
b*c^6 + 35028*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^6*c^(7/2) + 41832*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^4*c^(9/2) + 47880*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^6*a^2*b^2*c^(11/2) + 2016*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^6*a^3*c^(13/2) + 13860*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^7*c^3 +
 16128*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^5*c^4 + 44856*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^5*a^2*b^3*c^5 + 6048*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^5*a^3*b*c^6 + 4176*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^8*c^(5/2)
+ 2484*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^6*c^(7/2) + 25416*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^4*c^(9/2) + 6984*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^4*a^3*b^2*c^(11/2) + 576*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
4*a^4*c^(13/2) + 960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^9*c^2 - 576*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^7*c^3 + 9000*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^3*a^2*b^5*c^4 + 3888*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^
3*b^3*c^5 + 1152*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b*c^6 + 162*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^2*b^10*c^(3/2) - 360*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^2*a*b^8*c^(5/2) + 2016*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*
a^2*b^6*c^(7/2) + 936*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^4*c^(9/2)
```

```
 + 1044*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^2*c^(11/2) - 144*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*c^(13/2) + 18*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*b^11*c - 72*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^9*c^2 + 288*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^7*c^3 + 468*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*a^4*b^3*c^5 - 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b*
c^6 + b^12*sqrt(c) - 6*a*b^10*c^(3/2) + 24*a^2*b^8*c^(5/2) - 32*a^3*b^6*c^(
7/2) + 96*a^4*b^4*c^(9/2) - 60*a^5*b^2*c^(11/2) + 16*a^6*c^(13/2))/((2*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^2*c + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*b*sqrt(c) + b^2 - 2*a*c)^9*c^3*d^10)
```

3.1217 $\int (bd+2cdx)^5 \left(a+bx+cx^2\right)^{5/2} \, dx$

Optimal. Leaf size=98

$$\frac{16}{693}d^5\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{7/2}+\frac{8}{99}d^5\left(b^2-4ac\right)(b+2cx)^2\left(a+bx+cx^2\right)^{7/2}+\frac{2}{11}d^5(b+2cx)^4\left(a+bx+cx^2\right)^{7/2}$$

```
[Out] (16*(b^2 - 4*a*c)^2*d^5*(a + b*x + c*x^2)^(7/2))/693 + (8*(b^2 - 4*a*c)*d^5
*(b + 2*c*x)^2*(a + b*x + c*x^2)^(7/2))/99 + (2*d^5*(b + 2*c*x)^4*(a + b*x
+ c*x^2)^(7/2))/11
```

Rubi [A] time = 0.0508876, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {692, 629}

$$\frac{16}{693}d^5\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{7/2}+\frac{8}{99}d^5\left(b^2-4ac\right)(b+2cx)^2\left(a+bx+cx^2\right)^{7/2}+\frac{2}{11}d^5(b+2cx)^4\left(a+bx+cx^2\right)^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^5*(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (16*(b^2 - 4*a*c)^2*d^5*(a + b*x + c*x^2)^(7/2))/693 + (8*(b^2 - 4*a*c)*d^5
*(b + 2*c*x)^2*(a + b*x + c*x^2)^(7/2))/99 + (2*d^5*(b + 2*c*x)^4*(a + b*x
+ c*x^2)^(7/2))/11
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (bd+2cdx)^5\left(a+bx+cx^2\right)^{5/2}\,dx &= \frac{2}{11}d^5(b+2cx)^4\left(a+bx+cx^2\right)^{7/2}+\frac{1}{11}\left(4\left(b^2-4ac\right)d^2\right)\int (bd+2cdx)^3\left(a+bx+c\right.\\ &= \frac{8}{99}\left(b^2-4ac\right)d^5(b+2cx)^2\left(a+bx+cx^2\right)^{7/2}+\frac{2}{11}d^5(b+2cx)^4\left(a+bx+cx^2\right)^{7/2}+\\ &= \frac{16}{693}\left(b^2-4ac\right)^2d^5\left(a+bx+cx^2\right)^{7/2}+\frac{8}{99}\left(b^2-4ac\right)d^5(b+2cx)^2\left(a+bx+cx^2\right)^{7/2}\end{aligned}$$

Mathematica [A] time = 0.0940328, size = 92, normalized size = 0.94

$$\frac{2}{693}d^5(a+x(b+cx))^{7/2}\left(16c^2\left(8a^2-28acx^2+63c^2x^4\right)+8b^2c\left(203cx^2-22a\right)+224bc^2x\left(9cx^2-2a\right)+616b^3cx+99b^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^5*(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*d^5*(a + x*(b + c*x))^(7/2)*(99*b^4 + 616*b^3*c*x + 224*b*c^2*x*(-2*a +
9*c*x^2) + 8*b^2*c*(-22*a + 203*c*x^2) + 16*c^2*(8*a^2 - 28*a*c*x^2 + 63*c^
2*x^4)))/693
```

Maple [A] time = 0.046, size = 91, normalized size = 0.9

$$\frac{\left(2016\,c^4x^4+4032\,bc^3x^3-896\,x^2ac^3+3248\,x^2b^2c^2-896\,xbac^2+1232\,xb^3c+256\,a^2c^2-352\,acb^2+198\,b^4\right)d^5}{693}\left(cx^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^5*(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 2/693*(c*x^2+b*x+a)^(7/2)*(1008*c^4*x^4+2016*b*c^3*x^3-448*a*c^3*x^2+1624*b
^2*c^2*x^2-448*a*b*c^2*x+616*b^3*c*x+128*a^2*c^2-176*a*b^2*c+99*b^4)*d^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^5*(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.03472, size = 778, normalized size = 7.94

$$\frac{2}{693}\left(1008\,c^7d^5x^{10}+5040\,bc^6d^5x^9+56\left(191\,b^2c^5+46\,ac^6\right)d^5x^8+448\left(28\,b^3c^4+23\,abc^5\right)d^5x^7+\left(8835\,b^4c^3+17128\,a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^5*(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/693*(1008*c^7*d^5*x^10 + 5040*b*c^6*d^5*x^9 + 56*(191*b^2*c^5 + 46*a*c^6)
*d^5*x^8 + 448*(28*b^3*c^4 + 23*a*b*c^5)*d^5*x^7 + (8835*b^4*c^3 + 17128*a*
b^2*c^4 + 1808*a^2*c^5)*d^5*x^6 + (3769*b^5*c^2 + 15320*a*b^3*c^3 + 5424*a^
2*b*c^4)*d^5*x^5 + (913*b^6*c + 7889*a*b^4*c^2 + 6744*a^2*b^2*c^3 + 48*a^3*
c^4)*d^5*x^4 + (99*b^7 + 2266*a*b^5*c + 4448*a^2*b^3*c^2 + 96*a^3*b*c^3)*d^
5*x^3 + (297*a*b^6 + 1617*a^2*b^4*c + 136*a^3*b^2*c^2 - 64*a^4*c^3)*d^5*x^2
 + (297*a^2*b^5 + 88*a^3*b^3*c - 64*a^4*b*c^2)*d^5*x + (99*a^3*b^4 - 176*a^
4*b^2*c + 128*a^5*c^2)*d^5)*sqrt(c*x^2 + b*x + a)
```

Sympy [B] time = 15.4882, size = 913, normalized size = 9.32

$$\frac{256a^5c^2d^5\sqrt{a+bx+cx^2}}{693} - \frac{32a^4b^2cd^5\sqrt{a+bx+cx^2}}{63} - \frac{128a^4bc^2d^5x\sqrt{a+bx+cx^2}}{693} - \frac{128a^4c^3d^5x^2\sqrt{a+bx+cx^2}}{693} + \frac{2a^3b^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**5*(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] 256*a**5*c**2*d**5*sqrt(a + b*x + c*x**2)/693 - 32*a**4*b**2*c*d**5*sqrt(a
+ b*x + c*x**2)/63 - 128*a**4*b*c**2*d**5*x*sqrt(a + b*x + c*x**2)/693 - 12
8*a**4*c**3*d**5*x**2*sqrt(a + b*x + c*x**2)/693 + 2*a**3*b**4*d**5*sqrt(a
+ b*x + c*x**2)/7 + 16*a**3*b**3*c*d**5*x*sqrt(a + b*x + c*x**2)/63 + 272*a
**3*b**2*c**2*d**5*x**2*sqrt(a + b*x + c*x**2)/693 + 64*a**3*b*c**3*d**5*x*
*3*sqrt(a + b*x + c*x**2)/231 + 32*a**3*c**4*d**5*x**4*sqrt(a + b*x + c*x**
2)/231 + 6*a**2*b**5*d**5*x*sqrt(a + b*x + c*x**2)/7 + 14*a**2*b**4*c*d**5*
x**2*sqrt(a + b*x + c*x**2)/3 + 8896*a**2*b**3*c**2*d**5*x**3*sqrt(a + b*x
+ c*x**2)/693 + 4496*a**2*b**2*c**3*d**5*x**4*sqrt(a + b*x + c*x**2)/231 +
3616*a**2*b*c**4*d**5*x**5*sqrt(a + b*x + c*x**2)/231 + 3616*a**2*c**5*d**5
*x**6*sqrt(a + b*x + c*x**2)/693 + 6*a*b**6*d**5*x**2*sqrt(a + b*x + c*x**2
)/7 + 412*a*b**5*c*d**5*x**3*sqrt(a + b*x + c*x**2)/63 + 2254*a*b**4*c**2*d
**5*x**4*sqrt(a + b*x + c*x**2)/99 + 30640*a*b**3*c**3*d**5*x**5*sqrt(a + b
*x + c*x**2)/693 + 34256*a*b**2*c**4*d**5*x**6*sqrt(a + b*x + c*x**2)/693 +
 2944*a*b*c**5*d**5*x**7*sqrt(a + b*x + c*x**2)/99 + 736*a*c**6*d**5*x**8*s
qrt(a + b*x + c*x**2)/99 + 2*b**7*d**5*x**3*sqrt(a + b*x + c*x**2)/7 + 166*
b**6*c*d**5*x**4*sqrt(a + b*x + c*x**2)/63 + 7538*b**5*c**2*d**5*x**5*sqrt(
a + b*x + c*x**2)/693 + 5890*b**4*c**3*d**5*x**6*sqrt(a + b*x + c*x**2)/231
 + 3584*b**3*c**4*d**5*x**7*sqrt(a + b*x + c*x**2)/99 + 3056*b**2*c**5*d**5
*x**8*sqrt(a + b*x + c*x**2)/99 + 160*b*c**6*d**5*x**9*sqrt(a + b*x + c*x**
2)/11 + 32*c**7*d**5*x**10*sqrt(a + b*x + c*x**2)/11
```

Giac [B] time = 2.1129, size = 599, normalized size = 6.11

$$\frac{2}{693}\sqrt{cx^2+bx+a}\left(\left(\left(\left(\left(\left(\left(56\left(\left(18\left(c^7d^5x+5\,bc^6d^5\right)x+\frac{191\,b^2c^{15}d^5+46\,ac^{16}d^5}{c^{10}}\right)x+\frac{8\left(28\,b^3c^{14}d^5+23\,abc^{15}d^5\right)}{c^{10}}\right)x+\frac{883}{}\right.\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^5*(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/693*sqrt(c*x^2 + b*x + a)*(((((((56*((18*(c^7*d^5*x + 5*b*c^6*d^5)*x + (1
91*b^2*c^15*d^5 + 46*a*c^16*d^5)/c^10)*x + 8*(28*b^3*c^14*d^5 + 23*a*b*c^15
*d^5)/c^10)*x + (8835*b^4*c^13*d^5 + 17128*a*b^2*c^14*d^5 + 1808*a^2*c^15*d
^5)/c^10)*x + (3769*b^5*c^12*d^5 + 15320*a*b^3*c^13*d^5 + 5424*a^2*b*c^14*d
^5)/c^10)*x + (913*b^6*c^11*d^5 + 7889*a*b^4*c^12*d^5 + 6744*a^2*b^2*c^13*d
^5 + 48*a^3*c^14*d^5)/c^10)*x + (99*b^7*c^10*d^5 + 2266*a*b^5*c^11*d^5 + 44
48*a^2*b^3*c^12*d^5 + 96*a^3*b*c^13*d^5)/c^10)*x + (297*a*b^6*c^10*d^5 + 16
17*a^2*b^4*c^11*d^5 + 136*a^3*b^2*c^12*d^5 - 64*a^4*c^13*d^5)/c^10)*x + (29
7*a^2*b^5*c^10*d^5 + 88*a^3*b^3*c^11*d^5 - 64*a^4*b*c^12*d^5)/c^10)*x + (99
*a^3*b^4*c^10*d^5 - 176*a^4*b^2*c^11*d^5 + 128*a^5*c^12*d^5)/c^10)
```

3.1218 $\int (bd+2cdx)^4 \left(a+bx+cx^2\right)^{5/2} dx$

Optimal. Leaf size=249

$$-\frac{3d^4\left(b^2-4ac\right)^4(b+2cx)\sqrt{a+bx+cx^2}}{8192c^3}-\frac{d^4\left(b^2-4ac\right)^3(b+2cx)^3\sqrt{a+bx+cx^2}}{4096c^3}+\frac{d^4\left(b^2-4ac\right)^2(b+2cx)^5\sqrt{a+b}}{1024c^3}$$

```
[Out] (-3*(b^2 - 4*a*c)^4*d^4*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(8192*c^3) - ((b
^2 - 4*a*c)^3*d^4*(b + 2*c*x)^3*Sqrt[a + b*x + c*x^2])/(4096*c^3) + ((b^2 -
 4*a*c)^2*d^4*(b + 2*c*x)^5*Sqrt[a + b*x + c*x^2])/(1024*c^3) - ((b^2 - 4*a
*c)*d^4*(b + 2*c*x)^5*(a + b*x + c*x^2)^(3/2))/(128*c^2) + (d^4*(b + 2*c*x)
^5*(a + b*x + c*x^2)^(5/2))/(20*c) - (3*(b^2 - 4*a*c)^5*d^4*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16384*c^(7/2))
```

Rubi [A] time = 0.170779, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {685, 692, 621, 206}

$$-\frac{3d^4\left(b^2-4ac\right)^4(b+2cx)\sqrt{a+bx+cx^2}}{8192c^3}-\frac{d^4\left(b^2-4ac\right)^3(b+2cx)^3\sqrt{a+bx+cx^2}}{4096c^3}+\frac{d^4\left(b^2-4ac\right)^2(b+2cx)^5\sqrt{a+b}}{1024c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^4*(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-3*(b^2 - 4*a*c)^4*d^4*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(8192*c^3) - ((b
^2 - 4*a*c)^3*d^4*(b + 2*c*x)^3*Sqrt[a + b*x + c*x^2])/(4096*c^3) + ((b^2 -
 4*a*c)^2*d^4*(b + 2*c*x)^5*Sqrt[a + b*x + c*x^2])/(1024*c^3) - ((b^2 - 4*a
*c)*d^4*(b + 2*c*x)^5*(a + b*x + c*x^2)^(3/2))/(128*c^2) + (d^4*(b + 2*c*x)
^5*(a + b*x + c*x^2)^(5/2))/(20*c) - (3*(b^2 - 4*a*c)^5*d^4*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16384*c^(7/2))
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (bd+2cdx)^4\left(a+bx+cx^2\right)^{5/2}dx = \frac{d^4(b+2cx)^5\left(a+bx+cx^2\right)^{5/2}}{20c} - \frac{\left(b^2-4ac\right)\int (bd+2cdx)^4\left(a+bx+cx^2\right)^{3/2}dx}{8c}$$

$$= -\frac{\left(b^2-4ac\right)d^4(b+2cx)^5\left(a+bx+cx^2\right)^{3/2}}{128c^2} + \frac{d^4(b+2cx)^5\left(a+bx+cx^2\right)^{5/2}}{20c} + \frac{(3(\cdots}{}$$

$$= \frac{\left(b^2-4ac\right)^2d^4(b+2cx)^5\sqrt{a+bx+cx^2}}{1024c^3} - \frac{\left(b^2-4ac\right)d^4(b+2cx)^5\left(a+bx+cx^2\right)^{3/2}}{128c^2}$$

$$= -\frac{\left(b^2-4ac\right)^3d^4(b+2cx)^3\sqrt{a+bx+cx^2}}{4096c^3} + \frac{\left(b^2-4ac\right)^2d^4(b+2cx)^5\sqrt{a+bx+cx^2}}{1024c^3}$$

$$= -\frac{3\left(b^2-4ac\right)^4d^4(b+2cx)\sqrt{a+bx+cx^2}}{8192c^3} - \frac{\left(b^2-4ac\right)^3d^4(b+2cx)^3\sqrt{a+bx+cx^2}}{4096c^3}$$

$$= -\frac{3\left(b^2-4ac\right)^4d^4(b+2cx)\sqrt{a+bx+cx^2}}{8192c^3} - \frac{\left(b^2-4ac\right)^3d^4(b+2cx)^3\sqrt{a+bx+cx^2}}{4096c^3}$$

$$= -\frac{3\left(b^2-4ac\right)^4d^4(b+2cx)\sqrt{a+bx+cx^2}}{8192c^3} - \frac{\left(b^2-4ac\right)^3d^4(b+2cx)^3\sqrt{a+bx+cx^2}}{4096c^3}$$

Mathematica [A] time = 3.74033, size = 265, normalized size = 1.06

$$\frac{1}{5}d^4\left((b+2cx)^3(a+x(b+cx))^{7/2} - \frac{3}{2}c\left(a-\frac{b^2}{4c}\right)(b+2cx)\sqrt{a+x(b+cx)}\left(\frac{\left(b^2-4ac\right)\left(16c^2\left(33a^2+26acx^2+8c^2x^4\right)+8b^2\cdots\right.}{}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^4*(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (d^4*((b + 2*c*x)^3*(a + x*(b + c*x))^(7/2) - (3*(a - b^2/(4*c))*c*(b + 2*c
*x)*Sqrt[a + x*(b + c*x)]*((a + x*(b + c*x))^3 + ((b^2 - 4*a*c)*(15*b^4 - 4
0*b^3*c*x + 32*b*c^2*x*(13*a + 8*c*x^2) + 8*b^2*c*(-20*a + 11*c*x^2) + 16*c
^2*(33*a^2 + 26*a*c*x^2 + 8*c^2*x^4)))/(3072*c^3) - (5*Sqrt[4*a - b^2/c]*Sq
rt[c]*(a + x*(b + c*x))^3*ArcSinh[(b + 2*c*x)/(Sqrt[4*a - b^2/c]*Sqrt[c])])
/(2048*(b + 2*c*x)*((c*(a + x*(b + c*x)))/(-b^2 + 4*a*c))^(7/2))))/2))/5
```

Maple [B] time = 0.056, size = 920, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^4*(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] -3/32*d^4*c*b^2*a^2*(c*x^2+b*x+a)^(3/2)*x-3/16*d^4*c*b^2*a^3*(c*x^2+b*x+a)^
(1/2)*x-3/5*d^4*c^2*a*x*(c*x^2+b*x+a)^(7/2)+27/20*d^4*c*b^2*x*(c*x^2+b*x+a)
^(7/2)-1/512*d^4/c*b^6*(c*x^2+b*x+a)^(3/2)*x+3/256*d^4/c*b^5*(c*x^2+b*x+a)^
(3/2)*a+3/4096*d^4/c^2*b^8*(c*x^2+b*x+a)^(1/2)*x+3/128*d^4*b^4*(c*x^2+b*x+a
)^(3/2)*x*a+9/128*d^4*b^4*(c*x^2+b*x+a)^(1/2)*x*a^2-3/512*d^4/c^2*b^7*(c*x^
2+b*x+a)^(1/2)*a+12/5*d^4*c^2*b*x^2*(c*x^2+b*x+a)^(7/2)-3/10*d^4*c*b*a*(c*x
^2+b*x+a)^(7/2)-15/64*d^4*c^(1/2)*b^2*a^4*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x
+a)^(1/2))+15/128*d^4*b^4/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2
))*a^3-15/512*d^4*b^6/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a
^2+15/4096*d^4*b^8/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+1/
10*d^4*c^2*a^2*x*(c*x^2+b*x+a)^(5/2)+1/20*d^4*c*a^2*(c*x^2+b*x+a)^(5/2)*b+1
/8*d^4*c^2*a^3*(c*x^2+b*x+a)^(3/2)*x+1/16*d^4*c*a^3*(c*x^2+b*x+a)^(3/2)*b+3
/16*d^4*c^2*a^4*(c*x^2+b*x+a)^(1/2)*x+3/32*d^4*c*a^4*(c*x^2+b*x+a)^(1/2)*b+
9/256*d^4/c*b^5*(c*x^2+b*x+a)^(1/2)*a^2+11/40*d^4*b^3*(c*x^2+b*x+a)^(7/2)-3
/32*d^4*b^3*a^3*(c*x^2+b*x+a)^(1/2)+1/160*d^4*b^4*x*(c*x^2+b*x+a)^(5/2)-3/1
6384*d^4*b^10/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+3/16*d^4*
c^(3/2)*a^5*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+8/5*d^4*c^3*x^3*(c*
x^2+b*x+a)^(7/2)+1/320*d^4/c*b^5*(c*x^2+b*x+a)^(5/2)-1/1024*d^4/c^2*b^7*(c*
x^2+b*x+a)^(3/2)+3/8192*d^4/c^3*b^9*(c*x^2+b*x+a)^(1/2)-1/40*d^4*b^3*a*(c*x
^2+b*x+a)^(5/2)-3/64*d^4*b^3*a^2*(c*x^2+b*x+a)^(3/2)-1/20*d^4*c*b^2*a*x*(c*
x^2+b*x+a)^(5/2)-3/256*d^4/c*b^6*(c*x^2+b*x+a)^(1/2)*x*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4*(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.82941, size = 2192, normalized size = 8.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4*(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/163840*(15*(b^10 - 20*a*b^8*c + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 128
0*a^4*b^2*c^4 - 1024*a^5*c^5)*sqrt(c)*d^4*log(-8*c^2*x^2 - 8*b*c*x - b^2 -
4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(65536*c^10*d^4*x^
9 + 294912*b*c^9*d^4*x^8 + 6144*(89*b^2*c^8 + 28*a*c^9)*d^4*x^7 + 21504*(25
*b^3*c^7 + 28*a*b*c^8)*d^4*x^6 + 256*(1165*b^4*c^6 + 3280*a*b^2*c^7 + 496*a
^2*c^8)*d^4*x^5 + 128*(701*b^5*c^5 + 4640*a*b^3*c^6 + 2480*a^2*b*c^7)*d^4*x
^4 + 16*(731*b^6*c^4 + 13660*a*b^4*c^5 + 19600*a^2*b^2*c^6 + 320*a^3*c^7)*d
^4*x^3 + 8*(b^7*c^3 + 4372*a*b^5*c^4 + 19120*a^2*b^3*c^5 + 960*a^3*b*c^6)*d
^4*x^2 - 2*(5*b^8*c^2 - 88*a*b^6*c^3 - 16960*a^2*b^4*c^4 - 5760*a^3*b^2*c^5
 + 3840*a^4*c^6)*d^4*x + (15*b^9*c - 280*a*b^7*c^2 + 2048*a^2*b^5*c^3 + 448
```

```
0*a^3*b^3*c^4 - 3840*a^4*b*c^5)*d^4)*sqrt(c*x^2 + b*x + a))/c^4, 1/81920*(1
5*(b^10 - 20*a*b^8*c + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4
 - 1024*a^5*c^5)*sqrt(-c)*d^4*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*
sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(65536*c^10*d^4*x^9 + 294912*b*c^9*d^
4*x^8 + 6144*(89*b^2*c^8 + 28*a*c^9)*d^4*x^7 + 21504*(25*b^3*c^7 + 28*a*b*c
^8)*d^4*x^6 + 256*(1165*b^4*c^6 + 3280*a*b^2*c^7 + 496*a^2*c^8)*d^4*x^5 + 1
28*(701*b^5*c^5 + 4640*a*b^3*c^6 + 2480*a^2*b*c^7)*d^4*x^4 + 16*(731*b^6*c^
4 + 13660*a*b^4*c^5 + 19600*a^2*b^2*c^6 + 320*a^3*c^7)*d^4*x^3 + 8*(b^7*c^3
 + 4372*a*b^5*c^4 + 19120*a^2*b^3*c^5 + 960*a^3*b*c^6)*d^4*x^2 - 2*(5*b^8*c
^2 - 88*a*b^6*c^3 - 16960*a^2*b^4*c^4 - 5760*a^3*b^2*c^5 + 3840*a^4*c^6)*d^
4*x + (15*b^9*c - 280*a*b^7*c^2 + 2048*a^2*b^5*c^3 + 4480*a^3*b^3*c^4 - 384
0*a^4*b*c^5)*d^4)*sqrt(c*x^2 + b*x + a))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^4\left(\int a^2b^4\sqrt{a+bx+cx^2}\,dx+\int b^6x^2\sqrt{a+bx+cx^2}\,dx+\int 16c^6x^8\sqrt{a+bx+cx^2}\,dx+\int 2ab^5x\sqrt{a+bx+cx^2}\,dx+\int 3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**4*(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] d**4*(Integral(a**2*b**4*sqrt(a + b*x + c*x**2), x) + Integral(b**6*x**2*sq
rt(a + b*x + c*x**2), x) + Integral(16*c**6*x**8*sqrt(a + b*x + c*x**2), x)
 + Integral(2*a*b**5*x*sqrt(a + b*x + c*x**2), x) + Integral(32*a*c**5*x**6
*sqrt(a + b*x + c*x**2), x) + Integral(16*a**2*c**4*x**4*sqrt(a + b*x + c*x
**2), x) + Integral(64*b*c**5*x**7*sqrt(a + b*x + c*x**2), x) + Integral(10
4*b**2*c**4*x**6*sqrt(a + b*x + c*x**2), x) + Integral(88*b**3*c**3*x**5*sq
rt(a + b*x + c*x**2), x) + Integral(41*b**4*c**2*x**4*sqrt(a + b*x + c*x**2
), x) + Integral(10*b**5*c*x**3*sqrt(a + b*x + c*x**2), x) + Integral(96*a*
b*c**4*x**5*sqrt(a + b*x + c*x**2), x) + Integral(112*a*b**2*c**3*x**4*sqrt
(a + b*x + c*x**2), x) + Integral(64*a*b**3*c**2*x**3*sqrt(a + b*x + c*x**2
), x) + Integral(18*a*b**4*c*x**2*sqrt(a + b*x + c*x**2), x) + Integral(32*
a**2*b*c**3*x**3*sqrt(a + b*x + c*x**2), x) + Integral(24*a**2*b**2*c**2*x*
*2*sqrt(a + b*x + c*x**2), x) + Integral(8*a**2*b**3*c*x*sqrt(a + b*x + c*x
**2), x))
```

Giac [B] time = 1.20954, size = 738, normalized size = 2.96

$$\frac{1}{40960}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(2\left(4\left(2\left(16\left(2c^6d^4x+9bc^5d^4\right)x+\frac{3\left(89b^2c^{13}d^4+28ac^{14}d^4\right)}{c^9}\right)x+\frac{21\left(25b^3c^{12}d^4+28ab\right.}{c^9}\right.\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4*(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/40960*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(2*(4*(2*(16*(2*c^6*d^4*x + 9*b*c
^5*d^4)*x + 3*(89*b^2*c^13*d^4 + 28*a*c^14*d^4)/c^9)*x + 21*(25*b^3*c^12*d^
4 + 28*a*b*c^13*d^4)/c^9)*x + (1165*b^4*c^11*d^4 + 3280*a*b^2*c^12*d^4 + 49
6*a^2*c^13*d^4)/c^9)*x + (701*b^5*c^10*d^4 + 4640*a*b^3*c^11*d^4 + 2480*a^2
*b*c^12*d^4)/c^9)*x + (731*b^6*c^9*d^4 + 13660*a*b^4*c^10*d^4 + 19600*a^2*b
^2*c^11*d^4 + 320*a^3*c^12*d^4)/c^9)*x + (b^7*c^8*d^4 + 4372*a*b^5*c^9*d^4
+ 19120*a^2*b^3*c^10*d^4 + 960*a^3*b*c^11*d^4)/c^9)*x - (5*b^8*c^7*d^4 - 88
*a*b^6*c^8*d^4 - 16960*a^2*b^4*c^9*d^4 - 5760*a^3*b^2*c^10*d^4 + 3840*a^4*c
```

```
^11*d^4)/c^9)*x + (15*b^9*c^6*d^4 - 280*a*b^7*c^7*d^4 + 2048*a^2*b^5*c^8*d^
4 + 4480*a^3*b^3*c^9*d^4 - 3840*a^4*b*c^10*d^4)/c^9) + 3/16384*(b^10*d^4 -
20*a*b^8*c*d^4 + 160*a^2*b^6*c^2*d^4 - 640*a^3*b^4*c^3*d^4 + 1280*a^4*b^2*c
^4*d^4 - 1024*a^5*c^5*d^4)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*s
qrt(c) - b))/c^(7/2)
```

3.1219 $\int (bd+2cdx)^3 \left(a+bx+cx^2\right)^{5/2} \, dx$

Optimal. Leaf size=59

$$\frac{4}{63}d^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}+\frac{2}{9}d^3(b+2cx)^2\left(a+bx+cx^2\right)^{7/2}$$

```
[Out] (4*(b^2 - 4*a*c)*d^3*(a + b*x + c*x^2)^(7/2))/63 + (2*d^3*(b + 2*c*x)^2*(a
+ b*x + c*x^2)^(7/2))/9
```

Rubi [A] time = 0.0264323, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {692, 629}

$$\frac{4}{63}d^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}+\frac{2}{9}d^3(b+2cx)^2\left(a+bx+cx^2\right)^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^3*(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (4*(b^2 - 4*a*c)*d^3*(a + b*x + c*x^2)^(7/2))/63 + (2*d^3*(b + 2*c*x)^2*(a
+ b*x + c*x^2)^(7/2))/9
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (bd+2cdx)^3\left(a+bx+cx^2\right)^{5/2}\,dx &= \frac{2}{9}d^3(b+2cx)^2\left(a+bx+cx^2\right)^{7/2}+\frac{1}{9}\left(2\left(b^2-4ac\right)d^2\right)\int (bd+2cdx)\left(a+bx+cx^2\right)\\ &= \frac{4}{63}\left(b^2-4ac\right)d^3\left(a+bx+cx^2\right)^{7/2}+\frac{2}{9}d^3(b+2cx)^2\left(a+bx+cx^2\right)^{7/2}\end{aligned}$$

Mathematica [A] time = 0.0614551, size = 44, normalized size = 0.75

$$\frac{2}{63}d^3(a+x(b+cx))^{7/2}\left(4c\left(7cx^2-2a\right)+9b^2+28bcx\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^3*(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*d^3*(a + x*(b + c*x))^(7/2)*(9*b^2 + 28*b*c*x + 4*c*(-2*a + 7*c*x^2)))/6
3
```

Maple [A] time = 0.044, size = 41, normalized size = 0.7

$$-\frac{\left(-56\,c^2x^2-56\,bcx+16\,ac-18\,b^2\right)d^3}{63}\left(cx^2+bx+a\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^3*(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] -2/63*(c*x^2+b*x+a)^(7/2)*(-28*c^2*x^2-28*b*c*x+8*a*c-9*b^2)*d^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3*(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.11147, size = 467, normalized size = 7.92

$$\frac{2}{63}\left(28\,c^5d^3x^8+112\,bc^4d^3x^7+\left(177\,b^2c^3+76\,ac^4\right)d^3x^6+\left(139\,b^3c^2+228\,abc^3\right)d^3x^5+5\left(11\,b^4c+51\,ab^2c^2+12\,a^2c^3\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3*(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/63*(28*c^5*d^3*x^8 + 112*b*c^4*d^3*x^7 + (177*b^2*c^3 + 76*a*c^4)*d^3*x^6
 + (139*b^3*c^2 + 228*a*b*c^3)*d^3*x^5 + 5*(11*b^4*c + 51*a*b^2*c^2 + 12*a^
2*c^3)*d^3*x^4 + (9*b^5 + 130*a*b^3*c + 120*a^2*b*c^2)*d^3*x^3 + (27*a*b^4
+ 87*a^2*b^2*c + 4*a^3*c^2)*d^3*x^2 + (27*a^2*b^3 + 4*a^3*b*c)*d^3*x + (9*a
^3*b^2 - 8*a^4*c)*d^3)*sqrt(c*x^2 + b*x + a)
```

Sympy [B] time = 7.86453, size = 559, normalized size = 9.47

$$-\frac{16a^4cd^3\sqrt{a+bx+cx^2}}{63}+\frac{2a^3b^2d^3\sqrt{a+bx+cx^2}}{7}+\frac{8a^3bcd^3x\sqrt{a+bx+cx^2}}{63}+\frac{8a^3c^2d^3x^2\sqrt{a+bx+cx^2}}{63}+\frac{6a^2b^3d^3x\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**3*(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] -16*a**4*c*d**3*sqrt(a + b*x + c*x**2)/63 + 2*a**3*b**2*d**3*sqrt(a + b*x +
 c*x**2)/7 + 8*a**3*b*c*d**3*x*sqrt(a + b*x + c*x**2)/63 + 8*a**3*c**2*d**3
*x**2*sqrt(a + b*x + c*x**2)/63 + 6*a**2*b**3*d**3*x*sqrt(a + b*x + c*x**2)
/7 + 58*a**2*b**2*c*d**3*x**2*sqrt(a + b*x + c*x**2)/21 + 80*a**2*b*c**2*d*
*3*x**3*sqrt(a + b*x + c*x**2)/21 + 40*a**2*c**3*d**3*x**4*sqrt(a + b*x + c
*x**2)/21 + 6*a*b**4*d**3*x**2*sqrt(a + b*x + c*x**2)/7 + 260*a*b**3*c*d**3
*x**3*sqrt(a + b*x + c*x**2)/63 + 170*a*b**2*c**2*d**3*x**4*sqrt(a + b*x +
c*x**2)/21 + 152*a*b*c**3*d**3*x**5*sqrt(a + b*x + c*x**2)/21 + 152*a*c**4*
d**3*x**6*sqrt(a + b*x + c*x**2)/63 + 2*b**5*d**3*x**3*sqrt(a + b*x + c*x**
2)/7 + 110*b**4*c*d**3*x**4*sqrt(a + b*x + c*x**2)/63 + 278*b**3*c**2*d**3*
x**5*sqrt(a + b*x + c*x**2)/63 + 118*b**2*c**3*d**3*x**6*sqrt(a + b*x + c*x
**2)/21 + 32*b*c**4*d**3*x**7*sqrt(a + b*x + c*x**2)/9 + 8*c**5*d**3*x**8*s
qrt(a + b*x + c*x**2)/9
```

Giac [B] time = 1.17485, size = 389, normalized size = 6.59

$$\frac{2}{63}\sqrt{cx^2+bx+a}\left(\left(\left(\left(\left(\left(\left(28\left(c^5d^3x+4\,bc^4d^3\right)x+\frac{177\,b^2c^{11}d^3+76\,ac^{12}d^3}{c^8}\right)x+\frac{139\,b^3c^{10}d^3+228\,abc^{11}d^3}{c^8}\right)x+\frac{5\left(11\,b^4c^9d\right.}{}\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3*(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/63*sqrt(c*x^2 + b*x + a)*(((((((28*(c^5*d^3*x + 4*b*c^4*d^3)*x + (177*b^2
*c^11*d^3 + 76*a*c^12*d^3)/c^8)*x + (139*b^3*c^10*d^3 + 228*a*b*c^11*d^3)/c
^8)*x + 5*(11*b^4*c^9*d^3 + 51*a*b^2*c^10*d^3 + 12*a^2*c^11*d^3)/c^8)*x + (
9*b^5*c^8*d^3 + 130*a*b^3*c^9*d^3 + 120*a^2*b*c^10*d^3)/c^8)*x + (27*a*b^4*
c^8*d^3 + 87*a^2*b^2*c^9*d^3 + 4*a^3*c^10*d^3)/c^8)*x + (27*a^2*b^3*c^8*d^3
 + 4*a^3*b*c^9*d^3)/c^8)*x + (9*a^3*b^2*c^8*d^3 - 8*a^4*c^9*d^3)/c^8)
```

3.1220 $\int (bd + 2cdx)^2 \left(a + bx + cx^2\right)^{5/2} dx$

Optimal. Leaf size=207

$$-\frac{5d^2\left(b^2-4ac\right)^3(b+2cx)\sqrt{a+bx+cx^2}}{4096c^3}+\frac{5d^2\left(b^2-4ac\right)^2(b+2cx)^3\sqrt{a+bx+cx^2}}{2048c^3}-\frac{5d^2\left(b^2-4ac\right)(b+2cx)^3\left(a+b\right.}{384c^2}$$

```
[Out] (-5*(b^2 - 4*a*c)^3*d^2*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4096*c^3) + (5*
(b^2 - 4*a*c)^2*d^2*(b + 2*c*x)^3*Sqrt[a + b*x + c*x^2])/(2048*c^3) - (5*(b
^2 - 4*a*c)*d^2*(b + 2*c*x)^3*(a + b*x + c*x^2)^(3/2))/(384*c^2) + (d^2*(b
+ 2*c*x)^3*(a + b*x + c*x^2)^(5/2))/(16*c) - (5*(b^2 - 4*a*c)^4*d^2*ArcTanh
[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8192*c^(7/2))
```

Rubi [A] time = 0.124275, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {685, 692, 621, 206}

$$-\frac{5d^2\left(b^2-4ac\right)^3(b+2cx)\sqrt{a+bx+cx^2}}{4096c^3}+\frac{5d^2\left(b^2-4ac\right)^2(b+2cx)^3\sqrt{a+bx+cx^2}}{2048c^3}-\frac{5d^2\left(b^2-4ac\right)(b+2cx)^3\left(a+b\right.}{384c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^2*(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-5*(b^2 - 4*a*c)^3*d^2*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4096*c^3) + (5*
(b^2 - 4*a*c)^2*d^2*(b + 2*c*x)^3*Sqrt[a + b*x + c*x^2])/(2048*c^3) - (5*(b
^2 - 4*a*c)*d^2*(b + 2*c*x)^3*(a + b*x + c*x^2)^(3/2))/(384*c^2) + (d^2*(b
+ 2*c*x)^3*(a + b*x + c*x^2)^(5/2))/(16*c) - (5*(b^2 - 4*a*c)^4*d^2*ArcTanh
[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8192*c^(7/2))
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (bd+2cdx)^2\left(a+bx+cx^2\right)^{5/2}\,dx=\frac{d^2(b+2cx)^3\left(a+bx+cx^2\right)^{5/2}}{16c}-\frac{\left(5\left(b^2-4ac\right)\right)\int (bd+2cdx)^2\left(a+bx+cx^2\right)^{3/2}\,dx}{32c}$$

$$=-\frac{5\left(b^2-4ac\right)d^2(b+2cx)^3\left(a+bx+cx^2\right)^{3/2}}{384c^2}+\frac{d^2(b+2cx)^3\left(a+bx+cx^2\right)^{5/2}}{16c}+\frac{\left(5\right.}{}$$

$$=\frac{5\left(b^2-4ac\right)^2d^2(b+2cx)^3\sqrt{a+bx+cx^2}}{2048c^3}-\frac{5\left(b^2-4ac\right)d^2(b+2cx)^3\left(a+bx+cx^2\right.}{384c^2}$$

$$=-\frac{5\left(b^2-4ac\right)^3d^2(b+2cx)\sqrt{a+bx+cx^2}}{4096c^3}+\frac{5\left(b^2-4ac\right)^2d^2(b+2cx)^3\sqrt{a+bx+c}}{2048c^3}$$

$$=-\frac{5\left(b^2-4ac\right)^3d^2(b+2cx)\sqrt{a+bx+cx^2}}{4096c^3}+\frac{5\left(b^2-4ac\right)^2d^2(b+2cx)^3\sqrt{a+bx+c}}{2048c^3}$$

$$=-\frac{5\left(b^2-4ac\right)^3d^2(b+2cx)\sqrt{a+bx+cx^2}}{4096c^3}+\frac{5\left(b^2-4ac\right)^2d^2(b+2cx)^3\sqrt{a+bx+c}}{2048c^3}$$

Mathematica [A] time = 0.989312, size = 225, normalized size = 1.09

$$\frac{1}{4}d^2(b+2cx)\sqrt{a+x(b+cx)}\left(\frac{\left(b^2-4ac\right)\left(16c^2\left(33a^2+26acx^2+8c^2x^4\right)+8b^2c\left(11cx^2-20a\right)+32bc^2x\left(13a+8cx^2\right)-40b\right.}{3072c^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^2*(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (d^2*(b + 2*c*x)*Sqrt[a + x*(b + c*x)]*((a + x*(b + c*x))^3 + ((b^2 - 4*a*c
)*(15*b^4 - 40*b^3*c*x + 32*b*c^2*x*(13*a + 8*c*x^2) + 8*b^2*c*(-20*a + 11*
c*x^2) + 16*c^2*(33*a^2 + 26*a*c*x^2 + 8*c^2*x^4)))/(3072*c^3) - (5*Sqrt[4*
a - b^2/c]*Sqrt[c]*(a + x*(b + c*x))^3*ArcSinh[(b + 2*c*x)/(Sqrt[4*a - b^2/
c]*Sqrt[c])])/(2048*(b + 2*c*x)*((c*(a + x*(b + c*x)))/(-b^2 + 4*a*c))^(7/2
))))/4
```

Maple [B] time = 0.049, size = 634, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^2*(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 15/128*d^2*b^2*(c*x^2+b*x+a)^(1/2)*x*a^2+5/96*d^2*b^2*(c*x^2+b*x+a)^(3/2)*x
*a+15/256*d^2/c*b^3*(c*x^2+b*x+a)^(1/2)*a^2-15/1024*d^2/c^2*b^5*(c*x^2+b*x+
a)^(1/2)*a+5/2048*d^2/c^2*b^6*(c*x^2+b*x+a)^(1/2)*x-1/12*d^2*c*a*x*(c*x^2+b
*x+a)^(5/2)-5/48*d^2*c*a^2*(c*x^2+b*x+a)^(3/2)*x-5/32*d^2*c*a^3*(c*x^2+b*x+
a)^(1/2)*x-5/768*d^2/c*b^4*(c*x^2+b*x+a)^(3/2)*x+5/192*d^2/c*b^3*(c*x^2+b*x
+a)^(3/2)*a-15/512*d^2/c*b^4*(c*x^2+b*x+a)^(1/2)*x*a-5/1536*d^2/c^2*b^5*(c*
x^2+b*x+a)^(3/2)+5/4096*d^2/c^3*b^7*(c*x^2+b*x+a)^(1/2)+1/96*d^2/c*b^3*(c*x
^2+b*x+a)^(5/2)+1/2*d^2*c*x*(c*x^2+b*x+a)^(7/2)-1/24*d^2*a*(c*x^2+b*x+a)^(5
/2)*b-5/96*d^2*a^2*(c*x^2+b*x+a)^(3/2)*b-5/64*d^2*a^3*(c*x^2+b*x+a)^(1/2)*b
-5/32*d^2*c^(1/2)*a^4*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-5/8192*d^
2*b^8/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/48*d^2*b^2*x*(c
*x^2+b*x+a)^(5/2)+1/4*d^2*b*(c*x^2+b*x+a)^(7/2)+5/32*d^2*b^2/c^(1/2)*ln((1/
2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^3-15/256*d^2*b^4/c^(3/2)*ln((1/2*b+
c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+5/512*d^2*b^6/c^(5/2)*ln((1/2*b+c*x)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2*(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.12294, size = 1569, normalized size = 7.58

$$\left[\frac{15\left(b^8-16\,ab^6c+96\,a^2b^4c^2-256\,a^3b^2c^3+256\,a^4c^4\right)\sqrt{c}d^2\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2*(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/49152*(15*(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4
*c^4)*sqrt(c)*d^2*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*
(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(6144*c^8*d^2*x^7 + 21504*b*c^7*d^2*x^6 +
256*(109*b^2*c^6 + 68*a*c^7)*d^2*x^5 + 640*(25*b^3*c^5 + 68*a*b*c^6)*d^2*x^
4 + 16*(219*b^4*c^4 + 2248*a*b^2*c^5 + 944*a^2*c^6)*d^2*x^3 + 8*(b^5*c^3 +
1304*a*b^3*c^4 + 2832*a^2*b*c^5)*d^2*x^2 - 2*(5*b^6*c^2 - 68*a*b^4*c^3 - 49
44*a^2*b^2*c^4 - 960*a^3*c^5)*d^2*x + (15*b^7*c - 220*a*b^5*c^2 + 1168*a^2*
b^3*c^3 + 960*a^3*b*c^4)*d^2)*sqrt(c*x^2 + b*x + a))/c^4, 1/24576*(15*(b^8
- 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4)*sqrt(-c)*d^2
*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a
*c)) + 2*(6144*c^8*d^2*x^7 + 21504*b*c^7*d^2*x^6 + 256*(109*b^2*c^6 + 68*a*
c^7)*d^2*x^5 + 640*(25*b^3*c^5 + 68*a*b*c^6)*d^2*x^4 + 16*(219*b^4*c^4 + 22
48*a*b^2*c^5 + 944*a^2*c^6)*d^2*x^3 + 8*(b^5*c^3 + 1304*a*b^3*c^4 + 2832*a^
2*b*c^5)*d^2*x^2 - 2*(5*b^6*c^2 - 68*a*b^4*c^3 - 4944*a^2*b^2*c^4 - 960*a^3
*c^5)*d^2*x + (15*b^7*c - 220*a*b^5*c^2 + 1168*a^2*b^3*c^3 + 960*a^3*b*c^4)
```

```
*d^2)*sqrt(c*x^2 + b*x + a))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int a^2b^2\sqrt{a+bx+cx^2}\,dx+\int b^4x^2\sqrt{a+bx+cx^2}\,dx+\int 4c^4x^6\sqrt{a+bx+cx^2}\,dx+\int 2ab^3x\sqrt{a+bx+cx^2}\,dx+\int 8a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**2*(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] d**2*(Integral(a**2*b**2*sqrt(a + b*x + c*x**2), x) + Integral(b**4*x**2*sq
rt(a + b*x + c*x**2), x) + Integral(4*c**4*x**6*sqrt(a + b*x + c*x**2), x)
+ Integral(2*a*b**3*x*sqrt(a + b*x + c*x**2), x) + Integral(8*a*c**3*x**4*s
qrt(a + b*x + c*x**2), x) + Integral(4*a**2*c**2*x**2*sqrt(a + b*x + c*x**2
), x) + Integral(12*b*c**3*x**5*sqrt(a + b*x + c*x**2), x) + Integral(13*b*
*2*c**2*x**4*sqrt(a + b*x + c*x**2), x) + Integral(6*b**3*c*x**3*sqrt(a + b
*x + c*x**2), x) + Integral(16*a*b*c**2*x**3*sqrt(a + b*x + c*x**2), x) + I
ntegral(10*a*b**2*c*x**2*sqrt(a + b*x + c*x**2), x) + Integral(4*a**2*b*c*x
*sqrt(a + b*x + c*x**2), x))
```

Giac [B] time = 1.21522, size = 525, normalized size = 2.54

$$\frac{1}{12288}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(2\left(12\left(2\,c^4d^2x+7\,bc^3d^2\right)x+\frac{109\,b^2c^9d^2+68\,ac^{10}d^2}{c^7}\right)x+\frac{5\left(25\,b^3c^8d^2+68\,abc^9d^2\right)}{c^7}\right)x+\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2*(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/12288*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(2*(12*(2*c^4*d^2*x + 7*b*c^3*d^2
)*x + (109*b^2*c^9*d^2 + 68*a*c^10*d^2)/c^7)*x + 5*(25*b^3*c^8*d^2 + 68*a*b
*c^9*d^2)/c^7)*x + (219*b^4*c^7*d^2 + 2248*a*b^2*c^8*d^2 + 944*a^2*c^9*d^2)
/c^7)*x + (b^5*c^6*d^2 + 1304*a*b^3*c^7*d^2 + 2832*a^2*b*c^8*d^2)/c^7)*x -
(5*b^6*c^5*d^2 - 68*a*b^4*c^6*d^2 - 4944*a^2*b^2*c^7*d^2 - 960*a^3*c^8*d^2)
/c^7)*x + (15*b^7*c^4*d^2 - 220*a*b^5*c^5*d^2 + 1168*a^2*b^3*c^6*d^2 + 960*
a^3*b*c^7*d^2)/c^7) + 5/8192*(b^8*d^2 - 16*a*b^6*c*d^2 + 96*a^2*b^4*c^2*d^2
 - 256*a^3*b^2*c^3*d^2 + 256*a^4*c^4*d^2)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

3.1221 $\int (bd+2cdx)\left(a+bx+cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=19

$$\frac{2}{7}d\left(a+bx+cx^2\right)^{7/2}$$

```
[Out] (2*d*(a + b*x + c*x^2)^(7/2))/7
```

Rubi [A] time = 0.0066538, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {629}

$$\frac{2}{7}d\left(a+bx+cx^2\right)^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)*(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*d*(a + b*x + c*x^2)^(7/2))/7
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (bd+2cdx)\left(a+bx+cx^2\right)^{5/2}\,dx=\frac{2}{7}d\left(a+bx+cx^2\right)^{7/2}$$

Mathematica [A] time = 0.0127724, size = 18, normalized size = 0.95

$$\frac{2}{7}d(a+x(b+cx))^{7/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)*(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*d*(a + x*(b + c*x))^(7/2))/7
```

Maple [A] time = 0.041, size = 16, normalized size = 0.8

$$\frac{2d}{7}\left(cx^2+bx+a\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)*(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 2/7*d*(c*x^2+b*x+a)^(7/2)
```

Maxima [A] time = 1.09422, size = 20, normalized size = 1.05

$$\frac{2}{7}\left(cx^2+bx+a\right)^{\frac{7}{2}}d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)*(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*(c*x^2 + b*x + a)^(7/2)*d
```

Fricas [B] time = 2.8411, size = 207, normalized size = 10.89

$$\frac{2}{7}\left(c^3dx^6+3\,bc^2dx^5+3\left(b^2c+ac^2\right)dx^4+3\,a^2bdx+\left(b^3+6\,abc\right)dx^3+a^3d+3\left(ab^2+a^2c\right)dx^2\right)\sqrt{cx^2+bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)*(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7*(c^3*d*x^6 + 3*b*c^2*d*x^5 + 3*(b^2*c + a*c^2)*d*x^4 + 3*a^2*b*d*x + (b
^3 + 6*a*b*c)*d*x^3 + a^3*d + 3*(a*b^2 + a^2*c)*d*x^2)*sqrt(c*x^2 + b*x + a
)
```

Sympy [B] time = 4.38953, size = 260, normalized size = 13.68

$$\frac{2a^3d\sqrt{a+bx+cx^2}}{7}+\frac{6a^2bdx\sqrt{a+bx+cx^2}}{7}+\frac{6a^2cdx^2\sqrt{a+bx+cx^2}}{7}+\frac{6ab^2dx^2\sqrt{a+bx+cx^2}}{7}+\frac{12abcdx^3\sqrt{a+bx+cx}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)*(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] 2*a**3*d*sqrt(a + b*x + c*x**2)/7 + 6*a**2*b*d*x*sqrt(a + b*x + c*x**2)/7 +
 6*a**2*c*d*x**2*sqrt(a + b*x + c*x**2)/7 + 6*a*b**2*d*x**2*sqrt(a + b*x +
c*x**2)/7 + 12*a*b*c*d*x**3*sqrt(a + b*x + c*x**2)/7 + 6*a*c**2*d*x**4*sqrt
(a + b*x + c*x**2)/7 + 2*b**3*d*x**3*sqrt(a + b*x + c*x**2)/7 + 6*b**2*c*d*
x**4*sqrt(a + b*x + c*x**2)/7 + 6*b*c**2*d*x**5*sqrt(a + b*x + c*x**2)/7 +
2*c**3*d*x**6*sqrt(a + b*x + c*x**2)/7
```

Giac [B] time = 1.19344, size = 161, normalized size = 8.47

$$\frac{2}{7}\left(a^3d+\left(3\,a^2bd+\left(\left(\left(\left(c^3dx+3\,bc^2d\right)x+\frac{3\left(b^2c^7d+ac^8d\right)}{c^6}\right)x+\frac{b^3c^6d+6\,abc^7d}{c^6}\right)x+\frac{3\left(ab^2c^6d+a^2c^7d\right)}{c^6}\right)x\right)x\right)\sqrt{cx^2+bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)*(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/7*(a^3*d + (3*a^2*b*d + ((((c^3*d*x + 3*b*c^2*d)*x + 3*(b^2*c^7*d + a*c^8
*d)/c^6)*x + (b^3*c^6*d + 6*a*b*c^7*d)/c^6)*x + 3*(a*b^2*c^6*d + a^2*c^7*d)
/c^6)*x)*x)*sqrt(c*x^2 + b*x + a)
```

3.1222 $\int \frac{(a+bx+cx^2)^{5/2}}{bd+2cdx} \, dx$

Optimal. Leaf size=149

$$\frac{(b^2-4ac)^2\sqrt{a+bx+cx^2}}{32c^3d}-\frac{(b^2-4ac)(a+bx+cx^2)^{3/2}}{24c^2d}-\frac{(b^2-4ac)^{5/2}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{64c^{7/2}d}+\frac{(a+bx+cx^2)^{5/2}}{10cd}$$

```
[Out] ((b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2])/(32*c^3*d) - ((b^2 - 4*a*c)*(a + b*
x + c*x^2)^(3/2))/(24*c^2*d) + (a + b*x + c*x^2)^(5/2)/(10*c*d) - ((b^2 - 4
*a*c)^(5/2)*ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c]])/(6
4*c^(7/2)*d)
```

Rubi [A] time = 0.12067, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {685, 688, 205}

$$\frac{(b^2-4ac)^2\sqrt{a+bx+cx^2}}{32c^3d}-\frac{(b^2-4ac)(a+bx+cx^2)^{3/2}}{24c^2d}-\frac{(b^2-4ac)^{5/2}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{64c^{7/2}d}+\frac{(a+bx+cx^2)^{5/2}}{10cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x),x]
```

```
[Out] ((b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2])/(32*c^3*d) - ((b^2 - 4*a*c)*(a + b*
x + c*x^2)^(3/2))/(24*c^2*d) + (a + b*x + c*x^2)^(5/2)/(10*c*d) - ((b^2 - 4
*a*c)^(5/2)*ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c]])/(6
4*c^(7/2)*d)
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{5/2}}{bd+2cdx}\,dx = \frac{(a+bx+cx^2)^{5/2}}{10cd} - \frac{(b^2-4ac)\int \frac{(a+bx+cx^2)^{3/2}}{bd+2cdx}\,dx}{4c}$$
$$= -\frac{(b^2-4ac)(a+bx+cx^2)^{3/2}}{24c^2d} + \frac{(a+bx+cx^2)^{5/2}}{10cd} + \frac{(b^2-4ac)^2\int \frac{\sqrt{a+bx+cx^2}}{bd+2cdx}\,dx}{16c^2}$$
$$= \frac{(b^2-4ac)^2\sqrt{a+bx+cx^2}}{32c^3d} - \frac{(b^2-4ac)(a+bx+cx^2)^{3/2}}{24c^2d} + \frac{(a+bx+cx^2)^{5/2}}{10cd} - \frac{(b^2-4ac)^3}{}$$
$$= \frac{(b^2-4ac)^2\sqrt{a+bx+cx^2}}{32c^3d} - \frac{(b^2-4ac)(a+bx+cx^2)^{3/2}}{24c^2d} + \frac{(a+bx+cx^2)^{5/2}}{10cd} - \frac{(b^2-4ac)^3}{}$$
$$= \frac{(b^2-4ac)^2\sqrt{a+bx+cx^2}}{32c^3d} - \frac{(b^2-4ac)(a+bx+cx^2)^{3/2}}{24c^2d} + \frac{(a+bx+cx^2)^{5/2}}{10cd} - \frac{(b^2-4ac)^5}{}$$

Mathematica [A] time = 0.174842, size = 150, normalized size = 1.01

$$\frac{\frac{\sqrt{a+x(b+cx)}\left(16c^2(23a^2+11acx^2+3c^2x^4)+28b^2c(cx^2-5a)+16bc^2x(11a+6cx^2)-20b^3cx+15b^4\right)}{480c^3} - \frac{(b^2-4ac)^{5/2}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+x(b+cx)}}{\sqrt{b^2-4ac}}\right)}{64c^{7/2}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x),x]
```

```
[Out] ((Sqrt[a + x*(b + c*x)]*(15*b^4 - 20*b^3*c*x + 28*b^2*c*(-5*a + c*x^2) + 16
*b*c^2*x*(11*a + 6*c*x^2) + 16*c^2*(23*a^2 + 11*a*c*x^2 + 3*c^2*x^4)))/(480
*c^3) - ((b^2 - 4*a*c)^(5/2)*ArcTan[(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])/Sqrt[
b^2 - 4*a*c]])/(64*c^(7/2)))/d
```

Maple [B] time = 0.19, size = 660, normalized size = 4.4

$$\frac{1}{10\,cd}\left(\left(x+\frac{b}{2\,c}\right)^2 c+\frac{4\,ac-b^2}{4\,c}\right)^{\frac{5}{2}}+\frac{a}{6\,cd}\left(\left(x+\frac{b}{2\,c}\right)^2 c+\frac{4\,ac-b^2}{4\,c}\right)^{\frac{3}{2}}-\frac{b^2}{24\,c^2d}\left(\left(x+\frac{b}{2\,c}\right)^2 c+\frac{4\,ac-b^2}{4\,c}\right)^{\frac{3}{2}}+\frac{a^2}{4\,cd}\sqrt{4\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d),x)
```

```
[Out] 1/10/d/c*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(5/2)+1/6/d/c*((x+1/2*b/c)^2*c
+1/4*(4*a*c-b^2)/c)^(3/2)*a-1/24/d/c^2*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^
(3/2)*b^2+1/4/d/c*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2)*a^2-1/8/d/c^2*(4*
(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2)*a*b^2+1/64/d/c^3*(4*(x+1/2*b/c)^2*c+(4
*a*c-b^2)/c)^(1/2)*b^4-1/d/c/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/
2*((4*a*c-b^2)/c)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c
))*a^3+3/4/d/c^2/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-b^
2)/c)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))*a^2*b^2-3
/16/d/c^3/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^(
1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))*a*b^4+1/64/d/c^4
/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/2)*(4*(
x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))*b^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.46849, size = 852, normalized size = 5.72

$$\left[\frac{15\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-\frac{b^2-4\,ac}{c}}\log\left(-\frac{4\,c^2x^2+4\,bcx-b^2+8\,ac-4\,\sqrt{cx^2+bx+ac}\sqrt{-\frac{b^2-4\,ac}{c}}}{4\,c^2x^2+4\,bcx+b^2}\right)+4\left(48\,c^4x^4+96\,bc^3x^3+15\,b^4-140\,ab\right.}{1920\,c^3d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d),x, algorithm="fricas")
```

```
[Out] [1/1920*(15*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-(b^2 - 4*a*c)/c)*log(-(4*c
^2*x^2 + 4*b*c*x - b^2 + 8*a*c - 4*sqrt(c*x^2 + b*x + a)*c*sqrt(-(b^2 - 4*a
*c)/c))/(4*c^2*x^2 + 4*b*c*x + b^2)) + 4*(48*c^4*x^4 + 96*b*c^3*x^3 + 15*b^
4 - 140*a*b^2*c + 368*a^2*c^2 + 4*(7*b^2*c^2 + 44*a*c^3)*x^2 - 4*(5*b^3*c -
 44*a*b*c^2)*x)*sqrt(c*x^2 + b*x + a))/(c^3*d), 1/960*(15*(b^4 - 8*a*b^2*c
+ 16*a^2*c^2)*sqrt((b^2 - 4*a*c)/c)*arctan(1/2*sqrt((b^2 - 4*a*c)/c)/sqrt(c
*x^2 + b*x + a)) + 2*(48*c^4*x^4 + 96*b*c^3*x^3 + 15*b^4 - 140*a*b^2*c + 36
8*a^2*c^2 + 4*(7*b^2*c^2 + 44*a*c^3)*x^2 - 4*(5*b^3*c - 44*a*b*c^2)*x)*sqrt
(c*x^2 + b*x + a))/(c^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a^2\sqrt{a+bx+cx^2}}{b+2cx}\,dx+\int\frac{b^2x^2\sqrt{a+bx+cx^2}}{b+2cx}\,dx+\int\frac{c^2x^4\sqrt{a+bx+cx^2}}{b+2cx}\,dx+\int\frac{2abx\sqrt{a+bx+cx^2}}{b+2cx}\,dx+\int\frac{2acx^2\sqrt{a+bx+cx^2}}{b+2cx}\,dx+\int\frac{2bcx^3\sqrt{a+bx+cx^2}}{b+2cx}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(2*c*d*x+b*d),x)
```

```
[Out] (Integral(a**2*sqrt(a + b*x + c*x**2)/(b + 2*c*x), x) + Integral(b**2*x**2*
sqrt(a + b*x + c*x**2)/(b + 2*c*x), x) + Integral(c**2*x**4*sqrt(a + b*x +
c*x**2)/(b + 2*c*x), x) + Integral(2*a*b*x*sqrt(a + b*x + c*x**2)/(b + 2*c*
x), x) + Integral(2*a*c*x**2*sqrt(a + b*x + c*x**2)/(b + 2*c*x), x) + Integ
ral(2*b*c*x**3*sqrt(a + b*x + c*x**2)/(b + 2*c*x), x))/d
```

Giac [A] time = 1.18425, size = 320, normalized size = 2.15

$$\frac{1}{480}\sqrt{cx^2+bx+a}\left(4\left(\left(12\left(\frac{cx}{d}+\frac{2b}{d}\right)x+\frac{7b^2c^9d^5+44ac^{10}d^5}{c^{10}d^6}\right)x-\frac{5b^3c^8d^5-44abc^9d^5}{c^{10}d^6}\right)x+\frac{15b^4c^7d^5-140ab^2c^8d^5}{c^{10}d^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d),x, algorithm="giac")
```

```
[Out] 1/480*sqrt(c*x^2 + b*x + a)*(4*((12*(c*x/d + 2*b/d)*x + (7*b^2*c^9*d^5 + 44
*a*c^10*d^5)/(c^10*d^6))*x - (5*b^3*c^8*d^5 - 44*a*b*c^9*d^5)/(c^10*d^6))*x
 + (15*b^4*c^7*d^5 - 140*a*b^2*c^8*d^5 + 368*a^2*c^9*d^5)/(c^10*d^6)) - 1/3
2*(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*arctan(-(2*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))*c + b*sqrt(c))/sqrt(b^2*c - 4*a*c^2))/(sqrt(b^2*c -
4*a*c^2)*c^3*d)
```

3.1223 $\int \frac{(a+bx+cx^2)^{5/2}}{(bd+2cdx)^2}\,dx$

Optimal. Leaf size=153

$$-\frac{15(b^2-4ac)(b+2cx)\sqrt{a+bx+cx^2}}{256c^3d^2}+\frac{15(b^2-4ac)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{512c^{7/2}d^2}+\frac{5(b+2cx)(a+bx+cx^2)^{3/2}}{32c^2d^2}-\frac{(a+bx}{2cd^2(b}$$

```
[Out] (-15*(b^2 - 4*a*c)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(256*c^3*d^2) + (5*(b
 + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(32*c^2*d^2) - (a + b*x + c*x^2)^(5/2)/(
2*c*d^2*(b + 2*c*x)) + (15*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + b*x + c*x^2])])/(512*c^(7/2)*d^2)
```

Rubi [A] time = 0.064322, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {684, 612, 621, 206}

$$-\frac{15(b^2-4ac)(b+2cx)\sqrt{a+bx+cx^2}}{256c^3d^2}+\frac{15(b^2-4ac)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{512c^{7/2}d^2}+\frac{5(b+2cx)(a+bx+cx^2)^{3/2}}{32c^2d^2}-\frac{(a+bx}{2cd^2(b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^2,x]
```

```
[Out] (-15*(b^2 - 4*a*c)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(256*c^3*d^2) + (5*(b
 + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(32*c^2*d^2) - (a + b*x + c*x^2)^(5/2)/(
2*c*d^2*(b + 2*c*x)) + (15*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + b*x + c*x^2])])/(512*c^(7/2)*d^2)
```

Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(bd+2cdx)^2}\,dx = -\frac{\left(a+bx+cx^2\right)^{5/2}}{2cd^2(b+2cx)}+\frac{5\int \left(a+bx+cx^2\right)^{3/2}\,dx}{4cd^2}$$
$$=\frac{5(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{32c^2d^2}-\frac{\left(a+bx+cx^2\right)^{5/2}}{2cd^2(b+2cx)}-\frac{\left(15\left(b^2-4ac\right)\right)\int \sqrt{a+bx+cx^2}\,dx}{64c^2d^2}$$
$$=-\frac{15\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}}{256c^3d^2}+\frac{5(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{32c^2d^2}-\frac{\left(a+bx+cx^2\right)^{5/2}}{2cd^2(b+2cx)}+$$
$$=-\frac{15\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}}{256c^3d^2}+\frac{5(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{32c^2d^2}-\frac{\left(a+bx+cx^2\right)^{5/2}}{2cd^2(b+2cx)}+$$
$$=-\frac{15\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}}{256c^3d^2}+\frac{5(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{32c^2d^2}-\frac{\left(a+bx+cx^2\right)^{5/2}}{2cd^2(b+2cx)}+$$

Mathematica [C] time = 0.0519634, size = 97, normalized size = 0.63

$$-\frac{\left(b^2-4ac\right)^2\sqrt{a+x(b+cx)}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{64c^3d^2(b+2cx)\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^2,x]
```

```
[Out] -((b^2 - 4*a*c)^2*Sqrt[a + x*(b + c*x)]*Hypergeometric2F1[-5/2, -1/2, 1/2,
(b + 2*c*x)^2/(b^2 - 4*a*c)])/(64*c^3*d^2*(b + 2*c*x)*Sqrt[(c*(a + x*(b + c
*x)))/(-b^2 + 4*a*c)])
```

Maple [B] time = 0.192, size = 961, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^2,x)
```

```
[Out] -1/c/d^2/(4*a*c-b^2)/(x+1/2*b/c)*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(7/2)+
1/d^2/(4*a*c-b^2)*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(5/2)*x+1/2/c/d^2/(4*
a*c-b^2)*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(5/2)*b+5/4/d^2/(4*a*c-b^2)*((
x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(3/2)*x*a-5/16/c/d^2/(4*a*c-b^2)*((x+1/2*
b/c)^2*c+1/4*(4*a*c-b^2)/c)^(3/2)*x*b^2+5/8/c/d^2/(4*a*c-b^2)*((x+1/2*b/c)^
2*c+1/4*(4*a*c-b^2)/c)^(3/2)*b*a-5/32/c^2/d^2/(4*a*c-b^2)*((x+1/2*b/c)^2*c+
1/4*(4*a*c-b^2)/c)^(3/2)*b^3+15/8/d^2/(4*a*c-b^2)*((x+1/2*b/c)^2*c+1/4*(4*a
*c-b^2)/c)^(1/2)*x*a^2-15/16/c/d^2/(4*a*c-b^2)*((x+1/2*b/c)^2*c+1/4*(4*a*c-
b^2)/c)^(1/2)*x*a*b^2+15/128/c^2/d^2/(4*a*c-b^2)*((x+1/2*b/c)^2*c+1/4*(4*a*
c-b^2)/c)^(1/2)*x*b^4+15/16/c/d^2/(4*a*c-b^2)*((x+1/2*b/c)^2*c+1/4*(4*a*c-b
^2)/c)^(1/2)*b*a^2-15/32/c^2/d^2/(4*a*c-b^2)*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^
2)/c)^(1/2)*b^3*a+15/256/c^3/d^2/(4*a*c-b^2)*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^
```

```
2)/c)^(1/2)*b^5+15/8/c^(1/2)/d^2/(4*a*c-b^2)*ln((x+1/2*b/c)*c^(1/2)+((x+1/2
*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2))*a^3-45/32/c^(3/2)/d^2/(4*a*c-b^2)*ln((x
+1/2*b/c)*c^(1/2)+((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2))*b^2*a^2+45/128
/c^(5/2)/d^2/(4*a*c-b^2)*ln((x+1/2*b/c)*c^(1/2)+((x+1/2*b/c)^2*c+1/4*(4*a*c
-b^2)/c)^(1/2))*b^4*a-15/512/c^(7/2)/d^2/(4*a*c-b^2)*ln((x+1/2*b/c)*c^(1/2)
+((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2))*b^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.96526, size = 977, normalized size = 6.39

$$\left[\frac{15\left(b^5-8\,ab^3c+16\,a^2bc^2+2\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)x\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\right.}{1024\left(2\,c^5d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^2,x, algorithm="fricas")
```

```
[Out] [1/1024*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 2*(b^4*c - 8*a*b^2*c^2 + 16*a
^2*c^3)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)
*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(32*c^5*x^4 + 64*b*c^4*x^3 - 15*b^4*c + 1
00*a*b^2*c^2 - 128*a^2*c^3 + 12*(b^2*c^3 + 12*a*c^4)*x^2 - 4*(5*b^3*c^2 - 3
6*a*b*c^3)*x)*sqrt(c*x^2 + b*x + a))/(2*c^5*d^2*x + b*c^4*d^2), -1/512*(15*
(b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x)*s
qrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*
c*x + a*c)) - 2*(32*c^5*x^4 + 64*b*c^4*x^3 - 15*b^4*c + 100*a*b^2*c^2 - 128
*a^2*c^3 + 12*(b^2*c^3 + 12*a*c^4)*x^2 - 4*(5*b^3*c^2 - 36*a*b*c^3)*x)*sqrt
(c*x^2 + b*x + a))/(2*c^5*d^2*x + b*c^4*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a^2\sqrt{a+bx+cx^2}}{b^2+4bcx+4c^2x^2}\,dx+\int\frac{b^2x^2\sqrt{a+bx+cx^2}}{b^2+4bcx+4c^2x^2}\,dx+\int\frac{c^2x^4\sqrt{a+bx+cx^2}}{b^2+4bcx+4c^2x^2}\,dx+\int\frac{2abx\sqrt{a+bx+cx^2}}{b^2+4bcx+4c^2x^2}\,dx+\int\frac{2acx^2\sqrt{a+bx+cx^2}}{b^2+4bcx+4c^2x^2}\,dx+\int\frac{2bcx^3\sqrt{a+bx+cx^2}}{b^2+4bcx+4c^2x^2}\,d}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(2*c*d*x+b*d)**2,x)
```

```
[Out] (Integral(a**2*sqrt(a + b*x + c*x**2)/(b**2 + 4*b*c*x + 4*c**2*x**2), x) +
Integral(b**2*x**2*sqrt(a + b*x + c*x**2)/(b**2 + 4*b*c*x + 4*c**2*x**2), x
) + Integral(c**2*x**4*sqrt(a + b*x + c*x**2)/(b**2 + 4*b*c*x + 4*c**2*x**2
```

```
), x) + Integral(2*a*b*x*sqrt(a + b*x + c*x**2)/(b**2 + 4*b*c*x + 4*c**2*x*
*2), x) + Integral(2*a*c*x**2*sqrt(a + b*x + c*x**2)/(b**2 + 4*b*c*x + 4*c*
*2*x**2), x) + Integral(2*b*c*x**3*sqrt(a + b*x + c*x**2)/(b**2 + 4*b*c*x +
 4*c**2*x**2), x))/d**2
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^2,x, algorithm="giac")

[Out] Timed out
```

3.1224 $\int \frac{(a+bx+cx^2)^{5/2}}{(bd+2cdx)^3}\,dx$

Optimal. Leaf size=147

$$-\frac{5\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}{64c^3d^3}+\frac{5\left(b^2-4ac\right)^{3/2}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{128c^{7/2}d^3}+\frac{5\left(a+bx+cx^2\right)^{3/2}}{48c^2d^3}-\frac{\left(a+bx+cx^2\right)^{5/2}}{4cd^3(b+2cx)^2}$$

```
[Out] (-5*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])/(64*c^3*d^3) + (5*(a + b*x + c*x^2
)^(3/2))/(48*c^2*d^3) - (a + b*x + c*x^2)^(5/2)/(4*c*d^3*(b + 2*c*x)^2) + (
5*(b^2 - 4*a*c)^(3/2)*ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4
*a*c]])/(128*c^(7/2)*d^3)
```

Rubi [A] time = 0.111583, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {684, 685, 688, 205}

$$-\frac{5\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}{64c^3d^3}+\frac{5\left(b^2-4ac\right)^{3/2}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{128c^{7/2}d^3}+\frac{5\left(a+bx+cx^2\right)^{3/2}}{48c^2d^3}-\frac{\left(a+bx+cx^2\right)^{5/2}}{4cd^3(b+2cx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^3,x]
```

```
[Out] (-5*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])/(64*c^3*d^3) + (5*(a + b*x + c*x^2
)^(3/2))/(48*c^2*d^3) - (a + b*x + c*x^2)^(5/2)/(4*c*d^3*(b + 2*c*x)^2) + (
5*(b^2 - 4*a*c)^(3/2)*ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4
*a*c]])/(128*c^(7/2)*d^3)
```

Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(bd+2cdx)^3}\,dx = -\frac{\left(a+bx+cx^2\right)^{5/2}}{4cd^3(b+2cx)^2}+\frac{5\int \frac{\left(a+bx+cx^2\right)^{3/2}}{bd+2cdx}\,dx}{8cd^2}$$

$$=\frac{5\left(a+bx+cx^2\right)^{3/2}}{48c^2d^3}-\frac{\left(a+bx+cx^2\right)^{5/2}}{4cd^3(b+2cx)^2}-\frac{\left(5\left(b^2-4ac\right)\right)\int \frac{\sqrt{a+bx+cx^2}}{bd+2cdx}\,dx}{32c^2d^2}$$

$$=-\frac{5\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}{64c^3d^3}+\frac{5\left(a+bx+cx^2\right)^{3/2}}{48c^2d^3}-\frac{\left(a+bx+cx^2\right)^{5/2}}{4cd^3(b+2cx)^2}+\frac{\left(5\left(b^2-4ac\right)^2\right)\int}{128}$$

$$=-\frac{5\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}{64c^3d^3}+\frac{5\left(a+bx+cx^2\right)^{3/2}}{48c^2d^3}-\frac{\left(a+bx+cx^2\right)^{5/2}}{4cd^3(b+2cx)^2}+\frac{\left(5\left(b^2-4ac\right)^2\right)\mathrm{Su}}{}$$

$$=-\frac{5\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}{64c^3d^3}+\frac{5\left(a+bx+cx^2\right)^{3/2}}{48c^2d^3}-\frac{\left(a+bx+cx^2\right)^{5/2}}{4cd^3(b+2cx)^2}+\frac{5\left(b^2-4ac\right)^{3/2}\tan}{128c}$$

Mathematica [C] time = 0.0396225, size = 62, normalized size = 0.42

$$\frac{2(a+x(b+cx))^{7/2}\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};\frac{4c(a+x(b+cx))}{4ac-b^2}\right)}{7d^3\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^3,x]
```

```
[Out] (2*(a + x*(b + c*x))^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, (4*c*(a + x*(b +
c*x)))/(-b^2 + 4*a*c)])/(7*(b^2 - 4*a*c)^2*d^3)
```

Maple [B] time = 0.196, size = 840, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^3,x)
```

```
[Out] -1/4/d^3/c^2/(4*a*c-b^2)/(x+1/2*b/c)^2*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^
(7/2)+1/4/d^3/c/(4*a*c-b^2)*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(5/2)+5/12/
d^3/c/(4*a*c-b^2)*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(3/2)*a-5/48/d^3/c^2/
(4*a*c-b^2)*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(3/2)*b^2+5/8/d^3/c/(4*a*c-
b^2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2)*a^2-5/16/d^3/c^2/(4*a*c-b^2)*(
4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2)*a*b^2+5/128/d^3/c^3/(4*a*c-b^2)*(4*(
x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2)*b^4-5/2/d^3/c/(4*a*c-b^2)/((4*a*c-b^2)/
c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/2)*(4*(x+1/2*b/c)^2*c
+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))*a^3+15/8/d^3/c^2/(4*a*c-b^2)/((4*a*c-b^
2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/2)*(4*(x+1/2*b/c)^
```

```
2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))*a^2*b^2-15/32/d^3/c^3/(4*a*c-b^2)/((
4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/2)*(4*(x+1
/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))*a*b^4+5/128/d^3/c^4/(4*a*c-b
^2)/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/2)*(
4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))*b^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 7.40079, size = 1056, normalized size = 7.18

$$\left[-\frac{15\left(b^4-4\,ab^2c+4\left(b^2c^2-4\,ac^3\right)x^2+4\left(b^3c-4\,abc^2\right)x\right)\sqrt{-\frac{b^2-4\,ac}{c}}\log\left(-\frac{4\,c^2x^2+4\,bcx-b^2+8\,ac-4\,\sqrt{cx^2+bx+a}c\sqrt{-\frac{b^2-4\,ac}{c}}}{4\,c^2x^2+4\,bcx+b^2}\right)-4\left(32\right.}{768\left(4\,c^5d^3x^2+4\,bc^4d^3x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^3,x, algorithm="fricas")
```

```
[Out] [-1/768*(15*(b^4 - 4*a*b^2*c + 4*(b^2*c^2 - 4*a*c^3)*x^2 + 4*(b^3*c - 4*a*b
*c^2)*x)*sqrt(-(b^2 - 4*a*c)/c)*log(-(4*c^2*x^2 + 4*b*c*x - b^2 + 8*a*c - 4
*sqrt(c*x^2 + b*x + a)*c*sqrt(-(b^2 - 4*a*c)/c))/(4*c^2*x^2 + 4*b*c*x + b^2
)) - 4*(32*c^4*x^4 + 64*b*c^3*x^3 - 15*b^4 + 80*a*b^2*c - 48*a^2*c^2 - 8*(b
^2*c^2 - 28*a*c^3)*x^2 - 8*(5*b^3*c - 28*a*b*c^2)*x)*sqrt(c*x^2 + b*x + a))
/(4*c^5*d^3*x^2 + 4*b*c^4*d^3*x + b^2*c^3*d^3), -1/384*(15*(b^4 - 4*a*b^2*c
 + 4*(b^2*c^2 - 4*a*c^3)*x^2 + 4*(b^3*c - 4*a*b*c^2)*x)*sqrt((b^2 - 4*a*c)/
c)*arctan(1/2*sqrt((b^2 - 4*a*c)/c)/sqrt(c*x^2 + b*x + a)) - 2*(32*c^4*x^4
+ 64*b*c^3*x^3 - 15*b^4 + 80*a*b^2*c - 48*a^2*c^2 - 8*(b^2*c^2 - 28*a*c^3)*
x^2 - 8*(5*b^3*c - 28*a*b*c^2)*x)*sqrt(c*x^2 + b*x + a))/(4*c^5*d^3*x^2 + 4
*b*c^4*d^3*x + b^2*c^3*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a^2\sqrt{a+bx+cx^2}}{b^3+6b^2cx+12bc^2x^2+8c^3x^3}\,dx+\int\frac{b^2x^2\sqrt{a+bx+cx^2}}{b^3+6b^2cx+12bc^2x^2+8c^3x^3}\,dx+\int\frac{c^2x^4\sqrt{a+bx+cx^2}}{b^3+6b^2cx+12bc^2x^2+8c^3x^3}\,dx+\int\frac{2abx\sqrt{a+bx+cx^2}}{b^3+6b^2cx+12bc^2x^2+8c^3x^3}\,dx+\int\frac{2ac}{b^3+6b^2c}}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(2*c*d*x+b*d)**3,x)
```

```
[Out] (Integral(a**2*sqrt(a + b*x + c*x**2)/(b**3 + 6*b**2*c*x + 12*b*c**2*x**2 +
 8*c**3*x**3), x) + Integral(b**2*x**2*sqrt(a + b*x + c*x**2)/(b**3 + 6*b**
```

```
2*c*x + 12*b*c**2*x**2 + 8*c**3*x**3), x) + Integral(c**2*x**4*sqrt(a + b*x
 + c*x**2)/(b**3 + 6*b**2*c*x + 12*b*c**2*x**2 + 8*c**3*x**3), x) + Integra
l(2*a*b*x*sqrt(a + b*x + c*x**2)/(b**3 + 6*b**2*c*x + 12*b*c**2*x**2 + 8*c*
*3*x**3), x) + Integral(2*a*c*x**2*sqrt(a + b*x + c*x**2)/(b**3 + 6*b**2*c*
x + 12*b*c**2*x**2 + 8*c**3*x**3), x) + Integral(2*b*c*x**3*sqrt(a + b*x +
c*x**2)/(b**3 + 6*b**2*c*x + 12*b*c**2*x**2 + 8*c**3*x**3), x))/d**3
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1225 $\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(bd+2cdx)^4}\,dx$

Optimal. Leaf size=145

$$-\frac{5\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}d^4}-\frac{5\left(a+bx+cx^2\right)^{3/2}}{24c^2d^4(b+2cx)}+\frac{5(b+2cx)\sqrt{a+bx+cx^2}}{64c^3d^4}-\frac{\left(a+bx+cx^2\right)^{5/2}}{6cd^4(b+2cx)^3}$$

```
[Out] (5*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(64*c^3*d^4) - (5*(a + b*x + c*x^2)^(
3/2))/(24*c^2*d^4*(b + 2*c*x)) - (a + b*x + c*x^2)^(5/2)/(6*c*d^4*(b + 2*c*
x)^3) - (5*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^
2])])/(128*c^(7/2)*d^4)
```

Rubi [A] time = 0.0695159, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {684, 612, 621, 206}

$$-\frac{5\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}d^4}-\frac{5\left(a+bx+cx^2\right)^{3/2}}{24c^2d^4(b+2cx)}+\frac{5(b+2cx)\sqrt{a+bx+cx^2}}{64c^3d^4}-\frac{\left(a+bx+cx^2\right)^{5/2}}{6cd^4(b+2cx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^4,x]
```

```
[Out] (5*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(64*c^3*d^4) - (5*(a + b*x + c*x^2)^(
3/2))/(24*c^2*d^4*(b + 2*c*x)) - (a + b*x + c*x^2)^(5/2)/(6*c*d^4*(b + 2*c*
x)^3) - (5*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^
2])])/(128*c^(7/2)*d^4)
```

Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(bd+2cdx)^4}\,dx = -\frac{\left(a+bx+cx^2\right)^{5/2}}{6cd^4(b+2cx)^3}+\frac{5\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(bd+2cdx)^2}\,dx}{12cd^2}$$
$$= -\frac{5\left(a+bx+cx^2\right)^{3/2}}{24c^2d^4(b+2cx)}-\frac{\left(a+bx+cx^2\right)^{5/2}}{6cd^4(b+2cx)^3}+\frac{5\int \sqrt{a+bx+cx^2}\,dx}{16c^2d^4}$$
$$= \frac{5(b+2cx)\sqrt{a+bx+cx^2}}{64c^3d^4}-\frac{5\left(a+bx+cx^2\right)^{3/2}}{24c^2d^4(b+2cx)}-\frac{\left(a+bx+cx^2\right)^{5/2}}{6cd^4(b+2cx)^3}-\frac{\left(5\left(b^2-4ac\right)\right)\int \frac{1}{\sqrt{a+bx}}}{128c^3d^4}$$
$$= \frac{5(b+2cx)\sqrt{a+bx+cx^2}}{64c^3d^4}-\frac{5\left(a+bx+cx^2\right)^{3/2}}{24c^2d^4(b+2cx)}-\frac{\left(a+bx+cx^2\right)^{5/2}}{6cd^4(b+2cx)^3}-\frac{\left(5\left(b^2-4ac\right)\right)\mathrm{Subst}(}{64}$$
$$= \frac{5(b+2cx)\sqrt{a+bx+cx^2}}{64c^3d^4}-\frac{5\left(a+bx+cx^2\right)^{3/2}}{24c^2d^4(b+2cx)}-\frac{\left(a+bx+cx^2\right)^{5/2}}{6cd^4(b+2cx)^3}-\frac{5\left(b^2-4ac\right)\tanh^{-1}\Big(}{128c^{7/2}d^4}$$

Mathematica [C] time = 0.0474012, size = 97, normalized size = 0.67

$$-\frac{\left(b^2-4ac\right)^2\sqrt{a+x(b+cx)}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{192c^3d^4(b+2cx)^3\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^4,x]

[Out] -((b^2 - 4*a*c)^2*Sqrt[a + x*(b + c*x)]*Hypergeometric2F1[-5/2, -3/2, -1/2,
 (b + 2*c*x)^2/(b^2 - 4*a*c)])/(192*c^3*d^4*(b + 2*c*x)^3*Sqrt[(c*(a + x*(b
 + c*x)))/(-b^2 + 4*a*c)])
```

Maple [B] time = 0.196, size = 1022, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^4,x)

[Out] -1/12/d^4/c^3/(4*a*c-b^2)/(x+1/2*b/c)^3*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)
^(7/2)-4/3/d^4/c/(4*a*c-b^2)^2/(x+1/2*b/c)*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)
/c)^(7/2)+4/3/d^4/(4*a*c-b^2)^2*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(5/2)*x
+2/3/d^4/c/(4*a*c-b^2)^2*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(5/2)*b+5/3/d^
4/(4*a*c-b^2)^2*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(3/2)*x*a-5/12/d^4/c/(4
*a*c-b^2)^2*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(3/2)*x*b^2+5/6/d^4/c/(4*a*
c-b^2)^2*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(3/2)*b*a-5/24/d^4/c^2/(4*a*c-
b^2)^2*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(3/2)*b^3+5/2/d^4/(4*a*c-b^2)^2*
((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2)*x*a^2-5/4/d^4/c/(4*a*c-b^2)^2*((x
+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2)*x*a*b^2+5/32/d^4/c^2/(4*a*c-b^2)^2*(
(x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2)*x*b^4+5/4/d^4/c/(4*a*c-b^2)^2*((x+
```

```
1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2)*b*a^2-5/8/d^4/c^2/(4*a*c-b^2)^2*((x+1
/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2)*b^3*a+5/64/d^4/c^3/(4*a*c-b^2)^2*((x+1
/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2)*b^5+5/2/d^4/c^(1/2)/(4*a*c-b^2)^2*ln((
x+1/2*b/c)*c^(1/2)+((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2))*a^3-15/8/d^4/
c^(3/2)/(4*a*c-b^2)^2*ln((x+1/2*b/c)*c^(1/2)+((x+1/2*b/c)^2*c+1/4*(4*a*c-b^
2)/c)^(1/2))*b^2*a^2+15/32/d^4/c^(5/2)/(4*a*c-b^2)^2*ln((x+1/2*b/c)*c^(1/2)
+((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2))*b^4*a-5/128/d^4/c^(7/2)/(4*a*c-
b^2)^2*ln((x+1/2*b/c)*c^(1/2)+((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2))*b^
6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 14.8094, size = 1170, normalized size = 8.07

$$\left[-\frac{15\left(b^5-4\,ab^3c+8\left(b^2c^3-4\,ac^4\right)x^3+12\left(b^3c^2-4\,abc^3\right)x^2+6\left(b^4c-4\,ab^2c^2\right)x\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+\cdots}\right.}{768\left(8\,c^7d^4x\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^4,x, algorithm="fricas")
```

```
[Out] [-1/768*(15*(b^5 - 4*a*b^3*c + 8*(b^2*c^3 - 4*a*c^4)*x^3 + 12*(b^3*c^2 - 4*
a*b*c^3)*x^2 + 6*(b^4*c - 4*a*b^2*c^2)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x
- b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(48*c^5*x^
4 + 96*b*c^4*x^3 + 15*b^4*c - 40*a*b^2*c^2 - 32*a^2*c^3 + 32*(4*b^2*c^3 - 7
*a*c^4)*x^2 + 16*(5*b^3*c^2 - 14*a*b*c^3)*x)*sqrt(c*x^2 + b*x + a))/(8*c^7*
d^4*x^3 + 12*b*c^6*d^4*x^2 + 6*b^2*c^5*d^4*x + b^3*c^4*d^4), 1/384*(15*(b^5
 - 4*a*b^3*c + 8*(b^2*c^3 - 4*a*c^4)*x^3 + 12*(b^3*c^2 - 4*a*b*c^3)*x^2 + 6
*(b^4*c - 4*a*b^2*c^2)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x
+ b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(48*c^5*x^4 + 96*b*c^4*x^3 + 15*
b^4*c - 40*a*b^2*c^2 - 32*a^2*c^3 + 32*(4*b^2*c^3 - 7*a*c^4)*x^2 + 16*(5*b^
3*c^2 - 14*a*b*c^3)*x)*sqrt(c*x^2 + b*x + a))/(8*c^7*d^4*x^3 + 12*b*c^6*d^4
*x^2 + 6*b^2*c^5*d^4*x + b^3*c^4*d^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2\sqrt{a+bx+cx^2}}{b^4+8b^3cx+24b^2c^2x^2+32bc^3x^3+16c^4x^4}\,dx+\int \frac{b^2x^2\sqrt{a+bx+cx^2}}{b^4+8b^3cx+24b^2c^2x^2+32bc^3x^3+16c^4x^4}\,dx+\int \frac{c^2x^4\sqrt{a+bx+cx^2}}{b^4+8b^3cx+24b^2c^2x^2+32bc^3x^3+16c^4x^4}\,dx+\int \frac{\cdots}{b^4+8b^3cx\cdots}}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(2*c*d*x+b*d)**4,x)
```

```
[Out] (Integral(a**2*sqrt(a + b*x + c*x**2)/(b**4 + 8*b**3*c*x + 24*b**2*c**2*x**
2 + 32*b*c**3*x**3 + 16*c**4*x**4), x) + Integral(b**2*x**2*sqrt(a + b*x +
c*x**2)/(b**4 + 8*b**3*c*x + 24*b**2*c**2*x**2 + 32*b*c**3*x**3 + 16*c**4*x
**4), x) + Integral(c**2*x**4*sqrt(a + b*x + c*x**2)/(b**4 + 8*b**3*c*x + 2
4*b**2*c**2*x**2 + 32*b*c**3*x**3 + 16*c**4*x**4), x) + Integral(2*a*b*x*sq
rt(a + b*x + c*x**2)/(b**4 + 8*b**3*c*x + 24*b**2*c**2*x**2 + 32*b*c**3*x**
3 + 16*c**4*x**4), x) + Integral(2*a*c*x**2*sqrt(a + b*x + c*x**2)/(b**4 +
8*b**3*c*x + 24*b**2*c**2*x**2 + 32*b*c**3*x**3 + 16*c**4*x**4), x) + Integ
ral(2*b*c*x**3*sqrt(a + b*x + c*x**2)/(b**4 + 8*b**3*c*x + 24*b**2*c**2*x**
2 + 32*b*c**3*x**3 + 16*c**4*x**4), x))/d**4
```

Giac [B] time = 1.92642, size = 841, normalized size = 5.8

$$\frac{1}{64}\sqrt{cx^2+bx+a}\left(\frac{2x}{c^2d^4}+\frac{b}{c^3d^4}\right)+\frac{5\left(b^2-4ac\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{128c^{\frac{7}{2}}d^4}+\frac{36\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^4,x, algorithm="giac")
```

```
[Out] 1/64*sqrt(c*x^2 + b*x + a)*(2*x/(c^2*d^4) + b/(c^3*d^4)) + 5/128*(b^2 - 4*a
*c)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/(c^(7/2)*d^
4) + 1/192*(36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^4*c^(5/2) - 288*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^2*c^(7/2) + 576*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^4*a^2*c^(9/2) + 72*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^5
*c^2 - 576*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*c^3 + 1152*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c^4 + 66*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^2*b^6*c^(3/2) - 576*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^4*c^(5/2
) + 1440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^2*c^(7/2) - 768*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*c^(9/2) + 30*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))*b^7*c - 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^5*c^2 + 864*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^3*c^3 - 768*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))*a^3*b*c^4 + 7*b^8*sqrt(c) - 82*a*b^6*c^(3/2) + 348*a^2*b^4*c^
(5/2) - 640*a^3*b^2*c^(7/2) + 448*a^4*c^(9/2))/((2*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^2*c^(3/2) + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c + b^2*sqr
t(c) - 2*a*c^(3/2))^3*c^(5/2)*d^4)
```

3.1226 $\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(bd+2cdx)^5}\,dx$

Optimal. Leaf size=147

$$-\frac{15\sqrt{b^2-4ac}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{512c^{7/2}d^5}-\frac{5\left(a+bx+cx^2\right)^{3/2}}{64c^2d^5(b+2cx)^2}+\frac{15\sqrt{a+bx+cx^2}}{256c^3d^5}-\frac{\left(a+bx+cx^2\right)^{5/2}}{8cd^5(b+2cx)^4}$$

```
[Out] (15*Sqrt[a + b*x + c*x^2])/(256*c^3*d^5) - (5*(a + b*x + c*x^2)^(3/2))/(64*
c^2*d^5*(b + 2*c*x)^2) - (a + b*x + c*x^2)^(5/2)/(8*c*d^5*(b + 2*c*x)^4) -
(15*Sqrt[b^2 - 4*a*c]*ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4
*a*c]])/(512*c^(7/2)*d^5)
```

Rubi [A] time = 0.101916, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {684, 685, 688, 205}

$$-\frac{15\sqrt{b^2-4ac}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{512c^{7/2}d^5}-\frac{5\left(a+bx+cx^2\right)^{3/2}}{64c^2d^5(b+2cx)^2}+\frac{15\sqrt{a+bx+cx^2}}{256c^3d^5}-\frac{\left(a+bx+cx^2\right)^{5/2}}{8cd^5(b+2cx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^5,x]
```


```
[Out] (15*Sqrt[a + b*x + c*x^2])/(256*c^3*d^5) - (5*(a + b*x + c*x^2)^(3/2))/(64*
c^2*d^5*(b + 2*c*x)^2) - (a + b*x + c*x^2)^(5/2)/(8*c*d^5*(b + 2*c*x)^4) -
(15*Sqrt[b^2 - 4*a*c]*ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4
*a*c]])/(512*c^(7/2)*d^5)
```


Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(bd+2cdx)^5}\,dx = -\frac{\left(a+bx+cx^2\right)^{5/2}}{8cd^5(b+2cx)^4} + \frac{5\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(bd+2cdx)^3}\,dx}{16cd^2}$$

$$= -\frac{5\left(a+bx+cx^2\right)^{3/2}}{64c^2d^5(b+2cx)^2} - \frac{\left(a+bx+cx^2\right)^{5/2}}{8cd^5(b+2cx)^4} + \frac{15\int \frac{\sqrt{a+bx+cx^2}}{bd+2cdx}\,dx}{128c^2d^4}$$

$$= \frac{15\sqrt{a+bx+cx^2}}{256c^3d^5} - \frac{5\left(a+bx+cx^2\right)^{3/2}}{64c^2d^5(b+2cx)^2} - \frac{\left(a+bx+cx^2\right)^{5/2}}{8cd^5(b+2cx)^4} - \frac{\left(15\left(b^2-4ac\right)\right)\int \frac{1}{(bd+2cdx)\sqrt{a+b}}$$

$$= \frac{15\sqrt{a+bx+cx^2}}{256c^3d^5} - \frac{5\left(a+bx+cx^2\right)^{3/2}}{64c^2d^5(b+2cx)^2} - \frac{\left(a+bx+cx^2\right)^{5/2}}{8cd^5(b+2cx)^4} - \frac{\left(15\left(b^2-4ac\right)\right)\text{Subst}\left(\int \frac{}{2b^2ca}\right.}{12}$$

$$= \frac{15\sqrt{a+bx+cx^2}}{256c^3d^5} - \frac{5\left(a+bx+cx^2\right)^{3/2}}{64c^2d^5(b+2cx)^2} - \frac{\left(a+bx+cx^2\right)^{5/2}}{8cd^5(b+2cx)^4} - \frac{15\sqrt{b^2-4ac}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+}}{\sqrt{b^2-4ac}}\right.}{512c^{7/2}d^5}$$

Mathematica [C] time = 0.0373061, size = 62, normalized size = 0.42

$$\frac{2(a+x(b+cx))^{7/2}\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};\frac{4c(a+x(b+cx))}{4ac-b^2}\right)}{7d^5\left(b^2-4ac\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^5,x]
```

```
[Out] (2*(a + x*(b + c*x))^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, (4*c*(a + x*(b +
c*x)))/(-b^2 + 4*a*c)])/(7*(b^2 - 4*a*c)^3*d^5)
```

Maple [B] time = 0.198, size = 900, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^5,x)
```

```
[Out] -1/32/d^5/c^4/(4*a*c-b^2)/(x+1/2*b/c)^4*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)
^(7/2)-3/16/d^5/c^2/(4*a*c-b^2)^2/(x+1/2*b/c)^2*((x+1/2*b/c)^2*c+1/4*(4*a*c
-b^2)/c)^(7/2)+3/16/d^5/c/(4*a*c-b^2)^2*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)
^(5/2)+5/16/d^5/c/(4*a*c-b^2)^2*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(3/2)*a
-5/64/d^5/c^2/(4*a*c-b^2)^2*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(3/2)*b^2+1
5/32/d^5/c/(4*a*c-b^2)^2*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2)*a^2-15/64/
d^5/c^2/(4*a*c-b^2)^2*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2)*a*b^2+15/512/
d^5/c^3/(4*a*c-b^2)^2*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2)*b^4-15/8/d^5/
c/(4*a*c-b^2)^2/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-b^2
)/c)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))*a^3+45/32/
```

```
d^5/c^2/(4*a*c-b^2)^2/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*((4*a
*c-b^2)/c)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))*a^2*
b^2-45/128/d^5/c^3/(4*a*c-b^2)^2/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/
c+1/2*((4*a*c-b^2)/c)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2
*b/c))*a*b^4+15/512/d^5/c^4/(4*a*c-b^2)^2/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*
a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2
))/(x+1/2*b/c))*b^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 18.1351, size = 1165, normalized size = 7.93

$$\left[\frac{15\left(16c^4x^4+32bc^3x^3+24b^2c^2x^2+8b^3cx+b^4\right)\sqrt{-\frac{b^2-4ac}{c}}\log\left(-\frac{4c^2x^2+4bcx-b^2+8ac-4\sqrt{cx^2+bx+a}c\sqrt{-\frac{b^2-4ac}{c}}}{4c^2x^2+4bcx+b^2}\right)+4\left(128c^4x^4+\right.}{1024\left(16c^7d^5x^4+32bc^6d^5x^3+24b^2c^5d^5x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^5,x, algorithm="fricas")
```

```
[Out] [1/1024*(15*(16*c^4*x^4 + 32*b*c^3*x^3 + 24*b^2*c^2*x^2 + 8*b^3*c*x + b^4)*
sqrt(-(b^2 - 4*a*c)/c)*log(-(4*c^2*x^2 + 4*b*c*x - b^2 + 8*a*c - 4*sqrt(c*x
^2 + b*x + a)*c*sqrt(-(b^2 - 4*a*c)/c))/(4*c^2*x^2 + 4*b*c*x + b^2)) + 4*(1
28*c^4*x^4 + 256*b*c^3*x^3 + 15*b^4 - 20*a*b^2*c - 32*a^2*c^2 + 12*(19*b^2*
c^2 - 12*a*c^3)*x^2 + 4*(25*b^3*c - 36*a*b*c^2)*x)*sqrt(c*x^2 + b*x + a))/(
16*c^7*d^5*x^4 + 32*b*c^6*d^5*x^3 + 24*b^2*c^5*d^5*x^2 + 8*b^3*c^4*d^5*x +
b^4*c^3*d^5), 1/512*(15*(16*c^4*x^4 + 32*b*c^3*x^3 + 24*b^2*c^2*x^2 + 8*b^3
*c*x + b^4)*sqrt((b^2 - 4*a*c)/c)*arctan(1/2*sqrt((b^2 - 4*a*c)/c)/sqrt(c*x
^2 + b*x + a)) + 2*(128*c^4*x^4 + 256*b*c^3*x^3 + 15*b^4 - 20*a*b^2*c - 32*
a^2*c^2 + 12*(19*b^2*c^2 - 12*a*c^3)*x^2 + 4*(25*b^3*c - 36*a*b*c^2)*x)*sqr
t(c*x^2 + b*x + a))/(16*c^7*d^5*x^4 + 32*b*c^6*d^5*x^3 + 24*b^2*c^5*d^5*x^2
 + 8*b^3*c^4*d^5*x + b^4*c^3*d^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a^2\sqrt{a+bx+cx^2}}{b^5+10b^4cx+40b^3c^2x^2+80b^2c^3x^3+80bc^4x^4+32c^5x^5}\,dx+\int\frac{b^2x^2\sqrt{a+bx+cx^2}}{b^5+10b^4cx+40b^3c^2x^2+80b^2c^3x^3+80bc^4x^4+32c^5x^5}\,dx+\int\frac{c^2x^4\sqrt{a+bx+cx^2}}{b^5+10b^4cx+40b^3c^2x^2+80b^2c^3x^3+8\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(2*c*d*x+b*d)**5,x)
```

```
[Out] (Integral(a**2*sqrt(a + b*x + c*x**2)/(b**5 + 10*b**4*c*x + 40*b**3*c**2*x*
*2 + 80*b**2*c**3*x**3 + 80*b*c**4*x**4 + 32*c**5*x**5), x) + Integral(b**2
*x**2*sqrt(a + b*x + c*x**2)/(b**5 + 10*b**4*c*x + 40*b**3*c**2*x**2 + 80*b
**2*c**3*x**3 + 80*b*c**4*x**4 + 32*c**5*x**5), x) + Integral(c**2*x**4*sqr
t(a + b*x + c*x**2)/(b**5 + 10*b**4*c*x + 40*b**3*c**2*x**2 + 80*b**2*c**3*
x**3 + 80*b*c**4*x**4 + 32*c**5*x**5), x) + Integral(2*a*b*x*sqrt(a + b*x +
 c*x**2)/(b**5 + 10*b**4*c*x + 40*b**3*c**2*x**2 + 80*b**2*c**3*x**3 + 80*b
*c**4*x**4 + 32*c**5*x**5), x) + Integral(2*a*c*x**2*sqrt(a + b*x + c*x**2)
/(b**5 + 10*b**4*c*x + 40*b**3*c**2*x**2 + 80*b**2*c**3*x**3 + 80*b*c**4*x*
*4 + 32*c**5*x**5), x) + Integral(2*b*c*x**3*sqrt(a + b*x + c*x**2)/(b**5 +
 10*b**4*c*x + 40*b**3*c**2*x**2 + 80*b**2*c**3*x**3 + 80*b*c**4*x**4 + 32*
c**5*x**5), x))/d**5
```

Giac [B] time = 2.15639, size = 1014, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^5,x, algorithm="giac")
```

```
[Out] -1/96*d^2*(64*sqrt(-b^2*c*d^2/(2*c*d*x + b*d)^2 + 4*a*c^2*d^2/(2*c*d*x + b*
d)^2 + c)*(9*(b^6*c^7*d^10*sgn(1/(2*c*d*x + b*d))*sgn(c)*sgn(d) - 12*a*b^4*
c^8*d^10*sgn(1/(2*c*d*x + b*d))*sgn(c)*sgn(d) + 48*a^2*b^2*c^9*d^10*sgn(1/(
2*c*d*x + b*d))*sgn(c)*sgn(d) - 64*a^3*c^10*d^10*sgn(1/(2*c*d*x + b*d))*sgn
(c)*sgn(d))/(b^8*c^12*d^16 - 16*a*b^6*c^13*d^16 + 96*a^2*b^4*c^14*d^16 - 25
6*a^3*b^2*c^15*d^16 + 256*a^4*c^16*d^16) - 2*(b^8*c^9*d^14*sgn(1/(2*c*d*x +
 b*d))*sgn(c)*sgn(d) - 16*a*b^6*c^10*d^14*sgn(1/(2*c*d*x + b*d))*sgn(c)*sgn
(d) + 96*a^2*b^4*c^11*d^14*sgn(1/(2*c*d*x + b*d))*sgn(c)*sgn(d) - 256*a^3*b
^2*c^12*d^14*sgn(1/(2*c*d*x + b*d))*sgn(c)*sgn(d) + 256*a^4*c^13*d^14*sgn(1
/(2*c*d*x + b*d))*sgn(c)*sgn(d))/((b^8*c^12*d^16 - 16*a*b^6*c^13*d^16 + 96*
a^2*b^4*c^14*d^16 - 256*a^3*b^2*c^15*d^16 + 256*a^4*c^16*d^16)*(2*c*d*x + b
*d)^2*c^2*d^2))/((2*c*d*x + b*d)*c*d) + 480*abs(c)*log((sqrt(-b^2*c*d^2/(2*
c*d*x + b*d)^2 + 4*a*c^2*d^2/(2*c*d*x + b*d)^2 + c) + sqrt(-b^2*c^3*d^4 + 4
*a*c^4*d^4)/((2*c*d*x + b*d)*c*d))^2)*sgn(1/(2*c*d*x + b*d))*sgn(c)*sgn(d)/
((b^2*c^6 - 4*a*c^7)*sqrt(-b^2*c + 4*a*c^2)*d^8) - 3*sqrt(-b^2*c + 4*a*c^2)
*sgn(1/(2*c*d*x + b*d))*sgn(c)*sgn(d)/(((sqrt(-b^2*c*d^2/(2*c*d*x + b*d)^2
+ 4*a*c^2*d^2/(2*c*d*x + b*d)^2 + c) + sqrt(-b^2*c^3*d^4 + 4*a*c^4*d^4)/((2
*c*d*x + b*d)*c*d))^2 - c)*c^4*d^7))*abs(c)
```

3.1227 $\int \frac{(a+bx+cx^2)^{5/2}}{(bd+2cdx)^6} dx$

Optimal. Leaf size=139

$$-\frac{(a+bx+cx^2)^{3/2}}{24c^2d^6(b+2cx)^3}-\frac{\sqrt{a+bx+cx^2}}{32c^3d^6(b+2cx)}+\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{64c^{7/2}d^6}-\frac{(a+bx+cx^2)^{5/2}}{10cd^6(b+2cx)^5}$$

```
[Out] -Sqrt[a + b*x + c*x^2]/(32*c^3*d^6*(b + 2*c*x)) - (a + b*x + c*x^2)^(3/2)/(
24*c^2*d^6*(b + 2*c*x)^3) - (a + b*x + c*x^2)^(5/2)/(10*c*d^6*(b + 2*c*x)^5
) + ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(64*c^(7/2)*d^6)
```

Rubi [A] time = 0.0745528, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {684, 621, 206}

$$-\frac{(a+bx+cx^2)^{3/2}}{24c^2d^6(b+2cx)^3}-\frac{\sqrt{a+bx+cx^2}}{32c^3d^6(b+2cx)}+\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{64c^{7/2}d^6}-\frac{(a+bx+cx^2)^{5/2}}{10cd^6(b+2cx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^6,x]
```

```
[Out] -Sqrt[a + b*x + c*x^2]/(32*c^3*d^6*(b + 2*c*x)) - (a + b*x + c*x^2)^(3/2)/(
24*c^2*d^6*(b + 2*c*x)^3) - (a + b*x + c*x^2)^(5/2)/(10*c*d^6*(b + 2*c*x)^5
) + ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(64*c^(7/2)*d^6)
```

Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{5/2}}{(bd+2cdx)^6} dx = -\frac{(a+bx+cx^2)^{5/2}}{10cd^6(b+2cx)^5} + \frac{\int \frac{(a+bx+cx^2)^{3/2}}{(bd+2cdx)^4} dx}{4cd^2}$$
$$= -\frac{(a+bx+cx^2)^{3/2}}{24c^2d^6(b+2cx)^3} - \frac{(a+bx+cx^2)^{5/2}}{10cd^6(b+2cx)^5} + \frac{\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^2} dx}{16c^2d^4}$$
$$= -\frac{\sqrt{a+bx+cx^2}}{32c^3d^6(b+2cx)} - \frac{(a+bx+cx^2)^{3/2}}{24c^2d^6(b+2cx)^3} - \frac{(a+bx+cx^2)^{5/2}}{10cd^6(b+2cx)^5} + \frac{\int \frac{1}{\sqrt{a+bx+cx^2}} dx}{64c^3d^6}$$
$$= -\frac{\sqrt{a+bx+cx^2}}{32c^3d^6(b+2cx)} - \frac{(a+bx+cx^2)^{3/2}}{24c^2d^6(b+2cx)^3} - \frac{(a+bx+cx^2)^{5/2}}{10cd^6(b+2cx)^5} + \frac{\text{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{32c^3d^6}$$
$$= -\frac{\sqrt{a+bx+cx^2}}{32c^3d^6(b+2cx)} - \frac{(a+bx+cx^2)^{3/2}}{24c^2d^6(b+2cx)^3} - \frac{(a+bx+cx^2)^{5/2}}{10cd^6(b+2cx)^5} + \frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{64c^{7/2}d^6}$$

Mathematica [C] time = 0.0507466, size = 97, normalized size = 0.7

$$-\frac{(b^2-4ac)^2 \sqrt{a+x(b+cx)}\, {}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{320c^3d^6(b+2cx)^5\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^6,x]
```

```
[Out] -((b^2 - 4*a*c)^2*Sqrt[a + x*(b + c*x)]*Hypergeometric2F1[-5/2, -5/2, -3/2,
 (b + 2*c*x)^2/(b^2 - 4*a*c)])/(320*c^3*d^6*(b + 2*c*x)^5*Sqrt[(c*(a + x*(b
 + c*x)))/(-b^2 + 4*a*c)])
```

Maple [B] time = 0.204, size = 1080, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^6,x)
```

```
[Out] -1/80/d^6/c^5/(4*a*c-b^2)/(x+1/2*b/c)^5*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)
^(7/2)-1/30/d^6/c^3/(4*a*c-b^2)^2/(x+1/2*b/c)^3*((x+1/2*b/c)^2*c+1/4*(4*a*c
-b^2)/c)^(7/2)-8/15/d^6/c/(4*a*c-b^2)^3/(x+1/2*b/c)*((x+1/2*b/c)^2*c+1/4*(4
*a*c-b^2)/c)^(7/2)+8/15/d^6/(4*a*c-b^2)^3*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/
c)^(5/2)*x+4/15/d^6/c/(4*a*c-b^2)^3*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(5/
2)*b+2/3/d^6/(4*a*c-b^2)^3*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(3/2)*x*a-1/
6/d^6/c/(4*a*c-b^2)^3*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(3/2)*x*b^2+1/3/d
^6/c/(4*a*c-b^2)^3*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(3/2)*b*a-1/12/d^6/c
^2/(4*a*c-b^2)^3*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(3/2)*b^3+1/d^6/(4*a*c
-b^2)^3*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2)*x*a^2-1/2/d^6/c/(4*a*c-b^
2)^3*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2)*x*a*b^2+1/16/d^6/c^2/(4*a*c-
b^2)^3*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2)*x*b^4+1/2/d^6/c/(4*a*c-b^2
)^3*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2)*b*a^2-1/4/d^6/c^2/(4*a*c-b^2)
^3*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2)*b^3*a+1/32/d^6/c^3/(4*a*c-b^2)
^3*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2)*b^5+1/d^6/c^(1/2)/(4*a*c-b^2)^
```

```
3*ln((x+1/2*b/c)*c^(1/2)+((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2))*a^3-3/4
/d^6/c^(3/2)/(4*a*c-b^2)^3*ln((x+1/2*b/c)*c^(1/2)+((x+1/2*b/c)^2*c+1/4*(4*a
*c-b^2)/c)^(1/2))*b^2*a^2+3/16/d^6/c^(5/2)/(4*a*c-b^2)^3*ln((x+1/2*b/c)*c^(
1/2)+((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2))*b^4*a-1/64/d^6/c^(7/2)/(4*a
*c-b^2)^3*ln((x+1/2*b/c)*c^(1/2)+((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2))
*b^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 30.0327, size = 1237, normalized size = 8.9

$$\left[\frac{15\left(32\,c^5x^5+80\,bc^4x^4+80\,b^2c^3x^3+40\,b^3c^2x^2+10\,b^4cx+b^5\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{\cdots}\right.}{1920\left(32\,c^9d^6x^5+80\,bc^8d^6x^4+80\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^6,x, algorithm="fricas")
```

```
[Out] [1/1920*(15*(32*c^5*x^5 + 80*b*c^4*x^4 + 80*b^2*c^3*x^3 + 40*b^3*c^2*x^2 +
10*b^4*c*x + b^5)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b
*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(368*c^5*x^4 + 736*b*c^4*x^3 + 15*
b^4*c + 20*a*b^2*c^2 + 48*a^2*c^3 + 4*(127*b^2*c^3 + 44*a*c^4)*x^2 + 4*(35*
b^3*c^2 + 44*a*b*c^3)*x)*sqrt(c*x^2 + b*x + a))/(32*c^9*d^6*x^5 + 80*b*c^8*
d^6*x^4 + 80*b^2*c^7*d^6*x^3 + 40*b^3*c^6*d^6*x^2 + 10*b^4*c^5*d^6*x + b^5*
c^4*d^6), -1/960*(15*(32*c^5*x^5 + 80*b*c^4*x^4 + 80*b^2*c^3*x^3 + 40*b^3*c
^2*x^2 + 10*b^4*c*x + b^5)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x
 + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(368*c^5*x^4 + 736*b*c^4*x^3 +
15*b^4*c + 20*a*b^2*c^2 + 48*a^2*c^3 + 4*(127*b^2*c^3 + 44*a*c^4)*x^2 + 4*(
35*b^3*c^2 + 44*a*b*c^3)*x)*sqrt(c*x^2 + b*x + a))/(32*c^9*d^6*x^5 + 80*b*c
^8*d^6*x^4 + 80*b^2*c^7*d^6*x^3 + 40*b^3*c^6*d^6*x^2 + 10*b^4*c^5*d^6*x + b
^5*c^4*d^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(2*c*d*x+b*d)**6,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^6,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1228 $\int \frac{(a+bx+cx^2)^{5/2}}{(bd+2cdx)^7}\,dx$

Optimal. Leaf size=155

$$\frac{5\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{1024c^{7/2}d^7\sqrt{b^2-4ac}}-\frac{5\left(a+bx+cx^2\right)^{3/2}}{192c^2d^7(b+2cx)^4}-\frac{5\sqrt{a+bx+cx^2}}{512c^3d^7(b+2cx)^2}-\frac{\left(a+bx+cx^2\right)^{5/2}}{12cd^7(b+2cx)^6}$$

```
[Out] (-5*Sqrt[a + b*x + c*x^2])/(512*c^3*d^7*(b + 2*c*x)^2) - (5*(a + b*x + c*x^
2)^(3/2))/(192*c^2*d^7*(b + 2*c*x)^4) - (a + b*x + c*x^2)^(5/2)/(12*c*d^7*(
b + 2*c*x)^6) + (5*ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*
c]])/(1024*c^(7/2)*Sqrt[b^2 - 4*a*c]*d^7)
```

Rubi [A] time = 0.0988183, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {684, 688, 205}

$$\frac{5\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{1024c^{7/2}d^7\sqrt{b^2-4ac}}-\frac{5\left(a+bx+cx^2\right)^{3/2}}{192c^2d^7(b+2cx)^4}-\frac{5\sqrt{a+bx+cx^2}}{512c^3d^7(b+2cx)^2}-\frac{\left(a+bx+cx^2\right)^{5/2}}{12cd^7(b+2cx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^7,x]
```


```
[Out] (-5*Sqrt[a + b*x + c*x^2])/(512*c^3*d^7*(b + 2*c*x)^2) - (5*(a + b*x + c*x^
2)^(3/2))/(192*c^2*d^7*(b + 2*c*x)^4) - (a + b*x + c*x^2)^(5/2)/(12*c*d^7*(
b + 2*c*x)^6) + (5*ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*
c]])/(1024*c^(7/2)*Sqrt[b^2 - 4*a*c]*d^7)
```


Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(bd+2cdx)^7}\,dx = -\frac{\left(a+bx+cx^2\right)^{5/2}}{12cd^7(b+2cx)^6}+\frac{5\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(bd+2cdx)^5}\,dx}{24cd^2}$$
$$= -\frac{5\left(a+bx+cx^2\right)^{3/2}}{192c^2d^7(b+2cx)^4}-\frac{\left(a+bx+cx^2\right)^{5/2}}{12cd^7(b+2cx)^6}+\frac{5\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^3}\,dx}{128c^2d^4}$$
$$= -\frac{5\sqrt{a+bx+cx^2}}{512c^3d^7(b+2cx)^2}-\frac{5\left(a+bx+cx^2\right)^{3/2}}{192c^2d^7(b+2cx)^4}-\frac{\left(a+bx+cx^2\right)^{5/2}}{12cd^7(b+2cx)^6}+\frac{5\int \frac{1}{(bd+2cdx)\sqrt{a+bx+cx^2}}\,dx}{1024c^3d^6}$$
$$= -\frac{5\sqrt{a+bx+cx^2}}{512c^3d^7(b+2cx)^2}-\frac{5\left(a+bx+cx^2\right)^{3/2}}{192c^2d^7(b+2cx)^4}-\frac{\left(a+bx+cx^2\right)^{5/2}}{12cd^7(b+2cx)^6}+\frac{5\,\text{Subst}\left(\int \frac{1}{2b^2cd-8ac^2d+8c^2dx}\right.}{256c^2}$$
$$= -\frac{5\sqrt{a+bx+cx^2}}{512c^3d^7(b+2cx)^2}-\frac{5\left(a+bx+cx^2\right)^{3/2}}{192c^2d^7(b+2cx)^4}-\frac{\left(a+bx+cx^2\right)^{5/2}}{12cd^7(b+2cx)^6}+\frac{5\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{1024c^{7/2}\sqrt{b^2-4ac}d^7}$$

Mathematica [A] time = 0.385353, size = 233, normalized size = 1.5

$$\frac{-2c\left(8a^2c\left(5b^2+68bcx+68c^2x^2\right)+128a^3c^2+a\left(1144b^2c^2x^2+200b^3cx+15b^4+1888bc^3x^3+944c^4x^4\right)+x\left(848b^3c^2\right.\right.}{3072c^4d^7(b+2cx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^7,x]
```

```
[Out] (-2*c*(128*a^3*c^2 + 8*a^2*c*(5*b^2 + 68*b*c*x + 68*c^2*x^2) + a*(15*b^4 +
200*b^3*c*x + 1144*b^2*c^2*x^2 + 1888*b*c^3*x^3 + 944*c^4*x^4) + x*(15*b^5
+ 175*b^4*c*x + 848*b^3*c^2*x^2 + 1744*b^2*c^3*x^3 + 1584*b*c^4*x^4 + 528*c
^5*x^5)) - 15*(b + 2*c*x)^6*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*ArcT
anh[2*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]])/(3072*c^4*d^7*(b + 2*c*x
)^6*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.2, size = 960, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^7,x)
```

```
[Out] -1/192/d^7/c^6/(4*a*c-b^2)/(x+1/2*b/c)^6*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c
)^(7/2)-1/192/d^7/c^4/(4*a*c-b^2)^2/(x+1/2*b/c)^4*((x+1/2*b/c)^2*c+1/4*(4*a
*c-b^2)/c)^(7/2)-1/32/d^7/c^2/(4*a*c-b^2)^3/(x+1/2*b/c)^2*((x+1/2*b/c)^2*c+
1/4*(4*a*c-b^2)/c)^(7/2)+1/32/d^7/c/(4*a*c-b^2)^3*((x+1/2*b/c)^2*c+1/4*(4*a
*c-b^2)/c)^(5/2)+5/96/d^7/c/(4*a*c-b^2)^3*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/
c)^(3/2)*a-5/384/d^7/c^2/(4*a*c-b^2)^3*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^
(3/2)*b^2+5/64/d^7/c/(4*a*c-b^2)^3*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2)*
a^2-5/128/d^7/c^2/(4*a*c-b^2)^3*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2)*a*b
^2+5/1024/d^7/c^3/(4*a*c-b^2)^3*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2)*b^4
-5/16/d^7/c/(4*a*c-b^2)^3/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*(
(4*a*c-b^2)/c)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))*
a^3+15/64/d^7/c^2/(4*a*c-b^2)^3/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c
+1/2*((4*a*c-b^2)/c)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*
```

```
b/c))*a^2*b^2-15/256/d^7/c^3/(4*a*c-b^2)^3/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4
*a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/
2))/(x+1/2*b/c))*a*b^4+5/1024/d^7/c^4/(4*a*c-b^2)^3/((4*a*c-b^2)/c)^(1/2)*l
n((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^
2)/c)^(1/2))/(x+1/2*b/c))*b^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 46.172, size = 1985, normalized size = 12.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^7,x, algorithm="fricas")
```

```
[Out] [-1/6144*(15*(64*c^6*x^6 + 192*b*c^5*x^5 + 240*b^2*c^4*x^4 + 160*b^3*c^3*x^
3 + 60*b^4*c^2*x^2 + 12*b^5*c*x + b^6)*sqrt(-b^2*c + 4*a*c^2)*log(-(4*c^2*x
^2 + 4*b*c*x - b^2 + 8*a*c - 4*sqrt(-b^2*c + 4*a*c^2)*sqrt(c*x^2 + b*x + a)
)/(4*c^2*x^2 + 4*b*c*x + b^2)) + 4*(15*b^6*c - 20*a*b^4*c^2 - 32*a^2*b^2*c^
3 - 512*a^3*c^4 + 528*(b^2*c^5 - 4*a*c^6)*x^4 + 1056*(b^3*c^4 - 4*a*b*c^5)*
x^3 + 16*(43*b^4*c^3 - 146*a*b^2*c^4 - 104*a^2*c^5)*x^2 + 32*(5*b^5*c^2 - 7
*a*b^3*c^3 - 52*a^2*b*c^4)*x)*sqrt(c*x^2 + b*x + a))/(64*(b^2*c^10 - 4*a*c^
11)*d^7*x^6 + 192*(b^3*c^9 - 4*a*b*c^10)*d^7*x^5 + 240*(b^4*c^8 - 4*a*b^2*c
^9)*d^7*x^4 + 160*(b^5*c^7 - 4*a*b^3*c^8)*d^7*x^3 + 60*(b^6*c^6 - 4*a*b^4*c
^7)*d^7*x^2 + 12*(b^7*c^5 - 4*a*b^5*c^6)*d^7*x + (b^8*c^4 - 4*a*b^6*c^5)*d^
7), -1/3072*(15*(64*c^6*x^6 + 192*b*c^5*x^5 + 240*b^2*c^4*x^4 + 160*b^3*c^3
*x^3 + 60*b^4*c^2*x^2 + 12*b^5*c*x + b^6)*sqrt(b^2*c - 4*a*c^2)*arctan(1/2*
sqrt(b^2*c - 4*a*c^2)*sqrt(c*x^2 + b*x + a)/(c^2*x^2 + b*c*x + a*c)) + 2*(1
5*b^6*c - 20*a*b^4*c^2 - 32*a^2*b^2*c^3 - 512*a^3*c^4 + 528*(b^2*c^5 - 4*a*
c^6)*x^4 + 1056*(b^3*c^4 - 4*a*b*c^5)*x^3 + 16*(43*b^4*c^3 - 146*a*b^2*c^4
- 104*a^2*c^5)*x^2 + 32*(5*b^5*c^2 - 7*a*b^3*c^3 - 52*a^2*b*c^4)*x)*sqrt(c*
x^2 + b*x + a))/(64*(b^2*c^10 - 4*a*c^11)*d^7*x^6 + 192*(b^3*c^9 - 4*a*b*c^
10)*d^7*x^5 + 240*(b^4*c^8 - 4*a*b^2*c^9)*d^7*x^4 + 160*(b^5*c^7 - 4*a*b^3*
c^8)*d^7*x^3 + 60*(b^6*c^6 - 4*a*b^4*c^7)*d^7*x^2 + 12*(b^7*c^5 - 4*a*b^5*c
^6)*d^7*x + (b^8*c^4 - 4*a*b^6*c^5)*d^7)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(2*c*d*x+b*d)**7,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^7,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1229 $\int \frac{(a+bx+cx^2)^{5/2}}{(bd+2cdx)^8} dx$

Optimal. Leaf size=39

$$\frac{2(a+bx+cx^2)^{7/2}}{7d^8(b^2-4ac)(b+2cx)^7}$$

```
[Out] (2*(a + b*x + c*x^2)^(7/2))/(7*(b^2 - 4*a*c)*d^8*(b + 2*c*x)^7)
```

Rubi [A] time = 0.0157464, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {682}

$$\frac{2(a+bx+cx^2)^{7/2}}{7d^8(b^2-4ac)(b+2cx)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^8,x]

[Out] (2*(a + b*x + c*x^2)^(7/2))/(7*(b^2 - 4*a*c)*d^8*(b + 2*c*x)^7)
```

Rule 682

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c,
0] && EqQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{5/2}}{(bd+2cdx)^8} dx = \frac{2(a+bx+cx^2)^{7/2}}{7(b^2-4ac)d^8(b+2cx)^7}$$

Mathematica [A] time = 0.0271806, size = 38, normalized size = 0.97

$$\frac{2(a+x(b+cx))^{7/2}}{7d^8(b^2-4ac)(b+2cx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^8,x]

[Out] (2*(a + x*(b + c*x))^(7/2))/(7*(b^2 - 4*a*c)*d^8*(b + 2*c*x)^7)
```

Maple [A] time = 0.042, size = 38, normalized size = 1.

$$-\frac{2}{7(2cx+b)^7 d^8(4ac-b^2)}(cx^2+bx+a)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^8,x)
```

```
[Out] -2/7*(c*x^2+b*x+a)^(7/2)/(2*c*x+b)^7/d^8/(4*a*c-b^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 112.052, size = 563, normalized size = 14.44

$$\frac{2\left(c^3x^6 + 3\,bc^2x^5 + 3\left(b^2c + ac^2\right)x^4 + 3\,a^2bx + \left(b^3 + 6\,abc\right)x^3 + a^3}{7\left(128\left(b^2c^7 - 4\,ac^8\right)d^8x^7 + 448\left(b^3c^6 - 4\,abc^7\right)d^8x^6 + 672\left(b^4c^5 - 4\,ab^2c^6\right)d^8x^5 + 560\left(b^5c^4 - 4\,ab^3c^5\right)d^8x^4 + 280\left(b^6c^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^8,x, algorithm="fricas")
```

```
[Out] 2/7*(c^3*x^6 + 3*b*c^2*x^5 + 3*(b^2*c + a*c^2)*x^4 + 3*a^2*b*x + (b^3 + 6*a
*b*c)*x^3 + a^3 + 3*(a*b^2 + a^2*c)*x^2)*sqrt(c*x^2 + b*x + a)/(128*(b^2*c^
7 - 4*a*c^8)*d^8*x^7 + 448*(b^3*c^6 - 4*a*b*c^7)*d^8*x^6 + 672*(b^4*c^5 - 4
*a*b^2*c^6)*d^8*x^5 + 560*(b^5*c^4 - 4*a*b^3*c^5)*d^8*x^4 + 280*(b^6*c^3 -
4*a*b^4*c^4)*d^8*x^3 + 84*(b^7*c^2 - 4*a*b^5*c^3)*d^8*x^2 + 14*(b^8*c - 4*a
*b^6*c^2)*d^8*x + (b^9 - 4*a*b^7*c)*d^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(2*c*d*x+b*d)**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.51329, size = 1683, normalized size = 43.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^8,x, algorithm="giac")
```

[Out] 1/448*(448*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*c^(13/2) + 2688*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b*c^6 + 7392*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*b^2*c^(11/2) + 12320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^3*c^5 + 14000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^4*c^(9/2) - 1120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^2*c^(11/2) + 2240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*c^(13/2) + 11648*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^5*c^4 - 4480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^3*c^5 + 8960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b*c^6 + 7448*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^6*c^(7/2) - 7840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^4*c^(9/2) + 15680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b^2*c^(11/2) + 3752*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^7*c^3 - 7840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^5*c^4 + 15680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^3*c^5 + 1484*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^8*c^(5/2) - 4984*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^6*c^(7/2) + 10304*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^4*c^(9/2) - 1344*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^2*c^(11/2) + 1344*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*c^(13/2) + 448*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^9*c^2 - 2128*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^7*c^3 + 4928*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^5*c^4 - 2688*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^3*c^5 + 2688*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b*c^6 + 98*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^10*c^(3/2) - 616*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^8*c^(5/2) + 1736*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^6*c^(7/2) - 2016*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^4*c^(9/2) + 2016*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^2*c^(11/2) + 14*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^11*c - 112*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^9*c^2 + 392*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^7*c^3 - 672*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^5*c^4 + 672*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^3*c^5 + b^12*sqrt(c) - 10*a*b^10*c^(3/2) + 44*a^2*b^8*c^(5/2) - 104*a^3*b^6*c^(7/2) + 144*a^4*b^4*c^(9/2) - 96*a^5*b^2*c^(11/2) + 64*a^6*c^(13/2))/((2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*sqrt(c) + b^2 - 2*a*c)^7*c^4*d^8)

3.1230 $\int \frac{(a+bx+cx^2)^{5/2}}{(bd+2cdx)^9}\,dx$

Optimal. Leaf size=197

$$\frac{5\sqrt{a+bx+cx^2}}{4096c^3d^9\left(b^2-4ac\right)(b+2cx)^2}+\frac{5\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{8192c^{7/2}d^9\left(b^2-4ac\right)^{3/2}}-\frac{5\left(a+bx+cx^2\right)^{3/2}}{384c^2d^9(b+2cx)^6}-\frac{5\sqrt{a+bx+cx^2}}{2048c^3d^9(b+2cx)^4}-\frac{\left(a+bx+cx^2\right)^{5/2}}{16cd^9(b+2cx)^8}$$

```
[Out] (-5*Sqrt[a + b*x + c*x^2])/(2048*c^3*d^9*(b + 2*c*x)^4) + (5*Sqrt[a + b*x +
 c*x^2])/(4096*c^3*(b^2 - 4*a*c)*d^9*(b + 2*c*x)^2) - (5*(a + b*x + c*x^2)^
(3/2))/(384*c^2*d^9*(b + 2*c*x)^6) - (a + b*x + c*x^2)^(5/2)/(16*c*d^9*(b +
 2*c*x)^8) + (5*ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c]]
)/(8192*c^(7/2)*(b^2 - 4*a*c)^(3/2)*d^9)
```

Rubi [A] time = 0.149922, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {684, 693, 688, 205}

$$\frac{5\sqrt{a+bx+cx^2}}{4096c^3d^9\left(b^2-4ac\right)(b+2cx)^2}+\frac{5\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{8192c^{7/2}d^9\left(b^2-4ac\right)^{3/2}}-\frac{5\left(a+bx+cx^2\right)^{3/2}}{384c^2d^9(b+2cx)^6}-\frac{5\sqrt{a+bx+cx^2}}{2048c^3d^9(b+2cx)^4}-\frac{\left(a+bx+cx^2\right)^{5/2}}{16cd^9(b+2cx)^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^9,x]
```

```
[Out] (-5*Sqrt[a + b*x + c*x^2])/(2048*c^3*d^9*(b + 2*c*x)^4) + (5*Sqrt[a + b*x +
 c*x^2])/(4096*c^3*(b^2 - 4*a*c)*d^9*(b + 2*c*x)^2) - (5*(a + b*x + c*x^2)^
(3/2))/(384*c^2*d^9*(b + 2*c*x)^6) - (a + b*x + c*x^2)^(5/2)/(16*c*d^9*(b +
 2*c*x)^8) + (5*ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c]]
)/(8192*c^(7/2)*(b^2 - 4*a*c)^(3/2)*d^9)
```

Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
```

```
b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{5/2}}{(bd+2cdx)^9}\,dx = -\frac{(a+bx+cx^2)^{5/2}}{16cd^9(b+2cx)^8} + \frac{5\int \frac{(a+bx+cx^2)^{3/2}}{(bd+2cdx)^7}\,dx}{32cd^2}$$

$$= -\frac{5(a+bx+cx^2)^{3/2}}{384c^2d^9(b+2cx)^6} - \frac{(a+bx+cx^2)^{5/2}}{16cd^9(b+2cx)^8} + \frac{5\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^5}\,dx}{256c^2d^4}$$

$$= -\frac{5\sqrt{a+bx+cx^2}}{2048c^3d^9(b+2cx)^4} - \frac{5(a+bx+cx^2)^{3/2}}{384c^2d^9(b+2cx)^6} - \frac{(a+bx+cx^2)^{5/2}}{16cd^9(b+2cx)^8} + \frac{5\int \frac{1}{(bd+2cdx)^3\sqrt{a+bx+cx^2}}\,dx}{4096c^3d^6}$$

$$= -\frac{5\sqrt{a+bx+cx^2}}{2048c^3d^9(b+2cx)^4} + \frac{5\sqrt{a+bx+cx^2}}{4096c^3(b^2-4ac)d^9(b+2cx)^2} - \frac{5(a+bx+cx^2)^{3/2}}{384c^2d^9(b+2cx)^6} - \frac{(a+bx+cx^2)^{5}}{16cd^9(b+2cx}$$

$$= -\frac{5\sqrt{a+bx+cx^2}}{2048c^3d^9(b+2cx)^4} + \frac{5\sqrt{a+bx+cx^2}}{4096c^3(b^2-4ac)d^9(b+2cx)^2} - \frac{5(a+bx+cx^2)^{3/2}}{384c^2d^9(b+2cx)^6} - \frac{(a+bx+cx^2)^{5}}{16cd^9(b+2cx}$$

$$= -\frac{5\sqrt{a+bx+cx^2}}{2048c^3d^9(b+2cx)^4} + \frac{5\sqrt{a+bx+cx^2}}{4096c^3(b^2-4ac)d^9(b+2cx)^2} - \frac{5(a+bx+cx^2)^{3/2}}{384c^2d^9(b+2cx)^6} - \frac{(a+bx+cx^2)^{5}}{16cd^9(b+2cx}$$

Mathematica [C] time = 0.0399195, size = 62, normalized size = 0.31

$$\frac{2(a+x(b+cx))^{7/2}\,{}_2F_1\left(\frac{7}{2},5;\frac{9}{2};\frac{4c(a+x(b+cx))}{4ac-b^2}\right)}{7d^9\left(b^2-4ac\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^9,x]
```

```
[Out] (2*(a + x*(b + c*x))^(7/2)*Hypergeometric2F1[7/2, 5, 9/2, (4*c*(a + x*(b +
c*x)))/(-b^2 + 4*a*c)])/(7*(b^2 - 4*a*c)^5*d^9)
```

Maple [B] time = 0.225, size = 1020, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^9,x)
```

```
[Out] -1/1024/d^9/c^8/(4*a*c-b^2)/(x+1/2*b/c)^8*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/
c)^(7/2)+1/1536/d^9/c^6/(4*a*c-b^2)^2/(x+1/2*b/c)^6*((x+1/2*b/c)^2*c+1/4*(4
```

```
*a*c-b^2)/c)^(7/2)+1/1536/d^9/c^4/(4*a*c-b^2)^3/(x+1/2*b/c)^4*((x+1/2*b/c)^
2*c+1/4*(4*a*c-b^2)/c)^(7/2)+1/256/d^9/c^2/(4*a*c-b^2)^4/(x+1/2*b/c)^2*((x+
1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(7/2)-1/256/d^9/c/(4*a*c-b^2)^4*((x+1/2*b/c
)^2*c+1/4*(4*a*c-b^2)/c)^(5/2)-5/768/d^9/c/(4*a*c-b^2)^4*((x+1/2*b/c)^2*c+1
/4*(4*a*c-b^2)/c)^(3/2)*a+5/3072/d^9/c^2/(4*a*c-b^2)^4*((x+1/2*b/c)^2*c+1/4
*(4*a*c-b^2)/c)^(3/2)*b^2-5/512/d^9/c/(4*a*c-b^2)^4*(4*(x+1/2*b/c)^2*c+(4*a
*c-b^2)/c)^(1/2)*a^2+5/1024/d^9/c^2/(4*a*c-b^2)^4*(4*(x+1/2*b/c)^2*c+(4*a*c
-b^2)/c)^(1/2)*a*b^2-5/8192/d^9/c^3/(4*a*c-b^2)^4*(4*(x+1/2*b/c)^2*c+(4*a*c
-b^2)/c)^(1/2)*b^4+5/128/d^9/c/(4*a*c-b^2)^4/((4*a*c-b^2)/c)^(1/2)*ln((1/2*
(4*a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(
1/2))/(x+1/2*b/c))*a^3-15/512/d^9/c^2/(4*a*c-b^2)^4/((4*a*c-b^2)/c)^(1/2)*l
n((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^
2)/c)^(1/2))/(x+1/2*b/c))*a^2*b^2+15/2048/d^9/c^3/(4*a*c-b^2)^4/((4*a*c-b^2
)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/2)*(4*(x+1/2*b/c)^2
*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))*a*b^4-5/8192/d^9/c^4/(4*a*c-b^2)^4/((
4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/2)*(4*(x+1
/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))*b^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^9,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^9,x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(2*c*d*x+b*d)**9,x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^9,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1231 $\int \frac{(a+bx+cx^2)^{5/2}}{(bd+2cdx)^{10}}\,dx$

Optimal. Leaf size=79

$$\frac{4\left(a+bx+cx^2\right)^{7/2}}{63d^{10}\left(b^2-4ac\right)^2(b+2cx)^7}+\frac{2\left(a+bx+cx^2\right)^{7/2}}{9d^{10}\left(b^2-4ac\right)(b+2cx)^9}$$

```
[Out] (2*(a + b*x + c*x^2)^(7/2))/(9*(b^2 - 4*a*c)*d^10*(b + 2*c*x)^9) + (4*(a +
b*x + c*x^2)^(7/2))/(63*(b^2 - 4*a*c)^2*d^10*(b + 2*c*x)^7)
```

Rubi [A] time = 0.0336197, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {693, 682}

$$\frac{4\left(a+bx+cx^2\right)^{7/2}}{63d^{10}\left(b^2-4ac\right)^2(b+2cx)^7}+\frac{2\left(a+bx+cx^2\right)^{7/2}}{9d^{10}\left(b^2-4ac\right)(b+2cx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^10,x]
```

```
[Out] (2*(a + b*x + c*x^2)^(7/2))/(9*(b^2 - 4*a*c)*d^10*(b + 2*c*x)^9) + (4*(a +
b*x + c*x^2)^(7/2))/(63*(b^2 - 4*a*c)^2*d^10*(b + 2*c*x)^7)
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 682

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c,
0] && EqQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(bd+2cdx)^{10}}\,dx &= \frac{2\left(a+bx+cx^2\right)^{7/2}}{9\left(b^2-4ac\right)d^{10}(b+2cx)^9}+\frac{2\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(bd+2cdx)^8}\,dx}{9\left(b^2-4ac\right)d^2}\\ &= \frac{2\left(a+bx+cx^2\right)^{7/2}}{9\left(b^2-4ac\right)d^{10}(b+2cx)^9}+\frac{4\left(a+bx+cx^2\right)^{7/2}}{63\left(b^2-4ac\right)^2d^{10}(b+2cx)^7}\end{aligned}$$

Mathematica [A] time = 0.0550576, size = 62, normalized size = 0.78

$$\frac{2(a+x(b+cx))^{7/2}\left(4c\left(2cx^2-7a\right)+9b^2+8bcx\right)}{63d^{10}\left(b^2-4ac\right)^2(b+2cx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^10,x]
```

```
[Out] (2*(a + x*(b + c*x))^(7/2)*(9*b^2 + 8*b*c*x + 4*c*(-7*a + 2*c*x^2)))/(63*(b
^2 - 4*a*c)^2*d^10*(b + 2*c*x)^9)
```

Maple [A] time = 0.042, size = 70, normalized size = 0.9

$$-\frac{-16\,c^2x^2-16\,bcx+56\,ac-18\,b^2}{63\,\left(2\,cx+b\right)^9d^{10}\left(16\,a^2c^2-8\,acb^2+b^4\right)}\left(cx^2+bx+a\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^10,x)
```

```
[Out] -2/63*(-8*c^2*x^2-8*b*c*x+28*a*c-9*b^2)*(c*x^2+b*x+a)^(7/2)/(2*c*x+b)^9/d^1
0/(16*a^2*c^2-8*a*b^2*c+b^4)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^10,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(2*c*d*x+b*d)**10,x)
```

```
[Out] Timed out
```

Giac [B] time = 4.57814, size = 2496, normalized size = 31.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^10,x, algorithm="giac")
```

```
[Out] 1/2016*(4032*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^14*c^(15/2) + 28224*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^13*b*c^7 + 90048*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^12*b^2*c^(13/2) + 6720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*a*c^(
15/2) + 173376*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b^3*c^6 + 40320*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^11*a*b*c^7 + 225792*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^10*b^4*c^(11/2) + 100800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10
*a*b^2*c^(13/2) + 20160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^2*c^(15/2)
 + 212352*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^5*c^5 + 134400*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^9*a*b^3*c^6 + 100800*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^9*a^2*b*c^7 + 151200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^6*c^(9
/2) + 96768*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^4*c^(11/2) + 217728*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b^2*c^(13/2) + 12096*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^8*a^3*c^(15/2) + 84672*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^7*b^7*c^4 + 24192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^5*c^5 +
266112*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^3*c^6 + 48384*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^7*a^3*b*c^7 + 38304*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^6*b^8*c^(7/2) - 20160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^6*c^
(9/2) + 205632*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b^4*c^(11/2) + 725
76*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^3*b^2*c^(13/2) + 12096*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^6*a^4*c^(15/2) + 14112*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^5*b^9*c^3 - 24192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^7*c^
4 + 108864*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^5*c^5 + 48384*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^3*c^6 + 36288*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^5*a^4*b*c^7 + 4176*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^10*
c^(5/2) - 12960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^8*c^(7/2) + 43200
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^6*c^(9/2) + 8640*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^4*a^3*b^4*c^(11/2) + 43200*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^4*a^4*b^2*c^(13/2) + 1728*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*
a^5*c^(15/2) + 960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^11*c^2 - 4416*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^9*c^3 + 13824*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^3*a^2*b^7*c^4 - 6912*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a
^3*b^5*c^5 + 25920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^3*c^6 + 3456
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b*c^7 + 162*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^2*b^12*c^(3/2) - 1008*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2
*a*b^10*c^(5/2) + 3456*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^8*c^(7/2
) - 4608*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^6*c^(9/2) + 8640*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^4*c^(11/2) + 1728*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^2*a^5*b^2*c^(13/2) + 576*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^2*a^6*c^(15/2) + 18*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^13*c - 144*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^11*c^2 + 576*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))*a^2*b^9*c^3 - 1152*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^7*
c^4 + 1728*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^5*c^5 + 576*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))*a^6*b*c^7 + b^14*sqrt(c) - 10*a*b^12*c^(3/2) + 48
*a^2*b^10*c^(5/2) - 128*a^3*b^8*c^(7/2) + 224*a^4*b^6*c^(9/2) - 192*a^5*b^4
*c^(11/2) + 256*a^6*b^2*c^(13/2) - 64*a^7*c^(15/2))/((2*(sqrt(c)*x - sqrt(c
```

```
*x^2 + b*x + a))^2*c + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*sqrt(c) + b^
2 - 2*a*c)^9*c^4*d^10)
```

3.1232 $\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(bd+2cdx)^{11}}\,dx$

Optimal. Leaf size=239

$$\frac{3\sqrt{a+bx+cx^2}}{8192c^3d^{11}\left(b^2-4ac\right)^2(b+2cx)^2}+\frac{\sqrt{a+bx+cx^2}}{4096c^3d^{11}\left(b^2-4ac\right)(b+2cx)^4}+\frac{3\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{16384c^{7/2}d^{11}\left(b^2-4ac\right)^{5/2}}-\frac{\left(a+bx+cx^2\right)^{3/2}}{128c^2d^{11}(b+2cx)^8}$$

```
[Out] -Sqrt[a + b*x + c*x^2]/(1024*c^3*d^11*(b + 2*c*x)^6) + Sqrt[a + b*x + c*x^2
]/(4096*c^3*(b^2 - 4*a*c)*d^11*(b + 2*c*x)^4) + (3*Sqrt[a + b*x + c*x^2])/(
8192*c^3*(b^2 - 4*a*c)^2*d^11*(b + 2*c*x)^2) - (a + b*x + c*x^2)^(3/2)/(128
*c^2*d^11*(b + 2*c*x)^8) - (a + b*x + c*x^2)^(5/2)/(20*c*d^11*(b + 2*c*x)^1
0) + (3*ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c]])/(16384
*c^(7/2)*(b^2 - 4*a*c)^(5/2)*d^11)
```

Rubi [A] time = 0.184909, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {684, 693, 688, 205}

$$\frac{3\sqrt{a+bx+cx^2}}{8192c^3d^{11}\left(b^2-4ac\right)^2(b+2cx)^2}+\frac{\sqrt{a+bx+cx^2}}{4096c^3d^{11}\left(b^2-4ac\right)(b+2cx)^4}+\frac{3\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{16384c^{7/2}d^{11}\left(b^2-4ac\right)^{5/2}}-\frac{\left(a+bx+cx^2\right)^{3/2}}{128c^2d^{11}(b+2cx)^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^11,x]
```

```
[Out] -Sqrt[a + b*x + c*x^2]/(1024*c^3*d^11*(b + 2*c*x)^6) + Sqrt[a + b*x + c*x^2
]/(4096*c^3*(b^2 - 4*a*c)*d^11*(b + 2*c*x)^4) + (3*Sqrt[a + b*x + c*x^2])/(
8192*c^3*(b^2 - 4*a*c)^2*d^11*(b + 2*c*x)^2) - (a + b*x + c*x^2)^(3/2)/(128
*c^2*d^11*(b + 2*c*x)^8) - (a + b*x + c*x^2)^(5/2)/(20*c*d^11*(b + 2*c*x)^1
0) + (3*ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c]])/(16384
*c^(7/2)*(b^2 - 4*a*c)^(5/2)*d^11)
```

Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(bd+2cdx)^{11}}\,dx &= -\frac{\left(a+bx+cx^2\right)^{5/2}}{20cd^{11}(b+2cx)^{10}}+\frac{\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(bd+2cdx)^9}\,dx}{8cd^2}\\
&= -\frac{\left(a+bx+cx^2\right)^{3/2}}{128c^2d^{11}(b+2cx)^8}-\frac{\left(a+bx+cx^2\right)^{5/2}}{20cd^{11}(b+2cx)^{10}}+\frac{3\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^7}\,dx}{256c^2d^4}\\
&= -\frac{\sqrt{a+bx+cx^2}}{1024c^3d^{11}(b+2cx)^6}-\frac{\left(a+bx+cx^2\right)^{3/2}}{128c^2d^{11}(b+2cx)^8}-\frac{\left(a+bx+cx^2\right)^{5/2}}{20cd^{11}(b+2cx)^{10}}+\frac{\int \frac{1}{(bd+2cdx)^5\sqrt{a+bx+cx^2}}\,dx}{2048c^3d^6}\\
&= -\frac{\sqrt{a+bx+cx^2}}{1024c^3d^{11}(b+2cx)^6}+\frac{\sqrt{a+bx+cx^2}}{4096c^3\left(b^2-4ac\right)d^{11}(b+2cx)^4}-\frac{\left(a+bx+cx^2\right)^{3/2}}{128c^2d^{11}(b+2cx)^8}-\frac{\left(a+bx+cx\right.}{20cd^{11}(b+2\ldots}\\
&= -\frac{\sqrt{a+bx+cx^2}}{1024c^3d^{11}(b+2cx)^6}+\frac{\sqrt{a+bx+cx^2}}{4096c^3\left(b^2-4ac\right)d^{11}(b+2cx)^4}+\frac{3\sqrt{a+bx+cx^2}}{8192c^3\left(b^2-4ac\right)^2d^{11}(b+2cx)^2}-\\
&= -\frac{\sqrt{a+bx+cx^2}}{1024c^3d^{11}(b+2cx)^6}+\frac{\sqrt{a+bx+cx^2}}{4096c^3\left(b^2-4ac\right)d^{11}(b+2cx)^4}+\frac{3\sqrt{a+bx+cx^2}}{8192c^3\left(b^2-4ac\right)^2d^{11}(b+2cx)^2}-\\
&= -\frac{\sqrt{a+bx+cx^2}}{1024c^3d^{11}(b+2cx)^6}+\frac{\sqrt{a+bx+cx^2}}{4096c^3\left(b^2-4ac\right)d^{11}(b+2cx)^4}+\frac{3\sqrt{a+bx+cx^2}}{8192c^3\left(b^2-4ac\right)^2d^{11}(b+2cx)^2}-
\end{aligned}$$

Mathematica [C] time = 0.0372217, size = 62, normalized size = 0.26

$$\frac{2(a+x(b+cx))^{7/2}\,{}_2F_1\left(\frac{7}{2},6;\frac{9}{2};\frac{4c(a+x(b+cx))}{4ac-b^2}\right)}{7d^{11}\left(b^2-4ac\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^11,x]
```

```
[Out] (2*(a + x*(b + c*x))^(7/2)*Hypergeometric2F1[7/2, 6, 9/2, (4*c*(a + x*(b +
c*x)))/(-b^2 + 4*a*c)])/(7*(b^2 - 4*a*c)^6*d^11)
```

Maple [B] time = 0.262, size = 1080, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^11,x)
```

```
[Out] -1/5120/d^11/c^10/(4*a*c-b^2)/(x+1/2*b/c)^10*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^
2)/c)^(7/2)+3/10240/d^11/c^8/(4*a*c-b^2)^2/(x+1/2*b/c)^8*((x+1/2*b/c)^2*c+1
/4*(4*a*c-b^2)/c)^(7/2)-1/5120/d^11/c^6/(4*a*c-b^2)^3/(x+1/2*b/c)^6*((x+1/2
*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(7/2)-1/5120/d^11/c^4/(4*a*c-b^2)^4/(x+1/2*b/c
)^4*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(7/2)-3/2560/d^11/c^2/(4*a*c-b^2)^5
/(x+1/2*b/c)^2*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(7/2)+3/2560/d^11/c/(4*a
*c-b^2)^5*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(5/2)+1/512/d^11/c/(4*a*c-b^2
)^5*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(3/2)*a-1/2048/d^11/c^2/(4*a*c-b^2)
^5*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(3/2)*b^2+3/1024/d^11/c/(4*a*c-b^2)^
5*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2)*a^2-3/2048/d^11/c^2/(4*a*c-b^2)^5
*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2)*a*b^2+3/16384/d^11/c^3/(4*a*c-b^2)
^5*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2)*b^4-3/256/d^11/c/(4*a*c-b^2)^5/(
(4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/2)*(4*(x+
1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))*a^3+9/1024/d^11/c^2/(4*a*c-
b^2)^5/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/2
)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))*a^2*b^2-9/4096/d^11
/c^3/(4*a*c-b^2)^5/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-
b^2)/c)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))*a*b^4+3
/16384/d^11/c^4/(4*a*c-b^2)^5/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1
/2*((4*a*c-b^2)/c)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/
c))*b^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^11,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(2*c*d*x+b*d)**11,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^11,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1233 $\int \frac{(a+bx+cx^2)^{5/2}}{(bd+2cdx)^{12}} dx$

Optimal. Leaf size=118

$$\frac{16\left(a+bx+cx^2\right)^{7/2}}{693d^{12}\left(b^2-4ac\right)^3(b+2cx)^7}+\frac{8\left(a+bx+cx^2\right)^{7/2}}{99d^{12}\left(b^2-4ac\right)^2(b+2cx)^9}+\frac{2\left(a+bx+cx^2\right)^{7/2}}{11d^{12}\left(b^2-4ac\right)(b+2cx)^{11}}$$

```
[Out] (2*(a + b*x + c*x^2)^(7/2))/(11*(b^2 - 4*a*c)*d^12*(b + 2*c*x)^11) + (8*(a
+ b*x + c*x^2)^(7/2))/(99*(b^2 - 4*a*c)^2*d^12*(b + 2*c*x)^9) + (16*(a + b*
x + c*x^2)^(7/2))/(693*(b^2 - 4*a*c)^3*d^12*(b + 2*c*x)^7)
```

Rubi [A] time = 0.056215, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {693, 682}

$$\frac{16\left(a+bx+cx^2\right)^{7/2}}{693d^{12}\left(b^2-4ac\right)^3(b+2cx)^7}+\frac{8\left(a+bx+cx^2\right)^{7/2}}{99d^{12}\left(b^2-4ac\right)^2(b+2cx)^9}+\frac{2\left(a+bx+cx^2\right)^{7/2}}{11d^{12}\left(b^2-4ac\right)(b+2cx)^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^12,x]
```

```
[Out] (2*(a + b*x + c*x^2)^(7/2))/(11*(b^2 - 4*a*c)*d^12*(b + 2*c*x)^11) + (8*(a
+ b*x + c*x^2)^(7/2))/(99*(b^2 - 4*a*c)^2*d^12*(b + 2*c*x)^9) + (16*(a + b*
x + c*x^2)^(7/2))/(693*(b^2 - 4*a*c)^3*d^12*(b + 2*c*x)^7)
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 682

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c,
0] && EqQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(bd+2cdx)^{12}}\,dx = \frac{2\left(a+bx+cx^2\right)^{7/2}}{11\left(b^2-4ac\right)d^{12}(b+2cx)^{11}} + \frac{4\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(bd+2cdx)^{10}}\,dx}{11\left(b^2-4ac\right)d^2}$$

$$= \frac{2\left(a+bx+cx^2\right)^{7/2}}{11\left(b^2-4ac\right)d^{12}(b+2cx)^{11}} + \frac{8\left(a+bx+cx^2\right)^{7/2}}{99\left(b^2-4ac\right)^2 d^{12}(b+2cx)^9} + \frac{8\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(bd+2cdx)^8}\,dx}{99\left(b^2-4ac\right)^2 d^4}$$

$$= \frac{2\left(a+bx+cx^2\right)^{7/2}}{11\left(b^2-4ac\right)d^{12}(b+2cx)^{11}} + \frac{8\left(a+bx+cx^2\right)^{7/2}}{99\left(b^2-4ac\right)^2 d^{12}(b+2cx)^9} + \frac{16\left(a+bx+cx^2\right)^{7/2}}{693\left(b^2-4ac\right)^3 d^{12}(b+2cx)^7}$$

Mathematica [A] time = 0.0815025, size = 110, normalized size = 0.93

$$\frac{2(a+x(b+cx))^{7/2}\left(16c^2\left(63a^2-28acx^2+8c^2x^4\right)+8b^2c\left(38cx^2-77a\right)+64bc^2x\left(4cx^2-7a\right)+176b^3cx+99b^4\right)}{693d^{12}\left(b^2-4ac\right)^3(b+2cx)^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^12,x]
```

```
[Out] (2*(a + x*(b + c*x))^(7/2)*(99*b^4 + 176*b^3*c*x + 64*b*c^2*x*(-7*a + 4*c*x
^2) + 8*b^2*c*(-77*a + 38*c*x^2) + 16*c^2*(63*a^2 - 28*a*c*x^2 + 8*c^2*x^4)
))/(693*(b^2 - 4*a*c)^3*d^12*(b + 2*c*x)^11)
```

Maple [A] time = 0.048, size = 133, normalized size = 1.1

$$-\frac{256\,c^4x^4+512\,bc^3x^3-896\,ac^3x^2+608\,b^2c^2x^2-896\,abc^2x+352\,b^3cx+2016\,a^2c^2-1232\,acb^2+198\,b^4}{693\,(2\,cx+b)^{11}\,d^{12}\left(64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6\right)}\left(cx^2+bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^12,x)
```

```
[Out] -2/693*(128*c^4*x^4+256*b*c^3*x^3-448*a*c^3*x^2+304*b^2*c^2*x^2-448*a*b*c^2
*x+176*b^3*c*x+1008*a^2*c^2-616*a*b^2*c+99*b^4)*(c*x^2+b*x+a)^(7/2)/(2*c*x+
b)^11/d^12/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^12,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^12,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(2*c*d*x+b*d)**12,x)
```

```
[Out] Timed out
```

Giac [B] time = 11.9111, size = 3201, normalized size = 27.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^12,x, algorithm="giac")
```

```
[Out] 1/5544*(29568*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^16*c^(17/2) + 236544*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^15*b*c^8 + 868560*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^14*b^2*c^(15/2) + 73920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^14*a
*c^(17/2) + 1940400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*b^3*c^7 + 517440
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a*b*c^8 + 2953104*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^12*b^4*c^(13/2) + 1600368*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^12*a*b^2*c^(15/2) + 162624*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*a^2
*c^(17/2) + 3256176*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b^5*c^6 + 287548
8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a*b^3*c^7 + 975744*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^11*a^2*b*c^8 + 2709168*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^10*b^6*c^(11/2) + 3307920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a*b^4
*c^(13/2) + 2583504*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^2*b^2*c^(15/2)
 + 133056*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^3*c^(17/2) + 1755600*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^7*c^5 + 2513280*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^9*a*b^5*c^6 + 3973200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a
^2*b^3*c^7 + 665280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*b*c^8 + 91080
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^8*c^(9/2) + 1227600*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^8*a*b^6*c^(11/2) + 3944160*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^8*a^2*b^4*c^(13/2) + 1401840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^8*a^3*b^2*c^(15/2) + 95040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^4*c^(17
/2) + 387024*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^9*c^4 + 319968*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^7*a*b^7*c^5 + 2670624*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^7*a^2*b^5*c^6 + 1615680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a
^3*b^3*c^7 + 380160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*b*c^8 + 13648
8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^10*c^(7/2) - 20592*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^6*a*b^8*c^(9/2) + 1284624*(sqrt(c)*x - sqrt(c*x^2 + b
```

*x + a))^6*a^2*b^6*c^(11/2) + 1092960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6
*a^3*b^4*c^(13/2) + 641520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*b^2*c^
(15/2) + 19008*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^5*c^(17/2) + 39864*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^11*c^3 - 58080*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^5*a*b^9*c^4 + 460944*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a
^2*b^7*c^5 + 418176*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^5*c^6 + 594
000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b^3*c^7 + 57024*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^5*a^5*b*c^8 + 9460*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^4*b^12*c^(5/2) - 27720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^10*c^(7
/2) + 132000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^8*c^(9/2) + 64240*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^6*c^(11/2) + 330000*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^4*a^4*b^4*c^(13/2) + 66000*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^4*a^5*b^2*c^(15/2) + 3520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
4*a^6*c^(17/2) + 1760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^13*c^2 - 7920
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^11*c^3 + 31680*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^3*a^2*b^9*c^4 - 14080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^3*a^3*b^7*c^5 + 113520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^5*c^6
+ 36960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b^3*c^7 + 7040*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^3*a^6*b*c^8 + 242*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^2*b^14*c^(3/2) - 1496*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^12*c^(
5/2) + 6072*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^10*c^(7/2) - 8800*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^8*c^(9/2) + 24640*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^2*a^4*b^6*c^(11/2) + 8976*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^2*a^5*b^4*c^(13/2) + 6512*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^
6*b^2*c^(15/2) - 704*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^7*c^(17/2) + 2
2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^15*c - 176*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))*a*b^13*c^2 + 792*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^11*c
^3 - 1760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^9*c^4 + 3520*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))*a^4*b^7*c^5 + 2992*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*a^6*b^3*c^7 - 704*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7*b*c^8 + b^1
6*sqrt(c) - 10*a*b^14*c^(3/2) + 52*a^2*b^12*c^(5/2) - 152*a^3*b^10*c^(7/2)
+ 320*a^4*b^8*c^(9/2) - 320*a^5*b^6*c^(11/2) + 640*a^6*b^4*c^(13/2) - 304*a
^7*b^2*c^(15/2) + 64*a^8*c^(17/2))/((2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
2*c + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*sqrt(c) + b^2 - 2*a*c)^11*c^4
*d^12)

3.1234 $\int \frac{(bd+2cdx)^4}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=117

$$\frac{3}{4}d^4\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}+\frac{3d^4\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}}+\frac{1}{2}d^4(b+2cx)^3\sqrt{a+bx+cx^2}$$

```
[Out] (3*(b^2 - 4*a*c)*d^4*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/4 + (d^4*(b + 2*c*x
)^3*Sqrt[a + b*x + c*x^2])/2 + (3*(b^2 - 4*a*c)^2*d^4*ArcTanh[(b + 2*c*x)/(
2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c])
```

Rubi [A] time = 0.0547876, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {692, 621, 206}

$$\frac{3}{4}d^4\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}+\frac{3d^4\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}}+\frac{1}{2}d^4(b+2cx)^3\sqrt{a+bx+cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^4/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (3*(b^2 - 4*a*c)*d^4*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/4 + (d^4*(b + 2*c*x
)^3*Sqrt[a + b*x + c*x^2])/2 + (3*(b^2 - 4*a*c)^2*d^4*ArcTanh[(b + 2*c*x)/(
2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c])
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(bd+2cdx)^4}{\sqrt{a+bx+cx^2}}\,dx = \frac{1}{2}d^4(b+2cx)^3\sqrt{a+bx+cx^2}+\frac{1}{4}\left(3\left(b^2-4ac\right)d^2\right)\int \frac{(bd+2cdx)^2}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{3}{4}\left(b^2-4ac\right)d^4(b+2cx)\sqrt{a+bx+cx^2}+\frac{1}{2}d^4(b+2cx)^3\sqrt{a+bx+cx^2}+\frac{1}{8}\left(3\left(b^2-4ac\right)^2d^4\right)\int \frac{}{\sqrt{}}$$
$$= \frac{3}{4}\left(b^2-4ac\right)d^4(b+2cx)\sqrt{a+bx+cx^2}+\frac{1}{2}d^4(b+2cx)^3\sqrt{a+bx+cx^2}+\frac{1}{4}\left(3\left(b^2-4ac\right)^2d^4\right)\text{Sub}$$
$$= \frac{3}{4}\left(b^2-4ac\right)d^4(b+2cx)\sqrt{a+bx+cx^2}+\frac{1}{2}d^4(b+2cx)^3\sqrt{a+bx+cx^2}+\frac{3\left(b^2-4ac\right)^2d^4\tanh^{-1}\left(\right.}{8\sqrt{c}}$$

Mathematica [A] time = 0.106997, size = 100, normalized size = 0.85

$$d^4\left(\frac{1}{4}(b+2cx)\sqrt{a+x(b+cx)}\left(4c\left(2cx^2-3a\right)+5b^2+8bcx\right)+\frac{3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{8\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^4/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] d^4*(((b + 2*c*x)*Sqrt[a + x*(b + c*x)]*(5*b^2 + 8*b*c*x + 4*c*(-3*a + 2*c*
x^2)))/4 + (3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b
+ c*x)])])/(8*Sqrt[c]))
```

Maple [B] time = 0.053, size = 242, normalized size = 2.1

$$4\,d^4c^3x^3\sqrt{cx^2+bx+a}+6\,d^4c^2bx^2\sqrt{cx^2+bx+a}+\frac{9\,d^4b^2cx}{2}\sqrt{cx^2+bx+a}+\frac{5\,d^4b^3}{4}\sqrt{cx^2+bx+a}+\frac{3\,d^4b^4}{8}\ln\left(\left(\frac{b}{2}+cx\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^4/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 4*d^4*c^3*x^3*(c*x^2+b*x+a)^(1/2)+6*d^4*c^2*b*x^2*(c*x^2+b*x+a)^(1/2)+9/2*d
^4*c*b^2*x*(c*x^2+b*x+a)^(1/2)+5/4*d^4*b^3*(c*x^2+b*x+a)^(1/2)+3/8*d^4*b^4*
ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-3*d^4*c^(1/2)*b^2*a*ln(
(1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-3*d^4*c*b*a*(c*x^2+b*x+a)^(1/2)-6*
d^4*c^2*a*x*(c*x^2+b*x+a)^(1/2)+6*d^4*c^(3/2)*a^2*ln((1/2*b+c*x)/c^(1/2)+(c
*x^2+b*x+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.3668, size = 710, normalized size = 6.07

$$\left[\frac{3\left(b^4 - 8\,ab^2c + 16\,a^2c^2\right)\sqrt{c}d^4\log\left(-8\,c^2x^2 - 8\,bcx - b^2 - 4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c} - 4\,ac\right) + 4\left(16\,c^4d^4x^3 + 24\,bc^3\right.}{16\,c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c)*d^4*log(-8*c^2*x^2 - 8*b*c*
x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(16*c^4*
d^4*x^3 + 24*b*c^3*d^4*x^2 + 6*(3*b^2*c^2 - 4*a*c^3)*d^4*x + (5*b^3*c - 12*
a*b*c^2)*d^4)*sqrt(c*x^2 + b*x + a))/c, -1/8*(3*(b^4 - 8*a*b^2*c + 16*a^2*c
^2)*sqrt(-c)*d^4*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2
*x^2 + b*c*x + a*c)) - 2*(16*c^4*d^4*x^3 + 24*b*c^3*d^4*x^2 + 6*(3*b^2*c^2
- 4*a*c^3)*d^4*x + (5*b^3*c - 12*a*b*c^2)*d^4)*sqrt(c*x^2 + b*x + a))/c]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^4\left(\int\frac{b^4}{\sqrt{a+bx+cx^2}}\,dx + \int\frac{16c^4x^4}{\sqrt{a+bx+cx^2}}\,dx + \int\frac{32bc^3x^3}{\sqrt{a+bx+cx^2}}\,dx + \int\frac{24b^2c^2x^2}{\sqrt{a+bx+cx^2}}\,dx + \int\frac{8b^3cx}{\sqrt{a+bx+cx^2}}\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**4/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] d**4*(Integral(b**4/sqrt(a + b*x + c*x**2), x) + Integral(16*c**4*x**4/sqrt
(a + b*x + c*x**2), x) + Integral(32*b*c**3*x**3/sqrt(a + b*x + c*x**2), x)
 + Integral(24*b**2*c**2*x**2/sqrt(a + b*x + c*x**2), x) + Integral(8*b**3*
c*x/sqrt(a + b*x + c*x**2), x))
```

Giac [A] time = 1.17606, size = 215, normalized size = 1.84

$$\frac{1}{4}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\,c^3d^4x + 3\,bc^2d^4\right)x + \frac{3\left(3\,b^2c^4d^4 - 4\,ac^5d^4\right)}{c^3}\right)x + \frac{5\,b^3c^3d^4 - 12\,abc^4d^4}{c^3}\right) - \frac{3\left(b^4d^4 - 8\,ab^2cd^4 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^2 + b*x + a)*(2*(4*(2*c^3*d^4*x + 3*b*c^2*d^4)*x + 3*(3*b^2*c^
4*d^4 - 4*a*c^5*d^4)/c^3)*x + (5*b^3*c^3*d^4 - 12*a*b*c^4*d^4)/c^3) - 3/8*(
b^4*d^4 - 8*a*b^2*c*d^4 + 16*a^2*c^2*d^4)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))*sqrt(c) - b))/sqrt(c)
```

3.1235 $\int \frac{(bd+2cdx)^3}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=59

$$\frac{4}{3}d^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}+\frac{2}{3}d^3(b+2cx)^2\sqrt{a+bx+cx^2}$$

```
[Out] (4*(b^2 - 4*a*c)*d^3*Sqrt[a + b*x + c*x^2])/3 + (2*d^3*(b + 2*c*x)^2*Sqrt[a
 + b*x + c*x^2])/3
```

Rubi [A] time = 0.0259978, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {692, 629}

$$\frac{4}{3}d^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}+\frac{2}{3}d^3(b+2cx)^2\sqrt{a+bx+cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^3/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (4*(b^2 - 4*a*c)*d^3*Sqrt[a + b*x + c*x^2])/3 + (2*d^3*(b + 2*c*x)^2*Sqrt[a
 + b*x + c*x^2])/3
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(bd+2cdx)^3}{\sqrt{a+bx+cx^2}}\,dx &= \frac{2}{3}d^3(b+2cx)^2\sqrt{a+bx+cx^2}+\frac{1}{3}\left(2\left(b^2-4ac\right)d^2\right)\int \frac{bd+2cdx}{\sqrt{a+bx+cx^2}}\,dx\\ &= \frac{4}{3}\left(b^2-4ac\right)d^3\sqrt{a+bx+cx^2}+\frac{2}{3}d^3(b+2cx)^2\sqrt{a+bx+cx^2}\end{aligned}$$

Mathematica [A] time = 0.0355574, size = 43, normalized size = 0.73

$$\frac{2}{3}d^3\sqrt{a+x(b+cx)}\left(4c\left(cx^2-2a\right)+3b^2+4bcx\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^3/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*d^3*Sqrt[a + x*(b + c*x)]*(3*b^2 + 4*b*c*x + 4*c*(-2*a + c*x^2)))/3
```

Maple [A] time = 0.046, size = 41, normalized size = 0.7

$$-\frac{2 d^3 \left(-4 c^2 x^2-4 b c x+8 a c-3 b^2\right)}{3} \sqrt{c x^2+b x+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^3/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -2/3*d^3*(-4*c^2*x^2-4*b*c*x+8*a*c-3*b^2)*(c*x^2+b*x+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.84848, size = 107, normalized size = 1.81

$$\frac{2}{3} \left(4 c^2 d^3 x^2+4 b c d^3 x+\left(3 b^2-8 a c\right) d^3\right) \sqrt{c x^2+b x+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(4*c^2*d^3*x^2 + 4*b*c*d^3*x + (3*b^2 - 8*a*c)*d^3)*sqrt(c*x^2 + b*x +
a)
```

Sympy [A] time = 0.387048, size = 97, normalized size = 1.64

$$-\frac{16 a c d^3 \sqrt{a+b x+c x^2}}{3}+2 b^2 d^3 \sqrt{a+b x+c x^2}+\frac{8 b c d^3 x \sqrt{a+b x+c x^2}}{3}+\frac{8 c^2 d^3 x^2 \sqrt{a+b x+c x^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**3/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] -16*a*c*d**3*sqrt(a + b*x + c*x**2)/3 + 2*b**2*d**3*sqrt(a + b*x + c*x**2)
+ 8*b*c*d**3*x*sqrt(a + b*x + c*x**2)/3 + 8*c**2*d**3*x**2*sqrt(a + b*x + c
```

```
*x**2)/3
```

Giac [A] time = 1.18905, size = 78, normalized size = 1.32

$$\frac{2}{3}\sqrt{cx^2+bx+a}\left(4\left(c^2d^3x+bcd^3\right)x+\frac{3\,b^2c^2d^3-8\,ac^3d^3}{c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(c*x^2 + b*x + a)*(4*(c^2*d^3*x + b*c*d^3)*x + (3*b^2*c^2*d^3 - 8*a
*c^3*d^3)/c^2)
```

3.1236 $\int \frac{(bd+2cdx)^2}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=75

$$\frac{d^2\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}}+d^2(b+2cx)\sqrt{a+bx+cx^2}$$

```
[Out] d^2*(b + 2*c*x)*Sqrt[a + b*x + c*x^2] + ((b^2 - 4*a*c)*d^2*ArcTanh[(b + 2*c
*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c])
```

Rubi [A] time = 0.0297058, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {692, 621, 206}

$$\frac{d^2\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}}+d^2(b+2cx)\sqrt{a+bx+cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^2/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] d^2*(b + 2*c*x)*Sqrt[a + b*x + c*x^2] + ((b^2 - 4*a*c)*d^2*ArcTanh[(b + 2*c
*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c])
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(bd+2cdx)^2}{\sqrt{a+bx+cx^2}}\,dx = d^2(b+2cx)\sqrt{a+bx+cx^2} + \frac{1}{2}\left(\left(b^2-4ac\right)d^2\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx$$
$$= d^2(b+2cx)\sqrt{a+bx+cx^2} + \left(\left(b^2-4ac\right)d^2\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)$$
$$= d^2(b+2cx)\sqrt{a+bx+cx^2} + \frac{\left(b^2-4ac\right)d^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}}$$

Mathematica [A] time = 0.0551479, size = 71, normalized size = 0.95

$$d^2\left(\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{2\sqrt{c}} + (b+2cx)\sqrt{a+x(b+cx)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^2/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] d^2*((b + 2*c*x)*Sqrt[a + x*(b + c*x)] + ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)
/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(2*Sqrt[c]))
```

Maple [A] time = 0.047, size = 108, normalized size = 1.4

$$2\,d^2cx\sqrt{cx^2+bx+a} + d^2b\sqrt{cx^2+bx+a} + \frac{b^2d^2}{2}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}} + \sqrt{cx^2+bx+a}\right)\frac{1}{\sqrt{c}} - 2\,d^2\sqrt{c}a\ln\left(\frac{b/2+cx}{\sqrt{c}} + \sqrt{cx^2+\dots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^2/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 2*d^2*c*x*(c*x^2+b*x+a)^(1/2)+d^2*b*(c*x^2+b*x+a)^(1/2)+1/2*d^2*b^2*ln((1/2
*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-2*d^2*c^(1/2)*a*ln((1/2*b+c*x)
/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.02853, size = 462, normalized size = 6.16

$$\left[-\frac{\left(b^2-4\,ac\right)\sqrt{c}d^2\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right)-4\left(2\,c^2d^2x+bcd^2\right)\sqrt{cx^2+bx+a}}{4\,c}, \dots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2 - 4*a*c)*sqrt(c)*d^2*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*
x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(2*c^2*d^2*x + b*c*d^2)*sqr
t(c*x^2 + b*x + a))/c, -1/2*((b^2 - 4*a*c)*sqrt(-c)*d^2*arctan(1/2*sqrt(c*x
^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(2*c^2*d^2*
x + b*c*d^2)*sqrt(c*x^2 + b*x + a))/c]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int\frac{b^2}{\sqrt{a+bx+cx^2}}\,dx+\int\frac{4c^2x^2}{\sqrt{a+bx+cx^2}}\,dx+\int\frac{4bcx}{\sqrt{a+bx+cx^2}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**2/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] d**2*(Integral(b**2/sqrt(a + b*x + c*x**2), x) + Integral(4*c**2*x**2/sqrt(
a + b*x + c*x**2), x) + Integral(4*b*c*x/sqrt(a + b*x + c*x**2), x))
```

Giac [A] time = 1.15866, size = 105, normalized size = 1.4

$$\left(2\,cd^2x+bd^2\right)\sqrt{cx^2+bx+a}-\frac{\left(b^2d^2-4\,acd^2\right)\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] (2*c*d^2*x + b*d^2)*sqrt(c*x^2 + b*x + a) - 1/2*(b^2*d^2 - 4*a*c*d^2)*log(a
bs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/sqrt(c)
```

3.1237 $\int \frac{bd+2cdx}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=17

$$2d\sqrt{a+bx+cx^2}$$

```
[Out] 2*d*Sqrt[a + b*x + c*x^2]
```

Rubi [A] time = 0.0060926, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {629}

$$2d\sqrt{a+bx+cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] 2*d*Sqrt[a + b*x + c*x^2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{bd+2cdx}{\sqrt{a+bx+cx^2}}\,dx = 2d\sqrt{a+bx+cx^2}$$

Mathematica [A] time = 0.005941, size = 16, normalized size = 0.94

$$2d\sqrt{a+x(b+cx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] 2*d*Sqrt[a + x*(b + c*x)]
```

Maple [A] time = 0.041, size = 16, normalized size = 0.9

$$2\,d\sqrt{cx^2+bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 2*d*(c*x^2+b*x+a)^(1/2)
```

Maxima [A] time = 2.27746, size = 20, normalized size = 1.18

$$2\sqrt{cx^2+bx+a}d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(c*x^2 + b*x + a)*d
```

Fricas [A] time = 2.63262, size = 36, normalized size = 2.12

$$2\sqrt{cx^2+bx+a}d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(c*x^2 + b*x + a)*d
```

Sympy [A] time = 0.194262, size = 15, normalized size = 0.88

$$2d\sqrt{a+bx+cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] 2*d*sqrt(a + b*x + c*x**2)
```

Giac [A] time = 1.18096, size = 20, normalized size = 1.18

$$2\sqrt{cx^2+bx+a}d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(c*x^2 + b*x + a)*d
```

3.1238 $\int \frac{1}{(bd+2cdx)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=55

$$\frac{\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{\sqrt{c}d\sqrt{b^2-4ac}}$$

```
[Out] ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c]]/(Sqrt[c]*Sqrt[b
^2 - 4*a*c]*d)
```

Rubi [A] time = 0.0333378, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {688, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{\sqrt{c}d\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c]]/(Sqrt[c]*Sqrt[b
^2 - 4*a*c]*d)
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)\sqrt{a+bx+cx^2}}\,dx = (4c)\,\text{Subst}\left(\int \frac{1}{2b^2cd-8ac^2d+8c^2dx^2}\,dx, x, \sqrt{a+bx+cx^2}\right)$$
$$= \frac{\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{\sqrt{c}\sqrt{b^2-4ac}d}$$

Mathematica [A] time = 0.0339335, size = 54, normalized size = 0.98

$$\frac{\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+x(b+cx)}}{\sqrt{b^2-4ac}}\right)}{\sqrt{c}d\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ArcTan[(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])/Sqrt[b^2 - 4*a*c]]/(Sqrt[c]*Sqrt[b
^2 - 4*a*c]*d)
```

Maple [B] time = 0.192, size = 101, normalized size = 1.8

$$-\frac{1}{cd}\ln\left(\left(\frac{4\,ac-b^2}{2\,c}+\frac{1}{2}\sqrt{\frac{4\,ac-b^2}{c}}\sqrt{4\left(x+1/2\,\frac{b}{c}\right)^2c+\frac{4\,ac-b^2}{c}}\right)\left(x+\frac{b}{2\,c}\right)^{-1}\right)\frac{1}{\sqrt{\frac{4\,ac-b^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/d/c/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/2
)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.85191, size = 381, normalized size = 6.93

$$\left[-\frac{\sqrt{-b^2c+4\,ac^2}\log\left(-\frac{4\,c^2x^2+4\,bcx-b^2+8\,ac-4\,\sqrt{-b^2c+4\,ac^2}\sqrt{cx^2+bx+a}}{4\,c^2x^2+4\,bcx+b^2}\right)}{2\left(b^2c-4\,ac^2\right)d},-\frac{\arctan\left(\frac{\sqrt{b^2c-4\,ac^2}\sqrt{cx^2+bx+a}}{2\left(c^2x^2+bcx+ac\right)}\right)}{\sqrt{b^2c-4\,ac^2}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b^2*c + 4*a*c^2)*log(-(4*c^2*x^2 + 4*b*c*x - b^2 + 8*a*c - 4*sq
rt(-b^2*c + 4*a*c^2)*sqrt(c*x^2 + b*x + a))/(4*c^2*x^2 + 4*b*c*x + b^2))/((
b^2*c - 4*a*c^2)*d), -arctan(1/2*sqrt(b^2*c - 4*a*c^2)*sqrt(c*x^2 + b*x + a
)/(c^2*x^2 + b*c*x + a*c))/(sqrt(b^2*c - 4*a*c^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{b\sqrt{a+bx+cx^2}+2cx\sqrt{a+bx+cx^2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/(b*sqrt(a + b*x + c*x**2) + 2*c*x*sqrt(a + b*x + c*x**2)), x)/d
```

Giac [A] time = 1.15107, size = 88, normalized size = 1.6

$$\frac{2 \arctan\left(-\frac{2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)c+b\sqrt{c}}{\sqrt{b^2c-4\,ac^2}}\right)}{\sqrt{b^2c-4\,ac^2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*c + b*sqrt(c))/sqrt(b^2*c
- 4*a*c^2))/(sqrt(b^2*c - 4*a*c^2)*d)
```

3.1239 $\int \frac{1}{(bd+2cdx)^2\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=37

$$\frac{2\sqrt{a+bx+cx^2}}{d^2\left(b^2-4ac\right)(b+2cx)}$$

```
[Out] (2*Sqrt[a + b*x + c*x^2])/((b^2 - 4*a*c)*d^2*(b + 2*c*x))
```

Rubi [A] time = 0.0151481, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {682}

$$\frac{2\sqrt{a+bx+cx^2}}{d^2\left(b^2-4ac\right)(b+2cx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*Sqrt[a + b*x + c*x^2])/((b^2 - 4*a*c)*d^2*(b + 2*c*x))
```

Rule 682

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c,
0] && EqQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)^2\sqrt{a+bx+cx^2}}\,dx = \frac{2\sqrt{a+bx+cx^2}}{\left(b^2-4ac\right)d^2(b+2cx)}$$

Mathematica [A] time = 0.0148778, size = 36, normalized size = 0.97

$$\frac{2\sqrt{a+x(b+cx)}}{d^2\left(b^2-4ac\right)(b+2cx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*Sqrt[a + x*(b + c*x)])/((b^2 - 4*a*c)*d^2*(b + 2*c*x))
```

Maple [A] time = 0.045, size = 38, normalized size = 1.

$$-2\,\frac{\sqrt{cx^2+bx+a}}{(2\,cx+b)\,d^2\left(4\,ac-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^2/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -2*(c*x^2+b*x+a)^(1/2)/(2*c*x+b)/d^2/(4*a*c-b^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.04856, size = 101, normalized size = 2.73

$$\frac{2\sqrt{cx^2+bx+a}}{2\left(b^2c-4\,ac^2\right)d^2x+\left(b^3-4\,abc\right)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(c*x^2 + b*x + a)/(2*(b^2*c - 4*a*c^2)*d^2*x + (b^3 - 4*a*b*c)*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{b^2\sqrt{a+bx+cx^2}+4bcx\sqrt{a+bx+cx^2}+4c^2x^2\sqrt{a+bx+cx^2}}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**2/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/(b**2*sqrt(a + b*x + c*x**2) + 4*b*c*x*sqrt(a + b*x + c*x**2) +
4*c**2*x**2*sqrt(a + b*x + c*x**2)), x)/d**2
```

Giac [B] time = 1.17538, size = 194, normalized size = 5.24

$$-\frac{\sqrt{c}\mathrm{sgn}\left(\frac{1}{2\,cdx+bd}\right)\mathrm{sgn}\left(c\right)\mathrm{sgn}\left(d\right)}{b^2cd^2-4\,ac^2d^2}+\frac{\sqrt{-\frac{b^2cd^2}{(2\,cdx+bd)^2}+\frac{4\,ac^2d^2}{(2\,cdx+bd)^2}+cc^2}}{b^2c^3d^2\mathrm{sgn}\left(\frac{1}{2\,cdx+bd}\right)\mathrm{sgn}\left(c\right)\mathrm{sgn}\left(d\right)-4\,ac^4d^2\mathrm{sgn}\left(\frac{1}{2\,cdx+bd}\right)\mathrm{sgn}\left(c\right)\mathrm{sgn}\left(d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(c)*sgn(1/(2*c*d*x + b*d))*sgn(c)*sgn(d)/(b^2*c*d^2 - 4*a*c^2*d^2) + s
qrt(-b^2*c*d^2/(2*c*d*x + b*d)^2 + 4*a*c^2*d^2/(2*c*d*x + b*d)^2 + c)*c^2/(
b^2*c^3*d^2*sgn(1/(2*c*d*x + b*d))*sgn(c)*sgn(d) - 4*a*c^4*d^2*sgn(1/(2*c*d
*x + b*d))*sgn(c)*sgn(d))
```

3.1240 $\int \frac{1}{(bd+2cdx)^3\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=95

$$\frac{\sqrt{a+bx+cx^2}}{d^3\left(b^2-4ac\right)(b+2cx)^2}+\frac{\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{2\sqrt{c}d^3\left(b^2-4ac\right)^{3/2}}$$

```
[Out] Sqrt[a + b*x + c*x^2]/((b^2 - 4*a*c)*d^3*(b + 2*c*x)^2) + ArcTan[(2*Sqrt[c]
*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c]]/(2*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*d
^3)
```

Rubi [A] time = 0.055632, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {693, 688, 205}

$$\frac{\sqrt{a+bx+cx^2}}{d^3\left(b^2-4ac\right)(b+2cx)^2}+\frac{\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{2\sqrt{c}d^3\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] Sqrt[a + b*x + c*x^2]/((b^2 - 4*a*c)*d^3*(b + 2*c*x)^2) + ArcTan[(2*Sqrt[c]
*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c]]/(2*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*d
^3)
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)^3\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{a+bx+cx^2}}{\left(b^2-4ac\right)d^3(b+2cx)^2} + \frac{\int \frac{1}{(bd+2cdx)\sqrt{a+bx+cx^2}}\,dx}{2\left(b^2-4ac\right)d^2}$$
$$= \frac{\sqrt{a+bx+cx^2}}{\left(b^2-4ac\right)d^3(b+2cx)^2} + \frac{(2c)\,\text{Subst}\left(\int \frac{1}{2b^2cd-8ac^2d+8c^2dx^2}\,dx, x, \sqrt{a+bx+cx^2}\right)}{\left(b^2-4ac\right)d^2}$$
$$= \frac{\sqrt{a+bx+cx^2}}{\left(b^2-4ac\right)d^3(b+2cx)^2} + \frac{\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{2\sqrt{c}\left(b^2-4ac\right)^{3/2}d^3}$$

Mathematica [A] time = 0.284009, size = 107, normalized size = 1.13

$$\frac{\sqrt{a+x(b+cx)}\left(\frac{2\left(b^2-4ac\right)}{(b+2cx)^2}+\frac{\tanh^{-1}\left(2\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\right)}{\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}\right)}{2d^3\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*((2*(b^2 - 4*a*c))/(b + 2*c*x)^2 + ArcTanh[2*Sqrt[(c
*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]]/Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a
*c)]))/(2*(b^2 - 4*a*c)^2*d^3)
```

Maple [B] time = 0.192, size = 174, normalized size = 1.8

$$-\frac{1}{4c^2d^3\left(4ac-b^2\right)}\sqrt{\left(x+\frac{b}{2c}\right)^2c+\frac{4ac-b^2}{4c}}\left(x+\frac{b}{2c}\right)^{-2}+\frac{1}{2cd^3\left(4ac-b^2\right)}\ln\left(\left(\frac{4ac-b^2}{2c}+\frac{1}{2}\sqrt{\frac{4ac-b^2}{c}}\sqrt{4\left(x+1\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^3/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/4/d^3/c^2/(4*a*c-b^2)/(x+1/2*b/c)^2*((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^
(1/2)+1/2/d^3/c/(4*a*c-b^2)/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2
*((4*a*c-b^2)/c)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c)
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.18764, size = 936, normalized size = 9.85

$$\left[\frac{\left(4\,c^2x^2+4\,bcx+b^2\right)\sqrt{-b^2c+4\,ac^2}\log\left(-\frac{4\,c^2x^2+4\,bcx-b^2+8\,ac+4\,\sqrt{-b^2c+4\,ac^2}\sqrt{cx^2+bx+a}}{4\,c^2x^2+4\,bcx+b^2}\right)+4\left(b^2c-4\,ac^2\right)\sqrt{cx^2+bx+a}}{4\left(4\left(b^4c^3-8\,ab^2c^4+16\,a^2c^5\right)d^3x^2+4\left(b^5c^2-8\,ab^3c^3+16\,a^2bc^4\right)d^3x+\left(b^6c-8\,ab^4c^2+16\,a^2b^2c^3\right)d^3\right)},-\frac{}{2\left(4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((4*c^2*x^2 + 4*b*c*x + b^2)*sqrt(-b^2*c + 4*a*c^2)*log(-(4*c^2*x^2 +
4*b*c*x - b^2 + 8*a*c + 4*sqrt(-b^2*c + 4*a*c^2)*sqrt(c*x^2 + b*x + a))/(4*
c^2*x^2 + 4*b*c*x + b^2)) + 4*(b^2*c - 4*a*c^2)*sqrt(c*x^2 + b*x + a))/(4*(
b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d^3*x^2 + 4*(b^5*c^2 - 8*a*b^3*c^3 + 16
*a^2*b*c^4)*d^3*x + (b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^3), -1/2*((4*c
^2*x^2 + 4*b*c*x + b^2)*sqrt(b^2*c - 4*a*c^2)*arctan(1/2*sqrt(b^2*c - 4*a*c
^2)*sqrt(c*x^2 + b*x + a)/(c^2*x^2 + b*c*x + a*c)) - 2*(b^2*c - 4*a*c^2)*sq
rt(c*x^2 + b*x + a))/(4*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d^3*x^2 + 4*(b
^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^3*x + (b^6*c - 8*a*b^4*c^2 + 16*a^2*
b^2*c^3)*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{b^3\sqrt{a+bx+cx^2}+6b^2cx\sqrt{a+bx+cx^2}+12bc^2x^2\sqrt{a+bx+cx^2}+8c^3x^3\sqrt{a+bx+cx^2}}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**3/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/(b**3*sqrt(a + b*x + c*x**2) + 6*b**2*c*x*sqrt(a + b*x + c*x**2)
 + 12*b*c**2*x**2*sqrt(a + b*x + c*x**2) + 8*c**3*x**3*sqrt(a + b*x + c*x**
2)), x)/d**3
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1241 $\int \frac{1}{(bd+2cdx)^4\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=79

$$\frac{4\sqrt{a+bx+cx^2}}{3d^4\left(b^2-4ac\right)^2(b+2cx)}+\frac{2\sqrt{a+bx+cx^2}}{3d^4\left(b^2-4ac\right)(b+2cx)^3}$$

```
[Out] (2*Sqrt[a + b*x + c*x^2])/(3*(b^2 - 4*a*c)*d^4*(b + 2*c*x)^3) + (4*Sqrt[a +
 b*x + c*x^2])/(3*(b^2 - 4*a*c)^2*d^4*(b + 2*c*x))
```

Rubi [A] time = 0.0319232, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {693, 682}

$$\frac{4\sqrt{a+bx+cx^2}}{3d^4\left(b^2-4ac\right)^2(b+2cx)}+\frac{2\sqrt{a+bx+cx^2}}{3d^4\left(b^2-4ac\right)(b+2cx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^4*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*Sqrt[a + b*x + c*x^2])/(3*(b^2 - 4*a*c)*d^4*(b + 2*c*x)^3) + (4*Sqrt[a +
 b*x + c*x^2])/(3*(b^2 - 4*a*c)^2*d^4*(b + 2*c*x))
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 682

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c,
0] && EqQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(bd+2cdx)^4\sqrt{a+bx+cx^2}}\,dx &= \frac{2\sqrt{a+bx+cx^2}}{3\left(b^2-4ac\right)d^4(b+2cx)^3}+\frac{2\int \frac{1}{(bd+2cdx)^2\sqrt{a+bx+cx^2}}\,dx}{3\left(b^2-4ac\right)d^2}\\ &= \frac{2\sqrt{a+bx+cx^2}}{3\left(b^2-4ac\right)d^4(b+2cx)^3}+\frac{4\sqrt{a+bx+cx^2}}{3\left(b^2-4ac\right)^2d^4(b+2cx)}\end{aligned}$$

Mathematica [A] time = 0.0275424, size = 60, normalized size = 0.76

$$\frac{2\sqrt{a+x(b+cx)}\left(-4c\left(a-2cx^2\right)+3b^2+8bcx\right)}{3d^4\left(b^2-4ac\right)^2(b+2cx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^4*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*Sqrt[a + x*(b + c*x)]*(3*b^2 + 8*b*c*x - 4*c*(a - 2*c*x^2)))/(3*(b^2 - 4
*a*c)^2*d^4*(b + 2*c*x)^3)
```

Maple [A] time = 0.044, size = 70, normalized size = 0.9

$$-\frac{-16\,c^2x^2-16\,bcx+8\,ac-6\,b^2}{3\,(2\,cx+b)^3\,d^4\left(16\,a^2c^2-8\,acb^2+b^4\right)}\sqrt{cx^2+bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^4/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -2/3*(-8*c^2*x^2-8*b*c*x+4*a*c-3*b^2)*(c*x^2+b*x+a)^(1/2)/(2*c*x+b)^3/d^4/(
16*a^2*c^2-8*a*b^2*c+b^4)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^4/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 7.41221, size = 346, normalized size = 4.38

$$\frac{2\left(8\,c^2x^2+8\,bcx+3\,b^2-4\,ac\right)\sqrt{cx^2+bx+a}}{3\left(8\left(b^4c^3-8\,ab^2c^4+16\,a^2c^5\right)d^4x^3+12\left(b^5c^2-8\,ab^3c^3+16\,a^2bc^4\right)d^4x^2+6\left(b^6c-8\,ab^4c^2+16\,a^2b^2c^3\right)d^4x+\left(b^7-8\,ab^5c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^4/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(8*c^2*x^2 + 8*b*c*x + 3*b^2 - 4*a*c)*sqrt(c*x^2 + b*x + a)/(8*(b^4*c^3
 - 8*a*b^2*c^4 + 16*a^2*c^5)*d^4*x^3 + 12*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b
*c^4)*d^4*x^2 + 6*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^4*x + (b^7 - 8*a
*b^5*c + 16*a^2*b^3*c^2)*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{b^4\sqrt{a+bx+cx^2}+8b^3cx\sqrt{a+bx+cx^2}+24b^2c^2x^2\sqrt{a+bx+cx^2}+32bc^3x^3\sqrt{a+bx+cx^2}+16c^4x^4\sqrt{a+bx+cx^2}}\,dx}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**4/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/(b**4*sqrt(a + b*x + c*x**2) + 8*b**3*c*x*sqrt(a + b*x + c*x**2)
 + 24*b**2*c**2*x**2*sqrt(a + b*x + c*x**2) + 32*b*c**3*x**3*sqrt(a + b*x +
 c*x**2) + 16*c**4*x**4*sqrt(a + b*x + c*x**2)), x)/d**4
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^4/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1242 $\int \frac{(bd+2cdx)^4}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=102

$$6\sqrt{c}d^4\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)+12cd^4(b+2cx)\sqrt{a+bx+cx^2}-\frac{2d^4(b+2cx)^3}{\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*d^4*(b + 2*c*x)^3)/Sqrt[a + b*x + c*x^2] + 12*c*d^4*(b + 2*c*x)*Sqrt[a
+ b*x + c*x^2] + 6*Sqrt[c]*(b^2 - 4*a*c)*d^4*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]
*Sqrt[a + b*x + c*x^2])]
```

Rubi [A] time = 0.0533681, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {686, 692, 621, 206}

$$6\sqrt{c}d^4\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)+12cd^4(b+2cx)\sqrt{a+bx+cx^2}-\frac{2d^4(b+2cx)^3}{\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^4/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*d^4*(b + 2*c*x)^3)/Sqrt[a + b*x + c*x^2] + 12*c*d^4*(b + 2*c*x)*Sqrt[a
+ b*x + c*x^2] + 6*Sqrt[c]*(b^2 - 4*a*c)*d^4*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]
*Sqrt[a + b*x + c*x^2])]
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(bd+2cdx)^4}{\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2d^4(b+2cx)^3}{\sqrt{a+bx+cx^2}} + \left(12cd^2\right)\int \frac{(bd+2cdx)^2}{\sqrt{a+bx+cx^2}}\,dx$$

$$= -\frac{2d^4(b+2cx)^3}{\sqrt{a+bx+cx^2}} + 12cd^4(b+2cx)\sqrt{a+bx+cx^2} + \left(6c\left(b^2-4ac\right)d^4\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx$$

$$= -\frac{2d^4(b+2cx)^3}{\sqrt{a+bx+cx^2}} + 12cd^4(b+2cx)\sqrt{a+bx+cx^2} + \left(12c\left(b^2-4ac\right)d^4\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx\right.$$

$$= -\frac{2d^4(b+2cx)^3}{\sqrt{a+bx+cx^2}} + 12cd^4(b+2cx)\sqrt{a+bx+cx^2} + 6\sqrt{c}\left(b^2-4ac\right)d^4\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx}}\right.$$

Mathematica [A] time = 0.410818, size = 139, normalized size = 1.36

$$d^4\left(-\frac{6c^{3/2}(a+x(b+cx))^{3/2}\sinh^{-1}\left(\frac{b+2cx}{\sqrt{c}\sqrt{4a-\frac{b^2}{c}}}\right)}{\sqrt{4a-\frac{b^2}{c}}\left(\frac{c(a+x(b+cx))}{4ac-b^2}\right)^{3/2}}-\frac{2(b+2cx)\left(-2c\left(3a+cx^2\right)+b^2-2bcx\right)}{\sqrt{a+x(b+cx)}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^4/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] d^4*((-2*(b + 2*c*x)*(b^2 - 2*b*c*x - 2*c*(3*a + c*x^2)))/Sqrt[a + x*(b + c
*x)] - (6*c^(3/2)*(a + x*(b + c*x))^(3/2)*ArcSinh[(b + 2*c*x)/(Sqrt[4*a - b
^2/c]*Sqrt[c])])/(Sqrt[4*a - b^2/c]*((c*(a + x*(b + c*x)))/(-b^2 + 4*a*c))^
(3/2)))
```

Maple [B] time = 0.052, size = 340, normalized size = 3.3

$$-6\,\frac{cd^4b^4x}{\left(4\,ac-b^2\right)\sqrt{cx^2+bx+a}}+12\,\frac{cd^4b^3a}{\left(4\,ac-b^2\right)\sqrt{cx^2+bx+a}}+24\,\frac{c^2d^4b^2ax}{\left(4\,ac-b^2\right)\sqrt{cx^2+bx+a}}+8\,\frac{d^4c^3x^3}{\sqrt{cx^2+bx+a}}-3\,\frac{}{(4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^4/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -6*d^4*c*b^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+12*d^4*c*b^3*a/(4*a*c-b^2)/(
c*x^2+b*x+a)^(1/2)+24*d^4*c^2*b^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+8*d^4
*c^3*x^3/(c*x^2+b*x+a)^(1/2)-3*d^4*b^5/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-5*d^
4*b^3/(c*x^2+b*x+a)^(1/2)+12*d^4*c^2*b*x^2/(c*x^2+b*x+a)^(1/2)-6*d^4*c*b^2*
x/(c*x^2+b*x+a)^(1/2)+6*d^4*c^(1/2)*b^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))+12*d^4*c*b*a/(c*x^2+b*x+a)^(1/2)+24*d^4*c^2*a*x/(c*x^2+b*x+a)^(1/2
)-24*d^4*c^(3/2)*a*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.59705, size = 788, normalized size = 7.73

$$\left[-\frac{3\left(\left(b^2c-4\,ac^2\right)d^4x^2+\left(b^3-4\,abc\right)d^4x+\left(ab^2-4\,a^2c\right)d^4\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-\right.}{cx^2+bx+a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-(3*((b^2*c - 4*a*c^2)*d^4*x^2 + (b^3 - 4*a*b*c)*d^4*x + (a*b^2 - 4*a^2*c)
*d^4)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c
*x + b)*sqrt(c) - 4*a*c) - 2*(4*c^3*d^4*x^3 + 6*b*c^2*d^4*x^2 + 12*a*c^2*d^
4*x - (b^3 - 6*a*b*c)*d^4)*sqrt(c*x^2 + b*x + a))/(c*x^2 + b*x + a), -2*(3*
((b^2*c - 4*a*c^2)*d^4*x^2 + (b^3 - 4*a*b*c)*d^4*x + (a*b^2 - 4*a^2*c)*d^4)
*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 +
b*c*x + a*c)) - (4*c^3*d^4*x^3 + 6*b*c^2*d^4*x^2 + 12*a*c^2*d^4*x - (b^3 -
6*a*b*c)*d^4)*sqrt(c*x^2 + b*x + a))/(c*x^2 + b*x + a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^4\left(\int\frac{b^4}{a\sqrt{a+bx+cx^2}+bx\sqrt{a+bx+cx^2}+cx^2\sqrt{a+bx+cx^2}}\,dx+\int\frac{16c^4x^4}{a\sqrt{a+bx+cx^2}+bx\sqrt{a+bx+cx^2}+cx^2\sqrt{a+bx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**4/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] d**4*(Integral(b**4/(a*sqrt(a + b*x + c*x**2) + b*x*sqrt(a + b*x + c*x**2)
+ c*x**2*sqrt(a + b*x + c*x**2)), x) + Integral(16*c**4*x**4/(a*sqrt(a + b*
x + c*x**2) + b*x*sqrt(a + b*x + c*x**2) + c*x**2*sqrt(a + b*x + c*x**2)),
x) + Integral(32*b*c**3*x**3/(a*sqrt(a + b*x + c*x**2) + b*x*sqrt(a + b*x +
 c*x**2) + c*x**2*sqrt(a + b*x + c*x**2)), x) + Integral(24*b**2*c**2*x**2/
(a*sqrt(a + b*x + c*x**2) + b*x*sqrt(a + b*x + c*x**2) + c*x**2*sqrt(a + b*
x + c*x**2)), x) + Integral(8*b**3*c*x/(a*sqrt(a + b*x + c*x**2) + b*x*sqrt
(a + b*x + c*x**2) + c*x**2*sqrt(a + b*x + c*x**2)), x))
```

Giac [B] time = 1.21195, size = 336, normalized size = 3.29

$$-\frac{6\left(b^2cd^4-4\,ac^2d^4\right)\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{\sqrt{c}}+\frac{2\left(2\left(\left(\frac{2\left(b^2c^5d^4-4\,ac^6d^4\right)x}{b^2c^2-4\,ac^3}+\frac{3\left(b^3c^4d^4-4\,abc^5d^4\right)}{b^2c^2-4\,ac^3}\right)x+\frac{6\left(ab^2c^4d^4-4\right.}{b^2c^2-4\,a}\right.\right.}{\sqrt{cx^2+bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -6*(b^2*c*d^4 - 4*a*c^2*d^4)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*sqrt(c) - b))/sqrt(c) + 2*(2*((2*(b^2*c^5*d^4 - 4*a*c^6*d^4)*x/(b^2*c^2 -
4*a*c^3) + 3*(b^3*c^4*d^4 - 4*a*b*c^5*d^4)/(b^2*c^2 - 4*a*c^3))*x + 6*(a*b^
2*c^4*d^4 - 4*a^2*c^5*d^4)/(b^2*c^2 - 4*a*c^3))*x - (b^5*c^2*d^4 - 10*a*b^3
*c^3*d^4 + 24*a^2*b*c^4*d^4)/(b^2*c^2 - 4*a*c^3))/sqrt(c*x^2 + b*x + a)
```

3.1243 $\int \frac{(bd+2cdx)^3}{\left(a+bx+cx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=48

$$16cd^3\sqrt{a+bx+cx^2}-\frac{2d^3(b+2cx)^2}{\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*d^3*(b + 2*c*x)^2)/Sqrt[a + b*x + c*x^2] + 16*c*d^3*Sqrt[a + b*x + c*x^
2]
```

Rubi [A] time = 0.0229013, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {686, 629}

$$16cd^3\sqrt{a+bx+cx^2}-\frac{2d^3(b+2cx)^2}{\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^3/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*d^3*(b + 2*c*x)^2)/Sqrt[a + b*x + c*x^2] + 16*c*d^3*Sqrt[a + b*x + c*x^
2]
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(bd+2cdx)^3}{\left(a+bx+cx^2\right)^{3/2}} \, dx &= -\frac{2d^3(b+2cx)^2}{\sqrt{a+bx+cx^2}}+\left(8cd^2\right)\int \frac{bd+2cdx}{\sqrt{a+bx+cx^2}} \, dx\\ &= -\frac{2d^3(b+2cx)^2}{\sqrt{a+bx+cx^2}}+16cd^3\sqrt{a+bx+cx^2}\end{aligned}$$

Mathematica [A] time = 0.0287474, size = 40, normalized size = 0.83

$$\frac{d^3\left(8c\left(2a+cx^2\right)-2b^2+8bcx\right)}{\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^3/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (d^3*(-2*b^2 + 8*b*c*x + 8*c*(2*a + c*x^2)))/Sqrt[a + x*(b + c*x)]
```

Maple [A] time = 0.044, size = 41, normalized size = 0.9

$$2\,{\frac {{d}^{3} \left(4\,{c}^{2}{x}^{2}+4\,bcx+8\,ac-{b}^{2} \right) }{\sqrt {c{x}^{2}+bx+a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^3/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 2*d^3*(4*c^2*x^2+4*b*c*x+8*a*c-b^2)/(c*x^2+b*x+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.55734, size = 101, normalized size = 2.1

$$\frac{2 \left(4 \, c^{2} d^{3} x^{2} + 4 \, b c d^{3} x - {\left(b^{2} - 8 \, a c\right)} d^{3}\right)}{\sqrt{c x^{2} + b x + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(4*c^2*d^3*x^2 + 4*b*c*d^3*x - (b^2 - 8*a*c)*d^3)/sqrt(c*x^2 + b*x + a)
```

Sympy [A] time = 1.36838, size = 92, normalized size = 1.92

$$\frac{16 a c d^{3}}{\sqrt{a + b x + c x^{2}}} - \frac{2 b^{2} d^{3}}{\sqrt{a + b x + c x^{2}}} + \frac{8 b c d^{3} x}{\sqrt{a + b x + c x^{2}}} + \frac{8 c^{2} d^{3} x^{2}}{\sqrt{a + b x + c x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**3/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] 16*a*c*d**3/sqrt(a + b*x + c*x**2) - 2*b**2*d**3/sqrt(a + b*x + c*x**2) + 8
*b*c*d**3*x/sqrt(a + b*x + c*x**2) + 8*c**2*d**3*x**2/sqrt(a + b*x + c*x**2
```

)

Giac [B] time = 1.17413, size = 188, normalized size = 3.92

$$\frac{2\left(4\left(\frac{(b^2c^3d^3-4\,ac^4d^3)x}{b^2c-4\,ac^2}+\frac{b^3c^2d^3-4\,abc^3d^3}{b^2c-4\,ac^2}\right)x-\frac{b^4cd^3-12\,ab^2c^2d^3+32\,a^2c^3d^3}{b^2c-4\,ac^2}\right)}{\sqrt{cx^2+bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2*(4*((b^2*c^3*d^3 - 4*a*c^4*d^3)*x/(b^2*c - 4*a*c^2) + (b^3*c^2*d^3 - 4*a*
b*c^3*d^3)/(b^2*c - 4*a*c^2))*x - (b^4*c*d^3 - 12*a*b^2*c^2*d^3 + 32*a^2*c^
3*d^3)/(b^2*c - 4*a*c^2))/sqrt(c*x^2 + b*x + a)
```

3.1244 $\int \frac{(bd+2cdx)^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$4\sqrt{c}d^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)-\frac{2d^2(b+2cx)}{\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*d^2*(b + 2*c*x))/Sqrt[a + b*x + c*x^2] + 4*Sqrt[c]*d^2*ArcTanh[(b + 2*c
*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]
```

Rubi [A] time = 0.0269304, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {686, 621, 206}

$$4\sqrt{c}d^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)-\frac{2d^2(b+2cx)}{\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^2/(a + b*x + c*x^2)^(3/2),x]
```


```
[Out] (-2*d^2*(b + 2*c*x))/Sqrt[a + b*x + c*x^2] + 4*Sqrt[c]*d^2*ArcTanh[(b + 2*c
*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]
```


Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(bd+2cdx)^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2d^2(b+2cx)}{\sqrt{a+bx+cx^2}} + \left(4cd^2\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx$$
$$= -\frac{2d^2(b+2cx)}{\sqrt{a+bx+cx^2}} + \left(8cd^2\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)$$
$$= -\frac{2d^2(b+2cx)}{\sqrt{a+bx+cx^2}} + 4\sqrt{c}d^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)$$

Mathematica [A] time = 0.195149, size = 118, normalized size = 1.79

$$d^2\left(\frac{4\sqrt{c}\sqrt{a+x(b+cx)}\sinh^{-1}\left(\frac{b+2cx}{\sqrt{c}\sqrt{4a-\frac{b^2}{c}}}\right)}{\sqrt{4a-\frac{b^2}{c}}\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}} - \frac{2(b+2cx)}{\sqrt{a+x(b+cx)}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^2/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] d^2*((-2*(b + 2*c*x))/Sqrt[a + x*(b + c*x)] + (4*Sqrt[c]*Sqrt[a + x*(b + c*
x)]*ArcSinh[(b + 2*c*x)/(Sqrt[4*a - b^2/c]*Sqrt[c])])/(Sqrt[4*a - b^2/c]*Sq
rt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]))
```

Maple [A] time = 0.048, size = 72, normalized size = 1.1

$$-4\,\frac{cd^2x}{\sqrt{cx^2+bx+a}} - 2\,\frac{d^2b}{\sqrt{cx^2+bx+a}} + 4\,d^2\sqrt{c}\ln\left(\frac{b/2+cx}{\sqrt{c}} + \sqrt{cx^2+bx+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^2/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -4*d^2*c*x/(c*x^2+b*x+a)^(1/2)-2*d^2*b/(c*x^2+b*x+a)^(1/2)+4*d^2*c^(1/2)*ln
((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.47733, size = 517, normalized size = 7.83

$$\left[\frac{2\left(\left(cd^2x^2+bd^2x+ad^2\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right)-\left(2\,cd^2x+bd^2\right)\sqrt{cx^2+b\right.}{cx^2+bx+a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [2*((c*d^2*x^2 + b*d^2*x + a*d^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 -
4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - (2*c*d^2*x + b*d^2)*
sqrt(c*x^2 + b*x + a))/(c*x^2 + b*x + a), -2*(2*(c*d^2*x^2 + b*d^2*x + a*d^
2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2
+ b*c*x + a*c)) + (2*c*d^2*x + b*d^2)*sqrt(c*x^2 + b*x + a))/(c*x^2 + b*x +
 a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int\frac{b^2}{a\sqrt{a+bx+cx^2}+bx\sqrt{a+bx+cx^2}+cx^2\sqrt{a+bx+cx^2}}\,dx+\int\frac{4c^2x^2}{a\sqrt{a+bx+cx^2}+bx\sqrt{a+bx+cx^2}+cx^2\sqrt{a+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**2/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] d**2*(Integral(b**2/(a*sqrt(a + b*x + c*x**2) + b*x*sqrt(a + b*x + c*x**2)
+ c*x**2*sqrt(a + b*x + c*x**2)), x) + Integral(4*c**2*x**2/(a*sqrt(a + b*x
 + c*x**2) + b*x*sqrt(a + b*x + c*x**2) + c*x**2*sqrt(a + b*x + c*x**2)), x
) + Integral(4*b*c*x/(a*sqrt(a + b*x + c*x**2) + b*x*sqrt(a + b*x + c*x**2)
 + c*x**2*sqrt(a + b*x + c*x**2)), x))
```

Giac [B] time = 1.23638, size = 153, normalized size = 2.32

$$-4\,\sqrt{c}d^2\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)-\frac{2\left(\frac{2\left(b^2cd^2-4\,ac^2d^2\right)x}{b^2-4\,ac}+\frac{b^3d^2-4\,abcd^2}{b^2-4\,ac}\right)}{\sqrt{cx^2+bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -4*sqrt(c)*d^2*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))
 - 2*(2*(b^2*c*d^2 - 4*a*c^2*d^2)*x/(b^2 - 4*a*c) + (b^3*d^2 - 4*a*b*c*d^2)
/(b^2 - 4*a*c))/sqrt(c*x^2 + b*x + a)
```

3.1245 $\int \frac{bd+2cdx}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=17

$$-\frac{2d}{\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*d)/Sqrt[a + b*x + c*x^2]
```

Rubi [A] time = 0.0061819, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {629}

$$-\frac{2d}{\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*d)/Sqrt[a + b*x + c*x^2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{bd+2cdx}{\left(a+bx+cx^2\right)^{3/2}}\,dx=-\frac{2d}{\sqrt{a+bx+cx^2}}$$

Mathematica [A] time = 0.0069606, size = 16, normalized size = 0.94

$$-\frac{2d}{\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*d)/Sqrt[a + x*(b + c*x)]
```

Maple [A] time = 0.042, size = 16, normalized size = 0.9

$$-2\,\frac{d}{\sqrt{cx^2+bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -2*d/(c*x^2+b*x+a)^(1/2)
```

Maxima [A] time = 1.1367, size = 20, normalized size = 1.18

$$-\frac{2\,d}{\sqrt{cx^2+bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*d/sqrt(c*x^2 + b*x + a)
```

Fricas [A] time = 3.19535, size = 38, normalized size = 2.24

$$-\frac{2\,d}{\sqrt{cx^2+bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*d/sqrt(c*x^2 + b*x + a)
```

Sympy [A] time = 1.49275, size = 17, normalized size = 1.

$$-\frac{2d}{\sqrt{a+bx+cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] -2*d/sqrt(a + b*x + c*x**2)
```

Giac [B] time = 1.16329, size = 47, normalized size = 2.76

$$-\frac{2\left(b^2d-4\,acd\right)}{\sqrt{cx^2+bx+a}\left(b^2-4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(b^2*d - 4*a*c*d)/(sqrt(c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

3.1246 $\int \frac{1}{(bd+2cdx)\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=86

$$-\frac{2}{d\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{4\sqrt{c}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{d\left(b^2-4ac\right)^{3/2}}$$

```
[Out] -2/((b^2 - 4*a*c)*d*Sqrt[a + b*x + c*x^2]) - (4*Sqrt[c]*ArcTan[(2*Sqrt[c]*S
qrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*d)
```

Rubi [A] time = 0.0582978, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {687, 688, 205}

$$-\frac{2}{d\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{4\sqrt{c}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{d\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] -2/((b^2 - 4*a*c)*d*Sqrt[a + b*x + c*x^2]) - (4*Sqrt[c]*ArcTan[(2*Sqrt[c]*S
qrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*d)
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2}{\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}} - \frac{(4c)\int \frac{1}{(bd+2cdx)\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac}$$

$$= -\frac{2}{\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}} - \frac{\left(16c^2\right)\text{Subst}\left(\int \frac{1}{2b^2cd-8ac^2d+8c^2dx^2}\,dx, x, \sqrt{a+bx}\right.}{b^2-4ac}$$

$$= -\frac{2}{\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}} - \frac{4\sqrt{c}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}d}$$

Mathematica [C] time = 0.0284685, size = 60, normalized size = 0.7

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{4c(a+x(b+cx))}{4ac-b^2}\right)}{d\left(b^2-4ac\right)\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, (4*c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]
)/((b^2 - 4*a*c)*d*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.196, size = 158, normalized size = 1.8

$$2\,\frac{1}{d\left(4\,ac-b^2\right)}\frac{1}{\sqrt{\left(x+1/2\,\frac{b}{c}\right)^2c+1/4\,\frac{4\,ac-b^2}{c}}}-4\,\frac{1}{d\left(4\,ac-b^2\right)}\ln\left(\left(1/2\,\frac{4\,ac-b^2}{c}+1/2\,\sqrt{\frac{4\,ac-b^2}{c}}\sqrt{4\left(x+1/2\,\frac{b}{c}\right)^2c+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 2/d/(4*a*c-b^2)/((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2)-4/d/(4*a*c-b^2)/(
(4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/2)*(4*(x+
1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.43152, size = 695, normalized size = 8.08

$$\left[-\frac{2\left(\left(cx^2+bx+a\right)\sqrt{-\frac{c}{b^2-4\,ac}}\log\left(-\frac{4\,c^2x^2+4\,bcx-b^2+8\,ac+4\,\sqrt{cx^2+bx+a}\left(b^2-4\,ac\right)\sqrt{-\frac{c}{b^2-4\,ac}}}{4\,c^2x^2+4\,bcx+b^2}\right)+\sqrt{cx^2+bx+a}\right)}{\left(b^2c-4\,ac^2\right)dx^2+\left(b^3-4\,abc\right)dx+\left(ab^2-4\,a^2c\right)d},-\frac{2\left(2\left(cx^2+bx+a\right)\right.}{\left(b^2c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-2*((c*x^2 + b*x + a)*sqrt(-c/(b^2 - 4*a*c))*log(-(4*c^2*x^2 + 4*b*c*x - b
^2 + 8*a*c + 4*sqrt(c*x^2 + b*x + a)*(b^2 - 4*a*c)*sqrt(-c/(b^2 - 4*a*c)))/
(4*c^2*x^2 + 4*b*c*x + b^2)) + sqrt(c*x^2 + b*x + a))/((b^2*c - 4*a*c^2)*d*
x^2 + (b^3 - 4*a*b*c)*d*x + (a*b^2 - 4*a^2*c)*d), -2*(2*(c*x^2 + b*x + a)*s
qrt(c/(b^2 - 4*a*c))*arctan(-1/2*sqrt(c*x^2 + b*x + a)*(b^2 - 4*a*c)*sqrt(c
/(b^2 - 4*a*c))/(c^2*x^2 + b*c*x + a*c)) + sqrt(c*x^2 + b*x + a))/((b^2*c -
 4*a*c^2)*d*x^2 + (b^3 - 4*a*b*c)*d*x + (a*b^2 - 4*a^2*c)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{ab\sqrt{a+bx+cx^2}+2acx\sqrt{a+bx+cx^2}+b^2x\sqrt{a+bx+cx^2}+3bcx^2\sqrt{a+bx+cx^2}+2c^2x^3\sqrt{a+bx+cx^2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral(1/(a*b*sqrt(a + b*x + c*x**2) + 2*a*c*x*sqrt(a + b*x + c*x**2) + b
**2*x*sqrt(a + b*x + c*x**2) + 3*b*c*x**2*sqrt(a + b*x + c*x**2) + 2*c**2*x
**3*sqrt(a + b*x + c*x**2)), x)/d
```

Giac [B] time = 1.15744, size = 213, normalized size = 2.48

$$\frac{8\,c\arctan\left(\frac{2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)c+b\sqrt{c}}{\sqrt{b^2c-4\,ac^2}}\right)}{\sqrt{b^2c-4\,ac^2}\left(b^2d-4\,acd\right)}-\frac{2\left(b^4d-8\,ab^2cd+16\,a^2c^2d\right)}{\left(b^6d^2-12\,ab^4cd^2+48\,a^2b^2c^2d^2-64\,a^3c^3d^2\right)\sqrt{cx^2+bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 8*c*arctan((2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*c + b*sqrt(c))/sqrt(b^2*c
 - 4*a*c^2))/(sqrt(b^2*c - 4*a*c^2)*(b^2*d - 4*a*c*d)) - 2*(b^4*d - 8*a*b^2
*c*d + 16*a^2*c^2*d)/((b^6*d^2 - 12*a*b^4*c*d^2 + 48*a^2*b^2*c^2*d^2 - 64*a
^3*c^3*d^2)*sqrt(c*x^2 + b*x + a))
```

3.1247 $\int \frac{1}{(bd+2cdx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=76

$$-\frac{16c\sqrt{a+bx+cx^2}}{d^2\left(b^2-4ac\right)^2(b+2cx)}-\frac{2}{d^2\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}}$$

```
[Out] -2/((b^2 - 4*a*c)*d^2*(b + 2*c*x)*Sqrt[a + b*x + c*x^2]) - (16*c*Sqrt[a + b
*x + c*x^2])/((b^2 - 4*a*c)^2*d^2*(b + 2*c*x))
```

Rubi [A] time = 0.0340286, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {687, 682}

$$-\frac{16c\sqrt{a+bx+cx^2}}{d^2\left(b^2-4ac\right)^2(b+2cx)}-\frac{2}{d^2\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^2*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] -2/((b^2 - 4*a*c)*d^2*(b + 2*c*x)*Sqrt[a + b*x + c*x^2]) - (16*c*Sqrt[a + b
*x + c*x^2])/((b^2 - 4*a*c)^2*d^2*(b + 2*c*x))
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 682

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c,
0] && EqQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(bd+2cdx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx&=-\frac{2}{\left(b^2-4ac\right)d^2(b+2cx)\sqrt{a+bx+cx^2}}-\frac{(8c)\int \frac{1}{(bd+2cdx)^2\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac}\\&=-\frac{2}{\left(b^2-4ac\right)d^2(b+2cx)\sqrt{a+bx+cx^2}}-\frac{16c\sqrt{a+bx+cx^2}}{\left(b^2-4ac\right)^2d^2(b+2cx)}\end{aligned}$$

Mathematica [A] time = 0.0280347, size = 56, normalized size = 0.74

$$-\frac{2\left(4c\left(a+2cx^2\right)+b^2+8bcx\right)}{d^2\left(b^2-4ac\right)^2(b+2cx)\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^2*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(b^2 + 8*b*c*x + 4*c*(a + 2*c*x^2)))/((b^2 - 4*a*c)^2*d^2*(b + 2*c*x)*S
qrt[a + x*(b + c*x)])
```

Maple [A] time = 0.045, size = 68, normalized size = 0.9

$$-2\,{\frac {8\,{c}^{2}{x}^{2}+8\,bcx+4\,ac+{b}^{2}}{ \left(16\,{a}^{2}{c}^{2}-8\,ac{b}^{2}+{b}^{4} \right) {d}^{2} \left(2\,cx+b \right) \sqrt {c{x}^{2}+bx+a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^2/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -2*(8*c^2*x^2+8*b*c*x+4*a*c+b^2)/(2*c*x+b)/d^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(
c*x^2+b*x+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.74985, size = 327, normalized size = 4.3

$$-\frac{2 \left(8 \, c^{2} x^{2} + 8 \, b c x + b^{2} + 4 \, a c\right) \sqrt{c x^{2} + b x + a}}{2 \left(b^{4} c^{2} - 8 \, a b^{2} c^{3} + 16 \, a^{2} c^{4}\right) d^{2} x^{3} + 3 \left(b^{5} c - 8 \, a b^{3} c^{2} + 16 \, a^{2} b c^{3}\right) d^{2} x^{2} + \left(b^{6} - 6 \, a b^{4} c + 32 \, a^{3} c^{3}\right) d^{2} x + \left(a b^{5} - 8 \, a^{2} b^{3} c + 16 \, a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(8*c^2*x^2 + 8*b*c*x + b^2 + 4*a*c)*sqrt(c*x^2 + b*x + a)/(2*(b^4*c^2 -
8*a*b^2*c^3 + 16*a^2*c^4)*d^2*x^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*
d^2*x^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2*x + (a*b^5 - 8*a^2*b^3*c + 16*
a^3*b*c^2)*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a b^{2} \sqrt{a + b x + c x^{2}} + 4 a b c x \sqrt{a + b x + c x^{2}} + 4 a c^{2} x^{2} \sqrt{a + b x + c x^{2}} + b^{3} x \sqrt{a + b x + c x^{2}} + 5 b^{2} c x^{2} \sqrt{a + b x + c x^{2}} + 8 b c^{2} x^{3} \sqrt{a + b x + c x^{2}} + 4 c^{3} x^{4} \sqrt{a + b x + c x^{2}}}\, dx}{d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**2/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral(1/(a*b**2*sqrt(a + b*x + c*x**2) + 4*a*b*c*x*sqrt(a + b*x + c*x**2
) + 4*a*c**2*x**2*sqrt(a + b*x + c*x**2) + b**3*x*sqrt(a + b*x + c*x**2) +
5*b**2*c*x**2*sqrt(a + b*x + c*x**2) + 8*b*c**2*x**3*sqrt(a + b*x + c*x**2)
 + 4*c**3*x**4*sqrt(a + b*x + c*x**2)), x)/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2\,cdx + bd)^2 \left(cx^2 + bx + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((2*c*d*x + b*d)^2*(c*x^2 + b*x + a)^(3/2)), x)
```

3.1248 $\int \frac{1}{(bd+2cdx)^3(a+bx+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=132

$$-\frac{12c\sqrt{a+bx+cx^2}}{d^3(b^2-4ac)^2(b+2cx)^2}-\frac{2}{d^3(b^2-4ac)(b+2cx)^2\sqrt{a+bx+cx^2}}-\frac{6\sqrt{c}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{d^3(b^2-4ac)^{5/2}}$$

```
[Out] -2/((b^2 - 4*a*c)*d^3*(b + 2*c*x)^2*Sqrt[a + b*x + c*x^2]) - (12*c*Sqrt[a +
 b*x + c*x^2])/((b^2 - 4*a*c)^2*d^3*(b + 2*c*x)^2) - (6*Sqrt[c]*ArcTan[(2*S
qrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2)*d^3)
```

Rubi [A] time = 0.0845561, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {687, 693, 688, 205}

$$-\frac{12c\sqrt{a+bx+cx^2}}{d^3(b^2-4ac)^2(b+2cx)^2}-\frac{2}{d^3(b^2-4ac)(b+2cx)^2\sqrt{a+bx+cx^2}}-\frac{6\sqrt{c}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{d^3(b^2-4ac)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^3*(a + b*x + c*x^2)^(3/2)),x]
```


```
[Out] -2/((b^2 - 4*a*c)*d^3*(b + 2*c*x)^2*Sqrt[a + b*x + c*x^2]) - (12*c*Sqrt[a +
 b*x + c*x^2])/((b^2 - 4*a*c)^2*d^3*(b + 2*c*x)^2) - (6*Sqrt[c]*ArcTan[(2*S
qrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2)*d^3)
```


Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2}{\left(b^2-4ac\right)d^3(b+2cx)^2\sqrt{a+bx+cx^2}} - \frac{(12c)\int \frac{1}{(bd+2cdx)^3\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac}$$

$$= -\frac{2}{\left(b^2-4ac\right)d^3(b+2cx)^2\sqrt{a+bx+cx^2}} - \frac{12c\sqrt{a+bx+cx^2}}{\left(b^2-4ac\right)^2 d^3(b+2cx)^2} - \frac{(6c)\int \frac{}{(bd}}{(b^2}$$

$$= -\frac{2}{\left(b^2-4ac\right)d^3(b+2cx)^2\sqrt{a+bx+cx^2}} - \frac{12c\sqrt{a+bx+cx^2}}{\left(b^2-4ac\right)^2 d^3(b+2cx)^2} - \frac{\left(24c^2\right)\mathrm{Su}}{}$$

$$= -\frac{2}{\left(b^2-4ac\right)d^3(b+2cx)^2\sqrt{a+bx+cx^2}} - \frac{12c\sqrt{a+bx+cx^2}}{\left(b^2-4ac\right)^2 d^3(b+2cx)^2} - \frac{6\sqrt{c}\tan^{}}{(b^2}$$

Mathematica [C] time = 0.0285623, size = 60, normalized size = 0.45

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{4c(a+x(b+cx))}{4ac-b^2}\right)}{d^3\left(b^2-4ac\right)^2\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^3*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 2, 1/2, (4*c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]
)/((b^2 - 4*a*c)^2*d^3*Sqrt[a + x*(b + c*x)])
```

Maple [A] time = 0.196, size = 218, normalized size = 1.7

$$-\frac{1}{4\,c^2d^3\left(4\,ac-b^2\right)}\left(x+\frac{b}{2\,c}\right)^{-2}\frac{1}{\sqrt{\left(x+\frac{b}{2\,c}\right)^2c+\frac{4\,ac-b^2}{4\,c}}}-3\,\frac{1}{d^3\left(4\,ac-b^2\right)^2}\frac{1}{\sqrt{\left(x+1/2\,\frac{b}{c}\right)^2c+1/4\,\frac{4\,ac-b^2}{c}}}+6\,\frac{1}{d^3\left(4\,ac-b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^3/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -1/4/d^3/c^2/(4*a*c-b^2)/(x+1/2*b/c)^2/((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^
(1/2)-3/d^3/(4*a*c-b^2)^2/((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2)+6/d^3/(
4*a*c-b^2)^2/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-b^2)/c
)^(1/2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 13.6228, size = 1496, normalized size = 11.33

$$\left[\frac{3\left(4\,c^3x^4+8\,bc^2x^3+ab^2+\left(5\,b^2c+4\,ac^2\right)x^2+\left(b^3+4\,abc\right)x\right)\sqrt{-\frac{c}{b^2-4\,ac}}\log\left(-\frac{4\,c^2x^2+4\,bcx-b^2+8\,ac-4\,\sqrt{cx^2+bx+a}\left(b^2-4\right.}{4\,c^2x^2+4\,bcx+b^2}\right.}{4\left(b^4c^3-8\,ab^2c^4+16\,a^2c^5\right)d^3x^4+8\left(b^5c^2-8\,ab^3c^3+16\,a^2bc^4\right)d^3x^3+\left(5\,b^6c-36\,ab^4c^2+48\,a^2b^2c^3+64\,a^3c^4\right)d^3x^2+\left(b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [(3*(4*c^3*x^4 + 8*b*c^2*x^3 + a*b^2 + (5*b^2*c + 4*a*c^2)*x^2 + (b^3 + 4*a
*b*c)*x)*sqrt(-c/(b^2 - 4*a*c))*log(-(4*c^2*x^2 + 4*b*c*x - b^2 + 8*a*c - 4
*sqrt(c*x^2 + b*x + a)*(b^2 - 4*a*c)*sqrt(-c/(b^2 - 4*a*c)))/(4*c^2*x^2 + 4
*b*c*x + b^2)) - 2*(6*c^2*x^2 + 6*b*c*x + b^2 + 2*a*c)*sqrt(c*x^2 + b*x + a
))/(4*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d^3*x^4 + 8*(b^5*c^2 - 8*a*b^3*c
^3 + 16*a^2*b*c^4)*d^3*x^3 + (5*b^6*c - 36*a*b^4*c^2 + 48*a^2*b^2*c^3 + 64*
a^3*c^4)*d^3*x^2 + (b^7 - 4*a*b^5*c - 16*a^2*b^3*c^2 + 64*a^3*b*c^3)*d^3*x
+ (a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*d^3), -2*(3*(4*c^3*x^4 + 8*b*c^2*x
^3 + a*b^2 + (5*b^2*c + 4*a*c^2)*x^2 + (b^3 + 4*a*b*c)*x)*sqrt(c/(b^2 - 4*a
*c))*arctan(-1/2*sqrt(c*x^2 + b*x + a)*(b^2 - 4*a*c)*sqrt(c/(b^2 - 4*a*c))/
(c^2*x^2 + b*c*x + a*c)) + (6*c^2*x^2 + 6*b*c*x + b^2 + 2*a*c)*sqrt(c*x^2 +
 b*x + a))/(4*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d^3*x^4 + 8*(b^5*c^2 - 8
*a*b^3*c^3 + 16*a^2*b*c^4)*d^3*x^3 + (5*b^6*c - 36*a*b^4*c^2 + 48*a^2*b^2*c
^3 + 64*a^3*c^4)*d^3*x^2 + (b^7 - 4*a*b^5*c - 16*a^2*b^3*c^2 + 64*a^3*b*c^3
)*d^3*x + (a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{ab^3\sqrt{a+bx+cx^2}+6ab^2cx\sqrt{a+bx+cx^2}+12abc^2x^2\sqrt{a+bx+cx^2}+8ac^3x^3\sqrt{a+bx+cx^2}+b^4x\sqrt{a+bx+cx^2}+7b^3cx^2\sqrt{a+bx+cx^2}+18b^2c^2x^3\sqrt{a+bx+cx^2}+20bc^3x^4\sqrt{a+bx+c}}}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**3/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral(1/(a*b**3*sqrt(a + b*x + c*x**2) + 6*a*b**2*c*x*sqrt(a + b*x + c*x
**2) + 12*a*b*c**2*x**2*sqrt(a + b*x + c*x**2) + 8*a*c**3*x**3*sqrt(a + b*x
 + c*x**2) + b**4*x*sqrt(a + b*x + c*x**2) + 7*b**3*c*x**2*sqrt(a + b*x + c
*x**2) + 18*b**2*c**2*x**3*sqrt(a + b*x + c*x**2) + 20*b*c**3*x**4*sqrt(a +
 b*x + c*x**2) + 8*c**4*x**5*sqrt(a + b*x + c*x**2)), x)/d**3
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1249 $\int \frac{1}{(bd+2cdx)^4 \left(a+bx+cx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=118

$$-\frac{64c\sqrt{a+bx+cx^2}}{3d^4\left(b^2-4ac\right)^3(b+2cx)}-\frac{32c\sqrt{a+bx+cx^2}}{3d^4\left(b^2-4ac\right)^2(b+2cx)^3}-\frac{2}{d^4\left(b^2-4ac\right)(b+2cx)^3\sqrt{a+bx+cx^2}}$$

```
[Out] -2/((b^2 - 4*a*c)*d^4*(b + 2*c*x)^3*Sqrt[a + b*x + c*x^2]) - (32*c*Sqrt[a +
 b*x + c*x^2])/(3*(b^2 - 4*a*c)^2*d^4*(b + 2*c*x)^3) - (64*c*Sqrt[a + b*x +
 c*x^2])/(3*(b^2 - 4*a*c)^3*d^4*(b + 2*c*x))
```

Rubi [A] time = 0.0552308, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {687, 693, 682}

$$-\frac{64c\sqrt{a+bx+cx^2}}{3d^4\left(b^2-4ac\right)^3(b+2cx)}-\frac{32c\sqrt{a+bx+cx^2}}{3d^4\left(b^2-4ac\right)^2(b+2cx)^3}-\frac{2}{d^4\left(b^2-4ac\right)(b+2cx)^3\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^4*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] -2/((b^2 - 4*a*c)*d^4*(b + 2*c*x)^3*Sqrt[a + b*x + c*x^2]) - (32*c*Sqrt[a +
 b*x + c*x^2])/(3*(b^2 - 4*a*c)^2*d^4*(b + 2*c*x)^3) - (64*c*Sqrt[a + b*x +
 c*x^2])/(3*(b^2 - 4*a*c)^3*d^4*(b + 2*c*x))
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 682

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c,
0] && EqQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)^4\left(a+bx+cx^2\right)^{3/2}}dx = -\frac{2}{\left(b^2-4ac\right)d^4(b+2cx)^3\sqrt{a+bx+cx^2}} - \frac{(16c)\int \frac{1}{(bd+2cdx)^4\sqrt{a+bx+cx^2}}dx}{b^2-4ac}$$

$$= -\frac{2}{\left(b^2-4ac\right)d^4(b+2cx)^3\sqrt{a+bx+cx^2}} - \frac{32c\sqrt{a+bx+cx^2}}{3\left(b^2-4ac\right)^2 d^4(b+2cx)^3} - \frac{(32c)\int}{3}$$

$$= -\frac{2}{\left(b^2-4ac\right)d^4(b+2cx)^3\sqrt{a+bx+cx^2}} - \frac{32c\sqrt{a+bx+cx^2}}{3\left(b^2-4ac\right)^2 d^4(b+2cx)^3} - \frac{64c}{3\left(b^2-\right.}$$

Mathematica [A] time = 0.0502315, size = 108, normalized size = 0.92

$$-\frac{2\left(16c^2\left(-a^2+4acx^2+8c^2x^4\right)+8b^2c\left(3a+22cx^2\right)+64bc^2x\left(a+4cx^2\right)+48b^3cx+3b^4\right)}{3d^4\left(b^2-4ac\right)^3(b+2cx)^3\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^4*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(3*b^4 + 48*b^3*c*x + 64*b*c^2*x*(a + 4*c*x^2) + 8*b^2*c*(3*a + 22*c*x^
2) + 16*c^2*(-a^2 + 4*a*c*x^2 + 8*c^2*x^4)))/(3*(b^2 - 4*a*c)^3*d^4*(b + 2*
c*x)^3*Sqrt[a + x*(b + c*x)])
```

Maple [A] time = 0.048, size = 133, normalized size = 1.1

$$-\frac{-256\,c^4x^4-512\,bc^3x^3-128\,ac^3x^2-352\,b^2c^2x^2-128\,abc^2x-96\,b^3cx+32\,a^2c^2-48\,acb^2-6\,b^4}{3\left(64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6\right)d^4\left(2\,cx+b\right)^3}\frac{1}{\sqrt{cx^2+bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^4/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -2/3*(-128*c^4*x^4-256*b*c^3*x^3-64*a*c^3*x^2-176*b^2*c^2*x^2-64*a*b*c^2*x-
48*b^3*c*x+16*a^2*c^2-24*a*b^2*c-3*b^4)/(2*c*x+b)^3/d^4/(64*a^3*c^3-48*a^2*
b^2*c^2+12*a*b^4*c-b^6)/(c*x^2+b*x+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^4/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 27.7697, size = 824, normalized size = 6.98

$$-\frac{2\left(128\,c^{4}\cdots\right.}{3\left(8\left(b^{6}c^{4}-12\,ab^{4}c^{5}+48\,a^{2}b^{2}c^{6}-64\,a^{3}c^{7}\right)d^{4}x^{5}+20\left(b^{7}c^{3}-12\,ab^{5}c^{4}+48\,a^{2}b^{3}c^{5}-64\,a^{3}bc^{6}\right)d^{4}x^{4}+2\left(9\,b^{8}c^{2}-104\,ab^{6}\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^4/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(128*c^4*x^4 + 256*b*c^3*x^3 + 3*b^4 + 24*a*b^2*c - 16*a^2*c^2 + 16*(1
1*b^2*c^2 + 4*a*c^3)*x^2 + 16*(3*b^3*c + 4*a*b*c^2)*x)*sqrt(c*x^2 + b*x + a
)/(8*(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*d^4*x^5 + 20*(b
^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*d^4*x^4 + 2*(9*b^8*c
^2 - 104*a*b^6*c^3 + 384*a^2*b^4*c^4 - 384*a^3*b^2*c^5 - 256*a^4*c^6)*d^4*x
^3 + (7*b^9*c - 72*a*b^7*c^2 + 192*a^2*b^5*c^3 + 128*a^3*b^3*c^4 - 768*a^4*
b*c^5)*d^4*x^2 + (b^10 - 6*a*b^8*c - 24*a^2*b^6*c^2 + 224*a^3*b^4*c^3 - 384
*a^4*b^2*c^4)*d^4*x + (a*b^9 - 12*a^2*b^7*c + 48*a^3*b^5*c^2 - 64*a^4*b^3*c
^3)*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{ab^{4}\sqrt{a+bx+cx^{2}}+8ab^{3}cx\sqrt{a+bx+cx^{2}}+24ab^{2}c^{2}x^{2}\sqrt{a+bx+cx^{2}}+32abc^{3}x^{3}\sqrt{a+bx+cx^{2}}+16ac^{4}x^{4}\sqrt{a+bx+cx^{2}}+b^{5}x\sqrt{a+bx+cx^{2}}+9b^{4}cx^{2}\sqrt{a+bx+cx^{2}}+32b^{3}c^{2}x^{3}\sqrt{a+bx+\cdots}}}{d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**4/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral(1/(a*b**4*sqrt(a + b*x + c*x**2) + 8*a*b**3*c*x*sqrt(a + b*x + c*x
**2) + 24*a*b**2*c**2*x**2*sqrt(a + b*x + c*x**2) + 32*a*b*c**3*x**3*sqrt(a
 + b*x + c*x**2) + 16*a*c**4*x**4*sqrt(a + b*x + c*x**2) + b**5*x*sqrt(a +
b*x + c*x**2) + 9*b**4*c*x**2*sqrt(a + b*x + c*x**2) + 32*b**3*c**2*x**3*sq
rt(a + b*x + c*x**2) + 56*b**2*c**3*x**4*sqrt(a + b*x + c*x**2) + 48*b*c**4
*x**5*sqrt(a + b*x + c*x**2) + 16*c**5*x**6*sqrt(a + b*x + c*x**2)), x)/d**
4
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^4/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1250 $\int \frac{(bd+2cdx)^6}{\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=136

$$40c^{3/2}d^6\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)+80c^2d^6(b+2cx)\sqrt{a+bx+cx^2}-\frac{40cd^6(b+2cx)^3}{3\sqrt{a+bx+cx^2}}-\frac{2d^6(b+2cx)}{3\left(a+bx+cx^2\right.}$$

```
[Out] (-2*d^6*(b + 2*c*x)^5)/(3*(a + b*x + c*x^2)^(3/2)) - (40*c*d^6*(b + 2*c*x)^
3)/(3*Sqrt[a + b*x + c*x^2]) + 80*c^2*d^6*(b + 2*c*x)*Sqrt[a + b*x + c*x^2]
 + 40*c^(3/2)*(b^2 - 4*a*c)*d^6*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x
 + c*x^2])]
```

Rubi [A] time = 0.0771676, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {686, 692, 621, 206}

$$40c^{3/2}d^6\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)+80c^2d^6(b+2cx)\sqrt{a+bx+cx^2}-\frac{40cd^6(b+2cx)^3}{3\sqrt{a+bx+cx^2}}-\frac{2d^6(b+2cx)}{3\left(a+bx+cx^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^6/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*d^6*(b + 2*c*x)^5)/(3*(a + b*x + c*x^2)^(3/2)) - (40*c*d^6*(b + 2*c*x)^
3)/(3*Sqrt[a + b*x + c*x^2]) + 80*c^2*d^6*(b + 2*c*x)*Sqrt[a + b*x + c*x^2]
 + 40*c^(3/2)*(b^2 - 4*a*c)*d^6*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x
 + c*x^2])]
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(bd+2cdx)^6}{\left(a+bx+cx^2\right)^{5/2}}\,dx &= -\frac{2d^6(b+2cx)^5}{3\left(a+bx+cx^2\right)^{3/2}}+\frac{1}{3}\left(20cd^2\right)\int \frac{(bd+2cdx)^4}{\left(a+bx+cx^2\right)^{3/2}}\,dx\\
&= -\frac{2d^6(b+2cx)^5}{3\left(a+bx+cx^2\right)^{3/2}}-\frac{40cd^6(b+2cx)^3}{3\sqrt{a+bx+cx^2}}+\left(80c^2d^4\right)\int \frac{(bd+2cdx)^2}{\sqrt{a+bx+cx^2}}\,dx\\
&= -\frac{2d^6(b+2cx)^5}{3\left(a+bx+cx^2\right)^{3/2}}-\frac{40cd^6(b+2cx)^3}{3\sqrt{a+bx+cx^2}}+80c^2d^6(b+2cx)\sqrt{a+bx+cx^2}+\left(40c^2\left(b^2-4ac\right)d^6\right.\\
&= -\frac{2d^6(b+2cx)^5}{3\left(a+bx+cx^2\right)^{3/2}}-\frac{40cd^6(b+2cx)^3}{3\sqrt{a+bx+cx^2}}+80c^2d^6(b+2cx)\sqrt{a+bx+cx^2}+\left(80c^2\left(b^2-4ac\right)d^6\right.\\
&= -\frac{2d^6(b+2cx)^5}{3\left(a+bx+cx^2\right)^{3/2}}-\frac{40cd^6(b+2cx)^3}{3\sqrt{a+bx+cx^2}}+80c^2d^6(b+2cx)\sqrt{a+bx+cx^2}+40c^{3/2}\left(b^2-4ac\right)d^6\end{aligned}$$

Mathematica [A] time = 1.50591, size = 204, normalized size = 1.5

$$\frac{d^6\left(3(b+2cx)^5-\frac{5\left(b^2-4ac\right)\left(\sqrt{4a-\frac{b^2}{c}}(b+2cx)\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\left(4c\left(3a+4cx^2\right)+b^2+16bcx\right)-24c^{3/2}(a+x(b+cx))^2\sinh^{-1}\left(\frac{b+2cx}{\sqrt{c}\sqrt{4a-\frac{b^2}{c}}}\right)\right)}{\sqrt{4a-\frac{b^2}{c}}\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}\right)}{3(a+x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^6/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (d^6*(3*(b + 2*c*x)^5 - (5*(b^2 - 4*a*c)*(Sqrt[4*a - b^2/c]*(b + 2*c*x)*Sqr
t[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*(b^2 + 16*b*c*x + 4*c*(3*a + 4*c*x^
2)) - 24*c^(3/2)*(a + x*(b + c*x))^2*ArcSinh[(b + 2*c*x)/(Sqrt[4*a - b^2/c]
*Sqrt[c])]))/(Sqrt[4*a - b^2/c]*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)])
))/(3*(a + x*(b + c*x))^(3/2))
```

Maple [B] time = 0.063, size = 997, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^6/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 40*d^6*c^(3/2)*b^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+15/2*d^6*b^7
/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+20*d^6*c*b^3/(c*x^2+b*x+a)^(1/2)+32*d^6*c^
5*x^5/(c*x^2+b*x+a)^(3/2)-160*d^6*c^(5/2)*a*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b
*x+a)^(1/2))+120*d^6*c^2*b^6/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-60*d^6*c*b
^5*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)-480*d^6*c^2*b^5*a/(4*a*c-b^2)^2/(c*x^2
```

```
+b*x+a)^(1/2)+160/3*d^6*c^4*a*x^3/(c*x^2+b*x+a)^(3/2)+160*d^6*c^3*a*x/(c*x^
2+b*x+a)^(1/2)-80*d^6*c^2*a*b/(c*x^2+b*x+a)^(1/2)+160*d^6*c^2*b*a^2/(c*x^2+
b*x+a)^(3/2)-40/3*d^6*c^3*b^2*x^3/(c*x^2+b*x+a)^(3/2)-140*d^6*c^2*b^3*x^2/(
c*x^2+b*x+a)^(3/2)-65*d^6*c*b^4*x/(c*x^2+b*x+a)^(3/2)+60*d^6*c*b^7/(4*a*c-b
^2)^2/(c*x^2+b*x+a)^(1/2)-310/3*d^6*c*b^3*a/(c*x^2+b*x+a)^(3/2)-40*d^6*c^2*
b^2*x/(c*x^2+b*x+a)^(1/2)+20*d^6*c*b^5/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+80*d
^6*c^4*b*x^4/(c*x^2+b*x+a)^(3/2)+240*d^6*c^3*b*a*x^2/(c*x^2+b*x+a)^(3/2)+60
*d^6*c^2*b^2*a*x/(c*x^2+b*x+a)^(3/2)+120*d^6*c^2*b^3*a^2/(4*a*c-b^2)/(c*x^2
+b*x+a)^(3/2)+960*d^6*c^3*b^3*a^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)+15*d^6*
c*b^6/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+40*d^6*c^2*b^4/(4*a*c-b^2)/(c*x^2+b
*x+a)^(1/2)*x-80*d^6*c^2*a*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+41/6*d^6*b^5
/(c*x^2+b*x+a)^(3/2)-960*d^6*c^3*b^4*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-
160*d^6*c^3*a*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+240*d^6*c^3*b^2*a^2/(4*
a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+1920*d^6*c^4*b^2*a^2/(4*a*c-b^2)^2/(c*x^2+b*
x+a)^(1/2)*x-120*d^6*c^2*b^4*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^6/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 11.5415, size = 1482, normalized size = 10.9

$$\left[-\frac{2\left(30\left(\left(b^2c^3-4\,ac^4\right)d^6x^4+2\left(b^3c^2-4\,abc^3\right)d^6x^3+\left(b^4c-2\,ab^2c^2-8\,a^2c^3\right)d^6x^2+2\left(ab^3c-4\,a^2bc^2\right)d^6x+\left(a^2b^2c-\right.\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^6/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-2/3*(30*((b^2*c^3 - 4*a*c^4)*d^6*x^4 + 2*(b^3*c^2 - 4*a*b*c^3)*d^6*x^3 +
(b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^6*x^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*d^6*x
 + (a^2*b^2*c - 4*a^3*c^2)*d^6)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*
sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - (48*c^5*d^6*x^5 + 120*
b*c^4*d^6*x^4 + 40*(b^2*c^3 + 8*a*c^4)*d^6*x^3 - 60*(b^3*c^2 - 8*a*b*c^3)*d
^6*x^2 - 30*(b^4*c - 4*a*b^2*c^2 - 8*a^2*c^3)*d^6*x - (b^5 + 20*a*b^3*c - 1
20*a^2*b*c^2)*d^6)*sqrt(c*x^2 + b*x + a))/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x +
(b^2 + 2*a*c)*x^2 + a^2), -2/3*(60*((b^2*c^3 - 4*a*c^4)*d^6*x^4 + 2*(b^3*c^
2 - 4*a*b*c^3)*d^6*x^3 + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^6*x^2 + 2*(a*b
^3*c - 4*a^2*b*c^2)*d^6*x + (a^2*b^2*c - 4*a^3*c^2)*d^6)*sqrt(-c)*arctan(1/
2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - (48
*c^5*d^6*x^5 + 120*b*c^4*d^6*x^4 + 40*(b^2*c^3 + 8*a*c^4)*d^6*x^3 - 60*(b^3
*c^2 - 8*a*b*c^3)*d^6*x^2 - 30*(b^4*c - 4*a*b^2*c^2 - 8*a^2*c^3)*d^6*x - (b
^5 + 20*a*b^3*c - 120*a^2*b*c^2)*d^6)*sqrt(c*x^2 + b*x + a))/(c^2*x^4 + 2*b
*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**6/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20005, size = 713, normalized size = 5.24

$$-\frac{40\left(b^2c^2d^6-4\,ac^3d^6\right)\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{\sqrt{c}}+\frac{2\left(2\left(2\left(2\left(3\left(\frac{2\left(b^4c^8d^6-8\,ab^2c^9d^6+16\,a^2c^{10}d^6\right)x}{b^4c^3-8\,ab^2c^4+16\,a^2c^5}+\frac{5\left(b^5c^7d^6-8\,ab\cdots\right.}{b^4c^3-8\,a\cdots}\right.\right.\right.\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^6/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -40*(b^2*c^2*d^6 - 4*a*c^3*d^6)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))*sqrt(c) - b))/sqrt(c) + 2/3*(2*(2*(2*(3*(2*(b^4*c^8*d^6 - 8*a*b^2*c^9*d
^6 + 16*a^2*c^10*d^6)*x/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5) + 5*(b^5*c^7*d
^6 - 8*a*b^3*c^8*d^6 + 16*a^2*b*c^9*d^6)/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^
5))*x + 5*(b^6*c^6*d^6 - 48*a^2*b^2*c^8*d^6 + 128*a^3*c^9*d^6)/(b^4*c^3 - 8
*a*b^2*c^4 + 16*a^2*c^5))*x - 15*(b^7*c^5*d^6 - 16*a*b^5*c^6*d^6 + 80*a^2*b
^3*c^7*d^6 - 128*a^3*b*c^8*d^6)/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))*x - 1
5*(b^8*c^4*d^6 - 12*a*b^6*c^5*d^6 + 40*a^2*b^4*c^6*d^6 - 128*a^4*c^8*d^6)/(
b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))*x - (b^9*c^3*d^6 + 12*a*b^7*c^4*d^6 -
264*a^2*b^5*c^5*d^6 + 1280*a^3*b^3*c^6*d^6 - 1920*a^4*b*c^7*d^6)/(b^4*c^3 -
 8*a*b^2*c^4 + 16*a^2*c^5))/(c*x^2 + b*x + a)^(3/2)
```

3.1251 $\int \frac{(bd+2cdx)^5}{\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=84

$$\frac{256}{3}c^2d^5\sqrt{a+bx+cx^2}-\frac{32cd^5(b+2cx)^2}{3\sqrt{a+bx+cx^2}}-\frac{2d^5(b+2cx)^4}{3\left(a+bx+cx^2\right)^{3/2}}$$

```
[Out] (-2*d^5*(b + 2*c*x)^4)/(3*(a + b*x + c*x^2)^(3/2)) - (32*c*d^5*(b + 2*c*x)^
2)/(3*Sqrt[a + b*x + c*x^2]) + (256*c^2*d^5*Sqrt[a + b*x + c*x^2])/3
```

Rubi [A] time = 0.0439912, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {686, 629}

$$\frac{256}{3}c^2d^5\sqrt{a+bx+cx^2}-\frac{32cd^5(b+2cx)^2}{3\sqrt{a+bx+cx^2}}-\frac{2d^5(b+2cx)^4}{3\left(a+bx+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^5/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*d^5*(b + 2*c*x)^4)/(3*(a + b*x + c*x^2)^(3/2)) - (32*c*d^5*(b + 2*c*x)^
2)/(3*Sqrt[a + b*x + c*x^2]) + (256*c^2*d^5*Sqrt[a + b*x + c*x^2])/3
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bd+2cdx)^5}{\left(a+bx+cx^2\right)^{5/2}}\,dx &= -\frac{2d^5(b+2cx)^4}{3\left(a+bx+cx^2\right)^{3/2}}+\frac{1}{3}\left(16cd^2\right)\int \frac{(bd+2cdx)^3}{\left(a+bx+cx^2\right)^{3/2}}\,dx\\
&= -\frac{2d^5(b+2cx)^4}{3\left(a+bx+cx^2\right)^{3/2}}-\frac{32cd^5(b+2cx)^2}{3\sqrt{a+bx+cx^2}}+\frac{1}{3}\left(128c^2d^4\right)\int \frac{bd+2cdx}{\sqrt{a+bx+cx^2}}\,dx\\
&= -\frac{2d^5(b+2cx)^4}{3\left(a+bx+cx^2\right)^{3/2}}-\frac{32cd^5(b+2cx)^2}{3\sqrt{a+bx+cx^2}}+\frac{256}{3}c^2d^5\sqrt{a+bx+cx^2}
\end{aligned}$$

Mathematica [A] time = 0.0532253, size = 91, normalized size = 1.08

$$\frac{d^5\left(32c^2\left(8a^2+12acx^2+3c^2x^4\right)+16b^2c\left(3cx^2-2a\right)+192bc^2x\left(2a+cx^2\right)-48b^3cx-2b^4\right)}{3(a+x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^5/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (d^5*(-2*b^4 - 48*b^3*c*x + 192*b*c^2*x*(2*a + c*x^2) + 16*b^2*c*(-2*a + 3*
c*x^2) + 32*c^2*(8*a^2 + 12*a*c*x^2 + 3*c^2*x^4)))/(3*(a + x*(b + c*x))^(3/
2))
```

Maple [A] time = 0.046, size = 91, normalized size = 1.1

$$\frac{2\,d^5\left(48\,c^4x^4+96\,bc^3x^3+192\,ac^3x^2+24\,b^2c^2x^2+192\,abc^2x-24\,b^3cx+128\,a^2c^2-16\,acb^2-b^4\right)}{3}\left(cx^2+bx+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^5/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 2/3*d^5*(48*c^4*x^4+96*b*c^3*x^3+192*a*c^3*x^2+24*b^2*c^2*x^2+192*a*b*c^2*x
-24*b^3*c*x+128*a^2*c^2-16*a*b^2*c-b^4)/(c*x^2+b*x+a)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^5/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 8.91066, size = 302, normalized size = 3.6

$$\frac{2\left(48\,c^4d^5x^4+96\,bc^3d^5x^3+24\left(b^2c^2+8\,ac^3\right)d^5x^2-24\left(b^3c-8\,abc^2\right)d^5x-\left(b^4+16\,ab^2c-128\,a^2c^2\right)d^5\right)\sqrt{cx^2+bx+a}}{3\left(c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\,ac\right)x^2+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^5/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(48*c^4*d^5*x^4 + 96*b*c^3*d^5*x^3 + 24*(b^2*c^2 + 8*a*c^3)*d^5*x^2 - 2
4*(b^3*c - 8*a*b*c^2)*d^5*x - (b^4 + 16*a*b^2*c - 128*a^2*c^2)*d^5)*sqrt(c*
x^2 + b*x + a)/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)
```

Sympy [B] time = 2.62557, size = 615, normalized size = 7.32

$$\frac{256a^2c^2d^5}{3a\sqrt{a+bx+cx^2}+3bx\sqrt{a+bx+cx^2}+3cx^2\sqrt{a+bx+cx^2}}-\frac{32ab^2cd^5}{3a\sqrt{a+bx+cx^2}+3bx\sqrt{a+bx+cx^2}+3cx^2\sqrt{a+bx+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**5/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] 256*a**2*c**2*d**5/(3*a*sqrt(a + b*x + c*x**2) + 3*b*x*sqrt(a + b*x + c*x**
2) + 3*c*x**2*sqrt(a + b*x + c*x**2)) - 32*a*b**2*c*d**5/(3*a*sqrt(a + b*x
+ c*x**2) + 3*b*x*sqrt(a + b*x + c*x**2) + 3*c*x**2*sqrt(a + b*x + c*x**2))
 + 384*a*b*c**2*d**5*x/(3*a*sqrt(a + b*x + c*x**2) + 3*b*x*sqrt(a + b*x + c
*x**2) + 3*c*x**2*sqrt(a + b*x + c*x**2)) + 384*a*c**3*d**5*x**2/(3*a*sqrt(
a + b*x + c*x**2) + 3*b*x*sqrt(a + b*x + c*x**2) + 3*c*x**2*sqrt(a + b*x +
c*x**2)) - 2*b**4*d**5/(3*a*sqrt(a + b*x + c*x**2) + 3*b*x*sqrt(a + b*x + c
*x**2) + 3*c*x**2*sqrt(a + b*x + c*x**2)) - 48*b**3*c*d**5*x/(3*a*sqrt(a +
b*x + c*x**2) + 3*b*x*sqrt(a + b*x + c*x**2) + 3*c*x**2*sqrt(a + b*x + c*x*
*2)) + 48*b**2*c**2*d**5*x**2/(3*a*sqrt(a + b*x + c*x**2) + 3*b*x*sqrt(a +
b*x + c*x**2) + 3*c*x**2*sqrt(a + b*x + c*x**2)) + 192*b*c**3*d**5*x**3/(3*
a*sqrt(a + b*x + c*x**2) + 3*b*x*sqrt(a + b*x + c*x**2) + 3*c*x**2*sqrt(a +
 b*x + c*x**2)) + 96*c**4*d**5*x**4/(3*a*sqrt(a + b*x + c*x**2) + 3*b*x*sqr
t(a + b*x + c*x**2) + 3*c*x**2*sqrt(a + b*x + c*x**2))
```

Giac [B] time = 1.15636, size = 521, normalized size = 6.2

$$\frac{2\left(24\left(\left(2\left(\frac{\left(b^4c^6d^5-8\,ab^2c^7d^5+16\,a^2c^8d^5\right)x}{b^4c^2-8\,ab^2c^3+16\,a^2c^4}+\frac{2\left(b^5c^5d^5-8\,ab^3c^6d^5+16\,a^2bc^7d^5\right)}{b^4c^2-8\,ab^2c^3+16\,a^2c^4}\right)x+\frac{b^6c^4d^5-48\,a^2b^2c^6d^5+128\,a^3c^7d^5}{b^4c^2-8\,ab^2c^3+16\,a^2c^4}\right)x-\frac{b^7c^3d^5-16\,ab^5c^4d^5+80\,a^2}{b^4c^2-8\,ab^2c^3+}\right.\right.}{3\left(cx^2+bx+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^5/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(24*((2*((b^4*c^6*d^5 - 8*a*b^2*c^7*d^5 + 16*a^2*c^8*d^5)*x/(b^4*c^2 -
8*a*b^2*c^3 + 16*a^2*c^4) + 2*(b^5*c^5*d^5 - 8*a*b^3*c^6*d^5 + 16*a^2*b*c^7
*d^5)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))*x + (b^6*c^4*d^5 - 48*a^2*b^2*c
^6*d^5 + 128*a^3*c^7*d^5)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))*x - (b^7*c^
3*d^5 - 16*a*b^5*c^4*d^5 + 80*a^2*b^3*c^5*d^5 - 128*a^3*b*c^6*d^5)/(b^4*c^2
 - 8*a*b^2*c^3 + 16*a^2*c^4))*x - (b^8*c^2*d^5 + 8*a*b^6*c^3*d^5 - 240*a^2*
b^4*c^4*d^5 + 1280*a^3*b^2*c^5*d^5 - 2048*a^4*c^6*d^5)/(b^4*c^2 - 8*a*b^2*c
^3 + 16*a^2*c^4))/(c*x^2 + b*x + a)^(3/2)
```

3.1252 $\int \frac{(bd+2cdx)^4}{\left(a+bx+cx^2\right)^{5/2}} dx$

Optimal. Leaf size=96

$$16c^{3/2}d^4 \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) - \frac{8cd^4(b+2cx)}{\sqrt{a+bx+cx^2}} - \frac{2d^4(b+2cx)^3}{3\left(a+bx+cx^2\right)^{3/2}}$$

```
[Out] (-2*d^4*(b + 2*c*x)^3)/(3*(a + b*x + c*x^2)^(3/2)) - (8*c*d^4*(b + 2*c*x))/
Sqrt[a + b*x + c*x^2] + 16*c^(3/2)*d^4*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[
a + b*x + c*x^2])]
```

Rubi [A] time = 0.0499499, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {686, 621, 206}

$$16c^{3/2}d^4 \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) - \frac{8cd^4(b+2cx)}{\sqrt{a+bx+cx^2}} - \frac{2d^4(b+2cx)^3}{3\left(a+bx+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^4/(a + b*x + c*x^2)^(5/2),x]
```



```
[Out] (-2*d^4*(b + 2*c*x)^3)/(3*(a + b*x + c*x^2)^(3/2)) - (8*c*d^4*(b + 2*c*x))/
Sqrt[a + b*x + c*x^2] + 16*c^(3/2)*d^4*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[
a + b*x + c*x^2])]
```


Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(bd+2cdx)^4}{\left(a+bx+cx^2\right)^{5/2}}\,dx &= -\frac{2d^4(b+2cx)^3}{3\left(a+bx+cx^2\right)^{3/2}} + \left(4cd^2\right)\int \frac{(bd+2cdx)^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx \\ &= -\frac{2d^4(b+2cx)^3}{3\left(a+bx+cx^2\right)^{3/2}} - \frac{8cd^4(b+2cx)}{\sqrt{a+bx+cx^2}} + \left(16c^2d^4\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx \\ &= -\frac{2d^4(b+2cx)^3}{3\left(a+bx+cx^2\right)^{3/2}} - \frac{8cd^4(b+2cx)}{\sqrt{a+bx+cx^2}} + \left(32c^2d^4\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right) \\ &= -\frac{2d^4(b+2cx)^3}{3\left(a+bx+cx^2\right)^{3/2}} - \frac{8cd^4(b+2cx)}{\sqrt{a+bx+cx^2}} + 16c^{3/2}d^4\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.348217, size = 142, normalized size = 1.48

$$d^4\left(\frac{16c^{3/2}\sqrt{a+x(b+cx)}\sinh^{-1}\left(\frac{b+2cx}{\sqrt{c}\sqrt{4a-\frac{b^2}{c}}}\right)}{\sqrt{4a-\frac{b^2}{c}}\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}} - \frac{2(b+2cx)\left(4c\left(3a+4cx^2\right)+b^2+16bcx\right)}{3(a+x(b+cx))^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^4/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] d^4*((-2*(b + 2*c*x)*(b^2 + 16*b*c*x + 4*c*(3*a + 4*c*x^2)))/(3*(a + x*(b +
 c*x))^(3/2)) + (16*c^(3/2)*Sqrt[a + x*(b + c*x)]*ArcSinh[(b + 2*c*x)/(Sqrt
[4*a - b^2/c]*Sqrt[c])])/(Sqrt[4*a - b^2/c]*Sqrt[(c*(a + x*(b + c*x)))/(-b^
2 + 4*a*c)]))
```

Maple [B] time = 0.053, size = 531, normalized size = 5.5

$$-8\,\frac{c^2d^4b^2ax}{\left(4\,ac-b^2\right)\left(cx^2+bx+a\right)^{3/2}}-64\,\frac{c^3d^4b^2ax}{\left(4\,ac-b^2\right)^2\sqrt{cx^2+bx+a}}-24\,\frac{d^4bc^2x^2}{\left(cx^2+bx+a\right)^{3/2}}-18\,\frac{d^4b^2cx}{\left(cx^2+bx+a\right)^{3/2}}+8\,\frac{}{(4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^4/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] -8*d^4*c^2*b^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x-64*d^4*c^3*b^2*a/(4*a*c-
b^2)^2/(c*x^2+b*x+a)^(1/2)*x-24*d^4*c^2*b*x^2/(c*x^2+b*x+a)^(3/2)-18*d^4*c*
b^2*x/(c*x^2+b*x+a)^(3/2)+8*d^4*c*b^5/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)-16*
d^4*c*b*a/(c*x^2+b*x+a)^(3/2)+8*d^4*c*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+1
/3*d^4*b^3/(c*x^2+b*x+a)^(3/2)+2*d^4*c*b^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*
x+16*d^4*c^2*b^4/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-4*d^4*c*b^3*a/(4*a*c-b
^2)/(c*x^2+b*x+a)^(3/2)-32*d^4*c^2*b^3*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)+
16*d^4*c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-16/3*d^4*c^3*x^3/(c*x^2+b*
x+a)^(3/2)+d^4*b^5/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)-16*d^4*c^2*x/(c*x^2+b*x+
a)^(1/2)+8*d^4*c*b/(c*x^2+b*x+a)^(1/2)+16*d^4*c^(3/2)*ln((1/2*b+c*x)/c^(1/2
)+(c*x^2+b*x+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 8.80002, size = 979, normalized size = 10.2

$$\left[\frac{2\left(12\left(c^3d^4x^4+2\,bc^2d^4x^3+2\,abcd^4x+a^2cd^4+\left(b^2c+2\,ac^2\right)d^4x^2\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}(2\,cx+\right.}{3\left(c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [2/3*(12*(c^3*d^4*x^4 + 2*b*c^2*d^4*x^3 + 2*a*b*c*d^4*x + a^2*c*d^4 + (b^2*
c + 2*a*c^2)*d^4*x^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2
 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - (32*c^3*d^4*x^3 + 48*b*c^2*d^4*x
^2 + 6*(3*b^2*c + 4*a*c^2)*d^4*x + (b^3 + 12*a*b*c)*d^4)*sqrt(c*x^2 + b*x +
 a))/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2), -2/3*(24*(c
^3*d^4*x^4 + 2*b*c^2*d^4*x^3 + 2*a*b*c*d^4*x + a^2*c*d^4 + (b^2*c + 2*a*c^2
)*d^4*x^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(
c^2*x^2 + b*c*x + a*c)) + (32*c^3*d^4*x^3 + 48*b*c^2*d^4*x^2 + 6*(3*b^2*c +
 4*a*c^2)*d^4*x + (b^3 + 12*a*b*c)*d^4)*sqrt(c*x^2 + b*x + a))/(c^2*x^4 + 2
*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**4/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17013, size = 458, normalized size = 4.77

$$-\frac{8\,d^4\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{\sqrt{c}}-\frac{2\left(8\left(\frac{2\left(b^4c^3d^4-8\,ab^2c^4d^4+16\,a^2c^5d^4\right)x}{b^4c^2-8\,ab^2c^3+16\,a^2c^4}+\frac{3\left(b^5c^2d^4-8\,ab^3c^3d^4+16\,a^2bc^4d^4\right)}{b^4c^2-8\,ab^2c^3+16\,a^2c^4}\right)x+\frac{3\left(3\,b^6cd\right.}{}\right.}{3\left(cx^2+bx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -8*d^4*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/sqrt(c)
 - 1/3*(2*(8*(2*(b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + 16*a^2*c^5*d^4)*x/(b^4*c^2
 - 8*a*b^2*c^3 + 16*a^2*c^4) + 3*(b^5*c^2*d^4 - 8*a*b^3*c^3*d^4 + 16*a^2*b*
c^4*d^4)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))*x + 3*(3*b^6*c*d^4 - 20*a*b^
4*c^2*d^4 + 16*a^2*b^2*c^3*d^4 + 64*a^3*c^4*d^4)/(b^4*c^2 - 8*a*b^2*c^3 + 1
6*a^2*c^4))*x + (b^7*d^4 + 4*a*b^5*c*d^4 - 80*a^2*b^3*c^2*d^4 + 192*a^3*b*c
^3*d^4)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))/(c*x^2 + b*x + a)^(3/2)
```

3.1253 $\int \frac{(bd+2cdx)^3}{\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=52

$$-\frac{16cd^3}{3\sqrt{a+bx+cx^2}}-\frac{2d^3(b+2cx)^2}{3\left(a+bx+cx^2\right)^{3/2}}$$

```
[Out] (-2*d^3*(b + 2*c*x)^2)/(3*(a + b*x + c*x^2)^(3/2)) - (16*c*d^3)/(3*Sqrt[a +
 b*x + c*x^2])
```

Rubi [A] time = 0.0233337, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {686, 629}

$$-\frac{16cd^3}{3\sqrt{a+bx+cx^2}}-\frac{2d^3(b+2cx)^2}{3\left(a+bx+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^3/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*d^3*(b + 2*c*x)^2)/(3*(a + b*x + c*x^2)^(3/2)) - (16*c*d^3)/(3*Sqrt[a +
 b*x + c*x^2])
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(bd+2cdx)^3}{\left(a+bx+cx^2\right)^{5/2}}\,dx &= -\frac{2d^3(b+2cx)^2}{3\left(a+bx+cx^2\right)^{3/2}}+\frac{1}{3}\left(8cd^2\right)\int \frac{bd+2cdx}{\left(a+bx+cx^2\right)^{3/2}}\,dx\\ &= -\frac{2d^3(b+2cx)^2}{3\left(a+bx+cx^2\right)^{3/2}}-\frac{16cd^3}{3\sqrt{a+bx+cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0292933, size = 42, normalized size = 0.81

$$-\frac{2d^3\left(4c\left(2a+3cx^2\right)+b^2+12bcx\right)}{3(a+x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^3/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*d^3*(b^2 + 12*b*c*x + 4*c*(2*a + 3*c*x^2)))/(3*(a + x*(b + c*x))^(3/2))
```

Maple [A] time = 0.044, size = 39, normalized size = 0.8

$$-\frac{2d^{3}\left(12c^{2}x^{2}+12bcx+8ac+b^{2}\right)}{3}\left(cx^{2}+bx+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^3/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] -2/3*d^3*(12*c^2*x^2+12*b*c*x+8*a*c+b^2)/(c*x^2+b*x+a)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.68305, size = 186, normalized size = 3.58

$$-\frac{2\left(12c^{2}d^{3}x^{2}+12bcd^{3}x+\left(b^{2}+8ac\right)d^{3}\right)\sqrt{cx^{2}+bx+a}}{3\left(c^{2}x^{4}+2bcx^{3}+2abx+\left(b^{2}+2ac\right)x^{2}+a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(12*c^2*d^3*x^2 + 12*b*c*d^3*x + (b^2 + 8*a*c)*d^3)*sqrt(c*x^2 + b*x +
 a)/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)
```

Sympy [B] time = 1.99567, size = 264, normalized size = 5.08

$$-\frac{16acd^{3}}{3a\sqrt{a+bx+cx^{2}}+3bx\sqrt{a+bx+cx^{2}}+3cx^{2}\sqrt{a+bx+cx^{2}}}-\frac{2b^{2}d^{3}}{3a\sqrt{a+bx+cx^{2}}+3bx\sqrt{a+bx+cx^{2}}+3cx^{2}\sqrt{a+bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**3/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] -16*a*c*d**3/(3*a*sqrt(a + b*x + c*x**2) + 3*b*x*sqrt(a + b*x + c*x**2) + 3
*c*x**2*sqrt(a + b*x + c*x**2)) - 2*b**2*d**3/(3*a*sqrt(a + b*x + c*x**2) +
 3*b*x*sqrt(a + b*x + c*x**2) + 3*c*x**2*sqrt(a + b*x + c*x**2)) - 24*b*c*d
**3*x/(3*a*sqrt(a + b*x + c*x**2) + 3*b*x*sqrt(a + b*x + c*x**2) + 3*c*x**2
*sqrt(a + b*x + c*x**2)) - 24*c**2*d**3*x**2/(3*a*sqrt(a + b*x + c*x**2) +
3*b*x*sqrt(a + b*x + c*x**2) + 3*c*x**2*sqrt(a + b*x + c*x**2))
```

Giac [B] time = 1.14325, size = 275, normalized size = 5.29

$$-\frac{12\left(\frac{\left(b^4c^2d^3-8\,ab^2c^3d^3+16\,a^2c^4d^3\right)x}{b^4c^2-8\,ab^2c^3+16\,a^2c^4}+\frac{b^5cd^3-8\,ab^3c^2d^3+16\,a^2bc^3d^3}{b^4c^2-8\,ab^2c^3+16\,a^2c^4}\right)x+\frac{b^6d^3-48\,a^2b^2c^2d^3+128\,a^3c^3d^3}{b^4c^2-8\,ab^2c^3+16\,a^2c^4}}{3\left(cx^2+bx+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(12*((b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + 16*a^2*c^4*d^3)*x/(b^4*c^2 - 8*a
*b^2*c^3 + 16*a^2*c^4) + (b^5*c*d^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3)/(
b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))*x + (b^6*d^3 - 48*a^2*b^2*c^2*d^3 + 12
8*a^3*c^3*d^3)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))/(c*x^2 + b*x + a)^(3/2
)
```

3.1254 $\int \frac{(bd+2cdx)^2}{(a+bx+cx^2)^{5/2}} \, dx$

Optimal. Leaf size=39

$$-\frac{2d^2(b+2cx)^3}{3(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

```
[Out] (-2*d^2*(b + 2*c*x)^3)/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2))
```

Rubi [A] time = 0.0142451, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {682}

$$-\frac{2d^2(b+2cx)^3}{3(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^2/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*d^2*(b + 2*c*x)^3)/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2))
```

Rule 682

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c,
0] && EqQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(bd+2cdx)^2}{(a+bx+cx^2)^{5/2}} \, dx = -\frac{2d^2(b+2cx)^3}{3(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

Mathematica [A] time = 0.0205791, size = 38, normalized size = 0.97

$$-\frac{2d^2(b+2cx)^3}{3(b^2-4ac)(a+x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^2/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*d^2*(b + 2*c*x)^3)/(3*(b^2 - 4*a*c)*(a + x*(b + c*x))^(3/2))
```

Maple [A] time = 0.047, size = 38, normalized size = 1.

$$\frac{2\,d^2\,(2\,cx+b)^3}{12\,ac-3\,b^2}\left(cx^2+bx+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^2/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 2/3*(2*c*x+b)^3*d^2/(c*x^2+b*x+a)^(3/2)/(4*a*c-b^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.55427, size = 304, normalized size = 7.79

$$-\frac{2\left(8\,c^{3}d^{2}x^{3}+12\,bc^{2}d^{2}x^{2}+6\,b^{2}cd^{2}x+b^{3}d^{2}\right)\sqrt{cx^{2}+bx+a}}{3\left(\left(b^{2}c^{2}-4\,ac^{3}\right)x^{4}+a^{2}b^{2}-4\,a^{3}c+2\left(b^{3}c-4\,abc^{2}\right)x^{3}+\left(b^{4}-2\,ab^{2}c-8\,a^{2}c^{2}\right)x^{2}+2\left(ab^{3}-4\,a^{2}bc\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(8*c^3*d^2*x^3 + 12*b*c^2*d^2*x^2 + 6*b^2*c*d^2*x + b^3*d^2)*sqrt(c*x^
2 + b*x + a)/((b^2*c^2 - 4*a*c^3)*x^4 + a^2*b^2 - 4*a^3*c + 2*(b^3*c - 4*a*
b*c^2)*x^3 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*x^2 + 2*(a*b^3 - 4*a^2*b*c)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**2/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19467, size = 296, normalized size = 7.59

$$-\frac{2\left(2\left(\frac{2\left(b^{2}c^{3}d^{2}-4\,ac^{4}d^{2}\right)x}{b^{4}c^{2}-8\,ab^{2}c^{3}+16\,a^{2}c^{4}}+\frac{3\left(b^{3}c^{2}d^{2}-4\,abc^{3}d^{2}\right)}{b^{4}c^{2}-8\,ab^{2}c^{3}+16\,a^{2}c^{4}}\right)x+\frac{3\left(b^{4}cd^{2}-4\,ab^{2}c^{2}d^{2}\right)}{b^{4}c^{2}-8\,ab^{2}c^{3}+16\,a^{2}c^{4}}\right)x+\frac{b^{5}d^{2}-4\,ab^{3}cd^{2}}{b^{4}c^{2}-8\,ab^{2}c^{3}+16\,a^{2}c^{4}}}{3\left(cx^{2}+bx+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(2*(2*(2*(b^2*c^3*d^2 - 4*a*c^4*d^2)*x/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2
*c^4) + 3*(b^3*c^2*d^2 - 4*a*b*c^3*d^2)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4
))*x + 3*(b^4*c*d^2 - 4*a*b^2*c^2*d^2)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)
)*x + (b^5*d^2 - 4*a*b^3*c*d^2)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))/(c*x^
2 + b*x + a)^(3/2)
```

3.1255 $\int \frac{bd+2cdx}{(a+bx+cx^2)^{5/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{2d}{3\left(a+bx+cx^2\right)^{3/2}}$$

[Out] (-2*d)/(3*(a + b*x + c*x^2)^(3/2))

Rubi [A] time = 0.0062324, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {629}

$$-\frac{2d}{3\left(a+bx+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(b*d + 2*c*d*x)/(a + b*x + c*x^2)^(5/2),x]

[Out] (-2*d)/(3*(a + b*x + c*x^2)^(3/2))

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{bd+2cdx}{\left(a+bx+cx^2\right)^{5/2}}\,dx=-\frac{2d}{3\left(a+bx+cx^2\right)^{3/2}}$$

Mathematica [A] time = 0.0067048, size = 18, normalized size = 0.95

$$-\frac{2d}{3(a+x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*d + 2*c*d*x)/(a + b*x + c*x^2)^(5/2),x]

[Out] (-2*d)/(3*(a + x*(b + c*x))^(3/2))

Maple [A] time = 0.043, size = 16, normalized size = 0.8

$$-\frac{2d}{3}\left(cx^2+bx+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] -2/3*d/(c*x^2+b*x+a)^(3/2)
```

Maxima [A] time = 1.14394, size = 20, normalized size = 1.05

$$-\frac{2\,d}{3\,\left(cx^2+bx+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*d/(c*x^2 + b*x + a)^(3/2)
```

Fricas [B] time = 4.77405, size = 119, normalized size = 6.26

$$-\frac{2\,\sqrt{cx^2+bx+a}d}{3\,\left(c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\,ac\right)x^2+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(c*x^2 + b*x + a)*d/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)
*x^2 + a^2)
```

Sympy [B] time = 2.45053, size = 60, normalized size = 3.16

$$-\frac{2d}{3a\sqrt{a+bx+cx^2}+3bx\sqrt{a+bx+cx^2}+3cx^2\sqrt{a+bx+cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] -2*d/(3*a*sqrt(a + b*x + c*x**2) + 3*b*x*sqrt(a + b*x + c*x**2) + 3*c*x**2*
sqrt(a + b*x + c*x**2))
```

Giac [B] time = 1.15215, size = 86, normalized size = 4.53

$$-\frac{b^4d-8\,ab^2cd+16\,a^2c^2d}{3\,\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)\left(cx^2+bx+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(b^4*d - 8*a*b^2*c*d + 16*a^2*c^2*d)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*
c^4)*(c*x^2 + b*x + a)^(3/2))
```

3.1256 $\int \frac{1}{(bd+2cdx)\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=118

$$\frac{16c^{3/2}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{d\left(b^2-4ac\right)^{5/2}}+\frac{8c}{d\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}-\frac{2}{3d\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

```
[Out] -2/(3*(b^2 - 4*a*c)*d*(a + b*x + c*x^2)^(3/2)) + (8*c)/((b^2 - 4*a*c)^2*d*S
qrt[a + b*x + c*x^2]) + (16*c^(3/2)*ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2]
)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2)*d)
```

Rubi [A] time = 0.0767907, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {687, 688, 205}

$$\frac{16c^{3/2}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{d\left(b^2-4ac\right)^{5/2}}+\frac{8c}{d\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}-\frac{2}{3d\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)*(a + b*x + c*x^2)^(5/2)),x]
```

```
[Out] -2/(3*(b^2 - 4*a*c)*d*(a + b*x + c*x^2)^(3/2)) + (8*c)/((b^2 - 4*a*c)^2*d*S
qrt[a + b*x + c*x^2]) + (16*c^(3/2)*ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2]
)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2)*d)
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2}{3\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^{3/2}} - \frac{(4c)\int \frac{1}{(bd+2cdx)\left(a+bx+cx^2\right)^{3/2}}\,dx}{b^2-4ac}$$

$$= -\frac{2}{3\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^{3/2}} + \frac{8c}{\left(b^2-4ac\right)^2 d\sqrt{a+bx+cx^2}} + \frac{\left(16c^2\right)\int \frac{}{(bd+2cdx}}{\left(b^2-4\right.}$$

$$= -\frac{2}{3\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^{3/2}} + \frac{8c}{\left(b^2-4ac\right)^2 d\sqrt{a+bx+cx^2}} + \left(64c^3\right)\text{Subst}\left(\int\right.$$

$$= -\frac{2}{3\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^{3/2}} + \frac{8c}{\left(b^2-4ac\right)^2 d\sqrt{a+bx+cx^2}} + \frac{16c^{3/2}\tan^{-1}\left(2\sqrt{}\right.}{\left(b^2-4ac\right.}$$

Mathematica [C] time = 0.0360012, size = 62, normalized size = 0.53

$$-\frac{2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{4c(a+x(b+cx))}{4ac-b^2}\right)}{3d\left(b^2-4ac\right)(a+x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)*(a + b*x + c*x^2)^(5/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/2, 1, -1/2, (4*c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)
])/(3*(b^2 - 4*a*c)*d*(a + x*(b + c*x))^(3/2))
```

Maple [B] time = 0.192, size = 207, normalized size = 1.8

$$\frac{2}{3d\left(4ac-b^2\right)}\left(\left(x+\frac{b}{2c}\right)^2c+\frac{4ac-b^2}{4c}\right)^{-\frac{3}{2}}+8\frac{c}{d\left(4ac-b^2\right)^2}\frac{1}{\sqrt{\left(x+1/2\frac{b}{c}\right)^2c+1/4\frac{4ac-b^2}{c}}}-16\frac{c}{d\left(4ac-b^2\right)^2}\ln\left(\left(1/2\frac{4a}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 2/3/d/(4*a*c-b^2)/((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(3/2)+8/d*c/(4*a*c-b^
2)^2/((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2)-16/d*c/(4*a*c-b^2)^2/((4*a*c
-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/2)*(4*(x+1/2*b/
c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 7.54834, size = 1341, normalized size = 11.36

$$\left[\frac{2\left(12\left(c^3x^4+2\,bc^2x^3+2\,abcx+a^2c+\left(b^2c+2\,ac^2\right)x^2\right)\sqrt{-\frac{c}{b^2-4\,ac}}\log\left(-\frac{4\,c^2x^2+4\,bcx-b^2+8\,ac+4\,\sqrt{cx^2+bx+a}\left(b^2-4\,ac\right)\sqrt{-\frac{c}{b^2-4\,ac}}}{4\,c^2x^2+4\,bcx+b^2}\right)-\cdots}{3\left(\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)dx^4+2\left(b^5c-8\,ab^3c^2+16\,a^2bc^3\right)dx^3+\left(b^6-6\,ab^4c+32\,a^3c^3\right)dx^2+2\left(ab^5-8\,a^2b^3c\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [2/3*(12*(c^3*x^4 + 2*b*c^2*x^3 + 2*a*b*c*x + a^2*c + (b^2*c + 2*a*c^2)*x^2
)*sqrt(-c/(b^2 - 4*a*c))*log(-(4*c^2*x^2 + 4*b*c*x - b^2 + 8*a*c + 4*sqrt(c
*x^2 + b*x + a)*(b^2 - 4*a*c)*sqrt(-c/(b^2 - 4*a*c)))/(4*c^2*x^2 + 4*b*c*x
+ b^2)) + (12*c^2*x^2 + 12*b*c*x - b^2 + 16*a*c)*sqrt(c*x^2 + b*x + a))/((b
^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*
b*c^3)*d*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d*x^2 + 2*(a*b^5 - 8*a^2*b^3*
c + 16*a^3*b*c^2)*d*x + (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*d), 2/3*(24*(c
^3*x^4 + 2*b*c^2*x^3 + 2*a*b*c*x + a^2*c + (b^2*c + 2*a*c^2)*x^2)*sqrt(c/(b
^2 - 4*a*c))*arctan(-1/2*sqrt(c*x^2 + b*x + a)*(b^2 - 4*a*c)*sqrt(c/(b^2 -
4*a*c))/(c^2*x^2 + b*c*x + a*c)) + (12*c^2*x^2 + 12*b*c*x - b^2 + 16*a*c)*s
qrt(c*x^2 + b*x + a))/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*x^4 + 2*(b^5*
c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d*x^
2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d*x + (a^2*b^4 - 8*a^3*b^2*c + 1
6*a^4*c^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{1}{a^2b\sqrt{a+bx+cx^2}+2a^2cx\sqrt{a+bx+cx^2}+2ab^2x\sqrt{a+bx+cx^2}+6abcx^2\sqrt{a+bx+cx^2}+4ac^2x^3\sqrt{a+bx+cx^2}+b^3x^2\sqrt{a+bx+cx^2}+4b^2cx^3\sqrt{a+bx+cx^2}+5bc^2x^4\sqrt{a+bx+cx^2}+\cdots}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Integral(1/(a**2*b*sqrt(a + b*x + c*x**2) + 2*a**2*c*x*sqrt(a + b*x + c*x**
2) + 2*a*b**2*x*sqrt(a + b*x + c*x**2) + 6*a*b*c*x**2*sqrt(a + b*x + c*x**2
) + 4*a*c**2*x**3*sqrt(a + b*x + c*x**2) + b**3*x**2*sqrt(a + b*x + c*x**2)
 + 4*b**2*c*x**3*sqrt(a + b*x + c*x**2) + 5*b*c**2*x**4*sqrt(a + b*x + c*x*
*2) + 2*c**3*x**5*sqrt(a + b*x + c*x**2)), x)/d
```

Giac [B] time = 1.21558, size = 753, normalized size = 6.38

$$\frac{32\,c^2\arctan\left(-\frac{2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)c+b\sqrt{c}}{\sqrt{b^2c-4\,ac^2}}\right)}{\left(b^4d-8\,ab^2cd+16\,a^2c^2d\right)\sqrt{b^2c-4\,ac^2}}+\frac{12\left(\frac{\left(b^{16}c^2d^3-32\,ab^{14}c^3d^3+448\,a^2b^{12}c^4d^3-3584\,a^3b^{10}c^5d^3+17920\,a^4b^8c^6d^3-57344\,a^5b^6c^7d^3+11468\cdots\right.}{b^4c^2-8\,ab^2c^3+16\,a^2c^4}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 32*c^2*arctan(-(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*c + b*sqrt(c))/sqrt(b
^2*c - 4*a*c^2))/((b^4*d - 8*a*b^2*c*d + 16*a^2*c^2*d)*sqrt(b^2*c - 4*a*c^2
)) + 1/3*(12*((b^16*c^2*d^3 - 32*a*b^14*c^3*d^3 + 448*a^2*b^12*c^4*d^3 - 35
84*a^3*b^10*c^5*d^3 + 17920*a^4*b^8*c^6*d^3 - 57344*a^5*b^6*c^7*d^3 + 11468
8*a^6*b^4*c^8*d^3 - 131072*a^7*b^2*c^9*d^3 + 65536*a^8*c^10*d^3)*x/(b^4*c^2
 - 8*a*b^2*c^3 + 16*a^2*c^4) + (b^17*c*d^3 - 32*a*b^15*c^2*d^3 + 448*a^2*b^
13*c^3*d^3 - 3584*a^3*b^11*c^4*d^3 + 17920*a^4*b^9*c^5*d^3 - 57344*a^5*b^7*
c^6*d^3 + 114688*a^6*b^5*c^7*d^3 - 131072*a^7*b^3*c^8*d^3 + 65536*a^8*b*c^9
*d^3)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))*x - (b^18*d^3 - 48*a*b^16*c*d^3
 + 960*a^2*b^14*c^2*d^3 - 10752*a^3*b^12*c^3*d^3 + 75264*a^4*b^10*c^4*d^3 -
 344064*a^5*b^8*c^5*d^3 + 1032192*a^6*b^6*c^6*d^3 - 1966080*a^7*b^4*c^7*d^3
 + 2162688*a^8*b^2*c^8*d^3 - 1048576*a^9*c^9*d^3)/(b^4*c^2 - 8*a*b^2*c^3 +
16*a^2*c^4))/(c*x^2 + b*x + a)^(3/2)
```

3.1257 $\int \frac{1}{(bd+2cdx)^2\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=122

$$\frac{256c^2\sqrt{a+bx+cx^2}}{3d^2\left(b^2-4ac\right)^3(b+2cx)}+\frac{32c}{3d^2\left(b^2-4ac\right)^2(b+2cx)\sqrt{a+bx+cx^2}}-\frac{2}{3d^2\left(b^2-4ac\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}$$

```
[Out] -2/(3*(b^2 - 4*a*c)*d^2*(b + 2*c*x)*(a + b*x + c*x^2)^(3/2)) + (32*c)/(3*(b
^2 - 4*a*c)^2*d^2*(b + 2*c*x)*Sqrt[a + b*x + c*x^2]) + (256*c^2*Sqrt[a + b*
x + c*x^2])/(3*(b^2 - 4*a*c)^3*d^2*(b + 2*c*x))
```

Rubi [A] time = 0.0557404, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {687, 682}

$$\frac{256c^2\sqrt{a+bx+cx^2}}{3d^2\left(b^2-4ac\right)^3(b+2cx)}+\frac{32c}{3d^2\left(b^2-4ac\right)^2(b+2cx)\sqrt{a+bx+cx^2}}-\frac{2}{3d^2\left(b^2-4ac\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^2*(a + b*x + c*x^2)^(5/2)),x]
```

```
[Out] -2/(3*(b^2 - 4*a*c)*d^2*(b + 2*c*x)*(a + b*x + c*x^2)^(3/2)) + (32*c)/(3*(b
^2 - 4*a*c)^2*d^2*(b + 2*c*x)*Sqrt[a + b*x + c*x^2]) + (256*c^2*Sqrt[a + b*
x + c*x^2])/(3*(b^2 - 4*a*c)^3*d^2*(b + 2*c*x))
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 682

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c,
0] && EqQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(bd+2cdx)^2\left(a+bx+cx^2\right)^{5/2}}\,dx &= -\frac{2}{3\left(b^2-4ac\right)d^2(b+2cx)\left(a+bx+cx^2\right)^{3/2}}-\frac{(16c)\int \frac{1}{(bd+2cdx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx}{3\left(b^2-4ac\right)}\\ &= -\frac{2}{3\left(b^2-4ac\right)d^2(b+2cx)\left(a+bx+cx^2\right)^{3/2}}+\frac{32c}{3\left(b^2-4ac\right)^2d^2(b+2cx)\sqrt{a+bx}}\\ &= -\frac{2}{3\left(b^2-4ac\right)d^2(b+2cx)\left(a+bx+cx^2\right)^{3/2}}+\frac{32c}{3\left(b^2-4ac\right)^2d^2(b+2cx)\sqrt{a+bx}}\end{aligned}$$

Mathematica [A] time = 0.0516286, size = 108, normalized size = 0.89

$$\frac{32c^2\left(3a^2+12acx^2+8c^2x^4\right)+48b^2c\left(a+6cx^2\right)+128bc^2x\left(3a+4cx^2\right)+32b^3cx-2b^4}{3d^2\left(b^2-4ac\right)^3(b+2cx)(a+x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^2*(a + b*x + c*x^2)^(5/2)),x]
```

```
[Out] (-2*b^4 + 32*b^3*c*x + 128*b*c^2*x*(3*a + 4*c*x^2) + 48*b^2*c*(a + 6*c*x^2)
 + 32*c^2*(3*a^2 + 12*a*c*x^2 + 8*c^2*x^4))/(3*(b^2 - 4*a*c)^3*d^2*(b + 2*c
*x)*(a + x*(b + c*x))^(3/2))
```

Maple [A] time = 0.048, size = 133, normalized size = 1.1

$$-\frac{256\,c^4x^4+512\,bc^3x^3+384\,ac^3x^2+288\,b^2c^2x^2+384\,abc^2x+32\,b^3cx+96\,a^2c^2+48\,acb^2-2\,b^4}{(6\,cx+3\,b)\,d^2\left(64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6\right)}\left(cx^2+bx+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^2/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] -2/3*(128*c^4*x^4+256*b*c^3*x^3+192*a*c^3*x^2+144*b^2*c^2*x^2+192*a*b*c^2*x
+16*b^3*c*x+48*a^2*c^2+24*a*b^2*c-b^4)/(2*c*x+b)/d^2/(64*a^3*c^3-48*a^2*b^2
*c^2+12*a*b^4*c-b^6)/(c*x^2+b*x+a)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 22.3653, size = 794, normalized size = 6.51

$$\frac{2\left(128\,c^4x^4+256\right.}{3\left(2\left(b^6c^3-12\,ab^4c^4+48\,a^2b^2c^5-64\,a^3c^6\right)d^2x^5+5\left(b^7c^2-12\,ab^5c^3+48\,a^2b^3c^4-64\,a^3bc^5\right)d^2x^4+4\left(b^8c-11\,ab^6c^2+36\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(128*c^4*x^4 + 256*b*c^3*x^3 - b^4 + 24*a*b^2*c + 48*a^2*c^2 + 48*(3*b^
2*c^2 + 4*a*c^3)*x^2 + 16*(b^3*c + 12*a*b*c^2)*x)*sqrt(c*x^2 + b*x + a)/(2*
(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*d^2*x^5 + 5*(b^7*c^2
 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^2*x^4 + 4*(b^8*c - 11*a*
b^6*c^2 + 36*a^2*b^4*c^3 - 16*a^3*b^2*c^4 - 64*a^4*c^5)*d^2*x^3 + (b^9 - 6*
```

```
a*b^7*c - 24*a^2*b^5*c^2 + 224*a^3*b^3*c^3 - 384*a^4*b*c^4)*d^2*x^2 + 2*(a*
b^8 - 11*a^2*b^6*c + 36*a^3*b^4*c^2 - 16*a^4*b^2*c^3 - 64*a^5*c^4)*d^2*x +
(a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^2b^2\sqrt{a+bx+cx^2}+4a^2bcx\sqrt{a+bx+cx^2}+4a^2c^2x^2\sqrt{a+bx+cx^2}+2ab^3x\sqrt{a+bx+cx^2}+10ab^2cx^2\sqrt{a+bx+cx^2}+16abc^2x^3\sqrt{a+bx+cx^2}+8ac^3x^4\sqrt{a+bx+cx^2}+b^4x^2\sqrt{a+}}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**2/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Integral(1/(a**2*b**2*sqrt(a + b*x + c*x**2) + 4*a**2*b*c*x*sqrt(a + b*x +
c*x**2) + 4*a**2*c**2*x**2*sqrt(a + b*x + c*x**2) + 2*a*b**3*x*sqrt(a + b*x
 + c*x**2) + 10*a*b**2*c*x**2*sqrt(a + b*x + c*x**2) + 16*a*b*c**2*x**3*sqr
t(a + b*x + c*x**2) + 8*a*c**3*x**4*sqrt(a + b*x + c*x**2) + b**4*x**2*sqrt
(a + b*x + c*x**2) + 6*b**3*c*x**3*sqrt(a + b*x + c*x**2) + 13*b**2*c**2*x*
*4*sqrt(a + b*x + c*x**2) + 12*b*c**3*x**5*sqrt(a + b*x + c*x**2) + 4*c**4*
x**6*sqrt(a + b*x + c*x**2)), x)/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2\,cdx + bd)^2 \left(cx^2 + bx + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((2*c*d*x + b*d)^2*(c*x^2 + b*x + a)^(5/2)), x)
```

3.1258 $\int \frac{1}{(bd+2cdx)^3\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=176

$$\frac{80c^2\sqrt{a+bx+cx^2}}{d^3\left(b^2-4ac\right)^3(b+2cx)^2}+\frac{40c^{3/2}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{d^3\left(b^2-4ac\right)^{7/2}}+\frac{40c}{3d^3\left(b^2-4ac\right)^2(b+2cx)^2\sqrt{a+bx+cx^2}}-\frac{\cdots}{3d^3\left(b^2-4ac\right)(b+\cdots}$$

```
[Out] -2/(3*(b^2 - 4*a*c)*d^3*(b + 2*c*x)^2*(a + b*x + c*x^2)^(3/2)) + (40*c)/(3*
(b^2 - 4*a*c)^2*d^3*(b + 2*c*x)^2*Sqrt[a + b*x + c*x^2]) + (80*c^2*Sqrt[a +
 b*x + c*x^2])/((b^2 - 4*a*c)^3*d^3*(b + 2*c*x)^2) + (40*c^(3/2)*ArcTan[(2*
Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(7/2)*d^3
)
```

Rubi [A] time = 0.105792, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {687, 693, 688, 205}

$$\frac{80c^2\sqrt{a+bx+cx^2}}{d^3\left(b^2-4ac\right)^3(b+2cx)^2}+\frac{40c^{3/2}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{d^3\left(b^2-4ac\right)^{7/2}}+\frac{40c}{3d^3\left(b^2-4ac\right)^2(b+2cx)^2\sqrt{a+bx+cx^2}}-\frac{\cdots}{3d^3\left(b^2-4ac\right)(b+\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^3*(a + b*x + c*x^2)^(5/2)),x]
```


```
[Out] -2/(3*(b^2 - 4*a*c)*d^3*(b + 2*c*x)^2*(a + b*x + c*x^2)^(3/2)) + (40*c)/(3*
(b^2 - 4*a*c)^2*d^3*(b + 2*c*x)^2*Sqrt[a + b*x + c*x^2]) + (80*c^2*Sqrt[a +
 b*x + c*x^2])/((b^2 - 4*a*c)^3*d^3*(b + 2*c*x)^2) + (40*c^(3/2)*ArcTan[(2*
Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(7/2)*d^3
)
```


Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
```

```
b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)^3\left(a+bx+cx^2\right)^{5/2}}dx = -\frac{2}{3\left(b^2-4ac\right)d^3(b+2cx)^2\left(a+bx+cx^2\right)^{3/2}} - \frac{(20c)\int \frac{1}{(bd+2cdx)^3\left(a+bx+cx^2\right)^{3/2}}dx}{3\left(b^2-4ac\right)}$$

$$= -\frac{2}{3\left(b^2-4ac\right)d^3(b+2cx)^2\left(a+bx+cx^2\right)^{3/2}} + \frac{40c}{3\left(b^2-4ac\right)^2 d^3(b+2cx)^2\sqrt{a+}}$$

$$= -\frac{2}{3\left(b^2-4ac\right)d^3(b+2cx)^2\left(a+bx+cx^2\right)^{3/2}} + \frac{40c}{3\left(b^2-4ac\right)^2 d^3(b+2cx)^2\sqrt{a+}}$$

$$= -\frac{2}{3\left(b^2-4ac\right)d^3(b+2cx)^2\left(a+bx+cx^2\right)^{3/2}} + \frac{40c}{3\left(b^2-4ac\right)^2 d^3(b+2cx)^2\sqrt{a+}}$$

$$= -\frac{2}{3\left(b^2-4ac\right)d^3(b+2cx)^2\left(a+bx+cx^2\right)^{3/2}} + \frac{40c}{3\left(b^2-4ac\right)^2 d^3(b+2cx)^2\sqrt{a+}}$$

Mathematica [C] time = 0.0364234, size = 62, normalized size = 0.35

$$-\frac{2\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};\frac{4c(a+x(b+cx))}{4ac-b^2}\right)}{3d^3\left(b^2-4ac\right)^2(a+x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^3*(a + b*x + c*x^2)^(5/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/2, 2, -1/2, (4*c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)
])/(3*(b^2 - 4*a*c)^2*d^3*(a + x*(b + c*x))^(3/2))
```

Maple [A] time = 0.2, size = 267, normalized size = 1.5

$$-\frac{1}{4c^2d^3\left(4ac-b^2\right)}\left(x+\frac{b}{2c}\right)^{-2}\left(\left(x+\frac{b}{2c}\right)^2c+\frac{4ac-b^2}{4c}\right)^{-\frac{3}{2}} - \frac{5}{3d^3\left(4ac-b^2\right)^2}\left(\left(x+\frac{b}{2c}\right)^2c+\frac{4ac-b^2}{4c}\right)^{-\frac{3}{2}} - 20\frac{}{d^3(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^3/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] -1/4/d^3/c^2/(4*a*c-b^2)/(x+1/2*b/c)^2/((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^
(3/2)-5/3/d^3/(4*a*c-b^2)^2/((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(3/2)-20/d^
```

```
3*c/(4*a*c-b^2)^3/((x+1/2*b/c)^2*c+1/4*(4*a*c-b^2)/c)^(1/2)+40/d^3*c/(4*a*c
-b^2)^3/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/
2)*(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^3/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 40.8697, size = 2759, normalized size = 15.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^3/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-2/3*(30*(4*c^5*x^6 + 12*b*c^4*x^5 + a^2*b^2*c + (13*b^2*c^3 + 8*a*c^4)*x^
4 + 2*(3*b^3*c^2 + 8*a*b*c^3)*x^3 + (b^4*c + 10*a*b^2*c^2 + 4*a^2*c^3)*x^2
+ 2*(a*b^3*c + 2*a^2*b*c^2)*x)*sqrt(-c/(b^2 - 4*a*c))*log(-(4*c^2*x^2 + 4*b
*c*x - b^2 + 8*a*c - 4*sqrt(c*x^2 + b*x + a)*(b^2 - 4*a*c)*sqrt(-c/(b^2 - 4
*a*c)))/(4*c^2*x^2 + 4*b*c*x + b^2)) - (120*c^4*x^4 + 240*b*c^3*x^3 - b^4 +
 28*a*b^2*c + 24*a^2*c^2 + 20*(7*b^2*c^2 + 8*a*c^3)*x^2 + 20*(b^3*c + 8*a*b
*c^2)*x)*sqrt(c*x^2 + b*x + a))/(4*(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6
 - 64*a^3*c^7)*d^3*x^6 + 12*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a
^3*b*c^6)*d^3*x^5 + (13*b^8*c^2 - 148*a*b^6*c^3 + 528*a^2*b^4*c^4 - 448*a^3
*b^2*c^5 - 512*a^4*c^6)*d^3*x^4 + 2*(3*b^9*c - 28*a*b^7*c^2 + 48*a^2*b^5*c^
3 + 192*a^3*b^3*c^4 - 512*a^4*b*c^5)*d^3*x^3 + (b^10 - 2*a*b^8*c - 68*a^2*b
^6*c^2 + 368*a^3*b^4*c^3 - 448*a^4*b^2*c^4 - 256*a^5*c^5)*d^3*x^2 + 2*(a*b^
9 - 10*a^2*b^7*c + 24*a^3*b^5*c^2 + 32*a^4*b^3*c^3 - 128*a^5*b*c^4)*d^3*x +
 (a^2*b^8 - 12*a^3*b^6*c + 48*a^4*b^4*c^2 - 64*a^5*b^2*c^3)*d^3), 2/3*(60*(
4*c^5*x^6 + 12*b*c^4*x^5 + a^2*b^2*c + (13*b^2*c^3 + 8*a*c^4)*x^4 + 2*(3*b^
3*c^2 + 8*a*b*c^3)*x^3 + (b^4*c + 10*a*b^2*c^2 + 4*a^2*c^3)*x^2 + 2*(a*b^3*
c + 2*a^2*b*c^2)*x)*sqrt(c/(b^2 - 4*a*c))*arctan(-1/2*sqrt(c*x^2 + b*x + a)
*(b^2 - 4*a*c)*sqrt(c/(b^2 - 4*a*c))/(c^2*x^2 + b*c*x + a*c)) + (120*c^4*x^
4 + 240*b*c^3*x^3 - b^4 + 28*a*b^2*c + 24*a^2*c^2 + 20*(7*b^2*c^2 + 8*a*c^3
)*x^2 + 20*(b^3*c + 8*a*b*c^2)*x)*sqrt(c*x^2 + b*x + a))/(4*(b^6*c^4 - 12*a
*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*d^3*x^6 + 12*(b^7*c^3 - 12*a*b^5*c^
4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*d^3*x^5 + (13*b^8*c^2 - 148*a*b^6*c^3 +
528*a^2*b^4*c^4 - 448*a^3*b^2*c^5 - 512*a^4*c^6)*d^3*x^4 + 2*(3*b^9*c - 28*
a*b^7*c^2 + 48*a^2*b^5*c^3 + 192*a^3*b^3*c^4 - 512*a^4*b*c^5)*d^3*x^3 + (b^
10 - 2*a*b^8*c - 68*a^2*b^6*c^2 + 368*a^3*b^4*c^3 - 448*a^4*b^2*c^4 - 256*a
^5*c^5)*d^3*x^2 + 2*(a*b^9 - 10*a^2*b^7*c + 24*a^3*b^5*c^2 + 32*a^4*b^3*c^3
 - 128*a^5*b*c^4)*d^3*x + (a^2*b^8 - 12*a^3*b^6*c + 48*a^4*b^4*c^2 - 64*a^5
*b^2*c^3)*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{}{a^2b^3\sqrt{a+bx+cx^2}+6a^2b^2cx\sqrt{a+bx+cx^2}+12a^2bc^2x^2\sqrt{a+bx+cx^2}+8a^2c^3x^3\sqrt{a+bx+cx^2}+2ab^4x\sqrt{a+bx+cx^2}+14ab^3cx^2\sqrt{a+bx+cx^2}+36ab^2c^2x^3\sqrt{a+bx+cx^2}+40a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**3/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Integral(1/(a**2*b**3*sqrt(a + b*x + c*x**2) + 6*a**2*b**2*c*x*sqrt(a + b*x
 + c*x**2) + 12*a**2*b*c**2*x**2*sqrt(a + b*x + c*x**2) + 8*a**2*c**3*x**3*
sqrt(a + b*x + c*x**2) + 2*a*b**4*x*sqrt(a + b*x + c*x**2) + 14*a*b**3*c*x*
*2*sqrt(a + b*x + c*x**2) + 36*a*b**2*c**2*x**3*sqrt(a + b*x + c*x**2) + 40
*a*b*c**3*x**4*sqrt(a + b*x + c*x**2) + 16*a*c**4*x**5*sqrt(a + b*x + c*x**
2) + b**5*x**2*sqrt(a + b*x + c*x**2) + 8*b**4*c*x**3*sqrt(a + b*x + c*x**2
) + 25*b**3*c**2*x**4*sqrt(a + b*x + c*x**2) + 38*b**2*c**3*x**5*sqrt(a + b
*x + c*x**2) + 28*b*c**4*x**6*sqrt(a + b*x + c*x**2) + 8*c**5*x**7*sqrt(a +
 b*x + c*x**2)), x)/d**3
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^3/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1259 $\int \frac{1}{(bd+2cdx)^4\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=162

$$\frac{512c^2\sqrt{a+bx+cx^2}}{3d^4\left(b^2-4ac\right)^4(b+2cx)}+\frac{256c^2\sqrt{a+bx+cx^2}}{3d^4\left(b^2-4ac\right)^3(b+2cx)^3}+\frac{16c}{d^4\left(b^2-4ac\right)^2(b+2cx)^3\sqrt{a+bx+cx^2}}-\frac{2}{3d^4\left(b^2-4ac\right)(b+2cx)^3\left(a+bx+cx^2\right)^{3/2}}$$

```
[Out] -2/(3*(b^2 - 4*a*c)*d^4*(b + 2*c*x)^3*(a + b*x + c*x^2)^(3/2)) + (16*c)/((b
^2 - 4*a*c)^2*d^4*(b + 2*c*x)^3*Sqrt[a + b*x + c*x^2]) + (256*c^2*Sqrt[a +
b*x + c*x^2])/(3*(b^2 - 4*a*c)^3*d^4*(b + 2*c*x)^3) + (512*c^2*Sqrt[a + b*x
 + c*x^2])/(3*(b^2 - 4*a*c)^4*d^4*(b + 2*c*x))
```

Rubi [A] time = 0.0794517, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {687, 693, 682}

$$\frac{512c^2\sqrt{a+bx+cx^2}}{3d^4\left(b^2-4ac\right)^4(b+2cx)}+\frac{256c^2\sqrt{a+bx+cx^2}}{3d^4\left(b^2-4ac\right)^3(b+2cx)^3}+\frac{16c}{d^4\left(b^2-4ac\right)^2(b+2cx)^3\sqrt{a+bx+cx^2}}-\frac{2}{3d^4\left(b^2-4ac\right)(b+2cx)^3\left(a+bx+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^4*(a + b*x + c*x^2)^(5/2)),x]
```

```
[Out] -2/(3*(b^2 - 4*a*c)*d^4*(b + 2*c*x)^3*(a + b*x + c*x^2)^(3/2)) + (16*c)/((b
^2 - 4*a*c)^2*d^4*(b + 2*c*x)^3*Sqrt[a + b*x + c*x^2]) + (256*c^2*Sqrt[a +
b*x + c*x^2])/(3*(b^2 - 4*a*c)^3*d^4*(b + 2*c*x)^3) + (512*c^2*Sqrt[a + b*x
 + c*x^2])/(3*(b^2 - 4*a*c)^4*d^4*(b + 2*c*x))
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 682

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c,
0] && EqQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)^4\left(a+bx+cx^2\right)^{5/2}}dx = -\frac{2}{3\left(b^2-4ac\right)d^4(b+2cx)^3\left(a+bx+cx^2\right)^{3/2}} - \frac{(8c)\int \frac{1}{(bd+2cdx)^4\left(a+bx+cx^2\right)^{3/2}}dx}{b^2-4ac}$$

$$= -\frac{2}{3\left(b^2-4ac\right)d^4(b+2cx)^3\left(a+bx+cx^2\right)^{3/2}} + \frac{16c}{\left(b^2-4ac\right)^2 d^4(b+2cx)^3\sqrt{a+b}}$$

$$= -\frac{2}{3\left(b^2-4ac\right)d^4(b+2cx)^3\left(a+bx+cx^2\right)^{3/2}} + \frac{16c}{\left(b^2-4ac\right)^2 d^4(b+2cx)^3\sqrt{a+b}}$$

$$= -\frac{2}{3\left(b^2-4ac\right)d^4(b+2cx)^3\left(a+bx+cx^2\right)^{3/2}} + \frac{16c}{\left(b^2-4ac\right)^2 d^4(b+2cx)^3\sqrt{a+b}}$$

Mathematica [A] time = 0.0844971, size = 178, normalized size = 1.1

$$\frac{2\left(48b^2c^2\left(3a^2+44acx^2+72c^2x^4\right)+384bc^3x\left(a^2+8acx^2+8c^2x^4\right)+64c^3\left(6a^2cx^2-a^3+24ac^2x^4+16c^3x^6\right)+64b^3c^2\right.}{3d^4\left(b^2-4ac\right)^4(b+2cx)^3(a+x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^4*(a + b*x + c*x^2)^(5/2)),x]
```

```
[Out] (2*(-b^6 + 24*b^5*c*x + 64*b^3*c^2*x*(9*a + 28*c*x^2) + 12*b^4*c*(3*a + 34*
c*x^2) + 384*b*c^3*x*(a^2 + 8*a*c*x^2 + 8*c^2*x^4) + 48*b^2*c^2*(3*a^2 + 44
*a*c*x^2 + 72*c^2*x^4) + 64*c^3*(-a^3 + 6*a^2*c*x^2 + 24*a*c^2*x^4 + 16*c^3
*x^6)))/(3*(b^2 - 4*a*c)^4*d^4*(b + 2*c*x)^3*(a + x*(b + c*x))^(3/2))
```

Maple [A] time = 0.052, size = 218, normalized size = 1.4

$$-\frac{-2048\,c^6x^6-6144\,bc^5x^5-3072\,ac^5x^4-6912\,b^2c^4x^4-6144\,abc^4x^3-3584\,b^3c^3x^3-768\,a^2c^4x^2-4224\,ab^2c^3x^2-816}{3\left(256\,a^4c^4-256\,a^3b^2c^3+96\,a^2b^4c^2-16\,ab^6c+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^4/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] -2/3*(-1024*c^6*x^6-3072*b*c^5*x^5-1536*a*c^5*x^4-3456*b^2*c^4*x^4-3072*a*b
*c^4*x^3-1792*b^3*c^3*x^3-384*a^2*c^4*x^2-2112*a*b^2*c^3*x^2-408*b^4*c^2*x^
2-384*a^2*b*c^3*x-576*a*b^3*c^2*x-24*b^5*c*x+64*a^3*c^3-144*a^2*b^2*c^2-36*
a*b^4*c+b^6)/(2*c*x+b)^3/d^4/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16
*a*b^6*c+b^8)/(c*x^2+b*x+a)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^4/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 74.7853, size = 1494, normalized size = 9.22

$$3\left(8\left(b^8c^5-16\,ab^6c^6+96\,a^2b^4c^7-256\,a^3b^2c^8+256\,a^4c^9\right)d^4x^7+28\left(b^9c^4-16\,ab^7c^5+96\,a^2b^5c^6-256\,a^3b^3c^7+256\,a^4bc^8\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^4/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(1024*c^6*x^6 + 3072*b*c^5*x^5 - b^6 + 36*a*b^4*c + 144*a^2*b^2*c^2 - 6
4*a^3*c^3 + 384*(9*b^2*c^4 + 4*a*c^5)*x^4 + 256*(7*b^3*c^3 + 12*a*b*c^4)*x^
3 + 24*(17*b^4*c^2 + 88*a*b^2*c^3 + 16*a^2*c^4)*x^2 + 24*(b^5*c + 24*a*b^3*
c^2 + 16*a^2*b*c^3)*x)*sqrt(c*x^2 + b*x + a)/(8*(b^8*c^5 - 16*a*b^6*c^6 + 9
6*a^2*b^4*c^7 - 256*a^3*b^2*c^8 + 256*a^4*c^9)*d^4*x^7 + 28*(b^9*c^4 - 16*a
*b^7*c^5 + 96*a^2*b^5*c^6 - 256*a^3*b^3*c^7 + 256*a^4*b*c^8)*d^4*x^6 + 2*(1
9*b^10*c^3 - 296*a*b^8*c^4 + 1696*a^2*b^6*c^5 - 4096*a^3*b^4*c^6 + 2816*a^4
*b^2*c^7 + 2048*a^5*c^8)*d^4*x^5 + 5*(5*b^11*c^2 - 72*a*b^9*c^3 + 352*a^2*b
^7*c^4 - 512*a^3*b^5*c^5 - 768*a^4*b^3*c^6 + 2048*a^5*b*c^7)*d^4*x^4 + 4*(2
*b^12*c - 23*a*b^10*c^2 + 50*a^2*b^8*c^3 + 320*a^3*b^6*c^4 - 1600*a^4*b^4*c
^5 + 1792*a^5*b^2*c^6 + 512*a^6*c^7)*d^4*x^3 + (b^13 - 2*a*b^11*c - 116*a^2
*b^9*c^2 + 896*a^3*b^7*c^3 - 2176*a^4*b^5*c^4 + 512*a^5*b^3*c^5 + 3072*a^6*
b*c^6)*d^4*x^2 + 2*(a*b^12 - 13*a^2*b^10*c + 48*a^3*b^8*c^2 + 32*a^4*b^6*c^
3 - 512*a^5*b^4*c^4 + 768*a^6*b^2*c^5)*d^4*x + (a^2*b^11 - 16*a^3*b^9*c + 9
6*a^4*b^7*c^2 - 256*a^5*b^5*c^3 + 256*a^6*b^3*c^4)*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{}{a^2b^4\sqrt{a+bx+cx^2}+8a^2b^3cx\sqrt{a+bx+cx^2}+24a^2b^2c^2x^2\sqrt{a+bx+cx^2}+32a^2bc^3x^3\sqrt{a+bx+cx^2}+16a^2c^4x^4\sqrt{a+bx+cx^2}+2ab^5x\sqrt{a+bx+cx^2}+18ab^4cx^2\sqrt{a+bx+cx^2}+64ab^3\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**4/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Integral(1/(a**2*b**4*sqrt(a + b*x + c*x**2) + 8*a**2*b**3*c*x*sqrt(a + b*x
 + c*x**2) + 24*a**2*b**2*c**2*x**2*sqrt(a + b*x + c*x**2) + 32*a**2*b*c**3
*x**3*sqrt(a + b*x + c*x**2) + 16*a**2*c**4*x**4*sqrt(a + b*x + c*x**2) + 2
*a*b**5*x*sqrt(a + b*x + c*x**2) + 18*a*b**4*c*x**2*sqrt(a + b*x + c*x**2)
+ 64*a*b**3*c**2*x**3*sqrt(a + b*x + c*x**2) + 112*a*b**2*c**3*x**4*sqrt(a
+ b*x + c*x**2) + 96*a*b*c**4*x**5*sqrt(a + b*x + c*x**2) + 32*a*c**5*x**6*
sqrt(a + b*x + c*x**2) + b**6*x**2*sqrt(a + b*x + c*x**2) + 10*b**5*c*x**3*
sqrt(a + b*x + c*x**2) + 41*b**4*c**2*x**4*sqrt(a + b*x + c*x**2) + 88*b**3
*c**3*x**5*sqrt(a + b*x + c*x**2) + 104*b**2*c**4*x**6*sqrt(a + b*x + c*x**
2) + 64*b*c**5*x**7*sqrt(a + b*x + c*x**2) + 16*c**6*x**8*sqrt(a + b*x + c*
x**2)), x)/d**4
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^4/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1260 $\int \frac{1}{(a+bx)\sqrt{1+a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=27

$$-\frac{\tanh^{-1}\left(\sqrt{a^2+2abx+b^2x^2+1}\right)}{b}$$

```
[Out] -(ArcTanh[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]]/b)
```

Rubi [A] time = 0.0206604, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {688, 208}

$$-\frac{\tanh^{-1}\left(\sqrt{a^2+2abx+b^2x^2+1}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]),x]
```

```
[Out] -(ArcTanh[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]]/b)
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)\sqrt{1+a^2+2abx+b^2x^2}}\,dx &= \left(4b^2\right)\text{Subst}\left(\int \frac{1}{4a^2b^3-4\left(1+a^2\right)b^3+4b^3x^2}\,dx, x, \sqrt{1+a^2+2abx+b^2x^2}\right)\\ &= -\frac{\tanh^{-1}\left(\sqrt{1+a^2+2abx+b^2x^2}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0081073, size = 19, normalized size = 0.7

$$-\frac{\tanh^{-1}\left(\sqrt{(a+bx)^2+1}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]),x]
```

```
[Out] -(ArcTanh[Sqrt[1 + (a + b*x)^2]]/b)
```

Maple [A] time = 0.045, size = 24, normalized size = 0.9

$$-\frac{1}{b}\operatorname{Artanh}\left(\frac{1}{\sqrt{\left(x+\frac{a}{b}\right)^{2} b^{2}+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x)
```

```
[Out] -1/b*arctanh(1/((x+1/b*a)^2*b^2+1)^(1/2))
```

Maxima [A] time = 2.5732, size = 19, normalized size = 0.7

$$-\frac{\operatorname{arsinh}\left(\frac{1}{|bx+a|}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsinh(1/abs(b*x + a))/b
```

Fricas [B] time = 2.05115, size = 157, normalized size = 5.81

$$-\frac{\log\left(-bx-a+\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}+1\right)-\log\left(-bx-a+\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1) - log(-b*x - a + sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 1))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(a+bx)\sqrt{a^{2}+2abx+b^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
```

Giac [B] time = 1.23572, size = 120, normalized size = 4.44

$$\frac{\log\left(\frac{\left|-2\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)b-2\,a|b|-2\,|b|\right|}{\left|-2\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)b-2\,a|b|+2\,|b|\right|}\right)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] log(abs(-2*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*b - 2*a*abs(b) -
2*abs(b))/abs(-2*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*b - 2*a*abs
(b) + 2*abs(b)))/abs(b)
```

3.1261 $\int (bd+2cdx)^{5/2} \left(a+bx+cx^2\right) dx$

Optimal. Leaf size=55

$$\frac{(bd+2cdx)^{11/2}}{44c^2d^3}-\frac{\left(b^2-4ac\right)(bd+2cdx)^{7/2}}{28c^2d}$$

```
[Out] -((b^2 - 4*a*c)*(b*d + 2*c*d*x)^(7/2))/(28*c^2*d) + (b*d + 2*c*d*x)^(11/2)/
(44*c^2*d^3)
```

Rubi [A] time = 0.0238854, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$\frac{(bd+2cdx)^{11/2}}{44c^2d^3}-\frac{\left(b^2-4ac\right)(bd+2cdx)^{7/2}}{28c^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(5/2)*(a + b*x + c*x^2),x]
```

```
[Out] -((b^2 - 4*a*c)*(b*d + 2*c*d*x)^(7/2))/(28*c^2*d) + (b*d + 2*c*d*x)^(11/2)/
(44*c^2*d^3)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] && !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int (bd+2cdx)^{5/2}\left(a+bx+cx^2\right)dx &= \int \left(\frac{\left(-b^2+4ac\right)(bd+2cdx)^{5/2}}{4c}+\frac{(bd+2cdx)^{9/2}}{4cd^2}\right)dx\\ &= -\frac{\left(b^2-4ac\right)(bd+2cdx)^{7/2}}{28c^2d}+\frac{(bd+2cdx)^{11/2}}{44c^2d^3}\end{aligned}$$

Mathematica [A] time = 0.0436917, size = 45, normalized size = 0.82

$$\frac{\left(c\left(11a+7cx^2\right)-b^2+7bcx\right)(d(b+2cx))^{7/2}}{77c^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(5/2)*(a + b*x + c*x^2),x]
```

```
[Out] ((d*(b + 2*c*x))^(7/2)*(-b^2 + 7*b*c*x + c*(11*a + 7*c*x^2)))/(77*c^2*d)
```

Maple [A] time = 0.041, size = 46, normalized size = 0.8

$$\frac{(2cx+b)\left(7c^2x^2+7bcx+11ac-b^2\right)(2cdx+bd)^{\frac{5}{2}}}{77c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(5/2)*(c*x^2+b*x+a),x)
```

```
[Out] 1/77*(2*c*x+b)*(7*c^2*x^2+7*b*c*x+11*a*c-b^2)*(2*c*d*x+b*d)^(5/2)/c^2
```

Maxima [A] time = 1.098, size = 62, normalized size = 1.13

$$-\frac{11(2cdx+bd)^{\frac{7}{2}}\left(b^2-4ac\right)d^2-7(2cdx+bd)^{\frac{11}{2}}}{308c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(5/2)*(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] -1/308*(11*(2*c*d*x + b*d)^(7/2)*(b^2 - 4*a*c)*d^2 - 7*(2*c*d*x + b*d)^(11/
2))/(c^2*d^3)
```

Fricas [B] time = 1.98296, size = 265, normalized size = 4.82

$$\frac{\left(56c^5d^2x^5+140bc^4d^2x^4+2\left(59b^2c^3+44ac^4\right)d^2x^3+\left(37b^3c^2+132abc^3\right)d^2x^2+\left(b^4c+66ab^2c^2\right)d^2x-\left(b^5-11ab^3c\right)d^2}{77c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(5/2)*(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/77*(56*c^5*d^2*x^5 + 140*b*c^4*d^2*x^4 + 2*(59*b^2*c^3 + 44*a*c^4)*d^2*x^
3 + (37*b^3*c^2 + 132*a*b*c^3)*d^2*x^2 + (b^4*c + 66*a*b^2*c^2)*d^2*x - (b^
5 - 11*a*b^3*c)*d^2)*sqrt(2*c*d*x + b*d)/c^2
```

Sympy [A] time = 5.2453, size = 289, normalized size = 5.25

$$\begin{cases} \frac{ab^3d^2\sqrt{bd+2cdx}}{7c}+\frac{6ab^2d^2x\sqrt{bd+2cdx}}{7}+\frac{12abcd^2x^2\sqrt{bd+2cdx}}{7}+\frac{8ac^2d^2x^3\sqrt{bd+2cdx}}{7}-\frac{b^5d^2\sqrt{bd+2cdx}}{77c^2}+\frac{b^4d^2x\sqrt{bd+2cdx}}{77c}+\frac{37b^3d^2x^2\sqrt{bd+2cdx}}{77}+\frac{11}{} \\ (bd)^{\frac{5}{2}}\left(ax+\frac{bx^2}{2}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(5/2)*(c*x**2+b*x+a),x)
```

```
[Out] Piecewise((a*b**3*d**2*sqrt(b*d + 2*c*d*x)/(7*c) + 6*a*b**2*d**2*x*sqrt(b*d
 + 2*c*d*x)/7 + 12*a*b*c*d**2*x**2*sqrt(b*d + 2*c*d*x)/7 + 8*a*c**2*d**2*x*
*3*sqrt(b*d + 2*c*d*x)/7 - b**5*d**2*sqrt(b*d + 2*c*d*x)/(77*c**2) + b**4*d
**2*x*sqrt(b*d + 2*c*d*x)/(77*c) + 37*b**3*d**2*x**2*sqrt(b*d + 2*c*d*x)/77
 + 118*b**2*c*d**2*x**3*sqrt(b*d + 2*c*d*x)/77 + 20*b*c**2*d**2*x**4*sqrt(b
```

```
*d + 2*c*d*x)/11 + 8*c**3*d**2*x**5*sqrt(b*d + 2*c*d*x)/11, Ne(c, 0)), ((b*
d)**(5/2)*(a*x + b*x**2/2), True))
```

Giac [B] time = 1.14354, size = 508, normalized size = 9.24

$$\frac{4620\,(2\,cdx + bd)^{\frac{3}{2}}ab^2d - 1848\left(5\,(2\,cdx + bd)^{\frac{3}{2}}bd - 3\,(2\,cdx + bd)^{\frac{5}{2}}\right)ab - \frac{462\left(5\,(2\,cdx+bd)^{\frac{3}{2}}bd-3\,(2\,cdx+bd)^{\frac{5}{2}}\right)b^3}{c} + \frac{132\left(35\,(2\,cdx\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(5/2)*(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/13860*(4620*(2*c*d*x + b*d)^(3/2)*a*b^2*d - 1848*(5*(2*c*d*x + b*d)^(3/2)
*b*d - 3*(2*c*d*x + b*d)^(5/2))*a*b - 462*(5*(2*c*d*x + b*d)^(3/2)*b*d - 3*
(2*c*d*x + b*d)^(5/2))*b^3/c + 132*(35*(2*c*d*x + b*d)^(3/2)*b^2*d^2 - 42*(
2*c*d*x + b*d)^(5/2)*b*d + 15*(2*c*d*x + b*d)^(7/2))*a/d + 165*(35*(2*c*d*x
 + b*d)^(3/2)*b^2*d^2 - 42*(2*c*d*x + b*d)^(5/2)*b*d + 15*(2*c*d*x + b*d)^(
7/2))*b^2/(c*d) - 44*(105*(2*c*d*x + b*d)^(3/2)*b^3*d^3 - 189*(2*c*d*x + b*
d)^(5/2)*b^2*d^2 + 135*(2*c*d*x + b*d)^(7/2)*b*d - 35*(2*c*d*x + b*d)^(9/2)
)*b/(c*d^2) + (1155*(2*c*d*x + b*d)^(3/2)*b^4*d^4 - 2772*(2*c*d*x + b*d)^(5
/2)*b^3*d^3 + 2970*(2*c*d*x + b*d)^(7/2)*b^2*d^2 - 1540*(2*c*d*x + b*d)^(9/
2)*b*d + 315*(2*c*d*x + b*d)^(11/2))/(c*d^3))/c
```

3.1262 $\int (bd+2cdx)^{3/2} \left(a+bx+cx^2\right) \, dx$

Optimal. Leaf size=55

$$\frac{(bd+2cdx)^{9/2}}{36c^2d^3}-\frac{\left(b^2-4ac\right)(bd+2cdx)^{5/2}}{20c^2d}$$

```
[Out] -((b^2 - 4*a*c)*(b*d + 2*c*d*x)^(5/2))/(20*c^2*d) + (b*d + 2*c*d*x)^(9/2)/(
36*c^2*d^3)
```

Rubi [A] time = 0.0227016, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$\frac{(bd+2cdx)^{9/2}}{36c^2d^3}-\frac{\left(b^2-4ac\right)(bd+2cdx)^{5/2}}{20c^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2),x]
```

```
[Out] -((b^2 - 4*a*c)*(b*d + 2*c*d*x)^(5/2))/(20*c^2*d) + (b*d + 2*c*d*x)^(9/2)/(
36*c^2*d^3)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int (bd+2cdx)^{3/2}\left(a+bx+cx^2\right)\,dx &= \int \left(\frac{\left(-b^2+4ac\right)(bd+2cdx)^{3/2}}{4c}+\frac{(bd+2cdx)^{7/2}}{4cd^2}\right)dx\\ &= -\frac{\left(b^2-4ac\right)(bd+2cdx)^{5/2}}{20c^2d}+\frac{(bd+2cdx)^{9/2}}{36c^2d^3}\end{aligned}$$

Mathematica [A] time = 0.0292183, size = 45, normalized size = 0.82

$$\frac{\left(c\left(9a+5cx^2\right)-b^2+5bcx\right)(d(b+2cx))^{5/2}}{45c^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2),x]
```

```
[Out] ((d*(b + 2*c*x))^(5/2)*(-b^2 + 5*b*c*x + c*(9*a + 5*c*x^2)))/(45*c^2*d)
```

Maple [A] time = 0.043, size = 46, normalized size = 0.8

$$\frac{(2\,cx+b)\left(5\,c^{2}x^{2}+5\,bcx+9\,ac-b^{2}\right)}{45\,c^{2}}(2\,cdx+bd)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(3/2)*(c*x^2+b*x+a),x)
```

```
[Out] 1/45*(2*c*x+b)*(5*c^2*x^2+5*b*c*x+9*a*c-b^2)*(2*c*d*x+b*d)^(3/2)/c^2
```

Maxima [A] time = 1.11853, size = 62, normalized size = 1.13

$$-\frac{9\,(2\,cdx+bd)^{\frac{5}{2}}\left(b^{2}-4\,ac\right)d^{2}-5\,(2\,cdx+bd)^{\frac{9}{2}}}{180\,c^{2}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)*(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] -1/180*(9*(2*c*d*x + b*d)^(5/2)*(b^2 - 4*a*c)*d^2 - 5*(2*c*d*x + b*d)^(9/2)
)/(c^2*d^3)
```

Fricas [A] time = 2.0613, size = 194, normalized size = 3.53

$$\frac{\left(20\,c^{4}dx^{4}+40\,bc^{3}dx^{3}+3\left(7\,b^{2}c^{2}+12\,ac^{3}\right)dx^{2}+\left(b^{3}c+36\,abc^{2}\right)dx-\left(b^{4}-9\,ab^{2}c\right)d\right)\sqrt{2\,cdx+bd}}{45\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)*(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/45*(20*c^4*d*x^4 + 40*b*c^3*d*x^3 + 3*(7*b^2*c^2 + 12*a*c^3)*d*x^2 + (b^3
*c + 36*a*b*c^2)*d*x - (b^4 - 9*a*b^2*c)*d)*sqrt(2*c*d*x + b*d)/c^2
```

Sympy [A] time = 10.2782, size = 274, normalized size = 4.98

$$abd\left(\begin{cases} x\sqrt{bd} & \text{for}\: c=0\\ 0 & \text{for}\: d=0\\ \frac{(bd+2cdx)^{\frac{3}{2}}}{3cd} & \text{otherwise}\end{cases}\right)+\frac{a\left(-\frac{bd(bd+2cdx)^{\frac{3}{2}}}{3}+\frac{(bd+2cdx)^{\frac{5}{2}}}{5}\right)}{cd}+\frac{b^{2}\left(-\frac{bd(bd+2cdx)^{\frac{3}{2}}}{3}+\frac{(bd+2cdx)^{\frac{5}{2}}}{5}\right)}{2c^{2}d}+\frac{3b\left(\frac{b^{2}d^{2}(bd+2cdx)^{\frac{3}{2}}}{3}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(3/2)*(c*x**2+b*x+a),x)
```

```
[Out] a*b*d*Piecewise((x*sqrt(b*d), Eq(c, 0)), (0, Eq(d, 0)), ((b*d + 2*c*d*x)**(
3/2)/(3*c*d), True)) + a*(-b*d*(b*d + 2*c*d*x)**(3/2)/3 + (b*d + 2*c*d*x)**
(5/2)/5)/(c*d) + b**2*(-b*d*(b*d + 2*c*d*x)**(3/2)/3 + (b*d + 2*c*d*x)**(5/
2)/5)/(2*c**2*d) + 3*b*(b**2*d**2*(b*d + 2*c*d*x)**(3/2)/3 - 2*b*d*(b*d + 2
```

```
*c*d*x)**(5/2)/5 + (b*d + 2*c*d*x)**(7/2)/7)/(4*c**2*d**2) + (-b**3*d**3*(b
*d + 2*c*d*x)**(3/2)/3 + 3*b**2*d**2*(b*d + 2*c*d*x)**(5/2)/5 - 3*b*d*(b*d
+ 2*c*d*x)**(7/2)/7 + (b*d + 2*c*d*x)**(9/2)/9)/(4*c**2*d**3)
```

Giac [B] time = 1.17555, size = 308, normalized size = 5.6

$$\frac{420\,(2\,cdx+bd)^{\frac{3}{2}}ab - \frac{84\left(5\,(2\,cdx+bd)^{\frac{3}{2}}bd-3\,(2\,cdx+bd)^{\frac{5}{2}}\right)a}{d} - \frac{42\left(5\,(2\,cdx+bd)^{\frac{3}{2}}bd-3\,(2\,cdx+bd)^{\frac{5}{2}}\right)b^2}{cd} + \frac{9\left(35\,(2\,cdx+bd)^{\frac{3}{2}}b^2d^2-42\,(2\,cdx+bd)^{\frac{5}{2}}bd+15\right.}{cd^2}}{1260\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)*(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/1260*(420*(2*c*d*x + b*d)^(3/2)*a*b - 84*(5*(2*c*d*x + b*d)^(3/2)*b*d - 3
*(2*c*d*x + b*d)^(5/2))*a/d - 42*(5*(2*c*d*x + b*d)^(3/2)*b*d - 3*(2*c*d*x
+ b*d)^(5/2))*b^2/(c*d) + 9*(35*(2*c*d*x + b*d)^(3/2)*b^2*d^2 - 42*(2*c*d*x
 + b*d)^(5/2)*b*d + 15*(2*c*d*x + b*d)^(7/2))*b/(c*d^2) - (105*(2*c*d*x + b
*d)^(3/2)*b^3*d^3 - 189*(2*c*d*x + b*d)^(5/2)*b^2*d^2 + 135*(2*c*d*x + b*d)
^(7/2)*b*d - 35*(2*c*d*x + b*d)^(9/2))/(c*d^3))/c
```

3.1263 $\int \sqrt{bd+2cdx}\left(a+bx+cx^2\right)\,dx$

Optimal. Leaf size=55

$$\frac{(bd+2cdx)^{7/2}}{28c^2d^3}-\frac{\left(b^2-4ac\right)(bd+2cdx)^{3/2}}{12c^2d}$$

```
[Out] -((b^2 - 4*a*c)*(b*d + 2*c*d*x)^(3/2))/(12*c^2*d) + (b*d + 2*c*d*x)^(7/2)/(
28*c^2*d^3)
```

Rubi [A] time = 0.0221096, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$\frac{(bd+2cdx)^{7/2}}{28c^2d^3}-\frac{\left(b^2-4ac\right)(bd+2cdx)^{3/2}}{12c^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*d + 2*c*d*x]*(a + b*x + c*x^2),x]
```

```
[Out] -((b^2 - 4*a*c)*(b*d + 2*c*d*x)^(3/2))/(12*c^2*d) + (b*d + 2*c*d*x)^(7/2)/(
28*c^2*d^3)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \sqrt{bd+2cdx}\left(a+bx+cx^2\right)\,dx &= \int \left(\frac{\left(-b^2+4ac\right)\sqrt{bd+2cdx}}{4c}+\frac{(bd+2cdx)^{5/2}}{4cd^2}\right)dx\\ &= -\frac{\left(b^2-4ac\right)(bd+2cdx)^{3/2}}{12c^2d}+\frac{(bd+2cdx)^{7/2}}{28c^2d^3}\end{aligned}$$

Mathematica [A] time = 0.0235017, size = 45, normalized size = 0.82

$$\frac{\left(c\left(7a+3cx^2\right)-b^2+3bcx\right)(d(b+2cx))^{3/2}}{21c^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*d + 2*c*d*x]*(a + b*x + c*x^2),x]
```

```
[Out] ((d*(b + 2*c*x))^(3/2)*(-b^2 + 3*b*c*x + c*(7*a + 3*c*x^2)))/(21*c^2*d)
```

Maple [A] time = 0.042, size = 46, normalized size = 0.8

$$\frac{(2cx+b)\left(3c^2x^2+3bcx+7ac-b^2\right)}{21c^2}\sqrt{2cdx+bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(1/2)*(c*x^2+b*x+a),x)
```

```
[Out] 1/21*(2*c*x+b)*(3*c^2*x^2+3*b*c*x+7*a*c-b^2)*(2*c*d*x+b*d)^(1/2)/c^2
```

Maxima [A] time = 1.02077, size = 62, normalized size = 1.13

$$-\frac{7(2cdx+bd)^{\frac{3}{2}}\left(b^2-4ac\right)d^2-3(2cdx+bd)^{\frac{7}{2}}}{84c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)*(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] -1/84*(7*(2*c*d*x + b*d)^(3/2)*(b^2 - 4*a*c)*d^2 - 3*(2*c*d*x + b*d)^(7/2))
/(c^2*d^3)
```

Fricas [A] time = 1.90143, size = 128, normalized size = 2.33

$$\frac{\left(6c^3x^3+9bc^2x^2-b^3+7abc+\left(b^2c+14ac^2\right)x\right)\sqrt{2cdx+bd}}{21c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)*(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/21*(6*c^3*x^3 + 9*b*c^2*x^2 - b^3 + 7*a*b*c + (b^2*c + 14*a*c^2)*x)*sqrt(
2*c*d*x + b*d)/c^2
```

Sympy [A] time = 3.23593, size = 48, normalized size = 0.87

$$\frac{\frac{\left(4ac-b^2\right)(bd+2cdx)^{\frac{3}{2}}}{12c}+\frac{(bd+2cdx)^{\frac{7}{2}}}{28cd^2}}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(1/2)*(c*x**2+b*x+a),x)
```

```
[Out] ((4*a*c - b**2)*(b*d + 2*c*d*x)**(3/2)/(12*c) + (b*d + 2*c*d*x)**(7/2)/(28*
c*d**2))/(c*d)
```

Giac [B] time = 1.14305, size = 157, normalized size = 2.85

$$\frac{140\,(2\,cdx+bd)^{\frac{3}{2}}a-\frac{14\left(5\,(2\,cdx+bd)^{\frac{3}{2}}bd-3\,(2\,cdx+bd)^{\frac{5}{2}}\right)b}{cd}+\frac{35\,(2\,cdx+bd)^{\frac{3}{2}}b^2d^2-42\,(2\,cdx+bd)^{\frac{5}{2}}bd+15\,(2\,cdx+bd)^{\frac{7}{2}}}{cd^2}}{420\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)*(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/420*(140*(2*c*d*x + b*d)^(3/2)*a - 14*(5*(2*c*d*x + b*d)^(3/2)*b*d - 3*(2
*c*d*x + b*d)^(5/2))*b/(c*d) + (35*(2*c*d*x + b*d)^(3/2)*b^2*d^2 - 42*(2*c*
d*x + b*d)^(5/2)*b*d + 15*(2*c*d*x + b*d)^(7/2))/(c*d^2))/(c*d)
```

3.1264 $\int \frac{a+bx+cx^2}{\sqrt{bd+2cdx}}\,dx$

Optimal. Leaf size=55

$$\frac{(bd+2cdx)^{5/2}}{20c^2d^3}-\frac{\left(b^2-4ac\right)\sqrt{bd+2cdx}}{4c^2d}$$

```
[Out] -((b^2 - 4*a*c)*Sqrt[b*d + 2*c*d*x])/(4*c^2*d) + (b*d + 2*c*d*x)^(5/2)/(20*
c^2*d^3)
```

Rubi [A] time = 0.0219751, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$\frac{(bd+2cdx)^{5/2}}{20c^2d^3}-\frac{\left(b^2-4ac\right)\sqrt{bd+2cdx}}{4c^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/Sqrt[b*d + 2*c*d*x],x]
```

```
[Out] -((b^2 - 4*a*c)*Sqrt[b*d + 2*c*d*x])/(4*c^2*d) + (b*d + 2*c*d*x)^(5/2)/(20*
c^2*d^3)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{\sqrt{bd+2cdx}}\,dx&=\int \left(\frac{-b^2+4ac}{4c\sqrt{bd+2cdx}}+\frac{(bd+2cdx)^{3/2}}{4cd^2}\right)dx\\&=-\frac{\left(b^2-4ac\right)\sqrt{bd+2cdx}}{4c^2d}+\frac{(bd+2cdx)^{5/2}}{20c^2d^3}\end{aligned}$$

Mathematica [A] time = 0.0247459, size = 43, normalized size = 0.78

$$\frac{\left(c\left(5a+cx^2\right)-b^2+bcx\right)\sqrt{d(b+2cx)}}{5c^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/Sqrt[b*d + 2*c*d*x],x]
```

```
[Out] (Sqrt[d*(b + 2*c*x)]*(-b^2 + b*c*x + c*(5*a + c*x^2)))/(5*c^2*d)
```

Maple [A] time = 0.041, size = 44, normalized size = 0.8

$$\frac{(2\,cx+b)\left(c^2x^2+bcx+5\,ac-b^2\right)}{5\,c^2}\frac{1}{\sqrt{2\,cdx+bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(2*c*d*x+b*d)^(1/2),x)
```

```
[Out] 1/5*(2*c*x+b)*(c^2*x^2+b*c*x+5*a*c-b^2)/c^2/(2*c*d*x+b*d)^(1/2)
```

Maxima [B] time = 1.16641, size = 157, normalized size = 2.85

$$\frac{60\,\sqrt{2\,cdx+bd}a-\frac{10\left(3\,\sqrt{2\,cdx+bd}bd-(2\,cdx+bd)^{\frac{3}{2}}\right)b}{cd}+\frac{15\,\sqrt{2\,cdx+bd}b^2d^2-10\,(2\,cdx+bd)^{\frac{3}{2}}bd+3\,(2\,cdx+bd)^{\frac{5}{2}}}{cd^2}}{60\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/60*(60*sqrt(2*c*d*x + b*d)*a - 10*(3*sqrt(2*c*d*x + b*d)*b*d - (2*c*d*x +
 b*d)^(3/2))*b/(c*d) + (15*sqrt(2*c*d*x + b*d)*b^2*d^2 - 10*(2*c*d*x + b*d)
^(3/2)*b*d + 3*(2*c*d*x + b*d)^(5/2))/(c*d^2))/(c*d)
```

Fricas [A] time = 1.95994, size = 88, normalized size = 1.6

$$\frac{\left(c^2x^2+bcx-b^2+5\,ac\right)\sqrt{2\,cdx+bd}}{5\,c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*(c^2*x^2 + b*c*x - b^2 + 5*a*c)*sqrt(2*c*d*x + b*d)/(c^2*d)
```

Sympy [A] time = 12.0852, size = 258, normalized size = 4.69

$$\begin{cases} -\frac{\frac{ab}{\sqrt{bd+2cdx}}+\frac{a\left(-\frac{bd}{\sqrt{bd+2cdx}}-\sqrt{bd+2cdx}\right)}{d}+\frac{b^2\left(-\frac{bd}{\sqrt{bd+2cdx}}-\sqrt{bd+2cdx}\right)}{2cd}+\frac{3b\left(\frac{b^2d^2}{\sqrt{bd+2cdx}}+2bd\sqrt{bd+2cdx}-\frac{(bd+2cdx)^{\frac{3}{2}}}{3}\right)}{4cd^2}+\frac{-\frac{b^3d^3}{\sqrt{bd+2cdx}}-3b^2d^2\sqrt{bd+2cdx}+bd(bd+2cdx)^{\frac{3}{2}}-\frac{(bd+2cdx)^{\frac{5}{2}}}{5}}{4cd^3}}{c} \\ \frac{ax+\frac{bx^2}{2}}{\sqrt{bd}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(2*c*d*x+b*d)**(1/2),x)
```

```
[Out] Piecewise((-(a*b/sqrt(b*d + 2*c*d*x) + a*(-b*d/sqrt(b*d + 2*c*d*x) - sqrt(b
*d + 2*c*d*x))/d + b**2*(-b*d/sqrt(b*d + 2*c*d*x) - sqrt(b*d + 2*c*d*x))/(2
```

```
*c*d) + 3*b*(b**2*d**2/sqrt(b*d + 2*c*d*x) + 2*b*d*sqrt(b*d + 2*c*d*x) - (b
*d + 2*c*d*x)**(3/2)/3)/(4*c*d**2) + (-b**3*d**3/sqrt(b*d + 2*c*d*x) - 3*b*
*2*d**2*sqrt(b*d + 2*c*d*x) + b*d*(b*d + 2*c*d*x)**(3/2) - (b*d + 2*c*d*x)*
*(5/2)/5)/(4*c*d**3))/c, Ne(c, 0)), ((a*x + b*x**2/2)/sqrt(b*d), True))
```

Giac [B] time = 1.14201, size = 157, normalized size = 2.85

$$\frac{60\sqrt{2\,cdx+bd}a-\frac{10\left(3\sqrt{2\,cdx+bd}bd-(2\,cdx+bd)^{\frac{3}{2}}\right)b}{cd}+\frac{15\sqrt{2\,cdx+bd}b^2d^2-10\,(2\,cdx+bd)^{\frac{3}{2}}bd+3\,(2\,cdx+bd)^{\frac{5}{2}}}{cd^2}}{60\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^(1/2),x, algorithm="giac")
```

```
[Out] 1/60*(60*sqrt(2*c*d*x + b*d)*a - 10*(3*sqrt(2*c*d*x + b*d)*b*d - (2*c*d*x +
 b*d)^(3/2))*b/(c*d) + (15*sqrt(2*c*d*x + b*d)*b^2*d^2 - 10*(2*c*d*x + b*d)
^(3/2)*b*d + 3*(2*c*d*x + b*d)^(5/2))/(c*d^2))/(c*d)
```

3.1265 $\int \frac{a+bx+cx^2}{(bd+2cdx)^{3/2}}\,dx$

Optimal. Leaf size=55

$$\frac{b^2-4ac}{4c^2d\sqrt{bd+2cdx}}+\frac{(bd+2cdx)^{3/2}}{12c^2d^3}$$

```
[Out] (b^2 - 4*a*c)/(4*c^2*d*Sqrt[b*d + 2*c*d*x]) + (b*d + 2*c*d*x)^(3/2)/(12*c^2
*d^3)
```

Rubi [A] time = 0.0233488, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$\frac{b^2-4ac}{4c^2d\sqrt{bd+2cdx}}+\frac{(bd+2cdx)^{3/2}}{12c^2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(b*d + 2*c*d*x)^(3/2),x]
```

```
[Out] (b^2 - 4*a*c)/(4*c^2*d*Sqrt[b*d + 2*c*d*x]) + (b*d + 2*c*d*x)^(3/2)/(12*c^2
*d^3)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] && !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{(bd+2cdx)^{3/2}}\,dx &= \int \left(\frac{-b^2+4ac}{4c(bd+2cdx)^{3/2}}+\frac{\sqrt{bd+2cdx}}{4cd^2}\right)dx\\ &= \frac{b^2-4ac}{4c^2d\sqrt{bd+2cdx}}+\frac{(bd+2cdx)^{3/2}}{12c^2d^3}\end{aligned}$$

Mathematica [A] time = 0.0249194, size = 41, normalized size = 0.75

$$\frac{c\left(cx^2-3a\right)+b^2+bcx}{3c^2d\sqrt{d(b+2cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(b*d + 2*c*d*x)^(3/2),x]
```

```
[Out] (b^2 + b*c*x + c*(-3*a + c*x^2))/(3*c^2*d*Sqrt[d*(b + 2*c*x)])
```

Maple [A] time = 0.041, size = 46, normalized size = 0.8

$$-\frac{(2\,cx+b)\left(-c^{2}x^{2}-bcx+3\,ac-b^{2}\right)}{3\,c^{2}}(2\,cdx+bd)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(2*c*d*x+b*d)^(3/2),x)
```

```
[Out] -1/3*(2*c*x+b)*(-c^2*x^2-b*c*x+3*a*c-b^2)/c^2/(2*c*d*x+b*d)^(3/2)
```

Maxima [A] time = 1.0142, size = 69, normalized size = 1.25

$$\frac{\frac{3\left(b^{2}-4\,ac\right)}{\sqrt{2\,cdx+bd}c}+\frac{(2\,cdx+bd)^{\frac{3}{2}}}{cd^{2}}}{12\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(b^2 - 4*a*c)/(sqrt(2*c*d*x + b*d)*c) + (2*c*d*x + b*d)^(3/2)/(c*d^
2))/(c*d)
```

Fricas [A] time = 1.95868, size = 112, normalized size = 2.04

$$\frac{\left(c^{2}x^{2}+bcx+b^{2}-3\,ac\right)\sqrt{2\,cdx+bd}}{3\left(2\,c^{3}d^{2}x+bc^{2}d^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(c^2*x^2 + b*c*x + b^2 - 3*a*c)*sqrt(2*c*d*x + b*d)/(2*c^3*d^2*x + b*c^
2*d^2)
```

Sympy [A] time = 12.5489, size = 49, normalized size = 0.89

$$-\frac{4ac-b^{2}}{4c^{2}d\sqrt{bd+2cdx}}+\frac{(bd+2cdx)^{\frac{3}{2}}}{12c^{2}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(2*c*d*x+b*d)**(3/2),x)
```

```
[Out] -(4*a*c - b**2)/(4*c**2*d*sqrt(b*d + 2*c*d*x)) + (b*d + 2*c*d*x)**(3/2)/(12
*c**2*d**3)
```

Giac [A] time = 1.11439, size = 63, normalized size = 1.15

$$\frac{b^2-4\,ac}{4\,\sqrt{2\,cdx+bd}c^2d}+\frac{(2\,cdx+bd)^{\frac{3}{2}}}{12\,c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(b^2 - 4*a*c)/(sqrt(2*c*d*x + b*d)*c^2*d) + 1/12*(2*c*d*x + b*d)^(3/2)/
(c^2*d^3)
```

3.1266 $\int \frac{a+bx+cx^2}{(bd+2cdx)^{5/2}}\,dx$

Optimal. Leaf size=55

$$\frac{b^2-4ac}{12c^2d(bd+2cdx)^{3/2}}+\frac{\sqrt{bd+2cdx}}{4c^2d^3}$$

```
[Out] (b^2 - 4*a*c)/(12*c^2*d*(b*d + 2*c*d*x)^(3/2)) + Sqrt[b*d + 2*c*d*x]/(4*c^2
*d^3)
```

Rubi [A] time = 0.0216346, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$\frac{b^2-4ac}{12c^2d(bd+2cdx)^{3/2}}+\frac{\sqrt{bd+2cdx}}{4c^2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(b*d + 2*c*d*x)^(5/2),x]
```

```
[Out] (b^2 - 4*a*c)/(12*c^2*d*(b*d + 2*c*d*x)^(3/2)) + Sqrt[b*d + 2*c*d*x]/(4*c^2
*d^3)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{(bd+2cdx)^{5/2}}\,dx &= \int \left(\frac{-b^2+4ac}{4c(bd+2cdx)^{5/2}}+\frac{1}{4cd^2\sqrt{bd+2cdx}}\right)dx\\ &= \frac{b^2-4ac}{12c^2d(bd+2cdx)^{3/2}}+\frac{\sqrt{bd+2cdx}}{4c^2d^3}\end{aligned}$$

Mathematica [A] time = 0.0266623, size = 43, normalized size = 0.78

$$\frac{c\left(3cx^2-a\right)+b^2+3bcx}{3c^2d(d(b+2cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(b*d + 2*c*d*x)^(5/2),x]
```

```
[Out] (b^2 + 3*b*c*x + c*(-a + 3*c*x^2))/(3*c^2*d*(d*(b + 2*c*x))^(3/2))
```

Maple [A] time = 0.042, size = 45, normalized size = 0.8

$$-\frac{(2cx+b)\left(-3c^2x^2-3bcx+ac-b^2\right)}{3c^2}(2cdx+bd)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(2*c*d*x+b*d)^(5/2),x)
```

```
[Out] -1/3*(2*c*x+b)*(-3*c^2*x^2-3*b*c*x+a*c-b^2)/c^2/(2*c*d*x+b*d)^(5/2)
```

Maxima [A] time = 1.16885, size = 69, normalized size = 1.25

$$\frac{\frac{b^2-4ac}{(2cdx+bd)^{\frac{3}{2}}c}+\frac{3\sqrt{2cdx+bd}}{cd^2}}{12cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*((b^2 - 4*a*c)/((2*c*d*x + b*d)^(3/2)*c) + 3*sqrt(2*c*d*x + b*d)/(c*d^
2))/(c*d)
```

Fricas [A] time = 2.03218, size = 142, normalized size = 2.58

$$\frac{\left(3c^2x^2+3bcx+b^2-ac\right)\sqrt{2cdx+bd}}{3\left(4c^4d^3x^2+4bc^3d^3x+b^2c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*c^2*x^2 + 3*b*c*x + b^2 - a*c)*sqrt(2*c*d*x + b*d)/(4*c^4*d^3*x^2 +
4*b*c^3*d^3*x + b^2*c^2*d^3)
```

Sympy [A] time = 1.71848, size = 235, normalized size = 4.27

$$\begin{cases} -\frac{ac\sqrt{bd+2cdx}}{3b^2c^2d^3+12bc^3d^3x+12c^4d^3x^2}+\frac{b^2\sqrt{bd+2cdx}}{3b^2c^2d^3+12bc^3d^3x+12c^4d^3x^2}+\frac{3bcx\sqrt{bd+2cdx}}{3b^2c^2d^3+12bc^3d^3x+12c^4d^3x^2}+\frac{3c^2x^2\sqrt{bd+2cdx}}{3b^2c^2d^3+12bc^3d^3x+12c^4d^3x^2} & \text{for } c\neq 0 \\ \frac{ax+\frac{bx^2}{2}}{(bd)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(2*c*d*x+b*d)**(5/2),x)
```

```
[Out] Piecewise((-a*c*sqrt(b*d + 2*c*d*x)/(3*b**2*c**2*d**3 + 12*b*c**3*d**3*x +
12*c**4*d**3*x**2) + b**2*sqrt(b*d + 2*c*d*x)/(3*b**2*c**2*d**3 + 12*b*c**3
*d**3*x + 12*c**4*d**3*x**2) + 3*b*c*x*sqrt(b*d + 2*c*d*x)/(3*b**2*c**2*d**
3 + 12*b*c**3*d**3*x + 12*c**4*d**3*x**2) + 3*c**2*x**2*sqrt(b*d + 2*c*d*x)
```

```
/(3*b**2*c**2*d**3 + 12*b*c**3*d**3*x + 12*c**4*d**3*x**2), Ne(c, 0)), ((a*
x + b*x**2/2)/(b*d)**(5/2), True))
```

Giac [A] time = 1.15943, size = 63, normalized size = 1.15

$$\frac{b^2 - 4\,ac}{12\,(2\,cdx + bd)^{\frac{3}{2}}c^2d} + \frac{\sqrt{2\,cdx + bd}}{4\,c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^(5/2),x, algorithm="giac")
```

```
[Out] 1/12*(b^2 - 4*a*c)/((2*c*d*x + b*d)^(3/2)*c^2*d) + 1/4*sqrt(2*c*d*x + b*d)/
(c^2*d^3)
```

3.1267 $\int \frac{a+bx+cx^2}{(bd+2cdx)^{7/2}}\,dx$

Optimal. Leaf size=55

$$\frac{b^2-4ac}{20c^2d(bd+2cdx)^{5/2}}-\frac{1}{4c^2d^3\sqrt{bd+2cdx}}$$

```
[Out] (b^2 - 4*a*c)/(20*c^2*d*(b*d + 2*c*d*x)^(5/2)) - 1/(4*c^2*d^3*Sqrt[b*d + 2*
c*d*x])
```

Rubi [A] time = 0.0223381, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$\frac{b^2-4ac}{20c^2d(bd+2cdx)^{5/2}}-\frac{1}{4c^2d^3\sqrt{bd+2cdx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(b*d + 2*c*d*x)^(7/2),x]
```

```
[Out] (b^2 - 4*a*c)/(20*c^2*d*(b*d + 2*c*d*x)^(5/2)) - 1/(4*c^2*d^3*Sqrt[b*d + 2*
c*d*x])
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] && !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{(bd+2cdx)^{7/2}}\,dx &= \int \left(\frac{-b^2+4ac}{4c(bd+2cdx)^{7/2}}+\frac{1}{4cd^2(bd+2cdx)^{3/2}}\right)dx\\ &= \frac{b^2-4ac}{20c^2d(bd+2cdx)^{5/2}}-\frac{1}{4c^2d^3\sqrt{bd+2cdx}}\end{aligned}$$

Mathematica [A] time = 0.0293239, size = 44, normalized size = 0.8

$$\frac{-c\left(a+5cx^2\right)-b^2-5bcx}{5c^2d(d(b+2cx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(b*d + 2*c*d*x)^(7/2),x]
```

```
[Out] (-b^2 - 5*b*c*x - c*(a + 5*c*x^2))/(5*c^2*d*(d*(b + 2*c*x))^(5/2))
```

Maple [A] time = 0.046, size = 43, normalized size = 0.8

$$-\frac{(2cx+b)\left(5c^2x^2+5bcx+ac+b^2\right)}{5c^2}(2cdx+bd)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(2*c*d*x+b*d)^(7/2),x)
```

```
[Out] -1/5*(2*c*x+b)*(5*c^2*x^2+5*b*c*x+a*c+b^2)/c^2/(2*c*d*x+b*d)^(7/2)
```

Maxima [A] time = 1.026, size = 61, normalized size = 1.11

$$\frac{\left(b^2-4ac\right)d^2-5(2cdx+bd)^2}{20(2cdx+bd)^{\frac{5}{2}}c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/20*((b^2 - 4*a*c)*d^2 - 5*(2*c*d*x + b*d)^2)/((2*c*d*x + b*d)^(5/2)*c^2*d
^3)
```

Fricas [A] time = 2.01084, size = 171, normalized size = 3.11

$$-\frac{\left(5c^2x^2+5bcx+b^2+ac\right)\sqrt{2cdx+bd}}{5\left(8c^5d^4x^3+12bc^4d^4x^2+6b^2c^3d^4x+b^3c^2d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/5*(5*c^2*x^2 + 5*b*c*x + b^2 + a*c)*sqrt(2*c*d*x + b*d)/(8*c^5*d^4*x^3 +
 12*b*c^4*d^4*x^2 + 6*b^2*c^3*d^4*x + b^3*c^2*d^4)
```

Sympy [A] time = 4.4656, size = 298, normalized size = 5.42

$$\begin{cases} -\frac{ac\sqrt{bd+2cdx}}{5b^3c^2d^4+30b^2c^3d^4x+60bc^4d^4x^2+40c^5d^4x^3} - \frac{b^2\sqrt{bd+2cdx}}{5b^3c^2d^4+30b^2c^3d^4x+60bc^4d^4x^2+40c^5d^4x^3} - \frac{5bcx\sqrt{bd+2cdx}}{5b^3c^2d^4+30b^2c^3d^4x+60bc^4d^4x^2+40c^5d^4x^3} - \frac{5c^2x}{5b^3c^2d^4+30b^2c^3d} \\ \frac{ax+\frac{bx^2}{2}}{(bd)^{\frac{7}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(2*c*d*x+b*d)**(7/2),x)
```

```
[Out] Piecewise((-a*c*sqrt(b*d + 2*c*d*x)/(5*b**3*c**2*d**4 + 30*b**2*c**3*d**4*x
 + 60*b*c**4*d**4*x**2 + 40*c**5*d**4*x**3) - b**2*sqrt(b*d + 2*c*d*x)/(5*b
**3*c**2*d**4 + 30*b**2*c**3*d**4*x + 60*b*c**4*d**4*x**2 + 40*c**5*d**4*x*
*3) - 5*b*c*x*sqrt(b*d + 2*c*d*x)/(5*b**3*c**2*d**4 + 30*b**2*c**3*d**4*x +
```

```
 60*b*c**4*d**4*x**2 + 40*c**5*d**4*x**3) - 5*c**2*x**2*sqrt(b*d + 2*c*d*x)
/(5*b**3*c**2*d**4 + 30*b**2*c**3*d**4*x + 60*b*c**4*d**4*x**2 + 40*c**5*d*
*4*x**3), Ne(c, 0)), ((a*x + b*x**2/2)/(b*d)**(7/2), True))
```

Giac [A] time = 1.1477, size = 63, normalized size = 1.15

$$\frac{b^2d^2 - 4\,acd^2 - 5\,(2\,cdx + bd)^2}{20\,(2\,cdx + bd)^{\frac{5}{2}}c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^(7/2),x, algorithm="giac")
```

```
[Out] 1/20*(b^2*d^2 - 4*a*c*d^2 - 5*(2*c*d*x + b*d)^2)/((2*c*d*x + b*d)^(5/2)*c^2
*d^3)
```

3.1268 $\int \frac{a+bx+cx^2}{(bd+2cdx)^{9/2}} dx$

Optimal. Leaf size=55

$$\frac{b^2-4ac}{28c^2d(bd+2cdx)^{7/2}}-\frac{1}{12c^2d^3(bd+2cdx)^{3/2}}$$

```
[Out] (b^2 - 4*a*c)/(28*c^2*d*(b*d + 2*c*d*x)^(7/2)) - 1/(12*c^2*d^3*(b*d + 2*c*d
*x)^(3/2))
```

Rubi [A] time = 0.023352, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$\frac{b^2-4ac}{28c^2d(bd+2cdx)^{7/2}}-\frac{1}{12c^2d^3(bd+2cdx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(b*d + 2*c*d*x)^(9/2),x]
```

```
[Out] (b^2 - 4*a*c)/(28*c^2*d*(b*d + 2*c*d*x)^(7/2)) - 1/(12*c^2*d^3*(b*d + 2*c*d
*x)^(3/2))
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{(bd+2cdx)^{9/2}}\,dx &= \int \left(\frac{-b^2+4ac}{4c(bd+2cdx)^{9/2}}+\frac{1}{4cd^2(bd+2cdx)^{5/2}}\right)dx\\ &= \frac{b^2-4ac}{28c^2d(bd+2cdx)^{7/2}}-\frac{1}{12c^2d^3(bd+2cdx)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0372111, size = 51, normalized size = 0.93

$$-\frac{\left(c\left(3a+7cx^2\right)+b^2+7bcx\right)\sqrt{d(b+2cx)}}{21c^2d^5(b+2cx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(b*d + 2*c*d*x)^(9/2),x]
```

```
[Out] -(Sqrt[d*(b + 2*c*x)]*(b^2 + 7*b*c*x + c*(3*a + 7*c*x^2)))/(21*c^2*d^5*(b +
 2*c*x)^4)
```

Maple [A] time = 0.042, size = 44, normalized size = 0.8

$$-\frac{(2cx+b)\left(7c^2x^2+7bcx+3ac+b^2\right)(2cdx+bd)^{-\frac{9}{2}}}{21c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(2*c*d*x+b*d)^(9/2),x)
```

```
[Out] -1/21*(2*c*x+b)*(7*c^2*x^2+7*b*c*x+3*a*c+b^2)/c^2/(2*c*d*x+b*d)^(9/2)
```

Maxima [A] time = 1.0138, size = 62, normalized size = 1.13

$$\frac{3\left(b^2-4ac\right)d^2-7(2cdx+bd)^2}{84(2cdx+bd)^{\frac{7}{2}}c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/84*(3*(b^2 - 4*a*c)*d^2 - 7*(2*c*d*x + b*d)^2)/((2*c*d*x + b*d)^(7/2)*c^2
*d^3)
```

Fricas [B] time = 2.00594, size = 205, normalized size = 3.73

$$-\frac{\left(7c^2x^2+7bcx+b^2+3ac\right)\sqrt{2cdx+bd}}{21\left(16c^6d^5x^4+32bc^5d^5x^3+24b^2c^4d^5x^2+8b^3c^3d^5x+b^4c^2d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/21*(7*c^2*x^2 + 7*b*c*x + b^2 + 3*a*c)*sqrt(2*c*d*x + b*d)/(16*c^6*d^5*x
^4 + 32*b*c^5*d^5*x^3 + 24*b^2*c^4*d^5*x^2 + 8*b^3*c^3*d^5*x + b^4*c^2*d^5)
```

Sympy [A] time = 10.6182, size = 360, normalized size = 6.55

$$\begin{cases} -\frac{3ac\sqrt{bd+2cdx}}{21b^4c^2d^5+168b^3c^3d^5x+504b^2c^4d^5x^2+672bc^5d^5x^3+336c^6d^5x^4} - \frac{b^2\sqrt{bd+2cdx}}{21b^4c^2d^5+168b^3c^3d^5x+504b^2c^4d^5x^2+672bc^5d^5x^3+336c^6d^5x^4} - \frac{\cdots}{21b^4c^2d^5+168b^3c^3d^5\cdots} \\ \frac{ax+\frac{bx^2}{2}}{(bd)^{\frac{9}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(2*c*d*x+b*d)**(9/2),x)
```

```
[Out] Piecewise((-3*a*c*sqrt(b*d + 2*c*d*x)/(21*b**4*c**2*d**5 + 168*b**3*c**3*d*
*5*x + 504*b**2*c**4*d**5*x**2 + 672*b*c**5*d**5*x**3 + 336*c**6*d**5*x**4)
 - b**2*sqrt(b*d + 2*c*d*x)/(21*b**4*c**2*d**5 + 168*b**3*c**3*d**5*x + 504
*b**2*c**4*d**5*x**2 + 672*b*c**5*d**5*x**3 + 336*c**6*d**5*x**4) - 7*b*c*x
```

```
*sqrt(b*d + 2*c*d*x)/(21*b**4*c**2*d**5 + 168*b**3*c**3*d**5*x + 504*b**2*c
**4*d**5*x**2 + 672*b*c**5*d**5*x**3 + 336*c**6*d**5*x**4) - 7*c**2*x**2*sq
rt(b*d + 2*c*d*x)/(21*b**4*c**2*d**5 + 168*b**3*c**3*d**5*x + 504*b**2*c**4
*d**5*x**2 + 672*b*c**5*d**5*x**3 + 336*c**6*d**5*x**4), Ne(c, 0)), ((a*x +
 b*x**2/2)/(b*d)**(9/2), True))
```

Giac [A] time = 1.15317, size = 65, normalized size = 1.18

$$\frac{3\,b^2d^2-12\,acd^2-7\,(2\,cdx+bd)^2}{84\,(2\,cdx+bd)^{\frac{7}{2}}c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(2*c*d*x+b*d)^(9/2),x, algorithm="giac")
```

```
[Out] 1/84*(3*b^2*d^2 - 12*a*c*d^2 - 7*(2*c*d*x + b*d)^2)/((2*c*d*x + b*d)^(7/2)*
c^2*d^3)
```

3.1269 $\int (bd+2cdx)^{3/2} \left(a+bx+cx^2\right)^2 \, dx$

Optimal. Leaf size=88

$$-\frac{\left(b^2-4ac\right)(bd+2cdx)^{9/2}}{72c^3d^3}+\frac{\left(b^2-4ac\right)^2(bd+2cdx)^{5/2}}{80c^3d}+\frac{(bd+2cdx)^{13/2}}{208c^3d^5}$$

```
[Out] ((b^2 - 4*a*c)^2*(b*d + 2*c*d*x)^(5/2))/(80*c^3*d) - ((b^2 - 4*a*c)*(b*d + 2*c*d*x)^(9/2))/(72*c^3*d^3) + (b*d + 2*c*d*x)^(13/2)/(208*c^3*d^5)
```

Rubi [A] time = 0.0549081, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {683}

$$-\frac{\left(b^2-4ac\right)(bd+2cdx)^{9/2}}{72c^3d^3}+\frac{\left(b^2-4ac\right)^2(bd+2cdx)^{5/2}}{80c^3d}+\frac{(bd+2cdx)^{13/2}}{208c^3d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((b^2 - 4*a*c)^2*(b*d + 2*c*d*x)^(5/2))/(80*c^3*d) - ((b^2 - 4*a*c)*(b*d + 2*c*d*x)^(9/2))/(72*c^3*d^3) + (b*d + 2*c*d*x)^(13/2)/(208*c^3*d^5)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && IGtQ[p, 0] && !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int (bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^2\,dx &= \int \left(\frac{\left(-b^2+4ac\right)^2(bd+2cdx)^{3/2}}{16c^2}+\frac{\left(-b^2+4ac\right)(bd+2cdx)^{7/2}}{8c^2d^2}+\frac{(bd+2cdx)^{11/2}}{16c^2d^4}\right)dx\\ &= \frac{\left(b^2-4ac\right)^2(bd+2cdx)^{5/2}}{80c^3d}-\frac{\left(b^2-4ac\right)(bd+2cdx)^{9/2}}{72c^3d^3}+\frac{(bd+2cdx)^{13/2}}{208c^3d^5}\end{aligned}$$

Mathematica [A] time = 0.0608572, size = 63, normalized size = 0.72

$$\frac{\left(-130\left(b^2-4ac\right)(b+2cx)^2+117\left(b^2-4ac\right)^2+45(b+2cx)^4\right)(d(b+2cx))^{5/2}}{9360c^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((d*(b + 2*c*x))^(5/2)*(117*(b^2 - 4*a*c)^2 - 130*(b^2 - 4*a*c)*(b + 2*c*x)^2 + 45*(b + 2*c*x)^4))/(9360*c^3*d)
```

Maple [A] time = 0.043, size = 96, normalized size = 1.1

$$\frac{(2cx+b)\left(45c^4x^4+90bx^3c^3+130ac^3x^2+35b^2c^2x^2+130abc^2x-10b^3cx+117a^2c^2-26acb^2+2b^4\right)(2cdx+bd)^{\frac{3}{2}}}{585c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(3/2)*(c*x^2+b*x+a)^2,x)
```

```
[Out] 1/585*(2*c*x+b)*(45*c^4*x^4+90*b*c^3*x^3+130*a*c^3*x^2+35*b^2*c^2*x^2+130*a
*b*c^2*x-10*b^3*c*x+117*a^2*c^2-26*a*b^2*c+2*b^4)*(2*c*d*x+b*d)^(3/2)/c^3
```

Maxima [A] time = 1.05443, size = 109, normalized size = 1.24

$$-\frac{130(2cdx+bd)^{\frac{9}{2}}\left(b^2-4ac\right)d^2-117\left(b^4-8ab^2c+16a^2c^2\right)(2cdx+bd)^{\frac{5}{2}}d^4-45(2cdx+bd)^{\frac{13}{2}}}{9360c^3d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/9360*(130*(2*c*d*x + b*d)^(9/2)*(b^2 - 4*a*c)*d^2 - 117*(b^4 - 8*a*b^2*c
 + 16*a^2*c^2)*(2*c*d*x + b*d)^(5/2)*d^4 - 45*(2*c*d*x + b*d)^(13/2))/(c^3*
d^5)
```

Fricas [B] time = 2.02091, size = 377, normalized size = 4.28

$$\frac{\left(180c^6dx^6+540bc^5dx^5+5\left(109b^2c^4+104ac^5\right)dx^4+10\left(19b^3c^3+104abc^4\right)dx^3+3\left(b^4c^2+182ab^2c^3+156a^2c^4\right)dx^2-\cdots\right.}{585c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/585*(180*c^6*d*x^6 + 540*b*c^5*d*x^5 + 5*(109*b^2*c^4 + 104*a*c^5)*d*x^4
+ 10*(19*b^3*c^3 + 104*a*b*c^4)*d*x^3 + 3*(b^4*c^2 + 182*a*b^2*c^3 + 156*a^
2*c^4)*d*x^2 - 2*(b^5*c - 13*a*b^3*c^2 - 234*a^2*b*c^3)*d*x + (2*b^6 - 26*a
*b^4*c + 117*a^2*b^2*c^2)*d)*sqrt(2*c*d*x + b*d)/c^3
```

Sympy [A] time = 21.6616, size = 695, normalized size = 7.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(3/2)*(c*x**2+b*x+a)**2,x)
```

```
[Out] a**2*b*d*Piecewise((x*sqrt(b*d), Eq(c, 0)), (0, Eq(d, 0)), ((b*d + 2*c*d*x)
**(3/2)/(3*c*d), True)) + a**2*(-b*d*(b*d + 2*c*d*x)**(3/2)/3 + (b*d + 2*c*
```

```
d*x)**(5/2)/5)/(c*d) + a*b**2*(-b*d*(b*d + 2*c*d*x)**(3/2)/3 + (b*d + 2*c*d
*x)**(5/2)/5)/(c**2*d) + 3*a*b*(b**2*d**2*(b*d + 2*c*d*x)**(3/2)/3 - 2*b*d*
(b*d + 2*c*d*x)**(5/2)/5 + (b*d + 2*c*d*x)**(7/2)/7)/(2*c**2*d**2) + a*(-b*
*3*d**3*(b*d + 2*c*d*x)**(3/2)/3 + 3*b**2*d**2*(b*d + 2*c*d*x)**(5/2)/5 - 3
*b*d*(b*d + 2*c*d*x)**(7/2)/7 + (b*d + 2*c*d*x)**(9/2)/9)/(2*c**2*d**3) + b
**3*(b**2*d**2*(b*d + 2*c*d*x)**(3/2)/3 - 2*b*d*(b*d + 2*c*d*x)**(5/2)/5 +
(b*d + 2*c*d*x)**(7/2)/7)/(4*c**3*d**2) + b**2*(-b**3*d**3*(b*d + 2*c*d*x)*
*(3/2)/3 + 3*b**2*d**2*(b*d + 2*c*d*x)**(5/2)/5 - 3*b*d*(b*d + 2*c*d*x)**(7
/2)/7 + (b*d + 2*c*d*x)**(9/2)/9)/(2*c**3*d**3) + 5*b*(b**4*d**4*(b*d + 2*c
*d*x)**(3/2)/3 - 4*b**3*d**3*(b*d + 2*c*d*x)**(5/2)/5 + 6*b**2*d**2*(b*d +
2*c*d*x)**(7/2)/7 - 4*b*d*(b*d + 2*c*d*x)**(9/2)/9 + (b*d + 2*c*d*x)**(11/2
)/11)/(16*c**3*d**4) + (-b**5*d**5*(b*d + 2*c*d*x)**(3/2)/3 + b**4*d**4*(b*
d + 2*c*d*x)**(5/2) - 10*b**3*d**3*(b*d + 2*c*d*x)**(7/2)/7 + 10*b**2*d**2*
(b*d + 2*c*d*x)**(9/2)/9 - 5*b*d*(b*d + 2*c*d*x)**(11/2)/11 + (b*d + 2*c*d*
x)**(13/2)/13)/(16*c**3*d**5)
```

Giac [B] time = 1.16506, size = 783, normalized size = 8.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/720720*(240240*(2*c*d*x + b*d)^(3/2)*a^2*b - 48048*(5*(2*c*d*x + b*d)^(3/
2)*b*d - 3*(2*c*d*x + b*d)^(5/2))*a^2/d - 48048*(5*(2*c*d*x + b*d)^(3/2)*b*
d - 3*(2*c*d*x + b*d)^(5/2))*a*b^2/(c*d) + 1716*(35*(2*c*d*x + b*d)^(3/2)*b
^2*d^2 - 42*(2*c*d*x + b*d)^(5/2)*b*d + 15*(2*c*d*x + b*d)^(7/2))*b^3/(c^2*
d^2) + 10296*(35*(2*c*d*x + b*d)^(3/2)*b^2*d^2 - 42*(2*c*d*x + b*d)^(5/2)*b
*d + 15*(2*c*d*x + b*d)^(7/2))*a*b/(c*d^2) - 1144*(105*(2*c*d*x + b*d)^(3/2
)*b^3*d^3 - 189*(2*c*d*x + b*d)^(5/2)*b^2*d^2 + 135*(2*c*d*x + b*d)^(7/2)*b
*d - 35*(2*c*d*x + b*d)^(9/2))*b^2/(c^2*d^3) - 1144*(105*(2*c*d*x + b*d)^(3
/2)*b^3*d^3 - 189*(2*c*d*x + b*d)^(5/2)*b^2*d^2 + 135*(2*c*d*x + b*d)^(7/2)
*b*d - 35*(2*c*d*x + b*d)^(9/2))*a/(c*d^3) + 65*(1155*(2*c*d*x + b*d)^(3/2)
*b^4*d^4 - 2772*(2*c*d*x + b*d)^(5/2)*b^3*d^3 + 2970*(2*c*d*x + b*d)^(7/2)*
b^2*d^2 - 1540*(2*c*d*x + b*d)^(9/2)*b*d + 315*(2*c*d*x + b*d)^(11/2))*b/(c
^2*d^4) - 5*(3003*(2*c*d*x + b*d)^(3/2)*b^5*d^5 - 9009*(2*c*d*x + b*d)^(5/2
)*b^4*d^4 + 12870*(2*c*d*x + b*d)^(7/2)*b^3*d^3 - 10010*(2*c*d*x + b*d)^(9/
2)*b^2*d^2 + 4095*(2*c*d*x + b*d)^(11/2)*b*d - 693*(2*c*d*x + b*d)^(13/2))/
(c^2*d^5))/c
```

3.1270 $\int \sqrt{bd+2cdx}\left(a+bx+cx^2\right)^2\,dx$

Optimal. Leaf size=88

$$-\frac{\left(b^2-4ac\right)(bd+2cdx)^{7/2}}{56c^3d^3}+\frac{\left(b^2-4ac\right)^2(bd+2cdx)^{3/2}}{48c^3d}+\frac{(bd+2cdx)^{11/2}}{176c^3d^5}$$

```
[Out] ((b^2 - 4*a*c)^2*(b*d + 2*c*d*x)^(3/2))/(48*c^3*d) - ((b^2 - 4*a*c)*(b*d +
2*c*d*x)^(7/2))/(56*c^3*d^3) + (b*d + 2*c*d*x)^(11/2)/(176*c^3*d^5)
```

Rubi [A] time = 0.039476, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {683}

$$-\frac{\left(b^2-4ac\right)(bd+2cdx)^{7/2}}{56c^3d^3}+\frac{\left(b^2-4ac\right)^2(bd+2cdx)^{3/2}}{48c^3d}+\frac{(bd+2cdx)^{11/2}}{176c^3d^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*d + 2*c*d*x]*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((b^2 - 4*a*c)^2*(b*d + 2*c*d*x)^(3/2))/(48*c^3*d) - ((b^2 - 4*a*c)*(b*d +
2*c*d*x)^(7/2))/(56*c^3*d^3) + (b*d + 2*c*d*x)^(11/2)/(176*c^3*d^5)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \sqrt{bd+2cdx}\left(a+bx+cx^2\right)^2\,dx &= \int \left(\frac{\left(-b^2+4ac\right)^2\sqrt{bd+2cdx}}{16c^2}+\frac{\left(-b^2+4ac\right)(bd+2cdx)^{5/2}}{8c^2d^2}+\frac{(bd+2cdx)^{9/2}}{16c^2d^4}\right)dx\\ &= \frac{\left(b^2-4ac\right)^2(bd+2cdx)^{3/2}}{48c^3d}-\frac{\left(b^2-4ac\right)(bd+2cdx)^{7/2}}{56c^3d^3}+\frac{(bd+2cdx)^{11/2}}{176c^3d^5}\end{aligned}$$

Mathematica [A] time = 0.0439803, size = 92, normalized size = 1.05

$$\frac{\left(c^2\left(77a^2+66acx^2+21c^2x^4\right)+b^2c\left(15cx^2-22a\right)+6bc^2x\left(11a+7cx^2\right)-6b^3cx+2b^4\right)(d(b+2cx))^{3/2}}{231c^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*d + 2*c*d*x]*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((d*(b + 2*c*x))^(3/2)*(2*b^4 - 6*b^3*c*x + 6*b*c^2*x*(11*a + 7*c*x^2) + b^
2*c*(-22*a + 15*c*x^2) + c^2*(77*a^2 + 66*a*c*x^2 + 21*c^2*x^4)))/(231*c^3*
d)
```

Maple [A] time = 0.044, size = 96, normalized size = 1.1

$$\frac{(2\,cx+b)\left(21\,c^4x^4+42\,bx^3c^3+66\,ac^3x^2+15\,b^2c^2x^2+66\,abc^2x-6\,b^3cx+77\,a^2c^2-22\,acb^2+2\,b^4\right)}{231\,c^3}\sqrt{2\,cdx+bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(1/2)*(c*x^2+b*x+a)^2,x)
```

```
[Out] 1/231*(2*c*x+b)*(21*c^4*x^4+42*b*c^3*x^3+66*a*c^3*x^2+15*b^2*c^2*x^2+66*a*b
*c^2*x-6*b^3*c*x+77*a^2*c^2-22*a*b^2*c+2*b^4)*(2*c*d*x+b*d)^(1/2)/c^3
```

Maxima [A] time = 1.19286, size = 109, normalized size = 1.24

$$-\frac{66\,(2\,cdx+bd)^{\frac{7}{2}}\left(b^2-4\,ac\right)d^2-77\,\left(b^4-8\,ab^2c+16\,a^2c^2\right)(2\,cdx+bd)^{\frac{3}{2}}d^4-21\,(2\,cdx+bd)^{\frac{11}{2}}}{3696\,c^3d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3696*(66*(2*c*d*x + b*d)^(7/2)*(b^2 - 4*a*c)*d^2 - 77*(b^4 - 8*a*b^2*c +
 16*a^2*c^2)*(2*c*d*x + b*d)^(3/2)*d^4 - 21*(2*c*d*x + b*d)^(11/2))/(c^3*d^
5)
```

Fricas [A] time = 1.99735, size = 271, normalized size = 3.08

$$\frac{\left(42\,c^5x^5+105\,bc^4x^4+2\,b^5-22\,ab^3c+77\,a^2bc^2+12\,\left(6\,b^2c^3+11\,ac^4\right)x^3+3\,\left(b^3c^2+66\,abc^3\right)x^2-2\,\left(b^4c-11\,ab^2c^2-\cdots\right.\right.}{231\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/231*(42*c^5*x^5 + 105*b*c^4*x^4 + 2*b^5 - 22*a*b^3*c + 77*a^2*b*c^2 + 12*
(6*b^2*c^3 + 11*a*c^4)*x^3 + 3*(b^3*c^2 + 66*a*b*c^3)*x^2 - 2*(b^4*c - 11*a
*b^2*c^2 - 77*a^2*c^3)*x)*sqrt(2*c*d*x + b*d)/c^3
```

Sympy [A] time = 3.29871, size = 94, normalized size = 1.07

$$\frac{\frac{(bd+2cdx)^{\frac{3}{2}}\left(16a^2c^2-8ab^2c+b^4\right)}{48c^2}+\frac{\left(4ac-b^2\right)(bd+2cdx)^{\frac{7}{2}}}{56c^2d^2}+\frac{(bd+2cdx)^{\frac{11}{2}}}{176c^2d^4}}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(1/2)*(c*x**2+b*x+a)**2,x)
```

```
[Out] ((b*d + 2*c*d*x)**(3/2)*(16*a**2*c**2 - 8*a*b**2*c + b**4)/(48*c**2) + (4*a
*c - b**2)*(b*d + 2*c*d*x)**(7/2)/(56*c**2*d**2) + (b*d + 2*c*d*x)**(11/2)/
```

```
(176*c**2*d**4))/(c*d)
```

Giac [B] time = 1.14695, size = 471, normalized size = 5.35

$$\frac{18480\,(2\,cdx+bd)^{\frac{3}{2}}a^2-\frac{3696\left(5\,(2\,cdx+bd)^{\frac{3}{2}}bd-3\,(2\,cdx+bd)^{\frac{5}{2}}\right)ab}{cd}+\frac{132\left(35\,(2\,cdx+bd)^{\frac{3}{2}}b^2d^2-42\,(2\,cdx+bd)^{\frac{5}{2}}bd+15\,(2\,cdx+bd)^{\frac{7}{2}}\right)b^2}{c^2d^2}+\frac{264\left(35\,(2\,cd\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/55440*(18480*(2*c*d*x + b*d)^(3/2)*a^2 - 3696*(5*(2*c*d*x + b*d)^(3/2)*b*
d - 3*(2*c*d*x + b*d)^(5/2))*a*b/(c*d) + 132*(35*(2*c*d*x + b*d)^(3/2)*b^2*
d^2 - 42*(2*c*d*x + b*d)^(5/2)*b*d + 15*(2*c*d*x + b*d)^(7/2))*b^2/(c^2*d^2
) + 264*(35*(2*c*d*x + b*d)^(3/2)*b^2*d^2 - 42*(2*c*d*x + b*d)^(5/2)*b*d +
15*(2*c*d*x + b*d)^(7/2))*a/(c*d^2) - 44*(105*(2*c*d*x + b*d)^(3/2)*b^3*d^3
 - 189*(2*c*d*x + b*d)^(5/2)*b^2*d^2 + 135*(2*c*d*x + b*d)^(7/2)*b*d - 35*(
2*c*d*x + b*d)^(9/2))*b/(c^2*d^3) + (1155*(2*c*d*x + b*d)^(3/2)*b^4*d^4 - 2
772*(2*c*d*x + b*d)^(5/2)*b^3*d^3 + 2970*(2*c*d*x + b*d)^(7/2)*b^2*d^2 - 15
40*(2*c*d*x + b*d)^(9/2)*b*d + 315*(2*c*d*x + b*d)^(11/2))/(c^2*d^4))/(c*d)
```

3.1271 $\int \frac{(a+bx+cx^2)^2}{\sqrt{bd+2cdx}}\,dx$

Optimal. Leaf size=88

$$-\frac{(b^2-4ac)(bd+2cdx)^{5/2}}{40c^3d^3}+\frac{(b^2-4ac)^2\sqrt{bd+2cdx}}{16c^3d}+\frac{(bd+2cdx)^{9/2}}{144c^3d^5}$$

```
[Out] ((b^2 - 4*a*c)^2*Sqrt[b*d + 2*c*d*x])/(16*c^3*d) - ((b^2 - 4*a*c)*(b*d + 2*
c*d*x)^(5/2))/(40*c^3*d^3) + (b*d + 2*c*d*x)^(9/2)/(144*c^3*d^5)
```

Rubi [A] time = 0.0381218, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {683}

$$-\frac{(b^2-4ac)(bd+2cdx)^{5/2}}{40c^3d^3}+\frac{(b^2-4ac)^2\sqrt{bd+2cdx}}{16c^3d}+\frac{(bd+2cdx)^{9/2}}{144c^3d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/Sqrt[b*d + 2*c*d*x],x]
```

```
[Out] ((b^2 - 4*a*c)^2*Sqrt[b*d + 2*c*d*x])/(16*c^3*d) - ((b^2 - 4*a*c)*(b*d + 2*
c*d*x)^(5/2))/(40*c^3*d^3) + (b*d + 2*c*d*x)^(9/2)/(144*c^3*d^5)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx+cx^2)^2}{\sqrt{bd+2cdx}}\,dx &= \int \left(\frac{(-b^2+4ac)^2}{16c^2\sqrt{bd+2cdx}}+\frac{(-b^2+4ac)(bd+2cdx)^{3/2}}{8c^2d^2}+\frac{(bd+2cdx)^{7/2}}{16c^2d^4}\right)dx\\ &= \frac{(b^2-4ac)^2\sqrt{bd+2cdx}}{16c^3d}-\frac{(b^2-4ac)(bd+2cdx)^{5/2}}{40c^3d^3}+\frac{(bd+2cdx)^{9/2}}{144c^3d^5}\end{aligned}$$

Mathematica [A] time = 0.0426301, size = 92, normalized size = 1.05

$$\frac{\left(c^2\left(45a^2+18acx^2+5c^2x^4\right)+3b^2c\left(cx^2-6a\right)+2bc^2x\left(9a+5cx^2\right)-2b^3cx+2b^4\right)\sqrt{d(b+2cx)}}{45c^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/Sqrt[b*d + 2*c*d*x],x]
```

```
[Out] (Sqrt[d*(b + 2*c*x)]*(2*b^4 - 2*b^3*c*x + 3*b^2*c*(-6*a + c*x^2) + 2*b*c^2*
x*(9*a + 5*c*x^2) + c^2*(45*a^2 + 18*a*c*x^2 + 5*c^2*x^4)))/(45*c^3*d)
```

Maple [A] time = 0.044, size = 96, normalized size = 1.1

$$\frac{(2cx+b)\left(5c^4x^4+10bx^3c^3+18ac^3x^2+3b^2c^2x^2+18abc^2x-2b^3cx+45a^2c^2-18acb^2+2b^4\right)}{45c^3}\frac{1}{\sqrt{2cdx+bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^(1/2),x)
```

```
[Out] 1/45*(2*c*x+b)*(5*c^4*x^4+10*b*c^3*x^3+18*a*c^3*x^2+3*b^2*c^2*x^2+18*a*b*c^
2*x-2*b^3*c*x+45*a^2*c^2-18*a*b^2*c+2*b^4)/c^3/(2*c*d*x+b*d)^(1/2)
```

Maxima [B] time = 1.02153, size = 474, normalized size = 5.39

$$5040\sqrt{2cdx+bd}a^2-168a\left(\frac{10\left(3\sqrt{2cdx+bd}bd-(2cdx+bd)^{\frac{3}{2}}\right)b}{cd}-\frac{15\sqrt{2cdx+bd}b^2d^2-10(2cdx+bd)^{\frac{3}{2}}bd+3(2cdx+bd)^{\frac{5}{2}}}{cd^2}\right)+\frac{84\left(15\sqrt{2cdx+bd}b^2d^2-1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5040*(5040*sqrt(2*c*d*x + b*d)*a^2 - 168*a*(10*(3*sqrt(2*c*d*x + b*d)*b*d
 - (2*c*d*x + b*d)^(3/2))*b/(c*d) - (15*sqrt(2*c*d*x + b*d)*b^2*d^2 - 10*(2
*c*d*x + b*d)^(3/2)*b*d + 3*(2*c*d*x + b*d)^(5/2))/(c*d^2)) + 84*(15*sqrt(2
*c*d*x + b*d)*b^2*d^2 - 10*(2*c*d*x + b*d)^(3/2)*b*d + 3*(2*c*d*x + b*d)^(5
/2))*b^2/(c^2*d^2) - 36*(35*sqrt(2*c*d*x + b*d)*b^3*d^3 - 35*(2*c*d*x + b*d
)^(3/2)*b^2*d^2 + 21*(2*c*d*x + b*d)^(5/2)*b*d - 5*(2*c*d*x + b*d)^(7/2))*b
/(c^2*d^3) + (315*sqrt(2*c*d*x + b*d)*b^4*d^4 - 420*(2*c*d*x + b*d)^(3/2)*b
^3*d^3 + 378*(2*c*d*x + b*d)^(5/2)*b^2*d^2 - 180*(2*c*d*x + b*d)^(7/2)*b*d
+ 35*(2*c*d*x + b*d)^(9/2))/(c^2*d^4))/(c*d)
```

Fricas [A] time = 2.02639, size = 201, normalized size = 2.28

$$\frac{\left(5c^4x^4+10bc^3x^3+2b^4-18ab^2c+45a^2c^2+3\left(b^2c^2+6ac^3\right)x^2-2\left(b^3c-9abc^2\right)x\right)\sqrt{2cdx+bd}}{45c^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/45*(5*c^4*x^4 + 10*b*c^3*x^3 + 2*b^4 - 18*a*b^2*c + 45*a^2*c^2 + 3*(b^2*c
^2 + 6*a*c^3)*x^2 - 2*(b^3*c - 9*a*b*c^2)*x)*sqrt(2*c*d*x + b*d)/(c^3*d)
```

Sympy [A] time = 63.1151, size = 668, normalized size = 7.59

$$\begin{cases} -\frac{\frac{a^2b}{\sqrt{bd+2cdx}} + \frac{a^2\left(-\frac{bd}{\sqrt{bd+2cdx}} - \sqrt{bd+2cdx}\right)}{d} + \frac{ab^2\left(-\frac{bd}{\sqrt{bd+2cdx}} - \sqrt{bd+2cdx}\right)}{cd} + \frac{3ab\left(\frac{b^2d^2}{\sqrt{bd+2cdx}} + 2bd\sqrt{bd+2cdx} - \frac{(bd+2cdx)^{\frac{3}{2}}}{3}\right)}{2cd^2} + \frac{a\left(-\frac{b^3d^3}{\sqrt{bd+2cdx}} - 3b^2d^2\sqrt{bd+2cdx} + bd(bd+2cdx)^{\frac{3}{2}} - \frac{(bd+2}{}\right.}{2cd^3}}{} \\ \frac{\begin{cases} a^2x & \text{for } b = 0 \\ \frac{(a+bx)^3}{3b} & \text{otherwise} \end{cases}}{\sqrt{bd}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(2*c*d*x+b*d)**(1/2),x)
```

```
[Out] Piecewise((-(a**2*b/sqrt(b*d + 2*c*d*x) + a**2*(-b*d/sqrt(b*d + 2*c*d*x) -
sqrt(b*d + 2*c*d*x))/d + a*b**2*(-b*d/sqrt(b*d + 2*c*d*x) - sqrt(b*d + 2*c*
d*x))/(c*d) + 3*a*b*(b**2*d**2/sqrt(b*d + 2*c*d*x) + 2*b*d*sqrt(b*d + 2*c*d
*x) - (b*d + 2*c*d*x)**(3/2)/3)/(2*c*d**2) + a*(-b**3*d**3/sqrt(b*d + 2*c*d
*x) - 3*b**2*d**2*sqrt(b*d + 2*c*d*x) + b*d*(b*d + 2*c*d*x)**(3/2) - (b*d +
 2*c*d*x)**(5/2)/5)/(2*c*d**3) + b**3*(b**2*d**2/sqrt(b*d + 2*c*d*x) + 2*b*
d*sqrt(b*d + 2*c*d*x) - (b*d + 2*c*d*x)**(3/2)/3)/(4*c**2*d**2) + b**2*(-b*
*3*d**3/sqrt(b*d + 2*c*d*x) - 3*b**2*d**2*sqrt(b*d + 2*c*d*x) + b*d*(b*d +
2*c*d*x)**(3/2) - (b*d + 2*c*d*x)**(5/2)/5)/(2*c**2*d**3) + 5*b*(b**4*d**4/
sqrt(b*d + 2*c*d*x) + 4*b**3*d**3*sqrt(b*d + 2*c*d*x) - 2*b**2*d**2*(b*d +
2*c*d*x)**(3/2) + 4*b*d*(b*d + 2*c*d*x)**(5/2)/5 - (b*d + 2*c*d*x)**(7/2)/7
)/(16*c**2*d**4) + (-b**5*d**5/sqrt(b*d + 2*c*d*x) - 5*b**4*d**4*sqrt(b*d +
 2*c*d*x) + 10*b**3*d**3*(b*d + 2*c*d*x)**(3/2)/3 - 2*b**2*d**2*(b*d + 2*c*
d*x)**(5/2) + 5*b*d*(b*d + 2*c*d*x)**(7/2)/7 - (b*d + 2*c*d*x)**(9/2)/9)/(1
6*c**2*d**5))/c, Ne(c, 0)), (Piecewise((a**2*x, Eq(b, 0)), ((a + b*x)**3/(3
*b), True))/sqrt(b*d), True))
```

Giac [B] time = 1.13943, size = 471, normalized size = 5.35

$$\frac{5040\sqrt{2cdx+bd}\,a^2 - \frac{1680\left(3\sqrt{2cdx+bd}\,bd - (2cdx+bd)^{\frac{3}{2}}\right)ab}{cd} + \frac{84\left(15\sqrt{2cdx+bd}\,b^2d^2 - 10(2cdx+bd)^{\frac{3}{2}}bd + 3(2cdx+bd)^{\frac{5}{2}}\right)b^2}{c^2d^2} + \frac{168\left(15\sqrt{2cdx+bd}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5040*(5040*sqrt(2*c*d*x + b*d)*a^2 - 1680*(3*sqrt(2*c*d*x + b*d)*b*d - (2
*c*d*x + b*d)^(3/2))*a*b/(c*d) + 84*(15*sqrt(2*c*d*x + b*d)*b^2*d^2 - 10*(2
*c*d*x + b*d)^(3/2)*b*d + 3*(2*c*d*x + b*d)^(5/2))*b^2/(c^2*d^2) + 168*(15*
sqrt(2*c*d*x + b*d)*b^2*d^2 - 10*(2*c*d*x + b*d)^(3/2)*b*d + 3*(2*c*d*x + b
*d)^(5/2))*a/(c*d^2) - 36*(35*sqrt(2*c*d*x + b*d)*b^3*d^3 - 35*(2*c*d*x + b
*d)^(3/2)*b^2*d^2 + 21*(2*c*d*x + b*d)^(5/2)*b*d - 5*(2*c*d*x + b*d)^(7/2))
*b/(c^2*d^3) + (315*sqrt(2*c*d*x + b*d)*b^4*d^4 - 420*(2*c*d*x + b*d)^(3/2)
*b^3*d^3 + 378*(2*c*d*x + b*d)^(5/2)*b^2*d^2 - 180*(2*c*d*x + b*d)^(7/2)*b*
d + 35*(2*c*d*x + b*d)^(9/2))/(c^2*d^4))/(c*d)
```

3.1272 $\int \frac{(a+bx+cx^2)^2}{(bd+2cdx)^{3/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{(b^2-4ac)(bd+2cdx)^{3/2}}{24c^3d^3}-\frac{(b^2-4ac)^2}{16c^3d\sqrt{bd+2cdx}}+\frac{(bd+2cdx)^{7/2}}{112c^3d^5}$$

```
[Out] -(b^2 - 4*a*c)^2/(16*c^3*d*Sqrt[b*d + 2*c*d*x]) - ((b^2 - 4*a*c)*(b*d + 2*c
*d*x)^(3/2))/(24*c^3*d^3) + (b*d + 2*c*d*x)^(7/2)/(112*c^3*d^5)
```

Rubi [A] time = 0.0382175, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {683}

$$-\frac{(b^2-4ac)(bd+2cdx)^{3/2}}{24c^3d^3}-\frac{(b^2-4ac)^2}{16c^3d\sqrt{bd+2cdx}}+\frac{(bd+2cdx)^{7/2}}{112c^3d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x)^(3/2),x]
```

```
[Out] -(b^2 - 4*a*c)^2/(16*c^3*d*Sqrt[b*d + 2*c*d*x]) - ((b^2 - 4*a*c)*(b*d + 2*c
*d*x)^(3/2))/(24*c^3*d^3) + (b*d + 2*c*d*x)^(7/2)/(112*c^3*d^5)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx+cx^2)^2}{(bd+2cdx)^{3/2}}\,dx &= \int \left(\frac{(-b^2+4ac)^2}{16c^2(bd+2cdx)^{3/2}}+\frac{(-b^2+4ac)\sqrt{bd+2cdx}}{8c^2d^2}+\frac{(bd+2cdx)^{5/2}}{16c^2d^4}\right)dx\\ &= -\frac{(b^2-4ac)^2}{16c^3d\sqrt{bd+2cdx}}-\frac{(b^2-4ac)(bd+2cdx)^{3/2}}{24c^3d^3}+\frac{(bd+2cdx)^{7/2}}{112c^3d^5}\end{aligned}$$

Mathematica [A] time = 0.0382741, size = 91, normalized size = 1.03

$$\frac{c^2(-21a^2+14acx^2+3c^2x^4)+b^2c(14a+cx^2)+2bc^2x(7a+3cx^2)-2b^3cx-2b^4}{21c^3d\sqrt{d(b+2cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x)^(3/2),x]
```

```
[Out] (-2*b^4 - 2*b^3*c*x + b^2*c*(14*a + c*x^2) + 2*b*c^2*x*(7*a + 3*c*x^2) + c^
2*(-21*a^2 + 14*a*c*x^2 + 3*c^2*x^4))/(21*c^3*d*Sqrt[d*(b + 2*c*x)])
```

Maple [A] time = 0.044, size = 96, normalized size = 1.1

$$-\frac{(2\,cx+b)\left(-3\,c^{4}x^{4}-6\,bx^{3}c^{3}-14\,ac^{3}x^{2}-b^{2}c^{2}x^{2}-14\,abc^{2}x+2\,b^{3}cx+21\,a^{2}c^{2}-14\,acb^{2}+2\,b^{4}\right)}{21\,c^{3}}(2\,cdx+bd)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^(3/2),x)
```

```
[Out] -1/21*(2*c*x+b)*(-3*c^4*x^4-6*b*c^3*x^3-14*a*c^3*x^2-b^2*c^2*x^2-14*a*b*c^2
*x+2*b^3*c*x+21*a^2*c^2-14*a*b^2*c+2*b^4)/c^3/(2*c*d*x+b*d)^(3/2)
```

Maxima [A] time = 1.2528, size = 120, normalized size = 1.36

$$-\frac{\frac{21\left(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2}\right)}{\sqrt{2\,cdx+bd}c^{2}}+\frac{14\,(2\,cdx+bd)^{\frac{3}{2}}\left(b^{2}-4\,ac\right)d^{2}-3\,(2\,cdx+bd)^{\frac{7}{2}}}{c^{2}d^{4}}}{336\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/336*(21*(b^4 - 8*a*b^2*c + 16*a^2*c^2)/(sqrt(2*c*d*x + b*d)*c^2) + (14*(
2*c*d*x + b*d)^(3/2)*(b^2 - 4*a*c)*d^2 - 3*(2*c*d*x + b*d)^(7/2))/(c^2*d^4)
)/(c*d)
```

Fricas [A] time = 2.01115, size = 223, normalized size = 2.53

$$\frac{\left(3\,c^{4}x^{4}+6\,bc^{3}x^{3}-2\,b^{4}+14\,ab^{2}c-21\,a^{2}c^{2}+\left(b^{2}c^{2}+14\,ac^{3}\right)x^{2}-2\,\left(b^{3}c-7\,abc^{2}\right)x\right)\sqrt{2\,cdx+bd}}{21\,\left(2\,c^{4}d^{2}x+bc^{3}d^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/21*(3*c^4*x^4 + 6*b*c^3*x^3 - 2*b^4 + 14*a*b^2*c - 21*a^2*c^2 + (b^2*c^2
+ 14*a*c^3)*x^2 - 2*(b^3*c - 7*a*b*c^2)*x)*sqrt(2*c*d*x + b*d)/(2*c^4*d^2*x
 + b*c^3*d^2)
```

Sympy [A] time = 27.4823, size = 82, normalized size = 0.93

$$-\frac{\left(4ac-b^{2}\right)^{2}}{16c^{3}d\sqrt{bd+2cdx}}+\frac{\left(4ac-b^{2}\right)(bd+2cdx)^{\frac{3}{2}}}{24c^{3}d^{3}}+\frac{(bd+2cdx)^{\frac{7}{2}}}{112c^{3}d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(2*c*d*x+b*d)**(3/2),x)
```

```
[Out] -(4*a*c - b**2)**2/(16*c**3*d*sqrt(b*d + 2*c*d*x)) + (4*a*c - b**2)*(b*d +
2*c*d*x)**(3/2)/(24*c**3*d**3) + (b*d + 2*c*d*x)**(7/2)/(112*c**3*d**5)
```

Giac [A] time = 1.19754, size = 147, normalized size = 1.67

$$-\frac{b^4 - 8\,ab^2c + 16\,a^2c^2}{16\,\sqrt{2\,cdx + bd}c^3d} - \frac{14\,(2\,cdx + bd)^{\frac{3}{2}}b^2c^{18}d^{32} - 56\,(2\,cdx + bd)^{\frac{3}{2}}ac^{19}d^{32} - 3\,(2\,cdx + bd)^{\frac{7}{2}}c^{18}d^{30}}{336\,c^{21}d^{35}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^(3/2),x, algorithm="giac")
```

```
[Out] -1/16*(b^4 - 8*a*b^2*c + 16*a^2*c^2)/(sqrt(2*c*d*x + b*d)*c^3*d) - 1/336*(1
4*(2*c*d*x + b*d)^(3/2)*b^2*c^18*d^32 - 56*(2*c*d*x + b*d)^(3/2)*a*c^19*d^3
2 - 3*(2*c*d*x + b*d)^(7/2)*c^18*d^30)/(c^21*d^35)
```

3.1273 $\int \frac{(a+bx+cx^2)^2}{(bd+2cdx)^{5/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{(b^2-4ac)\sqrt{bd+2cdx}}{8c^3d^3}-\frac{(b^2-4ac)^2}{48c^3d(bd+2cdx)^{3/2}}+\frac{(bd+2cdx)^{5/2}}{80c^3d^5}$$

```
[Out] -(b^2 - 4*a*c)^2/(48*c^3*d*(b*d + 2*c*d*x)^(3/2)) - ((b^2 - 4*a*c)*Sqrt[b*d
 + 2*c*d*x])/(8*c^3*d^3) + (b*d + 2*c*d*x)^(5/2)/(80*c^3*d^5)
```

Rubi [A] time = 0.0403534, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {683}

$$-\frac{(b^2-4ac)\sqrt{bd+2cdx}}{8c^3d^3}-\frac{(b^2-4ac)^2}{48c^3d(bd+2cdx)^{3/2}}+\frac{(bd+2cdx)^{5/2}}{80c^3d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x)^(5/2),x]
```

```
[Out] -(b^2 - 4*a*c)^2/(48*c^3*d*(b*d + 2*c*d*x)^(3/2)) - ((b^2 - 4*a*c)*Sqrt[b*d
 + 2*c*d*x])/(8*c^3*d^3) + (b*d + 2*c*d*x)^(5/2)/(80*c^3*d^5)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx+cx^2)^2}{(bd+2cdx)^{5/2}}\,dx &= \int \left(\frac{(-b^2+4ac)^2}{16c^2(bd+2cdx)^{5/2}}+\frac{-b^2+4ac}{8c^2d^2\sqrt{bd+2cdx}}+\frac{(bd+2cdx)^{3/2}}{16c^2d^4}\right)dx\\ &= -\frac{(b^2-4ac)^2}{48c^3d(bd+2cdx)^{3/2}}-\frac{(b^2-4ac)\sqrt{bd+2cdx}}{8c^3d^3}+\frac{(bd+2cdx)^{5/2}}{80c^3d^5}\end{aligned}$$

Mathematica [A] time = 0.0398138, size = 91, normalized size = 1.03

$$\frac{c^2(-5a^2+30acx^2+3c^2x^4)+b^2c(10a-3cx^2)+6bc^2x(5a+cx^2)-6b^3cx-2b^4}{15c^3d(d(b+2cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x)^(5/2),x]
```

```
[Out] (-2*b^4 - 6*b^3*c*x + b^2*c*(10*a - 3*c*x^2) + 6*b*c^2*x*(5*a + c*x^2) + c^
2*(-5*a^2 + 30*a*c*x^2 + 3*c^2*x^4))/(15*c^3*d*(d*(b + 2*c*x))^(3/2))
```

Maple [A] time = 0.044, size = 96, normalized size = 1.1

$$-\frac{(2cx+b)\left(-3c^4x^4-6bx^3c^3-30ac^3x^2+3b^2c^2x^2-30abc^2x+6b^3cx+5a^2c^2-10acb^2+2b^4\right)}{15c^3}(2cdx+bd)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^(5/2),x)
```

```
[Out] -1/15*(2*c*x+b)*(-3*c^4*x^4-6*b*c^3*x^3-30*a*c^3*x^2+3*b^2*c^2*x^2-30*a*b*c
^2*x+6*b^3*c*x+5*a^2*c^2-10*a*b^2*c+2*b^4)/c^3/(2*c*d*x+b*d)^(5/2)
```

Maxima [A] time = 1.10376, size = 122, normalized size = 1.39

$$-\frac{\frac{5\left(b^4-8ab^2c+16a^2c^2\right)}{(2cdx+bd)^{\frac{3}{2}}c^2}+\frac{3\left(10\sqrt{2cdx+bd}\left(b^2-4ac\right)d^2-(2cdx+bd)^{\frac{5}{2}}\right)}{c^2d^4}}{240cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/240*(5*(b^4 - 8*a*b^2*c + 16*a^2*c^2)/((2*c*d*x + b*d)^(3/2)*c^2) + 3*(1
0*sqrt(2*c*d*x + b*d)*(b^2 - 4*a*c)*d^2 - (2*c*d*x + b*d)^(5/2))/(c^2*d^4))
/(c*d)
```

Fricas [A] time = 2.01993, size = 251, normalized size = 2.85

$$\frac{\left(3c^4x^4+6bc^3x^3-2b^4+10ab^2c-5a^2c^2-3\left(b^2c^2-10ac^3\right)x^2-6\left(b^3c-5abc^2\right)x\right)\sqrt{2cdx+bd}}{15\left(4c^5d^3x^2+4bc^4d^3x+b^2c^3d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*c^4*x^4 + 6*b*c^3*x^3 - 2*b^4 + 10*a*b^2*c - 5*a^2*c^2 - 3*(b^2*c^2
 - 10*a*c^3)*x^2 - 6*(b^3*c - 5*a*b*c^2)*x)*sqrt(2*c*d*x + b*d)/(4*c^5*d^3*
x^2 + 4*b*c^4*d^3*x + b^2*c^3*d^3)
```

Sympy [A] time = 49.9339, size = 82, normalized size = 0.93

$$-\frac{\left(4ac-b^2\right)^2}{48c^3d\left(bd+2cdx\right)^{\frac{3}{2}}}+\frac{\left(4ac-b^2\right)\sqrt{bd+2cdx}}{8c^3d^3}+\frac{\left(bd+2cdx\right)^{\frac{5}{2}}}{80c^3d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(2*c*d*x+b*d)**(5/2),x)
```

```
[Out] -(4*a*c - b**2)**2/(48*c**3*d*(b*d + 2*c*d*x)**(3/2)) + (4*a*c - b**2)*sqrt
(b*d + 2*c*d*x)/(8*c**3*d**3) + (b*d + 2*c*d*x)**(5/2)/(80*c**3*d**5)
```

Giac [A] time = 1.18923, size = 147, normalized size = 1.67

$$-\frac{b^4 - 8\,ab^2c + 16\,a^2c^2}{48\,(2\,cdx + bd)^{\frac{3}{2}}c^3d} - \frac{10\,\sqrt{2\,cdx + bd}b^2c^{12}d^{22} - 40\,\sqrt{2\,cdx + bd}ac^{13}d^{22} - (2\,cdx + bd)^{\frac{5}{2}}c^{12}d^{20}}{80\,c^{15}d^{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^(5/2),x, algorithm="giac")
```

```
[Out] -1/48*(b^4 - 8*a*b^2*c + 16*a^2*c^2)/((2*c*d*x + b*d)^(3/2)*c^3*d) - 1/80*(
10*sqrt(2*c*d*x + b*d)*b^2*c^12*d^22 - 40*sqrt(2*c*d*x + b*d)*a*c^13*d^22 -
 (2*c*d*x + b*d)^(5/2)*c^12*d^20)/(c^15*d^25)
```

3.1274 $\int \frac{(a+bx+cx^2)^2}{(bd+2cdx)^{7/2}}\,dx$

Optimal. Leaf size=88

$$\frac{b^2-4ac}{8c^3d^3\sqrt{bd+2cdx}}-\frac{(b^2-4ac)^2}{80c^3d(bd+2cdx)^{5/2}}+\frac{(bd+2cdx)^{3/2}}{48c^3d^5}$$

```
[Out] -(b^2 - 4*a*c)^2/(80*c^3*d*(b*d + 2*c*d*x)^(5/2)) + (b^2 - 4*a*c)/(8*c^3*d^
3*Sqrt[b*d + 2*c*d*x]) + (b*d + 2*c*d*x)^(3/2)/(48*c^3*d^5)
```

Rubi [A] time = 0.0383003, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {683}

$$\frac{b^2-4ac}{8c^3d^3\sqrt{bd+2cdx}}-\frac{(b^2-4ac)^2}{80c^3d(bd+2cdx)^{5/2}}+\frac{(bd+2cdx)^{3/2}}{48c^3d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x)^(7/2),x]
```

```
[Out] -(b^2 - 4*a*c)^2/(80*c^3*d*(b*d + 2*c*d*x)^(5/2)) + (b^2 - 4*a*c)/(8*c^3*d^
3*Sqrt[b*d + 2*c*d*x]) + (b*d + 2*c*d*x)^(3/2)/(48*c^3*d^5)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx+cx^2)^2}{(bd+2cdx)^{7/2}}\,dx &= \int \left(\frac{(-b^2+4ac)^2}{16c^2(bd+2cdx)^{7/2}}+\frac{-b^2+4ac}{8c^2d^2(bd+2cdx)^{3/2}}+\frac{\sqrt{bd+2cdx}}{16c^2d^4}\right)dx\\ &= -\frac{(b^2-4ac)^2}{80c^3d(bd+2cdx)^{5/2}}+\frac{b^2-4ac}{8c^3d^3\sqrt{bd+2cdx}}+\frac{(bd+2cdx)^{3/2}}{48c^3d^5}\end{aligned}$$

Mathematica [A] time = 0.0431497, size = 92, normalized size = 1.05

$$\frac{c^2(-3a^2-30acx^2+5c^2x^4)+3b^2c(5cx^2-2a)+10bc^2x(cx^2-3a)+10b^3cx+2b^4}{15c^3d(d(b+2cx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x)^(7/2),x]
```

```
[Out] (2*b^4 + 10*b^3*c*x + 10*b*c^2*x*(-3*a + c*x^2) + 3*b^2*c*(-2*a + 5*c*x^2)
+ c^2*(-3*a^2 - 30*a*c*x^2 + 5*c^2*x^4))/(15*c^3*d*(d*(b + 2*c*x))^(5/2))
```

Maple [A] time = 0.046, size = 96, normalized size = 1.1

$$-\frac{(2cx+b)\left(-5c^4x^4-10bx^3c^3+30ac^3x^2-15b^2c^2x^2+30abc^2x-10b^3cx+3a^2c^2+6acb^2-2b^4\right)}{15c^3}(2cdx+bd)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^(7/2),x)
```

```
[Out] -1/15*(2*c*x+b)*(-5*c^4*x^4-10*b*c^3*x^3+30*a*c^3*x^2-15*b^2*c^2*x^2+30*a*b
*c^2*x-10*b^3*c*x+3*a^2*c^2+6*a*b^2*c-2*b^4)/c^3/(2*c*d*x+b*d)^(7/2)
```

Maxima [A] time = 1.00077, size = 126, normalized size = 1.43

$$\frac{\frac{5(2cdx+bd)^{\frac{3}{2}}}{c^2d^4}+\frac{3\left(10(2cdx+bd)^2\left(b^2-4ac\right)-\left(b^4-8ab^2c+16a^2c^2\right)d^2\right)}{(2cdx+bd)^{\frac{5}{2}}c^2d^2}}{240cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/240*(5*(2*c*d*x + b*d)^(3/2)/(c^2*d^4) + 3*(10*(2*c*d*x + b*d)^2*(b^2 - 4
*a*c) - (b^4 - 8*a*b^2*c + 16*a^2*c^2)*d^2)/((2*c*d*x + b*d)^(5/2)*c^2*d^2)
)/(c*d)
```

Fricas [A] time = 2.01388, size = 281, normalized size = 3.19

$$\frac{\left(5c^4x^4+10bc^3x^3+2b^4-6ab^2c-3a^2c^2+15\left(b^2c^2-2ac^3\right)x^2+10\left(b^3c-3abc^2\right)x\right)\sqrt{2cdx+bd}}{15\left(8c^6d^4x^3+12bc^5d^4x^2+6b^2c^4d^4x+b^3c^3d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(5*c^4*x^4 + 10*b*c^3*x^3 + 2*b^4 - 6*a*b^2*c - 3*a^2*c^2 + 15*(b^2*c^
2 - 2*a*c^3)*x^2 + 10*(b^3*c - 3*a*b*c^2)*x)*sqrt(2*c*d*x + b*d)/(8*c^6*d^4
*x^3 + 12*b*c^5*d^4*x^2 + 6*b^2*c^4*d^4*x + b^3*c^3*d^4)
```

Sympy [A] time = 5.93285, size = 688, normalized size = 7.82

$$\begin{cases} -\frac{3a^2c^2\sqrt{bd+2cdx}}{15b^3c^3d^4+90b^2c^4d^4x+180bc^5d^4x^2+120c^6d^4x^3} - \frac{6ab^2c\sqrt{bd+2cdx}}{15b^3c^3d^4+90b^2c^4d^4x+180bc^5d^4x^2+120c^6d^4x^3} - \frac{30abc^2x\sqrt{bd+2cdx}}{15b^3c^3d^4+90b^2c^4d^4x+180bc^5d^4x^2+120c^6d^4x^3} - \frac{}{15} \\ \frac{a^2x+abx^2+\frac{b^2x^3}{3}}{(bd)^{\frac{7}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(2*c*d*x+b*d)**(7/2),x)
```

```
[Out] Piecewise((-3*a**2*c**2*sqrt(b*d + 2*c*d*x)/(15*b**3*c**3*d**4 + 90*b**2*c*
*4*d**4*x + 180*b*c**5*d**4*x**2 + 120*c**6*d**4*x**3) - 6*a*b**2*c*sqrt(b*
d + 2*c*d*x)/(15*b**3*c**3*d**4 + 90*b**2*c**4*d**4*x + 180*b*c**5*d**4*x**
2 + 120*c**6*d**4*x**3) - 30*a*b*c**2*x*sqrt(b*d + 2*c*d*x)/(15*b**3*c**3*d
**4 + 90*b**2*c**4*d**4*x + 180*b*c**5*d**4*x**2 + 120*c**6*d**4*x**3) - 30
*a*c**3*x**2*sqrt(b*d + 2*c*d*x)/(15*b**3*c**3*d**4 + 90*b**2*c**4*d**4*x +
 180*b*c**5*d**4*x**2 + 120*c**6*d**4*x**3) + 2*b**4*sqrt(b*d + 2*c*d*x)/(1
5*b**3*c**3*d**4 + 90*b**2*c**4*d**4*x + 180*b*c**5*d**4*x**2 + 120*c**6*d*
*4*x**3) + 10*b**3*c*x*sqrt(b*d + 2*c*d*x)/(15*b**3*c**3*d**4 + 90*b**2*c**
4*d**4*x + 180*b*c**5*d**4*x**2 + 120*c**6*d**4*x**3) + 15*b**2*c**2*x**2*s
qrt(b*d + 2*c*d*x)/(15*b**3*c**3*d**4 + 90*b**2*c**4*d**4*x + 180*b*c**5*d*
*4*x**2 + 120*c**6*d**4*x**3) + 10*b*c**3*x**3*sqrt(b*d + 2*c*d*x)/(15*b**3
*c**3*d**4 + 90*b**2*c**4*d**4*x + 180*b*c**5*d**4*x**2 + 120*c**6*d**4*x**
3) + 5*c**4*x**4*sqrt(b*d + 2*c*d*x)/(15*b**3*c**3*d**4 + 90*b**2*c**4*d**4
*x + 180*b*c**5*d**4*x**2 + 120*c**6*d**4*x**3), Ne(c, 0)), ((a**2*x + a*b*
x**2 + b**2*x**3/3)/(b*d)**(7/2), True))
```

Giac [A] time = 1.18467, size = 134, normalized size = 1.52

$$\frac{(2\,cdx+bd)^{\frac{3}{2}}}{48\,c^3d^5}-\frac{b^4d^2-8\,ab^2cd^2+16\,a^2c^2d^2-10\,(2\,cdx+bd)^2b^2+40\,(2\,cdx+bd)^2ac}{80\,(2\,cdx+bd)^{\frac{5}{2}}c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^(7/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*c*d*x + b*d)^(3/2)/(c^3*d^5) - 1/80*(b^4*d^2 - 8*a*b^2*c*d^2 + 16*a
^2*c^2*d^2 - 10*(2*c*d*x + b*d)^2*b^2 + 40*(2*c*d*x + b*d)^2*a*c)/((2*c*d*x
 + b*d)^(5/2)*c^3*d^3)
```

3.1275 $\int \frac{(a+bx+cx^2)^2}{(bd+2cdx)^{9/2}}\,dx$

Optimal. Leaf size=88

$$\frac{b^2-4ac}{24c^3d^3(bd+2cdx)^{3/2}}-\frac{\left(b^2-4ac\right)^2}{112c^3d(bd+2cdx)^{7/2}}+\frac{\sqrt{bd+2cdx}}{16c^3d^5}$$

```
[Out] -(b^2 - 4*a*c)^2/(112*c^3*d*(b*d + 2*c*d*x)^(7/2)) + (b^2 - 4*a*c)/(24*c^3*
d^3*(b*d + 2*c*d*x)^(3/2)) + Sqrt[b*d + 2*c*d*x]/(16*c^3*d^5)
```

Rubi [A] time = 0.0374583, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {683}

$$\frac{b^2-4ac}{24c^3d^3(bd+2cdx)^{3/2}}-\frac{\left(b^2-4ac\right)^2}{112c^3d(bd+2cdx)^{7/2}}+\frac{\sqrt{bd+2cdx}}{16c^3d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x)^(9/2),x]
```

```
[Out] -(b^2 - 4*a*c)^2/(112*c^3*d*(b*d + 2*c*d*x)^(7/2)) + (b^2 - 4*a*c)/(24*c^3*
d^3*(b*d + 2*c*d*x)^(3/2)) + Sqrt[b*d + 2*c*d*x]/(16*c^3*d^5)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx+cx^2\right)^2}{(bd+2cdx)^{9/2}}\,dx &= \int \left(\frac{\left(-b^2+4ac\right)^2}{16c^2(bd+2cdx)^{9/2}}+\frac{-b^2+4ac}{8c^2d^2(bd+2cdx)^{5/2}}+\frac{1}{16c^2d^4\sqrt{bd+2cdx}}\right)dx\\ &= -\frac{\left(b^2-4ac\right)^2}{112c^3d(bd+2cdx)^{7/2}}+\frac{b^2-4ac}{24c^3d^3(bd+2cdx)^{3/2}}+\frac{\sqrt{bd+2cdx}}{16c^3d^5}\end{aligned}$$

Mathematica [A] time = 0.0476852, size = 63, normalized size = 0.72

$$\frac{14\left(b^2-4ac\right)(b+2cx)^2-3\left(b^2-4ac\right)^2+21(b+2cx)^4}{336c^3d(d(b+2cx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x)^(9/2),x]
```

```
[Out] (-3*(b^2 - 4*a*c)^2 + 14*(b^2 - 4*a*c)*(b + 2*c*x)^2 + 21*(b + 2*c*x)^4)/(3
36*c^3*d*(d*(b + 2*c*x))^(7/2))
```

Maple [A] time = 0.049, size = 96, normalized size = 1.1

$$-\frac{(2cx+b)\left(-21c^4x^4-42bx^3c^3+14ac^3x^2-35b^2c^2x^2+14abc^2x-14b^3cx+3a^2c^2+2acb^2-2b^4\right)}{21c^3}(2cdx+bd)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^(9/2),x)
```

```
[Out] -1/21*(2*c*x+b)*(-21*c^4*x^4-42*b*c^3*x^3+14*a*c^3*x^2-35*b^2*c^2*x^2+14*a*
b*c^2*x-14*b^3*c*x+3*a^2*c^2+2*a*b^2*c-2*b^4)/c^3/(2*c*d*x+b*d)^(9/2)
```

Maxima [A] time = 0.993463, size = 124, normalized size = 1.41

$$\frac{\frac{21\sqrt{2cdx+bd}}{c^2d^4}+\frac{14(2cdx+bd)^2(b^2-4ac)-3(b^4-8ab^2c+16a^2c^2)d^2}{(2cdx+bd)^{\frac{7}{2}}c^2d^2}}{336cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/336*(21*sqrt(2*c*d*x + b*d)/(c^2*d^4) + (14*(2*c*d*x + b*d)^2*(b^2 - 4*a*
c) - 3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*d^2)/((2*c*d*x + b*d)^(7/2)*c^2*d^2))
/(c*d)
```

Fricas [A] time = 2.01866, size = 311, normalized size = 3.53

$$\frac{\left(21c^4x^4+42bc^3x^3+2b^4-2ab^2c-3a^2c^2+7\left(5b^2c^2-2ac^3\right)x^2+14\left(b^3c-abc^2\right)x\right)\sqrt{2cdx+bd}}{21\left(16c^7d^5x^4+32bc^6d^5x^3+24b^2c^5d^5x^2+8b^3c^4d^5x+b^4c^3d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/21*(21*c^4*x^4 + 42*b*c^3*x^3 + 2*b^4 - 2*a*b^2*c - 3*a^2*c^2 + 7*(5*b^2*
c^2 - 2*a*c^3)*x^2 + 14*(b^3*c - a*b*c^2)*x)*sqrt(2*c*d*x + b*d)/(16*c^7*d^
5*x^4 + 32*b*c^6*d^5*x^3 + 24*b^2*c^5*d^5*x^2 + 8*b^3*c^4*d^5*x + b^4*c^3*d
^5)
```

Sympy [A] time = 9.5047, size = 826, normalized size = 9.39

$$\begin{cases} -\frac{3a^2c^2\sqrt{bd+2cdx}}{21b^4c^3d^5+168b^3c^4d^5x+504b^2c^5d^5x^2+672bc^6d^5x^3+336c^7d^5x^4}-\frac{2ab^2c\sqrt{bd+2cdx}}{21b^4c^3d^5+168b^3c^4d^5x+504b^2c^5d^5x^2+672bc^6d^5x^3+336c^7d^5x^4}-\frac{14ab}{21b^4c^3d^5+168b^3c^4d^5x+50} \\ \frac{a^2x+abx^2+\frac{b^2x^3}{3}}{(bd)^{\frac{9}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(2*c*d*x+b*d)**(9/2),x)
```

```
[Out] Piecewise((-3*a**2*c**2*sqrt(b*d + 2*c*d*x)/(21*b**4*c**3*d**5 + 168*b**3*c
**4*d**5*x + 504*b**2*c**5*d**5*x**2 + 672*b*c**6*d**5*x**3 + 336*c**7*d**5
*x**4) - 2*a*b**2*c*sqrt(b*d + 2*c*d*x)/(21*b**4*c**3*d**5 + 168*b**3*c**4*
d**5*x + 504*b**2*c**5*d**5*x**2 + 672*b*c**6*d**5*x**3 + 336*c**7*d**5*x**
4) - 14*a*b*c**2*x*sqrt(b*d + 2*c*d*x)/(21*b**4*c**3*d**5 + 168*b**3*c**4*d
**5*x + 504*b**2*c**5*d**5*x**2 + 672*b*c**6*d**5*x**3 + 336*c**7*d**5*x**4
) - 14*a*c**3*x**2*sqrt(b*d + 2*c*d*x)/(21*b**4*c**3*d**5 + 168*b**3*c**4*d
**5*x + 504*b**2*c**5*d**5*x**2 + 672*b*c**6*d**5*x**3 + 336*c**7*d**5*x**4
) + 2*b**4*sqrt(b*d + 2*c*d*x)/(21*b**4*c**3*d**5 + 168*b**3*c**4*d**5*x +
504*b**2*c**5*d**5*x**2 + 672*b*c**6*d**5*x**3 + 336*c**7*d**5*x**4) + 14*b
**3*c*x*sqrt(b*d + 2*c*d*x)/(21*b**4*c**3*d**5 + 168*b**3*c**4*d**5*x + 504
*b**2*c**5*d**5*x**2 + 672*b*c**6*d**5*x**3 + 336*c**7*d**5*x**4) + 35*b**2
*c**2*x**2*sqrt(b*d + 2*c*d*x)/(21*b**4*c**3*d**5 + 168*b**3*c**4*d**5*x +
504*b**2*c**5*d**5*x**2 + 672*b*c**6*d**5*x**3 + 336*c**7*d**5*x**4) + 42*b
*c**3*x**3*sqrt(b*d + 2*c*d*x)/(21*b**4*c**3*d**5 + 168*b**3*c**4*d**5*x +
504*b**2*c**5*d**5*x**2 + 672*b*c**6*d**5*x**3 + 336*c**7*d**5*x**4) + 21*c
**4*x**4*sqrt(b*d + 2*c*d*x)/(21*b**4*c**3*d**5 + 168*b**3*c**4*d**5*x + 50
4*b**2*c**5*d**5*x**2 + 672*b*c**6*d**5*x**3 + 336*c**7*d**5*x**4), Ne(c, 0
)), ((a**2*x + a*b*x**2 + b**2*x**3/3)/(b*d)**(9/2), True))
```

Giac [A] time = 1.16398, size = 135, normalized size = 1.53

$$\frac{\sqrt{2\,cdx+bd}}{16\,c^3d^5}-\frac{3\,b^4d^2-24\,ab^2cd^2+48\,a^2c^2d^2-14\,(2\,cdx+bd)^2b^2+56\,(2\,cdx+bd)^2ac}{336\,(2\,cdx+bd)^{\frac{7}{2}}c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^(9/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2*c*d*x + b*d)/(c^3*d^5) - 1/336*(3*b^4*d^2 - 24*a*b^2*c*d^2 + 48
*a^2*c^2*d^2 - 14*(2*c*d*x + b*d)^2*b^2 + 56*(2*c*d*x + b*d)^2*a*c)/((2*c*d
*x + b*d)^(7/2)*c^3*d^3)
```

3.1276 $\int \frac{(a+bx+cx^2)^2}{(bd+2cdx)^{11/2}}\,dx$

Optimal. Leaf size=88

$$\frac{b^2-4ac}{40c^3d^3(bd+2cdx)^{5/2}}-\frac{(b^2-4ac)^2}{144c^3d(bd+2cdx)^{9/2}}-\frac{1}{16c^3d^5\sqrt{bd+2cdx}}$$

```
[Out] -(b^2 - 4*a*c)^2/(144*c^3*d*(b*d + 2*c*d*x)^(9/2)) + (b^2 - 4*a*c)/(40*c^3*
d^3*(b*d + 2*c*d*x)^(5/2)) - 1/(16*c^3*d^5*Sqrt[b*d + 2*c*d*x])
```

Rubi [A] time = 0.0372918, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {683}

$$\frac{b^2-4ac}{40c^3d^3(bd+2cdx)^{5/2}}-\frac{(b^2-4ac)^2}{144c^3d(bd+2cdx)^{9/2}}-\frac{1}{16c^3d^5\sqrt{bd+2cdx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x)^(11/2),x]
```

```
[Out] -(b^2 - 4*a*c)^2/(144*c^3*d*(b*d + 2*c*d*x)^(9/2)) + (b^2 - 4*a*c)/(40*c^3*
d^3*(b*d + 2*c*d*x)^(5/2)) - 1/(16*c^3*d^5*Sqrt[b*d + 2*c*d*x])
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx+cx^2)^2}{(bd+2cdx)^{11/2}}\,dx &= \int \left(\frac{(-b^2+4ac)^2}{16c^2(bd+2cdx)^{11/2}}+\frac{-b^2+4ac}{8c^2d^2(bd+2cdx)^{7/2}}+\frac{1}{16c^2d^4(bd+2cdx)^{3/2}}\right)dx\\ &= -\frac{(b^2-4ac)^2}{144c^3d(bd+2cdx)^{9/2}}+\frac{b^2-4ac}{40c^3d^3(bd+2cdx)^{5/2}}-\frac{1}{16c^3d^5\sqrt{bd+2cdx}}\end{aligned}$$

Mathematica [A] time = 0.0507299, size = 63, normalized size = 0.72

$$\frac{18(b^2-4ac)(b+2cx)^2-5(b^2-4ac)^2-45(b+2cx)^4}{720c^3d(d(b+2cx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(b*d + 2*c*d*x)^(11/2),x]
```

```
[Out] (-5*(b^2 - 4*a*c)^2 + 18*(b^2 - 4*a*c)*(b + 2*c*x)^2 - 45*(b + 2*c*x)^4)/(7
20*c^3*d*(d*(b + 2*c*x))^(9/2))
```

Maple [A] time = 0.044, size = 96, normalized size = 1.1

$$-\frac{(2cx+b)\left(45c^4x^4+90bx^3c^3+18ac^3x^2+63b^2c^2x^2+18abc^2x+18b^3cx+5a^2c^2+2acb^2+2b^4\right)}{45c^3}(2cdx+bd)^{-\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^(11/2),x)
```

```
[Out] -1/45*(2*c*x+b)*(45*c^4*x^4+90*b*c^3*x^3+18*a*c^3*x^2+63*b^2*c^2*x^2+18*a*b
*c^2*x+18*b^3*c*x+5*a^2*c^2+2*a*b^2*c+2*b^4)/c^3/(2*c*d*x+b*d)^(11/2)
```

Maxima [A] time = 1.07437, size = 109, normalized size = 1.24

$$\frac{18(2cdx+bd)^2\left(b^2-4ac\right)d^2-5\left(b^4-8ab^2c+16a^2c^2\right)d^4-45(2cdx+bd)^4}{720(2cdx+bd)^{\frac{9}{2}}c^3d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^(11/2),x, algorithm="maxima")
```

```
[Out] 1/720*(18*(2*c*d*x + b*d)^2*(b^2 - 4*a*c)*d^2 - 5*(b^4 - 8*a*b^2*c + 16*a^2
*c^2)*d^4 - 45*(2*c*d*x + b*d)^4)/((2*c*d*x + b*d)^(9/2)*c^3*d^5)
```

Fricas [B] time = 1.97052, size = 342, normalized size = 3.89

$$-\frac{\left(45c^4x^4+90bc^3x^3+2b^4+2ab^2c+5a^2c^2+9\left(7b^2c^2+2ac^3\right)x^2+18\left(b^3c+abc^2\right)x\right)\sqrt{2cdx+bd}}{45\left(32c^8d^6x^5+80bc^7d^6x^4+80b^2c^6d^6x^3+40b^3c^5d^6x^2+10b^4c^4d^6x+b^5c^3d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^(11/2),x, algorithm="fricas")
```

```
[Out] -1/45*(45*c^4*x^4 + 90*b*c^3*x^3 + 2*b^4 + 2*a*b^2*c + 5*a^2*c^2 + 9*(7*b^2
*c^2 + 2*a*c^3)*x^2 + 18*(b^3*c + a*b*c^2)*x)*sqrt(2*c*d*x + b*d)/(32*c^8*d
^6*x^5 + 80*b*c^7*d^6*x^4 + 80*b^2*c^6*d^6*x^3 + 40*b^3*c^5*d^6*x^2 + 10*b^
4*c^4*d^6*x + b^5*c^3*d^6)
```

Sympy [A] time = 20.5602, size = 966, normalized size = 10.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(2*c*d*x+b*d)**(11/2),x)
```

```
[Out] Piecewise((-5*a**2*c**2*sqrt(b*d + 2*c*d*x)/(45*b**5*c**3*d**6 + 450*b**4*c
**4*d**6*x + 1800*b**3*c**5*d**6*x**2 + 3600*b**2*c**6*d**6*x**3 + 3600*b*c
**7*d**6*x**4 + 1440*c**8*d**6*x**5) - 2*a*b**2*c*sqrt(b*d + 2*c*d*x)/(45*b
```

```
**5*c**3*d**6 + 450*b**4*c**4*d**6*x + 1800*b**3*c**5*d**6*x**2 + 3600*b**2
*c**6*d**6*x**3 + 3600*b*c**7*d**6*x**4 + 1440*c**8*d**6*x**5) - 18*a*b*c**
2*x*sqrt(b*d + 2*c*d*x)/(45*b**5*c**3*d**6 + 450*b**4*c**4*d**6*x + 1800*b*
*3*c**5*d**6*x**2 + 3600*b**2*c**6*d**6*x**3 + 3600*b*c**7*d**6*x**4 + 1440
*c**8*d**6*x**5) - 18*a*c**3*x**2*sqrt(b*d + 2*c*d*x)/(45*b**5*c**3*d**6 +
450*b**4*c**4*d**6*x + 1800*b**3*c**5*d**6*x**2 + 3600*b**2*c**6*d**6*x**3
+ 3600*b*c**7*d**6*x**4 + 1440*c**8*d**6*x**5) - 2*b**4*sqrt(b*d + 2*c*d*x)
/(45*b**5*c**3*d**6 + 450*b**4*c**4*d**6*x + 1800*b**3*c**5*d**6*x**2 + 360
0*b**2*c**6*d**6*x**3 + 3600*b*c**7*d**6*x**4 + 1440*c**8*d**6*x**5) - 18*b
**3*c*x*sqrt(b*d + 2*c*d*x)/(45*b**5*c**3*d**6 + 450*b**4*c**4*d**6*x + 180
0*b**3*c**5*d**6*x**2 + 3600*b**2*c**6*d**6*x**3 + 3600*b*c**7*d**6*x**4 +
1440*c**8*d**6*x**5) - 63*b**2*c**2*x**2*sqrt(b*d + 2*c*d*x)/(45*b**5*c**3*
d**6 + 450*b**4*c**4*d**6*x + 1800*b**3*c**5*d**6*x**2 + 3600*b**2*c**6*d**
6*x**3 + 3600*b*c**7*d**6*x**4 + 1440*c**8*d**6*x**5) - 90*b*c**3*x**3*sqrt
(b*d + 2*c*d*x)/(45*b**5*c**3*d**6 + 450*b**4*c**4*d**6*x + 1800*b**3*c**5*
d**6*x**2 + 3600*b**2*c**6*d**6*x**3 + 3600*b*c**7*d**6*x**4 + 1440*c**8*d*
*6*x**5) - 45*c**4*x**4*sqrt(b*d + 2*c*d*x)/(45*b**5*c**3*d**6 + 450*b**4*c
**4*d**6*x + 1800*b**3*c**5*d**6*x**2 + 3600*b**2*c**6*d**6*x**3 + 3600*b*c
**7*d**6*x**4 + 1440*c**8*d**6*x**5), Ne(c, 0)), ((a**2*x + a*b*x**2 + b**2
*x**3/3)/(b*d)**(11/2), True))
```

Giac [A] time = 1.16077, size = 134, normalized size = 1.52

$$-\frac{5\,b^4d^4-40\,ab^2cd^4+80\,a^2c^2d^4-18\,(2\,cdx+bd)^2b^2d^2+72\,(2\,cdx+bd)^2acd^2+45\,(2\,cdx+bd)^4}{720\,(2\,cdx+bd)^{\frac{9}{2}}c^3d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(2*c*d*x+b*d)^(11/2),x, algorithm="giac")
```

```
[Out] -1/720*(5*b^4*d^4 - 40*a*b^2*c*d^4 + 80*a^2*c^2*d^4 - 18*(2*c*d*x + b*d)^2*
b^2*d^2 + 72*(2*c*d*x + b*d)^2*a*c*d^2 + 45*(2*c*d*x + b*d)^4)/((2*c*d*x +
b*d)^(9/2)*c^3*d^5)
```

3.1277 $\int \sqrt{bd+2cdx}\left(a+bx+cx^2\right)^3\,dx$

Optimal. Leaf size=121

$$-\frac{3\left(b^2-4ac\right)(bd+2cdx)^{11/2}}{704c^4d^5}+\frac{3\left(b^2-4ac\right)^2(bd+2cdx)^{7/2}}{448c^4d^3}-\frac{\left(b^2-4ac\right)^3(bd+2cdx)^{3/2}}{192c^4d}+\frac{(bd+2cdx)^{15/2}}{960c^4d^7}$$

```
[Out] -((b^2 - 4*a*c)^3*(b*d + 2*c*d*x)^(3/2))/(192*c^4*d) + (3*(b^2 - 4*a*c)^2*(
b*d + 2*c*d*x)^(7/2))/(448*c^4*d^3) - (3*(b^2 - 4*a*c)*(b*d + 2*c*d*x)^(11/
2))/(704*c^4*d^5) + (b*d + 2*c*d*x)^(15/2)/(960*c^4*d^7)
```

Rubi [A] time = 0.0512324, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {683}

$$-\frac{3\left(b^2-4ac\right)(bd+2cdx)^{11/2}}{704c^4d^5}+\frac{3\left(b^2-4ac\right)^2(bd+2cdx)^{7/2}}{448c^4d^3}-\frac{\left(b^2-4ac\right)^3(bd+2cdx)^{3/2}}{192c^4d}+\frac{(bd+2cdx)^{15/2}}{960c^4d^7}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*d + 2*c*d*x]*(a + b*x + c*x^2)^3,x]
```

```
[Out] -((b^2 - 4*a*c)^3*(b*d + 2*c*d*x)^(3/2))/(192*c^4*d) + (3*(b^2 - 4*a*c)^2*(
b*d + 2*c*d*x)^(7/2))/(448*c^4*d^3) - (3*(b^2 - 4*a*c)*(b*d + 2*c*d*x)^(11/
2))/(704*c^4*d^5) + (b*d + 2*c*d*x)^(15/2)/(960*c^4*d^7)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] && !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\int \sqrt{bd+2cdx}\left(a+bx+cx^2\right)^3\,dx=\int\left(\frac{\left(-b^2+4ac\right)^3\sqrt{bd+2cdx}}{64c^3}+\frac{3\left(-b^2+4ac\right)^2(bd+2cdx)^{5/2}}{64c^3d^2}+\frac{3\left(-b^2+4ac\right)(b}{64c^3}\right.$$

$$=-\frac{\left(b^2-4ac\right)^3(bd+2cdx)^{3/2}}{192c^4d}+\frac{3\left(b^2-4ac\right)^2(bd+2cdx)^{7/2}}{448c^4d^3}-\frac{3\left(b^2-4ac\right)(bd+2}{704c^4d^5}$$

Mathematica [A] time = 0.0844362, size = 83, normalized size = 0.69

$$\frac{\left(-315\left(b^2-4ac\right)(b+2cx)^4+495\left(b^2-4ac\right)^2(b+2cx)^2-385\left(b^2-4ac\right)^3+77(b+2cx)^6\right)(d(b+2cx))^{3/2}}{73920c^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*d + 2*c*d*x]*(a + b*x + c*x^2)^3,x]
```

```
[Out] ((d*(b + 2*c*x))^(3/2)*(-385*(b^2 - 4*a*c)^3 + 495*(b^2 - 4*a*c)^2*(b + 2*c
*x)^2 - 315*(b^2 - 4*a*c)*(b + 2*c*x)^4 + 77*(b + 2*c*x)^6))/(73920*c^4*d)
```

Maple [A] time = 0.046, size = 174, normalized size = 1.4

$$\frac{(2\,cx+b)\left(77\,c^6x^6+231\,bc^5x^5+315\,ac^5x^4+210\,b^2c^4x^4+630\,abc^4x^3+35\,b^3c^3x^3+495\,a^2c^4x^2+225\,ab^2c^3x^2-15\,b^4c^2x^2\right.}{1155\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(1/2)*(c*x^2+b*x+a)^3,x)
```

```
[Out] 1/1155*(2*c*x+b)*(77*c^6*x^6+231*b*c^5*x^5+315*a*c^5*x^4+210*b^2*c^4*x^4+63
0*a*b*c^4*x^3+35*b^3*c^3*x^3+495*a^2*c^4*x^2+225*a*b^2*c^3*x^2-15*b^4*c^2*x
^2+495*a^2*b*c^3*x-90*a*b^3*c^2*x+6*b^5*c*x+385*a^3*c^3-165*a^2*b^2*c^2+30*
a*b^4*c-2*b^6)*(2*c*d*x+b*d)^(1/2)/c^4
```

Maxima [A] time = 1.0426, size = 171, normalized size = 1.41

$$-\frac{315\,(2\,cdx+bd)^{\frac{11}{2}}\left(b^2-4\,ac\right)d^2-495\left(b^4-8\,ab^2c+16\,a^2c^2\right)(2\,cdx+bd)^{\frac{7}{2}}d^4+385\left(b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3\right)(}{73920\,c^4d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)*(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/73920*(315*(2*c*d*x + b*d)^(11/2)*(b^2 - 4*a*c)*d^2 - 495*(b^4 - 8*a*b^2
*c + 16*a^2*c^2)*(2*c*d*x + b*d)^(7/2)*d^4 + 385*(b^6 - 12*a*b^4*c + 48*a^2
*b^2*c^2 - 64*a^3*c^3)*(2*c*d*x + b*d)^(3/2)*d^6 - 77*(2*c*d*x + b*d)^(15/2
))/(c^4*d^7)
```

Fricas [A] time = 2.03903, size = 462, normalized size = 3.82

$$\frac{\left(154\,c^7x^7+539\,bc^6x^6-2\,b^7+30\,ab^5c-165\,a^2b^3c^2+385\,a^3bc^3+21\left(31\,b^2c^5+30\,ac^6\right)x^5+35\left(8\,b^3c^4+45\,abc^5\right)x^4+5\,(\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)*(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/1155*(154*c^7*x^7 + 539*b*c^6*x^6 - 2*b^7 + 30*a*b^5*c - 165*a^2*b^3*c^2
+ 385*a^3*b*c^3 + 21*(31*b^2*c^5 + 30*a*c^6)*x^5 + 35*(8*b^3*c^4 + 45*a*b*c
^5)*x^4 + 5*(b^4*c^3 + 216*a*b^2*c^4 + 198*a^2*c^5)*x^3 - 3*(b^5*c^2 - 15*a
*b^3*c^3 - 495*a^2*b*c^4)*x^2 + (2*b^6*c - 30*a*b^4*c^2 + 165*a^2*b^2*c^3 +
 770*a^3*c^4)*x)*sqrt(2*c*d*x + b*d)/c^4
```

Sympy [A] time = 4.47442, size = 151, normalized size = 1.25

$$\frac{\frac{(bd+2cdx)^{\frac{3}{2}}\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}{192c^3}+\frac{(bd+2cdx)^{\frac{7}{2}}\left(48a^2c^2-24ab^2c+3b^4\right)}{448c^3d^2}+\frac{\left(12ac-3b^2\right)(bd+2cdx)^{\frac{11}{2}}}{704c^3d^4}+\frac{(bd+2cdx)^{\frac{15}{2}}}{960c^3d^6}}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(1/2)*(c*x**2+b*x+a)**3,x)
```

```
[Out] ((b*d + 2*c*d*x)**(3/2)*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b
**6)/(192*c**3) + (b*d + 2*c*d*x)**(7/2)*(48*a**2*c**2 - 24*a*b**2*c + 3*b*
*4)/(448*c**3*d**2) + (12*a*c - 3*b**2)*(b*d + 2*c*d*x)**(11/2)/(704*c**3*d
**4) + (b*d + 2*c*d*x)**(15/2)/(960*c**3*d**6))/(c*d)
```

Giac [B] time = 1.18686, size = 1049, normalized size = 8.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)*(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2882880*(960960*(2*c*d*x + b*d)^(3/2)*a^3 - 288288*(5*(2*c*d*x + b*d)^(3/
2)*b*d - 3*(2*c*d*x + b*d)^(5/2))*a^2*b/(c*d) + 20592*(35*(2*c*d*x + b*d)^(
3/2)*b^2*d^2 - 42*(2*c*d*x + b*d)^(5/2)*b*d + 15*(2*c*d*x + b*d)^(7/2))*a*b
^2/(c^2*d^2) + 20592*(35*(2*c*d*x + b*d)^(3/2)*b^2*d^2 - 42*(2*c*d*x + b*d)
^(5/2)*b*d + 15*(2*c*d*x + b*d)^(7/2))*a^2/(c*d^2) - 1144*(105*(2*c*d*x + b
*d)^(3/2)*b^3*d^3 - 189*(2*c*d*x + b*d)^(5/2)*b^2*d^2 + 135*(2*c*d*x + b*d)
^(7/2)*b*d - 35*(2*c*d*x + b*d)^(9/2))*b^3/(c^3*d^3) - 6864*(105*(2*c*d*x +
 b*d)^(3/2)*b^3*d^3 - 189*(2*c*d*x + b*d)^(5/2)*b^2*d^2 + 135*(2*c*d*x + b*
d)^(7/2)*b*d - 35*(2*c*d*x + b*d)^(9/2))*a*b/(c^2*d^3) + 156*(1155*(2*c*d*x
 + b*d)^(3/2)*b^4*d^4 - 2772*(2*c*d*x + b*d)^(5/2)*b^3*d^3 + 2970*(2*c*d*x
+ b*d)^(7/2)*b^2*d^2 - 1540*(2*c*d*x + b*d)^(9/2)*b*d + 315*(2*c*d*x + b*d)
^(11/2))*b^2/(c^3*d^4) + 156*(1155*(2*c*d*x + b*d)^(3/2)*b^4*d^4 - 2772*(2*
c*d*x + b*d)^(5/2)*b^3*d^3 + 2970*(2*c*d*x + b*d)^(7/2)*b^2*d^2 - 1540*(2*c
*d*x + b*d)^(9/2)*b*d + 315*(2*c*d*x + b*d)^(11/2))*a/(c^2*d^4) - 30*(3003*
(2*c*d*x + b*d)^(3/2)*b^5*d^5 - 9009*(2*c*d*x + b*d)^(5/2)*b^4*d^4 + 12870*
(2*c*d*x + b*d)^(7/2)*b^3*d^3 - 10010*(2*c*d*x + b*d)^(9/2)*b^2*d^2 + 4095*
(2*c*d*x + b*d)^(11/2)*b*d - 693*(2*c*d*x + b*d)^(13/2))*b/(c^3*d^5) + (150
15*(2*c*d*x + b*d)^(3/2)*b^6*d^6 - 54054*(2*c*d*x + b*d)^(5/2)*b^5*d^5 + 96
525*(2*c*d*x + b*d)^(7/2)*b^4*d^4 - 100100*(2*c*d*x + b*d)^(9/2)*b^3*d^3 +
61425*(2*c*d*x + b*d)^(11/2)*b^2*d^2 - 20790*(2*c*d*x + b*d)^(13/2)*b*d + 3
003*(2*c*d*x + b*d)^(15/2))/(c^3*d^6))/(c*d)
```

3.1278 $\int \frac{(a+bx+cx^2)^3}{\sqrt{bd+2cdx}}\,dx$

Optimal. Leaf size=121

$$-\frac{(b^2-4ac)(bd+2cdx)^{9/2}}{192c^4d^5}+\frac{3(b^2-4ac)^2(bd+2cdx)^{5/2}}{320c^4d^3}-\frac{(b^2-4ac)^3\sqrt{bd+2cdx}}{64c^4d}+\frac{(bd+2cdx)^{13/2}}{832c^4d^7}$$

```
[Out] -((b^2 - 4*a*c)^3*Sqrt[b*d + 2*c*d*x])/(64*c^4*d) + (3*(b^2 - 4*a*c)^2*(b*d
 + 2*c*d*x)^(5/2))/(320*c^4*d^3) - ((b^2 - 4*a*c)*(b*d + 2*c*d*x)^(9/2))/(1
92*c^4*d^5) + (b*d + 2*c*d*x)^(13/2)/(832*c^4*d^7)
```

Rubi [A] time = 0.0499577, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {683}

$$-\frac{(b^2-4ac)(bd+2cdx)^{9/2}}{192c^4d^5}+\frac{3(b^2-4ac)^2(bd+2cdx)^{5/2}}{320c^4d^3}-\frac{(b^2-4ac)^3\sqrt{bd+2cdx}}{64c^4d}+\frac{(bd+2cdx)^{13/2}}{832c^4d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/Sqrt[b*d + 2*c*d*x],x]
```

```
[Out] -((b^2 - 4*a*c)^3*Sqrt[b*d + 2*c*d*x])/(64*c^4*d) + (3*(b^2 - 4*a*c)^2*(b*d
 + 2*c*d*x)^(5/2))/(320*c^4*d^3) - ((b^2 - 4*a*c)*(b*d + 2*c*d*x)^(9/2))/(1
92*c^4*d^5) + (b*d + 2*c*d*x)^(13/2)/(832*c^4*d^7)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx+cx^2)^3}{\sqrt{bd+2cdx}}\,dx &= \int \left(\frac{(-b^2+4ac)^3}{64c^3\sqrt{bd+2cdx}}+\frac{3(-b^2+4ac)^2(bd+2cdx)^{3/2}}{64c^3d^2}+\frac{3(-b^2+4ac)(bd+2cdx)^{7/2}}{64c^3d^4}+\frac{(bd+2c\ldots}{64c^3\ldots}\right.\\ &= -\frac{(b^2-4ac)^3\sqrt{bd+2cdx}}{64c^4d}+\frac{3(b^2-4ac)^2(bd+2cdx)^{5/2}}{320c^4d^3}-\frac{(b^2-4ac)(bd+2cdx)^{9/2}}{192c^4d^5}+\frac{(bd+2c\ldots}{832c^4\ldots}\end{aligned}$$

Mathematica [A] time = 0.0761271, size = 83, normalized size = 0.69

$$\frac{\left(-65(b^2-4ac)(b+2cx)^4+117(b^2-4ac)^2(b+2cx)^2-195(b^2-4ac)^3+15(b+2cx)^6\right)\sqrt{d(b+2cx)}}{12480c^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/Sqrt[b*d + 2*c*d*x],x]
```

```
[Out] (Sqrt[d*(b + 2*c*x)]*(-195*(b^2 - 4*a*c)^3 + 117*(b^2 - 4*a*c)^2*(b + 2*c*x
)^2 - 65*(b^2 - 4*a*c)*(b + 2*c*x)^4 + 15*(b + 2*c*x)^6))/(12480*c^4*d)
```

Maple [A] time = 0.044, size = 174, normalized size = 1.4

$$\frac{(2\,cx+b)\left(15\,c^6x^6+45\,bc^5x^5+65\,ac^5x^4+40\,b^2c^4x^4+130\,abc^4x^3+5\,b^3c^3x^3+117\,a^2c^4x^2+39\,ab^2c^3x^2-3\,b^4c^2x^2+1\right.}{195\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^(1/2),x)
```

```
[Out] 1/195*(2*c*x+b)*(15*c^6*x^6+45*b*c^5*x^5+65*a*c^5*x^4+40*b^2*c^4*x^4+130*a*
b*c^4*x^3+5*b^3*c^3*x^3+117*a^2*c^4*x^2+39*a*b^2*c^3*x^2-3*b^4*c^2*x^2+117*
a^2*b*c^3*x-26*a*b^3*c^2*x+2*b^5*c*x+195*a^3*c^3-117*a^2*b^2*c^2+26*a*b^4*c
-2*b^6)/c^4/(2*c*d*x+b*d)^(1/2)
```

Maxima [B] time = 1.12018, size = 1050, normalized size = 8.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/960960*(960960*sqrt(2*c*d*x + b*d)*a^3 - 48048*a^2*(10*(3*sqrt(2*c*d*x +
b*d)*b*d - (2*c*d*x + b*d)^(3/2))*b/(c*d) - (15*sqrt(2*c*d*x + b*d)*b^2*d^2
 - 10*(2*c*d*x + b*d)^(3/2)*b*d + 3*(2*c*d*x + b*d)^(5/2))/(c*d^2)) + 572*a
*(84*(15*sqrt(2*c*d*x + b*d)*b^2*d^2 - 10*(2*c*d*x + b*d)^(3/2)*b*d + 3*(2*
c*d*x + b*d)^(5/2))*b^2/(c^2*d^2) - 36*(35*sqrt(2*c*d*x + b*d)*b^3*d^3 - 35
*(2*c*d*x + b*d)^(3/2)*b^2*d^2 + 21*(2*c*d*x + b*d)^(5/2)*b*d - 5*(2*c*d*x
+ b*d)^(7/2))*b/(c^2*d^3) + (315*sqrt(2*c*d*x + b*d)*b^4*d^4 - 420*(2*c*d*x
 + b*d)^(3/2)*b^3*d^3 + 378*(2*c*d*x + b*d)^(5/2)*b^2*d^2 - 180*(2*c*d*x +
b*d)^(7/2)*b*d + 35*(2*c*d*x + b*d)^(9/2))/(c^2*d^4)) - 3432*(35*sqrt(2*c*d
*x + b*d)*b^3*d^3 - 35*(2*c*d*x + b*d)^(3/2)*b^2*d^2 + 21*(2*c*d*x + b*d)^(
5/2)*b*d - 5*(2*c*d*x + b*d)^(7/2))*b^3/(c^3*d^3) + 572*(315*sqrt(2*c*d*x +
 b*d)*b^4*d^4 - 420*(2*c*d*x + b*d)^(3/2)*b^3*d^3 + 378*(2*c*d*x + b*d)^(5/
2)*b^2*d^2 - 180*(2*c*d*x + b*d)^(7/2)*b*d + 35*(2*c*d*x + b*d)^(9/2))*b^2/
(c^3*d^4) - 130*(693*sqrt(2*c*d*x + b*d)*b^5*d^5 - 1155*(2*c*d*x + b*d)^(3/
2)*b^4*d^4 + 1386*(2*c*d*x + b*d)^(5/2)*b^3*d^3 - 990*(2*c*d*x + b*d)^(7/2)
*b^2*d^2 + 385*(2*c*d*x + b*d)^(9/2)*b*d - 63*(2*c*d*x + b*d)^(11/2))*b/(c^
3*d^5) + 5*(3003*sqrt(2*c*d*x + b*d)*b^6*d^6 - 6006*(2*c*d*x + b*d)^(3/2)*b
^5*d^5 + 9009*(2*c*d*x + b*d)^(5/2)*b^4*d^4 - 8580*(2*c*d*x + b*d)^(7/2)*b^
3*d^3 + 5005*(2*c*d*x + b*d)^(9/2)*b^2*d^2 - 1638*(2*c*d*x + b*d)^(11/2)*b*
d + 231*(2*c*d*x + b*d)^(13/2))/(c^3*d^6))/(c*d)
```

Fricas [A] time = 2.02513, size = 363, normalized size = 3.

$$\frac{\left(15\,c^6x^6+45\,bc^5x^5-2\,b^6+26\,ab^4c-117\,a^2b^2c^2+195\,a^3c^3+5\left(8\,b^2c^4+13\,ac^5\right)x^4+5\left(b^3c^3+26\,abc^4\right)x^3-3\left(b^4c^2-\right.\right.}{195\,c^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/195*(15*c^6*x^6 + 45*b*c^5*x^5 - 2*b^6 + 26*a*b^4*c - 117*a^2*b^2*c^2 + 1
95*a^3*c^3 + 5*(8*b^2*c^4 + 13*a*c^5)*x^4 + 5*(b^3*c^3 + 26*a*b*c^4)*x^3 -
3*(b^4*c^2 - 13*a*b^2*c^3 - 39*a^2*c^4)*x^2 + (2*b^5*c - 26*a*b^3*c^2 + 117
*a^2*b*c^3)*x)*sqrt(2*c*d*x + b*d)/(c^4*d)
```

Sympy [A] time = 161.051, size = 1363, normalized size = 11.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(2*c*d*x+b*d)**(1/2),x)
```

```
[Out] Piecewise((-(a**3*b/sqrt(b*d + 2*c*d*x) + a**3*(-b*d/sqrt(b*d + 2*c*d*x) -
sqrt(b*d + 2*c*d*x))/d + 3*a**2*b**2*(-b*d/sqrt(b*d + 2*c*d*x) - sqrt(b*d +
 2*c*d*x))/(2*c*d) + 9*a**2*b*(b**2*d**2/sqrt(b*d + 2*c*d*x) + 2*b*d*sqrt(b
*d + 2*c*d*x) - (b*d + 2*c*d*x)**(3/2)/3)/(4*c*d**2) + 3*a**2*(-b**3*d**3/s
qrt(b*d + 2*c*d*x) - 3*b**2*d**2*sqrt(b*d + 2*c*d*x) + b*d*(b*d + 2*c*d*x)*
*(3/2) - (b*d + 2*c*d*x)**(5/2)/5)/(4*c*d**3) + 3*a*b**3*(b**2*d**2/sqrt(b*
d + 2*c*d*x) + 2*b*d*sqrt(b*d + 2*c*d*x) - (b*d + 2*c*d*x)**(3/2)/3)/(4*c**
2*d**2) + 3*a*b**2*(-b**3*d**3/sqrt(b*d + 2*c*d*x) - 3*b**2*d**2*sqrt(b*d +
 2*c*d*x) + b*d*(b*d + 2*c*d*x)**(3/2) - (b*d + 2*c*d*x)**(5/2)/5)/(2*c**2*
d**3) + 15*a*b*(b**4*d**4/sqrt(b*d + 2*c*d*x) + 4*b**3*d**3*sqrt(b*d + 2*c*
d*x) - 2*b**2*d**2*(b*d + 2*c*d*x)**(3/2) + 4*b*d*(b*d + 2*c*d*x)**(5/2)/5
- (b*d + 2*c*d*x)**(7/2)/7)/(16*c**2*d**4) + 3*a*(-b**5*d**5/sqrt(b*d + 2*c
*d*x) - 5*b**4*d**4*sqrt(b*d + 2*c*d*x) + 10*b**3*d**3*(b*d + 2*c*d*x)**(3/
2)/3 - 2*b**2*d**2*(b*d + 2*c*d*x)**(5/2) + 5*b*d*(b*d + 2*c*d*x)**(7/2)/7
- (b*d + 2*c*d*x)**(9/2)/9)/(16*c**2*d**5) + b**4*(-b**3*d**3/sqrt(b*d + 2*
c*d*x) - 3*b**2*d**2*sqrt(b*d + 2*c*d*x) + b*d*(b*d + 2*c*d*x)**(3/2) - (b*
d + 2*c*d*x)**(5/2)/5)/(8*c**3*d**3) + 5*b**3*(b**4*d**4/sqrt(b*d + 2*c*d*x
) + 4*b**3*d**3*sqrt(b*d + 2*c*d*x) - 2*b**2*d**2*(b*d + 2*c*d*x)**(3/2) +
4*b*d*(b*d + 2*c*d*x)**(5/2)/5 - (b*d + 2*c*d*x)**(7/2)/7)/(16*c**3*d**4) +
 9*b**2*(-b**5*d**5/sqrt(b*d + 2*c*d*x) - 5*b**4*d**4*sqrt(b*d + 2*c*d*x) +
 10*b**3*d**3*(b*d + 2*c*d*x)**(3/2)/3 - 2*b**2*d**2*(b*d + 2*c*d*x)**(5/2)
 + 5*b*d*(b*d + 2*c*d*x)**(7/2)/7 - (b*d + 2*c*d*x)**(9/2)/9)/(32*c**3*d**5
) + 7*b*(b**6*d**6/sqrt(b*d + 2*c*d*x) + 6*b**5*d**5*sqrt(b*d + 2*c*d*x) -
5*b**4*d**4*(b*d + 2*c*d*x)**(3/2) + 4*b**3*d**3*(b*d + 2*c*d*x)**(5/2) - 1
5*b**2*d**2*(b*d + 2*c*d*x)**(7/2)/7 + 2*b*d*(b*d + 2*c*d*x)**(9/2)/3 - (b*
d + 2*c*d*x)**(11/2)/11)/(64*c**3*d**6) + (-b**7*d**7/sqrt(b*d + 2*c*d*x) -
 7*b**6*d**6*sqrt(b*d + 2*c*d*x) + 7*b**5*d**5*(b*d + 2*c*d*x)**(3/2) - 7*b
**4*d**4*(b*d + 2*c*d*x)**(5/2) + 5*b**3*d**3*(b*d + 2*c*d*x)**(7/2) - 7*b*
*2*d**2*(b*d + 2*c*d*x)**(9/2)/3 + 7*b*d*(b*d + 2*c*d*x)**(11/2)/11 - (b*d
+ 2*c*d*x)**(13/2)/13)/(64*c**3*d**7))/c, Ne(c, 0)), (Piecewise((a**3*x, Eq
(b, 0)), ((a + b*x)**4/(4*b), True))/sqrt(b*d), True))
```

Giac [B] time = 1.17592, size = 1050, normalized size = 8.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^(1/2),x, algorithm="giac")
```

```
[Out] 1/960960*(960960*sqrt(2*c*d*x + b*d)*a^3 - 480480*(3*sqrt(2*c*d*x + b*d)*b*
d - (2*c*d*x + b*d)^(3/2))*a^2*b/(c*d) + 48048*(15*sqrt(2*c*d*x + b*d)*b^2*
d^2 - 10*(2*c*d*x + b*d)^(3/2)*b*d + 3*(2*c*d*x + b*d)^(5/2))*a*b^2/(c^2*d^
2) + 48048*(15*sqrt(2*c*d*x + b*d)*b^2*d^2 - 10*(2*c*d*x + b*d)^(3/2)*b*d +
 3*(2*c*d*x + b*d)^(5/2))*a^2/(c*d^2) - 3432*(35*sqrt(2*c*d*x + b*d)*b^3*d^
3 - 35*(2*c*d*x + b*d)^(3/2)*b^2*d^2 + 21*(2*c*d*x + b*d)^(5/2)*b*d - 5*(2*
c*d*x + b*d)^(7/2))*b^3/(c^3*d^3) - 20592*(35*sqrt(2*c*d*x + b*d)*b^3*d^3 -
 35*(2*c*d*x + b*d)^(3/2)*b^2*d^2 + 21*(2*c*d*x + b*d)^(5/2)*b*d - 5*(2*c*d
*x + b*d)^(7/2))*a*b/(c^2*d^3) + 572*(315*sqrt(2*c*d*x + b*d)*b^4*d^4 - 420
*(2*c*d*x + b*d)^(3/2)*b^3*d^3 + 378*(2*c*d*x + b*d)^(5/2)*b^2*d^2 - 180*(2
*c*d*x + b*d)^(7/2)*b*d + 35*(2*c*d*x + b*d)^(9/2))*b^2/(c^3*d^4) + 572*(31
5*sqrt(2*c*d*x + b*d)*b^4*d^4 - 420*(2*c*d*x + b*d)^(3/2)*b^3*d^3 + 378*(2*
c*d*x + b*d)^(5/2)*b^2*d^2 - 180*(2*c*d*x + b*d)^(7/2)*b*d + 35*(2*c*d*x +
b*d)^(9/2))*a/(c^2*d^4) - 130*(693*sqrt(2*c*d*x + b*d)*b^5*d^5 - 1155*(2*c*
d*x + b*d)^(3/2)*b^4*d^4 + 1386*(2*c*d*x + b*d)^(5/2)*b^3*d^3 - 990*(2*c*d*
x + b*d)^(7/2)*b^2*d^2 + 385*(2*c*d*x + b*d)^(9/2)*b*d - 63*(2*c*d*x + b*d)
^(11/2))*b/(c^3*d^5) + 5*(3003*sqrt(2*c*d*x + b*d)*b^6*d^6 - 6006*(2*c*d*x
+ b*d)^(3/2)*b^5*d^5 + 9009*(2*c*d*x + b*d)^(5/2)*b^4*d^4 - 8580*(2*c*d*x +
 b*d)^(7/2)*b^3*d^3 + 5005*(2*c*d*x + b*d)^(9/2)*b^2*d^2 - 1638*(2*c*d*x +
b*d)^(11/2)*b*d + 231*(2*c*d*x + b*d)^(13/2))/(c^3*d^6))/(c*d)
```

3.1279 $\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^{3/2}}\,dx$

Optimal. Leaf size=121

$$-\frac{3(b^2-4ac)(bd+2cdx)^{7/2}}{448c^4d^5}+\frac{(b^2-4ac)^2(bd+2cdx)^{3/2}}{64c^4d^3}+\frac{(b^2-4ac)^3}{64c^4d\sqrt{bd+2cdx}}+\frac{(bd+2cdx)^{11/2}}{704c^4d^7}$$

```
[Out] (b^2 - 4*a*c)^3/(64*c^4*d*Sqrt[b*d + 2*c*d*x]) + ((b^2 - 4*a*c)^2*(b*d + 2*
c*d*x)^(3/2))/(64*c^4*d^3) - (3*(b^2 - 4*a*c)*(b*d + 2*c*d*x)^(7/2))/(448*c
^4*d^5) + (b*d + 2*c*d*x)^(11/2)/(704*c^4*d^7)
```

Rubi [A] time = 0.049329, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {683}

$$-\frac{3(b^2-4ac)(bd+2cdx)^{7/2}}{448c^4d^5}+\frac{(b^2-4ac)^2(bd+2cdx)^{3/2}}{64c^4d^3}+\frac{(b^2-4ac)^3}{64c^4d\sqrt{bd+2cdx}}+\frac{(bd+2cdx)^{11/2}}{704c^4d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^(3/2),x]
```

```
[Out] (b^2 - 4*a*c)^3/(64*c^4*d*Sqrt[b*d + 2*c*d*x]) + ((b^2 - 4*a*c)^2*(b*d + 2*
c*d*x)^(3/2))/(64*c^4*d^3) - (3*(b^2 - 4*a*c)*(b*d + 2*c*d*x)^(7/2))/(448*c
^4*d^5) + (b*d + 2*c*d*x)^(11/2)/(704*c^4*d^7)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^{3/2}}\,dx=\int \left(\frac{(-b^2+4ac)^3}{64c^3(bd+2cdx)^{3/2}}+\frac{3(-b^2+4ac)^2\sqrt{bd+2cdx}}{64c^3d^2}+\frac{3(-b^2+4ac)(bd+2cdx)^{5/2}}{64c^3d^4}+\frac{(bd+2c}{64c^3}$$
$$=\frac{(b^2-4ac)^3}{64c^4d\sqrt{bd+2cdx}}+\frac{(b^2-4ac)^2(bd+2cdx)^{3/2}}{64c^4d^3}-\frac{3(b^2-4ac)(bd+2cdx)^{7/2}}{448c^4d^5}+\frac{(bd+2cdx)^{11/2}}{704c^4d^7}$$

Mathematica [A] time = 0.0681235, size = 83, normalized size = 0.69

$$\frac{-33(b^2-4ac)(b+2cx)^4+77(b^2-4ac)^2(b+2cx)^2+77(b^2-4ac)^3+7(b+2cx)^6}{4928c^4d\sqrt{d(b+2cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^(3/2),x]
```

```
[Out] (77*(b^2 - 4*a*c)^3 + 77*(b^2 - 4*a*c)^2*(b + 2*c*x)^2 - 33*(b^2 - 4*a*c)*(
b + 2*c*x)^4 + 7*(b + 2*c*x)^6)/(4928*c^4*d*Sqrt[d*(b + 2*c*x)])
```

Maple [A] time = 0.044, size = 173, normalized size = 1.4

$$-\frac{(2\,cx+b)\left(-7\,c^6x^6-21\,bc^5x^5-33\,ac^5x^4-18\,b^2c^4x^4-66\,abc^4x^3-b^3c^3x^3-77\,a^2c^4x^2-11\,ab^2c^3x^2+b^4c^2x^2-77\,a^2b\right.}{77\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^(3/2),x)
```

```
[Out] -1/77*(2*c*x+b)*(-7*c^6*x^6-21*b*c^5*x^5-33*a*c^5*x^4-18*b^2*c^4*x^4-66*a*b
*c^4*x^3-b^3*c^3*x^3-77*a^2*c^4*x^2-11*a*b^2*c^3*x^2+b^4*c^2*x^2-77*a^2*b*c
^3*x+22*a*b^3*c^2*x-2*b^5*c*x+77*a^3*c^3-77*a^2*b^2*c^2+22*a*b^4*c-2*b^6)/c
^4/(2*c*d*x+b*d)^(3/2)
```

Maxima [A] time = 1.07999, size = 184, normalized size = 1.52

$$\frac{\frac{77\left(b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3\right)}{\sqrt{2\,cdx+bd}c^3}-\frac{33\,(2\,cdx+bd)^{\frac{7}{2}}\left(b^2-4\,ac\right)d^2-77\left(b^4-8\,ab^2c+16\,a^2c^2\right)(2\,cdx+bd)^{\frac{3}{2}}d^4-7\,(2\,cdx+bd)^{\frac{11}{2}}}{c^3d^6}}{4928\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4928*(77*(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)/(sqrt(2*c*d*x +
 b*d)*c^3) - (33*(2*c*d*x + b*d)^(7/2)*(b^2 - 4*a*c)*d^2 - 77*(b^4 - 8*a*b^
2*c + 16*a^2*c^2)*(2*c*d*x + b*d)^(3/2)*d^4 - 7*(2*c*d*x + b*d)^(11/2))/(c^
3*d^6))/(c*d)
```

Fricas [A] time = 2.06958, size = 375, normalized size = 3.1

$$\frac{\left(7\,c^6x^6+21\,bc^5x^5+2\,b^6-22\,ab^4c+77\,a^2b^2c^2-77\,a^3c^3+3\left(6\,b^2c^4+11\,ac^5\right)x^4+\left(b^3c^3+66\,abc^4\right)x^3-\left(b^4c^2-11\,ab^2c\right.\right.}{77\left(2\,c^5d^2x+bc^4d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/77*(7*c^6*x^6 + 21*b*c^5*x^5 + 2*b^6 - 22*a*b^4*c + 77*a^2*b^2*c^2 - 77*a
^3*c^3 + 3*(6*b^2*c^4 + 11*a*c^5)*x^4 + (b^3*c^3 + 66*a*b*c^4)*x^3 - (b^4*c
^2 - 11*a*b^2*c^3 - 77*a^2*c^4)*x^2 + (2*b^5*c - 22*a*b^3*c^2 + 77*a^2*b*c^
3)*x)*sqrt(2*c*d*x + b*d)/(2*c^5*d^2*x + b*c^4*d^2)
```

Sympy [A] time = 68.8422, size = 128, normalized size = 1.06

$$-\frac{\left(4ac-b^2\right)^3}{64c^4d\sqrt{bd+2cdx}}+\frac{(bd+2cdx)^{\frac{3}{2}}\left(48a^2c^2-24ab^2c+3b^4\right)}{192c^4d^3}+\frac{\left(12ac-3b^2\right)(bd+2cdx)^{\frac{7}{2}}}{448c^4d^5}+\frac{(bd+2cdx)^{\frac{11}{2}}}{704c^4d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(2*c*d*x+b*d)**(3/2),x)
```

```
[Out] -(4*a*c - b**2)**3/(64*c**4*d*sqrt(b*d + 2*c*d*x)) + (b*d + 2*c*d*x)**(3/2)
*(48*a**2*c**2 - 24*a*b**2*c + 3*b**4)/(192*c**4*d**3) + (12*a*c - 3*b**2)*
(b*d + 2*c*d*x)**(7/2)/(448*c**4*d**5) + (b*d + 2*c*d*x)**(11/2)/(704*c**4*
d**7)
```

Giac [A] time = 1.16593, size = 252, normalized size = 2.08

$$\frac{b^6 - 12\,ab^4c + 48\,a^2b^2c^2 - 64\,a^3c^3}{64\,\sqrt{2\,cdx + bd}c^4d} + \frac{77\,(2\,cdx + bd)^{\frac{3}{2}}b^4c^{40}d^{74} - 616\,(2\,cdx + bd)^{\frac{3}{2}}ab^2c^{41}d^{74} + 1232\,(2\,cdx + bd)^{\frac{3}{2}}a^2c^{42}d^{74}}{4928\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^(3/2),x, algorithm="giac")
```

```
[Out] 1/64*(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)/(sqrt(2*c*d*x + b*d)*
c^4*d) + 1/4928*(77*(2*c*d*x + b*d)^(3/2)*b^4*c^40*d^74 - 616*(2*c*d*x + b*
d)^(3/2)*a*b^2*c^41*d^74 + 1232*(2*c*d*x + b*d)^(3/2)*a^2*c^42*d^74 - 33*(2
*c*d*x + b*d)^(7/2)*b^2*c^40*d^72 + 132*(2*c*d*x + b*d)^(7/2)*a*c^41*d^72 +
 7*(2*c*d*x + b*d)^(11/2)*c^40*d^70)/(c^44*d^77)
```

3.1280 $\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^{5/2}}\,dx$

Optimal. Leaf size=121

$$-\frac{3\left(b^2-4ac\right)(bd+2cdx)^{5/2}}{320c^4d^5}+\frac{3\left(b^2-4ac\right)^2\sqrt{bd+2cdx}}{64c^4d^3}+\frac{\left(b^2-4ac\right)^3}{192c^4d(bd+2cdx)^{3/2}}+\frac{(bd+2cdx)^{9/2}}{576c^4d^7}$$

```
[Out] (b^2 - 4*a*c)^3/(192*c^4*d*(b*d + 2*c*d*x)^(3/2)) + (3*(b^2 - 4*a*c)^2*Sqrt
[b*d + 2*c*d*x])/(64*c^4*d^3) - (3*(b^2 - 4*a*c)*(b*d + 2*c*d*x)^(5/2))/(32
0*c^4*d^5) + (b*d + 2*c*d*x)^(9/2)/(576*c^4*d^7)
```

Rubi [A] time = 0.049903, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {683}

$$-\frac{3\left(b^2-4ac\right)(bd+2cdx)^{5/2}}{320c^4d^5}+\frac{3\left(b^2-4ac\right)^2\sqrt{bd+2cdx}}{64c^4d^3}+\frac{\left(b^2-4ac\right)^3}{192c^4d(bd+2cdx)^{3/2}}+\frac{(bd+2cdx)^{9/2}}{576c^4d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^(5/2),x]
```

```
[Out] (b^2 - 4*a*c)^3/(192*c^4*d*(b*d + 2*c*d*x)^(3/2)) + (3*(b^2 - 4*a*c)^2*Sqrt
[b*d + 2*c*d*x])/(64*c^4*d^3) - (3*(b^2 - 4*a*c)*(b*d + 2*c*d*x)^(5/2))/(32
0*c^4*d^5) + (b*d + 2*c*d*x)^(9/2)/(576*c^4*d^7)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] && !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx+cx^2\right)^3}{(bd+2cdx)^{5/2}}\,dx &= \int \left(\frac{\left(-b^2+4ac\right)^3}{64c^3(bd+2cdx)^{5/2}}+\frac{3\left(-b^2+4ac\right)^2}{64c^3d^2\sqrt{bd+2cdx}}+\frac{3\left(-b^2+4ac\right)(bd+2cdx)^{3/2}}{64c^3d^4}+\frac{(bd+2cdx)^{7/2}}{64c^3d^6}\right.\\ &= \frac{\left(b^2-4ac\right)^3}{192c^4d(bd+2cdx)^{3/2}}+\frac{3\left(b^2-4ac\right)^2\sqrt{bd+2cdx}}{64c^4d^3}-\frac{3\left(b^2-4ac\right)(bd+2cdx)^{5/2}}{320c^4d^5}+\frac{(bd+2cdx}{576c^4d^7}\end{aligned}$$

Mathematica [A] time = 0.0692922, size = 83, normalized size = 0.69

$$\frac{-27\left(b^2-4ac\right)(b+2cx)^4+135\left(b^2-4ac\right)^2(b+2cx)^2+15\left(b^2-4ac\right)^3+5(b+2cx)^6}{2880c^4d(d(b+2cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^(5/2),x]
```

```
[Out] (15*(b^2 - 4*a*c)^3 + 135*(b^2 - 4*a*c)^2*(b + 2*c*x)^2 - 27*(b^2 - 4*a*c)*
(b + 2*c*x)^4 + 5*(b + 2*c*x)^6)/(2880*c^4*d*(d*(b + 2*c*x))^(3/2))
```

Maple [A] time = 0.043, size = 173, normalized size = 1.4

$$-\frac{(2\,cx+b)\left(-5\,c^6x^6-15\,bc^5x^5-27\,ac^5x^4-12\,b^2c^4x^4-54\,abc^4x^3+b^3c^3x^3-135\,a^2c^4x^2+27\,ab^2c^3x^2-3\,b^4c^2x^2-135\,a^2\right.}{45\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^(5/2),x)
```

```
[Out] -1/45*(2*c*x+b)*(-5*c^6*x^6-15*b*c^5*x^5-27*a*c^5*x^4-12*b^2*c^4*x^4-54*a*b
*c^4*x^3+b^3*c^3*x^3-135*a^2*c^4*x^2+27*a*b^2*c^3*x^2-3*b^4*c^2*x^2-135*a^2
*b*c^3*x+54*a*b^3*c^2*x-6*b^5*c*x+15*a^3*c^3-45*a^2*b^2*c^2+18*a*b^4*c-2*b^
6)/c^4/(2*c*d*x+b*d)^(5/2)
```

Maxima [A] time = 1.04897, size = 184, normalized size = 1.52

$$\frac{\frac{15\left(b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3\right)}{(2\,cdx+bd)^{\frac{3}{2}}c^3}-\frac{27\,(2\,cdx+bd)^{\frac{5}{2}}\left(b^2-4\,ac\right)d^2-135\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{2\,cdx+bd}d^4-5\,(2\,cdx+bd)^{\frac{9}{2}}}{c^3d^6}}{2880\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2880*(15*(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)/((2*c*d*x + b*d
)^(3/2)*c^3) - (27*(2*c*d*x + b*d)^(5/2)*(b^2 - 4*a*c)*d^2 - 135*(b^4 - 8*a
*b^2*c + 16*a^2*c^2)*sqrt(2*c*d*x + b*d)*d^4 - 5*(2*c*d*x + b*d)^(9/2))/(c^
3*d^6))/(c*d)
```

Fricas [A] time = 2.10276, size = 405, normalized size = 3.35

$$\frac{\left(5\,c^6x^6+15\,bc^5x^5+2\,b^6-18\,ab^4c+45\,a^2b^2c^2-15\,a^3c^3+3\left(4\,b^2c^4+9\,ac^5\right)x^4-\left(b^3c^3-54\,abc^4\right)x^3+3\left(b^4c^2-9\,ab^2c^3+\right.\right.}{45\left(4\,c^6d^3x^2+4\,bc^5d^3x+b^2c^4d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/45*(5*c^6*x^6 + 15*b*c^5*x^5 + 2*b^6 - 18*a*b^4*c + 45*a^2*b^2*c^2 - 15*a
^3*c^3 + 3*(4*b^2*c^4 + 9*a*c^5)*x^4 - (b^3*c^3 - 54*a*b*c^4)*x^3 + 3*(b^4*
c^2 - 9*a*b^2*c^3 + 45*a^2*c^4)*x^2 + 3*(2*b^5*c - 18*a*b^3*c^2 + 45*a^2*b*
c^3)*x)*sqrt(2*c*d*x + b*d)/(4*c^6*d^3*x^2 + 4*b*c^5*d^3*x + b^2*c^4*d^3)
```

Sympy [A] time = 95.0958, size = 128, normalized size = 1.06

$$-\frac{\left(4ac-b^2\right)^3}{192c^4d\,(bd+2cdx)^{\frac{3}{2}}}+\frac{\sqrt{bd+2cdx}\left(48a^2c^2-24ab^2c+3b^4\right)}{64c^4d^3}+\frac{\left(12ac-3b^2\right)(bd+2cdx)^{\frac{5}{2}}}{320c^4d^5}+\frac{(bd+2cdx)^{\frac{9}{2}}}{576c^4d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(2*c*d*x+b*d)**(5/2),x)
```

```
[Out] -(4*a*c - b**2)**3/(192*c**4*d*(b*d + 2*c*d*x)**(3/2)) + sqrt(b*d + 2*c*d*x
)*(48*a**2*c**2 - 24*a*b**2*c + 3*b**4)/(64*c**4*d**3) + (12*a*c - 3*b**2)*
(b*d + 2*c*d*x)**(5/2)/(320*c**4*d**5) + (b*d + 2*c*d*x)**(9/2)/(576*c**4*d
**7)
```

Giac [A] time = 1.18633, size = 252, normalized size = 2.08

$$\frac{b^6 - 12\,ab^4c + 48\,a^2b^2c^2 - 64\,a^3c^3}{192\,(2\,cdx + bd)^{\frac{3}{2}}c^4d} + \frac{135\,\sqrt{2\,cdx + bd}b^4c^{32}d^{60} - 1080\,\sqrt{2\,cdx + bd}ab^2c^{33}d^{60} + 2160\,\sqrt{2\,cdx + bd}a^2c^{34}d}{2880}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^(5/2),x, algorithm="giac")
```

```
[Out] 1/192*(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)/((2*c*d*x + b*d)^(3/
2)*c^4*d) + 1/2880*(135*sqrt(2*c*d*x + b*d)*b^4*c^32*d^60 - 1080*sqrt(2*c*d
*x + b*d)*a*b^2*c^33*d^60 + 2160*sqrt(2*c*d*x + b*d)*a^2*c^34*d^60 - 27*(2*
c*d*x + b*d)^(5/2)*b^2*c^32*d^58 + 108*(2*c*d*x + b*d)^(5/2)*a*c^33*d^58 +
5*(2*c*d*x + b*d)^(9/2)*c^32*d^56)/(c^36*d^63)
```

3.1281 $\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^{7/2}}\,dx$

Optimal. Leaf size=121

$$-\frac{(b^2-4ac)(bd+2cdx)^{3/2}}{64c^4d^5}-\frac{3(b^2-4ac)^2}{64c^4d^3\sqrt{bd+2cdx}}+\frac{(b^2-4ac)^3}{320c^4d(bd+2cdx)^{5/2}}+\frac{(bd+2cdx)^{7/2}}{448c^4d^7}$$

```
[Out] (b^2 - 4*a*c)^3/(320*c^4*d*(b*d + 2*c*d*x)^(5/2)) - (3*(b^2 - 4*a*c)^2)/(64
*c^4*d^3*Sqrt[b*d + 2*c*d*x]) - ((b^2 - 4*a*c)*(b*d + 2*c*d*x)^(3/2))/(64*c
^4*d^5) + (b*d + 2*c*d*x)^(7/2)/(448*c^4*d^7)
```

Rubi [A] time = 0.048796, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {683}

$$-\frac{(b^2-4ac)(bd+2cdx)^{3/2}}{64c^4d^5}-\frac{3(b^2-4ac)^2}{64c^4d^3\sqrt{bd+2cdx}}+\frac{(b^2-4ac)^3}{320c^4d(bd+2cdx)^{5/2}}+\frac{(bd+2cdx)^{7/2}}{448c^4d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^(7/2),x]
```

```
[Out] (b^2 - 4*a*c)^3/(320*c^4*d*(b*d + 2*c*d*x)^(5/2)) - (3*(b^2 - 4*a*c)^2)/(64
*c^4*d^3*Sqrt[b*d + 2*c*d*x]) - ((b^2 - 4*a*c)*(b*d + 2*c*d*x)^(3/2))/(64*c
^4*d^5) + (b*d + 2*c*d*x)^(7/2)/(448*c^4*d^7)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^{7/2}}\,dx &= \int \left(\frac{(-b^2+4ac)^3}{64c^3(bd+2cdx)^{7/2}}+\frac{3(-b^2+4ac)^2}{64c^3d^2(bd+2cdx)^{3/2}}+\frac{3(-b^2+4ac)\sqrt{bd+2cdx}}{64c^3d^4}+\frac{(bd+2cdx)^{5/2}}{64c^3d^6}\right) d\\ &= \frac{(b^2-4ac)^3}{320c^4d(bd+2cdx)^{5/2}}-\frac{3(b^2-4ac)^2}{64c^4d^3\sqrt{bd+2cdx}}-\frac{(b^2-4ac)(bd+2cdx)^{3/2}}{64c^4d^5}+\frac{(bd+2cdx)^{7/2}}{448c^4d^7}\end{aligned}$$

Mathematica [A] time = 0.0756387, size = 83, normalized size = 0.69

$$\frac{-35(b^2-4ac)(b+2cx)^4-105(b^2-4ac)^2(b+2cx)^2+7(b^2-4ac)^3+5(b+2cx)^6}{2240c^4d(d(b+2cx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^(7/2),x]
```

```
[Out] (7*(b^2 - 4*a*c)^3 - 105*(b^2 - 4*a*c)^2*(b + 2*c*x)^2 - 35*(b^2 - 4*a*c)*(
b + 2*c*x)^4 + 5*(b + 2*c*x)^6)/(2240*c^4*d*(d*(b + 2*c*x))^(5/2))
```

Maple [A] time = 0.047, size = 174, normalized size = 1.4

$$-\frac{(2cx+b)\left(-5c^6x^6-15bc^5x^5-35ac^5x^4-10b^2c^4x^4-70abc^4x^3+5b^3c^3x^3+105a^2c^4x^2-105ab^2c^3x^2+15b^4c^2x^2 \cdots\right.}{35c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^(7/2),x)
```

```
[Out] -1/35*(2*c*x+b)*(-5*c^6*x^6-15*b*c^5*x^5-35*a*c^5*x^4-10*b^2*c^4*x^4-70*a*b
*c^4*x^3+5*b^3*c^3*x^3+105*a^2*c^4*x^2-105*a*b^2*c^3*x^2+15*b^4*c^2*x^2+105
*a^2*b*c^3*x-70*a*b^3*c^2*x+10*b^5*c*x+7*a^3*c^3+21*a^2*b^2*c^2-14*a*b^4*c+
2*b^6)/c^4/(2*c*d*x+b*d)^(7/2)
```

Maxima [A] time = 1.0159, size = 192, normalized size = 1.59

$$-\frac{\frac{7\left(15\left(b^4-8ab^2c+16a^2c^2\right)(2cdx+bd)^2-\left(b^6-12ab^4c+48a^2b^2c^2-64a^3c^3\right)d^2\right)}{(2cdx+bd)^{\frac{5}{2}}c^3d^2}+\frac{5\left(7(2cdx+bd)^{\frac{3}{2}}\left(b^2-4ac\right)d^2-(2cdx+bd)^{\frac{7}{2}}\right)}{c^3d^6}}{2240cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/2240*(7*(15*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*(2*c*d*x + b*d)^2 - (b^6 - 12
*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*d^2)/((2*c*d*x + b*d)^(5/2)*c^3*d^2
) + 5*(7*(2*c*d*x + b*d)^(3/2)*(b^2 - 4*a*c)*d^2 - (2*c*d*x + b*d)^(7/2))/(
c^3*d^6))/(c*d)
```

Fricas [A] time = 1.95558, size = 435, normalized size = 3.6

$$\frac{\left(5c^6x^6+15bc^5x^5-2b^6+14ab^4c-21a^2b^2c^2-7a^3c^3+5\left(2b^2c^4+7ac^5\right)x^4-5\left(b^3c^3-14abc^4\right)x^3-15\left(b^4c^2-7ab^2 \cdots\right.\right.}{35\left(8c^7d^4x^3+12bc^6d^4x^2+6b^2c^5d^4x+b^3c^4d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/35*(5*c^6*x^6 + 15*b*c^5*x^5 - 2*b^6 + 14*a*b^4*c - 21*a^2*b^2*c^2 - 7*a^
3*c^3 + 5*(2*b^2*c^4 + 7*a*c^5)*x^4 - 5*(b^3*c^3 - 14*a*b*c^4)*x^3 - 15*(b^
4*c^2 - 7*a*b^2*c^3 + 7*a^2*c^4)*x^2 - 5*(2*b^5*c - 14*a*b^3*c^2 + 21*a^2*b
*c^3)*x)*sqrt(2*c*d*x + b*d)/(8*c^7*d^4*x^3 + 12*b*c^6*d^4*x^2 + 6*b^2*c^5*
d^4*x + b^3*c^4*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(2*c*d*x+b*d)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17302, size = 251, normalized size = 2.07

$$\frac{b^6d^2 - 12\,ab^4cd^2 + 48\,a^2b^2c^2d^2 - 64\,a^3c^3d^2 - 15\,(2\,cdx + bd)^2b^4 + 120\,(2\,cdx + bd)^2ab^2c - 240\,(2\,cdx + bd)^2a^2c^2}{320\,(2\,cdx + bd)^{\frac{5}{2}}c^4d^3} - \frac{7\,(2\,cd}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^(7/2),x, algorithm="giac")
```

```
[Out] 1/320*(b^6*d^2 - 12*a*b^4*c*d^2 + 48*a^2*b^2*c^2*d^2 - 64*a^3*c^3*d^2 - 15*
(2*c*d*x + b*d)^2*b^4 + 120*(2*c*d*x + b*d)^2*a*b^2*c - 240*(2*c*d*x + b*d)
^2*a^2*c^2)/((2*c*d*x + b*d)^(5/2)*c^4*d^3) - 1/448*(7*(2*c*d*x + b*d)^(3/2
)*b^2*c^24*d^44 - 28*(2*c*d*x + b*d)^(3/2)*a*c^25*d^44 - (2*c*d*x + b*d)^(7
/2)*c^24*d^42)/(c^28*d^49)
```

3.1282 $\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^{9/2}} dx$

Optimal. Leaf size=121

$$-\frac{(b^2-4ac)^2}{64c^4d^3(bd+2cdx)^{3/2}}-\frac{3(b^2-4ac)\sqrt{bd+2cdx}}{64c^4d^5}+\frac{(b^2-4ac)^3}{448c^4d(bd+2cdx)^{7/2}}+\frac{(bd+2cdx)^{5/2}}{320c^4d^7}$$

```
[Out] (b^2 - 4*a*c)^3/(448*c^4*d*(b*d + 2*c*d*x)^(7/2)) - (b^2 - 4*a*c)^2/(64*c^4
*d^3*(b*d + 2*c*d*x)^(3/2)) - (3*(b^2 - 4*a*c)*Sqrt[b*d + 2*c*d*x])/(64*c^4
*d^5) + (b*d + 2*c*d*x)^(5/2)/(320*c^4*d^7)
```

Rubi [A] time = 0.0504371, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {683}

$$-\frac{(b^2-4ac)^2}{64c^4d^3(bd+2cdx)^{3/2}}-\frac{3(b^2-4ac)\sqrt{bd+2cdx}}{64c^4d^5}+\frac{(b^2-4ac)^3}{448c^4d(bd+2cdx)^{7/2}}+\frac{(bd+2cdx)^{5/2}}{320c^4d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^(9/2),x]
```

```
[Out] (b^2 - 4*a*c)^3/(448*c^4*d*(b*d + 2*c*d*x)^(7/2)) - (b^2 - 4*a*c)^2/(64*c^4
*d^3*(b*d + 2*c*d*x)^(3/2)) - (3*(b^2 - 4*a*c)*Sqrt[b*d + 2*c*d*x])/(64*c^4
*d^5) + (b*d + 2*c*d*x)^(5/2)/(320*c^4*d^7)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^{9/2}} dx &= \int \left(\frac{(-b^2+4ac)^3}{64c^3(bd+2cdx)^{9/2}}+\frac{3(-b^2+4ac)^2}{64c^3d^2(bd+2cdx)^{5/2}}+\frac{3(-b^2+4ac)}{64c^3d^4\sqrt{bd+2cdx}}+\frac{(bd+2cdx)^{3/2}}{64c^3d^6}\right) dx\\ &= \frac{(b^2-4ac)^3}{448c^4d(bd+2cdx)^{7/2}}-\frac{(b^2-4ac)^2}{64c^4d^3(bd+2cdx)^{3/2}}-\frac{3(b^2-4ac)\sqrt{bd+2cdx}}{64c^4d^5}+\frac{(bd+2cdx)^{5/2}}{320c^4d^7}\end{aligned}$$

Mathematica [A] time = 0.0701657, size = 83, normalized size = 0.69

$$\frac{-105(b^2-4ac)(b+2cx)^4-35(b^2-4ac)^2(b+2cx)^2+5(b^2-4ac)^3+7(b+2cx)^6}{2240c^4d(d(b+2cx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^(9/2),x]
```

```
[Out] (5*(b^2 - 4*a*c)^3 - 35*(b^2 - 4*a*c)^2*(b + 2*c*x)^2 - 105*(b^2 - 4*a*c)*(
b + 2*c*x)^4 + 7*(b + 2*c*x)^6)/(2240*c^4*d*(d*(b + 2*c*x))^(7/2))
```

Maple [A] time = 0.045, size = 163, normalized size = 1.4

$$-\frac{(2\,cx+b)\left(-7\,c^6x^6-21\,bc^5x^5-105\,ac^5x^4-210\,abc^4x^3+35\,b^3c^3x^3+35\,a^2c^4x^2-175\,ab^2c^3x^2+35\,b^4c^2x^2+35\,a^2bc^3x-\right.}{35\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^(9/2),x)
```

```
[Out] -1/35*(2*c*x+b)*(-7*c^6*x^6-21*b*c^5*x^5-105*a*c^5*x^4-210*a*b*c^4*x^3+35*b
^3*c^3*x^3+35*a^2*c^4*x^2-175*a*b^2*c^3*x^2+35*b^4*c^2*x^2+35*a^2*b*c^3*x-7
0*a*b^3*c^2*x+14*b^5*c*x+5*a^3*c^3+5*a^2*b^2*c^2-10*a*b^4*c+2*b^6)/c^4/(2*c
*d*x+b*d)^(9/2)
```

Maxima [A] time = 1.14639, size = 192, normalized size = 1.59

$$-\frac{\frac{5\left(7\left(b^4-8\,ab^2c+16\,a^2c^2\right)(2\,cdx+bd)^2-\left(b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3\right)d^2\right)}{(2\,cdx+bd)^{\frac{7}{2}}c^3d^2}+\frac{7\left(15\,\sqrt{2\,cdx+bd}\left(b^2-4\,ac\right)d^2-(2\,cdx+bd)^{\frac{5}{2}}\right)}{c^3d^6}}{2240\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/2240*(5*(7*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*(2*c*d*x + b*d)^2 - (b^6 - 12*
a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*d^2)/((2*c*d*x + b*d)^(7/2)*c^3*d^2)
 + 7*(15*sqrt(2*c*d*x + b*d)*(b^2 - 4*a*c)*d^2 - (2*c*d*x + b*d)^(5/2))/(c^
3*d^6))/(c*d)
```

Fricas [A] time = 2.07601, size = 440, normalized size = 3.64

$$\frac{\left(7\,c^6x^6+21\,bc^5x^5+105\,ac^5x^4-2\,b^6+10\,ab^4c-5\,a^2b^2c^2-5\,a^3c^3-35\left(b^3c^3-6\,abc^4\right)x^3-35\left(b^4c^2-5\,ab^2c^3+a^2c^4\right)x^2-\right.}{35\left(16\,c^8d^5x^4+32\,bc^7d^5x^3+24\,b^2c^6d^5x^2+8\,b^3c^5d^5x+b^4c^4d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/35*(7*c^6*x^6 + 21*b*c^5*x^5 + 105*a*c^5*x^4 - 2*b^6 + 10*a*b^4*c - 5*a^2
*b^2*c^2 - 5*a^3*c^3 - 35*(b^3*c^3 - 6*a*b*c^4)*x^3 - 35*(b^4*c^2 - 5*a*b^2
*c^3 + a^2*c^4)*x^2 - 7*(2*b^5*c - 10*a*b^3*c^2 + 5*a^2*b*c^3)*x)*sqrt(2*c*
d*x + b*d)/(16*c^8*d^5*x^4 + 32*b*c^7*d^5*x^3 + 24*b^2*c^6*d^5*x^2 + 8*b^3*
c^5*d^5*x + b^4*c^4*d^5)
```

Sympy [A] time = 15.2915, size = 1394, normalized size = 11.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(2*c*d*x+b*d)**(9/2),x)
```

```
[Out] Piecewise((-5*a**3*c**3*sqrt(b*d + 2*c*d*x)/(35*b**4*c**4*d**5 + 280*b**3*c
**5*d**5*x + 840*b**2*c**6*d**5*x**2 + 1120*b*c**7*d**5*x**3 + 560*c**8*d**
5*x**4) - 5*a**2*b**2*c**2*sqrt(b*d + 2*c*d*x)/(35*b**4*c**4*d**5 + 280*b**
3*c**5*d**5*x + 840*b**2*c**6*d**5*x**2 + 1120*b*c**7*d**5*x**3 + 560*c**8*
d**5*x**4) - 35*a**2*b*c**3*x*sqrt(b*d + 2*c*d*x)/(35*b**4*c**4*d**5 + 280*
b**3*c**5*d**5*x + 840*b**2*c**6*d**5*x**2 + 1120*b*c**7*d**5*x**3 + 560*c*
*8*d**5*x**4) - 35*a**2*c**4*x**2*sqrt(b*d + 2*c*d*x)/(35*b**4*c**4*d**5 +
280*b**3*c**5*d**5*x + 840*b**2*c**6*d**5*x**2 + 1120*b*c**7*d**5*x**3 + 56
0*c**8*d**5*x**4) + 10*a*b**4*c*sqrt(b*d + 2*c*d*x)/(35*b**4*c**4*d**5 + 28
0*b**3*c**5*d**5*x + 840*b**2*c**6*d**5*x**2 + 1120*b*c**7*d**5*x**3 + 560*
c**8*d**5*x**4) + 70*a*b**3*c**2*x*sqrt(b*d + 2*c*d*x)/(35*b**4*c**4*d**5 +
 280*b**3*c**5*d**5*x + 840*b**2*c**6*d**5*x**2 + 1120*b*c**7*d**5*x**3 + 5
60*c**8*d**5*x**4) + 175*a*b**2*c**3*x**2*sqrt(b*d + 2*c*d*x)/(35*b**4*c**4
*d**5 + 280*b**3*c**5*d**5*x + 840*b**2*c**6*d**5*x**2 + 1120*b*c**7*d**5*x
**3 + 560*c**8*d**5*x**4) + 210*a*b*c**4*x**3*sqrt(b*d + 2*c*d*x)/(35*b**4*
c**4*d**5 + 280*b**3*c**5*d**5*x + 840*b**2*c**6*d**5*x**2 + 1120*b*c**7*d*
*5*x**3 + 560*c**8*d**5*x**4) + 105*a*c**5*x**4*sqrt(b*d + 2*c*d*x)/(35*b**
4*c**4*d**5 + 280*b**3*c**5*d**5*x + 840*b**2*c**6*d**5*x**2 + 1120*b*c**7*
d**5*x**3 + 560*c**8*d**5*x**4) - 2*b**6*sqrt(b*d + 2*c*d*x)/(35*b**4*c**4*
d**5 + 280*b**3*c**5*d**5*x + 840*b**2*c**6*d**5*x**2 + 1120*b*c**7*d**5*x*
*3 + 560*c**8*d**5*x**4) - 14*b**5*c*x*sqrt(b*d + 2*c*d*x)/(35*b**4*c**4*d*
*5 + 280*b**3*c**5*d**5*x + 840*b**2*c**6*d**5*x**2 + 1120*b*c**7*d**5*x**3
 + 560*c**8*d**5*x**4) - 35*b**4*c**2*x**2*sqrt(b*d + 2*c*d*x)/(35*b**4*c**
4*d**5 + 280*b**3*c**5*d**5*x + 840*b**2*c**6*d**5*x**2 + 1120*b*c**7*d**5*
x**3 + 560*c**8*d**5*x**4) - 35*b**3*c**3*x**3*sqrt(b*d + 2*c*d*x)/(35*b**4
*c**4*d**5 + 280*b**3*c**5*d**5*x + 840*b**2*c**6*d**5*x**2 + 1120*b*c**7*d
**5*x**3 + 560*c**8*d**5*x**4) + 21*b*c**5*x**5*sqrt(b*d + 2*c*d*x)/(35*b**
4*c**4*d**5 + 280*b**3*c**5*d**5*x + 840*b**2*c**6*d**5*x**2 + 1120*b*c**7*
d**5*x**3 + 560*c**8*d**5*x**4) + 7*c**6*x**6*sqrt(b*d + 2*c*d*x)/(35*b**4*
c**4*d**5 + 280*b**3*c**5*d**5*x + 840*b**2*c**6*d**5*x**2 + 1120*b*c**7*d*
*5*x**3 + 560*c**8*d**5*x**4), Ne(c, 0)), ((a**3*x + 3*a**2*b*x**2/2 + a*b*
*2*x**3 + b**3*x**4/4)/(b*d)**(9/2), True))
```

Giac [A] time = 1.20658, size = 251, normalized size = 2.07

$$\frac{b^6d^2 - 12\,ab^4cd^2 + 48\,a^2b^2c^2d^2 - 64\,a^3c^3d^2 - 7\,(2\,cdx + bd)^2b^4 + 56\,(2\,cdx + bd)^2ab^2c - 112\,(2\,cdx + bd)^2a^2c^2}{448\,(2\,cdx + bd)^{\frac{7}{2}}c^4d^3} - \frac{15\,\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^(9/2),x, algorithm="giac")
```

```
[Out] 1/448*(b^6*d^2 - 12*a*b^4*c*d^2 + 48*a^2*b^2*c^2*d^2 - 64*a^3*c^3*d^2 - 7*(
2*c*d*x + b*d)^2*b^4 + 56*(2*c*d*x + b*d)^2*a*b^2*c - 112*(2*c*d*x + b*d)^2
*a^2*c^2)/((2*c*d*x + b*d)^(7/2)*c^4*d^3) - 1/320*(15*sqrt(2*c*d*x + b*d)*b
^2*c^16*d^30 - 60*sqrt(2*c*d*x + b*d)*a*c^17*d^30 - (2*c*d*x + b*d)^(5/2)*c
^16*d^28)/(c^20*d^35)
```

3.1283 $\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^{11/2}}\,dx$

Optimal. Leaf size=121

$$-\frac{3\left(b^2-4ac\right)^2}{320c^4d^3(bd+2cdx)^{5/2}}+\frac{3\left(b^2-4ac\right)}{64c^4d^5\sqrt{bd+2cdx}}+\frac{\left(b^2-4ac\right)^3}{576c^4d(bd+2cdx)^{9/2}}+\frac{(bd+2cdx)^{3/2}}{192c^4d^7}$$

```
[Out] (b^2 - 4*a*c)^3/(576*c^4*d*(b*d + 2*c*d*x)^(9/2)) - (3*(b^2 - 4*a*c)^2)/(32
0*c^4*d^3*(b*d + 2*c*d*x)^(5/2)) + (3*(b^2 - 4*a*c))/(64*c^4*d^5*Sqrt[b*d +
 2*c*d*x]) + (b*d + 2*c*d*x)^(3/2)/(192*c^4*d^7)
```

Rubi [A] time = 0.0500006, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {683}

$$-\frac{3\left(b^2-4ac\right)^2}{320c^4d^3(bd+2cdx)^{5/2}}+\frac{3\left(b^2-4ac\right)}{64c^4d^5\sqrt{bd+2cdx}}+\frac{\left(b^2-4ac\right)^3}{576c^4d(bd+2cdx)^{9/2}}+\frac{(bd+2cdx)^{3/2}}{192c^4d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^(11/2),x]
```

```
[Out] (b^2 - 4*a*c)^3/(576*c^4*d*(b*d + 2*c*d*x)^(9/2)) - (3*(b^2 - 4*a*c)^2)/(32
0*c^4*d^3*(b*d + 2*c*d*x)^(5/2)) + (3*(b^2 - 4*a*c))/(64*c^4*d^5*Sqrt[b*d +
 2*c*d*x]) + (b*d + 2*c*d*x)^(3/2)/(192*c^4*d^7)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx+cx^2\right)^3}{(bd+2cdx)^{11/2}}\,dx &= \int \left(\frac{\left(-b^2+4ac\right)^3}{64c^3(bd+2cdx)^{11/2}}+\frac{3\left(-b^2+4ac\right)^2}{64c^3d^2(bd+2cdx)^{7/2}}+\frac{3\left(-b^2+4ac\right)}{64c^3d^4(bd+2cdx)^{3/2}}+\frac{\sqrt{bd+2cdx}}{64c^3d^6}\right)dx\\ &= \frac{\left(b^2-4ac\right)^3}{576c^4d(bd+2cdx)^{9/2}}-\frac{3\left(b^2-4ac\right)^2}{320c^4d^3(bd+2cdx)^{5/2}}+\frac{3\left(b^2-4ac\right)}{64c^4d^5\sqrt{bd+2cdx}}+\frac{(bd+2cdx)^{3/2}}{192c^4d^7}\end{aligned}$$

Mathematica [A] time = 0.0739362, size = 83, normalized size = 0.69

$$\frac{135\left(b^2-4ac\right)(b+2cx)^4-27\left(b^2-4ac\right)^2(b+2cx)^2+5\left(b^2-4ac\right)^3+15(b+2cx)^6}{2880c^4d(d(b+2cx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^(11/2),x]
```

```
[Out] (5*(b^2 - 4*a*c)^3 - 27*(b^2 - 4*a*c)^2*(b + 2*c*x)^2 + 135*(b^2 - 4*a*c)*(
b + 2*c*x)^4 + 15*(b + 2*c*x)^6)/(2880*c^4*d*(d*(b + 2*c*x))^(9/2))
```

Maple [A] time = 0.044, size = 174, normalized size = 1.4

$$-\frac{(2\,cx+b)\left(-15\,c^6x^6-45\,bc^5x^5+135\,ac^5x^4-90\,b^2c^4x^4+270\,abc^4x^3-105\,b^3c^3x^3+27\,a^2c^4x^2+189\,ab^2c^3x^2-63\,b^4c\right.}{45\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^(11/2),x)
```

```
[Out] -1/45*(2*c*x+b)*(-15*c^6*x^6-45*b*c^5*x^5+135*a*c^5*x^4-90*b^2*c^4*x^4+270*
a*b*c^4*x^3-105*b^3*c^3*x^3+27*a^2*c^4*x^2+189*a*b^2*c^3*x^2-63*b^4*c^2*x^2
+27*a^2*b*c^3*x+54*a*b^3*c^2*x-18*b^5*c*x+5*a^3*c^3+3*a^2*b^2*c^2+6*a*b^4*c
-2*b^6)/c^4/(2*c*d*x+b*d)^(11/2)
```

Maxima [A] time = 1.13138, size = 186, normalized size = 1.54

$$\frac{\frac{15\,(2\,cdx+bd)^{\frac{3}{2}}}{c^3d^6}+\frac{135\,(2\,cdx+bd)^4\left(b^2-4\,ac\right)-27\left(b^4-8\,ab^2c+16\,a^2c^2\right)(2\,cdx+bd)^2d^2+5\left(b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3\right)d^4}{(2\,cdx+bd)^{\frac{9}{2}}c^3d^4}}{2880\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^(11/2),x, algorithm="maxima")
```

```
[Out] 1/2880*(15*(2*c*d*x + b*d)^(3/2)/(c^3*d^6) + (135*(2*c*d*x + b*d)^4*(b^2 -
4*a*c) - 27*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*(2*c*d*x + b*d)^2*d^2 + 5*(b^6 -
 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*d^4)/((2*c*d*x + b*d)^(9/2)*c^3*
d^4))/(c*d)
```

Fricas [B] time = 2.06714, size = 498, normalized size = 4.12

$$\frac{\left(15\,c^6x^6+45\,bc^5x^5+2\,b^6-6\,ab^4c-3\,a^2b^2c^2-5\,a^3c^3+45\left(2\,b^2c^4-3\,ac^5\right)x^4+15\left(7\,b^3c^3-18\,abc^4\right)x^3+9\left(7\,b^4c^2-2\right.\right.}{45\left(32\,c^9d^6x^5+80\,bc^8d^6x^4+80\,b^2c^7d^6x^3+40\,b^3c^6d^6x^2+10\,b^4c^5d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^(11/2),x, algorithm="fricas")
```

```
[Out] 1/45*(15*c^6*x^6 + 45*b*c^5*x^5 + 2*b^6 - 6*a*b^4*c - 3*a^2*b^2*c^2 - 5*a^3
*c^3 + 45*(2*b^2*c^4 - 3*a*c^5)*x^4 + 15*(7*b^3*c^3 - 18*a*b*c^4)*x^3 + 9*(
7*b^4*c^2 - 21*a*b^2*c^3 - 3*a^2*c^4)*x^2 + 9*(2*b^5*c - 6*a*b^3*c^2 - 3*a^
2*b*c^3)*x)*sqrt(2*c*d*x + b*d)/(32*c^9*d^6*x^5 + 80*b*c^8*d^6*x^4 + 80*b^2
*c^7*d^6*x^3 + 40*b^3*c^6*d^6*x^2 + 10*b^4*c^5*d^6*x + b^5*c^4*d^6)
```

Sympy [A] time = 24.1072, size = 1731, normalized size = 14.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(2*c*d*x+b*d)**(11/2),x)
```

```
[Out] Piecewise((-5*a**3*c**3*sqrt(b*d + 2*c*d*x)/(45*b**5*c**4*d**6 + 450*b**4*c
**5*d**6*x + 1800*b**3*c**6*d**6*x**2 + 3600*b**2*c**7*d**6*x**3 + 3600*b*c
**8*d**6*x**4 + 1440*c**9*d**6*x**5) - 3*a**2*b**2*c**2*sqrt(b*d + 2*c*d*x)
/(45*b**5*c**4*d**6 + 450*b**4*c**5*d**6*x + 1800*b**3*c**6*d**6*x**2 + 360
0*b**2*c**7*d**6*x**3 + 3600*b*c**8*d**6*x**4 + 1440*c**9*d**6*x**5) - 27*a
**2*b*c**3*x*sqrt(b*d + 2*c*d*x)/(45*b**5*c**4*d**6 + 450*b**4*c**5*d**6*x
+ 1800*b**3*c**6*d**6*x**2 + 3600*b**2*c**7*d**6*x**3 + 3600*b*c**8*d**6*x*
*4 + 1440*c**9*d**6*x**5) - 27*a**2*c**4*x**2*sqrt(b*d + 2*c*d*x)/(45*b**5*
c**4*d**6 + 450*b**4*c**5*d**6*x + 1800*b**3*c**6*d**6*x**2 + 3600*b**2*c**
7*d**6*x**3 + 3600*b*c**8*d**6*x**4 + 1440*c**9*d**6*x**5) - 6*a*b**4*c*sqr
t(b*d + 2*c*d*x)/(45*b**5*c**4*d**6 + 450*b**4*c**5*d**6*x + 1800*b**3*c**6
*d**6*x**2 + 3600*b**2*c**7*d**6*x**3 + 3600*b*c**8*d**6*x**4 + 1440*c**9*d
**6*x**5) - 54*a*b**3*c**2*x*sqrt(b*d + 2*c*d*x)/(45*b**5*c**4*d**6 + 450*b
**4*c**5*d**6*x + 1800*b**3*c**6*d**6*x**2 + 3600*b**2*c**7*d**6*x**3 + 360
0*b*c**8*d**6*x**4 + 1440*c**9*d**6*x**5) - 189*a*b**2*c**3*x**2*sqrt(b*d +
 2*c*d*x)/(45*b**5*c**4*d**6 + 450*b**4*c**5*d**6*x + 1800*b**3*c**6*d**6*x
**2 + 3600*b**2*c**7*d**6*x**3 + 3600*b*c**8*d**6*x**4 + 1440*c**9*d**6*x**
5) - 270*a*b*c**4*x**3*sqrt(b*d + 2*c*d*x)/(45*b**5*c**4*d**6 + 450*b**4*c*
*5*d**6*x + 1800*b**3*c**6*d**6*x**2 + 3600*b**2*c**7*d**6*x**3 + 3600*b*c*
*8*d**6*x**4 + 1440*c**9*d**6*x**5) - 135*a*c**5*x**4*sqrt(b*d + 2*c*d*x)/(
45*b**5*c**4*d**6 + 450*b**4*c**5*d**6*x + 1800*b**3*c**6*d**6*x**2 + 3600*
b**2*c**7*d**6*x**3 + 3600*b*c**8*d**6*x**4 + 1440*c**9*d**6*x**5) + 2*b**6
*sqrt(b*d + 2*c*d*x)/(45*b**5*c**4*d**6 + 450*b**4*c**5*d**6*x + 1800*b**3*
c**6*d**6*x**2 + 3600*b**2*c**7*d**6*x**3 + 3600*b*c**8*d**6*x**4 + 1440*c*
*9*d**6*x**5) + 18*b**5*c*x*sqrt(b*d + 2*c*d*x)/(45*b**5*c**4*d**6 + 450*b*
*4*c**5*d**6*x + 1800*b**3*c**6*d**6*x**2 + 3600*b**2*c**7*d**6*x**3 + 3600
*b*c**8*d**6*x**4 + 1440*c**9*d**6*x**5) + 63*b**4*c**2*x**2*sqrt(b*d + 2*c
*d*x)/(45*b**5*c**4*d**6 + 450*b**4*c**5*d**6*x + 1800*b**3*c**6*d**6*x**2
+ 3600*b**2*c**7*d**6*x**3 + 3600*b*c**8*d**6*x**4 + 1440*c**9*d**6*x**5) +
 105*b**3*c**3*x**3*sqrt(b*d + 2*c*d*x)/(45*b**5*c**4*d**6 + 450*b**4*c**5*
d**6*x + 1800*b**3*c**6*d**6*x**2 + 3600*b**2*c**7*d**6*x**3 + 3600*b*c**8*
d**6*x**4 + 1440*c**9*d**6*x**5) + 90*b**2*c**4*x**4*sqrt(b*d + 2*c*d*x)/(4
5*b**5*c**4*d**6 + 450*b**4*c**5*d**6*x + 1800*b**3*c**6*d**6*x**2 + 3600*b
**2*c**7*d**6*x**3 + 3600*b*c**8*d**6*x**4 + 1440*c**9*d**6*x**5) + 45*b*c*
*5*x**5*sqrt(b*d + 2*c*d*x)/(45*b**5*c**4*d**6 + 450*b**4*c**5*d**6*x + 180
0*b**3*c**6*d**6*x**2 + 3600*b**2*c**7*d**6*x**3 + 3600*b*c**8*d**6*x**4 +
1440*c**9*d**6*x**5) + 15*c**6*x**6*sqrt(b*d + 2*c*d*x)/(45*b**5*c**4*d**6
+ 450*b**4*c**5*d**6*x + 1800*b**3*c**6*d**6*x**2 + 3600*b**2*c**7*d**6*x**
3 + 3600*b*c**8*d**6*x**4 + 1440*c**9*d**6*x**5), Ne(c, 0)), ((a**3*x + 3*a
**2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4)/(b*d)**(11/2), True))
```

Giac [A] time = 1.21828, size = 238, normalized size = 1.97

$$\frac{(2\,cdx+bd)^{\frac{3}{2}}}{192\,c^4d^7}+\frac{5\,b^6d^4-60\,ab^4cd^4+240\,a^2b^2c^2d^4-320\,a^3c^3d^4-27\,(2\,cdx+bd)^2b^4d^2+216\,(2\,cdx+bd)^2ab^2cd^2-432\,(}{2880\,(2\,cdx+bd)^{\frac{9}{2}}c^4d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^(11/2),x, algorithm="giac")
```

```
[Out] 1/192*(2*c*d*x + b*d)^(3/2)/(c^4*d^7) + 1/2880*(5*b^6*d^4 - 60*a*b^4*c*d^4
+ 240*a^2*b^2*c^2*d^4 - 320*a^3*c^3*d^4 - 27*(2*c*d*x + b*d)^2*b^4*d^2 + 21
```

```
6*(2*c*d*x + b*d)^2*a*b^2*c*d^2 - 432*(2*c*d*x + b*d)^2*a^2*c^2*d^2 + 135*(
2*c*d*x + b*d)^4*b^2 - 540*(2*c*d*x + b*d)^4*a*c)/((2*c*d*x + b*d)^(9/2)*c^
4*d^5)
```

3.1284 $\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^{13/2}}\,dx$

Optimal. Leaf size=121

$$-\frac{3(b^2-4ac)^2}{448c^4d^3(bd+2cdx)^{7/2}}+\frac{b^2-4ac}{64c^4d^5(bd+2cdx)^{3/2}}+\frac{(b^2-4ac)^3}{704c^4d(bd+2cdx)^{11/2}}+\frac{\sqrt{bd+2cdx}}{64c^4d^7}$$

```
[Out] (b^2 - 4*a*c)^3/(704*c^4*d*(b*d + 2*c*d*x)^(11/2)) - (3*(b^2 - 4*a*c)^2)/(4
48*c^4*d^3*(b*d + 2*c*d*x)^(7/2)) + (b^2 - 4*a*c)/(64*c^4*d^5*(b*d + 2*c*d*
x)^(3/2)) + Sqrt[b*d + 2*c*d*x]/(64*c^4*d^7)
```

Rubi [A] time = 0.0533011, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {683}

$$-\frac{3(b^2-4ac)^2}{448c^4d^3(bd+2cdx)^{7/2}}+\frac{b^2-4ac}{64c^4d^5(bd+2cdx)^{3/2}}+\frac{(b^2-4ac)^3}{704c^4d(bd+2cdx)^{11/2}}+\frac{\sqrt{bd+2cdx}}{64c^4d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^(13/2),x]
```

```
[Out] (b^2 - 4*a*c)^3/(704*c^4*d*(b*d + 2*c*d*x)^(11/2)) - (3*(b^2 - 4*a*c)^2)/(4
48*c^4*d^3*(b*d + 2*c*d*x)^(7/2)) + (b^2 - 4*a*c)/(64*c^4*d^5*(b*d + 2*c*d*
x)^(3/2)) + Sqrt[b*d + 2*c*d*x]/(64*c^4*d^7)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx+cx^2)^3}{(bd+2cdx)^{13/2}}\,dx &= \int \left(\frac{(-b^2+4ac)^3}{64c^3(bd+2cdx)^{13/2}}+\frac{3(-b^2+4ac)^2}{64c^3d^2(bd+2cdx)^{9/2}}+\frac{3(-b^2+4ac)}{64c^3d^4(bd+2cdx)^{5/2}}+\frac{1}{64c^3d^6\sqrt{bd+2cdx}}\right)\\ &= \frac{(b^2-4ac)^3}{704c^4d(bd+2cdx)^{11/2}}-\frac{3(b^2-4ac)^2}{448c^4d^3(bd+2cdx)^{7/2}}+\frac{b^2-4ac}{64c^4d^5(bd+2cdx)^{3/2}}+\frac{\sqrt{bd+2cdx}}{64c^4d^7}\end{aligned}$$

Mathematica [A] time = 0.0759858, size = 83, normalized size = 0.69

$$\frac{77(b^2-4ac)(b+2cx)^4-33(b^2-4ac)^2(b+2cx)^2+7(b^2-4ac)^3+77(b+2cx)^6}{4928c^4d(d(b+2cx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(b*d + 2*c*d*x)^(13/2),x]
```

```
[Out] (7*(b^2 - 4*a*c)^3 - 33*(b^2 - 4*a*c)^2*(b + 2*c*x)^2 + 77*(b^2 - 4*a*c)*(b
 + 2*c*x)^4 + 77*(b + 2*c*x)^6)/(4928*c^4*d*(d*(b + 2*c*x))^(11/2))
```

Maple [A] time = 0.045, size = 174, normalized size = 1.4

$$-\frac{(2cx+b)\left(-77c^6x^6-231bc^5x^5+77ac^5x^4-308b^2c^4x^4+154abc^4x^3-231b^3c^3x^3+33a^2c^4x^2+99ab^2c^3x^2-99b^4\right.}{77c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^(13/2),x)
```

```
[Out] -1/77*(2*c*x+b)*(-77*c^6*x^6-231*b*c^5*x^5+77*a*c^5*x^4-308*b^2*c^4*x^4+154
*a*b*c^4*x^3-231*b^3*c^3*x^3+33*a^2*c^4*x^2+99*a*b^2*c^3*x^2-99*b^4*c^2*x^2
+33*a^2*b*c^3*x+22*a*b^3*c^2*x-22*b^5*c*x+7*a^3*c^3+3*a^2*b^2*c^2+2*a*b^4*c
-2*b^6)/c^4/(2*c*d*x+b*d)^(13/2)
```

Maxima [A] time = 1.25101, size = 186, normalized size = 1.54

$$\frac{\frac{77\sqrt{2cdx+bd}}{c^3d^6}+\frac{77(2cdx+bd)^4\left(b^2-4ac\right)-33\left(b^4-8ab^2c+16a^2c^2\right)(2cdx+bd)^2d^2+7\left(b^6-12ab^4c+48a^2b^2c^2-64a^3c^3\right)d^4}{(2cdx+bd)^{\frac{11}{2}}c^3d^4}}{4928cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^(13/2),x, algorithm="maxima")
```

```
[Out] 1/4928*(77*sqrt(2*c*d*x + b*d)/(c^3*d^6) + (77*(2*c*d*x + b*d)^4*(b^2 - 4*a
*c) - 33*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*(2*c*d*x + b*d)^2*d^2 + 7*(b^6 - 12
*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*d^4)/((2*c*d*x + b*d)^(11/2)*c^3*d^
4))/(c*d)
```

Fricas [B] time = 2.16869, size = 528, normalized size = 4.36

$$\frac{\left(77c^6x^6+231bc^5x^5+2b^6-2ab^4c-3a^2b^2c^2-7a^3c^3+77\left(4b^2c^4-ac^5\right)x^4+77\left(3b^3c^3-2abc^4\right)x^3+33\left(3b^4c^2-3\right.\right.}{77\left(64c^{10}d^7x^6+192bc^9d^7x^5+240b^2c^8d^7x^4+160b^3c^7d^7x^3+60b^4c^6d^7x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^(13/2),x, algorithm="fricas")
```

```
[Out] 1/77*(77*c^6*x^6 + 231*b*c^5*x^5 + 2*b^6 - 2*a*b^4*c - 3*a^2*b^2*c^2 - 7*a^
3*c^3 + 77*(4*b^2*c^4 - a*c^5)*x^4 + 77*(3*b^3*c^3 - 2*a*b*c^4)*x^3 + 33*(3
*b^4*c^2 - 3*a*b^2*c^3 - a^2*c^4)*x^2 + 11*(2*b^5*c - 2*a*b^3*c^2 - 3*a^2*b
*c^3)*x)*sqrt(2*c*d*x + b*d)/(64*c^10*d^7*x^6 + 192*b*c^9*d^7*x^5 + 240*b^2
*c^8*d^7*x^4 + 160*b^3*c^7*d^7*x^3 + 60*b^4*c^6*d^7*x^2 + 12*b^5*c^5*d^7*x
+ b^6*c^4*d^7)
```

Sympy [A] time = 40.5587, size = 1975, normalized size = 16.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(2*c*d*x+b*d)**(13/2),x)
```

```
[Out] Piecewise((-7*a**3*c**3*sqrt(b*d + 2*c*d*x)/(77*b**6*c**4*d**7 + 924*b**5*c
**5*d**7*x + 4620*b**4*c**6*d**7*x**2 + 12320*b**3*c**7*d**7*x**3 + 18480*b
**2*c**8*d**7*x**4 + 14784*b*c**9*d**7*x**5 + 4928*c**10*d**7*x**6) - 3*a**
2*b**2*c**2*sqrt(b*d + 2*c*d*x)/(77*b**6*c**4*d**7 + 924*b**5*c**5*d**7*x +
 4620*b**4*c**6*d**7*x**2 + 12320*b**3*c**7*d**7*x**3 + 18480*b**2*c**8*d**
7*x**4 + 14784*b*c**9*d**7*x**5 + 4928*c**10*d**7*x**6) - 33*a**2*b*c**3*x*
sqrt(b*d + 2*c*d*x)/(77*b**6*c**4*d**7 + 924*b**5*c**5*d**7*x + 4620*b**4*c
**6*d**7*x**2 + 12320*b**3*c**7*d**7*x**3 + 18480*b**2*c**8*d**7*x**4 + 147
84*b*c**9*d**7*x**5 + 4928*c**10*d**7*x**6) - 33*a**2*c**4*x**2*sqrt(b*d +
2*c*d*x)/(77*b**6*c**4*d**7 + 924*b**5*c**5*d**7*x + 4620*b**4*c**6*d**7*x*
*2 + 12320*b**3*c**7*d**7*x**3 + 18480*b**2*c**8*d**7*x**4 + 14784*b*c**9*d
**7*x**5 + 4928*c**10*d**7*x**6) - 2*a*b**4*c*sqrt(b*d + 2*c*d*x)/(77*b**6*
c**4*d**7 + 924*b**5*c**5*d**7*x + 4620*b**4*c**6*d**7*x**2 + 12320*b**3*c*
*7*d**7*x**3 + 18480*b**2*c**8*d**7*x**4 + 14784*b*c**9*d**7*x**5 + 4928*c*
*10*d**7*x**6) - 22*a*b**3*c**2*x*sqrt(b*d + 2*c*d*x)/(77*b**6*c**4*d**7 +
924*b**5*c**5*d**7*x + 4620*b**4*c**6*d**7*x**2 + 12320*b**3*c**7*d**7*x**3
 + 18480*b**2*c**8*d**7*x**4 + 14784*b*c**9*d**7*x**5 + 4928*c**10*d**7*x**
6) - 99*a*b**2*c**3*x**2*sqrt(b*d + 2*c*d*x)/(77*b**6*c**4*d**7 + 924*b**5*
c**5*d**7*x + 4620*b**4*c**6*d**7*x**2 + 12320*b**3*c**7*d**7*x**3 + 18480*
b**2*c**8*d**7*x**4 + 14784*b*c**9*d**7*x**5 + 4928*c**10*d**7*x**6) - 154*
a*b*c**4*x**3*sqrt(b*d + 2*c*d*x)/(77*b**6*c**4*d**7 + 924*b**5*c**5*d**7*x
 + 4620*b**4*c**6*d**7*x**2 + 12320*b**3*c**7*d**7*x**3 + 18480*b**2*c**8*d
**7*x**4 + 14784*b*c**9*d**7*x**5 + 4928*c**10*d**7*x**6) - 77*a*c**5*x**4*
sqrt(b*d + 2*c*d*x)/(77*b**6*c**4*d**7 + 924*b**5*c**5*d**7*x + 4620*b**4*c
**6*d**7*x**2 + 12320*b**3*c**7*d**7*x**3 + 18480*b**2*c**8*d**7*x**4 + 147
84*b*c**9*d**7*x**5 + 4928*c**10*d**7*x**6) + 2*b**6*sqrt(b*d + 2*c*d*x)/(7
7*b**6*c**4*d**7 + 924*b**5*c**5*d**7*x + 4620*b**4*c**6*d**7*x**2 + 12320*
b**3*c**7*d**7*x**3 + 18480*b**2*c**8*d**7*x**4 + 14784*b*c**9*d**7*x**5 +
4928*c**10*d**7*x**6) + 22*b**5*c*x*sqrt(b*d + 2*c*d*x)/(77*b**6*c**4*d**7
+ 924*b**5*c**5*d**7*x + 4620*b**4*c**6*d**7*x**2 + 12320*b**3*c**7*d**7*x*
*3 + 18480*b**2*c**8*d**7*x**4 + 14784*b*c**9*d**7*x**5 + 4928*c**10*d**7*x
**6) + 99*b**4*c**2*x**2*sqrt(b*d + 2*c*d*x)/(77*b**6*c**4*d**7 + 924*b**5*
c**5*d**7*x + 4620*b**4*c**6*d**7*x**2 + 12320*b**3*c**7*d**7*x**3 + 18480*
b**2*c**8*d**7*x**4 + 14784*b*c**9*d**7*x**5 + 4928*c**10*d**7*x**6) + 231*
b**3*c**3*x**3*sqrt(b*d + 2*c*d*x)/(77*b**6*c**4*d**7 + 924*b**5*c**5*d**7*
x + 4620*b**4*c**6*d**7*x**2 + 12320*b**3*c**7*d**7*x**3 + 18480*b**2*c**8*
d**7*x**4 + 14784*b*c**9*d**7*x**5 + 4928*c**10*d**7*x**6) + 308*b**2*c**4*
x**4*sqrt(b*d + 2*c*d*x)/(77*b**6*c**4*d**7 + 924*b**5*c**5*d**7*x + 4620*b
**4*c**6*d**7*x**2 + 12320*b**3*c**7*d**7*x**3 + 18480*b**2*c**8*d**7*x**4
+ 14784*b*c**9*d**7*x**5 + 4928*c**10*d**7*x**6) + 231*b*c**5*x**5*sqrt(b*d
 + 2*c*d*x)/(77*b**6*c**4*d**7 + 924*b**5*c**5*d**7*x + 4620*b**4*c**6*d**7
*x**2 + 12320*b**3*c**7*d**7*x**3 + 18480*b**2*c**8*d**7*x**4 + 14784*b*c**
9*d**7*x**5 + 4928*c**10*d**7*x**6) + 77*c**6*x**6*sqrt(b*d + 2*c*d*x)/(77*
b**6*c**4*d**7 + 924*b**5*c**5*d**7*x + 4620*b**4*c**6*d**7*x**2 + 12320*b*
*3*c**7*d**7*x**3 + 18480*b**2*c**8*d**7*x**4 + 14784*b*c**9*d**7*x**5 + 49
28*c**10*d**7*x**6), Ne(c, 0)), ((a**3*x + 3*a**2*b*x**2/2 + a*b**2*x**3 +
b**3*x**4/4)/(b*d)**(13/2), True))
```

Giac [A] time = 1.20978, size = 238, normalized size = 1.97

$$\frac{\sqrt{2\,cdx+bd}}{64\,c^4d^7}+\frac{7\,b^6d^4-84\,ab^4cd^4+336\,a^2b^2c^2d^4-448\,a^3c^3d^4-33\,(2\,cdx+bd)^2b^4d^2+264\,(2\,cdx+bd)^2ab^2cd^2-52}{4928\,(2\,cdx+bd)^{\frac{11}{2}}c^4d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(2*c*d*x+b*d)^(13/2),x, algorithm="giac")
```

```
[Out] 1/64*sqrt(2*c*d*x + b*d)/(c^4*d^7) + 1/4928*(7*b^6*d^4 - 84*a*b^4*c*d^4 + 3
36*a^2*b^2*c^2*d^4 - 448*a^3*c^3*d^4 - 33*(2*c*d*x + b*d)^2*b^4*d^2 + 264*(
2*c*d*x + b*d)^2*a*b^2*c*d^2 - 528*(2*c*d*x + b*d)^2*a^2*c^2*d^2 + 77*(2*c*
d*x + b*d)^4*b^2 - 308*(2*c*d*x + b*d)^4*a*c)/((2*c*d*x + b*d)^(11/2)*c^4*d
^5)
```

3.1285 $\int \frac{(bd+2cdx)^{11/2}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=175

$$4d^5\left(b^2-4ac\right)^2\sqrt{bd+2cdx}+\frac{4}{5}d^3\left(b^2-4ac\right)(bd+2cdx)^{5/2}-2d^{11/2}\left(b^2-4ac\right)^{9/4}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-2d^{11/2}\left(b^2-4a\right.$$

```
[Out] 4*(b^2 - 4*a*c)^2*d^5*Sqrt[b*d + 2*c*d*x] + (4*(b^2 - 4*a*c)*d^3*(b*d + 2*c
*d*x)^(5/2))/5 + (4*d*(b*d + 2*c*d*x)^(9/2))/9 - 2*(b^2 - 4*a*c)^(9/4)*d^(1
1/2)*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])] - 2*(b^2 - 4
*a*c)^(9/4)*d^(11/2)*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[
d])]
```

Rubi [A] time = 0.214603, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {692, 694, 329, 212, 206, 203}

$$4d^5\left(b^2-4ac\right)^2\sqrt{bd+2cdx}+\frac{4}{5}d^3\left(b^2-4ac\right)(bd+2cdx)^{5/2}-2d^{11/2}\left(b^2-4ac\right)^{9/4}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-2d^{11/2}\left(b^2-4a\right.$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(11/2)/(a + b*x + c*x^2),x]
```


```
[Out] 4*(b^2 - 4*a*c)^2*d^5*Sqrt[b*d + 2*c*d*x] + (4*(b^2 - 4*a*c)*d^3*(b*d + 2*c
*d*x)^(5/2))/5 + (4*d*(b*d + 2*c*d*x)^(9/2))/9 - 2*(b^2 - 4*a*c)^(9/4)*d^(1
1/2)*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])] - 2*(b^2 - 4
*a*c)^(9/4)*d^(11/2)*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[
d])]
```


Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(bd+2cdx)^{11/2}}{a+bx+cx^2}\,dx = \frac{4}{9}d(bd+2cdx)^{9/2} + \left(\left(b^2-4ac\right)d^2\right)\int \frac{(bd+2cdx)^{7/2}}{a+bx+cx^2}\,dx$$
$$= \frac{4}{5}\left(b^2-4ac\right)d^3(bd+2cdx)^{5/2} + \frac{4}{9}d(bd+2cdx)^{9/2} + \left(\left(b^2-4ac\right)^2 d^4\right)\int \frac{(bd+2cdx)^{3/2}}{a+bx+cx^2}\,dx$$
$$= 4\left(b^2-4ac\right)^2 d^5\sqrt{bd+2cdx} + \frac{4}{5}\left(b^2-4ac\right)d^3(bd+2cdx)^{5/2} + \frac{4}{9}d(bd+2cdx)^{9/2} + \left(\left(b^2-4ac\right)\right.$$
$$= 4\left(b^2-4ac\right)^2 d^5\sqrt{bd+2cdx} + \frac{4}{5}\left(b^2-4ac\right)d^3(bd+2cdx)^{5/2} + \frac{4}{9}d(bd+2cdx)^{9/2} + \frac{\left(\left(b^2-4ac\right)\right.}{}$$
$$= 4\left(b^2-4ac\right)^2 d^5\sqrt{bd+2cdx} + \frac{4}{5}\left(b^2-4ac\right)d^3(bd+2cdx)^{5/2} + \frac{4}{9}d(bd+2cdx)^{9/2} + \frac{\left(\left(b^2-4ac\right)\right.}{}$$
$$= 4\left(b^2-4ac\right)^2 d^5\sqrt{bd+2cdx} + \frac{4}{5}\left(b^2-4ac\right)d^3(bd+2cdx)^{5/2} + \frac{4}{9}d(bd+2cdx)^{9/2} - \left(2\left(b^2-4ac\right.\right.$$
$$= 4\left(b^2-4ac\right)^2 d^5\sqrt{bd+2cdx} + \frac{4}{5}\left(b^2-4ac\right)d^3(bd+2cdx)^{5/2} + \frac{4}{9}d(bd+2cdx)^{9/2} - 2\left(b^2-4ac\right)$$

Mathematica [A] time = 0.196021, size = 157, normalized size = 0.9

$$\frac{4d(d(b+2cx))^{9/2}\left(\frac{1}{9}(b+2cx)^{9/2} - \left(4ac-b^2\right)\left(\frac{1}{5}(b+2cx)^{5/2} - \frac{1}{2}\left(4ac-b^2\right)\left(2\sqrt{b+2cx} - \sqrt[4]{b^2-4ac}\left(\tan^{-1}\left(\frac{\sqrt{b+2cx}}{\sqrt[4]{b^2-4ac}}\right) + \right.\right.\right.\right.}{(b+2cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(11/2)/(a + b*x + c*x^2),x]
```

```
[Out] (4*d*(d*(b + 2*c*x))^(9/2)*((b + 2*c*x)^(9/2)/9 - (-b^2 + 4*a*c)*((b + 2*c*
x)^(5/2)/5 - ((-b^2 + 4*a*c)*(2*Sqrt[b + 2*c*x] - (b^2 - 4*a*c)^(1/4)*(ArcT
an[Sqrt[b + 2*c*x]/(b^2 - 4*a*c)^(1/4)] + ArcTanh[Sqrt[b + 2*c*x]/(b^2 - 4*
a*c)^(1/4)])))/2)))/(b + 2*c*x)^(9/2)
```

Maple [B] time = 0.213, size = 1287, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(11/2)/(c*x^2+b*x+a),x)
```

```
[Out] 4/9*d*(2*c*d*x+b*d)^(9/2)-16/5*(2*c*d*x+b*d)^(5/2)*a*c*d^3+4/5*(2*c*d*x+b*d
)^(5/2)*b^2*d^3+64*a^2*c^2*d^5*(2*c*d*x+b*d)^(1/2)-32*a*b^2*c*d^5*(2*c*d*x+
b*d)^(1/2)+4*b^4*d^5*(2*c*d*x+b*d)^(1/2)-64*d^7/(4*a*c*d^2-b^2*d^2)^(3/4)*2
^(1/2)*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*a^3*
c^3+48*d^7/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(2^(1/2)/(4*a*c*d^2-b^2*
d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*a^2*b^2*c^2-12*d^7/(4*a*c*d^2-b^2*d^2)^(3
/4)*2^(1/2)*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)
*a*b^4*c+d^7/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(2^(1/2)/(4*a*c*d^2-b^
2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*b^6+64*d^7/(4*a*c*d^2-b^2*d^2)^(3/4)*2^
(1/2)*arctan(-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*a^3*
c^3-48*d^7/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(-2^(1/2)/(4*a*c*d^2-b^2
*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*a^2*b^2*c^2+12*d^7/(4*a*c*d^2-b^2*d^2)^(
3/4)*2^(1/2)*arctan(-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+
1)*a*b^4*c-d^7/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(-2^(1/2)/(4*a*c*d^2
-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*b^6-32*d^7/(4*a*c*d^2-b^2*d^2)^(3/4)
*2^(1/2)*ln((2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1
/2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*
d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)))*a^3*c^3+24*d^7/(4*a*c*d^
2-b^2*d^2)^(3/4)*2^(1/2)*ln((2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x
+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d-(4*a*c*d^2-b^2*
d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)))*a^2*b^2*
c^2-6*d^7/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*ln((2*c*d*x+b*d+(4*a*c*d^2-b^2*
d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+
b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^
2)^(1/2)))*a*b^4*c+1/2*d^7/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*ln((2*c*d*x+b*
d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)
^(1/2))/(2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+
(4*a*c*d^2-b^2*d^2)^(1/2)))*b^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(11/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.51011, size = 3090, normalized size = 17.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(11/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 4/45*(80*c^4*d^5*x^4 + 160*b*c^3*d^5*x^3 + 12*(13*b^2*c^2 - 12*a*c^3)*d^5*x
^2 + 4*(19*b^3*c - 36*a*b*c^2)*d^5*x + (59*b^4 - 396*a*b^2*c + 720*a^2*c^2)
*d^5)*sqrt(2*c*d*x + b*d) - 4*((b^18 - 36*a*b^16*c + 576*a^2*b^14*c^2 - 537
6*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c
^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 262144*a^9*c^9)*d^22)^(1/4)*
arctan((((b^18 - 36*a*b^16*c + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256
*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^
7 + 589824*a^8*b^2*c^8 - 262144*a^9*c^9)*d^22)^(3/4)*(b^4 - 8*a*b^2*c + 16*
a^2*c^2)*sqrt(2*c*d*x + b*d)*d^5 - ((b^18 - 36*a*b^16*c + 576*a^2*b^14*c^2
- 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*
b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 262144*a^9*c^9)*d^22)^(
3/4)*sqrt(2*(b^8*c - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4 + 256*
a^4*c^5)*d^11*x + (b^9 - 16*a*b^7*c + 96*a^2*b^5*c^2 - 256*a^3*b^3*c^3 + 25
6*a^4*b*c^4)*d^11 + sqrt((b^18 - 36*a*b^16*c + 576*a^2*b^14*c^2 - 5376*a^3*
b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 5
89824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 262144*a^9*c^9)*d^22)))/((b^18 - 3
6*a*b^16*c + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 12
9024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2
*c^8 - 262144*a^9*c^9)*d^22)) - ((b^18 - 36*a*b^16*c + 576*a^2*b^14*c^2 - 5
376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6
*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 262144*a^9*c^9)*d^22)^(1/4
)*log((b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(2*c*d*x + b*d)*d^5 + ((b^18 - 36*
a*b^16*c + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 1290
24*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c
^8 - 262144*a^9*c^9)*d^22)^(1/4)) + ((b^18 - 36*a*b^16*c + 576*a^2*b^14*c^2
 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6
*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 262144*a^9*c^9)*d^22)^
(1/4)*log((b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(2*c*d*x + b*d)*d^5 - ((b^18 -
 36*a*b^16*c + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 -
129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b
^2*c^8 - 262144*a^9*c^9)*d^22)^(1/4))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(11/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26744, size = 779, normalized size = 4.45

$$4\sqrt{2\,cdx+bd}b^4d^5-32\sqrt{2\,cdx+bd}ab^2cd^5+64\sqrt{2\,cdx+bd}a^2c^2d^5+\frac{4}{5}\,(2\,cdx+bd)^{\frac{5}{2}}b^2d^3-\frac{16}{5}\,(2\,cdx+bd)^{\frac{5}{2}}acd^3+\frac{4}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(11/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 4*sqrt(2*c*d*x + b*d)*b^4*d^5 - 32*sqrt(2*c*d*x + b*d)*a*b^2*c*d^5 + 64*sqr
t(2*c*d*x + b*d)*a^2*c^2*d^5 + 4/5*(2*c*d*x + b*d)^(5/2)*b^2*d^3 - 16/5*(2*
```

```
c*d*x + b*d)^(5/2)*a*c*d^3 + 4/9*(2*c*d*x + b*d)^(9/2)*d - 1/2*sqrt(2)*(b^4
*d^5 - 8*a*b^2*c*d^5 + 16*a^2*c^2*d^5)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*log(2*c
*d*x + b*d + sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqr
t(-b^2*d^2 + 4*a*c*d^2)) + 1/2*sqrt(2)*(b^4*d^5 - 8*a*b^2*c*d^5 + 16*a^2*c^
2*d^5)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*log(2*c*d*x + b*d - sqrt(2)*(-b^2*d^2 +
 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2)) - (sqrt
(2)*b^4*d^5 - 8*sqrt(2)*a*b^2*c*d^5 + 16*sqrt(2)*a^2*c^2*d^5)*(-b^2*d^2 + 4
*a*c*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) +
2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4)) - (sqrt(2)*b^4*d^5 - 8
*sqrt(2)*a*b^2*c*d^5 + 16*sqrt(2)*a^2*c^2*d^5)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)
*arctan(-1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) - 2*sqrt(2*c*d*x
 + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4))
```

3.1286 $\int \frac{(bd+2cdx)^{9/2}}{a+bx+cx^2} dx$

Optimal. Leaf size=147

$$\frac{4}{3}d^3\left(b^2-4ac\right)(bd+2cdx)^{3/2}+2d^{9/2}\left(b^2-4ac\right)^{7/4}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-2d^{9/2}\left(b^2-4ac\right)^{7/4}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)$$

```
[Out] (4*(b^2 - 4*a*c)*d^3*(b*d + 2*c*d*x)^(3/2))/3 + (4*d*(b*d + 2*c*d*x)^(7/2))
/7 + 2*(b^2 - 4*a*c)^(7/4)*d^(9/2)*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c
)^(1/4)*Sqrt[d])] - 2*(b^2 - 4*a*c)^(7/4)*d^(9/2)*ArcTanh[Sqrt[d*(b + 2*c*x
)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])]
```

Rubi [A] time = 0.138146, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {692, 694, 329, 298, 203, 206}

$$\frac{4}{3}d^3\left(b^2-4ac\right)(bd+2cdx)^{3/2}+2d^{9/2}\left(b^2-4ac\right)^{7/4}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-2d^{9/2}\left(b^2-4ac\right)^{7/4}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(9/2)/(a + b*x + c*x^2),x]
```

```
[Out] (4*(b^2 - 4*a*c)*d^3*(b*d + 2*c*d*x)^(3/2))/3 + (4*d*(b*d + 2*c*d*x)^(7/2))
/7 + 2*(b^2 - 4*a*c)^(7/4)*d^(9/2)*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c
)^(1/4)*Sqrt[d])] - 2*(b^2 - 4*a*c)^(7/4)*d^(9/2)*ArcTanh[Sqrt[d*(b + 2*c*x
)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
```

```
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(bd+2cdx)^{9/2}}{a+bx+cx^2}\,dx = \frac{4}{7}d(bd+2cdx)^{7/2} + \left(\left(b^2-4ac\right)d^2\right)\int \frac{(bd+2cdx)^{5/2}}{a+bx+cx^2}\,dx$$
$$= \frac{4}{3}\left(b^2-4ac\right)d^3(bd+2cdx)^{3/2} + \frac{4}{7}d(bd+2cdx)^{7/2} + \left(\left(b^2-4ac\right)^2 d^4\right)\int \frac{\sqrt{bd+2cdx}}{a+bx+cx^2}\,dx$$
$$= \frac{4}{3}\left(b^2-4ac\right)d^3(bd+2cdx)^{3/2} + \frac{4}{7}d(bd+2cdx)^{7/2} + \frac{\left(\left(b^2-4ac\right)^2 d^3\right)\text{Subst}\left(\int \frac{\sqrt{x}}{a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}}\,dx, x, bd \right.}{2c}$$
$$= \frac{4}{3}\left(b^2-4ac\right)d^3(bd+2cdx)^{3/2} + \frac{4}{7}d(bd+2cdx)^{7/2} + \frac{\left(\left(b^2-4ac\right)^2 d^3\right)\text{Subst}\left(\int \frac{x^2}{a-\frac{b^2}{4c}+\frac{x^4}{4cd^2}}\,dx, x, \sqrt{d} \right.}{c}$$
$$= \frac{4}{3}\left(b^2-4ac\right)d^3(bd+2cdx)^{3/2} + \frac{4}{7}d(bd+2cdx)^{7/2} - \left(2\left(b^2-4ac\right)^2 d^5\right)\text{Subst}\left(\int \frac{1}{\sqrt{b^2-4acd}-x^2}\right.$$
$$= \frac{4}{3}\left(b^2-4ac\right)d^3(bd+2cdx)^{3/2} + \frac{4}{7}d(bd+2cdx)^{7/2} + 2\left(b^2-4ac\right)^{7/4}d^{9/2}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right) - 2$$

Mathematica [A] time = 0.145131, size = 131, normalized size = 0.89

$$\frac{4(d(b+2cx))^{9/2}\left(\frac{1}{7}(b+2cx)^{7/2}-\frac{1}{6}\left(4ac-b^2\right)\left(3\left(b^2-4ac\right)^{3/4}\left(\tan^{-1}\left(\frac{\sqrt{b+2cx}}{\sqrt[4]{b^2-4ac}}\right)-\tanh^{-1}\left(\frac{\sqrt{b+2cx}}{\sqrt[4]{b^2-4ac}}\right)\right)+2(b+2cx)^{3/2}\right)\right)}{(b+2cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(9/2)/(a + b*x + c*x^2),x]
```

```
[Out] (4*(d*(b + 2*c*x))^(9/2)*((b + 2*c*x)^(7/2)/7 - ((-b^2 + 4*a*c)*(2*(b + 2*c
*x)^(3/2) + 3*(b^2 - 4*a*c)^(3/4)*(ArcTan[Sqrt[b + 2*c*x]/(b^2 - 4*a*c)^(1/
4)] - ArcTanh[Sqrt[b + 2*c*x]/(b^2 - 4*a*c)^(1/4)]))/6))/(b + 2*c*x)^(9/2)
```

Maple [B] time = 0.194, size = 922, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a),x)
```

```
[Out] 4/7*d*(2*c*d*x+b*d)^(7/2)-16/3*(2*c*d*x+b*d)^(3/2)*a*c*d^3+4/3*(2*c*d*x+b*d
)^(3/2)*b^2*d^3-16*d^5/(4*a*c*d^2-b^2*d^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(4
*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*a^2*c^2+8*d^5/(4*a*c*d^2-b^2
*d^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)
^(1/2)+1)*a*b^2*c-d^5/(4*a*c*d^2-b^2*d^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(4*
a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*b^4+8*d^5/(4*a*c*d^2-b^2*d^2)
^(1/4)*2^(1/2)*ln((2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2
)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)
*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)))*a^2*c^2-4*d^5/(4*a
*c*d^2-b^2*d^2)^(1/4)*2^(1/2)*ln((2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*
c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d+(4*a*c*d^2
-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)))*a*b
^2*c+1/2*d^5/(4*a*c*d^2-b^2*d^2)^(1/4)*2^(1/2)*ln((2*c*d*x+b*d-(4*a*c*d^2-b
^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d
*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2
*d^2)^(1/2)))*b^4+16*d^5/(4*a*c*d^2-b^2*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*a^2*c^2-8*d^5/(4*a*c*d^2-b^
2*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)
^(1/2)+1)*a*b^2*c+d^5/(4*a*c*d^2-b^2*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(4*a
*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.19676, size = 3245, normalized size = 22.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 8/21*(12*c^3*d^4*x^3 + 18*b*c^2*d^4*x^2 + 4*(4*b^2*c - 7*a*c^2)*d^4*x + (5*
b^3 - 14*a*b*c)*d^4)*sqrt(2*c*d*x + b*d) - 4*((b^14 - 28*a*b^12*c + 336*a^2
*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672
*a^6*b^2*c^6 - 16384*a^7*c^7)*d^18)^(1/4)*arctan(-(((b^14 - 28*a*b^12*c + 3
36*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 +
 28672*a^6*b^2*c^6 - 16384*a^7*c^7)*d^18)^(1/4)*(b^10 - 20*a*b^8*c + 160*a^
2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 1024*a^5*c^5)*sqrt(2*c*d*x
 + b*d)*d^13 + sqrt(2*(b^20*c - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3
*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 -
 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10 + 1048576
*a^10*c^11)*d^27*x + (b^21 - 40*a*b^19*c + 720*a^2*b^17*c^2 - 7680*a^3*b^15
*c^3 + 53760*a^4*b^13*c^4 - 258048*a^5*b^11*c^5 + 860160*a^6*b^9*c^6 - 1966
080*a^7*b^7*c^7 + 2949120*a^8*b^5*c^8 - 2621440*a^9*b^3*c^9 + 1048576*a^10*
```

```
b*c^10)*d^27 + sqrt((b^14 - 28*a*b^12*c + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c
^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 16384*a^7*c
^7)*d^18)*(b^14 - 28*a*b^12*c + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*
a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 16384*a^7*c^7)*d^18)*
((b^14 - 28*a*b^12*c + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c
^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 16384*a^7*c^7)*d^18)^(1/4))/((
b^14 - 28*a*b^12*c + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4
 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 16384*a^7*c^7)*d^18)) + ((b^14 -
 28*a*b^12*c + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 215
04*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 16384*a^7*c^7)*d^18)^(1/4)*log(-(b^10
- 20*a*b^8*c + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 1024*
a^5*c^5)*sqrt(2*c*d*x + b*d)*d^13 + ((b^14 - 28*a*b^12*c + 336*a^2*b^10*c^2
 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*
c^6 - 16384*a^7*c^7)*d^18)^(3/4)) - ((b^14 - 28*a*b^12*c + 336*a^2*b^10*c^2
 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*
c^6 - 16384*a^7*c^7)*d^18)^(1/4)*log(-(b^10 - 20*a*b^8*c + 160*a^2*b^6*c^2
- 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 1024*a^5*c^5)*sqrt(2*c*d*x + b*d)*d^
13 - ((b^14 - 28*a*b^12*c + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*
b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 16384*a^7*c^7)*d^18)^(3/4
))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(9/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25914, size = 612, normalized size = 4.16

$$\frac{4}{3}(2\,cdx+bd)^{\frac{3}{2}}b^2d^3-\frac{16}{3}(2\,cdx+bd)^{\frac{3}{2}}acd^3+\frac{4}{7}(2\,cdx+bd)^{\frac{7}{2}}d+\frac{1}{2}\sqrt{2}\left(b^2d^3-4\,acd^3\right)\left(-b^2d^2+4\,acd^2\right)^{\frac{3}{4}}\log\left(2\,cdx+bd\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 4/3*(2*c*d*x + b*d)^(3/2)*b^2*d^3 - 16/3*(2*c*d*x + b*d)^(3/2)*a*c*d^3 + 4/
7*(2*c*d*x + b*d)^(7/2)*d + 1/2*sqrt(2)*(b^2*d^3 - 4*a*c*d^3)*(-b^2*d^2 + 4
*a*c*d^2)^(3/4)*log(2*c*d*x + b*d + sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sq
rt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2)) - 1/2*sqrt(2)*(b^2*d^3 - 4*
a*c*d^3)*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*log(2*c*d*x + b*d - sqrt(2)*(-b^2*d^2
 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2)) - (sq
rt(2)*b^2*d^3 - 4*sqrt(2)*a*c*d^3)*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*arctan(1/2*
sqrt(2)*(sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^
2*d^2 + 4*a*c*d^2)^(1/4)) - (sqrt(2)*b^2*d^3 - 4*sqrt(2)*a*c*d^3)*(-b^2*d^2
 + 4*a*c*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/
4) - 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4))
```

3.1287 $\int \frac{(bd+2cdx)^{7/2}}{a+bx+cx^2} dx$

Optimal. Leaf size=145

$$4d^3\left(b^2-4ac\right)\sqrt{bd+2cdx}-2d^{7/2}\left(b^2-4ac\right)^{5/4}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-2d^{7/2}\left(b^2-4ac\right)^{5/4}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)+$$

```
[Out] 4*(b^2 - 4*a*c)*d^3*Sqrt[b*d + 2*c*d*x] + (4*d*(b*d + 2*c*d*x)^(5/2))/5 - 2
*(b^2 - 4*a*c)^(5/4)*d^(7/2)*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4
)*Sqrt[d])] - 2*(b^2 - 4*a*c)^(5/4)*d^(7/2)*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b
^2 - 4*a*c)^(1/4)*Sqrt[d])]
```

Rubi [A] time = 0.12059, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {692, 694, 329, 212, 206, 203}

$$4d^3\left(b^2-4ac\right)\sqrt{bd+2cdx}-2d^{7/2}\left(b^2-4ac\right)^{5/4}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-2d^{7/2}\left(b^2-4ac\right)^{5/4}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)+$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(7/2)/(a + b*x + c*x^2),x]
```

```
[Out] 4*(b^2 - 4*a*c)*d^3*Sqrt[b*d + 2*c*d*x] + (4*d*(b*d + 2*c*d*x)^(5/2))/5 - 2
*(b^2 - 4*a*c)^(5/4)*d^(7/2)*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4
)*Sqrt[d])] - 2*(b^2 - 4*a*c)^(5/4)*d^(7/2)*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b
^2 - 4*a*c)^(1/4)*Sqrt[d])]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
```

```
x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(bd+2cdx)^{7/2}}{a+bx+cx^2}\,dx = \frac{4}{5}d(bd+2cdx)^{5/2} + \left(\left(b^2-4ac\right)d^2\right)\int \frac{(bd+2cdx)^{3/2}}{a+bx+cx^2}\,dx$$

$$= 4\left(b^2-4ac\right)d^3\sqrt{bd+2cdx} + \frac{4}{5}d(bd+2cdx)^{5/2} + \left(\left(b^2-4ac\right)^2 d^4\right)\int \frac{1}{\sqrt{bd+2cdx}\left(a+bx+cx^2\right)}\,d$$

$$= 4\left(b^2-4ac\right)d^3\sqrt{bd+2cdx} + \frac{4}{5}d(bd+2cdx)^{5/2} + \frac{\left(\left(b^2-4ac\right)^2 d^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)}\,dx, x, bd\right.}{2c}$$

$$= 4\left(b^2-4ac\right)d^3\sqrt{bd+2cdx} + \frac{4}{5}d(bd+2cdx)^{5/2} + \frac{\left(\left(b^2-4ac\right)^2 d^3\right)\text{Subst}\left(\int \frac{1}{a-\frac{b^2}{4c}+\frac{x^4}{4cd^2}}\,dx, x, \sqrt{d(b\right.}{c}$$

$$= 4\left(b^2-4ac\right)d^3\sqrt{bd+2cdx} + \frac{4}{5}d(bd+2cdx)^{5/2} - \left(2\left(b^2-4ac\right)^{3/2} d^4\right)\text{Subst}\left(\int \frac{1}{\sqrt{b^2-4acd}-x^2}\,d\right.$$

$$= 4\left(b^2-4ac\right)d^3\sqrt{bd+2cdx} + \frac{4}{5}d(bd+2cdx)^{5/2} - 2\left(b^2-4ac\right)^{5/4} d^{7/2}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right) - 2\left(b^2\right.$$

Mathematica [A] time = 0.134548, size = 141, normalized size = 0.97

$$\frac{2d^3\sqrt{d(b+2cx)}\left(4\sqrt{b+2cx}\left(2c\left(cx^2-5a\right)+3b^2+2bcx\right)-5\left(b^2-4ac\right)^{5/4}\tan^{-1}\left(\frac{\sqrt{b+2cx}}{\sqrt[4]{b^2-4ac}}\right)-5\left(b^2-4ac\right)^{5/4}\tanh^{-1}\left(\frac{\sqrt{b+}}{\sqrt[4]{b^2-}}\right.\right.}{5\sqrt{b+2cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(7/2)/(a + b*x + c*x^2),x]
```

```
[Out] (2*d^3*Sqrt[d*(b + 2*c*x)]*(4*Sqrt[b + 2*c*x]*(3*b^2 + 2*b*c*x + 2*c*(-5*a
+ c*x^2)) - 5*(b^2 - 4*a*c)^(5/4)*ArcTan[Sqrt[b + 2*c*x]/(b^2 - 4*a*c)^(1/4
)] - 5*(b^2 - 4*a*c)^(5/4)*ArcTanh[Sqrt[b + 2*c*x]/(b^2 - 4*a*c)^(1/4)]))/(
5*Sqrt[b + 2*c*x])
```

Maple [B] time = 0.196, size = 922, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a),x)
```

```
[Out] 4/5*d*(2*c*d*x+b*d)^(5/2)-16*a*c*d^3*(2*c*d*x+b*d)^(1/2)+4*b^2*d^3*(2*c*d*x
+b*d)^(1/2)+16*d^5/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(2^(1/2)/(4*a*c*
d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*a^2*c^2-8*d^5/(4*a*c*d^2-b^2*d^2)
^(3/4)*2^(1/2)*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)
+1)*a*b^2*c+d^5/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(2^(1/2)/(4*a*c*d^2
-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*b^4-16*d^5/(4*a*c*d^2-b^2*d^2)^(3/4)
*2^(1/2)*arctan(-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*a
^2*c^2+8*d^5/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(-2^(1/2)/(4*a*c*d^2-b
^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*a*b^2*c-d^5/(4*a*c*d^2-b^2*d^2)^(3/4)*
2^(1/2)*arctan(-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*b^
4+8*d^5/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*ln((2*c*d*x+b*d+(4*a*c*d^2-b^2*d^
2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*
d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)
^(1/2)))*a^2*c^2-4*d^5/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*ln((2*c*d*x+b*d+(4
*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/
2))/(2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a
*c*d^2-b^2*d^2)^(1/2)))*a*b^2*c+1/2*d^5/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*l
n((2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c
*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(
1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)))*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.10053, size = 1671, normalized size = 11.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 8/5*(2*c^2*d^3*x^2 + 2*b*c*d^3*x + (3*b^2 - 10*a*c)*d^3)*sqrt(2*c*d*x + b*d
) + 4*((b^10 - 20*a*b^8*c + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^
2*c^4 - 1024*a^5*c^5)*d^14)^(1/4)*arctan(-(((b^10 - 20*a*b^8*c + 160*a^2*b^
6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 1024*a^5*c^5)*d^14)^(3/4)*sqrt
(2*c*d*x + b*d)*(b^2 - 4*a*c)*d^3 + ((b^10 - 20*a*b^8*c + 160*a^2*b^6*c^2 -
 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 1024*a^5*c^5)*d^14)^(3/4)*sqrt(2*(b^4
*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^7*x + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^7
 + sqrt((b^10 - 20*a*b^8*c + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b
^2*c^4 - 1024*a^5*c^5)*d^14)))/((b^10 - 20*a*b^8*c + 160*a^2*b^6*c^2 - 640*
a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 1024*a^5*c^5)*d^14)) + ((b^10 - 20*a*b^8*c
 + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 1024*a^5*c^5)*d^1
```

```
4)^(1/4)*log(-sqrt(2*c*d*x + b*d)*(b^2 - 4*a*c)*d^3 + ((b^10 - 20*a*b^8*c +
 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 1024*a^5*c^5)*d^14)
^(1/4)) - ((b^10 - 20*a*b^8*c + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^
4*b^2*c^4 - 1024*a^5*c^5)*d^14)^(1/4)*log(-sqrt(2*c*d*x + b*d)*(b^2 - 4*a*c
)*d^3 - ((b^10 - 20*a*b^8*c + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*
b^2*c^4 - 1024*a^5*c^5)*d^14)^(1/4))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(7/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19183, size = 612, normalized size = 4.22

$$4\sqrt{2cdx+bd}b^2d^3 - 16\sqrt{2cdx+bd}acd^3 + \frac{4}{5}(2cdx+bd)^{\frac{5}{2}}d - \frac{1}{2}\sqrt{2}\left(b^2d^3 - 4acd^3\right)\left(-b^2d^2+4acd^2\right)^{\frac{1}{4}}\log\left(2cdx+bd+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 4*sqrt(2*c*d*x + b*d)*b^2*d^3 - 16*sqrt(2*c*d*x + b*d)*a*c*d^3 + 4/5*(2*c*d
*x + b*d)^(5/2)*d - 1/2*sqrt(2)*(b^2*d^3 - 4*a*c*d^3)*(-b^2*d^2 + 4*a*c*d^2
)^(1/4)*log(2*c*d*x + b*d + sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d
*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2)) + 1/2*sqrt(2)*(b^2*d^3 - 4*a*c*d^3)
*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*log(2*c*d*x + b*d - sqrt(2)*(-b^2*d^2 + 4*a*c
*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2)) - (sqrt(2)*b^
2*d^3 - 4*sqrt(2)*a*c*d^3)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*arctan(1/2*sqrt(2)*
(sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 +
4*a*c*d^2)^(1/4)) - (sqrt(2)*b^2*d^3 - 4*sqrt(2)*a*c*d^3)*(-b^2*d^2 + 4*a*c
*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) - 2*s
qrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4))
```

3.1288 $\int \frac{(bd+2cdx)^{5/2}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=119

$$2d^{5/2}\left(b^2-4ac\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-2d^{5/2}\left(b^2-4ac\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)+\frac{4}{3}d(bd+2cdx)^{3/2}$$

```
[Out] (4*d*(b*d + 2*c*d*x)^(3/2))/3 + 2*(b^2 - 4*a*c)^(3/4)*d^(5/2)*ArcTan[Sqrt[d
*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])] - 2*(b^2 - 4*a*c)^(3/4)*d^(5/2
)*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])]
```

Rubi [A] time = 0.0956372, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {692, 694, 329, 298, 203, 206}

$$2d^{5/2}\left(b^2-4ac\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-2d^{5/2}\left(b^2-4ac\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)+\frac{4}{3}d(bd+2cdx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(5/2)/(a + b*x + c*x^2),x]
```

```
[Out] (4*d*(b*d + 2*c*d*x)^(3/2))/3 + 2*(b^2 - 4*a*c)^(3/4)*d^(5/2)*ArcTan[Sqrt[d
*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])] - 2*(b^2 - 4*a*c)^(3/4)*d^(5/2
)*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
```

```
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(bd+2cdx)^{5/2}}{a+bx+cx^2}\,dx = \frac{4}{3}d(bd+2cdx)^{3/2} + \left(\left(b^2-4ac\right)d^2\right)\int \frac{\sqrt{bd+2cdx}}{a+bx+cx^2}\,dx$$

$$= \frac{4}{3}d(bd+2cdx)^{3/2} + \frac{\left(\left(b^2-4ac\right)d\right)\text{Subst}\left(\int \frac{\sqrt{x}}{a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}}\,dx, x, bd+2cdx\right)}{2c}$$

$$= \frac{4}{3}d(bd+2cdx)^{3/2} + \frac{\left(\left(b^2-4ac\right)d\right)\text{Subst}\left(\int \frac{x^2}{a-\frac{b^2}{4c}+\frac{x^4}{4cd^2}}\,dx, x, \sqrt{d(b+2cx)}\right)}{c}$$

$$= \frac{4}{3}d(bd+2cdx)^{3/2} - \left(2\left(b^2-4ac\right)d^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{b^2-4ac}d-x^2}\,dx, x, \sqrt{d(b+2cx)}\right) + \left(2\left(b^2-4a\right.\right.$$

$$= \frac{4}{3}d(bd+2cdx)^{3/2} + 2\left(b^2-4ac\right)^{3/4}d^{5/2}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right) - 2\left(b^2-4ac\right)^{3/4}d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d(b}}{\sqrt[4]{b^2-}}\right.$$

Mathematica [A] time = 0.135752, size = 104, normalized size = 0.87

$$\frac{2(d(b+2cx))^{5/2}\left(3\left(b^2-4ac\right)^{3/4}\left(\tan^{-1}\left(\frac{\sqrt{b+2cx}}{\sqrt[4]{b^2-4ac}}\right)-\tanh^{-1}\left(\frac{\sqrt{b+2cx}}{\sqrt[4]{b^2-4ac}}\right)\right)+2(b+2cx)^{3/2}\right)}{3(b+2cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(5/2)/(a + b*x + c*x^2),x]
```

```
[Out] (2*(d*(b + 2*c*x))^(5/2)*(2*(b + 2*c*x)^(3/2) + 3*(b^2 - 4*a*c)^(3/4)*(ArcT
an[Sqrt[b + 2*c*x]/(b^2 - 4*a*c)^(1/4)] - ArcTanh[Sqrt[b + 2*c*x]/(b^2 - 4*
a*c)^(1/4)])))/(3*(b + 2*c*x)^(5/2))
```

Maple [B] time = 0.196, size = 582, normalized size = 4.9

$$\frac{4d}{3}(2cdx+bd)^{\frac{3}{2}} - 4\frac{d^3\sqrt{2}ac}{\sqrt[4]{4acd^2-b^2d^2}}\arctan\left(\frac{\sqrt{2}\sqrt{2cdx+bd}}{\sqrt[4]{4acd^2-b^2d^2}}+1\right) + d^3\sqrt{2}b^2\arctan\left(\sqrt{2}\sqrt{2cdx+bd}\frac{1}{\sqrt[4]{4acd^2-b^2d^2}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a),x)
```

```
[Out] 4/3*d*(2*c*d*x+b*d)^(3/2)-4*d^3/(4*a*c*d^2-b^2*d^2)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*a*c+d^3/(4*a*c*d^2-b
^2*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d
)^(1/2)+1)*b^2+4*d^3/(4*a*c*d^2-b^2*d^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(4*a
*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*a*c-d^3/(4*a*c*d^2-b^2*d^2)^(1
/4)*2^(1/2)*arctan(-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1
)*b^2-2*d^3/(4*a*c*d^2-b^2*d^2)^(1/4)*2^(1/2)*ln((2*c*d*x+b*d-(4*a*c*d^2-b^
2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*
x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*
d^2)^(1/2)))*a*c+1/2*d^3/(4*a*c*d^2-b^2*d^2)^(1/4)*2^(1/2)*ln((2*c*d*x+b*d-
(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(
1/2))/(2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4
*a*c*d^2-b^2*d^2)^(1/2)))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.04912, size = 1369, normalized size = 11.5

$$\frac{4}{3}\left(2\,cd^2x+bd^2\right)\sqrt{2\,cdx+bd}+4\,\left(\left(b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3\right)d^{10}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(\left(b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 4/3*(2*c*d^2*x + b*d^2)*sqrt(2*c*d*x + b*d) + 4*((b^6 - 12*a*b^4*c + 48*a^2
*b^2*c^2 - 64*a^3*c^3)*d^10)^(1/4)*arctan((((b^6 - 12*a*b^4*c + 48*a^2*b^2*
c^2 - 64*a^3*c^3)*d^10)^(1/4)*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(2*c*d*x +
 b*d)*d^7 - sqrt(2*(b^8*c - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4
 + 256*a^4*c^5)*d^15*x + (b^9 - 16*a*b^7*c + 96*a^2*b^5*c^2 - 256*a^3*b^3*c
^3 + 256*a^4*b*c^4)*d^15 + sqrt((b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3
*c^3)*d^10)*(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*d^10)*((b^6 -
12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*d^10)^(1/4))/((b^6 - 12*a*b^4*c +
 48*a^2*b^2*c^2 - 64*a^3*c^3)*d^10)) - ((b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2
- 64*a^3*c^3)*d^10)^(1/4)*log((b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(2*c*d*x +
 b*d)*d^7 + ((b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*d^10)^(3/4))
+ ((b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*d^10)^(1/4)*log((b^4 -
8*a*b^2*c + 16*a^2*c^2)*sqrt(2*c*d*x + b*d)*d^7 - ((b^6 - 12*a*b^4*c + 48*a
^2*b^2*c^2 - 64*a^3*c^3)*d^10)^(3/4))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(5/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23923, size = 478, normalized size = 4.02

$$-\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{3}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}+2\sqrt{2\,cdx+bd}\right)}{2\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}}\right)-\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{3}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(-b^2*d
^2 + 4*a*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4)
) - sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(-b
^2*d^2 + 4*a*c*d^2)^(1/4) - 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(
1/4)) + 1/2*sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*d*log(2*c*d*x + b*d + sqrt
(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*
c*d^2)) - 1/2*sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*d*log(2*c*d*x + b*d - sq
rt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*
a*c*d^2)) + 4/3*(2*c*d*x + b*d)^(3/2)*d
```

3.1289 $\int \frac{(bd+2cdx)^{3/2}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=117

$$-2d^{3/2}\sqrt[4]{b^2-4ac}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-2d^{3/2}\sqrt[4]{b^2-4ac}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)+4d\sqrt{bd+2cdx}$$

```
[Out] 4*d*Sqrt[b*d + 2*c*d*x] - 2*(b^2 - 4*a*c)^(1/4)*d^(3/2)*ArcTan[Sqrt[d*(b +
2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])] - 2*(b^2 - 4*a*c)^(1/4)*d^(3/2)*ArcT
anh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])]
```

Rubi [A] time = 0.0963116, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {692, 694, 329, 212, 206, 203}

$$-2d^{3/2}\sqrt[4]{b^2-4ac}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-2d^{3/2}\sqrt[4]{b^2-4ac}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)+4d\sqrt{bd+2cdx}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(3/2)/(a + b*x + c*x^2),x]
```

```
[Out] 4*d*Sqrt[b*d + 2*c*d*x] - 2*(b^2 - 4*a*c)^(1/4)*d^(3/2)*ArcTan[Sqrt[d*(b +
2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])] - 2*(b^2 - 4*a*c)^(1/4)*d^(3/2)*ArcT
anh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(bd+2cdx)^{3/2}}{a+bx+cx^2}\,dx = 4d\sqrt{bd+2cdx}+\left(\left(b^2-4ac\right)d^2\right)\int \frac{1}{\sqrt{bd+2cdx}\left(a+bx+cx^2\right)}\,dx$$

$$= 4d\sqrt{bd+2cdx}+\frac{\left(\left(b^2-4ac\right)d\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)}\,dx, x, bd+2cdx\right)}{2c}$$

$$= 4d\sqrt{bd+2cdx}+\frac{\left(\left(b^2-4ac\right)d\right)\text{Subst}\left(\int \frac{1}{a-\frac{b^2}{4c}+\frac{x^4}{4cd^2}}\,dx, x, \sqrt{d(b+2cx)}\right)}{c}$$

$$= 4d\sqrt{bd+2cdx}-\left(2\sqrt{b^2-4ac}d^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{b^2-4ac}d-x^2}\,dx, x, \sqrt{d(b+2cx)}\right)-\left(2\sqrt{b^2-4ac}d^2\right.$$

$$= 4d\sqrt{bd+2cdx}-2\sqrt[4]{b^2-4ac}d^{3/2}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)-2\sqrt[4]{b^2-4ac}d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)$$

Mathematica [A] time = 0.0476966, size = 113, normalized size = 0.97

$$\frac{2(d(b+2cx))^{3/2}\left(-\sqrt[4]{b^2-4ac}\tan^{-1}\left(\frac{\sqrt{b+2cx}}{\sqrt[4]{b^2-4ac}}\right)-\sqrt[4]{b^2-4ac}\tanh^{-1}\left(\frac{\sqrt{b+2cx}}{\sqrt[4]{b^2-4ac}}\right)+2\sqrt{b+2cx}\right)}{(b+2cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(3/2)/(a + b*x + c*x^2),x]
```

```
[Out] (2*(d*(b + 2*c*x))^(3/2)*(2*Sqrt[b + 2*c*x] - (b^2 - 4*a*c)^(1/4)*ArcTan[Sq
rt[b + 2*c*x]/(b^2 - 4*a*c)^(1/4)] - (b^2 - 4*a*c)^(1/4)*ArcTanh[Sqrt[b + 2
*c*x]/(b^2 - 4*a*c)^(1/4)]))/(b + 2*c*x)^(3/2)
```

Maple [B] time = 0.19, size = 582, normalized size = 5.

$$4d\sqrt{2cdx+bd}+4\frac{d^3\sqrt{2}ac}{\left(4acd^2-b^2d^2\right)^{3/4}}\arctan\left(-\frac{\sqrt{2}\sqrt{2cdx+bd}}{\sqrt[4]{4acd^2-b^2d^2}}+1\right)-d^3\sqrt{2}b^2\arctan\left(-\sqrt{2}\sqrt{2cdx+bd}\frac{1}{\sqrt[4]{4acd^2-b^2d^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a),x)
```

```
[Out] 4*d*(2*c*d*x+b*d)^(1/2)+4*d^3/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(-2^(
1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*a*c-d^3/(4*a*c*d^2-b^
2*d^2)^(3/4)*2^(1/2)*arctan(-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d
)^(1/2)+1)*b^2-2*d^3/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*ln((2*c*d*x+b*d+(4*a
*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)
)/(2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c
*d^2-b^2*d^2)^(1/2)))*a*c+1/2*d^3/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*ln((2*c
*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b
^2*d^2)^(1/2))/(2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2
^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)))*b^2-4*d^3/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1
/2)*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*a*c+d^3
/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)
*(2*c*d*x+b*d)^(1/2)+1)*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.8926, size = 509, normalized size = 4.35

$$4\sqrt{2\,cdx+bd}d-4\left(\left(b^2-4\,ac\right)d^6\right)^{\frac{1}{4}}\arctan\left(\frac{\left(\left(b^2-4\,ac\right)d^6\right)^{\frac{3}{4}}\sqrt{2\,cdx+bd}d-\left(\left(b^2-4\,ac\right)d^6\right)^{\frac{3}{4}}\sqrt{2\,cd^3x+bd^3+\sqrt{\left(b^2\right.}}}{\left(b^2-4\,ac\right)d^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 4*sqrt(2*c*d*x + b*d)*d - 4*((b^2 - 4*a*c)*d^6)^(1/4)*arctan((((b^2 - 4*a*c
)*d^6)^(3/4)*sqrt(2*c*d*x + b*d)*d - ((b^2 - 4*a*c)*d^6)^(3/4)*sqrt(2*c*d^3
*x + b*d^3 + sqrt((b^2 - 4*a*c)*d^6)))/((b^2 - 4*a*c)*d^6)) - ((b^2 - 4*a*c
)*d^6)^(1/4)*log(sqrt(2*c*d*x + b*d)*d + ((b^2 - 4*a*c)*d^6)^(1/4)) + ((b^2
 - 4*a*c)*d^6)^(1/4)*log(sqrt(2*c*d*x + b*d)*d - ((b^2 - 4*a*c)*d^6)^(1/4))
```

Sympy [A] time = 37.3032, size = 212, normalized size = 1.81

$$-16acd^3\operatorname{RootSum}\left(t^4\left(16384a^3c^3d^6-12288a^2b^2c^2d^6+3072ab^4cd^6-256b^6d^6\right)+1,\left(t\mapsto t\log\left(16tacd^2-4tb^2d^2+\sqrt{\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(3/2)/(c*x**2+b*x+a),x)
```

```
[Out] -16*a*c*d**3*RootSum(_t**4*(16384*a**3*c**3*d**6 - 12288*a**2*b**2*c**2*d**
6 + 3072*a*b**4*c*d**6 - 256*b**6*d**6) + 1, Lambda(_t, _t*log(16*_t*a*c*d*
*2 - 4*_t*b**2*d**2 + sqrt(b*d + 2*c*d*x)))) + 4*b**2*d**3*RootSum(_t**4*(1
6384*a**3*c**3*d**6 - 12288*a**2*b**2*c**2*d**6 + 3072*a*b**4*c*d**6 - 256*
b**6*d**6) + 1, Lambda(_t, _t*log(16*_t*a*c*d**2 - 4*_t*b**2*d**2 + sqrt(b*
d + 2*c*d*x)))) + 4*d*sqrt(b*d + 2*c*d*x)
```

Giac [B] time = 1.23847, size = 478, normalized size = 4.09

$$-\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}+2\sqrt{2\,cdx+bd}\right)}{2\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}}\right)-\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(-b^2*d
^2 + 4*a*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4)
) - sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(-b
^2*d^2 + 4*a*c*d^2)^(1/4) - 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(
1/4)) - 1/2*sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*d*log(2*c*d*x + b*d + sqrt
(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*
c*d^2)) + 1/2*sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*d*log(2*c*d*x + b*d - sq
rt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*
a*c*d^2)) + 4*sqrt(2*c*d*x + b*d)*d
```

3.1290 $\int \frac{\sqrt{bd+2cdx}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=101

$$\frac{2\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\sqrt[4]{b^2-4ac}}-\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\sqrt[4]{b^2-4ac}}$$

```
[Out] (2*Sqrt[d]*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(b^2
- 4*a*c)^(1/4) - (2*Sqrt[d]*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4
)*Sqrt[d])])/(b^2 - 4*a*c)^(1/4)
```

Rubi [A] time = 0.0722369, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {694, 329, 298, 203, 206}

$$\frac{2\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\sqrt[4]{b^2-4ac}}-\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\sqrt[4]{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*d + 2*c*d*x]/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d]*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(b^2
- 4*a*c)^(1/4) - (2*Sqrt[d]*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4
)*Sqrt[d])])/(b^2 - 4*a*c)^(1/4)
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{bd+2cdx}}{a+bx+cx^2}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}}\,dx, x, bd+2cdx\right)}{2cd}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{a-\frac{b^2}{4c}+\frac{x^4}{4cd^2}}\,dx, x, \sqrt{d(b+2cx)}\right)}{cd}$$
$$= -\left((2d)\,\text{Subst}\left(\int \frac{1}{\sqrt{b^2-4acd}-x^2}\,dx, x, \sqrt{d(b+2cx)}\right)\right) + (2d)\,\text{Subst}\left(\int \frac{1}{\sqrt{b^2-4acd}+x^2}\,dx, x, \sqrt{}\right.$$
$$= \frac{2\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)}{\sqrt[4]{b^2-4ac}} - \frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)}{\sqrt[4]{b^2-4ac}}$$

Mathematica [A] time = 0.0386442, size = 87, normalized size = 0.86

$$\frac{2\sqrt{d(b+2cx)}\left(\tan^{-1}\left(\frac{\sqrt{b+2cx}}{\sqrt[4]{b^2-4ac}}\right)-\tanh^{-1}\left(\frac{\sqrt{b+2cx}}{\sqrt[4]{b^2-4ac}}\right)\right)}{\sqrt[4]{b^2-4ac}\sqrt{b+2cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*d + 2*c*d*x]/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d*(b + 2*c*x)]*(ArcTan[Sqrt[b + 2*c*x]/(b^2 - 4*a*c)^(1/4)] - ArcTa
nh[Sqrt[b + 2*c*x]/(b^2 - 4*a*c)^(1/4)]))/((b^2 - 4*a*c)^(1/4)*Sqrt[b + 2*c
*x])
```

Maple [B] time = 0.193, size = 271, normalized size = 2.7

$$\frac{d\sqrt{2}}{2}\ln\left(\left(2\,cdx+bd-\sqrt[4]{4\,acd^2-b^2d^2}\sqrt{2\,cdx+bd}\sqrt{2}+\sqrt{4\,acd^2-b^2d^2}\right)\left(2\,cdx+bd+\sqrt[4]{4\,acd^2-b^2d^2}\sqrt{2\,cdx+bd}\sqrt{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a),x)
```

```
[Out] 1/2*d/(4*a*c*d^2-b^2*d^2)^(1/4)*2^(1/2)*ln((2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)
^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d+
(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(
1/2)))+d/(4*a*c*d^2-b^2*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^
2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)-d/(4*a*c*d^2-b^2*d^2)^(1/4)*2^(1/2)*arctan(
-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.00982, size = 518, normalized size = 5.13

$$-4\left(\frac{d^2}{b^2-4\,ac}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2\,cdx+bd}d\left(\frac{d^2}{b^2-4\,ac}\right)^{\frac{1}{4}}-\sqrt{2\,cd^3x+bd^3+\left(b^2-4\,ac\right)d^2\sqrt{\frac{d^2}{b^2-4\,ac}}}\left(\frac{d^2}{b^2-4\,ac}\right)^{\frac{1}{4}}}{d^2}\right)-\left(\frac{d^2}{b^2-4\,ac}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -4*(d^2/(b^2 - 4*a*c))^(1/4)*arctan(-(sqrt(2*c*d*x + b*d)*d*(d^2/(b^2 - 4*a
*c))^(1/4) - sqrt(2*c*d^3*x + b*d^3 + (b^2 - 4*a*c)*d^2*sqrt(d^2/(b^2 - 4*a
*c)))*(d^2/(b^2 - 4*a*c))^(1/4))/d^2) - (d^2/(b^2 - 4*a*c))^(1/4)*log((b^2
- 4*a*c)*(d^2/(b^2 - 4*a*c))^(3/4) + sqrt(2*c*d*x + b*d)*d) + (d^2/(b^2 - 4
*a*c))^(1/4)*log(-(b^2 - 4*a*c)*(d^2/(b^2 - 4*a*c))^(3/4) + sqrt(2*c*d*x +
b*d)*d)
```

Sympy [A] time = 4.54553, size = 65, normalized size = 0.64

$$4d\operatorname{RootSum}\left(t^4\left(1024acd^2-256b^2d^2\right)+1,\left(t\mapsto t\log\left(256t^3acd^2-64t^3b^2d^2+\sqrt{bd+2cdx}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(1/2)/(c*x**2+b*x+a),x)
```

```
[Out] 4*d*RootSum(_t**4*(1024*a*c*d**2 - 256*b**2*d**2) + 1, Lambda(_t, _t*log(25
6*_t**3*a*c*d**2 - 64*_t**3*b**2*d**2 + sqrt(b*d + 2*c*d*x))))
```

Giac [B] time = 1.14661, size = 531, normalized size = 5.26

$$-\frac{\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}+2\,\sqrt{2\,cdx+bd}\right)}{2\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}}\right)}{b^2d-4\,acd}-\frac{\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}\right.}{2\left(-b^2d^2+4\,ac\right.}\right.}{b^2d-4\,acd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2
 + 4*a*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4))/
(b^2*d - 4*a*c*d) - sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) - 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2
+ 4*a*c*d^2)^(1/4))/(b^2*d - 4*a*c*d) + (-b^2*d^2 + 4*a*c*d^2)^(3/4)*log(2*
c*d*x + b*d + sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sq
rt(-b^2*d^2 + 4*a*c*d^2))/(sqrt(2)*b^2*d - 4*sqrt(2)*a*c*d) - (-b^2*d^2 + 4
*a*c*d^2)^(3/4)*log(2*c*d*x + b*d - sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sq
rt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2))/(sqrt(2)*b^2*d - 4*sqrt(2)*
a*c*d)
```

3.1291 $\int \frac{1}{\sqrt{bd+2cdx}\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=101

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\sqrt{d}\left(b^2-4ac\right)^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\sqrt{d}\left(b^2-4ac\right)^{3/4}}$$

```
[Out] (-2*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*
c)^(3/4)*Sqrt[d]) - (2*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqr
t[d])])/((b^2 - 4*a*c)^(3/4)*Sqrt[d])
```

Rubi [A] time = 0.0774185, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {694, 329, 212, 206, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\sqrt{d}\left(b^2-4ac\right)^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\sqrt{d}\left(b^2-4ac\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[b*d + 2*c*d*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*
c)^(3/4)*Sqrt[d]) - (2*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqr
t[d])])/((b^2 - 4*a*c)^(3/4)*Sqrt[d])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{bd+2cdx}\left(a+bx+cx^2\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)}\,dx, x, bd+2cdx\right)}{2cd}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{a-\frac{b^2}{4c}+\frac{x^4}{4cd^2}}\,dx, x, \sqrt{d(b+2cx)}\right)}{cd}$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{b^2-4acd}-x^2}\,dx, x, \sqrt{d(b+2cx)}\right)}{\sqrt{b^2-4ac}} - \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{b^2-4acd}+x^2}\,dx, x, \sqrt{d(b+2
$$
$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)}{\left(b^2-4ac\right)^{3/4}\sqrt{d}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)}{\left(b^2-4ac\right)^{3/4}\sqrt{d}}$$

Mathematica [A] time = 0.0503212, size = 85, normalized size = 0.84

$$-\frac{2\sqrt{b+2cx}\left(\tan^{-1}\left(\frac{\sqrt{b+2cx}}{\sqrt[4]{b^2-4ac}}\right)+\tanh^{-1}\left(\frac{\sqrt{b+2cx}}{\sqrt[4]{b^2-4ac}}\right)\right)}{\left(b^2-4ac\right)^{3/4}\sqrt{d(b+2cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[b*d + 2*c*d*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*Sqrt[b + 2*c*x]*(ArcTan[Sqrt[b + 2*c*x]/(b^2 - 4*a*c)^(1/4)] + ArcTanh[
Sqrt[b + 2*c*x]/(b^2 - 4*a*c)^(1/4)]))/((b^2 - 4*a*c)^(3/4)*Sqrt[d*(b + 2*c
*x)])
```

Maple [B] time = 0.19, size = 271, normalized size = 2.7

$$\frac{d\sqrt{2}}{2}\ln\left(\left(2\,cdx+bd+\sqrt[4]{4\,acd^2-b^2d^2}\sqrt{2\,cdx+bd}\sqrt{2}+\sqrt{4\,acd^2-b^2d^2}\right)\left(2\,cdx+bd-\sqrt[4]{4\,acd^2-b^2d^2}\sqrt{2\,cdx+bd}\sqrt{2}+
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a),x)
```

```
[Out] 1/2*d/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*ln((2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)
^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d-
(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(
1/2)))+d/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^
2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)-d/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(
-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.26901, size = 1033, normalized size = 10.23

$$4\left(\frac{1}{\left(b^{6}-12\,ab^{4}c+48\,a^{2}b^{2}c^{2}-64\,a^{3}c^{3}\right)d^{2}}\right)^{\frac{1}{4}}\arctan\left(\left(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2}\right)\sqrt{\left(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2}\right)d^{2}\sqrt{\frac{1}{\left(b^{6}-12\,ab^{4}\right.}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 4*(1/((b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*d^2))^(1/4)*arctan((
b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt((b^4 - 8*a*b^2*c + 16*a^2*c^2)*d^2*sqrt(
1/((b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*d^2)) + 2*c*d*x + b*d)*
d*(1/((b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*d^2))^(3/4) - (b^4 -
 8*a*b^2*c + 16*a^2*c^2)*sqrt(2*c*d*x + b*d)*d*(1/((b^6 - 12*a*b^4*c + 48*a
^2*b^2*c^2 - 64*a^3*c^3)*d^2))^(3/4)) - (1/((b^6 - 12*a*b^4*c + 48*a^2*b^2*
c^2 - 64*a^3*c^3)*d^2))^(1/4)*log((b^2 - 4*a*c)*d*(1/((b^6 - 12*a*b^4*c + 4
8*a^2*b^2*c^2 - 64*a^3*c^3)*d^2))^(1/4) + sqrt(2*c*d*x + b*d)) + (1/((b^6 -
 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*d^2))^(1/4)*log(-(b^2 - 4*a*c)*d
*(1/((b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*d^2))^(1/4) + sqrt(2*
c*d*x + b*d))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d\left(b+2cx\right)}\left(a+bx+cx^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**(1/2)/(c*x**2+b*x+a),x)
```

```
[Out] Integral(1/(sqrt(d*(b + 2*c*x))*(a + b*x + c*x**2)), x)
```

Giac [B] time = 1.13272, size = 531, normalized size = 5.26

$$-\frac{\sqrt{2}\left(-b^{2}d^{2}+4\,acd^{2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-b^{2}d^{2}+4\,acd^{2}\right)^{\frac{1}{4}}+2\,\sqrt{2\,cdx+bd}\right)}{2\left(-b^{2}d^{2}+4\,acd^{2}\right)^{\frac{1}{4}}}\right)}{b^{2}d-4\,acd}-\frac{\sqrt{2}\left(-b^{2}d^{2}+4\,acd^{2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-b^{2}d^{2}+4\,acd^{2}\right)^{\frac{1}{4}}\right.}{2\left(-b^{2}d^{2}+4\,ac\right.}\right.}{b^{2}d-4\,acd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2
 + 4*a*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4))/
(b^2*d - 4*a*c*d) - sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) - 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2
+ 4*a*c*d^2)^(1/4))/(b^2*d - 4*a*c*d) - (-b^2*d^2 + 4*a*c*d^2)^(1/4)*log(2*
c*d*x + b*d + sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sq
rt(-b^2*d^2 + 4*a*c*d^2))/(sqrt(2)*b^2*d - 4*sqrt(2)*a*c*d) + (-b^2*d^2 + 4
*a*c*d^2)^(1/4)*log(2*c*d*x + b*d - sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sq
rt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2))/(sqrt(2)*b^2*d - 4*sqrt(2)*
a*c*d)
```

3.1292 $\int \frac{1}{(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=129

$$\frac{2\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{3/2}\left(b^2-4ac\right)^{5/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{3/2}\left(b^2-4ac\right)^{5/4}}+\frac{4}{d\left(b^2-4ac\right)\sqrt{bd+2cdx}}$$

```
[Out] 4/((b^2 - 4*a*c)*d*Sqrt[b*d + 2*c*d*x]) + (2*ArcTan[Sqrt[d*(b + 2*c*x)]/((b
^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(5/4)*d^(3/2)) - (2*ArcTanh[Sqr
t[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(5/4)*d^(3/
2))
```

Rubi [A] time = 0.108493, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {693, 694, 329, 298, 203, 206}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{3/2}\left(b^2-4ac\right)^{5/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{3/2}\left(b^2-4ac\right)^{5/4}}+\frac{4}{d\left(b^2-4ac\right)\sqrt{bd+2cdx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)),x]
```

```
[Out] 4/((b^2 - 4*a*c)*d*Sqrt[b*d + 2*c*d*x]) + (2*ArcTan[Sqrt[d*(b + 2*c*x)]/((b
^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(5/4)*d^(3/2)) - (2*ArcTanh[Sqr
t[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(5/4)*d^(3/
2))
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)}\,dx &= \frac{4}{\left(b^2-4ac\right)d\sqrt{bd+2cdx}}+\frac{\int \frac{\sqrt{bd+2cdx}}{a+bx+cx^2}\,dx}{\left(b^2-4ac\right)d^2}\\
&= \frac{4}{\left(b^2-4ac\right)d\sqrt{bd+2cdx}}+\frac{\text{Subst}\left(\int \frac{\sqrt{x}}{a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}}\,dx,x,bd+2cdx\right)}{2c\left(b^2-4ac\right)d^3}\\
&= \frac{4}{\left(b^2-4ac\right)d\sqrt{bd+2cdx}}+\frac{\text{Subst}\left(\int \frac{x^2}{a-\frac{b^2}{4c}+\frac{x^4}{4cd^2}}\,dx,x,\sqrt{d(b+2cx)}\right)}{c\left(b^2-4ac\right)d^3}\\
&= \frac{4}{\left(b^2-4ac\right)d\sqrt{bd+2cdx}}-\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{b^2-4ac}d-x^2}\,dx,x,\sqrt{d(b+2cx)}\right)}{\left(b^2-4ac\right)d}+\frac{2\,\text{Subst}\left(\right.}{}\\
&= \frac{4}{\left(b^2-4ac\right)d\sqrt{bd+2cdx}}+\frac{2\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)}{\left(b^2-4ac\right)^{5/4}d^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)}{\left(b^2-4ac\right)^{5/4}d^{3/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0447679, size = 54, normalized size = 0.42

$$
\frac{4\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{d\left(b^2-4ac\right)\sqrt{d(b+2cx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)),x]
```

```
[Out] (4*Hypergeometric2F1[-1/4, 1, 3/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/((b^2 - 4*
a*c)*d*Sqrt[d*(b + 2*c*x)])
```

Maple [B] time = 0.195, size = 341, normalized size = 2.6

$$-\frac{\sqrt{2}}{2\,d\left(4\,ac-b^{2}\right)}\ln\left(\left(2\,cdx+bd-\sqrt[4]{4\,acd^{2}-b^{2}d^{2}}\sqrt{2\,cdx+bd}\sqrt{2}+\sqrt{4\,acd^{2}-b^{2}d^{2}}\right)\left(2\,cdx+bd+\sqrt[4]{4\,acd^{2}-b^{2}d^{2}}\sqrt{2\,c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a),x)
```

```
[Out] -1/2/d/(4*a*c-b^2)/(4*a*c*d^2-b^2*d^2)^(1/4)*2^(1/2)*ln((2*c*d*x+b*d-(4*a*c
*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2))/
(2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d
^2-b^2*d^2)^(1/2)))-1/d/(4*a*c-b^2)/(4*a*c*d^2-b^2*d^2)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)+1/d/(4*a*c-b^2)/
(4*a*c*d^2-b^2*d^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)
*(2*c*d*x+b*d)^(1/2)+1)-4/d/(4*a*c-b^2)/(2*c*d*x+b*d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.38624, size = 1912, normalized size = 14.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] (4*(2*(b^2*c - 4*a*c^2)*d^2*x + (b^3 - 4*a*b*c)*d^2)*(1/((b^10 - 20*a*b^8*c
 + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 1024*a^5*c^5)*d^6
))^(1/4)*arctan(-sqrt((b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*d^4*
sqrt(1/((b^10 - 20*a*b^8*c + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b
^2*c^4 - 1024*a^5*c^5)*d^6)) + 2*c*d*x + b*d)*(b^2 - 4*a*c)*d*(1/((b^10 - 2
0*a*b^8*c + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 1024*a^5
*c^5)*d^6))^(1/4) + sqrt(2*c*d*x + b*d)*(b^2 - 4*a*c)*d*(1/((b^10 - 20*a*b^
8*c + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 1024*a^5*c^5)*
d^6))^(1/4)) - (2*(b^2*c - 4*a*c^2)*d^2*x + (b^3 - 4*a*b*c)*d^2)*(1/((b^10
- 20*a*b^8*c + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 1024*
a^5*c^5)*d^6))^(1/4)*log((b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c
^3 + 256*a^4*c^4)*d^5*(1/((b^10 - 20*a*b^8*c + 160*a^2*b^6*c^2 - 640*a^3*b^
4*c^3 + 1280*a^4*b^2*c^4 - 1024*a^5*c^5)*d^6))^(3/4) + sqrt(2*c*d*x + b*d))
 + (2*(b^2*c - 4*a*c^2)*d^2*x + (b^3 - 4*a*b*c)*d^2)*(1/((b^10 - 20*a*b^8*c
 + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 1024*a^5*c^5)*d^6
))^(1/4)*log(-(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^
4*c^4)*d^5*(1/((b^10 - 20*a*b^8*c + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 128
0*a^4*b^2*c^4 - 1024*a^5*c^5)*d^6))^(3/4) + sqrt(2*c*d*x + b*d)) + 4*sqrt(2
```

```
*c*d*x + b*d))/(2*(b^2*c - 4*a*c^2)*d^2*x + (b^3 - 4*a*b*c)*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d(b+2cx))^{\frac{3}{2}}\left(a+bx+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**(3/2)/(c*x**2+b*x+a),x)
```

```
[Out] Integral(1/((d*(b + 2*c*x))**(3/2)*(a + b*x + c*x**2)), x)
```

Giac [B] time = 1.15078, size = 671, normalized size = 5.2

$$-\frac{\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}+2\,\sqrt{2\,cdx+bd}\right)}{2\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}}\right)}{b^4d^3-8\,ab^2cd^3+16\,a^2c^2d^3}-\frac{\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}-2\,\sqrt{}\right.}{2\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}}\right.}{b^4d^3-8\,ab^2cd^3+16\,a^2c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2
 + 4*a*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4))/
(b^4*d^3 - 8*a*b^2*c*d^3 + 16*a^2*c^2*d^3) - sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)
^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) - 2*sqrt(2
*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4))/(b^4*d^3 - 8*a*b^2*c*d^3 + 16*
a^2*c^2*d^3) + (-b^2*d^2 + 4*a*c*d^2)^(3/4)*log(2*c*d*x + b*d + sqrt(2)*(-b
^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2))
/(sqrt(2)*b^4*d^3 - 8*sqrt(2)*a*b^2*c*d^3 + 16*sqrt(2)*a^2*c^2*d^3) - (-b^2
*d^2 + 4*a*c*d^2)^(3/4)*log(2*c*d*x + b*d - sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^
(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2))/(sqrt(2)*b^4*d^3 -
8*sqrt(2)*a*b^2*c*d^3 + 16*sqrt(2)*a^2*c^2*d^3) + 4/((b^2*d - 4*a*c*d)*sqrt
(2*c*d*x + b*d))
```

3.1293 $\int \frac{1}{(bd+2cdx)^{5/2}\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=131

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{5/2}\left(b^2-4ac\right)^{7/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{5/2}\left(b^2-4ac\right)^{7/4}}+\frac{4}{3d\left(b^2-4ac\right)(bd+2cdx)^{3/2}}$$

```
[Out] 4/(3*(b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(3/2)) - (2*ArcTan[Sqrt[d*(b + 2*c*x)]
/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(7/4)*d^(5/2)) - (2*ArcTanh
[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(7/4)*d
^(5/2))
```

Rubi [A] time = 0.107208, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {693, 694, 329, 212, 206, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{5/2}\left(b^2-4ac\right)^{7/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{5/2}\left(b^2-4ac\right)^{7/4}}+\frac{4}{3d\left(b^2-4ac\right)(bd+2cdx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^(5/2)*(a + b*x + c*x^2)),x]
```

```
[Out] 4/(3*(b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(3/2)) - (2*ArcTan[Sqrt[d*(b + 2*c*x)]
/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(7/4)*d^(5/2)) - (2*ArcTanh
[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(7/4)*d
^(5/2))
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)^{5/2}\left(a+bx+cx^2\right)}\,dx = \frac{4}{3\left(b^2-4ac\right)d(bd+2cdx)^{3/2}}+\frac{\int \frac{1}{\sqrt{bd+2cdx}\left(a+bx+cx^2\right)}\,dx}{\left(b^2-4ac\right)d^2}$$

$$= \frac{4}{3\left(b^2-4ac\right)d(bd+2cdx)^{3/2}}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)}\,dx,x,bd+2cdx\right)}{2c\left(b^2-4ac\right)d^3}$$

$$= \frac{4}{3\left(b^2-4ac\right)d(bd+2cdx)^{3/2}}+\frac{\text{Subst}\left(\int \frac{1}{a-\frac{b^2}{4c}+\frac{x^4}{4cd^2}}\,dx,x,\sqrt{d(b+2cx)}\right)}{c\left(b^2-4ac\right)d^3}$$

$$= \frac{4}{3\left(b^2-4ac\right)d(bd+2cdx)^{3/2}}-\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{b^2-4acd}-x^2}\,dx,x,\sqrt{d(b+2cx)}\right)}{\left(b^2-4ac\right)^{3/2}d^2}-\frac{2\,\text{Subs}}{}$$

$$= \frac{4}{3\left(b^2-4ac\right)d(bd+2cdx)^{3/2}}-\frac{2\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)}{\left(b^2-4ac\right)^{7/4}d^{5/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)}{\left(b^2-4ac\right)^{7/4}d^{5/2}}$$

Mathematica [C] time = 0.0528148, size = 56, normalized size = 0.43

$$\frac{4\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{3d\left(b^2-4ac\right)(d(b+2cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^(5/2)*(a + b*x + c*x^2)),x]
```

```
[Out] (4*Hypergeometric2F1[-3/4, 1, 1/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(3*(b^2 -
4*a*c)*d*(d*(b + 2*c*x))^(3/2))
```

Maple [B] time = 0.192, size = 341, normalized size = 2.6

$$-\frac{4}{3\,d\left(4\,ac-b^{2}\right)}\left(2\,cdx+bd\right)^{-\frac{3}{2}}-\frac{\sqrt{2}}{2\,d\left(4\,ac-b^{2}\right)}\ln\left(\left(2\,cdx+bd+\sqrt[4]{4\,acd^{2}-b^{2}d^{2}}\sqrt{2\,cdx+bd}\sqrt{2}+\sqrt{4\,acd^{2}-b^{2}d^{2}}\right)\right($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a),x)
```

```
[Out] -4/3/d/(4*a*c-b^2)/(2*c*d*x+b*d)^(3/2)-1/2/d/(4*a*c-b^2)/(4*a*c*d^2-b^2*d^2
)^(3/4)*2^(1/2)*ln((2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/
2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4
)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)))-1/d/(4*a*c-b^2)/(
4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(
2*c*d*x+b*d)^(1/2)+1)+1/d/(4*a*c-b^2)/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arc
tan(-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.34273, size = 2742, normalized size = 20.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/3*(12*(4*(b^2*c^2 - 4*a*c^3)*d^3*x^2 + 4*(b^3*c - 4*a*b*c^2)*d^3*x + (b^
4 - 4*a*b^2*c)*d^3)*(1/((b^14 - 28*a*b^12*c + 336*a^2*b^10*c^2 - 2240*a^3*b
^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 16384*a
^7*c^7)*d^10))^(1/4)*arctan(-(b^10 - 20*a*b^8*c + 160*a^2*b^6*c^2 - 640*a^3
*b^4*c^3 + 1280*a^4*b^2*c^4 - 1024*a^5*c^5)*sqrt((b^8 - 16*a*b^6*c + 96*a^2
*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4)*d^6*sqrt(1/((b^14 - 28*a*b^12*c +
 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5
 + 28672*a^6*b^2*c^6 - 16384*a^7*c^7)*d^10)) + 2*c*d*x + b*d)*d^7*(1/((b^14
 - 28*a*b^12*c + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 2
1504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 16384*a^7*c^7)*d^10))^(3/4) + (b^10
- 20*a*b^8*c + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 1024*
a^5*c^5)*sqrt(2*c*d*x + b*d)*d^7*(1/((b^14 - 28*a*b^12*c + 336*a^2*b^10*c^2
 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*
c^6 - 16384*a^7*c^7)*d^10))^(3/4)) + 3*(4*(b^2*c^2 - 4*a*c^3)*d^3*x^2 + 4*(
b^3*c - 4*a*b*c^2)*d^3*x + (b^4 - 4*a*b^2*c)*d^3)*(1/((b^14 - 28*a*b^12*c +
 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5
 + 28672*a^6*b^2*c^6 - 16384*a^7*c^7)*d^10))^(1/4)*log((b^4 - 8*a*b^2*c + 1
6*a^2*c^2)*d^3*(1/((b^14 - 28*a*b^12*c + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^
```

```
3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 16384*a^7*c^
7)*d^10))^(1/4) + sqrt(2*c*d*x + b*d)) - 3*(4*(b^2*c^2 - 4*a*c^3)*d^3*x^2 +
 4*(b^3*c - 4*a*b*c^2)*d^3*x + (b^4 - 4*a*b^2*c)*d^3)*(1/((b^14 - 28*a*b^12
*c + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4
*c^5 + 28672*a^6*b^2*c^6 - 16384*a^7*c^7)*d^10))^(1/4)*log(-(b^4 - 8*a*b^2*
c + 16*a^2*c^2)*d^3*(1/((b^14 - 28*a*b^12*c + 336*a^2*b^10*c^2 - 2240*a^3*b
^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 16384*a
^7*c^7)*d^10))^(1/4) + sqrt(2*c*d*x + b*d)) - 4*sqrt(2*c*d*x + b*d))/(4*(b^
2*c^2 - 4*a*c^3)*d^3*x^2 + 4*(b^3*c - 4*a*b*c^2)*d^3*x + (b^4 - 4*a*b^2*c)*
d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**(5/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.1978, size = 671, normalized size = 5.12

$$-\frac{\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}+2\,\sqrt{2\,cdx+bd}\right)}{2\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}}\right)}{b^4d^3-8\,ab^2cd^3+16\,a^2c^2d^3}-\frac{\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}-2\,\sqrt{\cdots}\right)}{2\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}}\right)}{b^4d^3-8\,ab^2cd^3+16\,a^2c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2
 + 4*a*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4))/
(b^4*d^3 - 8*a*b^2*c*d^3 + 16*a^2*c^2*d^3) - sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)
^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) - 2*sqrt(2
*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4))/(b^4*d^3 - 8*a*b^2*c*d^3 + 16*
a^2*c^2*d^3) - (-b^2*d^2 + 4*a*c*d^2)^(1/4)*log(2*c*d*x + b*d + sqrt(2)*(-b
^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2))
/(sqrt(2)*b^4*d^3 - 8*sqrt(2)*a*b^2*c*d^3 + 16*sqrt(2)*a^2*c^2*d^3) + (-b^2
*d^2 + 4*a*c*d^2)^(1/4)*log(2*c*d*x + b*d - sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^
(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2))/(sqrt(2)*b^4*d^3 -
8*sqrt(2)*a*b^2*c*d^3 + 16*sqrt(2)*a^2*c^2*d^3) + 4/3/((b^2*d - 4*a*c*d)*(2
*c*d*x + b*d)^(3/2))
```

3.1294 $\int \frac{1}{(bd+2cdx)^{7/2}\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=159

$$\frac{4}{d^3\left(b^2-4ac\right)^2\sqrt{bd+2cdx}}+\frac{2\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{7/2}\left(b^2-4ac\right)^{9/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{7/2}\left(b^2-4ac\right)^{9/4}}+\frac{4}{5d\left(b^2-4ac\right)(bd+2cdx)^{5/2}}$$

```
[Out] 4/(5*(b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(5/2)) + 4/((b^2 - 4*a*c)^2*d^3*Sqrt[b
*d + 2*c*d*x]) + (2*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d]
)])/((b^2 - 4*a*c)^(9/4)*d^(7/2)) - (2*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 -
4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(9/4)*d^(7/2))
```

Rubi [A] time = 0.131502, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {693, 694, 329, 298, 203, 206}

$$\frac{4}{d^3\left(b^2-4ac\right)^2\sqrt{bd+2cdx}}+\frac{2\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{7/2}\left(b^2-4ac\right)^{9/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{7/2}\left(b^2-4ac\right)^{9/4}}+\frac{4}{5d\left(b^2-4ac\right)(bd+2cdx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^(7/2)*(a + b*x + c*x^2)),x]
```

```
[Out] 4/(5*(b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(5/2)) + 4/((b^2 - 4*a*c)^2*d^3*Sqrt[b
*d + 2*c*d*x]) + (2*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d]
)])/((b^2 - 4*a*c)^(9/4)*d^(7/2)) - (2*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 -
4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(9/4)*d^(7/2))
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)^{7/2}\left(a+bx+cx^2\right)}\,dx = \frac{4}{5\left(b^2-4ac\right)d(bd+2cdx)^{5/2}}+\frac{\int \frac{1}{(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)}\,dx}{\left(b^2-4ac\right)d^2}$$

$$= \frac{4}{5\left(b^2-4ac\right)d(bd+2cdx)^{5/2}}+\frac{4}{\left(b^2-4ac\right)^2 d^3\sqrt{bd+2cdx}}+\frac{\int \frac{\sqrt{bd+2cdx}}{a+bx+cx^2}\,dx}{\left(b^2-4ac\right)^2 d^4}$$

$$= \frac{4}{5\left(b^2-4ac\right)d(bd+2cdx)^{5/2}}+\frac{4}{\left(b^2-4ac\right)^2 d^3\sqrt{bd+2cdx}}+\frac{\operatorname{Subst}\left(\int \frac{\sqrt{x}}{a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}}\,dx\right.}{2c\left(b^2-4ac\right)}$$

$$= \frac{4}{5\left(b^2-4ac\right)d(bd+2cdx)^{5/2}}+\frac{4}{\left(b^2-4ac\right)^2 d^3\sqrt{bd+2cdx}}+\frac{\operatorname{Subst}\left(\int \frac{x^2}{a-\frac{b^2}{4c}+\frac{x^4}{4cd^2}}\,dx\right.}{c\left(b^2-4ac\right.}$$

$$= \frac{4}{5\left(b^2-4ac\right)d(bd+2cdx)^{5/2}}+\frac{4}{\left(b^2-4ac\right)^2 d^3\sqrt{bd+2cdx}}-\frac{2\operatorname{Subst}\left(\int \frac{1}{\sqrt{b^2-4acd}-x^2}\right.}{\left(b^2-4a\right.}$$

$$= \frac{4}{5\left(b^2-4ac\right)d(bd+2cdx)^{5/2}}+\frac{4}{\left(b^2-4ac\right)^2 d^3\sqrt{bd+2cdx}}+\frac{2\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)}{\left(b^2-4ac\right)^{9/4} d^{7/2}}-$$

Mathematica [C] time = 0.0660311, size = 56, normalized size = 0.35

$$\frac{4\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{5d\left(b^2-4ac\right)(d(b+2cx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^(7/2)*(a + b*x + c*x^2)),x]
```

```
[Out] (4*Hypergeometric2F1[-5/4, 1, -1/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(5*(b^2 -
 4*a*c)*d*(d*(b + 2*c*x))^(5/2))
```

Maple [B] time = 0.197, size = 369, normalized size = 2.3

$$-\frac{4}{5d\left(4ac-b^2\right)}\left(2cdx+bd\right)^{-\frac{5}{2}}+4\frac{1}{d^3\left(4ac-b^2\right)^2\sqrt{2cdx+bd}}+\frac{\sqrt{2}}{2d^3\left(4ac-b^2\right)^2}\ln\left(\left(2cdx+bd-\sqrt[4]{4acd^2-b^2d^2}\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a),x)
```

```
[Out] -4/5/d/(4*a*c-b^2)/(2*c*d*x+b*d)^(5/2)+4/d^3/(4*a*c-b^2)^2/(2*c*d*x+b*d)^(1
/2)+1/2/d^3/(4*a*c-b^2)^2/(4*a*c*d^2-b^2*d^2)^(1/4)*2^(1/2)*ln((2*c*d*x+b*d
-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^
(1/2))/(2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(
4*a*c*d^2-b^2*d^2)^(1/2)))+1/d^3/(4*a*c-b^2)^2/(4*a*c*d^2-b^2*d^2)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)-1/d^3
/(4*a*c-b^2)^2/(4*a*c*d^2-b^2*d^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(4*a*c*d^2
-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.31129, size = 4031, normalized size = 25.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/5*(20*(8*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d^4*x^3 + 12*(b^5*c^2 - 8*
a*b^3*c^3 + 16*a^2*b*c^4)*d^4*x^2 + 6*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3
)*d^4*x + (b^7 - 8*a*b^5*c + 16*a^2*b^3*c^2)*d^4)*(1/((b^18 - 36*a*b^16*c +
 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8
*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 26214
4*a^9*c^9)*d^14))^(1/4)*arctan(sqrt((b^10 - 20*a*b^8*c + 160*a^2*b^6*c^2 -
640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 1024*a^5*c^5)*d^8*sqrt(1/((b^18 - 36*a
*b^16*c + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 12902
4*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^
8 - 262144*a^9*c^9)*d^14)) + 2*c*d*x + b*d)*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*
d^3*(1/((b^18 - 36*a*b^16*c + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*
a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7
 + 589824*a^8*b^2*c^8 - 262144*a^9*c^9)*d^14))^(1/4) - (b^4 - 8*a*b^2*c + 1
6*a^2*c^2)*sqrt(2*c*d*x + b*d)*d^3*(1/((b^18 - 36*a*b^16*c + 576*a^2*b^14*c
^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a
^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 262144*a^9*c^9)*d^14
```

```
))^(1/4)) + 5*(8*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d^4*x^3 + 12*(b^5*c^2
 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^4*x^2 + 6*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^
2*c^3)*d^4*x + (b^7 - 8*a*b^5*c + 16*a^2*b^3*c^2)*d^4)*(1/((b^18 - 36*a*b^1
6*c + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^
5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 -
262144*a^9*c^9)*d^14))^(1/4)*log((b^14 - 28*a*b^12*c + 336*a^2*b^10*c^2 - 2
240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6
- 16384*a^7*c^7)*d^11*(1/((b^18 - 36*a*b^16*c + 576*a^2*b^14*c^2 - 5376*a^3
*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 -
589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 262144*a^9*c^9)*d^14))^(3/4) + sq
rt(2*c*d*x + b*d)) - 5*(8*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d^4*x^3 + 12
*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^4*x^2 + 6*(b^6*c - 8*a*b^4*c^2 +
16*a^2*b^2*c^3)*d^4*x + (b^7 - 8*a*b^5*c + 16*a^2*b^3*c^2)*d^4)*(1/((b^18 -
 36*a*b^16*c + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 -
129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b
^2*c^8 - 262144*a^9*c^9)*d^14))^(1/4)*log(-(b^14 - 28*a*b^12*c + 336*a^2*b^
10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^
6*b^2*c^6 - 16384*a^7*c^7)*d^11*(1/((b^18 - 36*a*b^16*c + 576*a^2*b^14*c^2
- 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*
b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 262144*a^9*c^9)*d^14))^
(3/4) + sqrt(2*c*d*x + b*d)) - 8*(10*c^2*x^2 + 10*b*c*x + 3*b^2 - 2*a*c)*sq
rt(2*c*d*x + b*d))/(8*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d^4*x^3 + 12*(b^
5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^4*x^2 + 6*(b^6*c - 8*a*b^4*c^2 + 16*a
^2*b^2*c^3)*d^4*x + (b^7 - 8*a*b^5*c + 16*a^2*b^3*c^2)*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**(7/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20625, size = 817, normalized size = 5.14

$$-\frac{\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}+2\,\sqrt{2\,cdx+bd}\right)}{2\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}}\right)}{b^6d^5-12\,ab^4cd^5+48\,a^2b^2c^2d^5-64\,a^3c^3d^5}-\frac{\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}-2\,\sqrt{2\,cdx+bd}\right)}{2\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}}\right)}{b^6d^5-12\,ab^4cd^5+48\,a^2b^2c^2d^5-64\,a^3c^3d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2
 + 4*a*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4))/
(b^6*d^5 - 12*a*b^4*c*d^5 + 48*a^2*b^2*c^2*d^5 - 64*a^3*c^3*d^5) - sqrt(2)*
(-b^2*d^2 + 4*a*c*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2 + 4*a*c
*d^2)^(1/4) - 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4))/(b^6*d^5
 - 12*a*b^4*c*d^5 + 48*a^2*b^2*c^2*d^5 - 64*a^3*c^3*d^5) + (-b^2*d^2 + 4*a*
c*d^2)^(3/4)*log(2*c*d*x + b*d + sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(
```

```
2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2))/(sqrt(2)*b^6*d^5 - 12*sqrt(2)*
a*b^4*c*d^5 + 48*sqrt(2)*a^2*b^2*c^2*d^5 - 64*sqrt(2)*a^3*c^3*d^5) - (-b^2*
d^2 + 4*a*c*d^2)^(3/4)*log(2*c*d*x + b*d - sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(
1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2))/(sqrt(2)*b^6*d^5 - 1
2*sqrt(2)*a*b^4*c*d^5 + 48*sqrt(2)*a^2*b^2*c^2*d^5 - 64*sqrt(2)*a^3*c^3*d^5
) + 4/5*(b^2*d^2 - 4*a*c*d^2 + 5*(2*c*d*x + b*d)^2)/((b^4*d^3 - 8*a*b^2*c*d
^3 + 16*a^2*c^2*d^3)*(2*c*d*x + b*d)^(5/2))
```

3.1295 $\int \frac{(bd+2cdx)^{15/2}}{\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=210

$$52cd^7\left(b^2-4ac\right)^2\sqrt{bd+2cdx}+\frac{52}{5}cd^5\left(b^2-4ac\right)(bd+2cdx)^{5/2}-26cd^{15/2}\left(b^2-4ac\right)^{9/4}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-26cd^{15/2}$$

```
[Out] 52*c*(b^2 - 4*a*c)^2*d^7*Sqrt[b*d + 2*c*d*x] + (52*c*(b^2 - 4*a*c)*d^5*(b*d
 + 2*c*d*x)^(5/2))/5 + (52*c*d^3*(b*d + 2*c*d*x)^(9/2))/9 - (d*(b*d + 2*c*d
*x)^(13/2))/(a + b*x + c*x^2) - 26*c*(b^2 - 4*a*c)^(9/4)*d^(15/2)*ArcTan[Sq
rt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])] - 26*c*(b^2 - 4*a*c)^(9/4)
*d^(15/2)*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])]
```

Rubi [A] time = 0.179978, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {686, 692, 694, 329, 212, 206, 203}

$$52cd^7\left(b^2-4ac\right)^2\sqrt{bd+2cdx}+\frac{52}{5}cd^5\left(b^2-4ac\right)(bd+2cdx)^{5/2}-26cd^{15/2}\left(b^2-4ac\right)^{9/4}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-26cd^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(15/2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] 52*c*(b^2 - 4*a*c)^2*d^7*Sqrt[b*d + 2*c*d*x] + (52*c*(b^2 - 4*a*c)*d^5*(b*d
 + 2*c*d*x)^(5/2))/5 + (52*c*d^3*(b*d + 2*c*d*x)^(9/2))/9 - (d*(b*d + 2*c*d
*x)^(13/2))/(a + b*x + c*x^2) - 26*c*(b^2 - 4*a*c)^(9/4)*d^(15/2)*ArcTan[Sq
rt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])] - 26*c*(b^2 - 4*a*c)^(9/4)
*d^(15/2)*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])]
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
```

```
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(bd+2cdx)^{15/2}}{\left(a+bx+cx^2\right)^2}\,dx = -\frac{d(bd+2cdx)^{13/2}}{a+bx+cx^2} + \left(13cd^2\right)\int \frac{(bd+2cdx)^{11/2}}{a+bx+cx^2}\,dx$$

$$= \frac{52}{9}cd^3(bd+2cdx)^{9/2} - \frac{d(bd+2cdx)^{13/2}}{a+bx+cx^2} + \left(13c\left(b^2-4ac\right)d^4\right)\int \frac{(bd+2cdx)^{7/2}}{a+bx+cx^2}\,dx$$

$$= \frac{52}{5}c\left(b^2-4ac\right)d^5(bd+2cdx)^{5/2} + \frac{52}{9}cd^3(bd+2cdx)^{9/2} - \frac{d(bd+2cdx)^{13/2}}{a+bx+cx^2} + \left(13c\left(b^2-4ac\right)^2\right.$$

$$= 52c\left(b^2-4ac\right)^2 d^7\sqrt{bd+2cdx} + \frac{52}{5}c\left(b^2-4ac\right)d^5(bd+2cdx)^{5/2} + \frac{52}{9}cd^3(bd+2cdx)^{9/2} - \frac{d(b}{a}$$

$$= 52c\left(b^2-4ac\right)^2 d^7\sqrt{bd+2cdx} + \frac{52}{5}c\left(b^2-4ac\right)d^5(bd+2cdx)^{5/2} + \frac{52}{9}cd^3(bd+2cdx)^{9/2} - \frac{d(b}{a}$$

$$= 52c\left(b^2-4ac\right)^2 d^7\sqrt{bd+2cdx} + \frac{52}{5}c\left(b^2-4ac\right)d^5(bd+2cdx)^{5/2} + \frac{52}{9}cd^3(bd+2cdx)^{9/2} - \frac{d(b}{a}$$

$$= 52c\left(b^2-4ac\right)^2 d^7\sqrt{bd+2cdx} + \frac{52}{5}c\left(b^2-4ac\right)d^5(bd+2cdx)^{5/2} + \frac{52}{9}cd^3(bd+2cdx)^{9/2} - \frac{d(b}{a}$$

$$= 52c\left(b^2-4ac\right)^2 d^7\sqrt{bd+2cdx} + \frac{52}{5}c\left(b^2-4ac\right)d^5(bd+2cdx)^{5/2} + \frac{52}{9}cd^3(bd+2cdx)^{9/2} - \frac{d(b}{a}$$

Mathematica [A] time = 0.601822, size = 189, normalized size = 0.9

$$-\frac{(d(b+2cx))^{15/2}\left(-13\left(b^2-4ac\right)\left(3\left(b^2-4ac\right)\left(-30\left(b^2-4ac\right)\sqrt{b+2cx}-60c\sqrt[4]{b^2-4ac}(a+x(b+cx))\left(\tan^{-1}\left(\frac{\sqrt{b+2c}}{\sqrt[4]{b^2-4}}\right.\right.\right.\right.\right.}{90(b+2cx)^{15/2}(a+x(b+cx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(15/2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] -((d*(b + 2*c*x))^(15/2)*(-40*(b + 2*c*x)^(13/2) - 13*(b^2 - 4*a*c)*(8*(b +
 2*c*x)^(9/2) + 3*(b^2 - 4*a*c)*(-30*(b^2 - 4*a*c)*Sqrt[b + 2*c*x] + 24*(b
+ 2*c*x)^(5/2) - 60*c*(b^2 - 4*a*c)^(1/4)*(a + x*(b + c*x))*(ArcTan[Sqrt[b
+ 2*c*x]/(b^2 - 4*a*c)^(1/4)] + ArcTanh[Sqrt[b + 2*c*x]/(b^2 - 4*a*c)^(1/4)
])))))/(90*(b + 2*c*x)^(15/2)*(a + x*(b + c*x)))
```

Maple [B] time = 0.201, size = 1512, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(15/2)/(c*x^2+b*x+a)^2,x)
```

```
[Out] 16/9*c*d^3*(2*c*d*x+b*d)^(9/2)-128/5*c^2*d^5*(2*c*d*x+b*d)^(5/2)*a+32/5*c*d
^5*(2*c*d*x+b*d)^(5/2)*b^2+768*c^3*d^7*a^2*(2*c*d*x+b*d)^(1/2)-384*c^2*d^7*
a*b^2*(2*c*d*x+b*d)^(1/2)+48*c*d^7*b^4*(2*c*d*x+b*d)^(1/2)+256*c^4*d^9*(2*c
*d*x+b*d)^(1/2)/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c*d^2)*a^3-192*c^3*d^9*(2*c*
d*x+b*d)^(1/2)/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c*d^2)*a^2*b^2+48*c^2*d^9*(2*
c*d*x+b*d)^(1/2)/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c*d^2)*a*b^4-4*c*d^9*(2*c*d
*x+b*d)^(1/2)/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c*d^2)*b^6-832*c^4*d^9/(4*a*c*
d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*
x+b*d)^(1/2)+1)*a^3+624*c^3*d^9/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(2^
(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*a^2*b^2-156*c^2*d^9/
(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*
(2*c*d*x+b*d)^(1/2)+1)*a*b^4+13*c*d^9/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arc
tan(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*b^6+832*c^4*d^
9/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/
4)*(2*c*d*x+b*d)^(1/2)+1)*a^3-624*c^3*d^9/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)
*arctan(-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*a^2*b^2+1
56*c^2*d^9/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(-2^(1/2)/(4*a*c*d^2-b^2
*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*a*b^4-13*c*d^9/(4*a*c*d^2-b^2*d^2)^(3/4)
*2^(1/2)*arctan(-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*b
^6-416*c^4*d^9/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*ln((2*c*d*x+b*d+(4*a*c*d^2
-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c
*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b
^2*d^2)^(1/2)))*a^3+312*c^3*d^9/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*ln((2*c*d
*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2
*d^2)^(1/2))/(2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(
1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)))*a^2*b^2-78*c^2*d^9/(4*a*c*d^2-b^2*d^2)^(3/
4)*2^(1/2)*ln((2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^
(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*
c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)))*a*b^4+13/2*c*d^9/(4*a*
c*d^2-b^2*d^2)^(3/4)*2^(1/2)*ln((2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c
*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d-(4*a*c*d^2-
b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)))*b^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(15/2)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.28474, size = 3564, normalized size = 16.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(15/2)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/45*(2340*((b^18*c^4 - 36*a*b^16*c^5 + 576*a^2*b^14*c^6 - 5376*a^3*b^12*c
^7 + 32256*a^4*b^10*c^8 - 129024*a^5*b^8*c^9 + 344064*a^6*b^6*c^10 - 589824
*a^7*b^4*c^11 + 589824*a^8*b^2*c^12 - 262144*a^9*c^13)*d^30)^(1/4)*(c*x^2 +
 b*x + a)*arctan((((b^18*c^4 - 36*a*b^16*c^5 + 576*a^2*b^14*c^6 - 5376*a^3*
b^12*c^7 + 32256*a^4*b^10*c^8 - 129024*a^5*b^8*c^9 + 344064*a^6*b^6*c^10 -
589824*a^7*b^4*c^11 + 589824*a^8*b^2*c^12 - 262144*a^9*c^13)*d^30)^(3/4)*(b
^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(2*c*d*x + b*d)*d^7 - ((b^18*c^4 - 36*
a*b^16*c^5 + 576*a^2*b^14*c^6 - 5376*a^3*b^12*c^7 + 32256*a^4*b^10*c^8 - 12
9024*a^5*b^8*c^9 + 344064*a^6*b^6*c^10 - 589824*a^7*b^4*c^11 + 589824*a^8*b
^2*c^12 - 262144*a^9*c^13)*d^30)^(3/4)*sqrt(2*(b^8*c^3 - 16*a*b^6*c^4 + 96*
a^2*b^4*c^5 - 256*a^3*b^2*c^6 + 256*a^4*c^7)*d^15*x + (b^9*c^2 - 16*a*b^7*c
^3 + 96*a^2*b^5*c^4 - 256*a^3*b^3*c^5 + 256*a^4*b*c^6)*d^15 + sqrt((b^18*c^
4 - 36*a*b^16*c^5 + 576*a^2*b^14*c^6 - 5376*a^3*b^12*c^7 + 32256*a^4*b^10*c
^8 - 129024*a^5*b^8*c^9 + 344064*a^6*b^6*c^10 - 589824*a^7*b^4*c^11 + 58982
4*a^8*b^2*c^12 - 262144*a^9*c^13)*d^30)))/((b^18*c^4 - 36*a*b^16*c^5 + 576*
a^2*b^14*c^6 - 5376*a^3*b^12*c^7 + 32256*a^4*b^10*c^8 - 129024*a^5*b^8*c^9
+ 344064*a^6*b^6*c^10 - 589824*a^7*b^4*c^11 + 589824*a^8*b^2*c^12 - 262144*
a^9*c^13)*d^30)) + 585*((b^18*c^4 - 36*a*b^16*c^5 + 576*a^2*b^14*c^6 - 5376
*a^3*b^12*c^7 + 32256*a^4*b^10*c^8 - 129024*a^5*b^8*c^9 + 344064*a^6*b^6*c^
10 - 589824*a^7*b^4*c^11 + 589824*a^8*b^2*c^12 - 262144*a^9*c^13)*d^30)^(1/
4)*(c*x^2 + b*x + a)*log(13*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(2*c*d*x
 + b*d)*d^7 + 13*((b^18*c^4 - 36*a*b^16*c^5 + 576*a^2*b^14*c^6 - 5376*a^3*b
^12*c^7 + 32256*a^4*b^10*c^8 - 129024*a^5*b^8*c^9 + 344064*a^6*b^6*c^10 - 5
89824*a^7*b^4*c^11 + 589824*a^8*b^2*c^12 - 262144*a^9*c^13)*d^30)^(1/4)) -
585*((b^18*c^4 - 36*a*b^16*c^5 + 576*a^2*b^14*c^6 - 5376*a^3*b^12*c^7 + 322
56*a^4*b^10*c^8 - 129024*a^5*b^8*c^9 + 344064*a^6*b^6*c^10 - 589824*a^7*b^4
*c^11 + 589824*a^8*b^2*c^12 - 262144*a^9*c^13)*d^30)^(1/4)*(c*x^2 + b*x + a
)*log(13*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(2*c*d*x + b*d)*d^7 - 13*((
b^18*c^4 - 36*a*b^16*c^5 + 576*a^2*b^14*c^6 - 5376*a^3*b^12*c^7 + 32256*a^4
*b^10*c^8 - 129024*a^5*b^8*c^9 + 344064*a^6*b^6*c^10 - 589824*a^7*b^4*c^11
+ 589824*a^8*b^2*c^12 - 262144*a^9*c^13)*d^30)^(1/4)) - (1280*c^6*d^7*x^6 +
 3840*b*c^5*d^7*x^5 + 256*(22*b^2*c^4 - 13*a*c^5)*d^7*x^4 + 256*(19*b^3*c^3
 - 26*a*b*c^4)*d^7*x^3 + 96*(45*b^4*c^2 - 208*a*b^2*c^3 + 312*a^2*c^4)*d^7*
x^2 + 32*(79*b^5*c - 520*a*b^3*c^2 + 936*a^2*b*c^3)*d^7*x - (45*b^6 - 3068*
a*b^4*c + 20592*a^2*b^2*c^2 - 37440*a^3*c^3)*d^7)*sqrt(2*c*d*x + b*d))/(c*x
^2 + b*x + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(15/2)/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26822, size = 972, normalized size = 4.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(15/2)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 48*sqrt(2*c*d*x + b*d)*b^4*c*d^7 - 384*sqrt(2*c*d*x + b*d)*a*b^2*c^2*d^7 +
768*sqrt(2*c*d*x + b*d)*a^2*c^3*d^7 + 32/5*(2*c*d*x + b*d)^(5/2)*b^2*c*d^5
- 128/5*(2*c*d*x + b*d)^(5/2)*a*c^2*d^5 + 16/9*(2*c*d*x + b*d)^(9/2)*c*d^3
- 13/2*sqrt(2)*(b^4*c*d^7 - 8*a*b^2*c^2*d^7 + 16*a^2*c^3*d^7)*(-b^2*d^2 + 4
*a*c*d^2)^(1/4)*log(2*c*d*x + b*d + sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sq
rt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2)) + 13/2*sqrt(2)*(b^4*c*d^7 -
 8*a*b^2*c^2*d^7 + 16*a^2*c^3*d^7)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*log(2*c*d*x
 + b*d - sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b
^2*d^2 + 4*a*c*d^2)) - 13*(sqrt(2)*b^4*c*d^7 - 8*sqrt(2)*a*b^2*c^2*d^7 + 16
*sqrt(2)*a^2*c^3*d^7)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt
(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c
*d^2)^(1/4)) - 13*(sqrt(2)*b^4*c*d^7 - 8*sqrt(2)*a*b^2*c^2*d^7 + 16*sqrt(2)
*a^2*c^3*d^7)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(-b
^2*d^2 + 4*a*c*d^2)^(1/4) - 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(
1/4)) + 4*(sqrt(2*c*d*x + b*d)*b^6*c*d^9 - 12*sqrt(2*c*d*x + b*d)*a*b^4*c^2
*d^9 + 48*sqrt(2*c*d*x + b*d)*a^2*b^2*c^3*d^9 - 64*sqrt(2*c*d*x + b*d)*a^3*
c^4*d^9)/(b^2*d^2 - 4*a*c*d^2 - (2*c*d*x + b*d)^2)
```

3.1296 $\int \frac{(bd+2cdx)^{13/2}}{\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=181

$$\frac{44}{3}cd^5\left(b^2-4ac\right)(bd+2cdx)^{3/2}+22cd^{13/2}\left(b^2-4ac\right)^{7/4}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-22cd^{13/2}\left(b^2-4ac\right)^{7/4}\tanh^{-1}\left(\frac{\sqrt{d(b}}{\sqrt{d}\sqrt[4]{b}}\right.$$

```
[Out] (44*c*(b^2 - 4*a*c)*d^5*(b*d + 2*c*d*x)^(3/2))/3 + (44*c*d^3*(b*d + 2*c*d*x
)^(7/2))/7 - (d*(b*d + 2*c*d*x)^(11/2))/(a + b*x + c*x^2) + 22*c*(b^2 - 4*a
*c)^(7/4)*d^(13/2)*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])
] - 22*c*(b^2 - 4*a*c)^(7/4)*d^(13/2)*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4
*a*c)^(1/4)*Sqrt[d])]
```

Rubi [A] time = 0.159613, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {686, 692, 694, 329, 298, 203, 206}

$$\frac{44}{3}cd^5\left(b^2-4ac\right)(bd+2cdx)^{3/2}+22cd^{13/2}\left(b^2-4ac\right)^{7/4}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-22cd^{13/2}\left(b^2-4ac\right)^{7/4}\tanh^{-1}\left(\frac{\sqrt{d(b}}{\sqrt{d}\sqrt[4]{b}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(13/2)/(a + b*x + c*x^2)^2,x]
```


```
[Out] (44*c*(b^2 - 4*a*c)*d^5*(b*d + 2*c*d*x)^(3/2))/3 + (44*c*d^3*(b*d + 2*c*d*x
)^(7/2))/7 - (d*(b*d + 2*c*d*x)^(11/2))/(a + b*x + c*x^2) + 22*c*(b^2 - 4*a
*c)^(7/4)*d^(13/2)*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])
] - 22*c*(b^2 - 4*a*c)^(7/4)*d^(13/2)*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4
*a*c)^(1/4)*Sqrt[d])]
```


Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
```

```
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(bd+2cdx)^{13/2}}{\left(a+bx+cx^2\right)^2}\,dx = -\frac{d(bd+2cdx)^{11/2}}{a+bx+cx^2}+\left(11cd^2\right)\int \frac{(bd+2cdx)^{9/2}}{a+bx+cx^2}\,dx$$

$$= \frac{44}{7}cd^3(bd+2cdx)^{7/2}-\frac{d(bd+2cdx)^{11/2}}{a+bx+cx^2}+\left(11c\left(b^2-4ac\right)d^4\right)\int \frac{(bd+2cdx)^{5/2}}{a+bx+cx^2}\,dx$$

$$= \frac{44}{3}c\left(b^2-4ac\right)d^5(bd+2cdx)^{3/2}+\frac{44}{7}cd^3(bd+2cdx)^{7/2}-\frac{d(bd+2cdx)^{11/2}}{a+bx+cx^2}+\left(11c\left(b^2-4ac\right)^2d^6\right)$$

$$= \frac{44}{3}c\left(b^2-4ac\right)d^5(bd+2cdx)^{3/2}+\frac{44}{7}cd^3(bd+2cdx)^{7/2}-\frac{d(bd+2cdx)^{11/2}}{a+bx+cx^2}+\frac{1}{2}\left(11\left(b^2-4ac\right)^2d^5\right.$$

$$= \frac{44}{3}c\left(b^2-4ac\right)d^5(bd+2cdx)^{3/2}+\frac{44}{7}cd^3(bd+2cdx)^{7/2}-\frac{d(bd+2cdx)^{11/2}}{a+bx+cx^2}+\left(11\left(b^2-4ac\right)^2d^5\right)$$

$$= \frac{44}{3}c\left(b^2-4ac\right)d^5(bd+2cdx)^{3/2}+\frac{44}{7}cd^3(bd+2cdx)^{7/2}-\frac{d(bd+2cdx)^{11/2}}{a+bx+cx^2}-\left(22c\left(b^2-4ac\right)^2d^7\right)$$

$$= \frac{44}{3}c\left(b^2-4ac\right)d^5(bd+2cdx)^{3/2}+\frac{44}{7}cd^3(bd+2cdx)^{7/2}-\frac{d(bd+2cdx)^{11/2}}{a+bx+cx^2}+22c\left(b^2-4ac\right)^{7/4}d^{1}$$

Mathematica [C] time = 0.183084, size = 171, normalized size = 0.94

$$\frac{4d^5(d(b+2cx))^{3/2}\left(308c\left(4a^2c+a\left(-b^2+4bcx+4c^2x^2\right)-b^2x(b+cx)\right)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)+16c^2\left(-77a^2-11acx^2+3c\right.}{21(a+x(b+cx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(13/2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] (4*d^5*(d*(b + 2*c*x))^(3/2)*(-63*b^4 + 68*b^3*c*x + 16*b*c^2*x*(-11*a + 6*
c*x^2) + 4*b^2*c*(143*a + 29*c*x^2) + 16*c^2*(-77*a^2 - 11*a*c*x^2 + 3*c^2*
x^4) + 308*c*(4*a^2*c - b^2*x*(b + c*x) + a*(-b^2 + 4*b*c*x + 4*c^2*x^2))*H
ypergeometric2F1[3/4, 2, 7/4, (b + 2*c*x)^2/(b^2 - 4*a*c)]))/(21*(a + x*(b
+ c*x)))
```

Maple [B] time = 0.202, size = 1090, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(13/2)/(c*x^2+b*x+a)^2,x)
```

```
[Out] 16/7*c*d^3*(2*c*d*x+b*d)^(7/2)-128/3*c^2*d^5*(2*c*d*x+b*d)^(3/2)*a+32/3*c*d
^5*(2*c*d*x+b*d)^(3/2)*b^2-64*c^3*d^7*(2*c*d*x+b*d)^(3/2)/(4*c^2*d^2*x^2+4*
b*c*d^2*x+4*a*c*d^2)*a^2+32*c^2*d^7*(2*c*d*x+b*d)^(3/2)/(4*c^2*d^2*x^2+4*b*
c*d^2*x+4*a*c*d^2)*a*b^2-4*c*d^7*(2*c*d*x+b*d)^(3/2)/(4*c^2*d^2*x^2+4*b*c*d
^2*x+4*a*c*d^2)*b^4+88*c^3*d^7*2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*a^2*ln((2*
c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-
b^2*d^2)^(1/2))/(2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*
2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)))+176*c^3*d^7*2^(1/2)/(4*a*c*d^2-b^2*d^2)
^(1/4)*a^2*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)-
176*c^3*d^7*2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*a^2*arctan(-2^(1/2)/(4*a*c*d^
2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)-44*c^2*d^7*2^(1/2)/(4*a*c*d^2-b^2*d
^2)^(1/4)*a*b^2*ln((2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/
2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4
)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)))-88*c^2*d^7*2^(1/2
)/(4*a*c*d^2-b^2*d^2)^(1/4)*a*b^2*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*
(2*c*d*x+b*d)^(1/2)+1)+88*c^2*d^7*2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*a*b^2*a
rctan(-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)+11/2*c*d^7*
2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*b^4*ln((2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(
1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d+(4
*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/
2)))+11*c*d^7*2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*b^4*arctan(2^(1/2)/(4*a*c*d
^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)-11*c*d^7*2^(1/2)/(4*a*c*d^2-b^2*d^
2)^(1/4)*b^4*arctan(-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+
1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(13/2)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.30889, size = 3710, normalized size = 20.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(13/2)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/21*(924*((b^14*c^4 - 28*a*b^12*c^5 + 336*a^2*b^10*c^6 - 2240*a^3*b^8*c^7
 + 8960*a^4*b^6*c^8 - 21504*a^5*b^4*c^9 + 28672*a^6*b^2*c^10 - 16384*a^7*c^
11)*d^26)^(1/4)*(c*x^2 + b*x + a)*arctan(-(((b^14*c^4 - 28*a*b^12*c^5 + 336
*a^2*b^10*c^6 - 2240*a^3*b^8*c^7 + 8960*a^4*b^6*c^8 - 21504*a^5*b^4*c^9 + 2
8672*a^6*b^2*c^10 - 16384*a^7*c^11)*d^26)^(1/4)*(b^10*c^3 - 20*a*b^8*c^4 +
160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt(2
*c*d*x + b*d)*d^19 + sqrt(2*(b^20*c^7 - 40*a*b^18*c^8 + 720*a^2*b^16*c^9 -
7680*a^3*b^14*c^10 + 53760*a^4*b^12*c^11 - 258048*a^5*b^10*c^12 + 860160*a^
6*b^8*c^13 - 1966080*a^7*b^6*c^14 + 2949120*a^8*b^4*c^15 - 2621440*a^9*b^2*
c^16 + 1048576*a^10*c^17)*d^39*x + (b^21*c^6 - 40*a*b^19*c^7 + 720*a^2*b^17
*c^8 - 7680*a^3*b^15*c^9 + 53760*a^4*b^13*c^10 - 258048*a^5*b^11*c^11 + 860
160*a^6*b^9*c^12 - 1966080*a^7*b^7*c^13 + 2949120*a^8*b^5*c^14 - 2621440*a^
9*b^3*c^15 + 1048576*a^10*b*c^16)*d^39 + sqrt((b^14*c^4 - 28*a*b^12*c^5 + 3
36*a^2*b^10*c^6 - 2240*a^3*b^8*c^7 + 8960*a^4*b^6*c^8 - 21504*a^5*b^4*c^9 +
 28672*a^6*b^2*c^10 - 16384*a^7*c^11)*d^26)*(b^14*c^4 - 28*a*b^12*c^5 + 336
*a^2*b^10*c^6 - 2240*a^3*b^8*c^7 + 8960*a^4*b^6*c^8 - 21504*a^5*b^4*c^9 + 2
8672*a^6*b^2*c^10 - 16384*a^7*c^11)*d^26)*((b^14*c^4 - 28*a*b^12*c^5 + 336*
a^2*b^10*c^6 - 2240*a^3*b^8*c^7 + 8960*a^4*b^6*c^8 - 21504*a^5*b^4*c^9 + 28
672*a^6*b^2*c^10 - 16384*a^7*c^11)*d^26)^(1/4))/((b^14*c^4 - 28*a*b^12*c^5
+ 336*a^2*b^10*c^6 - 2240*a^3*b^8*c^7 + 8960*a^4*b^6*c^8 - 21504*a^5*b^4*c^
9 + 28672*a^6*b^2*c^10 - 16384*a^7*c^11)*d^26)) - 231*((b^14*c^4 - 28*a*b^1
2*c^5 + 336*a^2*b^10*c^6 - 2240*a^3*b^8*c^7 + 8960*a^4*b^6*c^8 - 21504*a^5*
b^4*c^9 + 28672*a^6*b^2*c^10 - 16384*a^7*c^11)*d^26)^(1/4)*(c*x^2 + b*x + a
)*log(-1331*(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 +
1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt(2*c*d*x + b*d)*d^19 + 1331*((b^14*c^4
 - 28*a*b^12*c^5 + 336*a^2*b^10*c^6 - 2240*a^3*b^8*c^7 + 8960*a^4*b^6*c^8 -
 21504*a^5*b^4*c^9 + 28672*a^6*b^2*c^10 - 16384*a^7*c^11)*d^26)^(3/4)) + 23
1*((b^14*c^4 - 28*a*b^12*c^5 + 336*a^2*b^10*c^6 - 2240*a^3*b^8*c^7 + 8960*a
^4*b^6*c^8 - 21504*a^5*b^4*c^9 + 28672*a^6*b^2*c^10 - 16384*a^7*c^11)*d^26)
^(1/4)*(c*x^2 + b*x + a)*log(-1331*(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c
^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt(2*c*d*x + b*d)
*d^19 - 1331*((b^14*c^4 - 28*a*b^12*c^5 + 336*a^2*b^10*c^6 - 2240*a^3*b^8*c
^7 + 8960*a^4*b^6*c^8 - 21504*a^5*b^4*c^9 + 28672*a^6*b^2*c^10 - 16384*a^7*
c^11)*d^26)^(3/4)) - (384*c^5*d^6*x^5 + 960*b*c^4*d^6*x^4 + 32*(41*b^2*c^3
- 44*a*c^4)*d^6*x^3 + 48*(21*b^3*c^2 - 44*a*b*c^3)*d^6*x^2 + 2*(115*b^4*c +
 88*a*b^2*c^2 - 1232*a^2*c^3)*d^6*x - (21*b^5 - 440*a*b^3*c + 1232*a^2*b*c^
2)*d^6)*sqrt(2*c*d*x + b*d))/(c*x^2 + b*x + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(13/2)/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31144, size = 767, normalized size = 4.24

$$\frac{32}{3}\,(2\,cdx+bd)^{\frac{3}{2}}b^2cd^5-\frac{128}{3}\,(2\,cdx+bd)^{\frac{3}{2}}ac^2d^5+\frac{16}{7}\,(2\,cdx+bd)^{\frac{7}{2}}cd^3+\frac{11}{2}\,\sqrt{2}\left(b^2cd^5-4\,ac^2d^5\right)\left(-b^2d^2+4\,acd^2\right)^{\frac{3}{4}}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(13/2)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 32/3*(2*c*d*x + b*d)^(3/2)*b^2*c*d^5 - 128/3*(2*c*d*x + b*d)^(3/2)*a*c^2*d^
5 + 16/7*(2*c*d*x + b*d)^(7/2)*c*d^3 + 11/2*sqrt(2)*(b^2*c*d^5 - 4*a*c^2*d^
5)*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*log(2*c*d*x + b*d + sqrt(2)*(-b^2*d^2 + 4*a
*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2)) - 11/2*sqrt
(2)*(b^2*c*d^5 - 4*a*c^2*d^5)*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*log(2*c*d*x + b*
d - sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^
2 + 4*a*c*d^2)) - 11*(sqrt(2)*b^2*c*d^5 - 4*sqrt(2)*a*c^2*d^5)*(-b^2*d^2 +
4*a*c*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) +
 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4)) - 11*(sqrt(2)*b^2*c*d
^5 - 4*sqrt(2)*a*c^2*d^5)*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) - 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 +
4*a*c*d^2)^(1/4)) + 4*((2*c*d*x + b*d)^(3/2)*b^4*c*d^7 - 8*(2*c*d*x + b*d)^
(3/2)*a*b^2*c^2*d^7 + 16*(2*c*d*x + b*d)^(3/2)*a^2*c^3*d^7)/(b^2*d^2 - 4*a*
c*d^2 - (2*c*d*x + b*d)^2)
```

3.1297 $\int \frac{(bd+2cdx)^{11/2}}{\left(a+bx+cx^2\right)^2} dx$

Optimal. Leaf size=179

$$36cd^5\left(b^2-4ac\right)\sqrt{bd+2cdx}-18cd^{11/2}\left(b^2-4ac\right)^{5/4}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-18cd^{11/2}\left(b^2-4ac\right)^{5/4}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4a}}\right.$$

```
[Out] 36*c*(b^2 - 4*a*c)*d^5*Sqrt[b*d + 2*c*d*x] + (36*c*d^3*(b*d + 2*c*d*x)^(5/2
))/5 - (d*(b*d + 2*c*d*x)^(9/2))/(a + b*x + c*x^2) - 18*c*(b^2 - 4*a*c)^(5/
4)*d^(11/2)*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])] - 18*
c*(b^2 - 4*a*c)^(5/4)*d^(11/2)*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(
1/4)*Sqrt[d])]
```

Rubi [A] time = 0.150475, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {686, 692, 694, 329, 212, 206, 203}

$$36cd^5\left(b^2-4ac\right)\sqrt{bd+2cdx}-18cd^{11/2}\left(b^2-4ac\right)^{5/4}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-18cd^{11/2}\left(b^2-4ac\right)^{5/4}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4a}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(11/2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] 36*c*(b^2 - 4*a*c)*d^5*Sqrt[b*d + 2*c*d*x] + (36*c*d^3*(b*d + 2*c*d*x)^(5/2
))/5 - (d*(b*d + 2*c*d*x)^(9/2))/(a + b*x + c*x^2) - 18*c*(b^2 - 4*a*c)^(5/
4)*d^(11/2)*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])] - 18*
c*(b^2 - 4*a*c)^(5/4)*d^(11/2)*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(
1/4)*Sqrt[d])]
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
```

```
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(bd+2cdx)^{11/2}}{\left(a+bx+cx^2\right)^2}\,dx = -\frac{d(bd+2cdx)^{9/2}}{a+bx+cx^2}+\left(9cd^2\right)\int \frac{(bd+2cdx)^{7/2}}{a+bx+cx^2}\,dx$$

$$= \frac{36}{5}cd^3(bd+2cdx)^{5/2}-\frac{d(bd+2cdx)^{9/2}}{a+bx+cx^2}+\left(9c\left(b^2-4ac\right)d^4\right)\int \frac{(bd+2cdx)^{3/2}}{a+bx+cx^2}\,dx$$

$$= 36c\left(b^2-4ac\right)d^5\sqrt{bd+2cdx}+\frac{36}{5}cd^3(bd+2cdx)^{5/2}-\frac{d(bd+2cdx)^{9/2}}{a+bx+cx^2}+\left(9c\left(b^2-4ac\right)^2d^6\right)\int$$

$$= 36c\left(b^2-4ac\right)d^5\sqrt{bd+2cdx}+\frac{36}{5}cd^3(bd+2cdx)^{5/2}-\frac{d(bd+2cdx)^{9/2}}{a+bx+cx^2}+\frac{1}{2}\left(9\left(b^2-4ac\right)^2d^5\right)$$

$$= 36c\left(b^2-4ac\right)d^5\sqrt{bd+2cdx}+\frac{36}{5}cd^3(bd+2cdx)^{5/2}-\frac{d(bd+2cdx)^{9/2}}{a+bx+cx^2}+\left(9\left(b^2-4ac\right)^2d^5\right)\mathrm{S}$$

$$= 36c\left(b^2-4ac\right)d^5\sqrt{bd+2cdx}+\frac{36}{5}cd^3(bd+2cdx)^{5/2}-\frac{d(bd+2cdx)^{9/2}}{a+bx+cx^2}-\left(18c\left(b^2-4ac\right)^{3/2}d^6\right.$$

$$= 36c\left(b^2-4ac\right)d^5\sqrt{bd+2cdx}+\frac{36}{5}cd^3(bd+2cdx)^{5/2}-\frac{d(bd+2cdx)^{9/2}}{a+bx+cx^2}-18c\left(b^2-4ac\right)^{5/4}d^{11/}$$

Mathematica [A] time = 0.364707, size = 167, normalized size = 0.93

$$-\frac{d(d(b+2cx))^{9/2}\left(-3\left(b^2-4ac\right)\left(-30\left(b^2-4ac\right)\sqrt{b+2cx}-60c\sqrt[4]{b^2-4ac}(a+x(b+cx))\left(\tan^{-1}\left(\frac{\sqrt{b+2cx}}{\sqrt[4]{b^2-4ac}}\right)+\tanh^{-1}\right.\right.\right.}{10(b+2cx)^{9/2}(a+x(b+cx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(11/2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] -(d*(d*(b + 2*c*x))^(9/2)*(-8*(b + 2*c*x)^(9/2) - 3*(b^2 - 4*a*c)*(-30*(b^2
 - 4*a*c)*Sqrt[b + 2*c*x] + 24*(b + 2*c*x)^(5/2) - 60*c*(b^2 - 4*a*c)^(1/4)
*(a + x*(b + c*x))*(ArcTan[Sqrt[b + 2*c*x]/(b^2 - 4*a*c)^(1/4)] + ArcTanh[S
qrt[b + 2*c*x]/(b^2 - 4*a*c)^(1/4)]))))/(10*(b + 2*c*x)^(9/2)*(a + x*(b + c
*x)))
```

Maple [B] time = 0.201, size = 1090, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(11/2)/(c*x^2+b*x+a)^2,x)
```

```
[Out] 16/5*c*d^3*(2*c*d*x+b*d)^(5/2)-128*c^2*d^5*a*(2*c*d*x+b*d)^(1/2)+32*c*d^5*b
^2*(2*c*d*x+b*d)^(1/2)-64*c^3*d^7*(2*c*d*x+b*d)^(1/2)/(4*c^2*d^2*x^2+4*b*c*
d^2*x+4*a*c*d^2)*a^2+32*c^2*d^7*(2*c*d*x+b*d)^(1/2)/(4*c^2*d^2*x^2+4*b*c*d^
2*x+4*a*c*d^2)*a*b^2-4*c*d^7*(2*c*d*x+b*d)^(1/2)/(4*c^2*d^2*x^2+4*b*c*d^2*x
+4*a*c*d^2)*b^4+144*c^3*d^7/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(2^(1/2
)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*a^2-72*c^2*d^7/(4*a*c*d^
2-b^2*d^2)^(3/4)*2^(1/2)*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+
b*d)^(1/2)+1)*a*b^2+9*c*d^7/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(2^(1/2
)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*b^4-144*c^3*d^7/(4*a*c*d
^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*
x+b*d)^(1/2)+1)*a^2+72*c^2*d^7/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(-2^
(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*a*b^2-9*c*d^7/(4*a*c
*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*
d*x+b*d)^(1/2)+1)*b^4+72*c^3*d^7/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*ln((2*c*
d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^
2*d^2)^(1/2))/(2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^
(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)))*a^2-36*c^2*d^7/(4*a*c*d^2-b^2*d^2)^(3/4)*
2^(1/2)*ln((2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/
2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d
*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)))*a*b^2+9/2*c*d^7/(4*a*c*d^
2-b^2*d^2)^(3/4)*2^(1/2)*ln((2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x
+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d-(4*a*c*d^2-b^2*
d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)))*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(11/2)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.17817, size = 2026, normalized size = 11.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(11/2)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/5*(180*((b^10*c^4 - 20*a*b^8*c^5 + 160*a^2*b^6*c^6 - 640*a^3*b^4*c^7 + 12
80*a^4*b^2*c^8 - 1024*a^5*c^9)*d^22)^(1/4)*(c*x^2 + b*x + a)*arctan(-(((b^1
0*c^4 - 20*a*b^8*c^5 + 160*a^2*b^6*c^6 - 640*a^3*b^4*c^7 + 1280*a^4*b^2*c^8
 - 1024*a^5*c^9)*d^22)^(3/4)*(b^2*c - 4*a*c^2)*sqrt(2*c*d*x + b*d)*d^5 + ((
b^10*c^4 - 20*a*b^8*c^5 + 160*a^2*b^6*c^6 - 640*a^3*b^4*c^7 + 1280*a^4*b^2*
c^8 - 1024*a^5*c^9)*d^22)^(3/4)*sqrt(2*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)
*d^11*x + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^11 + sqrt((b^10*c^4 - 20
*a*b^8*c^5 + 160*a^2*b^6*c^6 - 640*a^3*b^4*c^7 + 1280*a^4*b^2*c^8 - 1024*a^
5*c^9)*d^22)))/((b^10*c^4 - 20*a*b^8*c^5 + 160*a^2*b^6*c^6 - 640*a^3*b^4*c^
7 + 1280*a^4*b^2*c^8 - 1024*a^5*c^9)*d^22)) + 45*((b^10*c^4 - 20*a*b^8*c^5
+ 160*a^2*b^6*c^6 - 640*a^3*b^4*c^7 + 1280*a^4*b^2*c^8 - 1024*a^5*c^9)*d^22
)^(1/4)*(c*x^2 + b*x + a)*log(-9*(b^2*c - 4*a*c^2)*sqrt(2*c*d*x + b*d)*d^5
+ 9*((b^10*c^4 - 20*a*b^8*c^5 + 160*a^2*b^6*c^6 - 640*a^3*b^4*c^7 + 1280*a^
4*b^2*c^8 - 1024*a^5*c^9)*d^22)^(1/4)) - 45*((b^10*c^4 - 20*a*b^8*c^5 + 160
*a^2*b^6*c^6 - 640*a^3*b^4*c^7 + 1280*a^4*b^2*c^8 - 1024*a^5*c^9)*d^22)^(1/
4)*(c*x^2 + b*x + a)*log(-9*(b^2*c - 4*a*c^2)*sqrt(2*c*d*x + b*d)*d^5 - 9*(
(b^10*c^4 - 20*a*b^8*c^5 + 160*a^2*b^6*c^6 - 640*a^3*b^4*c^7 + 1280*a^4*b^2
*c^8 - 1024*a^5*c^9)*d^22)^(1/4)) + (64*c^4*d^5*x^4 + 128*b*c^3*d^5*x^3 + 4
8*(5*b^2*c^2 - 12*a*c^3)*d^5*x^2 + 16*(11*b^3*c - 36*a*b*c^2)*d^5*x - (5*b^
4 - 216*a*b^2*c + 720*a^2*c^2)*d^5)*sqrt(2*c*d*x + b*d))/(c*x^2 + b*x + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(11/2)/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26147, size = 767, normalized size = 4.28

$$32\sqrt{2\,cdx+bd}b^2cd^5 - 128\sqrt{2\,cdx+bd}ac^2d^5 + \frac{16}{5}(2\,cdx+bd)^{\frac{5}{2}}cd^3 - \frac{9}{2}\sqrt{2}\left(b^2cd^5 - 4\,ac^2d^5\right)\left(-b^2d^2 + 4\,acd^2\right)^{\frac{1}{4}}\log\Big(2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(11/2)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 32*sqrt(2*c*d*x + b*d)*b^2*c*d^5 - 128*sqrt(2*c*d*x + b*d)*a*c^2*d^5 + 16/5
*(2*c*d*x + b*d)^(5/2)*c*d^3 - 9/2*sqrt(2)*(b^2*c*d^5 - 4*a*c^2*d^5)*(-b^2*
d^2 + 4*a*c*d^2)^(1/4)*log(2*c*d*x + b*d + sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(
1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2)) + 9/2*sqrt(2)*(b^2*c
*d^5 - 4*a*c^2*d^5)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*log(2*c*d*x + b*d - sqrt(2
)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*
d^2)) - 9*(sqrt(2)*b^2*c*d^5 - 4*sqrt(2)*a*c^2*d^5)*(-b^2*d^2 + 4*a*c*d^2)^
(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) + 2*sqrt(2*c
```

```
*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4)) - 9*(sqrt(2)*b^2*c*d^5 - 4*sqrt(
2)*a*c^2*d^5)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(-b
^2*d^2 + 4*a*c*d^2)^(1/4) - 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(
1/4)) + 4*(sqrt(2*c*d*x + b*d)*b^4*c*d^7 - 8*sqrt(2*c*d*x + b*d)*a*b^2*c^2*
d^7 + 16*sqrt(2*c*d*x + b*d)*a^2*c^3*d^7)/(b^2*d^2 - 4*a*c*d^2 - (2*c*d*x +
 b*d)^2)
```

3.1298 $\int \frac{(bd+2cdx)^{9/2}}{(a+bx+cx^2)^2} dx$

Optimal. Leaf size=152

$$14cd^{9/2}\left(b^2-4ac\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-14cd^{9/2}\left(b^2-4ac\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-\frac{d(bd+2cdx)^{7/2}}{a+bx+cx^2}+\frac{28}{3}cd^3$$

```
[Out] (28*c*d^3*(b*d + 2*c*d*x)^(3/2))/3 - (d*(b*d + 2*c*d*x)^(7/2))/(a + b*x + c
*x^2) + 14*c*(b^2 - 4*a*c)^(3/4)*d^(9/2)*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 -
 4*a*c)^(1/4)*Sqrt[d])] - 14*c*(b^2 - 4*a*c)^(3/4)*d^(9/2)*ArcTanh[Sqrt[d*(
b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])]
```

Rubi [A] time = 0.12081, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {686, 692, 694, 329, 298, 203, 206}

$$14cd^{9/2}\left(b^2-4ac\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-14cd^{9/2}\left(b^2-4ac\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-\frac{d(bd+2cdx)^{7/2}}{a+bx+cx^2}+\frac{28}{3}cd^3$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(9/2)/(a + b*x + c*x^2)^2,x]
```


```
[Out] (28*c*d^3*(b*d + 2*c*d*x)^(3/2))/3 - (d*(b*d + 2*c*d*x)^(7/2))/(a + b*x + c
*x^2) + 14*c*(b^2 - 4*a*c)^(3/4)*d^(9/2)*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 -
 4*a*c)^(1/4)*Sqrt[d])] - 14*c*(b^2 - 4*a*c)^(3/4)*d^(9/2)*ArcTanh[Sqrt[d*(
b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])]
```


Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(bd+2cdx)^{9/2}}{\left(a+bx+cx^2\right)^2}\,dx = -\frac{d(bd+2cdx)^{7/2}}{a+bx+cx^2}+\left(7cd^2\right)\int \frac{(bd+2cdx)^{5/2}}{a+bx+cx^2}\,dx$$

$$= \frac{28}{3}cd^3(bd+2cdx)^{3/2}-\frac{d(bd+2cdx)^{7/2}}{a+bx+cx^2}+\left(7c\left(b^2-4ac\right)d^4\right)\int \frac{\sqrt{bd+2cdx}}{a+bx+cx^2}\,dx$$

$$= \frac{28}{3}cd^3(bd+2cdx)^{3/2}-\frac{d(bd+2cdx)^{7/2}}{a+bx+cx^2}+\frac{1}{2}\left(7\left(b^2-4ac\right)d^3\right)\text{Subst}\left(\int \frac{\sqrt{x}}{a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}}\,dx,x,bd+$$

$$= \frac{28}{3}cd^3(bd+2cdx)^{3/2}-\frac{d(bd+2cdx)^{7/2}}{a+bx+cx^2}+\left(7\left(b^2-4ac\right)d^3\right)\text{Subst}\left(\int \frac{x^2}{a-\frac{b^2}{4c}+\frac{x^4}{4cd^2}}\,dx,x,\sqrt{d(b+$$

$$= \frac{28}{3}cd^3(bd+2cdx)^{3/2}-\frac{d(bd+2cdx)^{7/2}}{a+bx+cx^2}-\left(14c\left(b^2-4ac\right)d^5\right)\text{Subst}\left(\int \frac{1}{\sqrt{b^2-4ac}d-x^2}\,dx,x,\sqrt{$$

$$= \frac{28}{3}cd^3(bd+2cdx)^{3/2}-\frac{d(bd+2cdx)^{7/2}}{a+bx+cx^2}+14c\left(b^2-4ac\right)^{3/4}d^{9/2}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)-14c\left(b^2-$$

Mathematica [C] time = 0.0910897, size = 92, normalized size = 0.61

$$-\frac{8d^3(d(b+2cx))^{3/2}\left(14c(a+x(b+cx))\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)-2c\left(7a+cx^2\right)+3b^2-2bcx\right)}{3(a+x(b+cx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(9/2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] (-8*d^3*(d*(b + 2*c*x))^(3/2)*(3*b^2 - 2*b*c*x - 2*c*(7*a + c*x^2) + 14*c*(
a + x*(b + c*x))*Hypergeometric2F1[3/4, 2, 7/4, (b + 2*c*x)^2/(b^2 - 4*a*c)
]))/(3*(a + x*(b + c*x)))
```

Maple [B] time = 0.2, size = 693, normalized size = 4.6

$$\frac{16\,cd^{3}}{3}(2\,cdx+bd)^{\frac{3}{2}}+16\,\frac{c^{2}d^{5}\,(2\,cdx+bd)^{3/2}\,a}{4\,c^{2}d^{2}x^{2}+4\,bcd^{2}x+4\,acd^{2}}-4\,\frac{cd^{5}\,(2\,cdx+bd)^{3/2}\,b^{2}}{4\,c^{2}d^{2}x^{2}+4\,bcd^{2}x+4\,acd^{2}}-14\,\frac{c^{2}d^{5}\sqrt{2}a}{\sqrt[4]{4\,acd^{2}-b^{2}d^{2}}}\ln\left(\frac{2\,cdx+}{2\,cdx+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a)^2,x)
```

```
[Out] 16/3*c*d^3*(2*c*d*x+b*d)^(3/2)+16*c^2*d^5*(2*c*d*x+b*d)^(3/2)/(4*c^2*d^2*x^
2+4*b*c*d^2*x+4*a*c*d^2)*a-4*c*d^5*(2*c*d*x+b*d)^(3/2)/(4*c^2*d^2*x^2+4*b*c
*d^2*x+4*a*c*d^2)*b^2-14*c^2*d^5*2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*a*ln((2*
c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-
b^2*d^2)^(1/2))/(2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*
2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)))-28*c^2*d^5*2^(1/2)/(4*a*c*d^2-b^2*d^2)^
(1/4)*a*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)+28*
c^2*d^5*2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*a*arctan(-2^(1/2)/(4*a*c*d^2-b^2*
d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)+7/2*c*d^5*2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/
4)*b^2*ln((2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2
)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*
x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)))+7*c*d^5*2^(1/2)/(4*a*c*d^2
-b^2*d^2)^(1/4)*b^2*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^
(1/2)+1)-7*c*d^5*2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*b^2*arctan(-2^(1/2)/(4*a
*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.06604, size = 1721, normalized size = 11.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(84*((b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*d^18)^(1/4)
*(c*x^2 + b*x + a)*arctan((((b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a
^3*c^7)*d^18)^(1/4)*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt(2*c*d*x + b*d
)*d^13 - sqrt(2*(b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10
 + 256*a^4*c^11)*d^27*x + (b^9*c^6 - 16*a*b^7*c^7 + 96*a^2*b^5*c^8 - 256*a^
3*b^3*c^9 + 256*a^4*b*c^10)*d^27 + sqrt((b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^
2*c^6 - 64*a^3*c^7)*d^18)*(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3
*c^7)*d^18)*((b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*d^18)^(
1/4))/((b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*d^18)) - 21*(
```

```
(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*d^18)^(1/4)*(c*x^2 +
 b*x + a)*log(343*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt(2*c*d*x + b*d)*
d^13 + 343*((b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*d^18)^(3
/4)) + 21*((b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*d^18)^(1/
4)*(c*x^2 + b*x + a)*log(343*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt(2*c*
d*x + b*d)*d^13 - 343*((b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^
7)*d^18)^(3/4)) + (32*c^3*d^4*x^3 + 48*b*c^2*d^4*x^2 + 2*(5*b^2*c + 28*a*c^
2)*d^4*x - (3*b^3 - 28*a*b*c)*d^4)*sqrt(2*c*d*x + b*d))/(c*x^2 + b*x + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(9/2)/(c*x**2+b*x+a)**2,x)

[Out] Timed out
```

Giac [B] time = 1.24153, size = 595, normalized size = 3.91

$$-7\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{3}{4}}cd^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}+2\sqrt{2\,cdx+bd}\right)}{2\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}}\right)-7\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{3}{4}}cd^3\arctan\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a)^2,x, algorithm="giac")

[Out] -7*sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c*d^3*arctan(1/2*sqrt(2)*(sqrt(2)*(
-b^2*d^2 + 4*a*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)
^(1/4)) - 7*sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c*d^3*arctan(-1/2*sqrt(2)*
(sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) - 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 +
4*a*c*d^2)^(1/4)) + 7/2*sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c*d^3*log(2*c*
d*x + b*d + sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt
(-b^2*d^2 + 4*a*c*d^2)) - 7/2*sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c*d^3*lo
g(2*c*d*x + b*d - sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d)
+ sqrt(-b^2*d^2 + 4*a*c*d^2)) + 16/3*(2*c*d*x + b*d)^(3/2)*c*d^3 + 4*((2*c*
d*x + b*d)^(3/2)*b^2*c*d^5 - 4*(2*c*d*x + b*d)^(3/2)*a*c^2*d^5)/(b^2*d^2 -
4*a*c*d^2 - (2*c*d*x + b*d)^2)
```

3.1299 $\int \frac{(bd+2cdx)^{7/2}}{\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=150

$$-10cd^{7/2}\sqrt[4]{b^2-4ac}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-10cd^{7/2}\sqrt[4]{b^2-4ac}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-\frac{d(bd+2cdx)^{5/2}}{a+bx+cx^2}+20cd^3\sqrt{bd}$$

```
[Out] 20*c*d^3*Sqrt[b*d + 2*c*d*x] - (d*(b*d + 2*c*d*x)^(5/2))/(a + b*x + c*x^2)
- 10*c*(b^2 - 4*a*c)^(1/4)*d^(7/2)*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c
)^(1/4)*Sqrt[d])] - 10*c*(b^2 - 4*a*c)^(1/4)*d^(7/2)*ArcTanh[Sqrt[d*(b + 2*
c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])]
```

Rubi [A] time = 0.119651, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {686, 692, 694, 329, 212, 206, 203}

$$-10cd^{7/2}\sqrt[4]{b^2-4ac}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-10cd^{7/2}\sqrt[4]{b^2-4ac}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-\frac{d(bd+2cdx)^{5/2}}{a+bx+cx^2}+20cd^3\sqrt{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(7/2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] 20*c*d^3*Sqrt[b*d + 2*c*d*x] - (d*(b*d + 2*c*d*x)^(5/2))/(a + b*x + c*x^2)
- 10*c*(b^2 - 4*a*c)^(1/4)*d^(7/2)*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c
)^(1/4)*Sqrt[d])] - 10*c*(b^2 - 4*a*c)^(1/4)*d^(7/2)*ArcTanh[Sqrt[d*(b + 2*
c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])]
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(bd+2cdx)^{7/2}}{\left(a+bx+cx^2\right)^2}\,dx = -\frac{d(bd+2cdx)^{5/2}}{a+bx+cx^2}+\left(5cd^2\right)\int \frac{(bd+2cdx)^{3/2}}{a+bx+cx^2}\,dx$$
$$= 20cd^3\sqrt{bd+2cdx}-\frac{d(bd+2cdx)^{5/2}}{a+bx+cx^2}+\left(5c\left(b^2-4ac\right)d^4\right)\int \frac{1}{\sqrt{bd+2cdx}\left(a+bx+cx^2\right)}\,dx$$
$$= 20cd^3\sqrt{bd+2cdx}-\frac{d(bd+2cdx)^{5/2}}{a+bx+cx^2}+\frac{1}{2}\left(5\left(b^2-4ac\right)d^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)}\,dx,x,bd$$
$$= 20cd^3\sqrt{bd+2cdx}-\frac{d(bd+2cdx)^{5/2}}{a+bx+cx^2}+\left(5\left(b^2-4ac\right)d^3\right)\text{Subst}\left(\int \frac{1}{a-\frac{b^2}{4c}+\frac{x^4}{4cd^2}}\,dx,x,\sqrt{d(b+2c$$
$$= 20cd^3\sqrt{bd+2cdx}-\frac{d(bd+2cdx)^{5/2}}{a+bx+cx^2}-\left(10c\sqrt{b^2-4ac}d^4\right)\text{Subst}\left(\int \frac{1}{\sqrt{b^2-4ac}d-x^2}\,dx,x,\sqrt{d(b$$
$$= 20cd^3\sqrt{bd+2cdx}-\frac{d(bd+2cdx)^{5/2}}{a+bx+cx^2}-10c\sqrt[4]{b^2-4ac}d^{7/2}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)-10c\sqrt[4]{b^2-4acd}$$

Mathematica [A] time = 0.193973, size = 168, normalized size = 1.12

$$-\frac{d^3\sqrt{d(b+2cx)}\left(\sqrt{b+2cx}\left(-4c\left(5a+4cx^2\right)+b^2-16bcx\right)+10c\sqrt[4]{b^2-4ac}(a+x(b+cx))\tan^{-1}\left(\frac{\sqrt{b+2cx}}{\sqrt[4]{b^2-4ac}}\right)+10c\sqrt[4]{b^2-4a}}{\sqrt{b+2cx}(a+x(b+cx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(7/2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] -((d^3*Sqrt[d*(b + 2*c*x)]*(Sqrt[b + 2*c*x]*(b^2 - 16*b*c*x - 4*c*(5*a + 4*
c*x^2)) + 10*c*(b^2 - 4*a*c)^(1/4)*(a + x*(b + c*x))*ArcTan[Sqrt[b + 2*c*x]
/(b^2 - 4*a*c)^(1/4)] + 10*c*(b^2 - 4*a*c)^(1/4)*(a + x*(b + c*x))*ArcTanh[
```

```
Sqrt[b + 2*c*x]/(b^2 - 4*a*c)^(1/4)]))/(Sqrt[b + 2*c*x]*(a + x*(b + c*x))))
```

Maple [B] time = 0.198, size = 693, normalized size = 4.6

$$16\,cd^3\sqrt{2\,cdx+bd}+16\,\frac{c^2d^5a\sqrt{2\,cdx+bd}}{4\,c^2d^2x^2+4\,bcd^2x+4\,acd^2}-4\,\frac{cd^5b^2\sqrt{2\,cdx+bd}}{4\,c^2d^2x^2+4\,bcd^2x+4\,acd^2}-20\,\frac{c^2d^5\sqrt{2}a}{\left(4\,acd^2-b^2d^2\right)^{3/4}}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^2,x)
```

```
[Out] 16*c*d^3*(2*c*d*x+b*d)^(1/2)+16*c^2*d^5*(2*c*d*x+b*d)^(1/2)/(4*c^2*d^2*x^2+
4*b*c*d^2*x+4*a*c*d^2)*a-4*c*d^5*(2*c*d*x+b*d)^(1/2)/(4*c^2*d^2*x^2+4*b*c*d
^2*x+4*a*c*d^2)*b^2-20*c^2*d^5/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(2^(
1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*a+5*c*d^5/(4*a*c*d^2-
b^2*d^2)^(3/4)*2^(1/2)*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*
d)^(1/2)+1)*b^2+20*c^2*d^5/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(-2^(1/2
)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*a-5*c*d^5/(4*a*c*d^2-b^2
*d^2)^(3/4)*2^(1/2)*arctan(-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)
^(1/2)+1)*b^2-10*c^2*d^5/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*ln((2*c*d*x+b*d+
(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(
1/2))/(2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4
*a*c*d^2-b^2*d^2)^(1/2)))*a+5/2*c*d^5/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*ln(
(2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d
^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/
2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.86165, size = 803, normalized size = 5.35

$$-\frac{20\,\left(\left(b^2c^4-4\,ac^5\right)d^{14}\right)^{\frac{1}{4}}\left(cx^2+bx+a\right)\arctan\left(\frac{\left(\left(b^2c^4-4\,ac^5\right)d^{14}\right)^{\frac{3}{4}}\sqrt{2\,cdx+bd}cd^3-\left(\left(b^2c^4-4\,ac^5\right)d^{14}\right)^{\frac{3}{4}}\sqrt{2\,c^3d^7x+bc^2d^7+\sqrt{\left(b^2c^4-4\,ac^5\right)d^{14}}}}{\left(b^2c^4-4\,ac^5\right)d^{14}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(20*((b^2*c^4 - 4*a*c^5)*d^14)^(1/4)*(c*x^2 + b*x + a)*arctan((((b^2*c^4 -
 4*a*c^5)*d^14)^(3/4)*sqrt(2*c*d*x + b*d)*c*d^3 - ((b^2*c^4 - 4*a*c^5)*d^14
)^(3/4)*sqrt(2*c^3*d^7*x + b*c^2*d^7 + sqrt((b^2*c^4 - 4*a*c^5)*d^14)))/((b
```

```
^2*c^4 - 4*a*c^5)*d^14)) + 5*((b^2*c^4 - 4*a*c^5)*d^14)^(1/4)*(c*x^2 + b*x
+ a)*log(5*sqrt(2*c*d*x + b*d)*c*d^3 + 5*((b^2*c^4 - 4*a*c^5)*d^14)^(1/4))
- 5*((b^2*c^4 - 4*a*c^5)*d^14)^(1/4)*(c*x^2 + b*x + a)*log(5*sqrt(2*c*d*x +
 b*d)*c*d^3 - 5*((b^2*c^4 - 4*a*c^5)*d^14)^(1/4)) - (16*c^2*d^3*x^2 + 16*b*
c*d^3*x - (b^2 - 20*a*c)*d^3)*sqrt(2*c*d*x + b*d))/(c*x^2 + b*x + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(7/2)/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19009, size = 595, normalized size = 3.97

$$-5\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}cd^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}+2\sqrt{2\,cdx+bd}\right)}{2\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}}\right)-5\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}cd^3\arctan\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -5*sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*c*d^3*arctan(1/2*sqrt(2)*(sqrt(2)*(
-b^2*d^2 + 4*a*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)
^(1/4)) - 5*sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*c*d^3*arctan(-1/2*sqrt(2)*
(sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) - 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 +
4*a*c*d^2)^(1/4)) - 5/2*sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*c*d^3*log(2*c*
d*x + b*d + sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt
(-b^2*d^2 + 4*a*c*d^2)) + 5/2*sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*c*d^3*lo
g(2*c*d*x + b*d - sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d)
+ sqrt(-b^2*d^2 + 4*a*c*d^2)) + 16*sqrt(2*c*d*x + b*d)*c*d^3 + 4*(sqrt(2*c*
d*x + b*d)*b^2*c*d^5 - 4*sqrt(2*c*d*x + b*d)*a*c^2*d^5)/(b^2*d^2 - 4*a*c*d^
2 - (2*c*d*x + b*d)^2)
```

3.1300 $\int \frac{(bd+2cdx)^{5/2}}{\left(a+bx+cx^2\right)^2} dx$

Optimal. Leaf size=131

$$\frac{6cd^{5/2}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\sqrt[4]{b^2-4ac}}-\frac{6cd^{5/2}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\sqrt[4]{b^2-4ac}}-\frac{d(bd+2cdx)^{3/2}}{a+bx+cx^2}$$

```
[Out] -((d*(b*d + 2*c*d*x)^(3/2))/(a + b*x + c*x^2)) + (6*c*d^(5/2)*ArcTan[Sqrt[d
*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(b^2 - 4*a*c)^(1/4) - (6*c*d^
(5/2)*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(b^2 - 4*
a*c)^(1/4)
```

Rubi [A] time = 0.0988734, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {686, 694, 329, 298, 203, 206}

$$\frac{6cd^{5/2}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\sqrt[4]{b^2-4ac}}-\frac{6cd^{5/2}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\sqrt[4]{b^2-4ac}}-\frac{d(bd+2cdx)^{3/2}}{a+bx+cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(5/2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] -((d*(b*d + 2*c*d*x)^(3/2))/(a + b*x + c*x^2)) + (6*c*d^(5/2)*ArcTan[Sqrt[d
*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(b^2 - 4*a*c)^(1/4) - (6*c*d^
(5/2)*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(b^2 - 4*
a*c)^(1/4)
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(bd+2cdx)^{5/2}}{\left(a+bx+cx^2\right)^2}\,dx &= -\frac{d(bd+2cdx)^{3/2}}{a+bx+cx^2}+\left(3cd^2\right)\int \frac{\sqrt{bd+2cdx}}{a+bx+cx^2}\,dx\\
&= -\frac{d(bd+2cdx)^{3/2}}{a+bx+cx^2}+\frac{1}{2}(3d)\,\mathrm{Subst}\left(\int \frac{\sqrt{x}}{a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}}\,dx,x,bd+2cdx\right)\\
&= -\frac{d(bd+2cdx)^{3/2}}{a+bx+cx^2}+(3d)\,\mathrm{Subst}\left(\int \frac{x^2}{a-\frac{b^2}{4c}+\frac{x^4}{4cd^2}}\,dx,x,\sqrt{d(b+2cx)}\right)\\
&= -\frac{d(bd+2cdx)^{3/2}}{a+bx+cx^2}-\left(6cd^3\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b^2-4ac}d-x^2}\,dx,x,\sqrt{d(b+2cx)}\right)+\left(6cd^3\right)\mathrm{Subst}\left(\int \frac{}{\sqrt{}}\right.\\
&= -\frac{d(bd+2cdx)^{3/2}}{a+bx+cx^2}+\frac{6cd^{5/2}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)}{\sqrt[4]{b^2-4ac}}-\frac{6cd^{5/2}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)}{\sqrt[4]{b^2-4ac}}
\end{aligned}$$

Mathematica [C] time = 0.0850247, size = 83, normalized size = 0.63

$$-\frac{4d(d(b+2cx))^{3/2}\left(4c(a+x(b+cx))\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)-4ac+b^2\right)}{\left(b^2-4ac\right)(a+x(b+cx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(5/2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] (-4*d*(d*(b + 2*c*x))^(3/2)*(b^2 - 4*a*c + 4*c*(a + x*(b + c*x))*Hypergeome
tric2F1[3/4, 2, 7/4, (b + 2*c*x)^2/(b^2 - 4*a*c)]))/((b^2 - 4*a*c)*(a + x*(
b + c*x)))
```

Maple [B] time = 0.222, size = 327, normalized size = 2.5

$$-4\,\frac{cd^3\,(2\,cdx+bd)^{3/2}}{4\,c^2d^2x^2+4\,bcd^2x+4\,acd^2}+\frac{3\,cd^3\sqrt{2}}{2}\ln\left(\left(2\,cdx+bd-\sqrt[4]{4\,acd^2-b^2d^2}\sqrt{2\,cdx+bd}\sqrt{2}+\sqrt{4\,acd^2-b^2d^2}\right)\left(2\,cdx+bd\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^2,x)
```

```
[Out] -4*c*d^3*(2*c*d*x+b*d)^(3/2)/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c*d^2)+3/2*c*d^
3*2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*ln((2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/
4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d+(4*a
*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)
))+3*c*d^3*2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*arctan(2^(1/2)/(4*a*c*d^2-b^2*
d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)-3*c*d^3*2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)
*arctan(-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.97312, size = 805, normalized size = 6.15

$$-\frac{12\left(\frac{c^4d^{10}}{b^2-4\,ac}\right)^{\frac{1}{4}}\left(cx^2+bx+a\right)\arctan\left(-\frac{\left(\frac{c^4d^{10}}{b^2-4\,ac}\right)^{\frac{1}{4}}\sqrt{2\,cdx+bd}c^3d^7-\sqrt{2\,c^7d^{15}x+bc^6d^{15}+\sqrt{\frac{c^4d^{10}}{b^2-4\,ac}}\left(b^2c^4-4\,ac^5\right)d^{10}}\left(\frac{c^4d^{10}}{b^2-4\,ac}\right)^{\frac{1}{4}}}{c^4d^{10}}\right)+3\left(\frac{c^4d^{10}}{b^2-4\,ac}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(12*(c^4*d^10/(b^2 - 4*a*c))^(1/4)*(c*x^2 + b*x + a)*arctan(-((c^4*d^10/(b
^2 - 4*a*c))^(1/4)*sqrt(2*c*d*x + b*d)*c^3*d^7 - sqrt(2*c^7*d^15*x + b*c^6*
d^15 + sqrt(c^4*d^10/(b^2 - 4*a*c))*(b^2*c^4 - 4*a*c^5)*d^10)*(c^4*d^10/(b^
2 - 4*a*c))^(1/4))/(c^4*d^10)) + 3*(c^4*d^10/(b^2 - 4*a*c))^(1/4)*(c*x^2 +
b*x + a)*log(27*sqrt(2*c*d*x + b*d)*c^3*d^7 + 27*(c^4*d^10/(b^2 - 4*a*c))^(
3/4)*(b^2 - 4*a*c)) - 3*(c^4*d^10/(b^2 - 4*a*c))^(1/4)*(c*x^2 + b*x + a)*lo
g(27*sqrt(2*c*d*x + b*d)*c^3*d^7 - 27*(c^4*d^10/(b^2 - 4*a*c))^(3/4)*(b^2 -
 4*a*c)) + (2*c*d^2*x + b*d^2)*sqrt(2*c*d*x + b*d))/(c*x^2 + b*x + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(5/2)/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26524, size = 593, normalized size = 4.53

$$\frac{4\,(2\,cdx+bd)^{\frac{3}{2}}cd^3}{b^2d^2-4\,acd^2-(2\,cdx+bd)^2}-\frac{3\,\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{3}{4}}cd\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}+2\,\sqrt{2\,cdx+bd}\right)}{2\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}}\right)}{b^2-4\,ac}-\frac{3\,\sqrt{2}\left(-b^2d^2+4\,acd\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 4*(2*c*d*x + b*d)^(3/2)*c*d^3/(b^2*d^2 - 4*a*c*d^2 - (2*c*d*x + b*d)^2) - 3
*sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c*d*arctan(1/2*sqrt(2)*(sqrt(2)*(-b^2
*d^2 + 4*a*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/
4))/(b^2 - 4*a*c) - 3*sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c*d*arctan(-1/2*
sqrt(2)*(sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) - 2*sqrt(2*c*d*x + b*d))/(-b^
2*d^2 + 4*a*c*d^2)^(1/4))/(b^2 - 4*a*c) + 3*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c*
d*log(2*c*d*x + b*d + sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b
*d) + sqrt(-b^2*d^2 + 4*a*c*d^2))/(sqrt(2)*b^2 - 4*sqrt(2)*a*c) - 3*(-b^2*d
^2 + 4*a*c*d^2)^(3/4)*c*d*log(2*c*d*x + b*d - sqrt(2)*(-b^2*d^2 + 4*a*c*d^2
)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2))/(sqrt(2)*b^2 - 4*
sqrt(2)*a*c)
```

3.1301 $\int \frac{(bd+2cdx)^{3/2}}{(a+bx+cx^2)^2}\,dx$

Optimal. Leaf size=131

$$-\frac{2cd^{3/2}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{(b^2-4ac)^{3/4}}-\frac{2cd^{3/2}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{(b^2-4ac)^{3/4}}-\frac{d\sqrt{bd+2cdx}}{a+bx+cx^2}$$

```
[Out] -((d*Sqrt[b*d + 2*c*d*x])/(a + b*x + c*x^2)) - (2*c*d^(3/2)*ArcTan[Sqrt[d*(
b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(b^2 - 4*a*c)^(3/4) - (2*c*d^(3
/2)*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(b^2 - 4*a*
c)^(3/4)
```

Rubi [A] time = 0.0993918, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {686, 694, 329, 212, 206, 203}

$$-\frac{2cd^{3/2}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{(b^2-4ac)^{3/4}}-\frac{2cd^{3/2}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{(b^2-4ac)^{3/4}}-\frac{d\sqrt{bd+2cdx}}{a+bx+cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(3/2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] -((d*Sqrt[b*d + 2*c*d*x])/(a + b*x + c*x^2)) - (2*c*d^(3/2)*ArcTan[Sqrt[d*(
b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(b^2 - 4*a*c)^(3/4) - (2*c*d^(3
/2)*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(b^2 - 4*a*
c)^(3/4)
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(bd+2cdx)^{3/2}}{\left(a+bx+cx^2\right)^2}\,dx &= -\frac{d\sqrt{bd+2cdx}}{a+bx+cx^2}+\left(cd^2\right)\int \frac{1}{\sqrt{bd+2cdx}\left(a+bx+cx^2\right)}\,dx\\
&= -\frac{d\sqrt{bd+2cdx}}{a+bx+cx^2}+\frac{1}{2}d\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)}\,dx,x,bd+2cdx\right)\\
&= -\frac{d\sqrt{bd+2cdx}}{a+bx+cx^2}+d\,\text{Subst}\left(\int \frac{1}{a-\frac{b^2}{4c}+\frac{x^4}{4cd^2}}\,dx,x,\sqrt{d(b+2cx)}\right)\\
&= -\frac{d\sqrt{bd+2cdx}}{a+bx+cx^2}-\frac{\left(2cd^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{b^2-4acd}-x^2}\,dx,x,\sqrt{d(b+2cx)}\right)}{\sqrt{b^2-4ac}}-\frac{\left(2cd^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{b^2-4acd}+x^2}\right.}{\sqrt{b^2-4a}}\\
&= -\frac{d\sqrt{bd+2cdx}}{a+bx+cx^2}-\frac{2cd^{3/2}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)}{\left(b^2-4ac\right)^{3/4}}-\frac{2cd^{3/2}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)}{\left(b^2-4ac\right)^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.23111, size = 126, normalized size = 0.96

$$d\sqrt{d(b+2cx)}\left(-\frac{2c\tan^{-1}\left(\frac{\sqrt{b+2cx}}{\sqrt[4]{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/4}\sqrt{b+2cx}}-\frac{2c\tanh^{-1}\left(\frac{\sqrt{b+2cx}}{\sqrt[4]{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/4}\sqrt{b+2cx}}-\frac{1}{a+x(b+cx)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(3/2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] d*Sqrt[d*(b + 2*c*x)]*(-(a + x*(b + c*x))^(-1) - (2*c*ArcTan[Sqrt[b + 2*c*x
]/(b^2 - 4*a*c)^(1/4)])/((b^2 - 4*a*c)^(3/4)*Sqrt[b + 2*c*x]) - (2*c*ArcTan
h[Sqrt[b + 2*c*x]/(b^2 - 4*a*c)^(1/4)])/((b^2 - 4*a*c)^(3/4)*Sqrt[b + 2*c*x
]))
```

Maple [B] time = 0.198, size = 326, normalized size = 2.5

$$-4\,\frac{cd^3\sqrt{2\,cdx+bd}}{4\,c^2d^2x^2+4\,bcd^2x+4\,acd^2}+\frac{cd^3\sqrt{2}}{2}\ln\left(\left(2\,cdx+bd+\sqrt[4]{4\,acd^2-b^2d^2}\sqrt{2\,cdx+bd}\sqrt{2}+\sqrt{4\,acd^2-b^2d^2}\right)\left(2\,cdx+bd\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^2,x)
```

```
[Out] -4*c*d^3*(2*c*d*x+b*d)^(1/2)/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c*d^2)+1/2*c*d^
3/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*ln((2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/
4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d-(4*a
*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)
))+c*d^3/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^
2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)-c*d^3/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arc
tan(-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.97755, size = 1197, normalized size = 9.14

$$4\left(\frac{c^4d^6}{b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3}\right)^{\frac{1}{4}}\left(cx^2+bx+a\right)\arctan\left(-\frac{\left(\frac{c^4d^6}{b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3}\right)^{\frac{3}{4}}\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)\sqrt{2\,cdx+bd}d-\left(\frac{c^4d^6}{b^6-12\,ab^4c+48\,a^2\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (4*(c^4*d^6/(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3))^(1/4)*(c*x^2
+ b*x + a)*arctan(-((c^4*d^6/(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^
3))^(3/4)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(2*c*d*x + b*d)*d - (c^4*d
^6/(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3))^(3/4)*sqrt(2*c^3*d^3*x
 + b*c^2*d^3 + sqrt(c^4*d^6/(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3
))*(b^4 - 8*a*b^2*c + 16*a^2*c^2))*(b^4 - 8*a*b^2*c + 16*a^2*c^2))/(c^4*d^6
)) - (c^4*d^6/(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3))^(1/4)*(c*x^
2 + b*x + a)*log(sqrt(2*c*d*x + b*d)*c*d + (c^4*d^6/(b^6 - 12*a*b^4*c + 48*
a^2*b^2*c^2 - 64*a^3*c^3))^(1/4)*(b^2 - 4*a*c)) + (c^4*d^6/(b^6 - 12*a*b^4*
c + 48*a^2*b^2*c^2 - 64*a^3*c^3))^(1/4)*(c*x^2 + b*x + a)*log(sqrt(2*c*d*x
+ b*d)*c*d - (c^4*d^6/(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3))^(1/
4)*(b^2 - 4*a*c)) - sqrt(2*c*d*x + b*d)*d)/(c*x^2 + b*x + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(3/2)/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17054, size = 591, normalized size = 4.51

$$\frac{4\sqrt{2\,cdx+bd}cd^3}{b^2d^2-4\,acd^2-(2\,cdx+bd)^2}-\frac{\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}cd\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}+2\,\sqrt{2\,cdx+bd}\right)}{2\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}}\right)}{b^2-4\,ac}-\frac{\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 4*sqrt(2*c*d*x + b*d)*c*d^3/(b^2*d^2 - 4*a*c*d^2 - (2*c*d*x + b*d)^2) - sqr
t(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*c*d*arctan(1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2
 + 4*a*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4))/
(b^2 - 4*a*c) - sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*c*d*arctan(-1/2*sqrt(2
)*(sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) - 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2
+ 4*a*c*d^2)^(1/4))/(b^2 - 4*a*c) - (-b^2*d^2 + 4*a*c*d^2)^(1/4)*c*d*log(2*
c*d*x + b*d + sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sq
rt(-b^2*d^2 + 4*a*c*d^2))/(sqrt(2)*b^2 - 4*sqrt(2)*a*c) + (-b^2*d^2 + 4*a*c
*d^2)^(1/4)*c*d*log(2*c*d*x + b*d - sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sq
rt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2))/(sqrt(2)*b^2 - 4*sqrt(2)*a*
c)
```

3.1302 $\int \frac{\sqrt{bd+2cdx}}{(a+bx+cx^2)^2} dx$

Optimal. Leaf size=143

$$-\frac{(bd+2cdx)^{3/2}}{d(b^2-4ac)(a+bx+cx^2)}-\frac{2c\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{(b^2-4ac)^{5/4}}+\frac{2c\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{(b^2-4ac)^{5/4}}$$

```
[Out] -((b*d + 2*c*d*x)^(3/2)/((b^2 - 4*a*c)*d*(a + b*x + c*x^2))) - (2*c*Sqrt[d]
*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(b^2 - 4*a*c)^(
5/4) + (2*c*Sqrt[d]*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d
])])/(b^2 - 4*a*c)^(5/4)
```

Rubi [A] time = 0.097171, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {687, 694, 329, 298, 203, 206}

$$-\frac{(bd+2cdx)^{3/2}}{d(b^2-4ac)(a+bx+cx^2)}-\frac{2c\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{(b^2-4ac)^{5/4}}+\frac{2c\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{(b^2-4ac)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*d + 2*c*d*x]/(a + b*x + c*x^2)^2,x]
```

```
[Out] -((b*d + 2*c*d*x)^(3/2)/((b^2 - 4*a*c)*d*(a + b*x + c*x^2))) - (2*c*Sqrt[d]
*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(b^2 - 4*a*c)^(
5/4) + (2*c*Sqrt[d]*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d
])])/(b^2 - 4*a*c)^(5/4)
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{bd+2cdx}}{\left(a+bx+cx^2\right)^2}\,dx = -\frac{(bd+2cdx)^{3/2}}{\left(b^2-4ac\right)d\left(a+bx+cx^2\right)} - \frac{c\int \frac{\sqrt{bd+2cdx}}{a+bx+cx^2}\,dx}{b^2-4ac}$$

$$= -\frac{(bd+2cdx)^{3/2}}{\left(b^2-4ac\right)d\left(a+bx+cx^2\right)} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}}\,dx, x, bd+2cdx\right)}{2\left(b^2-4ac\right)d}$$

$$= -\frac{(bd+2cdx)^{3/2}}{\left(b^2-4ac\right)d\left(a+bx+cx^2\right)} - \frac{\text{Subst}\left(\int \frac{x^2}{a-\frac{b^2}{4c}+\frac{x^4}{4cd^2}}\,dx, x, \sqrt{d(b+2cx)}\right)}{\left(b^2-4ac\right)d}$$

$$= -\frac{(bd+2cdx)^{3/2}}{\left(b^2-4ac\right)d\left(a+bx+cx^2\right)} + \frac{(2cd)\,\text{Subst}\left(\int \frac{1}{\sqrt{b^2-4ac}d-x^2}\,dx, x, \sqrt{d(b+2cx)}\right)}{b^2-4ac} - \frac{(2cd)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(bd+2cdx)^{3/2}}{\left(b^2-4ac\right)d\left(a+bx+cx^2\right)} - \frac{2c\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)}{\left(b^2-4ac\right)^{5/4}} + \frac{2c\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)}{\left(b^2-4ac\right)^{5/4}}$$

Mathematica [C] time = 0.0384849, size = 57, normalized size = 0.4

$$\frac{16c(d(b+2cx))^{3/2}\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{3d\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*d + 2*c*d*x]/(a + b*x + c*x^2)^2,x]
```

```
[Out] (16*c*(d*(b + 2*c*x))^(3/2)*Hypergeometric2F1[3/4, 2, 7/4, (b + 2*c*x)^2/(b
^2 - 4*a*c)])/(3*(b^2 - 4*a*c)^2*d)
```

Maple [B] time = 0.194, size = 344, normalized size = 2.4

$$4\frac{cd^3\left(2cdx+bd\right)^{3/2}}{\left(4acd^2-b^2d^2\right)\left(4c^2d^2x^2+4bcd^2x+4acd^2\right)} + \frac{cd^3\sqrt{2}}{2}\ln\left(\left(2cdx+bd-\sqrt[4]{4acd^2-b^2d^2}\sqrt{2cdx+bd}\sqrt{2}+\sqrt{4acd^2-b^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^2,x)
```

```
[Out] 4*c*d^3*(2*c*d*x+b*d)^(3/2)/(4*a*c*d^2-b^2*d^2)/(4*c^2*d^2*x^2+4*b*c*d^2*x+
4*a*c*d^2)+1/2*c*d^3/(4*a*c*d^2-b^2*d^2)^(5/4)*2^(1/2)*ln((2*c*d*x+b*d-(4*a
*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)
)/(2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c
*d^2-b^2*d^2)^(1/2)))+c*d^3/(4*a*c*d^2-b^2*d^2)^(5/4)*2^(1/2)*arctan(2^(1/2
)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)-c*d^3/(4*a*c*d^2-b^2*d^2
)^(5/4)*2^(1/2)*arctan(-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/
2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.19359, size = 2037, normalized size = 14.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(4*(c^4*d^2/(b^10 - 20*a*b^8*c + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*
a^4*b^2*c^4 - 1024*a^5*c^5))^(1/4)*(a*b^2 - 4*a^2*c + (b^2*c - 4*a*c^2)*x^2
 + (b^3 - 4*a*b*c)*x)*arctan(((b^2*c^3 - 4*a*c^4)*(c^4*d^2/(b^10 - 20*a*b^8
*c + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 1024*a^5*c^5))^
(1/4)*sqrt(2*c*d*x + b*d)*d - sqrt(2*c^7*d^3*x + b*c^6*d^3 + (b^6*c^4 - 12*
a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*sqrt(c^4*d^2/(b^10 - 20*a*b^8*c +
160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 1024*a^5*c^5))*d^2)*
(c^4*d^2/(b^10 - 20*a*b^8*c + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*
b^2*c^4 - 1024*a^5*c^5))^(1/4)*(b^2 - 4*a*c))/(c^4*d^2)) - (c^4*d^2/(b^10 -
 20*a*b^8*c + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 1024*a
^5*c^5))^(1/4)*(a*b^2 - 4*a^2*c + (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x
)*log(sqrt(2*c*d*x + b*d)*c^3*d + (b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*
a^3*b^2*c^3 + 256*a^4*c^4)*(c^4*d^2/(b^10 - 20*a*b^8*c + 160*a^2*b^6*c^2 -
640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 1024*a^5*c^5))^(3/4)) + (c^4*d^2/(b^10
 - 20*a*b^8*c + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 1024
*a^5*c^5))^(1/4)*(a*b^2 - 4*a^2*c + (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)
*x)*log(sqrt(2*c*d*x + b*d)*c^3*d - (b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 25
6*a^3*b^2*c^3 + 256*a^4*c^4)*(c^4*d^2/(b^10 - 20*a*b^8*c + 160*a^2*b^6*c^2
- 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 1024*a^5*c^5))^(3/4)) + sqrt(2*c*d*x
 + b*d)*(2*c*x + b))/(a*b^2 - 4*a^2*c + (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*
b*c)*x)
```

Sympy [B] time = 98.0098, size = 279, normalized size = 1.95

$$\frac{16cd^3\,(bd+2cdx)^{\frac{3}{2}}}{64a^2c^2d^4-32ab^2cd^4+16acd^2\,(bd+2cdx)^2+4b^4d^4-4b^2d^2\,(bd+2cdx)^2}+16cd^3\,\mathrm{RootSum}\left(t^4\left(67108864a^5c^5d^{10}-838\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(1/2)/(c*x**2+b*x+a)**2,x)
```

```
[Out] 16*c*d**3*(b*d + 2*c*d*x)**(3/2)/(64*a**2*c**2*d**4 - 32*a*b**2*c*d**4 + 16
*a*c*d**2*(b*d + 2*c*d*x)**2 + 4*b**4*d**4 - 4*b**2*d**2*(b*d + 2*c*d*x)**2
) + 16*c*d**3*RootSum(_t**4*(67108864*a**5*c**5*d**10 - 83886080*a**4*b**2*
c**4*d**10 + 41943040*a**3*b**4*c**3*d**10 - 10485760*a**2*b**6*c**2*d**10
+ 1310720*a*b**8*c*d**10 - 65536*b**10*d**10) + 1, Lambda(_t, _t*log(104857
6*_t**3*a**4*c**4*d**8 - 1048576*_t**3*a**3*b**2*c**3*d**8 + 393216*_t**3*a
**2*b**4*c**2*d**8 - 65536*_t**3*a*b**6*c*d**8 + 4096*_t**3*b**8*d**8 + sqr
t(b*d + 2*c*d*x))))
```

Giac [B] time = 1.23593, size = 680, normalized size = 4.76

$$\frac{\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{3}{4}}c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}+2\,\sqrt{2\,cdx+bd}\right)}{2\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}}\right)}{b^4d-8\,ab^2cd+16\,a^2c^2d}+\frac{\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{3}{4}}c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}-2\right.}{2\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}}\right.}{b^4d-8\,ab^2cd+16\,a^2c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(-b^2*d^
2 + 4*a*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4))
/(b^4*d - 8*a*b^2*c*d + 16*a^2*c^2*d) + sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(3/4
)*c*arctan(-1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) - 2*sqrt(2*c*
d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4))/(b^4*d - 8*a*b^2*c*d + 16*a^2*c^2
*d) - (-b^2*d^2 + 4*a*c*d^2)^(3/4)*c*log(2*c*d*x + b*d + sqrt(2)*(-b^2*d^2
+ 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2))/(sqrt(
2)*b^4*d - 8*sqrt(2)*a*b^2*c*d + 16*sqrt(2)*a^2*c^2*d) + (-b^2*d^2 + 4*a*c*
d^2)^(3/4)*c*log(2*c*d*x + b*d - sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(
2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2))/(sqrt(2)*b^4*d - 8*sqrt(2)*a*b
^2*c*d + 16*sqrt(2)*a^2*c^2*d) + 4*(2*c*d*x + b*d)^(3/2)*c*d/((b^2*d^2 - 4*
a*c*d^2 - (2*c*d*x + b*d)^2)*(b^2 - 4*a*c))
```

3.1303 $\int \frac{1}{\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=143

$$-\frac{\sqrt{bd+2cdx}}{d\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{6c\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\sqrt{d}\left(b^2-4ac\right)^{7/4}}+\frac{6c\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\sqrt{d}\left(b^2-4ac\right)^{7/4}}$$

```
[Out] -(Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)*d*(a + b*x + c*x^2))) + (6*c*ArcTan[Sq
rt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(7/4)*Sqrt
[d]) + (6*c*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b
^2 - 4*a*c)^(7/4)*Sqrt[d])
```

Rubi [A] time = 0.102449, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {687, 694, 329, 212, 206, 203}

$$-\frac{\sqrt{bd+2cdx}}{d\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{6c\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\sqrt{d}\left(b^2-4ac\right)^{7/4}}+\frac{6c\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\sqrt{d}\left(b^2-4ac\right)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[b*d + 2*c*d*x]*(a + b*x + c*x^2)^2),x]
```

```
[Out] -(Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)*d*(a + b*x + c*x^2))) + (6*c*ArcTan[Sq
rt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(7/4)*Sqrt
[d]) + (6*c*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b
^2 - 4*a*c)^(7/4)*Sqrt[d])
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^2}\,dx = -\frac{\sqrt{bd+2cdx}}{\left(b^2-4ac\right)d\left(a+bx+cx^2\right)} - \frac{(3c)\int \frac{1}{\sqrt{bd+2cdx}\left(a+bx+cx^2\right)}\,dx}{b^2-4ac}$$

$$= -\frac{\sqrt{bd+2cdx}}{\left(b^2-4ac\right)d\left(a+bx+cx^2\right)} - \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)}\,dx, x, bd+2cdx\right)}{2\left(b^2-4ac\right)d}$$

$$= -\frac{\sqrt{bd+2cdx}}{\left(b^2-4ac\right)d\left(a+bx+cx^2\right)} - \frac{3\,\text{Subst}\left(\int \frac{1}{a-\frac{b^2}{4c}+\frac{x^4}{4cd^2}}\,dx, x, \sqrt{d(b+2cx)}\right)}{\left(b^2-4ac\right)d}$$

$$= -\frac{\sqrt{bd+2cdx}}{\left(b^2-4ac\right)d\left(a+bx+cx^2\right)} + \frac{(6c)\,\text{Subst}\left(\int \frac{1}{\sqrt{b^2-4ac}d-x^2}\,dx, x, \sqrt{d(b+2cx)}\right)}{\left(b^2-4ac\right)^{3/2}} + \frac{(6c)}{}$$

$$= -\frac{\sqrt{bd+2cdx}}{\left(b^2-4ac\right)d\left(a+bx+cx^2\right)} + \frac{6c\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)}{\left(b^2-4ac\right)^{7/4}\sqrt{d}} + \frac{6c\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)}{\left(b^2-4ac\right)^{7/4}\sqrt{d}}$$

Mathematica [A] time = 0.163875, size = 167, normalized size = 1.17

$$\frac{-\left(b^2-4ac\right)(b+2cx)+6c\sqrt[4]{b^2-4ac}\sqrt{b+2cx}(a+x(b+cx))\tan^{-1}\left(\frac{\sqrt{b+2cx}}{\sqrt[4]{b^2-4ac}}\right)+6c\sqrt[4]{b^2-4ac}\sqrt{b+2cx}(a+x(b+cx))\tan}{\left(b^2-4ac\right)^2(a+x(b+cx))\sqrt{d(b+2cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[b*d + 2*c*d*x]*(a + b*x + c*x^2)^2),x]
```

```
[Out] (-((b^2 - 4*a*c)*(b + 2*c*x)) + 6*c*(b^2 - 4*a*c)^(1/4)*Sqrt[b + 2*c*x]*(a
+ x*(b + c*x))*ArcTan[Sqrt[b + 2*c*x]/(b^2 - 4*a*c)^(1/4)] + 6*c*(b^2 - 4*a
*c)^(1/4)*Sqrt[b + 2*c*x]*(a + x*(b + c*x))*ArcTanh[Sqrt[b + 2*c*x]/(b^2 -
4*a*c)^(1/4)])/((b^2 - 4*a*c)^2*Sqrt[d*(b + 2*c*x)]*(a + x*(b + c*x)))
```

Maple [B] time = 0.191, size = 345, normalized size = 2.4

$$4\frac{cd^{3}\sqrt{2\,cdx+bd}}{\left(4\,acd^{2}-b^{2}d^{2}\right)\left(4\,c^{2}d^{2}x^{2}+4\,bcd^{2}x+4\,acd^{2}\right)}+\frac{3\,cd^{3}\sqrt{2}}{2}\ln\left(\left(2\,cdx+bd+\sqrt[4]{4\,acd^{2}-b^{2}d^{2}}\sqrt{2\,cdx+bd}\sqrt{2}+\sqrt{4\,acd^{2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^2,x)
```

```
[Out] 4*c*d^3*(2*c*d*x+b*d)^(1/2)/(4*a*c*d^2-b^2*d^2)/(4*c^2*d^2*x^2+4*b*c*d^2*x+
4*a*c*d^2)+3/2*c*d^3/(4*a*c*d^2-b^2*d^2)^(7/4)*2^(1/2)*ln((2*c*d*x+b*d+(4*a
*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)
)/(2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c
*d^2-b^2*d^2)^(1/2)))+3*c*d^3/(4*a*c*d^2-b^2*d^2)^(7/4)*2^(1/2)*arctan(2^(1
/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)-3*c*d^3/(4*a*c*d^2-b^2
*d^2)^(7/4)*2^(1/2)*arctan(-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)
^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.22683, size = 2684, normalized size = 18.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (12*((b^2*c - 4*a*c^2)*d*x^2 + (b^3 - 4*a*b*c)*d*x + (a*b^2 - 4*a^2*c)*d)*(
c^4/((b^14 - 28*a*b^12*c + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b
^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 16384*a^7*c^7)*d^2))^(1/4)
*arctan(-((b^10 - 20*a*b^8*c + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4
*b^2*c^4 - 1024*a^5*c^5)*sqrt(2*c^3*d*x + b*c^2*d + (b^8 - 16*a*b^6*c + 96*
a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4)*sqrt(c^4/((b^14 - 28*a*b^12*c
+ 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^
5 + 28672*a^6*b^2*c^6 - 16384*a^7*c^7)*d^2))*d^2)*(c^4/((b^14 - 28*a*b^12*c
 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c
^5 + 28672*a^6*b^2*c^6 - 16384*a^7*c^7)*d^2))^(3/4)*d - (b^10*c - 20*a*b^8*
c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)*
sqrt(2*c*d*x + b*d)*(c^4/((b^14 - 28*a*b^12*c + 336*a^2*b^10*c^2 - 2240*a^3
*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 16384
*a^7*c^7)*d^2))^(3/4)*d)/c^4) + 3*((b^2*c - 4*a*c^2)*d*x^2 + (b^3 - 4*a*b*c
)*d*x + (a*b^2 - 4*a^2*c)*d)*(c^4/((b^14 - 28*a*b^12*c + 336*a^2*b^10*c^2 -
 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^
6 - 16384*a^7*c^7)*d^2))^(1/4)*log(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c^4/((
```

```
b^14 - 28*a*b^12*c + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4
 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 16384*a^7*c^7)*d^2))^(1/4)*d + 3
*sqrt(2*c*d*x + b*d)*c) - 3*((b^2*c - 4*a*c^2)*d*x^2 + (b^3 - 4*a*b*c)*d*x
+ (a*b^2 - 4*a^2*c)*d)*(c^4/((b^14 - 28*a*b^12*c + 336*a^2*b^10*c^2 - 2240*
a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 16
384*a^7*c^7)*d^2))^(1/4)*log(-3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c^4/((b^14
- 28*a*b^12*c + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21
504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 16384*a^7*c^7)*d^2))^(1/4)*d + 3*sqrt
(2*c*d*x + b*d)*c) - sqrt(2*c*d*x + b*d))/((b^2*c - 4*a*c^2)*d*x^2 + (b^3 -
 4*a*b*c)*d*x + (a*b^2 - 4*a^2*c)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d\left(b+2cx\right)}\left(a+bx+cx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**(1/2)/(c*x**2+b*x+a)**2,x)
```

```
[Out] Integral(1/(sqrt(d*(b + 2*c*x))*(a + b*x + c*x**2)**2), x)
```

Giac [B] time = 1.16489, size = 684, normalized size = 4.78

$$\frac{3\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}+2\sqrt{2\,cdx+bd}\right)}{2\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}}\right)}{b^4d-8\,ab^2cd+16\,a^2c^2d}+\frac{3\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}\cdots\right)}{2\left(-b^2d^2+4\,ac\cdots\right)}\right)}{b^4d-8\,ab^2cd+16\,a^2c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 3*sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(-b^2*
d^2 + 4*a*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4
))/(b^4*d - 8*a*b^2*c*d + 16*a^2*c^2*d) + 3*sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^
(1/4)*c*arctan(-1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) - 2*sqrt(
2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4))/(b^4*d - 8*a*b^2*c*d + 16*a^2
*c^2*d) + 3*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*c*log(2*c*d*x + b*d + sqrt(2)*(-b^
2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2))/
(sqrt(2)*b^4*d - 8*sqrt(2)*a*b^2*c*d + 16*sqrt(2)*a^2*c^2*d) - 3*(-b^2*d^2
+ 4*a*c*d^2)^(1/4)*c*log(2*c*d*x + b*d - sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/
4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2))/(sqrt(2)*b^4*d - 8*sqr
t(2)*a*b^2*c*d + 16*sqrt(2)*a^2*c^2*d) + 4*sqrt(2*c*d*x + b*d)*c*d/((b^2*d^
2 - 4*a*c*d^2 - (2*c*d*x + b*d)^2)*(b^2 - 4*a*c))
```

3.1304 $\int \frac{1}{(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=172

$$-\frac{10c\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{3/2}\left(b^2-4ac\right)^{9/4}}+\frac{10c\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{3/2}\left(b^2-4ac\right)^{9/4}}-\frac{1}{d\left(b^2-4ac\right)\left(a+bx+cx^2\right)\sqrt{bd+2cdx}}-\frac{20c}{d\left(b^2-4ac\right)^2\sqrt{bd+2cdx}}$$

```
[Out] (-20*c)/((b^2 - 4*a*c)^2*d*Sqrt[b*d + 2*c*d*x]) - 1/((b^2 - 4*a*c)*d*Sqrt[b
*d + 2*c*d*x]*(a + b*x + c*x^2)) - (10*c*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 -
 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(9/4)*d^(3/2)) + (10*c*ArcTanh[Sqrt
[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(9/4)*d^(3/2
))
```

Rubi [A] time = 0.123109, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {687, 693, 694, 329, 298, 203, 206}

$$-\frac{10c\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{3/2}\left(b^2-4ac\right)^{9/4}}+\frac{10c\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{3/2}\left(b^2-4ac\right)^{9/4}}-\frac{1}{d\left(b^2-4ac\right)\left(a+bx+cx^2\right)\sqrt{bd+2cdx}}-\frac{20c}{d\left(b^2-4ac\right)^2\sqrt{bd+2cdx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)^2),x]
```


```
[Out] (-20*c)/((b^2 - 4*a*c)^2*d*Sqrt[b*d + 2*c*d*x]) - 1/((b^2 - 4*a*c)*d*Sqrt[b
*d + 2*c*d*x]*(a + b*x + c*x^2)) - (10*c*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 -
 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(9/4)*d^(3/2)) + (10*c*ArcTanh[Sqrt
[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(9/4)*d^(3/2
))
```


Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
```

```
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^2}\,dx = -\frac{1}{\left(b^2-4ac\right)d\sqrt{bd+2cdx}\left(a+bx+cx^2\right)} - \frac{(5c)\int \frac{1}{(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)}\,dx}{b^2-4ac}$$

$$= -\frac{20c}{\left(b^2-4ac\right)^2 d\sqrt{bd+2cdx}} - \frac{1}{\left(b^2-4ac\right)d\sqrt{bd+2cdx}\left(a+bx+cx^2\right)} - \frac{(5c)\int \frac{\sqrt{bd\cdots}}{a+b\cdots}}{\left(b^2-4a\cdots\right.}$$

$$= -\frac{20c}{\left(b^2-4ac\right)^2 d\sqrt{bd+2cdx}} - \frac{1}{\left(b^2-4ac\right)d\sqrt{bd+2cdx}\left(a+bx+cx^2\right)} - \frac{5\,\mathrm{Subst}\left(\cdots\right.}{\cdots}$$

$$= -\frac{20c}{\left(b^2-4ac\right)^2 d\sqrt{bd+2cdx}} - \frac{1}{\left(b^2-4ac\right)d\sqrt{bd+2cdx}\left(a+bx+cx^2\right)} - \frac{5\,\mathrm{Subst}\left(\cdots\right.}{\cdots}$$

$$= -\frac{20c}{\left(b^2-4ac\right)^2 d\sqrt{bd+2cdx}} - \frac{1}{\left(b^2-4ac\right)d\sqrt{bd+2cdx}\left(a+bx+cx^2\right)} + \frac{(10c)\,\mathrm{Subs}\cdots}{\cdots}$$

$$= -\frac{20c}{\left(b^2-4ac\right)^2 d\sqrt{bd+2cdx}} - \frac{1}{\left(b^2-4ac\right)d\sqrt{bd+2cdx}\left(a+bx+cx^2\right)} - \frac{10c\tan^{-1}\cdots}{\left(b^2-4a\cdots\right.}$$

Mathematica [C] time = 0.0513546, size = 55, normalized size = 0.32

$$-\frac{16c\,{}_2F_1\left(-\frac{1}{4},2;\frac{3}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{d\left(b^2-4ac\right)^2\sqrt{d(b+2cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)^2),x]
```

```
[Out] (-16*c*Hypergeometric2F1[-1/4, 2, 3/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/((b^2
- 4*a*c)^2*d*Sqrt[d*(b + 2*c*x)])
```

Maple [B] time = 0.227, size = 404, normalized size = 2.4

$$-16\,{\frac {c}{d \left(4\,ac-{b}^{2} \right) ^{2}\sqrt {2\,cdx+bd}}}-4\,{\frac {c \left(2\,cdx+bd \right) ^{3/2}}{d \left(4\,ac-{b}^{2} \right) ^{2} \left(4\,{c}^{2}{d}^{2}{x}^{2}+4\,bc{d}^{2}x+4\,ac{d}^{2} \right) }}-{\frac {5\,c\sqrt {2}}{2\,d \left(4\,ac-{b}^{2} \right) ^{2}}\ln \left(\left(2\,cdx+bd-\sqrt [4]{4}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^2,x)
```

```
[Out] -16*c/d/(4*a*c-b^2)^2/(2*c*d*x+b*d)^(1/2)-4*c/d/(4*a*c-b^2)^2*(2*c*d*x+b*d)
^(3/2)/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c*d^2)-5/2*c/d/(4*a*c-b^2)^2*2^(1/2)/
(4*a*c*d^2-b^2*d^2)^(1/4)*ln((2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*
x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d+(4*a*c*d^2-b^2
*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)))-5*c/d/(
4*a*c-b^2)^2*2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*arctan(2^(1/2)/(4*a*c*d^2-b^
2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)+5*c/d/(4*a*c-b^2)^2*2^(1/2)/(4*a*c*d^2-
b^2*d^2)^(1/4)*arctan(-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2
)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.33261, size = 4028, normalized size = 23.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (20*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2*x^3 + 3*(b^5*c - 8*a*b^3*c^
2 + 16*a^2*b*c^3)*d^2*x^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2*x + (a*b^5 -
 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*(c^4/((b^18 - 36*a*b^16*c + 576*a^2*b^14*
c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*
a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 262144*a^9*c^9)*d^6
))^(1/4)*arctan((sqrt(2*c^7*d*x + b*c^6*d + (b^10*c^4 - 20*a*b^8*c^5 + 160*
a^2*b^6*c^6 - 640*a^3*b^4*c^7 + 1280*a^4*b^2*c^8 - 1024*a^5*c^9)*d^4*sqrt(c
^4/((b^18 - 36*a*b^16*c + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*
b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 5
89824*a^8*b^2*c^8 - 262144*a^9*c^9)*d^6)))*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*d
*(c^4/((b^18 - 36*a*b^16*c + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a
^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7
+ 589824*a^8*b^2*c^8 - 262144*a^9*c^9)*d^6))^(1/4) - (b^4*c^3 - 8*a*b^2*c^4
 + 16*a^2*c^5)*sqrt(2*c*d*x + b*d)*d*(c^4/((b^18 - 36*a*b^16*c + 576*a^2*b^
14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 3440
64*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 262144*a^9*c^9)*
d^6))^(1/4))/c^4) + 5*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2*x^3 + 3*(
b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2*x^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3
)*d^2*x + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*(c^4/((b^18 - 36*a*b^16
*c + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5
*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 2
62144*a^9*c^9)*d^6))^(1/4)*log(125*(b^14 - 28*a*b^12*c + 336*a^2*b^10*c^2 -
 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^
6 - 16384*a^7*c^7)*d^5*(c^4/((b^18 - 36*a*b^16*c + 576*a^2*b^14*c^2 - 5376*
a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6
 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 262144*a^9*c^9)*d^6))^(3/4) +
125*sqrt(2*c*d*x + b*d)*c^3) - 5*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^
2*x^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2*x^2 + (b^6 - 6*a*b^4*c +
 32*a^3*c^3)*d^2*x + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*(c^4/((b^18
- 36*a*b^16*c + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 -
 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*
b^2*c^8 - 262144*a^9*c^9)*d^6))^(1/4)*log(-125*(b^14 - 28*a*b^12*c + 336*a^
2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 2867
2*a^6*b^2*c^6 - 16384*a^7*c^7)*d^5*(c^4/((b^18 - 36*a*b^16*c + 576*a^2*b^14
*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064
*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 262144*a^9*c^9)*d^
6))^(3/4) + 125*sqrt(2*c*d*x + b*d)*c^3) - (20*c^2*x^2 + 20*b*c*x + b^2 + 1
6*a*c)*sqrt(2*c*d*x + b*d))/(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2*x^3
 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2*x^2 + (b^6 - 6*a*b^4*c + 32*a
^3*c^3)*d^2*x + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**(3/2)/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20927, size = 878, normalized size = 5.1

$$\frac{5\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{3}{4}}c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}+2\sqrt{2\,cdx+bd}\right)}{2\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}}\right)}{b^6d^3-12\,ab^4cd^3+48\,a^2b^2c^2d^3-64\,a^3c^3d^3}+\frac{5\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{3}{4}}c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,ac\right.\right.}{2\left(-b^2d^2\right.}\right.}{b^6d^3-12\,ab^4cd^3+48\,a^2b^2c^2d^3-64\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 5*sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(-b^2*
d^2 + 4*a*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4
))/(b^6*d^3 - 12*a*b^4*c*d^3 + 48*a^2*b^2*c^2*d^3 - 64*a^3*c^3*d^3) + 5*sqr
t(2)*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c*arctan(-1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2
+ 4*a*c*d^2)^(1/4) - 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4))/(
b^6*d^3 - 12*a*b^4*c*d^3 + 48*a^2*b^2*c^2*d^3 - 64*a^3*c^3*d^3) - 5*(-b^2*d
^2 + 4*a*c*d^2)^(3/4)*c*log(2*c*d*x + b*d + sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^
(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2))/(sqrt(2)*b^6*d^3 -
12*sqrt(2)*a*b^4*c*d^3 + 48*sqrt(2)*a^2*b^2*c^2*d^3 - 64*sqrt(2)*a^3*c^3*d^
3) + 5*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c*log(2*c*d*x + b*d - sqrt(2)*(-b^2*d^2
 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2))/(sqrt
(2)*b^6*d^3 - 12*sqrt(2)*a*b^4*c*d^3 + 48*sqrt(2)*a^2*b^2*c^2*d^3 - 64*sqrt
(2)*a^3*c^3*d^3) - 4*(4*b^2*c*d^2 - 16*a*c^2*d^2 - 5*(2*c*d*x + b*d)^2*c)/(
(b^4*d - 8*a*b^2*c*d + 16*a^2*c^2*d)*(sqrt(2*c*d*x + b*d)*b^2*d^2 - 4*sqrt(
2*c*d*x + b*d)*a*c*d^2 - (2*c*d*x + b*d)^(5/2)))
```

3.1305 $\int \frac{1}{(bd+2cdx)^{5/2}\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=174

$$\frac{14c\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{5/2}\left(b^2-4ac\right)^{11/4}}+\frac{14c\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{5/2}\left(b^2-4ac\right)^{11/4}}-\frac{1}{d\left(b^2-4ac\right)\left(a+bx+cx^2\right)(bd+2cdx)^{3/2}}-\frac{28c}{3d\left(b^2-4ac\right)^2(bd+2cd}$$

```
[Out] (-28*c)/(3*(b^2 - 4*a*c)^2*d*(b*d + 2*c*d*x)^(3/2)) - 1/((b^2 - 4*a*c)*d*(b
*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)) + (14*c*ArcTan[Sqrt[d*(b + 2*c*x)]/(
(b^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(11/4)*d^(5/2)) + (14*c*ArcTa
nh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(11/4
)*d^(5/2))
```

Rubi [A] time = 0.141952, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {687, 693, 694, 329, 212, 206, 203}

$$\frac{14c\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{5/2}\left(b^2-4ac\right)^{11/4}}+\frac{14c\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{5/2}\left(b^2-4ac\right)^{11/4}}-\frac{1}{d\left(b^2-4ac\right)\left(a+bx+cx^2\right)(bd+2cdx)^{3/2}}-\frac{28c}{3d\left(b^2-4ac\right)^2(bd+2cd}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^(5/2)*(a + b*x + c*x^2)^2),x]
```

```
[Out] (-28*c)/(3*(b^2 - 4*a*c)^2*d*(b*d + 2*c*d*x)^(3/2)) - 1/((b^2 - 4*a*c)*d*(b
*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)) + (14*c*ArcTan[Sqrt[d*(b + 2*c*x)]/(
(b^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(11/4)*d^(5/2)) + (14*c*ArcTa
nh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(11/4
)*d^(5/2))
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
```

```
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)^{5/2}\left(a+bx+cx^2\right)^2}\,dx = -\frac{1}{\left(b^2-4ac\right)d(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)} - \frac{(7c)\int \frac{1}{(bd+2cdx)^{5/2}\left(a+bx+cx^2\right)}\,dx}{b^2-4ac}$$

$$= -\frac{28c}{3\left(b^2-4ac\right)^2 d(bd+2cdx)^{3/2}} - \frac{1}{\left(b^2-4ac\right)d(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)} - \cdots$$

$$= -\frac{28c}{3\left(b^2-4ac\right)^2 d(bd+2cdx)^{3/2}} - \frac{1}{\left(b^2-4ac\right)d(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)} - \cdots$$

$$= -\frac{28c}{3\left(b^2-4ac\right)^2 d(bd+2cdx)^{3/2}} - \frac{1}{\left(b^2-4ac\right)d(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)} - \cdots$$

$$= -\frac{28c}{3\left(b^2-4ac\right)^2 d(bd+2cdx)^{3/2}} - \frac{1}{\left(b^2-4ac\right)d(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)} + \cdots$$

$$= -\frac{28c}{3\left(b^2-4ac\right)^2 d(bd+2cdx)^{3/2}} - \frac{1}{\left(b^2-4ac\right)d(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)} + \cdots$$

Mathematica [C] time = 0.0657688, size = 57, normalized size = 0.33

$$-\frac{16c\,{}_2F_1\left(-\frac{3}{4},2;\frac{1}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{3d\left(b^2-4ac\right)^2(d(b+2cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^(5/2)*(a + b*x + c*x^2)^2),x]
```

```
[Out] (-16*c*Hypergeometric2F1[-3/4, 2, 1/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(3*(b^
2 - 4*a*c)^2*d*(d*(b + 2*c*x))^(3/2))
```

Maple [B] time = 0.203, size = 404, normalized size = 2.3

$$-\frac{16\,c}{3\,d\left(4\,ac-b^2\right)^2}\left(2\,cdx+bd\right)^{-\frac{3}{2}}-4\,\frac{c\sqrt{2\,cdx+bd}}{d\left(4\,ac-b^2\right)^2\left(4\,c^2d^2x^2+4\,bcd^2x+4\,acd^2\right)}-\frac{7\,c\sqrt{2}}{2\,d\left(4\,ac-b^2\right)^2}\ln\left(\left(2\,cdx+bd+\sqrt[4]{4\,a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^2,x)
```

```
[Out] -16/3*c/d/(4*a*c-b^2)^2/(2*c*d*x+b*d)^(3/2)-4*c/d/(4*a*c-b^2)^2*(2*c*d*x+b*
d)^(1/2)/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c*d^2)-7/2*c/d/(4*a*c-b^2)^2/(4*a*c
*d^2-b^2*d^2)^(3/4)*2^(1/2)*ln((2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*
d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d-(4*a*c*d^2-b
^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)))-7*c/d
/(4*a*c-b^2)^2/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(2^(1/2)/(4*a*c*d^2-
b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)+7*c/d/(4*a*c-b^2)^2/(4*a*c*d^2-b^2*d^
2)^(3/4)*2^(1/2)*arctan(-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1
/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.36246, size = 5273, normalized size = 30.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(84*(4*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d^3*x^4 + 8*(b^5*c^2 - 8*a
*b^3*c^3 + 16*a^2*b*c^4)*d^3*x^3 + (5*b^6*c - 36*a*b^4*c^2 + 48*a^2*b^2*c^3
 + 64*a^3*c^4)*d^3*x^2 + (b^7 - 4*a*b^5*c - 16*a^2*b^3*c^2 + 64*a^3*b*c^3)*
d^3*x + (a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*d^3)*(c^4/((b^22 - 44*a*b^20
*c + 880*a^2*b^18*c^2 - 10560*a^3*b^16*c^3 + 84480*a^4*b^14*c^4 - 473088*a^
5*b^12*c^5 + 1892352*a^6*b^10*c^6 - 5406720*a^7*b^8*c^7 + 10813440*a^8*b^6*
c^8 - 14417920*a^9*b^4*c^9 + 11534336*a^10*b^2*c^10 - 4194304*a^11*c^11)*d^
10))^(1/4)*arctan(((b^16 - 32*a*b^14*c + 448*a^2*b^12*c^2 - 3584*a^3*b^10*c
^3 + 17920*a^4*b^8*c^4 - 57344*a^5*b^6*c^5 + 114688*a^6*b^4*c^6 - 131072*a^
7*b^2*c^7 + 65536*a^8*c^8)*sqrt((b^12 - 24*a*b^10*c + 240*a^2*b^8*c^2 - 128
0*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 + 4096*a^6*c^6)*d^6*sqr
t(c^4/((b^22 - 44*a*b^20*c + 880*a^2*b^18*c^2 - 10560*a^3*b^16*c^3 + 84480*
a^4*b^14*c^4 - 473088*a^5*b^12*c^5 + 1892352*a^6*b^10*c^6 - 5406720*a^7*b^8
*c^7 + 10813440*a^8*b^6*c^8 - 14417920*a^9*b^4*c^9 + 11534336*a^10*b^2*c^10
 - 4194304*a^11*c^11)*d^10)) + 2*c^3*d*x + b*c^2*d)*d^7*(c^4/((b^22 - 44*a*
b^20*c + 880*a^2*b^18*c^2 - 10560*a^3*b^16*c^3 + 84480*a^4*b^14*c^4 - 47308
8*a^5*b^12*c^5 + 1892352*a^6*b^10*c^6 - 5406720*a^7*b^8*c^7 + 10813440*a^8*
b^6*c^8 - 14417920*a^9*b^4*c^9 + 11534336*a^10*b^2*c^10 - 4194304*a^11*c^11
)*d^10))^(3/4) - (b^16*c - 32*a*b^14*c^2 + 448*a^2*b^12*c^3 - 3584*a^3*b^10
*c^4 + 17920*a^4*b^8*c^5 - 57344*a^5*b^6*c^6 + 114688*a^6*b^4*c^7 - 131072*
a^7*b^2*c^8 + 65536*a^8*c^9)*sqrt(2*c*d*x + b*d)*d^7*(c^4/((b^22 - 44*a*b^2
0*c + 880*a^2*b^18*c^2 - 10560*a^3*b^16*c^3 + 84480*a^4*b^14*c^4 - 473088*a
^5*b^12*c^5 + 1892352*a^6*b^10*c^6 - 5406720*a^7*b^8*c^7 + 10813440*a^8*b^6
*c^8 - 14417920*a^9*b^4*c^9 + 11534336*a^10*b^2*c^10 - 4194304*a^11*c^11)*d
^10))^(3/4))/c^4) - 21*(4*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d^3*x^4 + 8*
(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^3*x^3 + (5*b^6*c - 36*a*b^4*c^2 +
48*a^2*b^2*c^3 + 64*a^3*c^4)*d^3*x^2 + (b^7 - 4*a*b^5*c - 16*a^2*b^3*c^2 +
64*a^3*b*c^3)*d^3*x + (a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*d^3)*(c^4/((b^
22 - 44*a*b^20*c + 880*a^2*b^18*c^2 - 10560*a^3*b^16*c^3 + 84480*a^4*b^14*c
^4 - 473088*a^5*b^12*c^5 + 1892352*a^6*b^10*c^6 - 5406720*a^7*b^8*c^7 + 108
13440*a^8*b^6*c^8 - 14417920*a^9*b^4*c^9 + 11534336*a^10*b^2*c^10 - 4194304
*a^11*c^11)*d^10))^(1/4)*log(7*(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*
c^3)*d^3*(c^4/((b^22 - 44*a*b^20*c + 880*a^2*b^18*c^2 - 10560*a^3*b^16*c^3
+ 84480*a^4*b^14*c^4 - 473088*a^5*b^12*c^5 + 1892352*a^6*b^10*c^6 - 5406720
*a^7*b^8*c^7 + 10813440*a^8*b^6*c^8 - 14417920*a^9*b^4*c^9 + 11534336*a^10*
b^2*c^10 - 4194304*a^11*c^11)*d^10))^(1/4) + 7*sqrt(2*c*d*x + b*d)*c) + 21*
(4*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d^3*x^4 + 8*(b^5*c^2 - 8*a*b^3*c^3
+ 16*a^2*b*c^4)*d^3*x^3 + (5*b^6*c - 36*a*b^4*c^2 + 48*a^2*b^2*c^3 + 64*a^3
*c^4)*d^3*x^2 + (b^7 - 4*a*b^5*c - 16*a^2*b^3*c^2 + 64*a^3*b*c^3)*d^3*x + (
a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*d^3)*(c^4/((b^22 - 44*a*b^20*c + 880*
a^2*b^18*c^2 - 10560*a^3*b^16*c^3 + 84480*a^4*b^14*c^4 - 473088*a^5*b^12*c^
5 + 1892352*a^6*b^10*c^6 - 5406720*a^7*b^8*c^7 + 10813440*a^8*b^6*c^8 - 144
17920*a^9*b^4*c^9 + 11534336*a^10*b^2*c^10 - 4194304*a^11*c^11)*d^10))^(1/4
)*log(-7*(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*d^3*(c^4/((b^22 -
 44*a*b^20*c + 880*a^2*b^18*c^2 - 10560*a^3*b^16*c^3 + 84480*a^4*b^14*c^4 -
 473088*a^5*b^12*c^5 + 1892352*a^6*b^10*c^6 - 5406720*a^7*b^8*c^7 + 1081344
0*a^8*b^6*c^8 - 14417920*a^9*b^4*c^9 + 11534336*a^10*b^2*c^10 - 4194304*a^1
1*c^11)*d^10))^(1/4) + 7*sqrt(2*c*d*x + b*d)*c) + (28*c^2*x^2 + 28*b*c*x +
3*b^2 + 16*a*c)*sqrt(2*c*d*x + b*d))/(4*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5
)*d^3*x^4 + 8*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^3*x^3 + (5*b^6*c - 3
6*a*b^4*c^2 + 48*a^2*b^2*c^3 + 64*a^3*c^4)*d^3*x^2 + (b^7 - 4*a*b^5*c - 16*
a^2*b^3*c^2 + 64*a^3*b*c^3)*d^3*x + (a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*
d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**(5/2)/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20674, size = 872, normalized size = 5.01

$$\frac{7\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}+2\sqrt{2\,cdx+bd}\right)}{2\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}}\right)}{b^6d^3-12\,ab^4cd^3+48\,a^2b^2c^2d^3-64\,a^3c^3d^3}+\frac{7\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}\right.}{2\left(-b^2d^2+4\,ac\right.}\right.}{b^6d^3-12\,ab^4cd^3+48\,a^2b^2c^2d^3-64\,a^3c^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 7*sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(-b^2*
d^2 + 4*a*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4
))/(b^6*d^3 - 12*a*b^4*c*d^3 + 48*a^2*b^2*c^2*d^3 - 64*a^3*c^3*d^3) + 7*sqr
t(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*c*arctan(-1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2
+ 4*a*c*d^2)^(1/4) - 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4))/(
b^6*d^3 - 12*a*b^4*c*d^3 + 48*a^2*b^2*c^2*d^3 - 64*a^3*c^3*d^3) + 7*(-b^2*d
^2 + 4*a*c*d^2)^(1/4)*c*log(2*c*d*x + b*d + sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^
(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2))/(sqrt(2)*b^6*d^3 -
12*sqrt(2)*a*b^4*c*d^3 + 48*sqrt(2)*a^2*b^2*c^2*d^3 - 64*sqrt(2)*a^3*c^3*d^
3) - 7*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*c*log(2*c*d*x + b*d - sqrt(2)*(-b^2*d^2
 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2))/(sqrt
(2)*b^6*d^3 - 12*sqrt(2)*a*b^4*c*d^3 + 48*sqrt(2)*a^2*b^2*c^2*d^3 - 64*sqrt
(2)*a^3*c^3*d^3) + 4*sqrt(2*c*d*x + b*d)*c/((b^4*d - 8*a*b^2*c*d + 16*a^2*c
^2*d)*(b^2*d^2 - 4*a*c*d^2 - (2*c*d*x + b*d)^2)) - 16/3*c/((b^4*d - 8*a*b^2
*c*d + 16*a^2*c^2*d)*(2*c*d*x + b*d)^(3/2))
```

3.1306 $\int \frac{1}{(bd+2cdx)^{7/2}\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=203

$$-\frac{36c}{d^3\left(b^2-4ac\right)^3\sqrt{bd+2cdx}}-\frac{18c\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{7/2}\left(b^2-4ac\right)^{13/4}}+\frac{18c\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{7/2}\left(b^2-4ac\right)^{13/4}}-\frac{1}{d\left(b^2-4ac\right)\left(a+bx+cx^2\right)(bd+2}$$

```
[Out] (-36*c)/(5*(b^2 - 4*a*c)^2*d*(b*d + 2*c*d*x)^(5/2)) - (36*c)/((b^2 - 4*a*c)
^3*d^3*Sqrt[b*d + 2*c*d*x]) - 1/((b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(5/2)*(a +
 b*x + c*x^2)) - (18*c*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt
[d])])/((b^2 - 4*a*c)^(13/4)*d^(7/2)) + (18*c*ArcTanh[Sqrt[d*(b + 2*c*x)]/(
(b^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(13/4)*d^(7/2))
```

Rubi [A] time = 0.171861, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {687, 693, 694, 329, 298, 203, 206}

$$-\frac{36c}{d^3\left(b^2-4ac\right)^3\sqrt{bd+2cdx}}-\frac{18c\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{7/2}\left(b^2-4ac\right)^{13/4}}+\frac{18c\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{7/2}\left(b^2-4ac\right)^{13/4}}-\frac{1}{d\left(b^2-4ac\right)\left(a+bx+cx^2\right)(bd+2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^(7/2)*(a + b*x + c*x^2)^2),x]
```

```
[Out] (-36*c)/(5*(b^2 - 4*a*c)^2*d*(b*d + 2*c*d*x)^(5/2)) - (36*c)/((b^2 - 4*a*c)
^3*d^3*Sqrt[b*d + 2*c*d*x]) - 1/((b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(5/2)*(a +
 b*x + c*x^2)) - (18*c*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt
[d])])/((b^2 - 4*a*c)^(13/4)*d^(7/2)) + (18*c*ArcTanh[Sqrt[d*(b + 2*c*x)]/(
(b^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(13/4)*d^(7/2))
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
```

```
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)^{7/2}\left(a+bx+cx^2\right)^2}\,dx = -\frac{1}{\left(b^2-4ac\right)d(bd+2cdx)^{5/2}\left(a+bx+cx^2\right)} - \frac{(9c)\int \frac{1}{(bd+2cdx)^{7/2}\left(a+bx+cx^2\right)}\,dx}{b^2-4ac}$$

$$= -\frac{36c}{5\left(b^2-4ac\right)^2 d(bd+2cdx)^{5/2}} - \frac{1}{\left(b^2-4ac\right)d(bd+2cdx)^{5/2}\left(a+bx+cx^2\right)} - \frac{(9c)}{}$$

$$= -\frac{36c}{5\left(b^2-4ac\right)^2 d(bd+2cdx)^{5/2}} - \frac{36c}{\left(b^2-4ac\right)^3 d^3\sqrt{bd+2cdx}} - \frac{}{\left(b^2-4ac\right)d(bd+2}$$

$$= -\frac{36c}{5\left(b^2-4ac\right)^2 d(bd+2cdx)^{5/2}} - \frac{36c}{\left(b^2-4ac\right)^3 d^3\sqrt{bd+2cdx}} - \frac{}{\left(b^2-4ac\right)d(bd+2}$$

$$= -\frac{36c}{5\left(b^2-4ac\right)^2 d(bd+2cdx)^{5/2}} - \frac{36c}{\left(b^2-4ac\right)^3 d^3\sqrt{bd+2cdx}} - \frac{}{\left(b^2-4ac\right)d(bd+2}$$

$$= -\frac{36c}{5\left(b^2-4ac\right)^2 d(bd+2cdx)^{5/2}} - \frac{36c}{\left(b^2-4ac\right)^3 d^3\sqrt{bd+2cdx}} - \frac{}{\left(b^2-4ac\right)d(bd+2}$$

$$= -\frac{36c}{5\left(b^2-4ac\right)^2 d(bd+2cdx)^{5/2}} - \frac{36c}{\left(b^2-4ac\right)^3 d^3\sqrt{bd+2cdx}} - \frac{}{\left(b^2-4ac\right)d(bd+2}$$

Mathematica [C] time = 0.081183, size = 57, normalized size = 0.28

$$-\frac{16c\,{}_2F_1\left(-\frac{5}{4},2;-\frac{1}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{5d\left(b^2-4ac\right)^2(d(b+2cx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^(7/2)*(a + b*x + c*x^2)^2),x]
```

```
[Out] (-16*c*Hypergeometric2F1[-5/4, 2, -1/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(5*(b
^2 - 4*a*c)^2*d*(d*(b + 2*c*x))^(5/2))
```

Maple [B] time = 0.202, size = 433, normalized size = 2.1

$$-\frac{16c}{5d\left(4ac-b^2\right)^2}(2cdx+bd)^{-\frac{5}{2}}+32\frac{c}{d^3\left(4ac-b^2\right)^3\sqrt{2cdx+bd}}+4\frac{c(2cdx+bd)^{3/2}}{d^3\left(4ac-b^2\right)^3\left(4c^2d^2x^2+4bcd^2x+4acd^2\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^2,x)
```

```
[Out] -16/5*c/d/(4*a*c-b^2)^2/(2*c*d*x+b*d)^(5/2)+32*c/d^3/(4*a*c-b^2)^3/(2*c*d*x
+b*d)^(1/2)+4*c/d^3/(4*a*c-b^2)^3*(2*c*d*x+b*d)^(3/2)/(4*c^2*d^2*x^2+4*b*c*
d^2*x+4*a*c*d^2)+9/2*c/d^3/(4*a*c-b^2)^3*2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*
ln((2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*
c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^
(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)))+9*c/d^3/(4*a*c-b^2)^3*2^(1/2)/(4*
a*c*d^2-b^2*d^2)^(1/4)*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*
d)^(1/2)+1)-9*c/d^3/(4*a*c-b^2)^3*2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*arctan(
-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.10764, size = 7275, normalized size = 35.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

[Out] -1/5*(180*(8*(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*d^4*x^5 + 20*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*d^4*x^4 + 2*(9*b^8*c^2 - 104*a*b^6*c^3 + 384*a^2*b^4*c^4 - 384*a^3*b^2*c^5 - 256*a^4*c^6)*d^4*x^3 + (7*b^9*c - 72*a*b^7*c^2 + 192*a^2*b^5*c^3 + 128*a^3*b^3*c^4 - 768*a^4*b*c^5)*d^4*x^2 + (b^10 - 6*a*b^8*c - 24*a^2*b^6*c^2 + 224*a^3*b^4*c^3 - 384*a^4*b^2*c^4)*d^4*x + (a*b^9 - 12*a^2*b^7*c + 48*a^3*b^5*c^2 - 64*a^4*b^3*c^3)*d^4)*(c^4/((b^26 - 52*a*b^24*c + 1248*a^2*b^22*c^2 - 18304*a^3*b^20*c^3 + 183040*a^4*b^18*c^4 - 1317888*a^5*b^16*c^5 + 7028736*a^6*b^14*c^6 - 28114944*a^7*b^12*c^7 + 84344832*a^8*b^10*c^8 - 187432960*a^9*b^8*c^9 + 299892736*a^10*b^6*c^10 - 327155712*a^11*b^4*c^11 + 218103808*a^12*b^2*c^12 - 67108864*a^13*c^13)*d^14))^(1/4)*arctan(-(sqrt((b^14*c^4 - 28*a*b^12*c^5 + 336*a^2*b^10*c^6 - 2240*a^3*b^8*c^7 + 8960*a^4*b^6*c^8 - 21504*a^5*b^4*c^9 + 28672*a^6*b^2*c^10 - 16384*a^7*c^11)*d^8*sqrt(c^4/((b^26 - 52*a*b^24*c + 1248*a^2*b^22*c^2 - 18304*a^3*b^20*c^3 + 183040*a^4*b^18*c^4 - 1317888*a^5*b^16*c^5 + 7028736*a^6*b^14*c^6 - 28114944*a^7*b^12*c^7 + 84344832*a^8*b^10*c^8 - 187432960*a^9*b^8*c^9 + 299892736*a^10*b^6*c^10 - 327155712*a^11*b^4*c^11 + 218103808*a^12*b^2*c^12 - 67108864*a^13*c^13)*d^14)) + 2*c^7*d*x + b*c^6*d)*(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*d^3*(c^4/((b^26 - 52*a*b^24*c + 1248*a^2*b^22*c^2 - 18304*a^3*b^20*c^3 + 183040*a^4*b^18*c^4 - 1317888*a^5*b^16*c^5 + 7028736*a^6*b^14*c^6 - 28114944*a^7*b^12*c^7 + 84344832*a^8*b^10*c^8 - 187432960*a^9*b^8*c^9 + 299892736*a^10*b^6*c^10 - 327155712*a^11*b^4*c^11 + 218103808*a^12*b^2*c^12 - 67108864*a^13*c^13)*d^14))^(1/4) - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt(2*c*d*x + b*d)*d^3*(c^4/((b^26 - 52*a*b^24*c + 1248*a^2*b^22*c^2 - 18304*a^3*b^20*c^3 + 183040*a^4*b^18*c^4 - 1317888*a^5*b^16*c^5 + 7028736*a^6*b^14*c^6 - 28114944*a^7*b^12*c^7 + 84344832*a^8*b^10*c^8 - 187432960*a^9*b^8*c^9 + 299892736*a^10*b^6*c^10 - 327155712*a^11*b^4*c^11 + 218103808*a^12*b^2*c^12 - 67108864*a^13*c^13)*d^14))^(1/4))/c^4) - 45*(8*(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*d^4*x^5 + 20*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*d^4*x^4 + 2*(9*b^8*c^2 - 104*a*b^6*c^3 + 384*a^2*b^4*c^4 - 384*a^3*b^2*c^5 - 256*a^4*c^6)*d^4*x^3 + (7*b^9*c - 72*a*b^7*c^2 + 192*a^2*b^5*c^3 + 128*a^3*b^3*c^4 - 768*a^4*b*c^5)*d^4*x^2 + (b^10 - 6*a*b^8*c - 24*a^2*b^6*c^2 + 224*a^3*b^4*c^3 - 384*a^4*b^2*c^4)*d^4*x + (a*b^9 - 12*a^2*b^7*c + 48*a^3*b^5*c^2 - 64*a^4*b^3*c^3)*d^4)*(c^4/((b^26 - 52*a*b^24*c + 1248*a^2*b^22*c^2 - 18304*a^3*b^20*c^3 + 183040*a^4*b^18*c^4 - 1317888*a^5*b^16*c^5 + 7028736*a^6*b^14*c^6 - 28114944*a^7*b^12*c^7 + 84344832*a^8*b^10*c^8 - 187432960*a^9*b^8*c^9 + 299892736*a^10*b^6*c^10 - 327155712*a^11*b^4*c^11 + 218103808*a^12*b^2*c^12 - 67108864*a^13*c^13)*d^14))^(1/4)*log(729*(b^20 - 40*a*b^18*c + 720*a^2*b^16*c^2 - 7680*a^3*b^14*c^3 + 53760*a^4*b^12*c^4 - 258048*a^5*b^10*c^5 + 860160*a^6*b^8*c^6 - 1966080*a^7*b^6*c^7 + 2949120*a^8*b^4*c^8 - 2621440*a^9*b^2*c^9 + 1048576*a^10*c^10)*d^11*(c^4/((b^26 - 52*a*b^24*c + 1248*a^2*b^22*c^2 - 18304*a^3*b^20*c^3 + 183040*a^4*b^18*c^4 - 1317888*a^5*b^16*c^5 + 7028736*a^6*b^14*c^6 - 28114944*a^7*b^12*c^7 + 84344832*a^8*b^10*c^8 - 187432960*a^9*b^8*c^9 + 299892736*a^10*b^6*c^10 - 327155712*a^11*b^4*c^11 + 218103808*a^12*b^2*c^12 - 67108864*a^13*c^13)*d^14))^(3/4) + 729*sqrt(2*c*d*x + b*d)*c^3) + 45*(8*(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*d^4*x^5 + 20*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*d^4*x^4 + 2*(9*b^8*c^2 - 104*a*b^6*c^3 + 384*a^2*b^4*c^4 - 384*a^3*b^2*c^5 - 256*a^4*c^6)*d^4*x^3 + (7*b^9*c - 72*a*b^7*c^2 + 192*a^2*b^5*c^3 + 128*a^3*b^3*c^4 - 768*a^4*b*c^5)*d^4*x^2 + (b^10 - 6*a*b^8*c - 24*a^2*b^6*c^2 + 224*a^3*b^4*c^3 - 384*a^4*b^2*c^4)*d^4*x + (a*b^9 - 12*a^2*b^7*c + 48*a^3*b^5*c^2 - 64*a^4*b^3*c^3)*d^4)*(c^4/((b^26 - 52*a*b^24*c + 1248*a^2*b^22*c^2 - 18304*a^3*b^20*c^3 + 183040*a^4*b^18*c^4 - 1317888*a^5*b^16*c^5 + 7028736*a^6*b^14*c^6 - 28114944*a^7*b^12*c^7 + 84344832*a^8*b^10*c^8 - 187432960*a^9*b^8*c^9 + 299892736*a^10*b^6*c^10 - 327155712*a^11*b^4*c^11 + 218103808*a^12*b^2*c^12 - 67108864*a^13*c^13)*d^14))^(1/4)*log(-729*(b^20 - 40*a*b^18*c + 720*a^2*b^16*c^2 - 7680*a^3*b^14*c^3 + 53760*a^4*b^12*c^4 - 258048*a^5*b^10*c^5 + 860160*a^6*b^8*c^6 - 1966080*a^7*b^6*c^7 + 2949120*a^8*b^4*c^8 - 2621440*a^9*b^2*c^9 + 1048576*a^10*c^10)*

```
d^11*(c^4/((b^26 - 52*a*b^24*c + 1248*a^2*b^22*c^2 - 18304*a^3*b^20*c^3 + 1
83040*a^4*b^18*c^4 - 1317888*a^5*b^16*c^5 + 7028736*a^6*b^14*c^6 - 28114944
*a^7*b^12*c^7 + 84344832*a^8*b^10*c^8 - 187432960*a^9*b^8*c^9 + 299892736*a
^10*b^6*c^10 - 327155712*a^11*b^4*c^11 + 218103808*a^12*b^2*c^12 - 67108864
*a^13*c^13)*d^14))^(3/4) + 729*sqrt(2*c*d*x + b*d)*c^3) + (720*c^4*x^4 + 14
40*b*c^3*x^3 + 5*b^4 + 176*a*b^2*c - 64*a^2*c^2 + 72*(13*b^2*c^2 + 8*a*c^3)
*x^2 + 72*(3*b^3*c + 8*a*b*c^2)*x)*sqrt(2*c*d*x + b*d))/(8*(b^6*c^4 - 12*a*
b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*d^4*x^5 + 20*(b^7*c^3 - 12*a*b^5*c^4
 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*d^4*x^4 + 2*(9*b^8*c^2 - 104*a*b^6*c^3 +
384*a^2*b^4*c^4 - 384*a^3*b^2*c^5 - 256*a^4*c^6)*d^4*x^3 + (7*b^9*c - 72*a*
b^7*c^2 + 192*a^2*b^5*c^3 + 128*a^3*b^3*c^4 - 768*a^4*b*c^5)*d^4*x^2 + (b^1
0 - 6*a*b^8*c - 24*a^2*b^6*c^2 + 224*a^3*b^4*c^3 - 384*a^4*b^2*c^4)*d^4*x +
 (a*b^9 - 12*a^2*b^7*c + 48*a^3*b^5*c^2 - 64*a^4*b^3*c^3)*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**(7/2)/(c*x**2+b*x+a)**2,x)

[Out] Timed out
```

Giac [B] time = 1.21528, size = 1052, normalized size = 5.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^2,x, algorithm="giac")

[Out] 9*sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(-b^2*
d^2 + 4*a*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4
))/(b^8*d^5 - 16*a*b^6*c*d^5 + 96*a^2*b^4*c^2*d^5 - 256*a^3*b^2*c^3*d^5 + 2
56*a^4*c^4*d^5) + 9*sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c*arctan(-1/2*sqrt
(2)*(sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) - 2*sqrt(2*c*d*x + b*d))/(-b^2*d^
2 + 4*a*c*d^2)^(1/4))/(b^8*d^5 - 16*a*b^6*c*d^5 + 96*a^2*b^4*c^2*d^5 - 256*
a^3*b^2*c^3*d^5 + 256*a^4*c^4*d^5) - 9*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c*log(2
*c*d*x + b*d + sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + s
qrt(-b^2*d^2 + 4*a*c*d^2))/(sqrt(2)*b^8*d^5 - 16*sqrt(2)*a*b^6*c*d^5 + 96*s
qrt(2)*a^2*b^4*c^2*d^5 - 256*sqrt(2)*a^3*b^2*c^3*d^5 + 256*sqrt(2)*a^4*c^4*
d^5) + 9*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c*log(2*c*d*x + b*d - sqrt(2)*(-b^2*d
^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2))/(sq
rt(2)*b^8*d^5 - 16*sqrt(2)*a*b^6*c*d^5 + 96*sqrt(2)*a^2*b^4*c^2*d^5 - 256*s
qrt(2)*a^3*b^2*c^3*d^5 + 256*sqrt(2)*a^4*c^4*d^5) + 4*(2*c*d*x + b*d)^(3/2)
*c/((b^6*d^3 - 12*a*b^4*c*d^3 + 48*a^2*b^2*c^2*d^3 - 64*a^3*c^3*d^3)*(b^2*d
^2 - 4*a*c*d^2 - (2*c*d*x + b*d)^2)) - 16/5*(b^2*c*d^2 - 4*a*c^2*d^2 + 10*(
2*c*d*x + b*d)^2*c)/((b^6*d^3 - 12*a*b^4*c*d^3 + 48*a^2*b^2*c^2*d^3 - 64*a^
3*c^3*d^3)*(2*c*d*x + b*d)^(5/2))
```

3.1307 $\int \frac{(bd+2cdx)^{17/2}}{\left(a+bx+cx^2\right)^3}\,dx$

Optimal. Leaf size=222

$$110c^2d^7\left(b^2-4ac\right)(bd+2cdx)^{3/2}+165c^2d^{17/2}\left(b^2-4ac\right)^{7/4}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-165c^2d^{17/2}\left(b^2-4ac\right)^{7/4}\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{d}}\right.$$

```
[Out] 110*c^2*(b^2 - 4*a*c)*d^7*(b*d + 2*c*d*x)^(3/2) + (330*c^2*d^5*(b*d + 2*c*d
*x)^(7/2))/7 - (d*(b*d + 2*c*d*x)^(15/2))/(2*(a + b*x + c*x^2)^2) - (15*c*d
^3*(b*d + 2*c*d*x)^(11/2))/(2*(a + b*x + c*x^2)) + 165*c^2*(b^2 - 4*a*c)^(7
/4)*d^(17/2)*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])] - 16
5*c^2*(b^2 - 4*a*c)^(7/4)*d^(17/2)*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*
c)^(1/4)*Sqrt[d])]
```

Rubi [A] time = 0.19136, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {686, 692, 694, 329, 298, 203, 206}

$$110c^2d^7\left(b^2-4ac\right)(bd+2cdx)^{3/2}+165c^2d^{17/2}\left(b^2-4ac\right)^{7/4}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-165c^2d^{17/2}\left(b^2-4ac\right)^{7/4}\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{d}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(17/2)/(a + b*x + c*x^2)^3,x]
```

```
[Out] 110*c^2*(b^2 - 4*a*c)*d^7*(b*d + 2*c*d*x)^(3/2) + (330*c^2*d^5*(b*d + 2*c*d
*x)^(7/2))/7 - (d*(b*d + 2*c*d*x)^(15/2))/(2*(a + b*x + c*x^2)^2) - (15*c*d
^3*(b*d + 2*c*d*x)^(11/2))/(2*(a + b*x + c*x^2)) + 165*c^2*(b^2 - 4*a*c)^(7
/4)*d^(17/2)*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])] - 16
5*c^2*(b^2 - 4*a*c)^(7/4)*d^(17/2)*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*
c)^(1/4)*Sqrt[d])]
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
```

```
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(bd+2cdx)^{17/2}}{\left(a+bx+cx^2\right)^3}\,dx = -\frac{d(bd+2cdx)^{15/2}}{2\left(a+bx+cx^2\right)^2}+\frac{1}{2}\left(15cd^2\right)\int \frac{(bd+2cdx)^{13/2}}{\left(a+bx+cx^2\right)^2}\,dx$$

$$= -\frac{d(bd+2cdx)^{15/2}}{2\left(a+bx+cx^2\right)^2}-\frac{15cd^3(bd+2cdx)^{11/2}}{2\left(a+bx+cx^2\right)}+\frac{1}{2}\left(165c^2d^4\right)\int \frac{(bd+2cdx)^{9/2}}{a+bx+cx^2}\,dx$$

$$= \frac{330}{7}c^2d^5(bd+2cdx)^{7/2}-\frac{d(bd+2cdx)^{15/2}}{2\left(a+bx+cx^2\right)^2}-\frac{15cd^3(bd+2cdx)^{11/2}}{2\left(a+bx+cx^2\right)}+\frac{1}{2}\left(165c^2\left(b^2-4ac\right)d^6\right)$$

$$= 110c^2\left(b^2-4ac\right)d^7(bd+2cdx)^{3/2}+\frac{330}{7}c^2d^5(bd+2cdx)^{7/2}-\frac{d(bd+2cdx)^{15/2}}{2\left(a+bx+cx^2\right)^2}-\frac{15cd^3(bd+}{2\left(a+bx\right.}$$

$$= 110c^2\left(b^2-4ac\right)d^7(bd+2cdx)^{3/2}+\frac{330}{7}c^2d^5(bd+2cdx)^{7/2}-\frac{d(bd+2cdx)^{15/2}}{2\left(a+bx+cx^2\right)^2}-\frac{15cd^3(bd+}{2\left(a+bx\right.}$$

$$= 110c^2\left(b^2-4ac\right)d^7(bd+2cdx)^{3/2}+\frac{330}{7}c^2d^5(bd+2cdx)^{7/2}-\frac{d(bd+2cdx)^{15/2}}{2\left(a+bx+cx^2\right)^2}-\frac{15cd^3(bd+}{2\left(a+bx\right.}$$

$$= 110c^2\left(b^2-4ac\right)d^7(bd+2cdx)^{3/2}+\frac{330}{7}c^2d^5(bd+2cdx)^{7/2}-\frac{d(bd+2cdx)^{15/2}}{2\left(a+bx+cx^2\right)^2}-\frac{15cd^3(bd+}{2\left(a+bx\right.}$$

$$= 110c^2\left(b^2-4ac\right)d^7(bd+2cdx)^{3/2}+\frac{330}{7}c^2d^5(bd+2cdx)^{7/2}-\frac{d(bd+2cdx)^{15/2}}{2\left(a+bx+cx^2\right)^2}-\frac{15cd^3(bd+}{2\left(a+bx\right.}$$

Mathematica [C] time = 0.324156, size = 138, normalized size = 0.62

$$\frac{4(d(b+2cx))^{17/2}\left(\left(b^2-4ac\right)\left(77\left(\left(b^2-4ac\right)^2-16c^2(a+x(b+cx))^2\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)\right)-55\left(b^2-4ac\right)(b+2cx)^2+5(\cdots\right.\right.}{7(b+2cx)^7(a+x(b+cx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(17/2)/(a + b*x + c*x^2)^3,x]
```

```
[Out] (4*(d*(b + 2*c*x))^(17/2)*((b + 2*c*x)^6 + (b^2 - 4*a*c)*(-55*(b^2 - 4*a*c)
*(b + 2*c*x)^2 + 5*(b + 2*c*x)^4 + 77*((b^2 - 4*a*c)^2 - 16*c^2*(a + x*(b +
 c*x))^2*Hypergeometric2F1[3/4, 3, 7/4, (b + 2*c*x)^2/(b^2 - 4*a*c)]))))/(7
*(b + 2*c*x)^7*(a + x*(b + c*x))^2)
```

Maple [B] time = 0.219, size = 1310, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(17/2)/(c*x^2+b*x+a)^3,x)
```

```
[Out] 64/7*c^2*d^5*(2*c*d*x+b*d)^(7/2)-256*c^3*d^7*(2*c*d*x+b*d)^(3/2)*a+64*c^2*d
^7*(2*c*d*x+b*d)^(3/2)*b^2-864*c^4*d^9/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c*d^2
)^2*(2*c*d*x+b*d)^(7/2)*a^2+432*c^3*d^9/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c*d^
2)^2*(2*c*d*x+b*d)^(7/2)*a*b^2-54*c^2*d^9/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c*
d^2)^2*(2*c*d*x+b*d)^(7/2)*b^4-2944*c^5*d^11/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a
*c*d^2)^2*(2*c*d*x+b*d)^(3/2)*a^3+2208*c^4*d^11/(4*c^2*d^2*x^2+4*b*c*d^2*x+
4*a*c*d^2)^2*(2*c*d*x+b*d)^(3/2)*a^2*b^2-552*c^3*d^11/(4*c^2*d^2*x^2+4*b*c*
d^2*x+4*a*c*d^2)^2*(2*c*d*x+b*d)^(3/2)*a*b^4+46*c^2*d^11/(4*c^2*d^2*x^2+4*b
*c*d^2*x+4*a*c*d^2)^2*(2*c*d*x+b*d)^(3/2)*b^6+660*c^4*d^9*2^(1/2)/(4*a*c*d^
2-b^2*d^2)^(1/4)*a^2*ln((2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d
)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)
^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)))+1320*c^4*d^9
*2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*a^2*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(
1/4)*(2*c*d*x+b*d)^(1/2)+1)-1320*c^4*d^9*2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*
a^2*arctan(-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)-330*c^
3*d^9*2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*a*b^2*ln((2*c*d*x+b*d-(4*a*c*d^2-b^
2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*
x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*
d^2)^(1/2)))-660*c^3*d^9*2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*a*b^2*arctan(2^(
1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)+660*c^3*d^9*2^(1/2)/(
4*a*c*d^2-b^2*d^2)^(1/4)*a*b^2*arctan(-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2
*c*d*x+b*d)^(1/2)+1)+165/4*c^2*d^9*2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*b^4*ln
((2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*
d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1
/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)))+165/2*c^2*d^9*2^(1/2)/(4*a*c*d^2-b^
2*d^2)^(1/4)*b^4*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/
2)+1)-165/2*c^2*d^9*2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*b^4*arctan(-2^(1/2)/(
4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)
```

Maple [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(17/2)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.7111, size = 4231, normalized size = 19.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(17/2)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/14*(4620*((b^14*c^8 - 28*a*b^12*c^9 + 336*a^2*b^10*c^10 - 2240*a^3*b^8*c
^11 + 8960*a^4*b^6*c^12 - 21504*a^5*b^4*c^13 + 28672*a^6*b^2*c^14 - 16384*a
^7*c^15)*d^34)^(1/4)*(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a
^2)*arctan(-(((b^14*c^8 - 28*a*b^12*c^9 + 336*a^2*b^10*c^10 - 2240*a^3*b^8*
c^11 + 8960*a^4*b^6*c^12 - 21504*a^5*b^4*c^13 + 28672*a^6*b^2*c^14 - 16384*
a^7*c^15)*d^34)^(1/4)*(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*
b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)*sqrt(2*c*d*x + b*d)*d^25 + sqr
t(2*(b^20*c^13 - 40*a*b^18*c^14 + 720*a^2*b^16*c^15 - 7680*a^3*b^14*c^16 +
53760*a^4*b^12*c^17 - 258048*a^5*b^10*c^18 + 860160*a^6*b^8*c^19 - 1966080*
a^7*b^6*c^20 + 2949120*a^8*b^4*c^21 - 2621440*a^9*b^2*c^22 + 1048576*a^10*c
^23)*d^51*x + (b^21*c^12 - 40*a*b^19*c^13 + 720*a^2*b^17*c^14 - 7680*a^3*b^
15*c^15 + 53760*a^4*b^13*c^16 - 258048*a^5*b^11*c^17 + 860160*a^6*b^9*c^18
- 1966080*a^7*b^7*c^19 + 2949120*a^8*b^5*c^20 - 2621440*a^9*b^3*c^21 + 1048
576*a^10*b*c^22)*d^51 + sqrt((b^14*c^8 - 28*a*b^12*c^9 + 336*a^2*b^10*c^10
- 2240*a^3*b^8*c^11 + 8960*a^4*b^6*c^12 - 21504*a^5*b^4*c^13 + 28672*a^6*b^
2*c^14 - 16384*a^7*c^15)*d^34)*(b^14*c^8 - 28*a*b^12*c^9 + 336*a^2*b^10*c^1
0 - 2240*a^3*b^8*c^11 + 8960*a^4*b^6*c^12 - 21504*a^5*b^4*c^13 + 28672*a^6*
b^2*c^14 - 16384*a^7*c^15)*d^34)*((b^14*c^8 - 28*a*b^12*c^9 + 336*a^2*b^10*
c^10 - 2240*a^3*b^8*c^11 + 8960*a^4*b^6*c^12 - 21504*a^5*b^4*c^13 + 28672*a
^6*b^2*c^14 - 16384*a^7*c^15)*d^34)^(1/4))/((b^14*c^8 - 28*a*b^12*c^9 + 336
*a^2*b^10*c^10 - 2240*a^3*b^8*c^11 + 8960*a^4*b^6*c^12 - 21504*a^5*b^4*c^13
 + 28672*a^6*b^2*c^14 - 16384*a^7*c^15)*d^34)) - 1155*((b^14*c^8 - 28*a*b^1
2*c^9 + 336*a^2*b^10*c^10 - 2240*a^3*b^8*c^11 + 8960*a^4*b^6*c^12 - 21504*a
^5*b^4*c^13 + 28672*a^6*b^2*c^14 - 16384*a^7*c^15)*d^34)^(1/4)*(c^2*x^4 + 2
*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*log(-4492125*(b^10*c^6 - 20*a
*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5
*c^11)*sqrt(2*c*d*x + b*d)*d^25 + 4492125*((b^14*c^8 - 28*a*b^12*c^9 + 336*
a^2*b^10*c^10 - 2240*a^3*b^8*c^11 + 8960*a^4*b^6*c^12 - 21504*a^5*b^4*c^13
+ 28672*a^6*b^2*c^14 - 16384*a^7*c^15)*d^34)^(3/4)) + 1155*((b^14*c^8 - 28*
a*b^12*c^9 + 336*a^2*b^10*c^10 - 2240*a^3*b^8*c^11 + 8960*a^4*b^6*c^12 - 21
504*a^5*b^4*c^13 + 28672*a^6*b^2*c^14 - 16384*a^7*c^15)*d^34)^(1/4)*(c^2*x^
4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*log(-4492125*(b^10*c^6 -
 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 102
4*a^5*c^11)*sqrt(2*c*d*x + b*d)*d^25 - 4492125*((b^14*c^8 - 28*a*b^12*c^9 +
 336*a^2*b^10*c^10 - 2240*a^3*b^8*c^11 + 8960*a^4*b^6*c^12 - 21504*a^5*b^4*
c^13 + 28672*a^6*b^2*c^14 - 16384*a^7*c^15)*d^34)^(3/4)) - (1024*c^7*d^8*x^
7 + 3584*b*c^6*d^8*x^6 + 512*(13*b^2*c^5 - 10*a*c^6)*d^8*x^5 + 2560*(3*b^3*
c^4 - 5*a*b*c^5)*d^8*x^4 + 10*(423*b^4*c^3 - 312*a*b^2*c^4 - 1936*a^2*c^5)*
d^8*x^3 + (457*b^5*c^2 + 8120*a*b^3*c^3 - 29040*a^2*b*c^4)*d^8*x^2 - (203*b
^6*c - 3350*a*b^4*c^2 + 5280*a^2*b^2*c^3 + 12320*a^3*c^4)*d^8*x - (7*b^7 +
105*a*b^5*c - 2200*a^2*b^3*c^2 + 6160*a^3*b*c^3)*d^8)*sqrt(2*c*d*x + b*d))/
(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(17/2)/(c*x**2+b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.40116, size = 903, normalized size = 4.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(17/2)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 64*(2*c*d*x + b*d)^(3/2)*b^2*c^2*d^7 - 256*(2*c*d*x + b*d)^(3/2)*a*c^3*d^7
+ 64/7*(2*c*d*x + b*d)^(7/2)*c^2*d^5 - 165/2*sqrt(2)*(b^2*c^2*d^7 - 4*a*c^3
*d^7)*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2 +
4*a*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4)) - 1
65/2*sqrt(2)*(b^2*c^2*d^7 - 4*a*c^3*d^7)*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) - 2*sqrt(2*c*d*x + b*d
))/(-b^2*d^2 + 4*a*c*d^2)^(1/4)) + 165/4*sqrt(2)*(b^2*c^2*d^7 - 4*a*c^3*d^7
)*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*log(2*c*d*x + b*d + sqrt(2)*(-b^2*d^2 + 4*a*
c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2)) - 165/4*sqrt
(2)*(b^2*c^2*d^7 - 4*a*c^3*d^7)*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*log(2*c*d*x +
b*d - sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*
d^2 + 4*a*c*d^2)) + 2*(23*(2*c*d*x + b*d)^(3/2)*b^6*c^2*d^11 - 276*(2*c*d*x
 + b*d)^(3/2)*a*b^4*c^3*d^11 + 1104*(2*c*d*x + b*d)^(3/2)*a^2*b^2*c^4*d^11
- 1472*(2*c*d*x + b*d)^(3/2)*a^3*c^5*d^11 - 27*(2*c*d*x + b*d)^(7/2)*b^4*c^
2*d^9 + 216*(2*c*d*x + b*d)^(7/2)*a*b^2*c^3*d^9 - 432*(2*c*d*x + b*d)^(7/2)
*a^2*c^4*d^9)/(b^2*d^2 - 4*a*c*d^2 - (2*c*d*x + b*d)^2)^2
```

3.1308 $\int \frac{(bd+2cdx)^{15/2}}{\left(a+bx+cx^2\right)^3}\,dx$

Optimal. Leaf size=222

$$234c^2d^7\left(b^2-4ac\right)\sqrt{bd+2cdx}-117c^2d^{15/2}\left(b^2-4ac\right)^{5/4}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-117c^2d^{15/2}\left(b^2-4ac\right)^{5/4}\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{d}}\right.$$

```
[Out] 234*c^2*(b^2 - 4*a*c)*d^7*Sqrt[b*d + 2*c*d*x] + (234*c^2*d^5*(b*d + 2*c*d*x
)^(5/2))/5 - (d*(b*d + 2*c*d*x)^(13/2))/(2*(a + b*x + c*x^2)^2) - (13*c*d^3
*(b*d + 2*c*d*x)^(9/2))/(2*(a + b*x + c*x^2)) - 117*c^2*(b^2 - 4*a*c)^(5/4)
*d^(15/2)*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])] - 117*c
^2*(b^2 - 4*a*c)^(5/4)*d^(15/2)*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^
(1/4)*Sqrt[d])]
```

Rubi [A] time = 0.18192, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {686, 692, 694, 329, 212, 206, 203}

$$234c^2d^7\left(b^2-4ac\right)\sqrt{bd+2cdx}-117c^2d^{15/2}\left(b^2-4ac\right)^{5/4}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-117c^2d^{15/2}\left(b^2-4ac\right)^{5/4}\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{d}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(15/2)/(a + b*x + c*x^2)^3,x]
```


```
[Out] 234*c^2*(b^2 - 4*a*c)*d^7*Sqrt[b*d + 2*c*d*x] + (234*c^2*d^5*(b*d + 2*c*d*x
)^(5/2))/5 - (d*(b*d + 2*c*d*x)^(13/2))/(2*(a + b*x + c*x^2)^2) - (13*c*d^3
*(b*d + 2*c*d*x)^(9/2))/(2*(a + b*x + c*x^2)) - 117*c^2*(b^2 - 4*a*c)^(5/4)
*d^(15/2)*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])] - 117*c
^2*(b^2 - 4*a*c)^(5/4)*d^(15/2)*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^
(1/4)*Sqrt[d])]
```


Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
```

```
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(bd+2cdx)^{15/2}}{\left(a+bx+cx^2\right)^3}\,dx = -\frac{d(bd+2cdx)^{13/2}}{2\left(a+bx+cx^2\right)^2}+\frac{1}{2}\left(13cd^2\right)\int \frac{(bd+2cdx)^{11/2}}{\left(a+bx+cx^2\right)^2}\,dx$$

$$= -\frac{d(bd+2cdx)^{13/2}}{2\left(a+bx+cx^2\right)^2}-\frac{13cd^3(bd+2cdx)^{9/2}}{2\left(a+bx+cx^2\right)}+\frac{1}{2}\left(117c^2d^4\right)\int \frac{(bd+2cdx)^{7/2}}{a+bx+cx^2}\,dx$$

$$= \frac{234}{5}c^2d^5(bd+2cdx)^{5/2}-\frac{d(bd+2cdx)^{13/2}}{2\left(a+bx+cx^2\right)^2}-\frac{13cd^3(bd+2cdx)^{9/2}}{2\left(a+bx+cx^2\right)}+\frac{1}{2}\left(117c^2\left(b^2-4ac\right)d^6\right)\int \frac{(}{}$$

$$= 234c^2\left(b^2-4ac\right)d^7\sqrt{bd+2cdx}+\frac{234}{5}c^2d^5(bd+2cdx)^{5/2}-\frac{d(bd+2cdx)^{13/2}}{2\left(a+bx+cx^2\right)^2}-\frac{13cd^3(bd+2cdx}{2\left(a+bx+cx^2\right.}$$

$$= 234c^2\left(b^2-4ac\right)d^7\sqrt{bd+2cdx}+\frac{234}{5}c^2d^5(bd+2cdx)^{5/2}-\frac{d(bd+2cdx)^{13/2}}{2\left(a+bx+cx^2\right)^2}-\frac{13cd^3(bd+2cdx}{2\left(a+bx+cx^2\right.}$$

$$= 234c^2\left(b^2-4ac\right)d^7\sqrt{bd+2cdx}+\frac{234}{5}c^2d^5(bd+2cdx)^{5/2}-\frac{d(bd+2cdx)^{13/2}}{2\left(a+bx+cx^2\right)^2}-\frac{13cd^3(bd+2cdx}{2\left(a+bx+cx^2\right.}$$

$$= 234c^2\left(b^2-4ac\right)d^7\sqrt{bd+2cdx}+\frac{234}{5}c^2d^5(bd+2cdx)^{5/2}-\frac{d(bd+2cdx)^{13/2}}{2\left(a+bx+cx^2\right)^2}-\frac{13cd^3(bd+2cdx}{2\left(a+bx+cx^2\right.}$$

$$= 234c^2\left(b^2-4ac\right)d^7\sqrt{bd+2cdx}+\frac{234}{5}c^2d^5(bd+2cdx)^{5/2}-\frac{d(bd+2cdx)^{13/2}}{2\left(a+bx+cx^2\right)^2}-\frac{13cd^3(bd+2cdx}{2\left(a+bx+cx^2\right.}$$

Mathematica [A] time = 0.75566, size = 225, normalized size = 1.01

$$\frac{(d(b+2cx))^{15/2}\left(91\left(b^2-4ac\right)\left(-192\left(b^2-4ac\right)(b+2cx)^{5/2}+120\left(b^2-4ac\right)^2\sqrt{b+2cx}+60c\sqrt[4]{b^2-4ac}(a+x(b+cx)\right.\right.}{560(b+2cx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(15/2)/(a + b*x + c*x^2)^3,x]
```

```
[Out] ((d*(b + 2*c*x))^(15/2)*(448*(b + 2*c*x)^(13/2) + 91*(b^2 - 4*a*c)*(120*(b^
2 - 4*a*c)^2*Sqrt[b + 2*c*x] - 192*(b^2 - 4*a*c)*(b + 2*c*x)^(5/2) + 64*(b
+ 2*c*x)^(9/2) + 60*c*(b^2 - 4*a*c)^(1/4)*(a + x*(b + c*x))*(2*(b^2 - 4*a*c
)^(3/4)*Sqrt[b + 2*c*x] - 12*c*(a + x*(b + c*x))*(ArcTan[Sqrt[b + 2*c*x]/(b
^2 - 4*a*c)^(1/4)] + ArcTanh[Sqrt[b + 2*c*x]/(b^2 - 4*a*c)^(1/4)])))))/(560
*(b + 2*c*x)^(15/2)*(a + x*(b + c*x))^2)
```

Maple [B] time = 0.211, size = 1310, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(15/2)/(c*x^2+b*x+a)^3,x)
```

```
[Out] 64/5*c^2*d^5*(2*c*d*x+b*d)^(5/2)-768*c^3*d^7*a*(2*c*d*x+b*d)^(1/2)+192*c^2*
d^7*b^2*(2*c*d*x+b*d)^(1/2)-800*c^4*d^9/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c*d^
2)^2*(2*c*d*x+b*d)^(5/2)*a^2+400*c^3*d^9/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c*d
^2)^2*(2*c*d*x+b*d)^(5/2)*a*b^2-50*c^2*d^9/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c
*d^2)^2*(2*c*d*x+b*d)^(5/2)*b^4-2688*c^5*d^11/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*
a*c*d^2)^2*(2*c*d*x+b*d)^(1/2)*a^3+2016*c^4*d^11/(4*c^2*d^2*x^2+4*b*c*d^2*x
+4*a*c*d^2)^2*(2*c*d*x+b*d)^(1/2)*a^2*b^2-504*c^3*d^11/(4*c^2*d^2*x^2+4*b*c
*d^2*x+4*a*c*d^2)^2*(2*c*d*x+b*d)^(1/2)*a*b^4+42*c^2*d^11/(4*c^2*d^2*x^2+4*
b*c*d^2*x+4*a*c*d^2)^2*(2*c*d*x+b*d)^(1/2)*b^6+936*c^4*d^9/(4*a*c*d^2-b^2*d
^2)^(3/4)*2^(1/2)*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1
/2)+1)*a^2-468*c^3*d^9/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(2^(1/2)/(4*
a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*a*b^2+117/2*c^2*d^9/(4*a*c*d^
2-b^2*d^2)^(3/4)*2^(1/2)*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+
b*d)^(1/2)+1)*b^4-936*c^4*d^9/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(-2^(
1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*a^2+468*c^3*d^9/(4*a*
c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c
*d*x+b*d)^(1/2)+1)*a*b^2-117/2*c^2*d^9/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*ar
ctan(-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*b^4+468*c^4*
d^9/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*ln((2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(
1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d-(4
*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/
2)))*a^2-234*c^3*d^9/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*ln((2*c*d*x+b*d+(4*a
*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)
)/(2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c
*d^2-b^2*d^2)^(1/2)))*a*b^2+117/4*c^2*d^9/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)
*ln((2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a
*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)
^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)))*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(15/2)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.84988, size = 2502, normalized size = 11.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(15/2)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/10*(2340*((b^10*c^8 - 20*a*b^8*c^9 + 160*a^2*b^6*c^10 - 640*a^3*b^4*c^11
+ 1280*a^4*b^2*c^12 - 1024*a^5*c^13)*d^30)^(1/4)*(c^2*x^4 + 2*b*c*x^3 + 2*a
*b*x + (b^2 + 2*a*c)*x^2 + a^2)*arctan(-(((b^10*c^8 - 20*a*b^8*c^9 + 160*a^
2*b^6*c^10 - 640*a^3*b^4*c^11 + 1280*a^4*b^2*c^12 - 1024*a^5*c^13)*d^30)^(3
/4)*(b^2*c^2 - 4*a*c^3)*sqrt(2*c*d*x + b*d)*d^7 + ((b^10*c^8 - 20*a*b^8*c^9
 + 160*a^2*b^6*c^10 - 640*a^3*b^4*c^11 + 1280*a^4*b^2*c^12 - 1024*a^5*c^13)
*d^30)^(3/4)*sqrt(2*(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*d^15*x + (b^5*c^4
- 8*a*b^3*c^5 + 16*a^2*b*c^6)*d^15 + sqrt((b^10*c^8 - 20*a*b^8*c^9 + 160*a^
2*b^6*c^10 - 640*a^3*b^4*c^11 + 1280*a^4*b^2*c^12 - 1024*a^5*c^13)*d^30)))/
((b^10*c^8 - 20*a*b^8*c^9 + 160*a^2*b^6*c^10 - 640*a^3*b^4*c^11 + 1280*a^4*
b^2*c^12 - 1024*a^5*c^13)*d^30)) + 585*((b^10*c^8 - 20*a*b^8*c^9 + 160*a^2*
b^6*c^10 - 640*a^3*b^4*c^11 + 1280*a^4*b^2*c^12 - 1024*a^5*c^13)*d^30)^(1/4
)*(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*log(-117*(b^2*c
^2 - 4*a*c^3)*sqrt(2*c*d*x + b*d)*d^7 + 117*((b^10*c^8 - 20*a*b^8*c^9 + 160
*a^2*b^6*c^10 - 640*a^3*b^4*c^11 + 1280*a^4*b^2*c^12 - 1024*a^5*c^13)*d^30)
^(1/4)) - 585*((b^10*c^8 - 20*a*b^8*c^9 + 160*a^2*b^6*c^10 - 640*a^3*b^4*c^
11 + 1280*a^4*b^2*c^12 - 1024*a^5*c^13)*d^30)^(1/4)*(c^2*x^4 + 2*b*c*x^3 +
2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*log(-117*(b^2*c^2 - 4*a*c^3)*sqrt(2*c*d*
x + b*d)*d^7 - 117*((b^10*c^8 - 20*a*b^8*c^9 + 160*a^2*b^6*c^10 - 640*a^3*b
^4*c^11 + 1280*a^4*b^2*c^12 - 1024*a^5*c^13)*d^30)^(1/4)) + (512*c^6*d^7*x^
6 + 1536*b*c^5*d^7*x^5 + 512*(7*b^2*c^4 - 13*a*c^5)*d^7*x^4 + 512*(9*b^3*c^
3 - 26*a*b*c^4)*d^7*x^3 + 3*(641*b^4*c^2 - 520*a*b^2*c^3 - 5616*a^2*c^4)*d^
7*x^2 - (125*b^5*c - 5096*a*b^3*c^2 + 16848*a^2*b*c^3)*d^7*x - (5*b^6 + 65*
a*b^4*c - 2808*a^2*b^2*c^2 + 9360*a^3*c^3)*d^7)*sqrt(2*c*d*x + b*d))/(c^2*x
^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(15/2)/(c*x**2+b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.29784, size = 903, normalized size = 4.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(15/2)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 192*sqrt(2*c*d*x + b*d)*b^2*c^2*d^7 - 768*sqrt(2*c*d*x + b*d)*a*c^3*d^7 + 6
4/5*(2*c*d*x + b*d)^(5/2)*c^2*d^5 - 117/2*sqrt(2)*(b^2*c^2*d^7 - 4*a*c^3*d^
7)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2 + 4*a
*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4)) - 117/
2*sqrt(2)*(b^2*c^2*d^7 - 4*a*c^3*d^7)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*arctan(-
1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) - 2*sqrt(2*c*d*x + b*d))/
(-b^2*d^2 + 4*a*c*d^2)^(1/4)) - 117/4*sqrt(2)*(b^2*c^2*d^7 - 4*a*c^3*d^7)*(
-b^2*d^2 + 4*a*c*d^2)^(1/4)*log(2*c*d*x + b*d + sqrt(2)*(-b^2*d^2 + 4*a*c*d
^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2)) + 117/4*sqrt(2)
*(b^2*c^2*d^7 - 4*a*c^3*d^7)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*log(2*c*d*x + b*d
 - sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2
 + 4*a*c*d^2)) + 2*(21*sqrt(2*c*d*x + b*d)*b^6*c^2*d^11 - 252*sqrt(2*c*d*x
+ b*d)*a*b^4*c^3*d^11 + 1008*sqrt(2*c*d*x + b*d)*a^2*b^2*c^4*d^11 - 1344*sq
rt(2*c*d*x + b*d)*a^3*c^5*d^11 - 25*(2*c*d*x + b*d)^(5/2)*b^4*c^2*d^9 + 200
*(2*c*d*x + b*d)^(5/2)*a*b^2*c^3*d^9 - 400*(2*c*d*x + b*d)^(5/2)*a^2*c^4*d^
9)/(b^2*d^2 - 4*a*c*d^2 - (2*c*d*x + b*d)^2)^2
```

3.1309 $\int \frac{(bd+2cdx)^{13/2}}{(a+bx+cx^2)^3}\,dx$

Optimal. Leaf size=193

$$77c^2d^{13/2}\left(b^2-4ac\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-77c^2d^{13/2}\left(b^2-4ac\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-\frac{11cd^3(bd+2cdx)^{7/2}}{2\left(a+bx+cx^2\right)}-\frac{d}{2}$$

```
[Out] (154*c^2*d^5*(b*d + 2*c*d*x)^(3/2))/3 - (d*(b*d + 2*c*d*x)^(11/2))/(2*(a +
b*x + c*x^2)^2) - (11*c*d^3*(b*d + 2*c*d*x)^(7/2))/(2*(a + b*x + c*x^2)) +
77*c^2*(b^2 - 4*a*c)^(3/4)*d^(13/2)*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*
c)^(1/4)*Sqrt[d])] - 77*c^2*(b^2 - 4*a*c)^(3/4)*d^(13/2)*ArcTanh[Sqrt[d*(b
+ 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])]
```

Rubi [A] time = 0.148014, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {686, 692, 694, 329, 298, 203, 206}

$$77c^2d^{13/2}\left(b^2-4ac\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-77c^2d^{13/2}\left(b^2-4ac\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-\frac{11cd^3(bd+2cdx)^{7/2}}{2\left(a+bx+cx^2\right)}-\frac{d}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(13/2)/(a + b*x + c*x^2)^3,x]
```


```
[Out] (154*c^2*d^5*(b*d + 2*c*d*x)^(3/2))/3 - (d*(b*d + 2*c*d*x)^(11/2))/(2*(a +
b*x + c*x^2)^2) - (11*c*d^3*(b*d + 2*c*d*x)^(7/2))/(2*(a + b*x + c*x^2)) +
77*c^2*(b^2 - 4*a*c)^(3/4)*d^(13/2)*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*
c)^(1/4)*Sqrt[d])] - 77*c^2*(b^2 - 4*a*c)^(3/4)*d^(13/2)*ArcTanh[Sqrt[d*(b
+ 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])]
```


Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
```

```
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(bd+2cdx)^{13/2}}{\left(a+bx+cx^2\right)^3}\,dx &= -\frac{d(bd+2cdx)^{11/2}}{2\left(a+bx+cx^2\right)^2}+\frac{1}{2}\left(11cd^2\right)\int \frac{(bd+2cdx)^{9/2}}{\left(a+bx+cx^2\right)^2}\,dx \\
&= -\frac{d(bd+2cdx)^{11/2}}{2\left(a+bx+cx^2\right)^2}-\frac{11cd^3(bd+2cdx)^{7/2}}{2\left(a+bx+cx^2\right)}+\frac{1}{2}\left(77c^2d^4\right)\int \frac{(bd+2cdx)^{5/2}}{a+bx+cx^2}\,dx \\
&= \frac{154}{3}c^2d^5(bd+2cdx)^{3/2}-\frac{d(bd+2cdx)^{11/2}}{2\left(a+bx+cx^2\right)^2}-\frac{11cd^3(bd+2cdx)^{7/2}}{2\left(a+bx+cx^2\right)}+\frac{1}{2}\left(77c^2\left(b^2-4ac\right)d^6\right)\int \\
&= \frac{154}{3}c^2d^5(bd+2cdx)^{3/2}-\frac{d(bd+2cdx)^{11/2}}{2\left(a+bx+cx^2\right)^2}-\frac{11cd^3(bd+2cdx)^{7/2}}{2\left(a+bx+cx^2\right)}+\frac{1}{4}\left(77c\left(b^2-4ac\right)d^5\right)\mathrm{Sub} \\
&= \frac{154}{3}c^2d^5(bd+2cdx)^{3/2}-\frac{d(bd+2cdx)^{11/2}}{2\left(a+bx+cx^2\right)^2}-\frac{11cd^3(bd+2cdx)^{7/2}}{2\left(a+bx+cx^2\right)}+\frac{1}{2}\left(77c\left(b^2-4ac\right)d^5\right)\mathrm{Sub} \\
&= \frac{154}{3}c^2d^5(bd+2cdx)^{3/2}-\frac{d(bd+2cdx)^{11/2}}{2\left(a+bx+cx^2\right)^2}-\frac{11cd^3(bd+2cdx)^{7/2}}{2\left(a+bx+cx^2\right)}-\left(77c^2\left(b^2-4ac\right)d^7\right)\mathrm{Subs} \\
&= \frac{154}{3}c^2d^5(bd+2cdx)^{3/2}-\frac{d(bd+2cdx)^{11/2}}{2\left(a+bx+cx^2\right)^2}-\frac{11cd^3(bd+2cdx)^{7/2}}{2\left(a+bx+cx^2\right)}+77c^2\left(b^2-4ac\right)^{3/4}d^{13/2}\,\mathrm{ta}
\end{aligned}
$$

Mathematica [C] time = 0.160206, size = 145, normalized size = 0.75

$$
-\frac{4d^5(d(b+2cx))^{3/2}\left(-16c^2\left(77a^2+55acx^2+5c^2x^4\right)+1232c^2(a+x(b+cx))^2\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)+4b^2c\left(99a+25cx\right.\right.}{15(a+x(b+cx))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(13/2)/(a + b*x + c*x^2)^3,x]
```

```
[Out] (-4*d^5*(d*(b + 2*c*x))^(3/2)*(-27*b^4 + 180*b^3*c*x - 80*b*c^2*x*(11*a + 2
*c*x^2) + 4*b^2*c*(99*a + 25*c*x^2) - 16*c^2*(77*a^2 + 55*a*c*x^2 + 5*c^2*x
^4) + 1232*c^2*(a + x*(b + c*x))^2*Hypergeometric2F1[3/4, 3, 7/4, (b + 2*c*
x)^2/(b^2 - 4*a*c)]))/(15*(a + x*(b + c*x))^2)
```

Maple [B] time = 0.203, size = 857, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(13/2)/(c*x^2+b*x+a)^3,x)
```

```
[Out] 64/3*c^2*d^5*(2*c*d*x+b*d)^(3/2)+152*c^3*d^7/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a
*c*d^2)^2*(2*c*d*x+b*d)^(7/2)*a-38*c^2*d^7/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c
*d^2)^2*(2*c*d*x+b*d)^(7/2)*b^2+480*c^4*d^9/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*
c*d^2)^2*(2*c*d*x+b*d)^(3/2)*a^2-240*c^3*d^9/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a
*c*d^2)^2*(2*c*d*x+b*d)^(3/2)*a*b^2+30*c^2*d^9/(4*c^2*d^2*x^2+4*b*c*d^2*x+4
*a*c*d^2)^2*(2*c*d*x+b*d)^(3/2)*b^4-77*c^3*d^7*2^(1/2)/(4*a*c*d^2-b^2*d^2)^
(1/4)*a*ln((2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/
2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d
*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)))-154*c^3*d^7*2^(1/2)/(4*a*
c*d^2-b^2*d^2)^(1/4)*a*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*
d)^(1/2)+1)+154*c^3*d^7*2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*a*arctan(-2^(1/2)
/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)+77/4*c^2*d^7*2^(1/2)/(4*a
*c*d^2-b^2*d^2)^(1/4)*b^2*ln((2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*
x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d+(4*a*c*d^2-b^2
*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)))+77/2*c^
2*d^7*2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*b^2*arctan(2^(1/2)/(4*a*c*d^2-b^2*d
^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)-77/2*c^2*d^7*2^(1/2)/(4*a*c*d^2-b^2*d^2)^(
1/4)*b^2*arctan(-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(13/2)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.90145, size = 2148, normalized size = 11.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(13/2)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(924*((b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)*d^26)^(1
/4)*(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*arctan((((b^6
*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)*d^26)^(1/4)*(b^4*c^6 -
 8*a*b^2*c^7 + 16*a^2*c^8)*sqrt(2*c*d*x + b*d)*d^19 - sqrt(2*(b^8*c^13 - 16
*a*b^6*c^14 + 96*a^2*b^4*c^15 - 256*a^3*b^2*c^16 + 256*a^4*c^17)*d^39*x + (
b^9*c^12 - 16*a*b^7*c^13 + 96*a^2*b^5*c^14 - 256*a^3*b^3*c^15 + 256*a^4*b*c
^16)*d^39 + sqrt((b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)*d
^26)*(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)*d^26)*((b^6*c
^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)*d^26)^(1/4))/((b^6*c^8 -
 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)*d^26)) - 231*((b^6*c^8 - 12*
a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)*d^26)^(1/4)*(c^2*x^4 + 2*b*c*x^3
 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*log(456533*(b^4*c^6 - 8*a*b^2*c^7 + 1
6*a^2*c^8)*sqrt(2*c*d*x + b*d)*d^19 + 456533*((b^6*c^8 - 12*a*b^4*c^9 + 48*
a^2*b^2*c^10 - 64*a^3*c^11)*d^26)^(3/4)) + 231*((b^6*c^8 - 12*a*b^4*c^9 + 4
8*a^2*b^2*c^10 - 64*a^3*c^11)*d^26)^(1/4)*(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x +
(b^2 + 2*a*c)*x^2 + a^2)*log(456533*(b^4*c^6 - 8*a*b^2*c^7 + 16*a^2*c^8)*sq
rt(2*c*d*x + b*d)*d^19 - 456533*((b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10
- 64*a^3*c^11)*d^26)^(3/4)) + (256*c^5*d^6*x^5 + 640*b*c^4*d^6*x^4 + 2*(199
*b^2*c^3 + 484*a*c^4)*d^6*x^3 - (43*b^3*c^2 - 1452*a*b*c^3)*d^6*x^2 - (63*b
^4*c - 418*a*b^2*c^2 - 616*a^2*c^3)*d^6*x - (3*b^5 + 33*a*b^3*c - 308*a^2*b
*c^2)*d^6)*sqrt(2*c*d*x + b*d))/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a
*c)*x^2 + a^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(13/2)/(c*x**2+b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27027, size = 703, normalized size = 3.64

$$-\frac{77}{2}\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{3}{4}}c^2d^5\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}+2\sqrt{2\,cdx+bd}\right)}{2\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}}\right)-\frac{77}{2}\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{3}{4}}c^2d^5\,\varepsilon$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(13/2)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] -77/2*sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c^2*d^5*arctan(1/2*sqrt(2)*(sqrt
(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c
*d^2)^(1/4)) - 77/2*sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c^2*d^5*arctan(-1/
2*sqrt(2)*(sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) - 2*sqrt(2*c*d*x + b*d))/(-
b^2*d^2 + 4*a*c*d^2)^(1/4)) + 77/4*sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c^2
*d^5*log(2*c*d*x + b*d + sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x
+ b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2)) - 77/4*sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^
(3/4)*c^2*d^5*log(2*c*d*x + b*d - sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt
(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2)) + 64/3*(2*c*d*x + b*d)^(3/2)*
c^2*d^5 + 2*(15*(2*c*d*x + b*d)^(3/2)*b^4*c^2*d^9 - 120*(2*c*d*x + b*d)^(3/
```

```
2)*a*b^2*c^3*d^9 + 240*(2*c*d*x + b*d)^(3/2)*a^2*c^4*d^9 - 19*(2*c*d*x + b*
d)^(7/2)*b^2*c^2*d^7 + 76*(2*c*d*x + b*d)^(7/2)*a*c^3*d^7)/(b^2*d^2 - 4*a*c
*d^2 - (2*c*d*x + b*d)^2)^2
```

3.1310 $\int \frac{(bd+2cdx)^{11/2}}{(a+bx+cx^2)^3} dx$

Optimal. Leaf size=191

$$-45c^2d^{11/2}\sqrt[4]{b^2-4ac}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-45c^2d^{11/2}\sqrt[4]{b^2-4ac}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-\frac{9cd^3(bd+2cdx)^{5/2}}{2\left(a+bx+cx^2\right)}-\frac{d(\cdots}{2(\cdots}$$

```
[Out] 90*c^2*d^5*Sqrt[b*d + 2*c*d*x] - (d*(b*d + 2*c*d*x)^(9/2))/(2*(a + b*x + c*
x^2)^2) - (9*c*d^3*(b*d + 2*c*d*x)^(5/2))/(2*(a + b*x + c*x^2)) - 45*c^2*(b
^2 - 4*a*c)^(1/4)*d^(11/2)*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*
Sqrt[d])] - 45*c^2*(b^2 - 4*a*c)^(1/4)*d^(11/2)*ArcTanh[Sqrt[d*(b + 2*c*x)]
/((b^2 - 4*a*c)^(1/4)*Sqrt[d])]
```

Rubi [A] time = 0.147768, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {686, 692, 694, 329, 212, 206, 203}

$$-45c^2d^{11/2}\sqrt[4]{b^2-4ac}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-45c^2d^{11/2}\sqrt[4]{b^2-4ac}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)-\frac{9cd^3(bd+2cdx)^{5/2}}{2\left(a+bx+cx^2\right)}-\frac{d(\cdots}{2(\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(11/2)/(a + b*x + c*x^2)^3,x]
```

```
[Out] 90*c^2*d^5*Sqrt[b*d + 2*c*d*x] - (d*(b*d + 2*c*d*x)^(9/2))/(2*(a + b*x + c*
x^2)^2) - (9*c*d^3*(b*d + 2*c*d*x)^(5/2))/(2*(a + b*x + c*x^2)) - 45*c^2*(b
^2 - 4*a*c)^(1/4)*d^(11/2)*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*
Sqrt[d])] - 45*c^2*(b^2 - 4*a*c)^(1/4)*d^(11/2)*ArcTanh[Sqrt[d*(b + 2*c*x)]
/((b^2 - 4*a*c)^(1/4)*Sqrt[d])]
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
```

```
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(bd+2cdx)^{11/2}}{\left(a+bx+cx^2\right)^3}\,dx &= -\frac{d(bd+2cdx)^{9/2}}{2\left(a+bx+cx^2\right)^2}+\frac{1}{2}\left(9cd^2\right)\int \frac{(bd+2cdx)^{7/2}}{\left(a+bx+cx^2\right)^2}\,dx \\
&= -\frac{d(bd+2cdx)^{9/2}}{2\left(a+bx+cx^2\right)^2}-\frac{9cd^3(bd+2cdx)^{5/2}}{2\left(a+bx+cx^2\right)}+\frac{1}{2}\left(45c^2d^4\right)\int \frac{(bd+2cdx)^{3/2}}{a+bx+cx^2}\,dx \\
&= 90c^2d^5\sqrt{bd+2cdx}-\frac{d(bd+2cdx)^{9/2}}{2\left(a+bx+cx^2\right)^2}-\frac{9cd^3(bd+2cdx)^{5/2}}{2\left(a+bx+cx^2\right)}+\frac{1}{2}\left(45c^2\left(b^2-4ac\right)d^6\right)\int \frac{}{\sqrt{bd+}} \\
&= 90c^2d^5\sqrt{bd+2cdx}-\frac{d(bd+2cdx)^{9/2}}{2\left(a+bx+cx^2\right)^2}-\frac{9cd^3(bd+2cdx)^{5/2}}{2\left(a+bx+cx^2\right)}+\frac{1}{4}\left(45c\left(b^2-4ac\right)d^5\right)\text{Subst}\left(\int \right. \\
&= 90c^2d^5\sqrt{bd+2cdx}-\frac{d(bd+2cdx)^{9/2}}{2\left(a+bx+cx^2\right)^2}-\frac{9cd^3(bd+2cdx)^{5/2}}{2\left(a+bx+cx^2\right)}+\frac{1}{2}\left(45c\left(b^2-4ac\right)d^5\right)\text{Subst}\left(\int \right. \\
&= 90c^2d^5\sqrt{bd+2cdx}-\frac{d(bd+2cdx)^{9/2}}{2\left(a+bx+cx^2\right)^2}-\frac{9cd^3(bd+2cdx)^{5/2}}{2\left(a+bx+cx^2\right)}-\left(45c^2\sqrt{b^2-4ac}d^6\right)\text{Subst}\left(\left(\int \frac{}{\sqrt{b}}\right.\right. \\
&= 90c^2d^5\sqrt{bd+2cdx}-\frac{d(bd+2cdx)^{9/2}}{2\left(a+bx+cx^2\right)^2}-\frac{9cd^3(bd+2cdx)^{5/2}}{2\left(a+bx+cx^2\right)}-45c^2\sqrt[4]{b^2-4ac}d^{11/2}\tan^{-1}\left(\frac{\sqrt{d(b}}{\sqrt[4]{b^2-}}\right.
\end{aligned}$$

Mathematica [A] time = 0.349355, size = 226, normalized size = 1.18

$$-\frac{d^5\sqrt{d(b+2cx)}\left(\sqrt{b+2cx}\left(-4c^2\left(45a^2+81acx^2+32c^2x^4\right)+3b^2c\left(3a-37cx^2\right)-4bc^2x\left(81a+64cx^2\right)+17b^3cx+b^4\right)+\right.}{2\sqrt{b+2cx}(a+x(b+cx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(11/2)/(a + b*x + c*x^2)^3,x]
```

```
[Out] -(d^5*Sqrt[d*(b + 2*c*x)]*(Sqrt[b + 2*c*x]*(b^4 + 17*b^3*c*x + 3*b^2*c*(3*a
 - 37*c*x^2) - 4*b*c^2*x*(81*a + 64*c*x^2) - 4*c^2*(45*a^2 + 81*a*c*x^2 + 3
2*c^2*x^4)) + 90*c^2*(b^2 - 4*a*c)^(1/4)*(a + x*(b + c*x))^2*ArcTan[Sqrt[b
+ 2*c*x]/(b^2 - 4*a*c)^(1/4)] + 90*c^2*(b^2 - 4*a*c)^(1/4)*(a + x*(b + c*x)
)^2*ArcTanh[Sqrt[b + 2*c*x]/(b^2 - 4*a*c)^(1/4)]))/(2*Sqrt[b + 2*c*x]*(a +
x*(b + c*x))^2)
```

Maple [B] time = 0.199, size = 857, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(11/2)/(c*x^2+b*x+a)^3,x)
```

```
[Out] 64*c^2*d^5*(2*c*d*x+b*d)^(1/2)+136*c^3*d^7/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c
*d^2)^2*(2*c*d*x+b*d)^(5/2)*a-34*c^2*d^7/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c*d
^2)^2*(2*c*d*x+b*d)^(5/2)*b^2+416*c^4*d^9/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c*
d^2)^2*(2*c*d*x+b*d)^(1/2)*a^2-208*c^3*d^9/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c
*d^2)^2*(2*c*d*x+b*d)^(1/2)*a*b^2+26*c^2*d^9/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a
*c*d^2)^2*(2*c*d*x+b*d)^(1/2)*b^4-90*c^3*d^7/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1
/2)*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*a+45/2*
c^2*d^7/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^2
)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*b^2+90*c^3*d^7/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(
1/2)*arctan(-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*a-45/
2*c^2*d^7/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(-2^(1/2)/(4*a*c*d^2-b^2*
d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)*b^2-45*c^3*d^7/(4*a*c*d^2-b^2*d^2)^(3/4)*
2^(1/2)*ln((2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/
2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d
*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)))*a+45/4*c^2*d^7/(4*a*c*d^2
-b^2*d^2)^(3/4)*2^(1/2)*ln((2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+
b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d-(4*a*c*d^2-b^2*d
^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(11/2)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.7876, size = 1170, normalized size = 6.13

$$-\frac{180\left(\left(b^2c^8-4\,ac^9\right)d^{22}\right)^{\frac{1}{4}}\left(c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\,ac\right)x^2+a^2\right)\arctan\left(\frac{\left(\left(b^2c^8-4\,ac^9\right)d^{22}\right)^{\frac{3}{4}}\sqrt{2\,cdx+bd}\,c^2d^5-\left(\left(b^2c^8-4\,ac^9\right)\right.}{\left(b^2c^8-4\,ac\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(11/2)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(180*((b^2*c^8 - 4*a*c^9)*d^22)^(1/4)*(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x +
 (b^2 + 2*a*c)*x^2 + a^2)*arctan((((b^2*c^8 - 4*a*c^9)*d^22)^(3/4)*sqrt(2*c
*d*x + b*d)*c^2*d^5 - ((b^2*c^8 - 4*a*c^9)*d^22)^(3/4)*sqrt(2*c^5*d^11*x +
b*c^4*d^11 + sqrt((b^2*c^8 - 4*a*c^9)*d^22)))/((b^2*c^8 - 4*a*c^9)*d^22)) +
 45*((b^2*c^8 - 4*a*c^9)*d^22)^(1/4)*(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2
+ 2*a*c)*x^2 + a^2)*log(45*sqrt(2*c*d*x + b*d)*c^2*d^5 + 45*((b^2*c^8 - 4*a
*c^9)*d^22)^(1/4)) - 45*((b^2*c^8 - 4*a*c^9)*d^22)^(1/4)*(c^2*x^4 + 2*b*c*x
^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*log(45*sqrt(2*c*d*x + b*d)*c^2*d^5
- 45*((b^2*c^8 - 4*a*c^9)*d^22)^(1/4)) - (128*c^4*d^5*x^4 + 256*b*c^3*d^5*x
^3 + 3*(37*b^2*c^2 + 108*a*c^3)*d^5*x^2 - (17*b^3*c - 324*a*b*c^2)*d^5*x -
(b^4 + 9*a*b^2*c - 180*a^2*c^2)*d^5)*sqrt(2*c*d*x + b*d))/(c^2*x^4 + 2*b*c*
x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(11/2)/(c*x**2+b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25903, size = 703, normalized size = 3.68

$$-\frac{45}{2}\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}c^2d^5\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}+2\,\sqrt{2\,cdx+bd}\right)}{2\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}}\right)-\frac{45}{2}\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}c^2d^5\,\mathrm{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(11/2)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] -45/2*sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*c^2*d^5*arctan(1/2*sqrt(2)*(sqrt
(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c
*d^2)^(1/4)) - 45/2*sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*c^2*d^5*arctan(-1/
2*sqrt(2)*(sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) - 2*sqrt(2*c*d*x + b*d))/(-
b^2*d^2 + 4*a*c*d^2)^(1/4)) - 45/4*sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*c^2
*d^5*log(2*c*d*x + b*d + sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x
+ b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2)) + 45/4*sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^
(1/4)*c^2*d^5*log(2*c*d*x + b*d - sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt
(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2)) + 64*sqrt(2*c*d*x + b*d)*c^2*
d^5 + 2*(13*sqrt(2*c*d*x + b*d)*b^4*c^2*d^9 - 104*sqrt(2*c*d*x + b*d)*a*b^2
*c^3*d^9 + 208*sqrt(2*c*d*x + b*d)*a^2*c^4*d^9 - 17*(2*c*d*x + b*d)^(5/2)*b
^2*c^2*d^7 + 68*(2*c*d*x + b*d)^(5/2)*a*c^3*d^7)/(b^2*d^2 - 4*a*c*d^2 - (2*
c*d*x + b*d)^2)^2
```

3.1311 $\int \frac{(bd+2cdx)^{9/2}}{(a+bx+cx^2)^3}\,dx$

Optimal. Leaf size=170

$$\frac{21c^2d^{9/2}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\sqrt[4]{b^2-4ac}}-\frac{21c^2d^{9/2}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\sqrt[4]{b^2-4ac}}-\frac{7cd^3(bd+2cdx)^{3/2}}{2\left(a+bx+cx^2\right)}-\frac{d(bd+2cdx)^{7/2}}{2\left(a+bx+cx^2\right)^2}$$

```
[Out] -(d*(b*d + 2*c*d*x)^(7/2))/(2*(a + b*x + c*x^2)^2) - (7*c*d^3*(b*d + 2*c*d*
x)^(3/2))/(2*(a + b*x + c*x^2)) + (21*c^2*d^(9/2)*ArcTan[Sqrt[d*(b + 2*c*x)
]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(b^2 - 4*a*c)^(1/4) - (21*c^2*d^(9/2)*Arc
Tanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(b^2 - 4*a*c)^(1/4
)
```

Rubi [A] time = 0.123375, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {686, 694, 329, 298, 203, 206}

$$\frac{21c^2d^{9/2}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\sqrt[4]{b^2-4ac}}-\frac{21c^2d^{9/2}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\sqrt[4]{b^2-4ac}}-\frac{7cd^3(bd+2cdx)^{3/2}}{2\left(a+bx+cx^2\right)}-\frac{d(bd+2cdx)^{7/2}}{2\left(a+bx+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(9/2)/(a + b*x + c*x^2)^3,x]
```


```
[Out] -(d*(b*d + 2*c*d*x)^(7/2))/(2*(a + b*x + c*x^2)^2) - (7*c*d^3*(b*d + 2*c*d*
x)^(3/2))/(2*(a + b*x + c*x^2)) + (21*c^2*d^(9/2)*ArcTan[Sqrt[d*(b + 2*c*x)
]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(b^2 - 4*a*c)^(1/4) - (21*c^2*d^(9/2)*Arc
Tanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(b^2 - 4*a*c)^(1/4
)
```


Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(bd+2cdx)^{9/2}}{\left(a+bx+cx^2\right)^3}\,dx = -\frac{d(bd+2cdx)^{7/2}}{2\left(a+bx+cx^2\right)^2}+\frac{1}{2}\left(7cd^2\right)\int \frac{(bd+2cdx)^{5/2}}{\left(a+bx+cx^2\right)^2}\,dx$$

$$= -\frac{d(bd+2cdx)^{7/2}}{2\left(a+bx+cx^2\right)^2}-\frac{7cd^3(bd+2cdx)^{3/2}}{2\left(a+bx+cx^2\right)}+\frac{1}{2}\left(21c^2d^4\right)\int \frac{\sqrt{bd+2cdx}}{a+bx+cx^2}\,dx$$

$$= -\frac{d(bd+2cdx)^{7/2}}{2\left(a+bx+cx^2\right)^2}-\frac{7cd^3(bd+2cdx)^{3/2}}{2\left(a+bx+cx^2\right)}+\frac{1}{4}\left(21cd^3\right)\text{Subst}\left(\int \frac{\sqrt{x}}{a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}}\,dx, x, bd+2cdx\right)$$

$$= -\frac{d(bd+2cdx)^{7/2}}{2\left(a+bx+cx^2\right)^2}-\frac{7cd^3(bd+2cdx)^{3/2}}{2\left(a+bx+cx^2\right)}+\frac{1}{2}\left(21cd^3\right)\text{Subst}\left(\int \frac{x^2}{a-\frac{b^2}{4c}+\frac{x^4}{4cd^2}}\,dx, x, \sqrt{d(b+2cx)}\right.$$

$$= -\frac{d(bd+2cdx)^{7/2}}{2\left(a+bx+cx^2\right)^2}-\frac{7cd^3(bd+2cdx)^{3/2}}{2\left(a+bx+cx^2\right)}-\left(21c^2d^5\right)\text{Subst}\left(\int \frac{1}{\sqrt{b^2-4ac}d-x^2}\,dx, x, \sqrt{d(b+2c\right.$$

$$= -\frac{d(bd+2cdx)^{7/2}}{2\left(a+bx+cx^2\right)^2}-\frac{7cd^3(bd+2cdx)^{3/2}}{2\left(a+bx+cx^2\right)}+\frac{21c^2d^{9/2}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)}{\sqrt[4]{b^2-4ac}}-\frac{21c^2d^{9/2}\tanh^{-1}\left(\frac{\sqrt{d(b\cdots}}{\sqrt[4]{b^2-\cdots}}\right.}{\sqrt[4]{b^2-4ac}}$$

Mathematica [C] time = 0.154552, size = 119, normalized size = 0.7

$$\frac{4(d(b+2cx))^{9/2}\left(-112c^2(a+x(b+cx))^2\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)-5\left(b^2-4ac\right)(b+2cx)^2+7\left(b^2-4ac\right)^2\right)}{5\left(b^2-4ac\right)(b+2cx)^3(a+x(b+cx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(9/2)/(a + b*x + c*x^2)^3,x]
```

```
[Out] (4*(d*(b + 2*c*x))^(9/2)*(7*(b^2 - 4*a*c)^2 - 5*(b^2 - 4*a*c)*(b + 2*c*x)^2
 - 112*c^2*(a + x*(b + c*x))^2*Hypergeometric2F1[3/4, 3, 7/4, (b + 2*c*x)^2
/(b^2 - 4*a*c)]))/(5*(b^2 - 4*a*c)*(b + 2*c*x)^3*(a + x*(b + c*x))^2)
```

Maple [B] time = 0.199, size = 435, normalized size = 2.6

$$-22\frac{c^2d^5(2cdx+bd)^{7/2}}{(4c^2d^2x^2+4bcd^2x+4acd^2)^2}-56\frac{c^3d^7(2cdx+bd)^{3/2}a}{(4c^2d^2x^2+4bcd^2x+4acd^2)^2}+14\frac{c^2d^7(2cdx+bd)^{3/2}b^2}{(4c^2d^2x^2+4bcd^2x+4acd^2)^2}+\frac{21c^2d^5}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a)^3,x)
```

```
[Out] -22*c^2*d^5/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c*d^2)^2*(2*c*d*x+b*d)^(7/2)-56*
c^3*d^7/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c*d^2)^2*(2*c*d*x+b*d)^(3/2)*a+14*c^
2*d^7/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c*d^2)^2*(2*c*d*x+b*d)^(3/2)*b^2+21/4*
c^2*d^5*2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*ln((2*c*d*x+b*d-(4*a*c*d^2-b^2*d^
2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*
d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)
^(1/2)))+21/2*c^2*d^5*2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*arctan(2^(1/2)/(4*a
*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)-21/2*c^2*d^5*2^(1/2)/(4*a*c*d^
2-b^2*d^2)^(1/4)*arctan(-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1
/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.80276, size = 1138, normalized size = 6.69

$$-\frac{84\left(\frac{c^8d^{18}}{b^2-4ac}\right)^{\frac{1}{4}}\left(c^2x^4+2bcx^3+2abx+\left(b^2+2ac\right)x^2+a^2\right)\arctan\left(-\frac{\left(\frac{c^8d^{18}}{b^2-4ac}\right)^{\frac{1}{4}}\sqrt{2cdx+bd}c^6d^{13}-\sqrt{2c^{13}d^{27}x+bc^{12}d^{27}+\sqrt{\frac{c^8d^{18}}{b^2-4ac}}(b^2}}{c^8d^{18}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(84*(c^8*d^18/(b^2 - 4*a*c))^(1/4)*(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b
^2 + 2*a*c)*x^2 + a^2)*arctan(-((c^8*d^18/(b^2 - 4*a*c))^(1/4)*sqrt(2*c*d*x
 + b*d)*c^6*d^13 - sqrt(2*c^13*d^27*x + b*c^12*d^27 + sqrt(c^8*d^18/(b^2 -
4*a*c))*(b^2*c^8 - 4*a*c^9)*d^18)*(c^8*d^18/(b^2 - 4*a*c))^(1/4))/(c^8*d^18
)) + 21*(c^8*d^18/(b^2 - 4*a*c))^(1/4)*(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^
2 + 2*a*c)*x^2 + a^2)*log(9261*sqrt(2*c*d*x + b*d)*c^6*d^13 + 9261*(c^8*d^1
8/(b^2 - 4*a*c))^(3/4)*(b^2 - 4*a*c)) - 21*(c^8*d^18/(b^2 - 4*a*c))^(1/4)*(
c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*log(9261*sqrt(2*c*
d*x + b*d)*c^6*d^13 - 9261*(c^8*d^18/(b^2 - 4*a*c))^(3/4)*(b^2 - 4*a*c)) +
(22*c^3*d^4*x^3 + 33*b*c^2*d^4*x^2 + (13*b^2*c + 14*a*c^2)*d^4*x + (b^3 + 7
*a*b*c)*d^4)*sqrt(2*c*d*x + b*d))/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2
```

```
*a*c)*x^2 + a^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(9/2)/(c*x**2+b*x+a)**3,x)

[Out] Timed out
```

Giac [B] time = 1.269, size = 689, normalized size = 4.05

$$-\frac{21\left(-b^2d^2+4\,acd^2\right)^{\frac{3}{4}}c^2d^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}+2\,\sqrt{2\,cdx+bd}\right)}{2\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^2-4\,\sqrt{2}ac}-\frac{21\left(-b^2d^2+4\,acd^2\right)^{\frac{3}{4}}c^2d^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,a\right.\right.}{2\left(-b^2d^2\right.}\right.}{\sqrt{2}b^2-4\,\sqrt{2}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a)^3,x, algorithm="giac")

[Out] -21*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c^2*d^3*arctan(1/2*sqrt(2)*(sqrt(2)*(-b^2*
d^2 + 4*a*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4
))/(sqrt(2)*b^2 - 4*sqrt(2)*a*c) - 21*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c^2*d^3*
arctan(-1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) - 2*sqrt(2*c*d*x
+ b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4))/(sqrt(2)*b^2 - 4*sqrt(2)*a*c) + 21/2*
(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c^2*d^3*log(2*c*d*x + b*d + sqrt(2)*(-b^2*d^2
+ 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2))/(sqrt(
2)*b^2 - 4*sqrt(2)*a*c) - 21/2*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c^2*d^3*log(2*c
*d*x + b*d - sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqr
t(-b^2*d^2 + 4*a*c*d^2))/(sqrt(2)*b^2 - 4*sqrt(2)*a*c) + 2*(7*(2*c*d*x + b*
d)^(3/2)*b^2*c^2*d^7 - 28*(2*c*d*x + b*d)^(3/2)*a*c^3*d^7 - 11*(2*c*d*x + b
*d)^(7/2)*c^2*d^5)/(b^2*d^2 - 4*a*c*d^2 - (2*c*d*x + b*d)^2)^2
```

3.1312 $\int \frac{(bd+2cdx)^{7/2}}{\left(a+bx+cx^2\right)^3}\,dx$

Optimal. Leaf size=170

$$-\frac{5c^2d^{7/2}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/4}}-\frac{5c^2d^{7/2}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/4}}-\frac{5cd^3\sqrt{bd+2cdx}}{2\left(a+bx+cx^2\right)}-\frac{d(bd+2cdx)^{5/2}}{2\left(a+bx+cx^2\right)^2}$$

```
[Out] -(d*(b*d + 2*c*d*x)^(5/2))/(2*(a + b*x + c*x^2)^2) - (5*c*d^3*Sqrt[b*d + 2*
c*d*x])/(2*(a + b*x + c*x^2)) - (5*c^2*d^(7/2)*ArcTan[Sqrt[d*(b + 2*c*x)]/(
(b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(b^2 - 4*a*c)^(3/4) - (5*c^2*d^(7/2)*ArcTanh
[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(b^2 - 4*a*c)^(3/4)
```

Rubi [A] time = 0.124467, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {686, 694, 329, 212, 206, 203}

$$-\frac{5c^2d^{7/2}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/4}}-\frac{5c^2d^{7/2}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/4}}-\frac{5cd^3\sqrt{bd+2cdx}}{2\left(a+bx+cx^2\right)}-\frac{d(bd+2cdx)^{5/2}}{2\left(a+bx+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(7/2)/(a + b*x + c*x^2)^3,x]
```

```
[Out] -(d*(b*d + 2*c*d*x)^(5/2))/(2*(a + b*x + c*x^2)^2) - (5*c*d^3*Sqrt[b*d + 2*
c*d*x])/(2*(a + b*x + c*x^2)) - (5*c^2*d^(7/2)*ArcTan[Sqrt[d*(b + 2*c*x)]/(
(b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(b^2 - 4*a*c)^(3/4) - (5*c^2*d^(7/2)*ArcTanh
[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(b^2 - 4*a*c)^(3/4)
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(bd+2cdx)^{7/2}}{\left(a+bx+cx^2\right)^3}\,dx = -\frac{d(bd+2cdx)^{5/2}}{2\left(a+bx+cx^2\right)^2}+\frac{1}{2}\left(5cd^2\right)\int \frac{(bd+2cdx)^{3/2}}{\left(a+bx+cx^2\right)^2}\,dx$$

$$= -\frac{d(bd+2cdx)^{5/2}}{2\left(a+bx+cx^2\right)^2}-\frac{5cd^3\sqrt{bd+2cdx}}{2\left(a+bx+cx^2\right)}+\frac{1}{2}\left(5c^2d^4\right)\int \frac{1}{\sqrt{bd+2cdx}\left(a+bx+cx^2\right)}\,dx$$

$$= -\frac{d(bd+2cdx)^{5/2}}{2\left(a+bx+cx^2\right)^2}-\frac{5cd^3\sqrt{bd+2cdx}}{2\left(a+bx+cx^2\right)}+\frac{1}{4}\left(5cd^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)}\,dx,x,bd+2cdx$$

$$= -\frac{d(bd+2cdx)^{5/2}}{2\left(a+bx+cx^2\right)^2}-\frac{5cd^3\sqrt{bd+2cdx}}{2\left(a+bx+cx^2\right)}+\frac{1}{2}\left(5cd^3\right)\text{Subst}\left(\int \frac{1}{a-\frac{b^2}{4c}+\frac{x^4}{4cd^2}}\,dx,x,\sqrt{d(b+2cx)}\right)$$

$$= -\frac{d(bd+2cdx)^{5/2}}{2\left(a+bx+cx^2\right)^2}-\frac{5cd^3\sqrt{bd+2cdx}}{2\left(a+bx+cx^2\right)}-\frac{\left(5c^2d^4\right)\text{Subst}\left(\int \frac{1}{\sqrt{b^2-4ac}d-x^2}\,dx,x,\sqrt{d(b+2cx)}\right)}{\sqrt{b^2-4ac}}-\frac{(5c$$

$$= -\frac{d(bd+2cdx)^{5/2}}{2\left(a+bx+cx^2\right)^2}-\frac{5cd^3\sqrt{bd+2cdx}}{2\left(a+bx+cx^2\right)}-\frac{5c^2d^{7/2}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)}{\left(b^2-4ac\right)^{3/4}}-\frac{5c^2d^{7/2}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right.}{\left(b^2-4ac\right)^{3/4}}$$

Mathematica [A] time = 0.280626, size = 196, normalized size = 1.15

$$\frac{(d(b+2cx))^{7/2}\left(-64\left(b^2-4ac\right)^{3/4}(b+2cx)^{5/2}+40\left(b^2-4ac\right)^{7/4}\sqrt{b+2cx}+20c(a+x(b+cx))\left(2\left(b^2-4ac\right)^{3/4}\sqrt{b+2cx}\right.\right.}{48\left(b^2-4ac\right)^{3/4}(b+2cx)^{7/2}(a+x(b+cx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(7/2)/(a + b*x + c*x^2)^3,x]
```

```
[Out] ((d*(b + 2*c*x))^(7/2)*(40*(b^2 - 4*a*c)^(7/4)*Sqrt[b + 2*c*x] - 64*(b^2 -
4*a*c)^(3/4)*(b + 2*c*x)^(5/2) + 20*c*(a + x*(b + c*x))*(2*(b^2 - 4*a*c)^(3
/4)*Sqrt[b + 2*c*x] - 12*c*(a + x*(b + c*x))*(ArcTan[Sqrt[b + 2*c*x]/(b^2 -
 4*a*c)^(1/4)] + ArcTanh[Sqrt[b + 2*c*x]/(b^2 - 4*a*c)^(1/4)]))))/(48*(b^2
- 4*a*c)^(3/4)*(b + 2*c*x)^(7/2)*(a + x*(b + c*x))^2)
```

Maple [B] time = 0.224, size = 435, normalized size = 2.6

$$-18\frac{c^{2}d^{5}\left(2cdx+bd\right)^{5/2}}{\left(4c^{2}d^{2}x^{2}+4bcd^{2}x+4acd^{2}\right)^{2}}-40\frac{c^{3}d^{7}a\sqrt{2cdx+bd}}{\left(4c^{2}d^{2}x^{2}+4bcd^{2}x+4acd^{2}\right)^{2}}+10\frac{c^{2}d^{7}b^{2}\sqrt{2cdx+bd}}{\left(4c^{2}d^{2}x^{2}+4bcd^{2}x+4acd^{2}\right)^{2}}+\frac{5c^{2}d^{5}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^3,x)
```

```
[Out] -18*c^2*d^5/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c*d^2)^2*(2*c*d*x+b*d)^(5/2)-40*
c^3*d^7/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c*d^2)^2*(2*c*d*x+b*d)^(1/2)*a+10*c^
2*d^7/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c*d^2)^2*(2*c*d*x+b*d)^(1/2)*b^2+5/4*c
^2*d^5/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*ln((2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2
)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d
-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^
(1/2)))+5/2*c^2*d^5/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(2^(1/2)/(4*a*c
*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)-5/2*c^2*d^5/(4*a*c*d^2-b^2*d^2)^
(3/4)*2^(1/2)*arctan(-2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)
+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.75763, size = 1530, normalized size = 9.

$$20\left(\frac{c^{8}d^{14}}{b^{6}-12ab^{4}c+48a^{2}b^{2}c^{2}-64a^{3}c^{3}}\right)^{\frac{1}{4}}\left(c^{2}x^{4}+2bcx^{3}+2abx+\left(b^{2}+2ac\right)x^{2}+a^{2}\right)\arctan\left(-\frac{\left(\frac{c^{8}d^{14}}{b^{6}-12ab^{4}c+48a^{2}b^{2}c^{2}-64a^{3}c^{3}}\right)^{\frac{3}{4}}\left(b^{4}c^{2}-8ab^{2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(20*(c^8*d^14/(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3))^(1/4)*(
c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*arctan(-((c^8*d^14
/(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3))^(3/4)*(b^4*c^2 - 8*a*b^2
*c^3 + 16*a^2*c^4)*sqrt(2*c*d*x + b*d)*d^3 - (c^8*d^14/(b^6 - 12*a*b^4*c +
48*a^2*b^2*c^2 - 64*a^3*c^3))^(3/4)*sqrt(2*c^5*d^7*x + b*c^4*d^7 + sqrt(c^8
*d^14/(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3))*(b^4 - 8*a*b^2*c +
16*a^2*c^2))*(b^4 - 8*a*b^2*c + 16*a^2*c^2))/(c^8*d^14)) - 5*(c^8*d^14/(b^6
 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3))^(1/4)*(c^2*x^4 + 2*b*c*x^3 +
2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*log(5*sqrt(2*c*d*x + b*d)*c^2*d^3 + 5*(c
```

```
^8*d^14/(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3))^(1/4)*(b^2 - 4*a*
c)) + 5*(c^8*d^14/(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3))^(1/4)*(
c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*log(5*sqrt(2*c*d*x
 + b*d)*c^2*d^3 - 5*(c^8*d^14/(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c
^3))^(1/4)*(b^2 - 4*a*c)) - (9*c^2*d^3*x^2 + 9*b*c*d^3*x + (b^2 + 5*a*c)*d^
3)*sqrt(2*c*d*x + b*d))/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2
+ a^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(7/2)/(c*x**2+b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.30434, size = 689, normalized size = 4.05

$$-\frac{5\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}c^2d^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}+2\,\sqrt{2\,cdx+bd}\right)}{2\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^2-4\,\sqrt{2}ac}-\frac{5\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}c^2d^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right.\right.}{2\left(-b^2d^2+4\right.}\right.}{\sqrt{2}b^2-4\,\sqrt{2}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] -5*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*c^2*d^3*arctan(1/2*sqrt(2)*(sqrt(2)*(-b^2*d
^2 + 4*a*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4)
)/(sqrt(2)*b^2 - 4*sqrt(2)*a*c) - 5*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*c^2*d^3*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) - 2*sqrt(2*c*d*x +
b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4))/(sqrt(2)*b^2 - 4*sqrt(2)*a*c) - 5/2*(-b
^2*d^2 + 4*a*c*d^2)^(1/4)*c^2*d^3*log(2*c*d*x + b*d + sqrt(2)*(-b^2*d^2 + 4
*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2))/(sqrt(2)*
b^2 - 4*sqrt(2)*a*c) + 5/2*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*c^2*d^3*log(2*c*d*x
 + b*d - sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b
^2*d^2 + 4*a*c*d^2))/(sqrt(2)*b^2 - 4*sqrt(2)*a*c) + 2*(5*sqrt(2*c*d*x + b*
d)*b^2*c^2*d^7 - 20*sqrt(2*c*d*x + b*d)*a*c^3*d^7 - 9*(2*c*d*x + b*d)^(5/2)
*c^2*d^5)/(b^2*d^2 - 4*a*c*d^2 - (2*c*d*x + b*d)^2)^2
```

3.1313 $\int \frac{(bd+2cdx)^{5/2}}{(a+bx+cx^2)^3}\,dx$

Optimal. Leaf size=178

$$-\frac{3c^2d^{5/2}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/4}}+\frac{3c^2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/4}}-\frac{3cd(bd+2cdx)^{3/2}}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{d(bd+2cdx)^{3/2}}{2\left(a+bx+cx^2\right)^2}$$

```
[Out] -(d*(b*d + 2*c*d*x)^(3/2))/(2*(a + b*x + c*x^2)^2) - (3*c*d*(b*d + 2*c*d*x)
^(3/2))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)) - (3*c^2*d^(5/2)*ArcTan[Sqrt[d*
(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(b^2 - 4*a*c)^(5/4) + (3*c^2*d
^(5/2)*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(b^2 - 4
*a*c)^(5/4)
```

Rubi [A] time = 0.128111, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {686, 687, 694, 329, 298, 203, 206}

$$-\frac{3c^2d^{5/2}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/4}}+\frac{3c^2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/4}}-\frac{3cd(bd+2cdx)^{3/2}}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{d(bd+2cdx)^{3/2}}{2\left(a+bx+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(5/2)/(a + b*x + c*x^2)^3,x]
```


```
[Out] -(d*(b*d + 2*c*d*x)^(3/2))/(2*(a + b*x + c*x^2)^2) - (3*c*d*(b*d + 2*c*d*x)
^(3/2))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)) - (3*c^2*d^(5/2)*ArcTan[Sqrt[d*
(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(b^2 - 4*a*c)^(5/4) + (3*c^2*d
^(5/2)*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(b^2 - 4
*a*c)^(5/4)
```


Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
```

```
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(bd+2cdx)^{5/2}}{\left(a+bx+cx^2\right)^3}\,dx &= -\frac{d(bd+2cdx)^{3/2}}{2\left(a+bx+cx^2\right)^2}+\frac{1}{2}\left(3cd^2\right)\int \frac{\sqrt{bd+2cdx}}{\left(a+bx+cx^2\right)^2}\,dx\\
&= -\frac{d(bd+2cdx)^{3/2}}{2\left(a+bx+cx^2\right)^2}-\frac{3cd(bd+2cdx)^{3/2}}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\left(3c^2d^2\right)\int \frac{\sqrt{bd+2cdx}}{a+bx+cx^2}\,dx}{2\left(b^2-4ac\right)}\\
&= -\frac{d(bd+2cdx)^{3/2}}{2\left(a+bx+cx^2\right)^2}-\frac{3cd(bd+2cdx)^{3/2}}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{(3cd)\,\mathrm{Subst}\left(\int \frac{\sqrt{x}}{a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}}\,dx,x,bd+2cdx\right)}{4\left(b^2-4ac\right)}\\
&= -\frac{d(bd+2cdx)^{3/2}}{2\left(a+bx+cx^2\right)^2}-\frac{3cd(bd+2cdx)^{3/2}}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{(3cd)\,\mathrm{Subst}\left(\int \frac{x^2}{a-\frac{b^2}{4c}+\frac{x^4}{4cd^2}}\,dx,x,\sqrt{d(b+2cx)}\right)}{2\left(b^2-4ac\right)}\\
&= -\frac{d(bd+2cdx)^{3/2}}{2\left(a+bx+cx^2\right)^2}-\frac{3cd(bd+2cdx)^{3/2}}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\left(3c^2d^3\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b^2-4ac}d-x^2}\,dx,x,\sqrt{d(b+2
\end{aligned}
$$

$$
= -\frac{d(bd+2cdx)^{3/2}}{2\left(a+bx+cx^2\right)^2}-\frac{3cd(bd+2cdx)^{3/2}}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{3c^2d^{5/2}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)}{\left(b^2-4ac\right)^{5/4}}+\frac{3c^2d^{5/2}\tanh^{-1}}{\left(b^2-4a\right.}
$$

Mathematica [C] time = 0.117127, size = 77, normalized size = 0.43

$$\frac{64}{5}c^2d(d(b+2cx))^{3/2}\left(\frac{{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{\left(b^2-4ac\right)^2}-\frac{1}{16c^2(a+x(b+cx))^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(5/2)/(a + b*x + c*x^2)^3,x]
```

```
[Out] (64*c^2*d*(d*(b + 2*c*x))^(3/2)*(-1/(16*c^2*(a + x*(b + c*x))^2) + Hypergeo
metric2F1[3/4, 3, 7/4, (b + 2*c*x)^2/(b^2 - 4*a*c)]/(b^2 - 4*a*c)^2))/5
```

Maple [B] time = 0.198, size = 431, normalized size = 2.4

$$6\frac{c^2d^3(2cdx+bd)^{7/2}}{\left(4c^2d^2x^2+4bcd^2x+4acd^2\right)^2\left(4ac-b^2\right)}-2\frac{c^2d^5(2cdx+bd)^{3/2}}{\left(4c^2d^2x^2+4bcd^2x+4acd^2\right)^2}+\frac{3c^2d^3\sqrt{2}}{16ac-4b^2}\ln\left(\left(2cdx+bd-\sqrt[4]{4ac\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^3,x)
```

```
[Out] 6*c^2*d^3/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c*d^2)^2/(4*a*c-b^2)*(2*c*d*x+b*d)
^(7/2)-2*c^2*d^5/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c*d^2)^2*(2*c*d*x+b*d)^(3/2
)+3/4*c^2*d^3/(4*a*c-b^2)/(4*a*c*d^2-b^2*d^2)^(1/4)*2^(1/2)*ln((2*c*d*x+b*d
-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^
(1/2))/(2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(
4*a*c*d^2-b^2*d^2)^(1/2)))+3/2*c^2*d^3/(4*a*c-b^2)/(4*a*c*d^2-b^2*d^2)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)-3
/2*c^2*d^3/(4*a*c-b^2)/(4*a*c*d^2-b^2*d^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(4
*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.02724, size = 2589, normalized size = 14.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(12*(c^8*d^10/(b^10 - 20*a*b^8*c + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 +
 1280*a^4*b^2*c^4 - 1024*a^5*c^5))^(1/4)*((b^2*c^2 - 4*a*c^3)*x^4 + a^2*b^2
 - 4*a^3*c + 2*(b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*x^2
+ 2*(a*b^3 - 4*a^2*b*c)*x)*arctan(((c^8*d^10/(b^10 - 20*a*b^8*c + 160*a^2*b
^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 1024*a^5*c^5))^(1/4)*(b^2*c^6
 - 4*a*c^7)*sqrt(2*c*d*x + b*d)*d^7 - sqrt(2*c^13*d^15*x + b*c^12*d^15 + sq
rt(c^8*d^10/(b^10 - 20*a*b^8*c + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a
^4*b^2*c^4 - 1024*a^5*c^5))*(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*
a^3*c^11)*d^10)*(c^8*d^10/(b^10 - 20*a*b^8*c + 160*a^2*b^6*c^2 - 640*a^3*b^
4*c^3 + 1280*a^4*b^2*c^4 - 1024*a^5*c^5))^(1/4)*(b^2 - 4*a*c))/(c^8*d^10))
- 3*(c^8*d^10/(b^10 - 20*a*b^8*c + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280
*a^4*b^2*c^4 - 1024*a^5*c^5))^(1/4)*((b^2*c^2 - 4*a*c^3)*x^4 + a^2*b^2 - 4*
a^3*c + 2*(b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*x^2 + 2*(
a*b^3 - 4*a^2*b*c)*x)*log(27*sqrt(2*c*d*x + b*d)*c^6*d^7 + 27*(c^8*d^10/(b^
10 - 20*a*b^8*c + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 10
24*a^5*c^5))^(3/4)*(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 2
56*a^4*c^4)) + 3*(c^8*d^10/(b^10 - 20*a*b^8*c + 160*a^2*b^6*c^2 - 640*a^3*b
^4*c^3 + 1280*a^4*b^2*c^4 - 1024*a^5*c^5))^(1/4)*((b^2*c^2 - 4*a*c^3)*x^4 +
 a^2*b^2 - 4*a^3*c + 2*(b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 2*a*b^2*c - 8*a^2*c
^2)*x^2 + 2*(a*b^3 - 4*a^2*b*c)*x)*log(27*sqrt(2*c*d*x + b*d)*c^6*d^7 - 27*
(c^8*d^10/(b^10 - 20*a*b^8*c + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4
*b^2*c^4 - 1024*a^5*c^5))^(3/4)*(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^
3*b^2*c^3 + 256*a^4*c^4)) + (6*c^3*d^2*x^3 + 9*b*c^2*d^2*x^2 + (5*b^2*c - 2
*a*c^2)*d^2*x + (b^3 - a*b*c)*d^2)*sqrt(2*c*d*x + b*d))/((b^2*c^2 - 4*a*c^3
)*x^4 + a^2*b^2 - 4*a^3*c + 2*(b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 2*a*b^2*c -
8*a^2*c^2)*x^2 + 2*(a*b^3 - 4*a^2*b*c)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(5/2)/(c*x**2+b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25093, size = 765, normalized size = 4.3

$$\frac{3\left(-b^2d^2+4\,acd^2\right)^{\frac{3}{4}}c^2d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}+2\,\sqrt{2\,cdx+bd}\right)}{2\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^4-8\,\sqrt{2}ab^2c+16\,\sqrt{2}a^2c^2}+\frac{3\left(-b^2d^2+4\,acd^2\right)^{\frac{3}{4}}c^2d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}-\right.}{2\left(-b^2d^2+4\,acd^2\right.}\right.}{\sqrt{2}b^4-8\,\sqrt{2}ab^2c+16\,\sqrt{2}a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c^2*d*arctan(1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2
+ 4*a*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4))/(
sqrt(2)*b^4 - 8*sqrt(2)*a*b^2*c + 16*sqrt(2)*a^2*c^2) + 3*(-b^2*d^2 + 4*a*c
*d^2)^(3/4)*c^2*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)
 - 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4))/(sqrt(2)*b^4 - 8*sq
```

```
rt(2)*a*b^2*c + 16*sqrt(2)*a^2*c^2) - 3/2*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c^2*
d*log(2*c*d*x + b*d + sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b
*d) + sqrt(-b^2*d^2 + 4*a*c*d^2))/(sqrt(2)*b^4 - 8*sqrt(2)*a*b^2*c + 16*sqr
t(2)*a^2*c^2) + 3/2*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c^2*d*log(2*c*d*x + b*d -
sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 +
4*a*c*d^2))/(sqrt(2)*b^4 - 8*sqrt(2)*a*b^2*c + 16*sqrt(2)*a^2*c^2) - 2*((2*
c*d*x + b*d)^(3/2)*b^2*c^2*d^5 - 4*(2*c*d*x + b*d)^(3/2)*a*c^3*d^5 + 3*(2*c
*d*x + b*d)^(7/2)*c^2*d^3)/((b^2*d^2 - 4*a*c*d^2 - (2*c*d*x + b*d)^2)^2*(b^
2 - 4*a*c))
```

3.1314 $\int \frac{(bd+2cdx)^{3/2}}{(a+bx+cx^2)^3}\,dx$

Optimal. Leaf size=178

$$\frac{3c^2d^{3/2}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{(b^2-4ac)^{7/4}}+\frac{3c^2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{(b^2-4ac)^{7/4}}-\frac{cd\sqrt{bd+2cdx}}{2(b^2-4ac)(a+bx+cx^2)}-\frac{d\sqrt{bd+2cdx}}{2(a+bx+cx^2)^2}$$

```
[Out] -(d*Sqrt[b*d + 2*c*d*x])/(2*(a + b*x + c*x^2)^2) - (c*d*Sqrt[b*d + 2*c*d*x]
)/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + (3*c^2*d^(3/2)*ArcTan[Sqrt[d*(b + 2
*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(b^2 - 4*a*c)^(7/4) + (3*c^2*d^(3/2)
*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(b^2 - 4*a*c)^
(7/4)
```

Rubi [A] time = 0.128863, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {686, 687, 694, 329, 212, 206, 203}

$$\frac{3c^2d^{3/2}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{(b^2-4ac)^{7/4}}+\frac{3c^2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{(b^2-4ac)^{7/4}}-\frac{cd\sqrt{bd+2cdx}}{2(b^2-4ac)(a+bx+cx^2)}-\frac{d\sqrt{bd+2cdx}}{2(a+bx+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(3/2)/(a + b*x + c*x^2)^3,x]
```


```
[Out] -(d*Sqrt[b*d + 2*c*d*x])/(2*(a + b*x + c*x^2)^2) - (c*d*Sqrt[b*d + 2*c*d*x]
)/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + (3*c^2*d^(3/2)*ArcTan[Sqrt[d*(b + 2
*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(b^2 - 4*a*c)^(7/4) + (3*c^2*d^(3/2)
*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(b^2 - 4*a*c)^
(7/4)
```


Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
```

```
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(bd+2cdx)^{3/2}}{\left(a+bx+cx^2\right)^3}\,dx = -\frac{d\sqrt{bd+2cdx}}{2\left(a+bx+cx^2\right)^2}+\frac{1}{2}\left(cd^2\right)\int \frac{1}{\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^2}\,dx$$

$$= -\frac{d\sqrt{bd+2cdx}}{2\left(a+bx+cx^2\right)^2}-\frac{cd\sqrt{bd+2cdx}}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\left(3c^2d^2\right)\int \frac{1}{\sqrt{bd+2cdx}\left(a+bx+cx^2\right)}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{d\sqrt{bd+2cdx}}{2\left(a+bx+cx^2\right)^2}-\frac{cd\sqrt{bd+2cdx}}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{(3cd)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)}\,dx,x,bd+2\right.}{4\left(b^2-4ac\right)}$$

$$= -\frac{d\sqrt{bd+2cdx}}{2\left(a+bx+cx^2\right)^2}-\frac{cd\sqrt{bd+2cdx}}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{(3cd)\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{b^2}{4c}+\frac{x^4}{4cd^2}}\,dx,x,\sqrt{d(b+2c}\right.}{2\left(b^2-4ac\right)}$$

$$= -\frac{d\sqrt{bd+2cdx}}{2\left(a+bx+cx^2\right)^2}-\frac{cd\sqrt{bd+2cdx}}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\left(3c^2d^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b^2-4acd-x^2}}\,dx,x,\sqrt{d(b}\right.}{\left(b^2-4ac\right)^{3/2}}$$

$$= -\frac{d\sqrt{bd+2cdx}}{2\left(a+bx+cx^2\right)^2}-\frac{cd\sqrt{bd+2cdx}}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{3c^2d^{3/2}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)}{\left(b^2-4ac\right)^{7/4}}+\frac{3c^2d^{3/2}\tanh}{\left(b^2-\right.}$$

Mathematica [A] time = 0.378265, size = 171, normalized size = 0.96

$$\frac{(d(b+2cx))^{3/2}\left(\frac{8c(a+x(b+cx))\left(-\left(b^2-4ac\right)^{3/4}\sqrt{b+2cx}+6c(a+x(b+cx))\tan^{-1}\left(\frac{\sqrt{b+2cx}}{\sqrt[4]{b^2-4ac}}\right)+6c(a+x(b+cx))\tanh^{-1}\left(\frac{\sqrt{b+2cx}}{\sqrt[4]{b^2-4ac}}\right)\right)}{\left(b^2-4ac\right)^{7/4}}-8\sqrt{b+2cx}\right)}{16(b+2cx)^{3/2}(a+x(b+cx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(3/2)/(a + b*x + c*x^2)^3,x]
```

```
[Out] ((d*(b + 2*c*x))^(3/2)*(-8*Sqrt[b + 2*c*x] + (8*c*(a + x*(b + c*x))*(-((b^2
 - 4*a*c)^(3/4)*Sqrt[b + 2*c*x]) + 6*c*(a + x*(b + c*x))*ArcTan[Sqrt[b + 2*
c*x]/(b^2 - 4*a*c)^(1/4)] + 6*c*(a + x*(b + c*x))*ArcTanh[Sqrt[b + 2*c*x]/(
b^2 - 4*a*c)^(1/4)]))/(b^2 - 4*a*c)^(7/4)))/(16*(b + 2*c*x)^(3/2)*(a + x*(b
 + c*x))^2)
```

Maple [B] time = 0.2, size = 431, normalized size = 2.4

$$2\frac{c^2d^3(2cdx+bd)^{5/2}}{\left(4c^2d^2x^2+4bcd^2x+4acd^2\right)^2\left(4ac-b^2\right)}-6\frac{c^2d^5\sqrt{2cdx+bd}}{\left(4c^2d^2x^2+4bcd^2x+4acd^2\right)^2}+\frac{3c^2d^3\sqrt{2}}{16ac-4b^2}\ln\left(\left(2cdx+bd+\sqrt[4]{4acd^2-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^3,x)
```

```
[Out] 2*c^2*d^3/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c*d^2)^2/(4*a*c-b^2)*(2*c*d*x+b*d)
^(5/2)-6*c^2*d^5/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c*d^2)^2*(2*c*d*x+b*d)^(1/2
)+3/4*c^2*d^3/(4*a*c-b^2)/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*ln((2*c*d*x+b*d
+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^
(1/2))/(2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(
4*a*c*d^2-b^2*d^2)^(1/2)))+3/2*c^2*d^3/(4*a*c-b^2)/(4*a*c*d^2-b^2*d^2)^(3/4
)*2^(1/2)*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)-3
/2*c^2*d^3/(4*a*c-b^2)/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(-2^(1/2)/(4
*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.07073, size = 3089, normalized size = 17.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(12*(c^8*d^6/(b^14 - 28*a*b^12*c + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3
+ 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 16384*a^7*c^7)
)^(1/4)*((b^2*c^2 - 4*a*c^3)*x^4 + a^2*b^2 - 4*a^3*c + 2*(b^3*c - 4*a*b*c^2
)*x^3 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*x^2 + 2*(a*b^3 - 4*a^2*b*c)*x)*arctan
(((c^8*d^6/(b^14 - 28*a*b^12*c + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960
*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 16384*a^7*c^7))^(3/4
)*(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b
^2*c^6 - 1024*a^5*c^7)*sqrt(2*c*d*x + b*d)*d - (c^8*d^6/(b^14 - 28*a*b^12*c
 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c
^5 + 28672*a^6*b^2*c^6 - 16384*a^7*c^7))^(3/4)*(b^10 - 20*a*b^8*c + 160*a^2
*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 1024*a^5*c^5)*sqrt(2*c^5*d^
3*x + b*c^4*d^3 + sqrt(c^8*d^6/(b^14 - 28*a*b^12*c + 336*a^2*b^10*c^2 - 224
0*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 -
16384*a^7*c^7))*(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*
a^4*c^4)))/(c^8*d^6)) + 3*(c^8*d^6/(b^14 - 28*a*b^12*c + 336*a^2*b^10*c^2 -
 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^
6 - 16384*a^7*c^7))^(1/4)*((b^2*c^2 - 4*a*c^3)*x^4 + a^2*b^2 - 4*a^3*c + 2*
(b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*x^2 + 2*(a*b^3 - 4*
a^2*b*c)*x)*log(3*sqrt(2*c*d*x + b*d)*c^2*d + 3*(c^8*d^6/(b^14 - 28*a*b^12*
c + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*
c^5 + 28672*a^6*b^2*c^6 - 16384*a^7*c^7))^(1/4)*(b^4 - 8*a*b^2*c + 16*a^2*c
^2)) - 3*(c^8*d^6/(b^14 - 28*a*b^12*c + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3
 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 16384*a^7*c^7
))^(1/4)*((b^2*c^2 - 4*a*c^3)*x^4 + a^2*b^2 - 4*a^3*c + 2*(b^3*c - 4*a*b*c^
2)*x^3 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*x^2 + 2*(a*b^3 - 4*a^2*b*c)*x)*log(3
*sqrt(2*c*d*x + b*d)*c^2*d - 3*(c^8*d^6/(b^14 - 28*a*b^12*c + 336*a^2*b^10*
c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b
^2*c^6 - 16384*a^7*c^7))^(1/4)*(b^4 - 8*a*b^2*c + 16*a^2*c^2)) - (c^2*d*x^2
 + b*c*d*x + (b^2 - 3*a*c)*d)*sqrt(2*c*d*x + b*d))/((b^2*c^2 - 4*a*c^3)*x^4
 + a^2*b^2 - 4*a^3*c + 2*(b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 2*a*b^2*c - 8*a^2
*c^2)*x^2 + 2*(a*b^3 - 4*a^2*b*c)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(3/2)/(c*x**2+b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23079, size = 765, normalized size = 4.3

$$\frac{3\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}c^2d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}+2\sqrt{2\,cdx+bd}\right)}{2\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^4-8\sqrt{2}ab^2c+16\sqrt{2}a^2c^2}+\frac{3\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}c^2d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right.\right.}{2\left(-b^2d^2+4\right.}\right.}{\sqrt{2}b^4-8\sqrt{2}ab^2c+16\sqrt{2}a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*c^2*d*arctan(1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2
+ 4*a*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4))/(
sqrt(2)*b^4 - 8*sqrt(2)*a*b^2*c + 16*sqrt(2)*a^2*c^2) + 3*(-b^2*d^2 + 4*a*c
*d^2)^(1/4)*c^2*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)
 - 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4))/(sqrt(2)*b^4 - 8*sq
rt(2)*a*b^2*c + 16*sqrt(2)*a^2*c^2) + 3/2*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*c^2*
d*log(2*c*d*x + b*d + sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b
*d) + sqrt(-b^2*d^2 + 4*a*c*d^2))/(sqrt(2)*b^4 - 8*sqrt(2)*a*b^2*c + 16*sqr
t(2)*a^2*c^2) - 3/2*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*c^2*d*log(2*c*d*x + b*d -
sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 +
4*a*c*d^2))/(sqrt(2)*b^4 - 8*sqrt(2)*a*b^2*c + 16*sqrt(2)*a^2*c^2) - 2*(3*s
qrt(2*c*d*x + b*d)*b^2*c^2*d^5 - 12*sqrt(2*c*d*x + b*d)*a*c^3*d^5 + (2*c*d*
x + b*d)^(5/2)*c^2*d^3)/((b^2*d^2 - 4*a*c*d^2 - (2*c*d*x + b*d)^2)^2*(b^2 -
 4*a*c))
```

3.1315 $\int \frac{\sqrt{bd+2cdx}}{\left(a+bx+cx^2\right)^3}\,dx$

Optimal. Leaf size=192

$$\frac{5c^2\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{9/4}}-\frac{5c^2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{9/4}}+\frac{5c(bd+2cdx)^{3/2}}{2d\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{(bd+2cdx)^{3/2}}{2d\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

```
[Out] -(b*d + 2*c*d*x)^(3/2)/(2*(b^2 - 4*a*c)*d*(a + b*x + c*x^2)^2) + (5*c*(b*d
+ 2*c*d*x)^(3/2))/(2*(b^2 - 4*a*c)^2*d*(a + b*x + c*x^2)) + (5*c^2*Sqrt[d]*
ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(b^2 - 4*a*c)^(9
/4) - (5*c^2*Sqrt[d]*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[
d])])/(b^2 - 4*a*c)^(9/4)
```

Rubi [A] time = 0.129688, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {687, 694, 329, 298, 203, 206}

$$\frac{5c^2\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{9/4}}-\frac{5c^2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{9/4}}+\frac{5c(bd+2cdx)^{3/2}}{2d\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{(bd+2cdx)^{3/2}}{2d\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*d + 2*c*d*x]/(a + b*x + c*x^2)^3,x]
```

```
[Out] -(b*d + 2*c*d*x)^(3/2)/(2*(b^2 - 4*a*c)*d*(a + b*x + c*x^2)^2) + (5*c*(b*d
+ 2*c*d*x)^(3/2))/(2*(b^2 - 4*a*c)^2*d*(a + b*x + c*x^2)) + (5*c^2*Sqrt[d]*
ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(b^2 - 4*a*c)^(9
/4) - (5*c^2*Sqrt[d]*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[
d])])/(b^2 - 4*a*c)^(9/4)
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{bd+2cdx}}{\left(a+bx+cx^2\right)^3}\,dx = -\frac{(bd+2cdx)^{3/2}}{2\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^2}-\frac{(5c)\int \frac{\sqrt{bd+2cdx}}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{(bd+2cdx)^{3/2}}{2\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^2}+\frac{5c(bd+2cdx)^{3/2}}{2\left(b^2-4ac\right)^2d\left(a+bx+cx^2\right)}+\frac{\left(5c^2\right)\int \frac{\sqrt{bd+2cdx}}{a+bx+cx^2}\,dx}{2\left(b^2-4ac\right)^2}$$

$$= -\frac{(bd+2cdx)^{3/2}}{2\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^2}+\frac{5c(bd+2cdx)^{3/2}}{2\left(b^2-4ac\right)^2d\left(a+bx+cx^2\right)}+\frac{(5c)\,\mathrm{Subst}\left(\int \frac{\sqrt{x}}{a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}}\,dx,x\right.}{4\left(b^2-4ac\right)^2d}$$

$$= -\frac{(bd+2cdx)^{3/2}}{2\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^2}+\frac{5c(bd+2cdx)^{3/2}}{2\left(b^2-4ac\right)^2d\left(a+bx+cx^2\right)}+\frac{(5c)\,\mathrm{Subst}\left(\int \frac{x^2}{a-\frac{b^2}{4c}+\frac{x^4}{4cd^2}}\,dx,x\right.}{2\left(b^2-4ac\right)^2d}$$

$$= -\frac{(bd+2cdx)^{3/2}}{2\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^2}+\frac{5c(bd+2cdx)^{3/2}}{2\left(b^2-4ac\right)^2d\left(a+bx+cx^2\right)}-\frac{\left(5c^2d\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b^2-4acd}-x^2}\,d\right.}{\left(b^2-4ac\right)^2}$$

$$= -\frac{(bd+2cdx)^{3/2}}{2\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^2}+\frac{5c(bd+2cdx)^{3/2}}{2\left(b^2-4ac\right)^2d\left(a+bx+cx^2\right)}+\frac{5c^2\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)}{\left(b^2-4ac\right)^{9/4}}-$$

Mathematica [C] time = 0.045127, size = 59, normalized size = 0.31

$$-\frac{64c^2(d(b+2cx))^{3/2}\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{3d\left(b^2-4ac\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*d + 2*c*d*x]/(a + b*x + c*x^2)^3,x]
```

```
[Out] (-64*c^2*(d*(b + 2*c*x))^(3/2)*Hypergeometric2F1[3/4, 3, 7/4, (b + 2*c*x)^2
/(b^2 - 4*a*c)])/(3*(b^2 - 4*a*c)^3*d)
```

Maple [B] time = 0.194, size = 419, normalized size = 2.2

$$8\,{\frac {{c}^{2}{d}^{5} \left(2\,cdx+bd \right) ^{3/2}}{ \left(4\,ac{d}^{2}-{b}^{2}{d}^{2} \right) \left(4\,{c}^{2}{d}^{2}{x}^{2}+4\,bc{d}^{2}x+4\,ac{d}^{2} \right) ^{2}}}+10\,{\frac {{c}^{2}{d}^{5} \left(2\,cdx+bd \right) ^{3/2}}{ \left(4\,ac{d}^{2}-{b}^{2}{d}^{2} \right) ^{2} \left(4\,{c}^{2}{d}^{2}{x}^{2}+4\,bc{d}^{2}x+4\,ac{d}^{2} \right) }}+{\frac {5\,{c}^{2}{d}^{5}\sqrt {2}}{4}\ln \left(\left(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^3,x)
```

```
[Out] 8*c^2*d^5*(2*c*d*x+b*d)^(3/2)/(4*a*c*d^2-b^2*d^2)/(4*c^2*d^2*x^2+4*b*c*d^2*
x+4*a*c*d^2)^2+10*c^2*d^5/(4*a*c*d^2-b^2*d^2)^2*(2*c*d*x+b*d)^(3/2)/(4*c^2*
d^2*x^2+4*b*c*d^2*x+4*a*c*d^2)+5/4*c^2*d^5/(4*a*c*d^2-b^2*d^2)^(9/4)*2^(1/2
)*ln((2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*
a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d
)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)))+5/2*c^2*d^5/(4*a*c*d^2-b^2*d^2)
^(9/4)*2^(1/2)*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)
+1)-5/2*c^2*d^5/(4*a*c*d^2-b^2*d^2)^(9/4)*2^(1/2)*arctan(-2^(1/2)/(4*a*c*d^
2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.11057, size = 4201, normalized size = 21.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(20*(c^8*d^2/(b^18 - 36*a*b^16*c + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^
3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a
^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 262144*a^9*c^9))^(1/4)*(a^2*b^4 - 8*a^3*b
^2*c + 16*a^4*c^2 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8
*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*
b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*arctan(-((b^4*c^6 - 8*a*b^2*c^7 + 16*a
^2*c^8)*(c^8*d^2/(b^18 - 36*a*b^16*c + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3
 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^
7*b^4*c^7 + 589824*a^8*b^2*c^8 - 262144*a^9*c^9))^(1/4)*sqrt(2*c*d*x + b*d)
*d - sqrt(2*c^13*d^3*x + b*c^12*d^3 + (b^10*c^8 - 20*a*b^8*c^9 + 160*a^2*b^
6*c^10 - 640*a^3*b^4*c^11 + 1280*a^4*b^2*c^12 - 1024*a^5*c^13)*sqrt(c^8*d^2
```

```
/(b^18 - 36*a*b^16*c + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^1
0*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 5898
24*a^8*b^2*c^8 - 262144*a^9*c^9))*d^2)*(c^8*d^2/(b^18 - 36*a*b^16*c + 576*a
^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 +
 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 262144*a^9*
c^9))^(1/4)*(b^4 - 8*a*b^2*c + 16*a^2*c^2))/(c^8*d^2)) + 5*(c^8*d^2/(b^18 -
 36*a*b^16*c + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 -
129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b
^2*c^8 - 262144*a^9*c^9))^(1/4)*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^4*
c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3
)*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^
3*b*c^2)*x)*log(125*sqrt(2*c*d*x + b*d)*c^6*d + 125*(b^14 - 28*a*b^12*c + 3
36*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 +
 28672*a^6*b^2*c^6 - 16384*a^7*c^7)*(c^8*d^2/(b^18 - 36*a*b^16*c + 576*a^2*
b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 34
4064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 262144*a^9*c^9
))^(3/4)) - 5*(c^8*d^2/(b^18 - 36*a*b^16*c + 576*a^2*b^14*c^2 - 5376*a^3*b^
12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589
824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 262144*a^9*c^9))^(1/4)*(a^2*b^4 - 8*
a^3*b^2*c + 16*a^4*c^2 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*
c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 +
2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(125*sqrt(2*c*d*x + b*d)*c^6*d
 - 125*(b^14 - 28*a*b^12*c + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4
*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 16384*a^7*c^7)*(c^8*d^2/
(b^18 - 36*a*b^16*c + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10
*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 58982
4*a^8*b^2*c^8 - 262144*a^9*c^9))^(3/4)) - (10*c^3*x^3 + 15*b*c^2*x^2 - b^3
+ 9*a*b*c + 3*(b^2*c + 6*a*c^2)*x)*sqrt(2*c*d*x + b*d))/(a^2*b^4 - 8*a^3*b^
2*c + 16*a^4*c^2 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*
a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b
^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(1/2)/(c*x**2+b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27762, size = 871, normalized size = 4.54

$$-\frac{5\left(-b^{2}d^{2}+4\,acd^{2}\right)^{\frac{3}{4}}c^{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-b^{2}d^{2}+4\,acd^{2}\right)^{\frac{1}{4}}+2\,\sqrt{2\,cdx+bd}\right)}{2\left(-b^{2}d^{2}+4\,acd^{2}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^{6}d-12\,\sqrt{2}ab^{4}cd+48\,\sqrt{2}a^{2}b^{2}c^{2}d-64\,\sqrt{2}a^{3}c^{3}d}-\frac{5\left(-b^{2}d^{2}+4\,acd^{2}\right)^{\frac{3}{4}}c^{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-b^{2}d^{2}+4\,acd^{2}\right)^{\frac{1}{4}}-2\right.}{2\left(-b^{2}d^{2}+4\,acd^{2}\right)}\right.}{\sqrt{2}b^{6}d-12\,\sqrt{2}ab^{4}cd+48\,\sqrt{2}a^{2}b^{2}c^{2}d-64\,\sqrt{2}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] -5*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c^2*arctan(1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2 +
 4*a*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4))/(s
qrt(2)*b^6*d - 12*sqrt(2)*a*b^4*c*d + 48*sqrt(2)*a^2*b^2*c^2*d - 64*sqrt(2)
*a^3*c^3*d) - 5*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c^2*arctan(-1/2*sqrt(2)*(sqrt(
2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) - 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*
d^2)^(1/4))/(sqrt(2)*b^6*d - 12*sqrt(2)*a*b^4*c*d + 48*sqrt(2)*a^2*b^2*c^2*
d - 64*sqrt(2)*a^3*c^3*d) + 5/2*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c^2*log(2*c*d*
x + b*d + sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-
b^2*d^2 + 4*a*c*d^2))/(sqrt(2)*b^6*d - 12*sqrt(2)*a*b^4*c*d + 48*sqrt(2)*a^
2*b^2*c^2*d - 64*sqrt(2)*a^3*c^3*d) - 5/2*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c^2*
log(2*c*d*x + b*d - sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d
) + sqrt(-b^2*d^2 + 4*a*c*d^2))/(sqrt(2)*b^6*d - 12*sqrt(2)*a*b^4*c*d + 48*
sqrt(2)*a^2*b^2*c^2*d - 64*sqrt(2)*a^3*c^3*d) - 2*(9*(2*c*d*x + b*d)^(3/2)*
b^2*c^2*d^3 - 36*(2*c*d*x + b*d)^(3/2)*a*c^3*d^3 - 5*(2*c*d*x + b*d)^(7/2)*
c^2*d)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*(b^2*d^2 - 4*a*c*d^2 - (2*c*d*x + b*
d)^2)^2)
```

3.1316 $\int \frac{1}{\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^3}\,dx$

Optimal. Leaf size=192

$$-\frac{21c^2\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\sqrt{d}\left(b^2-4ac\right)^{11/4}}-\frac{21c^2\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\sqrt{d}\left(b^2-4ac\right)^{11/4}}+\frac{7c\sqrt{bd+2cdx}}{2d\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{\sqrt{bd+2cdx}}{2d\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

```
[Out] -Sqrt[b*d + 2*c*d*x]/(2*(b^2 - 4*a*c)*d*(a + b*x + c*x^2)^2) + (7*c*Sqrt[b*
d + 2*c*d*x])/(2*(b^2 - 4*a*c)^2*d*(a + b*x + c*x^2)) - (21*c^2*ArcTan[Sqrt
[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(11/4)*Sqrt[
d]) - (21*c^2*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(
(b^2 - 4*a*c)^(11/4)*Sqrt[d])
```

Rubi [A] time = 0.135095, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {687, 694, 329, 212, 206, 203}

$$-\frac{21c^2\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\sqrt{d}\left(b^2-4ac\right)^{11/4}}-\frac{21c^2\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{\sqrt{d}\left(b^2-4ac\right)^{11/4}}+\frac{7c\sqrt{bd+2cdx}}{2d\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{\sqrt{bd+2cdx}}{2d\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[b*d + 2*c*d*x]*(a + b*x + c*x^2)^3),x]
```

```
[Out] -Sqrt[b*d + 2*c*d*x]/(2*(b^2 - 4*a*c)*d*(a + b*x + c*x^2)^2) + (7*c*Sqrt[b*
d + 2*c*d*x])/(2*(b^2 - 4*a*c)^2*d*(a + b*x + c*x^2)) - (21*c^2*ArcTan[Sqrt
[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(11/4)*Sqrt[
d]) - (21*c^2*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/(
(b^2 - 4*a*c)^(11/4)*Sqrt[d])
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^3}\,dx = -\frac{\sqrt{bd+2cdx}}{2\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^2}-\frac{(7c)\int \frac{1}{\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{\sqrt{bd+2cdx}}{2\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^2}+\frac{7c\sqrt{bd+2cdx}}{2\left(b^2-4ac\right)^2d\left(a+bx+cx^2\right)}+\frac{\left(21c^2\right)\int \frac{1}{\sqrt{bd+\cdots}}}{2\left(b^2\cdots\right.}$$

$$= -\frac{\sqrt{bd+2cdx}}{2\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^2}+\frac{7c\sqrt{bd+2cdx}}{2\left(b^2-4ac\right)^2d\left(a+bx+cx^2\right)}+\frac{(21c)\,\mathrm{Subst}\left(\cdots\right.}{\cdots}$$

$$= -\frac{\sqrt{bd+2cdx}}{2\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^2}+\frac{7c\sqrt{bd+2cdx}}{2\left(b^2-4ac\right)^2d\left(a+bx+cx^2\right)}+\frac{(21c)\,\mathrm{Subst}\left(\cdots\right.}{\cdots}$$

$$= -\frac{\sqrt{bd+2cdx}}{2\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^2}+\frac{7c\sqrt{bd+2cdx}}{2\left(b^2-4ac\right)^2d\left(a+bx+cx^2\right)}-\frac{\left(21c^2\right)\mathrm{Subst}\cdots}{\cdots}$$

$$= -\frac{\sqrt{bd+2cdx}}{2\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^2}+\frac{7c\sqrt{bd+2cdx}}{2\left(b^2-4ac\right)^2d\left(a+bx+cx^2\right)}-\frac{21c^2\tan^{-1}\left(\cdots\right.}{\left(b^2-4ac\right)\cdots}$$

Mathematica [A] time = 0.379637, size = 179, normalized size = 0.93

$$\frac{c^2\left(\frac{\left(b^2-4ac\right)(b+2cx)\left(-c\left(11a+7cx^2\right)+b^2-7bcx\right)}{2c^2(a+x(b+cx))^2}+21\sqrt[4]{b^2-4ac}\sqrt{b+2cx}\tan^{-1}\left(\frac{\sqrt{b+2cx}}{\sqrt[4]{b^2-4ac}}\right)+21\sqrt[4]{b^2-4ac}\sqrt{b+2cx}\tanh^{-1}\left(\frac{\sqrt{b+2cx}}{\sqrt[4]{b^2-4a\cdots}}\right.\cdots\right.}{\left(4ac-b^2\right)^3\sqrt{d(b+2cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[b*d + 2*c*d*x]*(a + b*x + c*x^2)^3),x]
```

```
[Out] (c^2*(((b^2 - 4*a*c)*(b + 2*c*x)*(b^2 - 7*b*c*x - c*(11*a + 7*c*x^2)))/(2*c
^2*(a + x*(b + c*x))^2) + 21*(b^2 - 4*a*c)^(1/4)*Sqrt[b + 2*c*x]*ArcTan[Sqr
t[b + 2*c*x]/(b^2 - 4*a*c)^(1/4)] + 21*(b^2 - 4*a*c)^(1/4)*Sqrt[b + 2*c*x]*
ArcTanh[Sqrt[b + 2*c*x]/(b^2 - 4*a*c)^(1/4)]))/((-b^2 + 4*a*c)^3*Sqrt[d*(b
+ 2*c*x)])
```

Maple [B] time = 0.197, size = 419, normalized size = 2.2

$$8\,\frac{c^2d^5\sqrt{2\,cdx+bd}}{\left(4\,acd^2-b^2d^2\right)\left(4\,c^2d^2x^2+4\,bcd^2x+4\,acd^2\right)^2}+14\,\frac{c^2d^5\sqrt{2\,cdx+bd}}{\left(4\,acd^2-b^2d^2\right)^2\left(4\,c^2d^2x^2+4\,bcd^2x+4\,acd^2\right)}+\frac{21\,c^2d^5\sqrt{2}}{4}\ln\left(\left(2\,c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^3,x)
```

```
[Out] 8*c^2*d^5*(2*c*d*x+b*d)^(1/2)/(4*a*c*d^2-b^2*d^2)/(4*c^2*d^2*x^2+4*b*c*d^2*
x+4*a*c*d^2)^2+14*c^2*d^5/(4*a*c*d^2-b^2*d^2)^2*(2*c*d*x+b*d)^(1/2)/(4*c^2*
d^2*x^2+4*b*c*d^2*x+4*a*c*d^2)+21/4*c^2*d^5/(4*a*c*d^2-b^2*d^2)^(11/4)*2^(1
/2)*ln((2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(
4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b
*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2)))+21/2*c^2*d^5/(4*a*c*d^2-b^2*d
^2)^(11/4)*2^(1/2)*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(
1/2)+1)-21/2*c^2*d^5/(4*a*c*d^2-b^2*d^2)^(11/4)*2^(1/2)*arctan(-2^(1/2)/(4*
a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.02501, size = 4983, normalized size = 25.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(84*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*x^4 + 2*(b^5*c - 8*a*b^3*c^
2 + 16*a^2*b*c^3)*d*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d*x^2 + 2*(a*b^5 -
 8*a^2*b^3*c + 16*a^3*b*c^2)*d*x + (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*d)*
(c^8/((b^22 - 44*a*b^20*c + 880*a^2*b^18*c^2 - 10560*a^3*b^16*c^3 + 84480*a
^4*b^14*c^4 - 473088*a^5*b^12*c^5 + 1892352*a^6*b^10*c^6 - 5406720*a^7*b^8*
c^7 + 10813440*a^8*b^6*c^8 - 14417920*a^9*b^4*c^9 + 11534336*a^10*b^2*c^10
- 4194304*a^11*c^11)*d^2))^(1/4)*arctan(((b^16 - 32*a*b^14*c + 448*a^2*b^12
*c^2 - 3584*a^3*b^10*c^3 + 17920*a^4*b^8*c^4 - 57344*a^5*b^6*c^5 + 114688*a
```

```
^6*b^4*c^6 - 131072*a^7*b^2*c^7 + 65536*a^8*c^8)*sqrt(2*c^5*d*x + b*c^4*d +
 (b^12 - 24*a*b^10*c + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^
4 - 6144*a^5*b^2*c^5 + 4096*a^6*c^6)*sqrt(c^8/((b^22 - 44*a*b^20*c + 880*a^
2*b^18*c^2 - 10560*a^3*b^16*c^3 + 84480*a^4*b^14*c^4 - 473088*a^5*b^12*c^5
+ 1892352*a^6*b^10*c^6 - 5406720*a^7*b^8*c^7 + 10813440*a^8*b^6*c^8 - 14417
920*a^9*b^4*c^9 + 11534336*a^10*b^2*c^10 - 4194304*a^11*c^11)*d^2))*d^2)*(c
^8/((b^22 - 44*a*b^20*c + 880*a^2*b^18*c^2 - 10560*a^3*b^16*c^3 + 84480*a^4
*b^14*c^4 - 473088*a^5*b^12*c^5 + 1892352*a^6*b^10*c^6 - 5406720*a^7*b^8*c^
7 + 10813440*a^8*b^6*c^8 - 14417920*a^9*b^4*c^9 + 11534336*a^10*b^2*c^10 -
4194304*a^11*c^11)*d^2))^(3/4)*d - (b^16*c^2 - 32*a*b^14*c^3 + 448*a^2*b^12
*c^4 - 3584*a^3*b^10*c^5 + 17920*a^4*b^8*c^6 - 57344*a^5*b^6*c^7 + 114688*a
^6*b^4*c^8 - 131072*a^7*b^2*c^9 + 65536*a^8*c^10)*(c^8/((b^22 - 44*a*b^20*c
 + 880*a^2*b^18*c^2 - 10560*a^3*b^16*c^3 + 84480*a^4*b^14*c^4 - 473088*a^5*
b^12*c^5 + 1892352*a^6*b^10*c^6 - 5406720*a^7*b^8*c^7 + 10813440*a^8*b^6*c^
8 - 14417920*a^9*b^4*c^9 + 11534336*a^10*b^2*c^10 - 4194304*a^11*c^11)*d^2)
)^(3/4)*sqrt(2*c*d*x + b*d)*d)/c^8) - 21*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c
^4)*d*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*x^3 + (b^6 - 6*a*b^4*c
 + 32*a^3*c^3)*d*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d*x + (a^2*b^
4 - 8*a^3*b^2*c + 16*a^4*c^2)*d)*(c^8/((b^22 - 44*a*b^20*c + 880*a^2*b^18*c
^2 - 10560*a^3*b^16*c^3 + 84480*a^4*b^14*c^4 - 473088*a^5*b^12*c^5 + 189235
2*a^6*b^10*c^6 - 5406720*a^7*b^8*c^7 + 10813440*a^8*b^6*c^8 - 14417920*a^9*
b^4*c^9 + 11534336*a^10*b^2*c^10 - 4194304*a^11*c^11)*d^2))^(1/4)*log(21*sq
rt(2*c*d*x + b*d)*c^2 + 21*(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)
*(c^8/((b^22 - 44*a*b^20*c + 880*a^2*b^18*c^2 - 10560*a^3*b^16*c^3 + 84480*
a^4*b^14*c^4 - 473088*a^5*b^12*c^5 + 1892352*a^6*b^10*c^6 - 5406720*a^7*b^8
*c^7 + 10813440*a^8*b^6*c^8 - 14417920*a^9*b^4*c^9 + 11534336*a^10*b^2*c^10
 - 4194304*a^11*c^11)*d^2))^(1/4)*d) + 21*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*
c^4)*d*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*x^3 + (b^6 - 6*a*b^4*
c + 32*a^3*c^3)*d*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d*x + (a^2*b
^4 - 8*a^3*b^2*c + 16*a^4*c^2)*d)*(c^8/((b^22 - 44*a*b^20*c + 880*a^2*b^18*
c^2 - 10560*a^3*b^16*c^3 + 84480*a^4*b^14*c^4 - 473088*a^5*b^12*c^5 + 18923
52*a^6*b^10*c^6 - 5406720*a^7*b^8*c^7 + 10813440*a^8*b^6*c^8 - 14417920*a^9
*b^4*c^9 + 11534336*a^10*b^2*c^10 - 4194304*a^11*c^11)*d^2))^(1/4)*log(21*s
qrt(2*c*d*x + b*d)*c^2 - 21*(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3
)*(c^8/((b^22 - 44*a*b^20*c + 880*a^2*b^18*c^2 - 10560*a^3*b^16*c^3 + 84480
*a^4*b^14*c^4 - 473088*a^5*b^12*c^5 + 1892352*a^6*b^10*c^6 - 5406720*a^7*b^
8*c^7 + 10813440*a^8*b^6*c^8 - 14417920*a^9*b^4*c^9 + 11534336*a^10*b^2*c^1
0 - 4194304*a^11*c^11)*d^2))^(1/4)*d) + (7*c^2*x^2 + 7*b*c*x - b^2 + 11*a*c
)*sqrt(2*c*d*x + b*d))/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*x^4 + 2*(b^5
*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d*x
^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d*x + (a^2*b^4 - 8*a^3*b^2*c +
16*a^4*c^2)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**(1/2)/(c*x**2+b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19312, size = 871, normalized size = 4.54

$$-\frac{21\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}c^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}+2\sqrt{2\,cdx+bd}\right)}{2\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^6d-12\sqrt{2}ab^4cd+48\sqrt{2}a^2b^2c^2d-64\sqrt{2}a^3c^3d}-\frac{21\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}c^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-b^2d^2+4\,acd^2\right)^{\frac{1}{4}}\right.}{2\left(-b^2d^2+4\,ac\right.}\right.}{\sqrt{2}b^6d-12\sqrt{2}ab^4cd+48\sqrt{2}a^2b^2c^2d-64\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] -21*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*c^2*arctan(1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2
+ 4*a*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4))/(
sqrt(2)*b^6*d - 12*sqrt(2)*a*b^4*c*d + 48*sqrt(2)*a^2*b^2*c^2*d - 64*sqrt(2
)*a^3*c^3*d) - 21*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*c^2*arctan(-1/2*sqrt(2)*(sqr
t(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) - 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*
c*d^2)^(1/4))/(sqrt(2)*b^6*d - 12*sqrt(2)*a*b^4*c*d + 48*sqrt(2)*a^2*b^2*c^
2*d - 64*sqrt(2)*a^3*c^3*d) - 21/2*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*c^2*log(2*c
*d*x + b*d + sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqr
t(-b^2*d^2 + 4*a*c*d^2))/(sqrt(2)*b^6*d - 12*sqrt(2)*a*b^4*c*d + 48*sqrt(2)
*a^2*b^2*c^2*d - 64*sqrt(2)*a^3*c^3*d) + 21/2*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*
c^2*log(2*c*d*x + b*d - sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x +
 b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2))/(sqrt(2)*b^6*d - 12*sqrt(2)*a*b^4*c*d +
 48*sqrt(2)*a^2*b^2*c^2*d - 64*sqrt(2)*a^3*c^3*d) - 2*(11*sqrt(2*c*d*x + b*
d)*b^2*c^2*d^3 - 44*sqrt(2*c*d*x + b*d)*a*c^3*d^3 - 7*(2*c*d*x + b*d)^(5/2)
*c^2*d)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*(b^2*d^2 - 4*a*c*d^2 - (2*c*d*x + b
*d)^2)^2)
```

3.1317 $\int \frac{1}{(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^3}\,dx$

Optimal. Leaf size=223

$$\frac{45c^2\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{3/2}\left(b^2-4ac\right)^{13/4}}-\frac{45c^2\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{3/2}\left(b^2-4ac\right)^{13/4}}+\frac{90c^2}{d\left(b^2-4ac\right)^3\sqrt{bd+2cdx}}+\frac{9c}{2d\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)\sqrt{bd}}$$

```
[Out] (90*c^2)/((b^2 - 4*a*c)^3*d*Sqrt[b*d + 2*c*d*x]) - 1/(2*(b^2 - 4*a*c)*d*Sqr
t[b*d + 2*c*d*x]*(a + b*x + c*x^2)^2) + (9*c)/(2*(b^2 - 4*a*c)^2*d*Sqrt[b*d
 + 2*c*d*x]*(a + b*x + c*x^2)) + (45*c^2*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 -
 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(13/4)*d^(3/2)) - (45*c^2*ArcTanh[S
qrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(13/4)*d^
(3/2))
```

Rubi [A] time = 0.168262, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {687, 693, 694, 329, 298, 203, 206}

$$\frac{45c^2\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{3/2}\left(b^2-4ac\right)^{13/4}}-\frac{45c^2\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{3/2}\left(b^2-4ac\right)^{13/4}}+\frac{90c^2}{d\left(b^2-4ac\right)^3\sqrt{bd+2cdx}}+\frac{9c}{2d\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)\sqrt{bd}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)^3),x]
```

```
[Out] (90*c^2)/((b^2 - 4*a*c)^3*d*Sqrt[b*d + 2*c*d*x]) - 1/(2*(b^2 - 4*a*c)*d*Sqr
t[b*d + 2*c*d*x]*(a + b*x + c*x^2)^2) + (9*c)/(2*(b^2 - 4*a*c)^2*d*Sqrt[b*d
 + 2*c*d*x]*(a + b*x + c*x^2)) + (45*c^2*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 -
 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(13/4)*d^(3/2)) - (45*c^2*ArcTanh[S
qrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(13/4)*d^
(3/2))
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^3}dx = -\frac{1}{2\left(b^2-4ac\right)d\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^2} - \frac{(9c)\int \frac{1}{(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^2}dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{1}{2\left(b^2-4ac\right)d\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^2} + \frac{9c}{2\left(b^2-4ac\right)^2 d\sqrt{bd+2cdx}\left(a+\right.}$$

$$= \frac{90c^2}{\left(b^2-4ac\right)^3 d\sqrt{bd+2cdx}} - \frac{1}{2\left(b^2-4ac\right)d\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^2} + \frac{}{2\left(b^2\right.}$$

$$= \frac{90c^2}{\left(b^2-4ac\right)^3 d\sqrt{bd+2cdx}} - \frac{1}{2\left(b^2-4ac\right)d\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^2} + \frac{}{2\left(b^2\right.}$$

$$= \frac{90c^2}{\left(b^2-4ac\right)^3 d\sqrt{bd+2cdx}} - \frac{1}{2\left(b^2-4ac\right)d\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^2} + \frac{}{2\left(b^2\right.}$$

$$= \frac{90c^2}{\left(b^2-4ac\right)^3 d\sqrt{bd+2cdx}} - \frac{1}{2\left(b^2-4ac\right)d\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^2} + \frac{}{2\left(b^2\right.}$$

$$= \frac{90c^2}{\left(b^2-4ac\right)^3 d\sqrt{bd+2cdx}} - \frac{1}{2\left(b^2-4ac\right)d\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^2} + \frac{}{2\left(b^2\right.}$$

Mathematica [C] time = 0.0689023, size = 57, normalized size = 0.26

$$\frac{64c^2\,{}_2F_1\left(-\frac{1}{4},3;\frac{3}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{d\left(b^2-4ac\right)^3\sqrt{d(b+2cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)^3),x]
```

```
[Out] (64*c^2*Hypergeometric2F1[-1/4, 3, 3/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/((b^2
 - 4*a*c)^3*d*Sqrt[d*(b + 2*c*x)])
```

Maple [B] time = 0.209, size = 534, normalized size = 2.4

$$-64\frac{c^2}{d\left(4ac-b^2\right)^3\sqrt{2cdx+bd}} - 26\frac{c^2(2cdx+bd)^{7/2}}{d\left(4ac-b^2\right)^3\left(4c^2d^2x^2+4bcd^2x+4acd^2\right)^2} - 136\frac{c^3d(2cdx+bd)^{3/2}}{\left(4ac-b^2\right)^3\left(4c^2d^2x^2+4bcd^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^3,x)
```

```
[Out] -64*c^2/d/(4*a*c-b^2)^3/(2*c*d*x+b*d)^(1/2)-26*c^2/d/(4*a*c-b^2)^3/(4*c^2*d
^2*x^2+4*b*c*d^2*x+4*a*c*d^2)^2*(2*c*d*x+b*d)^(7/2)-136*c^3*d/(4*a*c-b^2)^3
/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c*d^2)^2*(2*c*d*x+b*d)^(3/2)*a+34*c^2*d/(4*
```

```
a*c-b^2)^3/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c*d^2)^2*(2*c*d*x+b*d)^(3/2)*b^2-
45/4*c^2/d/(4*a*c-b^2)^3*2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*ln((2*c*d*x+b*d-
(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(
1/2))/(2*c*d*x+b*d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4
*a*c*d^2-b^2*d^2)^(1/2)))-45/2*c^2/d/(4*a*c-b^2)^3*2^(1/2)/(4*a*c*d^2-b^2*d
^2)^(1/4)*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)+4
5/2*c^2/d/(4*a*c-b^2)^3*2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*arctan(-2^(1/2)/(
4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.47029, size = 7175, normalized size = 32.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(180*(2*(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*d^2*x^5
+ 5*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^2*x^4 + 4*(b
^8*c - 11*a*b^6*c^2 + 36*a^2*b^4*c^3 - 16*a^3*b^2*c^4 - 64*a^4*c^5)*d^2*x^3
 + (b^9 - 6*a*b^7*c - 24*a^2*b^5*c^2 + 224*a^3*b^3*c^3 - 384*a^4*b*c^4)*d^2
*x^2 + 2*(a*b^8 - 11*a^2*b^6*c + 36*a^3*b^4*c^2 - 16*a^4*b^2*c^3 - 64*a^5*c
^4)*d^2*x + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d^2)*(
c^8/((b^26 - 52*a*b^24*c + 1248*a^2*b^22*c^2 - 18304*a^3*b^20*c^3 + 183040*
a^4*b^18*c^4 - 1317888*a^5*b^16*c^5 + 7028736*a^6*b^14*c^6 - 28114944*a^7*b
^12*c^7 + 84344832*a^8*b^10*c^8 - 187432960*a^9*b^8*c^9 + 299892736*a^10*b^
6*c^10 - 327155712*a^11*b^4*c^11 + 218103808*a^12*b^2*c^12 - 67108864*a^13*
c^13)*d^6))^(1/4)*arctan(-(sqrt(2*c^13*d*x + b*c^12*d + (b^14*c^8 - 28*a*b^
12*c^9 + 336*a^2*b^10*c^10 - 2240*a^3*b^8*c^11 + 8960*a^4*b^6*c^12 - 21504*
a^5*b^4*c^13 + 28672*a^6*b^2*c^14 - 16384*a^7*c^15)*sqrt(c^8/((b^26 - 52*a*
b^24*c + 1248*a^2*b^22*c^2 - 18304*a^3*b^20*c^3 + 183040*a^4*b^18*c^4 - 131
7888*a^5*b^16*c^5 + 7028736*a^6*b^14*c^6 - 28114944*a^7*b^12*c^7 + 84344832
*a^8*b^10*c^8 - 187432960*a^9*b^8*c^9 + 299892736*a^10*b^6*c^10 - 327155712
*a^11*b^4*c^11 + 218103808*a^12*b^2*c^12 - 67108864*a^13*c^13)*d^6))*d^4)*(
b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*(c^8/((b^26 - 52*a*b^24*c +
 1248*a^2*b^22*c^2 - 18304*a^3*b^20*c^3 + 183040*a^4*b^18*c^4 - 1317888*a^5
*b^16*c^5 + 7028736*a^6*b^14*c^6 - 28114944*a^7*b^12*c^7 + 84344832*a^8*b^1
0*c^8 - 187432960*a^9*b^8*c^9 + 299892736*a^10*b^6*c^10 - 327155712*a^11*b^
4*c^11 + 218103808*a^12*b^2*c^12 - 67108864*a^13*c^13)*d^6))^(1/4)*d - (b^6
*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)*sqrt(2*c*d*x + b*d)*(c^8
/((b^26 - 52*a*b^24*c + 1248*a^2*b^22*c^2 - 18304*a^3*b^20*c^3 + 183040*a^4
*b^18*c^4 - 1317888*a^5*b^16*c^5 + 7028736*a^6*b^14*c^6 - 28114944*a^7*b^12
*c^7 + 84344832*a^8*b^10*c^8 - 187432960*a^9*b^8*c^9 + 299892736*a^10*b^6*c
^10 - 327155712*a^11*b^4*c^11 + 218103808*a^12*b^2*c^12 - 67108864*a^13*c^1
3)*d^6))^(1/4)*d)/c^8) - 45*(2*(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 6
```

```
4*a^3*c^6)*d^2*x^5 + 5*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*
c^5)*d^2*x^4 + 4*(b^8*c - 11*a*b^6*c^2 + 36*a^2*b^4*c^3 - 16*a^3*b^2*c^4 -
64*a^4*c^5)*d^2*x^3 + (b^9 - 6*a*b^7*c - 24*a^2*b^5*c^2 + 224*a^3*b^3*c^3 -
 384*a^4*b*c^4)*d^2*x^2 + 2*(a*b^8 - 11*a^2*b^6*c + 36*a^3*b^4*c^2 - 16*a^4
*b^2*c^3 - 64*a^5*c^4)*d^2*x + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 6
4*a^5*b*c^3)*d^2)*(c^8/((b^26 - 52*a*b^24*c + 1248*a^2*b^22*c^2 - 18304*a^3
*b^20*c^3 + 183040*a^4*b^18*c^4 - 1317888*a^5*b^16*c^5 + 7028736*a^6*b^14*c
^6 - 28114944*a^7*b^12*c^7 + 84344832*a^8*b^10*c^8 - 187432960*a^9*b^8*c^9
+ 299892736*a^10*b^6*c^10 - 327155712*a^11*b^4*c^11 + 218103808*a^12*b^2*c^
12 - 67108864*a^13*c^13)*d^6))^(1/4)*log(91125*(b^20 - 40*a*b^18*c + 720*a^
2*b^16*c^2 - 7680*a^3*b^14*c^3 + 53760*a^4*b^12*c^4 - 258048*a^5*b^10*c^5 +
 860160*a^6*b^8*c^6 - 1966080*a^7*b^6*c^7 + 2949120*a^8*b^4*c^8 - 2621440*a
^9*b^2*c^9 + 1048576*a^10*c^10)*(c^8/((b^26 - 52*a*b^24*c + 1248*a^2*b^22*c
^2 - 18304*a^3*b^20*c^3 + 183040*a^4*b^18*c^4 - 1317888*a^5*b^16*c^5 + 7028
736*a^6*b^14*c^6 - 28114944*a^7*b^12*c^7 + 84344832*a^8*b^10*c^8 - 18743296
0*a^9*b^8*c^9 + 299892736*a^10*b^6*c^10 - 327155712*a^11*b^4*c^11 + 2181038
08*a^12*b^2*c^12 - 67108864*a^13*c^13)*d^6))^(3/4)*d^5 + 91125*sqrt(2*c*d*x
 + b*d)*c^6) + 45*(2*(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)
*d^2*x^5 + 5*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^2*x
^4 + 4*(b^8*c - 11*a*b^6*c^2 + 36*a^2*b^4*c^3 - 16*a^3*b^2*c^4 - 64*a^4*c^5
)*d^2*x^3 + (b^9 - 6*a*b^7*c - 24*a^2*b^5*c^2 + 224*a^3*b^3*c^3 - 384*a^4*b
*c^4)*d^2*x^2 + 2*(a*b^8 - 11*a^2*b^6*c + 36*a^3*b^4*c^2 - 16*a^4*b^2*c^3 -
 64*a^5*c^4)*d^2*x + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^
3)*d^2)*(c^8/((b^26 - 52*a*b^24*c + 1248*a^2*b^22*c^2 - 18304*a^3*b^20*c^3
+ 183040*a^4*b^18*c^4 - 1317888*a^5*b^16*c^5 + 7028736*a^6*b^14*c^6 - 28114
944*a^7*b^12*c^7 + 84344832*a^8*b^10*c^8 - 187432960*a^9*b^8*c^9 + 29989273
6*a^10*b^6*c^10 - 327155712*a^11*b^4*c^11 + 218103808*a^12*b^2*c^12 - 67108
864*a^13*c^13)*d^6))^(1/4)*log(-91125*(b^20 - 40*a*b^18*c + 720*a^2*b^16*c^
2 - 7680*a^3*b^14*c^3 + 53760*a^4*b^12*c^4 - 258048*a^5*b^10*c^5 + 860160*a
^6*b^8*c^6 - 1966080*a^7*b^6*c^7 + 2949120*a^8*b^4*c^8 - 2621440*a^9*b^2*c^
9 + 1048576*a^10*c^10)*(c^8/((b^26 - 52*a*b^24*c + 1248*a^2*b^22*c^2 - 1830
4*a^3*b^20*c^3 + 183040*a^4*b^18*c^4 - 1317888*a^5*b^16*c^5 + 7028736*a^6*b
^14*c^6 - 28114944*a^7*b^12*c^7 + 84344832*a^8*b^10*c^8 - 187432960*a^9*b^8
*c^9 + 299892736*a^10*b^6*c^10 - 327155712*a^11*b^4*c^11 + 218103808*a^12*b
^2*c^12 - 67108864*a^13*c^13)*d^6))^(3/4)*d^5 + 91125*sqrt(2*c*d*x + b*d)*c
^6) + (180*c^4*x^4 + 360*b*c^3*x^3 - b^4 + 17*a*b^2*c + 128*a^2*c^2 + 27*(7
*b^2*c^2 + 12*a*c^3)*x^2 + 9*(b^3*c + 36*a*b*c^2)*x)*sqrt(2*c*d*x + b*d))/(
2*(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*d^2*x^5 + 5*(b^7*c
^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^2*x^4 + 4*(b^8*c - 11*
a*b^6*c^2 + 36*a^2*b^4*c^3 - 16*a^3*b^2*c^4 - 64*a^4*c^5)*d^2*x^3 + (b^9 -
6*a*b^7*c - 24*a^2*b^5*c^2 + 224*a^3*b^3*c^3 - 384*a^4*b*c^4)*d^2*x^2 + 2*(
a*b^8 - 11*a^2*b^6*c + 36*a^3*b^4*c^2 - 16*a^4*b^2*c^3 - 64*a^5*c^4)*d^2*x
+ (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**(3/2)/(c*x**2+b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2384, size = 1098, normalized size = 4.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] -45*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c^2*arctan(1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2
+ 4*a*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4))/(
sqrt(2)*b^8*d^3 - 16*sqrt(2)*a*b^6*c*d^3 + 96*sqrt(2)*a^2*b^4*c^2*d^3 - 256
*sqrt(2)*a^3*b^2*c^3*d^3 + 256*sqrt(2)*a^4*c^4*d^3) - 45*(-b^2*d^2 + 4*a*c*
d^2)^(3/4)*c^2*arctan(-1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) -
2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4))/(sqrt(2)*b^8*d^3 - 16*
sqrt(2)*a*b^6*c*d^3 + 96*sqrt(2)*a^2*b^4*c^2*d^3 - 256*sqrt(2)*a^3*b^2*c^3*
d^3 + 256*sqrt(2)*a^4*c^4*d^3) + 45/2*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c^2*log(
2*c*d*x + b*d + sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) +
sqrt(-b^2*d^2 + 4*a*c*d^2))/(sqrt(2)*b^8*d^3 - 16*sqrt(2)*a*b^6*c*d^3 + 96*
sqrt(2)*a^2*b^4*c^2*d^3 - 256*sqrt(2)*a^3*b^2*c^3*d^3 + 256*sqrt(2)*a^4*c^4
*d^3) - 45/2*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c^2*log(2*c*d*x + b*d - sqrt(2)*(
-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2
))/(sqrt(2)*b^8*d^3 - 16*sqrt(2)*a*b^6*c*d^3 + 96*sqrt(2)*a^2*b^4*c^2*d^3 -
 256*sqrt(2)*a^3*b^2*c^3*d^3 + 256*sqrt(2)*a^4*c^4*d^3) + 64*c^2/((b^6*d -
12*a*b^4*c*d + 48*a^2*b^2*c^2*d - 64*a^3*c^3*d)*sqrt(2*c*d*x + b*d)) - 2*(1
7*(2*c*d*x + b*d)^(3/2)*b^2*c^2*d^2 - 68*(2*c*d*x + b*d)^(3/2)*a*c^3*d^2 -
13*(2*c*d*x + b*d)^(7/2)*c^2)/((b^6*d - 12*a*b^4*c*d + 48*a^2*b^2*c^2*d - 6
4*a^3*c^3*d)*(b^2*d^2 - 4*a*c*d^2 - (2*c*d*x + b*d)^2)^2)
```

3.1318 $\int \frac{1}{(bd+2cdx)^{5/2}\left(a+bx+cx^2\right)^3}\,dx$

Optimal. Leaf size=225

$$-\frac{77c^2\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{5/2}\left(b^2-4ac\right)^{15/4}}-\frac{77c^2\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{5/2}\left(b^2-4ac\right)^{15/4}}+\frac{154c^2}{3d\left(b^2-4ac\right)^3(bd+2cdx)^{3/2}}+\frac{11c}{2d\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)(}$$

```
[Out] (154*c^2)/(3*(b^2 - 4*a*c)^3*d*(b*d + 2*c*d*x)^(3/2)) - 1/(2*(b^2 - 4*a*c)*
d*(b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)^2) + (11*c)/(2*(b^2 - 4*a*c)^2*d*
(b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)) - (77*c^2*ArcTan[Sqrt[d*(b + 2*c*x
)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(15/4)*d^(5/2)) - (77*c^2
*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)
^(15/4)*d^(5/2))
```

Rubi [A] time = 0.191198, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {687, 693, 694, 329, 212, 206, 203}

$$-\frac{77c^2\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{5/2}\left(b^2-4ac\right)^{15/4}}-\frac{77c^2\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{5/2}\left(b^2-4ac\right)^{15/4}}+\frac{154c^2}{3d\left(b^2-4ac\right)^3(bd+2cdx)^{3/2}}+\frac{11c}{2d\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)(}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^(5/2)*(a + b*x + c*x^2)^3),x]
```

```
[Out] (154*c^2)/(3*(b^2 - 4*a*c)^3*d*(b*d + 2*c*d*x)^(3/2)) - 1/(2*(b^2 - 4*a*c)*
d*(b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)^2) + (11*c)/(2*(b^2 - 4*a*c)^2*d*
(b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)) - (77*c^2*ArcTan[Sqrt[d*(b + 2*c*x
)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(15/4)*d^(5/2)) - (77*c^2
*ArcTanh[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)
^(15/4)*d^(5/2))
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)^{5/2}\left(a+bx+cx^2\right)^3}\,dx = -\frac{1}{2\left(b^2-4ac\right)d(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^2} - \frac{(11c)\int \frac{1}{(bd+2cdx)^{5/2}\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{1}{2\left(b^2-4ac\right)d(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^2} + \frac{11c}{2\left(b^2-4ac\right)^2 d(bd+2cdx)^{3/2}(}$$

$$= \frac{154c^2}{3\left(b^2-4ac\right)^3 d(bd+2cdx)^{3/2}} - \frac{1}{2\left(b^2-4ac\right)d(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^2} +$$

$$= \frac{154c^2}{3\left(b^2-4ac\right)^3 d(bd+2cdx)^{3/2}} - \frac{1}{2\left(b^2-4ac\right)d(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^2} +$$

$$= \frac{154c^2}{3\left(b^2-4ac\right)^3 d(bd+2cdx)^{3/2}} - \frac{1}{2\left(b^2-4ac\right)d(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^2} +$$

$$= \frac{154c^2}{3\left(b^2-4ac\right)^3 d(bd+2cdx)^{3/2}} - \frac{1}{2\left(b^2-4ac\right)d(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^2} +$$

$$= \frac{154c^2}{3\left(b^2-4ac\right)^3 d(bd+2cdx)^{3/2}} - \frac{1}{2\left(b^2-4ac\right)d(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^2} +$$

Mathematica [C] time = 0.0837658, size = 59, normalized size = 0.26

$$\frac{64c^2\, {}_2F_1\left(-\frac{3}{4},3;\frac{1}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{3d\left(b^2-4ac\right)^3 (d(b+2cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^(5/2)*(a + b*x + c*x^2)^3),x]
```

```
[Out] (64*c^2*Hypergeometric2F1[-3/4, 3, 1/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(3*(b
^2 - 4*a*c)^3*d*(d*(b + 2*c*x))^(3/2))
```

Maple [B] time = 0.206, size = 534, normalized size = 2.4

$$-\frac{64\,c^2}{3\,d\left(4\,ac-b^2\right)^3}(2\,cdx+bd)^{-\frac{3}{2}} - 30\,\frac{c^2\,(2\,cdx+bd)^{5/2}}{d\left(4\,ac-b^2\right)^3\left(4\,c^2d^2x^2+4\,bcd^2x+4\,acd^2\right)^2} - 152\,\frac{c^3d\sqrt{2\,cdx+bd}}{\left(4\,ac-b^2\right)^3\left(4\,c^2d^2x^2+4\,bc\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^3,x)
```

```
[Out] -64/3*c^2/d/(4*a*c-b^2)^3/(2*c*d*x+b*d)^(3/2)-30*c^2/d/(4*a*c-b^2)^3/(4*c^2
*d^2*x^2+4*b*c*d^2*x+4*a*c*d^2)^2*(2*c*d*x+b*d)^(5/2)-152*c^3*d/(4*a*c-b^2)
^3/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c*d^2)^2*(2*c*d*x+b*d)^(1/2)*a+38*c^2*d/(
```

```
4*a*c-b^2)^3/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c*d^2)^2*(2*c*d*x+b*d)^(1/2)*b^
2-77/4*c^2/d/(4*a*c-b^2)^3/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*ln((2*c*d*x+b*
d+(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)
^(1/2))/(2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+
(4*a*c*d^2-b^2*d^2)^(1/2)))-77/2*c^2/d/(4*a*c-b^2)^3/(4*a*c*d^2-b^2*d^2)^(3
/4)*2^(1/2)*arctan(2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)
+77/2*c^2/d/(4*a*c-b^2)^3/(4*a*c*d^2-b^2*d^2)^(3/4)*2^(1/2)*arctan(-2^(1/2)
/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.45367, size = 8675, normalized size = 38.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/6*(924*(4*(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*d^3*x^6
 + 12*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*d^3*x^5 + (1
3*b^8*c^2 - 148*a*b^6*c^3 + 528*a^2*b^4*c^4 - 448*a^3*b^2*c^5 - 512*a^4*c^6
)*d^3*x^4 + 2*(3*b^9*c - 28*a*b^7*c^2 + 48*a^2*b^5*c^3 + 192*a^3*b^3*c^4 -
512*a^4*b*c^5)*d^3*x^3 + (b^10 - 2*a*b^8*c - 68*a^2*b^6*c^2 + 368*a^3*b^4*c
^3 - 448*a^4*b^2*c^4 - 256*a^5*c^5)*d^3*x^2 + 2*(a*b^9 - 10*a^2*b^7*c + 24*
a^3*b^5*c^2 + 32*a^4*b^3*c^3 - 128*a^5*b*c^4)*d^3*x + (a^2*b^8 - 12*a^3*b^6
*c + 48*a^4*b^4*c^2 - 64*a^5*b^2*c^3)*d^3)*(c^8/((b^30 - 60*a*b^28*c + 1680
*a^2*b^26*c^2 - 29120*a^3*b^24*c^3 + 349440*a^4*b^22*c^4 - 3075072*a^5*b^20
*c^5 + 20500480*a^6*b^18*c^6 - 105431040*a^7*b^16*c^7 + 421724160*a^8*b^14*
c^8 - 1312030720*a^9*b^12*c^9 + 3148873728*a^10*b^10*c^10 - 5725224960*a^11
*b^8*c^11 + 7633633280*a^12*b^6*c^12 - 7046430720*a^13*b^4*c^13 + 402653184
0*a^14*b^2*c^14 - 1073741824*a^15*c^15)*d^10))^(1/4)*arctan(-((b^22 - 44*a*
b^20*c + 880*a^2*b^18*c^2 - 10560*a^3*b^16*c^3 + 84480*a^4*b^14*c^4 - 47308
8*a^5*b^12*c^5 + 1892352*a^6*b^10*c^6 - 5406720*a^7*b^8*c^7 + 10813440*a^8*
b^6*c^8 - 14417920*a^9*b^4*c^9 + 11534336*a^10*b^2*c^10 - 4194304*a^11*c^11
)*sqrt((b^16 - 32*a*b^14*c + 448*a^2*b^12*c^2 - 3584*a^3*b^10*c^3 + 17920*a
^4*b^8*c^4 - 57344*a^5*b^6*c^5 + 114688*a^6*b^4*c^6 - 131072*a^7*b^2*c^7 +
65536*a^8*c^8)*d^6*sqrt(c^8/((b^30 - 60*a*b^28*c + 1680*a^2*b^26*c^2 - 2912
0*a^3*b^24*c^3 + 349440*a^4*b^22*c^4 - 3075072*a^5*b^20*c^5 + 20500480*a^6*
b^18*c^6 - 105431040*a^7*b^16*c^7 + 421724160*a^8*b^14*c^8 - 1312030720*a^9
*b^12*c^9 + 3148873728*a^10*b^10*c^10 - 5725224960*a^11*b^8*c^11 + 76336332
80*a^12*b^6*c^12 - 7046430720*a^13*b^4*c^13 + 4026531840*a^14*b^2*c^14 - 10
73741824*a^15*c^15)*d^10)) + 2*c^5*d*x + b*c^4*d)*d^7*(c^8/((b^30 - 60*a*b^
28*c + 1680*a^2*b^26*c^2 - 29120*a^3*b^24*c^3 + 349440*a^4*b^22*c^4 - 30750
72*a^5*b^20*c^5 + 20500480*a^6*b^18*c^6 - 105431040*a^7*b^16*c^7 + 42172416
0*a^8*b^14*c^8 - 1312030720*a^9*b^12*c^9 + 3148873728*a^10*b^10*c^10 - 5725
224960*a^11*b^8*c^11 + 7633633280*a^12*b^6*c^12 - 7046430720*a^13*b^4*c^13
```

+ 4026531840*a^14*b^2*c^14 - 1073741824*a^15*c^15)*d^10))^(3/4) - (b^22*c^2 - 44*a*b^20*c^3 + 880*a^2*b^18*c^4 - 10560*a^3*b^16*c^5 + 84480*a^4*b^14*c^6 - 473088*a^5*b^12*c^7 + 1892352*a^6*b^10*c^8 - 5406720*a^7*b^8*c^9 + 10813440*a^8*b^6*c^10 - 14417920*a^9*b^4*c^11 + 11534336*a^10*b^2*c^12 - 4194304*a^11*c^13)*sqrt(2*c*d*x + b*d)*d^7*(c^8/((b^30 - 60*a*b^28*c + 1680*a^2*b^26*c^2 - 29120*a^3*b^24*c^3 + 349440*a^4*b^22*c^4 - 3075072*a^5*b^20*c^5 + 20500480*a^6*b^18*c^6 - 105431040*a^7*b^16*c^7 + 421724160*a^8*b^14*c^8 - 1312030720*a^9*b^12*c^9 + 3148873728*a^10*b^10*c^10 - 5725224960*a^11*b^8*c^11 + 7633633280*a^12*b^6*c^12 - 7046430720*a^13*b^4*c^13 + 4026531840*a^14*b^2*c^14 - 1073741824*a^15*c^15)*d^10))^(3/4))/c^8) + 231*(4*(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*d^3*x^6 + 12*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*d^3*x^5 + (13*b^8*c^2 - 148*a*b^6*c^3 + 528*a^2*b^4*c^4 - 448*a^3*b^2*c^5 - 512*a^4*c^6)*d^3*x^4 + 2*(3*b^9*c - 28*a*b^7*c^2 + 48*a^2*b^5*c^3 + 192*a^3*b^3*c^4 - 512*a^4*b*c^5)*d^3*x^3 + (b^10 - 2*a*b^8*c - 68*a^2*b^6*c^2 + 368*a^3*b^4*c^3 - 448*a^4*b^2*c^4 - 256*a^5*c^5)*d^3*x^2 + 2*(a*b^9 - 10*a^2*b^7*c + 24*a^3*b^5*c^2 + 32*a^4*b^3*c^3 - 128*a^5*b*c^4)*d^3*x + (a^2*b^8 - 12*a^3*b^6*c + 48*a^4*b^4*c^2 - 64*a^5*b^2*c^3)*d^3)*(c^8/((b^30 - 60*a*b^28*c + 1680*a^2*b^26*c^2 - 29120*a^3*b^24*c^3 + 349440*a^4*b^22*c^4 - 3075072*a^5*b^20*c^5 + 20500480*a^6*b^18*c^6 - 105431040*a^7*b^16*c^7 + 421724160*a^8*b^14*c^8 - 1312030720*a^9*b^12*c^9 + 3148873728*a^10*b^10*c^10 - 5725224960*a^11*b^8*c^11 + 7633633280*a^12*b^6*c^12 - 7046430720*a^13*b^4*c^13 + 4026531840*a^14*b^2*c^14 - 1073741824*a^15*c^15)*d^10))^(1/4)*log(77*(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4)*d^3*(c^8/((b^30 - 60*a*b^28*c + 1680*a^2*b^26*c^2 - 29120*a^3*b^24*c^3 + 349440*a^4*b^22*c^4 - 3075072*a^5*b^20*c^5 + 20500480*a^6*b^18*c^6 - 105431040*a^7*b^16*c^7 + 421724160*a^8*b^14*c^8 - 1312030720*a^9*b^12*c^9 + 3148873728*a^10*b^10*c^10 - 5725224960*a^11*b^8*c^11 + 7633633280*a^12*b^6*c^12 - 7046430720*a^13*b^4*c^13 + 4026531840*a^14*b^2*c^14 - 1073741824*a^15*c^15)*d^10))^(1/4) + 77*sqrt(2*c*d*x + b*d)*c^2) - 231*(4*(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*d^3*x^6 + 12*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*d^3*x^5 + (13*b^8*c^2 - 148*a*b^6*c^3 + 528*a^2*b^4*c^4 - 448*a^3*b^2*c^5 - 512*a^4*c^6)*d^3*x^4 + 2*(3*b^9*c - 28*a*b^7*c^2 + 48*a^2*b^5*c^3 + 192*a^3*b^3*c^4 - 512*a^4*b*c^5)*d^3*x^3 + (b^10 - 2*a*b^8*c - 68*a^2*b^6*c^2 + 368*a^3*b^4*c^3 - 448*a^4*b^2*c^4 - 256*a^5*c^5)*d^3*x^2 + 2*(a*b^9 - 10*a^2*b^7*c + 24*a^3*b^5*c^2 + 32*a^4*b^3*c^3 - 128*a^5*b*c^4)*d^3*x + (a^2*b^8 - 12*a^3*b^6*c + 48*a^4*b^4*c^2 - 64*a^5*b^2*c^3)*d^3)*(c^8/((b^30 - 60*a*b^28*c + 1680*a^2*b^26*c^2 - 29120*a^3*b^24*c^3 + 349440*a^4*b^22*c^4 - 3075072*a^5*b^20*c^5 + 20500480*a^6*b^18*c^6 - 105431040*a^7*b^16*c^7 + 421724160*a^8*b^14*c^8 - 1312030720*a^9*b^12*c^9 + 3148873728*a^10*b^10*c^10 - 5725224960*a^11*b^8*c^11 + 7633633280*a^12*b^6*c^12 - 7046430720*a^13*b^4*c^13 + 4026531840*a^14*b^2*c^14 - 1073741824*a^15*c^15)*d^10))^(1/4)*log(-77*(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4)*d^3*(c^8/((b^30 - 60*a*b^28*c + 1680*a^2*b^26*c^2 - 29120*a^3*b^24*c^3 + 349440*a^4*b^22*c^4 - 3075072*a^5*b^20*c^5 + 20500480*a^6*b^18*c^6 - 105431040*a^7*b^16*c^7 + 421724160*a^8*b^14*c^8 - 1312030720*a^9*b^12*c^9 + 3148873728*a^10*b^10*c^10 - 5725224960*a^11*b^8*c^11 + 7633633280*a^12*b^6*c^12 - 7046430720*a^13*b^4*c^13 + 4026531840*a^14*b^2*c^14 - 1073741824*a^15*c^15)*d^10))^(1/4) + 77*sqrt(2*c*d*x + b*d)*c^2) - (308*c^4*x^4 + 616*b*c^3*x^3 - 3*b^4 + 57*a*b^2*c + 128*a^2*c^2 + 11*(31*b^2*c^2 + 44*a*c^3)*x^2 + 11*(3*b^3*c + 44*a*b*c^2)*x)*sqrt(2*c*d*x + b*d))/(4*(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*d^3*x^6 + 12*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*d^3*x^5 + (13*b^8*c^2 - 148*a*b^6*c^3 + 528*a^2*b^4*c^4 - 448*a^3*b^2*c^5 - 512*a^4*c^6)*d^3*x^4 + 2*(3*b^9*c - 28*a*b^7*c^2 + 48*a^2*b^5*c^3 + 192*a^3*b^3*c^4 - 512*a^4*b*c^5)*d^3*x^3 + (b^10 - 2*a*b^8*c - 68*a^2*b^6*c^2 + 368*a^3*b^4*c^3 - 448*a^4*b^2*c^4 - 256*a^5*c^5)*d^3*x^2 + 2*(a*b^9 - 10*a^2*b^7*c + 24*a^3*b^5*c^2 + 32*a^4*b^3*c^3 - 128*a^5*b*c^4)*d^3*x + (a^2*b^8 - 12*a^3*b^6*c + 48*a^4*b^4*c^2 - 64*a^5*b^2*c^3)*d^3)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**(5/2)/(c*x**2+b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27741, size = 1098, normalized size = 4.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] -77*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*c^2*arctan(1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2
+ 4*a*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4))/(
sqrt(2)*b^8*d^3 - 16*sqrt(2)*a*b^6*c*d^3 + 96*sqrt(2)*a^2*b^4*c^2*d^3 - 256
*sqrt(2)*a^3*b^2*c^3*d^3 + 256*sqrt(2)*a^4*c^4*d^3) - 77*(-b^2*d^2 + 4*a*c*
d^2)^(1/4)*c^2*arctan(-1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) -
2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4))/(sqrt(2)*b^8*d^3 - 16*
sqrt(2)*a*b^6*c*d^3 + 96*sqrt(2)*a^2*b^4*c^2*d^3 - 256*sqrt(2)*a^3*b^2*c^3*
d^3 + 256*sqrt(2)*a^4*c^4*d^3) - 77/2*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*c^2*log(
2*c*d*x + b*d + sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) +
sqrt(-b^2*d^2 + 4*a*c*d^2))/(sqrt(2)*b^8*d^3 - 16*sqrt(2)*a*b^6*c*d^3 + 96*
sqrt(2)*a^2*b^4*c^2*d^3 - 256*sqrt(2)*a^3*b^2*c^3*d^3 + 256*sqrt(2)*a^4*c^4
*d^3) + 77/2*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*c^2*log(2*c*d*x + b*d - sqrt(2)*(
-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2
))/(sqrt(2)*b^8*d^3 - 16*sqrt(2)*a*b^6*c*d^3 + 96*sqrt(2)*a^2*b^4*c^2*d^3 -
 256*sqrt(2)*a^3*b^2*c^3*d^3 + 256*sqrt(2)*a^4*c^4*d^3) + 64/3*c^2/((b^6*d
- 12*a*b^4*c*d + 48*a^2*b^2*c^2*d - 64*a^3*c^3*d)*(2*c*d*x + b*d)^(3/2)) -
2*(19*sqrt(2*c*d*x + b*d)*b^2*c^2*d^2 - 76*sqrt(2*c*d*x + b*d)*a*c^3*d^2 -
15*(2*c*d*x + b*d)^(5/2)*c^2)/((b^6*d - 12*a*b^4*c*d + 48*a^2*b^2*c^2*d - 6
4*a^3*c^3*d)*(b^2*d^2 - 4*a*c*d^2 - (2*c*d*x + b*d)^2)^2)
```

3.1319 $\int \frac{1}{(bd+2cdx)^{7/2}(a+bx+cx^2)^3}\,dx$

Optimal. Leaf size=256

$$\frac{234c^2}{d^3\left(b^2-4ac\right)^4\sqrt{bd+2cdx}}+\frac{117c^2\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{7/2}\left(b^2-4ac\right)^{17/4}}-\frac{117c^2\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{7/2}\left(b^2-4ac\right)^{17/4}}+\frac{234c^2}{5d\left(b^2-4ac\right)^3(bd+2cdx)^{5/2}}+$$

```
[Out] (234*c^2)/(5*(b^2 - 4*a*c)^3*d*(b*d + 2*c*d*x)^(5/2)) + (234*c^2)/((b^2 - 4
*a*c)^4*d^3*Sqrt[b*d + 2*c*d*x]) - 1/(2*(b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(5/
2)*(a + b*x + c*x^2)^2) + (13*c)/(2*(b^2 - 4*a*c)^2*d*(b*d + 2*c*d*x)^(5/2)
*(a + b*x + c*x^2)) + (117*c^2*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1
/4)*Sqrt[d])])/((b^2 - 4*a*c)^(17/4)*d^(7/2)) - (117*c^2*ArcTanh[Sqrt[d*(b
+ 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(17/4)*d^(7/2))
```

Rubi [A] time = 0.227809, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {687, 693, 694, 329, 298, 203, 206}

$$\frac{234c^2}{d^3\left(b^2-4ac\right)^4\sqrt{bd+2cdx}}+\frac{117c^2\tan^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{7/2}\left(b^2-4ac\right)^{17/4}}-\frac{117c^2\tanh^{-1}\left(\frac{\sqrt{d(b+2cx)}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right)}{d^{7/2}\left(b^2-4ac\right)^{17/4}}+\frac{234c^2}{5d\left(b^2-4ac\right)^3(bd+2cdx)^{5/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^(7/2)*(a + b*x + c*x^2)^3),x]
```


```
[Out] (234*c^2)/(5*(b^2 - 4*a*c)^3*d*(b*d + 2*c*d*x)^(5/2)) + (234*c^2)/((b^2 - 4
*a*c)^4*d^3*Sqrt[b*d + 2*c*d*x]) - 1/(2*(b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(5/
2)*(a + b*x + c*x^2)^2) + (13*c)/(2*(b^2 - 4*a*c)^2*d*(b*d + 2*c*d*x)^(5/2)
*(a + b*x + c*x^2)) + (117*c^2*ArcTan[Sqrt[d*(b + 2*c*x)]/((b^2 - 4*a*c)^(1
/4)*Sqrt[d])])/((b^2 - 4*a*c)^(17/4)*d^(7/2)) - (117*c^2*ArcTanh[Sqrt[d*(b
+ 2*c*x)]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])])/((b^2 - 4*a*c)^(17/4)*d^(7/2))
```


Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)^{7/2}\left(a+bx+cx^2\right)^3}dx = -\frac{1}{2\left(b^2-4ac\right)d(bd+2cdx)^{5/2}\left(a+bx+cx^2\right)^2} - \frac{(13c)\int \frac{1}{(bd+2cdx)^{7/2}\left(a+bx+cx^2\right)^2}dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{1}{2\left(b^2-4ac\right)d(bd+2cdx)^{5/2}\left(a+bx+cx^2\right)^2} + \frac{13c}{2\left(b^2-4ac\right)^2 d(bd+2cdx)^{5/2}(}$$

$$= \frac{234c^2}{5\left(b^2-4ac\right)^3 d(bd+2cdx)^{5/2}} - \frac{1}{2\left(b^2-4ac\right)d(bd+2cdx)^{5/2}\left(a+bx+cx^2\right)^2} +$$

$$= \frac{234c^2}{5\left(b^2-4ac\right)^3 d(bd+2cdx)^{5/2}} + \frac{234c^2}{\left(b^2-4ac\right)^4 d^3\sqrt{bd+2cdx}} - \frac{}{2\left(b^2-4ac\right)d(bd}$$

$$= \frac{234c^2}{5\left(b^2-4ac\right)^3 d(bd+2cdx)^{5/2}} + \frac{234c^2}{\left(b^2-4ac\right)^4 d^3\sqrt{bd+2cdx}} - \frac{}{2\left(b^2-4ac\right)d(bd}$$

$$= \frac{234c^2}{5\left(b^2-4ac\right)^3 d(bd+2cdx)^{5/2}} + \frac{234c^2}{\left(b^2-4ac\right)^4 d^3\sqrt{bd+2cdx}} - \frac{}{2\left(b^2-4ac\right)d(bd}$$

$$= \frac{234c^2}{5\left(b^2-4ac\right)^3 d(bd+2cdx)^{5/2}} + \frac{234c^2}{\left(b^2-4ac\right)^4 d^3\sqrt{bd+2cdx}} - \frac{}{2\left(b^2-4ac\right)d(bd}$$

$$= \frac{234c^2}{5\left(b^2-4ac\right)^3 d(bd+2cdx)^{5/2}} + \frac{234c^2}{\left(b^2-4ac\right)^4 d^3\sqrt{bd+2cdx}} - \frac{}{2\left(b^2-4ac\right)d(bd}$$

Mathematica [C] time = 0.0903296, size = 59, normalized size = 0.23

$$\frac{64c^2\,{}_2F_1\left(-\frac{5}{4},3;-\frac{1}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{5d\left(b^2-4ac\right)^3(d(b+2cx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^(7/2)*(a + b*x + c*x^2)^3),x]
```

```
[Out] (64*c^2*Hypergeometric2F1[-5/4, 3, -1/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(5*(
b^2 - 4*a*c)^3*d*(d*(b + 2*c*x))^(5/2))
```

Maple [B] time = 0.21, size = 569, normalized size = 2.2

$$-\frac{64\,c^2}{5\,d\left(4\,ac-b^2\right)^3}(2\,cdx+bd)^{-\frac{5}{2}}+192\,\frac{c^2}{d^3\left(4\,ac-b^2\right)^4\sqrt{2\,cdx+bd}}+42\,\frac{c^2\,(2\,cdx+bd)^{7/2}}{d^3\left(4\,ac-b^2\right)^4\left(4\,c^2d^2x^2+4\,bcd^2x+4\,acd^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^3,x)
```

```
[Out] -64/5*c^2/d/(4*a*c-b^2)^3/(2*c*d*x+b*d)^(5/2)+192*c^2/d^3/(4*a*c-b^2)^4/(2*
c*d*x+b*d)^(1/2)+42*c^2/d^3/(4*a*c-b^2)^4/(4*c^2*d^2*x^2+4*b*c*d^2*x+4*a*c*
d^2)^2*(2*c*d*x+b*d)^(7/2)+200*c^3/d/(4*a*c-b^2)^4/(4*c^2*d^2*x^2+4*b*c*d^2
*x+4*a*c*d^2)^2*(2*c*d*x+b*d)^(3/2)*a-50*c^2/d/(4*a*c-b^2)^4/(4*c^2*d^2*x^2
+4*b*c*d^2*x+4*a*c*d^2)^2*(2*c*d*x+b*d)^(3/2)*b^2+117/4*c^2/d^3/(4*a*c-b^2)
^4*2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*ln((2*c*d*x+b*d-(4*a*c*d^2-b^2*d^2)^(1
/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2))/(2*c*d*x+b*d+(4*
a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)*2^(1/2)+(4*a*c*d^2-b^2*d^2)^(1/2
)))+117/2*c^2/d^3/(4*a*c-b^2)^4*2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*arctan(2^
(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)-117/2*c^2/d^3/(4*a*c
-b^2)^4*2^(1/2)/(4*a*c*d^2-b^2*d^2)^(1/4)*arctan(-2^(1/2)/(4*a*c*d^2-b^2*d^
2)^(1/4)*(2*c*d*x+b*d)^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.70787, size = 11348, normalized size = 44.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/10*(2340*(8*(b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8 +
 256*a^4*c^9)*d^4*x^7 + 28*(b^9*c^4 - 16*a*b^7*c^5 + 96*a^2*b^5*c^6 - 256*a
^3*b^3*c^7 + 256*a^4*b*c^8)*d^4*x^6 + 2*(19*b^10*c^3 - 296*a*b^8*c^4 + 1696
*a^2*b^6*c^5 - 4096*a^3*b^4*c^6 + 2816*a^4*b^2*c^7 + 2048*a^5*c^8)*d^4*x^5
+ 5*(5*b^11*c^2 - 72*a*b^9*c^3 + 352*a^2*b^7*c^4 - 512*a^3*b^5*c^5 - 768*a^
4*b^3*c^6 + 2048*a^5*b*c^7)*d^4*x^4 + 4*(2*b^12*c - 23*a*b^10*c^2 + 50*a^2*
b^8*c^3 + 320*a^3*b^6*c^4 - 1600*a^4*b^4*c^5 + 1792*a^5*b^2*c^6 + 512*a^6*c
^7)*d^4*x^3 + (b^13 - 2*a*b^11*c - 116*a^2*b^9*c^2 + 896*a^3*b^7*c^3 - 2176
*a^4*b^5*c^4 + 512*a^5*b^3*c^5 + 3072*a^6*b*c^6)*d^4*x^2 + 2*(a*b^12 - 13*a
^2*b^10*c + 48*a^3*b^8*c^2 + 32*a^4*b^6*c^3 - 512*a^5*b^4*c^4 + 768*a^6*b^2
*c^5)*d^4*x + (a^2*b^11 - 16*a^3*b^9*c + 96*a^4*b^7*c^2 - 256*a^5*b^5*c^3 +
 256*a^6*b^3*c^4)*d^4)*(c^8/((b^34 - 68*a*b^32*c + 2176*a^2*b^30*c^2 - 4352
0*a^3*b^28*c^3 + 609280*a^4*b^26*c^4 - 6336512*a^5*b^24*c^5 + 50692096*a^6*
b^22*c^6 - 318636032*a^7*b^20*c^7 + 1593180160*a^8*b^18*c^8 - 6372720640*a^
9*b^16*c^9 + 20392706048*a^10*b^14*c^10 - 51908706304*a^11*b^12*c^11 + 1038
17412608*a^12*b^10*c^12 - 159719096320*a^13*b^8*c^13 + 182536110080*a^14*b^
6*c^14 - 146028888064*a^15*b^4*c^15 + 73014444032*a^16*b^2*c^16 - 171798691
84*a^17*c^17)*d^14))^(1/4)*arctan((sqrt(2*c^13*d*x + b*c^12*d + (b^18*c^8 -
 36*a*b^16*c^9 + 576*a^2*b^14*c^10 - 5376*a^3*b^12*c^11 + 32256*a^4*b^10*c^
12 - 129024*a^5*b^8*c^13 + 344064*a^6*b^6*c^14 - 589824*a^7*b^4*c^15 + 5898
24*a^8*b^2*c^16 - 262144*a^9*c^17)*d^8*sqrt(c^8/((b^34 - 68*a*b^32*c + 2176
*a^2*b^30*c^2 - 43520*a^3*b^28*c^3 + 609280*a^4*b^26*c^4 - 6336512*a^5*b^24
*c^5 + 50692096*a^6*b^22*c^6 - 318636032*a^7*b^20*c^7 + 1593180160*a^8*b^18
*c^8 - 6372720640*a^9*b^16*c^9 + 20392706048*a^10*b^14*c^10 - 51908706304*a
```

^11*b^12*c^11 + 103817412608*a^12*b^10*c^12 - 159719096320*a^13*b^8*c^13 + 182536110080*a^14*b^6*c^14 - 146028888064*a^15*b^4*c^15 + 73014444032*a^16*b^2*c^16 - 17179869184*a^17*c^17)*d^14)))*(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4)*d^3*(c^8/((b^34 - 68*a*b^32*c + 2176*a^2*b^30*c^2 - 43520*a^3*b^28*c^3 + 609280*a^4*b^26*c^4 - 6336512*a^5*b^24*c^5 + 50692096*a^6*b^22*c^6 - 318636032*a^7*b^20*c^7 + 1593180160*a^8*b^18*c^8 - 6372720640*a^9*b^16*c^9 + 20392706048*a^10*b^14*c^10 - 51908706304*a^11*b^12*c^11 + 103817412608*a^12*b^10*c^12 - 159719096320*a^13*b^8*c^13 + 182536110080*a^14*b^6*c^14 - 146028888064*a^15*b^4*c^15 + 73014444032*a^16*b^2*c^16 - 17179869184*a^17*c^17)*d^14))^(1/4) - (b^8*c^6 - 16*a*b^6*c^7 + 96*a^2*b^4*c^8 - 256*a^3*b^2*c^9 + 256*a^4*c^10)*sqrt(2*c*d*x + b*d)*d^3*(c^8/((b^34 - 68*a*b^32*c + 2176*a^2*b^30*c^2 - 43520*a^3*b^28*c^3 + 609280*a^4*b^26*c^4 - 6336512*a^5*b^24*c^5 + 50692096*a^6*b^22*c^6 - 318636032*a^7*b^20*c^7 + 1593180160*a^8*b^18*c^8 - 6372720640*a^9*b^16*c^9 + 20392706048*a^10*b^14*c^10 - 51908706304*a^11*b^12*c^11 + 103817412608*a^12*b^10*c^12 - 159719096320*a^13*b^8*c^13 + 182536110080*a^14*b^6*c^14 - 146028888064*a^15*b^4*c^15 + 73014444032*a^16*b^2*c^16 - 17179869184*a^17*c^17)*d^14))^(1/4))/c^8) + 585*(8*(b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8 + 256*a^4*c^9)*d^4*x^7 + 28*(b^9*c^4 - 16*a*b^7*c^5 + 96*a^2*b^5*c^6 - 256*a^3*b^3*c^7 + 256*a^4*b*c^8)*d^4*x^6 + 2*(19*b^10*c^3 - 296*a*b^8*c^4 + 1696*a^2*b^6*c^5 - 4096*a^3*b^4*c^6 + 2816*a^4*b^2*c^7 + 2048*a^5*c^8)*d^4*x^5 + 5*(5*b^11*c^2 - 72*a*b^9*c^3 + 352*a^2*b^7*c^4 - 512*a^3*b^5*c^5 - 768*a^4*b^3*c^6 + 2048*a^5*b*c^7)*d^4*x^4 + 4*(2*b^12*c - 23*a*b^10*c^2 + 50*a^2*b^8*c^3 + 320*a^3*b^6*c^4 - 1600*a^4*b^4*c^5 + 1792*a^5*b^2*c^6 + 512*a^6*c^7)*d^4*x^3 + (b^13 - 2*a*b^11*c - 116*a^2*b^9*c^2 + 896*a^3*b^7*c^3 - 2176*a^4*b^5*c^4 + 512*a^5*b^3*c^5 + 3072*a^6*b*c^6)*d^4*x^2 + 2*(a*b^12 - 13*a^2*b^10*c + 48*a^3*b^8*c^2 + 32*a^4*b^6*c^3 - 512*a^5*b^4*c^4 + 768*a^6*b^2*c^5)*d^4*x + (a^2*b^11 - 16*a^3*b^9*c + 96*a^4*b^7*c^2 - 256*a^5*b^5*c^3 + 256*a^6*b^3*c^4)*d^4)*(c^8/((b^34 - 68*a*b^32*c + 2176*a^2*b^30*c^2 - 43520*a^3*b^28*c^3 + 609280*a^4*b^26*c^4 - 6336512*a^5*b^24*c^5 + 50692096*a^6*b^22*c^6 - 318636032*a^7*b^20*c^7 + 1593180160*a^8*b^18*c^8 - 6372720640*a^9*b^16*c^9 + 20392706048*a^10*b^14*c^10 - 51908706304*a^11*b^12*c^11 + 103817412608*a^12*b^10*c^12 - 159719096320*a^13*b^8*c^13 + 182536110080*a^14*b^6*c^14 - 146028888064*a^15*b^4*c^15 + 73014444032*a^16*b^2*c^16 - 17179869184*a^17*c^17)*d^14))^(1/4)*log(1601613*(b^26 - 52*a*b^24*c + 1248*a^2*b^22*c^2 - 18304*a^3*b^20*c^3 + 183040*a^4*b^18*c^4 - 1317888*a^5*b^16*c^5 + 7028736*a^6*b^14*c^6 - 28114944*a^7*b^12*c^7 + 84344832*a^8*b^10*c^8 - 187432960*a^9*b^8*c^9 + 299892736*a^10*b^6*c^10 - 327155712*a^11*b^4*c^11 + 218103808*a^12*b^2*c^12 - 67108864*a^13*c^13)*d^11*(c^8/((b^34 - 68*a*b^32*c + 2176*a^2*b^30*c^2 - 43520*a^3*b^28*c^3 + 609280*a^4*b^26*c^4 - 6336512*a^5*b^24*c^5 + 50692096*a^6*b^22*c^6 - 318636032*a^7*b^20*c^7 + 1593180160*a^8*b^18*c^8 - 6372720640*a^9*b^16*c^9 + 20392706048*a^10*b^14*c^10 - 51908706304*a^11*b^12*c^11 + 103817412608*a^12*b^10*c^12 - 159719096320*a^13*b^8*c^13 + 182536110080*a^14*b^6*c^14 - 146028888064*a^15*b^4*c^15 + 73014444032*a^16*b^2*c^16 - 17179869184*a^17*c^17)*d^14))^(3/4) + 1601613*sqrt(2*c*d*x + b*d)*c^6) - 585*(8*(b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8 + 256*a^4*c^9)*d^4*x^7 + 28*(b^9*c^4 - 16*a*b^7*c^5 + 96*a^2*b^5*c^6 - 256*a^3*b^3*c^7 + 256*a^4*b*c^8)*d^4*x^6 + 2*(19*b^10*c^3 - 296*a*b^8*c^4 + 1696*a^2*b^6*c^5 - 4096*a^3*b^4*c^6 + 2816*a^4*b^2*c^7 + 2048*a^5*c^8)*d^4*x^5 + 5*(5*b^11*c^2 - 72*a*b^9*c^3 + 352*a^2*b^7*c^4 - 512*a^3*b^5*c^5 - 768*a^4*b^3*c^6 + 2048*a^5*b*c^7)*d^4*x^4 + 4*(2*b^12*c - 23*a*b^10*c^2 + 50*a^2*b^8*c^3 + 320*a^3*b^6*c^4 - 1600*a^4*b^4*c^5 + 1792*a^5*b^2*c^6 + 512*a^6*c^7)*d^4*x^3 + (b^13 - 2*a*b^11*c - 116*a^2*b^9*c^2 + 896*a^3*b^7*c^3 - 2176*a^4*b^5*c^4 + 512*a^5*b^3*c^5 + 3072*a^6*b*c^6)*d^4*x^2 + 2*(a*b^12 - 13*a^2*b^10*c + 48*a^3*b^8*c^2 + 32*a^4*b^6*c^3 - 512*a^5*b^4*c^4 + 768*a^6*b^2*c^5)*d^4*x + (a^2*b^11 - 16*a^3*b^9*c + 96*a^4*b^7*c^2 - 256*a^5*b^5*c^3 + 256*a^6*b^3*c^4)*d^4)*(c^8/((b^34 - 68*a*b^32*c + 2176*a^2*b^30*c^2 - 43520*a^3*b^28*c^3 + 609280*a^4*b^26*c^4 - 6336512*a^5*b^24*c^5 + 50692096*a^6*b^22*c^6 - 318636032*a^7*b^20*c^7 + 1593180160*a^8*b^18*c^8 - 6372720640*a

```
^9*b^16*c^9 + 20392706048*a^10*b^14*c^10 - 51908706304*a^11*b^12*c^11 + 103
817412608*a^12*b^10*c^12 - 159719096320*a^13*b^8*c^13 + 182536110080*a^14*b
^6*c^14 - 146028888064*a^15*b^4*c^15 + 73014444032*a^16*b^2*c^16 - 17179869
184*a^17*c^17)*d^14))^(1/4)*log(-1601613*(b^26 - 52*a*b^24*c + 1248*a^2*b^2
2*c^2 - 18304*a^3*b^20*c^3 + 183040*a^4*b^18*c^4 - 1317888*a^5*b^16*c^5 + 7
028736*a^6*b^14*c^6 - 28114944*a^7*b^12*c^7 + 84344832*a^8*b^10*c^8 - 18743
2960*a^9*b^8*c^9 + 299892736*a^10*b^6*c^10 - 327155712*a^11*b^4*c^11 + 2181
03808*a^12*b^2*c^12 - 67108864*a^13*c^13)*d^11*(c^8/((b^34 - 68*a*b^32*c +
2176*a^2*b^30*c^2 - 43520*a^3*b^28*c^3 + 609280*a^4*b^26*c^4 - 6336512*a^5*
b^24*c^5 + 50692096*a^6*b^22*c^6 - 318636032*a^7*b^20*c^7 + 1593180160*a^8*
b^18*c^8 - 6372720640*a^9*b^16*c^9 + 20392706048*a^10*b^14*c^10 - 519087063
04*a^11*b^12*c^11 + 103817412608*a^12*b^10*c^12 - 159719096320*a^13*b^8*c^1
3 + 182536110080*a^14*b^6*c^14 - 146028888064*a^15*b^4*c^15 + 73014444032*a
^16*b^2*c^16 - 17179869184*a^17*c^17)*d^14))^(3/4) + 1601613*sqrt(2*c*d*x +
 b*d)*c^6) - (9360*c^6*x^6 + 28080*b*c^5*x^5 - 5*b^6 + 125*a*b^4*c + 2048*a
^2*b^2*c^2 - 512*a^3*c^3 + 2808*(11*b^2*c^4 + 6*a*c^5)*x^4 + 3744*(4*b^3*c^
3 + 9*a*b*c^4)*x^3 + 13*(221*b^4*c^2 + 1688*a*b^2*c^3 + 512*a^2*c^4)*x^2 +
13*(5*b^5*c + 392*a*b^3*c^2 + 512*a^2*b*c^3)*x)*sqrt(2*c*d*x + b*d))/(8*(b^
8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8 + 256*a^4*c^9)*d^4*
x^7 + 28*(b^9*c^4 - 16*a*b^7*c^5 + 96*a^2*b^5*c^6 - 256*a^3*b^3*c^7 + 256*a
^4*b*c^8)*d^4*x^6 + 2*(19*b^10*c^3 - 296*a*b^8*c^4 + 1696*a^2*b^6*c^5 - 409
6*a^3*b^4*c^6 + 2816*a^4*b^2*c^7 + 2048*a^5*c^8)*d^4*x^5 + 5*(5*b^11*c^2 -
72*a*b^9*c^3 + 352*a^2*b^7*c^4 - 512*a^3*b^5*c^5 - 768*a^4*b^3*c^6 + 2048*a
^5*b*c^7)*d^4*x^4 + 4*(2*b^12*c - 23*a*b^10*c^2 + 50*a^2*b^8*c^3 + 320*a^3*
b^6*c^4 - 1600*a^4*b^4*c^5 + 1792*a^5*b^2*c^6 + 512*a^6*c^7)*d^4*x^3 + (b^1
3 - 2*a*b^11*c - 116*a^2*b^9*c^2 + 896*a^3*b^7*c^3 - 2176*a^4*b^5*c^4 + 512
*a^5*b^3*c^5 + 3072*a^6*b*c^6)*d^4*x^2 + 2*(a*b^12 - 13*a^2*b^10*c + 48*a^3
*b^8*c^2 + 32*a^4*b^6*c^3 - 512*a^5*b^4*c^4 + 768*a^6*b^2*c^5)*d^4*x + (a^2
*b^11 - 16*a^3*b^9*c + 96*a^4*b^7*c^2 - 256*a^5*b^5*c^3 + 256*a^6*b^3*c^4)*
d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**(7/2)/(c*x**2+b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32044, size = 1293, normalized size = 5.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] -117*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c^2*arctan(1/2*sqrt(2)*(sqrt(2)*(-b^2*d^2
 + 4*a*c*d^2)^(1/4) + 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4))/
(sqrt(2)*b^10*d^5 - 20*sqrt(2)*a*b^8*c*d^5 + 160*sqrt(2)*a^2*b^6*c^2*d^5 -
640*sqrt(2)*a^3*b^4*c^3*d^5 + 1280*sqrt(2)*a^4*b^2*c^4*d^5 - 1024*sqrt(2)*a
^5*c^5*d^5) - 117*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c^2*arctan(-1/2*sqrt(2)*(sqr
```

t(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4) - 2*sqrt(2*c*d*x + b*d))/(-b^2*d^2 + 4*a*c*d^2)^(1/4))/(sqrt(2)*b^10*d^5 - 20*sqrt(2)*a*b^8*c*d^5 + 160*sqrt(2)*a^2*b^6*c^2*d^5 - 640*sqrt(2)*a^3*b^4*c^3*d^5 + 1280*sqrt(2)*a^4*b^2*c^4*d^5 - 1024*sqrt(2)*a^5*c^5*d^5) + 117/2*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c^2*log(2*c*d*x + b*d + sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2))/(sqrt(2)*b^10*d^5 - 20*sqrt(2)*a*b^8*c*d^5 + 160*sqrt(2)*a^2*b^6*c^2*d^5 - 640*sqrt(2)*a^3*b^4*c^3*d^5 + 1280*sqrt(2)*a^4*b^2*c^4*d^5 - 1024*sqrt(2)*a^5*c^5*d^5) - 117/2*(-b^2*d^2 + 4*a*c*d^2)^(3/4)*c^2*log(2*c*d*x + b*d - sqrt(2)*(-b^2*d^2 + 4*a*c*d^2)^(1/4)*sqrt(2*c*d*x + b*d) + sqrt(-b^2*d^2 + 4*a*c*d^2))/(sqrt(2)*b^10*d^5 - 20*sqrt(2)*a*b^8*c*d^5 + 160*sqrt(2)*a^2*b^6*c^2*d^5 - 640*sqrt(2)*a^3*b^4*c^3*d^5 + 1280*sqrt(2)*a^4*b^2*c^4*d^5 - 1024*sqrt(2)*a^5*c^5*d^5) - 2*(25*(2*c*d*x + b*d)^(3/2)*b^2*c^2*d^2 - 100*(2*c*d*x + b*d)^(3/2)*a*c^3*d^2 - 21*(2*c*d*x + b*d)^(7/2)*c^2)/((b^8*d^3 - 16*a*b^6*c*d^3 + 96*a^2*b^4*c^2*d^3 - 256*a^3*b^2*c^3*d^3 + 256*a^4*c^4*d^3)*(b^2*d^2 - 4*a*c*d^2 - (2*c*d*x + b*d)^2)^2) + 64/5*(b^2*c^2*d^2 - 4*a*c^3*d^2 + 15*(2*c*d*x + b*d)^2*c^2)/((b^8*d^3 - 16*a*b^6*c*d^3 + 96*a^2*b^4*c^2*d^3 - 256*a^3*b^2*c^3*d^3 + 256*a^4*c^4*d^3)*(2*c*d*x + b*d)^(5/2))

3.1320 $\int \frac{(1+2x)^{7/2}}{1+x+x^2}\,dx$

Optimal. Leaf size=183

$$\frac{4}{5}(2x+1)^{5/2}-12\sqrt{2x+1}-\frac{3\sqrt[4]{3}\log\left(2x-\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+\sqrt{3}+1\right)}{\sqrt{2}}+\frac{3\sqrt[4]{3}\log\left(2x+\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+\sqrt{3}+1\right)}{\sqrt{2}}-3\sqrt{2}\sqrt[4]{}$$

```
[Out] -12*Sqrt[1 + 2*x] + (4*(1 + 2*x)^(5/2))/5 - 3*Sqrt[2]*3^(1/4)*ArcTan[1 - (S
qrt[2]*Sqrt[1 + 2*x])/3^(1/4)] + 3*Sqrt[2]*3^(1/4)*ArcTan[1 + (Sqrt[2]*Sqrt
[1 + 2*x])/3^(1/4)] - (3*3^(1/4)*Log[1 + Sqrt[3] + 2*x - Sqrt[2]*3^(1/4)*Sq
rt[1 + 2*x]])/Sqrt[2] + (3*3^(1/4)*Log[1 + Sqrt[3] + 2*x + Sqrt[2]*3^(1/4)*
Sqrt[1 + 2*x]])/Sqrt[2]
```

Rubi [A] time = 0.17134, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {692, 694, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{4}{5}(2x+1)^{5/2}-12\sqrt{2x+1}-\frac{3\sqrt[4]{3}\log\left(2x-\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+\sqrt{3}+1\right)}{\sqrt{2}}+\frac{3\sqrt[4]{3}\log\left(2x+\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+\sqrt{3}+1\right)}{\sqrt{2}}-3\sqrt{2}\sqrt[4]{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^(7/2)/(1 + x + x^2),x]
```


```
[Out] -12*Sqrt[1 + 2*x] + (4*(1 + 2*x)^(5/2))/5 - 3*Sqrt[2]*3^(1/4)*ArcTan[1 - (S
qrt[2]*Sqrt[1 + 2*x])/3^(1/4)] + 3*Sqrt[2]*3^(1/4)*ArcTan[1 + (Sqrt[2]*Sqrt
[1 + 2*x])/3^(1/4)] - (3*3^(1/4)*Log[1 + Sqrt[3] + 2*x - Sqrt[2]*3^(1/4)*Sq
rt[1 + 2*x]])/Sqrt[2] + (3*3^(1/4)*Log[1 + Sqrt[3] + 2*x + Sqrt[2]*3^(1/4)*
Sqrt[1 + 2*x]])/Sqrt[2]
```


Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(1+2x)^{7/2}}{1+x+x^2}\,dx = \frac{4}{5}(1+2x)^{5/2} - 3\int \frac{(1+2x)^{3/2}}{1+x+x^2}\,dx$$
$$= -12\sqrt{1+2x} + \frac{4}{5}(1+2x)^{5/2} + 9\int \frac{1}{\sqrt{1+2x}\left(1+x+x^2\right)}\,dx$$
$$= -12\sqrt{1+2x} + \frac{4}{5}(1+2x)^{5/2} + \frac{9}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(\frac{3}{4}+\frac{x^2}{4}\right)}\,dx, x, 1+2x\right)$$
$$= -12\sqrt{1+2x} + \frac{4}{5}(1+2x)^{5/2} + 9\,\text{Subst}\left(\int \frac{1}{\frac{3}{4}+\frac{x^4}{4}}\,dx, x, \sqrt{1+2x}\right)$$
$$= -12\sqrt{1+2x} + \frac{4}{5}(1+2x)^{5/2} + \frac{1}{2}\left(3\sqrt{3}\right)\text{Subst}\left(\int \frac{\sqrt{3}-x^2}{\frac{3}{4}+\frac{x^4}{4}}\,dx, x, \sqrt{1+2x}\right) + \frac{1}{2}\left(3\sqrt{3}\right)\text{Subst}\left(\int \frac{\sqrt{3}\ \ldots}{\frac{3}{4}+\ldots}\right.$$
$$= -12\sqrt{1+2x} + \frac{4}{5}(1+2x)^{5/2} - \frac{\left(3\sqrt[4]{3}\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{3}+2x}{-\sqrt{3}-\sqrt{2}\sqrt[4]{3}x-x^2}\,dx, x, \sqrt{1+2x}\right)}{\sqrt{2}} - \frac{\left(3\sqrt[4]{3}\right)\text{Subst}\left(\int \frac{\sqrt{\ldots}}{-\sqrt{3}+\ldots}\right.}{}$$
$$= -12\sqrt{1+2x} + \frac{4}{5}(1+2x)^{5/2} - \frac{3\sqrt[4]{3}\log\left(1+\sqrt{3}+2x-\sqrt{2}\sqrt[4]{3}\sqrt{1+2x}\right)}{\sqrt{2}} + \frac{3\sqrt[4]{3}\log\left(1+\sqrt{3}+2x+\sqrt{2}\ldots\right.}{\sqrt{2}}$$
$$= -12\sqrt{1+2x} + \frac{4}{5}(1+2x)^{5/2} - 3\sqrt{2}\sqrt[4]{3}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{1+2x}}{\sqrt[4]{3}}\right) + 3\sqrt{2}\sqrt[4]{3}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{1+2x}}{\sqrt[4]{3}}\right) - \frac{3\ldots}{\ldots}$$

Mathematica [A] time = 0.0929894, size = 182, normalized size = 0.99

$$\frac{16}{5}\sqrt{2x+1}x^2 + \frac{16}{5}\sqrt{2x+1}x - \frac{56}{5}\sqrt{2x+1} - \frac{3\sqrt[4]{3}\log\left(2x-\sqrt[4]{3}\sqrt{4x+2}+\sqrt{3}+1\right)}{\sqrt{2}} + \frac{3\sqrt[4]{3}\log\left(2x+\sqrt[4]{3}\sqrt{4x+2}+\sqrt{3}+\ldots\right.}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^(7/2)/(1 + x + x^2),x]
```

```
[Out] (-56*Sqrt[1 + 2*x])/5 + (16*x*Sqrt[1 + 2*x])/5 + (16*x^2*Sqrt[1 + 2*x])/5 -
 3*Sqrt[2]*3^(1/4)*ArcTan[1 - Sqrt[2 + 4*x]/3^(1/4)] + 3*Sqrt[2]*3^(1/4)*Ar
cTan[1 + Sqrt[2 + 4*x]/3^(1/4)] - (3*3^(1/4)*Log[1 + Sqrt[3] + 2*x - 3^(1/4
)*Sqrt[2 + 4*x]])/Sqrt[2] + (3*3^(1/4)*Log[1 + Sqrt[3] + 2*x + 3^(1/4)*Sqrt
[2 + 4*x]])/Sqrt[2]
```

Maple [A] time = 0.046, size = 129, normalized size = 0.7

$$\frac{4}{5}(1+2x)^{\frac{5}{2}} - 12\sqrt{1+2x} + 3\sqrt[4]{3}\arctan\left(1+1/3\sqrt{2}\sqrt{1+2x}3^{3/4}\right)\sqrt{2} + 3\sqrt[4]{3}\arctan\left(-1+1/3\sqrt{2}\sqrt{1+2x}3^{3/4}\right)\sqrt{2} + \frac{3}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^(7/2)/(x^2+x+1),x)
```

```
[Out] 4/5*(1+2*x)^(5/2)-12*(1+2*x)^(1/2)+3*3^(1/4)*arctan(1+1/3*2^(1/2)*(1+2*x)^(
1/2)*3^(3/4))*2^(1/2)+3*3^(1/4)*arctan(-1+1/3*2^(1/2)*(1+2*x)^(1/2)*3^(3/4)
)*2^(1/2)+3/2*3^(1/4)*2^(1/2)*ln((1+2*x+3^(1/2)+3^(1/4)*2^(1/2)*(1+2*x)^(1/
2))/(1+2*x+3^(1/2)-3^(1/4)*2^(1/2)*(1+2*x)^(1/2)))
```

Maxima [A] time = 1.80035, size = 203, normalized size = 1.11

$$\frac{4}{5}\,(2\,x+1)^{\frac{5}{2}}+3\cdot 3^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}+2\,\sqrt{2\,x+1}\right)\right)+3\cdot 3^{\frac{1}{4}}\sqrt{2}\arctan\left(-\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}-2\,\sqrt{2\,x+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(7/2)/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] 4/5*(2*x + 1)^(5/2) + 3*3^(1/4)*sqrt(2)*arctan(1/6*3^(3/4)*sqrt(2)*(3^(1/4)
*sqrt(2) + 2*sqrt(2*x + 1))) + 3*3^(1/4)*sqrt(2)*arctan(-1/6*3^(3/4)*sqrt(2
)*(3^(1/4)*sqrt(2) - 2*sqrt(2*x + 1))) + 3/2*3^(1/4)*sqrt(2)*log(3^(1/4)*sq
rt(2)*sqrt(2*x + 1) + 2*x + sqrt(3) + 1) - 3/2*3^(1/4)*sqrt(2)*log(-3^(1/4)
*sqrt(2)*sqrt(2*x + 1) + 2*x + sqrt(3) + 1) - 12*sqrt(2*x + 1)
```

Fricas [A] time = 1.70997, size = 651, normalized size = 3.56

$$-6\cdot 3^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{3}\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{3^{\frac{1}{4}}\sqrt{2}\sqrt{2\,x+1}+2\,x+\sqrt{3}+1}-\frac{1}{3}\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{2\,x+1}-1\right)-6\cdot 3^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{3}\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(7/2)/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] -6*3^(1/4)*sqrt(2)*arctan(1/3*3^(3/4)*sqrt(2)*sqrt(3^(1/4)*sqrt(2)*sqrt(2*x
 + 1) + 2*x + sqrt(3) + 1) - 1/3*3^(3/4)*sqrt(2)*sqrt(2*x + 1) - 1) - 6*3^(
1/4)*sqrt(2)*arctan(1/3*3^(3/4)*sqrt(2)*sqrt(-3^(1/4)*sqrt(2)*sqrt(2*x + 1)
 + 2*x + sqrt(3) + 1) - 1/3*3^(3/4)*sqrt(2)*sqrt(2*x + 1) + 1) + 3/2*3^(1/4
)*sqrt(2)*log(3^(1/4)*sqrt(2)*sqrt(2*x + 1) + 2*x + sqrt(3) + 1) - 3/2*3^(1
/4)*sqrt(2)*log(-3^(1/4)*sqrt(2)*sqrt(2*x + 1) + 2*x + sqrt(3) + 1) + 8/5*(
2*x^2 + 2*x - 7)*sqrt(2*x + 1)
```

Sympy [A] time = 54.7249, size = 180, normalized size = 0.98

$$\frac{4\left(2x+1\right)^{\frac{5}{2}}}{5}-12\sqrt{2x+1}-\frac{3\sqrt{2}\sqrt[4]{3}\log\left(2x-\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+1+\sqrt{3}\right)}{2}+\frac{3\sqrt{2}\sqrt[4]{3}\log\left(2x+\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+1+\sqrt{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**(7/2)/(x**2+x+1),x)
```

```
[Out] 4*(2*x + 1)**(5/2)/5 - 12*sqrt(2*x + 1) - 3*sqrt(2)*3**(1/4)*log(2*x - sqrt
(2)*3**(1/4)*sqrt(2*x + 1) + 1 + sqrt(3))/2 + 3*sqrt(2)*3**(1/4)*log(2*x +
sqrt(2)*3**(1/4)*sqrt(2*x + 1) + 1 + sqrt(3))/2 + 3*sqrt(2)*3**(1/4)*atan(s
qrt(2)*3**(3/4)*sqrt(2*x + 1)/3 - 1) + 3*sqrt(2)*3**(1/4)*atan(sqrt(2)*3**(
3/4)*sqrt(2*x + 1)/3 + 1)
```

Giac [A] time = 1.18237, size = 186, normalized size = 1.02

$$\frac{4}{5}\,(2\,x+1)^{\frac{5}{2}}+3\cdot 12^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}+2\,\sqrt{2\,x+1}\right)\right)+3\cdot 12^{\frac{1}{4}}\arctan\left(-\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}-2\,\sqrt{2\,x+1}\right)\right)+\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(7/2)/(x^2+x+1),x, algorithm="giac")
```

```
[Out] 4/5*(2*x + 1)^(5/2) + 3*12^(1/4)*arctan(1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqrt(2
) + 2*sqrt(2*x + 1))) + 3*12^(1/4)*arctan(-1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqr
t(2) - 2*sqrt(2*x + 1))) + 3/2*12^(1/4)*log(3^(1/4)*sqrt(2)*sqrt(2*x + 1) +
 2*x + sqrt(3) + 1) - 3/2*12^(1/4)*log(-3^(1/4)*sqrt(2)*sqrt(2*x + 1) + 2*x
 + sqrt(3) + 1) - 12*sqrt(2*x + 1)
```

3.1321 $\int \frac{(1+2x)^{5/2}}{1+x+x^2}\,dx$

Optimal. Leaf size=170

$$\frac{4}{3}(2x+1)^{3/2} - \frac{3^{3/4}\log\left(2x-\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+\sqrt{3}+1\right)}{\sqrt{2}} + \frac{3^{3/4}\log\left(2x+\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+\sqrt{3}+1\right)}{\sqrt{2}} + \sqrt{2}3^{3/4}\tan^{-1}\left(1-\right.$$

```
[Out] (4*(1 + 2*x)^(3/2))/3 + Sqrt[2]*3^(3/4)*ArcTan[1 - (Sqrt[2]*Sqrt[1 + 2*x])/
3^(1/4)] - Sqrt[2]*3^(3/4)*ArcTan[1 + (Sqrt[2]*Sqrt[1 + 2*x])/3^(1/4)] - (3
^(3/4)*Log[1 + Sqrt[3] + 2*x - Sqrt[2]*3^(1/4)*Sqrt[1 + 2*x]])/Sqrt[2] + (3
^(3/4)*Log[1 + Sqrt[3] + 2*x + Sqrt[2]*3^(1/4)*Sqrt[1 + 2*x]])/Sqrt[2]
```

Rubi [A] time = 0.136914, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {692, 694, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{4}{3}(2x+1)^{3/2} - \frac{3^{3/4}\log\left(2x-\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+\sqrt{3}+1\right)}{\sqrt{2}} + \frac{3^{3/4}\log\left(2x+\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+\sqrt{3}+1\right)}{\sqrt{2}} + \sqrt{2}3^{3/4}\tan^{-1}\left(1-\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^(5/2)/(1 + x + x^2),x]
```


```
[Out] (4*(1 + 2*x)^(3/2))/3 + Sqrt[2]*3^(3/4)*ArcTan[1 - (Sqrt[2]*Sqrt[1 + 2*x])/
3^(1/4)] - Sqrt[2]*3^(3/4)*ArcTan[1 + (Sqrt[2]*Sqrt[1 + 2*x])/3^(1/4)] - (3
^(3/4)*Log[1 + Sqrt[3] + 2*x - Sqrt[2]*3^(1/4)*Sqrt[1 + 2*x]])/Sqrt[2] + (3
^(3/4)*Log[1 + Sqrt[3] + 2*x + Sqrt[2]*3^(1/4)*Sqrt[1 + 2*x]])/Sqrt[2]
```


Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+2x)^{5/2}}{1+x+x^2}\,dx &= \frac{4}{3}(1+2x)^{3/2} - 3\int \frac{\sqrt{1+2x}}{1+x+x^2}\,dx \\
&= \frac{4}{3}(1+2x)^{3/2} - \frac{3}{2}\,\text{Subst}\left(\int \frac{\sqrt{x}}{\frac{3}{4}+\frac{x^2}{4}}\,dx, x, 1+2x\right) \\
&= \frac{4}{3}(1+2x)^{3/2} - 3\,\text{Subst}\left(\int \frac{x^2}{\frac{3}{4}+\frac{x^4}{4}}\,dx, x, \sqrt{1+2x}\right) \\
&= \frac{4}{3}(1+2x)^{3/2} + \frac{3}{2}\,\text{Subst}\left(\int \frac{\sqrt{3}-x^2}{\frac{3}{4}+\frac{x^4}{4}}\,dx, x, \sqrt{1+2x}\right) - \frac{3}{2}\,\text{Subst}\left(\int \frac{\sqrt{3}+x^2}{\frac{3}{4}+\frac{x^4}{4}}\,dx, x, \sqrt{1+2x}\right) \\
&= \frac{4}{3}(1+2x)^{3/2} - 3\,\text{Subst}\left(\int \frac{1}{\sqrt{3}-\sqrt{2}\sqrt[4]{3}x+x^2}\,dx, x, \sqrt{1+2x}\right) - 3\,\text{Subst}\left(\int \frac{1}{\sqrt{3}+\sqrt{2}\sqrt[4]{3}x+x^2}\,d\right. \\
&= \frac{4}{3}(1+2x)^{3/2} - \frac{3^{3/4}\log\left(1+\sqrt{3}+2x-\sqrt{2}\sqrt[4]{3}\sqrt{1+2x}\right)}{\sqrt{2}} + \frac{3^{3/4}\log\left(1+\sqrt{3}+2x+\sqrt{2}\sqrt[4]{3}\sqrt{1+2x}\right)}{\sqrt{2}} \\
&= \frac{4}{3}(1+2x)^{3/2} + \sqrt{2}3^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{1+2x}}{\sqrt[4]{3}}\right) - \sqrt{2}3^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{1+2x}}{\sqrt[4]{3}}\right) - \frac{3^{3/4}\log\left(1+\sqrt{\right.}}{}
\end{aligned}
$$

Mathematica [C] time = 0.0104773, size = 34, normalized size = 0.2

$$
-\frac{4}{3}(2x+1)^{3/2}\left({}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\frac{1}{3}(2x+1)^2\right)-1\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^(5/2)/(1 + x + x^2),x]
```

```
[Out] (-4*(1 + 2*x)^(3/2)*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -(1 + 2*x)^2/3]))/
3
```

Maple [A] time = 0.043, size = 120, normalized size = 0.7

$$
\frac{4}{3}(1+2x)^{\frac{3}{2}} - 3^{\frac{3}{4}}\arctan\left(1+\frac{\sqrt{2}3^{\frac{3}{4}}}{3}\sqrt{1+2x}\right)\sqrt{2} - 3^{\frac{3}{4}}\arctan\left(-1+\frac{\sqrt{2}3^{\frac{3}{4}}}{3}\sqrt{1+2x}\right)\sqrt{2} - \frac{\sqrt{2}3^{\frac{3}{4}}}{2}\ln\left(\left(1+2x+\sqrt{3}-\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^(5/2)/(x^2+x+1),x)
```

```
[Out] 4/3*(1+2*x)^(3/2)-3^(3/4)*arctan(1+1/3*2^(1/2)*(1+2*x)^(1/2)*3^(3/4))*2^(1/
2)-3^(3/4)*arctan(-1+1/3*2^(1/2)*(1+2*x)^(1/2)*3^(3/4))*2^(1/2)-1/2*2^(1/2)
*3^(3/4)*ln((1+2*x+3^(1/2)-3^(1/4)*2^(1/2)*(1+2*x)^(1/2))/(1+2*x+3^(1/2)+3^
(1/4)*2^(1/2)*(1+2*x)^(1/2)))
```

Maxima [A] time = 1.91176, size = 190, normalized size = 1.12

$$
-3^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}+2\sqrt{2x+1}\right)\right) - 3^{\frac{3}{4}}\sqrt{2}\arctan\left(-\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}-2\sqrt{2x+1}\right)\right) + \frac{1}{2}\cdot 3^{\frac{3}{4}}\sqrt{2}\log\left(3^{\frac{1}{\cdot}}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(5/2)/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] -3^(3/4)*sqrt(2)*arctan(1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqrt(2) + 2*sqrt(2*x +
 1))) - 3^(3/4)*sqrt(2)*arctan(-1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqrt(2) - 2*sq
rt(2*x + 1))) + 1/2*3^(3/4)*sqrt(2)*log(3^(1/4)*sqrt(2)*sqrt(2*x + 1) + 2*x
 + sqrt(3) + 1) - 1/2*3^(3/4)*sqrt(2)*log(-3^(1/4)*sqrt(2)*sqrt(2*x + 1) +
2*x + sqrt(3) + 1) + 4/3*(2*x + 1)^(3/2)
```

Fricas [A] time = 1.67823, size = 682, normalized size = 4.01

$$2 \cdot 27^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{1}{9} \cdot 27^{\frac{1}{4}} \sqrt{2} \sqrt{27^{\frac{3}{4}} \sqrt{2} \sqrt{2x+1} + 18x + 9\sqrt{3} + 9} - \frac{1}{3} \cdot 27^{\frac{1}{4}} \sqrt{2} \sqrt{2x+1} - 1\right) + 2 \cdot 27^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{1}{27} \cdot 2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(5/2)/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] 2*27^(1/4)*sqrt(2)*arctan(1/9*27^(1/4)*sqrt(2)*sqrt(27^(3/4)*sqrt(2)*sqrt(2
*x + 1) + 18*x + 9*sqrt(3) + 9) - 1/3*27^(1/4)*sqrt(2)*sqrt(2*x + 1) - 1) +
 2*27^(1/4)*sqrt(2)*arctan(1/27*27^(1/4)*sqrt(2)*sqrt(-9*27^(3/4)*sqrt(2)*s
qrt(2*x + 1) + 162*x + 81*sqrt(3) + 81) - 1/3*27^(1/4)*sqrt(2)*sqrt(2*x + 1
) + 1) + 1/2*27^(1/4)*sqrt(2)*log(9*27^(3/4)*sqrt(2)*sqrt(2*x + 1) + 162*x
+ 81*sqrt(3) + 81) - 1/2*27^(1/4)*sqrt(2)*log(-9*27^(3/4)*sqrt(2)*sqrt(2*x
+ 1) + 162*x + 81*sqrt(3) + 81) + 4/3*(2*x + 1)^(3/2)
```

Sympy [A] time = 32.9937, size = 163, normalized size = 0.96

$$\frac{4(2x+1)^{\frac{3}{2}}}{3} - \frac{\sqrt{2} \cdot 3^{\frac{3}{4}} \log\left(2x - \sqrt{2}\sqrt[4]{3}\sqrt{2x+1} + 1 + \sqrt{3}\right)}{2} + \frac{\sqrt{2} \cdot 3^{\frac{3}{4}} \log\left(2x + \sqrt{2}\sqrt[4]{3}\sqrt{2x+1} + 1 + \sqrt{3}\right)}{2} - \sqrt{2} \cdot 3^{\frac{3}{4}} \operatorname{atan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**(5/2)/(x**2+x+1),x)
```

```
[Out] 4*(2*x + 1)**(3/2)/3 - sqrt(2)*3**(3/4)*log(2*x - sqrt(2)*3**(1/4)*sqrt(2*x
 + 1) + 1 + sqrt(3))/2 + sqrt(2)*3**(3/4)*log(2*x + sqrt(2)*3**(1/4)*sqrt(2
*x + 1) + 1 + sqrt(3))/2 - sqrt(2)*3**(3/4)*atan(sqrt(2)*3**(3/4)*sqrt(2*x
+ 1)/3 - 1) - sqrt(2)*3**(3/4)*atan(sqrt(2)*3**(3/4)*sqrt(2*x + 1)/3 + 1)
```

Giac [A] time = 1.19452, size = 174, normalized size = 1.02

$$\frac{4}{3}(2x+1)^{\frac{3}{2}} - 108^{\frac{1}{4}} \arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}} \sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2} + 2\sqrt{2x+1}\right)\right) - 108^{\frac{1}{4}} \arctan\left(-\frac{1}{6} \cdot 3^{\frac{3}{4}} \sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2} - 2\sqrt{2x+1}\right)\right) + \frac{1}{2} \cdot 108^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(5/2)/(x^2+x+1),x, algorithm="giac")
```

```
[Out] 4/3*(2*x + 1)^(3/2) - 108^(1/4)*arctan(1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqrt(2)
 + 2*sqrt(2*x + 1))) - 108^(1/4)*arctan(-1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqrt(
2) - 2*sqrt(2*x + 1))) + 1/2*108^(1/4)*log(3^(1/4)*sqrt(2)*sqrt(2*x + 1) +
2*x + sqrt(3) + 1) - 1/2*108^(1/4)*log(-3^(1/4)*sqrt(2)*sqrt(2*x + 1) + 2*x
 + sqrt(3) + 1)
```

3.1322 $\int \frac{(1+2x)^{3/2}}{1+x+x^2} \, dx$

Optimal. Leaf size=168

$$4\sqrt{2x+1}+\frac{\sqrt[4]{3}\log\left(2x-\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+\sqrt{3}+1\right)}{\sqrt{2}}-\frac{\sqrt[4]{3}\log\left(2x+\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+\sqrt{3}+1\right)}{\sqrt{2}}+\sqrt{2}\sqrt[4]{3}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{2}}{\sqrt[4]{}}\right.$$

```
[Out] 4*Sqrt[1 + 2*x] + Sqrt[2]*3^(1/4)*ArcTan[1 - (Sqrt[2]*Sqrt[1 + 2*x])/3^(1/4
)] - Sqrt[2]*3^(1/4)*ArcTan[1 + (Sqrt[2]*Sqrt[1 + 2*x])/3^(1/4)] + (3^(1/4)
*Log[1 + Sqrt[3] + 2*x - Sqrt[2]*3^(1/4)*Sqrt[1 + 2*x]])/Sqrt[2] - (3^(1/4)
*Log[1 + Sqrt[3] + 2*x + Sqrt[2]*3^(1/4)*Sqrt[1 + 2*x]])/Sqrt[2]
```

Rubi [A] time = 0.135155, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {692, 694, 329, 211, 1165, 628, 1162, 617, 204}

$$4\sqrt{2x+1}+\frac{\sqrt[4]{3}\log\left(2x-\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+\sqrt{3}+1\right)}{\sqrt{2}}-\frac{\sqrt[4]{3}\log\left(2x+\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+\sqrt{3}+1\right)}{\sqrt{2}}+\sqrt{2}\sqrt[4]{3}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{2}}{\sqrt[4]{}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^(3/2)/(1 + x + x^2),x]
```

```
[Out] 4*Sqrt[1 + 2*x] + Sqrt[2]*3^(1/4)*ArcTan[1 - (Sqrt[2]*Sqrt[1 + 2*x])/3^(1/4
)] - Sqrt[2]*3^(1/4)*ArcTan[1 + (Sqrt[2]*Sqrt[1 + 2*x])/3^(1/4)] + (3^(1/4)
*Log[1 + Sqrt[3] + 2*x - Sqrt[2]*3^(1/4)*Sqrt[1 + 2*x]])/Sqrt[2] - (3^(1/4)
*Log[1 + Sqrt[3] + 2*x + Sqrt[2]*3^(1/4)*Sqrt[1 + 2*x]])/Sqrt[2]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(1+2x)^{3/2}}{1+x+x^2}\,dx = 4\sqrt{1+2x} - 3\int \frac{1}{\sqrt{1+2x}\left(1+x+x^2\right)}\,dx$$
$$= 4\sqrt{1+2x} - \frac{3}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(\frac{3}{4}+\frac{x^2}{4}\right)}\,dx, x, 1+2x\right)$$
$$= 4\sqrt{1+2x} - 3\,\text{Subst}\left(\int \frac{1}{\frac{3}{4}+\frac{x^4}{4}}\,dx, x, \sqrt{1+2x}\right)$$
$$= 4\sqrt{1+2x} - \frac{1}{2}\sqrt{3}\,\text{Subst}\left(\int \frac{\sqrt{3}-x^2}{\frac{3}{4}+\frac{x^4}{4}}\,dx, x, \sqrt{1+2x}\right) - \frac{1}{2}\sqrt{3}\,\text{Subst}\left(\int \frac{\sqrt{3}+x^2}{\frac{3}{4}+\frac{x^4}{4}}\,dx, x, \sqrt{1+2x}\right)$$
$$= 4\sqrt{1+2x} + \frac{\sqrt[4]{3}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{3}+2x}{-\sqrt{3}-\sqrt{2}\sqrt[4]{3}x-x^2}\,dx, x, \sqrt{1+2x}\right)}{\sqrt{2}} + \frac{\sqrt[4]{3}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{3}-2x}{-\sqrt{3}+\sqrt{2}\sqrt[4]{3}x-x^2}\,dx, x, \sqrt{1+2x}\right.}{\sqrt{2}}$$
$$= 4\sqrt{1+2x} + \frac{\sqrt[4]{3}\log\left(1+\sqrt{3}+2x-\sqrt{2}\sqrt[4]{3}\sqrt{1+2x}\right)}{\sqrt{2}} - \frac{\sqrt[4]{3}\log\left(1+\sqrt{3}+2x+\sqrt{2}\sqrt[4]{3}\sqrt{1+2x}\right)}{\sqrt{2}} - \left(\sqrt{2}\right.$$
$$= 4\sqrt{1+2x} + \sqrt{2}\sqrt[4]{3}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{1+2x}}{\sqrt[4]{3}}\right) - \sqrt{2}\sqrt[4]{3}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{1+2x}}{\sqrt[4]{3}}\right) + \frac{\sqrt[4]{3}\log\left(1+\sqrt{3}+2x\right.}{\sqrt{2}}$$

Mathematica [A] time = 0.0399658, size = 148, normalized size = 0.88

$$4\sqrt{2x+1} + \frac{\sqrt[4]{3}\log\left(2x-\sqrt[4]{3}\sqrt{4x+2}+\sqrt{3}+1\right)}{\sqrt{2}} - \frac{\sqrt[4]{3}\log\left(2x+\sqrt[4]{3}\sqrt{4x+2}+\sqrt{3}+1\right)}{\sqrt{2}} + \sqrt{2}\sqrt[4]{3}\tan^{-1}\left(1-\frac{\sqrt{4x+2}}{\sqrt[4]{3}}\right) -$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^(3/2)/(1 + x + x^2),x]
```

```
[Out] 4*Sqrt[1 + 2*x] + Sqrt[2]*3^(1/4)*ArcTan[1 - Sqrt[2 + 4*x]/3^(1/4)] - Sqrt[
2]*3^(1/4)*ArcTan[1 + Sqrt[2 + 4*x]/3^(1/4)] + (3^(1/4)*Log[1 + Sqrt[3] + 2
*x - 3^(1/4)*Sqrt[2 + 4*x]])/Sqrt[2] - (3^(1/4)*Log[1 + Sqrt[3] + 2*x + 3^(
1/4)*Sqrt[2 + 4*x]])/Sqrt[2]
```

Maple [A] time = 0.041, size = 120, normalized size = 0.7

$$4\sqrt{1+2x} - \sqrt[4]{3}\arctan\left(1+\frac{\sqrt{2}3^{\frac{3}{4}}}{3}\sqrt{1+2x}\right)\sqrt{2} - \sqrt[4]{3}\arctan\left(-1+\frac{\sqrt{2}3^{\frac{3}{4}}}{3}\sqrt{1+2x}\right)\sqrt{2} - \frac{\sqrt[4]{3}\sqrt{2}}{2}\ln\left(\left(1+2x+\sqrt{3}+\sqrt[4]{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^(3/2)/(x^2+x+1),x)
```

```
[Out] 4*(1+2*x)^(1/2)-3^(1/4)*arctan(1+1/3*2^(1/2)*(1+2*x)^(1/2)*3^(3/4))*2^(1/2)
-3^(1/4)*arctan(-1+1/3*2^(1/2)*(1+2*x)^(1/2)*3^(3/4))*2^(1/2)-1/2*3^(1/4)*2
^(1/2)*ln((1+2*x+3^(1/2)+3^(1/4)*2^(1/2)*(1+2*x)^(1/2))/(1+2*x+3^(1/2)-3^(1
/4)*2^(1/2)*(1+2*x)^(1/2)))
```

Maxima [A] time = 1.82125, size = 190, normalized size = 1.13

$$-3^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}+2\sqrt{2x+1}\right)\right)-3^{\frac{1}{4}}\sqrt{2}\arctan\left(-\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}-2\sqrt{2x+1}\right)\right)-\frac{1}{2}\cdot 3^{\frac{1}{4}}\sqrt{2}\log\left(3^{\frac{1}{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(3/2)/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] -3^(1/4)*sqrt(2)*arctan(1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqrt(2) + 2*sqrt(2*x +
 1))) - 3^(1/4)*sqrt(2)*arctan(-1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqrt(2) - 2*sq
rt(2*x + 1))) - 1/2*3^(1/4)*sqrt(2)*log(3^(1/4)*sqrt(2)*sqrt(2*x + 1) + 2*x
 + sqrt(3) + 1) + 1/2*3^(1/4)*sqrt(2)*log(-3^(1/4)*sqrt(2)*sqrt(2*x + 1) +
2*x + sqrt(3) + 1) + 4*sqrt(2*x + 1)
```

Fricas [A] time = 1.60279, size = 622, normalized size = 3.7

$$2\cdot 3^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{3}\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{3^{\frac{1}{4}}\sqrt{2}\sqrt{2x+1}+2x+\sqrt{3}+1}-\frac{1}{3}\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{2x+1}-1\right)+2\cdot 3^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{3}\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(3/2)/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] 2*3^(1/4)*sqrt(2)*arctan(1/3*3^(3/4)*sqrt(2)*sqrt(3^(1/4)*sqrt(2)*sqrt(2*x
+ 1) + 2*x + sqrt(3) + 1) - 1/3*3^(3/4)*sqrt(2)*sqrt(2*x + 1) - 1) + 2*3^(1
/4)*sqrt(2)*arctan(1/3*3^(3/4)*sqrt(2)*sqrt(-3^(1/4)*sqrt(2)*sqrt(2*x + 1)
+ 2*x + sqrt(3) + 1) - 1/3*3^(3/4)*sqrt(2)*sqrt(2*x + 1) + 1) - 1/2*3^(1/4)
*sqrt(2)*log(3^(1/4)*sqrt(2)*sqrt(2*x + 1) + 2*x + sqrt(3) + 1) + 1/2*3^(1/
4)*sqrt(2)*log(-3^(1/4)*sqrt(2)*sqrt(2*x + 1) + 2*x + sqrt(3) + 1) + 4*sqrt
(2*x + 1)
```

Sympy [A] time = 19.481, size = 162, normalized size = 0.96

$$4\sqrt{2x+1}+\frac{\sqrt{2}\sqrt[4]{3}\log\left(2x-\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+1+\sqrt{3}\right)}{2}-\frac{\sqrt{2}\sqrt[4]{3}\log\left(2x+\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+1+\sqrt{3}\right)}{2}-\sqrt{2}\sqrt[4]{3}\operatorname{atan}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**(3/2)/(x**2+x+1),x)
```

```
[Out] 4*sqrt(2*x + 1) + sqrt(2)*3**(1/4)*log(2*x - sqrt(2)*3**(1/4)*sqrt(2*x + 1)
 + 1 + sqrt(3))/2 - sqrt(2)*3**(1/4)*log(2*x + sqrt(2)*3**(1/4)*sqrt(2*x +
1) + 1 + sqrt(3))/2 - sqrt(2)*3**(1/4)*atan(sqrt(2)*3**(3/4)*sqrt(2*x + 1)/
3 - 1) - sqrt(2)*3**(1/4)*atan(sqrt(2)*3**(3/4)*sqrt(2*x + 1)/3 + 1)
```

Giac [A] time = 1.17534, size = 174, normalized size = 1.04

$$-12^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}+2\sqrt{2x+1}\right)\right)-12^{\frac{1}{4}}\arctan\left(-\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}-2\sqrt{2x+1}\right)\right)-\frac{1}{2}\cdot 12^{\frac{1}{4}}\log\left(3^{\frac{1}{4}}\sqrt{2}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(3/2)/(x^2+x+1),x, algorithm="giac")
```

```
[Out] -12^(1/4)*arctan(1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqrt(2) + 2*sqrt(2*x + 1))) -
 12^(1/4)*arctan(-1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqrt(2) - 2*sqrt(2*x + 1)))
- 1/2*12^(1/4)*log(3^(1/4)*sqrt(2)*sqrt(2*x + 1) + 2*x + sqrt(3) + 1) + 1/2
*12^(1/4)*log(-3^(1/4)*sqrt(2)*sqrt(2*x + 1) + 2*x + sqrt(3) + 1) + 4*sqrt(
2*x + 1)
```

3.1323 $\int \frac{\sqrt{1+2x}}{1+x+x^2} \, dx$

Optimal. Leaf size=157

$$\frac{\log\left(2x-\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+\sqrt{3}+1\right)}{\sqrt{2}\sqrt[4]{3}}-\frac{\log\left(2x+\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+\sqrt{3}+1\right)}{\sqrt{2}\sqrt[4]{3}}-\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{2x+1}}{\sqrt[4]{3}}\right)}{\sqrt[4]{3}}+\frac{\sqrt{2}\tan^{-1}\left(\right.}{\sqrt[4]{3}}$$

```
[Out] -((Sqrt[2]*ArcTan[1 - (Sqrt[2]*Sqrt[1 + 2*x])/3^(1/4)])/3^(1/4)) + (Sqrt[2]
*ArcTan[1 + (Sqrt[2]*Sqrt[1 + 2*x])/3^(1/4)])/3^(1/4) + Log[1 + Sqrt[3] + 2
*x - Sqrt[2]*3^(1/4)*Sqrt[1 + 2*x]]/(Sqrt[2]*3^(1/4)) - Log[1 + Sqrt[3] + 2
*x + Sqrt[2]*3^(1/4)*Sqrt[1 + 2*x]]/(Sqrt[2]*3^(1/4))
```

Rubi [A] time = 0.120514, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {694, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(2x-\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+\sqrt{3}+1\right)}{\sqrt{2}\sqrt[4]{3}}-\frac{\log\left(2x+\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+\sqrt{3}+1\right)}{\sqrt{2}\sqrt[4]{3}}-\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{2x+1}}{\sqrt[4]{3}}\right)}{\sqrt[4]{3}}+\frac{\sqrt{2}\tan^{-1}\left(\right.}{\sqrt[4]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 2*x]/(1 + x + x^2),x]
```

```
[Out] -((Sqrt[2]*ArcTan[1 - (Sqrt[2]*Sqrt[1 + 2*x])/3^(1/4)])/3^(1/4)) + (Sqrt[2]
*ArcTan[1 + (Sqrt[2]*Sqrt[1 + 2*x])/3^(1/4)])/3^(1/4) + Log[1 + Sqrt[3] + 2
*x - Sqrt[2]*3^(1/4)*Sqrt[1 + 2*x]]/(Sqrt[2]*3^(1/4)) - Log[1 + Sqrt[3] + 2
*x + Sqrt[2]*3^(1/4)*Sqrt[1 + 2*x]]/(Sqrt[2]*3^(1/4))
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+2x}}{1+x+x^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{x}}{\frac{3}{4}+\frac{x^2}{4}}\,dx, x, 1+2x\right)$$

$$= \text{Subst}\left(\int \frac{x^2}{\frac{3}{4}+\frac{x^4}{4}}\,dx, x, \sqrt{1+2x}\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{3}-x^2}{\frac{3}{4}+\frac{x^4}{4}}\,dx, x, \sqrt{1+2x}\right)\right) + \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{3}+x^2}{\frac{3}{4}+\frac{x^4}{4}}\,dx, x, \sqrt{1+2x}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{3}+2x}{-\sqrt{3}-\sqrt{2}\sqrt[4]{3}x-x^2}\,dx, x, \sqrt{1+2x}\right)}{\sqrt{2}\sqrt[4]{3}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{3}-2x}{-\sqrt{3}+\sqrt{2}\sqrt[4]{3}x-x^2}\,dx, x, \sqrt{1+2x}\right)}{\sqrt{2}\sqrt[4]{3}} + \text{Subst}\left(\int \frac{}{\sqrt{3}-}\right.$$

$$= \frac{\log\left(1+\sqrt{3}+2x-\sqrt{2}\sqrt[4]{3}\sqrt{1+2x}\right)}{\sqrt{2}\sqrt[4]{3}} - \frac{\log\left(1+\sqrt{3}+2x+\sqrt{2}\sqrt[4]{3}\sqrt{1+2x}\right)}{\sqrt{2}\sqrt[4]{3}} + \frac{\sqrt{2}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x\right.}{\sqrt[4]{3}}$$

$$= -\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{1+2x}}{\sqrt[4]{3}}\right)}{\sqrt[4]{3}} + \frac{\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{1+2x}}{\sqrt[4]{3}}\right)}{\sqrt[4]{3}} + \frac{\log\left(1+\sqrt{3}+2x-\sqrt{2}\sqrt[4]{3}\sqrt{1+2x}\right)}{\sqrt{2}\sqrt[4]{3}} - \frac{\log\left(1\right.}{}$$

Mathematica [C] time = 0.0058424, size = 32, normalized size = 0.2

$$\frac{4}{9}(2x+1)^{3/2}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\frac{1}{3}(2x+1)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 2*x]/(1 + x + x^2),x]
```

```
[Out] (4*(1 + 2*x)^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -(1 + 2*x)^2/3])/9
```

Maple [A] time = 0.041, size = 111, normalized size = 0.7

$$\frac{\sqrt{2}3^{\frac{3}{4}}}{3}\arctan\left(1+\frac{\sqrt{2}3^{\frac{3}{4}}}{3}\sqrt{1+2x}\right)+\frac{\sqrt{2}3^{\frac{3}{4}}}{3}\arctan\left(-1+\frac{\sqrt{2}3^{\frac{3}{4}}}{3}\sqrt{1+2x}\right)+\frac{\sqrt{2}3^{\frac{3}{4}}}{6}\ln\left(\left(1+2x+\sqrt{3}-\sqrt[4]{3}\sqrt{2}\sqrt{1+2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^(1/2)/(x^2+x+1),x)
```

```
[Out] 1/3*3^(3/4)*arctan(1+1/3*2^(1/2)*(1+2*x)^(1/2)*3^(3/4))*2^(1/2)+1/3*3^(3/4)
*arctan(-1+1/3*2^(1/2)*(1+2*x)^(1/2)*3^(3/4))*2^(1/2)+1/6*2^(1/2)*3^(3/4)*l
n((1+2*x+3^(1/2)-3^(1/4)*2^(1/2)*(1+2*x)^(1/2))/(1+2*x+3^(1/2)+3^(1/4)*2^(1
/2)*(1+2*x)^(1/2)))
```

Maxima [A] time = 2.18092, size = 178, normalized size = 1.13

$$\frac{1}{3}\cdot 3^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}+2\sqrt{2x+1}\right)\right)+\frac{1}{3}\cdot 3^{\frac{3}{4}}\sqrt{2}\arctan\left(-\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}-2\sqrt{2x+1}\right)\right)-\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(1/2)/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] 1/3*3^(3/4)*sqrt(2)*arctan(1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqrt(2) + 2*sqrt(2*
x + 1))) + 1/3*3^(3/4)*sqrt(2)*arctan(-1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqrt(2)
 - 2*sqrt(2*x + 1))) - 1/6*3^(3/4)*sqrt(2)*log(3^(1/4)*sqrt(2)*sqrt(2*x + 1
) + 2*x + sqrt(3) + 1) + 1/6*3^(3/4)*sqrt(2)*log(-3^(1/4)*sqrt(2)*sqrt(2*x
+ 1) + 2*x + sqrt(3) + 1)
```

Fricas [A] time = 1.59657, size = 621, normalized size = 3.96

$$-\frac{2}{3}\cdot 3^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{1}{3}\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{3^{\frac{1}{4}}\sqrt{2}\sqrt{2x+1}+2x+\sqrt{3}+1}-\frac{1}{3}\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{2x+1}-1\right)-\frac{2}{3}\cdot 3^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(1/2)/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] -2/3*3^(3/4)*sqrt(2)*arctan(1/3*3^(3/4)*sqrt(2)*sqrt(3^(1/4)*sqrt(2)*sqrt(2
*x + 1) + 2*x + sqrt(3) + 1) - 1/3*3^(3/4)*sqrt(2)*sqrt(2*x + 1) - 1) - 2/3
*3^(3/4)*sqrt(2)*arctan(1/6*3^(3/4)*sqrt(2)*sqrt(-4*3^(1/4)*sqrt(2)*sqrt(2*
x + 1) + 8*x + 4*sqrt(3) + 4) - 1/3*3^(3/4)*sqrt(2)*sqrt(2*x + 1) + 1) - 1/
6*3^(3/4)*sqrt(2)*log(4*3^(1/4)*sqrt(2)*sqrt(2*x + 1) + 8*x + 4*sqrt(3) + 4
) + 1/6*3^(3/4)*sqrt(2)*log(-4*3^(1/4)*sqrt(2)*sqrt(2*x + 1) + 8*x + 4*sqrt
(3) + 4)
```

Sympy [A] time = 2.76604, size = 155, normalized size = 0.99

$$\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\log\left(2x-\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+1+\sqrt{3}\right)}{6}-\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\log\left(2x+\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+1+\sqrt{3}\right)}{6}+\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\operatorname{atan}\left(\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\sqrt{2x+1}}{3}-\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**(1/2)/(x**2+x+1),x)
```

```
[Out] sqrt(2)*3**(3/4)*log(2*x - sqrt(2)*3**(1/4)*sqrt(2*x + 1) + 1 + sqrt(3))/6
- sqrt(2)*3**(3/4)*log(2*x + sqrt(2)*3**(1/4)*sqrt(2*x + 1) + 1 + sqrt(3))/
6 + sqrt(2)*3**(3/4)*atan(sqrt(2)*3**(3/4)*sqrt(2*x + 1)/3 - 1)/3 + sqrt(2)
*3**(3/4)*atan(sqrt(2)*3**(3/4)*sqrt(2*x + 1)/3 + 1)/3
```

Giac [A] time = 1.16337, size = 162, normalized size = 1.03

$$\frac{1}{3}\cdot 108^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}+2\sqrt{2x+1}\right)\right)+\frac{1}{3}\cdot 108^{\frac{1}{4}}\arctan\left(-\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}-2\sqrt{2x+1}\right)\right)-\frac{1}{6}\cdot 108^{\frac{1}{4}}\log\left(3^{\frac{1}{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(1/2)/(x^2+x+1),x, algorithm="giac")
```

```
[Out] 1/3*108^(1/4)*arctan(1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqrt(2) + 2*sqrt(2*x + 1)
)) + 1/3*108^(1/4)*arctan(-1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqrt(2) - 2*sqrt(2*
x + 1))) - 1/6*108^(1/4)*log(3^(1/4)*sqrt(2)*sqrt(2*x + 1) + 2*x + sqrt(3)
+ 1) + 1/6*108^(1/4)*log(-3^(1/4)*sqrt(2)*sqrt(2*x + 1) + 2*x + sqrt(3) + 1
)
```

3.1324 $\int \frac{1}{\sqrt{1+2x}\left(1+x+x^2\right)}\,dx$

Optimal. Leaf size=157

$$-\frac{\log\left(2x-\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+\sqrt{3}+1\right)}{\sqrt{2}3^{3/4}}+\frac{\log\left(2x+\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+\sqrt{3}+1\right)}{\sqrt{2}3^{3/4}}-\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{2x+1}}{\sqrt[4]{3}}\right)}{3^{3/4}}+\frac{\sqrt{2}\tan^{-1}\left(\right.}{3}$$

```
[Out] -((Sqrt[2]*ArcTan[1 - (Sqrt[2]*Sqrt[1 + 2*x])/3^(1/4)])/3^(3/4)) + (Sqrt[2]
*ArcTan[1 + (Sqrt[2]*Sqrt[1 + 2*x])/3^(1/4)])/3^(3/4) - Log[1 + Sqrt[3] + 2
*x - Sqrt[2]*3^(1/4)*Sqrt[1 + 2*x]]/(Sqrt[2]*3^(3/4)) + Log[1 + Sqrt[3] + 2
*x + Sqrt[2]*3^(1/4)*Sqrt[1 + 2*x]]/(Sqrt[2]*3^(3/4))
```

Rubi [A] time = 0.119428, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {694, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(2x-\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+\sqrt{3}+1\right)}{\sqrt{2}3^{3/4}}+\frac{\log\left(2x+\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+\sqrt{3}+1\right)}{\sqrt{2}3^{3/4}}-\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{2x+1}}{\sqrt[4]{3}}\right)}{3^{3/4}}+\frac{\sqrt{2}\tan^{-1}\left(\right.}{3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + 2*x]*(1 + x + x^2)),x]
```

```
[Out] -((Sqrt[2]*ArcTan[1 - (Sqrt[2]*Sqrt[1 + 2*x])/3^(1/4)])/3^(3/4)) + (Sqrt[2]
*ArcTan[1 + (Sqrt[2]*Sqrt[1 + 2*x])/3^(1/4)])/3^(3/4) - Log[1 + Sqrt[3] + 2
*x - Sqrt[2]*3^(1/4)*Sqrt[1 + 2*x]]/(Sqrt[2]*3^(3/4)) + Log[1 + Sqrt[3] + 2
*x + Sqrt[2]*3^(1/4)*Sqrt[1 + 2*x]]/(Sqrt[2]*3^(3/4))
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{1+2x}\left(1+x+x^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(\frac{3}{4}+\frac{x^2}{4}\right)}\,dx, x, 1+2x\right)\\
&= \text{Subst}\left(\int \frac{1}{\frac{3}{4}+\frac{x^4}{4}}\,dx, x, \sqrt{1+2x}\right)\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{3}-x^2}{\frac{3}{4}+\frac{x^4}{4}}\,dx, x, \sqrt{1+2x}\right)}{2\sqrt{3}} + \frac{\text{Subst}\left(\int \frac{\sqrt{3}+x^2}{\frac{3}{4}+\frac{x^4}{4}}\,dx, x, \sqrt{1+2x}\right)}{2\sqrt{3}}\\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{3}+2x}{-\sqrt{3}-\sqrt{2}\sqrt[4]{3}x-x^2}\,dx, x, \sqrt{1+2x}\right)}{\sqrt{2}3^{3/4}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{3}-2x}{-\sqrt{3}+\sqrt{2}\sqrt[4]{3}x-x^2}\,dx, x, \sqrt{1+2x}\right)}{\sqrt{2}3^{3/4}} + \text{Sub}\\
&= -\frac{\log\left(1+\sqrt{3}+2x-\sqrt{2}\sqrt[4]{3}\sqrt{1+2x}\right)}{\sqrt{2}3^{3/4}} + \frac{\log\left(1+\sqrt{3}+2x+\sqrt{2}\sqrt[4]{3}\sqrt{1+2x}\right)}{\sqrt{2}3^{3/4}} + \frac{\sqrt{2}\,\text{Subst}\left(\right.}{}\\
&= -\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{1+2x}}{\sqrt[4]{3}}\right)}{3^{3/4}} + \frac{\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{1+2x}}{\sqrt[4]{3}}\right)}{3^{3/4}} - \frac{\log\left(1+\sqrt{3}+2x-\sqrt{2}\sqrt[4]{3}\sqrt{1+2x}\right.}{\sqrt{2}3^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.026307, size = 108, normalized size = 0.69

$$\frac{-\log\left(2x-\sqrt[4]{3}\sqrt{4x+2}+\sqrt{3}+1\right)+\log\left(2x+\sqrt[4]{3}\sqrt{4x+2}+\sqrt{3}+1\right)-2\tan^{-1}\left(1-\frac{\sqrt{4x+2}}{\sqrt[4]{3}}\right)+2\tan^{-1}\left(\frac{\sqrt{4x+2}}{\sqrt[4]{3}}+1\right)}{\sqrt{2}3^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + 2*x]*(1 + x + x^2)),x]
```

```
[Out] (-2*ArcTan[1 - Sqrt[2 + 4*x]/3^(1/4)] + 2*ArcTan[1 + Sqrt[2 + 4*x]/3^(1/4)]
 - Log[1 + Sqrt[3] + 2*x - 3^(1/4)*Sqrt[2 + 4*x]] + Log[1 + Sqrt[3] + 2*x +
 3^(1/4)*Sqrt[2 + 4*x]])/(Sqrt[2]*3^(3/4))
```

Maple [A] time = 0.043, size = 111, normalized size = 0.7

$$\frac{\sqrt[4]{3}\sqrt{2}}{3}\arctan\left(-1+\frac{\sqrt{2}3^{\frac{3}{4}}}{3}\sqrt{1+2x}\right)+\frac{\sqrt[4]{3}\sqrt{2}}{6}\ln\left(\left(1+2x+\sqrt{3}+\sqrt[4]{3}\sqrt{2}\sqrt{1+2x}\right)\left(1+2x+\sqrt{3}-\sqrt[4]{3}\sqrt{2}\sqrt{1+2x}\right)^{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2*x)^(1/2)/(x^2+x+1),x)
```

```
[Out] 1/3*3^(1/4)*arctan(-1+1/3*2^(1/2)*(1+2*x)^(1/2)*3^(3/4))*2^(1/2)+1/6*3^(1/4
)*2^(1/2)*ln((1+2*x+3^(1/2)+3^(1/4)*2^(1/2)*(1+2*x)^(1/2))/(1+2*x+3^(1/2)-3
^(1/4)*2^(1/2)*(1+2*x)^(1/2)))+1/3*3^(1/4)*arctan(1+1/3*2^(1/2)*(1+2*x)^(1/
2)*3^(3/4))*2^(1/2)
```

Maxima [A] time = 1.88842, size = 178, normalized size = 1.13

$$\frac{1}{3}\cdot3^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{6}\cdot3^{\frac{3}{4}}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}+2\sqrt{2x+1}\right)\right)+\frac{1}{3}\cdot3^{\frac{1}{4}}\sqrt{2}\arctan\left(-\frac{1}{6}\cdot3^{\frac{3}{4}}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}-2\sqrt{2x+1}\right)\right)+\frac{1}{6}\cdot3^{\frac{1}{4}}\sqrt{2}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(1/2)/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] 1/3*3^(1/4)*sqrt(2)*arctan(1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqrt(2) + 2*sqrt(2*
x + 1))) + 1/3*3^(1/4)*sqrt(2)*arctan(-1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqrt(2)
 - 2*sqrt(2*x + 1))) + 1/6*3^(1/4)*sqrt(2)*log(3^(1/4)*sqrt(2)*sqrt(2*x + 1
) + 2*x + sqrt(3) + 1) - 1/6*3^(1/4)*sqrt(2)*log(-3^(1/4)*sqrt(2)*sqrt(2*x
+ 1) + 2*x + sqrt(3) + 1)
```

Fricas [A] time = 1.62952, size = 676, normalized size = 4.31

$$-\frac{2}{27}\cdot27^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{1}{9}\cdot27^{\frac{1}{4}}\sqrt{2}\sqrt{27^{\frac{3}{4}}\sqrt{2}\sqrt{2x+1}+18x+9\sqrt{3}+9}-\frac{1}{3}\cdot27^{\frac{1}{4}}\sqrt{2}\sqrt{2x+1}-1\right)-\frac{2}{27}\cdot27^{\frac{3}{4}}\sqrt{2}\,\mathrm{arctan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(1/2)/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] -2/27*27^(3/4)*sqrt(2)*arctan(1/9*27^(1/4)*sqrt(2)*sqrt(27^(3/4)*sqrt(2)*sq
rt(2*x + 1) + 18*x + 9*sqrt(3) + 9) - 1/3*27^(1/4)*sqrt(2)*sqrt(2*x + 1) -
1) - 2/27*27^(3/4)*sqrt(2)*arctan(1/54*27^(1/4)*sqrt(2)*sqrt(-36*27^(3/4)*s
qrt(2)*sqrt(2*x + 1) + 648*x + 324*sqrt(3) + 324) - 1/3*27^(1/4)*sqrt(2)*sq
rt(2*x + 1) + 1) + 1/54*27^(3/4)*sqrt(2)*log(36*27^(3/4)*sqrt(2)*sqrt(2*x +
```

```
1) + 648*x + 324*sqrt(3) + 324) - 1/54*27^(3/4)*sqrt(2)*log(-36*27^(3/4)*s
qrt(2)*sqrt(2*x + 1) + 648*x + 324*sqrt(3) + 324)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x+1}\,(x^2+x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)**(1/2)/(x**2+x+1),x)
```

```
[Out] Integral(1/(sqrt(2*x + 1)*(x**2 + x + 1)), x)
```

Giac [A] time = 1.22032, size = 162, normalized size = 1.03

$$\frac{1}{3}\cdot 12^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}+2\sqrt{2x+1}\right)\right)+\frac{1}{3}\cdot 12^{\frac{1}{4}}\arctan\left(-\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}-2\sqrt{2x+1}\right)\right)+\frac{1}{6}\cdot 12^{\frac{1}{4}}\log\left(3^{\frac{1}{4}}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(1/2)/(x^2+x+1),x, algorithm="giac")
```

```
[Out] 1/3*12^(1/4)*arctan(1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqrt(2) + 2*sqrt(2*x + 1))
) + 1/3*12^(1/4)*arctan(-1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqrt(2) - 2*sqrt(2*x
+ 1))) + 1/6*12^(1/4)*log(3^(1/4)*sqrt(2)*sqrt(2*x + 1) + 2*x + sqrt(3) + 1
) - 1/6*12^(1/4)*log(-3^(1/4)*sqrt(2)*sqrt(2*x + 1) + 2*x + sqrt(3) + 1)
```

3.1325 $\int \frac{1}{(1+2x)^{3/2}(1+x+x^2)}\,dx$

Optimal. Leaf size=180

$$-\frac{4}{3\sqrt{2x+1}}-\frac{\log\left(2x-\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+\sqrt{3}+1\right)}{3\sqrt{2}\sqrt[4]{3}}+\frac{\log\left(2x+\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+\sqrt{3}+1\right)}{3\sqrt{2}\sqrt[4]{3}}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{2x+1}}{\sqrt[4]{3}}\right)}{3\sqrt[4]{3}}-$$

```
[Out] -4/(3*Sqrt[1 + 2*x]) + (Sqrt[2]*ArcTan[1 - (Sqrt[2]*Sqrt[1 + 2*x])/3^(1/4)]
)/(3*3^(1/4)) - (Sqrt[2]*ArcTan[1 + (Sqrt[2]*Sqrt[1 + 2*x])/3^(1/4)])/(3*3^
(1/4)) - Log[1 + Sqrt[3] + 2*x - Sqrt[2]*3^(1/4)*Sqrt[1 + 2*x]]/(3*Sqrt[2]*
3^(1/4)) + Log[1 + Sqrt[3] + 2*x + Sqrt[2]*3^(1/4)*Sqrt[1 + 2*x]]/(3*Sqrt[2
]*3^(1/4))
```

Rubi [A] time = 0.13923, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {693, 694, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{4}{3\sqrt{2x+1}}-\frac{\log\left(2x-\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+\sqrt{3}+1\right)}{3\sqrt{2}\sqrt[4]{3}}+\frac{\log\left(2x+\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+\sqrt{3}+1\right)}{3\sqrt{2}\sqrt[4]{3}}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{2x+1}}{\sqrt[4]{3}}\right)}{3\sqrt[4]{3}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + 2*x)^(3/2)*(1 + x + x^2)),x]
```

```
[Out] -4/(3*Sqrt[1 + 2*x]) + (Sqrt[2]*ArcTan[1 - (Sqrt[2]*Sqrt[1 + 2*x])/3^(1/4)]
)/(3*3^(1/4)) - (Sqrt[2]*ArcTan[1 + (Sqrt[2]*Sqrt[1 + 2*x])/3^(1/4)])/(3*3^
(1/4)) - Log[1 + Sqrt[3] + 2*x - Sqrt[2]*3^(1/4)*Sqrt[1 + 2*x]]/(3*Sqrt[2]*
3^(1/4)) + Log[1 + Sqrt[3] + 2*x + Sqrt[2]*3^(1/4)*Sqrt[1 + 2*x]]/(3*Sqrt[2
]*3^(1/4))
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(1+2x)^{3/2}\left(1+x+x^2\right)}\,dx = -\frac{4}{3\sqrt{1+2x}} - \frac{1}{3}\int \frac{\sqrt{1+2x}}{1+x+x^2}\,dx$$
$$= -\frac{4}{3\sqrt{1+2x}} - \frac{1}{6}\text{Subst}\left(\int \frac{\sqrt{x}}{\frac{3}{4}+\frac{x^2}{4}}\,dx, x, 1+2x\right)$$
$$= -\frac{4}{3\sqrt{1+2x}} - \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{\frac{3}{4}+\frac{x^4}{4}}\,dx, x, \sqrt{1+2x}\right)$$
$$= -\frac{4}{3\sqrt{1+2x}} + \frac{1}{6}\text{Subst}\left(\int \frac{\sqrt{3}-x^2}{\frac{3}{4}+\frac{x^4}{4}}\,dx, x, \sqrt{1+2x}\right) - \frac{1}{6}\text{Subst}\left(\int \frac{\sqrt{3}+x^2}{\frac{3}{4}+\frac{x^4}{4}}\,dx, x, \sqrt{1\right.$$
$$= -\frac{4}{3\sqrt{1+2x}} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{3}-\sqrt{2}\sqrt[4]{3}x+x^2}\,dx, x, \sqrt{1+2x}\right) - \frac{1}{3}\text{Subst}\left(\int \frac{}{\sqrt{3}+\sqrt{}}\right.$$
$$= -\frac{4}{3\sqrt{1+2x}} - \frac{\log\left(1+\sqrt{3}+2x-\sqrt{2}\sqrt[4]{3}\sqrt{1+2x}\right)}{3\sqrt{2}\sqrt[4]{3}} + \frac{\log\left(1+\sqrt{3}+2x+\sqrt{2}\sqrt[4]{3}\sqrt{1+\right.}{3\sqrt{2}\sqrt[4]{3}}$$
$$= -\frac{4}{3\sqrt{1+2x}} + \frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{1+2x}}{\sqrt[4]{3}}\right)}{3\sqrt[4]{3}} - \frac{\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{1+2x}}{\sqrt[4]{3}}\right)}{3\sqrt[4]{3}} - \frac{\log\left(1+\sqrt{3}+2\right.}{3\sqrt{}}$$

Mathematica [C] time = 0.0076582, size = 32, normalized size = 0.18

$$-\frac{4\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\frac{1}{3}(2x+1)^2\right)}{3\sqrt{2x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + 2*x)^(3/2)*(1 + x + x^2)),x]
```

```
[Out] (-4*Hypergeometric2F1[-1/4, 1, 3/4, -(1 + 2*x)^2/3])/(3*Sqrt[1 + 2*x])
```

Maple [A] time = 0.047, size = 120, normalized size = 0.7

$$-\frac{\sqrt{2}3^{\frac{3}{4}}}{9}\arctan\left(1+\frac{\sqrt{2}3^{\frac{3}{4}}}{3}\sqrt{1+2x}\right) - \frac{\sqrt{2}3^{\frac{3}{4}}}{9}\arctan\left(-1+\frac{\sqrt{2}3^{\frac{3}{4}}}{3}\sqrt{1+2x}\right) - \frac{\sqrt{2}3^{\frac{3}{4}}}{18}\ln\left(\left(1+2x+\sqrt{3}-\sqrt[4]{3}\sqrt{2}\sqrt{1+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2*x)^(3/2)/(x^2+x+1),x)
```

```
[Out] -1/9*3^(3/4)*arctan(1+1/3*2^(1/2)*(1+2*x)^(1/2)*3^(3/4))*2^(1/2)-1/9*3^(3/4
)*arctan(-1+1/3*2^(1/2)*(1+2*x)^(1/2)*3^(3/4))*2^(1/2)-1/18*2^(1/2)*3^(3/4)
*ln((1+2*x+3^(1/2)-3^(1/4)*2^(1/2)*(1+2*x)^(1/2))/(1+2*x+3^(1/2)+3^(1/4)*2^
(1/2)*(1+2*x)^(1/2)))-4/3/(1+2*x)^(1/2)
```

Maxima [A] time = 1.87757, size = 190, normalized size = 1.06

$$-\frac{1}{9}\cdot 3^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}+2\sqrt{2x+1}\right)\right) - \frac{1}{9}\cdot 3^{\frac{3}{4}}\sqrt{2}\arctan\left(-\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}-2\sqrt{2x+1}\right)\right) + \frac{1}{18}\cdot 3^{\frac{3}{4}}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(3/2)/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] -1/9*3^(3/4)*sqrt(2)*arctan(1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqrt(2) + 2*sqrt(2
*x + 1))) - 1/9*3^(3/4)*sqrt(2)*arctan(-1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqrt(2
) - 2*sqrt(2*x + 1))) + 1/18*3^(3/4)*sqrt(2)*log(3^(1/4)*sqrt(2)*sqrt(2*x +
 1) + 2*x + sqrt(3) + 1) - 1/18*3^(3/4)*sqrt(2)*log(-3^(1/4)*sqrt(2)*sqrt(2
*x + 1) + 2*x + sqrt(3) + 1) - 4/3/sqrt(2*x + 1)
```

Fricas [A] time = 1.61084, size = 706, normalized size = 3.92

$$4 \cdot 3^{\frac{3}{4}} \sqrt{2}(2x+1) \arctan\left(\frac{1}{3} \cdot 3^{\frac{3}{4}} \sqrt{2} \sqrt{3^{\frac{1}{4}} \sqrt{2} \sqrt{2x+1} + 2x + \sqrt{3} + 1} - \frac{1}{3} \cdot 3^{\frac{3}{4}} \sqrt{2} \sqrt{2x+1} - 1\right) + 4 \cdot 3^{\frac{3}{4}} \sqrt{2}(2x+1) \arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(3/2)/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] 1/18*(4*3^(3/4)*sqrt(2)*(2*x + 1)*arctan(1/3*3^(3/4)*sqrt(2)*sqrt(3^(1/4)*s
qrt(2)*sqrt(2*x + 1) + 2*x + sqrt(3) + 1) - 1/3*3^(3/4)*sqrt(2)*sqrt(2*x +
1) - 1) + 4*3^(3/4)*sqrt(2)*(2*x + 1)*arctan(1/6*3^(3/4)*sqrt(2)*sqrt(-4*3^
(1/4)*sqrt(2)*sqrt(2*x + 1) + 8*x + 4*sqrt(3) + 4) - 1/3*3^(3/4)*sqrt(2)*sq
rt(2*x + 1) + 1) + 3^(3/4)*sqrt(2)*(2*x + 1)*log(4*3^(1/4)*sqrt(2)*sqrt(2*x
 + 1) + 8*x + 4*sqrt(3) + 4) - 3^(3/4)*sqrt(2)*(2*x + 1)*log(-4*3^(1/4)*sqr
t(2)*sqrt(2*x + 1) + 8*x + 4*sqrt(3) + 4) - 24*sqrt(2*x + 1))/(2*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x+1)^{\frac{3}{2}} \left(x^2+x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)**(3/2)/(x**2+x+1),x)
```

```
[Out] Integral(1/((2*x + 1)**(3/2)*(x**2 + x + 1)), x)
```

Giac [A] time = 1.10591, size = 174, normalized size = 0.97

$$-\frac{1}{9} \cdot 108^{\frac{1}{4}} \arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}} \sqrt{2}\left(3^{\frac{1}{4}} \sqrt{2} + 2\sqrt{2x+1}\right)\right) - \frac{1}{9} \cdot 108^{\frac{1}{4}} \arctan\left(-\frac{1}{6} \cdot 3^{\frac{3}{4}} \sqrt{2}\left(3^{\frac{1}{4}} \sqrt{2} - 2\sqrt{2x+1}\right)\right) + \frac{1}{18} \cdot 108^{\frac{1}{4}} \log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(3/2)/(x^2+x+1),x, algorithm="giac")
```

```
[Out] -1/9*108^(1/4)*arctan(1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqrt(2) + 2*sqrt(2*x + 1
))) - 1/9*108^(1/4)*arctan(-1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqrt(2) - 2*sqrt(2
*x + 1))) + 1/18*108^(1/4)*log(3^(1/4)*sqrt(2)*sqrt(2*x + 1) + 2*x + sqrt(3
```

```
) + 1) - 1/18*108^(1/4)*log(-3^(1/4)*sqrt(2)*sqrt(2*x + 1) + 2*x + sqrt(3)
+ 1) - 4/3/sqrt(2*x + 1)
```

3.1326 $\int \frac{1}{(1+2x)^{5/2}(1+x+x^2)}\,dx$

Optimal. Leaf size=180

$$-\frac{4}{9(2x+1)^{3/2}}+\frac{\log\left(2x-\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+\sqrt{3}+1\right)}{3\sqrt{2}3^{3/4}}-\frac{\log\left(2x+\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+\sqrt{3}+1\right)}{3\sqrt{2}3^{3/4}}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{2x+1}}{\sqrt[4]{3}}\right)}{3\ 3^{3/4}}-$$

```
[Out] -4/(9*(1 + 2*x)^(3/2)) + (Sqrt[2]*ArcTan[1 - (Sqrt[2]*Sqrt[1 + 2*x])/3^(1/4
)])/(3*3^(3/4)) - (Sqrt[2]*ArcTan[1 + (Sqrt[2]*Sqrt[1 + 2*x])/3^(1/4)])/(3*
3^(3/4)) + Log[1 + Sqrt[3] + 2*x - Sqrt[2]*3^(1/4)*Sqrt[1 + 2*x]]/(3*Sqrt[2
]*3^(3/4)) - Log[1 + Sqrt[3] + 2*x + Sqrt[2]*3^(1/4)*Sqrt[1 + 2*x]]/(3*Sqrt
[2]*3^(3/4))
```

Rubi [A] time = 0.13279, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {693, 694, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{4}{9(2x+1)^{3/2}}+\frac{\log\left(2x-\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+\sqrt{3}+1\right)}{3\sqrt{2}3^{3/4}}-\frac{\log\left(2x+\sqrt{2}\sqrt[4]{3}\sqrt{2x+1}+\sqrt{3}+1\right)}{3\sqrt{2}3^{3/4}}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{2x+1}}{\sqrt[4]{3}}\right)}{3\ 3^{3/4}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + 2*x)^(5/2)*(1 + x + x^2)),x]
```



```
[Out] -4/(9*(1 + 2*x)^(3/2)) + (Sqrt[2]*ArcTan[1 - (Sqrt[2]*Sqrt[1 + 2*x])/3^(1/4
)])/(3*3^(3/4)) - (Sqrt[2]*ArcTan[1 + (Sqrt[2]*Sqrt[1 + 2*x])/3^(1/4)])/(3*
3^(3/4)) + Log[1 + Sqrt[3] + 2*x - Sqrt[2]*3^(1/4)*Sqrt[1 + 2*x]]/(3*Sqrt[2
]*3^(3/4)) - Log[1 + Sqrt[3] + 2*x + Sqrt[2]*3^(1/4)*Sqrt[1 + 2*x]]/(3*Sqrt
[2]*3^(3/4))
```


Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(1+2x)^{5/2}\left(1+x+x^2\right)}\,dx = -\frac{4}{9(1+2x)^{3/2}}-\frac{1}{3}\int \frac{1}{\sqrt{1+2x}\left(1+x+x^2\right)}\,dx$$
$$= -\frac{4}{9(1+2x)^{3/2}}-\frac{1}{6}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\left(\frac{3}{4}+\frac{x^2}{4}\right)}\,dx,x,1+2x\right)$$
$$= -\frac{4}{9(1+2x)^{3/2}}-\frac{1}{3}\,\mathrm{Subst}\left(\int \frac{1}{\frac{3}{4}+\frac{x^4}{4}}\,dx,x,\sqrt{1+2x}\right)$$
$$= -\frac{4}{9(1+2x)^{3/2}}-\frac{\mathrm{Subst}\left(\int \frac{\sqrt{3}-x^2}{\frac{3}{4}+\frac{x^4}{4}}\,dx,x,\sqrt{1+2x}\right)}{6\sqrt{3}}-\frac{\mathrm{Subst}\left(\int \frac{\sqrt{3}+x^2}{\frac{3}{4}+\frac{x^4}{4}}\,dx,x,\sqrt{1+2x}\right)}{6\sqrt{3}}$$
$$= -\frac{4}{9(1+2x)^{3/2}}+\frac{\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{3}+2x}{-\sqrt{3}-\sqrt{2}\sqrt[4]{3}x-x^2}\,dx,x,\sqrt{1+2x}\right)}{3\sqrt{2}3^{3/4}}+\frac{\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{3}-2x}{-\sqrt{3}+\sqrt{2}\sqrt[4]{3}x-x^2}\,dx,x\right.}{3\sqrt{2}3^{3/4}}$$
$$= -\frac{4}{9(1+2x)^{3/2}}+\frac{\log\left(1+\sqrt{3}+2x-\sqrt{2}\sqrt[4]{3}\sqrt{1+2x}\right)}{3\sqrt{2}3^{3/4}}-\frac{\log\left(1+\sqrt{3}+2x+\sqrt{2}\sqrt[4]{3}\sqrt{1+2\right.}}{3\sqrt{2}3^{3/4}}$$
$$= -\frac{4}{9(1+2x)^{3/2}}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{1+2x}}{\sqrt[4]{3}}\right)}{3\ 3^{3/4}}-\frac{\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{1+2x}}{\sqrt[4]{3}}\right)}{3\ 3^{3/4}}+\frac{\log\left(1+\sqrt{3}+2x\right.}{3\sqrt{}}$$

Mathematica [C] time = 0.008174, size = 32, normalized size = 0.18

$$-\frac{4\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\frac{1}{3}(2x+1)^2\right)}{9(2x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + 2*x)^(5/2)*(1 + x + x^2)),x]
```

```
[Out] (-4*Hypergeometric2F1[-3/4, 1, 1/4, -(1 + 2*x)^2/3])/(9*(1 + 2*x)^(3/2))
```

Maple [A] time = 0.046, size = 120, normalized size = 0.7

$$-\frac{\sqrt[4]{3}\sqrt{2}}{9}\arctan\left(1+\frac{\sqrt{2}3^{\frac{3}{4}}}{3}\sqrt{1+2x}\right)-\frac{\sqrt[4]{3}\sqrt{2}}{9}\arctan\left(-1+\frac{\sqrt{2}3^{\frac{3}{4}}}{3}\sqrt{1+2x}\right)-\frac{\sqrt[4]{3}\sqrt{2}}{18}\ln\left(\left(1+2x+\sqrt{3}+\sqrt[4]{3}\sqrt{2}\sqrt{1+2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2*x)^(5/2)/(x^2+x+1),x)
```

```
[Out] -1/9*3^(1/4)*arctan(1+1/3*2^(1/2)*(1+2*x)^(1/2)*3^(3/4))*2^(1/2)-1/9*3^(1/4
)*arctan(-1+1/3*2^(1/2)*(1+2*x)^(1/2)*3^(3/4))*2^(1/2)-1/18*3^(1/4)*2^(1/2)
*ln((1+2*x+3^(1/2)+3^(1/4)*2^(1/2)*(1+2*x)^(1/2))/(1+2*x+3^(1/2)-3^(1/4)*2^
(1/2)*(1+2*x)^(1/2)))-4/9/(1+2*x)^(3/2)
```

Maxima [A] time = 2.01312, size = 190, normalized size = 1.06

$$-\frac{1}{9}\cdot 3^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}+2\sqrt{2x+1}\right)\right)-\frac{1}{9}\cdot 3^{\frac{1}{4}}\sqrt{2}\arctan\left(-\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}-2\sqrt{2x+1}\right)\right)-\frac{1}{18}\cdot 3^{\frac{1}{4}}\sqrt{2}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(5/2)/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] -1/9*3^(1/4)*sqrt(2)*arctan(1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqrt(2) + 2*sqrt(2
*x + 1))) - 1/9*3^(1/4)*sqrt(2)*arctan(-1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqrt(2
) - 2*sqrt(2*x + 1))) - 1/18*3^(1/4)*sqrt(2)*log(3^(1/4)*sqrt(2)*sqrt(2*x +
 1) + 2*x + sqrt(3) + 1) + 1/18*3^(1/4)*sqrt(2)*log(-3^(1/4)*sqrt(2)*sqrt(2
*x + 1) + 2*x + sqrt(3) + 1) - 4/9/(2*x + 1)^(3/2)
```

Fricas [B] time = 1.68823, size = 811, normalized size = 4.51

$$4 \cdot 27^{\frac{3}{4}} \sqrt{2}\left(4 x^2 + 4 x + 1\right) \arctan\left(\frac{1}{9} \cdot 27^{\frac{1}{4}} \sqrt{2}\sqrt{27^{\frac{3}{4}} \sqrt{2}\sqrt{2 x + 1} + 18 x + 9 \sqrt{3} + 9} - \frac{1}{3} \cdot 27^{\frac{1}{4}} \sqrt{2}\sqrt{2 x + 1} - 1\right) + 4 \cdot 27^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(5/2)/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] 1/162*(4*27^(3/4)*sqrt(2)*(4*x^2 + 4*x + 1)*arctan(1/9*27^(1/4)*sqrt(2)*sqr
t(27^(3/4)*sqrt(2)*sqrt(2*x + 1) + 18*x + 9*sqrt(3) + 9) - 1/3*27^(1/4)*sqr
t(2)*sqrt(2*x + 1) - 1) + 4*27^(3/4)*sqrt(2)*(4*x^2 + 4*x + 1)*arctan(1/54*
27^(1/4)*sqrt(2)*sqrt(-36*27^(3/4)*sqrt(2)*sqrt(2*x + 1) + 648*x + 324*sqrt
(3) + 324) - 1/3*27^(1/4)*sqrt(2)*sqrt(2*x + 1) + 1) - 27^(3/4)*sqrt(2)*(4*
x^2 + 4*x + 1)*log(36*27^(3/4)*sqrt(2)*sqrt(2*x + 1) + 648*x + 324*sqrt(3)
+ 324) + 27^(3/4)*sqrt(2)*(4*x^2 + 4*x + 1)*log(-36*27^(3/4)*sqrt(2)*sqrt(2
*x + 1) + 648*x + 324*sqrt(3) + 324) - 72*sqrt(2*x + 1))/(4*x^2 + 4*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x + 1)^{\frac{5}{2}} \left(x^2 + x + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)**(5/2)/(x**2+x+1),x)
```

```
[Out] Integral(1/((2*x + 1)**(5/2)*(x**2 + x + 1)), x)
```

Giac [A] time = 1.17082, size = 174, normalized size = 0.97

$$-\frac{1}{9} \cdot 12^{\frac{1}{4}} \arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}} \sqrt{2}\left(3^{\frac{1}{4}} \sqrt{2} + 2 \sqrt{2 x + 1}\right)\right) - \frac{1}{9} \cdot 12^{\frac{1}{4}} \arctan\left(-\frac{1}{6} \cdot 3^{\frac{3}{4}} \sqrt{2}\left(3^{\frac{1}{4}} \sqrt{2} - 2 \sqrt{2 x + 1}\right)\right) - \frac{1}{18} \cdot 12^{\frac{1}{4}} \log($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(5/2)/(x^2+x+1),x, algorithm="giac")
```

```
[Out] -1/9*12^(1/4)*arctan(1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqrt(2) + 2*sqrt(2*x + 1)
)) - 1/9*12^(1/4)*arctan(-1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqrt(2) - 2*sqrt(2*x
```

```
+ 1))) - 1/18*12^(1/4)*log(3^(1/4)*sqrt(2)*sqrt(2*x + 1) + 2*x + sqrt(3) +
1) + 1/18*12^(1/4)*log(-3^(1/4)*sqrt(2)*sqrt(2*x + 1) + 2*x + sqrt(3) + 1)
- 4/9/(2*x + 1)^(3/2)
```

3.1327 $\int (bd+2cdx)^{7/2}\sqrt{a+bx+cx^2}\,dx$

Optimal. Leaf size=227

$$-\frac{5d^{7/2}\left(b^2-4ac\right)^{13/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{231c^2\sqrt{a+bx+cx^2}}-\frac{10d^3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}}{231c}-$$

```
[Out] (-10*(b^2 - 4*a*c)^2*d^3*Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/(231*c)
 - (2*(b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(77*c) +
 ((b*d + 2*c*d*x)^(9/2)*Sqrt[a + b*x + c*x^2])/(11*c*d) - (5*(b^2 - 4*a*c)^
(13/4)*d^(7/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSi
n[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(231*c^2*Sqrt[a
+ b*x + c*x^2])
```

Rubi [A] time = 0.207217, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {685, 692, 691, 689, 221}

$$-\frac{5d^{7/2}\left(b^2-4ac\right)^{13/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{231c^2\sqrt{a+bx+cx^2}}-\frac{10d^3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}}{231c}-\frac{2d\left(b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(7/2)*Sqrt[a + b*x + c*x^2],x]
```


```
[Out] (-10*(b^2 - 4*a*c)^2*d^3*Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/(231*c)
 - (2*(b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(77*c) +
 ((b*d + 2*c*d*x)^(9/2)*Sqrt[a + b*x + c*x^2])/(11*c*d) - (5*(b^2 - 4*a*c)^
(13/4)*d^(7/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSi
n[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(231*c^2*Sqrt[a
+ b*x + c*x^2])
```


Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int (bd+2cdx)^{7/2}\sqrt{a+bx+cx^2}\,dx = \frac{(bd+2cdx)^{9/2}\sqrt{a+bx+cx^2}}{11cd} - \frac{\left(b^2-4ac\right)\int \frac{(bd+2cdx)^{7/2}}{\sqrt{a+bx+cx^2}}\,dx}{22c}$$

$$= -\frac{2\left(b^2-4ac\right)d(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}}{77c} + \frac{(bd+2cdx)^{9/2}\sqrt{a+bx+cx^2}}{11cd} - \frac{\left(5\left(b^2\right.\right.}{}$$

$$= -\frac{10\left(b^2-4ac\right)^2 d^3\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{231c} - \frac{2\left(b^2-4ac\right)d(bd+2cdx)^{5/2}\sqrt{a+bx}}{77c}$$

$$= -\frac{10\left(b^2-4ac\right)^2 d^3\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{231c} - \frac{2\left(b^2-4ac\right)d(bd+2cdx)^{5/2}\sqrt{a+bx}}{77c}$$

$$= -\frac{10\left(b^2-4ac\right)^2 d^3\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{231c} - \frac{2\left(b^2-4ac\right)d(bd+2cdx)^{5/2}\sqrt{a+bx}}{77c}$$

$$= -\frac{10\left(b^2-4ac\right)^2 d^3\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{231c} - \frac{2\left(b^2-4ac\right)d(bd+2cdx)^{5/2}\sqrt{a+bx}}{77c}$$

Mathematica [C] time = 0.40669, size = 155, normalized size = 0.68

$$\frac{4\sqrt{a+x(b+cx)}(d(b+2cx))^{7/2}\left(7(b+2cx)^2(a+x(b+cx))-10c\left(a-\frac{b^2}{4c}\right)\left(\frac{\left(b^2-4ac\right){}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{4c\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}+2(a+x(b+cx))\right)\right)}{77(b+2cx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(7/2)*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (4*(d*(b + 2*c*x))^(7/2)*Sqrt[a + x*(b + c*x)]*(7*(b + 2*c*x)^2*(a + x*(b +
 c*x)) - 10*(a - b^2/(4*c))*c*(2*(a + x*(b + c*x)) + ((b^2 - 4*a*c)*Hyperge
```

```
ometric2F1[-1/2, 1/4, 5/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(4*c*Sqrt[(c*(a +
x*(b + c*x)))/(-b^2 + 4*a*c)]))))/(77*(b + 2*c*x)^3)
```

Maple [B] time = 0.381, size = 798, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(7/2)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/462*(d*(2*c*x+b))^(1/2)*(c*x^2+b*x+a)^(1/2)*d^3*(1344*x^7*c^7+4704*x^6*b*
c^6+1728*x^5*a*c^6+6624*x^5*b^2*c^5+320*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*
c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/
2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*(
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a^3
*c^3-240*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2
))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2
))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-
4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a^2*b^2*c^2+60*(-4*a*c+b^2)^(1/2)*
((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c
+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)
*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1
/2),2^(1/2))*a*b^4*c-5*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4
*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4
*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+
b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*b^6+4320*x^4*a*b*c^5
+4800*x^4*b^3*c^4-256*x^3*a^2*c^5+4448*x^3*a*b^2*c^4+1844*x^3*b^4*c^3-384*x
^2*a^2*b*c^4+2352*x^2*a*b^3*c^3+318*x^2*b^5*c^2-640*x*a^3*c^4+288*x*a^2*b^2
*c^3+516*x*a*b^4*c^2+10*x*b^6*c-320*a^3*b*c^3+208*a^2*b^3*c^2+10*a*b^5*c)/c
^2/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (2\,cdx + bd)^{\frac{7}{2}} \sqrt{cx^2 + bx + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(7/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^(7/2)*sqrt(c*x^2 + b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(8\,c^3d^3x^3 + 12\,bc^2d^3x^2 + 6\,b^2cd^3x + b^3d^3\right)\sqrt{2\,cdx + bd}\sqrt{cx^2 + bx + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(7/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((8*c^3*d^3*x^3 + 12*b*c^2*d^3*x^2 + 6*b^2*c*d^3*x + b^3*d^3)*sqrt(
2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(7/2)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (2\,cdx+bd)^{\frac{7}{2}}\sqrt{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(7/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^(7/2)*sqrt(c*x^2 + b*x + a), x)
```

3.1328 $\int (bd + 2cdx)^{3/2}\sqrt{a + bx + cx^2}\,dx$

Optimal. Leaf size=180

$$-\frac{d^{3/2}\left(b^2-4ac\right)^{9/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{21c^2\sqrt{a+bx+cx^2}}-\frac{2d\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}}{21c}+\frac{\sqrt{a-}}{}$$

```
[Out] (-2*(b^2 - 4*a*c)*d*Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/(21*c) + ((b
*d + 2*c*d*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(7*c*d) - ((b^2 - 4*a*c)^(9/4)*d
^(3/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b
*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(21*c^2*Sqrt[a + b*x + c
*x^2])
```

Rubi [A] time = 0.148845, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {685, 692, 691, 689, 221}

$$-\frac{d^{3/2}\left(b^2-4ac\right)^{9/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{21c^2\sqrt{a+bx+cx^2}}-\frac{2d\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}}{21c}+\frac{\sqrt{a+bx+c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(3/2)*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (-2*(b^2 - 4*a*c)*d*Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/(21*c) + ((b
*d + 2*c*d*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(7*c*d) - ((b^2 - 4*a*c)^(9/4)*d
^(3/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b
*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(21*c^2*Sqrt[a + b*x + c
*x^2])
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
```

```
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int (bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}\,dx = \frac{(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}}{7cd} - \frac{\left(b^2-4ac\right)\int \frac{(bd+2cdx)^{3/2}}{\sqrt{a+bx+cx^2}}\,dx}{14c}$$

$$= -\frac{2\left(b^2-4ac\right)d\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{21c} + \frac{(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}}{7cd} - \frac{\left(\left(b^2-4\right.\right.}{}$$

$$= -\frac{2\left(b^2-4ac\right)d\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{21c} + \frac{(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}}{7cd} - \frac{\left(\left(b^2-4\right.\right.}{}$$

$$= -\frac{2\left(b^2-4ac\right)d\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{21c} + \frac{(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}}{7cd} - \frac{\left(\left(b^2-4\right.\right.}{}$$

$$= -\frac{2\left(b^2-4ac\right)d\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{21c} + \frac{(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}}{7cd} - \frac{\left(b^2-4a\right.}{}$$

Mathematica [C] time = 0.137379, size = 110, normalized size = 0.61

$$\frac{1}{14}d\sqrt{a+x(b+cx)}\sqrt{d(b+2cx)}\left(\frac{\left(b^2-4ac\right)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{c\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}+8(a+x(b+cx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(3/2)*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (d*Sqrt[d*(b + 2*c*x)]*Sqrt[a + x*(b + c*x)]*(8*(a + x*(b + c*x)) + ((b^2 -
 4*a*c)*Hypergeometric2F1[-1/2, 1/4, 5/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(c*
Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)])))/14
```

Maple [B] time = 0.207, size = 564, normalized size = 3.1

$$-\frac{d}{42 c^2 \left(2 c^2 x^3 + 3 bcx^2 + 2 acx + b^2 x + ab\right)} \sqrt{d (2 cx + b)} \sqrt{cx^2 + bx + a} \left(-48 x^5 c^5 + 16 \sqrt{\frac{b + 2 cx + \sqrt{-4 ac + b^2}}{\sqrt{-4 ac + b^2}}} \sqrt{-\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(3/2)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/42*(d*(2*c*x+b))^(1/2)*(c*x^2+b*x+a)^(1/2)*d*(-48*x^5*c^5+16*((b+2*c*x+(
-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2)
)^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(
1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)
)*(-4*a*c+b^2)^(1/2)*a^2*c^2-8*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(
1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))
/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2)^(1/2)*a*b^2*c+((b+
2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2
)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*Ell
ipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),
2^(1/2))*(-4*a*c+b^2)^(1/2)*b^4-120*x^4*b*c^4-80*x^3*a*c^4-100*x^3*b^2*c^3-
120*x^2*a*b*c^3-30*x^2*b^3*c^2-32*x*a^2*c^3-44*x*a*b^2*c^2-2*x*b^4*c-16*a^2
*b*c^2-2*a*b^3*c)/c^2/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (2\,cdx + bd)^{\frac{3}{2}} \sqrt{cx^2 + bx + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^(3/2)*sqrt(c*x^2 + b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((2\,cdx + bd)^{\frac{3}{2}} \sqrt{cx^2 + bx + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((2*c*d*x + b*d)^(3/2)*sqrt(c*x^2 + b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d\,(b + 2cx))^{\frac{3}{2}} \sqrt{a + bx + cx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(3/2)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d*(b + 2*c*x))**(3/2)*sqrt(a + b*x + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (2\,cdx + bd)^{\frac{3}{2}} \sqrt{cx^2 + bx + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^(3/2)*sqrt(c*x^2 + b*x + a), x)
```

3.1329 $\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{bd+2cdx}} dx$

Optimal. Leaf size=137

$$\frac{\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}}{3cd} - \frac{(b^2-4ac)^{5/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{3c^2\sqrt{d}\sqrt{a+bx+cx^2}}$$

```
[Out] (Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/(3*c*d) - ((b^2 - 4*a*c)^(5/4)*
Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*
c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(3*c^2*Sqrt[d]*Sqrt[a + b*x + c
*x^2])
```

Rubi [A] time = 0.120008, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {685, 691, 689, 221}

$$\frac{\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}}{3cd} - \frac{(b^2-4ac)^{5/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{3c^2\sqrt{d}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/Sqrt[b*d + 2*c*d*x],x]
```

```
[Out] (Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/(3*c*d) - ((b^2 - 4*a*c)^(5/4)*
Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*
c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(3*c^2*Sqrt[d]*Sqrt[a + b*x + c
*x^2])
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
```

```
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{bd+2cdx}}\,dx = \frac{\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{3cd} - \frac{\left(b^2-4ac\right)\int \frac{1}{\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}\,dx}{6c}$$

$$= \frac{\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{3cd} - \frac{\left(\left(b^2-4ac\right)\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\int \frac{1}{\sqrt{bd+2cdx}\sqrt{-\frac{ac}{b^2-4ac}-\frac{bcx}{b^2-4ac}-\frac{c^2x^2}{b^2-4ac}}}\,dx}{6c\sqrt{a+bx+cx^2}}$$

$$= \frac{\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{3cd} - \frac{\left(\left(b^2-4ac\right)\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{\left(b^2-4ac\right)d^2}}}\,dx, x, \sqrt{bd+2cdx}\right)}{3c^2d\sqrt{a+bx+cx^2}}$$

$$= \frac{\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{3cd} - \frac{\left(b^2-4ac\right)^{5/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{3c^2\sqrt{d}\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 0.0575454, size = 91, normalized size = 0.66

$$\frac{\sqrt{a+x(b+cx)}\sqrt{d(b+2cx)}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{2cd\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/Sqrt[b*d + 2*c*d*x],x]
```

```
[Out] (Sqrt[d*(b + 2*c*x)]*Sqrt[a + x*(b + c*x)]*Hypergeometric2F1[-1/2, 1/4, 5/4
, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(2*c*d*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4
*a*c)])
```

Maple [B] time = 0.232, size = 364, normalized size = 2.7

$$\frac{1}{6d\left(2c^2x^3+3bcx^2+2acx+b^2x+ab\right)c^2}\sqrt{cx^2+bx+a}\sqrt{d(2cx+b)}\left(4\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}\sqrt{-\frac{2cx+b}{\sqrt{-4ac+b^2}}}\sqrt{\frac{-b}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^(1/2),x)
```

```
[Out] 1/6*(c*x^2+b*x+a)^(1/2)*(d*(2*c*x+b))^(1/2)/d*(4*((b+2*c*x+(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*
```

```
c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(
-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2)^
(1/2)*a*c-((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+
b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1
/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^
(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2)^(1/2)*b^2+4*c^3*x^3+6*b*c^2*x^2+4*x*a*c
^2+2*x*b^2*c+2*a*b*c)/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{\sqrt{2\,cdx + bd}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/sqrt(2*c*d*x + b*d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2 + bx + a}}{\sqrt{2\,cdx + bd}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)/sqrt(2*c*d*x + b*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx + cx^2}}{\sqrt{d\,(b + 2cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(2*c*d*x+b*d)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/sqrt(d*(b + 2*c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{\sqrt{2\,cdx + bd}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/sqrt(2*c*d*x + b*d), x)
```

3.1330 $\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^{5/2}}\,dx$

Optimal. Leaf size=137

$$\frac{\sqrt[4]{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{3c^2d^{5/2}\sqrt{a+bx+cx^2}}-\frac{\sqrt{a+bx+cx^2}}{3cd(bd+2cdx)^{3/2}}$$

```
[Out] -Sqrt[a + b*x + c*x^2]/(3*c*d*(b*d + 2*c*d*x)^(3/2)) + ((b^2 - 4*a*c)^(1/4)
*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2
*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(3*c^2*d^(5/2)*Sqrt[a + b*x +
c*x^2])
```

Rubi [A] time = 0.117037, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {684, 691, 689, 221}

$$\frac{\sqrt[4]{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{3c^2d^{5/2}\sqrt{a+bx+cx^2}}-\frac{\sqrt{a+bx+cx^2}}{3cd(bd+2cdx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(b*d + 2*c*d*x)^(5/2),x]
```


```
[Out] -Sqrt[a + b*x + c*x^2]/(3*c*d*(b*d + 2*c*d*x)^(3/2)) + ((b^2 - 4*a*c)^(1/4)
*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2
*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(3*c^2*d^(5/2)*Sqrt[a + b*x +
c*x^2])
```


Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^{5/2}}\,dx = -\frac{\sqrt{a+bx+cx^2}}{3cd(bd+2cdx)^{3/2}}+\frac{\int \frac{1}{\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}\,dx}{6cd^2}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{3cd(bd+2cdx)^{3/2}}+\frac{\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\int \frac{1}{\sqrt{bd+2cdx}\sqrt{-\frac{ac}{b^2-4ac}-\frac{bcx}{b^2-4ac}-\frac{c^2x^2}{b^2-4ac}}}\,dx}{6cd^2\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{3cd(bd+2cdx)^{3/2}}+\frac{\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{(b^2-4ac)d^2}}}\,dx, x, \sqrt{bd+2cdx}\right)}{3c^2d^3\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{3cd(bd+2cdx)^{3/2}}+\frac{\sqrt[4]{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{3c^2d^{5/2}\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 0.0535975, size = 91, normalized size = 0.66

$$-\frac{\sqrt{a+x(b+cx)}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{6cd\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}(d(b+2cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(b*d + 2*c*d*x)^(5/2),x]
```

```
[Out] -(Sqrt[a + x*(b + c*x)]*Hypergeometric2F1[-3/4, -1/2, 1/4, (b + 2*c*x)^2/(b
^2 - 4*a*c)])/(6*c*d*(d*(b + 2*c*x))^(3/2)*Sqrt[(c*(a + x*(b + c*x)))/(-b^2
 + 4*a*c)])
```

Maple [B] time = 0.312, size = 319, normalized size = 2.3

$$\frac{1}{6\,d^3\,(2\,cx+b)^2\,c^2}\left(2\,\sqrt{-4\,ac+b^2}\sqrt{\frac{b+2\,cx+\sqrt{-4\,ac+b^2}}{\sqrt{-4\,ac+b^2}}}\sqrt{-\frac{2\,cx+b}{\sqrt{-4\,ac+b^2}}}\sqrt{\frac{-b-2\,cx+\sqrt{-4\,ac+b^2}}{\sqrt{-4\,ac+b^2}}}\text{EllipticF}\left(1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^(5/2),x)
```

```
[Out] 1/6*(2*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))
^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))
/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*
a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x*c+(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4
*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^
```

```
(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/
2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*
b-2*c^2*x^2-2*b*c*x-2*a*c)/d^3*(d*(2*c*x+b))^(1/2)/(c*x^2+b*x+a)^(1/2)/(2*c
*x+b)^2/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+bx+a}}{(2\,cdx+bd)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/(2*c*d*x + b*d)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,cdx+bd}\sqrt{cx^2+bx+a}}{8\,c^3d^3x^3+12\,bc^2d^3x^2+6\,b^2cd^3x+b^3d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(8*c^3*d^3*x^3 + 12*b*c^
2*d^3*x^2 + 6*b^2*c*d^3*x + b^3*d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d\,(b+2cx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(2*c*d*x+b*d)**(5/2),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(d*(b + 2*c*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+bx+a}}{(2\,cdx+bd)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/(2*c*d*x + b*d)^(5/2), x)
```

3.1331 $\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^{9/2}}\,dx$

Optimal. Leaf size=184

$$\frac{\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{21c^2d^{9/2}\left(b^2-4ac\right)^{3/4}\sqrt{a+bx+cx^2}}+\frac{2\sqrt{a+bx+cx^2}}{21cd^3\left(b^2-4ac\right)(bd+2cdx)^{3/2}}-\frac{\sqrt{a+bx+cx^2}}{7cd(bd+2cdx)^{7/2}}$$

```
[Out] -Sqrt[a + b*x + c*x^2]/(7*c*d*(b*d + 2*c*d*x)^(7/2)) + (2*Sqrt[a + b*x + c*
x^2])/(21*c*(b^2 - 4*a*c)*d^3*(b*d + 2*c*d*x)^(3/2)) + (Sqrt[-((c*(a + b*x
+ c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*
c)^(1/4)*Sqrt[d])], -1])/(21*c^2*(b^2 - 4*a*c)^(3/4)*d^(9/2)*Sqrt[a + b*x +
 c*x^2])
```

Rubi [A] time = 0.155442, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {684, 693, 691, 689, 221}

$$\frac{\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{21c^2d^{9/2}\left(b^2-4ac\right)^{3/4}\sqrt{a+bx+cx^2}}+\frac{2\sqrt{a+bx+cx^2}}{21cd^3\left(b^2-4ac\right)(bd+2cdx)^{3/2}}-\frac{\sqrt{a+bx+cx^2}}{7cd(bd+2cdx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(b*d + 2*c*d*x)^(9/2),x]
```


```
[Out] -Sqrt[a + b*x + c*x^2]/(7*c*d*(b*d + 2*c*d*x)^(7/2)) + (2*Sqrt[a + b*x + c*
x^2])/(21*c*(b^2 - 4*a*c)*d^3*(b*d + 2*c*d*x)^(3/2)) + (Sqrt[-((c*(a + b*x
+ c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*
c)^(1/4)*Sqrt[d])], -1])/(21*c^2*(b^2 - 4*a*c)^(3/4)*d^(9/2)*Sqrt[a + b*x +
 c*x^2])
```


Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
```

```
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^{9/2}}\,dx = -\frac{\sqrt{a+bx+cx^2}}{7cd(bd+2cdx)^{7/2}} + \frac{\int \frac{1}{(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}}\,dx}{14cd^2}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{7cd(bd+2cdx)^{7/2}} + \frac{2\sqrt{a+bx+cx^2}}{21c\left(b^2-4ac\right)d^3(bd+2cdx)^{3/2}} + \frac{\int \frac{1}{\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}\,dx}{42c\left(b^2-4ac\right)d^4}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{7cd(bd+2cdx)^{7/2}} + \frac{2\sqrt{a+bx+cx^2}}{21c\left(b^2-4ac\right)d^3(bd+2cdx)^{3/2}} + \frac{\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\int \frac{1}{\sqrt{bd+2cdx}\sqrt{-\frac{ac}{b^2-4ac}-\frac{bcx}{b^2-4ac}-\cdots}}}{42c\left(b^2-4ac\right)d^4\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{7cd(bd+2cdx)^{7/2}} + \frac{2\sqrt{a+bx+cx^2}}{21c\left(b^2-4ac\right)d^3(bd+2cdx)^{3/2}} + \frac{\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{\left(b^2-4ac\right)d^2}}}\,dx, \cdots\right.}{21c^2\left(b^2-4ac\right)d^5\sqrt{a+bx+\cdots}}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{7cd(bd+2cdx)^{7/2}} + \frac{2\sqrt{a+bx+cx^2}}{21c\left(b^2-4ac\right)d^3(bd+2cdx)^{3/2}} + \frac{\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{21c^2\left(b^2-4ac\right)^{3/4}d^{9/2}\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 0.0728355, size = 99, normalized size = 0.54

$$-\frac{\sqrt{a+x(b+cx)}\sqrt{d(b+2cx)}\,{}_2F_1\left(-\frac{7}{4},-\frac{1}{2};-\frac{3}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{14cd^5(b+2cx)^4\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(b*d + 2*c*d*x)^(9/2),x]
```

```
[Out] -(Sqrt[d*(b + 2*c*x)]*Sqrt[a + x*(b + c*x)]*Hypergeometric2F1[-7/4, -1/2, -
3/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(14*c*d^5*(b + 2*c*x)^4*Sqrt[(c*(a + x*(
b + c*x)))/(-b^2 + 4*a*c)])
```

Maple [B] time = 0.268, size = 659, normalized size = 3.6

$$-\frac{1}{42 d^5 \left(4 a c - b^2\right) (2 c x + b)^4 c^2} \left(8 \sqrt{-4 a c + b^2} \sqrt{\frac{b + 2 c x + \sqrt{-4 a c + b^2}}{\sqrt{-4 a c + b^2}}} \sqrt{-\frac{2 c x + b}{\sqrt{-4 a c + b^2}}} \sqrt{\frac{-b - 2 c x + \sqrt{-4 a c + b^2}}{\sqrt{-4 a c + b^2}}} \mathrm{E}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^(9/2),x)
```

```
[Out] -1/42*(8*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2
))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2
))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-
4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^3*c^3+12*(-4*a*c+b^2)^(1/2)*((b+
2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2
)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*Ell
ipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),
2^(1/2))*x^2*b*c^2+6*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a
*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a
*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x*b^2*c+(-4*a*c+b^2)^(
1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-
4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^
(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)
*2^(1/2),2^(1/2))*b^3+16*c^4*x^4+32*b*c^3*x^3+40*x^2*a*c^3+14*x^2*b^2*c^2+4
0*b*a*c^2*x-2*b^3*c*x+24*a^2*c^2-2*a*c*b^2)/d^5*(d*(2*c*x+b))^(1/2)/(c*x^2+
b*x+a)^(1/2)/(4*a*c-b^2)/(2*c*x+b)^4/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{(2\,cdx + bd)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/(2*c*d*x + b*d)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{2\,cdx + bd}\sqrt{cx^2 + bx + a}}{32\,c^5d^5x^5 + 80\,bc^4d^5x^4 + 80\,b^2c^3d^5x^3 + 40\,b^3c^2d^5x^2 + 10\,b^4cd^5x + b^5d^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^(9/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(32*c^5*d^5*x^5 + 80*b*c
^4*d^5*x^4 + 80*b^2*c^3*d^5*x^3 + 40*b^3*c^2*d^5*x^2 + 10*b^4*c*d^5*x + b^5
*d^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(2*c*d*x+b*d)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+bx+a}}{(2\,cdx+bd)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/(2*c*d*x + b*d)^(9/2), x)
```

3.1332 $\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^{13/2}}\,dx$

Optimal. Leaf size=231

$$\frac{5\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{231c^2d^{13/2}\left(b^2-4ac\right)^{7/4}\sqrt{a+bx+cx^2}}+\frac{10\sqrt{a+bx+cx^2}}{231cd^5\left(b^2-4ac\right)^2(bd+2cdx)^{3/2}}+\frac{2\sqrt{a+bx+cx^2}}{77cd^3\left(b^2-4ac\right)(bd+2cdx)}$$

```
[Out] -Sqrt[a + b*x + c*x^2]/(11*c*d*(b*d + 2*c*d*x)^(11/2)) + (2*Sqrt[a + b*x +
c*x^2])/(77*c*(b^2 - 4*a*c)*d^3*(b*d + 2*c*d*x)^(7/2)) + (10*Sqrt[a + b*x +
 c*x^2])/(231*c*(b^2 - 4*a*c)^2*d^5*(b*d + 2*c*d*x)^(3/2)) + (5*Sqrt[-((c*(
a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^
2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(231*c^2*(b^2 - 4*a*c)^(7/4)*d^(13/2)*Sqrt
[a + b*x + c*x^2])
```

Rubi [A] time = 0.186901, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {684, 693, 691, 689, 221}

$$\frac{5\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{231c^2d^{13/2}\left(b^2-4ac\right)^{7/4}\sqrt{a+bx+cx^2}}+\frac{10\sqrt{a+bx+cx^2}}{231cd^5\left(b^2-4ac\right)^2(bd+2cdx)^{3/2}}+\frac{2\sqrt{a+bx+cx^2}}{77cd^3\left(b^2-4ac\right)(bd+2cdx)^{7/2}}-\frac{}{11}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(b*d + 2*c*d*x)^(13/2),x]
```


```
[Out] -Sqrt[a + b*x + c*x^2]/(11*c*d*(b*d + 2*c*d*x)^(11/2)) + (2*Sqrt[a + b*x +
c*x^2])/(77*c*(b^2 - 4*a*c)*d^3*(b*d + 2*c*d*x)^(7/2)) + (10*Sqrt[a + b*x +
 c*x^2])/(231*c*(b^2 - 4*a*c)^2*d^5*(b*d + 2*c*d*x)^(3/2)) + (5*Sqrt[-((c*(
a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^
2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(231*c^2*(b^2 - 4*a*c)^(7/4)*d^(13/2)*Sqrt
[a + b*x + c*x^2])
```


Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^{13/2}}\,dx = -\frac{\sqrt{a+bx+cx^2}}{11cd(bd+2cdx)^{11/2}}+\frac{\int \frac{1}{(bd+2cdx)^{9/2}\sqrt{a+bx+cx^2}}\,dx}{22cd^2}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{11cd(bd+2cdx)^{11/2}}+\frac{2\sqrt{a+bx+cx^2}}{77c\left(b^2-4ac\right)d^3(bd+2cdx)^{7/2}}+\frac{5\int \frac{1}{(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}}\,dx}{154c\left(b^2-4ac\right)d^4}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{11cd(bd+2cdx)^{11/2}}+\frac{2\sqrt{a+bx+cx^2}}{77c\left(b^2-4ac\right)d^3(bd+2cdx)^{7/2}}+\frac{10\sqrt{a+bx+cx^2}}{231c\left(b^2-4ac\right)^2d^5(bd+2cdx)^{3/2}}+\frac{5}{\cdots}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{11cd(bd+2cdx)^{11/2}}+\frac{2\sqrt{a+bx+cx^2}}{77c\left(b^2-4ac\right)d^3(bd+2cdx)^{7/2}}+\frac{10\sqrt{a+bx+cx^2}}{231c\left(b^2-4ac\right)^2d^5(bd+2cdx)^{3/2}}+\frac{\left(5\cdots\right.}{\cdots}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{11cd(bd+2cdx)^{11/2}}+\frac{2\sqrt{a+bx+cx^2}}{77c\left(b^2-4ac\right)d^3(bd+2cdx)^{7/2}}+\frac{10\sqrt{a+bx+cx^2}}{231c\left(b^2-4ac\right)^2d^5(bd+2cdx)^{3/2}}+\frac{\left(5\cdots\right.}{\cdots}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{11cd(bd+2cdx)^{11/2}}+\frac{2\sqrt{a+bx+cx^2}}{77c\left(b^2-4ac\right)d^3(bd+2cdx)^{7/2}}+\frac{10\sqrt{a+bx+cx^2}}{231c\left(b^2-4ac\right)^2d^5(bd+2cdx)^{3/2}}+\frac{5\cdots}{\cdots}$$

Mathematica [C] time = 0.0775007, size = 99, normalized size = 0.43

$$-\frac{\sqrt{a+x(b+cx)}\sqrt{d(b+2cx)}\,{}_2F_1\left(-\frac{11}{4},-\frac{1}{2};-\frac{7}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{22cd^7(b+2cx)^6\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(b*d + 2*c*d*x)^(13/2),x]
```

```
[Out] -(Sqrt[d*(b + 2*c*x)]*Sqrt[a + x*(b + c*x)]*Hypergeometric2F1[-11/4, -1/2,
-7/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(22*c*d^7*(b + 2*c*x)^6*Sqrt[(c*(a + x*
(b + c*x)))/(-b^2 + 4*a*c)])
```

Maple [B] time = 0.274, size = 1016, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^(13/2),x)
```

```
[Out] 1/462*(160*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1
/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/
(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^5*c^5+400*(-4*a*c+b^2)^(1/2)*(
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+
b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*
EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/
2),2^(1/2))*x^4*b*c^4+400*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/
(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+
(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a
*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^3*b^2*c^3+200*(
-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(
-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c
+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^2*b^3*c^2+320*x^6*c^6+50*(-4*a*c+b^2)^(1/2
)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a
*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/
2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^
(1/2),2^(1/2))*x*b^4*c+960*x^5*b*c^5+5*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c
+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2
)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((
b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*b^5+
128*x^4*a*c^5+1168*x^4*b^2*c^4+256*x^3*a*b*c^4+736*x^3*b^3*c^3-864*x^2*a^2*
c^4+624*x^2*a*b^2*c^3+198*x^2*b^4*c^2-864*x*a^2*b*c^3+496*x*a*b^3*c^2-10*x*
b^5*c-672*a^3*c^3+288*a^2*b^2*c^2-10*a*b^4*c)/d^7*(d*(2*c*x+b))^(1/2)/(c*x^
2+b*x+a)^(1/2)/(4*a*c-b^2)^2/(2*c*x+b)^6/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{(2\,cdx + bd)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^(13/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/(2*c*d*x + b*d)^(13/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,cdx + bd}\sqrt{cx^2 + bx + a}}{128\,c^7d^7x^7 + 448\,bc^6d^7x^6 + 672\,b^2c^5d^7x^5 + 560\,b^3c^4d^7x^4 + 280\,b^4c^3d^7x^3 + 84\,b^5c^2d^7x^2 + 14\,b^6cd^7x + b^7d^7},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^(13/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(128*c^7*d^7*x^7 + 448*b
*c^6*d^7*x^6 + 672*b^2*c^5*d^7*x^5 + 560*b^3*c^4*d^7*x^4 + 280*b^4*c^3*d^7*
x^3 + 84*b^5*c^2*d^7*x^2 + 14*b^6*c*d^7*x + b^7*d^7), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(2*c*d*x+b*d)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{(2\,cdx + bd)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^(13/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/(2*c*d*x + b*d)^(13/2), x)
```

3.1333 $\int (bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}\,dx$

Optimal. Leaf size=279

$$\frac{d^{5/2}\left(b^2-4ac\right)^{11/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{15c^2\sqrt{a+bx+cx^2}}-\frac{d^{5/2}\left(b^2-4ac\right)^{11/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{bd+}}{\sqrt[4]{b^2-4}}\right.\right.}{15c^2\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*(b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(45*c) + (
(b*d + 2*c*d*x)^(7/2)*Sqrt[a + b*x + c*x^2])/(9*c*d) - ((b^2 - 4*a*c)^(11/4
)*d^(5/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqr
t[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(15*c^2*Sqrt[a + b*x
+ c*x^2]) + ((b^2 - 4*a*c)^(11/4)*d^(5/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[
d])], -1])/(15*c^2*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.267596, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {685, 692, 691, 690, 307, 221, 1199, 424}

$$\frac{d^{5/2}\left(b^2-4ac\right)^{11/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{15c^2\sqrt{a+bx+cx^2}}-\frac{d^{5/2}\left(b^2-4ac\right)^{11/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-\right.}{15c^2\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(5/2)*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (-2*(b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(45*c) + (
(b*d + 2*c*d*x)^(7/2)*Sqrt[a + b*x + c*x^2])/(9*c*d) - ((b^2 - 4*a*c)^(11/4
)*d^(5/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqr
t[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(15*c^2*Sqrt[a + b*x
+ c*x^2]) + ((b^2 - 4*a*c)^(11/4)*d^(5/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[
d])], -1])/(15*c^2*Sqrt[a + b*x + c*x^2])
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int (bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}\,dx = \frac{(bd+2cdx)^{7/2}\sqrt{a+bx+cx^2}}{9cd} - \frac{(b^2-4ac)\int \frac{(bd+2cdx)^{5/2}}{\sqrt{a+bx+cx^2}}dx}{18c}$$

$$= -\frac{2(b^2-4ac)d(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{45c} + \frac{(bd+2cdx)^{7/2}\sqrt{a+bx+cx^2}}{9cd} - \frac{((}{}$$

$$= -\frac{2(b^2-4ac)d(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{45c} + \frac{(bd+2cdx)^{7/2}\sqrt{a+bx+cx^2}}{9cd} - $$

$$= -\frac{2(b^2-4ac)d(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{45c} + \frac{(bd+2cdx)^{7/2}\sqrt{a+bx+cx^2}}{9cd} - $$

$$= -\frac{2(b^2-4ac)d(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{45c} + \frac{(bd+2cdx)^{7/2}\sqrt{a+bx+cx^2}}{9cd} + $$

$$= -\frac{2(b^2-4ac)d(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{45c} + \frac{(bd+2cdx)^{7/2}\sqrt{a+bx+cx^2}}{9cd} + $$

$$= -\frac{2(b^2-4ac)d(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{45c} + \frac{(bd+2cdx)^{7/2}\sqrt{a+bx+cx^2}}{9cd} - $$

Mathematica [C] time = 0.220034, size = 110, normalized size = 0.39

$$\frac{1}{18}d\sqrt{a+x(b+cx)}(d(b+2cx))^{3/2}\left(\frac{(b^2-4ac)\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{c\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}+8(a+x(b+cx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(5/2)*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (d*(d*(b + 2*c*x))^(3/2)*Sqrt[a + x*(b + c*x)]*(8*(a + x*(b + c*x)) + ((b^2
 - 4*a*c)*Hypergeometric2F1[-1/2, 3/4, 7/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(
c*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)])))/18
```

Maple [B] time = 0.211, size = 703, normalized size = 2.5

$$-\frac{d^2}{90c^2(2c^2x^3+3bcx^2+2acx+b^2x+ab)}\sqrt{d(2cx+b)}\sqrt{cx^2+bx+a}\left(-160x^6c^6-480x^5bc^5+192\sqrt{\frac{b+2cx+\sqrt{-}}{\sqrt{-4ac+}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(5/2)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/90*(d*(2*c*x+b))^(1/2)*(c*x^2+b*x+a)^(1/2)*d^2*(-160*x^6*c^6-480*x^5*b*c
^5+192*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/
(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
)^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/
2)*2^(1/2),2^(1/2))*a^3*c^3-144*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^
(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^
(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2)
)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a^2*b^2*c^2+36*((b+2*c*x+(-4*a
*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1
/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*
((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a*
b^4*c-3*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)
/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2
))^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1
/2)*2^(1/2),2^(1/2))*b^6-224*x^4*a*c^5-544*x^4*b^2*c^4-448*x^3*a*b*c^4-288*
x^3*b^3*c^3-64*x^2*a^2*c^4-304*x^2*a*b^2*c^3-70*x^2*b^4*c^2-64*x*a^2*b*c^3-
80*x*a*b^3*c^2-6*x*b^5*c-16*a^2*b^2*c^2-6*a*b^4*c)/c^2/(2*c^2*x^3+3*b*c*x^2
+2*a*c*x+b^2*x+a*b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (2\,cdx + bd)^{\frac{5}{2}} \sqrt{cx^2 + bx + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(5/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^(5/2)*sqrt(c*x^2 + b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(4\,c^2d^2x^2 + 4\,bcd^2x + b^2d^2\right)\sqrt{2\,cdx + bd}\sqrt{cx^2 + bx + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(5/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((4*c^2*d^2*x^2 + 4*b*c*d^2*x + b^2*d^2)*sqrt(2*c*d*x + b*d)*sqrt(c
*x^2 + b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d\,(b + 2cx))^{\frac{5}{2}}\,\sqrt{a + bx + cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(5/2)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d*(b + 2*c*x))**(5/2)*sqrt(a + b*x + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (2\,cdx + bd)^{\frac{5}{2}}\sqrt{cx^2 + bx + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(5/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^(5/2)*sqrt(c*x^2 + b*x + a), x)
```

3.1334 $\int \sqrt{bd+2cdx}\sqrt{a+bx+cx^2}\,dx$

Optimal. Leaf size=236

$$\frac{\sqrt{d}\left(b^2-4ac\right)^{7/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{5c^2\sqrt{a+bx+cx^2}}-\frac{\sqrt{d}\left(b^2-4ac\right)^{7/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\right|}{5c^2\sqrt{a+bx+cx^2}}$$

```
[Out] ((b*d + 2*c*d*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*c*d) - ((b^2 - 4*a*c)^(7/4
)*Sqrt[d]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqr
t[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(5*c^2*Sqrt[a + b*x +
 c*x^2]) + ((b^2 - 4*a*c)^(7/4)*Sqrt[d]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d]
)], -1])/(5*c^2*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.215207, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {685, 691, 690, 307, 221, 1199, 424}

$$\frac{\sqrt{d}\left(b^2-4ac\right)^{7/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\right|-1\right)}{5c^2\sqrt{a+bx+cx^2}}-\frac{\sqrt{d}\left(b^2-4ac\right)^{7/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\right|-1\right)}{5c^2\sqrt{a+bx+cx^2}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((b*d + 2*c*d*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*c*d) - ((b^2 - 4*a*c)^(7/4
)*Sqrt[d]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqr
t[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(5*c^2*Sqrt[a + b*x +
 c*x^2]) + ((b^2 - 4*a*c)^(7/4)*Sqrt[d]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d]
)], -1])/(5*c^2*Sqrt[a + b*x + c*x^2])
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
```

```
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{bd+2cdx}\sqrt{a+bx+cx^2}\,dx = \frac{(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{5cd} - \frac{\left(b^2-4ac\right)\int \frac{\sqrt{bd+2cdx}}{\sqrt{a+bx+cx^2}}\,dx}{10c}$$

$$= \frac{(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{5cd} - \frac{\left(\left(b^2-4ac\right)\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\int \frac{\sqrt{bd+2cdx}}{\sqrt{-\frac{ac}{b^2-4ac}-\frac{bcx}{b^2-4ac}-\frac{c^2x^2}{b^2-4ac}}}}{10c\sqrt{a+bx+cx^2}}$$

$$= \frac{(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{5cd} - \frac{\left(\left(b^2-4ac\right)\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{\left(b^2-4ac\right)d^2}}}\,d\right.}{5c^2d\sqrt{a+bx+cx^2}}$$

$$= \frac{(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{5cd} + \frac{\left(\left(b^2-4ac\right)^{3/2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{\left(b^2-4ac\right)d}}}\right.}{5c^2\sqrt{a+bx+cx^2}}$$

$$= \frac{(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{5cd} + \frac{\left(b^2-4ac\right)^{7/4}\sqrt{d}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\right.}{5c^2\sqrt{a+bx+cx^2}}$$

$$= \frac{(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{5cd} - \frac{\left(b^2-4ac\right)^{7/4}\sqrt{d}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\right.}{5c^2\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 0.0502355, size = 91, normalized size = 0.39

$$\frac{\sqrt{a+x(b+cx)}(d(b+2cx))^{3/2}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{6cd\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((d*(b + 2*c*x))^(3/2)*Sqrt[a + x*(b + c*x)]*Hypergeometric2F1[-1/2, 3/4, 7
/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(6*c*d*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 +
 4*a*c)])
```

Maple [B] time = 0.207, size = 492, normalized size = 2.1

$$\frac{1}{\left(20\,c^2x^3+30\,bcx^2+20\,acx+10\,b^2x+10\,ab\right)c^2}\sqrt{d\,(2\,cx+b)}\sqrt{cx^2+bx+a}\left(16\sqrt{\frac{b+2\,cx+\sqrt{-4\,ac+b^2}}{\sqrt{-4\,ac+b^2}}}\sqrt{-\frac{2\,cx+b}{\sqrt{-4\,ac+b}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(1/2)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/10*(d*(2*c*x+b))^(1/2)*(c*x^2+b*x+a)^(1/2)*(16*((b+2*c*x+(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*
c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(
-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a^2*c^2-8*((b
+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^
2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*El
lipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)
,2^(1/2))*a*b^2*c+((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(
-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c
+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(1/2))^(1/2)*2^(1/2),2^(1/2))*b^4+8*c^4*x^4+16*b*c^3*x^3+8*x^2*a*c^3+10*x^
2*b^2*c^2+8*b*a*c^2*x+2*b^3*c*x+2*a*c*b^2)/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2
*x+a*b)/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{2\,cdx+bd}\sqrt{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{2\,cdx+bd}\sqrt{cx^2+bx+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\,(b+2cx)}\sqrt{a+bx+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(1/2)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(d*(b + 2*c*x))*sqrt(a + b*x + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{2\,cdx+bd}\sqrt{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a), x)
```

3.1335 $\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^{3/2}}\,dx$

Optimal. Leaf size=229

$$-\frac{\left(b^2-4ac\right)^{3/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\,\sqrt[4]{b^2-4ac}}\right),-1\right)}{c^2d^{3/2}\sqrt{a+bx+cx^2}}+\frac{\left(b^2-4ac\right)^{3/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{c^2d^{3/2}\sqrt{a+bx+cx^2}}$$

```
[Out] -(Sqrt[a + b*x + c*x^2]/(c*d*Sqrt[b*d + 2*c*d*x])) + ((b^2 - 4*a*c)^(3/4)*S
qrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c
*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(c^2*d^(3/2)*Sqrt[a + b*x + c*x^
2]) - ((b^2 - 4*a*c)^(3/4)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ell
ipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(c^2
*d^(3/2)*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.212812, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {684, 691, 690, 307, 221, 1199, 424}

$$-\frac{\left(b^2-4ac\right)^{3/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{c^2d^{3/2}\sqrt{a+bx+cx^2}}+\frac{\left(b^2-4ac\right)^{3/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{c^2d^{3/2}\sqrt{a+bx+cx^2}}-\frac{\sqrt{a+}}{cd\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(b*d + 2*c*d*x)^(3/2),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(c*d*Sqrt[b*d + 2*c*d*x])) + ((b^2 - 4*a*c)^(3/4)*S
qrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c
*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(c^2*d^(3/2)*Sqrt[a + b*x + c*x^
2]) - ((b^2 - 4*a*c)^(3/4)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ell
ipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(c^2
*d^(3/2)*Sqrt[a + b*x + c*x^2])
```

Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
```

```
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^{3/2}}\,dx = -\frac{\sqrt{a+bx+cx^2}}{cd\sqrt{bd+2cdx}} + \frac{\int \frac{\sqrt{bd+2cdx}}{\sqrt{a+bx+cx^2}}\,dx}{2cd^2}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{cd\sqrt{bd+2cdx}} + \frac{\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\int \frac{\sqrt{bd+2cdx}}{\sqrt{-\frac{ac}{b^2-4ac}-\frac{bcx}{b^2-4ac}-\frac{c^2x^2}{b^2-4ac}}}\,dx}{2cd^2\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{cd\sqrt{bd+2cdx}} + \frac{\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{\left(b^2-4ac\right)d^2}}}\,dx, x, \sqrt{bd+2cdx}\right)}{c^2d^3\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{cd\sqrt{bd+2cdx}} - \frac{\left(\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{\left(b^2-4ac\right)d^2}}}\,dx, x, \sqrt{bd+2cdx}\right)}{c^2d^2\sqrt{a+bx+cx^2}} +$$

$$= -\frac{\sqrt{a+bx+cx^2}}{cd\sqrt{bd+2cdx}} - \frac{\left(b^2-4ac\right)^{3/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{c^2d^{3/2}\sqrt{a+bx+cx^2}} +$$

$$= -\frac{\sqrt{a+bx+cx^2}}{cd\sqrt{bd+2cdx}} + \frac{\left(b^2-4ac\right)^{3/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{c^2d^{3/2}\sqrt{a+bx+cx^2}} -$$

Mathematica [C] time = 0.0455986, size = 91, normalized size = 0.4

$$-\frac{\sqrt{a+x(b+cx)}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{2cd\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\sqrt{d(b+2cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(b*d + 2*c*d*x)^(3/2),x]
```

```
[Out] -(Sqrt[a + x*(b + c*x)]*Hypergeometric2F1[-1/2, -1/4, 3/4, (b + 2*c*x)^2/(b
^2 - 4*a*c)])/(2*c*d*Sqrt[d*(b + 2*c*x)]*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 +
 4*a*c)])
```

Maple [A] time = 0.314, size = 325, normalized size = 1.4

$$\frac{1}{2\,d^2\left(2\,c^2x^3+3\,bcx^2+2\,acx+b^2x+ab\right)c^2}\sqrt{cx^2+bx+a}\sqrt{d\,(2\,cx+b)}\left(4\,\mathrm{EllipticE}\left(1/2\,\sqrt{\frac{b+2\,cx+\sqrt{-4\,ac+b^2}}{\sqrt{-4\,ac+b^2}}}\sqrt{2},\sqrt{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^(3/2),x)
```

```
[Out] 1/2*(c*x^2+b*x+a)^(1/2)*(d*(2*c*x+b))^(1/2)*(4*EllipticE(1/2*((b+2*c*x+(-4*
a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a*c*((b+2*c*x+(-
4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))
^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)-EllipticE(1
/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))
*b^2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-
4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^
(1/2)-2*c^2*x^2-2*b*c*x-2*a*c)/d^2/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)/
c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+bx+a}}{(2\,cdx+bd)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/(2*c*d*x + b*d)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{2\,cdx+bd}\sqrt{cx^2+bx+a}}{4\,c^2d^2x^2+4\,bcd^2x+b^2d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(4*c^2*d^2*x^2 + 4*b*c*d
^2*x + b^2*d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d(b+2cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(2*c*d*x+b*d)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(d*(b + 2*c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+bx+a}}{(2\,cdx+bd)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/(2*c*d*x + b*d)^(3/2), x)
```

3.1336 $\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^{7/2}}\,dx$

Optimal. Leaf size=283

$$\frac{\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{5c^2d^{7/2}\sqrt[4]{b^2-4ac}\sqrt{a+bx+cx^2}}-\frac{\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{5c^2d^{7/2}\sqrt[4]{b^2-4ac}\sqrt{a+bx+cx^2}}+\frac{2\sqrt{a+bx+cx^2}}{5cd^3\left(b^2-4ac\right)\sqrt{bd+2c}}$$

```
[Out] -Sqrt[a + b*x + c*x^2]/(5*c*d*(b*d + 2*c*d*x)^(5/2)) + (2*Sqrt[a + b*x + c*
x^2])/(5*c*(b^2 - 4*a*c)*d^3*Sqrt[b*d + 2*c*d*x]) - (Sqrt[-((c*(a + b*x + c
*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^
(1/4)*Sqrt[d])], -1])/(5*c^2*(b^2 - 4*a*c)^(1/4)*d^(7/2)*Sqrt[a + b*x + c*x
^2]) + (Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[
b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(5*c^2*(b^2 - 4*a*c)^(1
/4)*d^(7/2)*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.251851, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {684, 693, 691, 690, 307, 221, 1199, 424}

$$\frac{\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{5c^2d^{7/2}\sqrt[4]{b^2-4ac}\sqrt{a+bx+cx^2}}-\frac{\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{5c^2d^{7/2}\sqrt[4]{b^2-4ac}\sqrt{a+bx+cx^2}}+\frac{2\sqrt{a+bx+cx^2}}{5cd^3\left(b^2-4ac\right)\sqrt{bd+2cdx}}-\frac{}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(b*d + 2*c*d*x)^(7/2),x]
```

```
[Out] -Sqrt[a + b*x + c*x^2]/(5*c*d*(b*d + 2*c*d*x)^(5/2)) + (2*Sqrt[a + b*x + c*
x^2])/(5*c*(b^2 - 4*a*c)*d^3*Sqrt[b*d + 2*c*d*x]) - (Sqrt[-((c*(a + b*x + c
*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^
(1/4)*Sqrt[d])], -1])/(5*c^2*(b^2 - 4*a*c)^(1/4)*d^(7/2)*Sqrt[a + b*x + c*x
^2]) + (Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[
b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(5*c^2*(b^2 - 4*a*c)^(1
/4)*d^(7/2)*Sqrt[a + b*x + c*x^2])
```

Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^{7/2}}\,dx = -\frac{\sqrt{a+bx+cx^2}}{5cd(bd+2cdx)^{5/2}} + \frac{\int \frac{1}{(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}dx}{10cd^2}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{5cd(bd+2cdx)^{5/2}} + \frac{2\sqrt{a+bx+cx^2}}{5c\left(b^2-4ac\right)d^3\sqrt{bd+2cdx}} - \frac{\int \frac{\sqrt{bd+2cdx}}{\sqrt{a+bx+cx^2}}dx}{10c\left(b^2-4ac\right)d^4}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{5cd(bd+2cdx)^{5/2}} + \frac{2\sqrt{a+bx+cx^2}}{5c\left(b^2-4ac\right)d^3\sqrt{bd+2cdx}} - \frac{\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\int \frac{\sqrt{bd+2cdx}}{\sqrt{-\frac{ac}{b^2-4ac}-\frac{bcx}{b^2-4ac}-\frac{c^2x^2}{b^2-4ac}}}dx}{10c\left(b^2-4ac\right)d^4\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{5cd(bd+2cdx)^{5/2}} + \frac{2\sqrt{a+bx+cx^2}}{5c\left(b^2-4ac\right)d^3\sqrt{bd+2cdx}} - \frac{\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{\left(b^2-4ac\right)d^2}}}dx, x, \sqrt{\right.}{5c^2\left(b^2-4ac\right)d^5\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{5cd(bd+2cdx)^{5/2}} + \frac{2\sqrt{a+bx+cx^2}}{5c\left(b^2-4ac\right)d^3\sqrt{bd+2cdx}} + \frac{\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{\left(b^2-4ac\right)d^2}}}dx, x, \sqrt{\right.}{5c^2\sqrt{b^2-4ac}d^4\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{5cd(bd+2cdx)^{5/2}} + \frac{2\sqrt{a+bx+cx^2}}{5c\left(b^2-4ac\right)d^3\sqrt{bd+2cdx}} + \frac{\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{5c^2\sqrt[4]{b^2-4ac}d^{7/2}\sqrt{a+bx+cx^2}} -$$

$$= -\frac{\sqrt{a+bx+cx^2}}{5cd(bd+2cdx)^{5/2}} + \frac{2\sqrt{a+bx+cx^2}}{5c\left(b^2-4ac\right)d^3\sqrt{bd+2cdx}} - \frac{\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{5c^2\sqrt[4]{b^2-4ac}d^{7/2}\sqrt{a+bx+cx^2}} +$$

Mathematica [C] time = 0.0589823, size = 91, normalized size = 0.32

$$-\frac{\sqrt{a+x(b+cx)}\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{2};-\frac{1}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{10cd\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}(d(b+2cx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(b*d + 2*c*d*x)^(7/2),x]
```

```
[Out] -(Sqrt[a + x*(b + c*x)]*Hypergeometric2F1[-5/4, -1/2, -1/4, (b + 2*c*x)^2/(
b^2 - 4*a*c)])/(10*c*d*(d*(b + 2*c*x))^(5/2)*Sqrt[(c*(a + x*(b + c*x)))/(-b
^2 + 4*a*c)])
```

Maple [B] time = 0.277, size = 874, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^(7/2),x)
```

```
[Out] 1/10*(16*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1
/2)*2^(1/2),2^(1/2))*x^2*a*c^3*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(
1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(1/2))^(1/2)-4*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2
))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^2*b^2*c^2*((b+2*c*x+(-4*a*c
+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2
)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)+16*EllipticE(1/2
*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x
*a*b*c^2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b
)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/
2))^(1/2)-4*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))
^(1/2)*2^(1/2),2^(1/2))*x*b^3*c*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^
(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^
(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)+4*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^
2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a*b^2*c-((b+2*c*x+(-4*a*c+b
^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*
((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+
2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*b^4-16
*c^4*x^4-32*b*c^3*x^3-24*x^2*a*c^3-18*x^2*b^2*c^2-24*b*a*c^2*x-2*b^3*c*x-8*
a^2*c^2-2*a*c*b^2)*(d*(2*c*x+b))^(1/2)/d^4/(c*x^2+b*x+a)^(1/2)/(4*a*c-b^2)/
(2*c*x+b)^3/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{(2\,cdx + bd)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/(2*c*d*x + b*d)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,cdx + bd}\sqrt{cx^2 + bx + a}}{16\,c^4d^4x^4 + 32\,bc^3d^4x^3 + 24\,b^2c^2d^4x^2 + 8\,b^3cd^4x + b^4d^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(16*c^4*d^4*x^4 + 32*b*c
^3*d^4*x^3 + 24*b^2*c^2*d^4*x^2 + 8*b^3*c*d^4*x + b^4*d^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx + cx^2}}{(d\,(b + 2cx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(2*c*d*x+b*d)**(7/2),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(d*(b + 2*c*x))**(7/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{(2\,cdx + bd)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(2*c*d*x+b*d)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/(2*c*d*x + b*d)^(7/2), x)
```

3.1337 $\int (bd+2cdx)^{7/2}\left(a+bx+cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=274

$$\frac{d^{7/2}\left(b^2-4ac\right)^{17/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{462c^3\sqrt{a+bx+cx^2}}+\frac{d^3\left(b^2-4ac\right)^3\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}}{231c^2}-\frac{\left(b^2\right.}{}$$

```
[Out] ((b^2 - 4*a*c)^3*d^3*Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/(231*c^2) +
 ((b^2 - 4*a*c)^2*d*(b*d + 2*c*d*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(385*c^2)
- ((b^2 - 4*a*c)*(b*d + 2*c*d*x)^(9/2)*Sqrt[a + b*x + c*x^2])/(110*c^2*d) +
 ((b*d + 2*c*d*x)^(9/2)*(a + b*x + c*x^2)^(3/2))/(15*c*d) + ((b^2 - 4*a*c)^
(17/4)*d^(7/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSi
n[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(462*c^3*Sqrt[a
+ b*x + c*x^2])
```

Rubi [A] time = 0.223885, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {685, 692, 691, 689, 221}

$$\frac{d^3\left(b^2-4ac\right)^3\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}}{231c^2}+\frac{d^{7/2}\left(b^2-4ac\right)^{17/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{462c^3\sqrt{a+bx+cx^2}}-\frac{\left(b^2-4ac\right)\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(7/2)*(a + b*x + c*x^2)^(3/2),x]
```


```
[Out] ((b^2 - 4*a*c)^3*d^3*Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/(231*c^2) +
 ((b^2 - 4*a*c)^2*d*(b*d + 2*c*d*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(385*c^2)
- ((b^2 - 4*a*c)*(b*d + 2*c*d*x)^(9/2)*Sqrt[a + b*x + c*x^2])/(110*c^2*d) +
 ((b*d + 2*c*d*x)^(9/2)*(a + b*x + c*x^2)^(3/2))/(15*c*d) + ((b^2 - 4*a*c)^
(17/4)*d^(7/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSi
n[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(462*c^3*Sqrt[a
+ b*x + c*x^2])
```


Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int (bd+2cdx)^{7/2}\left(a+bx+cx^2\right)^{3/2}\,dx = \frac{(bd+2cdx)^{9/2}\left(a+bx+cx^2\right)^{3/2}}{15cd} - \frac{\left(b^2-4ac\right)\int (bd+2cdx)^{7/2}\sqrt{a+bx+cx^2}\,dx}{10c}$$

$$= -\frac{\left(b^2-4ac\right)(bd+2cdx)^{9/2}\sqrt{a+bx+cx^2}}{110c^2d} + \frac{(bd+2cdx)^{9/2}\left(a+bx+cx^2\right)^{3/2}}{15cd} + \frac{(b}{}$$

$$= \frac{\left(b^2-4ac\right)^2 d(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}}{385c^2} - \frac{\left(b^2-4ac\right)(bd+2cdx)^{9/2}\sqrt{a+bx+}}{110c^2d}$$

$$= \frac{\left(b^2-4ac\right)^3 d^3\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{231c^2} + \frac{\left(b^2-4ac\right)^2 d(bd+2cdx)^{5/2}\sqrt{a+bx+}}{385c^2}$$

$$= \frac{\left(b^2-4ac\right)^3 d^3\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{231c^2} + \frac{\left(b^2-4ac\right)^2 d(bd+2cdx)^{5/2}\sqrt{a+bx+}}{385c^2}$$

$$= \frac{\left(b^2-4ac\right)^3 d^3\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{231c^2} + \frac{\left(b^2-4ac\right)^2 d(bd+2cdx)^{5/2}\sqrt{a+bx+}}{385c^2}$$

$$= \frac{\left(b^2-4ac\right)^3 d^3\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{231c^2} + \frac{\left(b^2-4ac\right)^2 d(bd+2cdx)^{5/2}\sqrt{a+bx+}}{385c^2}$$

Mathematica [C] time = 0.411386, size = 161, normalized size = 0.59

$$\frac{4\sqrt{a+x(b+cx)}(d(b+2cx))^{7/2}\left(11(b+2cx)^2(a+x(b+cx))^2-10c\left(a-\frac{b^2}{4c}\right)\left(2(a+x(b+cx))^2-\frac{\left(b^2-4ac\right)^2\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{16c^2\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}\right.\right.}{165(b+2cx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(7/2)*(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (4*(d*(b + 2*c*x))^(7/2)*Sqrt[a + x*(b + c*x)]*(11*(b + 2*c*x)^2*(a + x*(b
+ c*x))^2 - 10*(a - b^2/(4*c))*c*(2*(a + x*(b + c*x))^2 - ((b^2 - 4*a*c)^2*
Hypergeometric2F1[-3/2, 1/4, 5/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(16*c^2*Sqr
t[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]))))/(165*(b + 2*c*x)^3)
```

Maple [B] time = 0.271, size = 1057, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(7/2)*(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 1/4620*(d*(2*c*x+b))^(1/2)*(c*x^2+b*x+a)^(1/2)*d^3*(44352*x^8*b*c^8+25088*x
^7*a*c^8+82432*x^7*b^2*c^7+81536*x^6*b^3*c^6+16768*x^5*a^2*c^7+45736*x^5*b^
4*c^5+13988*x^4*b^5*c^4-1024*x^3*a^3*c^6+1852*x^3*b^6*c^3-10*x^2*b^7*c^2-25
60*x*a^4*c^5-10*x*b^8*c-1280*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2
))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c
*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-
4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a^3*b^2*c^3-12
80*a^4*b*c^4+1152*a^3*b^3*c^3+140*a^2*b^5*c^2-10*a*b^7*c+87808*x^6*a*b*c^7+
123328*x^5*a*b^2*c^6+41920*x^4*a^2*b*c^6+88800*x^4*a*b^3*c^5+42688*x^3*a^2*
b^2*c^5+33728*x^3*a*b^4*c^4-1536*x^2*a^3*b*c^5+22112*x^2*a^2*b^3*c^4+5696*x
^2*a*b^5*c^3+1792*x*a^3*b^2*c^4+4856*x*a^2*b^4*c^3+140*x*a*b^6*c^2+480*(-4*
a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2
*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^
2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1
/2))^(1/2)*2^(1/2),2^(1/2))*a^2*b^4*c^2-80*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4
*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^
(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/
2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*
a*b^6*c+9856*x^9*c^9+5*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4
*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4
*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+
b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*b^8+1280*(-4*a*c+b^2
)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)
/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2
))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1
/2)*2^(1/2),2^(1/2))*a^4*c^4)/c^3/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (2\,cdx + bd)^{\frac{7}{2}}\left(cx^2 + bx + a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(7/2)*(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^(7/2)*(c*x^2 + b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\Big(\Big(8\,c^4d^3x^5+20\,bc^3d^3x^4+ab^3d^3+2\left(9\,b^2c^2+4\,ac^3\right)d^3x^3+\left(7\,b^3c+12\,abc^2\right)d^3x^2+\left(b^4+6\,ab^2c\right)d^3x\Big)\sqrt{2\,cdx+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(7/2)*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((8*c^4*d^3*x^5 + 20*b*c^3*d^3*x^4 + a*b^3*d^3 + 2*(9*b^2*c^2 + 4*a
*c^3)*d^3*x^3 + (7*b^3*c + 12*a*b*c^2)*d^3*x^2 + (b^4 + 6*a*b^2*c)*d^3*x)*s
qrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(7/2)*(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (2\,cdx+bd)^{\frac{7}{2}}\left(cx^2+bx+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(7/2)*(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^(7/2)*(c*x^2 + b*x + a)^(3/2), x)
```

3.1338 $\int (bd+2cdx)^{3/2} \left(a+bx+cx^2\right)^{3/2} dx$

Optimal. Leaf size=227

$$\frac{d^{3/2}\left(b^2-4ac\right)^{13/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{154c^3\sqrt{a+bx+cx^2}}+\frac{d\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}}{77c^2}-\frac{3\left(b^2\right.}{}$$

```
[Out] ((b^2 - 4*a*c)^2*d*Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/(77*c^2) - (3
*(b^2 - 4*a*c)*(b*d + 2*c*d*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(154*c^2*d) + (
(b*d + 2*c*d*x)^(5/2)*(a + b*x + c*x^2)^(3/2))/(11*c*d) + ((b^2 - 4*a*c)^(1
3/4)*d^(3/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[
Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(154*c^3*Sqrt[a +
b*x + c*x^2])
```

Rubi [A] time = 0.183112, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {685, 692, 691, 689, 221}

$$\frac{d^{3/2}\left(b^2-4ac\right)^{13/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{154c^3\sqrt{a+bx+cx^2}}+\frac{d\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}}{77c^2}-\frac{3\left(b^2-4ac\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] ((b^2 - 4*a*c)^2*d*Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/(77*c^2) - (3
*(b^2 - 4*a*c)*(b*d + 2*c*d*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(154*c^2*d) + (
(b*d + 2*c*d*x)^(5/2)*(a + b*x + c*x^2)^(3/2))/(11*c*d) + ((b^2 - 4*a*c)^(1
3/4)*d^(3/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[
Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(154*c^3*Sqrt[a +
b*x + c*x^2])
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int (bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{3/2}dx = \frac{(bd+2cdx)^{5/2}\left(a+bx+cx^2\right)^{3/2}}{11cd} - \frac{\left(3\left(b^2-4ac\right)\right)\int (bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}\,d}{22c}$$

$$= -\frac{3\left(b^2-4ac\right)(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}}{154c^2d} + \frac{(bd+2cdx)^{5/2}\left(a+bx+cx^2\right)^{3/2}}{11cd} + $$

$$= \frac{\left(b^2-4ac\right)^2 d\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{77c^2} - \frac{3\left(b^2-4ac\right)(bd+2cdx)^{5/2}\sqrt{a+bx+c}}{154c^2d}$$

$$= \frac{\left(b^2-4ac\right)^2 d\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{77c^2} - \frac{3\left(b^2-4ac\right)(bd+2cdx)^{5/2}\sqrt{a+bx+c}}{154c^2d}$$

$$= \frac{\left(b^2-4ac\right)^2 d\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{77c^2} - \frac{3\left(b^2-4ac\right)(bd+2cdx)^{5/2}\sqrt{a+bx+c}}{154c^2d}$$

$$= \frac{\left(b^2-4ac\right)^2 d\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{77c^2} - \frac{3\left(b^2-4ac\right)(bd+2cdx)^{5/2}\sqrt{a+bx+c}}{154c^2d}$$

Mathematica [C] time = 0.0892676, size = 117, normalized size = 0.52

$$\frac{2}{11}d\sqrt{a+x(b+cx)}\sqrt{d(b+2cx)}\left(2(a+x(b+cx))^2-\frac{\left(b^2-4ac\right)^2\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{16c^2\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*d*Sqrt[d*(b + 2*c*x)]*Sqrt[a + x*(b + c*x)]*(2*(a + x*(b + c*x))^2 - ((b
^2 - 4*a*c)^2*Hypergeometric2F1[-3/2, 1/4, 5/4, (b + 2*c*x)^2/(b^2 - 4*a*c)
```

```
])/(16*c^2*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]))))/11
```

Maple [B] time = 0.214, size = 796, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(3/2)*(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -1/308*(d*(2*c*x+b))^(1/2)*(c*x^2+b*x+a)^(1/2)*d*(-224*x^7*c^7-784*x^6*b*c^
6-640*x^5*a*c^6-1016*x^5*b^2*c^5+64*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*(
(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2
*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a^3*c^3
-48*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1
/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-
4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c
+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a^2*b^2*c^2+12*(-4*a*c+b^2)^(1/2)*((b+2
*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)
^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*Elli
pticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2
^(1/2))*a*b^4*c-(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^
2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*b^6-1600*x^4*a*b*c^5-580*x^
4*b^3*c^4-544*x^3*a^2*c^5-1328*x^3*a*b^2*c^4-124*x^3*b^4*c^3-816*x^2*a^2*b*
c^4-392*x^2*a*b^3*c^3+2*x^2*b^5*c^2-128*x*a^3*c^4-312*x*a^2*b^2*c^3-20*x*a*
b^4*c^2+2*x*b^6*c-64*a^3*b*c^3-20*a^2*b^3*c^2+2*a*b^5*c)/c^3/(2*c^2*x^3+3*b
*c*x^2+2*a*c*x+b^2*x+a*b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (2\,cdx + bd)^{\frac{3}{2}} \left(cx^2 + bx + a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)*(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^(3/2)*(c*x^2 + b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(2\,c^2dx^3 + 3\,bcdx^2 + abd + \left(b^2 + 2\,ac\right)dx\right)\sqrt{2\,cdx + bd}\sqrt{cx^2 + bx + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((2*c^2*d*x^3 + 3*b*c*d*x^2 + a*b*d + (b^2 + 2*a*c)*d*x)*sqrt(2*c*d
*x + b*d)*sqrt(c*x^2 + b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d(b+2cx))^{\frac{3}{2}}\left(a+bx+cx^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(3/2)*(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d*(b + 2*c*x))**(3/2)*(a + b*x + c*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (2\,cdx+bd)^{\frac{3}{2}}\left(cx^2+bx+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)*(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^(3/2)*(c*x^2 + b*x + a)^(3/2), x)
```

3.1339 $\int \frac{(a+bx+cx^2)^{3/2}}{\sqrt{bd+2cdx}}\,dx$

Optimal. Leaf size=182

$$\frac{(b^2-4ac)^{9/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{14c^3\sqrt{d}\sqrt{a+bx+cx^2}}-\frac{(b^2-4ac)\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}}{14c^2d}+\frac{(a+bx+cx^2)}{}$$

```
[Out] -((b^2 - 4*a*c)*Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/(14*c^2*d) + (Sq
rt[b*d + 2*c*d*x]*(a + b*x + c*x^2)^(3/2))/(7*c*d) + ((b^2 - 4*a*c)^(9/4)*S
qrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c
*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(14*c^3*Sqrt[d]*Sqrt[a + b*x + c
*x^2])
```

Rubi [A] time = 0.149949, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {685, 691, 689, 221}

$$-\frac{(b^2-4ac)\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}}{14c^2d}+\frac{(b^2-4ac)^{9/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{14c^3\sqrt{d}\sqrt{a+bx+cx^2}}+\frac{(a+bx+cx^2)^{3/2}}{7cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/Sqrt[b*d + 2*c*d*x],x]
```



```
[Out] -((b^2 - 4*a*c)*Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/(14*c^2*d) + (Sq
rt[b*d + 2*c*d*x]*(a + b*x + c*x^2)^(3/2))/(7*c*d) + ((b^2 - 4*a*c)^(9/4)*S
qrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c
*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(14*c^3*Sqrt[d]*Sqrt[a + b*x + c
*x^2])
```


Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
```

```
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{\sqrt{bd+2cdx}}\,dx = \frac{\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2}}{7cd} - \frac{\left(3\left(b^2-4ac\right)\right)\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{bd+2cdx}}\,dx}{14c}$$

$$= -\frac{\left(b^2-4ac\right)\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{14c^2d} + \frac{\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2}}{7cd} + \frac{\left(b^2-4ac\right)^2\int \frac{}{\sqrt{bd+2cd}}}{28c^2}$$

$$= -\frac{\left(b^2-4ac\right)\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{14c^2d} + \frac{\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2}}{7cd} + \frac{\left(\left(b^2-4ac\right)^2\sqrt{-\frac{c(a+}{b^2}}\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{14c^2d} + \frac{\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2}}{7cd} + \frac{\left(\left(b^2-4ac\right)^2\sqrt{-\frac{c(a+}{b^2}}\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{14c^2d} + \frac{\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2}}{7cd} + \frac{\left(b^2-4ac\right)^{9/4}\sqrt{-\frac{c(a+}{b}}}{14c^3}$$

Mathematica [C] time = 0.0705426, size = 99, normalized size = 0.54

$$-\frac{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\sqrt{d(b+2cx)}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{8c^2d\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/Sqrt[b*d + 2*c*d*x],x]
```

```
[Out] -((b^2 - 4*a*c)*Sqrt[d*(b + 2*c*x)]*Sqrt[a + x*(b + c*x)]*Hypergeometric2F1
[-3/2, 1/4, 5/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(8*c^2*d*Sqrt[(c*(a + x*(b +
 c*x)))/(-b^2 + 4*a*c)])
```

Maple [B] time = 0.207, size = 566, normalized size = 3.1

$$\frac{1}{28\,d\left(2\,c^2x^3+3\,bcx^2+2\,acx+b^2x+ab\right)c^3}\sqrt{cx^2+bx+a}\sqrt{d\,(2\,cx+b)}\left(8\,x^5c^5+16\sqrt{\frac{b+2\,cx+\sqrt{-4\,ac+b^2}}{\sqrt{-4\,ac+b^2}}}\sqrt{-\frac{2\,cx}{\sqrt{-4\,ac}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(1/2),x)
```

```
[Out] 1/28*(c*x^2+b*x+a)^(1/2)*(d*(2*c*x+b))^(1/2)/d*(8*x^5*c^5+16*((b+2*c*x+(-4*
a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(
1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2
*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*(
-4*a*c+b^2)^(1/2)*a^2*c^2-8*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2
))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2
))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-
4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2)^(1/2)*a*b^2*c+((b+2*c
*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(
1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*Ellipt
icF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(
1/2))*(-4*a*c+b^2)^(1/2)*b^4+20*x^4*b*c^4+32*x^3*a*c^4+12*x^3*b^2*c^3+48*x^
2*a*b*c^3-2*x^2*b^3*c^2+24*x*a^2*c^3+12*x*a*b^2*c^2-2*x*b^4*c+12*a^2*b*c^2-
2*a*b^3*c)/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}{\sqrt{2\,cdx + bd}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)/sqrt(2*c*d*x + b*d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}{\sqrt{2\,cdx + bd}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^(3/2)/sqrt(2*c*d*x + b*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx + cx^2\right)^{\frac{3}{2}}}{\sqrt{d\,(b + 2cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(2*c*d*x+b*d)**(1/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)/sqrt(d*(b + 2*c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}{\sqrt{2\,cdx + bd}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)/sqrt(2*c*d*x + b*d), x)
```

3.1340 $\int \frac{(a+bx+cx^2)^{3/2}}{(bd+2cdx)^{5/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{\left(b^2-4ac\right)^{5/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{6c^3d^{5/2}\sqrt{a+bx+cx^2}}+\frac{\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}}{6c^2d^3}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3cd(bd+2cdx)^{3/2}}$$

```
[Out] (Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/(6*c^2*d^3) - (a + b*x + c*x^2)
^(3/2)/(3*c*d*(b*d + 2*c*d*x)^(3/2)) - ((b^2 - 4*a*c)^(5/4)*Sqrt[-((c*(a +
b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 -
4*a*c)^(1/4)*Sqrt[d])], -1])/(6*c^3*d^(5/2)*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.143503, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {684, 685, 691, 689, 221}

$$-\frac{\left(b^2-4ac\right)^{5/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{6c^3d^{5/2}\sqrt{a+bx+cx^2}}+\frac{\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}}{6c^2d^3}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3cd(bd+2cdx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^(5/2),x]
```


```
[Out] (Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/(6*c^2*d^3) - (a + b*x + c*x^2)
^(3/2)/(3*c*d*(b*d + 2*c*d*x)^(3/2)) - ((b^2 - 4*a*c)^(5/4)*Sqrt[-((c*(a +
b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 -
4*a*c)^(1/4)*Sqrt[d])], -1])/(6*c^3*d^(5/2)*Sqrt[a + b*x + c*x^2])
```


Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(bd+2cdx)^{5/2}}\,dx = -\frac{\left(a+bx+cx^2\right)^{3/2}}{3cd(bd+2cdx)^{3/2}}+\frac{\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{bd+2cdx}}\,dx}{2cd^2}$$

$$=\frac{\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{6c^2d^3}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3cd(bd+2cdx)^{3/2}}-\frac{\left(b^2-4ac\right)\int \frac{1}{\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}\,dx}{12c^2d^2}$$

$$=\frac{\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{6c^2d^3}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3cd(bd+2cdx)^{3/2}}-\frac{\left(\left(b^2-4ac\right)\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\int \frac{1}{\sqrt{bd+2cdx}\sqrt{-\frac{ac}{b^2-4a}}}}{12c^2d^2\sqrt{a+bx+cx^2}}$$

$$=\frac{\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{6c^2d^3}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3cd(bd+2cdx)^{3/2}}-\frac{\left(\left(b^2-4ac\right)\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{}{(b^2-}}}\right.}{6c^3d^3\sqrt{a+bx+cx^2}}$$

$$=\frac{\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{6c^2d^3}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3cd(bd+2cdx)^{3/2}}-\frac{\left(b^2-4ac\right)^{5/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2c}}{\sqrt[4]{b^2-4ac}}\right.\right.}{6c^3d^{5/2}\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 0.0578706, size = 99, normalized size = 0.57

$$\frac{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};\frac{1}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{24c^2d\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}(d(b+2cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^(5/2),x]
```

```
[Out] ((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]*Hypergeometric2F1[-3/2, -3/4, 1/4, (b
+ 2*c*x)^2/(b^2 - 4*a*c)])/(24*c^2*d*(d*(b + 2*c*x))^(3/2)*Sqrt[(c*(a + x*(
b + c*x)))/(-b^2 + 4*a*c)])
```

Maple [B] time = 0.216, size = 626, normalized size = 3.6

$$\frac{1}{12\,d^3\,(2\,cx+b)^2\,c^3}\left(8\,\sqrt{\frac{b+2\,cx+\sqrt{-4\,ac+b^2}}{\sqrt{-4\,ac+b^2}}}\sqrt{-\frac{2\,cx+b}{\sqrt{-4\,ac+b^2}}}\sqrt{\frac{-b-2\,cx+\sqrt{-4\,ac+b^2}}{\sqrt{-4\,ac+b^2}}}\mathrm{EllipticF}\left(1/2\,\sqrt{\frac{b+2\,cx}{\sqrt{-}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(5/2),x)
```

```
[Out] 1/12*(8*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)
/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2
))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1
/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2)^(1/2)*x*a*c^2-2*(-4*a*c+b^2)^(1/2)*((b+2*
c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^
(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*Ellip
ticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^
(1/2))*x*b^2*c+4*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-
(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^
(1/2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2)^(1/2)*a*b*c-(-4*a*c+b^2)^(1/2)*(
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+
b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*
EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/
2),2^(1/2))*b^3+4*c^4*x^4+8*b*c^3*x^3+6*x^2*b^2*c^2+2*b^3*c*x-4*a^2*c^2+2*a
*c*b^2)/d^3*(d*(2*c*x+b))^(1/2)/(c*x^2+b*x+a)^(1/2)/(2*c*x+b)^2/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{3}{2}}}{(2\,cdx+bd)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)/(2*c*d*x + b*d)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{2\,cdx+bd}\left(cx^2+bx+a\right)^{\frac{3}{2}}}{8\,c^3d^3x^3+12\,bc^2d^3x^2+6\,b^2cd^3x+b^3d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*c*d*x + b*d)*(c*x^2 + b*x + a)^(3/2)/(8*c^3*d^3*x^3 + 12*b*
c^2*d^3*x^2 + 6*b^2*c*d^3*x + b^3*d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx+cx^2\right)^{\frac{3}{2}}}{\left(d\left(b+2cx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(2*c*d*x+b*d)**(5/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)/(d*(b + 2*c*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{3}{2}}}{\left(2\,cdx+bd\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)/(2*c*d*x + b*d)^(5/2), x)
```

3.1341 $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(bd+2cdx)^{9/2}}\,dx$

Optimal. Leaf size=174

$$\frac{\sqrt[4]{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{14c^3d^{9/2}\sqrt{a+bx+cx^2}}-\frac{\sqrt{a+bx+cx^2}}{14c^2d^3(bd+2cdx)^{3/2}}-\frac{\left(a+bx+cx^2\right)^{3/2}}{7cd(bd+2cdx)^{7/2}}$$

```
[Out] -Sqrt[a + b*x + c*x^2]/(14*c^2*d^3*(b*d + 2*c*d*x)^(3/2)) - (a + b*x + c*x^
2)^(3/2)/(7*c*d*(b*d + 2*c*d*x)^(7/2)) + ((b^2 - 4*a*c)^(1/4)*Sqrt[-((c*(a
+ b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2
- 4*a*c)^(1/4)*Sqrt[d])], -1])/(14*c^3*d^(9/2)*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.14244, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {684, 691, 689, 221}

$$\frac{\sqrt[4]{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{14c^3d^{9/2}\sqrt{a+bx+cx^2}}-\frac{\sqrt{a+bx+cx^2}}{14c^2d^3(bd+2cdx)^{3/2}}-\frac{\left(a+bx+cx^2\right)^{3/2}}{7cd(bd+2cdx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^(9/2),x]
```

```
[Out] -Sqrt[a + b*x + c*x^2]/(14*c^2*d^3*(b*d + 2*c*d*x)^(3/2)) - (a + b*x + c*x^
2)^(3/2)/(7*c*d*(b*d + 2*c*d*x)^(7/2)) + ((b^2 - 4*a*c)^(1/4)*Sqrt[-((c*(a
+ b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2
- 4*a*c)^(1/4)*Sqrt[d])], -1])/(14*c^3*d^(9/2)*Sqrt[a + b*x + c*x^2])
```

Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(bd+2cdx)^{9/2}}\,dx = -\frac{\left(a+bx+cx^2\right)^{3/2}}{7cd(bd+2cdx)^{7/2}}+\frac{3\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^{5/2}}\,dx}{14cd^2}$$

$$=-\frac{\sqrt{a+bx+cx^2}}{14c^2d^3(bd+2cdx)^{3/2}}-\frac{\left(a+bx+cx^2\right)^{3/2}}{7cd(bd+2cdx)^{7/2}}+\frac{\int \frac{1}{\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}\,dx}{28c^2d^4}$$

$$=-\frac{\sqrt{a+bx+cx^2}}{14c^2d^3(bd+2cdx)^{3/2}}-\frac{\left(a+bx+cx^2\right)^{3/2}}{7cd(bd+2cdx)^{7/2}}+\frac{\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\int \frac{1}{\sqrt{bd+2cdx}\sqrt{-\frac{ac}{b^2-4ac}-\frac{bcx}{b^2-4ac}-\frac{c^2x^2}{b^2-4ac}}}\,dx}{28c^2d^4\sqrt{a+bx+cx^2}}$$

$$=-\frac{\sqrt{a+bx+cx^2}}{14c^2d^3(bd+2cdx)^{3/2}}-\frac{\left(a+bx+cx^2\right)^{3/2}}{7cd(bd+2cdx)^{7/2}}+\frac{\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{\left(b^2-4ac\right)d^2}}}\,dx,x,\sqrt{bd\cdots}\right)}{14c^3d^5\sqrt{a+bx+cx^2}}$$

$$=-\frac{\sqrt{a+bx+cx^2}}{14c^2d^3(bd+2cdx)^{3/2}}-\frac{\left(a+bx+cx^2\right)^{3/2}}{7cd(bd+2cdx)^{7/2}}+\frac{\sqrt[4]{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-\cdots\right)}{14c^3d^{9/2}\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 0.0820332, size = 107, normalized size = 0.61

$$\frac{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\sqrt{d(b+2cx)}\,{}_2F_1\left(-\frac{7}{4},-\frac{3}{2};-\frac{3}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{56c^2d^5(b+2cx)^4\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^(9/2),x]
```

```
[Out] ((b^2 - 4*a*c)*Sqrt[d*(b + 2*c*x)]*Sqrt[a + x*(b + c*x)]*Hypergeometric2F1[
-7/4, -3/2, -3/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(56*c^2*d^5*(b + 2*c*x)^4*S
qrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)])
```

Maple [B] time = 0.22, size = 678, normalized size = 3.9

$$\frac{1}{28\,d^5\left(2\,c^2x^3+3\,bcx^2+2\,acx+b^2x+ab\right)(2\,cx+b)^3\,c^3}\sqrt{cx^2+bx+a}\sqrt{d\,(2\,cx+b)}\left(8\sqrt{-4\,ac+b^2}\sqrt{\frac{b+2\,cx+\sqrt{-4\,ac+\cdots}}{\sqrt{-4\,ac+b^2}}}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(9/2),x)
```

```
[Out] 1/28*(c*x^2+b*x+a)^(1/2)*(d*(2*c*x+b))^(1/2)*(8*(-4*a*c+b^2)^(1/2)*((b+2*c*
x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1
```

```
/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*Ellipti
cF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1
/2))*x^3*c^3+12*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^
2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^2*b*c^2+6*(-4*a*c+b^2)^(1
/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4
*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(
1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*
2^(1/2),2^(1/2))*x*b^2*c+(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(
-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(
-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*
c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*b^3-12*c^4*x^4-24*
b*c^3*x^3-16*x^2*a*c^3-14*x^2*b^2*c^2-16*b*a*c^2*x-2*b^3*c*x-4*a^2*c^2-2*a*
c*b^2)/d^5/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)/(2*c*x+b)^3/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{3}{2}}}{(2\,cdx+bd)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)/(2*c*d*x + b*d)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,cdx+bd}\left(cx^2+bx+a\right)^{\frac{3}{2}}}{32\,c^5d^5x^5+80\,bc^4d^5x^4+80\,b^2c^3d^5x^3+40\,b^3c^2d^5x^2+10\,b^4cd^5x+b^5d^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(9/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*c*d*x + b*d)*(c*x^2 + b*x + a)^(3/2)/(32*c^5*d^5*x^5 + 80*b
*c^4*d^5*x^4 + 80*b^2*c^3*d^5*x^3 + 40*b^3*c^2*d^5*x^2 + 10*b^4*c*d^5*x + b
^5*d^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(2*c*d*x+b*d)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}{\left(2\,cdx + bd\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)/(2*c*d*x + b*d)^(9/2), x)
```

3.1342 $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(bd+2cdx)^{13/2}}\,dx$

Optimal. Leaf size=221

$$\frac{\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{154c^3d^{13/2}\left(b^2-4ac\right)^{3/4}\sqrt{a+bx+cx^2}}+\frac{\sqrt{a+bx+cx^2}}{77c^2d^5\left(b^2-4ac\right)(bd+2cdx)^{3/2}}-\frac{3\sqrt{a+bx+cx^2}}{154c^2d^3(bd+2cdx)^{7/2}}-\frac{(a+\ldots}{11cd(\ldots}$$

```
[Out] (-3*Sqrt[a + b*x + c*x^2])/(154*c^2*d^3*(b*d + 2*c*d*x)^(7/2)) + Sqrt[a + b
*x + c*x^2]/(77*c^2*(b^2 - 4*a*c)*d^5*(b*d + 2*c*d*x)^(3/2)) - (a + b*x + c
*x^2)^(3/2)/(11*c*d*(b*d + 2*c*d*x)^(11/2)) + (Sqrt[-((c*(a + b*x + c*x^2))
/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*
Sqrt[d])], -1])/(154*c^3*(b^2 - 4*a*c)^(3/4)*d^(13/2)*Sqrt[a + b*x + c*x^2]
)
```

Rubi [A] time = 0.18009, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {684, 693, 691, 689, 221}

$$\frac{\sqrt{a+bx+cx^2}}{77c^2d^5\left(b^2-4ac\right)(bd+2cdx)^{3/2}}+\frac{\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{154c^3d^{13/2}\left(b^2-4ac\right)^{3/4}\sqrt{a+bx+cx^2}}-\frac{3\sqrt{a+bx+cx^2}}{154c^2d^3(bd+2cdx)^{7/2}}-\frac{\left(a+bx+cx\ldots\right.}{11cd(bd+2c\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^(13/2),x]
```

```
[Out] (-3*Sqrt[a + b*x + c*x^2])/(154*c^2*d^3*(b*d + 2*c*d*x)^(7/2)) + Sqrt[a + b
*x + c*x^2]/(77*c^2*(b^2 - 4*a*c)*d^5*(b*d + 2*c*d*x)^(3/2)) - (a + b*x + c
*x^2)^(3/2)/(11*c*d*(b*d + 2*c*d*x)^(11/2)) + (Sqrt[-((c*(a + b*x + c*x^2))
/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*
Sqrt[d])], -1])/(154*c^3*(b^2 - 4*a*c)^(3/4)*d^(13/2)*Sqrt[a + b*x + c*x^2]
)
```

Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(bd+2cdx)^{13/2}}\,dx = -\frac{\left(a+bx+cx^2\right)^{3/2}}{11cd(bd+2cdx)^{11/2}}+\frac{3\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^{9/2}}\,dx}{22cd^2}$$

$$= -\frac{3\sqrt{a+bx+cx^2}}{154c^2d^3(bd+2cdx)^{7/2}}-\frac{\left(a+bx+cx^2\right)^{3/2}}{11cd(bd+2cdx)^{11/2}}+\frac{3\int \frac{1}{(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}}\,dx}{308c^2d^4}$$

$$= -\frac{3\sqrt{a+bx+cx^2}}{154c^2d^3(bd+2cdx)^{7/2}}+\frac{\sqrt{a+bx+cx^2}}{77c^2\left(b^2-4ac\right)d^5(bd+2cdx)^{3/2}}-\frac{\left(a+bx+cx^2\right)^{3/2}}{11cd(bd+2cdx)^{11/2}}+\frac{\int \frac{}{\sqrt{bd+2cdx}\ldots}}{308c^2\left(b\ldots\right.}$$

$$= -\frac{3\sqrt{a+bx+cx^2}}{154c^2d^3(bd+2cdx)^{7/2}}+\frac{\sqrt{a+bx+cx^2}}{77c^2\left(b^2-4ac\right)d^5(bd+2cdx)^{3/2}}-\frac{\left(a+bx+cx^2\right)^{3/2}}{11cd(bd+2cdx)^{11/2}}+\frac{\sqrt{-\frac{c\left(a+bx\ldots\right.}{b^2-4\ldots}}\ldots}{30\ldots}$$

$$= -\frac{3\sqrt{a+bx+cx^2}}{154c^2d^3(bd+2cdx)^{7/2}}+\frac{\sqrt{a+bx+cx^2}}{77c^2\left(b^2-4ac\right)d^5(bd+2cdx)^{3/2}}-\frac{\left(a+bx+cx^2\right)^{3/2}}{11cd(bd+2cdx)^{11/2}}+\frac{\sqrt{-\frac{c\left(a+bx\ldots\right.}{b^2-4\ldots}}\ldots}{\ldots}$$

$$= -\frac{3\sqrt{a+bx+cx^2}}{154c^2d^3(bd+2cdx)^{7/2}}+\frac{\sqrt{a+bx+cx^2}}{77c^2\left(b^2-4ac\right)d^5(bd+2cdx)^{3/2}}-\frac{\left(a+bx+cx^2\right)^{3/2}}{11cd(bd+2cdx)^{11/2}}+\frac{\sqrt{-\frac{c\left(a+bx\ldots\right.}{b^2-4\ldots}}\ldots}{154c^3\left(b\ldots\right.}$$

Mathematica [C] time = 0.086596, size = 107, normalized size = 0.48

$$\frac{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\sqrt{d(b+2cx)}\,{}_2F_1\left(-\frac{11}{4},-\frac{3}{2};-\frac{7}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{88c^2d^7(b+2cx)^6\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^(13/2),x]
```

```
[Out] ((b^2 - 4*a*c)*Sqrt[d*(b + 2*c*x)]*Sqrt[a + x*(b + c*x)]*Hypergeometric2F1[
-11/4, -3/2, -7/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(88*c^2*d^7*(b + 2*c*x)^6*
Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)])
```

Maple [B] time = 0.218, size = 1046, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(13/2),x)
```

```
[Out] -1/308*(c*x^2+b*x+a)^(1/2)*(d*(2*c*x+b))^(1/2)*(32*(-4*a*c+b^2)^(1/2)*((b+2
*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)
^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*Elli
pticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2
^(1/2))*x^5*c^5+80*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c
+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c
+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)
^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^4*b*c^4+80*(-4*a*c+b^2
)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)
/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2
))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1
/2)*2^(1/2),2^(1/2))*x^3*b^2*c^3+40*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*(
(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2
*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^2*b^3
*c^2+64*x^6*c^6+10*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c
+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c
+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)
^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x*b^4*c+192*x^5*b*c^5+(-
4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-
(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^
(1/2))^(1/2)*2^(1/2),2^(1/2))*b^5+272*x^4*a*c^5+172*x^4*b^2*c^4+544*x^3*a*b
*c^4+24*x^3*b^3*c^3+320*x^2*a^2*c^4+248*x^2*a*b^2*c^3-22*x^2*b^4*c^2+320*x*
a^2*b*c^3-24*x*a*b^3*c^2-2*x*b^5*c+112*a^3*c^3-4*a^2*b^2*c^2-2*a*b^4*c)/d^7
/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)/(2*c*x+b)^5/(4*a*c-b^2)/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}{(2\,cdx + bd)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(13/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)/(2*c*d*x + b*d)^(13/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,cdx+bd}\left(cx^2+bx+a\right)^{\frac{3}{2}}}{128\,c^7d^7x^7+448\,bc^6d^7x^6+672\,b^2c^5d^7x^5+560\,b^3c^4d^7x^4+280\,b^4c^3d^7x^3+84\,b^5c^2d^7x^2+14\,b^6cd^7x+b^7d^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(13/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*c*d*x + b*d)*(c*x^2 + b*x + a)^(3/2)/(128*c^7*d^7*x^7 + 448
*b*c^6*d^7*x^6 + 672*b^2*c^5*d^7*x^5 + 560*b^3*c^4*d^7*x^4 + 280*b^4*c^3*d^
7*x^3 + 84*b^5*c^2*d^7*x^2 + 14*b^6*c*d^7*x + b^7*d^7), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(2*c*d*x+b*d)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{3}{2}}}{\left(2\,cdx+bd\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(13/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)/(2*c*d*x + b*d)^(13/2), x)
```

3.1343 $\int \frac{(a+bx+cx^2)^{3/2}}{(bd+2cdx)^{17/2}}\,dx$

Optimal. Leaf size=268

$$\frac{\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{462c^3d^{17/2}(b^2-4ac)^{7/4}\sqrt{a+bx+cx^2}}+\frac{\sqrt{a+bx+cx^2}}{231c^2d^7(b^2-4ac)^2(bd+2cdx)^{3/2}}+\frac{\sqrt{a+bx+cx^2}}{385c^2d^5(b^2-4ac)(bd+2cd$$

```
[Out] -Sqrt[a + b*x + c*x^2]/(110*c^2*d^3*(b*d + 2*c*d*x)^(11/2)) + Sqrt[a + b*x
+ c*x^2]/(385*c^2*(b^2 - 4*a*c)*d^5*(b*d + 2*c*d*x)^(7/2)) + Sqrt[a + b*x +
 c*x^2]/(231*c^2*(b^2 - 4*a*c)^2*d^7*(b*d + 2*c*d*x)^(3/2)) - (a + b*x + c*
x^2)^(3/2)/(15*c*d*(b*d + 2*c*d*x)^(15/2)) + (Sqrt[-((c*(a + b*x + c*x^2))/
(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*S
qrt[d])], -1])/(462*c^3*(b^2 - 4*a*c)^(7/4)*d^(17/2)*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.218567, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {684, 693, 691, 689, 221}

$$\frac{\sqrt{a+bx+cx^2}}{231c^2d^7(b^2-4ac)^2(bd+2cdx)^{3/2}}+\frac{\sqrt{a+bx+cx^2}}{385c^2d^5(b^2-4ac)(bd+2cdx)^{7/2}}+\frac{\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{462c^3d^{17/2}(b^2-4ac)^{7/4}\sqrt{a+bx+cx^2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^(17/2),x]
```


```
[Out] -Sqrt[a + b*x + c*x^2]/(110*c^2*d^3*(b*d + 2*c*d*x)^(11/2)) + Sqrt[a + b*x
+ c*x^2]/(385*c^2*(b^2 - 4*a*c)*d^5*(b*d + 2*c*d*x)^(7/2)) + Sqrt[a + b*x +
 c*x^2]/(231*c^2*(b^2 - 4*a*c)^2*d^7*(b*d + 2*c*d*x)^(3/2)) - (a + b*x + c*
x^2)^(3/2)/(15*c*d*(b*d + 2*c*d*x)^(15/2)) + (Sqrt[-((c*(a + b*x + c*x^2))/
(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*S
qrt[d])], -1])/(462*c^3*(b^2 - 4*a*c)^(7/4)*d^(17/2)*Sqrt[a + b*x + c*x^2])
```


Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(bd+2cdx)^{17/2}}\,dx = -\frac{\left(a+bx+cx^2\right)^{3/2}}{15cd(bd+2cdx)^{15/2}}+\frac{\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^{13/2}}\,dx}{10cd^2}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{110c^2d^3(bd+2cdx)^{11/2}}-\frac{\left(a+bx+cx^2\right)^{3/2}}{15cd(bd+2cdx)^{15/2}}+\frac{\int \frac{1}{(bd+2cdx)^{9/2}\sqrt{a+bx+cx^2}}\,dx}{220c^2d^4}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{110c^2d^3(bd+2cdx)^{11/2}}+\frac{\sqrt{a+bx+cx^2}}{385c^2\left(b^2-4ac\right)d^5(bd+2cdx)^{7/2}}-\frac{\left(a+bx+cx^2\right)^{3/2}}{15cd(bd+2cdx)^{15/2}}+\frac{\int \frac{1}{(bd+2c\ldots}}{308c\ldots}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{110c^2d^3(bd+2cdx)^{11/2}}+\frac{\sqrt{a+bx+cx^2}}{385c^2\left(b^2-4ac\right)d^5(bd+2cdx)^{7/2}}+\frac{\sqrt{a+bx+cx^2}}{231c^2\left(b^2-4ac\right)^2d^7(bd+2cdx\ldots}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{110c^2d^3(bd+2cdx)^{11/2}}+\frac{\sqrt{a+bx+cx^2}}{385c^2\left(b^2-4ac\right)d^5(bd+2cdx)^{7/2}}+\frac{\sqrt{a+bx+cx^2}}{231c^2\left(b^2-4ac\right)^2d^7(bd+2cdx\ldots}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{110c^2d^3(bd+2cdx)^{11/2}}+\frac{\sqrt{a+bx+cx^2}}{385c^2\left(b^2-4ac\right)d^5(bd+2cdx)^{7/2}}+\frac{\sqrt{a+bx+cx^2}}{231c^2\left(b^2-4ac\right)^2d^7(bd+2cdx\ldots}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{110c^2d^3(bd+2cdx)^{11/2}}+\frac{\sqrt{a+bx+cx^2}}{385c^2\left(b^2-4ac\right)d^5(bd+2cdx)^{7/2}}+\frac{\sqrt{a+bx+cx^2}}{231c^2\left(b^2-4ac\right)^2d^7(bd+2cdx\ldots}$$

Mathematica [C] time = 0.0937255, size = 107, normalized size = 0.4

$$\frac{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\sqrt{d(b+2cx)}\,{}_2F_1\left(-\frac{15}{4},-\frac{3}{2};-\frac{11}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{120c^2d^9(b+2cx)^8\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^(17/2),x]
```

```
[Out] ((b^2 - 4*a*c)*Sqrt[d*(b + 2*c*x)]*Sqrt[a + x*(b + c*x)]*Hypergeometric2F1[
-15/4, -3/2, -11/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(120*c^2*d^9*(b + 2*c*x)^
8*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)])
```

Maple [B] time = 0.284, size = 1431, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(17/2),x)
```

```
[Out] 1/4620*(c*x^2+b*x+a)^(1/2)*(d*(2*c*x+b))^(1/2)*(2240*((b+2*c*x+(-4*a*c+b^2)
^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-
b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c
*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b
^2)^(1/2)*x^6*b*c^6+3360*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^
(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/
(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a
*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2)^(1/2)*x^5*b^2*c^5+1280*x
^8*c^8-10*x*b^7*c+70*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2
)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*
a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b
^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2)^(1/2)*x*b^6*c+420*((b+2*c*x+
(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2
))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF
(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2
))*(-4*a*c+b^2)^(1/2)*x^2*b^5*c^2+2800*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*
c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*
c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2
)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2)^(1/2)*x^4*
b^3*c^4+1400*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c
*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2
))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2)^(1/2)*x^3*b^4*c^3+640*((b+2*c*x+(-4*
a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(
1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2
*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*(
-4*a*c+b^2)^(1/2)*x^7*c^7+4596*x^4*b^4*c^4+8576*x^5*b^3*c^5+512*x^6*a*c^7-8
384*x^4*a^2*c^6-12544*x^2*a^3*c^5+5120*x^7*b*c^7+872*x^3*b^5*c^3-150*x^2*b^
6*c^2+8832*x^6*b^2*c^6-4928*a^4*c^4+1792*a^3*b^2*c^3-20*a^2*b^4*c^2-10*a*b^
6*c-160*x*a*b^5*c^2+5*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/
2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4
*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2)^(1/2)*b^7+1536*x^5*a*b*c^6+
6112*x^4*a*b^2*c^5-16768*x^3*a^2*b*c^5+9664*x^3*a*b^3*c^4-3168*x^2*a^2*b^2*
c^4+4416*x^2*a*b^4*c^3-12544*x*a^3*b*c^4+5216*x*a^2*b^3*c^3)/d^9/(2*c^2*x^3
+3*b*c*x^2+2*a*c*x+b^2*x+a*b)/(2*c*x+b)^7/c^3/(4*a*c-b^2)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)^{\frac{3}{2}}}{(2\,cdx + bd)^{\frac{17}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(17/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)/(2*c*d*x + b*d)^(17/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,cdx + bd}(cx^2 + bx + a)^{\frac{3}{2}}}{512\,c^9d^9x^9 + 2304\,bc^8d^9x^8 + 4608\,b^2c^7d^9x^7 + 5376\,b^3c^6d^9x^6 + 4032\,b^4c^5d^9x^5 + 2016\,b^5c^4d^9x^4 + 672\,b^6c^3d^9x^3 \cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(17/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*c*d*x + b*d)*(c*x^2 + b*x + a)^(3/2)/(512*c^9*d^9*x^9 + 230
4*b*c^8*d^9*x^8 + 4608*b^2*c^7*d^9*x^7 + 5376*b^3*c^6*d^9*x^6 + 4032*b^4*c^
5*d^9*x^5 + 2016*b^5*c^4*d^9*x^4 + 672*b^6*c^3*d^9*x^3 + 144*b^7*c^2*d^9*x^
2 + 18*b^8*c*d^9*x + b^9*d^9), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(2*c*d*x+b*d)**(17/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)^{\frac{3}{2}}}{(2\,cdx + bd)^{\frac{17}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(17/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)/(2*c*d*x + b*d)^(17/2), x)
```

3.1344 $\int (bd+2cdx)^{5/2}\left(a+bx+cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=326

$$-\frac{d^{5/2}\left(b^2-4ac\right)^{15/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{130c^3\sqrt{a+bx+cx^2}}+\frac{d^{5/2}\left(b^2-4ac\right)^{15/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{bc}\cdots}{\sqrt[4]{b^2}\cdots}\right.\right.}{130c^3\sqrt{a+bx+cx^2}}$$

```
[Out] ((b^2 - 4*a*c)^2*d*(b*d + 2*c*d*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(195*c^2) -
 ((b^2 - 4*a*c)*(b*d + 2*c*d*x)^(7/2)*Sqrt[a + b*x + c*x^2])/(78*c^2*d) + (
(b*d + 2*c*d*x)^(7/2)*(a + b*x + c*x^2)^(3/2))/(13*c*d) + ((b^2 - 4*a*c)^(1
5/4)*d^(5/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[
Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(130*c^3*Sqrt[a +
b*x + c*x^2]) - ((b^2 - 4*a*c)^(15/4)*d^(5/2)*Sqrt[-((c*(a + b*x + c*x^2))/
(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*S
qrt[d])], -1])/(130*c^3*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.29612, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {685, 692, 691, 690, 307, 221, 1199, 424}

$$-\frac{d^{5/2}\left(b^2-4ac\right)^{15/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{130c^3\sqrt{a+bx+cx^2}}+\frac{d^{5/2}\left(b^2-4ac\right)^{15/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|\cdots\right.}{130c^3\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(5/2)*(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] ((b^2 - 4*a*c)^2*d*(b*d + 2*c*d*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(195*c^2) -
 ((b^2 - 4*a*c)*(b*d + 2*c*d*x)^(7/2)*Sqrt[a + b*x + c*x^2])/(78*c^2*d) + (
(b*d + 2*c*d*x)^(7/2)*(a + b*x + c*x^2)^(3/2))/(13*c*d) + ((b^2 - 4*a*c)^(1
5/4)*d^(5/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[
Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(130*c^3*Sqrt[a +
b*x + c*x^2]) - ((b^2 - 4*a*c)^(15/4)*d^(5/2)*Sqrt[-((c*(a + b*x + c*x^2))/
(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*S
qrt[d])], -1])/(130*c^3*Sqrt[a + b*x + c*x^2])
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
```

```
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int (bd+2cdx)^{5/2}\left(a+bx+cx^2\right)^{3/2}dx=\frac{(bd+2cdx)^{7/2}\left(a+bx+cx^2\right)^{3/2}}{13cd}-\frac{\left(3\left(b^2-4ac\right)\right)\int (bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}$$

$$=-\frac{\left(b^2-4ac\right)(bd+2cdx)^{7/2}\sqrt{a+bx+cx^2}}{78c^2d}+\frac{(bd+2cdx)^{7/2}\left(a+bx+cx^2\right)^{3/2}}{13cd}+$$

$$=\frac{\left(b^2-4ac\right)^2d(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{195c^2}-\frac{\left(b^2-4ac\right)(bd+2cdx)^{7/2}\sqrt{a+bx}}{78c^2d}$$

$$=\frac{\left(b^2-4ac\right)^2d(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{195c^2}-\frac{\left(b^2-4ac\right)(bd+2cdx)^{7/2}\sqrt{a+bx}}{78c^2d}$$

$$=\frac{\left(b^2-4ac\right)^2d(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{195c^2}-\frac{\left(b^2-4ac\right)(bd+2cdx)^{7/2}\sqrt{a+bx}}{78c^2d}$$

$$=\frac{\left(b^2-4ac\right)^2d(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{195c^2}-\frac{\left(b^2-4ac\right)(bd+2cdx)^{7/2}\sqrt{a+bx}}{78c^2d}$$

$$=\frac{\left(b^2-4ac\right)^2d(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{195c^2}-\frac{\left(b^2-4ac\right)(bd+2cdx)^{7/2}\sqrt{a+bx}}{78c^2d}$$

$$=\frac{\left(b^2-4ac\right)^2d(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{195c^2}-\frac{\left(b^2-4ac\right)(bd+2cdx)^{7/2}\sqrt{a+bx}}{78c^2d}$$

Mathematica [C] time = 0.154901, size = 117, normalized size = 0.36

$$\frac{2}{13}d\sqrt{a+x(b+cx)}(d(b+2cx))^{3/2}\left(2(a+x(b+cx))^2-\frac{\left(b^2-4ac\right)^2{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{16c^2\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(5/2)*(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*d*(d*(b + 2*c*x))^(3/2)*Sqrt[a + x*(b + c*x)]*(2*(a + x*(b + c*x))^2 - (
(b^2 - 4*a*c)^2*Hypergeometric2F1[-3/2, 3/4, 7/4, (b + 2*c*x)^2/(b^2 - 4*a*
c)])/(16*c^2*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)])))/13
```

Maple [B] time = 0.214, size = 938, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(5/2)*(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -1/780*(d*(2*c*x+b))^(1/2)*(c*x^2+b*x+a)^(1/2)*d^2*(-960*x^8*c^8+6*x*b^7*c+
3*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a
*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/
2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^
(1/2),2^(1/2))*b^8-1916*x^4*b^4*c^4-4800*x^5*b^3*c^5-2560*x^6*a*c^7-1856*x^
4*a^2*c^6-256*x^2*a^3*c^5-3840*x^7*b*c^7-312*x^3*b^5*c^3+10*x^2*b^6*c^2-608
0*x^6*b^2*c^6-64*a^3*b^2*c^3-68*a^2*b^4*c^2+6*a*b^6*c-64*x*a*b^5*c^2-7680*x
^5*a*b*c^6-8672*x^4*a*b^2*c^5-3712*x^3*a^2*b*c^5-4544*x^3*a*b^3*c^4-2592*x^
2*a^2*b^2*c^4-1056*x^2*a*b^4*c^3-256*x*a^3*b*c^4-736*x*a^2*b^3*c^3+768*((b+
2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2
)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*Ell
ipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),
2^(1/2))*a^4*c^4-768*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2
)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*
a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b
^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a^3*b^2*c^3+288*((b+2*c*x+(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2
*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+
(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a^2*b^4*c^2-
48*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*
a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1
/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2
^(1/2),2^(1/2))*a*b^6*c)/c^3/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (2\,cdx + bd)^{\frac{5}{2}} \left(cx^2 + bx + a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(5/2)*(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^(5/2)*(c*x^2 + b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(4\,c^3d^2x^4 + 8\,bc^2d^2x^3 + ab^2d^2 + \left(5\,b^2c + 4\,ac^2\right)d^2x^2 + \left(b^3 + 4\,abc\right)d^2x\right)\sqrt{2\,cdx + bd}\sqrt{cx^2 + bx + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(5/2)*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((4*c^3*d^2*x^4 + 8*b*c^2*d^2*x^3 + a*b^2*d^2 + (5*b^2*c + 4*a*c^2)
*d^2*x^2 + (b^3 + 4*a*b*c)*d^2*x)*sqrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(5/2)*(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (2\,cdx + bd)^{\frac{5}{2}}\left(cx^2 + bx + a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(5/2)*(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^(5/2)*(c*x^2 + b*x + a)^(3/2), x)
```

3.1345 $\int \sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2} dx$

Optimal. Leaf size=281

$$-\frac{\sqrt{d}\left(b^2-4ac\right)^{11/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{30c^3\sqrt{a+bx+cx^2}}-\frac{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}(bd+2cdx)^{3/2}}{30c^2d}+\frac{\sqrt{d}\left(b^2-\right.}{}$$

```
[Out] -((b^2 - 4*a*c)*(b*d + 2*c*d*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(30*c^2*d) + (
(b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)^(3/2))/(9*c*d) + ((b^2 - 4*a*c)^(11
/4)*Sqrt[d]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[S
qrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(30*c^3*Sqrt[a + b*
x + c*x^2]) - ((b^2 - 4*a*c)^(11/4)*Sqrt[d]*Sqrt[-((c*(a + b*x + c*x^2))/(b
^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqr
t[d])], -1])/(30*c^3*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.248778, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {685, 691, 690, 307, 221, 1199, 424}

$$-\frac{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}(bd+2cdx)^{3/2}}{30c^2d}-\frac{\sqrt{d}\left(b^2-4ac\right)^{11/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{30c^3\sqrt{a+bx+cx^2}}+\frac{\sqrt{d}\left(b^2-4ac\right)^{11}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*d + 2*c*d*x]*(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] -((b^2 - 4*a*c)*(b*d + 2*c*d*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(30*c^2*d) + (
(b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)^(3/2))/(9*c*d) + ((b^2 - 4*a*c)^(11
/4)*Sqrt[d]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[S
qrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(30*c^3*Sqrt[a + b*
x + c*x^2]) - ((b^2 - 4*a*c)^(11/4)*Sqrt[d]*Sqrt[-((c*(a + b*x + c*x^2))/(b
^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqr
t[d])], -1])/(30*c^3*Sqrt[a + b*x + c*x^2])
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2} dx = \frac{(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9cd} - \frac{\left(b^2-4ac\right)\int \sqrt{bd+2cdx}\sqrt{a+bx+cx^2}\,dx}{6c}$$

$$= -\frac{\left(b^2-4ac\right)(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{30c^2d} + \frac{(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9cd} + \frac{\left(b^2\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{30c^2d} + \frac{(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9cd} + \frac{\left(\left(b^2\right.\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{30c^2d} + \frac{(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9cd} + \frac{\left(\left(b^2\right.\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{30c^2d} + \frac{(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9cd} - \frac{\left(\left(b^2\right.\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{30c^2d} + \frac{(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9cd} - \frac{\left(b^2\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{30c^2d} + \frac{(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9cd} + \frac{\left(b^2\right.}{}$$

Mathematica [C] time = 0.0585438, size = 99, normalized size = 0.35

$$-\frac{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}(d(b+2cx))^{3/2}\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{24c^2d\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*d + 2*c*d*x]*(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] -((b^2 - 4*a*c)*(d*(b + 2*c*x))^(3/2)*Sqrt[a + x*(b + c*x)]*Hypergeometric2
F1[-3/2, 3/4, 7/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(24*c^2*d*Sqrt[(c*(a + x*(
b + c*x)))/(-b^2 + 4*a*c)])
```

Maple [B] time = 0.209, size = 700, normalized size = 2.5

$$\frac{1}{180\,c^3\left(2\,c^2x^3+3\,bcx^2+2\,acx+b^2x+ab\right)}\sqrt{d\left(2\,cx+b\right)}\sqrt{cx^2+bx+a}\left(80\,x^6c^6+240\,x^5bc^5+192\sqrt{\frac{b+2\,cx+\sqrt{-4\,ac+b^2}}{\sqrt{-4\,ac+b^2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(1/2)*(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 1/180*(d*(2*c*x+b))^(1/2)*(c*x^2+b*x+a)^(1/2)*(80*x^6*c^6+240*x^5*b*c^5+192
*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*
c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2
)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(
1/2),2^(1/2))*a^3*c^3-144*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))
^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))
/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*
a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a^2*b^2*c^2+36*((b+2*c*x+(-4*a*c+b^2
)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((
-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*
c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a*b^4*c-
3*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a
*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/
2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^
(1/2),2^(1/2))*b^6+256*x^4*a*c^5+236*x^4*b^2*c^4+512*x^3*a*b*c^4+72*x^3*b^3
*c^3+176*x^2*a^2*c^4+296*x^2*a*b^2*c^3-10*x^2*b^4*c^2+176*x*a^2*b*c^3+40*x*
a*b^3*c^2-6*x*b^5*c+44*a^2*b^2*c^2-6*a*b^4*c)/c^3/(2*c^2*x^3+3*b*c*x^2+2*a*
c*x+b^2*x+a*b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{2\,cdx+bd}\left(cx^2+bx+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)*(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*c*d*x + b*d)*(c*x^2 + b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{2\,cdx+bd}\left(cx^2+bx+a\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*c*d*x + b*d)*(c*x^2 + b*x + a)^(3/2), x)
```

Sympy [A] time = 9.77876, size = 264, normalized size = 0.94

$$\frac{a\left(bd+2cdx\right)^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{(bd+2cdx)^2e^{i\pi}}{4cd^2\,\mathrm{polar\_lift}\left(a-\frac{b^2}{4c}\right)}\right)\sqrt{\mathrm{polar\_lift}\left(a-\frac{b^2}{4c}\right)}}{4cd\Gamma\left(\frac{7}{4}\right)}-\frac{b^2\left(bd+2cdx\right)^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{(bd+2c\cdots}{4cd^2\,\mathrm{polar\_}\cdots}\right.}{16c^2d\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(1/2)*(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] a*(b*d + 2*c*d*x)**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), (b*d + 2*c*d
*x)**2*exp_polar(I*pi)/(4*c*d**2*polar_lift(a - b**2/(4*c))))*sqrt(polar_li
ft(a - b**2/(4*c)))/(4*c*d*gamma(7/4)) - b**2*(b*d + 2*c*d*x)**(3/2)*gamma(
3/4)*hyper((-1/2, 3/4), (7/4,), (b*d + 2*c*d*x)**2*exp_polar(I*pi)/(4*c*d**
2*polar_lift(a - b**2/(4*c))))*sqrt(polar_lift(a - b**2/(4*c)))/(16*c**2*d*
gamma(7/4)) + (b*d + 2*c*d*x)**(7/2)*gamma(7/4)*hyper((-1/2, 7/4), (11/4,),
 (b*d + 2*c*d*x)**2*exp_polar(I*pi)/(4*c*d**2*polar_lift(a - b**2/(4*c))))*
sqrt(polar_lift(a - b**2/(4*c)))/(16*c**2*d**3*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{2\,cdx + bd}\left(cx^2 + bx + a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)*(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(2*c*d*x + b*d)*(c*x^2 + b*x + a)^(3/2), x)
```

3.1346 $\int \frac{(a+bx+cx^2)^{3/2}}{(bd+2cdx)^{3/2}}\,dx$

Optimal. Leaf size=271

$$\frac{3\left(b^2-4ac\right)^{7/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{10c^3d^{3/2}\sqrt{a+bx+cx^2}}-\frac{3\left(b^2-4ac\right)^{7/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\right|}{10c^3d^{3/2}\sqrt{a+bx+cx^2}}$$

```
[Out] (3*(b*d + 2*c*d*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(10*c^2*d^3) - (a + b*x + c
*x^2)^(3/2)/(c*d*Sqrt[b*d + 2*c*d*x]) - (3*(b^2 - 4*a*c)^(7/4)*Sqrt[-((c*(a
 + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2
 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(10*c^3*d^(3/2)*Sqrt[a + b*x + c*x^2]) + (3
*(b^2 - 4*a*c)^(7/4)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF
[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(10*c^3*d^
(3/2)*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.248365, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {684, 685, 691, 690, 307, 221, 1199, 424}

$$\frac{3\left(b^2-4ac\right)^{7/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\right|-1\right)}{10c^3d^{3/2}\sqrt{a+bx+cx^2}}-\frac{3\left(b^2-4ac\right)^{7/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\right|-1\right)}{10c^3d^{3/2}\sqrt{a+bx+cx^2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^(3/2),x]
```

```
[Out] (3*(b*d + 2*c*d*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(10*c^2*d^3) - (a + b*x + c
*x^2)^(3/2)/(c*d*Sqrt[b*d + 2*c*d*x]) - (3*(b^2 - 4*a*c)^(7/4)*Sqrt[-((c*(a
 + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2
 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(10*c^3*d^(3/2)*Sqrt[a + b*x + c*x^2]) + (3
*(b^2 - 4*a*c)^(7/4)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF
[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(10*c^3*d^
(3/2)*Sqrt[a + b*x + c*x^2])
```

Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(bd+2cdx)^{3/2}}\,dx = -\frac{\left(a+bx+cx^2\right)^{3/2}}{cd\sqrt{bd+2cdx}} + \frac{3\int \sqrt{bd+2cdx}\sqrt{a+bx+cx^2}\,dx}{2cd^2}$$

$$= \frac{3(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{10c^2d^3} - \frac{\left(a+bx+cx^2\right)^{3/2}}{cd\sqrt{bd+2cdx}} - \frac{\left(3\left(b^2-4ac\right)\right)\int \frac{\sqrt{bd+2cdx}}{\sqrt{a+bx+cx^2}}\,dx}{20c^2d^2}$$

$$= \frac{3(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{10c^2d^3} - \frac{\left(a+bx+cx^2\right)^{3/2}}{cd\sqrt{bd+2cdx}} - \frac{\left(3\left(b^2-4ac\right)\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\int \frac{\sqrt{}}{\sqrt{-\frac{ac}{b^2-4a}}}}{20c^2d^2\sqrt{a+bx+cx^2}}$$

$$= \frac{3(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{10c^2d^3} - \frac{\left(a+bx+cx^2\right)^{3/2}}{cd\sqrt{bd+2cdx}} - \frac{\left(3\left(b^2-4ac\right)\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int\right.}{10c^3d^3\sqrt{a+b}}$$

$$= \frac{3(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{10c^2d^3} - \frac{\left(a+bx+cx^2\right)^{3/2}}{cd\sqrt{bd+2cdx}} + \frac{\left(3\left(b^2-4ac\right)^{3/2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}}{10c^3d^2\sqrt{a+}}$$

$$= \frac{3(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{10c^2d^3} - \frac{\left(a+bx+cx^2\right)^{3/2}}{cd\sqrt{bd+2cdx}} + \frac{3\left(b^2-4ac\right)^{7/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\right.}{10c^3d^{3/2}\sqrt{a+bx+cx^2}}$$

$$= \frac{3(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{10c^2d^3} - \frac{\left(a+bx+cx^2\right)^{3/2}}{cd\sqrt{bd+2cdx}} - \frac{3\left(b^2-4ac\right)^{7/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\right.}{10c^3d^{3/2}\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 0.0560629, size = 99, normalized size = 0.37

$$\frac{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{8c^2d\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\sqrt{d(b+2cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^(3/2),x]
```

```
[Out] ((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]*Hypergeometric2F1[-3/2, -1/4, 3/4, (b
+ 2*c*x)^2/(b^2 - 4*a*c)])/(8*c^2*d*Sqrt[d*(b + 2*c*x)]*Sqrt[(c*(a + x*(b +
 c*x)))/(-b^2 + 4*a*c)])
```

Maple [B] time = 0.22, size = 504, normalized size = 1.9

$$\frac{1}{20\,d^2\left(2\,c^2x^3+3\,bcx^2+2\,acx+b^2x+ab\right)c^3}\sqrt{cx^2+bx+a}\sqrt{d\left(2\,cx+b\right)}\left(48\sqrt{\frac{b+2\,cx+\sqrt{-4\,ac+b^2}}{\sqrt{-4\,ac+b^2}}}\sqrt{-\frac{2\,cx+b}{\sqrt{-4\,ac+b^2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(3/2),x)
```

```
[Out] 1/20*(c*x^2+b*x+a)^(1/2)*(d*(2*c*x+b))^(1/2)*(48*((b+2*c*x+(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*
c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(
-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a^2*c^2-24*((
b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b
^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*E
llipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2
),2^(1/2))*a*b^2*c+3*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2
)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*
a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b
^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*b^4+4*c^4*x^4+8*b*c^3*x^3-16*x^2*a*c^3+10
*x^2*b^2*c^2-16*b*a*c^2*x+6*b^3*c*x-20*a^2*c^2+6*a*c*b^2)/d^2/(2*c^2*x^3+3*
b*c*x^2+2*a*c*x+b^2*x+a*b)/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{3}{2}}}{\left(2\,cdx+bd\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)/(2*c*d*x + b*d)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{2\,cdx+bd}\left(cx^2+bx+a\right)^{\frac{3}{2}}}{4\,c^2d^2x^2+4\,bcd^2x+b^2d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*c*d*x + b*d)*(c*x^2 + b*x + a)^(3/2)/(4*c^2*d^2*x^2 + 4*b*c
*d^2*x + b^2*d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx+cx^2\right)^{\frac{3}{2}}}{\left(d\left(b+2cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(2*c*d*x+b*d)**(3/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)/(d*(b + 2*c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}{\left(2\,cdx + bd\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)/(2*c*d*x + b*d)^(3/2), x)
```

3.1347 $\int \frac{(a+bx+cx^2)^{3/2}}{(bd+2cdx)^{7/2}}\,dx$

Optimal. Leaf size=273

$$-\frac{3\left(b^2-4ac\right)^{3/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\mathrm{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{10c^3d^{7/2}\sqrt{a+bx+cx^2}}+\frac{3\left(b^2-4ac\right)^{3/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-\right.}{10c^3d^{7/2}\sqrt{a+bx+cx^2}}$$

```
[Out] (-3*Sqrt[a + b*x + c*x^2])/(10*c^2*d^3*Sqrt[b*d + 2*c*d*x]) - (a + b*x + c*
x^2)^(3/2)/(5*c*d*(b*d + 2*c*d*x)^(5/2)) + (3*(b^2 - 4*a*c)^(3/4)*Sqrt[-((c
*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d*x]/((
b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(10*c^3*d^(7/2)*Sqrt[a + b*x + c*x^2]) -
 (3*(b^2 - 4*a*c)^(3/4)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellipt
icF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(10*c^3
*d^(7/2)*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.243723, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {684, 691, 690, 307, 221, 1199, 424}

$$-\frac{3\left(b^2-4ac\right)^{3/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{10c^3d^{7/2}\sqrt{a+bx+cx^2}}+\frac{3\left(b^2-4ac\right)^{3/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{10c^3d^{7/2}\sqrt{a+bx+cx^2}}-\frac{3}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^(7/2),x]
```

```
[Out] (-3*Sqrt[a + b*x + c*x^2])/(10*c^2*d^3*Sqrt[b*d + 2*c*d*x]) - (a + b*x + c*
x^2)^(3/2)/(5*c*d*(b*d + 2*c*d*x)^(5/2)) + (3*(b^2 - 4*a*c)^(3/4)*Sqrt[-((c
*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d*x]/((
b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(10*c^3*d^(7/2)*Sqrt[a + b*x + c*x^2]) -
 (3*(b^2 - 4*a*c)^(3/4)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellipt
icF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(10*c^3
*d^(7/2)*Sqrt[a + b*x + c*x^2])
```

Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{3/2}}{(bd+2cdx)^{7/2}}\,dx = -\frac{(a+bx+cx^2)^{3/2}}{5cd(bd+2cdx)^{5/2}} + \frac{3\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^{3/2}}dx}{10cd^2}$$

$$= -\frac{3\sqrt{a+bx+cx^2}}{10c^2d^3\sqrt{bd+2cdx}} - \frac{(a+bx+cx^2)^{3/2}}{5cd(bd+2cdx)^{5/2}} + \frac{3\int \frac{\sqrt{bd+2cdx}}{\sqrt{a+bx+cx^2}}dx}{20c^2d^4}$$

$$= -\frac{3\sqrt{a+bx+cx^2}}{10c^2d^3\sqrt{bd+2cdx}} - \frac{(a+bx+cx^2)^{3/2}}{5cd(bd+2cdx)^{5/2}} + \frac{\left(3\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\int \frac{\sqrt{bd+2cdx}}{\sqrt{-\frac{ac}{b^2-4ac}-\frac{bcx}{b^2-4ac}-\frac{c^2x^2}{b^2-4ac}}}dx}{20c^2d^4\sqrt{a+bx+cx^2}}$$

$$= -\frac{3\sqrt{a+bx+cx^2}}{10c^2d^3\sqrt{bd+2cdx}} - \frac{(a+bx+cx^2)^{3/2}}{5cd(bd+2cdx)^{5/2}} + \frac{\left(3\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{(b^2-4ac)d^2}}}dx, x, \sqrt{bd}\right.}{10c^3d^5\sqrt{a+bx+cx^2}}$$

$$= -\frac{3\sqrt{a+bx+cx^2}}{10c^2d^3\sqrt{bd+2cdx}} - \frac{(a+bx+cx^2)^{3/2}}{5cd(bd+2cdx)^{5/2}} - \frac{\left(3\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{(b^2-4ac)d^2}}}\right.}{10c^3d^4\sqrt{a+bx+cx^2}}$$

$$= -\frac{3\sqrt{a+bx+cx^2}}{10c^2d^3\sqrt{bd+2cdx}} - \frac{(a+bx+cx^2)^{3/2}}{5cd(bd+2cdx)^{5/2}} - \frac{3(b^2-4ac)^{3/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\right|}{10c^3d^{7/2}\sqrt{a+bx+cx^2}}$$

$$= -\frac{3\sqrt{a+bx+cx^2}}{10c^2d^3\sqrt{bd+2cdx}} - \frac{(a+bx+cx^2)^{3/2}}{5cd(bd+2cdx)^{5/2}} + \frac{3(b^2-4ac)^{3/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\right|}{10c^3d^{7/2}\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 0.0603235, size = 99, normalized size = 0.36

$$\frac{(b^2-4ac)\sqrt{a+x(b+cx)}\,{}_2F_1\left(-\frac{3}{2},-\frac{5}{4};-\frac{1}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{40c^2d\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}(d(b+2cx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^(7/2),x]
```

```
[Out] ((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]*Hypergeometric2F1[-3/2, -5/4, -1/4, (b
 + 2*c*x)^2/(b^2 - 4*a*c)])/(40*c^2*d*(d*(b + 2*c*x))^(5/2)*Sqrt[(c*(a + x*
(b + c*x)))/(-b^2 + 4*a*c)])
```

Maple [B] time = 0.223, size = 893, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(7/2),x)
```

```
[Out] 1/20*(c*x^2+b*x+a)^(1/2)*(d*(2*c*x+b))^(1/2)*(48*EllipticE(1/2*((b+2*c*x+(-
4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^2*a*c^3*((b+
```

```
2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2
)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)-12*
EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/
2),2^(1/2))*x^2*b^2*c^2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(
1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(
-4*a*c+b^2)^(1/2))^(1/2)+48*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4
*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x*a*b*c^2*((b+2*c*x+(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2
*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)-12*EllipticE(1/2*((b+2*c
*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x*b^3*c*(
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+
b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)+
12*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*
a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1
/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2
^(1/2),2^(1/2))*a*b^2*c-3*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))
^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))
/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*
a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*b^4-28*c^4*x^4-56*b*c^3*x^3-32*x^2*a
*c^3-34*x^2*b^2*c^2-32*b*a*c^2*x-6*b^3*c*x-4*a^2*c^2-6*a*c*b^2)/d^4/(2*c^2*
x^3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)/(2*c*x+b)^2/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2+bx+a)^{\frac{3}{2}}}{(2\,cdx+bd)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)/(2*c*d*x + b*d)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,cdx+bd}(cx^2+bx+a)^{\frac{3}{2}}}{16\,c^4d^4x^4+32\,bc^3d^4x^3+24\,b^2c^2d^4x^2+8\,b^3cd^4x+b^4d^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*c*d*x + b*d)*(c*x^2 + b*x + a)^(3/2)/(16*c^4*d^4*x^4 + 32*b
*c^3*d^4*x^3 + 24*b^2*c^2*d^4*x^2 + 8*b^3*c*d^4*x + b^4*d^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx+cx^2)^{\frac{3}{2}}}{(d\,(b+2cx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(2*c*d*x+b*d)**(7/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)/(d*(b + 2*c*x))**(7/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}{(2\,cdx + bd)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)/(2*c*d*x + b*d)^(7/2), x)
```

3.1348 $\int \frac{(a+bx+cx^2)^{3/2}}{(bd+2cdx)^{11/2}}\,dx$

Optimal. Leaf size=320

$$\frac{\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{30c^3d^{11/2}\sqrt[4]{b^2-4ac}\sqrt{a+bx+cx^2}}+\frac{\sqrt{a+bx+cx^2}}{15c^2d^5\left(b^2-4ac\right)\sqrt{bd+2cdx}}-\frac{\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\right|-1\right)}{30c^3d^{11/2}\sqrt[4]{b^2-4ac}\sqrt{a+bx+cx^2}}$$

```
[Out] -Sqrt[a + b*x + c*x^2]/(30*c^2*d^3*(b*d + 2*c*d*x)^(5/2)) + Sqrt[a + b*x +
c*x^2]/(15*c^2*(b^2 - 4*a*c)*d^5*Sqrt[b*d + 2*c*d*x]) - (a + b*x + c*x^2)^(
3/2)/(9*c*d*(b*d + 2*c*d*x)^(9/2)) - (Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4
*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])]
, -1])/(30*c^3*(b^2 - 4*a*c)^(1/4)*d^(11/2)*Sqrt[a + b*x + c*x^2]) + (Sqrt[
-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x
]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(30*c^3*(b^2 - 4*a*c)^(1/4)*d^(11/2)
*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.291262, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {684, 693, 691, 690, 307, 221, 1199, 424}

$$\frac{\sqrt{a+bx+cx^2}}{15c^2d^5\left(b^2-4ac\right)\sqrt{bd+2cdx}}+\frac{\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\right|-1\right)}{30c^3d^{11/2}\sqrt[4]{b^2-4ac}\sqrt{a+bx+cx^2}}-\frac{\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\right|-1\right)}{30c^3d^{11/2}\sqrt[4]{b^2-4ac}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^(11/2),x]
```

```
[Out] -Sqrt[a + b*x + c*x^2]/(30*c^2*d^3*(b*d + 2*c*d*x)^(5/2)) + Sqrt[a + b*x +
c*x^2]/(15*c^2*(b^2 - 4*a*c)*d^5*Sqrt[b*d + 2*c*d*x]) - (a + b*x + c*x^2)^(
3/2)/(9*c*d*(b*d + 2*c*d*x)^(9/2)) - (Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4
*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])]
, -1])/(30*c^3*(b^2 - 4*a*c)^(1/4)*d^(11/2)*Sqrt[a + b*x + c*x^2]) + (Sqrt[
-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x
]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(30*c^3*(b^2 - 4*a*c)^(1/4)*d^(11/2)
*Sqrt[a + b*x + c*x^2])
```

Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
```

```
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(bd+2cdx)^{11/2}}\,dx = -\frac{\left(a+bx+cx^2\right)^{3/2}}{9cd(bd+2cdx)^{9/2}} + \frac{\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^{7/2}}\,dx}{6cd^2}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{30c^2d^3(bd+2cdx)^{5/2}} - \frac{\left(a+bx+cx^2\right)^{3/2}}{9cd(bd+2cdx)^{9/2}} + \frac{\int \frac{1}{(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}\,dx}{60c^2d^4}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{30c^2d^3(bd+2cdx)^{5/2}} + \frac{\sqrt{a+bx+cx^2}}{15c^2\left(b^2-4ac\right)d^5\sqrt{bd+2cdx}} - \frac{\left(a+bx+cx^2\right)^{3/2}}{9cd(bd+2cdx)^{9/2}} - \frac{\int \frac{\sqrt{bd+2cd\cdots}}{\sqrt{a+bx+c\cdots}}}{60c^2\left(b^2-4\cdots\right.}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{30c^2d^3(bd+2cdx)^{5/2}} + \frac{\sqrt{a+bx+cx^2}}{15c^2\left(b^2-4ac\right)d^5\sqrt{bd+2cdx}} - \frac{\left(a+bx+cx^2\right)^{3/2}}{9cd(bd+2cdx)^{9/2}} - \frac{\sqrt{-\frac{c\left(a+bx+c\cdots\right.}{b^2-4ac}}\cdots}{60c^2\left(b^2\cdots\right.}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{30c^2d^3(bd+2cdx)^{5/2}} + \frac{\sqrt{a+bx+cx^2}}{15c^2\left(b^2-4ac\right)d^5\sqrt{bd+2cdx}} - \frac{\left(a+bx+cx^2\right)^{3/2}}{9cd(bd+2cdx)^{9/2}} - \frac{\sqrt{-\frac{c\left(a+bx+c\cdots\right.}{b^2-4ac}}\cdots}{\cdots}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{30c^2d^3(bd+2cdx)^{5/2}} + \frac{\sqrt{a+bx+cx^2}}{15c^2\left(b^2-4ac\right)d^5\sqrt{bd+2cdx}} - \frac{\left(a+bx+cx^2\right)^{3/2}}{9cd(bd+2cdx)^{9/2}} + \frac{\sqrt{-\frac{c\left(a+bx+c\cdots\right.}{b^2-4ac}}\cdots}{\cdots}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{30c^2d^3(bd+2cdx)^{5/2}} + \frac{\sqrt{a+bx+cx^2}}{15c^2\left(b^2-4ac\right)d^5\sqrt{bd+2cdx}} - \frac{\left(a+bx+cx^2\right)^{3/2}}{9cd(bd+2cdx)^{9/2}} + \frac{\sqrt{-\frac{c\left(a+bx+c\cdots\right.}{b^2-4ac}}\cdots}{30c^3\sqrt[4]{b^2\cdots}}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{30c^2d^3(bd+2cdx)^{5/2}} + \frac{\sqrt{a+bx+cx^2}}{15c^2\left(b^2-4ac\right)d^5\sqrt{bd+2cdx}} - \frac{\left(a+bx+cx^2\right)^{3/2}}{9cd(bd+2cdx)^{9/2}} - \frac{\sqrt{-\frac{c\left(a+bx+c\cdots\right.}{b^2-4ac}}\cdots}{30c^3\sqrt[4]{b^2\cdots}}$$

Mathematica [C] time = 0.0862667, size = 107, normalized size = 0.33

$$\frac{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\sqrt{d(b+2cx)}\,{}_2F_1\left(-\frac{9}{4},-\frac{3}{2};-\frac{5}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{72c^2d^6(b+2cx)^5\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(b*d + 2*c*d*x)^(11/2),x]
```

```
[Out] ((b^2 - 4*a*c)*Sqrt[d*(b + 2*c*x)]*Sqrt[a + x*(b + c*x)]*Hypergeometric2F1[
-9/4, -3/2, -5/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(72*c^2*d^6*(b + 2*c*x)^5*S
qrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)])
```

Maple [B] time = 0.29, size = 1501, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(11/2),x)
```

```
[Out] 1/180*(c*x^2+b*x+a)^(1/2)*(d*(2*c*x+b))^(1/2)*(-192*x^6*c^6+12*((b+2*c*x+(-
4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))
^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1
/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))
*a*b^4*c-736*x^3*a*b*c^4-424*x^2*a*b^2*c^3-256*x*a^2*b*c^3-56*x*a*b^3*c^2+3
84*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^
(1/2),2^(1/2))*x^3*a*b*c^4*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
)^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2)
)/(-4*a*c+b^2)^(1/2))^(1/2)+288*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))
/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^2*a*b^2*c^3*((b+2*c*x+(-4*a*c
+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2
)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)+96*EllipticE(1/2
*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x
*a*b^3*c^2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x
+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(
1/2))^(1/2)-576*x^5*b*c^5-368*x^4*a*c^5-628*x^4*b^2*c^4-296*x^3*b^3*c^3-256
*x^2*a^2*c^4-58*x^2*b^4*c^2-6*x*b^5*c-3*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a
*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a
*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*b^6-24*EllipticE(1/2*(
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x*b
^5*c*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-
4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^
(1/2)+192*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(
1/2)*2^(1/2),2^(1/2))*x^4*a*c^5*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^
(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^
(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)-48*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^4*b^2*c^4*((b+2*c*x+(-4*a
*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1
/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)-96*EllipticE(1
/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))
*x^3*b^3*c^3*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c
*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(1/2))^(1/2)-72*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(
1/2))^(1/2)*2^(1/2),2^(1/2))*x^2*b^4*c^2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*
a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*
a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)-6*a*b^4*c-4*a^2*b^2*c^2-80*a^3*c^
3)/d^6/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)/(2*c*x+b)^4/(4*a*c-b^2)/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{3}{2}}}{\left(2\,cdx+bd\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)/(2*c*d*x + b*d)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,cdx+bd}\left(cx^2+bx+a\right)^{\frac{3}{2}}}{64\,c^6d^6x^6+192\,bc^5d^6x^5+240\,b^2c^4d^6x^4+160\,b^3c^3d^6x^3+60\,b^4c^2d^6x^2+12\,b^5cd^6x+b^6d^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(11/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*c*d*x + b*d)*(c*x^2 + b*x + a)^(3/2)/(64*c^6*d^6*x^6 + 192*
b*c^5*d^6*x^5 + 240*b^2*c^4*d^6*x^4 + 160*b^3*c^3*d^6*x^3 + 60*b^4*c^2*d^6*
x^2 + 12*b^5*c*d^6*x + b^6*d^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(2*c*d*x+b*d)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{3}{2}}}{(2\,cdx+bd)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(2*c*d*x+b*d)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)/(2*c*d*x + b*d)^(11/2), x)
```

3.1349 $\int (bd+2cdx)^{7/2}\left(a+bx+cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=321

$$-\frac{5d^{7/2}\left(b^2-4ac\right)^{21/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{17556c^4\sqrt{a+bx+cx^2}}-\frac{5d^3\left(b^2-4ac\right)^4\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}}{8778c^3}-\frac{d(b}{}$$

```
[Out] (-5*(b^2 - 4*a*c)^4*d^3*Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/(8778*c^
3) - ((b^2 - 4*a*c)^3*d*(b*d + 2*c*d*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(2926*
c^3) + ((b^2 - 4*a*c)^2*(b*d + 2*c*d*x)^(9/2)*Sqrt[a + b*x + c*x^2])/(836*c
^3*d) - ((b^2 - 4*a*c)*(b*d + 2*c*d*x)^(9/2)*(a + b*x + c*x^2)^(3/2))/(114*
c^2*d) + ((b*d + 2*c*d*x)^(9/2)*(a + b*x + c*x^2)^(5/2))/(19*c*d) - (5*(b^2
 - 4*a*c)^(21/4)*d^(7/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellip
ticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(17556
*c^4*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.295517, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {685, 692, 691, 689, 221}

$$-\frac{5d^3\left(b^2-4ac\right)^4\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}}{8778c^3}-\frac{5d^{7/2}\left(b^2-4ac\right)^{21/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{17556c^4\sqrt{a+bx+cx^2}}-\frac{d\left(b^2-4ac\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(7/2)*(a + b*x + c*x^2)^(5/2),x]
```


```
[Out] (-5*(b^2 - 4*a*c)^4*d^3*Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/(8778*c^
3) - ((b^2 - 4*a*c)^3*d*(b*d + 2*c*d*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(2926*
c^3) + ((b^2 - 4*a*c)^2*(b*d + 2*c*d*x)^(9/2)*Sqrt[a + b*x + c*x^2])/(836*c
^3*d) - ((b^2 - 4*a*c)*(b*d + 2*c*d*x)^(9/2)*(a + b*x + c*x^2)^(3/2))/(114*
c^2*d) + ((b*d + 2*c*d*x)^(9/2)*(a + b*x + c*x^2)^(5/2))/(19*c*d) - (5*(b^2
 - 4*a*c)^(21/4)*d^(7/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellip
ticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(17556
*c^4*Sqrt[a + b*x + c*x^2])
```


Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
```

```
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int (bd+2cdx)^{7/2}\left(a+bx+cx^2\right)^{5/2}dx = \frac{(bd+2cdx)^{9/2}\left(a+bx+cx^2\right)^{5/2}}{19cd} - \frac{\left(5\left(b^2-4ac\right)\right)\int (bd+2cdx)^{7/2}\left(a+bx+cx\right.}{38c}$$

$$= -\frac{\left(b^2-4ac\right)(bd+2cdx)^{9/2}\left(a+bx+cx^2\right)^{3/2}}{114c^2d} + \frac{(bd+2cdx)^{9/2}\left(a+bx+cx^2\right)^{5/}}{19cd}$$

$$= \frac{\left(b^2-4ac\right)^2(bd+2cdx)^{9/2}\sqrt{a+bx+cx^2}}{836c^3d} - \frac{\left(b^2-4ac\right)(bd+2cdx)^{9/2}\left(a+bx+\right.}{114c^2d}$$

$$= -\frac{\left(b^2-4ac\right)^3d(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}}{2926c^3} + \frac{\left(b^2-4ac\right)^2(bd+2cdx)^{9/2}\sqrt{a+}}{836c^3d}$$

$$= -\frac{5\left(b^2-4ac\right)^4d^3\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{8778c^3} - \frac{\left(b^2-4ac\right)^3d(bd+2cdx)^{5/2}\sqrt{a}}{2926c^3}$$

$$= -\frac{5\left(b^2-4ac\right)^4d^3\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{8778c^3} - \frac{\left(b^2-4ac\right)^3d(bd+2cdx)^{5/2}\sqrt{a}}{2926c^3}$$

$$= -\frac{5\left(b^2-4ac\right)^4d^3\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{8778c^3} - \frac{\left(b^2-4ac\right)^3d(bd+2cdx)^{5/2}\sqrt{a}}{2926c^3}$$

$$= -\frac{5\left(b^2-4ac\right)^4d^3\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{8778c^3} - \frac{\left(b^2-4ac\right)^3d(bd+2cdx)^{5/2}\sqrt{a}}{2926c^3}$$

Mathematica [C] time = 0.410117, size = 161, normalized size = 0.5

$$\frac{4\sqrt{a+x(b+cx)}(d(b+2cx))^{7/2}\left(3(b+2cx)^2(a+x(b+cx))^3-2c\left(a-\frac{b^2}{4c}\right)\left(\frac{(b^2-4ac)^3\,{}_2F_1\left(-\frac{5}{2},\frac{1}{4};\frac{5}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{64c^3\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}+2(a+x(b+cx))^3\right)\right)}{57(b+2cx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(7/2)*(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (4*(d*(b + 2*c*x))^(7/2)*Sqrt[a + x*(b + c*x)]*(3*(b + 2*c*x)^2*(a + x*(b +
 c*x))^3 - 2*(a - b^2/(4*c))*c*(2*(a + x*(b + c*x))^3 + ((b^2 - 4*a*c)^3*Hy
pergeometric2F1[-5/2, 1/4, 5/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(64*c^3*Sqrt[
(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]))))/(57*(b + 2*c*x)^3)
```

Maple [B] time = 0.26, size = 1344, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(7/2)*(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 1/35112*(d*(2*c*x+b))^(1/2)*(c*x^2+b*x+a)^(1/2)*d^3*(-5120*a^5*b*c^5+5888*a
^4*b^3*c^4+1256*a^3*b^5*c^3-180*a^2*b^7*c^2+10*a*b^9*c+325248*x^10*b*c^10+5
120*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1
/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-
4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c
+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a^5*c^5-6400*(-4*a*c+b^2)^(1/2)*((b+2*c
*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(
1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*Ellipt
icF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(
1/2))*a^4*b^2*c^4+59136*x^11*c^11-800*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+
b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)
*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b
+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a^2*b
^6*c^2+100*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1
/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/
(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a*b^8*c+216832*x^9*a*c^10+758912
*x^9*b^2*c^9+975744*x^8*b^3*c^8+277760*x^7*a^2*c^9+749168*x^7*b^4*c^7+34535
2*x^6*b^5*c^6+126208*x^5*a^3*c^8+89168*x^5*b^6*c^5+10036*x^4*b^7*c^4-4096*x
^3*a^4*c^7-4*x^3*b^8*c^3+10*x^2*b^9*c^2-10240*x*a^5*c^6+10*x*b^10*c+3200*(-
4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-
(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^
(1/2))^(1/2)*2^(1/2),2^(1/2))*a^3*b^4*c^3+1812608*x^7*a*b^2*c^8+305336*x^4*
a*b^5*c^5+319616*x^3*a^3*b^2*c^6+1790656*x^6*a*b^3*c^7+1363584*x^5*a^2*b^2*
c^7+1002096*x^5*a*b^4*c^6+9728*x*a^4*b^2*c^5+972160*x^6*a^2*b*c^8-192*x^2*a
*b^7*c^3+315520*x^4*a^3*b*c^7+978560*x^4*a^2*b^3*c^6+1248*x*a^2*b^6*c^3+361
12*x*a^3*b^4*c^4-6144*x^2*a^4*b*c^6+163904*x^2*a^3*b^3*c^5-180*x*a*b^8*c^2+
369424*x^3*a^2*b^4*c^5+40208*x^3*a*b^6*c^4+61656*x^2*a^2*b^5*c^4+975744*x^8
*a*b*c^9-5*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1
/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/
```

```
(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*b^10)/c^4/(2*c^2*x^3+3*b*c*x^2+2
*a*c*x+b^2*x+a*b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (2\,cdx+bd)^{\frac{7}{2}}\left(cx^2+bx+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(7/2)*(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^(7/2)*(c*x^2 + b*x + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(8\,c^5d^3x^7+28\,bc^4d^3x^6+2\left(19\,b^2c^3+8\,ac^4\right)d^3x^5+a^2b^3d^3+5\left(5\,b^3c^2+8\,abc^3\right)d^3x^4+4\left(2\,b^4c+9\,ab^2c^2+2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(7/2)*(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((8*c^5*d^3*x^7 + 28*b*c^4*d^3*x^6 + 2*(19*b^2*c^3 + 8*a*c^4)*d^3*x
^5 + a^2*b^3*d^3 + 5*(5*b^3*c^2 + 8*a*b*c^3)*d^3*x^4 + 4*(2*b^4*c + 9*a*b^2
*c^2 + 2*a^2*c^3)*d^3*x^3 + (b^5 + 14*a*b^3*c + 12*a^2*b*c^2)*d^3*x^2 + 2*(
a*b^4 + 3*a^2*b^2*c)*d^3*x)*sqrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(7/2)*(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (2\,cdx+bd)^{\frac{7}{2}}\left(cx^2+bx+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(7/2)*(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^(7/2)*(c*x^2 + b*x + a)^(5/2), x)
```

3.1350 $\int (bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=274

$$-\frac{d^{3/2}\left(b^2-4ac\right)^{17/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{924c^4\sqrt{a+bx+cx^2}}-\frac{d\left(b^2-4ac\right)^3\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}}{462c^3}+\frac{\left(b^2-4\right.}{}$$

```
[Out] -((b^2 - 4*a*c)^3*d*Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/(462*c^3) +
((b^2 - 4*a*c)^2*(b*d + 2*c*d*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(308*c^3*d) -
 ((b^2 - 4*a*c)*(b*d + 2*c*d*x)^(5/2)*(a + b*x + c*x^2)^(3/2))/(66*c^2*d) +
 ((b*d + 2*c*d*x)^(5/2)*(a + b*x + c*x^2)^(5/2))/(15*c*d) - ((b^2 - 4*a*c)^
(17/4)*d^(3/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSi
n[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(924*c^4*Sqrt[a
+ b*x + c*x^2])
```

Rubi [A] time = 0.22119, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {685, 692, 691, 689, 221}

$$-\frac{d^{3/2}\left(b^2-4ac\right)^{17/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{924c^4\sqrt{a+bx+cx^2}}-\frac{d\left(b^2-4ac\right)^3\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}}{462c^3}+\frac{\left(b^2-4ac\right)^2\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] -((b^2 - 4*a*c)^3*d*Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/(462*c^3) +
((b^2 - 4*a*c)^2*(b*d + 2*c*d*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(308*c^3*d) -
 ((b^2 - 4*a*c)*(b*d + 2*c*d*x)^(5/2)*(a + b*x + c*x^2)^(3/2))/(66*c^2*d) +
 ((b*d + 2*c*d*x)^(5/2)*(a + b*x + c*x^2)^(5/2))/(15*c*d) - ((b^2 - 4*a*c)^
(17/4)*d^(3/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSi
n[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(924*c^4*Sqrt[a
+ b*x + c*x^2])
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int (bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{5/2}dx=\frac{(bd+2cdx)^{5/2}\left(a+bx+cx^2\right)^{5/2}}{15cd}-\frac{\left(b^2-4ac\right)\int (bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{3/}}{6c}$$

$$=-\frac{\left(b^2-4ac\right)(bd+2cdx)^{5/2}\left(a+bx+cx^2\right)^{3/2}}{66c^2d}+\frac{(bd+2cdx)^{5/2}\left(a+bx+cx^2\right)^{5/}}{15cd}$$

$$=\frac{\left(b^2-4ac\right)^2(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}}{308c^3d}-\frac{\left(b^2-4ac\right)(bd+2cdx)^{5/2}\left(a+bx+\right.}{66c^2d}$$

$$=-\frac{\left(b^2-4ac\right)^3d\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{462c^3}+\frac{\left(b^2-4ac\right)^2(bd+2cdx)^{5/2}\sqrt{a+bx}}{308c^3d}$$

$$=-\frac{\left(b^2-4ac\right)^3d\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{462c^3}+\frac{\left(b^2-4ac\right)^2(bd+2cdx)^{5/2}\sqrt{a+bx}}{308c^3d}$$

$$=-\frac{\left(b^2-4ac\right)^3d\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{462c^3}+\frac{\left(b^2-4ac\right)^2(bd+2cdx)^{5/2}\sqrt{a+bx}}{308c^3d}$$

$$=-\frac{\left(b^2-4ac\right)^3d\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{462c^3}+\frac{\left(b^2-4ac\right)^2(bd+2cdx)^{5/2}\sqrt{a+bx}}{308c^3d}$$

Mathematica [C] time = 0.119294, size = 117, normalized size = 0.43

$$\frac{2}{15}d\sqrt{a+x(b+cx)}\sqrt{d(b+2cx)}\left(\frac{\left(b^2-4ac\right)^3{}_2F_1\left(-\frac{5}{2},\frac{1}{4};\frac{5}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{64c^3\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}+2(a+x(b+cx))^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*d*Sqrt[d*(b + 2*c*x)]*Sqrt[a + x*(b + c*x)]*(2*(a + x*(b + c*x))^3 + ((b
^2 - 4*a*c)^3*Hypergeometric2F1[-5/2, 1/4, 5/4, (b + 2*c*x)^2/(b^2 - 4*a*c)
])/(64*c^3*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)])))/15
```

Maple [B] time = 0.222, size = 1055, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(3/2)*(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] -1/9240*(d*(2*c*x+b))^(1/2)*(c*x^2+b*x+a)^(1/2)*d*(-22176*x^8*b*c^8-19264*x
^7*a*c^8-39536*x^7*b^2*c^7-34888*x^6*b^3*c^6-27584*x^5*a^2*c^7-15248*x^5*b^
4*c^5-2644*x^4*b^5*c^4-15808*x^3*a^3*c^6+4*x^3*b^6*c^3-10*x^2*b^7*c^2-2560*
x*a^4*c^5-10*x*b^8*c-1280*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/
(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+
(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a
*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a^3*b^2*c^3-1280*
a^4*b*c^4-696*a^3*b^3*c^3+140*a^2*b^5*c^2-10*a*b^7*c-67424*x^6*a*b*c^7-8734
4*x^5*a*b^2*c^6-68960*x^4*a^2*b*c^6-49800*x^4*a*b^3*c^5-57104*x^3*a^2*b^2*c
^5-10624*x^3*a*b^4*c^4-23712*x^2*a^3*b*c^5-16696*x^2*a^2*b^3*c^4+152*x^2*a*
b^5*c^3-9296*x*a^3*b^2*c^4-688*x*a^2*b^4*c^3+140*x*a*b^6*c^2+480*(-4*a*c+b^
2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b
)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/
2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(
1/2)*2^(1/2),2^(1/2))*a^2*b^4*c^2-80*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b
^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*
((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+
2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a*b^6*
c-4928*x^9*c^9+5*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b
^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b
^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*b^8+1280*(-4*a*c+b^2)^(1/2
)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a
*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/
2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^
(1/2),2^(1/2))*a^4*c^4)/c^4/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (2\,cdx+bd)^{\frac{3}{2}}\left(cx^2+bx+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)*(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^(3/2)*(c*x^2 + b*x + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(2\,c^3dx^5 + 5\,bc^2dx^4 + 4\,\left(b^2c + ac^2\right)dx^3 + a^2bd + \left(b^3 + 6\,abc\right)dx^2 + 2\,\left(ab^2 + a^2c\right)dx\right)\sqrt{2\,cdx + bd}\sqrt{cx^2 + bx +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)*(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((2*c^3*d*x^5 + 5*b*c^2*d*x^4 + 4*(b^2*c + a*c^2)*d*x^3 + a^2*b*d +
 (b^3 + 6*a*b*c)*d*x^2 + 2*(a*b^2 + a^2*c)*d*x)*sqrt(2*c*d*x + b*d)*sqrt(c*
x^2 + b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(3/2)*(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (2\,cdx + bd)^{\frac{3}{2}}\left(cx^2 + bx + a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)*(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^(3/2)*(c*x^2 + b*x + a)^(5/2), x)
```

3.1351 $\int \frac{(a+bx+cx^2)^{5/2}}{\sqrt{bd+2cdx}}\,dx$

Optimal. Leaf size=229

$$-\frac{5\left(b^2-4ac\right)^{13/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{308c^4\sqrt{d}\sqrt{a+bx+cx^2}}+\frac{5\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}}{308c^3d}-\frac{5\left(b^2-4a\right.}{}$$

```
[Out] (5*(b^2 - 4*a*c)^2*Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/(308*c^3*d) -
 (5*(b^2 - 4*a*c)*Sqrt[b*d + 2*c*d*x]*(a + b*x + c*x^2)^(3/2))/(154*c^2*d)
+ (Sqrt[b*d + 2*c*d*x]*(a + b*x + c*x^2)^(5/2))/(11*c*d) - (5*(b^2 - 4*a*c)
^(13/4)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[
b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(308*c^4*Sqrt[d]*Sqrt[a
 + b*x + c*x^2])
```

Rubi [A] time = 0.187023, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {685, 691, 689, 221}

$$\frac{5\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}}{308c^3d}-\frac{5\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}\sqrt{bd+2cdx}}{154c^2d}-\frac{5\left(b^2-4ac\right)^{13/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\right.}{308c^4\sqrt{d}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/Sqrt[b*d + 2*c*d*x],x]
```


```
[Out] (5*(b^2 - 4*a*c)^2*Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/(308*c^3*d) -
 (5*(b^2 - 4*a*c)*Sqrt[b*d + 2*c*d*x]*(a + b*x + c*x^2)^(3/2))/(154*c^2*d)
+ (Sqrt[b*d + 2*c*d*x]*(a + b*x + c*x^2)^(5/2))/(11*c*d) - (5*(b^2 - 4*a*c)
^(13/4)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[
b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(308*c^4*Sqrt[d]*Sqrt[a
 + b*x + c*x^2])
```


Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{5/2}}{\sqrt{bd+2cdx}}\,dx = \frac{\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{5/2}}{11cd} - \frac{\left(5\left(b^2-4ac\right)\right)\int \frac{\left(a+bx+cx^2\right)^{3/2}}{\sqrt{bd+2cdx}}\,dx}{22c}$$

$$= -\frac{5\left(b^2-4ac\right)\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2}}{154c^2d} + \frac{\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{5/2}}{11cd} + \frac{\left(15\left(b^2-4ac\cdots\right.\right.}{\cdots}$$

$$= \frac{5\left(b^2-4ac\right)^2\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{308c^3d} - \frac{5\left(b^2-4ac\right)\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2}}{154c^2d} + \frac{\sqrt{bd\cdots}}{\cdots}$$

$$= \frac{5\left(b^2-4ac\right)^2\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{308c^3d} - \frac{5\left(b^2-4ac\right)\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2}}{154c^2d} + \frac{\sqrt{bd\cdots}}{\cdots}$$

$$= \frac{5\left(b^2-4ac\right)^2\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{308c^3d} - \frac{5\left(b^2-4ac\right)\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2}}{154c^2d} + \frac{\sqrt{bd\cdots}}{\cdots}$$

$$= \frac{5\left(b^2-4ac\right)^2\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{308c^3d} - \frac{5\left(b^2-4ac\right)\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2}}{154c^2d} + \frac{\sqrt{bd\cdots}}{\cdots}$$

Mathematica [C] time = 0.0801335, size = 101, normalized size = 0.44

$$\frac{\left(b^2-4ac\right)^2\sqrt{a+x(b+cx)}\sqrt{d(b+2cx)}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{4};\frac{5}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{32c^3d\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/Sqrt[b*d + 2*c*d*x],x]
```

```
[Out] ((b^2 - 4*a*c)^2*Sqrt[d*(b + 2*c*x)]*Sqrt[a + x*(b + c*x)]*Hypergeometric2F
1[-5/2, 1/4, 5/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(32*c^3*d*Sqrt[(c*(a + x*(b
 + c*x)))/(-b^2 + 4*a*c)])
```

Maple [B] time = 0.243, size = 798, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(1/2),x)
```

```
[Out] 1/616*(c*x^2+b*x+a)^(1/2)*(d*(2*c*x+b))^(1/2)/d*(112*x^7*c^7+392*x^6*b*c^6+
496*x^5*a*c^6+464*x^5*b^2*c^5+320*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)
^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-
b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c
*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a^3*c^3-2
40*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/
2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4
*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a^2*b^2*c^2+60*(-4*a*c+b^2)^(1/2)*((b+2*
c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^
(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*Ellip
ticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^
(1/2))*a*b^4*c-5*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b
^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b
^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*b^6+1240*x^4*a*b*c^5+180*x
^4*b^3*c^4+976*x^3*a^2*c^5+752*x^3*a*b^2*c^4-4*x^3*b^4*c^3+1464*x^2*a^2*b*c
^4-112*x^2*a*b^3*c^3+10*x^2*b^5*c^2+592*x*a^3*c^4+288*x*a^2*b^2*c^3-100*x*a
*b^4*c^2+10*x*b^6*c+296*a^3*b*c^3-100*a^2*b^3*c^2+10*a*b^5*c)/c^4/(2*c^2*x^
3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx + a\right)^{\frac{5}{2}}}{\sqrt{2\,cdx + bd}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)/sqrt(2*c*d*x + b*d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2x^4 + 2\,bcx^3 + 2\,abx + \left(b^2 + 2\,ac\right)x^2 + a^2\right)\sqrt{cx^2 + bx + a}}{\sqrt{2\,cdx + bd}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*sqrt(c*x
^2 + b*x + a)/sqrt(2*c*d*x + b*d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(2*c*d*x+b*d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx + a\right)^{\frac{5}{2}}}{\sqrt{2\,cdx + bd}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)/sqrt(2*c*d*x + b*d), x)
```

3.1352 $\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(bd+2cdx)^{5/2}}\,dx$

Optimal. Leaf size=219

$$\frac{5\left(b^2-4ac\right)^{9/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{84c^4d^{5/2}\sqrt{a+bx+cx^2}}-\frac{5\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}}{84c^3d^3}+\frac{5\left(a+bx+c\right.}{4}$$

```
[Out] (-5*(b^2 - 4*a*c)*Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/(84*c^3*d^3) +
 (5*Sqrt[b*d + 2*c*d*x]*(a + b*x + c*x^2)^(3/2))/(42*c^2*d^3) - (a + b*x +
c*x^2)^(5/2)/(3*c*d*(b*d + 2*c*d*x)^(3/2)) + (5*(b^2 - 4*a*c)^(9/4)*Sqrt[-(
(c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/
((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(84*c^4*d^(5/2)*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.183386, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {684, 685, 691, 689, 221}

$$-\frac{5\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}}{84c^3d^3}+\frac{5\left(b^2-4ac\right)^{9/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{84c^4d^{5/2}\sqrt{a+bx+cx^2}}+\frac{5\left(a+bx+cx^2\right)^{3/2}}{42c^2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^(5/2),x]
```

```
[Out] (-5*(b^2 - 4*a*c)*Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/(84*c^3*d^3) +
 (5*Sqrt[b*d + 2*c*d*x]*(a + b*x + c*x^2)^(3/2))/(42*c^2*d^3) - (a + b*x +
c*x^2)^(5/2)/(3*c*d*(b*d + 2*c*d*x)^(3/2)) + (5*(b^2 - 4*a*c)^(9/4)*Sqrt[-(
(c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/
((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(84*c^4*d^(5/2)*Sqrt[a + b*x + c*x^2])
```

Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
```

```
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(bd+2cdx)^{5/2}}\,dx = -\frac{\left(a+bx+cx^2\right)^{5/2}}{3cd(bd+2cdx)^{3/2}}+\frac{5\int \frac{\left(a+bx+cx^2\right)^{3/2}}{\sqrt{bd+2cdx}}\,dx}{6cd^2}$$

$$=\frac{5\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2}}{42c^2d^3}-\frac{\left(a+bx+cx^2\right)^{5/2}}{3cd(bd+2cdx)^{3/2}}-\frac{\left(5\left(b^2-4ac\right)\right)\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{bd+2cdx}}\,dx}{28c^2d^2}$$

$$=-\frac{5\left(b^2-4ac\right)\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{84c^3d^3}+\frac{5\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2}}{42c^2d^3}-\frac{\left(a+bx+cx^2\right)}{3cd(bd+2cdx}$$

$$=-\frac{5\left(b^2-4ac\right)\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{84c^3d^3}+\frac{5\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2}}{42c^2d^3}-\frac{\left(a+bx+cx^2\right)}{3cd(bd+2cdx}$$

$$=-\frac{5\left(b^2-4ac\right)\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{84c^3d^3}+\frac{5\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2}}{42c^2d^3}-\frac{\left(a+bx+cx^2\right)}{3cd(bd+2cdx}$$

$$=-\frac{5\left(b^2-4ac\right)\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{84c^3d^3}+\frac{5\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2}}{42c^2d^3}-\frac{\left(a+bx+cx^2\right)}{3cd(bd+2cdx}$$

Mathematica [C] time = 0.0628769, size = 101, normalized size = 0.46

$$-\frac{\left(b^2-4ac\right)^2\sqrt{a+x(b+cx)}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{4};\frac{1}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{96c^3d\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}(d(b+2cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^(5/2),x]
```

```
[Out] -((b^2 - 4*a*c)^2*Sqrt[a + x*(b + c*x)]*Hypergeometric2F1[-5/2, -3/4, 1/4,
(b + 2*c*x)^2/(b^2 - 4*a*c)])/(96*c^3*d*(d*(b + 2*c*x))^(3/2)*Sqrt[(c*(a +
```

```
x*(b + c*x)))/(-b^2 + 4*a*c)])
```

Maple [B] time = 0.245, size = 1000, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(5/2),x)
```

```
[Out] 1/168*(24*x^6*c^6+160*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/
2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4
*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2)^(1/2)*x*a^2*c^3-80*((b+2*c*
x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1
/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*Ellipti
cF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1
/2))*(-4*a*c+b^2)^(1/2)*x*a*b^2*c^2+10*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c
+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2
)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((
b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x*b^
4*c+72*x^5*b*c^5+80*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)
*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a
*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^
2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2)^(1/2)*a^2*b*c^2-40*((b+2*c*x+
(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2
))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF
(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2
))*(-4*a*c+b^2)^(1/2)*a*b^3*c+5*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-
2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x
+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*b^5+152*x^4
*a*c^5+52*x^4*b^2*c^4+304*x^3*a*b*c^4-16*x^3*b^3*c^3+72*x^2*a^2*c^4+192*x^2
*a*b^2*c^3-30*x^2*b^4*c^2+72*x*a^2*b*c^3+40*x*a*b^3*c^2-10*x*b^5*c-56*a^3*c
^3+60*a^2*b^2*c^2-10*a*b^4*c)/d^3*(d*(2*c*x+b))^(1/2)/(c*x^2+b*x+a)^(1/2)/(
2*c*x+b)^2/c^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{5}{2}}}{\left(2\,cdx+bd\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)/(2*c*d*x + b*d)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\,ac\right)x^2+a^2\right)\sqrt{2\,cdx+bd}\sqrt{cx^2+bx+a}}{8\,c^3d^3x^3+12\,bc^2d^3x^2+6\,b^2cd^3x+b^3d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*sqrt(2*c
*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(8*c^3*d^3*x^3 + 12*b*c^2*d^3*x^2 + 6*b^2
*c*d^3*x + b^3*d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx + cx^2\right)^{\frac{5}{2}}}{\left(d\left(b + 2cx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(2*c*d*x+b*d)**(5/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(5/2)/(d*(b + 2*c*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx + a\right)^{\frac{5}{2}}}{\left(2\,cdx + bd\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)/(2*c*d*x + b*d)^(5/2), x)
```

3.1353 $\int \frac{(a+bx+cx^2)^{5/2}}{(bd+2cdx)^{9/2}}\,dx$

Optimal. Leaf size=211

$$-\frac{5\left(b^2-4ac\right)^{5/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{84c^4d^{9/2}\sqrt{a+bx+cx^2}}-\frac{5\left(a+bx+cx^2\right)^{3/2}}{42c^2d^3(bd+2cdx)^{3/2}}+\frac{5\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}}{84c^3d^5}-$$

```
[Out] (5*Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/(84*c^3*d^5) - (5*(a + b*x +
c*x^2)^(3/2))/(42*c^2*d^3*(b*d + 2*c*d*x)^(3/2)) - (a + b*x + c*x^2)^(5/2)/
(7*c*d*(b*d + 2*c*d*x)^(7/2)) - (5*(b^2 - 4*a*c)^(5/4)*Sqrt[-((c*(a + b*x +
 c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c
)^(1/4)*Sqrt[d])], -1])/(84*c^4*d^(9/2)*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.179425, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {684, 685, 691, 689, 221}

$$-\frac{5\left(b^2-4ac\right)^{5/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{84c^4d^{9/2}\sqrt{a+bx+cx^2}}-\frac{5\left(a+bx+cx^2\right)^{3/2}}{42c^2d^3(bd+2cdx)^{3/2}}+\frac{5\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}}{84c^3d^5}-\frac{(a+b}{7cd(b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^(9/2),x]
```

```
[Out] (5*Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/(84*c^3*d^5) - (5*(a + b*x +
c*x^2)^(3/2))/(42*c^2*d^3*(b*d + 2*c*d*x)^(3/2)) - (a + b*x + c*x^2)^(5/2)/
(7*c*d*(b*d + 2*c*d*x)^(7/2)) - (5*(b^2 - 4*a*c)^(5/4)*Sqrt[-((c*(a + b*x +
 c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c
)^(1/4)*Sqrt[d])], -1])/(84*c^4*d^(9/2)*Sqrt[a + b*x + c*x^2])
```

Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
```

```
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(bd+2cdx)^{9/2}}\,dx = -\frac{\left(a+bx+cx^2\right)^{5/2}}{7cd(bd+2cdx)^{7/2}}+\frac{5\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(bd+2cdx)^{5/2}}\,dx}{14cd^2}$$

$$= -\frac{5\left(a+bx+cx^2\right)^{3/2}}{42c^2d^3(bd+2cdx)^{3/2}}-\frac{\left(a+bx+cx^2\right)^{5/2}}{7cd(bd+2cdx)^{7/2}}+\frac{5\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{bd+2cdx}}\,dx}{28c^2d^4}$$

$$= \frac{5\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{84c^3d^5}-\frac{5\left(a+bx+cx^2\right)^{3/2}}{42c^2d^3(bd+2cdx)^{3/2}}-\frac{\left(a+bx+cx^2\right)^{5/2}}{7cd(bd+2cdx)^{7/2}}-\frac{\left(5\left(b^2-4ac\right)\right)\int}{16}$$

$$= \frac{5\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{84c^3d^5}-\frac{5\left(a+bx+cx^2\right)^{3/2}}{42c^2d^3(bd+2cdx)^{3/2}}-\frac{\left(a+bx+cx^2\right)^{5/2}}{7cd(bd+2cdx)^{7/2}}-\frac{\left(5\left(b^2-4ac\right)\sqrt{}\right.}{}$$

$$= \frac{5\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{84c^3d^5}-\frac{5\left(a+bx+cx^2\right)^{3/2}}{42c^2d^3(bd+2cdx)^{3/2}}-\frac{\left(a+bx+cx^2\right)^{5/2}}{7cd(bd+2cdx)^{7/2}}-\frac{\left(5\left(b^2-4ac\right)\sqrt{}\right.}{}$$

$$= \frac{5\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}{84c^3d^5}-\frac{5\left(a+bx+cx^2\right)^{3/2}}{42c^2d^3(bd+2cdx)^{3/2}}-\frac{\left(a+bx+cx^2\right)^{5/2}}{7cd(bd+2cdx)^{7/2}}-\frac{5\left(b^2-4ac\right)^{5/4}\sqrt{}}{}$$

Mathematica [C] time = 0.0867505, size = 109, normalized size = 0.52

$$-\frac{\left(b^2-4ac\right)^2\sqrt{a+x(b+cx)}\sqrt{d(b+2cx)}\,{}_2F_1\left(-\frac{5}{2},-\frac{7}{4};-\frac{3}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{224c^3d^5(b+2cx)^4\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^(9/2),x]
```

```
[Out] -((b^2 - 4*a*c)^2*Sqrt[d*(b + 2*c*x)]*Sqrt[a + x*(b + c*x)]*Hypergeometric2
F1[-5/2, -7/4, -3/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(224*c^3*d^5*(b + 2*c*x)
```

```
^4*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)])
```

Maple [B] time = 0.22, size = 1310, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(9/2),x)
```

```
[Out] 1/168*(c*x^2+b*x+a)^(1/2)*(d*(2*c*x+b))^(1/2)*(160*((b+2*c*x+(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-
2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x
+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2
)^(1/2)*x^3*a*c^4-40*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a
*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a
*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^3*b^2*c^3+240*((b+2*
c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^
(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*Ellip
ticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^
(1/2))*(-4*a*c+b^2)^(1/2)*x^2*a*b*c^3-60*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a
*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1
/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*
((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^
2*b^3*c^2+56*x^6*c^6+120*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^
(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/
(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a
*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2)^(1/2)*x*a*b^2*c^2-30*(-4
*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(
2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b
^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(
1/2))^(1/2)*2^(1/2),2^(1/2))*x*b^4*c+168*x^5*b*c^5+20*((b+2*c*x+(-4*a*c+b^2
)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((
-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*
c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+
b^2)^(1/2)*a*b^3*c-5*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a
*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a
*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*b^5-72*x^4*a*c^5+228*x
^4*b^2*c^4-144*x^3*a*b*c^4+176*x^3*b^3*c^3-152*x^2*a^2*c^4-32*x^2*a*b^2*c^3
+70*x^2*b^4*c^2-152*x*a^2*b*c^3+40*x*a*b^3*c^2+10*x*b^5*c-24*a^3*c^3-20*a^2
*b^2*c^2+10*a*b^4*c)/d^5/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)/(2*c*x+b)^
3/c^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{5}{2}}}{\left(2\,cdx+bd\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)/(2*c*d*x + b*d)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\,ac\right)x^2+a^2\right)\sqrt{2\,cdx+bd}\sqrt{cx^2+bx+a}}{32\,c^5d^5x^5+80\,bc^4d^5x^4+80\,b^2c^3d^5x^3+40\,b^3c^2d^5x^2+10\,b^4cd^5x+b^5d^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*sqrt(2*c
*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(32*c^5*d^5*x^5 + 80*b*c^4*d^5*x^4 + 80*b
^2*c^3*d^5*x^3 + 40*b^3*c^2*d^5*x^2 + 10*b^4*c*d^5*x + b^5*d^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(2*c*d*x+b*d)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{5}{2}}}{\left(2\,cdx+bd\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)/(2*c*d*x + b*d)^(9/2), x)
```

3.1354 $\int \frac{(a+bx+cx^2)^{5/2}}{(bd+2cdx)^{13/2}}\,dx$

Optimal. Leaf size=211

$$\frac{5\sqrt[4]{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{308c^4d^{13/2}\sqrt{a+bx+cx^2}}-\frac{5\left(a+bx+cx^2\right)^{3/2}}{154c^2d^3(bd+2cdx)^{7/2}}-\frac{5\sqrt{a+bx+cx^2}}{308c^3d^5(bd+2cdx)^{3/2}}-\frac{(a+b}{11cd(bd}$$

```
[Out] (-5*Sqrt[a + b*x + c*x^2])/(308*c^3*d^5*(b*d + 2*c*d*x)^(3/2)) - (5*(a + b*
x + c*x^2)^(3/2))/(154*c^2*d^3*(b*d + 2*c*d*x)^(7/2)) - (a + b*x + c*x^2)^(
5/2)/(11*c*d*(b*d + 2*c*d*x)^(11/2)) + (5*(b^2 - 4*a*c)^(1/4)*Sqrt[-((c*(a
+ b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2
- 4*a*c)^(1/4)*Sqrt[d])], -1])/(308*c^4*d^(13/2)*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.178951, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {684, 691, 689, 221}

$$\frac{5\sqrt[4]{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{308c^4d^{13/2}\sqrt{a+bx+cx^2}}-\frac{5\left(a+bx+cx^2\right)^{3/2}}{154c^2d^3(bd+2cdx)^{7/2}}-\frac{5\sqrt{a+bx+cx^2}}{308c^3d^5(bd+2cdx)^{3/2}}-\frac{(a+bx+cx^2}{11cd(bd+2cdx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^(13/2),x]
```


```
[Out] (-5*Sqrt[a + b*x + c*x^2])/(308*c^3*d^5*(b*d + 2*c*d*x)^(3/2)) - (5*(a + b*
x + c*x^2)^(3/2))/(154*c^2*d^3*(b*d + 2*c*d*x)^(7/2)) - (a + b*x + c*x^2)^(
5/2)/(11*c*d*(b*d + 2*c*d*x)^(11/2)) + (5*(b^2 - 4*a*c)^(1/4)*Sqrt[-((c*(a
+ b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2
- 4*a*c)^(1/4)*Sqrt[d])], -1])/(308*c^4*d^(13/2)*Sqrt[a + b*x + c*x^2])
```


Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
```

```
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(bd+2cdx)^{13/2}}\,dx = -\frac{\left(a+bx+cx^2\right)^{5/2}}{11cd(bd+2cdx)^{11/2}}+\frac{5\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(bd+2cdx)^{9/2}}\,dx}{22cd^2}$$

$$= -\frac{5\left(a+bx+cx^2\right)^{3/2}}{154c^2d^3(bd+2cdx)^{7/2}}-\frac{\left(a+bx+cx^2\right)^{5/2}}{11cd(bd+2cdx)^{11/2}}+\frac{15\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^{5/2}}\,dx}{308c^2d^4}$$

$$= -\frac{5\sqrt{a+bx+cx^2}}{308c^3d^5(bd+2cdx)^{3/2}}-\frac{5\left(a+bx+cx^2\right)^{3/2}}{154c^2d^3(bd+2cdx)^{7/2}}-\frac{\left(a+bx+cx^2\right)^{5/2}}{11cd(bd+2cdx)^{11/2}}+\frac{5\int \frac{1}{\sqrt{bd+2cdx}\sqrt{a+b}}}{616c^3d^6}$$

$$= -\frac{5\sqrt{a+bx+cx^2}}{308c^3d^5(bd+2cdx)^{3/2}}-\frac{5\left(a+bx+cx^2\right)^{3/2}}{154c^2d^3(bd+2cdx)^{7/2}}-\frac{\left(a+bx+cx^2\right)^{5/2}}{11cd(bd+2cdx)^{11/2}}+\frac{\left(5\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right.}{61}$$

$$= -\frac{5\sqrt{a+bx+cx^2}}{308c^3d^5(bd+2cdx)^{3/2}}-\frac{5\left(a+bx+cx^2\right)^{3/2}}{154c^2d^3(bd+2cdx)^{7/2}}-\frac{\left(a+bx+cx^2\right)^{5/2}}{11cd(bd+2cdx)^{11/2}}+\frac{\left(5\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right.}{}$$

$$= -\frac{5\sqrt{a+bx+cx^2}}{308c^3d^5(bd+2cdx)^{3/2}}-\frac{5\left(a+bx+cx^2\right)^{3/2}}{154c^2d^3(bd+2cdx)^{7/2}}-\frac{\left(a+bx+cx^2\right)^{5/2}}{11cd(bd+2cdx)^{11/2}}+\frac{5\sqrt[4]{b^2-4ac}\sqrt{-\frac{c}{}}}{308}$$

Mathematica [C] time = 0.0913246, size = 109, normalized size = 0.52

$$-\frac{\left(b^2-4ac\right)^2\sqrt{a+x(b+cx)}\sqrt{d(b+2cx)}\,{}_2F_1\left(-\frac{11}{4},-\frac{5}{2};-\frac{7}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{352c^3d^7(b+2cx)^6\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^(13/2),x]
```

```
[Out] -((b^2 - 4*a*c)^2*Sqrt[d*(b + 2*c*x)]*Sqrt[a + x*(b + c*x)]*Hypergeometric2
F1[-11/4, -5/2, -7/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(352*c^3*d^7*(b + 2*c*x
)^6*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)])
```

Maple [B] time = 0.222, size = 1035, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(13/2),x)
```

```
[Out] 1/616*(c*x^2+b*x+a)^(1/2)*(d*(2*c*x+b))^(1/2)*(160*(-4*a*c+b^2)^(1/2)*((b+2
*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)
^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*Elli
pticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2
^(1/2))*x^5*c^5+400*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*
c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*
c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2
)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^4*b*c^4+400*(-4*a*c+b
^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+
b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1
/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^
(1/2)*2^(1/2),2^(1/2))*x^3*b^2*c^3+200*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c
+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2
)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((
b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^2*
b^3*c^2-296*x^6*c^6+50*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4
*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4
*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+
b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x*b^4*c-888*x^5*b*c^
5+5*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1
/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-
4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c
+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*b^5-488*x^4*a*c^5-988*x^4*b^2*c^4-976*x
^3*a*b*c^4-496*x^3*b^3*c^3-248*x^2*a^2*c^4-608*x^2*a*b^2*c^3-110*x^2*b^4*c^
2-248*x*a^2*b*c^3-120*x*a*b^3*c^2-10*x*b^5*c-56*a^3*c^3-20*a^2*b^2*c^2-10*a
*b^4*c)/d^7/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)/(2*c*x+b)^5/c^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2+bx+a)^{\frac{5}{2}}}{(2\,cdx+bd)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(13/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)/(2*c*d*x + b*d)^(13/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\,ac\right)x^2+a^2\right)\sqrt{2\,cdx+bd}\sqrt{cx^2+bx+a}}{128\,c^7d^7x^7+448\,bc^6d^7x^6+672\,b^2c^5d^7x^5+560\,b^3c^4d^7x^4+280\,b^4c^3d^7x^3+84\,b^5c^2d^7x^2+14\,b^6cd^7x+b^7d^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(13/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*sqrt(2*c
*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(128*c^7*d^7*x^7 + 448*b*c^6*d^7*x^6 + 67
2*b^2*c^5*d^7*x^5 + 560*b^3*c^4*d^7*x^4 + 280*b^4*c^3*d^7*x^3 + 84*b^5*c^2*
d^7*x^2 + 14*b^6*c*d^7*x + b^7*d^7), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(2*c*d*x+b*d)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{5}{2}}}{\left(2\,cdx+bd\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(13/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)/(2*c*d*x + b*d)^(13/2), x)
```

3.1355 $\int \frac{(a+bx+cx^2)^{5/2}}{(bd+2cdx)^{17/2}}\,dx$

Optimal. Leaf size=258

$$\frac{\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{924c^4d^{17/2}(b^2-4ac)^{3/4}\sqrt{a+bx+cx^2}}+\frac{\sqrt{a+bx+cx^2}}{462c^3d^7(b^2-4ac)(bd+2cdx)^{3/2}}-\frac{(a+bx+cx^2)^{3/2}}{66c^2d^3(bd+2cdx)^{11/2}}-\frac{\sqrt{a}}{308c^3d^5}$$

```
[Out] -Sqrt[a + b*x + c*x^2]/(308*c^3*d^5*(b*d + 2*c*d*x)^(7/2)) + Sqrt[a + b*x +
 c*x^2]/(462*c^3*(b^2 - 4*a*c)*d^7*(b*d + 2*c*d*x)^(3/2)) - (a + b*x + c*x^
2)^(3/2)/(66*c^2*d^3*(b*d + 2*c*d*x)^(11/2)) - (a + b*x + c*x^2)^(5/2)/(15*
c*d*(b*d + 2*c*d*x)^(15/2)) + (Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]
*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/
(924*c^4*(b^2 - 4*a*c)^(3/4)*d^(17/2)*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.213809, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {684, 693, 691, 689, 221}

$$\frac{\sqrt{a+bx+cx^2}}{462c^3d^7(b^2-4ac)(bd+2cdx)^{3/2}}+\frac{\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{924c^4d^{17/2}(b^2-4ac)^{3/4}\sqrt{a+bx+cx^2}}-\frac{(a+bx+cx^2)^{3/2}}{66c^2d^3(bd+2cdx)^{11/2}}-\frac{\sqrt{a+bx+c}}{308c^3d^5(bd+2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^(17/2),x]
```


```
[Out] -Sqrt[a + b*x + c*x^2]/(308*c^3*d^5*(b*d + 2*c*d*x)^(7/2)) + Sqrt[a + b*x +
 c*x^2]/(462*c^3*(b^2 - 4*a*c)*d^7*(b*d + 2*c*d*x)^(3/2)) - (a + b*x + c*x^
2)^(3/2)/(66*c^2*d^3*(b*d + 2*c*d*x)^(11/2)) - (a + b*x + c*x^2)^(5/2)/(15*
c*d*(b*d + 2*c*d*x)^(15/2)) + (Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]
*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/
(924*c^4*(b^2 - 4*a*c)^(3/4)*d^(17/2)*Sqrt[a + b*x + c*x^2])
```


Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(bd+2cdx)^{17/2}}\,dx = -\frac{\left(a+bx+cx^2\right)^{5/2}}{15cd(bd+2cdx)^{15/2}}+\frac{\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(bd+2cdx)^{13/2}}\,dx}{6cd^2}$$

$$= -\frac{\left(a+bx+cx^2\right)^{3/2}}{66c^2d^3(bd+2cdx)^{11/2}}-\frac{\left(a+bx+cx^2\right)^{5/2}}{15cd(bd+2cdx)^{15/2}}+\frac{\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^{9/2}}\,dx}{44c^2d^4}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{308c^3d^5(bd+2cdx)^{7/2}}-\frac{\left(a+bx+cx^2\right)^{3/2}}{66c^2d^3(bd+2cdx)^{11/2}}-\frac{\left(a+bx+cx^2\right)^{5/2}}{15cd(bd+2cdx)^{15/2}}+\frac{\int \frac{1}{(bd+2cdx)^{5/2}\sqrt{a+b}}}{616c^3d^6}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{308c^3d^5(bd+2cdx)^{7/2}}+\frac{\sqrt{a+bx+cx^2}}{462c^3\left(b^2-4ac\right)d^7(bd+2cdx)^{3/2}}-\frac{\left(a+bx+cx^2\right)^{3/2}}{66c^2d^3(bd+2cdx)^{11/2}}-\frac{(a}{15c}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{308c^3d^5(bd+2cdx)^{7/2}}+\frac{\sqrt{a+bx+cx^2}}{462c^3\left(b^2-4ac\right)d^7(bd+2cdx)^{3/2}}-\frac{\left(a+bx+cx^2\right)^{3/2}}{66c^2d^3(bd+2cdx)^{11/2}}-\frac{(a}{15c}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{308c^3d^5(bd+2cdx)^{7/2}}+\frac{\sqrt{a+bx+cx^2}}{462c^3\left(b^2-4ac\right)d^7(bd+2cdx)^{3/2}}-\frac{\left(a+bx+cx^2\right)^{3/2}}{66c^2d^3(bd+2cdx)^{11/2}}-\frac{(a}{15c}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{308c^3d^5(bd+2cdx)^{7/2}}+\frac{\sqrt{a+bx+cx^2}}{462c^3\left(b^2-4ac\right)d^7(bd+2cdx)^{3/2}}-\frac{\left(a+bx+cx^2\right)^{3/2}}{66c^2d^3(bd+2cdx)^{11/2}}-\frac{(a}{15c}$$

Mathematica [C] time = 0.0950798, size = 109, normalized size = 0.42

$$-\frac{\left(b^2-4ac\right)^2\sqrt{a+x(b+cx)}\sqrt{d(b+2cx)}\,{}_2F_1\left(-\frac{15}{4},-\frac{5}{2};-\frac{11}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{480c^3d^9(b+2cx)^8\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^(17/2),x]
```

```
[Out] -((b^2 - 4*a*c)^2*Sqrt[d*(b + 2*c*x)]*Sqrt[a + x*(b + c*x)]*Hypergeometric2
F1[-15/4, -5/2, -11/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(480*c^3*d^9*(b + 2*c*
x)^8*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)])
```

Maple [B] time = 0.234, size = 1431, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(17/2),x)
```

```
[Out] -1/9240*(c*x^2+b*x+a)^(1/2)*(d*(2*c*x+b))^(1/2)*(2240*((b+2*c*x+(-4*a*c+b^2
)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((
-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*
c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+
b^2)^(1/2)*x^6*b*c^6+3360*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))
^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))
/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*
a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2)^(1/2)*x^5*b^2*c^5+1280*
x^8*c^8-10*x*b^7*c+70*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/
2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4
*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2)^(1/2)*x*b^6*c+420*((b+2*c*x
+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/
2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*Elliptic
F(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/
2))*(-4*a*c+b^2)^(1/2)*x^2*b^5*c^2+2800*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a
*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a
*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2)^(1/2)*x^4
*b^3*c^4+1400*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*
c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2
)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/
2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2)^(1/2)*x^3*b^4*c^3+640*((b+2*c*x+(-4
*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^
(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/
2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*
(-4*a*c+b^2)^(1/2)*x^7*c^7-948*x^4*b^4*c^4+3032*x^5*b^3*c^5+7904*x^6*a*c^7+
13792*x^4*a^2*c^6+9632*x^2*a^3*c^5+5120*x^7*b*c^7-976*x^3*b^5*c^3-150*x^2*b
^6*c^2+6984*x^6*b^2*c^6+2464*a^4*c^4-56*a^3*b^2*c^3-20*a^2*b^4*c^2-10*a*b^6
*c-160*x*a*b^5*c^2+5*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2
)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*
a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b
^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2)^(1/2)*b^7+23712*x^5*a*b*c^6+
22744*x^4*a*b^2*c^5+27584*x^3*a^2*b*c^5+5968*x^3*a*b^3*c^4+13464*x^2*a^2*b^
2*c^4-1128*x^2*a*b^4*c^3+9632*x*a^3*b*c^4-328*x*a^2*b^3*c^3)/d^9/(2*c^2*x^3
+3*b*c*x^2+2*a*c*x+b^2*x+a*b)/(2*c*x+b)^7/(4*a*c-b^2)/c^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)^{\frac{5}{2}}}{(2\,cdx + bd)^{\frac{17}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(17/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)/(2*c*d*x + b*d)^(17/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2x^4 + 2\,bcx^3 + 2\,abx + \left(b^2 + 2\,ac\right)x^2 + a^2\right)\sqrt{2\,cdx + bd}\sqrt{cx^2 + bx + a}}{512\,c^9d^9x^9 + 2304\,bc^8d^9x^8 + 4608\,b^2c^7d^9x^7 + 5376\,b^3c^6d^9x^6 + 4032\,b^4c^5d^9x^5 + 2016\,b^5c^4d^9x^4 + 672\,b^6c^3d^9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(17/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*sqrt(2*c
*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(512*c^9*d^9*x^9 + 2304*b*c^8*d^9*x^8 + 4
608*b^2*c^7*d^9*x^7 + 5376*b^3*c^6*d^9*x^6 + 4032*b^4*c^5*d^9*x^5 + 2016*b^
5*c^4*d^9*x^4 + 672*b^6*c^3*d^9*x^3 + 144*b^7*c^2*d^9*x^2 + 18*b^8*c*d^9*x
+ b^9*d^9), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(2*c*d*x+b*d)**(17/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)^{\frac{5}{2}}}{(2\,cdx + bd)^{\frac{17}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(17/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)/(2*c*d*x + b*d)^(17/2), x)
```

3.1356 $\int \frac{(a+bx+cx^2)^{5/2}}{(bd+2cdx)^{21/2}} dx$

Optimal. Leaf size=305

$$\frac{5\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{17556c^4d^{21/2}(b^2-4ac)^{7/4}\sqrt{a+bx+cx^2}}+\frac{5\sqrt{a+bx+cx^2}}{8778c^3d^9(b^2-4ac)^2(bd+2cdx)^{3/2}}+\frac{\sqrt{a+bx+cx^2}}{2926c^3d^7(b^2-4ac)(bd+2c}$$

```
[Out] -Sqrt[a + b*x + c*x^2]/(836*c^3*d^5*(b*d + 2*c*d*x)^(11/2)) + Sqrt[a + b*x
+ c*x^2]/(2926*c^3*(b^2 - 4*a*c)*d^7*(b*d + 2*c*d*x)^(7/2)) + (5*Sqrt[a + b
*x + c*x^2])/(8778*c^3*(b^2 - 4*a*c)^2*d^9*(b*d + 2*c*d*x)^(3/2)) - (a + b*
x + c*x^2)^(3/2)/(114*c^2*d^3*(b*d + 2*c*d*x)^(15/2)) - (a + b*x + c*x^2)^(
5/2)/(19*c*d*(b*d + 2*c*d*x)^(19/2)) + (5*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[
d])], -1])/(17556*c^4*(b^2 - 4*a*c)^(7/4)*d^(21/2)*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.262961, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {684, 693, 691, 689, 221}

$$\frac{5\sqrt{a+bx+cx^2}}{8778c^3d^9(b^2-4ac)^2(bd+2cdx)^{3/2}}+\frac{\sqrt{a+bx+cx^2}}{2926c^3d^7(b^2-4ac)(bd+2cdx)^{7/2}}+\frac{5\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{17556c^4d^{21/2}(b^2-4ac)^{7/4}\sqrt{a+bx+cx^2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^(21/2),x]
```


```
[Out] -Sqrt[a + b*x + c*x^2]/(836*c^3*d^5*(b*d + 2*c*d*x)^(11/2)) + Sqrt[a + b*x
+ c*x^2]/(2926*c^3*(b^2 - 4*a*c)*d^7*(b*d + 2*c*d*x)^(7/2)) + (5*Sqrt[a + b
*x + c*x^2])/(8778*c^3*(b^2 - 4*a*c)^2*d^9*(b*d + 2*c*d*x)^(3/2)) - (a + b*
x + c*x^2)^(3/2)/(114*c^2*d^3*(b*d + 2*c*d*x)^(15/2)) - (a + b*x + c*x^2)^(
5/2)/(19*c*d*(b*d + 2*c*d*x)^(19/2)) + (5*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[
d])], -1])/(17556*c^4*(b^2 - 4*a*c)^(7/4)*d^(21/2)*Sqrt[a + b*x + c*x^2])
```


Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
```

```
IntegerQ[(m + 2*p + 3)/2])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(bd+2cdx)^{21/2}}\,dx = -\frac{\left(a+bx+cx^2\right)^{5/2}}{19cd(bd+2cdx)^{19/2}}+\frac{5\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(bd+2cdx)^{17/2}}\,dx}{38cd^2}$$

$$= -\frac{\left(a+bx+cx^2\right)^{3/2}}{114c^2d^3(bd+2cdx)^{15/2}}-\frac{\left(a+bx+cx^2\right)^{5/2}}{19cd(bd+2cdx)^{19/2}}+\frac{\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^{13/2}}\,dx}{76c^2d^4}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{836c^3d^5(bd+2cdx)^{11/2}}-\frac{\left(a+bx+cx^2\right)^{3/2}}{114c^2d^3(bd+2cdx)^{15/2}}-\frac{\left(a+bx+cx^2\right)^{5/2}}{19cd(bd+2cdx)^{19/2}}+\frac{\int \frac{1}{(bd+2cdx)^{9/2}\sqrt{a\cdots}}}{1672c^3\cdots}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{836c^3d^5(bd+2cdx)^{11/2}}+\frac{\sqrt{a+bx+cx^2}}{2926c^3\left(b^2-4ac\right)d^7(bd+2cdx)^{7/2}}-\frac{\left(a+bx+cx^2\right)^{3/2}}{114c^2d^3(bd+2cdx)^{15/2}}-\cdots$$

$$= -\frac{\sqrt{a+bx+cx^2}}{836c^3d^5(bd+2cdx)^{11/2}}+\frac{\sqrt{a+bx+cx^2}}{2926c^3\left(b^2-4ac\right)d^7(bd+2cdx)^{7/2}}+\frac{5\sqrt{a+bx+cx^2}}{8778c^3\left(b^2-4ac\right)^2d^9(bd\cdots}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{836c^3d^5(bd+2cdx)^{11/2}}+\frac{\sqrt{a+bx+cx^2}}{2926c^3\left(b^2-4ac\right)d^7(bd+2cdx)^{7/2}}+\frac{5\sqrt{a+bx+cx^2}}{8778c^3\left(b^2-4ac\right)^2d^9(bd\cdots}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{836c^3d^5(bd+2cdx)^{11/2}}+\frac{\sqrt{a+bx+cx^2}}{2926c^3\left(b^2-4ac\right)d^7(bd+2cdx)^{7/2}}+\frac{5\sqrt{a+bx+cx^2}}{8778c^3\left(b^2-4ac\right)^2d^9(bd\cdots}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{836c^3d^5(bd+2cdx)^{11/2}}+\frac{\sqrt{a+bx+cx^2}}{2926c^3\left(b^2-4ac\right)d^7(bd+2cdx)^{7/2}}+\frac{5\sqrt{a+bx+cx^2}}{8778c^3\left(b^2-4ac\right)^2d^9(bd\cdots}$$

Mathematica [C] time = 0.0846049, size = 109, normalized size = 0.36

$$-\frac{(b^2-4ac)^2\sqrt{a+x(b+cx)}\,{}_2F_1\left(-\frac{19}{4},-\frac{5}{2};-\frac{15}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{608c^3d^9(b+2cx)^8\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}(d(b+2cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^(21/2),x]
```

```
[Out] -((b^2 - 4*a*c)^2*Sqrt[a + x*(b + c*x)]*Hypergeometric2F1[-19/4, -5/2, -15/
4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(608*c^3*d^9*(b + 2*c*x)^8*(d*(b + 2*c*x))
^(3/2)*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)])
```

Maple [B] time = 0.287, size = 1843, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(21/2),x)
```

```
[Out] 1/35112*(c*x^2+b*x+a)^(1/2)*(d*(2*c*x+b))^(1/2)*(-29568*a^5*c^5+24904*x^5*b
^5*c^5-190*x^2*b^8*c^2+25600*x^9*b*c^9+1940*x^4*b^6*c^4-10*x*b^9*c+59672*x^
6*b^4*c^6+74752*x^7*b^3*c^7-1616*x^3*b^7*c^3+57088*x^8*b^2*c^8+2048*x^8*a*c
^9-63104*x^6*a^2*c^8-138880*x^4*a^3*c^7-108416*x^2*a^4*c^6+9856*a^4*b^2*c^4
-56*a^3*b^4*c^3-20*a^2*b^6*c^2-10*a*b^8*c+2560*(-4*a*c+b^2)^(1/2)*((b+2*c*x
+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/
2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*Elliptic
F(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/
2))*x^9*c^9+5*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)
^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2
))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*b^9+720*(-4*a*c+b^2)^(1/2)*((
b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b
^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*E
llipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2
),2^(1/2))*x^2*b^7*c^2+90*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/
(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+
(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a
*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x*b^8*c+10080*(-4
*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(
2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b
^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(
1/2))^(1/2)*2^(1/2),2^(1/2))*x^4*b^5*c^4+3360*(-4*a*c+b^2)^(1/2)*((b+2*c*x+
(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2
))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF
(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2
))*x^3*b^6*c^3+26880*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a
*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a
*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^6*b^3*c^6+20160*(-4*
a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2
*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^
2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1
/2))^(1/2)*2^(1/2),2^(1/2))*x^5*b^4*c^5+11520*(-4*a*c+b^2)^(1/2)*((b+2*c*x+
```

```
(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2
))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF
(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2
))*x^8*b*c^8+23040*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c
+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c
+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)
^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^7*b^2*c^7-189312*x^5*a
^2*b*c^7+108992*x^5*a*b^3*c^6-132480*x^4*a^2*b^2*c^6+101240*x^4*a*b^4*c^5-2
77760*x^3*a^3*b*c^6+50560*x^3*a^2*b^3*c^5+30384*x^3*a*b^5*c^4-99904*x^2*a^3
*b^2*c^5+56424*x^2*a^2*b^4*c^4-1808*x^2*a*b^6*c^3-108416*x*a^4*b*c^5+38976*
x*a^3*b^3*c^4-408*x*a^2*b^5*c^3-200*x*a*b^7*c^2+8192*x^7*a*b*c^8+45888*x^6*
a*b^2*c^7+5120*x^10*c^10)/d^11/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)/(2*c
*x+b)^9/(4*a*c-b^2)^2/c^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{5}{2}}}{\left(2\,cdx+bd\right)^{\frac{21}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(21/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)/(2*c*d*x + b*d)^(21/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\,ac\right)x^2+\right.}{2048\,c^{11}d^{11}x^{11}+11264\,bc^{10}d^{11}x^{10}+28160\,b^2c^9d^{11}x^9+42240\,b^3c^8d^{11}x^8+42240\,b^4c^7d^{11}x^7+29568\,b^5c^6d^{1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(21/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*sqrt(2*c
*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(2048*c^11*d^11*x^11 + 11264*b*c^10*d^11*
x^10 + 28160*b^2*c^9*d^11*x^9 + 42240*b^3*c^8*d^11*x^8 + 42240*b^4*c^7*d^11
*x^7 + 29568*b^5*c^6*d^11*x^6 + 14784*b^6*c^5*d^11*x^5 + 5280*b^7*c^4*d^11*
x^4 + 1320*b^8*c^3*d^11*x^3 + 220*b^9*c^2*d^11*x^2 + 22*b^10*c*d^11*x + b^1
1*d^11), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(2*c*d*x+b*d)**(21/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{5}{2}}}{\left(2\,cdx+bd\right)^{\frac{21}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(21/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)/(2*c*d*x + b*d)^(21/2), x)
```

3.1357 $\int (bd+2cdx)^{5/2}\left(a+bx+cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=373

$$\frac{d^{5/2}\left(b^2-4ac\right)^{19/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{884c^4\sqrt{a+bx+cx^2}}-\frac{d^{5/2}\left(b^2-4ac\right)^{19/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{bd+}}{\sqrt[4]{b^2-4}}\right.\right.}{884c^4\sqrt{a+bx+cx^2}}$$

```
[Out] -((b^2 - 4*a*c)^3*d*(b*d + 2*c*d*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(1326*c^3)
 + (5*(b^2 - 4*a*c)^2*(b*d + 2*c*d*x)^(7/2)*Sqrt[a + b*x + c*x^2])/(2652*c^
3*d) - (5*(b^2 - 4*a*c)*(b*d + 2*c*d*x)^(7/2)*(a + b*x + c*x^2)^(3/2))/(442
*c^2*d) + ((b*d + 2*c*d*x)^(7/2)*(a + b*x + c*x^2)^(5/2))/(17*c*d) - ((b^2
- 4*a*c)^(19/4)*d^(5/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellipt
icE[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(884*c^
4*Sqrt[a + b*x + c*x^2]) + ((b^2 - 4*a*c)^(19/4)*d^(5/2)*Sqrt[-((c*(a + b*x
 + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a
*c)^(1/4)*Sqrt[d])], -1])/(884*c^4*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.372943, antiderivative size = 373, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {685, 692, 691, 690, 307, 221, 1199, 424}

$$\frac{d^{5/2}\left(b^2-4ac\right)^{19/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{884c^4\sqrt{a+bx+cx^2}}-\frac{d^{5/2}\left(b^2-4ac\right)^{19/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-\right.}{884c^4\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(5/2)*(a + b*x + c*x^2)^(5/2),x]
```


```
[Out] -((b^2 - 4*a*c)^3*d*(b*d + 2*c*d*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(1326*c^3)
 + (5*(b^2 - 4*a*c)^2*(b*d + 2*c*d*x)^(7/2)*Sqrt[a + b*x + c*x^2])/(2652*c^
3*d) - (5*(b^2 - 4*a*c)*(b*d + 2*c*d*x)^(7/2)*(a + b*x + c*x^2)^(3/2))/(442
*c^2*d) + ((b*d + 2*c*d*x)^(7/2)*(a + b*x + c*x^2)^(5/2))/(17*c*d) - ((b^2
- 4*a*c)^(19/4)*d^(5/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellipt
icE[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(884*c^
4*Sqrt[a + b*x + c*x^2]) + ((b^2 - 4*a*c)^(19/4)*d^(5/2)*Sqrt[-((c*(a + b*x
 + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a
*c)^(1/4)*Sqrt[d])], -1])/(884*c^4*Sqrt[a + b*x + c*x^2])
```


Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
```

```
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int (bd+2cdx)^{5/2}\left(a+bx+cx^2\right)^{5/2} dx = \frac{(bd+2cdx)^{7/2}\left(a+bx+cx^2\right)^{5/2}}{17cd} - \frac{\left(5\left(b^2-4ac\right)\right)\int (bd+2cdx)^{5/2}\left(a+bx+cx\right.}{34c}$$

$$= -\frac{5\left(b^2-4ac\right)(bd+2cdx)^{7/2}\left(a+bx+cx^2\right)^{3/2}}{442c^2d} + \frac{(bd+2cdx)^{7/2}\left(a+bx+cx^2\right)}{17cd}$$

$$= \frac{5\left(b^2-4ac\right)^2(bd+2cdx)^{7/2}\sqrt{a+bx+cx^2}}{2652c^3d} - \frac{5\left(b^2-4ac\right)(bd+2cdx)^{7/2}\left(a+b\right.}{442c^2d}$$

$$= -\frac{\left(b^2-4ac\right)^3 d(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{1326c^3} + \frac{5\left(b^2-4ac\right)^2(bd+2cdx)^{7/2}\sqrt{a}}{2652c^3d}$$

$$= -\frac{\left(b^2-4ac\right)^3 d(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{1326c^3} + \frac{5\left(b^2-4ac\right)^2(bd+2cdx)^{7/2}\sqrt{a}}{2652c^3d}$$

$$= -\frac{\left(b^2-4ac\right)^3 d(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{1326c^3} + \frac{5\left(b^2-4ac\right)^2(bd+2cdx)^{7/2}\sqrt{a}}{2652c^3d}$$

$$= -\frac{\left(b^2-4ac\right)^3 d(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{1326c^3} + \frac{5\left(b^2-4ac\right)^2(bd+2cdx)^{7/2}\sqrt{a}}{2652c^3d}$$

$$= -\frac{\left(b^2-4ac\right)^3 d(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{1326c^3} + \frac{5\left(b^2-4ac\right)^2(bd+2cdx)^{7/2}\sqrt{a}}{2652c^3d}$$

$$= -\frac{\left(b^2-4ac\right)^3 d(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{1326c^3} + \frac{5\left(b^2-4ac\right)^2(bd+2cdx)^{7/2}\sqrt{a}}{2652c^3d}$$

Mathematica [C] time = 0.185608, size = 117, normalized size = 0.31

$$\frac{2}{17}d\sqrt{a+x(b+cx)}(d(b+2cx))^{3/2}\left(\frac{\left(b^2-4ac\right)^3 {}_2F_1\left(-\frac{5}{2},\frac{3}{4};\frac{7}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{64c^3\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}+2(a+x(b+cx))^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(5/2)*(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*d*(d*(b + 2*c*x))^(3/2)*Sqrt[a + x*(b + c*x)]*(2*(a + x*(b + c*x))^3 + (
(b^2 - 4*a*c)^3*Hypergeometric2F1[-5/2, 3/4, 7/4, (b + 2*c*x)^2/(b^2 - 4*a*
c)])/(64*c^3*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)])))/17
```

Maple [B] time = 0.227, size = 1190, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(5/2)*(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] -1/5304*(d*(2*c*x+b))^(1/2)*(c*x^2+b*x+a)^(1/2)*d^2*(-11112*x^5*b^5*c^5-10*
x^2*b^8*c^2-24960*x^9*b*c^9-1516*x^4*b^6*c^4-6*x*b^9*c-34168*x^6*b^4*c^6-56
064*x^7*b^3*c^7-51456*x^8*b^2*c^8-18816*x^8*a*c^9-25216*x^6*a^2*c^8-12416*x
^4*a^3*c^7-1024*x^2*a^4*c^6-256*a^4*b^2*c^4-520*a^3*b^4*c^3+92*a^2*b^6*c^2-
6*a*b^8*c-3*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*
x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^
(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
)^(1/2)*2^(1/2),2^(1/2))*b^10+3072*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^
2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a^5*c^5-75648*x^5*a^2*b*c^7
-93888*x^5*a*b^3*c^6-85248*x^4*a^2*b^2*c^6-37368*x^4*a*b^4*c^5-24832*x^3*a^
3*b*c^6-44416*x^3*a^2*b^3*c^5-6064*x^3*a*b^5*c^4-17600*x^2*a^3*b^2*c^5-1005
6*x^2*a^2*b^4*c^4+160*x^2*a*b^6*c^3-1024*x*a^4*b*c^5-5184*x*a^3*b^3*c^4-456
*x*a^2*b^5*c^3+88*x*a*b^7*c^2-75264*x^7*a*b*c^8-119104*x^6*a*b^2*c^7+1920*(
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+
b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*
EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/
2),2^(1/2))*a^3*b^4*c^3-480*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2
))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2
))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-
4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a^2*b^6*c^2+60*((b+2*c*x+(-4*a*c+b
^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*
((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+
2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a*b^8*
c-3840*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/
(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
)^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/
2)*2^(1/2),2^(1/2))*a^4*b^2*c^4-4992*x^10*c^10)/c^4/(2*c^2*x^3+3*b*c*x^2+2*
a*c*x+b^2*x+a*b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (2\,cdx + bd)^{\frac{5}{2}} \left(cx^2 + bx + a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(5/2)*(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^(5/2)*(c*x^2 + b*x + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$\text{integral}\Big(\Big(4\,c^4d^2x^6 + 12\,bc^3d^2x^5 + \left(13\,b^2c^2 + 8\,ac^3\right)d^2x^4 + a^2b^2d^2 + 2\left(3\,b^3c + 8\,abc^2\right)d^2x^3 + \left(b^4 + 10\,ab^2c + 4\,a^2c^2\right)d^2x^2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(5/2)*(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((4*c^4*d^2*x^6 + 12*b*c^3*d^2*x^5 + (13*b^2*c^2 + 8*a*c^3)*d^2*x^4
 + a^2*b^2*d^2 + 2*(3*b^3*c + 8*a*b*c^2)*d^2*x^3 + (b^4 + 10*a*b^2*c + 4*a^
```

```
2*c^2)*d^2*x^2 + 2*(a*b^3 + 2*a^2*b*c)*d^2*x)*sqrt(2*c*d*x + b*d)*sqrt(c*x^
2 + b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(5/2)*(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (2\,cdx + bd)^{\frac{5}{2}} \left(cx^2 + bx + a\right)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(5/2)*(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^(5/2)*(c*x^2 + b*x + a)^(5/2), x)
```

3.1358 $\int \sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=328

$$\frac{\sqrt{d}\left(b^2-4ac\right)^{15/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{156c^4\sqrt{a+bx+cx^2}}+\frac{\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}(bd+2cdx)^{3/2}}{156c^3d}-\frac{5\left(b^2-4\right.}{}$$

```
[Out] ((b^2 - 4*a*c)^2*(b*d + 2*c*d*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(156*c^3*d) -
 (5*(b^2 - 4*a*c)*(b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)^(3/2))/(234*c^2*d
) + ((b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)^(5/2))/(13*c*d) - ((b^2 - 4*a*
c)^(15/4)*Sqrt[d]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[Ar
cSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(156*c^4*Sqrt
[a + b*x + c*x^2]) + ((b^2 - 4*a*c)^(15/4)*Sqrt[d]*Sqrt[-((c*(a + b*x + c*x
^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1
/4)*Sqrt[d])], -1])/(156*c^4*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.298775, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {685, 691, 690, 307, 221, 1199, 424}

$$\frac{\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}(bd+2cdx)^{3/2}}{156c^3d}-\frac{5\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}(bd+2cdx)^{3/2}}{234c^2d}+\frac{\sqrt{d}\left(b^2-4ac\right)^{15/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}}{156c^4\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*d + 2*c*d*x]*(a + b*x + c*x^2)^(5/2),x]
```


```
[Out] ((b^2 - 4*a*c)^2*(b*d + 2*c*d*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(156*c^3*d) -
 (5*(b^2 - 4*a*c)*(b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)^(3/2))/(234*c^2*d
) + ((b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)^(5/2))/(13*c*d) - ((b^2 - 4*a*
c)^(15/4)*Sqrt[d]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[Ar
cSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(156*c^4*Sqrt
[a + b*x + c*x^2]) + ((b^2 - 4*a*c)^(15/4)*Sqrt[d]*Sqrt[-((c*(a + b*x + c*x
^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1
/4)*Sqrt[d])], -1])/(156*c^4*Sqrt[a + b*x + c*x^2])
```


Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{5/2}\,dx = \frac{(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{5/2}}{13cd} - \frac{\left(5\left(b^2-4ac\right)\right)\int \sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2}\,d}{26c}$$

$$= -\frac{5\left(b^2-4ac\right)(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{234c^2d} + \frac{(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{5/2}}{13cd} +$$

$$= \frac{\left(b^2-4ac\right)^2(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{156c^3d} - \frac{5\left(b^2-4ac\right)(bd+2cdx)^{3/2}\left(a+bx+cx\right.}{234c^2d}$$

$$= \frac{\left(b^2-4ac\right)^2(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{156c^3d} - \frac{5\left(b^2-4ac\right)(bd+2cdx)^{3/2}\left(a+bx+cx\right.}{234c^2d}$$

$$= \frac{\left(b^2-4ac\right)^2(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{156c^3d} - \frac{5\left(b^2-4ac\right)(bd+2cdx)^{3/2}\left(a+bx+cx\right.}{234c^2d}$$

$$= \frac{\left(b^2-4ac\right)^2(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{156c^3d} - \frac{5\left(b^2-4ac\right)(bd+2cdx)^{3/2}\left(a+bx+cx\right.}{234c^2d}$$

$$= \frac{\left(b^2-4ac\right)^2(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{156c^3d} - \frac{5\left(b^2-4ac\right)(bd+2cdx)^{3/2}\left(a+bx+cx\right.}{234c^2d}$$

$$= \frac{\left(b^2-4ac\right)^2(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{156c^3d} - \frac{5\left(b^2-4ac\right)(bd+2cdx)^{3/2}\left(a+bx+cx\right.}{234c^2d}$$

Mathematica [C] time = 0.067064, size = 101, normalized size = 0.31

$$\frac{\left(b^2-4ac\right)^2\sqrt{a+x(b+cx)}(d(b+2cx))^{3/2}\,{}_2F_1\left(-\frac{5}{2},\frac{3}{4};\frac{7}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{96c^3d\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*d + 2*c*d*x]*(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] ((b^2 - 4*a*c)^2*(d*(b + 2*c*x))^(3/2)*Sqrt[a + x*(b + c*x)]*Hypergeometric
2F1[-5/2, 3/4, 7/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(96*c^3*d*Sqrt[(c*(a + x*
(b + c*x)))/(-b^2 + 4*a*c)])
```

Maple [B] time = 0.219, size = 924, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(1/2)*(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 1/936*(d*(2*c*x+b))^(1/2)*(c*x^2+b*x+a)^(1/2)*(288*x^8*c^8+6*x*b^7*c+3*((b+
2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2
)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*Ell
ipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),
2^(1/2))*b^8+268*x^4*b^4*c^4+1128*x^5*b^3*c^5+1184*x^6*a*c^7+1888*x^4*a^2*c
^6+992*x^2*a^3*c^5+1152*x^7*b*c^7+10*x^2*b^6*c^2+1720*x^6*b^2*c^6+248*a^3*b
^2*c^3-68*a^2*b^4*c^2+6*a*b^6*c-64*x*a*b^5*c^2+3552*x^5*a*b*c^6+3496*x^4*a*
b^2*c^5+3776*x^3*a^2*b*c^5+1072*x^3*a*b^3*c^4+2088*x^2*a^2*b^2*c^4-120*x^2*
a*b^4*c^3+992*x*a^3*b*c^4+200*x*a^2*b^3*c^3+768*((b+2*c*x+(-4*a*c+b^2)^(1/2
))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c
*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(-
4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a^4*c^4-768*((
b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b
^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*E
llipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2
),2^(1/2))*a^3*b^2*c^3+288*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
)^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2)
)/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4
*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a^2*b^4*c^2-48*((b+2*c*x+(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*(
(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2
*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a*b^6*c
)/c^4/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{2\,cdx+bd}\left(cx^2+bx+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)*(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*c*d*x + b*d)*(c*x^2 + b*x + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\,ac\right)x^2+a^2\right)\sqrt{2\,cdx+bd}\sqrt{cx^2+bx+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)*(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*sqrt(2*c
*d*x + b*d)*sqrt(c*x^2 + b*x + a), x)
```

Sympy [A] time = 47.0422, size = 539, normalized size = 1.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(1/2)*(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] a**2*(b*d + 2*c*d*x)**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), (b*d + 2*
c*d*x)**2*exp_polar(I*pi)/(4*c*d**2*polar_lift(a - b**2/(4*c))))*sqrt(polar
_lift(a - b**2/(4*c)))/(4*c*d*gamma(7/4)) - a*b**2*(b*d + 2*c*d*x)**(3/2)*g
amma(3/4)*hyper((-1/2, 3/4), (7/4,), (b*d + 2*c*d*x)**2*exp_polar(I*pi)/(4*
c*d**2*polar_lift(a - b**2/(4*c))))*sqrt(polar_lift(a - b**2/(4*c)))/(8*c**
2*d*gamma(7/4)) + a*(b*d + 2*c*d*x)**(7/2)*gamma(7/4)*hyper((-1/2, 7/4), (1
1/4,), (b*d + 2*c*d*x)**2*exp_polar(I*pi)/(4*c*d**2*polar_lift(a - b**2/(4*
c))))*sqrt(polar_lift(a - b**2/(4*c)))/(8*c**2*d**3*gamma(11/4)) + b**4*(b*
d + 2*c*d*x)**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), (b*d + 2*c*d*x)**
2*exp_polar(I*pi)/(4*c*d**2*polar_lift(a - b**2/(4*c))))*sqrt(polar_lift(a
- b**2/(4*c)))/(64*c**3*d*gamma(7/4)) - b**2*(b*d + 2*c*d*x)**(7/2)*gamma(7
/4)*hyper((-1/2, 7/4), (11/4,), (b*d + 2*c*d*x)**2*exp_polar(I*pi)/(4*c*d**
2*polar_lift(a - b**2/(4*c))))*sqrt(polar_lift(a - b**2/(4*c)))/(32*c**3*d*
*3*gamma(11/4)) + (b*d + 2*c*d*x)**(11/2)*gamma(11/4)*hyper((-1/2, 11/4), (
15/4,), (b*d + 2*c*d*x)**2*exp_polar(I*pi)/(4*c*d**2*polar_lift(a - b**2/(4
*c))))*sqrt(polar_lift(a - b**2/(4*c)))/(64*c**3*d**5*gamma(15/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{2\,cdx + bd}\left(cx^2 + bx + a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)*(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(2*c*d*x + b*d)*(c*x^2 + b*x + a)^(5/2), x)
```

3.1359 $\int \frac{(a+bx+cx^2)^{5/2}}{(bd+2cdx)^{3/2}}\,dx$

Optimal. Leaf size=316

$$-\frac{(b^2-4ac)^{11/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{12c^4d^{3/2}\sqrt{a+bx+cx^2}}-\frac{(b^2-4ac)\sqrt{a+bx+cx^2}(bd+2cdx)^{3/2}}{12c^3d^3}+\frac{(b^2-4a}{}$$

```
[Out] -((b^2 - 4*a*c)*(b*d + 2*c*d*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(12*c^3*d^3) +
 (5*(b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)^(3/2))/(18*c^2*d^3) - (a + b*x
+ c*x^2)^(5/2)/(c*d*Sqrt[b*d + 2*c*d*x]) + ((b^2 - 4*a*c)^(11/4)*Sqrt[-((c*
(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d*x]/((b
^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(12*c^4*d^(3/2)*Sqrt[a + b*x + c*x^2]) -
((b^2 - 4*a*c)^(11/4)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Elliptic
F[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(12*c^4*d
^(3/2)*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.292052, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {684, 685, 691, 690, 307, 221, 1199, 424}

$$-\frac{(b^2-4ac)\sqrt{a+bx+cx^2}(bd+2cdx)^{3/2}}{12c^3d^3}-\frac{(b^2-4ac)^{11/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{12c^4d^{3/2}\sqrt{a+bx+cx^2}}+\frac{(b^2-4ac)^{11/4}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^(3/2),x]
```

```
[Out] -((b^2 - 4*a*c)*(b*d + 2*c*d*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(12*c^3*d^3) +
 (5*(b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)^(3/2))/(18*c^2*d^3) - (a + b*x
+ c*x^2)^(5/2)/(c*d*Sqrt[b*d + 2*c*d*x]) + ((b^2 - 4*a*c)^(11/4)*Sqrt[-((c*
(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d*x]/((b
^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(12*c^4*d^(3/2)*Sqrt[a + b*x + c*x^2]) -
((b^2 - 4*a*c)^(11/4)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Elliptic
F[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(12*c^4*d
^(3/2)*Sqrt[a + b*x + c*x^2])
```

Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
```

```
lQ[m] && IntegerQ[2*p]
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(bd+2cdx)^{3/2}}\,dx = -\frac{\left(a+bx+cx^2\right)^{5/2}}{cd\sqrt{bd+2cdx}} + \frac{5\int\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2}dx}{2cd^2}$$

$$= \frac{5(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{18c^2d^3} - \frac{\left(a+bx+cx^2\right)^{5/2}}{cd\sqrt{bd+2cdx}} - \frac{\left(5\left(b^2-4ac\right)\right)\int\sqrt{bd+2cdx}\sqrt{a+b}}{12c^2d^2}$$

$$= -\frac{\left(b^2-4ac\right)(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{12c^3d^3} + \frac{5(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{18c^2d^3} - \frac{\left(a+bx+c\right.}{cd\sqrt{bd+2}}$$

$$= -\frac{\left(b^2-4ac\right)(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{12c^3d^3} + \frac{5(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{18c^2d^3} - \frac{\left(a+bx+c\right.}{cd\sqrt{bd+2}}$$

$$= -\frac{\left(b^2-4ac\right)(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{12c^3d^3} + \frac{5(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{18c^2d^3} - \frac{\left(a+bx+c\right.}{cd\sqrt{bd+2}}$$

$$= -\frac{\left(b^2-4ac\right)(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{12c^3d^3} + \frac{5(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{18c^2d^3} - \frac{\left(a+bx+c\right.}{cd\sqrt{bd+2}}$$

$$= -\frac{\left(b^2-4ac\right)(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{12c^3d^3} + \frac{5(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{18c^2d^3} - \frac{\left(a+bx+c\right.}{cd\sqrt{bd+2}}$$

$$= -\frac{\left(b^2-4ac\right)(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{12c^3d^3} + \frac{5(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{18c^2d^3} - \frac{\left(a+bx+c\right.}{cd\sqrt{bd+2}}$$

Mathematica [C] time = 0.0536455, size = 101, normalized size = 0.32

$$-\frac{\left(b^2-4ac\right)^2\sqrt{a+x(b+cx)}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{4};\frac{3}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{32c^3d\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\sqrt{d(b+2cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^(3/2),x]
```

```
[Out] -((b^2 - 4*a*c)^2*Sqrt[a + x*(b + c*x)]*Hypergeometric2F1[-5/2, -1/4, 3/4,
(b + 2*c*x)^2/(b^2 - 4*a*c)])/(32*c^3*d*Sqrt[d*(b + 2*c*x)]*Sqrt[(c*(a + x*
(b + c*x)))/(-b^2 + 4*a*c)])
```

Maple [B] time = 0.227, size = 700, normalized size = 2.2

$$\frac{1}{72\,c^4d^2\left(2\,c^2x^3+3\,bcx^2+2\,acx+b^2x+ab\right)}\sqrt{cx^2+bx+a}\sqrt{d\left(2\,cx+b\right)}\left(8\,x^6c^6+24\,x^5bc^5+192\sqrt{\frac{b+2\,cx+\sqrt{-4\,ac}}{\sqrt{-4\,ac+b^2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(3/2),x)
```

```
[Out] 1/72*(c*x^2+b*x+a)^(1/2)*(d*(2*c*x+b))^(1/2)*(8*x^6*c^6+24*x^5*b*c^5+192*((
b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b
^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*E
llipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2
),2^(1/2))*a^3*c^3-144*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1
/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-
4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c
+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a^2*b^2*c^2+36*((b+2*c*x+(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-
2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x
+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a*b^4*c-3*(
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+
b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*
EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/
2),2^(1/2))*b^6+40*x^4*a*c^5+20*x^4*b^2*c^4+80*x^3*a*b*c^4-40*x^2*a^2*c^4+8
0*x^2*a*b^2*c^3-10*x^2*b^4*c^2-40*x*a^2*b*c^3+40*x*a*b^3*c^2-6*x*b^5*c-72*a
^3*c^3+44*a^2*b^2*c^2-6*a*b^4*c)/d^2/c^4/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*x
+a*b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{5}{2}}}{\left(2\,cdx+bd\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)/(2*c*d*x + b*d)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\,ac\right)x^2+a^2\right)\sqrt{2\,cdx+bd}\sqrt{cx^2+bx+a}}{4\,c^2d^2x^2+4\,bcd^2x+b^2d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*sqrt(2*c
*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(4*c^2*d^2*x^2 + 4*b*c*d^2*x + b^2*d^2),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx+cx^2\right)^{\frac{5}{2}}}{\left(d\left(b+2cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(2*c*d*x+b*d)**(3/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(5/2)/(d*(b + 2*c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)^{\frac{5}{2}}}{(2\,cdx + bd)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)/(2*c*d*x + b*d)^(3/2), x)
```

3.1360 $\int \frac{(a+bx+cx^2)^{5/2}}{(bd+2cdx)^{7/2}}\,dx$

Optimal. Leaf size=310

$$\frac{3\left(b^2-4ac\right)^{7/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{20c^4d^{7/2}\sqrt{a+bx+cx^2}}-\frac{3\left(b^2-4ac\right)^{7/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{20c^4d^{7/2}\sqrt{a+bx+cx^2}}$$

```
[Out] (3*(b*d + 2*c*d*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(20*c^3*d^5) - (a + b*x + c
*x^2)^(3/2)/(2*c^2*d^3*Sqrt[b*d + 2*c*d*x]) - (a + b*x + c*x^2)^(5/2)/(5*c*
d*(b*d + 2*c*d*x)^(5/2)) - (3*(b^2 - 4*a*c)^(7/4)*Sqrt[-((c*(a + b*x + c*x^
2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/
4)*Sqrt[d])], -1])/(20*c^4*d^(7/2)*Sqrt[a + b*x + c*x^2]) + (3*(b^2 - 4*a*c
)^(7/4)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[
b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(20*c^4*d^(7/2)*Sqrt[a
+ b*x + c*x^2])
```

Rubi [A] time = 0.285641, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {684, 685, 691, 690, 307, 221, 1199, 424}

$$\frac{3\left(b^2-4ac\right)^{7/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{20c^4d^{7/2}\sqrt{a+bx+cx^2}}-\frac{3\left(b^2-4ac\right)^{7/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{20c^4d^{7/2}\sqrt{a+bx+cx^2}}-\frac{(a}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^(7/2),x]
```

```
[Out] (3*(b*d + 2*c*d*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(20*c^3*d^5) - (a + b*x + c
*x^2)^(3/2)/(2*c^2*d^3*Sqrt[b*d + 2*c*d*x]) - (a + b*x + c*x^2)^(5/2)/(5*c*
d*(b*d + 2*c*d*x)^(5/2)) - (3*(b^2 - 4*a*c)^(7/4)*Sqrt[-((c*(a + b*x + c*x^
2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/
4)*Sqrt[d])], -1])/(20*c^4*d^(7/2)*Sqrt[a + b*x + c*x^2]) + (3*(b^2 - 4*a*c
)^(7/4)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[
b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(20*c^4*d^(7/2)*Sqrt[a
+ b*x + c*x^2])
```

Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
```

```
lQ[m] && IntegerQ[2*p]
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(bd+2cdx)^{7/2}}dx = -\frac{\left(a+bx+cx^2\right)^{5/2}}{5cd(bd+2cdx)^{5/2}} + \frac{\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(bd+2cdx)^{3/2}}dx}{2cd^2}$$

$$= -\frac{\left(a+bx+cx^2\right)^{3/2}}{2c^2d^3\sqrt{bd+2cdx}} - \frac{\left(a+bx+cx^2\right)^{5/2}}{5cd(bd+2cdx)^{5/2}} + \frac{3\int \sqrt{bd+2cdx}\sqrt{a+bx+cx^2}\,dx}{4c^2d^4}$$

$$= \frac{3(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{20c^3d^5} - \frac{\left(a+bx+cx^2\right)^{3/2}}{2c^2d^3\sqrt{bd+2cdx}} - \frac{\left(a+bx+cx^2\right)^{5/2}}{5cd(bd+2cdx)^{5/2}} - \frac{\left(3\left(b^2-4ac\right)\right)\int \frac{\sqrt{bd}}{\sqrt{a+}}}{40c^3d^4}$$

$$= \frac{3(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{20c^3d^5} - \frac{\left(a+bx+cx^2\right)^{3/2}}{2c^2d^3\sqrt{bd+2cdx}} - \frac{\left(a+bx+cx^2\right)^{5/2}}{5cd(bd+2cdx)^{5/2}} - \frac{\left(3\left(b^2-4ac\right)\sqrt{-\frac{c(a}{b}}\right.}{40c}$$

$$= \frac{3(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{20c^3d^5} - \frac{\left(a+bx+cx^2\right)^{3/2}}{2c^2d^3\sqrt{bd+2cdx}} - \frac{\left(a+bx+cx^2\right)^{5/2}}{5cd(bd+2cdx)^{5/2}} - \frac{\left(3\left(b^2-4ac\right)\sqrt{-\frac{c(a}{b}}\right.}{}$$

$$= \frac{3(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{20c^3d^5} - \frac{\left(a+bx+cx^2\right)^{3/2}}{2c^2d^3\sqrt{bd+2cdx}} - \frac{\left(a+bx+cx^2\right)^{5/2}}{5cd(bd+2cdx)^{5/2}} + \frac{\left(3\left(b^2-4ac\right)^{3/2}\sqrt{-}\right.}{}$$

$$= \frac{3(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{20c^3d^5} - \frac{\left(a+bx+cx^2\right)^{3/2}}{2c^2d^3\sqrt{bd+2cdx}} - \frac{\left(a+bx+cx^2\right)^{5/2}}{5cd(bd+2cdx)^{5/2}} + \frac{3\left(b^2-4ac\right)^{7/4}\sqrt{-\frac{c(}{}}}{20c}$$

$$= \frac{3(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{20c^3d^5} - \frac{\left(a+bx+cx^2\right)^{3/2}}{2c^2d^3\sqrt{bd+2cdx}} - \frac{\left(a+bx+cx^2\right)^{5/2}}{5cd(bd+2cdx)^{5/2}} - \frac{3\left(b^2-4ac\right)^{7/4}\sqrt{-\frac{c(}{}}}{20c}$$

Mathematica [C] time = 0.0647175, size = 101, normalized size = 0.33

$$-\frac{\left(b^2-4ac\right)^2\sqrt{a+x(b+cx)}\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{4};-\frac{1}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{160c^3d\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}(d(b+2cx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^(7/2),x]
```

```
[Out] -((b^2 - 4*a*c)^2*Sqrt[a + x*(b + c*x)]*Hypergeometric2F1[-5/2, -5/4, -1/4,
 (b + 2*c*x)^2/(b^2 - 4*a*c)])/(160*c^3*d*(d*(b + 2*c*x))^(5/2)*Sqrt[(c*(a
+ x*(b + c*x)))/(-b^2 + 4*a*c)])
```

Maple [B] time = 0.228, size = 1362, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(7/2),x)
```

```
[Out] 1/40*(c*x^2+b*x+a)^(1/2)*(d*(2*c*x+b))^(1/2)*(192*EllipticE(1/2*((b+2*c*x+(
-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^2*a^2*c^4*(
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+
b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)-
96*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^
(1/2),2^(1/2))*x^2*a*b^2*c^3*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/
2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(1/2))^(1/2)+12*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2)
)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^2*b^4*c^2*((b+2*c*x+(-4*a*c+
b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)
*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)+8*x^6*c^6+192*Ell
ipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),
2^(1/2))*x*a^2*b*c^3*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2
)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*
a*c+b^2)^(1/2))^(1/2)-96*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*
c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x*a*b^3*c^2*((b+2*c*x+(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*
c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)+12*EllipticE(1/2*((b+2*c*
x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x*b^5*c*((
b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b
^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)+2
4*x^5*b*c^5+48*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2
*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^
2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1
/2))^(1/2)*2^(1/2),2^(1/2))*a^2*b^2*c^2-24*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-
4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-
4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c
+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a*b^4*c+3*((b+2*c*x
+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/
2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*Elliptic
E(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/
2))*b^6-88*x^4*a*c^5+52*x^4*b^2*c^4-176*x^3*a*b*c^4+64*x^3*b^3*c^3-104*x^2*
a^2*c^4-80*x^2*a*b^2*c^3+34*x^2*b^4*c^2-104*x*a^2*b*c^3+8*x*a*b^3*c^2+6*x*b
^5*c-8*a^3*c^3-20*a^2*b^2*c^2+6*a*b^4*c)/d^4/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b
^2*x+a*b)/(2*c*x+b)^2/c^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{5}{2}}}{(2\,cdx+bd)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)/(2*c*d*x + b*d)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\,ac\right)x^2+a^2\right)\sqrt{2\,cdx+bd}\sqrt{cx^2+bx+a}}{16\,c^4d^4x^4+32\,bc^3d^4x^3+24\,b^2c^2d^4x^2+8\,b^3cd^4x+b^4d^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*sqrt(2*c
*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(16*c^4*d^4*x^4 + 32*b*c^3*d^4*x^3 + 24*b
^2*c^2*d^4*x^2 + 8*b^3*c*d^4*x + b^4*d^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx+cx^2\right)^{\frac{5}{2}}}{\left(d\left(b+2cx\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(2*c*d*x+b*d)**(7/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(5/2)/(d*(b + 2*c*x))**(7/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{5}{2}}}{\left(2\,cdx+bd\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)/(2*c*d*x + b*d)^(7/2), x)
```

3.1361 $\int \frac{(a+bx+cx^2)^{5/2}}{(bd+2cdx)^{11/2}}\,dx$

Optimal. Leaf size=310

$$-\frac{(b^2-4ac)^{3/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\,\sqrt[4]{b^2-4ac}}\right),-1\right)}{12c^4d^{11/2}\sqrt{a+bx+cx^2}}+\frac{(b^2-4ac)^{3/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-\right.}{12c^4d^{11/2}\sqrt{a+bx+cx^2}}$$

```
[Out] -Sqrt[a + b*x + c*x^2]/(12*c^3*d^5*Sqrt[b*d + 2*c*d*x]) - (a + b*x + c*x^2)
^(3/2)/(18*c^2*d^3*(b*d + 2*c*d*x)^(5/2)) - (a + b*x + c*x^2)^(5/2)/(9*c*d*
(b*d + 2*c*d*x)^(9/2)) + ((b^2 - 4*a*c)^(3/4)*Sqrt[-((c*(a + b*x + c*x^2))/
(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*S
qrt[d])], -1])/(12*c^4*d^(11/2)*Sqrt[a + b*x + c*x^2]) - ((b^2 - 4*a*c)^(3/
4)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d +
 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(12*c^4*d^(11/2)*Sqrt[a + b*
x + c*x^2])
```

Rubi [A] time = 0.287238, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {684, 691, 690, 307, 221, 1199, 424}

$$-\frac{(b^2-4ac)^{3/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{12c^4d^{11/2}\sqrt{a+bx+cx^2}}+\frac{(b^2-4ac)^{3/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{12c^4d^{11/2}\sqrt{a+bx+cx^2}}-\frac{}{18}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^(11/2),x]
```

```
[Out] -Sqrt[a + b*x + c*x^2]/(12*c^3*d^5*Sqrt[b*d + 2*c*d*x]) - (a + b*x + c*x^2)
^(3/2)/(18*c^2*d^3*(b*d + 2*c*d*x)^(5/2)) - (a + b*x + c*x^2)^(5/2)/(9*c*d*
(b*d + 2*c*d*x)^(9/2)) + ((b^2 - 4*a*c)^(3/4)*Sqrt[-((c*(a + b*x + c*x^2))/
(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*S
qrt[d])], -1])/(12*c^4*d^(11/2)*Sqrt[a + b*x + c*x^2]) - ((b^2 - 4*a*c)^(3/
4)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d +
 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(12*c^4*d^(11/2)*Sqrt[a + b*
x + c*x^2])
```

Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{5/2}}{(bd+2cdx)^{11/2}}dx = -\frac{(a+bx+cx^2)^{5/2}}{9cd(bd+2cdx)^{9/2}} + \frac{5\int \frac{(a+bx+cx^2)^{3/2}}{(bd+2cdx)^{7/2}}dx}{18cd^2}$$

$$= -\frac{(a+bx+cx^2)^{3/2}}{18c^2d^3(bd+2cdx)^{5/2}} - \frac{(a+bx+cx^2)^{5/2}}{9cd(bd+2cdx)^{9/2}} + \frac{\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^{3/2}}dx}{12c^2d^4}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{12c^3d^5\sqrt{bd+2cdx}} - \frac{(a+bx+cx^2)^{3/2}}{18c^2d^3(bd+2cdx)^{5/2}} - \frac{(a+bx+cx^2)^{5/2}}{9cd(bd+2cdx)^{9/2}} + \frac{\int \frac{\sqrt{bd+2cdx}}{\sqrt{a+bx+cx^2}}dx}{24c^3d^6}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{12c^3d^5\sqrt{bd+2cdx}} - \frac{(a+bx+cx^2)^{3/2}}{18c^2d^3(bd+2cdx)^{5/2}} - \frac{(a+bx+cx^2)^{5/2}}{9cd(bd+2cdx)^{9/2}} + \frac{\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\int \frac{}{\sqrt{-\frac{ac}{b^2-4}}}}{24c^3d^6\sqrt{a+b}}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{12c^3d^5\sqrt{bd+2cdx}} - \frac{(a+bx+cx^2)^{3/2}}{18c^2d^3(bd+2cdx)^{5/2}} - \frac{(a+bx+cx^2)^{5/2}}{9cd(bd+2cdx)^{9/2}} + \frac{\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\mathrm{Subst}\Big(}{12c^4d}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{12c^3d^5\sqrt{bd+2cdx}} - \frac{(a+bx+cx^2)^{3/2}}{18c^2d^3(bd+2cdx)^{5/2}} - \frac{(a+bx+cx^2)^{5/2}}{9cd(bd+2cdx)^{9/2}} - \frac{\Big(\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx}{b^2-4ac}}}{}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{12c^3d^5\sqrt{bd+2cdx}} - \frac{(a+bx+cx^2)^{3/2}}{18c^2d^3(bd+2cdx)^{5/2}} - \frac{(a+bx+cx^2)^{5/2}}{9cd(bd+2cdx)^{9/2}} - \frac{(b^2-4ac)^{3/4}\sqrt{-\frac{c(a+bx+}{b^2-4}}}{12c^4d^{11/}}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{12c^3d^5\sqrt{bd+2cdx}} - \frac{(a+bx+cx^2)^{3/2}}{18c^2d^3(bd+2cdx)^{5/2}} - \frac{(a+bx+cx^2)^{5/2}}{9cd(bd+2cdx)^{9/2}} + \frac{(b^2-4ac)^{3/4}\sqrt{-\frac{c(a+bx+}{b^2-4}}}{12c^4d^{11/}}$$

Mathematica [C] time = 0.086722, size = 109, normalized size = 0.35

$$-\frac{(b^2-4ac)^2\sqrt{a+x(b+cx)}\sqrt{d(b+2cx)}\,{}_2F_1\left(-\frac{5}{2},-\frac{9}{4};-\frac{5}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{288c^3d^6(b+2cx)^5\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^(11/2),x]
```

```
[Out] -((b^2 - 4*a*c)^2*Sqrt[d*(b + 2*c*x)]*Sqrt[a + x*(b + c*x)]*Hypergeometric2
F1[-5/2, -9/4, -5/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(288*c^3*d^6*(b + 2*c*x)
^5*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)])
```

Maple [B] time = 0.24, size = 1489, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(11/2),x)
```

```
[Out] 1/72*(c*x^2+b*x+a)^(1/2)*(d*(2*c*x+b))^(1/2)*(-120*x^6*c^6+12*((b+2*c*x+(-4
*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^
(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/
2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*
a*b^4*c-304*x^3*a*b*c^4-208*x^2*a*b^2*c^3-40*x*a^2*b*c^3-56*x*a*b^3*c^2+384
*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1
/2),2^(1/2))*x^3*a*b*c^4*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^
(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/
(-4*a*c+b^2)^(1/2))^(1/2)+288*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(
-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^2*a*b^2*c^3*((b+2*c*x+(-4*a*c+b
^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*
((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)+96*EllipticE(1/2*(
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x*a
*b^3*c^2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b
)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/
2))^(1/2)-360*x^5*b*c^5-152*x^4*a*c^5-412*x^4*b^2*c^4-224*x^3*b^3*c^3-40*x^
2*a^2*c^4-58*x^2*b^4*c^2-6*x*b^5*c-3*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+
b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^
(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*b^6-24*EllipticE(1/2*((b+
2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x*b^5*
c*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a
*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/
2)+192*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2
)*2^(1/2),2^(1/2))*x^4*a*c^5*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/
2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(1/2))^(1/2)-48*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2)
)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^4*b^2*c^4*((b+2*c*x+(-4*a*c+
b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)
*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)-96*EllipticE(1/2*
((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^
3*b^3*c^3*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+
b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1
/2))^(1/2)-72*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2
))^(1/2)*2^(1/2),2^(1/2))*x^2*b^4*c^2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c
+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c
+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)-6*a*b^4*c-4*a^2*b^2*c^2-8*a^3*c^3)/d
^6/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)/(2*c*x+b)^4/c^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx + a\right)^{\frac{5}{2}}}{\left(2\,cdx + bd\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)/(2*c*d*x + b*d)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2x^4 + 2\,bcx^3 + 2\,abx + \left(b^2 + 2\,ac\right)x^2 + a^2\right)\sqrt{2\,cdx + bd}\sqrt{cx^2 + bx + a}}{64\,c^6d^6x^6 + 192\,bc^5d^6x^5 + 240\,b^2c^4d^6x^4 + 160\,b^3c^3d^6x^3 + 60\,b^4c^2d^6x^2 + 12\,b^5cd^6x + b^6d^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(11/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*sqrt(2*c
*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(64*c^6*d^6*x^6 + 192*b*c^5*d^6*x^5 + 240
*b^2*c^4*d^6*x^4 + 160*b^3*c^3*d^6*x^3 + 60*b^4*c^2*d^6*x^2 + 12*b^5*c*d^6*
x + b^6*d^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(2*c*d*x+b*d)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{5}{2}}}{\left(2\,cdx+bd\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)/(2*c*d*x + b*d)^(11/2), x)
```

3.1362 $\int \frac{(a+bx+cx^2)^{5/2}}{(bd+2cdx)^{15/2}} dx$

Optimal. Leaf size=357

$$\frac{\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{156c^4d^{15/2}\sqrt[4]{b^2-4ac}\sqrt{a+bx+cx^2}}+\frac{\sqrt{a+bx+cx^2}}{78c^3d^7(b^2-4ac)\sqrt{bd+2cdx}}-\frac{\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|\right.}{156c^4d^{15/2}\sqrt[4]{b^2-4ac}\sqrt{a+bx+cx^2}}$$

```
[Out] -Sqrt[a + b*x + c*x^2]/(156*c^3*d^5*(b*d + 2*c*d*x)^(5/2)) + Sqrt[a + b*x +
 c*x^2]/(78*c^3*(b^2 - 4*a*c)*d^7*Sqrt[b*d + 2*c*d*x]) - (5*(a + b*x + c*x^
2)^(3/2))/(234*c^2*d^3*(b*d + 2*c*d*x)^(9/2)) - (a + b*x + c*x^2)^(5/2)/(13
*c*d*(b*d + 2*c*d*x)^(13/2)) - (Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))
]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])
/(156*c^4*(b^2 - 4*a*c)^(1/4)*d^(15/2)*Sqrt[a + b*x + c*x^2]) + (Sqrt[-((c*
(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b
^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(156*c^4*(b^2 - 4*a*c)^(1/4)*d^(15/2)*Sqr
t[a + b*x + c*x^2])
```

Rubi [A] time = 0.337042, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {684, 693, 691, 690, 307, 221, 1199, 424}

$$\frac{\sqrt{a+bx+cx^2}}{78c^3d^7(b^2-4ac)\sqrt{bd+2cdx}}+\frac{\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{156c^4d^{15/2}\sqrt[4]{b^2-4ac}\sqrt{a+bx+cx^2}}-\frac{\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{156c^4d^{15/2}\sqrt[4]{b^2-4ac}\sqrt{a+bx+cx^2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^(15/2),x]
```


```
[Out] -Sqrt[a + b*x + c*x^2]/(156*c^3*d^5*(b*d + 2*c*d*x)^(5/2)) + Sqrt[a + b*x +
 c*x^2]/(78*c^3*(b^2 - 4*a*c)*d^7*Sqrt[b*d + 2*c*d*x]) - (5*(a + b*x + c*x^
2)^(3/2))/(234*c^2*d^3*(b*d + 2*c*d*x)^(9/2)) - (a + b*x + c*x^2)^(5/2)/(13
*c*d*(b*d + 2*c*d*x)^(13/2)) - (Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))
]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])
/(156*c^4*(b^2 - 4*a*c)^(1/4)*d^(15/2)*Sqrt[a + b*x + c*x^2]) + (Sqrt[-((c*
(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b
^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(156*c^4*(b^2 - 4*a*c)^(1/4)*d^(15/2)*Sqr
t[a + b*x + c*x^2])
```


Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
```

```
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(bd+2cdx)^{15/2}}\,dx = -\frac{\left(a+bx+cx^2\right)^{5/2}}{13cd(bd+2cdx)^{13/2}} + \frac{5\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(bd+2cdx)^{11/2}}dx}{26cd^2}$$

$$= -\frac{5\left(a+bx+cx^2\right)^{3/2}}{234c^2d^3(bd+2cdx)^{9/2}} - \frac{\left(a+bx+cx^2\right)^{5/2}}{13cd(bd+2cdx)^{13/2}} + \frac{5\int \frac{\sqrt{a+bx+cx^2}}{(bd+2cdx)^{7/2}}dx}{156c^2d^4}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{156c^3d^5(bd+2cdx)^{5/2}} - \frac{5\left(a+bx+cx^2\right)^{3/2}}{234c^2d^3(bd+2cdx)^{9/2}} - \frac{\left(a+bx+cx^2\right)^{5/2}}{13cd(bd+2cdx)^{13/2}} + \frac{\int \frac{1}{(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}}{312c^3d^6}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{156c^3d^5(bd+2cdx)^{5/2}} + \frac{\sqrt{a+bx+cx^2}}{78c^3\left(b^2-4ac\right)d^7\sqrt{bd+2cdx}} - \frac{5\left(a+bx+cx^2\right)^{3/2}}{234c^2d^3(bd+2cdx)^{9/2}} - \frac{(a+bx}{13cd(bd+}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{156c^3d^5(bd+2cdx)^{5/2}} + \frac{\sqrt{a+bx+cx^2}}{78c^3\left(b^2-4ac\right)d^7\sqrt{bd+2cdx}} - \frac{5\left(a+bx+cx^2\right)^{3/2}}{234c^2d^3(bd+2cdx)^{9/2}} - \frac{(a+bx}{13cd(bd+}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{156c^3d^5(bd+2cdx)^{5/2}} + \frac{\sqrt{a+bx+cx^2}}{78c^3\left(b^2-4ac\right)d^7\sqrt{bd+2cdx}} - \frac{5\left(a+bx+cx^2\right)^{3/2}}{234c^2d^3(bd+2cdx)^{9/2}} - \frac{(a+bx}{13cd(bd+}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{156c^3d^5(bd+2cdx)^{5/2}} + \frac{\sqrt{a+bx+cx^2}}{78c^3\left(b^2-4ac\right)d^7\sqrt{bd+2cdx}} - \frac{5\left(a+bx+cx^2\right)^{3/2}}{234c^2d^3(bd+2cdx)^{9/2}} - \frac{(a+bx}{13cd(bd+}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{156c^3d^5(bd+2cdx)^{5/2}} + \frac{\sqrt{a+bx+cx^2}}{78c^3\left(b^2-4ac\right)d^7\sqrt{bd+2cdx}} - \frac{5\left(a+bx+cx^2\right)^{3/2}}{234c^2d^3(bd+2cdx)^{9/2}} - \frac{(a+bx}{13cd(bd+}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{156c^3d^5(bd+2cdx)^{5/2}} + \frac{\sqrt{a+bx+cx^2}}{78c^3\left(b^2-4ac\right)d^7\sqrt{bd+2cdx}} - \frac{5\left(a+bx+cx^2\right)^{3/2}}{234c^2d^3(bd+2cdx)^{9/2}} - \frac{(a+bx}{13cd(bd+}$$

Mathematica [C] time = 0.0876682, size = 109, normalized size = 0.31

$$-\frac{\left(b^2-4ac\right)^2\sqrt{a+x(b+cx)}\sqrt{d(b+2cx)}\,{}_2F_1\left(-\frac{13}{4},-\frac{5}{2};-\frac{9}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{416c^3d^8(b+2cx)^7\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/(b*d + 2*c*d*x)^(15/2),x]
```

```
[Out] -((b^2 - 4*a*c)^2*Sqrt[d*(b + 2*c*x)]*Sqrt[a + x*(b + c*x)]*Hypergeometric2
F1[-13/4, -5/2, -9/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(416*c^3*d^8*(b + 2*c*x
)^7*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)])
```

Maple [B] time = 0.33, size = 2125, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(15/2),x)
```

```
[Out] 1/936*(c*x^2+b*x+a)^(1/2)*(d*(2*c*x+b))^(1/2)*(-768*x^8*c^8-6*x*b^7*c+720*E
llipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2
),2^(1/2))*x^2*a*b^4*c^3*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))
^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/
(-4*a*c+b^2)^(1/2))^(1/2)+144*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(
-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x*a*b^5*c^2*((-b-2*c*x+(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b
^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)+2304*EllipticE(1/2*(
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^5
*a*b*c^6*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*((b+2*c*x
+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/
2))^(1/2)+2880*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/
2))^(1/2)*2^(1/2),2^(1/2))*x^4*a*b^2*c^5*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4
*a*c+b^2)^(1/2))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1
/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)+1920*EllipticE(1/2*((b+2*c*x+(-4*
a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^3*a*b^3*c^4*((
-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*((b+2*c*x+(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)-3
*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*
c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2
)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(
1/2),2^(1/2))*b^8+768*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b
^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^6*a*c^7*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/
(-4*a*c+b^2)^(1/2))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))
^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)-192*EllipticE(1/2*((b+2*c*x+(-
4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^6*b^2*c^6*((
-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*((b+2*c*x+(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)-5
76*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^
(1/2),2^(1/2))*x^5*b^3*c^5*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2
))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b
)/(-4*a*c+b^2)^(1/2))^(1/2)-720*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))
/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^4*b^4*c^4*((-b-2*c*x+(-4*a*c+
b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c
+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)-480*EllipticE(1/2*
((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^
3*b^5*c^3*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*((b+2*c*
x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1
/2))^(1/2)-180*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/
2))^(1/2)*2^(1/2),2^(1/2))*x^2*b^6*c^2*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a
*c+b^2)^(1/2))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2
)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)-36*EllipticE(1/2*((b+2*c*x+(-4*a*c+
b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x*b^7*c*((-b-2*c*x+(
-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/
(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)-1828*x^4*b^
4*c^4-4056*x^5*b^3*c^5-1760*x^6*a*c^7-1888*x^4*a^2*c^6-1184*x^2*a^3*c^5-307
2*x^7*b*c^7-480*x^3*b^5*c^3-82*x^2*b^6*c^2-4936*x^6*b^2*c^6-288*a^4*c^4-8*a
^3*b^2*c^3-4*a^2*b^4*c^2-6*a*b^6*c-80*x*a*b^5*c^2-5280*x^5*a*b*c^6-5656*x^4
*a*b^2*c^5-3776*x^3*a^2*b*c^5-2512*x^3*a*b^3*c^4-1944*x^2*a^2*b^2*c^4-456*x
^2*a*b^4*c^3-1184*x*a^3*b*c^4-56*x*a^2*b^3*c^3+12*((b+2*c*x+(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2
*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+
(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a*b^6*c)/d^8
/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)/(2*c*x+b)^6/(4*a*c-b^2)/c^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)^{\frac{5}{2}}}{(2\,cdx + bd)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(15/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)/(2*c*d*x + b*d)^(15/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c^2x^4 + 2\,bcx^3 + 2\,abx + \left(b^2 + 2\,ac\right)x^2 + a^2\right)\sqrt{2\,cdx + bd}\sqrt{cx^2 + bx + a}}{256\,c^8d^8x^8 + 1024\,bc^7d^8x^7 + 1792\,b^2c^6d^8x^6 + 1792\,b^3c^5d^8x^5 + 1120\,b^4c^4d^8x^4 + 448\,b^5c^3d^8x^3 + 112\,b^6c^2d^8x^2 + \cdots}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(15/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*sqrt(2*c
*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(256*c^8*d^8*x^8 + 1024*b*c^7*d^8*x^7 + 1
792*b^2*c^6*d^8*x^6 + 1792*b^3*c^5*d^8*x^5 + 1120*b^4*c^4*d^8*x^4 + 448*b^5
*c^3*d^8*x^3 + 112*b^6*c^2*d^8*x^2 + 16*b^7*c*d^8*x + b^8*d^8), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(2*c*d*x+b*d)**(15/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)^{\frac{5}{2}}}{(2\,cdx + bd)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(2*c*d*x+b*d)^(15/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)/(2*c*d*x + b*d)^(15/2), x)
```

3.1363 $\int \frac{(bd+2cdx)^{7/2}}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=174

$$\frac{10d^{7/2}\left(b^2-4ac\right)^{9/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{21c\sqrt{a+bx+cx^2}}+\frac{20}{21}d^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}+\frac{4}{7}$$

```
[Out] (20*(b^2 - 4*a*c)*d^3*Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/21 + (4*d*
(b*d + 2*c*d*x)^(5/2)*Sqrt[a + b*x + c*x^2])/7 + (10*(b^2 - 4*a*c)^(9/4)*d^
(7/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*
d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(21*c*Sqrt[a + b*x + c*x^
2])
```

Rubi [A] time = 0.144182, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {692, 691, 689, 221}

$$\frac{20}{21}d^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}+\frac{10d^{7/2}\left(b^2-4ac\right)^{9/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{21c\sqrt{a+bx+cx^2}}+\frac{4}{7}d\sqrt{a+}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(7/2)/Sqrt[a + b*x + c*x^2],x]
```



```
[Out] (20*(b^2 - 4*a*c)*d^3*Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/21 + (4*d*
(b*d + 2*c*d*x)^(5/2)*Sqrt[a + b*x + c*x^2])/7 + (10*(b^2 - 4*a*c)^(9/4)*d^
(7/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*
d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(21*c*Sqrt[a + b*x + c*x^
2])
```


Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
```

```
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(bd+2cdx)^{7/2}}{\sqrt{a+bx+cx^2}}dx = \frac{4}{7}d(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}+\frac{1}{7}\left(5\left(b^2-4ac\right)d^2\right)\int \frac{(bd+2cdx)^{3/2}}{\sqrt{a+bx+cx^2}}dx$$
$$= \frac{20}{21}\left(b^2-4ac\right)d^3\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}+\frac{4}{7}d(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}+\frac{1}{21}\left(5\left(b^2-4ac\right)^2\right.$$
$$= \frac{20}{21}\left(b^2-4ac\right)d^3\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}+\frac{4}{7}d(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}+\frac{\left(5\left(b^2-4ac\right)^2d^4\right.}{}$$
$$= \frac{20}{21}\left(b^2-4ac\right)d^3\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}+\frac{4}{7}d(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}+\frac{\left(10\left(b^2-4ac\right)^2d^3\right.}{}$$
$$= \frac{20}{21}\left(b^2-4ac\right)d^3\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}+\frac{4}{7}d(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}+\frac{10\left(b^2-4ac\right)^{9/4}d}{}$$

Mathematica [C] time = 0.181379, size = 166, normalized size = 0.95

$$\frac{2d^3\sqrt{d(b+2cx)}\left(8c\left(-5a^2c+2a\left(b^2-bcx-c^2x^2\right)+x\left(5b^2cx+2b^3+6bc^2x^2+3c^3x^3\right)\right)+5\left(b^2-4ac\right)^2\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\,{}_2F_1\left(\right.\right.}{21c\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(7/2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*d^3*Sqrt[d*(b + 2*c*x)]*(8*c*(-5*a^2*c + 2*a*(b^2 - b*c*x - c^2*x^2) + x
*(2*b^3 + 5*b^2*c*x + 6*b*c^2*x^2 + 3*c^3*x^3)) + 5*(b^2 - 4*a*c)^2*Sqrt[(c
*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*Hypergeometric2F1[1/4, 1/2, 5/4, (b + 2
*c*x)^2/(b^2 - 4*a*c)]))/(21*c*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.232, size = 567, normalized size = 3.3

$$\frac{d^3}{21\,c\left(2\,c^2x^3+3\,bcx^2+2\,acx+b^2x+ab\right)}\sqrt{d\left(2\,cx+b\right)}\sqrt{cx^2+bx+a}\left(96\,x^5c^5+80\,\sqrt{\frac{b+2\,cx+\sqrt{-4\,ac+b^2}}{\sqrt{-4\,ac+b^2}}}\sqrt{-\frac{2\,cx+}{\sqrt{-4\,ac}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/21*(d*(2*c*x+b))^(1/2)*(c*x^2+b*x+a)^(1/2)*d^3*(96*x^5*c^5+80*((b+2*c*x+(
-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2)
)^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(
1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)
)*(-4*a*c+b^2)^(1/2)*a^2*c^2-40*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^
(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^
(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2)
)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2)^(1/2)*a*b^2*c+5*(
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+
b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*
EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/
2),2^(1/2))*(-4*a*c+b^2)^(1/2)*b^4+240*x^4*b*c^4-64*x^3*a*c^4+256*x^3*b^2*c
^3-96*x^2*a*b*c^3+144*x^2*b^3*c^2-160*x*a^2*c^3+32*x*a*b^2*c^2+32*x*b^4*c-8
0*a^2*b*c^2+32*a*b^3*c)/c/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx + bd)^{\frac{7}{2}}}{\sqrt{cx^2 + bx + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^(7/2)/sqrt(c*x^2 + b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(8\,c^3d^3x^3 + 12\,bc^2d^3x^2 + 6\,b^2cd^3x + b^3d^3\right)\sqrt{2\,cdx + bd}}{\sqrt{cx^2 + bx + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((8*c^3*d^3*x^3 + 12*b*c^2*d^3*x^2 + 6*b^2*c*d^3*x + b^3*d^3)*sqrt(
2*c*d*x + b*d)/sqrt(c*x^2 + b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(7/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx + bd)^{\frac{7}{2}}}{\sqrt{cx^2 + bx + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^(7/2)/sqrt(c*x^2 + b*x + a), x)
```

3.1364 $\int \frac{(bd+2cdx)^{3/2}}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=132

$$\frac{2d^{3/2}\left(b^2-4ac\right)^{5/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{3c\sqrt{a+bx+cx^2}}+\frac{4}{3}d\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}$$

```
[Out] (4*d*Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/3 + (2*(b^2 - 4*a*c)^(5/4)*
d^(3/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[
b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(3*c*Sqrt[a + b*x + c*x
^2])
```

Rubi [A] time = 0.116392, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {692, 691, 689, 221}

$$\frac{2d^{3/2}\left(b^2-4ac\right)^{5/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{3c\sqrt{a+bx+cx^2}}+\frac{4}{3}d\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(3/2)/Sqrt[a + b*x + c*x^2],x]
```


```
[Out] (4*d*Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/3 + (2*(b^2 - 4*a*c)^(5/4)*
d^(3/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[
b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(3*c*Sqrt[a + b*x + c*x
^2])
```


Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
```

```
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(bd+2cdx)^{3/2}}{\sqrt{a+bx+cx^2}}\,dx = \frac{4}{3}d\sqrt{bd+2cdx}\sqrt{a+bx+cx^2} + \frac{1}{3}\left(\left(b^2-4ac\right)d^2\right)\int \frac{1}{\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{4}{3}d\sqrt{bd+2cdx}\sqrt{a+bx+cx^2} + \frac{\left(\left(b^2-4ac\right)d^2\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\int \frac{1}{\sqrt{bd+2cdx}\sqrt{-\frac{ac}{b^2-4ac}-\frac{bcx}{b^2-4ac}-\frac{c^2x^2}{b^2-4ac}}}\,dx}{3\sqrt{a+bx+cx^2}}$$

$$= \frac{4}{3}d\sqrt{bd+2cdx}\sqrt{a+bx+cx^2} + \frac{\left(2\left(b^2-4ac\right)d\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{\left(b^2-4ac\right)d^2}}}\,dx, x, \sqrt{bd+\right.}{3c\sqrt{a+bx+cx^2}}$$

$$= \frac{4}{3}d\sqrt{bd+2cdx}\sqrt{a+bx+cx^2} + \frac{2\left(b^2-4ac\right)^{5/4}d^{3/2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{3c\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 0.109234, size = 111, normalized size = 0.84

$$\frac{2d\sqrt{d(b+2cx)}\left(\left(b^2-4ac\right)\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)+2c(a+x(b+cx))\right)}{3c\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(3/2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*d*Sqrt[d*(b + 2*c*x)]*(2*c*(a + x*(b + c*x)) + (b^2 - 4*a*c)*Sqrt[(c*(a
+ x*(b + c*x)))/(-b^2 + 4*a*c)]*Hypergeometric2F1[1/4, 1/2, 5/4, (b + 2*c*x
)^2/(b^2 - 4*a*c)]))/(3*c*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.218, size = 362, normalized size = 2.7

$$-\frac{d}{3c\left(2c^2x^3+3bcx^2+2acx+b^2x+ab\right)}\sqrt{d(2cx+b)}\sqrt{cx^2+bx+a}\left(4\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}\sqrt{-\frac{2cx+b}{\sqrt{-4ac+b^2}}}\sqrt{\frac{-b-\right.}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/3*(d*(2*c*x+b))^(1/2)*(c*x^2+b*x+a)^(1/2)*d*(4*((b+2*c*x+(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2
*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+
```

```
(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2)
^(1/2)*a*c-((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x
+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(
1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))
^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2)^(1/2)*b^2-8*c^3*x^3-12*b*c^2*x^2-8*x*a
*c^2-4*x*b^2*c-4*a*b*c)/c/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx+bd)^{\frac{3}{2}}}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^(3/2)/sqrt(c*x^2 + b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(2\,cdx+bd)^{\frac{3}{2}}}{\sqrt{cx^2+bx+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((2*c*d*x + b*d)^(3/2)/sqrt(c*x^2 + b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\,(b+2cx))^{\frac{3}{2}}}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(3/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d*(b + 2*c*x))**(3/2)/sqrt(a + b*x + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx+bd)^{\frac{3}{2}}}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^(3/2)/sqrt(c*x^2 + b*x + a), x)
```

3.1365 $\int \frac{1}{\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=97

$$\frac{2\sqrt[4]{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{c\sqrt{d}\sqrt{a+bx+cx^2}}$$

```
[Out] (2*(b^2 - 4*a*c)^(1/4)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellipti
cF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(c*Sqrt[
d]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.0918113, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {691, 689, 221}

$$\frac{2\sqrt[4]{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{c\sqrt{d}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*(b^2 - 4*a*c)^(1/4)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellipti
cF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(c*Sqrt[
d]*Sqrt[a + b*x + c*x^2])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\int \frac{1}{\sqrt{bd+2cdx}\sqrt{-\frac{ac}{b^2-4ac}-\frac{bcx}{b^2-4ac}-\frac{c^2x^2}{b^2-4ac}}}\,dx}{\sqrt{a+bx+cx^2}}$$

$$= \frac{\left(2\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{(b^2-4ac)d^2}}}\,dx, x, \sqrt{bd+2cdx}\right)}{cd\sqrt{a+bx+cx^2}}$$

$$= \frac{2\sqrt[4]{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{c\sqrt{d}\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 0.0592199, size = 89, normalized size = 0.92

$$\frac{2\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\sqrt{d(b+2cx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{cd\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*Sqrt[d*(b + 2*c*x)]*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*Hypergeom
etric2F1[1/4, 1/2, 5/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(c*d*Sqrt[a + x*(b +
c*x)])
```

Maple [B] time = 0.212, size = 189, normalized size = 2.

$$\frac{1}{cd\left(2c^2x^3+3bcx^2+2acx+b^2x+ab\right)}\text{EllipticF}\left(\frac{\sqrt{2}}{2}\sqrt{\left(b+2cx+\sqrt{-4ac+b^2}\right)\frac{1}{\sqrt{-4ac+b^2}}},\sqrt{2}\right)\sqrt{\left(-b-2cx+\sqrt{\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/
2),2^(1/2))*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c
*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^
(1/2))^(1/2)*(-4*a*c+b^2)^(1/2)*(d*(2*c*x+b))^(1/2)*(c*x^2+b*x+a)^(1/2)/c/d
/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,cdx+bd}\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,cdx + bd}\sqrt{cx^2 + bx + a}}{2\,c^2dx^3 + 3\,bcdx^2 + abd + \left(b^2 + 2\,ac\right)dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(2*c^2*d*x^3 + 3*b*c*d*x
^2 + a*b*d + (b^2 + 2*a*c)*d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d\,(b + 2cx)}\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d*(b + 2*c*x))*sqrt(a + b*x + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,cdx + bd}\sqrt{cx^2 + bx + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a)), x)
```

3.1366 $\int \frac{1}{(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=144

$$\frac{2\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{3cd^{5/2}\left(b^2-4ac\right)^{3/4}\sqrt{a+bx+cx^2}}+\frac{4\sqrt{a+bx+cx^2}}{3d\left(b^2-4ac\right)(bd+2cdx)^{3/2}}$$

```
[Out] (4*Sqrt[a + b*x + c*x^2])/(3*(b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(3/2)) + (2*Sq
rt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*
d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(3*c*(b^2 - 4*a*c)^(3/4)*d^(5/2)*
Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.116817, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {693, 691, 689, 221}

$$\frac{2\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{3cd^{5/2}\left(b^2-4ac\right)^{3/4}\sqrt{a+bx+cx^2}}+\frac{4\sqrt{a+bx+cx^2}}{3d\left(b^2-4ac\right)(bd+2cdx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^(5/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (4*Sqrt[a + b*x + c*x^2])/(3*(b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(3/2)) + (2*Sq
rt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*
d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(3*c*(b^2 - 4*a*c)^(3/4)*d^(5/2)*
Sqrt[a + b*x + c*x^2])
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
```

```
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}}dx = \frac{4\sqrt{a+bx+cx^2}}{3(b^2-4ac)d(bd+2cdx)^{3/2}} + \frac{\int \frac{1}{\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}dx}{3(b^2-4ac)d^2}$$

$$= \frac{4\sqrt{a+bx+cx^2}}{3(b^2-4ac)d(bd+2cdx)^{3/2}} + \frac{\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\int \frac{1}{\sqrt{bd+2cdx}\sqrt{-\frac{ac}{b^2-4ac}-\frac{bcx}{b^2-4ac}-\frac{c^2x^2}{b^2-4ac}}}dx}{3(b^2-4ac)d^2\sqrt{a+bx+cx^2}}$$

$$= \frac{4\sqrt{a+bx+cx^2}}{3(b^2-4ac)d(bd+2cdx)^{3/2}} + \frac{\left(2\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{(b^2-4ac)d^2}}}dx, x, \sqrt{bd+\right.}{3c(b^2-4ac)d^3\sqrt{a+bx+cx^2}}$$

$$= \frac{4\sqrt{a+bx+cx^2}}{3(b^2-4ac)d(bd+2cdx)^{3/2}} + \frac{2\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{3c(b^2-4ac)^{3/4}d^{5/2}\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 0.0512377, size = 91, normalized size = 0.63

$$-\frac{2\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{3cd\sqrt{a+x(b+cx)}(d(b+2cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^(5/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-2*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*Hypergeometric2F1[-3/4, 1/2,
 1/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(3*c*d*(d*(b + 2*c*x))^(3/2)*Sqrt[a + x
*(b + c*x)])
```

Maple [B] time = 0.217, size = 362, normalized size = 2.5

$$-\frac{1}{3d^3(2c^2x^3+3bcx^2+2acx+b^2x+ab)(4ac-b^2)c(2cx+b)}\sqrt{d(2cx+b)}\sqrt{cx^2+bx+a}\left(2\sqrt{-4ac+b^2}\sqrt{\frac{b+2cx+}{\sqrt{-4}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/3*(d*(2*c*x+b))^(1/2)*(c*x^2+b*x+a)^(1/2)*(2*(-4*a*c+b^2)^(1/2)*((b+2*c*
x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1
```

```
/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*Ellipti
cF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1
/2))*x*c+(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2
))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2
))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-
4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*b+4*c^2*x^2+4*b*c*x+4*a*c)/d^3/(2*
c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)/(4*a*c-b^2)/c/(2*c*x+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2\,cdx+bd)^{\frac{5}{2}}\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*c*d*x + b*d)^(5/2)*sqrt(c*x^2 + b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{2\,cdx+bd}\sqrt{cx^2+bx+a}}{8\,c^4d^3x^5+20\,bc^3d^3x^4+ab^3d^3+2\left(9\,b^2c^2+4\,ac^3\right)d^3x^3+\left(7\,b^3c+12\,abc^2\right)d^3x^2+\left(b^4+6\,ab^2c\right)d^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(8*c^4*d^3*x^5 + 20*b*c^
3*d^3*x^4 + a*b^3*d^3 + 2*(9*b^2*c^2 + 4*a*c^3)*d^3*x^3 + (7*b^3*c + 12*a*b
*c^2)*d^3*x^2 + (b^4 + 6*a*b^2*c)*d^3*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d\,(b+2cx))^{\frac{5}{2}}\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**(5/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/((d*(b + 2*c*x))**(5/2)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2\,cdx+bd)^{\frac{5}{2}}\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((2*c*d*x + b*d)^(5/2)*sqrt(c*x^2 + b*x + a)), x)
```

3.1367 $\int \frac{1}{(bd+2cdx)^{9/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=188

$$\frac{10\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{21cd^{9/2}\left(b^2-4ac\right)^{7/4}\sqrt{a+bx+cx^2}}+\frac{20\sqrt{a+bx+cx^2}}{21d^3\left(b^2-4ac\right)^2(bd+2cdx)^{3/2}}+\frac{4\sqrt{a+bx+cx^2}}{7d\left(b^2-4ac\right)(bd+2cdx)^{7/2}}$$

```
[Out] (4*Sqrt[a + b*x + c*x^2])/(7*(b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(7/2)) + (20*S
qrt[a + b*x + c*x^2])/(21*(b^2 - 4*a*c)^2*d^3*(b*d + 2*c*d*x)^(3/2)) + (10*
Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*
c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(21*c*(b^2 - 4*a*c)^(7/4)*d^(9/
2)*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.146826, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {693, 691, 689, 221}

$$\frac{20\sqrt{a+bx+cx^2}}{21d^3\left(b^2-4ac\right)^2(bd+2cdx)^{3/2}}+\frac{10\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{21cd^{9/2}\left(b^2-4ac\right)^{7/4}\sqrt{a+bx+cx^2}}+\frac{4\sqrt{a+bx+cx^2}}{7d\left(b^2-4ac\right)(bd+2cdx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^(9/2)*Sqrt[a + b*x + c*x^2]),x]
```


```
[Out] (4*Sqrt[a + b*x + c*x^2])/(7*(b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(7/2)) + (20*S
qrt[a + b*x + c*x^2])/(21*(b^2 - 4*a*c)^2*d^3*(b*d + 2*c*d*x)^(3/2)) + (10*
Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*
c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(21*c*(b^2 - 4*a*c)^(7/4)*d^(9/
2)*Sqrt[a + b*x + c*x^2])
```


Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
```

```
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)^{9/2}\sqrt{a+bx+cx^2}}\,dx = \frac{4\sqrt{a+bx+cx^2}}{7\left(b^2-4ac\right)d(bd+2cdx)^{7/2}} + \frac{5\int \frac{1}{(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}}\,dx}{7\left(b^2-4ac\right)d^2}$$

$$= \frac{4\sqrt{a+bx+cx^2}}{7\left(b^2-4ac\right)d(bd+2cdx)^{7/2}} + \frac{20\sqrt{a+bx+cx^2}}{21\left(b^2-4ac\right)^2 d^3(bd+2cdx)^{3/2}} + \frac{5\int \frac{1}{\sqrt{bd+2cdx}\sqrt{a+bx+}}}{21\left(b^2-4ac\right)^2}$$

$$= \frac{4\sqrt{a+bx+cx^2}}{7\left(b^2-4ac\right)d(bd+2cdx)^{7/2}} + \frac{20\sqrt{a+bx+cx^2}}{21\left(b^2-4ac\right)^2 d^3(bd+2cdx)^{3/2}} + \frac{\left(5\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)}{21\left(b^2-\right.}$$

$$= \frac{4\sqrt{a+bx+cx^2}}{7\left(b^2-4ac\right)d(bd+2cdx)^{7/2}} + \frac{20\sqrt{a+bx+cx^2}}{21\left(b^2-4ac\right)^2 d^3(bd+2cdx)^{3/2}} + \frac{\left(10\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right.}{21c}$$

$$= \frac{4\sqrt{a+bx+cx^2}}{7\left(b^2-4ac\right)d(bd+2cdx)^{7/2}} + \frac{20\sqrt{a+bx+cx^2}}{21\left(b^2-4ac\right)^2 d^3(bd+2cdx)^{3/2}} + \frac{10\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F}{21c\left(b^2-4ac\right)}$$

Mathematica [C] time = 0.0747777, size = 99, normalized size = 0.53

$$-\frac{2\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\sqrt{d(b+2cx)}\,{}_2F_1\left(-\frac{7}{4},\frac{1}{2};-\frac{3}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{7cd^5(b+2cx)^4\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^(9/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-2*Sqrt[d*(b + 2*c*x)]*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*Hypergeo
metric2F1[-7/4, 1/2, -3/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(7*c*d^5*(b + 2*c*
x)^4*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.224, size = 691, normalized size = 3.7

$$\frac{1}{21\,d^5\left(2\,c^2x^3+3\,bcx^2+2\,acx+b^2x+ab\right)\left(4\,ac-b^2\right)^2(2\,cx+b)^3\,c}\sqrt{d\,(2\,cx+b)}\sqrt{cx^2+bx+a}\left(40\,\sqrt{-4\,ac+b^2}\sqrt{\frac{b+2\,cx}{\sqrt{-}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/21*(d*(2*c*x+b))^(1/2)*(c*x^2+b*x+a)^(1/2)*(40*(-4*a*c+b^2)^(1/2)*((b+2*c
*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(
1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*Ellipt
icF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(
1/2))*x^3*c^3+60*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b
^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b
^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^2*b*c^2+30*(-4*a*c+b^2)^
(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(
-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))
^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2
)*2^(1/2),2^(1/2))*x*b^2*c+5*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2
))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c
*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-
4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*b^3+80*c^4*x^4
+160*b*c^3*x^3+32*x^2*a*c^3+112*x^2*b^2*c^2+32*b*a*c^2*x+32*b^3*c*x-48*a^2*
c^2+32*a*c*b^2)/d^5/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)/(4*a*c-b^2)^2/(
2*c*x+b)^3/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2\,cdx+bd)^{\frac{9}{2}}\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*c*d*x + b*d)^(9/2)*sqrt(c*x^2 + b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{2\,cdx+bd}\sqrt{cx^2+bx+a}}{32\,c^6d^5x^7+112\,bc^5d^5x^6+ab^5d^5+32\left(5\,b^2c^4+ac^5\right)d^5x^5+40\left(3\,b^3c^3+2\,abc^4\right)d^5x^4+10\left(5\,b^4c^2+8\,ab^2c^3\right)d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(32*c^6*d^5*x^7 + 112*b*
c^5*d^5*x^6 + a*b^5*d^5 + 32*(5*b^2*c^4 + a*c^5)*d^5*x^5 + 40*(3*b^3*c^3 +
2*a*b*c^4)*d^5*x^4 + 10*(5*b^4*c^2 + 8*a*b^2*c^3)*d^5*x^3 + (11*b^5*c + 40*
a*b^3*c^2)*d^5*x^2 + (b^6 + 10*a*b^4*c)*d^5*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**(9/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2\,cdx + bd)^{\frac{9}{2}}\sqrt{cx^2 + bx + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((2*c*d*x + b*d)^(9/2)*sqrt(c*x^2 + b*x + a)), x)
```

3.1368 $\int \frac{(bd+2cdx)^{9/2}}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=273

$$-\frac{14d^{9/2}\left(b^2-4ac\right)^{11/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{15c\sqrt{a+bx+cx^2}}+\frac{28}{45}d^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}(bd+2cdx)^{3/2}$$

```
[Out] (28*(b^2 - 4*a*c)*d^3*(b*d + 2*c*d*x)^(3/2)*Sqrt[a + b*x + c*x^2])/45 + (4*
d*(b*d + 2*c*d*x)^(7/2)*Sqrt[a + b*x + c*x^2])/9 + (14*(b^2 - 4*a*c)^(11/4)
*d^(9/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt
[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(15*c*Sqrt[a + b*x + c
*x^2]) - (14*(b^2 - 4*a*c)^(11/4)*d^(9/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[
d])], -1])/(15*c*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.25192, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {692, 691, 690, 307, 221, 1199, 424}

$$\frac{28}{45}d^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}(bd+2cdx)^{3/2}-\frac{14d^{9/2}\left(b^2-4ac\right)^{11/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{15c\sqrt{a+bx+cx^2}}+\frac{14d^{9/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(9/2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (28*(b^2 - 4*a*c)*d^3*(b*d + 2*c*d*x)^(3/2)*Sqrt[a + b*x + c*x^2])/45 + (4*
d*(b*d + 2*c*d*x)^(7/2)*Sqrt[a + b*x + c*x^2])/9 + (14*(b^2 - 4*a*c)^(11/4)
*d^(9/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt
[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(15*c*Sqrt[a + b*x + c
*x^2]) - (14*(b^2 - 4*a*c)^(11/4)*d^(9/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[
d])], -1])/(15*c*Sqrt[a + b*x + c*x^2])
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(bd+2cdx)^{9/2}}{\sqrt{a+bx+cx^2}}\,dx = \frac{4}{9}d(bd+2cdx)^{7/2}\sqrt{a+bx+cx^2} + \frac{1}{9}\left(7\left(b^2-4ac\right)d^2\right)\int \frac{(bd+2cdx)^{5/2}}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{28}{45}\left(b^2-4ac\right)d^3(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2} + \frac{4}{9}d(bd+2cdx)^{7/2}\sqrt{a+bx+cx^2} + \frac{1}{15}\left(7\left(b^2-\right.\right.$$

$$= \frac{28}{45}\left(b^2-4ac\right)d^3(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2} + \frac{4}{9}d(bd+2cdx)^{7/2}\sqrt{a+bx+cx^2} + \frac{\left(7\left(b^2-4ac\right.\right.}{}$$

$$= \frac{28}{45}\left(b^2-4ac\right)d^3(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2} + \frac{4}{9}d(bd+2cdx)^{7/2}\sqrt{a+bx+cx^2} + \frac{\left(14\left(b^2-4a\right.\right.}{}$$

$$= \frac{28}{45}\left(b^2-4ac\right)d^3(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2} + \frac{4}{9}d(bd+2cdx)^{7/2}\sqrt{a+bx+cx^2} - \frac{\left(14\left(b^2-4a\right.\right.}{}$$

$$= \frac{28}{45}\left(b^2-4ac\right)d^3(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2} + \frac{4}{9}d(bd+2cdx)^{7/2}\sqrt{a+bx+cx^2} - \frac{14\left(b^2-4ac\right.}{}$$

$$= \frac{28}{45}\left(b^2-4ac\right)d^3(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2} + \frac{4}{9}d(bd+2cdx)^{7/2}\sqrt{a+bx+cx^2} + \frac{14\left(b^2-4ac\right.}{}$$

Mathematica [C] time = 0.201675, size = 167, normalized size = 0.61

$$\frac{2d^3(d(b+2cx))^{3/2}\left(8c\left(-7a^2c+a\left(3b^2-2bcx-2c^2x^2\right)+x\left(8b^2cx+3b^3+10bc^2x^2+5c^3x^3\right)\right)+7\left(b^2-4ac\right)^2\sqrt{\frac{c(a+x(b+}{4ac-b}}\right.}{45c\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(9/2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*d^3*(d*(b + 2*c*x))^(3/2)*(8*c*(-7*a^2*c + a*(3*b^2 - 2*b*c*x - 2*c^2*x^
2) + x*(3*b^3 + 8*b^2*c*x + 10*b*c^2*x^2 + 5*c^3*x^3)) + 7*(b^2 - 4*a*c)^2*
Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*Hypergeometric2F1[1/2, 3/4, 7/4,
 (b + 2*c*x)^2/(b^2 - 4*a*c)]))/(45*c*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.23, size = 703, normalized size = 2.6

$$\frac{d^4}{45\,c\left(2\,c^2x^3+3\,bcx^2+2\,acx+b^2x+ab\right)}\sqrt{d\left(2\,cx+b\right)}\sqrt{cx^2+bx+a}\left(320\,x^6c^6+960\,x^5bc^5+1344\sqrt{\frac{b+2\,cx+\sqrt{-4\,a}}{\sqrt{-4\,ac+b}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/45*(d*(2*c*x+b))^(1/2)*(c*x^2+b*x+a)^(1/2)*d^4*(320*x^6*c^6+960*x^5*b*c^5
+1344*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(
-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))
^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2
)*2^(1/2),2^(1/2))*a^3*c^3-1008*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^
(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^
(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2)
)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a^2*b^2*c^2+252*((b+2*c*x+(-4*
a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(
1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2
*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a
*b^4*c-21*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+
b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1
/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^
(1/2)*2^(1/2),2^(1/2))*b^6-128*x^4*a*c^5+1232*x^4*b^2*c^4-256*x^3*a*b*c^4+8
64*x^3*b^3*c^3-448*x^2*a^2*c^4+32*x^2*a*b^2*c^3+320*x^2*b^4*c^2-448*x*a^2*b
*c^3+160*x*a*b^3*c^2+48*x*b^5*c-112*a^2*b^2*c^2+48*a*b^4*c)/c/(2*c^2*x^3+3*
b*c*x^2+2*a*c*x+b^2*x+a*b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx+bd)^{\frac{9}{2}}}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^(9/2)/sqrt(c*x^2 + b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(16\,c^4d^4x^4+32\,bc^3d^4x^3+24\,b^2c^2d^4x^2+8\,b^3cd^4x+b^4d^4\right)\sqrt{2\,cdx+bd}}{\sqrt{cx^2+bx+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((16*c^4*d^4*x^4 + 32*b*c^3*d^4*x^3 + 24*b^2*c^2*d^4*x^2 + 8*b^3*c*
d^4*x + b^4*d^4)*sqrt(2*c*d*x + b*d)/sqrt(c*x^2 + b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(9/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx + bd)^{\frac{9}{2}}}{\sqrt{cx^2 + bx + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^(9/2)/sqrt(c*x^2 + b*x + a), x)
```

3.1369 $\int \frac{(bd+2cdx)^{5/2}}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=231

$$-\frac{6d^{5/2}\left(b^2-4ac\right)^{7/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\,\sqrt[4]{b^2-4ac}}\right),-1\right)}{5c\sqrt{a+bx+cx^2}}+\frac{6d^{5/2}\left(b^2-4ac\right)^{7/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{bd+2}}{\sqrt[4]{b^2-4a}}\right.\right.}{5c\sqrt{a+bx+cx^2}}$$

```
[Out] (4*d*(b*d + 2*c*d*x)^(3/2)*Sqrt[a + b*x + c*x^2])/5 + (6*(b^2 - 4*a*c)^(7/4
)*d^(5/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqr
t[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(5*c*Sqrt[a + b*x + c
*x^2]) - (6*(b^2 - 4*a*c)^(7/4)*d^(5/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d]
)], -1])/(5*c*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.215393, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {692, 691, 690, 307, 221, 1199, 424}

$$-\frac{6d^{5/2}\left(b^2-4ac\right)^{7/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{5c\sqrt{a+bx+cx^2}}+\frac{6d^{5/2}\left(b^2-4ac\right)^{7/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-\right.}{5c\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(5/2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (4*d*(b*d + 2*c*d*x)^(3/2)*Sqrt[a + b*x + c*x^2])/5 + (6*(b^2 - 4*a*c)^(7/4
)*d^(5/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqr
t[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(5*c*Sqrt[a + b*x + c
*x^2]) - (6*(b^2 - 4*a*c)^(7/4)*d^(5/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d]
)], -1])/(5*c*Sqrt[a + b*x + c*x^2])
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
```

```
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(bd+2cdx)^{5/2}}{\sqrt{a+bx+cx^2}}\,dx = \frac{4}{5}d(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}+\frac{1}{5}\left(3\left(b^2-4ac\right)d^2\right)\int \frac{\sqrt{bd+2cdx}}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{4}{5}d(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}+\frac{\left(3\left(b^2-4ac\right)d^2\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\int \frac{\sqrt{bd+2cdx}}{\sqrt{-\frac{ac}{b^2-4ac}-\frac{bcx}{b^2-4ac}-\frac{c^2x^2}{b^2-4ac}}}\,dx}{5\sqrt{a+bx+cx^2}}$$

$$= \frac{4}{5}d(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}+\frac{\left(6\left(b^2-4ac\right)d\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{\left(b^2-4ac\right)d^2}}}\,dx,x,\sqrt{\ldots}\right)}{5c\sqrt{a+bx+cx^2}}$$

$$= \frac{4}{5}d(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}-\frac{\left(6\left(b^2-4ac\right)^{3/2}d^2\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{\left(b^2-4ac\right)d^2}}}\,dx,\ldots\right)}{5c\sqrt{a+bx+cx^2}}$$

$$= \frac{4}{5}d(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}-\frac{6\left(b^2-4ac\right)^{7/4}d^{5/2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{5c\sqrt{a+bx+cx^2}}$$

$$= \frac{4}{5}d(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}+\frac{6\left(b^2-4ac\right)^{7/4}d^{5/2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{5c\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 0.126672, size = 111, normalized size = 0.48

$$\frac{2d(d(b+2cx))^{3/2}\left(\left(b^2-4ac\right)\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)+2c(a+x(b+cx))\right)}{5c\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(5/2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*d*(d*(b + 2*c*x))^(3/2)*(2*c*(a + x*(b + c*x)) + (b^2 - 4*a*c)*Sqrt[(c*(
a + x*(b + c*x)))/(-b^2 + 4*a*c)]*Hypergeometric2F1[1/2, 3/4, 7/4, (b + 2*c
*x)^2/(b^2 - 4*a*c)]))/(5*c*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.208, size = 496, normalized size = 2.2

$$-\frac{d^2}{5c\left(2c^2x^3+3bcx^2+2acx+b^2x+ab\right)}\sqrt{d(2cx+b)}\sqrt{cx^2+bx+a}\left(48\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}\sqrt{-\frac{2cx+b}{\sqrt{-4ac+b^2}}}\sqrt{\frac{-b}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/5*(d*(2*c*x+b))^(1/2)*(c*x^2+b*x+a)^(1/2)*d^2*(48*((b+2*c*x+(-4*a*c+b^2)
^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-
b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c
*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a^2*c^2-2
4*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a
*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/
2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^
(1/2),2^(1/2))*a*b^2*c+3*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^
(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/
(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a
*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*b^4-16*c^4*x^4-32*b*c^3*x^3-16*x^2*a*
c^3-20*x^2*b^2*c^2-16*b*a*c^2*x-4*b^3*c*x-4*a*c*b^2)/c/(2*c^2*x^3+3*b*c*x^2
+2*a*c*x+b^2*x+a*b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(2cdx+bd)^{\frac{5}{2}}}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^(5/2)/sqrt(c*x^2 + b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(4c^2d^2x^2+4bcd^2x+b^2d^2\right)\sqrt{2cdx+bd}}{\sqrt{cx^2+bx+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((4*c^2*d^2*x^2 + 4*b*c*d^2*x + b^2*d^2)*sqrt(2*c*d*x + b*d)/sqrt(c
*x^2 + b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\,(b+2cx))^{\frac{5}{2}}}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(5/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d*(b + 2*c*x))**(5/2)/sqrt(a + b*x + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx+bd)^{\frac{5}{2}}}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^(5/2)/sqrt(c*x^2 + b*x + a), x)
```

3.1370 $\int \frac{\sqrt{bd+2cdx}}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=195

$$\frac{2\sqrt{d}\left(b^2-4ac\right)^{3/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{c\sqrt{a+bx+cx^2}}-\frac{2\sqrt{d}\left(b^2-4ac\right)^{3/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4a}}\right.\right.}{c\sqrt{a+bx+cx^2}}$$

```
[Out] (2*(b^2 - 4*a*c)^(3/4)*Sqrt[d]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]
*EllipticE[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/
(c*Sqrt[a + b*x + c*x^2]) - (2*(b^2 - 4*a*c)^(3/4)*Sqrt[d]*Sqrt[-((c*(a + b
*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4
*a*c)^(1/4)*Sqrt[d])], -1])/(c*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.179107, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {691, 690, 307, 221, 1199, 424}

$$\frac{2\sqrt{d}\left(b^2-4ac\right)^{3/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{c\sqrt{a+bx+cx^2}}-\frac{2\sqrt{d}\left(b^2-4ac\right)^{3/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{c\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*d + 2*c*d*x]/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*(b^2 - 4*a*c)^(3/4)*Sqrt[d]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]
*EllipticE[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/
(c*Sqrt[a + b*x + c*x^2]) - (2*(b^2 - 4*a*c)^(3/4)*Sqrt[d]*Sqrt[-((c*(a + b
*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4
*a*c)^(1/4)*Sqrt[d])], -1])/(c*Sqrt[a + b*x + c*x^2])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{bd+2cdx}}{\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\int \frac{\sqrt{bd+2cdx}}{\sqrt{-\frac{ac}{b^2-4ac}-\frac{bcx}{b^2-4ac}-\frac{c^2x^2}{b^2-4ac}}}\,dx}{\sqrt{a+bx+cx^2}}$$

$$= \frac{\left(2\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{(b^2-4ac)d^2}}}\,dx, x, \sqrt{bd+2cdx}\right)}{cd\sqrt{a+bx+cx^2}}$$

$$= -\frac{\left(2\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{(b^2-4ac)d^2}}}\,dx, x, \sqrt{bd+2cdx}\right)}{c\sqrt{a+bx+cx^2}} + \frac{\left(2\sqrt{b^2-4ac}\sqrt{-\frac{c(a\cdots}{b\cdots}}\cdots\right.}{\cdots}$$

$$= -\frac{2\left(b^2-4ac\right)^{3/4}\sqrt{d}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{c\sqrt{a+bx+cx^2}} + \frac{\left(2\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Su}\cdots}{c\sqrt{a}\cdots}$$

$$= \frac{2\left(b^2-4ac\right)^{3/4}\sqrt{d}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{c\sqrt{a+bx+cx^2}} - \frac{2\left(b^2-4ac\right)^{3/4}\sqrt{d}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\cdots}{c\sqrt{a+bx}\cdots}$$

Mathematica [C] time = 0.0466139, size = 91, normalized size = 0.47

$$\frac{2\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}(d(b+2cx))^{3/2}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{3cd\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*d + 2*c*d*x]/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*(d*(b + 2*c*x))^(3/2)*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*Hyperge
ometric2F1[1/2, 3/4, 7/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(3*c*d*Sqrt[a + x*(
b + c*x)])
```

Maple [A] time = 0.193, size = 186, normalized size = 1.

$$\frac{4\,ac-b^{2}}{c\left(2\,c^{2}x^{3}+3\,bcx^{2}+2\,acx+b^{2}x+ab\right)}\sqrt{d\left(2\,cx+b\right)}\sqrt{cx^{2}+bx+a}\sqrt{\left(b+2\,cx+\sqrt{-4\,ac+b^{2}}\right)\frac{1}{\sqrt{-4\,ac+b^{2}}}}\sqrt{-(2\,cx+b)-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] (d*(2*c*x+b))^(1/2)*(c*x^2+b*x+a)^(1/2)*(4*a*c-b^2)*((b+2*c*x+(-4*a*c+b^2)^
(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b
-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*
x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))/c/(2*c^2*x
^3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,cdx+bd}}{\sqrt{cx^{2}+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*c*d*x + b*d)/sqrt(c*x^2 + b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,cdx+bd}}{\sqrt{cx^{2}+bx+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*c*d*x + b*d)/sqrt(c*x^2 + b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\,(b+2cx)}}{\sqrt{a+bx+cx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(d*(b + 2*c*x))/sqrt(a + b*x + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,cdx+bd}}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(2*c*d*x + b*d)/sqrt(c*x^2 + b*x + a), x)
```

3.1371 $\int \frac{1}{(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=237

$$\frac{2\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{cd^{3/2}\sqrt[4]{b^2-4ac}\sqrt{a+bx+cx^2}}-\frac{2\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{cd^{3/2}\sqrt[4]{b^2-4ac}\sqrt{a+bx+cx^2}}+\frac{4\sqrt{a+bx+cx^2}}{d\left(b^2-4ac\right)\sqrt{bd+2cd}}$$

```
[Out] (4*Sqrt[a + b*x + c*x^2])/((b^2 - 4*a*c)*d*Sqrt[b*d + 2*c*d*x]) - (2*Sqrt[-
((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d*x]
/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(c*(b^2 - 4*a*c)^(1/4)*d^(3/2)*Sqrt[a
 + b*x + c*x^2]) + (2*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Elliptic
F[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(c*(b^2 -
 4*a*c)^(1/4)*d^(3/2)*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.209, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {693, 691, 690, 307, 221, 1199, 424}

$$\frac{2\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{cd^{3/2}\sqrt[4]{b^2-4ac}\sqrt{a+bx+cx^2}}-\frac{2\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{cd^{3/2}\sqrt[4]{b^2-4ac}\sqrt{a+bx+cx^2}}+\frac{4\sqrt{a+bx+cx^2}}{d\left(b^2-4ac\right)\sqrt{bd+2cdx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (4*Sqrt[a + b*x + c*x^2])/((b^2 - 4*a*c)*d*Sqrt[b*d + 2*c*d*x]) - (2*Sqrt[-
((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d*x]
/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(c*(b^2 - 4*a*c)^(1/4)*d^(3/2)*Sqrt[a
 + b*x + c*x^2]) + (2*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Elliptic
F[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(c*(b^2 -
 4*a*c)^(1/4)*d^(3/2)*Sqrt[a + b*x + c*x^2])
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}dx = \frac{4\sqrt{a+bx+cx^2}}{(b^2-4ac)\,d\sqrt{bd+2cdx}} - \frac{\int \frac{\sqrt{bd+2cdx}}{\sqrt{a+bx+cx^2}}dx}{(b^2-4ac)\,d^2}$$

$$= \frac{4\sqrt{a+bx+cx^2}}{(b^2-4ac)\,d\sqrt{bd+2cdx}} - \frac{\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\int \frac{\sqrt{bd+2cdx}}{\sqrt{-\frac{ac}{b^2-4ac}-\frac{bcx}{b^2-4ac}-\frac{c^2x^2}{b^2-4ac}}}dx}{(b^2-4ac)\,d^2\sqrt{a+bx+cx^2}}$$

$$= \frac{4\sqrt{a+bx+cx^2}}{(b^2-4ac)\,d\sqrt{bd+2cdx}} - \frac{\left(2\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{(b^2-4ac)d^2}}}dx, x, \sqrt{bd+2c\ldots}\right.}{c\,(b^2-4ac)\,d^3\sqrt{a+bx+cx^2}}$$

$$= \frac{4\sqrt{a+bx+cx^2}}{(b^2-4ac)\,d\sqrt{bd+2cdx}} + \frac{\left(2\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{(b^2-4ac)d^2}}}dx, x, \sqrt{bd+2c\ldots}\right.}{c\sqrt{b^2-4ac}d^2\sqrt{a+bx+cx^2}}$$

$$= \frac{4\sqrt{a+bx+cx^2}}{(b^2-4ac)\,d\sqrt{bd+2cdx}} + \frac{2\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{c\sqrt[4]{b^2-4ac}d^{3/2}\sqrt{a+bx+cx^2}} - \frac{\left(2\sqrt{-\frac{c(a+b\ldots}{b^2\ldots}}\right.}{\ldots}$$

$$= \frac{4\sqrt{a+bx+cx^2}}{(b^2-4ac)\,d\sqrt{bd+2cdx}} - \frac{2\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{c\sqrt[4]{b^2-4ac}d^{3/2}\sqrt{a+bx+cx^2}} + \frac{2\sqrt{-\frac{c(a+bx\ldots}{b^2\ldots}}}{c\sqrt[4]{b\ldots}}$$

Mathematica [C] time = 0.0442038, size = 89, normalized size = 0.38

$$-\frac{2\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{cd\sqrt{a+x(b+cx)}\sqrt{d(b+2cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-2*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*Hypergeometric2F1[-1/4, 1/2,
 3/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(c*d*Sqrt[d*(b + 2*c*x)]*Sqrt[a + x*(b
+ c*x)])
```

Maple [A] time = 0.251, size = 336, normalized size = 1.4

$$\frac{1}{cd^2\left(2c^2x^3+3bcx^2+2acx+b^2x+ab\right)\left(4ac-b^2\right)}\sqrt{d(2cx+b)}\sqrt{cx^2+bx+a}\left(4\,\text{EllipticE}\left(1/2\sqrt{\frac{b+2cx+\sqrt{-4ac+b\ldots}}{\sqrt{-4ac+b^2}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] (d*(2*c*x+b))^(1/2)*(c*x^2+b*x+a)^(1/2)*(4*EllipticE(1/2*((b+2*c*x+(-4*a*c+
b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a*c*((b+2*c*x+(-4*a*
```

```
c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/
2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)-EllipticE(1/2*(
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*b^2
*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*
c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2
)-4*c^2*x^2-4*b*c*x-4*a*c)/c/d^2/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)/(4
*a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2\,cdx+bd)^{\frac{3}{2}}\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*c*d*x + b*d)^(3/2)*sqrt(c*x^2 + b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,cdx+bd}\sqrt{cx^2+bx+a}}{4\,c^3d^2x^4+8\,bc^2d^2x^3+ab^2d^2+\left(5\,b^2c+4\,ac^2\right)d^2x^2+\left(b^3+4\,abc\right)d^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(4*c^3*d^2*x^4 + 8*b*c^2
*d^2*x^3 + a*b^2*d^2 + (5*b^2*c + 4*a*c^2)*d^2*x^2 + (b^3 + 4*a*b*c)*d^2*x)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d\,(b+2cx))^{\frac{3}{2}}\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**(3/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/((d*(b + 2*c*x))**(3/2)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2\,cdx+bd)^{\frac{3}{2}}\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")

[Out] integrate(1/((2*c*d*x + b*d)^(3/2)*sqrt(c*x^2 + b*x + a)), x)
```

3.1372 $\int \frac{1}{(bd+2cdx)^{7/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=287

$$\frac{6\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{5cd^{7/2}\left(b^2-4ac\right)^{5/4}\sqrt{a+bx+cx^2}}+\frac{12\sqrt{a+bx+cx^2}}{5d^3\left(b^2-4ac\right)^2\sqrt{bd+2cdx}}-\frac{6\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cx}}{\sqrt[4]{b^2-4ac}\sqrt{}}\right.\right.}{5cd^{7/2}\left(b^2-4ac\right)^{5/4}\sqrt{a+bx+}}$$

```
[Out] (4*Sqrt[a + b*x + c*x^2])/(5*(b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(5/2)) + (12*S
qrt[a + b*x + c*x^2])/(5*(b^2 - 4*a*c)^2*d^3*Sqrt[b*d + 2*c*d*x]) - (6*Sqrt
[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d*
x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(5*c*(b^2 - 4*a*c)^(5/4)*d^(7/2)*Sq
rt[a + b*x + c*x^2]) + (6*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Elli
pticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(5*c*
(b^2 - 4*a*c)^(5/4)*d^(7/2)*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.248231, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {693, 691, 690, 307, 221, 1199, 424}

$$\frac{12\sqrt{a+bx+cx^2}}{5d^3\left(b^2-4ac\right)^2\sqrt{bd+2cdx}}+\frac{6\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{5cd^{7/2}\left(b^2-4ac\right)^{5/4}\sqrt{a+bx+cx^2}}-\frac{6\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{5cd^{7/2}\left(b^2-4ac\right)^{5/4}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^(7/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (4*Sqrt[a + b*x + c*x^2])/(5*(b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(5/2)) + (12*S
qrt[a + b*x + c*x^2])/(5*(b^2 - 4*a*c)^2*d^3*Sqrt[b*d + 2*c*d*x]) - (6*Sqrt
[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d*
x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(5*c*(b^2 - 4*a*c)^(5/4)*d^(7/2)*Sq
rt[a + b*x + c*x^2]) + (6*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Elli
pticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(5*c*
(b^2 - 4*a*c)^(5/4)*d^(7/2)*Sqrt[a + b*x + c*x^2])
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)^{7/2}\sqrt{a+bx+cx^2}}dx = \frac{4\sqrt{a+bx+cx^2}}{5(b^2-4ac)d(bd+2cdx)^{5/2}} + \frac{3\int \frac{1}{(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}dx}{5(b^2-4ac)d^2}$$

$$= \frac{4\sqrt{a+bx+cx^2}}{5(b^2-4ac)d(bd+2cdx)^{5/2}} + \frac{12\sqrt{a+bx+cx^2}}{5(b^2-4ac)^2 d^3\sqrt{bd+2cdx}} - \frac{3\int \frac{\sqrt{bd+2cdx}}{\sqrt{a+bx+cx^2}}dx}{5(b^2-4ac)^2 d^4}$$

$$= \frac{4\sqrt{a+bx+cx^2}}{5(b^2-4ac)d(bd+2cdx)^{5/2}} + \frac{12\sqrt{a+bx+cx^2}}{5(b^2-4ac)^2 d^3\sqrt{bd+2cdx}} - \frac{\left(3\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\cdots}{5(b^2-4ac\cdots}$$

$$= \frac{4\sqrt{a+bx+cx^2}}{5(b^2-4ac)d(bd+2cdx)^{5/2}} + \frac{12\sqrt{a+bx+cx^2}}{5(b^2-4ac)^2 d^3\sqrt{bd+2cdx}} - \frac{\left(6\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\cdots}{5c(\cdots}$$

$$= \frac{4\sqrt{a+bx+cx^2}}{5(b^2-4ac)d(bd+2cdx)^{5/2}} + \frac{12\sqrt{a+bx+cx^2}}{5(b^2-4ac)^2 d^3\sqrt{bd+2cdx}} + \frac{\left(6\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\cdots}{5c(b\cdots}$$

$$= \frac{4\sqrt{a+bx+cx^2}}{5(b^2-4ac)d(bd+2cdx)^{5/2}} + \frac{12\sqrt{a+bx+cx^2}}{5(b^2-4ac)^2 d^3\sqrt{bd+2cdx}} + \frac{6\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F(\cdots}{5c(b^2-4ac)^{5\cdots}}$$

$$= \frac{4\sqrt{a+bx+cx^2}}{5(b^2-4ac)d(bd+2cdx)^{5/2}} + \frac{12\sqrt{a+bx+cx^2}}{5(b^2-4ac)^2 d^3\sqrt{bd+2cdx}} - \frac{6\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E(\cdots}{5c(b^2-4ac)^{5\cdots}}$$

Mathematica [C] time = 0.0613787, size = 91, normalized size = 0.32

$$-\frac{2\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{2};-\frac{1}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{5cd\sqrt{a+x(b+cx)}(d(b+2cx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^(7/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-2*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*Hypergeometric2F1[-5/4, 1/2,
 -1/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(5*c*d*(d*(b + 2*c*x))^(5/2)*Sqrt[a +
x*(b + c*x)])
```

Maple [B] time = 0.258, size = 874, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/5*(48*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1
/2)*2^(1/2),2^(1/2))*x^2*a*c^3*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(
1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(1/2))^(1/2)-12*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^2*b^2*c^2*((b+2*c*x+(-4*a*
c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/
2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)+48*EllipticE(1/
2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*
x*a*b*c^2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+
b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1
/2))^(1/2)-12*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2
))^(1/2)*2^(1/2),2^(1/2))*x*b^3*c*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2
)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2
)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)+12*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c
+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c
+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)
^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a*b^2*c-3*((b+2*c*x+(-4*
a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(
1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2
*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*b
^4-48*c^4*x^4-96*b*c^3*x^3-32*x^2*a*c^3-64*x^2*b^2*c^2-32*b*a*c^2*x-16*b^3*
c*x+16*a^2*c^2-16*a*c*b^2)*(d*(2*c*x+b))^(1/2)/d^4/(c*x^2+b*x+a)^(1/2)/(2*c
*x+b)^3/(4*a*c-b^2)^2/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2\,cdx+bd)^{\frac{7}{2}}\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*c*d*x + b*d)^(7/2)*sqrt(c*x^2 + b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,cdx+bd}\sqrt{cx^2+bx+a}}{16\,c^5d^4x^6+48\,bc^4d^4x^5+ab^4d^4+8\left(7\,b^2c^3+2\,ac^4\right)d^4x^4+32\left(b^3c^2+abc^3\right)d^4x^3+3\left(3\,b^4c+8\,ab^2c^2\right)d^4x^2+\left(b^5\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(16*c^5*d^4*x^6 + 48*b*c
^4*d^4*x^5 + a*b^4*d^4 + 8*(7*b^2*c^3 + 2*a*c^4)*d^4*x^4 + 32*(b^3*c^2 + a*
b*c^3)*d^4*x^3 + 3*(3*b^4*c + 8*a*b^2*c^2)*d^4*x^2 + (b^5 + 8*a*b^3*c)*d^4*
x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d\,(b+2cx))^{\frac{7}{2}}\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**(7/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/((d*(b + 2*c*x))**(7/2)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2\,cdx+bd)^{\frac{7}{2}}\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((2*c*d*x + b*d)^(7/2)*sqrt(c*x^2 + b*x + a)), x)
```

3.1373 $\int \frac{(3-2x)^{3/2}}{\sqrt{1-3x+x^2}} \, dx$

Optimal. Leaf size=79

$$-\frac{2\ 5^{3/4}\sqrt{-x^2+3x-1}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{3-2x}}{\sqrt[4]{5}}\right),-1\right)}{3\sqrt{x^2-3x+1}}-\frac{4}{3}\sqrt{3-2x}\sqrt{x^2-3x+1}$$

```
[Out] (-4*Sqrt[3 - 2*x]*Sqrt[1 - 3*x + x^2])/3 - (2*5^(3/4)*Sqrt[-1 + 3*x - x^2]*
EllipticF[ArcSin[Sqrt[3 - 2*x]/5^(1/4)], -1])/(3*Sqrt[1 - 3*x + x^2])
```

Rubi [A] time = 0.0391474, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {692, 691, 689, 221}

$$-\frac{4}{3}\sqrt{3-2x}\sqrt{x^2-3x+1}-\frac{2\ 5^{3/4}\sqrt{-x^2+3x-1}F\left(\sin^{-1}\left(\frac{\sqrt{3-2x}}{\sqrt[4]{5}}\right)\middle|-1\right)}{3\sqrt{x^2-3x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 2*x)^(3/2)/Sqrt[1 - 3*x + x^2],x]
```

```
[Out] (-4*Sqrt[3 - 2*x]*Sqrt[1 - 3*x + x^2])/3 - (2*5^(3/4)*Sqrt[-1 + 3*x - x^2]*
EllipticF[ArcSin[Sqrt[3 - 2*x]/5^(1/4)], -1])/(3*Sqrt[1 - 3*x + x^2])
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

b/a] && GtQ[a, 0]

Rubi steps

$$\int \frac{(3-2x)^{3/2}}{\sqrt{1-3x+x^2}}\,dx = -\frac{4}{3}\sqrt{3-2x}\sqrt{1-3x+x^2}+\frac{5}{3}\int \frac{1}{\sqrt{3-2x}\sqrt{1-3x+x^2}}\,dx$$
$$= -\frac{4}{3}\sqrt{3-2x}\sqrt{1-3x+x^2}+\frac{\left(\sqrt{5}\sqrt{-1+3x-x^2}\right)\int \frac{1}{\sqrt{3-2x}\sqrt{-\frac{1}{5}+\frac{3x}{5}-\frac{x^2}{5}}}\,dx}{3\sqrt{1-3x+x^2}}$$
$$= -\frac{4}{3}\sqrt{3-2x}\sqrt{1-3x+x^2}-\frac{\left(2\sqrt{5}\sqrt{-1+3x-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{5}}}\,dx,x,\sqrt{3-2x}\right)}{3\sqrt{1-3x+x^2}}$$
$$= -\frac{4}{3}\sqrt{3-2x}\sqrt{1-3x+x^2}-\frac{2\,5^{3/4}\sqrt{-1+3x-x^2}F\left(\sin^{-1}\left(\frac{\sqrt{3-2x}}{\sqrt[4]{5}}\right)\middle|-1\right)}{3\sqrt{1-3x+x^2}}$$

Mathematica [C] time = 0.0276804, size = 76, normalized size = 0.96

$$-\frac{2\sqrt{3-2x}\left(\sqrt{5}\sqrt{-x^2+3x-1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{1}{5}(3-2x)^2\right)+2x^2-6x+2\right)}{3\sqrt{x^2-3x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 2*x)^(3/2)/Sqrt[1 - 3*x + x^2],x]
```

```
[Out] (-2*Sqrt[3 - 2*x]*(2 - 6*x + 2*x^2 + Sqrt[5]*Sqrt[-1 + 3*x - x^2]*Hypergeom
etric2F1[1/4, 1/2, 5/4, (3 - 2*x)^2/5]))/(3*Sqrt[1 - 3*x + x^2])
```

Maple [A] time = 0.167, size = 118, normalized size = 1.5

$$\frac{1}{6x^3-27x^2+33x-9}\sqrt{3-2x}\sqrt{x^2-3x+1}\left(\sqrt{\left(-2x+3+\sqrt{5}\right)\sqrt{5}}\sqrt{(-3+2x)\sqrt{5}}\sqrt{\left(2x-3+\sqrt{5}\right)\sqrt{5}}\text{EllipticF}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-2*x)^(3/2)/(x^2-3*x+1)^(1/2),x)
```

```
[Out] 1/3*(3-2*x)^(1/2)*(x^2-3*x+1)^(1/2)*(((-2*x+3+5^(1/2))*5^(1/2))^(1/2)*((-3+
2*x)*5^(1/2))^(1/2)*((2*x-3+5^(1/2))*5^(1/2))^(1/2)*EllipticF(1/10*2^(1/2)*
5^(1/2)*((-2*x+3+5^(1/2))*5^(1/2))^(1/2),2^(1/2))-8*x^3+36*x^2-44*x+12)/(2*
x^3-9*x^2+11*x-3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-2x+3)^{\frac{3}{2}}}{\sqrt{x^2-3x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)^(3/2)/(x^2-3*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-2*x + 3)^(3/2)/sqrt(x^2 - 3*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-2x+3)^{\frac{3}{2}}}{\sqrt{x^2-3x+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)^(3/2)/(x^2-3*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((-2*x + 3)^(3/2)/sqrt(x^2 - 3*x + 1), x)
```

Sympy [A] time = 10.5448, size = 41, normalized size = 0.52

$$\frac{\sqrt{5}i\,(3-2x)^{\frac{5}{2}}\,\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{(3-2x)^2}{5}\right)}{10\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)**(3/2)/(x**2-3*x+1)**(1/2),x)
```

```
[Out] sqrt(5)*I*(3 - 2*x)**(5/2)*gamma(5/4)*hyper((1/2, 5/4), (9/4,), (3 - 2*x)**
2/5)/(10*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-2x+3)^{\frac{3}{2}}}{\sqrt{x^2-3x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)^(3/2)/(x^2-3*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-2*x + 3)^(3/2)/sqrt(x^2 - 3*x + 1), x)
```

3.1374 $\int \frac{1}{\sqrt{3-2x}\sqrt{1-3x+x^2}}\,dx$

Optimal. Leaf size=51

$$-\frac{2\sqrt{-x^2+3x-1}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{3-2x}}{\sqrt[4]{5}}\right),-1\right)}{\sqrt[4]{5}\sqrt{x^2-3x+1}}$$

```
[Out] (-2*Sqrt[-1 + 3*x - x^2]*EllipticF[ArcSin[Sqrt[3 - 2*x]/5^(1/4)], -1])/(5^(
1/4)*Sqrt[1 - 3*x + x^2])
```

Rubi [A] time = 0.0240225, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {691, 689, 221}

$$-\frac{2\sqrt{-x^2+3x-1}F\left(\sin^{-1}\left(\frac{\sqrt{3-2x}}{\sqrt[4]{5}}\right)\middle|-1\right)}{\sqrt[4]{5}\sqrt{x^2-3x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - 2*x]*Sqrt[1 - 3*x + x^2]),x]
```

```
[Out] (-2*Sqrt[-1 + 3*x - x^2]*EllipticF[ArcSin[Sqrt[3 - 2*x]/5^(1/4)], -1])/(5^(
1/4)*Sqrt[1 - 3*x + x^2])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-2x}\sqrt{1-3x+x^2}}\,dx = \frac{\sqrt{-1+3x-x^2}\int \frac{1}{\sqrt{3-2x}\sqrt{-\frac{1}{5}+\frac{3x}{5}-\frac{x^2}{5}}}\,dx}{\sqrt{5}\sqrt{1-3x+x^2}}$$
$$= -\frac{\left(2\sqrt{-1+3x-x^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{5}}}\,dx, x, \sqrt{3-2x}\right)}{\sqrt{5}\sqrt{1-3x+x^2}}$$
$$= -\frac{2\sqrt{-1+3x-x^2}F\left(\sin^{-1}\left(\frac{\sqrt{3-2x}}{\sqrt[4]{5}}\right)\middle|-1\right)}{\sqrt[4]{5}\sqrt{1-3x+x^2}}$$

Mathematica [C] time = 0.0138915, size = 63, normalized size = 1.24

$$-\frac{2\sqrt{3-2x}\sqrt{-x^2+3x-1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{1}{5}(3-2x)^2\right)}{\sqrt{5}\sqrt{x^2-3x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - 2*x]*Sqrt[1 - 3*x + x^2]),x]
```

```
[Out] (-2*Sqrt[3 - 2*x]*Sqrt[-1 + 3*x - x^2]*Hypergeometric2F1[1/4, 1/2, 5/4, (3
 - 2*x)^2/5])/(Sqrt[5]*Sqrt[1 - 3*x + x^2])
```

Maple [B] time = 0.162, size = 102, normalized size = 2.

$$\frac{1}{10x^3-45x^2+55x-15}\sqrt{3-2x}\sqrt{x^2-3x+1}\sqrt{\left(-2x+3+\sqrt{5}\right)\sqrt{5}}\sqrt{(-3+2x)\sqrt{5}}\sqrt{\left(2x-3+\sqrt{5}\right)\sqrt{5}}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}}{1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-2*x)^(1/2)/(x^2-3*x+1)^(1/2),x)
```

```
[Out] 1/5*(3-2*x)^(1/2)*(x^2-3*x+1)^(1/2)*((-2*x+3+5^(1/2))*5^(1/2))^(1/2)*((-3+2
*x)*5^(1/2))^(1/2)*((2*x-3+5^(1/2))*5^(1/2))^(1/2)*EllipticF(1/10*2^(1/2)*5
^(1/2)*((-2*x+3+5^(1/2))*5^(1/2))^(1/2),2^(1/2))/(2*x^3-9*x^2+11*x-3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-3x+1}\sqrt{-2x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/(x^2-3*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - 3*x + 1)*sqrt(-2*x + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{x^2-3\,x+1}\sqrt{-2\,x+3}}{2\,x^3-9\,x^2+11\,x-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/(x^2-3*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^2 - 3*x + 1)*sqrt(-2*x + 3)/(2*x^3 - 9*x^2 + 11*x - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3-2x}\sqrt{x^2-3x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)**(1/2)/(x**2-3*x+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(3 - 2*x)*sqrt(x**2 - 3*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-3\,x+1}\sqrt{-2\,x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/(x^2-3*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 - 3*x + 1)*sqrt(-2*x + 3)), x)
```

3.1375 $\int \frac{1}{(3-2x)^{5/2}\sqrt{1-3x+x^2}}\,dx$

Optimal. Leaf size=79

$$-\frac{2\sqrt{-x^2+3x-1}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{3-2x}}{\sqrt[4]{5}}\right),-1\right)}{15\sqrt[4]{5}\sqrt{x^2-3x+1}}-\frac{4\sqrt{x^2-3x+1}}{15(3-2x)^{3/2}}$$

```
[Out] (-4*Sqrt[1 - 3*x + x^2])/(15*(3 - 2*x)^(3/2)) - (2*Sqrt[-1 + 3*x - x^2]*Ell
ipticF[ArcSin[Sqrt[3 - 2*x]/5^(1/4)], -1])/(15*5^(1/4)*Sqrt[1 - 3*x + x^2])
```

Rubi [A] time = 0.0344653, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {693, 691, 689, 221}

$$-\frac{4\sqrt{x^2-3x+1}}{15(3-2x)^{3/2}}-\frac{2\sqrt{-x^2+3x-1}\,F\left(\sin^{-1}\left(\frac{\sqrt{3-2x}}{\sqrt[4]{5}}\right)\middle|-1\right)}{15\sqrt[4]{5}\sqrt{x^2-3x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 2*x)^(5/2)*Sqrt[1 - 3*x + x^2]),x]
```

```
[Out] (-4*Sqrt[1 - 3*x + x^2])/(15*(3 - 2*x)^(3/2)) - (2*Sqrt[-1 + 3*x - x^2]*Ell
ipticF[ArcSin[Sqrt[3 - 2*x]/5^(1/4)], -1])/(15*5^(1/4)*Sqrt[1 - 3*x + x^2])
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-2x)^{5/2}\sqrt{1-3x+x^2}}\,dx &= -\frac{4\sqrt{1-3x+x^2}}{15(3-2x)^{3/2}}+\frac{1}{15}\int \frac{1}{\sqrt{3-2x}\sqrt{1-3x+x^2}}\,dx\\
&= -\frac{4\sqrt{1-3x+x^2}}{15(3-2x)^{3/2}}+\frac{\sqrt{-1+3x-x^2}\int \frac{1}{\sqrt{3-2x}\sqrt{-\frac{1}{5}+\frac{3x}{5}-\frac{x^2}{5}}}\,dx}{15\sqrt{5}\sqrt{1-3x+x^2}}\\
&= -\frac{4\sqrt{1-3x+x^2}}{15(3-2x)^{3/2}}-\frac{\left(2\sqrt{-1+3x-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{5}}}\,dx,x,\sqrt{3-2x}\right)}{15\sqrt{5}\sqrt{1-3x+x^2}}\\
&= -\frac{4\sqrt{1-3x+x^2}}{15(3-2x)^{3/2}}-\frac{2\sqrt{-1+3x-x^2}F\left(\sin^{-1}\left(\frac{\sqrt{3-2x}}{\sqrt[4]{5}}\right)\middle|-1\right)}{15\sqrt[4]{5}\sqrt{1-3x+x^2}}
\end{aligned}$$

Mathematica [C] time = 0.0147454, size = 65, normalized size = 0.82

$$\frac{2\sqrt{-x^2+3x-1}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};\frac{1}{5}(3-2x)^2\right)}{3\sqrt{5}(3-2x)^{3/2}\sqrt{x^2-3x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 2*x)^(5/2)*Sqrt[1 - 3*x + x^2]),x]
```

```
[Out] (2*Sqrt[-1 + 3*x - x^2]*Hypergeometric2F1[-3/4, 1/2, 1/4, (3 - 2*x)^2/5])/(
3*Sqrt[5]*(3 - 2*x)^(3/2)*Sqrt[1 - 3*x + x^2])
```

Maple [B] time = 0.206, size = 172, normalized size = 2.2

$$\frac{1}{75\,(-3+2\,x)^2}\left(2\,\sqrt{\left(-2\,x+3+\sqrt{5}\right)\sqrt{5}}\sqrt{(-3+2\,x)\,\sqrt{5}}\sqrt{\left(2\,x-3+\sqrt{5}\right)\sqrt{5}}\text{EllipticF}\left(1/10\,\sqrt{2}\sqrt{5}\sqrt{\left(-2\,x+3+\sqrt{5}\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-2*x)^(5/2)/(x^2-3*x+1)^(1/2),x)
```

```
[Out] 1/75*(2*((-2*x+3+5^(1/2))*5^(1/2))^(1/2)*((-3+2*x)*5^(1/2))^(1/2)*((2*x-3+5
^(1/2))*5^(1/2))^(1/2)*EllipticF(1/10*2^(1/2)*5^(1/2)*((-2*x+3+5^(1/2))*5^(
1/2))^(1/2),2^(1/2))*x-3*((-2*x+3+5^(1/2))*5^(1/2))^(1/2)*((-3+2*x)*5^(1/2)
)^(1/2)*((2*x-3+5^(1/2))*5^(1/2))^(1/2)*EllipticF(1/10*2^(1/2)*5^(1/2)*((-2
*x+3+5^(1/2))*5^(1/2))^(1/2),2^(1/2))-20*x^2+60*x-20)/(x^2-3*x+1)^(1/2)*(3-
2*x)^(1/2)/(-3+2*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-3\,x+1}(-2\,x+3)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(5/2)/(x^2-3*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - 3*x + 1)*(-2*x + 3)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{x^2-3x+1}\sqrt{-2x+3}}{8x^5-60x^4+170x^3-225x^2+135x-27}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(5/2)/(x^2-3*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^2 - 3*x + 1)*sqrt(-2*x + 3)/(8*x^5 - 60*x^4 + 170*x^3 - 22
5*x^2 + 135*x - 27), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3-2x)^{\frac{5}{2}}\sqrt{x^2-3x+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)**(5/2)/(x**2-3*x+1)**(1/2),x)
```

```
[Out] Integral(1/((3 - 2*x)**(5/2)*sqrt(x**2 - 3*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-3x+1}(-2x+3)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(5/2)/(x^2-3*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 - 3*x + 1)*(-2*x + 3)^(5/2)), x)
```

3.1376 $\int \frac{(3-2x)^{5/2}}{\sqrt{1-3x+x^2}}\,dx$

Optimal. Leaf size=128

$$\frac{6\sqrt[4]{5}\sqrt{-x^2+3x-1}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{3-2x}}{\sqrt[4]{5}}\right),-1\right)}{\sqrt{x^2-3x+1}}-\frac{4}{5}\sqrt{x^2-3x+1}(3-2x)^{3/2}-\frac{6\sqrt[4]{5}\sqrt{-x^2+3x-1}E\left(\sin^{-1}\left(\frac{\sqrt{3-2x}}{\sqrt[4]{5}}\right)\right|}{\sqrt{x^2-3x+1}}$$

```
[Out] (-4*(3 - 2*x)^(3/2)*Sqrt[1 - 3*x + x^2])/5 - (6*5^(1/4)*Sqrt[-1 + 3*x - x^2
]*EllipticE[ArcSin[Sqrt[3 - 2*x]/5^(1/4)], -1])/Sqrt[1 - 3*x + x^2] + (6*5^
(1/4)*Sqrt[-1 + 3*x - x^2]*EllipticF[ArcSin[Sqrt[3 - 2*x]/5^(1/4)], -1])/Sq
rt[1 - 3*x + x^2]
```

Rubi [A] time = 0.0711873, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {692, 691, 690, 307, 221, 1181, 21, 424}

$$-\frac{4}{5}\sqrt{x^2-3x+1}(3-2x)^{3/2}+\frac{6\sqrt[4]{5}\sqrt{-x^2+3x-1}F\left(\sin^{-1}\left(\frac{\sqrt{3-2x}}{\sqrt[4]{5}}\right)\right|-1\right)}{\sqrt{x^2-3x+1}}-\frac{6\sqrt[4]{5}\sqrt{-x^2+3x-1}E\left(\sin^{-1}\left(\frac{\sqrt{3-2x}}{\sqrt[4]{5}}\right)\right|-1\right)}{\sqrt{x^2-3x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 2*x)^(5/2)/Sqrt[1 - 3*x + x^2],x]
```

```
[Out] (-4*(3 - 2*x)^(3/2)*Sqrt[1 - 3*x + x^2])/5 - (6*5^(1/4)*Sqrt[-1 + 3*x - x^2
]*EllipticE[ArcSin[Sqrt[3 - 2*x]/5^(1/4)], -1])/Sqrt[1 - 3*x + x^2] + (6*5^
(1/4)*Sqrt[-1 + 3*x - x^2]*EllipticF[ArcSin[Sqrt[3 - 2*x]/5^(1/4)], -1])/Sq
rt[1 - 3*x + x^2]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(3-2x)^{5/2}}{\sqrt{1-3x+x^2}}\,dx = -\frac{4}{5}(3-2x)^{3/2}\sqrt{1-3x+x^2} + 3\int \frac{\sqrt{3-2x}}{\sqrt{1-3x+x^2}}\,dx$$

$$= -\frac{4}{5}(3-2x)^{3/2}\sqrt{1-3x+x^2} + \frac{\left(3\sqrt{-1+3x-x^2}\right)\int \frac{\sqrt{3-2x}}{\sqrt{-\frac{1}{5}+\frac{3x}{5}-\frac{x^2}{5}}}\,dx}{\sqrt{5}\sqrt{1-3x+x^2}}$$

$$= -\frac{4}{5}(3-2x)^{3/2}\sqrt{1-3x+x^2} - \frac{\left(6\sqrt{-1+3x-x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{5}}}\,dx, x, \sqrt{3-2x}\right)}{\sqrt{5}\sqrt{1-3x+x^2}}$$

$$= -\frac{4}{5}(3-2x)^{3/2}\sqrt{1-3x+x^2} + \frac{\left(6\sqrt{-1+3x-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{5}}}\,dx, x, \sqrt{3-2x}\right)}{\sqrt{1-3x+x^2}} - \frac{\left(6\sqrt{-1+3x\right.}{}$$

$$= -\frac{4}{5}(3-2x)^{3/2}\sqrt{1-3x+x^2} + \frac{6\sqrt[4]{5}\sqrt{-1+3x-x^2}F\left(\sin^{-1}\left(\frac{\sqrt{3-2x}}{\sqrt[4]{5}}\right)\middle|-1\right)}{\sqrt{1-3x+x^2}} - \frac{\left(6\sqrt{-1+3x-x^2}\right)\text{Su}}{}$$

$$= -\frac{4}{5}(3-2x)^{3/2}\sqrt{1-3x+x^2} + \frac{6\sqrt[4]{5}\sqrt{-1+3x-x^2}F\left(\sin^{-1}\left(\frac{\sqrt{3-2x}}{\sqrt[4]{5}}\right)\middle|-1\right)}{\sqrt{1-3x+x^2}} - \frac{\left(6\sqrt{-1+3x-x^2}\right)\text{Su}}{\sqrt{}}$$

$$= -\frac{4}{5}(3-2x)^{3/2}\sqrt{1-3x+x^2} - \frac{6\sqrt[4]{5}\sqrt{-1+3x-x^2}E\left(\sin^{-1}\left(\frac{\sqrt{3-2x}}{\sqrt[4]{5}}\right)\middle|-1\right)}{\sqrt{1-3x+x^2}} + \frac{6\sqrt[4]{5}\sqrt{-1+3x-x^2}F}{\sqrt{1-3}}$$

Mathematica [C] time = 0.0233063, size = 76, normalized size = 0.59

$$-\frac{2(3-2x)^{3/2}\left(\sqrt{5}\sqrt{-x^2+3x-1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{1}{5}(3-2x)^2\right)+2x^2-6x+2\right)}{5\sqrt{x^2-3x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 2*x)^(5/2)/Sqrt[1 - 3*x + x^2],x]
```

```
[Out] (-2*(3 - 2*x)^(3/2)*(2 - 6*x + 2*x^2 + Sqrt[5]*Sqrt[-1 + 3*x - x^2]*Hyperge
ometric2F1[1/2, 3/4, 7/4, (3 - 2*x)^2/5]))/(5*Sqrt[1 - 3*x + x^2])
```

Maple [A] time = 0.252, size = 127, normalized size = 1.

$$-\frac{1}{10x^3-45x^2+55x-15}\sqrt{3-2x}\sqrt{x^2-3x+1}\left(3\sqrt{\left(-2x+3+\sqrt{5}\right)\sqrt{5}}\sqrt{(-3+2x)\sqrt{5}}\sqrt{\left(2x-3+\sqrt{5}\right)\sqrt{5}}\text{Ellipti}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-2*x)^(5/2)/(x^2-3*x+1)^(1/2),x)
```

```
[Out] -1/5*(3-2*x)^(1/2)*(x^2-3*x+1)^(1/2)*(3*((-2*x+3+5^(1/2))*5^(1/2))^(1/2)*((
-3+2*x)*5^(1/2))^(1/2)*((2*x-3+5^(1/2))*5^(1/2))^(1/2)*EllipticE(1/10*2^(1/
2)*5^(1/2)*((-2*x+3+5^(1/2))*5^(1/2))^(1/2),2^(1/2))*5^(1/2)-16*x^4+96*x^3-
```

```
196*x^2+156*x-36)/(2*x^3-9*x^2+11*x-3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-2x+3)^{\frac{5}{2}}}{\sqrt{x^2-3x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)^(5/2)/(x^2-3*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-2*x + 3)^(5/2)/sqrt(x^2 - 3*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(4x^2-12x+9)\sqrt{-2x+3}}{\sqrt{x^2-3x+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)^(5/2)/(x^2-3*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((4*x^2 - 12*x + 9)*sqrt(-2*x + 3)/sqrt(x^2 - 3*x + 1), x)
```

Sympy [A] time = 25.7578, size = 41, normalized size = 0.32

$$\frac{\sqrt{5}i\,(3-2x)^{\frac{7}{2}}\,\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{(3-2x)^2}{5}\right)}{10\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)**(5/2)/(x**2-3*x+1)**(1/2),x)
```

```
[Out] sqrt(5)*I*(3 - 2*x)**(7/2)*gamma(7/4)*hyper((1/2, 7/4), (11/4,), (3 - 2*x)*
*2/5)/(10*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-2x+3)^{\frac{5}{2}}}{\sqrt{x^2-3x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)^(5/2)/(x^2-3*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-2*x + 3)^(5/2)/sqrt(x^2 - 3*x + 1), x)
```

3.1377 $\int \frac{\sqrt{3-2x}}{\sqrt{1-3x+x^2}}\,dx$

Optimal. Leaf size=103

$$\frac{2\sqrt[4]{5}\sqrt{-x^2+3x-1}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{3-2x}}{\sqrt[4]{5}}\right),-1\right)}{\sqrt{x^2-3x+1}}-\frac{2\sqrt[4]{5}\sqrt{-x^2+3x-1}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-2x}}{\sqrt[4]{5}}\right)\middle|-1\right)}{\sqrt{x^2-3x+1}}$$

```
[Out] (-2*5^(1/4)*Sqrt[-1 + 3*x - x^2]*EllipticE[ArcSin[Sqrt[3 - 2*x]/5^(1/4)], -
1])/Sqrt[1 - 3*x + x^2] + (2*5^(1/4)*Sqrt[-1 + 3*x - x^2]*EllipticF[ArcSin[
Sqrt[3 - 2*x]/5^(1/4)], -1])/Sqrt[1 - 3*x + x^2]
```

Rubi [A] time = 0.0523656, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {691, 690, 307, 221, 1181, 21, 424}

$$\frac{2\sqrt[4]{5}\sqrt{-x^2+3x-1}\,F\left(\sin^{-1}\left(\frac{\sqrt{3-2x}}{\sqrt[4]{5}}\right)\middle|-1\right)}{\sqrt{x^2-3x+1}}-\frac{2\sqrt[4]{5}\sqrt{-x^2+3x-1}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-2x}}{\sqrt[4]{5}}\right)\middle|-1\right)}{\sqrt{x^2-3x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - 2*x]/Sqrt[1 - 3*x + x^2],x]
```

```
[Out] (-2*5^(1/4)*Sqrt[-1 + 3*x - x^2]*EllipticE[ArcSin[Sqrt[3 - 2*x]/5^(1/4)], -
1])/Sqrt[1 - 3*x + x^2] + (2*5^(1/4)*Sqrt[-1 + 3*x - x^2]*EllipticF[ArcSin[
Sqrt[3 - 2*x]/5^(1/4)], -1])/Sqrt[1 - 3*x + x^2]
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{3-2x}}{\sqrt{1-3x+x^2}}\,dx = \frac{\sqrt{-1+3x-x^2}\int \frac{\sqrt{3-2x}}{\sqrt{-\frac{1}{5}+\frac{3x}{5}-\frac{x^2}{5}}}\,dx}{\sqrt{5}\sqrt{1-3x+x^2}}$$

$$= -\frac{\left(2\sqrt{-1+3x-x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{5}}}\,dx, x, \sqrt{3-2x}\right)}{\sqrt{5}\sqrt{1-3x+x^2}}$$

$$= \frac{\left(2\sqrt{-1+3x-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{5}}}\,dx, x, \sqrt{3-2x}\right)}{\sqrt{1-3x+x^2}} - \frac{\left(2\sqrt{-1+3x-x^2}\right)\text{Subst}\left(\int \frac{1+\frac{x^2}{\sqrt{5}}}{\sqrt{1-\frac{x^4}{5}}}\,dx, x, \sqrt{3\cdots}\right.}{\sqrt{1-3x+x^2}}$$

$$= \frac{2\sqrt[4]{5}\sqrt{-1+3x-x^2}F\left(\sin^{-1}\left(\frac{\sqrt{3-2x}}{\sqrt[4]{5}}\right)\middle|-1\right)}{\sqrt{1-3x+x^2}} - \frac{\left(2\sqrt{-1+3x-x^2}\right)\text{Subst}\left(\int \frac{1+\frac{x^2}{\sqrt{5}}}{\sqrt{\frac{1}{\sqrt{5}}-\frac{x^2}{5}}\sqrt{\frac{1}{\sqrt{5}}+\frac{x^2}{5}}}\,dx, x, \sqrt{3\cdots}\right.}{\sqrt{5}\sqrt{1-3x+x^2}}$$

$$= \frac{2\sqrt[4]{5}\sqrt{-1+3x-x^2}F\left(\sin^{-1}\left(\frac{\sqrt{3-2x}}{\sqrt[4]{5}}\right)\middle|-1\right)}{\sqrt{1-3x+x^2}} - \frac{\left(2\sqrt{-1+3x-x^2}\right)\text{Subst}\left(\int \frac{\sqrt{\frac{1}{\sqrt{5}}+\frac{x^2}{5}}}{\sqrt{\frac{1}{\sqrt{5}}-\frac{x^2}{5}}}\,dx, x, \sqrt{3-2x}\right)}{\sqrt{1-3x+x^2}}$$

$$= -\frac{2\sqrt[4]{5}\sqrt{-1+3x-x^2}E\left(\sin^{-1}\left(\frac{\sqrt{3-2x}}{\sqrt[4]{5}}\right)\middle|-1\right)}{\sqrt{1-3x+x^2}} + \frac{2\sqrt[4]{5}\sqrt{-1+3x-x^2}F\left(\sin^{-1}\left(\frac{\sqrt{3-2x}}{\sqrt[4]{5}}\right)\middle|-1\right)}{\sqrt{1-3x+x^2}}$$

Mathematica [C] time = 0.0133973, size = 65, normalized size = 0.63

$$-\frac{2(3-2x)^{3/2}\sqrt{-x^2+3x-1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{1}{5}(3-2x)^2\right)}{3\sqrt{5}\sqrt{x^2-3x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - 2*x]/Sqrt[1 - 3*x + x^2],x]
```

```
[Out] (-2*(3 - 2*x)^(3/2)*Sqrt[-1 + 3*x - x^2]*Hypergeometric2F1[1/2, 3/4, 7/4, (
3 - 2*x)^2/5])/(3*Sqrt[5]*Sqrt[1 - 3*x + x^2])
```

Maple [A] time = 0.1, size = 105, normalized size = 1.

$$-\frac{\sqrt{5}}{10\,x^{3}-45\,x^{2}+55\,x-15}\sqrt{3-2\,x}\sqrt{x^{2}-3\,x+1}\sqrt{\left(-2\,x+3+\sqrt{5}\right)\sqrt{5}}\sqrt{\left(-3+2\,x\right)\sqrt{5}}\sqrt{\left(2\,x-3+\sqrt{5}\right)\sqrt{5}}\sqrt{5}\mathrm{EllipticE}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-2*x)^(1/2)/(x^2-3*x+1)^(1/2),x)
```

```
[Out] -1/5*(3-2*x)^(1/2)*(x^2-3*x+1)^(1/2)*((-2*x+3+5^(1/2))*5^(1/2))^(1/2)*((-3+
2*x)*5^(1/2))^(1/2)*((2*x-3+5^(1/2))*5^(1/2))^(1/2)*5^(1/2)*EllipticE(1/10*
2^(1/2)*5^(1/2)*((-2*x+3+5^(1/2))*5^(1/2))^(1/2),2^(1/2))/(2*x^3-9*x^2+11*x
-3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-2\,x+3}}{\sqrt{x^{2}-3\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)^(1/2)/(x^2-3*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-2*x + 3)/sqrt(x^2 - 3*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-2\,x+3}}{\sqrt{x^{2}-3\,x+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)^(1/2)/(x^2-3*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-2*x + 3)/sqrt(x^2 - 3*x + 1), x)
```

Sympy [A] time = 3.5848, size = 41, normalized size = 0.4

$$\frac{\sqrt{5}i\left(3-2x\right)^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{\left(3-2x\right)^{2}}{5}\right)}{10\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)**(1/2)/(x**2-3*x+1)**(1/2),x)
```

```
[Out] sqrt(5)*I*(3 - 2*x)**(3/2)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), (3 - 2*x)**
2/5)/(10*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-2x+3}}{\sqrt{x^2-3x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)^(1/2)/(x^2-3*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-2*x + 3)/sqrt(x^2 - 3*x + 1), x)
```

3.1378 $\int \frac{1}{(3-2x)^{3/2}\sqrt{1-3x+x^2}}\,dx$

Optimal. Leaf size=128

$$-\frac{2\sqrt{-x^2+3x-1}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{3-2x}}{\sqrt[4]{5}}\right),-1\right)}{5^{3/4}\sqrt{x^2-3x+1}}-\frac{4\sqrt{x^2-3x+1}}{5\sqrt{3-2x}}+\frac{2\sqrt{-x^2+3x-1}E\left(\sin^{-1}\left(\frac{\sqrt{3-2x}}{\sqrt[4]{5}}\right)\middle|-1\right)}{5^{3/4}\sqrt{x^2-3x+1}}$$

```
[Out] (-4*Sqrt[1 - 3*x + x^2])/(5*Sqrt[3 - 2*x]) + (2*Sqrt[-1 + 3*x - x^2]*Ellipt
icE[ArcSin[Sqrt[3 - 2*x]/5^(1/4)], -1])/(5^(3/4)*Sqrt[1 - 3*x + x^2]) - (2*
Sqrt[-1 + 3*x - x^2]*EllipticF[ArcSin[Sqrt[3 - 2*x]/5^(1/4)], -1])/(5^(3/4)
*Sqrt[1 - 3*x + x^2])
```

Rubi [A] time = 0.0681879, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {693, 691, 690, 307, 221, 1181, 21, 424}

$$-\frac{4\sqrt{x^2-3x+1}}{5\sqrt{3-2x}}-\frac{2\sqrt{-x^2+3x-1}F\left(\sin^{-1}\left(\frac{\sqrt{3-2x}}{\sqrt[4]{5}}\right)\middle|-1\right)}{5^{3/4}\sqrt{x^2-3x+1}}+\frac{2\sqrt{-x^2+3x-1}E\left(\sin^{-1}\left(\frac{\sqrt{3-2x}}{\sqrt[4]{5}}\right)\middle|-1\right)}{5^{3/4}\sqrt{x^2-3x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 2*x)^(3/2)*Sqrt[1 - 3*x + x^2]),x]
```

```
[Out] (-4*Sqrt[1 - 3*x + x^2])/(5*Sqrt[3 - 2*x]) + (2*Sqrt[-1 + 3*x - x^2]*Ellipt
icE[ArcSin[Sqrt[3 - 2*x]/5^(1/4)], -1])/(5^(3/4)*Sqrt[1 - 3*x + x^2]) - (2*
Sqrt[-1 + 3*x - x^2]*EllipticF[ArcSin[Sqrt[3 - 2*x]/5^(1/4)], -1])/(5^(3/4)
*Sqrt[1 - 3*x + x^2])
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(3-2x)^{3/2}\sqrt{1-3x+x^2}}\,dx = -\frac{4\sqrt{1-3x+x^2}}{5\sqrt{3-2x}} - \frac{1}{5}\int \frac{\sqrt{3-2x}}{\sqrt{1-3x+x^2}}\,dx$$

$$= -\frac{4\sqrt{1-3x+x^2}}{5\sqrt{3-2x}} - \frac{\sqrt{-1+3x-x^2}\int \frac{\sqrt{3-2x}}{\sqrt{-\frac{1}{5}+\frac{3x}{5}-\frac{x^2}{5}}}\,dx}{5\sqrt{5}\sqrt{1-3x+x^2}}$$

$$= -\frac{4\sqrt{1-3x+x^2}}{5\sqrt{3-2x}} + \frac{\left(2\sqrt{-1+3x-x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{5}}}\,dx, x, \sqrt{3-2x}\right)}{5\sqrt{5}\sqrt{1-3x+x^2}}$$

$$= -\frac{4\sqrt{1-3x+x^2}}{5\sqrt{3-2x}} - \frac{\left(2\sqrt{-1+3x-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{5}}}\,dx, x, \sqrt{3-2x}\right)}{5\sqrt{1-3x+x^2}} + \frac{\left(2\sqrt{-1+3x\right.}{}$$

$$= -\frac{4\sqrt{1-3x+x^2}}{5\sqrt{3-2x}} - \frac{2\sqrt{-1+3x-x^2}F\left(\sin^{-1}\left(\frac{\sqrt{3-2x}}{\sqrt[4]{5}}\right)\middle| -1\right)}{5^{3/4}\sqrt{1-3x+x^2}} + \frac{\left(2\sqrt{-1+3x-x^2}\right)\text{Subs}}{5\sqrt{}}$$

$$= -\frac{4\sqrt{1-3x+x^2}}{5\sqrt{3-2x}} - \frac{2\sqrt{-1+3x-x^2}F\left(\sin^{-1}\left(\frac{\sqrt{3-2x}}{\sqrt[4]{5}}\right)\middle| -1\right)}{5^{3/4}\sqrt{1-3x+x^2}} + \frac{\left(2\sqrt{-1+3x-x^2}\right)\text{Subs}}{5\sqrt{1}}$$

$$= -\frac{4\sqrt{1-3x+x^2}}{5\sqrt{3-2x}} + \frac{2\sqrt{-1+3x-x^2}E\left(\sin^{-1}\left(\frac{\sqrt{3-2x}}{\sqrt[4]{5}}\right)\middle| -1\right)}{5^{3/4}\sqrt{1-3x+x^2}} - \frac{2\sqrt{-1+3x-x^2}F\left(\sin^{-1}\right.}{5^{3/4}\sqrt{1-3x}}$$

Mathematica [C] time = 0.0129933, size = 63, normalized size = 0.49

$$\frac{2\sqrt{-x^2+3x-1}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};\frac{1}{5}(3-2x)^2\right)}{\sqrt{5}\sqrt{3-2x}\sqrt{x^2-3x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 2*x)^(3/2)*Sqrt[1 - 3*x + x^2]),x]
```

```
[Out] (2*Sqrt[-1 + 3*x - x^2]*Hypergeometric2F1[-1/4, 1/2, 3/4, (3 - 2*x)^2/5])/(
Sqrt[5]*Sqrt[3 - 2*x]*Sqrt[1 - 3*x + x^2])
```

Maple [A] time = 0.174, size = 116, normalized size = 0.9

$$\frac{1}{50x^3-225x^2+275x-75}\sqrt{3-2x}\sqrt{x^2-3x+1}\left(\sqrt{\left(-2x+3+\sqrt{5}\right)\sqrt{5}}\sqrt{(-3+2x)\sqrt{5}}\sqrt{\left(2x-3+\sqrt{5}\right)\sqrt{5}}\text{Ellipti}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-2*x)^(3/2)/(x^2-3*x+1)^(1/2),x)
```

```
[Out] 1/25*(3-2*x)^(1/2)*(x^2-3*x+1)^(1/2)*(((-2*x+3+5^(1/2))*5^(1/2))^(1/2)*((-3
+2*x)*5^(1/2))^(1/2)*((2*x-3+5^(1/2))*5^(1/2))^(1/2)*EllipticE(1/10*2^(1/2)
```

```
*5^(1/2)*((-2*x+3+5^(1/2))*5^(1/2))^(1/2),2^(1/2))*5^(1/2)+20*x^2-60*x+20)/
(2*x^3-9*x^2+11*x-3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-3x+1}(-2x+3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(3/2)/(x^2-3*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - 3*x + 1)*(-2*x + 3)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2-3x+1}\sqrt{-2x+3}}{4x^4-24x^3+49x^2-39x+9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(3/2)/(x^2-3*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - 3*x + 1)*sqrt(-2*x + 3)/(4*x^4 - 24*x^3 + 49*x^2 - 39*x
 + 9), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3-2x)^{\frac{3}{2}}\sqrt{x^2-3x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)**(3/2)/(x**2-3*x+1)**(1/2),x)
```

```
[Out] Integral(1/((3 - 2*x)**(3/2)*sqrt(x**2 - 3*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-3x+1}(-2x+3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(3/2)/(x^2-3*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 - 3*x + 1)*(-2*x + 3)^(3/2)), x)
```

3.1379 $\int \frac{(bd+2cdx)^{11/2}}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=205

$$\frac{60d^{11/2}\left(b^2-4ac\right)^{9/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{7\sqrt{a+bx+cx^2}}+\frac{120}{7}cd^5\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}$$

```
[Out] (-2*d*(b*d + 2*c*d*x)^(9/2))/Sqrt[a + b*x + c*x^2] + (120*c*(b^2 - 4*a*c)*d
^5*Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/7 + (72*c*d^3*(b*d + 2*c*d*x)
^(5/2)*Sqrt[a + b*x + c*x^2])/7 + (60*(b^2 - 4*a*c)^(9/4)*d^(11/2)*Sqrt[-((
c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/(
(b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(7*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.175973, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {686, 692, 691, 689, 221}

$$\frac{120}{7}cd^5\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}+\frac{60d^{11/2}\left(b^2-4ac\right)^{9/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{7\sqrt{a+bx+cx^2}}+\frac{72}{7}cd^3$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(11/2)/(a + b*x + c*x^2)^(3/2),x]
```


```
[Out] (-2*d*(b*d + 2*c*d*x)^(9/2))/Sqrt[a + b*x + c*x^2] + (120*c*(b^2 - 4*a*c)*d
^5*Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/7 + (72*c*d^3*(b*d + 2*c*d*x)
^(5/2)*Sqrt[a + b*x + c*x^2])/7 + (60*(b^2 - 4*a*c)^(9/4)*d^(11/2)*Sqrt[-((
c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/(
(b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(7*Sqrt[a + b*x + c*x^2])
```


Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
```

```
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(bd+2cdx)^{11/2}}{\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2d(bd+2cdx)^{9/2}}{\sqrt{a+bx+cx^2}} + \left(18cd^2\right)\int \frac{(bd+2cdx)^{7/2}}{\sqrt{a+bx+cx^2}}\,dx$$

$$= -\frac{2d(bd+2cdx)^{9/2}}{\sqrt{a+bx+cx^2}} + \frac{72}{7}cd^3(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2} + \frac{1}{7}\left(90c\left(b^2-4ac\right)d^4\right)\int \frac{(bd+2cdx)}{\sqrt{a+bx+}}$$

$$= -\frac{2d(bd+2cdx)^{9/2}}{\sqrt{a+bx+cx^2}} + \frac{120}{7}c\left(b^2-4ac\right)d^5\sqrt{bd+2cdx}\sqrt{a+bx+cx^2} + \frac{72}{7}cd^3(bd+2cdx)^{5/2}\sqrt{a+}$$

$$= -\frac{2d(bd+2cdx)^{9/2}}{\sqrt{a+bx+cx^2}} + \frac{120}{7}c\left(b^2-4ac\right)d^5\sqrt{bd+2cdx}\sqrt{a+bx+cx^2} + \frac{72}{7}cd^3(bd+2cdx)^{5/2}\sqrt{a+}$$

$$= -\frac{2d(bd+2cdx)^{9/2}}{\sqrt{a+bx+cx^2}} + \frac{120}{7}c\left(b^2-4ac\right)d^5\sqrt{bd+2cdx}\sqrt{a+bx+cx^2} + \frac{72}{7}cd^3(bd+2cdx)^{5/2}\sqrt{a+}$$

$$= -\frac{2d(bd+2cdx)^{9/2}}{\sqrt{a+bx+cx^2}} + \frac{120}{7}c\left(b^2-4ac\right)d^5\sqrt{bd+2cdx}\sqrt{a+bx+cx^2} + \frac{72}{7}cd^3(bd+2cdx)^{5/2}\sqrt{a+}$$

Mathematica [C] time = 0.197787, size = 172, normalized size = 0.84

$$\frac{2d^5\sqrt{d(b+2cx)}\left(16c^2\left(-15a^2-6acx^2+2c^2x^4\right)+30\left(b^2-4ac\right)^2\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)+24b^2c\left(4a+3cx^2\right)+\right.}{7\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(11/2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*d^5*Sqrt[d*(b + 2*c*x)]*(-7*b^4 + 40*b^3*c*x + 32*b*c^2*x*(-3*a + 2*c*x^
2) + 24*b^2*c*(4*a + 3*c*x^2) + 16*c^2*(-15*a^2 - 6*a*c*x^2 + 2*c^2*x^4) +
30*(b^2 - 4*a*c)^2*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*Hypergeometri
c2F1[1/4, 1/2, 5/4, (b + 2*c*x)^2/(b^2 - 4*a*c)]))/(7*Sqrt[a + x*(b + c*x)]
```

)

Maple [B] time = 0.343, size = 569, normalized size = 2.8

$$\frac{2 d^5}{14 c^2 x^3 + 21 b c x^2 + 14 a c x + 7 b^2 x + 7 a b} \sqrt{d (2 c x + b)} \sqrt{c x^2 + b x + a} \left(64 x^5 c^5 + 240 \sqrt{\frac{b + 2 c x + \sqrt{-4 a c + b^2}}{\sqrt{-4 a c + b^2}}} \sqrt{-\frac{2}{\sqrt{-}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(11/2)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 2/7*(d*(2*c*x+b))^(1/2)*(c*x^2+b*x+a)^(1/2)*d^5*(64*x^5*c^5+240*((b+2*c*x+(
-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2)
)^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(
1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)
)*(-4*a*c+b^2)^(1/2)*a^2*c^2-120*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)
^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2
))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2)^(1/2)*a*b^2*c+15
*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*
c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2
)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(
1/2),2^(1/2))*(-4*a*c+b^2)^(1/2)*b^4+160*x^4*b*c^4-192*x^3*a*c^4+208*x^3*b^
2*c^3-288*x^2*a*b*c^3+152*x^2*b^3*c^2-480*x*a^2*c^3+96*x*a*b^2*c^2+26*x*b^4
*c-240*a^2*b*c^2+96*a*b^3*c-7*b^5)/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2 c d x + b d)^{\frac{11}{2}}}{(c x^2 + b x + a)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(11/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^(11/2)/(c*x^2 + b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(32 c^5 d^5 x^5 + 80 b c^4 d^5 x^4 + 80 b^2 c^3 d^5 x^3 + 40 b^3 c^2 d^5 x^2 + 10 b^4 c d^5 x + b^5 d^5\right) \sqrt{2 c d x + b d} \sqrt{c x^2 + b x + a}}{c^2 x^4 + 2 b c x^3 + 2 a b x + \left(b^2 + 2 a c\right) x^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(11/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((32*c^5*d^5*x^5 + 80*b*c^4*d^5*x^4 + 80*b^2*c^3*d^5*x^3 + 40*b^3*c
^2*d^5*x^2 + 10*b^4*c*d^5*x + b^5*d^5)*sqrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x
 + a)/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(11/2)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx+bd)^{\frac{11}{2}}}{(cx^2+bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(11/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^(11/2)/(c*x^2 + b*x + a)^(3/2), x)
```

3.1380 $\int \frac{(bd+2cdx)^{7/2}}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=162

$$\frac{20d^{7/2}\left(b^2-4ac\right)^{5/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{3\sqrt{a+bx+cx^2}}+\frac{40}{3}cd^3\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}-\frac{2d(bd+2c}{\sqrt{a+bx+}}$$

```
[Out] (-2*d*(b*d + 2*c*d*x)^(5/2))/Sqrt[a + b*x + c*x^2] + (40*c*d^3*Sqrt[b*d + 2
*c*d*x]*Sqrt[a + b*x + c*x^2])/3 + (20*(b^2 - 4*a*c)^(5/4)*d^(7/2)*Sqrt[-((
c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/(
(b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(3*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.142299, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {686, 692, 691, 689, 221}

$$\frac{20d^{7/2}\left(b^2-4ac\right)^{5/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{3\sqrt{a+bx+cx^2}}+\frac{40}{3}cd^3\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}-\frac{2d(bd+2cdx)^{5/2}}{\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(7/2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*d*(b*d + 2*c*d*x)^(5/2))/Sqrt[a + b*x + c*x^2] + (40*c*d^3*Sqrt[b*d + 2
*c*d*x]*Sqrt[a + b*x + c*x^2])/3 + (20*(b^2 - 4*a*c)^(5/4)*d^(7/2)*Sqrt[-((
c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/(
(b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(3*Sqrt[a + b*x + c*x^2])
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
```

```
(c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(bd+2cdx)^{7/2}}{\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2d(bd+2cdx)^{5/2}}{\sqrt{a+bx+cx^2}} + \left(10cd^2\right)\int \frac{(bd+2cdx)^{3/2}}{\sqrt{a+bx+cx^2}}\,dx$$

$$= -\frac{2d(bd+2cdx)^{5/2}}{\sqrt{a+bx+cx^2}} + \frac{40}{3}cd^3\sqrt{bd+2cdx}\sqrt{a+bx+cx^2} + \frac{1}{3}\left(10c\left(b^2-4ac\right)d^4\right)\int \frac{1}{\sqrt{bd+2cdx}\sqrt{\ldots}}$$

$$= -\frac{2d(bd+2cdx)^{5/2}}{\sqrt{a+bx+cx^2}} + \frac{40}{3}cd^3\sqrt{bd+2cdx}\sqrt{a+bx+cx^2} + \frac{\left(10c\left(b^2-4ac\right)d^4\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\int \ldots}{3\sqrt{a+bx+\ldots}}$$

$$= -\frac{2d(bd+2cdx)^{5/2}}{\sqrt{a+bx+cx^2}} + \frac{40}{3}cd^3\sqrt{bd+2cdx}\sqrt{a+bx+cx^2} + \frac{\left(20\left(b^2-4ac\right)d^3\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\mathrm{Subs}\ldots}{3\sqrt{a+b\ldots}}$$

$$= -\frac{2d(bd+2cdx)^{5/2}}{\sqrt{a+bx+cx^2}} + \frac{40}{3}cd^3\sqrt{bd+2cdx}\sqrt{a+bx+cx^2} + \frac{20\left(b^2-4ac\right)^{5/4}d^{7/2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\ldots\right.}{3\sqrt{a+bx+cx^2}\ldots}$$

Mathematica [C] time = 0.126666, size = 122, normalized size = 0.75

$$\frac{2d^3\sqrt{d(b+2cx)}\left(10\left(b^2-4ac\right)\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)+4c\left(5a+2cx^2\right)-3b^2+8bcx\right)}{3\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(7/2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*d^3*Sqrt[d*(b + 2*c*x)]*(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2) + 10*(b^
2 - 4*a*c)*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*Hypergeometric2F1[1/4
, 1/2, 5/4, (b + 2*c*x)^2/(b^2 - 4*a*c)]))/(3*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.302, size = 366, normalized size = 2.3

$$-\frac{2d^3}{6c^2x^3+9bcx^2+6acx+3b^2x+3ab}\sqrt{d(2cx+b)}\sqrt{cx^2+bx+a}\left(20\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}\sqrt{-\frac{2cx+b}{\sqrt{-4ac+b^2}}}\sqrt{-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -2/3*(d*(2*c*x+b))^(1/2)*(c*x^2+b*x+a)^(1/2)*d^3*(20*((b+2*c*x+(-4*a*c+b^2)
^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-
b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c
*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b
^2)^(1/2)*a*c-5*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(
2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b
^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(
1/2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2)^(1/2)*b^2-16*c^3*x^3-24*b*c^2*x^2
-40*x*a*c^2-2*x*b^2*c-20*a*b*c+3*b^3)/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*x+a*
b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx+bd)^{\frac{7}{2}}}{\left(cx^2+bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^(7/2)/(c*x^2 + b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(8\,c^3d^3x^3+12\,bc^2d^3x^2+6\,b^2cd^3x+b^3d^3\right)\sqrt{2\,cdx+bd}\sqrt{cx^2+bx+a}}{c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\,ac\right)x^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((8*c^3*d^3*x^3 + 12*b*c^2*d^3*x^2 + 6*b^2*c*d^3*x + b^3*d^3)*sqrt(
2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2
+ 2*a*c)*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(7/2)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx+bd)^{\frac{7}{2}}}{\left(cx^2+bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^(7/2)/(c*x^2 + b*x + a)^(3/2), x)
```

3.1381 $\int \frac{(bd+2cdx)^{3/2}}{\left(a+bx+cx^2\right)^{3/2}} dx$

Optimal. Leaf size=125

$$\frac{4d^{3/2}\sqrt[4]{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{\sqrt{a+bx+cx^2}}-\frac{2d\sqrt{bd+2cdx}}{\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*d*Sqrt[b*d + 2*c*d*x])/Sqrt[a + b*x + c*x^2] + (4*(b^2 - 4*a*c)^(1/4)*d
^(3/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b
*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/Sqrt[a + b*x + c*x^2]
```

Rubi [A] time = 0.113233, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {686, 691, 689, 221}

$$\frac{4d^{3/2}\sqrt[4]{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{\sqrt{a+bx+cx^2}}-\frac{2d\sqrt{bd+2cdx}}{\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(3/2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*d*Sqrt[b*d + 2*c*d*x])/Sqrt[a + b*x + c*x^2] + (4*(b^2 - 4*a*c)^(1/4)*d
^(3/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b
*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/Sqrt[a + b*x + c*x^2]
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(bd+2cdx)^{3/2}}{\left(a+bx+cx^2\right)^{3/2}}\,dx &= -\frac{2d\sqrt{bd+2cdx}}{\sqrt{a+bx+cx^2}}+\left(2cd^2\right)\int \frac{1}{\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}\,dx\\ &= -\frac{2d\sqrt{bd+2cdx}}{\sqrt{a+bx+cx^2}}+\frac{\left(2cd^2\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\int \frac{1}{\sqrt{bd+2cdx}\sqrt{-\frac{ac}{b^2-4ac}-\frac{bcx}{b^2-4ac}-\frac{c^2x^2}{b^2-4ac}}}\,dx}{\sqrt{a+bx+cx^2}}\\ &= -\frac{2d\sqrt{bd+2cdx}}{\sqrt{a+bx+cx^2}}+\frac{\left(4d\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{\left(b^2-4ac\right)d^2}}}\,dx,x,\sqrt{bd+2cdx}\right)}{\sqrt{a+bx+cx^2}}\\ &= -\frac{2d\sqrt{bd+2cdx}}{\sqrt{a+bx+cx^2}}+\frac{4\sqrt[4]{b^2-4ac}\,d^{3/2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{\sqrt{a+bx+cx^2}}\end{aligned}$$

Mathematica [C] time = 0.0651446, size = 88, normalized size = 0.7

$$\frac{2d\sqrt{d(b+2cx)}\left(2\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)-1\right)}{\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(3/2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*d*Sqrt[d*(b + 2*c*x)]*(-1 + 2*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]
*Hypergeometric2F1[1/4, 1/2, 5/4, (b + 2*c*x)^2/(b^2 - 4*a*c)]))/Sqrt[a + x
*(b + c*x)]
```

Maple [A] time = 0.282, size = 194, normalized size = 1.6

$$2\,\frac{\sqrt{d\left(2\,cx+b\right)}\sqrt{cx^2+bx+ad}}{2\,c^2x^3+3\,bcx^2+2\,acx+b^2x+ab}\left(\text{EllipticF}\left(1/2\sqrt{\frac{b+2\,cx+\sqrt{-4\,ac+b^2}}{\sqrt{-4\,ac+b^2}}}\sqrt{2},\sqrt{2}\right)\sqrt{-4\,ac+b^2}\sqrt{\frac{b+2\,cx+\sqrt{-4\,ac}}{\sqrt{-4\,ac+b^2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 2*(d*(2*c*x+b))^(1/2)*(c*x^2+b*x+a)^(1/2)*d*(EllipticF(1/2*((b+2*c*x+(-4*a*
c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2)^(1/2)
*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*
c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2
)-2*c*x-b)/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx + bd)^{\frac{3}{2}}}{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^(3/2)/(c*x^2 + b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(2\,cdx + bd)^{\frac{3}{2}}\sqrt{cx^2 + bx + a}}{c^2x^4 + 2\,bcx^3 + 2\,abx + \left(b^2 + 2\,ac\right)x^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((2*c*d*x + b*d)^(3/2)*sqrt(c*x^2 + b*x + a)/(c^2*x^4 + 2*b*c*x^3 +
 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(3/2)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx + bd)^{\frac{3}{2}}}{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^(3/2)/(c*x^2 + b*x + a)^(3/2), x)
```

3.1382 $\int \frac{1}{\sqrt{bd+2cdx}(a+bx+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{4\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{\sqrt{d}\left(b^2-4ac\right)^{3/4}\sqrt{a+bx+cx^2}}-\frac{2\sqrt{bd+2cdx}}{d\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*Sqrt[b*d + 2*c*d*x])/((b^2 - 4*a*c)*d*Sqrt[a + b*x + c*x^2]) - (4*Sqrt[
-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x
]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/((b^2 - 4*a*c)^(3/4)*Sqrt[d]*Sqrt[a
+ b*x + c*x^2])
```

Rubi [A] time = 0.112691, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {687, 691, 689, 221}

$$-\frac{2\sqrt{bd+2cdx}}{d\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{4\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{\sqrt{d}\left(b^2-4ac\right)^{3/4}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[b*d + 2*c*d*x]*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*Sqrt[b*d + 2*c*d*x])/((b^2 - 4*a*c)*d*Sqrt[a + b*x + c*x^2]) - (4*Sqrt[
-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x
]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/((b^2 - 4*a*c)^(3/4)*Sqrt[d]*Sqrt[a
+ b*x + c*x^2])
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
```

```
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2\sqrt{bd+2cdx}}{\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}} - \frac{(2c)\int \frac{1}{\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac}$$

$$= -\frac{2\sqrt{bd+2cdx}}{\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}} - \frac{\left(2c\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\int \frac{1}{\sqrt{bd+2cdx}\sqrt{-\frac{ac}{b^2-4ac}-\frac{bcx}{b^2-4ac}-\frac{c^2x^2}{b^2-4ac}}}\,dx}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$= -\frac{2\sqrt{bd+2cdx}}{\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}} - \frac{\left(4\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{\left(b^2-4ac\right)d^2}}}\,dx, x, \sqrt{bd+2cdx}\right)}{\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}}$$

$$= -\frac{2\sqrt{bd+2cdx}}{\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}} - \frac{4\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{\left(b^2-4ac\right)^{3/4}\sqrt{d}\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 0.0704735, size = 100, normalized size = 0.73

$$-\frac{2\sqrt{d(b+2cx)}\left(2\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)+1\right)}{d\left(b^2-4ac\right)\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[b*d + 2*c*d*x]*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d*(b + 2*c*x)]*(1 + 2*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*H
ypergeometric2F1[1/4, 1/2, 5/4, (b + 2*c*x)^2/(b^2 - 4*a*c)]))/((b^2 - 4*a*
c)*d*Sqrt[a + x*(b + c*x)])
```

Maple [A] time = 0.226, size = 206, normalized size = 1.5

$$2\,\frac{\sqrt{cx^2+bx+a}\sqrt{d\,(2\,cx+b)}}{d\left(4\,ac-b^2\right)\left(2\,c^2x^3+3\,bcx^2+2\,acx+b^2x+ab\right)}\left(\text{EllipticF}\left(1/2\,\sqrt{\frac{b+2\,cx+\sqrt{-4\,ac+b^2}}{\sqrt{-4\,ac+b^2}}}\sqrt{2},\sqrt{2}\right)\sqrt{-4\,ac+b^2}\sqrt{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 2*(EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^
(1/2),2^(1/2))*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2
```

```
)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2
)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)+2*c*x+b)*(c*x^2+b*x+a)^(1/2)*(d*(2*c*x+b
))^(1/2)/(4*a*c-b^2)/d/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,cdx + bd}\left(cx^2 + bx + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*c*d*x + b*d)*(c*x^2 + b*x + a)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,cdx + bd}\sqrt{cx^2 + bx + a}}{2\,c^3dx^5 + 5\,bc^2dx^4 + 4\,\left(b^2c + ac^2\right)dx^3 + a^2bd + \left(b^3 + 6\,abc\right)dx^2 + 2\,\left(ab^2 + a^2c\right)dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(2*c^3*d*x^5 + 5*b*c^2*d
*x^4 + 4*(b^2*c + a*c^2)*d*x^3 + a^2*b*d + (b^3 + 6*a*b*c)*d*x^2 + 2*(a*b^2
 + a^2*c)*d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d\,(b + 2cx)}\,\left(a + bx + cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**(1/2)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral(1/(sqrt(d*(b + 2*c*x))*(a + b*x + c*x**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,cdx + bd}\left(cx^2 + bx + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*c*d*x + b*d)*(c*x^2 + b*x + a)^(3/2)), x)
```

3.1383 $\int \frac{1}{(bd+2cdx)^{5/2}(a+bx+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=184

$$-\frac{20\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{3d^{5/2}\left(b^2-4ac\right)^{7/4}\sqrt{a+bx+cx^2}}-\frac{40c\sqrt{a+bx+cx^2}}{3d\left(b^2-4ac\right)^2(bd+2cdx)^{3/2}}-\frac{2}{d\left(b^2-4ac\right)\sqrt{a+bx+cx^2}(bd$$

```
[Out] -2/((b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(3/2)*Sqrt[a + b*x + c*x^2]) - (40*c*Sq
rt[a + b*x + c*x^2])/(3*(b^2 - 4*a*c)^2*d*(b*d + 2*c*d*x)^(3/2)) - (20*Sqrt
[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*
x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(3*(b^2 - 4*a*c)^(7/4)*d^(5/2)*Sqrt
[a + b*x + c*x^2])
```

Rubi [A] time = 0.143852, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {687, 693, 691, 689, 221}

$$-\frac{20\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{3d^{5/2}\left(b^2-4ac\right)^{7/4}\sqrt{a+bx+cx^2}}-\frac{40c\sqrt{a+bx+cx^2}}{3d\left(b^2-4ac\right)^2(bd+2cdx)^{3/2}}-\frac{2}{d\left(b^2-4ac\right)\sqrt{a+bx+cx^2}(bd+2cdx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^(5/2)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] -2/((b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(3/2)*Sqrt[a + b*x + c*x^2]) - (40*c*Sq
rt[a + b*x + c*x^2])/(3*(b^2 - 4*a*c)^2*d*(b*d + 2*c*d*x)^(3/2)) - (20*Sqrt
[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*
x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(3*(b^2 - 4*a*c)^(7/4)*d^(5/2)*Sqrt
[a + b*x + c*x^2])
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)^{5/2}\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2}{\left(b^2-4ac\right)d(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}} - \frac{(10c)\int \frac{1}{(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac}$$

$$= -\frac{2}{\left(b^2-4ac\right)d(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}} - \frac{40c\sqrt{a+bx+cx^2}}{3\left(b^2-4ac\right)^2 d(bd+2cdx)^{3/2}} - \frac{(10\cdots}{\cdots}$$

$$= -\frac{2}{\left(b^2-4ac\right)d(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}} - \frac{40c\sqrt{a+bx+cx^2}}{3\left(b^2-4ac\right)^2 d(bd+2cdx)^{3/2}} - \frac{\left(10\cdots\right.}{\cdots}$$

$$= -\frac{2}{\left(b^2-4ac\right)d(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}} - \frac{40c\sqrt{a+bx+cx^2}}{3\left(b^2-4ac\right)^2 d(bd+2cdx)^{3/2}} - \frac{\left(20\cdots\right.}{\cdots}$$

$$= -\frac{2}{\left(b^2-4ac\right)d(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}} - \frac{40c\sqrt{a+bx+cx^2}}{3\left(b^2-4ac\right)^2 d(bd+2cdx)^{3/2}} - \frac{20\cdots}{\cdots}$$

Mathematica [C] time = 0.0619869, size = 98, normalized size = 0.53

$$\frac{8\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{1}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{3d\left(b^2-4ac\right)\sqrt{a+x(b+cx)}(d(b+2cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^(5/2)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (8*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*Hypergeometric2F1[-3/4, 3/2,
1/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(3*(b^2 - 4*a*c)*d*(d*(b + 2*c*x))^(3/2)
```

```
*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.23, size = 365, normalized size = 2.

$$-\frac{2}{3 d^3 \left(2 c^2 x^3+3 b c x^2+2 a c x+b^2 x+a b\right) \left(4 a c-b^2\right)^2 (2 c x+b)}\sqrt{d (2 c x+b)}\sqrt{c x^2+b x+a}\left(10 \sqrt{-4 a c+b^2}\sqrt{\frac{b+2 c}{\sqrt{}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -2/3*(d*(2*c*x+b))^(1/2)*(c*x^2+b*x+a)^(1/2)*(10*(-4*a*c+b^2)^(1/2)*((b+2*c
*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(
1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*Ellipt
icF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(
1/2))*x*c+5*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(
1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))
/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*b+20*c^2*x^2+20*b*c*x+8*a*c+3*b
^2)/d^3/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)/(4*a*c-b^2)^2/(2*c*x+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2 c d x+b d)^{\frac{5}{2}} \left(c x^2+b x+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*c*d*x + b*d)^(5/2)*(c*x^2 + b*x + a)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2 c d x+b d}\sqrt{c x^2+b x+a}}{8 c^5 d^3 x^7+28 b c^4 d^3 x^6+2 \left(19 b^2 c^3+8 a c^4\right) d^3 x^5+a^2 b^3 d^3+5 \left(5 b^3 c^2+8 a b c^3\right) d^3 x^4+4 \left(2 b^4 c+9 a b^2 c^2+2 a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(8*c^5*d^3*x^7 + 28*b*c^
4*d^3*x^6 + 2*(19*b^2*c^3 + 8*a*c^4)*d^3*x^5 + a^2*b^3*d^3 + 5*(5*b^3*c^2 +
 8*a*b*c^3)*d^3*x^4 + 4*(2*b^4*c + 9*a*b^2*c^2 + 2*a^2*c^3)*d^3*x^3 + (b^5
+ 14*a*b^3*c + 12*a^2*b*c^2)*d^3*x^2 + 2*(a*b^4 + 3*a^2*b^2*c)*d^3*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d\,(b+2cx))^{\frac{5}{2}}\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**(5/2)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral(1/((d*(b + 2*c*x))**(5/2)*(a + b*x + c*x**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2\,cdx+bd)^{\frac{5}{2}}\left(cx^2+bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((2*c*d*x + b*d)^(5/2)*(c*x^2 + b*x + a)^(3/2)), x)
```

3.1384 $\int \frac{(bd+2cdx)^{9/2}}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=258

$$-\frac{84d^{9/2}\left(b^2-4ac\right)^{7/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{5\sqrt{a+bx+cx^2}}+\frac{84d^{9/2}\left(b^2-4ac\right)^{7/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\cdots\right.\right.}{5\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*d*(b*d + 2*c*d*x)^(7/2))/Sqrt[a + b*x + c*x^2] + (56*c*d^3*(b*d + 2*c*d
*x)^(3/2)*Sqrt[a + b*x + c*x^2])/5 + (84*(b^2 - 4*a*c)^(7/4)*d^(9/2)*Sqrt[-
((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d*x]
/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(5*Sqrt[a + b*x + c*x^2]) - (84*(b^2
- 4*a*c)^(7/4)*d^(9/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellipti
cF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(5*Sqrt[
a + b*x + c*x^2])
```

Rubi [A] time = 0.246077, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {686, 692, 691, 690, 307, 221, 1199, 424}

$$-\frac{84d^{9/2}\left(b^2-4ac\right)^{7/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{5\sqrt{a+bx+cx^2}}+\frac{84d^{9/2}\left(b^2-4ac\right)^{7/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cx\cdots}}{\sqrt[4]{b^2-4ac}\cdots}\right.\right.}{5\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(9/2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*d*(b*d + 2*c*d*x)^(7/2))/Sqrt[a + b*x + c*x^2] + (56*c*d^3*(b*d + 2*c*d
*x)^(3/2)*Sqrt[a + b*x + c*x^2])/5 + (84*(b^2 - 4*a*c)^(7/4)*d^(9/2)*Sqrt[-
((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d*x]
/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(5*Sqrt[a + b*x + c*x^2]) - (84*(b^2
- 4*a*c)^(7/4)*d^(9/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellipti
cF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(5*Sqrt[
a + b*x + c*x^2])
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(bd+2cdx)^{9/2}}{\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2d(bd+2cdx)^{7/2}}{\sqrt{a+bx+cx^2}} + \left(14cd^2\right)\int \frac{(bd+2cdx)^{5/2}}{\sqrt{a+bx+cx^2}}\,dx$$

$$= -\frac{2d(bd+2cdx)^{7/2}}{\sqrt{a+bx+cx^2}} + \frac{56}{5}cd^3(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2} + \frac{1}{5}\left(42c\left(b^2-4ac\right)d^4\right)\int \frac{\sqrt{bd+}}{\sqrt{a+bx}}$$

$$= -\frac{2d(bd+2cdx)^{7/2}}{\sqrt{a+bx+cx^2}} + \frac{56}{5}cd^3(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2} + \frac{\left(42c\left(b^2-4ac\right)d^4\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right.}{5\sqrt{a+bx}}$$

$$= -\frac{2d(bd+2cdx)^{7/2}}{\sqrt{a+bx+cx^2}} + \frac{56}{5}cd^3(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2} + \frac{\left(84\left(b^2-4ac\right)d^3\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)}{5\sqrt{}}$$

$$= -\frac{2d(bd+2cdx)^{7/2}}{\sqrt{a+bx+cx^2}} + \frac{56}{5}cd^3(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2} - \frac{\left(84\left(b^2-4ac\right)^{3/2}d^4\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right.}{5\sqrt{}}$$

$$= -\frac{2d(bd+2cdx)^{7/2}}{\sqrt{a+bx+cx^2}} + \frac{56}{5}cd^3(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2} - \frac{84\left(b^2-4ac\right)^{7/4}d^{9/2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}}{5\sqrt{a+bx}}$$

$$= -\frac{2d(bd+2cdx)^{7/2}}{\sqrt{a+bx+cx^2}} + \frac{56}{5}cd^3(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2} + \frac{84\left(b^2-4ac\right)^{7/4}d^{9/2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}}{5\sqrt{a+bx}}$$

Mathematica [C] time = 0.146354, size = 122, normalized size = 0.47

$$-\frac{8d^3(d(b+2cx))^{3/2}\left(7\left(b^2-4ac\right)\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)-2\left(c\left(cx^2-7a\right)+2b^2+bcx\right)\right)}{5\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(9/2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-8*d^3*(d*(b + 2*c*x))^(3/2)*(-2*(2*b^2 + b*c*x + c*(-7*a + c*x^2)) + 7*(b
^2 - 4*a*c)*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*Hypergeometric2F1[3/
4, 3/2, 7/4, (b + 2*c*x)^2/(b^2 - 4*a*c)]))/(5*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.277, size = 498, normalized size = 1.9

$$-\frac{2\,d^4}{10\,c^2x^3+15\,bcx^2+10\,acx+5\,b^2x+5\,ab}\left(336\sqrt{\frac{b+2\,cx+\sqrt{-4\,ac+b^2}}{\sqrt{-4\,ac+b^2}}}\sqrt{-\frac{2\,cx+b}{\sqrt{-4\,ac+b^2}}}\sqrt{\frac{-b-2\,cx+\sqrt{-4\,ac+b^2}}{\sqrt{-4\,ac+b^2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -2/5*d^4*(336*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*
c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2
```

```
)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/
2))^(1/2)*2^(1/2),2^(1/2))*a^2*c^2-168*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*
c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*
c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2
)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a*b^2*c+21*((b+2*c*x+(-
4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))
^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1
/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))
*b^4-32*c^4*x^4-64*b*c^3*x^3-112*x^2*a*c^3-20*x^2*b^2*c^2-112*b*a*c^2*x+12*
b^3*c*x-28*a*c*b^2+5*b^4)*(c*x^2+b*x+a)^(1/2)*(d*(2*c*x+b))^(1/2)/(2*c^2*x^
3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx + bd)^{\frac{9}{2}}}{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^(9/2)/(c*x^2 + b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(16\,c^4d^4x^4 + 32\,bc^3d^4x^3 + 24\,b^2c^2d^4x^2 + 8\,b^3cd^4x + b^4d^4\right)\sqrt{2\,cdx + bd}\sqrt{cx^2 + bx + a}}{c^2x^4 + 2\,bcx^3 + 2\,abx + \left(b^2 + 2\,ac\right)x^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((16*c^4*d^4*x^4 + 32*b*c^3*d^4*x^3 + 24*b^2*c^2*d^4*x^2 + 8*b^3*c*
d^4*x + b^4*d^4)*sqrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(c^2*x^4 + 2*b*c
*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(9/2)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx + bd)^{\frac{9}{2}}}{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^(9/2)/(c*x^2 + b*x + a)^(3/2), x)
```

3.1385 $\int \frac{(bd+2cdx)^{5/2}}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=219

$$-\frac{12d^{5/2}\left(b^2-4ac\right)^{3/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{\sqrt{a+bx+cx^2}}+\frac{12d^{5/2}\left(b^2-4ac\right)^{3/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{bd}}{\sqrt[4]{b^2}}\right.\right.}{\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*d*(b*d + 2*c*d*x)^(3/2))/Sqrt[a + b*x + c*x^2] + (12*(b^2 - 4*a*c)^(3/4
)*d^(5/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqr
t[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/Sqrt[a + b*x + c*x^2]
 - (12*(b^2 - 4*a*c)^(3/4)*d^(5/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*
c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -
1])/Sqrt[a + b*x + c*x^2]
```

Rubi [A] time = 0.204948, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {686, 691, 690, 307, 221, 1199, 424}

$$-\frac{12d^{5/2}\left(b^2-4ac\right)^{3/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{\sqrt{a+bx+cx^2}}+\frac{12d^{5/2}\left(b^2-4ac\right)^{3/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\right|}{\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(5/2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*d*(b*d + 2*c*d*x)^(3/2))/Sqrt[a + b*x + c*x^2] + (12*(b^2 - 4*a*c)^(3/4
)*d^(5/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqr
t[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/Sqrt[a + b*x + c*x^2]
 - (12*(b^2 - 4*a*c)^(3/4)*d^(5/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*
c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -
1])/Sqrt[a + b*x + c*x^2]
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
```

```
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(bd+2cdx)^{5/2}}{\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2d(bd+2cdx)^{3/2}}{\sqrt{a+bx+cx^2}} + \left(6cd^2\right)\int \frac{\sqrt{bd+2cdx}}{\sqrt{a+bx+cx^2}}\,dx$$

$$= -\frac{2d(bd+2cdx)^{3/2}}{\sqrt{a+bx+cx^2}} + \frac{\left(6cd^2\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\int \frac{\sqrt{bd+2cdx}}{\sqrt{-\frac{ac}{b^2-4ac}-\frac{bcx}{b^2-4ac}-\frac{c^2x^2}{b^2-4ac}}}\,dx}{\sqrt{a+bx+cx^2}}$$

$$= -\frac{2d(bd+2cdx)^{3/2}}{\sqrt{a+bx+cx^2}} + \frac{\left(12d\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{\left(b^2-4ac\right)d^2}}}\,dx, x, \sqrt{bd+2cdx}\right)}{\sqrt{a+bx+cx^2}}$$

$$= -\frac{2d(bd+2cdx)^{3/2}}{\sqrt{a+bx+cx^2}} - \frac{\left(12\sqrt{b^2-4ac}d^2\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{\left(b^2-4ac\right)d^2}}}\,dx, x, \sqrt{bd+2cd\right.}{\sqrt{a+bx+cx^2}}$$

$$= -\frac{2d(bd+2cdx)^{3/2}}{\sqrt{a+bx+cx^2}} - \frac{12\left(b^2-4ac\right)^{3/4}d^{5/2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{\sqrt{a+bx+cx^2}} + \frac{\left(12\sqrt{b\right.}{}$$

$$= -\frac{2d(bd+2cdx)^{3/2}}{\sqrt{a+bx+cx^2}} + \frac{12\left(b^2-4ac\right)^{3/4}d^{5/2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{\sqrt{a+bx+cx^2}} - \frac{12\left(b^2\right.}{}$$

Mathematica [C] time = 0.0730151, size = 88, normalized size = 0.4

$$-\frac{4d(d(b+2cx))^{3/2}\left(2\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)-1\right)}{\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(5/2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-4*d*(d*(b + 2*c*x))^(3/2)*(-1 + 2*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*
c)]*Hypergeometric2F1[3/4, 3/2, 7/4, (b + 2*c*x)^2/(b^2 - 4*a*c)]))/Sqrt[a
+ x*(b + c*x)]
```

Maple [A] time = 0.219, size = 323, normalized size = 1.5

$$2\frac{d^2\sqrt{cx^2+bx+a}\sqrt{d(2cx+b)}}{2c^2x^3+3bcx^2+2acx+b^2x+ab}\left(12\,\mathrm{EllipticE}\left(1/2\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}\sqrt{2},\sqrt{2}\right)ac\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}\sqrt{\cdot}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 2*(12*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)
*2^(1/2),2^(1/2))*a*c*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/
2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4
*a*c+b^2)^(1/2))^(1/2)-3*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*
c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*b^2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*
a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*
a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)-4*c^2*x^2-4*b*c*x-b^2)*d^2*(c*x^2
+b*x+a)^(1/2)*(d*(2*c*x+b))^(1/2)/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(2\,cdx+bd)^{\frac{5}{2}}}{\left(cx^2+bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^(5/2)/(c*x^2 + b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(4\,c^2d^2x^2+4\,bcd^2x+b^2d^2\right)\sqrt{2\,cdx+bd}\sqrt{cx^2+bx+a}}{c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\,ac\right)x^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((4*c^2*d^2*x^2 + 4*b*c*d^2*x + b^2*d^2)*sqrt(2*c*d*x + b*d)*sqrt(c
*x^2 + b*x + a)/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(5/2)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx + bd)^{\frac{5}{2}}}{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^(5/2)/(c*x^2 + b*x + a)^(3/2), x)
```

3.1386 $\int \frac{\sqrt{bd+2cdx}}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=231

$$-\frac{4\sqrt{d}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{\sqrt[4]{b^2-4ac}\sqrt{a+bx+cx^2}}-\frac{2(bd+2cdx)^{3/2}}{d\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{4\sqrt{d}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\right|-1\right)}{\sqrt[4]{b^2-4ac}\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*(b*d + 2*c*d*x)^(3/2))/((b^2 - 4*a*c)*d*Sqrt[a + b*x + c*x^2]) + (4*Sqr
t[d]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d
 + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/((b^2 - 4*a*c)^(1/4)*Sqrt[
a + b*x + c*x^2]) - (4*Sqrt[d]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]
*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/
((b^2 - 4*a*c)^(1/4)*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.206103, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {687, 691, 690, 307, 221, 1199, 424}

$$-\frac{2(bd+2cdx)^{3/2}}{d\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{4\sqrt{d}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\right|-1\right)}{\sqrt[4]{b^2-4ac}\sqrt{a+bx+cx^2}}+\frac{4\sqrt{d}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\right|-1\right)}{\sqrt[4]{b^2-4ac}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*d + 2*c*d*x]/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*(b*d + 2*c*d*x)^(3/2))/((b^2 - 4*a*c)*d*Sqrt[a + b*x + c*x^2]) + (4*Sqr
t[d]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d
 + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/((b^2 - 4*a*c)^(1/4)*Sqrt[
a + b*x + c*x^2]) - (4*Sqrt[d]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]
*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/
((b^2 - 4*a*c)^(1/4)*Sqrt[a + b*x + c*x^2])
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{bd+2cdx}}{\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2(bd+2cdx)^{3/2}}{\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}} + \frac{(2c)\int \frac{\sqrt{bd+2cdx}}{\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac}$$

$$= -\frac{2(bd+2cdx)^{3/2}}{\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}} + \frac{\left(2c\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\int \frac{\sqrt{bd+2cdx}}{\sqrt{-\frac{ac}{b^2-4ac}-\frac{bcx}{b^2-4ac}-\frac{c^2x^2}{b^2-4ac}}}\,dx}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$= -\frac{2(bd+2cdx)^{3/2}}{\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}} + \frac{\left(4\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{\left(b^2-4ac\right)d^2}}}\,dx, x, \sqrt{bd+2cdx}\right)}{\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}}$$

$$= -\frac{2(bd+2cdx)^{3/2}}{\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}} - \frac{\left(4\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{\left(b^2-4ac\right)d^2}}}\,dx, x, \sqrt{bd+2cdx}\right)}{\sqrt{b^2-4ac}\sqrt{a+bx+cx^2}} + -$$

$$= -\frac{2(bd+2cdx)^{3/2}}{\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}} - \frac{4\sqrt{d}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{\sqrt[4]{b^2-4ac}\sqrt{a+bx+cx^2}} + \frac{\left(4\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right.}{}$$

$$= -\frac{2(bd+2cdx)^{3/2}}{\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}} + \frac{4\sqrt{d}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{\sqrt[4]{b^2-4ac}\sqrt{a+bx+cx^2}} - \frac{4\sqrt{d}\sqrt{-\frac{c\left(a+bx+\right.}{b^2-4a}}}{\sqrt[4]{b^2}}$$

Mathematica [C] time = 0.0621653, size = 98, normalized size = 0.42

$$-\frac{8\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}(d(b+2cx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{3d\left(b^2-4ac\right)\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*d + 2*c*d*x]/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-8*(d*(b + 2*c*x))^(3/2)*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*Hyperg
eometric2F1[3/4, 3/2, 7/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(3*(b^2 - 4*a*c)*d
*Sqrt[a + x*(b + c*x)])
```

Maple [A] time = 0.212, size = 332, normalized size = 1.4

$$-2\,\frac{\sqrt{d\,(2\,cx+b)}\sqrt{cx^2+bx+a}}{\left(2\,c^2x^3+3\,bcx^2+2\,acx+b^2x+ab\right)\left(4\,ac-b^2\right)}\left(4\,\text{EllipticE}\left(1/2\sqrt{\frac{b+2\,cx+\sqrt{-4\,ac+b^2}}{\sqrt{-4\,ac+b^2}}}\sqrt{2},\sqrt{2}\right)ac\sqrt{\frac{b+2\,cx+\sqrt{}}{\sqrt{-4\,ac}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -2*(d*(2*c*x+b))^(1/2)*(c*x^2+b*x+a)^(1/2)*(4*EllipticE(1/2*((b+2*c*x+(-4*a
*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a*c*((b+2*c*x+(-4
```

```
*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^
(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)-EllipticE(1/
2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*
b^2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4
*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(
1/2)-4*c^2*x^2-4*b*c*x-b^2)/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*x+a*b)/(4*a*c-
b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,cdx + bd}}{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*c*d*x + b*d)/(c*x^2 + b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,cdx + bd}\sqrt{cx^2 + bx + a}}{c^2x^4 + 2\,bcx^3 + 2\,abx + \left(b^2 + 2\,ac\right)x^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(c^2*x^4 + 2*b*c*x^3 + 2
*a*b*x + (b^2 + 2*a*c)*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\,(b + 2cx)}}{\left(a + bx + cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(1/2)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral(sqrt(d*(b + 2*c*x))/(a + b*x + c*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,cdx + bd}}{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(2*c*d*x + b*d)/(c*x^2 + b*x + a)^(3/2), x)
```

3.1387 $\int \frac{1}{(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=274

$$-\frac{12\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{d^{3/2}\left(b^2-4ac\right)^{5/4}\sqrt{a+bx+cx^2}}+\frac{12\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{d^{3/2}\left(b^2-4ac\right)^{5/4}\sqrt{a+bx+cx^2}}-\frac{24c\sqrt{a+bx}}{d\left(b^2-4ac\right)^2\sqrt{b}}$$

```
[Out] -2/((b^2 - 4*a*c)*d*Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2]) - (24*c*Sqrt
[a + b*x + c*x^2])/((b^2 - 4*a*c)^2*d*Sqrt[b*d + 2*c*d*x]) + (12*Sqrt[-((c*
(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d*x]/((b
^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/((b^2 - 4*a*c)^(5/4)*d^(3/2)*Sqrt[a + b*x
 + c*x^2]) - (12*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[Arc
Sin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/((b^2 - 4*a*c)
^(5/4)*d^(3/2)*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.244904, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {687, 693, 691, 690, 307, 221, 1199, 424}

$$-\frac{12\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{d^{3/2}\left(b^2-4ac\right)^{5/4}\sqrt{a+bx+cx^2}}+\frac{12\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{d^{3/2}\left(b^2-4ac\right)^{5/4}\sqrt{a+bx+cx^2}}-\frac{24c\sqrt{a+bx+cx^2}}{d\left(b^2-4ac\right)^2\sqrt{bd+2cd}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] -2/((b^2 - 4*a*c)*d*Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2]) - (24*c*Sqrt
[a + b*x + c*x^2])/((b^2 - 4*a*c)^2*d*Sqrt[b*d + 2*c*d*x]) + (12*Sqrt[-((c*
(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d*x]/((b
^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/((b^2 - 4*a*c)^(5/4)*d^(3/2)*Sqrt[a + b*x
 + c*x^2]) - (12*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[Arc
Sin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/((b^2 - 4*a*c)
^(5/4)*d^(3/2)*Sqrt[a + b*x + c*x^2])
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
```

```
IntegerQ[(m + 2*p + 3)/2])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2}{\left(b^2-4ac\right)d\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}} - \frac{(6c)\int \frac{1}{(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac}$$

$$= -\frac{2}{\left(b^2-4ac\right)d\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}} - \frac{24c\sqrt{a+bx+cx^2}}{\left(b^2-4ac\right)^2 d\sqrt{bd+2cdx}} + \frac{(6c)\int}{(b^2}$$

$$= -\frac{2}{\left(b^2-4ac\right)d\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}} - \frac{24c\sqrt{a+bx+cx^2}}{\left(b^2-4ac\right)^2 d\sqrt{bd+2cdx}} + \frac{\left(6c\sqrt{}\right.}{}$$

$$= -\frac{2}{\left(b^2-4ac\right)d\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}} - \frac{24c\sqrt{a+bx+cx^2}}{\left(b^2-4ac\right)^2 d\sqrt{bd+2cdx}} + \frac{\left(12\sqrt{}\right.}{}$$

$$= -\frac{2}{\left(b^2-4ac\right)d\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}} - \frac{24c\sqrt{a+bx+cx^2}}{\left(b^2-4ac\right)^2 d\sqrt{bd+2cdx}} - \frac{\left(12\sqrt{}\right.}{}$$

$$= -\frac{2}{\left(b^2-4ac\right)d\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}} - \frac{24c\sqrt{a+bx+cx^2}}{\left(b^2-4ac\right)^2 d\sqrt{bd+2cdx}} - \frac{12\sqrt{-}}{}$$

$$= -\frac{2}{\left(b^2-4ac\right)d\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}} - \frac{24c\sqrt{a+bx+cx^2}}{\left(b^2-4ac\right)^2 d\sqrt{bd+2cdx}} + \frac{12\sqrt{-}}{}$$

Mathematica [C] time = 0.0557143, size = 96, normalized size = 0.35

$$\frac{8\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{d\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\sqrt{d(b+2cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (8*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*Hypergeometric2F1[-1/4, 3/2,
3/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/((b^2 - 4*a*c)*d*Sqrt[d*(b + 2*c*x)]*Sqr
t[a + x*(b + c*x)])
```

Maple [A] time = 0.254, size = 339, normalized size = 1.2

$$2\,\frac{\sqrt{d\left(2\,cx+b\right)}\sqrt{cx^2+bx+a}}{d^2\left(2\,c^2x^3+3\,bcx^2+2\,acx+b^2x+ab\right)\left(4\,ac-b^2\right)^2}\left(12\,\mathrm{EllipticE}\left(1/2\,\sqrt{\frac{b+2\,cx+\sqrt{-4\,ac+b^2}}{\sqrt{-4\,ac+b^2}}}\sqrt{2},\sqrt{2}\right)ac\sqrt{\frac{b+2\,c}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 2*(d*(2*c*x+b))^(1/2)*(c*x^2+b*x+a)^(1/2)*(12*EllipticE(1/2*((b+2*c*x+(-4*a
*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a*c*((b+2*c*x+(-4
*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^
(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)-3*EllipticE(
1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)
)*b^2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(
-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))
^(1/2)-12*c^2*x^2-12*b*c*x-8*a*c-b^2)/d^2/(2*c^2*x^3+3*b*c*x^2+2*a*c*x+b^2*
x+a*b)/(4*a*c-b^2)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2\,cdx + bd)^{\frac{3}{2}}\left(cx^2 + bx + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*c*d*x + b*d)^(3/2)*(c*x^2 + b*x + a)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,cdx + bd}\sqrt{cx^2 + bx + a}}{4\,c^4d^2x^6 + 12\,bc^3d^2x^5 + \left(13\,b^2c^2 + 8\,ac^3\right)d^2x^4 + a^2b^2d^2 + 2\left(3\,b^3c + 8\,abc^2\right)d^2x^3 + \left(b^4 + 10\,ab^2c + 4\,a^2c^2\right)d^2x^2 \cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(4*c^4*d^2*x^6 + 12*b*c^
3*d^2*x^5 + (13*b^2*c^2 + 8*a*c^3)*d^2*x^4 + a^2*b^2*d^2 + 2*(3*b^3*c + 8*a
*b*c^2)*d^2*x^3 + (b^4 + 10*a*b^2*c + 4*a^2*c^2)*d^2*x^2 + 2*(a*b^3 + 2*a^2
*b*c)*d^2*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d\,(b + 2cx))^{\frac{3}{2}}\left(a + bx + cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**(3/2)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral(1/((d*(b + 2*c*x))**(3/2)*(a + b*x + c*x**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2\,cdx + bd)^{\frac{3}{2}}\left(cx^2 + bx + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((2*c*d*x + b*d)^(3/2)*(c*x^2 + b*x + a)^(3/2)), x)
```

3.1388 $\int \frac{1}{(bd+2cdx)^{7/2}\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=325

$$-\frac{84\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{5d^{7/2}\left(b^2-4ac\right)^{9/4}\sqrt{a+bx+cx^2}}-\frac{168c\sqrt{a+bx+cx^2}}{5d^3\left(b^2-4ac\right)^3\sqrt{bd+2cdx}}+\frac{84\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cx}}{\sqrt[4]{b^2-4ac}}\right.\right.}{5d^{7/2}\left(b^2-4ac\right)^{9/4}\sqrt{a+bx+}}$$

```
[Out] -2/((b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(5/2)*Sqrt[a + b*x + c*x^2]) - (56*c*Sq
rt[a + b*x + c*x^2])/(5*(b^2 - 4*a*c)^2*d*(b*d + 2*c*d*x)^(5/2)) - (168*c*S
qrt[a + b*x + c*x^2])/(5*(b^2 - 4*a*c)^3*d^3*Sqrt[b*d + 2*c*d*x]) + (84*Sqr
t[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d
*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(5*(b^2 - 4*a*c)^(9/4)*d^(7/2)*Sqr
t[a + b*x + c*x^2]) - (84*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Elli
pticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(5*(b
^2 - 4*a*c)^(9/4)*d^(7/2)*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.293854, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {687, 693, 691, 690, 307, 221, 1199, 424}

$$-\frac{168c\sqrt{a+bx+cx^2}}{5d^3\left(b^2-4ac\right)^3\sqrt{bd+2cdx}}-\frac{84\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{5d^{7/2}\left(b^2-4ac\right)^{9/4}\sqrt{a+bx+cx^2}}+\frac{84\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right.}{5d^{7/2}\left(b^2-4ac\right)^{9/4}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^(7/2)*(a + b*x + c*x^2)^(3/2)),x]
```



```
[Out] -2/((b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(5/2)*Sqrt[a + b*x + c*x^2]) - (56*c*Sq
rt[a + b*x + c*x^2])/(5*(b^2 - 4*a*c)^2*d*(b*d + 2*c*d*x)^(5/2)) - (168*c*S
qrt[a + b*x + c*x^2])/(5*(b^2 - 4*a*c)^3*d^3*Sqrt[b*d + 2*c*d*x]) + (84*Sqr
t[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d
*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(5*(b^2 - 4*a*c)^(9/4)*d^(7/2)*Sqr
t[a + b*x + c*x^2]) - (84*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Elli
pticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(5*(b
^2 - 4*a*c)^(9/4)*d^(7/2)*Sqrt[a + b*x + c*x^2])
```


Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
```

```
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)^{7/2}\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2}{\left(b^2-4ac\right)d(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}} - \frac{(14c)\int \frac{1}{(bd+2cdx)^{7/2}\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac}$$

$$= -\frac{2}{\left(b^2-4ac\right)d(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}} - \frac{56c\sqrt{a+bx+cx^2}}{5\left(b^2-4ac\right)^2 d(bd+2cdx)^{5/2}} - \frac{(42}{}$$

$$= -\frac{2}{\left(b^2-4ac\right)d(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}} - \frac{56c\sqrt{a+bx+cx^2}}{5\left(b^2-4ac\right)^2 d(bd+2cdx)^{5/2}} - \frac{}{5(}$$

$$= -\frac{2}{\left(b^2-4ac\right)d(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}} - \frac{56c\sqrt{a+bx+cx^2}}{5\left(b^2-4ac\right)^2 d(bd+2cdx)^{5/2}} - \frac{}{5(}$$

$$= -\frac{2}{\left(b^2-4ac\right)d(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}} - \frac{56c\sqrt{a+bx+cx^2}}{5\left(b^2-4ac\right)^2 d(bd+2cdx)^{5/2}} - \frac{}{5(}$$

$$= -\frac{2}{\left(b^2-4ac\right)d(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}} - \frac{56c\sqrt{a+bx+cx^2}}{5\left(b^2-4ac\right)^2 d(bd+2cdx)^{5/2}} - \frac{}{5(}$$

$$= -\frac{2}{\left(b^2-4ac\right)d(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}} - \frac{56c\sqrt{a+bx+cx^2}}{5\left(b^2-4ac\right)^2 d(bd+2cdx)^{5/2}} - \frac{}{5(}$$

$$= -\frac{2}{\left(b^2-4ac\right)d(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}} - \frac{56c\sqrt{a+bx+cx^2}}{5\left(b^2-4ac\right)^2 d(bd+2cdx)^{5/2}} - \frac{}{5(}$$

Mathematica [C] time = 0.0699083, size = 98, normalized size = 0.3

$$\frac{8\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\,{}_2F_1\left(-\frac{5}{4},\frac{3}{2};-\frac{1}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{5d\left(b^2-4ac\right)\sqrt{a+x(b+cx)}(d(b+2cx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^(7/2)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (8*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*Hypergeometric2F1[-5/4, 3/2,
-1/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(5*(b^2 - 4*a*c)*d*(d*(b + 2*c*x))^(5/2
)*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.237, size = 876, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -2/5*(336*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(
1/2)*2^(1/2),2^(1/2))*x^2*a*c^3*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^
(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^
(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)-84*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^2*b^2*c^2*((b+2*c*x+(-4*a
*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1
/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)+336*EllipticE(
1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)
)*x*a*b*c^2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*
x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^
(1/2))^(1/2)-84*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1
/2))^(1/2)*2^(1/2),2^(1/2))*x*b^3*c*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b
^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b
^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)+84*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a
*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a
*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a*b^2*c-21*((b+2*c*x+(
-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2)
)^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(
1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)
)*b^4-336*c^4*x^4-672*b*c^3*x^3-224*x^2*a*c^3-448*x^2*b^2*c^2-224*b*a*c^2*x
-112*b^3*c*x+32*a^2*c^2-72*a*c*b^2-5*b^4)*(d*(2*c*x+b))^(1/2)/d^4/(c*x^2+b*
x+a)^(1/2)/(2*c*x+b)^3/(4*a*c-b^2)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2\,cdx+bd)^{\frac{7}{2}}\left(cx^2+bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*c*d*x + b*d)^(7/2)*(c*x^2 + b*x + a)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,cdx+bd}}{16\,c^6d^4x^8+64\,bc^5d^4x^7+8\left(13\,b^2c^4+4\,ac^5\right)d^4x^6+a^2b^4d^4+8\left(11\,b^3c^3+12\,abc^4\right)d^4x^5+\left(41\,b^4c^2+112\,ab^2c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(16*c^6*d^4*x^8 + 64*b*c
^5*d^4*x^7 + 8*(13*b^2*c^4 + 4*a*c^5)*d^4*x^6 + a^2*b^4*d^4 + 8*(11*b^3*c^3
 + 12*a*b*c^4)*d^4*x^5 + (41*b^4*c^2 + 112*a*b^2*c^3 + 16*a^2*c^4)*d^4*x^4
+ 2*(5*b^5*c + 32*a*b^3*c^2 + 16*a^2*b*c^3)*d^4*x^3 + (b^6 + 18*a*b^4*c + 2
4*a^2*b^2*c^2)*d^4*x^2 + 2*(a*b^5 + 4*a^2*b^3*c)*d^4*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**(7/2)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2\,cdx+bd)^{\frac{7}{2}}\left(cx^2+bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((2*c*d*x + b*d)^(7/2)*(c*x^2 + b*x + a)^(3/2)), x)
```

3.1389 $\int \frac{(bd+2cdx)^{15/2}}{\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=247

$$\frac{520cd^{15/2}\left(b^2-4ac\right)^{9/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{7\sqrt{a+bx+cx^2}}+\frac{1040}{7}c^2d^7\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\sqrt{bd+2}$$

```
[Out] (-2*d*(b*d + 2*c*d*x)^(13/2))/(3*(a + b*x + c*x^2)^(3/2)) - (52*c*d^3*(b*d
+ 2*c*d*x)^(9/2))/(3*Sqrt[a + b*x + c*x^2]) + (1040*c^2*(b^2 - 4*a*c)*d^7*S
qrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/7 + (624*c^2*d^5*(b*d + 2*c*d*x)^
(5/2)*Sqrt[a + b*x + c*x^2])/7 + (520*c*(b^2 - 4*a*c)^(9/4)*d^(15/2)*Sqrt[-
((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]
/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(7*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.202617, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {686, 692, 691, 689, 221}

$$\frac{1040}{7}c^2d^7\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}+\frac{520cd^{15/2}\left(b^2-4ac\right)^{9/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{7\sqrt{a+bx+cx^2}}+6$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(15/2)/(a + b*x + c*x^2)^(5/2),x]
```


```
[Out] (-2*d*(b*d + 2*c*d*x)^(13/2))/(3*(a + b*x + c*x^2)^(3/2)) - (52*c*d^3*(b*d
+ 2*c*d*x)^(9/2))/(3*Sqrt[a + b*x + c*x^2]) + (1040*c^2*(b^2 - 4*a*c)*d^7*S
qrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2])/7 + (624*c^2*d^5*(b*d + 2*c*d*x)^
(5/2)*Sqrt[a + b*x + c*x^2])/7 + (520*c*(b^2 - 4*a*c)^(9/4)*d^(15/2)*Sqrt[-
((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]
/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(7*Sqrt[a + b*x + c*x^2])
```


Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(bd+2cdx)^{15/2}}{\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2d(bd+2cdx)^{13/2}}{3\left(a+bx+cx^2\right)^{3/2}}+\frac{1}{3}\left(26cd^2\right)\int \frac{(bd+2cdx)^{11/2}}{\left(a+bx+cx^2\right)^{3/2}}\,dx$$

$$= -\frac{2d(bd+2cdx)^{13/2}}{3\left(a+bx+cx^2\right)^{3/2}}-\frac{52cd^3(bd+2cdx)^{9/2}}{3\sqrt{a+bx+cx^2}}+\left(156c^2d^4\right)\int \frac{(bd+2cdx)^{7/2}}{\sqrt{a+bx+cx^2}}\,dx$$

$$= -\frac{2d(bd+2cdx)^{13/2}}{3\left(a+bx+cx^2\right)^{3/2}}-\frac{52cd^3(bd+2cdx)^{9/2}}{3\sqrt{a+bx+cx^2}}+\frac{624}{7}c^2d^5(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}+\frac{1}{7}\left(780c^2\right.$$

$$= -\frac{2d(bd+2cdx)^{13/2}}{3\left(a+bx+cx^2\right)^{3/2}}-\frac{52cd^3(bd+2cdx)^{9/2}}{3\sqrt{a+bx+cx^2}}+\frac{1040}{7}c^2\left(b^2-4ac\right)d^7\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}+$$

$$= -\frac{2d(bd+2cdx)^{13/2}}{3\left(a+bx+cx^2\right)^{3/2}}-\frac{52cd^3(bd+2cdx)^{9/2}}{3\sqrt{a+bx+cx^2}}+\frac{1040}{7}c^2\left(b^2-4ac\right)d^7\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}+$$

$$= -\frac{2d(bd+2cdx)^{13/2}}{3\left(a+bx+cx^2\right)^{3/2}}-\frac{52cd^3(bd+2cdx)^{9/2}}{3\sqrt{a+bx+cx^2}}+\frac{1040}{7}c^2\left(b^2-4ac\right)d^7\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}+$$

$$= -\frac{2d(bd+2cdx)^{13/2}}{3\left(a+bx+cx^2\right)^{3/2}}-\frac{52cd^3(bd+2cdx)^{9/2}}{3\sqrt{a+bx+cx^2}}+\frac{1040}{7}c^2\left(b^2-4ac\right)d^7\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}+$$

Mathematica [C] time = 0.324416, size = 252, normalized size = 1.02

$$2d^7\sqrt{d(b+2cx)}\left(16b^2c^2\left(156a^2+117acx^2+116c^2x^4\right)+32bc^3x\left(-273a^2-104acx^2+36c^2x^4\right)-32c^3\left(273a^2cx^2+195a^3\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(15/2)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*d^7*Sqrt[d*(b + 2*c*x)]*(-7*b^6 - 266*b^5*c*x + 16*b^3*c^2*x*(221*a + 11
2*c*x^2) + 2*b^4*c*(-91*a + 219*c*x^2) + 32*b*c^3*x*(-273*a^2 - 104*a*c*x^2
 + 36*c^2*x^4) + 16*b^2*c^2*(156*a^2 + 117*a*c*x^2 + 116*c^2*x^4) - 32*c^3*
(195*a^3 + 273*a^2*c*x^2 + 52*a*c^2*x^4 - 12*c^3*x^6) + 780*c*(b^2 - 4*a*c)
^2*(a + x*(b + c*x))*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*Hypergeomet
ric2F1[1/4, 1/2, 5/4, (b + 2*c*x)^2/(b^2 - 4*a*c)]))/(21*(a + x*(b + c*x))^
(3/2))
```

Maple [B] time = 0.313, size = 1473, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(15/2)/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 2/21*(6240*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^
(1/2)*2^(1/2),2^(1/2))*x^2*a^2*c^4*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2
)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((
-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)+390*EllipticF(1/2*((
b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^2*
b^4*c^2*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
)^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2)
)/(-4*a*c+b^2)^(1/2))^(1/2)+390*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))
/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x*b^5*c*(-4*a*c+b^2)^(1/2)*((b+
2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2
)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)-832
0*x^4*a*b*c^5+416*x^3*a*b^2*c^4-26208*x^2*a^2*b*c^4+8944*x^2*a*b^3*c^3-3744
*x*a^2*b^2*c^3+3172*x*a*b^4*c^2+6240*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x*a^2*b*c^3*(-4*a*c+b^2)^(
1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-
4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^
(1/2)-3120*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^
(1/2)*2^(1/2),2^(1/2))*x*a*b^3*c^2*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2
)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((
-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)-3120*EllipticF(1/2*(
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^2
*a*b^2*c^3*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1
/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(1/2))^(1/2)-7*b^7-6240*a^3*b*c^3+2496*a^2*b^3*c^2-182*a*
b^5*c+390*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/
2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(
-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a*b^4*c+6240*(-4*a*c+b^2)^(1/2)*(
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+
b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*
EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/
2),2^(1/2))*a^3*c^3+2688*x^6*b*c^6-3120*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*
c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/
2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*(
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a^2
*b^2*c^2-3328*x^5*a*c^6+4864*x^5*b^2*c^5+5440*x^4*b^3*c^4-17472*x^3*a^2*c^5
+2668*x^3*b^4*c^3-94*x^2*b^5*c^2-12480*x*a^3*c^4-280*x*b^6*c+768*x^7*c^7)*d
^7*(d*(2*c*x+b))^(1/2)/(2*c*x+b)/(c*x^2+b*x+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx+bd)^{\frac{15}{2}}}{\left(cx^2+bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(15/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^(15/2)/(c*x^2 + b*x + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(128\,c^7d^7x^7+448\,bc^6d^7x^6+672\,b^2c^5d^7x^5+560\,b^3c^4d^7x^4+280\,b^4c^3d^7x^3+84\,b^5c^2d^7x^2+14\,b^6cd^7x+b^7d^7\right)\sqrt{2}}{c^3x^6+3\,bc^2x^5+3\left(b^2c+ac^2\right)x^4+3\,a^2bx+\left(b^3+6\,abc\right)x^3+a^3+3\left(ab^2+a^2c\right)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(15/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((128*c^7*d^7*x^7 + 448*b*c^6*d^7*x^6 + 672*b^2*c^5*d^7*x^5 + 560*b
^3*c^4*d^7*x^4 + 280*b^4*c^3*d^7*x^3 + 84*b^5*c^2*d^7*x^2 + 14*b^6*c*d^7*x
+ b^7*d^7)*sqrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(c^3*x^6 + 3*b*c^2*x^5
 + 3*(b^2*c + a*c^2)*x^4 + 3*a^2*b*x + (b^3 + 6*a*b*c)*x^3 + a^3 + 3*(a*b^2
 + a^2*c)*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(15/2)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx+bd)^{\frac{15}{2}}}{\left(cx^2+bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(15/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^(15/2)/(c*x^2 + b*x + a)^(5/2), x)
```

3.1390 $\int \frac{(bd+2cdx)^{11/2}}{\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=196

$$\frac{40cd^{11/2}\left(b^2-4ac\right)^{5/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{\sqrt{a+bx+cx^2}}+80c^2d^5\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}-\frac{12cd^3(bd}{\sqrt{a+}}$$

```
[Out] (-2*d*(b*d + 2*c*d*x)^(9/2))/(3*(a + b*x + c*x^2)^(3/2)) - (12*c*d^3*(b*d +
 2*c*d*x)^(5/2))/Sqrt[a + b*x + c*x^2] + 80*c^2*d^5*Sqrt[b*d + 2*c*d*x]*Sqr
t[a + b*x + c*x^2] + (40*c*(b^2 - 4*a*c)^(5/4)*d^(11/2)*Sqrt[-((c*(a + b*x
+ c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*
c)^(1/4)*Sqrt[d])], -1])/Sqrt[a + b*x + c*x^2]
```

Rubi [A] time = 0.166264, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {686, 692, 691, 689, 221}

$$\frac{40cd^{11/2}\left(b^2-4ac\right)^{5/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{\sqrt{a+bx+cx^2}}+80c^2d^5\sqrt{a+bx+cx^2}\sqrt{bd+2cdx}-\frac{12cd^3(bd+2cdx}{\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(11/2)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*d*(b*d + 2*c*d*x)^(9/2))/(3*(a + b*x + c*x^2)^(3/2)) - (12*c*d^3*(b*d +
 2*c*d*x)^(5/2))/Sqrt[a + b*x + c*x^2] + 80*c^2*d^5*Sqrt[b*d + 2*c*d*x]*Sqr
t[a + b*x + c*x^2] + (40*c*(b^2 - 4*a*c)^(5/4)*d^(11/2)*Sqrt[-((c*(a + b*x
+ c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*
c)^(1/4)*Sqrt[d])], -1])/Sqrt[a + b*x + c*x^2]
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(bd+2cdx)^{11/2}}{\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2d(bd+2cdx)^{9/2}}{3\left(a+bx+cx^2\right)^{3/2}} + \left(6cd^2\right)\int \frac{(bd+2cdx)^{7/2}}{\left(a+bx+cx^2\right)^{3/2}}\,dx$$

$$= -\frac{2d(bd+2cdx)^{9/2}}{3\left(a+bx+cx^2\right)^{3/2}} - \frac{12cd^3(bd+2cdx)^{5/2}}{\sqrt{a+bx+cx^2}} + \left(60c^2d^4\right)\int \frac{(bd+2cdx)^{3/2}}{\sqrt{a+bx+cx^2}}\,dx$$

$$= -\frac{2d(bd+2cdx)^{9/2}}{3\left(a+bx+cx^2\right)^{3/2}} - \frac{12cd^3(bd+2cdx)^{5/2}}{\sqrt{a+bx+cx^2}} + 80c^2d^5\sqrt{bd+2cdx}\sqrt{a+bx+cx^2} + \left(20c^2\left(b^2-4\right.\right.$$

$$= -\frac{2d(bd+2cdx)^{9/2}}{3\left(a+bx+cx^2\right)^{3/2}} - \frac{12cd^3(bd+2cdx)^{5/2}}{\sqrt{a+bx+cx^2}} + 80c^2d^5\sqrt{bd+2cdx}\sqrt{a+bx+cx^2} + \frac{\left(20c^2\left(b^2-4\right.\right.}{}$$

$$= -\frac{2d(bd+2cdx)^{9/2}}{3\left(a+bx+cx^2\right)^{3/2}} - \frac{12cd^3(bd+2cdx)^{5/2}}{\sqrt{a+bx+cx^2}} + 80c^2d^5\sqrt{bd+2cdx}\sqrt{a+bx+cx^2} + \frac{\left(40c\left(b^2-4a\right.\right.}{}$$

$$= -\frac{2d(bd+2cdx)^{9/2}}{3\left(a+bx+cx^2\right)^{3/2}} - \frac{12cd^3(bd+2cdx)^{5/2}}{\sqrt{a+bx+cx^2}} + 80c^2d^5\sqrt{bd+2cdx}\sqrt{a+bx+cx^2} + \frac{40c\left(b^2-4ac\right.}{}$$

Mathematica [C] time = 0.228164, size = 201, normalized size = 1.03

$$\frac{2d^5\sqrt{d(b+2cx)}\left(-60c\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\left(4a^2c+a\left(-b^2+4bcx+4c^2x^2\right)-b^2x(b+cx)\right)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)+8c^2\left(15a^2+21a\right.\right.}{3(a+x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(11/2)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*d^5*Sqrt[d*(b + 2*c*x)]*(-b^4 - 26*b^3*c*x + 6*b^2*c*(-3*a + c*x^2) + 8*
b*c^2*x*(21*a + 8*c*x^2) + 8*c^2*(15*a^2 + 21*a*c*x^2 + 4*c^2*x^4) - 60*c*S
```

```
qrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*(4*a^2*c - b^2*x*(b + c*x) + a*(-
b^2 + 4*b*c*x + 4*c^2*x^2))*Hypergeometric2F1[1/4, 1/2, 5/4, (b + 2*c*x)^2/
(b^2 - 4*a*c)]))/(3*(a + x*(b + c*x))^(3/2))
```

Maple [B] time = 0.314, size = 958, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(11/2)/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] -2/3*(120*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(
1/2)*2^(1/2),2^(1/2))*x^2*a*c^3*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(1/2))^(1/2)*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)-30*EllipticF(1/2*((b+2*
c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^2*b^2*
c^2*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-4*a*c+b^2)^(
1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-
4*a*c+b^2)^(1/2))^(1/2)+120*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4
*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x*a*b*c^2*((-b-2*c*x+(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)-30*El
lipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)
,2^(1/2))*x*b^3*c*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*
(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*
(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)-64*x^5*c^5+120*((b+2*c*x+(-4*a*c+b^2)
^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-
b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c
*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b
^2)^(1/2)*a^2*c^2-30*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2
)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*
a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b
^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2)^(1/2)*a*b^2*c-160*x^4*b*c^4-
336*x^3*a*c^4-76*x^3*b^2*c^3-504*x^2*a*b*c^3+46*x^2*b^3*c^2-240*x*a^2*c^3-1
32*x*a*b^2*c^2+28*x*b^4*c-120*a^2*b*c^2+18*a*b^3*c+b^5)*d^5*(d*(2*c*x+b))^(
1/2)/(2*c*x+b)/(c*x^2+b*x+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx + bd)^{\frac{11}{2}}}{\left(cx^2 + bx + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(11/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^(11/2)/(c*x^2 + b*x + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(32\,c^5d^5x^5 + 80\,bc^4d^5x^4 + 80\,b^2c^3d^5x^3 + 40\,b^3c^2d^5x^2 + 10\,b^4cd^5x + b^5d^5\right)\sqrt{2\,cdx + bd}\sqrt{cx^2 + bx + a}}{c^3x^6 + 3\,bc^2x^5 + 3\left(b^2c + ac^2\right)x^4 + 3\,a^2bx + \left(b^3 + 6\,abc\right)x^3 + a^3 + 3\left(ab^2 + a^2c\right)x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(11/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((32*c^5*d^5*x^5 + 80*b*c^4*d^5*x^4 + 80*b^2*c^3*d^5*x^3 + 40*b^3*c
^2*d^5*x^2 + 10*b^4*c*d^5*x + b^5*d^5)*sqrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x
 + a)/(c^3*x^6 + 3*b*c^2*x^5 + 3*(b^2*c + a*c^2)*x^4 + 3*a^2*b*x + (b^3 + 6
*a*b*c)*x^3 + a^3 + 3*(a*b^2 + a^2*c)*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(11/2)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx + bd)^{\frac{11}{2}}}{\left(cx^2 + bx + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(11/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^(11/2)/(c*x^2 + b*x + a)^(5/2), x)
```

3.1391 $\int \frac{(bd+2cdx)^{7/2}}{\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=165

$$\frac{40cd^{7/2}\sqrt[4]{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{3\sqrt{a+bx+cx^2}}-\frac{20cd^3\sqrt{bd+2cdx}}{3\sqrt{a+bx+cx^2}}-\frac{2d(bd+2cdx)^{5/2}}{3\left(a+bx+cx^2\right)^{3/2}}$$

```
[Out] (-2*d*(b*d + 2*c*d*x)^(5/2))/(3*(a + b*x + c*x^2)^(3/2)) - (20*c*d^3*Sqrt[b
*d + 2*c*d*x])/(3*Sqrt[a + b*x + c*x^2]) + (40*c*(b^2 - 4*a*c)^(1/4)*d^(7/2
)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d +
2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(3*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.136209, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {686, 691, 689, 221}

$$\frac{40cd^{7/2}\sqrt[4]{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{3\sqrt{a+bx+cx^2}}-\frac{20cd^3\sqrt{bd+2cdx}}{3\sqrt{a+bx+cx^2}}-\frac{2d(bd+2cdx)^{5/2}}{3\left(a+bx+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(7/2)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*d*(b*d + 2*c*d*x)^(5/2))/(3*(a + b*x + c*x^2)^(3/2)) - (20*c*d^3*Sqrt[b
*d + 2*c*d*x])/(3*Sqrt[a + b*x + c*x^2]) + (40*c*(b^2 - 4*a*c)^(1/4)*d^(7/2
)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d +
2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(3*Sqrt[a + b*x + c*x^2])
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
```

```
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(bd+2cdx)^{7/2}}{\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2d(bd+2cdx)^{5/2}}{3\left(a+bx+cx^2\right)^{3/2}}+\frac{1}{3}\left(10cd^2\right)\int \frac{(bd+2cdx)^{3/2}}{\left(a+bx+cx^2\right)^{3/2}}\,dx$$

$$= -\frac{2d(bd+2cdx)^{5/2}}{3\left(a+bx+cx^2\right)^{3/2}}-\frac{20cd^3\sqrt{bd+2cdx}}{3\sqrt{a+bx+cx^2}}+\frac{1}{3}\left(20c^2d^4\right)\int \frac{1}{\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}\,dx$$

$$= -\frac{2d(bd+2cdx)^{5/2}}{3\left(a+bx+cx^2\right)^{3/2}}-\frac{20cd^3\sqrt{bd+2cdx}}{3\sqrt{a+bx+cx^2}}+\frac{\left(20c^2d^4\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\int \frac{1}{\sqrt{bd+2cdx}\sqrt{-\frac{ac}{b^2-4ac}-\frac{bcx}{b^2-4ac}-\frac{c}{b^2}}}}{3\sqrt{a+bx+cx^2}}$$

$$= -\frac{2d(bd+2cdx)^{5/2}}{3\left(a+bx+cx^2\right)^{3/2}}-\frac{20cd^3\sqrt{bd+2cdx}}{3\sqrt{a+bx+cx^2}}+\frac{\left(40cd^3\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{\left(b^2-4ac\right)d^2}}}\,dx,x,\right.}{3\sqrt{a+bx+cx^2}}$$

$$= -\frac{2d(bd+2cdx)^{5/2}}{3\left(a+bx+cx^2\right)^{3/2}}-\frac{20cd^3\sqrt{bd+2cdx}}{3\sqrt{a+bx+cx^2}}+\frac{40c\sqrt[4]{b^2-4ac}\,d^{7/2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{}}\right.\right.}{3\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 0.132878, size = 122, normalized size = 0.74

$$-\frac{2d^3\sqrt{d(b+2cx)}\left(-20c(a+x(b+cx))\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)+2c\left(5a+7cx^2\right)+b^2+14bcx\right)}{3(a+x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(7/2)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*d^3*Sqrt[d*(b + 2*c*x)]*(b^2 + 14*b*c*x + 2*c*(5*a + 7*c*x^2) - 20*c*(a
 + x*(b + c*x))*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*Hypergeometric2F
1[1/4, 1/2, 5/4, (b + 2*c*x)^2/(b^2 - 4*a*c)]))/(3*(a + x*(b + c*x))^(3/2))
```

Maple [B] time = 0.286, size = 479, normalized size = 2.9

$$\frac{2\,d^3}{6\,cx+3\,b}\left(10\,\text{EllipticF}\left(1/2\,\sqrt{\frac{b+2\,cx+\sqrt{-4\,ac+b^2}}{\sqrt{-4\,ac+b^2}}}\sqrt{2},\sqrt{2}\right)x^2c^2\sqrt{\frac{b+2\,cx+\sqrt{-4\,ac+b^2}}{\sqrt{-4\,ac+b^2}}}\sqrt{-\frac{2\,cx+b}{\sqrt{-4\,ac+b^2}}}\sqrt{\frac{-b-2\,c}{\sqrt{}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 2/3*(10*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/
2)*2^(1/2),2^(1/2))*x^2*c^2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2
))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2
))/(-4*a*c+b^2)^(1/2))^(1/2)*(-4*a*c+b^2)^(1/2)+10*EllipticF(1/2*((b+2*c*x+
(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x*b*c*((b+2*
c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^
(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-4*a
*c+b^2)^(1/2)+10*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-
(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^
(1/2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2)^(1/2)*a*c-28*c^3*x^3-42*b*c^2*x^
2-20*x*a*c^2-16*x*b^2*c-10*a*b*c-b^3)*d^3*(d*(2*c*x+b))^(1/2)/(2*c*x+b)/(c*
x^2+b*x+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx+bd)^{\frac{7}{2}}}{\left(cx^2+bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^(7/2)/(c*x^2 + b*x + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(8\,c^3d^3x^3+12\,bc^2d^3x^2+6\,b^2cd^3x+b^3d^3\right)\sqrt{2\,cdx+bd}\sqrt{cx^2+bx+a}}{c^3x^6+3\,bc^2x^5+3\,\left(b^2c+ac^2\right)x^4+3\,a^2bx+\left(b^3+6\,abc\right)x^3+a^3+3\,\left(ab^2+a^2c\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((8*c^3*d^3*x^3 + 12*b*c^2*d^3*x^2 + 6*b^2*c*d^3*x + b^3*d^3)*sqrt(
2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(c^3*x^6 + 3*b*c^2*x^5 + 3*(b^2*c + a*
c^2)*x^4 + 3*a^2*b*x + (b^3 + 6*a*b*c)*x^3 + a^3 + 3*(a*b^2 + a^2*c)*x^2),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(7/2)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx + bd)^{\frac{7}{2}}}{\left(cx^2 + bx + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(7/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^(7/2)/(c*x^2 + b*x + a)^(5/2), x)
```

3.1392 $\int \frac{(bd+2cdx)^{3/2}}{\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=173

$$-\frac{8cd^{3/2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{3\left(b^2-4ac\right)^{3/4}\sqrt{a+bx+cx^2}}-\frac{4cd\sqrt{bd+2cdx}}{3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2d\sqrt{bd+2cdx}}{3\left(a+bx+cx^2\right)^{3/2}}$$

```
[Out] (-2*d*Sqrt[b*d + 2*c*d*x])/(3*(a + b*x + c*x^2)^(3/2)) - (4*c*d*Sqrt[b*d +
2*c*d*x])/(3*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) - (8*c*d^(3/2)*Sqrt[-((c*
(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b
^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(3*(b^2 - 4*a*c)^(3/4)*Sqrt[a + b*x + c*x
^2])
```

Rubi [A] time = 0.138313, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {686, 687, 691, 689, 221}

$$-\frac{8cd^{3/2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{3\left(b^2-4ac\right)^{3/4}\sqrt{a+bx+cx^2}}-\frac{4cd\sqrt{bd+2cdx}}{3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2d\sqrt{bd+2cdx}}{3\left(a+bx+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(3/2)/(a + b*x + c*x^2)^(5/2),x]
```


```
[Out] (-2*d*Sqrt[b*d + 2*c*d*x])/(3*(a + b*x + c*x^2)^(3/2)) - (4*c*d*Sqrt[b*d +
2*c*d*x])/(3*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) - (8*c*d^(3/2)*Sqrt[-((c*
(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b
^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(3*(b^2 - 4*a*c)^(3/4)*Sqrt[a + b*x + c*x
^2])
```


Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
```

```
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(bd+2cdx)^{3/2}}{\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2d\sqrt{bd+2cdx}}{3\left(a+bx+cx^2\right)^{3/2}}+\frac{1}{3}\left(2cd^2\right)\int \frac{1}{\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2}}\,dx$$

$$= -\frac{2d\sqrt{bd+2cdx}}{3\left(a+bx+cx^2\right)^{3/2}}-\frac{4cd\sqrt{bd+2cdx}}{3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{\left(4c^2d^2\right)\int \frac{1}{\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}\,dx}{3\left(b^2-4ac\right)}$$

$$= -\frac{2d\sqrt{bd+2cdx}}{3\left(a+bx+cx^2\right)^{3/2}}-\frac{4cd\sqrt{bd+2cdx}}{3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{\left(4c^2d^2\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\int \frac{1}{\sqrt{bd+2cdx}\sqrt{-\frac{ac}{b^2-4ac}-}}}{3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$= -\frac{2d\sqrt{bd+2cdx}}{3\left(a+bx+cx^2\right)^{3/2}}-\frac{4cd\sqrt{bd+2cdx}}{3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{\left(8cd\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{\left(b^2-4ac\right)d^2}}}\right.}{3\left(b^2-4ac\right)\sqrt{a+bx+c}}$$

$$= -\frac{2d\sqrt{bd+2cdx}}{3\left(a+bx+cx^2\right)^{3/2}}-\frac{4cd\sqrt{bd+2cdx}}{3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{8cd^{3/2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\right.}{3\left(b^2-4ac\right)^{3/4}\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 0.16264, size = 129, normalized size = 0.75

$$-\frac{2d\sqrt{d(b+2cx)}\left(4c(a+x(b+cx))\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)+2c\left(cx^2-a\right)+b^2+2bcx\right)}{3\left(b^2-4ac\right)(a+x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(3/2)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*d*Sqrt[d*(b + 2*c*x)]*(b^2 + 2*b*c*x + 2*c*(-a + c*x^2) + 4*c*(a + x*(b
 + c*x))*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*Hypergeometric2F1[1/4,
1/2, 5/4, (b + 2*c*x)^2/(b^2 - 4*a*c)]))/(3*(b^2 - 4*a*c)*(a + x*(b + c*x))
^(3/2))
```

Maple [B] time = 0.279, size = 487, normalized size = 2.8

$$\frac{2d}{\left(12ac-3b^2\right)(2cx+b)}\left(2\,\mathrm{EllipticF}\left(1/2\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}\sqrt{2},\sqrt{2}\right)x^2c^2\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}\sqrt{-\frac{2cx}{\sqrt{-4ac}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 2/3*(2*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2
)*2^(1/2),2^(1/2))*x^2*c^2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
)^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2)
)/(-4*a*c+b^2)^(1/2))^(1/2)*(-4*a*c+b^2)^(1/2)+2*EllipticF(1/2*((b+2*c*x+(-
4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x*b*c*((b+2*c*
x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1
/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-4*a*c
+b^2)^(1/2)+2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*
c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2
)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/
2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2)^(1/2)*a*c+4*c^3*x^3+6*b*c^2*x^2-4*x
*a*c^2+4*x*b^2*c-2*a*b*c+b^3)*d*(d*(2*c*x+b))^(1/2)/(4*a*c-b^2)/(2*c*x+b)/(
c*x^2+b*x+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx+bd)^{\frac{3}{2}}}{\left(cx^2+bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^(3/2)/(c*x^2 + b*x + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(2\,cdx+bd)^{\frac{3}{2}}\sqrt{cx^2+bx+a}}{c^3x^6+3\,bc^2x^5+3\left(b^2c+ac^2\right)x^4+3\,a^2bx+\left(b^3+6\,abc\right)x^3+a^3+3\left(ab^2+a^2c\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((2*c*d*x + b*d)^(3/2)*sqrt(c*x^2 + b*x + a)/(c^3*x^6 + 3*b*c^2*x^5
 + 3*(b^2*c + a*c^2)*x^4 + 3*a^2*b*x + (b^3 + 6*a*b*c)*x^3 + a^3 + 3*(a*b^2
 + a^2*c)*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(3/2)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx + bd)^{\frac{3}{2}}}{\left(cx^2 + bx + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^(3/2)/(c*x^2 + b*x + a)^(5/2), x)
```

3.1393 $\int \frac{1}{\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=187

$$\frac{40c\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{3\sqrt{d}\left(b^2-4ac\right)^{7/4}\sqrt{a+bx+cx^2}}+\frac{20c\sqrt{bd+2cdx}}{3d\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}-\frac{2\sqrt{bd+2cdx}}{3d\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

```
[Out] (-2*Sqrt[b*d + 2*c*d*x])/(3*(b^2 - 4*a*c)*d*(a + b*x + c*x^2)^(3/2)) + (20*
c*Sqrt[b*d + 2*c*d*x])/(3*(b^2 - 4*a*c)^2*d*Sqrt[a + b*x + c*x^2]) + (40*c*
Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*
c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(3*(b^2 - 4*a*c)^(7/4)*Sqrt[d]*
Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.140025, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {687, 691, 689, 221}

$$\frac{20c\sqrt{bd+2cdx}}{3d\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}-\frac{2\sqrt{bd+2cdx}}{3d\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}+\frac{40c\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{3\sqrt{d}\left(b^2-4ac\right)^{7/4}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[b*d + 2*c*d*x]*(a + b*x + c*x^2)^(5/2)),x]
```

```
[Out] (-2*Sqrt[b*d + 2*c*d*x])/(3*(b^2 - 4*a*c)*d*(a + b*x + c*x^2)^(3/2)) + (20*
c*Sqrt[b*d + 2*c*d*x])/(3*(b^2 - 4*a*c)^2*d*Sqrt[a + b*x + c*x^2]) + (40*c*
Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*
c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(3*(b^2 - 4*a*c)^(7/4)*Sqrt[d]*
Sqrt[a + b*x + c*x^2])
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
```

```
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2\sqrt{bd+2cdx}}{3\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^{3/2}} - \frac{(10c)\int \frac{1}{\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2}}\,dx}{3\left(b^2-4ac\right)}$$

$$= -\frac{2\sqrt{bd+2cdx}}{3\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^{3/2}} + \frac{20c\sqrt{bd+2cdx}}{3\left(b^2-4ac\right)^2 d\sqrt{a+bx+cx^2}} + \frac{\left(20c^2\right)\int \frac{}{\sqrt{bd+2}}}{3\left(b^2-\right.}$$

$$= -\frac{2\sqrt{bd+2cdx}}{3\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^{3/2}} + \frac{20c\sqrt{bd+2cdx}}{3\left(b^2-4ac\right)^2 d\sqrt{a+bx+cx^2}} + \frac{\left(20c^2\sqrt{-\frac{c(a+b}{b^2-}}\right.}{3}$$

$$= -\frac{2\sqrt{bd+2cdx}}{3\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^{3/2}} + \frac{20c\sqrt{bd+2cdx}}{3\left(b^2-4ac\right)^2 d\sqrt{a+bx+cx^2}} + \frac{\left(40c\sqrt{-\frac{c(a+bx}{b^2-4}}\right.}{}$$

$$= -\frac{2\sqrt{bd+2cdx}}{3\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^{3/2}} + \frac{20c\sqrt{bd+2cdx}}{3\left(b^2-4ac\right)^2 d\sqrt{a+bx+cx^2}} + \frac{40c\sqrt{-\frac{c(a+bx+}{b^2-4a}}}{3\left(b^2-4\right.}$$

Mathematica [C] time = 0.167233, size = 132, normalized size = 0.71

$$-\frac{2\sqrt{d(b+2cx)}\left(-20c(a+x(b+cx))\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)-2c\left(7a+5cx^2\right)+b^2-10bcx\right)}{3d\left(b^2-4ac\right)^2(a+x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[b*d + 2*c*d*x]*(a + b*x + c*x^2)^(5/2)),x]
```

```
[Out] (-2*Sqrt[d*(b + 2*c*x)]*(b^2 - 10*b*c*x - 2*c*(7*a + 5*c*x^2) - 20*c*(a + x
*(b + c*x))*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*Hypergeometric2F1[1/
4, 1/2, 5/4, (b + 2*c*x)^2/(b^2 - 4*a*c)]))/(3*(b^2 - 4*a*c)^2*d*(a + x*(b
+ c*x))^(3/2))
```

Maple [B] time = 0.235, size = 491, normalized size = 2.6

$$\frac{2}{3d(2cx+b)\left(4ac-b^2\right)^2}\sqrt{d(2cx+b)}\left(10\,\mathrm{EllipticF}\left(1/2\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}\sqrt{2},\sqrt{2}\right)x^2c^2\sqrt{\frac{b+2cx+\sqrt{-4ac+b}}{\sqrt{-4ac+b^2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 2/3*(d*(2*c*x+b))^(1/2)*(10*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4
*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^2*c^2*((b+2*c*x+(-4*a*c+b^2)^(1/2
))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c
*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-4*a*c+b^2)^(1/2)+10*Elli
pticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2
^(1/2))*x*b*c*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*
c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2
)^(1/2))^(1/2)*(-4*a*c+b^2)^(1/2)+10*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+
b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^
(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2)^(1/2)*a*c+20
*c^3*x^3+30*b*c^2*x^2+28*x*a*c^2+8*x*b^2*c+14*a*b*c-b^3)/d/(2*c*x+b)/(4*a*c
-b^2)^2/(c*x^2+b*x+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,cdx + bd}\left(cx^2 + bx + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*c*d*x + b*d)*(c*x^2 + b*x + a)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{2\,cdx + bd}\sqrt{cx^2 + bx + a}}{2\,c^4dx^7 + 7\,bc^3dx^6 + 3\,\left(3\,b^2c^2 + 2\,ac^3\right)dx^5 + 5\,\left(b^3c + 3\,abc^2\right)dx^4 + a^3bd + \left(b^4 + 12\,ab^2c + 6\,a^2c^2\right)dx^3 + 3\,(ab}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(2*c^4*d*x^7 + 7*b*c^3*d
*x^6 + 3*(3*b^2*c^2 + 2*a*c^3)*d*x^5 + 5*(b^3*c + 3*a*b*c^2)*d*x^4 + a^3*b*
d + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d*x^3 + 3*(a*b^3 + 3*a^2*b*c)*d*x^2 + (3
*a^2*b^2 + 2*a^3*c)*d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d\,(b + 2cx)}\,\left(a + bx + cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**(1/2)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Integral(1/(sqrt(d*(b + 2*c*x))*(a + b*x + c*x**2)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,cdx + bd}\left(cx^2 + bx + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*c*d*x + b*d)*(c*x^2 + b*x + a)^(5/2)), x)
```

3.1394 $\int \frac{1}{(bd+2cdx)^{5/2}\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=228

$$\frac{40c\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{d^{5/2}\left(b^2-4ac\right)^{11/4}\sqrt{a+bx+cx^2}}+\frac{80c^2\sqrt{a+bx+cx^2}}{d\left(b^2-4ac\right)^3(bd+2cdx)^{3/2}}+\frac{12c}{d\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}(bd+\dots}$$

```
[Out] -2/(3*(b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)^(3/2)) + (12*
c)/((b^2 - 4*a*c)^2*d*(b*d + 2*c*d*x)^(3/2)*Sqrt[a + b*x + c*x^2]) + (80*c^
2*Sqrt[a + b*x + c*x^2])/((b^2 - 4*a*c)^3*d*(b*d + 2*c*d*x)^(3/2)) + (40*c*
Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*
c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/((b^2 - 4*a*c)^(11/4)*d^(5/2)*S
qrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.176612, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {687, 693, 691, 689, 221}

$$\frac{80c^2\sqrt{a+bx+cx^2}}{d\left(b^2-4ac\right)^3(bd+2cdx)^{3/2}}+\frac{40c\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{d^{5/2}\left(b^2-4ac\right)^{11/4}\sqrt{a+bx+cx^2}}+\frac{12c}{d\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}(bd+2cdx)\dots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^(5/2)*(a + b*x + c*x^2)^(5/2)),x]
```

```
[Out] -2/(3*(b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)^(3/2)) + (12*
c)/((b^2 - 4*a*c)^2*d*(b*d + 2*c*d*x)^(3/2)*Sqrt[a + b*x + c*x^2]) + (80*c^
2*Sqrt[a + b*x + c*x^2])/((b^2 - 4*a*c)^3*d*(b*d + 2*c*d*x)^(3/2)) + (40*c*
Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*
c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/((b^2 - 4*a*c)^(11/4)*d^(5/2)*S
qrt[a + b*x + c*x^2])
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)^{5/2}\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2}{3\left(b^2-4ac\right)d(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{3/2}} - \frac{(6c)\int \frac{1}{(bd+2cdx)^{5/2}\left(a+bx+cx^2\right)^{3/2}}\,dx}{b^2-4ac}$$

$$= -\frac{2}{3\left(b^2-4ac\right)d(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{3/2}} + \frac{12c}{\left(b^2-4ac\right)^2 d(bd+2cdx)^{3/2}\sqrt{a}}$$

$$= -\frac{2}{3\left(b^2-4ac\right)d(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{3/2}} + \frac{12c}{\left(b^2-4ac\right)^2 d(bd+2cdx)^{3/2}\sqrt{a}}$$

$$= -\frac{2}{3\left(b^2-4ac\right)d(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{3/2}} + \frac{12c}{\left(b^2-4ac\right)^2 d(bd+2cdx)^{3/2}\sqrt{a}}$$

$$= -\frac{2}{3\left(b^2-4ac\right)d(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{3/2}} + \frac{12c}{\left(b^2-4ac\right)^2 d(bd+2cdx)^{3/2}\sqrt{a}}$$

$$= -\frac{2}{3\left(b^2-4ac\right)d(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{3/2}} + \frac{12c}{\left(b^2-4ac\right)^2 d(bd+2cdx)^{3/2}\sqrt{a}}$$

Mathematica [C] time = 0.07151, size = 99, normalized size = 0.43

$$-\frac{32c\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\;{}_2F_1\left(-\frac{3}{4},\frac{5}{2};\frac{1}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{3d\left(b^2-4ac\right)^2\sqrt{a+x(b+cx)}(d(b+2cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^(5/2)*(a + b*x + c*x^2)^(5/2)),x]
```

```
[Out] (-32*c*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*Hypergeometric2F1[-3/4, 5
/2, 1/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(3*(b^2 - 4*a*c)^2*d*(d*(b + 2*c*x))
^(3/2)*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.24, size = 797, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] -2/3*(60*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2
))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2
))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-
4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^3*c^3+90*(-4*a*c+b^2)^(1/2)*((b+
2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2
)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*Ell
ipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),
2^(1/2))*x^2*b*c^2+60*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/
2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4
*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2)^(1/2)*x*a*c^2+30*(-4*a*c+b^
2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b
)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/
2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(
1/2)*2^(1/2),2^(1/2))*x*b^2*c+30*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)
^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2
))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*(-4*a*c+b^2)^(1/2)*a*b*c+120*
c^4*x^4+240*b*c^3*x^3+168*x^2*a*c^3+138*x^2*b^2*c^2+168*b*a*c^2*x+18*b^3*c*
x+32*a^2*c^2+26*a*c*b^2-b^4)*(d*(2*c*x+b))^(1/2)/d^3/(2*c*x+b)^2/(4*a*c-b^2
)^3/(c*x^2+b*x+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2\,cdx + bd)^{\frac{5}{2}}\left(cx^2 + bx + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*c*d*x + b*d)^(5/2)*(c*x^2 + b*x + a)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{}{8\,c^6d^3x^9 + 36\,bc^5d^3x^8 + 6\left(11\,b^2c^4 + 4\,ac^5\right)d^3x^7 + 21\left(3\,b^3c^3 + 4\,abc^4\right)d^3x^6 + a^3b^3d^3 + 3\left(11\,b^4c^2 + 38\,ab^2c^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(8*c^6*d^3*x^9 + 36*b*c^
5*d^3*x^8 + 6*(11*b^2*c^4 + 4*a*c^5)*d^3*x^7 + 21*(3*b^3*c^3 + 4*a*b*c^4)*d
^3*x^6 + a^3*b^3*d^3 + 3*(11*b^4*c^2 + 38*a*b^2*c^3 + 8*a^2*c^4)*d^3*x^5 +
3*(3*b^5*c + 25*a*b^3*c^2 + 20*a^2*b*c^3)*d^3*x^4 + (b^6 + 24*a*b^4*c + 54*
a^2*b^2*c^2 + 8*a^3*c^3)*d^3*x^3 + 3*(a*b^5 + 7*a^2*b^3*c + 4*a^3*b*c^2)*d^
3*x^2 + 3*(a^2*b^4 + 2*a^3*b^2*c)*d^3*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d\,(b+2cx))^{\frac{5}{2}}\left(a+bx+cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**(5/2)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Integral(1/((d*(b + 2*c*x))**(5/2)*(a + b*x + c*x**2)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2\,cdx+bd)^{\frac{5}{2}}\left(cx^2+bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((2*c*d*x + b*d)^(5/2)*(c*x^2 + b*x + a)^(5/2)), x)
```

3.1395 $\int \frac{(bd+2cdx)^{13/2}}{(a+bx+cx^2)^{5/2}}\,dx$

Optimal. Leaf size=299

$$-\frac{616cd^{13/2}\left(b^2-4ac\right)^{7/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{5\sqrt{a+bx+cx^2}}+\frac{616cd^{13/2}\left(b^2-4ac\right)^{7/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\text{s}\right.}{5\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*d*(b*d + 2*c*d*x)^(11/2))/(3*(a + b*x + c*x^2)^(3/2)) - (44*c*d^3*(b*d
+ 2*c*d*x)^(7/2))/(3*Sqrt[a + b*x + c*x^2]) + (1232*c^2*d^5*(b*d + 2*c*d*x)
^(3/2)*Sqrt[a + b*x + c*x^2])/15 + (616*c*(b^2 - 4*a*c)^(7/4)*d^(13/2)*Sqrt
[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d*
x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(5*Sqrt[a + b*x + c*x^2]) - (616*c*
(b^2 - 4*a*c)^(7/4)*d^(13/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*E
llipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(5
*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.286027, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {686, 692, 691, 690, 307, 221, 1199, 424}

$$-\frac{616cd^{13/2}\left(b^2-4ac\right)^{7/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{5\sqrt{a+bx+cx^2}}+\frac{616cd^{13/2}\left(b^2-4ac\right)^{7/4}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{}}{\sqrt[4]{}}\right.\right.}{5\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(13/2)/(a + b*x + c*x^2)^(5/2),x]
```


```
[Out] (-2*d*(b*d + 2*c*d*x)^(11/2))/(3*(a + b*x + c*x^2)^(3/2)) - (44*c*d^3*(b*d
+ 2*c*d*x)^(7/2))/(3*Sqrt[a + b*x + c*x^2]) + (1232*c^2*d^5*(b*d + 2*c*d*x)
^(3/2)*Sqrt[a + b*x + c*x^2])/15 + (616*c*(b^2 - 4*a*c)^(7/4)*d^(13/2)*Sqrt
[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d*
x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(5*Sqrt[a + b*x + c*x^2]) - (616*c*
(b^2 - 4*a*c)^(7/4)*d^(13/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*E
llipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/(5
*Sqrt[a + b*x + c*x^2])
```


Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
```

```
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(bd+2cdx)^{13/2}}{\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2d(bd+2cdx)^{11/2}}{3\left(a+bx+cx^2\right)^{3/2}} + \frac{1}{3}\left(22cd^2\right)\int \frac{(bd+2cdx)^{9/2}}{\left(a+bx+cx^2\right)^{3/2}}\,dx$$

$$= -\frac{2d(bd+2cdx)^{11/2}}{3\left(a+bx+cx^2\right)^{3/2}} - \frac{44cd^3(bd+2cdx)^{7/2}}{3\sqrt{a+bx+cx^2}} + \frac{1}{3}\left(308c^2d^4\right)\int \frac{(bd+2cdx)^{5/2}}{\sqrt{a+bx+cx^2}}\,dx$$

$$= -\frac{2d(bd+2cdx)^{11/2}}{3\left(a+bx+cx^2\right)^{3/2}} - \frac{44cd^3(bd+2cdx)^{7/2}}{3\sqrt{a+bx+cx^2}} + \frac{1232}{15}c^2d^5(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2} + \frac{1}{5}\left(3\right.$$

$$= -\frac{2d(bd+2cdx)^{11/2}}{3\left(a+bx+cx^2\right)^{3/2}} - \frac{44cd^3(bd+2cdx)^{7/2}}{3\sqrt{a+bx+cx^2}} + \frac{1232}{15}c^2d^5(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2} + \frac{\left(308\right.}{}$$

$$= -\frac{2d(bd+2cdx)^{11/2}}{3\left(a+bx+cx^2\right)^{3/2}} - \frac{44cd^3(bd+2cdx)^{7/2}}{3\sqrt{a+bx+cx^2}} + \frac{1232}{15}c^2d^5(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2} + \frac{\left(616\right.}{}$$

$$= -\frac{2d(bd+2cdx)^{11/2}}{3\left(a+bx+cx^2\right)^{3/2}} - \frac{44cd^3(bd+2cdx)^{7/2}}{3\sqrt{a+bx+cx^2}} + \frac{1232}{15}c^2d^5(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2} - \frac{\left(616\right.}{}$$

$$= -\frac{2d(bd+2cdx)^{11/2}}{3\left(a+bx+cx^2\right)^{3/2}} - \frac{44cd^3(bd+2cdx)^{7/2}}{3\sqrt{a+bx+cx^2}} + \frac{1232}{15}c^2d^5(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2} - \frac{616c}{}$$

$$= -\frac{2d(bd+2cdx)^{11/2}}{3\left(a+bx+cx^2\right)^{3/2}} - \frac{44cd^3(bd+2cdx)^{7/2}}{3\sqrt{a+bx+cx^2}} + \frac{1232}{15}c^2d^5(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2} + \frac{616c}{}$$

Mathematica [C] time = 0.228671, size = 202, normalized size = 0.68

$$\frac{4d^5(d(b+2cx))^{3/2}\left(616c\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\left(4a^2c+a\left(-b^2+4bcx+4c^2x^2\right)-b^2x(b+cx)\right){}_2F_1\left(\frac{3}{4},\frac{5}{2};\frac{7}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)+16c^2\left(-77\right.\right.}{15(a+x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(13/2)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (4*d^5*(d*(b + 2*c*x))^(3/2)*(-41*b^4 + 156*b^3*c*x + 48*b*c^2*x*(-11*a + 2
*c*x^2) + 4*b^2*c*(121*a + 51*c*x^2) + 16*c^2*(-77*a^2 - 33*a*c*x^2 + 3*c^2
*x^4) + 616*c*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*(4*a^2*c - b^2*x*(
b + c*x) + a*(-b^2 + 4*b*c*x + 4*c^2*x^2))*Hypergeometric2F1[3/4, 5/2, 7/4,
 (b + 2*c*x)^2/(b^2 - 4*a*c)]))/(15*(a + x*(b + c*x))^(3/2))
```

Maple [B] time = 0.292, size = 1328, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*d*x+b*d)^(13/2)/(c*x^2+b*x+a)^(5/2),x)

[Out] -2/15*(d*(2*c*x+b))^(1/2)*(7392*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^2*a^2*c^4*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)-3696*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^2*a*b^2*c^3*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)+462*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^2*b^4*c^2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)-384*x^6*c^6+7392*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x*a^2*b*c^3*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)-3696*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x*a*b^3*c^2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)+462*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x*b^5*c*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)-1152*x^5*b*c^5+7392*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a^3*c^3-3696*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a^2*b^2*c^2+462*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a*b^4*c-3168*x^4*a*c^5-648*x^4*b^2*c^4-6336*x^3*a*b*c^4+624*x^3*b^3*c^3-2464*x^2*a^2*c^4-3520*x^2*a*b^2*c^3+674*x^2*b^4*c^2-2464*x*a^2*b*c^3-352*x*a*b^3*c^2+170*x*b^5*c-616*a^2*b^2*c^2+110*a*b^4*c+5*b^6)*d^6/(2*c*x+b)/(c*x^2+b*x+a)^(3/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx+bd)^{\frac{13}{2}}}{\left(cx^2+bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*d*x+b*d)^(13/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")

[Out] integrate((2*c*d*x + b*d)^(13/2)/(c*x^2 + b*x + a)^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(64\,c^6d^6x^6+192\,bc^5d^6x^5+240\,b^2c^4d^6x^4+160\,b^3c^3d^6x^3+60\,b^4c^2d^6x^2+12\,b^5cd^6x+b^6d^6\right)\sqrt{2\,cdx+bd}\sqrt{cx}}{c^3x^6+3\,bc^2x^5+3\left(b^2c+ac^2\right)x^4+3\,a^2bx+\left(b^3+6\,abc\right)x^3+a^3+3\left(ab^2+a^2c\right)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(13/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((64*c^6*d^6*x^6 + 192*b*c^5*d^6*x^5 + 240*b^2*c^4*d^6*x^4 + 160*b^
3*c^3*d^6*x^3 + 60*b^4*c^2*d^6*x^2 + 12*b^5*c*d^6*x + b^6*d^6)*sqrt(2*c*d*x
 + b*d)*sqrt(c*x^2 + b*x + a)/(c^3*x^6 + 3*b*c^2*x^5 + 3*(b^2*c + a*c^2)*x^
4 + 3*a^2*b*x + (b^3 + 6*a*b*c)*x^3 + a^3 + 3*(a*b^2 + a^2*c)*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(13/2)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx+bd)^{\frac{13}{2}}}{\left(cx^2+bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(13/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^(13/2)/(c*x^2 + b*x + a)^(5/2), x)
```

3.1396 $\int \frac{(bd+2cdx)^{9/2}}{\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=258

$$-\frac{56cd^{9/2}\left(b^2-4ac\right)^{3/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{\sqrt{a+bx+cx^2}}+\frac{56cd^{9/2}\left(b^2-4ac\right)^{3/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{}}{\sqrt[4]{}}\right.\right.}{\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*d*(b*d + 2*c*d*x)^(7/2))/(3*(a + b*x + c*x^2)^(3/2)) - (28*c*d^3*(b*d +
 2*c*d*x)^(3/2))/(3*Sqrt[a + b*x + c*x^2]) + (56*c*(b^2 - 4*a*c)^(3/4)*d^(9
/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d
+ 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/Sqrt[a + b*x + c*x^2] - (56
*c*(b^2 - 4*a*c)^(3/4)*d^(9/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]
*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/
Sqrt[a + b*x + c*x^2]
```

Rubi [A] time = 0.237905, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {686, 691, 690, 307, 221, 1199, 424}

$$-\frac{56cd^{9/2}\left(b^2-4ac\right)^{3/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{\sqrt{a+bx+cx^2}}+\frac{56cd^{9/2}\left(b^2-4ac\right)^{3/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right.\right.}{\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(9/2)/(a + b*x + c*x^2)^(5/2),x]
```


```
[Out] (-2*d*(b*d + 2*c*d*x)^(7/2))/(3*(a + b*x + c*x^2)^(3/2)) - (28*c*d^3*(b*d +
 2*c*d*x)^(3/2))/(3*Sqrt[a + b*x + c*x^2]) + (56*c*(b^2 - 4*a*c)^(3/4)*d^(9
/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d
+ 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/Sqrt[a + b*x + c*x^2] - (56
*c*(b^2 - 4*a*c)^(3/4)*d^(9/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]
*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/
Sqrt[a + b*x + c*x^2]
```


Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(bd+2cdx)^{9/2}}{\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2d(bd+2cdx)^{7/2}}{3\left(a+bx+cx^2\right)^{3/2}}+\frac{1}{3}\left(14cd^2\right)\int \frac{(bd+2cdx)^{5/2}}{\left(a+bx+cx^2\right)^{3/2}}\,dx$$

$$= -\frac{2d(bd+2cdx)^{7/2}}{3\left(a+bx+cx^2\right)^{3/2}}-\frac{28cd^3(bd+2cdx)^{3/2}}{3\sqrt{a+bx+cx^2}}+\left(28c^2d^4\right)\int \frac{\sqrt{bd+2cdx}}{\sqrt{a+bx+cx^2}}\,dx$$

$$= -\frac{2d(bd+2cdx)^{7/2}}{3\left(a+bx+cx^2\right)^{3/2}}-\frac{28cd^3(bd+2cdx)^{3/2}}{3\sqrt{a+bx+cx^2}}+\frac{\left(28c^2d^4\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\int \frac{\sqrt{bd+2cdx}}{\sqrt{-\frac{ac}{b^2-4ac}-\frac{bcx}{b^2-4ac}-\frac{c^2x^2}{b^2-4ac}}}\,d}{\sqrt{a+bx+cx^2}}$$

$$= -\frac{2d(bd+2cdx)^{7/2}}{3\left(a+bx+cx^2\right)^{3/2}}-\frac{28cd^3(bd+2cdx)^{3/2}}{3\sqrt{a+bx+cx^2}}+\frac{\left(56cd^3\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{\left(b^2-4ac\right)d^2}}}\,dx,\right.}{\sqrt{a+bx+cx^2}}$$

$$= -\frac{2d(bd+2cdx)^{7/2}}{3\left(a+bx+cx^2\right)^{3/2}}-\frac{28cd^3(bd+2cdx)^{3/2}}{3\sqrt{a+bx+cx^2}}-\frac{\left(56c\sqrt{b^2-4ac}d^4\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-\frac{}{(b}}}\right.}{\sqrt{a+bx+cx^2}}$$

$$= -\frac{2d(bd+2cdx)^{7/2}}{3\left(a+bx+cx^2\right)^{3/2}}-\frac{28cd^3(bd+2cdx)^{3/2}}{3\sqrt{a+bx+cx^2}}-\frac{56c\left(b^2-4ac\right)^{3/4}d^{9/2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{b}}{\sqrt[4]{b^2}}\right.\right.}{\sqrt{a+bx+cx^2}}$$

$$= -\frac{2d(bd+2cdx)^{7/2}}{3\left(a+bx+cx^2\right)^{3/2}}-\frac{28cd^3(bd+2cdx)^{3/2}}{3\sqrt{a+bx+cx^2}}+\frac{56c\left(b^2-4ac\right)^{3/4}d^{9/2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{b}}{\sqrt[4]{b^2}}\right.\right.}{\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 0.132319, size = 122, normalized size = 0.47

$$-\frac{16d^3(d(b+2cx))^{3/2}\left(14c(a+x(b+cx))\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\,{}_2F_1\left(\frac{3}{4},\frac{5}{2};\frac{7}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)-c\left(7a+3cx^2\right)+b^2-3bcx\right)}{3(a+x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(9/2)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-16*d^3*(d*(b + 2*c*x))^(3/2)*(b^2 - 3*b*c*x - c*(7*a + 3*c*x^2) + 14*c*(a
 + x*(b + c*x))*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*Hypergeometric2F
1[3/4, 5/2, 7/4, (b + 2*c*x)^2/(b^2 - 4*a*c)]))/(3*(a + x*(b + c*x))^(3/2))
```

Maple [B] time = 0.229, size = 859, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a)^(5/2),x)
```

[Out] 2/3*(d*(2*c*x+b))^(1/2)*(168*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^2*a*c^3*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)-42*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^2*b^2*c^2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)+168*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x*a*b*c^2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)-42*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x*b^3*c*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)+168*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a^2*c^2-42*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a*b^2*c-72*c^4*x^4-144*b*c^3*x^3-56*x^2*a*c^3-94*x^2*b^2*c^2-56*b*a*c^2*x-22*b^3*c*x-14*a*c*b^2-b^4)*d^4/(2*c*x+b)/(c*x^2+b*x+a)^(3/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx+bd)^{\frac{9}{2}}}{\left(cx^2+bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")

[Out] integrate((2*c*d*x + b*d)^(9/2)/(c*x^2 + b*x + a)^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(16\,c^4d^4x^4+32\,bc^3d^4x^3+24\,b^2c^2d^4x^2+8\,b^3cd^4x+b^4d^4\right)\sqrt{2\,cdx+bd}\sqrt{cx^2+bx+a}}{c^3x^6+3\,bc^2x^5+3\left(b^2c+ac^2\right)x^4+3\,a^2bx+\left(b^3+6\,abc\right)x^3+a^3+3\left(ab^2+a^2c\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")

[Out] integral((16*c^4*d^4*x^4 + 32*b*c^3*d^4*x^3 + 24*b^2*c^2*d^4*x^2 + 8*b^3*c*d^4*x + b^4*d^4)*sqrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(c^3*x^6 + 3*b*c^2*x^5 + 3*(b^2*c + a*c^2)*x^4 + 3*a^2*b*x + (b^3 + 6*a*b*c)*x^3 + a^3 + 3*(a*b^2 + a^2*c)*x^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(9/2)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx+bd)^{\frac{9}{2}}}{\left(cx^2+bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(9/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^(9/2)/(c*x^2 + b*x + a)^(5/2), x)
```

3.1397 $\int \frac{(bd+2cdx)^{5/2}}{\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=264

$$-\frac{8cd^{5/2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{\sqrt[4]{b^2-4ac}\sqrt{a+bx+cx^2}}+\frac{8cd^{5/2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{\sqrt[4]{b^2-4ac}\sqrt{a+bx+cx^2}}-\frac{4cd(b}{\left(b^2-4a\right.}$$

```
[Out] (-2*d*(b*d + 2*c*d*x)^(3/2))/(3*(a + b*x + c*x^2)^(3/2)) - (4*c*d*(b*d + 2*
c*d*x)^(3/2))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (8*c*d^(5/2)*Sqrt[-((
c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d*x]/(
(b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/((b^2 - 4*a*c)^(1/4)*Sqrt[a + b*x + c*x
^2]) - (8*c*d^(5/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[
ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/((b^2 - 4*a
*c)^(1/4)*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.237248, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {686, 687, 691, 690, 307, 221, 1199, 424}

$$-\frac{8cd^{5/2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{\sqrt[4]{b^2-4ac}\sqrt{a+bx+cx^2}}+\frac{8cd^{5/2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{\sqrt[4]{b^2-4ac}\sqrt{a+bx+cx^2}}-\frac{4cd(bd+2cc}{\left(b^2-4ac\right)\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^(5/2)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*d*(b*d + 2*c*d*x)^(3/2))/(3*(a + b*x + c*x^2)^(3/2)) - (4*c*d*(b*d + 2*
c*d*x)^(3/2))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (8*c*d^(5/2)*Sqrt[-((
c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d*x]/(
(b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/((b^2 - 4*a*c)^(1/4)*Sqrt[a + b*x + c*x
^2]) - (8*c*d^(5/2)*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[
ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/((b^2 - 4*a
*c)^(1/4)*Sqrt[a + b*x + c*x^2])
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(bd+2cdx)^{5/2}}{\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2d(bd+2cdx)^{3/2}}{3\left(a+bx+cx^2\right)^{3/2}} + \left(2cd^2\right)\int \frac{\sqrt{bd+2cdx}}{\left(a+bx+cx^2\right)^{3/2}}\,dx$$

$$= -\frac{2d(bd+2cdx)^{3/2}}{3\left(a+bx+cx^2\right)^{3/2}} - \frac{4cd(bd+2cdx)^{3/2}}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\left(4c^2d^2\right)\int \frac{\sqrt{bd+2cdx}}{\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac}$$

$$= -\frac{2d(bd+2cdx)^{3/2}}{3\left(a+bx+cx^2\right)^{3/2}} - \frac{4cd(bd+2cdx)^{3/2}}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\left(4c^2d^2\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\int \frac{\sqrt{bd+2cdx}}{\sqrt{-\frac{ac}{b^2-4ac}-\frac{bcx}{b^2-4ac}-}}$$

$$= -\frac{2d(bd+2cdx)^{3/2}}{3\left(a+bx+cx^2\right)^{3/2}} - \frac{4cd(bd+2cdx)^{3/2}}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\left(8cd\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{\left(b^2-4ac\right.}}}\right.}{\left(b^2-4ac\right)\sqrt{a+bx+}}$$

$$= -\frac{2d(bd+2cdx)^{3/2}}{3\left(a+bx+cx^2\right)^{3/2}} - \frac{4cd(bd+2cdx)^{3/2}}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} - \frac{\left(8cd^2\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{\left(b^2-4a\right.}}}\right.}{\sqrt{b^2-4ac}\sqrt{a+bx+}}$$

$$= -\frac{2d(bd+2cdx)^{3/2}}{3\left(a+bx+cx^2\right)^{3/2}} - \frac{4cd(bd+2cdx)^{3/2}}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} - \frac{8cd^{5/2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right.\right.}{\sqrt[4]{b^2-4ac}\sqrt{a+bx+cx^2}}$$

$$= -\frac{2d(bd+2cdx)^{3/2}}{3\left(a+bx+cx^2\right)^{3/2}} - \frac{4cd(bd+2cdx)^{3/2}}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{8cd^{5/2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right.\right.}{\sqrt[4]{b^2-4ac}\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 0.116073, size = 116, normalized size = 0.44

$$-\frac{4d(d(b+2cx))^{3/2}\left(8c(a+x(b+cx))\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\,{}_2F_1\left(\frac{3}{4},\frac{5}{2};\frac{7}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)-4ac+b^2\right)}{3\left(b^2-4ac\right)(a+x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^(5/2)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-4*d*(d*(b + 2*c*x))^(3/2)*(b^2 - 4*a*c + 8*c*(a + x*(b + c*x))*Sqrt[(c*(a
 + x*(b + c*x)))/(-b^2 + 4*a*c)]*Hypergeometric2F1[3/4, 5/2, 7/4, (b + 2*c*
x)^2/(b^2 - 4*a*c)]))/(3*(b^2 - 4*a*c)*(a + x*(b + c*x))^(3/2))
```

Maple [B] time = 0.227, size = 871, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] -2/3*(d*(2*c*x+b))^(1/2)*(24*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-
4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^2*a*c^3*((b+2*c*x+(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-
2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)-6*EllipticE(1/2*((b+2*c
*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^2*b^2*c
^2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*
a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1
/2)+24*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2
)*2^(1/2),2^(1/2))*x*a*b*c^2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/
2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(1/2))^(1/2)-6*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))
/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x*b^3*c*((b+2*c*x+(-4*a*c+b^2)^
(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b
-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)+24*((b+2*c*x+(-4*a*c+b
^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*
((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+
2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a^2*c^
2-6*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4
*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(
1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*
2^(1/2),2^(1/2))*a*b^2*c-24*c^4*x^4-48*b*c^3*x^3-8*x^2*a*c^3-34*x^2*b^2*c^2
-8*b*a*c^2*x-10*b^3*c*x-2*a*c*b^2-b^4)*d^2/(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x
+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx + bd)^{\frac{5}{2}}}{\left(cx^2 + bx + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^(5/2)/(c*x^2 + b*x + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(4\,c^2d^2x^2 + 4\,bcd^2x + b^2d^2\right)\sqrt{2\,cdx + bd}\sqrt{cx^2 + bx + a}}{c^3x^6 + 3\,bc^2x^5 + 3\left(b^2c + ac^2\right)x^4 + 3\,a^2bx + \left(b^3 + 6\,abc\right)x^3 + a^3 + 3\left(ab^2 + a^2c\right)x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((4*c^2*d^2*x^2 + 4*b*c*d^2*x + b^2*d^2)*sqrt(2*c*d*x + b*d)*sqrt(c
*x^2 + b*x + a)/(c^3*x^6 + 3*b*c^2*x^5 + 3*(b^2*c + a*c^2)*x^4 + 3*a^2*b*x
+ (b^3 + 6*a*b*c)*x^3 + a^3 + 3*(a*b^2 + a^2*c)*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(5/2)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx + bd)^{\frac{5}{2}}}{\left(cx^2 + bx + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(5/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^(5/2)/(c*x^2 + b*x + a)^(5/2), x)
```

3.1398 $\int \frac{\sqrt{bd+2cdx}}{(a+bx+cx^2)^{5/2}}\,dx$

Optimal. Leaf size=278

$$\frac{8c\sqrt{d}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{(b^2-4ac)^{5/4}\sqrt{a+bx+cx^2}}+\frac{4c(bd+2cdx)^{3/2}}{d(b^2-4ac)^2\sqrt{a+bx+cx^2}}-\frac{2(bd+2cdx)^{3/2}}{3d(b^2-4ac)(a+bx+cx^2)^{3/2}}-$$

```
[Out] (-2*(b*d + 2*c*d*x)^(3/2))/(3*(b^2 - 4*a*c)*d*(a + b*x + c*x^2)^(3/2)) + (4
*c*(b*d + 2*c*d*x)^(3/2))/((b^2 - 4*a*c)^2*d*Sqrt[a + b*x + c*x^2]) - (8*c*
Sqrt[d]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[
b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/((b^2 - 4*a*c)^(5/4)*Sq
rt[a + b*x + c*x^2]) + (8*c*Sqrt[d]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a
*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])],
-1])/((b^2 - 4*a*c)^(5/4)*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.247288, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {687, 691, 690, 307, 221, 1199, 424}

$$\frac{4c(bd+2cdx)^{3/2}}{d(b^2-4ac)^2\sqrt{a+bx+cx^2}}-\frac{2(bd+2cdx)^{3/2}}{3d(b^2-4ac)(a+bx+cx^2)^{3/2}}+\frac{8c\sqrt{d}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{(b^2-4ac)^{5/4}\sqrt{a+bx+cx^2}}-\frac{8c\sqrt{d}\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*d + 2*c*d*x]/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*(b*d + 2*c*d*x)^(3/2))/(3*(b^2 - 4*a*c)*d*(a + b*x + c*x^2)^(3/2)) + (4
*c*(b*d + 2*c*d*x)^(3/2))/((b^2 - 4*a*c)^2*d*Sqrt[a + b*x + c*x^2]) - (8*c*
Sqrt[d]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[
b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/((b^2 - 4*a*c)^(5/4)*Sq
rt[a + b*x + c*x^2]) + (8*c*Sqrt[d]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a
*c))]*EllipticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])],
-1])/((b^2 - 4*a*c)^(5/4)*Sqrt[a + b*x + c*x^2])
```

Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{bd+2cdx}}{\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2(bd+2cdx)^{3/2}}{3\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^{3/2}} - \frac{(2c)\int \frac{\sqrt{bd+2cdx}}{\left(a+bx+cx^2\right)^{3/2}}\,dx}{b^2-4ac}$$

$$= -\frac{2(bd+2cdx)^{3/2}}{3\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^{3/2}} + \frac{4c(bd+2cdx)^{3/2}}{\left(b^2-4ac\right)^2 d\sqrt{a+bx+cx^2}} - \frac{\left(4c^2\right)\int \frac{\sqrt{bd+2cdx}}{\sqrt{a+bx+cx^2}}\,dx}{\left(b^2-4ac\right)^2}$$

$$= -\frac{2(bd+2cdx)^{3/2}}{3\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^{3/2}} + \frac{4c(bd+2cdx)^{3/2}}{\left(b^2-4ac\right)^2 d\sqrt{a+bx+cx^2}} - \frac{\left(4c^2\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\int \frac{\cdots}{\sqrt{-\frac{ac\cdots}{b^2-\cdots}}}}{\left(b^2-4ac\right)^2\sqrt{a+b\cdots}}$$

$$= -\frac{2(bd+2cdx)^{3/2}}{3\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^{3/2}} + \frac{4c(bd+2cdx)^{3/2}}{\left(b^2-4ac\right)^2 d\sqrt{a+bx+cx^2}} - \frac{\left(8c\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int\cdots\right.}{\left(b^2-4ac\right)^2\cdots}$$

$$= -\frac{2(bd+2cdx)^{3/2}}{3\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^{3/2}} + \frac{4c(bd+2cdx)^{3/2}}{\left(b^2-4ac\right)^2 d\sqrt{a+bx+cx^2}} + \frac{\left(8c\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int\cdots\right.}{\left(b^2-4ac\right)^{3/\cdots}}$$

$$= -\frac{2(bd+2cdx)^{3/2}}{3\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^{3/2}} + \frac{4c(bd+2cdx)^{3/2}}{\left(b^2-4ac\right)^2 d\sqrt{a+bx+cx^2}} + \frac{8c\sqrt{d}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\cdots\right.}{\left(b^2-4ac\right)^{5/4}\sqrt{a+\cdots}}$$

$$= -\frac{2(bd+2cdx)^{3/2}}{3\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^{3/2}} + \frac{4c(bd+2cdx)^{3/2}}{\left(b^2-4ac\right)^2 d\sqrt{a+bx+cx^2}} - \frac{8c\sqrt{d}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\cdots\right.}{\left(b^2-4ac\right)^{5/4}\sqrt{a+\cdots}}$$

Mathematica [C] time = 0.0589341, size = 99, normalized size = 0.36

$$\frac{32c\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}(d(b+2cx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{5}{2};\frac{7}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{3d\left(b^2-4ac\right)^2\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*d + 2*c*d*x]/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (32*c*(d*(b + 2*c*x))^(3/2)*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*Hype
rgeometric2F1[3/4, 5/2, 7/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(3*(b^2 - 4*a*c)
^2*d*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.217, size = 866, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] -2/3*(24*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1
/2)*2^(1/2),2^(1/2))*x^2*a*c^3*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(
1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(1/2))^(1/2)-6*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2
))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^2*b^2*c^2*((b+2*c*x+(-4*a*c
+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2
)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)+24*EllipticE(1/2
*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x
*a*b*c^2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b
)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/
2))^(1/2)-6*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))
^(1/2)*2^(1/2),2^(1/2))*x*b^3*c*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^
(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^
(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)+24*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b
^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b
^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a^2*c^2-6*((b+2*c*x+(-4*a*
c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/
2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*(
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a*b
^2*c-24*c^4*x^4-48*b*c^3*x^3-40*x^2*a*c^3-26*x^2*b^2*c^2-40*b*a*c^2*x-2*b^3
*c*x-10*a*c*b^2+b^4)*(d*(2*c*x+b))^(1/2)/(4*a*c-b^2)^2/(2*c*x+b)/(c*x^2+b*x
+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,cdx + bd}}{\left(cx^2 + bx + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*c*d*x + b*d)/(c*x^2 + b*x + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,cdx + bd}\sqrt{cx^2 + bx + a}}{c^3x^6 + 3\,bc^2x^5 + 3\left(b^2c + ac^2\right)x^4 + 3\,a^2bx + \left(b^3 + 6\,abc\right)x^3 + a^3 + 3\left(ab^2 + a^2c\right)x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(c^3*x^6 + 3*b*c^2*x^5 +
 3*(b^2*c + a*c^2)*x^4 + 3*a^2*b*x + (b^3 + 6*a*b*c)*x^3 + a^3 + 3*(a*b^2 +
 a^2*c)*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**(1/2)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,cdx+bd}}{\left(cx^2+bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^(1/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(2*c*d*x + b*d)/(c*x^2 + b*x + a)^(5/2), x)
```

3.1399 $\int \frac{1}{(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=325

$$\frac{56c\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{bd+2cdx}}{\sqrt{d}\sqrt[4]{b^2-4ac}}\right),-1\right)}{d^{3/2}\left(b^2-4ac\right)^{9/4}\sqrt{a+bx+cx^2}}+\frac{112c^2\sqrt{a+bx+cx^2}}{d\left(b^2-4ac\right)^3\sqrt{bd+2cdx}}-\frac{56c\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{bd+2}}{\sqrt[4]{b^2-4a}}\right.\right.}{d^{3/2}\left(b^2-4ac\right)^{9/4}\sqrt{a+bx+}}$$

```
[Out] -2/(3*(b^2 - 4*a*c)*d*Sqrt[b*d + 2*c*d*x]*(a + b*x + c*x^2)^(3/2)) + (28*c)
/(3*(b^2 - 4*a*c)^2*d*Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2]) + (112*c^2
*Sqrt[a + b*x + c*x^2])/((b^2 - 4*a*c)^3*d*Sqrt[b*d + 2*c*d*x]) - (56*c*Sqr
t[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d
*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/((b^2 - 4*a*c)^(9/4)*d^(3/2)*Sqrt[
a + b*x + c*x^2]) + (56*c*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Elli
pticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/((b^2
 - 4*a*c)^(9/4)*d^(3/2)*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.283973, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {687, 693, 691, 690, 307, 221, 1199, 424}

$$\frac{112c^2\sqrt{a+bx+cx^2}}{d\left(b^2-4ac\right)^3\sqrt{bd+2cdx}}+\frac{56c\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-1\right)}{d^{3/2}\left(b^2-4ac\right)^{9/4}\sqrt{a+bx+cx^2}}-\frac{56c\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{bd+2cxd}}{\sqrt[4]{b^2-4ac}\sqrt{d}}\right)\middle|-\right.}{d^{3/2}\left(b^2-4ac\right)^{9/4}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)^(5/2)),x]
```


```
[Out] -2/(3*(b^2 - 4*a*c)*d*Sqrt[b*d + 2*c*d*x]*(a + b*x + c*x^2)^(3/2)) + (28*c)
/(3*(b^2 - 4*a*c)^2*d*Sqrt[b*d + 2*c*d*x]*Sqrt[a + b*x + c*x^2]) + (112*c^2
*Sqrt[a + b*x + c*x^2])/((b^2 - 4*a*c)^3*d*Sqrt[b*d + 2*c*d*x]) - (56*c*Sqr
t[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[b*d + 2*c*d
*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/((b^2 - 4*a*c)^(9/4)*d^(3/2)*Sqrt[
a + b*x + c*x^2]) + (56*c*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Elli
pticF[ArcSin[Sqrt[b*d + 2*c*d*x]/((b^2 - 4*a*c)^(1/4)*Sqrt[d])], -1])/((b^2
 - 4*a*c)^(9/4)*d^(3/2)*Sqrt[a + b*x + c*x^2])
```


Rule 687

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] - Dist[(2*c*e*(m + 2*p + 3))/(e*(p + 1)*(b^2 - 4*a*c)),
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0
] && LtQ[p, -1] &&  !GtQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
```

```
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Dist[Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]/Sqrt[a + b*x + c*
x^2], Int[(d + e*x)^m/Sqrt[-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) -
 (c^2*x^2)/(b^2 - 4*a*c)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2}{3\left(b^2-4ac\right)d\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2}} - \frac{(14c)\int \frac{1}{(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}}{3\left(b^2-4ac\right)}$$

$$= -\frac{2}{3\left(b^2-4ac\right)d\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2}} + \frac{28c}{3\left(b^2-4ac\right)^2 d\sqrt{bd+2cdx}\sqrt{a}}$$

$$= -\frac{2}{3\left(b^2-4ac\right)d\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2}} + \frac{28c}{3\left(b^2-4ac\right)^2 d\sqrt{bd+2cdx}\sqrt{a}}$$

$$= -\frac{2}{3\left(b^2-4ac\right)d\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2}} + \frac{28c}{3\left(b^2-4ac\right)^2 d\sqrt{bd+2cdx}\sqrt{a}}$$

$$= -\frac{2}{3\left(b^2-4ac\right)d\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2}} + \frac{28c}{3\left(b^2-4ac\right)^2 d\sqrt{bd+2cdx}\sqrt{a}}$$

$$= -\frac{2}{3\left(b^2-4ac\right)d\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2}} + \frac{28c}{3\left(b^2-4ac\right)^2 d\sqrt{bd+2cdx}\sqrt{a}}$$

$$= -\frac{2}{3\left(b^2-4ac\right)d\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2}} + \frac{28c}{3\left(b^2-4ac\right)^2 d\sqrt{bd+2cdx}\sqrt{a}}$$

$$= -\frac{2}{3\left(b^2-4ac\right)d\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2}} + \frac{28c}{3\left(b^2-4ac\right)^2 d\sqrt{bd+2cdx}\sqrt{a}}$$

Mathematica [C] time = 0.0577227, size = 97, normalized size = 0.3

$$-\frac{32c\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\,{}_2F_1\left(-\frac{1}{4},\frac{5}{2};\frac{3}{4};\frac{(b+2cx)^2}{b^2-4ac}\right)}{d\left(b^2-4ac\right)^2\sqrt{a+x(b+cx)}\sqrt{d(b+2cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*d + 2*c*d*x)^(3/2)*(a + b*x + c*x^2)^(5/2)),x]
```

```
[Out] (-32*c*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*Hypergeometric2F1[-1/4, 5
/2, 3/4, (b + 2*c*x)^2/(b^2 - 4*a*c)])/((b^2 - 4*a*c)^2*d*Sqrt[d*(b + 2*c*x
)]*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.239, size = 877, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 2/3*(168*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1
/2)*2^(1/2),2^(1/2))*x^2*a*c^3*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(
1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(1/2))^(1/2)-42*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*x^2*b^2*c^2*((b+2*c*x+(-4*a*
c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/
2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)+168*EllipticE(1
/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))
*x*a*b*c^2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x
+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(
1/2))^(1/2)-42*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/
2))^(1/2)*2^(1/2),2^(1/2))*x*b^3*c*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^
2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)+168*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a
*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*((-b-2*c*x+(-4*a
*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2))*a^2*c^2-42*((b+2*c*x+(
-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-(2*c*x+b)/(-4*a*c+b^2)^(1/2)
)^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*EllipticE(
1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)
)*a*b^2*c-168*c^4*x^4-336*b*c^3*x^3-280*x^2*a*c^3-182*x^2*b^2*c^2-280*b*a*c
^2*x-14*b^3*c*x-96*a^2*c^2-22*a*c*b^2+b^4)*(d*(2*c*x+b))^(1/2)/d^2/(4*a*c-b
^2)^3/(2*c*x+b)/(c*x^2+b*x+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2\,cdx+bd)^{\frac{3}{2}}\left(cx^2+bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*c*d*x + b*d)^(3/2)*(c*x^2 + b*x + a)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,cdx+bd}\sqrt{cx^2+}}{4\,c^5d^2x^8+16\,bc^4d^2x^7+\left(25\,b^2c^3+12\,ac^4\right)d^2x^6+\left(19\,b^3c^2+36\,abc^3\right)d^2x^5+a^3b^2d^2+\left(7\,b^4c+39\,ab^2c^2+12\,a^2c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*c*d*x + b*d)*sqrt(c*x^2 + b*x + a)/(4*c^5*d^2*x^8 + 16*b*c^
4*d^2*x^7 + (25*b^2*c^3 + 12*a*c^4)*d^2*x^6 + (19*b^3*c^2 + 36*a*b*c^3)*d^2
*x^5 + a^3*b^2*d^2 + (7*b^4*c + 39*a*b^2*c^2 + 12*a^2*c^3)*d^2*x^4 + (b^5 +
 18*a*b^3*c + 24*a^2*b*c^2)*d^2*x^3 + (3*a*b^4 + 15*a^2*b^2*c + 4*a^3*c^2)*
d^2*x^2 + (3*a^2*b^3 + 4*a^3*b*c)*d^2*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d(b+2cx))^{\frac{3}{2}}\left(a+bx+cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)**(3/2)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Integral(1/((d*(b + 2*c*x))**(3/2)*(a + b*x + c*x**2)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2\,cdx+bd)^{\frac{3}{2}}\left(cx^2+bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*c*d*x+b*d)^(3/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((2*c*d*x + b*d)^(3/2)*(c*x^2 + b*x + a)^(5/2)), x)
```

3.1400 $\int \frac{(ce+dex)^{11/2}}{\sqrt{1-c^2-2cdx-d^2x^2}}\,dx$

Optimal. Leaf size=170

$$\frac{30e^{11/2}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{ce+dex}}{\sqrt{e}}\right),-1\right)}{77d}-\frac{30e^5\sqrt{-c^2-2cdx-d^2x^2+1}\sqrt{ce+dex}}{77d}-\frac{18e^3\sqrt{-c^2-2cdx-d^2x^2+1}(ce+dex)}{77d}$$

```
[Out] (-30*e^5*Sqrt[c*e + d*e*x]*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2])/(77*d) - (18*
e^3*(c*e + d*e*x)^(5/2)*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2])/(77*d) - (2*e*(c
*e + d*e*x)^(9/2)*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2])/(11*d) + (30*e^(11/2)*
EllipticF[ArcSin[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/(77*d)
```

Rubi [A] time = 0.145616, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {692, 689, 221}

$$-\frac{30e^5\sqrt{-c^2-2cdx-d^2x^2+1}\sqrt{ce+dex}}{77d}-\frac{18e^3\sqrt{-c^2-2cdx-d^2x^2+1}(ce+dex)^{5/2}}{77d}-\frac{2e\sqrt{-c^2-2cdx-d^2x^2+1}(ce+de}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(11/2)/Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2],x]
```

```
[Out] (-30*e^5*Sqrt[c*e + d*e*x]*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2])/(77*d) - (18*
e^3*(c*e + d*e*x)^(5/2)*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2])/(77*d) - (2*e*(c
*e + d*e*x)^(9/2)*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2])/(11*d) + (30*e^(11/2)*
EllipticF[ArcSin[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/(77*d)
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ce+dex)^{11/2}}{\sqrt{1-c^2-2cdx-d^2x^2}}dx = -\frac{2e(ce+dex)^{9/2}\sqrt{1-c^2-2cdx-d^2x^2}}{11d} + \frac{1}{11}\left(9e^2\right)\int \frac{(ce+dex)^{7/2}}{\sqrt{1-c^2-2cdx-d^2x^2}}dx$$

$$= -\frac{18e^3(ce+dex)^{5/2}\sqrt{1-c^2-2cdx-d^2x^2}}{77d} - \frac{2e(ce+dex)^{9/2}\sqrt{1-c^2-2cdx-d^2x^2}}{11d} + \frac{1}{77}$$

$$= -\frac{30e^5\sqrt{ce+dex}\sqrt{1-c^2-2cdx-d^2x^2}}{77d} - \frac{18e^3(ce+dex)^{5/2}\sqrt{1-c^2-2cdx-d^2x^2}}{77d} - 2e$$

$$= -\frac{30e^5\sqrt{ce+dex}\sqrt{1-c^2-2cdx-d^2x^2}}{77d} - \frac{18e^3(ce+dex)^{5/2}\sqrt{1-c^2-2cdx-d^2x^2}}{77d} - 2e$$

$$= -\frac{30e^5\sqrt{ce+dex}\sqrt{1-c^2-2cdx-d^2x^2}}{77d} - \frac{18e^3(ce+dex)^{5/2}\sqrt{1-c^2-2cdx-d^2x^2}}{77d} - 2e$$

Mathematica [C] time = 0.101802, size = 125, normalized size = 0.74

$$-\frac{2e^5\sqrt{e(c+dx)}\left(\sqrt{-c^2-2cdx-d^2x^2+1}\left(c^2\left(42d^2x^2+9\right)+28c^3dx+7c^4+2cdx\left(14d^2x^2+9\right)+7d^4x^4+9d^2x^2+15\right)\right.}{77d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(11/2)/Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2],x]
```

```
[Out] (-2*e^5*Sqrt[e*(c + d*x)]*(Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2]*(15 + 7*c^4 +
28*c^3*d*x + 9*d^2*x^2 + 7*d^4*x^4 + 2*c*d*x*(9 + 14*d^2*x^2) + c^2*(9 + 42
*d^2*x^2)) - 15*Hypergeometric2F1[1/4, 1/2, 5/4, (c + d*x)^2]))/(77*d)
```

Maple [B] time = 0.384, size = 641, normalized size = 3.8

$$-\frac{e^5}{2310\,d\left(x^3d^3+3\,x^2cd^2+3\,xc^2d+c^3-dx-c\right)}\sqrt{e\,(dx+c)}\sqrt{-d^2x^2-2\,cdx-c^2+1}\left(-900\,c+120\,c^5+924\,\sqrt{2\,dx+2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(11/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x)
```

```
[Out] -1/2310*(e*(d*x+c))^(1/2)*(-d^2*x^2-2*c*d*x-c^2+1)^(1/2)*e^5*(-900*c+120*c^
5+924*(2*d*x+2*c+2)^(1/2)*(-2*d*x-2*c+2)^(1/2)*(d*x+c)^(1/2)*EllipticE(1/2*
(-2*d*x-2*c+2)^(1/2),2^(1/2))*c^3-924*(2*d*x+2*c+2)^(1/2)*(-d*x-c)^(1/2)*(-
2*d*x-2*c+2)^(1/2)*EllipticE(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2))*c+924*(2*d*x+
2*c+2)^(1/2)*(-2*d*x-2*c+2)^(1/2)*(d*x+c)^(1/2)*EllipticE(1/2*(-2*d*x-2*c+2
)^(1/2),2^(1/2))*c-900*d*x+2940*x^6*c*d^6+8820*x^5*c^2*d^5+14700*x^4*c^3*d^
4+14700*x^3*c^4*d^3+600*x^4*c*d^4+8820*x^2*c^5*d^2+1200*x^3*c^2*d^3+2940*x*
c^6*d+1200*x^2*c^3*d^2+600*x*c^4*d+1080*x^2*c*d^2+420*c^7+65*(2*d*x+2*c+2)^
(1/2)*(-d*x-c)^(1/2)*(-2*d*x-2*c+2)^(1/2)*EllipticF(1/2*(2*d*x+2*c+2)^(1/2)
,2^(1/2))+1155*(2*d*x+2*c+2)^(1/2)*(-d*x-c)^(1/2)*(-2*d*x-2*c+2)^(1/2)*Elli
pticE(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2))-385*(2*d*x+2*c+2)^(1/2)*(-2*d*x-2*c+
2)^(1/2)*(d*x+c)^(1/2)*EllipticF(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2))-1155*(2*
d*x+2*c+2)^(1/2)*(-2*d*x-2*c+2)^(1/2)*(d*x+c)^(1/2)*EllipticE(1/2*(-2*d*x-2
*c+2)^(1/2),2^(1/2))+360*c^3+1080*x*c^2*d-924*(2*d*x+2*c+2)^(1/2)*(-d*x-c)^
(1/2)*(-2*d*x-2*c+2)^(1/2)*EllipticE(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2))*c^3+4
```

```
20*x^7*d^7+120*x^5*d^5+360*x^3*d^3)/d/(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3-d*
x-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^{\frac{11}{2}}}{\sqrt{-d^2x^2-2\,cdx-c^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(11/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((d*e*x + c*e)^(11/2)/sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(d^5e^5x^5+5\,cd^4e^5x^4+10\,c^2d^3e^5x^3+10\,c^3d^2e^5x^2+5\,c^4de^5x+c^5e^5\right)\sqrt{-d^2x^2-2\,cdx-c^2+1}\sqrt{dex+ce}}{d^2x^2+2\,cdx+c^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(11/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(-(d^5*e^5*x^5 + 5*c*d^4*e^5*x^4 + 10*c^2*d^3*e^5*x^3 + 10*c^3*d^2*
e^5*x^2 + 5*c^4*d*e^5*x + c^5*e^5)*sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1)*sqrt(
d*e*x + c*e)/(d^2*x^2 + 2*c*d*x + c^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(11/2)/(-d**2*x**2-2*c*d*x-c**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^{\frac{11}{2}}}{\sqrt{-d^2x^2-2\,cdx-c^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(11/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((d*e*x + c*e)^(11/2)/sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1), x)
```

3.1401 $\int \frac{(ce+dex)^{7/2}}{\sqrt{1-c^2-2cdx-d^2x^2}}\,dx$

Optimal. Leaf size=124

$$\frac{10e^{7/2}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{ce+dex}}{\sqrt{e}}\right),-1\right)}{21d}-\frac{10e^3\sqrt{-c^2-2cdx-d^2x^2+1}\sqrt{ce+dex}}{21d}-\frac{2e\sqrt{-c^2-2cdx-d^2x^2+1}(ce+dex)}{7d}$$

```
[Out] (-10*e^3*Sqrt[c*e + d*e*x]*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2])/(21*d) - (2*e
*(c*e + d*e*x)^(5/2)*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2])/(7*d) + (10*e^(7/2)
*EllipticF[ArcSin[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/(21*d)
```

Rubi [A] time = 0.0952692, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {692, 689, 221}

$$-\frac{10e^3\sqrt{-c^2-2cdx-d^2x^2+1}\sqrt{ce+dex}}{21d}-\frac{2e\sqrt{-c^2-2cdx-d^2x^2+1}(ce+dex)^{5/2}}{7d}+\frac{10e^{7/2}F\left(\sin^{-1}\left(\frac{\sqrt{ce+dxe}}{\sqrt{e}}\right)\middle|-1\right)}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(7/2)/Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2],x]
```

```
[Out] (-10*e^3*Sqrt[c*e + d*e*x]*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2])/(21*d) - (2*e
*(c*e + d*e*x)^(5/2)*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2])/(7*d) + (10*e^(7/2)
*EllipticF[ArcSin[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/(21*d)
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ce+dex)^{7/2}}{\sqrt{1-c^2-2cdx-d^2x^2}}\,dx = -\frac{2e(ce+dex)^{5/2}\sqrt{1-c^2-2cdx-d^2x^2}}{7d} + \frac{1}{7}\left(5e^2\right)\int \frac{(ce+dex)^{3/2}}{\sqrt{1-c^2-2cdx-d^2x^2}}\,dx$$
$$= -\frac{10e^3\sqrt{ce+dex}\sqrt{1-c^2-2cdx-d^2x^2}}{21d} - \frac{2e(ce+dex)^{5/2}\sqrt{1-c^2-2cdx-d^2x^2}}{7d} + \frac{1}{21}\left(5e^4\right.$$
$$= -\frac{10e^3\sqrt{ce+dex}\sqrt{1-c^2-2cdx-d^2x^2}}{21d} - \frac{2e(ce+dex)^{5/2}\sqrt{1-c^2-2cdx-d^2x^2}}{7d} + \frac{\left(10e^3\right)\mathrm{S}}{}$$
$$= -\frac{10e^3\sqrt{ce+dex}\sqrt{1-c^2-2cdx-d^2x^2}}{21d} - \frac{2e(ce+dex)^{5/2}\sqrt{1-c^2-2cdx-d^2x^2}}{7d} + \frac{10e^{7/2}F}{}$$

Mathematica [C] time = 0.063168, size = 86, normalized size = 0.69

$$-\frac{2e^3\sqrt{e(c+dx)}\left(\sqrt{-c^2-2cdx-d^2x^2+1}\left(3c^2+6cdx+3d^2x^2+5\right)-5\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};(c+dx)^2\right)\right)}{21d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(7/2)/Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2],x]
```

```
[Out] (-2*e^3*Sqrt[e*(c + d*x)]*(Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2]*(5 + 3*c^2 + 6
*c*d*x + 3*d^2*x^2) - 5*Hypergeometric2F1[1/4, 1/2, 5/4, (c + d*x)^2]))/(21
*d)
```

Maple [B] time = 0.179, size = 463, normalized size = 3.7

$$\frac{e^3}{210\,d\left(x^3d^3+3\,x^2cd^2+3\,xc^2d+c^3-dx-c\right)}\sqrt{e\,(dx+c)}\sqrt{-d^2x^2-2\,cdx-c^2+1}\left(-60\,x^5d^5-300\,x^4cd^4-600\,x^3c^2d^3-6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(7/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x)
```

```
[Out] 1/210*(e*(d*x+c))^(1/2)*(-d^2*x^2-2*c*d*x-c^2+1)^(1/2)*e^3*(-60*x^5*d^5-300
*x^4*c*d^4-600*x^3*c^2*d^3-600*x^2*c^3*d^2-40*x^3*d^3-300*x*c^4*d+84*(2*d*x
+2*c+2)^(1/2)*(-d*x-c)^(1/2)*(-2*d*x-2*c+2)^(1/2)*EllipticE(1/2*(2*d*x+2*c+
2)^(1/2),2^(1/2))*c-84*(2*d*x+2*c+2)^(1/2)*(-2*d*x-2*c+2)^(1/2)*(d*x+c)^(1/
2)*EllipticE(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2))*c-120*x^2*c*d^2-60*c^5-105*(
2*d*x+2*c+2)^(1/2)*(-d*x-c)^(1/2)*(-2*d*x-2*c+2)^(1/2)*EllipticE(1/2*(2*d*x
+2*c+2)^(1/2),2^(1/2))+105*(2*d*x+2*c+2)^(1/2)*(-2*d*x-2*c+2)^(1/2)*(d*x+c)
^(1/2)*EllipticE(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2))-15*(2*d*x+2*c+2)^(1/2)*(
-d*x-c)^(1/2)*(-2*d*x-2*c+2)^(1/2)*EllipticF(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2
))+35*(2*d*x+2*c+2)^(1/2)*(-2*d*x-2*c+2)^(1/2)*(d*x+c)^(1/2)*EllipticF(1/2*
(-2*d*x-2*c+2)^(1/2),2^(1/2))-120*x*c^2*d-40*c^3+100*d*x+100*c)/d/(d^3*x^3+
3*c*d^2*x^2+3*c^2*d*x+c^3-d*x-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^{\frac{7}{2}}}{\sqrt{-d^2x^2-2\,cdx-c^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate((d*e*x + c*e)^(7/2)/sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(d^3e^3x^3+3\,cd^2e^3x^2+3\,c^2de^3x+c^3e^3\right)\sqrt{-d^2x^2-2\,cdx-c^2+1}\sqrt{dex+ce}}{d^2x^2+2\,cdx+c^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(-(d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)*sqrt(-d
^2*x^2 - 2*c*d*x - c^2 + 1)*sqrt(d*e*x + c*e)/(d^2*x^2 + 2*c*d*x + c^2 - 1)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(7/2)/(-d**2*x**2-2*c*d*x-c**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^{\frac{7}{2}}}{\sqrt{-d^2x^2-2\,cdx-c^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((d*e*x + c*e)^(7/2)/sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1), x)
```

3.1402 $\int \frac{(ce+dex)^{3/2}}{\sqrt{1-c^2-2cdx-d^2x^2}}\,dx$

Optimal. Leaf size=78

$$\frac{2e^{3/2}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{ce+dex}}{\sqrt{e}}\right),-1\right)}{3d}-\frac{2e\sqrt{-c^2-2cdx-d^2x^2+1}\sqrt{ce+dex}}{3d}$$

```
[Out] (-2*e*Sqrt[c*e + d*e*x]*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2])/(3*d) + (2*e^(3/
2)*EllipticF[ArcSin[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/(3*d)
```

Rubi [A] time = 0.0587986, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {692, 689, 221}

$$\frac{2e^{3/2}F\left(\sin^{-1}\left(\frac{\sqrt{ce+dxe}}{\sqrt{e}}\right)\middle|-1\right)}{3d}-\frac{2e\sqrt{-c^2-2cdx-d^2x^2+1}\sqrt{ce+dex}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(3/2)/Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2],x]
```


```
[Out] (-2*e*Sqrt[c*e + d*e*x]*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2])/(3*d) + (2*e^(3/
2)*EllipticF[ArcSin[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/(3*d)
```


Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ce+dex)^{3/2}}{\sqrt{1-c^2-2cdx-d^2x^2}}\,dx = -\frac{2e\sqrt{ce+dex}\sqrt{1-c^2-2cdx-d^2x^2}}{3d} + \frac{1}{3}e^2\int \frac{1}{\sqrt{ce+dex}\sqrt{1-c^2-2cdx-d^2x^2}}\,dx$$

$$= -\frac{2e\sqrt{ce+dex}\sqrt{1-c^2-2cdx-d^2x^2}}{3d} + \frac{(2e)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{e^2}}}\,dx, x, \sqrt{ce+dex}\right)}{3d}$$

$$= -\frac{2e\sqrt{ce+dex}\sqrt{1-c^2-2cdx-d^2x^2}}{3d} + \frac{2e^{3/2}F\left(\sin^{-1}\left(\frac{\sqrt{ce+dex}}{\sqrt{e}}\right)\middle|-1\right)}{3d}$$

Mathematica [C] time = 0.0200724, size = 54, normalized size = 0.69

$$-\frac{2e\sqrt{e(c+dx)}\left(\sqrt{1-(c+dx)^2} - {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};(c+dx)^2\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(3/2)/Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2],x]
```

```
[Out] (-2*e*Sqrt[e*(c + d*x)]*(Sqrt[1 - (c + d*x)^2] - Hypergeometric2F1[1/4, 1/2
, 5/4, (c + d*x)^2]))/(3*d)
```

Maple [B] time = 0.172, size = 308, normalized size = 4.

$$-\frac{e}{6d\left(x^3d^3+3x^2cd^2+3xc^2d+c^3-dx-c\right)}\sqrt{e(dx+c)}\sqrt{-d^2x^2-2cdx-c^2+1}\left(4x^3d^3+12x^2cd^2+\sqrt{2dx+2c+2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(3/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x)
```

```
[Out] -1/6*(e*(d*x+c))^(1/2)*(-d^2*x^2-2*c*d*x-c^2+1)^(1/2)*e*(4*x^3*d^3+12*x^2*c
*d^2+(2*d*x+2*c+2)^(1/2)*(-d*x-c)^(1/2)*(-2*d*x-2*c+2)^(1/2)*EllipticF(1/2*
(2*d*x+2*c+2)^(1/2),2^(1/2))+3*(2*d*x+2*c+2)^(1/2)*(-d*x-c)^(1/2)*(-2*d*x-2
*c+2)^(1/2)*EllipticE(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2))-(2*d*x+2*c+2)^(1/2)*
(-2*d*x-2*c+2)^(1/2)*(d*x+c)^(1/2)*EllipticF(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/
2))-3*(2*d*x+2*c+2)^(1/2)*(-2*d*x-2*c+2)^(1/2)*(d*x+c)^(1/2)*EllipticE(1/2*
(-2*d*x-2*c+2)^(1/2),2^(1/2))+12*x*c^2*d+4*c^3-4*d*x-4*c)/d/(d^3*x^3+3*c*d^
2*x^2+3*c^2*d*x+c^3-d*x-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^{\frac{3}{2}}}{\sqrt{-d^2x^2-2\,cdx-c^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate((d*e*x + c*e)^(3/2)/sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-d^2x^2 - 2\,cdx - c^2 + 1}(dex + ce)^{\frac{3}{2}}}{d^2x^2 + 2\,cdx + c^2 - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(-sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1)*(d*e*x + c*e)^(3/2)/(d^2*x^2 +
 2*c*d*x + c^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e(c + dx))^{\frac{3}{2}}}{\sqrt{-(c + dx - 1)(c + dx + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(3/2)/(-d**2*x**2-2*c*d*x-c**2+1)**(1/2),x)
```

```
[Out] Integral((e*(c + d*x))**(3/2)/sqrt(-(c + d*x - 1)*(c + d*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex + ce)^{\frac{3}{2}}}{\sqrt{-d^2x^2 - 2\,cdx - c^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((d*e*x + c*e)^(3/2)/sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1), x)
```

3.1403 $\int \frac{1}{\sqrt{ce+dex}\sqrt{1-c^2-2cdx-d^2x^2}}\,dx$

Optimal. Leaf size=31

$$\frac{2\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{ce+dex}}{\sqrt{e}}\right),-1\right)}{d\sqrt{e}}$$

```
[Out] (2*EllipticF[ArcSin[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/(d*Sqrt[e])
```

Rubi [A] time = 0.0291083, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {689, 221}

$$\frac{2F\left(\sin^{-1}\left(\frac{\sqrt{ce+dxe}}{\sqrt{e}}\right)\middle|-1\right)}{d\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*e + d*e*x]*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2]),x]
```

```
[Out] (2*EllipticF[ArcSin[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/(d*Sqrt[e])
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{ce+dex}\sqrt{1-c^2-2cdx-d^2x^2}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{e^2}}}\,dx, x, \sqrt{ce+dex}\right)}{de}\\ &= \frac{2F\left(\sin^{-1}\left(\frac{\sqrt{ce+dex}}{\sqrt{e}}\right)\middle|-1\right)}{d\sqrt{e}}\end{aligned}$$

Mathematica [C] time = 0.0164825, size = 38, normalized size = 1.23

$$\frac{2(c+dx)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};(c+dx)^2\right)}{d\sqrt{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*e + d*e*x]*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2]),x]
```

```
[Out] (2*(c + d*x)*Hypergeometric2F1[1/4, 1/2, 5/4, (c + d*x)^2])/(d*Sqrt[e*(c +
d*x)])
```

Maple [B] time = 0.23, size = 209, normalized size = 6.7

$$\frac{1}{6\,de\left(x^3d^3+3\,x^2cd^2+3\,xc^2d+c^3-dx-c\right)}\sqrt{e\left(dx+c\right)}\sqrt{-d^2x^2-2\,cdx-c^2+1}\sqrt{-2\,dx-2\,c+2}\sqrt{2\,dx+2\,c+2}\left(5\,\sqrt{dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)^(1/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x)
```

```
[Out] 1/6*(e*(d*x+c))^(1/2)*(-d^2*x^2-2*c*d*x-c^2+1)^(1/2)*(-2*d*x-2*c+2)^(1/2)*(
2*d*x+2*c+2)^(1/2)*(5*(d*x+c)^(1/2)*EllipticF(1/2*(-2*d*x-2*c+2)^(1/2),2^(1
/2))+3*(d*x+c)^(1/2)*EllipticE(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2))-(-d*x-c)^(
1/2)*EllipticF(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2))-3*(-d*x-c)^(1/2)*EllipticE(
1/2*(2*d*x+2*c+2)^(1/2),2^(1/2)))/d/e/(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3-d*
x-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-d^2x^2-2\,cdx-c^2+1}\sqrt{dex+ce}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^(1/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(1/(sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1)*sqrt(d*e*x + c*e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-d^2x^2-2\,cdx-c^2+1}\sqrt{dex+ce}}{d^3ex^3+3\,cd^2ex^2+\left(3\,c^2-1\right)dex+\left(c^3-c\right)e},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^(1/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral(-sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1)*sqrt(d*e*x + c*e)/(d^3*e*x^3 +
 3*c*d^2*e*x^2 + (3*c^2 - 1)*d*e*x + (c^3 - c)*e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e\left(c+dx\right)}\sqrt{-\left(c+dx-1\right)\left(c+dx+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)**(1/2)/(-d**2*x**2-2*c*d*x-c**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(e*(c + d*x))*sqrt(-(c + d*x - 1)*(c + d*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-d^2x^2-2\,cdx-c^2+1}\sqrt{dex+ce}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^(1/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(1/(sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1)*sqrt(d*e*x + c*e)), x)
```

3.1404 $\int \frac{1}{(ce+dex)^{5/2}\sqrt{1-c^2-2cdx-d^2x^2}}\,dx$

Optimal. Leaf size=80

$$\frac{2\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{ce+dex}}{\sqrt{e}}\right),-1\right)}{3de^{5/2}}-\frac{2\sqrt{-c^2-2cdx-d^2x^2+1}}{3de(ce+dex)^{3/2}}$$

```
[Out] (-2*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2])/(3*d*e*(c*e + d*e*x)^(3/2)) + (2*Ell
ipticF[ArcSin[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/(3*d*e^(5/2))
```

Rubi [A] time = 0.0579532, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {693, 689, 221}

$$\frac{2F\left(\sin^{-1}\left(\frac{\sqrt{ce+dxe}}{\sqrt{e}}\right)\middle|-1\right)}{3de^{5/2}}-\frac{2\sqrt{-c^2-2cdx-d^2x^2+1}}{3de(ce+dex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*e + d*e*x)^(5/2)*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2]),x]
```

```
[Out] (-2*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2])/(3*d*e*(c*e + d*e*x)^(3/2)) + (2*Ell
ipticF[ArcSin[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/(3*d*e^(5/2))
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(ce+dex)^{5/2}\sqrt{1-c^2-2cdx-d^2x^2}}dx = -\frac{2\sqrt{1-c^2-2cdx-d^2x^2}}{3de(ce+dex)^{3/2}} + \frac{\int \frac{1}{\sqrt{ce+dex}\sqrt{1-c^2-2cdx-d^2x^2}}dx}{3e^2}$$

$$= -\frac{2\sqrt{1-c^2-2cdx-d^2x^2}}{3de(ce+dex)^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{e^2}}}dx, x, \sqrt{ce+dex}\right)}{3de^3}$$

$$= -\frac{2\sqrt{1-c^2-2cdx-d^2x^2}}{3de(ce+dex)^{3/2}} + \frac{2F\left(\sin^{-1}\left(\frac{\sqrt{ce+dex}}{\sqrt{e}}\right)\middle|-1\right)}{3de^{5/2}}$$

Mathematica [C] time = 0.0201096, size = 40, normalized size = 0.5

$$-\frac{2(c+dx)\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};(c+dx)^2\right)}{3d(e(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*e + d*e*x)^(5/2)*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2]),x]
```

```
[Out] (-2*(c + d*x)*Hypergeometric2F1[-3/4, 1/2, 1/4, (c + d*x)^2])/(3*d*(e*(c +
d*x))^(5/2))
```

Maple [B] time = 0.286, size = 494, normalized size = 6.2

$$\frac{1}{6\,e^3\,(dx+c)^2\left(d^2x^2+2\,cdx+c^2-1\right)d}\left(5\,\sqrt{-2\,dx-2\,c+2}\sqrt{dx+c}\sqrt{2\,dx+2\,c+2}\text{EllipticF}\left(1/2\,\sqrt{-2\,dx-2\,c+2},\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)^(5/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x)
```

```
[Out] 1/6*(5*(-2*d*x-2*c+2)^(1/2)*(d*x+c)^(1/2)*(2*d*x+2*c+2)^(1/2)*EllipticF(1/2
*(-2*d*x-2*c+2)^(1/2),2^(1/2))*x*d+3*(-2*d*x-2*c+2)^(1/2)*(d*x+c)^(1/2)*(2*
d*x+2*c+2)^(1/2)*EllipticE(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2))*x*d+3*(-2*d*x-
2*c+2)^(1/2)*(2*d*x+2*c+2)^(1/2)*(-d*x-c)^(1/2)*EllipticF(1/2*(2*d*x+2*c+2)
^(1/2),2^(1/2))*x*d-3*(-2*d*x-2*c+2)^(1/2)*(2*d*x+2*c+2)^(1/2)*(-d*x-c)^(1/
2)*EllipticE(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2))*x*d+5*(-2*d*x-2*c+2)^(1/2)*(d
*x+c)^(1/2)*(2*d*x+2*c+2)^(1/2)*EllipticF(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2))
*c+3*(2*d*x+2*c+2)^(1/2)*(-2*d*x-2*c+2)^(1/2)*(d*x+c)^(1/2)*EllipticE(1/2*(
-2*d*x-2*c+2)^(1/2),2^(1/2))*c+3*(-2*d*x-2*c+2)^(1/2)*(2*d*x+2*c+2)^(1/2)*(
-d*x-c)^(1/2)*EllipticF(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2))*c-3*(2*d*x+2*c+2)^
(1/2)*(-d*x-c)^(1/2)*(-2*d*x-2*c+2)^(1/2)*EllipticE(1/2*(2*d*x+2*c+2)^(1/2)
,2^(1/2))*c-4*d^2*x^2-8*c*d*x-4*c^2+4)/e^3*(-d^2*x^2-2*c*d*x-c^2+1)^(1/2)*(
e*(d*x+c))^(1/2)/(d*x+c)^2/(d^2*x^2+2*c*d*x+c^2-1)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-d^2x^2-2\,cdx-c^2+1}(dex+ce)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^(5/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(1/(sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1)*(d*e*x + c*e)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-d^2x^2-2\,cdx-c^2+1}\sqrt{dex+ce}}{d^5e^3x^5+5\,cd^4e^3x^4+\left(10\,c^2-1\right)d^3e^3x^3+\left(10\,c^3-3\,c\right)d^2e^3x^2+\left(5\,c^4-3\,c^2\right)de^3x+\left(c^5-c^3\right)e^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^(5/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral(-sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1)*sqrt(d*e*x + c*e)/(d^5*e^3*x^5
 + 5*c*d^4*e^3*x^4 + (10*c^2 - 1)*d^3*e^3*x^3 + (10*c^3 - 3*c)*d^2*e^3*x^2
+ (5*c^4 - 3*c^2)*d*e^3*x + (c^5 - c^3)*e^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\left(c+dx\right)\right)^{\frac{5}{2}}\sqrt{-\left(c+dx-1\right)\left(c+dx+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)**(5/2)/(-d**2*x**2-2*c*d*x-c**2+1)**(1/2),x)
```

```
[Out] Integral(1/((e*(c + d*x))**(5/2)*sqrt(-(c + d*x - 1)*(c + d*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-d^2x^2-2\,cdx-c^2+1}\left(dex+ce\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^(5/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(1/(sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1)*(d*e*x + c*e)^(5/2)), x)
```

3.1405 $\int \frac{1}{(ce+dex)^{9/2}\sqrt{1-c^2-2cdx-d^2x^2}}\,dx$

Optimal. Leaf size=126

$$\frac{10\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{ce+dex}}{\sqrt{e}}\right),-1\right)}{21de^{9/2}}-\frac{10\sqrt{-c^2-2cdx-d^2x^2+1}}{21de^3(ce+dex)^{3/2}}-\frac{2\sqrt{-c^2-2cdx-d^2x^2+1}}{7de(ce+dex)^{7/2}}$$

```
[Out] (-2*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2])/(7*d*e*(c*e + d*e*x)^(7/2)) - (10*Sq
rt[1 - c^2 - 2*c*d*x - d^2*x^2])/(21*d*e^3*(c*e + d*e*x)^(3/2)) + (10*Ellip
ticF[ArcSin[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/(21*d*e^(9/2))
```

Rubi [A] time = 0.0858568, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {693, 689, 221}

$$-\frac{10\sqrt{-c^2-2cdx-d^2x^2+1}}{21de^3(ce+dex)^{3/2}}-\frac{2\sqrt{-c^2-2cdx-d^2x^2+1}}{7de(ce+dex)^{7/2}}+\frac{10F\left(\sin^{-1}\left(\frac{\sqrt{ce+dxe}}{\sqrt{e}}\right)\middle|-1\right)}{21de^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*e + d*e*x)^(9/2)*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2]),x]
```

```
[Out] (-2*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2])/(7*d*e*(c*e + d*e*x)^(7/2)) - (10*Sq
rt[1 - c^2 - 2*c*d*x - d^2*x^2])/(21*d*e^3*(c*e + d*e*x)^(3/2)) + (10*Ellip
ticF[ArcSin[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/(21*d*e^(9/2))
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(ce+dex)^{9/2}\sqrt{1-c^2-2cdx-d^2x^2}}\,dx = -\frac{2\sqrt{1-c^2-2cdx-d^2x^2}}{7de(ce+dex)^{7/2}} + \frac{5\int \frac{1}{(ce+dex)^{5/2}\sqrt{1-c^2-2cdx-d^2x^2}}dx}{7e^2}$$

$$= -\frac{2\sqrt{1-c^2-2cdx-d^2x^2}}{7de(ce+dex)^{7/2}} - \frac{10\sqrt{1-c^2-2cdx-d^2x^2}}{21de^3(ce+dex)^{3/2}} + \frac{5\int \frac{1}{\sqrt{ce+dex}\sqrt{1-c^2-2cdx-}}}{21e^4}$$

$$= -\frac{2\sqrt{1-c^2-2cdx-d^2x^2}}{7de(ce+dex)^{7/2}} - \frac{10\sqrt{1-c^2-2cdx-d^2x^2}}{21de^3(ce+dex)^{3/2}} + \frac{10\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{e^2}}}d\right.}{21de^5}$$

$$= -\frac{2\sqrt{1-c^2-2cdx-d^2x^2}}{7de(ce+dex)^{7/2}} - \frac{10\sqrt{1-c^2-2cdx-d^2x^2}}{21de^3(ce+dex)^{3/2}} + \frac{10F\left(\sin^{-1}\left(\frac{\sqrt{ce+dex}}{\sqrt{e}}\right)\right|}{21de^{9/2}}$$

Mathematica [C] time = 0.0233428, size = 40, normalized size = 0.32

$$-\frac{2(c+dx)\,{}_2F_1\left(-\frac{7}{4},\frac{1}{2};-\frac{3}{4};(c+dx)^2\right)}{7d(e(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*e + d*e*x)^(9/2)*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2]),x]
```

```
[Out] (-2*(c + d*x)*Hypergeometric2F1[-7/4, 1/2, -3/4, (c + d*x)^2])/(7*d*(e*(c +
 d*x))^(9/2))
```

Maple [B] time = 0.254, size = 1002, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)^(9/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x)
```

```
[Out] 1/42*(12+8*c^2+21*(2*d*x+2*c+2)^(1/2)*(-2*d*x-2*c+2)^(1/2)*(d*x+c)^(1/2)*El
lipticE(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2))*c^3+75*(2*d*x+2*c+2)^(1/2)*(-d*x-
c)^(1/2)*(-2*d*x-2*c+2)^(1/2)*EllipticF(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2))*x^
2*c*d^2-63*(2*d*x+2*c+2)^(1/2)*(-d*x-c)^(1/2)*(-2*d*x-2*c+2)^(1/2)*Elliptic
E(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2))*x^2*c*d^2+105*(2*d*x+2*c+2)^(1/2)*(-2*d*
x-2*c+2)^(1/2)*(d*x+c)^(1/2)*EllipticF(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2))*x^
2*c*d^2+63*(2*d*x+2*c+2)^(1/2)*(-2*d*x-2*c+2)^(1/2)*(d*x+c)^(1/2)*EllipticE
(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2))*x^2*c*d^2+75*(2*d*x+2*c+2)^(1/2)*(-d*x-c
)^(1/2)*(-2*d*x-2*c+2)^(1/2)*EllipticF(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2))*x*c
^2*d-63*(2*d*x+2*c+2)^(1/2)*(-d*x-c)^(1/2)*(-2*d*x-2*c+2)^(1/2)*EllipticE(1
/2*(2*d*x+2*c+2)^(1/2),2^(1/2))*x*c^2*d+105*(2*d*x+2*c+2)^(1/2)*(-2*d*x-2*c
+2)^(1/2)*(d*x+c)^(1/2)*EllipticF(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2))*x*c^2*d
+63*(2*d*x+2*c+2)^(1/2)*(-2*d*x-2*c+2)^(1/2)*(d*x+c)^(1/2)*EllipticE(1/2*(-
2*d*x-2*c+2)^(1/2),2^(1/2))*x*c^2*d-20*c^4+25*(2*d*x+2*c+2)^(1/2)*(-d*x-c)^
(1/2)*(-2*d*x-2*c+2)^(1/2)*EllipticF(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2))*x^3*d
^3+8*d^2*x^2-80*x^3*c*d^3-80*x*c^3*d+35*(2*d*x+2*c+2)^(1/2)*(-2*d*x-2*c+2)^
(1/2)*(d*x+c)^(1/2)*EllipticF(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2))*c^3+16*c*d*
x+25*(2*d*x+2*c+2)^(1/2)*(-d*x-c)^(1/2)*(-2*d*x-2*c+2)^(1/2)*EllipticF(1/2*
```

```
(2*d*x+2*c+2)^(1/2),2^(1/2))*c^3-120*c^2*d^2*x^2-21*(2*d*x+2*c+2)^(1/2)*(-d
*x-c)^(1/2)*(-2*d*x-2*c+2)^(1/2)*EllipticE(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2))
*c^3-20*d^4*x^4+21*(2*d*x+2*c+2)^(1/2)*(-2*d*x-2*c+2)^(1/2)*(d*x+c)^(1/2)*E
llipticE(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2))*x^3*d^3-21*(2*d*x+2*c+2)^(1/2)*(
-d*x-c)^(1/2)*(-2*d*x-2*c+2)^(1/2)*EllipticE(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2
))*x^3*d^3+35*(2*d*x+2*c+2)^(1/2)*(-2*d*x-2*c+2)^(1/2)*(d*x+c)^(1/2)*Ellipt
icF(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2))*x^3*d^3)/e^5*(-d^2*x^2-2*c*d*x-c^2+1)
^(1/2)*(e*(d*x+c))^(1/2)/(d*x+c)^4/(d^2*x^2+2*c*d*x+c^2-1)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-d^2x^2 - 2\,cdx - c^2 + 1}(dex + ce)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^(9/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(1/(sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1)*(d*e*x + c*e)^(9/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-d^2x^2 - 2\,cdx - c^2 + 1}\sqrt{dex + ce}}{d^7e^5x^7 + 7\,cd^6e^5x^6 + \left(21\,c^2 - 1\right)d^5e^5x^5 + 5\left(7\,c^3 - c\right)d^4e^5x^4 + 5\left(7\,c^4 - 2\,c^2\right)d^3e^5x^3 + \left(21\,c^5 - 10\,c^3\right)d^2e^5x^2 \cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^(9/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral(-sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1)*sqrt(d*e*x + c*e)/(d^7*e^5*x^7
 + 7*c*d^6*e^5*x^6 + (21*c^2 - 1)*d^5*e^5*x^5 + 5*(7*c^3 - c)*d^4*e^5*x^4 +
 5*(7*c^4 - 2*c^2)*d^3*e^5*x^3 + (21*c^5 - 10*c^3)*d^2*e^5*x^2 + (7*c^6 - 5
*c^4)*d*e^5*x + (c^7 - c^5)*e^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)**(9/2)/(-d**2*x**2-2*c*d*x-c**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-d^2x^2 - 2\,cdx - c^2 + 1}(dex + ce)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^(9/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(1/(sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1)*(d*e*x + c*e)^(9/2)), x)
```

3.1406 $\int \frac{1}{(ce+dex)^{13/2}\sqrt{1-c^2-2cdx-d^2x^2}}\,dx$

Optimal. Leaf size=172

$$\frac{30\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{ce+dex}}{\sqrt{e}}\right),-1\right)}{77de^{13/2}}-\frac{30\sqrt{-c^2-2cdx-d^2x^2+1}}{77de^5(ce+dex)^{3/2}}-\frac{18\sqrt{-c^2-2cdx-d^2x^2+1}}{77de^3(ce+dex)^{7/2}}-\frac{2\sqrt{-c^2-2cdx-d^2x^2}}{11de(ce+dex)^{11/2}}$$

```
[Out] (-2*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2])/(11*d*e*(c*e + d*e*x)^(11/2)) - (18*
Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2])/(77*d*e^3*(c*e + d*e*x)^(7/2)) - (30*Sqr
t[1 - c^2 - 2*c*d*x - d^2*x^2])/(77*d*e^5*(c*e + d*e*x)^(3/2)) + (30*Ellipt
icF[ArcSin[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/(77*d*e^(13/2))
```

Rubi [A] time = 0.118387, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {693, 689, 221}

$$-\frac{30\sqrt{-c^2-2cdx-d^2x^2+1}}{77de^5(ce+dex)^{3/2}}-\frac{18\sqrt{-c^2-2cdx-d^2x^2+1}}{77de^3(ce+dex)^{7/2}}-\frac{2\sqrt{-c^2-2cdx-d^2x^2+1}}{11de(ce+dex)^{11/2}}+\frac{30F\left(\sin^{-1}\left(\frac{\sqrt{ce+dxe}}{\sqrt{e}}\right)\middle|-1\right)}{77de^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*e + d*e*x)^(13/2)*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2]),x]
```

```
[Out] (-2*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2])/(11*d*e*(c*e + d*e*x)^(11/2)) - (18*
Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2])/(77*d*e^3*(c*e + d*e*x)^(7/2)) - (30*Sqr
t[1 - c^2 - 2*c*d*x - d^2*x^2])/(77*d*e^5*(c*e + d*e*x)^(3/2)) + (30*Ellipt
icF[ArcSin[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/(77*d*e^(13/2))
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 689

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_
Symbol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[1/Sqrt[Simp[1 - (
b^2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2
- 4*a*c), 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(ce+dex)^{13/2}\sqrt{1-c^2-2cdx-d^2x^2}}\,dx = -\frac{2\sqrt{1-c^2-2cdx-d^2x^2}}{11de(ce+dex)^{11/2}} + \frac{9\int \frac{1}{(ce+dex)^{9/2}\sqrt{1-c^2-2cdx-d^2x^2}}dx}{11e^2}$$

$$= -\frac{2\sqrt{1-c^2-2cdx-d^2x^2}}{11de(ce+dex)^{11/2}} - \frac{18\sqrt{1-c^2-2cdx-d^2x^2}}{77de^3(ce+dex)^{7/2}} + \frac{45\int \frac{1}{(ce+dex)^{5/2}\sqrt{1-c^2-}}}{77e^4}$$

$$= -\frac{2\sqrt{1-c^2-2cdx-d^2x^2}}{11de(ce+dex)^{11/2}} - \frac{18\sqrt{1-c^2-2cdx-d^2x^2}}{77de^3(ce+dex)^{7/2}} - \frac{30\sqrt{1-c^2-2cdx-}}{77de^5(ce+dex)^{3/}}$$

$$= -\frac{2\sqrt{1-c^2-2cdx-d^2x^2}}{11de(ce+dex)^{11/2}} - \frac{18\sqrt{1-c^2-2cdx-d^2x^2}}{77de^3(ce+dex)^{7/2}} - \frac{30\sqrt{1-c^2-2cdx-}}{77de^5(ce+dex)^{3/}}$$

$$= -\frac{2\sqrt{1-c^2-2cdx-d^2x^2}}{11de(ce+dex)^{11/2}} - \frac{18\sqrt{1-c^2-2cdx-d^2x^2}}{77de^3(ce+dex)^{7/2}} - \frac{30\sqrt{1-c^2-2cdx-}}{77de^5(ce+dex)^{3/}}$$

Mathematica [C] time = 0.0259742, size = 40, normalized size = 0.23

$$-\frac{2(c+dx)\,_2F_1\left(-\frac{11}{4},\frac{1}{2};-\frac{7}{4};(c+dx)^2\right)}{11d(e(c+dx))^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*e + d*e*x)^(13/2)*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2]),x]
```

```
[Out] (-2*(c + d*x)*Hypergeometric2F1[-11/4, 1/2, -7/4, (c + d*x)^2])/(11*d*(e*(c
 + d*x))^(13/2))
```

Maple [B] time = 0.298, size = 1532, normalized size = 8.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)^(13/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x)
```

```
[Out] 1/462*(84+24*c^2-231*(2*d*x+2*c+2)^(1/2)*(-d*x-c)^(1/2)*(-2*d*x-2*c+2)^(1/2
)*EllipticE(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2))*c^5+385*(2*d*x+2*c+2)^(1/2)*(-
2*d*x-2*c+2)^(1/2)*(d*x+c)^(1/2)*EllipticF(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2)
)*c^5+72*c^4+295*(2*d*x+2*c+2)^(1/2)*(-d*x-c)^(1/2)*(-2*d*x-2*c+2)^(1/2)*El
lipticF(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2))*x^5*d^5+231*(2*d*x+2*c+2)^(1/2)*(-
2*d*x-2*c+2)^(1/2)*(d*x+c)^(1/2)*EllipticE(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2)
)*c^5+24*d^2*x^2-180*c^6-180*d^6*x^6-2700*x^2*c^4*d^2-3600*x^3*c^3*d^3-2700
*x^4*c^2*d^4-1080*x*c^5*d-1080*x^5*c*d^5-231*(2*d*x+2*c+2)^(1/2)*(-d*x-c)^(
1/2)*(-2*d*x-2*c+2)^(1/2)*EllipticE(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2))*x^5*d^
5+385*(2*d*x+2*c+2)^(1/2)*(-2*d*x-2*c+2)^(1/2)*(d*x+c)^(1/2)*EllipticF(1/2*
(-2*d*x-2*c+2)^(1/2),2^(1/2))*x^5*d^5+231*(2*d*x+2*c+2)^(1/2)*(-2*d*x-2*c+2
)^(1/2)*(d*x+c)^(1/2)*EllipticE(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2))*x^5*d^5+2
88*x^3*c*d^3+288*x*c^3*d+295*(2*d*x+2*c+2)^(1/2)*(-d*x-c)^(1/2)*(-2*d*x-2*c
+2)^(1/2)*EllipticF(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2))*c^5+48*c*d*x+432*c^2*d
^2*x^2+72*d^4*x^4+1155*(2*d*x+2*c+2)^(1/2)*(-2*d*x-2*c+2)^(1/2)*(d*x+c)^(1/
2)*EllipticE(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2))*x^4*c*d^4-2310*(2*d*x+2*c+2)
```

```
^(1/2)*(-d*x-c)^(1/2)*(-2*d*x-2*c+2)^(1/2)*EllipticE(1/2*(2*d*x+2*c+2)^(1/2
),2^(1/2))*x^3*c^2*d^3+3850*(2*d*x+2*c+2)^(1/2)*(-2*d*x-2*c+2)^(1/2)*(d*x+c
)^(1/2)*EllipticF(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2))*x^3*c^2*d^3+2310*(2*d*x
+2*c+2)^(1/2)*(-2*d*x-2*c+2)^(1/2)*(d*x+c)^(1/2)*EllipticE(1/2*(-2*d*x-2*c+
2)^(1/2),2^(1/2))*x^3*c^2*d^3+2950*(2*d*x+2*c+2)^(1/2)*(-d*x-c)^(1/2)*(-2*d
*x-2*c+2)^(1/2)*EllipticF(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2))*x^2*c^3*d^2-2310
*(2*d*x+2*c+2)^(1/2)*(-d*x-c)^(1/2)*(-2*d*x-2*c+2)^(1/2)*EllipticE(1/2*(2*d
*x+2*c+2)^(1/2),2^(1/2))*x^2*c^3*d^2+3850*(2*d*x+2*c+2)^(1/2)*(-2*d*x-2*c+2
)^(1/2)*(d*x+c)^(1/2)*EllipticF(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2))*x^2*c^3*d
^2+2310*(2*d*x+2*c+2)^(1/2)*(-2*d*x-2*c+2)^(1/2)*(d*x+c)^(1/2)*EllipticE(1/
2*(-2*d*x-2*c+2)^(1/2),2^(1/2))*x^2*c^3*d^2+1475*(2*d*x+2*c+2)^(1/2)*(-d*x-
c)^(1/2)*(-2*d*x-2*c+2)^(1/2)*EllipticF(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2))*x*
c^4*d-1155*(2*d*x+2*c+2)^(1/2)*(-d*x-c)^(1/2)*(-2*d*x-2*c+2)^(1/2)*Elliptic
E(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2))*x*c^4*d+1925*(2*d*x+2*c+2)^(1/2)*(-2*d*x
-2*c+2)^(1/2)*(d*x+c)^(1/2)*EllipticF(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2))*x*c
^4*d+1155*(2*d*x+2*c+2)^(1/2)*(-2*d*x-2*c+2)^(1/2)*(d*x+c)^(1/2)*EllipticE(
1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2))*x*c^4*d+1475*(2*d*x+2*c+2)^(1/2)*(-d*x-c)
^(1/2)*(-2*d*x-2*c+2)^(1/2)*EllipticF(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2))*x^4*
c*d^4+1925*(2*d*x+2*c+2)^(1/2)*(-2*d*x-2*c+2)^(1/2)*(d*x+c)^(1/2)*EllipticF
(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2))*x^4*c*d^4+2950*(2*d*x+2*c+2)^(1/2)*(-d*x
-c)^(1/2)*(-2*d*x-2*c+2)^(1/2)*EllipticF(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2))*x
^3*c^2*d^3-1155*(2*d*x+2*c+2)^(1/2)*(-d*x-c)^(1/2)*(-2*d*x-2*c+2)^(1/2)*Ell
ipticE(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2))*x^4*c*d^4)/e^7*(-d^2*x^2-2*c*d*x-c^
2+1)^(1/2)*(e*(d*x+c))^(1/2)/(d*x+c)^6/(d^2*x^2+2*c*d*x+c^2-1)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-d^2x^2 - 2\,cdx - c^2 + 1}(dex + ce)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^(13/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate(1/(sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1)*(d*e*x + c*e)^(13/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-d^2x^2 - 2\,cdx - c^2 + 1}\sqrt{dex}}{d^9e^7x^9 + 9\,cd^8e^7x^8 + \left(36\,c^2 - 1\right)d^7e^7x^7 + 7\left(12\,c^3 - c\right)d^6e^7x^6 + 21\left(6\,c^4 - c^2\right)d^5e^7x^5 + 7\left(18\,c^5 - 5\,c^3\right)d^4e^7x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^(13/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm=
"fricas")
```

```
[Out] integral(-sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1)*sqrt(d*e*x + c*e)/(d^9*e^7*x^9
 + 9*c*d^8*e^7*x^8 + (36*c^2 - 1)*d^7*e^7*x^7 + 7*(12*c^3 - c)*d^6*e^7*x^6
+ 21*(6*c^4 - c^2)*d^5*e^7*x^5 + 7*(18*c^5 - 5*c^3)*d^4*e^7*x^4 + 7*(12*c^6
 - 5*c^4)*d^3*e^7*x^3 + 3*(12*c^7 - 7*c^5)*d^2*e^7*x^2 + (9*c^8 - 7*c^6)*d*
e^7*x + (c^9 - c^7)*e^7), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)**(13/2)/(-d**2*x**2-2*c*d*x-c**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-d^2x^2 - 2\,cdx - c^2 + 1}(dex + ce)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^(13/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate(1/(sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1)*(d*e*x + c*e)^(13/2)), x)
```

3.1407 $\int \frac{(ce+dex)^{9/2}}{\sqrt{1-c^2-2cdx-d^2x^2}}\,dx$

Optimal. Leaf size=157

$$-\frac{14e^{9/2}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{ce+dex}}{\sqrt{e}}\right),-1\right)}{15d}-\frac{14e^3\sqrt{-c^2-2cdx-d^2x^2+1}(ce+dex)^{3/2}}{45d}-\frac{2e\sqrt{-c^2-2cdx-d^2x^2+1}(ce+}{9d}$$

```
[Out] (-14*e^3*(c*e + d*e*x)^(3/2)*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2])/(45*d) - (2
*e*(c*e + d*e*x)^(7/2)*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2])/(9*d) + (14*e^(9/
2)*EllipticE[ArcSin[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/(15*d) - (14*e^(9/2)*E
llipticF[ArcSin[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/(15*d)
```

Rubi [A] time = 0.134437, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {692, 690, 307, 221, 1199, 424}

$$-\frac{14e^3\sqrt{-c^2-2cdx-d^2x^2+1}(ce+dex)^{3/2}}{45d}-\frac{2e\sqrt{-c^2-2cdx-d^2x^2+1}(ce+dex)^{7/2}}{9d}-\frac{14e^{9/2}F\left(\sin^{-1}\left(\frac{\sqrt{ce+dxe}}{\sqrt{e}}\right)\middle|-1\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(9/2)/Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2],x]
```

```
[Out] (-14*e^3*(c*e + d*e*x)^(3/2)*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2])/(45*d) - (2
*e*(c*e + d*e*x)^(7/2)*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2])/(9*d) + (14*e^(9/
2)*EllipticE[ArcSin[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/(15*d) - (14*e^(9/2)*E
llipticF[ArcSin[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/(15*d)
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ce+dex)^{9/2}}{\sqrt{1-c^2-2cdx-d^2x^2}}\,dx = -\frac{2e(ce+dex)^{7/2}\sqrt{1-c^2-2cdx-d^2x^2}}{9d}+\frac{1}{9}\left(7e^2\right)\int \frac{(ce+dex)^{5/2}}{\sqrt{1-c^2-2cdx-d^2x^2}}\,dx$$

$$= -\frac{14e^3(ce+dex)^{3/2}\sqrt{1-c^2-2cdx-d^2x^2}}{45d}-\frac{2e(ce+dex)^{7/2}\sqrt{1-c^2-2cdx-d^2x^2}}{9d}+\frac{1}{15}(7\ldots$$

$$= -\frac{14e^3(ce+dex)^{3/2}\sqrt{1-c^2-2cdx-d^2x^2}}{45d}-\frac{2e(ce+dex)^{7/2}\sqrt{1-c^2-2cdx-d^2x^2}}{9d}+\frac{(14e^3\ldots}{\ldots}$$

$$= -\frac{14e^3(ce+dex)^{3/2}\sqrt{1-c^2-2cdx-d^2x^2}}{45d}-\frac{2e(ce+dex)^{7/2}\sqrt{1-c^2-2cdx-d^2x^2}}{9d}-\frac{(14e^4\ldots}{\ldots}$$

$$= -\frac{14e^3(ce+dex)^{3/2}\sqrt{1-c^2-2cdx-d^2x^2}}{45d}-\frac{2e(ce+dex)^{7/2}\sqrt{1-c^2-2cdx-d^2x^2}}{9d}-\frac{14e^{9/2}\ldots}{\ldots}$$

$$= -\frac{14e^3(ce+dex)^{3/2}\sqrt{1-c^2-2cdx-d^2x^2}}{45d}-\frac{2e(ce+dex)^{7/2}\sqrt{1-c^2-2cdx-d^2x^2}}{9d}+\frac{14e^{9/2}\ldots}{\ldots}$$

Mathematica [C] time = 0.0798781, size = 86, normalized size = 0.55

$$-\frac{2e^3(e(c+dx))^{3/2}\left(\sqrt{-c^2-2cdx-d^2x^2+1}\left(5c^2+10cdx+5d^2x^2+7\right)-7\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};(c+dx)^2\right)\right)}{45d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(9/2)/Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2],x]
```

```
[Out] (-2*e^3*(e*(c + d*x))^(3/2)*(Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2]*(7 + 5*c^2 +
 10*c*d*x + 5*d^2*x^2) - 7*Hypergeometric2F1[1/2, 3/4, 7/4, (c + d*x)^2]))/
(45*d)
```

Maple [B] time = 0.205, size = 500, normalized size = 3.2

$$\frac{e^4}{90\,d\left(x^3d^3+3\,x^2cd^2+3\,xc^2d+c^3-dx-c\right)}\sqrt{e\,(dx+c)}\sqrt{-d^2x^2-2\,cdx-c^2+1}\left(-20\,d^6x^6-120\,x^5cd^5-300\,x^4c^2d^4-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(9/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x)
```

```
[Out] 1/90*(e*(d*x+c))^(1/2)*(-d^2*x^2-2*c*d*x-c^2+1)^(1/2)*e^4*(-20*d^6*x^6-120*
x^5*c*d^5-300*x^4*c^2*d^4-400*x^3*c^3*d^3-8*d^4*x^4-300*x^2*c^4*d^2-32*x^3*
c*d^3-120*x*x*c^5*d+36*(2*d*x+2*c+2)^(1/2)*(-d*x-c)^(1/2)*(-2*d*x-2*c+2)^(1/2
)*EllipticE(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2))*c^2-36*(2*d*x+2*c+2)^(1/2)*(-2
*d*x-2*c+2)^(1/2)*(d*x+c)^(1/2)*EllipticE(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2))
*c^2-48*c^2*d^2*x^2-20*c^6-32*x*c^3*d+15*(2*d*x+2*c+2)^(1/2)*(-d*x-c)^(1/2)
*(-2*d*x-2*c+2)^(1/2)*EllipticF(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2))+33*(2*d*x+
2*c+2)^(1/2)*(-d*x-c)^(1/2)*(-2*d*x-2*c+2)^(1/2)*EllipticE(1/2*(2*d*x+2*c+2
)^(1/2),2^(1/2))+15*(2*d*x+2*c+2)^(1/2)*(-2*d*x-2*c+2)^(1/2)*(d*x+c)^(1/2)*
EllipticF(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2))+9*(2*d*x+2*c+2)^(1/2)*(-2*d*x-2
*c+2)^(1/2)*(d*x+c)^(1/2)*EllipticE(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2))+28*d^
2*x^2-8*c^4+56*c*d*x+28*c^2)/d/(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3-d*x-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^{\frac{9}{2}}}{\sqrt{-d^2x^2-2\,cdx-c^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(9/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate((d*e*x + c*e)^(9/2)/sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(d^4e^4x^4+4\,cd^3e^4x^3+6\,c^2d^2e^4x^2+4\,c^3de^4x+c^4e^4\right)\sqrt{-d^2x^2-2\,cdx-c^2+1}\sqrt{dex+ce}}{d^2x^2+2\,cdx+c^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(9/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(-(d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*
x + c^4*e^4)*sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1)*sqrt(d*e*x + c*e)/(d^2*x^2
+ 2*c*d*x + c^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(9/2)/(-d**2*x**2-2*c*d*x-c**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex + ce)^{\frac{9}{2}}}{\sqrt{-d^2x^2 - 2\,cdx - c^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(9/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((d*e*x + c*e)^(9/2)/sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1), x)
```

3.1408 $\int \frac{(ce+dex)^{5/2}}{\sqrt{1-c^2-2cdx-d^2x^2}}\,dx$

Optimal. Leaf size=111

$$-\frac{6e^{5/2}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{ce+dex}}{\sqrt{e}}\right),-1\right)}{5d}-\frac{2e\sqrt{-c^2-2cdx-d^2x^2+1}(ce+dex)^{3/2}}{5d}+\frac{6e^{5/2}E\left(\sin^{-1}\left(\frac{\sqrt{ce+dxe}}{\sqrt{e}}\right)\middle|-1\right)}{5d}$$

```
[Out] (-2*e*(c*e + d*e*x)^(3/2)*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2])/(5*d) + (6*e^(
5/2)*EllipticE[ArcSin[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/(5*d) - (6*e^(5/2)*E
llipticF[ArcSin[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/(5*d)
```

Rubi [A] time = 0.090528, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {692, 690, 307, 221, 1199, 424}

$$-\frac{2e\sqrt{-c^2-2cdx-d^2x^2+1}(ce+dex)^{3/2}}{5d}-\frac{6e^{5/2}F\left(\sin^{-1}\left(\frac{\sqrt{ce+dxe}}{\sqrt{e}}\right)\middle|-1\right)}{5d}+\frac{6e^{5/2}E\left(\sin^{-1}\left(\frac{\sqrt{ce+dxe}}{\sqrt{e}}\right)\middle|-1\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(5/2)/Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2],x]
```

```
[Out] (-2*e*(c*e + d*e*x)^(3/2)*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2])/(5*d) + (6*e^(
5/2)*EllipticE[ArcSin[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/(5*d) - (6*e^(5/2)*E
llipticF[ArcSin[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/(5*d)
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ce+dex)^{5/2}}{\sqrt{1-c^2-2cdx-d^2x^2}}\,dx = -\frac{2e(ce+dex)^{3/2}\sqrt{1-c^2-2cdx-d^2x^2}}{5d}+\frac{1}{5}\left(3e^2\right)\int \frac{\sqrt{ce+dex}}{\sqrt{1-c^2-2cdx-d^2x^2}}\,dx$$

$$= -\frac{2e(ce+dex)^{3/2}\sqrt{1-c^2-2cdx-d^2x^2}}{5d}+\frac{(6e)\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{e^2}}}\,dx, x, \sqrt{ce+dex}\right)}{5d}$$

$$= -\frac{2e(ce+dex)^{3/2}\sqrt{1-c^2-2cdx-d^2x^2}}{5d}-\frac{\left(6e^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{e^2}}}\,dx, x, \sqrt{ce+dex}\right)}{5d}+\frac{\left(6e^2\right)}{}$$

$$= -\frac{2e(ce+dex)^{3/2}\sqrt{1-c^2-2cdx-d^2x^2}}{5d}-\frac{6e^{5/2}F\left(\sin^{-1}\left(\frac{\sqrt{ce+dex}}{\sqrt{e}}\right)\middle|-1\right)}{5d}+\frac{\left(6e^2\right)\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{2e(ce+dex)^{3/2}\sqrt{1-c^2-2cdx-d^2x^2}}{5d}+\frac{6e^{5/2}E\left(\sin^{-1}\left(\frac{\sqrt{ce+dex}}{\sqrt{e}}\right)\middle|-1\right)}{5d}-\frac{6e^{5/2}F\left(\sin^{-1}\left(\right.\right.}{5d}$$

Mathematica [C] time = 0.0293227, size = 54, normalized size = 0.49

$$-\frac{2e(e(c+dx))^{3/2}\left(\sqrt{1-(c+dx)^2}-{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};(c+dx)^2\right)\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(5/2)/Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2],x]
```

```
[Out] (-2*e*(e*(c + d*x))^(3/2)*(Sqrt[1 - (c + d*x)^2] - Hypergeometric2F1[1/2, 3
/4, 7/4, (c + d*x)^2]))/(5*d)
```

Maple [B] time = 0.176, size = 333, normalized size = 3.

$$\frac{e^2}{30\,d\left(x^3d^3+3\,x^2cd^2+3\,xc^2d+c^3-dx-c\right)}\sqrt{e\,(dx+c)}\sqrt{-d^2x^2-2\,cdx-c^2+1}\left(-12\,d^4x^4-48\,x^3cd^3-72\,c^2d^2x^2-48\,xc\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(5/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x)
```

```
[Out] 1/30*(e*(d*x+c))^(1/2)*(-d^2*x^2-2*c*d*x-c^2+1)^(1/2)*e^2*(-12*d^4*x^4-48*x
^3*c*d^3-72*c^2*d^2*x^2-48*x*c^3*d+5*(2*d*x+2*c+2)^(1/2)*(-2*d*x-2*c+2)^(1/
2)*(d*x+c)^(1/2)*EllipticF(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2))+3*(2*d*x+2*c+2
)^(1/2)*(-2*d*x-2*c+2)^(1/2)*(d*x+c)^(1/2)*EllipticE(1/2*(-2*d*x-2*c+2)^(1/
2),2^(1/2))+5*(2*d*x+2*c+2)^(1/2)*(-d*x-c)^(1/2)*(-2*d*x-2*c+2)^(1/2)*Ellip
ticF(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2))+15*(2*d*x+2*c+2)^(1/2)*(-d*x-c)^(1/2)
*(-2*d*x-2*c+2)^(1/2)*EllipticE(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2))+12*d^2*x^2
-12*c^4+24*c*d*x+12*c^2)/d/(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3-d*x-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^{\frac{5}{2}}}{\sqrt{-d^2x^2-2\,cdx-c^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate((d*e*x + c*e)^(5/2)/sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(d^2e^2x^2+2\,cde^2x+c^2e^2\right)\sqrt{-d^2x^2-2\,cdx-c^2+1}\sqrt{dex+ce}}{d^2x^2+2\,cdx+c^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(-(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)*sqrt(-d^2*x^2 - 2*c*d*x - c
^2 + 1)*sqrt(d*e*x + c*e)/(d^2*x^2 + 2*c*d*x + c^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\,(c+dx))^{\frac{5}{2}}}{\sqrt{-\,(c+dx-1)\,(c+dx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(5/2)/(-d**2*x**2-2*c*d*x-c**2+1)**(1/2),x)
```

```
[Out] Integral((e*(c + d*x))**(5/2)/sqrt(-(c + d*x - 1)*(c + d*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex + ce)^{\frac{5}{2}}}{\sqrt{-d^2x^2 - 2\,cdx - c^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((d*e*x + c*e)^(5/2)/sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1), x)
```

3.1409 $\int \frac{\sqrt{ce+dex}}{\sqrt{1-c^2-2cdx-d^2x^2}}\,dx$

Optimal. Leaf size=63

$$\frac{2\sqrt{e}E\left(\sin^{-1}\left(\frac{\sqrt{ce+dxe}}{\sqrt{e}}\right)\middle|-1\right)}{d}-\frac{2\sqrt{e}\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{ce+dex}}{\sqrt{e}}\right),-1\right)}{d}$$

```
[Out] (2*Sqrt[e]*EllipticE[ArcSin[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/d - (2*Sqrt[e]
*EllipticF[ArcSin[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/d
```

Rubi [A] time = 0.0553057, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {690, 307, 221, 1199, 424}

$$\frac{2\sqrt{e}E\left(\sin^{-1}\left(\frac{\sqrt{ce+dxe}}{\sqrt{e}}\right)\middle|-1\right)}{d}-\frac{2\sqrt{e}F\left(\sin^{-1}\left(\frac{\sqrt{ce+dxe}}{\sqrt{e}}\right)\middle|-1\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*e + d*e*x]/Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2],x]
```

```
[Out] (2*Sqrt[e]*EllipticE[ArcSin[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/d - (2*Sqrt[e]
*EllipticF[ArcSin[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/d
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{ce+dex}}{\sqrt{1-c^2-2cdx-d^2x^2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{e^2}}}\,dx, x, \sqrt{ce+dex}\right)}{de}$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{e^2}}}\,dx, x, \sqrt{ce+dex}\right)}{d} + \frac{2\,\text{Subst}\left(\int \frac{1+\frac{x^2}{e}}{\sqrt{1-\frac{x^4}{e^2}}}\,dx, x, \sqrt{ce+dex}\right)}{d}$$

$$= -\frac{2\sqrt{e}F\left(\sin^{-1}\left(\frac{\sqrt{ce+dex}}{\sqrt{e}}\right)\middle|-1\right)}{d} + \frac{2\,\text{Subst}\left(\int \frac{\sqrt{1+\frac{x^2}{e}}}{\sqrt{1-\frac{x^2}{e}}}\,dx, x, \sqrt{ce+dex}\right)}{d}$$

$$= \frac{2\sqrt{e}E\left(\sin^{-1}\left(\frac{\sqrt{ce+dex}}{\sqrt{e}}\right)\middle|-1\right)}{d} - \frac{2\sqrt{e}F\left(\sin^{-1}\left(\frac{\sqrt{ce+dex}}{\sqrt{e}}\right)\middle|-1\right)}{d}$$

Mathematica [C] time = 0.0165099, size = 40, normalized size = 0.63

$$\frac{2(c+dx)\sqrt{e(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};(c+dx)^2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*e + d*e*x]/Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2],x]
```

```
[Out] (2*(c + d*x)*Sqrt[e*(c + d*x)]*Hypergeometric2F1[1/2, 3/4, 7/4, (c + d*x)^2
])/(3*d)
```

Maple [B] time = 0.17, size = 204, normalized size = 3.2

$$\frac{1}{6\,d\left(x^3d^3+3\,x^2cd^2+3\,xc^2d+c^3-dx-c\right)}\sqrt{e\,(dx+c)}\sqrt{-d^2x^2-2\,cdx-c^2+1}\sqrt{2\,dx+2\,c+2}\sqrt{-2\,dx-2\,c+2}\left(\sqrt{-dx-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(1/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x)
```

```
[Out] 1/6*(e*(d*x+c))^(1/2)*(-d^2*x^2-2*c*d*x-c^2+1)^(1/2)*(2*d*x+2*c+2)^(1/2)*(-
2*d*x-2*c+2)^(1/2)*((-d*x-c)^(1/2)*EllipticF(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2
))+3*(-d*x-c)^(1/2)*EllipticE(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2))+(d*x+c)^(1/2
)*EllipticF(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2))+3*(d*x+c)^(1/2)*EllipticE(1/2
*(-2*d*x-2*c+2)^(1/2),2^(1/2)))/d/(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3-d*x-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dex+ce}}{\sqrt{-d^2x^2-2\,cdx-c^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(1/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(d*e*x + c*e)/sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-d^2x^2-2\,cdx-c^2+1}\sqrt{dex+ce}}{d^2x^2+2\,cdx+c^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(1/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(-sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1)*sqrt(d*e*x + c*e)/(d^2*x^2 + 2
*c*d*x + c^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{e\,(c+dx)}}{\sqrt{-\,(c+dx-1)\,(c+dx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(1/2)/(-d**2*x**2-2*c*d*x-c**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(e*(c + d*x))/sqrt(-(c + d*x - 1)*(c + d*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{dex+ce}}{\sqrt{-d^2x^2-2\,cdx-c^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(1/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(d*e*x + c*e)/sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1), x)
```

3.1410 $\int \frac{1}{(ce+dex)^{3/2}\sqrt{1-c^2-2cdx-d^2x^2}}\,dx$

Optimal. Leaf size=107

$$\frac{2\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{ce+dex}}{\sqrt{e}}\right),-1\right)}{de^{3/2}}-\frac{2\sqrt{-c^2-2cdx-d^2x^2+1}}{de\sqrt{ce+dex}}-\frac{2E\left(\sin^{-1}\left(\frac{\sqrt{ce+dxe}}{\sqrt{e}}\right)\middle|-1\right)}{de^{3/2}}$$

```
[Out] (-2*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2])/(d*e*Sqrt[c*e + d*e*x]) - (2*Ellipti
cE[ArcSin[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/(d*e^(3/2)) + (2*EllipticF[ArcSi
n[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/(d*e^(3/2))
```

Rubi [A] time = 0.0851578, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {693, 690, 307, 221, 1199, 424}

$$-\frac{2\sqrt{-c^2-2cdx-d^2x^2+1}}{de\sqrt{ce+dex}}+\frac{2F\left(\sin^{-1}\left(\frac{\sqrt{ce+dxe}}{\sqrt{e}}\right)\middle|-1\right)}{de^{3/2}}-\frac{2E\left(\sin^{-1}\left(\frac{\sqrt{ce+dxe}}{\sqrt{e}}\right)\middle|-1\right)}{de^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*e + d*e*x)^(3/2)*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2]),x]
```

```
[Out] (-2*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2])/(d*e*Sqrt[c*e + d*e*x]) - (2*Ellipti
cE[ArcSin[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/(d*e^(3/2)) + (2*EllipticF[ArcSi
n[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/(d*e^(3/2))
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(ce+dex)^{3/2}\sqrt{1-c^2-2cdx-d^2x^2}}\,dx = -\frac{2\sqrt{1-c^2-2cdx-d^2x^2}}{de\sqrt{ce+dex}} - \frac{\int \frac{\sqrt{ce+dex}}{\sqrt{1-c^2-2cdx-d^2x^2}}\,dx}{e^2}$$

$$= -\frac{2\sqrt{1-c^2-2cdx-d^2x^2}}{de\sqrt{ce+dex}} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{e^2}}}\,dx, x, \sqrt{ce+dex}\right)}{de^3}$$

$$= -\frac{2\sqrt{1-c^2-2cdx-d^2x^2}}{de\sqrt{ce+dex}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{e^2}}}\,dx, x, \sqrt{ce+dex}\right)}{de^2} - \frac{2\,\text{Subs}}{}$$

$$= -\frac{2\sqrt{1-c^2-2cdx-d^2x^2}}{de\sqrt{ce+dex}} + \frac{2F\left(\sin^{-1}\left(\frac{\sqrt{ce+dex}}{\sqrt{e}}\right)\middle|-1\right)}{de^{3/2}} - \frac{2\,\text{Subst}\left(\int \frac{\sqrt{1+\frac{x^2}{e}}}{\sqrt{1-\frac{x^2}{e}}}\right.}{d}$$

$$= -\frac{2\sqrt{1-c^2-2cdx-d^2x^2}}{de\sqrt{ce+dex}} - \frac{2E\left(\sin^{-1}\left(\frac{\sqrt{ce+dex}}{\sqrt{e}}\right)\middle|-1\right)}{de^{3/2}} + \frac{2F\left(\sin^{-1}\left(\frac{\sqrt{ce+de}}{\sqrt{e}}\right.\right.}{de^{3/2}}$$

Mathematica [C] time = 0.0159298, size = 38, normalized size = 0.36

$$-\frac{2(c+dx)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};(c+dx)^2\right)}{d(e(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*e + d*e*x)^(3/2)*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2]),x]
```

```
[Out] (-2*(c + d*x)*Hypergeometric2F1[-1/4, 1/2, 3/4, (c + d*x)^2])/(d*(e*(c + d*
x))^(3/2))
```

Maple [B] time = 0.288, size = 292, normalized size = 2.7

$$\frac{1}{6\,de^2\left(x^3d^3+3\,x^2cd^2+3\,xc^2d+c^3-dx-c\right)}\sqrt{e\,(dx+c)}\sqrt{-d^2x^2-2\,cdx-c^2+1}\left(\sqrt{2\,dx+2\,c+2}\sqrt{-dx-c}\sqrt{-2\,dx-2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)^(3/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x)
```

```
[Out] 1/6*(e*(d*x+c))^(1/2)*(-d^2*x^2-2*c*d*x-c^2+1)^(1/2)*((2*d*x+2*c+2)^(1/2)*(
-d*x-c)^(1/2)*(-2*d*x-2*c+2)^(1/2)*EllipticF(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2
))-9*(2*d*x+2*c+2)^(1/2)*(-d*x-c)^(1/2)*(-2*d*x-2*c+2)^(1/2)*EllipticE(1/2*
(2*d*x+2*c+2)^(1/2),2^(1/2))+(2*d*x+2*c+2)^(1/2)*(-2*d*x-2*c+2)^(1/2)*(d*x+
c)^(1/2)*EllipticF(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2))+3*(2*d*x+2*c+2)^(1/2)*
(-2*d*x-2*c+2)^(1/2)*(d*x+c)^(1/2)*EllipticE(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/
2))-12*d^2*x^2-24*c*d*x-12*c^2+12)/d/e^2/(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3
-d*x-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-d^2x^2 - 2\,cdx - c^2 + 1}(dex + ce)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^(3/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(1/(sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1)*(d*e*x + c*e)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-d^2x^2 - 2\,cdx - c^2 + 1}\sqrt{dex + ce}}{d^4e^2x^4 + 4\,cd^3e^2x^3 + \left(6\,c^2 - 1\right)d^2e^2x^2 + 2\left(2\,c^3 - c\right)de^2x + \left(c^4 - c^2\right)e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^(3/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral(-sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1)*sqrt(d*e*x + c*e)/(d^4*e^2*x^4
 + 4*c*d^3*e^2*x^3 + (6*c^2 - 1)*d^2*e^2*x^2 + 2*(2*c^3 - c)*d*e^2*x + (c^4
 - c^2)*e^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\left(c + dx\right)\right)^{\frac{3}{2}}\sqrt{-\left(c + dx - 1\right)\left(c + dx + 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)**(3/2)/(-d**2*x**2-2*c*d*x-c**2+1)**(1/2),x)
```

```
[Out] Integral(1/((e*(c + d*x))**(3/2)*sqrt(-(c + d*x - 1)*(c + d*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-d^2x^2 - 2\,cdx - c^2 + 1}(dex + ce)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^(3/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(1/(sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1)*(d*e*x + c*e)^(3/2)), x)
```

3.1411 $\int \frac{1}{(ce+dex)^{7/2}\sqrt{1-c^2-2cdx-d^2x^2}}\,dx$

Optimal. Leaf size=159

$$\frac{6\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{ce+dex}}{\sqrt{e}}\right),-1\right)}{5de^{7/2}}-\frac{6\sqrt{-c^2-2cdx-d^2x^2+1}}{5de^3\sqrt{ce+dex}}-\frac{2\sqrt{-c^2-2cdx-d^2x^2+1}}{5de(ce+dex)^{5/2}}-\frac{6E\left(\sin^{-1}\left(\frac{\sqrt{ce+dxe}}{\sqrt{e}}\right)\middle|-1\right)}{5de^{7/2}}$$

```
[Out] (-2*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2])/(5*d*e*(c*e + d*e*x)^(5/2)) - (6*Sqr
t[1 - c^2 - 2*c*d*x - d^2*x^2])/(5*d*e^3*Sqrt[c*e + d*e*x]) - (6*EllipticE[
ArcSin[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/(5*d*e^(7/2)) + (6*EllipticF[ArcSin
[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/(5*d*e^(7/2))
```

Rubi [A] time = 0.118039, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {693, 690, 307, 221, 1199, 424}

$$-\frac{6\sqrt{-c^2-2cdx-d^2x^2+1}}{5de^3\sqrt{ce+dex}}-\frac{2\sqrt{-c^2-2cdx-d^2x^2+1}}{5de(ce+dex)^{5/2}}+\frac{6F\left(\sin^{-1}\left(\frac{\sqrt{ce+dxe}}{\sqrt{e}}\right)\middle|-1\right)}{5de^{7/2}}-\frac{6E\left(\sin^{-1}\left(\frac{\sqrt{ce+dxe}}{\sqrt{e}}\right)\middle|-1\right)}{5de^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*e + d*e*x)^(7/2)*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2]),x]
```

```
[Out] (-2*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2])/(5*d*e*(c*e + d*e*x)^(5/2)) - (6*Sqr
t[1 - c^2 - 2*c*d*x - d^2*x^2])/(5*d*e^3*Sqrt[c*e + d*e*x]) - (6*EllipticE[
ArcSin[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/(5*d*e^(7/2)) + (6*EllipticF[ArcSin
[Sqrt[c*e + d*e*x]/Sqrt[e]], -1])/(5*d*e^(7/2))
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 690

```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Dist[(4*Sqrt[-(c/(b^2 - 4*a*c))])/e, Subst[Int[x^2/Sqrt[Simp[1 - (b^
2*x^4)/(d^2*(b^2 - 4*a*c)), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 -
4*a*c), 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(ce+dex)^{7/2}\sqrt{1-c^2-2cdx-d^2x^2}}\,dx = -\frac{2\sqrt{1-c^2-2cdx-d^2x^2}}{5de(ce+dex)^{5/2}}+\frac{3\int \frac{1}{(ce+dex)^{3/2}\sqrt{1-c^2-2cdx-d^2x^2}}\,dx}{5e^2}$$

$$= -\frac{2\sqrt{1-c^2-2cdx-d^2x^2}}{5de(ce+dex)^{5/2}}-\frac{6\sqrt{1-c^2-2cdx-d^2x^2}}{5de^3\sqrt{ce+dex}}-\frac{3\int \frac{\sqrt{ce+dex}}{\sqrt{1-c^2-2cdx-d^2x^2}}}{5e^4}$$

$$= -\frac{2\sqrt{1-c^2-2cdx-d^2x^2}}{5de(ce+dex)^{5/2}}-\frac{6\sqrt{1-c^2-2cdx-d^2x^2}}{5de^3\sqrt{ce+dex}}-\frac{6\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{e^2}}}\right.}{5de}$$

$$= -\frac{2\sqrt{1-c^2-2cdx-d^2x^2}}{5de(ce+dex)^{5/2}}-\frac{6\sqrt{1-c^2-2cdx-d^2x^2}}{5de^3\sqrt{ce+dex}}+\frac{6\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{e^2}}}\right.}{5de}$$

$$= -\frac{2\sqrt{1-c^2-2cdx-d^2x^2}}{5de(ce+dex)^{5/2}}-\frac{6\sqrt{1-c^2-2cdx-d^2x^2}}{5de^3\sqrt{ce+dex}}+\frac{6F\left(\sin^{-1}\left(\frac{\sqrt{ce+dex}}{\sqrt{e}}\right)\right.}{5de^{7/2}}$$

$$= -\frac{2\sqrt{1-c^2-2cdx-d^2x^2}}{5de(ce+dex)^{5/2}}-\frac{6\sqrt{1-c^2-2cdx-d^2x^2}}{5de^3\sqrt{ce+dex}}-\frac{6E\left(\sin^{-1}\left(\frac{\sqrt{ce+dex}}{\sqrt{e}}\right)\right.}{5de^{7/2}}$$

Mathematica [C] time = 0.0229086, size = 40, normalized size = 0.25

$$-\frac{2(c+dx)\,{}_2F_1\left(-\frac{5}{4},\frac{1}{2};-\frac{1}{4};(c+dx)^2\right)}{5d(e(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*e + d*e*x)^(7/2)*Sqrt[1 - c^2 - 2*c*d*x - d^2*x^2]),x]
```

```
[Out] (-2*(c + d*x)*Hypergeometric2F1[-5/4, 1/2, -1/4, (c + d*x)^2])/(5*d*(e*(c +
 d*x))^(7/2))
```

Maple [B] time = 0.258, size = 768, normalized size = 4.8

$$\frac{1}{30\,e^4\,(dx+c)^3\,(d^2x^2+2\,cdx+c^2-1)\,d}\left(5\,\mathrm{EllipticF}\left(1/2\,\sqrt{2\,dx+2\,c+2},\sqrt{2}\right)x^2d^2\sqrt{2\,dx+2\,c+2}\sqrt{-dx-c}\sqrt{-2\,dx-2\,c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)^(7/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x)
```

```
[Out] 1/30*(5*EllipticF(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2))*x^2*d^2*(2*d*x+2*c+2)^(1
/2)*(-d*x-c)^(1/2)*(-2*d*x-2*c+2)^(1/2)-33*EllipticE(1/2*(2*d*x+2*c+2)^(1/2
),2^(1/2))*x^2*d^2*(2*d*x+2*c+2)^(1/2)*(-d*x-c)^(1/2)*(-2*d*x-2*c+2)^(1/2)+
5*EllipticF(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2))*x^2*d^2*(2*d*x+2*c+2)^(1/2)*(
d*x+c)^(1/2)*(-2*d*x-2*c+2)^(1/2)+15*EllipticE(1/2*(-2*d*x-2*c+2)^(1/2),2^(
1/2))*x^2*d^2*(2*d*x+2*c+2)^(1/2)*(d*x+c)^(1/2)*(-2*d*x-2*c+2)^(1/2)-36*d^4
*x^4+10*EllipticF(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2))*x*c*d*(2*d*x+2*c+2)^(1/2
)*(-d*x-c)^(1/2)*(-2*d*x-2*c+2)^(1/2)-66*EllipticE(1/2*(2*d*x+2*c+2)^(1/2),
2^(1/2))*x*c*d*(2*d*x+2*c+2)^(1/2)*(-d*x-c)^(1/2)*(-2*d*x-2*c+2)^(1/2)+10*E
llipticF(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2))*x*c*d*(2*d*x+2*c+2)^(1/2)*(d*x+c
)^(1/2)*(-2*d*x-2*c+2)^(1/2)+30*EllipticE(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2))
*x*c*d*(2*d*x+2*c+2)^(1/2)*(d*x+c)^(1/2)*(-2*d*x-2*c+2)^(1/2)-144*x^3*c*d^3
+5*EllipticF(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2))*c^2*(2*d*x+2*c+2)^(1/2)*(-d*x
-c)^(1/2)*(-2*d*x-2*c+2)^(1/2)-33*(2*d*x+2*c+2)^(1/2)*(-d*x-c)^(1/2)*(-2*d*
x-2*c+2)^(1/2)*EllipticE(1/2*(2*d*x+2*c+2)^(1/2),2^(1/2))*c^2+5*EllipticF(1
/2*(-2*d*x-2*c+2)^(1/2),2^(1/2))*c^2*(2*d*x+2*c+2)^(1/2)*(d*x+c)^(1/2)*(-2*
d*x-2*c+2)^(1/2)+15*(2*d*x+2*c+2)^(1/2)*(-2*d*x-2*c+2)^(1/2)*(d*x+c)^(1/2)*
EllipticE(1/2*(-2*d*x-2*c+2)^(1/2),2^(1/2))*c^2-216*c^2*d^2*x^2-144*x*c^3*d
+24*d^2*x^2-36*c^4+48*c*d*x+24*c^2+12)*(-d^2*x^2-2*c*d*x-c^2+1)^(1/2)*(e*(d
*x+c))^(1/2)/e^4/(d*x+c)^3/(d^2*x^2+2*c*d*x+c^2-1)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-d^2x^2-2\,cdx-c^2+1}(dex+ce)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^(7/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(1/(sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1)*(d*e*x + c*e)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-d^2x^2-2\,cdx-c^2+1}\sqrt{dex+ce}}{d^6e^4x^6+6\,cd^5e^4x^5+\left(15\,c^2-1\right)d^4e^4x^4+4\left(5\,c^3-c\right)d^3e^4x^3+3\left(5\,c^4-2\,c^2\right)d^2e^4x^2+2\left(3\,c^5-2\,c^3\right)de^4x+\left(c^6\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^(7/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral(-sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1)*sqrt(d*e*x + c*e)/(d^6*e^4*x^6
 + 6*c*d^5*e^4*x^5 + (15*c^2 - 1)*d^4*e^4*x^4 + 4*(5*c^3 - c)*d^3*e^4*x^3 +
```

```
3*(5*c^4 - 2*c^2)*d^2*e^4*x^2 + 2*(3*c^5 - 2*c^3)*d*e^4*x + (c^6 - c^4)*e^
4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)**(7/2)/(-d**2*x**2-2*c*d*x-c**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-d^2x^2 - 2\,cdx - c^2 + 1}(dex + ce)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^(7/2)/(-d^2*x^2-2*c*d*x-c^2+1)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(1/(sqrt(-d^2*x^2 - 2*c*d*x - c^2 + 1)*(d*e*x + c*e)^(7/2)), x)
```

3.1412 $\int \frac{(a+bx+cx^2)^{4/3}}{(bd+2cdx)^{11/3}} dx$

Optimal. Leaf size=320

$$-\frac{3\sqrt[3]{a+bx+cx^2}(d(b+2cx))^{4/3}}{16c^2d^5(b^2-4ac)} - \frac{9(a+bx+cx^2)^{7/3}}{4d^3(b^2-4ac)^2(bd+2cdx)^{2/3}} + \frac{9(a+bx+cx^2)^{4/3}(d(b+2cx))^{4/3}}{16cd^5(b^2-4ac)^2} + \frac{3(a+bx+}{4d(b^2-4ac)(bd}$$

```
[Out] (-3*(d*(b + 2*c*x))^(4/3)*(a + b*x + c*x^2)^(1/3))/(16*c^2*(b^2 - 4*a*c)*d^
5) + (9*(d*(b + 2*c*x))^(4/3)*(a + b*x + c*x^2)^(4/3))/(16*c*(b^2 - 4*a*c)^
2*d^5) + (3*(a + b*x + c*x^2)^(7/3))/(4*(b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(8/
3)) - (9*(a + b*x + c*x^2)^(7/3))/(4*(b^2 - 4*a*c)^2*d^3*(b*d + 2*c*d*x)^(2
/3)) - (Sqrt[3]*ArcTan[(1 + (2^(1/3)*(d*(b + 2*c*x))^(2/3))/(c^(1/3)*d^(2/3
)*(a + b*x + c*x^2)^(1/3)))/Sqrt[3]])/(16*2^(2/3)*c^(7/3)*d^(11/3)) - (3*Lo
g[(d*(b + 2*c*x))^(2/3) - 2^(2/3)*c^(1/3)*d^(2/3)*(a + b*x + c*x^2)^(1/3)])
/(32*2^(2/3)*c^(7/3)*d^(11/3))
```

Rubi [A] time = 1.28214, antiderivative size = 468, normalized size of antiderivative = 1.46, number of steps used = 14, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {693, 694, 279, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{3\sqrt[3]{a+bx+cx^2}(d(b+2cx))^{4/3}}{16c^2d^5(b^2-4ac)} - \frac{9(a+bx+cx^2)^{7/3}}{4d^3(b^2-4ac)^2(bd+2cdx)^{2/3}} + \frac{9(a+bx+cx^2)^{4/3}(d(b+2cx))^{4/3}}{16cd^5(b^2-4ac)^2} + \frac{3(a+bx+}{4d(b^2-4ac)(bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(4/3)/(b*d + 2*c*d*x)^(11/3),x]
```

```
[Out] (-3*(d*(b + 2*c*x))^(4/3)*(a + b*x + c*x^2)^(1/3))/(16*c^2*(b^2 - 4*a*c)*d^
5) + (9*(d*(b + 2*c*x))^(4/3)*(a + b*x + c*x^2)^(4/3))/(16*c*(b^2 - 4*a*c)^
2*d^5) + (3*(a + b*x + c*x^2)^(7/3))/(4*(b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(8/
3)) - (9*(a + b*x + c*x^2)^(7/3))/(4*(b^2 - 4*a*c)^2*d^3*(b*d + 2*c*d*x)^(2
/3)) - (Sqrt[3]*ArcTan[(c^(1/3)*d^(2/3) + (2^(1/3)*(d*(b + 2*c*x))^(2/3))/(
a + b*x + c*x^2)^(1/3))/(Sqrt[3]*c^(1/3)*d^(2/3))])/(16*2^(2/3)*c^(7/3)*d^(
11/3)) - Log[-((2^(1/3)*(d*(b + 2*c*x))^(2/3) - 2*c^(1/3)*d^(2/3)*(a + b*x
+ c*x^2)^(1/3))/(a + b*x + c*x^2)^(1/3))]/(16*2^(2/3)*c^(7/3)*d^(11/3)) + L
og[((d*(b + 2*c*x))^(4/3) + 2^(2/3)*c^(1/3)*d^(2/3)*(d*(b + 2*c*x))^(2/3)*(
a + b*x + c*x^2)^(1/3) + 2*2^(1/3)*c^(2/3)*d^(4/3)*(a + b*x + c*x^2)^(2/3))
/(a + b*x + c*x^2)^(2/3)]/(32*2^(2/3)*c^(7/3)*d^(11/3))
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{4/3}}{(bd+2cdx)^{11/3}}\,dx = \frac{3\left(a+bx+cx^2\right)^{7/3}}{4\left(b^2-4ac\right)d(bd+2cdx)^{8/3}} - \frac{3\int \frac{\left(a+bx+cx^2\right)^{4/3}}{(bd+2cdx)^{5/3}}\,dx}{4\left(b^2-4ac\right)d^2}$$

$$= \frac{3\left(a+bx+cx^2\right)^{7/3}}{4\left(b^2-4ac\right)d(bd+2cdx)^{8/3}} - \frac{9\left(a+bx+cx^2\right)^{7/3}}{4\left(b^2-4ac\right)^2 d^3(bd+2cdx)^{2/3}} + \frac{9\int \sqrt[3]{bd+2cdx}\left(a+bx+c\right.}{2\left(b^2-4ac\right)^2 d^4}$$

$$= \frac{3\left(a+bx+cx^2\right)^{7/3}}{4\left(b^2-4ac\right)d(bd+2cdx)^{8/3}} - \frac{9\left(a+bx+cx^2\right)^{7/3}}{4\left(b^2-4ac\right)^2 d^3(bd+2cdx)^{2/3}} + \frac{9\,\mathrm{Subst}\left(\int \sqrt[3]{x}\left(a-\frac{b^2}{4c}+\frac{x^2}{4c}\right.\right.}{4c\left(b^2-4a\right.}$$

$$= \frac{9(d(b+2cx))^{4/3}\left(a+bx+cx^2\right)^{4/3}}{16c\left(b^2-4ac\right)^2 d^5} + \frac{3\left(a+bx+cx^2\right)^{7/3}}{4\left(b^2-4ac\right)d(bd+2cdx)^{8/3}} - \frac{9\left(a+bx+cx^2\right)^{7/3}}{4\left(b^2-4ac\right)^2 d^3(bd+2c\right.}$$

$$= -\frac{3(d(b+2cx))^{4/3}\sqrt[3]{a+bx+cx^2}}{16c^2\left(b^2-4ac\right)d^5} + \frac{9(d(b+2cx))^{4/3}\left(a+bx+cx^2\right)^{4/3}}{16c\left(b^2-4ac\right)^2 d^5} + \frac{3\left(a+bx+cx^2\right)}{4\left(b^2-4ac\right)d(bd+2\right.}$$

$$= -\frac{3(d(b+2cx))^{4/3}\sqrt[3]{a+bx+cx^2}}{16c^2\left(b^2-4ac\right)d^5} + \frac{9(d(b+2cx))^{4/3}\left(a+bx+cx^2\right)^{4/3}}{16c\left(b^2-4ac\right)^2 d^5} + \frac{3\left(a+bx+cx^2\right)}{4\left(b^2-4ac\right)d(bd+2\right.}$$

$$= -\frac{3(d(b+2cx))^{4/3}\sqrt[3]{a+bx+cx^2}}{16c^2\left(b^2-4ac\right)d^5} + \frac{9(d(b+2cx))^{4/3}\left(a+bx+cx^2\right)^{4/3}}{16c\left(b^2-4ac\right)^2 d^5} + \frac{3\left(a+bx+cx^2\right)}{4\left(b^2-4ac\right)d(bd+2\right.}$$

$$= -\frac{3(d(b+2cx))^{4/3}\sqrt[3]{a+bx+cx^2}}{16c^2\left(b^2-4ac\right)d^5} + \frac{9(d(b+2cx))^{4/3}\left(a+bx+cx^2\right)^{4/3}}{16c\left(b^2-4ac\right)^2 d^5} + \frac{3\left(a+bx+cx^2\right)}{4\left(b^2-4ac\right)d(bd+2\right.}$$

$$= -\frac{3(d(b+2cx))^{4/3}\sqrt[3]{a+bx+cx^2}}{16c^2\left(b^2-4ac\right)d^5} + \frac{9(d(b+2cx))^{4/3}\left(a+bx+cx^2\right)^{4/3}}{16c\left(b^2-4ac\right)^2 d^5} + \frac{3\left(a+bx+cx^2\right)}{4\left(b^2-4ac\right)d(bd+2\right.}$$

$$= -\frac{3(d(b+2cx))^{4/3}\sqrt[3]{a+bx+cx^2}}{16c^2\left(b^2-4ac\right)d^5} + \frac{9(d(b+2cx))^{4/3}\left(a+bx+cx^2\right)^{4/3}}{16c\left(b^2-4ac\right)^2 d^5} + \frac{3\left(a+bx+cx^2\right)}{4\left(b^2-4ac\right)d(bd+2\right.}$$

$$= -\frac{3(d(b+2cx))^{4/3}\sqrt[3]{a+bx+cx^2}}{16c^2\left(b^2-4ac\right)d^5} + \frac{9(d(b+2cx))^{4/3}\left(a+bx+cx^2\right)^{4/3}}{16c\left(b^2-4ac\right)^2 d^5} + \frac{3\left(a+bx+cx^2\right)}{4\left(b^2-4ac\right)d(bd+2\right.}$$

$$= -\frac{3(d(b+2cx))^{4/3}\sqrt[3]{a+bx+cx^2}}{16c^2\left(b^2-4ac\right)d^5} + \frac{9(d(b+2cx))^{4/3}\left(a+bx+cx^2\right)^{4/3}}{16c\left(b^2-4ac\right)^2 d^5} + \frac{3\left(a+bx+cx^2\right)}{4\left(b^2-4ac\right)d(bd+2\right.}$$

Mathematica [C] time = 0.0947293, size = 104, normalized size = 0.32

$$\frac{3\left(b^2-4ac\right)\sqrt[3]{a+x(b+cx)}\,{}_2F_1\left(-\frac{4}{3},-\frac{4}{3};-\frac{1}{3};\frac{(b+2cx)^2}{b^2-4ac}\right)}{64\ 2^{2/3}c^2d\sqrt[3]{\frac{c(a+x(b+cx))}{4ac-b^2}}(d(b+2cx))^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(4/3)/(b*d + 2*c*d*x)^(11/3),x]
```

```
[Out] (3*(b^2 - 4*a*c)*(a + x*(b + c*x))^(1/3)*Hypergeometric2F1[-4/3, -4/3, -1/3
, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(64*2^(2/3)*c^2*d*(d*(b + 2*c*x))^(8/3)*((c
*(a + x*(b + c*x)))/(-b^2 + 4*a*c))^(1/3))
```

Maple [F] time = 1.25, size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{\frac{4}{3}} (2\,cdx+bd)^{-\frac{11}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(11/3),x)
```

```
[Out] int((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(11/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{4}{3}}}{(2\,cdx+bd)^{\frac{11}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(11/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(4/3)/(2*c*d*x + b*d)^(11/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(11/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(4/3)/(2*c*d*x+b*d)**(11/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)^{\frac{4}{3}}}{(2\,cdx + bd)^{\frac{11}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(11/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(4/3)/(2*c*d*x + b*d)^(11/3), x)
```

3.1413 $\int \frac{(a+bx+cx^2)^{4/3}}{(bd+2cdx)^{17/3}}\,dx$

Optimal. Leaf size=44

$$\frac{3(a+bx+cx^2)^{7/3}}{7d(b^2-4ac)(bd+2cdx)^{14/3}}$$

```
[Out] (3*(a + b*x + c*x^2)^(7/3))/(7*(b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(14/3))
```

Rubi [A] time = 0.0177368, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {682}

$$\frac{3(a+bx+cx^2)^{7/3}}{7d(b^2-4ac)(bd+2cdx)^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(4/3)/(b*d + 2*c*d*x)^(17/3),x]
```

```
[Out] (3*(a + b*x + c*x^2)^(7/3))/(7*(b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(14/3))
```

Rule 682

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c,
0] && EqQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{4/3}}{(bd+2cdx)^{17/3}}\,dx = \frac{3(a+bx+cx^2)^{7/3}}{7(b^2-4ac)d(bd+2cdx)^{14/3}}$$

Mathematica [A] time = 0.0500632, size = 50, normalized size = 1.14

$$\frac{3(a+x(b+cx))^{7/3}\sqrt[3]{d(b+2cx)}}{7d^6(b^2-4ac)(b+2cx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(4/3)/(b*d + 2*c*d*x)^(17/3),x]
```

```
[Out] (3*(d*(b + 2*c*x))^(1/3)*(a + x*(b + c*x))^(7/3))/(7*(b^2 - 4*a*c)*d^6*(b +
 2*c*x)^5)
```

Maple [A] time = 0.046, size = 44, normalized size = 1.

$$-\frac{6\,cx+3\,b}{28\,ac-7\,b^{2}}\left(cx^{2}+bx+a\right)^{\frac{7}{3}}\left(2\,cdx+bd\right)^{-\frac{17}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(17/3),x)
```

```
[Out] -3/7*(2*c*x+b)*(c*x^2+b*x+a)^(7/3)/(4*a*c-b^2)/(2*c*d*x+b*d)^(17/3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2}+bx+a\right)^{\frac{4}{3}}}{\left(2\,cdx+bd\right)^{\frac{17}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(17/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(4/3)/(2*c*d*x + b*d)^(17/3), x)
```

Fricas [B] time = 2.95271, size = 416, normalized size = 9.45

$$\frac{3\left(c^{2}x^{4}+2\,bcx^{3}+2\,abx+\left(b^{2}+2\,ac\right)x^{2}+a^{2}\right)\left(2\,cdx+bd\right)^{\frac{1}{3}}\left(cx^{2}+bx+a\right)^{\frac{1}{3}}}{7\left(32\left(b^{2}c^{5}-4\,ac^{6}\right)d^{6}x^{5}+80\left(b^{3}c^{4}-4\,abc^{5}\right)d^{6}x^{4}+80\left(b^{4}c^{3}-4\,ab^{2}c^{4}\right)d^{6}x^{3}+40\left(b^{5}c^{2}-4\,ab^{3}c^{3}\right)d^{6}x^{2}+10\left(b^{6}c-4\,ab\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(17/3),x, algorithm="fricas")
```

```
[Out] 3/7*(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*(2*c*d*x + b*
d)^(1/3)*(c*x^2 + b*x + a)^(1/3)/(32*(b^2*c^5 - 4*a*c^6)*d^6*x^5 + 80*(b^3*
c^4 - 4*a*b*c^5)*d^6*x^4 + 80*(b^4*c^3 - 4*a*b^2*c^4)*d^6*x^3 + 40*(b^5*c^2
 - 4*a*b^3*c^3)*d^6*x^2 + 10*(b^6*c - 4*a*b^4*c^2)*d^6*x + (b^7 - 4*a*b^5*c
)*d^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(4/3)/(2*c*d*x+b*d)**(17/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx + a\right)^{\frac{4}{3}}}{\left(2\,cdx + bd\right)^{\frac{17}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(17/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(4/3)/(2*c*d*x + b*d)^(17/3), x)
```

3.1414 $\int \frac{(a+bx+cx^2)^{4/3}}{(bd+2cdx)^{23/3}}\,dx$

Optimal. Leaf size=89

$$\frac{9\left(a+bx+cx^2\right)^{7/3}}{70d^3\left(b^2-4ac\right)^2(bd+2cdx)^{14/3}}+\frac{3\left(a+bx+cx^2\right)^{7/3}}{10d\left(b^2-4ac\right)(bd+2cdx)^{20/3}}$$

```
[Out] (3*(a + b*x + c*x^2)^(7/3))/(10*(b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(20/3)) + (
9*(a + b*x + c*x^2)^(7/3))/(70*(b^2 - 4*a*c)^2*d^3*(b*d + 2*c*d*x)^(14/3))
```

Rubi [A] time = 0.0395993, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {693, 682}

$$\frac{9\left(a+bx+cx^2\right)^{7/3}}{70d^3\left(b^2-4ac\right)^2(bd+2cdx)^{14/3}}+\frac{3\left(a+bx+cx^2\right)^{7/3}}{10d\left(b^2-4ac\right)(bd+2cdx)^{20/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(4/3)/(b*d + 2*c*d*x)^(23/3),x]
```

```
[Out] (3*(a + b*x + c*x^2)^(7/3))/(10*(b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(20/3)) + (
9*(a + b*x + c*x^2)^(7/3))/(70*(b^2 - 4*a*c)^2*d^3*(b*d + 2*c*d*x)^(14/3))
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 682

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c,
0] && EqQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx+cx^2\right)^{4/3}}{(bd+2cdx)^{23/3}}\,dx &= \frac{3\left(a+bx+cx^2\right)^{7/3}}{10\left(b^2-4ac\right)d(bd+2cdx)^{20/3}}+\frac{3\int \frac{\left(a+bx+cx^2\right)^{4/3}}{(bd+2cdx)^{17/3}}\,dx}{10\left(b^2-4ac\right)d^2}\\ &= \frac{3\left(a+bx+cx^2\right)^{7/3}}{10\left(b^2-4ac\right)d(bd+2cdx)^{20/3}}+\frac{9\left(a+bx+cx^2\right)^{7/3}}{70\left(b^2-4ac\right)^2d^3(bd+2cdx)^{14/3}}\end{aligned}$$

Mathematica [A] time = 0.0776626, size = 74, normalized size = 0.83

$$\frac{3(a+x(b+cx))^{7/3}\left(2c\left(3cx^2-7a\right)+5b^2+6bcx\right)\sqrt[3]{d(b+2cx)}}{35d^8\left(b^2-4ac\right)^2(b+2cx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(4/3)/(b*d + 2*c*d*x)^(23/3),x]
```

```
[Out] (3*(d*(b + 2*c*x))^(1/3)*(a + x*(b + c*x))^(7/3)*(5*b^2 + 6*b*c*x + 2*c*(-7
*a + 3*c*x^2)))/(35*(b^2 - 4*a*c)^2*d^8*(b + 2*c*x)^7)
```

Maple [A] time = 0.044, size = 76, normalized size = 0.9

$$-\frac{(6\,cx+3\,b)\left(-6\,c^2x^2-6\,bcx+14\,ac-5\,b^2\right)}{560\,a^2c^2-280\,acb^2+35\,b^4}\left(cx^2+bx+a\right)^{\frac{7}{3}}(2\,cdx+bd)^{-\frac{23}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(23/3),x)
```

```
[Out] -3/35*(2*c*x+b)*(c*x^2+b*x+a)^(7/3)*(-6*c^2*x^2-6*b*c*x+14*a*c-5*b^2)/(16*a
^2*c^2-8*a*b^2*c+b^4)/(2*c*d*x+b*d)^(23/3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{4}{3}}}{(2\,cdx+bd)^{\frac{23}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(23/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(4/3)/(2*c*d*x + b*d)^(23/3), x)
```

Fricas [B] time = 3.58114, size = 868, normalized size = 9.75

$$\frac{3\left(6\,c^4x^6+18\,bc^3x^5+\left(23\,b^2c^2-2\,ac^3\right)x^4+5\,a^2b^2\cdots\right.}{35\left(128\left(b^4c^7-8\,ab^2c^8+16\,a^2c^9\right)d^8x^7+448\left(b^5c^6-8\,ab^3c^7+16\,a^2bc^8\right)d^8x^6+672\left(b^6c^5-8\,ab^4c^6+16\,a^2b^2c^7\right)d^8x^5+56\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(23/3),x, algorithm="fricas")
```

```
[Out] 3/35*(6*c^4*x^6 + 18*b*c^3*x^5 + (23*b^2*c^2 - 2*a*c^3)*x^4 + 5*a^2*b^2 - 1
4*a^3*c + 4*(4*b^3*c - a*b*c^2)*x^3 + (5*b^4 + 8*a*b^2*c - 22*a^2*c^2)*x^2
+ 2*(5*a*b^3 - 11*a^2*b*c)*x)*(2*c*d*x + b*d)^(1/3)*(c*x^2 + b*x + a)^(1/3)
/(128*(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*d^8*x^7 + 448*(b^5*c^6 - 8*a*b^3
*c^7 + 16*a^2*b*c^8)*d^8*x^6 + 672*(b^6*c^5 - 8*a*b^4*c^6 + 16*a^2*b^2*c^7)
```

```
*d^8*x^5 + 560*(b^7*c^4 - 8*a*b^5*c^5 + 16*a^2*b^3*c^6)*d^8*x^4 + 280*(b^8*
c^3 - 8*a*b^6*c^4 + 16*a^2*b^4*c^5)*d^8*x^3 + 84*(b^9*c^2 - 8*a*b^7*c^3 + 1
6*a^2*b^5*c^4)*d^8*x^2 + 14*(b^10*c - 8*a*b^8*c^2 + 16*a^2*b^6*c^3)*d^8*x +
 (b^11 - 8*a*b^9*c + 16*a^2*b^7*c^2)*d^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(4/3)/(2*c*d*x+b*d)**(23/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx + a\right)^{\frac{4}{3}}}{(2\,cdx + bd)^{\frac{23}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(23/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(4/3)/(2*c*d*x + b*d)^(23/3), x)
```

3.1415 $\int \frac{(a+bx+cx^2)^{4/3}}{(bd+2cdx)^{29/3}}\,dx$

Optimal. Leaf size=133

$$\frac{27\left(a+bx+cx^2\right)^{7/3}}{455d^5\left(b^2-4ac\right)^3(bd+2cdx)^{14/3}}+\frac{9\left(a+bx+cx^2\right)^{7/3}}{65d^3\left(b^2-4ac\right)^2(bd+2cdx)^{20/3}}+\frac{3\left(a+bx+cx^2\right)^{7/3}}{13d\left(b^2-4ac\right)(bd+2cdx)^{26/3}}$$

```
[Out] (3*(a + b*x + c*x^2)^(7/3))/(13*(b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(26/3)) + (
9*(a + b*x + c*x^2)^(7/3))/(65*(b^2 - 4*a*c)^2*d^3*(b*d + 2*c*d*x)^(20/3))
+ (27*(a + b*x + c*x^2)^(7/3))/(455*(b^2 - 4*a*c)^3*d^5*(b*d + 2*c*d*x)^(14
/3))
```

Rubi [A] time = 0.0658108, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {693, 682}

$$\frac{27\left(a+bx+cx^2\right)^{7/3}}{455d^5\left(b^2-4ac\right)^3(bd+2cdx)^{14/3}}+\frac{9\left(a+bx+cx^2\right)^{7/3}}{65d^3\left(b^2-4ac\right)^2(bd+2cdx)^{20/3}}+\frac{3\left(a+bx+cx^2\right)^{7/3}}{13d\left(b^2-4ac\right)(bd+2cdx)^{26/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(4/3)/(b*d + 2*c*d*x)^(29/3),x]
```

```
[Out] (3*(a + b*x + c*x^2)^(7/3))/(13*(b^2 - 4*a*c)*d*(b*d + 2*c*d*x)^(26/3)) + (
9*(a + b*x + c*x^2)^(7/3))/(65*(b^2 - 4*a*c)^2*d^3*(b*d + 2*c*d*x)^(20/3))
+ (27*(a + b*x + c*x^2)^(7/3))/(455*(b^2 - 4*a*c)^3*d^5*(b*d + 2*c*d*x)^(14
/3))
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 682

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*c*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(e*(p + 1)
*(b^2 - 4*a*c)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c,
0] && EqQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{4/3}}{(bd+2cdx)^{29/3}}\,dx = \frac{3(a+bx+cx^2)^{7/3}}{13(b^2-4ac)\,d(bd+2cdx)^{26/3}} + \frac{6\int \frac{(a+bx+cx^2)^{4/3}}{(bd+2cdx)^{23/3}}\,dx}{13(b^2-4ac)\,d^2}$$
$$= \frac{3(a+bx+cx^2)^{7/3}}{13(b^2-4ac)\,d(bd+2cdx)^{26/3}} + \frac{9(a+bx+cx^2)^{7/3}}{65(b^2-4ac)^2 d^3(bd+2cdx)^{20/3}} + \frac{9\int \frac{(a+bx+cx^2)^{4/3}}{(bd+2cdx)^{17/3}}\,dx}{65(b^2-4ac)^2 d^4}$$
$$= \frac{3(a+bx+cx^2)^{7/3}}{13(b^2-4ac)\,d(bd+2cdx)^{26/3}} + \frac{9(a+bx+cx^2)^{7/3}}{65(b^2-4ac)^2 d^3(bd+2cdx)^{20/3}} + \frac{27(a+bx+cx^2}{455(b^2-4ac)^3 d^5(bd+}$$

Mathematica [A] time = 0.104086, size = 122, normalized size = 0.92

$$\frac{3(a+x(b+cx))^{7/3}\left(16c^2\left(35a^2-21acx^2+9c^2x^4\right)+4b^2c\left(75cx^2-91a\right)+48bc^2x\left(6cx^2-7a\right)+156b^3cx+65b^4\right)}{455d^9\left(b^2-4ac\right)^3(b+2cx)^8(d(b+2cx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(4/3)/(b*d + 2*c*d*x)^(29/3),x]
```

```
[Out] (3*(a + x*(b + c*x))^(7/3)*(65*b^4 + 156*b^3*c*x + 48*b*c^2*x*(-7*a + 6*c*x
^2) + 4*b^2*c*(-91*a + 75*c*x^2) + 16*c^2*(35*a^2 - 21*a*c*x^2 + 9*c^2*x^4)
))/(455*(b^2 - 4*a*c)^3*d^9*(b + 2*c*x)^8*(d*(b + 2*c*x))^(2/3))
```

Maple [A] time = 0.048, size = 139, normalized size = 1.1

$$-\frac{(6cx+3b)\left(144c^4x^4+288bc^3x^3-336x^2ac^3+300x^2b^2c^2-336bac^2x+156b^3cx+560a^2c^2-364acb^2+65b^4\right)}{29120a^3c^3-21840a^2b^2c^2+5460ab^4c-455b^6}\,($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(29/3),x)
```

```
[Out] -3/455*(2*c*x+b)*(c*x^2+b*x+a)^(7/3)*(144*c^4*x^4+288*b*c^3*x^3-336*a*c^3*x
^2+300*b^2*c^2*x^2-336*a*b*c^2*x+156*b^3*c*x+560*a^2*c^2-364*a*b^2*c+65*b^4
)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(2*c*d*x+b*d)^(29/3)
```

Maxima [B] time = 2.55788, size = 1057, normalized size = 7.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(29/3),x, algorithm="maxima")
```

```
[Out] 3/455*(144*c^6*d^(1/3)*x^8 + 576*b*c^5*d^(1/3)*x^7 + 12*(85*b^2*c^4*d^(1/3)
 - 4*a*c^5*d^(1/3))*x^6 + 65*a^2*b^4*d^(1/3) - 364*a^3*b^2*c*d^(1/3) + 560*
a^4*c^2*d^(1/3) + 36*(29*b^3*c^3*d^(1/3) - 4*a*b*c^4*d^(1/3))*x^5 + (677*b^
4*c^2*d^(1/3) - 196*a*b^2*c^3*d^(1/3) + 32*a^2*c^4*d^(1/3))*x^4 + 2*(143*b^
5*c*d^(1/3) - 76*a*b^3*c^2*d^(1/3) + 32*a^2*b*c^3*d^(1/3))*x^3 + (65*b^6*d^
```

```
(1/3) + 78*a*b^4*c*d^(1/3) - 540*a^2*b^2*c^2*d^(1/3) + 784*a^3*c^3*d^(1/3))
*x^2 + 2*(65*a*b^5*d^(1/3) - 286*a^2*b^3*c*d^(1/3) + 392*a^3*b*c^2*d^(1/3))
*x)*(c*x^2 + b*x + a)^(1/3)/((b^14*d^10 - 12*a*b^12*c*d^10 + 48*a^2*b^10*c^
2*d^10 - 64*a^3*b^8*c^3*d^10 + 256*(b^6*c^8*d^10 - 12*a*b^4*c^9*d^10 + 48*a
^2*b^2*c^10*d^10 - 64*a^3*c^11*d^10)*x^8 + 1024*(b^7*c^7*d^10 - 12*a*b^5*c^
8*d^10 + 48*a^2*b^3*c^9*d^10 - 64*a^3*b*c^10*d^10)*x^7 + 1792*(b^8*c^6*d^10
 - 12*a*b^6*c^7*d^10 + 48*a^2*b^4*c^8*d^10 - 64*a^3*b^2*c^9*d^10)*x^6 + 179
2*(b^9*c^5*d^10 - 12*a*b^7*c^6*d^10 + 48*a^2*b^5*c^7*d^10 - 64*a^3*b^3*c^8*
d^10)*x^5 + 1120*(b^10*c^4*d^10 - 12*a*b^8*c^5*d^10 + 48*a^2*b^6*c^6*d^10 -
 64*a^3*b^4*c^7*d^10)*x^4 + 448*(b^11*c^3*d^10 - 12*a*b^9*c^4*d^10 + 48*a^2
*b^7*c^5*d^10 - 64*a^3*b^5*c^6*d^10)*x^3 + 112*(b^12*c^2*d^10 - 12*a*b^10*c
^3*d^10 + 48*a^2*b^8*c^4*d^10 - 64*a^3*b^6*c^5*d^10)*x^2 + 16*(b^13*c*d^10
- 12*a*b^11*c^2*d^10 + 48*a^2*b^9*c^3*d^10 - 64*a^3*b^7*c^4*d^10)*x)*(2*c*x
 + b)^(2/3))
```

Fricas [B] time = 4.3618, size = 1536, normalized size = 11.55

$$\overline{455\left(512\left(b^6c^9-12\,ab^4c^{10}+48\,a^2b^2c^{11}-64\,a^3c^{12}\right)d^{10}x^9+2304\left(b^7c^8-12\,ab^5c^9+48\,a^2b^3c^{10}-64\,a^3bc^{11}\right)d^{10}x^8+4608\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(29/3),x, algorithm="fricas")
```

```
[Out] 3/455*(144*c^6*x^8 + 576*b*c^5*x^7 + 12*(85*b^2*c^4 - 4*a*c^5)*x^6 + 65*a^2
*b^4 - 364*a^3*b^2*c + 560*a^4*c^2 + 36*(29*b^3*c^3 - 4*a*b*c^4)*x^5 + (677
*b^4*c^2 - 196*a*b^2*c^3 + 32*a^2*c^4)*x^4 + 2*(143*b^5*c - 76*a*b^3*c^2 +
32*a^2*b*c^3)*x^3 + (65*b^6 + 78*a*b^4*c - 540*a^2*b^2*c^2 + 784*a^3*c^3)*x
^2 + 2*(65*a*b^5 - 286*a^2*b^3*c + 392*a^3*b*c^2)*x)*(2*c*d*x + b*d)^(1/3)*
(c*x^2 + b*x + a)^(1/3)/(512*(b^6*c^9 - 12*a*b^4*c^10 + 48*a^2*b^2*c^11 - 6
4*a^3*c^12)*d^10*x^9 + 2304*(b^7*c^8 - 12*a*b^5*c^9 + 48*a^2*b^3*c^10 - 64*
a^3*b*c^11)*d^10*x^8 + 4608*(b^8*c^7 - 12*a*b^6*c^8 + 48*a^2*b^4*c^9 - 64*a
^3*b^2*c^10)*d^10*x^7 + 5376*(b^9*c^6 - 12*a*b^7*c^7 + 48*a^2*b^5*c^8 - 64*
a^3*b^3*c^9)*d^10*x^6 + 4032*(b^10*c^5 - 12*a*b^8*c^6 + 48*a^2*b^6*c^7 - 64
*a^3*b^4*c^8)*d^10*x^5 + 2016*(b^11*c^4 - 12*a*b^9*c^5 + 48*a^2*b^7*c^6 - 6
4*a^3*b^5*c^7)*d^10*x^4 + 672*(b^12*c^3 - 12*a*b^10*c^4 + 48*a^2*b^8*c^5 -
64*a^3*b^6*c^6)*d^10*x^3 + 144*(b^13*c^2 - 12*a*b^11*c^3 + 48*a^2*b^9*c^4 -
 64*a^3*b^7*c^5)*d^10*x^2 + 18*(b^14*c - 12*a*b^12*c^2 + 48*a^2*b^10*c^3 -
64*a^3*b^8*c^4)*d^10*x + (b^15 - 12*a*b^13*c + 48*a^2*b^11*c^2 - 64*a^3*b^9
*c^3)*d^10)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(4/3)/(2*c*d*x+b*d)**(29/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{4}{3}}}{\left(2\,cdx+bd\right)^{\frac{29}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(29/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(4/3)/(2*c*d*x + b*d)^(29/3), x)
```

3.1416 $\int \frac{(a+bx+cx^2)^{4/3}}{(bd+2cdx)^{2/3}}\,dx$

Optimal. Leaf size=597

$$\frac{(b^2-4ac)\left(-4ac+b^2-(b+2cx)^2\right)\sqrt[3]{d(b+2cx)}\left(2\sqrt[3]{c}d^{2/3}-\frac{\sqrt[3]{2}(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}\right)\sqrt{\frac{\frac{2^{2/3}\sqrt[3]{c}d^{2/3}(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}+\frac{(d(b+2cx))^{4/3}}{(a+bx+cx^2)^{2/3}}+2\sqrt[3]{2}c^{2/3}d^{4/3}}{\left(2^{2/3}\sqrt[3]{c}d^{2/3}-\frac{(1+\sqrt{3})(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}\right)^2}}\,\text{Ellip}}{72\sqrt[4]{3}c^{10/3}d^{5/3}\left(a+bx+cx^2\right)^{2/3}\sqrt{-\frac{(d(b+2cx))^{2/3}\left(2^{2/3}\sqrt[3]{c}d^{2/3}-\frac{(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}\right)}{\sqrt[3]{a+bx+cx^2}\left(2^{2/3}\sqrt[3]{c}d^{2/3}-\frac{(1+\sqrt{3})(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}\right)^2}}}$$

```
[Out] -((b^2 - 4*a*c)*(d*(b + 2*c*x))^(1/3)*(a + b*x + c*x^2)^(1/3))/(9*c^2*d) +
((d*(b + 2*c*x))^(1/3)*(a + b*x + c*x^2)^(4/3))/(6*c*d) + ((b^2 - 4*a*c)*(d
*(b + 2*c*x))^(1/3)*(b^2 - 4*a*c - (b + 2*c*x)^2)*(2*c^(1/3)*d^(2/3) - (2^(
1/3)*(d*(b + 2*c*x))^(2/3))/(a + b*x + c*x^2)^(1/3))*Sqrt[(2*2^(1/3)*c^(2/3
)*d^(4/3) + (d*(b + 2*c*x))^(4/3)/(a + b*x + c*x^2)^(2/3) + (2^(2/3)*c^(1/3
)*d^(2/3)*(d*(b + 2*c*x))^(2/3))/(a + b*x + c*x^2)^(1/3))/(2^(2/3)*c^(1/3)*
d^(2/3) - ((1 + Sqrt[3])*(d*(b + 2*c*x))^(2/3))/(a + b*x + c*x^2)^(1/3))^2]
*EllipticF[ArcCos[(2^(2/3)*c^(1/3)*d^(2/3) - ((1 - Sqrt[3])*(d*(b + 2*c*x))
^(2/3))/(a + b*x + c*x^2)^(1/3))/(2^(2/3)*c^(1/3)*d^(2/3) - ((1 + Sqrt[3])*
(d*(b + 2*c*x))^(2/3))/(a + b*x + c*x^2)^(1/3))], (2 + Sqrt[3])/4])/(72*3^(
1/4)*c^(10/3)*d^(5/3)*(a + b*x + c*x^2)^(2/3)*Sqrt[-(((d*(b + 2*c*x))^(2/3)
*(2^(2/3)*c^(1/3)*d^(2/3) - (d*(b + 2*c*x))^(2/3)/(a + b*x + c*x^2)^(1/3)))
/((a + b*x + c*x^2)^(1/3)*(2^(2/3)*c^(1/3)*d^(2/3) - ((1 + Sqrt[3])*(d*(b +
 2*c*x))^(2/3))/(a + b*x + c*x^2)^(1/3))^2))])
```

Rubi [A] time = 2.52024, antiderivative size = 597, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {694, 279, 329, 241, 225}

$$\frac{(b^2-4ac)\left(-4ac+b^2-(b+2cx)^2\right)\sqrt[3]{d(b+2cx)}\left(2\sqrt[3]{c}d^{2/3}-\frac{\sqrt[3]{2}(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}\right)\sqrt{\frac{\frac{2^{2/3}\sqrt[3]{c}d^{2/3}(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}+\frac{(d(b+2cx))^{4/3}}{(a+bx+cx^2)^{2/3}}+2\sqrt[3]{2}c^{2/3}d^{4/3}}{\left(2^{2/3}\sqrt[3]{c}d^{2/3}-\frac{(1+\sqrt{3})(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}\right)^2}}\,F\left(\text{co}\right.}{72\sqrt[4]{3}c^{10/3}d^{5/3}\left(a+bx+cx^2\right)^{2/3}\sqrt{-\frac{(d(b+2cx))^{2/3}\left(2^{2/3}\sqrt[3]{c}d^{2/3}-\frac{(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}\right)}{\sqrt[3]{a+bx+cx^2}\left(2^{2/3}\sqrt[3]{c}d^{2/3}-\frac{(1+\sqrt{3})(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(4/3)/(b*d + 2*c*d*x)^(2/3),x]
```



```
[Out] -((b^2 - 4*a*c)*(d*(b + 2*c*x))^(1/3)*(a + b*x + c*x^2)^(1/3))/(9*c^2*d) +
((d*(b + 2*c*x))^(1/3)*(a + b*x + c*x^2)^(4/3))/(6*c*d) + ((b^2 - 4*a*c)*(d
*(b + 2*c*x))^(1/3)*(b^2 - 4*a*c - (b + 2*c*x)^2)*(2*c^(1/3)*d^(2/3) - (2^(
1/3)*(d*(b + 2*c*x))^(2/3))/(a + b*x + c*x^2)^(1/3))*Sqrt[(2*2^(1/3)*c^(2/3
)*d^(4/3) + (d*(b + 2*c*x))^(4/3)/(a + b*x + c*x^2)^(2/3) + (2^(2/3)*c^(1/3
)*d^(2/3)*(d*(b + 2*c*x))^(2/3))/(a + b*x + c*x^2)^(1/3))/(2^(2/3)*c^(1/3)*
d^(2/3) - ((1 + Sqrt[3])*(d*(b + 2*c*x))^(2/3))/(a + b*x + c*x^2)^(1/3))^2]
*EllipticF[ArcCos[(2^(2/3)*c^(1/3)*d^(2/3) - ((1 - Sqrt[3])*(d*(b + 2*c*x))
^(2/3))/(a + b*x + c*x^2)^(1/3))/(2^(2/3)*c^(1/3)*d^(2/3) - ((1 + Sqrt[3])*
(d*(b + 2*c*x))^(2/3))/(a + b*x + c*x^2)^(1/3))], (2 + Sqrt[3])/4])/(72*3^(
1/4)*c^(10/3)*d^(5/3)*(a + b*x + c*x^2)^(2/3)*Sqrt[-(((d*(b + 2*c*x))^(2/3)
```

```
*(2^(2/3)*c^(1/3)*d^(2/3) - (d*(b + 2*c*x))^(2/3)/(a + b*x + c*x^2)^(1/3)))
/((a + b*x + c*x^2)^(1/3)*(2^(2/3)*c^(1/3)*d^(2/3) - ((1 + Sqrt[3])*(d*(b +
 2*c*x))^(2/3))/(a + b*x + c*x^2)^(1/3))^2))])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{4/3}}{(bd+2cdx)^{2/3}}dx = \frac{\text{Subst}\left(\int \frac{\left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)^{4/3}}{x^{2/3}}dx, x, bd+2cdx\right)}{2cd}$$

$$= \frac{\sqrt[3]{d(b+2cx)}\,(a+bx+cx^2)^{4/3}}{6cd} - \frac{(b^2-4ac)\,\text{Subst}\left(\int \frac{\sqrt[3]{a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}}}{x^{2/3}}dx, x, bd+2cdx\right)}{9c^2d}$$

$$= -\frac{(b^2-4ac)\sqrt[3]{d(b+2cx)}\sqrt[3]{a+bx+cx^2}}{9c^2d} + \frac{\sqrt[3]{d(b+2cx)}\,(a+bx+cx^2)^{4/3}}{6cd} + \frac{(b^2-4ac)^2\,\text{Subst}}{}$$

$$= -\frac{(b^2-4ac)\sqrt[3]{d(b+2cx)}\sqrt[3]{a+bx+cx^2}}{9c^2d} + \frac{\sqrt[3]{d(b+2cx)}\,(a+bx+cx^2)^{4/3}}{6cd} + \frac{(b^2-4ac)^2\,\text{Subst}}{}$$

$$= -\frac{(b^2-4ac)\sqrt[3]{d(b+2cx)}\sqrt[3]{a+bx+cx^2}}{9c^2d} + \frac{\sqrt[3]{d(b+2cx)}\,(a+bx+cx^2)^{4/3}}{6cd} + \frac{(b^2-4ac)^2\,\text{Subst}}{18c^3d\sqrt{\frac{a-}{a+x(}}}$$

$$= -\frac{(b^2-4ac)\sqrt[3]{d(b+2cx)}\sqrt[3]{a+bx+cx^2}}{9c^2d} + \frac{\sqrt[3]{d(b+2cx)}\,(a+bx+cx^2)^{4/3}}{6cd} + \frac{(b^2-4ac)^2\sqrt[3]{d(b+}}{}$$

Mathematica [C] time = 0.0767884, size = 104, normalized size = 0.17

$$-\frac{3(b^2-4ac)\sqrt[3]{a+x(b+cx)}\sqrt[3]{d(b+2cx)}\,{}_2F_1\left(-\frac{4}{3},\frac{1}{6};\frac{7}{6};\frac{(b+2cx)^2}{b^2-4ac}\right)}{8\,2^{2/3}c^2d\sqrt[3]{\frac{c(a+x(b+cx))}{4ac-b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(4/3)/(b*d + 2*c*d*x)^(2/3),x]
```

```
[Out] (-3*(b^2 - 4*a*c)*(d*(b + 2*c*x))^(1/3)*(a + x*(b + c*x))^(1/3)*Hypergeomet
ric2F1[-4/3, 1/6, 7/6, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(8*2^(2/3)*c^2*d*((c*(
a + x*(b + c*x)))/(-b^2 + 4*a*c))^(1/3))
```

Maple [F] time = 0.091, size = 0, normalized size = 0.

$$\int (cx^2+bx+a)^{\frac{4}{3}}(2\,cdx+bd)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(2/3),x)
```

```
[Out] int((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{4}{3}}}{\left(2\,cdx+bd\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(4/3)/(2*c*d*x + b*d)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(cx^2+bx+a\right)^{\frac{4}{3}}}{\left(2\,cdx+bd\right)^{\frac{2}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^(4/3)/(2*c*d*x + b*d)^(2/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx+cx^2\right)^{\frac{4}{3}}}{\left(d\,(b+2cx)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(4/3)/(2*c*d*x+b*d)**(2/3),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(4/3)/(d*(b + 2*c*x))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{4}{3}}}{\left(2\,cdx+bd\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(4/3)/(2*c*d*x + b*d)^(2/3), x)
```

3.1417 $\int \frac{(a+bx+cx^2)^{4/3}}{(bd+2cdx)^{8/3}}\,dx$

Optimal. Leaf size=581

$$-\frac{\left(-4ac+b^2-(b+2cx)^2\right)\sqrt[3]{d(b+2cx)}\left(2\sqrt[3]{c}d^{2/3}-\frac{\sqrt[3]{2}(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}\right)\sqrt{\frac{\frac{2^{2/3}\sqrt[3]{c}d^{2/3}(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}+\frac{(d(b+2cx))^{4/3}}{(a+bx+cx^2)^{2/3}}+2\sqrt[3]{2}c^{2/3}d^{4/3}}{\left(2^{2/3}\sqrt[3]{c}d^{2/3}-\frac{(1+\sqrt{3})(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}\right)^2}}\,\text{EllipticF}\left(\cos\cdots\right.}{40\sqrt[4]{3}c^{10/3}d^{11/3}\left(a+bx+cx^2\right)^{2/3}\sqrt{-\frac{(d(b+2cx))^{2/3}\left(2^{2/3}\sqrt[3]{c}d^{2/3}-\frac{(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}\right)}{\sqrt[3]{a+bx+cx^2}\left(2^{2/3}\sqrt[3]{c}d^{2/3}-\frac{(1+\sqrt{3})(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}\right)^2}}}$$

```
[Out] ((d*(b + 2*c*x))^(1/3)*(a + b*x + c*x^2)^(1/3))/(5*c^2*d^3) - (3*(a + b*x +
 c*x^2)^(4/3))/(10*c*d*(d*(b + 2*c*x))^(5/3)) - ((d*(b + 2*c*x))^(1/3)*(b^2
 - 4*a*c - (b + 2*c*x)^2)*(2*c^(1/3)*d^(2/3) - (2^(1/3)*(d*(b + 2*c*x))^(2/
3))/(a + b*x + c*x^2)^(1/3))*Sqrt[(2*2^(1/3)*c^(2/3)*d^(4/3) + (d*(b + 2*c*
x))^(4/3)/(a + b*x + c*x^2)^(2/3) + (2^(2/3)*c^(1/3)*d^(2/3)*(d*(b + 2*c*x)
)^(2/3))/(a + b*x + c*x^2)^(1/3))/(2^(2/3)*c^(1/3)*d^(2/3) - ((1 + Sqrt[3])
*(d*(b + 2*c*x))^(2/3))/(a + b*x + c*x^2)^(1/3))^2]*EllipticF[ArcCos[(2^(2/
3)*c^(1/3)*d^(2/3) - ((1 - Sqrt[3])*(d*(b + 2*c*x))^(2/3))/(a + b*x + c*x^2
)^(1/3))/(2^(2/3)*c^(1/3)*d^(2/3) - ((1 + Sqrt[3])*(d*(b + 2*c*x))^(2/3))/(
a + b*x + c*x^2)^(1/3))], (2 + Sqrt[3])/4])/(40*3^(1/4)*c^(10/3)*d^(11/3)*(
a + b*x + c*x^2)^(2/3)*Sqrt[-(((d*(b + 2*c*x))^(2/3)*(2^(2/3)*c^(1/3)*d^(2/
3) - (d*(b + 2*c*x))^(2/3)/(a + b*x + c*x^2)^(1/3)))/((a + b*x + c*x^2)^(1/
3)*(2^(2/3)*c^(1/3)*d^(2/3) - ((1 + Sqrt[3])*(d*(b + 2*c*x))^(2/3))/(a + b*
x + c*x^2)^(1/3))^2))])
```

Rubi [A] time = 1.94999, antiderivative size = 581, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {694, 277, 279, 329, 241, 225}

$$-\frac{\left(-4ac+b^2-(b+2cx)^2\right)\sqrt[3]{d(b+2cx)}\left(2\sqrt[3]{c}d^{2/3}-\frac{\sqrt[3]{2}(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}\right)\sqrt{\frac{\frac{2^{2/3}\sqrt[3]{c}d^{2/3}(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}+\frac{(d(b+2cx))^{4/3}}{(a+bx+cx^2)^{2/3}}+2\sqrt[3]{2}c^{2/3}d^{4/3}}{\left(2^{2/3}\sqrt[3]{c}d^{2/3}-\frac{(1+\sqrt{3})(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}\right)^2}}\,F\left(\cos^{-1}\left(\frac{2^{2/3}\cdots}{2^{2/3}\cdots}\right.\right.}{40\sqrt[4]{3}c^{10/3}d^{11/3}\left(a+bx+cx^2\right)^{2/3}\sqrt{-\frac{(d(b+2cx))^{2/3}\left(2^{2/3}\sqrt[3]{c}d^{2/3}-\frac{(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}\right)}{\sqrt[3]{a+bx+cx^2}\left(2^{2/3}\sqrt[3]{c}d^{2/3}-\frac{(1+\sqrt{3})(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(4/3)/(b*d + 2*c*d*x)^(8/3),x]
```

```
[Out] ((d*(b + 2*c*x))^(1/3)*(a + b*x + c*x^2)^(1/3))/(5*c^2*d^3) - (3*(a + b*x +
 c*x^2)^(4/3))/(10*c*d*(d*(b + 2*c*x))^(5/3)) - ((d*(b + 2*c*x))^(1/3)*(b^2
 - 4*a*c - (b + 2*c*x)^2)*(2*c^(1/3)*d^(2/3) - (2^(1/3)*(d*(b + 2*c*x))^(2/
3))/(a + b*x + c*x^2)^(1/3))*Sqrt[(2*2^(1/3)*c^(2/3)*d^(4/3) + (d*(b + 2*c*
x))^(4/3)/(a + b*x + c*x^2)^(2/3) + (2^(2/3)*c^(1/3)*d^(2/3)*(d*(b + 2*c*x)
)^(2/3))/(a + b*x + c*x^2)^(1/3))/(2^(2/3)*c^(1/3)*d^(2/3) - ((1 + Sqrt[3])
*(d*(b + 2*c*x))^(2/3))/(a + b*x + c*x^2)^(1/3))^2]*EllipticF[ArcCos[(2^(2/
3)*c^(1/3)*d^(2/3) - ((1 - Sqrt[3])*(d*(b + 2*c*x))^(2/3))/(a + b*x + c*x^2
)^(1/3))/(2^(2/3)*c^(1/3)*d^(2/3) - ((1 + Sqrt[3])*(d*(b + 2*c*x))^(2/3))/(
a + b*x + c*x^2)^(1/3))], (2 + Sqrt[3])/4])/(40*3^(1/4)*c^(10/3)*d^(11/3)*(
a + b*x + c*x^2)^(2/3)*Sqrt[-(((d*(b + 2*c*x))^(2/3)*(2^(2/3)*c^(1/3)*d^(2/
```

```
3) - (d*(b + 2*c*x))^(2/3)/(a + b*x + c*x^2)^(1/3)))/((a + b*x + c*x^2)^(1/
3)*(2^(2/3)*c^(1/3)*d^(2/3) - ((1 + Sqrt[3])*(d*(b + 2*c*x))^(2/3))/(a + b*
x + c*x^2)^(1/3))^2))])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{4/3}}{(bd+2cdx)^{8/3}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)^{4/3}}{x^{8/3}}\,dx,x,bd+2cdx\right)}{2cd}$$

$$= -\frac{3\left(a+bx+cx^2\right)^{4/3}}{10cd(d(b+2cx))^{5/3}} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}}}{x^{2/3}}\,dx,x,bd+2cdx\right)}{5c^2d^3}$$

$$= \frac{\sqrt[3]{d(b+2cx)}\sqrt[3]{a+bx+cx^2}}{5c^2d^3} - \frac{3\left(a+bx+cx^2\right)^{4/3}}{10cd(d(b+2cx))^{5/3}} - \frac{\left(b^2-4ac\right)\text{Subst}\left(\int \frac{1}{x^{2/3}\left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)^{2/3}}\,dx,x,\right.}{30c^3d^3}$$

$$= \frac{\sqrt[3]{d(b+2cx)}\sqrt[3]{a+bx+cx^2}}{5c^2d^3} - \frac{3\left(a+bx+cx^2\right)^{4/3}}{10cd(d(b+2cx))^{5/3}} - \frac{\left(b^2-4ac\right)\text{Subst}\left(\int \frac{1}{\left(a-\frac{b^2}{4c}+\frac{x^6}{4cd^2}\right)^{2/3}}\,dx,x,\sqrt[3]{a}\right.}{10c^3d^3}$$

$$= \frac{\sqrt[3]{d(b+2cx)}\sqrt[3]{a+bx+cx^2}}{5c^2d^3} - \frac{3\left(a+bx+cx^2\right)^{4/3}}{10cd(d(b+2cx))^{5/3}} - \frac{\left(b^2-4ac\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^6}{4cd^2}}}\,dx,x,\frac{\sqrt[3]{d(b+2c}}{\sqrt[6]{a+x(b+}}\right.}{10c^3d^3\sqrt{\frac{a-\frac{b^2}{4c}}{a+x(b+cx)}}\sqrt{a+x(b+cx)}}$$

$$= \frac{\sqrt[3]{d(b+2cx)}\sqrt[3]{a+bx+cx^2}}{5c^2d^3} - \frac{3\left(a+bx+cx^2\right)^{4/3}}{10cd(d(b+2cx))^{5/3}} - \frac{\left(b^2-4ac\right)\sqrt[3]{d(b+2cx)}\left(2^{2/3}\sqrt[3]{c}d^{2/3}-\frac{(d(b+2}{\sqrt[3]{a+x}}\right.}{5\ 2^{2/3}\sqrt[4]{3}c^{10/3}d^{11/3}\sqrt{\frac{4a-}{a+x(}}}$$

Mathematica [C] time = 0.0645814, size = 104, normalized size = 0.18

$$\frac{3\left(b^2-4ac\right)\sqrt[3]{a+x(b+cx)}\,{}_2F_1\left(-\frac{4}{3},-\frac{5}{6};\frac{1}{6};\frac{(b+2cx)^2}{b^2-4ac}\right)}{40\ 2^{2/3}c^2d\sqrt[3]{\frac{c(a+x(b+cx))}{4ac-b^2}}(d(b+2cx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(4/3)/(b*d + 2*c*d*x)^(8/3),x]
```

```
[Out] (3*(b^2 - 4*a*c)*(a + x*(b + c*x))^(1/3)*Hypergeometric2F1[-4/3, -5/6, 1/6,
 (b + 2*c*x)^2/(b^2 - 4*a*c)])/(40*2^(2/3)*c^2*d*(d*(b + 2*c*x))^(5/3)*((c*
(a + x*(b + c*x)))/(-b^2 + 4*a*c))^(1/3))
```

Maple [F] time = 0.776, size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{\frac{4}{3}}(2\,cdx+bd)^{-\frac{8}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(8/3),x)
```

```
[Out] int((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(8/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{4}{3}}}{\left(2\,cdx+bd\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(8/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(4/3)/(2*c*d*x + b*d)^(8/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(2\,cdx+bd\right)^{\frac{1}{3}}\left(cx^2+bx+a\right)^{\frac{4}{3}}}{8\,c^3d^3x^3+12\,bc^2d^3x^2+6\,b^2cd^3x+b^3d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(8/3),x, algorithm="fricas")
```

```
[Out] integral((2*c*d*x + b*d)^(1/3)*(c*x^2 + b*x + a)^(4/3)/(8*c^3*d^3*x^3 + 12*
b*c^2*d^3*x^2 + 6*b^2*c*d^3*x + b^3*d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx+cx^2\right)^{\frac{4}{3}}}{\left(d\left(b+2cx\right)\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(4/3)/(2*c*d*x+b*d)**(8/3),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(4/3)/(d*(b + 2*c*x))**(8/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{4}{3}}}{\left(2\,cdx+bd\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(8/3),x, algorithm="giac")

[Out] integrate((c*x^2 + b*x + a)^(4/3)/(2*c*d*x + b*d)^(8/3), x)
```

3.1418 $\int \frac{(a+bx+cx^2)^{4/3}}{(bd+2cdx)^{14/3}}\,dx$

Optimal. Leaf size=591

$$\frac{3^{3/4}\left(-4ac+b^2-(b+2cx)^2\right)\sqrt[3]{d(b+2cx)}\left(2\sqrt[3]{c}d^{2/3}-\frac{\sqrt[3]{2}(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}\right)\sqrt{\frac{\frac{2^{2/3}\sqrt[3]{c}d^{2/3}(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}+\frac{(d(b+2cx))^{4/3}}{(a+bx+cx^2)^{2/3}}+2\sqrt[3]{2}c^{2/3}d^{4/3}}{\left(2^{2/3}\sqrt[3]{c}d^{2/3}-\frac{(1+\sqrt{3})(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}\right)^2}}\,\text{Elliptic}}{440c^{10/3}d^{17/3}\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{2/3}\sqrt{-\frac{(d(b+2cx))^{2/3}\left(2^{2/3}\sqrt[3]{c}d^{2/3}-\frac{(d(b+2cx))}{\sqrt[3]{a+bx+c}}\right.}{\sqrt[3]{a+bx+cx^2}\left(2^{2/3}\sqrt[3]{c}d^{2/3}-\frac{(1+\sqrt{3})(d(b+2c}{\sqrt[3]{a+bx+cx}}\right.}}}$$

```
[Out] (-3*(a + b*x + c*x^2)^(1/3))/(55*c^2*d^3*(d*(b + 2*c*x))^(5/3)) - (3*(a + b
*x + c*x^2)^(4/3))/(22*c*d*(d*(b + 2*c*x))^(11/3)) + (3^(3/4)*(d*(b + 2*c*x
))^(1/3)*(b^2 - 4*a*c - (b + 2*c*x)^2)*(2*c^(1/3)*d^(2/3) - (2^(1/3)*(d*(b
+ 2*c*x))^(2/3))/(a + b*x + c*x^2)^(1/3))*Sqrt[(2*2^(1/3)*c^(2/3)*d^(4/3) +
 (d*(b + 2*c*x))^(4/3)/(a + b*x + c*x^2)^(2/3) + (2^(2/3)*c^(1/3)*d^(2/3)*(
d*(b + 2*c*x))^(2/3))/(a + b*x + c*x^2)^(1/3))/(2^(2/3)*c^(1/3)*d^(2/3) - (
(1 + Sqrt[3])*(d*(b + 2*c*x))^(2/3))/(a + b*x + c*x^2)^(1/3))^2]*EllipticF[
ArcCos[(2^(2/3)*c^(1/3)*d^(2/3) - ((1 - Sqrt[3])*(d*(b + 2*c*x))^(2/3))/(a
+ b*x + c*x^2)^(1/3))/(2^(2/3)*c^(1/3)*d^(2/3) - ((1 + Sqrt[3])*(d*(b + 2*c
*x))^(2/3))/(a + b*x + c*x^2)^(1/3))], (2 + Sqrt[3])/4])/(440*c^(10/3)*(b^2
 - 4*a*c)*d^(17/3)*(a + b*x + c*x^2)^(2/3)*Sqrt[-(((d*(b + 2*c*x))^(2/3)*(2
^(2/3)*c^(1/3)*d^(2/3) - (d*(b + 2*c*x))^(2/3)/(a + b*x + c*x^2)^(1/3)))/((
a + b*x + c*x^2)^(1/3)*(2^(2/3)*c^(1/3)*d^(2/3) - ((1 + Sqrt[3])*(d*(b + 2*
c*x))^(2/3))/(a + b*x + c*x^2)^(1/3))^2))])
```

Rubi [A] time = 2.00472, antiderivative size = 591, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {694, 277, 329, 241, 225}

$$\frac{3^{3/4}\left(-4ac+b^2-(b+2cx)^2\right)\sqrt[3]{d(b+2cx)}\left(2\sqrt[3]{c}d^{2/3}-\frac{\sqrt[3]{2}(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}\right)\sqrt{\frac{\frac{2^{2/3}\sqrt[3]{c}d^{2/3}(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}+\frac{(d(b+2cx))^{4/3}}{(a+bx+cx^2)^{2/3}}+2\sqrt[3]{2}c^{2/3}d^{4/3}}{\left(2^{2/3}\sqrt[3]{c}d^{2/3}-\frac{(1+\sqrt{3})(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}\right)^2}}\,F\left(\cos^{-1}\right.}{440c^{10/3}d^{17/3}\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{2/3}\sqrt{-\frac{(d(b+2cx))^{2/3}\left(2^{2/3}\sqrt[3]{c}d^{2/3}-\frac{(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}\right)}{\sqrt[3]{a+bx+cx^2}\left(2^{2/3}\sqrt[3]{c}d^{2/3}-\frac{(1+\sqrt{3})(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(4/3)/(b*d + 2*c*d*x)^(14/3),x]
```

```
[Out] (-3*(a + b*x + c*x^2)^(1/3))/(55*c^2*d^3*(d*(b + 2*c*x))^(5/3)) - (3*(a + b
*x + c*x^2)^(4/3))/(22*c*d*(d*(b + 2*c*x))^(11/3)) + (3^(3/4)*(d*(b + 2*c*x
))^(1/3)*(b^2 - 4*a*c - (b + 2*c*x)^2)*(2*c^(1/3)*d^(2/3) - (2^(1/3)*(d*(b
+ 2*c*x))^(2/3))/(a + b*x + c*x^2)^(1/3))*Sqrt[(2*2^(1/3)*c^(2/3)*d^(4/3) +
 (d*(b + 2*c*x))^(4/3)/(a + b*x + c*x^2)^(2/3) + (2^(2/3)*c^(1/3)*d^(2/3)*(
d*(b + 2*c*x))^(2/3))/(a + b*x + c*x^2)^(1/3))/(2^(2/3)*c^(1/3)*d^(2/3) - (
(1 + Sqrt[3])*(d*(b + 2*c*x))^(2/3))/(a + b*x + c*x^2)^(1/3))^2]*EllipticF[
ArcCos[(2^(2/3)*c^(1/3)*d^(2/3) - ((1 - Sqrt[3])*(d*(b + 2*c*x))^(2/3))/(a
+ b*x + c*x^2)^(1/3))/(2^(2/3)*c^(1/3)*d^(2/3) - ((1 + Sqrt[3])*(d*(b + 2*c
*x))^(2/3))/(a + b*x + c*x^2)^(1/3))], (2 + Sqrt[3])/4])/(440*c^(10/3)*(b^2
 - 4*a*c)*d^(17/3)*(a + b*x + c*x^2)^(2/3)*Sqrt[-(((d*(b + 2*c*x))^(2/3)*(2
```

```
^(2/3)*c^(1/3)*d^(2/3) - (d*(b + 2*c*x))^(2/3)/(a + b*x + c*x^2)^(1/3)))/((
a + b*x + c*x^2)^(1/3)*(2^(2/3)*c^(1/3)*d^(2/3) - ((1 + Sqrt[3])*(d*(b + 2*
c*x))^(2/3))/(a + b*x + c*x^2)^(1/3))^2))])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{4/3}}{(bd+2cdx)^{14/3}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)^{4/3}}{x^{14/3}}\,dx, x, bd+2cdx\right)}{2cd}$$

$$= -\frac{3(a+bx+cx^2)^{4/3}}{22cd(d(b+2cx))^{11/3}} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}}}{x^{8/3}}\,dx, x, bd+2cdx\right)}{11c^2d^3}$$

$$= -\frac{3\sqrt[3]{a+bx+cx^2}}{55c^2d^3(d(b+2cx))^{5/3}} - \frac{3(a+bx+cx^2)^{4/3}}{22cd(d(b+2cx))^{11/3}} + \frac{\text{Subst}\left(\int \frac{1}{x^{2/3}\left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)^{2/3}}\,dx, x, bd+2cdx\right)}{110c^3d^5}$$

$$= -\frac{3\sqrt[3]{a+bx+cx^2}}{55c^2d^3(d(b+2cx))^{5/3}} - \frac{3(a+bx+cx^2)^{4/3}}{22cd(d(b+2cx))^{11/3}} + \frac{3\,\text{Subst}\left(\int \frac{1}{\left(a-\frac{b^2}{4c}+\frac{x^6}{4cd^2}\right)^{2/3}}\,dx, x, \sqrt[3]{d(b+2cx)}\right)}{110c^3d^5}$$

$$= -\frac{3\sqrt[3]{a+bx+cx^2}}{55c^2d^3(d(b+2cx))^{5/3}} - \frac{3(a+bx+cx^2)^{4/3}}{22cd(d(b+2cx))^{11/3}} + \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^6}{4cd^2}}}\,dx, x, \frac{\sqrt[3]{d(b+2cx)}}{\sqrt[6]{a+x(b+cx)}}\right)}{110c^3d^5\sqrt{\frac{a-\frac{b^2}{4c}}{a+x(b+cx)}}\sqrt{a+x(b+cx)}}$$

$$= -\frac{3\sqrt[3]{a+bx+cx^2}}{55c^2d^3(d(b+2cx))^{5/3}} - \frac{3(a+bx+cx^2)^{4/3}}{22cd(d(b+2cx))^{11/3}} + \frac{3^{3/4}\sqrt[3]{d(b+2cx)}\left(2^{2/3}\sqrt[3]{c}d^{2/3}-\frac{(d(b+2cx))^{2/3}}{\sqrt[3]{a+x(b+cx)}}\right)}{55\,2^{2/3}c^{10/3}d^{17/3}\sqrt{\frac{4a-\frac{b^2}{c}}{a+x(b+cx)}}}$$

Mathematica [C] time = 0.0912694, size = 112, normalized size = 0.19

$$\frac{3(b^2-4ac)\sqrt[3]{a+x(b+cx)}\sqrt[3]{d(b+2cx)}\,{}_2F_1\left(-\frac{11}{6},-\frac{4}{3};-\frac{5}{6};\frac{(b+2cx)^2}{b^2-4ac}\right)}{88\,2^{2/3}c^2d^5(b+2cx)^4\sqrt[3]{\frac{c(a+x(b+cx))}{4ac-b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(4/3)/(b*d + 2*c*d*x)^(14/3),x]
```

```
[Out] (3*(b^2 - 4*a*c)*(d*(b + 2*c*x))^(1/3)*(a + x*(b + c*x))^(1/3)*Hypergeometr
ic2F1[-11/6, -4/3, -5/6, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(88*2^(2/3)*c^2*d^5*
(b + 2*c*x)^4*((c*(a + x*(b + c*x)))/(-b^2 + 4*a*c))^(1/3))
```

Maple [F] time = 1.164, size = 0, normalized size = 0.

$$\int (cx^2+bx+a)^{\frac{4}{3}}(2\,cdx+bd)^{-\frac{14}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(14/3),x)
```

```
[Out] int((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(14/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{4}{3}}}{\left(2\,cdx+bd\right)^{\frac{14}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(14/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(4/3)/(2*c*d*x + b*d)^(14/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(2\,cdx+bd\right)^{\frac{1}{3}}\left(cx^2+bx+a\right)^{\frac{4}{3}}}{32\,c^5d^5x^5+80\,bc^4d^5x^4+80\,b^2c^3d^5x^3+40\,b^3c^2d^5x^2+10\,b^4cd^5x+b^5d^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(14/3),x, algorithm="fricas")
```

```
[Out] integral((2*c*d*x + b*d)^(1/3)*(c*x^2 + b*x + a)^(4/3)/(32*c^5*d^5*x^5 + 80
*b*c^4*d^5*x^4 + 80*b^2*c^3*d^5*x^3 + 40*b^3*c^2*d^5*x^2 + 10*b^4*c*d^5*x +
 b^5*d^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(4/3)/(2*c*d*x+b*d)**(14/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{4}{3}}}{\left(2\,cdx+bd\right)^{\frac{14}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(14/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(4/3)/(2*c*d*x + b*d)^(14/3), x)
```

3.1419 $\int \frac{(a+bx+cx^2)^{4/3}}{(bd+2cdx)^{20/3}}\,dx$

Optimal. Leaf size=637

$$\frac{3\,3^{3/4}\left(-4ac+b^2-(b+2cx)^2\right)\sqrt[3]{d(b+2cx)}\left(2\sqrt[3]{c}d^{2/3}-\frac{\sqrt[3]{2}(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}\right)\sqrt{\frac{\frac{2^{2/3}\sqrt[3]{c}d^{2/3}(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}+\frac{(d(b+2cx))^{4/3}}{(a+bx+cx^2)^{2/3}}+2\sqrt[3]{2}c^{2/3}d^{4/3}}{\left(2^{2/3}\sqrt[3]{c}d^{2/3}-\frac{(1+\sqrt{3})(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}\right)^2}}\,\text{Ellipt}}{7480c^{10/3}d^{23/3}\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{2/3}\sqrt{-\frac{(d(b+2cx))^{2/3}\left(2^{2/3}\sqrt[3]{c}d^{2/3}-\frac{(d(b+2c}{\sqrt[3]{a+b}}\right.}{\sqrt[3]{a+bx+cx^2}\left(2^{2/3}\sqrt[3]{c}d^{2/3}-\frac{(1+\sqrt{3})(d(b+}{\sqrt[3]{a+bx}}\right.}}}$$

```
[Out] (-3*(a + b*x + c*x^2)^(1/3))/(187*c^2*d^3*(d*(b + 2*c*x))^(11/3)) + (6*(a +
 b*x + c*x^2)^(1/3))/(935*c^2*(b^2 - 4*a*c)*d^5*(d*(b + 2*c*x))^(5/3)) - (3
*(a + b*x + c*x^2)^(4/3))/(34*c*d*(d*(b + 2*c*x))^(17/3)) + (3*3^(3/4)*(d*(
b + 2*c*x))^(1/3)*(b^2 - 4*a*c - (b + 2*c*x)^2)*(2*c^(1/3)*d^(2/3) - (2^(1/
3)*(d*(b + 2*c*x))^(2/3))/(a + b*x + c*x^2)^(1/3))*Sqrt[(2*2^(1/3)*c^(2/3)*
d^(4/3) + (d*(b + 2*c*x))^(4/3)/(a + b*x + c*x^2)^(2/3) + (2^(2/3)*c^(1/3)*
d^(2/3)*(d*(b + 2*c*x))^(2/3))/(a + b*x + c*x^2)^(1/3))/(2^(2/3)*c^(1/3)*d^
(2/3) - ((1 + Sqrt[3])*(d*(b + 2*c*x))^(2/3))/(a + b*x + c*x^2)^(1/3))^2]*E
llipticF[ArcCos[(2^(2/3)*c^(1/3)*d^(2/3) - ((1 - Sqrt[3])*(d*(b + 2*c*x))^(
2/3))/(a + b*x + c*x^2)^(1/3))/(2^(2/3)*c^(1/3)*d^(2/3) - ((1 + Sqrt[3])*(d
*(b + 2*c*x))^(2/3))/(a + b*x + c*x^2)^(1/3))], (2 + Sqrt[3])/4])/(7480*c^(
10/3)*(b^2 - 4*a*c)^2*d^(23/3)*(a + b*x + c*x^2)^(2/3)*Sqrt[-(((d*(b + 2*c*
x))^(2/3)*(2^(2/3)*c^(1/3)*d^(2/3) - (d*(b + 2*c*x))^(2/3)/(a + b*x + c*x^2
)^(1/3)))/((a + b*x + c*x^2)^(1/3)*(2^(2/3)*c^(1/3)*d^(2/3) - ((1 + Sqrt[3]
)*(d*(b + 2*c*x))^(2/3))/(a + b*x + c*x^2)^(1/3))^2))])
```

Rubi [A] time = 2.1399, antiderivative size = 637, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {694, 277, 325, 329, 241, 225}

$$\frac{6\sqrt[3]{a+bx+cx^2}}{935c^2d^5\left(b^2-4ac\right)(d(b+2cx))^{5/3}}+\frac{3\,3^{3/4}\left(-4ac+b^2-(b+2cx)^2\right)\sqrt[3]{d(b+2cx)}\left(2\sqrt[3]{c}d^{2/3}-\frac{\sqrt[3]{2}(d(b+2cx))^{2/3}}{\sqrt[3]{a+bx+cx^2}}\right)\sqrt{\frac{2^{2/3}\sqrt[3]{c}d^{2/3}}{\sqrt[3]{a+}}}}{7480c^{10/3}d^{23/3}\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(4/3)/(b*d + 2*c*d*x)^(20/3),x]
```

```
[Out] (-3*(a + b*x + c*x^2)^(1/3))/(187*c^2*d^3*(d*(b + 2*c*x))^(11/3)) + (6*(a +
 b*x + c*x^2)^(1/3))/(935*c^2*(b^2 - 4*a*c)*d^5*(d*(b + 2*c*x))^(5/3)) - (3
*(a + b*x + c*x^2)^(4/3))/(34*c*d*(d*(b + 2*c*x))^(17/3)) + (3*3^(3/4)*(d*(
b + 2*c*x))^(1/3)*(b^2 - 4*a*c - (b + 2*c*x)^2)*(2*c^(1/3)*d^(2/3) - (2^(1/
3)*(d*(b + 2*c*x))^(2/3))/(a + b*x + c*x^2)^(1/3))*Sqrt[(2*2^(1/3)*c^(2/3)*
d^(4/3) + (d*(b + 2*c*x))^(4/3)/(a + b*x + c*x^2)^(2/3) + (2^(2/3)*c^(1/3)*
d^(2/3)*(d*(b + 2*c*x))^(2/3))/(a + b*x + c*x^2)^(1/3))/(2^(2/3)*c^(1/3)*d^
(2/3) - ((1 + Sqrt[3])*(d*(b + 2*c*x))^(2/3))/(a + b*x + c*x^2)^(1/3))^2]*E
llipticF[ArcCos[(2^(2/3)*c^(1/3)*d^(2/3) - ((1 - Sqrt[3])*(d*(b + 2*c*x))^(
2/3))/(a + b*x + c*x^2)^(1/3))/(2^(2/3)*c^(1/3)*d^(2/3) - ((1 + Sqrt[3])*(d
```

```
*(b + 2*c*x))^(2/3))/(a + b*x + c*x^2)^(1/3))], (2 + Sqrt[3])/4])/(7480*c^(
10/3)*(b^2 - 4*a*c)^2*d^(23/3)*(a + b*x + c*x^2)^(2/3)*Sqrt[-(((d*(b + 2*c*
x))^(2/3)*(2^(2/3)*c^(1/3)*d^(2/3) - (d*(b + 2*c*x))^(2/3)/(a + b*x + c*x^2
)^(1/3)))/((a + b*x + c*x^2)^(1/3)*(2^(2/3)*c^(1/3)*d^(2/3) - ((1 + Sqrt[3]
)*(d*(b + 2*c*x))^(2/3))/(a + b*x + c*x^2)^(1/3))^2))])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{4/3}}{(bd+2cdx)^{20/3}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)^{4/3}}{x^{20/3}}\,dx, x, bd+2cdx\right)}{2cd}$$

$$= -\frac{3\left(a+bx+cx^2\right)^{4/3}}{34cd(d(b+2cx))^{17/3}} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}}}{x^{14/3}}\,dx, x, bd+2cdx\right)}{17c^2d^3}$$

$$= -\frac{3\sqrt[3]{a+bx+cx^2}}{187c^2d^3(d(b+2cx))^{11/3}} - \frac{3\left(a+bx+cx^2\right)^{4/3}}{34cd(d(b+2cx))^{17/3}} + \frac{\text{Subst}\left(\int \frac{1}{x^{8/3}\left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)^{2/3}}\,dx, x, bd+2cd\right.}{374c^3d^5}$$

$$= -\frac{3\sqrt[3]{a+bx+cx^2}}{187c^2d^3(d(b+2cx))^{11/3}} + \frac{6\sqrt[3]{a+bx+cx^2}}{935c^2\left(b^2-4ac\right)d^5(d(b+2cx))^{5/3}} - \frac{3\left(a+bx+cx^2\right)^{4/3}}{34cd(d(b+2cx))^{17/3}} + \frac{3\,\text{S}}{}$$

$$= -\frac{3\sqrt[3]{a+bx+cx^2}}{187c^2d^3(d(b+2cx))^{11/3}} + \frac{6\sqrt[3]{a+bx+cx^2}}{935c^2\left(b^2-4ac\right)d^5(d(b+2cx))^{5/3}} - \frac{3\left(a+bx+cx^2\right)^{4/3}}{34cd(d(b+2cx))^{17/3}} + \frac{9\,\text{S}}{}$$

$$= -\frac{3\sqrt[3]{a+bx+cx^2}}{187c^2d^3(d(b+2cx))^{11/3}} + \frac{6\sqrt[3]{a+bx+cx^2}}{935c^2\left(b^2-4ac\right)d^5(d(b+2cx))^{5/3}} - \frac{3\left(a+bx+cx^2\right)^{4/3}}{34cd(d(b+2cx))^{17/3}} + \frac{}{187}$$

$$= -\frac{3\sqrt[3]{a+bx+cx^2}}{187c^2d^3(d(b+2cx))^{11/3}} + \frac{6\sqrt[3]{a+bx+cx^2}}{935c^2\left(b^2-4ac\right)d^5(d(b+2cx))^{5/3}} - \frac{3\left(a+bx+cx^2\right)^{4/3}}{34cd(d(b+2cx))^{17/3}} + \frac{3\;3}{}$$

Mathematica [C] time = 0.107573, size = 112, normalized size = 0.18

$$\frac{3\left(b^2-4ac\right)\sqrt[3]{a+x(b+cx)}\sqrt[3]{d(b+2cx)}\,{}_2F_1\left(-\frac{17}{6},-\frac{4}{3};-\frac{11}{6};\frac{(b+2cx)^2}{b^2-4ac}\right)}{136\;2^{2/3}c^2d^7(b+2cx)^6\sqrt[3]{\frac{c(a+x(b+cx))}{4ac-b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(4/3)/(b*d + 2*c*d*x)^(20/3),x]
```

```
[Out] (3*(b^2 - 4*a*c)*(d*(b + 2*c*x))^(1/3)*(a + x*(b + c*x))^(1/3)*Hypergeometr
ic2F1[-17/6, -4/3, -11/6, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(136*2^(2/3)*c^2*d^
7*(b + 2*c*x)^6*((c*(a + x*(b + c*x)))/(-b^2 + 4*a*c))^(1/3))
```

Maple [F] time = 1.153, size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{\frac{4}{3}} (2\,cdx+bd)^{-\frac{20}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(20/3),x)
```

```
[Out] int((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(20/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{4}{3}}}{(2\,cdx+bd)^{\frac{20}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(20/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(4/3)/(2*c*d*x + b*d)^(20/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(2\,cdx+bd)^{\frac{1}{3}}\left(cx^2+bx+a\right)^{\frac{4}{3}}}{128\,c^7d^7x^7+448\,bc^6d^7x^6+672\,b^2c^5d^7x^5+560\,b^3c^4d^7x^4+280\,b^4c^3d^7x^3+84\,b^5c^2d^7x^2+14\,b^6cd^7x+b^7d^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(20/3),x, algorithm="fricas")
```

```
[Out] integral((2*c*d*x + b*d)^(1/3)*(c*x^2 + b*x + a)^(4/3)/(128*c^7*d^7*x^7 + 4
48*b*c^6*d^7*x^6 + 672*b^2*c^5*d^7*x^5 + 560*b^3*c^4*d^7*x^4 + 280*b^4*c^3*
d^7*x^3 + 84*b^5*c^2*d^7*x^2 + 14*b^6*c*d^7*x + b^7*d^7), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(4/3)/(2*c*d*x+b*d)**(20/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx + a\right)^{\frac{4}{3}}}{(2\,cdx + bd)^{\frac{20}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(20/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(4/3)/(2*c*d*x + b*d)^(20/3), x)
```

3.1420 $\int \frac{\left(a+bx+cx^2\right)^{4/3}}{(bd+2cdx)^{4/3}}\,dx$

Optimal. Leaf size=99

$$\frac{3\left(b^2-4ac\right)\sqrt[3]{a+bx+cx^2}\,{}_2F_1\left(-\frac{4}{3},-\frac{1}{6};\frac{5}{6};\frac{(b+2cx)^2}{b^2-4ac}\right)}{8c^2d\sqrt[3]{1-\frac{(b+2cx)^2}{b^2-4ac}}\sqrt[3]{d(b+2cx)}}$$

```
[Out] (3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(1/3)*Hypergeometric2F1[-4/3, -1/6, 5/6,
 (b + 2*c*x)^2/(b^2 - 4*a*c)])/(8*c^2*d*(d*(b + 2*c*x))^(1/3)*(1 - (b + 2*c
*x)^2/(b^2 - 4*a*c))^(1/3))
```

Rubi [A] time = 0.124247, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {694, 365, 364}

$$\frac{3\left(b^2-4ac\right)\sqrt[3]{a+bx+cx^2}\,{}_2F_1\left(-\frac{4}{3},-\frac{1}{6};\frac{5}{6};\frac{(b+2cx)^2}{b^2-4ac}\right)}{8c^2d\sqrt[3]{1-\frac{(b+2cx)^2}{b^2-4ac}}\sqrt[3]{d(b+2cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(4/3)/(b*d + 2*c*d*x)^(4/3),x]
```

```
[Out] (3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(1/3)*Hypergeometric2F1[-4/3, -1/6, 5/6,
 (b + 2*c*x)^2/(b^2 - 4*a*c)])/(8*c^2*d*(d*(b + 2*c*x))^(1/3)*(1 - (b + 2*c
*x)^2/(b^2 - 4*a*c))^(1/3))
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{4/3}}{(bd+2cdx)^{4/3}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)^{4/3}}{x^{4/3}}dx, x, bd+2cdx\right)}{2cd}$$

$$= \frac{\left(\left(a-\frac{b^2}{4c}\right)\sqrt[3]{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{4\left(a-\frac{b^2}{4c}\right)cd^2}\right)^{4/3}}{x^{4/3}}dx, x, bd+2cdx\right)}{\sqrt[3]{2}cd\sqrt[3]{4+\frac{(bd+2cdx)^2}{\left(a-\frac{b^2}{4c}\right)cd^2}}}$$

$$= \frac{3\left(b^2-4ac\right)\sqrt[3]{a+bx+cx^2}\,{}_2F_1\left(-\frac{4}{3},-\frac{1}{6};\frac{5}{6};\frac{(b+2cx)^2}{b^2-4ac}\right)}{8c^2d\sqrt[3]{d(b+2cx)}\sqrt[3]{1-\frac{(b+2cx)^2}{b^2-4ac}}}$$

Mathematica [A] time = 0.0669767, size = 104, normalized size = 1.05

$$\frac{3\left(b^2-4ac\right)\sqrt[3]{a+x(b+cx)}\,{}_2F_1\left(-\frac{4}{3},-\frac{1}{6};\frac{5}{6};\frac{(b+2cx)^2}{b^2-4ac}\right)}{8\,2^{2/3}c^2d\sqrt[3]{\frac{c(a+x(b+cx))}{4ac-b^2}}\sqrt[3]{d(b+2cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(4/3)/(b*d + 2*c*d*x)^(4/3),x]
```

```
[Out] (3*(b^2 - 4*a*c)*(a + x*(b + c*x))^(1/3)*Hypergeometric2F1[-4/3, -1/6, 5/6,
 (b + 2*c*x)^2/(b^2 - 4*a*c)])/(8*2^(2/3)*c^2*d*(d*(b + 2*c*x))^(1/3)*((c*(
a + x*(b + c*x)))/(-b^2 + 4*a*c))^(1/3))
```

Maple [F] time = 0.413, size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{\frac{4}{3}}(2\,cdx+bd)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(4/3),x)
```

```
[Out] int((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{4}{3}}}{(2\,cdx+bd)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(4/3),x, algorithm="maxima")
```

[Out] integrate((c*x^2 + b*x + a)^(4/3)/(2*c*d*x + b*d)^(4/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(2\,cdx + bd)^{\frac{2}{3}}\left(cx^2 + bx + a\right)^{\frac{4}{3}}}{4\,c^2d^2x^2 + 4\,bcd^2x + b^2d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(4/3),x, algorithm="fricas")

[Out] integral((2*c*d*x + b*d)^(2/3)*(c*x^2 + b*x + a)^(4/3)/(4*c^2*d^2*x^2 + 4*b
*c*d^2*x + b^2*d^2), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx + cx^2\right)^{\frac{4}{3}}}{(d\,(b + 2cx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)**(4/3)/(2*c*d*x+b*d)**(4/3),x)

[Out] Integral((a + b*x + c*x**2)**(4/3)/(d*(b + 2*c*x))**(4/3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx + a\right)^{\frac{4}{3}}}{(2\,cdx + bd)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(4/3),x, algorithm="giac")

[Out] integrate((c*x^2 + b*x + a)^(4/3)/(2*c*d*x + b*d)^(4/3), x)

3.1421 $\int \frac{(a+bx+cx^2)^{4/3}}{(bd+2cdx)^{10/3}} dx$

Optimal. Leaf size=99

$$\frac{3(b^2-4ac)\sqrt[3]{a+bx+cx^2}\,{}_2F_1\left(-\frac{4}{3},-\frac{7}{6};-\frac{1}{6};\frac{(b+2cx)^2}{b^2-4ac}\right)}{56c^2d\sqrt[3]{1-\frac{(b+2cx)^2}{b^2-4ac}}(d(b+2cx))^{7/3}}$$

```
[Out] (3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(1/3)*Hypergeometric2F1[-4/3, -7/6, -1/6
, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(56*c^2*d*(d*(b + 2*c*x))^(7/3)*(1 - (b + 2
*c*x)^2/(b^2 - 4*a*c))^(1/3))
```

Rubi [A] time = 0.112751, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {694, 365, 364}

$$\frac{3(b^2-4ac)\sqrt[3]{a+bx+cx^2}\,{}_2F_1\left(-\frac{4}{3},-\frac{7}{6};-\frac{1}{6};\frac{(b+2cx)^2}{b^2-4ac}\right)}{56c^2d\sqrt[3]{1-\frac{(b+2cx)^2}{b^2-4ac}}(d(b+2cx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(4/3)/(b*d + 2*c*d*x)^(10/3),x]
```


```
[Out] (3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(1/3)*Hypergeometric2F1[-4/3, -7/6, -1/6
, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(56*c^2*d*(d*(b + 2*c*x))^(7/3)*(1 - (b + 2
*c*x)^2/(b^2 - 4*a*c))^(1/3))
```


Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{4/3}}{(bd+2cdx)^{10/3}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)^{4/3}}{x^{10/3}}\,dx, x, bd+2cdx\right)}{2cd}$$

$$= \frac{\left(\left(a-\frac{b^2}{4c}\right)\sqrt[3]{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{4\left(a-\frac{b^2}{4c}\right)cd^2}\right)^{4/3}}{x^{10/3}}\,dx, x, bd+2cdx\right)}{\sqrt[3]{2}cd\sqrt[3]{4+\frac{(bd+2cdx)^2}{\left(a-\frac{b^2}{4c}\right)cd^2}}}$$

$$= \frac{3(b^2-4ac)\sqrt[3]{a+bx+cx^2}\,{}_2F_1\left(-\frac{4}{3},-\frac{7}{6};-\frac{1}{6};\frac{(b+2cx)^2}{b^2-4ac}\right)}{56c^2d(d(b+2cx))^{7/3}\sqrt[3]{1-\frac{(b+2cx)^2}{b^2-4ac}}}$$

Mathematica [A] time = 0.0679253, size = 104, normalized size = 1.05

$$\frac{3(b^2-4ac)\sqrt[3]{a+x(b+cx)}\,{}_2F_1\left(-\frac{4}{3},-\frac{7}{6};-\frac{1}{6};\frac{(b+2cx)^2}{b^2-4ac}\right)}{56\,2^{2/3}c^2d\sqrt[3]{\frac{c(a+x(b+cx))}{4ac-b^2}}(d(b+2cx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(4/3)/(b*d + 2*c*d*x)^(10/3),x]
```

```
[Out] (3*(b^2 - 4*a*c)*(a + x*(b + c*x))^(1/3)*Hypergeometric2F1[-4/3, -7/6, -1/6
, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(56*2^(2/3)*c^2*d*(d*(b + 2*c*x))^(7/3)*((c
*(a + x*(b + c*x)))/(-b^2 + 4*a*c))^(1/3))
```

Maple [F] time = 1.148, size = 0, normalized size = 0.

$$\int (cx^2+bx+a)^{\frac{4}{3}} (2\,cdx+bd)^{-\frac{10}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(10/3),x)
```

```
[Out] int((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(10/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2+bx+a)^{\frac{4}{3}}}{(2\,cdx+bd)^{\frac{10}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(10/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(4/3)/(2*c*d*x + b*d)^(10/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(2\,cdx+bd)^{\frac{2}{3}}\left(cx^2+bx+a\right)^{\frac{4}{3}}}{16\,c^4d^4x^4+32\,bc^3d^4x^3+24\,b^2c^2d^4x^2+8\,b^3cd^4x+b^4d^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(10/3),x, algorithm="fricas")
```

```
[Out] integral((2*c*d*x + b*d)^(2/3)*(c*x^2 + b*x + a)^(4/3)/(16*c^4*d^4*x^4 + 32
*b*c^3*d^4*x^3 + 24*b^2*c^2*d^4*x^2 + 8*b^3*c*d^4*x + b^4*d^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(4/3)/(2*c*d*x+b*d)**(10/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{4}{3}}}{(2\,cdx+bd)^{\frac{10}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(10/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(4/3)/(2*c*d*x + b*d)^(10/3), x)
```

3.1422 $\int \frac{(a+bx+cx^2)^{4/3}}{(bd+2cdx)^{16/3}}\,dx$

Optimal. Leaf size=99

$$\frac{3(b^2-4ac)\sqrt[3]{a+bx+cx^2}\,{}_2F_1\left(-\frac{13}{6},-\frac{4}{3};-\frac{7}{6};\frac{(b+2cx)^2}{b^2-4ac}\right)}{104c^2d\sqrt[3]{1-\frac{(b+2cx)^2}{b^2-4ac}}(d(b+2cx))^{13/3}}$$

```
[Out] (3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(1/3)*Hypergeometric2F1[-13/6, -4/3, -7/
6, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(104*c^2*d*(d*(b + 2*c*x))^(13/3)*(1 - (b
+ 2*c*x)^2/(b^2 - 4*a*c))^(1/3))
```

Rubi [A] time = 0.116381, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {694, 365, 364}

$$\frac{3(b^2-4ac)\sqrt[3]{a+bx+cx^2}\,{}_2F_1\left(-\frac{13}{6},-\frac{4}{3};-\frac{7}{6};\frac{(b+2cx)^2}{b^2-4ac}\right)}{104c^2d\sqrt[3]{1-\frac{(b+2cx)^2}{b^2-4ac}}(d(b+2cx))^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(4/3)/(b*d + 2*c*d*x)^(16/3),x]
```

```
[Out] (3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(1/3)*Hypergeometric2F1[-13/6, -4/3, -7/
6, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(104*c^2*d*(d*(b + 2*c*x))^(13/3)*(1 - (b
+ 2*c*x)^2/(b^2 - 4*a*c))^(1/3))
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{4/3}}{(bd+2cdx)^{16/3}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)^{4/3}}{x^{16/3}}dx, x, bd+2cdx\right)}{2cd}$$

$$= \frac{\left(\left(a-\frac{b^2}{4c}\right)\sqrt[3]{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{4\left(a-\frac{b^2}{4c}\right)cd^2}\right)^{4/3}}{x^{16/3}}dx, x, bd+2cdx\right)}{\sqrt[3]{2}cd\sqrt[3]{4+\frac{(bd+2cdx)^2}{\left(a-\frac{b^2}{4c}\right)cd^2}}}$$

$$= \frac{3\left(b^2-4ac\right)\sqrt[3]{a+bx+cx^2}\,{}_2F_1\left(-\frac{13}{6},-\frac{4}{3};-\frac{7}{6};\frac{(b+2cx)^2}{b^2-4ac}\right)}{104c^2d(d(b+2cx))^{13/3}\sqrt[3]{1-\frac{(b+2cx)^2}{b^2-4ac}}}$$

Mathematica [A] time = 0.0898834, size = 112, normalized size = 1.13

$$\frac{3\left(b^2-4ac\right)\sqrt[3]{a+x(b+cx)}(d(b+2cx))^{2/3}\,{}_2F_1\left(-\frac{13}{6},-\frac{4}{3};-\frac{7}{6};\frac{(b+2cx)^2}{b^2-4ac}\right)}{104\ 2^{2/3}c^2d^6(b+2cx)^5\sqrt[3]{\frac{c(a+x(b+cx))}{4ac-b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(4/3)/(b*d + 2*c*d*x)^(16/3),x]
```

```
[Out] (3*(b^2 - 4*a*c)*(d*(b + 2*c*x))^(2/3)*(a + x*(b + c*x))^(1/3)*Hypergeometr
ic2F1[-13/6, -4/3, -7/6, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(104*2^(2/3)*c^2*d^6
*(b + 2*c*x)^5*((c*(a + x*(b + c*x)))/(-b^2 + 4*a*c))^(1/3))
```

Maple [F] time = 1.184, size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{\frac{4}{3}}(2\,cdx+bd)^{-\frac{16}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(16/3),x)
```

```
[Out] int((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(16/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{4}{3}}}{(2\,cdx+bd)^{\frac{16}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(16/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(4/3)/(2*c*d*x + b*d)^(16/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(2\,cdx+bd)^{\frac{2}{3}}\left(cx^2+bx+a\right)^{\frac{4}{3}}}{64\,c^6d^6x^6+192\,bc^5d^6x^5+240\,b^2c^4d^6x^4+160\,b^3c^3d^6x^3+60\,b^4c^2d^6x^2+12\,b^5cd^6x+b^6d^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(16/3),x, algorithm="fricas")
```

```
[Out] integral((2*c*d*x + b*d)^(2/3)*(c*x^2 + b*x + a)^(4/3)/(64*c^6*d^6*x^6 + 19
2*b*c^5*d^6*x^5 + 240*b^2*c^4*d^6*x^4 + 160*b^3*c^3*d^6*x^3 + 60*b^4*c^2*d^
6*x^2 + 12*b^5*c*d^6*x + b^6*d^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(4/3)/(2*c*d*x+b*d)**(16/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{4}{3}}}{(2\,cdx+bd)^{\frac{16}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(2*c*d*x+b*d)^(16/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(4/3)/(2*c*d*x + b*d)^(16/3), x)
```

3.1423 $\int (bd+2cdx)^m \left(a+bx+cx^2\right)^3\,dx$

Optimal. Leaf size=141

$$\frac{3\left(b^2-4ac\right)^2(bd+2cdx)^{m+3}}{128c^4d^3(m+3)}-\frac{3\left(b^2-4ac\right)(bd+2cdx)^{m+5}}{128c^4d^5(m+5)}-\frac{\left(b^2-4ac\right)^3(bd+2cdx)^{m+1}}{128c^4d(m+1)}+\frac{(bd+2cdx)^{m+7}}{128c^4d^7(m+7)}$$

```
[Out] -((b^2 - 4*a*c)^3*(b*d + 2*c*d*x)^(1 + m))/(128*c^4*d*(1 + m)) + (3*(b^2 -
4*a*c)^2*(b*d + 2*c*d*x)^(3 + m))/(128*c^4*d^3*(3 + m)) - (3*(b^2 - 4*a*c)*
(b*d + 2*c*d*x)^(5 + m))/(128*c^4*d^5*(5 + m)) + (b*d + 2*c*d*x)^(7 + m)/(1
28*c^4*d^7*(7 + m))
```

Rubi [A] time = 0.0786644, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$\frac{3\left(b^2-4ac\right)^2(bd+2cdx)^{m+3}}{128c^4d^3(m+3)}-\frac{3\left(b^2-4ac\right)(bd+2cdx)^{m+5}}{128c^4d^5(m+5)}-\frac{\left(b^2-4ac\right)^3(bd+2cdx)^{m+1}}{128c^4d(m+1)}+\frac{(bd+2cdx)^{m+7}}{128c^4d^7(m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^m*(a + b*x + c*x^2)^3,x]
```

```
[Out] -((b^2 - 4*a*c)^3*(b*d + 2*c*d*x)^(1 + m))/(128*c^4*d*(1 + m)) + (3*(b^2 -
4*a*c)^2*(b*d + 2*c*d*x)^(3 + m))/(128*c^4*d^3*(3 + m)) - (3*(b^2 - 4*a*c)*
(b*d + 2*c*d*x)^(5 + m))/(128*c^4*d^5*(5 + m)) + (b*d + 2*c*d*x)^(7 + m)/(1
28*c^4*d^7*(7 + m))
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] && !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\int (bd+2cdx)^m\left(a+bx+cx^2\right)^3\,dx=\int\left(\frac{\left(-b^2+4ac\right)^3(bd+2cdx)^m}{64c^3}+\frac{3\left(-b^2+4ac\right)^2(bd+2cdx)^{2+m}}{64c^3d^2}+\frac{3\left(-b^2+4ac\right.}{6}\right.$$
$$=-\frac{\left(b^2-4ac\right)^3(bd+2cdx)^{1+m}}{128c^4d(1+m)}+\frac{3\left(b^2-4ac\right)^2(bd+2cdx)^{3+m}}{128c^4d^3(3+m)}-\frac{3\left(b^2-4ac\right)(bd}{128c^4d^5(5}$$

Mathematica [A] time = 0.0910315, size = 103, normalized size = 0.73

$$\frac{(b+2cx)\left(-\frac{3\left(b^2-4ac\right)(b+2cx)^4}{m+5}+\frac{3\left(b^2-4ac\right)^2(b+2cx)^2}{m+3}-\frac{\left(b^2-4ac\right)^3}{m+1}+\frac{(b+2cx)^6}{m+7}\right)(d(b+2cx))^m}{128c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^m*(a + b*x + c*x^2)^3,x]
```

```
[Out] ((b + 2*c*x)*(d*(b + 2*c*x))^m*(-((b^2 - 4*a*c)^3/(1 + m)) + (3*(b^2 - 4*a*
c)^2*(b + 2*c*x)^2)/(3 + m) - (3*(b^2 - 4*a*c)*(b + 2*c*x)^4)/(5 + m) + (b
+ 2*c*x)^6/(7 + m)))/(128*c^4)
```

Maple [B] time = 0.048, size = 653, normalized size = 4.6

$$(2\,cdx+bd)^{m}\left(4\,c^{6}m^{3}x^{6}+12\,bc^{5}m^{3}x^{5}+36\,c^{6}m^{2}x^{6}+12\,ac^{5}m^{3}x^{4}+12\,b^{2}c^{4}m^{3}x^{4}+108\,bc^{5}m^{2}x^{5}+92\,c^{6}mx^{6}+24\,abc^{4}m^{3}x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^m*(c*x^2+b*x+a)^3,x)
```

```
[Out] 1/8*(2*c*d*x+b*d)^m*(4*c^6*m^3*x^6+12*b*c^5*m^3*x^5+36*c^6*m^2*x^6+12*a*c^5
*m^3*x^4+12*b^2*c^4*m^3*x^4+108*b*c^5*m^2*x^5+92*c^6*m*x^6+24*a*b*c^4*m^3*x
^3+132*a*c^5*m^2*x^4+4*b^3*c^3*m^3*x^3+102*b^2*c^4*m^2*x^4+276*b*c^5*m*x^5+
60*c^6*x^6+12*a^2*c^4*m^3*x^2+12*a*b^2*c^3*m^3*x^2+264*a*b*c^4*m^2*x^3+372*
a*c^5*m*x^4+24*b^3*c^3*m^2*x^3+252*b^2*c^4*m*x^4+180*b*c^5*x^5+12*a^2*b*c^3
*m^3*x+156*a^2*c^4*m^2*x^2+120*a*b^2*c^3*m^2*x^2+744*a*b*c^4*m*x^3+252*a*c^
5*x^4-6*b^4*c^2*m^2*x^2+44*b^3*c^3*m*x^3+162*b^2*c^4*x^4+4*a^3*c^3*m^3+156*
a^2*b*c^3*m^2*x+564*a^2*c^4*m*x^2-12*a*b^3*c^2*m^2*x+276*a*b^2*c^3*m*x^2+50
4*a*b*c^4*x^3-18*b^4*c^2*m*x^2+24*b^3*c^3*x^3+60*a^3*c^3*m^2-6*a^2*b^2*c^2*
m^2+564*a^2*b*c^3*m*x+420*a^2*c^4*x^2-96*a*b^3*c^2*m*x+168*a*b^2*c^3*x^2+6*
b^5*c*m*x-12*b^4*c^2*x^2+284*a^3*c^3*m-72*a^2*b^2*c^2*m+420*a^2*b*c^3*x+6*a
*b^4*c*m-84*a*b^3*c^2*x+6*b^5*c*x+420*a^3*c^3-210*a^2*b^2*c^2+42*a*b^4*c-3*
b^6)*(2*c*x+b)/c^4/(m^4+16*m^3+86*m^2+176*m+105)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m*(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.19783, size = 1474, normalized size = 10.45

$$\left(4\,a^{3}bc^{3}m^{3}+8\left(c^{7}m^{3}+9\,c^{7}m^{2}+23\,c^{7}m+15\,c^{7}\right)x^{7}-3\,b^{7}+42\,ab^{5}c-210\,a^{2}b^{3}c^{2}+420\,a^{3}bc^{3}+28\left(bc^{6}m^{3}+9\,bc^{6}m^{2}+2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m*(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(4*a^3*b*c^3*m^3 + 8*(c^7*m^3 + 9*c^7*m^2 + 23*c^7*m + 15*c^7)*x^7 - 3*
b^7 + 42*a*b^5*c - 210*a^2*b^3*c^2 + 420*a^3*b*c^3 + 28*(b*c^6*m^3 + 9*b*c^
6*m^2 + 23*b*c^6*m + 15*b*c^6)*x^6 + 12*(42*b^2*c^5 + 42*a*c^6 + (3*b^2*c^5
 + 2*a*c^6)*m^3 + 2*(13*b^2*c^5 + 11*a*c^6)*m^2 + (65*b^2*c^5 + 62*a*c^6)*m
)*x^5 + 10*(21*b^3*c^4 + 126*a*b*c^5 + 2*(b^3*c^4 + 3*a*b*c^5)*m^3 + 3*(5*b
```

```
^3*c^4 + 22*a*b*c^5)*m^2 + 2*(17*b^3*c^4 + 93*a*b*c^5)*m)*x^4 + 4*(210*a*b^
2*c^4 + 210*a^2*c^5 + (b^4*c^3 + 12*a*b^2*c^4 + 6*a^2*c^5)*m^3 + 3*(b^4*c^3
 + 42*a*b^2*c^4 + 26*a^2*c^5)*m^2 + 2*(b^4*c^3 + 162*a*b^2*c^4 + 141*a^2*c^
5)*m)*x^3 - 6*(a^2*b^3*c^2 - 10*a^3*b*c^3)*m^2 + 6*(210*a^2*b*c^4 + 2*(a*b^
3*c^3 + 3*a^2*b*c^4)*m^3 - (b^5*c^2 - 16*a*b^3*c^3 - 78*a^2*b*c^4)*m^2 - (b
^5*c^2 - 14*a*b^3*c^3 - 282*a^2*b*c^4)*m)*x^2 + 2*(3*a*b^5*c - 36*a^2*b^3*c
^2 + 142*a^3*b*c^3)*m + 2*(420*a^3*c^4 + 2*(3*a^2*b^2*c^3 + 2*a^3*c^4)*m^3
- 6*(a*b^4*c^2 - 12*a^2*b^2*c^3 - 10*a^3*c^4)*m^2 + (3*b^6*c - 42*a*b^4*c^2
 + 210*a^2*b^2*c^3 + 284*a^3*c^4)*m)*x)*(2*c*d*x + b*d)^m/(c^4*m^4 + 16*c^4
*m^3 + 86*c^4*m^2 + 176*c^4*m + 105*c^4)
```

Sympy [A] time = 28.5141, size = 10271, normalized size = 72.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**m*(c*x**2+b*x+a)**3,x)
```

```
[Out] Piecewise(((b*d)**m*(a**3*x + 3*a**2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4),
 Eq(c, 0)), (-320*a**3*b**4*c**3/(3840*b**10*c**4*d**7 + 46080*b**9*c**5*d*
*7*x + 230400*b**8*c**6*d**7*x**2 + 614400*b**7*c**7*d**7*x**3 + 921600*b**
6*c**8*d**7*x**4 + 737280*b**5*c**9*d**7*x**5 + 245760*b**4*c**10*d**7*x**6
) - 96*a**2*b**6*c**2/(3840*b**10*c**4*d**7 + 46080*b**9*c**5*d**7*x + 2304
00*b**8*c**6*d**7*x**2 + 614400*b**7*c**7*d**7*x**3 + 921600*b**6*c**8*d**7
*x**4 + 737280*b**5*c**9*d**7*x**5 + 245760*b**4*c**10*d**7*x**6) - 1152*a*
*2*b**5*c**3*x/(3840*b**10*c**4*d**7 + 46080*b**9*c**5*d**7*x + 230400*b**8
*c**6*d**7*x**2 + 614400*b**7*c**7*d**7*x**3 + 921600*b**6*c**8*d**7*x**4 +
 737280*b**5*c**9*d**7*x**5 + 245760*b**4*c**10*d**7*x**6) + 3840*a**2*b**3
*c**5*x**3/(3840*b**10*c**4*d**7 + 46080*b**9*c**5*d**7*x + 230400*b**8*c**
6*d**7*x**2 + 614400*b**7*c**7*d**7*x**3 + 921600*b**6*c**8*d**7*x**4 + 737
280*b**5*c**9*d**7*x**5 + 245760*b**4*c**10*d**7*x**6) + 5760*a**2*b**2*c**
6*x**4/(3840*b**10*c**4*d**7 + 46080*b**9*c**5*d**7*x + 230400*b**8*c**6*d*
*7*x**2 + 614400*b**7*c**7*d**7*x**3 + 921600*b**6*c**8*d**7*x**4 + 737280*
b**5*c**9*d**7*x**5 + 245760*b**4*c**10*d**7*x**6) + 4608*a**2*b*c**7*x**5/
(3840*b**10*c**4*d**7 + 46080*b**9*c**5*d**7*x + 230400*b**8*c**6*d**7*x**2
 + 614400*b**7*c**7*d**7*x**3 + 921600*b**6*c**8*d**7*x**4 + 737280*b**5*c*
*9*d**7*x**5 + 245760*b**4*c**10*d**7*x**6) + 1536*a**2*c**8*x**6/(3840*b**
10*c**4*d**7 + 46080*b**9*c**5*d**7*x + 230400*b**8*c**6*d**7*x**2 + 614400
*b**7*c**7*d**7*x**3 + 921600*b**6*c**8*d**7*x**4 + 737280*b**5*c**9*d**7*x
**5 + 245760*b**4*c**10*d**7*x**6) + 3840*a*b**5*c**4*x**3/(3840*b**10*c**4
*d**7 + 46080*b**9*c**5*d**7*x + 230400*b**8*c**6*d**7*x**2 + 614400*b**7*c
**7*d**7*x**3 + 921600*b**6*c**8*d**7*x**4 + 737280*b**5*c**9*d**7*x**5 + 2
45760*b**4*c**10*d**7*x**6) + 11520*a*b**4*c**5*x**4/(3840*b**10*c**4*d**7
+ 46080*b**9*c**5*d**7*x + 230400*b**8*c**6*d**7*x**2 + 614400*b**7*c**7*d*
*7*x**3 + 921600*b**6*c**8*d**7*x**4 + 737280*b**5*c**9*d**7*x**5 + 245760*
b**4*c**10*d**7*x**6) + 11520*a*b**3*c**6*x**5/(3840*b**10*c**4*d**7 + 4608
0*b**9*c**5*d**7*x + 230400*b**8*c**6*d**7*x**2 + 614400*b**7*c**7*d**7*x**
3 + 921600*b**6*c**8*d**7*x**4 + 737280*b**5*c**9*d**7*x**5 + 245760*b**4*c
**10*d**7*x**6) + 3840*a*b**2*c**7*x**6/(3840*b**10*c**4*d**7 + 46080*b**9*
c**5*d**7*x + 230400*b**8*c**6*d**7*x**2 + 614400*b**7*c**7*d**7*x**3 + 921
600*b**6*c**8*d**7*x**4 + 737280*b**5*c**9*d**7*x**5 + 245760*b**4*c**10*d*
*7*x**6) + 30*b**10*log(b/(2*c) + x)/(3840*b**10*c**4*d**7 + 46080*b**9*c**
5*d**7*x + 230400*b**8*c**6*d**7*x**2 + 614400*b**7*c**7*d**7*x**3 + 921600
*b**6*c**8*d**7*x**4 + 737280*b**5*c**9*d**7*x**5 + 245760*b**4*c**10*d**7*
x**6) + 11*b**10/(3840*b**10*c**4*d**7 + 46080*b**9*c**5*d**7*x + 230400*b*
*8*c**6*d**7*x**2 + 614400*b**7*c**7*d**7*x**3 + 921600*b**6*c**8*d**7*x**4
```

```
 + 737280*b**5*c**9*d**7*x**5 + 245760*b**4*c**10*d**7*x**6) + 360*b**9*c*x
*log(b/(2*c) + x)/(3840*b**10*c**4*d**7 + 46080*b**9*c**5*d**7*x + 230400*b
**8*c**6*d**7*x**2 + 614400*b**7*c**7*d**7*x**3 + 921600*b**6*c**8*d**7*x**
4 + 737280*b**5*c**9*d**7*x**5 + 245760*b**4*c**10*d**7*x**6) + 72*b**9*c*x
/(3840*b**10*c**4*d**7 + 46080*b**9*c**5*d**7*x + 230400*b**8*c**6*d**7*x**
2 + 614400*b**7*c**7*d**7*x**3 + 921600*b**6*c**8*d**7*x**4 + 737280*b**5*c
**9*d**7*x**5 + 245760*b**4*c**10*d**7*x**6) + 1800*b**8*c**2*x**2*log(b/(2
*c) + x)/(3840*b**10*c**4*d**7 + 46080*b**9*c**5*d**7*x + 230400*b**8*c**6*
d**7*x**2 + 614400*b**7*c**7*d**7*x**3 + 921600*b**6*c**8*d**7*x**4 + 73728
0*b**5*c**9*d**7*x**5 + 245760*b**4*c**10*d**7*x**6) + 4800*b**7*c**3*x**3*
log(b/(2*c) + x)/(3840*b**10*c**4*d**7 + 46080*b**9*c**5*d**7*x + 230400*b*
*8*c**6*d**7*x**2 + 614400*b**7*c**7*d**7*x**3 + 921600*b**6*c**8*d**7*x**4
 + 737280*b**5*c**9*d**7*x**5 + 245760*b**4*c**10*d**7*x**6) - 1200*b**7*c*
*3*x**3/(3840*b**10*c**4*d**7 + 46080*b**9*c**5*d**7*x + 230400*b**8*c**6*d
**7*x**2 + 614400*b**7*c**7*d**7*x**3 + 921600*b**6*c**8*d**7*x**4 + 737280
*b**5*c**9*d**7*x**5 + 245760*b**4*c**10*d**7*x**6) + 7200*b**6*c**4*x**4*l
og(b/(2*c) + x)/(3840*b**10*c**4*d**7 + 46080*b**9*c**5*d**7*x + 230400*b**
8*c**6*d**7*x**2 + 614400*b**7*c**7*d**7*x**3 + 921600*b**6*c**8*d**7*x**4
+ 737280*b**5*c**9*d**7*x**5 + 245760*b**4*c**10*d**7*x**6) - 3240*b**6*c**
4*x**4/(3840*b**10*c**4*d**7 + 46080*b**9*c**5*d**7*x + 230400*b**8*c**6*d*
*7*x**2 + 614400*b**7*c**7*d**7*x**3 + 921600*b**6*c**8*d**7*x**4 + 737280*
b**5*c**9*d**7*x**5 + 245760*b**4*c**10*d**7*x**6) + 5760*b**5*c**5*x**5*lo
g(b/(2*c) + x)/(3840*b**10*c**4*d**7 + 46080*b**9*c**5*d**7*x + 230400*b**8
*c**6*d**7*x**2 + 614400*b**7*c**7*d**7*x**3 + 921600*b**6*c**8*d**7*x**4 +
 737280*b**5*c**9*d**7*x**5 + 245760*b**4*c**10*d**7*x**6) - 3168*b**5*c**5
*x**5/(3840*b**10*c**4*d**7 + 46080*b**9*c**5*d**7*x + 230400*b**8*c**6*d**
7*x**2 + 614400*b**7*c**7*d**7*x**3 + 921600*b**6*c**8*d**7*x**4 + 737280*b
**5*c**9*d**7*x**5 + 245760*b**4*c**10*d**7*x**6) + 1920*b**4*c**6*x**6*log
(b/(2*c) + x)/(3840*b**10*c**4*d**7 + 46080*b**9*c**5*d**7*x + 230400*b**8*
c**6*d**7*x**2 + 614400*b**7*c**7*d**7*x**3 + 921600*b**6*c**8*d**7*x**4 +
737280*b**5*c**9*d**7*x**5 + 245760*b**4*c**10*d**7*x**6) - 1056*b**4*c**6*
x**6/(3840*b**10*c**4*d**7 + 46080*b**9*c**5*d**7*x + 230400*b**8*c**6*d**7
*x**2 + 614400*b**7*c**7*d**7*x**3 + 921600*b**6*c**8*d**7*x**4 + 737280*b*
*5*c**9*d**7*x**5 + 245760*b**4*c**10*d**7*x**6), Eq(m, -7)), (-64*a**3*b**
2*c**3/(512*b**6*c**4*d**5 + 4096*b**5*c**5*d**5*x + 12288*b**4*c**6*d**5*x
**2 + 16384*b**3*c**7*d**5*x**3 + 8192*b**2*c**8*d**5*x**4) - 32*a**2*b**4*
c**2/(512*b**6*c**4*d**5 + 4096*b**5*c**5*d**5*x + 12288*b**4*c**6*d**5*x**
2 + 16384*b**3*c**7*d**5*x**3 + 8192*b**2*c**8*d**5*x**4) - 256*a**2*b**3*c
**3*x/(512*b**6*c**4*d**5 + 4096*b**5*c**5*d**5*x + 12288*b**4*c**6*d**5*x*
*2 + 16384*b**3*c**7*d**5*x**3 + 8192*b**2*c**8*d**5*x**4) + 512*a**2*b*c**
5*x**3/(512*b**6*c**4*d**5 + 4096*b**5*c**5*d**5*x + 12288*b**4*c**6*d**5*x
**2 + 16384*b**3*c**7*d**5*x**3 + 8192*b**2*c**8*d**5*x**4) + 256*a**2*c**6
*x**4/(512*b**6*c**4*d**5 + 4096*b**5*c**5*d**5*x + 12288*b**4*c**6*d**5*x*
*2 + 16384*b**3*c**7*d**5*x**3 + 8192*b**2*c**8*d**5*x**4) + 48*a*b**6*c*lo
g(b/(2*c) + x)/(512*b**6*c**4*d**5 + 4096*b**5*c**5*d**5*x + 12288*b**4*c**
6*d**5*x**2 + 16384*b**3*c**7*d**5*x**3 + 8192*b**2*c**8*d**5*x**4) + 28*a*
b**6*c/(512*b**6*c**4*d**5 + 4096*b**5*c**5*d**5*x + 12288*b**4*c**6*d**5*x
**2 + 16384*b**3*c**7*d**5*x**3 + 8192*b**2*c**8*d**5*x**4) + 384*a*b**5*c*
*2*x*log(b/(2*c) + x)/(512*b**6*c**4*d**5 + 4096*b**5*c**5*d**5*x + 12288*b
**4*c**6*d**5*x**2 + 16384*b**3*c**7*d**5*x**3 + 8192*b**2*c**8*d**5*x**4)
+ 128*a*b**5*c**2*x/(512*b**6*c**4*d**5 + 4096*b**5*c**5*d**5*x + 12288*b**
4*c**6*d**5*x**2 + 16384*b**3*c**7*d**5*x**3 + 8192*b**2*c**8*d**5*x**4) +
1152*a*b**4*c**3*x**2*log(b/(2*c) + x)/(512*b**6*c**4*d**5 + 4096*b**5*c**5
*d**5*x + 12288*b**4*c**6*d**5*x**2 + 16384*b**3*c**7*d**5*x**3 + 8192*b**2
*c**8*d**5*x**4) + 1536*a*b**3*c**4*x**3*log(b/(2*c) + x)/(512*b**6*c**4*d*
*5 + 4096*b**5*c**5*d**5*x + 12288*b**4*c**6*d**5*x**2 + 16384*b**3*c**7*d*
*5*x**3 + 8192*b**2*c**8*d**5*x**4) - 256*a*b**3*c**4*x**3/(512*b**6*c**4*d
**5 + 4096*b**5*c**5*d**5*x + 12288*b**4*c**6*d**5*x**2 + 16384*b**3*c**7*d
**5*x**3 + 8192*b**2*c**8*d**5*x**4) + 768*a*b**2*c**5*x**4*log(b/(2*c) + x
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)/(512*b**6*c**4*d**5 + 4096*b**5*c**5*d**5*x + 12288*b**4*c**6*d**5*x**2 +
16384*b**3*c**7*d**5*x**3 + 8192*b**2*c**8*d**5*x**4) - 128*a*b**2*c**5*x*
*4/(512*b**6*c**4*d**5 + 4096*b**5*c**5*d**5*x + 12288*b**4*c**6*d**5*x**2
+ 16384*b**3*c**7*d**5*x**3 + 8192*b**2*c**8*d**5*x**4) - 12*b**8*log(b/(2*
c) + x)/(512*b**6*c**4*d**5 + 4096*b**5*c**5*d**5*x + 12288*b**4*c**6*d**5*
x**2 + 16384*b**3*c**7*d**5*x**3 + 8192*b**2*c**8*d**5*x**4) - 7*b**8/(512*
b**6*c**4*d**5 + 4096*b**5*c**5*d**5*x + 12288*b**4*c**6*d**5*x**2 + 16384*
b**3*c**7*d**5*x**3 + 8192*b**2*c**8*d**5*x**4) - 96*b**7*c*x*log(b/(2*c) +
x)/(512*b**6*c**4*d**5 + 4096*b**5*c**5*d**5*x + 12288*b**4*c**6*d**5*x**2
+ 16384*b**3*c**7*d**5*x**3 + 8192*b**2*c**8*d**5*x**4) - 32*b**7*c*x/(512
*b**6*c**4*d**5 + 4096*b**5*c**5*d**5*x + 12288*b**4*c**6*d**5*x**2 + 16384
*b**3*c**7*d**5*x**3 + 8192*b**2*c**8*d**5*x**4) - 288*b**6*c**2*x**2*log(b
/(2*c) + x)/(512*b**6*c**4*d**5 + 4096*b**5*c**5*d**5*x + 12288*b**4*c**6*d
5*x2 + 16384*b**3*c**7*d**5*x**3 + 8192*b**2*c**8*d**5*x**4) - 384*b**5
*c**3*x**3*log(b/(2*c) + x)/(512*b**6*c**4*d**5 + 4096*b**5*c**5*d**5*x + 1
2288*b**4*c**6*d**5*x**2 + 16384*b**3*c**7*d**5*x**3 + 8192*b**2*c**8*d**5*
x**4) + 192*b**5*c**3*x**3/(512*b**6*c**4*d**5 + 4096*b**5*c**5*d**5*x + 12
288*b**4*c**6*d**5*x**2 + 16384*b**3*c**7*d**5*x**3 + 8192*b**2*c**8*d**5*x
4) - 192*b4*c**4*x**4*log(b/(2*c) + x)/(512*b**6*c**4*d**5 + 4096*b**5*
c**5*d**5*x + 12288*b**4*c**6*d**5*x**2 + 16384*b**3*c**7*d**5*x**3 + 8192*
b**2*c**8*d**5*x**4) + 416*b**4*c**4*x**4/(512*b**6*c**4*d**5 + 4096*b**5*c
5*d5*x + 12288*b**4*c**6*d**5*x**2 + 16384*b**3*c**7*d**5*x**3 + 8192*b
2*c8*d**5*x**4) + 384*b**3*c**5*x**5/(512*b**6*c**4*d**5 + 4096*b**5*c*
*5*d**5*x + 12288*b**4*c**6*d**5*x**2 + 16384*b**3*c**7*d**5*x**3 + 8192*b*
*2*c**8*d**5*x**4) + 128*b**2*c**6*x**6/(512*b**6*c**4*d**5 + 4096*b**5*c**
5*d**5*x + 12288*b**4*c**6*d**5*x**2 + 16384*b**3*c**7*d**5*x**3 + 8192*b**
2*c**8*d**5*x**4), Eq(m, -5)), (-64*a**3*c**3/(256*b**2*c**4*d**3 + 1024*b*
c**5*d**3*x + 1024*c**6*d**3*x**2) + 96*a**2*b**2*c**2*log(b/(2*c) + x)/(25
6*b**2*c**4*d**3 + 1024*b*c**5*d**3*x + 1024*c**6*d**3*x**2) + 48*a**2*b**2
*c**2/(256*b**2*c**4*d**3 + 1024*b*c**5*d**3*x + 1024*c**6*d**3*x**2) + 384
*a**2*b*c**3*x*log(b/(2*c) + x)/(256*b**2*c**4*d**3 + 1024*b*c**5*d**3*x +
1024*c**6*d**3*x**2) + 384*a**2*c**4*x**2*log(b/(2*c) + x)/(256*b**2*c**4*d
3 + 1024*b*c5*d**3*x + 1024*c**6*d**3*x**2) - 48*a*b**4*c*log(b/(2*c) +
x)/(256*b**2*c**4*d**3 + 1024*b*c**5*d**3*x + 1024*c**6*d**3*x**2) - 72*a*
b**4*c/(256*b**2*c**4*d**3 + 1024*b*c**5*d**3*x + 1024*c**6*d**3*x**2) - 19
2*a*b**3*c**2*x*log(b/(2*c) + x)/(256*b**2*c**4*d**3 + 1024*b*c**5*d**3*x +
1024*c**6*d**3*x**2) - 192*a*b**3*c**2*x/(256*b**2*c**4*d**3 + 1024*b*c**5
*d**3*x + 1024*c**6*d**3*x**2) - 192*a*b**2*c**3*x**2*log(b/(2*c) + x)/(256
*b**2*c**4*d**3 + 1024*b*c**5*d**3*x + 1024*c**6*d**3*x**2) + 384*a*b*c**4*
x**3/(256*b**2*c**4*d**3 + 1024*b*c**5*d**3*x + 1024*c**6*d**3*x**2) + 192*
a*c**5*x**4/(256*b**2*c**4*d**3 + 1024*b*c**5*d**3*x + 1024*c**6*d**3*x**2)
+ 6*b**6*log(b/(2*c) + x)/(256*b**2*c**4*d**3 + 1024*b*c**5*d**3*x + 1024*
c**6*d**3*x**2) + 9*b**6/(256*b**2*c**4*d**3 + 1024*b*c**5*d**3*x + 1024*c*
*6*d**3*x**2) + 24*b**5*c*x*log(b/(2*c) + x)/(256*b**2*c**4*d**3 + 1024*b*c
5*d3*x + 1024*c**6*d**3*x**2) + 24*b**5*c*x/(256*b**2*c**4*d**3 + 1024*
b*c**5*d**3*x + 1024*c**6*d**3*x**2) + 24*b**4*c**2*x**2*log(b/(2*c) + x)/(
256*b**2*c**4*d**3 + 1024*b*c**5*d**3*x + 1024*c**6*d**3*x**2) - 16*b**3*c*
*3*x**3/(256*b**2*c**4*d**3 + 1024*b*c**5*d**3*x + 1024*c**6*d**3*x**2) + 7
2*b**2*c**4*x**4/(256*b**2*c**4*d**3 + 1024*b*c**5*d**3*x + 1024*c**6*d**3*
x**2) + 96*b*c**5*x**5/(256*b**2*c**4*d**3 + 1024*b*c**5*d**3*x + 1024*c**6
*d**3*x**2) + 32*c**6*x**6/(256*b**2*c**4*d**3 + 1024*b*c**5*d**3*x + 1024*
c**6*d**3*x**2), Eq(m, -3)), (a**3*log(b/(2*c) + x)/(2*c*d) - 3*a**2*b**2*l
og(b/(2*c) + x)/(8*c**2*d) + 3*a**2*b*x/(4*c*d) + 3*a**2*x**2/(4*d) + 3*a*b
4*log(b/(2*c) + x)/(32*c3*d) - 3*a*b**3*x/(16*c**2*d) + 3*a*b**2*x**2/(
16*c*d) + 3*a*b*x**3/(4*d) + 3*a*c*x**4/(8*d) - b**6*log(b/(2*c) + x)/(128*
c**4*d) + b**5*x/(64*c**3*d) - b**4*x**2/(64*c**2*d) + b**3*x**3/(48*c*d) +
7*b**2*x**4/(32*d) + b*c*x**5/(4*d) + c**2*x**6/(12*d), Eq(m, -1)), (4*a**
3*b*c**3*m**3*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m*
*2 + 1408*c**4*m + 840*c**4) + 60*a**3*b*c**3*m**2*(b*d + 2*c*d*x)**m/(8*c*

*4*m**4 + 128*c**4*m**3 + 688*c**4*m**2 + 1408*c**4*m + 840*c**4) + 284*a**
3*b*c**3*m*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m**2
+ 1408*c**4*m + 840*c**4) + 420*a**3*b*c**3*(b*d + 2*c*d*x)**m/(8*c**4*m**4
 + 128*c**4*m**3 + 688*c**4*m**2 + 1408*c**4*m + 840*c**4) + 8*a**3*c**4*m*
*3*x*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m**2 + 1408
*c**4*m + 840*c**4) + 120*a**3*c**4*m**2*x*(b*d + 2*c*d*x)**m/(8*c**4*m**4
+ 128*c**4*m**3 + 688*c**4*m**2 + 1408*c**4*m + 840*c**4) + 568*a**3*c**4*m
x(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m**2 + 1408*c
4*m + 840*c4) + 840*a**3*c**4*x*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c
4*m3 + 688*c**4*m**2 + 1408*c**4*m + 840*c**4) - 6*a**2*b**3*c**2*m**2*
(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m**2 + 1408*c**4
*m + 840*c**4) - 72*a**2*b**3*c**2*m*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*
c**4*m**3 + 688*c**4*m**2 + 1408*c**4*m + 840*c**4) - 210*a**2*b**3*c**2*(b
*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m**2 + 1408*c**4*m
 + 840*c**4) + 12*a**2*b**2*c**3*m**3*x*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 1
28*c**4*m**3 + 688*c**4*m**2 + 1408*c**4*m + 840*c**4) + 144*a**2*b**2*c**3
*m**2*x*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m**2 + 1
408*c**4*m + 840*c**4) + 420*a**2*b**2*c**3*m*x*(b*d + 2*c*d*x)**m/(8*c**4*
m**4 + 128*c**4*m**3 + 688*c**4*m**2 + 1408*c**4*m + 840*c**4) + 36*a**2*b*
c**4*m**3*x**2*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m
2 + 1408*c4*m + 840*c**4) + 468*a**2*b*c**4*m**2*x**2*(b*d + 2*c*d*x)**
m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m**2 + 1408*c**4*m + 840*c**4) +
1692*a**2*b*c**4*m*x**2*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 + 6
88*c**4*m**2 + 1408*c**4*m + 840*c**4) + 1260*a**2*b*c**4*x**2*(b*d + 2*c*d
*x)**m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m**2 + 1408*c**4*m + 840*c**
4) + 24*a**2*c**5*m**3*x**3*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3
 + 688*c**4*m**2 + 1408*c**4*m + 840*c**4) + 312*a**2*c**5*m**2*x**3*(b*d +
 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m**2 + 1408*c**4*m + 8
40*c**4) + 1128*a**2*c**5*m*x**3*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4
*m**3 + 688*c**4*m**2 + 1408*c**4*m + 840*c**4) + 840*a**2*c**5*x**3*(b*d +
 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m**2 + 1408*c**4*m + 8
40*c**4) + 6*a*b**5*c*m*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 + 6
88*c**4*m**2 + 1408*c**4*m + 840*c**4) + 42*a*b**5*c*(b*d + 2*c*d*x)**m/(8*
c**4*m**4 + 128*c**4*m**3 + 688*c**4*m**2 + 1408*c**4*m + 840*c**4) - 12*a*
b**4*c**2*m**2*x*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c**4
*m**2 + 1408*c**4*m + 840*c**4) - 84*a*b**4*c**2*m*x*(b*d + 2*c*d*x)**m/(8*
c**4*m**4 + 128*c**4*m**3 + 688*c**4*m**2 + 1408*c**4*m + 840*c**4) + 12*a*
b**3*c**3*m**3*x**2*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c
4*m2 + 1408*c**4*m + 840*c**4) + 96*a*b**3*c**3*m**2*x**2*(b*d + 2*c*d*
x)**m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m**2 + 1408*c**4*m + 840*c**4
) + 84*a*b**3*c**3*m*x**2*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 +
 688*c**4*m**2 + 1408*c**4*m + 840*c**4) + 48*a*b**2*c**4*m**3*x**3*(b*d +
2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m**2 + 1408*c**4*m + 84
0*c**4) + 504*a*b**2*c**4*m**2*x**3*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c
4*m3 + 688*c**4*m**2 + 1408*c**4*m + 840*c**4) + 1296*a*b**2*c**4*m*x**
3*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m**2 + 1408*c*
*4*m + 840*c**4) + 840*a*b**2*c**4*x**3*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 1
28*c**4*m**3 + 688*c**4*m**2 + 1408*c**4*m + 840*c**4) + 60*a*b*c**5*m**3*x
4*(b*d + 2*c*d*x)m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m**2 + 1408*
c**4*m + 840*c**4) + 660*a*b*c**5*m**2*x**4*(b*d + 2*c*d*x)**m/(8*c**4*m**4
 + 128*c**4*m**3 + 688*c**4*m**2 + 1408*c**4*m + 840*c**4) + 1860*a*b*c**5*
m*x**4*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m**2 + 14
08*c**4*m + 840*c**4) + 1260*a*b*c**5*x**4*(b*d + 2*c*d*x)**m/(8*c**4*m**4
+ 128*c**4*m**3 + 688*c**4*m**2 + 1408*c**4*m + 840*c**4) + 24*a*c**6*m**3*
x**5*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m**2 + 1408
*c**4*m + 840*c**4) + 264*a*c**6*m**2*x**5*(b*d + 2*c*d*x)**m/(8*c**4*m**4
+ 128*c**4*m**3 + 688*c**4*m**2 + 1408*c**4*m + 840*c**4) + 744*a*c**6*m*x*
5(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m**2 + 1408*c
4*m + 840*c4) + 504*a*c**6*x**5*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c

```
**4*m**3 + 688*c**4*m**2 + 1408*c**4*m + 840*c**4) - 3*b**7*(b*d + 2*c*d*x)
**m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m**2 + 1408*c**4*m + 840*c**4)
+ 6*b**6*c*m*x*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m
**2 + 1408*c**4*m + 840*c**4) - 6*b**5*c**2*m**2*x**2*(b*d + 2*c*d*x)**m/(8
*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m**2 + 1408*c**4*m + 840*c**4) - 6*b*
*5*c**2*m*x**2*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m
**2 + 1408*c**4*m + 840*c**4) + 4*b**4*c**3*m**3*x**3*(b*d + 2*c*d*x)**m/(8
*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m**2 + 1408*c**4*m + 840*c**4) + 12*b
**4*c**3*m**2*x**3*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c*
*4*m**2 + 1408*c**4*m + 840*c**4) + 8*b**4*c**3*m*x**3*(b*d + 2*c*d*x)**m/(
8*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m**2 + 1408*c**4*m + 840*c**4) + 20*
b**3*c**4*m**3*x**4*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c
**4*m**2 + 1408*c**4*m + 840*c**4) + 150*b**3*c**4*m**2*x**4*(b*d + 2*c*d*x
)**m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m**2 + 1408*c**4*m + 840*c**4)
 + 340*b**3*c**4*m*x**4*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 + 6
88*c**4*m**2 + 1408*c**4*m + 840*c**4) + 210*b**3*c**4*x**4*(b*d + 2*c*d*x)
**m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m**2 + 1408*c**4*m + 840*c**4)
+ 36*b**2*c**5*m**3*x**5*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 +
688*c**4*m**2 + 1408*c**4*m + 840*c**4) + 312*b**2*c**5*m**2*x**5*(b*d + 2*
c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m**2 + 1408*c**4*m + 840*
c**4) + 780*b**2*c**5*m*x**5*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**
3 + 688*c**4*m**2 + 1408*c**4*m + 840*c**4) + 504*b**2*c**5*x**5*(b*d + 2*c
*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m**2 + 1408*c**4*m + 840*c
**4) + 28*b*c**6*m**3*x**6*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3
+ 688*c**4*m**2 + 1408*c**4*m + 840*c**4) + 252*b*c**6*m**2*x**6*(b*d + 2*c
*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m**2 + 1408*c**4*m + 840*c
**4) + 644*b*c**6*m*x**6*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 +
688*c**4*m**2 + 1408*c**4*m + 840*c**4) + 420*b*c**6*x**6*(b*d + 2*c*d*x)**
m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m**2 + 1408*c**4*m + 840*c**4) +
8*c**7*m**3*x**7*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c**4
*m**2 + 1408*c**4*m + 840*c**4) + 72*c**7*m**2*x**7*(b*d + 2*c*d*x)**m/(8*c
**4*m**4 + 128*c**4*m**3 + 688*c**4*m**2 + 1408*c**4*m + 840*c**4) + 184*c*
*7*m*x**7*(b*d + 2*c*d*x)**m/(8*c**4*m**4 + 128*c**4*m**3 + 688*c**4*m**2 +
 1408*c**4*m + 840*c**4) + 120*c**7*x**7*(b*d + 2*c*d*x)**m/(8*c**4*m**4 +
128*c**4*m**3 + 688*c**4*m**2 + 1408*c**4*m + 840*c**4), True))
```

Giac [B] time = 1.19964, size = 2033, normalized size = 14.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m*(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*(2*c*d*x + b*d)^m*c^7*m^3*x^7 + 28*(2*c*d*x + b*d)^m*b*c^6*m^3*x^6 +
 72*(2*c*d*x + b*d)^m*c^7*m^2*x^7 + 36*(2*c*d*x + b*d)^m*b^2*c^5*m^3*x^5 +
24*(2*c*d*x + b*d)^m*a*c^6*m^3*x^5 + 252*(2*c*d*x + b*d)^m*b*c^6*m^2*x^6 +
184*(2*c*d*x + b*d)^m*c^7*m*x^7 + 20*(2*c*d*x + b*d)^m*b^3*c^4*m^3*x^4 + 60
*(2*c*d*x + b*d)^m*a*b*c^5*m^3*x^4 + 312*(2*c*d*x + b*d)^m*b^2*c^5*m^2*x^5
+ 264*(2*c*d*x + b*d)^m*a*c^6*m^2*x^5 + 644*(2*c*d*x + b*d)^m*b*c^6*m*x^6 +
 120*(2*c*d*x + b*d)^m*c^7*x^7 + 4*(2*c*d*x + b*d)^m*b^4*c^3*m^3*x^3 + 48*(
2*c*d*x + b*d)^m*a*b^2*c^4*m^3*x^3 + 24*(2*c*d*x + b*d)^m*a^2*c^5*m^3*x^3 +
 150*(2*c*d*x + b*d)^m*b^3*c^4*m^2*x^4 + 660*(2*c*d*x + b*d)^m*a*b*c^5*m^2*
x^4 + 780*(2*c*d*x + b*d)^m*b^2*c^5*m*x^5 + 744*(2*c*d*x + b*d)^m*a*c^6*m*x
^5 + 420*(2*c*d*x + b*d)^m*b*c^6*x^6 + 12*(2*c*d*x + b*d)^m*a*b^3*c^3*m^3*x
^2 + 36*(2*c*d*x + b*d)^m*a^2*b*c^4*m^3*x^2 + 12*(2*c*d*x + b*d)^m*b^4*c^3*
m^2*x^3 + 504*(2*c*d*x + b*d)^m*a*b^2*c^4*m^2*x^3 + 312*(2*c*d*x + b*d)^m*a
```

^2*c^5*m^2*x^3 + 340*(2*c*d*x + b*d)^m*b^3*c^4*m*x^4 + 1860*(2*c*d*x + b*d)
^m*a*b*c^5*m*x^4 + 504*(2*c*d*x + b*d)^m*b^2*c^5*x^5 + 504*(2*c*d*x + b*d)^
m*a*c^6*x^5 + 12*(2*c*d*x + b*d)^m*a^2*b^2*c^3*m^3*x + 8*(2*c*d*x + b*d)^m*
a^3*c^4*m^3*x - 6*(2*c*d*x + b*d)^m*b^5*c^2*m^2*x^2 + 96*(2*c*d*x + b*d)^m*
a*b^3*c^3*m^2*x^2 + 468*(2*c*d*x + b*d)^m*a^2*b*c^4*m^2*x^2 + 8*(2*c*d*x +
b*d)^m*b^4*c^3*m*x^3 + 1296*(2*c*d*x + b*d)^m*a*b^2*c^4*m*x^3 + 1128*(2*c*d
*x + b*d)^m*a^2*c^5*m*x^3 + 210*(2*c*d*x + b*d)^m*b^3*c^4*x^4 + 1260*(2*c*d
*x + b*d)^m*a*b*c^5*x^4 + 4*(2*c*d*x + b*d)^m*a^3*b*c^3*m^3 - 12*(2*c*d*x +
 b*d)^m*a*b^4*c^2*m^2*x + 144*(2*c*d*x + b*d)^m*a^2*b^2*c^3*m^2*x + 120*(2*
c*d*x + b*d)^m*a^3*c^4*m^2*x - 6*(2*c*d*x + b*d)^m*b^5*c^2*m*x^2 + 84*(2*c*
d*x + b*d)^m*a*b^3*c^3*m*x^2 + 1692*(2*c*d*x + b*d)^m*a^2*b*c^4*m*x^2 + 840
*(2*c*d*x + b*d)^m*a*b^2*c^4*x^3 + 840*(2*c*d*x + b*d)^m*a^2*c^5*x^3 - 6*(2
*c*d*x + b*d)^m*a^2*b^3*c^2*m^2 + 60*(2*c*d*x + b*d)^m*a^3*b*c^3*m^2 + 6*(2
*c*d*x + b*d)^m*b^6*c*m*x - 84*(2*c*d*x + b*d)^m*a*b^4*c^2*m*x + 420*(2*c*d
*x + b*d)^m*a^2*b^2*c^3*m*x + 568*(2*c*d*x + b*d)^m*a^3*c^4*m*x + 1260*(2*c
*d*x + b*d)^m*a^2*b*c^4*x^2 + 6*(2*c*d*x + b*d)^m*a*b^5*c*m - 72*(2*c*d*x +
 b*d)^m*a^2*b^3*c^2*m + 284*(2*c*d*x + b*d)^m*a^3*b*c^3*m + 840*(2*c*d*x +
b*d)^m*a^3*c^4*x - 3*(2*c*d*x + b*d)^m*b^7 + 42*(2*c*d*x + b*d)^m*a*b^5*c -
 210*(2*c*d*x + b*d)^m*a^2*b^3*c^2 + 420*(2*c*d*x + b*d)^m*a^3*b*c^3)/(c^4*
m^4 + 16*c^4*m^3 + 86*c^4*m^2 + 176*c^4*m + 105*c^4)

3.1424 $\int (bd+2cdx)^m \left(a+bx+cx^2\right)^2 \, dx$

Optimal. Leaf size=103

$$-\frac{\left(b^2-4ac\right)(bd+2cdx)^{m+3}}{16c^3d^3(m+3)}+\frac{\left(b^2-4ac\right)^2(bd+2cdx)^{m+1}}{32c^3d(m+1)}+\frac{(bd+2cdx)^{m+5}}{32c^3d^5(m+5)}$$

```
[Out] ((b^2 - 4*a*c)^2*(b*d + 2*c*d*x)^(1 + m))/(32*c^3*d*(1 + m)) - ((b^2 - 4*a*
c)*(b*d + 2*c*d*x)^(3 + m))/(16*c^3*d^3*(3 + m)) + (b*d + 2*c*d*x)^(5 + m)/
(32*c^3*d^5*(5 + m))
```

Rubi [A] time = 0.0505594, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {683}

$$-\frac{\left(b^2-4ac\right)(bd+2cdx)^{m+3}}{16c^3d^3(m+3)}+\frac{\left(b^2-4ac\right)^2(bd+2cdx)^{m+1}}{32c^3d(m+1)}+\frac{(bd+2cdx)^{m+5}}{32c^3d^5(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^m*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((b^2 - 4*a*c)^2*(b*d + 2*c*d*x)^(1 + m))/(32*c^3*d*(1 + m)) - ((b^2 - 4*a*
c)*(b*d + 2*c*d*x)^(3 + m))/(16*c^3*d^3*(3 + m)) + (b*d + 2*c*d*x)^(5 + m)/
(32*c^3*d^5*(5 + m))
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int (bd+2cdx)^m \left(a+bx+cx^2\right)^2 \, dx &= \int \left(\frac{\left(-b^2+4ac\right)^2(bd+2cdx)^m}{16c^2}+\frac{\left(-b^2+4ac\right)(bd+2cdx)^{2+m}}{8c^2d^2}+\frac{(bd+2cdx)^{4+m}}{16c^2d^4}\right.\\ &= \frac{\left(b^2-4ac\right)^2(bd+2cdx)^{1+m}}{32c^3d(1+m)}-\frac{\left(b^2-4ac\right)(bd+2cdx)^{3+m}}{16c^3d^3(3+m)}+\frac{(bd+2cdx)^{5+m}}{32c^3d^5(5+m)}\end{aligned}$$

Mathematica [A] time = 0.0470792, size = 77, normalized size = 0.75

$$\frac{(b+2cx)\left(-\frac{2\left(b^2-4ac\right)(b+2cx)^2}{m+3}+\frac{\left(b^2-4ac\right)^2}{m+1}+\frac{(b+2cx)^4}{m+5}\right)(d(b+2cx))^m}{32c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^m*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((b + 2*c*x)*(d*(b + 2*c*x))^m*((b^2 - 4*a*c)^2/(1 + m) - (2*(b^2 - 4*a*c)*
(b + 2*c*x)^2)/(3 + m) + (b + 2*c*x)^4/(5 + m)))/(32*c^3)
```

Maple [B] time = 0.046, size = 255, normalized size = 2.5

$$\frac{\left(2c^4m^2x^4 + 4bc^3m^2x^3 + 8c^4mx^4 + 4ac^3m^2x^2 + 2b^2c^2m^2x^2 + 16bc^3mx^3 + 6c^4x^4 + 4abc^2m^2x + 24ac^3mx^2 + 6b^2c^2mx^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^m*(c*x^2+b*x+a)^2,x)
```

```
[Out] 1/4*(2*c*x+b)*(2*c^4*m^2*x^4+4*b*c^3*m^2*x^3+8*c^4*m*x^4+4*a*c^3*m^2*x^2+2*
b^2*c^2*m^2*x^2+16*b*c^3*m*x^3+6*c^4*x^4+4*a*b*c^2*m^2*x+24*a*c^3*m*x^2+6*b
^2*c^2*m*x^2+12*b*c^3*x^3+2*a^2*c^2*m^2+24*a*b*c^2*m*x+20*a*c^3*x^2-2*b^3*c
*m*x+4*b^2*c^2*x^2+16*a^2*c^2*m-2*a*b^2*c*m+20*a*b*c^2*x-2*b^3*c*x+30*a^2*c
^2-10*a*b^2*c+b^4)*(2*c*d*x+b*d)^m/c^3/(m^3+9*m^2+23*m+15)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.21289, size = 641, normalized size = 6.22

$$\frac{\left(2a^2bc^2m^2 + 4\left(c^5m^2 + 4c^5m + 3c^5\right)x^5 + b^5 - 10ab^3c + 30a^2bc^2 + 10\left(bc^4m^2 + 4bc^4m + 3bc^4\right)x^4 + 4\left(5b^2c^3 + 10ac^4 +\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^2*b*c^2*m^2 + 4*(c^5*m^2 + 4*c^5*m + 3*c^5)*x^5 + b^5 - 10*a*b^3*c
 + 30*a^2*b*c^2 + 10*(b*c^4*m^2 + 4*b*c^4*m + 3*b*c^4)*x^4 + 4*(5*b^2*c^3 +
 10*a*c^4 + 2*(b^2*c^3 + a*c^4)*m^2 + (7*b^2*c^3 + 12*a*c^4)*m)*x^3 + 2*(30
*a*b*c^3 + (b^3*c^2 + 6*a*b*c^3)*m^2 + (b^3*c^2 + 36*a*b*c^3)*m)*x^2 - 2*(a
*b^3*c - 8*a^2*b*c^2)*m + 2*(30*a^2*c^3 + 2*(a*b^2*c^2 + a^2*c^3)*m^2 - (b^
4*c - 10*a*b^2*c^2 - 16*a^2*c^3)*m)*x)*(2*c*d*x + b*d)^m/(c^3*m^3 + 9*c^3*m
^2 + 23*c^3*m + 15*c^3)
```

Sympy [A] time = 6.70394, size = 3434, normalized size = 33.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**m*(c*x**2+b*x+a)**2,x)
```

```
[Out] Piecewise(((b*d)**m*(a**2*x + a*b*x**2 + b**2*x**3/3), Eq(c, 0)), (-48*a**2
*b**2*c**2/(384*b**6*c**3*d**5 + 3072*b**5*c**4*d**5*x + 9216*b**4*c**5*d**
5*x**2 + 12288*b**3*c**6*d**5*x**3 + 6144*b**2*c**7*d**5*x**4) - 16*a*b**4*
c/(384*b**6*c**3*d**5 + 3072*b**5*c**4*d**5*x + 9216*b**4*c**5*d**5*x**2 +
12288*b**3*c**6*d**5*x**3 + 6144*b**2*c**7*d**5*x**4) - 128*a*b**3*c**2*x/(
384*b**6*c**3*d**5 + 3072*b**5*c**4*d**5*x + 9216*b**4*c**5*d**5*x**2 + 122
88*b**3*c**6*d**5*x**3 + 6144*b**2*c**7*d**5*x**4) + 256*a*b*c**4*x**3/(384
*b**6*c**3*d**5 + 3072*b**5*c**4*d**5*x + 9216*b**4*c**5*d**5*x**2 + 12288*
b**3*c**6*d**5*x**3 + 6144*b**2*c**7*d**5*x**4) + 128*a*c**5*x**4/(384*b**6
*c**3*d**5 + 3072*b**5*c**4*d**5*x + 9216*b**4*c**5*d**5*x**2 + 12288*b**3*
c**6*d**5*x**3 + 6144*b**2*c**7*d**5*x**4) + 12*b**6*log(b/(2*c) + x)/(384*
b**6*c**3*d**5 + 3072*b**5*c**4*d**5*x + 9216*b**4*c**5*d**5*x**2 + 12288*b
**3*c**6*d**5*x**3 + 6144*b**2*c**7*d**5*x**4) + 7*b**6/(384*b**6*c**3*d**5
 + 3072*b**5*c**4*d**5*x + 9216*b**4*c**5*d**5*x**2 + 12288*b**3*c**6*d**5*
x**3 + 6144*b**2*c**7*d**5*x**4) + 96*b**5*c*x*log(b/(2*c) + x)/(384*b**6*c
**3*d**5 + 3072*b**5*c**4*d**5*x + 9216*b**4*c**5*d**5*x**2 + 12288*b**3*c*
*6*d**5*x**3 + 6144*b**2*c**7*d**5*x**4) + 32*b**5*c*x/(384*b**6*c**3*d**5
+ 3072*b**5*c**4*d**5*x + 9216*b**4*c**5*d**5*x**2 + 12288*b**3*c**6*d**5*x
**3 + 6144*b**2*c**7*d**5*x**4) + 288*b**4*c**2*x**2*log(b/(2*c) + x)/(384*
b**6*c**3*d**5 + 3072*b**5*c**4*d**5*x + 9216*b**4*c**5*d**5*x**2 + 12288*b
**3*c**6*d**5*x**3 + 6144*b**2*c**7*d**5*x**4) + 384*b**3*c**3*x**3*log(b/(
2*c) + x)/(384*b**6*c**3*d**5 + 3072*b**5*c**4*d**5*x + 9216*b**4*c**5*d**5
*x**2 + 12288*b**3*c**6*d**5*x**3 + 6144*b**2*c**7*d**5*x**4) - 64*b**3*c**
3*x**3/(384*b**6*c**3*d**5 + 3072*b**5*c**4*d**5*x + 9216*b**4*c**5*d**5*x*
*2 + 12288*b**3*c**6*d**5*x**3 + 6144*b**2*c**7*d**5*x**4) + 192*b**2*c**4*
x**4*log(b/(2*c) + x)/(384*b**6*c**3*d**5 + 3072*b**5*c**4*d**5*x + 9216*b*
*4*c**5*d**5*x**2 + 12288*b**3*c**6*d**5*x**3 + 6144*b**2*c**7*d**5*x**4) -
 32*b**2*c**4*x**4/(384*b**6*c**3*d**5 + 3072*b**5*c**4*d**5*x + 9216*b**4*
c**5*d**5*x**2 + 12288*b**3*c**6*d**5*x**3 + 6144*b**2*c**7*d**5*x**4), Eq(
m, -5)), (-8*a**2*c**2/(32*b**2*c**3*d**3 + 128*b*c**4*d**3*x + 128*c**5*d*
*3*x**2) + 8*a*b**2*c*log(b/(2*c) + x)/(32*b**2*c**3*d**3 + 128*b*c**4*d**3
*x + 128*c**5*d**3*x**2) + 4*a*b**2*c/(32*b**2*c**3*d**3 + 128*b*c**4*d**3*
x + 128*c**5*d**3*x**2) + 32*a*b*c**2*x*log(b/(2*c) + x)/(32*b**2*c**3*d**3
 + 128*b*c**4*d**3*x + 128*c**5*d**3*x**2) + 32*a*c**3*x**2*log(b/(2*c) + x
)/(32*b**2*c**3*d**3 + 128*b*c**4*d**3*x + 128*c**5*d**3*x**2) - 2*b**4*log
(b/(2*c) + x)/(32*b**2*c**3*d**3 + 128*b*c**4*d**3*x + 128*c**5*d**3*x**2)
- 3*b**4/(32*b**2*c**3*d**3 + 128*b*c**4*d**3*x + 128*c**5*d**3*x**2) - 8*b
**3*c*x*log(b/(2*c) + x)/(32*b**2*c**3*d**3 + 128*b*c**4*d**3*x + 128*c**5*
d**3*x**2) - 8*b**3*c*x/(32*b**2*c**3*d**3 + 128*b*c**4*d**3*x + 128*c**5*d
**3*x**2) - 8*b**2*c**2*x**2*log(b/(2*c) + x)/(32*b**2*c**3*d**3 + 128*b*c*
*4*d**3*x + 128*c**5*d**3*x**2) + 16*b*c**3*x**3/(32*b**2*c**3*d**3 + 128*b
*c**4*d**3*x + 128*c**5*d**3*x**2) + 8*c**4*x**4/(32*b**2*c**3*d**3 + 128*b
*c**4*d**3*x + 128*c**5*d**3*x**2), Eq(m, -3)), (a**2*log(b/(2*c) + x)/(2*c
*d) - a*b**2*log(b/(2*c) + x)/(4*c**2*d) + a*b*x/(2*c*d) + a*x**2/(2*d) + b
**4*log(b/(2*c) + x)/(32*c**3*d) - b**3*x/(16*c**2*d) + b**2*x**2/(16*c*d)
+ b*x**3/(4*d) + c*x**4/(8*d), Eq(m, -1)), (2*a**2*b*c**2*m**2*(b*d + 2*c*d
*x)**m/(4*c**3*m**3 + 36*c**3*m**2 + 92*c**3*m + 60*c**3) + 16*a**2*b*c**2*
m*(b*d + 2*c*d*x)**m/(4*c**3*m**3 + 36*c**3*m**2 + 92*c**3*m + 60*c**3) + 3
0*a**2*b*c**2*(b*d + 2*c*d*x)**m/(4*c**3*m**3 + 36*c**3*m**2 + 92*c**3*m +
60*c**3) + 4*a**2*c**3*m**2*x*(b*d + 2*c*d*x)**m/(4*c**3*m**3 + 36*c**3*m**
2 + 92*c**3*m + 60*c**3) + 32*a**2*c**3*m*x*(b*d + 2*c*d*x)**m/(4*c**3*m**3
 + 36*c**3*m**2 + 92*c**3*m + 60*c**3) + 60*a**2*c**3*x*(b*d + 2*c*d*x)**m/
(4*c**3*m**3 + 36*c**3*m**2 + 92*c**3*m + 60*c**3) - 2*a*b**3*c*m*(b*d + 2*
c*d*x)**m/(4*c**3*m**3 + 36*c**3*m**2 + 92*c**3*m + 60*c**3) - 10*a*b**3*c*
(b*d + 2*c*d*x)**m/(4*c**3*m**3 + 36*c**3*m**2 + 92*c**3*m + 60*c**3) + 4*a
```

```
*b**2*c**2*m**2*x*(b*d + 2*c*d*x)**m/(4*c**3*m**3 + 36*c**3*m**2 + 92*c**3*
m + 60*c**3) + 20*a*b**2*c**2*m*x*(b*d + 2*c*d*x)**m/(4*c**3*m**3 + 36*c**3
*m**2 + 92*c**3*m + 60*c**3) + 12*a*b*c**3*m**2*x**2*(b*d + 2*c*d*x)**m/(4*
c**3*m**3 + 36*c**3*m**2 + 92*c**3*m + 60*c**3) + 72*a*b*c**3*m*x**2*(b*d +
 2*c*d*x)**m/(4*c**3*m**3 + 36*c**3*m**2 + 92*c**3*m + 60*c**3) + 60*a*b*c*
*3*x**2*(b*d + 2*c*d*x)**m/(4*c**3*m**3 + 36*c**3*m**2 + 92*c**3*m + 60*c**
3) + 8*a*c**4*m**2*x**3*(b*d + 2*c*d*x)**m/(4*c**3*m**3 + 36*c**3*m**2 + 92
*c**3*m + 60*c**3) + 48*a*c**4*m*x**3*(b*d + 2*c*d*x)**m/(4*c**3*m**3 + 36*
c**3*m**2 + 92*c**3*m + 60*c**3) + 40*a*c**4*x**3*(b*d + 2*c*d*x)**m/(4*c**
3*m**3 + 36*c**3*m**2 + 92*c**3*m + 60*c**3) + b**5*(b*d + 2*c*d*x)**m/(4*c
**3*m**3 + 36*c**3*m**2 + 92*c**3*m + 60*c**3) - 2*b**4*c*m*x*(b*d + 2*c*d*
x)**m/(4*c**3*m**3 + 36*c**3*m**2 + 92*c**3*m + 60*c**3) + 2*b**3*c**2*m**2
*x**2*(b*d + 2*c*d*x)**m/(4*c**3*m**3 + 36*c**3*m**2 + 92*c**3*m + 60*c**3)
 + 2*b**3*c**2*m*x**2*(b*d + 2*c*d*x)**m/(4*c**3*m**3 + 36*c**3*m**2 + 92*c
**3*m + 60*c**3) + 8*b**2*c**3*m**2*x**3*(b*d + 2*c*d*x)**m/(4*c**3*m**3 +
36*c**3*m**2 + 92*c**3*m + 60*c**3) + 28*b**2*c**3*m*x**3*(b*d + 2*c*d*x)**
m/(4*c**3*m**3 + 36*c**3*m**2 + 92*c**3*m + 60*c**3) + 20*b**2*c**3*x**3*(b
*d + 2*c*d*x)**m/(4*c**3*m**3 + 36*c**3*m**2 + 92*c**3*m + 60*c**3) + 10*b*
c**4*m**2*x**4*(b*d + 2*c*d*x)**m/(4*c**3*m**3 + 36*c**3*m**2 + 92*c**3*m +
 60*c**3) + 40*b*c**4*m*x**4*(b*d + 2*c*d*x)**m/(4*c**3*m**3 + 36*c**3*m**2
 + 92*c**3*m + 60*c**3) + 30*b*c**4*x**4*(b*d + 2*c*d*x)**m/(4*c**3*m**3 +
36*c**3*m**2 + 92*c**3*m + 60*c**3) + 4*c**5*m**2*x**5*(b*d + 2*c*d*x)**m/(
4*c**3*m**3 + 36*c**3*m**2 + 92*c**3*m + 60*c**3) + 16*c**5*m*x**5*(b*d + 2
*c*d*x)**m/(4*c**3*m**3 + 36*c**3*m**2 + 92*c**3*m + 60*c**3) + 12*c**5*x**
5*(b*d + 2*c*d*x)**m/(4*c**3*m**3 + 36*c**3*m**2 + 92*c**3*m + 60*c**3), Tr
ue))
```

Giac [B] time = 1.20724, size = 879, normalized size = 8.53

$$\frac{4\,(2\,cdx+bd)^{m}c^{5}m^{2}x^{5}+10\,(2\,cdx+bd)^{m}bc^{4}m^{2}x^{4}+16\,(2\,cdx+bd)^{m}c^{5}mx^{5}+8\,(2\,cdx+bd)^{m}b^{2}c^{3}m^{2}x^{3}+8\,(2\,cdx+bd)^{m}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(4*(2*c*d*x + b*d)^m*c^5*m^2*x^5 + 10*(2*c*d*x + b*d)^m*b*c^4*m^2*x^4 +
 16*(2*c*d*x + b*d)^m*c^5*m*x^5 + 8*(2*c*d*x + b*d)^m*b^2*c^3*m^2*x^3 + 8*(
2*c*d*x + b*d)^m*a*c^4*m^2*x^3 + 40*(2*c*d*x + b*d)^m*b*c^4*m*x^4 + 12*(2*c
*d*x + b*d)^m*c^5*x^5 + 2*(2*c*d*x + b*d)^m*b^3*c^2*m^2*x^2 + 12*(2*c*d*x +
 b*d)^m*a*b*c^3*m^2*x^2 + 28*(2*c*d*x + b*d)^m*b^2*c^3*m*x^3 + 48*(2*c*d*x
+ b*d)^m*a*c^4*m*x^3 + 30*(2*c*d*x + b*d)^m*b*c^4*x^4 + 4*(2*c*d*x + b*d)^m
*a*b^2*c^2*m^2*x + 4*(2*c*d*x + b*d)^m*a^2*c^3*m^2*x + 2*(2*c*d*x + b*d)^m*
b^3*c^2*m*x^2 + 72*(2*c*d*x + b*d)^m*a*b*c^3*m*x^2 + 20*(2*c*d*x + b*d)^m*b
^2*c^3*x^3 + 40*(2*c*d*x + b*d)^m*a*c^4*x^3 + 2*(2*c*d*x + b*d)^m*a^2*b*c^2
*m^2 - 2*(2*c*d*x + b*d)^m*b^4*c*m*x + 20*(2*c*d*x + b*d)^m*a*b^2*c^2*m*x +
 32*(2*c*d*x + b*d)^m*a^2*c^3*m*x + 60*(2*c*d*x + b*d)^m*a*b*c^3*x^2 - 2*(2
*c*d*x + b*d)^m*a*b^3*c*m + 16*(2*c*d*x + b*d)^m*a^2*b*c^2*m + 60*(2*c*d*x
+ b*d)^m*a^2*c^3*x + (2*c*d*x + b*d)^m*b^5 - 10*(2*c*d*x + b*d)^m*a*b^3*c +
 30*(2*c*d*x + b*d)^m*a^2*b*c^2)/(c^3*m^3 + 9*c^3*m^2 + 23*c^3*m + 15*c^3)
```

3.1425 $\int (bd+2cdx)^m \left(a+bx+cx^2\right) dx$

Optimal. Leaf size=65

$$\frac{(bd+2cdx)^{m+3}}{8c^2d^3(m+3)}-\frac{\left(b^2-4ac\right)(bd+2cdx)^{m+1}}{8c^2d(m+1)}$$

```
[Out] -((b^2 - 4*a*c)*(b*d + 2*c*d*x)^(1 + m))/(8*c^2*d*(1 + m)) + (b*d + 2*c*d*x
)^(3 + m)/(8*c^2*d^3*(3 + m))
```

Rubi [A] time = 0.0284689, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {683}

$$\frac{(bd+2cdx)^{m+3}}{8c^2d^3(m+3)}-\frac{\left(b^2-4ac\right)(bd+2cdx)^{m+1}}{8c^2d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^m*(a + b*x + c*x^2),x]
```

```
[Out] -((b^2 - 4*a*c)*(b*d + 2*c*d*x)^(1 + m))/(8*c^2*d*(1 + m)) + (b*d + 2*c*d*x
)^(3 + m)/(8*c^2*d^3*(3 + m))
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int (bd+2cdx)^m\left(a+bx+cx^2\right)dx&=\int\left(\frac{\left(-b^2+4ac\right)(bd+2cdx)^m}{4c}+\frac{(bd+2cdx)^{2+m}}{4cd^2}\right)dx\\&=-\frac{\left(b^2-4ac\right)(bd+2cdx)^{1+m}}{8c^2d(1+m)}+\frac{(bd+2cdx)^{3+m}}{8c^2d^3(3+m)}\end{aligned}$$

Mathematica [A] time = 0.0334382, size = 64, normalized size = 0.98

$$\frac{(b+2cx)\left(2c\left(a(m+3)+c(m+1)x^2\right)-b^2+2bc(m+1)x\right)(d(b+2cx))^m}{4c^2(m+1)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^m*(a + b*x + c*x^2),x]
```

```
[Out] ((b + 2*c*x)*(d*(b + 2*c*x))^m*(-b^2 + 2*b*c*(1 + m)*x + 2*c*(a*(3 + m) + c
*(1 + m)*x^2)))/(4*c^2*(1 + m)*(3 + m))
```

Maple [A] time = 0.041, size = 76, normalized size = 1.2

$$\frac{(2\,cdx+bd)^{m}\left(2\,c^{2}mx^{2}+2\,bcmx+2\,c^{2}x^{2}+2\,acm+2\,bcx+6\,ac-b^{2}\right)(2\,cx+b)}{4\,c^{2}\left(m^{2}+4\,m+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^m*(c*x^2+b*x+a),x)
```

```
[Out] 1/4*(2*c*d*x+b*d)^m*(2*c^2*m*x^2+2*b*c*m*x+2*c^2*x^2+2*a*c*m+2*b*c*x+6*a*c-
b^2)*(2*c*x+b)/c^2/(m^2+4*m+3)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m*(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.22908, size = 223, normalized size = 3.43

$$\frac{\left(2\,abcm+4\left(c^{3}m+c^{3}\right)x^{3}-b^{3}+6\,abc+6\left(bc^{2}m+bc^{2}\right)x^{2}+2\left(6\,ac^{2}+\left(b^{2}c+2\,ac^{2}\right)m\right)x\right)(2\,cdx+bd)^{m}}{4\left(c^{2}m^{2}+4\,c^{2}m+3\,c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m*(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b*c*m + 4*(c^3*m + c^3)*x^3 - b^3 + 6*a*b*c + 6*(b*c^2*m + b*c^2)*
x^2 + 2*(6*a*c^2 + (b^2*c + 2*a*c^2)*m)*x)*(2*c*d*x + b*d)^m/(c^2*m^2 + 4*c
^2*m + 3*c^2)
```

Sympy [A] time = 3.5383, size = 707, normalized size = 10.88

$$\begin{cases} (bd)^{m}\left(ax+\frac{bx^{2}}{2}\right) \\ -\frac{4ac}{16b^{2}c^{2}d^{3}+64bc^{3}d^{3}x+64c^{4}d^{3}x^{2}}+\frac{2b^{2}\log\left(\frac{b}{2c}+x\right)}{16b^{2}c^{2}d^{3}+64bc^{3}d^{3}x+64c^{4}d^{3}x^{2}}+\frac{b^{2}}{16b^{2}c^{2}d^{3}+64bc^{3}d^{3}x+64c^{4}d^{3}x^{2}}+\frac{8bcx\log\left(\frac{b}{2c}+x\right)}{16b^{2}c^{2}d^{3}+64bc^{3}d^{3}x+64c^{4}d^{3}x^{2}}+\frac{8c^{2}x^{2}\log(}{16b^{2}c^{2}d^{3}+64bc^{3}} \\ \frac{a\log\left(\frac{b}{2c}+x\right)}{2cd}-\frac{b^{2}\log\left(\frac{b}{2c}+x\right)}{8c^{2}d}+\frac{bx}{4cd}+\frac{x^{2}}{4d} \\ \frac{2abcm(bd+2cdx)^{m}}{4c^{2}m^{2}+16c^{2}m+12c^{2}}+\frac{6abc(bd+2cdx)^{m}}{4c^{2}m^{2}+16c^{2}m+12c^{2}}+\frac{4ac^{2}mx(bd+2cdx)^{m}}{4c^{2}m^{2}+16c^{2}m+12c^{2}}+\frac{12ac^{2}x(bd+2cdx)^{m}}{4c^{2}m^{2}+16c^{2}m+12c^{2}}-\frac{b^{3}(bd+2cdx)^{m}}{4c^{2}m^{2}+16c^{2}m+12c^{2}}+\frac{2b^{2}cmx(bd+2cdx)^{m}}{4c^{2}m^{2}+16c^{2}m+12c^{2}}+\frac{6bc^{2}mx^{2}(}{4c^{2}m^{2}+16} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**m*(c*x**2+b*x+a),x)
```

```
[Out] Piecewise(((b*d)**m*(a*x + b*x**2/2), Eq(c, 0)), (-4*a*c/(16*b**2*c**2*d**3
 + 64*b*c**3*d**3*x + 64*c**4*d**3*x**2) + 2*b**2*log(b/(2*c) + x)/(16*b**2
```

```
*c**2*d**3 + 64*b*c**3*d**3*x + 64*c**4*d**3*x**2) + b**2/(16*b**2*c**2*d**
3 + 64*b*c**3*d**3*x + 64*c**4*d**3*x**2) + 8*b*c*x*log(b/(2*c) + x)/(16*b*
*2*c**2*d**3 + 64*b*c**3*d**3*x + 64*c**4*d**3*x**2) + 8*c**2*x**2*log(b/(2
*c) + x)/(16*b**2*c**2*d**3 + 64*b*c**3*d**3*x + 64*c**4*d**3*x**2), Eq(m,
-3)), (a*log(b/(2*c) + x)/(2*c*d) - b**2*log(b/(2*c) + x)/(8*c**2*d) + b*x/
(4*c*d) + x**2/(4*d), Eq(m, -1)), (2*a*b*c*m*(b*d + 2*c*d*x)**m/(4*c**2*m**
2 + 16*c**2*m + 12*c**2) + 6*a*b*c*(b*d + 2*c*d*x)**m/(4*c**2*m**2 + 16*c**
2*m + 12*c**2) + 4*a*c**2*m*x*(b*d + 2*c*d*x)**m/(4*c**2*m**2 + 16*c**2*m +
 12*c**2) + 12*a*c**2*x*(b*d + 2*c*d*x)**m/(4*c**2*m**2 + 16*c**2*m + 12*c*
*2) - b**3*(b*d + 2*c*d*x)**m/(4*c**2*m**2 + 16*c**2*m + 12*c**2) + 2*b**2*
c*m*x*(b*d + 2*c*d*x)**m/(4*c**2*m**2 + 16*c**2*m + 12*c**2) + 6*b*c**2*m*x
**2*(b*d + 2*c*d*x)**m/(4*c**2*m**2 + 16*c**2*m + 12*c**2) + 6*b*c**2*x**2*
(b*d + 2*c*d*x)**m/(4*c**2*m**2 + 16*c**2*m + 12*c**2) + 4*c**3*m*x**3*(b*d
 + 2*c*d*x)**m/(4*c**2*m**2 + 16*c**2*m + 12*c**2) + 4*c**3*x**3*(b*d + 2*c
*d*x)**m/(4*c**2*m**2 + 16*c**2*m + 12*c**2), True))
```

Giac [B] time = 1.16209, size = 282, normalized size = 4.34

$$\frac{4\,(2\,cdx+bd)^m c^3 m x^3 + 6\,(2\,cdx+bd)^m bc^2 m x^2 + 4\,(2\,cdx+bd)^m c^3 x^3 + 2\,(2\,cdx+bd)^m b^2 c m x + 4\,(2\,cdx+bd)^m ac^2 mx}{4\left(c^2 m^2 + 4\,c^2 m\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m*(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*(4*(2*c*d*x + b*d)^m*c^3*m*x^3 + 6*(2*c*d*x + b*d)^m*b*c^2*m*x^2 + 4*(2
*c*d*x + b*d)^m*c^3*x^3 + 2*(2*c*d*x + b*d)^m*b^2*c*m*x + 4*(2*c*d*x + b*d)
^m*a*c^2*m*x + 6*(2*c*d*x + b*d)^m*b*c^2*x^2 + 2*(2*c*d*x + b*d)^m*a*b*c*m
+ 12*(2*c*d*x + b*d)^m*a*c^2*x - (2*c*d*x + b*d)^m*b^3 + 6*(2*c*d*x + b*d)^
m*a*b*c)/(c^2*m^2 + 4*c^2*m + 3*c^2)
```

3.1426 $\int \frac{(bd+2cdx)^m}{a+bx+cx^2}\,dx$

Optimal. Leaf size=67

$$-\frac{2(d(b+2cx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{d(m+1)\left(b^2-4ac\right)}$$

```
[Out] (-2*(d*(b + 2*c*x))^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (b +
 2*c*x)^2/(b^2 - 4*a*c)])/((b^2 - 4*a*c)*d*(1 + m))
```

Rubi [A] time = 0.0437272, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {694, 364}

$$-\frac{2(d(b+2cx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{d(m+1)\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^m/(a + b*x + c*x^2),x]
```

```
[Out] (-2*(d*(b + 2*c*x))^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (b +
 2*c*x)^2/(b^2 - 4*a*c)])/((b^2 - 4*a*c)*d*(1 + m))
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(bd+2cdx)^m}{a+bx+cx^2}\,dx &= \frac{\text{Subst}\left(\int \frac{x^m}{a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}}\,dx,x,bd+2cdx\right)}{2cd}\\ &= -\frac{2(d(b+2cx))^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{\left(b^2-4ac\right)d(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0378771, size = 68, normalized size = 1.01

$$-\frac{2(b+2cx)(d(b+2cx))^m\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{(m+1)\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^m/(a + b*x + c*x^2),x]
```

```
[Out] (-2*(b + 2*c*x)*(d*(b + 2*c*x))^m*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2
, (b + 2*c*x)^2/(b^2 - 4*a*c)])/((b^2 - 4*a*c)*(1 + m))
```

Maple [F] time = 1.204, size = 0, normalized size = 0.

$$\int \frac{(2\,cdx+bd)^{m}}{c x^{2}+bx+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^m/(c*x^2+b*x+a),x)
```

```
[Out] int((2*c*d*x+b*d)^m/(c*x^2+b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx+bd)^{m}}{c x^{2}+bx+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^m/(c*x^2 + b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(2\,cdx+bd)^{m}}{c x^{2}+bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral((2*c*d*x + b*d)^m/(c*x^2 + b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\,(b+2cx))^{m}}{a+bx+cx^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**m/(c*x**2+b*x+a),x)
```

```
[Out] Integral((d*(b + 2*c*x))**m/(a + b*x + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx + bd)^m}{cx^2 + bx + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^m/(c*x^2 + b*x + a), x)
```

3.1427 $\int \frac{(bd+2cdx)^m}{\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=68

$$\frac{8c(d(b+2cx))^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{d(m+1)\left(b^2-4ac\right)^2}$$

```
[Out] (8*c*(d*(b + 2*c*x))^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, (b
+ 2*c*x)^2/(b^2 - 4*a*c)])/((b^2 - 4*a*c)^2*d*(1 + m))
```

Rubi [A] time = 0.0466202, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {694, 364}

$$\frac{8c(d(b+2cx))^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{d(m+1)\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^m/(a + b*x + c*x^2)^2,x]
```

```
[Out] (8*c*(d*(b + 2*c*x))^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, (b
+ 2*c*x)^2/(b^2 - 4*a*c)])/((b^2 - 4*a*c)^2*d*(1 + m))
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(bd+2cdx)^m}{\left(a+bx+cx^2\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{x^m}{\left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)^2}\,dx,x,bd+2cdx\right)}{2cd}\\ &= \frac{8c(d(b+2cx))^{1+m}\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{\left(b^2-4ac\right)^2 d(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0370594, size = 69, normalized size = 1.01

$$\frac{8c(b+2cx)(d(b+2cx))^m\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{(m+1)\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^m/(a + b*x + c*x^2)^2,x]
```

```
[Out] (8*c*(b + 2*c*x)*(d*(b + 2*c*x))^m*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/
2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/((b^2 - 4*a*c)^2*(1 + m))
```

Maple [F] time = 1.211, size = 0, normalized size = 0.

$$\int \frac{(2\,cdx+bd)^m}{\left(cx^2+bx+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^m/(c*x^2+b*x+a)^2,x)
```

```
[Out] int((2*c*d*x+b*d)^m/(c*x^2+b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx+bd)^m}{\left(cx^2+bx+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^m/(c*x^2 + b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(2\,cdx+bd)^m}{c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\,ac\right)x^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((2*c*d*x + b*d)^m/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x
^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**m/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx + bd)^m}{\left(cx^2 + bx + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^m/(c*x^2 + b*x + a)^2, x)
```

3.1428 $\int \frac{(bd+2cdx)^m}{\left(a+bx+cx^2\right)^3}\,dx$

Optimal. Leaf size=70

$$-\frac{32c^2(d(b+2cx))^{m+1}\,{}_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{d(m+1)\left(b^2-4ac\right)^3}$$

```
[Out] (-32*c^2*(d*(b + 2*c*x))^(1 + m)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2,
 (b + 2*c*x)^2/(b^2 - 4*a*c)])/((b^2 - 4*a*c)^3*d*(1 + m))
```

Rubi [A] time = 0.0516068, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {694, 364}

$$-\frac{32c^2(d(b+2cx))^{m+1}\,{}_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{d(m+1)\left(b^2-4ac\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^m/(a + b*x + c*x^2)^3,x]
```

```
[Out] (-32*c^2*(d*(b + 2*c*x))^(1 + m)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2,
 (b + 2*c*x)^2/(b^2 - 4*a*c)])/((b^2 - 4*a*c)^3*d*(1 + m))
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(bd+2cdx)^m}{\left(a+bx+cx^2\right)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{x^m}{\left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)^3}\,dx, x, bd+2cdx\right)}{2cd}\\
&= -\frac{32c^2(d(b+2cx))^{1+m}\,{}_2F_1\left(3,\frac{1+m}{2};\frac{3+m}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{\left(b^2-4ac\right)^3 d(1+m)}
\end{aligned}$$

Mathematica [A] time = 0.0440508, size = 71, normalized size = 1.01

$$-\frac{32c^2(b+2cx)(d(b+2cx))^m\,{}_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{(m+1)\left(b^2-4ac\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^m/(a + b*x + c*x^2)^3,x]
```

```
[Out] (-32*c^2*(b + 2*c*x)*(d*(b + 2*c*x))^m*Hypergeometric2F1[3, (1 + m)/2, (3 +
 m)/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/((b^2 - 4*a*c)^3*(1 + m))
```

Maple [F] time = 1.237, size = 0, normalized size = 0.

$$\int \frac{(2\,cdx+bd)^m}{\left(cx^2+bx+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^m/(c*x^2+b*x+a)^3,x)
```

```
[Out] int((2*c*d*x+b*d)^m/(c*x^2+b*x+a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx+bd)^m}{\left(cx^2+bx+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^m/(c*x^2 + b*x + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(2\,cdx+bd)^m}{c^3x^6+3\,bc^2x^5+3\left(b^2c+ac^2\right)x^4+3\,a^2bx+\left(b^3+6\,abc\right)x^3+a^3+3\left(ab^2+a^2c\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral((2*c*d*x + b*d)^m/(c^3*x^6 + 3*b*c^2*x^5 + 3*(b^2*c + a*c^2)*x^4 +
 3*a^2*b*x + (b^3 + 6*a*b*c)*x^3 + a^3 + 3*(a*b^2 + a^2*c)*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**m/(c*x**2+b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx + bd)^m}{\left(cx^2 + bx + a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^m/(c*x^2 + b*x + a)^3, x)
```

3.1429 $\int (bd+2cdx)^m \left(a+bx+cx^2\right)^{5/2} dx$

Optimal. Leaf size=82

$$-\frac{2\left(a+bx+cx^2\right)^{7/2}(bd+2cdx)^{m+1}{}_2F_1\left(1,\frac{m+8}{2};\frac{m+3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{d(m+1)\left(b^2-4ac\right)}$$

```
[Out] (-2*(b*d + 2*c*d*x)^(1 + m)*(a + b*x + c*x^2)^(7/2)*Hypergeometric2F1[1, (8
 + m)/2, (3 + m)/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/((b^2 - 4*a*c)*d*(1 + m))
```

Rubi [A] time = 0.119357, antiderivative size = 114, normalized size of antiderivative = 1.39, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {694, 365, 364}

$$\frac{\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}(d(b+2cx))^{m+1}{}_2F_1\left(-\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{32c^3d(m+1)\sqrt{1-\frac{(b+2cx)^2}{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^m*(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] ((b^2 - 4*a*c)^2*(d*(b + 2*c*x))^(1 + m)*Sqrt[a + b*x + c*x^2]*Hypergeometr
ic2F1[-5/2, (1 + m)/2, (3 + m)/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(32*c^3*d*(
1 + m)*Sqrt[1 - (b + 2*c*x)^2/(b^2 - 4*a*c)])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (bd+2cdx)^m \left(a+bx+cx^2\right)^{5/2} dx = \frac{\text{Subst}\left(\int x^m \left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)^{5/2} dx, x, bd+2cdx\right)}{2cd}$$

$$= \frac{\left(\left(a-\frac{b^2}{4c}\right)^2 \sqrt{a+bx+cx^2}\right) \text{Subst}\left(\int x^m \left(1+\frac{x^2}{4\left(a-\frac{b^2}{4c}\right)cd^2}\right)^{5/2} dx, x, bd+2cdx\right)}{cd\sqrt{4+\frac{(bd+2cdx)^2}{\left(a-\frac{b^2}{4c}\right)cd^2}}}$$

$$= \frac{\left(b^2-4ac\right)^2 (d(b+2cx))^{1+m}\sqrt{a+bx+cx^2}\, {}_2F_1\left(-\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{32c^3d(1+m)\sqrt{1-\frac{(b+2cx)^2}{b^2-4ac}}}$$

Mathematica [A] time = 0.0841635, size = 115, normalized size = 1.4

$$\frac{\left(b^2-4ac\right)^2 (b+2cx)\sqrt{a+x(b+cx)}(d(b+2cx))^m\, {}_2F_1\left(-\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{64c^3(m+1)\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^m*(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] ((b^2 - 4*a*c)^2*(b + 2*c*x)*(d*(b + 2*c*x))^m*Sqrt[a + x*(b + c*x)]*Hyperg
eometric2F1[-5/2, (1 + m)/2, (3 + m)/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(64*c
^3*(1 + m)*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)])
```

Maple [F] time = 1.167, size = 0, normalized size = 0.

$$\int (2\,cdx+bd)^m \left(cx^2+bx+a\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^m*(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] int((2*c*d*x+b*d)^m*(c*x^2+b*x+a)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{\frac{5}{2}} (2\,cdx+bd)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m*(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)*(2*c*d*x + b*d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(c^2x^4 + 2\,bcx^3 + 2\,abx + \left(b^2 + 2\,ac\right)x^2 + a^2\right)\sqrt{cx^2 + bx + a}(2\,cdx + bd)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m*(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*sqrt(c*x
^2 + b*x + a)*(2*c*d*x + b*d)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**m*(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx + a\right)^{\frac{5}{2}}(2\,cdx + bd)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m*(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)*(2*c*d*x + b*d)^m, x)
```

3.1430 $\int (bd+2cdx)^m \left(a+bx+cx^2\right)^{3/2} dx$

Optimal. Leaf size=98

$$-\frac{\left(4a-\frac{b^2}{c}+\frac{(b+2cx)^2}{c}\right)^{5/2}(bd+2cdx)^{m+1}{}_2F_1\left(1,\frac{m+6}{2};\frac{m+3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{16d(m+1)\left(b^2-4ac\right)}$$

```
[Out] -((b*d + 2*c*d*x)^(1 + m)*(4*a - b^2/c + (b + 2*c*x)^2/c)^(5/2)*Hypergeomet
ric2F1[1, (6 + m)/2, (3 + m)/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(16*(b^2 - 4*
a*c)*d*(1 + m))
```

Rubi [A] time = 0.11964, antiderivative size = 112, normalized size of antiderivative = 1.14, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {694, 365, 364}

$$-\frac{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}(d(b+2cx))^{m+1}{}_2F_1\left(-\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{8c^2d(m+1)\sqrt{1-\frac{(b+2cx)^2}{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^m*(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] -((b^2 - 4*a*c)*(d*(b + 2*c*x))^(1 + m)*Sqrt[a + b*x + c*x^2]*Hypergeometri
c2F1[-3/2, (1 + m)/2, (3 + m)/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(8*c^2*d*(1
+ m)*Sqrt[1 - (b + 2*c*x)^2/(b^2 - 4*a*c)])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (bd+2cdx)^m \left(a+bx+cx^2\right)^{3/2} dx = \frac{\text{Subst}\left(\int x^m \left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)^{3/2} dx, x, bd+2cdx\right)}{2cd}$$

$$= \frac{\left(\left(a-\frac{b^2}{4c}\right)\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int x^m \left(1+\frac{x^2}{4\left(a-\frac{b^2}{4c}\right)cd^2}\right)^{3/2} dx, x, bd+2cdx\right)}{cd\sqrt{4+\frac{(bd+2cdx)^2}{\left(a-\frac{b^2}{4c}\right)cd^2}}}$$

$$= -\frac{\left(b^2-4ac\right)(d(b+2cx))^{1+m}\sqrt{a+bx+cx^2}\ {}_2F_1\left(-\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{8c^2d(1+m)\sqrt{1-\frac{(b+2cx)^2}{b^2-4ac}}}$$

Mathematica [A] time = 0.0641194, size = 113, normalized size = 1.15

$$-\frac{\left(b^2-4ac\right)(b+2cx)\sqrt{a+x(b+cx)}(d(b+2cx))^m\ {}_2F_1\left(-\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{16c^2(m+1)\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^m*(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] -((b^2 - 4*a*c)*(b + 2*c*x)*(d*(b + 2*c*x))^m*Sqrt[a + x*(b + c*x)]*Hyperge
ometric2F1[-3/2, (1 + m)/2, (3 + m)/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(16*c^
2*(1 + m)*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)])
```

Maple [F] time = 1.196, size = 0, normalized size = 0.

$$\int (2\,cdx+bd)^m \left(cx^2+bx+a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^m*(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] int((2*c*d*x+b*d)^m*(c*x^2+b*x+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{\frac{3}{2}} (2\,cdx+bd)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m*(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)*(2*c*d*x + b*d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2 + bx + a\right)^{\frac{3}{2}}(2\,cdx + bd)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^(3/2)*(2*c*d*x + b*d)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d\,(b + 2cx))^m \left(a + bx + cx^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**m*(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d*(b + 2*c*x))**m*(a + b*x + c*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx + a\right)^{\frac{3}{2}}(2\,cdx + bd)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m*(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)*(2*c*d*x + b*d)^m, x)
```

3.1431 $\int (bd+2cdx)^m \sqrt{a+bx+cx^2}\,dx$

Optimal. Leaf size=98

$$-\frac{\left(4a-\frac{b^2}{c}+\frac{(b+2cx)^2}{c}\right)^{3/2}(bd+2cdx)^{m+1}\,{}_2F_1\left(1,\frac{m+4}{2};\frac{m+3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{4d(m+1)\left(b^2-4ac\right)}$$

```
[Out] -((b*d + 2*c*d*x)^(1 + m)*(4*a - b^2/c + (b + 2*c*x)^2/c)^(3/2)*Hypergeomet
ric2F1[1, (4 + m)/2, (3 + m)/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(4*(b^2 - 4*a
*c)*d*(1 + m))
```

Rubi [A] time = 0.108348, antiderivative size = 104, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {694, 365, 364}

$$\frac{\sqrt{a+bx+cx^2}(d(b+2cx))^{m+1}\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{2cd(m+1)\sqrt{1-\frac{(b+2cx)^2}{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^m*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((d*(b + 2*c*x))^(1 + m)*Sqrt[a + b*x + c*x^2]*Hypergeometric2F1[-1/2, (1 +
 m)/2, (3 + m)/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(2*c*d*(1 + m)*Sqrt[1 - (b
+ 2*c*x)^2/(b^2 - 4*a*c)])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (bd+2cdx)^m\sqrt{a+bx+cx^2}\,dx = \frac{\text{Subst}\left(\int x^m\sqrt{a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}}\,dx,x,bd+2cdx\right)}{2cd}$$
$$= \frac{\sqrt{a+bx+cx^2}\,\text{Subst}\left(\int x^m\sqrt{1+\frac{x^2}{4\left(a-\frac{b^2}{4c}\right)cd^2}}\,dx,x,bd+2cdx\right)}{cd\sqrt{4+\frac{(bd+2cdx)^2}{\left(a-\frac{b^2}{4c}\right)cd^2}}}$$
$$= \frac{(d(b+2cx))^{1+m}\sqrt{a+bx+cx^2}\,{}_2F_1\left(-\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{2cd(1+m)\sqrt{1-\frac{(b+2cx)^2}{b^2-4ac}}}$$

Mathematica [A] time = 0.054965, size = 105, normalized size = 1.07

$$\frac{(b+2cx)\sqrt{a+x(b+cx)}(d(b+2cx))^m\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{4c(m+1)\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^m*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((b + 2*c*x)*(d*(b + 2*c*x))^m*Sqrt[a + x*(b + c*x)]*Hypergeometric2F1[-1/2
, (1 + m)/2, (3 + m)/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(4*c*(1 + m)*Sqrt[(c*
(a + x*(b + c*x)))/(-b^2 + 4*a*c)])
```

Maple [F] time = 1.16, size = 0, normalized size = 0.

$$\int (2\,cdx+bd)^m\sqrt{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^m*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] int((2*c*d*x+b*d)^m*(c*x^2+b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2+bx+a}(2\,cdx+bd)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(2*c*d*x + b*d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^2 + bx + a}(2\,cdx + bd)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*(2*c*d*x + b*d)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d(b + 2cx))^m \sqrt{a + bx + cx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**m*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d*(b + 2*c*x))**m*sqrt(a + b*x + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + bx + a}(2\,cdx + bd)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(2*c*d*x + b*d)^m, x)
```

3.1432 $\int \frac{(bd+2cdx)^m}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=96

$$-\frac{\sqrt{4a-\frac{b^2}{c}+\frac{(b+2cx)^2}{c}}\,(bd+2cdx)^{m+1}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{d(m+1)\left(b^2-4ac\right)}$$

```
[Out] -(((b*d + 2*c*d*x)^(1 + m)*Sqrt[4*a - b^2/c + (b + 2*c*x)^2/c]*Hypergeometr
ic2F1[1, (2 + m)/2, (3 + m)/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/((b^2 - 4*a*c)
*d*(1 + m)))
```

Rubi [A] time = 0.106837, antiderivative size = 104, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {694, 365, 364}

$$\frac{\sqrt{1-\frac{(b+2cx)^2}{b^2-4ac}}\,(d(b+2cx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{2cd(m+1)\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^m/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((d*(b + 2*c*x))^(1 + m)*Sqrt[1 - (b + 2*c*x)^2/(b^2 - 4*a*c)]*Hypergeometr
ic2F1[1/2, (1 + m)/2, (3 + m)/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(2*c*d*(1 +
m)*Sqrt[a + b*x + c*x^2])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(bd+2cdx)^m}{\sqrt{a+bx+cx^2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^m}{\sqrt{a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}}}\,dx, x, bd+2cdx\right)}{2cd}$$

$$= \frac{\sqrt{4+\frac{(bd+2cdx)^2}{\left(a-\frac{b^2}{4c}\right)cd^2}}\,\text{Subst}\left(\int \frac{x^m}{\sqrt{1+\frac{x^2}{4\left(a-\frac{b^2}{4c}\right)cd^2}}}\,dx, x, bd+2cdx\right)}{4cd\sqrt{a+bx+cx^2}}$$

$$= \frac{(d(b+2cx))^{1+m}\sqrt{1-\frac{(b+2cx)^2}{b^2-4ac}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{2cd(1+m)\sqrt{a+bx+cx^2}}$$

Mathematica [A] time = 0.046378, size = 102, normalized size = 1.06

$$\frac{(b+2cx)\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}(d(b+2cx))^m\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{c(m+1)\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^m/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((b + 2*c*x)*(d*(b + 2*c*x))^m*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*H
ypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(
c*(1 + m)*Sqrt[a + x*(b + c*x)])
```

Maple [F] time = 1.159, size = 0, normalized size = 0.

$$\int (2\,cdx+bd)^m \frac{1}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^m/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] int((2*c*d*x+b*d)^m/(c*x^2+b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx+bd)^m}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^m/sqrt(c*x^2 + b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(2\,cdx+bd)^m}{\sqrt{cx^2+bx+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((2*c*d*x + b*d)^m/sqrt(c*x^2 + b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(d\,(b+2cx))^m}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**m/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d*(b + 2*c*x))**m/sqrt(a + b*x + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(2\,cdx+bd)^m}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^m/sqrt(c*x^2 + b*x + a), x)
```

3.1433 $\int \frac{(bd+2cdx)^m}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{4(bd+2cdx)^{m+1}\,{}_2F_1\left(1,\frac{m}{2};\frac{m+3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{d(m+1)\left(b^2-4ac\right)\sqrt{4a-\frac{b^2}{c}+\frac{(b+2cx)^2}{c}}}$$

```
[Out] (-4*(b*d + 2*c*d*x)^(1 + m)*Hypergeometric2F1[1, m/2, (3 + m)/2, (b + 2*c*x
)^2/(b^2 - 4*a*c)])/((b^2 - 4*a*c)*d*(1 + m)*Sqrt[4*a - b^2/c + (b + 2*c*x)
^2/c])
```

Rubi [A] time = 0.114332, antiderivative size = 109, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {694, 365, 364}

$$-\frac{2\sqrt{1-\frac{(b+2cx)^2}{b^2-4ac}}(d(b+2cx))^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{d(m+1)\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^m/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*(d*(b + 2*c*x))^(1 + m)*Sqrt[1 - (b + 2*c*x)^2/(b^2 - 4*a*c)]*Hypergeom
etric2F1[3/2, (1 + m)/2, (3 + m)/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/((b^2 - 4
*a*c)*d*(1 + m)*Sqrt[a + b*x + c*x^2])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(bd+2cdx)^m}{(a+bx+cx^2)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^m}{\left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)^{3/2}}\,dx, x, bd+2cdx\right)}{2cd}$$

$$= \frac{\sqrt{4+\frac{(bd+2cdx)^2}{\left(a-\frac{b^2}{4c}\right)cd^2}}\,\text{Subst}\left(\int \frac{x^m}{\left(1+\frac{x^2}{4\left(a-\frac{b^2}{4c}\right)cd^2}\right)^{3/2}}\,dx, x, bd+2cdx\right)}{4\left(a-\frac{b^2}{4c}\right)cd\sqrt{a+bx+cx^2}}$$

$$= -\frac{2(d(b+2cx))^{1+m}\sqrt{1-\frac{(b+2cx)^2}{b^2-4ac}}\,{}_2F_1\left(\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{\left(b^2-4ac\right)d(1+m)\sqrt{a+bx+cx^2}}$$

Mathematica [A] time = 0.0526937, size = 110, normalized size = 1.17

$$-\frac{4(b+2cx)\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}(d(b+2cx))^m\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{(m+1)\left(b^2-4ac\right)\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^m/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-4*(b + 2*c*x)*(d*(b + 2*c*x))^m*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)
]*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, (b + 2*c*x)^2/(b^2 - 4*a*c)]
)/((b^2 - 4*a*c)*(1 + m)*Sqrt[a + x*(b + c*x)])
```

Maple [F] time = 1.164, size = 0, normalized size = 0.

$$\int (2\,cdx+bd)^m \left(cx^2+bx+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^m/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] int((2*c*d*x+b*d)^m/(c*x^2+b*x+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx+bd)^m}{\left(cx^2+bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^m/(c*x^2 + b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+bx+a}(2\,cdx+bd)^m}{c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\,ac\right)x^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*(2*c*d*x + b*d)^m/(c^2*x^4 + 2*b*c*x^3 + 2*a
*b*x + (b^2 + 2*a*c)*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\,(b+2cx))^m}{\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**m/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d*(b + 2*c*x))**m/(a + b*x + c*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx+bd)^m}{\left(cx^2+bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^m/(c*x^2 + b*x + a)^(3/2), x)
```

3.1434 $\int \frac{(bd+2cdx)^m}{\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=96

$$-\frac{16(bd+2cdx)^{m+1}\,{}_2F_1\left(1,\frac{m-2}{2};\frac{m+3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{d(m+1)\left(b^2-4ac\right)\left(4a-\frac{b^2}{c}+\frac{(b+2cx)^2}{c}\right)^{3/2}}$$

```
[Out] (-16*(b*d + 2*c*d*x)^(1 + m)*Hypergeometric2F1[1, (-2 + m)/2, (3 + m)/2, (b
 + 2*c*x)^2/(b^2 - 4*a*c)])/((b^2 - 4*a*c)*d*(1 + m)*(4*a - b^2/c + (b + 2*
c*x)^2/c)^(3/2))
```

Rubi [A] time = 0.116954, antiderivative size = 110, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {694, 365, 364}

$$\frac{8c\sqrt{1-\frac{(b+2cx)^2}{b^2-4ac}}(d(b+2cx))^{m+1}\,{}_2F_1\left(\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{d(m+1)\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^m/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (8*c*(d*(b + 2*c*x))^(1 + m)*Sqrt[1 - (b + 2*c*x)^2/(b^2 - 4*a*c)]*Hypergeo
metric2F1[5/2, (1 + m)/2, (3 + m)/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/((b^2 -
4*a*c)^2*d*(1 + m)*Sqrt[a + b*x + c*x^2])
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(bd+2cdx)^m}{\left(a+bx+cx^2\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^m}{\left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)^{5/2}}\,dx, x, bd+2cdx\right)}{2cd}$$

$$= \frac{\sqrt{4+\frac{(bd+2cdx)^2}{\left(a-\frac{b^2}{4c}\right)cd^2}}\,\text{Subst}\left(\int \frac{x^m}{\left(1+\frac{x^2}{4\left(a-\frac{b^2}{4c}\right)cd^2}\right)^{5/2}}\,dx, x, bd+2cdx\right)}{4\left(a-\frac{b^2}{4c}\right)^2 cd\sqrt{a+bx+cx^2}}$$

$$= \frac{8c(d(b+2cx))^{1+m}\sqrt{1-\frac{(b+2cx)^2}{b^2-4ac}}\,{}_2F_1\left(\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{\left(b^2-4ac\right)^2 d(1+m)\sqrt{a+bx+cx^2}}$$

Mathematica [A] time = 0.0493093, size = 111, normalized size = 1.16

$$\frac{16c(b+2cx)\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}(d(b+2cx))^m\,{}_2F_1\left(\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{(m+1)\left(b^2-4ac\right)^2\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^m/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (16*c*(b + 2*c*x)*(d*(b + 2*c*x))^m*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*
c)]*Hypergeometric2F1[5/2, (1 + m)/2, (3 + m)/2, (b + 2*c*x)^2/(b^2 - 4*a*c
)])/((b^2 - 4*a*c)^2*(1 + m)*Sqrt[a + x*(b + c*x)])
```

Maple [F] time = 1.163, size = 0, normalized size = 0.

$$\int (2\,cdx+bd)^m \left(cx^2+bx+a\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^m/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] int((2*c*d*x+b*d)^m/(c*x^2+b*x+a)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx+bd)^m}{\left(cx^2+bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^m/(c*x^2 + b*x + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2 + bx + a}(2\,cdx + bd)^m}{c^3x^6 + 3\,bc^2x^5 + 3\left(b^2c + ac^2\right)x^4 + 3\,a^2bx + \left(b^3 + 6\,abc\right)x^3 + a^3 + 3\left(ab^2 + a^2c\right)x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*(2*c*d*x + b*d)^m/(c^3*x^6 + 3*b*c^2*x^5 + 3
*(b^2*c + a*c^2)*x^4 + 3*a^2*b*x + (b^3 + 6*a*b*c)*x^3 + a^3 + 3*(a*b^2 + a
^2*c)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\,(b + 2cx))^m}{\left(a + bx + cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**m/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Integral((d*(b + 2*c*x))**m/(a + b*x + c*x**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cdx + bd)^m}{\left(cx^2 + bx + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^m/(c*x^2 + b*x + a)^(5/2), x)
```

3.1435 $\int (bd+2cdx)^m \left(a+bx+cx^2\right)^p \, dx$

Optimal. Leaf size=107

$$-\frac{2\left(\frac{1}{4}\left(4a-\frac{b^2}{c}\right)+\frac{(b+2cx)^2}{4c}\right)^{p+1}(bd+2cdx)^{m+1}\,{}_2F_1\left(1,\frac{1}{2}(m+2p+3);\frac{m+3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{d(m+1)\left(b^2-4ac\right)}$$

```
[Out] (-2*(b*d + 2*c*d*x)^(1 + m)*((4*a - b^2/c)/4 + (b + 2*c*x)^2/(4*c))^(1 + p)
*Hypergeometric2F1[1, (3 + m + 2*p)/2, (3 + m)/2, (b + 2*c*x)^2/(b^2 - 4*a*
c)])/((b^2 - 4*a*c)*d*(1 + m))
```

Rubi [A] time = 0.074085, antiderivative size = 102, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {694, 365, 364}

$$\frac{\left(a+bx+cx^2\right)^p\left(1-\frac{(b+2cx)^2}{b^2-4ac}\right)^{-p}(d(b+2cx))^{m+1}\,{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{2cd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^m*(a + b*x + c*x^2)^p,x]
```

```
[Out] ((d*(b + 2*c*x))^(1 + m)*(a + b*x + c*x^2)^p*Hypergeometric2F1[(1 + m)/2, -
p, (3 + m)/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(2*c*d*(1 + m)*(1 - (b + 2*c*x)
^2/(b^2 - 4*a*c))^p)
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (bd+2cdx)^m \left(a+bx+cx^2\right)^p dx = \frac{\text{Subst}\left(\int x^m \left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)^p dx, x, bd+2cdx\right)}{2cd}$$
$$= \frac{\left(2^{-1+2p}\left(a+bx+cx^2\right)^p \left(4+\frac{(bd+2cdx)^2}{\left(a-\frac{b^2}{4c}\right)cd^2}\right)^{-p}\right) \text{Subst}\left(\int x^m \left(1+\frac{x^2}{4\left(a-\frac{b^2}{4c}\right)cd^2}\right)^p dx, x, bd +\right.}{cd}$$
$$= \frac{2^{-1+2p}(d(b+2cx))^{1+m}\left(a+bx+cx^2\right)^p \left(4-\frac{4(b+2cx)^2}{b^2-4ac}\right)^{-p} {}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{cd(1+m)}$$

Mathematica [A] time = 0.0606184, size = 107, normalized size = 1.

$$\frac{2^{-2p-1}(b+2cx)(a+x(b+cx))^p \left(\frac{c(a+x(b+cx))}{4ac-b^2}\right)^{-p} (d(b+2cx))^m {}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{c(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^m*(a + b*x + c*x^2)^p,x]
```

```
[Out] (2^(-1 - 2*p)*(b + 2*c*x)*(d*(b + 2*c*x))^m*(a + x*(b + c*x))^p*Hypergeomet
ric2F1[(1 + m)/2, -p, (3 + m)/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(c*(1 + m)*(
(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c))^p)
```

Maple [F] time = 1.272, size = 0, normalized size = 0.

$$\int (2\,cdx+bd)^m \left(cx^2+bx+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^m*(c*x^2+b*x+a)^p,x)
```

```
[Out] int((2*c*d*x+b*d)^m*(c*x^2+b*x+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (2\,cdx+bd)^m \left(cx^2+bx+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^m*(c*x^2 + b*x + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((2\,cdx+bd)^m \left(cx^2+bx+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

```
[Out] integral((2*c*d*x + b*d)^m*(c*x^2 + b*x + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**m*(c*x**2+b*x+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (2\,cdx + bd)^m \left(cx^2 + bx + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^m*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^m*(c*x^2 + b*x + a)^p, x)
```

3.1436 $\int (bd+2cdx)^5 \left(a+bx+cx^2\right)^p \, dx$

Optimal. Leaf size=121

$$\frac{2d^5\left(b^2-4ac\right)(b+2cx)^2\left(a+bx+cx^2\right)^{p+1}}{(p+2)(p+3)}+\frac{2d^5\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{p+1}}{(p+1)(p+2)(p+3)}+\frac{d^5(b+2cx)^4\left(a+bx+cx^2\right)^{p+1}}{p+3}$$

```
[Out] (2*(b^2 - 4*a*c)^2*d^5*(a + b*x + c*x^2)^(1 + p))/((1 + p)*(2 + p)*(3 + p))
 + (2*(b^2 - 4*a*c)*d^5*(b + 2*c*x)^2*(a + b*x + c*x^2)^(1 + p))/((2 + p)*(
3 + p)) + (d^5*(b + 2*c*x)^4*(a + b*x + c*x^2)^(1 + p))/(3 + p)
```

Rubi [A] time = 0.0730149, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {692, 629}

$$\frac{2d^5\left(b^2-4ac\right)(b+2cx)^2\left(a+bx+cx^2\right)^{p+1}}{(p+2)(p+3)}+\frac{2d^5\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{p+1}}{(p+1)(p+2)(p+3)}+\frac{d^5(b+2cx)^4\left(a+bx+cx^2\right)^{p+1}}{p+3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^5*(a + b*x + c*x^2)^p,x]
```

```
[Out] (2*(b^2 - 4*a*c)^2*d^5*(a + b*x + c*x^2)^(1 + p))/((1 + p)*(2 + p)*(3 + p))
 + (2*(b^2 - 4*a*c)*d^5*(b + 2*c*x)^2*(a + b*x + c*x^2)^(1 + p))/((2 + p)*(
3 + p)) + (d^5*(b + 2*c*x)^4*(a + b*x + c*x^2)^(1 + p))/(3 + p)
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (bd+2cdx)^5\left(a+bx+cx^2\right)^p\,dx=\frac{d^5(b+2cx)^4\left(a+bx+cx^2\right)^{1+p}}{3+p}+\frac{\left(2\left(b^2-4ac\right)d^2\right)\int (bd+2cdx)^3\left(a+bx+cx^2\right)^p\,dx}{3+p}$$

$$=\frac{2\left(b^2-4ac\right)d^5(b+2cx)^2\left(a+bx+cx^2\right)^{1+p}}{(2+p)(3+p)}+\frac{d^5(b+2cx)^4\left(a+bx+cx^2\right)^{1+p}}{3+p}+$$

$$=\frac{2\left(b^2-4ac\right)^2d^5\left(a+bx+cx^2\right)^{1+p}}{(1+p)(2+p)(3+p)}+\frac{2\left(b^2-4ac\right)d^5(b+2cx)^2\left(a+bx+cx^2\right)^{1+p}}{(2+p)(3+p)}+$$

Mathematica [A] time = 0.100672, size = 145, normalized size = 1.2

$$\frac{d^5(a+x(b+cx))^{p+1}\left(16c^2\left(2a^2-2ac(p+1)x^2+c^2\left(p^2+3p+2\right)x^4\right)-8b^2c\left(a(p+3)-c\left(3p^2+10p+7\right)x^2\right)-32bc^2(\right.}{(p+1)(p+2)(p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^5*(a + b*x + c*x^2)^p,x]
```

```
[Out] (d^5*(a + x*(b + c*x))^(1 + p)*(b^4*(6 + 5*p + p^2) + 8*b^3*c*(3 + 4*p + p^
2)*x - 32*b*c^2*(1 + p)*x*(a - c*(2 + p)*x^2) - 8*b^2*c*(a*(3 + p) - c*(7 +
 10*p + 3*p^2)*x^2) + 16*c^2*(2*a^2 - 2*a*c*(1 + p)*x^2 + c^2*(2 + 3*p + p^
2)*x^4)))/((1 + p)*(2 + p)*(3 + p))
```

Maple [A] time = 0.051, size = 233, normalized size = 1.9

$$\frac{\left(cx^2+bx+a\right)^{1+p}d^5\left(16c^4p^2x^4+32bc^3p^2x^3+48c^4px^4+24b^2c^2p^2x^2+96bc^3px^3+32c^4x^4-32ac^3px^2+8b^3cp^2x+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^5*(c*x^2+b*x+a)^p,x)
```

```
[Out] (c*x^2+b*x+a)^(1+p)*d^5*(16*c^4*p^2*x^4+32*b*c^3*p^2*x^3+48*c^4*p*x^4+24*b^
2*c^2*p^2*x^2+96*b*c^3*p*x^3+32*c^4*x^4-32*a*c^3*p*x^2+8*b^3*c*p^2*x+80*b^2
*c^2*p*x^2+64*b*c^3*x^3-32*a*b*c^2*p*x-32*a*c^3*x^2+b^4*p^2+32*b^3*c*p*x+56
*b^2*c^2*x^2-8*a*b^2*c*p-32*a*b*c^2*x+5*b^4*p+24*b^3*c*x+32*a^2*c^2-24*a*b^
2*c+6*b^4)/(p^3+6*p^2+11*p+6)
```

Maxima [B] time = 1.28906, size = 398, normalized size = 3.29

$$\frac{\left(16\left(p^2+3p+2\right)c^5d^5x^6+48\left(p^2+3p+2\right)bc^4d^5x^5+\left(p^2+5p+6\right)ab^4d^5-8a^2b^2cd^5\left(p+3\right)+32a^3c^2d^5+8\left(\left(7p^2+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^5*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] (16*(p^2 + 3*p + 2)*c^5*d^5*x^6 + 48*(p^2 + 3*p + 2)*b*c^4*d^5*x^5 + (p^2 +
 5*p + 6)*a*b^4*d^5 - 8*a^2*b^2*c*d^5*(p + 3) + 32*a^3*c^2*d^5 + 8*((7*p^2
+ 22*p + 15)*b^2*c^3*d^5 + 2*(p^2 + p)*a*c^4*d^5)*x^4 + 16*((2*p^2 + 7*p +
5)*b^3*c^2*d^5 + 2*(p^2 + p)*a*b*c^3*d^5)*x^3 + ((9*p^2 + 37*p + 30)*b^4*c*
d^5 + 8*(3*p^2 + 5*p)*a*b^2*c^2*d^5 - 32*a^2*c^3*d^5*p)*x^2 + ((p^2 + 5*p +
 6)*b^5*d^5 + 8*(p^2 + 3*p)*a*b^3*c*d^5 - 32*a^2*b*c^2*d^5*p)*x)*(c*x^2 + b
*x + a)^p/(p^3 + 6*p^2 + 11*p + 6)
```

Fricas [B] time = 2.22443, size = 828, normalized size = 6.84

$$\frac{\left(ab^4d^5p^2+\left(5ab^4-8a^2b^2c\right)d^5p+16\left(c^5d^5p^2+3c^5d^5p+2c^5d^5\right)x^6+2\left(3ab^4-12a^2b^2c+16a^3c^2\right)d^5+48\left(bc^4d^5p^2+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^5*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

```
[Out] (a*b^4*d^5*p^2 + (5*a*b^4 - 8*a^2*b^2*c)*d^5*p + 16*(c^5*d^5*p^2 + 3*c^5*d^
5*p + 2*c^5*d^5)*x^6 + 2*(3*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2)*d^5 + 48*(b*
c^4*d^5*p^2 + 3*b*c^4*d^5*p + 2*b*c^4*d^5)*x^5 + 8*(15*b^2*c^3*d^5 + (7*b^2
*c^3 + 2*a*c^4)*d^5*p^2 + 2*(11*b^2*c^3 + a*c^4)*d^5*p)*x^4 + 16*(5*b^3*c^2
*d^5 + 2*(b^3*c^2 + a*b*c^3)*d^5*p^2 + (7*b^3*c^2 + 2*a*b*c^3)*d^5*p)*x^3 +
 (30*b^4*c*d^5 + 3*(3*b^4*c + 8*a*b^2*c^2)*d^5*p^2 + (37*b^4*c + 40*a*b^2*c
^2 - 32*a^2*c^3)*d^5*p)*x^2 + (6*b^5*d^5 + (b^5 + 8*a*b^3*c)*d^5*p^2 + (5*b
^5 + 24*a*b^3*c - 32*a^2*b*c^2)*d^5*p)*x)*(c*x^2 + b*x + a)^p/(p^3 + 6*p^2
+ 11*p + 6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**5*(c*x**2+b*x+a)**p,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21724, size = 1184, normalized size = 9.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^5*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

```
[Out] (16*(c*x^2 + b*x + a)^p*c^5*d^5*p^2*x^6 + 48*(c*x^2 + b*x + a)^p*b*c^4*d^5*
p^2*x^5 + 48*(c*x^2 + b*x + a)^p*c^5*d^5*p*x^6 + 56*(c*x^2 + b*x + a)^p*b^2
*c^3*d^5*p^2*x^4 + 16*(c*x^2 + b*x + a)^p*a*c^4*d^5*p^2*x^4 + 144*(c*x^2 +
b*x + a)^p*b*c^4*d^5*p*x^5 + 32*(c*x^2 + b*x + a)^p*c^5*d^5*x^6 + 32*(c*x^2
 + b*x + a)^p*b^3*c^2*d^5*p^2*x^3 + 32*(c*x^2 + b*x + a)^p*a*b*c^3*d^5*p^2*
x^3 + 176*(c*x^2 + b*x + a)^p*b^2*c^3*d^5*p*x^4 + 16*(c*x^2 + b*x + a)^p*a*
c^4*d^5*p*x^4 + 96*(c*x^2 + b*x + a)^p*b*c^4*d^5*x^5 + 9*(c*x^2 + b*x + a)^
p*b^4*c*d^5*p^2*x^2 + 24*(c*x^2 + b*x + a)^p*a*b^2*c^2*d^5*p^2*x^2 + 112*(c
*x^2 + b*x + a)^p*b^3*c^2*d^5*p*x^3 + 32*(c*x^2 + b*x + a)^p*a*b*c^3*d^5*p*
x^3 + 120*(c*x^2 + b*x + a)^p*b^2*c^3*d^5*x^4 + (c*x^2 + b*x + a)^p*b^5*d^5
*p^2*x + 8*(c*x^2 + b*x + a)^p*a*b^3*c*d^5*p^2*x + 37*(c*x^2 + b*x + a)^p*b
^4*c*d^5*p*x^2 + 40*(c*x^2 + b*x + a)^p*a*b^2*c^2*d^5*p*x^2 - 32*(c*x^2 + b
*x + a)^p*a^2*c^3*d^5*p*x^2 + 80*(c*x^2 + b*x + a)^p*b^3*c^2*d^5*x^3 + (c*x
^2 + b*x + a)^p*a*b^4*d^5*p^2 + 5*(c*x^2 + b*x + a)^p*b^5*d^5*p*x + 24*(c*x
^2 + b*x + a)^p*a*b^3*c*d^5*p*x - 32*(c*x^2 + b*x + a)^p*a^2*b*c^2*d^5*p*x
+ 30*(c*x^2 + b*x + a)^p*b^4*c*d^5*x^2 + 5*(c*x^2 + b*x + a)^p*a*b^4*d^5*p
- 8*(c*x^2 + b*x + a)^p*a^2*b^2*c*d^5*p + 6*(c*x^2 + b*x + a)^p*b^5*d^5*x +
 6*(c*x^2 + b*x + a)^p*a*b^4*d^5 - 24*(c*x^2 + b*x + a)^p*a^2*b^2*c*d^5 + 3
2*(c*x^2 + b*x + a)^p*a^3*c^2*d^5)/(p^3 + 6*p^2 + 11*p + 6)
```

3.1437 $\int (bd+2cdx)^4 \left(a+bx+cx^2\right)^p\,dx$

Optimal. Leaf size=90

$$-\frac{2d^4(b+2cx)^5\left(\frac{1}{4}\left(4a-\frac{b^2}{c}\right)+\frac{(b+2cx)^2}{4c}\right)^{p+1}{}_2F_1\left(1,p+\frac{7}{2};\frac{7}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{5\left(b^2-4ac\right)}$$

```
[Out] (-2*d^4*(b + 2*c*x)^5*((4*a - b^2/c)/4 + (b + 2*c*x)^2/(4*c))^(1 + p)*Hyper
geometric2F1[1, 7/2 + p, 7/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(5*(b^2 - 4*a*c
))
```

Rubi [A] time = 0.0695637, antiderivative size = 85, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {694, 365, 364}

$$\frac{d^4(b+2cx)^5\left(a+bx+cx^2\right)^p\left(1-\frac{(b+2cx)^2}{b^2-4ac}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{10c}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^4*(a + b*x + c*x^2)^p,x]
```

```
[Out] (d^4*(b + 2*c*x)^5*(a + b*x + c*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, (b +
 2*c*x)^2/(b^2 - 4*a*c)])/(10*c*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^p)
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (bd+2cdx)^4\left(a+bx+cx^2\right)^p\,dx = \frac{\text{Subst}\left(\int x^4\left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)^p dx, x, bd+2cdx\right)}{2cd}$$

$$= \frac{\left(2^{-1+2p}\left(a+bx+cx^2\right)^p\left(4+\frac{(bd+2cdx)^2}{\left(a-\frac{b^2}{4c}\right)cd^2}\right)^{-p}\right)\text{Subst}\left(\int x^4\left(1+\frac{x^2}{4\left(a-\frac{b^2}{4c}\right)cd^2}\right)^p dx, x, bd+\right.}{cd}$$

$$= \frac{2^{-1+2p}d^4(b+2cx)^5\left(a+bx+cx^2\right)^p\left(4-\frac{4(b+2cx)^2}{b^2-4ac}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{5c}$$

Mathematica [A] time = 0.0396398, size = 92, normalized size = 1.02

$$\frac{d^4 2^{-2p-1}(b+2cx)^5(a+x(b+cx))^p\left(\frac{c(a+x(b+cx))}{4ac-b^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{5c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^4*(a + b*x + c*x^2)^p,x]
```

```
[Out] (2^(-1 - 2*p)*d^4*(b + 2*c*x)^5*(a + x*(b + c*x))^p*Hypergeometric2F1[5/2,
-p, 7/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(5*c*((c*(a + x*(b + c*x)))/(-b^2 +
4*a*c))^p)
```

Maple [F] time = 1.141, size = 0, normalized size = 0.

$$\int (2\,cdx+bd)^4\left(cx^2+bx+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^4*(c*x^2+b*x+a)^p,x)
```

```
[Out] int((2*c*d*x+b*d)^4*(c*x^2+b*x+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (2\,cdx+bd)^4\left(cx^2+bx+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^4*(c*x^2 + b*x + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(16\,c^4d^4x^4+32\,bc^3d^4x^3+24\,b^2c^2d^4x^2+8\,b^3cd^4x+b^4d^4\right)\left(cx^2+bx+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

```
[Out] integral((16*c^4*d^4*x^4 + 32*b*c^3*d^4*x^3 + 24*b^2*c^2*d^4*x^2 + 8*b^3*c*
d^4*x + b^4*d^4)*(c*x^2 + b*x + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**4*(c*x**2+b*x+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (2\,cdx + bd)^4 \left(cx^2 + bx + a\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^4*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^4*(c*x^2 + b*x + a)^p, x)
```

3.1438 $\int (bd+2cdx)^3 \left(a+bx+cx^2\right)^p \, dx$

Optimal. Leaf size=68

$$\frac{d^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{p+1}}{(p+1)(p+2)}+\frac{d^3(b+2cx)^2\left(a+bx+cx^2\right)^{p+1}}{p+2}$$

```
[Out] ((b^2 - 4*a*c)*d^3*(a + b*x + c*x^2)^(1 + p))/((1 + p)*(2 + p)) + (d^3*(b +
 2*c*x)^2*(a + b*x + c*x^2)^(1 + p))/(2 + p)
```

Rubi [A] time = 0.0266789, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {692, 629}

$$\frac{d^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{p+1}}{(p+1)(p+2)}+\frac{d^3(b+2cx)^2\left(a+bx+cx^2\right)^{p+1}}{p+2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^3*(a + b*x + c*x^2)^p,x]
```

```
[Out] ((b^2 - 4*a*c)*d^3*(a + b*x + c*x^2)^(1 + p))/((1 + p)*(2 + p)) + (d^3*(b +
 2*c*x)^2*(a + b*x + c*x^2)^(1 + p))/(2 + p)
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (bd+2cdx)^3\left(a+bx+cx^2\right)^p\,dx &= \frac{d^3(b+2cx)^2\left(a+bx+cx^2\right)^{1+p}}{2+p}+\frac{\left(\left(b^2-4ac\right)d^2\right)\int (bd+2cdx)\left(a+bx+cx^2\right)^p\,dx}{2+p}\\ &= \frac{\left(b^2-4ac\right)d^3\left(a+bx+cx^2\right)^{1+p}}{(1+p)(2+p)}+\frac{d^3(b+2cx)^2\left(a+bx+cx^2\right)^{1+p}}{2+p}\end{aligned}$$

Mathematica [A] time = 0.0498266, size = 58, normalized size = 0.85

$$\frac{d^3(a+x(b+cx))^{p+1}\left(4c\left(c(p+1)x^2-a\right)+b^2(p+2)+4bc(p+1)x\right)}{(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^3*(a + b*x + c*x^2)^p,x]
```

```
[Out] (d^3*(a + x*(b + c*x))^(1 + p)*(b^2*(2 + p) + 4*b*c*(1 + p)*x + 4*c*(-a + c
*(1 + p)*x^2)))/((1 + p)*(2 + p))
```

Maple [A] time = 0.047, size = 74, normalized size = 1.1

$$-\frac{\left(cx^2+bx+a\right)^{1+p}\left(-4c^2px^2-4bcpx-4c^2x^2-b^2p-4bcx+4ac-2b^2\right)d^3}{p^2+3p+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^3*(c*x^2+b*x+a)^p,x)
```

```
[Out] -(c*x^2+b*x+a)^(1+p)*(-4*c^2*p*x^2-4*b*c*p*x-4*c^2*x^2-b^2*p-4*b*c*x+4*a*c-
2*b^2)*d^3/(p^2+3*p+2)
```

Maxima [A] time = 1.21346, size = 166, normalized size = 2.44

$$\frac{\left(4c^3d^3(p+1)x^4+8bc^2d^3(p+1)x^3+ab^2d^3(p+2)-4a^2cd^3+\left(b^2cd^3(5p+6)+4ac^2d^3p\right)x^2+\left(b^3d^3(p+2)+4abc\right.}{p^2+3p+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] (4*c^3*d^3*(p + 1)*x^4 + 8*b*c^2*d^3*(p + 1)*x^3 + a*b^2*d^3*(p + 2) - 4*a^
2*c*d^3 + (b^2*c*d^3*(5*p + 6) + 4*a*c^2*d^3*p)*x^2 + (b^3*d^3*(p + 2) + 4*
a*b*c*d^3*p)*x)*(c*x^2 + b*x + a)^p/(p^2 + 3*p + 2)
```

Fricas [B] time = 2.18462, size = 309, normalized size = 4.54

$$\frac{\left(ab^2d^3p+4\left(c^3d^3p+c^3d^3\right)x^4+2\left(ab^2-2a^2c\right)d^3+8\left(bc^2d^3p+bc^2d^3\right)x^3+\left(6b^2cd^3+\left(5b^2c+4ac^2\right)d^3p\right)x^2+\left(2b^3d^3\right.\right.}{p^2+3p+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

```
[Out] (a*b^2*d^3*p + 4*(c^3*d^3*p + c^3*d^3)*x^4 + 2*(a*b^2 - 2*a^2*c)*d^3 + 8*(b
*c^2*d^3*p + b*c^2*d^3)*x^3 + (6*b^2*c*d^3 + (5*b^2*c + 4*a*c^2)*d^3*p)*x^2
 + (2*b^3*d^3 + (b^3 + 4*a*b*c)*d^3*p)*x)*(c*x^2 + b*x + a)^p/(p^2 + 3*p +
2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**3*(c*x**2+b*x+a)**p,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22823, size = 417, normalized size = 6.13

$$\frac{4\left(cx^2+bx+a\right)^p c^3d^3px^4+8\left(cx^2+bx+a\right)^p bc^2d^3px^3+4\left(cx^2+bx+a\right)^p c^3d^3x^4+5\left(cx^2+bx+a\right)^p b^2cd^3px^2+4\left(cx^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^3*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

```
[Out] (4*(c*x^2 + b*x + a)^p*c^3*d^3*p*x^4 + 8*(c*x^2 + b*x + a)^p*b*c^2*d^3*p*x^
3 + 4*(c*x^2 + b*x + a)^p*c^3*d^3*x^4 + 5*(c*x^2 + b*x + a)^p*b^2*c*d^3*p*x
^2 + 4*(c*x^2 + b*x + a)^p*a*c^2*d^3*p*x^2 + 8*(c*x^2 + b*x + a)^p*b*c^2*d^
3*x^3 + (c*x^2 + b*x + a)^p*b^3*d^3*p*x + 4*(c*x^2 + b*x + a)^p*a*b*c*d^3*p
*x + 6*(c*x^2 + b*x + a)^p*b^2*c*d^3*x^2 + (c*x^2 + b*x + a)^p*a*b^2*d^3*p
+ 2*(c*x^2 + b*x + a)^p*b^3*d^3*x + 2*(c*x^2 + b*x + a)^p*a*b^2*d^3 - 4*(c*
x^2 + b*x + a)^p*a^2*c*d^3)/(p^2 + 3*p + 2)
```

3.1439 $\int (bd+2cdx)^2 \left(a+bx+cx^2\right)^p \, dx$

Optimal. Leaf size=90

$$-\frac{2d^2(b+2cx)^3\left(\frac{1}{4}\left(4a-\frac{b^2}{c}\right)+\frac{(b+2cx)^2}{4c}\right)^{p+1}\,{}_2F_1\left(1,p+\frac{5}{2};\frac{5}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{3\left(b^2-4ac\right)}$$

```
[Out] (-2*d^2*(b + 2*c*x)^3*((4*a - b^2/c)/4 + (b + 2*c*x)^2/(4*c))^(1 + p)*Hyper
geometric2F1[1, 5/2 + p, 5/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(3*(b^2 - 4*a*c
))
```

Rubi [A] time = 0.0691887, antiderivative size = 85, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {694, 365, 364}

$$\frac{d^2(b+2cx)^3\left(a+bx+cx^2\right)^p\left(1-\frac{(b+2cx)^2}{b^2-4ac}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)^2*(a + b*x + c*x^2)^p,x]
```

```
[Out] (d^2*(b + 2*c*x)^3*(a + b*x + c*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, (b +
 2*c*x)^2/(b^2 - 4*a*c)])/(6*c*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^p)
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (bd+2cdx)^2\left(a+bx+cx^2\right)^p\,dx = \frac{\text{Subst}\left(\int x^2\left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)^p dx, x, bd+2cdx\right)}{2cd}$$
$$= \frac{\left(2^{-1+2p}\left(a+bx+cx^2\right)^p\left(4+\frac{(bd+2cdx)^2}{\left(a-\frac{b^2}{4c}\right)cd^2}\right)^{-p}\right)\text{Subst}\left(\int x^2\left(1+\frac{x^2}{4\left(a-\frac{b^2}{4c}\right)cd^2}\right)^p dx, x, bd+\right.}{cd}$$
$$= \frac{2^{-1+2p}d^2(b+2cx)^3\left(a+bx+cx^2\right)^p\left(4-\frac{4(b+2cx)^2}{b^2-4ac}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{3c}$$

Mathematica [A] time = 0.0343164, size = 92, normalized size = 1.02

$$\frac{d^2 2^{-2p-1}(b+2cx)^3(a+x(b+cx))^p\left(\frac{c(a+x(b+cx))}{4ac-b^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)^2*(a + b*x + c*x^2)^p,x]
```

```
[Out] (2^(-1 - 2*p)*d^2*(b + 2*c*x)^3*(a + x*(b + c*x))^p*Hypergeometric2F1[3/2,
-p, 5/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(3*c*((c*(a + x*(b + c*x)))/(-b^2 +
4*a*c))^p)
```

Maple [F] time = 1.095, size = 0, normalized size = 0.

$$\int (2\,cdx+bd)^2\left(cx^2+bx+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)^2*(c*x^2+b*x+a)^p,x)
```

```
[Out] int((2*c*d*x+b*d)^2*(c*x^2+b*x+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (2\,cdx+bd)^2\left(cx^2+bx+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((2*c*d*x + b*d)^2*(c*x^2 + b*x + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(4\,c^2d^2x^2+4\,bcd^2x+b^2d^2\right)\left(cx^2+bx+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

```
[Out] integral((4*c^2*d^2*x^2 + 4*b*c*d^2*x + b^2*d^2)*(c*x^2 + b*x + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)**2*(c*x**2+b*x+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (2\,cdx + bd)^2 \left(cx^2 + bx + a\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)^2*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

```
[Out] integrate((2*c*d*x + b*d)^2*(c*x^2 + b*x + a)^p, x)
```

3.1440 $\int (bd+2cdx)\left(a+bx+cx^2\right)^p\,dx$

Optimal. Leaf size=21

$$\frac{d\left(a+bx+cx^2\right)^{p+1}}{p+1}$$

```
[Out] (d*(a + b*x + c*x^2)^(1 + p))/(1 + p)
```

Rubi [A] time = 0.0055643, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {629}

$$\frac{d\left(a+bx+cx^2\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b*d + 2*c*d*x)*(a + b*x + c*x^2)^p,x]

[Out] (d*(a + b*x + c*x^2)^(1 + p))/(1 + p)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (bd+2cdx)\left(a+bx+cx^2\right)^p\,dx=\frac{d\left(a+bx+cx^2\right)^{1+p}}{1+p}$$

Mathematica [A] time = 0.0066071, size = 20, normalized size = 0.95

$$\frac{d(a+x(b+cx))^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*d + 2*c*d*x)*(a + b*x + c*x^2)^p,x]

[Out] (d*(a + x*(b + c*x))^(1 + p))/(1 + p)
```

Maple [A] time = 0.042, size = 22, normalized size = 1.1

$$\frac{d\left(cx^2+bx+a\right)^{1+p}}{1+p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*d*x+b*d)*(c*x^2+b*x+a)^p,x)
```

```
[Out] d*(c*x^2+b*x+a)^(1+p)/(1+p)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.43547, size = 72, normalized size = 3.43

$$\frac{\left(cdx^2 + bdx + ad\right)\left(cx^2 + bx + a\right)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

```
[Out] (c*d*x^2 + b*d*x + a*d)*(c*x^2 + b*x + a)^p/(p + 1)
```

Sympy [B] time = 90.2458, size = 112, normalized size = 5.33

$$\begin{cases} \frac{ad\left(a+bx+cx^2\right)^p}{p+1} + \frac{bdx\left(a+bx+cx^2\right)^p}{p+1} + \frac{cdx^2\left(a+bx+cx^2\right)^p}{p+1} & \text{for } p \neq -1 \\ d\log\left(\frac{b}{2c} + x - \frac{\sqrt{-4ac+b^2}}{2c}\right) + d\log\left(\frac{b}{2c} + x + \frac{\sqrt{-4ac+b^2}}{2c}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)*(c*x**2+b*x+a)**p,x)
```

```
[Out] Piecewise((a*d*(a + b*x + c*x**2)**p/(p + 1) + b*d*x*(a + b*x + c*x**2)**p/
(p + 1) + c*d*x**2*(a + b*x + c*x**2)**p/(p + 1), Ne(p, -1)), (d*log(b/(2*c
) + x - sqrt(-4*a*c + b**2)/(2*c)) + d*log(b/(2*c) + x + sqrt(-4*a*c + b**2
)/(2*c)), True))
```

Giac [B] time = 1.23159, size = 76, normalized size = 3.62

$$\frac{\left(cx^2 + bx + a\right)^p cdx^2 + \left(cx^2 + bx + a\right)^p bdx + \left(cx^2 + bx + a\right)^p ad}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*d*x+b*d)*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

```
[Out] ((c*x^2 + b*x + a)^p*c*d*x^2 + (c*x^2 + b*x + a)^p*b*d*x + (c*x^2 + b*x + a
)^p*a*d)/(p + 1)
```

3.1441 $\int \frac{(a+bx+cx^2)^p}{bd+2cdx} dx$

Optimal. Leaf size=63

$$\frac{(a+bx+cx^2)^{p+1} {}_2F_1\left(1,p+1;p+2;1-\frac{(b+2cx)^2}{b^2-4ac}\right)}{d(p+1)(b^2-4ac)}$$

```
[Out] ((a + b*x + c*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (b + 2*c*
x)^2/(b^2 - 4*a*c)])/((b^2 - 4*a*c)*d*(1 + p))
```

Rubi [A] time = 0.102314, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {694, 266, 65}

$$\frac{(a+bx+cx^2)^{p+1} {}_2F_1\left(1,p+1;p+2;1-\frac{(b+2cx)^2}{b^2-4ac}\right)}{d(p+1)(b^2-4ac)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^p/(b*d + 2*c*d*x),x]
```

```
[Out] ((a + b*x + c*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (b + 2*c*
x)^2/(b^2 - 4*a*c)])/((b^2 - 4*a*c)*d*(1 + p))
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^p}{bd+2cdx}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)^p}{x}\,dx, x, bd+2cdx\right)}{2cd}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(a-\frac{b^2}{4c}+\frac{x}{4cd^2}\right)^p}{x}\,dx, x, (bd+2cdx)^2\right)}{4cd}$$
$$= \frac{(a+x(b+cx))^{1+p}\,{}_2F_1\left(1,1+p;2+p;1-\frac{(b+2cx)^2}{b^2-4ac}\right)}{(b^2-4ac)\,d(1+p)}$$

Mathematica [A] time = 0.0269272, size = 64, normalized size = 1.02

$$\frac{(a+x(b+cx))^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{4c(a+x(b+cx))}{4ac-b^2}\right)}{d(p+1)\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^p/(b*d + 2*c*d*x),x]
```

```
[Out] ((a + x*(b + c*x))^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (4*c*(a + x*(
b + c*x)))/(-b^2 + 4*a*c)])/((b^2 - 4*a*c)*d*(1 + p))
```

Maple [F] time = 1.157, size = 0, normalized size = 0.

$$\int \frac{(cx^2+bx+a)^p}{2\,cdx+bd}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^p/(2*c*d*x+b*d),x)
```

```
[Out] int((c*x^2+b*x+a)^p/(2*c*d*x+b*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2+bx+a)^p}{2\,cdx+bd}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(2*c*d*x+b*d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^p/(2*c*d*x + b*d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+bx+a\right)^p}{2\,cdx+bd},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(2*c*d*x+b*d),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^p/(2*c*d*x + b*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\left(a+bx+cx^2\right)^p}{b+2cx}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**p/(2*c*d*x+b*d),x)
```

```
[Out] Integral((a + b*x + c*x**2)**p/(b + 2*c*x), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^p}{2\,cdx+bd}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(2*c*d*x+b*d),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^p/(2*c*d*x + b*d), x)
```

3.1442 $\int \frac{(a+bx+cx^2)^p}{(bd+2cdx)^2} dx$

Optimal. Leaf size=88

$$\frac{2\left(\frac{1}{4}\left(4a-\frac{b^2}{c}\right)+\frac{(b+2cx)^2}{4c}\right)^{p+1} {}_2F_1\left(1,p+\frac{1}{2};\frac{1}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{d^2\left(b^2-4ac\right)(b+2cx)}$$

```
[Out] (2*((4*a - b^2/c)/4 + (b + 2*c*x)^2/(4*c))^(1 + p)*Hypergeometric2F1[1, 1/2
 + p, 1/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/((b^2 - 4*a*c)*d^2*(b + 2*c*x))
```

Rubi [A] time = 0.0671433, antiderivative size = 85, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {694, 365, 364}

$$-\frac{\left(a+bx+cx^2\right)^p\left(1-\frac{(b+2cx)^2}{b^2-4ac}\right)^{-p} {}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{2cd^2(b+2cx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^p/(b*d + 2*c*d*x)^2,x]
```

```
[Out] -((a + b*x + c*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, (b + 2*c*x)^2/(b^2 -
 4*a*c)])/(2*c*d^2*(b + 2*c*x)*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^p)
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^p}{(bd+2cdx)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)^p}{x^2}\,dx, x, bd+2cdx\right)}{2cd}$$

$$= \frac{\left(2^{-1+2p}\left(a+bx+cx^2\right)^p\left(4+\frac{(bd+2cdx)^2}{\left(a-\frac{b^2}{4c}\right)cd^2}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{4\left(a-\frac{b^2}{4c}\right)cd^2}\right)^p}{x^2}\,dx, x, bd+2cdx\right)}{cd}$$

$$= -\frac{2^{-1+2p}\left(a+bx+cx^2\right)^p\left(4-\frac{4(b+2cx)^2}{b^2-4ac}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{cd^2(b+2cx)}$$

Mathematica [A] time = 0.0393938, size = 90, normalized size = 1.02

$$-\frac{2^{-2p-1}(a+x(b+cx))^p\left(\frac{c(a+x(b+cx))}{4ac-b^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{cd^2(b+2cx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^p/(b*d + 2*c*d*x)^2,x]
```

```
[Out] -((2^(-1 - 2*p)*(a + x*(b + c*x))^p*Hypergeometric2F1[-1/2, -p, 1/2, (b + 2
*c*x)^2/(b^2 - 4*a*c)])/(c*d^2*(b + 2*c*x)*((c*(a + x*(b + c*x)))/(-b^2 + 4
*a*c))^p))
```

Maple [F] time = 1.168, size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^p}{\left(2\,cdx+bd\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^p/(2*c*d*x+b*d)^2,x)
```

```
[Out] int((c*x^2+b*x+a)^p/(2*c*d*x+b*d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^p}{\left(2\,cdx+bd\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(2*c*d*x+b*d)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^p/(2*c*d*x + b*d)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+bx+a\right)^p}{4\,c^2d^2x^2+4\,bcd^2x+b^2d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(2*c*d*x+b*d)^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^p/(4*c^2*d^2*x^2 + 4*b*c*d^2*x + b^2*d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**p/(2*c*d*x+b*d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^p}{(2\,cdx+bd)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(2*c*d*x+b*d)^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^p/(2*c*d*x + b*d)^2, x)
```

3.1443 $\int \frac{(a+bx+cx^2)^p}{(bd+2cdx)^3} dx$

Optimal. Leaf size=63

$$\frac{(a+bx+cx^2)^{p+1} {}_2F_1\left(2,p+1;p+2;1-\frac{(b+2cx)^2}{b^2-4ac}\right)}{d^3(p+1)\left(b^2-4ac\right)^2}$$

```
[Out] ((a + b*x + c*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 - (b + 2*c*
x)^2/(b^2 - 4*a*c)])/((b^2 - 4*a*c)^2*d^3*(1 + p))
```

Rubi [A] time = 0.101622, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {694, 266, 65}

$$\frac{(a+bx+cx^2)^{p+1} {}_2F_1\left(2,p+1;p+2;1-\frac{(b+2cx)^2}{b^2-4ac}\right)}{d^3(p+1)\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^p/(b*d + 2*c*d*x)^3,x]
```

```
[Out] ((a + b*x + c*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 - (b + 2*c*
x)^2/(b^2 - 4*a*c)])/((b^2 - 4*a*c)^2*d^3*(1 + p))
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^p}{(bd+2cdx)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)^p}{x^3}\,dx, x, bd+2cdx\right)}{2cd}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a-\frac{b^2}{4c}+\frac{x}{4cd^2}\right)^p}{x^2}\,dx, x, (bd+2cdx)^2\right)}{4cd}$$

$$= \frac{(a+x(b+cx))^{1+p}\,{}_2F_1\left(2,1+p;2+p;1-\frac{(b+2cx)^2}{b^2-4ac}\right)}{\left(b^2-4ac\right)^2 d^3(1+p)}$$

Mathematica [A] time = 0.0376386, size = 64, normalized size = 1.02

$$\frac{(a+x(b+cx))^{p+1}\,{}_2F_1\left(2,p+1;p+2;\frac{4c(a+x(b+cx))}{4ac-b^2}\right)}{d^3(p+1)\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^p/(b*d + 2*c*d*x)^3,x]
```

```
[Out] ((a + x*(b + c*x))^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (4*c*(a + x*(
b + c*x)))/(-b^2 + 4*a*c)])/((b^2 - 4*a*c)^2*d^3*(1 + p))
```

Maple [F] time = 1.158, size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^p}{\left(2\,cdx+bd\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^p/(2*c*d*x+b*d)^3,x)
```

```
[Out] int((c*x^2+b*x+a)^p/(2*c*d*x+b*d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^p}{\left(2\,cdx+bd\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(2*c*d*x+b*d)^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^p/(2*c*d*x + b*d)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+bx+a\right)^p}{8\,c^3d^3x^3+12\,bc^2d^3x^2+6\,b^2cd^3x+b^3d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(2*c*d*x+b*d)^3,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^p/(8*c^3*d^3*x^3 + 12*b*c^2*d^3*x^2 + 6*b^2*c*d^
3*x + b^3*d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**p/(2*c*d*x+b*d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^p}{(2\,cdx+bd)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(2*c*d*x+b*d)^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^p/(2*c*d*x + b*d)^3, x)
```

3.1444 $\int \frac{(a+bx+cx^2)^p}{(bd+2cdx)^4}\,dx$

Optimal. Leaf size=90

$$\frac{2\left(\frac{1}{4}\left(4a-\frac{b^2}{c}\right)+\frac{(b+2cx)^2}{4c}\right)^{p+1}{}_2F_1\left(1,p-\frac{1}{2};-\frac{1}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{3d^4\left(b^2-4ac\right)(b+2cx)^3}$$

```
[Out] (2*((4*a - b^2/c)/4 + (b + 2*c*x)^2/(4*c))^(1 + p)*Hypergeometric2F1[1, -1/
2 + p, -1/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(3*(b^2 - 4*a*c)*d^4*(b + 2*c*x)
^3)
```

Rubi [A] time = 0.0682533, antiderivative size = 85, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {694, 365, 364}

$$-\frac{\left(a+bx+cx^2\right)^p\left(1-\frac{(b+2cx)^2}{b^2-4ac}\right)^{-p}{}_2F_1\left(-\frac{3}{2},-p;-\frac{1}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{6cd^4(b+2cx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^p/(b*d + 2*c*d*x)^4,x]
```

```
[Out] -((a + b*x + c*x^2)^p*Hypergeometric2F1[-3/2, -p, -1/2, (b + 2*c*x)^2/(b^2
- 4*a*c)])/(6*c*d^4*(b + 2*c*x)^3*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^p)
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^p}{(bd+2cdx)^4}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)^p}{x^4}\,dx, x, bd+2cdx\right)}{2cd}$$

$$= \frac{\left(2^{-1+2p}(a+bx+cx^2)^p\left(4+\frac{(bd+2cdx)^2}{\left(a-\frac{b^2}{4c}\right)cd^2}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{4\left(a-\frac{b^2}{4c}\right)cd^2}\right)^p}{x^4}\,dx, x, bd+2cdx\right)}{cd}$$

$$= -\frac{2^{-1+2p}(a+bx+cx^2)^p\left(4-\frac{4(b+2cx)^2}{b^2-4ac}\right)^{-p}{}_2F_1\left(-\frac{3}{2},-p;-\frac{1}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{3cd^4(b+2cx)^3}$$

Mathematica [A] time = 0.0436174, size = 92, normalized size = 1.02

$$-\frac{2^{-2p-1}(a+x(b+cx))^p\left(\frac{c(a+x(b+cx))}{4ac-b^2}\right)^{-p}{}_2F_1\left(-\frac{3}{2},-p;-\frac{1}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{3cd^4(b+2cx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^p/(b*d + 2*c*d*x)^4,x]
```

```
[Out] -(2^(-1 - 2*p)*(a + x*(b + c*x))^p*Hypergeometric2F1[-3/2, -p, -1/2, (b + 2
*c*x)^2/(b^2 - 4*a*c)])/(3*c*d^4*(b + 2*c*x)^3*((c*(a + x*(b + c*x)))/(-b^2
 + 4*a*c))^p)
```

Maple [F] time = 1.174, size = 0, normalized size = 0.

$$\int \frac{(cx^2+bx+a)^p}{(2\,cdx+bd)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^p/(2*c*d*x+b*d)^4,x)
```

```
[Out] int((c*x^2+b*x+a)^p/(2*c*d*x+b*d)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2+bx+a)^p}{(2\,cdx+bd)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(2*c*d*x+b*d)^4,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^p/(2*c*d*x + b*d)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+bx+a\right)^p}{16\,c^4d^4x^4+32\,bc^3d^4x^3+24\,b^2c^2d^4x^2+8\,b^3cd^4x+b^4d^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(2*c*d*x+b*d)^4,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^p/(16*c^4*d^4*x^4 + 32*b*c^3*d^4*x^3 + 24*b^2*c^
2*d^4*x^2 + 8*b^3*c*d^4*x + b^4*d^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**p/(2*c*d*x+b*d)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^p}{\left(2\,cdx+bd\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(2*c*d*x+b*d)^4,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^p/(2*c*d*x + b*d)^4, x)
```

3.1445 $\int \frac{(a+bx+cx^2)^p}{(bd+2cdx)^5} dx$

Optimal. Leaf size=63

$$\frac{(a+bx+cx^2)^{p+1} {}_2F_1\left(3,p+1;p+2;1-\frac{(b+2cx)^2}{b^2-4ac}\right)}{d^5(p+1)\left(b^2-4ac\right)^3}$$

```
[Out] ((a + b*x + c*x^2)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, 1 - (b + 2*c*
x)^2/(b^2 - 4*a*c)])/((b^2 - 4*a*c)^3*d^5*(1 + p))
```

Rubi [A] time = 0.105499, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {694, 266, 65}

$$\frac{(a+bx+cx^2)^{p+1} {}_2F_1\left(3,p+1;p+2;1-\frac{(b+2cx)^2}{b^2-4ac}\right)}{d^5(p+1)\left(b^2-4ac\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^p/(b*d + 2*c*d*x)^5,x]
```

```
[Out] ((a + b*x + c*x^2)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, 1 - (b + 2*c*
x)^2/(b^2 - 4*a*c)])/((b^2 - 4*a*c)^3*d^5*(1 + p))
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^p}{(bd+2cdx)^5}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)^p}{x^5}\,dx, x, bd+2cdx\right)}{2cd}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(a-\frac{b^2}{4c}+\frac{x}{4cd^2}\right)^p}{x^3}\,dx, x, (bd+2cdx)^2\right)}{4cd}$$
$$= \frac{(a+x(b+cx))^{1+p}\,{}_2F_1\left(3,1+p;2+p;1-\frac{(b+2cx)^2}{b^2-4ac}\right)}{(b^2-4ac)^3\,d^5(1+p)}$$

Mathematica [A] time = 0.0430908, size = 64, normalized size = 1.02

$$\frac{(a+x(b+cx))^{p+1}\,{}_2F_1\left(3,p+1;p+2;\frac{4c(a+x(b+cx))}{4ac-b^2}\right)}{d^5(p+1)\left(b^2-4ac\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^p/(b*d + 2*c*d*x)^5,x]
```

```
[Out] ((a + x*(b + c*x))^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, (4*c*(a + x*(
b + c*x)))/(-b^2 + 4*a*c)])/((b^2 - 4*a*c)^3*d^5*(1 + p))
```

Maple [F] time = 1.23, size = 0, normalized size = 0.

$$\int \frac{(cx^2+bx+a)^p}{(2\,cdx+bd)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^p/(2*c*d*x+b*d)^5,x)
```

```
[Out] int((c*x^2+b*x+a)^p/(2*c*d*x+b*d)^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2+bx+a)^p}{(2\,cdx+bd)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(2*c*d*x+b*d)^5,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^p/(2*c*d*x + b*d)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+bx+a\right)^p}{32\,c^5d^5x^5+80\,bc^4d^5x^4+80\,b^2c^3d^5x^3+40\,b^3c^2d^5x^2+10\,b^4cd^5x+b^5d^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(2*c*d*x+b*d)^5,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^p/(32*c^5*d^5*x^5 + 80*b*c^4*d^5*x^4 + 80*b^2*c^
3*d^5*x^3 + 40*b^3*c^2*d^5*x^2 + 10*b^4*c*d^5*x + b^5*d^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**p/(2*c*d*x+b*d)**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^p}{(2\,cdx+bd)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(2*c*d*x+b*d)^5,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^p/(2*c*d*x + b*d)^5, x)
```

3.1446 $\int \frac{(a+bx+cx^2)^p}{(bd+2cdx)^6}\,dx$

Optimal. Leaf size=90

$$\frac{2\left(\frac{1}{4}\left(4a-\frac{b^2}{c}\right)+\frac{(b+2cx)^2}{4c}\right)^{p+1}{}_2F_1\left(1,p-\frac{3}{2};-\frac{3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{5d^6\left(b^2-4ac\right)(b+2cx)^5}$$

```
[Out] (2*((4*a - b^2/c)/4 + (b + 2*c*x)^2/(4*c))^(1 + p)*Hypergeometric2F1[1, -3/
2 + p, -3/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(5*(b^2 - 4*a*c)*d^6*(b + 2*c*x)
^5)
```

Rubi [A] time = 0.0679028, antiderivative size = 85, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {694, 365, 364}

$$-\frac{\left(a+bx+cx^2\right)^p\left(1-\frac{(b+2cx)^2}{b^2-4ac}\right)^{-p}{}_2F_1\left(-\frac{5}{2},-p;-\frac{3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{10cd^6(b+2cx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^p/(b*d + 2*c*d*x)^6,x]
```

```
[Out] -((a + b*x + c*x^2)^p*Hypergeometric2F1[-5/2, -p, -3/2, (b + 2*c*x)^2/(b^2
 - 4*a*c)])/(10*c*d^6*(b + 2*c*x)^5*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^p)
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^p}{(bd+2cdx)^6}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-\frac{b^2}{4c}+\frac{x^2}{4cd^2}\right)^p}{x^6}\,dx, x, bd+2cdx\right)}{2cd}$$

$$= \frac{\left(2^{-1+2p}\left(a+bx+cx^2\right)^p\left(4+\frac{(bd+2cdx)^2}{\left(a-\frac{b^2}{4c}\right)cd^2}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{4\left(a-\frac{b^2}{4c}\right)cd^2}\right)^p}{x^6}\,dx, x, bd+2cdx\right)}{cd}$$

$$= -\frac{2^{-1+2p}\left(a+bx+cx^2\right)^p\left(4-\frac{4(b+2cx)^2}{b^2-4ac}\right)^{-p}\,{}_2F_1\left(-\frac{5}{2},-p;-\frac{3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{5cd^6(b+2cx)^5}$$

Mathematica [A] time = 0.0475862, size = 92, normalized size = 1.02

$$-\frac{2^{-2p-1}(a+x(b+cx))^p\left(\frac{c(a+x(b+cx))}{4ac-b^2}\right)^{-p}\,{}_2F_1\left(-\frac{5}{2},-p;-\frac{3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{5cd^6(b+2cx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^p/(b*d + 2*c*d*x)^6,x]
```

```
[Out] -(2^(-1 - 2*p)*(a + x*(b + c*x))^p*Hypergeometric2F1[-5/2, -p, -3/2, (b + 2
*c*x)^2/(b^2 - 4*a*c)])/(5*c*d^6*(b + 2*c*x)^5*((c*(a + x*(b + c*x)))/(-b^2
 + 4*a*c))^p)
```

Maple [F] time = 1.259, size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^p}{\left(2\,cdx+bd\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^p/(2*c*d*x+b*d)^6,x)
```

```
[Out] int((c*x^2+b*x+a)^p/(2*c*d*x+b*d)^6,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^p}{\left(2\,cdx+bd\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(2*c*d*x+b*d)^6,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^p/(2*c*d*x + b*d)^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+bx+a\right)^p}{64\,c^6d^6x^6+192\,bc^5d^6x^5+240\,b^2c^4d^6x^4+160\,b^3c^3d^6x^3+60\,b^4c^2d^6x^2+12\,b^5cd^6x+b^6d^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(2*c*d*x+b*d)^6,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^p/(64*c^6*d^6*x^6 + 192*b*c^5*d^6*x^5 + 240*b^2*
c^4*d^6*x^4 + 160*b^3*c^3*d^6*x^3 + 60*b^4*c^2*d^6*x^2 + 12*b^5*c*d^6*x + b
^6*d^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**p/(2*c*d*x+b*d)**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^p}{(2\,cdx+bd)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(2*c*d*x+b*d)^6,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^p/(2*c*d*x + b*d)^6, x)
```

3.1447 $\int \frac{1+x}{(-3+2x+x^2)^{2/3}}\,dx$

Optimal. Leaf size=16

$$\frac{3}{2}\sqrt[3]{x^2+2x-3}$$

```
[Out] (3*(-3 + 2*x + x^2)^(1/3))/2
```

Rubi [A] time = 0.0039318, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {629}

$$\frac{3}{2}\sqrt[3]{x^2+2x-3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(-3 + 2*x + x^2)^(2/3),x]
```

```
[Out] (3*(-3 + 2*x + x^2)^(1/3))/2
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1+x}{(-3+2x+x^2)^{2/3}}\,dx = \frac{3}{2}\sqrt[3]{-3+2x+x^2}$$

Mathematica [A] time = 0.0038345, size = 16, normalized size = 1.

$$\frac{3}{2}\sqrt[3]{x^2+2x-3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(-3 + 2*x + x^2)^(2/3),x]
```

```
[Out] (3*(-3 + 2*x + x^2)^(1/3))/2
```

Maple [A] time = 0.04, size = 19, normalized size = 1.2

$$\frac{(9+3x)(-1+x)}{2}\left(x^2+2x-3\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(x^2+2*x-3)^(2/3),x)

[Out] 3/2*(3+x)*(-1+x)/(x^2+2*x-3)^(2/3)
```

Maxima [A] time = 1.17557, size = 16, normalized size = 1.

$$\frac{3}{2} \left(x^2 + 2\,x - 3\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+2*x-3)^(2/3),x, algorithm="maxima")

[Out] 3/2*(x^2 + 2*x - 3)^(1/3)
```

Fricas [A] time = 2.02086, size = 36, normalized size = 2.25

$$\frac{3}{2} \left(x^2 + 2\,x - 3\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+2*x-3)^(2/3),x, algorithm="fricas")

[Out] 3/2*(x^2 + 2*x - 3)^(1/3)
```

Sympy [A] time = 0.15836, size = 14, normalized size = 0.88

$$\frac{3\sqrt[3]{x^2 + 2x - 3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**2+2*x-3)**(2/3),x)

[Out] 3*(x**2 + 2*x - 3)**(1/3)/2
```

Giac [A] time = 1.21356, size = 16, normalized size = 1.

$$\frac{3}{2} \left(x^2 + 2\,x - 3\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+2*x-3)^(2/3),x, algorithm="giac")

[Out] 3/2*(x^2 + 2*x - 3)^(1/3)
```

3.1448 $\int \frac{b+cx}{(a+2bx+cx^2)^{3/7}} dx$

Optimal. Leaf size=19

$$\frac{7}{8}\left(a+2bx+cx^2\right)^{4/7}$$

[Out] (7*(a + 2*b*x + c*x^2)^(4/7))/8

Rubi [A] time = 0.0064562, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {629}

$$\frac{7}{8}\left(a+2bx+cx^2\right)^{4/7}$$

Antiderivative was successfully verified.

[In] Int[(b + c*x)/(a + 2*b*x + c*x^2)^(3/7),x]

[Out] (7*(a + 2*b*x + c*x^2)^(4/7))/8

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{b+cx}{\left(a+2bx+cx^2\right)^{3/7}}\,dx=\frac{7}{8}\left(a+2bx+cx^2\right)^{4/7}$$

Mathematica [A] time = 0.011354, size = 19, normalized size = 1.

$$\frac{7}{8}(a+x(2b+cx))^{4/7}$$

Antiderivative was successfully verified.

[In] Integrate[(b + c*x)/(a + 2*b*x + c*x^2)^(3/7),x]

[Out] (7*(a + x*(2*b + c*x))^(4/7))/8

Maple [A] time = 0.044, size = 16, normalized size = 0.8

$$\frac{7}{8}\left(cx^2+2\,bx+a\right)^{\frac{4}{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x+b)/(c*x^2+2*b*x+a)^(3/7),x)

[Out] 7/8*(c*x^2+2*b*x+a)^(4/7)
```

Maxima [A] time = 1.28466, size = 20, normalized size = 1.05

$$\frac{7}{8}\left(cx^2 + 2\,bx + a\right)^{\frac{4}{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+b)/(c*x^2+2*b*x+a)^(3/7),x, algorithm="maxima")

[Out] 7/8*(c*x^2 + 2*b*x + a)^(4/7)
```

Fricas [A] time = 2.00528, size = 42, normalized size = 2.21

$$\frac{7}{8}\left(cx^2 + 2\,bx + a\right)^{\frac{4}{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+b)/(c*x^2+2*b*x+a)^(3/7),x, algorithm="fricas")

[Out] 7/8*(c*x^2 + 2*b*x + a)^(4/7)
```

Sympy [A] time = 0.371145, size = 17, normalized size = 0.89

$$\frac{7\left(a + 2bx + cx^2\right)^{\frac{4}{7}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+b)/(c*x**2+2*b*x+a)**(3/7),x)

[Out] 7*(a + 2*b*x + c*x**2)**(4/7)/8
```

Giac [A] time = 1.13929, size = 20, normalized size = 1.05

$$\frac{7}{8}\left(cx^2 + 2\,bx + a\right)^{\frac{4}{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+b)/(c*x^2+2*b*x+a)^(3/7),x, algorithm="giac")

[Out] 7/8*(c*x^2 + 2*b*x + a)^(4/7)
```

3.1449 $\int (1+x)^m \left(1+2x+x^2\right)^n \, dx$

Optimal. Leaf size=26

$$\frac{(x+1)^{m+1}\left(x^2+2x+1\right)^n}{m+2n+1}$$

```
[Out] ((1 + x)^(1 + m)*(1 + 2*x + x^2)^n)/(1 + m + 2*n)
```

Rubi [A] time = 0.0108419, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {644, 32}

$$\frac{(x+1)^{m+1}\left(x^2+2x+1\right)^n}{m+2n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^m*(1 + 2*x + x^2)^n,x]
```

```
[Out] ((1 + x)^(1 + m)*(1 + 2*x + x^2)^n)/(1 + m + 2*n)
```

Rule 644

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[(a + b*x + c*x^2)^p/(d + e*x)^(2*p), Int[(d + e*x)^(m + 2*p),
x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !Integ
erQ[p] && EqQ[2*c*d - b*e, 0] &&  !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (1+x)^m \left(1+2x+x^2\right)^n \, dx &= \left((1+x)^{-2n}\left(1+2x+x^2\right)^n\right)\int (1+x)^{m+2n}\, dx\\ &= \frac{(1+x)^{1+m}\left(1+2x+x^2\right)^n}{1+m+2n}\end{aligned}$$

Mathematica [A] time = 0.0107284, size = 23, normalized size = 0.88

$$\frac{(x+1)^{m+1}\left((x+1)^2\right)^n}{m+2n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^m*(1 + 2*x + x^2)^n,x]
```

```
[Out] ((1 + x)^(1 + m)*((1 + x)^2)^n)/(1 + m + 2*n)
```

Maple [A] time = 0.04, size = 27, normalized size = 1.

$$\frac{(1+x)^{1+m}\left(x^{2}+2x+1\right)^{n}}{1+m+2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^m*(x^2+2*x+1)^n,x)
```

```
[Out] (1+x)^(1+m)*(x^2+2*x+1)^n/(1+m+2*n)
```

Maxima [A] time = 1.31031, size = 36, normalized size = 1.38

$$\frac{(x+1)e^{(m\log(x+1)+2n\log(x+1))}}{m+2n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^m*(x^2+2*x+1)^n,x, algorithm="maxima")
```

```
[Out] (x + 1)*e^(m*log(x + 1) + 2*n*log(x + 1))/(m + 2*n + 1)
```

Fricas [A] time = 2.18157, size = 63, normalized size = 2.42

$$\frac{(x+1)^{m}(x+1)^{2n}(x+1)}{m+2n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^m*(x^2+2*x+1)^n,x, algorithm="fricas")
```

```
[Out] (x + 1)^m*(x + 1)^(2*n)*(x + 1)/(m + 2*n + 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**m*(x**2+2*x+1)**n,x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.23872, size = 50, normalized size = 1.92

$$\frac{(x+1)^{m}(x+1)^{2n}x+(x+1)^{m}(x+1)^{2n}}{m+2n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^m*(x^2+2*x+1)^n,x, algorithm="giac")
```

```
[Out] ((x + 1)^m*(x + 1)^(2*n)*x + (x + 1)^m*(x + 1)^(2*n))/(m + 2*n + 1)
```

3.1450 $\int \left(\frac{be}{2c}+ex\right)^m \left(\frac{b^2}{4c}+bx+cx^2\right)^n \, dx$

Optimal. Leaf size=50

$$\frac{\left(\frac{b^2}{4c}+bx+cx^2\right)^n \left(\frac{be}{2c}+ex\right)^{m+1}}{e(m+2n+1)}$$

```
[Out] (((b*e)/(2*c) + e*x)^(1 + m)*(b^2/(4*c) + b*x + c*x^2)^n)/(e*(1 + m + 2*n))
```

Rubi [A] time = 0.0205833, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {644, 32}

$$\frac{\left(\frac{b^2}{4c}+bx+cx^2\right)^n \left(\frac{be}{2c}+ex\right)^{m+1}}{e(m+2n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((b*e)/(2*c) + e*x)^m*(b^2/(4*c) + b*x + c*x^2)^n,x]
```

```
[Out] (((b*e)/(2*c) + e*x)^(1 + m)*(b^2/(4*c) + b*x + c*x^2)^n)/(e*(1 + m + 2*n))
```

Rule 644

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[(a + b*x + c*x^2)^p/(d + e*x)^(2*p), Int[(d + e*x)^(m + 2*p),
x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !Integ
erQ[p] && EqQ[2*c*d - b*e, 0] &&  !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \left(\frac{be}{2c}+ex\right)^m \left(\frac{b^2}{4c}+bx+cx^2\right)^n \, dx &= \left(\left(\frac{be}{2c}+ex\right)^{-2n} \left(\frac{b^2}{4c}+bx+cx^2\right)^n\right) \int \left(\frac{be}{2c}+ex\right)^{m+2n} \, dx \\ &= \frac{\left(\frac{be}{2c}+ex\right)^{1+m} \left(\frac{b^2}{4c}+bx+cx^2\right)^n}{e(1+m+2n)} \end{aligned}$$

Mathematica [A] time = 0.0271691, size = 54, normalized size = 1.08

$$\frac{2^{-2n-1}(b+2cx)\left(\frac{(b+2cx)^2}{c}\right)^n \left(\frac{be}{2c}+ex\right)^m}{c(m+2n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*e)/(2*c) + e*x)^m*(b^2/(4*c) + b*x + c*x^2)^n,x]
```

```
[Out] (2^(-1 - 2*n)*(b + 2*c*x)*((b + 2*c*x)^2/c)^n*((b*e)/(2*c) + e*x)^m)/(c*(1
+ m + 2*n))
```

Maple [A] time = 0.044, size = 58, normalized size = 1.2

$$\frac{2\,cx+b}{2\,c\,(1+m+2\,n)}\left(\frac{e\,(2\,cx+b)}{2\,c}\right)^{m}\left(\frac{4\,c^{2}x^{2}+4\,bcx+b^{2}}{4\,c}\right)^{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/2*b*e/c+e*x)^m*(1/4*b^2/c+b*x+c*x^2)^n,x)
```

```
[Out] 1/2*(2*c*x+b)/c/(1+m+2*n)*(1/2*e*(2*c*x+b)/c)^m*(1/4*(4*c^2*x^2+4*b*c*x+b^2
)/c)^n
```

Maxima [A] time = 1.66517, size = 107, normalized size = 2.14

$$\frac{(2\,ce^{m}x+be^{m})c^{-m-n-1}e^{(m\log(2\,cx+b)+2\,n\log(2\,cx+b))}}{\left(2^{2\,n+2}n+2^{2\,n+1}\right)2^{m}+2^{m+2\,n+1}m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/2*b*e/c+e*x)^m*(1/4/c*b^2+b*x+c*x^2)^n,x, algorithm="maxima")
```

```
[Out] (2*c*e^m*x + b*e^m)*c^(-m - n - 1)*e^(m*log(2*c*x + b) + 2*n*log(2*c*x + b)
)/((2^(2*n + 2)*n + 2^(2*n + 1))*2^m + 2^(m + 2*n + 1)*m)
```

Fricas [A] time = 2.19233, size = 150, normalized size = 3.

$$\frac{(2\,cx+b)\left(\frac{2\,cex+be}{2\,c}\right)^{m}e^{\left(2\,n\log\left(\frac{2\,cex+be}{2\,c}\right)+n\log\left(\frac{c}{e^{2}}\right)\right)}}{2\,(cm+2\,cn+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/2*b*e/c+e*x)^m*(1/4/c*b^2+b*x+c*x^2)^n,x, algorithm="fricas")
```

```
[Out] 1/2*(2*c*x + b)*(1/2*(2*c*e*x + b*e)/c)^m*e^(2*n*log(1/2*(2*c*e*x + b*e)/c)
 + n*log(c/e^2))/(c*m + 2*c*n + c)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/2*b*e/c+e*x)**m*(1/4/c*b**2+b*x+c*x**2)**n,x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.18686, size = 140, normalized size = 2.8

$$\frac{2\,cxe^{\left(-m\log(2)-2\,n\log(2)+m\log(2\,cx+b)+2\,n\log(2\,cx+b)-m\log(c)-n\log(c)+m\right)}+be^{\left(-m\log(2)-2\,n\log(2)+m\log(2\,cx+b)+2\,n\log(2\,cx+b)-m\log(c)-n\log\right.}}{2\,(cm+2\,cn+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/2*b*e/c+e*x)^m*(1/4/c*b^2+b*x+c*x^2)^n,x, algorithm="giac")
```

```
[Out] 1/2*(2*c*x*e^(-m*log(2) - 2*n*log(2) + m*log(2*c*x + b) + 2*n*log(2*c*x + b
) - m*log(c) - n*log(c) + m) + b*e^(-m*log(2) - 2*n*log(2) + m*log(2*c*x +
b) + 2*n*log(2*c*x + b) - m*log(c) - n*log(c) + m))/(c*m + 2*c*n + c)
```

3.1451 $\int (d+ex)^4 \left(a^2+2abx+b^2x^2\right) dx$

Optimal. Leaf size=65

$$-\frac{b(d+ex)^6(bd-ae)}{3e^3}+\frac{(d+ex)^5(bd-ae)^2}{5e^3}+\frac{b^2(d+ex)^7}{7e^3}$$

```
[Out] ((b*d - a*e)^2*(d + e*x)^5)/(5*e^3) - (b*(b*d - a*e)*(d + e*x)^6)/(3*e^3) +
 (b^2*(d + e*x)^7)/(7*e^3)
```

Rubi [A] time = 0.0977775, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {27, 43}

$$-\frac{b(d+ex)^6(bd-ae)}{3e^3}+\frac{(d+ex)^5(bd-ae)^2}{5e^3}+\frac{b^2(d+ex)^7}{7e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4*(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] ((b*d - a*e)^2*(d + e*x)^5)/(5*e^3) - (b*(b*d - a*e)*(d + e*x)^6)/(3*e^3) +
 (b^2*(d + e*x)^7)/(7*e^3)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^4 \left(a^2+2abx+b^2x^2\right) dx &= \int (a+bx)^2(d+ex)^4 \, dx \\
&= \int \left(\frac{(-bd+ae)^2(d+ex)^4}{e^2}-\frac{2b(bd-ae)(d+ex)^5}{e^2}+\frac{b^2(d+ex)^6}{e^2}\right) dx \\
&= \frac{(bd-ae)^2(d+ex)^5}{5e^3}-\frac{b(bd-ae)(d+ex)^6}{3e^3}+\frac{b^2(d+ex)^7}{7e^3}
\end{aligned}$$

Mathematica [B] time = 0.0261923, size = 148, normalized size = 2.28

$$\frac{1}{5}e^2x^5\left(a^2e^2+8abde+6b^2d^2\right)+dex^4\left(a^2e^2+3abde+b^2d^2\right)+\frac{1}{3}d^2x^3\left(6a^2e^2+8abde+b^2d^2\right)+a^2d^4x+ad^3x^2(2ae+bc$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4*(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] a^2*d^4*x + a*d^3*(b*d + 2*a*e)*x^2 + (d^2*(b^2*d^2 + 8*a*b*d*e + 6*a^2*e^2
)*x^3)/3 + d*e*(b^2*d^2 + 3*a*b*d*e + a^2*e^2)*x^4 + (e^2*(6*b^2*d^2 + 8*a*
b*d*e + a^2*e^2)*x^5)/5 + (b*e^3*(2*b*d + a*e)*x^6)/3 + (b^2*e^4*x^7)/7
```

Maple [B] time = 0.04, size = 163, normalized size = 2.5

$$\frac{e^4b^2x^7}{7}+\frac{\left(2\,e^4ab+4\,de^3b^2\right)x^6}{6}+\frac{\left(a^2e^4+8\,de^3ab+6\,d^2e^2b^2\right)x^5}{5}+\frac{\left(4\,de^3a^2+12\,d^2e^2ab+4\,d^3eb^2\right)x^4}{4}+\frac{\left(6\,d^2e^2a^2+8\,d^3e\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2),x)
```

```
[Out] 1/7*e^4*b^2*x^7+1/6*(2*a*b*e^4+4*b^2*d*e^3)*x^6+1/5*(a^2*e^4+8*a*b*d*e^3+6*
b^2*d^2*e^2)*x^5+1/4*(4*a^2*d*e^3+12*a*b*d^2*e^2+4*b^2*d^3*e)*x^4+1/3*(6*a^
2*d^2*e^2+8*a*b*d^3*e+b^2*d^4)*x^3+1/2*(4*a^2*d^3*e+2*a*b*d^4)*x^2+d^4*a^2*
x
```

Maxima [B] time = 1.14135, size = 211, normalized size = 3.25

$$\frac{1}{7}\,b^2e^4x^7+a^2d^4x+\frac{1}{3}\left(2\,b^2de^3+abe^4\right)x^6+\frac{1}{5}\left(6\,b^2d^2e^2+8\,abde^3+a^2e^4\right)x^5+\left(b^2d^3e+3\,abd^2e^2+a^2de^3\right)x^4+\frac{1}{3}\left(b^2d^4+8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

```
[Out] 1/7*b^2*e^4*x^7 + a^2*d^4*x + 1/3*(2*b^2*d*e^3 + a*b*e^4)*x^6 + 1/5*(6*b^2*
d^2*e^2 + 8*a*b*d*e^3 + a^2*e^4)*x^5 + (b^2*d^3*e + 3*a*b*d^2*e^2 + a^2*d*e
^3)*x^4 + 1/3*(b^2*d^4 + 8*a*b*d^3*e + 6*a^2*d^2*e^2)*x^3 + (a*b*d^4 + 2*a^
2*d^3*e)*x^2
```

Fricas [B] time = 1.70062, size = 363, normalized size = 5.58

$$\frac{1}{7}x^7e^4b^2+\frac{2}{3}x^6e^3db^2+\frac{1}{3}x^6e^4ba+\frac{6}{5}x^5e^2d^2b^2+\frac{8}{5}x^5e^3dba+\frac{1}{5}x^5e^4a^2+x^4ed^3b^2+3x^4e^2d^2ba+x^4e^3da^2+\frac{1}{3}x^3d^4b^2+\frac{8}{3}x^3e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*e^4*b^2 + 2/3*x^6*e^3*d*b^2 + 1/3*x^6*e^4*b*a + 6/5*x^5*e^2*d^2*b^2
 + 8/5*x^5*e^3*d*b*a + 1/5*x^5*e^4*a^2 + x^4*e*d^3*b^2 + 3*x^4*e^2*d^2*b*a
+ x^4*e^3*d*a^2 + 1/3*x^3*d^4*b^2 + 8/3*x^3*e*d^3*b*a + 2*x^3*e^2*d^2*a^2 +
 x^2*d^4*b*a + 2*x^2*e*d^3*a^2 + x*d^4*a^2
```

Sympy [B] time = 0.100004, size = 168, normalized size = 2.58

$$a^2d^4x+\frac{b^2e^4x^7}{7}+x^6\left(\frac{abe^4}{3}+\frac{2b^2de^3}{3}\right)+x^5\left(\frac{a^2e^4}{5}+\frac{8abde^3}{5}+\frac{6b^2d^2e^2}{5}\right)+x^4\left(a^2de^3+3abd^2e^2+b^2d^3e\right)+x^3\left(2a^2d^2e^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(b**2*x**2+2*a*b*x+a**2),x)
```

```
[Out] a**2*d**4*x + b**2*e**4*x**7/7 + x**6*(a*b*e**4/3 + 2*b**2*d*e**3/3) + x**5
*(a**2*e**4/5 + 8*a*b*d*e**3/5 + 6*b**2*d**2*e**2/5) + x**4*(a**2*d*e**3 +
3*a*b*d**2*e**2 + b**2*d**3*e) + x**3*(2*a**2*d**2*e**2 + 8*a*b*d**3*e/3 +
b**2*d**4/3) + x**2*(2*a**2*d**3*e + a*b*d**4)
```

Giac [B] time = 1.18998, size = 221, normalized size = 3.4

$$\frac{1}{7}\,b^2x^7e^4 + \frac{2}{3}\,b^2dx^6e^3 + \frac{6}{5}\,b^2d^2x^5e^2 + b^2d^3x^4e + \frac{1}{3}\,b^2d^4x^3 + \frac{1}{3}\,abx^6e^4 + \frac{8}{5}\,abdx^5e^3 + 3\,abd^2x^4e^2 + \frac{8}{3}\,abd^3x^3e + abd^4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

```
[Out] 1/7*b^2*x^7*e^4 + 2/3*b^2*d*x^6*e^3 + 6/5*b^2*d^2*x^5*e^2 + b^2*d^3*x^4*e +
 1/3*b^2*d^4*x^3 + 1/3*a*b*x^6*e^4 + 8/5*a*b*d*x^5*e^3 + 3*a*b*d^2*x^4*e^2
+ 8/3*a*b*d^3*x^3*e + a*b*d^4*x^2 + 1/5*a^2*x^5*e^4 + a^2*d*x^4*e^3 + 2*a^2
*d^2*x^3*e^2 + 2*a^2*d^3*x^2*e + a^2*d^4*x
```

3.1452 $\int (d+ex)^3 \left(a^2+2abx+b^2x^2\right) dx$

Optimal. Leaf size=65

$$-\frac{2b(d+ex)^5(bd-ae)}{5e^3}+\frac{(d+ex)^4(bd-ae)^2}{4e^3}+\frac{b^2(d+ex)^6}{6e^3}$$

```
[Out] ((b*d - a*e)^2*(d + e*x)^4)/(4*e^3) - (2*b*(b*d - a*e)*(d + e*x)^5)/(5*e^3)
 + (b^2*(d + e*x)^6)/(6*e^3)
```

Rubi [A] time = 0.0679921, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {27, 43}

$$-\frac{2b(d+ex)^5(bd-ae)}{5e^3}+\frac{(d+ex)^4(bd-ae)^2}{4e^3}+\frac{b^2(d+ex)^6}{6e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2),x]
```


```
[Out] ((b*d - a*e)^2*(d + e*x)^4)/(4*e^3) - (2*b*(b*d - a*e)*(d + e*x)^5)/(5*e^3)
 + (b^2*(d + e*x)^6)/(6*e^3)
```


Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^3\left(a^2+2abx+b^2x^2\right)dx &= \int (a+bx)^2(d+ex)^3\,dx\\ &= \int \left(\frac{(-bd+ae)^2(d+ex)^3}{e^2}-\frac{2b(bd-ae)(d+ex)^4}{e^2}+\frac{b^2(d+ex)^5}{e^2}\right)dx\\ &= \frac{(bd-ae)^2(d+ex)^4}{4e^3}-\frac{2b(bd-ae)(d+ex)^5}{5e^3}+\frac{b^2(d+ex)^6}{6e^3}\end{aligned}$$

Mathematica [A] time = 0.0177003, size = 122, normalized size = 1.88

$$\frac{1}{4}ex^4\left(a^2e^2+6abde+3b^2d^2\right)+\frac{1}{3}dx^3\left(3a^2e^2+6abde+b^2d^2\right)+a^2d^3x+\frac{1}{2}ad^2x^2(3ae+2bd)+\frac{1}{5}be^2x^5(2ae+3bd)+\frac{1}{6}b^2e^3$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] a^2*d^3*x + (a*d^2*(2*b*d + 3*a*e)*x^2)/2 + (d*(b^2*d^2 + 6*a*b*d*e + 3*a^2
*e^2)*x^3)/3 + (e*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2)*x^4)/4 + (b*e^2*(3*b*d
+ 2*a*e)*x^5)/5 + (b^2*e^3*x^6)/6
```

Maple [B] time = 0.039, size = 125, normalized size = 1.9

$$\frac{e^3b^2x^6}{6} + \frac{(2\,abe^3 + 3\,de^2b^2)\,x^5}{5} + \frac{(a^2e^3 + 6\,de^2ab + 3\,d^2eb^2)\,x^4}{4} + \frac{(3\,de^2a^2 + 6\,d^2eab + d^3b^2)\,x^3}{3} + \frac{(3\,d^2ea^2 + 2\,d^3ab)\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2),x)
```

```
[Out] 1/6*e^3*b^2*x^6+1/5*(2*a*b*e^3+3*b^2*d*e^2)*x^5+1/4*(a^2*e^3+6*a*b*d*e^2+3*
b^2*d^2*e)*x^4+1/3*(3*a^2*d*e^2+6*a*b*d^2*e+b^2*d^3)*x^3+1/2*(3*a^2*d^2*e+2
*a*b*d^3)*x^2+d^3*a^2*x
```

Maxima [B] time = 1.13521, size = 167, normalized size = 2.57

$$\frac{1}{6}\,b^2e^3x^6 + a^2d^3x + \frac{1}{5}\left(3\,b^2de^2 + 2\,abe^3\right)x^5 + \frac{1}{4}\left(3\,b^2d^2e + 6\,abde^2 + a^2e^3\right)x^4 + \frac{1}{3}\left(b^2d^3 + 6\,abd^2e + 3\,a^2de^2\right)x^3 + \frac{1}{2}\left(2\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

```
[Out] 1/6*b^2*e^3*x^6 + a^2*d^3*x + 1/5*(3*b^2*d*e^2 + 2*a*b*e^3)*x^5 + 1/4*(3*b^
2*d^2*e + 6*a*b*d*e^2 + a^2*e^3)*x^4 + 1/3*(b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d
*e^2)*x^3 + 1/2*(2*a*b*d^3 + 3*a^2*d^2*e)*x^2
```

Fricas [B] time = 1.76083, size = 285, normalized size = 4.38

$$\frac{1}{6}x^6e^3b^2 + \frac{3}{5}x^5e^2db^2 + \frac{2}{5}x^5e^3ba + \frac{3}{4}x^4ed^2b^2 + \frac{3}{2}x^4e^2dba + \frac{1}{4}x^4e^3a^2 + \frac{1}{3}x^3d^3b^2 + 2x^3ed^2ba + x^3e^2da^2 + x^2d^3ba + \frac{3}{2}x^2e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*e^3*b^2 + 3/5*x^5*e^2*d*b^2 + 2/5*x^5*e^3*b*a + 3/4*x^4*e*d^2*b^2 +
 3/2*x^4*e^2*d*b*a + 1/4*x^4*e^3*a^2 + 1/3*x^3*d^3*b^2 + 2*x^3*e*d^2*b*a +
x^3*e^2*d*a^2 + x^2*d^3*b*a + 3/2*x^2*e*d^2*a^2 + x*d^3*a^2
```

Sympy [B] time = 0.081116, size = 133, normalized size = 2.05

$$a^2d^3x + \frac{b^2e^3x^6}{6} + x^5\left(\frac{2abe^3}{5} + \frac{3b^2de^2}{5}\right) + x^4\left(\frac{a^2e^3}{4} + \frac{3abde^2}{2} + \frac{3b^2d^2e}{4}\right) + x^3\left(a^2de^2 + 2abd^2e + \frac{b^2d^3}{3}\right) + x^2\left(\frac{3a^2d^2e}{2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(b**2*x**2+2*a*b*x+a**2),x)
```

```
[Out] a**2*d**3*x + b**2*e**3*x**6/6 + x**5*(2*a*b*e**3/5 + 3*b**2*d*e**2/5) + x*
*4*(a**2*e**3/4 + 3*a*b*d*e**2/2 + 3*b**2*d**2*e/4) + x**3*(a**2*d*e**2 + 2
*a*b*d**2*e + b**2*d**3/3) + x**2*(3*a**2*d**2*e/2 + a*b*d**3)
```

Giac [B] time = 1.15857, size = 171, normalized size = 2.63

$$\frac{1}{6}b^2x^6e^3 + \frac{3}{5}b^2dx^5e^2 + \frac{3}{4}b^2d^2x^4e + \frac{1}{3}b^2d^3x^3 + \frac{2}{5}abx^5e^3 + \frac{3}{2}abdx^4e^2 + 2\,abd^2x^3e + abd^3x^2 + \frac{1}{4}a^2x^4e^3 + a^2dx^3e^2 + \frac{3}{2}a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

```
[Out] 1/6*b^2*x^6*e^3 + 3/5*b^2*d*x^5*e^2 + 3/4*b^2*d^2*x^4*e + 1/3*b^2*d^3*x^3 +
 2/5*a*b*x^5*e^3 + 3/2*a*b*d*x^4*e^2 + 2*a*b*d^2*x^3*e + a*b*d^3*x^2 + 1/4*
a^2*x^4*e^3 + a^2*d*x^3*e^2 + 3/2*a^2*d^2*x^2*e + a^2*d^3*x
```

3.1453 $\int (d+ex)^2 \left(a^2+2abx+b^2x^2\right) \, dx$

Optimal. Leaf size=65

$$\frac{e(a+bx)^4(bd-ae)}{2b^3}+\frac{(a+bx)^3(bd-ae)^2}{3b^3}+\frac{e^2(a+bx)^5}{5b^3}$$

```
[Out] ((b*d - a*e)^2*(a + b*x)^3)/(3*b^3) + (e*(b*d - a*e)*(a + b*x)^4)/(2*b^3) +
 (e^2*(a + b*x)^5)/(5*b^3)
```

Rubi [A] time = 0.07521, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {27, 43}

$$\frac{e(a+bx)^4(bd-ae)}{2b^3}+\frac{(a+bx)^3(bd-ae)^2}{3b^3}+\frac{e^2(a+bx)^5}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] ((b*d - a*e)^2*(a + b*x)^3)/(3*b^3) + (e*(b*d - a*e)*(a + b*x)^4)/(2*b^3) +
 (e^2*(a + b*x)^5)/(5*b^3)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^2 \left(a^2+2abx+b^2x^2\right) \, dx &= \int (a+bx)^2(d+ex)^2 \, dx\\
&= \int \left(\frac{(bd-ae)^2(a+bx)^2}{b^2}+\frac{2e(bd-ae)(a+bx)^3}{b^2}+\frac{e^2(a+bx)^4}{b^2}\right) dx\\
&= \frac{(bd-ae)^2(a+bx)^3}{3b^3}+\frac{e(bd-ae)(a+bx)^4}{2b^3}+\frac{e^2(a+bx)^5}{5b^3}
\end{aligned}$$

Mathematica [A] time = 0.0136899, size = 79, normalized size = 1.22

$$\frac{1}{3}x^3\left(a^2e^2+4abde+b^2d^2\right)+a^2d^2x+\frac{1}{2}bex^4(ae+bd)+adx^2(ae+bd)+\frac{1}{5}b^2e^2x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] a^2*d^2*x + a*d*(b*d + a*e)*x^2 + ((b^2*d^2 + 4*a*b*d*e + a^2*e^2)*x^3)/3 +
 (b*e*(b*d + a*e)*x^4)/2 + (b^2*e^2*x^5)/5
```

Maple [A] time = 0.039, size = 87, normalized size = 1.3

$$\frac{b^2e^2x^5}{5}+\frac{\left(2\,e^2ab+2\,b^2de\right)x^4}{4}+\frac{\left(a^2e^2+4\,deab+b^2d^2\right)x^3}{3}+\frac{\left(2\,dea^2+2\,d^2ab\right)x^2}{2}+a^2d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2),x)
```

```
[Out] 1/5*b^2*e^2*x^5+1/4*(2*a*b*e^2+2*b^2*d*e)*x^4+1/3*(a^2*e^2+4*a*b*d*e+b^2*d^
2)*x^3+1/2*(2*a^2*d*e+2*a*b*d^2)*x^2+a^2*d^2*x
```

Maxima [A] time = 1.1589, size = 109, normalized size = 1.68

$$\frac{1}{5}\,b^2e^2x^5+a^2d^2x+\frac{1}{2}\left(b^2de+abe^2\right)x^4+\frac{1}{3}\left(b^2d^2+4\,abde+a^2e^2\right)x^3+\left(abd^2+a^2de\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

```
[Out] 1/5*b^2*e^2*x^5 + a^2*d^2*x + 1/2*(b^2*d*e + a*b*e^2)*x^4 + 1/3*(b^2*d^2 +
4*a*b*d*e + a^2*e^2)*x^3 + (a*b*d^2 + a^2*d*e)*x^2
```

Fricas [A] time = 1.77938, size = 198, normalized size = 3.05

$$\frac{1}{5}x^5e^2b^2+\frac{1}{2}x^4edb^2+\frac{1}{2}x^4e^2ba+\frac{1}{3}x^3d^2b^2+\frac{4}{3}x^3edba+\frac{1}{3}x^3e^2a^2+x^2d^2ba+x^2eda^2+xd^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

```
[Out] 1/5*x^5*e^2*b^2 + 1/2*x^4*e*d*b^2 + 1/2*x^4*e^2*b*a + 1/3*x^3*d^2*b^2 + 4/3
*x^3*e*d*b*a + 1/3*x^3*e^2*a^2 + x^2*d^2*b*a + x^2*e*d*a^2 + x*d^2*a^2
```

Sympy [A] time = 0.079999, size = 87, normalized size = 1.34

$$a^2d^2x+\frac{b^2e^2x^5}{5}+x^4\left(\frac{abe^2}{2}+\frac{b^2de}{2}\right)+x^3\left(\frac{a^2e^2}{3}+\frac{4abde}{3}+\frac{b^2d^2}{3}\right)+x^2\left(a^2de+abd^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(b**2*x**2+2*a*b*x+a**2),x)
```

```
[Out] a**2*d**2*x + b**2*e**2*x**5/5 + x**4*(a*b*e**2/2 + b**2*d*e/2) + x**3*(a**
2*e**2/3 + 4*a*b*d*e/3 + b**2*d**2/3) + x**2*(a**2*d*e + a*b*d**2)
```

Giac [A] time = 1.16609, size = 120, normalized size = 1.85

$$\frac{1}{5}b^2x^5e^2 + \frac{1}{2}b^2dx^4e + \frac{1}{3}b^2d^2x^3 + \frac{1}{2}abx^4e^2 + \frac{4}{3}abdx^3e + abd^2x^2 + \frac{1}{3}a^2x^3e^2 + a^2dx^2e + a^2d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

```
[Out] 1/5*b^2*x^5*e^2 + 1/2*b^2*d*x^4*e + 1/3*b^2*d^2*x^3 + 1/2*a*b*x^4*e^2 + 4/3
*a*b*d*x^3*e + a*b*d^2*x^2 + 1/3*a^2*x^3*e^2 + a^2*d*x^2*e + a^2*d^2*x
```

3.1454 $\int (d+ex)\left(a^2+2abx+b^2x^2\right)\,dx$

Optimal. Leaf size=38

$$\frac{(a+bx)^3(bd-ae)}{3b^2}+\frac{e(a+bx)^4}{4b^2}$$

```
[Out] ((b*d - a*e)*(a + b*x)^3)/(3*b^2) + (e*(a + b*x)^4)/(4*b^2)
```

Rubi [A] time = 0.0297702, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {27, 43}

$$\frac{(a+bx)^3(bd-ae)}{3b^2}+\frac{e(a+bx)^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] ((b*d - a*e)*(a + b*x)^3)/(3*b^2) + (e*(a + b*x)^4)/(4*b^2)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(a^2+2abx+b^2x^2\right)\,dx &= \int (a+bx)^2(d+ex)\,dx\\ &= \int \left(\frac{(bd-ae)(a+bx)^2}{b}+\frac{e(a+bx)^3}{b}\right)dx\\ &= \frac{(bd-ae)(a+bx)^3}{3b^2}+\frac{e(a+bx)^4}{4b^2}\end{aligned}$$

Mathematica [A] time = 0.0098354, size = 46, normalized size = 1.21

$$\frac{1}{12}x\left(6a^2(2d+ex)+4abx(3d+2ex)+b^2x^2(4d+3ex)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] (x*(6*a^2*(2*d + e*x) + 4*a*b*x*(3*d + 2*e*x) + b^2*x^2*(4*d + 3*e*x)))/12
```

Maple [A] time = 0.039, size = 49, normalized size = 1.3

$$\frac{b^2ex^4}{4}+\frac{\left(2\,aeb+b^2d\right)x^3}{3}+\frac{\left(a^2e+2\,abd\right)x^2}{2}+a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(b^2*x^2+2*a*b*x+a^2),x)
```

```
[Out] 1/4*b^2*e*x^4+1/3*(2*a*b*e+b^2*d)*x^3+1/2*(a^2*e+2*a*b*d)*x^2+a^2*d*x
```

Maxima [A] time = 1.18585, size = 65, normalized size = 1.71

$$\frac{1}{4}\,b^2ex^4+a^2dx+\frac{1}{3}\,\left(b^2d+2\,abe\right)x^3+\frac{1}{2}\,\left(2\,abd+a^2e\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*e*x^4 + a^2*d*x + 1/3*(b^2*d + 2*a*b*e)*x^3 + 1/2*(2*a*b*d + a^2*e)
*x^2
```

Fricas [A] time = 1.69772, size = 115, normalized size = 3.03

$$\frac{1}{4}x^4eb^2+\frac{1}{3}x^3db^2+\frac{2}{3}x^3eba+x^2dba+\frac{1}{2}x^2ea^2+xda^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*e*b^2 + 1/3*x^3*d*b^2 + 2/3*x^3*e*b*a + x^2*d*b*a + 1/2*x^2*e*a^2 +
 x*d*a^2
```

Sympy [A] time = 0.074636, size = 49, normalized size = 1.29

$$a^2dx+\frac{b^2ex^4}{4}+x^3\left(\frac{2abe}{3}+\frac{b^2d}{3}\right)+x^2\left(\frac{a^2e}{2}+abd\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b**2*x**2+2*a*b*x+a**2),x)
```

```
[Out] a**2*d*x + b**2*e*x**4/4 + x**3*(2*a*b*e/3 + b**2*d/3) + x**2*(a**2*e/2 + a
*b*d)
```

Giac [A] time = 1.16536, size = 70, normalized size = 1.84

$$\frac{1}{4}\,b^2x^4e+\frac{1}{3}\,b^2dx^3+\frac{2}{3}\,abx^3e+abdx^2+\frac{1}{2}\,a^2x^2e+a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

```
[Out] 1/4*b^2*x^4*e + 1/3*b^2*d*x^3 + 2/3*a*b*x^3*e + a*b*d*x^2 + 1/2*a^2*x^2*e +
 a^2*d*x
```

3.1455 $\int \left(a^2+2abx+b^2x^2\right)\,dx$

Optimal. Leaf size=22

$$a^2x+abx^2+\frac{b^2x^3}{3}$$

```
[Out] a^2*x + a*b*x^2 + (b^2*x^3)/3
```

Rubi [A] time = 0.004962, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$a^2x+abx^2+\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[a^2 + 2*a*b*x + b^2*x^2,x]
```

```
[Out] a^2*x + a*b*x^2 + (b^2*x^3)/3
```

Rubi steps

$$\int \left(a^2+2abx+b^2x^2\right)\,dx=a^2x+abx^2+\frac{b^2x^3}{3}$$

Mathematica [A] time = 0.0000511, size = 22, normalized size = 1.

$$a^2x+abx^2+\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^2 + 2*a*b*x + b^2*x^2,x]
```

```
[Out] a^2*x + a*b*x^2 + (b^2*x^3)/3
```

Maple [A] time = 0.038, size = 21, normalized size = 1.

$$a^2x+abx^2+\frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b^2*x^2+2*a*b*x+a^2,x)
```

```
[Out] a^2*x+a*b*x^2+1/3*b^2*x^3
```

Maxima [A] time = 1.17979, size = 27, normalized size = 1.23

$$\frac{1}{3}b^2x^3 + abx^2 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^2*x^2+2*a*b*x+a^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

Fricas [A] time = 1.64184, size = 42, normalized size = 1.91

$$\frac{1}{3}x^3b^2 + x^2ba + xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^2*x^2+2*a*b*x+a^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3*b^2 + x^2*b*a + x*a^2
```

Sympy [A] time = 0.064795, size = 19, normalized size = 0.86

$$a^2x + abx^2 + \frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b**2*x**2+2*a*b*x+a**2,x)
```

```
[Out] a**2*x + a*b*x**2 + b**2*x**3/3
```

Giac [A] time = 1.11468, size = 27, normalized size = 1.23

$$\frac{1}{3}b^2x^3 + abx^2 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^2*x^2+2*a*b*x+a^2,x, algorithm="giac")
```

```
[Out] 1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

3.1456 $\int \frac{a^2+2abx+b^2x^2}{d+ex}\,dx$

Optimal. Leaf size=50

$$-\frac{bx(bd-ae)}{e^2}+\frac{(bd-ae)^2\log(d+ex)}{e^3}+\frac{(a+bx)^2}{2e}$$

```
[Out] -((b*(b*d - a*e)*x)/e^2) + (a + b*x)^2/(2*e) + ((b*d - a*e)^2*Log[d + e*x])
/e^3
```

Rubi [A] time = 0.021483, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {27, 43}

$$-\frac{bx(bd-ae)}{e^2}+\frac{(bd-ae)^2\log(d+ex)}{e^3}+\frac{(a+bx)^2}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x),x]
```

```
[Out] -((b*(b*d - a*e)*x)/e^2) + (a + b*x)^2/(2*e) + ((b*d - a*e)^2*Log[d + e*x])
/e^3
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx+b^2x^2}{d+ex}\,dx &= \int \frac{(a+bx)^2}{d+ex}\,dx\\ &= \int \left(-\frac{b(bd-ae)}{e^2}+\frac{b(a+bx)}{e}+\frac{(-bd+ae)^2}{e^2(d+ex)}\right)dx\\ &= -\frac{b(bd-ae)x}{e^2}+\frac{(a+bx)^2}{2e}+\frac{(bd-ae)^2\log(d+ex)}{e^3}\end{aligned}$$

Mathematica [A] time = 0.0182524, size = 43, normalized size = 0.86

$$\frac{bex(4ae-2bd+bex)+2(bd-ae)^2\log(d+ex)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x),x]
```

```
[Out] (b*e*x*(-2*b*d + 4*a*e + b*e*x) + 2*(b*d - a*e)^2*Log[d + e*x])/(2*e^3)
```

Maple [A] time = 0.04, size = 74, normalized size = 1.5

$$\frac{b^2x^2}{2e}+2\frac{abx}{e}-\frac{b^2dx}{e^2}+\frac{\ln(ex+d)\,a^2}{e}-2\frac{\ln(ex+d)\,abd}{e^2}+\frac{d^2\ln(ex+d)\,b^2}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)/(e*x+d),x)
```

```
[Out] 1/2/e*x^2*b^2+2*b/e*a*x-1/e^2*b^2*d*x+1/e*ln(e*x+d)*a^2-2/e^2*ln(e*x+d)*a*b
*d+d^2/e^3*ln(e*x+d)*b^2
```

Maxima [A] time = 1.19115, size = 81, normalized size = 1.62

$$\frac{b^2ex^2-2\left(b^2d-2\,abe\right)x}{2\,e^2}+\frac{\left(b^2d^2-2\,abde+a^2e^2\right)\log(ex+d)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*e*x^2 - 2*(b^2*d - 2*a*b*e)*x)/e^2 + (b^2*d^2 - 2*a*b*d*e + a^2*e^
2)*log(e*x + d)/e^3
```

Fricas [A] time = 1.64159, size = 135, normalized size = 2.7

$$\frac{b^2e^2x^2-2\left(b^2de-2\,abe^2\right)x+2\left(b^2d^2-2\,abde+a^2e^2\right)\log(ex+d)}{2\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*e^2*x^2 - 2*(b^2*d*e - 2*a*b*e^2)*x + 2*(b^2*d^2 - 2*a*b*d*e + a^2
*e^2)*log(e*x + d))/e^3
```

Sympy [A] time = 0.401215, size = 44, normalized size = 0.88

$$\frac{b^2x^2}{2e}+\frac{x\left(2abe-b^2d\right)}{e^2}+\frac{(ae-bd)^2\log(d+ex)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)/(e*x+d),x)
```

```
[Out] b**2*x**2/(2*e) + x*(2*a*b*e - b**2*d)/e**2 + (a*e - b*d)**2*log(d + e*x)/e
**3
```

Giac [A] time = 1.14471, size = 82, normalized size = 1.64

$$\left(b^2d^2 - 2\,abde + a^2e^2\right)e^{(-3)}\log\left(|xe + d|\right) + \frac{1}{2}\left(b^2x^2e - 2\,b^2dx + 4\,abxe\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d),x, algorithm="giac")
```

```
[Out] (b^2*d^2 - 2*a*b*d*e + a^2*e^2)*e^(-3)*log(abs(x*e + d)) + 1/2*(b^2*x^2*e -
 2*b^2*d*x + 4*a*b*x*e)*e^(-2)
```

3.1457 $\int \frac{a^2+2abx+b^2x^2}{(d+ex)^2} dx$

Optimal. Leaf size=51

$$-\frac{(bd-ae)^2}{e^3(d+ex)}-\frac{2b(bd-ae)\log(d+ex)}{e^3}+\frac{b^2x}{e^2}$$

```
[Out] (b^2*x)/e^2 - (b*d - a*e)^2/(e^3*(d + e*x)) - (2*b*(b*d - a*e)*Log[d + e*x]
)/e^3
```

Rubi [A] time = 0.0477378, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {27, 43}

$$-\frac{(bd-ae)^2}{e^3(d+ex)}-\frac{2b(bd-ae)\log(d+ex)}{e^3}+\frac{b^2x}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^2,x]
```

```
[Out] (b^2*x)/e^2 - (b*d - a*e)^2/(e^3*(d + e*x)) - (2*b*(b*d - a*e)*Log[d + e*x]
)/e^3
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx+b^2x^2}{(d+ex)^2}\,dx &= \int \frac{(a+bx)^2}{(d+ex)^2}\,dx\\ &= \int \left(\frac{b^2}{e^2}+\frac{(-bd+ae)^2}{e^2(d+ex)^2}-\frac{2b(bd-ae)}{e^2(d+ex)}\right)dx\\ &= \frac{b^2x}{e^2}-\frac{(bd-ae)^2}{e^3(d+ex)}-\frac{2b(bd-ae)\log(d+ex)}{e^3}\end{aligned}$$

Mathematica [A] time = 0.0369414, size = 47, normalized size = 0.92

$$\frac{-\frac{(bd-ae)^2}{d+ex}+2b(ae-bd)\log(d+ex)+b^2ex}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^2,x]
```

```
[Out] (b^2*e*x - (b*d - a*e)^2/(d + e*x) + 2*b*(-(b*d) + a*e)*Log[d + e*x])/e^3
```

Maple [A] time = 0.046, size = 86, normalized size = 1.7

$$\frac{b^2x}{e^2}+2\,\frac{b\ln(ex+d)\,a}{e^2}-2\,\frac{b^2\ln(ex+d)\,d}{e^3}-\frac{a^2}{e\,(ex+d)}+2\,\frac{abd}{e^2\,(ex+d)}-\frac{b^2d^2}{e^3\,(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^2,x)
```

```
[Out] b^2*x/e^2+2*b/e^2*ln(e*x+d)*a-2*b^2/e^3*ln(e*x+d)*d-1/e/(e*x+d)*a^2+2/e^2/(
e*x+d)*d*a*b-1/e^3/(e*x+d)*b^2*d^2
```

Maxima [A] time = 1.18164, size = 90, normalized size = 1.76

$$\frac{b^2x}{e^2}-\frac{b^2d^2-2\,abde+a^2e^2}{e^4x+de^3}-\frac{2\left(b^2d-abe\right)\log(ex+d)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] b^2*x/e^2 - (b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(e^4*x + d*e^3) - 2*(b^2*d - a*
b*e)*log(e*x + d)/e^3
```

Fricas [A] time = 1.74924, size = 184, normalized size = 3.61

$$\frac{b^2e^2x^2+b^2dex-b^2d^2+2\,abde-a^2e^2-2\left(b^2d^2-abde+\left(b^2de-abe^2\right)x\right)\log(ex+d)}{e^4x+de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] (b^2*e^2*x^2 + b^2*d*e*x - b^2*d^2 + 2*a*b*d*e - a^2*e^2 - 2*(b^2*d^2 - a*b
*d*e + (b^2*d*e - a*b*e^2)*x)*log(e*x + d))/(e^4*x + d*e^3)
```

Sympy [A] time = 0.553262, size = 60, normalized size = 1.18

$$\frac{b^2x}{e^2}+\frac{2b\,(ae-bd)\log(d+ex)}{e^3}-\frac{a^2e^2-2abde+b^2d^2}{de^3+e^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)/(e*x+d)**2,x)
```

```
[Out] b**2*x/e**2 + 2*b*(a*e - b*d)*log(d + e*x)/e**3 - (a**2*e**2 - 2*a*b*d*e +
b**2*d**2)/(d*e**3 + e**4*x)
```

Giac [B] time = 1.14451, size = 150, normalized size = 2.94

$$-2\left(e^{(-1)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^2}\right)-\frac{de^{(-1)}}{xe+d}\right)abe^{(-1)}+\left(2\,de^{(-3)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^2}\right)+(xe+d)e^{(-3)}-\frac{d^2e^{(-3)}}{xe+d}\right)b^2-\frac{a^2e^{(-1)}}{xe+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -2*(e^(-1)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2) - d*e^(-1)/(x*e + d))*a*b*e
^(-1) + (2*d*e^(-3)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2) + (x*e + d)*e^(-3)
 - d^2*e^(-3)/(x*e + d))*b^2 - a^2*e^(-1)/(x*e + d)
```

3.1458 $\int \frac{a^2+2abx+b^2x^2}{(d+ex)^3} dx$

Optimal. Leaf size=59

$$\frac{2b(bd-ae)}{e^3(d+ex)} - \frac{(bd-ae)^2}{2e^3(d+ex)^2} + \frac{b^2\log(d+ex)}{e^3}$$

```
[Out] -(b*d - a*e)^2/(2*e^3*(d + e*x)^2) + (2*b*(b*d - a*e))/(e^3*(d + e*x)) + (b
^2*Log[d + e*x])/e^3
```

Rubi [A] time = 0.0379732, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {27, 43}

$$\frac{2b(bd-ae)}{e^3(d+ex)} - \frac{(bd-ae)^2}{2e^3(d+ex)^2} + \frac{b^2\log(d+ex)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^3,x]
```

```
[Out] -(b*d - a*e)^2/(2*e^3*(d + e*x)^2) + (2*b*(b*d - a*e))/(e^3*(d + e*x)) + (b
^2*Log[d + e*x])/e^3
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a^2+2abx+b^2x^2}{(d+ex)^3}\,dx &= \int \frac{(a+bx)^2}{(d+ex)^3}\,dx \\
&= \int \left(\frac{(-bd+ae)^2}{e^2(d+ex)^3} - \frac{2b(bd-ae)}{e^2(d+ex)^2} + \frac{b^2}{e^2(d+ex)}\right) dx \\
&= -\frac{(bd-ae)^2}{2e^3(d+ex)^2} + \frac{2b(bd-ae)}{e^3(d+ex)} + \frac{b^2\log(d+ex)}{e^3}
\end{aligned}$$

Mathematica [A] time = 0.0239186, size = 48, normalized size = 0.81

$$\frac{\frac{(bd-ae)(ae+3bd+4bex)}{(d+ex)^2} + 2b^2\log(d+ex)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^3,x]
```

```
[Out] (((b*d - a*e)*(3*b*d + a*e + 4*b*e*x))/(d + e*x)^2 + 2*b^2*Log[d + e*x])/(2
*e^3)
```

Maple [A] time = 0.046, size = 92, normalized size = 1.6

$$-\frac{a^2}{2e(ex+d)^2}+\frac{abd}{e^2(ex+d)^2}-\frac{b^2d^2}{2e^3(ex+d)^2}+\frac{b^2\ln(ex+d)}{e^3}-2\frac{ab}{e^2(ex+d)}+2\frac{b^2d}{e^3(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^3,x)
```

```
[Out] -1/2/e/(e*x+d)^2*a^2+1/e^2/(e*x+d)^2*a*b*d-1/2/e^3/(e*x+d)^2*b^2*d^2+b^2*ln
(e*x+d)/e^3-2*b/e^2/(e*x+d)*a+2*b^2/e^3/(e*x+d)*d
```

Maxima [A] time = 1.14444, size = 108, normalized size = 1.83

$$\frac{3b^2d^2-2abde-a^2e^2+4\left(b^2de-abe^2\right)x}{2\left(e^5x^2+2de^4x+d^2e^3\right)}+\frac{b^2\log(ex+d)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(3*b^2*d^2 - 2*a*b*d*e - a^2*e^2 + 4*(b^2*d*e - a*b*e^2)*x)/(e^5*x^2 +
2*d*e^4*x + d^2*e^3) + b^2*log(e*x + d)/e^3
```

Fricas [A] time = 1.6027, size = 205, normalized size = 3.47

$$\frac{3b^2d^2-2abde-a^2e^2+4\left(b^2de-abe^2\right)x+2\left(b^2e^2x^2+2b^2dex+b^2d^2\right)\log(ex+d)}{2\left(e^5x^2+2de^4x+d^2e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*b^2*d^2 - 2*a*b*d*e - a^2*e^2 + 4*(b^2*d*e - a*b*e^2)*x + 2*(b^2*e^2
*x^2 + 2*b^2*d*e*x + b^2*d^2)*log(e*x + d))/(e^5*x^2 + 2*d*e^4*x + d^2*e^3)
```

Sympy [A] time = 0.795259, size = 80, normalized size = 1.36

$$\frac{b^2\log(d+ex)}{e^3}-\frac{a^2e^2+2abde-3b^2d^2+x\left(4abe^2-4b^2de\right)}{2d^2e^3+4de^4x+2e^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)/(e*x+d)**3,x)
```

```
[Out] b**2*log(d + e*x)/e**3 - (a**2*e**2 + 2*a*b*d*e - 3*b**2*d**2 + x*(4*a*b*e*
*2 - 4*b**2*d*e))/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2)
```

Giac [A] time = 1.19965, size = 93, normalized size = 1.58

$$b^2e^{(-3)}\log\left(|xe+d|\right)+\frac{\left(4\left(b^2d-abe\right)x+\left(3\,b^2d^2-2\,abde-a^2e^2\right)e^{(-1)}\right)e^{(-2)}}{2\,(xe+d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] b^2*e^(-3)*log(abs(x*e + d)) + 1/2*(4*(b^2*d - a*b*e)*x + (3*b^2*d^2 - 2*a*
b*d*e - a^2*e^2)*e^(-1))*e^(-2)/(x*e + d)^2
```

3.1459 $\int \frac{a^2+2abx+b^2x^2}{(d+ex)^4} dx$

Optimal. Leaf size=28

$$\frac{(a+bx)^3}{3(d+ex)^3(bd-ae)}$$

```
[Out] (a + b*x)^3/(3*(b*d - a*e)*(d + e*x)^3)
```

Rubi [A] time = 0.0054238, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {27, 37}

$$\frac{(a+bx)^3}{3(d+ex)^3(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^4,x]
```

```
[Out] (a + b*x)^3/(3*(b*d - a*e)*(d + e*x)^3)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx+b^2x^2}{(d+ex)^4}\,dx &= \int \frac{(a+bx)^2}{(d+ex)^4}\,dx\\ &= \frac{(a+bx)^3}{3(bd-ae)(d+ex)^3}\end{aligned}$$

Mathematica [A] time = 0.0248631, size = 53, normalized size = 1.89

$$-\frac{a^2e^2+abe(d+3ex)+b^2\left(d^2+3dex+3e^2x^2\right)}{3e^3(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^4,x]
```

```
[Out] -(a^2*e^2 + a*b*e*(d + 3*e*x) + b^2*(d^2 + 3*d*e*x + 3*e^2*x^2))/(3*e^3*(d
+ e*x)^3)
```

Maple [B] time = 0.045, size = 71, normalized size = 2.5

$$-\frac{a^2e^2 - 2\,abde + b^2d^2}{3\,e^3\,(ex+d)^3} - \frac{(ae-bd)\,b}{e^3\,(ex+d)^2} - \frac{b^2}{e^3\,(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^4,x)
```

```
[Out] -1/3*(a^2*e^2-2*a*b*d*e+b^2*d^2)/e^3/(e*x+d)^3-(a*e-b*d)*b/e^3/(e*x+d)^2-b^
2/e^3/(e*x+d)
```

Maxima [B] time = 1.10879, size = 113, normalized size = 4.04

$$-\frac{3\,b^2e^2x^2 + b^2d^2 + abde + a^2e^2 + 3\left(b^2de + abe^2\right)x}{3\left(e^6x^3 + 3\,de^5x^2 + 3\,d^2e^4x + d^3e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^2*e^2*x^2 + b^2*d^2 + a*b*d*e + a^2*e^2 + 3*(b^2*d*e + a*b*e^2)*x
)/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3)
```

Fricas [B] time = 1.68778, size = 170, normalized size = 6.07

$$-\frac{3\,b^2e^2x^2 + b^2d^2 + abde + a^2e^2 + 3\left(b^2de + abe^2\right)x}{3\left(e^6x^3 + 3\,de^5x^2 + 3\,d^2e^4x + d^3e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*e^2*x^2 + b^2*d^2 + a*b*d*e + a^2*e^2 + 3*(b^2*d*e + a*b*e^2)*x
)/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3)
```

Sympy [B] time = 0.868145, size = 88, normalized size = 3.14

$$-\frac{a^2e^2 + abde + b^2d^2 + 3b^2e^2x^2 + x\left(3abe^2 + 3b^2de\right)}{3d^3e^3 + 9d^2e^4x + 9de^5x^2 + 3e^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)/(e*x+d)**4,x)
```

```
[Out] -(a**2*e**2 + a*b*d*e + b**2*d**2 + 3*b**2*e**2*x**2 + x*(3*a*b*e**2 + 3*b*
*2*d*e))/(3*d**3*e**3 + 9*d**2*e**4*x + 9*d*e**5*x**2 + 3*e**6*x**3)
```

Giac [B] time = 1.17025, size = 78, normalized size = 2.79

$$-\frac{\left(3 b^2 x^2 e^2 + 3 b^2 d x e + b^2 d^2 + 3 a b x e^2 + a b d e + a^2 e^2\right) e^{(-3)}}{3 (x e + d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*x^2*e^2 + 3*b^2*d*x*e + b^2*d^2 + 3*a*b*x*e^2 + a*b*d*e + a^2*e
^2)*e^(-3)/(x*e + d)^3
```

3.1460 $\int \frac{a^2+2abx+b^2x^2}{(d+ex)^5} dx$

Optimal. Leaf size=65

$$\frac{2b(bd-ae)}{3e^3(d+ex)^3} - \frac{(bd-ae)^2}{4e^3(d+ex)^4} - \frac{b^2}{2e^3(d+ex)^2}$$

```
[Out] -(b*d - a*e)^2/(4*e^3*(d + e*x)^4) + (2*b*(b*d - a*e))/(3*e^3*(d + e*x)^3)
- b^2/(2*e^3*(d + e*x)^2)
```

Rubi [A] time = 0.0371152, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {27, 43}

$$\frac{2b(bd-ae)}{3e^3(d+ex)^3} - \frac{(bd-ae)^2}{4e^3(d+ex)^4} - \frac{b^2}{2e^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^5,x]
```

```
[Out] -(b*d - a*e)^2/(4*e^3*(d + e*x)^4) + (2*b*(b*d - a*e))/(3*e^3*(d + e*x)^3)
- b^2/(2*e^3*(d + e*x)^2)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a^2+2abx+b^2x^2}{(d+ex)^5} dx &= \int \frac{(a+bx)^2}{(d+ex)^5} dx \\
&= \int \left(\frac{(-bd+ae)^2}{e^2(d+ex)^5} - \frac{2b(bd-ae)}{e^2(d+ex)^4} + \frac{b^2}{e^2(d+ex)^3}\right) dx \\
&= -\frac{(bd-ae)^2}{4e^3(d+ex)^4} + \frac{2b(bd-ae)}{3e^3(d+ex)^3} - \frac{b^2}{2e^3(d+ex)^2}
\end{aligned}$$

Mathematica [A] time = 0.0202525, size = 55, normalized size = 0.85

$$-\frac{3a^2e^2+2abe(d+4ex)+b^2\left(d^2+4dex+6e^2x^2\right)}{12e^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^5,x]
```

```
[Out] -(3*a^2*e^2 + 2*a*b*e*(d + 4*e*x) + b^2*(d^2 + 4*d*e*x + 6*e^2*x^2))/(12*e^
3*(d + e*x)^4)
```

Maple [A] time = 0.045, size = 71, normalized size = 1.1

$$-\frac{(2\,ae-2\,bd)\,b}{3\,e^3\,(ex+d)^3}-\frac{a^2e^2-2\,abde+b^2d^2}{4\,e^3\,(ex+d)^4}-\frac{b^2}{2\,e^3\,(ex+d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^5,x)
```

```
[Out] -2/3*(a*e-b*d)*b/e^3/(e*x+d)^3-1/4*(a^2*e^2-2*a*b*d*e+b^2*d^2)/e^3/(e*x+d)^
4-1/2*b^2/e^3/(e*x+d)^2
```

Maxima [A] time = 1.17613, size = 132, normalized size = 2.03

$$-\frac{6\,b^2e^2x^2+b^2d^2+2\,abde+3\,a^2e^2+4\left(b^2de+2\,abe^2\right)x}{12\left(e^7x^4+4\,de^6x^3+6\,d^2e^5x^2+4\,d^3e^4x+d^4e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(6*b^2*e^2*x^2 + b^2*d^2 + 2*a*b*d*e + 3*a^2*e^2 + 4*(b^2*d*e + 2*a*b
*e^2)*x)/(e^7*x^4 + 4*d*e^6*x^3 + 6*d^2*e^5*x^2 + 4*d^3*e^4*x + d^4*e^3)
```

Fricas [A] time = 1.68925, size = 201, normalized size = 3.09

$$-\frac{6\,b^2e^2x^2+b^2d^2+2\,abde+3\,a^2e^2+4\left(b^2de+2\,abe^2\right)x}{12\left(e^7x^4+4\,de^6x^3+6\,d^2e^5x^2+4\,d^3e^4x+d^4e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^5,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^2*e^2*x^2 + b^2*d^2 + 2*a*b*d*e + 3*a^2*e^2 + 4*(b^2*d*e + 2*a*b
*e^2)*x)/(e^7*x^4 + 4*d*e^6*x^3 + 6*d^2*e^5*x^2 + 4*d^3*e^4*x + d^4*e^3)
```

Sympy [A] time = 1.10241, size = 104, normalized size = 1.6

$$-\frac{3a^2e^2+2abde+b^2d^2+6b^2e^2x^2+x\left(8abe^2+4b^2de\right)}{12d^4e^3+48d^3e^4x+72d^2e^5x^2+48de^6x^3+12e^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)/(e*x+d)**5,x)
```

```
[Out] -(3*a**2*e**2 + 2*a*b*d*e + b**2*d**2 + 6*b**2*e**2*x**2 + x*(8*a*b*e**2 +
4*b**2*d*e))/(12*d**4*e**3 + 48*d**3*e**4*x + 72*d**2*e**5*x**2 + 48*d*e**6
*x**3 + 12*e**7*x**4)
```

Giac [A] time = 1.14012, size = 130, normalized size = 2.

$$-\frac{1}{12}\left(\frac{6\,b^2e^{(-2)}}{(xe+d)^2}-\frac{8\,b^2de^{(-2)}}{(xe+d)^3}+\frac{3\,b^2d^2e^{(-2)}}{(xe+d)^4}+\frac{8\,abe^{(-1)}}{(xe+d)^3}-\frac{6\,abde^{(-1)}}{(xe+d)^4}+\frac{3\,a^2}{(xe+d)^4}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] -1/12*(6*b^2*e^(-2)/(x*e + d)^2 - 8*b^2*d*e^(-2)/(x*e + d)^3 + 3*b^2*d^2*e^
(-2)/(x*e + d)^4 + 8*a*b*e^(-1)/(x*e + d)^3 - 6*a*b*d*e^(-1)/(x*e + d)^4 +
3*a^2/(x*e + d)^4)*e^(-1)
```

3.1461 $\int \frac{a^2+2abx+b^2x^2}{(d+ex)^6} dx$

Optimal. Leaf size=65

$$\frac{b(bd-ae)}{2e^3(d+ex)^4}-\frac{(bd-ae)^2}{5e^3(d+ex)^5}-\frac{b^2}{3e^3(d+ex)^3}$$

```
[Out] -(b*d - a*e)^2/(5*e^3*(d + e*x)^5) + (b*(b*d - a*e))/(2*e^3*(d + e*x)^4) -
b^2/(3*e^3*(d + e*x)^3)
```

Rubi [A] time = 0.0364159, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {27, 43}

$$\frac{b(bd-ae)}{2e^3(d+ex)^4}-\frac{(bd-ae)^2}{5e^3(d+ex)^5}-\frac{b^2}{3e^3(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^6,x]
```

```
[Out] -(b*d - a*e)^2/(5*e^3*(d + e*x)^5) + (b*(b*d - a*e))/(2*e^3*(d + e*x)^4) -
b^2/(3*e^3*(d + e*x)^3)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a^2+2abx+b^2x^2}{(d+ex)^6}\,dx &= \int \frac{(a+bx)^2}{(d+ex)^6}\,dx\\
&= \int \left(\frac{(-bd+ae)^2}{e^2(d+ex)^6}-\frac{2b(bd-ae)}{e^2(d+ex)^5}+\frac{b^2}{e^2(d+ex)^4}\right)dx\\
&= -\frac{(bd-ae)^2}{5e^3(d+ex)^5}+\frac{b(bd-ae)}{2e^3(d+ex)^4}-\frac{b^2}{3e^3(d+ex)^3}
\end{aligned}$$

Mathematica [A] time = 0.0268726, size = 55, normalized size = 0.85

$$-\frac{6a^2e^2+3abe(d+5ex)+b^2\left(d^2+5dex+10e^2x^2\right)}{30e^3(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^6,x]
```

```
[Out] -(6*a^2*e^2 + 3*a*b*e*(d + 5*e*x) + b^2*(d^2 + 5*d*e*x + 10*e^2*x^2))/(30*e
^3*(d + e*x)^5)
```

Maple [A] time = 0.045, size = 71, normalized size = 1.1

$$-\frac{b^2}{3\,e^3\,(ex+d)^3}-\frac{(ae-bd)\,b}{2\,e^3\,(ex+d)^4}-\frac{a^2e^2-2\,abde+b^2d^2}{5\,e^3\,(ex+d)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^6,x)
```

```
[Out] -1/3*b^2/e^3/(e*x+d)^3-1/2*(a*e-b*d)*b/e^3/(e*x+d)^4-1/5*(a^2*e^2-2*a*b*d*e
+b^2*d^2)/e^3/(e*x+d)^5
```

Maxima [A] time = 1.24846, size = 147, normalized size = 2.26

$$-\frac{10\,b^2e^2x^2+b^2d^2+3\,abde+6\,a^2e^2+5\left(b^2de+3\,abe^2\right)x}{30\left(e^8x^5+5\,de^7x^4+10\,d^2e^6x^3+10\,d^3e^5x^2+5\,d^4e^4x+d^5e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^6,x, algorithm="maxima")
```

```
[Out] -1/30*(10*b^2*e^2*x^2 + b^2*d^2 + 3*a*b*d*e + 6*a^2*e^2 + 5*(b^2*d*e + 3*a*
b*e^2)*x)/(e^8*x^5 + 5*d*e^7*x^4 + 10*d^2*e^6*x^3 + 10*d^3*e^5*x^2 + 5*d^4*
e^4*x + d^5*e^3)
```

Fricas [A] time = 1.76496, size = 227, normalized size = 3.49

$$-\frac{10\,b^2e^2x^2+b^2d^2+3\,abde+6\,a^2e^2+5\left(b^2de+3\,abe^2\right)x}{30\left(e^8x^5+5\,de^7x^4+10\,d^2e^6x^3+10\,d^3e^5x^2+5\,d^4e^4x+d^5e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^6,x, algorithm="fricas")
```

```
[Out] -1/30*(10*b^2*e^2*x^2 + b^2*d^2 + 3*a*b*d*e + 6*a^2*e^2 + 5*(b^2*d*e + 3*a*
b*e^2)*x)/(e^8*x^5 + 5*d*e^7*x^4 + 10*d^2*e^6*x^3 + 10*d^3*e^5*x^2 + 5*d^4*
e^4*x + d^5*e^3)
```

Sympy [B] time = 1.28312, size = 116, normalized size = 1.78

$$-\frac{6a^2e^2+3abde+b^2d^2+10b^2e^2x^2+x\left(15abe^2+5b^2de\right)}{30d^5e^3+150d^4e^4x+300d^3e^5x^2+300d^2e^6x^3+150de^7x^4+30e^8x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)/(e*x+d)**6,x)
```

```
[Out] -(6*a**2*e**2 + 3*a*b*d*e + b**2*d**2 + 10*b**2*e**2*x**2 + x*(15*a*b*e**2
+ 5*b**2*d*e))/(30*d**5*e**3 + 150*d**4*e**4*x + 300*d**3*e**5*x**2 + 300*d
**2*e**6*x**3 + 150*d*e**7*x**4 + 30*e**8*x**5)
```

Giac [A] time = 1.14132, size = 81, normalized size = 1.25

$$-\frac{\left(10\, b^2 x^2 e^2 + 5\, b^2 d x e + b^2 d^2 + 15\, a b x e^2 + 3\, a b d e + 6\, a^2 e^2\right) e^{(-3)}}{30\, (x e + d)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^6,x, algorithm="giac")
```

```
[Out] -1/30*(10*b^2*x^2*e^2 + 5*b^2*d*x*e + b^2*d^2 + 15*a*b*x*e^2 + 3*a*b*d*e +
6*a^2*e^2)*e^(-3)/(x*e + d)^5
```

3.1462 $\int \frac{a^2+2abx+b^2x^2}{(d+ex)^7} dx$

Optimal. Leaf size=65

$$\frac{2b(bd-ae)}{5e^3(d+ex)^5}-\frac{(bd-ae)^2}{6e^3(d+ex)^6}-\frac{b^2}{4e^3(d+ex)^4}$$

```
[Out] -(b*d - a*e)^2/(6*e^3*(d + e*x)^6) + (2*b*(b*d - a*e))/(5*e^3*(d + e*x)^5)
- b^2/(4*e^3*(d + e*x)^4)
```

Rubi [A] time = 0.0365427, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {27, 43}

$$\frac{2b(bd-ae)}{5e^3(d+ex)^5}-\frac{(bd-ae)^2}{6e^3(d+ex)^6}-\frac{b^2}{4e^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^7,x]
```

```
[Out] -(b*d - a*e)^2/(6*e^3*(d + e*x)^6) + (2*b*(b*d - a*e))/(5*e^3*(d + e*x)^5)
- b^2/(4*e^3*(d + e*x)^4)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx+b^2x^2}{(d+ex)^7}\,dx &= \int \frac{(a+bx)^2}{(d+ex)^7}\,dx\\ &= \int \left(\frac{(-bd+ae)^2}{e^2(d+ex)^7}-\frac{2b(bd-ae)}{e^2(d+ex)^6}+\frac{b^2}{e^2(d+ex)^5}\right)dx\\ &= -\frac{(bd-ae)^2}{6e^3(d+ex)^6}+\frac{2b(bd-ae)}{5e^3(d+ex)^5}-\frac{b^2}{4e^3(d+ex)^4}\end{aligned}$$

Mathematica [A] time = 0.0211527, size = 55, normalized size = 0.85

$$-\frac{10a^2e^2+4abe(d+6ex)+b^2\left(d^2+6dex+15e^2x^2\right)}{60e^3(d+ex)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^7,x]
```

```
[Out] -(10*a^2*e^2 + 4*a*b*e*(d + 6*e*x) + b^2*(d^2 + 6*d*e*x + 15*e^2*x^2))/(60*
e^3*(d + e*x)^6)
```

Maple [A] time = 0.045, size = 71, normalized size = 1.1

$$-\frac{a^2e^2-2\,abde+b^2d^2}{6\,e^3\,(ex+d)^6}-\frac{b^2}{4\,e^3\,(ex+d)^4}-\frac{(2\,ae-2\,bd)\,b}{5\,e^3\,(ex+d)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^7,x)
```

```
[Out] -1/6*(a^2*e^2-2*a*b*d*e+b^2*d^2)/e^3/(e*x+d)^6-1/4*b^2/e^3/(e*x+d)^4-2/5*(a
*e-b*d)*b/e^3/(e*x+d)^5
```

Maxima [B] time = 1.20014, size = 162, normalized size = 2.49

$$-\frac{15\,b^2e^2x^2+b^2d^2+4\,abde+10\,a^2e^2+6\left(b^2de+4\,abe^2\right)x}{60\left(e^9x^6+6\,de^8x^5+15\,d^2e^7x^4+20\,d^3e^6x^3+15\,d^4e^5x^2+6\,d^5e^4x+d^6e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(15*b^2*e^2*x^2 + b^2*d^2 + 4*a*b*d*e + 10*a^2*e^2 + 6*(b^2*d*e + 4*a
*b*e^2)*x)/(e^9*x^6 + 6*d*e^8*x^5 + 15*d^2*e^7*x^4 + 20*d^3*e^6*x^3 + 15*d^
4*e^5*x^2 + 6*d^5*e^4*x + d^6*e^3)
```

Fricas [B] time = 1.77717, size = 251, normalized size = 3.86

$$-\frac{15\,b^2e^2x^2+b^2d^2+4\,abde+10\,a^2e^2+6\left(b^2de+4\,abe^2\right)x}{60\left(e^9x^6+6\,de^8x^5+15\,d^2e^7x^4+20\,d^3e^6x^3+15\,d^4e^5x^2+6\,d^5e^4x+d^6e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^7,x, algorithm="fricas")
```

```
[Out] -1/60*(15*b^2*e^2*x^2 + b^2*d^2 + 4*a*b*d*e + 10*a^2*e^2 + 6*(b^2*d*e + 4*a
*b*e^2)*x)/(e^9*x^6 + 6*d*e^8*x^5 + 15*d^2*e^7*x^4 + 20*d^3*e^6*x^3 + 15*d^
4*e^5*x^2 + 6*d^5*e^4*x + d^6*e^3)
```

Sympy [B] time = 1.57847, size = 128, normalized size = 1.97

$$-\frac{10a^2e^2+4abde+b^2d^2+15b^2e^2x^2+x\left(24abe^2+6b^2de\right)}{60d^6e^3+360d^5e^4x+900d^4e^5x^2+1200d^3e^6x^3+900d^2e^7x^4+360de^8x^5+60e^9x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)/(e*x+d)**7,x)
```

```
[Out] -(10*a**2*e**2 + 4*a*b*d*e + b**2*d**2 + 15*b**2*e**2*x**2 + x*(24*a*b*e**2
 + 6*b**2*d*e))/(60*d**6*e**3 + 360*d**5*e**4*x + 900*d**4*e**5*x**2 + 1200
*d**3*e**6*x**3 + 900*d**2*e**7*x**4 + 360*d*e**8*x**5 + 60*e**9*x**6)
```

Giac [A] time = 1.1518, size = 81, normalized size = 1.25

$$-\frac{\left(15\,b^2x^2e^2+6\,b^2dxe+b^2d^2+24\,abxe^2+4\,abde+10\,a^2e^2\right)e^{(-3)}}{60\,(xe+d)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^7,x, algorithm="giac")
```

```
[Out] -1/60*(15*b^2*x^2*e^2 + 6*b^2*d*x*e + b^2*d^2 + 24*a*b*x*e^2 + 4*a*b*d*e +
10*a^2*e^2)*e^(-3)/(x*e + d)^6
```

3.1463 $\int (d+ex)^6 \left(a^2+2abx+b^2x^2\right)^2 \, dx$

Optimal. Leaf size=119

$$-\frac{2b^3(d+ex)^{10}(bd-ae)}{5e^5}+\frac{2b^2(d+ex)^9(bd-ae)^2}{3e^5}-\frac{b(d+ex)^8(bd-ae)^3}{2e^5}+\frac{(d+ex)^7(bd-ae)^4}{7e^5}+\frac{b^4(d+ex)^{11}}{11e^5}$$

```
[Out] ((b*d - a*e)^4*(d + e*x)^7)/(7*e^5) - (b*(b*d - a*e)^3*(d + e*x)^8)/(2*e^5)
 + (2*b^2*(b*d - a*e)^2*(d + e*x)^9)/(3*e^5) - (2*b^3*(b*d - a*e)*(d + e*x)
^10)/(5*e^5) + (b^4*(d + e*x)^11)/(11*e^5)
```

Rubi [A] time = 0.266867, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$-\frac{2b^3(d+ex)^{10}(bd-ae)}{5e^5}+\frac{2b^2(d+ex)^9(bd-ae)^2}{3e^5}-\frac{b(d+ex)^8(bd-ae)^3}{2e^5}+\frac{(d+ex)^7(bd-ae)^4}{7e^5}+\frac{b^4(d+ex)^{11}}{11e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^6*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] ((b*d - a*e)^4*(d + e*x)^7)/(7*e^5) - (b*(b*d - a*e)^3*(d + e*x)^8)/(2*e^5)
 + (2*b^2*(b*d - a*e)^2*(d + e*x)^9)/(3*e^5) - (2*b^3*(b*d - a*e)*(d + e*x)
^10)/(5*e^5) + (b^4*(d + e*x)^11)/(11*e^5)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^6 \left(a^2+2abx+b^2x^2\right)^2 \, dx &= \int (a+bx)^4(d+ex)^6 \, dx\\ &= \int \left(\frac{(-bd+ae)^4(d+ex)^6}{e^4}-\frac{4b(bd-ae)^3(d+ex)^7}{e^4}+\frac{6b^2(bd-ae)^2(d+ex)^8}{e^4}-\frac{4b^3(b\right.\\ &= \frac{(bd-ae)^4(d+ex)^7}{7e^5}-\frac{b(bd-ae)^3(d+ex)^8}{2e^5}+\frac{2b^2(bd-ae)^2(d+ex)^9}{3e^5}-\frac{2b^3(bd-ae)}{5e^5}\end{aligned}$$

Mathematica [B] time = 0.0612715, size = 398, normalized size = 3.34

$$\frac{1}{3}b^2e^4x^9\left(2a^2e^2+8abde+5b^2d^2\right)+\frac{1}{2}be^3x^8\left(9a^2bde^2+a^3e^3+15ab^2d^2e+5b^3d^3\right)+\frac{1}{7}e^2x^7\left(90a^2b^2d^2e^2+24a^3bde^3+a^4e^4\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^6*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] a^4*d^6*x + a^3*d^5*(2*b*d + 3*a*e)*x^2 + a^2*d^4*(2*b^2*d^2 + 8*a*b*d*e +
5*a^2*e^2)*x^3 + a*d^3*(b^3*d^3 + 9*a*b^2*d^2*e + 15*a^2*b*d*e^2 + 5*a^3*e^
3)*x^4 + (d^2*(b^4*d^4 + 24*a*b^3*d^3*e + 90*a^2*b^2*d^2*e^2 + 80*a^3*b*d*e
^3 + 15*a^4*e^4)*x^5)/5 + d*e*(b^4*d^4 + 10*a*b^3*d^3*e + 20*a^2*b^2*d^2*e^
2 + 10*a^3*b*d*e^3 + a^4*e^4)*x^6 + (e^2*(15*b^4*d^4 + 80*a*b^3*d^3*e + 90*
a^2*b^2*d^2*e^2 + 24*a^3*b*d*e^3 + a^4*e^4)*x^7)/7 + (b*e^3*(5*b^3*d^3 + 15
*a*b^2*d^2*e + 9*a^2*b*d*e^2 + a^3*e^3)*x^8)/2 + (b^2*e^4*(5*b^2*d^2 + 8*a*
b*d*e + 2*a^2*e^2)*x^9)/3 + (b^3*e^5*(3*b*d + 2*a*e)*x^10)/5 + (b^4*e^6*x^1
1)/11
```

Maple [B] time = 0.045, size = 427, normalized size = 3.6

$$\frac{e^6b^4x^{11}}{11} + \frac{\left(4\,e^6ab^3 + 6\,de^5b^4\right)x^{10}}{10} + \frac{\left(6\,e^6b^2a^2 + 24\,de^5ab^3 + 15\,d^2e^4b^4\right)x^9}{9} + \frac{\left(4\,e^6a^3b + 36\,de^5b^2a^2 + 60\,d^2e^4ab^3 + 20\,c\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^6*(b^2*x^2+2*a*b*x+a^2)^2,x)
```

```
[Out] 1/11*e^6*b^4*x^11+1/10*(4*a*b^3*e^6+6*b^4*d*e^5)*x^10+1/9*(6*a^2*b^2*e^6+24
*a*b^3*d*e^5+15*b^4*d^2*e^4)*x^9+1/8*(4*a^3*b*e^6+36*a^2*b^2*d*e^5+60*a*b^3
*d^2*e^4+20*b^4*d^3*e^3)*x^8+1/7*(a^4*e^6+24*a^3*b*d*e^5+90*a^2*b^2*d^2*e^4
+80*a*b^3*d^3*e^3+15*b^4*d^4*e^2)*x^7+1/6*(6*a^4*d*e^5+60*a^3*b*d^2*e^4+120
*a^2*b^2*d^3*e^3+60*a*b^3*d^4*e^2+6*b^4*d^5*e)*x^6+1/5*(15*a^4*d^2*e^4+80*a
^3*b*d^3*e^3+90*a^2*b^2*d^4*e^2+24*a*b^3*d^5*e+b^4*d^6)*x^5+1/4*(20*a^4*d^3
*e^3+60*a^3*b*d^4*e^2+36*a^2*b^2*d^5*e+4*a*b^3*d^6)*x^4+1/3*(15*a^4*d^4*e^2
+24*a^3*b*d^5*e+6*a^2*b^2*d^6)*x^3+1/2*(6*a^4*d^5*e+4*a^3*b*d^6)*x^2+d^6*a^
4*x
```

Maxima [B] time = 1.17021, size = 564, normalized size = 4.74

$$\frac{1}{11}\,b^4e^6x^{11} + a^4d^6x + \frac{1}{5}\left(3\,b^4de^5 + 2\,ab^3e^6\right)x^{10} + \frac{1}{3}\left(5\,b^4d^2e^4 + 8\,ab^3de^5 + 2\,a^2b^2e^6\right)x^9 + \frac{1}{2}\left(5\,b^4d^3e^3 + 15\,ab^3d^2e^4 + 9\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^4*e^6*x^11 + a^4*d^6*x + 1/5*(3*b^4*d*e^5 + 2*a*b^3*e^6)*x^10 + 1/3*
(5*b^4*d^2*e^4 + 8*a*b^3*d*e^5 + 2*a^2*b^2*e^6)*x^9 + 1/2*(5*b^4*d^3*e^3 +
15*a*b^3*d^2*e^4 + 9*a^2*b^2*d*e^5 + a^3*b*e^6)*x^8 + 1/7*(15*b^4*d^4*e^2 +
 80*a*b^3*d^3*e^3 + 90*a^2*b^2*d^2*e^4 + 24*a^3*b*d*e^5 + a^4*e^6)*x^7 + (b
^4*d^5*e + 10*a*b^3*d^4*e^2 + 20*a^2*b^2*d^3*e^3 + 10*a^3*b*d^2*e^4 + a^4*d
*e^5)*x^6 + 1/5*(b^4*d^6 + 24*a*b^3*d^5*e + 90*a^2*b^2*d^4*e^2 + 80*a^3*b*d
^3*e^3 + 15*a^4*d^2*e^4)*x^5 + (a*b^3*d^6 + 9*a^2*b^2*d^5*e + 15*a^3*b*d^4*
e^2 + 5*a^4*d^3*e^3)*x^4 + (2*a^2*b^2*d^6 + 8*a^3*b*d^5*e + 5*a^4*d^4*e^2)*
x^3 + (2*a^3*b*d^6 + 3*a^4*d^5*e)*x^2
```

Fricas [B] time = 1.47675, size = 992, normalized size = 8.34

$$\frac{1}{11}x^{11}e^6b^4 + \frac{3}{5}x^{10}e^5db^4 + \frac{2}{5}x^{10}e^6b^3a + \frac{5}{3}x^9e^4d^2b^4 + \frac{8}{3}x^9e^5db^3a + \frac{2}{3}x^9e^6b^2a^2 + \frac{5}{2}x^8e^3d^3b^4 + \frac{15}{2}x^8e^4d^2b^3a + \frac{9}{2}x^8e^5db^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*e^6*b^4 + 3/5*x^10*e^5*d*b^4 + 2/5*x^10*e^6*b^3*a + 5/3*x^9*e^4*d
^2*b^4 + 8/3*x^9*e^5*d*b^3*a + 2/3*x^9*e^6*b^2*a^2 + 5/2*x^8*e^3*d^3*b^4 +
15/2*x^8*e^4*d^2*b^3*a + 9/2*x^8*e^5*d*b^2*a^2 + 1/2*x^8*e^6*b*a^3 + 15/7*x
^7*e^2*d^4*b^4 + 80/7*x^7*e^3*d^3*b^3*a + 90/7*x^7*e^4*d^2*b^2*a^2 + 24/7*x
^7*e^5*d*b*a^3 + 1/7*x^7*e^6*a^4 + x^6*e*d^5*b^4 + 10*x^6*e^2*d^4*b^3*a + 2
0*x^6*e^3*d^3*b^2*a^2 + 10*x^6*e^4*d^2*b*a^3 + x^6*e^5*d*a^4 + 1/5*x^5*d^6*
b^4 + 24/5*x^5*e*d^5*b^3*a + 18*x^5*e^2*d^4*b^2*a^2 + 16*x^5*e^3*d^3*b*a^3
+ 3*x^5*e^4*d^2*a^4 + x^4*d^6*b^3*a + 9*x^4*e*d^5*b^2*a^2 + 15*x^4*e^2*d^4*
b*a^3 + 5*x^4*e^3*d^3*a^4 + 2*x^3*d^6*b^2*a^2 + 8*x^3*e*d^5*b*a^3 + 5*x^3*e
^2*d^4*a^4 + 2*x^2*d^6*b*a^3 + 3*x^2*e*d^5*a^4 + x*d^6*a^4
```

Sympy [B] time = 0.131337, size = 462, normalized size = 3.88

$$a^4d^6x + \frac{b^4e^6x^{11}}{11} + x^{10}\left(\frac{2ab^3e^6}{5} + \frac{3b^4de^5}{5}\right) + x^9\left(\frac{2a^2b^2e^6}{3} + \frac{8ab^3de^5}{3} + \frac{5b^4d^2e^4}{3}\right) + x^8\left(\frac{a^3be^6}{2} + \frac{9a^2b^2de^5}{2} + \frac{15ab^3d^2e^4}{2} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**6*(b**2*x**2+2*a*b*x+a**2)**2,x)
```

```
[Out] a**4*d**6*x + b**4*e**6*x**11/11 + x**10*(2*a*b**3*e**6/5 + 3*b**4*d*e**5/5
) + x**9*(2*a**2*b**2*e**6/3 + 8*a*b**3*d*e**5/3 + 5*b**4*d**2*e**4/3) + x*
*8*(a**3*b*e**6/2 + 9*a**2*b**2*d*e**5/2 + 15*a*b**3*d**2*e**4/2 + 5*b**4*d
**3*e**3/2) + x**7*(a**4*e**6/7 + 24*a**3*b*d*e**5/7 + 90*a**2*b**2*d**2*e*
*4/7 + 80*a*b**3*d**3*e**3/7 + 15*b**4*d**4*e**2/7) + x**6*(a**4*d*e**5 + 1
0*a**3*b*d**2*e**4 + 20*a**2*b**2*d**3*e**3 + 10*a*b**3*d**4*e**2 + b**4*d*
*5*e) + x**5*(3*a**4*d**2*e**4 + 16*a**3*b*d**3*e**3 + 18*a**2*b**2*d**4*e*
*2 + 24*a*b**3*d**5*e/5 + b**4*d**6/5) + x**4*(5*a**4*d**3*e**3 + 15*a**3*b
*d**4*e**2 + 9*a**2*b**2*d**5*e + a*b**3*d**6) + x**3*(5*a**4*d**4*e**2 + 8
*a**3*b*d**5*e + 2*a**2*b**2*d**6) + x**2*(3*a**4*d**5*e + 2*a**3*b*d**6)
```

Giac [B] time = 1.09952, size = 608, normalized size = 5.11

$$\frac{1}{11}b^4x^{11}e^6 + \frac{3}{5}b^4dx^{10}e^5 + \frac{5}{3}b^4d^2x^9e^4 + \frac{5}{2}b^4d^3x^8e^3 + \frac{15}{7}b^4d^4x^7e^2 + b^4d^5x^6e + \frac{1}{5}b^4d^6x^5 + \frac{2}{5}ab^3x^{10}e^6 + \frac{8}{3}ab^3dx^9e^5 + \frac{15}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/11*b^4*x^11*e^6 + 3/5*b^4*d*x^10*e^5 + 5/3*b^4*d^2*x^9*e^4 + 5/2*b^4*d^3*
x^8*e^3 + 15/7*b^4*d^4*x^7*e^2 + b^4*d^5*x^6*e + 1/5*b^4*d^6*x^5 + 2/5*a*b^
3*x^10*e^6 + 8/3*a*b^3*d*x^9*e^5 + 15/2*a*b^3*d^2*x^8*e^4 + 80/7*a*b^3*d^3*
x^7*e^3 + 10*a*b^3*d^4*x^6*e^2 + 24/5*a*b^3*d^5*x^5*e + a*b^3*d^6*x^4 + 2/3
*a^2*b^2*x^9*e^6 + 9/2*a^2*b^2*d*x^8*e^5 + 90/7*a^2*b^2*d^2*x^7*e^4 + 20*a^
2*b^2*d^3*x^6*e^3 + 18*a^2*b^2*d^4*x^5*e^2 + 9*a^2*b^2*d^5*x^4*e + 2*a^2*b^
2*d^6*x^3 + 1/2*a^3*b*x^8*e^6 + 24/7*a^3*b*d*x^7*e^5 + 10*a^3*b*d^2*x^6*e^4
 + 16*a^3*b*d^3*x^5*e^3 + 15*a^3*b*d^4*x^4*e^2 + 8*a^3*b*d^5*x^3*e + 2*a^3*
b*d^6*x^2 + 1/7*a^4*x^7*e^6 + a^4*d*x^6*e^5 + 3*a^4*d^2*x^5*e^4 + 5*a^4*d^3
*x^4*e^3 + 5*a^4*d^4*x^3*e^2 + 3*a^4*d^5*x^2*e + a^4*d^6*x
```

3.1464 $\int (d+ex)^5 \left(a^2+2abx+b^2x^2\right)^2 \, dx$

Optimal. Leaf size=119

$$-\frac{4b^3(d+ex)^9(bd-ae)}{9e^5}+\frac{3b^2(d+ex)^8(bd-ae)^2}{4e^5}-\frac{4b(d+ex)^7(bd-ae)^3}{7e^5}+\frac{(d+ex)^6(bd-ae)^4}{6e^5}+\frac{b^4(d+ex)^{10}}{10e^5}$$

```
[Out] ((b*d - a*e)^4*(d + e*x)^6)/(6*e^5) - (4*b*(b*d - a*e)^3*(d + e*x)^7)/(7*e^
5) + (3*b^2*(b*d - a*e)^2*(d + e*x)^8)/(4*e^5) - (4*b^3*(b*d - a*e)*(d + e*
x)^9)/(9*e^5) + (b^4*(d + e*x)^10)/(10*e^5)
```

Rubi [A] time = 0.208124, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$-\frac{4b^3(d+ex)^9(bd-ae)}{9e^5}+\frac{3b^2(d+ex)^8(bd-ae)^2}{4e^5}-\frac{4b(d+ex)^7(bd-ae)^3}{7e^5}+\frac{(d+ex)^6(bd-ae)^4}{6e^5}+\frac{b^4(d+ex)^{10}}{10e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] ((b*d - a*e)^4*(d + e*x)^6)/(6*e^5) - (4*b*(b*d - a*e)^3*(d + e*x)^7)/(7*e^
5) + (3*b^2*(b*d - a*e)^2*(d + e*x)^8)/(4*e^5) - (4*b^3*(b*d - a*e)*(d + e*
x)^9)/(9*e^5) + (b^4*(d + e*x)^10)/(10*e^5)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^5\left(a^2+2abx+b^2x^2\right)^2\,dx &= \int (a+bx)^4(d+ex)^5\,dx\\
&= \int \left(\frac{(-bd+ae)^4(d+ex)^5}{e^4}-\frac{4b(bd-ae)^3(d+ex)^6}{e^4}+\frac{6b^2(bd-ae)^2(d+ex)^7}{e^4}-\frac{4l}{}\right.\\
&= \frac{(bd-ae)^4(d+ex)^6}{6e^5}-\frac{4b(bd-ae)^3(d+ex)^7}{7e^5}+\frac{3b^2(bd-ae)^2(d+ex)^8}{4e^5}-\frac{4b^3(bd}{}
\end{aligned}$$

Mathematica [B] time = 0.048776, size = 350, normalized size = 2.94

$$\frac{1}{4}b^2e^3x^8\left(3a^2e^2+10abde+5b^2d^2\right)+\frac{2}{7}be^2x^7\left(15a^2bde^2+2a^3e^3+20ab^2d^2e+5b^3d^3\right)+\frac{1}{6}ex^6\left(60a^2b^2d^2e^2+20a^3bde^3+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] a^4*d^5*x + (a^3*d^4*(4*b*d + 5*a*e)*x^2)/2 + (2*a^2*d^3*(3*b^2*d^2 + 10*a*
b*d*e + 5*a^2*e^2)*x^3)/3 + (a*d^2*(2*b^3*d^3 + 15*a*b^2*d^2*e + 20*a^2*b*d
*e^2 + 5*a^3*e^3)*x^4)/2 + (d*(b^4*d^4 + 20*a*b^3*d^3*e + 60*a^2*b^2*d^2*e^
2 + 40*a^3*b*d*e^3 + 5*a^4*e^4)*x^5)/5 + (e*(5*b^4*d^4 + 40*a*b^3*d^3*e + 6
0*a^2*b^2*d^2*e^2 + 20*a^3*b*d*e^3 + a^4*e^4)*x^6)/6 + (2*b*e^2*(5*b^3*d^3
+ 20*a*b^2*d^2*e + 15*a^2*b*d*e^2 + 2*a^3*e^3)*x^7)/7 + (b^2*e^3*(5*b^2*d^2
 + 10*a*b*d*e + 3*a^2*e^2)*x^8)/4 + (b^3*e^4*(5*b*d + 4*a*e)*x^9)/9 + (b^4*
e^5*x^10)/10
```

Maple [B] time = 0.041, size = 361, normalized size = 3.

$$\frac{e^5b^4x^{10}}{10}+\frac{\left(4\,ab^3e^5+5\,de^4b^4\right)x^9}{9}+\frac{\left(6\,e^5b^2a^2+20\,de^4ab^3+10\,d^2e^3b^4\right)x^8}{8}+\frac{\left(4\,e^5a^3b+30\,de^4b^2a^2+40\,d^2e^3ab^3+10\,d^3e^2b\right.}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^2,x)
```

```
[Out] 1/10*e^5*b^4*x^10+1/9*(4*a*b^3*e^5+5*b^4*d*e^4)*x^9+1/8*(6*a^2*b^2*e^5+20*a
*b^3*d*e^4+10*b^4*d^2*e^3)*x^8+1/7*(4*a^3*b*e^5+30*a^2*b^2*d*e^4+40*a*b^3*d
^2*e^3+10*b^4*d^3*e^2)*x^7+1/6*(a^4*e^5+20*a^3*b*d*e^4+60*a^2*b^2*d^2*e^3+4
0*a*b^3*d^3*e^2+5*b^4*d^4*e)*x^6+1/5*(5*a^4*d*e^4+40*a^3*b*d^2*e^3+60*a^2*b
^2*d^3*e^2+20*a*b^3*d^4*e+b^4*d^5)*x^5+1/4*(10*a^4*d^2*e^3+40*a^3*b*d^3*e^2
+30*a^2*b^2*d^4*e+4*a*b^3*d^5)*x^4+1/3*(10*a^4*d^3*e^2+20*a^3*b*d^4*e+6*a^2
*b^2*d^5)*x^3+1/2*(5*a^4*d^4*e+4*a^3*b*d^5)*x^2+d^5*a^4*x
```

Maxima [B] time = 1.14052, size = 486, normalized size = 4.08

$$\frac{1}{10}\,b^4e^5x^{10}+a^4d^5x+\frac{1}{9}\left(5\,b^4de^4+4\,ab^3e^5\right)x^9+\frac{1}{4}\left(5\,b^4d^2e^3+10\,ab^3de^4+3\,a^2b^2e^5\right)x^8+\frac{2}{7}\left(5\,b^4d^3e^2+20\,ab^3d^2e^3+15\,a^2b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/10*b^4*e^5*x^10 + a^4*d^5*x + 1/9*(5*b^4*d*e^4 + 4*a*b^3*e^5)*x^9 + 1/4*(
5*b^4*d^2*e^3 + 10*a*b^3*d*e^4 + 3*a^2*b^2*e^5)*x^8 + 2/7*(5*b^4*d^3*e^2 +
20*a*b^3*d^2*e^3 + 15*a^2*b^2*d*e^4 + 2*a^3*b*e^5)*x^7 + 1/6*(5*b^4*d^4*e +
 40*a*b^3*d^3*e^2 + 60*a^2*b^2*d^2*e^3 + 20*a^3*b*d*e^4 + a^4*e^5)*x^6 + 1/
5*(b^4*d^5 + 20*a*b^3*d^4*e + 60*a^2*b^2*d^3*e^2 + 40*a^3*b*d^2*e^3 + 5*a^4
*d*e^4)*x^5 + 1/2*(2*a*b^3*d^5 + 15*a^2*b^2*d^4*e + 20*a^3*b*d^3*e^2 + 5*a^
4*d^2*e^3)*x^4 + 2/3*(3*a^2*b^2*d^5 + 10*a^3*b*d^4*e + 5*a^4*d^3*e^2)*x^3 +
 1/2*(4*a^3*b*d^5 + 5*a^4*d^4*e)*x^2
```

Fricas [B] time = 1.48742, size = 853, normalized size = 7.17

$$\frac{1}{10}x^{10}e^5b^4+\frac{5}{9}x^9e^4db^4+\frac{4}{9}x^9e^5b^3a+\frac{5}{4}x^8e^3d^2b^4+\frac{5}{2}x^8e^4db^3a+\frac{3}{4}x^8e^5b^2a^2+\frac{10}{7}x^7e^2d^3b^4+\frac{40}{7}x^7e^3d^2b^3a+\frac{30}{7}x^7e^4db^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*e^5*b^4 + 5/9*x^9*e^4*d*b^4 + 4/9*x^9*e^5*b^3*a + 5/4*x^8*e^3*d^2
*b^4 + 5/2*x^8*e^4*d*b^3*a + 3/4*x^8*e^5*b^2*a^2 + 10/7*x^7*e^2*d^3*b^4 + 4
0/7*x^7*e^3*d^2*b^3*a + 30/7*x^7*e^4*d*b^2*a^2 + 4/7*x^7*e^5*b*a^3 + 5/6*x^
6*e*d^4*b^4 + 20/3*x^6*e^2*d^3*b^3*a + 10*x^6*e^3*d^2*b^2*a^2 + 10/3*x^6*e^
4*d*b*a^3 + 1/6*x^6*e^5*a^4 + 1/5*x^5*d^5*b^4 + 4*x^5*e*d^4*b^3*a + 12*x^5*
e^2*d^3*b^2*a^2 + 8*x^5*e^3*d^2*b*a^3 + x^5*e^4*d*a^4 + x^4*d^5*b^3*a + 15/
2*x^4*e*d^4*b^2*a^2 + 10*x^4*e^2*d^3*b*a^3 + 5/2*x^4*e^3*d^2*a^4 + 2*x^3*d^
5*b^2*a^2 + 20/3*x^3*e*d^4*b*a^3 + 10/3*x^3*e^2*d^3*a^4 + 2*x^2*d^5*b*a^3 +
 5/2*x^2*e*d^4*a^4 + x*d^5*a^4
```

Sympy [B] time = 0.120092, size = 401, normalized size = 3.37

$$a^4d^5x + \frac{b^4e^5x^{10}}{10} + x^9\left(\frac{4ab^3e^5}{9} + \frac{5b^4de^4}{9}\right) + x^8\left(\frac{3a^2b^2e^5}{4} + \frac{5ab^3de^4}{2} + \frac{5b^4d^2e^3}{4}\right) + x^7\left(\frac{4a^3be^5}{7} + \frac{30a^2b^2de^4}{7} + \frac{40ab^3d^2e}{7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5*(b**2*x**2+2*a*b*x+a**2)**2,x)
```

```
[Out] a**4*d**5*x + b**4*e**5*x**10/10 + x**9*(4*a*b**3*e**5/9 + 5*b**4*d*e**4/9)
 + x**8*(3*a**2*b**2*e**5/4 + 5*a*b**3*d*e**4/2 + 5*b**4*d**2*e**3/4) + x**
7*(4*a**3*b*e**5/7 + 30*a**2*b**2*d*e**4/7 + 40*a*b**3*d**2*e**3/7 + 10*b**
4*d**3*e**2/7) + x**6*(a**4*e**5/6 + 10*a**3*b*d*e**4/3 + 10*a**2*b**2*d**2
*e**3 + 20*a*b**3*d**3*e**2/3 + 5*b**4*d**4*e/6) + x**5*(a**4*d*e**4 + 8*a*
*3*b*d**2*e**3 + 12*a**2*b**2*d**3*e**2 + 4*a*b**3*d**4*e + b**4*d**5/5) +
x**4*(5*a**4*d**2*e**3/2 + 10*a**3*b*d**3*e**2 + 15*a**2*b**2*d**4*e/2 + a*
b**3*d**5) + x**3*(10*a**4*d**3*e**2/3 + 20*a**3*b*d**4*e/3 + 2*a**2*b**2*d
**5) + x**2*(5*a**4*d**4*e/2 + 2*a**3*b*d**5)
```

Giac [B] time = 1.10441, size = 514, normalized size = 4.32

$$\frac{1}{10}b^4x^{10}e^5 + \frac{5}{9}b^4dx^9e^4 + \frac{5}{4}b^4d^2x^8e^3 + \frac{10}{7}b^4d^3x^7e^2 + \frac{5}{6}b^4d^4x^6e + \frac{1}{5}b^4d^5x^5 + \frac{4}{9}ab^3x^9e^5 + \frac{5}{2}ab^3dx^8e^4 + \frac{40}{7}ab^3d^2x^7e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/10*b^4*x^10*e^5 + 5/9*b^4*d*x^9*e^4 + 5/4*b^4*d^2*x^8*e^3 + 10/7*b^4*d^3*
x^7*e^2 + 5/6*b^4*d^4*x^6*e + 1/5*b^4*d^5*x^5 + 4/9*a*b^3*x^9*e^5 + 5/2*a*b
^3*d*x^8*e^4 + 40/7*a*b^3*d^2*x^7*e^3 + 20/3*a*b^3*d^3*x^6*e^2 + 4*a*b^3*d^
4*x^5*e + a*b^3*d^5*x^4 + 3/4*a^2*b^2*x^8*e^5 + 30/7*a^2*b^2*d*x^7*e^4 + 10
*a^2*b^2*d^2*x^6*e^3 + 12*a^2*b^2*d^3*x^5*e^2 + 15/2*a^2*b^2*d^4*x^4*e + 2*
a^2*b^2*d^5*x^3 + 4/7*a^3*b*x^7*e^5 + 10/3*a^3*b*d*x^6*e^4 + 8*a^3*b*d^2*x^
5*e^3 + 10*a^3*b*d^3*x^4*e^2 + 20/3*a^3*b*d^4*x^3*e + 2*a^3*b*d^5*x^2 + 1/6
*a^4*x^6*e^5 + a^4*d*x^5*e^4 + 5/2*a^4*d^2*x^4*e^3 + 10/3*a^4*d^3*x^3*e^2 +
 5/2*a^4*d^4*x^2*e + a^4*d^5*x
```

3.1465 $\int (d+ex)^4 \left(a^2+2abx+b^2x^2\right)^2 \, dx$

Optimal. Leaf size=119

$$\frac{e^3(a+bx)^8(bd-ae)}{2b^5}+\frac{6e^2(a+bx)^7(bd-ae)^2}{7b^5}+\frac{2e(a+bx)^6(bd-ae)^3}{3b^5}+\frac{(a+bx)^5(bd-ae)^4}{5b^5}+\frac{e^4(a+bx)^9}{9b^5}$$

```
[Out] ((b*d - a*e)^4*(a + b*x)^5)/(5*b^5) + (2*e*(b*d - a*e)^3*(a + b*x)^6)/(3*b^
5) + (6*e^2*(b*d - a*e)^2*(a + b*x)^7)/(7*b^5) + (e^3*(b*d - a*e)*(a + b*x)
^8)/(2*b^5) + (e^4*(a + b*x)^9)/(9*b^5)
```

Rubi [A] time = 0.164931, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$\frac{e^3(a+bx)^8(bd-ae)}{2b^5}+\frac{6e^2(a+bx)^7(bd-ae)^2}{7b^5}+\frac{2e(a+bx)^6(bd-ae)^3}{3b^5}+\frac{(a+bx)^5(bd-ae)^4}{5b^5}+\frac{e^4(a+bx)^9}{9b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] ((b*d - a*e)^4*(a + b*x)^5)/(5*b^5) + (2*e*(b*d - a*e)^3*(a + b*x)^6)/(3*b^
5) + (6*e^2*(b*d - a*e)^2*(a + b*x)^7)/(7*b^5) + (e^3*(b*d - a*e)*(a + b*x)
^8)/(2*b^5) + (e^4*(a + b*x)^9)/(9*b^5)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^4 \left(a^2+2abx+b^2x^2\right)^2 \, dx &= \int (a+bx)^4(d+ex)^4 \, dx\\ &= \int \left(\frac{(bd-ae)^4(a+bx)^4}{b^4}+\frac{4e(bd-ae)^3(a+bx)^5}{b^4}+\frac{6e^2(bd-ae)^2(a+bx)^6}{b^4}+\frac{4e^3(bd\\ &= \frac{(bd-ae)^4(a+bx)^5}{5b^5}+\frac{2e(bd-ae)^3(a+bx)^6}{3b^5}+\frac{6e^2(bd-ae)^2(a+bx)^7}{7b^5}+\frac{e^3(bd-ae}{2b}\end{aligned}$$

Mathematica [B] time = 0.0385267, size = 273, normalized size = 2.29

$$\frac{2}{7}b^2e^2x^7\left(3a^2e^2+8abde+3b^2d^2\right)+\frac{2}{3}bex^6\left(6a^2bde^2+a^3e^3+6ab^2d^2e+b^3d^3\right)+\frac{1}{5}x^5\left(36a^2b^2d^2e^2+16a^3bde^3+a^4e^4+16a\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] a^4*d^4*x + 2*a^3*d^3*(b*d + a*e)*x^2 + (2*a^2*d^2*(3*b^2*d^2 + 8*a*b*d*e +
 3*a^2*e^2)*x^3)/3 + a*d*(b^3*d^3 + 6*a*b^2*d^2*e + 6*a^2*b*d*e^2 + a^3*e^3
)*x^4 + ((b^4*d^4 + 16*a*b^3*d^3*e + 36*a^2*b^2*d^2*e^2 + 16*a^3*b*d*e^3 +
a^4*e^4)*x^5)/5 + (2*b*e*(b^3*d^3 + 6*a*b^2*d^2*e + 6*a^2*b*d*e^2 + a^3*e^3
)*x^6)/3 + (2*b^2*e^2*(3*b^2*d^2 + 8*a*b*d*e + 3*a^2*e^2)*x^7)/7 + (b^3*e^3
*(b*d + a*e)*x^8)/2 + (b^4*e^4*x^9)/9
```

Maple [B] time = 0.04, size = 295, normalized size = 2.5

$$\frac{b^4e^4x^9}{9}+\frac{\left(4\,e^4ab^3+4\,b^4de^3\right)x^8}{8}+\frac{\left(6\,e^4b^2a^2+16\,de^3ab^3+6\,d^2e^2b^4\right)x^7}{7}+\frac{\left(4\,e^4a^3b+24\,de^3b^2a^2+24\,d^2e^2ab^3+4\,d^3eb^4\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^2,x)
```

```
[Out] 1/9*b^4*e^4*x^9+1/8*(4*a*b^3*e^4+4*b^4*d*e^3)*x^8+1/7*(6*a^2*b^2*e^4+16*a*b
^3*d*e^3+6*b^4*d^2*e^2)*x^7+1/6*(4*a^3*b*e^4+24*a^2*b^2*d*e^3+24*a*b^3*d^2*
e^2+4*b^4*d^3*e)*x^6+1/5*(a^4*e^4+16*a^3*b*d*e^3+36*a^2*b^2*d^2*e^2+16*a*b^
3*d^3*e+b^4*d^4)*x^5+1/4*(4*a^4*d*e^3+24*a^3*b*d^2*e^2+24*a^2*b^2*d^3*e+4*a
*b^3*d^4)*x^4+1/3*(6*a^4*d^2*e^2+16*a^3*b*d^3*e+6*a^2*b^2*d^4)*x^3+1/2*(4*a
^4*d^3*e+4*a^3*b*d^4)*x^2+a^4*d^4*x
```

Maxima [B] time = 1.09669, size = 385, normalized size = 3.24

$$\frac{1}{9}\,b^4e^4x^9+a^4d^4x+\frac{1}{2}\left(b^4de^3+ab^3e^4\right)x^8+\frac{2}{7}\left(3\,b^4d^2e^2+8\,ab^3de^3+3\,a^2b^2e^4\right)x^7+\frac{2}{3}\left(b^4d^3e+6\,ab^3d^2e^2+6\,a^2b^2de^3+a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^4*e^4*x^9 + a^4*d^4*x + 1/2*(b^4*d*e^3 + a*b^3*e^4)*x^8 + 2/7*(3*b^4*
d^2*e^2 + 8*a*b^3*d*e^3 + 3*a^2*b^2*e^4)*x^7 + 2/3*(b^4*d^3*e + 6*a*b^3*d^2
*e^2 + 6*a^2*b^2*d*e^3 + a^3*b*e^4)*x^6 + 1/5*(b^4*d^4 + 16*a*b^3*d^3*e + 3
6*a^2*b^2*d^2*e^2 + 16*a^3*b*d*e^3 + a^4*e^4)*x^5 + (a*b^3*d^4 + 6*a^2*b^2*
d^3*e + 6*a^3*b*d^2*e^2 + a^4*d*e^3)*x^4 + 2/3*(3*a^2*b^2*d^4 + 8*a^3*b*d^3
*e + 3*a^4*d^2*e^2)*x^3 + 2*(a^3*b*d^4 + a^4*d^3*e)*x^2
```

Fricas [B] time = 1.51175, size = 680, normalized size = 5.71

$$\frac{1}{9}x^9e^4b^4+\frac{1}{2}x^8e^3db^4+\frac{1}{2}x^8e^4b^3a+\frac{6}{7}x^7e^2d^2b^4+\frac{16}{7}x^7e^3db^3a+\frac{6}{7}x^7e^4b^2a^2+\frac{2}{3}x^6ed^3b^4+4x^6e^2d^2b^3a+4x^6e^3db^2a^2+\frac{2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*e^4*b^4 + 1/2*x^8*e^3*d*b^4 + 1/2*x^8*e^4*b^3*a + 6/7*x^7*e^2*d^2*b
^4 + 16/7*x^7*e^3*d*b^3*a + 6/7*x^7*e^4*b^2*a^2 + 2/3*x^6*e*d^3*b^4 + 4*x^6
```

```
*e^2*d^2*b^3*a + 4*x^6*e^3*d*b^2*a^2 + 2/3*x^6*e^4*b*a^3 + 1/5*x^5*d^4*b^4
+ 16/5*x^5*e*d^3*b^3*a + 36/5*x^5*e^2*d^2*b^2*a^2 + 16/5*x^5*e^3*d*b*a^3 +
1/5*x^5*e^4*a^4 + x^4*d^4*b^3*a + 6*x^4*e*d^3*b^2*a^2 + 6*x^4*e^2*d^2*b*a^3
 + x^4*e^3*d*a^4 + 2*x^3*d^4*b^2*a^2 + 16/3*x^3*e*d^3*b*a^3 + 2*x^3*e^2*d^2
*a^4 + 2*x^2*d^4*b*a^3 + 2*x^2*e*d^3*a^4 + x*d^4*a^4
```

Sympy [B] time = 0.115406, size = 318, normalized size = 2.67

$$a^4d^4x + \frac{b^4e^4x^9}{9} + x^8\left(\frac{ab^3e^4}{2} + \frac{b^4de^3}{2}\right) + x^7\left(\frac{6a^2b^2e^4}{7} + \frac{16ab^3de^3}{7} + \frac{6b^4d^2e^2}{7}\right) + x^6\left(\frac{2a^3be^4}{3} + 4a^2b^2de^3 + 4ab^3d^2e^2 + \frac{2b^4d}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(b**2*x**2+2*a*b*x+a**2)**2,x)
```

```
[Out] a**4*d**4*x + b**4*e**4*x**9/9 + x**8*(a*b**3*e**4/2 + b**4*d*e**3/2) + x**
7*(6*a**2*b**2*e**4/7 + 16*a*b**3*d*e**3/7 + 6*b**4*d**2*e**2/7) + x**6*(2*
a**3*b*e**4/3 + 4*a**2*b**2*d*e**3 + 4*a*b**3*d**2*e**2 + 2*b**4*d**3*e/3)
+ x**5*(a**4*e**4/5 + 16*a**3*b*d*e**3/5 + 36*a**2*b**2*d**2*e**2/5 + 16*a*
b**3*d**3*e/5 + b**4*d**4/5) + x**4*(a**4*d*e**3 + 6*a**3*b*d**2*e**2 + 6*a
**2*b**2*d**3*e + a*b**3*d**4) + x**3*(2*a**4*d**2*e**2 + 16*a**3*b*d**3*e/
3 + 2*a**2*b**2*d**4) + x**2*(2*a**4*d**3*e + 2*a**3*b*d**4)
```

Giac [B] time = 1.1317, size = 420, normalized size = 3.53

$$\frac{1}{9}b^4x^9e^4 + \frac{1}{2}b^4dx^8e^3 + \frac{6}{7}b^4d^2x^7e^2 + \frac{2}{3}b^4d^3x^6e + \frac{1}{5}b^4d^4x^5 + \frac{1}{2}ab^3x^8e^4 + \frac{16}{7}ab^3dx^7e^3 + 4\,ab^3d^2x^6e^2 + \frac{16}{5}ab^3d^3x^5e + ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/9*b^4*x^9*e^4 + 1/2*b^4*d*x^8*e^3 + 6/7*b^4*d^2*x^7*e^2 + 2/3*b^4*d^3*x^6
*e + 1/5*b^4*d^4*x^5 + 1/2*a*b^3*x^8*e^4 + 16/7*a*b^3*d*x^7*e^3 + 4*a*b^3*d
^2*x^6*e^2 + 16/5*a*b^3*d^3*x^5*e + a*b^3*d^4*x^4 + 6/7*a^2*b^2*x^7*e^4 + 4
*a^2*b^2*d*x^6*e^3 + 36/5*a^2*b^2*d^2*x^5*e^2 + 6*a^2*b^2*d^3*x^4*e + 2*a^2
*b^2*d^4*x^3 + 2/3*a^3*b*x^6*e^4 + 16/5*a^3*b*d*x^5*e^3 + 6*a^3*b*d^2*x^4*e
^2 + 16/3*a^3*b*d^3*x^3*e + 2*a^3*b*d^4*x^2 + 1/5*a^4*x^5*e^4 + a^4*d*x^4*e
^3 + 2*a^4*d^2*x^3*e^2 + 2*a^4*d^3*x^2*e + a^4*d^4*x
```

3.1466 $\int (d+ex)^3 \left(a^2+2abx+b^2x^2\right)^2 \, dx$

Optimal. Leaf size=92

$$\frac{3e^2(a+bx)^7(bd-ae)}{7b^4}+\frac{e(a+bx)^6(bd-ae)^2}{2b^4}+\frac{(a+bx)^5(bd-ae)^3}{5b^4}+\frac{e^3(a+bx)^8}{8b^4}$$

```
[Out] ((b*d - a*e)^3*(a + b*x)^5)/(5*b^4) + (e*(b*d - a*e)^2*(a + b*x)^6)/(2*b^4)
 + (3*e^2*(b*d - a*e)*(a + b*x)^7)/(7*b^4) + (e^3*(a + b*x)^8)/(8*b^4)
```

Rubi [A] time = 0.130013, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$\frac{3e^2(a+bx)^7(bd-ae)}{7b^4}+\frac{e(a+bx)^6(bd-ae)^2}{2b^4}+\frac{(a+bx)^5(bd-ae)^3}{5b^4}+\frac{e^3(a+bx)^8}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] ((b*d - a*e)^3*(a + b*x)^5)/(5*b^4) + (e*(b*d - a*e)^2*(a + b*x)^6)/(2*b^4)
 + (3*e^2*(b*d - a*e)*(a + b*x)^7)/(7*b^4) + (e^3*(a + b*x)^8)/(8*b^4)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^3 \left(a^2+2abx+b^2x^2\right)^2 \, dx &= \int (a+bx)^4(d+ex)^3 \, dx\\ &= \int \left(\frac{(bd-ae)^3(a+bx)^4}{b^3}+\frac{3e(bd-ae)^2(a+bx)^5}{b^3}+\frac{3e^2(bd-ae)(a+bx)^6}{b^3}+\frac{e^3(a\right.\\ &= \frac{(bd-ae)^3(a+bx)^5}{5b^4}+\frac{e(bd-ae)^2(a+bx)^6}{2b^4}+\frac{3e^2(bd-ae)(a+bx)^7}{7b^4}+\frac{e^3(a+bx}{8b^4}\end{aligned}$$

Mathematica [B] time = 0.0311339, size = 217, normalized size = 2.36

$$\frac{1}{2}b^2ex^6\left(2a^2e^2+4abde+b^2d^2\right)+\frac{1}{5}bx^5\left(18a^2bde^2+4a^3e^3+12ab^2d^2e+b^3d^3\right)+\frac{1}{4}ax^4\left(12a^2bde^2+a^3e^3+18ab^2d^2e+4\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] a^4*d^3*x + (a^3*d^2*(4*b*d + 3*a*e)*x^2)/2 + a^2*d*(2*b^2*d^2 + 4*a*b*d*e
+ a^2*e^2)*x^3 + (a*(4*b^3*d^3 + 18*a*b^2*d^2*e + 12*a^2*b*d*e^2 + a^3*e^3)
*x^4)/4 + (b*(b^3*d^3 + 12*a*b^2*d^2*e + 18*a^2*b*d*e^2 + 4*a^3*e^3)*x^5)/5
 + (b^2*e*(b^2*d^2 + 4*a*b*d*e + 2*a^2*e^2)*x^6)/2 + (b^3*e^2*(3*b*d + 4*a*
e)*x^7)/7 + (b^4*e^3*x^8)/8
```

Maple [B] time = 0.041, size = 229, normalized size = 2.5

$$\frac{e^3b^4x^8}{8}+\frac{\left(4\,e^3ab^3+3\,de^2b^4\right)x^7}{7}+\frac{\left(6\,e^3b^2a^2+12\,de^2ab^3+3\,d^2eb^4\right)x^6}{6}+\frac{\left(4\,e^3a^3b+18\,de^2b^2a^2+12\,d^2eab^3+d^3b^4\right)x^5}{5}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^2,x)
```

```
[Out] 1/8*e^3*b^4*x^8+1/7*(4*a*b^3*e^3+3*b^4*d*e^2)*x^7+1/6*(6*a^2*b^2*e^3+12*a*b
^3*d*e^2+3*b^4*d^2*e)*x^6+1/5*(4*a^3*b*e^3+18*a^2*b^2*d*e^2+12*a*b^3*d^2*e+
b^4*d^3)*x^5+1/4*(a^4*e^3+12*a^3*b*d*e^2+18*a^2*b^2*d^2*e+4*a*b^3*d^3)*x^4+
1/3*(3*a^4*d*e^2+12*a^3*b*d^2*e+6*a^2*b^2*d^3)*x^3+1/2*(3*a^4*d^2*e+4*a^3*b
*d^3)*x^2+a^4*d^3*x
```

Maxima [B] time = 1.18706, size = 304, normalized size = 3.3

$$\frac{1}{8}\,b^4e^3x^8+a^4d^3x+\frac{1}{7}\left(3\,b^4de^2+4\,ab^3e^3\right)x^7+\frac{1}{2}\left(b^4d^2e+4\,ab^3de^2+2\,a^2b^2e^3\right)x^6+\frac{1}{5}\left(b^4d^3+12\,ab^3d^2e+18\,a^2b^2de^2+4\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^4*e^3*x^8 + a^4*d^3*x + 1/7*(3*b^4*d*e^2 + 4*a*b^3*e^3)*x^7 + 1/2*(b^
4*d^2*e + 4*a*b^3*d*e^2 + 2*a^2*b^2*e^3)*x^6 + 1/5*(b^4*d^3 + 12*a*b^3*d^2*
e + 18*a^2*b^2*d*e^2 + 4*a^3*b*e^3)*x^5 + 1/4*(4*a*b^3*d^3 + 18*a^2*b^2*d^2
*e + 12*a^3*b*d*e^2 + a^4*e^3)*x^4 + (2*a^2*b^2*d^3 + 4*a^3*b*d^2*e + a^4*d
*e^2)*x^3 + 1/2*(4*a^3*b*d^3 + 3*a^4*d^2*e)*x^2
```

Fricas [B] time = 1.51139, size = 520, normalized size = 5.65

$$\frac{1}{8}x^8e^3b^4+\frac{3}{7}x^7e^2db^4+\frac{4}{7}x^7e^3b^3a+\frac{1}{2}x^6ed^2b^4+2x^6e^2db^3a+x^6e^3b^2a^2+\frac{1}{5}x^5d^3b^4+\frac{12}{5}x^5ed^2b^3a+\frac{18}{5}x^5e^2db^2a^2+\frac{4}{5}x^5e^3ba$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*e^3*b^4 + 3/7*x^7*e^2*d*b^4 + 4/7*x^7*e^3*b^3*a + 1/2*x^6*e*d^2*b^4
 + 2*x^6*e^2*d*b^3*a + x^6*e^3*b^2*a^2 + 1/5*x^5*d^3*b^4 + 12/5*x^5*e*d^2*b
^3*a + 18/5*x^5*e^2*d*b^2*a^2 + 4/5*x^5*e^3*b*a^3 + x^4*d^3*b^3*a + 9/2*x^4
*e*d^2*b^2*a^2 + 3*x^4*e^2*d*b*a^3 + 1/4*x^4*e^3*a^4 + 2*x^3*d^3*b^2*a^2 +
4*x^3*e*d^2*b*a^3 + x^3*e^2*d*a^4 + 2*x^2*d^3*b*a^3 + 3/2*x^2*e*d^2*a^4 + x
*d^3*a^4
```

Sympy [B] time = 0.109806, size = 243, normalized size = 2.64

$$a^4d^3x+\frac{b^4e^3x^8}{8}+x^7\left(\frac{4ab^3e^3}{7}+\frac{3b^4de^2}{7}\right)+x^6\left(a^2b^2e^3+2ab^3de^2+\frac{b^4d^2e}{2}\right)+x^5\left(\frac{4a^3be^3}{5}+\frac{18a^2b^2de^2}{5}+\frac{12ab^3d^2e}{5}+\frac{b^4}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(b**2*x**2+2*a*b*x+a**2)**2,x)
```

```
[Out] a**4*d**3*x + b**4*e**3*x**8/8 + x**7*(4*a*b**3*e**3/7 + 3*b**4*d*e**2/7) +
 x**6*(a**2*b**2*e**3 + 2*a*b**3*d*e**2 + b**4*d**2*e/2) + x**5*(4*a**3*b*e
**3/5 + 18*a**2*b**2*d*e**2/5 + 12*a*b**3*d**2*e/5 + b**4*d**3/5) + x**4*(a
**4*e**3/4 + 3*a**3*b*d*e**2 + 9*a**2*b**2*d**2*e/2 + a*b**3*d**3) + x**3*(
a**4*d*e**2 + 4*a**3*b*d**2*e + 2*a**2*b**2*d**3) + x**2*(3*a**4*d**2*e/2 +
 2*a**3*b*d**3)
```

Giac [B] time = 1.16648, size = 324, normalized size = 3.52

$$\frac{1}{8}b^4x^8e^3+\frac{3}{7}b^4dx^7e^2+\frac{1}{2}b^4d^2x^6e+\frac{1}{5}b^4d^3x^5+\frac{4}{7}ab^3x^7e^3+2\,ab^3dx^6e^2+\frac{12}{5}ab^3d^2x^5e+ab^3d^3x^4+a^2b^2x^6e^3+\frac{18}{5}a^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*b^4*x^8*e^3 + 3/7*b^4*d*x^7*e^2 + 1/2*b^4*d^2*x^6*e + 1/5*b^4*d^3*x^5 +
 4/7*a*b^3*x^7*e^3 + 2*a*b^3*d*x^6*e^2 + 12/5*a*b^3*d^2*x^5*e + a*b^3*d^3*x
^4 + a^2*b^2*x^6*e^3 + 18/5*a^2*b^2*d*x^5*e^2 + 9/2*a^2*b^2*d^2*x^4*e + 2*a
^2*b^2*d^3*x^3 + 4/5*a^3*b*x^5*e^3 + 3*a^3*b*d*x^4*e^2 + 4*a^3*b*d^2*x^3*e
+ 2*a^3*b*d^3*x^2 + 1/4*a^4*x^4*e^3 + a^4*d*x^3*e^2 + 3/2*a^4*d^2*x^2*e + a
^4*d^3*x
```

3.1467 $\int (d+ex)^2 \left(a^2+2abx+b^2x^2\right)^2 \, dx$

Optimal. Leaf size=65

$$\frac{e(a+bx)^6(bd-ae)}{3b^3}+\frac{(a+bx)^5(bd-ae)^2}{5b^3}+\frac{e^2(a+bx)^7}{7b^3}$$

```
[Out] ((b*d - a*e)^2*(a + b*x)^5)/(5*b^3) + (e*(b*d - a*e)*(a + b*x)^6)/(3*b^3) +
 (e^2*(a + b*x)^7)/(7*b^3)
```

Rubi [A] time = 0.086089, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$\frac{e(a+bx)^6(bd-ae)}{3b^3}+\frac{(a+bx)^5(bd-ae)^2}{5b^3}+\frac{e^2(a+bx)^7}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] ((b*d - a*e)^2*(a + b*x)^5)/(5*b^3) + (e*(b*d - a*e)*(a + b*x)^6)/(3*b^3) +
 (e^2*(a + b*x)^7)/(7*b^3)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^2\left(a^2+2abx+b^2x^2\right)^2\,dx &= \int (a+bx)^4(d+ex)^2\,dx\\
&= \int \left(\frac{(bd-ae)^2(a+bx)^4}{b^2}+\frac{2e(bd-ae)(a+bx)^5}{b^2}+\frac{e^2(a+bx)^6}{b^2}\right)dx\\
&= \frac{(bd-ae)^2(a+bx)^5}{5b^3}+\frac{e(bd-ae)(a+bx)^6}{3b^3}+\frac{e^2(a+bx)^7}{7b^3}
\end{aligned}$$

Mathematica [B] time = 0.0256762, size = 148, normalized size = 2.28

$$\frac{1}{5}b^2x^5\left(6a^2e^2+8abde+b^2d^2\right)+abx^4\left(a^2e^2+3abde+b^2d^2\right)+\frac{1}{3}a^2x^3\left(a^2e^2+8abde+6b^2d^2\right)+a^3dx^2(ae+2bd)+a^4d^2x+$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] a^4*d^2*x + a^3*d*(2*b*d + a*e)*x^2 + (a^2*(6*b^2*d^2 + 8*a*b*d*e + a^2*e^2
)*x^3)/3 + a*b*(b^2*d^2 + 3*a*b*d*e + a^2*e^2)*x^4 + (b^2*(b^2*d^2 + 8*a*b*
d*e + 6*a^2*e^2)*x^5)/5 + (b^3*e*(b*d + 2*a*e)*x^6)/3 + (b^4*e^2*x^7)/7
```

Maple [B] time = 0.04, size = 163, normalized size = 2.5

$$\frac{e^2b^4x^7}{7}+\frac{\left(4\,e^2ab^3+2\,deb^4\right)x^6}{6}+\frac{\left(6\,e^2b^2a^2+8\,deab^3+b^4d^2\right)x^5}{5}+\frac{\left(4\,e^2a^3b+12\,deb^2a^2+4\,d^2ab^3\right)x^4}{4}+\frac{\left(e^2a^4+8\,dea^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^2,x)
```

```
[Out] 1/7*e^2*b^4*x^7+1/6*(4*a*b^3*e^2+2*b^4*d*e)*x^6+1/5*(6*a^2*b^2*e^2+8*a*b^3*
d*e+b^4*d^2)*x^5+1/4*(4*a^3*b*e^2+12*a^2*b^2*d*e+4*a*b^3*d^2)*x^4+1/3*(a^4*
e^2+8*a^3*b*d*e+6*a^2*b^2*d^2)*x^3+1/2*(2*a^4*d*e+4*a^3*b*d^2)*x^2+a^4*d^2*
x
```

Maxima [B] time = 1.18639, size = 211, normalized size = 3.25

$$\frac{1}{7}\,b^4e^2x^7+a^4d^2x+\frac{1}{3}\left(b^4de+2\,ab^3e^2\right)x^6+\frac{1}{5}\left(b^4d^2+8\,ab^3de+6\,a^2b^2e^2\right)x^5+\left(ab^3d^2+3\,a^2b^2de+a^3be^2\right)x^4+\frac{1}{3}\left(6\,a^2b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^4*e^2*x^7 + a^4*d^2*x + 1/3*(b^4*d*e + 2*a*b^3*e^2)*x^6 + 1/5*(b^4*d^
2 + 8*a*b^3*d*e + 6*a^2*b^2*e^2)*x^5 + (a*b^3*d^2 + 3*a^2*b^2*d*e + a^3*b*e
^2)*x^4 + 1/3*(6*a^2*b^2*d^2 + 8*a^3*b*d*e + a^4*e^2)*x^3 + (2*a^3*b*d^2 +
a^4*d*e)*x^2
```

Fricas [B] time = 1.50431, size = 363, normalized size = 5.58

$$\frac{1}{7}x^7e^2b^4+\frac{1}{3}x^6edb^4+\frac{2}{3}x^6e^2b^3a+\frac{1}{5}x^5d^2b^4+\frac{8}{5}x^5edb^3a+\frac{6}{5}x^5e^2b^2a^2+x^4d^2b^3a+3x^4edb^2a^2+x^4e^2ba^3+2x^3d^2b^2a^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*e^2*b^4 + 1/3*x^6*e*d*b^4 + 2/3*x^6*e^2*b^3*a + 1/5*x^5*d^2*b^4 + 8
/5*x^5*e*d*b^3*a + 6/5*x^5*e^2*b^2*a^2 + x^4*d^2*b^3*a + 3*x^4*e*d*b^2*a^2
+ x^4*e^2*b*a^3 + 2*x^3*d^2*b^2*a^2 + 8/3*x^3*e*d*b*a^3 + 1/3*x^3*e^2*a^4 +
 2*x^2*d^2*b*a^3 + x^2*e*d*a^4 + x*d^2*a^4
```

Sympy [B] time = 0.097381, size = 168, normalized size = 2.58

$$a^4d^2x+\frac{b^4e^2x^7}{7}+x^6\left(\frac{2ab^3e^2}{3}+\frac{b^4de}{3}\right)+x^5\left(\frac{6a^2b^2e^2}{5}+\frac{8ab^3de}{5}+\frac{b^4d^2}{5}\right)+x^4\left(a^3be^2+3a^2b^2de+ab^3d^2\right)+x^3\left(\frac{a^4e^2}{3}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(b**2*x**2+2*a*b*x+a**2)**2,x)
```

```
[Out] a**4*d**2*x + b**4*e**2*x**7/7 + x**6*(2*a*b**3*e**2/3 + b**4*d*e/3) + x**5
*(6*a**2*b**2*e**2/5 + 8*a*b**3*d*e/5 + b**4*d**2/5) + x**4*(a**3*b*e**2 +
3*a**2*b**2*d*e + a*b**3*d**2) + x**3*(a**4*e**2/3 + 8*a**3*b*d*e/3 + 2*a**
2*b**2*d**2) + x**2*(a**4*d*e + 2*a**3*b*d**2)
```

Giac [B] time = 1.12583, size = 230, normalized size = 3.54

$$\frac{1}{7}b^4x^7e^2 + \frac{1}{3}b^4dx^6e + \frac{1}{5}b^4d^2x^5 + \frac{2}{3}ab^3x^6e^2 + \frac{8}{5}ab^3dx^5e + ab^3d^2x^4 + \frac{6}{5}a^2b^2x^5e^2 + 3a^2b^2dx^4e + 2a^2b^2d^2x^3 + a^3bx^4e^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/7*b^4*x^7*e^2 + 1/3*b^4*d*x^6*e + 1/5*b^4*d^2*x^5 + 2/3*a*b^3*x^6*e^2 + 8
/5*a*b^3*d*x^5*e + a*b^3*d^2*x^4 + 6/5*a^2*b^2*x^5*e^2 + 3*a^2*b^2*d*x^4*e
+ 2*a^2*b^2*d^2*x^3 + a^3*b*x^4*e^2 + 8/3*a^3*b*d*x^3*e + 2*a^3*b*d^2*x^2 +
 1/3*a^4*x^3*e^2 + a^4*d*x^2*e + a^4*d^2*x
```

3.1468 $\int (d+ex)\left(a^2+2abx+b^2x^2\right)^2\,dx$

Optimal. Leaf size=38

$$\frac{(a+bx)^5(bd-ae)}{5b^2}+\frac{e(a+bx)^6}{6b^2}$$

```
[Out] ((b*d - a*e)*(a + b*x)^5)/(5*b^2) + (e*(a + b*x)^6)/(6*b^2)
```

Rubi [A] time = 0.0158858, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {27, 43}

$$\frac{(a+bx)^5(bd-ae)}{5b^2}+\frac{e(a+bx)^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] ((b*d - a*e)*(a + b*x)^5)/(5*b^2) + (e*(a + b*x)^6)/(6*b^2)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(a^2+2abx+b^2x^2\right)^2\,dx &= \int (a+bx)^4(d+ex)\,dx\\ &= \int \left(\frac{(bd-ae)(a+bx)^4}{b}+\frac{e(a+bx)^5}{b}\right)dx\\ &= \frac{(bd-ae)(a+bx)^5}{5b^2}+\frac{e(a+bx)^6}{6b^2}\end{aligned}$$

Mathematica [B] time = 0.0161652, size = 84, normalized size = 2.21

$$\frac{1}{30}x\left(15a^2b^2x^2(4d+3ex)+20a^3bx(3d+2ex)+15a^4(2d+ex)+6ab^3x^3(5d+4ex)+b^4x^4(6d+5ex)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] (x*(15*a^4*(2*d + e*x) + 20*a^3*b*x*(3*d + 2*e*x) + 15*a^2*b^2*x^2*(4*d + 3
*e*x) + 6*a*b^3*x^3*(5*d + 4*e*x) + b^4*x^4*(6*d + 5*e*x)))/30
```

Maple [B] time = 0.039, size = 97, normalized size = 2.6

$$\frac{eb^4x^6}{6}+\frac{\left(4\,eab^3+db^4\right)x^5}{5}+\frac{\left(6\,a^2eb^2+4\,adb^3\right)x^4}{4}+\frac{\left(4\,ea^3b+6\,db^2a^2\right)x^3}{3}+\frac{\left(ea^4+4\,da^3b\right)x^2}{2}+a^4dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^2,x)
```

```
[Out] 1/6*e*b^4*x^6+1/5*(4*a*b^3*e+b^4*d)*x^5+1/4*(6*a^2*b^2*e+4*a*b^3*d)*x^4+1/3
*(4*a^3*b*e+6*a^2*b^2*d)*x^3+1/2*(a^4*e+4*a^3*b*d)*x^2+a^4*d*x
```

Maxima [B] time = 1.1702, size = 130, normalized size = 3.42

$$\frac{1}{6}\,b^4ex^6+a^4dx+\frac{1}{5}\left(b^4d+4\,ab^3e\right)x^5+\frac{1}{2}\left(2\,ab^3d+3\,a^2b^2e\right)x^4+\frac{2}{3}\left(3\,a^2b^2d+2\,a^3be\right)x^3+\frac{1}{2}\left(4\,a^3bd+a^4e\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^4*e*x^6 + a^4*d*x + 1/5*(b^4*d + 4*a*b^3*e)*x^5 + 1/2*(2*a*b^3*d + 3*
a^2*b^2*e)*x^4 + 2/3*(3*a^2*b^2*d + 2*a^3*b*e)*x^3 + 1/2*(4*a^3*b*d + a^4*e
)*x^2
```

Fricas [B] time = 1.52879, size = 217, normalized size = 5.71

$$\frac{1}{6}x^6eb^4+\frac{1}{5}x^5db^4+\frac{4}{5}x^5eb^3a+x^4db^3a+\frac{3}{2}x^4eb^2a^2+2x^3db^2a^2+\frac{4}{3}x^3eba^3+2x^2dba^3+\frac{1}{2}x^2ea^4+xda^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*x^6*e*b^4 + 1/5*x^5*d*b^4 + 4/5*x^5*e*b^3*a + x^4*d*b^3*a + 3/2*x^4*e*b
^2*a^2 + 2*x^3*d*b^2*a^2 + 4/3*x^3*e*b*a^3 + 2*x^2*d*b*a^3 + 1/2*x^2*e*a^4
+ x*d*a^4
```

Sympy [B] time = 0.076732, size = 100, normalized size = 2.63

$$a^4dx+\frac{b^4ex^6}{6}+x^5\left(\frac{4ab^3e}{5}+\frac{b^4d}{5}\right)+x^4\left(\frac{3a^2b^2e}{2}+ab^3d\right)+x^3\left(\frac{4a^3be}{3}+2a^2b^2d\right)+x^2\left(\frac{a^4e}{2}+2a^3bd\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b**2*x**2+2*a*b*x+a**2)**2,x)
```

```
[Out] a**4*d*x + b**4*e*x**6/6 + x**5*(4*a*b**3*e/5 + b**4*d/5) + x**4*(3*a**2*b*
*2*e/2 + a*b**3*d) + x**3*(4*a**3*b*e/3 + 2*a**2*b**2*d) + x**2*(a**4*e/2 +
 2*a**3*b*d)
```

Giac [B] time = 1.15537, size = 138, normalized size = 3.63

$$\frac{1}{6}\,b^4x^6e+\frac{1}{5}\,b^4dx^5+\frac{4}{5}\,ab^3x^5e+ab^3dx^4+\frac{3}{2}\,a^2b^2x^4e+2\,a^2b^2dx^3+\frac{4}{3}\,a^3bx^3e+2\,a^3bdx^2+\frac{1}{2}\,a^4x^2e+a^4dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*b^4*x^6*e + 1/5*b^4*d*x^5 + 4/5*a*b^3*x^5*e + a*b^3*d*x^4 + 3/2*a^2*b^2
*x^4*e + 2*a^2*b^2*d*x^3 + 4/3*a^3*b*x^3*e + 2*a^3*b*d*x^2 + 1/2*a^4*x^2*e
+ a^4*d*x
```

3.1469 $\int \left(a^2+2abx+b^2x^2\right)^2 \, dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^5}{5b}$$

[Out] (a + b*x)^5/(5*b)

Rubi [A] time = 0.0023447, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {27, 32}

$$\frac{(a+bx)^5}{5b}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^2,x]

[Out] (a + b*x)^5/(5*b)

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(a^2+2abx+b^2x^2\right)^2 \, dx &= \int (a+bx)^4 \, dx\\ &= \frac{(a+bx)^5}{5b}\end{aligned}$$

Mathematica [A] time = 0.001292, size = 14, normalized size = 1.

$$\frac{(a+bx)^5}{5b}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2,x]

[Out] (a + b*x)^5/(5*b)

Maple [B] time = 0.039, size = 43, normalized size = 3.1

$$\frac{b^4x^5}{5} + ab^3x^4 + 2\,b^2a^2x^3 + 2\,a^3bx^2 + a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^2,x)
```

```
[Out] 1/5*b^4*x^5+a*b^3*x^4+2*b^2*a^2*x^3+2*a^3*b*x^2+a^4*x
```

Maxima [B] time = 1.16626, size = 72, normalized size = 5.14

$$\frac{1}{5}\,b^4x^5 + ab^3x^4 + \frac{4}{3}\,a^2b^2x^3 + a^4x + \frac{2}{3}\left(b^2x^3 + 3\,abx^2\right)a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^4*x^5 + a*b^3*x^4 + 4/3*a^2*b^2*x^3 + a^4*x + 2/3*(b^2*x^3 + 3*a*b*x^
2)*a^2
```

Fricas [B] time = 1.39525, size = 85, normalized size = 6.07

$$\frac{1}{5}x^5b^4 + x^4b^3a + 2x^3b^2a^2 + 2x^2ba^3 + xa^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*b^4 + x^4*b^3*a + 2*x^3*b^2*a^2 + 2*x^2*b*a^3 + x*a^4
```

Sympy [B] time = 0.065736, size = 42, normalized size = 3.

$$a^4x + 2a^3bx^2 + 2a^2b^2x^3 + ab^3x^4 + \frac{b^4x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**2,x)
```

```
[Out] a**4*x + 2*a**3*b*x**2 + 2*a**2*b**2*x**3 + a*b**3*x**4 + b**4*x**5/5
```

Giac [B] time = 1.14409, size = 57, normalized size = 4.07

$$\frac{1}{5}\,b^4x^5 + ab^3x^4 + 2\,a^2b^2x^3 + 2\,a^3bx^2 + a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/5*b^4*x^5 + a*b^3*x^4 + 2*a^2*b^2*x^3 + 2*a^3*b*x^2 + a^4*x
```

3.1470 $\int \frac{(a^2+2abx+b^2x^2)^2}{d+ex}\,dx$

Optimal. Leaf size=98

$$-\frac{bx(bd-ae)^3}{e^4}+\frac{(a+bx)^2(bd-ae)^2}{2e^3}-\frac{(a+bx)^3(bd-ae)}{3e^2}+\frac{(bd-ae)^4\log(d+ex)}{e^5}+\frac{(a+bx)^4}{4e}$$

```
[Out] -((b*(b*d - a*e)^3*x)/e^4) + ((b*d - a*e)^2*(a + b*x)^2)/(2*e^3) - ((b*d -
a*e)*(a + b*x)^3)/(3*e^2) + (a + b*x)^4/(4*e) + ((b*d - a*e)^4*Log[d + e*x]
)/e^5
```

Rubi [A] time = 0.0423296, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$-\frac{bx(bd-ae)^3}{e^4}+\frac{(a+bx)^2(bd-ae)^2}{2e^3}-\frac{(a+bx)^3(bd-ae)}{3e^2}+\frac{(bd-ae)^4\log(d+ex)}{e^5}+\frac{(a+bx)^4}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x),x]
```

```
[Out] -((b*(b*d - a*e)^3*x)/e^4) + ((b*d - a*e)^2*(a + b*x)^2)/(2*e^3) - ((b*d -
a*e)*(a + b*x)^3)/(3*e^2) + (a + b*x)^4/(4*e) + ((b*d - a*e)^4*Log[d + e*x]
)/e^5
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx+b^2x^2\right)^2}{d+ex}\,dx &= \int \frac{(a+bx)^4}{d+ex}\,dx\\ &= \int \left(-\frac{b(bd-ae)^3}{e^4}+\frac{b(bd-ae)^2(a+bx)}{e^3}-\frac{b(bd-ae)(a+bx)^2}{e^2}+\frac{b(a+bx)^3}{e}+\frac{(-bd+ae)}{e^4(d+ex}\right.\\ &= -\frac{b(bd-ae)^3x}{e^4}+\frac{(bd-ae)^2(a+bx)^2}{2e^3}-\frac{(bd-ae)(a+bx)^3}{3e^2}+\frac{(a+bx)^4}{4e}+\frac{(bd-ae)^4\log(d}{e^5}\end{aligned}$$

Mathematica [A] time = 0.0452689, size = 115, normalized size = 1.17

$$\frac{bex\left(36a^2be^2(ex-2d)+48a^3e^3+8ab^2e\left(6d^2-3dex+2e^2x^2\right)+b^3\left(6d^2ex-12d^3-4de^2x^2+3e^3x^3\right)\right)+12(bd-ae)^4\text{lo}}{12e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x),x]
```

```
[Out] (b*e*x*(48*a^3*e^3 + 36*a^2*b*e^2*(-2*d + e*x) + 8*a*b^2*e*(6*d^2 - 3*d*e*x
 + 2*e^2*x^2) + b^3*(-12*d^3 + 6*d^2*e*x - 4*d*e^2*x^2 + 3*e^3*x^3)) + 12*(
b*d - a*e)^4*Log[d + e*x])/(12*e^5)
```

Maple [B] time = 0.042, size = 209, normalized size = 2.1

$$\frac{b^4x^4}{4e}+\frac{4ab^3x^3}{3e}-\frac{b^4x^3d}{3e^2}+3\frac{b^2x^2a^2}{e}-2\frac{b^3x^2ad}{e^2}+\frac{b^4x^2d^2}{2e^3}+4\frac{xa^3b}{e}-6\frac{b^2a^2dx}{e^2}+4\frac{d^2ab^3x}{e^3}-\frac{b^4d^3x}{e^4}+\frac{\ln(ex+d)a^4}{e}-4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d),x)
```

```
[Out] 1/4*b^4/e*x^4+4/3*b^3/e*x^3*a-1/3*b^4/e^2*x^3*d+3*b^2/e*x^2*a^2-2*b^3/e^2*x
^2*a*d+1/2*b^4/e^3*x^2*d^2+4*b/e*a^3*x-6*b^2/e^2*a^2*d*x+4*b^3/e^3*a*d^2*x-
b^4/e^4*d^3*x+1/e*ln(e*x+d)*a^4-4/e^2*ln(e*x+d)*a^3*b*d+6/e^3*ln(e*x+d)*d^2
*b^2*a^2-4/e^4*ln(e*x+d)*d^3*a*b^3+1/e^5*ln(e*x+d)*b^4*d^4
```

Maxima [A] time = 1.15193, size = 239, normalized size = 2.44

$$\frac{3b^4e^3x^4-4\left(b^4de^2-4ab^3e^3\right)x^3+6\left(b^4d^2e-4ab^3de^2+6a^2b^2e^3\right)x^2-12\left(b^4d^3-4ab^3d^2e+6a^2b^2de^2-4a^3be^3\right)x}{12e^4}+\frac{(b^4d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/12*(3*b^4*e^3*x^4 - 4*(b^4*d*e^2 - 4*a*b^3*e^3)*x^3 + 6*(b^4*d^2*e - 4*a*
b^3*d*e^2 + 6*a^2*b^2*e^3)*x^2 - 12*(b^4*d^3 - 4*a*b^3*d^2*e + 6*a^2*b^2*d*
e^2 - 4*a^3*b*e^3)*x)/e^4 + (b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 -
4*a^3*b*d*e^3 + a^4*e^4)*log(e*x + d)/e^5
```

Fricas [A] time = 1.78922, size = 369, normalized size = 3.77

$$\frac{3b^4e^4x^4-4\left(b^4de^3-4ab^3e^4\right)x^3+6\left(b^4d^2e^2-4ab^3de^3+6a^2b^2e^4\right)x^2-12\left(b^4d^3e-4ab^3d^2e^2+6a^2b^2de^3-4a^3be^4\right)x+12}{12e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*e^4*x^4 - 4*(b^4*d*e^3 - 4*a*b^3*e^4)*x^3 + 6*(b^4*d^2*e^2 - 4*
a*b^3*d*e^3 + 6*a^2*b^2*e^4)*x^2 - 12*(b^4*d^3*e - 4*a*b^3*d^2*e^2 + 6*a^2*
b^2*d*e^3 - 4*a^3*b*e^4)*x + 12*(b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^
2 - 4*a^3*b*d*e^3 + a^4*e^4)*log(e*x + d))/e^5
```

Sympy [A] time = 0.706945, size = 134, normalized size = 1.37

$$\frac{b^4x^4}{4e} + \frac{x^3\left(4ab^3e - b^4d\right)}{3e^2} + \frac{x^2\left(6a^2b^2e^2 - 4ab^3de + b^4d^2\right)}{2e^3} + \frac{x\left(4a^3be^3 - 6a^2b^2de^2 + 4ab^3d^2e - b^4d^3\right)}{e^4} + \frac{(ae - bd)^4\log(d + ex)}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**2/(e*x+d),x)
```

```
[Out] b**4*x**4/(4*e) + x**3*(4*a*b**3*e - b**4*d)/(3*e**2) + x**2*(6*a**2*b**2*e
**2 - 4*a*b**3*d*e + b**4*d**2)/(2*e**3) + x*(4*a**3*b*e**3 - 6*a**2*b**2*d
*e**2 + 4*a*b**3*d**2*e - b**4*d**3)/e**4 + (a*e - b*d)**4*log(d + e*x)/e**
5
```

Giac [A] time = 1.15053, size = 238, normalized size = 2.43

$$\left(b^4d^4 - 4\,ab^3d^3e + 6\,a^2b^2d^2e^2 - 4\,a^3bde^3 + a^4e^4\right)e^{(-5)}\log\left(|xe + d|\right) + \frac{1}{12}\left(3\,b^4x^4e^3 - 4\,b^4dx^3e^2 + 6\,b^4d^2x^2e - 12\,b^4d^3x + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d),x, algorithm="giac")
```

```
[Out] (b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4)*e^(
-5)*log(abs(x*e + d)) + 1/12*(3*b^4*x^4*e^3 - 4*b^4*d*x^3*e^2 + 6*b^4*d^2*x
^2*e - 12*b^4*d^3*x + 16*a*b^3*x^3*e^3 - 24*a*b^3*d*x^2*e^2 + 48*a*b^3*d^2*
x*e + 36*a^2*b^2*x^2*e^3 - 72*a^2*b^2*d*x*e^2 + 48*a^3*b*x*e^3)*e^(-4)
```

3.1471 $\int \frac{\left(a^2+2abx+b^2x^2\right)^2}{(d+ex)^2} dx$

Optimal. Leaf size=104

$$-\frac{2b^3(d+ex)^2(bd-ae)}{e^5}+\frac{6b^2x(bd-ae)^2}{e^4}-\frac{(bd-ae)^4}{e^5(d+ex)}-\frac{4b(bd-ae)^3\log(d+ex)}{e^5}+\frac{b^4(d+ex)^3}{3e^5}$$

```
[Out] (6*b^2*(b*d - a*e)^2*x)/e^4 - (b*d - a*e)^4/(e^5*(d + e*x)) - (2*b^3*(b*d -
 a*e)*(d + e*x)^2)/e^5 + (b^4*(d + e*x)^3)/(3*e^5) - (4*b*(b*d - a*e)^3*Log
[d + e*x])/e^5
```

Rubi [A] time = 0.100024, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$-\frac{2b^3(d+ex)^2(bd-ae)}{e^5}+\frac{6b^2x(bd-ae)^2}{e^4}-\frac{(bd-ae)^4}{e^5(d+ex)}-\frac{4b(bd-ae)^3\log(d+ex)}{e^5}+\frac{b^4(d+ex)^3}{3e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^2,x]
```

```
[Out] (6*b^2*(b*d - a*e)^2*x)/e^4 - (b*d - a*e)^4/(e^5*(d + e*x)) - (2*b^3*(b*d -
 a*e)*(d + e*x)^2)/e^5 + (b^4*(d + e*x)^3)/(3*e^5) - (4*b*(b*d - a*e)^3*Log
[d + e*x])/e^5
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx+b^2x^2\right)^2}{(d+ex)^2} dx &= \int \frac{(a+bx)^4}{(d+ex)^2} dx \\
&= \int \left(\frac{6b^2(bd-ae)^2}{e^4}+\frac{(-bd+ae)^4}{e^4(d+ex)^2}-\frac{4b(bd-ae)^3}{e^4(d+ex)}-\frac{4b^3(bd-ae)(d+ex)}{e^4}+\frac{b^4(d+ex)^2}{e^4}\right) dx \\
&= \frac{6b^2(bd-ae)^2x}{e^4}-\frac{(bd-ae)^4}{e^5(d+ex)}-\frac{2b^3(bd-ae)(d+ex)^2}{e^5}+\frac{b^4(d+ex)^3}{3e^5}-\frac{4b(bd-ae)^3\log(d+e
\end{aligned}$$

Mathematica [A] time = 0.0605836, size = 165, normalized size = 1.59

$$\frac{18a^2b^2e^2\left(-d^2+dex+e^2x^2\right)+12a^3bde^3-3a^4e^4+6ab^3e\left(-4d^2ex+2d^3-3de^2x^2+e^3x^3\right)-12b(d+ex)(bd-ae)^3\log(d+}{3e^5(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^2,x]
```

```
[Out] (12*a^3*b*d*e^3 - 3*a^4*e^4 + 18*a^2*b^2*e^2*(-d^2 + d*e*x + e^2*x^2) + 6*a
*b^3*e*(2*d^3 - 4*d^2*e*x - 3*d*e^2*x^2 + e^3*x^3) + b^4*(-3*d^4 + 9*d^3*e*
x + 6*d^2*e^2*x^2 - 2*d*e^3*x^3 + e^4*x^4) - 12*b*(b*d - a*e)^3*(d + e*x)*L
og[d + e*x])/(3*e^5*(d + e*x))
```

Maple [B] time = 0.049, size = 230, normalized size = 2.2

$$\frac{b^4x^3}{3\,e^2}+2\,\frac{b^3x^2a}{e^2}-\frac{b^4x^2d}{e^3}+6\,\frac{a^2b^2x}{e^2}-8\,\frac{adb^3x}{e^3}+3\,\frac{b^4d^2x}{e^4}+4\,\frac{b\ln(ex+d)\,a^3}{e^2}-12\,\frac{b^2\ln(ex+d)\,a^2d}{e^3}+12\,\frac{b^3\ln(ex+d)}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^2,x)
```

```
[Out] 1/3*b^4/e^2*x^3+2*b^3/e^2*x^2*a-b^4/e^3*x^2*d+6*b^2/e^2*a^2*x-8*b^3/e^3*a*d
*x+3*b^4/e^4*d^2*x+4*b/e^2*ln(e*x+d)*a^3-12*b^2/e^3*ln(e*x+d)*a^2*d+12*b^3/
e^4*ln(e*x+d)*a*d^2-4*b^4/e^5*ln(e*x+d)*d^3-1/e/(e*x+d)*a^4+4/e^2/(e*x+d)*d
*a^3*b-6/e^3/(e*x+d)*d^2*b^2*a^2+4/e^4/(e*x+d)*d^3*a*b^3-1/e^5/(e*x+d)*b^4*
d^4
```

Maxima [A] time = 1.16265, size = 247, normalized size = 2.38

$$-\frac{b^4d^4-4\,ab^3d^3e+6\,a^2b^2d^2e^2-4\,a^3bde^3+a^4e^4}{e^6x+de^5}+\frac{b^4e^2x^3-3\left(b^4de-2\,ab^3e^2\right)x^2+3\left(3\,b^4d^2-8\,ab^3de+6\,a^2b^2e^2\right)x}{3\,e^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4)/(e
^6*x + d*e^5) + 1/3*(b^4*e^2*x^3 - 3*(b^4*d*e - 2*a*b^3*e^2)*x^2 + 3*(3*b^4
*d^2 - 8*a*b^3*d*e + 6*a^2*b^2*e^2)*x)/e^4 - 4*(b^4*d^3 - 3*a*b^3*d^2*e + 3
*a^2*b^2*d*e^2 - a^3*b*e^3)*log(e*x + d)/e^5
```

Fricas [B] time = 1.94949, size = 540, normalized size = 5.19

$$\frac{b^4e^4x^4-3\,b^4d^4+12\,ab^3d^3e-18\,a^2b^2d^2e^2+12\,a^3bde^3-3\,a^4e^4-2\left(b^4de^3-3\,ab^3e^4\right)x^3+6\left(b^4d^2e^2-3\,ab^3de^3+3\,a^2b^2e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*e^4*x^4 - 3*b^4*d^4 + 12*a*b^3*d^3*e - 18*a^2*b^2*d^2*e^2 + 12*a^3
*b*d*e^3 - 3*a^4*e^4 - 2*(b^4*d*e^3 - 3*a*b^3*e^4)*x^3 + 6*(b^4*d^2*e^2 - 3
*a*b^3*d*e^3 + 3*a^2*b^2*e^4)*x^2 + 3*(3*b^4*d^3*e - 8*a*b^3*d^2*e^2 + 6*a^
2*b^2*d*e^3)*x - 12*(b^4*d^4 - 3*a*b^3*d^3*e + 3*a^2*b^2*d^2*e^2 - a^3*b*d*
```

```
e^3 + (b^4*d^3*e - 3*a*b^3*d^2*e^2 + 3*a^2*b^2*d*e^3 - a^3*b*e^4)*x)*log(e*
x + d))/(e^6*x + d*e^5)
```

Sympy [A] time = 1.10256, size = 151, normalized size = 1.45

$$\frac{b^4x^3}{3e^2}+\frac{4b\,(ae-bd)^3\log{(d+ex)}}{e^5}-\frac{a^4e^4-4a^3bde^3+6a^2b^2d^2e^2-4ab^3d^3e+b^4d^4}{de^5+e^6x}+\frac{x^2\left(2ab^3e-b^4d\right)}{e^3}+\frac{x\left(6a^2b^2e^2-8ab^3\right.}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**2/(e*x+d)**2,x)
```

```
[Out] b**4*x**3/(3*e**2) + 4*b*(a*e - b*d)**3*log(d + e*x)/e**5 - (a**4*e**4 - 4*
a**3*b*d*e**3 + 6*a**2*b**2*d**2*e**2 - 4*a*b**3*d**3*e + b**4*d**4)/(d*e**
5 + e**6*x) + x**2*(2*a*b**3*e - b**4*d)/e**3 + x*(6*a**2*b**2*e**2 - 8*a*b
**3*d*e + 3*b**4*d**2)/e**4
```

Giac [B] time = 1.18258, size = 323, normalized size = 3.11

$$\frac{1}{3}\left(b^4-\frac{6\left(b^4de-ab^3e^2\right)e^{(-1)}}{xe+d}+\frac{18\left(b^4d^2e^2-2\,ab^3de^3+a^2b^2e^4\right)e^{(-2)}}{(xe+d)^2}\right)(xe+d)^3e^{(-5)}+4\left(b^4d^3-3\,ab^3d^2e+3\,a^2b^2de^2-a^3b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^4 - 6*(b^4*d*e - a*b^3*e^2)*e^(-1)/(x*e + d) + 18*(b^4*d^2*e^2 - 2*a
*b^3*d*e^3 + a^2*b^2*e^4)*e^(-2)/(x*e + d)^2)*(x*e + d)^3*e^(-5) + 4*(b^4*d
^3 - 3*a*b^3*d^2*e + 3*a^2*b^2*d*e^2 - a^3*b*e^3)*e^(-5)*log(abs(x*e + d)*e
^(-1)/(x*e + d)^2) - (b^4*d^4*e^3/(x*e + d) - 4*a*b^3*d^3*e^4/(x*e + d) + 6
*a^2*b^2*d^2*e^5/(x*e + d) - 4*a^3*b*d*e^6/(x*e + d) + a^4*e^7/(x*e + d))*e
^(-8)
```

3.1472 $\int \frac{\left(a^2+2abx+b^2x^2\right)^2}{(d+ex)^3}\,dx$

Optimal. Leaf size=103

$$-\frac{b^3x(3bd-4ae)}{e^4}+\frac{6b^2(bd-ae)^2\log(d+ex)}{e^5}+\frac{4b(bd-ae)^3}{e^5(d+ex)}-\frac{(bd-ae)^4}{2e^5(d+ex)^2}+\frac{b^4x^2}{2e^3}$$

```
[Out] -((b^3*(3*b*d - 4*a*e)*x)/e^4) + (b^4*x^2)/(2*e^3) - (b*d - a*e)^4/(2*e^5*(
d + e*x)^2) + (4*b*(b*d - a*e)^3)/(e^5*(d + e*x)) + (6*b^2*(b*d - a*e)^2*Lo
g[d + e*x])/e^5
```

Rubi [A] time = 0.0852951, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$-\frac{b^3x(3bd-4ae)}{e^4}+\frac{6b^2(bd-ae)^2\log(d+ex)}{e^5}+\frac{4b(bd-ae)^3}{e^5(d+ex)}-\frac{(bd-ae)^4}{2e^5(d+ex)^2}+\frac{b^4x^2}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^3,x]
```

```
[Out] -((b^3*(3*b*d - 4*a*e)*x)/e^4) + (b^4*x^2)/(2*e^3) - (b*d - a*e)^4/(2*e^5*(
d + e*x)^2) + (4*b*(b*d - a*e)^3)/(e^5*(d + e*x)) + (6*b^2*(b*d - a*e)^2*Lo
g[d + e*x])/e^5
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx+b^2x^2\right)^2}{(d+ex)^3}\,dx &= \int \frac{(a+bx)^4}{(d+ex)^3}\,dx\\
&= \int \left(-\frac{b^3(3bd-4ae)}{e^4}+\frac{b^4x}{e^3}+\frac{(-bd+ae)^4}{e^4(d+ex)^3}-\frac{4b(bd-ae)^3}{e^4(d+ex)^2}+\frac{6b^2(bd-ae)^2}{e^4(d+ex)}\right)dx\\
&= -\frac{b^3(3bd-4ae)x}{e^4}+\frac{b^4x^2}{2e^3}-\frac{(bd-ae)^4}{2e^5(d+ex)^2}+\frac{4b(bd-ae)^3}{e^5(d+ex)}+\frac{6b^2(bd-ae)^2\log(d+ex)}{e^5}
\end{aligned}$$

Mathematica [A] time = 0.0562678, size = 167, normalized size = 1.62

$$\frac{6a^2b^2de^2(3d+4ex)-4a^3be^3(d+2ex)-a^4e^4+4ab^3e\left(-4d^2ex-5d^3+4de^2x^2+2e^3x^3\right)+12b^2(d+ex)^2(bd-ae)^2\log(d}{2e^5(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^3,x]
```

```
[Out] (-(a^4*e^4) - 4*a^3*b*e^3*(d + 2*e*x) + 6*a^2*b^2*d*e^2*(3*d + 4*e*x) + 4*a
*b^3*e*(-5*d^3 - 4*d^2*e*x + 4*d*e^2*x^2 + 2*e^3*x^3) + b^4*(7*d^4 + 2*d^3*
e*x - 11*d^2*e^2*x^2 - 4*d*e^3*x^3 + e^4*x^4) + 12*b^2*(b*d - a*e)^2*(d + e
*x)^2*Log[d + e*x])/(2*e^5*(d + e*x)^2)
```

Maple [B] time = 0.049, size = 245, normalized size = 2.4

$$\frac{b^4x^2}{2e^3}+4\frac{ab^3x}{e^3}-3\frac{b^4xd}{e^4}-\frac{a^4}{2e(ex+d)^2}+2\frac{da^3b}{e^2(ex+d)^2}-3\frac{b^2d^2a^2}{e^3(ex+d)^2}+2\frac{d^3ab^3}{e^4(ex+d)^2}-\frac{b^4d^4}{2e^5(ex+d)^2}+6\frac{b^2\ln(ex+d}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^3,x)
```

```
[Out] 1/2*b^4*x^2/e^3+4*b^3/e^3*a*x-3*b^4/e^4*x*d-1/2/e/(e*x+d)^2*a^4+2/e^2/(e*x+
d)^2*d*a^3*b-3/e^3/(e*x+d)^2*d^2*b^2*a^2+2/e^4/(e*x+d)^2*d^3*a*b^3-1/2/e^5/
(e*x+d)^2*b^4*d^4+6*b^2/e^3*ln(e*x+d)*a^2-12*b^3/e^4*ln(e*x+d)*a*d+6*b^4/e^
5*ln(e*x+d)*d^2-4*b/e^2/(e*x+d)*a^3+12*b^2/e^3/(e*x+d)*a^2*d-12*b^3/e^4/(e*
x+d)*a*d^2+4*b^4/e^5/(e*x+d)*d^3
```

Maxima [A] time = 1.11434, size = 258, normalized size = 2.5

$$\frac{7b^4d^4-20ab^3d^3e+18a^2b^2d^2e^2-4a^3bde^3-a^4e^4+8\left(b^4d^3e-3ab^3d^2e^2+3a^2b^2de^3-a^3be^4\right)x}{2\left(e^7x^2+2de^6x+d^2e^5\right)}+\frac{b^4ex^2-2\left(3b^4d-4ab\right.}{2e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(7*b^4*d^4 - 20*a*b^3*d^3*e + 18*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 - a^4*
e^4 + 8*(b^4*d^3*e - 3*a*b^3*d^2*e^2 + 3*a^2*b^2*d*e^3 - a^3*b*e^4)*x)/(e^7
*x^2 + 2*d*e^6*x + d^2*e^5) + 1/2*(b^4*e*x^2 - 2*(3*b^4*d - 4*a*b^3*e)*x)/e
^4 + 6*(b^4*d^2 - 2*a*b^3*d*e + a^2*b^2*e^2)*log(e*x + d)/e^5
```

Fricas [B] time = 2.03556, size = 586, normalized size = 5.69

$$\frac{b^4e^4x^4+7b^4d^4-20ab^3d^3e+18a^2b^2d^2e^2-4a^3bde^3-a^4e^4-4\left(b^4de^3-2ab^3e^4\right)x^3-\left(11b^4d^2e^2-16ab^3de^3\right)x^2+2\left(b^4d^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*e^4*x^4 + 7*b^4*d^4 - 20*a*b^3*d^3*e + 18*a^2*b^2*d^2*e^2 - 4*a^3*
b*d*e^3 - a^4*e^4 - 4*(b^4*d*e^3 - 2*a*b^3*e^4)*x^3 - (11*b^4*d^2*e^2 - 16*
a*b^3*d*e^3)*x^2 + 2*(b^4*d^3*e - 8*a*b^3*d^2*e^2 + 12*a^2*b^2*d*e^3 - 4*a^
```

```
3*b*e^4)*x + 12*(b^4*d^4 - 2*a*b^3*d^3*e + a^2*b^2*d^2*e^2 + (b^4*d^2*e^2 -
 2*a*b^3*d*e^3 + a^2*b^2*e^4)*x^2 + 2*(b^4*d^3*e - 2*a*b^3*d^2*e^2 + a^2*b^
2*d*e^3)*x)*log(e*x + d))/(e^7*x^2 + 2*d*e^6*x + d^2*e^5)
```

Sympy [A] time = 1.70429, size = 184, normalized size = 1.79

$$\frac{b^4x^2}{2e^3} + \frac{6b^2(ae-bd)^2\log(d+ex)}{e^5} - \frac{a^4e^4 + 4a^3bde^3 - 18a^2b^2d^2e^2 + 20ab^3d^3e - 7b^4d^4 + x\left(8a^3be^4 - 24a^2b^2de^3 + 24ab\right.}{2d^2e^5 + 4de^6x + 2e^7x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**2/(e*x+d)**3,x)
```

```
[Out] b**4*x**2/(2*e**3) + 6*b**2*(a*e - b*d)**2*log(d + e*x)/e**5 - (a**4*e**4 +
 4*a**3*b*d*e**3 - 18*a**2*b**2*d**2*e**2 + 20*a*b**3*d**3*e - 7*b**4*d**4
+ x*(8*a**3*b*e**4 - 24*a**2*b**2*d*e**3 + 24*a*b**3*d**2*e**2 - 8*b**4*d**
3*e))/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + x*(4*a*b**3*e - 3*b**4*d)/
e**4
```

Giac [A] time = 1.13883, size = 236, normalized size = 2.29

$$6\left(b^4d^2 - 2\,ab^3de + a^2b^2e^2\right)e^{(-5)}\log(|xe+d|) + \frac{1}{2}\left(b^4x^2e^3 - 6\,b^4dxe^2 + 8\,ab^3xe^3\right)e^{(-6)} + \frac{\left(7\,b^4d^4 - 20\,ab^3d^3e + 18\,a^2b^2d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 6*(b^4*d^2 - 2*a*b^3*d*e + a^2*b^2*e^2)*e^(-5)*log(abs(x*e + d)) + 1/2*(b^4
*x^2*e^3 - 6*b^4*d*x*e^2 + 8*a*b^3*x*e^3)*e^(-6) + 1/2*(7*b^4*d^4 - 20*a*b^
3*d^3*e + 18*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 - a^4*e^4 + 8*(b^4*d^3*e - 3*a
*b^3*d^2*e^2 + 3*a^2*b^2*d*e^3 - a^3*b*e^4)*x)*e^(-5)/(x*e + d)^2
```

3.1473 $\int \frac{(a^2+2abx+b^2x^2)^2}{(d+ex)^4} dx$

Optimal. Leaf size=103

$$-\frac{6b^2(bd-ae)^2}{e^5(d+ex)}-\frac{4b^3(bd-ae)\log(d+ex)}{e^5}+\frac{2b(bd-ae)^3}{e^5(d+ex)^2}-\frac{(bd-ae)^4}{3e^5(d+ex)^3}+\frac{b^4x}{e^4}$$

```
[Out] (b^4*x)/e^4 - (b*d - a*e)^4/(3*e^5*(d + e*x)^3) + (2*b*(b*d - a*e)^3)/(e^5*
(d + e*x)^2) - (6*b^2*(b*d - a*e)^2)/(e^5*(d + e*x)) - (4*b^3*(b*d - a*e)*L
og[d + e*x])/e^5
```

Rubi [A] time = 0.0798661, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$-\frac{6b^2(bd-ae)^2}{e^5(d+ex)}-\frac{4b^3(bd-ae)\log(d+ex)}{e^5}+\frac{2b(bd-ae)^3}{e^5(d+ex)^2}-\frac{(bd-ae)^4}{3e^5(d+ex)^3}+\frac{b^4x}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^4,x]
```

```
[Out] (b^4*x)/e^4 - (b*d - a*e)^4/(3*e^5*(d + e*x)^3) + (2*b*(b*d - a*e)^3)/(e^5*
(d + e*x)^2) - (6*b^2*(b*d - a*e)^2)/(e^5*(d + e*x)) - (4*b^3*(b*d - a*e)*L
og[d + e*x])/e^5
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx+b^2x^2)^2}{(d+ex)^4}\,dx &= \int \frac{(a+bx)^4}{(d+ex)^4}\,dx\\ &= \int \left(\frac{b^4}{e^4}+\frac{(-bd+ae)^4}{e^4(d+ex)^4}-\frac{4b(bd-ae)^3}{e^4(d+ex)^3}+\frac{6b^2(bd-ae)^2}{e^4(d+ex)^2}-\frac{4b^3(bd-ae)}{e^4(d+ex)}\right)dx\\ &= \frac{b^4x}{e^4}-\frac{(bd-ae)^4}{3e^5(d+ex)^3}+\frac{2b(bd-ae)^3}{e^5(d+ex)^2}-\frac{6b^2(bd-ae)^2}{e^5(d+ex)}-\frac{4b^3(bd-ae)\log(d+ex)}{e^5}\end{aligned}$$

Mathematica [A] time = 0.0746552, size = 163, normalized size = 1.58

$$-\frac{6a^2b^2e^2(d^2+3dex+3e^2x^2)+2a^3be^3(d+3ex)+a^4e^4-2ab^3de(11d^2+27dex+18e^2x^2)+12b^3(d+ex)^3(bd-ae)\log(d}{3e^5(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^4,x]
```

```
[Out] -(a^4*e^4 + 2*a^3*b*e^3*(d + 3*e*x) + 6*a^2*b^2*e^2*(d^2 + 3*d*e*x + 3*e^2*
x^2) - 2*a*b^3*d*e*(11*d^2 + 27*d*e*x + 18*e^2*x^2) + b^4*(13*d^4 + 27*d^3*
e*x + 9*d^2*e^2*x^2 - 9*d*e^3*x^3 - 3*e^4*x^4) + 12*b^3*(b*d - a*e)*(d + e*
x)^3*Log[d + e*x])/(3*e^5*(d + e*x)^3)
```

Maple [B] time = 0.05, size = 255, normalized size = 2.5

$$\frac{b^4x}{e^4} - \frac{a^4}{3\,e\,(ex+d)^3} + \frac{4\,da^3b}{3\,e^2\,(ex+d)^3} - 2\,\frac{b^2d^2a^2}{e^3\,(ex+d)^3} + \frac{4\,d^3ab^3}{3\,e^4\,(ex+d)^3} - \frac{b^4d^4}{3\,e^5\,(ex+d)^3} - 2\,\frac{a^3b}{e^2\,(ex+d)^2} + 6\,\frac{b^2a^2d}{e^3\,(ex+d)^2} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^4,x)
```

```
[Out] b^4*x/e^4-1/3/e/(e*x+d)^3*a^4+4/3/e^2/(e*x+d)^3*d*a^3*b-2/e^3/(e*x+d)^3*d^2
*b^2*a^2+4/3/e^4/(e*x+d)^3*d^3*a*b^3-1/3/e^5/(e*x+d)^3*b^4*d^4-2*b/e^2/(e*x
+d)^2*a^3+6*b^2/e^3/(e*x+d)^2*a^2*d-6*b^3/e^4/(e*x+d)^2*a*d^2+2*b^4/e^5/(e*
x+d)^2*d^3+4*b^3/e^4*ln(e*x+d)*a-4*b^4/e^5*ln(e*x+d)*d-6*b^2/e^3/(e*x+d)*a^
2+12*b^3/e^4/(e*x+d)*a*d-6*b^4/e^5/(e*x+d)*d^2
```

Maxima [A] time = 1.04734, size = 271, normalized size = 2.63

$$\frac{b^4x}{e^4} - \frac{13\,b^4d^4 - 22\,ab^3d^3e + 6\,a^2b^2d^2e^2 + 2\,a^3bde^3 + a^4e^4 + 18\left(b^4d^2e^2 - 2\,ab^3de^3 + a^2b^2e^4\right)x^2 + 6\left(5\,b^4d^3e - 9\,ab^3d^2e^2\right.}{3\left(e^8x^3 + 3\,de^7x^2 + 3\,d^2e^6x + d^3e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] b^4*x/e^4 - 1/3*(13*b^4*d^4 - 22*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 + 2*a^3*b*
d*e^3 + a^4*e^4 + 18*(b^4*d^2*e^2 - 2*a*b^3*d*e^3 + a^2*b^2*e^4)*x^2 + 6*(5
*b^4*d^3*e - 9*a*b^3*d^2*e^2 + 3*a^2*b^2*d*e^3 + a^3*b*e^4)*x)/(e^8*x^3 + 3
*d*e^7*x^2 + 3*d^2*e^6*x + d^3*e^5) - 4*(b^4*d - a*b^3*e)*log(e*x + d)/e^5
```

Fricas [B] time = 2.03765, size = 581, normalized size = 5.64

$$\frac{3\,b^4e^4x^4 + 9\,b^4de^3x^3 - 13\,b^4d^4 + 22\,ab^3d^3e - 6\,a^2b^2d^2e^2 - 2\,a^3bde^3 - a^4e^4 - 9\left(b^4d^2e^2 - 4\,ab^3de^3 + 2\,a^2b^2e^4\right)x^2 - 3\left(9\,b\right.}{3\left(e^8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b^4*e^4*x^4 + 9*b^4*d*e^3*x^3 - 13*b^4*d^4 + 22*a*b^3*d^3*e - 6*a^2*
b^2*d^2*e^2 - 2*a^3*b*d*e^3 - a^4*e^4 - 9*(b^4*d^2*e^2 - 4*a*b^3*d*e^3 + 2*
a^2*b^2*e^4)*x^2 - 3*(9*b^4*d^3*e - 18*a*b^3*d^2*e^2 + 6*a^2*b^2*d*e^3 + 2*
```

```
a^3*b*e^4)*x - 12*(b^4*d^4 - a*b^3*d^3*e + (b^4*d*e^3 - a*b^3*e^4)*x^3 + 3*
(b^4*d^2*e^2 - a*b^3*d*e^3)*x^2 + 3*(b^4*d^3*e - a*b^3*d^2*e^2)*x)*log(e*x
+ d))/(e^8*x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3*e^5)
```

Sympy [B] time = 2.67267, size = 209, normalized size = 2.03

$$\frac{b^4x}{e^4}+\frac{4b^3\left(ae-bd\right)\log\left(d+ex\right)}{e^5}-\frac{a^4e^4+2a^3bde^3+6a^2b^2d^2e^2-22ab^3d^3e+13b^4d^4+x^2\left(18a^2b^2e^4-36ab^3de^3+18b^4d^2e\right.}{3d^3e^5+9d^2e^6x+9de^7x^2+3e^8x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**2/(e*x+d)**4,x)
```

```
[Out] b**4*x/e**4 + 4*b**3*(a*e - b*d)*log(d + e*x)/e**5 - (a**4*e**4 + 2*a**3*b*
d*e**3 + 6*a**2*b**2*d**2*e**2 - 22*a*b**3*d**3*e + 13*b**4*d**4 + x**2*(18
*a**2*b**2*e**4 - 36*a*b**3*d*e**3 + 18*b**4*d**2*e**2) + x*(6*a**3*b*e**4
+ 18*a**2*b**2*d*e**3 - 54*a*b**3*d**2*e**2 + 30*b**4*d**3*e))/(3*d**3*e**5
 + 9*d**2*e**6*x + 9*d*e**7*x**2 + 3*e**8*x**3)
```

Giac [A] time = 1.17378, size = 230, normalized size = 2.23

$$b^4xe^{(-4)}-4\left(b^4d-ab^3e\right)e^{(-5)}\log\left(|xe+d|\right)-\frac{\left(13\,b^4d^4-22\,ab^3d^3e+6\,a^2b^2d^2e^2+2\,a^3bde^3+a^4e^4+18\left(b^4d^2e^2-2\,ab^3de^3\right.\right.}{3\left(xe+d\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] b^4*x*e^(-4) - 4*(b^4*d - a*b^3*e)*e^(-5)*log(abs(x*e + d)) - 1/3*(13*b^4*d
^4 - 22*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 + 2*a^3*b*d*e^3 + a^4*e^4 + 18*(b^4
*d^2*e^2 - 2*a*b^3*d*e^3 + a^2*b^2*e^4)*x^2 + 6*(5*b^4*d^3*e - 9*a*b^3*d^2*
e^2 + 3*a^2*b^2*d*e^3 + a^3*b*e^4)*x)*e^(-5)/(x*e + d)^3
```

3.1474 $\int \frac{\left(a^2+2abx+b^2x^2\right)^2}{(d+ex)^5}\,dx$

Optimal. Leaf size=111

$$\frac{4b^3(bd-ae)}{e^5(d+ex)}-\frac{3b^2(bd-ae)^2}{e^5(d+ex)^2}+\frac{4b(bd-ae)^3}{3e^5(d+ex)^3}-\frac{(bd-ae)^4}{4e^5(d+ex)^4}+\frac{b^4\log(d+ex)}{e^5}$$

```
[Out] -(b*d - a*e)^4/(4*e^5*(d + e*x)^4) + (4*b*(b*d - a*e)^3)/(3*e^5*(d + e*x)^3
) - (3*b^2*(b*d - a*e)^2)/(e^5*(d + e*x)^2) + (4*b^3*(b*d - a*e))/(e^5*(d +
 e*x)) + (b^4*Log[d + e*x])/e^5
```

Rubi [A] time = 0.0765808, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$\frac{4b^3(bd-ae)}{e^5(d+ex)}-\frac{3b^2(bd-ae)^2}{e^5(d+ex)^2}+\frac{4b(bd-ae)^3}{3e^5(d+ex)^3}-\frac{(bd-ae)^4}{4e^5(d+ex)^4}+\frac{b^4\log(d+ex)}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^5,x]
```


```
[Out] -(b*d - a*e)^4/(4*e^5*(d + e*x)^4) + (4*b*(b*d - a*e)^3)/(3*e^5*(d + e*x)^3
) - (3*b^2*(b*d - a*e)^2)/(e^5*(d + e*x)^2) + (4*b^3*(b*d - a*e))/(e^5*(d +
 e*x)) + (b^4*Log[d + e*x])/e^5
```


Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx+b^2x^2\right)^2}{(d+ex)^5}\,dx &= \int \frac{(a+bx)^4}{(d+ex)^5}\,dx\\ &= \int \left(\frac{(-bd+ae)^4}{e^4(d+ex)^5}-\frac{4b(bd-ae)^3}{e^4(d+ex)^4}+\frac{6b^2(bd-ae)^2}{e^4(d+ex)^3}-\frac{4b^3(bd-ae)}{e^4(d+ex)^2}+\frac{b^4}{e^4(d+ex)}\right)dx\\ &= -\frac{(bd-ae)^4}{4e^5(d+ex)^4}+\frac{4b(bd-ae)^3}{3e^5(d+ex)^3}-\frac{3b^2(bd-ae)^2}{e^5(d+ex)^2}+\frac{4b^3(bd-ae)}{e^5(d+ex)}+\frac{b^4\log(d+ex)}{e^5}\end{aligned}$$

Mathematica [A] time = 0.0649222, size = 119, normalized size = 1.07

$$\frac{\frac{(bd-ae)\left(a^2be^2(7d+16ex)+3a^3e^3+ab^2e\left(13d^2+40dex+36e^2x^2\right)+b^3\left(88d^2ex+25d^3+108de^2x^2+48e^3x^3\right)\right)}{(d+ex)^4}+12b^4\log(d+ex)}{12e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^5,x]
```

```
[Out] (((b*d - a*e)*(3*a^3*e^3 + a^2*b*e^2*(7*d + 16*e*x) + a*b^2*e*(13*d^2 + 40*
d*e*x + 36*e^2*x^2) + b^3*(25*d^3 + 88*d^2*e*x + 108*d*e^2*x^2 + 48*e^3*x^3
)))/(d + e*x)^4 + 12*b^4*Log[d + e*x])/(12*e^5)
```

Maple [B] time = 0.047, size = 260, normalized size = 2.3

$$-\frac{4\,a^3b}{3\,e^2\,(ex+d)^3}+4\,\frac{b^2a^2d}{e^3\,(ex+d)^3}-4\,\frac{d^2ab^3}{e^4\,(ex+d)^3}+\frac{4\,b^4d^3}{3\,e^5\,(ex+d)^3}-\frac{a^4}{4\,e\,(ex+d)^4}+\frac{a^3bd}{e^2\,(ex+d)^4}-\frac{3\,b^2a^2d^2}{2\,e^3\,(ex+d)^4}+\frac{ab^3d^3}{e^4\,(ex+\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^5,x)
```

```
[Out] -4/3*b/e^2/(e*x+d)^3*a^3+4*b^2/e^3/(e*x+d)^3*a^2*d-4*b^3/e^4/(e*x+d)^3*a*d^
2+4/3*b^4/e^5/(e*x+d)^3*d^3-1/4/e/(e*x+d)^4*a^4+1/e^2/(e*x+d)^4*a^3*b*d-3/2
/e^3/(e*x+d)^4*d^2*b^2*a^2+1/e^4/(e*x+d)^4*d^3*a*b^3-1/4/e^5/(e*x+d)^4*b^4*
d^4-3*b^2/e^3/(e*x+d)^2*a^2+6*b^3/e^4/(e*x+d)^2*a*d-3*b^4/e^5/(e*x+d)^2*d^2
+b^4*ln(e*x+d)/e^5-4*b^3/e^4/(e*x+d)*a+4*b^4/e^5/(e*x+d)*d
```

Maxima [B] time = 1.22636, size = 297, normalized size = 2.68

$$\frac{25\,b^4d^4-12\,ab^3d^3e-6\,a^2b^2d^2e^2-4\,a^3bde^3-3\,a^4e^4+48\left(b^4de^3-ab^3e^4\right)x^3+36\left(3\,b^4d^2e^2-2\,ab^3de^3-a^2b^2e^4\right)x^2+8\,(11\ldots}{12\left(e^9x^4+4\,de^8x^3+6\,d^2e^7x^2+4\,d^3e^6x+d^4e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^5,x, algorithm="maxima")
```

```
[Out] 1/12*(25*b^4*d^4 - 12*a*b^3*d^3*e - 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 - 3*a
^4*e^4 + 48*(b^4*d*e^3 - a*b^3*e^4)*x^3 + 36*(3*b^4*d^2*e^2 - 2*a*b^3*d*e^3
 - a^2*b^2*e^4)*x^2 + 8*(11*b^4*d^3*e - 6*a*b^3*d^2*e^2 - 3*a^2*b^2*d*e^3 -
 2*a^3*b*e^4)*x)/(e^9*x^4 + 4*d*e^8*x^3 + 6*d^2*e^7*x^2 + 4*d^3*e^6*x + d^4
*e^5) + b^4*log(e*x + d)/e^5
```

Fricas [B] time = 1.99869, size = 544, normalized size = 4.9

$$\frac{25\,b^4d^4-12\,ab^3d^3e-6\,a^2b^2d^2e^2-4\,a^3bde^3-3\,a^4e^4+48\left(b^4de^3-ab^3e^4\right)x^3+36\left(3\,b^4d^2e^2-2\,ab^3de^3-a^2b^2e^4\right)x^2+8\,(11\ldots}{12\left(e^9x^4+4\,de^8x^3+6\,d^2e^7x^2\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(25*b^4*d^4 - 12*a*b^3*d^3*e - 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 - 3*a
^4*e^4 + 48*(b^4*d*e^3 - a*b^3*e^4)*x^3 + 36*(3*b^4*d^2*e^2 - 2*a*b^3*d*e^3
 - a^2*b^2*e^4)*x^2 + 8*(11*b^4*d^3*e - 6*a*b^3*d^2*e^2 - 3*a^2*b^2*d*e^3 -
```

```
 2*a^3*b*e^4)*x + 12*(b^4*e^4*x^4 + 4*b^4*d*e^3*x^3 + 6*b^4*d^2*e^2*x^2 + 4
*b^4*d^3*e*x + b^4*d^4)*log(e*x + d))/(e^9*x^4 + 4*d*e^8*x^3 + 6*d^2*e^7*x^
2 + 4*d^3*e^6*x + d^4*e^5)
```

Sympy [B] time = 3.70738, size = 230, normalized size = 2.07

$$\frac{b^4 \log(d+ex)}{e^5} - \frac{3a^4e^4 + 4a^3bde^3 + 6a^2b^2d^2e^2 + 12ab^3d^3e - 25b^4d^4 + x^3\left(48ab^3e^4 - 48b^4de^3\right) + x^2\left(36a^2b^2e^4 + 72ab^3\right.}{12d^4e^5 + 48d^3e^6x + 72d^2e^7x^2 + 48de^8x^3 +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**2/(e*x+d)**5,x)
```

```
[Out] b**4*log(d + e*x)/e**5 - (3*a**4*e**4 + 4*a**3*b*d*e**3 + 6*a**2*b**2*d**2*
e**2 + 12*a*b**3*d**3*e - 25*b**4*d**4 + x**3*(48*a*b**3*e**4 - 48*b**4*d*e
**3) + x**2*(36*a**2*b**2*e**4 + 72*a*b**3*d*e**3 - 108*b**4*d**2*e**2) + x
*(16*a**3*b*e**4 + 24*a**2*b**2*d*e**3 + 48*a*b**3*d**2*e**2 - 88*b**4*d**3
*e))/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 +
12*e**9*x**4)
```

Giac [B] time = 1.17618, size = 377, normalized size = 3.4

$$-b^4e^{(-5)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^2}\right) + \frac{1}{12}\left(\frac{48\,b^4de^{15}}{xe+d} - \frac{36\,b^4d^2e^{15}}{(xe+d)^2} + \frac{16\,b^4d^3e^{15}}{(xe+d)^3} - \frac{3\,b^4d^4e^{15}}{(xe+d)^4} - \frac{48\,ab^3e^{16}}{xe+d} + \frac{72\,ab^3de^{16}}{(xe+d)^2} - \frac{48\,ab^3}{(xe+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] -b^4*e^(-5)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2) + 1/12*(48*b^4*d*e^15/(x*e
 + d) - 36*b^4*d^2*e^15/(x*e + d)^2 + 16*b^4*d^3*e^15/(x*e + d)^3 - 3*b^4*d
^4*e^15/(x*e + d)^4 - 48*a*b^3*e^16/(x*e + d) + 72*a*b^3*d*e^16/(x*e + d)^2
 - 48*a*b^3*d^2*e^16/(x*e + d)^3 + 12*a*b^3*d^3*e^16/(x*e + d)^4 - 36*a^2*b
^2*e^17/(x*e + d)^2 + 48*a^2*b^2*d*e^17/(x*e + d)^3 - 18*a^2*b^2*d^2*e^17/(
x*e + d)^4 - 16*a^3*b*e^18/(x*e + d)^3 + 12*a^3*b*d*e^18/(x*e + d)^4 - 3*a^
4*e^19/(x*e + d)^4)*e^(-20)
```

3.1475 $\int \frac{\left(a^2+2abx+b^2x^2\right)^2}{(d+ex)^6}\,dx$

Optimal. Leaf size=28

$$\frac{(a+bx)^5}{5(d+ex)^5(bd-ae)}$$

```
[Out] (a + b*x)^5/(5*(b*d - a*e)*(d + e*x)^5)
```

Rubi [A] time = 0.004715, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 37}

$$\frac{(a+bx)^5}{5(d+ex)^5(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^6,x]
```

```
[Out] (a + b*x)^5/(5*(b*d - a*e)*(d + e*x)^5)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx+b^2x^2\right)^2}{(d+ex)^6}\,dx &= \int \frac{(a+bx)^4}{(d+ex)^6}\,dx\\ &= \frac{(a+bx)^5}{5(bd-ae)(d+ex)^5}\end{aligned}$$

Mathematica [B] time = 0.0535618, size = 140, normalized size = 5.

$$-\frac{a^2b^2e^2\left(d^2+5dex+10e^2x^2\right)+a^3be^3(d+5ex)+a^4e^4+ab^3e\left(5d^2ex+d^3+10de^2x^2+10e^3x^3\right)+b^4\left(10d^2e^2x^2+5d^3ex+\right.}{5e^5(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^6,x]
```

```
[Out] -(a^4*e^4 + a^3*b*e^3*(d + 5*e*x) + a^2*b^2*e^2*(d^2 + 5*d*e*x + 10*e^2*x^2
) + a*b^3*e*(d^3 + 5*d^2*e*x + 10*d*e^2*x^2 + 10*e^3*x^3) + b^4*(d^4 + 5*d^
3*e*x + 10*d^2*e^2*x^2 + 10*d*e^3*x^3 + 5*e^4*x^4))/(5*e^5*(d + e*x)^5)
```

Maple [B] time = 0.046, size = 186, normalized size = 6.6

$$-2\,\frac{b^2\left(a^2e^2-2\,abde+b^2d^2\right)}{e^5\left(ex+d\right)^3}-\frac{b\left(a^3e^3-3\,a^2bde^2+3\,ab^2d^2e-b^3d^3\right)}{e^5\left(ex+d\right)^4}-2\,\frac{b^3\left(ae-bd\right)}{e^5\left(ex+d\right)^2}-\frac{b^4}{e^5\left(ex+d\right)}-\frac{a^4e^4-4\,a^3bde^3+\dots}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^6,x)
```

```
[Out] -2*b^2*(a^2*e^2-2*a*b*d*e+b^2*d^2)/e^5/(e*x+d)^3-b*(a^3*e^3-3*a^2*b*d*e^2+3
*a*b^2*d^2*e-b^3*d^3)/e^5/(e*x+d)^4-2*b^3*(a*e-b*d)/e^5/(e*x+d)^2-b^4/e^5/(
e*x+d)-1/5*(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-4*a*b^3*d^3*e+b^4*d^4)/
e^5/(e*x+d)^5
```

Maxima [B] time = 1.16982, size = 290, normalized size = 10.36

$$-\frac{5\,b^4e^4x^4+b^4d^4+ab^3d^3e+a^2b^2d^2e^2+a^3bde^3+a^4e^4+10\left(b^4de^3+ab^3e^4\right)x^3+10\left(b^4d^2e^2+ab^3de^3+a^2b^2e^4\right)x^2+5\left(\dots\right)}{5\left(e^{10}x^5+5\,de^9x^4+10\,d^2e^8x^3+10\,d^3e^7x^2+5\,d^4e^6x+d^5e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^6,x, algorithm="maxima")
```

```
[Out] -1/5*(5*b^4*e^4*x^4 + b^4*d^4 + a*b^3*d^3*e + a^2*b^2*d^2*e^2 + a^3*b*d*e^3
 + a^4*e^4 + 10*(b^4*d*e^3 + a*b^3*e^4)*x^3 + 10*(b^4*d^2*e^2 + a*b^3*d*e^3
 + a^2*b^2*e^4)*x^2 + 5*(b^4*d^3*e + a*b^3*d^2*e^2 + a^2*b^2*d*e^3 + a^3*b*
e^4)*x)/(e^10*x^5 + 5*d*e^9*x^4 + 10*d^2*e^8*x^3 + 10*d^3*e^7*x^2 + 5*d^4*e
^6*x + d^5*e^5)
```

Fricas [B] time = 1.92392, size = 428, normalized size = 15.29

$$-\frac{5\,b^4e^4x^4+b^4d^4+ab^3d^3e+a^2b^2d^2e^2+a^3bde^3+a^4e^4+10\left(b^4de^3+ab^3e^4\right)x^3+10\left(b^4d^2e^2+ab^3de^3+a^2b^2e^4\right)x^2+5\left(\dots\right)}{5\left(e^{10}x^5+5\,de^9x^4+10\,d^2e^8x^3+10\,d^3e^7x^2+5\,d^4e^6x+d^5e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^6,x, algorithm="fricas")
```

```
[Out] -1/5*(5*b^4*e^4*x^4 + b^4*d^4 + a*b^3*d^3*e + a^2*b^2*d^2*e^2 + a^3*b*d*e^3
 + a^4*e^4 + 10*(b^4*d*e^3 + a*b^3*e^4)*x^3 + 10*(b^4*d^2*e^2 + a*b^3*d*e^3
 + a^2*b^2*e^4)*x^2 + 5*(b^4*d^3*e + a*b^3*d^2*e^2 + a^2*b^2*d*e^3 + a^3*b*
e^4)*x)/(e^10*x^5 + 5*d*e^9*x^4 + 10*d^2*e^8*x^3 + 10*d^3*e^7*x^2 + 5*d^4*e
^6*x + d^5*e^5)
```

Sympy [B] time = 6.00661, size = 233, normalized size = 8.32

$$-\frac{a^4e^4 + a^3bde^3 + a^2b^2d^2e^2 + ab^3d^3e + b^4d^4 + 5b^4e^4x^4 + x^3\left(10ab^3e^4 + 10b^4de^3\right) + x^2\left(10a^2b^2e^4 + 10ab^3de^3 + 10b^4d^2e^2\right) + \cdots}{5d^5e^5 + 25d^4e^6x + 50d^3e^7x^2 + 50d^2e^8x^3 + 25de^9x^4 + 5e^{10}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**2/(e*x+d)**6,x)
```

```
[Out] -(a**4*e**4 + a**3*b*d*e**3 + a**2*b**2*d**2*e**2 + a*b**3*d**3*e + b**4*d*
*4 + 5*b**4*e**4*x**4 + x**3*(10*a*b**3*e**4 + 10*b**4*d*e**3) + x**2*(10*a
**2*b**2*e**4 + 10*a*b**3*d*e**3 + 10*b**4*d**2*e**2) + x*(5*a**3*b*e**4 +
5*a**2*b**2*d*e**3 + 5*a*b**3*d**2*e**2 + 5*b**4*d**3*e))/(5*d**5*e**5 + 25
*d**4*e**6*x + 50*d**3*e**7*x**2 + 50*d**2*e**8*x**3 + 25*d*e**9*x**4 + 5*e
**10*x**5)
```

Giac [B] time = 1.171, size = 230, normalized size = 8.21

$$-\frac{\left(5\,b^4x^4e^4 + 10\,b^4dx^3e^3 + 10\,b^4d^2x^2e^2 + 5\,b^4d^3xe + b^4d^4 + 10\,ab^3x^3e^4 + 10\,ab^3dx^2e^3 + 5\,ab^3d^2xe^2 + ab^3d^3e + 10\,a^2b^2x^2e^4 \cdots\right.}{5\,(xe + d)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^6,x, algorithm="giac")
```

```
[Out] -1/5*(5*b^4*x^4*e^4 + 10*b^4*d*x^3*e^3 + 10*b^4*d^2*x^2*e^2 + 5*b^4*d^3*x*e
 + b^4*d^4 + 10*a*b^3*x^3*e^4 + 10*a*b^3*d*x^2*e^3 + 5*a*b^3*d^2*x*e^2 + a*
b^3*d^3*e + 10*a^2*b^2*x^2*e^4 + 5*a^2*b^2*d*x*e^3 + a^2*b^2*d^2*e^2 + 5*a^
3*b*x*e^4 + a^3*b*d*e^3 + a^4*e^4)*e^(-5)/(x*e + d)^5
```

3.1476 $\int \frac{(a^2+2abx+b^2x^2)^2}{(d+ex)^7} dx$

Optimal. Leaf size=58

$$\frac{b(a+bx)^5}{30(d+ex)^5(bd-ae)^2}+\frac{(a+bx)^5}{6(d+ex)^6(bd-ae)}$$

```
[Out] (a + b*x)^5/(6*(b*d - a*e)*(d + e*x)^6) + (b*(a + b*x)^5)/(30*(b*d - a*e)^2
*(d + e*x)^5)
```

Rubi [A] time = 0.0142256, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {27, 45, 37}

$$\frac{b(a+bx)^5}{30(d+ex)^5(bd-ae)^2}+\frac{(a+bx)^5}{6(d+ex)^6(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^7,x]
```

```
[Out] (a + b*x)^5/(6*(b*d - a*e)*(d + e*x)^6) + (b*(a + b*x)^5)/(30*(b*d - a*e)^2
*(d + e*x)^5)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^2}{(d+ex)^7}\,dx=\int \frac{(a+bx)^4}{(d+ex)^7}\,dx$$
$$=\frac{(a+bx)^5}{6(bd-ae)(d+ex)^6}+\frac{b\int \frac{(a+bx)^4}{(d+ex)^6}\,dx}{6(bd-ae)}$$
$$=\frac{(a+bx)^5}{6(bd-ae)(d+ex)^6}+\frac{b(a+bx)^5}{30(bd-ae)^2(d+ex)^5}$$

Mathematica [B] time = 0.0451627, size = 144, normalized size = 2.48

$$-\frac{3a^2b^2e^2\left(d^2+6dex+15e^2x^2\right)+4a^3be^3(d+6ex)+5a^4e^4+2ab^3e\left(6d^2ex+d^3+15de^2x^2+20e^3x^3\right)+b^4\left(15d^2e^2x^2+6d^3\right.}{30e^5(d+ex)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^7,x]
```

```
[Out] -(5*a^4*e^4 + 4*a^3*b*e^3*(d + 6*e*x) + 3*a^2*b^2*e^2*(d^2 + 6*d*e*x + 15*e
^2*x^2) + 2*a*b^3*e*(d^3 + 6*d^2*e*x + 15*d*e^2*x^2 + 20*e^3*x^3) + b^4*(d^
4 + 6*d^3*e*x + 15*d^2*e^2*x^2 + 20*d*e^3*x^3 + 15*e^4*x^4))/(30*e^5*(d + e
*x)^6)
```

Maple [B] time = 0.045, size = 186, normalized size = 3.2

$$-\frac{4\,b^3\,(ae-bd)}{3\,e^5\,(ex+d)^3}-\frac{a^4e^4-4\,a^3bde^3+6\,d^2e^2b^2a^2-4\,d^3eab^3+b^4d^4}{6\,e^5\,(ex+d)^6}-\frac{3\,b^2\left(a^2e^2-2\,abde+b^2d^2\right)}{2\,e^5\,(ex+d)^4}-\frac{b^4}{2\,e^5\,(ex+d)^2}-\frac{4\,b\left(a^3e^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^7,x)
```

```
[Out] -4/3*b^3*(a*e-b*d)/e^5/(e*x+d)^3-1/6*(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*e
^2-4*a*b^3*d^3*e+b^4*d^4)/e^5/(e*x+d)^6-3/2*b^2*(a^2*e^2-2*a*b*d*e+b^2*d^2)
/e^5/(e*x+d)^4-1/2*b^4/e^5/(e*x+d)^2-4/5*b*(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d
^2*e-b^3*d^3)/e^5/(e*x+d)^5
```

Maxima [B] time = 1.18201, size = 319, normalized size = 5.5

$$-\frac{15\,b^4e^4x^4+b^4d^4+2\,ab^3d^3e+3\,a^2b^2d^2e^2+4\,a^3bde^3+5\,a^4e^4+20\left(b^4de^3+2\,ab^3e^4\right)x^3+15\left(b^4d^2e^2+2\,ab^3de^3+3\,a^2b^2e\right.}{30\left(e^{11}x^6+6\,de^{10}x^5+15\,d^2e^9x^4+20\,d^3e^8x^3+15\,d^4e^7x^2+6\,d^5e^6x+d^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^7,x, algorithm="maxima")
```

```
[Out] -1/30*(15*b^4*e^4*x^4 + b^4*d^4 + 2*a*b^3*d^3*e + 3*a^2*b^2*d^2*e^2 + 4*a^3
*b*d*e^3 + 5*a^4*e^4 + 20*(b^4*d*e^3 + 2*a*b^3*e^4)*x^3 + 15*(b^4*d^2*e^2 +
 2*a*b^3*d*e^3 + 3*a^2*b^2*e^4)*x^2 + 6*(b^4*d^3*e + 2*a*b^3*d^2*e^2 + 3*a^
2*b^2*d*e^3 + 4*a^3*b*e^4)*x)/(e^11*x^6 + 6*d*e^10*x^5 + 15*d^2*e^9*x^4 + 2
```

```
0*d^3*e^8*x^3 + 15*d^4*e^7*x^2 + 6*d^5*e^6*x + d^6*e^5)
```

Fricas [B] time = 1.63437, size = 482, normalized size = 8.31

$$-\frac{15\,b^4e^4x^4+b^4d^4+2\,ab^3d^3e+3\,a^2b^2d^2e^2+4\,a^3bde^3+5\,a^4e^4+20\left(b^4de^3+2\,ab^3e^4\right)x^3+15\left(b^4d^2e^2+2\,ab^3de^3+3\,a^2\right.}{30\left(e^{11}x^6+6\,de^{10}x^5+15\,d^2e^9x^4+20\,d^3e^8x^3+15\,d^4e^7x^2+6\,d^5e^6x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^7,x, algorithm="fricas")
```

```
[Out] -1/30*(15*b^4*e^4*x^4 + b^4*d^4 + 2*a*b^3*d^3*e + 3*a^2*b^2*d^2*e^2 + 4*a^3
*b*d*e^3 + 5*a^4*e^4 + 20*(b^4*d*e^3 + 2*a*b^3*e^4)*x^3 + 15*(b^4*d^2*e^2 +
 2*a*b^3*d*e^3 + 3*a^2*b^2*e^4)*x^2 + 6*(b^4*d^3*e + 2*a*b^3*d^2*e^2 + 3*a^
2*b^2*d*e^3 + 4*a^3*b*e^4)*x)/(e^11*x^6 + 6*d*e^10*x^5 + 15*d^2*e^9*x^4 + 2
0*d^3*e^8*x^3 + 15*d^4*e^7*x^2 + 6*d^5*e^6*x + d^6*e^5)
```

Sympy [B] time = 11.0187, size = 252, normalized size = 4.34

$$-\frac{5a^4e^4+4a^3bde^3+3a^2b^2d^2e^2+2ab^3d^3e+b^4d^4+15b^4e^4x^4+x^3\left(40ab^3e^4+20b^4de^3\right)+x^2\left(45a^2b^2e^4+30ab^3de^3+15\right.}{30d^6e^5+180d^5e^6x+450d^4e^7x^2+600d^3e^8x^3+450d^2e^9x^4+180de^{10}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**2/(e*x+d)**7,x)
```

```
[Out] -(5*a**4*e**4 + 4*a**3*b*d*e**3 + 3*a**2*b**2*d**2*e**2 + 2*a*b**3*d**3*e +
 b**4*d**4 + 15*b**4*e**4*x**4 + x**3*(40*a*b**3*e**4 + 20*b**4*d*e**3) + x
**2*(45*a**2*b**2*e**4 + 30*a*b**3*d*e**3 + 15*b**4*d**2*e**2) + x*(24*a**3
*b*e**4 + 18*a**2*b**2*d*e**3 + 12*a*b**3*d**2*e**2 + 6*b**4*d**3*e))/(30*d
**6*e**5 + 180*d**5*e**6*x + 450*d**4*e**7*x**2 + 600*d**3*e**8*x**3 + 450*
d**2*e**9*x**4 + 180*d*e**10*x**5 + 30*e**11*x**6)
```

Giac [B] time = 1.21412, size = 235, normalized size = 4.05

$$-\frac{\left(15\,b^4x^4e^4+20\,b^4dx^3e^3+15\,b^4d^2x^2e^2+6\,b^4d^3xe+b^4d^4+40\,ab^3x^3e^4+30\,ab^3dx^2e^3+12\,ab^3d^2xe^2+2\,ab^3d^3e+45\,a\right.}{30\,(xe+d)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^7,x, algorithm="giac")
```

```
[Out] -1/30*(15*b^4*x^4*e^4 + 20*b^4*d*x^3*e^3 + 15*b^4*d^2*x^2*e^2 + 6*b^4*d^3*x
*e + b^4*d^4 + 40*a*b^3*x^3*e^4 + 30*a*b^3*d*x^2*e^3 + 12*a*b^3*d^2*x*e^2 +
 2*a*b^3*d^3*e + 45*a^2*b^2*x^2*e^4 + 18*a^2*b^2*d*x*e^3 + 3*a^2*b^2*d^2*e^
2 + 24*a^3*b*x*e^4 + 4*a^3*b*d*e^3 + 5*a^4*e^4)*e^(-5)/(x*e + d)^6
```

3.1477 $\int \frac{(a^2+2abx+b^2x^2)^2}{(d+ex)^8} dx$

Optimal. Leaf size=89

$$\frac{b^2(a+bx)^5}{105(d+ex)^5(bd-ae)^3}+\frac{b(a+bx)^5}{21(d+ex)^6(bd-ae)^2}+\frac{(a+bx)^5}{7(d+ex)^7(bd-ae)}$$

```
[Out] (a + b*x)^5/(7*(b*d - a*e)*(d + e*x)^7) + (b*(a + b*x)^5)/(21*(b*d - a*e)^2
*(d + e*x)^6) + (b^2*(a + b*x)^5)/(105*(b*d - a*e)^3*(d + e*x)^5)
```

Rubi [A] time = 0.0214929, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {27, 45, 37}

$$\frac{b^2(a+bx)^5}{105(d+ex)^5(bd-ae)^3}+\frac{b(a+bx)^5}{21(d+ex)^6(bd-ae)^2}+\frac{(a+bx)^5}{7(d+ex)^7(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^8,x]
```

```
[Out] (a + b*x)^5/(7*(b*d - a*e)*(d + e*x)^7) + (b*(a + b*x)^5)/(21*(b*d - a*e)^2
*(d + e*x)^6) + (b^2*(a + b*x)^5)/(105*(b*d - a*e)^3*(d + e*x)^5)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^2}{(d+ex)^8}\,dx = \int \frac{(a+bx)^4}{(d+ex)^8}\,dx$$
$$= \frac{(a+bx)^5}{7(bd-ae)(d+ex)^7} + \frac{(2b)\int \frac{(a+bx)^4}{(d+ex)^7}\,dx}{7(bd-ae)}$$
$$= \frac{(a+bx)^5}{7(bd-ae)(d+ex)^7} + \frac{b(a+bx)^5}{21(bd-ae)^2(d+ex)^6} + \frac{b^2\int \frac{(a+bx)^4}{(d+ex)^6}\,dx}{21(bd-ae)^2}$$
$$= \frac{(a+bx)^5}{7(bd-ae)(d+ex)^7} + \frac{b(a+bx)^5}{21(bd-ae)^2(d+ex)^6} + \frac{b^2(a+bx)^5}{105(bd-ae)^3(d+ex)^5}$$

Mathematica [A] time = 0.050105, size = 144, normalized size = 1.62

$$-\frac{6a^2b^2e^2\left(d^2+7dex+21e^2x^2\right)+10a^3be^3(d+7ex)+15a^4e^4+3ab^3e\left(7d^2ex+d^3+21de^2x^2+35e^3x^3\right)+b^4\left(21d^2e^2x^2 \right.}{105e^5(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^8,x]
```

```
[Out] -(15*a^4*e^4 + 10*a^3*b*e^3*(d + 7*e*x) + 6*a^2*b^2*e^2*(d^2 + 7*d*e*x + 21
*e^2*x^2) + 3*a*b^3*e*(d^3 + 7*d^2*e*x + 21*d*e^2*x^2 + 35*e^3*x^3) + b^4*(
d^4 + 7*d^3*e*x + 21*d^2*e^2*x^2 + 35*d*e^3*x^3 + 35*e^4*x^4))/(105*e^5*(d
+ e*x)^7)
```

Maple [B] time = 0.046, size = 186, normalized size = 2.1

$$-\frac{b^4}{3\,e^5\,(ex+d)^3} - \frac{2\,b\left(a^3e^3-3\,a^2bde^2+3\,ab^2d^2e-b^3d^3\right)}{3\,e^5\,(ex+d)^6} - \frac{b^3\,(ae-bd)}{e^5\,(ex+d)^4} - \frac{a^4e^4-4\,a^3bde^3+6\,d^2e^2b^2a^2-4\,d^3eab^3+b^4}{7\,e^5\,(ex+d)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^8,x)
```

```
[Out] -1/3*b^4/e^5/(e*x+d)^3-2/3*b*(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d^2*e-b^3*d^3)/
e^5/(e*x+d)^6-b^3*(a*e-b*d)/e^5/(e*x+d)^4-1/7*(a^4*e^4-4*a^3*b*d*e^3+6*a^2*
b^2*d^2*e^2-4*a*b^3*d^3*e+b^4*d^4)/e^5/(e*x+d)^7-6/5*b^2*(a^2*e^2-2*a*b*d*e
+b^2*d^2)/e^5/(e*x+d)^5
```

Maxima [B] time = 1.1831, size = 333, normalized size = 3.74

$$-\frac{35\,b^4e^4x^4+b^4d^4+3\,ab^3d^3e+6\,a^2b^2d^2e^2+10\,a^3bde^3+15\,a^4e^4+35\left(b^4de^3+3\,ab^3e^4\right)x^3+21\left(b^4d^2e^2+3\,ab^3de^3+6\right.}{105\left(e^{12}x^7+7\,de^{11}x^6+21\,d^2e^{10}x^5+35\,d^3e^9x^4+35\,d^4e^8x^3+21\,d^5e^7x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^8,x, algorithm="maxima")
```

```
[Out] -1/105*(35*b^4*e^4*x^4 + b^4*d^4 + 3*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 + 10*a
^3*b*d*e^3 + 15*a^4*e^4 + 35*(b^4*d*e^3 + 3*a*b^3*e^4)*x^3 + 21*(b^4*d^2*e^
```

```
2 + 3*a*b^3*d*e^3 + 6*a^2*b^2*e^4)*x^2 + 7*(b^4*d^3*e + 3*a*b^3*d^2*e^2 + 6
*a^2*b^2*d*e^3 + 10*a^3*b*e^4)*x)/(e^12*x^7 + 7*d*e^11*x^6 + 21*d^2*e^10*x^
5 + 35*d^3*e^9*x^4 + 35*d^4*e^8*x^3 + 21*d^5*e^7*x^2 + 7*d^6*e^6*x + d^7*e^
5)
```

Fricas [B] time = 1.76871, size = 512, normalized size = 5.75

$$-\frac{35\,b^4e^4x^4 + b^4d^4 + 3\,ab^3d^3e + 6\,a^2b^2d^2e^2 + 10\,a^3bde^3 + 15\,a^4e^4 + 35\,\left(b^4de^3 + 3\,ab^3e^4\right)x^3 + 21\,\left(b^4d^2e^2 + 3\,ab^3de^3 + 6\,a^2b \cdots\right.}{105\,\left(e^{12}x^7 + 7\,de^{11}x^6 + 21\,d^2e^{10}x^5 + 35\,d^3e^9x^4 + 35\,d^4e^8x^3 + 21\,d^5e^7x^2 + 7\,d^6 \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^8,x, algorithm="fricas")
```

```
[Out] -1/105*(35*b^4*e^4*x^4 + b^4*d^4 + 3*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 + 10*a
^3*b*d*e^3 + 15*a^4*e^4 + 35*(b^4*d*e^3 + 3*a*b^3*e^4)*x^3 + 21*(b^4*d^2*e^
2 + 3*a*b^3*d*e^3 + 6*a^2*b^2*e^4)*x^2 + 7*(b^4*d^3*e + 3*a*b^3*d^2*e^2 + 6
*a^2*b^2*d*e^3 + 10*a^3*b*e^4)*x)/(e^12*x^7 + 7*d*e^11*x^6 + 21*d^2*e^10*x^
5 + 35*d^3*e^9*x^4 + 35*d^4*e^8*x^3 + 21*d^5*e^7*x^2 + 7*d^6*e^6*x + d^7*e^
5)
```

Sympy [B] time = 16.6001, size = 264, normalized size = 2.97

$$-\frac{15a^4e^4 + 10a^3bde^3 + 6a^2b^2d^2e^2 + 3ab^3d^3e + b^4d^4 + 35b^4e^4x^4 + x^3\left(105ab^3e^4 + 35b^4de^3\right) + x^2\left(126a^2b^2e^4 + 63ab^3de^3 + 2 \cdots\right.}{105d^7e^5 + 735d^6e^6x + 2205d^5e^7x^2 + 3675d^4e^8x^3 + 3675d^3e^9x^4 + 2205d^2e^{10}x^5 + 7 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**2/(e*x+d)**8,x)
```

```
[Out] -(15*a**4*e**4 + 10*a**3*b*d*e**3 + 6*a**2*b**2*d**2*e**2 + 3*a*b**3*d**3*e
 + b**4*d**4 + 35*b**4*e**4*x**4 + x**3*(105*a*b**3*e**4 + 35*b**4*d*e**3)
+ x**2*(126*a**2*b**2*e**4 + 63*a*b**3*d*e**3 + 21*b**4*d**2*e**2) + x*(70*
a**3*b*e**4 + 42*a**2*b**2*d*e**3 + 21*a*b**3*d**2*e**2 + 7*b**4*d**3*e))/(
105*d**7*e**5 + 735*d**6*e**6*x + 2205*d**5*e**7*x**2 + 3675*d**4*e**8*x**3
 + 3675*d**3*e**9*x**4 + 2205*d**2*e**10*x**5 + 735*d*e**11*x**6 + 105*e**1
2*x**7)
```

Giac [B] time = 1.10957, size = 235, normalized size = 2.64

$$-\frac{\left(35\,b^4x^4e^4 + 35\,b^4dx^3e^3 + 21\,b^4d^2x^2e^2 + 7\,b^4d^3xe + b^4d^4 + 105\,ab^3x^3e^4 + 63\,ab^3dx^2e^3 + 21\,ab^3d^2xe^2 + 3\,ab^3d^3e + 126\,a^2 \cdots\right.}{105\,(xe + d)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^8,x, algorithm="giac")
```

```
[Out] -1/105*(35*b^4*x^4*e^4 + 35*b^4*d*x^3*e^3 + 21*b^4*d^2*x^2*e^2 + 7*b^4*d^3*
x*e + b^4*d^4 + 105*a*b^3*x^3*e^4 + 63*a*b^3*d*x^2*e^3 + 21*a*b^3*d^2*x*e^2
 + 3*a*b^3*d^3*e + 126*a^2*b^2*x^2*e^4 + 42*a^2*b^2*d*x*e^3 + 6*a^2*b^2*d^2
*e^2 + 70*a^3*b*x*e^4 + 10*a^3*b*d*e^3 + 15*a^4*e^4)*e^(-5)/(x*e + d)^7
```

3.1478 $\int \frac{\left(a^2+2abx+b^2x^2\right)^2}{(d+ex)^9}\,dx$

Optimal. Leaf size=117

$$\frac{4b^3(bd-ae)}{5e^5(d+ex)^5}-\frac{b^2(bd-ae)^2}{e^5(d+ex)^6}+\frac{4b(bd-ae)^3}{7e^5(d+ex)^7}-\frac{(bd-ae)^4}{8e^5(d+ex)^8}-\frac{b^4}{4e^5(d+ex)^4}$$

```
[Out] -(b*d - a*e)^4/(8*e^5*(d + e*x)^8) + (4*b*(b*d - a*e)^3)/(7*e^5*(d + e*x)^7
) - (b^2*(b*d - a*e)^2)/(e^5*(d + e*x)^6) + (4*b^3*(b*d - a*e))/(5*e^5*(d +
 e*x)^5) - b^4/(4*e^5*(d + e*x)^4)
```

Rubi [A] time = 0.0713198, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$\frac{4b^3(bd-ae)}{5e^5(d+ex)^5}-\frac{b^2(bd-ae)^2}{e^5(d+ex)^6}+\frac{4b(bd-ae)^3}{7e^5(d+ex)^7}-\frac{(bd-ae)^4}{8e^5(d+ex)^8}-\frac{b^4}{4e^5(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^9,x]
```

```
[Out] -(b*d - a*e)^4/(8*e^5*(d + e*x)^8) + (4*b*(b*d - a*e)^3)/(7*e^5*(d + e*x)^7
) - (b^2*(b*d - a*e)^2)/(e^5*(d + e*x)^6) + (4*b^3*(b*d - a*e))/(5*e^5*(d +
 e*x)^5) - b^4/(4*e^5*(d + e*x)^4)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx+b^2x^2\right)^2}{(d+ex)^9}\,dx &= \int \frac{(a+bx)^4}{(d+ex)^9}\,dx\\ &= \int \left(\frac{(-bd+ae)^4}{e^4(d+ex)^9}-\frac{4b(bd-ae)^3}{e^4(d+ex)^8}+\frac{6b^2(bd-ae)^2}{e^4(d+ex)^7}-\frac{4b^3(bd-ae)}{e^4(d+ex)^6}+\frac{b^4}{e^4(d+ex)^5}\right)dx\\ &= -\frac{(bd-ae)^4}{8e^5(d+ex)^8}+\frac{4b(bd-ae)^3}{7e^5(d+ex)^7}-\frac{b^2(bd-ae)^2}{e^5(d+ex)^6}+\frac{4b^3(bd-ae)}{5e^5(d+ex)^5}-\frac{b^4}{4e^5(d+ex)^4}\end{aligned}$$

Mathematica [A] time = 0.0466586, size = 144, normalized size = 1.23

$$-\frac{10a^2b^2e^2\left(d^2+8dex+28e^2x^2\right)+20a^3be^3(d+8ex)+35a^4e^4+4ab^3e\left(8d^2ex+d^3+28de^2x^2+56e^3x^3\right)+b^4\left(28d^2e^2x^2\right.}{280e^5(d+ex)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^9,x]
```

```
[Out] -(35*a^4*e^4 + 20*a^3*b*e^3*(d + 8*e*x) + 10*a^2*b^2*e^2*(d^2 + 8*d*e*x + 2
8*e^2*x^2) + 4*a*b^3*e*(d^3 + 8*d^2*e*x + 28*d*e^2*x^2 + 56*e^3*x^3) + b^4*
(d^4 + 8*d^3*e*x + 28*d^2*e^2*x^2 + 56*d*e^3*x^3 + 70*e^4*x^4))/(280*e^5*(d
 + e*x)^8)
```

Maple [A] time = 0.046, size = 186, normalized size = 1.6

$$-\frac{b^2\left(a^2e^2-2\,abde+b^2d^2\right)}{e^5\left(ex+d\right)^6}-\frac{b^4}{4\,e^5\left(ex+d\right)^4}-\frac{a^4e^4-4\,a^3bde^3+6\,d^2e^2b^2a^2-4\,d^3eab^3+b^4d^4}{8\,e^5\left(ex+d\right)^8}-\frac{4\,b\left(a^3e^3-3\,a^2bde^2+3\,ab\right.}{7\,e^5\left(ex+d\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^9,x)
```

```
[Out] -b^2*(a^2*e^2-2*a*b*d*e+b^2*d^2)/e^5/(e*x+d)^6-1/4*b^4/e^5/(e*x+d)^4-1/8*(a
^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-4*a*b^3*d^3*e+b^4*d^4)/e^5/(e*x+d)^8
-4/7*b*(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d^2*e-b^3*d^3)/e^5/(e*x+d)^7-4/5*b^3*
(a*e-b*d)/e^5/(e*x+d)^5
```

Maxima [B] time = 1.15478, size = 348, normalized size = 2.97

$$-\frac{70\,b^4e^4x^4+b^4d^4+4\,ab^3d^3e+10\,a^2b^2d^2e^2+20\,a^3bde^3+35\,a^4e^4+56\left(b^4de^3+4\,ab^3e^4\right)x^3+28\left(b^4d^2e^2+4\,ab^3de^3+10\,a\right.}{280\left(e^{13}x^8+8\,de^{12}x^7+28\,d^2e^{11}x^6+56\,d^3e^{10}x^5+70\,d^4e^9x^4+56\,d^5e^8x^3+28\,d^6e^7\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^9,x, algorithm="maxima")
```

```
[Out] -1/280*(70*b^4*e^4*x^4 + b^4*d^4 + 4*a*b^3*d^3*e + 10*a^2*b^2*d^2*e^2 + 20*
a^3*b*d*e^3 + 35*a^4*e^4 + 56*(b^4*d*e^3 + 4*a*b^3*e^4)*x^3 + 28*(b^4*d^2*e
^2 + 4*a*b^3*d*e^3 + 10*a^2*b^2*e^4)*x^2 + 8*(b^4*d^3*e + 4*a*b^3*d^2*e^2 +
 10*a^2*b^2*d*e^3 + 20*a^3*b*e^4)*x)/(e^13*x^8 + 8*d*e^12*x^7 + 28*d^2*e^11
*x^6 + 56*d^3*e^10*x^5 + 70*d^4*e^9*x^4 + 56*d^5*e^8*x^3 + 28*d^6*e^7*x^2 +
 8*d^7*e^6*x + d^8*e^5)
```

Fricas [B] time = 1.65301, size = 540, normalized size = 4.62

$$-\frac{70\,b^4e^4x^4+b^4d^4+4\,ab^3d^3e+10\,a^2b^2d^2e^2+20\,a^3bde^3+35\,a^4e^4+56\left(b^4de^3+4\,ab^3e^4\right)x^3+28\left(b^4d^2e^2+4\,ab^3de^3+10\,a\right.}{280\left(e^{13}x^8+8\,de^{12}x^7+28\,d^2e^{11}x^6+56\,d^3e^{10}x^5+70\,d^4e^9x^4+56\,d^5e^8x^3+28\,d^6e^7\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^9,x, algorithm="fricas")
```

```
[Out] -1/280*(70*b^4*e^4*x^4 + b^4*d^4 + 4*a*b^3*d^3*e + 10*a^2*b^2*d^2*e^2 + 20*
a^3*b*d*e^3 + 35*a^4*e^4 + 56*(b^4*d*e^3 + 4*a*b^3*e^4)*x^3 + 28*(b^4*d^2*e
```

```
^2 + 4*a*b^3*d*e^3 + 10*a^2*b^2*e^4)*x^2 + 8*(b^4*d^3*e + 4*a*b^3*d^2*e^2 +
 10*a^2*b^2*d*e^3 + 20*a^3*b*e^4)*x)/(e^13*x^8 + 8*d*e^12*x^7 + 28*d^2*e^11
*x^6 + 56*d^3*e^10*x^5 + 70*d^4*e^9*x^4 + 56*d^5*e^8*x^3 + 28*d^6*e^7*x^2 +
 8*d^7*e^6*x + d^8*e^5)
```

Sympy [B] time = 23.1461, size = 275, normalized size = 2.35

$$-\frac{35a^4e^4 + 20a^3bde^3 + 10a^2b^2d^2e^2 + 4ab^3d^3e + b^4d^4 + 70b^4e^4x^4 + x^3\left(224ab^3e^4 + 56b^4de^3\right) + x^2\left(280a^2b^2e^4 + 112ab^3d\right.}{280d^8e^5 + 2240d^7e^6x + 7840d^6e^7x^2 + 15680d^5e^8x^3 + 19600d^4e^9x^4 + 15680d^3e^{10}x^5 + 78}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**2/(e*x+d)**9,x)
```

```
[Out] -(35*a**4*e**4 + 20*a**3*b*d*e**3 + 10*a**2*b**2*d**2*e**2 + 4*a*b**3*d**3*
e + b**4*d**4 + 70*b**4*e**4*x**4 + x**3*(224*a*b**3*e**4 + 56*b**4*d*e**3)
 + x**2*(280*a**2*b**2*e**4 + 112*a*b**3*d*e**3 + 28*b**4*d**2*e**2) + x*(1
60*a**3*b*e**4 + 80*a**2*b**2*d*e**3 + 32*a*b**3*d**2*e**2 + 8*b**4*d**3*e)
)/(280*d**8*e**5 + 2240*d**7*e**6*x + 7840*d**6*e**7*x**2 + 15680*d**5*e**8
*x**3 + 19600*d**4*e**9*x**4 + 15680*d**3*e**10*x**5 + 7840*d**2*e**11*x**6
 + 2240*d*e**12*x**7 + 280*e**13*x**8)
```

Giac [A] time = 1.09208, size = 235, normalized size = 2.01

$$-\frac{\left(70\,b^4x^4e^4 + 56\,b^4dx^3e^3 + 28\,b^4d^2x^2e^2 + 8\,b^4d^3xe + b^4d^4 + 224\,ab^3x^3e^4 + 112\,ab^3dx^2e^3 + 32\,ab^3d^2xe^2 + 4\,ab^3d^3e + 2\right.}{280\,(xe + d)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^9,x, algorithm="giac")
```

```
[Out] -1/280*(70*b^4*x^4*e^4 + 56*b^4*d*x^3*e^3 + 28*b^4*d^2*x^2*e^2 + 8*b^4*d^3*
x*e + b^4*d^4 + 224*a*b^3*x^3*e^4 + 112*a*b^3*d*x^2*e^3 + 32*a*b^3*d^2*x*e^
2 + 4*a*b^3*d^3*e + 280*a^2*b^2*x^2*e^4 + 80*a^2*b^2*d*x*e^3 + 10*a^2*b^2*d
^2*e^2 + 160*a^3*b*x*e^4 + 20*a^3*b*d*e^3 + 35*a^4*e^4)*e^(-5)/(x*e + d)^8
```

3.1479 $\int \frac{\left(a^2+2abx+b^2x^2\right)^2}{(d+ex)^{10}}\,dx$

Optimal. Leaf size=119

$$\frac{2b^3(bd-ae)}{3e^5(d+ex)^6}-\frac{6b^2(bd-ae)^2}{7e^5(d+ex)^7}+\frac{b(bd-ae)^3}{2e^5(d+ex)^8}-\frac{(bd-ae)^4}{9e^5(d+ex)^9}-\frac{b^4}{5e^5(d+ex)^5}$$

```
[Out] -(b*d - a*e)^4/(9*e^5*(d + e*x)^9) + (b*(b*d - a*e)^3)/(2*e^5*(d + e*x)^8)
- (6*b^2*(b*d - a*e)^2)/(7*e^5*(d + e*x)^7) + (2*b^3*(b*d - a*e))/(3*e^5*(d
 + e*x)^6) - b^4/(5*e^5*(d + e*x)^5)
```

Rubi [A] time = 0.06821, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$\frac{2b^3(bd-ae)}{3e^5(d+ex)^6}-\frac{6b^2(bd-ae)^2}{7e^5(d+ex)^7}+\frac{b(bd-ae)^3}{2e^5(d+ex)^8}-\frac{(bd-ae)^4}{9e^5(d+ex)^9}-\frac{b^4}{5e^5(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^10,x]
```

```
[Out] -(b*d - a*e)^4/(9*e^5*(d + e*x)^9) + (b*(b*d - a*e)^3)/(2*e^5*(d + e*x)^8)
- (6*b^2*(b*d - a*e)^2)/(7*e^5*(d + e*x)^7) + (2*b^3*(b*d - a*e))/(3*e^5*(d
 + e*x)^6) - b^4/(5*e^5*(d + e*x)^5)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx+b^2x^2\right)^2}{(d+ex)^{10}}\,dx &= \int \frac{(a+bx)^4}{(d+ex)^{10}}\,dx\\ &= \int \left(\frac{(-bd+ae)^4}{e^4(d+ex)^{10}}-\frac{4b(bd-ae)^3}{e^4(d+ex)^9}+\frac{6b^2(bd-ae)^2}{e^4(d+ex)^8}-\frac{4b^3(bd-ae)}{e^4(d+ex)^7}+\frac{b^4}{e^4(d+ex)^6}\right)dx\\ &= -\frac{(bd-ae)^4}{9e^5(d+ex)^9}+\frac{b(bd-ae)^3}{2e^5(d+ex)^8}-\frac{6b^2(bd-ae)^2}{7e^5(d+ex)^7}+\frac{2b^3(bd-ae)}{3e^5(d+ex)^6}-\frac{b^4}{5e^5(d+ex)^5}\end{aligned}$$

Mathematica [A] time = 0.0512886, size = 144, normalized size = 1.21

$$-\frac{15a^2b^2e^2\left(d^2+9dex+36e^2x^2\right)+35a^3be^3(d+9ex)+70a^4e^4+5ab^3e\left(9d^2ex+d^3+36de^2x^2+84e^3x^3\right)+b^4\left(36d^2e^2x^2+\right.}{630e^5(d+ex)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^10,x]
```

```
[Out] -(70*a^4*e^4 + 35*a^3*b*e^3*(d + 9*e*x) + 15*a^2*b^2*e^2*(d^2 + 9*d*e*x + 3
6*e^2*x^2) + 5*a*b^3*e*(d^3 + 9*d^2*e*x + 36*d*e^2*x^2 + 84*e^3*x^3) + b^4*
(d^4 + 9*d^3*e*x + 36*d^2*e^2*x^2 + 84*d*e^3*x^3 + 126*e^4*x^4))/(630*e^5*(
d + e*x)^9)
```

Maple [A] time = 0.046, size = 186, normalized size = 1.6

$$-\frac{a^4e^4-4a^3bde^3+6d^2e^2b^2a^2-4d^3eab^3+b^4d^4}{9e^5(ex+d)^9}-\frac{2b^3(ae-bd)}{3e^5(ex+d)^6}-\frac{b(a^3e^3-3a^2bde^2+3ab^2d^2e-b^3d^3)}{2e^5(ex+d)^8}-\frac{6b^2(a^2e^2-}{7e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^10,x)
```

```
[Out] -1/9*(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-4*a*b^3*d^3*e+b^4*d^4)/e^5/(e
*x+d)^9-2/3*b^3*(a*e-b*d)/e^5/(e*x+d)^6-1/2*b*(a^3*e^3-3*a^2*b*d*e^2+3*a*b^
2*d^2*e-b^3*d^3)/e^5/(e*x+d)^8-6/7*b^2*(a^2*e^2-2*a*b*d*e+b^2*d^2)/e^5/(e*x
+d)^7-1/5*b^4/e^5/(e*x+d)^5
```

Maxima [B] time = 1.14001, size = 363, normalized size = 3.05

$$-\frac{126b^4e^4x^4+b^4d^4+5ab^3d^3e+15a^2b^2d^2e^2+35a^3bde^3+70a^4e^4+84(b^4de^3+5ab^3e^4)x^3+36(b^4d^2e^2+5ab^3de^3+}{630(e^{14}x^9+9de^{13}x^8+36d^2e^{12}x^7+84d^3e^{11}x^6+126d^4e^{10}x^5+126d^5e^9x^4+84d^6e^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^10,x, algorithm="maxima")
```

```
[Out] -1/630*(126*b^4*e^4*x^4 + b^4*d^4 + 5*a*b^3*d^3*e + 15*a^2*b^2*d^2*e^2 + 35
*a^3*b*d*e^3 + 70*a^4*e^4 + 84*(b^4*d*e^3 + 5*a*b^3*e^4)*x^3 + 36*(b^4*d^2*
e^2 + 5*a*b^3*d*e^3 + 15*a^2*b^2*e^4)*x^2 + 9*(b^4*d^3*e + 5*a*b^3*d^2*e^2
+ 15*a^2*b^2*d*e^3 + 35*a^3*b*e^4)*x)/(e^14*x^9 + 9*d*e^13*x^8 + 36*d^2*e^1
2*x^7 + 84*d^3*e^11*x^6 + 126*d^4*e^10*x^5 + 126*d^5*e^9*x^4 + 84*d^6*e^8*x
^3 + 36*d^7*e^7*x^2 + 9*d^8*e^6*x + d^9*e^5)
```

Fricas [B] time = 1.69008, size = 568, normalized size = 4.77

$$-\frac{126b^4e^4x^4+b^4d^4+5ab^3d^3e+15a^2b^2d^2e^2+35a^3bde^3+70a^4e^4+84(b^4de^3+5ab^3e^4)x^3+36(b^4d^2e^2+5ab^3de^3+}{630(e^{14}x^9+9de^{13}x^8+36d^2e^{12}x^7+84d^3e^{11}x^6+126d^4e^{10}x^5+126d^5e^9x^4+84d^6e^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^10,x, algorithm="fricas")
```

```
[Out] -1/630*(126*b^4*e^4*x^4 + b^4*d^4 + 5*a*b^3*d^3*e + 15*a^2*b^2*d^2*e^2 + 35
*a^3*b*d*e^3 + 70*a^4*e^4 + 84*(b^4*d*e^3 + 5*a*b^3*e^4)*x^3 + 36*(b^4*d^2*
```

```
e^2 + 5*a*b^3*d*e^3 + 15*a^2*b^2*e^4)*x^2 + 9*(b^4*d^3*e + 5*a*b^3*d^2*e^2
+ 15*a^2*b^2*d*e^3 + 35*a^3*b*e^4)*x)/(e^14*x^9 + 9*d*e^13*x^8 + 36*d^2*e^1
2*x^7 + 84*d^3*e^11*x^6 + 126*d^4*e^10*x^5 + 126*d^5*e^9*x^4 + 84*d^6*e^8*x
^3 + 36*d^7*e^7*x^2 + 9*d^8*e^6*x + d^9*e^5)
```

Sympy [B] time = 44.3185, size = 287, normalized size = 2.41

$$-\frac{70a^4e^4 + 35a^3bde^3 + 15a^2b^2d^2e^2 + 5ab^3d^3e + b^4d^4 + 126b^4e^4x^4 + x^3\left(420ab^3e^4 + 84b^4de^3\right) + x^2\left(540a^2b^2e^4 + 180ab^3de^3\right.}{630d^9e^5 + 5670d^8e^6x + 22680d^7e^7x^2 + 52920d^6e^8x^3 + 79380d^5e^9x^4 + 79380d^4e^{10}x^5 + 52920d^3e^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**2/(e*x+d)**10,x)
```

```
[Out] -(70*a**4*e**4 + 35*a**3*b*d*e**3 + 15*a**2*b**2*d**2*e**2 + 5*a*b**3*d**3*
e + b**4*d**4 + 126*b**4*e**4*x**4 + x**3*(420*a*b**3*e**4 + 84*b**4*d*e**3
) + x**2*(540*a**2*b**2*e**4 + 180*a*b**3*d*e**3 + 36*b**4*d**2*e**2) + x*(
315*a**3*b*e**4 + 135*a**2*b**2*d*e**3 + 45*a*b**3*d**2*e**2 + 9*b**4*d**3*
e))/(630*d**9*e**5 + 5670*d**8*e**6*x + 22680*d**7*e**7*x**2 + 52920*d**6*e
**8*x**3 + 79380*d**5*e**9*x**4 + 79380*d**4*e**10*x**5 + 52920*d**3*e**11*
x**6 + 22680*d**2*e**12*x**7 + 5670*d*e**13*x**8 + 630*e**14*x**9)
```

Giac [A] time = 1.19851, size = 235, normalized size = 1.97

$$-\frac{\left(126\,b^4x^4e^4 + 84\,b^4dx^3e^3 + 36\,b^4d^2x^2e^2 + 9\,b^4d^3xe + b^4d^4 + 420\,ab^3x^3e^4 + 180\,ab^3dx^2e^3 + 45\,ab^3d^2xe^2 + 5\,ab^3d^3e + 540\right.}{630\,(xe+d)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^10,x, algorithm="giac")
```

```
[Out] -1/630*(126*b^4*x^4*e^4 + 84*b^4*d*x^3*e^3 + 36*b^4*d^2*x^2*e^2 + 9*b^4*d^3
*x*e + b^4*d^4 + 420*a*b^3*x^3*e^4 + 180*a*b^3*d*x^2*e^3 + 45*a*b^3*d^2*x*e
^2 + 5*a*b^3*d^3*e + 540*a^2*b^2*x^2*e^4 + 135*a^2*b^2*d*x*e^3 + 15*a^2*b^2
*d^2*e^2 + 315*a^3*b*x*e^4 + 35*a^3*b*d*e^3 + 70*a^4*e^4)*e^(-5)/(x*e + d)^
9
```

3.1480 $\int \frac{\left(a^2+2abx+b^2x^2\right)^2}{(d+ex)^{11}}\,dx$

Optimal. Leaf size=119

$$\frac{4b^3(bd-ae)}{7e^5(d+ex)^7}-\frac{3b^2(bd-ae)^2}{4e^5(d+ex)^8}+\frac{4b(bd-ae)^3}{9e^5(d+ex)^9}-\frac{(bd-ae)^4}{10e^5(d+ex)^{10}}-\frac{b^4}{6e^5(d+ex)^6}$$

```
[Out] -(b*d - a*e)^4/(10*e^5*(d + e*x)^10) + (4*b*(b*d - a*e)^3)/(9*e^5*(d + e*x)
^9) - (3*b^2*(b*d - a*e)^2)/(4*e^5*(d + e*x)^8) + (4*b^3*(b*d - a*e))/(7*e^
5*(d + e*x)^7) - b^4/(6*e^5*(d + e*x)^6)
```

Rubi [A] time = 0.0682365, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$\frac{4b^3(bd-ae)}{7e^5(d+ex)^7}-\frac{3b^2(bd-ae)^2}{4e^5(d+ex)^8}+\frac{4b(bd-ae)^3}{9e^5(d+ex)^9}-\frac{(bd-ae)^4}{10e^5(d+ex)^{10}}-\frac{b^4}{6e^5(d+ex)^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^11,x]
```

```
[Out] -(b*d - a*e)^4/(10*e^5*(d + e*x)^10) + (4*b*(b*d - a*e)^3)/(9*e^5*(d + e*x)
^9) - (3*b^2*(b*d - a*e)^2)/(4*e^5*(d + e*x)^8) + (4*b^3*(b*d - a*e))/(7*e^
5*(d + e*x)^7) - b^4/(6*e^5*(d + e*x)^6)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx+b^2x^2\right)^2}{(d+ex)^{11}}\,dx &= \int \frac{(a+bx)^4}{(d+ex)^{11}}\,dx\\ &= \int \left(\frac{(-bd+ae)^4}{e^4(d+ex)^{11}}-\frac{4b(bd-ae)^3}{e^4(d+ex)^{10}}+\frac{6b^2(bd-ae)^2}{e^4(d+ex)^9}-\frac{4b^3(bd-ae)}{e^4(d+ex)^8}+\frac{b^4}{e^4(d+ex)^7}\right)dx\\ &= -\frac{(bd-ae)^4}{10e^5(d+ex)^{10}}+\frac{4b(bd-ae)^3}{9e^5(d+ex)^9}-\frac{3b^2(bd-ae)^2}{4e^5(d+ex)^8}+\frac{4b^3(bd-ae)}{7e^5(d+ex)^7}-\frac{b^4}{6e^5(d+ex)^6}\end{aligned}$$

Mathematica [A] time = 0.0459065, size = 144, normalized size = 1.21

$$-\frac{21a^2b^2e^2\left(d^2+10dex+45e^2x^2\right)+56a^3be^3(d+10ex)+126a^4e^4+6ab^3e\left(10d^2ex+d^3+45de^2x^2+120e^3x^3\right)+b^4\left(45\right.}{1260e^5(d+ex)^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^11,x]
```

```
[Out] -(126*a^4*e^4 + 56*a^3*b*e^3*(d + 10*e*x) + 21*a^2*b^2*e^2*(d^2 + 10*d*e*x
+ 45*e^2*x^2) + 6*a*b^3*e*(d^3 + 10*d^2*e*x + 45*d*e^2*x^2 + 120*e^3*x^3) +
 b^4*(d^4 + 10*d^3*e*x + 45*d^2*e^2*x^2 + 120*d*e^3*x^3 + 210*e^4*x^4))/(12
60*e^5*(d + e*x)^10)
```

Maple [A] time = 0.045, size = 186, normalized size = 1.6

$$-\frac{4\,b\left(a^3e^3-3\,a^2bde^2+3\,ab^2d^2e-b^3d^3\right)}{9\,e^5\left(ex+d\right)^9}-\frac{b^4}{6\,e^5\left(ex+d\right)^6}-\frac{3\,b^2\left(a^2e^2-2\,abde+b^2d^2\right)}{4\,e^5\left(ex+d\right)^8}-\frac{4\,b^3\left(ae-bd\right)}{7\,e^5\left(ex+d\right)^7}-\frac{a^4e^4-4\,a^3bde^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^11,x)
```

```
[Out] -4/9*b*(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d^2*e-b^3*d^3)/e^5/(e*x+d)^9-1/6*b^4/
e^5/(e*x+d)^6-3/4*b^2*(a^2*e^2-2*a*b*d*e+b^2*d^2)/e^5/(e*x+d)^8-4/7*b^3*(a*
e-b*d)/e^5/(e*x+d)^7-1/10*(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-4*a*b^3*
d^3*e+b^4*d^4)/e^5/(e*x+d)^10
```

Maxima [B] time = 1.06797, size = 378, normalized size = 3.18

$$-\frac{210\,b^4e^4x^4+b^4d^4+6\,ab^3d^3e+21\,a^2b^2d^2e^2+56\,a^3bde^3+126\,a^4e^4+120\left(b^4de^3+6\,ab^3e^4\right)x^3+45\left(b^4d^2e^2+6\,ab^3de^3+\right.}{1260\left(e^{15}x^{10}+10\,de^{14}x^9+45\,d^2e^{13}x^8+120\,d^3e^{12}x^7+210\,d^4e^{11}x^6+252\,d^5e^{10}x^5+210\,d^6e^9x^4+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^11,x, algorithm="maxima")
```

```
[Out] -1/1260*(210*b^4*e^4*x^4 + b^4*d^4 + 6*a*b^3*d^3*e + 21*a^2*b^2*d^2*e^2 + 5
6*a^3*b*d*e^3 + 126*a^4*e^4 + 120*(b^4*d*e^3 + 6*a*b^3*e^4)*x^3 + 45*(b^4*d
^2*e^2 + 6*a*b^3*d*e^3 + 21*a^2*b^2*e^4)*x^2 + 10*(b^4*d^3*e + 6*a*b^3*d^2*
e^2 + 21*a^2*b^2*d*e^3 + 56*a^3*b*e^4)*x)/(e^15*x^10 + 10*d*e^14*x^9 + 45*d
^2*e^13*x^8 + 120*d^3*e^12*x^7 + 210*d^4*e^11*x^6 + 252*d^5*e^10*x^5 + 210*
d^6*e^9*x^4 + 120*d^7*e^8*x^3 + 45*d^8*e^7*x^2 + 10*d^9*e^6*x + d^10*e^5)
```

Fricas [B] time = 1.70903, size = 608, normalized size = 5.11

$$-\frac{210\,b^4e^4x^4+b^4d^4+6\,ab^3d^3e+21\,a^2b^2d^2e^2+56\,a^3bde^3+126\,a^4e^4+120\left(b^4de^3+6\,ab^3e^4\right)x^3+45\left(b^4d^2e^2+6\,ab^3de^3+\right.}{1260\left(e^{15}x^{10}+10\,de^{14}x^9+45\,d^2e^{13}x^8+120\,d^3e^{12}x^7+210\,d^4e^{11}x^6+252\,d^5e^{10}x^5+210\,d^6e^9x^4+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^11,x, algorithm="fricas")
```

```
[Out] -1/1260*(210*b^4*e^4*x^4 + b^4*d^4 + 6*a*b^3*d^3*e + 21*a^2*b^2*d^2*e^2 + 5
6*a^3*b*d*e^3 + 126*a^4*e^4 + 120*(b^4*d*e^3 + 6*a*b^3*e^4)*x^3 + 45*(b^4*d
```

```
^2*e^2 + 6*a*b^3*d*e^3 + 21*a^2*b^2*e^4)*x^2 + 10*(b^4*d^3*e + 6*a*b^3*d^2*
e^2 + 21*a^2*b^2*d*e^3 + 56*a^3*b*e^4)*x)/(e^15*x^10 + 10*d*e^14*x^9 + 45*d
^2*e^13*x^8 + 120*d^3*e^12*x^7 + 210*d^4*e^11*x^6 + 252*d^5*e^10*x^5 + 210*
d^6*e^9*x^4 + 120*d^7*e^8*x^3 + 45*d^8*e^7*x^2 + 10*d^9*e^6*x + d^10*e^5)
```

Sympy [B] time = 69.0312, size = 299, normalized size = 2.51

$$-\frac{126a^4e^4 + 56a^3bde^3 + 21a^2b^2d^2e^2 + 6ab^3d^3e + b^4d^4 + 210b^4e^4x^4 + x^3\left(720ab^3e^4 + 120b^4de^3\right) + x^2\left(945a^2b^2e^4 + 270a\right.}{1260d^{10}e^5 + 12600d^9e^6x + 56700d^8e^7x^2 + 151200d^7e^8x^3 + 264600d^6e^9x^4 + 317520d^5e^{10}x^5 + 264600d^4e^{11}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**2/(e*x+d)**11,x)
```

```
[Out] -(126*a**4*e**4 + 56*a**3*b*d*e**3 + 21*a**2*b**2*d**2*e**2 + 6*a*b**3*d**3
*e + b**4*d**4 + 210*b**4*e**4*x**4 + x**3*(720*a*b**3*e**4 + 120*b**4*d*e*
*3) + x**2*(945*a**2*b**2*e**4 + 270*a*b**3*d*e**3 + 45*b**4*d**2*e**2) + x
*(560*a**3*b*e**4 + 210*a**2*b**2*d*e**3 + 60*a*b**3*d**2*e**2 + 10*b**4*d*
*3*e))/(1260*d**10*e**5 + 12600*d**9*e**6*x + 56700*d**8*e**7*x**2 + 151200
*d**7*e**8*x**3 + 264600*d**6*e**9*x**4 + 317520*d**5*e**10*x**5 + 264600*d
**4*e**11*x**6 + 151200*d**3*e**12*x**7 + 56700*d**2*e**13*x**8 + 12600*d*e
**14*x**9 + 1260*e**15*x**10)
```

Giac [A] time = 1.12953, size = 235, normalized size = 1.97

$$-\frac{\left(210\, b^4x^4e^4 + 120\, b^4dx^3e^3 + 45\, b^4d^2x^2e^2 + 10\, b^4d^3xe + b^4d^4 + 720\, ab^3x^3e^4 + 270\, ab^3dx^2e^3 + 60\, ab^3d^2xe^2 + 6\, ab^3d^3e\right.}{1260\,(xe + d)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^11,x, algorithm="giac")
```

```
[Out] -1/1260*(210*b^4*x^4*e^4 + 120*b^4*d*x^3*e^3 + 45*b^4*d^2*x^2*e^2 + 10*b^4*
d^3*x*e + b^4*d^4 + 720*a*b^3*x^3*e^4 + 270*a*b^3*d*x^2*e^3 + 60*a*b^3*d^2*
x*e^2 + 6*a*b^3*d^3*e + 945*a^2*b^2*x^2*e^4 + 210*a^2*b^2*d*x*e^3 + 21*a^2*
b^2*d^2*e^2 + 560*a^3*b*x*e^4 + 56*a^3*b*d*e^3 + 126*a^4*e^4)*e^(-5)/(x*e +
 d)^10
```

3.1481 $\int (d+ex)^8 \left(a^2+2abx+b^2x^2\right)^3 \, dx$

Optimal. Leaf size=173

$$-\frac{3b^5(d+ex)^{14}(bd-ae)}{7e^7}+\frac{15b^4(d+ex)^{13}(bd-ae)^2}{13e^7}-\frac{5b^3(d+ex)^{12}(bd-ae)^3}{3e^7}+\frac{15b^2(d+ex)^{11}(bd-ae)^4}{11e^7}-\frac{3b(d+ex)^{10}(}{5e^7}$$

```
[Out] ((b*d - a*e)^6*(d + e*x)^9)/(9*e^7) - (3*b*(b*d - a*e)^5*(d + e*x)^10)/(5*e
^7) + (15*b^2*(b*d - a*e)^4*(d + e*x)^11)/(11*e^7) - (5*b^3*(b*d - a*e)^3*(
d + e*x)^12)/(3*e^7) + (15*b^4*(b*d - a*e)^2*(d + e*x)^13)/(13*e^7) - (3*b^
5*(b*d - a*e)*(d + e*x)^14)/(7*e^7) + (b^6*(d + e*x)^15)/(15*e^7)
```

Rubi [A] time = 0.516546, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$-\frac{3b^5(d+ex)^{14}(bd-ae)}{7e^7}+\frac{15b^4(d+ex)^{13}(bd-ae)^2}{13e^7}-\frac{5b^3(d+ex)^{12}(bd-ae)^3}{3e^7}+\frac{15b^2(d+ex)^{11}(bd-ae)^4}{11e^7}-\frac{3b(d+ex)^{10}(}{5e^7}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^8*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] ((b*d - a*e)^6*(d + e*x)^9)/(9*e^7) - (3*b*(b*d - a*e)^5*(d + e*x)^10)/(5*e
^7) + (15*b^2*(b*d - a*e)^4*(d + e*x)^11)/(11*e^7) - (5*b^3*(b*d - a*e)^3*(
d + e*x)^12)/(3*e^7) + (15*b^4*(b*d - a*e)^2*(d + e*x)^13)/(13*e^7) - (3*b^
5*(b*d - a*e)*(d + e*x)^14)/(7*e^7) + (b^6*(d + e*x)^15)/(15*e^7)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^8\left(a^2+2abx+b^2x^2\right)^3 dx &= \int (a+bx)^6(d+ex)^8 dx\\ &= \int \left(\frac{(-bd+ae)^6(d+ex)^8}{e^6}-\frac{6b(bd-ae)^5(d+ex)^9}{e^6}+\frac{15b^2(bd-ae)^4(d+ex)^{10}}{e^6}-20b\right.\\ &= \frac{(bd-ae)^6(d+ex)^9}{9e^7}-\frac{3b(bd-ae)^5(d+ex)^{10}}{5e^7}+\frac{15b^2(bd-ae)^4(d+ex)^{11}}{11e^7}-\frac{5b^3(bd-}{}\end{aligned}$$

Mathematica [B] time = 0.111101, size = 771, normalized size = 4.46

$$\frac{1}{13}b^4e^6x^{13}\left(15a^2e^2+48abde+28b^2d^2\right)+\frac{1}{3}b^3e^5x^{12}\left(30a^2bde^2+5a^3e^3+42ab^2d^2e+14b^3d^3\right)+\frac{1}{11}b^2e^4x^{11}\left(420a^2b^2d^2e^2+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^8*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] a^6*d^8*x + a^5*d^7*(3*b*d + 4*a*e)*x^2 + (a^4*d^6*(15*b^2*d^2 + 48*a*b*d*e
 + 28*a^2*e^2)*x^3)/3 + a^3*d^5*(5*b^3*d^3 + 30*a*b^2*d^2*e + 42*a^2*b*d*e^
2 + 14*a^3*e^3)*x^4 + (a^2*d^4*(15*b^4*d^4 + 160*a*b^3*d^3*e + 420*a^2*b^2*
d^2*e^2 + 336*a^3*b*d*e^3 + 70*a^4*e^4)*x^5)/5 + (a*d^3*(3*b^5*d^5 + 60*a*b
^4*d^4*e + 280*a^2*b^3*d^3*e^2 + 420*a^3*b^2*d^2*e^3 + 210*a^4*b*d*e^4 + 28
*a^5*e^5)*x^6)/3 + (d^2*(b^6*d^6 + 48*a*b^5*d^5*e + 420*a^2*b^4*d^4*e^2 + 1
120*a^3*b^3*d^3*e^3 + 1050*a^4*b^2*d^2*e^4 + 336*a^5*b*d*e^5 + 28*a^6*e^6)*
x^7)/7 + d*e*(b^6*d^6 + 21*a*b^5*d^5*e + 105*a^2*b^4*d^4*e^2 + 175*a^3*b^3*
d^3*e^3 + 105*a^4*b^2*d^2*e^4 + 21*a^5*b*d*e^5 + a^6*e^6)*x^8 + (e^2*(28*b^
6*d^6 + 336*a*b^5*d^5*e + 1050*a^2*b^4*d^4*e^2 + 1120*a^3*b^3*d^3*e^3 + 420
*a^4*b^2*d^2*e^4 + 48*a^5*b*d*e^5 + a^6*e^6)*x^9)/9 + (b*e^3*(28*b^5*d^5 +
210*a*b^4*d^4*e + 420*a^2*b^3*d^3*e^2 + 280*a^3*b^2*d^2*e^3 + 60*a^4*b*d*e^
4 + 3*a^5*e^5)*x^10)/5 + (b^2*e^4*(70*b^4*d^4 + 336*a*b^3*d^3*e + 420*a^2*b
^2*d^2*e^2 + 160*a^3*b*d*e^3 + 15*a^4*e^4)*x^11)/11 + (b^3*e^5*(14*b^3*d^3
+ 42*a*b^2*d^2*e + 30*a^2*b*d*e^2 + 5*a^3*e^3)*x^12)/3 + (b^4*e^6*(28*b^2*d
^2 + 48*a*b*d*e + 15*a^2*e^2)*x^13)/13 + (b^5*e^7*(4*b*d + 3*a*e)*x^14)/7 +
 (b^6*e^8*x^15)/15
```

Maple [B] time = 0.042, size = 803, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^8*(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] 1/15*e^8*b^6*x^15+1/14*(6*a*b^5*e^8+8*b^6*d*e^7)*x^14+1/13*(15*a^2*b^4*e^8+
48*a*b^5*d*e^7+28*b^6*d^2*e^6)*x^13+1/12*(20*a^3*b^3*e^8+120*a^2*b^4*d*e^7+
168*a*b^5*d^2*e^6+56*b^6*d^3*e^5)*x^12+1/11*(15*a^4*b^2*e^8+160*a^3*b^3*d*e
^7+420*a^2*b^4*d^2*e^6+336*a*b^5*d^3*e^5+70*b^6*d^4*e^4)*x^11+1/10*(6*a^5*b
*e^8+120*a^4*b^2*d*e^7+560*a^3*b^3*d^2*e^6+840*a^2*b^4*d^3*e^5+420*a*b^5*d^
4*e^4+56*b^6*d^5*e^3)*x^10+1/9*(a^6*e^8+48*a^5*b*d*e^7+420*a^4*b^2*d^2*e^6+
1120*a^3*b^3*d^3*e^5+1050*a^2*b^4*d^4*e^4+336*a*b^5*d^5*e^3+28*b^6*d^6*e^2)
*x^9+1/8*(8*a^6*d*e^7+168*a^5*b*d^2*e^6+840*a^4*b^2*d^3*e^5+1400*a^3*b^3*d^
4*e^4+840*a^2*b^4*d^5*e^3+168*a*b^5*d^6*e^2+8*b^6*d^7*e)*x^8+1/7*(28*a^6*d^
2*e^6+336*a^5*b*d^3*e^5+1050*a^4*b^2*d^4*e^4+1120*a^3*b^3*d^5*e^3+420*a^2*b
^4*d^6*e^2+48*a*b^5*d^7*e+b^6*d^8)*x^7+1/6*(56*a^6*d^3*e^5+420*a^5*b*d^4*e^
4+840*a^4*b^2*d^5*e^3+560*a^3*b^3*d^6*e^2+120*a^2*b^4*d^7*e+6*a*b^5*d^8)*x^
6+1/5*(70*a^6*d^4*e^4+336*a^5*b*d^5*e^3+420*a^4*b^2*d^6*e^2+160*a^3*b^3*d^7
*e+15*a^2*b^4*d^8)*x^5+1/4*(56*a^6*d^5*e^3+168*a^5*b*d^6*e^2+120*a^4*b^2*d^
7*e+20*a^3*b^3*d^8)*x^4+1/3*(28*a^6*d^6*e^2+48*a^5*b*d^7*e+15*a^4*b^2*d^8)*
x^3+1/2*(8*a^6*d^7*e+6*a^5*b*d^8)*x^2+d^8*a^6*x
```

Maxima [B] time = 1.08672, size = 1076, normalized size = 6.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^8*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/15*b^6*e^8*x^15 + a^6*d^8*x + 1/7*(4*b^6*d*e^7 + 3*a*b^5*e^8)*x^14 + 1/13
*(28*b^6*d^2*e^6 + 48*a*b^5*d*e^7 + 15*a^2*b^4*e^8)*x^13 + 1/3*(14*b^6*d^3*
e^5 + 42*a*b^5*d^2*e^6 + 30*a^2*b^4*d*e^7 + 5*a^3*b^3*e^8)*x^12 + 1/11*(70*
b^6*d^4*e^4 + 336*a*b^5*d^3*e^5 + 420*a^2*b^4*d^2*e^6 + 160*a^3*b^3*d*e^7 +
 15*a^4*b^2*e^8)*x^11 + 1/5*(28*b^6*d^5*e^3 + 210*a*b^5*d^4*e^4 + 420*a^2*b
^4*d^3*e^5 + 280*a^3*b^3*d^2*e^6 + 60*a^4*b^2*d*e^7 + 3*a^5*b*e^8)*x^10 + 1
/9*(28*b^6*d^6*e^2 + 336*a*b^5*d^5*e^3 + 1050*a^2*b^4*d^4*e^4 + 1120*a^3*b^
3*d^3*e^5 + 420*a^4*b^2*d^2*e^6 + 48*a^5*b*d*e^7 + a^6*e^8)*x^9 + (b^6*d^7*
e + 21*a*b^5*d^6*e^2 + 105*a^2*b^4*d^5*e^3 + 175*a^3*b^3*d^4*e^4 + 105*a^4*
b^2*d^3*e^5 + 21*a^5*b*d^2*e^6 + a^6*d*e^7)*x^8 + 1/7*(b^6*d^8 + 48*a*b^5*d
^7*e + 420*a^2*b^4*d^6*e^2 + 1120*a^3*b^3*d^5*e^3 + 1050*a^4*b^2*d^4*e^4 +
336*a^5*b*d^3*e^5 + 28*a^6*d^2*e^6)*x^7 + 1/3*(3*a*b^5*d^8 + 60*a^2*b^4*d^7
*e + 280*a^3*b^3*d^6*e^2 + 420*a^4*b^2*d^5*e^3 + 210*a^5*b*d^4*e^4 + 28*a^6
*d^3*e^5)*x^6 + 1/5*(15*a^2*b^4*d^8 + 160*a^3*b^3*d^7*e + 420*a^4*b^2*d^6*e
^2 + 336*a^5*b*d^5*e^3 + 70*a^6*d^4*e^4)*x^5 + (5*a^3*b^3*d^8 + 30*a^4*b^2*
d^7*e + 42*a^5*b*d^6*e^2 + 14*a^6*d^5*e^3)*x^4 + 1/3*(15*a^4*b^2*d^8 + 48*a
^5*b*d^7*e + 28*a^6*d^6*e^2)*x^3 + (3*a^5*b*d^8 + 4*a^6*d^7*e)*x^2
```

Fricas [B] time = 1.50474, size = 1967, normalized size = 11.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^8*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*e^8*b^6 + 4/7*x^14*e^7*d*b^6 + 3/7*x^14*e^8*b^5*a + 28/13*x^13*e^
6*d^2*b^6 + 48/13*x^13*e^7*d*b^5*a + 15/13*x^13*e^8*b^4*a^2 + 14/3*x^12*e^5
*d^3*b^6 + 14*x^12*e^6*d^2*b^5*a + 10*x^12*e^7*d*b^4*a^2 + 5/3*x^12*e^8*b^3
*a^3 + 70/11*x^11*e^4*d^4*b^6 + 336/11*x^11*e^5*d^3*b^5*a + 420/11*x^11*e^6
*d^2*b^4*a^2 + 160/11*x^11*e^7*d*b^3*a^3 + 15/11*x^11*e^8*b^2*a^4 + 28/5*x^
10*e^3*d^5*b^6 + 42*x^10*e^4*d^4*b^5*a + 84*x^10*e^5*d^3*b^4*a^2 + 56*x^10*
e^6*d^2*b^3*a^3 + 12*x^10*e^7*d*b^2*a^4 + 3/5*x^10*e^8*b*a^5 + 28/9*x^9*e^2
*d^6*b^6 + 112/3*x^9*e^3*d^5*b^5*a + 350/3*x^9*e^4*d^4*b^4*a^2 + 1120/9*x^9
*e^5*d^3*b^3*a^3 + 140/3*x^9*e^6*d^2*b^2*a^4 + 16/3*x^9*e^7*d*b*a^5 + 1/9*x
^9*e^8*a^6 + x^8*e*d^7*b^6 + 21*x^8*e^2*d^6*b^5*a + 105*x^8*e^3*d^5*b^4*a^2
 + 175*x^8*e^4*d^4*b^3*a^3 + 105*x^8*e^5*d^3*b^2*a^4 + 21*x^8*e^6*d^2*b*a^5
 + x^8*e^7*d*a^6 + 1/7*x^7*d^8*b^6 + 48/7*x^7*e*d^7*b^5*a + 60*x^7*e^2*d^6*
b^4*a^2 + 160*x^7*e^3*d^5*b^3*a^3 + 150*x^7*e^4*d^4*b^2*a^4 + 48*x^7*e^5*d^
3*b*a^5 + 4*x^7*e^6*d^2*a^6 + x^6*d^8*b^5*a + 20*x^6*e*d^7*b^4*a^2 + 280/3*
x^6*e^2*d^6*b^3*a^3 + 140*x^6*e^3*d^5*b^2*a^4 + 70*x^6*e^4*d^4*b*a^5 + 28/3
*x^6*e^5*d^3*a^6 + 3*x^5*d^8*b^4*a^2 + 32*x^5*e*d^7*b^3*a^3 + 84*x^5*e^2*d^
6*b^2*a^4 + 336/5*x^5*e^3*d^5*b*a^5 + 14*x^5*e^4*d^4*a^6 + 5*x^4*d^8*b^3*a^
3 + 30*x^4*e*d^7*b^2*a^4 + 42*x^4*e^2*d^6*b*a^5 + 14*x^4*e^3*d^5*a^6 + 5*x^
3*d^8*b^2*a^4 + 16*x^3*e*d^7*b*a^5 + 28/3*x^3*e^2*d^6*a^6 + 3*x^2*d^8*b*a^5
 + 4*x^2*e*d^7*a^6 + x*d^8*a^6
```

Sympy [B] time = 0.215769, size = 884, normalized size = 5.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**8*(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] a**6*d**8*x + b**6*e**8*x**15/15 + x**14*(3*a*b**5*e**8/7 + 4*b**6*d*e**7/7
) + x**13*(15*a**2*b**4*e**8/13 + 48*a*b**5*d*e**7/13 + 28*b**6*d**2*e**6/1
3) + x**12*(5*a**3*b**3*e**8/3 + 10*a**2*b**4*d*e**7 + 14*a*b**5*d**2*e**6
+ 14*b**6*d**3*e**5/3) + x**11*(15*a**4*b**2*e**8/11 + 160*a**3*b**3*d*e**7
/11 + 420*a**2*b**4*d**2*e**6/11 + 336*a*b**5*d**3*e**5/11 + 70*b**6*d**4*e
**4/11) + x**10*(3*a**5*b*e**8/5 + 12*a**4*b**2*d*e**7 + 56*a**3*b**3*d**2*
e**6 + 84*a**2*b**4*d**3*e**5 + 42*a*b**5*d**4*e**4 + 28*b**6*d**5*e**3/5)
+ x**9*(a**6*e**8/9 + 16*a**5*b*d*e**7/3 + 140*a**4*b**2*d**2*e**6/3 + 1120
*a**3*b**3*d**3*e**5/9 + 350*a**2*b**4*d**4*e**4/3 + 112*a*b**5*d**5*e**3/3
 + 28*b**6*d**6*e**2/9) + x**8*(a**6*d*e**7 + 21*a**5*b*d**2*e**6 + 105*a**
4*b**2*d**3*e**5 + 175*a**3*b**3*d**4*e**4 + 105*a**2*b**4*d**5*e**3 + 21*a
*b**5*d**6*e**2 + b**6*d**7*e) + x**7*(4*a**6*d**2*e**6 + 48*a**5*b*d**3*e*
*5 + 150*a**4*b**2*d**4*e**4 + 160*a**3*b**3*d**5*e**3 + 60*a**2*b**4*d**6*
e**2 + 48*a*b**5*d**7*e/7 + b**6*d**8/7) + x**6*(28*a**6*d**3*e**5/3 + 70*a
**5*b*d**4*e**4 + 140*a**4*b**2*d**5*e**3 + 280*a**3*b**3*d**6*e**2/3 + 20*
a**2*b**4*d**7*e + a*b**5*d**8) + x**5*(14*a**6*d**4*e**4 + 336*a**5*b*d**5
*e**3/5 + 84*a**4*b**2*d**6*e**2 + 32*a**3*b**3*d**7*e + 3*a**2*b**4*d**8)
+ x**4*(14*a**6*d**5*e**3 + 42*a**5*b*d**6*e**2 + 30*a**4*b**2*d**7*e + 5*a
**3*b**3*d**8) + x**3*(28*a**6*d**6*e**2/3 + 16*a**5*b*d**7*e + 5*a**4*b**2
*d**8) + x**2*(4*a**6*d**7*e + 3*a**5*b*d**8)
```

Giac [B] time = 1.11774, size = 1166, normalized size = 6.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^8*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/15*b^6*x^15*e^8 + 4/7*b^6*d*x^14*e^7 + 28/13*b^6*d^2*x^13*e^6 + 14/3*b^6*
d^3*x^12*e^5 + 70/11*b^6*d^4*x^11*e^4 + 28/5*b^6*d^5*x^10*e^3 + 28/9*b^6*d^
6*x^9*e^2 + b^6*d^7*x^8*e + 1/7*b^6*d^8*x^7 + 3/7*a*b^5*x^14*e^8 + 48/13*a*
b^5*d*x^13*e^7 + 14*a*b^5*d^2*x^12*e^6 + 336/11*a*b^5*d^3*x^11*e^5 + 42*a*b
^5*d^4*x^10*e^4 + 112/3*a*b^5*d^5*x^9*e^3 + 21*a*b^5*d^6*x^8*e^2 + 48/7*a*b
^5*d^7*x^7*e + a*b^5*d^8*x^6 + 15/13*a^2*b^4*x^13*e^8 + 10*a^2*b^4*d*x^12*e
^7 + 420/11*a^2*b^4*d^2*x^11*e^6 + 84*a^2*b^4*d^3*x^10*e^5 + 350/3*a^2*b^4*
d^4*x^9*e^4 + 105*a^2*b^4*d^5*x^8*e^3 + 60*a^2*b^4*d^6*x^7*e^2 + 20*a^2*b^4
*d^7*x^6*e + 3*a^2*b^4*d^8*x^5 + 5/3*a^3*b^3*x^12*e^8 + 160/11*a^3*b^3*d*x^
11*e^7 + 56*a^3*b^3*d^2*x^10*e^6 + 1120/9*a^3*b^3*d^3*x^9*e^5 + 175*a^3*b^3
*d^4*x^8*e^4 + 160*a^3*b^3*d^5*x^7*e^3 + 280/3*a^3*b^3*d^6*x^6*e^2 + 32*a^3
*b^3*d^7*x^5*e + 5*a^3*b^3*d^8*x^4 + 15/11*a^4*b^2*x^11*e^8 + 12*a^4*b^2*d*
x^10*e^7 + 140/3*a^4*b^2*d^2*x^9*e^6 + 105*a^4*b^2*d^3*x^8*e^5 + 150*a^4*b^
2*d^4*x^7*e^4 + 140*a^4*b^2*d^5*x^6*e^3 + 84*a^4*b^2*d^6*x^5*e^2 + 30*a^4*b
^2*d^7*x^4*e + 5*a^4*b^2*d^8*x^3 + 3/5*a^5*b*x^10*e^8 + 16/3*a^5*b*d*x^9*e^
7 + 21*a^5*b*d^2*x^8*e^6 + 48*a^5*b*d^3*x^7*e^5 + 70*a^5*b*d^4*x^6*e^4 + 33
6/5*a^5*b*d^5*x^5*e^3 + 42*a^5*b*d^6*x^4*e^2 + 16*a^5*b*d^7*x^3*e + 3*a^5*b
*d^8*x^2 + 1/9*a^6*x^9*e^8 + a^6*d*x^8*e^7 + 4*a^6*d^2*x^7*e^6 + 28/3*a^6*d
^3*x^6*e^5 + 14*a^6*d^4*x^5*e^4 + 14*a^6*d^5*x^4*e^3 + 28/3*a^6*d^6*x^3*e^2
 + 4*a^6*d^7*x^2*e + a^6*d^8*x
```

3.1482 $\int (d+ex)^7 \left(a^2+2abx+b^2x^2\right)^3 \, dx$

Optimal. Leaf size=173

$$-\frac{6b^5(d+ex)^{13}(bd-ae)}{13e^7}+\frac{5b^4(d+ex)^{12}(bd-ae)^2}{4e^7}-\frac{20b^3(d+ex)^{11}(bd-ae)^3}{11e^7}+\frac{3b^2(d+ex)^{10}(bd-ae)^4}{2e^7}-\frac{2b(d+ex)^9(b\cdots}{3e^7}$$

```
[Out] ((b*d - a*e)^6*(d + e*x)^8)/(8*e^7) - (2*b*(b*d - a*e)^5*(d + e*x)^9)/(3*e^
7) + (3*b^2*(b*d - a*e)^4*(d + e*x)^10)/(2*e^7) - (20*b^3*(b*d - a*e)^3*(d
+ e*x)^11)/(11*e^7) + (5*b^4*(b*d - a*e)^2*(d + e*x)^12)/(4*e^7) - (6*b^5*(
b*d - a*e)*(d + e*x)^13)/(13*e^7) + (b^6*(d + e*x)^14)/(14*e^7)
```

Rubi [A] time = 0.432571, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$-\frac{6b^5(d+ex)^{13}(bd-ae)}{13e^7}+\frac{5b^4(d+ex)^{12}(bd-ae)^2}{4e^7}-\frac{20b^3(d+ex)^{11}(bd-ae)^3}{11e^7}+\frac{3b^2(d+ex)^{10}(bd-ae)^4}{2e^7}-\frac{2b(d+ex)^9(b\cdots}{3e^7}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^7*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```


```
[Out] ((b*d - a*e)^6*(d + e*x)^8)/(8*e^7) - (2*b*(b*d - a*e)^5*(d + e*x)^9)/(3*e^
7) + (3*b^2*(b*d - a*e)^4*(d + e*x)^10)/(2*e^7) - (20*b^3*(b*d - a*e)^3*(d
+ e*x)^11)/(11*e^7) + (5*b^4*(b*d - a*e)^2*(d + e*x)^12)/(4*e^7) - (6*b^5*(
b*d - a*e)*(d + e*x)^13)/(13*e^7) + (b^6*(d + e*x)^14)/(14*e^7)
```


Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^7\left(a^2+2abx+b^2x^2\right)^3 dx &= \int (a+bx)^6(d+ex)^7 dx\\ &= \int \left(\frac{(-bd+ae)^6(d+ex)^7}{e^6}-\frac{6b(bd-ae)^5(d+ex)^8}{e^6}+\frac{15b^2(bd-ae)^4(d+ex)^9}{e^6}-\frac{20b\cdots}{}\right.\\ &= \frac{(bd-ae)^6(d+ex)^8}{8e^7}-\frac{2b(bd-ae)^5(d+ex)^9}{3e^7}+\frac{3b^2(bd-ae)^4(d+ex)^{10}}{2e^7}-\frac{20b^3(bd-\cdots}{}\end{aligned}$$

Mathematica [B] time = 0.0957017, size = 684, normalized size = 3.95

$$\frac{1}{4}b^4e^5x^{12}\left(5a^2e^2+14abde+7b^2d^2\right)+\frac{1}{11}b^3e^4x^{11}\left(105a^2bde^2+20a^3e^3+126ab^2d^2e+35b^3d^3\right)+\frac{1}{2}b^2e^3x^{10}\left(63a^2b^2d^2e^2+2\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^7*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] a^6*d^7*x + (a^5*d^6*(6*b*d + 7*a*e)*x^2)/2 + a^4*d^5*(5*b^2*d^2 + 14*a*b*d
*e + 7*a^2*e^2)*x^3 + (a^3*d^4*(20*b^3*d^3 + 105*a*b^2*d^2*e + 126*a^2*b*d*
e^2 + 35*a^3*e^3)*x^4)/4 + a^2*d^3*(3*b^4*d^4 + 28*a*b^3*d^3*e + 63*a^2*b^2
*d^2*e^2 + 42*a^3*b*d*e^3 + 7*a^4*e^4)*x^5 + (a*d^2*(2*b^5*d^5 + 35*a*b^4*d
^4*e + 140*a^2*b^3*d^3*e^2 + 175*a^3*b^2*d^2*e^3 + 70*a^4*b*d*e^4 + 7*a^5*e
^5)*x^6)/2 + (d*(b^6*d^6 + 42*a*b^5*d^5*e + 315*a^2*b^4*d^4*e^2 + 700*a^3*b
^3*d^3*e^3 + 525*a^4*b^2*d^2*e^4 + 126*a^5*b*d*e^5 + 7*a^6*e^6)*x^7)/7 + (e
*(7*b^6*d^6 + 126*a*b^5*d^5*e + 525*a^2*b^4*d^4*e^2 + 700*a^3*b^3*d^3*e^3 +
 315*a^4*b^2*d^2*e^4 + 42*a^5*b*d*e^5 + a^6*e^6)*x^8)/8 + (b*e^2*(7*b^5*d^5
 + 70*a*b^4*d^4*e + 175*a^2*b^3*d^3*e^2 + 140*a^3*b^2*d^2*e^3 + 35*a^4*b*d*
e^4 + 2*a^5*e^5)*x^9)/3 + (b^2*e^3*(7*b^4*d^4 + 42*a*b^3*d^3*e + 63*a^2*b^2
*d^2*e^2 + 28*a^3*b*d*e^3 + 3*a^4*e^4)*x^10)/2 + (b^3*e^4*(35*b^3*d^3 + 126
*a*b^2*d^2*e + 105*a^2*b*d*e^2 + 20*a^3*e^3)*x^11)/11 + (b^4*e^5*(7*b^2*d^2
 + 14*a*b*d*e + 5*a^2*e^2)*x^12)/4 + (b^5*e^6*(7*b*d + 6*a*e)*x^13)/13 + (b
^6*e^7*x^14)/14
```

Maple [B] time = 0.04, size = 709, normalized size = 4.1

$$\frac{e^7 b^6 x^{14}}{14} + \frac{\left(6\,e^7 a b^5 + 7\,d e^6 b^6\right) x^{13}}{13} + \frac{\left(15\,e^7 a^2 b^4 + 42\,d e^6 a b^5 + 21\,d^2 e^5 b^6\right) x^{12}}{12} + \frac{\left(20\,e^7 a^3 b^3 + 105\,d e^6 a^2 b^4 + 126\,d^2 e^5 a b^5\right.}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^7*(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] 1/14*e^7*b^6*x^14+1/13*(6*a*b^5*e^7+7*b^6*d*e^6)*x^13+1/12*(15*a^2*b^4*e^7+
42*a*b^5*d*e^6+21*b^6*d^2*e^5)*x^12+1/11*(20*a^3*b^3*e^7+105*a^2*b^4*d*e^6+
126*a*b^5*d^2*e^5+35*b^6*d^3*e^4)*x^11+1/10*(15*a^4*b^2*e^7+140*a^3*b^3*d*e
^6+315*a^2*b^4*d^2*e^5+210*a*b^5*d^3*e^4+35*b^6*d^4*e^3)*x^10+1/9*(6*a^5*b*
e^7+105*a^4*b^2*d*e^6+420*a^3*b^3*d^2*e^5+525*a^2*b^4*d^3*e^4+210*a*b^5*d^4
*e^3+21*b^6*d^5*e^2)*x^9+1/8*(a^6*e^7+42*a^5*b*d*e^6+315*a^4*b^2*d^2*e^5+70
0*a^3*b^3*d^3*e^4+525*a^2*b^4*d^4*e^3+126*a*b^5*d^5*e^2+7*b^6*d^6*e)*x^8+1/
7*(7*a^6*d*e^6+126*a^5*b*d^2*e^5+525*a^4*b^2*d^3*e^4+700*a^3*b^3*d^4*e^3+31
5*a^2*b^4*d^5*e^2+42*a*b^5*d^6*e+b^6*d^7)*x^7+1/6*(21*a^6*d^2*e^5+210*a^5*b
*d^3*e^4+525*a^4*b^2*d^4*e^3+420*a^3*b^3*d^5*e^2+105*a^2*b^4*d^6*e+6*a*b^5*
d^7)*x^6+1/5*(35*a^6*d^3*e^4+210*a^5*b*d^4*e^3+315*a^4*b^2*d^5*e^2+140*a^3*
b^3*d^6*e+15*a^2*b^4*d^7)*x^5+1/4*(35*a^6*d^4*e^3+126*a^5*b*d^5*e^2+105*a^4
*b^2*d^6*e+20*a^3*b^3*d^7)*x^4+1/3*(21*a^6*d^5*e^2+42*a^5*b*d^6*e+15*a^4*b^
2*d^7)*x^3+1/2*(7*a^6*d^6*e+6*a^5*b*d^7)*x^2+d^7*a^6*x
```

Maxima [B] time = 1.14744, size = 953, normalized size = 5.51

$$\frac{1}{14}\,b^6 e^7 x^{14} + a^6 d^7 x + \frac{1}{13}\left(7\,b^6 d e^6 + 6\,a b^5 e^7\right)x^{13} + \frac{1}{4}\left(7\,b^6 d^2 e^5 + 14\,a b^5 d e^6 + 5\,a^2 b^4 e^7\right)x^{12} + \frac{1}{11}\left(35\,b^6 d^3 e^4 + 126\,a b^5 d^2 e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/14*b^6*e^7*x^14 + a^6*d^7*x + 1/13*(7*b^6*d*e^6 + 6*a*b^5*e^7)*x^13 + 1/4
*(7*b^6*d^2*e^5 + 14*a*b^5*d*e^6 + 5*a^2*b^4*e^7)*x^12 + 1/11*(35*b^6*d^3*e
```

```
^4 + 126*a*b^5*d^2*e^5 + 105*a^2*b^4*d*e^6 + 20*a^3*b^3*e^7)*x^11 + 1/2*(7*
b^6*d^4*e^3 + 42*a*b^5*d^3*e^4 + 63*a^2*b^4*d^2*e^5 + 28*a^3*b^3*d*e^6 + 3*
a^4*b^2*e^7)*x^10 + 1/3*(7*b^6*d^5*e^2 + 70*a*b^5*d^4*e^3 + 175*a^2*b^4*d^3
*e^4 + 140*a^3*b^3*d^2*e^5 + 35*a^4*b^2*d*e^6 + 2*a^5*b*e^7)*x^9 + 1/8*(7*b
^6*d^6*e + 126*a*b^5*d^5*e^2 + 525*a^2*b^4*d^4*e^3 + 700*a^3*b^3*d^3*e^4 +
315*a^4*b^2*d^2*e^5 + 42*a^5*b*d*e^6 + a^6*e^7)*x^8 + 1/7*(b^6*d^7 + 42*a*b
^5*d^6*e + 315*a^2*b^4*d^5*e^2 + 700*a^3*b^3*d^4*e^3 + 525*a^4*b^2*d^3*e^4
+ 126*a^5*b*d^2*e^5 + 7*a^6*d*e^6)*x^7 + 1/2*(2*a*b^5*d^7 + 35*a^2*b^4*d^6*
e + 140*a^3*b^3*d^5*e^2 + 175*a^4*b^2*d^4*e^3 + 70*a^5*b*d^3*e^4 + 7*a^6*d^
2*e^5)*x^6 + (3*a^2*b^4*d^7 + 28*a^3*b^3*d^6*e + 63*a^4*b^2*d^5*e^2 + 42*a^
5*b*d^4*e^3 + 7*a^6*d^3*e^4)*x^5 + 1/4*(20*a^3*b^3*d^7 + 105*a^4*b^2*d^6*e
+ 126*a^5*b*d^5*e^2 + 35*a^6*d^4*e^3)*x^4 + (5*a^4*b^2*d^7 + 14*a^5*b*d^6*e
 + 7*a^6*d^5*e^2)*x^3 + 1/2*(6*a^5*b*d^7 + 7*a^6*d^6*e)*x^2
```

Fricas [B] time = 1.5536, size = 1736, normalized size = 10.03

$$\frac{1}{14}x^{14}e^7b^6 + \frac{7}{13}x^{13}e^6db^6 + \frac{6}{13}x^{13}e^7b^5a + \frac{7}{4}x^{12}e^5d^2b^6 + \frac{7}{2}x^{12}e^6db^5a + \frac{5}{4}x^{12}e^7b^4a^2 + \frac{35}{11}x^{11}e^4d^3b^6 + \frac{126}{11}x^{11}e^5d^2b^5a + \frac{105}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*e^7*b^6 + 7/13*x^13*e^6*d*b^6 + 6/13*x^13*e^7*b^5*a + 7/4*x^12*e^
5*d^2*b^6 + 7/2*x^12*e^6*d*b^5*a + 5/4*x^12*e^7*b^4*a^2 + 35/11*x^11*e^4*d^
3*b^6 + 126/11*x^11*e^5*d^2*b^5*a + 105/11*x^11*e^6*d*b^4*a^2 + 20/11*x^11*
e^7*b^3*a^3 + 7/2*x^10*e^3*d^4*b^6 + 21*x^10*e^4*d^3*b^5*a + 63/2*x^10*e^5*
d^2*b^4*a^2 + 14*x^10*e^6*d*b^3*a^3 + 3/2*x^10*e^7*b^2*a^4 + 7/3*x^9*e^2*d^
5*b^6 + 70/3*x^9*e^3*d^4*b^5*a + 175/3*x^9*e^4*d^3*b^4*a^2 + 140/3*x^9*e^5*
d^2*b^3*a^3 + 35/3*x^9*e^6*d*b^2*a^4 + 2/3*x^9*e^7*b*a^5 + 7/8*x^8*e*d^6*b^
6 + 63/4*x^8*e^2*d^5*b^5*a + 525/8*x^8*e^3*d^4*b^4*a^2 + 175/2*x^8*e^4*d^3*
b^3*a^3 + 315/8*x^8*e^5*d^2*b^2*a^4 + 21/4*x^8*e^6*d*b*a^5 + 1/8*x^8*e^7*a^
6 + 1/7*x^7*d^7*b^6 + 6*x^7*e*d^6*b^5*a + 45*x^7*e^2*d^5*b^4*a^2 + 100*x^7*
e^3*d^4*b^3*a^3 + 75*x^7*e^4*d^3*b^2*a^4 + 18*x^7*e^5*d^2*b*a^5 + x^7*e^6*d
*a^6 + x^6*d^7*b^5*a + 35/2*x^6*e*d^6*b^4*a^2 + 70*x^6*e^2*d^5*b^3*a^3 + 17
5/2*x^6*e^3*d^4*b^2*a^4 + 35*x^6*e^4*d^3*b*a^5 + 7/2*x^6*e^5*d^2*a^6 + 3*x^
5*d^7*b^4*a^2 + 28*x^5*e*d^6*b^3*a^3 + 63*x^5*e^2*d^5*b^2*a^4 + 42*x^5*e^3*
d^4*b*a^5 + 7*x^5*e^4*d^3*a^6 + 5*x^4*d^7*b^3*a^3 + 105/4*x^4*e*d^6*b^2*a^4
 + 63/2*x^4*e^2*d^5*b*a^5 + 35/4*x^4*e^3*d^4*a^6 + 5*x^3*d^7*b^2*a^4 + 14*x
^3*e*d^6*b*a^5 + 7*x^3*e^2*d^5*a^6 + 3*x^2*d^7*b*a^5 + 7/2*x^2*e*d^6*a^6 +
x*d^7*a^6
```

Sympy [B] time = 0.204006, size = 796, normalized size = 4.6

$$a^6d^7x + \frac{b^6e^7x^{14}}{14} + x^{13}\left(\frac{6ab^5e^7}{13} + \frac{7b^6de^6}{13}\right) + x^{12}\left(\frac{5a^2b^4e^7}{4} + \frac{7ab^5de^6}{2} + \frac{7b^6d^2e^5}{4}\right) + x^{11}\left(\frac{20a^3b^3e^7}{11} + \frac{105a^2b^4de^6}{11} + \frac{126ab}{11}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**7*(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] a**6*d**7*x + b**6*e**7*x**14/14 + x**13*(6*a*b**5*e**7/13 + 7*b**6*d*e**6/
13) + x**12*(5*a**2*b**4*e**7/4 + 7*a*b**5*d*e**6/2 + 7*b**6*d**2*e**5/4) +
 x**11*(20*a**3*b**3*e**7/11 + 105*a**2*b**4*d*e**6/11 + 126*a*b**5*d**2*e*
*5/11 + 35*b**6*d**3*e**4/11) + x**10*(3*a**4*b**2*e**7/2 + 14*a**3*b**3*d*
```

```
e**6 + 63*a**2*b**4*d**2*e**5/2 + 21*a*b**5*d**3*e**4 + 7*b**6*d**4*e**3/2)
 + x**9*(2*a**5*b*e**7/3 + 35*a**4*b**2*d*e**6/3 + 140*a**3*b**3*d**2*e**5/
3 + 175*a**2*b**4*d**3*e**4/3 + 70*a*b**5*d**4*e**3/3 + 7*b**6*d**5*e**2/3)
 + x**8*(a**6*e**7/8 + 21*a**5*b*d*e**6/4 + 315*a**4*b**2*d**2*e**5/8 + 175
*a**3*b**3*d**3*e**4/2 + 525*a**2*b**4*d**4*e**3/8 + 63*a*b**5*d**5*e**2/4
+ 7*b**6*d**6*e/8) + x**7*(a**6*d*e**6 + 18*a**5*b*d**2*e**5 + 75*a**4*b**2
*d**3*e**4 + 100*a**3*b**3*d**4*e**3 + 45*a**2*b**4*d**5*e**2 + 6*a*b**5*d*
*6*e + b**6*d**7/7) + x**6*(7*a**6*d**2*e**5/2 + 35*a**5*b*d**3*e**4 + 175*
a**4*b**2*d**4*e**3/2 + 70*a**3*b**3*d**5*e**2 + 35*a**2*b**4*d**6*e/2 + a*
b**5*d**7) + x**5*(7*a**6*d**3*e**4 + 42*a**5*b*d**4*e**3 + 63*a**4*b**2*d*
*5*e**2 + 28*a**3*b**3*d**6*e + 3*a**2*b**4*d**7) + x**4*(35*a**6*d**4*e**3
/4 + 63*a**5*b*d**5*e**2/2 + 105*a**4*b**2*d**6*e/4 + 5*a**3*b**3*d**7) + x
**3*(7*a**6*d**5*e**2 + 14*a**5*b*d**6*e + 5*a**4*b**2*d**7) + x**2*(7*a**6
*d**6*e/2 + 3*a**5*b*d**7)
```

Giac [B] time = 1.18343, size = 1030, normalized size = 5.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/14*b^6*x^14*e^7 + 7/13*b^6*d*x^13*e^6 + 7/4*b^6*d^2*x^12*e^5 + 35/11*b^6*
d^3*x^11*e^4 + 7/2*b^6*d^4*x^10*e^3 + 7/3*b^6*d^5*x^9*e^2 + 7/8*b^6*d^6*x^8
*e + 1/7*b^6*d^7*x^7 + 6/13*a*b^5*x^13*e^7 + 7/2*a*b^5*d*x^12*e^6 + 126/11*
a*b^5*d^2*x^11*e^5 + 21*a*b^5*d^3*x^10*e^4 + 70/3*a*b^5*d^4*x^9*e^3 + 63/4*
a*b^5*d^5*x^8*e^2 + 6*a*b^5*d^6*x^7*e + a*b^5*d^7*x^6 + 5/4*a^2*b^4*x^12*e^
7 + 105/11*a^2*b^4*d*x^11*e^6 + 63/2*a^2*b^4*d^2*x^10*e^5 + 175/3*a^2*b^4*d
^3*x^9*e^4 + 525/8*a^2*b^4*d^4*x^8*e^3 + 45*a^2*b^4*d^5*x^7*e^2 + 35/2*a^2*
b^4*d^6*x^6*e + 3*a^2*b^4*d^7*x^5 + 20/11*a^3*b^3*x^11*e^7 + 14*a^3*b^3*d*x
^10*e^6 + 140/3*a^3*b^3*d^2*x^9*e^5 + 175/2*a^3*b^3*d^3*x^8*e^4 + 100*a^3*b
^3*d^4*x^7*e^3 + 70*a^3*b^3*d^5*x^6*e^2 + 28*a^3*b^3*d^6*x^5*e + 5*a^3*b^3*
d^7*x^4 + 3/2*a^4*b^2*x^10*e^7 + 35/3*a^4*b^2*d*x^9*e^6 + 315/8*a^4*b^2*d^2
*x^8*e^5 + 75*a^4*b^2*d^3*x^7*e^4 + 175/2*a^4*b^2*d^4*x^6*e^3 + 63*a^4*b^2*
d^5*x^5*e^2 + 105/4*a^4*b^2*d^6*x^4*e + 5*a^4*b^2*d^7*x^3 + 2/3*a^5*b*x^9*e
^7 + 21/4*a^5*b*d*x^8*e^6 + 18*a^5*b*d^2*x^7*e^5 + 35*a^5*b*d^3*x^6*e^4 + 4
2*a^5*b*d^4*x^5*e^3 + 63/2*a^5*b*d^5*x^4*e^2 + 14*a^5*b*d^6*x^3*e + 3*a^5*b
*d^7*x^2 + 1/8*a^6*x^8*e^7 + a^6*d*x^7*e^6 + 7/2*a^6*d^2*x^6*e^5 + 7*a^6*d^
3*x^5*e^4 + 35/4*a^6*d^4*x^4*e^3 + 7*a^6*d^5*x^3*e^2 + 7/2*a^6*d^6*x^2*e +
a^6*d^7*x
```

3.1483 $\int (d+ex)^6 \left(a^2+2abx+b^2x^2\right)^3 \, dx$

Optimal. Leaf size=171

$$\frac{e^5(a+bx)^{12}(bd-ae)}{2b^7}+\frac{15e^4(a+bx)^{11}(bd-ae)^2}{11b^7}+\frac{2e^3(a+bx)^{10}(bd-ae)^3}{b^7}+\frac{5e^2(a+bx)^9(bd-ae)^4}{3b^7}+\frac{3e(a+bx)^8(bd-}{4b^7}$$

```
[Out] ((b*d - a*e)^6*(a + b*x)^7)/(7*b^7) + (3*e*(b*d - a*e)^5*(a + b*x)^8)/(4*b^
7) + (5*e^2*(b*d - a*e)^4*(a + b*x)^9)/(3*b^7) + (2*e^3*(b*d - a*e)^3*(a +
b*x)^10)/b^7 + (15*e^4*(b*d - a*e)^2*(a + b*x)^11)/(11*b^7) + (e^5*(b*d - a
*e)*(a + b*x)^12)/(2*b^7) + (e^6*(a + b*x)^13)/(13*b^7)
```

Rubi [A] time = 0.358183, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$\frac{e^5(a+bx)^{12}(bd-ae)}{2b^7}+\frac{15e^4(a+bx)^{11}(bd-ae)^2}{11b^7}+\frac{2e^3(a+bx)^{10}(bd-ae)^3}{b^7}+\frac{5e^2(a+bx)^9(bd-ae)^4}{3b^7}+\frac{3e(a+bx)^8(bd-}{4b^7}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^6*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] ((b*d - a*e)^6*(a + b*x)^7)/(7*b^7) + (3*e*(b*d - a*e)^5*(a + b*x)^8)/(4*b^
7) + (5*e^2*(b*d - a*e)^4*(a + b*x)^9)/(3*b^7) + (2*e^3*(b*d - a*e)^3*(a +
b*x)^10)/b^7 + (15*e^4*(b*d - a*e)^2*(a + b*x)^11)/(11*b^7) + (e^5*(b*d - a
*e)*(a + b*x)^12)/(2*b^7) + (e^6*(a + b*x)^13)/(13*b^7)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^6 \left(a^2+2abx+b^2x^2\right)^3 \, dx &= \int (a+bx)^6(d+ex)^6 \, dx\\ &= \int \left(\frac{(bd-ae)^6(a+bx)^6}{b^6}+\frac{6e(bd-ae)^5(a+bx)^7}{b^6}+\frac{15e^2(bd-ae)^4(a+bx)^8}{b^6}+\frac{20e^3(}{}\right.\\ &= \frac{(bd-ae)^6(a+bx)^7}{7b^7}+\frac{3e(bd-ae)^5(a+bx)^8}{4b^7}+\frac{5e^2(bd-ae)^4(a+bx)^9}{3b^7}+\frac{2e^3(bd-a}{}\end{aligned}$$

Mathematica [B] time = 0.0817923, size = 573, normalized size = 3.35

$$\frac{3}{11}b^4e^4x^{11}\left(5a^2e^2+12abde+5b^2d^2\right)+b^3e^3x^{10}\left(9a^2bde^2+2a^3e^3+9ab^2d^2e+2b^3d^3\right)+\frac{5}{3}b^2e^2x^9\left(15a^2b^2d^2e^2+8a^3bde^3+a\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^6*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] a^6*d^6*x + 3*a^5*d^5*(b*d + a*e)*x^2 + a^4*d^4*(5*b^2*d^2 + 12*a*b*d*e + 5
*a^2*e^2)*x^3 + (5*a^3*d^3*(2*b^3*d^3 + 9*a*b^2*d^2*e + 9*a^2*b*d*e^2 + 2*a
^3*e^3)*x^4)/2 + 3*a^2*d^2*(b^4*d^4 + 8*a*b^3*d^3*e + 15*a^2*b^2*d^2*e^2 +
8*a^3*b*d*e^3 + a^4*e^4)*x^5 + a*d*(b^5*d^5 + 15*a*b^4*d^4*e + 50*a^2*b^3*d
^3*e^2 + 50*a^3*b^2*d^2*e^3 + 15*a^4*b*d*e^4 + a^5*e^5)*x^6 + ((b^6*d^6 + 3
6*a*b^5*d^5*e + 225*a^2*b^4*d^4*e^2 + 400*a^3*b^3*d^3*e^3 + 225*a^4*b^2*d^2
*e^4 + 36*a^5*b*d*e^5 + a^6*e^6)*x^7)/7 + (3*b*e*(b^5*d^5 + 15*a*b^4*d^4*e
+ 50*a^2*b^3*d^3*e^2 + 50*a^3*b^2*d^2*e^3 + 15*a^4*b*d*e^4 + a^5*e^5)*x^8)/
4 + (5*b^2*e^2*(b^4*d^4 + 8*a*b^3*d^3*e + 15*a^2*b^2*d^2*e^2 + 8*a^3*b*d*e^
3 + a^4*e^4)*x^9)/3 + b^3*e^3*(2*b^3*d^3 + 9*a*b^2*d^2*e + 9*a^2*b*d*e^2 +
2*a^3*e^3)*x^10 + (3*b^4*e^4*(5*b^2*d^2 + 12*a*b*d*e + 5*a^2*e^2)*x^11)/11
+ (b^5*e^5*(b*d + a*e)*x^12)/2 + (b^6*e^6*x^13)/13
```

Maple [B] time = 0.04, size = 615, normalized size = 3.6

$$\frac{b^6e^6x^{13}}{13}+\frac{\left(6\,e^6ab^5+6\,de^5b^6\right)x^{12}}{12}+\frac{\left(15\,e^6a^2b^4+36\,de^5ab^5+15\,d^2e^4b^6\right)x^{11}}{11}+\frac{\left(20\,e^6a^3b^3+90\,de^5a^2b^4+90\,d^2e^4ab^5+\right.}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^6*(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] 1/13*b^6*e^6*x^13+1/12*(6*a*b^5*e^6+6*b^6*d*e^5)*x^12+1/11*(15*a^2*b^4*e^6+
36*a*b^5*d*e^5+15*b^6*d^2*e^4)*x^11+1/10*(20*a^3*b^3*e^6+90*a^2*b^4*d*e^5+9
0*a*b^5*d^2*e^4+20*b^6*d^3*e^3)*x^10+1/9*(15*a^4*b^2*e^6+120*a^3*b^3*d*e^5+
225*a^2*b^4*d^2*e^4+120*a*b^5*d^3*e^3+15*b^6*d^4*e^2)*x^9+1/8*(6*a^5*b*e^6+
90*a^4*b^2*d*e^5+300*a^3*b^3*d^2*e^4+300*a^2*b^4*d^3*e^3+90*a*b^5*d^4*e^2+6
*b^6*d^5*e)*x^8+1/7*(a^6*e^6+36*a^5*b*d*e^5+225*a^4*b^2*d^2*e^4+400*a^3*b^3
*d^3*e^3+225*a^2*b^4*d^4*e^2+36*a*b^5*d^5*e+b^6*d^6)*x^7+1/6*(6*a^6*d*e^5+9
0*a^5*b*d^2*e^4+300*a^4*b^2*d^3*e^3+300*a^3*b^3*d^4*e^2+90*a^2*b^4*d^5*e+6*
a*b^5*d^6)*x^6+1/5*(15*a^6*d^2*e^4+120*a^5*b*d^3*e^3+225*a^4*b^2*d^4*e^2+12
0*a^3*b^3*d^5*e+15*a^2*b^4*d^6)*x^5+1/4*(20*a^6*d^3*e^3+90*a^5*b*d^4*e^2+90
*a^4*b^2*d^5*e+20*a^3*b^3*d^6)*x^4+1/3*(15*a^6*d^4*e^2+36*a^5*b*d^5*e+15*a^
4*b^2*d^6)*x^3+1/2*(6*a^6*d^5*e+6*a^5*b*d^6)*x^2+d^6*a^6*x
```

Maxima [B] time = 1.04237, size = 809, normalized size = 4.73

$$\frac{1}{13}\,b^6e^6x^{13}+a^6d^6x+\frac{1}{2}\left(b^6de^5+ab^5e^6\right)x^{12}+\frac{3}{11}\left(5\,b^6d^2e^4+12\,ab^5de^5+5\,a^2b^4e^6\right)x^{11}+\left(2\,b^6d^3e^3+9\,ab^5d^2e^4+9\,a^2b^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b^6*e^6*x^13 + a^6*d^6*x + 1/2*(b^6*d*e^5 + a*b^5*e^6)*x^12 + 3/11*(5*
b^6*d^2*e^4 + 12*a*b^5*d*e^5 + 5*a^2*b^4*e^6)*x^11 + (2*b^6*d^3*e^3 + 9*a*b
^5*d^2*e^4 + 9*a^2*b^4*d*e^5 + 2*a^3*b^3*e^6)*x^10 + 5/3*(b^6*d^4*e^2 + 8*a
*b^5*d^3*e^3 + 15*a^2*b^4*d^2*e^4 + 8*a^3*b^3*d*e^5 + a^4*b^2*e^6)*x^9 + 3/
4*(b^6*d^5*e + 15*a*b^5*d^4*e^2 + 50*a^2*b^4*d^3*e^3 + 50*a^3*b^3*d^2*e^4 +
 15*a^4*b^2*d*e^5 + a^5*b*e^6)*x^8 + 1/7*(b^6*d^6 + 36*a*b^5*d^5*e + 225*a^
```

```
2*b^4*d^4*e^2 + 400*a^3*b^3*d^3*e^3 + 225*a^4*b^2*d^2*e^4 + 36*a^5*b*d*e^5
+ a^6*e^6)*x^7 + (a*b^5*d^6 + 15*a^2*b^4*d^5*e + 50*a^3*b^3*d^4*e^2 + 50*a^
4*b^2*d^3*e^3 + 15*a^5*b*d^2*e^4 + a^6*d*e^5)*x^6 + 3*(a^2*b^4*d^6 + 8*a^3*
b^3*d^5*e + 15*a^4*b^2*d^4*e^2 + 8*a^5*b*d^3*e^3 + a^6*d^2*e^4)*x^5 + 5/2*(
2*a^3*b^3*d^6 + 9*a^4*b^2*d^5*e + 9*a^5*b*d^4*e^2 + 2*a^6*d^3*e^3)*x^4 + (5
*a^4*b^2*d^6 + 12*a^5*b*d^5*e + 5*a^6*d^4*e^2)*x^3 + 3*(a^5*b*d^6 + a^6*d^5
*e)*x^2
```

Fricas [B] time = 1.51522, size = 1473, normalized size = 8.61

$$\frac{1}{13}x^{13}e^6b^6 + \frac{1}{2}x^{12}e^5db^6 + \frac{1}{2}x^{12}e^6b^5a + \frac{15}{11}x^{11}e^4d^2b^6 + \frac{36}{11}x^{11}e^5db^5a + \frac{15}{11}x^{11}e^6b^4a^2 + 2x^{10}e^3d^3b^6 + 9x^{10}e^4d^2b^5a + 9x^{10}e^5d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*e^6*b^6 + 1/2*x^12*e^5*d*b^6 + 1/2*x^12*e^6*b^5*a + 15/11*x^11*e^
4*d^2*b^6 + 36/11*x^11*e^5*d*b^5*a + 15/11*x^11*e^6*b^4*a^2 + 2*x^10*e^3*d^
3*b^6 + 9*x^10*e^4*d^2*b^5*a + 9*x^10*e^5*d*b^4*a^2 + 2*x^10*e^6*b^3*a^3 +
5/3*x^9*e^2*d^4*b^6 + 40/3*x^9*e^3*d^3*b^5*a + 25*x^9*e^4*d^2*b^4*a^2 + 40/
3*x^9*e^5*d*b^3*a^3 + 5/3*x^9*e^6*b^2*a^4 + 3/4*x^8*e*d^5*b^6 + 45/4*x^8*e^
2*d^4*b^5*a + 75/2*x^8*e^3*d^3*b^4*a^2 + 75/2*x^8*e^4*d^2*b^3*a^3 + 45/4*x^
8*e^5*d*b^2*a^4 + 3/4*x^8*e^6*b*a^5 + 1/7*x^7*d^6*b^6 + 36/7*x^7*e*d^5*b^5*
a + 225/7*x^7*e^2*d^4*b^4*a^2 + 400/7*x^7*e^3*d^3*b^3*a^3 + 225/7*x^7*e^4*d
^2*b^2*a^4 + 36/7*x^7*e^5*d*b*a^5 + 1/7*x^7*e^6*a^6 + x^6*d^6*b^5*a + 15*x^
6*e*d^5*b^4*a^2 + 50*x^6*e^2*d^4*b^3*a^3 + 50*x^6*e^3*d^3*b^2*a^4 + 15*x^6*
e^4*d^2*b*a^5 + x^6*e^5*d*a^6 + 3*x^5*d^6*b^4*a^2 + 24*x^5*e*d^5*b^3*a^3 +
45*x^5*e^2*d^4*b^2*a^4 + 24*x^5*e^3*d^3*b*a^5 + 3*x^5*e^4*d^2*a^6 + 5*x^4*d
^6*b^3*a^3 + 45/2*x^4*e*d^5*b^2*a^4 + 45/2*x^4*e^2*d^4*b*a^5 + 5*x^4*e^3*d^
3*a^6 + 5*x^3*d^6*b^2*a^4 + 12*x^3*e*d^5*b*a^5 + 5*x^3*e^2*d^4*a^6 + 3*x^2*
d^6*b*a^5 + 3*x^2*e*d^5*a^6 + x*d^6*a^6
```

Sympy [B] time = 0.159113, size = 677, normalized size = 3.96

$$a^6d^6x + \frac{b^6e^6x^{13}}{13} + x^{12}\left(\frac{ab^5e^6}{2} + \frac{b^6de^5}{2}\right) + x^{11}\left(\frac{15a^2b^4e^6}{11} + \frac{36ab^5de^5}{11} + \frac{15b^6d^2e^4}{11}\right) + x^{10}\left(2a^3b^3e^6 + 9a^2b^4de^5 + 9ab^5d^2e^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**6*(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] a**6*d**6*x + b**6*e**6*x**13/13 + x**12*(a*b**5*e**6/2 + b**6*d*e**5/2) +
x**11*(15*a**2*b**4*e**6/11 + 36*a*b**5*d*e**5/11 + 15*b**6*d**2*e**4/11) +
 x**10*(2*a**3*b**3*e**6 + 9*a**2*b**4*d*e**5 + 9*a*b**5*d**2*e**4 + 2*b**6
*d**3*e**3) + x**9*(5*a**4*b**2*e**6/3 + 40*a**3*b**3*d*e**5/3 + 25*a**2*b*
*4*d**2*e**4 + 40*a*b**5*d**3*e**3/3 + 5*b**6*d**4*e**2/3) + x**8*(3*a**5*b
*e**6/4 + 45*a**4*b**2*d*e**5/4 + 75*a**3*b**3*d**2*e**4/2 + 75*a**2*b**4*d
**3*e**3/2 + 45*a*b**5*d**4*e**2/4 + 3*b**6*d**5*e/4) + x**7*(a**6*e**6/7 +
 36*a**5*b*d*e**5/7 + 225*a**4*b**2*d**2*e**4/7 + 400*a**3*b**3*d**3*e**3/7
 + 225*a**2*b**4*d**4*e**2/7 + 36*a*b**5*d**5*e/7 + b**6*d**6/7) + x**6*(a*
*6*d*e**5 + 15*a**5*b*d**2*e**4 + 50*a**4*b**2*d**3*e**3 + 50*a**3*b**3*d**
4*e**2 + 15*a**2*b**4*d**5*e + a*b**5*d**6) + x**5*(3*a**6*d**2*e**4 + 24*a
**5*b*d**3*e**3 + 45*a**4*b**2*d**4*e**2 + 24*a**3*b**3*d**5*e + 3*a**2*b**
4*d**6) + x**4*(5*a**6*d**3*e**3 + 45*a**5*b*d**4*e**2/2 + 45*a**4*b**2*d**
```

```
5*e/2 + 5*a**3*b**3*d**6) + x**3*(5*a**6*d**4*e**2 + 12*a**5*b*d**5*e + 5*a
**4*b**2*d**6) + x**2*(3*a**6*d**5*e + 3*a**5*b*d**6)
```

Giac [B] time = 1.10845, size = 892, normalized size = 5.22

$$\frac{1}{13}\,b^6x^{13}e^6+\frac{1}{2}\,b^6dx^{12}e^5+\frac{15}{11}\,b^6d^2x^{11}e^4+2\,b^6d^3x^{10}e^3+\frac{5}{3}\,b^6d^4x^9e^2+\frac{3}{4}\,b^6d^5x^8e+\frac{1}{7}\,b^6d^6x^7+\frac{1}{2}\,ab^5x^{12}e^6+\frac{36}{11}\,ab^5dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/13*b^6*x^13*e^6 + 1/2*b^6*d*x^12*e^5 + 15/11*b^6*d^2*x^11*e^4 + 2*b^6*d^3
*x^10*e^3 + 5/3*b^6*d^4*x^9*e^2 + 3/4*b^6*d^5*x^8*e + 1/7*b^6*d^6*x^7 + 1/2
*a*b^5*x^12*e^6 + 36/11*a*b^5*d*x^11*e^5 + 9*a*b^5*d^2*x^10*e^4 + 40/3*a*b^
5*d^3*x^9*e^3 + 45/4*a*b^5*d^4*x^8*e^2 + 36/7*a*b^5*d^5*x^7*e + a*b^5*d^6*x
^6 + 15/11*a^2*b^4*x^11*e^6 + 9*a^2*b^4*d*x^10*e^5 + 25*a^2*b^4*d^2*x^9*e^4
 + 75/2*a^2*b^4*d^3*x^8*e^3 + 225/7*a^2*b^4*d^4*x^7*e^2 + 15*a^2*b^4*d^5*x^
6*e + 3*a^2*b^4*d^6*x^5 + 2*a^3*b^3*x^10*e^6 + 40/3*a^3*b^3*d*x^9*e^5 + 75/
2*a^3*b^3*d^2*x^8*e^4 + 400/7*a^3*b^3*d^3*x^7*e^3 + 50*a^3*b^3*d^4*x^6*e^2
+ 24*a^3*b^3*d^5*x^5*e + 5*a^3*b^3*d^6*x^4 + 5/3*a^4*b^2*x^9*e^6 + 45/4*a^4
*b^2*d*x^8*e^5 + 225/7*a^4*b^2*d^2*x^7*e^4 + 50*a^4*b^2*d^3*x^6*e^3 + 45*a^
4*b^2*d^4*x^5*e^2 + 45/2*a^4*b^2*d^5*x^4*e + 5*a^4*b^2*d^6*x^3 + 3/4*a^5*b*
x^8*e^6 + 36/7*a^5*b*d*x^7*e^5 + 15*a^5*b*d^2*x^6*e^4 + 24*a^5*b*d^3*x^5*e^
3 + 45/2*a^5*b*d^4*x^4*e^2 + 12*a^5*b*d^5*x^3*e + 3*a^5*b*d^6*x^2 + 1/7*a^6
*x^7*e^6 + a^6*d*x^6*e^5 + 3*a^6*d^2*x^5*e^4 + 5*a^6*d^3*x^4*e^3 + 5*a^6*d^
4*x^3*e^2 + 3*a^6*d^5*x^2*e + a^6*d^6*x
```

3.1484 $\int (d+ex)^5 \left(a^2+2abx+b^2x^2\right)^3 \, dx$

Optimal. Leaf size=143

$$\frac{5e^4(a+bx)^{11}(bd-ae)}{11b^6}+\frac{e^3(a+bx)^{10}(bd-ae)^2}{b^6}+\frac{10e^2(a+bx)^9(bd-ae)^3}{9b^6}+\frac{5e(a+bx)^8(bd-ae)^4}{8b^6}+\frac{(a+bx)^7(bd-ae)^5}{7b^6}$$

```
[Out] ((b*d - a*e)^5*(a + b*x)^7)/(7*b^6) + (5*e*(b*d - a*e)^4*(a + b*x)^8)/(8*b^
6) + (10*e^2*(b*d - a*e)^3*(a + b*x)^9)/(9*b^6) + (e^3*(b*d - a*e)^2*(a + b
*x)^10)/b^6 + (5*e^4*(b*d - a*e)*(a + b*x)^11)/(11*b^6) + (e^5*(a + b*x)^12
)/(12*b^6)
```

Rubi [A] time = 0.305659, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$\frac{5e^4(a+bx)^{11}(bd-ae)}{11b^6}+\frac{e^3(a+bx)^{10}(bd-ae)^2}{b^6}+\frac{10e^2(a+bx)^9(bd-ae)^3}{9b^6}+\frac{5e(a+bx)^8(bd-ae)^4}{8b^6}+\frac{(a+bx)^7(bd-ae)^5}{7b^6}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] ((b*d - a*e)^5*(a + b*x)^7)/(7*b^6) + (5*e*(b*d - a*e)^4*(a + b*x)^8)/(8*b^
6) + (10*e^2*(b*d - a*e)^3*(a + b*x)^9)/(9*b^6) + (e^3*(b*d - a*e)^2*(a + b
*x)^10)/b^6 + (5*e^4*(b*d - a*e)*(a + b*x)^11)/(11*b^6) + (e^5*(a + b*x)^12
)/(12*b^6)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^5 \left(a^2+2abx+b^2x^2\right)^3 \, dx &= \int (a+bx)^6(d+ex)^5 \, dx\\ &= \int \left(\frac{(bd-ae)^5(a+bx)^6}{b^5}+\frac{5e(bd-ae)^4(a+bx)^7}{b^5}+\frac{10e^2(bd-ae)^3(a+bx)^8}{b^5}+\frac{10e^3(\right.\\ &= \frac{(bd-ae)^5(a+bx)^7}{7b^6}+\frac{5e(bd-ae)^4(a+bx)^8}{8b^6}+\frac{10e^2(bd-ae)^3(a+bx)^9}{9b^6}+\frac{e^3(bd-a\end{aligned}$$

Mathematica [B] time = 0.0709639, size = 501, normalized size = 3.5

$$\frac{1}{2}b^4e^3x^{10}\left(3a^2e^2+6abde+2b^2d^2\right)+\frac{5}{9}b^3e^2x^9\left(15a^2bde^2+4a^3e^3+12ab^2d^2e+2b^3d^3\right)+\frac{5}{8}b^2ex^8\left(30a^2b^2d^2e^2+20a^3bde^3+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] a^6*d^5*x + (a^5*d^4*(6*b*d + 5*a*e)*x^2)/2 + (5*a^4*d^3*(3*b^2*d^2 + 6*a*b
*d*e + 2*a^2*e^2)*x^3)/3 + (5*a^3*d^2*(4*b^3*d^3 + 15*a*b^2*d^2*e + 12*a^2*
b*d*e^2 + 2*a^3*e^3)*x^4)/4 + a^2*d*(3*b^4*d^4 + 20*a*b^3*d^3*e + 30*a^2*b^
2*d^2*e^2 + 12*a^3*b*d*e^3 + a^4*e^4)*x^5 + (a*(6*b^5*d^5 + 75*a*b^4*d^4*e
+ 200*a^2*b^3*d^3*e^2 + 150*a^3*b^2*d^2*e^3 + 30*a^4*b*d*e^4 + a^5*e^5)*x^6
)/6 + (b*(b^5*d^5 + 30*a*b^4*d^4*e + 150*a^2*b^3*d^3*e^2 + 200*a^3*b^2*d^2*
e^3 + 75*a^4*b*d*e^4 + 6*a^5*e^5)*x^7)/7 + (5*b^2*e*(b^4*d^4 + 12*a*b^3*d^3
*e + 30*a^2*b^2*d^2*e^2 + 20*a^3*b*d*e^3 + 3*a^4*e^4)*x^8)/8 + (5*b^3*e^2*(
2*b^3*d^3 + 12*a*b^2*d^2*e + 15*a^2*b*d*e^2 + 4*a^3*e^3)*x^9)/9 + (b^4*e^3*
(2*b^2*d^2 + 6*a*b*d*e + 3*a^2*e^2)*x^10)/2 + (b^5*e^4*(5*b*d + 6*a*e)*x^11
)/11 + (b^6*e^5*x^12)/12
```

Maple [B] time = 0.039, size = 521, normalized size = 3.6

$$\frac{b^6e^5x^{12}}{12} + \frac{\left(6\,e^5ab^5 + 5\,de^4b^6\right)x^{11}}{11} + \frac{\left(15\,e^5a^2b^4 + 30\,de^4ab^5 + 10\,d^2e^3b^6\right)x^{10}}{10} + \frac{\left(20\,e^5a^3b^3 + 75\,de^4a^2b^4 + 60\,d^2e^3ab^5 + \right.}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] 1/12*b^6*e^5*x^12+1/11*(6*a*b^5*e^5+5*b^6*d*e^4)*x^11+1/10*(15*a^2*b^4*e^5+
30*a*b^5*d*e^4+10*b^6*d^2*e^3)*x^10+1/9*(20*a^3*b^3*e^5+75*a^2*b^4*d*e^4+60
*a*b^5*d^2*e^3+10*b^6*d^3*e^2)*x^9+1/8*(15*a^4*b^2*e^5+100*a^3*b^3*d*e^4+15
0*a^2*b^4*d^2*e^3+60*a*b^5*d^3*e^2+5*b^6*d^4*e)*x^8+1/7*(6*a^5*b*e^5+75*a^4
*b^2*d*e^4+200*a^3*b^3*d^2*e^3+150*a^2*b^4*d^3*e^2+30*a*b^5*d^4*e+b^6*d^5)*
x^7+1/6*(a^6*e^5+30*a^5*b*d*e^4+150*a^4*b^2*d^2*e^3+200*a^3*b^3*d^3*e^2+75*
a^2*b^4*d^4*e+6*a*b^5*d^5)*x^6+1/5*(5*a^6*d*e^4+60*a^5*b*d^2*e^3+150*a^4*b^
2*d^3*e^2+100*a^3*b^3*d^4*e+15*a^2*b^4*d^5)*x^5+1/4*(10*a^6*d^2*e^3+60*a^5*
b*d^3*e^2+75*a^4*b^2*d^4*e+20*a^3*b^3*d^5)*x^4+1/3*(10*a^6*d^3*e^2+30*a^5*b
*d^4*e+15*a^4*b^2*d^5)*x^3+1/2*(5*a^6*d^4*e+6*a^5*b*d^5)*x^2+d^5*a^6*x
```

Maxima [B] time = 1.19909, size = 698, normalized size = 4.88

$$\frac{1}{12}\,b^6e^5x^{12} + a^6d^5x + \frac{1}{11}\left(5\,b^6de^4 + 6\,ab^5e^5\right)x^{11} + \frac{1}{2}\left(2\,b^6d^2e^3 + 6\,ab^5de^4 + 3\,a^2b^4e^5\right)x^{10} + \frac{5}{9}\left(2\,b^6d^3e^2 + 12\,ab^5d^2e^3 + 1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/12*b^6*e^5*x^12 + a^6*d^5*x + 1/11*(5*b^6*d*e^4 + 6*a*b^5*e^5)*x^11 + 1/2
*(2*b^6*d^2*e^3 + 6*a*b^5*d*e^4 + 3*a^2*b^4*e^5)*x^10 + 5/9*(2*b^6*d^3*e^2
+ 12*a*b^5*d^2*e^3 + 15*a^2*b^4*d*e^4 + 4*a^3*b^3*e^5)*x^9 + 5/8*(b^6*d^4*e
 + 12*a*b^5*d^3*e^2 + 30*a^2*b^4*d^2*e^3 + 20*a^3*b^3*d*e^4 + 3*a^4*b^2*e^5
)*x^8 + 1/7*(b^6*d^5 + 30*a*b^5*d^4*e + 150*a^2*b^4*d^3*e^2 + 200*a^3*b^3*d
^2*e^3 + 75*a^4*b^2*d*e^4 + 6*a^5*b*e^5)*x^7 + 1/6*(6*a*b^5*d^5 + 75*a^2*b^
4*d^4*e + 200*a^3*b^3*d^3*e^2 + 150*a^4*b^2*d^2*e^3 + 30*a^5*b*d*e^4 + a^6*
e^5)*x^6 + (3*a^2*b^4*d^5 + 20*a^3*b^3*d^4*e + 30*a^4*b^2*d^3*e^2 + 12*a^5*
b*d^2*e^3 + a^6*d*e^4)*x^5 + 5/4*(4*a^3*b^3*d^5 + 15*a^4*b^2*d^4*e + 12*a^5
```

```
*b*d^3*e^2 + 2*a^6*d^2*e^3)*x^4 + 5/3*(3*a^4*b^2*d^5 + 6*a^5*b*d^4*e + 2*a^
6*d^3*e^2)*x^3 + 1/2*(6*a^5*b*d^5 + 5*a^6*d^4*e)*x^2
```

Fricas [B] time = 1.52875, size = 1251, normalized size = 8.75

$$\frac{1}{12}x^{12}e^5b^6 + \frac{5}{11}x^{11}e^4db^6 + \frac{6}{11}x^{11}e^5b^5a + x^{10}e^3d^2b^6 + 3x^{10}e^4db^5a + \frac{3}{2}x^{10}e^5b^4a^2 + \frac{10}{9}x^9e^2d^3b^6 + \frac{20}{3}x^9e^3d^2b^5a + \frac{25}{3}x^9e^4db$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*e^5*b^6 + 5/11*x^11*e^4*d*b^6 + 6/11*x^11*e^5*b^5*a + x^10*e^3*d^
2*b^6 + 3*x^10*e^4*d*b^5*a + 3/2*x^10*e^5*b^4*a^2 + 10/9*x^9*e^2*d^3*b^6 +
20/3*x^9*e^3*d^2*b^5*a + 25/3*x^9*e^4*d*b^4*a^2 + 20/9*x^9*e^5*b^3*a^3 + 5/
8*x^8*e*d^4*b^6 + 15/2*x^8*e^2*d^3*b^5*a + 75/4*x^8*e^3*d^2*b^4*a^2 + 25/2*
x^8*e^4*d*b^3*a^3 + 15/8*x^8*e^5*b^2*a^4 + 1/7*x^7*d^5*b^6 + 30/7*x^7*e*d^4
*b^5*a + 150/7*x^7*e^2*d^3*b^4*a^2 + 200/7*x^7*e^3*d^2*b^3*a^3 + 75/7*x^7*e
^4*d*b^2*a^4 + 6/7*x^7*e^5*b*a^5 + x^6*d^5*b^5*a + 25/2*x^6*e*d^4*b^4*a^2 +
 100/3*x^6*e^2*d^3*b^3*a^3 + 25*x^6*e^3*d^2*b^2*a^4 + 5*x^6*e^4*d*b*a^5 + 1
/6*x^6*e^5*a^6 + 3*x^5*d^5*b^4*a^2 + 20*x^5*e*d^4*b^3*a^3 + 30*x^5*e^2*d^3*
b^2*a^4 + 12*x^5*e^3*d^2*b*a^5 + x^5*e^4*d*a^6 + 5*x^4*d^5*b^3*a^3 + 75/4*x
^4*e*d^4*b^2*a^4 + 15*x^4*e^2*d^3*b*a^5 + 5/2*x^4*e^3*d^2*a^6 + 5*x^3*d^5*b
^2*a^4 + 10*x^3*e*d^4*b*a^5 + 10/3*x^3*e^2*d^3*a^6 + 3*x^2*d^5*b*a^5 + 5/2*
x^2*e*d^4*a^6 + x*d^5*a^6
```

Sympy [B] time = 0.214697, size = 580, normalized size = 4.06

$$a^6d^5x + \frac{b^6e^5x^{12}}{12} + x^{11}\left(\frac{6ab^5e^5}{11} + \frac{5b^6de^4}{11}\right) + x^{10}\left(\frac{3a^2b^4e^5}{2} + 3ab^5de^4 + b^6d^2e^3\right) + x^9\left(\frac{20a^3b^3e^5}{9} + \frac{25a^2b^4de^4}{3} + \frac{20ab^5d^2e^3}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5*(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] a**6*d**5*x + b**6*e**5*x**12/12 + x**11*(6*a*b**5*e**5/11 + 5*b**6*d*e**4/
11) + x**10*(3*a**2*b**4*e**5/2 + 3*a*b**5*d*e**4 + b**6*d**2*e**3) + x**9*
(20*a**3*b**3*e**5/9 + 25*a**2*b**4*d*e**4/3 + 20*a*b**5*d**2*e**3/3 + 10*b
**6*d**3*e**2/9) + x**8*(15*a**4*b**2*e**5/8 + 25*a**3*b**3*d*e**4/2 + 75*a
**2*b**4*d**2*e**3/4 + 15*a*b**5*d**3*e**2/2 + 5*b**6*d**4*e/8) + x**7*(6*a
**5*b*e**5/7 + 75*a**4*b**2*d*e**4/7 + 200*a**3*b**3*d**2*e**3/7 + 150*a**2
*b**4*d**3*e**2/7 + 30*a*b**5*d**4*e/7 + b**6*d**5/7) + x**6*(a**6*e**5/6 +
 5*a**5*b*d*e**4 + 25*a**4*b**2*d**2*e**3 + 100*a**3*b**3*d**3*e**2/3 + 25*
a**2*b**4*d**4*e/2 + a*b**5*d**5) + x**5*(a**6*d*e**4 + 12*a**5*b*d**2*e**3
 + 30*a**4*b**2*d**3*e**2 + 20*a**3*b**3*d**4*e + 3*a**2*b**4*d**5) + x**4*
(5*a**6*d**2*e**3/2 + 15*a**5*b*d**3*e**2 + 75*a**4*b**2*d**4*e/4 + 5*a**3*
b**3*d**5) + x**3*(10*a**6*d**3*e**2/3 + 10*a**5*b*d**4*e + 5*a**4*b**2*d**
5) + x**2*(5*a**6*d**4*e/2 + 3*a**5*b*d**5)
```

Giac [B] time = 1.13697, size = 753, normalized size = 5.27

$$\frac{1}{12}b^6x^{12}e^5 + \frac{5}{11}b^6dx^{11}e^4 + b^6d^2x^{10}e^3 + \frac{10}{9}b^6d^3x^9e^2 + \frac{5}{8}b^6d^4x^8e + \frac{1}{7}b^6d^5x^7 + \frac{6}{11}ab^5x^{11}e^5 + 3\,ab^5dx^{10}e^4 + \frac{20}{3}ab^5d^2x^9e^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/12*b^6*x^12*e^5 + 5/11*b^6*d*x^11*e^4 + b^6*d^2*x^10*e^3 + 10/9*b^6*d^3*x
^9*e^2 + 5/8*b^6*d^4*x^8*e + 1/7*b^6*d^5*x^7 + 6/11*a*b^5*x^11*e^5 + 3*a*b^
5*d*x^10*e^4 + 20/3*a*b^5*d^2*x^9*e^3 + 15/2*a*b^5*d^3*x^8*e^2 + 30/7*a*b^5
*d^4*x^7*e + a*b^5*d^5*x^6 + 3/2*a^2*b^4*x^10*e^5 + 25/3*a^2*b^4*d*x^9*e^4
+ 75/4*a^2*b^4*d^2*x^8*e^3 + 150/7*a^2*b^4*d^3*x^7*e^2 + 25/2*a^2*b^4*d^4*x
^6*e + 3*a^2*b^4*d^5*x^5 + 20/9*a^3*b^3*x^9*e^5 + 25/2*a^3*b^3*d*x^8*e^4 +
200/7*a^3*b^3*d^2*x^7*e^3 + 100/3*a^3*b^3*d^3*x^6*e^2 + 20*a^3*b^3*d^4*x^5*
e + 5*a^3*b^3*d^5*x^4 + 15/8*a^4*b^2*x^8*e^5 + 75/7*a^4*b^2*d*x^7*e^4 + 25*
a^4*b^2*d^2*x^6*e^3 + 30*a^4*b^2*d^3*x^5*e^2 + 75/4*a^4*b^2*d^4*x^4*e + 5*a
^4*b^2*d^5*x^3 + 6/7*a^5*b*x^7*e^5 + 5*a^5*b*d*x^6*e^4 + 12*a^5*b*d^2*x^5*e
^3 + 15*a^5*b*d^3*x^4*e^2 + 10*a^5*b*d^4*x^3*e + 3*a^5*b*d^5*x^2 + 1/6*a^6*
x^6*e^5 + a^6*d*x^5*e^4 + 5/2*a^6*d^2*x^4*e^3 + 10/3*a^6*d^3*x^3*e^2 + 5/2*
a^6*d^4*x^2*e + a^6*d^5*x
```

3.1485 $\int (d+ex)^4 \left(a^2+2abx+b^2x^2\right)^3 \, dx$

Optimal. Leaf size=119

$$\frac{2e^3(a+bx)^{10}(bd-ae)}{5b^5}+\frac{2e^2(a+bx)^9(bd-ae)^2}{3b^5}+\frac{e(a+bx)^8(bd-ae)^3}{2b^5}+\frac{(a+bx)^7(bd-ae)^4}{7b^5}+\frac{e^4(a+bx)^{11}}{11b^5}$$

```
[Out] ((b*d - a*e)^4*(a + b*x)^7)/(7*b^5) + (e*(b*d - a*e)^3*(a + b*x)^8)/(2*b^5)
 + (2*e^2*(b*d - a*e)^2*(a + b*x)^9)/(3*b^5) + (2*e^3*(b*d - a*e)*(a + b*x)
^10)/(5*b^5) + (e^4*(a + b*x)^11)/(11*b^5)
```

Rubi [A] time = 0.251798, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$\frac{2e^3(a+bx)^{10}(bd-ae)}{5b^5}+\frac{2e^2(a+bx)^9(bd-ae)^2}{3b^5}+\frac{e(a+bx)^8(bd-ae)^3}{2b^5}+\frac{(a+bx)^7(bd-ae)^4}{7b^5}+\frac{e^4(a+bx)^{11}}{11b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] ((b*d - a*e)^4*(a + b*x)^7)/(7*b^5) + (e*(b*d - a*e)^3*(a + b*x)^8)/(2*b^5)
 + (2*e^2*(b*d - a*e)^2*(a + b*x)^9)/(3*b^5) + (2*e^3*(b*d - a*e)*(a + b*x)
^10)/(5*b^5) + (e^4*(a + b*x)^11)/(11*b^5)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (d+ex)^4 \left(a^2+2abx+b^2x^2\right)^3 \, dx = \int (a+bx)^6(d+ex)^4 \, dx$$
$$= \int \left(\frac{(bd-ae)^4(a+bx)^6}{b^4}+\frac{4e(bd-ae)^3(a+bx)^7}{b^4}+\frac{6e^2(bd-ae)^2(a+bx)^8}{b^4}+\frac{4e^3(bd\ldots\right.$$
$$= \frac{(bd-ae)^4(a+bx)^7}{7b^5}+\frac{e(bd-ae)^3(a+bx)^8}{2b^5}+\frac{2e^2(bd-ae)^2(a+bx)^9}{3b^5}+\frac{2e^3(bd-ae\ldots}{5b\ldots}$$

Mathematica [B] time = 0.061379, size = 398, normalized size = 3.34

$$\frac{1}{3}b^4e^2x^9\left(5a^2e^2+8abde+2b^2d^2\right)+\frac{1}{2}b^3ex^8\left(15a^2bde^2+5a^3e^3+9ab^2d^2e+b^3d^3\right)+\frac{1}{7}b^2x^7\left(90a^2b^2d^2e^2+80a^3bde^3+15a^4\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] a^6*d^4*x + a^5*d^3*(3*b*d + 2*a*e)*x^2 + a^4*d^2*(5*b^2*d^2 + 8*a*b*d*e +
2*a^2*e^2)*x^3 + a^3*d*(5*b^3*d^3 + 15*a*b^2*d^2*e + 9*a^2*b*d*e^2 + a^3*e^
3)*x^4 + (a^2*(15*b^4*d^4 + 80*a*b^3*d^3*e + 90*a^2*b^2*d^2*e^2 + 24*a^3*b*
d*e^3 + a^4*e^4)*x^5)/5 + a*b*(b^4*d^4 + 10*a*b^3*d^3*e + 20*a^2*b^2*d^2*e^
2 + 10*a^3*b*d*e^3 + a^4*e^4)*x^6 + (b^2*(b^4*d^4 + 24*a*b^3*d^3*e + 90*a^2
*b^2*d^2*e^2 + 80*a^3*b*d*e^3 + 15*a^4*e^4)*x^7)/7 + (b^3*e*(b^3*d^3 + 9*a*
b^2*d^2*e + 15*a^2*b*d*e^2 + 5*a^3*e^3)*x^8)/2 + (b^4*e^2*(2*b^2*d^2 + 8*a*
b*d*e + 5*a^2*e^2)*x^9)/3 + (b^5*e^3*(2*b*d + 3*a*e)*x^10)/5 + (b^6*e^4*x^1
1)/11
```

Maple [B] time = 0.04, size = 427, normalized size = 3.6

$$\frac{e^4b^6x^{11}}{11} + \frac{\left(6\,e^4ab^5 + 4\,de^3b^6\right)x^{10}}{10} + \frac{\left(15\,e^4a^2b^4 + 24\,de^3ab^5 + 6\,d^2e^2b^6\right)x^9}{9} + \frac{\left(20\,e^4a^3b^3 + 60\,de^3a^2b^4 + 36\,d^2e^2ab^5 + 4\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] 1/11*e^4*b^6*x^11+1/10*(6*a*b^5*e^4+4*b^6*d*e^3)*x^10+1/9*(15*a^2*b^4*e^4+2
4*a*b^5*d*e^3+6*b^6*d^2*e^2)*x^9+1/8*(20*a^3*b^3*e^4+60*a^2*b^4*d*e^3+36*a*
b^5*d^2*e^2+4*b^6*d^3*e)*x^8+1/7*(15*a^4*b^2*e^4+80*a^3*b^3*d*e^3+90*a^2*b^
4*d^2*e^2+24*a*b^5*d^3*e+b^6*d^4)*x^7+1/6*(6*a^5*b*e^4+60*a^4*b^2*d*e^3+120
*a^3*b^3*d^2*e^2+60*a^2*b^4*d^3*e+6*a*b^5*d^4)*x^6+1/5*(a^6*e^4+24*a^5*b*d*
e^3+90*a^4*b^2*d^2*e^2+80*a^3*b^3*d^3*e+15*a^2*b^4*d^4)*x^5+1/4*(4*a^6*d*e^
3+36*a^5*b*d^2*e^2+60*a^4*b^2*d^3*e+20*a^3*b^3*d^4)*x^4+1/3*(6*a^6*d^2*e^2+
24*a^5*b*d^3*e+15*a^4*b^2*d^4)*x^3+1/2*(4*a^6*d^3*e+6*a^5*b*d^4)*x^2+d^4*a^
6*x
```

Maxima [B] time = 1.1732, size = 564, normalized size = 4.74

$$\frac{1}{11}\,b^6e^4x^{11} + a^6d^4x + \frac{1}{5}\left(2\,b^6de^3 + 3\,ab^5e^4\right)x^{10} + \frac{1}{3}\left(2\,b^6d^2e^2 + 8\,ab^5de^3 + 5\,a^2b^4e^4\right)x^9 + \frac{1}{2}\left(b^6d^3e + 9\,ab^5d^2e^2 + 15\,a^2b^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/11*b^6*e^4*x^11 + a^6*d^4*x + 1/5*(2*b^6*d*e^3 + 3*a*b^5*e^4)*x^10 + 1/3*
(2*b^6*d^2*e^2 + 8*a*b^5*d*e^3 + 5*a^2*b^4*e^4)*x^9 + 1/2*(b^6*d^3*e + 9*a*
b^5*d^2*e^2 + 15*a^2*b^4*d*e^3 + 5*a^3*b^3*e^4)*x^8 + 1/7*(b^6*d^4 + 24*a*b
^5*d^3*e + 90*a^2*b^4*d^2*e^2 + 80*a^3*b^3*d*e^3 + 15*a^4*b^2*e^4)*x^7 + (a
*b^5*d^4 + 10*a^2*b^4*d^3*e + 20*a^3*b^3*d^2*e^2 + 10*a^4*b^2*d*e^3 + a^5*b
*e^4)*x^6 + 1/5*(15*a^2*b^4*d^4 + 80*a^3*b^3*d^3*e + 90*a^4*b^2*d^2*e^2 + 2
4*a^5*b*d*e^3 + a^6*e^4)*x^5 + (5*a^3*b^3*d^4 + 15*a^4*b^2*d^3*e + 9*a^5*b*
d^2*e^2 + a^6*d*e^3)*x^4 + (5*a^4*b^2*d^4 + 8*a^5*b*d^3*e + 2*a^6*d^2*e^2)*
x^3 + (3*a^5*b*d^4 + 2*a^6*d^3*e)*x^2
```

Fricas [B] time = 1.50303, size = 992, normalized size = 8.34

$$\frac{1}{11}x^{11}e^4b^6 + \frac{2}{5}x^{10}e^3db^6 + \frac{3}{5}x^{10}e^4b^5a + \frac{2}{3}x^9e^2d^2b^6 + \frac{8}{3}x^9e^3db^5a + \frac{5}{3}x^9e^4b^4a^2 + \frac{1}{2}x^8ed^3b^6 + \frac{9}{2}x^8e^2d^2b^5a + \frac{15}{2}x^8e^3db^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*e^4*b^6 + 2/5*x^10*e^3*d*b^6 + 3/5*x^10*e^4*b^5*a + 2/3*x^9*e^2*d
^2*b^6 + 8/3*x^9*e^3*d*b^5*a + 5/3*x^9*e^4*b^4*a^2 + 1/2*x^8*e*d^3*b^6 + 9/
2*x^8*e^2*d^2*b^5*a + 15/2*x^8*e^3*d*b^4*a^2 + 5/2*x^8*e^4*b^3*a^3 + 1/7*x^
7*d^4*b^6 + 24/7*x^7*e*d^3*b^5*a + 90/7*x^7*e^2*d^2*b^4*a^2 + 80/7*x^7*e^3*
d*b^3*a^3 + 15/7*x^7*e^4*b^2*a^4 + x^6*d^4*b^5*a + 10*x^6*e*d^3*b^4*a^2 + 2
0*x^6*e^2*d^2*b^3*a^3 + 10*x^6*e^3*d*b^2*a^4 + x^6*e^4*b*a^5 + 3*x^5*d^4*b^
4*a^2 + 16*x^5*e*d^3*b^3*a^3 + 18*x^5*e^2*d^2*b^2*a^4 + 24/5*x^5*e^3*d*b*a^
5 + 1/5*x^5*e^4*a^6 + 5*x^4*d^4*b^3*a^3 + 15*x^4*e*d^3*b^2*a^4 + 9*x^4*e^2*
d^2*b*a^5 + x^4*e^3*d*a^6 + 5*x^3*d^4*b^2*a^4 + 8*x^3*e*d^3*b*a^5 + 2*x^3*e
^2*d^2*a^6 + 3*x^2*d^4*b*a^5 + 2*x^2*e*d^3*a^6 + x*d^4*a^6
```

Sympy [B] time = 0.150309, size = 462, normalized size = 3.88

$$a^6d^4x + \frac{b^6e^4x^{11}}{11} + x^{10}\left(\frac{3ab^5e^4}{5} + \frac{2b^6de^3}{5}\right) + x^9\left(\frac{5a^2b^4e^4}{3} + \frac{8ab^5de^3}{3} + \frac{2b^6d^2e^2}{3}\right) + x^8\left(\frac{5a^3b^3e^4}{2} + \frac{15a^2b^4de^3}{2} + \frac{9ab^5d^2e^2}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] a**6*d**4*x + b**6*e**4*x**11/11 + x**10*(3*a*b**5*e**4/5 + 2*b**6*d*e**3/5
) + x**9*(5*a**2*b**4*e**4/3 + 8*a*b**5*d*e**3/3 + 2*b**6*d**2*e**2/3) + x*
*8*(5*a**3*b**3*e**4/2 + 15*a**2*b**4*d*e**3/2 + 9*a*b**5*d**2*e**2/2 + b**
6*d**3*e/2) + x**7*(15*a**4*b**2*e**4/7 + 80*a**3*b**3*d*e**3/7 + 90*a**2*b
**4*d**2*e**2/7 + 24*a*b**5*d**3*e/7 + b**6*d**4/7) + x**6*(a**5*b*e**4 + 1
0*a**4*b**2*d*e**3 + 20*a**3*b**3*d**2*e**2 + 10*a**2*b**4*d**3*e + a*b**5*
d**4) + x**5*(a**6*e**4/5 + 24*a**5*b*d*e**3/5 + 18*a**4*b**2*d**2*e**2 + 1
6*a**3*b**3*d**3*e + 3*a**2*b**4*d**4) + x**4*(a**6*d*e**3 + 9*a**5*b*d**2*
e**2 + 15*a**4*b**2*d**3*e + 5*a**3*b**3*d**4) + x**3*(2*a**6*d**2*e**2 + 8
*a**5*b*d**3*e + 5*a**4*b**2*d**4) + x**2*(2*a**6*d**3*e + 3*a**5*b*d**4)
```

Giac [B] time = 1.12307, size = 616, normalized size = 5.18

$$\frac{1}{11}b^6x^{11}e^4 + \frac{2}{5}b^6dx^{10}e^3 + \frac{2}{3}b^6d^2x^9e^2 + \frac{1}{2}b^6d^3x^8e + \frac{1}{7}b^6d^4x^7 + \frac{3}{5}ab^5x^{10}e^4 + \frac{8}{3}ab^5dx^9e^3 + \frac{9}{2}ab^5d^2x^8e^2 + \frac{24}{7}ab^5d^3x^7e +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/11*b^6*x^11*e^4 + 2/5*b^6*d*x^10*e^3 + 2/3*b^6*d^2*x^9*e^2 + 1/2*b^6*d^3*
x^8*e + 1/7*b^6*d^4*x^7 + 3/5*a*b^5*x^10*e^4 + 8/3*a*b^5*d*x^9*e^3 + 9/2*a*
b^5*d^2*x^8*e^2 + 24/7*a*b^5*d^3*x^7*e + a*b^5*d^4*x^6 + 5/3*a^2*b^4*x^9*e^
4 + 15/2*a^2*b^4*d*x^8*e^3 + 90/7*a^2*b^4*d^2*x^7*e^2 + 10*a^2*b^4*d^3*x^6*
e + 3*a^2*b^4*d^4*x^5 + 5/2*a^3*b^3*x^8*e^4 + 80/7*a^3*b^3*d*x^7*e^3 + 20*a
^3*b^3*d^2*x^6*e^2 + 16*a^3*b^3*d^3*x^5*e + 5*a^3*b^3*d^4*x^4 + 15/7*a^4*b^
2*x^7*e^4 + 10*a^4*b^2*d*x^6*e^3 + 18*a^4*b^2*d^2*x^5*e^2 + 15*a^4*b^2*d^3*
x^4*e + 5*a^4*b^2*d^4*x^3 + a^5*b*x^6*e^4 + 24/5*a^5*b*d*x^5*e^3 + 9*a^5*b*
d^2*x^4*e^2 + 8*a^5*b*d^3*x^3*e + 3*a^5*b*d^4*x^2 + 1/5*a^6*x^5*e^4 + a^6*d
*x^4*e^3 + 2*a^6*d^2*x^3*e^2 + 2*a^6*d^3*x^2*e + a^6*d^4*x
```

3.1486 $\int (d+ex)^3 \left(a^2+2abx+b^2x^2\right)^3 \, dx$

Optimal. Leaf size=92

$$\frac{e^2(a+bx)^9(bd-ae)}{3b^4}+\frac{3e(a+bx)^8(bd-ae)^2}{8b^4}+\frac{(a+bx)^7(bd-ae)^3}{7b^4}+\frac{e^3(a+bx)^{10}}{10b^4}$$

```
[Out] ((b*d - a*e)^3*(a + b*x)^7)/(7*b^4) + (3*e*(b*d - a*e)^2*(a + b*x)^8)/(8*b^
4) + (e^2*(b*d - a*e)*(a + b*x)^9)/(3*b^4) + (e^3*(a + b*x)^10)/(10*b^4)
```

Rubi [A] time = 0.191523, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$\frac{e^2(a+bx)^9(bd-ae)}{3b^4}+\frac{3e(a+bx)^8(bd-ae)^2}{8b^4}+\frac{(a+bx)^7(bd-ae)^3}{7b^4}+\frac{e^3(a+bx)^{10}}{10b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] ((b*d - a*e)^3*(a + b*x)^7)/(7*b^4) + (3*e*(b*d - a*e)^2*(a + b*x)^8)/(8*b^
4) + (e^2*(b*d - a*e)*(a + b*x)^9)/(3*b^4) + (e^3*(a + b*x)^10)/(10*b^4)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^3 \left(a^2+2abx+b^2x^2\right)^3 \, dx &= \int (a+bx)^6(d+ex)^3 \, dx\\ &= \int \left(\frac{(bd-ae)^3(a+bx)^6}{b^3}+\frac{3e(bd-ae)^2(a+bx)^7}{b^3}+\frac{3e^2(bd-ae)(a+bx)^8}{b^3}+\frac{e^3(a\right.\\ &= \frac{(bd-ae)^3(a+bx)^7}{7b^4}+\frac{3e(bd-ae)^2(a+bx)^8}{8b^4}+\frac{e^2(bd-ae)(a+bx)^9}{3b^4}+\frac{e^3(a+bx}{10b^4}\end{aligned}$$

Mathematica [B] time = 0.0813186, size = 276, normalized size = 3.

$$\frac{1}{840}x\left(210a^4b^2x^2\left(45d^2ex+20d^3+36de^2x^2+10e^3x^3\right)+120a^3b^3x^3\left(84d^2ex+35d^3+70de^2x^2+20e^3x^3\right)+45a^2b^4x^4\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] (x*(210*a^6*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) + 252*a^5*b*x*(10*d
^3 + 20*d^2*e*x + 15*d*e^2*x^2 + 4*e^3*x^3) + 210*a^4*b^2*x^2*(20*d^3 + 45*
d^2*e*x + 36*d*e^2*x^2 + 10*e^3*x^3) + 120*a^3*b^3*x^3*(35*d^3 + 84*d^2*e*x
 + 70*d*e^2*x^2 + 20*e^3*x^3) + 45*a^2*b^4*x^4*(56*d^3 + 140*d^2*e*x + 120*
d*e^2*x^2 + 35*e^3*x^3) + 10*a*b^5*x^5*(84*d^3 + 216*d^2*e*x + 189*d*e^2*x^
2 + 56*e^3*x^3) + b^6*x^6*(120*d^3 + 315*d^2*e*x + 280*d*e^2*x^2 + 84*e^3*x
^3)))/840
```

Maple [B] time = 0.04, size = 333, normalized size = 3.6

$$\frac{e^3b^6x^{10}}{10}+\frac{\left(6\,e^3ab^5+3\,de^2b^6\right)x^9}{9}+\frac{\left(15\,e^3a^2b^4+18\,de^2ab^5+3\,d^2eb^6\right)x^8}{8}+\frac{\left(20\,e^3a^3b^3+45\,de^2a^2b^4+18\,d^2eab^5+d^3b^6\right)x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] 1/10*e^3*b^6*x^10+1/9*(6*a*b^5*e^3+3*b^6*d*e^2)*x^9+1/8*(15*a^2*b^4*e^3+18*
a*b^5*d*e^2+3*b^6*d^2*e)*x^8+1/7*(20*a^3*b^3*e^3+45*a^2*b^4*d*e^2+18*a*b^5*
d^2*e+b^6*d^3)*x^7+1/6*(15*a^4*b^2*e^3+60*a^3*b^3*d*e^2+45*a^2*b^4*d^2*e+6*
a*b^5*d^3)*x^6+1/5*(6*a^5*b*e^3+45*a^4*b^2*d*e^2+60*a^3*b^3*d^2*e+15*a^2*b^
4*d^3)*x^5+1/4*(a^6*e^3+18*a^5*b*d*e^2+45*a^4*b^2*d^2*e+20*a^3*b^3*d^3)*x^4
+1/3*(3*a^6*d*e^2+18*a^5*b*d^2*e+15*a^4*b^2*d^3)*x^3+1/2*(3*a^6*d^2*e+6*a^5
*b*d^3)*x^2+d^3*a^6*x
```

Maxima [B] time = 1.07234, size = 441, normalized size = 4.79

$$\frac{1}{10}\,b^6e^3x^{10}+a^6d^3x+\frac{1}{3}\left(b^6de^2+2\,ab^5e^3\right)x^9+\frac{3}{8}\left(b^6d^2e+6\,ab^5de^2+5\,a^2b^4e^3\right)x^8+\frac{1}{7}\left(b^6d^3+18\,ab^5d^2e+45\,a^2b^4de^2+20\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/10*b^6*e^3*x^10 + a^6*d^3*x + 1/3*(b^6*d*e^2 + 2*a*b^5*e^3)*x^9 + 3/8*(b^
6*d^2*e + 6*a*b^5*d*e^2 + 5*a^2*b^4*e^3)*x^8 + 1/7*(b^6*d^3 + 18*a*b^5*d^2*
e + 45*a^2*b^4*d*e^2 + 20*a^3*b^3*e^3)*x^7 + 1/2*(2*a*b^5*d^3 + 15*a^2*b^4*
d^2*e + 20*a^3*b^3*d*e^2 + 5*a^4*b^2*e^3)*x^6 + 3/5*(5*a^2*b^4*d^3 + 20*a^3
*b^3*d^2*e + 15*a^4*b^2*d*e^2 + 2*a^5*b*e^3)*x^5 + 1/4*(20*a^3*b^3*d^3 + 45
*a^4*b^2*d^2*e + 18*a^5*b*d*e^2 + a^6*e^3)*x^4 + (5*a^4*b^2*d^3 + 6*a^5*b*d
^2*e + a^6*d*e^2)*x^3 + 3/2*(2*a^5*b*d^3 + a^6*d^2*e)*x^2
```

Fricas [B] time = 1.62168, size = 776, normalized size = 8.43

$$\frac{1}{10}x^{10}e^3b^6+\frac{1}{3}x^9e^2db^6+\frac{2}{3}x^9e^3b^5a+\frac{3}{8}x^8ed^2b^6+\frac{9}{4}x^8e^2db^5a+\frac{15}{8}x^8e^3b^4a^2+\frac{1}{7}x^7d^3b^6+\frac{18}{7}x^7ed^2b^5a+\frac{45}{7}x^7e^2db^4a^2+\frac{20}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*e^3*b^6 + 1/3*x^9*e^2*d*b^6 + 2/3*x^9*e^3*b^5*a + 3/8*x^8*e*d^2*b
^6 + 9/4*x^8*e^2*d*b^5*a + 15/8*x^8*e^3*b^4*a^2 + 1/7*x^7*d^3*b^6 + 18/7*x^
```

```
7*e*d^2*b^5*a + 45/7*x^7*e^2*d*b^4*a^2 + 20/7*x^7*e^3*b^3*a^3 + x^6*d^3*b^5
*a + 15/2*x^6*e*d^2*b^4*a^2 + 10*x^6*e^2*d*b^3*a^3 + 5/2*x^6*e^3*b^2*a^4 +
3*x^5*d^3*b^4*a^2 + 12*x^5*e*d^2*b^3*a^3 + 9*x^5*e^2*d*b^2*a^4 + 6/5*x^5*e^
3*b*a^5 + 5*x^4*d^3*b^3*a^3 + 45/4*x^4*e*d^2*b^2*a^4 + 9/2*x^4*e^2*d*b*a^5
+ 1/4*x^4*e^3*a^6 + 5*x^3*d^3*b^2*a^4 + 6*x^3*e*d^2*b*a^5 + x^3*e^2*d*a^6 +
 3*x^2*d^3*b*a^5 + 3/2*x^2*e*d^2*a^6 + x*d^3*a^6
```

Sympy [B] time = 0.125044, size = 364, normalized size = 3.96

$$a^6d^3x + \frac{b^6e^3x^{10}}{10} + x^9\left(\frac{2ab^5e^3}{3} + \frac{b^6de^2}{3}\right) + x^8\left(\frac{15a^2b^4e^3}{8} + \frac{9ab^5de^2}{4} + \frac{3b^6d^2e}{8}\right) + x^7\left(\frac{20a^3b^3e^3}{7} + \frac{45a^2b^4de^2}{7} + \frac{18ab^5d^2}{7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] a**6*d**3*x + b**6*e**3*x**10/10 + x**9*(2*a*b**5*e**3/3 + b**6*d*e**2/3) +
 x**8*(15*a**2*b**4*e**3/8 + 9*a*b**5*d*e**2/4 + 3*b**6*d**2*e/8) + x**7*(2
0*a**3*b**3*e**3/7 + 45*a**2*b**4*d*e**2/7 + 18*a*b**5*d**2*e/7 + b**6*d**3
/7) + x**6*(5*a**4*b**2*e**3/2 + 10*a**3*b**3*d*e**2 + 15*a**2*b**4*d**2*e/
2 + a*b**5*d**3) + x**5*(6*a**5*b*e**3/5 + 9*a**4*b**2*d*e**2 + 12*a**3*b**
3*d**2*e + 3*a**2*b**4*d**3) + x**4*(a**6*e**3/4 + 9*a**5*b*d*e**2/2 + 45*a
**4*b**2*d**2*e/4 + 5*a**3*b**3*d**3) + x**3*(a**6*d*e**2 + 6*a**5*b*d**2*e
 + 5*a**4*b**2*d**3) + x**2*(3*a**6*d**2*e/2 + 3*a**5*b*d**3)
```

Giac [B] time = 1.1808, size = 479, normalized size = 5.21

$$\frac{1}{10}b^6x^{10}e^3 + \frac{1}{3}b^6dx^9e^2 + \frac{3}{8}b^6d^2x^8e + \frac{1}{7}b^6d^3x^7 + \frac{2}{3}ab^5x^9e^3 + \frac{9}{4}ab^5dx^8e^2 + \frac{18}{7}ab^5d^2x^7e + ab^5d^3x^6 + \frac{15}{8}a^2b^4x^8e^3 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/10*b^6*x^10*e^3 + 1/3*b^6*d*x^9*e^2 + 3/8*b^6*d^2*x^8*e + 1/7*b^6*d^3*x^7
 + 2/3*a*b^5*x^9*e^3 + 9/4*a*b^5*d*x^8*e^2 + 18/7*a*b^5*d^2*x^7*e + a*b^5*d
^3*x^6 + 15/8*a^2*b^4*x^8*e^3 + 45/7*a^2*b^4*d*x^7*e^2 + 15/2*a^2*b^4*d^2*x
^6*e + 3*a^2*b^4*d^3*x^5 + 20/7*a^3*b^3*x^7*e^3 + 10*a^3*b^3*d*x^6*e^2 + 12
*a^3*b^3*d^2*x^5*e + 5*a^3*b^3*d^3*x^4 + 5/2*a^4*b^2*x^6*e^3 + 9*a^4*b^2*d*
x^5*e^2 + 45/4*a^4*b^2*d^2*x^4*e + 5*a^4*b^2*d^3*x^3 + 6/5*a^5*b*x^5*e^3 +
9/2*a^5*b*d*x^4*e^2 + 6*a^5*b*d^2*x^3*e + 3*a^5*b*d^3*x^2 + 1/4*a^6*x^4*e^3
 + a^6*d*x^3*e^2 + 3/2*a^6*d^2*x^2*e + a^6*d^3*x
```

3.1487 $\int (d+ex)^2 \left(a^2+2abx+b^2x^2\right)^3 \, dx$

Optimal. Leaf size=65

$$\frac{e(a+bx)^8(bd-ae)}{4b^3}+\frac{(a+bx)^7(bd-ae)^2}{7b^3}+\frac{e^2(a+bx)^9}{9b^3}$$

```
[Out] ((b*d - a*e)^2*(a + b*x)^7)/(7*b^3) + (e*(b*d - a*e)*(a + b*x)^8)/(4*b^3) +
 (e^2*(a + b*x)^9)/(9*b^3)
```

Rubi [A] time = 0.135757, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$\frac{e(a+bx)^8(bd-ae)}{4b^3}+\frac{(a+bx)^7(bd-ae)^2}{7b^3}+\frac{e^2(a+bx)^9}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] ((b*d - a*e)^2*(a + b*x)^7)/(7*b^3) + (e*(b*d - a*e)*(a + b*x)^8)/(4*b^3) +
 (e^2*(a + b*x)^9)/(9*b^3)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^2 \left(a^2+2abx+b^2x^2\right)^3 \, dx &= \int (a+bx)^6(d+ex)^2 \, dx\\
&= \int \left(\frac{(bd-ae)^2(a+bx)^6}{b^2}+\frac{2e(bd-ae)(a+bx)^7}{b^2}+\frac{e^2(a+bx)^8}{b^2}\right) dx\\
&= \frac{(bd-ae)^2(a+bx)^7}{7b^3}+\frac{e(bd-ae)(a+bx)^8}{4b^3}+\frac{e^2(a+bx)^9}{9b^3}
\end{aligned}$$

Mathematica [B] time = 0.067054, size = 199, normalized size = 3.06

$$\frac{1}{252}x\left(126a^4b^2x^2\left(10d^2+15dex+6e^2x^2\right)+84a^3b^3x^3\left(15d^2+24dex+10e^2x^2\right)+36a^2b^4x^4\left(21d^2+35dex+15e^2x^2\right)+126\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] (x*(84*a^6*(3*d^2 + 3*d*e*x + e^2*x^2) + 126*a^5*b*x*(6*d^2 + 8*d*e*x + 3*e
^2*x^2) + 126*a^4*b^2*x^2*(10*d^2 + 15*d*e*x + 6*e^2*x^2) + 84*a^3*b^3*x^3*
(15*d^2 + 24*d*e*x + 10*e^2*x^2) + 36*a^2*b^4*x^4*(21*d^2 + 35*d*e*x + 15*e
^2*x^2) + 9*a*b^5*x^5*(28*d^2 + 48*d*e*x + 21*e^2*x^2) + b^6*x^6*(36*d^2 +
63*d*e*x + 28*e^2*x^2)))/252
```

Maple [B] time = 0.041, size = 239, normalized size = 3.7

$$\frac{e^2b^6x^9}{9}+\frac{\left(6\,e^2ab^5+2\,deb^6\right)x^8}{8}+\frac{\left(15\,e^2a^2b^4+12\,deab^5+b^6d^2\right)x^7}{7}+\frac{\left(20\,e^2a^3b^3+30\,dea^2b^4+6\,d^2ab^5\right)x^6}{6}+\frac{\left(15\,e^2a^4b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] 1/9*e^2*b^6*x^9+1/8*(6*a*b^5*e^2+2*b^6*d*e)*x^8+1/7*(15*a^2*b^4*e^2+12*a*b^
5*d*e+b^6*d^2)*x^7+1/6*(20*a^3*b^3*e^2+30*a^2*b^4*d*e+6*a*b^5*d^2)*x^6+1/5*
(15*a^4*b^2*e^2+40*a^3*b^3*d*e+15*a^2*b^4*d^2)*x^5+1/4*(6*a^5*b*e^2+30*a^4*
b^2*d*e+20*a^3*b^3*d^2)*x^4+1/3*(a^6*e^2+12*a^5*b*d*e+15*a^4*b^2*d^2)*x^3+1
/2*(2*a^6*d*e+6*a^5*b*d^2)*x^2+d^2*a^6*x
```

Maxima [B] time = 1.09423, size = 316, normalized size = 4.86

$$\frac{1}{9}\,b^6e^2x^9+a^6d^2x+\frac{1}{4}\left(b^6de+3\,ab^5e^2\right)x^8+\frac{1}{7}\left(b^6d^2+12\,ab^5de+15\,a^2b^4e^2\right)x^7+\frac{1}{3}\left(3\,ab^5d^2+15\,a^2b^4de+10\,a^3b^3e^2\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/9*b^6*e^2*x^9 + a^6*d^2*x + 1/4*(b^6*d*e + 3*a*b^5*e^2)*x^8 + 1/7*(b^6*d^
2 + 12*a*b^5*d*e + 15*a^2*b^4*e^2)*x^7 + 1/3*(3*a*b^5*d^2 + 15*a^2*b^4*d*e
+ 10*a^3*b^3*e^2)*x^6 + (3*a^2*b^4*d^2 + 8*a^3*b^3*d*e + 3*a^4*b^2*e^2)*x^5
 + 1/2*(10*a^3*b^3*d^2 + 15*a^4*b^2*d*e + 3*a^5*b*e^2)*x^4 + 1/3*(15*a^4*b^
2*d^2 + 12*a^5*b*d*e + a^6*e^2)*x^3 + (3*a^5*b*d^2 + a^6*d*e)*x^2
```

Fricas [B] time = 1.55068, size = 539, normalized size = 8.29

$$\frac{1}{9}x^9e^2b^6+\frac{1}{4}x^8edb^6+\frac{3}{4}x^8e^2b^5a+\frac{1}{7}x^7d^2b^6+\frac{12}{7}x^7edb^5a+\frac{15}{7}x^7e^2b^4a^2+x^6d^2b^5a+5x^6edb^4a^2+\frac{10}{3}x^6e^2b^3a^3+3x^5d^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*e^2*b^6 + 1/4*x^8*e*d*b^6 + 3/4*x^8*e^2*b^5*a + 1/7*x^7*d^2*b^6 + 1
2/7*x^7*e*d*b^5*a + 15/7*x^7*e^2*b^4*a^2 + x^6*d^2*b^5*a + 5*x^6*e*d*b^4*a^
2 + 10/3*x^6*e^2*b^3*a^3 + 3*x^5*d^2*b^4*a^2 + 8*x^5*e*d*b^3*a^3 + 3*x^5*e^
2*b^2*a^4 + 5*x^4*d^2*b^3*a^3 + 15/2*x^4*e*d*b^2*a^4 + 3/2*x^4*e^2*b*a^5 +
5*x^3*d^2*b^2*a^4 + 4*x^3*e*d*b*a^5 + 1/3*x^3*e^2*a^6 + 3*x^2*d^2*b*a^5 + x
^2*e*d*a^6 + x*d^2*a^6
```

Sympy [B] time = 0.121035, size = 252, normalized size = 3.88

$$a^6d^2x + \frac{b^6e^2x^9}{9} + x^8\left(\frac{3ab^5e^2}{4} + \frac{b^6de}{4}\right) + x^7\left(\frac{15a^2b^4e^2}{7} + \frac{12ab^5de}{7} + \frac{b^6d^2}{7}\right) + x^6\left(\frac{10a^3b^3e^2}{3} + 5a^2b^4de + ab^5d^2\right) + x^5\left(3a^4b^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] a**6*d**2*x + b**6*e**2*x**9/9 + x**8*(3*a*b**5*e**2/4 + b**6*d*e/4) + x**7
*(15*a**2*b**4*e**2/7 + 12*a*b**5*d*e/7 + b**6*d**2/7) + x**6*(10*a**3*b**3
*e**2/3 + 5*a**2*b**4*d*e + a*b**5*d**2) + x**5*(3*a**4*b**2*e**2 + 8*a**3*
b**3*d*e + 3*a**2*b**4*d**2) + x**4*(3*a**5*b*e**2/2 + 15*a**4*b**2*d*e/2 +
 5*a**3*b**3*d**2) + x**3*(a**6*e**2/3 + 4*a**5*b*d*e + 5*a**4*b**2*d**2) +
 x**2*(a**6*d*e + 3*a**5*b*d**2)
```

Giac [B] time = 1.17611, size = 342, normalized size = 5.26

$$\frac{1}{9}b^6x^9e^2 + \frac{1}{4}b^6dx^8e + \frac{1}{7}b^6d^2x^7 + \frac{3}{4}ab^5x^8e^2 + \frac{12}{7}ab^5dx^7e + ab^5d^2x^6 + \frac{15}{7}a^2b^4x^7e^2 + 5a^2b^4dx^6e + 3a^2b^4d^2x^5 + \frac{10}{3}a^3b^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/9*b^6*x^9*e^2 + 1/4*b^6*d*x^8*e + 1/7*b^6*d^2*x^7 + 3/4*a*b^5*x^8*e^2 + 1
2/7*a*b^5*d*x^7*e + a*b^5*d^2*x^6 + 15/7*a^2*b^4*x^7*e^2 + 5*a^2*b^4*d*x^6*
e + 3*a^2*b^4*d^2*x^5 + 10/3*a^3*b^3*x^6*e^2 + 8*a^3*b^3*d*x^5*e + 5*a^3*b^
3*d^2*x^4 + 3*a^4*b^2*x^5*e^2 + 15/2*a^4*b^2*d*x^4*e + 5*a^4*b^2*d^2*x^3 +
3/2*a^5*b*x^4*e^2 + 4*a^5*b*d*x^3*e + 3*a^5*b*d^2*x^2 + 1/3*a^6*x^3*e^2 + a
^6*d*x^2*e + a^6*d^2*x
```

3.1488 $\int (d+ex) \left(a^2+2abx+b^2x^2\right)^3 \, dx$

Optimal. Leaf size=38

$$\frac{(a+bx)^7(bd-ae)}{7b^2}+\frac{e(a+bx)^8}{8b^2}$$

```
[Out] ((b*d - a*e)*(a + b*x)^7)/(7*b^2) + (e*(a + b*x)^8)/(8*b^2)
```

Rubi [A] time = 0.0163301, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {27, 43}

$$\frac{(a+bx)^7(bd-ae)}{7b^2}+\frac{e(a+bx)^8}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] ((b*d - a*e)*(a + b*x)^7)/(7*b^2) + (e*(a + b*x)^8)/(8*b^2)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(a^2+2abx+b^2x^2\right)^3\,dx &= \int (a+bx)^6(d+ex)\,dx\\ &= \int \left(\frac{(bd-ae)(a+bx)^6}{b}+\frac{e(a+bx)^7}{b}\right)dx\\ &= \frac{(bd-ae)(a+bx)^7}{7b^2}+\frac{e(a+bx)^8}{8b^2}\end{aligned}$$

Mathematica [B] time = 0.0357605, size = 122, normalized size = 3.21

$$\frac{1}{56}x\left(70a^4b^2x^2(4d+3ex)+56a^3b^3x^3(5d+4ex)+28a^2b^4x^4(6d+5ex)+56a^5bx(3d+2ex)+28a^6(2d+ex)+8ab^5x^5(7\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] (x*(28*a^6*(2*d + e*x) + 56*a^5*b*x*(3*d + 2*e*x) + 70*a^4*b^2*x^2*(4*d + 3
*e*x) + 56*a^3*b^3*x^3*(5*d + 4*e*x) + 28*a^2*b^4*x^4*(6*d + 5*e*x) + 8*a*b
^5*x^5*(7*d + 6*e*x) + b^6*x^6*(8*d + 7*e*x)))/56
```

Maple [B] time = 0.04, size = 145, normalized size = 3.8

$$\frac{eb^6x^8}{8}+\frac{(6\,eab^5+db^6)\,x^7}{7}+\frac{(15\,ea^2b^4+6\,dab^5)\,x^6}{6}+\frac{(20\,ea^3b^3+15\,da^2b^4)\,x^5}{5}+\frac{(15\,ea^4b^2+20\,da^3b^3)\,x^4}{4}+\frac{(6\,ea^5b+15}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] 1/8*e*b^6*x^8+1/7*(6*a*b^5*e+b^6*d)*x^7+1/6*(15*a^2*b^4*e+6*a*b^5*d)*x^6+1/
5*(20*a^3*b^3*e+15*a^2*b^4*d)*x^5+1/4*(15*a^4*b^2*e+20*a^3*b^3*d)*x^4+1/3*(
6*a^5*b*e+15*a^4*b^2*d)*x^3+1/2*(a^6*e+6*a^5*b*d)*x^2+d*a^6*x
```

Maxima [B] time = 1.14097, size = 192, normalized size = 5.05

$$\frac{1}{8}\,b^6ex^8+a^6dx+\frac{1}{7}\left(b^6d+6\,ab^5e\right)x^7+\frac{1}{2}\left(2\,ab^5d+5\,a^2b^4e\right)x^6+\left(3\,a^2b^4d+4\,a^3b^3e\right)x^5+\frac{5}{4}\left(4\,a^3b^3d+3\,a^4b^2e\right)x^4+\left(5\,a^4b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*b^6*e*x^8 + a^6*d*x + 1/7*(b^6*d + 6*a*b^5*e)*x^7 + 1/2*(2*a*b^5*d + 5*
a^2*b^4*e)*x^6 + (3*a^2*b^4*d + 4*a^3*b^3*e)*x^5 + 5/4*(4*a^3*b^3*d + 3*a^4
*b^2*e)*x^4 + (5*a^4*b^2*d + 2*a^5*b*e)*x^3 + 1/2*(6*a^5*b*d + a^6*e)*x^2
```

Fricas [B] time = 1.48967, size = 316, normalized size = 8.32

$$\frac{1}{8}x^8eb^6+\frac{1}{7}x^7db^6+\frac{6}{7}x^7eb^5a+x^6db^5a+\frac{5}{2}x^6eb^4a^2+3x^5db^4a^2+4x^5eb^3a^3+5x^4db^3a^3+\frac{15}{4}x^4eb^2a^4+5x^3db^2a^4+2x^3eba^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*e*b^6 + 1/7*x^7*d*b^6 + 6/7*x^7*e*b^5*a + x^6*d*b^5*a + 5/2*x^6*e*b
^4*a^2 + 3*x^5*d*b^4*a^2 + 4*x^5*e*b^3*a^3 + 5*x^4*d*b^3*a^3 + 15/4*x^4*e*b
^2*a^4 + 5*x^3*d*b^2*a^4 + 2*x^3*e*b*a^5 + 3*x^2*d*b*a^5 + 1/2*x^2*e*a^6 +
x*d*a^6
```

Sympy [B] time = 0.096173, size = 148, normalized size = 3.89

$$a^6dx+\frac{b^6ex^8}{8}+x^7\left(\frac{6ab^5e}{7}+\frac{b^6d}{7}\right)+x^6\left(\frac{5a^2b^4e}{2}+ab^5d\right)+x^5\left(4a^3b^3e+3a^2b^4d\right)+x^4\left(\frac{15a^4b^2e}{4}+5a^3b^3d\right)+x^3\left(2a^5be+5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] a**6*d*x + b**6*e*x**8/8 + x**7*(6*a*b**5*e/7 + b**6*d/7) + x**6*(5*a**2*b*
*4*e/2 + a*b**5*d) + x**5*(4*a**3*b**3*e + 3*a**2*b**4*d) + x**4*(15*a**4*b
**2*e/4 + 5*a**3*b**3*d) + x**3*(2*a**5*b*e + 5*a**4*b**2*d) + x**2*(a**6*e
/2 + 3*a**5*b*d)
```

Giac [B] time = 1.14366, size = 205, normalized size = 5.39

$$\frac{1}{8}\,b^6x^8e + \frac{1}{7}\,b^6dx^7 + \frac{6}{7}\,ab^5x^7e + ab^5dx^6 + \frac{5}{2}\,a^2b^4x^6e + 3\,a^2b^4dx^5 + 4\,a^3b^3x^5e + 5\,a^3b^3dx^4 + \frac{15}{4}\,a^4b^2x^4e + 5\,a^4b^2dx^3 + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*b^6*x^8*e + 1/7*b^6*d*x^7 + 6/7*a*b^5*x^7*e + a*b^5*d*x^6 + 5/2*a^2*b^4
*x^6*e + 3*a^2*b^4*d*x^5 + 4*a^3*b^3*x^5*e + 5*a^3*b^3*d*x^4 + 15/4*a^4*b^2
*x^4*e + 5*a^4*b^2*d*x^3 + 2*a^5*b*x^3*e + 3*a^5*b*d*x^2 + 1/2*a^6*x^2*e +
a^6*d*x
```

3.1489 $\int \left(a^2+2abx+b^2x^2\right)^3\,dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^7}{7b}$$

```
[Out] (a + b*x)^7/(7*b)
```

Rubi [A] time = 0.0021366, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {27, 32}

$$\frac{(a+bx)^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] (a + b*x)^7/(7*b)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(a^2+2abx+b^2x^2\right)^3\,dx &= \int (a+bx)^6\,dx\\ &= \frac{(a+bx)^7}{7b}\end{aligned}$$

Mathematica [A] time = 0.0010708, size = 14, normalized size = 1.

$$\frac{(a+bx)^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] (a + b*x)^7/(7*b)
```

Maple [B] time = 0.04, size = 65, normalized size = 4.6

$$\frac{b^6x^7}{7} + ab^5x^6 + 3\,a^2b^4x^5 + 5\,a^3b^3x^4 + 5\,a^4b^2x^3 + 3\,a^5bx^2 + a^6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] 1/7*b^6*x^7+a*b^5*x^6+3*a^2*b^4*x^5+5*a^3*b^3*x^4+5*a^4*b^2*x^3+3*a^5*b*x^2
+a^6*x
```

Maxima [B] time = 1.15505, size = 131, normalized size = 9.36

$$\frac{1}{7}\,b^6x^7 + ab^5x^6 + \frac{12}{5}\,a^2b^4x^5 + 2\,a^3b^3x^4 + a^6x + \left(b^2x^3 + 3\,abx^2\right)a^4 + \frac{1}{5}\left(3\,b^4x^5 + 15\,ab^3x^4 + 20\,a^2b^2x^3\right)a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/7*b^6*x^7 + a*b^5*x^6 + 12/5*a^2*b^4*x^5 + 2*a^3*b^3*x^4 + a^6*x + (b^2*x
^3 + 3*a*b*x^2)*a^4 + 1/5*(3*b^4*x^5 + 15*a*b^3*x^4 + 20*a^2*b^2*x^3)*a^2
```

Fricas [B] time = 1.57417, size = 128, normalized size = 9.14

$$\frac{1}{7}x^7b^6 + x^6b^5a + 3x^5b^4a^2 + 5x^4b^3a^3 + 5x^3b^2a^4 + 3x^2ba^5 + xa^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b^6 + x^6*b^5*a + 3*x^5*b^4*a^2 + 5*x^4*b^3*a^3 + 5*x^3*b^2*a^4 + 3
*x^2*b*a^5 + x*a^6
```

Sympy [B] time = 0.085929, size = 66, normalized size = 4.71

$$a^6x + 3a^5bx^2 + 5a^4b^2x^3 + 5a^3b^3x^4 + 3a^2b^4x^5 + ab^5x^6 + \frac{b^6x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] a**6*x + 3*a**5*b*x**2 + 5*a**4*b**2*x**3 + 5*a**3*b**3*x**4 + 3*a**2*b**4*
x**5 + a*b**5*x**6 + b**6*x**7/7
```

Giac [B] time = 1.14211, size = 86, normalized size = 6.14

$$\frac{1}{7}\,b^6x^7 + ab^5x^6 + 3\,a^2b^4x^5 + 5\,a^3b^3x^4 + 5\,a^4b^2x^3 + 3\,a^5bx^2 + a^6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/7*b^6*x^7 + a*b^5*x^6 + 3*a^2*b^4*x^5 + 5*a^3*b^3*x^4 + 5*a^4*b^2*x^3 + 3
*a^5*b*x^2 + a^6*x
```

3.1490 $\int \frac{(a^2+2abx+b^2x^2)^3}{d+ex}\,dx$

Optimal. Leaf size=146

$$-\frac{bx(bd-ae)^5}{e^6}+\frac{(a+bx)^2(bd-ae)^4}{2e^5}-\frac{(a+bx)^3(bd-ae)^3}{3e^4}+\frac{(a+bx)^4(bd-ae)^2}{4e^3}-\frac{(a+bx)^5(bd-ae)}{5e^2}+\frac{(bd-ae)^6\log}{e^7}$$

```
[Out] -((b*(b*d - a*e)^5*x)/e^6) + ((b*d - a*e)^4*(a + b*x)^2)/(2*e^5) - ((b*d -
a*e)^3*(a + b*x)^3)/(3*e^4) + ((b*d - a*e)^2*(a + b*x)^4)/(4*e^3) - ((b*d -
 a*e)*(a + b*x)^5)/(5*e^2) + (a + b*x)^6/(6*e) + ((b*d - a*e)^6*Log[d + e*x
])/e^7
```

Rubi [A] time = 0.0642648, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$-\frac{bx(bd-ae)^5}{e^6}+\frac{(a+bx)^2(bd-ae)^4}{2e^5}-\frac{(a+bx)^3(bd-ae)^3}{3e^4}+\frac{(a+bx)^4(bd-ae)^2}{4e^3}-\frac{(a+bx)^5(bd-ae)}{5e^2}+\frac{(bd-ae)^6\log}{e^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x),x]
```

```
[Out] -((b*(b*d - a*e)^5*x)/e^6) + ((b*d - a*e)^4*(a + b*x)^2)/(2*e^5) - ((b*d -
a*e)^3*(a + b*x)^3)/(3*e^4) + ((b*d - a*e)^2*(a + b*x)^4)/(4*e^3) - ((b*d -
 a*e)*(a + b*x)^5)/(5*e^2) + (a + b*x)^6/(6*e) + ((b*d - a*e)^6*Log[d + e*x
])/e^7
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx+b^2x^2\right)^3}{d+ex}\,dx &= \int \frac{(a+bx)^6}{d+ex}\,dx \\
&= \int \left(-\frac{b(bd-ae)^5}{e^6}+\frac{b(bd-ae)^4(a+bx)}{e^5}-\frac{b(bd-ae)^3(a+bx)^2}{e^4}+\frac{b(bd-ae)^2(a+bx)^3}{e^3}-\right. \\
&= -\frac{b(bd-ae)^5x}{e^6}+\frac{(bd-ae)^4(a+bx)^2}{2e^5}-\frac{(bd-ae)^3(a+bx)^3}{3e^4}+\frac{(bd-ae)^2(a+bx)^4}{4e^3}-\frac{(bd-}{}
\end{aligned}$$

Mathematica [A] time = 0.0867949, size = 230, normalized size = 1.58

$$\frac{bex\left(75a^2b^3e^2\left(6d^2ex-12d^3-4de^2x^2+3e^3x^3\right)+200a^3b^2e^3\left(6d^2-3dex+2e^2x^2\right)+450a^4be^4(ex-2d)+360a^5e^5+6a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x),x]
```

```
[Out] (b*e*x*(360*a^5*e^5 + 450*a^4*b*e^4*(-2*d + e*x) + 200*a^3*b^2*e^3*(6*d^2 -
 3*d*e*x + 2*e^2*x^2) + 75*a^2*b^3*e^2*(-12*d^3 + 6*d^2*e*x - 4*d*e^2*x^2 +
 3*e^3*x^3) + 6*a*b^4*e*(60*d^4 - 30*d^3*e*x + 20*d^2*e^2*x^2 - 15*d*e^3*x^
3 + 12*e^4*x^4) + b^5*(-60*d^5 + 30*d^4*e*x - 20*d^3*e^2*x^2 + 15*d^2*e^3*x
^3 - 12*d*e^4*x^4 + 10*e^5*x^5)) + 60*(b*d - a*e)^6*Log[d + e*x])/(60*e^7)
```

Maple [B] time = 0.043, size = 412, normalized size = 2.8

$$-6\,\frac{\ln(ex+d)\,ab^5d^5}{e^6}-\frac{3\,b^5x^4ad}{2\,e^2}-5\,\frac{b^4x^3a^2d}{e^2}-20\,\frac{\ln(ex+d)\,a^3b^3d^3}{e^4}+15\,\frac{\ln(ex+d)\,a^2b^4d^4}{e^5}+\frac{\ln(ex+d)\,a^6}{e}+\frac{b^6x^6}{6\,e}+\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d),x)
```

```
[Out] -6/e^6*ln(e*x+d)*a*b^5*d^5-3/2*b^5/e^2*x^4*a*d-5*b^4/e^2*x^3*a^2*d-20/e^4*l
n(e*x+d)*a^3*b^3*d^3+15/e^5*ln(e*x+d)*a^2*b^4*d^4+1/e*ln(e*x+d)*a^6+1/6*b^6
/e*x^6+6/5*b^5/e*x^5*a-b^6/e^6*d^5*x+1/4*b^6/e^3*x^4*d^2+20/3*b^3/e*x^3*a^3
-1/3*b^6/e^4*x^3*d^3+15/2*b^2/e*x^2*a^4+1/2*b^6/e^5*x^2*d^4+6*b/e*a^5*x-1/5
*b^6/e^2*x^5*d+15/4*b^4/e*x^4*a^2+1/e^7*ln(e*x+d)*d^6*b^6-6/e^2*ln(e*x+d)*a
^5*b*d+15/e^3*ln(e*x+d)*d^2*a^4*b^2-15*b^4/e^4*a^2*d^3*x+6*b^5/e^5*a*d^4*x+
2*b^5/e^3*x^3*a*d^2-10*b^3/e^2*x^2*a^3*d+15/2*b^4/e^3*x^2*a^2*d^2-3*b^5/e^4
*x^2*a*d^3-15*b^2/e^2*a^4*d*x+20*b^3/e^3*a^3*d^2*x
```

Maxima [B] time = 1.14834, size = 471, normalized size = 3.23

$$\frac{10\,b^6e^5x^6-12\left(b^6de^4-6\,ab^5e^5\right)x^5+15\left(b^6d^2e^3-6\,ab^5de^4+15\,a^2b^4e^5\right)x^4-20\left(b^6d^3e^2-6\,ab^5d^2e^3+15\,a^2b^4de^4-20\,a^3b^3\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/60*(10*b^6*e^5*x^6 - 12*(b^6*d*e^4 - 6*a*b^5*e^5)*x^5 + 15*(b^6*d^2*e^3 -
 6*a*b^5*d*e^4 + 15*a^2*b^4*e^5)*x^4 - 20*(b^6*d^3*e^2 - 6*a*b^5*d^2*e^3 +
15*a^2*b^4*d*e^4 - 20*a^3*b^3*e^5)*x^3 + 30*(b^6*d^4*e - 6*a*b^5*d^3*e^2 +
15*a^2*b^4*d^2*e^3 - 20*a^3*b^3*d*e^4 + 15*a^4*b^2*e^5)*x^2 - 60*(b^6*d^5 -
 6*a*b^5*d^4*e + 15*a^2*b^4*d^3*e^2 - 20*a^3*b^3*d^2*e^3 + 15*a^4*b^2*d*e^4
 - 6*a^5*b*e^5)*x)/e^6 + (b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20
*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 + a^6*e^6)*log(e*x +
d)/e^7
```

Fricas [B] time = 1.82455, size = 726, normalized size = 4.97

$$\frac{10\,b^6e^6x^6-12\left(b^6de^5-6\,ab^5e^6\right)x^5+15\left(b^6d^2e^4-6\,ab^5de^5+15\,a^2b^4e^6\right)x^4-20\left(b^6d^3e^3-6\,ab^5d^2e^4+15\,a^2b^4de^5-20\,a^3b^3\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^6*e^6*x^6 - 12*(b^6*d*e^5 - 6*a*b^5*e^6)*x^5 + 15*(b^6*d^2*e^4 -
 6*a*b^5*d*e^5 + 15*a^2*b^4*e^6)*x^4 - 20*(b^6*d^3*e^3 - 6*a*b^5*d^2*e^4 +
15*a^2*b^4*d*e^5 - 20*a^3*b^3*e^6)*x^3 + 30*(b^6*d^4*e^2 - 6*a*b^5*d^3*e^3
+ 15*a^2*b^4*d^2*e^4 - 20*a^3*b^3*d*e^5 + 15*a^4*b^2*e^6)*x^2 - 60*(b^6*d^5
*e - 6*a*b^5*d^4*e^2 + 15*a^2*b^4*d^3*e^3 - 20*a^3*b^3*d^2*e^4 + 15*a^4*b^2
*d*e^5 - 6*a^5*b*e^6)*x + 60*(b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2
- 20*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 + a^6*e^6)*log(e*
x + d))/e^7
```

Sympy [B] time = 1.60546, size = 286, normalized size = 1.96

$$\frac{b^6x^6}{6e} + \frac{x^5\left(6ab^5e - b^6d\right)}{5e^2} + \frac{x^4\left(15a^2b^4e^2 - 6ab^5de + b^6d^2\right)}{4e^3} + \frac{x^3\left(20a^3b^3e^3 - 15a^2b^4de^2 + 6ab^5d^2e - b^6d^3\right)}{3e^4} + \frac{x^2\left(15a^4b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d),x)
```

```
[Out] b**6*x**6/(6*e) + x**5*(6*a*b**5*e - b**6*d)/(5*e**2) + x**4*(15*a**2*b**4*
e**2 - 6*a*b**5*d*e + b**6*d**2)/(4*e**3) + x**3*(20*a**3*b**3*e**3 - 15*a*
*2*b**4*d*e**2 + 6*a*b**5*d**2*e - b**6*d**3)/(3*e**4) + x**2*(15*a**4*b**2
*e**4 - 20*a**3*b**3*d*e**3 + 15*a**2*b**4*d**2*e**2 - 6*a*b**5*d**3*e + b*
*6*d**4)/(2*e**5) + x*(6*a**5*b*e**5 - 15*a**4*b**2*d*e**4 + 20*a**3*b**3*d
**2*e**3 - 15*a**2*b**4*d**3*e**2 + 6*a*b**5*d**4*e - b**6*d**5)/e**6 + (a*
e - b*d)**6*log(d + e*x)/e**7
```

Giac [B] time = 1.1816, size = 478, normalized size = 3.27

$$\left(b^6d^6 - 6\,ab^5d^5e + 15\,a^2b^4d^4e^2 - 20\,a^3b^3d^3e^3 + 15\,a^4b^2d^2e^4 - 6\,a^5bde^5 + a^6e^6\right)e^{(-7)}\log\left(|xe + d|\right) + \frac{1}{60}\left(10\,b^6x^6e^5 - 12\,b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d),x, algorithm="giac")
```

```
[Out] (b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 + 15*a^4
*b^2*d^2*e^4 - 6*a^5*b*d*e^5 + a^6*e^6)*e^(-7)*log(abs(x*e + d)) + 1/60*(10
*b^6*x^6*e^5 - 12*b^6*d*x^5*e^4 + 15*b^6*d^2*x^4*e^3 - 20*b^6*d^3*x^3*e^2 +
 30*b^6*d^4*x^2*e - 60*b^6*d^5*x + 72*a*b^5*x^5*e^5 - 90*a*b^5*d*x^4*e^4 +
120*a*b^5*d^2*x^3*e^3 - 180*a*b^5*d^3*x^2*e^2 + 360*a*b^5*d^4*x*e + 225*a^2
*b^4*x^4*e^5 - 300*a^2*b^4*d*x^3*e^4 + 450*a^2*b^4*d^2*x^2*e^3 - 900*a^2*b^
4*d^3*x*e^2 + 400*a^3*b^3*x^3*e^5 - 600*a^3*b^3*d*x^2*e^4 + 1200*a^3*b^3*d^
2*x*e^3 + 450*a^4*b^2*x^2*e^5 - 900*a^4*b^2*d*x*e^4 + 360*a^5*b*x*e^5)*e^(-
6)
```

3.1491 $\int \frac{\left(a^2+2abx+b^2x^2\right)^3}{(d+ex)^2}\,dx$

Optimal. Leaf size=156

$$-\frac{3b^5(d+ex)^4(bd-ae)}{2e^7}+\frac{5b^4(d+ex)^3(bd-ae)^2}{e^7}-\frac{10b^3(d+ex)^2(bd-ae)^3}{e^7}+\frac{15b^2x(bd-ae)^4}{e^6}-\frac{(bd-ae)^6}{e^7(d+ex)}-\frac{6b(bd-ae)}{}$$

```
[Out] (15*b^2*(b*d - a*e)^4*x)/e^6 - (b*d - a*e)^6/(e^7*(d + e*x)) - (10*b^3*(b*d
 - a*e)^3*(d + e*x)^2)/e^7 + (5*b^4*(b*d - a*e)^2*(d + e*x)^3)/e^7 - (3*b^5
*(b*d - a*e)*(d + e*x)^4)/(2*e^7) + (b^6*(d + e*x)^5)/(5*e^7) - (6*b*(b*d -
 a*e)^5*Log[d + e*x])/e^7
```

Rubi [A] time = 0.212552, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$-\frac{3b^5(d+ex)^4(bd-ae)}{2e^7}+\frac{5b^4(d+ex)^3(bd-ae)^2}{e^7}-\frac{10b^3(d+ex)^2(bd-ae)^3}{e^7}+\frac{15b^2x(bd-ae)^4}{e^6}-\frac{(bd-ae)^6}{e^7(d+ex)}-\frac{6b(bd-ae)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^2,x]
```

```
[Out] (15*b^2*(b*d - a*e)^4*x)/e^6 - (b*d - a*e)^6/(e^7*(d + e*x)) - (10*b^3*(b*d
 - a*e)^3*(d + e*x)^2)/e^7 + (5*b^4*(b*d - a*e)^2*(d + e*x)^3)/e^7 - (3*b^5
*(b*d - a*e)*(d + e*x)^4)/(2*e^7) + (b^6*(d + e*x)^5)/(5*e^7) - (6*b*(b*d -
 a*e)^5*Log[d + e*x])/e^7
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx+b^2x^2\right)^3}{(d+ex)^2}\,dx &= \int \frac{(a+bx)^6}{(d+ex)^2}\,dx\\ &= \int \left(\frac{15b^2(bd-ae)^4}{e^6}+\frac{(-bd+ae)^6}{e^6(d+ex)^2}-\frac{6b(bd-ae)^5}{e^6(d+ex)}-\frac{20b^3(bd-ae)^3(d+ex)}{e^6}+\frac{15b^4(bd-ae)^2}{e^6}\right.\\ &= \frac{15b^2(bd-ae)^4x}{e^6}-\frac{(bd-ae)^6}{e^7(d+ex)}-\frac{10b^3(bd-ae)^3(d+ex)^2}{e^7}+\frac{5b^4(bd-ae)^2(d+ex)^3}{e^7}-\frac{3b^5(bd}{}\end{aligned}$$

Mathematica [A] time = 0.103336, size = 302, normalized size = 1.94

$$\frac{50a^2b^4e^2\left(6d^2e^2x^2+9d^3ex-3d^4-2de^3x^3+e^4x^4\right)+100a^3b^3e^3\left(-4d^2ex+2d^3-3de^2x^2+e^3x^3\right)+150a^4b^2e^4\left(-d^2+dex+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^2,x]
```

```
[Out] (60*a^5*b*d*e^5 - 10*a^6*e^6 + 150*a^4*b^2*e^4*(-d^2 + d*e*x + e^2*x^2) + 1
00*a^3*b^3*e^3*(2*d^3 - 4*d^2*e*x - 3*d*e^2*x^2 + e^3*x^3) + 50*a^2*b^4*e^2
*(-3*d^4 + 9*d^3*e*x + 6*d^2*e^2*x^2 - 2*d*e^3*x^3 + e^4*x^4) + 5*a*b^5*e*(
12*d^5 - 48*d^4*e*x - 30*d^3*e^2*x^2 + 10*d^2*e^3*x^3 - 5*d*e^4*x^4 + 3*e^5
*x^5) + b^6*(-10*d^6 + 50*d^5*e*x + 30*d^4*e^2*x^2 - 10*d^3*e^3*x^3 + 5*d^2
*e^4*x^4 - 3*d*e^5*x^5 + 2*e^6*x^6) - 60*b*(b*d - a*e)^5*(d + e*x)*Log[d +
e*x])/(10*e^7*(d + e*x))
```

Maple [B] time = 0.049, size = 440, normalized size = 2.8

$$-4\frac{b^5x^3ad}{e^3}+20\frac{a^3b^3d^3}{e^4(ex+d)}-15\frac{a^2b^4d^4}{e^5(ex+d)}-15\frac{b^4x^2a^2d}{e^3}+6\frac{ab^5d^5}{e^6(ex+d)}-30\frac{b^2\ln(ex+d)a^4d}{e^3}+60\frac{b^3\ln(ex+d)a^3d}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^2,x)
```

```
[Out] -4*b^5/e^3*x^3*a*d+20/e^4/(e*x+d)*a^3*b^3*d^3-15/e^5/(e*x+d)*a^2*b^4*d^4-15
*b^4/e^3*x^2*a^2*d+6/e^6/(e*x+d)*a*b^5*d^5-30*b^2/e^3*ln(e*x+d)*a^4*d+60*b^
3/e^4*ln(e*x+d)*a^3*d^2-60*b^4/e^5*ln(e*x+d)*a^2*d^3+30*b^5/e^6*ln(e*x+d)*a
*d^4+6/e^2/(e*x+d)*d*a^5*b-15/e^3/(e*x+d)*d^2*a^4*b^2+1/5*b^6/e^2*x^5-1/e/(
e*x+d)*a^6+9*b^5/e^4*x^2*a*d^2-40*b^3/e^3*a^3*d*x+45*b^4/e^4*d^2*a^2*x-24*b
^5/e^5*a*d^3*x+5*b^6/e^6*d^4*x+3/2*b^5/e^2*x^4*a-1/2*b^6/e^3*x^4*d+5*b^4/e^
2*x^3*a^2+b^6/e^4*x^3*d^2+10*b^3/e^2*x^2*a^3-2*b^6/e^5*x^2*d^3+15*b^2/e^2*a
^4*x+6*b/e^2*ln(e*x+d)*a^5-6*b^6/e^7*ln(e*x+d)*d^5-1/e^7/(e*x+d)*d^6*b^6
```

Maxima [B] time = 1.20369, size = 482, normalized size = 3.09

$$-\frac{b^6d^6-6\,ab^5d^5e+15\,a^2b^4d^4e^2-20\,a^3b^3d^3e^3+15\,a^4b^2d^2e^4-6\,a^5bde^5+a^6e^6}{e^8x+de^7}+\frac{2\,b^6e^4x^5-5\left(b^6de^3-3\,ab^5e^4\right)x^4+10}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 + 15*a^
4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 + a^6*e^6)/(e^8*x + d*e^7) + 1/10*(2*b^6*e^4*
x^5 - 5*(b^6*d*e^3 - 3*a*b^5*e^4)*x^4 + 10*(b^6*d^2*e^2 - 4*a*b^5*d*e^3 + 5
*a^2*b^4*e^4)*x^3 - 10*(2*b^6*d^3*e - 9*a*b^5*d^2*e^2 + 15*a^2*b^4*d*e^3 -
10*a^3*b^3*e^4)*x^2 + 10*(5*b^6*d^4 - 24*a*b^5*d^3*e + 45*a^2*b^4*d^2*e^2 -
 40*a^3*b^3*d*e^3 + 15*a^4*b^2*e^4)*x)/e^6 - 6*(b^6*d^5 - 5*a*b^5*d^4*e + 1
0*a^2*b^4*d^3*e^2 - 10*a^3*b^3*d^2*e^3 + 5*a^4*b^2*d*e^4 - a^5*b*e^5)*log(e
*x + d)/e^7
```

Fricas [B] time = 1.81108, size = 1018, normalized size = 6.53

$$\frac{2\,b^6e^6x^6-10\,b^6d^6+60\,ab^5d^5e-150\,a^2b^4d^4e^2+200\,a^3b^3d^3e^3-150\,a^4b^2d^2e^4+60\,a^5bde^5-10\,a^6e^6-3\left(b^6de^5-5\,ab^5e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/10*(2*b^6*e^6*x^6 - 10*b^6*d^6 + 60*a*b^5*d^5*e - 150*a^2*b^4*d^4*e^2 + 2
00*a^3*b^3*d^3*e^3 - 150*a^4*b^2*d^2*e^4 + 60*a^5*b*d*e^5 - 10*a^6*e^6 - 3*
(b^6*d*e^5 - 5*a*b^5*e^6)*x^5 + 5*(b^6*d^2*e^4 - 5*a*b^5*d*e^5 + 10*a^2*b^4
*e^6)*x^4 - 10*(b^6*d^3*e^3 - 5*a*b^5*d^2*e^4 + 10*a^2*b^4*d*e^5 - 10*a^3*b
^3*e^6)*x^3 + 30*(b^6*d^4*e^2 - 5*a*b^5*d^3*e^3 + 10*a^2*b^4*d^2*e^4 - 10*a
^3*b^3*d*e^5 + 5*a^4*b^2*e^6)*x^2 + 10*(5*b^6*d^5*e - 24*a*b^5*d^4*e^2 + 45
*a^2*b^4*d^3*e^3 - 40*a^3*b^3*d^2*e^4 + 15*a^4*b^2*d*e^5)*x - 60*(b^6*d^6 -
 5*a*b^5*d^5*e + 10*a^2*b^4*d^4*e^2 - 10*a^3*b^3*d^3*e^3 + 5*a^4*b^2*d^2*e^
4 - a^5*b*d*e^5 + (b^6*d^5*e - 5*a*b^5*d^4*e^2 + 10*a^2*b^4*d^3*e^3 - 10*a^
3*b^3*d^2*e^4 + 5*a^4*b^2*d*e^5 - a^5*b*e^6)*x)*log(e*x + d))/(e^8*x + d*e^
7)
```

Sympy [B] time = 1.83105, size = 303, normalized size = 1.94

$$\frac{b^6x^5}{5e^2} + \frac{6b\,(ae-bd)^5\log{(d+ex)}}{e^7} - \frac{a^6e^6-6a^5bde^5+15a^4b^2d^2e^4-20a^3b^3d^3e^3+15a^2b^4d^4e^2-6ab^5d^5e+b^6d^6}{de^7+e^8x} + \frac{x^4\left(3ab^5 \right.}{2\!\!}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**2,x)
```

```
[Out] b**6*x**5/(5*e**2) + 6*b*(a*e - b*d)**5*log(d + e*x)/e**7 - (a**6*e**6 - 6*
a**5*b*d*e**5 + 15*a**4*b**2*d**2*e**4 - 20*a**3*b**3*d**3*e**3 + 15*a**2*b
**4*d**4*e**2 - 6*a*b**5*d**5*e + b**6*d**6)/(d*e**7 + e**8*x) + x**4*(3*a*
b**5*e - b**6*d)/(2*e**3) + x**3*(5*a**2*b**4*e**2 - 4*a*b**5*d*e + b**6*d*
*2)/e**4 + x**2*(10*a**3*b**3*e**3 - 15*a**2*b**4*d*e**2 + 9*a*b**5*d**2*e
- 2*b**6*d**3)/e**5 + x*(15*a**4*b**2*e**4 - 40*a**3*b**3*d*e**3 + 45*a**2*
b**4*d**2*e**2 - 24*a*b**5*d**3*e + 5*b**6*d**4)/e**6
```

Giac [B] time = 1.18799, size = 579, normalized size = 3.71

$$\frac{1}{10}\left(2\,b^6 - \frac{15\left(b^6de-ab^5e^2\right)e^{(-1)}}{xe+d} + \frac{50\left(b^6d^2e^2-2\,ab^5de^3+a^2b^4e^4\right)e^{(-2)}}{(xe+d)^2} - \frac{100\left(b^6d^3e^3-3\,ab^5d^2e^4+3\,a^2b^4de^5-a^3b^3e^6\right)e}{(xe+d)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/10*(2*b^6 - 15*(b^6*d*e - a*b^5*e^2)*e^(-1)/(x*e + d) + 50*(b^6*d^2*e^2 -
 2*a*b^5*d*e^3 + a^2*b^4*e^4)*e^(-2)/(x*e + d)^2 - 100*(b^6*d^3*e^3 - 3*a*b
^5*d^2*e^4 + 3*a^2*b^4*d*e^5 - a^3*b^3*e^6)*e^(-3)/(x*e + d)^3 + 150*(b^6*d
^4*e^4 - 4*a*b^5*d^3*e^5 + 6*a^2*b^4*d^2*e^6 - 4*a^3*b^3*d*e^7 + a^4*b^2*e^
8)*e^(-4)/(x*e + d)^4)*(x*e + d)^5*e^(-7) + 6*(b^6*d^5 - 5*a*b^5*d^4*e + 10
*a^2*b^4*d^3*e^2 - 10*a^3*b^3*d^2*e^3 + 5*a^4*b^2*d*e^4 - a^5*b*e^5)*e^(-7)
*log(abs(x*e + d)*e^(-1)/(x*e + d)^2) - (b^6*d^6*e^5/(x*e + d) - 6*a*b^5*d^
5*e^6/(x*e + d) + 15*a^2*b^4*d^4*e^7/(x*e + d) - 20*a^3*b^3*d^3*e^8/(x*e +
d) + 15*a^4*b^2*d^2*e^9/(x*e + d) - 6*a^5*b*d*e^10/(x*e + d) + a^6*e^11/(x*
e + d))*e^(-12)
```

3.1492 $\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^3}\,dx$

Optimal. Leaf size=158

$$-\frac{2b^5(d+ex)^3(bd-ae)}{e^7}+\frac{15b^4(d+ex)^2(bd-ae)^2}{2e^7}-\frac{20b^3x(bd-ae)^3}{e^6}+\frac{15b^2(bd-ae)^4\log(d+ex)}{e^7}+\frac{6b(bd-ae)^5}{e^7(d+ex)}-\frac{}{2}$$

```
[Out] (-20*b^3*(b*d - a*e)^3*x)/e^6 - (b*d - a*e)^6/(2*e^7*(d + e*x)^2) + (6*b*(b
*d - a*e)^5)/(e^7*(d + e*x)) + (15*b^4*(b*d - a*e)^2*(d + e*x)^2)/(2*e^7) -
 (2*b^5*(b*d - a*e)*(d + e*x)^3)/e^7 + (b^6*(d + e*x)^4)/(4*e^7) + (15*b^2*
(b*d - a*e)^4*Log[d + e*x])/e^7
```

Rubi [A] time = 0.179208, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$-\frac{2b^5(d+ex)^3(bd-ae)}{e^7}+\frac{15b^4(d+ex)^2(bd-ae)^2}{2e^7}-\frac{20b^3x(bd-ae)^3}{e^6}+\frac{15b^2(bd-ae)^4\log(d+ex)}{e^7}+\frac{6b(bd-ae)^5}{e^7(d+ex)}-\frac{}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^3,x]
```

```
[Out] (-20*b^3*(b*d - a*e)^3*x)/e^6 - (b*d - a*e)^6/(2*e^7*(d + e*x)^2) + (6*b*(b
*d - a*e)^5)/(e^7*(d + e*x)) + (15*b^4*(b*d - a*e)^2*(d + e*x)^2)/(2*e^7) -
 (2*b^5*(b*d - a*e)*(d + e*x)^3)/e^7 + (b^6*(d + e*x)^4)/(4*e^7) + (15*b^2*
(b*d - a*e)^4*Log[d + e*x])/e^7
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^3}\,dx &= \int \frac{(a+bx)^6}{(d+ex)^3}\,dx\\ &= \int \left(-\frac{20b^3(bd-ae)^3}{e^6}+\frac{(-bd+ae)^6}{e^6(d+ex)^3}-\frac{6b(bd-ae)^5}{e^6(d+ex)^2}+\frac{15b^2(bd-ae)^4}{e^6(d+ex)}+\frac{15b^4(bd-ae)^2(d}{e^6}\right.\\ &= -\frac{20b^3(bd-ae)^3x}{e^6}-\frac{(bd-ae)^6}{2e^7(d+ex)^2}+\frac{6b(bd-ae)^5}{e^7(d+ex)}+\frac{15b^4(bd-ae)^2(d+ex)^2}{2e^7}-\frac{2b^5(bd-a}{e}\end{aligned}$$

Mathematica [A] time = 0.108777, size = 303, normalized size = 1.92

$$30a^2b^4e^2\left(-11d^2e^2x^2+2d^3ex+7d^4-4de^3x^3+e^4x^4\right)+40a^3b^3e^3\left(-4d^2ex-5d^3+4de^2x^2+2e^3x^3\right)+30a^4b^2de^4(3d+4$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^3,x]
```

```
[Out] (-2*a^6*e^6 - 12*a^5*b*e^5*(d + 2*e*x) + 30*a^4*b^2*d*e^4*(3*d + 4*e*x) + 4
0*a^3*b^3*e^3*(-5*d^3 - 4*d^2*e*x + 4*d*e^2*x^2 + 2*e^3*x^3) + 30*a^2*b^4*e
^2*(7*d^4 + 2*d^3*e*x - 11*d^2*e^2*x^2 - 4*d*e^3*x^3 + e^4*x^4) + 4*a*b^5*e
*(-27*d^5 + 6*d^4*e*x + 63*d^3*e^2*x^2 + 20*d^2*e^3*x^3 - 5*d*e^4*x^4 + 2*e
^5*x^5) + b^6*(22*d^6 - 16*d^5*e*x - 68*d^4*e^2*x^2 - 20*d^3*e^3*x^3 + 5*d^
2*e^4*x^4 - 2*d*e^5*x^5 + e^6*x^6) + 60*b^2*(b*d - a*e)^4*(d + e*x)^2*Log[d
 + e*x])/(4*e^7*(d + e*x)^2)
```

Maple [B] time = 0.052, size = 464, normalized size = 2.9

$$\frac{b^6x^4}{4\,e^3}-\frac{a^6}{2\,e\,(ex+d)^2}-9\,\frac{b^5x^2ad}{e^4}-45\,\frac{a^2b^4dx}{e^4}-\frac{15\,d^2a^4b^2}{2\,e^3\,(ex+d)^2}+10\,\frac{a^3b^3d^3}{e^4\,(ex+d)^2}-\frac{15\,a^2b^4d^4}{2\,e^5\,(ex+d)^2}+3\,\frac{ab^5d^5}{e^6\,(ex+d)^2}-60\,\frac{b^3\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^3,x)
```

```
[Out] 1/4*b^6/e^3*x^4-1/2/e/(e*x+d)^2*a^6-9*b^5/e^4*x^2*a*d-45*b^4/e^4*a^2*d*x-15
/2/e^3/(e*x+d)^2*d^2*a^4*b^2+10/e^4/(e*x+d)^2*a^3*b^3*d^3-15/2/e^5/(e*x+d)^
2*a^2*b^4*d^4+3/e^6/(e*x+d)^2*a*b^5*d^5-60*b^3/e^4*ln(e*x+d)*a^3*d+90*b^4/e
^5*ln(e*x+d)*d^2*a^2-60*b^5/e^6*ln(e*x+d)*a*d^3+30*b^2/e^3/(e*x+d)*a^4*d-60
*b^3/e^4/(e*x+d)*a^3*d^2+60*b^4/e^5/(e*x+d)*a^2*d^3-30*b^5/e^6/(e*x+d)*a*d^
4+36*b^5/e^5*a*d^2*x+3/e^2/(e*x+d)^2*d*a^5*b+2*b^5/e^3*x^3*a-b^6/e^4*x^3*d+
15/2*b^4/e^3*x^2*a^2+3*b^6/e^5*x^2*d^2+20*b^3/e^3*a^3*x-10*b^6/e^6*d^3*x-1/
2/e^7/(e*x+d)^2*d^6*b^6+15*b^2/e^3*ln(e*x+d)*a^4+15*b^6/e^7*ln(e*x+d)*d^4-6
*b/e^2/(e*x+d)*a^5+6*b^6/e^7/(e*x+d)*d^5
```

Maxima [B] time = 1.16169, size = 491, normalized size = 3.11

$$\frac{11\,b^6d^6-54\,ab^5d^5e+105\,a^2b^4d^4e^2-100\,a^3b^3d^3e^3+45\,a^4b^2d^2e^4-6\,a^5bde^5-a^6e^6+12\left(b^6d^5e-5\,ab^5d^4e^2+10\,a^2b^4d^3e^3\right.}{2\left(e^9x^2+2\,de^8x+d^2e^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(11*b^6*d^6 - 54*a*b^5*d^5*e + 105*a^2*b^4*d^4*e^2 - 100*a^3*b^3*d^3*e^
3 + 45*a^4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 - a^6*e^6 + 12*(b^6*d^5*e - 5*a*b^5*
d^4*e^2 + 10*a^2*b^4*d^3*e^3 - 10*a^3*b^3*d^2*e^4 + 5*a^4*b^2*d*e^5 - a^5*b
*e^6)*x)/(e^9*x^2 + 2*d*e^8*x + d^2*e^7) + 1/4*(b^6*e^3*x^4 - 4*(b^6*d*e^2
- 2*a*b^5*e^3)*x^3 + 6*(2*b^6*d^2*e - 6*a*b^5*d*e^2 + 5*a^2*b^4*e^3)*x^2 -
4*(10*b^6*d^3 - 36*a*b^5*d^2*e + 45*a^2*b^4*d*e^2 - 20*a^3*b^3*e^3)*x)/e^6
+ 15*(b^6*d^4 - 4*a*b^5*d^3*e + 6*a^2*b^4*d^2*e^2 - 4*a^3*b^3*d*e^3 + a^4*b
^2*e^4)*log(e*x + d)/e^7
```

Fricas [B] time = 1.75028, size = 1110, normalized size = 7.03

$$\frac{b^6e^6x^6+22\,b^6d^6-108\,ab^5d^5e+210\,a^2b^4d^4e^2-200\,a^3b^3d^3e^3+90\,a^4b^2d^2e^4-12\,a^5bde^5-2\,a^6e^6-2\left(b^6de^5-4\,ab^5e^6\right)x^5+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^6*e^6*x^6 + 22*b^6*d^6 - 108*a*b^5*d^5*e + 210*a^2*b^4*d^4*e^2 - 200
*a^3*b^3*d^3*e^3 + 90*a^4*b^2*d^2*e^4 - 12*a^5*b*d*e^5 - 2*a^6*e^6 - 2*(b^6
*d*e^5 - 4*a*b^5*e^6)*x^5 + 5*(b^6*d^2*e^4 - 4*a*b^5*d*e^5 + 6*a^2*b^4*e^6)
*x^4 - 20*(b^6*d^3*e^3 - 4*a*b^5*d^2*e^4 + 6*a^2*b^4*d*e^5 - 4*a^3*b^3*e^6)
*x^3 - 2*(34*b^6*d^4*e^2 - 126*a*b^5*d^3*e^3 + 165*a^2*b^4*d^2*e^4 - 80*a^3
*b^3*d*e^5)*x^2 - 4*(4*b^6*d^5*e - 6*a*b^5*d^4*e^2 - 15*a^2*b^4*d^3*e^3 + 4
0*a^3*b^3*d^2*e^4 - 30*a^4*b^2*d*e^5 + 6*a^5*b*e^6)*x + 60*(b^6*d^6 - 4*a*b
^5*d^5*e + 6*a^2*b^4*d^4*e^2 - 4*a^3*b^3*d^3*e^3 + a^4*b^2*d^2*e^4 + (b^6*d
^4*e^2 - 4*a*b^5*d^3*e^3 + 6*a^2*b^4*d^2*e^4 - 4*a^3*b^3*d*e^5 + a^4*b^2*e^
6)*x^2 + 2*(b^6*d^5*e - 4*a*b^5*d^4*e^2 + 6*a^2*b^4*d^3*e^3 - 4*a^3*b^3*d^2
*e^4 + a^4*b^2*d*e^5)*x)*log(e*x + d))/(e^9*x^2 + 2*d*e^8*x + d^2*e^7)
```

Sympy [B] time = 5.16562, size = 335, normalized size = 2.12

$$\frac{b^6x^4}{4e^3} + \frac{15b^2\left(ae-bd\right)^4\log\left(d+ex\right)}{e^7} - \frac{a^6e^6+6a^5bde^5-45a^4b^2d^2e^4+100a^3b^3d^3e^3-105a^2b^4d^4e^2+54ab^5d^5e-11b^6d^6}{2d^2e^7+4d\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**3,x)
```

```
[Out] b**6*x**4/(4*e**3) + 15*b**2*(a*e - b*d)**4*log(d + e*x)/e**7 - (a**6*e**6
+ 6*a**5*b*d*e**5 - 45*a**4*b**2*d**2*e**4 + 100*a**3*b**3*d**3*e**3 - 105*
a**2*b**4*d**4*e**2 + 54*a*b**5*d**5*e - 11*b**6*d**6 + x*(12*a**5*b*e**6 -
 60*a**4*b**2*d*e**5 + 120*a**3*b**3*d**2*e**4 - 120*a**2*b**4*d**3*e**3 +
60*a*b**5*d**4*e**2 - 12*b**6*d**5*e))/(2*d**2*e**7 + 4*d*e**8*x + 2*e**9*x
**2) + x**3*(2*a*b**5*e - b**6*d)/e**4 + x**2*(15*a**2*b**4*e**2 - 18*a*b**
5*d*e + 6*b**6*d**2)/(2*e**5) + x*(20*a**3*b**3*e**3 - 45*a**2*b**4*d*e**2
+ 36*a*b**5*d**2*e - 10*b**6*d**3)/e**6
```

Giac [B] time = 1.14652, size = 460, normalized size = 2.91

$$15\left(b^6d^4-4\,ab^5d^3e+6\,a^2b^4d^2e^2-4\,a^3b^3de^3+a^4b^2e^4\right)e^{(-7)}\log\left(|xe+d|\right)+\frac{1}{4}\left(b^6x^4e^9-4\,b^6dx^3e^8+12\,b^6d^2x^2e^7-40\,b^6\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 15*(b^6*d^4 - 4*a*b^5*d^3*e + 6*a^2*b^4*d^2*e^2 - 4*a^3*b^3*d*e^3 + a^4*b^2
*e^4)*e^(-7)*log(abs(x*e + d)) + 1/4*(b^6*x^4*e^9 - 4*b^6*d*x^3*e^8 + 12*b^
6*d^2*x^2*e^7 - 40*b^6*d^3*x*e^6 + 8*a*b^5*x^3*e^9 - 36*a*b^5*d*x^2*e^8 + 1
44*a*b^5*d^2*x*e^7 + 30*a^2*b^4*x^2*e^9 - 180*a^2*b^4*d*x*e^8 + 80*a^3*b^3*
x*e^9)*e^(-12) + 1/2*(11*b^6*d^6 - 54*a*b^5*d^5*e + 105*a^2*b^4*d^4*e^2 - 1
00*a^3*b^3*d^3*e^3 + 45*a^4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 - a^6*e^6 + 12*(b^6
*d^5*e - 5*a*b^5*d^4*e^2 + 10*a^2*b^4*d^3*e^3 - 10*a^3*b^3*d^2*e^4 + 5*a^4*
b^2*d*e^5 - a^5*b*e^6)*x)*e^(-7)/(x*e + d)^2
```

3.1493 $\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^4} dx$

Optimal. Leaf size=156

$$-\frac{3b^5(d+ex)^2(bd-ae)}{e^7}+\frac{15b^4x(bd-ae)^2}{e^6}-\frac{15b^2(bd-ae)^4}{e^7(d+ex)}-\frac{20b^3(bd-ae)^3\log(d+ex)}{e^7}+\frac{3b(bd-ae)^5}{e^7(d+ex)^2}-\frac{(bd-ae)^6}{3e^7(d+ex)^3}$$

```
[Out] (15*b^4*(b*d - a*e)^2*x)/e^6 - (b*d - a*e)^6/(3*e^7*(d + e*x)^3) + (3*b*(b*
d - a*e)^5)/(e^7*(d + e*x)^2) - (15*b^2*(b*d - a*e)^4)/(e^7*(d + e*x)) - (3
*b^5*(b*d - a*e)*(d + e*x)^2)/e^7 + (b^6*(d + e*x)^3)/(3*e^7) - (20*b^3*(b*
d - a*e)^3*Log[d + e*x])/e^7
```

Rubi [A] time = 0.160576, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$-\frac{3b^5(d+ex)^2(bd-ae)}{e^7}+\frac{15b^4x(bd-ae)^2}{e^6}-\frac{15b^2(bd-ae)^4}{e^7(d+ex)}-\frac{20b^3(bd-ae)^3\log(d+ex)}{e^7}+\frac{3b(bd-ae)^5}{e^7(d+ex)^2}-\frac{(bd-ae)^6}{3e^7(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^4,x]
```

```
[Out] (15*b^4*(b*d - a*e)^2*x)/e^6 - (b*d - a*e)^6/(3*e^7*(d + e*x)^3) + (3*b*(b*
d - a*e)^5)/(e^7*(d + e*x)^2) - (15*b^2*(b*d - a*e)^4)/(e^7*(d + e*x)) - (3
*b^5*(b*d - a*e)*(d + e*x)^2)/e^7 + (b^6*(d + e*x)^3)/(3*e^7) - (20*b^3*(b*
d - a*e)^3*Log[d + e*x])/e^7
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^4}dx &= \int \frac{(a+bx)^6}{(d+ex)^4}dx\\ &= \int \left(\frac{15b^4(bd-ae)^2}{e^6}+\frac{(-bd+ae)^6}{e^6(d+ex)^4}-\frac{6b(bd-ae)^5}{e^6(d+ex)^3}+\frac{15b^2(bd-ae)^4}{e^6(d+ex)^2}-\frac{20b^3(bd-ae)^3}{e^6(d+ex)}-\frac{6b^5(\ldots}{}\right.\\ &= \frac{15b^4(bd-ae)^2x}{e^6}-\frac{(bd-ae)^6}{3e^7(d+ex)^3}+\frac{3b(bd-ae)^5}{e^7(d+ex)^2}-\frac{15b^2(bd-ae)^4}{e^7(d+ex)}-\frac{3b^5(bd-ae)(d+ex)^2}{e^7}+\ldots\end{aligned}$$

Mathematica [A] time = 0.11547, size = 302, normalized size = 1.94

$$15a^2b^4e^2\left(-9d^2e^2x^2-27d^3ex-13d^4+9de^3x^3+3e^4x^4\right)+10a^3b^3de^3\left(11d^2+27dex+18e^2x^2\right)-15a^4b^2e^4\left(d^2+3dex+3e^{\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^4,x]
```

```
[Out] (-(a^6*e^6) - 3*a^5*b*e^5*(d + 3*e*x) - 15*a^4*b^2*e^4*(d^2 + 3*d*e*x + 3*e
^2*x^2) + 10*a^3*b^3*d*e^3*(11*d^2 + 27*d*e*x + 18*e^2*x^2) + 15*a^2*b^4*e^
2*(-13*d^4 - 27*d^3*e*x - 9*d^2*e^2*x^2 + 9*d*e^3*x^3 + 3*e^4*x^4) + 3*a*b^
5*e*(47*d^5 + 81*d^4*e*x - 9*d^3*e^2*x^2 - 63*d^2*e^3*x^3 - 15*d*e^4*x^4 +
3*e^5*x^5) + b^6*(-37*d^6 - 51*d^5*e*x + 39*d^4*e^2*x^2 + 73*d^3*e^3*x^3 +
15*d^2*e^4*x^4 - 3*d*e^5*x^5 + e^6*x^6) - 60*b^3*(b*d - a*e)^3*(d + e*x)^3*
Log[d + e*x])/(3*e^7*(d + e*x)^3)
```

Maple [B] time = 0.051, size = 483, normalized size = 3.1

$$-30\frac{a^3b^3d^2}{e^4(ex+d)^2}-5\frac{a^2b^4d^4}{e^5(ex+d)^3}+60\frac{a^3b^3d}{e^4(ex+d)}-90\frac{a^2b^4d^2}{e^5(ex+d)}+60\frac{ab^5d^3}{e^6(ex+d)}+20\frac{b^3\ln(ex+d)a^3}{e^4}-20\frac{b^6\ln(ex}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^4,x)
```

```
[Out] -30*b^3/e^4/(e*x+d)^2*a^3*d^2-5/e^5/(e*x+d)^3*a^2*b^4*d^4+60*b^3/e^4/(e*x+d
)*a^3*d-90*b^4/e^5/(e*x+d)*d^2*a^2+60*b^5/e^6/(e*x+d)*a*d^3+20*b^3/e^4*ln(e
*x+d)*a^3-20*b^6/e^7*ln(e*x+d)*d^3-15*b^2/e^3/(e*x+d)*a^4-15*b^6/e^7/(e*x+d
)*d^4+3*b^5/e^4*x^2*a-2*b^6/e^5*x^2*d+15*b^4/e^4*a^2*x+10*b^6/e^6*d^2*x-1/3
/e^7/(e*x+d)^3*d^6*b^6-3*b/e^2/(e*x+d)^2*a^5+3*b^6/e^7/(e*x+d)^2*d^5+20/3/e
^4/(e*x+d)^3*d^3*a^3*b^3+2/e^6/(e*x+d)^3*a*b^5*d^5+15*b^2/e^3/(e*x+d)^2*a^4
*d-5/e^3/(e*x+d)^3*d^2*a^4*b^2-24*b^5/e^5*a*d*x+2/e^2/(e*x+d)^3*d*a^5*b-1/3
/e/(e*x+d)^3*a^6+1/3*b^6/e^4*x^3+30*b^4/e^5/(e*x+d)^2*a^2*d^3-15*b^5/e^6/(e
*x+d)^2*a*d^4-60*b^4/e^5*ln(e*x+d)*a^2*d+60*b^5/e^6*ln(e*x+d)*a*d^2
```

Maxima [B] time = 1.05284, size = 505, normalized size = 3.24

$$-\frac{37\,b^6d^6-141\,ab^5d^5e+195\,a^2b^4d^4e^2-110\,a^3b^3d^3e^3+15\,a^4b^2d^2e^4+3\,a^5bde^5+a^6e^6+45\left(b^6d^4e^2-4\,ab^5d^3e^3+6\,a^2b\right.}{3\left(e^{10}x^3+3\,de^9x^2+3\,d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(37*b^6*d^6 - 141*a*b^5*d^5*e + 195*a^2*b^4*d^4*e^2 - 110*a^3*b^3*d^3*
e^3 + 15*a^4*b^2*d^2*e^4 + 3*a^5*b*d*e^5 + a^6*e^6 + 45*(b^6*d^4*e^2 - 4*a*
b^5*d^3*e^3 + 6*a^2*b^4*d^2*e^4 - 4*a^3*b^3*d*e^5 + a^4*b^2*e^6)*x^2 + 9*(9
*b^6*d^5*e - 35*a*b^5*d^4*e^2 + 50*a^2*b^4*d^3*e^3 - 30*a^3*b^3*d^2*e^4 + 5
*a^4*b^2*d*e^5 + a^5*b*e^6)*x)/(e^10*x^3 + 3*d*e^9*x^2 + 3*d^2*e^8*x + d^3*
e^7) + 1/3*(b^6*e^2*x^3 - 3*(2*b^6*d*e - 3*a*b^5*e^2)*x^2 + 3*(10*b^6*d^2 -
 24*a*b^5*d*e + 15*a^2*b^4*e^2)*x)/e^6 - 20*(b^6*d^3 - 3*a*b^5*d^2*e + 3*a^
2*b^4*d*e^2 - a^3*b^3*e^3)*log(e*x + d)/e^7
```

Fricas [B] time = 1.76712, size = 1161, normalized size = 7.44

$$\frac{b^6e^6x^6-37\,b^6d^6+141\,ab^5d^5e-195\,a^2b^4d^4e^2+110\,a^3b^3d^3e^3-15\,a^4b^2d^2e^4-3\,a^5bde^5-a^6e^6-3\left(b^6de^5-3\,ab^5e^6\right)x^5+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(b^6*e^6*x^6 - 37*b^6*d^6 + 141*a*b^5*d^5*e - 195*a^2*b^4*d^4*e^2 + 110
*a^3*b^3*d^3*e^3 - 15*a^4*b^2*d^2*e^4 - 3*a^5*b*d*e^5 - a^6*e^6 - 3*(b^6*d*
e^5 - 3*a*b^5*e^6)*x^5 + 15*(b^6*d^2*e^4 - 3*a*b^5*d*e^5 + 3*a^2*b^4*e^6)*x
^4 + (73*b^6*d^3*e^3 - 189*a*b^5*d^2*e^4 + 135*a^2*b^4*d*e^5)*x^3 + 3*(13*b
^6*d^4*e^2 - 9*a*b^5*d^3*e^3 - 45*a^2*b^4*d^2*e^4 + 60*a^3*b^3*d*e^5 - 15*a
^4*b^2*e^6)*x^2 - 3*(17*b^6*d^5*e - 81*a*b^5*d^4*e^2 + 135*a^2*b^4*d^3*e^3
- 90*a^3*b^3*d^2*e^4 + 15*a^4*b^2*d*e^5 + 3*a^5*b*e^6)*x - 60*(b^6*d^6 - 3*
a*b^5*d^5*e + 3*a^2*b^4*d^4*e^2 - a^3*b^3*d^3*e^3 + (b^6*d^3*e^3 - 3*a*b^5*
d^2*e^4 + 3*a^2*b^4*d*e^5 - a^3*b^3*e^6)*x^3 + 3*(b^6*d^4*e^2 - 3*a*b^5*d^3
*e^3 + 3*a^2*b^4*d^2*e^4 - a^3*b^3*d*e^5)*x^2 + 3*(b^6*d^5*e - 3*a*b^5*d^4*
e^2 + 3*a^2*b^4*d^3*e^3 - a^3*b^3*d^2*e^4)*x)*log(e*x + d))/(e^10*x^3 + 3*d
*e^9*x^2 + 3*d^2*e^8*x + d^3*e^7)
```

Sympy [B] time = 6.69639, size = 364, normalized size = 2.33

$$\frac{b^6x^3}{3e^4} + \frac{20b^3\left(ae-bd\right)^3\log\left(d+ex\right)}{e^7} - \frac{a^6e^6+3a^5bde^5+15a^4b^2d^2e^4-110a^3b^3d^3e^3+195a^2b^4d^4e^2-141ab^5d^5e+37b^6d^6+\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**4,x)
```

```
[Out] b**6*x**3/(3*e**4) + 20*b**3*(a*e - b*d)**3*log(d + e*x)/e**7 - (a**6*e**6
+ 3*a**5*b*d*e**5 + 15*a**4*b**2*d**2*e**4 - 110*a**3*b**3*d**3*e**3 + 195*
a**2*b**4*d**4*e**2 - 141*a*b**5*d**5*e + 37*b**6*d**6 + x**2*(45*a**4*b**2
*e**6 - 180*a**3*b**3*d*e**5 + 270*a**2*b**4*d**2*e**4 - 180*a*b**5*d**3*e*
*3 + 45*b**6*d**4*e**2) + x*(9*a**5*b*e**6 + 45*a**4*b**2*d*e**5 - 270*a**3
*b**3*d**2*e**4 + 450*a**2*b**4*d**3*e**3 - 315*a*b**5*d**4*e**2 + 81*b**6*
d**5*e))/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + x**
2*(3*a*b**5*e - 2*b**6*d)/e**5 + x*(15*a**2*b**4*e**2 - 24*a*b**5*d*e + 10*
b**6*d**2)/e**6
```

Giac [B] time = 1.11391, size = 452, normalized size = 2.9

$$-20\left(b^6d^3-3\,ab^5d^2e+3\,a^2b^4de^2-a^3b^3e^3\right)e^{(-7)}\log\left(|xe+d|\right)+\frac{1}{3}\left(b^6x^3e^8-6\,b^6dx^2e^7+30\,b^6d^2xe^6+9\,ab^5x^2e^8-72\,ab^5dx\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -20*(b^6*d^3 - 3*a*b^5*d^2*e + 3*a^2*b^4*d*e^2 - a^3*b^3*e^3)*e^(-7)*log(ab
s(x*e + d)) + 1/3*(b^6*x^3*e^8 - 6*b^6*d*x^2*e^7 + 30*b^6*d^2*x*e^6 + 9*a*b
^5*x^2*e^8 - 72*a*b^5*d*x*e^7 + 45*a^2*b^4*x*e^8)*e^(-12) - 1/3*(37*b^6*d^6
 - 141*a*b^5*d^5*e + 195*a^2*b^4*d^4*e^2 - 110*a^3*b^3*d^3*e^3 + 15*a^4*b^2
*d^2*e^4 + 3*a^5*b*d*e^5 + a^6*e^6 + 45*(b^6*d^4*e^2 - 4*a*b^5*d^3*e^3 + 6*
a^2*b^4*d^2*e^4 - 4*a^3*b^3*d*e^5 + a^4*b^2*e^6)*x^2 + 9*(9*b^6*d^5*e - 35*
a*b^5*d^4*e^2 + 50*a^2*b^4*d^3*e^3 - 30*a^3*b^3*d^2*e^4 + 5*a^4*b^2*d*e^5 +
 a^5*b*e^6)*x)*e^(-7)/(x*e + d)^3
```

3.1494 $\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^5} dx$

Optimal. Leaf size=155

$$-\frac{b^5x(5bd-6ae)}{e^6}+\frac{20b^3(bd-ae)^3}{e^7(d+ex)}-\frac{15b^2(bd-ae)^4}{2e^7(d+ex)^2}+\frac{15b^4(bd-ae)^2\log(d+ex)}{e^7}+\frac{2b(bd-ae)^5}{e^7(d+ex)^3}-\frac{(bd-ae)^6}{4e^7(d+ex)^4}+\frac{b^6 ...}{2e ...}$$

```
[Out] -((b^5*(5*b*d - 6*a*e)*x)/e^6) + (b^6*x^2)/(2*e^5) - (b*d - a*e)^6/(4*e^7*(
d + e*x)^4) + (2*b*(b*d - a*e)^5)/(e^7*(d + e*x)^3) - (15*b^2*(b*d - a*e)^4
)/(2*e^7*(d + e*x)^2) + (20*b^3*(b*d - a*e)^3)/(e^7*(d + e*x)) + (15*b^4*(b
*d - a*e)^2*Log[d + e*x])/e^7
```

Rubi [A] time = 0.146443, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$-\frac{b^5x(5bd-6ae)}{e^6}+\frac{20b^3(bd-ae)^3}{e^7(d+ex)}-\frac{15b^2(bd-ae)^4}{2e^7(d+ex)^2}+\frac{15b^4(bd-ae)^2\log(d+ex)}{e^7}+\frac{2b(bd-ae)^5}{e^7(d+ex)^3}-\frac{(bd-ae)^6}{4e^7(d+ex)^4}+\frac{b^6 ...}{2e ...}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^5,x]
```

```
[Out] -((b^5*(5*b*d - 6*a*e)*x)/e^6) + (b^6*x^2)/(2*e^5) - (b*d - a*e)^6/(4*e^7*(
d + e*x)^4) + (2*b*(b*d - a*e)^5)/(e^7*(d + e*x)^3) - (15*b^2*(b*d - a*e)^4
)/(2*e^7*(d + e*x)^2) + (20*b^3*(b*d - a*e)^3)/(e^7*(d + e*x)) + (15*b^4*(b
*d - a*e)^2*Log[d + e*x])/e^7
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^5} dx &= \int \frac{(a+bx)^6}{(d+ex)^5} dx \\ &= \int \left(-\frac{b^5(5bd-6ae)}{e^6}+\frac{b^6x}{e^5}+\frac{(-bd+ae)^6}{e^6(d+ex)^5}-\frac{6b(bd-ae)^5}{e^6(d+ex)^4}+\frac{15b^2(bd-ae)^4}{e^6(d+ex)^3}-\frac{20b^3(bd-a ...}{e^6(d+ex ...}\right. \\ &= -\frac{b^5(5bd-6ae)x}{e^6}+\frac{b^6x^2}{2e^5}-\frac{(bd-ae)^6}{4e^7(d+ex)^4}+\frac{2b(bd-ae)^5}{e^7(d+ex)^3}-\frac{15b^2(bd-ae)^4}{2e^7(d+ex)^2}+\frac{20b^3(bd-ae ...}{e^7(d+ex) ...}\end{aligned}$$

Mathematica [A] time = 0.121313, size = 301, normalized size = 1.94

$$-\frac{-5a^2b^4de^2\left(88d^2ex+25d^3+108de^2x^2+48e^3x^3\right)+20a^3b^3e^3\left(4d^2ex+d^3+6de^2x^2+4e^3x^3\right)+5a^4b^2e^4\left(d^2+4dex+ ...\right.}{...}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^5,x]
```

```
[Out] -(a^6*e^6 + 2*a^5*b*e^5*(d + 4*e*x) + 5*a^4*b^2*e^4*(d^2 + 4*d*e*x + 6*e^2*
x^2) + 20*a^3*b^3*e^3*(d^3 + 4*d^2*e*x + 6*d*e^2*x^2 + 4*e^3*x^3) - 5*a^2*b
^4*d*e^2*(25*d^3 + 88*d^2*e*x + 108*d*e^2*x^2 + 48*e^3*x^3) + 2*a*b^5*e*(77
*d^5 + 248*d^4*e*x + 252*d^3*e^2*x^2 + 48*d^2*e^3*x^3 - 48*d*e^4*x^4 - 12*e
^5*x^5) - b^6*(57*d^6 + 168*d^5*e*x + 132*d^4*e^2*x^2 - 32*d^3*e^3*x^3 - 68
*d^2*e^4*x^4 - 12*d*e^5*x^5 + 2*e^6*x^6) - 60*b^4*(b*d - a*e)^2*(d + e*x)^4
*Log[d + e*x])/(4*e^7*(d + e*x)^4)
```

Maple [B] time = 0.052, size = 498, normalized size = 3.2

$$60\frac{a^2b^4d}{e^5(ex+d)} - 60\frac{ab^5d^2}{e^6(ex+d)} + \frac{b^6x^2}{2e^5} - 45\frac{b^4d^2a^2}{e^5(ex+d)^2} + 30\frac{ab^5d^3}{e^6(ex+d)^2} - 30\frac{b^5\ln(ex+d)ad}{e^6} + 30\frac{a^3b^3d}{e^4(ex+d)^2} - 20\frac{}{e^4(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^5,x)
```

```
[Out] 60*b^4/e^5/(e*x+d)*a^2*d-60*b^5/e^6/(e*x+d)*a*d^2+1/2*b^6*x^2/e^5-45*b^4/e^
5/(e*x+d)^2*d^2*a^2+30*b^5/e^6/(e*x+d)^2*a*d^3-30*b^5/e^6*ln(e*x+d)*a*d+30*
b^3/e^4/(e*x+d)^2*a^3*d-20*b^3/e^4/(e*x+d)^3*a^3*d^2+20*b^4/e^5/(e*x+d)^3*a
^2*d^3-10*b^5/e^6/(e*x+d)^3*a*d^4+3/2/e^2/(e*x+d)^4*d*a^5*b-15/4/e^3/(e*x+d
)^4*d^2*a^4*b^2+5/e^4/(e*x+d)^4*d^3*a^3*b^3-15/4/e^5/(e*x+d)^4*d^4*a^2*b^4+
3/2/e^6/(e*x+d)^4*a*b^5*d^5-1/4/e^7/(e*x+d)^4*d^6*b^6-15/2*b^2/e^3/(e*x+d)^
2*a^4-15/2*b^6/e^7/(e*x+d)^2*d^4+6*b^5/e^5*a*x-5*b^6/e^6*x*d-2*b/e^2/(e*x+d
)^3*a^5+2*b^6/e^7/(e*x+d)^3*d^5+15*b^4/e^5*ln(e*x+d)*a^2+15*b^6/e^7*ln(e*x+
d)*d^2-20*b^3/e^4/(e*x+d)*a^3+20*b^6/e^7/(e*x+d)*d^3-1/4/e/(e*x+d)^4*a^6+10
*b^2/e^3/(e*x+d)^3*a^4*d
```

Maxima [B] time = 1.08471, size = 522, normalized size = 3.37

$$\frac{57\,b^6d^6 - 154\,ab^5d^5e + 125\,a^2b^4d^4e^2 - 20\,a^3b^3d^3e^3 - 5\,a^4b^2d^2e^4 - 2\,a^5bde^5 - a^6e^6 + 80\left(b^6d^3e^3 - 3\,ab^5d^2e^4 + 3\,a^2b^4de^5 - \right.}{4\left(e^{11}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^5,x, algorithm="maxima")
```

```
[Out] 1/4*(57*b^6*d^6 - 154*a*b^5*d^5*e + 125*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^
3 - 5*a^4*b^2*d^2*e^4 - 2*a^5*b*d*e^5 - a^6*e^6 + 80*(b^6*d^3*e^3 - 3*a*b^5
*d^2*e^4 + 3*a^2*b^4*d*e^5 - a^3*b^3*e^6)*x^3 + 30*(7*b^6*d^4*e^2 - 20*a*b^
5*d^3*e^3 + 18*a^2*b^4*d^2*e^4 - 4*a^3*b^3*d*e^5 - a^4*b^2*e^6)*x^2 + 4*(47
*b^6*d^5*e - 130*a*b^5*d^4*e^2 + 110*a^2*b^4*d^3*e^3 - 20*a^3*b^3*d^2*e^4 -
 5*a^4*b^2*d*e^5 - 2*a^5*b*e^6)*x)/(e^11*x^4 + 4*d*e^10*x^3 + 6*d^2*e^9*x^2
 + 4*d^3*e^8*x + d^4*e^7) + 1/2*(b^6*e*x^2 - 2*(5*b^6*d - 6*a*b^5*e)*x)/e^6
 + 15*(b^6*d^2 - 2*a*b^5*d*e + a^2*b^4*e^2)*log(e*x + d)/e^7
```

Fricas [B] time = 1.83083, size = 1158, normalized size = 7.47

$$\frac{2\,b^6e^6x^6+57\,b^6d^6-154\,ab^5d^5e+125\,a^2b^4d^4e^2-20\,a^3b^3d^3e^3-5\,a^4b^2d^2e^4-2\,a^5bde^5-a^6e^6-12\left(b^6de^5-2\,ab^5e^6\right)x^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^5,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^6*e^6*x^6 + 57*b^6*d^6 - 154*a*b^5*d^5*e + 125*a^2*b^4*d^4*e^2 - 2
0*a^3*b^3*d^3*e^3 - 5*a^4*b^2*d^2*e^4 - 2*a^5*b*d*e^5 - a^6*e^6 - 12*(b^6*d
*e^5 - 2*a*b^5*e^6)*x^5 - 4*(17*b^6*d^2*e^4 - 24*a*b^5*d*e^5)*x^4 - 16*(2*b
^6*d^3*e^3 + 6*a*b^5*d^2*e^4 - 15*a^2*b^4*d*e^5 + 5*a^3*b^3*e^6)*x^3 + 6*(2
2*b^6*d^4*e^2 - 84*a*b^5*d^3*e^3 + 90*a^2*b^4*d^2*e^4 - 20*a^3*b^3*d*e^5 -
5*a^4*b^2*e^6)*x^2 + 4*(42*b^6*d^5*e - 124*a*b^5*d^4*e^2 + 110*a^2*b^4*d^3*
e^3 - 20*a^3*b^3*d^2*e^4 - 5*a^4*b^2*d*e^5 - 2*a^5*b*e^6)*x + 60*(b^6*d^6 -
 2*a*b^5*d^5*e + a^2*b^4*d^4*e^2 + (b^6*d^2*e^4 - 2*a*b^5*d*e^5 + a^2*b^4*e
^6)*x^4 + 4*(b^6*d^3*e^3 - 2*a*b^5*d^2*e^4 + a^2*b^4*d*e^5)*x^3 + 6*(b^6*d^
4*e^2 - 2*a*b^5*d^3*e^3 + a^2*b^4*d^2*e^4)*x^2 + 4*(b^6*d^5*e - 2*a*b^5*d^4
*e^2 + a^2*b^4*d^3*e^3)*x)*log(e*x + d))/(e^11*x^4 + 4*d*e^10*x^3 + 6*d^2*e
^9*x^2 + 4*d^3*e^8*x + d^4*e^7)
```

Sympy [B] time = 13.3998, size = 393, normalized size = 2.54

$$\frac{b^6x^2}{2e^5}+\frac{15b^4\left(ae-bd\right)^2\log\left(d+ex\right)}{e^7}-\frac{a^6e^6+2a^5bde^5+5a^4b^2d^2e^4+20a^3b^3d^3e^3-125a^2b^4d^4e^2+154ab^5d^5e-57b^6d^6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**5,x)
```

```
[Out] b**6*x**2/(2*e**5) + 15*b**4*(a*e - b*d)**2*log(d + e*x)/e**7 - (a**6*e**6
+ 2*a**5*b*d*e**5 + 5*a**4*b**2*d**2*e**4 + 20*a**3*b**3*d**3*e**3 - 125*a*
*2*b**4*d**4*e**2 + 154*a*b**5*d**5*e - 57*b**6*d**6 + x**3*(80*a**3*b**3*e
**6 - 240*a**2*b**4*d*e**5 + 240*a*b**5*d**2*e**4 - 80*b**6*d**3*e**3) + x*
*2*(30*a**4*b**2*e**6 + 120*a**3*b**3*d*e**5 - 540*a**2*b**4*d**2*e**4 + 60
0*a*b**5*d**3*e**3 - 210*b**6*d**4*e**2) + x*(8*a**5*b*e**6 + 20*a**4*b**2*
d*e**5 + 80*a**3*b**3*d**2*e**4 - 440*a**2*b**4*d**3*e**3 + 520*a*b**5*d**4
*e**2 - 188*b**6*d**5*e))/(4*d**4*e**7 + 16*d**3*e**8*x + 24*d**2*e**9*x**2
 + 16*d*e**10*x**3 + 4*e**11*x**4) + x*(6*a*b**5*e - 5*b**6*d)/e**6
```

Giac [B] time = 1.14891, size = 694, normalized size = 4.48

$$\frac{1}{2}\left(b^6-\frac{12\left(b^6de-ab^5e^2\right)e^{(-1)}}{xe+d}\right)(xe+d)^2e^{(-7)}-15\left(b^6d^2-2\,ab^5de+a^2b^4e^2\right)e^{(-7)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^2}\right)+\frac{1}{4}\left(\frac{80\,b^6d^3e^{29}}{xe+d}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] 1/2*(b^6 - 12*(b^6*d*e - a*b^5*e^2)*e^(-1)/(x*e + d))*(x*e + d)^2*e^(-7) -
15*(b^6*d^2 - 2*a*b^5*d*e + a^2*b^4*e^2)*e^(-7)*log(abs(x*e + d)*e^(-1)/(x*
e + d)^2) + 1/4*(80*b^6*d^3*e^29/(x*e + d) - 30*b^6*d^4*e^29/(x*e + d)^2 +
```

```
8*b^6*d^5*e^29/(x*e + d)^3 - b^6*d^6*e^29/(x*e + d)^4 - 240*a*b^5*d^2*e^30/
(x*e + d) + 120*a*b^5*d^3*e^30/(x*e + d)^2 - 40*a*b^5*d^4*e^30/(x*e + d)^3
+ 6*a*b^5*d^5*e^30/(x*e + d)^4 + 240*a^2*b^4*d*e^31/(x*e + d) - 180*a^2*b^4
*d^2*e^31/(x*e + d)^2 + 80*a^2*b^4*d^3*e^31/(x*e + d)^3 - 15*a^2*b^4*d^4*e^
31/(x*e + d)^4 - 80*a^3*b^3*e^32/(x*e + d) + 120*a^3*b^3*d*e^32/(x*e + d)^2
 - 80*a^3*b^3*d^2*e^32/(x*e + d)^3 + 20*a^3*b^3*d^3*e^32/(x*e + d)^4 - 30*a
^4*b^2*e^33/(x*e + d)^2 + 40*a^4*b^2*d*e^33/(x*e + d)^3 - 15*a^4*b^2*d^2*e^
33/(x*e + d)^4 - 8*a^5*b*e^34/(x*e + d)^3 + 6*a^5*b*d*e^34/(x*e + d)^4 - a^
6*e^35/(x*e + d)^4)*e^(-36)
```

3.1495 $\int \frac{\left(a^2+2abx+b^2x^2\right)^3}{(d+ex)^6}\,dx$

Optimal. Leaf size=155

$$-\frac{15b^4(bd-ae)^2}{e^7(d+ex)}+\frac{10b^3(bd-ae)^3}{e^7(d+ex)^2}-\frac{5b^2(bd-ae)^4}{e^7(d+ex)^3}-\frac{6b^5(bd-ae)\log(d+ex)}{e^7}+\frac{3b(bd-ae)^5}{2e^7(d+ex)^4}-\frac{(bd-ae)^6}{5e^7(d+ex)^5}+\frac{b^6x}{e^6}$$

```
[Out] (b^6*x)/e^6 - (b*d - a*e)^6/(5*e^7*(d + e*x)^5) + (3*b*(b*d - a*e)^5)/(2*e^
7*(d + e*x)^4) - (5*b^2*(b*d - a*e)^4)/(e^7*(d + e*x)^3) + (10*b^3*(b*d - a
*e)^3)/(e^7*(d + e*x)^2) - (15*b^4*(b*d - a*e)^2)/(e^7*(d + e*x)) - (6*b^5*
(b*d - a*e)*Log[d + e*x])/e^7
```

Rubi [A] time = 0.140043, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$-\frac{15b^4(bd-ae)^2}{e^7(d+ex)}+\frac{10b^3(bd-ae)^3}{e^7(d+ex)^2}-\frac{5b^2(bd-ae)^4}{e^7(d+ex)^3}-\frac{6b^5(bd-ae)\log(d+ex)}{e^7}+\frac{3b(bd-ae)^5}{2e^7(d+ex)^4}-\frac{(bd-ae)^6}{5e^7(d+ex)^5}+\frac{b^6x}{e^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^6,x]
```

```
[Out] (b^6*x)/e^6 - (b*d - a*e)^6/(5*e^7*(d + e*x)^5) + (3*b*(b*d - a*e)^5)/(2*e^
7*(d + e*x)^4) - (5*b^2*(b*d - a*e)^4)/(e^7*(d + e*x)^3) + (10*b^3*(b*d - a
*e)^3)/(e^7*(d + e*x)^2) - (15*b^4*(b*d - a*e)^2)/(e^7*(d + e*x)) - (6*b^5*
(b*d - a*e)*Log[d + e*x])/e^7
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx+b^2x^2\right)^3}{(d+ex)^6}\,dx &= \int \frac{(a+bx)^6}{(d+ex)^6}\,dx\\ &= \int \left(\frac{b^6}{e^6}+\frac{(-bd+ae)^6}{e^6(d+ex)^6}-\frac{6b(bd-ae)^5}{e^6(d+ex)^5}+\frac{15b^2(bd-ae)^4}{e^6(d+ex)^4}-\frac{20b^3(bd-ae)^3}{e^6(d+ex)^3}+\frac{15b^4(bd-ae)^2}{e^6(d+ex)^2}\right.\\ &= \frac{b^6x}{e^6}-\frac{(bd-ae)^6}{5e^7(d+ex)^5}+\frac{3b(bd-ae)^5}{2e^7(d+ex)^4}-\frac{5b^2(bd-ae)^4}{e^7(d+ex)^3}+\frac{10b^3(bd-ae)^3}{e^7(d+ex)^2}-\frac{15b^4(bd-ae)^2}{e^7(d+ex)}-\end{aligned}$$

Mathematica [A] time = 0.124327, size = 297, normalized size = 1.92

$$-\frac{30a^2b^4e^2\left(10d^2e^2x^2+5d^3ex+d^4+10de^3x^3+5e^4x^4\right)+10a^3b^3e^3\left(5d^2ex+d^3+10de^2x^2+10e^3x^3\right)+5a^4b^2e^4\left(d^2+5d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^6,x]
```

```
[Out] -(2*a^6*e^6 + 3*a^5*b*e^5*(d + 5*e*x) + 5*a^4*b^2*e^4*(d^2 + 5*d*e*x + 10*e
^2*x^2) + 10*a^3*b^3*e^3*(d^3 + 5*d^2*e*x + 10*d*e^2*x^2 + 10*e^3*x^3) + 30
*a^2*b^4*e^2*(d^4 + 5*d^3*e*x + 10*d^2*e^2*x^2 + 10*d*e^3*x^3 + 5*e^4*x^4)
- a*b^5*d*e*(137*d^4 + 625*d^3*e*x + 1100*d^2*e^2*x^2 + 900*d*e^3*x^3 + 300
*e^4*x^4) + b^6*(87*d^6 + 375*d^5*e*x + 600*d^4*e^2*x^2 + 400*d^3*e^3*x^3 +
 50*d^2*e^4*x^4 - 50*d*e^5*x^5 - 10*e^6*x^6) + 60*b^5*(b*d - a*e)*(d + e*x)
^5*Log[d + e*x])/(10*e^7*(d + e*x)^5)
```

Maple [B] time = 0.052, size = 508, normalized size = 3.3

$$\frac{6\,da^5b}{5\,e^2\,(ex+d)^5}-3\,\frac{d^2a^4b^2}{e^3\,(ex+d)^5}+4\,\frac{a^3d^3b^3}{e^4\,(ex+d)^5}-3\,\frac{d^4a^2b^4}{e^5\,(ex+d)^5}+30\,\frac{a^2b^4d}{e^5\,(ex+d)^2}-30\,\frac{ab^5d^2}{e^6\,(ex+d)^2}+20\,\frac{a^3b^3d}{e^4\,(ex+d)^3}-30$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^6,x)
```

```
[Out] 6/5/e^2/(e*x+d)^5*d*a^5*b-3/e^3/(e*x+d)^5*d^2*a^4*b^2+4/e^4/(e*x+d)^5*d^3*a
^3*b^3-3/e^5/(e*x+d)^5*d^4*a^2*b^4+30*b^4/e^5/(e*x+d)^2*a^2*d-30*b^5/e^6/(e
*x+d)^2*a*d^2+20*b^3/e^4/(e*x+d)^3*a^3*d-30*b^4/e^5/(e*x+d)^3*d^2*a^2+20*b^
5/e^6/(e*x+d)^3*a*d^3+15/2*b^2/e^3/(e*x+d)^4*a^4*d-15*b^3/e^4/(e*x+d)^4*a^3
*d^2+15*b^4/e^5/(e*x+d)^4*a^2*d^3-15/2*b^5/e^6/(e*x+d)^4*a*d^4+b^6*x/e^6-5*
b^6/e^7/(e*x+d)^3*d^4-3/2*b/e^2/(e*x+d)^4*a^5+3/2*b^6/e^7/(e*x+d)^4*d^5+6*b
^5/e^6*ln(e*x+d)*a-6*b^6/e^7*ln(e*x+d)*d-10*b^3/e^4/(e*x+d)^2*a^3+10*b^6/e^
7/(e*x+d)^2*d^3-15*b^4/e^5/(e*x+d)*a^2-15*b^6/e^7/(e*x+d)*d^2-1/5/e^7/(e*x+
d)^5*d^6*b^6+6/5/e^6/(e*x+d)^5*d^5*a*b^5+30*b^5/e^6/(e*x+d)*a*d-1/5/e/(e*x+
d)^5*a^6-5*b^2/e^3/(e*x+d)^3*a^4
```

Maxima [B] time = 1.09464, size = 536, normalized size = 3.46

$$\frac{b^6x}{e^6}-\frac{87\,b^6d^6-137\,ab^5d^5e+30\,a^2b^4d^4e^2+10\,a^3b^3d^3e^3+5\,a^4b^2d^2e^4+3\,a^5bde^5+2\,a^6e^6+150\,\left(b^6d^2e^4-2\,ab^5de^5+a^2b^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^6,x, algorithm="maxima")
```

```
[Out] b^6*x/e^6 - 1/10*(87*b^6*d^6 - 137*a*b^5*d^5*e + 30*a^2*b^4*d^4*e^2 + 10*a^
3*b^3*d^3*e^3 + 5*a^4*b^2*d^2*e^4 + 3*a^5*b*d*e^5 + 2*a^6*e^6 + 150*(b^6*d^
2*e^4 - 2*a*b^5*d*e^5 + a^2*b^4*e^6)*x^4 + 100*(5*b^6*d^3*e^3 - 9*a*b^5*d^2
*e^4 + 3*a^2*b^4*d*e^5 + a^3*b^3*e^6)*x^3 + 50*(13*b^6*d^4*e^2 - 22*a*b^5*d
^3*e^3 + 6*a^2*b^4*d^2*e^4 + 2*a^3*b^3*d*e^5 + a^4*b^2*e^6)*x^2 + 5*(77*b^6
*d^5*e - 125*a*b^5*d^4*e^2 + 30*a^2*b^4*d^3*e^3 + 10*a^3*b^3*d^2*e^4 + 5*a^
4*b^2*d*e^5 + 3*a^5*b*e^6)*x)/(e^12*x^5 + 5*d*e^11*x^4 + 10*d^2*e^10*x^3 +
10*d^3*e^9*x^2 + 5*d^4*e^8*x + d^5*e^7) - 6*(b^6*d - a*b^5*e)*log(e*x + d)/
e^7
```

Fricas [B] time = 1.88706, size = 1095, normalized size = 7.06

$$\frac{10\,b^6e^6x^6+50\,b^6de^5x^5-87\,b^6d^6+137\,ab^5d^5e-30\,a^2b^4d^4e^2-10\,a^3b^3d^3e^3-5\,a^4b^2d^2e^4-3\,a^5bde^5-2\,a^6e^6-50\,\left(b^6d^2\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^6,x, algorithm="fricas")
```

```
[Out] 1/10*(10*b^6*e^6*x^6 + 50*b^6*d*e^5*x^5 - 87*b^6*d^6 + 137*a*b^5*d^5*e - 30
*a^2*b^4*d^4*e^2 - 10*a^3*b^3*d^3*e^3 - 5*a^4*b^2*d^2*e^4 - 3*a^5*b*d*e^5 -
 2*a^6*e^6 - 50*(b^6*d^2*e^4 - 6*a*b^5*d*e^5 + 3*a^2*b^4*e^6)*x^4 - 100*(4*
b^6*d^3*e^3 - 9*a*b^5*d^2*e^4 + 3*a^2*b^4*d*e^5 + a^3*b^3*e^6)*x^3 - 50*(12
*b^6*d^4*e^2 - 22*a*b^5*d^3*e^3 + 6*a^2*b^4*d^2*e^4 + 2*a^3*b^3*d*e^5 + a^4
*b^2*e^6)*x^2 - 5*(75*b^6*d^5*e - 125*a*b^5*d^4*e^2 + 30*a^2*b^4*d^3*e^3 +
10*a^3*b^3*d^2*e^4 + 5*a^4*b^2*d*e^5 + 3*a^5*b*e^6)*x - 60*(b^6*d^6 - a*b^5
*d^5*e + (b^6*d*e^5 - a*b^5*e^6)*x^5 + 5*(b^6*d^2*e^4 - a*b^5*d*e^5)*x^4 +
10*(b^6*d^3*e^3 - a*b^5*d^2*e^4)*x^3 + 10*(b^6*d^4*e^2 - a*b^5*d^3*e^3)*x^2
 + 5*(b^6*d^5*e - a*b^5*d^4*e^2)*x)*log(e*x + d))/(e^12*x^5 + 5*d*e^11*x^4
+ 10*d^2*e^10*x^3 + 10*d^3*e^9*x^2 + 5*d^4*e^8*x + d^5*e^7)
```

Sympy [B] time = 36.0141, size = 420, normalized size = 2.71

$$\frac{b^6x}{e^6}+\frac{6b^5\left(ae-bd\right)\log\left(d+ex\right)}{e^7}-\frac{2a^6e^6+3a^5bde^5+5a^4b^2d^2e^4+10a^3b^3d^3e^3+30a^2b^4d^4e^2-137ab^5d^5e+87b^6d^6+x\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**6,x)
```

```
[Out] b**6*x/e**6 + 6*b**5*(a*e - b*d)*log(d + e*x)/e**7 - (2*a**6*e**6 + 3*a**5*
b*d*e**5 + 5*a**4*b**2*d**2*e**4 + 10*a**3*b**3*d**3*e**3 + 30*a**2*b**4*d*
*4*e**2 - 137*a*b**5*d**5*e + 87*b**6*d**6 + x**4*(150*a**2*b**4*e**6 - 300
*a*b**5*d*e**5 + 150*b**6*d**2*e**4) + x**3*(100*a**3*b**3*e**6 + 300*a**2*
b**4*d*e**5 - 900*a*b**5*d**2*e**4 + 500*b**6*d**3*e**3) + x**2*(50*a**4*b*
*2*e**6 + 100*a**3*b**3*d*e**5 + 300*a**2*b**4*d**2*e**4 - 1100*a*b**5*d**3
*e**3 + 650*b**6*d**4*e**2) + x*(15*a**5*b*e**6 + 25*a**4*b**2*d*e**5 + 50*
a**3*b**3*d**2*e**4 + 150*a**2*b**4*d**3*e**3 - 625*a*b**5*d**4*e**2 + 385*
b**6*d**5*e))/(10*d**5*e**7 + 50*d**4*e**8*x + 100*d**3*e**9*x**2 + 100*d**
2*e**10*x**3 + 50*d*e**11*x**4 + 10*e**12*x**5)
```

Giac [B] time = 1.23668, size = 447, normalized size = 2.88

$$b^6xe^{(-6)}-6\left(b^6d-ab^5e\right)e^{(-7)}\log\left(|xe+d|\right)-\frac{\left(87\,b^6d^6-137\,ab^5d^5e+30\,a^2b^4d^4e^2+10\,a^3b^3d^3e^3+5\,a^4b^2d^2e^4+3\,a^5bde\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^6,x, algorithm="giac")
```

```
[Out] b^6*x*e^(-6) - 6*(b^6*d - a*b^5*e)*e^(-7)*log(abs(x*e + d)) - 1/10*(87*b^6*
d^6 - 137*a*b^5*d^5*e + 30*a^2*b^4*d^4*e^2 + 10*a^3*b^3*d^3*e^3 + 5*a^4*b^2
*d^2*e^4 + 3*a^5*b*d*e^5 + 2*a^6*e^6 + 150*(b^6*d^2*e^4 - 2*a*b^5*d*e^5 + a
```

```
^2*b^4*e^6)*x^4 + 100*(5*b^6*d^3*e^3 - 9*a*b^5*d^2*e^4 + 3*a^2*b^4*d*e^5 +
a^3*b^3*e^6)*x^3 + 50*(13*b^6*d^4*e^2 - 22*a*b^5*d^3*e^3 + 6*a^2*b^4*d^2*e^
4 + 2*a^3*b^3*d*e^5 + a^4*b^2*e^6)*x^2 + 5*(77*b^6*d^5*e - 125*a*b^5*d^4*e^
2 + 30*a^2*b^4*d^3*e^3 + 10*a^3*b^3*d^2*e^4 + 5*a^4*b^2*d*e^5 + 3*a^5*b*e^6
)*x)*e^(-7)/(x*e + d)^5
```

3.1496 $\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^7} dx$

Optimal. Leaf size=167

$$\frac{6b^5(bd-ae)}{e^7(d+ex)} - \frac{15b^4(bd-ae)^2}{2e^7(d+ex)^2} + \frac{20b^3(bd-ae)^3}{3e^7(d+ex)^3} - \frac{15b^2(bd-ae)^4}{4e^7(d+ex)^4} + \frac{6b(bd-ae)^5}{5e^7(d+ex)^5} - \frac{(bd-ae)^6}{6e^7(d+ex)^6} + \frac{b^6\log(d+ex)}{e^7}$$

```
[Out] -(b*d - a*e)^6/(6*e^7*(d + e*x)^6) + (6*b*(b*d - a*e)^5)/(5*e^7*(d + e*x)^5
) - (15*b^2*(b*d - a*e)^4)/(4*e^7*(d + e*x)^4) + (20*b^3*(b*d - a*e)^3)/(3*
e^7*(d + e*x)^3) - (15*b^4*(b*d - a*e)^2)/(2*e^7*(d + e*x)^2) + (6*b^5*(b*d
 - a*e))/(e^7*(d + e*x)) + (b^6*Log[d + e*x])/e^7
```

Rubi [A] time = 0.131526, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$\frac{6b^5(bd-ae)}{e^7(d+ex)} - \frac{15b^4(bd-ae)^2}{2e^7(d+ex)^2} + \frac{20b^3(bd-ae)^3}{3e^7(d+ex)^3} - \frac{15b^2(bd-ae)^4}{4e^7(d+ex)^4} + \frac{6b(bd-ae)^5}{5e^7(d+ex)^5} - \frac{(bd-ae)^6}{6e^7(d+ex)^6} + \frac{b^6\log(d+ex)}{e^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^7,x]
```

```
[Out] -(b*d - a*e)^6/(6*e^7*(d + e*x)^6) + (6*b*(b*d - a*e)^5)/(5*e^7*(d + e*x)^5
) - (15*b^2*(b*d - a*e)^4)/(4*e^7*(d + e*x)^4) + (20*b^3*(b*d - a*e)^3)/(3*
e^7*(d + e*x)^3) - (15*b^4*(b*d - a*e)^2)/(2*e^7*(d + e*x)^2) + (6*b^5*(b*d
 - a*e))/(e^7*(d + e*x)) + (b^6*Log[d + e*x])/e^7
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^7} dx &= \int \frac{(a+bx)^6}{(d+ex)^7} dx \\ &= \int \left(\frac{(-bd+ae)^6}{e^6(d+ex)^7} - \frac{6b(bd-ae)^5}{e^6(d+ex)^6} + \frac{15b^2(bd-ae)^4}{e^6(d+ex)^5} - \frac{20b^3(bd-ae)^3}{e^6(d+ex)^4} + \frac{15b^4(bd-ae)^2}{e^6(d+ex)^3} - \right. \\ &= -\frac{(bd-ae)^6}{6e^7(d+ex)^6} + \frac{6b(bd-ae)^5}{5e^7(d+ex)^5} - \frac{15b^2(bd-ae)^4}{4e^7(d+ex)^4} + \frac{20b^3(bd-ae)^3}{3e^7(d+ex)^3} - \frac{15b^4(bd-ae)^2}{2e^7(d+ex)^2} + \end{aligned}$$

Mathematica [A] time = 0.125531, size = 233, normalized size = 1.4

$$\frac{\frac{(bd-ae)\left(a^2b^3e^2\left(282d^2ex+57d^3+525de^2x^2+400e^3x^3\right)+a^3b^2e^3\left(37d^2+162dex+225e^2x^2\right)+2a^4be^4(11d+36ex)+10a^5e^5+ab^4e\left(975d^2e^2x^2+462d^3ex+87d^4+1000de^3\right.}{(d+ex)^6}}{60e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^7,x]
```

```
[Out] (((b*d - a*e)*(10*a^5*e^5 + 2*a^4*b*e^4*(11*d + 36*e*x) + a^3*b^2*e^3*(37*d
^2 + 162*d*e*x + 225*e^2*x^2) + a^2*b^3*e^2*(57*d^3 + 282*d^2*e*x + 525*d*e
^2*x^2 + 400*e^3*x^3) + a*b^4*e*(87*d^4 + 462*d^3*e*x + 975*d^2*e^2*x^2 + 1
000*d*e^3*x^3 + 450*e^4*x^4) + b^5*(147*d^5 + 822*d^4*e*x + 1875*d^3*e^2*x^
2 + 2200*d^2*e^3*x^3 + 1350*d*e^4*x^4 + 360*e^5*x^5)))/(d + e*x)^6 + 60*b^6
*Log[d + e*x])/(60*e^7)
```

Maple [B] time = 0.048, size = 513, normalized size = 3.1

$$\frac{b^6\ln(ex+d)}{e^7} - 12\frac{a^3b^3d^2}{e^4(ex+d)^5} + 12\frac{a^2b^4d^3}{e^5(ex+d)^5} + 15\frac{a^3b^3d}{e^4(ex+d)^4} - \frac{45a^2b^4d^2}{2e^5(ex+d)^4} + 15\frac{ab^5d^3}{e^6(ex+d)^4} + 15\frac{ab^5d}{e^6(ex+d)^2} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^7,x)
```

```
[Out] b^6*ln(e*x+d)/e^7-12*b^3/e^4/(e*x+d)^5*a^3*d^2+12*b^4/e^5/(e*x+d)^5*a^2*d^3
+15*b^3/e^4/(e*x+d)^4*a^3*d-45/2*b^4/e^5/(e*x+d)^4*d^2*a^2+15*b^5/e^6/(e*x+
d)^4*a*d^3+15*b^5/e^6/(e*x+d)^2*a*d+1/e^2/(e*x+d)^6*a^5*b*d-5/2/e^3/(e*x+d)
^6*d^2*a^4*b^2+10/3/e^4/(e*x+d)^6*a^3*b^3*d^3-5/2/e^5/(e*x+d)^6*a^2*b^4*d^4
+1/e^6/(e*x+d)^6*a*b^5*d^5-6*b^5/e^6/(e*x+d)^5*a*d^4-1/6/e^7/(e*x+d)^6*d^6*
b^6-15/4*b^2/e^3/(e*x+d)^4*a^4-15/4*b^6/e^7/(e*x+d)^4*d^4-6/5*b/e^2/(e*x+d)
^5*a^5+6/5*b^6/e^7/(e*x+d)^5*d^5+20*b^4/e^5/(e*x+d)^3*a^2*d-20*b^5/e^6/(e*x
+d)^3*a*d^2-1/6/e/(e*x+d)^6*a^6-15/2*b^4/e^5/(e*x+d)^2*a^2-15/2*b^6/e^7/(e*
x+d)^2*d^2-6*b^5/e^6/(e*x+d)*a+6*b^6/e^7/(e*x+d)*d-20/3*b^3/e^4/(e*x+d)^3*a
^3+20/3*b^6/e^7/(e*x+d)^3*d^3+6*b^2/e^3/(e*x+d)^5*a^4*d
```

Maxima [B] time = 1.27061, size = 562, normalized size = 3.37

$$\frac{147\,b^6d^6 - 60\,ab^5d^5e - 30\,a^2b^4d^4e^2 - 20\,a^3b^3d^3e^3 - 15\,a^4b^2d^2e^4 - 12\,a^5bde^5 - 10\,a^6e^6 + 360\left(b^6de^5 - ab^5e^6\right)x^5 + 450\left(3\,b^6 \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^7,x, algorithm="maxima")
```

```
[Out] 1/60*(147*b^6*d^6 - 60*a*b^5*d^5*e - 30*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^
3 - 15*a^4*b^2*d^2*e^4 - 12*a^5*b*d*e^5 - 10*a^6*e^6 + 360*(b^6*d*e^5 - a*b
^5*e^6)*x^5 + 450*(3*b^6*d^2*e^4 - 2*a*b^5*d*e^5 - a^2*b^4*e^6)*x^4 + 200*(
11*b^6*d^3*e^3 - 6*a*b^5*d^2*e^4 - 3*a^2*b^4*d*e^5 - 2*a^3*b^3*e^6)*x^3 + 7
5*(25*b^6*d^4*e^2 - 12*a*b^5*d^3*e^3 - 6*a^2*b^4*d^2*e^4 - 4*a^3*b^3*d*e^5
- 3*a^4*b^2*e^6)*x^2 + 6*(137*b^6*d^5*e - 60*a*b^5*d^4*e^2 - 30*a^2*b^4*d^3
*e^3 - 20*a^3*b^3*d^2*e^4 - 15*a^4*b^2*d*e^5 - 12*a^5*b*e^6)*x)/(e^13*x^6 +
 6*d*e^12*x^5 + 15*d^2*e^11*x^4 + 20*d^3*e^10*x^3 + 15*d^4*e^9*x^2 + 6*d^5*
e^8*x + d^6*e^7) + b^6*log(e*x + d)/e^7
```

Fricas [B] time = 1.84461, size = 1022, normalized size = 6.12

$$\frac{147\,b^6d^6 - 60\,ab^5d^5e - 30\,a^2b^4d^4e^2 - 20\,a^3b^3d^3e^3 - 15\,a^4b^2d^2e^4 - 12\,a^5bde^5 - 10\,a^6e^6 + 360\,\left(b^6de^5 - ab^5e^6\right)x^5 + 450\,(3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^7,x, algorithm="fricas")
```

```
[Out] 1/60*(147*b^6*d^6 - 60*a*b^5*d^5*e - 30*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^
3 - 15*a^4*b^2*d^2*e^4 - 12*a^5*b*d*e^5 - 10*a^6*e^6 + 360*(b^6*d*e^5 - a*b
^5*e^6)*x^5 + 450*(3*b^6*d^2*e^4 - 2*a*b^5*d*e^5 - a^2*b^4*e^6)*x^4 + 200*(
11*b^6*d^3*e^3 - 6*a*b^5*d^2*e^4 - 3*a^2*b^4*d*e^5 - 2*a^3*b^3*e^6)*x^3 + 7
5*(25*b^6*d^4*e^2 - 12*a*b^5*d^3*e^3 - 6*a^2*b^4*d^2*e^4 - 4*a^3*b^3*d*e^5
- 3*a^4*b^2*e^6)*x^2 + 6*(137*b^6*d^5*e - 60*a*b^5*d^4*e^2 - 30*a^2*b^4*d^3
*e^3 - 20*a^3*b^3*d^2*e^4 - 15*a^4*b^2*d*e^5 - 12*a^5*b*e^6)*x + 60*(b^6*e^
6*x^6 + 6*b^6*d*e^5*x^5 + 15*b^6*d^2*e^4*x^4 + 20*b^6*d^3*e^3*x^3 + 15*b^6*
d^4*e^2*x^2 + 6*b^6*d^5*e*x + b^6*d^6)*log(e*x + d))/(e^13*x^6 + 6*d*e^12*x
^5 + 15*d^2*e^11*x^4 + 20*d^3*e^10*x^3 + 15*d^4*e^9*x^2 + 6*d^5*e^8*x + d^6
*e^7)
```

Sympy [B] time = 96.2197, size = 439, normalized size = 2.63

$$\frac{b^6\log\left(d+ex\right)}{e^7} - \frac{10a^6e^6 + 12a^5bde^5 + 15a^4b^2d^2e^4 + 20a^3b^3d^3e^3 + 30a^2b^4d^4e^2 + 60ab^5d^5e - 147b^6d^6 + x^5\left(360ab^5e^6 -\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**7,x)
```

```
[Out] b**6*log(d + e*x)/e**7 - (10*a**6*e**6 + 12*a**5*b*d*e**5 + 15*a**4*b**2*d*
*2*e**4 + 20*a**3*b**3*d**3*e**3 + 30*a**2*b**4*d**4*e**2 + 60*a*b**5*d**5*
e - 147*b**6*d**6 + x**5*(360*a*b**5*e**6 - 360*b**6*d*e**5) + x**4*(450*a*
*2*b**4*e**6 + 900*a*b**5*d*e**5 - 1350*b**6*d**2*e**4) + x**3*(400*a**3*b*
*3*e**6 + 600*a**2*b**4*d*e**5 + 1200*a*b**5*d**2*e**4 - 2200*b**6*d**3*e**
3) + x**2*(225*a**4*b**2*e**6 + 300*a**3*b**3*d*e**5 + 450*a**2*b**4*d**2*e
**4 + 900*a*b**5*d**3*e**3 - 1875*b**6*d**4*e**2) + x*(72*a**5*b*e**6 + 90*
a**4*b**2*d*e**5 + 120*a**3*b**3*d**2*e**4 + 180*a**2*b**4*d**3*e**3 + 360*
a*b**5*d**4*e**2 - 822*b**6*d**5*e))/(60*d**6*e**7 + 360*d**5*e**8*x + 900*
d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x
**5 + 60*e**13*x**6)
```

Giac [B] time = 1.15095, size = 458, normalized size = 2.74

$$b^6e^{(-7)}\log\left(|xe+d|\right) + \frac{\left(360\left(b^6de^4 - ab^5e^5\right)x^5 + 450\left(3\,b^6d^2e^3 - 2\,ab^5de^4 - a^2b^4e^5\right)x^4 + 200\left(11\,b^6d^3e^2 - 6\,ab^5d^2e^3 - 3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^7,x, algorithm="giac")
```

```
[Out] b^6*e^(-7)*log(abs(x*e + d)) + 1/60*(360*(b^6*d*e^4 - a*b^5*e^5)*x^5 + 450*
(3*b^6*d^2*e^3 - 2*a*b^5*d*e^4 - a^2*b^4*e^5)*x^4 + 200*(11*b^6*d^3*e^2 - 6
```

```
*a*b^5*d^2*e^3 - 3*a^2*b^4*d*e^4 - 2*a^3*b^3*e^5)*x^3 + 75*(25*b^6*d^4*e -
12*a*b^5*d^3*e^2 - 6*a^2*b^4*d^2*e^3 - 4*a^3*b^3*d*e^4 - 3*a^4*b^2*e^5)*x^2
 + 6*(137*b^6*d^5 - 60*a*b^5*d^4*e - 30*a^2*b^4*d^3*e^2 - 20*a^3*b^3*d^2*e^
3 - 15*a^4*b^2*d*e^4 - 12*a^5*b*e^5)*x + (147*b^6*d^6 - 60*a*b^5*d^5*e - 30
*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 - 15*a^4*b^2*d^2*e^4 - 12*a^5*b*d*e^5
 - 10*a^6*e^6)*e^(-1))*e^(-6)/(x*e + d)^6
```

3.1497 $\int \frac{\left(a^2+2abx+b^2x^2\right)^3}{(d+ex)^8} dx$

Optimal. Leaf size=28

$$\frac{(a+bx)^7}{7(d+ex)^7(bd-ae)}$$

```
[Out] (a + b*x)^7/(7*(b*d - a*e)*(d + e*x)^7)
```

Rubi [A] time = 0.0046189, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 37}

$$\frac{(a+bx)^7}{7(d+ex)^7(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^8,x]
```

```
[Out] (a + b*x)^7/(7*(b*d - a*e)*(d + e*x)^7)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx+b^2x^2\right)^3}{(d+ex)^8} dx &= \int \frac{(a+bx)^6}{(d+ex)^8} dx\\ &= \frac{(a+bx)^7}{7(bd-ae)(d+ex)^7}\end{aligned}$$

Mathematica [B] time = 0.0989133, size = 271, normalized size = 9.68

$$-\frac{a^2b^4e^2\left(21d^2e^2x^2+7d^3ex+d^4+35de^3x^3+35e^4x^4\right)+a^3b^3e^3\left(7d^2ex+d^3+21de^2x^2+35e^3x^3\right)+a^4b^2e^4\left(d^2+7dex+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^8,x]
```

```
[Out] -(a^6*e^6 + a^5*b*e^5*(d + 7*e*x) + a^4*b^2*e^4*(d^2 + 7*d*e*x + 21*e^2*x^2
) + a^3*b^3*e^3*(d^3 + 7*d^2*e*x + 21*d*e^2*x^2 + 35*e^3*x^3) + a^2*b^4*e^2
*(d^4 + 7*d^3*e*x + 21*d^2*e^2*x^2 + 35*d*e^3*x^3 + 35*e^4*x^4) + a*b^5*e*(
d^5 + 7*d^4*e*x + 21*d^3*e^2*x^2 + 35*d^2*e^3*x^3 + 35*d*e^4*x^4 + 21*e^5*x
^5) + b^6*(d^6 + 7*d^5*e*x + 21*d^4*e^2*x^2 + 35*d^3*e^3*x^3 + 35*d^2*e^4*x
^4 + 21*d*e^5*x^5 + 7*e^6*x^6))/(7*e^7*(d + e*x)^7)
```

Maple [B] time = 0.048, size = 357, normalized size = 12.8

$$-5\,\frac{b^4\left(a^2e^2-2\,abde+b^2d^2\right)}{e^7\left(ex+d\right)^3}-\frac{b\left(a^5e^5-5\,a^4bde^4+10\,a^3b^2d^2e^3-10\,a^2b^3d^3e^2+5\,ab^4d^4e-b^5d^5\right)}{e^7\left(ex+d\right)^6}-5\,\frac{b^3\left(a^3e^3-3\,a^2bde^2\ \cdots\right.}{e^7\left(ex\ \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^8,x)
```

```
[Out] -5*b^4*(a^2*e^2-2*a*b*d*e+b^2*d^2)/e^7/(e*x+d)^3-b*(a^5*e^5-5*a^4*b*d*e^4+1
0*a^3*b^2*d^2*e^3-10*a^2*b^3*d^3*e^2+5*a*b^4*d^4*e-b^5*d^5)/e^7/(e*x+d)^6-5
*b^3*(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d^2*e-b^3*d^3)/e^7/(e*x+d)^4-3*b^5*(a*e
-b*d)/e^7/(e*x+d)^2-1/7*(a^6*e^6-6*a^5*b*d*e^5+15*a^4*b^2*d^2*e^4-20*a^3*b^
3*d^3*e^3+15*a^2*b^4*d^4*e^2-6*a*b^5*d^5*e+b^6*d^6)/e^7/(e*x+d)^7-b^6/e^7/(
e*x+d)-3*b^2*(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-4*a*b^3*d^3*e+b^4*d^4
)/e^7/(e*x+d)^5
```

Maxima [B] time = 1.2449, size = 537, normalized size = 19.18

$$-\frac{7\,b^6e^6x^6+b^6d^6+ab^5d^5e+a^2b^4d^4e^2+a^3b^3d^3e^3+a^4b^2d^2e^4+a^5bde^5+a^6e^6+21\left(b^6de^5+ab^5e^6\right)x^5+35\left(b^6d^2e^4+ab^5d\ \cdots\right.}{7\left(e^{14}x^7+7\ \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^8,x, algorithm="maxima")
```

```
[Out] -1/7*(7*b^6*e^6*x^6 + b^6*d^6 + a*b^5*d^5*e + a^2*b^4*d^4*e^2 + a^3*b^3*d^3
*e^3 + a^4*b^2*d^2*e^4 + a^5*b*d*e^5 + a^6*e^6 + 21*(b^6*d*e^5 + a*b^5*e^6)
*x^5 + 35*(b^6*d^2*e^4 + a*b^5*d*e^5 + a^2*b^4*e^6)*x^4 + 35*(b^6*d^3*e^3 +
 a*b^5*d^2*e^4 + a^2*b^4*d*e^5 + a^3*b^3*e^6)*x^3 + 21*(b^6*d^4*e^2 + a*b^5
*d^3*e^3 + a^2*b^4*d^2*e^4 + a^3*b^3*d*e^5 + a^4*b^2*e^6)*x^2 + 7*(b^6*d^5*
e + a*b^5*d^4*e^2 + a^2*b^4*d^3*e^3 + a^3*b^3*d^2*e^4 + a^4*b^2*d*e^5 + a^5
*b*e^6)*x)/(e^14*x^7 + 7*d*e^13*x^6 + 21*d^2*e^12*x^5 + 35*d^3*e^11*x^4 + 3
5*d^4*e^10*x^3 + 21*d^5*e^9*x^2 + 7*d^6*e^8*x + d^7*e^7)
```

Fricas [B] time = 1.74631, size = 787, normalized size = 28.11

$$-\frac{7\,b^6e^6x^6+b^6d^6+ab^5d^5e+a^2b^4d^4e^2+a^3b^3d^3e^3+a^4b^2d^2e^4+a^5bde^5+a^6e^6+21\left(b^6de^5+ab^5e^6\right)x^5+35\left(b^6d^2e^4+ab^5d\ \cdots\right.}{7\left(e^{14}x^7+7\ \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^8,x, algorithm="fricas")
```

```
[Out] -1/7*(7*b^6*e^6*x^6 + b^6*d^6 + a*b^5*d^5*e + a^2*b^4*d^4*e^2 + a^3*b^3*d^3
*e^3 + a^4*b^2*d^2*e^4 + a^5*b*d*e^5 + a^6*e^6 + 21*(b^6*d*e^5 + a*b^5*e^6)
*x^5 + 35*(b^6*d^2*e^4 + a*b^5*d*e^5 + a^2*b^4*e^6)*x^4 + 35*(b^6*d^3*e^3 +
 a*b^5*d^2*e^4 + a^2*b^4*d*e^5 + a^3*b^3*e^6)*x^3 + 21*(b^6*d^4*e^2 + a*b^5
*d^3*e^3 + a^2*b^4*d^2*e^4 + a^3*b^3*d*e^5 + a^4*b^2*e^6)*x^2 + 7*(b^6*d^5*
e + a*b^5*d^4*e^2 + a^2*b^4*d^3*e^3 + a^3*b^3*d^2*e^4 + a^4*b^2*d*e^5 + a^5
*b*e^6)*x)/(e^14*x^7 + 7*d*e^13*x^6 + 21*d^2*e^12*x^5 + 35*d^3*e^11*x^4 + 3
5*d^4*e^10*x^3 + 21*d^5*e^9*x^2 + 7*d^6*e^8*x + d^7*e^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.13582, size = 467, normalized size = 16.68

$$-\frac{(7\,b^6x^6e^6 + 21\,b^6dx^5e^5 + 35\,b^6d^2x^4e^4 + 35\,b^6d^3x^3e^3 + 21\,b^6d^4x^2e^2 + 7\,b^6d^5xe + b^6d^6 + 21\,ab^5x^5e^6 + 35\,ab^5dx^4e^5 + 35}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^8,x, algorithm="giac")
```

```
[Out] -1/7*(7*b^6*x^6*e^6 + 21*b^6*d*x^5*e^5 + 35*b^6*d^2*x^4*e^4 + 35*b^6*d^3*x^
3*e^3 + 21*b^6*d^4*x^2*e^2 + 7*b^6*d^5*x*e + b^6*d^6 + 21*a*b^5*x^5*e^6 + 3
5*a*b^5*d*x^4*e^5 + 35*a*b^5*d^2*x^3*e^4 + 21*a*b^5*d^3*x^2*e^3 + 7*a*b^5*d
^4*x*e^2 + a*b^5*d^5*e + 35*a^2*b^4*x^4*e^6 + 35*a^2*b^4*d*x^3*e^5 + 21*a^2
*b^4*d^2*x^2*e^4 + 7*a^2*b^4*d^3*x*e^3 + a^2*b^4*d^4*e^2 + 35*a^3*b^3*x^3*e
^6 + 21*a^3*b^3*d*x^2*e^5 + 7*a^3*b^3*d^2*x*e^4 + a^3*b^3*d^3*e^3 + 21*a^4*
b^2*x^2*e^6 + 7*a^4*b^2*d*x*e^5 + a^4*b^2*d^2*e^4 + 7*a^5*b*x*e^6 + a^5*b*d
*e^5 + a^6*e^6)*e^(-7)/(x*e + d)^7
```

3.1498 $\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^9} dx$

Optimal. Leaf size=58

$$\frac{b(a+bx)^7}{56(d+ex)^7(bd-ae)^2}+\frac{(a+bx)^7}{8(d+ex)^8(bd-ae)}$$

```
[Out] (a + b*x)^7/(8*(b*d - a*e)*(d + e*x)^8) + (b*(a + b*x)^7)/(56*(b*d - a*e)^2
*(d + e*x)^7)
```

Rubi [A] time = 0.011517, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {27, 45, 37}

$$\frac{b(a+bx)^7}{56(d+ex)^7(bd-ae)^2}+\frac{(a+bx)^7}{8(d+ex)^8(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^9,x]
```

```
[Out] (a + b*x)^7/(8*(b*d - a*e)*(d + e*x)^8) + (b*(a + b*x)^7)/(56*(b*d - a*e)^2
*(d + e*x)^7)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^9}\,dx = \int \frac{(a+bx)^6}{(d+ex)^9}\,dx$$
$$= \frac{(a+bx)^7}{8(bd-ae)(d+ex)^8} + \frac{b\int \frac{(a+bx)^6}{(d+ex)^8}\,dx}{8(bd-ae)}$$
$$= \frac{(a+bx)^7}{8(bd-ae)(d+ex)^8} + \frac{b(a+bx)^7}{56(bd-ae)^2(d+ex)^7}$$

Mathematica [B] time = 0.0932927, size = 277, normalized size = 4.78

$$-\frac{3a^2b^4e^2\left(28d^2e^2x^2+8d^3ex+d^4+56de^3x^3+70e^4x^4\right)+4a^3b^3e^3\left(8d^2ex+d^3+28de^2x^2+56e^3x^3\right)+5a^4b^2e^4\left(d^2+8d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^9,x]
```

```
[Out] -(7*a^6*e^6 + 6*a^5*b*e^5*(d + 8*e*x) + 5*a^4*b^2*e^4*(d^2 + 8*d*e*x + 28*e
^2*x^2) + 4*a^3*b^3*e^3*(d^3 + 8*d^2*e*x + 28*d*e^2*x^2 + 56*e^3*x^3) + 3*a
^2*b^4*e^2*(d^4 + 8*d^3*e*x + 28*d^2*e^2*x^2 + 56*d*e^3*x^3 + 70*e^4*x^4) +
 2*a*b^5*e*(d^5 + 8*d^4*e*x + 28*d^3*e^2*x^2 + 56*d^2*e^3*x^3 + 70*d*e^4*x^
4 + 56*e^5*x^5) + b^6*(d^6 + 8*d^5*e*x + 28*d^4*e^2*x^2 + 56*d^3*e^3*x^3 +
70*d^2*e^4*x^4 + 56*d*e^5*x^5 + 28*e^6*x^6))/(56*e^7*(d + e*x)^8)
```

Maple [B] time = 0.047, size = 357, normalized size = 6.2

$$-2\,\frac{b^5(ae-bd)}{e^7(ex+d)^3} - \frac{5\,b^2\left(a^4e^4-4\,a^3bde^3+6\,d^2e^2b^2a^2-4\,ab^3d^3e+b^4d^4\right)}{2\,e^7(ex+d)^6} - \frac{15\,b^4\left(a^2e^2-2\,abde+b^2d^2\right)}{4\,e^7(ex+d)^4} - \frac{b^6}{2\,e^7(ex+d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^9,x)
```

```
[Out] -2*b^5*(a*e-b*d)/e^7/(e*x+d)^3-5/2*b^2*(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2
*e^2-4*a*b^3*d^3*e+b^4*d^4)/e^7/(e*x+d)^6-15/4*b^4*(a^2*e^2-2*a*b*d*e+b^2*d
^2)/e^7/(e*x+d)^4-1/2*b^6/e^7/(e*x+d)^2-6/7*b*(a^5*e^5-5*a^4*b*d*e^4+10*a^3
*b^2*d^2*e^3-10*a^2*b^3*d^3*e^2+5*a*b^4*d^4*e-b^5*d^5)/e^7/(e*x+d)^7-1/8*(a
^6*e^6-6*a^5*b*d*e^5+15*a^4*b^2*d^2*e^4-20*a^3*b^3*d^3*e^3+15*a^2*b^4*d^4*e
^2-6*a*b^5*d^5*e+b^6*d^6)/e^7/(e*x+d)^8-4*b^3*(a^3*e^3-3*a^2*b*d*e^2+3*a*b^
2*d^2*e-b^3*d^3)/e^7/(e*x+d)^5
```

Maxima [B] time = 1.2714, size = 581, normalized size = 10.02

$$-\frac{28\,b^6e^6x^6+b^6d^6+2\,ab^5d^5e+3\,a^2b^4d^4e^2+4\,a^3b^3d^3e^3+5\,a^4b^2d^2e^4+6\,a^5bde^5+7\,a^6e^6+56\left(b^6de^5+2\,ab^5e^6\right)x^5+7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^9,x, algorithm="maxima")
```

```
[Out] -1/56*(28*b^6*e^6*x^6 + b^6*d^6 + 2*a*b^5*d^5*e + 3*a^2*b^4*d^4*e^2 + 4*a^3
*b^3*d^3*e^3 + 5*a^4*b^2*d^2*e^4 + 6*a^5*b*d*e^5 + 7*a^6*e^6 + 56*(b^6*d*e^
5 + 2*a*b^5*e^6)*x^5 + 70*(b^6*d^2*e^4 + 2*a*b^5*d*e^5 + 3*a^2*b^4*e^6)*x^4
 + 56*(b^6*d^3*e^3 + 2*a*b^5*d^2*e^4 + 3*a^2*b^4*d*e^5 + 4*a^3*b^3*e^6)*x^3
 + 28*(b^6*d^4*e^2 + 2*a*b^5*d^3*e^3 + 3*a^2*b^4*d^2*e^4 + 4*a^3*b^3*d*e^5
+ 5*a^4*b^2*e^6)*x^2 + 8*(b^6*d^5*e + 2*a*b^5*d^4*e^2 + 3*a^2*b^4*d^3*e^3 +
 4*a^3*b^3*d^2*e^4 + 5*a^4*b^2*d*e^5 + 6*a^5*b*e^6)*x)/(e^15*x^8 + 8*d*e^14
*x^7 + 28*d^2*e^13*x^6 + 56*d^3*e^12*x^5 + 70*d^4*e^11*x^4 + 56*d^5*e^10*x^
3 + 28*d^6*e^9*x^2 + 8*d^7*e^8*x + d^8*e^7)
```

Fricas [B] time = 1.57478, size = 871, normalized size = 15.02

$$-\frac{28\,b^6e^6x^6 + b^6d^6 + 2\,ab^5d^5e + 3\,a^2b^4d^4e^2 + 4\,a^3b^3d^3e^3 + 5\,a^4b^2d^2e^4 + 6\,a^5bde^5 + 7\,a^6e^6 + 56\left(b^6de^5 + 2\,ab^5e^6\right)x^5 + 70\left(b\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^9,x, algorithm="fricas")
```

```
[Out] -1/56*(28*b^6*e^6*x^6 + b^6*d^6 + 2*a*b^5*d^5*e + 3*a^2*b^4*d^4*e^2 + 4*a^3
*b^3*d^3*e^3 + 5*a^4*b^2*d^2*e^4 + 6*a^5*b*d*e^5 + 7*a^6*e^6 + 56*(b^6*d*e^
5 + 2*a*b^5*e^6)*x^5 + 70*(b^6*d^2*e^4 + 2*a*b^5*d*e^5 + 3*a^2*b^4*e^6)*x^4
 + 56*(b^6*d^3*e^3 + 2*a*b^5*d^2*e^4 + 3*a^2*b^4*d*e^5 + 4*a^3*b^3*e^6)*x^3
 + 28*(b^6*d^4*e^2 + 2*a*b^5*d^3*e^3 + 3*a^2*b^4*d^2*e^4 + 4*a^3*b^3*d*e^5
+ 5*a^4*b^2*e^6)*x^2 + 8*(b^6*d^5*e + 2*a*b^5*d^4*e^2 + 3*a^2*b^4*d^3*e^3 +
 4*a^3*b^3*d^2*e^4 + 5*a^4*b^2*d*e^5 + 6*a^5*b*e^6)*x)/(e^15*x^8 + 8*d*e^14
*x^7 + 28*d^2*e^13*x^6 + 56*d^3*e^12*x^5 + 70*d^4*e^11*x^4 + 56*d^5*e^10*x^
3 + 28*d^6*e^9*x^2 + 8*d^7*e^8*x + d^8*e^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**9,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19786, size = 475, normalized size = 8.19

$$-\frac{\left(28\,b^6x^6e^6 + 56\,b^6dx^5e^5 + 70\,b^6d^2x^4e^4 + 56\,b^6d^3x^3e^3 + 28\,b^6d^4x^2e^2 + 8\,b^6d^5xe + b^6d^6 + 112\,ab^5x^5e^6 + 140\,ab^5dx^4e^5 + 1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^9,x, algorithm="giac")
```

```
[Out] -1/56*(28*b^6*x^6*e^6 + 56*b^6*d*x^5*e^5 + 70*b^6*d^2*x^4*e^4 + 56*b^6*d^3*
x^3*e^3 + 28*b^6*d^4*x^2*e^2 + 8*b^6*d^5*x*e + b^6*d^6 + 112*a*b^5*x^5*e^6
+ 140*a*b^5*d*x^4*e^5 + 112*a*b^5*d^2*x^3*e^4 + 56*a*b^5*d^3*x^2*e^3 + 16*a
```

```
*b^5*d^4*x*e^2 + 2*a*b^5*d^5*e + 210*a^2*b^4*x^4*e^6 + 168*a^2*b^4*d*x^3*e^
5 + 84*a^2*b^4*d^2*x^2*e^4 + 24*a^2*b^4*d^3*x*e^3 + 3*a^2*b^4*d^4*e^2 + 224
*a^3*b^3*x^3*e^6 + 112*a^3*b^3*d*x^2*e^5 + 32*a^3*b^3*d^2*x*e^4 + 4*a^3*b^3
*d^3*e^3 + 140*a^4*b^2*x^2*e^6 + 40*a^4*b^2*d*x*e^5 + 5*a^4*b^2*d^2*e^4 + 4
8*a^5*b*x*e^6 + 6*a^5*b*d*e^5 + 7*a^6*e^6)*e^(-7)/(x*e + d)^8
```

3.1499 $\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^{10}} dx$

Optimal. Leaf size=89

$$\frac{b^2(a+bx)^7}{252(d+ex)^7(bd-ae)^3}+\frac{b(a+bx)^7}{36(d+ex)^8(bd-ae)^2}+\frac{(a+bx)^7}{9(d+ex)^9(bd-ae)}$$

```
[Out] (a + b*x)^7/(9*(b*d - a*e)*(d + e*x)^9) + (b*(a + b*x)^7)/(36*(b*d - a*e)^2
*(d + e*x)^8) + (b^2*(a + b*x)^7)/(252*(b*d - a*e)^3*(d + e*x)^7)
```

Rubi [A] time = 0.0205223, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {27, 45, 37}

$$\frac{b^2(a+bx)^7}{252(d+ex)^7(bd-ae)^3}+\frac{b(a+bx)^7}{36(d+ex)^8(bd-ae)^2}+\frac{(a+bx)^7}{9(d+ex)^9(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^10,x]
```

```
[Out] (a + b*x)^7/(9*(b*d - a*e)*(d + e*x)^9) + (b*(a + b*x)^7)/(36*(b*d - a*e)^2
*(d + e*x)^8) + (b^2*(a + b*x)^7)/(252*(b*d - a*e)^3*(d + e*x)^7)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^{10}}\,dx = \int \frac{(a+bx)^6}{(d+ex)^{10}}\,dx$$
$$= \frac{(a+bx)^7}{9(bd-ae)(d+ex)^9} + \frac{(2b)\int \frac{(a+bx)^6}{(d+ex)^9}\,dx}{9(bd-ae)}$$
$$= \frac{(a+bx)^7}{9(bd-ae)(d+ex)^9} + \frac{b(a+bx)^7}{36(bd-ae)^2(d+ex)^8} + \frac{b^2\int \frac{(a+bx)^6}{(d+ex)^8}\,dx}{36(bd-ae)^2}$$
$$= \frac{(a+bx)^7}{9(bd-ae)(d+ex)^9} + \frac{b(a+bx)^7}{36(bd-ae)^2(d+ex)^8} + \frac{b^2(a+bx)^7}{252(bd-ae)^3(d+ex)^7}$$

Mathematica [B] time = 0.0953959, size = 277, normalized size = 3.11

$$-\frac{6a^2b^4e^2\left(36d^2e^2x^2+9d^3ex+d^4+84de^3x^3+126e^4x^4\right)+10a^3b^3e^3\left(9d^2ex+d^3+36de^2x^2+84e^3x^3\right)+15a^4b^2e^4\left(d^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^10,x]
```

```
[Out] -(28*a^6*e^6 + 21*a^5*b*e^5*(d + 9*e*x) + 15*a^4*b^2*e^4*(d^2 + 9*d*e*x + 3
6*e^2*x^2) + 10*a^3*b^3*e^3*(d^3 + 9*d^2*e*x + 36*d*e^2*x^2 + 84*e^3*x^3) +
 6*a^2*b^4*e^2*(d^4 + 9*d^3*e*x + 36*d^2*e^2*x^2 + 84*d*e^3*x^3 + 126*e^4*x
^4) + 3*a*b^5*e*(d^5 + 9*d^4*e*x + 36*d^3*e^2*x^2 + 84*d^2*e^3*x^3 + 126*d*
e^4*x^4 + 126*e^5*x^5) + b^6*(d^6 + 9*d^5*e*x + 36*d^4*e^2*x^2 + 84*d^3*e^3
*x^3 + 126*d^2*e^4*x^4 + 126*d*e^5*x^5 + 84*e^6*x^6))/(252*e^7*(d + e*x)^9)
```

Maple [B] time = 0.049, size = 357, normalized size = 4.

$$-\frac{b^6}{3\,e^7\,(ex+d)^3} - \frac{10\,b^3\left(a^3e^3-3\,a^2bde^2+3\,ab^2d^2e-b^3d^3\right)}{3\,e^7\,(ex+d)^6} - \frac{3\,b^5\,(ae-bd)}{2\,e^7\,(ex+d)^4} - \frac{15\,b^2\left(a^4e^4-4\,a^3bde^3+6\,d^2e^2b^2a^2-4\,a\right.}{7\,e^7\,(ex+d)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^10,x)
```

```
[Out] -1/3*b^6/e^7/(e*x+d)^3-10/3*b^3*(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d^2*e-b^3*d^
3)/e^7/(e*x+d)^6-3/2*b^5*(a*e-b*d)/e^7/(e*x+d)^4-15/7*b^2*(a^4*e^4-4*a^3*b*
d*e^3+6*a^2*b^2*d^2*e^2-4*a*b^3*d^3*e+b^4*d^4)/e^7/(e*x+d)^7-1/9*(a^6*e^6-6
*a^5*b*d*e^5+15*a^4*b^2*d^2*e^4-20*a^3*b^3*d^3*e^3+15*a^2*b^4*d^4*e^2-6*a*b
^5*d^5*e+b^6*d^6)/e^7/(e*x+d)^9-3/4*b*(a^5*e^5-5*a^4*b*d*e^4+10*a^3*b^2*d^2
*e^3-10*a^2*b^3*d^3*e^2+5*a*b^4*d^4*e-b^5*d^5)/e^7/(e*x+d)^8-3*b^4*(a^2*e^2
-2*a*b*d*e+b^2*d^2)/e^7/(e*x+d)^5
```

Maxima [B] time = 1.21728, size = 595, normalized size = 6.69

$$-\frac{84\,b^6e^6x^6+b^6d^6+3\,ab^5d^5e+6\,a^2b^4d^4e^2+10\,a^3b^3d^3e^3+15\,a^4b^2d^2e^4+21\,a^5bde^5+28\,a^6e^6+126\left(b^6de^5+3\,ab^5e^6\right)x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^10,x, algorithm="maxima")
```

```
[Out] -1/252*(84*b^6*e^6*x^6 + b^6*d^6 + 3*a*b^5*d^5*e + 6*a^2*b^4*d^4*e^2 + 10*a
^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 + 21*a^5*b*d*e^5 + 28*a^6*e^6 + 126*(b^
6*d*e^5 + 3*a*b^5*e^6)*x^5 + 126*(b^6*d^2*e^4 + 3*a*b^5*d*e^5 + 6*a^2*b^4*e
^6)*x^4 + 84*(b^6*d^3*e^3 + 3*a*b^5*d^2*e^4 + 6*a^2*b^4*d*e^5 + 10*a^3*b^3*
e^6)*x^3 + 36*(b^6*d^4*e^2 + 3*a*b^5*d^3*e^3 + 6*a^2*b^4*d^2*e^4 + 10*a^3*b
^3*d*e^5 + 15*a^4*b^2*e^6)*x^2 + 9*(b^6*d^5*e + 3*a*b^5*d^4*e^2 + 6*a^2*b^4
*d^3*e^3 + 10*a^3*b^3*d^2*e^4 + 15*a^4*b^2*d*e^5 + 21*a^5*b*e^6)*x)/(e^16*x
^9 + 9*d*e^15*x^8 + 36*d^2*e^14*x^7 + 84*d^3*e^13*x^6 + 126*d^4*e^12*x^5 +
126*d^5*e^11*x^4 + 84*d^6*e^10*x^3 + 36*d^7*e^9*x^2 + 9*d^8*e^8*x + d^9*e^7
)
```

Fricas [B] time = 1.78445, size = 915, normalized size = 10.28

$$-\frac{84\,b^6e^6x^6 + b^6d^6 + 3\,ab^5d^5e + 6\,a^2b^4d^4e^2 + 10\,a^3b^3d^3e^3 + 15\,a^4b^2d^2e^4 + 21\,a^5bde^5 + 28\,a^6e^6 + 126\left(b^6de^5 + 3\,ab^5e^6\right)x^5 + \cdots}{252\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^10,x, algorithm="fricas")
```

```
[Out] -1/252*(84*b^6*e^6*x^6 + b^6*d^6 + 3*a*b^5*d^5*e + 6*a^2*b^4*d^4*e^2 + 10*a
^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 + 21*a^5*b*d*e^5 + 28*a^6*e^6 + 126*(b^
6*d*e^5 + 3*a*b^5*e^6)*x^5 + 126*(b^6*d^2*e^4 + 3*a*b^5*d*e^5 + 6*a^2*b^4*e
^6)*x^4 + 84*(b^6*d^3*e^3 + 3*a*b^5*d^2*e^4 + 6*a^2*b^4*d*e^5 + 10*a^3*b^3*
e^6)*x^3 + 36*(b^6*d^4*e^2 + 3*a*b^5*d^3*e^3 + 6*a^2*b^4*d^2*e^4 + 10*a^3*b
^3*d*e^5 + 15*a^4*b^2*e^6)*x^2 + 9*(b^6*d^5*e + 3*a*b^5*d^4*e^2 + 6*a^2*b^4
*d^3*e^3 + 10*a^3*b^3*d^2*e^4 + 15*a^4*b^2*d*e^5 + 21*a^5*b*e^6)*x)/(e^16*x
^9 + 9*d*e^15*x^8 + 36*d^2*e^14*x^7 + 84*d^3*e^13*x^6 + 126*d^4*e^12*x^5 +
126*d^5*e^11*x^4 + 84*d^6*e^10*x^3 + 36*d^7*e^9*x^2 + 9*d^8*e^8*x + d^9*e^7
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**10,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.11716, size = 475, normalized size = 5.34

$$-\frac{\left(84\,b^6x^6e^6 + 126\,b^6dx^5e^5 + 126\,b^6d^2x^4e^4 + 84\,b^6d^3x^3e^3 + 36\,b^6d^4x^2e^2 + 9\,b^6d^5xe + b^6d^6 + 378\,ab^5x^5e^6 + 378\,ab^5dx^4e^5 + \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^10,x, algorithm="giac")
```

```
[Out] -1/252*(84*b^6*x^6*e^6 + 126*b^6*d*x^5*e^5 + 126*b^6*d^2*x^4*e^4 + 84*b^6*d
^3*x^3*e^3 + 36*b^6*d^4*x^2*e^2 + 9*b^6*d^5*x*e + b^6*d^6 + 378*a*b^5*x^5*e
^6 + 378*a*b^5*d*x^4*e^5 + 252*a*b^5*d^2*x^3*e^4 + 108*a*b^5*d^3*x^2*e^3 +
27*a*b^5*d^4*x*e^2 + 3*a*b^5*d^5*e + 756*a^2*b^4*x^4*e^6 + 504*a^2*b^4*d*x^
3*e^5 + 216*a^2*b^4*d^2*x^2*e^4 + 54*a^2*b^4*d^3*x*e^3 + 6*a^2*b^4*d^4*e^2
+ 840*a^3*b^3*x^3*e^6 + 360*a^3*b^3*d*x^2*e^5 + 90*a^3*b^3*d^2*x*e^4 + 10*a
^3*b^3*d^3*e^3 + 540*a^4*b^2*x^2*e^6 + 135*a^4*b^2*d*x*e^5 + 15*a^4*b^2*d^2
*e^4 + 189*a^5*b*x*e^6 + 21*a^5*b*d*e^5 + 28*a^6*e^6)*e^(-7)/(x*e + d)^9
```

3.1500 $\int \frac{\left(a^2+2abx+b^2x^2\right)^3}{(d+ex)^{11}}\,dx$

Optimal. Leaf size=120

$$\frac{b^3(a+bx)^7}{840(d+ex)^7(bd-ae)^4}+\frac{b^2(a+bx)^7}{120(d+ex)^8(bd-ae)^3}+\frac{b(a+bx)^7}{30(d+ex)^9(bd-ae)^2}+\frac{(a+bx)^7}{10(d+ex)^{10}(bd-ae)}$$

```
[Out] (a + b*x)^7/(10*(b*d - a*e)*(d + e*x)^10) + (b*(a + b*x)^7)/(30*(b*d - a*e)
^2*(d + e*x)^9) + (b^2*(a + b*x)^7)/(120*(b*d - a*e)^3*(d + e*x)^8) + (b^3*
(a + b*x)^7)/(840*(b*d - a*e)^4*(d + e*x)^7)
```

Rubi [A] time = 0.0350822, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {27, 45, 37}

$$\frac{b^3(a+bx)^7}{840(d+ex)^7(bd-ae)^4}+\frac{b^2(a+bx)^7}{120(d+ex)^8(bd-ae)^3}+\frac{b(a+bx)^7}{30(d+ex)^9(bd-ae)^2}+\frac{(a+bx)^7}{10(d+ex)^{10}(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^11,x]
```

```
[Out] (a + b*x)^7/(10*(b*d - a*e)*(d + e*x)^10) + (b*(a + b*x)^7)/(30*(b*d - a*e)
^2*(d + e*x)^9) + (b^2*(a + b*x)^7)/(120*(b*d - a*e)^3*(d + e*x)^8) + (b^3*
(a + b*x)^7)/(840*(b*d - a*e)^4*(d + e*x)^7)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^{11}}\,dx = \int \frac{(a+bx)^6}{(d+ex)^{11}}\,dx$$
$$= \frac{(a+bx)^7}{10(bd-ae)(d+ex)^{10}} + \frac{(3b)\int \frac{(a+bx)^6}{(d+ex)^{10}}\,dx}{10(bd-ae)}$$
$$= \frac{(a+bx)^7}{10(bd-ae)(d+ex)^{10}} + \frac{b(a+bx)^7}{30(bd-ae)^2(d+ex)^9} + \frac{b^2\int \frac{(a+bx)^6}{(d+ex)^9}\,dx}{15(bd-ae)^2}$$
$$= \frac{(a+bx)^7}{10(bd-ae)(d+ex)^{10}} + \frac{b(a+bx)^7}{30(bd-ae)^2(d+ex)^9} + \frac{b^2(a+bx)^7}{120(bd-ae)^3(d+ex)^8} + \frac{b^3\int \frac{(a+bx)^6}{(d+ex)^8}\,dx}{120(bd-ae)^3}$$
$$= \frac{(a+bx)^7}{10(bd-ae)(d+ex)^{10}} + \frac{b(a+bx)^7}{30(bd-ae)^2(d+ex)^9} + \frac{b^2(a+bx)^7}{120(bd-ae)^3(d+ex)^8} + \frac{b^3(a+b}{840(bd-ae)^4}$$

Mathematica [B] time = 0.0952054, size = 277, normalized size = 2.31

$$-\frac{10a^2b^4e^2\left(45d^2e^2x^2+10d^3ex+d^4+120de^3x^3+210e^4x^4\right)+20a^3b^3e^3\left(10d^2ex+d^3+45de^2x^2+120e^3x^3\right)+35a^4b^2e^4}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^11,x]
```

```
[Out] -(84*a^6*e^6 + 56*a^5*b*e^5*(d + 10*e*x) + 35*a^4*b^2*e^4*(d^2 + 10*d*e*x +
 45*e^2*x^2) + 20*a^3*b^3*e^3*(d^3 + 10*d^2*e*x + 45*d*e^2*x^2 + 120*e^3*x^
3) + 10*a^2*b^4*e^2*(d^4 + 10*d^3*e*x + 45*d^2*e^2*x^2 + 120*d*e^3*x^3 + 21
0*e^4*x^4) + 4*a*b^5*e*(d^5 + 10*d^4*e*x + 45*d^3*e^2*x^2 + 120*d^2*e^3*x^3
 + 210*d*e^4*x^4 + 252*e^5*x^5) + b^6*(d^6 + 10*d^5*e*x + 45*d^4*e^2*x^2 +
120*d^3*e^3*x^3 + 210*d^2*e^4*x^4 + 252*d*e^5*x^5 + 210*e^6*x^6))/(840*e^7*
(d + e*x)^10)
```

Maple [B] time = 0.048, size = 357, normalized size = 3.

$$-\frac{5\,b^4\left(a^2e^2-2\,abde+b^2d^2\right)}{2\,e^7\,(ex+d)^6} - \frac{b^6}{4\,e^7\,(ex+d)^4} - \frac{e^6a^6-6\,a^5bde^5+15\,d^2e^4a^4b^2-20\,a^3b^3d^3e^3+15\,a^2b^4d^4e^2-6\,ab^5d^5e+}{10\,e^7\,(ex+d)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^11,x)
```

```
[Out] -5/2*b^4*(a^2*e^2-2*a*b*d*e+b^2*d^2)/e^7/(e*x+d)^6-1/4*b^6/e^7/(e*x+d)^4-1/
10*(a^6*e^6-6*a^5*b*d*e^5+15*a^4*b^2*d^2*e^4-20*a^3*b^3*d^3*e^3+15*a^2*b^4*
d^4*e^2-6*a*b^5*d^5*e+b^6*d^6)/e^7/(e*x+d)^10-20/7*b^3*(a^3*e^3-3*a^2*b*d*e
^2+3*a*b^2*d^2*e-b^3*d^3)/e^7/(e*x+d)^7-2/3*b*(a^5*e^5-5*a^4*b*d*e^4+10*a^3
*b^2*d^2*e^3-10*a^2*b^3*d^3*e^2+5*a*b^4*d^4*e-b^5*d^5)/e^7/(e*x+d)^9-15/8*b
^2*(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-4*a*b^3*d^3*e+b^4*d^4)/e^7/(e*x
+d)^8-6/5*b^5*(a*e-b*d)/e^7/(e*x+d)^5
```

Maxima [B] time = 1.30075, size = 610, normalized size = 5.08

$$-\frac{210\,b^6e^6x^6+b^6d^6+4\,ab^5d^5e+10\,a^2b^4d^4e^2+20\,a^3b^3d^3e^3+35\,a^4b^2d^2e^4+56\,a^5bde^5+84\,a^6e^6+252\left(b^6de^5+4\,ab^5e^{}\right.}{840\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^11,x, algorithm="maxima")
```

```
[Out] -1/840*(210*b^6*e^6*x^6 + b^6*d^6 + 4*a*b^5*d^5*e + 10*a^2*b^4*d^4*e^2 + 20
*a^3*b^3*d^3*e^3 + 35*a^4*b^2*d^2*e^4 + 56*a^5*b*d*e^5 + 84*a^6*e^6 + 252*(
b^6*d*e^5 + 4*a*b^5*e^6)*x^5 + 210*(b^6*d^2*e^4 + 4*a*b^5*d*e^5 + 10*a^2*b^
4*e^6)*x^4 + 120*(b^6*d^3*e^3 + 4*a*b^5*d^2*e^4 + 10*a^2*b^4*d*e^5 + 20*a^3
*b^3*e^6)*x^3 + 45*(b^6*d^4*e^2 + 4*a*b^5*d^3*e^3 + 10*a^2*b^4*d^2*e^4 + 20
*a^3*b^3*d*e^5 + 35*a^4*b^2*e^6)*x^2 + 10*(b^6*d^5*e + 4*a*b^5*d^4*e^2 + 10
*a^2*b^4*d^3*e^3 + 20*a^3*b^3*d^2*e^4 + 35*a^4*b^2*d*e^5 + 56*a^5*b*e^6)*x)
/(e^17*x^10 + 10*d*e^16*x^9 + 45*d^2*e^15*x^8 + 120*d^3*e^14*x^7 + 210*d^4*
e^13*x^6 + 252*d^5*e^12*x^5 + 210*d^6*e^11*x^4 + 120*d^7*e^10*x^3 + 45*d^8*
e^9*x^2 + 10*d^9*e^8*x + d^10*e^7)
```

Fricas [B] time = 1.74229, size = 960, normalized size = 8.

$$-\frac{210\,b^6e^6x^6 + b^6d^6 + 4\,ab^5d^5e + 10\,a^2b^4d^4e^2 + 20\,a^3b^3d^3e^3 + 35\,a^4b^2d^2e^4 + 56\,a^5bde^5 + 84\,a^6e^6 + 252\left(b^6de^5 + 4\,ab^5e^6\right)x^5}{840\,(e^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^11,x, algorithm="fricas")
```

```
[Out] -1/840*(210*b^6*e^6*x^6 + b^6*d^6 + 4*a*b^5*d^5*e + 10*a^2*b^4*d^4*e^2 + 20
*a^3*b^3*d^3*e^3 + 35*a^4*b^2*d^2*e^4 + 56*a^5*b*d*e^5 + 84*a^6*e^6 + 252*(
b^6*d*e^5 + 4*a*b^5*e^6)*x^5 + 210*(b^6*d^2*e^4 + 4*a*b^5*d*e^5 + 10*a^2*b^
4*e^6)*x^4 + 120*(b^6*d^3*e^3 + 4*a*b^5*d^2*e^4 + 10*a^2*b^4*d*e^5 + 20*a^3
*b^3*e^6)*x^3 + 45*(b^6*d^4*e^2 + 4*a*b^5*d^3*e^3 + 10*a^2*b^4*d^2*e^4 + 20
*a^3*b^3*d*e^5 + 35*a^4*b^2*e^6)*x^2 + 10*(b^6*d^5*e + 4*a*b^5*d^4*e^2 + 10
*a^2*b^4*d^3*e^3 + 20*a^3*b^3*d^2*e^4 + 35*a^4*b^2*d*e^5 + 56*a^5*b*e^6)*x)
/(e^17*x^10 + 10*d*e^16*x^9 + 45*d^2*e^15*x^8 + 120*d^3*e^14*x^7 + 210*d^4*
e^13*x^6 + 252*d^5*e^12*x^5 + 210*d^6*e^11*x^4 + 120*d^7*e^10*x^3 + 45*d^8*
e^9*x^2 + 10*d^9*e^8*x + d^10*e^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**11,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.14104, size = 475, normalized size = 3.96

$$-\frac{\left(210\,b^6x^6e^6 + 252\,b^6dx^5e^5 + 210\,b^6d^2x^4e^4 + 120\,b^6d^3x^3e^3 + 45\,b^6d^4x^2e^2 + 10\,b^6d^5xe + b^6d^6 + 1008\,ab^5x^5e^6 + 840\,ab^5dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^11,x, algorithm="giac")
```

```
[Out] -1/840*(210*b^6*x^6*e^6 + 252*b^6*d*x^5*e^5 + 210*b^6*d^2*x^4*e^4 + 120*b^6
*d^3*x^3*e^3 + 45*b^6*d^4*x^2*e^2 + 10*b^6*d^5*x*e + b^6*d^6 + 1008*a*b^5*x
^5*e^6 + 840*a*b^5*d*x^4*e^5 + 480*a*b^5*d^2*x^3*e^4 + 180*a*b^5*d^3*x^2*e^
3 + 40*a*b^5*d^4*x*e^2 + 4*a*b^5*d^5*e + 2100*a^2*b^4*x^4*e^6 + 1200*a^2*b^
4*d*x^3*e^5 + 450*a^2*b^4*d^2*x^2*e^4 + 100*a^2*b^4*d^3*x*e^3 + 10*a^2*b^4*
d^4*e^2 + 2400*a^3*b^3*x^3*e^6 + 900*a^3*b^3*d*x^2*e^5 + 200*a^3*b^3*d^2*x*
e^4 + 20*a^3*b^3*d^3*e^3 + 1575*a^4*b^2*x^2*e^6 + 350*a^4*b^2*d*x*e^5 + 35*
a^4*b^2*d^2*e^4 + 560*a^5*b*x*e^6 + 56*a^5*b*d*e^5 + 84*a^6*e^6)*e^(-7)/(x*
e + d)^10
```

3.1501 $\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^{12}} dx$

Optimal. Leaf size=170

$$\frac{b^5(bd-ae)}{e^7(d+ex)^6}-\frac{15b^4(bd-ae)^2}{7e^7(d+ex)^7}+\frac{5b^3(bd-ae)^3}{2e^7(d+ex)^8}-\frac{5b^2(bd-ae)^4}{3e^7(d+ex)^9}+\frac{3b(bd-ae)^5}{5e^7(d+ex)^{10}}-\frac{(bd-ae)^6}{11e^7(d+ex)^{11}}-\frac{b^6}{5e^7(d+ex)^5}$$

```
[Out] -(b*d - a*e)^6/(11*e^7*(d + e*x)^11) + (3*b*(b*d - a*e)^5)/(5*e^7*(d + e*x)
^10) - (5*b^2*(b*d - a*e)^4)/(3*e^7*(d + e*x)^9) + (5*b^3*(b*d - a*e)^3)/(2
*e^7*(d + e*x)^8) - (15*b^4*(b*d - a*e)^2)/(7*e^7*(d + e*x)^7) + (b^5*(b*d
- a*e))/(e^7*(d + e*x)^6) - b^6/(5*e^7*(d + e*x)^5)
```

Rubi [A] time = 0.128661, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$\frac{b^5(bd-ae)}{e^7(d+ex)^6}-\frac{15b^4(bd-ae)^2}{7e^7(d+ex)^7}+\frac{5b^3(bd-ae)^3}{2e^7(d+ex)^8}-\frac{5b^2(bd-ae)^4}{3e^7(d+ex)^9}+\frac{3b(bd-ae)^5}{5e^7(d+ex)^{10}}-\frac{(bd-ae)^6}{11e^7(d+ex)^{11}}-\frac{b^6}{5e^7(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^12,x]
```

```
[Out] -(b*d - a*e)^6/(11*e^7*(d + e*x)^11) + (3*b*(b*d - a*e)^5)/(5*e^7*(d + e*x)
^10) - (5*b^2*(b*d - a*e)^4)/(3*e^7*(d + e*x)^9) + (5*b^3*(b*d - a*e)^3)/(2
*e^7*(d + e*x)^8) - (15*b^4*(b*d - a*e)^2)/(7*e^7*(d + e*x)^7) + (b^5*(b*d
- a*e))/(e^7*(d + e*x)^6) - b^6/(5*e^7*(d + e*x)^5)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^{12}}\,dx &= \int \frac{(a+bx)^6}{(d+ex)^{12}}\,dx\\ &= \int \left(\frac{(-bd+ae)^6}{e^6(d+ex)^{12}}-\frac{6b(bd-ae)^5}{e^6(d+ex)^{11}}+\frac{15b^2(bd-ae)^4}{e^6(d+ex)^{10}}-\frac{20b^3(bd-ae)^3}{e^6(d+ex)^9}+\frac{15b^4(bd-ae)^2}{e^6(d+ex)^8}-\frac{6b^5(\ldots}{e^6(\ldots}\right.\\ &= -\frac{(bd-ae)^6}{11e^7(d+ex)^{11}}+\frac{3b(bd-ae)^5}{5e^7(d+ex)^{10}}-\frac{5b^2(bd-ae)^4}{3e^7(d+ex)^9}+\frac{5b^3(bd-ae)^3}{2e^7(d+ex)^8}-\frac{15b^4(bd-ae)^2}{7e^7(d+ex)^7}+\frac{b^5(bd\ldots}{e^7(d+\ldots}\end{aligned}$$

Mathematica [A] time = 0.0949331, size = 277, normalized size = 1.63

$$-\frac{15a^2b^4e^2\left(55d^2e^2x^2+11d^3ex+d^4+165de^3x^3+330e^4x^4\right)+35a^3b^3e^3\left(11d^2ex+d^3+55de^2x^2+165e^3x^3\right)+70a^4b^2e^4\left(d^2\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^12,x]
```

```
[Out] -(210*a^6*e^6 + 126*a^5*b*e^5*(d + 11*e*x) + 70*a^4*b^2*e^4*(d^2 + 11*d*e*x
 + 55*e^2*x^2) + 35*a^3*b^3*e^3*(d^3 + 11*d^2*e*x + 55*d*e^2*x^2 + 165*e^3*
x^3) + 15*a^2*b^4*e^2*(d^4 + 11*d^3*e*x + 55*d^2*e^2*x^2 + 165*d*e^3*x^3 +
330*e^4*x^4) + 5*a*b^5*e*(d^5 + 11*d^4*e*x + 55*d^3*e^2*x^2 + 165*d^2*e^3*x
^3 + 330*d*e^4*x^4 + 462*e^5*x^5) + b^6*(d^6 + 11*d^5*e*x + 55*d^4*e^2*x^2
+ 165*d^3*e^3*x^3 + 330*d^2*e^4*x^4 + 462*d*e^5*x^5 + 462*e^6*x^6))/(2310*e
^7*(d + e*x)^11)
```

Maple [B] time = 0.054, size = 357, normalized size = 2.1

$$-\frac{b^5(ae-bd)}{e^7(ex+d)^6}-\frac{3b\left(a^5e^5-5a^4bde^4+10a^3b^2d^2e^3-10a^2b^3d^3e^2+5ab^4d^4e-b^5d^5\right)}{5e^7(ex+d)^{10}}-\frac{15b^4\left(a^2e^2-2abde+b^2d^2\right)}{7e^7(ex+d)^7}-e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^12,x)
```

```
[Out] -b^5*(a*e-b*d)/e^7/(e*x+d)^6-3/5*b*(a^5*e^5-5*a^4*b*d*e^4+10*a^3*b^2*d^2*e^
3-10*a^2*b^3*d^3*e^2+5*a*b^4*d^4*e-b^5*d^5)/e^7/(e*x+d)^10-15/7*b^4*(a^2*e^
2-2*a*b*d*e+b^2*d^2)/e^7/(e*x+d)^7-1/11*(a^6*e^6-6*a^5*b*d*e^5+15*a^4*b^2*d
^2*e^4-20*a^3*b^3*d^3*e^3+15*a^2*b^4*d^4*e^2-6*a*b^5*d^5*e+b^6*d^6)/e^7/(e*
x+d)^11-5/3*b^2*(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-4*a*b^3*d^3*e+b^4*
d^4)/e^7/(e*x+d)^9-5/2*b^3*(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d^2*e-b^3*d^3)/e^
7/(e*x+d)^8-1/5*b^6/e^7/(e*x+d)^5
```

Maxima [B] time = 1.10339, size = 625, normalized size = 3.68

$$-\frac{462b^6e^6x^6+b^6d^6+5ab^5d^5e+15a^2b^4d^4e^2+35a^3b^3d^3e^3+70a^4b^2d^2e^4+126a^5bde^5+210a^6e^6+462\left(b^6de^5+5ab\right.}{2310\left(e^{18}x^{1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^12,x, algorithm="maxima")
```

```
[Out] -1/2310*(462*b^6*e^6*x^6 + b^6*d^6 + 5*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 + 3
5*a^3*b^3*d^3*e^3 + 70*a^4*b^2*d^2*e^4 + 126*a^5*b*d*e^5 + 210*a^6*e^6 + 46
2*(b^6*d*e^5 + 5*a*b^5*e^6)*x^5 + 330*(b^6*d^2*e^4 + 5*a*b^5*d*e^5 + 15*a^2
*b^4*e^6)*x^4 + 165*(b^6*d^3*e^3 + 5*a*b^5*d^2*e^4 + 15*a^2*b^4*d*e^5 + 35*
a^3*b^3*e^6)*x^3 + 55*(b^6*d^4*e^2 + 5*a*b^5*d^3*e^3 + 15*a^2*b^4*d^2*e^4 +
 35*a^3*b^3*d*e^5 + 70*a^4*b^2*e^6)*x^2 + 11*(b^6*d^5*e + 5*a*b^5*d^4*e^2 +
 15*a^2*b^4*d^3*e^3 + 35*a^3*b^3*d^2*e^4 + 70*a^4*b^2*d*e^5 + 126*a^5*b*e^6
)*x)/(e^18*x^11 + 11*d*e^17*x^10 + 55*d^2*e^16*x^9 + 165*d^3*e^15*x^8 + 330
*d^4*e^14*x^7 + 462*d^5*e^13*x^6 + 462*d^6*e^12*x^5 + 330*d^7*e^11*x^4 + 16
5*d^8*e^10*x^3 + 55*d^9*e^9*x^2 + 11*d^10*e^8*x + d^11*e^7)
```

Fricas [B] time = 1.74227, size = 994, normalized size = 5.85

$$-\frac{462b^6e^6x^6+b^6d^6+5ab^5d^5e+15a^2b^4d^4e^2+35a^3b^3d^3e^3+70a^4b^2d^2e^4+126a^5bde^5+210a^6e^6+462\left(b^6de^5+5ab\right.}{2310\left(e^{18}x^{1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^12,x, algorithm="fricas")
```

```
[Out] -1/2310*(462*b^6*e^6*x^6 + b^6*d^6 + 5*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 + 3
5*a^3*b^3*d^3*e^3 + 70*a^4*b^2*d^2*e^4 + 126*a^5*b*d*e^5 + 210*a^6*e^6 + 46
2*(b^6*d*e^5 + 5*a*b^5*e^6)*x^5 + 330*(b^6*d^2*e^4 + 5*a*b^5*d*e^5 + 15*a^2
*b^4*e^6)*x^4 + 165*(b^6*d^3*e^3 + 5*a*b^5*d^2*e^4 + 15*a^2*b^4*d*e^5 + 35*
a^3*b^3*e^6)*x^3 + 55*(b^6*d^4*e^2 + 5*a*b^5*d^3*e^3 + 15*a^2*b^4*d^2*e^4 +
 35*a^3*b^3*d*e^5 + 70*a^4*b^2*e^6)*x^2 + 11*(b^6*d^5*e + 5*a*b^5*d^4*e^2 +
 15*a^2*b^4*d^3*e^3 + 35*a^3*b^3*d^2*e^4 + 70*a^4*b^2*d*e^5 + 126*a^5*b*e^6
)*x)/(e^18*x^11 + 11*d*e^17*x^10 + 55*d^2*e^16*x^9 + 165*d^3*e^15*x^8 + 330
*d^4*e^14*x^7 + 462*d^5*e^13*x^6 + 462*d^6*e^12*x^5 + 330*d^7*e^11*x^4 + 16
5*d^8*e^10*x^3 + 55*d^9*e^9*x^2 + 11*d^10*e^8*x + d^11*e^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**12,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16728, size = 475, normalized size = 2.79

$$-\frac{(462\,b^6x^6e^6 + 462\,b^6dx^5e^5 + 330\,b^6d^2x^4e^4 + 165\,b^6d^3x^3e^3 + 55\,b^6d^4x^2e^2 + 11\,b^6d^5xe + b^6d^6 + 2310\,ab^5x^5e^6 + 1650\,ab^5d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^12,x, algorithm="giac")
```

```
[Out] -1/2310*(462*b^6*x^6*e^6 + 462*b^6*d*x^5*e^5 + 330*b^6*d^2*x^4*e^4 + 165*b^
6*d^3*x^3*e^3 + 55*b^6*d^4*x^2*e^2 + 11*b^6*d^5*x*e + b^6*d^6 + 2310*a*b^5*
x^5*e^6 + 1650*a*b^5*d*x^4*e^5 + 825*a*b^5*d^2*x^3*e^4 + 275*a*b^5*d^3*x^2*
e^3 + 55*a*b^5*d^4*x*e^2 + 5*a*b^5*d^5*e + 4950*a^2*b^4*x^4*e^6 + 2475*a^2*
b^4*d*x^3*e^5 + 825*a^2*b^4*d^2*x^2*e^4 + 165*a^2*b^4*d^3*x*e^3 + 15*a^2*b^
4*d^4*e^2 + 5775*a^3*b^3*x^3*e^6 + 1925*a^3*b^3*d*x^2*e^5 + 385*a^3*b^3*d^2
*x*e^4 + 35*a^3*b^3*d^3*e^3 + 3850*a^4*b^2*x^2*e^6 + 770*a^4*b^2*d*x*e^5 +
70*a^4*b^2*d^2*e^4 + 1386*a^5*b*x*e^6 + 126*a^5*b*d*e^5 + 210*a^6*e^6)*e^(-
7)/(x*e + d)^11
```

3.1502 $\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^{13}}\,dx$

Optimal. Leaf size=173

$$\frac{6b^5(bd-ae)}{7e^7(d+ex)^7}-\frac{15b^4(bd-ae)^2}{8e^7(d+ex)^8}+\frac{20b^3(bd-ae)^3}{9e^7(d+ex)^9}-\frac{3b^2(bd-ae)^4}{2e^7(d+ex)^{10}}+\frac{6b(bd-ae)^5}{11e^7(d+ex)^{11}}-\frac{(bd-ae)^6}{12e^7(d+ex)^{12}}-\frac{b^6}{6e^7(d+ex)^6}$$

```
[Out] -(b*d - a*e)^6/(12*e^7*(d + e*x)^12) + (6*b*(b*d - a*e)^5)/(11*e^7*(d + e*x
)^11) - (3*b^2*(b*d - a*e)^4)/(2*e^7*(d + e*x)^10) + (20*b^3*(b*d - a*e)^3)
/(9*e^7*(d + e*x)^9) - (15*b^4*(b*d - a*e)^2)/(8*e^7*(d + e*x)^8) + (6*b^5*
(b*d - a*e))/(7*e^7*(d + e*x)^7) - b^6/(6*e^7*(d + e*x)^6)
```

Rubi [A] time = 0.118325, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$\frac{6b^5(bd-ae)}{7e^7(d+ex)^7}-\frac{15b^4(bd-ae)^2}{8e^7(d+ex)^8}+\frac{20b^3(bd-ae)^3}{9e^7(d+ex)^9}-\frac{3b^2(bd-ae)^4}{2e^7(d+ex)^{10}}+\frac{6b(bd-ae)^5}{11e^7(d+ex)^{11}}-\frac{(bd-ae)^6}{12e^7(d+ex)^{12}}-\frac{b^6}{6e^7(d+ex)^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^13,x]
```


```
[Out] -(b*d - a*e)^6/(12*e^7*(d + e*x)^12) + (6*b*(b*d - a*e)^5)/(11*e^7*(d + e*x
)^11) - (3*b^2*(b*d - a*e)^4)/(2*e^7*(d + e*x)^10) + (20*b^3*(b*d - a*e)^3)
/(9*e^7*(d + e*x)^9) - (15*b^4*(b*d - a*e)^2)/(8*e^7*(d + e*x)^8) + (6*b^5*
(b*d - a*e))/(7*e^7*(d + e*x)^7) - b^6/(6*e^7*(d + e*x)^6)
```


Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^{13}}\,dx &= \int \frac{(a+bx)^6}{(d+ex)^{13}}\,dx\\ &= \int \left(\frac{(-bd+ae)^6}{e^6(d+ex)^{13}}-\frac{6b(bd-ae)^5}{e^6(d+ex)^{12}}+\frac{15b^2(bd-ae)^4}{e^6(d+ex)^{11}}-\frac{20b^3(bd-ae)^3}{e^6(d+ex)^{10}}+\frac{15b^4(bd-ae)^2}{e^6(d+ex)^9}-\ldots\\ &= -\frac{(bd-ae)^6}{12e^7(d+ex)^{12}}+\frac{6b(bd-ae)^5}{11e^7(d+ex)^{11}}-\frac{3b^2(bd-ae)^4}{2e^7(d+ex)^{10}}+\frac{20b^3(bd-ae)^3}{9e^7(d+ex)^9}-\frac{15b^4(bd-ae)^2}{8e^7(d+ex)^8}+\ldots\end{aligned}$$

Mathematica [A] time = 0.0915019, size = 277, normalized size = 1.6

$$-\frac{21a^2b^4e^2\left(66d^2e^2x^2+12d^3ex+d^4+220de^3x^3+495e^4x^4\right)+56a^3b^3e^3\left(12d^2ex+d^3+66de^2x^2+220e^3x^3\right)+126a^4b^2\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^13,x]
```

```
[Out] -(462*a^6*e^6 + 252*a^5*b*e^5*(d + 12*e*x) + 126*a^4*b^2*e^4*(d^2 + 12*d*e*
x + 66*e^2*x^2) + 56*a^3*b^3*e^3*(d^3 + 12*d^2*e*x + 66*d*e^2*x^2 + 220*e^3
*x^3) + 21*a^2*b^4*e^2*(d^4 + 12*d^3*e*x + 66*d^2*e^2*x^2 + 220*d*e^3*x^3 +
 495*e^4*x^4) + 6*a*b^5*e*(d^5 + 12*d^4*e*x + 66*d^3*e^2*x^2 + 220*d^2*e^3*
x^3 + 495*d*e^4*x^4 + 792*e^5*x^5) + b^6*(d^6 + 12*d^5*e*x + 66*d^4*e^2*x^2
 + 220*d^3*e^3*x^3 + 495*d^2*e^4*x^4 + 792*d*e^5*x^5 + 924*e^6*x^6))/(5544*
e^7*(d + e*x)^12)
```

Maple [B] time = 0.047, size = 357, normalized size = 2.1

$$-\frac{20\,b^3\left(a^3e^3-3\,a^2bde^2+3\,ab^2d^2e-b^3d^3\right)}{9\,e^7\,(ex+d)^9}-\frac{b^6}{6\,e^7\,(ex+d)^6}-\frac{3\,b^2\left(a^4e^4-4\,a^3bde^3+6\,d^2e^2b^2a^2-4\,ab^3d^3e+b^4d^4\right)}{2\,e^7\,(ex+d)^{10}}-\frac{6\,b^5}{7\,e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^13,x)
```

```
[Out] -20/9*b^3*(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d^2*e-b^3*d^3)/e^7/(e*x+d)^9-1/6*b
^6/e^7/(e*x+d)^6-3/2*b^2*(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-4*a*b^3*d
^3*e+b^4*d^4)/e^7/(e*x+d)^10-6/7*b^5*(a*e-b*d)/e^7/(e*x+d)^7-1/12*(a^6*e^6-
6*a^5*b*d*e^5+15*a^4*b^2*d^2*e^4-20*a^3*b^3*d^3*e^3+15*a^2*b^4*d^4*e^2-6*a*
b^5*d^5*e+b^6*d^6)/e^7/(e*x+d)^12-6/11*b*(a^5*e^5-5*a^4*b*d*e^4+10*a^3*b^2*
d^2*e^3-10*a^2*b^3*d^3*e^2+5*a*b^4*d^4*e-b^5*d^5)/e^7/(e*x+d)^11-15/8*b^4*(
a^2*e^2-2*a*b*d*e+b^2*d^2)/e^7/(e*x+d)^8
```

Maxima [B] time = 1.27283, size = 640, normalized size = 3.7

$$-\frac{924\,b^6e^6x^6+b^6d^6+6\,ab^5d^5e+21\,a^2b^4d^4e^2+56\,a^3b^3d^3e^3+126\,a^4b^2d^2e^4+252\,a^5bde^5+462\,a^6e^6+792\left(b^6de^5+6\,ab^5e\right.}{5544\left(e^{19}x^{12}+12\,de^{1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^13,x, algorithm="maxima")
```

```
[Out] -1/5544*(924*b^6*e^6*x^6 + b^6*d^6 + 6*a*b^5*d^5*e + 21*a^2*b^4*d^4*e^2 + 5
6*a^3*b^3*d^3*e^3 + 126*a^4*b^2*d^2*e^4 + 252*a^5*b*d*e^5 + 462*a^6*e^6 + 7
92*(b^6*d*e^5 + 6*a*b^5*e^6)*x^5 + 495*(b^6*d^2*e^4 + 6*a*b^5*d*e^5 + 21*a^
2*b^4*e^6)*x^4 + 220*(b^6*d^3*e^3 + 6*a*b^5*d^2*e^4 + 21*a^2*b^4*d*e^5 + 56
*a^3*b^3*e^6)*x^3 + 66*(b^6*d^4*e^2 + 6*a*b^5*d^3*e^3 + 21*a^2*b^4*d^2*e^4
+ 56*a^3*b^3*d*e^5 + 126*a^4*b^2*e^6)*x^2 + 12*(b^6*d^5*e + 6*a*b^5*d^4*e^2
 + 21*a^2*b^4*d^3*e^3 + 56*a^3*b^3*d^2*e^4 + 126*a^4*b^2*d*e^5 + 252*a^5*b*
e^6)*x)/(e^19*x^12 + 12*d*e^18*x^11 + 66*d^2*e^17*x^10 + 220*d^3*e^16*x^9 +
 495*d^4*e^15*x^8 + 792*d^5*e^14*x^7 + 924*d^6*e^13*x^6 + 792*d^7*e^12*x^5
+ 495*d^8*e^11*x^4 + 220*d^9*e^10*x^3 + 66*d^10*e^9*x^2 + 12*d^11*e^8*x + d
^12*e^7)
```

Fricas [B] time = 1.76447, size = 1026, normalized size = 5.93

$$-\frac{924\,b^6e^6x^6+b^6d^6+6\,ab^5d^5e+21\,a^2b^4d^4e^2+56\,a^3b^3d^3e^3+126\,a^4b^2d^2e^4+252\,a^5bde^5+462\,a^6e^6+792\left(b^6de^5+6\,a\right.}{5544\left(e^{19}x^{12}+12\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^13,x, algorithm="fricas")
```

```
[Out] -1/5544*(924*b^6*e^6*x^6 + b^6*d^6 + 6*a*b^5*d^5*e + 21*a^2*b^4*d^4*e^2 + 5
6*a^3*b^3*d^3*e^3 + 126*a^4*b^2*d^2*e^4 + 252*a^5*b*d*e^5 + 462*a^6*e^6 + 7
92*(b^6*d*e^5 + 6*a*b^5*e^6)*x^5 + 495*(b^6*d^2*e^4 + 6*a*b^5*d*e^5 + 21*a^
2*b^4*e^6)*x^4 + 220*(b^6*d^3*e^3 + 6*a*b^5*d^2*e^4 + 21*a^2*b^4*d*e^5 + 56
*a^3*b^3*e^6)*x^3 + 66*(b^6*d^4*e^2 + 6*a*b^5*d^3*e^3 + 21*a^2*b^4*d^2*e^4
+ 56*a^3*b^3*d*e^5 + 126*a^4*b^2*e^6)*x^2 + 12*(b^6*d^5*e + 6*a*b^5*d^4*e^2
 + 21*a^2*b^4*d^3*e^3 + 56*a^3*b^3*d^2*e^4 + 126*a^4*b^2*d*e^5 + 252*a^5*b*
e^6)*x)/(e^19*x^12 + 12*d*e^18*x^11 + 66*d^2*e^17*x^10 + 220*d^3*e^16*x^9 +
 495*d^4*e^15*x^8 + 792*d^5*e^14*x^7 + 924*d^6*e^13*x^6 + 792*d^7*e^12*x^5
+ 495*d^8*e^11*x^4 + 220*d^9*e^10*x^3 + 66*d^10*e^9*x^2 + 12*d^11*e^8*x + d
^12*e^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**13,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17238, size = 475, normalized size = 2.75

$$-\frac{\left(924\,b^6x^6e^6+792\,b^6dx^5e^5+495\,b^6d^2x^4e^4+220\,b^6d^3x^3e^3+66\,b^6d^4x^2e^2+12\,b^6d^5xe+b^6d^6+4752\,ab^5x^5e^6+2970\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^13,x, algorithm="giac")
```

```
[Out] -1/5544*(924*b^6*x^6*e^6 + 792*b^6*d*x^5*e^5 + 495*b^6*d^2*x^4*e^4 + 220*b^
6*d^3*x^3*e^3 + 66*b^6*d^4*x^2*e^2 + 12*b^6*d^5*x*e + b^6*d^6 + 4752*a*b^5*
x^5*e^6 + 2970*a*b^5*d*x^4*e^5 + 1320*a*b^5*d^2*x^3*e^4 + 396*a*b^5*d^3*x^2
*e^3 + 72*a*b^5*d^4*x*e^2 + 6*a*b^5*d^5*e + 10395*a^2*b^4*x^4*e^6 + 4620*a^
2*b^4*d*x^3*e^5 + 1386*a^2*b^4*d^2*x^2*e^4 + 252*a^2*b^4*d^3*x*e^3 + 21*a^2
*b^4*d^4*e^2 + 12320*a^3*b^3*x^3*e^6 + 3696*a^3*b^3*d*x^2*e^5 + 672*a^3*b^3
*d^2*x*e^4 + 56*a^3*b^3*d^3*e^3 + 8316*a^4*b^2*x^2*e^6 + 1512*a^4*b^2*d*x*e
^5 + 126*a^4*b^2*d^2*e^4 + 3024*a^5*b*x*e^6 + 252*a^5*b*d*e^5 + 462*a^6*e^6
)*e^(-7)/(x*e + d)^12
```

3.1503 $\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^{14}}\,dx$

Optimal. Leaf size=171

$$\frac{3b^5(bd-ae)}{4e^7(d+ex)^8}-\frac{5b^4(bd-ae)^2}{3e^7(d+ex)^9}+\frac{2b^3(bd-ae)^3}{e^7(d+ex)^{10}}-\frac{15b^2(bd-ae)^4}{11e^7(d+ex)^{11}}+\frac{b(bd-ae)^5}{2e^7(d+ex)^{12}}-\frac{(bd-ae)^6}{13e^7(d+ex)^{13}}-\frac{b^6}{7e^7(d+ex)^7}$$

```
[Out] -(b*d - a*e)^6/(13*e^7*(d + e*x)^13) + (b*(b*d - a*e)^5)/(2*e^7*(d + e*x)^1
2) - (15*b^2*(b*d - a*e)^4)/(11*e^7*(d + e*x)^11) + (2*b^3*(b*d - a*e)^3)/(
e^7*(d + e*x)^10) - (5*b^4*(b*d - a*e)^2)/(3*e^7*(d + e*x)^9) + (3*b^5*(b*d
 - a*e))/(4*e^7*(d + e*x)^8) - b^6/(7*e^7*(d + e*x)^7)
```

Rubi [A] time = 0.13074, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$\frac{3b^5(bd-ae)}{4e^7(d+ex)^8}-\frac{5b^4(bd-ae)^2}{3e^7(d+ex)^9}+\frac{2b^3(bd-ae)^3}{e^7(d+ex)^{10}}-\frac{15b^2(bd-ae)^4}{11e^7(d+ex)^{11}}+\frac{b(bd-ae)^5}{2e^7(d+ex)^{12}}-\frac{(bd-ae)^6}{13e^7(d+ex)^{13}}-\frac{b^6}{7e^7(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^14,x]
```

```
[Out] -(b*d - a*e)^6/(13*e^7*(d + e*x)^13) + (b*(b*d - a*e)^5)/(2*e^7*(d + e*x)^1
2) - (15*b^2*(b*d - a*e)^4)/(11*e^7*(d + e*x)^11) + (2*b^3*(b*d - a*e)^3)/(
e^7*(d + e*x)^10) - (5*b^4*(b*d - a*e)^2)/(3*e^7*(d + e*x)^9) + (3*b^5*(b*d
 - a*e))/(4*e^7*(d + e*x)^8) - b^6/(7*e^7*(d + e*x)^7)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^{14}}\,dx &= \int \frac{(a+bx)^6}{(d+ex)^{14}}\,dx\\ &= \int \left(\frac{(-bd+ae)^6}{e^6(d+ex)^{14}}-\frac{6b(bd-ae)^5}{e^6(d+ex)^{13}}+\frac{15b^2(bd-ae)^4}{e^6(d+ex)^{12}}-\frac{20b^3(bd-ae)^3}{e^6(d+ex)^{11}}+\frac{15b^4(bd-ae)^2}{e^6(d+ex)^{10}}-\frac{6b^5(\ldots}{e^6(\ldots}\right.\\ &= -\frac{(bd-ae)^6}{13e^7(d+ex)^{13}}+\frac{b(bd-ae)^5}{2e^7(d+ex)^{12}}-\frac{15b^2(bd-ae)^4}{11e^7(d+ex)^{11}}+\frac{2b^3(bd-ae)^3}{e^7(d+ex)^{10}}-\frac{5b^4(bd-ae)^2}{3e^7(d+ex)^9}+\frac{3b^5(b\ldots}{4e^7(d\ldots}\end{aligned}$$

Mathematica [A] time = 0.0982966, size = 277, normalized size = 1.62

$$-\frac{28a^2b^4e^2\left(78d^2e^2x^2+13d^3ex+d^4+286de^3x^3+715e^4x^4\right)+84a^3b^3e^3\left(13d^2ex+d^3+78de^2x^2+286e^3x^3\right)+210a^4b^2e^4\left(\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^14,x]
```

```
[Out] -(924*a^6*e^6 + 462*a^5*b*e^5*(d + 13*e*x) + 210*a^4*b^2*e^4*(d^2 + 13*d*e*
x + 78*e^2*x^2) + 84*a^3*b^3*e^3*(d^3 + 13*d^2*e*x + 78*d*e^2*x^2 + 286*e^3
*x^3) + 28*a^2*b^4*e^2*(d^4 + 13*d^3*e*x + 78*d^2*e^2*x^2 + 286*d*e^3*x^3 +
 715*e^4*x^4) + 7*a*b^5*e*(d^5 + 13*d^4*e*x + 78*d^3*e^2*x^2 + 286*d^2*e^3*
x^3 + 715*d*e^4*x^4 + 1287*e^5*x^5) + b^6*(d^6 + 13*d^5*e*x + 78*d^4*e^2*x^
2 + 286*d^3*e^3*x^3 + 715*d^2*e^4*x^4 + 1287*d*e^5*x^5 + 1716*e^6*x^6))/(12
012*e^7*(d + e*x)^13)
```

Maple [B] time = 0.048, size = 357, normalized size = 2.1

$$-2\,\frac{b^3\left(a^3e^3-3\,a^2bde^2+3\,ab^2d^2e-b^3d^3\right)}{e^7\,(ex+d)^{10}}-\frac{b^6}{7\,e^7\,(ex+d)^7}-\frac{b\left(a^5e^5-5\,a^4bde^4+10\,a^3b^2d^2e^3-10\,a^2b^3d^3e^2+5\,ab^4d^4e\right.}{2\,e^7\,(ex+d)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^14,x)
```

```
[Out] -2*b^3*(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d^2*e-b^3*d^3)/e^7/(e*x+d)^10-1/7*b^6
/e^7/(e*x+d)^7-1/2*b*(a^5*e^5-5*a^4*b*d*e^4+10*a^3*b^2*d^2*e^3-10*a^2*b^3*d
^3*e^2+5*a*b^4*d^4*e-b^5*d^5)/e^7/(e*x+d)^12-5/3*b^4*(a^2*e^2-2*a*b*d*e+b^2
*d^2)/e^7/(e*x+d)^9-15/11*b^2*(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-4*a*
b^3*d^3*e+b^4*d^4)/e^7/(e*x+d)^11-1/13*(a^6*e^6-6*a^5*b*d*e^5+15*a^4*b^2*d^
2*e^4-20*a^3*b^3*d^3*e^3+15*a^2*b^4*d^4*e^2-6*a*b^5*d^5*e+b^6*d^6)/e^7/(e*x
+d)^13-3/4*b^5*(a*e-b*d)/e^7/(e*x+d)^8
```

Maxima [B] time = 1.14424, size = 655, normalized size = 3.83

$$-\frac{1716\,b^6e^6x^6+b^6d^6+7\,ab^5d^5e+28\,a^2b^4d^4e^2+84\,a^3b^3d^3e^3+210\,a^4b^2d^2e^4+462\,a^5bde^5+924\,a^6e^6+1287\left(b^6de^5+\right.}{12012\left(e^{20}x^{13}+13\,de^{19}x^{12}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^14,x, algorithm="maxima")
```

```
[Out] -1/12012*(1716*b^6*e^6*x^6 + b^6*d^6 + 7*a*b^5*d^5*e + 28*a^2*b^4*d^4*e^2 +
 84*a^3*b^3*d^3*e^3 + 210*a^4*b^2*d^2*e^4 + 462*a^5*b*d*e^5 + 924*a^6*e^6 +
 1287*(b^6*d*e^5 + 7*a*b^5*e^6)*x^5 + 715*(b^6*d^2*e^4 + 7*a*b^5*d*e^5 + 28
*a^2*b^4*e^6)*x^4 + 286*(b^6*d^3*e^3 + 7*a*b^5*d^2*e^4 + 28*a^2*b^4*d*e^5 +
 84*a^3*b^3*e^6)*x^3 + 78*(b^6*d^4*e^2 + 7*a*b^5*d^3*e^3 + 28*a^2*b^4*d^2*e
^4 + 84*a^3*b^3*d*e^5 + 210*a^4*b^2*e^6)*x^2 + 13*(b^6*d^5*e + 7*a*b^5*d^4*
e^2 + 28*a^2*b^4*d^3*e^3 + 84*a^3*b^3*d^2*e^4 + 210*a^4*b^2*d*e^5 + 462*a^5
*b*e^6)*x)/(e^20*x^13 + 13*d*e^19*x^12 + 78*d^2*e^18*x^11 + 286*d^3*e^17*x^
10 + 715*d^4*e^16*x^9 + 1287*d^5*e^15*x^8 + 1716*d^6*e^14*x^7 + 1716*d^7*e^
13*x^6 + 1287*d^8*e^12*x^5 + 715*d^9*e^11*x^4 + 286*d^10*e^10*x^3 + 78*d^11
*e^9*x^2 + 13*d^12*e^8*x + d^13*e^7)
```

Fricas [B] time = 1.75834, size = 1064, normalized size = 6.22

$$-\frac{1716\,b^6e^6x^6 + b^6d^6 + 7\,ab^5d^5e + 28\,a^2b^4d^4e^2 + 84\,a^3b^3d^3e^3 + 210\,a^4b^2d^2e^4 + 462\,a^5bde^5 + 924\,a^6e^6 + 1287\,\left(b^6de^5 + 7\,ab\right.}{12012\,\left(e^{20}x^{13} + 13\,de^{19}x^{12} + 7\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^14,x, algorithm="fricas")
```

```
[Out] -1/12012*(1716*b^6*e^6*x^6 + b^6*d^6 + 7*a*b^5*d^5*e + 28*a^2*b^4*d^4*e^2 +
 84*a^3*b^3*d^3*e^3 + 210*a^4*b^2*d^2*e^4 + 462*a^5*b*d*e^5 + 924*a^6*e^6 +
 1287*(b^6*d*e^5 + 7*a*b^5*e^6)*x^5 + 715*(b^6*d^2*e^4 + 7*a*b^5*d*e^5 + 28
*a^2*b^4*e^6)*x^4 + 286*(b^6*d^3*e^3 + 7*a*b^5*d^2*e^4 + 28*a^2*b^4*d*e^5 +
 84*a^3*b^3*e^6)*x^3 + 78*(b^6*d^4*e^2 + 7*a*b^5*d^3*e^3 + 28*a^2*b^4*d^2*e
^4 + 84*a^3*b^3*d*e^5 + 210*a^4*b^2*e^6)*x^2 + 13*(b^6*d^5*e + 7*a*b^5*d^4*
e^2 + 28*a^2*b^4*d^3*e^3 + 84*a^3*b^3*d^2*e^4 + 210*a^4*b^2*d*e^5 + 462*a^5
*b*e^6)*x)/(e^20*x^13 + 13*d*e^19*x^12 + 78*d^2*e^18*x^11 + 286*d^3*e^17*x^
10 + 715*d^4*e^16*x^9 + 1287*d^5*e^15*x^8 + 1716*d^6*e^14*x^7 + 1716*d^7*e^
13*x^6 + 1287*d^8*e^12*x^5 + 715*d^9*e^11*x^4 + 286*d^10*e^10*x^3 + 78*d^11
*e^9*x^2 + 13*d^12*e^8*x + d^13*e^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**14,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.13461, size = 475, normalized size = 2.78

$$-\frac{\left(1716\,b^6x^6e^6 + 1287\,b^6dx^5e^5 + 715\,b^6d^2x^4e^4 + 286\,b^6d^3x^3e^3 + 78\,b^6d^4x^2e^2 + 13\,b^6d^5xe + b^6d^6 + 9009\,ab^5x^5e^6 + 5005\,ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^14,x, algorithm="giac")
```

```
[Out] -1/12012*(1716*b^6*x^6*e^6 + 1287*b^6*d*x^5*e^5 + 715*b^6*d^2*x^4*e^4 + 286
*b^6*d^3*x^3*e^3 + 78*b^6*d^4*x^2*e^2 + 13*b^6*d^5*x*e + b^6*d^6 + 9009*a*b
^5*x^5*e^6 + 5005*a*b^5*d*x^4*e^5 + 2002*a*b^5*d^2*x^3*e^4 + 546*a*b^5*d^3*
x^2*e^3 + 91*a*b^5*d^4*x*e^2 + 7*a*b^5*d^5*e + 20020*a^2*b^4*x^4*e^6 + 8008
*a^2*b^4*d*x^3*e^5 + 2184*a^2*b^4*d^2*x^2*e^4 + 364*a^2*b^4*d^3*x*e^3 + 28*
a^2*b^4*d^4*e^2 + 24024*a^3*b^3*x^3*e^6 + 6552*a^3*b^3*d*x^2*e^5 + 1092*a^3
*b^3*d^2*x*e^4 + 84*a^3*b^3*d^3*e^3 + 16380*a^4*b^2*x^2*e^6 + 2730*a^4*b^2*
d*x*e^5 + 210*a^4*b^2*d^2*e^4 + 6006*a^5*b*x*e^6 + 462*a^5*b*d*e^5 + 924*a^
6*e^6)*e^(-7)/(x*e + d)^13
```

3.1504 $\int \frac{\left(a^2+2abx+b^2x^2\right)^3}{(d+ex)^{15}}\,dx$

Optimal. Leaf size=173

$$\frac{2b^5(bd-ae)}{3e^7(d+ex)^9}-\frac{3b^4(bd-ae)^2}{2e^7(d+ex)^{10}}+\frac{20b^3(bd-ae)^3}{11e^7(d+ex)^{11}}-\frac{5b^2(bd-ae)^4}{4e^7(d+ex)^{12}}+\frac{6b(bd-ae)^5}{13e^7(d+ex)^{13}}-\frac{(bd-ae)^6}{14e^7(d+ex)^{14}}-\frac{b^6}{8e^7(d+ex)^8}$$

```
[Out] -(b*d - a*e)^6/(14*e^7*(d + e*x)^14) + (6*b*(b*d - a*e)^5)/(13*e^7*(d + e*x
)^13) - (5*b^2*(b*d - a*e)^4)/(4*e^7*(d + e*x)^12) + (20*b^3*(b*d - a*e)^3)
/(11*e^7*(d + e*x)^11) - (3*b^4*(b*d - a*e)^2)/(2*e^7*(d + e*x)^10) + (2*b^
5*(b*d - a*e))/(3*e^7*(d + e*x)^9) - b^6/(8*e^7*(d + e*x)^8)
```

Rubi [A] time = 0.115182, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$\frac{2b^5(bd-ae)}{3e^7(d+ex)^9}-\frac{3b^4(bd-ae)^2}{2e^7(d+ex)^{10}}+\frac{20b^3(bd-ae)^3}{11e^7(d+ex)^{11}}-\frac{5b^2(bd-ae)^4}{4e^7(d+ex)^{12}}+\frac{6b(bd-ae)^5}{13e^7(d+ex)^{13}}-\frac{(bd-ae)^6}{14e^7(d+ex)^{14}}-\frac{b^6}{8e^7(d+ex)^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^15,x]
```


```
[Out] -(b*d - a*e)^6/(14*e^7*(d + e*x)^14) + (6*b*(b*d - a*e)^5)/(13*e^7*(d + e*x
)^13) - (5*b^2*(b*d - a*e)^4)/(4*e^7*(d + e*x)^12) + (20*b^3*(b*d - a*e)^3)
/(11*e^7*(d + e*x)^11) - (3*b^4*(b*d - a*e)^2)/(2*e^7*(d + e*x)^10) + (2*b^
5*(b*d - a*e))/(3*e^7*(d + e*x)^9) - b^6/(8*e^7*(d + e*x)^8)
```


Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx+b^2x^2\right)^3}{(d+ex)^{15}}\,dx &= \int \frac{(a+bx)^6}{(d+ex)^{15}}\,dx\\ &= \int \left(\frac{(-bd+ae)^6}{e^6(d+ex)^{15}}-\frac{6b(bd-ae)^5}{e^6(d+ex)^{14}}+\frac{15b^2(bd-ae)^4}{e^6(d+ex)^{13}}-\frac{20b^3(bd-ae)^3}{e^6(d+ex)^{12}}+\frac{15b^4(bd-ae)^2}{e^6(d+ex)^{11}}-\right.\\ &= -\frac{(bd-ae)^6}{14e^7(d+ex)^{14}}+\frac{6b(bd-ae)^5}{13e^7(d+ex)^{13}}-\frac{5b^2(bd-ae)^4}{4e^7(d+ex)^{12}}+\frac{20b^3(bd-ae)^3}{11e^7(d+ex)^{11}}-\frac{3b^4(bd-ae)^2}{2e^7(d+ex)^{10}}+\end{aligned}$$

Mathematica [A] time = 0.094877, size = 277, normalized size = 1.6

$$-\frac{36a^2b^4e^2\left(91d^2e^2x^2+14d^3ex+d^4+364de^3x^3+1001e^4x^4\right)+120a^3b^3e^3\left(14d^2ex+d^3+91de^2x^2+364e^3x^3\right)+330a^4}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^15,x]
```

```
[Out] -(1716*a^6*e^6 + 792*a^5*b*e^5*(d + 14*e*x) + 330*a^4*b^2*e^4*(d^2 + 14*d*e
*x + 91*e^2*x^2) + 120*a^3*b^3*e^3*(d^3 + 14*d^2*e*x + 91*d*e^2*x^2 + 364*e
^3*x^3) + 36*a^2*b^4*e^2*(d^4 + 14*d^3*e*x + 91*d^2*e^2*x^2 + 364*d*e^3*x^3
 + 1001*e^4*x^4) + 8*a*b^5*e*(d^5 + 14*d^4*e*x + 91*d^3*e^2*x^2 + 364*d^2*e
^3*x^3 + 1001*d*e^4*x^4 + 2002*e^5*x^5) + b^6*(d^6 + 14*d^5*e*x + 91*d^4*e^
2*x^2 + 364*d^3*e^3*x^3 + 1001*d^2*e^4*x^4 + 2002*d*e^5*x^5 + 3003*e^6*x^6)
)/(24024*e^7*(d + e*x)^14)
```

Maple [B] time = 0.048, size = 357, normalized size = 2.1

$$-\frac{3\,b^4\left(a^2e^2-2\,abde+b^2d^2\right)}{2\,e^7\,(ex+d)^{10}}-\frac{2\,b^5\,(ae-bd)}{3\,e^7\,(ex+d)^9}-\frac{5\,b^2\left(a^4e^4-4\,a^3bde^3+6\,d^2e^2b^2a^2-4\,ab^3d^3e+b^4d^4\right)}{4\,e^7\,(ex+d)^{12}}-\frac{e^6a^6-6\,a^5bde^5+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^15,x)
```

```
[Out] -3/2*b^4*(a^2*e^2-2*a*b*d*e+b^2*d^2)/e^7/(e*x+d)^10-2/3*b^5*(a*e-b*d)/e^7/(
e*x+d)^9-5/4*b^2*(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-4*a*b^3*d^3*e+b^4
*d^4)/e^7/(e*x+d)^12-1/14*(a^6*e^6-6*a^5*b*d*e^5+15*a^4*b^2*d^2*e^4-20*a^3*
b^3*d^3*e^3+15*a^2*b^4*d^4*e^2-6*a*b^5*d^5*e+b^6*d^6)/e^7/(e*x+d)^14-20/11*
b^3*(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d^2*e-b^3*d^3)/e^7/(e*x+d)^11-6/13*b*(a^
5*e^5-5*a^4*b*d*e^4+10*a^3*b^2*d^2*e^3-10*a^2*b^3*d^3*e^2+5*a*b^4*d^4*e-b^5
*d^5)/e^7/(e*x+d)^13-1/8*b^6/e^7/(e*x+d)^8
```

Maxima [B] time = 1.24489, size = 670, normalized size = 3.87

$$-\frac{3003\,b^6e^6x^6+b^6d^6+8\,ab^5d^5e+36\,a^2b^4d^4e^2+120\,a^3b^3d^3e^3+330\,a^4b^2d^2e^4+792\,a^5bde^5+1716\,a^6e^6+2002\left(b^6de^5+8\cdots\right.}{24024\left(e^{21}x^{14}+14\,de^{20}x^{13}+91\,d^2e^{19}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^15,x, algorithm="maxima")
```

```
[Out] -1/24024*(3003*b^6*e^6*x^6 + b^6*d^6 + 8*a*b^5*d^5*e + 36*a^2*b^4*d^4*e^2 +
 120*a^3*b^3*d^3*e^3 + 330*a^4*b^2*d^2*e^4 + 792*a^5*b*d*e^5 + 1716*a^6*e^6
 + 2002*(b^6*d*e^5 + 8*a*b^5*e^6)*x^5 + 1001*(b^6*d^2*e^4 + 8*a*b^5*d*e^5 +
 36*a^2*b^4*e^6)*x^4 + 364*(b^6*d^3*e^3 + 8*a*b^5*d^2*e^4 + 36*a^2*b^4*d*e^
5 + 120*a^3*b^3*e^6)*x^3 + 91*(b^6*d^4*e^2 + 8*a*b^5*d^3*e^3 + 36*a^2*b^4*d
^2*e^4 + 120*a^3*b^3*d*e^5 + 330*a^4*b^2*e^6)*x^2 + 14*(b^6*d^5*e + 8*a*b^5
*d^4*e^2 + 36*a^2*b^4*d^3*e^3 + 120*a^3*b^3*d^2*e^4 + 330*a^4*b^2*d*e^5 + 7
92*a^5*b*e^6)*x)/(e^21*x^14 + 14*d*e^20*x^13 + 91*d^2*e^19*x^12 + 364*d^3*e
^18*x^11 + 1001*d^4*e^17*x^10 + 2002*d^5*e^16*x^9 + 3003*d^6*e^15*x^8 + 343
2*d^7*e^14*x^7 + 3003*d^8*e^13*x^6 + 2002*d^9*e^12*x^5 + 1001*d^10*e^11*x^4
 + 364*d^11*e^10*x^3 + 91*d^12*e^9*x^2 + 14*d^13*e^8*x + d^14*e^7)
```

Fricas [B] time = 1.7311, size = 1104, normalized size = 6.38

$$-\frac{3003\,b^6e^6x^6 + b^6d^6 + 8\,ab^5d^5e + 36\,a^2b^4d^4e^2 + 120\,a^3b^3d^3e^3 + 330\,a^4b^2d^2e^4 + 792\,a^5bde^5 + 1716\,a^6e^6 + 2002\,\left(b^6de^5\right.}{24024\,\left(e^{21}x^{14} + 14\,de^{20}x^{13} + 91\,d^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^15,x, algorithm="fricas")
```

```
[Out] -1/24024*(3003*b^6*e^6*x^6 + b^6*d^6 + 8*a*b^5*d^5*e + 36*a^2*b^4*d^4*e^2 +
 120*a^3*b^3*d^3*e^3 + 330*a^4*b^2*d^2*e^4 + 792*a^5*b*d*e^5 + 1716*a^6*e^6
 + 2002*(b^6*d*e^5 + 8*a*b^5*e^6)*x^5 + 1001*(b^6*d^2*e^4 + 8*a*b^5*d*e^5 +
 36*a^2*b^4*e^6)*x^4 + 364*(b^6*d^3*e^3 + 8*a*b^5*d^2*e^4 + 36*a^2*b^4*d*e^
5 + 120*a^3*b^3*e^6)*x^3 + 91*(b^6*d^4*e^2 + 8*a*b^5*d^3*e^3 + 36*a^2*b^4*d
^2*e^4 + 120*a^3*b^3*d*e^5 + 330*a^4*b^2*e^6)*x^2 + 14*(b^6*d^5*e + 8*a*b^5
*d^4*e^2 + 36*a^2*b^4*d^3*e^3 + 120*a^3*b^3*d^2*e^4 + 330*a^4*b^2*d*e^5 + 7
92*a^5*b*e^6)*x)/(e^21*x^14 + 14*d*e^20*x^13 + 91*d^2*e^19*x^12 + 364*d^3*e
^18*x^11 + 1001*d^4*e^17*x^10 + 2002*d^5*e^16*x^9 + 3003*d^6*e^15*x^8 + 343
2*d^7*e^14*x^7 + 3003*d^8*e^13*x^6 + 2002*d^9*e^12*x^5 + 1001*d^10*e^11*x^4
 + 364*d^11*e^10*x^3 + 91*d^12*e^9*x^2 + 14*d^13*e^8*x + d^14*e^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**15,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.15519, size = 475, normalized size = 2.75

$$-\frac{\left(3003\,b^6x^6e^6 + 2002\,b^6dx^5e^5 + 1001\,b^6d^2x^4e^4 + 364\,b^6d^3x^3e^3 + 91\,b^6d^4x^2e^2 + 14\,b^6d^5xe + b^6d^6 + 16016\,ab^5x^5e^6 + 80\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^15,x, algorithm="giac")
```

```
[Out] -1/24024*(3003*b^6*x^6*e^6 + 2002*b^6*d*x^5*e^5 + 1001*b^6*d^2*x^4*e^4 + 36
4*b^6*d^3*x^3*e^3 + 91*b^6*d^4*x^2*e^2 + 14*b^6*d^5*x*e + b^6*d^6 + 16016*a
*b^5*x^5*e^6 + 8008*a*b^5*d*x^4*e^5 + 2912*a*b^5*d^2*x^3*e^4 + 728*a*b^5*d^
3*x^2*e^3 + 112*a*b^5*d^4*x*e^2 + 8*a*b^5*d^5*e + 36036*a^2*b^4*x^4*e^6 + 1
3104*a^2*b^4*d*x^3*e^5 + 3276*a^2*b^4*d^2*x^2*e^4 + 504*a^2*b^4*d^3*x*e^3 +
 36*a^2*b^4*d^4*e^2 + 43680*a^3*b^3*x^3*e^6 + 10920*a^3*b^3*d*x^2*e^5 + 168
0*a^3*b^3*d^2*x*e^4 + 120*a^3*b^3*d^3*e^3 + 30030*a^4*b^2*x^2*e^6 + 4620*a^
4*b^2*d*x*e^5 + 330*a^4*b^2*d^2*e^4 + 11088*a^5*b*x*e^6 + 792*a^5*b*d*e^5 +
 1716*a^6*e^6)*e^(-7)/(x*e + d)^14
```

3.1505 $\int \frac{(d+ex)^5}{a^2+2abx+b^2x^2}\,dx$

Optimal. Leaf size=131

$$\frac{5e^4(a+bx)^3(bd-ae)}{3b^6}+\frac{5e^3(a+bx)^2(bd-ae)^2}{b^6}+\frac{10e^2x(bd-ae)^3}{b^5}-\frac{(bd-ae)^5}{b^6(a+bx)}+\frac{5e(bd-ae)^4\log(a+bx)}{b^6}+\frac{e^5(a+bx)^4}{4b^6}$$

```
[Out] (10*e^2*(b*d - a*e)^3*x)/b^5 - (b*d - a*e)^5/(b^6*(a + b*x)) + (5*e^3*(b*d
- a*e)^2*(a + b*x)^2)/b^6 + (5*e^4*(b*d - a*e)*(a + b*x)^3)/(3*b^6) + (e^5*
(a + b*x)^4)/(4*b^6) + (5*e*(b*d - a*e)^4*Log[a + b*x])/b^6
```

Rubi [A] time = 0.146367, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$\frac{5e^4(a+bx)^3(bd-ae)}{3b^6}+\frac{5e^3(a+bx)^2(bd-ae)^2}{b^6}+\frac{10e^2x(bd-ae)^3}{b^5}-\frac{(bd-ae)^5}{b^6(a+bx)}+\frac{5e(bd-ae)^4\log(a+bx)}{b^6}+\frac{e^5(a+bx)^4}{4b^6}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5/(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] (10*e^2*(b*d - a*e)^3*x)/b^5 - (b*d - a*e)^5/(b^6*(a + b*x)) + (5*e^3*(b*d
- a*e)^2*(a + b*x)^2)/b^6 + (5*e^4*(b*d - a*e)*(a + b*x)^3)/(3*b^6) + (e^5*
(a + b*x)^4)/(4*b^6) + (5*e*(b*d - a*e)^4*Log[a + b*x])/b^6
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^5}{a^2+2abx+b^2x^2}\,dx &= \int \frac{(d+ex)^5}{(a+bx)^2}\,dx\\ &= \int \left(\frac{10e^2(bd-ae)^3}{b^5}+\frac{(bd-ae)^5}{b^5(a+bx)^2}+\frac{5e(bd-ae)^4}{b^5(a+bx)}+\frac{10e^3(bd-ae)^2(a+bx)}{b^5}+\frac{5e^4(bd-ae)(a+b\right.\\ &= \frac{10e^2(bd-ae)^3x}{b^5}-\frac{(bd-ae)^5}{b^6(a+bx)}+\frac{5e^3(bd-ae)^2(a+bx)^2}{b^6}+\frac{5e^4(bd-ae)(a+bx)^3}{3b^6}+\frac{e^5(a+bx)^4}{4b^6}\end{aligned}$$

Mathematica [A] time = 0.0783108, size = 230, normalized size = 1.76

$$\frac{10a^2b^3e^2\left(-24d^2ex-12d^3+12de^2x^2+e^3x^3\right)+30a^3b^2e^3\left(4d^2+6dex-e^2x^2\right)-12a^4be^4(5d+4ex)+12a^5e^5-5ab^4e\left(36d^2\right.}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5/(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] (12*a^5*e^5 - 12*a^4*b*e^4*(5*d + 4*e*x) + 30*a^3*b^2*e^3*(4*d^2 + 6*d*e*x
- e^2*x^2) + 10*a^2*b^3*e^2*(-12*d^3 - 24*d^2*e*x + 12*d*e^2*x^2 + e^3*x^3)
 - 5*a*b^4*e*(-12*d^4 - 24*d^3*e*x + 36*d^2*e^2*x^2 + 8*d*e^3*x^3 + e^4*x^4
) + b^5*(-12*d^5 + 120*d^3*e^2*x^2 + 60*d^2*e^3*x^3 + 20*d*e^4*x^4 + 3*e^5*
x^5) + 60*e*(b*d - a*e)^4*(a + b*x)*Log[a + b*x])/(12*b^6*(a + b*x))
```

Maple [B] time = 0.048, size = 326, normalized size = 2.5

$$\frac{e^5x^4}{4\,b^2}-\frac{2\,e^5x^3a}{3\,b^3}+\frac{5\,e^4x^3d}{3\,b^2}+\frac{3\,e^5x^2a^2}{2\,b^4}-5\,\frac{e^4x^2ad}{b^3}+5\,\frac{e^3x^2d^2}{b^2}-4\,\frac{a^3e^5x}{b^5}+15\,\frac{a^2de^4x}{b^4}-20\,\frac{ad^2e^3x}{b^3}+10\,\frac{d^3e^2x}{b^2}+5\,\frac{e^5\ln(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2),x)
```

```
[Out] 1/4*e^5/b^2*x^4-2/3*e^5/b^3*x^3*a+5/3*e^4/b^2*x^3*d+3/2*e^5/b^4*x^2*a^2-5*e
^4/b^3*x^2*a*d+5*e^3/b^2*x^2*d^2-4*e^5/b^5*a^3*x+15*e^4/b^4*a^2*d*x-20*e^3/
b^3*a*d^2*x+10*e^2/b^2*d^3*x+5/b^6*e^5*ln(b*x+a)*a^4-20/b^5*e^4*ln(b*x+a)*a
^3*d+30/b^4*e^3*ln(b*x+a)*d^2*a^2-20/b^3*e^2*ln(b*x+a)*a*d^3+5/b^2*e*ln(b*x
+a)*d^4+1/b^6/(b*x+a)*a^5*e^5-5/b^5/(b*x+a)*a^4*d*e^4+10/b^4/(b*x+a)*a^3*d^
2*e^3-10/b^3/(b*x+a)*a^2*d^3*e^2+5/b^2/(b*x+a)*a*d^4*e-1/b/(b*x+a)*d^5
```

Maxima [B] time = 1.1607, size = 358, normalized size = 2.73

$$-\frac{b^5d^5-5\,ab^4d^4e+10\,a^2b^3d^3e^2-10\,a^3b^2d^2e^3+5\,a^4bde^4-a^5e^5}{b^7x+ab^6}+\frac{3\,b^3e^5x^4+4\left(5\,b^3de^4-2\,ab^2e^5\right)x^3+6\left(10\,b^3d^2e^3-1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

```
[Out] -(b^5*d^5 - 5*a*b^4*d^4*e + 10*a^2*b^3*d^3*e^2 - 10*a^3*b^2*d^2*e^3 + 5*a^4
*b*d*e^4 - a^5*e^5)/(b^7*x + a*b^6) + 1/12*(3*b^3*e^5*x^4 + 4*(5*b^3*d*e^4
- 2*a*b^2*e^5)*x^3 + 6*(10*b^3*d^2*e^3 - 10*a*b^2*d*e^4 + 3*a^2*b*e^5)*x^2
+ 12*(10*b^3*d^3*e^2 - 20*a*b^2*d^2*e^3 + 15*a^2*b*d*e^4 - 4*a^3*e^5)*x)/b^
5 + 5*(b^4*d^4*e - 4*a*b^3*d^3*e^2 + 6*a^2*b^2*d^2*e^3 - 4*a^3*b*d*e^4 + a^
4*e^5)*log(b*x + a)/b^6
```

Fricas [B] time = 1.81274, size = 767, normalized size = 5.85

$$\frac{3\,b^5e^5x^5-12\,b^5d^5+60\,ab^4d^4e-120\,a^2b^3d^3e^2+120\,a^3b^2d^2e^3-60\,a^4bde^4+12\,a^5e^5+5\left(4\,b^5de^4-ab^4e^5\right)x^4+10\left(6\,b^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^5*e^5*x^5 - 12*b^5*d^5 + 60*a*b^4*d^4*e - 120*a^2*b^3*d^3*e^2 + 1
20*a^3*b^2*d^2*e^3 - 60*a^4*b*d*e^4 + 12*a^5*e^5 + 5*(4*b^5*d*e^4 - a*b^4*e
^5)*x^4 + 10*(6*b^5*d^2*e^3 - 4*a*b^4*d*e^4 + a^2*b^3*e^5)*x^3 + 30*(4*b^5*
d^3*e^2 - 6*a*b^4*d^2*e^3 + 4*a^2*b^3*d*e^4 - a^3*b^2*e^5)*x^2 + 12*(10*a*b
^4*d^3*e^2 - 20*a^2*b^3*d^2*e^3 + 15*a^3*b^2*d*e^4 - 4*a^4*b*e^5)*x + 60*(a
*b^4*d^4*e - 4*a^2*b^3*d^3*e^2 + 6*a^3*b^2*d^2*e^3 - 4*a^4*b*d*e^4 + a^5*e^
5 + (b^5*d^4*e - 4*a*b^4*d^3*e^2 + 6*a^2*b^3*d^2*e^3 - 4*a^3*b^2*d*e^4 + a^
4*b*e^5)*x)*log(b*x + a))/(b^7*x + a*b^6)
```

Sympy [A] time = 1.20108, size = 224, normalized size = 1.71

$$\frac{a^5e^5-5a^4bde^4+10a^3b^2d^2e^3-10a^2b^3d^3e^2+5ab^4d^4e-b^5d^5}{ab^6+b^7x}+\frac{e^5x^4}{4b^2}-\frac{x^3\left(2ae^5-5bde^4\right)}{3b^3}+\frac{x^2\left(3a^2e^5-10abde^4+10b^2d^2e\right.}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5/(b**2*x**2+2*a*b*x+a**2),x)
```

```
[Out] (a**5*e**5 - 5*a**4*b*d*e**4 + 10*a**3*b**2*d**2*e**3 - 10*a**2*b**3*d**3*e
**2 + 5*a*b**4*d**4*e - b**5*d**5)/(a*b**6 + b**7*x) + e**5*x**4/(4*b**2) -
 x**3*(2*a*e**5 - 5*b*d*e**4)/(3*b**3) + x**2*(3*a**2*e**5 - 10*a*b*d*e**4
+ 10*b**2*d**2*e**3)/(2*b**4) - x*(4*a**3*e**5 - 15*a**2*b*d*e**4 + 20*a*b*
*2*d**2*e**3 - 10*b**3*d**3*e**2)/b**5 + 5*e*(a*e - b*d)**4*log(a + b*x)/b*
*6
```

Giac [B] time = 1.15016, size = 347, normalized size = 2.65

$$\frac{5\left(b^4d^4e-4\,ab^3d^3e^2+6\,a^2b^2d^2e^3-4\,a^3bde^4+a^4e^5\right)\log\left(|bx+a|\right)}{b^6}-\frac{b^5d^5-5\,ab^4d^4e+10\,a^2b^3d^3e^2-10\,a^3b^2d^2e^3+5\,a^4bd}{(bx+a)b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

```
[Out] 5*(b^4*d^4*e - 4*a*b^3*d^3*e^2 + 6*a^2*b^2*d^2*e^3 - 4*a^3*b*d*e^4 + a^4*e^
5)*log(abs(b*x + a))/b^6 - (b^5*d^5 - 5*a*b^4*d^4*e + 10*a^2*b^3*d^3*e^2 -
10*a^3*b^2*d^2*e^3 + 5*a^4*b*d*e^4 - a^5*e^5)/((b*x + a)*b^6) + 1/12*(3*b^6
*x^4*e^5 + 20*b^6*d*x^3*e^4 + 60*b^6*d^2*x^2*e^3 + 120*b^6*d^3*x*e^2 - 8*a*
b^5*x^3*e^5 - 60*a*b^5*d*x^2*e^4 - 240*a*b^5*d^2*x*e^3 + 18*a^2*b^4*x^2*e^5
 + 180*a^2*b^4*d*x*e^4 - 48*a^3*b^3*x*e^5)/b^8
```

3.1506 $\int \frac{(d+ex)^4}{a^2+2abx+b^2x^2}\,dx$

Optimal. Leaf size=104

$$\frac{2e^3(a+bx)^2(bd-ae)}{b^5}+\frac{6e^2x(bd-ae)^2}{b^4}-\frac{(bd-ae)^4}{b^5(a+bx)}+\frac{4e(bd-ae)^3\log(a+bx)}{b^5}+\frac{e^4(a+bx)^3}{3b^5}$$

```
[Out] (6*e^2*(b*d - a*e)^2*x)/b^4 - (b*d - a*e)^4/(b^5*(a + b*x)) + (2*e^3*(b*d -
 a*e)*(a + b*x)^2)/b^5 + (e^4*(a + b*x)^3)/(3*b^5) + (4*e*(b*d - a*e)^3*Log
[a + b*x])/b^5
```

Rubi [A] time = 0.102178, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$\frac{2e^3(a+bx)^2(bd-ae)}{b^5}+\frac{6e^2x(bd-ae)^2}{b^4}-\frac{(bd-ae)^4}{b^5(a+bx)}+\frac{4e(bd-ae)^3\log(a+bx)}{b^5}+\frac{e^4(a+bx)^3}{3b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] (6*e^2*(b*d - a*e)^2*x)/b^4 - (b*d - a*e)^4/(b^5*(a + b*x)) + (2*e^3*(b*d -
 a*e)*(a + b*x)^2)/b^5 + (e^4*(a + b*x)^3)/(3*b^5) + (4*e*(b*d - a*e)^3*Log
[a + b*x])/b^5
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^4}{a^2+2abx+b^2x^2}\,dx &= \int \frac{(d+ex)^4}{(a+bx)^2}\,dx\\ &= \int \left(\frac{6e^2(bd-ae)^2}{b^4}+\frac{(bd-ae)^4}{b^4(a+bx)^2}+\frac{4e(bd-ae)^3}{b^4(a+bx)}+\frac{4e^3(bd-ae)(a+bx)}{b^4}+\frac{e^4(a+bx)^2}{b^4}\right)dx\\ &= \frac{6e^2(bd-ae)^2x}{b^4}-\frac{(bd-ae)^4}{b^5(a+bx)}+\frac{2e^3(bd-ae)(a+bx)^2}{b^5}+\frac{e^4(a+bx)^3}{3b^5}+\frac{4e(bd-ae)^3\log(a+b}{b^5}\end{aligned}$$

Mathematica [A] time = 0.0637961, size = 166, normalized size = 1.6

$$\frac{6a^2b^2e^2\left(-3d^2-4dex+e^2x^2\right)+3a^3be^3(4d+3ex)-3a^4e^4-2ab^3e\left(-9d^2ex-6d^3+9de^2x^2+e^3x^3\right)-12e(a+bx)(ae-}{3b^5(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] (-3*a^4*e^4 + 3*a^3*b*e^3*(4*d + 3*e*x) + 6*a^2*b^2*e^2*(-3*d^2 - 4*d*e*x +
 e^2*x^2) - 2*a*b^3*e*(-6*d^3 - 9*d^2*e*x + 9*d*e^2*x^2 + e^3*x^3) + b^4*(-
3*d^4 + 18*d^2*e^2*x^2 + 6*d*e^3*x^3 + e^4*x^4) - 12*e*(-(b*d) + a*e)^3*(a
+ b*x)*Log[a + b*x])/(3*b^5*(a + b*x))
```

Maple [B] time = 0.048, size = 230, normalized size = 2.2

$$\frac{e^4x^3}{3\,b^2}-\frac{e^4x^2a}{b^3}+2\,\frac{e^3x^2d}{b^2}+3\,\frac{a^2e^4x}{b^4}-8\,\frac{ade^3x}{b^3}+6\,\frac{d^2e^2x}{b^2}-4\,\frac{e^4\ln(bx+a)\,a^3}{b^5}+12\,\frac{e^3\ln(bx+a)\,a^2d}{b^4}-12\,\frac{e^2\ln(bx+a)\,ad^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2),x)
```

```
[Out] 1/3*e^4/b^2*x^3-e^4/b^3*x^2*a+2*e^3/b^2*x^2*d+3*e^4/b^4*a^2*x-8*e^3/b^3*a*d
*x+6*e^2/b^2*d^2*x-4/b^5*e^4*ln(b*x+a)*a^3+12/b^4*e^3*ln(b*x+a)*a^2*d-12/b^
3*e^2*ln(b*x+a)*a*d^2+4/b^2*e*ln(b*x+a)*d^3-1/b^5/(b*x+a)*a^4*e^4+4/b^4/(b*
x+a)*a^3*d*e^3-6/b^3/(b*x+a)*d^2*e^2*a^2+4/b^2/(b*x+a)*a*d^3*e-1/b/(b*x+a)*
d^4
```

Maple [A] time = 1.1323, size = 248, normalized size = 2.38

$$-\frac{b^4d^4-4\,ab^3d^3e+6\,a^2b^2d^2e^2-4\,a^3bde^3+a^4e^4}{b^6x+ab^5}+\frac{b^2e^4x^3+3\left(2\,b^2de^3-abe^4\right)x^2+3\left(6\,b^2d^2e^2-8\,abde^3+3\,a^2e^4\right)x}{3\,b^4}+\frac{4(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

```
[Out] -(b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4)/(b
^6*x + a*b^5) + 1/3*(b^2*e^4*x^3 + 3*(2*b^2*d*e^3 - a*b*e^4)*x^2 + 3*(6*b^2
*d^2*e^2 - 8*a*b*d*e^3 + 3*a^2*e^4)*x)/b^4 + 4*(b^3*d^3*e - 3*a*b^2*d^2*e^2
 + 3*a^2*b*d*e^3 - a^3*e^4)*log(b*x + a)/b^5
```

Fricas [B] time = 1.75765, size = 540, normalized size = 5.19

$$\frac{b^4e^4x^4-3\,b^4d^4+12\,ab^3d^3e-18\,a^2b^2d^2e^2+12\,a^3bde^3-3\,a^4e^4+2\left(3\,b^4de^3-ab^3e^4\right)x^3+6\left(3\,b^4d^2e^2-3\,ab^3de^3+a^2b^2e^4\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*e^4*x^4 - 3*b^4*d^4 + 12*a*b^3*d^3*e - 18*a^2*b^2*d^2*e^2 + 12*a^3
*b*d*e^3 - 3*a^4*e^4 + 2*(3*b^4*d*e^3 - a*b^3*e^4)*x^3 + 6*(3*b^4*d^2*e^2 -
 3*a*b^3*d*e^3 + a^2*b^2*e^4)*x^2 + 3*(6*a*b^3*d^2*e^2 - 8*a^2*b^2*d*e^3 +
3*a^3*b*e^4)*x + 12*(a*b^3*d^3*e - 3*a^2*b^2*d^2*e^2 + 3*a^3*b*d*e^3 - a^4*
```

```
e^4 + (b^4*d^3*e - 3*a*b^3*d^2*e^2 + 3*a^2*b^2*d*e^3 - a^3*b*e^4)*x)*log(b*
x + a))/(b^6*x + a*b^5)
```

Sympy [A] time = 0.978266, size = 151, normalized size = 1.45

$$-\frac{a^4e^4-4a^3bde^3+6a^2b^2d^2e^2-4ab^3d^3e+b^4d^4}{ab^5+b^6x}+\frac{e^4x^3}{3b^2}-\frac{x^2\left(ae^4-2bde^3\right)}{b^3}+\frac{x\left(3a^2e^4-8abde^3+6b^2d^2e^2\right)}{b^4}-\frac{4e\left(ae-b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(b**2*x**2+2*a*b*x+a**2),x)
```

```
[Out] -(a**4*e**4 - 4*a**3*b*d*e**3 + 6*a**2*b**2*d**2*e**2 - 4*a*b**3*d**3*e + b
**4*d**4)/(a*b**5 + b**6*x) + e**4*x**3/(3*b**2) - x**2*(a*e**4 - 2*b*d*e**
3)/b**3 + x*(3*a**2*e**4 - 8*a*b*d*e**3 + 6*b**2*d**2*e**2)/b**4 - 4*e*(a*e
 - b*d)**3*log(a + b*x)/b**5
```

Giac [A] time = 1.19414, size = 240, normalized size = 2.31

$$\frac{4\left(b^3d^3e-3\,ab^2d^2e^2+3\,a^2bde^3-a^3e^4\right)\log\left(|bx+a|\right)}{b^5}+\frac{b^4x^3e^4+6\,b^4dx^2e^3+18\,b^4d^2xe^2-3\,ab^3x^2e^4-24\,ab^3dxe^3+9\,a}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

```
[Out] 4*(b^3*d^3*e - 3*a*b^2*d^2*e^2 + 3*a^2*b*d*e^3 - a^3*e^4)*log(abs(b*x + a))
/b^5 + 1/3*(b^4*x^3*e^4 + 6*b^4*d*x^2*e^3 + 18*b^4*d^2*x*e^2 - 3*a*b^3*x^2*
e^4 - 24*a*b^3*d*x*e^3 + 9*a^2*b^2*x*e^4)/b^6 - (b^4*d^4 - 4*a*b^3*d^3*e +
6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4)/((b*x + a)*b^5)
```

3.1507 $\int \frac{(d+ex)^3}{a^2+2abx+b^2x^2}\,dx$

Optimal. Leaf size=75

$$\frac{e^2x(3bd-2ae)}{b^3}-\frac{(bd-ae)^3}{b^4(a+bx)}+\frac{3e(bd-ae)^2\log(a+bx)}{b^4}+\frac{e^3x^2}{2b^2}$$

```
[Out] (e^2*(3*b*d - 2*a*e)*x)/b^3 + (e^3*x^2)/(2*b^2) - (b*d - a*e)^3/(b^4*(a + b
*x)) + (3*e*(b*d - a*e)^2*Log[a + b*x])/b^4
```

Rubi [A] time = 0.0641327, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$\frac{e^2x(3bd-2ae)}{b^3}-\frac{(bd-ae)^3}{b^4(a+bx)}+\frac{3e(bd-ae)^2\log(a+bx)}{b^4}+\frac{e^3x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] (e^2*(3*b*d - 2*a*e)*x)/b^3 + (e^3*x^2)/(2*b^2) - (b*d - a*e)^3/(b^4*(a + b
*x)) + (3*e*(b*d - a*e)^2*Log[a + b*x])/b^4
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^3}{a^2+2abx+b^2x^2}\,dx &= \int \frac{(d+ex)^3}{(a+bx)^2}\,dx\\ &= \int \left(\frac{e^2(3bd-2ae)}{b^3}+\frac{e^3x}{b^2}+\frac{(bd-ae)^3}{b^3(a+bx)^2}+\frac{3e(bd-ae)^2}{b^3(a+bx)}\right)dx\\ &= \frac{e^2(3bd-2ae)x}{b^3}+\frac{e^3x^2}{2b^2}-\frac{(bd-ae)^3}{b^4(a+bx)}+\frac{3e(bd-ae)^2\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.0519787, size = 72, normalized size = 0.96

$$\frac{2be^2x(3bd-2ae)-\frac{2(bd-ae)^3}{a+bx}+6e(bd-ae)^2\log(a+bx)+b^2e^3x^2}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] (2*b*e^2*(3*b*d - 2*a*e)*x + b^2*e^3*x^2 - (2*(b*d - a*e)^3)/(a + b*x) + 6*
e*(b*d - a*e)^2*Log[a + b*x])/(2*b^4)
```

Maple [B] time = 0.048, size = 149, normalized size = 2.

$$\frac{e^3x^2}{2\,b^2}-2\,\frac{ae^3x}{b^3}+3\,\frac{e^2xd}{b^2}+3\,\frac{e^3\ln(bx+a)\,a^2}{b^4}-6\,\frac{e^2\ln(bx+a)\,ad}{b^3}+3\,\frac{e\ln(bx+a)\,d^2}{b^2}+\frac{a^3e^3}{b^4\,(bx+a)}-3\,\frac{a^2de^2}{b^3\,(bx+a)}+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2),x)
```

```
[Out] 1/2*e^3*x^2/b^2-2*e^3/b^3*a*x+3*e^2/b^2*x*d+3/b^4*e^3*ln(b*x+a)*a^2-6/b^3*e
^2*ln(b*x+a)*a*d+3/b^2*e*ln(b*x+a)*d^2+1/b^4/(b*x+a)*a^3*e^3-3/b^3/(b*x+a)*
a^2*d*e^2+3/b^2/(b*x+a)*a*d^2*e-1/b/(b*x+a)*d^3
```

Maxima [A] time = 1.11418, size = 159, normalized size = 2.12

$$-\frac{b^3d^3-3\,ab^2d^2e+3\,a^2bde^2-a^3e^3}{b^5x+ab^4}+\frac{be^3x^2+2\left(3\,bde^2-2\,ae^3\right)x}{2\,b^3}+\frac{3\left(b^2d^2e-2\,abde^2+a^2e^3\right)\log(bx+a)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

```
[Out] -(b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3)/(b^5*x + a*b^4) + 1/2*
(b*e^3*x^2 + 2*(3*b*d*e^2 - 2*a*e^3)*x)/b^3 + 3*(b^2*d^2*e - 2*a*b*d*e^2 +
a^2*e^3)*log(b*x + a)/b^4
```

Fricas [B] time = 1.72801, size = 354, normalized size = 4.72

$$\frac{b^3e^3x^3-2\,b^3d^3+6\,ab^2d^2e-6\,a^2bde^2+2\,a^3e^3+3\left(2\,b^3de^2-ab^2e^3\right)x^2+2\left(3\,ab^2de^2-2\,a^2be^3\right)x+6\left(ab^2d^2e-2\,a^2bde^2\right.}{2\left(b^5x+ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*e^3*x^3 - 2*b^3*d^3 + 6*a*b^2*d^2*e - 6*a^2*b*d*e^2 + 2*a^3*e^3 +
3*(2*b^3*d*e^2 - a*b^2*e^3)*x^2 + 2*(3*a*b^2*d*e^2 - 2*a^2*b*e^3)*x + 6*(a*
b^2*d^2*e - 2*a^2*b*d*e^2 + a^3*e^3 + (b^3*d^2*e - 2*a*b^2*d*e^2 + a^2*b*e^
3)*x)*log(b*x + a))/(b^5*x + a*b^4)
```

Sympy [A] time = 0.773353, size = 100, normalized size = 1.33

$$\frac{a^3e^3-3a^2bde^2+3ab^2d^2e-b^3d^3}{ab^4+b^5x}+\frac{e^3x^2}{2b^2}-\frac{x\left(2ae^3-3bde^2\right)}{b^3}+\frac{3e\left(ae-bd\right)^2\log\left(a+bx\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(b**2*x**2+2*a*b*x+a**2),x)
```

```
[Out] (a**3*e**3 - 3*a**2*b*d*e**2 + 3*a*b**2*d**2*e - b**3*d**3)/(a*b**4 + b**5*
x) + e**3*x**2/(2*b**2) - x*(2*a*e**3 - 3*b*d*e**2)/b**3 + 3*e*(a*e - b*d)*
*2*log(a + b*x)/b**4
```

Giac [A] time = 1.11367, size = 154, normalized size = 2.05

$$\frac{3\left(b^2d^2e - 2\,abde^2 + a^2e^3\right)\log\left(|bx + a|\right)}{b^4} + \frac{b^2x^2e^3 + 6\,b^2dxe^2 - 4\,abxe^3}{2\,b^4} - \frac{b^3d^3 - 3\,ab^2d^2e + 3\,a^2bde^2 - a^3e^3}{(bx + a)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

```
[Out] 3*(b^2*d^2*e - 2*a*b*d*e^2 + a^2*e^3)*log(abs(b*x + a))/b^4 + 1/2*(b^2*x^2*
e^3 + 6*b^2*d*x*e^2 - 4*a*b*x*e^3)/b^4 - (b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b
*d*e^2 - a^3*e^3)/((b*x + a)*b^4)
```

3.1508 $\int \frac{(d+ex)^2}{a^2+2abx+b^2x^2} dx$

Optimal. Leaf size=51

$$-\frac{(bd-ae)^2}{b^3(a+bx)}+\frac{2e(bd-ae)\log(a+bx)}{b^3}+\frac{e^2x}{b^2}$$

```
[Out] (e^2*x)/b^2 - (b*d - a*e)^2/(b^3*(a + b*x)) + (2*e*(b*d - a*e)*Log[a + b*x]
)/b^3
```

Rubi [A] time = 0.0393365, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$-\frac{(bd-ae)^2}{b^3(a+bx)}+\frac{2e(bd-ae)\log(a+bx)}{b^3}+\frac{e^2x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] (e^2*x)/b^2 - (b*d - a*e)^2/(b^3*(a + b*x)) + (2*e*(b*d - a*e)*Log[a + b*x]
)/b^3
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2}{a^2+2abx+b^2x^2}\,dx &= \int \frac{(d+ex)^2}{(a+bx)^2}\,dx\\ &= \int \left(\frac{e^2}{b^2}+\frac{(bd-ae)^2}{b^2(a+bx)^2}+\frac{2e(bd-ae)}{b^2(a+bx)}\right)dx\\ &= \frac{e^2x}{b^2}-\frac{(bd-ae)^2}{b^3(a+bx)}+\frac{2e(bd-ae)\log(a+bx)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.0371345, size = 47, normalized size = 0.92

$$\frac{-\frac{(bd-ae)^2}{a+bx}+2e(bd-ae)\log(a+bx)+be^2x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a^2 + 2*a*b*x + b^2*x^2),x]

[Out] (b*e^2*x - (b*d - a*e)^2/(a + b*x) + 2*e*(b*d - a*e)*Log[a + b*x])/b^3
```

Maple [A] time = 0.047, size = 86, normalized size = 1.7

$$\frac{e^2x}{b^2}-2\,\frac{e^2\ln(bx+a)\,a}{b^3}+2\,\frac{e\ln(bx+a)\,d}{b^2}-\frac{a^2e^2}{b^3\,(bx+a)}+2\,\frac{ade}{b^2\,(bx+a)}-\frac{d^2}{b\,(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2),x)

[Out] e^2*x/b^2-2/b^3*e^2*ln(b*x+a)*a+2/b^2*e*ln(b*x+a)*d-1/b^3/(b*x+a)*a^2*e^2+2
/b^2/(b*x+a)*a*d*e-1/b/(b*x+a)*d^2
```

Maxima [A] time = 1.13578, size = 90, normalized size = 1.76

$$\frac{e^2x}{b^2}-\frac{b^2d^2-2\,abde+a^2e^2}{b^4x+ab^3}+\frac{2\left(bde-ae^2\right)\log(bx+a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")

[Out] e^2*x/b^2 - (b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(b^4*x + a*b^3) + 2*(b*d*e - a*
e^2)*log(b*x + a)/b^3
```

Fricas [A] time = 1.67671, size = 184, normalized size = 3.61

$$\frac{b^2e^2x^2+abe^2x-b^2d^2+2\,abde-a^2e^2+2\left(abde-a^2e^2+\left(b^2de-abe^2\right)x\right)\log(bx+a)}{b^4x+ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")

[Out] (b^2*e^2*x^2 + a*b*e^2*x - b^2*d^2 + 2*a*b*d*e - a^2*e^2 + 2*(a*b*d*e - a^2
*e^2 + (b^2*d*e - a*b*e^2)*x)*log(b*x + a))/(b^4*x + a*b^3)
```

Sympy [A] time = 0.592445, size = 60, normalized size = 1.18

$$-\frac{a^2e^2-2abde+b^2d^2}{ab^3+b^4x}+\frac{e^2x}{b^2}-\frac{2e\,(ae-bd)\log(a+bx)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(b**2*x**2+2*a*b*x+a**2),x)
```

```
[Out] -(a**2*e**2 - 2*a*b*d*e + b**2*d**2)/(a*b**3 + b**4*x) + e**2*x/b**2 - 2*e*
(a*e - b*d)*log(a + b*x)/b**3
```

Giac [A] time = 1.24279, size = 86, normalized size = 1.69

$$\frac{xe^2}{b^2} + \frac{2\left(bde - ae^2\right)\log\left(|bx + a|\right)}{b^3} - \frac{b^2d^2 - 2\,abde + a^2e^2}{(bx + a)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

```
[Out] x*e^2/b^2 + 2*(b*d*e - a*e^2)*log(abs(b*x + a))/b^3 - (b^2*d^2 - 2*a*b*d*e
+ a^2*e^2)/((b*x + a)*b^3)
```

3.1509 $\int \frac{d+ex}{a^2+2abx+b^2x^2} dx$

Optimal. Leaf size=32

$$\frac{e\log(a+bx)}{b^2}-\frac{bd-ae}{b^2(a+bx)}$$

```
[Out] -((b*d - a*e)/(b^2*(a + b*x))) + (e*Log[a + b*x])/b^2
```

Rubi [A] time = 0.0216175, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {27, 43}

$$\frac{e\log(a+bx)}{b^2}-\frac{bd-ae}{b^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] -((b*d - a*e)/(b^2*(a + b*x))) + (e*Log[a + b*x])/b^2
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex}{a^2+2abx+b^2x^2}\,dx &= \int \frac{d+ex}{(a+bx)^2}\,dx\\ &= \int \left(\frac{bd-ae}{b(a+bx)^2}+\frac{e}{b(a+bx)}\right)dx\\ &= -\frac{bd-ae}{b^2(a+bx)}+\frac{e\log(a+bx)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.0110032, size = 31, normalized size = 0.97

$$\frac{ae-bd}{b^2(a+bx)}+\frac{e\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] (-(b*d) + a*e)/(b^2*(a + b*x)) + (e*Log[a + b*x])/b^2
```

Maple [A] time = 0.043, size = 39, normalized size = 1.2

$$\frac{e\ln\left(bx+a\right)}{b^2}+\frac{ae}{b^2\left(bx+a\right)}-\frac{d}{b\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(b^2*x^2+2*a*b*x+a^2),x)
```

```
[Out] e*ln(b*x+a)/b^2+1/b^2/(b*x+a)*a*e-1/b/(b*x+a)*d
```

Maxima [A] time = 1.15544, size = 47, normalized size = 1.47

$$-\frac{bd-ae}{b^3x+ab^2}+\frac{e\log\left(bx+a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

```
[Out] -(b*d - a*e)/(b^3*x + a*b^2) + e*log(b*x + a)/b^2
```

Fricas [A] time = 1.69713, size = 80, normalized size = 2.5

$$-\frac{bd-ae-(bex+ae)\log\left(bx+a\right)}{b^3x+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

```
[Out] -(b*d - a*e - (b*e*x + a*e)*log(b*x + a))/(b^3*x + a*b^2)
```

Sympy [A] time = 0.391281, size = 27, normalized size = 0.84

$$\frac{ae-bd}{ab^2+b^3x}+\frac{e\log\left(a+bx\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(b**2*x**2+2*a*b*x+a**2),x)
```

```
[Out] (a*e - b*d)/(a*b**2 + b**3*x) + e*log(a + b*x)/b**2
```

Giac [A] time = 1.17031, size = 47, normalized size = 1.47

$$\frac{e\log\left(|bx+a|\right)}{b^2}-\frac{bd-ae}{(bx+a)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

```
[Out] e*log(abs(b*x + a))/b^2 - (b*d - a*e)/((b*x + a)*b^2)
```

3.1510 $\int \frac{1}{a^2+2abx+b^2x^2} dx$

Optimal. Leaf size=12

$$-\frac{1}{b(a+bx)}$$

```
[Out] -(1/(b*(a + b*x)))
```

Rubi [A] time = 0.0022466, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {27, 32}

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(-1),x]
```

```
[Out] -(1/(b*(a + b*x)))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a^2+2abx+b^2x^2} dx &= \int \frac{1}{(a+bx)^2} dx\\ &= -\frac{1}{b(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.0029372, size = 12, normalized size = 1.

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(-1),x]
```

```
[Out] -(1/(b*(a + b*x)))
```

Maple [A] time = 0.041, size = 13, normalized size = 1.1

$$-\frac{1}{b\,(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^2+2*a*b*x+a^2),x)
```

```
[Out] -1/b/(b*x+a)
```

Maxima [A] time = 1.14039, size = 18, normalized size = 1.5

$$-\frac{1}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

```
[Out] -1/(b^2*x + a*b)
```

Fricas [A] time = 1.70483, size = 24, normalized size = 2.

$$-\frac{1}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

```
[Out] -1/(b^2*x + a*b)
```

Sympy [A] time = 0.305257, size = 10, normalized size = 0.83

$$-\frac{1}{ab+b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**2+2*a*b*x+a**2),x)
```

```
[Out] -1/(a*b + b**2*x)
```

Giac [A] time = 1.16519, size = 16, normalized size = 1.33

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")

[Out] -1/((b*x + a)*b)
```

3.1511 $\int \frac{1}{(d+ex)\left(a^2+2abx+b^2x^2\right)}\,dx$

Optimal. Leaf size=57

$$-\frac{1}{(a+bx)(bd-ae)}-\frac{e\log(a+bx)}{(bd-ae)^2}+\frac{e\log(d+ex)}{(bd-ae)^2}$$

```
[Out] -(1/((b*d - a*e)*(a + b*x))) - (e*Log[a + b*x])/(b*d - a*e)^2 + (e*Log[d +
e*x])/(b*d - a*e)^2
```

Rubi [A] time = 0.0351234, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 44}

$$-\frac{1}{(a+bx)(bd-ae)}-\frac{e\log(a+bx)}{(bd-ae)^2}+\frac{e\log(d+ex)}{(bd-ae)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

```
[Out] -(1/((b*d - a*e)*(a + b*x))) - (e*Log[a + b*x])/(b*d - a*e)^2 + (e*Log[d +
e*x])/(b*d - a*e)^2
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)\left(a^2+2abx+b^2x^2\right)}\,dx &= \int \frac{1}{(a+bx)^2(d+ex)}\,dx\\ &= \int \left(\frac{b}{(bd-ae)(a+bx)^2}-\frac{be}{(bd-ae)^2(a+bx)}+\frac{e^2}{(bd-ae)^2(d+ex)}\right)dx\\ &= -\frac{1}{(bd-ae)(a+bx)}-\frac{e\log(a+bx)}{(bd-ae)^2}+\frac{e\log(d+ex)}{(bd-ae)^2}\end{aligned}$$

Mathematica [A] time = 0.0249867, size = 53, normalized size = 0.93

$$\frac{e(a+bx)\log(d+ex)-e(a+bx)\log(a+bx)+ae-bd}{(a+bx)(bd-ae)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

```
[Out] (-(b*d) + a*e - e*(a + b*x)*Log[a + b*x] + e*(a + b*x)*Log[d + e*x])/((b*d
- a*e)^2*(a + b*x))
```

Maple [A] time = 0.057, size = 57, normalized size = 1.

$$\frac{e\ln\left(ex+d\right)}{\left(ae-bd\right)^{2}}+\frac{1}{\left(ae-bd\right)\left(bx+a\right)}-\frac{e\ln\left(bx+a\right)}{\left(ae-bd\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2),x)
```

```
[Out] e/(a*e-b*d)^2*ln(e*x+d)+1/(a*e-b*d)/(b*x+a)-e/(a*e-b*d)^2*ln(b*x+a)
```

Maxima [A] time = 1.11861, size = 124, normalized size = 2.18

$$-\frac{e\log\left(bx+a\right)}{b^{2}d^{2}-2\,abde+a^{2}e^{2}}+\frac{e\log\left(ex+d\right)}{b^{2}d^{2}-2\,abde+a^{2}e^{2}}-\frac{1}{abd-a^{2}e+\left(b^{2}d-abe\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

```
[Out] -e*log(b*x + a)/(b^2*d^2 - 2*a*b*d*e + a^2*e^2) + e*log(e*x + d)/(b^2*d^2 -
 2*a*b*d*e + a^2*e^2) - 1/(a*b*d - a^2*e + (b^2*d - a*b*e)*x)
```

Fricas [A] time = 1.76115, size = 200, normalized size = 3.51

$$-\frac{bd-ae+\left(bex+ae\right)\log\left(bx+a\right)-\left(bex+ae\right)\log\left(ex+d\right)}{ab^{2}d^{2}-2\,a^{2}bde+a^{3}e^{2}+\left(b^{3}d^{2}-2\,ab^{2}de+a^{2}be^{2}\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

```
[Out] -(b*d - a*e + (b*e*x + a*e)*log(b*x + a) - (b*e*x + a*e)*log(e*x + d))/(a*b
^2*d^2 - 2*a^2*b*d*e + a^3*e^2 + (b^3*d^2 - 2*a*b^2*d*e + a^2*b*e^2)*x)
```

Sympy [B] time = 0.829572, size = 233, normalized size = 4.09

$$\frac{e\log\left(x+\frac{-\frac{a^{3}e^{4}}{(ae-bd)^{2}}+\frac{3a^{2}bde^{3}}{(ae-bd)^{2}}-\frac{3ab^{2}d^{2}e^{2}}{(ae-bd)^{2}}+ae^{2}+\frac{b^{3}d^{3}e}{(ae-bd)^{2}}+bde}{2be^{2}}\right)}{\left(ae-bd\right)^{2}}-\frac{e\log\left(x+\frac{\frac{a^{3}e^{4}}{(ae-bd)^{2}}-\frac{3a^{2}bde^{3}}{(ae-bd)^{2}}+\frac{3ab^{2}d^{2}e^{2}}{(ae-bd)^{2}}+ae^{2}-\frac{b^{3}d^{3}e}{(ae-bd)^{2}}+bde}{2be^{2}}\right)}{\left(ae-bd\right)^{2}}+\frac{1}{a^{2}e-abd+x\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(b**2*x**2+2*a*b*x+a**2),x)
```

```
[Out] e*log(x + (-a**3*e**4/(a*e - b*d)**2 + 3*a**2*b*d*e**3/(a*e - b*d)**2 - 3*a
*b**2*d**2*e**2/(a*e - b*d)**2 + a*e**2 + b**3*d**3*e/(a*e - b*d)**2 + b*d*
e)/(2*b*e**2))/(a*e - b*d)**2 - e*log(x + (a**3*e**4/(a*e - b*d)**2 - 3*a**
2*b*d*e**3/(a*e - b*d)**2 + 3*a*b**2*d**2*e**2/(a*e - b*d)**2 + a*e**2 - b*
*3*d**3*e/(a*e - b*d)**2 + b*d*e)/(2*b*e**2))/(a*e - b*d)**2 + 1/(a**2*e -
a*b*d + x*(a*b*e - b**2*d))
```

Giac [A] time = 1.14648, size = 128, normalized size = 2.25

$$-\frac{be\log\left(|bx+a|\right)}{b^3d^2-2\,ab^2de+a^2be^2}+\frac{e^2\log\left(|xe+d|\right)}{b^2d^2e-2\,abde^2+a^2e^3}-\frac{1}{(bd-ae)(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

```
[Out] -b*e*log(abs(b*x + a))/(b^3*d^2 - 2*a*b^2*d*e + a^2*b*e^2) + e^2*log(abs(x*
e + d))/(b^2*d^2*e - 2*a*b*d*e^2 + a^2*e^3) - 1/((b*d - a*e)*(b*x + a))
```

3.1512 $\int \frac{1}{(d+ex)^2\left(a^2+2abx+b^2x^2\right)}\,dx$

Optimal. Leaf size=81

$$-\frac{b}{(a+bx)(bd-ae)^2}-\frac{e}{(d+ex)(bd-ae)^2}-\frac{2be\log(a+bx)}{(bd-ae)^3}+\frac{2be\log(d+ex)}{(bd-ae)^3}$$

```
[Out] -(b/((b*d - a*e)^2*(a + b*x))) - e/((b*d - a*e)^2*(d + e*x)) - (2*b*e*Log[a
 + b*x])/(b*d - a*e)^3 + (2*b*e*Log[d + e*x])/(b*d - a*e)^3
```

Rubi [A] time = 0.0533884, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 44}

$$-\frac{b}{(a+bx)(bd-ae)^2}-\frac{e}{(d+ex)(bd-ae)^2}-\frac{2be\log(a+bx)}{(bd-ae)^3}+\frac{2be\log(d+ex)}{(bd-ae)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

```
[Out] -(b/((b*d - a*e)^2*(a + b*x))) - e/((b*d - a*e)^2*(d + e*x)) - (2*b*e*Log[a
 + b*x])/(b*d - a*e)^3 + (2*b*e*Log[d + e*x])/(b*d - a*e)^3
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)^2\left(a^2+2abx+b^2x^2\right)}\,dx &= \int \frac{1}{(a+bx)^2(d+ex)^2}\,dx\\ &= \int \left(\frac{b^2}{(bd-ae)^2(a+bx)^2}-\frac{2b^2e}{(bd-ae)^3(a+bx)}+\frac{e^2}{(bd-ae)^2(d+ex)^2}+\frac{2be^2}{(bd-ae)^3(\cdots}\right.\\ &= -\frac{b}{(bd-ae)^2(a+bx)}-\frac{e}{(bd-ae)^2(d+ex)}-\frac{2be\log(a+bx)}{(bd-ae)^3}+\frac{2be\log(d+ex)}{(bd-ae)^3}\end{aligned}$$

Mathematica [A] time = 0.0681853, size = 66, normalized size = 0.81

$$\frac{\frac{b(ae-bd)}{a+bx}+\frac{e(ae-bd)}{d+ex}-2be\log(a+bx)+2be\log(d+ex)}{(bd-ae)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

```
[Out] ((b*(-(b*d) + a*e))/(a + b*x) + (e*(-(b*d) + a*e))/(d + e*x) - 2*b*e*Log[a
+ b*x] + 2*b*e*Log[d + e*x])/(b*d - a*e)^3
```

Maple [A] time = 0.052, size = 82, normalized size = 1.

$$-\frac{e}{(ae-bd)^2(ex+d)}-2\frac{be\ln(ex+d)}{(ae-bd)^3}-\frac{b}{(ae-bd)^2(bx+a)}+2\frac{be\ln(bx+a)}{(ae-bd)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2),x)
```

```
[Out] -e/(a*e-b*d)^2/(e*x+d)-2*e/(a*e-b*d)^3*b*ln(e*x+d)-b/(a*e-b*d)^2/(b*x+a)+2*
e/(a*e-b*d)^3*b*ln(b*x+a)
```

Maxima [B] time = 1.02128, size = 281, normalized size = 3.47

$$-\frac{2\,be\log(bx+a)}{b^3d^3-3\,ab^2d^2e+3\,a^2bde^2-a^3e^3}+\frac{2\,be\log(ex+d)}{b^3d^3-3\,ab^2d^2e+3\,a^2bde^2-a^3e^3}-\frac{2\,bex+}{ab^2d^3-2\,a^2bd^2e+a^3de^2+\left(b^3d^2e-2\,ab^2de^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

```
[Out] -2*b*e*log(b*x + a)/(b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3) + 2
*b*e*log(e*x + d)/(b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3) - (2*
b*e*x + b*d + a*e)/(a*b^2*d^3 - 2*a^2*b*d^2*e + a^3*d*e^2 + (b^3*d^2*e - 2*
a*b^2*d*e^2 + a^2*b*e^3)*x^2 + (b^3*d^3 - a*b^2*d^2*e - a^2*b*d*e^2 + a^3*e
^3)*x)
```

Fricas [B] time = 1.8012, size = 486, normalized size = 6.

$$-\frac{b^2d^2-a^2e^2+2\left(b^2de-abe^2\right)x+2\left(b^2e^2x^2+abde+\left(b^2de+abe^2\right)x\right)\log(bx+a)-2\left(b^2e^2x^2+abde+\left(b^2de+abe^2\right)x\right)\mathrm{l}}{ab^3d^4-3\,a^2b^2d^3e+3\,a^3bd^2e^2-a^4de^3+\left(b^4d^3e-3\,ab^3d^2e^2+3\,a^2b^2de^3-a^3be^4\right)x^2+\left(b^4d^4-2\,ab^3d^3e+2\,a^3bde^3-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

```
[Out] -(b^2*d^2 - a^2*e^2 + 2*(b^2*d*e - a*b*e^2)*x + 2*(b^2*e^2*x^2 + a*b*d*e +
(b^2*d*e + a*b*e^2)*x)*log(b*x + a) - 2*(b^2*e^2*x^2 + a*b*d*e + (b^2*d*e +
 a*b*e^2)*x)*log(e*x + d))/(a*b^3*d^4 - 3*a^2*b^2*d^3*e + 3*a^3*b*d^2*e^2 -
 a^4*d*e^3 + (b^4*d^3*e - 3*a*b^3*d^2*e^2 + 3*a^2*b^2*d*e^3 - a^3*b*e^4)*x^
2 + (b^4*d^4 - 2*a*b^3*d^3*e + 2*a^3*b*d*e^3 - a^4*e^4)*x)
```

Sympy [B] time = 1.32834, size = 405, normalized size = 5.

$$-\frac{2be\log\left(x+\frac{-\frac{2a^4be^5}{(ae-bd)^3}+\frac{8a^3b^2de^4}{(ae-bd)^3}-\frac{12a^2b^3d^2e^3}{(ae-bd)^3}+\frac{8ab^4d^3e^2}{(ae-bd)^3}+2abe^2-\frac{2b^5d^4e}{(ae-bd)^3}+2b^2de}{4b^2e^2}\right)}{(ae-bd)^3}+\frac{2be\log\left(x+\frac{\frac{2a^4be^5}{(ae-bd)^3}-\frac{8a^3b^2de^4}{(ae-bd)^3}+\frac{12a^2b^3d^2e^3}{(ae-bd)^3}-\frac{8ab^4d^3e^2}{(ae-bd)^3}+2abe^2-}{4b^2e^2}\right.}{(ae-bd)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(b**2*x**2+2*a*b*x+a**2),x)
```

```
[Out] -2*b*e*log(x + (-2*a**4*b*e**5/(a*e - b*d)**3 + 8*a**3*b**2*d*e**4/(a*e - b
*d)**3 - 12*a**2*b**3*d**2*e**3/(a*e - b*d)**3 + 8*a*b**4*d**3*e**2/(a*e -
b*d)**3 + 2*a*b*e**2 - 2*b**5*d**4*e/(a*e - b*d)**3 + 2*b**2*d*e)/(4*b**2*e
**2))/(a*e - b*d)**3 + 2*b*e*log(x + (2*a**4*b*e**5/(a*e - b*d)**3 - 8*a**3
*b**2*d*e**4/(a*e - b*d)**3 + 12*a**2*b**3*d**2*e**3/(a*e - b*d)**3 - 8*a*b
**4*d**3*e**2/(a*e - b*d)**3 + 2*a*b*e**2 + 2*b**5*d**4*e/(a*e - b*d)**3 +
2*b**2*d*e)/(4*b**2*e**2))/(a*e - b*d)**3 - (a*e + b*d + 2*b*e*x)/(a**3*d*e
**2 - 2*a**2*b*d**2*e + a*b**2*d**3 + x**2*(a**2*b*e**3 - 2*a*b**2*d*e**2 +
 b**3*d**2*e) + x*(a**3*e**3 - a**2*b*d*e**2 - a*b**2*d**2*e + b**3*d**3))
```

Giac [A] time = 1.21836, size = 211, normalized size = 2.6

$$-\frac{2\,be^2\log\left(\left|b-\frac{bd}{xe+d}+\frac{ae}{xe+d}\right|\right)}{b^3d^3e-3\,ab^2d^2e^2+3\,a^2bde^3-a^3e^4}-\frac{e^3}{\left(b^2d^2e^2-2\,abde^3+a^2e^4\right)(xe+d)}-\frac{b^2e}{(bd-ae)^3\left(b-\frac{bd}{xe+d}+\frac{ae}{xe+d}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

```
[Out] -2*b*e^2*log(abs(b - b*d/(x*e + d) + a*e/(x*e + d)))/(b^3*d^3*e - 3*a*b^2*d
^2*e^2 + 3*a^2*b*d*e^3 - a^3*e^4) - e^3/((b^2*d^2*e^2 - 2*a*b*d*e^3 + a^2*e
^4)*(x*e + d)) - b^2*e/((b*d - a*e)^3*(b - b*d/(x*e + d) + a*e/(x*e + d)))
```

3.1513 $\int \frac{1}{(d+ex)^3(a^2+2abx+b^2x^2)}\,dx$

Optimal. Leaf size=110

$$-\frac{b^2}{(a+bx)(bd-ae)^3}-\frac{3b^2e\log(a+bx)}{(bd-ae)^4}+\frac{3b^2e\log(d+ex)}{(bd-ae)^4}-\frac{2be}{(d+ex)(bd-ae)^3}-\frac{e}{2(d+ex)^2(bd-ae)^2}$$

```
[Out] -(b^2/((b*d - a*e)^3*(a + b*x))) - e/(2*(b*d - a*e)^2*(d + e*x)^2) - (2*b*e
)/((b*d - a*e)^3*(d + e*x)) - (3*b^2*e*Log[a + b*x])/(b*d - a*e)^4 + (3*b^2
*e*Log[d + e*x])/(b*d - a*e)^4
```

Rubi [A] time = 0.0786082, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 44}

$$-\frac{b^2}{(a+bx)(bd-ae)^3}-\frac{3b^2e\log(a+bx)}{(bd-ae)^4}+\frac{3b^2e\log(d+ex)}{(bd-ae)^4}-\frac{2be}{(d+ex)(bd-ae)^3}-\frac{e}{2(d+ex)^2(bd-ae)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

```
[Out] -(b^2/((b*d - a*e)^3*(a + b*x))) - e/(2*(b*d - a*e)^2*(d + e*x)^2) - (2*b*e
)/((b*d - a*e)^3*(d + e*x)) - (3*b^2*e*Log[a + b*x])/(b*d - a*e)^4 + (3*b^2
*e*Log[d + e*x])/(b*d - a*e)^4
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)^3\left(a^2+2abx+b^2x^2\right)}\,dx &= \int \frac{1}{(a+bx)^2(d+ex)^3}\,dx\\ &= \int \left(\frac{b^3}{(bd-ae)^3(a+bx)^2}-\frac{3b^3e}{(bd-ae)^4(a+bx)}+\frac{e^2}{(bd-ae)^2(d+ex)^3}+\frac{2be^2}{(bd-ae)^3(d+\right.\\ &= -\frac{b^2}{(bd-ae)^3(a+bx)}-\frac{e}{2(bd-ae)^2(d+ex)^2}-\frac{2be}{(bd-ae)^3(d+ex)}-\frac{3b^2e\log(a+bx)}{(bd-ae)^4}\end{aligned}$$

Mathematica [A] time = 0.0995215, size = 97, normalized size = 0.88

$$-\frac{\frac{2b^2(bd-ae)}{a+bx}+6b^2e\log(a+bx)+\frac{4be(bd-ae)}{d+ex}+\frac{e(bd-ae)^2}{(d+ex)^2}-6b^2e\log(d+ex)}{2(bd-ae)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

```
[Out] -((2*b^2*(b*d - a*e))/(a + b*x) + (e*(b*d - a*e)^2)/(d + e*x)^2 + (4*b*e*(b
*d - a*e))/(d + e*x) + 6*b^2*e*Log[a + b*x] - 6*b^2*e*Log[d + e*x])/(2*(b*d
 - a*e)^4)
```

Maple [A] time = 0.055, size = 108, normalized size = 1.

$$-\frac{e}{2\,(ae-bd)^2\,(ex+d)^2}+3\,\frac{b^2e\ln(ex+d)}{(ae-bd)^4}+2\,\frac{be}{(ae-bd)^3\,(ex+d)}+\frac{b^2}{(ae-bd)^3\,(bx+a)}-3\,\frac{b^2e\ln(bx+a)}{(ae-bd)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2),x)
```

```
[Out] -1/2*e/(a*e-b*d)^2/(e*x+d)^2+3*e/(a*e-b*d)^4*b^2*ln(e*x+d)+2*e/(a*e-b*d)^3*
b/(e*x+d)+b^2/(a*e-b*d)^3/(b*x+a)-3*e/(a*e-b*d)^4*b^2*ln(b*x+a)
```

Maxima [B] time = 1.16565, size = 521, normalized size = 4.74

$$-\frac{3\,b^2e\log(bx+a)}{b^4d^4-4\,ab^3d^3e+6\,a^2b^2d^2e^2-4\,a^3bde^3+a^4e^4}+\frac{3\,b^2e\log(ex+d)}{b^4d^4-4\,ab^3d^3e+6\,a^2b^2d^2e^2-4\,a^3bde^3+a^4e^4}-\frac{\quad}{2\left(ab^3d^5-3\,a^2b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

```
[Out] -3*b^2*e*log(b*x + a)/(b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*
b*d*e^3 + a^4*e^4) + 3*b^2*e*log(e*x + d)/(b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*
b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4) - 1/2*(6*b^2*e^2*x^2 + 2*b^2*d^2 + 5
*a*b*d*e - a^2*e^2 + 3*(3*b^2*d*e + a*b*e^2)*x)/(a*b^3*d^5 - 3*a^2*b^2*d^4*
e + 3*a^3*b*d^3*e^2 - a^4*d^2*e^3 + (b^4*d^3*e^2 - 3*a*b^3*d^2*e^3 + 3*a^2*
b^2*d*e^4 - a^3*b*e^5)*x^3 + (2*b^4*d^4*e - 5*a*b^3*d^3*e^2 + 3*a^2*b^2*d^2
*e^3 + a^3*b*d*e^4 - a^4*e^5)*x^2 + (b^4*d^5 - a*b^3*d^4*e - 3*a^2*b^2*d^3*
e^2 + 5*a^3*b*d^2*e^3 - 2*a^4*d*e^4)*x)
```

Fricas [B] time = 1.78047, size = 991, normalized size = 9.01

$$-\frac{2\,b^3d^3+3\,ab^2d^2e-6\,a^2bde^2+a^3e^3+6\left(b^3de^2-ab^2e^3\right)x^2+3\left(3\,b^3d^2e-2\,ab^2de^2-a^2be^3\right)x+6\left(b^3e^3x^3+ab^2d^2e+\right.}{2\left(ab^4d^6-4\,a^2b^3d^5e+6\,a^3b^2d^4e^2-4\,a^4bd^3e^3+a^5d^2e^4+\left(b^5d^4e^2-4\,ab^4d^3e^3+6\,a^2b^3d^2e^4-4\,a^3b^2de^5+a^4be^6\right)x^3+\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*d^3 + 3*a*b^2*d^2*e - 6*a^2*b*d*e^2 + a^3*e^3 + 6*(b^3*d*e^2 -
a*b^2*e^3)*x^2 + 3*(3*b^3*d^2*e - 2*a*b^2*d*e^2 - a^2*b*e^3)*x + 6*(b^3*e^3
*x^3 + a*b^2*d^2*e + (2*b^3*d*e^2 + a*b^2*e^3)*x^2 + (b^3*d^2*e + 2*a*b^2*d
```

```
*e^2)*x)*log(b*x + a) - 6*(b^3*e^3*x^3 + a*b^2*d^2*e + (2*b^3*d*e^2 + a*b^2
*e^3)*x^2 + (b^3*d^2*e + 2*a*b^2*d*e^2)*x)*log(e*x + d))/(a*b^4*d^6 - 4*a^2
*b^3*d^5*e + 6*a^3*b^2*d^4*e^2 - 4*a^4*b*d^3*e^3 + a^5*d^2*e^4 + (b^5*d^4*e
^2 - 4*a*b^4*d^3*e^3 + 6*a^2*b^3*d^2*e^4 - 4*a^3*b^2*d*e^5 + a^4*b*e^6)*x^3
 + (2*b^5*d^5*e - 7*a*b^4*d^4*e^2 + 8*a^2*b^3*d^3*e^3 - 2*a^3*b^2*d^2*e^4 -
 2*a^4*b*d*e^5 + a^5*e^6)*x^2 + (b^5*d^6 - 2*a*b^4*d^5*e - 2*a^2*b^3*d^4*e^
2 + 8*a^3*b^2*d^3*e^3 - 7*a^4*b*d^2*e^4 + 2*a^5*d*e^5)*x)
```

Sympy [B] time = 2.1331, size = 632, normalized size = 5.75

$$\frac{3b^2e\log\left(x+\frac{-\frac{3a^5b^2e^6}{(ae-bd)^4}+\frac{15a^4b^3de^5}{(ae-bd)^4}-\frac{30a^3b^4d^2e^4}{(ae-bd)^4}+\frac{30a^2b^5d^3e^3}{(ae-bd)^4}-\frac{15ab^6d^4e^2}{(ae-bd)^4}+3ab^2e^2+\frac{3b^7d^5e}{(ae-bd)^4}+3b^3de}{6b^3e^2}\right)}{(ae-bd)^4}-\frac{3b^2e\log\left(x+\frac{\frac{3a^5b^2e^6}{(ae-bd)^4}-\frac{15a^4b^3de^5}{(ae-bd)^4}+\frac{30a^3b^4d^2e^4}{(ae-bd)^4}-30a^2\cdots}{\cdots}\right)}{(ae-bd)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(b**2*x**2+2*a*b*x+a**2),x)
```

```
[Out] 3*b**2*e*log(x + (-3*a**5*b**2*e**6/(a*e - b*d)**4 + 15*a**4*b**3*d*e**5/(a
*e - b*d)**4 - 30*a**3*b**4*d**2*e**4/(a*e - b*d)**4 + 30*a**2*b**5*d**3*e*
*3/(a*e - b*d)**4 - 15*a*b**6*d**4*e**2/(a*e - b*d)**4 + 3*a*b**2*e**2 + 3*
b**7*d**5*e/(a*e - b*d)**4 + 3*b**3*d*e)/(6*b**3*e**2))/(a*e - b*d)**4 - 3*
b**2*e*log(x + (3*a**5*b**2*e**6/(a*e - b*d)**4 - 15*a**4*b**3*d*e**5/(a*e
- b*d)**4 + 30*a**3*b**4*d**2*e**4/(a*e - b*d)**4 - 30*a**2*b**5*d**3*e**3/
(a*e - b*d)**4 + 15*a*b**6*d**4*e**2/(a*e - b*d)**4 + 3*a*b**2*e**2 - 3*b**
7*d**5*e/(a*e - b*d)**4 + 3*b**3*d*e)/(6*b**3*e**2))/(a*e - b*d)**4 + (-a**
2*e**2 + 5*a*b*d*e + 2*b**2*d**2 + 6*b**2*e**2*x**2 + x*(3*a*b*e**2 + 9*b**
2*d*e))/(2*a**4*d**2*e**3 - 6*a**3*b*d**3*e**2 + 6*a**2*b**2*d**4*e - 2*a*b
**3*d**5 + x**3*(2*a**3*b*e**5 - 6*a**2*b**2*d*e**4 + 6*a*b**3*d**2*e**3 -
2*b**4*d**3*e**2) + x**2*(2*a**4*e**5 - 2*a**3*b*d*e**4 - 6*a**2*b**2*d**2*
e**3 + 10*a*b**3*d**3*e**2 - 4*b**4*d**4*e) + x*(4*a**4*d*e**4 - 10*a**3*b*
d**2*e**3 + 6*a**2*b**2*d**3*e**2 + 2*a*b**3*d**4*e - 2*b**4*d**5))
```

Giac [B] time = 1.16789, size = 335, normalized size = 3.05

$$-\frac{3\,b^3e\log\left(|bx+a|\right)}{b^5d^4-4\,ab^4d^3e+6\,a^2b^3d^2e^2-4\,a^3b^2de^3+a^4be^4}+\frac{3\,b^2e^2\log\left(|xe+d|\right)}{b^4d^4e-4\,ab^3d^3e^2+6\,a^2b^2d^2e^3-4\,a^3bde^4+a^4e^5}-\frac{2\,b^3d^3+3\,ab^2d^2\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

```
[Out] -3*b^3*e*log(abs(b*x + a))/(b^5*d^4 - 4*a*b^4*d^3*e + 6*a^2*b^3*d^2*e^2 - 4
*a^3*b^2*d*e^3 + a^4*b*e^4) + 3*b^2*e^2*log(abs(x*e + d))/(b^4*d^4*e - 4*a*
b^3*d^3*e^2 + 6*a^2*b^2*d^2*e^3 - 4*a^3*b*d*e^4 + a^4*e^5) - 1/2*(2*b^3*d^3
 + 3*a*b^2*d^2*e - 6*a^2*b*d*e^2 + a^3*e^3 + 6*(b^3*d*e^2 - a*b^2*e^3)*x^2
+ 3*(3*b^3*d^2*e - 2*a*b^2*d*e^2 - a^2*b*e^3)*x)/((b*d - a*e)^4*(b*x + a)*(
x*e + d)^2)
```

3.1514 $\int \frac{1}{(d+ex)^4(a^2+2abx+b^2x^2)}\,dx$

Optimal. Leaf size=133

$$-\frac{b^3}{(a+bx)(bd-ae)^4}-\frac{3b^2e}{(d+ex)(bd-ae)^4}-\frac{4b^3e\log(a+bx)}{(bd-ae)^5}+\frac{4b^3e\log(d+ex)}{(bd-ae)^5}-\frac{be}{(d+ex)^2(bd-ae)^3}-\frac{e}{3(d+ex)^3(bd}$$

```
[Out] -(b^3/((b*d - a*e)^4*(a + b*x))) - e/(3*(b*d - a*e)^2*(d + e*x)^3) - (b*e)/
((b*d - a*e)^3*(d + e*x)^2) - (3*b^2*e)/((b*d - a*e)^4*(d + e*x)) - (4*b^3*
e*Log[a + b*x])/(b*d - a*e)^5 + (4*b^3*e*Log[d + e*x])/(b*d - a*e)^5
```

Rubi [A] time = 0.107646, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 44}

$$-\frac{b^3}{(a+bx)(bd-ae)^4}-\frac{3b^2e}{(d+ex)(bd-ae)^4}-\frac{4b^3e\log(a+bx)}{(bd-ae)^5}+\frac{4b^3e\log(d+ex)}{(bd-ae)^5}-\frac{be}{(d+ex)^2(bd-ae)^3}-\frac{e}{3(d+ex)^3(bd}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^4*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

```
[Out] -(b^3/((b*d - a*e)^4*(a + b*x))) - e/(3*(b*d - a*e)^2*(d + e*x)^3) - (b*e)/
((b*d - a*e)^3*(d + e*x)^2) - (3*b^2*e)/((b*d - a*e)^4*(d + e*x)) - (4*b^3*
e*Log[a + b*x])/(b*d - a*e)^5 + (4*b^3*e*Log[d + e*x])/(b*d - a*e)^5
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^4\left(a^2+2abx+b^2x^2\right)}\,dx=\int \frac{1}{(a+bx)^2(d+ex)^4}\,dx$$
$$=\int \left(\frac{b^4}{(bd-ae)^4(a+bx)^2}-\frac{4b^4e}{(bd-ae)^5(a+bx)}+\frac{e^2}{(bd-ae)^2(d+ex)^4}+\frac{2be}{(bd-ae)^3(}\right.$$
$$=-\frac{b^3}{(bd-ae)^4(a+bx)}-\frac{e}{3(bd-ae)^2(d+ex)^3}-\frac{be}{(bd-ae)^3(d+ex)^2}-\frac{3b^2e}{(bd-ae)^4(d}$$

Mathematica [A] time = 0.124966, size = 120, normalized size = 0.9

$$\frac{-\frac{3b^3(bd-ae)}{a+bx}-\frac{9b^2e(bd-ae)}{d+ex}-12b^3e\log(a+bx)-\frac{3be(bd-ae)^2}{(d+ex)^2}+\frac{e(ae-bd)^3}{(d+ex)^3}+12b^3e\log(d+ex)}{3(bd-ae)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^4*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

```
[Out] ((-3*b^3*(b*d - a*e))/(a + b*x) + (e*(-(b*d) + a*e)^3)/(d + e*x)^3 - (3*b*e
*(b*d - a*e)^2)/(d + e*x)^2 - (9*b^2*e*(b*d - a*e))/(d + e*x) - 12*b^3*e*Lo
g[a + b*x] + 12*b^3*e*Log[d + e*x])/(3*(b*d - a*e)^5)
```

Maple [A] time = 0.055, size = 131, normalized size = 1.

$$-\frac{e}{3\,(ae-bd)^2\,(ex+d)^3}-4\,\frac{eb^3\ln(ex+d)}{(ae-bd)^5}-3\,\frac{b^2e}{(ae-bd)^4\,(ex+d)}+\frac{be}{(ae-bd)^3\,(ex+d)^2}-\frac{b^3}{(ae-bd)^4\,(bx+a)}+4\,\frac{eb^3\ln(\ldots}{(ae-\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^4/(b^2*x^2+2*a*b*x+a^2),x)
```

```
[Out] -1/3*e/(a*e-b*d)^2/(e*x+d)^3-4*e/(a*e-b*d)^5*b^3*ln(e*x+d)-3*e/(a*e-b*d)^4*
b^2/(e*x+d)+e/(a*e-b*d)^3*b/(e*x+d)^2-b^3/(a*e-b*d)^4/(b*x+a)+4*e/(a*e-b*d)
^5*b^3*ln(b*x+a)
```

Maxima [B] time = 1.33082, size = 809, normalized size = 6.08

$$-\frac{4\,b^3e\log(bx+a)}{b^5d^5-5\,ab^4d^4e+10\,a^2b^3d^3e^2-10\,a^3b^2d^2e^3+5\,a^4bde^4-a^5e^5}+\frac{4\,b^3e\log(ex+d)}{b^5d^5-5\,ab^4d^4e+10\,a^2b^3d^3e^2-10\,a^3b^2d^2e^3+5\,a^4bde^4\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

```
[Out] -4*b^3*e*log(b*x + a)/(b^5*d^5 - 5*a*b^4*d^4*e + 10*a^2*b^3*d^3*e^2 - 10*a^
3*b^2*d^2*e^3 + 5*a^4*b*d*e^4 - a^5*e^5) + 4*b^3*e*log(e*x + d)/(b^5*d^5 -
5*a*b^4*d^4*e + 10*a^2*b^3*d^3*e^2 - 10*a^3*b^2*d^2*e^3 + 5*a^4*b*d*e^4 - a
^5*e^5) - 1/3*(12*b^3*e^3*x^3 + 3*b^3*d^3 + 13*a*b^2*d^2*e - 5*a^2*b*d*e^2
+ a^3*e^3 + 6*(5*b^3*d*e^2 + a*b^2*e^3)*x^2 + 2*(11*b^3*d^2*e + 8*a*b^2*d*e
^2 - a^2*b*e^3)*x)/(a*b^4*d^7 - 4*a^2*b^3*d^6*e + 6*a^3*b^2*d^5*e^2 - 4*a^4
*b*d^4*e^3 + a^5*d^3*e^4 + (b^5*d^4*e^3 - 4*a*b^4*d^3*e^4 + 6*a^2*b^3*d^2*e
^5 - 4*a^3*b^2*d*e^6 + a^4*b*e^7)*x^4 + (3*b^5*d^5*e^2 - 11*a*b^4*d^4*e^3 +
 14*a^2*b^3*d^3*e^4 - 6*a^3*b^2*d^2*e^5 - a^4*b*d*e^6 + a^5*e^7)*x^3 + 3*(b
^5*d^6*e - 3*a*b^4*d^5*e^2 + 2*a^2*b^3*d^4*e^3 + 2*a^3*b^2*d^3*e^4 - 3*a^4*
b*d^2*e^5 + a^5*d*e^6)*x^2 + (b^5*d^7 - a*b^4*d^6*e - 6*a^2*b^3*d^5*e^2 + 1
4*a^3*b^2*d^4*e^3 - 11*a^4*b*d^3*e^4 + 3*a^5*d^2*e^5)*x)
```

Fricas [B] time = 1.85781, size = 1501, normalized size = 11.29

$$-\frac{3\,b^4d^4+10\,ab^3d^3e-18\,a^2b^2d^2e^2+6\,a^3bde^3-a^4e^4+12\left(b^4de^3-ab^3e^4\right)x^3+6\left(5\,b^4d^2e^2-4\,ab^3de^3-a^2b^2e^4\right)x\ldots}{3\left(ab^5d^8-5\,a^2b^4d^7e+10\,a^3b^3d^6e^2-10\,a^4b^2d^5e^3+5\,a^5bd^4e^4-a^6d^3e^5+\left(b^6d^5e^3-5\,ab^5d^4e^4+10\,a^2b^4d^3e^5-10\,a^3b^3d^2e^{\ldots}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^4*d^4 + 10*a*b^3*d^3*e - 18*a^2*b^2*d^2*e^2 + 6*a^3*b*d*e^3 - a^4
*e^4 + 12*(b^4*d*e^3 - a*b^3*e^4)*x^3 + 6*(5*b^4*d^2*e^2 - 4*a*b^3*d*e^3 -
a^2*b^2*e^4)*x^2 + 2*(11*b^4*d^3*e - 3*a*b^3*d^2*e^2 - 9*a^2*b^2*d*e^3 + a^
3*b*e^4)*x + 12*(b^4*e^4*x^4 + a*b^3*d^3*e + (3*b^4*d*e^3 + a*b^3*e^4)*x^3
+ 3*(b^4*d^2*e^2 + a*b^3*d*e^3)*x^2 + (b^4*d^3*e + 3*a*b^3*d^2*e^2)*x)*log(
b*x + a) - 12*(b^4*e^4*x^4 + a*b^3*d^3*e + (3*b^4*d*e^3 + a*b^3*e^4)*x^3 +
3*(b^4*d^2*e^2 + a*b^3*d*e^3)*x^2 + (b^4*d^3*e + 3*a*b^3*d^2*e^2)*x)*log(e*
x + d))/(a*b^5*d^8 - 5*a^2*b^4*d^7*e + 10*a^3*b^3*d^6*e^2 - 10*a^4*b^2*d^5*
e^3 + 5*a^5*b*d^4*e^4 - a^6*d^3*e^5 + (b^6*d^5*e^3 - 5*a*b^5*d^4*e^4 + 10*a
^2*b^4*d^3*e^5 - 10*a^3*b^3*d^2*e^6 + 5*a^4*b^2*d*e^7 - a^5*b*e^8)*x^4 + (3
*b^6*d^6*e^2 - 14*a*b^5*d^5*e^3 + 25*a^2*b^4*d^4*e^4 - 20*a^3*b^3*d^3*e^5 +
 5*a^4*b^2*d^2*e^6 + 2*a^5*b*d*e^7 - a^6*e^8)*x^3 + 3*(b^6*d^7*e - 4*a*b^5*
d^6*e^2 + 5*a^2*b^4*d^5*e^3 - 5*a^4*b^2*d^3*e^5 + 4*a^5*b*d^2*e^6 - a^6*d*e
^7)*x^2 + (b^6*d^8 - 2*a*b^5*d^7*e - 5*a^2*b^4*d^6*e^2 + 20*a^3*b^3*d^5*e^3
 - 25*a^4*b^2*d^4*e^4 + 14*a^5*b*d^3*e^5 - 3*a^6*d^2*e^6)*x)
```

Sympy [B] time = 3.7665, size = 881, normalized size = 6.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**4/(b**2*x**2+2*a*b*x+a**2),x)
```

```
[Out] -4*b**3*e*log(x + (-4*a**6*b**3*e**7/(a*e - b*d)**5 + 24*a**5*b**4*d*e**6/(
a*e - b*d)**5 - 60*a**4*b**5*d**2*e**5/(a*e - b*d)**5 + 80*a**3*b**6*d**3*e
**4/(a*e - b*d)**5 - 60*a**2*b**7*d**4*e**3/(a*e - b*d)**5 + 24*a*b**8*d**5
*e**2/(a*e - b*d)**5 + 4*a*b**3*e**2 - 4*b**9*d**6*e/(a*e - b*d)**5 + 4*b**
4*d*e)/(8*b**4*e**2))/(a*e - b*d)**5 + 4*b**3*e*log(x + (4*a**6*b**3*e**7/(
a*e - b*d)**5 - 24*a**5*b**4*d*e**6/(a*e - b*d)**5 + 60*a**4*b**5*d**2*e**5
/(a*e - b*d)**5 - 80*a**3*b**6*d**3*e**4/(a*e - b*d)**5 + 60*a**2*b**7*d**4
*e**3/(a*e - b*d)**5 - 24*a*b**8*d**5*e**2/(a*e - b*d)**5 + 4*a*b**3*e**2 +
 4*b**9*d**6*e/(a*e - b*d)**5 + 4*b**4*d*e)/(8*b**4*e**2))/(a*e - b*d)**5 -
 (a**3*e**3 - 5*a**2*b*d*e**2 + 13*a*b**2*d**2*e + 3*b**3*d**3 + 12*b**3*e*
*3*x**3 + x**2*(6*a*b**2*e**3 + 30*b**3*d*e**2) + x*(-2*a**2*b*e**3 + 16*a*
b**2*d*e**2 + 22*b**3*d**2*e))/(3*a**5*d**3*e**4 - 12*a**4*b*d**4*e**3 + 18
*a**3*b**2*d**5*e**2 - 12*a**2*b**3*d**6*e + 3*a*b**4*d**7 + x**4*(3*a**4*b
*e**7 - 12*a**3*b**2*d*e**6 + 18*a**2*b**3*d**2*e**5 - 12*a*b**4*d**3*e**4
+ 3*b**5*d**4*e**3) + x**3*(3*a**5*e**7 - 3*a**4*b*d*e**6 - 18*a**3*b**2*d*
*2*e**5 + 42*a**2*b**3*d**3*e**4 - 33*a*b**4*d**4*e**3 + 9*b**5*d**5*e**2)
+ x**2*(9*a**5*d*e**6 - 27*a**4*b*d**2*e**5 + 18*a**3*b**2*d**3*e**4 + 18*a
**2*b**3*d**4*e**3 - 27*a*b**4*d**5*e**2 + 9*b**5*d**6*e) + x*(9*a**5*d**2*
e**5 - 33*a**4*b*d**3*e**4 + 42*a**3*b**2*d**4*e**3 - 18*a**2*b**3*d**5*e**
2 - 3*a*b**4*d**6*e + 3*b**5*d**7))
```

Giac [B] time = 1.12195, size = 455, normalized size = 3.42

$$-\frac{4\,b^4 e \log\left(|bx+a|\right)}{b^6 d^5 - 5\,ab^5d^4e + 10\,a^2b^4d^3e^2 - 10\,a^3b^3d^2e^3 + 5\,a^4b^2de^4 - a^5be^5} + \frac{4\,b^3e^2 \log\left(|xe+d|\right)}{b^5d^5e - 5\,ab^4d^4e^2 + 10\,a^2b^3d^3e^3 - 10\,a^3b^2d^2e^4 + 5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

```
[Out] -4*b^4*e*log(abs(b*x + a))/(b^6*d^5 - 5*a*b^5*d^4*e + 10*a^2*b^4*d^3*e^2 -
10*a^3*b^3*d^2*e^3 + 5*a^4*b^2*d*e^4 - a^5*b*e^5) + 4*b^3*e^2*log(abs(x*e +
 d))/(b^5*d^5*e - 5*a*b^4*d^4*e^2 + 10*a^2*b^3*d^3*e^3 - 10*a^3*b^2*d^2*e^4
 + 5*a^4*b*d*e^5 - a^5*e^6) - 1/3*(3*b^4*d^4 + 10*a*b^3*d^3*e - 18*a^2*b^2*
d^2*e^2 + 6*a^3*b*d*e^3 - a^4*e^4 + 12*(b^4*d*e^3 - a*b^3*e^4)*x^3 + 6*(5*b
^4*d^2*e^2 - 4*a*b^3*d*e^3 - a^2*b^2*e^4)*x^2 + 2*(11*b^4*d^3*e - 3*a*b^3*d
^2*e^2 - 9*a^2*b^2*d*e^3 + a^3*b*e^4)*x)/((b*d - a*e)^5*(b*x + a)*(x*e + d)
^3)
```

3.1515 $\int \frac{(d+ex)^6}{\left(a^2+2abx+b^2x^2\right)^2} dx$

Optimal. Leaf size=156

$$\frac{3e^5(a+bx)^2(bd-ae)}{b^7}+\frac{15e^4x(bd-ae)^2}{b^6}-\frac{15e^2(bd-ae)^4}{b^7(a+bx)}+\frac{20e^3(bd-ae)^3\log(a+bx)}{b^7}-\frac{3e(bd-ae)^5}{b^7(a+bx)^2}-\frac{(bd-ae)^6}{3b^7(a+bx)^3}$$

```
[Out] (15*e^4*(b*d - a*e)^2*x)/b^6 - (b*d - a*e)^6/(3*b^7*(a + b*x)^3) - (3*e*(b*
d - a*e)^5)/(b^7*(a + b*x)^2) - (15*e^2*(b*d - a*e)^4)/(b^7*(a + b*x)) + (3
*e^5*(b*d - a*e)*(a + b*x)^2)/b^7 + (e^6*(a + b*x)^3)/(3*b^7) + (20*e^3*(b*
d - a*e)^3*Log[a + b*x])/b^7
```

Rubi [A] time = 0.179918, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$\frac{3e^5(a+bx)^2(bd-ae)}{b^7}+\frac{15e^4x(bd-ae)^2}{b^6}-\frac{15e^2(bd-ae)^4}{b^7(a+bx)}+\frac{20e^3(bd-ae)^3\log(a+bx)}{b^7}-\frac{3e(bd-ae)^5}{b^7(a+bx)^2}-\frac{(bd-ae)^6}{3b^7(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^6/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] (15*e^4*(b*d - a*e)^2*x)/b^6 - (b*d - a*e)^6/(3*b^7*(a + b*x)^3) - (3*e*(b*
d - a*e)^5)/(b^7*(a + b*x)^2) - (15*e^2*(b*d - a*e)^4)/(b^7*(a + b*x)) + (3
*e^5*(b*d - a*e)*(a + b*x)^2)/b^7 + (e^6*(a + b*x)^3)/(3*b^7) + (20*e^3*(b*
d - a*e)^3*Log[a + b*x])/b^7
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^6}{\left(a^2+2abx+b^2x^2\right)^2} dx &= \int \frac{(d+ex)^6}{(a+bx)^4} dx \\
&= \int \left(\frac{15e^4(bd-ae)^2}{b^6}+\frac{(bd-ae)^6}{b^6(a+bx)^4}+\frac{6e(bd-ae)^5}{b^6(a+bx)^3}+\frac{15e^2(bd-ae)^4}{b^6(a+bx)^2}+\frac{20e^3(bd-ae)^3}{b^6(a+bx)}+\frac{6\ldots}{\ldots}\right. \\
&= \frac{15e^4(bd-ae)^2x}{b^6}-\frac{(bd-ae)^6}{3b^7(a+bx)^3}-\frac{3e(bd-ae)^5}{b^7(a+bx)^2}-\frac{15e^2(bd-ae)^4}{b^7(a+bx)}+\frac{3e^5(bd-ae)(a+bx)^2}{b^7}
\end{aligned}$$

Mathematica [A] time = 0.11873, size = 301, normalized size = 1.93

$$\frac{3a^2b^4e^2\left(-45d^2e^2x^2+90d^3ex-5d^4-63de^3x^3+5e^4x^4\right)+a^3b^3e^3\left(-405d^2ex+110d^3-27de^2x^2+73e^3x^3\right)+3a^4b^2e^4\left(\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^6/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] (-37*a^6*e^6 + 3*a^5*b*e^5*(47*d - 17*e*x) + 3*a^4*b^2*e^4*(-65*d^2 + 81*d*
e*x + 13*e^2*x^2) + a^3*b^3*e^3*(110*d^3 - 405*d^2*e*x - 27*d*e^2*x^2 + 73*
e^3*x^3) + 3*a^2*b^4*e^2*(-5*d^4 + 90*d^3*e*x - 45*d^2*e^2*x^2 - 63*d*e^3*x
^3 + 5*e^4*x^4) - 3*a*b^5*e*(d^5 + 15*d^4*e*x - 60*d^3*e^2*x^2 - 45*d^2*e^3
*x^3 + 15*d*e^4*x^4 + e^5*x^5) + b^6*(-d^6 - 9*d^5*e*x - 45*d^4*e^2*x^2 + 4
5*d^2*e^4*x^4 + 9*d*e^5*x^5 + e^6*x^6) - 60*e^3*(-(b*d) + a*e)^3*(a + b*x)^
3*Log[a + b*x])/(3*b^7*(a + b*x)^3)
```

Maple [B] time = 0.051, size = 483, normalized size = 3.1

$$-15\,{\frac {{e}^{5}{a}^{4}d}{{b}^{6} \left(bx+a \right) ^{2}}}+2\,{\frac {a{d}^{5}e}{{b}^{2} \left(bx+a \right) ^{3}}}+60\,{\frac {{e}^{5}\ln \left(bx+a \right) {a}^{2}d}{{b}^{6}}}-60\,{\frac {{e}^{4}\ln \left(bx+a \right) a{d}^{2}}{{b}^{5}}}+30\,{\frac {{e}^{4}{a}^{3}{d}^{2}}{{b}^{5} \left(bx+a \right) ^{2}}}-30\,{\frac {{a}^{2}{e}^{3}{d}^{3}}{{b}^{4} \left(bx+a \right) ^{2}}}+15\,\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^6/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

```
[Out] -15/b^6*e^5/(b*x+a)^2*a^4*d+2/b^2/(b*x+a)^3*a*d^5*e+60/b^6*e^5*ln(b*x+a)*a^
2*d-60/b^5*e^4*ln(b*x+a)*a*d^2+30/b^5*e^4/(b*x+a)^2*a^3*d^2-30/b^4*e^3/(b*x
+a)^2*a^2*d^3+15/b^3*e^2/(b*x+a)^2*a*d^4+2/b^6/(b*x+a)^3*a^5*d*e^5-5/b^5/(b
*x+a)^3*d^2*e^4*a^4+20/3/b^4/(b*x+a)^3*a^3*d^3*e^3-5/b^3/(b*x+a)^3*a^2*d^4*
e^2+60/b^6*e^5/(b*x+a)*a^3*d-90/b^5*e^4/(b*x+a)*d^2*a^2+60/b^4*e^3/(b*x+a)*
a*d^3-2*e^6/b^5*x^2*a+3*e^5/b^4*x^2*d+10*e^6/b^6*a^2*x+15*e^4/b^4*d^2*x+3/b
^7*e^6/(b*x+a)^2*a^5-3/b^2*e/(b*x+a)^2*d^5-1/3/b^7/(b*x+a)^3*e^6*a^6-20/b^7
*e^6*ln(b*x+a)*a^3+20/b^4*e^3*ln(b*x+a)*d^3-15/b^7*e^6/(b*x+a)*a^4-15/b^3*e
^2/(b*x+a)*d^4+1/3*e^6/b^4*x^3-1/3/b/(b*x+a)^3*d^6-24*e^5/b^5*a*d*x
```

Maxima [B] time = 1.25855, size = 505, normalized size = 3.24

$$-\frac{b^{6} d^{6} + 3 \, a b^{5} d^{5} e + 15 \, a^{2} b^{4} d^{4} e^{2} - 110 \, a^{3} b^{3} d^{3} e^{3} + 195 \, a^{4} b^{2} d^{2} e^{4} - 141 \, a^{5} b d e^{5} + 37 \, a^{6} e^{6} + 45 \, {\left(b^{6} d^{4} e^{2} - 4 \, a b^{5} d^{3} e^{3} + 6 \, a^{2} b^{4} d^{2} \ldots\right.}}{3 \, {\left(b^{10} x^{3} + 3 \, a b^{9} x^{2} + 3 \, a^{2} b^{8} \ldots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(b^6*d^6 + 3*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 110*a^3*b^3*d^3*e^3 +
195*a^4*b^2*d^2*e^4 - 141*a^5*b*d*e^5 + 37*a^6*e^6 + 45*(b^6*d^4*e^2 - 4*a*
b^5*d^3*e^3 + 6*a^2*b^4*d^2*e^4 - 4*a^3*b^3*d*e^5 + a^4*b^2*e^6)*x^2 + 9*(b
^6*d^5*e + 5*a*b^5*d^4*e^2 - 30*a^2*b^4*d^3*e^3 + 50*a^3*b^3*d^2*e^4 - 35*a
^4*b^2*d*e^5 + 9*a^5*b*e^6)*x)/(b^10*x^3 + 3*a*b^9*x^2 + 3*a^2*b^8*x + a^3*
b^7) + 1/3*(b^2*e^6*x^3 + 3*(3*b^2*d*e^5 - 2*a*b*e^6)*x^2 + 3*(15*b^2*d^2*e
^4 - 24*a*b*d*e^5 + 10*a^2*e^6)*x)/b^6 + 20*(b^3*d^3*e^3 - 3*a*b^2*d^2*e^4
+ 3*a^2*b*d*e^5 - a^3*e^6)*log(b*x + a)/b^7
```

Fricas [B] time = 1.79495, size = 1161, normalized size = 7.44

$$\frac{b^{6} e^{6} x^{6} - b^{6} d^{6} - 3 \, a b^{5} d^{5} e - 15 \, a^{2} b^{4} d^{4} e^{2} + 110 \, a^{3} b^{3} d^{3} e^{3} - 195 \, a^{4} b^{2} d^{2} e^{4} + 141 \, a^{5} b d e^{5} - 37 \, a^{6} e^{6} + 3 \, {\left(3 \, b^{6} d e^{5} - a b^{5} e^{6}\right)} x^{5} + 15 \ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^6*e^6*x^6 - b^6*d^6 - 3*a*b^5*d^5*e - 15*a^2*b^4*d^4*e^2 + 110*a^3*b
^3*d^3*e^3 - 195*a^4*b^2*d^2*e^4 + 141*a^5*b*d*e^5 - 37*a^6*e^6 + 3*(3*b^6*
d*e^5 - a*b^5*e^6)*x^5 + 15*(3*b^6*d^2*e^4 - 3*a*b^5*d*e^5 + a^2*b^4*e^6)*x
^4 + (135*a*b^5*d^2*e^4 - 189*a^2*b^4*d*e^5 + 73*a^3*b^3*e^6)*x^3 - 3*(15*b
^6*d^4*e^2 - 60*a*b^5*d^3*e^3 + 45*a^2*b^4*d^2*e^4 + 9*a^3*b^3*d*e^5 - 13*a
^4*b^2*e^6)*x^2 - 3*(3*b^6*d^5*e + 15*a*b^5*d^4*e^2 - 90*a^2*b^4*d^3*e^3 +
135*a^3*b^3*d^2*e^4 - 81*a^4*b^2*d*e^5 + 17*a^5*b*e^6)*x + 60*(a^3*b^3*d^3*
e^3 - 3*a^4*b^2*d^2*e^4 + 3*a^5*b*d*e^5 - a^6*e^6 + (b^6*d^3*e^3 - 3*a*b^5*
d^2*e^4 + 3*a^2*b^4*d*e^5 - a^3*b^3*e^6)*x^3 + 3*(a*b^5*d^3*e^3 - 3*a^2*b^4
*d^2*e^4 + 3*a^3*b^3*d*e^5 - a^4*b^2*e^6)*x^2 + 3*(a^2*b^4*d^3*e^3 - 3*a^3*
b^3*d^2*e^4 + 3*a^4*b^2*d*e^5 - a^5*b*e^6)*x)*log(b*x + a))/(b^10*x^3 + 3*a
*b^9*x^2 + 3*a^2*b^8*x + a^3*b^7)
```

Sympy [B] time = 5.61823, size = 364, normalized size = 2.33

$$-\frac{37a^6e^6 - 141a^5bde^5 + 195a^4b^2d^2e^4 - 110a^3b^3d^3e^3 + 15a^2b^4d^4e^2 + 3ab^5d^5e + b^6d^6 + x^2\left(45a^4b^2e^6 - 180a^3b^3de^5 + 270\right.}{3a^3b^7 + 9a^2b^8x +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**6/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

```
[Out] -(37*a**6*e**6 - 141*a**5*b*d*e**5 + 195*a**4*b**2*d**2*e**4 - 110*a**3*b**
3*d**3*e**3 + 15*a**2*b**4*d**4*e**2 + 3*a*b**5*d**5*e + b**6*d**6 + x**2*(
45*a**4*b**2*e**6 - 180*a**3*b**3*d*e**5 + 270*a**2*b**4*d**2*e**4 - 180*a*
b**5*d**3*e**3 + 45*b**6*d**4*e**2) + x*(81*a**5*b*e**6 - 315*a**4*b**2*d*e
**5 + 450*a**3*b**3*d**2*e**4 - 270*a**2*b**4*d**3*e**3 + 45*a*b**5*d**4*e*
*2 + 9*b**6*d**5*e))/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10
*x**3) + e**6*x**3/(3*b**4) - x**2*(2*a*e**6 - 3*b*d*e**5)/b**5 + x*(10*a**
2*e**6 - 24*a*b*d*e**5 + 15*b**2*d**2*e**4)/b**6 - 20*e**3*(a*e - b*d)**3*l
og(a + b*x)/b**7
```

Giac [B] time = 1.15243, size = 450, normalized size = 2.88

$$\frac{20\left(b^3d^3e^3 - 3\,ab^2d^2e^4 + 3\,a^2bde^5 - a^3e^6\right)\log\left(|bx + a|\right)}{b^7} - \frac{b^6d^6 + 3\,ab^5d^5e + 15\,a^2b^4d^4e^2 - 110\,a^3b^3d^3e^3 + 195\,a^4b^2d^2e^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

```
[Out] 20*(b^3*d^3*e^3 - 3*a*b^2*d^2*e^4 + 3*a^2*b*d*e^5 - a^3*e^6)*log(abs(b*x +
a))/b^7 - 1/3*(b^6*d^6 + 3*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 110*a^3*b^3*d
^3*e^3 + 195*a^4*b^2*d^2*e^4 - 141*a^5*b*d*e^5 + 37*a^6*e^6 + 45*(b^6*d^4*e
^2 - 4*a*b^5*d^3*e^3 + 6*a^2*b^4*d^2*e^4 - 4*a^3*b^3*d*e^5 + a^4*b^2*e^6)*x
^2 + 9*(b^6*d^5*e + 5*a*b^5*d^4*e^2 - 30*a^2*b^4*d^3*e^3 + 50*a^3*b^3*d^2*e
^4 - 35*a^4*b^2*d*e^5 + 9*a^5*b*e^6)*x)/((b*x + a)^3*b^7) + 1/3*(b^8*x^3*e^
6 + 9*b^8*d*x^2*e^5 + 45*b^8*d^2*x*e^4 - 6*a*b^7*x^2*e^6 - 72*a*b^7*d*x*e^5
 + 30*a^2*b^6*x*e^6)/b^12
```

3.1516 $\int \frac{(d+ex)^5}{\left(a^2+2abx+b^2x^2\right)^2}\,dx$

Optimal. Leaf size=129

$$\frac{e^4x(5bd-4ae)}{b^5}-\frac{10e^2(bd-ae)^3}{b^6(a+bx)}+\frac{10e^3(bd-ae)^2\log(a+bx)}{b^6}-\frac{5e(bd-ae)^4}{2b^6(a+bx)^2}-\frac{(bd-ae)^5}{3b^6(a+bx)^3}+\frac{e^5x^2}{2b^4}$$

```
[Out] (e^4*(5*b*d - 4*a*e)*x)/b^5 + (e^5*x^2)/(2*b^4) - (b*d - a*e)^5/(3*b^6*(a +
 b*x)^3) - (5*e*(b*d - a*e)^4)/(2*b^6*(a + b*x)^2) - (10*e^2*(b*d - a*e)^3)
/(b^6*(a + b*x)) + (10*e^3*(b*d - a*e)^2*Log[a + b*x])/b^6
```

Rubi [A] time = 0.124658, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$\frac{e^4x(5bd-4ae)}{b^5}-\frac{10e^2(bd-ae)^3}{b^6(a+bx)}+\frac{10e^3(bd-ae)^2\log(a+bx)}{b^6}-\frac{5e(bd-ae)^4}{2b^6(a+bx)^2}-\frac{(bd-ae)^5}{3b^6(a+bx)^3}+\frac{e^5x^2}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```



```
[Out] (e^4*(5*b*d - 4*a*e)*x)/b^5 + (e^5*x^2)/(2*b^4) - (b*d - a*e)^5/(3*b^6*(a +
 b*x)^3) - (5*e*(b*d - a*e)^4)/(2*b^6*(a + b*x)^2) - (10*e^2*(b*d - a*e)^3)
/(b^6*(a + b*x)) + (10*e^3*(b*d - a*e)^2*Log[a + b*x])/b^6
```


Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^5}{\left(a^2+2abx+b^2x^2\right)^2}\,dx &= \int \frac{(d+ex)^5}{(a+bx)^4}\,dx\\ &= \int \left(\frac{e^4(5bd-4ae)}{b^5}+\frac{e^5x}{b^4}+\frac{(bd-ae)^5}{b^5(a+bx)^4}+\frac{5e(bd-ae)^4}{b^5(a+bx)^3}+\frac{10e^2(bd-ae)^3}{b^5(a+bx)^2}+\frac{10e^3(bd-ae)^2}{b^5(a+bx)}\right)\\ &= \frac{e^4(5bd-4ae)x}{b^5}+\frac{e^5x^2}{2b^4}-\frac{(bd-ae)^5}{3b^6(a+bx)^3}-\frac{5e(bd-ae)^4}{2b^6(a+bx)^2}-\frac{10e^2(bd-ae)^3}{b^6(a+bx)}+\frac{10e^3(bd-ae)^2\log}{b^6}\end{aligned}$$

Mathematica [A] time = 0.0818163, size = 228, normalized size = 1.77

$$-a^2b^3e^2\left(-270d^2ex+20d^3+90de^2x^2+63e^3x^3\right)+a^3b^2e^3\left(110d^2-270dex-9e^2x^2\right)+a^4be^4(81ex-130d)+47a^5e^5-5ab$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] (47*a^5*e^5 + a^4*b*e^4*(-130*d + 81*e*x) + a^3*b^2*e^3*(110*d^2 - 270*d*e*
x - 9*e^2*x^2) - a^2*b^3*e^2*(20*d^3 - 270*d^2*e*x + 90*d*e^2*x^2 + 63*e^3*
x^3) - 5*a*b^4*e*(d^4 + 12*d^3*e*x - 36*d^2*e^2*x^2 - 18*d*e^3*x^3 + 3*e^4*
x^4) + b^5*(-2*d^5 - 15*d^4*e*x - 60*d^3*e^2*x^2 + 30*d*e^4*x^4 + 3*e^5*x^5
) + 60*e^3*(b*d - a*e)^2*(a + b*x)^3*Log[a + b*x])/(6*b^6*(a + b*x)^3)
```

Maple [B] time = 0.049, size = 361, normalized size = 2.8

$$\frac{e^5x^2}{2\,b^4}-4\,\frac{ae^5x}{b^5}+5\,\frac{e^4xd}{b^4}-\frac{5\,a^4e^5}{2\,b^6\,(bx+a)^2}+10\,\frac{e^4a^3d}{b^5\,(bx+a)^2}-15\,\frac{e^3d^2a^2}{b^4\,(bx+a)^2}+10\,\frac{ae^2d^3}{b^3\,(bx+a)^2}-\frac{5\,ed^4}{2\,b^2\,(bx+a)^2}+\frac{a}{3\,b^6\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

```
[Out] 1/2*e^5*x^2/b^4-4*e^5/b^5*a*x+5*e^4/b^4*x*d-5/2/b^6*e^5/(b*x+a)^2*a^4+10/b^
5*e^4/(b*x+a)^2*a^3*d-15/b^4*e^3/(b*x+a)^2*d^2*a^2+10/b^3*e^2/(b*x+a)^2*a*d
^3-5/2/b^2*e/(b*x+a)^2*d^4+1/3/b^6/(b*x+a)^3*a^5*e^5-5/3/b^5/(b*x+a)^3*a^4*
d*e^4+10/3/b^4/(b*x+a)^3*a^3*d^2*e^3-10/3/b^3/(b*x+a)^3*a^2*d^3*e^2+5/3/b^2
/(b*x+a)^3*a*d^4*e-1/3/b/(b*x+a)^3*d^5+10/b^6*e^5*ln(b*x+a)*a^2-20/b^5*e^4*
ln(b*x+a)*a*d+10/b^4*e^3*ln(b*x+a)*d^2+10/b^6*e^5/(b*x+a)*a^3-30/b^5*e^4/(b
*x+a)*a^2*d+30/b^4*e^3/(b*x+a)*a*d^2-10/b^3*e^2/(b*x+a)*d^3
```

Maxima [B] time = 1.14849, size = 379, normalized size = 2.94

$$-\frac{2\,b^5d^5+5\,ab^4d^4e+20\,a^2b^3d^3e^2-110\,a^3b^2d^2e^3+130\,a^4bde^4-47\,a^5e^5+60\left(b^5d^3e^2-3\,ab^4d^2e^3+3\,a^2b^3de^4-a^3b^2e^5\right.}{6\left(b^9x^3+3\,ab^8x^2+3\,a^2b^7x+a^3b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*b^5*d^5 + 5*a*b^4*d^4*e + 20*a^2*b^3*d^3*e^2 - 110*a^3*b^2*d^2*e^3
+ 130*a^4*b*d*e^4 - 47*a^5*e^5 + 60*(b^5*d^3*e^2 - 3*a*b^4*d^2*e^3 + 3*a^2*
b^3*d*e^4 - a^3*b^2*e^5)*x^2 + 15*(b^5*d^4*e + 4*a*b^4*d^3*e^2 - 18*a^2*b^3
*d^2*e^3 + 20*a^3*b^2*d*e^4 - 7*a^4*b*e^5)*x)/(b^9*x^3 + 3*a*b^8*x^2 + 3*a^
2*b^7*x + a^3*b^6) + 1/2*(b*e^5*x^2 + 2*(5*b*d*e^4 - 4*a*e^5)*x)/b^5 + 10*(
b^2*d^2*e^3 - 2*a*b*d*e^4 + a^2*e^5)*log(b*x + a)/b^6
```

Fricas [B] time = 1.66791, size = 867, normalized size = 6.72

$$\frac{3\,b^5e^5x^5-2\,b^5d^5-5\,ab^4d^4e-20\,a^2b^3d^3e^2+110\,a^3b^2d^2e^3-130\,a^4bde^4+47\,a^5e^5+15\left(2\,b^5de^4-ab^4e^5\right)x^4+9\left(10\,ab^4d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^5*e^5*x^5 - 2*b^5*d^5 - 5*a*b^4*d^4*e - 20*a^2*b^3*d^3*e^2 + 110*a
^3*b^2*d^2*e^3 - 130*a^4*b*d*e^4 + 47*a^5*e^5 + 15*(2*b^5*d*e^4 - a*b^4*e^5
)*x^4 + 9*(10*a*b^4*d*e^4 - 7*a^2*b^3*e^5)*x^3 - 3*(20*b^5*d^3*e^2 - 60*a*b
^4*d^2*e^3 + 30*a^2*b^3*d*e^4 + 3*a^3*b^2*e^5)*x^2 - 3*(5*b^5*d^4*e + 20*a*
b^4*d^3*e^2 - 90*a^2*b^3*d^2*e^3 + 90*a^3*b^2*d*e^4 - 27*a^4*b*e^5)*x + 60*
(a^3*b^2*d^2*e^3 - 2*a^4*b*d*e^4 + a^5*e^5 + (b^5*d^2*e^3 - 2*a*b^4*d*e^4 +
 a^2*b^3*e^5)*x^3 + 3*(a*b^4*d^2*e^3 - 2*a^2*b^3*d*e^4 + a^3*b^2*e^5)*x^2 +
 3*(a^2*b^3*d^2*e^3 - 2*a^3*b^2*d*e^4 + a^4*b*e^5)*x)*log(b*x + a))/(b^9*x^
3 + 3*a*b^8*x^2 + 3*a^2*b^7*x + a^3*b^6)
```

Sympy [B] time = 3.74392, size = 282, normalized size = 2.19

$$\frac{47a^5e^5 - 130a^4bde^4 + 110a^3b^2d^2e^3 - 20a^2b^3d^3e^2 - 5ab^4d^4e - 2b^5d^5 + x^2\left(60a^3b^2e^5 - 180a^2b^3de^4 + 180ab^4d^2e^3 - 60b^5d^3e^2 \cdots}{6a^3b^6 + 18a^2b^7x + 18ab^8x^2 + 6b^9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

```
[Out] (47*a**5*e**5 - 130*a**4*b*d*e**4 + 110*a**3*b**2*d**2*e**3 - 20*a**2*b**3*
d**3*e**2 - 5*a*b**4*d**4*e - 2*b**5*d**5 + x**2*(60*a**3*b**2*e**5 - 180*a
**2*b**3*d*e**4 + 180*a*b**4*d**2*e**3 - 60*b**5*d**3*e**2) + x*(105*a**4*b
*e**5 - 300*a**3*b**2*d*e**4 + 270*a**2*b**3*d**2*e**3 - 60*a*b**4*d**3*e**
2 - 15*b**5*d**4*e))/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**
9*x**3) + e**5*x**2/(2*b**4) - x*(4*a*e**5 - 5*b*d*e**4)/b**5 + 10*e**3*(a*
e - b*d)**2*log(a + b*x)/b**6
```

Giac [A] time = 1.12406, size = 332, normalized size = 2.57

$$\frac{10\left(b^2d^2e^3 - 2\,abde^4 + a^2e^5\right)\log\left(|bx + a|\right)}{b^6} + \frac{b^4x^2e^5 + 10\,b^4dxe^4 - 8\,ab^3xe^5}{2\,b^8} - \frac{2\,b^5d^5 + 5\,ab^4d^4e + 20\,a^2b^3d^3e^2 - 110\,a^3b^2d \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

```
[Out] 10*(b^2*d^2*e^3 - 2*a*b*d*e^4 + a^2*e^5)*log(abs(b*x + a))/b^6 + 1/2*(b^4*x
^2*e^5 + 10*b^4*d*x*e^4 - 8*a*b^3*x*e^5)/b^8 - 1/6*(2*b^5*d^5 + 5*a*b^4*d^4
*e + 20*a^2*b^3*d^3*e^2 - 110*a^3*b^2*d^2*e^3 + 130*a^4*b*d*e^4 - 47*a^5*e^
5 + 60*(b^5*d^3*e^2 - 3*a*b^4*d^2*e^3 + 3*a^2*b^3*d*e^4 - a^3*b^2*e^5)*x^2
+ 15*(b^5*d^4*e + 4*a*b^4*d^3*e^2 - 18*a^2*b^3*d^2*e^3 + 20*a^3*b^2*d*e^4 -
 7*a^4*b*e^5)*x)/((b*x + a)^3*b^6)
```

3.1517 $\int \frac{(d+ex)^4}{\left(a^2+2abx+b^2x^2\right)^2}\,dx$

Optimal. Leaf size=103

$$-\frac{6e^2(bd-ae)^2}{b^5(a+bx)}+\frac{4e^3(bd-ae)\log(a+bx)}{b^5}-\frac{2e(bd-ae)^3}{b^5(a+bx)^2}-\frac{(bd-ae)^4}{3b^5(a+bx)^3}+\frac{e^4x}{b^4}$$

```
[Out] (e^4*x)/b^4 - (b*d - a*e)^4/(3*b^5*(a + b*x)^3) - (2*e*(b*d - a*e)^3)/(b^5*
(a + b*x)^2) - (6*e^2*(b*d - a*e)^2)/(b^5*(a + b*x)) + (4*e^3*(b*d - a*e)*L
og[a + b*x])/b^5
```

Rubi [A] time = 0.0884639, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$-\frac{6e^2(bd-ae)^2}{b^5(a+bx)}+\frac{4e^3(bd-ae)\log(a+bx)}{b^5}-\frac{2e(bd-ae)^3}{b^5(a+bx)^2}-\frac{(bd-ae)^4}{3b^5(a+bx)^3}+\frac{e^4x}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] (e^4*x)/b^4 - (b*d - a*e)^4/(3*b^5*(a + b*x)^3) - (2*e*(b*d - a*e)^3)/(b^5*
(a + b*x)^2) - (6*e^2*(b*d - a*e)^2)/(b^5*(a + b*x)) + (4*e^3*(b*d - a*e)*L
og[a + b*x])/b^5
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^4}{\left(a^2+2abx+b^2x^2\right)^2}\,dx &= \int \frac{(d+ex)^4}{(a+bx)^4}\,dx\\ &= \int \left(\frac{e^4}{b^4}+\frac{(bd-ae)^4}{b^4(a+bx)^4}+\frac{4e(bd-ae)^3}{b^4(a+bx)^3}+\frac{6e^2(bd-ae)^2}{b^4(a+bx)^2}+\frac{4e^3(bd-ae)}{b^4(a+bx)}\right)dx\\ &= \frac{e^4x}{b^4}-\frac{(bd-ae)^4}{3b^5(a+bx)^3}-\frac{2e(bd-ae)^3}{b^5(a+bx)^2}-\frac{6e^2(bd-ae)^2}{b^5(a+bx)}+\frac{4e^3(bd-ae)\log(a+bx)}{b^5}\end{aligned}$$

Mathematica [A] time = 0.0606032, size = 166, normalized size = 1.61

$$\frac{-3a^2b^2e^2\left(2d^2-18dex+3e^2x^2\right)+a^3be^3(22d-27ex)-13a^4e^4+ab^3e\left(-18d^2ex-2d^3+36de^2x^2+9e^3x^3\right)-12e^3(a+b}{3b^5(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] (-13*a^4*e^4 + a^3*b*e^3*(22*d - 27*e*x) - 3*a^2*b^2*e^2*(2*d^2 - 18*d*e*x
+ 3*e^2*x^2) + a*b^3*e*(-2*d^3 - 18*d^2*e*x + 36*d*e^2*x^2 + 9*e^3*x^3) - b
^4*(d^4 + 6*d^3*e*x + 18*d^2*e^2*x^2 - 3*e^4*x^4) - 12*e^3*(-(b*d) + a*e)*(
a + b*x)^3*Log[a + b*x])/(3*b^5*(a + b*x)^3)
```

Maple [B] time = 0.049, size = 255, normalized size = 2.5

$$\frac{e^4x}{b^4} + 2\frac{a^3e^4}{b^5(bx+a)^2} - 6\frac{a^2e^3d}{b^4(bx+a)^2} + 6\frac{ad^2e^2}{b^3(bx+a)^2} - 2\frac{ed^3}{b^2(bx+a)^2} - \frac{a^4e^4}{3b^5(bx+a)^3} + \frac{4a^3de^3}{3b^4(bx+a)^3} - 2\frac{d^2e^2a^2}{b^3(bx+a)^3} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

```
[Out] e^4*x/b^4+2/b^5*e^4/(b*x+a)^2*a^3-6/b^4*e^3/(b*x+a)^2*a^2*d+6/b^3*e^2/(b*x+
a)^2*a*d^2-2/b^2*e/(b*x+a)^2*d^3-1/3/b^5/(b*x+a)^3*a^4*e^4+4/3/b^4/(b*x+a)^
3*a^3*d*e^3-2/b^3/(b*x+a)^3*d^2*e^2*a^2+4/3/b^2/(b*x+a)^3*a*d^3*e-1/3/b/(b*
x+a)^3*d^4-4/b^5*e^4*ln(b*x+a)*a+4/b^4*e^3*ln(b*x+a)*d-6/b^5*e^4/(b*x+a)*a^
2+12/b^4*e^3/(b*x+a)*a*d-6/b^3*e^2/(b*x+a)*d^2
```

Maxima [A] time = 1.12441, size = 271, normalized size = 2.63

$$\frac{e^4x}{b^4} - \frac{b^4d^4 + 2\,ab^3d^3e + 6\,a^2b^2d^2e^2 - 22\,a^3bde^3 + 13\,a^4e^4 + 18\left(b^4d^2e^2 - 2\,ab^3de^3 + a^2b^2e^4\right)x^2 + 6\left(b^4d^3e + 3\,ab^3d^2e^2 - 9\right.}{3\left(b^8x^3 + 3\,ab^7x^2 + 3\,a^2b^6x + a^3b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

```
[Out] e^4*x/b^4 - 1/3*(b^4*d^4 + 2*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 22*a^3*b*d*e
^3 + 13*a^4*e^4 + 18*(b^4*d^2*e^2 - 2*a*b^3*d*e^3 + a^2*b^2*e^4)*x^2 + 6*(b
^4*d^3*e + 3*a*b^3*d^2*e^2 - 9*a^2*b^2*d*e^3 + 5*a^3*b*e^4)*x)/(b^8*x^3 + 3
*a*b^7*x^2 + 3*a^2*b^6*x + a^3*b^5) + 4*(b*d*e^3 - a*e^4)*log(b*x + a)/b^5
```

Fricas [B] time = 1.69989, size = 581, normalized size = 5.64

$$\frac{3\,b^4e^4x^4 + 9\,ab^3e^4x^3 - b^4d^4 - 2\,ab^3d^3e - 6\,a^2b^2d^2e^2 + 22\,a^3bde^3 - 13\,a^4e^4 - 9\left(2\,b^4d^2e^2 - 4\,ab^3de^3 + a^2b^2e^4\right)x^2 - 3\left(2\,b^4d^3\right.}{3\left(b^8x^3 +\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b^4*e^4*x^4 + 9*a*b^3*e^4*x^3 - b^4*d^4 - 2*a*b^3*d^3*e - 6*a^2*b^2*
d^2*e^2 + 22*a^3*b*d*e^3 - 13*a^4*e^4 - 9*(2*b^4*d^2*e^2 - 4*a*b^3*d*e^3 +
a^2*b^2*e^4)*x^2 - 3*(2*b^4*d^3*e + 6*a*b^3*d^2*e^2 - 18*a^2*b^2*d*e^3 + 9*
```

```
a^3*b*e^4)*x + 12*(a^3*b*d*e^3 - a^4*e^4 + (b^4*d*e^3 - a*b^3*e^4)*x^3 + 3*
(a*b^3*d*e^3 - a^2*b^2*e^4)*x^2 + 3*(a^2*b^2*d*e^3 - a^3*b*e^4)*x)*log(b*x
+ a))/(b^8*x^3 + 3*a*b^7*x^2 + 3*a^2*b^6*x + a^3*b^5)
```

Sympy [B] time = 2.46601, size = 209, normalized size = 2.03

$$-\frac{13a^4e^4 - 22a^3bde^3 + 6a^2b^2d^2e^2 + 2ab^3d^3e + b^4d^4 + x^2\left(18a^2b^2e^4 - 36ab^3de^3 + 18b^4d^2e^2\right) + x\left(30a^3be^4 - 54a^2b^2de^3 + \right.}{3a^3b^5 + 9a^2b^6x + 9ab^7x^2 + 3b^8x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

```
[Out] -(13*a**4*e**4 - 22*a**3*b*d*e**3 + 6*a**2*b**2*d**2*e**2 + 2*a*b**3*d**3*e
 + b**4*d**4 + x**2*(18*a**2*b**2*e**4 - 36*a*b**3*d*e**3 + 18*b**4*d**2*e*
*2) + x*(30*a**3*b*e**4 - 54*a**2*b**2*d*e**3 + 18*a*b**3*d**2*e**2 + 6*b**
4*d**3*e))/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) + e*
*4*x/b**4 - 4*e**3*(a*e - b*d)*log(a + b*x)/b**5
```

Giac [A] time = 1.14, size = 225, normalized size = 2.18

$$\frac{xe^4}{b^4} + \frac{4\left(bde^3 - ae^4\right)\log\left(|bx + a|\right)}{b^5} - \frac{b^4d^4 + 2\,ab^3d^3e + 6\,a^2b^2d^2e^2 - 22\,a^3bde^3 + 13\,a^4e^4 + 18\left(b^4d^2e^2 - 2\,ab^3de^3 + a^2b^2\right.}{3\,(bx + a)^3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

```
[Out] x*e^4/b^4 + 4*(b*d*e^3 - a*e^4)*log(abs(b*x + a))/b^5 - 1/3*(b^4*d^4 + 2*a*
b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 22*a^3*b*d*e^3 + 13*a^4*e^4 + 18*(b^4*d^2*e
^2 - 2*a*b^3*d*e^3 + a^2*b^2*e^4)*x^2 + 6*(b^4*d^3*e + 3*a*b^3*d^2*e^2 - 9*
a^2*b^2*d*e^3 + 5*a^3*b*e^4)*x)/((b*x + a)^3*b^5)
```

3.1518 $\int \frac{(d+ex)^3}{\left(a^2+2abx+b^2x^2\right)^2}\,dx$

Optimal. Leaf size=86

$$-\frac{3e^2(bd-ae)}{b^4(a+bx)}-\frac{3e(bd-ae)^2}{2b^4(a+bx)^2}-\frac{(bd-ae)^3}{3b^4(a+bx)^3}+\frac{e^3\log(a+bx)}{b^4}$$

```
[Out] -(b*d - a*e)^3/(3*b^4*(a + b*x)^3) - (3*e*(b*d - a*e)^2)/(2*b^4*(a + b*x)^2
) - (3*e^2*(b*d - a*e))/(b^4*(a + b*x)) + (e^3*Log[a + b*x])/b^4
```

Rubi [A] time = 0.0595502, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$-\frac{3e^2(bd-ae)}{b^4(a+bx)}-\frac{3e(bd-ae)^2}{2b^4(a+bx)^2}-\frac{(bd-ae)^3}{3b^4(a+bx)^3}+\frac{e^3\log(a+bx)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] -(b*d - a*e)^3/(3*b^4*(a + b*x)^3) - (3*e*(b*d - a*e)^2)/(2*b^4*(a + b*x)^2
) - (3*e^2*(b*d - a*e))/(b^4*(a + b*x)) + (e^3*Log[a + b*x])/b^4
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^3}{\left(a^2+2abx+b^2x^2\right)^2}\,dx &= \int \frac{(d+ex)^3}{(a+bx)^4}\,dx\\ &= \int \left(\frac{(bd-ae)^3}{b^3(a+bx)^4}+\frac{3e(bd-ae)^2}{b^3(a+bx)^3}+\frac{3e^2(bd-ae)}{b^3(a+bx)^2}+\frac{e^3}{b^3(a+bx)}\right)dx\\ &= -\frac{(bd-ae)^3}{3b^4(a+bx)^3}-\frac{3e(bd-ae)^2}{2b^4(a+bx)^2}-\frac{3e^2(bd-ae)}{b^4(a+bx)}+\frac{e^3\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.0414583, size = 80, normalized size = 0.93

$$\frac{6e^3\log(a+bx)-\frac{(bd-ae)\left(11a^2e^2+abe(5d+27ex)+b^2\left(2d^2+9dex+18e^2x^2\right)\right)}{(a+bx)^3}}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] (-(((b*d - a*e)*(11*a^2*e^2 + a*b*e*(5*d + 27*e*x) + b^2*(2*d^2 + 9*d*e*x +
 18*e^2*x^2)))/(a + b*x)^3) + 6*e^3*Log[a + b*x])/(6*b^4)
```

Maple [B] time = 0.046, size = 166, normalized size = 1.9

$$-\frac{3a^2e^3}{2b^4(bx+a)^2}+3\frac{ade^2}{b^3(bx+a)^2}-\frac{3d^2e}{2b^2(bx+a)^2}+\frac{a^3e^3}{3b^4(bx+a)^3}-\frac{de^2a^2}{b^3(bx+a)^3}+\frac{d^2ea}{b^2(bx+a)^3}-\frac{d^3}{3b(bx+a)^3}+\frac{e^3\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

```
[Out] -3/2*e^3/b^4/(b*x+a)^2*a^2+3*e^2/b^3/(b*x+a)^2*a*d-3/2*e/b^2/(b*x+a)^2*d^2+
1/3/b^4/(b*x+a)^3*a^3*e^3-1/b^3/(b*x+a)^3*d*e^2*a^2+1/b^2/(b*x+a)^3*a*d^2*e
-1/3/b/(b*x+a)^3*d^3+e^3*ln(b*x+a)/b^4+3/b^4*e^3/(b*x+a)*a-3/b^3*e^2/(b*x+a
)*d
```

Maxima [A] time = 1.20806, size = 192, normalized size = 2.23

$$-\frac{2b^3d^3+3ab^2d^2e+6a^2bde^2-11a^3e^3+18\left(b^3de^2-ab^2e^3\right)x^2+9\left(b^3d^2e+2ab^2de^2-3a^2be^3\right)x}{6\left(b^7x^3+3ab^6x^2+3a^2b^5x+a^3b^4\right)}+\frac{e^3\log(bx+a)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*b^3*d^3 + 3*a*b^2*d^2*e + 6*a^2*b*d*e^2 - 11*a^3*e^3 + 18*(b^3*d*e^
2 - a*b^2*e^3)*x^2 + 9*(b^3*d^2*e + 2*a*b^2*d*e^2 - 3*a^2*b*e^3)*x)/(b^7*x^
3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4) + e^3*log(b*x + a)/b^4
```

Fricas [B] time = 1.74703, size = 360, normalized size = 4.19

$$-\frac{2b^3d^3+3ab^2d^2e+6a^2bde^2-11a^3e^3+18\left(b^3de^2-ab^2e^3\right)x^2+9\left(b^3d^2e+2ab^2de^2-3a^2be^3\right)x-6\left(b^3e^3x^3+3ab^2e^3\right.}{6\left(b^7x^3+3ab^6x^2+3a^2b^5x+a^3b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b^3*d^3 + 3*a*b^2*d^2*e + 6*a^2*b*d*e^2 - 11*a^3*e^3 + 18*(b^3*d*e^
2 - a*b^2*e^3)*x^2 + 9*(b^3*d^2*e + 2*a*b^2*d*e^2 - 3*a^2*b*e^3)*x - 6*(b^3
*e^3*x^3 + 3*a*b^2*e^3*x^2 + 3*a^2*b*e^3*x + a^3*e^3)*log(b*x + a))/(b^7*x^
3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4)
```

Sympy [A] time = 1.52887, size = 148, normalized size = 1.72

$$\frac{11a^3e^3-6a^2bde^2-3ab^2d^2e-2b^3d^3+x^2\left(18ab^2e^3-18b^3de^2\right)+x\left(27a^2be^3-18ab^2de^2-9b^3d^2e\right)}{6a^3b^4+18a^2b^5x+18ab^6x^2+6b^7x^3}+\frac{e^3\log\left(a+bx\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

```
[Out] (11*a**3*e**3 - 6*a**2*b*d*e**2 - 3*a*b**2*d**2*e - 2*b**3*d**3 + x**2*(18*
a*b**2*e**3 - 18*b**3*d*e**2) + x*(27*a**2*b*e**3 - 18*a*b**2*d*e**2 - 9*b*
*3*d**2*e))/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) +
 e**3*log(a + b*x)/b**4
```

Giac [A] time = 1.16156, size = 153, normalized size = 1.78

$$\frac{e^3\log\left(|bx+a|\right)}{b^4}-\frac{18\left(b^2de^2-abe^3\right)x^2+9\left(b^2d^2e+2\,abde^2-3\,a^2e^3\right)x+\frac{2\,b^3d^3+3\,ab^2d^2e+6\,a^2bde^2-11\,a^3e^3}{b}}{6\left(bx+a\right)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

```
[Out] e^3*log(abs(b*x + a))/b^4 - 1/6*(18*(b^2*d*e^2 - a*b*e^3)*x^2 + 9*(b^2*d^2*
e + 2*a*b*d*e^2 - 3*a^2*e^3)*x + (2*b^3*d^3 + 3*a*b^2*d^2*e + 6*a^2*b*d*e^2
 - 11*a^3*e^3)/b)/((b*x + a)^3*b^3)
```

3.1519 $\int \frac{(d+ex)^2}{\left(a^2+2abx+b^2x^2\right)^2}\,dx$

Optimal. Leaf size=28

$$-\frac{(d+ex)^3}{3(a+bx)^3(bd-ae)}$$

```
[Out] -(d + e*x)^3/(3*(b*d - a*e)*(a + b*x)^3)
```

Rubi [A] time = 0.0046141, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 37}

$$-\frac{(d+ex)^3}{3(a+bx)^3(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] -(d + e*x)^3/(3*(b*d - a*e)*(a + b*x)^3)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2}{\left(a^2+2abx+b^2x^2\right)^2}\,dx &= \int \frac{(d+ex)^2}{(a+bx)^4}\,dx\\ &= -\frac{(d+ex)^3}{3(bd-ae)(a+bx)^3}\end{aligned}$$

Mathematica [A] time = 0.0238475, size = 53, normalized size = 1.89

$$-\frac{a^2e^2+abe(d+3ex)+b^2\left(d^2+3dex+3e^2x^2\right)}{3b^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] -(a^2*e^2 + a*b*e*(d + 3*e*x) + b^2*(d^2 + 3*d*e*x + 3*e^2*x^2))/(3*b^3*(a
+ b*x)^3)
```

Maple [B] time = 0.046, size = 70, normalized size = 2.5

$$\frac{e(ae-bd)}{b^3(bx+a)^2}-\frac{a^2e^2-2abde+b^2d^2}{3b^3(bx+a)^3}-\frac{e^2}{b^3(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

```
[Out] e*(a*e-b*d)/b^3/(b*x+a)^2-1/3*(a^2*e^2-2*a*b*d*e+b^2*d^2)/b^3/(b*x+a)^3-e^2
/b^3/(b*x+a)
```

Maxima [B] time = 1.12975, size = 113, normalized size = 4.04

$$-\frac{3b^2e^2x^2+b^2d^2+abde+a^2e^2+3\left(b^2de+abe^2\right)x}{3\left(b^6x^3+3ab^5x^2+3a^2b^4x+a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^2*e^2*x^2 + b^2*d^2 + a*b*d*e + a^2*e^2 + 3*(b^2*d*e + a*b*e^2)*x
)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3*b^3)
```

Fricas [B] time = 1.74185, size = 170, normalized size = 6.07

$$-\frac{3b^2e^2x^2+b^2d^2+abde+a^2e^2+3\left(b^2de+abe^2\right)x}{3\left(b^6x^3+3ab^5x^2+3a^2b^4x+a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*e^2*x^2 + b^2*d^2 + a*b*d*e + a^2*e^2 + 3*(b^2*d*e + a*b*e^2)*x
)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3*b^3)
```

Sympy [B] time = 0.958547, size = 88, normalized size = 3.14

$$-\frac{a^2e^2+abde+b^2d^2+3b^2e^2x^2+x\left(3abe^2+3b^2de\right)}{3a^3b^3+9a^2b^4x+9ab^5x^2+3b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

```
[Out] -(a**2*e**2 + a*b*d*e + b**2*d**2 + 3*b**2*e**2*x**2 + x*(3*a*b*e**2 + 3*b*
*2*d*e))/(3*a**3*b**3 + 9*a**2*b**4*x + 9*a*b**5*x**2 + 3*b**6*x**3)
```

Giac [B] time = 1.21924, size = 78, normalized size = 2.79

$$-\frac{3\,b^2x^2e^2 + 3\,b^2dxe + b^2d^2 + 3\,abxe^2 + abde + a^2e^2}{3\,(bx+a)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*x^2*e^2 + 3*b^2*d*x*e + b^2*d^2 + 3*a*b*x*e^2 + a*b*d*e + a^2*e
^2)/((b*x + a)^3*b^3)
```

3.1520 $\int \frac{d+ex}{(a^2+2abx+b^2x^2)^2}\,dx$

Optimal. Leaf size=38

$$-\frac{bd-ae}{3b^2(a+bx)^3}-\frac{e}{2b^2(a+bx)^2}$$

```
[Out] -(b*d - a*e)/(3*b^2*(a + b*x)^3) - e/(2*b^2*(a + b*x)^2)
```

Rubi [A] time = 0.0219057, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {27, 43}

$$-\frac{bd-ae}{3b^2(a+bx)^3}-\frac{e}{2b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] -(b*d - a*e)/(3*b^2*(a + b*x)^3) - e/(2*b^2*(a + b*x)^2)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex}{\left(a^2+2abx+b^2x^2\right)^2}\,dx &= \int \frac{d+ex}{(a+bx)^4}\,dx\\
&= \int \left(\frac{bd-ae}{b(a+bx)^4}+\frac{e}{b(a+bx)^3}\right)dx\\
&= -\frac{bd-ae}{3b^2(a+bx)^3}-\frac{e}{2b^2(a+bx)^2}
\end{aligned}$$

Mathematica [A] time = 0.0083979, size = 27, normalized size = 0.71

$$-\frac{ae+2bd+3bex}{6b^2(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] -(2*b*d + a*e + 3*b*e*x)/(6*b^2*(a + b*x)^3)
```

Maple [A] time = 0.043, size = 35, normalized size = 0.9

$$-\frac{e}{2\,b^2\,(bx+a)^2}-\frac{-ae+bd}{3\,b^2\,(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

```
[Out] -1/2*e/b^2/(b*x+a)^2-1/3*(-a*e+b*d)/b^2/(b*x+a)^3
```

Maxima [A] time = 2.02562, size = 68, normalized size = 1.79

$$-\frac{3\,bex+2\,bd+ae}{6\,\left(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*b*e*x + 2*b*d + a*e)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2
)
```

Fricas [A] time = 1.69501, size = 105, normalized size = 2.76

$$-\frac{3\,bex+2\,bd+ae}{6\,\left(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b*e*x + 2*b*d + a*e)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2
)
```

Sympy [A] time = 0.517849, size = 53, normalized size = 1.39

$$-\frac{ae+2bd+3bex}{6a^3b^2+18a^2b^3x+18ab^4x^2+6b^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

```
[Out] -(a*e + 2*b*d + 3*b*e*x)/(6*a**3*b**2 + 18*a**2*b**3*x + 18*a*b**4*x**2 + 6
*b**5*x**3)
```

Giac [A] time = 1.17718, size = 36, normalized size = 0.95

$$-\frac{3\,bxe+2\,bd+ae}{6\,(bx+a)^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*b*x*e + 2*b*d + a*e)/((b*x + a)^3*b^2)
```

3.1521 $\int \frac{1}{(a^2+2abx+b^2x^2)^2} dx$

Optimal. Leaf size=14

$$-\frac{1}{3b(a+bx)^3}$$

```
[Out] -1/(3*b*(a + b*x)^3)
```

Rubi [A] time = 0.0023577, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {27, 32}

$$-\frac{1}{3b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(-2),x]
```

```
[Out] -1/(3*b*(a + b*x)^3)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(a^2+2abx+b^2x^2\right)^2} dx &= \int \frac{1}{(a+bx)^4} dx \\ &= -\frac{1}{3b(a+bx)^3} \end{aligned}$$

Mathematica [A] time = 0.0027517, size = 14, normalized size = 1.

$$-\frac{1}{3b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(-2),x]
```

```
[Out] -1/(3*b*(a + b*x)^3)
```

Maple [A] time = 0.043, size = 13, normalized size = 0.9

$$-\frac{1}{3\,b\,(bx+a)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

```
[Out] -1/3/b/(b*x+a)^3
```

Maxima [B] time = 1.38165, size = 47, normalized size = 3.36

$$-\frac{1}{3\left(b^{4}x^{3}+3\,ab^{3}x^{2}+3\,a^{2}b^{2}x+a^{3}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/3/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b)
```

Fricas [B] time = 1.70062, size = 70, normalized size = 5.

$$-\frac{1}{3\left(b^{4}x^{3}+3\,ab^{3}x^{2}+3\,a^{2}b^{2}x+a^{3}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b)
```

Sympy [B] time = 0.395112, size = 37, normalized size = 2.64

$$-\frac{1}{3a^{3}b+9a^{2}b^{2}x+9ab^{3}x^{2}+3b^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

```
[Out] -1/(3*a**3*b + 9*a**2*b**2*x + 9*a*b**3*x**2 + 3*b**4*x**3)
```

Giac [A] time = 1.20085, size = 16, normalized size = 1.14

$$-\frac{1}{3\,(bx+a)^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

```
[Out] -1/3/((b*x + a)^3*b)
```

3.1522 $\int \frac{1}{(d+ex)\left(a^2+2abx+b^2x^2\right)^2}\,dx$

Optimal. Leaf size=107

$$-\frac{e^2}{(a+bx)(bd-ae)^3}-\frac{e^3\log(a+bx)}{(bd-ae)^4}+\frac{e^3\log(d+ex)}{(bd-ae)^4}+\frac{e}{2(a+bx)^2(bd-ae)^2}-\frac{1}{3(a+bx)^3(bd-ae)}$$

```
[Out] -1/(3*(b*d - a*e)*(a + b*x)^3) + e/(2*(b*d - a*e)^2*(a + b*x)^2) - e^2/((b*
d - a*e)^3*(a + b*x)) - (e^3*Log[a + b*x])/(b*d - a*e)^4 + (e^3*Log[d + e*x
])/(b*d - a*e)^4
```

Rubi [A] time = 0.0680284, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 44}

$$-\frac{e^2}{(a+bx)(bd-ae)^3}-\frac{e^3\log(a+bx)}{(bd-ae)^4}+\frac{e^3\log(d+ex)}{(bd-ae)^4}+\frac{e}{2(a+bx)^2(bd-ae)^2}-\frac{1}{3(a+bx)^3(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

```
[Out] -1/(3*(b*d - a*e)*(a + b*x)^3) + e/(2*(b*d - a*e)^2*(a + b*x)^2) - e^2/((b*
d - a*e)^3*(a + b*x)) - (e^3*Log[a + b*x])/(b*d - a*e)^4 + (e^3*Log[d + e*x
])/(b*d - a*e)^4
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)\left(a^2+2abx+b^2x^2\right)^2}\,dx &= \int \frac{1}{(a+bx)^4(d+ex)}\,dx\\ &= \int \left(\frac{b}{(bd-ae)(a+bx)^4}-\frac{be}{(bd-ae)^2(a+bx)^3}+\frac{be^2}{(bd-ae)^3(a+bx)^2}-\frac{be^3}{(bd-ae)^4(a+\right.\\ &= -\frac{1}{3(bd-ae)(a+bx)^3}+\frac{e}{2(bd-ae)^2(a+bx)^2}-\frac{e^2}{(bd-ae)^3(a+bx)}-\frac{e^3\log(a+bx)}{(bd-ae)^4}\end{aligned}$$

Mathematica [A] time = 0.0448062, size = 107, normalized size = 1.

$$-\frac{e^2}{(a+bx)(bd-ae)^3}-\frac{e^3\log(a+bx)}{(bd-ae)^4}+\frac{e^3\log(d+ex)}{(bd-ae)^4}+\frac{e}{2(a+bx)^2(bd-ae)^2}+\frac{1}{3(a+bx)^3(ae-bd)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

```
[Out] 1/(3*(-(b*d) + a*e)*(a + b*x)^3) + e/(2*(b*d - a*e)^2*(a + b*x)^2) - e^2/((
b*d - a*e)^3*(a + b*x)) - (e^3*Log[a + b*x])/(b*d - a*e)^4 + (e^3*Log[d + e
*x])/(b*d - a*e)^4
```

Maple [A] time = 0.05, size = 103, normalized size = 1.

$$\frac{e^3\ln(ex+d)}{(ae-bd)^4}+\frac{1}{(3ae-3bd)(bx+a)^3}+\frac{e}{2(ae-bd)^2(bx+a)^2}+\frac{e^2}{(ae-bd)^3(bx+a)}-\frac{e^3\ln(bx+a)}{(ae-bd)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

```
[Out] e^3/(a*e-b*d)^4*ln(e*x+d)+1/3/(a*e-b*d)/(b*x+a)^3+1/2*e/(a*e-b*d)^2/(b*x+a)
^2+e^2/(a*e-b*d)^3/(b*x+a)-e^3/(a*e-b*d)^4*ln(b*x+a)
```

Maxima [B] time = 1.21259, size = 487, normalized size = 4.55

$$-\frac{e^3\log(bx+a)}{b^4d^4-4ab^3d^3e+6a^2b^2d^2e^2-4a^3bde^3+a^4e^4}+\frac{e^3\log(ex+d)}{b^4d^4-4ab^3d^3e+6a^2b^2d^2e^2-4a^3bde^3+a^4e^4}-\frac{}{6\left(a^3b^3d^3-3a^4b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

```
[Out] -e^3*log(b*x + a)/(b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*
e^3 + a^4*e^4) + e^3*log(e*x + d)/(b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*
e^2 - 4*a^3*b*d*e^3 + a^4*e^4) - 1/6*(6*b^2*e^2*x^2 + 2*b^2*d^2 - 7*a*b*d*e
 + 11*a^2*e^2 - 3*(b^2*d*e - 5*a*b*e^2)*x)/(a^3*b^3*d^3 - 3*a^4*b^2*d^2*e +
 3*a^5*b*d*e^2 - a^6*e^3 + (b^6*d^3 - 3*a*b^5*d^2*e + 3*a^2*b^4*d*e^2 - a^3
*b^3*e^3)*x^3 + 3*(a*b^5*d^3 - 3*a^2*b^4*d^2*e + 3*a^3*b^3*d*e^2 - a^4*b^2*
e^3)*x^2 + 3*(a^2*b^4*d^3 - 3*a^3*b^3*d^2*e + 3*a^4*b^2*d*e^2 - a^5*b*e^3)*
x)
```

Fricas [B] time = 1.76155, size = 856, normalized size = 8.

$$-\frac{2b^3d^3-9ab^2d^2e+18a^2bde^2-11a^3e^3+6\left(b^3de^2-ab^2e^3\right)x^2-3\left(b^3d^2e-6ab^2de^2+5a^2be^3\right)x+6\left(b^3e^3\right.}{6\left(a^3b^4d^4-4a^4b^3d^3e+6a^5b^2d^2e^2-4a^6bde^3+a^7e^4+\left(b^7d^4-4ab^6d^3e+6a^2b^5d^2e^2-4a^3b^4de^3+a^4b^3e^4\right)x^3+3\left(ab\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b^3*d^3 - 9*a*b^2*d^2*e + 18*a^2*b*d*e^2 - 11*a^3*e^3 + 6*(b^3*d*e^
2 - a*b^2*e^3)*x^2 - 3*(b^3*d^2*e - 6*a*b^2*d*e^2 + 5*a^2*b*e^3)*x + 6*(b^3
*e^3*x^3 + 3*a*b^2*e^3*x^2 + 3*a^2*b*e^3*x + a^3*e^3)*log(b*x + a) - 6*(b^3
```

```
*e^3*x^3 + 3*a*b^2*e^3*x^2 + 3*a^2*b*e^3*x + a^3*e^3)*log(e*x + d))/(a^3*b^
4*d^4 - 4*a^4*b^3*d^3*e + 6*a^5*b^2*d^2*e^2 - 4*a^6*b*d*e^3 + a^7*e^4 + (b^
7*d^4 - 4*a*b^6*d^3*e + 6*a^2*b^5*d^2*e^2 - 4*a^3*b^4*d*e^3 + a^4*b^3*e^4)*
x^3 + 3*(a*b^6*d^4 - 4*a^2*b^5*d^3*e + 6*a^3*b^4*d^2*e^2 - 4*a^4*b^3*d*e^3
+ a^5*b^2*e^4)*x^2 + 3*(a^2*b^5*d^4 - 4*a^3*b^4*d^3*e + 6*a^4*b^3*d^2*e^2 -
 4*a^5*b^2*d*e^3 + a^6*b*e^4)*x)
```

Sympy [B] time = 2.20426, size = 570, normalized size = 5.33

$$\frac{e^{3}\log\left(x+\frac{-\frac{a^{5}e^{8}}{(ae-bd)^{4}}+\frac{5a^{4}bde^{7}}{(ae-bd)^{4}}-\frac{10a^{3}b^{2}d^{2}e^{6}}{(ae-bd)^{4}}+\frac{10a^{2}b^{3}d^{3}e^{5}}{(ae-bd)^{4}}-\frac{5ab^{4}d^{4}e^{4}}{(ae-bd)^{4}}+ae^{4}+\frac{b^{5}d^{5}e^{3}}{(ae-bd)^{4}}+bde^{3}}{2be^{4}}\right)}{(ae-bd)^{4}}-\frac{e^{3}\log\left(x+\frac{\frac{a^{5}e^{8}}{(ae-bd)^{4}}-\frac{5a^{4}bde^{7}}{(ae-bd)^{4}}+\frac{10a^{3}b^{2}d^{2}e^{6}}{(ae-bd)^{4}}-\frac{10a^{2}b^{3}d^{3}e^{5}}{(ae-bd)^{4}}+\frac{5ab^{4}d^{4}e^{4}}{(ae-bd)^{4}}\cdots}{2be^{4}}\right.}{(ae-bd)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

```
[Out] e**3*log(x + (-a**5*e**8/(a*e - b*d)**4 + 5*a**4*b*d*e**7/(a*e - b*d)**4 -
10*a**3*b**2*d**2*e**6/(a*e - b*d)**4 + 10*a**2*b**3*d**3*e**5/(a*e - b*d)*
*4 - 5*a*b**4*d**4*e**4/(a*e - b*d)**4 + a*e**4 + b**5*d**5*e**3/(a*e - b*d
)**4 + b*d*e**3)/(2*b*e**4))/(a*e - b*d)**4 - e**3*log(x + (a**5*e**8/(a*e
- b*d)**4 - 5*a**4*b*d*e**7/(a*e - b*d)**4 + 10*a**3*b**2*d**2*e**6/(a*e -
b*d)**4 - 10*a**2*b**3*d**3*e**5/(a*e - b*d)**4 + 5*a*b**4*d**4*e**4/(a*e -
 b*d)**4 + a*e**4 - b**5*d**5*e**3/(a*e - b*d)**4 + b*d*e**3)/(2*b*e**4))/(
a*e - b*d)**4 + (11*a**2*e**2 - 7*a*b*d*e + 2*b**2*d**2 + 6*b**2*e**2*x**2
+ x*(15*a*b*e**2 - 3*b**2*d*e))/(6*a**6*e**3 - 18*a**5*b*d*e**2 + 18*a**4*b
**2*d**2*e - 6*a**3*b**3*d**3 + x**3*(6*a**3*b**3*e**3 - 18*a**2*b**4*d*e**
2 + 18*a*b**5*d**2*e - 6*b**6*d**3) + x**2*(18*a**4*b**2*e**3 - 54*a**3*b**
3*d*e**2 + 54*a**2*b**4*d**2*e - 18*a*b**5*d**3) + x*(18*a**5*b*e**3 - 54*a
**4*b**2*d*e**2 + 54*a**3*b**3*d**2*e - 18*a**2*b**4*d**3))
```

Giac [B] time = 1.17333, size = 316, normalized size = 2.95

$$-\frac{be^{3}\log\left(|bx+a|\right)}{b^{5}d^{4}-4\,ab^{4}d^{3}e+6\,a^{2}b^{3}d^{2}e^{2}-4\,a^{3}b^{2}de^{3}+a^{4}be^{4}}+\frac{e^{4}\log\left(|xe+d|\right)}{b^{4}d^{4}e-4\,ab^{3}d^{3}e^{2}+6\,a^{2}b^{2}d^{2}e^{3}-4\,a^{3}bde^{4}+a^{4}e^{5}}-\frac{2\,b^{3}d^{3}-9\,ab^{2}d^{2}\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

```
[Out] -b*e^3*log(abs(b*x + a))/(b^5*d^4 - 4*a*b^4*d^3*e + 6*a^2*b^3*d^2*e^2 - 4*a
^3*b^2*d*e^3 + a^4*b*e^4) + e^4*log(abs(x*e + d))/(b^4*d^4*e - 4*a*b^3*d^3*
e^2 + 6*a^2*b^2*d^2*e^3 - 4*a^3*b*d*e^4 + a^4*e^5) - 1/6*(2*b^3*d^3 - 9*a*b
^2*d^2*e + 18*a^2*b*d*e^2 - 11*a^3*e^3 + 6*(b^3*d*e^2 - a*b^2*e^3)*x^2 - 3*
(b^3*d^2*e - 6*a*b^2*d*e^2 + 5*a^2*b*e^3)*x)/((b*d - a*e)^4*(b*x + a)^3)
```

3.1523 $\int \frac{1}{(d+ex)^2\left(a^2+2abx+b^2x^2\right)^2}\,dx$

Optimal. Leaf size=132

$$-\frac{e^3}{(d+ex)(bd-ae)^4}-\frac{3be^2}{(a+bx)(bd-ae)^4}-\frac{4be^3\log(a+bx)}{(bd-ae)^5}+\frac{4be^3\log(d+ex)}{(bd-ae)^5}+\frac{be}{(a+bx)^2(bd-ae)^3}-\frac{b}{3(a+bx)^3(b\cdots}$$

```
[Out] -b/(3*(b*d - a*e)^2*(a + b*x)^3) + (b*e)/((b*d - a*e)^3*(a + b*x)^2) - (3*b
*e^2)/((b*d - a*e)^4*(a + b*x)) - e^3/((b*d - a*e)^4*(d + e*x)) - (4*b*e^3*
Log[a + b*x])/(b*d - a*e)^5 + (4*b*e^3*Log[d + e*x])/(b*d - a*e)^5
```

Rubi [A] time = 0.101166, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 44}

$$-\frac{e^3}{(d+ex)(bd-ae)^4}-\frac{3be^2}{(a+bx)(bd-ae)^4}-\frac{4be^3\log(a+bx)}{(bd-ae)^5}+\frac{4be^3\log(d+ex)}{(bd-ae)^5}+\frac{be}{(a+bx)^2(bd-ae)^3}-\frac{b}{3(a+bx)^3(b\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

```
[Out] -b/(3*(b*d - a*e)^2*(a + b*x)^3) + (b*e)/((b*d - a*e)^3*(a + b*x)^2) - (3*b
*e^2)/((b*d - a*e)^4*(a + b*x)) - e^3/((b*d - a*e)^4*(d + e*x)) - (4*b*e^3*
Log[a + b*x])/(b*d - a*e)^5 + (4*b*e^3*Log[d + e*x])/(b*d - a*e)^5
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a^2+2abx+b^2x^2\right)^2}\,dx=\int \frac{1}{(a+bx)^4(d+ex)^2}\,dx$$

$$=\int \left(\frac{b^2}{(bd-ae)^2(a+bx)^4}-\frac{2b^2e}{(bd-ae)^3(a+bx)^3}+\frac{3b^2e^2}{(bd-ae)^4(a+bx)^2}-\frac{4b\cdots}{(bd-ae\cdots}\right.$$

$$=-\frac{b}{3(bd-ae)^2(a+bx)^3}+\frac{be}{(bd-ae)^3(a+bx)^2}-\frac{3be^2}{(bd-ae)^4(a+bx)}-\frac{e^3}{(bd-ae)^4(\cdots}$$

Mathematica [A] time = 0.124786, size = 121, normalized size = 0.92

$$\frac{\frac{3e^3(ae-bd)}{d+ex}-\frac{9be^2(bd-ae)}{a+bx}+\frac{3be(bd-ae)^2}{(a+bx)^2}-\frac{b(bd-ae)^3}{(a+bx)^3}-12be^3\log(a+bx)+12be^3\log(d+ex)}{3(bd-ae)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

```
[Out] (-((b*(b*d - a*e)^3)/(a + b*x)^3) + (3*b*e*(b*d - a*e)^2)/(a + b*x)^2 - (9*
b*e^2*(b*d - a*e))/(a + b*x) + (3*e^3*(-(b*d) + a*e))/(d + e*x) - 12*b*e^3*
Log[a + b*x] + 12*b*e^3*Log[d + e*x])/(3*(b*d - a*e)^5)
```

Maple [A] time = 0.056, size = 132, normalized size = 1.

$$-\frac{e^3}{(ae-bd)^4(ex+d)}-4\frac{e^3b\ln(ex+d)}{(ae-bd)^5}-\frac{b}{3(ae-bd)^2(bx+a)^3}+4\frac{e^3b\ln(bx+a)}{(ae-bd)^5}-3\frac{be^2}{(ae-bd)^4(bx+a)}-\frac{be}{(ae-bd)^3(b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

```
[Out] -e^3/(a*e-b*d)^4/(e*x+d)-4*e^3/(a*e-b*d)^5*b*ln(e*x+d)-1/3*b/(a*e-b*d)^2/(b
*x+a)^3+4*e^3/(a*e-b*d)^5*b*ln(b*x+a)-3*b/(a*e-b*d)^4*e^2/(b*x+a)-b/(a*e-b*
d)^3*e/(b*x+a)^2
```

Maxima [B] time = 1.11037, size = 807, normalized size = 6.11

$$-\frac{4\,be^3\log(bx+a)}{b^5d^5-5\,ab^4d^4e+10\,a^2b^3d^3e^2-10\,a^3b^2d^2e^3+5\,a^4bde^4-a^5e^5}+\frac{4\,be^3\log(ex+d)}{b^5d^5-5\,ab^4d^4e+10\,a^2b^3d^3e^2-10\,a^3b^2d^2e^3+5\,a^4bde^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

```
[Out] -4*b*e^3*log(b*x + a)/(b^5*d^5 - 5*a*b^4*d^4*e + 10*a^2*b^3*d^3*e^2 - 10*a^
3*b^2*d^2*e^3 + 5*a^4*b*d*e^4 - a^5*e^5) + 4*b*e^3*log(e*x + d)/(b^5*d^5 -
5*a*b^4*d^4*e + 10*a^2*b^3*d^3*e^2 - 10*a^3*b^2*d^2*e^3 + 5*a^4*b*d*e^4 - a
^5*e^5) - 1/3*(12*b^3*e^3*x^3 + b^3*d^3 - 5*a*b^2*d^2*e + 13*a^2*b*d*e^2 +
3*a^3*e^3 + 6*(b^3*d*e^2 + 5*a*b^2*e^3)*x^2 - 2*(b^3*d^2*e - 8*a*b^2*d*e^2
- 11*a^2*b*e^3)*x)/(a^3*b^4*d^5 - 4*a^4*b^3*d^4*e + 6*a^5*b^2*d^3*e^2 - 4*a
^6*b*d^2*e^3 + a^7*d*e^4 + (b^7*d^4*e - 4*a*b^6*d^3*e^2 + 6*a^2*b^5*d^2*e^3
 - 4*a^3*b^4*d*e^4 + a^4*b^3*e^5)*x^4 + (b^7*d^5 - a*b^6*d^4*e - 6*a^2*b^5*
d^3*e^2 + 14*a^3*b^4*d^2*e^3 - 11*a^4*b^3*d*e^4 + 3*a^5*b^2*e^5)*x^3 + 3*(a
*b^6*d^5 - 3*a^2*b^5*d^4*e + 2*a^3*b^4*d^3*e^2 + 2*a^4*b^3*d^2*e^3 - 3*a^5*
b^2*d*e^4 + a^6*b*e^5)*x^2 + (3*a^2*b^5*d^5 - 11*a^3*b^4*d^4*e + 14*a^4*b^3
*d^3*e^2 - 6*a^5*b^2*d^2*e^3 - a^6*b*d*e^4 + a^7*e^5)*x)
```

Fricas [B] time = 1.81873, size = 1501, normalized size = 11.37

$$-\frac{b^4d^4-6\,ab^3d^3e+18\,a^2b^2d^2e^2-10\,a^3bde^3-3\,a^4e^4+12\left(b^4de^3-ab^3e^4\right)x^3+6\left(b^4d^2e^2+4\,ab^3de^3-5\,a^2b^2e^4\right)x}{3\left(a^3b^5d^6-5\,a^4b^4d^5e+10\,a^5b^3d^4e^2-10\,a^6b^2d^3e^3+5\,a^7bd^2e^4-a^8de^5+\left(b^8d^5e-5\,ab^7d^4e^2+10\,a^2b^6d^3e^3-10\,a^3b^5d^2e^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(b^4*d^4 - 6*a*b^3*d^3*e + 18*a^2*b^2*d^2*e^2 - 10*a^3*b*d*e^3 - 3*a^4
*e^4 + 12*(b^4*d*e^3 - a*b^3*e^4)*x^3 + 6*(b^4*d^2*e^2 + 4*a*b^3*d*e^3 - 5*
a^2*b^2*e^4)*x^2 - 2*(b^4*d^3*e - 9*a*b^3*d^2*e^2 - 3*a^2*b^2*d*e^3 + 11*a^
3*b*e^4)*x + 12*(b^4*e^4*x^4 + a^3*b*d*e^3 + (b^4*d*e^3 + 3*a*b^3*e^4)*x^3
+ 3*(a*b^3*d*e^3 + a^2*b^2*e^4)*x^2 + (3*a^2*b^2*d*e^3 + a^3*b*e^4)*x)*log(
b*x + a) - 12*(b^4*e^4*x^4 + a^3*b*d*e^3 + (b^4*d*e^3 + 3*a*b^3*e^4)*x^3 +
3*(a*b^3*d*e^3 + a^2*b^2*e^4)*x^2 + (3*a^2*b^2*d*e^3 + a^3*b*e^4)*x)*log(e*
x + d))/(a^3*b^5*d^6 - 5*a^4*b^4*d^5*e + 10*a^5*b^3*d^4*e^2 - 10*a^6*b^2*d^
3*e^3 + 5*a^7*b*d^2*e^4 - a^8*d*e^5 + (b^8*d^5*e - 5*a*b^7*d^4*e^2 + 10*a^2
*b^6*d^3*e^3 - 10*a^3*b^5*d^2*e^4 + 5*a^4*b^4*d*e^5 - a^5*b^3*e^6)*x^4 + (b
^8*d^6 - 2*a*b^7*d^5*e - 5*a^2*b^6*d^4*e^2 + 20*a^3*b^5*d^3*e^3 - 25*a^4*b^
4*d^2*e^4 + 14*a^5*b^3*d*e^5 - 3*a^6*b^2*e^6)*x^3 + 3*(a*b^7*d^6 - 4*a^2*b^
6*d^5*e + 5*a^3*b^5*d^4*e^2 - 5*a^5*b^3*d^2*e^4 + 4*a^6*b^2*d*e^5 - a^7*b*e
^6)*x^2 + (3*a^2*b^6*d^6 - 14*a^3*b^5*d^5*e + 25*a^4*b^4*d^4*e^2 - 20*a^5*b
^3*d^3*e^3 + 5*a^6*b^2*d^2*e^4 + 2*a^7*b*d*e^5 - a^8*e^6)*x)
```

Sympy [B] time = 3.59816, size = 881, normalized size = 6.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

```
[Out] -4*b*e**3*log(x + (-4*a**6*b*e**9/(a*e - b*d)**5 + 24*a**5*b**2*d*e**8/(a*e
 - b*d)**5 - 60*a**4*b**3*d**2*e**7/(a*e - b*d)**5 + 80*a**3*b**4*d**3*e**6
/(a*e - b*d)**5 - 60*a**2*b**5*d**4*e**5/(a*e - b*d)**5 + 24*a*b**6*d**5*e*
*4/(a*e - b*d)**5 + 4*a*b*e**4 - 4*b**7*d**6*e**3/(a*e - b*d)**5 + 4*b**2*d
*e**3)/(8*b**2*e**4))/(a*e - b*d)**5 + 4*b*e**3*log(x + (4*a**6*b*e**9/(a*e
 - b*d)**5 - 24*a**5*b**2*d*e**8/(a*e - b*d)**5 + 60*a**4*b**3*d**2*e**7/(a
*e - b*d)**5 - 80*a**3*b**4*d**3*e**6/(a*e - b*d)**5 + 60*a**2*b**5*d**4*e*
*5/(a*e - b*d)**5 - 24*a*b**6*d**5*e**4/(a*e - b*d)**5 + 4*a*b*e**4 + 4*b**
7*d**6*e**3/(a*e - b*d)**5 + 4*b**2*d*e**3)/(8*b**2*e**4))/(a*e - b*d)**5 -
 (3*a**3*e**3 + 13*a**2*b*d*e**2 - 5*a*b**2*d**2*e + b**3*d**3 + 12*b**3*e*
*3*x**3 + x**2*(30*a*b**2*e**3 + 6*b**3*d*e**2) + x*(22*a**2*b*e**3 + 16*a*
b**2*d*e**2 - 2*b**3*d**2*e))/(3*a**7*d*e**4 - 12*a**6*b*d**2*e**3 + 18*a**
5*b**2*d**3*e**2 - 12*a**4*b**3*d**4*e + 3*a**3*b**4*d**5 + x**4*(3*a**4*b*
*3*e**5 - 12*a**3*b**4*d*e**4 + 18*a**2*b**5*d**2*e**3 - 12*a*b**6*d**3*e**
2 + 3*b**7*d**4*e) + x**3*(9*a**5*b**2*e**5 - 33*a**4*b**3*d*e**4 + 42*a**3
*b**4*d**2*e**3 - 18*a**2*b**5*d**3*e**2 - 3*a*b**6*d**4*e + 3*b**7*d**5) +
 x**2*(9*a**6*b*e**5 - 27*a**5*b**2*d*e**4 + 18*a**4*b**3*d**2*e**3 + 18*a*
*3*b**4*d**3*e**2 - 27*a**2*b**5*d**4*e + 9*a*b**6*d**5) + x*(3*a**7*e**5 -
 3*a**6*b*d*e**4 - 18*a**5*b**2*d**2*e**3 + 42*a**4*b**3*d**3*e**2 - 33*a**
3*b**4*d**4*e + 9*a**2*b**5*d**5))
```

Giac [B] time = 1.23866, size = 377, normalized size = 2.86

$$-\frac{4\,be^4\log\left(\left|b-\frac{bd}{xe+d}+\frac{ae}{xe+d}\right|\right)}{b^5d^5e-5\,ab^4d^4e^2+10\,a^2b^3d^3e^3-10\,a^3b^2d^2e^4+5\,a^4bde^5-a^5e^6}-\frac{e^7}{\left(b^4d^4e^4-4\,ab^3d^3e^5+6\,a^2b^2d^2e^6-4\,a^3bde^7+a^4e^8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

```
[Out] -4*b*e^4*log(abs(b - b*d/(x*e + d) + a*e/(x*e + d)))/(b^5*d^5*e - 5*a*b^4*d
^4*e^2 + 10*a^2*b^3*d^3*e^3 - 10*a^3*b^2*d^2*e^4 + 5*a^4*b*d*e^5 - a^5*e^6)
 - e^7/((b^4*d^4*e^4 - 4*a*b^3*d^3*e^5 + 6*a^2*b^2*d^2*e^6 - 4*a^3*b*d*e^7
+ a^4*e^8)*(x*e + d)) - 1/3*(13*b^4*e^3 - 30*(b^4*d*e^4 - a*b^3*e^5)*e^(-1)
/(x*e + d) + 18*(b^4*d^2*e^5 - 2*a*b^3*d*e^6 + a^2*b^2*e^7)*e^(-2)/(x*e + d
)^2)/((b*d - a*e)^5*(b - b*d/(x*e + d) + a*e/(x*e + d))^3)
```

3.1524 $\int \frac{1}{(d+ex)^3(a^2+2abx+b^2x^2)^2} dx$

Optimal. Leaf size=170

$$-\frac{6b^2e^2}{(a+bx)(bd-ae)^5}-\frac{10b^2e^3\log(a+bx)}{(bd-ae)^6}+\frac{10b^2e^3\log(d+ex)}{(bd-ae)^6}+\frac{3b^2e}{2(a+bx)^2(bd-ae)^4}-\frac{b^2}{3(a+bx)^3(bd-ae)^3}-\frac{}{(d+e}$$

```
[Out] -b^2/(3*(b*d - a*e)^3*(a + b*x)^3) + (3*b^2*e)/(2*(b*d - a*e)^4*(a + b*x)^2
) - (6*b^2*e^2)/((b*d - a*e)^5*(a + b*x)) - e^3/(2*(b*d - a*e)^4*(d + e*x)^
2) - (4*b*e^3)/((b*d - a*e)^5*(d + e*x)) - (10*b^2*e^3*Log[a + b*x])/(b*d -
 a*e)^6 + (10*b^2*e^3*Log[d + e*x])/(b*d - a*e)^6
```

Rubi [A] time = 0.148281, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 44}

$$-\frac{6b^2e^2}{(a+bx)(bd-ae)^5}-\frac{10b^2e^3\log(a+bx)}{(bd-ae)^6}+\frac{10b^2e^3\log(d+ex)}{(bd-ae)^6}+\frac{3b^2e}{2(a+bx)^2(bd-ae)^4}-\frac{b^2}{3(a+bx)^3(bd-ae)^3}-\frac{}{(d+e}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

```
[Out] -b^2/(3*(b*d - a*e)^3*(a + b*x)^3) + (3*b^2*e)/(2*(b*d - a*e)^4*(a + b*x)^2
) - (6*b^2*e^2)/((b*d - a*e)^5*(a + b*x)) - e^3/(2*(b*d - a*e)^4*(d + e*x)^
2) - (4*b*e^3)/((b*d - a*e)^5*(d + e*x)) - (10*b^2*e^3*Log[a + b*x])/(b*d -
 a*e)^6 + (10*b^2*e^3*Log[d + e*x])/(b*d - a*e)^6
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(a^2+2abx+b^2x^2\right)^2}\,dx=\int \frac{1}{(a+bx)^4(d+ex)^3}\,dx$$
$$=\int \left(\frac{b^3}{(bd-ae)^3(a+bx)^4}-\frac{3b^3e}{(bd-ae)^4(a+bx)^3}+\frac{6b^3e^2}{(bd-ae)^5(a+bx)^2}-\frac{10}{(bd-ae}\right.$$
$$=-\frac{b^2}{3(bd-ae)^3(a+bx)^3}+\frac{3b^2e}{2(bd-ae)^4(a+bx)^2}-\frac{6b^2e^2}{(bd-ae)^5(a+bx)}-\frac{}{2(bd-ae}$$

Mathematica [A] time = 0.214391, size = 154, normalized size = 0.91

$$-\frac{\frac{36b^2e^2(bd-ae)}{a+bx}-\frac{9b^2e(bd-ae)^2}{(a+bx)^2}+\frac{2b^2(bd-ae)^3}{(a+bx)^3}+60b^2e^3\log(a+bx)+\frac{24be^3(bd-ae)}{d+ex}+\frac{3e^3(bd-ae)^2}{(d+ex)^2}-60b^2e^3\log(d+ex)}{6(bd-ae)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

```
[Out] -((2*b^2*(b*d - a*e)^3)/(a + b*x)^3 - (9*b^2*e*(b*d - a*e)^2)/(a + b*x)^2 +
 (36*b^2*e^2*(b*d - a*e))/(a + b*x) + (3*e^3*(b*d - a*e)^2)/(d + e*x)^2 + (
24*b*e^3*(b*d - a*e))/(d + e*x) + 60*b^2*e^3*Log[a + b*x] - 60*b^2*e^3*Log[
d + e*x])/(6*(b*d - a*e)^6)
```

Maple [A] time = 0.055, size = 165, normalized size = 1.

$$-\frac{e^3}{2\,(ae-bd)^4\,(ex+d)^2}+10\,\frac{b^2e^3\ln(ex+d)}{(ae-bd)^6}+4\,\frac{e^3b}{(ae-bd)^5\,(ex+d)}+\frac{b^2}{3\,(ae-bd)^3\,(bx+a)^3}-10\,\frac{b^2e^3\ln(bx+a)}{(ae-bd)^6}+6\,\frac{}{(ae}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

```
[Out] -1/2*e^3/(a*e-b*d)^4/(e*x+d)^2+10*e^3/(a*e-b*d)^6*b^2*ln(e*x+d)+4*e^3/(a*e-
b*d)^5*b/(e*x+d)+1/3*b^2/(a*e-b*d)^3/(b*x+a)^3-10*e^3/(a*e-b*d)^6*b^2*ln(b*
x+a)+6*b^2/(a*e-b*d)^5*e^2/(b*x+a)+3/2*b^2/(a*e-b*d)^4*e/(b*x+a)^2
```

Maxima [B] time = 1.36047, size = 1200, normalized size = 7.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

```
[Out] -10*b^2*e^3*log(b*x + a)/(b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20
*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 + a^6*e^6) + 10*b^2*e
^3*log(e*x + d)/(b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*
d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 + a^6*e^6) - 1/6*(60*b^4*e^4*x
^4 + 2*b^4*d^4 - 13*a*b^3*d^3*e + 47*a^2*b^2*d^2*e^2 + 27*a^3*b*d*e^3 - 3*a
^4*e^4 + 30*(3*b^4*d*e^3 + 5*a*b^3*e^4)*x^3 + 10*(2*b^4*d^2*e^2 + 23*a*b^3*
d*e^3 + 11*a^2*b^2*e^4)*x^2 - 5*(b^4*d^3*e - 11*a*b^3*d^2*e^2 - 35*a^2*b^2*
d*e^3 - 3*a^3*b*e^4)*x)/(a^3*b^5*d^7 - 5*a^4*b^4*d^6*e + 10*a^5*b^3*d^5*e^2
 - 10*a^6*b^2*d^4*e^3 + 5*a^7*b*d^3*e^4 - a^8*d^2*e^5 + (b^8*d^5*e^2 - 5*a*
b^7*d^4*e^3 + 10*a^2*b^6*d^3*e^4 - 10*a^3*b^5*d^2*e^5 + 5*a^4*b^4*d*e^6 - a
^5*b^3*e^7)*x^5 + (2*b^8*d^6*e - 7*a*b^7*d^5*e^2 + 5*a^2*b^6*d^4*e^3 + 10*a
^3*b^5*d^3*e^4 - 20*a^4*b^4*d^2*e^5 + 13*a^5*b^3*d*e^6 - 3*a^6*b^2*e^7)*x^4
 + (b^8*d^7 + a*b^7*d^6*e - 17*a^2*b^6*d^5*e^2 + 35*a^3*b^5*d^4*e^3 - 25*a^
4*b^4*d^3*e^4 - a^5*b^3*d^2*e^5 + 9*a^6*b^2*d*e^6 - 3*a^7*b*e^7)*x^3 + (3*a
*b^7*d^7 - 9*a^2*b^6*d^6*e + a^3*b^5*d^5*e^2 + 25*a^4*b^4*d^4*e^3 - 35*a^5*
b^3*d^3*e^4 + 17*a^6*b^2*d^2*e^5 - a^7*b*d*e^6 - a^8*e^7)*x^2 + (3*a^2*b^6*
d^7 - 13*a^3*b^5*d^6*e + 20*a^4*b^4*d^5*e^2 - 10*a^5*b^3*d^4*e^3 - 5*a^6*b^
2*d^3*e^4 + 7*a^7*b*d^2*e^5 - 2*a^8*d*e^6)*x)
```

Fricas [B] time = 2.03205, size = 2313, normalized size = 13.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b^5*d^5 - 15*a*b^4*d^4*e + 60*a^2*b^3*d^3*e^2 - 20*a^3*b^2*d^2*e^3
- 30*a^4*b*d*e^4 + 3*a^5*e^5 + 60*(b^5*d*e^4 - a*b^4*e^5)*x^4 + 30*(3*b^5*d
^2*e^3 + 2*a*b^4*d*e^4 - 5*a^2*b^3*e^5)*x^3 + 10*(2*b^5*d^3*e^2 + 21*a*b^4*
d^2*e^3 - 12*a^2*b^3*d*e^4 - 11*a^3*b^2*e^5)*x^2 - 5*(b^5*d^4*e - 12*a*b^4*
d^3*e^2 - 24*a^2*b^3*d^2*e^3 + 32*a^3*b^2*d*e^4 + 3*a^4*b*e^5)*x + 60*(b^5*
e^5*x^5 + a^3*b^2*d^2*e^3 + (2*b^5*d*e^4 + 3*a*b^4*e^5)*x^4 + (b^5*d^2*e^3
+ 6*a*b^4*d*e^4 + 3*a^2*b^3*e^5)*x^3 + (3*a*b^4*d^2*e^3 + 6*a^2*b^3*d*e^4 +
 a^3*b^2*e^5)*x^2 + (3*a^2*b^3*d^2*e^3 + 2*a^3*b^2*d*e^4)*x)*log(b*x + a) -
 60*(b^5*e^5*x^5 + a^3*b^2*d^2*e^3 + (2*b^5*d*e^4 + 3*a*b^4*e^5)*x^4 + (b^5
*d^2*e^3 + 6*a*b^4*d*e^4 + 3*a^2*b^3*e^5)*x^3 + (3*a*b^4*d^2*e^3 + 6*a^2*b^
3*d*e^4 + a^3*b^2*e^5)*x^2 + (3*a^2*b^3*d^2*e^3 + 2*a^3*b^2*d*e^4)*x)*log(e
*x + d))/(a^3*b^6*d^8 - 6*a^4*b^5*d^7*e + 15*a^5*b^4*d^6*e^2 - 20*a^6*b^3*d
^5*e^3 + 15*a^7*b^2*d^4*e^4 - 6*a^8*b*d^3*e^5 + a^9*d^2*e^6 + (b^9*d^6*e^2
- 6*a*b^8*d^5*e^3 + 15*a^2*b^7*d^4*e^4 - 20*a^3*b^6*d^3*e^5 + 15*a^4*b^5*d^
2*e^6 - 6*a^5*b^4*d*e^7 + a^6*b^3*e^8)*x^5 + (2*b^9*d^7*e - 9*a*b^8*d^6*e^2
 + 12*a^2*b^7*d^5*e^3 + 5*a^3*b^6*d^4*e^4 - 30*a^4*b^5*d^3*e^5 + 33*a^5*b^4
*d^2*e^6 - 16*a^6*b^3*d*e^7 + 3*a^7*b^2*e^8)*x^4 + (b^9*d^8 - 18*a^2*b^7*d^
6*e^2 + 52*a^3*b^6*d^5*e^3 - 60*a^4*b^5*d^4*e^4 + 24*a^5*b^4*d^3*e^5 + 10*a
^6*b^3*d^2*e^6 - 12*a^7*b^2*d*e^7 + 3*a^8*b*e^8)*x^3 + (3*a*b^8*d^8 - 12*a^
2*b^7*d^7*e + 10*a^3*b^6*d^6*e^2 + 24*a^4*b^5*d^5*e^3 - 60*a^5*b^4*d^4*e^4
+ 52*a^6*b^3*d^3*e^5 - 18*a^7*b^2*d^2*e^6 + a^9*e^8)*x^2 + (3*a^2*b^7*d^8 -
 16*a^3*b^6*d^7*e + 33*a^4*b^5*d^6*e^2 - 30*a^5*b^4*d^5*e^3 + 5*a^6*b^3*d^4
*e^4 + 12*a^7*b^2*d^3*e^5 - 9*a^8*b*d^2*e^6 + 2*a^9*d*e^7)*x)
```

Sympy [B] time = 5.09614, size = 1217, normalized size = 7.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

```
[Out] 10*b**2*e**3*log(x + (-10*a**7*b**2*e**10/(a*e - b*d)**6 + 70*a**6*b**3*d*e
**9/(a*e - b*d)**6 - 210*a**5*b**4*d**2*e**8/(a*e - b*d)**6 + 350*a**4*b**5
*d**3*e**7/(a*e - b*d)**6 - 350*a**3*b**6*d**4*e**6/(a*e - b*d)**6 + 210*a*
*2*b**7*d**5*e**5/(a*e - b*d)**6 - 70*a*b**8*d**6*e**4/(a*e - b*d)**6 + 10*
a*b**2*e**4 + 10*b**9*d**7*e**3/(a*e - b*d)**6 + 10*b**3*d*e**3)/(20*b**3*e
**4))/(a*e - b*d)**6 - 10*b**2*e**3*log(x + (10*a**7*b**2*e**10/(a*e - b*d)
**6 - 70*a**6*b**3*d*e**9/(a*e - b*d)**6 + 210*a**5*b**4*d**2*e**8/(a*e - b
*d)**6 - 350*a**4*b**5*d**3*e**7/(a*e - b*d)**6 + 350*a**3*b**6*d**4*e**6/(
a*e - b*d)**6 - 210*a**2*b**7*d**5*e**5/(a*e - b*d)**6 + 70*a*b**8*d**6*e**
4/(a*e - b*d)**6 + 10*a*b**2*e**4 - 10*b**9*d**7*e**3/(a*e - b*d)**6 + 10*b
**3*d*e**3)/(20*b**3*e**4))/(a*e - b*d)**6 + (-3*a**4*e**4 + 27*a**3*b*d*e*
*3 + 47*a**2*b**2*d**2*e**2 - 13*a*b**3*d**3*e + 2*b**4*d**4 + 60*b**4*e**4
*x**4 + x**3*(150*a*b**3*e**4 + 90*b**4*d*e**3) + x**2*(110*a**2*b**2*e**4
+ 230*a*b**3*d*e**3 + 20*b**4*d**2*e**2) + x*(15*a**3*b*e**4 + 175*a**2*b**
2*d*e**3 + 55*a*b**3*d**2*e**2 - 5*b**4*d**3*e))/(6*a**8*d**2*e**5 - 30*a**
7*b*d**3*e**4 + 60*a**6*b**2*d**4*e**3 - 60*a**5*b**3*d**5*e**2 + 30*a**4*b
**4*d**6*e - 6*a**3*b**5*d**7 + x**5*(6*a**5*b**3*e**7 - 30*a**4*b**4*d*e**
6 + 60*a**3*b**5*d**2*e**5 - 60*a**2*b**6*d**3*e**4 + 30*a*b**7*d**4*e**3 -
 6*b**8*d**5*e**2) + x**4*(18*a**6*b**2*e**7 - 78*a**5*b**3*d*e**6 + 120*a*
*4*b**4*d**2*e**5 - 60*a**3*b**5*d**3*e**4 - 30*a**2*b**6*d**4*e**3 + 42*a*
b**7*d**5*e**2 - 12*b**8*d**6*e) + x**3*(18*a**7*b*e**7 - 54*a**6*b**2*d*e*
```

```
*6 + 6*a**5*b**3*d**2*e**5 + 150*a**4*b**4*d**3*e**4 - 210*a**3*b**5*d**4*e
**3 + 102*a**2*b**6*d**5*e**2 - 6*a*b**7*d**6*e - 6*b**8*d**7) + x**2*(6*a*
*8*e**7 + 6*a**7*b*d*e**6 - 102*a**6*b**2*d**2*e**5 + 210*a**5*b**3*d**3*e*
*4 - 150*a**4*b**4*d**4*e**3 - 6*a**3*b**5*d**5*e**2 + 54*a**2*b**6*d**6*e
- 18*a*b**7*d**7) + x*(12*a**8*d*e**6 - 42*a**7*b*d**2*e**5 + 30*a**6*b**2*
d**3*e**4 + 60*a**5*b**3*d**4*e**3 - 120*a**4*b**4*d**5*e**2 + 78*a**3*b**5
*d**6*e - 18*a**2*b**6*d**7))
```

Giac [B] time = 1.12635, size = 587, normalized size = 3.45

$$-\frac{10\,b^3e^3\log\left(|bx+a|\right)}{b^7d^6-6\,ab^6d^5e+15\,a^2b^5d^4e^2-20\,a^3b^4d^3e^3+15\,a^4b^3d^2e^4-6\,a^5b^2de^5+a^6be^6}+\frac{10\,b^2e^4\,\mathrm{lo}}{b^6d^6e-6\,ab^5d^5e^2+15\,a^2b^4d^4e^3-20\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

```
[Out] -10*b^3*e^3*log(abs(b*x + a))/(b^7*d^6 - 6*a*b^6*d^5*e + 15*a^2*b^5*d^4*e^2
 - 20*a^3*b^4*d^3*e^3 + 15*a^4*b^3*d^2*e^4 - 6*a^5*b^2*d*e^5 + a^6*b*e^6) +
 10*b^2*e^4*log(abs(x*e + d))/(b^6*d^6*e - 6*a*b^5*d^5*e^2 + 15*a^2*b^4*d^4
*e^3 - 20*a^3*b^3*d^3*e^4 + 15*a^4*b^2*d^2*e^5 - 6*a^5*b*d*e^6 + a^6*e^7) -
 1/6*(2*b^5*d^5 - 15*a*b^4*d^4*e + 60*a^2*b^3*d^3*e^2 - 20*a^3*b^2*d^2*e^3
- 30*a^4*b*d*e^4 + 3*a^5*e^5 + 60*(b^5*d*e^4 - a*b^4*e^5)*x^4 + 30*(3*b^5*d
^2*e^3 + 2*a*b^4*d*e^4 - 5*a^2*b^3*e^5)*x^3 + 10*(2*b^5*d^3*e^2 + 21*a*b^4*
d^2*e^3 - 12*a^2*b^3*d*e^4 - 11*a^3*b^2*e^5)*x^2 - 5*(b^5*d^4*e - 12*a*b^4*
d^3*e^2 - 24*a^2*b^3*d^2*e^3 + 32*a^3*b^2*d*e^4 + 3*a^4*b*e^5)*x)/((b*d - a
*e)^6*(b*x + a)^3*(x*e + d)^2)
```

3.1525 $\int \frac{(d+ex)^8}{(a^2+2abx+b^2x^2)^3}\,dx$

Optimal. Leaf size=208

$$\frac{4e^7(a+bx)^2(bd-ae)}{b^9}+\frac{28e^6x(bd-ae)^2}{b^8}-\frac{70e^4(bd-ae)^4}{b^9(a+bx)}-\frac{28e^3(bd-ae)^5}{b^9(a+bx)^2}-\frac{28e^2(bd-ae)^6}{3b^9(a+bx)^3}+\frac{56e^5(bd-ae)^3\log(a+$$

```
[Out] (28*e^6*(b*d - a*e)^2*x)/b^8 - (b*d - a*e)^8/(5*b^9*(a + b*x)^5) - (2*e*(b*
d - a*e)^7)/(b^9*(a + b*x)^4) - (28*e^2*(b*d - a*e)^6)/(3*b^9*(a + b*x)^3)
- (28*e^3*(b*d - a*e)^5)/(b^9*(a + b*x)^2) - (70*e^4*(b*d - a*e)^4)/(b^9*(a
 + b*x)) + (4*e^7*(b*d - a*e)*(a + b*x)^2)/b^9 + (e^8*(a + b*x)^3)/(3*b^9)
+ (56*e^5*(b*d - a*e)^3*Log[a + b*x])/b^9
```

Rubi [A] time = 0.311493, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$\frac{4e^7(a+bx)^2(bd-ae)}{b^9}+\frac{28e^6x(bd-ae)^2}{b^8}-\frac{70e^4(bd-ae)^4}{b^9(a+bx)}-\frac{28e^3(bd-ae)^5}{b^9(a+bx)^2}-\frac{28e^2(bd-ae)^6}{3b^9(a+bx)^3}+\frac{56e^5(bd-ae)^3\log(a+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^8/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] (28*e^6*(b*d - a*e)^2*x)/b^8 - (b*d - a*e)^8/(5*b^9*(a + b*x)^5) - (2*e*(b*
d - a*e)^7)/(b^9*(a + b*x)^4) - (28*e^2*(b*d - a*e)^6)/(3*b^9*(a + b*x)^3)
- (28*e^3*(b*d - a*e)^5)/(b^9*(a + b*x)^2) - (70*e^4*(b*d - a*e)^4)/(b^9*(a
 + b*x)) + (4*e^7*(b*d - a*e)*(a + b*x)^2)/b^9 + (e^8*(a + b*x)^3)/(3*b^9)
+ (56*e^5*(b*d - a*e)^3*Log[a + b*x])/b^9
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^8}{\left(a^2+2abx+b^2x^2\right)^3}\,dx &= \int \frac{(d+ex)^8}{(a+bx)^6}\,dx\\ &= \int \left(\frac{28e^6(bd-ae)^2}{b^8}+\frac{(bd-ae)^8}{b^8(a+bx)^6}+\frac{8e(bd-ae)^7}{b^8(a+bx)^5}+\frac{28e^2(bd-ae)^6}{b^8(a+bx)^4}+\frac{56e^3(bd-ae)^5}{b^8(a+bx)^3}+\frac{7}{}\right.\\ &= \frac{28e^6(bd-ae)^2x}{b^8}-\frac{(bd-ae)^8}{5b^9(a+bx)^5}-\frac{2e(bd-ae)^7}{b^9(a+bx)^4}-\frac{28e^2(bd-ae)^6}{3b^9(a+bx)^3}-\frac{28e^3(bd-ae)^5}{b^9(a+bx)^2}-\frac{70e}{b^9}\end{aligned}$$

Mathematica [A] time = 0.170463, size = 195, normalized size = 0.94

$$\frac{15be^6x\left(21a^2e^2-48abde+28b^2d^2\right)+15b^2e^7x^2(4bd-3ae)-\frac{1050e^4(bd-ae)^4}{a+bx}+\frac{420e^3(ae-bd)^5}{(a+bx)^2}-\frac{140e^2(bd-ae)^6}{(a+bx)^3}+840e^5(bd-ae)^3\log \ldots}{15b^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^8/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] (15*b*e^6*(28*b^2*d^2 - 48*a*b*d*e + 21*a^2*e^2)*x + 15*b^2*e^7*(4*b*d - 3*
a*e)*x^2 + 5*b^3*e^8*x^3 - (3*(b*d - a*e)^8)/(a + b*x)^5 + (30*e*(-(b*d) +
a*e)^7)/(a + b*x)^4 - (140*e^2*(b*d - a*e)^6)/(a + b*x)^3 + (420*e^3*(-(b*d
) + a*e)^5)/(a + b*x)^2 - (1050*e^4*(b*d - a*e)^4)/(a + b*x) + 840*e^5*(b*d
 - a*e)^3*Log[a + b*x])/(15*b^9)
```

Maple [B] time = 0.057, size = 820, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^8/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] -56/b^9*e^8*ln(b*x+a)*a^3-28/b^4*e^3/(b*x+a)^2*d^5+56/b^6*e^5*ln(b*x+a)*d^3
+2/b^9*e^8/(b*x+a)^4*a^7-2/b^2*e/(b*x+a)^4*d^7-70/b^9*e^8/(b*x+a)*a^4-70/b^
5*e^4/(b*x+a)*d^4-1/5/b^9/(b*x+a)^5*a^8*e^8-28/3/b^9*e^8/(b*x+a)^3*a^6-28/3
/b^3*e^2/(b*x+a)^3*d^6-3*e^8/b^7*x^2*a+4*e^7/b^6*x^2*d+21*e^8/b^8*a^2*x+28*
e^6/b^6*d^2*x+28/b^9*e^8/(b*x+a)^2*a^5+1/3*e^8/b^6*x^3-1/5/b/(b*x+a)^5*d^8+
56/5/b^6/(b*x+a)^5*a^5*d^3*e^5-14/b^5/(b*x+a)^5*a^4*d^4*e^4+56/5/b^4/(b*x+a
)^5*a^3*d^5*e^3+280/b^6*e^5/(b*x+a)*a*d^3-48*e^7/b^7*a*d*x+42/b^7*e^6/(b*x+
a)^4*a^5*d^2-70/b^6*e^5/(b*x+a)^4*a^4*d^3+70/b^5*e^4/(b*x+a)^4*a^3*d^4-42/b
^4*e^3/(b*x+a)^4*a^2*d^5+14/b^3*e^2/(b*x+a)^4*a*d^6+280/b^8*e^7/(b*x+a)*a^3
*d-420/b^7*e^6/(b*x+a)*d^2*a^2-28/5/b^3/(b*x+a)^5*a^2*d^6*e^2+8/5/b^2/(b*x+
a)^5*a*d^7*e+56/b^8*e^7/(b*x+a)^3*a^5*d-140/b^7*e^6/(b*x+a)^3*d^2*a^4+560/3
/b^6*e^5/(b*x+a)^3*a^3*d^3-140/b^5*e^4/(b*x+a)^3*a^2*d^4+56/b^4*e^3/(b*x+a)
^3*a*d^5+168/b^8*e^7*ln(b*x+a)*a^2*d-140/b^8*e^7/(b*x+a)^2*a^4*d+280/b^7*e^
6/(b*x+a)^2*a^3*d^2-280/b^6*e^5/(b*x+a)^2*a^2*d^3+140/b^5*e^4/(b*x+a)^2*a*d
^4+8/5/b^8/(b*x+a)^5*a^7*d*e^7-28/5/b^7/(b*x+a)^5*a^6*d^2*e^6-168/b^7*e^6*l
n(b*x+a)*a*d^2-14/b^8*e^7/(b*x+a)^4*a^6*d
```

Maxima [B] time = 1.28623, size = 845, normalized size = 4.06

$$-\frac{3\,b^8d^8+6\,ab^7d^7e+14\,a^2b^6d^6e^2+42\,a^3b^5d^5e^3+210\,a^4b^4d^4e^4-1918\,a^5b^3d^3e^5+3654\,a^6b^2d^2e^6-2754\,a^7bde^7+743\,a^8e^8 \ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^8/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/15*(3*b^8*d^8 + 6*a*b^7*d^7*e + 14*a^2*b^6*d^6*e^2 + 42*a^3*b^5*d^5*e^3
+ 210*a^4*b^4*d^4*e^4 - 1918*a^5*b^3*d^3*e^5 + 3654*a^6*b^2*d^2*e^6 - 2754*
a^7*b*d*e^7 + 743*a^8*e^8 + 1050*(b^8*d^4*e^4 - 4*a*b^7*d^3*e^5 + 6*a^2*b^6
*d^2*e^6 - 4*a^3*b^5*d*e^7 + a^4*b^4*e^8)*x^4 + 420*(b^8*d^5*e^3 + 5*a*b^7*
```

```
d^4*e^4 - 30*a^2*b^6*d^3*e^5 + 50*a^3*b^5*d^2*e^6 - 35*a^4*b^4*d*e^7 + 9*a^
5*b^3*e^8)*x^3 + 140*(b^8*d^6*e^2 + 3*a*b^7*d^5*e^3 + 15*a^2*b^6*d^4*e^4 -
110*a^3*b^5*d^3*e^5 + 195*a^4*b^4*d^2*e^6 - 141*a^5*b^3*d*e^7 + 37*a^6*b^2*
e^8)*x^2 + 10*(3*b^8*d^7*e + 7*a*b^7*d^6*e^2 + 21*a^2*b^6*d^5*e^3 + 105*a^3
*b^5*d^4*e^4 - 875*a^4*b^4*d^3*e^5 + 1617*a^5*b^3*d^2*e^6 - 1197*a^6*b^2*d*
e^7 + 319*a^7*b*e^8)*x)/(b^14*x^5 + 5*a*b^13*x^4 + 10*a^2*b^12*x^3 + 10*a^3
*b^11*x^2 + 5*a^4*b^10*x + a^5*b^9) + 1/3*(b^2*e^8*x^3 + 3*(4*b^2*d*e^7 - 3
*a*b*e^8)*x^2 + 3*(28*b^2*d^2*e^6 - 48*a*b*d*e^7 + 21*a^2*e^8)*x)/b^8 + 56*
(b^3*d^3*e^5 - 3*a*b^2*d^2*e^6 + 3*a^2*b*d*e^7 - a^3*e^8)*log(b*x + a)/b^9
```

Fricas [B] time = 1.86638, size = 1952, normalized size = 9.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^8/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(5*b^8*e^8*x^8 - 3*b^8*d^8 - 6*a*b^7*d^7*e - 14*a^2*b^6*d^6*e^2 - 42*a
^3*b^5*d^5*e^3 - 210*a^4*b^4*d^4*e^4 + 1918*a^5*b^3*d^3*e^5 - 3654*a^6*b^2*
d^2*e^6 + 2754*a^7*b*d*e^7 - 743*a^8*e^8 + 20*(3*b^8*d*e^7 - a*b^7*e^8)*x^7
 + 140*(3*b^8*d^2*e^6 - 3*a*b^7*d*e^7 + a^2*b^6*e^8)*x^6 + 25*(84*a*b^7*d^2
*e^6 - 120*a^2*b^6*d*e^7 + 47*a^3*b^5*e^8)*x^5 - 25*(42*b^8*d^4*e^4 - 168*a
*b^7*d^3*e^5 + 84*a^2*b^6*d^2*e^6 + 96*a^3*b^5*d*e^7 - 67*a^4*b^4*e^8)*x^4
- 10*(42*b^8*d^5*e^3 + 210*a*b^7*d^4*e^4 - 1260*a^2*b^6*d^3*e^5 + 1680*a^3*
b^5*d^2*e^6 - 780*a^4*b^4*d*e^7 + 85*a^5*b^3*e^8)*x^3 - 10*(14*b^8*d^6*e^2
+ 42*a*b^7*d^5*e^3 + 210*a^2*b^6*d^4*e^4 - 1540*a^3*b^5*d^3*e^5 + 2520*a^4*
b^4*d^2*e^6 - 1620*a^5*b^3*d*e^7 + 365*a^6*b^2*e^8)*x^2 - 5*(6*b^8*d^7*e +
14*a*b^7*d^6*e^2 + 42*a^2*b^6*d^5*e^3 + 210*a^3*b^5*d^4*e^4 - 1750*a^4*b^4*
d^3*e^5 + 3150*a^5*b^3*d^2*e^6 - 2250*a^6*b^2*d*e^7 + 575*a^7*b*e^8)*x + 84
0*(a^5*b^3*d^3*e^5 - 3*a^6*b^2*d^2*e^6 + 3*a^7*b*d*e^7 - a^8*e^8 + (b^8*d^3
*e^5 - 3*a*b^7*d^2*e^6 + 3*a^2*b^6*d*e^7 - a^3*b^5*e^8)*x^5 + 5*(a*b^7*d^3*
e^5 - 3*a^2*b^6*d^2*e^6 + 3*a^3*b^5*d*e^7 - a^4*b^4*e^8)*x^4 + 10*(a^2*b^6*
d^3*e^5 - 3*a^3*b^5*d^2*e^6 + 3*a^4*b^4*d*e^7 - a^5*b^3*e^8)*x^3 + 10*(a^3*
b^5*d^3*e^5 - 3*a^4*b^4*d^2*e^6 + 3*a^5*b^3*d*e^7 - a^6*b^2*e^8)*x^2 + 5*(a
^4*b^4*d^3*e^5 - 3*a^5*b^3*d^2*e^6 + 3*a^6*b^2*d*e^7 - a^7*b*e^8)*x)*log(b*
x + a))/(b^14*x^5 + 5*a*b^13*x^4 + 10*a^2*b^12*x^3 + 10*a^3*b^11*x^2 + 5*a^
4*b^10*x + a^5*b^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**8/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17656, size = 737, normalized size = 3.54

$$\frac{56\left(b^3d^3e^5 - 3\,ab^2d^2e^6 + 3\,a^2bde^7 - a^3e^8\right)\log\left(|bx + a|\right)}{b^9} - \frac{3\,b^8d^8 + 6\,ab^7d^7e + 14\,a^2b^6d^6e^2 + 42\,a^3b^5d^5e^3 + 210\,a^4b^4d^4e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^8/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] 56*(b^3*d^3*e^5 - 3*a*b^2*d^2*e^6 + 3*a^2*b*d*e^7 - a^3*e^8)*log(abs(b*x +
a))/b^9 - 1/15*(3*b^8*d^8 + 6*a*b^7*d^7*e + 14*a^2*b^6*d^6*e^2 + 42*a^3*b^5
*d^5*e^3 + 210*a^4*b^4*d^4*e^4 - 1918*a^5*b^3*d^3*e^5 + 3654*a^6*b^2*d^2*e^
6 - 2754*a^7*b*d*e^7 + 743*a^8*e^8 + 1050*(b^8*d^4*e^4 - 4*a*b^7*d^3*e^5 +
6*a^2*b^6*d^2*e^6 - 4*a^3*b^5*d*e^7 + a^4*b^4*e^8)*x^4 + 420*(b^8*d^5*e^3 +
 5*a*b^7*d^4*e^4 - 30*a^2*b^6*d^3*e^5 + 50*a^3*b^5*d^2*e^6 - 35*a^4*b^4*d*e
^7 + 9*a^5*b^3*e^8)*x^3 + 140*(b^8*d^6*e^2 + 3*a*b^7*d^5*e^3 + 15*a^2*b^6*d
^4*e^4 - 110*a^3*b^5*d^3*e^5 + 195*a^4*b^4*d^2*e^6 - 141*a^5*b^3*d*e^7 + 37
*a^6*b^2*e^8)*x^2 + 10*(3*b^8*d^7*e + 7*a*b^7*d^6*e^2 + 21*a^2*b^6*d^5*e^3
+ 105*a^3*b^5*d^4*e^4 - 875*a^4*b^4*d^3*e^5 + 1617*a^5*b^3*d^2*e^6 - 1197*a
^6*b^2*d*e^7 + 319*a^7*b*e^8)*x)/((b*x + a)^5*b^9) + 1/3*(b^12*x^3*e^8 + 12
*b^12*d*x^2*e^7 + 84*b^12*d^2*x*e^6 - 9*a*b^11*x^2*e^8 - 144*a*b^11*d*x*e^7
 + 63*a^2*b^10*x*e^8)/b^18
```

3.1526 $\int \frac{(d+ex)^7}{\left(a^2+2abx+b^2x^2\right)^3}\,dx$

Optimal. Leaf size=181

$$\frac{e^6x(7bd-6ae)}{b^7}-\frac{35e^4(bd-ae)^3}{b^8(a+bx)}-\frac{35e^3(bd-ae)^4}{2b^8(a+bx)^2}-\frac{7e^2(bd-ae)^5}{b^8(a+bx)^3}+\frac{21e^5(bd-ae)^2\log(a+bx)}{b^8}-\frac{7e(bd-ae)^6}{4b^8(a+bx)^4}-\frac{(bd}{5b^8}$$

```
[Out] (e^6*(7*b*d - 6*a*e)*x)/b^7 + (e^7*x^2)/(2*b^6) - (b*d - a*e)^7/(5*b^8*(a +
 b*x)^5) - (7*e*(b*d - a*e)^6)/(4*b^8*(a + b*x)^4) - (7*e^2*(b*d - a*e)^5)/
(b^8*(a + b*x)^3) - (35*e^3*(b*d - a*e)^4)/(2*b^8*(a + b*x)^2) - (35*e^4*(b
*d - a*e)^3)/(b^8*(a + b*x)) + (21*e^5*(b*d - a*e)^2*Log[a + b*x])/b^8
```

Rubi [A] time = 0.234766, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$\frac{e^6x(7bd-6ae)}{b^7}-\frac{35e^4(bd-ae)^3}{b^8(a+bx)}-\frac{35e^3(bd-ae)^4}{2b^8(a+bx)^2}-\frac{7e^2(bd-ae)^5}{b^8(a+bx)^3}+\frac{21e^5(bd-ae)^2\log(a+bx)}{b^8}-\frac{7e(bd-ae)^6}{4b^8(a+bx)^4}-\frac{(bd}{5b^8}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^7/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] (e^6*(7*b*d - 6*a*e)*x)/b^7 + (e^7*x^2)/(2*b^6) - (b*d - a*e)^7/(5*b^8*(a +
 b*x)^5) - (7*e*(b*d - a*e)^6)/(4*b^8*(a + b*x)^4) - (7*e^2*(b*d - a*e)^5)/
(b^8*(a + b*x)^3) - (35*e^3*(b*d - a*e)^4)/(2*b^8*(a + b*x)^2) - (35*e^4*(b
*d - a*e)^3)/(b^8*(a + b*x)) + (21*e^5*(b*d - a*e)^2*Log[a + b*x])/b^8
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^7}{\left(a^2+2abx+b^2x^2\right)^3}\,dx &= \int \frac{(d+ex)^7}{(a+bx)^6}\,dx\\
&= \int \left(\frac{e^6(7bd-6ae)}{b^7}+\frac{e^7x}{b^6}+\frac{(bd-ae)^7}{b^7(a+bx)^6}+\frac{7e(bd-ae)^6}{b^7(a+bx)^5}+\frac{21e^2(bd-ae)^5}{b^7(a+bx)^4}+\frac{35e^3(bd-ae)}{b^7(a+bx)^3}\right.\\
&= \frac{e^6(7bd-6ae)x}{b^7}+\frac{e^7x^2}{2b^6}-\frac{(bd-ae)^7}{5b^8(a+bx)^5}-\frac{7e(bd-ae)^6}{4b^8(a+bx)^4}-\frac{7e^2(bd-ae)^5}{b^8(a+bx)^3}-\frac{35e^3(bd-ae)^4}{2b^8(a+bx)^2}
\end{aligned}$$

Mathematica [B] time = 0.148749, size = 389, normalized size = 2.15

$$-a^2b^5e^2\left(1400d^3e^2x^2-6300d^2e^3x^3+175d^4ex+14d^5+700de^4x^4+500e^5x^5\right)-5a^3b^4e^3\left(-1540d^2e^2x^2+140d^3ex+7d\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^7/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] (459*a^7*e^7 + 3*a^6*b*e^6*(-406*d + 625*e*x) + a^5*b^2*e^5*(959*d^2 - 5250
*d*e*x + 2700*e^2*x^2) + 5*a^4*b^3*e^4*(-28*d^3 + 875*d^2*e*x - 1680*d*e^2*
x^2 + 260*e^3*x^3) - 5*a^3*b^4*e^3*(7*d^4 + 140*d^3*e*x - 1540*d^2*e^2*x^2
+ 1120*d*e^3*x^3 + 80*e^4*x^4) - a^2*b^5*e^2*(14*d^5 + 175*d^4*e*x + 1400*d
^3*e^2*x^2 - 6300*d^2*e^3*x^3 + 700*d*e^4*x^4 + 500*e^5*x^5) - 7*a*b^6*e*(d
^6 + 10*d^5*e*x + 50*d^4*e^2*x^2 + 200*d^3*e^3*x^3 - 300*d^2*e^4*x^4 - 100*
d*e^5*x^5 + 10*e^6*x^6) - b^7*(4*d^7 + 35*d^6*e*x + 140*d^5*e^2*x^2 + 350*d
^4*e^3*x^3 + 700*d^3*e^4*x^4 - 140*d*e^6*x^6 - 10*e^7*x^7) + 420*e^5*(b*d -
 a*e)^2*(a + b*x)^5*Log[a + b*x])/(20*b^8*(a + b*x)^5)
```

Maple [B] time = 0.056, size = 656, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^7/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] 21/b^8*e^7*ln(b*x+a)*a^2+21/b^6*e^5*ln(b*x+a)*d^2+35/b^8*e^7/(b*x+a)*a^3-35
/b^5*e^4/(b*x+a)*d^3-6*e^7/b^7*a*x+7*e^6/b^6*x*d-35/2/b^8*e^7/(b*x+a)^2*a^4
-35/2/b^4*e^3/(b*x+a)^2*d^4+1/5/b^8/(b*x+a)^5*a^7*e^7+7/b^8*e^7/(b*x+a)^3*a
^5-7/b^3*e^2/(b*x+a)^3*d^5-7/4/b^8*e^7/(b*x+a)^4*a^6-7/4/b^2*e/(b*x+a)^4*d^
6-35/b^7*e^6/(b*x+a)^3*a^4*d+70/b^6*e^5/(b*x+a)^3*a^3*d^2+1/2*e^7*x^2/b^6-1
/5/b/(b*x+a)^5*d^7-42/b^7*e^6*ln(b*x+a)*a*d+35/b^5*e^4/(b*x+a)^4*a^3*d^3-10
5/4/b^4*e^3/(b*x+a)^4*a^2*d^4+21/2/b^3*e^2/(b*x+a)^4*a*d^5-7/5/b^7/(b*x+a)^
5*a^6*d*e^6+21/5/b^6/(b*x+a)^5*a^5*d^2*e^5-7/b^5/(b*x+a)^5*a^4*d^3*e^4+7/b^
4/(b*x+a)^5*a^3*d^4*e^3-21/5/b^3/(b*x+a)^5*a^2*d^5*e^2+7/5/b^2/(b*x+a)^5*a*
d^6*e+70/b^7*e^6/(b*x+a)^2*a^3*d-105/b^6*e^5/(b*x+a)^2*a^2*d^2+70/b^5*e^4/(
b*x+a)^2*a*d^3-105/b^7*e^6/(b*x+a)*a^2*d+105/b^6*e^5/(b*x+a)*a*d^2-70/b^5*e
^4/(b*x+a)^3*a^2*d^3+35/b^4*e^3/(b*x+a)^3*a*d^4+21/2/b^7*e^6/(b*x+a)^4*a^5*
d-105/4/b^6*e^5/(b*x+a)^4*d^2*a^4
```

Maxima [B] time = 1.29536, size = 680, normalized size = 3.76

$$-\frac{4\,b^7d^7 + 7\,ab^6d^6e + 14\,a^2b^5d^5e^2 + 35\,a^3b^4d^4e^3 + 140\,a^4b^3d^3e^4 - 959\,a^5b^2d^2e^5 + 1218\,a^6bde^6 - 459\,a^7e^7 + 700\,\left(b^7d^3e^4 - \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/20*(4*b^7*d^7 + 7*a*b^6*d^6*e + 14*a^2*b^5*d^5*e^2 + 35*a^3*b^4*d^4*e^3
+ 140*a^4*b^3*d^3*e^4 - 959*a^5*b^2*d^2*e^5 + 1218*a^6*b*d*e^6 - 459*a^7*e^
7 + 700*(b^7*d^3*e^4 - 3*a*b^6*d^2*e^5 + 3*a^2*b^5*d*e^6 - a^3*b^4*e^7)*x^4
 + 350*(b^7*d^4*e^3 + 4*a*b^6*d^3*e^4 - 18*a^2*b^5*d^2*e^5 + 20*a^3*b^4*d*e
^6 - 7*a^4*b^3*e^7)*x^3 + 70*(2*b^7*d^5*e^2 + 5*a*b^6*d^4*e^3 + 20*a^2*b^5*
d^3*e^4 - 110*a^3*b^4*d^2*e^5 + 130*a^4*b^3*d*e^6 - 47*a^5*b^2*e^7)*x^2 + 3
5*(b^7*d^6*e + 2*a*b^6*d^5*e^2 + 5*a^2*b^5*d^4*e^3 + 20*a^3*b^4*d^3*e^4 - 1
25*a^4*b^3*d^2*e^5 + 154*a^5*b^2*d*e^6 - 57*a^6*b*e^7)*x)/(b^13*x^5 + 5*a*b
```

```
^12*x^4 + 10*a^2*b^11*x^3 + 10*a^3*b^10*x^2 + 5*a^4*b^9*x + a^5*b^8) + 1/2*
(b*e^7*x^2 + 2*(7*b*d*e^6 - 6*a*e^7)*x)/b^7 + 21*(b^2*d^2*e^5 - 2*a*b*d*e^6
 + a^2*e^7)*log(b*x + a)/b^8
```

Fricas [B] time = 1.72278, size = 1517, normalized size = 8.38

$$10\,b^7e^7x^7 - 4\,b^7d^7 - 7\,ab^6d^6e - 14\,a^2b^5d^5e^2 - 35\,a^3b^4d^4e^3 - 140\,a^4b^3d^3e^4 + 959\,a^5b^2d^2e^5 - 1218\,a^6bde^6 + 459\,a^7e^7 + 7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/20*(10*b^7*e^7*x^7 - 4*b^7*d^7 - 7*a*b^6*d^6*e - 14*a^2*b^5*d^5*e^2 - 35*
a^3*b^4*d^4*e^3 - 140*a^4*b^3*d^3*e^4 + 959*a^5*b^2*d^2*e^5 - 1218*a^6*b*d*
e^6 + 459*a^7*e^7 + 70*(2*b^7*d*e^6 - a*b^6*e^7)*x^6 + 100*(7*a*b^6*d*e^6 -
 5*a^2*b^5*e^7)*x^5 - 100*(7*b^7*d^3*e^4 - 21*a*b^6*d^2*e^5 + 7*a^2*b^5*d*e
^6 + 4*a^3*b^4*e^7)*x^4 - 50*(7*b^7*d^4*e^3 + 28*a*b^6*d^3*e^4 - 126*a^2*b^
5*d^2*e^5 + 112*a^3*b^4*d*e^6 - 26*a^4*b^3*e^7)*x^3 - 10*(14*b^7*d^5*e^2 +
35*a*b^6*d^4*e^3 + 140*a^2*b^5*d^3*e^4 - 770*a^3*b^4*d^2*e^5 + 840*a^4*b^3*
d*e^6 - 270*a^5*b^2*e^7)*x^2 - 5*(7*b^7*d^6*e + 14*a*b^6*d^5*e^2 + 35*a^2*b
^5*d^4*e^3 + 140*a^3*b^4*d^3*e^4 - 875*a^4*b^3*d^2*e^5 + 1050*a^5*b^2*d*e^6
 - 375*a^6*b*e^7)*x + 420*(a^5*b^2*d^2*e^5 - 2*a^6*b*d*e^6 + a^7*e^7 + (b^7
*d^2*e^5 - 2*a*b^6*d*e^6 + a^2*b^5*e^7)*x^5 + 5*(a*b^6*d^2*e^5 - 2*a^2*b^5*
d*e^6 + a^3*b^4*e^7)*x^4 + 10*(a^2*b^5*d^2*e^5 - 2*a^3*b^4*d*e^6 + a^4*b^3*
e^7)*x^3 + 10*(a^3*b^4*d^2*e^5 - 2*a^4*b^3*d*e^6 + a^5*b^2*e^7)*x^2 + 5*(a^
4*b^3*d^2*e^5 - 2*a^5*b^2*d*e^6 + a^6*b*e^7)*x)*log(b*x + a))/(b^13*x^5 + 5
*a*b^12*x^4 + 10*a^2*b^11*x^3 + 10*a^3*b^10*x^2 + 5*a^4*b^9*x + a^5*b^8)
```

Sympy [B] time = 111.638, size = 522, normalized size = 2.88

$$459a^7e^7 - 1218a^6bde^6 + 959a^5b^2d^2e^5 - 140a^4b^3d^3e^4 - 35a^3b^4d^4e^3 - 14a^2b^5d^5e^2 - 7ab^6d^6e - 4b^7d^7 + x^4\left(700a^3b^4e^7 - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**7/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] (459*a**7*e**7 - 1218*a**6*b*d*e**6 + 959*a**5*b**2*d**2*e**5 - 140*a**4*b*
*3*d**3*e**4 - 35*a**3*b**4*d**4*e**3 - 14*a**2*b**5*d**5*e**2 - 7*a*b**6*d
**6*e - 4*b**7*d**7 + x**4*(700*a**3*b**4*e**7 - 2100*a**2*b**5*d*e**6 + 21
00*a*b**6*d**2*e**5 - 700*b**7*d**3*e**4) + x**3*(2450*a**4*b**3*e**7 - 700
0*a**3*b**4*d*e**6 + 6300*a**2*b**5*d**2*e**5 - 1400*a*b**6*d**3*e**4 - 350
*b**7*d**4*e**3) + x**2*(3290*a**5*b**2*e**7 - 9100*a**4*b**3*d*e**6 + 7700
*a**3*b**4*d**2*e**5 - 1400*a**2*b**5*d**3*e**4 - 350*a*b**6*d**4*e**3 - 14
0*b**7*d**5*e**2) + x*(1995*a**6*b*e**7 - 5390*a**5*b**2*d*e**6 + 4375*a**4
*b**3*d**2*e**5 - 700*a**3*b**4*d**3*e**4 - 175*a**2*b**5*d**4*e**3 - 70*a*
b**6*d**5*e**2 - 35*b**7*d**6*e))/(20*a**5*b**8 + 100*a**4*b**9*x + 200*a**
3*b**10*x**2 + 200*a**2*b**11*x**3 + 100*a*b**12*x**4 + 20*b**13*x**5) + e*
*7*x**2/(2*b**6) - x*(6*a*e**7 - 7*b*d*e**6)/b**7 + 21*e**5*(a*e - b*d)**2*
log(a + b*x)/b**8
```

Giac [B] time = 1.1476, size = 583, normalized size = 3.22

$$\frac{21\left(b^2d^2e^5-2\,abde^6+a^2e^7\right)\log\left(|bx+a|\right)}{b^8}+\frac{b^6x^2e^7+14\,b^6dxe^6-12\,ab^5xe^7}{2\,b^{12}}-\frac{4\,b^7d^7+7\,ab^6d^6e+14\,a^2b^5d^5e^2+35\,a^3b^4d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] 21*(b^2*d^2*e^5 - 2*a*b*d*e^6 + a^2*e^7)*log(abs(b*x + a))/b^8 + 1/2*(b^6*x
^2*e^7 + 14*b^6*d*x*e^6 - 12*a*b^5*x*e^7)/b^12 - 1/20*(4*b^7*d^7 + 7*a*b^6*
d^6*e + 14*a^2*b^5*d^5*e^2 + 35*a^3*b^4*d^4*e^3 + 140*a^4*b^3*d^3*e^4 - 959
*a^5*b^2*d^2*e^5 + 1218*a^6*b*d*e^6 - 459*a^7*e^7 + 700*(b^7*d^3*e^4 - 3*a*
b^6*d^2*e^5 + 3*a^2*b^5*d*e^6 - a^3*b^4*e^7)*x^4 + 350*(b^7*d^4*e^3 + 4*a*b
^6*d^3*e^4 - 18*a^2*b^5*d^2*e^5 + 20*a^3*b^4*d*e^6 - 7*a^4*b^3*e^7)*x^3 + 7
0*(2*b^7*d^5*e^2 + 5*a*b^6*d^4*e^3 + 20*a^2*b^5*d^3*e^4 - 110*a^3*b^4*d^2*e
^5 + 130*a^4*b^3*d*e^6 - 47*a^5*b^2*e^7)*x^2 + 35*(b^7*d^6*e + 2*a*b^6*d^5*
e^2 + 5*a^2*b^5*d^4*e^3 + 20*a^3*b^4*d^3*e^4 - 125*a^4*b^3*d^2*e^5 + 154*a^
5*b^2*d*e^6 - 57*a^6*b*e^7)*x)/((b*x + a)^5*b^8)
```

3.1527 $\int \frac{(d+ex)^6}{\left(a^2+2abx+b^2x^2\right)^3}\,dx$

Optimal. Leaf size=155

$$-\frac{15e^4(bd-ae)^2}{b^7(a+bx)}-\frac{10e^3(bd-ae)^3}{b^7(a+bx)^2}-\frac{5e^2(bd-ae)^4}{b^7(a+bx)^3}+\frac{6e^5(bd-ae)\log(a+bx)}{b^7}-\frac{3e(bd-ae)^5}{2b^7(a+bx)^4}-\frac{(bd-ae)^6}{5b^7(a+bx)^5}+\frac{e^6x}{b^6}$$

```
[Out] (e^6*x)/b^6 - (b*d - a*e)^6/(5*b^7*(a + b*x)^5) - (3*e*(b*d - a*e)^5)/(2*b^
7*(a + b*x)^4) - (5*e^2*(b*d - a*e)^4)/(b^7*(a + b*x)^3) - (10*e^3*(b*d - a
*e)^3)/(b^7*(a + b*x)^2) - (15*e^4*(b*d - a*e)^2)/(b^7*(a + b*x)) + (6*e^5*
(b*d - a*e)*Log[a + b*x])/b^7
```

Rubi [A] time = 0.169131, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$-\frac{15e^4(bd-ae)^2}{b^7(a+bx)}-\frac{10e^3(bd-ae)^3}{b^7(a+bx)^2}-\frac{5e^2(bd-ae)^4}{b^7(a+bx)^3}+\frac{6e^5(bd-ae)\log(a+bx)}{b^7}-\frac{3e(bd-ae)^5}{2b^7(a+bx)^4}-\frac{(bd-ae)^6}{5b^7(a+bx)^5}+\frac{e^6x}{b^6}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^6/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] (e^6*x)/b^6 - (b*d - a*e)^6/(5*b^7*(a + b*x)^5) - (3*e*(b*d - a*e)^5)/(2*b^
7*(a + b*x)^4) - (5*e^2*(b*d - a*e)^4)/(b^7*(a + b*x)^3) - (10*e^3*(b*d - a
*e)^3)/(b^7*(a + b*x)^2) - (15*e^4*(b*d - a*e)^2)/(b^7*(a + b*x)) + (6*e^5*
(b*d - a*e)*Log[a + b*x])/b^7
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^6}{\left(a^2+2abx+b^2x^2\right)^3}\,dx &= \int \frac{(d+ex)^6}{(a+bx)^6}\,dx\\ &= \int \left(\frac{e^6}{b^6}+\frac{(bd-ae)^6}{b^6(a+bx)^6}+\frac{6e(bd-ae)^5}{b^6(a+bx)^5}+\frac{15e^2(bd-ae)^4}{b^6(a+bx)^4}+\frac{20e^3(bd-ae)^3}{b^6(a+bx)^3}+\frac{15e^4(bd-ae)^2}{b^6(a+bx)^2}\right.\\ &= \frac{e^6x}{b^6}-\frac{(bd-ae)^6}{5b^7(a+bx)^5}-\frac{3e(bd-ae)^5}{2b^7(a+bx)^4}-\frac{5e^2(bd-ae)^4}{b^7(a+bx)^3}-\frac{10e^3(bd-ae)^3}{b^7(a+bx)^2}-\frac{15e^4(bd-ae)^2}{b^7(a+bx)}+\end{aligned}$$

Mathematica [A] time = 0.162551, size = 300, normalized size = 1.94

$$-\frac{5a^2b^4e^2\left(60d^2e^2x^2+10d^3ex+d^4-180de^3x^3+10e^4x^4\right)+10a^3b^3e^3\left(15d^2ex+d^3-110de^2x^2+40e^3x^3\right)+5a^4b^2e^4\left(6d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^6/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] -(87*a^6*e^6 + a^5*b*e^5*(-137*d + 375*e*x) + 5*a^4*b^2*e^4*(6*d^2 - 125*d*
e*x + 120*e^2*x^2) + 10*a^3*b^3*e^3*(d^3 + 15*d^2*e*x - 110*d*e^2*x^2 + 40*
e^3*x^3) + 5*a^2*b^4*e^2*(d^4 + 10*d^3*e*x + 60*d^2*e^2*x^2 - 180*d*e^3*x^3
 + 10*e^4*x^4) + a*b^5*e*(3*d^5 + 25*d^4*e*x + 100*d^3*e^2*x^2 + 300*d^2*e^
3*x^3 - 300*d*e^4*x^4 - 50*e^5*x^5) + b^6*(2*d^6 + 15*d^5*e*x + 50*d^4*e^2*
x^2 + 100*d^3*e^3*x^3 + 150*d^2*e^4*x^4 - 10*e^6*x^6) + 60*e^5*(-(b*d) + a*
e)*(a + b*x)^5*Log[a + b*x])/(10*b^7*(a + b*x)^5)
```

Maple [B] time = 0.052, size = 508, normalized size = 3.3

$$-3\,\frac{d^2e^4a^4}{b^5\,(bx+a)^5}+4\,\frac{a^3d^3e^3}{b^4\,(bx+a)^5}-3\,\frac{a^2d^4e^2}{b^3\,(bx+a)^5}+20\,\frac{a^3e^5d}{b^6\,(bx+a)^3}+30\,\frac{e^5ad}{b^6\,(bx+a)}-\frac{15\,e^5a^4d}{2\,b^6\,(bx+a)^4}+15\,\frac{e^4a^3d^2}{b^5\,(bx+a)^4}-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^6/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] -3/b^5/(b*x+a)^5*d^2*e^4*a^4+4/b^4/(b*x+a)^5*a^3*d^3*e^3-3/b^3/(b*x+a)^5*a^
2*d^4*e^2+20/b^6*e^5/(b*x+a)^3*a^3*d+30/b^6*e^5/(b*x+a)*a*d-15/2/b^6*e^5/(b
*x+a)^4*a^4*d+15/b^5*e^4/(b*x+a)^4*a^3*d^2-15/b^4*e^3/(b*x+a)^4*a^2*d^3+15/
2/b^3*e^2/(b*x+a)^4*a*d^4+6/5/b^2/(b*x+a)^5*a*d^5*e-30/b^6*e^5/(b*x+a)^2*a^
2*d+30/b^5*e^4/(b*x+a)^2*a*d^2+6/5/b^6/(b*x+a)^5*a^5*d*e^5-6/b^7*e^6*ln(b*x
+a)*a+6/b^6*e^5*ln(b*x+a)*d-15/b^7*e^6/(b*x+a)*a^2-15/b^5*e^4/(b*x+a)*d^2+1
0/b^7*e^6/(b*x+a)^2*a^3-10/b^4*e^3/(b*x+a)^2*d^3-1/5/b^7/(b*x+a)^5*a^6*e^6-
5/b^7*e^6/(b*x+a)^3*a^4-5/b^3*e^2/(b*x+a)^3*d^4+3/2/b^7*e^6/(b*x+a)^4*a^5-3
/2/b^2*e/(b*x+a)^4*d^5-30/b^5*e^4/(b*x+a)^3*a^2*d^2+20/b^4*e^3/(b*x+a)^3*a*
d^3-1/5/b/(b*x+a)^5*d^6+e^6*x/b^6
```

Maxima [B] time = 1.25274, size = 536, normalized size = 3.46

$$\frac{e^6x}{b^6}-\frac{2\,b^6d^6+3\,ab^5d^5e+5\,a^2b^4d^4e^2+10\,a^3b^3d^3e^3+30\,a^4b^2d^2e^4-137\,a^5bde^5+87\,a^6e^6+150\left(b^6d^2e^4-2\,ab^5de^5+a^2b^4e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] e^6*x/b^6 - 1/10*(2*b^6*d^6 + 3*a*b^5*d^5*e + 5*a^2*b^4*d^4*e^2 + 10*a^3*b^
3*d^3*e^3 + 30*a^4*b^2*d^2*e^4 - 137*a^5*b*d*e^5 + 87*a^6*e^6 + 150*(b^6*d^
2*e^4 - 2*a*b^5*d*e^5 + a^2*b^4*e^6)*x^4 + 100*(b^6*d^3*e^3 + 3*a*b^5*d^2*e
^4 - 9*a^2*b^4*d*e^5 + 5*a^3*b^3*e^6)*x^3 + 50*(b^6*d^4*e^2 + 2*a*b^5*d^3*e
^3 + 6*a^2*b^4*d^2*e^4 - 22*a^3*b^3*d*e^5 + 13*a^4*b^2*e^6)*x^2 + 5*(3*b^6*
d^5*e + 5*a*b^5*d^4*e^2 + 10*a^2*b^4*d^3*e^3 + 30*a^3*b^3*d^2*e^4 - 125*a^4
*b^2*d*e^5 + 77*a^5*b*e^6)*x)/(b^12*x^5 + 5*a*b^11*x^4 + 10*a^2*b^10*x^3 +
10*a^3*b^9*x^2 + 5*a^4*b^8*x + a^5*b^7) + 6*(b*d*e^5 - a*e^6)*log(b*x + a)/
b^7
```

Fricas [B] time = 1.92112, size = 1095, normalized size = 7.06

$$\frac{10\,b^6e^6x^6 + 50\,ab^5e^6x^5 - 2\,b^6d^6 - 3\,ab^5d^5e - 5\,a^2b^4d^4e^2 - 10\,a^3b^3d^3e^3 - 30\,a^4b^2d^2e^4 + 137\,a^5bde^5 - 87\,a^6e^6 - 50\,\left(3\,b^6 \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/10*(10*b^6*e^6*x^6 + 50*a*b^5*e^6*x^5 - 2*b^6*d^6 - 3*a*b^5*d^5*e - 5*a^2
*b^4*d^4*e^2 - 10*a^3*b^3*d^3*e^3 - 30*a^4*b^2*d^2*e^4 + 137*a^5*b*d*e^5 -
87*a^6*e^6 - 50*(3*b^6*d^2*e^4 - 6*a*b^5*d*e^5 + a^2*b^4*e^6)*x^4 - 100*(b^
6*d^3*e^3 + 3*a*b^5*d^2*e^4 - 9*a^2*b^4*d*e^5 + 4*a^3*b^3*e^6)*x^3 - 50*(b^
6*d^4*e^2 + 2*a*b^5*d^3*e^3 + 6*a^2*b^4*d^2*e^4 - 22*a^3*b^3*d*e^5 + 12*a^4
*b^2*e^6)*x^2 - 5*(3*b^6*d^5*e + 5*a*b^5*d^4*e^2 + 10*a^2*b^4*d^3*e^3 + 30*
a^3*b^3*d^2*e^4 - 125*a^4*b^2*d*e^5 + 75*a^5*b*e^6)*x + 60*(a^5*b*d*e^5 - a
^6*e^6 + (b^6*d*e^5 - a*b^5*e^6)*x^5 + 5*(a*b^5*d*e^5 - a^2*b^4*e^6)*x^4 +
10*(a^2*b^4*d*e^5 - a^3*b^3*e^6)*x^3 + 10*(a^3*b^3*d*e^5 - a^4*b^2*e^6)*x^2
 + 5*(a^4*b^2*d*e^5 - a^5*b*e^6)*x)*log(b*x + a))/(b^12*x^5 + 5*a*b^11*x^4
+ 10*a^2*b^10*x^3 + 10*a^3*b^9*x^2 + 5*a^4*b^8*x + a^5*b^7)
```

Sympy [B] time = 37.5701, size = 420, normalized size = 2.71

$$-\frac{87a^6e^6 - 137a^5bde^5 + 30a^4b^2d^2e^4 + 10a^3b^3d^3e^3 + 5a^2b^4d^4e^2 + 3ab^5d^5e + 2b^6d^6 + x^4\left(150a^2b^4e^6 - 300ab^5de^5 + 150b \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**6/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] -(87*a**6*e**6 - 137*a**5*b*d*e**5 + 30*a**4*b**2*d**2*e**4 + 10*a**3*b**3*
d**3*e**3 + 5*a**2*b**4*d**4*e**2 + 3*a*b**5*d**5*e + 2*b**6*d**6 + x**4*(1
50*a**2*b**4*e**6 - 300*a*b**5*d*e**5 + 150*b**6*d**2*e**4) + x**3*(500*a**
3*b**3*e**6 - 900*a**2*b**4*d*e**5 + 300*a*b**5*d**2*e**4 + 100*b**6*d**3*e
**3) + x**2*(650*a**4*b**2*e**6 - 1100*a**3*b**3*d*e**5 + 300*a**2*b**4*d**
2*e**4 + 100*a*b**5*d**3*e**3 + 50*b**6*d**4*e**2) + x*(385*a**5*b*e**6 - 6
25*a**4*b**2*d*e**5 + 150*a**3*b**3*d**2*e**4 + 50*a**2*b**4*d**3*e**3 + 25
*a*b**5*d**4*e**2 + 15*b**6*d**5*e))/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a
**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) + e*
*6*x/b**6 - 6*e**5*(a*e - b*d)*log(a + b*x)/b**7
```

Giac [B] time = 1.15733, size = 443, normalized size = 2.86

$$\frac{xe^6}{b^6} + \frac{6\left(bde^5 - ae^6\right)\log\left(|bx + a|\right)}{b^7} - \frac{2\,b^6d^6 + 3\,ab^5d^5e + 5\,a^2b^4d^4e^2 + 10\,a^3b^3d^3e^3 + 30\,a^4b^2d^2e^4 - 137\,a^5bde^5 + 87\,a^6e \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] x*e^6/b^6 + 6*(b*d*e^5 - a*e^6)*log(abs(b*x + a))/b^7 - 1/10*(2*b^6*d^6 + 3
*a*b^5*d^5*e + 5*a^2*b^4*d^4*e^2 + 10*a^3*b^3*d^3*e^3 + 30*a^4*b^2*d^2*e^4
- 137*a^5*b*d*e^5 + 87*a^6*e^6 + 150*(b^6*d^2*e^4 - 2*a*b^5*d*e^5 + a^2*b^4
```

```
*e^6)*x^4 + 100*(b^6*d^3*e^3 + 3*a*b^5*d^2*e^4 - 9*a^2*b^4*d*e^5 + 5*a^3*b^
3*e^6)*x^3 + 50*(b^6*d^4*e^2 + 2*a*b^5*d^3*e^3 + 6*a^2*b^4*d^2*e^4 - 22*a^3
*b^3*d*e^5 + 13*a^4*b^2*e^6)*x^2 + 5*(3*b^6*d^5*e + 5*a*b^5*d^4*e^2 + 10*a^
2*b^4*d^3*e^3 + 30*a^3*b^3*d^2*e^4 - 125*a^4*b^2*d*e^5 + 77*a^5*b*e^6)*x)/(
(b*x + a)^5*b^7)
```

3.1528 $\int \frac{(d+ex)^5}{\left(a^2+2abx+b^2x^2\right)^3}\,dx$

Optimal. Leaf size=138

$$-\frac{5e^4(bd-ae)}{b^6(a+bx)}-\frac{5e^3(bd-ae)^2}{b^6(a+bx)^2}-\frac{10e^2(bd-ae)^3}{3b^6(a+bx)^3}-\frac{5e(bd-ae)^4}{4b^6(a+bx)^4}-\frac{(bd-ae)^5}{5b^6(a+bx)^5}+\frac{e^5\log(a+bx)}{b^6}$$

```
[Out] -(b*d - a*e)^5/(5*b^6*(a + b*x)^5) - (5*e*(b*d - a*e)^4)/(4*b^6*(a + b*x)^4
) - (10*e^2*(b*d - a*e)^3)/(3*b^6*(a + b*x)^3) - (5*e^3*(b*d - a*e)^2)/(b^6
*(a + b*x)^2) - (5*e^4*(b*d - a*e))/(b^6*(a + b*x)) + (e^5*Log[a + b*x])/b^
6
```

Rubi [A] time = 0.116599, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$-\frac{5e^4(bd-ae)}{b^6(a+bx)}-\frac{5e^3(bd-ae)^2}{b^6(a+bx)^2}-\frac{10e^2(bd-ae)^3}{3b^6(a+bx)^3}-\frac{5e(bd-ae)^4}{4b^6(a+bx)^4}-\frac{(bd-ae)^5}{5b^6(a+bx)^5}+\frac{e^5\log(a+bx)}{b^6}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] -(b*d - a*e)^5/(5*b^6*(a + b*x)^5) - (5*e*(b*d - a*e)^4)/(4*b^6*(a + b*x)^4
) - (10*e^2*(b*d - a*e)^3)/(3*b^6*(a + b*x)^3) - (5*e^3*(b*d - a*e)^2)/(b^6
*(a + b*x)^2) - (5*e^4*(b*d - a*e))/(b^6*(a + b*x)) + (e^5*Log[a + b*x])/b^
6
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(d+ex)^5}{\left(a^2+2abx+b^2x^2\right)^3}\,dx=\int \frac{(d+ex)^5}{(a+bx)^6}\,dx$$
$$=\int \left(\frac{(bd-ae)^5}{b^5(a+bx)^6}+\frac{5e(bd-ae)^4}{b^5(a+bx)^5}+\frac{10e^2(bd-ae)^3}{b^5(a+bx)^4}+\frac{10e^3(bd-ae)^2}{b^5(a+bx)^3}+\frac{5e^4(bd-ae)}{b^5(a+bx)^2}+\ldots\right.$$
$$=-\frac{(bd-ae)^5}{5b^6(a+bx)^5}-\frac{5e(bd-ae)^4}{4b^6(a+bx)^4}-\frac{10e^2(bd-ae)^3}{3b^6(a+bx)^3}-\frac{5e^3(bd-ae)^2}{b^6(a+bx)^2}-\frac{5e^4(bd-ae)}{b^6(a+bx)}+\frac{e^5\log(\ldots}{\ldots}$$

Mathematica [A] time = 0.0886145, size = 171, normalized size = 1.24

$$\frac{e^5\log(a+bx)}{b^6}-\frac{(bd-ae)\left(a^2b^2e^2\left(47d^2+325dex+1100e^2x^2\right)+a^3be^3(77d+625ex)+137a^4e^4+ab^3e\left(175d^2ex+27\ldots\right.\right.}{60b^6(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] -((b*d - a*e)*(137*a^4*e^4 + a^3*b*e^3*(77*d + 625*e*x) + a^2*b^2*e^2*(47*d
^2 + 325*d*e*x + 1100*e^2*x^2) + a*b^3*e*(27*d^3 + 175*d^2*e*x + 500*d*e^2*
x^2 + 900*e^3*x^3) + b^4*(12*d^4 + 75*d^3*e*x + 200*d^2*e^2*x^2 + 300*d*e^3
*x^3 + 300*e^4*x^4)))/(60*b^6*(a + b*x)^5) + (e^5*Log[a + b*x])/b^6
```

Maple [B] time = 0.052, size = 377, normalized size = 2.7

$$-5\frac{a^2e^5}{b^6(bx+a)^2}+10\frac{de^4a}{b^5(bx+a)^2}-5\frac{d^2e^3}{b^4(bx+a)^2}+\frac{a^5e^5}{5b^6(bx+a)^5}-\frac{de^4a^4}{b^5(bx+a)^5}+2\frac{d^2e^3a^3}{b^4(bx+a)^5}-2\frac{d^3e^2a^2}{b^3(bx+a)^5}+\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] -5*e^5/b^6/(b*x+a)^2*a^2+10*e^4/b^5/(b*x+a)^2*a*d-5*e^3/b^4/(b*x+a)^2*d^2+1
/5/b^6/(b*x+a)^5*a^5*e^5-1/b^5/(b*x+a)^5*d*e^4*a^4+2/b^4/(b*x+a)^5*d^2*e^3*
a^3-2/b^3/(b*x+a)^5*d^3*e^2*a^2+1/b^2/(b*x+a)^5*a*d^4*e-1/5/b/(b*x+a)^5*d^5
+10/3*e^5/b^6/(b*x+a)^3*a^3-10*e^4/b^5/(b*x+a)^3*a^2*d+10*e^3/b^4/(b*x+a)^3
*a*d^2-10/3*e^2/b^3/(b*x+a)^3*d^3+e^5*ln(b*x+a)/b^6-5/4*e^5/b^6/(b*x+a)^4*a
^4+5*e^4/b^5/(b*x+a)^4*a^3*d-15/2*e^3/b^4/(b*x+a)^4*a^2*d^2+5*e^2/b^3/(b*x+
a)^4*a*d^3-5/4*e/b^2/(b*x+a)^4*d^4+5/b^6*e^5/(b*x+a)*a-5/b^5*e^4/(b*x+a)*d
```

Maxima [B] time = 1.18595, size = 419, normalized size = 3.04

$$-\frac{12\,b^5d^5+15\,ab^4d^4e+20\,a^2b^3d^3e^2+30\,a^3b^2d^2e^3+60\,a^4bde^4-137\,a^5e^5+300\left(b^5de^4-ab^4e^5\right)x^4+300\left(b^5d^2e^3+2\,ab^4d\ldots\right.}{60\left(b^{11}x^5+5\,ab^{10}x^4+\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/60*(12*b^5*d^5 + 15*a*b^4*d^4*e + 20*a^2*b^3*d^3*e^2 + 30*a^3*b^2*d^2*e^
3 + 60*a^4*b*d*e^4 - 137*a^5*e^5 + 300*(b^5*d*e^4 - a*b^4*e^5)*x^4 + 300*(b
^5*d^2*e^3 + 2*a*b^4*d*e^4 - 3*a^2*b^3*e^5)*x^3 + 100*(2*b^5*d^3*e^2 + 3*a*
b^4*d^2*e^3 + 6*a^2*b^3*d*e^4 - 11*a^3*b^2*e^5)*x^2 + 25*(3*b^5*d^4*e + 4*a
*b^4*d^3*e^2 + 6*a^2*b^3*d^2*e^3 + 12*a^3*b^2*d*e^4 - 25*a^4*b*e^5)*x)/(b^1
1*x^5 + 5*a*b^10*x^4 + 10*a^2*b^9*x^3 + 10*a^3*b^8*x^2 + 5*a^4*b^7*x + a^5*
b^6) + e^5*log(b*x + a)/b^6
```

Fricas [B] time = 2.09062, size = 775, normalized size = 5.62

$$-\frac{12\,b^5d^5+15\,ab^4d^4e+20\,a^2b^3d^3e^2+30\,a^3b^2d^2e^3+60\,a^4bde^4-137\,a^5e^5+300\left(b^5de^4-ab^4e^5\right)x^4+300\left(b^5d^2e^3+2\,ab^4d\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/60*(12*b^5*d^5 + 15*a*b^4*d^4*e + 20*a^2*b^3*d^3*e^2 + 30*a^3*b^2*d^2*e^
3 + 60*a^4*b*d*e^4 - 137*a^5*e^5 + 300*(b^5*d*e^4 - a*b^4*e^5)*x^4 + 300*(b
^5*d^2*e^3 + 2*a*b^4*d*e^4 - 3*a^2*b^3*e^5)*x^3 + 100*(2*b^5*d^3*e^2 + 3*a*
b^4*d^2*e^3 + 6*a^2*b^3*d*e^4 - 11*a^3*b^2*e^5)*x^2 + 25*(3*b^5*d^4*e + 4*a
*b^4*d^3*e^2 + 6*a^2*b^3*d^2*e^3 + 12*a^3*b^2*d*e^4 - 25*a^4*b*e^5)*x - 60*
(b^5*e^5*x^5 + 5*a*b^4*e^5*x^4 + 10*a^2*b^3*e^5*x^3 + 10*a^3*b^2*e^5*x^2 +
5*a^4*b*e^5*x + a^5*e^5)*log(b*x + a))/(b^11*x^5 + 5*a*b^10*x^4 + 10*a^2*b^
9*x^3 + 10*a^3*b^8*x^2 + 5*a^4*b^7*x + a^5*b^6)
```

Sympy [B] time = 14.3819, size = 326, normalized size = 2.36

$$\frac{137a^5e^5 - 60a^4bde^4 - 30a^3b^2d^2e^3 - 20a^2b^3d^3e^2 - 15ab^4d^4e - 12b^5d^5 + x^4\left(300ab^4e^5 - 300b^5de^4\right) + x^3\left(900a^2b^3e^5 - 60\right.}{60a^5b^6 + 300a^4b^7x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] (137*a**5*e**5 - 60*a**4*b*d*e**4 - 30*a**3*b**2*d**2*e**3 - 20*a**2*b**3*d
**3*e**2 - 15*a*b**4*d**4*e - 12*b**5*d**5 + x**4*(300*a*b**4*e**5 - 300*b*
*5*d*e**4) + x**3*(900*a**2*b**3*e**5 - 600*a*b**4*d*e**4 - 300*b**5*d**2*e
**3) + x**2*(1100*a**3*b**2*e**5 - 600*a**2*b**3*d*e**4 - 300*a*b**4*d**2*e
**3 - 200*b**5*d**3*e**2) + x*(625*a**4*b*e**5 - 300*a**3*b**2*d*e**4 - 150
*a**2*b**3*d**2*e**3 - 100*a*b**4*d**3*e**2 - 75*b**5*d**4*e))/(60*a**5*b**
6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10
*x**4 + 60*b**11*x**5) + e**5*log(a + b*x)/b**6
```

Giac [A] time = 1.1774, size = 335, normalized size = 2.43

$$\frac{e^5\log\left(|bx+a|\right)}{b^6} - \frac{300\left(b^4de^4 - ab^3e^5\right)x^4 + 300\left(b^4d^2e^3 + 2\,ab^3de^4 - 3\,a^2b^2e^5\right)x^3 + 100\left(2\,b^4d^3e^2 + 3\,ab^3d^2e^3 + 6\,a^2b^2d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] e^5*log(abs(b*x + a))/b^6 - 1/60*(300*(b^4*d*e^4 - a*b^3*e^5)*x^4 + 300*(b^
4*d^2*e^3 + 2*a*b^3*d*e^4 - 3*a^2*b^2*e^5)*x^3 + 100*(2*b^4*d^3*e^2 + 3*a*b
^3*d^2*e^3 + 6*a^2*b^2*d*e^4 - 11*a^3*b*e^5)*x^2 + 25*(3*b^4*d^4*e + 4*a*b^
3*d^3*e^2 + 6*a^2*b^2*d^2*e^3 + 12*a^3*b*d*e^4 - 25*a^4*e^5)*x + (12*b^5*d^
5 + 15*a*b^4*d^4*e + 20*a^2*b^3*d^3*e^2 + 30*a^3*b^2*d^2*e^3 + 60*a^4*b*d*e
^4 - 137*a^5*e^5)/b)/((b*x + a)^5*b^5)
```

3.1529 $\int \frac{(d+ex)^4}{\left(a^2+2abx+b^2x^2\right)^3}\,dx$

Optimal. Leaf size=28

$$-\frac{(d+ex)^5}{5(a+bx)^5(bd-ae)}$$

```
[Out] -(d + e*x)^5/(5*(b*d - a*e)*(a + b*x)^5)
```

Rubi [A] time = 0.0047047, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 37}

$$-\frac{(d+ex)^5}{5(a+bx)^5(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] -(d + e*x)^5/(5*(b*d - a*e)*(a + b*x)^5)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^4}{\left(a^2+2abx+b^2x^2\right)^3}\,dx &= \int \frac{(d+ex)^4}{(a+bx)^6}\,dx\\ &= -\frac{(d+ex)^5}{5(bd-ae)(a+bx)^5}\end{aligned}$$

Mathematica [B] time = 0.048172, size = 140, normalized size = 5.

$$-\frac{a^2b^2e^2\left(d^2+5dex+10e^2x^2\right)+a^3be^3(d+5ex)+a^4e^4+ab^3e\left(5d^2ex+d^3+10de^2x^2+10e^3x^3\right)+b^4\left(10d^2e^2x^2+5d^3ex+\right.}{5b^5(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] -(a^4*e^4 + a^3*b*e^3*(d + 5*e*x) + a^2*b^2*e^2*(d^2 + 5*d*e*x + 10*e^2*x^2
) + a*b^3*e*(d^3 + 5*d^2*e*x + 10*d*e^2*x^2 + 10*e^3*x^3) + b^4*(d^4 + 5*d^
3*e*x + 10*d^2*e^2*x^2 + 10*d*e^3*x^3 + 5*e^4*x^4))/(5*b^5*(a + b*x)^5)
```

Maple [B] time = 0.046, size = 185, normalized size = 6.6

$$2\,\frac{e^3\,(ae-bd)}{b^5\,(bx+a)^2}-\frac{e^4a^4-4\,a^3bde^3+6\,d^2e^2a^2b^2-4\,d^3eab^3+d^4b^4}{5\,b^5\,(bx+a)^5}-2\,\frac{e^2\left(a^2e^2-2\,abde+b^2d^2\right)}{b^5\,(bx+a)^3}-\frac{e^4}{b^5\,(bx+a)}+\frac{e\left(a^3e^3-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] 2*e^3*(a*e-b*d)/b^5/(b*x+a)^2-1/5*(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-
4*a*b^3*d^3*e+b^4*d^4)/b^5/(b*x+a)^5-2*e^2*(a^2*e^2-2*a*b*d*e+b^2*d^2)/b^5/
(b*x+a)^3-e^4/b^5/(b*x+a)+e*(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d^2*e-b^3*d^3)/b
^5/(b*x+a)^4
```

Maple [B] time = 1.18791, size = 290, normalized size = 10.36

$$-\frac{5\,b^4e^4x^4+b^4d^4+ab^3d^3e+a^2b^2d^2e^2+a^3bde^3+a^4e^4+10\left(b^4de^3+ab^3e^4\right)x^3+10\left(b^4d^2e^2+ab^3de^3+a^2b^2e^4\right)x^2+5\,(}{5\left(b^{10}x^5+5\,ab^9x^4+10\,a^2b^8x^3+10\,a^3b^7x^2+5\,a^4b^6x+a^5b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/5*(5*b^4*e^4*x^4 + b^4*d^4 + a*b^3*d^3*e + a^2*b^2*d^2*e^2 + a^3*b*d*e^3
 + a^4*e^4 + 10*(b^4*d*e^3 + a*b^3*e^4)*x^3 + 10*(b^4*d^2*e^2 + a*b^3*d*e^3
 + a^2*b^2*e^4)*x^2 + 5*(b^4*d^3*e + a*b^3*d^2*e^2 + a^2*b^2*d*e^3 + a^3*b*
e^4)*x)/(b^10*x^5 + 5*a*b^9*x^4 + 10*a^2*b^8*x^3 + 10*a^3*b^7*x^2 + 5*a^4*b
^6*x + a^5*b^5)
```

Fricas [B] time = 2.02882, size = 428, normalized size = 15.29

$$-\frac{5\,b^4e^4x^4+b^4d^4+ab^3d^3e+a^2b^2d^2e^2+a^3bde^3+a^4e^4+10\left(b^4de^3+ab^3e^4\right)x^3+10\left(b^4d^2e^2+ab^3de^3+a^2b^2e^4\right)x^2+5\,(}{5\left(b^{10}x^5+5\,ab^9x^4+10\,a^2b^8x^3+10\,a^3b^7x^2+5\,a^4b^6x+a^5b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/5*(5*b^4*e^4*x^4 + b^4*d^4 + a*b^3*d^3*e + a^2*b^2*d^2*e^2 + a^3*b*d*e^3
 + a^4*e^4 + 10*(b^4*d*e^3 + a*b^3*e^4)*x^3 + 10*(b^4*d^2*e^2 + a*b^3*d*e^3
 + a^2*b^2*e^4)*x^2 + 5*(b^4*d^3*e + a*b^3*d^2*e^2 + a^2*b^2*d*e^3 + a^3*b*
e^4)*x)/(b^10*x^5 + 5*a*b^9*x^4 + 10*a^2*b^8*x^3 + 10*a^3*b^7*x^2 + 5*a^4*b
^6*x + a^5*b^5)
```

Sympy [B] time = 5.9946, size = 233, normalized size = 8.32

$$-\frac{a^4e^4 + a^3bde^3 + a^2b^2d^2e^2 + ab^3d^3e + b^4d^4 + 5b^4e^4x^4 + x^3\left(10ab^3e^4 + 10b^4de^3\right) + x^2\left(10a^2b^2e^4 + 10ab^3de^3 + 10b^4d^2e^2\right) + \ldots}{5a^5b^5 + 25a^4b^6x + 50a^3b^7x^2 + 50a^2b^8x^3 + 25ab^9x^4 + 5b^{10}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] -(a**4*e**4 + a**3*b*d*e**3 + a**2*b**2*d**2*e**2 + a*b**3*d**3*e + b**4*d*
*4 + 5*b**4*e**4*x**4 + x**3*(10*a*b**3*e**4 + 10*b**4*d*e**3) + x**2*(10*a
**2*b**2*e**4 + 10*a*b**3*d*e**3 + 10*b**4*d**2*e**2) + x*(5*a**3*b*e**4 +
5*a**2*b**2*d*e**3 + 5*a*b**3*d**2*e**2 + 5*b**4*d**3*e))/(5*a**5*b**5 + 25
*a**4*b**6*x + 50*a**3*b**7*x**2 + 50*a**2*b**8*x**3 + 25*a*b**9*x**4 + 5*b
**10*x**5)
```

Giac [B] time = 1.19766, size = 230, normalized size = 8.21

$$-\frac{5\,b^4x^4e^4 + 10\,b^4dx^3e^3 + 10\,b^4d^2x^2e^2 + 5\,b^4d^3xe + b^4d^4 + 10\,ab^3x^3e^4 + 10\,ab^3dx^2e^3 + 5\,ab^3d^2xe^2 + ab^3d^3e + 10\,a^2b^2x^2e^4 + \ldots}{5\,(bx+a)^5b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] -1/5*(5*b^4*x^4*e^4 + 10*b^4*d*x^3*e^3 + 10*b^4*d^2*x^2*e^2 + 5*b^4*d^3*x*e
 + b^4*d^4 + 10*a*b^3*x^3*e^4 + 10*a*b^3*d*x^2*e^3 + 5*a*b^3*d^2*x*e^2 + a*
b^3*d^3*e + 10*a^2*b^2*x^2*e^4 + 5*a^2*b^2*d*x*e^3 + a^2*b^2*d^2*e^2 + 5*a^
3*b*x*e^4 + a^3*b*d*e^3 + a^4*e^4)/((b*x + a)^5*b^5)
```

3.1530 $\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^3} dx$

Optimal. Leaf size=58

$$\frac{e(d+ex)^4}{20(a+bx)^4(bd-ae)^2}-\frac{(d+ex)^4}{5(a+bx)^5(bd-ae)}$$

```
[Out] -(d + e*x)^4/(5*(b*d - a*e)*(a + b*x)^5) + (e*(d + e*x)^4)/(20*(b*d - a*e)^
2*(a + b*x)^4)
```

Rubi [A] time = 0.0110566, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {27, 45, 37}

$$\frac{e(d+ex)^4}{20(a+bx)^4(bd-ae)^2}-\frac{(d+ex)^4}{5(a+bx)^5(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] -(d + e*x)^4/(5*(b*d - a*e)*(a + b*x)^5) + (e*(d + e*x)^4)/(20*(b*d - a*e)^
2*(a + b*x)^4)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^3}\,dx = \int \frac{(d+ex)^3}{(a+bx)^6}\,dx$$
$$= -\frac{(d+ex)^4}{5(bd-ae)(a+bx)^5} - \frac{e\int \frac{(d+ex)^3}{(a+bx)^5}\,dx}{5(bd-ae)}$$
$$= -\frac{(d+ex)^4}{5(bd-ae)(a+bx)^5} + \frac{e(d+ex)^4}{20(bd-ae)^2(a+bx)^4}$$

Mathematica [A] time = 0.0359294, size = 97, normalized size = 1.67

$$-\frac{a^2be^2(2d+5ex)+a^3e^3+ab^2e\left(3d^2+10dex+10e^2x^2\right)+b^3\left(15d^2ex+4d^3+20de^2x^2+10e^3x^3\right)}{20b^4(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] -(a^3*e^3 + a^2*b*e^2*(2*d + 5*e*x) + a*b^2*e*(3*d^2 + 10*d*e*x + 10*e^2*x^
2) + b^3*(4*d^3 + 15*d^2*e*x + 20*d*e^2*x^2 + 10*e^3*x^3))/(20*b^4*(a + b*x
)^5)
```

Maple [B] time = 0.046, size = 121, normalized size = 2.1

$$-\frac{e^3}{2\,b^4\,(bx+a)^2} - \frac{-a^3e^3+3\,a^2bde^2-3\,ab^2d^2e+b^3d^3}{5\,b^4\,(bx+a)^5} + \frac{e^2\,(ae-bd)}{b^4\,(bx+a)^3} - \frac{3\,e\left(a^2e^2-2\,abde+b^2d^2\right)}{4\,b^4\,(bx+a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] -1/2*e^3/b^4/(b*x+a)^2-1/5*(-a^3*e^3+3*a^2*b*d*e^2-3*a*b^2*d^2*e+b^3*d^3)/b
^4/(b*x+a)^5+e^2*(a*e-b*d)/b^4/(b*x+a)^3-3/4*e*(a^2*e^2-2*a*b*d*e+b^2*d^2)/
b^4/(b*x+a)^4
```

Maxima [B] time = 1.16924, size = 216, normalized size = 3.72

$$-\frac{10\,b^3e^3x^3+4\,b^3d^3+3\,ab^2d^2e+2\,a^2bde^2+a^3e^3+10\left(2\,b^3de^2+ab^2e^3\right)x^2+5\left(3\,b^3d^2e+2\,ab^2de^2+a^2be^3\right)x}{20\left(b^9x^5+5\,ab^8x^4+10\,a^2b^7x^3+10\,a^3b^6x^2+5\,a^4b^5x+a^5b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/20*(10*b^3*e^3*x^3 + 4*b^3*d^3 + 3*a*b^2*d^2*e + 2*a^2*b*d*e^2 + a^3*e^3
 + 10*(2*b^3*d*e^2 + a*b^2*e^3)*x^2 + 5*(3*b^3*d^2*e + 2*a*b^2*d*e^2 + a^2*
b*e^3)*x)/(b^9*x^5 + 5*a*b^8*x^4 + 10*a^2*b^7*x^3 + 10*a^3*b^6*x^2 + 5*a^4*
b^5*x + a^5*b^4)
```

Fricas [B] time = 1.80828, size = 328, normalized size = 5.66

$$-\frac{10\,b^3e^3x^3+4\,b^3d^3+3\,ab^2d^2e+2\,a^2bde^2+a^3e^3+10\left(2\,b^3de^2+ab^2e^3\right)x^2+5\left(3\,b^3d^2e+2\,ab^2de^2+a^2be^3\right)x}{20\left(b^9x^5+5\,ab^8x^4+10\,a^2b^7x^3+10\,a^3b^6x^2+5\,a^4b^5x+a^5b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/20*(10*b^3*e^3*x^3 + 4*b^3*d^3 + 3*a*b^2*d^2*e + 2*a^2*b*d*e^2 + a^3*e^3
 + 10*(2*b^3*d*e^2 + a*b^2*e^3)*x^2 + 5*(3*b^3*d^2*e + 2*a*b^2*d*e^2 + a^2*
b*e^3)*x)/(b^9*x^5 + 5*a*b^8*x^4 + 10*a^2*b^7*x^3 + 10*a^3*b^6*x^2 + 5*a^4*
b^5*x + a^5*b^4)
```

Sympy [B] time = 2.68908, size = 170, normalized size = 2.93

$$-\frac{a^3e^3+2a^2bde^2+3ab^2d^2e+4b^3d^3+10b^3e^3x^3+x^2\left(10ab^2e^3+20b^3de^2\right)+x\left(5a^2be^3+10ab^2de^2+15b^3d^2e\right)}{20a^5b^4+100a^4b^5x+200a^3b^6x^2+200a^2b^7x^3+100ab^8x^4+20b^9x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] -(a**3*e**3 + 2*a**2*b*d*e**2 + 3*a*b**2*d**2*e + 4*b**3*d**3 + 10*b**3*e**
3*x**3 + x**2*(10*a*b**2*e**3 + 20*b**3*d*e**2) + x*(5*a**2*b*e**3 + 10*a*b
**2*d*e**2 + 15*b**3*d**2*e))/(20*a**5*b**4 + 100*a**4*b**5*x + 200*a**3*b*
*6*x**2 + 200*a**2*b**7*x**3 + 100*a*b**8*x**4 + 20*b**9*x**5)
```

Giac [B] time = 1.1305, size = 147, normalized size = 2.53

$$-\frac{10\,b^3x^3e^3+20\,b^3dx^2e^2+15\,b^3d^2xe+4\,b^3d^3+10\,ab^2x^2e^3+10\,ab^2dxe^2+3\,ab^2d^2e+5\,a^2bxe^3+2\,a^2bde^2+a^3e^3}{20\,(bx+a)^5b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] -1/20*(10*b^3*x^3*e^3 + 20*b^3*d*x^2*e^2 + 15*b^3*d^2*x*e + 4*b^3*d^3 + 10*
a*b^2*x^2*e^3 + 10*a*b^2*d*x*e^2 + 3*a*b^2*d^2*e + 5*a^2*b*x*e^3 + 2*a^2*b*
d*e^2 + a^3*e^3)/((b*x + a)^5*b^4)
```

3.1531 $\int \frac{(d+ex)^2}{\left(a^2+2abx+b^2x^2\right)^3}\,dx$

Optimal. Leaf size=65

$$-\frac{e(bd-ae)}{2b^3(a+bx)^4}-\frac{(bd-ae)^2}{5b^3(a+bx)^5}-\frac{e^2}{3b^3(a+bx)^3}$$

```
[Out] -(b*d - a*e)^2/(5*b^3*(a + b*x)^5) - (e*(b*d - a*e))/(2*b^3*(a + b*x)^4) -
e^2/(3*b^3*(a + b*x)^3)
```

Rubi [A] time = 0.0401324, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$-\frac{e(bd-ae)}{2b^3(a+bx)^4}-\frac{(bd-ae)^2}{5b^3(a+bx)^5}-\frac{e^2}{3b^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] -(b*d - a*e)^2/(5*b^3*(a + b*x)^5) - (e*(b*d - a*e))/(2*b^3*(a + b*x)^4) -
e^2/(3*b^3*(a + b*x)^3)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2}{\left(a^2+2abx+b^2x^2\right)^3}\,dx &= \int \frac{(d+ex)^2}{(a+bx)^6}\,dx\\
&= \int \left(\frac{(bd-ae)^2}{b^2(a+bx)^6}+\frac{2e(bd-ae)}{b^2(a+bx)^5}+\frac{e^2}{b^2(a+bx)^4}\right)dx\\
&= -\frac{(bd-ae)^2}{5b^3(a+bx)^5}-\frac{e(bd-ae)}{2b^3(a+bx)^4}-\frac{e^2}{3b^3(a+bx)^3}
\end{aligned}$$

Mathematica [A] time = 0.0239548, size = 57, normalized size = 0.88

$$-\frac{a^2e^2+abe(3d+5ex)+b^2\left(6d^2+15dex+10e^2x^2\right)}{30b^3(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] -(a^2*e^2 + a*b*e*(3*d + 5*e*x) + b^2*(6*d^2 + 15*d*e*x + 10*e^2*x^2))/(30*
b^3*(a + b*x)^5)
```

Maple [A] time = 0.045, size = 71, normalized size = 1.1

$$-\frac{a^2e^2 - 2\,abde + b^2d^2}{5\,b^3\,(bx+a)^5} - \frac{e^2}{3\,b^3\,(bx+a)^3} + \frac{e\,(ae-bd)}{2\,b^3\,(bx+a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] -1/5*(a^2*e^2-2*a*b*d*e+b^2*d^2)/b^3/(b*x+a)^5-1/3*e^2/b^3/(b*x+a)^3+1/2*e*
(a*e-b*d)/b^3/(b*x+a)^4
```

Maxima [A] time = 1.08138, size = 147, normalized size = 2.26

$$-\frac{10\,b^2e^2x^2 + 6\,b^2d^2 + 3\,abde + a^2e^2 + 5\left(3\,b^2de + abe^2\right)x}{30\left(b^8x^5 + 5\,ab^7x^4 + 10\,a^2b^6x^3 + 10\,a^3b^5x^2 + 5\,a^4b^4x + a^5b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/30*(10*b^2*e^2*x^2 + 6*b^2*d^2 + 3*a*b*d*e + a^2*e^2 + 5*(3*b^2*d*e + a*
b*e^2)*x)/(b^8*x^5 + 5*a*b^7*x^4 + 10*a^2*b^6*x^3 + 10*a^3*b^5*x^2 + 5*a^4*
b^4*x + a^5*b^3)
```

Fricas [A] time = 1.5739, size = 227, normalized size = 3.49

$$-\frac{10\,b^2e^2x^2 + 6\,b^2d^2 + 3\,abde + a^2e^2 + 5\left(3\,b^2de + abe^2\right)x}{30\left(b^8x^5 + 5\,ab^7x^4 + 10\,a^2b^6x^3 + 10\,a^3b^5x^2 + 5\,a^4b^4x + a^5b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/30*(10*b^2*e^2*x^2 + 6*b^2*d^2 + 3*a*b*d*e + a^2*e^2 + 5*(3*b^2*d*e + a*
b*e^2)*x)/(b^8*x^5 + 5*a*b^7*x^4 + 10*a^2*b^6*x^3 + 10*a^3*b^5*x^2 + 5*a^4*
b^4*x + a^5*b^3)
```

Sympy [B] time = 1.30671, size = 116, normalized size = 1.78

$$-\frac{a^2e^2 + 3abde + 6b^2d^2 + 10b^2e^2x^2 + x\left(5abe^2 + 15b^2de\right)}{30a^5b^3 + 150a^4b^4x + 300a^3b^5x^2 + 300a^2b^6x^3 + 150ab^7x^4 + 30b^8x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] -(a**2*e**2 + 3*a*b*d*e + 6*b**2*d**2 + 10*b**2*e**2*x**2 + x*(5*a*b*e**2 +
 15*b**2*d*e))/(30*a**5*b**3 + 150*a**4*b**4*x + 300*a**3*b**5*x**2 + 300*a
**2*b**6*x**3 + 150*a*b**7*x**4 + 30*b**8*x**5)
```

Giac [A] time = 1.18109, size = 81, normalized size = 1.25

$$-\frac{10\,b^2x^2e^2 + 15\,b^2dxe + 6\,b^2d^2 + 5\,abxe^2 + 3\,abde + a^2e^2}{30\,(bx+a)^5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] -1/30*(10*b^2*x^2*e^2 + 15*b^2*d*x*e + 6*b^2*d^2 + 5*a*b*x*e^2 + 3*a*b*d*e
+ a^2*e^2)/((b*x + a)^5*b^3)
```

3.1532 $\int \frac{d+ex}{\left(a^2+2abx+b^2x^2\right)^3}\,dx$

Optimal. Leaf size=38

$$-\frac{bd-ae}{5b^2(a+bx)^5}-\frac{e}{4b^2(a+bx)^4}$$

```
[Out] -(b*d - a*e)/(5*b^2*(a + b*x)^5) - e/(4*b^2*(a + b*x)^4)
```

Rubi [A] time = 0.024086, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {27, 43}

$$-\frac{bd-ae}{5b^2(a+bx)^5}-\frac{e}{4b^2(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] -(b*d - a*e)/(5*b^2*(a + b*x)^5) - e/(4*b^2*(a + b*x)^4)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex}{\left(a^2+2abx+b^2x^2\right)^3}\,dx &= \int \frac{d+ex}{(a+bx)^6}\,dx\\ &= \int \left(\frac{bd-ae}{b(a+bx)^6}+\frac{e}{b(a+bx)^5}\right)dx\\ &= -\frac{bd-ae}{5b^2(a+bx)^5}-\frac{e}{4b^2(a+bx)^4}\end{aligned}$$

Mathematica [A] time = 0.0094128, size = 27, normalized size = 0.71

$$-\frac{ae+4bd+5bex}{20b^2(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] -(4*b*d + a*e + 5*b*e*x)/(20*b^2*(a + b*x)^5)
```

Maple [A] time = 0.045, size = 35, normalized size = 0.9

$$-\frac{-ae+bd}{5\,b^{2}\left(bx+a\right)^{5}}-\frac{e}{4\,b^{2}\left(bx+a\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] -1/5*(-a*e+b*d)/b^2/(b*x+a)^5-1/4*e/b^2/(b*x+a)^4
```

Maxima [B] time = 1.14417, size = 97, normalized size = 2.55

$$-\frac{5\,bex+4\,bd+ae}{20\,\left(b^{7}x^{5}+5\,ab^{6}x^{4}+10\,a^{2}b^{5}x^{3}+10\,a^{3}b^{4}x^{2}+5\,a^{4}b^{3}x+a^{5}b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/20*(5*b*e*x + 4*b*d + a*e)/(b^7*x^5 + 5*a*b^6*x^4 + 10*a^2*b^5*x^3 + 10*
a^3*b^4*x^2 + 5*a^4*b^3*x + a^5*b^2)
```

Fricas [B] time = 1.50123, size = 153, normalized size = 4.03

$$-\frac{5\,bex+4\,bd+ae}{20\,\left(b^{7}x^{5}+5\,ab^{6}x^{4}+10\,a^{2}b^{5}x^{3}+10\,a^{3}b^{4}x^{2}+5\,a^{4}b^{3}x+a^{5}b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/20*(5*b*e*x + 4*b*d + a*e)/(b^7*x^5 + 5*a*b^6*x^4 + 10*a^2*b^5*x^3 + 10*
a^3*b^4*x^2 + 5*a^4*b^3*x + a^5*b^2)
```

Sympy [B] time = 0.772541, size = 76, normalized size = 2.

$$-\frac{ae+4bd+5bex}{20a^{5}b^{2}+100a^{4}b^{3}x+200a^{3}b^{4}x^{2}+200a^{2}b^{5}x^{3}+100ab^{6}x^{4}+20b^{7}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] -(a*e + 4*b*d + 5*b*e*x)/(20*a**5*b**2 + 100*a**4*b**3*x + 200*a**3*b**4*x*
*2 + 200*a**2*b**5*x**3 + 100*a*b**6*x**4 + 20*b**7*x**5)
```

Giac [A] time = 1.13497, size = 36, normalized size = 0.95

$$-\frac{5\,bxe + 4\,bd + ae}{20\,(bx + a)^5 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] -1/20*(5*b*x*e + 4*b*d + a*e)/((b*x + a)^5*b^2)
```

3.1533 $\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^3}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{5b(a+bx)^5}$$

[Out] -1/(5*b*(a + b*x)^5)

Rubi [A] time = 0.0022241, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {27, 32}

$$-\frac{1}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(-3),x]
```

```
[Out] -1/(5*b*(a + b*x)^5)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^3}\,dx &= \int \frac{1}{(a+bx)^6}\,dx\\ &= -\frac{1}{5b(a+bx)^5}\end{aligned}$$

Mathematica [A] time = 0.0032114, size = 14, normalized size = 1.

$$-\frac{1}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(-3),x]
```

```
[Out] -1/(5*b*(a + b*x)^5)
```

Maple [A] time = 0.041, size = 13, normalized size = 0.9

$$-\frac{1}{5\,b\,(bx+a)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] -1/5/b/(b*x+a)^5
```

Maxima [B] time = 1.11459, size = 77, normalized size = 5.5

$$-\frac{1}{5\left(b^{6}x^{5}+5\,ab^{5}x^{4}+10\,a^{2}b^{4}x^{3}+10\,a^{3}b^{3}x^{2}+5\,a^{4}b^{2}x+a^{5}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/5/(b^6*x^5 + 5*a*b^5*x^4 + 10*a^2*b^4*x^3 + 10*a^3*b^3*x^2 + 5*a^4*b^2*x
 + a^5*b)
```

Fricas [B] time = 1.49589, size = 116, normalized size = 8.29

$$-\frac{1}{5\left(b^{6}x^{5}+5\,ab^{5}x^{4}+10\,a^{2}b^{4}x^{3}+10\,a^{3}b^{3}x^{2}+5\,a^{4}b^{2}x+a^{5}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/5/(b^6*x^5 + 5*a*b^5*x^4 + 10*a^2*b^4*x^3 + 10*a^3*b^3*x^2 + 5*a^4*b^2*x
 + a^5*b)
```

Sympy [B] time = 0.558109, size = 61, normalized size = 4.36

$$-\frac{1}{5a^{5}b+25a^{4}b^{2}x+50a^{3}b^{3}x^{2}+50a^{2}b^{4}x^{3}+25ab^{5}x^{4}+5b^{6}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] -1/(5*a**5*b + 25*a**4*b**2*x + 50*a**3*b**3*x**2 + 50*a**2*b**4*x**3 + 25*
a*b**5*x**4 + 5*b**6*x**5)
```

Giac [A] time = 1.12057, size = 16, normalized size = 1.14

$$-\frac{1}{5\,(bx+a)^{5}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] -1/5/((b*x + a)^5*b)
```

3.1534 $\int \frac{1}{(d+ex)\left(a^2+2abx+b^2x^2\right)^3}\,dx$

Optimal. Leaf size=155

$$-\frac{e^4}{(a+bx)(bd-ae)^5}+\frac{e^3}{2(a+bx)^2(bd-ae)^4}-\frac{e^2}{3(a+bx)^3(bd-ae)^3}-\frac{e^5\log(a+bx)}{(bd-ae)^6}+\frac{e^5\log(d+ex)}{(bd-ae)^6}+\frac{e}{4(a+bx)^4(bd}$$

```
[Out] -1/(5*(b*d - a*e)*(a + b*x)^5) + e/(4*(b*d - a*e)^2*(a + b*x)^4) - e^2/(3*(
b*d - a*e)^3*(a + b*x)^3) + e^3/(2*(b*d - a*e)^4*(a + b*x)^2) - e^4/((b*d -
 a*e)^5*(a + b*x)) - (e^5*Log[a + b*x])/(b*d - a*e)^6 + (e^5*Log[d + e*x])/
(b*d - a*e)^6
```

Rubi [A] time = 0.113813, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 44}

$$-\frac{e^4}{(a+bx)(bd-ae)^5}+\frac{e^3}{2(a+bx)^2(bd-ae)^4}-\frac{e^2}{3(a+bx)^3(bd-ae)^3}-\frac{e^5\log(a+bx)}{(bd-ae)^6}+\frac{e^5\log(d+ex)}{(bd-ae)^6}+\frac{e}{4(a+bx)^4(bd}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```


```
[Out] -1/(5*(b*d - a*e)*(a + b*x)^5) + e/(4*(b*d - a*e)^2*(a + b*x)^4) - e^2/(3*(
b*d - a*e)^3*(a + b*x)^3) + e^3/(2*(b*d - a*e)^4*(a + b*x)^2) - e^4/((b*d -
 a*e)^5*(a + b*x)) - (e^5*Log[a + b*x])/(b*d - a*e)^6 + (e^5*Log[d + e*x])/
(b*d - a*e)^6
```


Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a^2+2abx+b^2x^2\right)^3}\,dx=\int \frac{1}{(a+bx)^6(d+ex)}\,dx$$

$$=\int \left(\frac{b}{(bd-ae)(a+bx)^6}-\frac{be}{(bd-ae)^2(a+bx)^5}+\frac{be^2}{(bd-ae)^3(a+bx)^4}-\frac{be^3}{(bd-ae)^4(}\right.$$

$$=-\frac{1}{5(bd-ae)(a+bx)^5}+\frac{e}{4(bd-ae)^2(a+bx)^4}-\frac{e^2}{3(bd-ae)^3(a+bx)^3}+\frac{}{2(bd-ae}$$

Mathematica [A] time = 0.076062, size = 152, normalized size = 0.98

$$\frac{30e^3(a+bx)^3(bd-ae)^2+60e^4(a+bx)^4(ae-bd)+20e^2(a+bx)^2(ae-bd)^3+60e^5(a+bx)^5\log(d+ex)+15e(a+bx)(}{60(a+bx)^5(bd-ae)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

```
[Out] (-12*(b*d - a*e)^5 + 15*e*(b*d - a*e)^4*(a + b*x) + 20*e^2*(-(b*d) + a*e)^3
*(a + b*x)^2 + 30*e^3*(b*d - a*e)^2*(a + b*x)^3 + 60*e^4*(-(b*d) + a*e)*(a
+ b*x)^4 - 60*e^5*(a + b*x)^5*Log[a + b*x] + 60*e^5*(a + b*x)^5*Log[d + e*x
])/(60*(b*d - a*e)^6*(a + b*x)^5)
```

Maple [A] time = 0.055, size = 147, normalized size = 1.

$$\frac{e^5\ln(ex+d)}{(ae-bd)^6}+\frac{1}{(5ae-5bd)(bx+a)^5}+\frac{e}{4(ae-bd)^2(bx+a)^4}+\frac{e^2}{3(ae-bd)^3(bx+a)^3}+\frac{e^3}{2(ae-bd)^4(bx+a)^2}+\frac{}{(ae-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] e^5/(a*e-b*d)^6*ln(e*x+d)+1/5/(a*e-b*d)/(b*x+a)^5+1/4*e/(a*e-b*d)^2/(b*x+a)
^4+1/3*e^2/(a*e-b*d)^3/(b*x+a)^3+1/2*e^3/(a*e-b*d)^4/(b*x+a)^2+e^4/(a*e-b*d
)^5/(b*x+a)-e^5/(a*e-b*d)^6*ln(b*x+a)
```

Maxima [B] time = 1.31708, size = 1087, normalized size = 7.01

$$-\frac{e^5\log(bx+a)}{b^6d^6-6ab^5d^5e+15a^2b^4d^4e^2-20a^3b^3d^3e^3+15a^4b^2d^2e^4-6a^5bde^5+a^6e^6}+\frac{e^5\log(ex}{b^6d^6-6ab^5d^5e+15a^2b^4d^4e^2-20a^3b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] -e^5*log(b*x + a)/(b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^
3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 + a^6*e^6) + e^5*log(e*x + d
)/(b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 + 15*a
^4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 + a^6*e^6) - 1/60*(60*b^4*e^4*x^4 + 12*b^4*d
^4 - 63*a*b^3*d^3*e + 137*a^2*b^2*d^2*e^2 - 163*a^3*b*d*e^3 + 137*a^4*e^4 -
 30*(b^4*d*e^3 - 9*a*b^3*e^4)*x^3 + 10*(2*b^4*d^2*e^2 - 13*a*b^3*d*e^3 + 47
*a^2*b^2*e^4)*x^2 - 5*(3*b^4*d^3*e - 17*a*b^3*d^2*e^2 + 43*a^2*b^2*d*e^3 -
77*a^3*b*e^4)*x)/(a^5*b^5*d^5 - 5*a^6*b^4*d^4*e + 10*a^7*b^3*d^3*e^2 - 10*a
^8*b^2*d^2*e^3 + 5*a^9*b*d*e^4 - a^10*e^5 + (b^10*d^5 - 5*a*b^9*d^4*e + 10*
a^2*b^8*d^3*e^2 - 10*a^3*b^7*d^2*e^3 + 5*a^4*b^6*d*e^4 - a^5*b^5*e^5)*x^5 +
 5*(a*b^9*d^5 - 5*a^2*b^8*d^4*e + 10*a^3*b^7*d^3*e^2 - 10*a^4*b^6*d^2*e^3 +
 5*a^5*b^5*d*e^4 - a^6*b^4*e^5)*x^4 + 10*(a^2*b^8*d^5 - 5*a^3*b^7*d^4*e + 1
0*a^4*b^6*d^3*e^2 - 10*a^5*b^5*d^2*e^3 + 5*a^6*b^4*d*e^4 - a^7*b^3*e^5)*x^3
 + 10*(a^3*b^7*d^5 - 5*a^4*b^6*d^4*e + 10*a^5*b^5*d^3*e^2 - 10*a^6*b^4*d^2*
e^3 + 5*a^7*b^3*d*e^4 - a^8*b^2*e^5)*x^2 + 5*(a^4*b^6*d^5 - 5*a^5*b^5*d^4*e
 + 10*a^6*b^4*d^3*e^2 - 10*a^7*b^3*d^2*e^3 + 5*a^8*b^2*d*e^4 - a^9*b*e^5)*x
)
```

Fricas [B] time = 1.70883, size = 1894, normalized size = 12.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/60*(12*b^5*d^5 - 75*a*b^4*d^4*e + 200*a^2*b^3*d^3*e^2 - 300*a^3*b^2*d^2*
e^3 + 300*a^4*b*d*e^4 - 137*a^5*e^5 + 60*(b^5*d*e^4 - a*b^4*e^5)*x^4 - 30*(
b^5*d^2*e^3 - 10*a*b^4*d*e^4 + 9*a^2*b^3*e^5)*x^3 + 10*(2*b^5*d^3*e^2 - 15*
a*b^4*d^2*e^3 + 60*a^2*b^3*d*e^4 - 47*a^3*b^2*e^5)*x^2 - 5*(3*b^5*d^4*e - 2
0*a*b^4*d^3*e^2 + 60*a^2*b^3*d^2*e^3 - 120*a^3*b^2*d*e^4 + 77*a^4*b*e^5)*x
+ 60*(b^5*e^5*x^5 + 5*a*b^4*e^5*x^4 + 10*a^2*b^3*e^5*x^3 + 10*a^3*b^2*e^5*x
^2 + 5*a^4*b*e^5*x + a^5*e^5)*log(b*x + a) - 60*(b^5*e^5*x^5 + 5*a*b^4*e^5*
x^4 + 10*a^2*b^3*e^5*x^3 + 10*a^3*b^2*e^5*x^2 + 5*a^4*b*e^5*x + a^5*e^5)*lo
g(e*x + d))/(a^5*b^6*d^6 - 6*a^6*b^5*d^5*e + 15*a^7*b^4*d^4*e^2 - 20*a^8*b^
3*d^3*e^3 + 15*a^9*b^2*d^2*e^4 - 6*a^10*b*d*e^5 + a^11*e^6 + (b^11*d^6 - 6*
a*b^10*d^5*e + 15*a^2*b^9*d^4*e^2 - 20*a^3*b^8*d^3*e^3 + 15*a^4*b^7*d^2*e^4
 - 6*a^5*b^6*d*e^5 + a^6*b^5*e^6)*x^5 + 5*(a*b^10*d^6 - 6*a^2*b^9*d^5*e + 1
5*a^3*b^8*d^4*e^2 - 20*a^4*b^7*d^3*e^3 + 15*a^5*b^6*d^2*e^4 - 6*a^6*b^5*d*e
^5 + a^7*b^4*e^6)*x^4 + 10*(a^2*b^9*d^6 - 6*a^3*b^8*d^5*e + 15*a^4*b^7*d^4*
e^2 - 20*a^5*b^6*d^3*e^3 + 15*a^6*b^5*d^2*e^4 - 6*a^7*b^4*d*e^5 + a^8*b^3*e
^6)*x^3 + 10*(a^3*b^8*d^6 - 6*a^4*b^7*d^5*e + 15*a^5*b^6*d^4*e^2 - 20*a^6*b
^5*d^3*e^3 + 15*a^7*b^4*d^2*e^4 - 6*a^8*b^3*d*e^5 + a^9*b^2*e^6)*x^2 + 5*(a
^4*b^7*d^6 - 6*a^5*b^6*d^5*e + 15*a^6*b^5*d^4*e^2 - 20*a^7*b^4*d^3*e^3 + 15
*a^8*b^3*d^2*e^4 - 6*a^9*b^2*d*e^5 + a^10*b*e^6)*x)
```

Sympy [B] time = 4.85619, size = 1081, normalized size = 6.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] e**5*log(x + (-a**7*e**12/(a*e - b*d)**6 + 7*a**6*b*d*e**11/(a*e - b*d)**6
- 21*a**5*b**2*d**2*e**10/(a*e - b*d)**6 + 35*a**4*b**3*d**3*e**9/(a*e - b*
d)**6 - 35*a**3*b**4*d**4*e**8/(a*e - b*d)**6 + 21*a**2*b**5*d**5*e**7/(a*e
 - b*d)**6 - 7*a*b**6*d**6*e**6/(a*e - b*d)**6 + a*e**6 + b**7*d**7*e**5/(a
*e - b*d)**6 + b*d*e**5)/(2*b*e**6))/(a*e - b*d)**6 - e**5*log(x + (a**7*e*
*12/(a*e - b*d)**6 - 7*a**6*b*d*e**11/(a*e - b*d)**6 + 21*a**5*b**2*d**2*e*
*10/(a*e - b*d)**6 - 35*a**4*b**3*d**3*e**9/(a*e - b*d)**6 + 35*a**3*b**4*d
**4*e**8/(a*e - b*d)**6 - 21*a**2*b**5*d**5*e**7/(a*e - b*d)**6 + 7*a*b**6*
d**6*e**6/(a*e - b*d)**6 + a*e**6 - b**7*d**7*e**5/(a*e - b*d)**6 + b*d*e**
5)/(2*b*e**6))/(a*e - b*d)**6 + (137*a**4*e**4 - 163*a**3*b*d*e**3 + 137*a*
*2*b**2*d**2*e**2 - 63*a*b**3*d**3*e + 12*b**4*d**4 + 60*b**4*e**4*x**4 + x
**3*(270*a*b**3*e**4 - 30*b**4*d*e**3) + x**2*(470*a**2*b**2*e**4 - 130*a*b
**3*d*e**3 + 20*b**4*d**2*e**2) + x*(385*a**3*b*e**4 - 215*a**2*b**2*d*e**3
 + 85*a*b**3*d**2*e**2 - 15*b**4*d**3*e))/(60*a**10*e**5 - 300*a**9*b*d*e**
4 + 600*a**8*b**2*d**2*e**3 - 600*a**7*b**3*d**3*e**2 + 300*a**6*b**4*d**4*
e - 60*a**5*b**5*d**5 + x**5*(60*a**5*b**5*e**5 - 300*a**4*b**6*d*e**4 + 60
0*a**3*b**7*d**2*e**3 - 600*a**2*b**8*d**3*e**2 + 300*a*b**9*d**4*e - 60*b*
*10*d**5) + x**4*(300*a**6*b**4*e**5 - 1500*a**5*b**5*d*e**4 + 3000*a**4*b*
*6*d**2*e**3 - 3000*a**3*b**7*d**3*e**2 + 1500*a**2*b**8*d**4*e - 300*a*b**
9*d**5) + x**3*(600*a**7*b**3*e**5 - 3000*a**6*b**4*d*e**4 + 6000*a**5*b**5
*d**2*e**3 - 6000*a**4*b**6*d**3*e**2 + 3000*a**3*b**7*d**4*e - 600*a**2*b*
*8*d**5) + x**2*(600*a**8*b**2*e**5 - 3000*a**7*b**3*d*e**4 + 6000*a**6*b**
4*d**2*e**3 - 6000*a**5*b**5*d**3*e**2 + 3000*a**4*b**6*d**4*e - 600*a**3*b
**7*d**5) + x*(300*a**9*b*e**5 - 1500*a**8*b**2*d*e**4 + 3000*a**7*b**3*d**
2*e**3 - 3000*a**6*b**4*d**3*e**2 + 1500*a**5*b**5*d**4*e - 300*a**4*b**6*d
```

```
**5))
```

Giac [B] time = 1.12238, size = 568, normalized size = 3.66

$$-\frac{be^5\log\left(|bx+a|\right)}{b^7d^6-6\,ab^6d^5e+15\,a^2b^5d^4e^2-20\,a^3b^4d^3e^3+15\,a^4b^3d^2e^4-6\,a^5b^2de^5+a^6be^6}+\frac{e^6\log\left(\right.}{b^6d^6e-6\,ab^5d^5e^2+15\,a^2b^4d^4e^3-20\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] -b*e^5*log(abs(b*x + a))/(b^7*d^6 - 6*a*b^6*d^5*e + 15*a^2*b^5*d^4*e^2 - 20
*a^3*b^4*d^3*e^3 + 15*a^4*b^3*d^2*e^4 - 6*a^5*b^2*d*e^5 + a^6*b*e^6) + e^6*
log(abs(x*e + d))/(b^6*d^6*e - 6*a*b^5*d^5*e^2 + 15*a^2*b^4*d^4*e^3 - 20*a^
3*b^3*d^3*e^4 + 15*a^4*b^2*d^2*e^5 - 6*a^5*b*d*e^6 + a^6*e^7) - 1/60*(12*b^
5*d^5 - 75*a*b^4*d^4*e + 200*a^2*b^3*d^3*e^2 - 300*a^3*b^2*d^2*e^3 + 300*a^
4*b*d*e^4 - 137*a^5*e^5 + 60*(b^5*d*e^4 - a*b^4*e^5)*x^4 - 30*(b^5*d^2*e^3
- 10*a*b^4*d*e^4 + 9*a^2*b^3*e^5)*x^3 + 10*(2*b^5*d^3*e^2 - 15*a*b^4*d^2*e^
3 + 60*a^2*b^3*d*e^4 - 47*a^3*b^2*e^5)*x^2 - 5*(3*b^5*d^4*e - 20*a*b^4*d^3*
e^2 + 60*a^2*b^3*d^2*e^3 - 120*a^3*b^2*d*e^4 + 77*a^4*b*e^5)*x)/((b*d - a*e
)^6*(b*x + a)^5)
```

3.1535 $\int \frac{1}{(d+ex)^2(a^2+2abx+b^2x^2)^3}\,dx$

Optimal. Leaf size=181

$$-\frac{e^5}{(d+ex)(bd-ae)^6}-\frac{5be^4}{(a+bx)(bd-ae)^6}+\frac{2be^3}{(a+bx)^2(bd-ae)^5}-\frac{be^2}{(a+bx)^3(bd-ae)^4}-\frac{6be^5\log(a+bx)}{(bd-ae)^7}+\frac{6be^5\log(d}{(bd-a}$$

```
[Out] -b/(5*(b*d - a*e)^2*(a + b*x)^5) + (b*e)/(2*(b*d - a*e)^3*(a + b*x)^4) - (b
*e^2)/((b*d - a*e)^4*(a + b*x)^3) + (2*b*e^3)/((b*d - a*e)^5*(a + b*x)^2) -
 (5*b*e^4)/((b*d - a*e)^6*(a + b*x)) - e^5/((b*d - a*e)^6*(d + e*x)) - (6*b
*e^5*Log[a + b*x])/(b*d - a*e)^7 + (6*b*e^5*Log[d + e*x])/(b*d - a*e)^7
```

Rubi [A] time = 0.183939, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 44}

$$-\frac{e^5}{(d+ex)(bd-ae)^6}-\frac{5be^4}{(a+bx)(bd-ae)^6}+\frac{2be^3}{(a+bx)^2(bd-ae)^5}-\frac{be^2}{(a+bx)^3(bd-ae)^4}-\frac{6be^5\log(a+bx)}{(bd-ae)^7}+\frac{6be^5\log(d}{(bd-a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

```
[Out] -b/(5*(b*d - a*e)^2*(a + b*x)^5) + (b*e)/(2*(b*d - a*e)^3*(a + b*x)^4) - (b
*e^2)/((b*d - a*e)^4*(a + b*x)^3) + (2*b*e^3)/((b*d - a*e)^5*(a + b*x)^2) -
 (5*b*e^4)/((b*d - a*e)^6*(a + b*x)) - e^5/((b*d - a*e)^6*(d + e*x)) - (6*b
*e^5*Log[a + b*x])/(b*d - a*e)^7 + (6*b*e^5*Log[d + e*x])/(b*d - a*e)^7
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)^2\left(a^2+2abx+b^2x^2\right)^3}\,dx &= \int \frac{1}{(a+bx)^6(d+ex)^2}\,dx\\ &= \int \left(\frac{b^2}{(bd-ae)^2(a+bx)^6}-\frac{2b^2e}{(bd-ae)^3(a+bx)^5}+\frac{3b^2e^2}{(bd-ae)^4(a+bx)^4}-\frac{4l}{(bd-ae)}\right.\\ &= -\frac{b}{5(bd-ae)^2(a+bx)^5}+\frac{be}{2(bd-ae)^3(a+bx)^4}-\frac{be^2}{(bd-ae)^4(a+bx)^3}+\frac{2}{(bd-ae)}\end{aligned}$$

Mathematica [A] time = 0.132863, size = 167, normalized size = 0.92

$$\frac{\frac{10e^5(ae-bd)}{d+ex}-\frac{50be^4(bd-ae)}{a+bx}+\frac{20be^3(bd-ae)^2}{(a+bx)^2}-\frac{10be^2(bd-ae)^3}{(a+bx)^3}+\frac{5be(bd-ae)^4}{(a+bx)^4}-\frac{2b(bd-ae)^5}{(a+bx)^5}-60be^5\log(a+bx)+60be^5\log(d+ex)}{10(bd-ae)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

```
[Out] ((-2*b*(b*d - a*e)^5)/(a + b*x)^5 + (5*b*e*(b*d - a*e)^4)/(a + b*x)^4 - (10
*b*e^2*(b*d - a*e)^3)/(a + b*x)^3 + (20*b*e^3*(b*d - a*e)^2)/(a + b*x)^2 -
(50*b*e^4*(b*d - a*e))/(a + b*x) + (10*e^5*(-(b*d) + a*e))/(d + e*x) - 60*b
*e^5*Log[a + b*x] + 60*b*e^5*Log[d + e*x])/(10*(b*d - a*e)^7)
```

Maple [A] time = 0.059, size = 178, normalized size = 1.

$$-\frac{e^5}{(ae-bd)^6(ex+d)}-6\frac{e^5b\ln(ex+d)}{(ae-bd)^7}-\frac{b}{5(ae-bd)^2(bx+a)^5}+6\frac{e^5b\ln(bx+a)}{(ae-bd)^7}-5\frac{be^4}{(ae-bd)^6(bx+a)}-2\frac{be^{3}}{(ae-bd)^5 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] -e^5/(a*e-b*d)^6/(e*x+d)-6*e^5/(a*e-b*d)^7*b*ln(e*x+d)-1/5*b/(a*e-b*d)^2/(b
*x+a)^5+6*e^5/(a*e-b*d)^7*b*ln(b*x+a)-5*b/(a*e-b*d)^6*e^4/(b*x+a)-2*b/(a*e-
b*d)^5*e^3/(b*x+a)^2-b/(a*e-b*d)^4*e^2/(b*x+a)^3-1/2*b/(a*e-b*d)^3*e/(b*x+a
)^4
```

Maxima [B] time = 1.39119, size = 1558, normalized size = 8.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] -6*b*e^5*log(b*x + a)/(b^7*d^7 - 7*a*b^6*d^6*e + 21*a^2*b^5*d^5*e^2 - 35*a^
3*b^4*d^4*e^3 + 35*a^4*b^3*d^3*e^4 - 21*a^5*b^2*d^2*e^5 + 7*a^6*b*d*e^6 - a
^7*e^7) + 6*b*e^5*log(e*x + d)/(b^7*d^7 - 7*a*b^6*d^6*e + 21*a^2*b^5*d^5*e^
2 - 35*a^3*b^4*d^4*e^3 + 35*a^4*b^3*d^3*e^4 - 21*a^5*b^2*d^2*e^5 + 7*a^6*b*
d*e^6 - a^7*e^7) - 1/10*(60*b^5*e^5*x^5 + 2*b^5*d^5 - 13*a*b^4*d^4*e + 37*a
^2*b^3*d^3*e^2 - 63*a^3*b^2*d^2*e^3 + 87*a^4*b*d*e^4 + 10*a^5*e^5 + 30*(b^5
*d*e^4 + 9*a*b^4*e^5)*x^4 - 10*(b^5*d^2*e^3 - 14*a*b^4*d*e^4 - 47*a^2*b^3*e
^5)*x^3 + 5*(b^5*d^3*e^2 - 9*a*b^4*d^2*e^3 + 51*a^2*b^3*d*e^4 + 77*a^3*b^2*
e^5)*x^2 - (3*b^5*d^4*e - 22*a*b^4*d^3*e^2 + 78*a^2*b^3*d^2*e^3 - 222*a^3*b
^2*d*e^4 - 137*a^4*b*e^5)*x)/(a^5*b^6*d^7 - 6*a^6*b^5*d^6*e + 15*a^7*b^4*d^
5*e^2 - 20*a^8*b^3*d^4*e^3 + 15*a^9*b^2*d^3*e^4 - 6*a^10*b*d^2*e^5 + a^11*d
*e^6 + (b^11*d^6*e - 6*a*b^10*d^5*e^2 + 15*a^2*b^9*d^4*e^3 - 20*a^3*b^8*d^3
*e^4 + 15*a^4*b^7*d^2*e^5 - 6*a^5*b^6*d*e^6 + a^6*b^5*e^7)*x^6 + (b^11*d^7
- a*b^10*d^6*e - 15*a^2*b^9*d^5*e^2 + 55*a^3*b^8*d^4*e^3 - 85*a^4*b^7*d^3*e
^4 + 69*a^5*b^6*d^2*e^5 - 29*a^6*b^5*d*e^6 + 5*a^7*b^4*e^7)*x^5 + 5*(a*b^10
*d^7 - 4*a^2*b^9*d^6*e + 3*a^3*b^8*d^5*e^2 + 10*a^4*b^7*d^4*e^3 - 25*a^5*b^
6*d^3*e^4 + 24*a^6*b^5*d^2*e^5 - 11*a^7*b^4*d*e^6 + 2*a^8*b^3*e^7)*x^4 + 10
*(a^2*b^9*d^7 - 5*a^3*b^8*d^6*e + 9*a^4*b^7*d^5*e^2 - 5*a^5*b^6*d^4*e^3 - 5
*a^6*b^5*d^3*e^4 + 9*a^7*b^4*d^2*e^5 - 5*a^8*b^3*d*e^6 + a^9*b^2*e^7)*x^3 +
 5*(2*a^3*b^8*d^7 - 11*a^4*b^7*d^6*e + 24*a^5*b^6*d^5*e^2 - 25*a^6*b^5*d^4*
e^3 + 10*a^7*b^4*d^3*e^4 + 3*a^8*b^3*d^2*e^5 - 4*a^9*b^2*d*e^6 + a^10*b*e^7
)*x^2 + (5*a^4*b^7*d^7 - 29*a^5*b^6*d^6*e + 69*a^6*b^5*d^5*e^2 - 85*a^7*b^4
*d^4*e^3 + 55*a^8*b^3*d^3*e^4 - 15*a^9*b^2*d^2*e^5 - a^10*b*d*e^6 + a^11*e^
```

```
7)*x)
```

Fricas [B] time = 1.88807, size = 2889, normalized size = 15.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/10*(2*b^6*d^6 - 15*a*b^5*d^5*e + 50*a^2*b^4*d^4*e^2 - 100*a^3*b^3*d^3*e^
3 + 150*a^4*b^2*d^2*e^4 - 77*a^5*b*d*e^5 - 10*a^6*e^6 + 60*(b^6*d*e^5 - a*b
^5*e^6)*x^5 + 30*(b^6*d^2*e^4 + 8*a*b^5*d*e^5 - 9*a^2*b^4*e^6)*x^4 - 10*(b^
6*d^3*e^3 - 15*a*b^5*d^2*e^4 - 33*a^2*b^4*d*e^5 + 47*a^3*b^3*e^6)*x^3 + 5*(
b^6*d^4*e^2 - 10*a*b^5*d^3*e^3 + 60*a^2*b^4*d^2*e^4 + 26*a^3*b^3*d*e^5 - 77
*a^4*b^2*e^6)*x^2 - (3*b^6*d^5*e - 25*a*b^5*d^4*e^2 + 100*a^2*b^4*d^3*e^3 -
 300*a^3*b^3*d^2*e^4 + 85*a^4*b^2*d*e^5 + 137*a^5*b*e^6)*x + 60*(b^6*e^6*x^
6 + a^5*b*d*e^5 + (b^6*d*e^5 + 5*a*b^5*e^6)*x^5 + 5*(a*b^5*d*e^5 + 2*a^2*b^
4*e^6)*x^4 + 10*(a^2*b^4*d*e^5 + a^3*b^3*e^6)*x^3 + 5*(2*a^3*b^3*d*e^5 + a^
4*b^2*e^6)*x^2 + (5*a^4*b^2*d*e^5 + a^5*b*e^6)*x)*log(b*x + a) - 60*(b^6*e^
6*x^6 + a^5*b*d*e^5 + (b^6*d*e^5 + 5*a*b^5*e^6)*x^5 + 5*(a*b^5*d*e^5 + 2*a^
2*b^4*e^6)*x^4 + 10*(a^2*b^4*d*e^5 + a^3*b^3*e^6)*x^3 + 5*(2*a^3*b^3*d*e^5
+ a^4*b^2*e^6)*x^2 + (5*a^4*b^2*d*e^5 + a^5*b*e^6)*x)*log(e*x + d))/(a^5*b^
7*d^8 - 7*a^6*b^6*d^7*e + 21*a^7*b^5*d^6*e^2 - 35*a^8*b^4*d^5*e^3 + 35*a^9*
b^3*d^4*e^4 - 21*a^10*b^2*d^3*e^5 + 7*a^11*b*d^2*e^6 - a^12*d*e^7 + (b^12*d
^7*e - 7*a*b^11*d^6*e^2 + 21*a^2*b^10*d^5*e^3 - 35*a^3*b^9*d^4*e^4 + 35*a^4
*b^8*d^3*e^5 - 21*a^5*b^7*d^2*e^6 + 7*a^6*b^6*d*e^7 - a^7*b^5*e^8)*x^6 + (b
^12*d^8 - 2*a*b^11*d^7*e - 14*a^2*b^10*d^6*e^2 + 70*a^3*b^9*d^5*e^3 - 140*a
^4*b^8*d^4*e^4 + 154*a^5*b^7*d^3*e^5 - 98*a^6*b^6*d^2*e^6 + 34*a^7*b^5*d*e^
7 - 5*a^8*b^4*e^8)*x^5 + 5*(a*b^11*d^8 - 5*a^2*b^10*d^7*e + 7*a^3*b^9*d^6*e
^2 + 7*a^4*b^8*d^5*e^3 - 35*a^5*b^7*d^4*e^4 + 49*a^6*b^6*d^3*e^5 - 35*a^7*b
^5*d^2*e^6 + 13*a^8*b^4*d*e^7 - 2*a^9*b^3*e^8)*x^4 + 10*(a^2*b^10*d^8 - 6*a
^3*b^9*d^7*e + 14*a^4*b^8*d^6*e^2 - 14*a^5*b^7*d^5*e^3 + 14*a^7*b^5*d^3*e^5
 - 14*a^8*b^4*d^2*e^6 + 6*a^9*b^3*d*e^7 - a^10*b^2*e^8)*x^3 + 5*(2*a^3*b^9*
d^8 - 13*a^4*b^8*d^7*e + 35*a^5*b^7*d^6*e^2 - 49*a^6*b^6*d^5*e^3 + 35*a^7*b
^5*d^4*e^4 - 7*a^8*b^4*d^3*e^5 - 7*a^9*b^3*d^2*e^6 + 5*a^10*b^2*d*e^7 - a^1
1*b*e^8)*x^2 + (5*a^4*b^8*d^8 - 34*a^5*b^7*d^7*e + 98*a^6*b^6*d^6*e^2 - 154
*a^7*b^5*d^5*e^3 + 140*a^8*b^4*d^4*e^4 - 70*a^9*b^3*d^3*e^5 + 14*a^10*b^2*d
^2*e^6 + 2*a^11*b*d*e^7 - a^12*e^8)*x)
```

Sympy [B] time = 9.33575, size = 1516, normalized size = 8.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] -6*b*e**5*log(x + (-6*a**8*b*e**13/(a*e - b*d)**7 + 48*a**7*b**2*d*e**12/(a
*e - b*d)**7 - 168*a**6*b**3*d**2*e**11/(a*e - b*d)**7 + 336*a**5*b**4*d**3
*e**10/(a*e - b*d)**7 - 420*a**4*b**5*d**4*e**9/(a*e - b*d)**7 + 336*a**3*b
**6*d**5*e**8/(a*e - b*d)**7 - 168*a**2*b**7*d**6*e**7/(a*e - b*d)**7 + 48*
a*b**8*d**7*e**6/(a*e - b*d)**7 + 6*a*b*e**6 - 6*b**9*d**8*e**5/(a*e - b*d)
**7 + 6*b**2*d*e**5)/(12*b**2*e**6))/(a*e - b*d)**7 + 6*b*e**5*log(x + (6*a
```

```
**8*b*e**13/(a*e - b*d)**7 - 48*a**7*b**2*d*e**12/(a*e - b*d)**7 + 168*a**6
*b**3*d**2*e**11/(a*e - b*d)**7 - 336*a**5*b**4*d**3*e**10/(a*e - b*d)**7 +
 420*a**4*b**5*d**4*e**9/(a*e - b*d)**7 - 336*a**3*b**6*d**5*e**8/(a*e - b*
d)**7 + 168*a**2*b**7*d**6*e**7/(a*e - b*d)**7 - 48*a*b**8*d**7*e**6/(a*e -
 b*d)**7 + 6*a*b*e**6 + 6*b**9*d**8*e**5/(a*e - b*d)**7 + 6*b**2*d*e**5)/(1
2*b**2*e**6))/(a*e - b*d)**7 - (10*a**5*e**5 + 87*a**4*b*d*e**4 - 63*a**3*b
**2*d**2*e**3 + 37*a**2*b**3*d**3*e**2 - 13*a*b**4*d**4*e + 2*b**5*d**5 + 6
0*b**5*e**5*x**5 + x**4*(270*a*b**4*e**5 + 30*b**5*d*e**4) + x**3*(470*a**2
*b**3*e**5 + 140*a*b**4*d*e**4 - 10*b**5*d**2*e**3) + x**2*(385*a**3*b**2*e
**5 + 255*a**2*b**3*d*e**4 - 45*a*b**4*d**2*e**3 + 5*b**5*d**3*e**2) + x*(1
37*a**4*b*e**5 + 222*a**3*b**2*d*e**4 - 78*a**2*b**3*d**2*e**3 + 22*a*b**4*
d**3*e**2 - 3*b**5*d**4*e))/(10*a**11*d*e**6 - 60*a**10*b*d**2*e**5 + 150*a
**9*b**2*d**3*e**4 - 200*a**8*b**3*d**4*e**3 + 150*a**7*b**4*d**5*e**2 - 60
*a**6*b**5*d**6*e + 10*a**5*b**6*d**7 + x**6*(10*a**6*b**5*e**7 - 60*a**5*b
**6*d*e**6 + 150*a**4*b**7*d**2*e**5 - 200*a**3*b**8*d**3*e**4 + 150*a**2*b
**9*d**4*e**3 - 60*a*b**10*d**5*e**2 + 10*b**11*d**6*e) + x**5*(50*a**7*b**
4*e**7 - 290*a**6*b**5*d*e**6 + 690*a**5*b**6*d**2*e**5 - 850*a**4*b**7*d**
3*e**4 + 550*a**3*b**8*d**4*e**3 - 150*a**2*b**9*d**5*e**2 - 10*a*b**10*d**
6*e + 10*b**11*d**7) + x**4*(100*a**8*b**3*e**7 - 550*a**7*b**4*d*e**6 + 12
00*a**6*b**5*d**2*e**5 - 1250*a**5*b**6*d**3*e**4 + 500*a**4*b**7*d**4*e**3
 + 150*a**3*b**8*d**5*e**2 - 200*a**2*b**9*d**6*e + 50*a*b**10*d**7) + x**3
*(100*a**9*b**2*e**7 - 500*a**8*b**3*d*e**6 + 900*a**7*b**4*d**2*e**5 - 500
*a**6*b**5*d**3*e**4 - 500*a**5*b**6*d**4*e**3 + 900*a**4*b**7*d**5*e**2 -
500*a**3*b**8*d**6*e + 100*a**2*b**9*d**7) + x**2*(50*a**10*b*e**7 - 200*a*
*9*b**2*d*e**6 + 150*a**8*b**3*d**2*e**5 + 500*a**7*b**4*d**3*e**4 - 1250*a
**6*b**5*d**4*e**3 + 1200*a**5*b**6*d**5*e**2 - 550*a**4*b**7*d**6*e + 100*
a**3*b**8*d**7) + x*(10*a**11*e**7 - 10*a**10*b*d*e**6 - 150*a**9*b**2*d**2
*e**5 + 550*a**8*b**3*d**3*e**4 - 850*a**7*b**4*d**4*e**3 + 690*a**6*b**5*d
**5*e**2 - 290*a**5*b**6*d**6*e + 50*a**4*b**7*d**7))
```

Giac [B] time = 1.15345, size = 609, normalized size = 3.36

$$-\frac{6\,be^6\log\left(\left|b-\frac{bd}{xe+d}+\frac{ae}{xe+d}\right|\right)}{b^7d^7e-7\,ab^6d^6e^2+21\,a^2b^5d^5e^3-35\,a^3b^4d^4e^4+35\,a^4b^3d^3e^5-21\,a^5b^2d^2e^6+7\,a^6bde^7-a^7e^8}-\frac{\cdots}{\left(b^6d^6e^6-6\,ab^5d^5e^7+15\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] -6*b*e^6*log(abs(b - b*d/(x*e + d) + a*e/(x*e + d)))/(b^7*d^7*e - 7*a*b^6*d
^6*e^2 + 21*a^2*b^5*d^5*e^3 - 35*a^3*b^4*d^4*e^4 + 35*a^4*b^3*d^3*e^5 - 21*
a^5*b^2*d^2*e^6 + 7*a^6*b*d*e^7 - a^7*e^8) - e^11/((b^6*d^6*e^6 - 6*a*b^5*d
^5*e^7 + 15*a^2*b^4*d^4*e^8 - 20*a^3*b^3*d^3*e^9 + 15*a^4*b^2*d^2*e^10 - 6*
a^5*b*d*e^11 + a^6*e^12)*(x*e + d)) - 1/10*(87*b^6*e^5 - 385*(b^6*d*e^6 - a
*b^5*e^7)*e^(-1)/(x*e + d) + 650*(b^6*d^2*e^7 - 2*a*b^5*d*e^8 + a^2*b^4*e^9
)*e^(-2)/(x*e + d)^2 - 500*(b^6*d^3*e^8 - 3*a*b^5*d^2*e^9 + 3*a^2*b^4*d*e^1
0 - a^3*b^3*e^11)*e^(-3)/(x*e + d)^3 + 150*(b^6*d^4*e^9 - 4*a*b^5*d^3*e^10
+ 6*a^2*b^4*d^2*e^11 - 4*a^3*b^3*d*e^12 + a^4*b^2*e^13)*e^(-4)/(x*e + d)^4)
/((b*d - a*e)^7*(b - b*d/(x*e + d) + a*e/(x*e + d))^5)
```

3.1536 $\int \frac{1}{(d+ex)^3(a^2+2abx+b^2x^2)^3}\,dx$

Optimal. Leaf size=220

$$-\frac{15b^2e^4}{(a+bx)(bd-ae)^7}+\frac{5b^2e^3}{(a+bx)^2(bd-ae)^6}-\frac{2b^2e^2}{(a+bx)^3(bd-ae)^5}-\frac{21b^2e^5\log(a+bx)}{(bd-ae)^8}+\frac{21b^2e^5\log(d+ex)}{(bd-ae)^8}+\frac{}{4(a+b}$$

```
[Out] -b^2/(5*(b*d - a*e)^3*(a + b*x)^5) + (3*b^2*e)/(4*(b*d - a*e)^4*(a + b*x)^4
) - (2*b^2*e^2)/((b*d - a*e)^5*(a + b*x)^3) + (5*b^2*e^3)/((b*d - a*e)^6*(a
 + b*x)^2) - (15*b^2*e^4)/((b*d - a*e)^7*(a + b*x)) - e^5/(2*(b*d - a*e)^6*
(d + e*x)^2) - (6*b*e^5)/((b*d - a*e)^7*(d + e*x)) - (21*b^2*e^5*Log[a + b*
x])/(b*d - a*e)^8 + (21*b^2*e^5*Log[d + e*x])/(b*d - a*e)^8
```

Rubi [A] time = 0.251798, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 44}

$$-\frac{15b^2e^4}{(a+bx)(bd-ae)^7}+\frac{5b^2e^3}{(a+bx)^2(bd-ae)^6}-\frac{2b^2e^2}{(a+bx)^3(bd-ae)^5}-\frac{21b^2e^5\log(a+bx)}{(bd-ae)^8}+\frac{21b^2e^5\log(d+ex)}{(bd-ae)^8}+\frac{}{4(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

```
[Out] -b^2/(5*(b*d - a*e)^3*(a + b*x)^5) + (3*b^2*e)/(4*(b*d - a*e)^4*(a + b*x)^4
) - (2*b^2*e^2)/((b*d - a*e)^5*(a + b*x)^3) + (5*b^2*e^3)/((b*d - a*e)^6*(a
 + b*x)^2) - (15*b^2*e^4)/((b*d - a*e)^7*(a + b*x)) - e^5/(2*(b*d - a*e)^6*
(d + e*x)^2) - (6*b*e^5)/((b*d - a*e)^7*(d + e*x)) - (21*b^2*e^5*Log[a + b*
x])/(b*d - a*e)^8 + (21*b^2*e^5*Log[d + e*x])/(b*d - a*e)^8
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)^3\left(a^2+2abx+b^2x^2\right)^3}\,dx &= \int \frac{1}{(a+bx)^6(d+ex)^3}\,dx\\ &= \int \left(\frac{b^3}{(bd-ae)^3(a+bx)^6}-\frac{3b^3e}{(bd-ae)^4(a+bx)^5}+\frac{6b^3e^2}{(bd-ae)^5(a+bx)^4}-\frac{10}{(bd-ae}\right.\\ &= -\frac{b^2}{5(bd-ae)^3(a+bx)^5}+\frac{3b^2e}{4(bd-ae)^4(a+bx)^4}-\frac{2b^2e^2}{(bd-ae)^5(a+bx)^3}+\frac{5}{(bd-ae}\end{aligned}$$

Mathematica [A] time = 0.167433, size = 204, normalized size = 0.93

$$-\frac{\frac{300b^2e^4(bd-ae)}{a+bx}-\frac{100b^2e^3(bd-ae)^2}{(a+bx)^2}+\frac{40b^2e^2(bd-ae)^3}{(a+bx)^3}-\frac{15b^2e(bd-ae)^4}{(a+bx)^4}+\frac{4b^2(bd-ae)^5}{(a+bx)^5}+420b^2e^5\log(a+bx)+\frac{120be^5(bd-ae)}{d+ex}+\frac{10e^5(bd-ae)^2}{(d+ex)^2}\cdots}{20(bd-ae)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

```
[Out] -((4*b^2*(b*d - a*e)^5)/(a + b*x)^5 - (15*b^2*e*(b*d - a*e)^4)/(a + b*x)^4
+ (40*b^2*e^2*(b*d - a*e)^3)/(a + b*x)^3 - (100*b^2*e^3*(b*d - a*e)^2)/(a +
 b*x)^2 + (300*b^2*e^4*(b*d - a*e))/(a + b*x) + (10*e^5*(b*d - a*e)^2)/(d +
 e*x)^2 + (120*b*e^5*(b*d - a*e))/(d + e*x) + 420*b^2*e^5*Log[a + b*x] - 42
0*b^2*e^5*Log[d + e*x])/(20*(b*d - a*e)^8)
```

Maple [A] time = 0.059, size = 215, normalized size = 1.

$$-\frac{e^5}{2\,(ae-bd)^6\,(ex+d)^2}+21\,\frac{e^5b^2\ln(ex+d)}{(ae-bd)^8}+6\,\frac{e^5b}{(ae-bd)^7\,(ex+d)}+\frac{b^2}{5\,(ae-bd)^3\,(bx+a)^5}-21\,\frac{e^5b^2\ln(bx+a)}{(ae-bd)^8}+15\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] -1/2*e^5/(a*e-b*d)^6/(e*x+d)^2+21*e^5/(a*e-b*d)^8*b^2*ln(e*x+d)+6*e^5/(a*e-
b*d)^7*b/(e*x+d)+1/5*b^2/(a*e-b*d)^3/(b*x+a)^5-21*e^5/(a*e-b*d)^8*b^2*ln(b*
x+a)+15*b^2/(a*e-b*d)^7*e^4/(b*x+a)+5*b^2/(a*e-b*d)^6*e^3/(b*x+a)^2+2*b^2/(
a*e-b*d)^5*e^2/(b*x+a)^3+3/4*b^2/(a*e-b*d)^4*e/(b*x+a)^4
```

Maxima [B] time = 1.8885, size = 2103, normalized size = 9.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] -21*b^2*e^5*log(b*x + a)/(b^8*d^8 - 8*a*b^7*d^7*e + 28*a^2*b^6*d^6*e^2 - 56
*a^3*b^5*d^5*e^3 + 70*a^4*b^4*d^4*e^4 - 56*a^5*b^3*d^3*e^5 + 28*a^6*b^2*d^2
*e^6 - 8*a^7*b*d*e^7 + a^8*e^8) + 21*b^2*e^5*log(e*x + d)/(b^8*d^8 - 8*a*b^
7*d^7*e + 28*a^2*b^6*d^6*e^2 - 56*a^3*b^5*d^5*e^3 + 70*a^4*b^4*d^4*e^4 - 56
*a^5*b^3*d^3*e^5 + 28*a^6*b^2*d^2*e^6 - 8*a^7*b*d*e^7 + a^8*e^8) - 1/20*(42
0*b^6*e^6*x^6 + 4*b^6*d^6 - 31*a*b^5*d^5*e + 109*a^2*b^4*d^4*e^2 - 241*a^3*
b^3*d^3*e^3 + 459*a^4*b^2*d^2*e^4 + 130*a^5*b*d*e^5 - 10*a^6*e^6 + 630*(b^6
*d*e^5 + 3*a*b^5*e^6)*x^5 + 70*(2*b^6*d^2*e^4 + 41*a*b^5*d*e^5 + 47*a^2*b^4
*e^6)*x^4 - 35*(b^6*d^3*e^3 - 19*a*b^5*d^2*e^4 - 145*a^2*b^4*d*e^5 - 77*a^3
*b^3*e^6)*x^3 + 7*(2*b^6*d^4*e^2 - 23*a*b^5*d^3*e^3 + 177*a^2*b^4*d^2*e^4 +
 607*a^3*b^3*d*e^5 + 137*a^4*b^2*e^6)*x^2 - 7*(b^6*d^5*e - 9*a*b^5*d^4*e^2
+ 41*a^2*b^4*d^3*e^3 - 159*a^3*b^3*d^2*e^4 - 224*a^4*b^2*d*e^5 - 10*a^5*b*e
^6)*x)/(a^5*b^7*d^9 - 7*a^6*b^6*d^8*e + 21*a^7*b^5*d^7*e^2 - 35*a^8*b^4*d^6
*e^3 + 35*a^9*b^3*d^5*e^4 - 21*a^10*b^2*d^4*e^5 + 7*a^11*b*d^3*e^6 - a^12*d
^2*e^7 + (b^12*d^7*e^2 - 7*a*b^11*d^6*e^3 + 21*a^2*b^10*d^5*e^4 - 35*a^3*b^
9*d^4*e^5 + 35*a^4*b^8*d^3*e^6 - 21*a^5*b^7*d^2*e^7 + 7*a^6*b^6*d*e^8 - a^7
```

```
*b^5*e^9)*x^7 + (2*b^12*d^8*e - 9*a*b^11*d^7*e^2 + 7*a^2*b^10*d^6*e^3 + 35*
a^3*b^9*d^5*e^4 - 105*a^4*b^8*d^4*e^5 + 133*a^5*b^7*d^3*e^6 - 91*a^6*b^6*d^
2*e^7 + 33*a^7*b^5*d*e^8 - 5*a^8*b^4*e^9)*x^6 + (b^12*d^9 + 3*a*b^11*d^8*e
- 39*a^2*b^10*d^7*e^2 + 105*a^3*b^9*d^6*e^3 - 105*a^4*b^8*d^5*e^4 - 21*a^5*
b^7*d^4*e^5 + 147*a^6*b^6*d^3*e^6 - 141*a^7*b^5*d^2*e^7 + 60*a^8*b^4*d*e^8
- 10*a^9*b^3*e^9)*x^5 + 5*(a*b^11*d^9 - 3*a^2*b^10*d^8*e - 5*a^3*b^9*d^7*e^
2 + 35*a^4*b^8*d^6*e^3 - 63*a^5*b^7*d^5*e^4 + 49*a^6*b^6*d^4*e^5 - 7*a^7*b^
5*d^3*e^6 - 15*a^8*b^4*d^2*e^7 + 10*a^9*b^3*d*e^8 - 2*a^10*b^2*e^9)*x^4 + 5
*(2*a^2*b^10*d^9 - 10*a^3*b^9*d^8*e + 15*a^4*b^8*d^7*e^2 + 7*a^5*b^7*d^6*e^
3 - 49*a^6*b^6*d^5*e^4 + 63*a^7*b^5*d^4*e^5 - 35*a^8*b^4*d^3*e^6 + 5*a^9*b^
3*d^2*e^7 + 3*a^10*b^2*d*e^8 - a^11*b*e^9)*x^3 + (10*a^3*b^9*d^9 - 60*a^4*b
^8*d^8*e + 141*a^5*b^7*d^7*e^2 - 147*a^6*b^6*d^6*e^3 + 21*a^7*b^5*d^5*e^4 +
 105*a^8*b^4*d^4*e^5 - 105*a^9*b^3*d^3*e^6 + 39*a^10*b^2*d^2*e^7 - 3*a^11*b
*d*e^8 - a^12*e^9)*x^2 + (5*a^4*b^8*d^9 - 33*a^5*b^7*d^8*e + 91*a^6*b^6*d^7
*e^2 - 133*a^7*b^5*d^6*e^3 + 105*a^8*b^4*d^5*e^4 - 35*a^9*b^3*d^4*e^5 - 7*a
^10*b^2*d^3*e^6 + 9*a^11*b*d^2*e^7 - 2*a^12*d*e^8)*x)
```

Fricas [B] time = 2.14151, size = 4157, normalized size = 18.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/20*(4*b^7*d^7 - 35*a*b^6*d^6*e + 140*a^2*b^5*d^5*e^2 - 350*a^3*b^4*d^4*e
^3 + 700*a^4*b^3*d^3*e^4 - 329*a^5*b^2*d^2*e^5 - 140*a^6*b*d*e^6 + 10*a^7*e
^7 + 420*(b^7*d*e^6 - a*b^6*e^7)*x^6 + 630*(b^7*d^2*e^5 + 2*a*b^6*d*e^6 - 3
*a^2*b^5*e^7)*x^5 + 70*(2*b^7*d^3*e^4 + 39*a*b^6*d^2*e^5 + 6*a^2*b^5*d*e^6
- 47*a^3*b^4*e^7)*x^4 - 35*(b^7*d^4*e^3 - 20*a*b^6*d^3*e^4 - 126*a^2*b^5*d^
2*e^5 + 68*a^3*b^4*d*e^6 + 77*a^4*b^3*e^7)*x^3 + 7*(2*b^7*d^5*e^2 - 25*a*b^
6*d^4*e^3 + 200*a^2*b^5*d^3*e^4 + 430*a^3*b^4*d^2*e^5 - 470*a^4*b^3*d*e^6 -
 137*a^5*b^2*e^7)*x^2 - 7*(b^7*d^6*e - 10*a*b^6*d^5*e^2 + 50*a^2*b^5*d^4*e^
3 - 200*a^3*b^4*d^3*e^4 - 65*a^4*b^3*d^2*e^5 + 214*a^5*b^2*d*e^6 + 10*a^6*b
*e^7)*x + 420*(b^7*e^7*x^7 + a^5*b^2*d^2*e^5 + (2*b^7*d*e^6 + 5*a*b^6*e^7)*
x^6 + (b^7*d^2*e^5 + 10*a*b^6*d*e^6 + 10*a^2*b^5*e^7)*x^5 + 5*(a*b^6*d^2*e^
5 + 4*a^2*b^5*d*e^6 + 2*a^3*b^4*e^7)*x^4 + 5*(2*a^2*b^5*d^2*e^5 + 4*a^3*b^4
*d*e^6 + a^4*b^3*e^7)*x^3 + (10*a^3*b^4*d^2*e^5 + 10*a^4*b^3*d*e^6 + a^5*b^
2*e^7)*x^2 + (5*a^4*b^3*d^2*e^5 + 2*a^5*b^2*d*e^6)*x)*log(b*x + a) - 420*(b
^7*e^7*x^7 + a^5*b^2*d^2*e^5 + (2*b^7*d*e^6 + 5*a*b^6*e^7)*x^6 + (b^7*d^2*e
^5 + 10*a*b^6*d*e^6 + 10*a^2*b^5*e^7)*x^5 + 5*(a*b^6*d^2*e^5 + 4*a^2*b^5*d*
e^6 + 2*a^3*b^4*e^7)*x^4 + 5*(2*a^2*b^5*d^2*e^5 + 4*a^3*b^4*d*e^6 + a^4*b^3
*e^7)*x^3 + (10*a^3*b^4*d^2*e^5 + 10*a^4*b^3*d*e^6 + a^5*b^2*e^7)*x^2 + (5*
a^4*b^3*d^2*e^5 + 2*a^5*b^2*d*e^6)*x)*log(e*x + d))/(a^5*b^8*d^10 - 8*a^6*b
^7*d^9*e + 28*a^7*b^6*d^8*e^2 - 56*a^8*b^5*d^7*e^3 + 70*a^9*b^4*d^6*e^4 - 5
6*a^10*b^3*d^5*e^5 + 28*a^11*b^2*d^4*e^6 - 8*a^12*b*d^3*e^7 + a^13*d^2*e^8
+ (b^13*d^8*e^2 - 8*a*b^12*d^7*e^3 + 28*a^2*b^11*d^6*e^4 - 56*a^3*b^10*d^5*
e^5 + 70*a^4*b^9*d^4*e^6 - 56*a^5*b^8*d^3*e^7 + 28*a^6*b^7*d^2*e^8 - 8*a^7*
b^6*d*e^9 + a^8*b^5*e^10)*x^7 + (2*b^13*d^9*e - 11*a*b^12*d^8*e^2 + 16*a^2*
b^11*d^7*e^3 + 28*a^3*b^10*d^6*e^4 - 140*a^4*b^9*d^5*e^5 + 238*a^5*b^8*d^4*
e^6 - 224*a^6*b^7*d^3*e^7 + 124*a^7*b^6*d^2*e^8 - 38*a^8*b^5*d*e^9 + 5*a^9*
b^4*e^10)*x^6 + (b^13*d^10 + 2*a*b^12*d^9*e - 42*a^2*b^11*d^8*e^2 + 144*a^3
*b^10*d^7*e^3 - 210*a^4*b^9*d^6*e^4 + 84*a^5*b^8*d^5*e^5 + 168*a^6*b^7*d^4*
e^6 - 288*a^7*b^6*d^3*e^7 + 201*a^8*b^5*d^2*e^8 - 70*a^9*b^4*d*e^9 + 10*a^1
0*b^3*e^10)*x^5 + 5*(a*b^12*d^10 - 4*a^2*b^11*d^9*e - 2*a^3*b^10*d^8*e^2 +
40*a^4*b^9*d^7*e^3 - 98*a^5*b^8*d^6*e^4 + 112*a^6*b^7*d^5*e^5 - 56*a^7*b^6*
d^4*e^6 - 8*a^8*b^5*d^3*e^7 + 25*a^9*b^4*d^2*e^8 - 12*a^10*b^3*d*e^9 + 2*a^
```

```
11*b^2*e^10)*x^4 + 5*(2*a^2*b^11*d^10 - 12*a^3*b^10*d^9*e + 25*a^4*b^9*d^8*
e^2 - 8*a^5*b^8*d^7*e^3 - 56*a^6*b^7*d^6*e^4 + 112*a^7*b^6*d^5*e^5 - 98*a^8
*b^5*d^4*e^6 + 40*a^9*b^4*d^3*e^7 - 2*a^10*b^3*d^2*e^8 - 4*a^11*b^2*d*e^9 +
 a^12*b*e^10)*x^3 + (10*a^3*b^10*d^10 - 70*a^4*b^9*d^9*e + 201*a^5*b^8*d^8*
e^2 - 288*a^6*b^7*d^7*e^3 + 168*a^7*b^6*d^6*e^4 + 84*a^8*b^5*d^5*e^5 - 210*
a^9*b^4*d^4*e^6 + 144*a^10*b^3*d^3*e^7 - 42*a^11*b^2*d^2*e^8 + 2*a^12*b*d*e
^9 + a^13*e^10)*x^2 + (5*a^4*b^9*d^10 - 38*a^5*b^8*d^9*e + 124*a^6*b^7*d^8*
e^2 - 224*a^7*b^6*d^7*e^3 + 238*a^8*b^5*d^6*e^4 - 140*a^9*b^4*d^5*e^5 + 28*
a^10*b^3*d^4*e^6 + 16*a^11*b^2*d^3*e^7 - 11*a^12*b*d^2*e^8 + 2*a^13*d*e^9)*
x)
```

Sympy [B] time = 16.3101, size = 1974, normalized size = 8.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] 21*b**2*e**5*log(x + (-21*a**9*b**2*e**14/(a*e - b*d)**8 + 189*a**8*b**3*d*
e**13/(a*e - b*d)**8 - 756*a**7*b**4*d**2*e**12/(a*e - b*d)**8 + 1764*a**6*
b**5*d**3*e**11/(a*e - b*d)**8 - 2646*a**5*b**6*d**4*e**10/(a*e - b*d)**8 +
 2646*a**4*b**7*d**5*e**9/(a*e - b*d)**8 - 1764*a**3*b**8*d**6*e**8/(a*e -
b*d)**8 + 756*a**2*b**9*d**7*e**7/(a*e - b*d)**8 - 189*a*b**10*d**8*e**6/(a
*e - b*d)**8 + 21*a*b**2*e**6 + 21*b**11*d**9*e**5/(a*e - b*d)**8 + 21*b**3
*d*e**5)/(42*b**3*e**6))/(a*e - b*d)**8 - 21*b**2*e**5*log(x + (21*a**9*b**
2*e**14/(a*e - b*d)**8 - 189*a**8*b**3*d*e**13/(a*e - b*d)**8 + 756*a**7*b*
*4*d**2*e**12/(a*e - b*d)**8 - 1764*a**6*b**5*d**3*e**11/(a*e - b*d)**8 + 2
646*a**5*b**6*d**4*e**10/(a*e - b*d)**8 - 2646*a**4*b**7*d**5*e**9/(a*e - b
*d)**8 + 1764*a**3*b**8*d**6*e**8/(a*e - b*d)**8 - 756*a**2*b**9*d**7*e**7/
(a*e - b*d)**8 + 189*a*b**10*d**8*e**6/(a*e - b*d)**8 + 21*a*b**2*e**6 - 21
*b**11*d**9*e**5/(a*e - b*d)**8 + 21*b**3*d*e**5)/(42*b**3*e**6))/(a*e - b*
d)**8 + (-10*a**6*e**6 + 130*a**5*b*d*e**5 + 459*a**4*b**2*d**2*e**4 - 241*
a**3*b**3*d**3*e**3 + 109*a**2*b**4*d**4*e**2 - 31*a*b**5*d**5*e + 4*b**6*d
**6 + 420*b**6*e**6*x**6 + x**5*(1890*a*b**5*e**6 + 630*b**6*d*e**5) + x**4
*(3290*a**2*b**4*e**6 + 2870*a*b**5*d*e**5 + 140*b**6*d**2*e**4) + x**3*(26
95*a**3*b**3*e**6 + 5075*a**2*b**4*d*e**5 + 665*a*b**5*d**2*e**4 - 35*b**6*
d**3*e**3) + x**2*(959*a**4*b**2*e**6 + 4249*a**3*b**3*d*e**5 + 1239*a**2*b
**4*d**2*e**4 - 161*a*b**5*d**3*e**3 + 14*b**6*d**4*e**2) + x*(70*a**5*b*e*
*6 + 1568*a**4*b**2*d*e**5 + 1113*a**3*b**3*d**2*e**4 - 287*a**2*b**4*d**3*
e**3 + 63*a*b**5*d**4*e**2 - 7*b**6*d**5*e))/(20*a**12*d**2*e**7 - 140*a**1
1*b*d**3*e**6 + 420*a**10*b**2*d**4*e**5 - 700*a**9*b**3*d**5*e**4 + 700*a*
*8*b**4*d**6*e**3 - 420*a**7*b**5*d**7*e**2 + 140*a**6*b**6*d**8*e - 20*a**
5*b**7*d**9 + x**7*(20*a**7*b**5*e**9 - 140*a**6*b**6*d*e**8 + 420*a**5*b**
7*d**2*e**7 - 700*a**4*b**8*d**3*e**6 + 700*a**3*b**9*d**4*e**5 - 420*a**2*
b**10*d**5*e**4 + 140*a*b**11*d**6*e**3 - 20*b**12*d**7*e**2) + x**6*(100*a
**8*b**4*e**9 - 660*a**7*b**5*d*e**8 + 1820*a**6*b**6*d**2*e**7 - 2660*a**5
*b**7*d**3*e**6 + 2100*a**4*b**8*d**4*e**5 - 700*a**3*b**9*d**5*e**4 - 140*
a**2*b**10*d**6*e**3 + 180*a*b**11*d**7*e**2 - 40*b**12*d**8*e) + x**5*(200
*a**9*b**3*e**9 - 1200*a**8*b**4*d*e**8 + 2820*a**7*b**5*d**2*e**7 - 2940*a
**6*b**6*d**3*e**6 + 420*a**5*b**7*d**4*e**5 + 2100*a**4*b**8*d**5*e**4 - 2
100*a**3*b**9*d**6*e**3 + 780*a**2*b**10*d**7*e**2 - 60*a*b**11*d**8*e - 20
*b**12*d**9) + x**4*(200*a**10*b**2*e**9 - 1000*a**9*b**3*d*e**8 + 1500*a**
8*b**4*d**2*e**7 + 700*a**7*b**5*d**3*e**6 - 4900*a**6*b**6*d**4*e**5 + 630
0*a**5*b**7*d**5*e**4 - 3500*a**4*b**8*d**6*e**3 + 500*a**3*b**9*d**7*e**2
+ 300*a**2*b**10*d**8*e - 100*a*b**11*d**9) + x**3*(100*a**11*b*e**9 - 300*
a**10*b**2*d*e**8 - 500*a**9*b**3*d**2*e**7 + 3500*a**8*b**4*d**3*e**6 - 63
```

```
00*a**7*b**5*d**4*e**5 + 4900*a**6*b**6*d**5*e**4 - 700*a**5*b**7*d**6*e**3
 - 1500*a**4*b**8*d**7*e**2 + 1000*a**3*b**9*d**8*e - 200*a**2*b**10*d**9)
+ x**2*(20*a**12*e**9 + 60*a**11*b*d*e**8 - 780*a**10*b**2*d**2*e**7 + 2100
*a**9*b**3*d**3*e**6 - 2100*a**8*b**4*d**4*e**5 - 420*a**7*b**5*d**5*e**4 +
 2940*a**6*b**6*d**6*e**3 - 2820*a**5*b**7*d**7*e**2 + 1200*a**4*b**8*d**8*
e - 200*a**3*b**9*d**9) + x*(40*a**12*d*e**8 - 180*a**11*b*d**2*e**7 + 140*
a**10*b**2*d**3*e**6 + 700*a**9*b**3*d**4*e**5 - 2100*a**8*b**4*d**5*e**4 +
 2660*a**7*b**5*d**6*e**3 - 1820*a**6*b**6*d**7*e**2 + 660*a**5*b**7*d**8*e
 - 100*a**4*b**8*d**9))
```

Giac [B] time = 1.22258, size = 907, normalized size = 4.12

$$-\frac{21\,b^3e^5\log\left(|bx+a|\right)}{b^9d^8-8\,ab^8d^7e+28\,a^2b^7d^6e^2-56\,a^3b^6d^5e^3+70\,a^4b^5d^4e^4-56\,a^5b^4d^3e^5+28\,a^6b^3d^2e^6-8\,a^7b^2de^7+a^8be^8}+\frac{}{b^8d^8e\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] -21*b^3*e^5*log(abs(b*x + a))/(b^9*d^8 - 8*a*b^8*d^7*e + 28*a^2*b^7*d^6*e^2
 - 56*a^3*b^6*d^5*e^3 + 70*a^4*b^5*d^4*e^4 - 56*a^5*b^4*d^3*e^5 + 28*a^6*b^
3*d^2*e^6 - 8*a^7*b^2*d*e^7 + a^8*b*e^8) + 21*b^2*e^6*log(abs(x*e + d))/(b^
8*d^8*e - 8*a*b^7*d^7*e^2 + 28*a^2*b^6*d^6*e^3 - 56*a^3*b^5*d^5*e^4 + 70*a^
4*b^4*d^4*e^5 - 56*a^5*b^3*d^3*e^6 + 28*a^6*b^2*d^2*e^7 - 8*a^7*b*d*e^8 + a
^8*e^9) - 1/20*(4*b^7*d^7 - 35*a*b^6*d^6*e + 140*a^2*b^5*d^5*e^2 - 350*a^3*
b^4*d^4*e^3 + 700*a^4*b^3*d^3*e^4 - 329*a^5*b^2*d^2*e^5 - 140*a^6*b*d*e^6 +
 10*a^7*e^7 + 420*(b^7*d*e^6 - a*b^6*e^7)*x^6 + 630*(b^7*d^2*e^5 + 2*a*b^6*
d*e^6 - 3*a^2*b^5*e^7)*x^5 + 70*(2*b^7*d^3*e^4 + 39*a*b^6*d^2*e^5 + 6*a^2*b
^5*d*e^6 - 47*a^3*b^4*e^7)*x^4 - 35*(b^7*d^4*e^3 - 20*a*b^6*d^3*e^4 - 126*a
^2*b^5*d^2*e^5 + 68*a^3*b^4*d*e^6 + 77*a^4*b^3*e^7)*x^3 + 7*(2*b^7*d^5*e^2
- 25*a*b^6*d^4*e^3 + 200*a^2*b^5*d^3*e^4 + 430*a^3*b^4*d^2*e^5 - 470*a^4*b^
3*d*e^6 - 137*a^5*b^2*e^7)*x^2 - 7*(b^7*d^6*e - 10*a*b^6*d^5*e^2 + 50*a^2*b
^5*d^4*e^3 - 200*a^3*b^4*d^3*e^4 - 65*a^4*b^3*d^2*e^5 + 214*a^5*b^2*d*e^6 +
 10*a^6*b*e^7)*x)/((b*d - a*e)^8*(b*x + a)^5*(x*e + d)^2)
```

3.1537 $\int (d+ex)\left(9+12x+4x^2\right)^3\,dx$

Optimal. Leaf size=31

$$\frac{1}{28}(2x+3)^7(2d-3e)+\frac{1}{32}e(2x+3)^8$$

```
[Out] ((2*d - 3*e)*(3 + 2*x)^7)/28 + (e*(3 + 2*x)^8)/32
```

Rubi [A] time = 0.0115803, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {27, 43}

$$\frac{1}{28}(2x+3)^7(2d-3e)+\frac{1}{32}e(2x+3)^8$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(9 + 12*x + 4*x^2)^3,x]
```

```
[Out] ((2*d - 3*e)*(3 + 2*x)^7)/28 + (e*(3 + 2*x)^8)/32
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(9+12x+4x^2\right)^3\,dx &= \int (3+2x)^6(d+ex)\,dx\\ &= \int \left(\frac{1}{2}(2d-3e)(3+2x)^6+\frac{1}{2}e(3+2x)^7\right)dx\\ &= \frac{1}{28}(2d-3e)(3+2x)^7+\frac{1}{32}e(3+2x)^8\end{aligned}$$

Mathematica [B] time = 0.0150449, size = 81, normalized size = 2.61

$$\frac{64}{7}x^7(d+9e)+24x^6(4d+15e)+432x^5(d+2e)+135x^4(8d+9e)+324x^3(5d+3e)+\frac{729}{2}x^2(4d+e)+729dx+8ex^8$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(9 + 12*x + 4*x^2)^3,x]
```

```
[Out] 729*d*x + (729*(4*d + e)*x^2)/2 + 324*(5*d + 3*e)*x^3 + 135*(8*d + 9*e)*x^4
 + 432*(d + 2*e)*x^5 + 24*(4*d + 15*e)*x^6 + (64*(d + 9*e)*x^7)/7 + 8*e*x^8
```

Maple [B] time = 0.04, size = 84, normalized size = 2.7

$$8\,e x^{8}+\frac{\left(64\,d+576\,e\right)x^{7}}{7}+\frac{\left(576\,d+2160\,e\right)x^{6}}{6}+\frac{\left(2160\,d+4320\,e\right)x^{5}}{5}+\frac{\left(4320\,d+4860\,e\right)x^{4}}{4}+\frac{\left(4860\,d+2916\,e\right)x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(4*x^2+12*x+9)^3,x)
```

```
[Out] 8*e*x^8+1/7*(64*d+576*e)*x^7+1/6*(576*d+2160*e)*x^6+1/5*(2160*d+4320*e)*x^5
+1/4*(4320*d+4860*e)*x^4+1/3*(4860*d+2916*e)*x^3+1/2*(2916*d+729*e)*x^2+729
*d*x
```

Maxima [B] time = 1.09033, size = 104, normalized size = 3.35

$$8\,e x^{8}+\frac{64}{7}\,(d+9\,e)x^{7}+24\,(4\,d+15\,e)x^{6}+432\,(d+2\,e)x^{5}+135\,(8\,d+9\,e)x^{4}+324\,(5\,d+3\,e)x^{3}+\frac{729}{2}\,(4\,d+e)x^{2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^2+12*x+9)^3,x, algorithm="maxima")
```

```
[Out] 8*e*x^8 + 64/7*(d + 9*e)*x^7 + 24*(4*d + 15*e)*x^6 + 432*(d + 2*e)*x^5 + 13
5*(8*d + 9*e)*x^4 + 324*(5*d + 3*e)*x^3 + 729/2*(4*d + e)*x^2 + 729*d*x
```

Fricas [B] time = 1.34382, size = 231, normalized size = 7.45

$$8x^{8}e+\frac{576}{7}x^{7}e+\frac{64}{7}x^{7}d+360x^{6}e+96x^{6}d+864x^{5}e+432x^{5}d+1215x^{4}e+1080x^{4}d+972x^{3}e+1620x^{3}d+\frac{729}{2}x^{2}e+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^2+12*x+9)^3,x, algorithm="fricas")
```

```
[Out] 8*x^8*e + 576/7*x^7*e + 64/7*x^7*d + 360*x^6*e + 96*x^6*d + 864*x^5*e + 432
*x^5*d + 1215*x^4*e + 1080*x^4*d + 972*x^3*e + 1620*x^3*d + 729/2*x^2*e + 1
458*x^2*d + 729*x*d
```

Sympy [B] time = 0.089094, size = 76, normalized size = 2.45

$$729dx+8ex^{8}+x^{7}\left(\frac{64d}{7}+\frac{576e}{7}\right)+x^{6}\left(96d+360e\right)+x^{5}\left(432d+864e\right)+x^{4}\left(1080d+1215e\right)+x^{3}\left(1620d+972e\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x**2+12*x+9)**3,x)
```

```
[Out] 729*d*x + 8*e*x**8 + x**7*(64*d/7 + 576*e/7) + x**6*(96*d + 360*e) + x**5*(
432*d + 864*e) + x**4*(1080*d + 1215*e) + x**3*(1620*d + 972*e) + x**2*(145
8*d + 729*e/2)
```

Giac [B] time = 1.13701, size = 122, normalized size = 3.94

$$8\,x^8e+\frac{64}{7}\,dx^7+\frac{576}{7}\,x^7e+96\,dx^6+360\,x^6e+432\,dx^5+864\,x^5e+1080\,dx^4+1215\,x^4e+1620\,dx^3+972\,x^3e+1458\,dx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^2+12*x+9)^3,x, algorithm="giac")
```

```
[Out] 8*x^8*e + 64/7*d*x^7 + 576/7*x^7*e + 96*d*x^6 + 360*x^6*e + 432*d*x^5 + 864
*x^5*e + 1080*d*x^4 + 1215*x^4*e + 1620*d*x^3 + 972*x^3*e + 1458*d*x^2 + 72
9/2*x^2*e + 729*d*x
```

3.1538 $\int (d+ex)\left(9+12x+4x^2\right)^2\,dx$

Optimal. Leaf size=31

$$\frac{1}{20}(2x+3)^5(2d-3e)+\frac{1}{24}e(2x+3)^6$$

```
[Out] ((2*d - 3*e)*(3 + 2*x)^5)/20 + (e*(3 + 2*x)^6)/24
```

Rubi [A] time = 0.0108392, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {27, 43}

$$\frac{1}{20}(2x+3)^5(2d-3e)+\frac{1}{24}e(2x+3)^6$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(9 + 12*x + 4*x^2)^2,x]
```

```
[Out] ((2*d - 3*e)*(3 + 2*x)^5)/20 + (e*(3 + 2*x)^6)/24
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(9+12x+4x^2\right)^2\,dx &= \int (3+2x)^4(d+ex)\,dx\\ &= \int \left(\frac{1}{2}(2d-3e)(3+2x)^4+\frac{1}{2}e(3+2x)^5\right)dx\\ &= \frac{1}{20}(2d-3e)(3+2x)^5+\frac{1}{24}e(3+2x)^6\end{aligned}$$

Mathematica [A] time = 0.0092227, size = 59, normalized size = 1.9

$$\frac{16}{5}x^5(d+6e)+6x^4(4d+9e)+72x^3(d+e)+\frac{27}{2}x^2(8d+3e)+81dx+\frac{8ex^6}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(9 + 12*x + 4*x^2)^2,x]
```

```
[Out] 81*d*x + (27*(8*d + 3*e)*x^2)/2 + 72*(d + e)*x^3 + 6*(4*d + 9*e)*x^4 + (16*
(d + 6*e)*x^5)/5 + (8*e*x^6)/3
```

Maple [B] time = 0.039, size = 60, normalized size = 1.9

$$\frac{8\,e x^{6}}{3}+\frac{\left(16\,d +96\,e \right) x^{5}}{5}+\frac{\left(96\,d +216\,e \right) x^{4}}{4}+\frac{\left(216\,d +216\,e \right) x^{3}}{3}+\frac{\left(216\,d +81\,e \right) x^{2}}{2}+81\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(4*x^2+12*x+9)^2,x)
```

```
[Out] 8/3*e*x^6+1/5*(16*d+96*e)*x^5+1/4*(96*d+216*e)*x^4+1/3*(216*d+216*e)*x^3+1/
2*(216*d+81*e)*x^2+81*d*x
```

Maxima [A] time = 1.16255, size = 72, normalized size = 2.32

$$\frac{8}{3}\,e x^{6}+\frac{16}{5}\,{\left(d+6\,e\right)}x^{5}+6\,{\left(4\,d+9\,e\right)}x^{4}+72\,{\left(d+e\right)}x^{3}+\frac{27}{2}\,{\left(8\,d+3\,e\right)}x^{2}+81\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^2+12*x+9)^2,x, algorithm="maxima")
```

```
[Out] 8/3*e*x^6 + 16/5*(d + 6*e)*x^5 + 6*(4*d + 9*e)*x^4 + 72*(d + e)*x^3 + 27/2*
(8*d + 3*e)*x^2 + 81*d*x
```

Fricas [B] time = 1.33961, size = 155, normalized size = 5.

$$\frac{8}{3}x^{6}e+\frac{96}{5}x^{5}e+\frac{16}{5}x^{5}d+54x^{4}e+24x^{4}d+72x^{3}e+72x^{3}d+\frac{81}{2}x^{2}e+108x^{2}d+81xd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^2+12*x+9)^2,x, algorithm="fricas")
```

```
[Out] 8/3*x^6*e + 96/5*x^5*e + 16/5*x^5*d + 54*x^4*e + 24*x^4*d + 72*x^3*e + 72*x
^3*d + 81/2*x^2*e + 108*x^2*d + 81*x*d
```

Sympy [B] time = 0.077843, size = 58, normalized size = 1.87

$$81dx+\frac{8ex^{6}}{3}+x^{5}\left(\frac{16d}{5}+\frac{96e}{5}\right)+x^{4}\left(24d+54e\right)+x^{3}\left(72d+72e\right)+x^{2}\left(108d+\frac{81e}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x**2+12*x+9)**2,x)
```

```
[Out] 81*d*x + 8*e*x**6/3 + x**5*(16*d/5 + 96*e/5) + x**4*(24*d + 54*e) + x**3*(7
2*d + 72*e) + x**2*(108*d + 81*e/2)
```

Giac [B] time = 1.18605, size = 86, normalized size = 2.77

$$\frac{8}{3}x^6e + \frac{16}{5}dx^5 + \frac{96}{5}x^5e + 24\,dx^4 + 54\,x^4e + 72\,dx^3 + 72\,x^3e + 108\,dx^2 + \frac{81}{2}x^2e + 81\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^2+12*x+9)^2,x, algorithm="giac")
```

```
[Out] 8/3*x^6*e + 16/5*d*x^5 + 96/5*x^5*e + 24*d*x^4 + 54*x^4*e + 72*d*x^3 + 72*x
^3*e + 108*d*x^2 + 81/2*x^2*e + 81*d*x
```

3.1539 $\int (d+ex)\left(9+12x+4x^2\right)\,dx$

Optimal. Leaf size=31

$$\frac{1}{12}(2x+3)^3(2d-3e)+\frac{1}{16}e(2x+3)^4$$

```
[Out] ((2*d - 3*e)*(3 + 2*x)^3)/12 + (e*(3 + 2*x)^4)/16
```

Rubi [A] time = 0.0206185, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {27, 43}

$$\frac{1}{12}(2x+3)^3(2d-3e)+\frac{1}{16}e(2x+3)^4$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(9 + 12*x + 4*x^2),x]

[Out] ((2*d - 3*e)*(3 + 2*x)^3)/12 + (e*(3 + 2*x)^4)/16
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(9+12x+4x^2\right)\,dx &= \int (3+2x)^2(d+ex)\,dx\\ &= \int \left(\frac{1}{2}(2d-3e)(3+2x)^2+\frac{1}{2}e(3+2x)^3\right)dx\\ &= \frac{1}{12}(2d-3e)(3+2x)^3+\frac{1}{16}e(3+2x)^4\end{aligned}$$

Mathematica [A] time = 0.005751, size = 36, normalized size = 1.16

$$\frac{4}{3}x^3(d+3e)+\frac{3}{2}x^2(4d+3e)+9dx+ex^4$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(9 + 12*x + 4*x^2),x]

[Out] 9*d*x + (3*(4*d + 3*e)*x^2)/2 + (4*(d + 3*e)*x^3)/3 + e*x^4
```

Maple [A] time = 0.038, size = 35, normalized size = 1.1

$$ex^4 + \frac{(4d + 12e)x^3}{3} + \frac{(12d + 9e)x^2}{2} + 9dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(4*x^2+12*x+9),x)
```

```
[Out] e*x^4+1/3*(4*d+12*e)*x^3+1/2*(12*d+9*e)*x^2+9*d*x
```

Maxima [A] time = 1.11375, size = 43, normalized size = 1.39

$$ex^4 + \frac{4}{3}(d + 3e)x^3 + \frac{3}{2}(4d + 3e)x^2 + 9dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^2+12*x+9),x, algorithm="maxima")
```

```
[Out] e*x^4 + 4/3*(d + 3*e)*x^3 + 3/2*(4*d + 3*e)*x^2 + 9*d*x
```

Fricas [A] time = 1.36334, size = 80, normalized size = 2.58

$$x^4e + 4x^3e + \frac{4}{3}x^3d + \frac{9}{2}x^2e + 6x^2d + 9xd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^2+12*x+9),x, algorithm="fricas")
```

```
[Out] x^4*e + 4*x^3*e + 4/3*x^3*d + 9/2*x^2*e + 6*x^2*d + 9*x*d
```

Sympy [A] time = 0.077802, size = 32, normalized size = 1.03

$$9dx + ex^4 + x^3\left(\frac{4d}{3} + 4e\right) + x^2\left(6d + \frac{9e}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x**2+12*x+9),x)
```

```
[Out] 9*d*x + e*x**4 + x**3*(4*d/3 + 4*e) + x**2*(6*d + 9*e/2)
```

Giac [A] time = 1.11945, size = 50, normalized size = 1.61

$$x^4e + \frac{4}{3}dx^3 + 4x^3e + 6dx^2 + \frac{9}{2}x^2e + 9dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^2+12*x+9),x, algorithm="giac")
```

```
[Out] x^4*e + 4/3*d*x^3 + 4*x^3*e + 6*d*x^2 + 9/2*x^2*e + 9*d*x
```

3.1540 $\int \frac{d+ex}{9+12x+4x^2}\,dx$

Optimal. Leaf size=30

$$\frac{1}{4}e\log(2x+3)-\frac{2d-3e}{4(2x+3)}$$

```
[Out] -(2*d - 3*e)/(4*(3 + 2*x)) + (e*Log[3 + 2*x])/4
```

Rubi [A] time = 0.0168794, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {27, 43}

$$\frac{1}{4}e\log(2x+3)-\frac{2d-3e}{4(2x+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(9 + 12*x + 4*x^2),x]
```

```
[Out] -(2*d - 3*e)/(4*(3 + 2*x)) + (e*Log[3 + 2*x])/4
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex}{9+12x+4x^2}\,dx &= \int \frac{d+ex}{(3+2x)^2}\,dx\\ &= \int \left(\frac{2d-3e}{2(3+2x)^2}+\frac{e}{2(3+2x)}\right)dx\\ &= -\frac{2d-3e}{4(3+2x)}+\frac{1}{4}e\log(3+2x)\end{aligned}$$

Mathematica [A] time = 0.0066698, size = 30, normalized size = 1.

$$\frac{3e-2d}{4(2x+3)}+\frac{1}{4}e\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(9 + 12*x + 4*x^2),x]
```

```
[Out] (-2*d + 3*e)/(4*(3 + 2*x)) + (e*Log[3 + 2*x])/4
```

Maple [A] time = 0.044, size = 31, normalized size = 1.

$$\frac{e\ln(3+2x)}{4}-\frac{d}{6+4x}+\frac{3e}{12+8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(4*x^2+12*x+9),x)
```

```
[Out] 1/4*e*ln(3+2*x)-1/2/(3+2*x)*d+3/4*e/(3+2*x)
```

Maxima [A] time = 1.10739, size = 35, normalized size = 1.17

$$\frac{1}{4}e\log(2x+3)-\frac{2d-3e}{4(2x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(4*x^2+12*x+9),x, algorithm="maxima")
```

```
[Out] 1/4*e*log(2*x + 3) - 1/4*(2*d - 3*e)/(2*x + 3)
```

Fricas [A] time = 1.37845, size = 76, normalized size = 2.53

$$\frac{(2ex+3e)\log(2x+3)-2d+3e}{4(2x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(4*x^2+12*x+9),x, algorithm="fricas")
```

```
[Out] 1/4*((2*e*x + 3*e)*log(2*x + 3) - 2*d + 3*e)/(2*x + 3)
```

Sympy [A] time = 0.335154, size = 20, normalized size = 0.67

$$\frac{e\log(2x+3)}{4}-\frac{2d-3e}{8x+12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(4*x**2+12*x+9),x)
```

```
[Out] e*log(2*x + 3)/4 - (2*d - 3*e)/(8*x + 12)
```

Giac [A] time = 1.13272, size = 39, normalized size = 1.3

$$\frac{1}{4}e\log(|2x+3|)-\frac{2d-3e}{4(2x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(4*x^2+12*x+9),x, algorithm="giac")
```

```
[Out] 1/4*e*log(abs(2*x + 3)) - 1/4*(2*d - 3*e)/(2*x + 3)
```

3.1541 $\int \frac{d+ex}{(9+12x+4x^2)^2} dx$

Optimal. Leaf size=31

$$-\frac{2d-3e}{12(2x+3)^3}-\frac{e}{8(2x+3)^2}$$

```
[Out] -(2*d - 3*e)/(12*(3 + 2*x)^3) - e/(8*(3 + 2*x)^2)
```

Rubi [A] time = 0.0162679, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {27, 43}

$$-\frac{2d-3e}{12(2x+3)^3}-\frac{e}{8(2x+3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(9 + 12*x + 4*x^2)^2,x]
```

```
[Out] -(2*d - 3*e)/(12*(3 + 2*x)^3) - e/(8*(3 + 2*x)^2)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex}{(9+12x+4x^2)^2}\,dx &= \int \frac{d+ex}{(3+2x)^4}\,dx\\ &= \int \left(\frac{2d-3e}{2(3+2x)^4}+\frac{e}{2(3+2x)^3}\right) dx\\ &= -\frac{2d-3e}{12(3+2x)^3}-\frac{e}{8(3+2x)^2}\end{aligned}$$

Mathematica [A] time = 0.0077466, size = 22, normalized size = 0.71

$$-\frac{4d+6ex+3e}{24(2x+3)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(9 + 12*x + 4*x^2)^2,x]
```

```
[Out] -(4*d + 3*e + 6*e*x)/(24*(3 + 2*x)^3)
```

Maple [A] time = 0.044, size = 28, normalized size = 0.9

$$-\frac{1}{3\,(3+2\,x)^{3}}\left(\frac{d}{2}-\frac{3\,e}{4}\right)-\frac{e}{8\,(3+2\,x)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(4*x^2+12*x+9)^2,x)
```

```
[Out] -1/3*(1/2*d-3/4*e)/(3+2*x)^3-1/8*e/(3+2*x)^2
```

Maxima [A] time = 1.15613, size = 41, normalized size = 1.32

$$-\frac{6\,ex+4\,d+3\,e}{24\,\left(8\,x^{3}+36\,x^{2}+54\,x+27\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(4*x^2+12*x+9)^2,x, algorithm="maxima")
```

```
[Out] -1/24*(6*e*x + 4*d + 3*e)/(8*x^3 + 36*x^2 + 54*x + 27)
```

Fricas [A] time = 1.39814, size = 76, normalized size = 2.45

$$-\frac{6\,ex+4\,d+3\,e}{24\,\left(8\,x^{3}+36\,x^{2}+54\,x+27\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(4*x^2+12*x+9)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(6*e*x + 4*d + 3*e)/(8*x^3 + 36*x^2 + 54*x + 27)
```

Sympy [A] time = 0.374043, size = 27, normalized size = 0.87

$$-\frac{4d+6ex+3e}{192x^{3}+864x^{2}+1296x+648}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(4*x**2+12*x+9)**2,x)
```

```
[Out] -(4*d + 6*e*x + 3*e)/(192*x**3 + 864*x**2 + 1296*x + 648)
```

<u>Giac</u> [A] time = 1.19541, size = 30, normalized size = 0.97

$$-\frac{6\,xe + 4\,d + 3\,e}{24\,(2\,x + 3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(4*x^2+12*x+9)^2,x, algorithm="giac")
```

```
[Out] -1/24*(6*x*e + 4*d + 3*e)/(2*x + 3)^3
```

3.1542 $\int \frac{d+ex}{(9+12x+4x^2)^3}\,dx$

Optimal. Leaf size=31

$$-\frac{2d-3e}{20(2x+3)^5}-\frac{e}{16(2x+3)^4}$$

```
[Out] -(2*d - 3*e)/(20*(3 + 2*x)^5) - e/(16*(3 + 2*x)^4)
```

Rubi [A] time = 0.0165174, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {27, 43}

$$-\frac{2d-3e}{20(2x+3)^5}-\frac{e}{16(2x+3)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(9 + 12*x + 4*x^2)^3,x]
```

```
[Out] -(2*d - 3*e)/(20*(3 + 2*x)^5) - e/(16*(3 + 2*x)^4)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex}{(9+12x+4x^2)^3}\,dx &= \int \frac{d+ex}{(3+2x)^6}\,dx\\ &= \int \left(\frac{2d-3e}{2(3+2x)^6}+\frac{e}{2(3+2x)^5}\right)dx\\ &= -\frac{2d-3e}{20(3+2x)^5}-\frac{e}{16(3+2x)^4}\end{aligned}$$

Mathematica [A] time = 0.0074101, size = 22, normalized size = 0.71

$$-\frac{8d+e(10x+3)}{80(2x+3)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(9 + 12*x + 4*x^2)^3,x]
```

```
[Out] -(8*d + e*(3 + 10*x))/(80*(3 + 2*x)^5)
```

Maple [A] time = 0.045, size = 28, normalized size = 0.9

$$-\frac{e}{16\,(3+2\,x)^4}-\frac{1}{5\,(3+2\,x)^5}\left(\frac{d}{2}-\frac{3\,e}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(4*x^2+12*x+9)^3,x)
```

```
[Out] -1/16*e/(3+2*x)^4-1/5*(1/2*d-3/4*e)/(3+2*x)^5
```

Maxima [A] time = 1.0152, size = 54, normalized size = 1.74

$$-\frac{10\,ex+8\,d+3\,e}{80\,\left(32\,x^5+240\,x^4+720\,x^3+1080\,x^2+810\,x+243\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(4*x^2+12*x+9)^3,x, algorithm="maxima")
```

```
[Out] -1/80*(10*e*x + 8*d + 3*e)/(32*x^5 + 240*x^4 + 720*x^3 + 1080*x^2 + 810*x +
 243)
```

Fricas [A] time = 1.67236, size = 111, normalized size = 3.58

$$-\frac{10\,ex+8\,d+3\,e}{80\,\left(32\,x^5+240\,x^4+720\,x^3+1080\,x^2+810\,x+243\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(4*x^2+12*x+9)^3,x, algorithm="fricas")
```

```
[Out] -1/80*(10*e*x + 8*d + 3*e)/(32*x^5 + 240*x^4 + 720*x^3 + 1080*x^2 + 810*x +
 243)
```

Sympy [A] time = 0.496349, size = 37, normalized size = 1.19

$$-\frac{8d+10ex+3e}{2560x^5+19200x^4+57600x^3+86400x^2+64800x+19440}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(4*x**2+12*x+9)**3,x)
```

```
[Out] -(8*d + 10*e*x + 3*e)/(2560*x**5 + 19200*x**4 + 57600*x**3 + 86400*x**2 + 6
4800*x + 19440)
```

Giac [A] time = 1.13009, size = 30, normalized size = 0.97

$$-\frac{10\,xe+8\,d+3\,e}{80\,(2\,x+3)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(4*x^2+12*x+9)^3,x, algorithm="giac")
```

```
[Out] -1/80*(10*x*e + 8*d + 3*e)/(2*x + 3)^5
```

3.1543 $\int (d+ex)^4\sqrt{a^2+2abx+b^2x^2}\,dx$

Optimal. Leaf size=92

$$\frac{b\sqrt{a^2+2abx+b^2x^2}(d+ex)^6}{6e^2(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}(d+ex)^5(bd-ae)}{5e^2(a+bx)}$$

```
[Out] -((b*d - a*e)*(d + e*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^2*(a + b*x))
+ (b*(d + e*x)^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*e^2*(a + b*x))
```

Rubi [A] time = 0.0400595, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$\frac{b\sqrt{a^2+2abx+b^2x^2}(d+ex)^6}{6e^2(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}(d+ex)^5(bd-ae)}{5e^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] -((b*d - a*e)*(d + e*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^2*(a + b*x))
+ (b*(d + e*x)^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*e^2*(a + b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^4\sqrt{a^2+2abx+b^2x^2}\,dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(ab+b^2x\right)(d+ex)^4\,dx}{ab+b^2x}\\
&= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b(bd-ae)(d+ex)^4}{e}+\frac{b^2(d+ex)^5}{e}\right)dx}{ab+b^2x}\\
&= -\frac{(bd-ae)(d+ex)^5\sqrt{a^2+2abx+b^2x^2}}{5e^2(a+bx)}+\frac{b(d+ex)^6\sqrt{a^2+2abx+b^2x^2}}{6e^2(a+bx)}
\end{aligned}$$

Mathematica [A] time = 0.0378056, size = 111, normalized size = 1.21

$$\frac{x\sqrt{(a+bx)^2}\left(6a\left(10d^2e^2x^2+10d^3ex+5d^4+5de^3x^3+e^4x^4\right)+bx\left(45d^2e^2x^2+40d^3ex+15d^4+24de^3x^3+5e^4x^4\right)\right)}{30(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (x*Sqrt[(a + b*x)^2]*(6*a*(5*d^4 + 10*d^3*e*x + 10*d^2*e^2*x^2 + 5*d*e^3*x^
3 + e^4*x^4) + b*x*(15*d^4 + 40*d^3*e*x + 45*d^2*e^2*x^2 + 24*d*e^3*x^3 + 5
*e^4*x^4)))/(30*(a + b*x))
```

Maple [A] time = 0.042, size = 114, normalized size = 1.2

$$\frac{x\left(5\,be^4x^5+6\,x^4ae^4+24\,x^4bde^3+30\,x^3ade^3+45\,x^3bd^2e^2+60\,x^2ad^2e^2+40\,x^2bd^3e+60\,xad^3e+15\,xbd^4+30\,ad^4\right)}{30\,bx+30\,a}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*((b*x+a)^2)^(1/2),x)
```

```
[Out] 1/30*x*(5*b*e^4*x^5+6*a*e^4*x^4+24*b*d*e^3*x^4+30*a*d*e^3*x^3+45*b*d^2*e^2*
x^3+60*a*d^2*e^2*x^2+40*b*d^3*e*x^2+60*a*d^3*e*x+15*b*d^4*x+30*a*d^4)*((b*x
+a)^2)^(1/2)/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.76527, size = 212, normalized size = 2.3

$$\frac{1}{6}\,be^4x^6+ad^4x+\frac{1}{5}\left(4\,bde^3+ae^4\right)x^5+\frac{1}{2}\left(3\,bd^2e^2+2\,ade^3\right)x^4+\frac{2}{3}\left(2\,bd^3e+3\,ad^2e^2\right)x^3+\frac{1}{2}\left(bd^4+4\,ad^3e\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*b*e^4*x^6 + a*d^4*x + 1/5*(4*b*d*e^3 + a*e^4)*x^5 + 1/2*(3*b*d^2*e^2 +
2*a*d*e^3)*x^4 + 2/3*(2*b*d^3*e + 3*a*d^2*e^2)*x^3 + 1/2*(b*d^4 + 4*a*d^3*e
)*x^2
```

Sympy [A] time = 0.118562, size = 100, normalized size = 1.09

$$ad^4x+\frac{be^4x^6}{6}+x^5\left(\frac{ae^4}{5}+\frac{4bde^3}{5}\right)+x^4\left(ade^3+\frac{3bd^2e^2}{2}\right)+x^3\left(2ad^2e^2+\frac{4bd^3e}{3}\right)+x^2\left(2ad^3e+\frac{bd^4}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*((b*x+a)**2)**(1/2),x)
```

```
[Out] a*d**4*x + b*e**4*x**6/6 + x**5*(a*e**4/5 + 4*b*d*e**3/5) + x**4*(a*d*e**3
+ 3*b*d**2*e**2/2) + x**3*(2*a*d**2*e**2 + 4*b*d**3*e/3) + x**2*(2*a*d**3*e
 + b*d**4/2)
```

Giac [B] time = 1.24279, size = 207, normalized size = 2.25

$$\frac{1}{6}\,bx^6e^4\mathrm{sgn}\left(bx+a\right)+\frac{4}{5}\,bdx^5e^3\mathrm{sgn}\left(bx+a\right)+\frac{3}{2}\,bd^2x^4e^2\mathrm{sgn}\left(bx+a\right)+\frac{4}{3}\,bd^3x^3e\mathrm{sgn}\left(bx+a\right)+\frac{1}{2}\,bd^4x^2\mathrm{sgn}\left(bx+a\right)+\frac{1}{5}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*b*x^6*e^4*sgn(b*x + a) + 4/5*b*d*x^5*e^3*sgn(b*x + a) + 3/2*b*d^2*x^4*e
^2*sgn(b*x + a) + 4/3*b*d^3*x^3*e*sgn(b*x + a) + 1/2*b*d^4*x^2*sgn(b*x + a)
 + 1/5*a*x^5*e^4*sgn(b*x + a) + a*d*x^4*e^3*sgn(b*x + a) + 2*a*d^2*x^3*e^2*
sgn(b*x + a) + 2*a*d^3*x^2*e*sgn(b*x + a) + a*d^4*x*sgn(b*x + a)
```

3.1544 $\int (d+ex)^3\sqrt{a^2+2abx+b^2x^2}\,dx$

Optimal. Leaf size=92

$$\frac{b\sqrt{a^2+2abx+b^2x^2}(d+ex)^5}{5e^2(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}(d+ex)^4(bd-ae)}{4e^2(a+bx)}$$

```
[Out] -((b*d - a*e)*(d + e*x)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*e^2*(a + b*x))
+ (b*(d + e*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^2*(a + b*x))
```

Rubi [A] time = 0.0358736, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$\frac{b\sqrt{a^2+2abx+b^2x^2}(d+ex)^5}{5e^2(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}(d+ex)^4(bd-ae)}{4e^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] -((b*d - a*e)*(d + e*x)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*e^2*(a + b*x))
+ (b*(d + e*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^2*(a + b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^3\sqrt{a^2+2abx+b^2x^2}\,dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(ab+b^2x\right)(d+ex)^3\,dx}{ab+b^2x}\\
&= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b(bd-ae)(d+ex)^3}{e}+\frac{b^2(d+ex)^4}{e}\right)dx}{ab+b^2x}\\
&= -\frac{(bd-ae)(d+ex)^4\sqrt{a^2+2abx+b^2x^2}}{4e^2(a+bx)}+\frac{b(d+ex)^5\sqrt{a^2+2abx+b^2x^2}}{5e^2(a+bx)}
\end{aligned}$$

Mathematica [A] time = 0.0282263, size = 89, normalized size = 0.97

$$\frac{x\sqrt{(a+bx)^2}\left(5a\left(6d^2ex+4d^3+4de^2x^2+e^3x^3\right)+bx\left(20d^2ex+10d^3+15de^2x^2+4e^3x^3\right)\right)}{20(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (x*Sqrt[(a + b*x)^2]*(5*a*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) + b*x
*(10*d^3 + 20*d^2*e*x + 15*d*e^2*x^2 + 4*e^3*x^3)))/(20*(a + b*x))
```

Maple [A] time = 0.042, size = 90, normalized size = 1.

$$\frac{x\left(4\,b e^3 x^4+5\,x^3 a e^3+15\,x^3 b d e^2+20\,a d e^2 x^2+20\,b d^2 e x^2+30\,x a d^2 e+10\,x b d^3+20\,a d^3\right)\sqrt{(bx+a)^2}}{20\,bx+20\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*((b*x+a)^2)^(1/2),x)
```

```
[Out] 1/20*x*(4*b*e^3*x^4+5*a*e^3*x^3+15*b*d*e^2*x^3+20*a*d*e^2*x^2+20*b*d^2*e*x^
2+30*a*d^2*e*x+10*b*d^3*x+20*a*d^3)*((b*x+a)^2)^(1/2)/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.85812, size = 150, normalized size = 1.63

$$\frac{1}{5}\,b e^3 x^5+a d^3 x+\frac{1}{4}\left(3\,b d e^2+a e^3\right)x^4+\left(b d^2 e+a d e^2\right)x^3+\frac{1}{2}\left(b d^3+3\,a d^2 e\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*b*e^3*x^5 + a*d^3*x + 1/4*(3*b*d*e^2 + a*e^3)*x^4 + (b*d^2*e + a*d*e^2)
*x^3 + 1/2*(b*d^3 + 3*a*d^2*e)*x^2
```

Sympy [A] time = 0.131303, size = 73, normalized size = 0.79

$$ad^3x+\frac{be^3x^5}{5}+x^4\left(\frac{ae^3}{4}+\frac{3bde^2}{4}\right)+x^3\left(ade^2+bd^2e\right)+x^2\left(\frac{3ad^2e}{2}+\frac{bd^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*((b*x+a)**2)**(1/2),x)
```

```
[Out] a*d**3*x + b*e**3*x**5/5 + x**4*(a*e**3/4 + 3*b*d*e**2/4) + x**3*(a*d*e**2
+ b*d**2*e) + x**2*(3*a*d**2*e/2 + b*d**3/2)
```

Giac [A] time = 1.16381, size = 159, normalized size = 1.73

$$\frac{1}{5}\,bx^5e^3\mathrm{sgn}\,(bx+a)+\frac{3}{4}\,bdx^4e^2\mathrm{sgn}\,(bx+a)+bd^2x^3e\mathrm{sgn}\,(bx+a)+\frac{1}{2}\,bd^3x^2\mathrm{sgn}\,(bx+a)+\frac{1}{4}\,ax^4e^3\mathrm{sgn}\,(bx+a)+adx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*b*x^5*e^3*sgn(b*x + a) + 3/4*b*d*x^4*e^2*sgn(b*x + a) + b*d^2*x^3*e*sgn
(b*x + a) + 1/2*b*d^3*x^2*sgn(b*x + a) + 1/4*a*x^4*e^3*sgn(b*x + a) + a*d*x
^3*e^2*sgn(b*x + a) + 3/2*a*d^2*x^2*e*sgn(b*x + a) + a*d^3*x*sgn(b*x + a)
```

3.1545 $\int (d+ex)^2\sqrt{a^2+2abx+b^2x^2}\,dx$

Optimal. Leaf size=92

$$\frac{b\sqrt{a^2+2abx+b^2x^2}(d+ex)^4}{4e^2(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}(d+ex)^3(bd-ae)}{3e^2(a+bx)}$$

```
[Out] -((b*d - a*e)*(d + e*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^2*(a + b*x))
+ (b*(d + e*x)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*e^2*(a + b*x))
```

Rubi [A] time = 0.0489773, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$\frac{b\sqrt{a^2+2abx+b^2x^2}(d+ex)^4}{4e^2(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}(d+ex)^3(bd-ae)}{3e^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] -((b*d - a*e)*(d + e*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^2*(a + b*x))
+ (b*(d + e*x)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*e^2*(a + b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^2\sqrt{a^2+2abx+b^2x^2}\,dx&=\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(ab+b^2x\right)(d+ex)^2\,dx}{ab+b^2x}\\&=\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b(bd-ae)(d+ex)^2}{e}+\frac{b^2(d+ex)^3}{e}\right)dx}{ab+b^2x}\\&=-\frac{(bd-ae)(d+ex)^3\sqrt{a^2+2abx+b^2x^2}}{3e^2(a+bx)}+\frac{b(d+ex)^4\sqrt{a^2+2abx+b^2x^2}}{4e^2(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.0219973, size = 67, normalized size = 0.73

$$\frac{x\sqrt{(a+bx)^2}\left(4a\left(3d^2+3dex+e^2x^2\right)+bx\left(6d^2+8dex+3e^2x^2\right)\right)}{12(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (x*Sqrt[(a + b*x)^2]*(4*a*(3*d^2 + 3*d*e*x + e^2*x^2) + b*x*(6*d^2 + 8*d*e*
x + 3*e^2*x^2)))/(12*(a + b*x))
```

Maple [A] time = 0.041, size = 66, normalized size = 0.7

$$\frac{x\left(3\,be^2x^3+4\,x^2ae^2+8\,x^2bde+12\,adex+6\,bd^2x+12\,ad^2\right)}{12\,bx+12\,a}\sqrt{(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*((b*x+a)^2)^(1/2),x)
```

```
[Out] 1/12*x*(3*b*e^2*x^3+4*a*e^2*x^2+8*b*d*e*x^2+12*a*d*e*x+6*b*d^2*x+12*a*d^2)*
((b*x+a)^2)^(1/2)/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64003, size = 109, normalized size = 1.18

$$\frac{1}{4}\,be^2x^4+ad^2x+\frac{1}{3}\left(2\,bde+ae^2\right)x^3+\frac{1}{2}\left(bd^2+2\,ade\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*b*e^2*x^4 + a*d^2*x + 1/3*(2*b*d*e + a*e^2)*x^3 + 1/2*(b*d^2 + 2*a*d*e)
*x^2
```

Sympy [A] time = 0.111704, size = 49, normalized size = 0.53

$$ad^2x+\frac{be^2x^4}{4}+x^3\left(\frac{ae^2}{3}+\frac{2bde}{3}\right)+x^2\left(ade+\frac{bd^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*((b*x+a)**2)**(1/2),x)
```

```
[Out] a*d**2*x + b*e**2*x**4/4 + x**3*(a*e**2/3 + 2*b*d*e/3) + x**2*(a*d*e + b*d*
*2/2)
```

Giac [A] time = 1.16699, size = 115, normalized size = 1.25

$$\frac{1}{4}bx^4e^2\mathrm{sgn}\left(bx+a\right)+\frac{2}{3}bdx^3e\mathrm{sgn}\left(bx+a\right)+\frac{1}{2}bd^2x^2\mathrm{sgn}\left(bx+a\right)+\frac{1}{3}ax^3e^2\mathrm{sgn}\left(bx+a\right)+adx^2e\mathrm{sgn}\left(bx+a\right)+ad^2x\mathrm{sgn}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*b*x^4*e^2*sgn(b*x + a) + 2/3*b*d*x^3*e*sgn(b*x + a) + 1/2*b*d^2*x^2*sgn
(b*x + a) + 1/3*a*x^3*e^2*sgn(b*x + a) + a*d*x^2*e*sgn(b*x + a) + a*d^2*x*s
gn(b*x + a)
```

3.1546 $\int (d+ex)\sqrt{a^2+2abx+b^2x^2}\,dx$

Optimal. Leaf size=69

$$\frac{(a+bx)\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{2b^2}+\frac{e\left(a^2+2abx+b^2x^2\right)^{3/2}}{3b^2}$$

```
[Out] ((b*d - a*e)*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*b^2) + (e*(a^2 + 2
*a*b*x + b^2*x^2)^(3/2))/(3*b^2)
```

Rubi [A] time = 0.0208345, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {640, 609}

$$\frac{(a+bx)\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{2b^2}+\frac{e\left(a^2+2abx+b^2x^2\right)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ((b*d - a*e)*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*b^2) + (e*(a^2 + 2
*a*b*x + b^2*x^2)^(3/2))/(3*b^2)
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int (d+ex)\sqrt{a^2+2abx+b^2x^2}\,dx &= \frac{e\left(a^2+2abx+b^2x^2\right)^{3/2}}{3b^2}+\frac{\left(2b^2d-2abe\right)\int \sqrt{a^2+2abx+b^2x^2}\,dx}{2b^2}\\ &= \frac{(bd-ae)(a+bx)\sqrt{a^2+2abx+b^2x^2}}{2b^2}+\frac{e\left(a^2+2abx+b^2x^2\right)^{3/2}}{3b^2}\end{aligned}$$

Mathematica [A] time = 0.0172388, size = 45, normalized size = 0.65

$$\frac{x\sqrt{(a+bx)^2}(3a(2d+ex)+bx(3d+2ex))}{6(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (x*Sqrt[(a + b*x)^2]*(3*a*(2*d + e*x) + b*x*(3*d + 2*e*x)))/(6*(a + b*x))
```

Maple [A] time = 0.041, size = 42, normalized size = 0.6

$$\frac{x\left(2\,be x^{2}+3\,aex+3\,bdx+6\,ad\right)\sqrt{\left(bx+a\right)^{2}}}{6\,bx+6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*((b*x+a)^2)^(1/2),x)
```

```
[Out] 1/6*x*(2*b*e*x^2+3*a*e*x+3*b*d*x+6*a*d)*((b*x+a)^2)^(1/2)/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51045, size = 58, normalized size = 0.84

$$\frac{1}{3}\,bex^{3}+adx+\frac{1}{2}\,(bd+ae)x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*b*e*x^3 + a*d*x + 1/2*(b*d + a*e)*x^2
```

Sympy [A] time = 0.115544, size = 26, normalized size = 0.38

$$adx+\frac{bex^{3}}{3}+x^{2}\left(\frac{ae}{2}+\frac{bd}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*((b*x+a)**2)**(1/2),x)
```

```
[Out] a*d*x + b*e*x**3/3 + x**2*(a*e/2 + b*d/2)
```

Giac [A] time = 1.12668, size = 70, normalized size = 1.01

$$\frac{1}{3}\,bx^{3}e\mathrm{sgn}\left(bx+a\right)+\frac{1}{2}\,bdx^{2}\mathrm{sgn}\left(bx+a\right)+\frac{1}{2}\,ax^{2}e\mathrm{sgn}\left(bx+a\right)+adx\mathrm{sgn}\left(bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*b*x^3*e*sgn(b*x + a) + 1/2*b*d*x^2*sgn(b*x + a) + 1/2*a*x^2*e*sgn(b*x +
 a) + a*d*x*sgn(b*x + a)
```

3.1547 $\int \sqrt{a^2+2abx+b^2x^2}\,dx$

Optimal. Leaf size=32

$$\frac{(a+bx)\sqrt{a^2+2abx+b^2x^2}}{2b}$$

```
[Out] ((a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*b)
```

Rubi [A] time = 0.005164, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {609}

$$\frac{(a+bx)\sqrt{a^2+2abx+b^2x^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]

[Out] ((a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*b)
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \sqrt{a^2+2abx+b^2x^2}\,dx=\frac{(a+bx)\sqrt{a^2+2abx+b^2x^2}}{2b}$$

Mathematica [A] time = 0.008513, size = 30, normalized size = 0.94

$$\frac{x\sqrt{(a+bx)^2}(2a+bx)}{2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]

[Out] (x*Sqrt[(a + b*x)^2]*(2*a + b*x))/(2*(a + b*x))
```

Maple [A] time = 0.041, size = 27, normalized size = 0.8

$$\frac{x\,(bx+2\,a)}{2\,bx+2\,a}\sqrt{(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2),x)
```

```
[Out] 1/2*x*(b*x+2*a)*((b*x+a)^2)^(1/2)/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.45479, size = 23, normalized size = 0.72

$$\frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*x
```

Sympy [A] time = 0.111854, size = 8, normalized size = 0.25

$$ax + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2),x)
```

```
[Out] a*x + b*x**2/2
```

Giac [A] time = 1.19896, size = 45, normalized size = 1.41

$$\frac{1}{2}\left(bx^2 + 2\,ax\right)\operatorname{sgn}\left(bx + a\right) + \frac{a^2\operatorname{sgn}\left(bx + a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*sgn(b*x + a) + 1/2*a^2*sgn(b*x + a)/b
```

3.1548 $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{d+ex} dx$

Optimal. Leaf size=80

$$\frac{bx\sqrt{a^2+2abx+b^2x^2}}{e(a+bx)} - \frac{\sqrt{a^2+2abx+b^2x^2}(bd-ae)\log(d+ex)}{e^2(a+bx)}$$

```
[Out] (b*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e*(a + b*x)) - ((b*d - a*e)*Sqrt[a^2 +
 2*a*b*x + b^2*x^2]*Log[d + e*x])/(e^2*(a + b*x))
```

Rubi [A] time = 0.0373813, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$\frac{bx\sqrt{a^2+2abx+b^2x^2}}{e(a+bx)} - \frac{\sqrt{a^2+2abx+b^2x^2}(bd-ae)\log(d+ex)}{e^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x),x]
```

```
[Out] (b*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e*(a + b*x)) - ((b*d - a*e)*Sqrt[a^2 +
 2*a*b*x + b^2*x^2]*Log[d + e*x])/(e^2*(a + b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx+b^2x^2}}{d+ex} dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{ab+b^2x}{d+ex} dx}{ab+b^2x}\\ &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{b^2}{e} - \frac{b(bd-ae)}{e(d+ex)}\right) dx}{ab+b^2x}\\ &= \frac{bx\sqrt{a^2+2abx+b^2x^2}}{e(a+bx)} - \frac{(bd-ae)\sqrt{a^2+2abx+b^2x^2}\log(d+ex)}{e^2(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.0156397, size = 42, normalized size = 0.52

$$\frac{\sqrt{(a+bx)^2}((ae-bd)\log(d+ex)+bex)}{e^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x),x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(b*e*x + (-(b*d) + a*e)*Log[d + e*x]))/(e^2*(a + b*x))
```

Maple [C] time = 0.201, size = 44, normalized size = 0.6

$$\frac{\operatorname{csgn}(bx+a)(\ln(bxe+bd)ae-\ln(bxe+bd)bd+bxe+ae)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)/(e*x+d),x)
```

```
[Out] csgn(b*x+a)*(ln(b*e*x+b*d)*a*e-ln(b*e*x+b*d)*b*d+b*x*e+a*e)/e^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54714, size = 54, normalized size = 0.68

$$\frac{bex-(bd-ae)\log(ex+d)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] (b*e*x - (b*d - a*e)*log(e*x + d))/e^2
```

Sympy [A] time = 0.585905, size = 20, normalized size = 0.25

$$\frac{bx}{e}+\frac{(ae-bd)\log(d+ex)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)/(e*x+d),x)
```

```
[Out] b*x/e + (a*e - b*d)*log(d + e*x)/e**2
```

Giac [A] time = 1.20215, size = 61, normalized size = 0.76

$$bxe^{(-1)}\operatorname{sgn}(bx+a)-(bd\operatorname{sgn}(bx+a)-ae\operatorname{sgn}(bx+a))e^{(-2)}\log(|xe+d|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] b*x*e^(-1)*sgn(b*x + a) - (b*d*sgn(b*x + a) - a*e*sgn(b*x + a))*e^(-2)*log(
abs(x*e + d))
```

3.1549 $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^2} dx$

Optimal. Leaf size=85

$$\frac{\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{e^2(a+bx)(d+ex)}+\frac{b\sqrt{a^2+2abx+b^2x^2}\log(d+ex)}{e^2(a+bx)}$$

```
[Out] ((b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^2*(a + b*x)*(d + e*x)) + (b*
Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Log[d + e*x])/(e^2*(a + b*x))
```

Rubi [A] time = 0.0405717, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$\frac{\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{e^2(a+bx)(d+ex)}+\frac{b\sqrt{a^2+2abx+b^2x^2}\log(d+ex)}{e^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^2,x]
```

```
[Out] ((b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^2*(a + b*x)*(d + e*x)) + (b*
Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Log[d + e*x])/(e^2*(a + b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^2} dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{ab+b^2x}{(d+ex)^2} dx}{ab+b^2x}\\
&= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b(bd-ae)}{e(d+ex)^2}+\frac{b^2}{e(d+ex)}\right) dx}{ab+b^2x}\\
&= \frac{(bd-ae)\sqrt{a^2+2abx+b^2x^2}}{e^2(a+bx)(d+ex)}+\frac{b\sqrt{a^2+2abx+b^2x^2}\log(d+ex)}{e^2(a+bx)}
\end{aligned}$$

Mathematica [A] time = 0.0194631, size = 50, normalized size = 0.59

$$\frac{\sqrt{(a+bx)^2}(-ae+b(d+ex)\log(d+ex)+bd)}{e^2(a+bx)(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^2,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(b*d - a*e + b*(d + e*x)*Log[d + e*x]))/(e^2*(a + b*x)*(
d + e*x))
```

Maple [C] time = 0.227, size = 51, normalized size = 0.6

$$\frac{\mathrm{csgn}\,(bx+a)\,(\ln{(bxe+bd)}\,xbe+\ln{(bxe+bd)}\,bd-ae+bd)}{e^2\,(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)/(e*x+d)^2,x)
```

```
[Out] csgn(b*x+a)*(ln(b*e*x+b*d)*x*b*e+ln(b*e*x+b*d)*b*d-a*e+b*d)/e^2/(e*x+d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57975, size = 78, normalized size = 0.92

$$\frac{bd-ae+(bex+bd)\log{(ex+d)}}{e^3x+de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] (b*d - a*e + (b*e*x + b*d)*log(e*x + d))/(e^3*x + d*e^2)
```

Sympy [A] time = 0.379246, size = 27, normalized size = 0.32

$$\frac{b\log{(d+ex)}}{e^2}-\frac{ae-bd}{de^2+e^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)/(e*x+d)**2,x)
```

```
[Out] b*log(d + e*x)/e**2 - (a*e - b*d)/(d*e**2 + e**3*x)
```

Giac [A] time = 1.22229, size = 69, normalized size = 0.81

$$be^{(-2)}\log\left(|xe+d|\right)\operatorname{sgn}\left(bx+a\right)+\frac{\left(bd\operatorname{sgn}\left(bx+a\right)-ae\operatorname{sgn}\left(bx+a\right)\right)e^{(-2)}}{xe+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] b*e^(-2)*log(abs(x*e + d))*sgn(b*x + a) + (b*d*sgn(b*x + a) - a*e*sgn(b*x +
 a))*e^(-2)/(x*e + d)
```

3.1550 $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^3}\,dx$

Optimal. Leaf size=46

$$\frac{(a+bx)\sqrt{a^2+2abx+b^2x^2}}{2(d+ex)^2(bd-ae)}$$

```
[Out] ((a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*(b*d - a*e)*(d + e*x)^2)
```

Rubi [A] time = 0.0198825, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 37}

$$\frac{(a+bx)\sqrt{a^2+2abx+b^2x^2}}{2(d+ex)^2(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^3,x]
```

```
[Out] ((a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*(b*d - a*e)*(d + e*x)^2)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^3}\,dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{ab+b^2x}{(d+ex)^3}\,dx}{ab+b^2x}\\ &= \frac{(a+bx)\sqrt{a^2+2abx+b^2x^2}}{2(bd-ae)(d+ex)^2}\end{aligned}$$

Mathematica [A] time = 0.0163794, size = 44, normalized size = 0.96

$$-\frac{\sqrt{(a+bx)^2}(ae+b(d+2ex))}{2e^2(a+bx)(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^3,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(a*e + b*(d + 2*e*x)))/(2*e^2*(a + b*x)*(d + e*x)^2)
```

Maple [A] time = 0.041, size = 41, normalized size = 0.9

$$-\frac{2\,bxe+ae+bd}{2\,\left(ex+d\right)^{2}e^{2}\left(bx+a\right)}\sqrt{\left(bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)/(e*x+d)^3,x)
```

```
[Out] -1/2*(2*b*e*x+a*e+b*d)*((b*x+a)^2)^(1/2)/(e*x+d)^2/e^2/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55283, size = 81, normalized size = 1.76

$$-\frac{2\,bex+bd+ae}{2\left(e^{4}x^{2}+2\,de^{3}x+d^{2}e^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*e*x + b*d + a*e)/(e^4*x^2 + 2*d*e^3*x + d^2*e^2)
```

Sympy [A] time = 0.451762, size = 39, normalized size = 0.85

$$-\frac{ae+bd+2bex}{2d^{2}e^{2}+4de^{3}x+2e^{4}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)/(e*x+d)**3,x)
```

```
[Out] -(a*e + b*d + 2*b*e*x)/(2*d**2*e**2 + 4*d*e**3*x + 2*e**4*x**2)
```

<u>Giac</u> [A] time = 1.13743, size = 59, normalized size = 1.28

$$-\frac{(2\,bxe\mathrm{sgn}\,(bx+a)+bd\mathrm{sgn}\,(bx+a)+ae\mathrm{sgn}\,(bx+a))e^{(-2)}}{2\,(xe+d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x*e*sgn(b*x + a) + b*d*sgn(b*x + a) + a*e*sgn(b*x + a))*e^(-2)/(x
*e + d)^2
```

3.1551 $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^4} dx$

Optimal. Leaf size=92

$$\frac{\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{3e^2(a+bx)(d+ex)^3}-\frac{b\sqrt{a^2+2abx+b^2x^2}}{2e^2(a+bx)(d+ex)^2}$$

```
[Out] ((b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^2*(a + b*x)*(d + e*x)^3) -
 (b*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*e^2*(a + b*x)*(d + e*x)^2)
```

Rubi [A] time = 0.0420961, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$\frac{\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{3e^2(a+bx)(d+ex)^3}-\frac{b\sqrt{a^2+2abx+b^2x^2}}{2e^2(a+bx)(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^4,x]
```

```
[Out] ((b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^2*(a + b*x)*(d + e*x)^3) -
 (b*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*e^2*(a + b*x)*(d + e*x)^2)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^4} dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{ab+b^2x}{(d+ex)^4} dx}{ab+b^2x}\\ &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b(bd-ae)}{e(d+ex)^4}+\frac{b^2}{e(d+ex)^3}\right) dx}{ab+b^2x}\\ &= \frac{(bd-ae)\sqrt{a^2+2abx+b^2x^2}}{3e^2(a+bx)(d+ex)^3}-\frac{b\sqrt{a^2+2abx+b^2x^2}}{2e^2(a+bx)(d+ex)^2}\end{aligned}$$

Mathematica [A] time = 0.0177096, size = 45, normalized size = 0.49

$$-\frac{\sqrt{(a+bx)^2}(2ae+b(d+3ex))}{6e^2(a+bx)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^4,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(2*a*e + b*(d + 3*e*x)))/(6*e^2*(a + b*x)*(d + e*x)^3)
```

Maple [A] time = 0.042, size = 42, normalized size = 0.5

$$-\frac{3\,bxe+2\,ae+bd}{6\,e^{2}\left(ex+d\right)^{3}\left(bx+a\right)}\sqrt{\left(bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)/(e*x+d)^4,x)
```

```
[Out] -1/6/e^2*(3*b*e*x+2*a*e+b*d)*((b*x+a)^2)^(1/2)/(e*x+d)^3/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54475, size = 105, normalized size = 1.14

$$-\frac{3\,bex+bd+2\,ae}{6\left(e^{5}x^{3}+3\,de^{4}x^{2}+3\,d^{2}e^{3}x+d^{3}e^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b*e*x + b*d + 2*a*e)/(e^5*x^3 + 3*d*e^4*x^2 + 3*d^2*e^3*x + d^3*e^2
)
```

Sympy [A] time = 0.536321, size = 53, normalized size = 0.58

$$-\frac{2ae+bd+3bex}{6d^{3}e^{2}+18d^{2}e^{3}x+18de^{4}x^{2}+6e^{5}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] -(2*a*e + b*d + 3*b*e*x)/(6*d**3*e**2 + 18*d**2*e**3*x + 18*d*e**4*x**2 + 6
*e**5*x**3)
```

Giac [A] time = 1.22576, size = 61, normalized size = 0.66

$$-\frac{(3\,bxe\mathrm{sgn}\,(bx+a)+bd\mathrm{sgn}\,(bx+a)+2\,ae\mathrm{sgn}\,(bx+a))e^{(-2)}}{6\,(xe+d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*b*x*e*sgn(b*x + a) + b*d*sgn(b*x + a) + 2*a*e*sgn(b*x + a))*e^(-2)/
(x*e + d)^3
```

3.1552 $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^5}\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{4e^2(a+bx)(d+ex)^4}-\frac{b\sqrt{a^2+2abx+b^2x^2}}{3e^2(a+bx)(d+ex)^3}$$

```
[Out] ((b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*e^2*(a + b*x)*(d + e*x)^4) -
 (b*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^2*(a + b*x)*(d + e*x)^3)
```

Rubi [A] time = 0.0413642, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$\frac{\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{4e^2(a+bx)(d+ex)^4}-\frac{b\sqrt{a^2+2abx+b^2x^2}}{3e^2(a+bx)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^5,x]
```

```
[Out] ((b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*e^2*(a + b*x)*(d + e*x)^4) -
 (b*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^2*(a + b*x)*(d + e*x)^3)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^5}\,dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{ab+b^2x}{(d+ex)^5}\,dx}{ab+b^2x}\\
&= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b(bd-ae)}{e(d+ex)^5}+\frac{b^2}{e(d+ex)^4}\right)dx}{ab+b^2x}\\
&= \frac{(bd-ae)\sqrt{a^2+2abx+b^2x^2}}{4e^2(a+bx)(d+ex)^4}-\frac{b\sqrt{a^2+2abx+b^2x^2}}{3e^2(a+bx)(d+ex)^3}
\end{aligned}$$

Mathematica [A] time = 0.0180968, size = 45, normalized size = 0.49

$$-\frac{\sqrt{(a+bx)^2}(3ae+b(d+4ex))}{12e^2(a+bx)(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^5,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(3*a*e + b*(d + 4*e*x)))/(12*e^2*(a + b*x)*(d + e*x)^4)
```

Maple [A] time = 0.044, size = 42, normalized size = 0.5

$$-\frac{4\,bxe+3\,ae+bd}{12\,e^{2}\left(ex+d\right)^{4}\left(bx+a\right)}\sqrt{\left(bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)/(e*x+d)^5,x)
```

```
[Out] -1/12/e^2*(4*b*e*x+3*a*e+b*d)*((b*x+a)^2)^(1/2)/(e*x+d)^4/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/(e*x+d)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49467, size = 128, normalized size = 1.39

$$-\frac{4\,bex+bd+3\,ae}{12\,\left(e^{6}x^{4}+4\,de^{5}x^{3}+6\,d^{2}e^{4}x^{2}+4\,d^{3}e^{3}x+d^{4}e^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/(e*x+d)^5,x, algorithm="fricas")
```

```
[Out] -1/12*(4*b*e*x + b*d + 3*a*e)/(e^6*x^4 + 4*d*e^5*x^3 + 6*d^2*e^4*x^2 + 4*d^
3*e^3*x + d^4*e^2)
```

Sympy [A] time = 0.653448, size = 65, normalized size = 0.71

$$-\frac{3ae+bd+4bex}{12d^{4}e^{2}+48d^{3}e^{3}x+72d^{2}e^{4}x^{2}+48de^{5}x^{3}+12e^{6}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)/(e*x+d)**5,x)
```

```
[Out] -(3*a*e + b*d + 4*b*e*x)/(12*d**4*e**2 + 48*d**3*e**3*x + 72*d**2*e**4*x**2
 + 48*d*e**5*x**3 + 12*e**6*x**4)
```

Giac [A] time = 1.16593, size = 61, normalized size = 0.66

$$-\frac{(4\,bxe\mathrm{sgn}\,(bx+a)+bd\mathrm{sgn}\,(bx+a)+3\,ae\mathrm{sgn}\,(bx+a))e^{(-2)}}{12\,(xe+d)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] -1/12*(4*b*x*e*sgn(b*x + a) + b*d*sgn(b*x + a) + 3*a*e*sgn(b*x + a))*e^(-2)
/(x*e + d)^4
```

3.1553 $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^6}\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{5e^2(a+bx)(d+ex)^5}-\frac{b\sqrt{a^2+2abx+b^2x^2}}{4e^2(a+bx)(d+ex)^4}$$

```
[Out] ((b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^2*(a + b*x)*(d + e*x)^5) -
 (b*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*e^2*(a + b*x)*(d + e*x)^4)
```

Rubi [A] time = 0.042384, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$\frac{\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{5e^2(a+bx)(d+ex)^5}-\frac{b\sqrt{a^2+2abx+b^2x^2}}{4e^2(a+bx)(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^6,x]
```

```
[Out] ((b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^2*(a + b*x)*(d + e*x)^5) -
 (b*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*e^2*(a + b*x)*(d + e*x)^4)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^6}\,dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{ab+b^2x}{(d+ex)^6}\,dx}{ab+b^2x}\\ &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b(bd-ae)}{e(d+ex)^6}+\frac{b^2}{e(d+ex)^5}\right)dx}{ab+b^2x}\\ &= \frac{(bd-ae)\sqrt{a^2+2abx+b^2x^2}}{5e^2(a+bx)(d+ex)^5}-\frac{b\sqrt{a^2+2abx+b^2x^2}}{4e^2(a+bx)(d+ex)^4}\end{aligned}$$

Mathematica [A] time = 0.019011, size = 45, normalized size = 0.49

$$-\frac{\sqrt{(a+bx)^2}(4ae+b(d+5ex))}{20e^2(a+bx)(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^6,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(4*a*e + b*(d + 5*e*x)))/(20*e^2*(a + b*x)*(d + e*x)^5)
```

Maple [A] time = 0.043, size = 42, normalized size = 0.5

$$-\frac{5\,bxe+4\,ae+bd}{20\,e^{2}\,(ex+d)^{5}\,(bx+a)}\sqrt{(bx+a)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)/(e*x+d)^6,x)
```

```
[Out] -1/20/e^2*(5*b*e*x+4*a*e+b*d)*((b*x+a)^2)^(1/2)/(e*x+d)^5/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/(e*x+d)^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59135, size = 153, normalized size = 1.66

$$-\frac{5\,bex+bd+4\,ae}{20\,\left(e^{7}x^{5}+5\,de^{6}x^{4}+10\,d^{2}e^{5}x^{3}+10\,d^{3}e^{4}x^{2}+5\,d^{4}e^{3}x+d^{5}e^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/(e*x+d)^6,x, algorithm="fricas")
```

```
[Out] -1/20*(5*b*e*x + b*d + 4*a*e)/(e^7*x^5 + 5*d*e^6*x^4 + 10*d^2*e^5*x^3 + 10*
d^3*e^4*x^2 + 5*d^4*e^3*x + d^5*e^2)
```

Sympy [A] time = 0.753685, size = 76, normalized size = 0.83

$$-\frac{4ae+bd+5bex}{20d^{5}e^{2}+100d^{4}e^{3}x+200d^{3}e^{4}x^{2}+200d^{2}e^{5}x^{3}+100de^{6}x^{4}+20e^{7}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)/(e*x+d)**6,x)
```

```
[Out] -(4*a*e + b*d + 5*b*e*x)/(20*d**5*e**2 + 100*d**4*e**3*x + 200*d**3*e**4*x*
*2 + 200*d**2*e**5*x**3 + 100*d*e**6*x**4 + 20*e**7*x**5)
```

Giac [A] time = 1.26325, size = 61, normalized size = 0.66

$$-\frac{(5\,bxe\mathrm{sgn}\,(bx+a)+bd\mathrm{sgn}\,(bx+a)+4\,ae\mathrm{sgn}\,(bx+a))e^{(-2)}}{20\,(xe+d)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/(e*x+d)^6,x, algorithm="giac")
```

```
[Out] -1/20*(5*b*x*e*sgn(b*x + a) + b*d*sgn(b*x + a) + 4*a*e*sgn(b*x + a))*e^(-2)
/(x*e + d)^5
```

3.1554 $\int (d+ex)^5 \left(a^2+2abx+b^2x^2\right)^{3/2} dx$

Optimal. Leaf size=200

$$\frac{b^3\sqrt{a^2+2abx+b^2x^2}(d+ex)^9}{9e^4(a+bx)}-\frac{3b^2\sqrt{a^2+2abx+b^2x^2}(d+ex)^8(bd-ae)}{8e^4(a+bx)}+\frac{3b\sqrt{a^2+2abx+b^2x^2}(d+ex)^7(bd-ae)^2}{7e^4(a+bx)}-$$

```
[Out] -((b*d - a*e)^3*(d + e*x)^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*e^4*(a + b*x)
) + (3*b*(b*d - a*e)^2*(d + e*x)^7*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*e^4*(a
 + b*x)) - (3*b^2*(b*d - a*e)*(d + e*x)^8*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(8
*e^4*(a + b*x)) + (b^3*(d + e*x)^9*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(9*e^4*(a
 + b*x))
```

Rubi [A] time = 0.197707, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$\frac{b^3\sqrt{a^2+2abx+b^2x^2}(d+ex)^9}{9e^4(a+bx)}-\frac{3b^2\sqrt{a^2+2abx+b^2x^2}(d+ex)^8(bd-ae)}{8e^4(a+bx)}+\frac{3b\sqrt{a^2+2abx+b^2x^2}(d+ex)^7(bd-ae)^2}{7e^4(a+bx)}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```


```
[Out] -((b*d - a*e)^3*(d + e*x)^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*e^4*(a + b*x)
) + (3*b*(b*d - a*e)^2*(d + e*x)^7*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*e^4*(a
 + b*x)) - (3*b^2*(b*d - a*e)*(d + e*x)^8*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(8
*e^4*(a + b*x)) + (b^3*(d + e*x)^9*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(9*e^4*(a
 + b*x))
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^5 \left(a^2+2abx+b^2x^2\right)^{3/2} dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(ab+b^2x\right)^3 (d+ex)^5 \, dx}{b^2\left(ab+b^2x\right)} \\
&= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^3(bd-ae)^3(d+ex)^5}{e^3}+\frac{3b^4(bd-ae)^2(d+ex)^6}{e^3}-\frac{3b^5(bd-ae)(d+ex)^7}{e^3}+\frac{b^6(d+}{e^3}\right.}{b^2\left(ab+b^2x\right)} \\
&= -\frac{(bd-ae)^3(d+ex)^6\sqrt{a^2+2abx+b^2x^2}}{6e^4(a+bx)}+\frac{3b(bd-ae)^2(d+ex)^7\sqrt{a^2+2abx+b^2x}}{7e^4(a+bx)}
\end{aligned}$$

Mathematica [A] time = 0.0930337, size = 259, normalized size = 1.3

$$\frac{x\sqrt{(a+bx)^2}\left(36a^2bx\left(105d^3e^2x^2+84d^2e^3x^3+70d^4ex+21d^5+35de^4x^4+6e^5x^5\right)+84a^3\left(20d^3e^2x^2+15d^2e^3x^3+15d^4\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[(a + b*x)^2]*(84*a^3*(6*d^5 + 15*d^4*e*x + 20*d^3*e^2*x^2 + 15*d^2*
e^3*x^3 + 6*d*e^4*x^4 + e^5*x^5) + 36*a^2*b*x*(21*d^5 + 70*d^4*e*x + 105*d^
3*e^2*x^2 + 84*d^2*e^3*x^3 + 35*d*e^4*x^4 + 6*e^5*x^5) + 9*a*b^2*x^2*(56*d^
5 + 210*d^4*e*x + 336*d^3*e^2*x^2 + 280*d^2*e^3*x^3 + 120*d*e^4*x^4 + 21*e^
5*x^5) + b^3*x^3*(126*d^5 + 504*d^4*e*x + 840*d^3*e^2*x^2 + 720*d^2*e^3*x^3
 + 315*d*e^4*x^4 + 56*e^5*x^5)))/(504*(a + b*x))
```

Maple [B] time = 0.156, size = 322, normalized size = 1.6

$$\frac{x\left(56b^3e^5x^8+189x^7b^2ae^5+315x^7b^3de^4+216x^6ba^2e^5+1080x^6b^2ade^4+720x^6b^3d^2e^3+84x^5a^3e^5+1260x^5ba^2de^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] 1/504*x*(56*b^3*e^5*x^8+189*a*b^2*e^5*x^7+315*b^3*d*e^4*x^7+216*a^2*b*e^5*x
^6+1080*a*b^2*d*e^4*x^6+720*b^3*d^2*e^3*x^6+84*a^3*e^5*x^5+1260*a^2*b*d*e^4
*x^5+2520*a*b^2*d^2*e^3*x^5+840*b^3*d^3*e^2*x^5+504*a^3*d*e^4*x^4+3024*a^2*
b*d^2*e^3*x^4+3024*a*b^2*d^3*e^2*x^4+504*b^3*d^4*e*x^4+1260*a^3*d^2*e^3*x^3
+3780*a^2*b*d^3*e^2*x^3+1890*a*b^2*d^4*e*x^3+126*b^3*d^5*x^3+1680*a^3*d^3*e
^2*x^2+2520*a^2*b*d^4*e*x^2+504*a*b^2*d^5*x^2+1260*a^3*d^4*e*x+756*a^2*b*d^
5*x+504*a^3*d^5)*((b*x+a)^2)^(3/2)/(b*x+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51822, size = 585, normalized size = 2.92

$$\frac{1}{9}b^3e^5x^9+a^3d^5x+\frac{1}{8}\left(5b^3de^4+3ab^2e^5\right)x^8+\frac{1}{7}\left(10b^3d^2e^3+15ab^2de^4+3a^2be^5\right)x^7+\frac{1}{6}\left(10b^3d^3e^2+30ab^2d^2e^3+15\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/9*b^3*e^5*x^9 + a^3*d^5*x + 1/8*(5*b^3*d*e^4 + 3*a*b^2*e^5)*x^8 + 1/7*(10
*b^3*d^2*e^3 + 15*a*b^2*d*e^4 + 3*a^2*b*e^5)*x^7 + 1/6*(10*b^3*d^3*e^2 + 30
*a*b^2*d^2*e^3 + 15*a^2*b*d*e^4 + a^3*e^5)*x^6 + (b^3*d^4*e + 6*a*b^2*d^3*e
^2 + 6*a^2*b*d^2*e^3 + a^3*d*e^4)*x^5 + 1/4*(b^3*d^5 + 15*a*b^2*d^4*e + 30*
a^2*b*d^3*e^2 + 10*a^3*d^2*e^3)*x^4 + 1/3*(3*a*b^2*d^5 + 15*a^2*b*d^4*e + 1
0*a^3*d^3*e^2)*x^3 + 1/2*(3*a^2*b*d^5 + 5*a^3*d^4*e)*x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d+ex)^5 \left((a+bx)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x)**5*((a + b*x)**2)**(3/2), x)
```

Giac [B] time = 1.19842, size = 587, normalized size = 2.94

$$\frac{1}{9}b^3x^9e^5\mathrm{sgn}(bx+a)+\frac{5}{8}b^3dx^8e^4\mathrm{sgn}(bx+a)+\frac{10}{7}b^3d^2x^7e^3\mathrm{sgn}(bx+a)+\frac{5}{3}b^3d^3x^6e^2\mathrm{sgn}(bx+a)+b^3d^4x^5e\mathrm{sgn}(bx+a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/9*b^3*x^9*e^5*sgn(b*x + a) + 5/8*b^3*d*x^8*e^4*sgn(b*x + a) + 10/7*b^3*d^
2*x^7*e^3*sgn(b*x + a) + 5/3*b^3*d^3*x^6*e^2*sgn(b*x + a) + b^3*d^4*x^5*e*s
gn(b*x + a) + 1/4*b^3*d^5*x^4*sgn(b*x + a) + 3/8*a*b^2*x^8*e^5*sgn(b*x + a)
 + 15/7*a*b^2*d*x^7*e^4*sgn(b*x + a) + 5*a*b^2*d^2*x^6*e^3*sgn(b*x + a) + 6
*a*b^2*d^3*x^5*e^2*sgn(b*x + a) + 15/4*a*b^2*d^4*x^4*e*sgn(b*x + a) + a*b^2
*d^5*x^3*sgn(b*x + a) + 3/7*a^2*b*x^7*e^5*sgn(b*x + a) + 5/2*a^2*b*d*x^6*e^
4*sgn(b*x + a) + 6*a^2*b*d^2*x^5*e^3*sgn(b*x + a) + 15/2*a^2*b*d^3*x^4*e^2*
sgn(b*x + a) + 5*a^2*b*d^4*x^3*e*sgn(b*x + a) + 3/2*a^2*b*d^5*x^2*sgn(b*x +
 a) + 1/6*a^3*x^6*e^5*sgn(b*x + a) + a^3*d*x^5*e^4*sgn(b*x + a) + 5/2*a^3*d
^2*x^4*e^3*sgn(b*x + a) + 10/3*a^3*d^3*x^3*e^2*sgn(b*x + a) + 5/2*a^3*d^4*x
^2*e*sgn(b*x + a) + a^3*d^5*x*sgn(b*x + a)
```

3.1555 $\int (d+ex)^4 \left(a^2+2abx+b^2x^2\right)^{3/2} dx$

Optimal. Leaf size=200

$$\frac{b^3\sqrt{a^2+2abx+b^2x^2}(d+ex)^8}{8e^4(a+bx)} - \frac{3b^2\sqrt{a^2+2abx+b^2x^2}(d+ex)^7(bd-ae)}{7e^4(a+bx)} + \frac{b\sqrt{a^2+2abx+b^2x^2}(d+ex)^6(bd-ae)^2}{2e^4(a+bx)} -$$

```
[Out] -((b*d - a*e)^3*(d + e*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^4*(a + b*x)
) + (b*(b*d - a*e)^2*(d + e*x)^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*e^4*(a +
 b*x)) - (3*b^2*(b*d - a*e)*(d + e*x)^7*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*e
^4*(a + b*x)) + (b^3*(d + e*x)^8*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(8*e^4*(a +
 b*x))
```

Rubi [A] time = 0.159973, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$\frac{b^3\sqrt{a^2+2abx+b^2x^2}(d+ex)^8}{8e^4(a+bx)} - \frac{3b^2\sqrt{a^2+2abx+b^2x^2}(d+ex)^7(bd-ae)}{7e^4(a+bx)} + \frac{b\sqrt{a^2+2abx+b^2x^2}(d+ex)^6(bd-ae)^2}{2e^4(a+bx)} -$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```


```
[Out] -((b*d - a*e)^3*(d + e*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^4*(a + b*x)
) + (b*(b*d - a*e)^2*(d + e*x)^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*e^4*(a +
 b*x)) - (3*b^2*(b*d - a*e)*(d + e*x)^7*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*e
^4*(a + b*x)) + (b^3*(d + e*x)^8*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(8*e^4*(a +
 b*x))
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^4 \left(a^2+2abx+b^2x^2\right)^{3/2} dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(ab+b^2x\right)^3 (d+ex)^4 dx}{b^2\left(ab+b^2x\right)} \\
&= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^3(bd-ae)^3(d+ex)^4}{e^3} + \frac{3b^4(bd-ae)^2(d+ex)^5}{e^3} - \frac{3b^5(bd-ae)(d+ex)^6}{e^3} + \frac{b}{}\right.}{b^2\left(ab+b^2x\right)} \\
&= -\frac{(bd-ae)^3(d+ex)^5\sqrt{a^2+2abx+b^2x^2}}{5e^4(a+bx)} + \frac{b(bd-ae)^2(d+ex)^6\sqrt{a^2+2abx+b}}{2e^4(a+bx)}
\end{aligned}$$

Mathematica [A] time = 0.076116, size = 215, normalized size = 1.08

$$\frac{x\sqrt{(a+bx)^2}\left(28a^2bx\left(45d^2e^2x^2+40d^3ex+15d^4+24de^3x^3+5e^4x^4\right)+56a^3\left(10d^2e^2x^2+10d^3ex+5d^4+5de^3x^3+e^4x^4\right)\cdots\right.}{280(a\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[(a + b*x)^2]*(56*a^3*(5*d^4 + 10*d^3*e*x + 10*d^2*e^2*x^2 + 5*d*e^3
*x^3 + e^4*x^4) + 28*a^2*b*x*(15*d^4 + 40*d^3*e*x + 45*d^2*e^2*x^2 + 24*d*e
^3*x^3 + 5*e^4*x^4) + 8*a*b^2*x^2*(35*d^4 + 105*d^3*e*x + 126*d^2*e^2*x^2 +
 70*d*e^3*x^3 + 15*e^4*x^4) + b^3*x^3*(70*d^4 + 224*d^3*e*x + 280*d^2*e^2*x
^2 + 160*d*e^3*x^3 + 35*e^4*x^4)))/(280*(a + b*x))
```

Maple [A] time = 0.154, size = 264, normalized size = 1.3

$$\frac{x\left(35\,b^3e^4x^7+120\,x^6b^2ae^4+160\,x^6b^3de^3+140\,x^5ba^2e^4+560\,x^5b^2ade^3+280\,x^5b^3d^2e^2+56\,x^4a^3e^4+672\,x^4ba^2de^3+100\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] 1/280*x*(35*b^3*e^4*x^7+120*a*b^2*e^4*x^6+160*b^3*d*e^3*x^6+140*a^2*b*e^4*x
^5+560*a*b^2*d*e^3*x^5+280*b^3*d^2*e^2*x^5+56*a^3*e^4*x^4+672*a^2*b*d*e^3*x
^4+1008*a*b^2*d^2*e^2*x^4+224*b^3*d^3*e*x^4+280*a^3*d*e^3*x^3+1260*a^2*b*d^
2*e^2*x^3+840*a*b^2*d^3*e*x^3+70*b^3*d^4*x^3+560*a^3*d^2*e^2*x^2+1120*a^2*b
*d^3*e*x^2+280*a*b^2*d^4*x^2+560*a^3*d^3*e*x+420*a^2*b*d^4*x+280*a^3*d^4)*(
(b*x+a)^2)^(3/2)/(b*x+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57092, size = 471, normalized size = 2.36

$$\frac{1}{8}\,b^3e^4x^8+a^3d^4x+\frac{1}{7}\left(4\,b^3de^3+3\,ab^2e^4\right)x^7+\frac{1}{2}\left(2\,b^3d^2e^2+4\,ab^2de^3+a^2be^4\right)x^6+\frac{1}{5}\left(4\,b^3d^3e+18\,ab^2d^2e^2+12\,a^2bde^3+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*b^3*e^4*x^8 + a^3*d^4*x + 1/7*(4*b^3*d*e^3 + 3*a*b^2*e^4)*x^7 + 1/2*(2*
b^3*d^2*e^2 + 4*a*b^2*d*e^3 + a^2*b*e^4)*x^6 + 1/5*(4*b^3*d^3*e + 18*a*b^2*
```

```
d^2*e^2 + 12*a^2*b*d*e^3 + a^3*e^4)*x^5 + 1/4*(b^3*d^4 + 12*a*b^2*d^3*e + 1
8*a^2*b*d^2*e^2 + 4*a^3*d*e^3)*x^4 + (a*b^2*d^4 + 4*a^2*b*d^3*e + 2*a^3*d^2
*e^2)*x^3 + 1/2*(3*a^2*b*d^4 + 4*a^3*d^3*e)*x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d+ex)^4 \left((a+bx)^2\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x)**4*((a + b*x)**2)**(3/2), x)
```

Giac [B] time = 1.17076, size = 482, normalized size = 2.41

$$\frac{1}{8}b^3x^8e^4\text{sgn}(bx+a)+\frac{4}{7}b^3dx^7e^3\text{sgn}(bx+a)+b^3d^2x^6e^2\text{sgn}(bx+a)+\frac{4}{5}b^3d^3x^5e\text{sgn}(bx+a)+\frac{1}{4}b^3d^4x^4\text{sgn}(bx+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*b^3*x^8*e^4*sgn(b*x + a) + 4/7*b^3*d*x^7*e^3*sgn(b*x + a) + b^3*d^2*x^6
*e^2*sgn(b*x + a) + 4/5*b^3*d^3*x^5*e*sgn(b*x + a) + 1/4*b^3*d^4*x^4*sgn(b*
x + a) + 3/7*a*b^2*x^7*e^4*sgn(b*x + a) + 2*a*b^2*d*x^6*e^3*sgn(b*x + a) +
18/5*a*b^2*d^2*x^5*e^2*sgn(b*x + a) + 3*a*b^2*d^3*x^4*e*sgn(b*x + a) + a*b^
2*d^4*x^3*sgn(b*x + a) + 1/2*a^2*b*x^6*e^4*sgn(b*x + a) + 12/5*a^2*b*d*x^5*
e^3*sgn(b*x + a) + 9/2*a^2*b*d^2*x^4*e^2*sgn(b*x + a) + 4*a^2*b*d^3*x^3*e*s
gn(b*x + a) + 3/2*a^2*b*d^4*x^2*sgn(b*x + a) + 1/5*a^3*x^5*e^4*sgn(b*x + a)
 + a^3*d*x^4*e^3*sgn(b*x + a) + 2*a^3*d^2*x^3*e^2*sgn(b*x + a) + 2*a^3*d^3*
x^2*e*sgn(b*x + a) + a^3*d^4*x*sgn(b*x + a)
```

3.1556 $\int (d+ex)^3 \left(a^2+2abx+b^2x^2\right)^{3/2} dx$

Optimal. Leaf size=172

$$\frac{e^2\sqrt{a^2+2abx+b^2x^2}(a+bx)^5(bd-ae)}{2b^4}+\frac{3e\sqrt{a^2+2abx+b^2x^2}(a+bx)^4(bd-ae)^2}{5b^4}+\frac{\sqrt{a^2+2abx+b^2x^2}(a+bx)^3(bd-ae)}{4b^4}$$

```
[Out] ((b*d - a*e)^3*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*b^4) + (3*e*(b
*d - a*e)^2*(a + b*x)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*b^4) + (e^2*(b*d
- a*e)*(a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*b^4) + (e^3*(a + b*x)^
6*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*b^4)
```

Rubi [A] time = 0.131752, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$\frac{e^2\sqrt{a^2+2abx+b^2x^2}(a+bx)^5(bd-ae)}{2b^4}+\frac{3e\sqrt{a^2+2abx+b^2x^2}(a+bx)^4(bd-ae)^2}{5b^4}+\frac{\sqrt{a^2+2abx+b^2x^2}(a+bx)^3(bd-ae)}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```


```
[Out] ((b*d - a*e)^3*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*b^4) + (3*e*(b
*d - a*e)^2*(a + b*x)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*b^4) + (e^2*(b*d
- a*e)*(a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*b^4) + (e^3*(a + b*x)^
6*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*b^4)
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^3 \left(a^2+2abx+b^2x^2\right)^{3/2} dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(ab+b^2x\right)^3 (d+ex)^3 dx}{b^2\left(ab+b^2x\right)}\\
&= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{(bd-ae)^3\left(ab+b^2x\right)^3}{b^3}+\frac{3e(bd-ae)^2\left(ab+b^2x\right)^4}{b^4}+\frac{3e^2(bd-ae)\left(ab+b^2x\right)^5}{b^5}+\frac{e^3(}{}\right.}{b^2\left(ab+b^2x\right)}\\
&= \frac{(bd-ae)^3(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{4b^4}+\frac{3e(bd-ae)^2(a+bx)^4\sqrt{a^2+2abx+b^2x^2}}{5b^4}
\end{aligned}$$

Mathematica [A] time = 0.0619149, size = 171, normalized size = 0.99

$$\frac{x\sqrt{(a+bx)^2}\left(21a^2bx\left(20d^2ex+10d^3+15de^2x^2+4e^3x^3\right)+35a^3\left(6d^2ex+4d^3+4de^2x^2+e^3x^3\right)+7ab^2x^2\left(45d^2ex+20\right.\right.}{140(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[(a + b*x)^2]*(35*a^3*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) +
21*a^2*b*x*(10*d^3 + 20*d^2*e*x + 15*d*e^2*x^2 + 4*e^3*x^3) + 7*a*b^2*x^2*(
20*d^3 + 45*d^2*e*x + 36*d*e^2*x^2 + 10*e^3*x^3) + b^3*x^3*(35*d^3 + 84*d^2
*e*x + 70*d*e^2*x^2 + 20*e^3*x^3)))/(140*(a + b*x))
```

Maple [A] time = 0.155, size = 206, normalized size = 1.2

$$\frac{x\left(20\,b^3e^3x^6+70\,x^5b^2ae^3+70\,x^5b^3de^2+84\,x^4ba^2e^3+252\,x^4b^2ade^2+84\,x^4b^3d^2e+35\,x^3a^3e^3+315\,x^3ba^2de^2+315\,x^3\right.}{140\,(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] 1/140*x*(20*b^3*e^3*x^6+70*a*b^2*e^3*x^5+70*b^3*d*e^2*x^5+84*a^2*b*e^3*x^4+
252*a*b^2*d*e^2*x^4+84*b^3*d^2*e*x^4+35*a^3*e^3*x^3+315*a^2*b*d*e^2*x^3+315
*a*b^2*d^2*e*x^3+35*b^3*d^3*x^3+140*a^3*d*e^2*x^2+420*a^2*b*d^2*e*x^2+140*a
*b^2*d^3*x^2+210*a^3*d^2*e*x+210*a^2*b*d^3*x+140*a^3*d^3)*((b*x+a)^2)^(3/2)
/(b*x+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5251, size = 344, normalized size = 2.

$$\frac{1}{7}\,b^3e^3x^7+a^3d^3x+\frac{1}{2}\left(b^3de^2+ab^2e^3\right)x^6+\frac{3}{5}\left(b^3d^2e+3\,ab^2de^2+a^2be^3\right)x^5+\frac{1}{4}\left(b^3d^3+9\,ab^2d^2e+9\,a^2bde^2+a^3e^3\right)x^4+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/7*b^3*e^3*x^7 + a^3*d^3*x + 1/2*(b^3*d*e^2 + a*b^2*e^3)*x^6 + 3/5*(b^3*d^
2*e + 3*a*b^2*d*e^2 + a^2*b*e^3)*x^5 + 1/4*(b^3*d^3 + 9*a*b^2*d^2*e + 9*a^2
*b*d*e^2 + a^3*e^3)*x^4 + (a*b^2*d^3 + 3*a^2*b*d^2*e + a^3*d*e^2)*x^3 + 3/2
```

```
*(a^2*b*d^3 + a^3*d^2*e)*x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d+ex)^3 \left((a+bx)^2\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x)**3*((a + b*x)**2)**(3/2), x)
```

Giac [B] time = 1.15484, size = 378, normalized size = 2.2

$$\frac{1}{7}b^3x^7e^3\text{sgn}(bx+a) + \frac{1}{2}b^3dx^6e^2\text{sgn}(bx+a) + \frac{3}{5}b^3d^2x^5e\text{sgn}(bx+a) + \frac{1}{4}b^3d^3x^4\text{sgn}(bx+a) + \frac{1}{2}ab^2x^6e^3\text{sgn}(bx+a) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/7*b^3*x^7*e^3*sgn(b*x + a) + 1/2*b^3*d*x^6*e^2*sgn(b*x + a) + 3/5*b^3*d^2
*x^5*e*sgn(b*x + a) + 1/4*b^3*d^3*x^4*sgn(b*x + a) + 1/2*a*b^2*x^6*e^3*sgn(
b*x + a) + 9/5*a*b^2*d*x^5*e^2*sgn(b*x + a) + 9/4*a*b^2*d^2*x^4*e*sgn(b*x +
 a) + a*b^2*d^3*x^3*sgn(b*x + a) + 3/5*a^2*b*x^5*e^3*sgn(b*x + a) + 9/4*a^2
*b*d*x^4*e^2*sgn(b*x + a) + 3*a^2*b*d^2*x^3*e*sgn(b*x + a) + 3/2*a^2*b*d^3*
x^2*sgn(b*x + a) + 1/4*a^3*x^4*e^3*sgn(b*x + a) + a^3*d*x^3*e^2*sgn(b*x + a
) + 3/2*a^3*d^2*x^2*e*sgn(b*x + a) + a^3*d^3*x*sgn(b*x + a)
```

3.1557 $\int (d+ex)^2 \left(a^2+2abx+b^2x^2\right)^{3/2} dx$

Optimal. Leaf size=114

$$\frac{2e\left(a^2+2abx+b^2x^2\right)^{5/2}(bd-ae)}{5b^3}+\frac{(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}(bd-ae)^2}{4b^3}+\frac{e^2(a+bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{6b^3}$$

```
[Out] ((b*d - a*e)^2*(a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/(4*b^3) + (2*e*(b
*d - a*e)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/(5*b^3) + (e^2*(a + b*x)*(a^2 +
2*a*b*x + b^2*x^2)^(5/2))/(6*b^3)
```

Rubi [A] time = 0.0499471, antiderivative size = 125, normalized size of antiderivative = 1.1, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {645}

$$\frac{2e\sqrt{a^2+2abx+b^2x^2}(a+bx)^4(bd-ae)}{5b^3}+\frac{\sqrt{a^2+2abx+b^2x^2}(a+bx)^3(bd-ae)^2}{4b^3}+\frac{e^2\sqrt{a^2+2abx+b^2x^2}(a+bx)^5}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] ((b*d - a*e)^2*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*b^3) + (2*e*(b
*d - a*e)*(a + b*x)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*b^3) + (e^2*(a + b*
x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*b^3)
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[ExpandLinearProduct[(b/2 + c*x)^(2*p), (d + e*x)^m, b/2, c
, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&
!IntegerQ[p] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 0] && EqQ[m - 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^2\left(a^2+2abx+b^2x^2\right)^{3/2}dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int\left(\frac{(bd-ae)^2\left(ab+b^2x\right)^3}{b^2}+\frac{2e(bd-ae)\left(ab+b^2x\right)^4}{b^3}+\frac{e^2\left(ab+b^2x\right)^5}{b^4}\right)dx}{b^2\left(ab+b^2x\right)}\\ &= \frac{(bd-ae)^2(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{4b^3}+\frac{2e(bd-ae)(a+bx)^4\sqrt{a^2+2abx+b^2}}{5b^3}\end{aligned}$$

Mathematica [A] time = 0.0449353, size = 127, normalized size = 1.11

$$\frac{x\sqrt{(a+bx)^2}\left(15a^2bx\left(6d^2+8dex+3e^2x^2\right)+20a^3\left(3d^2+3dex+e^2x^2\right)+6ab^2x^2\left(10d^2+15dex+6e^2x^2\right)+b^3x^3\left(15d^2\right.\right.}{60(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[(a + b*x)^2]*(20*a^3*(3*d^2 + 3*d*e*x + e^2*x^2) + 15*a^2*b*x*(6*d^
2 + 8*d*e*x + 3*e^2*x^2) + 6*a*b^2*x^2*(10*d^2 + 15*d*e*x + 6*e^2*x^2) + b^
3*x^3*(15*d^2 + 24*d*e*x + 10*e^2*x^2)))/(60*(a + b*x))
```

Maple [A] time = 0.154, size = 148, normalized size = 1.3

$$\frac{x\left(10\,b^3e^2x^5+36\,x^4e^2b^2a+24\,x^4deb^3+45\,x^3e^2ba^2+90\,x^3deb^2a+15\,x^3d^2b^3+20\,x^2e^2a^3+120\,x^2deba^2+60\,x^2ab^2d^2+6\right.}{60\,(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] 1/60*x*(10*b^3*e^2*x^5+36*a*b^2*e^2*x^4+24*b^3*d*e*x^4+45*a^2*b*e^2*x^3+90*
a*b^2*d*e*x^3+15*b^3*d^2*x^3+20*a^3*e^2*x^2+120*a^2*b*d*e*x^2+60*a*b^2*d^2*
x^2+60*a^3*d*e*x+90*a^2*b*d^2*x+60*a^3*d^2)*((b*x+a)^2)^(3/2)/(b*x+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.44664, size = 266, normalized size = 2.33

$$\frac{1}{6}\,b^3e^2x^6+a^3d^2x+\frac{1}{5}\left(2\,b^3de+3\,ab^2e^2\right)x^5+\frac{1}{4}\left(b^3d^2+6\,ab^2de+3\,a^2be^2\right)x^4+\frac{1}{3}\left(3\,ab^2d^2+6\,a^2bde+a^3e^2\right)x^3+\frac{1}{2}\left(3\,a^2ba\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*b^3*e^2*x^6 + a^3*d^2*x + 1/5*(2*b^3*d*e + 3*a*b^2*e^2)*x^5 + 1/4*(b^3*
d^2 + 6*a*b^2*d*e + 3*a^2*b*e^2)*x^4 + 1/3*(3*a*b^2*d^2 + 6*a^2*b*d*e + a^3
*e^2)*x^3 + 1/2*(3*a^2*b*d^2 + 2*a^3*d*e)*x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d+ex)^2\left((a+bx)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x)**2*((a + b*x)**2)**(3/2), x)
```

Giac [A] time = 1.173, size = 273, normalized size = 2.39

$$\frac{1}{6}\,b^3x^6e^2\mathrm{sgn}\,(bx+a)+\frac{2}{5}\,b^3dx^5e\mathrm{sgn}\,(bx+a)+\frac{1}{4}\,b^3d^2x^4\mathrm{sgn}\,(bx+a)+\frac{3}{5}\,ab^2x^5e^2\mathrm{sgn}\,(bx+a)+\frac{3}{2}\,ab^2dx^4e\mathrm{sgn}\,(bx+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/6*b^3*x^6*e^2*sgn(b*x + a) + 2/5*b^3*d*x^5*e*sgn(b*x + a) + 1/4*b^3*d^2*x
^4*sgn(b*x + a) + 3/5*a*b^2*x^5*e^2*sgn(b*x + a) + 3/2*a*b^2*d*x^4*e*sgn(b*
x + a) + a*b^2*d^2*x^3*sgn(b*x + a) + 3/4*a^2*b*x^4*e^2*sgn(b*x + a) + 2*a^
2*b*d*x^3*e*sgn(b*x + a) + 3/2*a^2*b*d^2*x^2*sgn(b*x + a) + 1/3*a^3*x^3*e^2
*sgn(b*x + a) + a^3*d*x^2*e*sgn(b*x + a) + a^3*d^2*x*sgn(b*x + a)
```

3.1558 $\int (d+ex)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=69

$$\frac{(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}(bd-ae)}{4b^2}+\frac{e\left(a^2+2abx+b^2x^2\right)^{5/2}}{5b^2}$$

```
[Out] ((b*d - a*e)*(a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/(4*b^2) + (e*(a^2 +
 2*a*b*x + b^2*x^2)^(5/2))/(5*b^2)
```

Rubi [A] time = 0.0211858, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {640, 609}

$$\frac{(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}(bd-ae)}{4b^2}+\frac{e\left(a^2+2abx+b^2x^2\right)^{5/2}}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] ((b*d - a*e)*(a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/(4*b^2) + (e*(a^2 +
 2*a*b*x + b^2*x^2)^(5/2))/(5*b^2)
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx &= \frac{e\left(a^2+2abx+b^2x^2\right)^{5/2}}{5b^2}+\frac{\left(2b^2d-2abe\right)\int \left(a^2+2abx+b^2x^2\right)^{3/2}\,dx}{2b^2}\\ &= \frac{(bd-ae)(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{4b^2}+\frac{e\left(a^2+2abx+b^2x^2\right)^{5/2}}{5b^2}\end{aligned}$$

Mathematica [A] time = 0.032854, size = 83, normalized size = 1.2

$$\frac{x\sqrt{(a+bx)^2}\left(10a^2bx(3d+2ex)+10a^3(2d+ex)+5ab^2x^2(4d+3ex)+b^3x^3(5d+4ex)\right)}{20(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[(a + b*x)^2]*(10*a^3*(2*d + e*x) + 10*a^2*b*x*(3*d + 2*e*x) + 5*a*b
^2*x^2*(4*d + 3*e*x) + b^3*x^3*(5*d + 4*e*x)))/(20*(a + b*x))
```

Maple [A] time = 0.178, size = 90, normalized size = 1.3

$$\frac{x\left(4\,e b^3 x^4+15\,x^3 e b^2 a+5\,x^3 d b^3+20\,a^2 b e x^2+20\,a b^2 d x^2+10\,a^3 e x+30\,x d b a^2+20\,d a^3\right)}{20\,(bx+a)^3}\left((bx+a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] 1/20*x*(4*b^3*e*x^4+15*a*b^2*e*x^3+5*b^3*d*x^3+20*a^2*b*e*x^2+20*a*b^2*d*x^
2+10*a^3*e*x+30*a^2*b*d*x+20*a^3*d)*((b*x+a)^2)^(3/2)/(b*x+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52932, size = 150, normalized size = 2.17

$$\frac{1}{5}\,b^3 e x^5+a^3 d x+\frac{1}{4}\left(b^3 d+3\,a b^2 e\right)x^4+\left(a b^2 d+a^2 b e\right)x^3+\frac{1}{2}\left(3\,a^2 b d+a^3 e\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*b^3*e*x^5 + a^3*d*x + 1/4*(b^3*d + 3*a*b^2*e)*x^4 + (a*b^2*d + a^2*b*e)
*x^3 + 1/2*(3*a^2*b*d + a^3*e)*x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d+ex)\left((a+bx)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x)*((a + b*x)**2)**(3/2), x)
```

Giac [B] time = 1.13498, size = 167, normalized size = 2.42

$$\frac{1}{5}b^3x^5e\mathrm{sgn}(bx+a)+\frac{1}{4}b^3dx^4\mathrm{sgn}(bx+a)+\frac{3}{4}ab^2x^4e\mathrm{sgn}(bx+a)+ab^2dx^3\mathrm{sgn}(bx+a)+a^2bx^3e\mathrm{sgn}(bx+a)+\frac{3}{2}a^2bdx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/5*b^3*x^5*e*sgn(b*x + a) + 1/4*b^3*d*x^4*sgn(b*x + a) + 3/4*a*b^2*x^4*e*s
gn(b*x + a) + a*b^2*d*x^3*sgn(b*x + a) + a^2*b*x^3*e*sgn(b*x + a) + 3/2*a^2
*b*d*x^2*sgn(b*x + a) + 1/2*a^3*x^2*e*sgn(b*x + a) + a^3*d*x*sgn(b*x + a)
```

3.1559 $\int \left(a^2+2abx+b^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=32

$$\frac{(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{4b}$$

```
[Out] ((a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/(4*b)
```

Rubi [A] time = 0.0049789, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {609}

$$\frac{(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] ((a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/(4*b)
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{4b}$$

Mathematica [A] time = 0.0088064, size = 23, normalized size = 0.72

$$\frac{(a+bx)\left((a+bx)^2\right)^{3/2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] ((a + b*x)*((a + b*x)^2)^(3/2))/(4*b)
```

Maple [A] time = 0.039, size = 49, normalized size = 1.5

$$\frac{x\left(b^3x^3+4\,ab^2x^2+6\,a^2bx+4\,a^3\right)}{4\,(bx+a)^3}\left((bx+a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] 1/4*x*(b^3*x^3+4*a*b^2*x^2+6*a^2*b*x+4*a^3)*((b*x+a)^2)^(3/2)/(b*x+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.42429, size = 66, normalized size = 2.06

$$\frac{1}{4}\,b^3x^4+ab^2x^3+\frac{3}{2}\,a^2bx^2+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2+2abx+b^2x^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x + b**2*x**2)**(3/2), x)
```

Giac [B] time = 1.11532, size = 93, normalized size = 2.91

$$\frac{1}{4}\,b^3x^4\mathrm{sgn}\left(bx+a\right)+ab^2x^3\mathrm{sgn}\left(bx+a\right)+\frac{3}{2}\,a^2bx^2\mathrm{sgn}\left(bx+a\right)+a^3x\mathrm{sgn}\left(bx+a\right)+\frac{a^4\mathrm{sgn}\left(bx+a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*b^3*x^4*sgn(b*x + a) + a*b^2*x^3*sgn(b*x + a) + 3/2*a^2*b*x^2*sgn(b*x +
 a) + a^3*x*sgn(b*x + a) + 1/4*a^4*sgn(b*x + a)/b
```

3.1560 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=166

$$\frac{bx\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{e^3(a+bx)}-\frac{(a+bx)\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{2e^2}-\frac{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3\log(d+ex)}{e^4(a+bx)}+$$

```
[Out] (b*(b*d - a*e)^2*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^3*(a + b*x)) - ((b*d -
 a*e)*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*e^2) + ((a + b*x)^2*Sqrt[
a^2 + 2*a*b*x + b^2*x^2])/(3*e) - ((b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x
^2]*Log[d + e*x])/(e^4*(a + b*x))
```

Rubi [A] time = 0.0678044, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$\frac{bx\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{e^3(a+bx)}-\frac{(a+bx)\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{2e^2}-\frac{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3\log(d+ex)}{e^4(a+bx)}+$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] (b*(b*d - a*e)^2*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^3*(a + b*x)) - ((b*d -
 a*e)*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*e^2) + ((a + b*x)^2*Sqrt[
a^2 + 2*a*b*x + b^2*x^2])/(3*e) - ((b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x
^2]*Log[d + e*x])/(e^4*(a + b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{d+ex}\,dx&=\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^3}{d+ex}\,dx}{b^2\left(ab+b^2x\right)}\\&=\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{b^4(bd-ae)^2}{e^3}-\frac{b^3(bd-ae)\left(ab+b^2x\right)}{e^2}+\frac{b^2\left(ab+b^2x\right)^2}{e}-\frac{b^3(bd-ae)^3}{e^3(d+ex)}\right)dx}{b^2\left(ab+b^2x\right)}\\&=\frac{b(bd-ae)^2x\sqrt{a^2+2abx+b^2x^2}}{e^3(a+bx)}-\frac{(bd-ae)(a+bx)\sqrt{a^2+2abx+b^2x^2}}{2e^2}+\frac{(a+bx)^2\sqrt{a^2}}{}\end{aligned}$$

Mathematica [A] time = 0.0464589, size = 92, normalized size = 0.55

$$\frac{\sqrt{(a+bx)^2}\left(bex\left(18a^2e^2+9abe(ex-2d)+b^2\left(6d^2-3dex+2e^2x^2\right)\right)-6(bd-ae)^3\log(d+ex)\right)}{6e^4(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(b*e*x*(18*a^2*e^2 + 9*a*b*e*(-2*d + e*x) + b^2*(6*d^2 -
 3*d*e*x + 2*e^2*x^2)) - 6*(b*d - a*e)^3*Log[d + e*x]))/(6*e^4*(a + b*x))
```

Maple [A] time = 0.199, size = 149, normalized size = 0.9

$$\frac{2\,x^3b^3e^3+9\,x^2ab^2e^3-3\,x^2b^3de^2+6\,\ln(ex+d)\,a^3e^3-18\,\ln(ex+d)\,a^2bde^2+18\,\ln(ex+d)\,ab^2d^2e-6\,\ln(ex+d)\,b^3d^3+}{6\,(bx+a)^3\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d),x)
```

```
[Out] 1/6*((b*x+a)^2)^(3/2)*(2*x^3*b^3*e^3+9*x^2*a*b^2*e^3-3*x^2*b^3*d*e^2+6*ln(e
*x+d)*a^3*e^3-18*ln(e*x+d)*a^2*b*d*e^2+18*ln(e*x+d)*a*b^2*d^2*e-6*ln(e*x+d)
*b^3*d^3+18*x*a^2*b*e^3-18*x*a*b^2*d*e^2+6*x*b^3*d^2*e)/(b*x+a)^3/e^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.61041, size = 238, normalized size = 1.43

$$\frac{2\,b^3e^3x^3-3\left(b^3de^2-3\,ab^2e^3\right)x^2+6\left(b^3d^2e-3\,ab^2de^2+3\,a^2be^3\right)x-6\left(b^3d^3-3\,ab^2d^2e+3\,a^2bde^2-a^3e^3\right)\log(ex+d)}{6\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*e^3*x^3 - 3*(b^3*d*e^2 - 3*a*b^2*e^3)*x^2 + 6*(b^3*d^2*e - 3*a*b
^2*d*e^2 + 3*a^2*b*e^3)*x - 6*(b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^
3*e^3)*log(e*x + d))/e^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((a+bx)^2\right)^{\frac{3}{2}}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/(e*x+d),x)
```

```
[Out] Integral(((a + b*x)**2)**(3/2)/(d + e*x), x)
```

Giac [A] time = 1.21871, size = 234, normalized size = 1.41

$$-\left(b^3d^3\mathrm{sgn}\,(bx+a) - 3\,ab^2d^2e\mathrm{sgn}\,(bx+a) + 3\,a^2bde^2\mathrm{sgn}\,(bx+a) - a^3e^3\mathrm{sgn}\,(bx+a)\right)e^{(-4)}\log\left(|xe+d|\right) + \frac{1}{6}\left(2\,b^3x^3e^2\mathrm{s}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] -(b^3*d^3*sgn(b*x + a) - 3*a*b^2*d^2*e*sgn(b*x + a) + 3*a^2*b*d*e^2*sgn(b*x
 + a) - a^3*e^3*sgn(b*x + a))*e^(-4)*log(abs(x*e + d)) + 1/6*(2*b^3*x^3*e^2
*sgn(b*x + a) - 3*b^3*d*x^2*e*sgn(b*x + a) + 6*b^3*d^2*x*sgn(b*x + a) + 9*a
*b^2*x^2*e^2*sgn(b*x + a) - 18*a*b^2*d*x*e*sgn(b*x + a) + 18*a^2*b*x*e^2*sg
n(b*x + a))*e^(-3)
```

3.1561 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{(d+ex)^2} \, dx$

Optimal. Leaf size=183

$$-\frac{b^2x\sqrt{a^2+2abx+b^2x^2}(2bd-3ae)}{e^3(a+bx)}+\frac{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3}{e^4(a+bx)(d+ex)}+\frac{3b\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2\log(d+ex)}{e^4(a+bx)}+\frac{b^3x^2\cdots}{\cdots}$$

```
[Out] -((b^2*(2*b*d - 3*a*e)*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^3*(a + b*x))) +
(b^3*x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*e^2*(a + b*x)) + ((b*d - a*e)^3*
Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^4*(a + b*x)*(d + e*x)) + (3*b*(b*d - a*e)
^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Log[d + e*x])/(e^4*(a + b*x))
```

Rubi [A] time = 0.0922766, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$-\frac{b^2x\sqrt{a^2+2abx+b^2x^2}(2bd-3ae)}{e^3(a+bx)}+\frac{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3}{e^4(a+bx)(d+ex)}+\frac{3b\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2\log(d+ex)}{e^4(a+bx)}+\frac{b^3x^2\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^2,x]
```

```
[Out] -((b^2*(2*b*d - 3*a*e)*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^3*(a + b*x))) +
(b^3*x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*e^2*(a + b*x)) + ((b*d - a*e)^3*
Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^4*(a + b*x)*(d + e*x)) + (3*b*(b*d - a*e)
^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Log[d + e*x])/(e^4*(a + b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{(d+ex)^2} \, dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^3}{(d+ex)^2} \, dx}{b^2\left(ab+b^2x\right)} \\
&= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^5(2bd-3ae)}{e^3}+\frac{b^6x}{e^2}-\frac{b^3(bd-ae)^3}{e^3(d+ex)^2}+\frac{3b^4(bd-ae)^2}{e^3(d+ex)}\right) dx}{b^2\left(ab+b^2x\right)} \\
&= -\frac{b^2(2bd-3ae)x\sqrt{a^2+2abx+b^2x^2}}{e^3(a+bx)}+\frac{b^3x^2\sqrt{a^2+2abx+b^2x^2}}{2e^2(a+bx)}+\frac{(bd-ae)^3\sqrt{a^2+2abx+\cdots}}{e^4(a+bx)(d+ex)}
\end{aligned}$$

Mathematica [A] time = 0.0900179, size = 132, normalized size = 0.72

$$\frac{\sqrt{(a+bx)^2}\left(6a^2bde^2-2a^3e^3+6ab^2e\left(-d^2+dex+e^2x^2\right)+6b(d+ex)(bd-ae)^2\log(d+ex)+b^3\left(-4d^2ex+2d^3-3de^2\right.\right.}{2e^4(a+bx)(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^2,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(6*a^2*b*d*e^2 - 2*a^3*e^3 + 6*a*b^2*e*(-d^2 + d*e*x + e
^2*x^2) + b^3*(2*d^3 - 4*d^2*e*x - 3*d*e^2*x^2 + e^3*x^3) + 6*b*(b*d - a*e)
^2*(d + e*x)*Log[d + e*x]))/(2*e^4*(a + b*x)*(d + e*x))
```

Maple [A] time = 0.202, size = 216, normalized size = 1.2

$$\frac{x^3b^3e^3+6\ln(ex+d)xa^2be^3-12\ln(ex+d)xab^2de^2+6\ln(ex+d)xb^3d^2e+6x^2ab^2e^3-3x^2b^3de^2+6\ln(ex+d)a^2}{2(bx+a)^3e^4(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^2,x)
```

```
[Out] 1/2*((b*x+a)^2)^(3/2)*(x^3*b^3*e^3+6*ln(e*x+d)*x*a^2*b*e^3-12*ln(e*x+d)*x*a
*b^2*d*e^2+6*ln(e*x+d)*x*b^3*d^2*e+6*x^2*a*b^2*e^3-3*x^2*b^3*d*e^2+6*ln(e*x
+d)*a^2*b*d*e^2-12*ln(e*x+d)*a*b^2*d^2*e+6*ln(e*x+d)*b^3*d^3+6*x*a*b^2*d*e^
2-4*x*b^3*d^2*e-2*a^3*e^3+6*d*e^2*a^2*b-6*a*b^2*d^2*e+2*b^3*d^3)/(b*x+a)^3/
e^4/(e*x+d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49036, size = 354, normalized size = 1.93

$$\frac{b^3e^3x^3+2b^3d^3-6ab^2d^2e+6a^2bde^2-2a^3e^3-3\left(b^3de^2-2ab^2e^3\right)x^2-2\left(2b^3d^2e-3ab^2de^2\right)x+6\left(b^3d^3-2ab^2d^2e+\right.}{2\left(e^5x+de^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*e^3*x^3 + 2*b^3*d^3 - 6*a*b^2*d^2*e + 6*a^2*b*d*e^2 - 2*a^3*e^3 -
3*(b^3*d*e^2 - 2*a*b^2*e^3)*x^2 - 2*(2*b^3*d^2*e - 3*a*b^2*d*e^2)*x + 6*(b^
3*d^3 - 2*a*b^2*d^2*e + a^2*b*d*e^2 + (b^3*d^2*e - 2*a*b^2*d*e^2 + a^2*b*e^
```

```
3)*x)*log(e*x + d))/(e^5*x + d*e^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17053, size = 236, normalized size = 1.29

$$3\left(b^3d^2\mathrm{sgn}\left(bx+a\right)-2\,ab^2de\mathrm{sgn}\left(bx+a\right)+a^2be^2\mathrm{sgn}\left(bx+a\right)\right)e^{(-4)}\log\left(|xe+d|\right)+\frac{1}{2}\left(b^3x^2e^2\mathrm{sgn}\left(bx+a\right)-4\,b^3dxe\mathrm{sgn}\left(b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 3*(b^3*d^2*sgn(b*x + a) - 2*a*b^2*d*e*sgn(b*x + a) + a^2*b*e^2*sgn(b*x + a)
)*e^(-4)*log(abs(x*e + d)) + 1/2*(b^3*x^2*e^2*sgn(b*x + a) - 4*b^3*d*x*e*sg
n(b*x + a) + 6*a*b^2*x*e^2*sgn(b*x + a))*e^(-4) + (b^3*d^3*sgn(b*x + a) - 3
*a*b^2*d^2*e*sgn(b*x + a) + 3*a^2*b*d*e^2*sgn(b*x + a) - a^3*e^3*sgn(b*x +
a))*e^(-4)/(x*e + d)
```

3.1562 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{(d+ex)^3} dx$

Optimal. Leaf size=186

$$-\frac{3b\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{e^4(a+bx)(d+ex)}+\frac{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3}{2e^4(a+bx)(d+ex)^2}-\frac{3b^2\sqrt{a^2+2abx+b^2x^2}(bd-ae)\log(d+ex)}{e^4(a+bx)}+\frac{b^3x\ldots}{\ldots}$$

```
[Out] (b^3*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^3*(a + b*x)) + ((b*d - a*e)^3*Sqrt
[a^2 + 2*a*b*x + b^2*x^2])/(2*e^4*(a + b*x)*(d + e*x)^2) - (3*b*(b*d - a*e)
^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^4*(a + b*x)*(d + e*x)) - (3*b^2*(b*d -
 a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Log[d + e*x])/(e^4*(a + b*x))
```

Rubi [A] time = 0.0856378, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$-\frac{3b\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{e^4(a+bx)(d+ex)}+\frac{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3}{2e^4(a+bx)(d+ex)^2}-\frac{3b^2\sqrt{a^2+2abx+b^2x^2}(bd-ae)\log(d+ex)}{e^4(a+bx)}+\frac{b^3x\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^3,x]
```

```
[Out] (b^3*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^3*(a + b*x)) + ((b*d - a*e)^3*Sqrt
[a^2 + 2*a*b*x + b^2*x^2])/(2*e^4*(a + b*x)*(d + e*x)^2) - (3*b*(b*d - a*e)
^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^4*(a + b*x)*(d + e*x)) - (3*b^2*(b*d -
 a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Log[d + e*x])/(e^4*(a + b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{(d+ex)^3}\,dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^3}{(d+ex)^3}\,dx}{b^2\left(ab+b^2x\right)}\\
&= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{b^6}{e^3}-\frac{b^3(bd-ae)^3}{e^3(d+ex)^3}+\frac{3b^4(bd-ae)^2}{e^3(d+ex)^2}-\frac{3b^5(bd-ae)}{e^3(d+ex)}\right)dx}{b^2\left(ab+b^2x\right)}\\
&= \frac{b^3x\sqrt{a^2+2abx+b^2x^2}}{e^3(a+bx)}+\frac{(bd-ae)^3\sqrt{a^2+2abx+b^2x^2}}{2e^4(a+bx)(d+ex)^2}-\frac{3b(bd-ae)^2\sqrt{a^2+2abx+b^2\ldots}}{e^4(a+bx)(d+ex)}
\end{aligned}$$

Mathematica [A] time = 0.0680947, size = 131, normalized size = 0.7

$$-\frac{\sqrt{(a+bx)^2}\left(3a^2be^2(d+2ex)+a^3e^3-3ab^2de(3d+4ex)+6b^2(d+ex)^2(bd-ae)\log(d+ex)+b^3\left(4d^2ex+5d^3-4de^2x^2\right.\right.}{2e^4(a+bx)(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^3,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(a^3*e^3 + 3*a^2*b*e^2*(d + 2*e*x) - 3*a*b^2*d*e*(3*d +
 4*e*x) + b^3*(5*d^3 + 4*d^2*e*x - 4*d*e^2*x^2 - 2*e^3*x^3) + 6*b^2*(b*d -
a*e)*(d + e*x)^2*Log[d + e*x]))/(2*e^4*(a + b*x)*(d + e*x)^2)
```

Maple [A] time = 0.202, size = 219, normalized size = 1.2

$$\frac{6\ln(ex+d)x^2ab^2e^3-6\ln(ex+d)x^2b^3de^2+2x^3b^3e^3+12\ln(ex+d)xab^2de^2-12\ln(ex+d)xb^3d^2e+4x^2b^3de^2+6\,\mathrm{l}}{2\,(bx+a)^3e^4(ex}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^3,x)
```

```
[Out] 1/2*((b*x+a)^2)^(3/2)*(6*ln(e*x+d)*x^2*a*b^2*e^3-6*ln(e*x+d)*x^2*b^3*d*e^2+
2*x^3*b^3*e^3+12*ln(e*x+d)*x*a*b^2*d*e^2-12*ln(e*x+d)*x*b^3*d^2*e+4*x^2*b^3
*d*e^2+6*ln(e*x+d)*a*b^2*d^2*e-6*ln(e*x+d)*b^3*d^3-6*x*a^2*b*e^3+12*x*a*b^2
*d*e^2-4*x*b^3*d^2*e-a^3*e^3-3*d*e^2*a^2*b+9*a*b^2*d^2*e-5*b^3*d^3)/(b*x+a)
^3/e^4/(e*x+d)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50743, size = 375, normalized size = 2.02

$$\frac{2b^3e^3x^3+4b^3de^2x^2-5b^3d^3+9ab^2d^2e-3a^2bde^2-a^3e^3-2\left(2b^3d^2e-6ab^2de^2+3a^2be^3\right)x-6\left(b^3d^3-ab^2d^2e+\left(b^3de^2\right.\right.}{2\left(e^6x^2+2de^5x+d^2e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*e^3*x^3 + 4*b^3*d*e^2*x^2 - 5*b^3*d^3 + 9*a*b^2*d^2*e - 3*a^2*b*
d*e^2 - a^3*e^3 - 2*(2*b^3*d^2*e - 6*a*b^2*d*e^2 + 3*a^2*b*e^3)*x - 6*(b^3*
d^3 - a*b^2*d^2*e + (b^3*d*e^2 - a*b^2*e^3)*x^2 + 2*(b^3*d^2*e - a*b^2*d*e^
```

```
2)*x)*log(e*x + d))/(e^6*x^2 + 2*d*e^5*x + d^2*e^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21325, size = 230, normalized size = 1.24

$$b^3xe^{(-3)}\text{sgn}(bx+a) - 3\left(b^3d\text{sgn}(bx+a) - ab^2e\text{sgn}(bx+a)\right)e^{(-4)}\log(|xe+d|) - \frac{\left(5\,b^3d^3\text{sgn}(bx+a) - 9\,ab^2d^2e\text{sgn}(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] b^3*x*e^(-3)*sgn(b*x + a) - 3*(b^3*d*sgn(b*x + a) - a*b^2*e*sgn(b*x + a))*e
^(-4)*log(abs(x*e + d)) - 1/2*(5*b^3*d^3*sgn(b*x + a) - 9*a*b^2*d^2*e*sgn(b
*x + a) + 3*a^2*b*d*e^2*sgn(b*x + a) + a^3*e^3*sgn(b*x + a) + 6*(b^3*d^2*e*
sgn(b*x + a) - 2*a*b^2*d*e^2*sgn(b*x + a) + a^2*b*e^3*sgn(b*x + a))*x)*e^(-
4)/(x*e + d)^2
```

3.1563 $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=194

$$\frac{3b^2\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{e^4(a+bx)(d+ex)}-\frac{3b\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{2e^4(a+bx)(d+ex)^2}+\frac{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3}{3e^4(a+bx)(d+ex)^3}+\frac{b^3\sqrt{a^2+2abx+b^2x}}{e^4(a+b}$$

```
[Out] ((b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^4*(a + b*x)*(d + e*x)^3)
 - (3*b*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*e^4*(a + b*x)*(d +
e*x)^2) + (3*b^2*(b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^4*(a + b*x)*
(d + e*x)) + (b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Log[d + e*x])/(e^4*(a + b*x
))
```

Rubi [A] time = 0.0883131, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$\frac{3b^2\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{e^4(a+bx)(d+ex)}-\frac{3b\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{2e^4(a+bx)(d+ex)^2}+\frac{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3}{3e^4(a+bx)(d+ex)^3}+\frac{b^3\sqrt{a^2+2abx+b^2x}}{e^4(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^4,x]
```

```
[Out] ((b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^4*(a + b*x)*(d + e*x)^3)
 - (3*b*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*e^4*(a + b*x)*(d +
e*x)^2) + (3*b^2*(b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^4*(a + b*x)*
(d + e*x)) + (b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Log[d + e*x])/(e^4*(a + b*x
))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{(d+ex)^4}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^3}{(d+ex)^4}\,dx}{b^2\left(ab+b^2x\right)}$$
$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^3(bd-ae)^3}{e^3(d+ex)^4}+\frac{3b^4(bd-ae)^2}{e^3(d+ex)^3}-\frac{3b^5(bd-ae)}{e^3(d+ex)^2}+\frac{b^6}{e^3(d+ex)}\right)dx}{b^2\left(ab+b^2x\right)}$$
$$= \frac{(bd-ae)^3\sqrt{a^2+2abx+b^2x^2}}{3e^4(a+bx)(d+ex)^3} - \frac{3b(bd-ae)^2\sqrt{a^2+2abx+b^2x^2}}{2e^4(a+bx)(d+ex)^2} + \frac{3b^2(bd-ae)\sqrt{a^2+2}}{e^4(a+bx)(d+}$$

Mathematica [A] time = 0.058323, size = 104, normalized size = 0.54

$$\frac{\sqrt{(a+bx)^2}\left((bd-ae)\left(2a^2e^2+abe(5d+9ex)+b^2\left(11d^2+27dex+18e^2x^2\right)\right)+6b^3(d+ex)^3\log(d+ex)\right)}{6e^4(a+bx)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^4,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*((b*d - a*e)*(2*a^2*e^2 + a*b*e*(5*d + 9*e*x) + b^2*(11*
d^2 + 27*d*e*x + 18*e^2*x^2)) + 6*b^3*(d + e*x)^3*Log[d + e*x]))/(6*e^4*(a
+ b*x)*(d + e*x)^3)
```

Maple [A] time = 0.199, size = 186, normalized size = 1.

$$\frac{6\ln(ex+d)x^3b^3e^3+18\ln(ex+d)x^2b^3de^2+18\ln(ex+d)xb^3d^2e-18x^2ab^2e^3+18x^2b^3de^2+6\ln(ex+d)b^3d^3-9}{6(bx+a)^3e^4(ex+d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^4,x)
```

```
[Out] 1/6*((b*x+a)^2)^(3/2)*(6*ln(e*x+d)*x^3*b^3*e^3+18*ln(e*x+d)*x^2*b^3*d*e^2+1
8*ln(e*x+d)*x*b^3*d^2*e-18*x^2*a*b^2*e^3+18*x^2*b^3*d*e^2+6*ln(e*x+d)*b^3*d
^3-9*x*a^2*b*e^3-18*x*a*b^2*d*e^2+27*x*b^3*d^2*e-2*a^3*e^3-3*d*e^2*a^2*b-6*
a*b^2*d^2*e+11*b^3*d^3)/(b*x+a)^3/e^4/(e*x+d)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50294, size = 359, normalized size = 1.85

$$\frac{11\,b^3d^3 - 6\,ab^2d^2e - 3\,a^2bde^2 - 2\,a^3e^3 + 18\left(b^3de^2 - ab^2e^3\right)x^2 + 9\left(3\,b^3d^2e - 2\,ab^2de^2 - a^2be^3\right)x + 6\left(b^3e^3x^3 + 3\,b^3de^2x^2 + \right.}{6\left(e^7x^3 + 3\,de^6x^2 + 3\,d^2e^5x + d^3e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(11*b^3*d^3 - 6*a*b^2*d^2*e - 3*a^2*b*d*e^2 - 2*a^3*e^3 + 18*(b^3*d*e^2
 - a*b^2*e^3)*x^2 + 9*(3*b^3*d^2*e - 2*a*b^2*d*e^2 - a^2*b*e^3)*x + 6*(b^3*
e^3*x^3 + 3*b^3*d*e^2*x^2 + 3*b^3*d^2*e*x + b^3*d^3)*log(e*x + d))/(e^7*x^3
 + 3*d*e^6*x^2 + 3*d^2*e^5*x + d^3*e^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/(e*x+d)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13174, size = 239, normalized size = 1.23

$$b^3e^{(-4)}\log\left(|xe + d|\right)\mathrm{sgn}\left(bx + a\right) + \frac{\left(18\left(b^3de\mathrm{sgn}\left(bx + a\right) - ab^2e^2\mathrm{sgn}\left(bx + a\right)\right)x^2 + 9\left(3\,b^3d^2\mathrm{sgn}\left(bx + a\right) - 2\,ab^2de\mathrm{sgn}\left(bx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] b^3*e^(-4)*log(abs(x*e + d))*sgn(b*x + a) + 1/6*(18*(b^3*d*e*sgn(b*x + a) -
 a*b^2*e^2*sgn(b*x + a))*x^2 + 9*(3*b^3*d^2*sgn(b*x + a) - 2*a*b^2*d*e*sgn(
b*x + a) - a^2*b*e^2*sgn(b*x + a))*x + (11*b^3*d^3*sgn(b*x + a) - 6*a*b^2*d
^2*e*sgn(b*x + a) - 3*a^2*b*d*e^2*sgn(b*x + a) - 2*a^3*e^3*sgn(b*x + a))*e^
(-1))*e^(-3)/(x*e + d)^3
```

3.1564 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{(d+ex)^5} \, dx$

Optimal. Leaf size=48

$$\frac{(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{4(d+ex)^4(bd-ae)}$$

```
[Out] ((a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*(b*d - a*e)*(d + e*x)^4)
```

Rubi [A] time = 0.0190633, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 37}

$$\frac{(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{4(d+ex)^4(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^5,x]
```

```
[Out] ((a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*(b*d - a*e)*(d + e*x)^4)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{(d+ex)^5} \, dx &= \frac{\sqrt{a^2+2abx+b^2x^2} \int \frac{\left(ab+b^2x\right)^3}{(d+ex)^5} \, dx}{b^2\left(ab+b^2x\right)}\\
&= \frac{(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{4(bd-ae)(d+ex)^4}
\end{aligned}$$

Mathematica [B] time = 0.0424282, size = 109, normalized size = 2.27

$$-\frac{\sqrt{(a+bx)^2}\left(a^2be^2(d+4ex)+a^3e^3+ab^2e\left(d^2+4dex+6e^2x^2\right)+b^3\left(4d^2ex+d^3+6de^2x^2+4e^3x^3\right)\right)}{4e^4(a+bx)(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^5,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(a^3*e^3 + a^2*b*e^2*(d + 4*e*x) + a*b^2*e*(d^2 + 4*d*e
*x + 6*e^2*x^2) + b^3*(d^3 + 4*d^2*e*x + 6*d*e^2*x^2 + 4*e^3*x^3)))/(4*e^4*
(a + b*x)*(d + e*x)^4)
```

Maple [B] time = 0.157, size = 128, normalized size = 2.7

$$-\frac{4\,x^3b^3e^3+6\,x^2ab^2e^3+6\,x^2b^3de^2+4\,xa^2be^3+4\,xab^2de^2+4\,xb^3d^2e+a^3e^3+de^2a^2b+ab^2d^2e+b^3d^3}{4\,(ex+d)^4\,e^4\,(bx+a)^3}\left((bx+a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^5,x)
```

```
[Out] -1/4*(4*b^3*e^3*x^3+6*a*b^2*e^3*x^2+6*b^3*d*e^2*x^2+4*a^2*b*e^3*x+4*a*b^2*d
*e^2*x+4*b^3*d^2*e*x+a^3*e^3+a^2*b*d*e^2+a*b^2*d^2*e+b^3*d^3)*((b*x+a)^2)^(
3/2)/(e*x+d)^4/e^4/(b*x+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.53743, size = 284, normalized size = 5.92

$$-\frac{4\,b^3e^3x^3+b^3d^3+ab^2d^2e+a^2bde^2+a^3e^3+6\left(b^3de^2+ab^2e^3\right)x^2+4\left(b^3d^2e+ab^2de^2+a^2be^3\right)x}{4\left(e^8x^4+4\,de^7x^3+6\,d^2e^6x^2+4\,d^3e^5x+d^4e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^5,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^3*e^3*x^3 + b^3*d^3 + a*b^2*d^2*e + a^2*b*d*e^2 + a^3*e^3 + 6*(b^
3*d*e^2 + a*b^2*e^3)*x^2 + 4*(b^3*d^2*e + a*b^2*d*e^2 + a^2*b*e^3)*x)/(e^8*
x^4 + 4*d*e^7*x^3 + 6*d^2*e^6*x^2 + 4*d^3*e^5*x + d^4*e^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/(e*x+d)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18077, size = 224, normalized size = 4.67

$$-\frac{\left(4\,b^{3}x^{3}e^{3}\mathrm{sgn}\left(bx+a\right)+6\,b^{3}dx^{2}e^{2}\mathrm{sgn}\left(bx+a\right)+4\,b^{3}d^{2}xe\mathrm{sgn}\left(bx+a\right)+b^{3}d^{3}\mathrm{sgn}\left(bx+a\right)+6\,ab^{2}x^{2}e^{3}\mathrm{sgn}\left(bx+a\right)+\cdots\right.}{4\left(xe+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] -1/4*(4*b^3*x^3*e^3*sgn(b*x + a) + 6*b^3*d*x^2*e^2*sgn(b*x + a) + 4*b^3*d^2
*x*e*sgn(b*x + a) + b^3*d^3*sgn(b*x + a) + 6*a*b^2*x^2*e^3*sgn(b*x + a) + 4
*a*b^2*d*x*e^2*sgn(b*x + a) + a*b^2*d^2*e*sgn(b*x + a) + 4*a^2*b*x*e^3*sgn(
b*x + a) + a^2*b*d*e^2*sgn(b*x + a) + a^3*e^3*sgn(b*x + a))*e^(-4)/(x*e + d
)^4
```

3.1565 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{(d+ex)^6} dx$

Optimal. Leaf size=98

$$\frac{b\sqrt{a^2+2abx+b^2x^2}(a+bx)^3}{20(d+ex)^4(bd-ae)^2}+\frac{\sqrt{a^2+2abx+b^2x^2}(a+bx)^3}{5(d+ex)^5(bd-ae)}$$

```
[Out] ((a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*(b*d - a*e)*(d + e*x)^5) + (
b*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(20*(b*d - a*e)^2*(d + e*x)^4)
```

Rubi [A] time = 0.0382047, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {646, 45, 37}

$$\frac{b\sqrt{a^2+2abx+b^2x^2}(a+bx)^3}{20(d+ex)^4(bd-ae)^2}+\frac{\sqrt{a^2+2abx+b^2x^2}(a+bx)^3}{5(d+ex)^5(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^6,x]
```

```
[Out] ((a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*(b*d - a*e)*(d + e*x)^5) + (
b*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(20*(b*d - a*e)^2*(d + e*x)^4)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{(d+ex)^6}\,dx=\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^3}{(d+ex)^6}\,dx}{b^2\left(ab+b^2x\right)}$$

$$=\frac{(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{5(bd-ae)(d+ex)^5}+\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^3}{(d+ex)^5}\,dx}{5b(bd-ae)\left(ab+b^2x\right)}$$

$$=\frac{(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{5(bd-ae)(d+ex)^5}+\frac{b(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{20(bd-ae)^2(d+ex)^4}$$

Mathematica [A] time = 0.0474105, size = 112, normalized size = 1.14

$$-\frac{\sqrt{(a+bx)^2}\left(3a^2be^2(d+5ex)+4a^3e^3+2ab^2e\left(d^2+5dex+10e^2x^2\right)+b^3\left(5d^2ex+d^3+10de^2x^2+10e^3x^3\right)\right)}{20e^4(a+bx)(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^6,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(4*a^3*e^3 + 3*a^2*b*e^2*(d + 5*e*x) + 2*a*b^2*e*(d^2 +
 5*d*e*x + 10*e^2*x^2) + b^3*(d^3 + 5*d^2*e*x + 10*d*e^2*x^2 + 10*e^3*x^3))
)/(20*e^4*(a + b*x)*(d + e*x)^5)
```

Maple [A] time = 0.155, size = 131, normalized size = 1.3

$$-\frac{10\,x^3b^3e^3+20\,x^2ab^2e^3+10\,x^2b^3de^2+15\,xa^2be^3+10\,xab^2de^2+5\,xb^3d^2e+4\,a^3e^3+3\,de^2a^2b+2\,ab^2d^2e+b^3d^3}{20\,e^4\,(ex+d)^5\,(bx+a)^3}\left((bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^6,x)
```

```
[Out] -1/20/e^4*(10*b^3*e^3*x^3+20*a*b^2*e^3*x^2+10*b^3*d*e^2*x^2+15*a^2*b*e^3*x+
10*a*b^2*d*e^2*x+5*b^3*d^2*e*x+4*a^3*e^3+3*a^2*b*d*e^2+2*a*b^2*d^2*e+b^3*d^
3)*((b*x+a)^2)^(3/2)/(e*x+d)^5/(b*x+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.59604, size = 328, normalized size = 3.35

$$-\frac{10\,b^3e^3x^3+b^3d^3+2\,ab^2d^2e+3\,a^2bde^2+4\,a^3e^3+10\left(b^3de^2+2\,ab^2e^3\right)x^2+5\left(b^3d^2e+2\,ab^2de^2+3\,a^2be^3\right)x}{20\left(e^9x^5+5\,de^8x^4+10\,d^2e^7x^3+10\,d^3e^6x^2+5\,d^4e^5x+d^5e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^6,x, algorithm="fricas")
```

```
[Out] -1/20*(10*b^3*e^3*x^3 + b^3*d^3 + 2*a*b^2*d^2*e + 3*a^2*b*d*e^2 + 4*a^3*e^3
 + 10*(b^3*d*e^2 + 2*a*b^2*e^3)*x^2 + 5*(b^3*d^2*e + 2*a*b^2*d*e^2 + 3*a^2*
b*e^3)*x)/(e^9*x^5 + 5*d*e^8*x^4 + 10*d^2*e^7*x^3 + 10*d^3*e^6*x^2 + 5*d^4*
e^5*x + d^5*e^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/(e*x+d)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18876, size = 228, normalized size = 2.33

$$-\frac{\left(10\, b^3 x^3 e^3 \mathrm{sgn}\left(bx+a\right) + 10\, b^3 d x^2 e^2 \mathrm{sgn}\left(bx+a\right) + 5\, b^3 d^2 x e \mathrm{sgn}\left(bx+a\right) + b^3 d^3 \mathrm{sgn}\left(bx+a\right) + 20\, a b^2 x^2 e^3 \mathrm{sgn}\left(bx+a\right) + \cdots\right.}{20\,(xe\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^6,x, algorithm="giac")
```

```
[Out] -1/20*(10*b^3*x^3*e^3*sgn(b*x + a) + 10*b^3*d*x^2*e^2*sgn(b*x + a) + 5*b^3*
d^2*x*e*sgn(b*x + a) + b^3*d^3*sgn(b*x + a) + 20*a*b^2*x^2*e^3*sgn(b*x + a)
 + 10*a*b^2*d*x*e^2*sgn(b*x + a) + 2*a*b^2*d^2*e*sgn(b*x + a) + 15*a^2*b*x*
e^3*sgn(b*x + a) + 3*a^2*b*d*e^2*sgn(b*x + a) + 4*a^3*e^3*sgn(b*x + a))*e^(
-4)/(x*e + d)^5
```

3.1566 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{(d+ex)^7}\,dx$

Optimal. Leaf size=143

$$\frac{b^2(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{60(d+ex)^4(bd-ae)^3}+\frac{b(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{15(d+ex)^5(bd-ae)^2}+\frac{(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{6(d+ex)^6(bd-ae)}$$

```
[Out] ((a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/(6*(b*d - a*e)*(d + e*x)^6) + (
b*(a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/(15*(b*d - a*e)^2*(d + e*x)^5)
 + (b^2*(a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/(60*(b*d - a*e)^3*(d + e
*x)^4)
```

Rubi [A] time = 0.0861064, antiderivative size = 200, normalized size of antiderivative = 1.4, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$-\frac{b^3\sqrt{a^2+2abx+b^2x^2}}{3e^4(a+bx)(d+ex)^3}+\frac{3b^2\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{4e^4(a+bx)(d+ex)^4}-\frac{3b\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{5e^4(a+bx)(d+ex)^5}+\frac{\sqrt{a^2+2abx+b^2x^2}(bd}{6e^4(a+bx)(d+ex}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^7,x]
```

```
[Out] ((b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*e^4*(a + b*x)*(d + e*x)^6)
 - (3*b*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^4*(a + b*x)*(d +
e*x)^5) + (3*b^2*(b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*e^4*(a + b*x
)*(d + e*x)^4) - (b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^4*(a + b*x)*(d +
e*x)^3)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{(d+ex)^7}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^3}{(d+ex)^7}\,dx}{b^2\left(ab+b^2x\right)}$$
$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^3(bd-ae)^3}{e^3(d+ex)^7}+\frac{3b^4(bd-ae)^2}{e^3(d+ex)^6}-\frac{3b^5(bd-ae)}{e^3(d+ex)^5}+\frac{b^6}{e^3(d+ex)^4}\right)dx}{b^2\left(ab+b^2x\right)}$$
$$= \frac{(bd-ae)^3\sqrt{a^2+2abx+b^2x^2}}{6e^4(a+bx)(d+ex)^6}-\frac{3b(bd-ae)^2\sqrt{a^2+2abx+b^2x^2}}{5e^4(a+bx)(d+ex)^5}+\frac{3b^2(bd-ae)\sqrt{a^2+2abx}}{4e^4(a+bx)(d+ex}$$

Mathematica [A] time = 0.0435135, size = 112, normalized size = 0.78

$$-\frac{\sqrt{(a+bx)^2}\left(6a^2be^2(d+6ex)+10a^3e^3+3ab^2e\left(d^2+6dex+15e^2x^2\right)+b^3\left(6d^2ex+d^3+15de^2x^2+20e^3x^3\right)\right)}{60e^4(a+bx)(d+ex)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^7,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(10*a^3*e^3 + 6*a^2*b*e^2*(d + 6*e*x) + 3*a*b^2*e*(d^2
+ 6*d*e*x + 15*e^2*x^2) + b^3*(d^3 + 6*d^2*e*x + 15*d*e^2*x^2 + 20*e^3*x^3)
))/(60*e^4*(a + b*x)*(d + e*x)^6)
```

Maple [A] time = 0.159, size = 131, normalized size = 0.9

$$-\frac{20x^3b^3e^3+45x^2ab^2e^3+15x^2b^3de^2+36xa^2be^3+18xab^2de^2+6xb^3d^2e+10a^3e^3+6de^2a^2b+3ab^2d^2e+b^3d^3}{60e^4(ex+d)^6(bx+a)^3}\left((bx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^7,x)
```

```
[Out] -1/60/e^4*(20*b^3*e^3*x^3+45*a*b^2*e^3*x^2+15*b^3*d*e^2*x^2+36*a^2*b*e^3*x+
18*a*b^2*d*e^2*x+6*b^3*d^2*e*x+10*a^3*e^3+6*a^2*b*d*e^2+3*a*b^2*d^2*e+b^3*d
^3)*((b*x+a)^2)^(3/2)/(e*x+d)^6/(b*x+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.84614, size = 354, normalized size = 2.48

$$-\frac{20b^3e^3x^3+b^3d^3+3ab^2d^2e+6a^2bde^2+10a^3e^3+15\left(b^3de^2+3ab^2e^3\right)x^2+6\left(b^3d^2e+3ab^2de^2+6a^2be^3\right)x}{60\left(e^{10}x^6+6de^9x^5+15d^2e^8x^4+20d^3e^7x^3+15d^4e^6x^2+6d^5e^5x+d^6e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^7,x, algorithm="fricas")
```

```
[Out] -1/60*(20*b^3*e^3*x^3 + b^3*d^3 + 3*a*b^2*d^2*e + 6*a^2*b*d*e^2 + 10*a^3*e^
3 + 15*(b^3*d*e^2 + 3*a*b^2*e^3)*x^2 + 6*(b^3*d^2*e + 3*a*b^2*d*e^2 + 6*a^2
*b*e^3)*x)/(e^10*x^6 + 6*d*e^9*x^5 + 15*d^2*e^8*x^4 + 20*d^3*e^7*x^3 + 15*d
^4*e^6*x^2 + 6*d^5*e^5*x + d^6*e^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/(e*x+d)**7,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14024, size = 228, normalized size = 1.59

$$-\frac{\left(20\, b^3 x^3 e^3 \mathrm{sgn}\,(bx+a) + 15\, b^3 d x^2 e^2 \mathrm{sgn}\,(bx+a) + 6\, b^3 d^2 x e \mathrm{sgn}\,(bx+a) + b^3 d^3 \mathrm{sgn}\,(bx+a) + 45\, a b^2 x^2 e^3 \mathrm{sgn}\,(bx+a)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^7,x, algorithm="giac")
```

```
[Out] -1/60*(20*b^3*x^3*e^3*sgn(b*x + a) + 15*b^3*d*x^2*e^2*sgn(b*x + a) + 6*b^3*
d^2*x*e*sgn(b*x + a) + b^3*d^3*sgn(b*x + a) + 45*a*b^2*x^2*e^3*sgn(b*x + a)
 + 18*a*b^2*d*x*e^2*sgn(b*x + a) + 3*a*b^2*d^2*e*sgn(b*x + a) + 36*a^2*b*x*
e^3*sgn(b*x + a) + 6*a^2*b*d*e^2*sgn(b*x + a) + 10*a^3*e^3*sgn(b*x + a))*e^
(-4)/(x*e + d)^6
```

3.1567 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{(d+ex)^8} dx$

Optimal. Leaf size=200

$$-\frac{b^3\sqrt{a^2+2abx+b^2x^2}}{4e^4(a+bx)(d+ex)^4}+\frac{3b^2\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{5e^4(a+bx)(d+ex)^5}-\frac{b\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{2e^4(a+bx)(d+ex)^6}+\frac{\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{7e^4(a+bx)(d+ex)^7}$$

```
[Out] ((b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*e^4*(a + b*x)*(d + e*x)^7)
 - (b*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*e^4*(a + b*x)*(d + e*
x)^6) + (3*b^2*(b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^4*(a + b*x)*
(d + e*x)^5) - (b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*e^4*(a + b*x)*(d + e*
x)^4)
```

Rubi [A] time = 0.0885224, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$-\frac{b^3\sqrt{a^2+2abx+b^2x^2}}{4e^4(a+bx)(d+ex)^4}+\frac{3b^2\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{5e^4(a+bx)(d+ex)^5}-\frac{b\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{2e^4(a+bx)(d+ex)^6}+\frac{\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{7e^4(a+bx)(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^8,x]
```

```
[Out] ((b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*e^4*(a + b*x)*(d + e*x)^7)
 - (b*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*e^4*(a + b*x)*(d + e*
x)^6) + (3*b^2*(b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^4*(a + b*x)*
(d + e*x)^5) - (b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*e^4*(a + b*x)*(d + e*
x)^4)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{(d+ex)^8}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^3}{(d+ex)^8}\,dx}{b^2\left(ab+b^2x\right)}$$

$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^3(bd-ae)^3}{e^3(d+ex)^8}+\frac{3b^4(bd-ae)^2}{e^3(d+ex)^7}-\frac{3b^5(bd-ae)}{e^3(d+ex)^6}+\frac{b^6}{e^3(d+ex)^5}\right)dx}{b^2\left(ab+b^2x\right)}$$

$$= \frac{(bd-ae)^3\sqrt{a^2+2abx+b^2x^2}}{7e^4(a+bx)(d+ex)^7} - \frac{b(bd-ae)^2\sqrt{a^2+2abx+b^2x^2}}{2e^4(a+bx)(d+ex)^6} + \frac{3b^2(bd-ae)\sqrt{a^2+2a}}{5e^4(a+bx)(d+}$$

Mathematica [A] time = 0.0413052, size = 112, normalized size = 0.56

$$-\frac{\sqrt{(a+bx)^2}\left(10a^2be^2(d+7ex)+20a^3e^3+4ab^2e\left(d^2+7dex+21e^2x^2\right)+b^3\left(7d^2ex+d^3+21de^2x^2+35e^3x^3\right)\right)}{140e^4(a+bx)(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^8,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(20*a^3*e^3 + 10*a^2*b*e^2*(d + 7*e*x) + 4*a*b^2*e*(d^2
 + 7*d*e*x + 21*e^2*x^2) + b^3*(d^3 + 7*d^2*e*x + 21*d*e^2*x^2 + 35*e^3*x^3
)))/(140*e^4*(a + b*x)*(d + e*x)^7)
```

Maple [A] time = 0.155, size = 131, normalized size = 0.7

$$-\frac{35\,x^3b^3e^3+84\,x^2ab^2e^3+21\,x^2b^3de^2+70\,xa^2be^3+28\,xab^2de^2+7\,xb^3d^2e+20\,a^3e^3+10\,de^2a^2b+4\,ab^2d^2e+b^3d^3}{140\,e^4\,(ex+d)^7\,(bx+a)^3}\left((l\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^8,x)
```

```
[Out] -1/140/e^4*(35*b^3*e^3*x^3+84*a*b^2*e^3*x^2+21*b^3*d*e^2*x^2+70*a^2*b*e^3*x
+28*a*b^2*d*e^2*x+7*b^3*d^2*e*x+20*a^3*e^3+10*a^2*b*d*e^2+4*a*b^2*d^2*e+b^3
*d^3)*((b*x+a)^2)^(3/2)/(e*x+d)^7/(b*x+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.85414, size = 382, normalized size = 1.91

$$-\frac{35\,b^3e^3x^3+b^3d^3+4\,ab^2d^2e+10\,a^2bde^2+20\,a^3e^3+21\left(b^3de^2+4\,ab^2e^3\right)x^2+7\left(b^3d^2e+4\,ab^2de^2+10\,a^2be^3\right)x}{140\left(e^{11}x^7+7\,de^{10}x^6+21\,d^2e^9x^5+35\,d^3e^8x^4+35\,d^4e^7x^3+21\,d^5e^6x^2+7\,d^6e^5x+d^7e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^8,x, algorithm="fricas")
```

```
[Out] -1/140*(35*b^3*e^3*x^3 + b^3*d^3 + 4*a*b^2*d^2*e + 10*a^2*b*d*e^2 + 20*a^3*
e^3 + 21*(b^3*d*e^2 + 4*a*b^2*e^3)*x^2 + 7*(b^3*d^2*e + 4*a*b^2*d*e^2 + 10*
a^2*b*e^3)*x)/(e^11*x^7 + 7*d*e^10*x^6 + 21*d^2*e^9*x^5 + 35*d^3*e^8*x^4 +
35*d^4*e^7*x^3 + 21*d^5*e^6*x^2 + 7*d^6*e^5*x + d^7*e^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/(e*x+d)**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18464, size = 228, normalized size = 1.14

$$-\frac{\left(35\,b^3x^3e^3\mathrm{sgn}\left(bx+a\right)+21\,b^3dx^2e^2\mathrm{sgn}\left(bx+a\right)+7\,b^3d^2xe\mathrm{sgn}\left(bx+a\right)+b^3d^3\mathrm{sgn}\left(bx+a\right)+84\,ab^2x^2e^3\mathrm{sgn}\left(bx+a\right)+\right.}{140\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^8,x, algorithm="giac")
```

```
[Out] -1/140*(35*b^3*x^3*e^3*sgn(b*x + a) + 21*b^3*d*x^2*e^2*sgn(b*x + a) + 7*b^3
*d^2*x*e*sgn(b*x + a) + b^3*d^3*sgn(b*x + a) + 84*a*b^2*x^2*e^3*sgn(b*x + a
) + 28*a*b^2*d*x*e^2*sgn(b*x + a) + 4*a*b^2*d^2*e*sgn(b*x + a) + 70*a^2*b*x
*e^3*sgn(b*x + a) + 10*a^2*b*d*e^2*sgn(b*x + a) + 20*a^3*e^3*sgn(b*x + a))*
e^(-4)/(x*e + d)^7
```

3.1568 $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^9} dx$

Optimal. Leaf size=200

$$-\frac{b^3\sqrt{a^2+2abx+b^2x^2}}{5e^4(a+bx)(d+ex)^5}+\frac{b^2\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{2e^4(a+bx)(d+ex)^6}-\frac{3b\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{7e^4(a+bx)(d+ex)^7}+\frac{\sqrt{a^2+2abx+b^2x^2}(bd-}{8e^4(a+bx)(d+ex)^{}}$$

```
[Out] ((b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(8*e^4*(a + b*x)*(d + e*x)^8)
 - (3*b*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*e^4*(a + b*x)*(d +
e*x)^7) + (b^2*(b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*e^4*(a + b*x)*
(d + e*x)^6) - (b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^4*(a + b*x)*(d + e*
x)^5)
```

Rubi [A] time = 0.0868989, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$-\frac{b^3\sqrt{a^2+2abx+b^2x^2}}{5e^4(a+bx)(d+ex)^5}+\frac{b^2\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{2e^4(a+bx)(d+ex)^6}-\frac{3b\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{7e^4(a+bx)(d+ex)^7}+\frac{\sqrt{a^2+2abx+b^2x^2}(bd-}{8e^4(a+bx)(d+ex)^{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^9,x]
```

```
[Out] ((b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(8*e^4*(a + b*x)*(d + e*x)^8)
 - (3*b*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*e^4*(a + b*x)*(d +
e*x)^7) + (b^2*(b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*e^4*(a + b*x)*
(d + e*x)^6) - (b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^4*(a + b*x)*(d + e*
x)^5)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^9}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(ab+b^2x)^3}{(d+ex)^9}\,dx}{b^2(ab+b^2x)}$$
$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^3(bd-ae)^3}{e^3(d+ex)^9}+\frac{3b^4(bd-ae)^2}{e^3(d+ex)^8}-\frac{3b^5(bd-ae)}{e^3(d+ex)^7}+\frac{b^6}{e^3(d+ex)^6}\right)dx}{b^2(ab+b^2x)}$$
$$= \frac{(bd-ae)^3\sqrt{a^2+2abx+b^2x^2}}{8e^4(a+bx)(d+ex)^8} - \frac{3b(bd-ae)^2\sqrt{a^2+2abx+b^2x^2}}{7e^4(a+bx)(d+ex)^7} + \frac{b^2(bd-ae)\sqrt{a^2+2abx}}{2e^4(a+bx)(d+ex}$$

Mathematica [A] time = 0.046937, size = 112, normalized size = 0.56

$$-\frac{\sqrt{(a+bx)^2}\left(15a^2be^2(d+8ex)+35a^3e^3+5ab^2e\left(d^2+8dex+28e^2x^2\right)+b^3\left(8d^2ex+d^3+28de^2x^2+56e^3x^3\right)\right)}{280e^4(a+bx)(d+ex)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^9,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(35*a^3*e^3 + 15*a^2*b*e^2*(d + 8*e*x) + 5*a*b^2*e*(d^2
 + 8*d*e*x + 28*e^2*x^2) + b^3*(d^3 + 8*d^2*e*x + 28*d*e^2*x^2 + 56*e^3*x^3
)))/(280*e^4*(a + b*x)*(d + e*x)^8)
```

Maple [A] time = 0.155, size = 131, normalized size = 0.7

$$-\frac{56x^3b^3e^3+140x^2ab^2e^3+28x^2b^3de^2+120xa^2be^3+40xab^2de^2+8xb^3d^2e+35a^3e^3+15de^2a^2b+5ab^2d^2e+b^3d^3}{280e^4(ex+d)^8(bx+a)^3}\left((b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^9,x)
```

```
[Out] -1/280/e^4*(56*b^3*e^3*x^3+140*a*b^2*e^3*x^2+28*b^3*d*e^2*x^2+120*a^2*b*e^3
*x+40*a*b^2*d*e^2*x+8*b^3*d^2*e*x+35*a^3*e^3+15*a^2*b*d*e^2+5*a*b^2*d^2*e+b
^3*d^3)*((b*x+a)^2)^(3/2)/(e*x+d)^8/(b*x+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.84164, size = 406, normalized size = 2.03

$$-\frac{56b^3e^3x^3+b^3d^3+5ab^2d^2e+15a^2bde^2+35a^3e^3+28\left(b^3de^2+5ab^2e^3\right)x^2+8\left(b^3d^2e+5ab^2de^2+15a^2be^3\right)x}{280\left(e^{12}x^8+8de^{11}x^7+28d^2e^{10}x^6+56d^3e^9x^5+70d^4e^8x^4+56d^5e^7x^3+28d^6e^6x^2+8d^7e^5x+d^8e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^9,x, algorithm="fricas")
```

```
[Out] -1/280*(56*b^3*e^3*x^3 + b^3*d^3 + 5*a*b^2*d^2*e + 15*a^2*b*d*e^2 + 35*a^3*
e^3 + 28*(b^3*d*e^2 + 5*a*b^2*e^3)*x^2 + 8*(b^3*d^2*e + 5*a*b^2*d*e^2 + 15*
a^2*b*e^3)*x)/(e^12*x^8 + 8*d*e^11*x^7 + 28*d^2*e^10*x^6 + 56*d^3*e^9*x^5 +
 70*d^4*e^8*x^4 + 56*d^5*e^7*x^3 + 28*d^6*e^6*x^2 + 8*d^7*e^5*x + d^8*e^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/(e*x+d)**9,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17929, size = 228, normalized size = 1.14

$$-\frac{\left(56\,b^3x^3e^3\mathrm{sgn}\left(bx+a\right)+28\,b^3dx^2e^2\mathrm{sgn}\left(bx+a\right)+8\,b^3d^2xe\mathrm{sgn}\left(bx+a\right)+b^3d^3\mathrm{sgn}\left(bx+a\right)+140\,ab^2x^2e^3\mathrm{sgn}\left(bx+\right.\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^9,x, algorithm="giac")
```

```
[Out] -1/280*(56*b^3*x^3*e^3*sgn(b*x + a) + 28*b^3*d*x^2*e^2*sgn(b*x + a) + 8*b^3
*d^2*x*e*sgn(b*x + a) + b^3*d^3*sgn(b*x + a) + 140*a*b^2*x^2*e^3*sgn(b*x +
a) + 40*a*b^2*d*x*e^2*sgn(b*x + a) + 5*a*b^2*d^2*e*sgn(b*x + a) + 120*a^2*b
*x*e^3*sgn(b*x + a) + 15*a^2*b*d*e^2*sgn(b*x + a) + 35*a^3*e^3*sgn(b*x + a)
)*e^(-4)/(x*e + d)^8
```

3.1569 $\int (d+ex)^5 \left(a^2+2abx+b^2x^2\right)^{5/2} dx$

Optimal. Leaf size=266

$$\frac{e^4\sqrt{a^2+2abx+b^2x^2}(a+bx)^9(bd-ae)}{2b^6}+\frac{10e^3\sqrt{a^2+2abx+b^2x^2}(a+bx)^8(bd-ae)^2}{9b^6}+\frac{5e^2\sqrt{a^2+2abx+b^2x^2}(a+bx)^7(b}{4b^6}$$

```
[Out] ((b*d - a*e)^5*(a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*b^6) + (5*e*(b
*d - a*e)^4*(a + b*x)^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*b^6) + (5*e^2*(b*
d - a*e)^3*(a + b*x)^7*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*b^6) + (10*e^3*(b*
d - a*e)^2*(a + b*x)^8*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(9*b^6) + (e^4*(b*d -
 a*e)*(a + b*x)^9*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*b^6) + (e^5*(a + b*x)^1
0*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(11*b^6)
```

Rubi [A] time = 0.302778, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$\frac{e^4\sqrt{a^2+2abx+b^2x^2}(a+bx)^9(bd-ae)}{2b^6}+\frac{10e^3\sqrt{a^2+2abx+b^2x^2}(a+bx)^8(bd-ae)^2}{9b^6}+\frac{5e^2\sqrt{a^2+2abx+b^2x^2}(a+bx)^7(b}{4b^6}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] ((b*d - a*e)^5*(a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*b^6) + (5*e*(b
*d - a*e)^4*(a + b*x)^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*b^6) + (5*e^2*(b*
d - a*e)^3*(a + b*x)^7*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*b^6) + (10*e^3*(b*
d - a*e)^2*(a + b*x)^8*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(9*b^6) + (e^4*(b*d -
 a*e)*(a + b*x)^9*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*b^6) + (e^5*(a + b*x)^1
0*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(11*b^6)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (d+ex)^5 \left(a^2+2abx+b^2x^2\right)^{5/2} dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(ab+b^2x\right)^5 (d+ex)^5 dx}{b^4\left(ab+b^2x\right)}$$

$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{(bd-ae)^5\left(ab+b^2x\right)^5}{b^5} + \frac{5e(bd-ae)^4\left(ab+b^2x\right)^6}{b^6} + \frac{10e^2(bd-ae)^3\left(ab+b^2x\right)^7}{b^7}\right.}{b^4\left(ab+b^2x\right)}$$

$$= \frac{(bd-ae)^5(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{6b^6} + \frac{5e(bd-ae)^4(a+bx)^6\sqrt{a^2+2abx+b}}{7b^6}$$

Mathematica [A] time = 0.132222, size = 385, normalized size = 1.45

$$\frac{x\sqrt{(a+bx)^2}\left(165a^3b^2x^2\left(336d^3e^2x^2+280d^2e^3x^3+210d^4ex+56d^5+120de^4x^4+21e^5x^5\right)+55a^2b^3x^3\left(840d^3e^2x^2+72\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[(a + b*x)^2]*(462*a^5*(6*d^5 + 15*d^4*e*x + 20*d^3*e^2*x^2 + 15*d^2
*e^3*x^3 + 6*d*e^4*x^4 + e^5*x^5) + 330*a^4*b*x*(21*d^5 + 70*d^4*e*x + 105*
d^3*e^2*x^2 + 84*d^2*e^3*x^3 + 35*d*e^4*x^4 + 6*e^5*x^5) + 165*a^3*b^2*x^2*
(56*d^5 + 210*d^4*e*x + 336*d^3*e^2*x^2 + 280*d^2*e^3*x^3 + 120*d*e^4*x^4 +
 21*e^5*x^5) + 55*a^2*b^3*x^3*(126*d^5 + 504*d^4*e*x + 840*d^3*e^2*x^2 + 72
0*d^2*e^3*x^3 + 315*d*e^4*x^4 + 56*e^5*x^5) + 11*a*b^4*x^4*(252*d^5 + 1050*
d^4*e*x + 1800*d^3*e^2*x^2 + 1575*d^2*e^3*x^3 + 700*d*e^4*x^4 + 126*e^5*x^5
) + b^5*x^5*(462*d^5 + 1980*d^4*e*x + 3465*d^3*e^2*x^2 + 3080*d^2*e^3*x^3 +
 1386*d*e^4*x^4 + 252*e^5*x^5)))/(2772*(a + b*x))
```

Maple [B] time = 0.157, size = 506, normalized size = 1.9

$$\frac{x\left(252\,b^5e^5x^{10}+1386\,x^9ab^4e^5+1386\,x^9b^5de^4+3080\,x^8a^2b^3e^5+7700\,x^8ab^4de^4+3080\,x^8b^5d^2e^3+3465\,x^7a^3b^2e^5+17\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] 1/2772*x*(252*b^5*e^5*x^10+1386*a*b^4*e^5*x^9+1386*b^5*d*e^4*x^9+3080*a^2*b
^3*e^5*x^8+7700*a*b^4*d*e^4*x^8+3080*b^5*d^2*e^3*x^8+3465*a^3*b^2*e^5*x^7+1
7325*a^2*b^3*d*e^4*x^7+17325*a*b^4*d^2*e^3*x^7+3465*b^5*d^3*e^2*x^7+1980*a^
4*b*e^5*x^6+19800*a^3*b^2*d*e^4*x^6+39600*a^2*b^3*d^2*e^3*x^6+19800*a*b^4*d
^3*e^2*x^6+1980*b^5*d^4*e*x^6+462*a^5*e^5*x^5+11550*a^4*b*d*e^4*x^5+46200*a
^3*b^2*d^2*e^3*x^5+46200*a^2*b^3*d^3*e^2*x^5+11550*a*b^4*d^4*e*x^5+462*b^5*
d^5*x^5+2772*a^5*d*e^4*x^4+27720*a^4*b*d^2*e^3*x^4+55440*a^3*b^2*d^3*e^2*x^
4+27720*a^2*b^3*d^4*e*x^4+2772*a*b^4*d^5*x^4+6930*a^5*d^2*e^3*x^3+34650*a^4
*b*d^3*e^2*x^3+34650*a^3*b^2*d^4*e*x^3+6930*a^2*b^3*d^5*x^3+9240*a^5*d^3*e^
2*x^2+23100*a^4*b*d^4*e*x^2+9240*a^3*b^2*d^5*x^2+6930*a^5*d^4*e*x+6930*a^4*
b*d^5*x+2772*a^5*d^5)*((b*x+a)^2)^(5/2)/(b*x+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.85984, size = 883, normalized size = 3.32

$$\frac{1}{11}b^5e^5x^{11} + a^5d^5x + \frac{1}{2}\left(b^5de^4 + ab^4e^5\right)x^{10} + \frac{5}{9}\left(2\,b^5d^2e^3 + 5\,ab^4de^4 + 2\,a^2b^3e^5\right)x^9 + \frac{5}{4}\left(b^5d^3e^2 + 5\,ab^4d^2e^3 + 5\,a^2b^3de^4 + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/11*b^5*e^5*x^11 + a^5*d^5*x + 1/2*(b^5*d*e^4 + a*b^4*e^5)*x^10 + 5/9*(2*b
^5*d^2*e^3 + 5*a*b^4*d*e^4 + 2*a^2*b^3*e^5)*x^9 + 5/4*(b^5*d^3*e^2 + 5*a*b^
4*d^2*e^3 + 5*a^2*b^3*d*e^4 + a^3*b^2*e^5)*x^8 + 5/7*(b^5*d^4*e + 10*a*b^4*
d^3*e^2 + 20*a^2*b^3*d^2*e^3 + 10*a^3*b^2*d*e^4 + a^4*b*e^5)*x^7 + 1/6*(b^5
*d^5 + 25*a*b^4*d^4*e + 100*a^2*b^3*d^3*e^2 + 100*a^3*b^2*d^2*e^3 + 25*a^4*
b*d*e^4 + a^5*e^5)*x^6 + (a*b^4*d^5 + 10*a^2*b^3*d^4*e + 20*a^3*b^2*d^3*e^2
 + 10*a^4*b*d^2*e^3 + a^5*d*e^4)*x^5 + 5/2*(a^2*b^3*d^5 + 5*a^3*b^2*d^4*e +
 5*a^4*b*d^3*e^2 + a^5*d^2*e^3)*x^4 + 5/3*(2*a^3*b^2*d^5 + 5*a^4*b*d^4*e +
2*a^5*d^3*e^2)*x^3 + 5/2*(a^4*b*d^5 + a^5*d^4*e)*x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d + ex)^5 \left((a + bx)^2\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Integral((d + e*x)**5*((a + b*x)**2)**(5/2), x)
```

Giac [B] time = 1.30252, size = 926, normalized size = 3.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/11*b^5*x^11*e^5*sgn(b*x + a) + 1/2*b^5*d*x^10*e^4*sgn(b*x + a) + 10/9*b^5
*d^2*x^9*e^3*sgn(b*x + a) + 5/4*b^5*d^3*x^8*e^2*sgn(b*x + a) + 5/7*b^5*d^4*
x^7*e*sgn(b*x + a) + 1/6*b^5*d^5*x^6*sgn(b*x + a) + 1/2*a*b^4*x^10*e^5*sgn(
b*x + a) + 25/9*a*b^4*d*x^9*e^4*sgn(b*x + a) + 25/4*a*b^4*d^2*x^8*e^3*sgn(b
```

*x + a) + 50/7*a*b^4*d^3*x^7*e^2*sgn(b*x + a) + 25/6*a*b^4*d^4*x^6*e*sgn(b*x + a) + a*b^4*d^5*x^5*sgn(b*x + a) + 10/9*a^2*b^3*x^9*e^5*sgn(b*x + a) + 25/4*a^2*b^3*d*x^8*e^4*sgn(b*x + a) + 100/7*a^2*b^3*d^2*x^7*e^3*sgn(b*x + a) + 50/3*a^2*b^3*d^3*x^6*e^2*sgn(b*x + a) + 10*a^2*b^3*d^4*x^5*e*sgn(b*x + a) + 5/2*a^2*b^3*d^5*x^4*sgn(b*x + a) + 5/4*a^3*b^2*x^8*e^5*sgn(b*x + a) + 50/7*a^3*b^2*d*x^7*e^4*sgn(b*x + a) + 50/3*a^3*b^2*d^2*x^6*e^3*sgn(b*x + a) + 20*a^3*b^2*d^3*x^5*e^2*sgn(b*x + a) + 25/2*a^3*b^2*d^4*x^4*e*sgn(b*x + a) + 10/3*a^3*b^2*d^5*x^3*sgn(b*x + a) + 5/7*a^4*b*x^7*e^5*sgn(b*x + a) + 25/6*a^4*b*d*x^6*e^4*sgn(b*x + a) + 10*a^4*b*d^2*x^5*e^3*sgn(b*x + a) + 25/2*a^4*b*d^3*x^4*e^2*sgn(b*x + a) + 25/3*a^4*b*d^4*x^3*e*sgn(b*x + a) + 5/2*a^4*b*d^5*x^2*sgn(b*x + a) + 1/6*a^5*x^6*e^5*sgn(b*x + a) + a^5*d*x^5*e^4*sgn(b*x + a) + 5/2*a^5*d^2*x^4*e^3*sgn(b*x + a) + 10/3*a^5*d^3*x^3*e^2*sgn(b*x + a) + 5/2*a^5*d^4*x^2*e*sgn(b*x + a) + a^5*d^5*x*sgn(b*x + a)

3.1570 $\int (d+ex)^4 \left(a^2+2abx+b^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=219

$$\frac{4e^3\sqrt{a^2+2abx+b^2x^2}(a+bx)^8(bd-ae)}{9b^5}+\frac{3e^2\sqrt{a^2+2abx+b^2x^2}(a+bx)^7(bd-ae)^2}{4b^5}+\frac{4e\sqrt{a^2+2abx+b^2x^2}(a+bx)^6(bd}{7b^5}$$

```
[Out] ((b*d - a*e)^4*(a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*b^5) + (4*e*(b
*d - a*e)^3*(a + b*x)^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*b^5) + (3*e^2*(b*
d - a*e)^2*(a + b*x)^7*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*b^5) + (4*e^3*(b*d
 - a*e)*(a + b*x)^8*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(9*b^5) + (e^4*(a + b*x)
^9*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(10*b^5)
```

Rubi [A] time = 0.0896996, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {645}

$$\frac{4e^3\sqrt{a^2+2abx+b^2x^2}(a+bx)^8(bd-ae)}{9b^5}+\frac{3e^2\sqrt{a^2+2abx+b^2x^2}(a+bx)^7(bd-ae)^2}{4b^5}+\frac{4e\sqrt{a^2+2abx+b^2x^2}(a+bx)^6(bd}{7b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] ((b*d - a*e)^4*(a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*b^5) + (4*e*(b
*d - a*e)^3*(a + b*x)^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*b^5) + (3*e^2*(b*
d - a*e)^2*(a + b*x)^7*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*b^5) + (4*e^3*(b*d
 - a*e)*(a + b*x)^8*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(9*b^5) + (e^4*(a + b*x)
^9*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(10*b^5)
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[ExpandLinearProduct[(b/2 + c*x)^(2*p), (d + e*x)^m, b/2, c
, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&
!IntegerQ[p] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 0] && EqQ[m - 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^4\left(a^2+2abx+b^2x^2\right)^{5/2}dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int\left(\frac{(bd-ae)^4\left(ab+b^2x\right)^5}{b^4}+\frac{4e(bd-ae)^3\left(ab+b^2x\right)^6}{b^5}+\frac{6e^2(bd-ae)^2\left(ab+b^2x\right)^7}{b^6}+\frac{4e}{}\right.}{b^4\left(ab+b^2x\right)}\\ &= \frac{(bd-ae)^4(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{6b^5}+\frac{4e(bd-ae)^3(a+bx)^6\sqrt{a^2+2abx+b^2x^2}}{7b^5}\end{aligned}$$

Mathematica [A] time = 0.107724, size = 319, normalized size = 1.46

$$\frac{x\sqrt{(a+bx)^2}\left(120a^3b^2x^2\left(126d^2e^2x^2+105d^3ex+35d^4+70de^3x^3+15e^4x^4\right)+45a^2b^3x^3\left(280d^2e^2x^2+224d^3ex+70d^4+1\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[(a + b*x)^2]*(252*a^5*(5*d^4 + 10*d^3*e*x + 10*d^2*e^2*x^2 + 5*d*e^
3*x^3 + e^4*x^4) + 210*a^4*b*x*(15*d^4 + 40*d^3*e*x + 45*d^2*e^2*x^2 + 24*d
*e^3*x^3 + 5*e^4*x^4) + 120*a^3*b^2*x^2*(35*d^4 + 105*d^3*e*x + 126*d^2*e^2
*x^2 + 70*d*e^3*x^3 + 15*e^4*x^4) + 45*a^2*b^3*x^3*(70*d^4 + 224*d^3*e*x +
280*d^2*e^2*x^2 + 160*d*e^3*x^3 + 35*e^4*x^4) + 10*a*b^4*x^4*(126*d^4 + 420
*d^3*e*x + 540*d^2*e^2*x^2 + 315*d*e^3*x^3 + 70*e^4*x^4) + b^5*x^5*(210*d^4
 + 720*d^3*e*x + 945*d^2*e^2*x^2 + 560*d*e^3*x^3 + 126*e^4*x^4)))/(1260*(a
+ b*x))
```

Maple [B] time = 0.157, size = 414, normalized size = 1.9

$$x\left(126\,e^4b^5x^9+700\,x^8e^4ab^4+560\,x^8de^3b^5+1575\,x^7e^4a^2b^3+3150\,x^7de^3ab^4+945\,x^7d^2e^2b^5+1800\,x^6e^4a^3b^2+7200\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] 1/1260*x*(126*b^5*e^4*x^9+700*a*b^4*e^4*x^8+560*b^5*d*e^3*x^8+1575*a^2*b^3*
e^4*x^7+3150*a*b^4*d*e^3*x^7+945*b^5*d^2*e^2*x^7+1800*a^3*b^2*e^4*x^6+7200*
a^2*b^3*d*e^3*x^6+5400*a*b^4*d^2*e^2*x^6+720*b^5*d^3*e*x^6+1050*a^4*b*e^4*x
^5+8400*a^3*b^2*d*e^3*x^5+12600*a^2*b^3*d^2*e^2*x^5+4200*a*b^4*d^3*e*x^5+21
0*b^5*d^4*x^5+252*a^5*e^4*x^4+5040*a^4*b*d*e^3*x^4+15120*a^3*b^2*d^2*e^2*x^
4+10080*a^2*b^3*d^3*e*x^4+1260*a*b^4*d^4*x^4+1260*a^5*d*e^3*x^3+9450*a^4*b*
d^2*e^2*x^3+12600*a^3*b^2*d^3*e*x^3+3150*a^2*b^3*d^4*x^3+2520*a^5*d^2*e^2*x
^2+8400*a^4*b*d^3*e*x^2+4200*a^3*b^2*d^4*x^2+2520*a^5*d^3*e*x+3150*a^4*b*d^
4*x+1260*a^5*d^4)*((b*x+a)^2)^(5/2)/(b*x+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.60779, size = 760, normalized size = 3.47

$$\frac{1}{10}\,b^5e^4x^{10}+a^5d^4x+\frac{1}{9}\left(4\,b^5de^3+5\,ab^4e^4\right)x^9+\frac{1}{4}\left(3\,b^5d^2e^2+10\,ab^4de^3+5\,a^2b^3e^4\right)x^8+\frac{2}{7}\left(2\,b^5d^3e+15\,ab^4d^2e^2+20\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/10*b^5*e^4*x^10 + a^5*d^4*x + 1/9*(4*b^5*d*e^3 + 5*a*b^4*e^4)*x^9 + 1/4*(
3*b^5*d^2*e^2 + 10*a*b^4*d*e^3 + 5*a^2*b^3*e^4)*x^8 + 2/7*(2*b^5*d^3*e + 15
```

```
*a*b^4*d^2*e^2 + 20*a^2*b^3*d*e^3 + 5*a^3*b^2*e^4)*x^7 + 1/6*(b^5*d^4 + 20*
a*b^4*d^3*e + 60*a^2*b^3*d^2*e^2 + 40*a^3*b^2*d*e^3 + 5*a^4*b*e^4)*x^6 + 1/
5*(5*a*b^4*d^4 + 40*a^2*b^3*d^3*e + 60*a^3*b^2*d^2*e^2 + 20*a^4*b*d*e^3 + a
^5*e^4)*x^5 + 1/2*(5*a^2*b^3*d^4 + 20*a^3*b^2*d^3*e + 15*a^4*b*d^2*e^2 + 2*
a^5*d*e^3)*x^4 + 2/3*(5*a^3*b^2*d^4 + 10*a^4*b*d^3*e + 3*a^5*d^2*e^2)*x^3 +
 1/2*(5*a^4*b*d^4 + 4*a^5*d^3*e)*x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d+ex)^4 \left((a+bx)^2\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Integral((d + e*x)**4*((a + b*x)**2)**(5/2), x)
```

Giac [B] time = 1.24433, size = 761, normalized size = 3.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/10*b^5*x^10*e^4*sgn(b*x + a) + 4/9*b^5*d*x^9*e^3*sgn(b*x + a) + 3/4*b^5*d
^2*x^8*e^2*sgn(b*x + a) + 4/7*b^5*d^3*x^7*e*sgn(b*x + a) + 1/6*b^5*d^4*x^6*
sgn(b*x + a) + 5/9*a*b^4*x^9*e^4*sgn(b*x + a) + 5/2*a*b^4*d*x^8*e^3*sgn(b*x
 + a) + 30/7*a*b^4*d^2*x^7*e^2*sgn(b*x + a) + 10/3*a*b^4*d^3*x^6*e*sgn(b*x
+ a) + a*b^4*d^4*x^5*sgn(b*x + a) + 5/4*a^2*b^3*x^8*e^4*sgn(b*x + a) + 40/7
*a^2*b^3*d*x^7*e^3*sgn(b*x + a) + 10*a^2*b^3*d^2*x^6*e^2*sgn(b*x + a) + 8*a
^2*b^3*d^3*x^5*e*sgn(b*x + a) + 5/2*a^2*b^3*d^4*x^4*sgn(b*x + a) + 10/7*a^3
*b^2*x^7*e^4*sgn(b*x + a) + 20/3*a^3*b^2*d*x^6*e^3*sgn(b*x + a) + 12*a^3*b^
2*d^2*x^5*e^2*sgn(b*x + a) + 10*a^3*b^2*d^3*x^4*e*sgn(b*x + a) + 10/3*a^3*b
^2*d^4*x^3*sgn(b*x + a) + 5/6*a^4*b*x^6*e^4*sgn(b*x + a) + 4*a^4*b*d*x^5*e^
3*sgn(b*x + a) + 15/2*a^4*b*d^2*x^4*e^2*sgn(b*x + a) + 20/3*a^4*b*d^3*x^3*e
*sgn(b*x + a) + 5/2*a^4*b*d^4*x^2*sgn(b*x + a) + 1/5*a^5*x^5*e^4*sgn(b*x +
a) + a^5*d*x^4*e^3*sgn(b*x + a) + 2*a^5*d^2*x^3*e^2*sgn(b*x + a) + 2*a^5*d^
3*x^2*e*sgn(b*x + a) + a^5*d^4*x*sgn(b*x + a)
```

3.1571 $\int (d+ex)^3 \left(a^2+2abx+b^2x^2\right)^{5/2} dx$

Optimal. Leaf size=172

$$\frac{3e^2\sqrt{a^2+2abx+b^2x^2}(a+bx)^7(bd-ae)}{8b^4}+\frac{3e\sqrt{a^2+2abx+b^2x^2}(a+bx)^6(bd-ae)^2}{7b^4}+\frac{\sqrt{a^2+2abx+b^2x^2}(a+bx)^5(bd}{6b^4}$$

```
[Out] ((b*d - a*e)^3*(a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*b^4) + (3*e*(b
*d - a*e)^2*(a + b*x)^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*b^4) + (3*e^2*(b*
d - a*e)*(a + b*x)^7*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(8*b^4) + (e^3*(a + b*x
)^8*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(9*b^4)
```

Rubi [A] time = 0.180173, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$\frac{3e^2\sqrt{a^2+2abx+b^2x^2}(a+bx)^7(bd-ae)}{8b^4}+\frac{3e\sqrt{a^2+2abx+b^2x^2}(a+bx)^6(bd-ae)^2}{7b^4}+\frac{\sqrt{a^2+2abx+b^2x^2}(a+bx)^5(bd}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] ((b*d - a*e)^3*(a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*b^4) + (3*e*(b
*d - a*e)^2*(a + b*x)^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*b^4) + (3*e^2*(b*
d - a*e)*(a + b*x)^7*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(8*b^4) + (e^3*(a + b*x
)^8*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(9*b^4)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^3\left(a^2+2abx+b^2x^2\right)^{5/2}dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(ab+b^2x\right)^5(d+ex)^3\,dx}{b^4\left(ab+b^2x\right)}\\
&= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{(bd-ae)^3\left(ab+b^2x\right)^5}{b^3}+\frac{3e(bd-ae)^2\left(ab+b^2x\right)^6}{b^4}+\frac{3e^2(bd-ae)\left(ab+b^2x\right)^7}{b^5}+\right.}{b^4\left(ab+b^2x\right)}\\
&= \frac{(bd-ae)^3(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{6b^4}+\frac{3e(bd-ae)^2(a+bx)^6\sqrt{a^2+2abx+b}}{7b^4}
\end{aligned}$$

Mathematica [A] time = 0.0872429, size = 253, normalized size = 1.47

$$\frac{x\sqrt{(a+bx)^2}\left(84a^3b^2x^2\left(45d^2ex+20d^3+36de^2x^2+10e^3x^3\right)+36a^2b^3x^3\left(84d^2ex+35d^3+70de^2x^2+20e^3x^3\right)+126a^4bx\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[(a + b*x)^2]*(126*a^5*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) +
 126*a^4*b*x*(10*d^3 + 20*d^2*e*x + 15*d*e^2*x^2 + 4*e^3*x^3) + 84*a^3*b^2*
x^2*(20*d^3 + 45*d^2*e*x + 36*d*e^2*x^2 + 10*e^3*x^3) + 36*a^2*b^3*x^3*(35*
d^3 + 84*d^2*e*x + 70*d*e^2*x^2 + 20*e^3*x^3) + 9*a*b^4*x^4*(56*d^3 + 140*d
^2*e*x + 120*d*e^2*x^2 + 35*e^3*x^3) + b^5*x^5*(84*d^3 + 216*d^2*e*x + 189*
d*e^2*x^2 + 56*e^3*x^3)))/(504*(a + b*x))
```

Maple [B] time = 0.156, size = 322, normalized size = 1.9

$$\frac{x\left(56\,e^3b^5x^8+315\,x^7e^3ab^4+189\,x^7de^2b^5+720\,x^6e^3a^2b^3+1080\,x^6de^2ab^4+216\,x^6d^2eb^5+840\,x^5e^3a^3b^2+2520\,x^5de^2a^2b^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] 1/504*x*(56*b^5*e^3*x^8+315*a*b^4*e^3*x^7+189*b^5*d*e^2*x^7+720*a^2*b^3*e^3
*x^6+1080*a*b^4*d*e^2*x^6+216*b^5*d^2*e*x^6+840*a^3*b^2*e^3*x^5+2520*a^2*b^
3*d*e^2*x^5+1260*a*b^4*d^2*e*x^5+84*b^5*d^3*x^5+504*a^4*b*e^3*x^4+3024*a^3*
b^2*d*e^2*x^4+3024*a^2*b^3*d^2*e*x^4+504*a*b^4*d^3*x^4+126*a^5*e^3*x^3+1890
*a^4*b*d*e^2*x^3+3780*a^3*b^2*d^2*e*x^3+1260*a^2*b^3*d^3*x^3+504*a^5*d*e^2*
x^2+2520*a^4*b*d^2*e*x^2+1680*a^3*b^2*d^3*x^2+756*a^5*d^2*e*x+1260*a^4*b*d^
3*x+504*a^5*d^3)*((b*x+a)^2)^(5/2)/(b*x+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.49215, size = 585, normalized size = 3.4

$$\frac{1}{9}\,b^5e^3x^9+a^5d^3x+\frac{1}{8}\left(3\,b^5de^2+5\,ab^4e^3\right)x^8+\frac{1}{7}\left(3\,b^5d^2e+15\,ab^4de^2+10\,a^2b^3e^3\right)x^7+\frac{1}{6}\left(b^5d^3+15\,ab^4d^2e+30\,a^2b^3de^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/9*b^5*e^3*x^9 + a^5*d^3*x + 1/8*(3*b^5*d*e^2 + 5*a*b^4*e^3)*x^8 + 1/7*(3*
b^5*d^2*e + 15*a*b^4*d*e^2 + 10*a^2*b^3*e^3)*x^7 + 1/6*(b^5*d^3 + 15*a*b^4*
d^2*e + 30*a^2*b^3*d*e^2 + 10*a^3*b^2*e^3)*x^6 + (a*b^4*d^3 + 6*a^2*b^3*d^2
*e + 6*a^3*b^2*d*e^2 + a^4*b*e^3)*x^5 + 1/4*(10*a^2*b^3*d^3 + 30*a^3*b^2*d^
2*e + 15*a^4*b*d*e^2 + a^5*e^3)*x^4 + 1/3*(10*a^3*b^2*d^3 + 15*a^4*b*d^2*e
+ 3*a^5*d*e^2)*x^3 + 1/2*(5*a^4*b*d^3 + 3*a^5*d^2*e)*x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d+ex)^3 \left((a+bx)^2\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Integral((d + e*x)**3*((a + b*x)**2)**(5/2), x)
```

Giac [B] time = 1.19104, size = 595, normalized size = 3.46

$$\frac{1}{9}b^5x^9e^3\mathrm{sgn}(bx+a) + \frac{3}{8}b^5dx^8e^2\mathrm{sgn}(bx+a) + \frac{3}{7}b^5d^2x^7e\mathrm{sgn}(bx+a) + \frac{1}{6}b^5d^3x^6\mathrm{sgn}(bx+a) + \frac{5}{8}ab^4x^8e^3\mathrm{sgn}(bx+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/9*b^5*x^9*e^3*sgn(b*x + a) + 3/8*b^5*d*x^8*e^2*sgn(b*x + a) + 3/7*b^5*d^2
*x^7*e*sgn(b*x + a) + 1/6*b^5*d^3*x^6*sgn(b*x + a) + 5/8*a*b^4*x^8*e^3*sgn(
b*x + a) + 15/7*a*b^4*d*x^7*e^2*sgn(b*x + a) + 5/2*a*b^4*d^2*x^6*e*sgn(b*x
+ a) + a*b^4*d^3*x^5*sgn(b*x + a) + 10/7*a^2*b^3*x^7*e^3*sgn(b*x + a) + 5*a
^2*b^3*d*x^6*e^2*sgn(b*x + a) + 6*a^2*b^3*d^2*x^5*e*sgn(b*x + a) + 5/2*a^2*
b^3*d^3*x^4*sgn(b*x + a) + 5/3*a^3*b^2*x^6*e^3*sgn(b*x + a) + 6*a^3*b^2*d*x
^5*e^2*sgn(b*x + a) + 15/2*a^3*b^2*d^2*x^4*e*sgn(b*x + a) + 10/3*a^3*b^2*d^
3*x^3*sgn(b*x + a) + a^4*b*x^5*e^3*sgn(b*x + a) + 15/4*a^4*b*d*x^4*e^2*sgn(
b*x + a) + 5*a^4*b*d^2*x^3*e*sgn(b*x + a) + 5/2*a^4*b*d^3*x^2*sgn(b*x + a)
+ 1/4*a^5*x^4*e^3*sgn(b*x + a) + a^5*d*x^3*e^2*sgn(b*x + a) + 3/2*a^5*d^2*x
^2*e*sgn(b*x + a) + a^5*d^3*x*sgn(b*x + a)
```

3.1572 $\int (d+ex)^2 \left(a^2+2abx+b^2x^2\right)^{5/2} dx$

Optimal. Leaf size=125

$$\frac{2e\sqrt{a^2+2abx+b^2x^2}(a+bx)^6(bd-ae)}{7b^3}+\frac{\sqrt{a^2+2abx+b^2x^2}(a+bx)^5(bd-ae)^2}{6b^3}+\frac{e^2\sqrt{a^2+2abx+b^2x^2}(a+bx)^7}{8b^3}$$

```
[Out] ((b*d - a*e)^2*(a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*b^3) + (2*e*(b
*d - a*e)*(a + b*x)^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*b^3) + (e^2*(a + b*
x)^7*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(8*b^3)
```

Rubi [A] time = 0.139914, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$\frac{2e\sqrt{a^2+2abx+b^2x^2}(a+bx)^6(bd-ae)}{7b^3}+\frac{\sqrt{a^2+2abx+b^2x^2}(a+bx)^5(bd-ae)^2}{6b^3}+\frac{e^2\sqrt{a^2+2abx+b^2x^2}(a+bx)^7}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] ((b*d - a*e)^2*(a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*b^3) + (2*e*(b
*d - a*e)*(a + b*x)^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*b^3) + (e^2*(a + b*
x)^7*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(8*b^3)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^2 \left(a^2+2abx+b^2x^2\right)^{5/2} dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(ab+b^2x\right)^5 (d+ex)^2\,dx}{b^4\left(ab+b^2x\right)}\\
&= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{(bd-ae)^2\left(ab+b^2x\right)^5}{b^2}+\frac{2e(bd-ae)\left(ab+b^2x\right)^6}{b^3}+\frac{e^2\left(ab+b^2x\right)^7}{b^4}\right)dx}{b^4\left(ab+b^2x\right)}\\
&= \frac{(bd-ae)^2(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{6b^3}+\frac{2e(bd-ae)(a+bx)^6\sqrt{a^2+2abx+b^2x^2}}{7b^3}
\end{aligned}$$

Mathematica [A] time = 0.0676425, size = 187, normalized size = 1.5

$$\frac{x\sqrt{(a+bx)^2}\left(56a^3b^2x^2\left(10d^2+15dex+6e^2x^2\right)+28a^2b^3x^3\left(15d^2+24dex+10e^2x^2\right)+70a^4bx\left(6d^2+8dex+3e^2x^2\right)+\right.}{168(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[(a + b*x)^2]*(56*a^5*(3*d^2 + 3*d*e*x + e^2*x^2) + 70*a^4*b*x*(6*d^
2 + 8*d*e*x + 3*e^2*x^2) + 56*a^3*b^2*x^2*(10*d^2 + 15*d*e*x + 6*e^2*x^2) +
 28*a^2*b^3*x^3*(15*d^2 + 24*d*e*x + 10*e^2*x^2) + 8*a*b^4*x^4*(21*d^2 + 35
*d*e*x + 15*e^2*x^2) + b^5*x^5*(28*d^2 + 48*d*e*x + 21*e^2*x^2)))/(168*(a +
 b*x))
```

Maple [B] time = 0.153, size = 230, normalized size = 1.8

$$\frac{x\left(21\,e^2b^5x^7+120\,x^6e^2ab^4+48\,x^6deb^5+280\,x^5e^2a^2b^3+280\,x^5deab^4+28\,x^5d^2b^5+336\,a^3b^2e^2x^4+672\,a^2b^3dex^4+16\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] 1/168*x*(21*b^5*e^2*x^7+120*a*b^4*e^2*x^6+48*b^5*d*e*x^6+280*a^2*b^3*e^2*x^
5+280*a*b^4*d*e*x^5+28*b^5*d^2*x^5+336*a^3*b^2*e^2*x^4+672*a^2*b^3*d*e*x^4+
168*a*b^4*d^2*x^4+210*a^4*b*e^2*x^3+840*a^3*b^2*d*e*x^3+420*a^2*b^3*d^2*x^3
+56*a^5*e^2*x^2+560*a^4*b*d*e*x^2+560*a^3*b^2*d^2*x^2+168*a^5*d*e*x+420*a^4
*b*d^2*x+168*a^5*d^2)*((b*x+a)^2)^(5/2)/(b*x+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.59418, size = 417, normalized size = 3.34

$$\frac{1}{8}\,b^5e^2x^8+a^5d^2x+\frac{1}{7}\left(2\,b^5de+5\,ab^4e^2\right)x^7+\frac{1}{6}\left(b^5d^2+10\,ab^4de+10\,a^2b^3e^2\right)x^6+\left(ab^4d^2+4\,a^2b^3de+2\,a^3b^2e^2\right)x^5+\frac{5}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/8*b^5*e^2*x^8 + a^5*d^2*x + 1/7*(2*b^5*d*e + 5*a*b^4*e^2)*x^7 + 1/6*(b^5*
d^2 + 10*a*b^4*d*e + 10*a^2*b^3*e^2)*x^6 + (a*b^4*d^2 + 4*a^2*b^3*d*e + 2*a
```

```
^3*b^2*e^2)*x^5 + 5/4*(2*a^2*b^3*d^2 + 4*a^3*b^2*d*e + a^4*b*e^2)*x^4 + 1/3
*(10*a^3*b^2*d^2 + 10*a^4*b*d*e + a^5*e^2)*x^3 + 1/2*(5*a^4*b*d^2 + 2*a^5*d
*e)*x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d+ex)^2 \left((a+bx)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Integral((d + e*x)**2*((a + b*x)**2)**(5/2), x)
```

Giac [B] time = 1.14295, size = 432, normalized size = 3.46

$$\frac{1}{8}\,b^5x^8e^2\mathrm{sgn}\left(bx+a\right)+\frac{2}{7}\,b^5dx^7e\mathrm{sgn}\left(bx+a\right)+\frac{1}{6}\,b^5d^2x^6\mathrm{sgn}\left(bx+a\right)+\frac{5}{7}\,ab^4x^7e^2\mathrm{sgn}\left(bx+a\right)+\frac{5}{3}\,ab^4dx^6e\mathrm{sgn}\left(bx+a\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/8*b^5*x^8*e^2*sgn(b*x + a) + 2/7*b^5*d*x^7*e*sgn(b*x + a) + 1/6*b^5*d^2*x
^6*sgn(b*x + a) + 5/7*a*b^4*x^7*e^2*sgn(b*x + a) + 5/3*a*b^4*d*x^6*e*sgn(b*
x + a) + a*b^4*d^2*x^5*sgn(b*x + a) + 5/3*a^2*b^3*x^6*e^2*sgn(b*x + a) + 4*
a^2*b^3*d*x^5*e*sgn(b*x + a) + 5/2*a^2*b^3*d^2*x^4*sgn(b*x + a) + 2*a^3*b^2
*x^5*e^2*sgn(b*x + a) + 5*a^3*b^2*d*x^4*e*sgn(b*x + a) + 10/3*a^3*b^2*d^2*x
^3*sgn(b*x + a) + 5/4*a^4*b*x^4*e^2*sgn(b*x + a) + 10/3*a^4*b*d*x^3*e*sgn(b
*x + a) + 5/2*a^4*b*d^2*x^2*sgn(b*x + a) + 1/3*a^5*x^3*e^2*sgn(b*x + a) + a
^5*d*x^2*e*sgn(b*x + a) + a^5*d^2*x*sgn(b*x + a)
```

3.1573 $\int (d+ex)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=69

$$\frac{(a+bx)\left(a^2+2abx+b^2x^2\right)^{5/2}(bd-ae)}{6b^2}+\frac{e\left(a^2+2abx+b^2x^2\right)^{7/2}}{7b^2}$$

```
[Out] ((b*d - a*e)*(a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/(6*b^2) + (e*(a^2 +
 2*a*b*x + b^2*x^2)^(7/2))/(7*b^2)
```

Rubi [A] time = 0.0235983, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {640, 609}

$$\frac{(a+bx)\left(a^2+2abx+b^2x^2\right)^{5/2}(bd-ae)}{6b^2}+\frac{e\left(a^2+2abx+b^2x^2\right)^{7/2}}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] ((b*d - a*e)*(a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/(6*b^2) + (e*(a^2 +
 2*a*b*x + b^2*x^2)^(7/2))/(7*b^2)
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx &= \frac{e\left(a^2+2abx+b^2x^2\right)^{7/2}}{7b^2}+\frac{\left(2b^2d-2abe\right)\int \left(a^2+2abx+b^2x^2\right)^{5/2}\,dx}{2b^2}\\ &= \frac{(bd-ae)(a+bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{6b^2}+\frac{e\left(a^2+2abx+b^2x^2\right)^{7/2}}{7b^2}\end{aligned}$$

Mathematica [A] time = 0.0453663, size = 121, normalized size = 1.75

$$\frac{x\sqrt{(a+bx)^2}\left(35a^3b^2x^2(4d+3ex)+21a^2b^3x^3(5d+4ex)+35a^4bx(3d+2ex)+21a^5(2d+ex)+7ab^4x^4(6d+5ex)+b^5\right.}{42(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[(a + b*x)^2]*(21*a^5*(2*d + e*x) + 35*a^4*b*x*(3*d + 2*e*x) + 35*a^
3*b^2*x^2*(4*d + 3*e*x) + 21*a^2*b^3*x^3*(5*d + 4*e*x) + 7*a*b^4*x^4*(6*d +
 5*e*x) + b^5*x^5*(7*d + 6*e*x)))/(42*(a + b*x))
```

Maple [B] time = 0.16, size = 138, normalized size = 2.

$$\frac{x\left(6\,eb^5x^6+35\,x^5eab^4+7\,x^5db^5+84\,a^2b^3ex^4+42\,ab^4dx^4+105\,x^3ea^3b^2+105\,x^3da^2b^3+70\,x^2ea^4b+140\,x^2da^3b^2+21\,x\right.}{42\,(bx+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] 1/42*x*(6*b^5*e*x^6+35*a*b^4*e*x^5+7*b^5*d*x^5+84*a^2*b^3*e*x^4+42*a*b^4*d*
x^4+105*a^3*b^2*e*x^3+105*a^2*b^3*d*x^3+70*a^4*b*e*x^2+140*a^3*b^2*d*x^2+21
*a^5*e*x+105*a^4*b*d*x+42*a^5*d)*((b*x+a)^2)^(5/2)/(b*x+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51008, size = 247, normalized size = 3.58

$$\frac{1}{7}\,b^5ex^7+a^5dx+\frac{1}{6}\left(b^5d+5\,ab^4e\right)x^6+\left(ab^4d+2\,a^2b^3e\right)x^5+\frac{5}{2}\left(a^2b^3d+a^3b^2e\right)x^4+\frac{5}{3}\left(2\,a^3b^2d+a^4be\right)x^3+\frac{1}{2}\left(5\,a^4bd+a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/7*b^5*e*x^7 + a^5*d*x + 1/6*(b^5*d + 5*a*b^4*e)*x^6 + (a*b^4*d + 2*a^2*b^
3*e)*x^5 + 5/2*(a^2*b^3*d + a^3*b^2*e)*x^4 + 5/3*(2*a^3*b^2*d + a^4*b*e)*x^
3 + 1/2*(5*a^4*b*d + a^5*e)*x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d+ex)\left((a+bx)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Integral((d + e*x)*((a + b*x)**2)**(5/2), x)
```

Giac [B] time = 1.24066, size = 269, normalized size = 3.9

$$\frac{1}{7}b^5x^7e\text{sgn}(bx+a)+\frac{1}{6}b^5dx^6\text{sgn}(bx+a)+\frac{5}{6}ab^4x^6e\text{sgn}(bx+a)+ab^4dx^5\text{sgn}(bx+a)+2a^2b^3x^5e\text{sgn}(bx+a)+\frac{5}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/7*b^5*x^7*e*sgn(b*x + a) + 1/6*b^5*d*x^6*sgn(b*x + a) + 5/6*a*b^4*x^6*e*s
gn(b*x + a) + a*b^4*d*x^5*sgn(b*x + a) + 2*a^2*b^3*x^5*e*sgn(b*x + a) + 5/2
*a^2*b^3*d*x^4*sgn(b*x + a) + 5/2*a^3*b^2*x^4*e*sgn(b*x + a) + 10/3*a^3*b^2
*d*x^3*sgn(b*x + a) + 5/3*a^4*b*x^3*e*sgn(b*x + a) + 5/2*a^4*b*d*x^2*sgn(b*
x + a) + 1/2*a^5*x^2*e*sgn(b*x + a) + a^5*d*x*sgn(b*x + a)
```

3.1574 $\int \left(a^2+2abx+b^2x^2\right)^{5/2} dx$

Optimal. Leaf size=32

$$\frac{(a+bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{6b}$$

```
[Out] ((a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/(6*b)
```

Rubi [A] time = 0.0055318, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {609}

$$\frac{(a+bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] ((a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/(6*b)
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \left(a^2+2abx+b^2x^2\right)^{5/2} dx = \frac{(a+bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{6b}$$

Mathematica [A] time = 0.0128349, size = 23, normalized size = 0.72

$$\frac{(a+bx)\left((a+bx)^2\right)^{5/2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] ((a + b*x)*((a + b*x)^2)^(5/2))/(6*b)
```

Maple [B] time = 0.042, size = 71, normalized size = 2.2

$$\frac{x\left(b^5x^5+6\,ab^4x^4+15\,a^2b^3x^3+20\,a^3b^2x^2+15\,a^4bx+6\,a^5\right)}{6\,(bx+a)^5}\left((bx+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] 1/6*x*(b^5*x^5+6*a*b^4*x^4+15*a^2*b^3*x^3+20*a^3*b^2*x^2+15*a^4*b*x+6*a^5)*
((b*x+a)^2)^(5/2)/(b*x+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53539, size = 116, normalized size = 3.62

$$\frac{1}{6}\,b^5x^6 + ab^4x^5 + \frac{5}{2}\,a^2b^3x^4 + \frac{10}{3}\,a^3b^2x^3 + \frac{5}{2}\,a^4bx^2 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/2*a^4*b*x^
2 + a^5*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2 + 2abx + b^2x^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x + b**2*x**2)**(5/2), x)
```

Giac [B] time = 1.17674, size = 139, normalized size = 4.34

$$\frac{1}{6}\,b^5x^6\mathrm{sgn}\,(bx + a) + ab^4x^5\mathrm{sgn}\,(bx + a) + \frac{5}{2}\,a^2b^3x^4\mathrm{sgn}\,(bx + a) + \frac{10}{3}\,a^3b^2x^3\mathrm{sgn}\,(bx + a) + \frac{5}{2}\,a^4bx^2\mathrm{sgn}\,(bx + a) + a^5x\mathrm{s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*b^5*x^6*sgn(b*x + a) + a*b^4*x^5*sgn(b*x + a) + 5/2*a^2*b^3*x^4*sgn(b*x
 + a) + 10/3*a^3*b^2*x^3*sgn(b*x + a) + 5/2*a^4*b*x^2*sgn(b*x + a) + a^5*x*
sgn(b*x + a) + 1/6*a^6*sgn(b*x + a)/b
```

3.1575 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=254

$$\frac{bx\sqrt{a^2+2abx+b^2x^2}(bd-ae)^4}{e^5(a+bx)}-\frac{(a+bx)\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3}{2e^4}+\frac{(a+bx)^2\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{3e^3}-\frac{(a+l}{}$$

```
[Out] (b*(b*d - a*e)^4*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^5*(a + b*x)) - ((b*d -
 a*e)^3*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*e^4) + ((b*d - a*e)^2*(
a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^3) - ((b*d - a*e)*(a + b*x)^
3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*e^2) + ((a + b*x)^4*Sqrt[a^2 + 2*a*b*x
+ b^2*x^2])/(5*e) - ((b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Log[d + e*
x])/(e^6*(a + b*x))
```

Rubi [A] time = 0.125486, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$\frac{bx\sqrt{a^2+2abx+b^2x^2}(bd-ae)^4}{e^5(a+bx)}-\frac{(a+bx)\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3}{2e^4}+\frac{(a+bx)^2\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{3e^3}-\frac{(a+l}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x),x]
```


```
[Out] (b*(b*d - a*e)^4*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^5*(a + b*x)) - ((b*d -
 a*e)^3*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*e^4) + ((b*d - a*e)^2*(
a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^3) - ((b*d - a*e)*(a + b*x)^
3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*e^2) + ((a + b*x)^4*Sqrt[a^2 + 2*a*b*x
+ b^2*x^2])/(5*e) - ((b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Log[d + e*
x])/(e^6*(a + b*x))
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{d+ex}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^5}{d+ex}\,dx}{b^4\left(ab+b^2x\right)}$$

$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{b^6(bd-ae)^4}{e^5} - \frac{b^5(bd-ae)^3\left(ab+b^2x\right)}{e^4} + \frac{b^4(bd-ae)^2\left(ab+b^2x\right)^2}{e^3} - \frac{b^3(bd-ae)\left(ab+b^2x\right)^3}{e^2}\right.}{b^4\left(ab+b^2x\right)}$$

$$= \frac{b(bd-ae)^4x\sqrt{a^2+2abx+b^2x^2}}{e^5(a+bx)} - \frac{(bd-ae)^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}{2e^4} + \frac{(bd-ae)^2(a}{}$$

Mathematica [A] time = 0.0963047, size = 185, normalized size = 0.73

$$\frac{\sqrt{(a+bx)^2}\left(bex\left(100a^2b^2e^2\left(6d^2-3dex+2e^2x^2\right)+300a^3be^3(ex-2d)+300a^4e^4+25ab^3e\left(6d^2ex-12d^3-4de^2x^2+3\right.\right.\right.}{60e^6(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x),x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(b*e*x*(300*a^4*e^4 + 300*a^3*b*e^3*(-2*d + e*x) + 100*a
^2*b^2*e^2*(6*d^2 - 3*d*e*x + 2*e^2*x^2) + 25*a*b^3*e*(-12*d^3 + 6*d^2*e*x
- 4*d*e^2*x^2 + 3*e^3*x^3) + b^4*(60*d^4 - 30*d^3*e*x + 20*d^2*e^2*x^2 - 15
*d*e^3*x^3 + 12*e^4*x^4)) - 60*(b*d - a*e)^5*Log[d + e*x]))/(60*e^6*(a + b*
x))
```

Maple [A] time = 0.197, size = 318, normalized size = 1.3

$$\frac{12\,x^5b^5e^5+75\,x^4ab^4e^5-15\,x^4b^5de^4+200\,x^3a^2b^3e^5-100\,x^3ab^4de^4+20\,x^3b^5d^2e^3+300\,x^2a^3b^2e^5-300\,x^2a^2b^3de^4+15}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d),x)
```

```
[Out] 1/60*((b*x+a)^2)^(5/2)*(12*x^5*b^5*e^5+75*x^4*a*b^4*e^5-15*x^4*b^5*d*e^4+20
0*x^3*a^2*b^3*e^5-100*x^3*a*b^4*d*e^4+20*x^3*b^5*d^2*e^3+300*x^2*a^3*b^2*e^
5-300*x^2*a^2*b^3*d*e^4+150*x^2*a*b^4*d^2*e^3-30*x^2*b^5*d^3*e^2+60*ln(e*x+
d)*a^5*e^5-300*ln(e*x+d)*a^4*b*d*e^4+600*ln(e*x+d)*a^3*b^2*d^2*e^3-600*ln(e
*x+d)*a^2*b^3*d^3*e^2+300*ln(e*x+d)*a*b^4*d^4*e-60*ln(e*x+d)*b^5*d^5+300*x*
a^4*b*e^5-600*x*a^3*b^2*d*e^4+600*x*a^2*b^3*d^2*e^3-300*x*a*b^4*d^3*e^2+60*
x*b^5*d^4*e)/(b*x+a)^5/e^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69253, size = 537, normalized size = 2.11

$$\frac{12\,b^5e^5x^5 - 15\left(b^5de^4 - 5\,ab^4e^5\right)x^4 + 20\left(b^5d^2e^3 - 5\,ab^4de^4 + 10\,a^2b^3e^5\right)x^3 - 30\left(b^5d^3e^2 - 5\,ab^4d^2e^3 + 10\,a^2b^3de^4 - 10\,a^3b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/60*(12*b^5*e^5*x^5 - 15*(b^5*d*e^4 - 5*a*b^4*e^5)*x^4 + 20*(b^5*d^2*e^3 -
 5*a*b^4*d*e^4 + 10*a^2*b^3*e^5)*x^3 - 30*(b^5*d^3*e^2 - 5*a*b^4*d^2*e^3 +
10*a^2*b^3*d*e^4 - 10*a^3*b^2*e^5)*x^2 + 60*(b^5*d^4*e - 5*a*b^4*d^3*e^2 +
10*a^2*b^3*d^2*e^3 - 10*a^3*b^2*d*e^4 + 5*a^4*b*e^5)*x - 60*(b^5*d^5 - 5*a*
b^4*d^4*e + 10*a^2*b^3*d^3*e^2 - 10*a^3*b^2*d^2*e^3 + 5*a^4*b*d*e^4 - a^5*e
^5)*log(e*x + d))/e^6
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24678, size = 520, normalized size = 2.05

$$-\left(b^5d^5\mathrm{sgn}\left(bx+a\right) - 5\,ab^4d^4e\mathrm{sgn}\left(bx+a\right) + 10\,a^2b^3d^3e^2\mathrm{sgn}\left(bx+a\right) - 10\,a^3b^2d^2e^3\mathrm{sgn}\left(bx+a\right) + 5\,a^4bde^4\mathrm{sgn}\left(bx+a\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] -(b^5*d^5*sgn(b*x + a) - 5*a*b^4*d^4*e*sgn(b*x + a) + 10*a^2*b^3*d^3*e^2*sg
n(b*x + a) - 10*a^3*b^2*d^2*e^3*sgn(b*x + a) + 5*a^4*b*d*e^4*sgn(b*x + a) -
 a^5*e^5*sgn(b*x + a))*e^(-6)*log(abs(x*e + d)) + 1/60*(12*b^5*x^5*e^4*sgn(
b*x + a) - 15*b^5*d*x^4*e^3*sgn(b*x + a) + 20*b^5*d^2*x^3*e^2*sgn(b*x + a)
 - 30*b^5*d^3*x^2*e*sgn(b*x + a) + 60*b^5*d^4*x*sgn(b*x + a) + 75*a*b^4*x^4*
e^4*sgn(b*x + a) - 100*a*b^4*d*x^3*e^3*sgn(b*x + a) + 150*a*b^4*d^2*x^2*e^2
*sgn(b*x + a) - 300*a*b^4*d^3*x*e*sgn(b*x + a) + 200*a^2*b^3*x^3*e^4*sgn(b*
x + a) - 300*a^2*b^3*d*x^2*e^3*sgn(b*x + a) + 600*a^2*b^3*d^2*x*e^2*sgn(b*x
 + a) + 300*a^3*b^2*x^2*e^4*sgn(b*x + a) - 600*a^3*b^2*d*x*e^3*sgn(b*x + a)
 + 300*a^4*b*x*e^4*sgn(b*x + a))*e^(-5)
```

3.1576 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=292

$$\frac{b^5\sqrt{a^2+2abx+b^2x^2}(d+ex)^4}{4e^6(a+bx)}-\frac{5b^4\sqrt{a^2+2abx+b^2x^2}(d+ex)^3(bd-ae)}{3e^6(a+bx)}+\frac{5b^3\sqrt{a^2+2abx+b^2x^2}(d+ex)^2(bd-ae)^2}{e^6(a+bx)}$$

```
[Out] (-10*b^2*(b*d - a*e)^3*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^5*(a + b*x)) + (
(b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*x)*(d + e*x)) + (5
*b^3*(b*d - a*e)^2*(d + e*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*x
)) - (5*b^4*(b*d - a*e)*(d + e*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^6*(
a + b*x)) + (b^5*(d + e*x)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*e^6*(a + b*x
)) + (5*b*(b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Log[d + e*x])/(e^6*(a
 + b*x))
```

Rubi [A] time = 0.210584, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$\frac{b^5\sqrt{a^2+2abx+b^2x^2}(d+ex)^4}{4e^6(a+bx)}-\frac{5b^4\sqrt{a^2+2abx+b^2x^2}(d+ex)^3(bd-ae)}{3e^6(a+bx)}+\frac{5b^3\sqrt{a^2+2abx+b^2x^2}(d+ex)^2(bd-ae)^2}{e^6(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^2,x]
```


```
[Out] (-10*b^2*(b*d - a*e)^3*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^5*(a + b*x)) + (
(b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*x)*(d + e*x)) + (5
*b^3*(b*d - a*e)^2*(d + e*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*x
)) - (5*b^4*(b*d - a*e)*(d + e*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^6*(
a + b*x)) + (b^5*(d + e*x)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*e^6*(a + b*x
)) + (5*b*(b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Log[d + e*x])/(e^6*(a
 + b*x))
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{(d+ex)^2}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^5}{(d+ex)^2}\,dx}{b^4\left(ab+b^2x\right)}$$

$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{10b^7(bd-ae)^3}{e^5} - \frac{b^5(bd-ae)^5}{e^5(d+ex)^2} + \frac{5b^6(bd-ae)^4}{e^5(d+ex)} + \frac{10b^8(bd-ae)^2(d+ex)}{e^5} - \frac{5b^9(bd-ae)(d+\right.}{e^5}}{b^4\left(ab+b^2x\right)}$$

$$= -\frac{10b^2(bd-ae)^3x\sqrt{a^2+2abx+b^2x^2}}{e^5(a+bx)} + \frac{(bd-ae)^5\sqrt{a^2+2abx+b^2x^2}}{e^6(a+bx)(d+ex)} + \frac{5b^3(bd-ae)^2(d+}{e^6(}$$

Mathematica [A] time = 0.160589, size = 246, normalized size = 0.84

$$\frac{\sqrt{(a+bx)^2}\left(60a^2b^3e^2\left(-4d^2ex+2d^3-3de^2x^2+e^3x^3\right)+120a^3b^2e^3\left(-d^2+dex+e^2x^2\right)+60a^4bde^4-12a^5e^5+20ab^4e\left(6d^2\right.}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^2,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(60*a^4*b*d*e^4 - 12*a^5*e^5 + 120*a^3*b^2*e^3*(-d^2 + d
*e*x + e^2*x^2) + 60*a^2*b^3*e^2*(2*d^3 - 4*d^2*e*x - 3*d*e^2*x^2 + e^3*x^3
) + 20*a*b^4*e*(-3*d^4 + 9*d^3*e*x + 6*d^2*e^2*x^2 - 2*d*e^3*x^3 + e^4*x^4)
 + b^5*(12*d^5 - 48*d^4*e*x - 30*d^3*e^2*x^2 + 10*d^2*e^3*x^3 - 5*d*e^4*x^4
 + 3*e^5*x^5) + 60*b*(b*d - a*e)^4*(d + e*x)*Log[d + e*x]))/(12*e^6*(a + b*
x)*(d + e*x))
```

Maple [B] time = 0.203, size = 456, normalized size = 1.6

$$\frac{360\ln(ex+d)xa^2b^3d^2e^3-240\ln(ex+d)xab^4d^3e^2-240\ln(ex+d)xa^3b^2de^4-12a^5e^5+12b^5d^5+60de^4a^4b-48xb^5d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^2,x)
```

```
[Out] 1/12*((b*x+a)^2)^(5/2)*(360*ln(e*x+d)*x*a^2*b^3*d^2*e^3-240*ln(e*x+d)*x*a*b
^4*d^3*e^2-240*ln(e*x+d)*x*a^3*b^2*d*e^4-12*a^5*e^5+12*b^5*d^5+60*d*e^4*a^4
*b-48*x*b^5*d^4*e+20*x^4*a*b^4*e^5-5*x^4*b^5*d*e^4+60*x^3*a^2*b^3*e^5+10*x^
3*b^5*d^2*e^3+120*x^2*a^3*b^2*e^5-30*x^2*b^5*d^3*e^2+60*ln(e*x+d)*x*a^4*b*e
^5+60*ln(e*x+d)*x*b^5*d^4*e+120*x^2*a*b^4*d^2*e^3-180*x^2*a^2*b^3*d*e^4-40*
x^3*a*b^4*d*e^4+60*ln(e*x+d)*a^4*b*d*e^4+120*a^2*b^3*d^3*e^2-120*a^3*b^2*d^
2*e^3+60*ln(e*x+d)*b^5*d^5+3*x^5*b^5*e^5+180*x*a*b^4*d^3*e^2-240*ln(e*x+d)*
a^3*b^2*d^2*e^3+360*ln(e*x+d)*a^2*b^3*d^3*e^2-240*ln(e*x+d)*a*b^4*d^4*e+120
*x*a^3*b^2*d*e^4-240*x*a^2*b^3*d^2*e^3-60*a*b^4*d^4*e)/(b*x+a)^5/e^6/(e*x+d
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.50243, size = 767, normalized size = 2.63

$$\frac{3\,b^5e^5x^5 + 12\,b^5d^5 - 60\,ab^4d^4e + 120\,a^2b^3d^3e^2 - 120\,a^3b^2d^2e^3 + 60\,a^4bde^4 - 12\,a^5e^5 - 5\left(b^5de^4 - 4\,ab^4e^5\right)x^4 + 10\left(b^5d^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")

[Out] 1/12*(3*b^5*e^5*x^5 + 12*b^5*d^5 - 60*a*b^4*d^4*e + 120*a^2*b^3*d^3*e^2 - 1
20*a^3*b^2*d^2*e^3 + 60*a^4*b*d*e^4 - 12*a^5*e^5 - 5*(b^5*d*e^4 - 4*a*b^4*e
^5)*x^4 + 10*(b^5*d^2*e^3 - 4*a*b^4*d*e^4 + 6*a^2*b^3*e^5)*x^3 - 30*(b^5*d^
3*e^2 - 4*a*b^4*d^2*e^3 + 6*a^2*b^3*d*e^4 - 4*a^3*b^2*e^5)*x^2 - 12*(4*b^5*
d^4*e - 15*a*b^4*d^3*e^2 + 20*a^2*b^3*d^2*e^3 - 10*a^3*b^2*d*e^4)*x + 60*(b
^5*d^5 - 4*a*b^4*d^4*e + 6*a^2*b^3*d^3*e^2 - 4*a^3*b^2*d^2*e^3 + a^4*b*d*e^
4 + (b^5*d^4*e - 4*a*b^4*d^3*e^2 + 6*a^2*b^3*d^2*e^3 - 4*a^3*b^2*d*e^4 + a^
4*b*e^5)*x)*log(e*x + d))/(e^7*x + d*e^6)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**2,x)

[Out] Timed out

Giac [A] time = 1.21518, size = 516, normalized size = 1.77

$$5\left(b^5d^4\mathrm{sgn}\left(bx+a\right) - 4\,ab^4d^3e\mathrm{sgn}\left(bx+a\right) + 6\,a^2b^3d^2e^2\mathrm{sgn}\left(bx+a\right) - 4\,a^3b^2de^3\mathrm{sgn}\left(bx+a\right) + a^4be^4\mathrm{sgn}\left(bx+a\right)\right)e^{(-6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")

[Out] 5*(b^5*d^4*sgn(b*x + a) - 4*a*b^4*d^3*e*sgn(b*x + a) + 6*a^2*b^3*d^2*e^2*sg
n(b*x + a) - 4*a^3*b^2*d*e^3*sgn(b*x + a) + a^4*b*e^4*sgn(b*x + a))*e^(-6)*
log(abs(x*e + d)) + 1/12*(3*b^5*x^4*e^6*sgn(b*x + a) - 8*b^5*d*x^3*e^5*sgn(
b*x + a) + 18*b^5*d^2*x^2*e^4*sgn(b*x + a) - 48*b^5*d^3*x*e^3*sgn(b*x + a)
+ 20*a*b^4*x^3*e^6*sgn(b*x + a) - 60*a*b^4*d*x^2*e^5*sgn(b*x + a) + 180*a*b
^4*d^2*x*e^4*sgn(b*x + a) + 60*a^2*b^3*x^2*e^6*sgn(b*x + a) - 240*a^2*b^3*d
*x*e^5*sgn(b*x + a) + 120*a^3*b^2*x*e^6*sgn(b*x + a))*e^(-8) + (b^5*d^5*sgn
(b*x + a) - 5*a*b^4*d^4*e*sgn(b*x + a) + 10*a^2*b^3*d^3*e^2*sgn(b*x + a) -
10*a^3*b^2*d^2*e^3*sgn(b*x + a) + 5*a^4*b*d*e^4*sgn(b*x + a) - a^5*e^5*sgn(
b*x + a))*e^(-6)/(x*e + d)

3.1577 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^3} dx$

Optimal. Leaf size=295

$$\frac{b^5\sqrt{a^2+2abx+b^2x^2}(d+ex)^3}{3e^6(a+bx)} - \frac{5b^4\sqrt{a^2+2abx+b^2x^2}(d+ex)^2(bd-ae)}{2e^6(a+bx)} + \frac{10b^3x\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{e^5(a+bx)} - 5b\sqrt{a^2}$$

```
[Out] (10*b^3*(b*d - a*e)^2*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^5*(a + b*x)) + ((
b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*e^6*(a + b*x)*(d + e*x)^2) -
 (5*b*(b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*x)*(d + e*x)
) - (5*b^4*(b*d - a*e)*(d + e*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*e^6*(a
 + b*x)) + (b^5*(d + e*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^6*(a + b*x)
) - (10*b^2*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Log[d + e*x])/(e^6*
(a + b*x))
```

Rubi [A] time = 0.189606, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$\frac{b^5\sqrt{a^2+2abx+b^2x^2}(d+ex)^3}{3e^6(a+bx)} - \frac{5b^4\sqrt{a^2+2abx+b^2x^2}(d+ex)^2(bd-ae)}{2e^6(a+bx)} + \frac{10b^3x\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{e^5(a+bx)} - 5b\sqrt{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^3,x]
```


```
[Out] (10*b^3*(b*d - a*e)^2*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^5*(a + b*x)) + ((
b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*e^6*(a + b*x)*(d + e*x)^2) -
 (5*b*(b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*x)*(d + e*x)
) - (5*b^4*(b*d - a*e)*(d + e*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*e^6*(a
 + b*x)) + (b^5*(d + e*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^6*(a + b*x)
) - (10*b^2*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Log[d + e*x])/(e^6*
(a + b*x))
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^3}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(ab+b^2x)^5}{(d+ex)^3}\,dx}{b^4(ab+b^2x)}$$

$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{10b^8(bd-ae)^2}{e^5} - \frac{b^5(bd-ae)^5}{e^5(d+ex)^3} + \frac{5b^6(bd-ae)^4}{e^5(d+ex)^2} - \frac{10b^7(bd-ae)^3}{e^5(d+ex)} - \frac{5b^9(bd-ae)(d+ex)}{e^5}\right.}{b^4(ab+b^2x)}$$

$$= \frac{10b^3(bd-ae)^2x\sqrt{a^2+2abx+b^2x^2}}{e^5(a+bx)} + \frac{(bd-ae)^5\sqrt{a^2+2abx+b^2x^2}}{2e^6(a+bx)(d+ex)^2} - \frac{5b(bd-ae)^4\sqrt{a^2}}{e^6(a+bx)}$$

Mathematica [A] time = 0.113645, size = 248, normalized size = 0.84

$$\frac{\sqrt{(a+bx)^2}\left(30a^2b^3e^2\left(-4d^2ex-5d^3+4de^2x^2+2e^3x^3\right)+30a^3b^2de^3(3d+4ex)-15a^4be^4(d+2ex)-3a^5e^5+15ab^4e\left(-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^3,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(-3*a^5*e^5 - 15*a^4*b*e^4*(d + 2*e*x) + 30*a^3*b^2*d*e^
3*(3*d + 4*e*x) + 30*a^2*b^3*e^2*(-5*d^3 - 4*d^2*e*x + 4*d*e^2*x^2 + 2*e^3*
x^3) + 15*a*b^4*e*(7*d^4 + 2*d^3*e*x - 11*d^2*e^2*x^2 - 4*d*e^3*x^3 + e^4*x
^4) + b^5*(-27*d^5 + 6*d^4*e*x + 63*d^3*e^2*x^2 + 20*d^2*e^3*x^3 - 5*d*e^4*
x^4 + 2*e^5*x^5) - 60*b^2*(b*d - a*e)^3*(d + e*x)^2*Log[d + e*x]))/(6*e^6*(
a + b*x)*(d + e*x)^2)
```

Maple [B] time = 0.203, size = 502, normalized size = 1.7

$$\frac{-360\ln(ex+d)xa^2b^3d^2e^3+360\ln(ex+d)xab^4d^3e^2+120\ln(ex+d)xa^3b^2de^4-180\ln(ex+d)x^2a^2b^3de^4+180\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^3,x)
```

```
[Out] 1/6*((b*x+a)^2)^(5/2)*(-360*ln(e*x+d)*x*a^2*b^3*d^2*e^3+360*ln(e*x+d)*x*a*b
^4*d^3*e^2+120*ln(e*x+d)*x*a^3*b^2*d*e^4-180*ln(e*x+d)*x^2*a^2*b^3*d*e^4+18
0*ln(e*x+d)*x^2*a*b^4*d^2*e^3-60*ln(e*x+d)*x^2*b^5*d^3*e^2+60*ln(e*x+d)*x^2
*a^3*b^2*e^5-3*a^5*e^5-27*b^5*d^5-15*d*e^4*a^4*b+6*x*b^5*d^4*e+15*x^4*a*b^4
*e^5-5*x^4*b^5*d*e^4+60*x^3*a^2*b^3*e^5+20*x^3*b^5*d^2*e^3+63*x^2*b^5*d^3*e
^2-30*x*a^4*b*e^5-120*ln(e*x+d)*x*b^5*d^4*e-165*x^2*a*b^4*d^2*e^3+120*x^2*a
^2*b^3*d*e^4-60*x^3*a*b^4*d*e^4-150*a^2*b^3*d^3*e^2+90*a^3*b^2*d^2*e^3-60*l
n(e*x+d)*b^5*d^5+2*x^5*b^5*e^5+30*x*a*b^4*d^3*e^2+60*ln(e*x+d)*a^3*b^2*d^2*
e^3-180*ln(e*x+d)*a^2*b^3*d^3*e^2+180*ln(e*x+d)*a*b^4*d^4*e+120*x*a^3*b^2*d
*e^4-120*x*a^2*b^3*d^2*e^3+105*a*b^4*d^4*e)/(b*x+a)^5/e^6/(e*x+d)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62572, size = 840, normalized size = 2.85

$$\frac{2\,b^5e^5x^5 - 27\,b^5d^5 + 105\,ab^4d^4e - 150\,a^2b^3d^3e^2 + 90\,a^3b^2d^2e^3 - 15\,a^4bde^4 - 3\,a^5e^5 - 5\left(b^5de^4 - 3\,ab^4e^5\right)x^4 + 20\left(b^5d^2e^3 - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^5*e^5*x^5 - 27*b^5*d^5 + 105*a*b^4*d^4*e - 150*a^2*b^3*d^3*e^2 + 9
0*a^3*b^2*d^2*e^3 - 15*a^4*b*d*e^4 - 3*a^5*e^5 - 5*(b^5*d*e^4 - 3*a*b^4*e^5
)*x^4 + 20*(b^5*d^2*e^3 - 3*a*b^4*d*e^4 + 3*a^2*b^3*e^5)*x^3 + 3*(21*b^5*d^
3*e^2 - 55*a*b^4*d^2*e^3 + 40*a^2*b^3*d*e^4)*x^2 + 6*(b^5*d^4*e + 5*a*b^4*d
^3*e^2 - 20*a^2*b^3*d^2*e^3 + 20*a^3*b^2*d*e^4 - 5*a^4*b*e^5)*x - 60*(b^5*d
^5 - 3*a*b^4*d^4*e + 3*a^2*b^3*d^3*e^2 - a^3*b^2*d^2*e^3 + (b^5*d^3*e^2 - 3
*a*b^4*d^2*e^3 + 3*a^2*b^3*d*e^4 - a^3*b^2*e^5)*x^2 + 2*(b^5*d^4*e - 3*a*b^
4*d^3*e^2 + 3*a^2*b^3*d^2*e^3 - a^3*b^2*d*e^4)*x)*log(e*x + d))/(e^8*x^2 +
2*d*e^7*x + d^2*e^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14085, size = 508, normalized size = 1.72

$$-10\left(b^5d^3\mathrm{sgn}\left(bx+a\right) - 3\,ab^4d^2e\mathrm{sgn}\left(bx+a\right) + 3\,a^2b^3de^2\mathrm{sgn}\left(bx+a\right) - a^3b^2e^3\mathrm{sgn}\left(bx+a\right)\right)e^{(-6)}\log\left(|xe+d|\right) + \frac{1}{6}\left(2\,b^5x^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] -10*(b^5*d^3*sgn(b*x + a) - 3*a*b^4*d^2*e*sgn(b*x + a) + 3*a^2*b^3*d*e^2*sg
n(b*x + a) - a^3*b^2*e^3*sgn(b*x + a))*e^(-6)*log(abs(x*e + d)) + 1/6*(2*b^
5*x^3*e^6*sgn(b*x + a) - 9*b^5*d*x^2*e^5*sgn(b*x + a) + 36*b^5*d^2*x*e^4*sg
n(b*x + a) + 15*a*b^4*x^2*e^6*sgn(b*x + a) - 90*a*b^4*d*x*e^5*sgn(b*x + a)
+ 60*a^2*b^3*x*e^6*sgn(b*x + a))*e^(-9) - 1/2*(9*b^5*d^5*sgn(b*x + a) - 35*
a*b^4*d^4*e*sgn(b*x + a) + 50*a^2*b^3*d^3*e^2*sgn(b*x + a) - 30*a^3*b^2*d^2
*e^3*sgn(b*x + a) + 5*a^4*b*d*e^4*sgn(b*x + a) + a^5*e^5*sgn(b*x + a) + 10*
(b^5*d^4*e*sgn(b*x + a) - 4*a*b^4*d^3*e^2*sgn(b*x + a) + 6*a^2*b^3*d^2*e^3*
sgn(b*x + a) - 4*a^3*b^2*d*e^4*sgn(b*x + a) + a^4*b*e^5*sgn(b*x + a))*x)*e^
(-6)/(x*e + d)^2
```

3.1578 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=292

$$-\frac{b^4x\sqrt{a^2+2abx+b^2x^2}(4bd-5ae)}{e^5(a+bx)}+\frac{10b^2\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3}{e^6(a+bx)(d+ex)}-\frac{5b\sqrt{a^2+2abx+b^2x^2}(bd-ae)^4}{2e^6(a+bx)(d+ex)^2}+\frac{\sqrt{a^2+2}}{3e^6(}$$

```
[Out] -((b^4*(4*b*d - 5*a*e)*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^5*(a + b*x))) +
(b^5*x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*e^4*(a + b*x)) + ((b*d - a*e)^5*
Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^6*(a + b*x)*(d + e*x)^3) - (5*b*(b*d -
a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*e^6*(a + b*x)*(d + e*x)^2) + (10*b
^2*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*x)*(d + e*x)) +
 (10*b^3*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Log[d + e*x])/(e^6*(a
+ b*x))
```

Rubi [A] time = 0.157445, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$-\frac{b^4x\sqrt{a^2+2abx+b^2x^2}(4bd-5ae)}{e^5(a+bx)}+\frac{10b^2\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3}{e^6(a+bx)(d+ex)}-\frac{5b\sqrt{a^2+2abx+b^2x^2}(bd-ae)^4}{2e^6(a+bx)(d+ex)^2}+\frac{\sqrt{a^2+2}}{3e^6(}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^4,x]
```

```
[Out] -((b^4*(4*b*d - 5*a*e)*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^5*(a + b*x))) +
(b^5*x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*e^4*(a + b*x)) + ((b*d - a*e)^5*
Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^6*(a + b*x)*(d + e*x)^3) - (5*b*(b*d -
a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*e^6*(a + b*x)*(d + e*x)^2) + (10*b
^2*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*x)*(d + e*x)) +
 (10*b^3*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Log[d + e*x])/(e^6*(a
+ b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^4}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(ab+b^2x)^5}{(d+ex)^4}\,dx}{b^4(ab+b^2x)}$$
$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^9(4bd-5ae)}{e^5}+\frac{b^{10}x}{e^4}-\frac{b^5(bd-ae)^5}{e^5(d+ex)^4}+\frac{5b^6(bd-ae)^4}{e^5(d+ex)^3}-\frac{10b^7(bd-ae)^3}{e^5(d+ex)^2}+\frac{10b^8(bd-ae)^2}{e^5(d+ex)}\right.}{b^4(ab+b^2x)}$$
$$= -\frac{b^4(4bd-5ae)x\sqrt{a^2+2abx+b^2x^2}}{e^5(a+bx)}+\frac{b^5x^2\sqrt{a^2+2abx+b^2x^2}}{2e^4(a+bx)}+\frac{(bd-ae)^5\sqrt{a^2+2abx+}}{3e^6(a+bx)(d+ex)^3}$$

Mathematica [A] time = 0.118378, size = 247, normalized size = 0.85

$$\sqrt{(a+bx)^2}\left(10a^2b^3de^2\left(11d^2+27dex+18e^2x^2\right)-20a^3b^2e^3\left(d^2+3dex+3e^2x^2\right)-5a^4be^4(d+3ex)-2a^5e^5+10ab^4e\left(-9d\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^4,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(-2*a^5*e^5 - 5*a^4*b*e^4*(d + 3*e*x) - 20*a^3*b^2*e^3*(
d^2 + 3*d*e*x + 3*e^2*x^2) + 10*a^2*b^3*d*e^2*(11*d^2 + 27*d*e*x + 18*e^2*x
^2) + 10*a*b^4*e*(-13*d^4 - 27*d^3*e*x - 9*d^2*e^2*x^2 + 9*d*e^3*x^3 + 3*e^
4*x^4) + b^5*(47*d^5 + 81*d^4*e*x - 9*d^3*e^2*x^2 - 63*d^2*e^3*x^3 - 15*d*e
^4*x^4 + 3*e^5*x^5) + 60*b^3*(b*d - a*e)^2*(d + e*x)^3*Log[d + e*x]))/(6*e^
6*(a + b*x)*(d + e*x)^3)
```

Maple [B] time = 0.2, size = 502, normalized size = 1.7

$$180\ln(ex+d)\,xa^2b^3d^2e^3-360\ln(ex+d)\,xab^4d^3e^2+180\ln(ex+d)\,x^2a^2b^3de^4-360\ln(ex+d)\,x^2ab^4d^2e^3+180\ln(ex$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^4,x)
```

```
[Out] 1/6*((b*x+a)^2)^(5/2)*(180*ln(e*x+d)*x*a^2*b^3*d^2*e^3-360*ln(e*x+d)*x*a*b^
4*d^3*e^2+180*ln(e*x+d)*x^2*a^2*b^3*d*e^4-360*ln(e*x+d)*x^2*a*b^4*d^2*e^3+1
80*ln(e*x+d)*x^2*b^5*d^3*e^2-2*a^5*e^5+47*b^5*d^5-5*d*e^4*a^4*b+81*x*b^5*d^
4*e+30*x^4*a*b^4*e^5-15*x^4*b^5*d*e^4-63*x^3*b^5*d^2*e^3-60*x^2*a^3*b^2*e^5
-9*x^2*b^5*d^3*e^2-15*x*a^4*b*e^5+180*ln(e*x+d)*x*b^5*d^4*e-90*x^2*a*b^4*d^
2*e^3+180*x^2*a^2*b^3*d*e^4+90*x^3*a*b^4*d*e^4+110*a^2*b^3*d^3*e^2-20*a^3*b
^2*d^2*e^3+60*ln(e*x+d)*b^5*d^5+3*x^5*b^5*e^5-270*x*a*b^4*d^3*e^2+60*ln(e*x
+d)*a^2*b^3*d^3*e^2-120*ln(e*x+d)*a*b^4*d^4*e-60*x*a^3*b^2*d*e^4+270*x*a^2*
b^3*d^2*e^3+60*ln(e*x+d)*x^3*a^2*b^3*e^5+60*ln(e*x+d)*x^3*b^5*d^2*e^3-120*l
n(e*x+d)*x^3*a*b^4*d*e^4-130*a*b^4*d^4*e)/(b*x+a)^5/e^6/(e*x+d)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.61037, size = 867, normalized size = 2.97

$$\frac{3\,b^5e^5x^5 + 47\,b^5d^5 - 130\,ab^4d^4e + 110\,a^2b^3d^3e^2 - 20\,a^3b^2d^2e^3 - 5\,a^4bde^4 - 2\,a^5e^5 - 15\left(b^5de^4 - 2\,ab^4e^5\right)x^4 - 9\left(7\,b^5d^2e^{\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^5*e^5*x^5 + 47*b^5*d^5 - 130*a*b^4*d^4*e + 110*a^2*b^3*d^3*e^2 - 2
0*a^3*b^2*d^2*e^3 - 5*a^4*b*d*e^4 - 2*a^5*e^5 - 15*(b^5*d*e^4 - 2*a*b^4*e^5
)*x^4 - 9*(7*b^5*d^2*e^3 - 10*a*b^4*d*e^4)*x^3 - 3*(3*b^5*d^3*e^2 + 30*a*b^
4*d^2*e^3 - 60*a^2*b^3*d*e^4 + 20*a^3*b^2*e^5)*x^2 + 3*(27*b^5*d^4*e - 90*a
*b^4*d^3*e^2 + 90*a^2*b^3*d^2*e^3 - 20*a^3*b^2*d*e^4 - 5*a^4*b*e^5)*x + 60*
(b^5*d^5 - 2*a*b^4*d^4*e + a^2*b^3*d^3*e^2 + (b^5*d^2*e^3 - 2*a*b^4*d*e^4 +
 a^2*b^3*e^5)*x^3 + 3*(b^5*d^3*e^2 - 2*a*b^4*d^2*e^3 + a^2*b^3*d*e^4)*x^2 +
 3*(b^5*d^4*e - 2*a*b^4*d^3*e^2 + a^2*b^3*d^2*e^3)*x)*log(e*x + d))/(e^9*x^
3 + 3*d*e^8*x^2 + 3*d^2*e^7*x + d^3*e^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25927, size = 506, normalized size = 1.73

$$10\left(b^5d^2\mathrm{sgn}\left(bx+a\right) - 2\,ab^4de\mathrm{sgn}\left(bx+a\right) + a^2b^3e^2\mathrm{sgn}\left(bx+a\right)\right)e^{(-6)}\log\left(|xe+d|\right) + \frac{1}{2}\left(b^5x^2e^4\mathrm{sgn}\left(bx+a\right) - 8\,b^5dxe^{3}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 10*(b^5*d^2*sgn(b*x + a) - 2*a*b^4*d*e*sgn(b*x + a) + a^2*b^3*e^2*sgn(b*x +
 a))*e^(-6)*log(abs(x*e + d)) + 1/2*(b^5*x^2*e^4*sgn(b*x + a) - 8*b^5*d*x*e
^3*sgn(b*x + a) + 10*a*b^4*x*e^4*sgn(b*x + a))*e^(-8) + 1/6*(47*b^5*d^5*sgn
(b*x + a) - 130*a*b^4*d^4*e*sgn(b*x + a) + 110*a^2*b^3*d^3*e^2*sgn(b*x + a)
 - 20*a^3*b^2*d^2*e^3*sgn(b*x + a) - 5*a^4*b*d*e^4*sgn(b*x + a) - 2*a^5*e^5
*sgn(b*x + a) + 60*(b^5*d^3*e^2*sgn(b*x + a) - 3*a*b^4*d^2*e^3*sgn(b*x + a)
 + 3*a^2*b^3*d*e^4*sgn(b*x + a) - a^3*b^2*e^5*sgn(b*x + a))*x^2 + 15*(7*b^5
*d^4*e*sgn(b*x + a) - 20*a*b^4*d^3*e^2*sgn(b*x + a) + 18*a^2*b^3*d^2*e^3*sg
n(b*x + a) - 4*a^3*b^2*d*e^4*sgn(b*x + a) - a^4*b*e^5*sgn(b*x + a))*x)*e^(-
6)/(x*e + d)^3
```

3.1579 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{(d+ex)^5} dx$

Optimal. Leaf size=292

$$-\frac{10b^3\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{e^6(a+bx)(d+ex)}+\frac{5b^2\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3}{e^6(a+bx)(d+ex)^2}-\frac{5b\sqrt{a^2+2abx+b^2x^2}(bd-ae)^4}{3e^6(a+bx)(d+ex)^3}+\frac{\sqrt{a^2+2abx}}{4e^6(a+b}$$

```
[Out] (b^5*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^5*(a + b*x)) + ((b*d - a*e)^5*Sqrt
[a^2 + 2*a*b*x + b^2*x^2])/(4*e^6*(a + b*x)*(d + e*x)^4) - (5*b*(b*d - a*e)
^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^6*(a + b*x)*(d + e*x)^3) + (5*b^2*(b
*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*x)*(d + e*x)^2) - (1
0*b^3*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*x)*(d + e*x)
) - (5*b^4*(b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Log[d + e*x])/(e^6*(a
+ b*x))
```

Rubi [A] time = 0.148387, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$-\frac{10b^3\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{e^6(a+bx)(d+ex)}+\frac{5b^2\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3}{e^6(a+bx)(d+ex)^2}-\frac{5b\sqrt{a^2+2abx+b^2x^2}(bd-ae)^4}{3e^6(a+bx)(d+ex)^3}+\frac{\sqrt{a^2+2abx}}{4e^6(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^5,x]
```


```
[Out] (b^5*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^5*(a + b*x)) + ((b*d - a*e)^5*Sqrt
[a^2 + 2*a*b*x + b^2*x^2])/(4*e^6*(a + b*x)*(d + e*x)^4) - (5*b*(b*d - a*e)
^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^6*(a + b*x)*(d + e*x)^3) + (5*b^2*(b
*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*x)*(d + e*x)^2) - (1
0*b^3*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*x)*(d + e*x)
) - (5*b^4*(b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Log[d + e*x])/(e^6*(a
+ b*x))
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{(d+ex)^5}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^5}{(d+ex)^5}\,dx}{b^4\left(ab+b^2x\right)}$$

$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{b^{10}}{e^5}-\frac{b^5(bd-ae)^5}{e^5(d+ex)^5}+\frac{5b^6(bd-ae)^4}{e^5(d+ex)^4}-\frac{10b^7(bd-ae)^3}{e^5(d+ex)^3}+\frac{10b^8(bd-ae)^2}{e^5(d+ex)^2}-\frac{5b^9(bd-ae)}{e^5(d+ex)}\right.}{b^4\left(ab+b^2x\right)}$$

$$= \frac{b^5x\sqrt{a^2+2abx+b^2x^2}}{e^5(a+bx)}+\frac{(bd-ae)^5\sqrt{a^2+2abx+b^2x^2}}{4e^6(a+bx)(d+ex)^4}-\frac{5b(bd-ae)^4\sqrt{a^2+2abx+b^2}}{3e^6(a+bx)(d+ex)^3}$$

Mathematica [A] time = 0.128634, size = 243, normalized size = 0.83

$$-\frac{\sqrt{(a+bx)^2}\left(30a^2b^3e^2\left(4d^2ex+d^3+6de^2x^2+4e^3x^3\right)+10a^3b^2e^3\left(d^2+4dex+6e^2x^2\right)+5a^4be^4(d+4ex)+3a^5e^5-5a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^5,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(3*a^5*e^5 + 5*a^4*b*e^4*(d + 4*e*x) + 10*a^3*b^2*e^3*(
d^2 + 4*d*e*x + 6*e^2*x^2) + 30*a^2*b^3*e^2*(d^3 + 4*d^2*e*x + 6*d*e^2*x^2
+ 4*e^3*x^3) - 5*a*b^4*d*e*(25*d^3 + 88*d^2*e*x + 108*d*e^2*x^2 + 48*e^3*x^
3) + b^5*(77*d^5 + 248*d^4*e*x + 252*d^3*e^2*x^2 + 48*d^2*e^3*x^3 - 48*d*e^
4*x^4 - 12*e^5*x^5) + 60*b^4*(b*d - a*e)*(d + e*x)^4*Log[d + e*x]))/(12*e^6
*(a + b*x)*(d + e*x)^4)
```

Maple [B] time = 0.232, size = 458, normalized size = 1.6

$$\frac{240\ln(ex+d)xab^4d^3e^2+360\ln(ex+d)x^2ab^4d^2e^3-360\ln(ex+d)x^2b^5d^3e^2-3a^5e^5-77b^5d^5-5de^4a^4b-248xb}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^5,x)
```

```
[Out] 1/12*((b*x+a)^2)^(5/2)*(240*ln(e*x+d)*x*a*b^4*d^3*e^2+360*ln(e*x+d)*x^2*a*b
^4*d^2*e^3-360*ln(e*x+d)*x^2*b^5*d^3*e^2-3*a^5*e^5-77*b^5*d^5-5*d*e^4*a^4*b
-248*x*b^5*d^4*e+48*x^4*b^5*d*e^4-120*x^3*a^2*b^3*e^5-48*x^3*b^5*d^2*e^3-60
*x^2*a^3*b^2*e^5-252*x^2*b^5*d^3*e^2-20*x*a^4*b*e^5-240*ln(e*x+d)*x*b^5*d^4
*e+540*x^2*a*b^4*d^2*e^3-180*x^2*a^2*b^3*d*e^4+240*x^3*a*b^4*d*e^4-30*a^2*b
^3*d^3*e^2-10*a^3*b^2*d^2*e^3-60*ln(e*x+d)*b^5*d^5+12*x^5*b^5*e^5+440*x*a*b
^4*d^3*e^2+60*ln(e*x+d)*a*b^4*d^4*e-40*x*a^3*b^2*d*e^4-120*x*a^2*b^3*d^2*e^
3-240*ln(e*x+d)*x^3*b^5*d^2*e^3+60*ln(e*x+d)*x^4*a*b^4*e^5-60*ln(e*x+d)*x^4
*b^5*d*e^4+240*ln(e*x+d)*x^3*a*b^4*d*e^4+125*a*b^4*d^4*e)/(b*x+a)^5/e^6/(e*
x+d)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62253, size = 838, normalized size = 2.87

$$\frac{12\,b^5e^5x^5 + 48\,b^5de^4x^4 - 77\,b^5d^5 + 125\,ab^4d^4e - 30\,a^2b^3d^3e^2 - 10\,a^3b^2d^2e^3 - 5\,a^4bde^4 - 3\,a^5e^5 - 24\left(2\,b^5d^2e^3 - 10\,ab^4de^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(12*b^5*e^5*x^5 + 48*b^5*d*e^4*x^4 - 77*b^5*d^5 + 125*a*b^4*d^4*e - 30
*a^2*b^3*d^3*e^2 - 10*a^3*b^2*d^2*e^3 - 5*a^4*b*d*e^4 - 3*a^5*e^5 - 24*(2*b
^5*d^2*e^3 - 10*a*b^4*d*e^4 + 5*a^2*b^3*e^5)*x^3 - 12*(21*b^5*d^3*e^2 - 45*
a*b^4*d^2*e^3 + 15*a^2*b^3*d*e^4 + 5*a^3*b^2*e^5)*x^2 - 4*(62*b^5*d^4*e - 1
10*a*b^4*d^3*e^2 + 30*a^2*b^3*d^2*e^3 + 10*a^3*b^2*d*e^4 + 5*a^4*b*e^5)*x -
 60*(b^5*d^5 - a*b^4*d^4*e + (b^5*d*e^4 - a*b^4*e^5)*x^4 + 4*(b^5*d^2*e^3 -
 a*b^4*d*e^4)*x^3 + 6*(b^5*d^3*e^2 - a*b^4*d^2*e^3)*x^2 + 4*(b^5*d^4*e - a*
b^4*d^3*e^2)*x)*log(e*x + d))/(e^10*x^4 + 4*d*e^9*x^3 + 6*d^2*e^8*x^2 + 4*d
^3*e^7*x + d^4*e^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21613, size = 500, normalized size = 1.71

$$b^5xe^{(-5)}\mathrm{sgn}\,(bx+a) - 5\left(b^5d\mathrm{sgn}\,(bx+a) - ab^4e\mathrm{sgn}\,(bx+a)\right)e^{(-6)}\log\left(|xe+d|\right) - \frac{\left(77\,b^5d^5\mathrm{sgn}\,(bx+a) - 125\,ab^4d^4e\mathrm{sgn}\,(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] b^5*x*e^(-5)*sgn(b*x + a) - 5*(b^5*d*sgn(b*x + a) - a*b^4*e*sgn(b*x + a))*e
^(-6)*log(abs(x*e + d)) - 1/12*(77*b^5*d^5*sgn(b*x + a) - 125*a*b^4*d^4*e*s
gn(b*x + a) + 30*a^2*b^3*d^3*e^2*sgn(b*x + a) + 10*a^3*b^2*d^2*e^3*sgn(b*x
+ a) + 5*a^4*b*d*e^4*sgn(b*x + a) + 3*a^5*e^5*sgn(b*x + a) + 120*(b^5*d^2*e
^3*sgn(b*x + a) - 2*a*b^4*d*e^4*sgn(b*x + a) + a^2*b^3*e^5*sgn(b*x + a))*x^
3 + 60*(5*b^5*d^3*e^2*sgn(b*x + a) - 9*a*b^4*d^2*e^3*sgn(b*x + a) + 3*a^2*b
^3*d*e^4*sgn(b*x + a) + a^3*b^2*e^5*sgn(b*x + a))*x^2 + 20*(13*b^5*d^4*e*sg
n(b*x + a) - 22*a*b^4*d^3*e^2*sgn(b*x + a) + 6*a^2*b^3*d^2*e^3*sgn(b*x + a)
 + 2*a^3*b^2*d*e^4*sgn(b*x + a) + a^4*b*e^5*sgn(b*x + a))*x)*e^(-6)/(x*e +
d)^4
```

3.1580 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^6}\,dx$

Optimal. Leaf size=300

$$\frac{5b^4\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{e^6(a+bx)(d+ex)}-\frac{5b^3\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{e^6(a+bx)(d+ex)^2}+\frac{10b^2\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3}{3e^6(a+bx)(d+ex)^3}-\frac{5b\sqrt{a^2+2a}}{4e^6(a}$$

```
[Out] ((b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^6*(a + b*x)*(d + e*x)^5)
 - (5*b*(b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*e^6*(a + b*x)*(d +
e*x)^4) + (10*b^2*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^6*(a +
b*x)*(d + e*x)^3) - (5*b^3*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^
6*(a + b*x)*(d + e*x)^2) + (5*b^4*(b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]
)/(e^6*(a + b*x)*(d + e*x)) + (b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Log[d + e*
x])/(e^6*(a + b*x))
```

Rubi [A] time = 0.143804, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$\frac{5b^4\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{e^6(a+bx)(d+ex)}-\frac{5b^3\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{e^6(a+bx)(d+ex)^2}+\frac{10b^2\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3}{3e^6(a+bx)(d+ex)^3}-\frac{5b\sqrt{a^2+2a}}{4e^6(a}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^6,x]
```

```
[Out] ((b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^6*(a + b*x)*(d + e*x)^5)
 - (5*b*(b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*e^6*(a + b*x)*(d +
e*x)^4) + (10*b^2*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^6*(a +
b*x)*(d + e*x)^3) - (5*b^3*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^
6*(a + b*x)*(d + e*x)^2) + (5*b^4*(b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]
)/(e^6*(a + b*x)*(d + e*x)) + (b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Log[d + e*
x])/(e^6*(a + b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{(d+ex)^6}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^5}{(d+ex)^6}\,dx}{b^4\left(ab+b^2x\right)}$$

$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^5(bd-ae)^5}{e^5(d+ex)^6}+\frac{5b^6(bd-ae)^4}{e^5(d+ex)^5}-\frac{10b^7(bd-ae)^3}{e^5(d+ex)^4}+\frac{10b^8(bd-ae)^2}{e^5(d+ex)^3}-\frac{5b^9(bd-ae)}{e^5(d+ex)^2}+\frac{b^{10}}{e^5(d+\dots}\right.}{b^4\left(ab+b^2x\right)}$$

$$= \frac{(bd-ae)^5\sqrt{a^2+2abx+b^2x^2}}{5e^6(a+bx)(d+ex)^5}-\frac{5b(bd-ae)^4\sqrt{a^2+2abx+b^2x^2}}{4e^6(a+bx)(d+ex)^4}+\frac{10b^2(bd-ae)^3\sqrt{a^2+2a\dots}}{3e^6(a+bx)(d+e\dots}$$

Mathematica [A] time = 0.129879, size = 196, normalized size = 0.65

$$\frac{\sqrt{(a+bx)^2}\left((bd-ae)\left(a^2b^2e^2\left(47d^2+175dex+200e^2x^2\right)+3a^3be^3(9d+25ex)+12a^4e^4+ab^3e\left(325d^2ex+77d^3+500de^{2}\dots\right.\right.\right.}{60e^6(a+bx)(d+e\dots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^6,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*((b*d - a*e)*(12*a^4*e^4 + 3*a^3*b*e^3*(9*d + 25*e*x) +
a^2*b^2*e^2*(47*d^2 + 175*d*e*x + 200*e^2*x^2) + a*b^3*e*(77*d^3 + 325*d^2*
e*x + 500*d*e^2*x^2 + 300*e^3*x^3) + b^4*(137*d^4 + 625*d^3*e*x + 1100*d^2*
e^2*x^2 + 900*d*e^3*x^3 + 300*e^4*x^4)) + 60*b^5*(d + e*x)^5*Log[d + e*x]))
/(60*e^6*(a + b*x)*(d + e*x)^5)
```

Maple [A] time = 0.2, size = 383, normalized size = 1.3

$$\frac{600\,\ln(ex+d)\,x^2b^5d^3e^2-12\,a^5e^5+137\,b^5d^5-15\,de^4a^4b+60\,\ln(ex+d)\,x^5b^5e^5+625\,xb^5d^4e-300\,x^4ab^4e^5+300\,x^4b^5d\dots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^6,x)
```

```
[Out] 1/60*((b*x+a)^2)^(5/2)*(600*ln(e*x+d)*x^2*b^5*d^3*e^2-12*a^5*e^5+137*b^5*d^
5-15*d*e^4*a^4*b+60*ln(e*x+d)*x^5*b^5*e^5+625*x*b^5*d^4*e-300*x^4*a*b^4*e^5
+300*x^4*b^5*d*e^4-300*x^3*a^2*b^3*e^5+900*x^3*b^5*d^2*e^3-200*x^2*a^3*b^2*
e^5+1100*x^2*b^5*d^3*e^2-75*x*a^4*b*e^5+300*ln(e*x+d)*x*b^5*d^4*e-600*x^2*a
*b^4*d^2*e^3-300*x^2*a^2*b^3*d*e^4-600*x^3*a*b^4*d*e^4-30*a^2*b^3*d^3*e^2-2
0*a^3*b^2*d^2*e^3+60*ln(e*x+d)*b^5*d^5-300*x*a*b^4*d^3*e^2-100*x*a^3*b^2*d*
e^4-150*x*a^2*b^3*d^2*e^3+600*ln(e*x+d)*x^3*b^5*d^2*e^3+300*ln(e*x+d)*x^4*b
^5*d*e^4-60*a*b^4*d^4*e)/(b*x+a)^5/e^6/(e*x+d)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56002, size = 774, normalized size = 2.58

$$\frac{137\,b^5d^5-60\,ab^4d^4e-30\,a^2b^3d^3e^2-20\,a^3b^2d^2e^3-15\,a^4bde^4-12\,a^5e^5+300\left(b^5de^4-ab^4e^5\right)x^4+300\left(3\,b^5d^2e^3-2\,ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^6,x, algorithm="fricas")
```

```
[Out] 1/60*(137*b^5*d^5 - 60*a*b^4*d^4*e - 30*a^2*b^3*d^3*e^2 - 20*a^3*b^2*d^2*e^
3 - 15*a^4*b*d*e^4 - 12*a^5*e^5 + 300*(b^5*d*e^4 - a*b^4*e^5)*x^4 + 300*(3*
b^5*d^2*e^3 - 2*a*b^4*d*e^4 - a^2*b^3*e^5)*x^3 + 100*(11*b^5*d^3*e^2 - 6*a*
b^4*d^2*e^3 - 3*a^2*b^3*d*e^4 - 2*a^3*b^2*e^5)*x^2 + 25*(25*b^5*d^4*e - 12*
a*b^4*d^3*e^2 - 6*a^2*b^3*d^2*e^3 - 4*a^3*b^2*d*e^4 - 3*a^4*b*e^5)*x + 60*(
b^5*e^5*x^5 + 5*b^5*d*e^4*x^4 + 10*b^5*d^2*e^3*x^3 + 10*b^5*d^3*e^2*x^2 + 5
*b^5*d^4*e*x + b^5*d^5)*log(e*x + d))/(e^11*x^5 + 5*d*e^10*x^4 + 10*d^2*e^9
*x^3 + 10*d^3*e^8*x^2 + 5*d^4*e^7*x + d^5*e^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15964, size = 510, normalized size = 1.7

$$b^5e^{(-6)}\log\left(|xe+d|\right)\mathrm{sgn}\left(bx+a\right)+\frac{\left(300\left(b^5de^3\mathrm{sgn}\left(bx+a\right)-ab^4e^4\mathrm{sgn}\left(bx+a\right)\right)x^4+300\left(3\,b^5d^2e^2\mathrm{sgn}\left(bx+a\right)-2\,ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^6,x, algorithm="giac")
```

```
[Out] b^5*e^(-6)*log(abs(x*e + d))*sgn(b*x + a) + 1/60*(300*(b^5*d*e^3*sgn(b*x +
a) - a*b^4*e^4*sgn(b*x + a))*x^4 + 300*(3*b^5*d^2*e^2*sgn(b*x + a) - 2*a*b^
4*d*e^3*sgn(b*x + a) - a^2*b^3*e^4*sgn(b*x + a))*x^3 + 100*(11*b^5*d^3*e*sg
n(b*x + a) - 6*a*b^4*d^2*e^2*sgn(b*x + a) - 3*a^2*b^3*d*e^3*sgn(b*x + a) -
2*a^3*b^2*e^4*sgn(b*x + a))*x^2 + 25*(25*b^5*d^4*sgn(b*x + a) - 12*a*b^4*d^
3*e*sgn(b*x + a) - 6*a^2*b^3*d^2*e^2*sgn(b*x + a) - 4*a^3*b^2*d*e^3*sgn(b*x
 + a) - 3*a^4*b*e^4*sgn(b*x + a))*x + (137*b^5*d^5*sgn(b*x + a) - 60*a*b^4*
d^4*e*sgn(b*x + a) - 30*a^2*b^3*d^3*e^2*sgn(b*x + a) - 20*a^3*b^2*d^2*e^3*s
gn(b*x + a) - 15*a^4*b*d*e^4*sgn(b*x + a) - 12*a^5*e^5*sgn(b*x + a))*e^(-1)
)*e^(-5)/(x*e + d)^5
```

3.1581 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{(d+ex)^7} dx$

Optimal. Leaf size=48

$$\frac{(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{6(d+ex)^6(bd-ae)}$$

```
[Out] ((a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*(b*d - a*e)*(d + e*x)^6)
```

Rubi [A] time = 0.0194524, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 37}

$$\frac{(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{6(d+ex)^6(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^7,x]
```

```
[Out] ((a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*(b*d - a*e)*(d + e*x)^6)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{(d+ex)^7} dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^5}{(d+ex)^7} dx}{b^4\left(ab+b^2x\right)}\\ &= \frac{(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{6(bd-ae)(d+ex)^6}\end{aligned}$$

Mathematica [B] time = 0.0784895, size = 218, normalized size = 4.54

$$-\frac{\sqrt{(a+bx)^2}\left(a^2b^3e^2\left(6d^2ex+d^3+15de^2x^2+20e^3x^3\right)+a^3b^2e^3\left(d^2+6dex+15e^2x^2\right)+a^4be^4(d+6ex)+a^5e^5+ab^4e\left(15d\right.\right.}{6e^6(a+bx)(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^7,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(a^5*e^5 + a^4*b*e^4*(d + 6*e*x) + a^3*b^2*e^3*(d^2 + 6
*d*e*x + 15*e^2*x^2) + a^2*b^3*e^2*(d^3 + 6*d^2*e*x + 15*d*e^2*x^2 + 20*e^3
*x^3) + a*b^4*e*(d^4 + 6*d^3*e*x + 15*d^2*e^2*x^2 + 20*d*e^3*x^3 + 15*e^4*x
^4) + b^5*(d^5 + 6*d^4*e*x + 15*d^3*e^2*x^2 + 20*d^2*e^3*x^3 + 15*d*e^4*x^4
 + 6*e^5*x^5)))/(6*e^6*(a + b*x)*(d + e*x)^6)
```

Maple [B] time = 0.155, size = 283, normalized size = 5.9

$$-\frac{6\,x^5b^5e^5+15\,x^4ab^4e^5+15\,x^4b^5de^4+20\,x^3a^2b^3e^5+20\,x^3ab^4de^4+20\,x^3b^5d^2e^3+15\,x^2a^3b^2e^5+15\,x^2a^2b^3de^4+15\,x^2a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^7,x)
```

```
[Out] -1/6*(6*b^5*e^5*x^5+15*a*b^4*e^5*x^4+15*b^5*d*e^4*x^4+20*a^2*b^3*e^5*x^3+20
*a*b^4*d*e^4*x^3+20*b^5*d^2*e^3*x^3+15*a^3*b^2*e^5*x^2+15*a^2*b^3*d*e^4*x^2
+15*a*b^4*d^2*e^3*x^2+15*b^5*d^3*e^2*x^2+6*a^4*b*e^5*x+6*a^3*b^2*d*e^4*x+6*
a^2*b^3*d^2*e^3*x+6*a*b^4*d^3*e^2*x+6*b^5*d^4*e*x+a^5*e^5+a^4*b*d*e^4+a^3*b
^2*d^2*e^3+a^2*b^3*d^3*e^2+a*b^4*d^4*e+b^5*d^5)*((b*x+a)^2)^(5/2)/(e*x+d)^6
/e^6/(b*x+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.64554, size = 595, normalized size = 12.4

$$-\frac{6\,b^5e^5x^5+b^5d^5+ab^4d^4e+a^2b^3d^3e^2+a^3b^2d^2e^3+a^4bde^4+a^5e^5+15\left(b^5de^4+ab^4e^5\right)x^4+20\left(b^5d^2e^3+ab^4de^4+a^2b\right.}{6\left(e^{12}x^6+6\,de^{11}x^5+15\,d^2e^{10}x^4+20\,d^3e^9x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^7,x, algorithm="fricas")
```

```
[Out] -1/6*(6*b^5*e^5*x^5 + b^5*d^5 + a*b^4*d^4*e + a^2*b^3*d^3*e^2 + a^3*b^2*d^2
*e^3 + a^4*b*d*e^4 + a^5*e^5 + 15*(b^5*d*e^4 + a*b^4*e^5)*x^4 + 20*(b^5*d^2
*e^3 + a*b^4*d*e^4 + a^2*b^3*e^5)*x^3 + 15*(b^5*d^3*e^2 + a*b^4*d^2*e^3 + a
^2*b^3*d*e^4 + a^3*b^2*e^5)*x^2 + 6*(b^5*d^4*e + a*b^4*d^3*e^2 + a^2*b^3*d^
2*e^3 + a^3*b^2*d*e^4 + a^4*b*e^5)*x)/(e^12*x^6 + 6*d*e^11*x^5 + 15*d^2*e^1
0*x^4 + 20*d^3*e^9*x^3 + 15*d^4*e^8*x^2 + 6*d^5*e^7*x + d^6*e^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**7,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21132, size = 508, normalized size = 10.58

$$-\frac{\left(6\,b^5x^5e^5\mathrm{sgn}\left(bx+a\right)+15\,b^5dx^4e^4\mathrm{sgn}\left(bx+a\right)+20\,b^5d^2x^3e^3\mathrm{sgn}\left(bx+a\right)+15\,b^5d^3x^2e^2\mathrm{sgn}\left(bx+a\right)+6\,b^5d^4xe\mathrm{sgn}\left(bx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^7,x, algorithm="giac")
```

```
[Out] -1/6*(6*b^5*x^5*e^5*sgn(b*x + a) + 15*b^5*d*x^4*e^4*sgn(b*x + a) + 20*b^5*d
^2*x^3*e^3*sgn(b*x + a) + 15*b^5*d^3*x^2*e^2*sgn(b*x + a) + 6*b^5*d^4*x*e*s
gn(b*x + a) + b^5*d^5*sgn(b*x + a) + 15*a*b^4*x^4*e^5*sgn(b*x + a) + 20*a*b
^4*d*x^3*e^4*sgn(b*x + a) + 15*a*b^4*d^2*x^2*e^3*sgn(b*x + a) + 6*a*b^4*d^3
*x*e^2*sgn(b*x + a) + a*b^4*d^4*e*sgn(b*x + a) + 20*a^2*b^3*x^3*e^5*sgn(b*x
 + a) + 15*a^2*b^3*d*x^2*e^4*sgn(b*x + a) + 6*a^2*b^3*d^2*x*e^3*sgn(b*x + a
) + a^2*b^3*d^3*e^2*sgn(b*x + a) + 15*a^3*b^2*x^2*e^5*sgn(b*x + a) + 6*a^3*
b^2*d*x*e^4*sgn(b*x + a) + a^3*b^2*d^2*e^3*sgn(b*x + a) + 6*a^4*b*x*e^5*sgn
(b*x + a) + a^4*b*d*e^4*sgn(b*x + a) + a^5*e^5*sgn(b*x + a))*e^(-6)/(x*e +
d)^6
```

3.1582 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{(d+ex)^8}\,dx$

Optimal. Leaf size=98

$$\frac{b\sqrt{a^2+2abx+b^2x^2}(a+bx)^5}{42(d+ex)^6(bd-ae)^2}+\frac{\sqrt{a^2+2abx+b^2x^2}(a+bx)^5}{7(d+ex)^7(bd-ae)}$$

```
[Out] ((a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*(b*d - a*e)*(d + e*x)^7) + (
b*(a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(42*(b*d - a*e)^2*(d + e*x)^6)
```

Rubi [A] time = 0.0360393, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {646, 45, 37}

$$\frac{b\sqrt{a^2+2abx+b^2x^2}(a+bx)^5}{42(d+ex)^6(bd-ae)^2}+\frac{\sqrt{a^2+2abx+b^2x^2}(a+bx)^5}{7(d+ex)^7(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^8,x]
```

```
[Out] ((a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*(b*d - a*e)*(d + e*x)^7) + (
b*(a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(42*(b*d - a*e)^2*(d + e*x)^6)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{(d+ex)^8}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^5}{(d+ex)^8}\,dx}{b^4\left(ab+b^2x\right)}$$
$$= \frac{(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{7(bd-ae)(d+ex)^7} + \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^5}{(d+ex)^7}\,dx}{7b^3(bd-ae)\left(ab+b^2x\right)}$$
$$= \frac{(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{7(bd-ae)(d+ex)^7} + \frac{b(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{42(bd-ae)^2(d+ex)^6}$$

Mathematica [B] time = 0.0796329, size = 223, normalized size = 2.28

$$-\frac{\sqrt{(a+bx)^2}\left(3a^2b^3e^2\left(7d^2ex+d^3+21de^2x^2+35e^3x^3\right)+4a^3b^2e^3\left(d^2+7dex+21e^2x^2\right)+5a^4be^4(d+7ex)+6a^5e^5+2ab^4\right.}{42e^6(a+bx)(d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^8,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(6*a^5*e^5 + 5*a^4*b*e^4*(d + 7*e*x) + 4*a^3*b^2*e^3*(d
^2 + 7*d*e*x + 21*e^2*x^2) + 3*a^2*b^3*e^2*(d^3 + 7*d^2*e*x + 21*d*e^2*x^2
+ 35*e^3*x^3) + 2*a*b^4*e*(d^4 + 7*d^3*e*x + 21*d^2*e^2*x^2 + 35*d*e^3*x^3
+ 35*e^4*x^4) + b^5*(d^5 + 7*d^4*e*x + 21*d^3*e^2*x^2 + 35*d^2*e^3*x^3 + 35
*d*e^4*x^4 + 21*e^5*x^5)))/(42*e^6*(a + b*x)*(d + e*x)^7)
```

Maple [B] time = 0.157, size = 288, normalized size = 2.9

$$-\frac{21x^5b^5e^5+70x^4ab^4e^5+35x^4b^5de^4+105x^3a^2b^3e^5+70x^3ab^4de^4+35x^3b^5d^2e^3+84x^2a^3b^2e^5+63x^2a^2b^3de^4+42x^2a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^8,x)
```

```
[Out] -1/42/e^6*(21*b^5*e^5*x^5+70*a*b^4*e^5*x^4+35*b^5*d*e^4*x^4+105*a^2*b^3*e^5
*x^3+70*a*b^4*d*e^4*x^3+35*b^5*d^2*e^3*x^3+84*a^3*b^2*e^5*x^2+63*a^2*b^3*d*
e^4*x^2+42*a*b^4*d^2*e^3*x^2+21*b^5*d^3*e^2*x^2+35*a^4*b*e^5*x+28*a^3*b^2*d
*e^4*x+21*a^2*b^3*d^2*e^3*x+14*a*b^4*d^3*e^2*x+7*b^5*d^4*e*x+6*a^5*e^5+5*a^
4*b*d*e^4+4*a^3*b^2*d^2*e^3+3*a^2*b^3*d^3*e^2+2*a*b^4*d^4*e+b^5*d^5)*((b*x+
a)^2)^(5/2)/(e*x+d)^7/(b*x+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.58109, size = 663, normalized size = 6.77

$$-\frac{21\,b^5e^5x^5 + b^5d^5 + 2\,ab^4d^4e + 3\,a^2b^3d^3e^2 + 4\,a^3b^2d^2e^3 + 5\,a^4bde^4 + 6\,a^5e^5 + 35\left(b^5de^4 + 2\,ab^4e^5\right)x^4 + 35\left(b^5d^2e^3 + 2\ldots\right.}{42\left(e^{13}x^7 + 7\,de^{12}x^6 + 21\,d^2e^{11}x^5 + \ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^8,x, algorithm="fricas")
```

```
[Out] -1/42*(21*b^5*e^5*x^5 + b^5*d^5 + 2*a*b^4*d^4*e + 3*a^2*b^3*d^3*e^2 + 4*a^3
*b^2*d^2*e^3 + 5*a^4*b*d*e^4 + 6*a^5*e^5 + 35*(b^5*d*e^4 + 2*a*b^4*e^5)*x^4
 + 35*(b^5*d^2*e^3 + 2*a*b^4*d*e^4 + 3*a^2*b^3*e^5)*x^3 + 21*(b^5*d^3*e^2 +
 2*a*b^4*d^2*e^3 + 3*a^2*b^3*d*e^4 + 4*a^3*b^2*e^5)*x^2 + 7*(b^5*d^4*e + 2*
a*b^4*d^3*e^2 + 3*a^2*b^3*d^2*e^3 + 4*a^3*b^2*d*e^4 + 5*a^4*b*e^5)*x)/(e^13
*x^7 + 7*d*e^12*x^6 + 21*d^2*e^11*x^5 + 35*d^3*e^10*x^4 + 35*d^4*e^9*x^3 +
21*d^5*e^8*x^2 + 7*d^6*e^7*x + d^7*e^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20155, size = 514, normalized size = 5.24

$$-\frac{\left(21\,b^5x^5e^5\mathrm{sgn}\left(bx+a\right) + 35\,b^5dx^4e^4\mathrm{sgn}\left(bx+a\right) + 35\,b^5d^2x^3e^3\mathrm{sgn}\left(bx+a\right) + 21\,b^5d^3x^2e^2\mathrm{sgn}\left(bx+a\right) + 7\,b^5d^4xe\mathrm{sgn}\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^8,x, algorithm="giac")
```

```
[Out] -1/42*(21*b^5*x^5*e^5*sgn(b*x + a) + 35*b^5*d*x^4*e^4*sgn(b*x + a) + 35*b^5
*d^2*x^3*e^3*sgn(b*x + a) + 21*b^5*d^3*x^2*e^2*sgn(b*x + a) + 7*b^5*d^4*x*e
*sgn(b*x + a) + b^5*d^5*sgn(b*x + a) + 70*a*b^4*x^4*e^5*sgn(b*x + a) + 70*a
*b^4*d*x^3*e^4*sgn(b*x + a) + 42*a*b^4*d^2*x^2*e^3*sgn(b*x + a) + 14*a*b^4*
d^3*x*e^2*sgn(b*x + a) + 2*a*b^4*d^4*e*sgn(b*x + a) + 105*a^2*b^3*x^3*e^5*s
gn(b*x + a) + 63*a^2*b^3*d*x^2*e^4*sgn(b*x + a) + 21*a^2*b^3*d^2*x*e^3*sgn(
b*x + a) + 3*a^2*b^3*d^3*e^2*sgn(b*x + a) + 84*a^3*b^2*x^2*e^5*sgn(b*x + a)
 + 28*a^3*b^2*d*x*e^4*sgn(b*x + a) + 4*a^3*b^2*d^2*e^3*sgn(b*x + a) + 35*a^
4*b*x*e^5*sgn(b*x + a) + 5*a^4*b*d*e^4*sgn(b*x + a) + 6*a^5*e^5*sgn(b*x + a
))*e^(-6)/(x*e + d)^7
```

3.1583 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{(d+ex)^9}\,dx$

Optimal. Leaf size=149

$$\frac{b^2\sqrt{a^2+2abx+b^2x^2}(a+bx)^5}{168(d+ex)^6(bd-ae)^3}+\frac{b\sqrt{a^2+2abx+b^2x^2}(a+bx)^5}{28(d+ex)^7(bd-ae)^2}+\frac{\sqrt{a^2+2abx+b^2x^2}(a+bx)^5}{8(d+ex)^8(bd-ae)}$$

```
[Out] ((a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(8*(b*d - a*e)*(d + e*x)^8) + (
b*(a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(28*(b*d - a*e)^2*(d + e*x)^7)
 + (b^2*(a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(168*(b*d - a*e)^3*(d +
e*x)^6)
```

Rubi [A] time = 0.0510391, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {646, 45, 37}

$$\frac{b^2\sqrt{a^2+2abx+b^2x^2}(a+bx)^5}{168(d+ex)^6(bd-ae)^3}+\frac{b\sqrt{a^2+2abx+b^2x^2}(a+bx)^5}{28(d+ex)^7(bd-ae)^2}+\frac{\sqrt{a^2+2abx+b^2x^2}(a+bx)^5}{8(d+ex)^8(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^9,x]
```

```
[Out] ((a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(8*(b*d - a*e)*(d + e*x)^8) + (
b*(a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(28*(b*d - a*e)^2*(d + e*x)^7)
 + (b^2*(a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(168*(b*d - a*e)^3*(d +
e*x)^6)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{(d+ex)^9}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^5}{(d+ex)^9}\,dx}{b^4\left(ab+b^2x\right)}$$

$$= \frac{(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{8(bd-ae)(d+ex)^8} + \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^5}{(d+ex)^8}\,dx}{4b^3(bd-ae)\left(ab+b^2x\right)}$$

$$= \frac{(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{8(bd-ae)(d+ex)^8} + \frac{b(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{28(bd-ae)^2(d+ex)^7} + \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(}{}}{28b^2(bd-ae)^2\left(ab\right.}$$

$$= \frac{(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{8(bd-ae)(d+ex)^8} + \frac{b(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{28(bd-ae)^2(d+ex)^7} + \frac{b^2(a+bx)^5\sqrt{a^2+2abx}}{168(bd-ae)^3(d+e}$$

Mathematica [A] time = 0.0803123, size = 223, normalized size = 1.5

$$-\frac{\sqrt{(a+bx)^2}\left(6a^2b^3e^2\left(8d^2ex+d^3+28de^2x^2+56e^3x^3\right)+10a^3b^2e^3\left(d^2+8dex+28e^2x^2\right)+15a^4be^4(d+8ex)+21a^5e^5}{168e^6(a+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^9,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(21*a^5*e^5 + 15*a^4*b*e^4*(d + 8*e*x) + 10*a^3*b^2*e^3
*(d^2 + 8*d*e*x + 28*e^2*x^2) + 6*a^2*b^3*e^2*(d^3 + 8*d^2*e*x + 28*d*e^2*x
^2 + 56*e^3*x^3) + 3*a*b^4*e*(d^4 + 8*d^3*e*x + 28*d^2*e^2*x^2 + 56*d*e^3*x
^3 + 70*e^4*x^4) + b^5*(d^5 + 8*d^4*e*x + 28*d^3*e^2*x^2 + 56*d^2*e^3*x^3 +
 70*d*e^4*x^4 + 56*e^5*x^5)))/(168*e^6*(a + b*x)*(d + e*x)^8)
```

Maple [B] time = 0.158, size = 288, normalized size = 1.9

$$-\frac{56x^5b^5e^5+210x^4ab^4e^5+70x^4b^5de^4+336x^3a^2b^3e^5+168x^3ab^4de^4+56x^3b^5d^2e^3+280x^2a^3b^2e^5+168x^2a^2b^3de^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^9,x)
```

```
[Out] -1/168/e^6*(56*b^5*e^5*x^5+210*a*b^4*e^5*x^4+70*b^5*d*e^4*x^4+336*a^2*b^3*e
^5*x^3+168*a*b^4*d*e^4*x^3+56*b^5*d^2*e^3*x^3+280*a^3*b^2*e^5*x^2+168*a^2*b
^3*d*e^4*x^2+84*a*b^4*d^2*e^3*x^2+28*b^5*d^3*e^2*x^2+120*a^4*b*e^5*x+80*a^3
*b^2*d*e^4*x+48*a^2*b^3*d^2*e^3*x+24*a*b^4*d^3*e^2*x+8*b^5*d^4*e*x+21*a^5*e
^5+15*a^4*b*d*e^4+10*a^3*b^2*d^2*e^3+6*a^2*b^3*d^3*e^2+3*a*b^4*d^4*e+b^5*d^
5)*((b*x+a)^2)^(5/2)/(e*x+d)^8/(b*x+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.62102, size = 697, normalized size = 4.68

$$-\frac{56\,b^5e^5x^5 + b^5d^5 + 3\,ab^4d^4e + 6\,a^2b^3d^3e^2 + 10\,a^3b^2d^2e^3 + 15\,a^4bde^4 + 21\,a^5e^5 + 70\left(b^5de^4 + 3\,ab^4e^5\right)x^4 + 56\left(b^5d^2e^3 + 3\right.}{168\left(e^{14}x^8 + 8\,de^{13}x^7 + 28\,d^2e^{12}x^6 + 56\,d^3e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^9,x, algorithm="fricas")
```

```
[Out] -1/168*(56*b^5*e^5*x^5 + b^5*d^5 + 3*a*b^4*d^4*e + 6*a^2*b^3*d^3*e^2 + 10*a
^3*b^2*d^2*e^3 + 15*a^4*b*d*e^4 + 21*a^5*e^5 + 70*(b^5*d*e^4 + 3*a*b^4*e^5)
*x^4 + 56*(b^5*d^2*e^3 + 3*a*b^4*d*e^4 + 6*a^2*b^3*e^5)*x^3 + 28*(b^5*d^3*e
^2 + 3*a*b^4*d^2*e^3 + 6*a^2*b^3*d*e^4 + 10*a^3*b^2*e^5)*x^2 + 8*(b^5*d^4*e
 + 3*a*b^4*d^3*e^2 + 6*a^2*b^3*d^2*e^3 + 10*a^3*b^2*d*e^4 + 15*a^4*b*e^5)*x
)/(e^14*x^8 + 8*d*e^13*x^7 + 28*d^2*e^12*x^6 + 56*d^3*e^11*x^5 + 70*d^4*e^1
0*x^4 + 56*d^5*e^9*x^3 + 28*d^6*e^8*x^2 + 8*d^7*e^7*x + d^8*e^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**9,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17226, size = 514, normalized size = 3.45

$$-\frac{\left(56\,b^5x^5e^5\mathrm{sgn}\left(bx+a\right) + 70\,b^5dx^4e^4\mathrm{sgn}\left(bx+a\right) + 56\,b^5d^2x^3e^3\mathrm{sgn}\left(bx+a\right) + 28\,b^5d^3x^2e^2\mathrm{sgn}\left(bx+a\right) + 8\,b^5d^4xe\mathrm{sgn}\left(b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^9,x, algorithm="giac")
```

```
[Out] -1/168*(56*b^5*x^5*e^5*sgn(b*x + a) + 70*b^5*d*x^4*e^4*sgn(b*x + a) + 56*b^
5*d^2*x^3*e^3*sgn(b*x + a) + 28*b^5*d^3*x^2*e^2*sgn(b*x + a) + 8*b^5*d^4*x*
e*sgn(b*x + a) + b^5*d^5*sgn(b*x + a) + 210*a*b^4*x^4*e^5*sgn(b*x + a) + 16
8*a*b^4*d*x^3*e^4*sgn(b*x + a) + 84*a*b^4*d^2*x^2*e^3*sgn(b*x + a) + 24*a*b
^4*d^3*x*e^2*sgn(b*x + a) + 3*a*b^4*d^4*e*sgn(b*x + a) + 336*a^2*b^3*x^3*e^
5*sgn(b*x + a) + 168*a^2*b^3*d*x^2*e^4*sgn(b*x + a) + 48*a^2*b^3*d^2*x*e^3*
sgn(b*x + a) + 6*a^2*b^3*d^3*e^2*sgn(b*x + a) + 280*a^3*b^2*x^2*e^5*sgn(b*x
 + a) + 80*a^3*b^2*d*x*e^4*sgn(b*x + a) + 10*a^3*b^2*d^2*e^3*sgn(b*x + a) +
 120*a^4*b*x*e^5*sgn(b*x + a) + 15*a^4*b*d*e^4*sgn(b*x + a) + 21*a^5*e^5*sg
n(b*x + a))*e^(-6)/(x*e + d)^8
```

3.1584 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{(d+ex)^{10}}\,dx$

Optimal. Leaf size=200

$$\frac{b^3\sqrt{a^2+2abx+b^2x^2}(a+bx)^5}{504(d+ex)^6(bd-ae)^4}+\frac{b^2\sqrt{a^2+2abx+b^2x^2}(a+bx)^5}{84(d+ex)^7(bd-ae)^3}+\frac{b\sqrt{a^2+2abx+b^2x^2}(a+bx)^5}{24(d+ex)^8(bd-ae)^2}+\frac{\sqrt{a^2+2abx+b^2x^2}}{9(d+ex)^9(bd}$$

```
[Out] ((a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(9*(b*d - a*e)*(d + e*x)^9) + (
b*(a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(24*(b*d - a*e)^2*(d + e*x)^8)
 + (b^2*(a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(84*(b*d - a*e)^3*(d + e
*x)^7) + (b^3*(a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(504*(b*d - a*e)^4
*(d + e*x)^6)
```

Rubi [A] time = 0.0678561, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {646, 45, 37}

$$\frac{b^3\sqrt{a^2+2abx+b^2x^2}(a+bx)^5}{504(d+ex)^6(bd-ae)^4}+\frac{b^2\sqrt{a^2+2abx+b^2x^2}(a+bx)^5}{84(d+ex)^7(bd-ae)^3}+\frac{b\sqrt{a^2+2abx+b^2x^2}(a+bx)^5}{24(d+ex)^8(bd-ae)^2}+\frac{\sqrt{a^2+2abx+b^2x^2}}{9(d+ex)^9(bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^10,x]
```

```
[Out] ((a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(9*(b*d - a*e)*(d + e*x)^9) + (
b*(a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(24*(b*d - a*e)^2*(d + e*x)^8)
 + (b^2*(a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(84*(b*d - a*e)^3*(d + e
*x)^7) + (b^3*(a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(504*(b*d - a*e)^4
*(d + e*x)^6)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{(d+ex)^{10}}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^5}{(d+ex)^{10}}\,dx}{b^4\left(ab+b^2x\right)}$$

$$= \frac{(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{9(bd-ae)(d+ex)^9} + \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^5}{(d+ex)^9}\,dx}{3b^3(bd-ae)\left(ab+b^2x\right)}$$

$$= \frac{(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{9(bd-ae)(d+ex)^9} + \frac{b(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{24(bd-ae)^2(d+ex)^8} + \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(ab+b}{(d+e}}{12b^2(bd-ae)^2\left(ab+b\right.}$$

$$= \frac{(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{9(bd-ae)(d+ex)^9} + \frac{b(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{24(bd-ae)^2(d+ex)^8} + \frac{b^2(a+bx)^5\sqrt{a^2+2abx+}}{84(bd-ae)^3(d+ex)^7}$$

$$= \frac{(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{9(bd-ae)(d+ex)^9} + \frac{b(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{24(bd-ae)^2(d+ex)^8} + \frac{b^2(a+bx)^5\sqrt{a^2+2abx+}}{84(bd-ae)^3(d+ex)^7}$$

Mathematica [A] time = 0.096909, size = 223, normalized size = 1.12

$$-\frac{\sqrt{(a+bx)^2}\left(10a^2b^3e^2\left(9d^2ex+d^3+36de^2x^2+84e^3x^3\right)+20a^3b^2e^3\left(d^2+9dex+36e^2x^2\right)+35a^4be^4(d+9ex)+56a^5e^5+\right.}{504e^6(a+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^10,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(56*a^5*e^5 + 35*a^4*b*e^4*(d + 9*e*x) + 20*a^3*b^2*e^3
*(d^2 + 9*d*e*x + 36*e^2*x^2) + 10*a^2*b^3*e^2*(d^3 + 9*d^2*e*x + 36*d*e^2*
x^2 + 84*e^3*x^3) + 4*a*b^4*e*(d^4 + 9*d^3*e*x + 36*d^2*e^2*x^2 + 84*d*e^3*
x^3 + 126*e^4*x^4) + b^5*(d^5 + 9*d^4*e*x + 36*d^3*e^2*x^2 + 84*d^2*e^3*x^3
 + 126*d*e^4*x^4 + 126*e^5*x^5)))/(504*e^6*(a + b*x)*(d + e*x)^9)
```

Maple [A] time = 0.155, size = 288, normalized size = 1.4

$$-\frac{126\,x^5b^5e^5+504\,x^4ab^4e^5+126\,x^4b^5de^4+840\,x^3a^2b^3e^5+336\,x^3ab^4de^4+84\,x^3b^5d^2e^3+720\,x^2a^3b^2e^5+360\,x^2a^2b^3de^4+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^10,x)
```

```
[Out] -1/504/e^6*(126*b^5*e^5*x^5+504*a*b^4*e^5*x^4+126*b^5*d*e^4*x^4+840*a^2*b^3
*e^5*x^3+336*a*b^4*d*e^4*x^3+84*b^5*d^2*e^3*x^3+720*a^3*b^2*e^5*x^2+360*a^2
*b^3*d*e^4*x^2+144*a*b^4*d^2*e^3*x^2+36*b^5*d^3*e^2*x^2+315*a^4*b*e^5*x+180
*a^3*b^2*d*e^4*x+90*a^2*b^3*d^2*e^3*x+36*a*b^4*d^3*e^2*x+9*b^5*d^4*e*x+56*a
^5*e^5+35*a^4*b*d*e^4+20*a^3*b^2*d^2*e^3+10*a^2*b^3*d^3*e^2+4*a*b^4*d^4*e+b
^5*d^5)*((b*x+a)^2)^(5/2)/(e*x+d)^9/(b*x+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.62802, size = 732, normalized size = 3.66

$$-\frac{126\, b^5e^5x^5 + b^5d^5 + 4\, ab^4d^4e + 10\, a^2b^3d^3e^2 + 20\, a^3b^2d^2e^3 + 35\, a^4bde^4 + 56\, a^5e^5 + 126\,\left(b^5de^4 + 4\, ab^4e^5\right)x^4 + 84\,\left(b^5d^2\right.}{504\,\left(e^{15}x^9 + 9\, de^{14}x^8 + 36\, d^2e^{13}x^7 + 84\, d^3e^{1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^10,x, algorithm="fricas")
```

```
[Out] -1/504*(126*b^5*e^5*x^5 + b^5*d^5 + 4*a*b^4*d^4*e + 10*a^2*b^3*d^3*e^2 + 20
*a^3*b^2*d^2*e^3 + 35*a^4*b*d*e^4 + 56*a^5*e^5 + 126*(b^5*d*e^4 + 4*a*b^4*e
^5)*x^4 + 84*(b^5*d^2*e^3 + 4*a*b^4*d*e^4 + 10*a^2*b^3*e^5)*x^3 + 36*(b^5*d
^3*e^2 + 4*a*b^4*d^2*e^3 + 10*a^2*b^3*d*e^4 + 20*a^3*b^2*e^5)*x^2 + 9*(b^5*
d^4*e + 4*a*b^4*d^3*e^2 + 10*a^2*b^3*d^2*e^3 + 20*a^3*b^2*d*e^4 + 35*a^4*b*
e^5)*x)/(e^15*x^9 + 9*d*e^14*x^8 + 36*d^2*e^13*x^7 + 84*d^3*e^12*x^6 + 126*
d^4*e^11*x^5 + 126*d^5*e^10*x^4 + 84*d^6*e^9*x^3 + 36*d^7*e^8*x^2 + 9*d^8*e
^7*x + d^9*e^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**10,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20285, size = 514, normalized size = 2.57

$$-\frac{\left(126\, b^5x^5e^5\mathrm{sgn}\left(bx + a\right) + 126\, b^5dx^4e^4\mathrm{sgn}\left(bx + a\right) + 84\, b^5d^2x^3e^3\mathrm{sgn}\left(bx + a\right) + 36\, b^5d^3x^2e^2\mathrm{sgn}\left(bx + a\right) + 9\, b^5d^4xes\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^10,x, algorithm="giac")
```

```
[Out] -1/504*(126*b^5*x^5*e^5*sgn(b*x + a) + 126*b^5*d*x^4*e^4*sgn(b*x + a) + 84*
b^5*d^2*x^3*e^3*sgn(b*x + a) + 36*b^5*d^3*x^2*e^2*sgn(b*x + a) + 9*b^5*d^4*
x*e*sgn(b*x + a) + b^5*d^5*sgn(b*x + a) + 504*a*b^4*x^4*e^5*sgn(b*x + a) +
336*a*b^4*d*x^3*e^4*sgn(b*x + a) + 144*a*b^4*d^2*x^2*e^3*sgn(b*x + a) + 36*
a*b^4*d^3*x*e^2*sgn(b*x + a) + 4*a*b^4*d^4*e*sgn(b*x + a) + 840*a^2*b^3*x^3
*e^5*sgn(b*x + a) + 360*a^2*b^3*d*x^2*e^4*sgn(b*x + a) + 90*a^2*b^3*d^2*x*e
^3*sgn(b*x + a) + 10*a^2*b^3*d^3*e^2*sgn(b*x + a) + 720*a^3*b^2*x^2*e^5*sgn
(b*x + a) + 180*a^3*b^2*d*x*e^4*sgn(b*x + a) + 20*a^3*b^2*d^2*e^3*sgn(b*x +
```

```
 a) + 315*a^4*b*x*e^5*sgn(b*x + a) + 35*a^4*b*d*e^4*sgn(b*x + a) + 56*a^5*e
^5*sgn(b*x + a))*e^(-6)/(x*e + d)^9
```

3.1585 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{(d+ex)^{11}}\,dx$

Optimal. Leaf size=308

$$-\frac{b^5\sqrt{a^2+2abx+b^2x^2}}{5e^6(a+bx)(d+ex)^5}+\frac{5b^4\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{6e^6(a+bx)(d+ex)^6}-\frac{10b^3\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{7e^6(a+bx)(d+ex)^7}+\frac{5b^2\sqrt{a^2+2abx+b^2x^2}}{4e^6(a+bx)(d+ex)}$$

```
[Out] ((b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(10*e^6*(a + b*x)*(d + e*x)^1
0) - (5*b*(b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(9*e^6*(a + b*x)*(d
+ e*x)^9) + (5*b^2*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*e^6*(a +
 b*x)*(d + e*x)^8) - (10*b^3*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(
7*e^6*(a + b*x)*(d + e*x)^7) + (5*b^4*(b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*
x^2])/(6*e^6*(a + b*x)*(d + e*x)^6) - (b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(
5*e^6*(a + b*x)*(d + e*x)^5)
```

Rubi [A] time = 0.140081, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$-\frac{b^5\sqrt{a^2+2abx+b^2x^2}}{5e^6(a+bx)(d+ex)^5}+\frac{5b^4\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{6e^6(a+bx)(d+ex)^6}-\frac{10b^3\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{7e^6(a+bx)(d+ex)^7}+\frac{5b^2\sqrt{a^2+2abx+b^2x^2}}{4e^6(a+bx)(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^11,x]
```


```
[Out] ((b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(10*e^6*(a + b*x)*(d + e*x)^1
0) - (5*b*(b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(9*e^6*(a + b*x)*(d
+ e*x)^9) + (5*b^2*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*e^6*(a +
 b*x)*(d + e*x)^8) - (10*b^3*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(
7*e^6*(a + b*x)*(d + e*x)^7) + (5*b^4*(b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*
x^2])/(6*e^6*(a + b*x)*(d + e*x)^6) - (b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(
5*e^6*(a + b*x)*(d + e*x)^5)
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{11}}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(ab+b^2x)^5}{(d+ex)^{11}}\,dx}{b^4(ab+b^2x)}$$
$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^5(bd-ae)^5}{e^5(d+ex)^{11}} + \frac{5b^6(bd-ae)^4}{e^5(d+ex)^{10}} - \frac{10b^7(bd-ae)^3}{e^5(d+ex)^9} + \frac{10b^8(bd-ae)^2}{e^5(d+ex)^8} - \frac{5b^9(bd-ae)}{e^5(d+ex)^7} + \frac{b^1}{e^5(d+}\right.}{b^4(ab+b^2x)}$$
$$= \frac{(bd-ae)^5\sqrt{a^2+2abx+b^2x^2}}{10e^6(a+bx)(d+ex)^{10}} - \frac{5b(bd-ae)^4\sqrt{a^2+2abx+b^2x^2}}{9e^6(a+bx)(d+ex)^9} + \frac{5b^2(bd-ae)^3\sqrt{a^2+2ab}}{4e^6(a+bx)(d+e}$$

Mathematica [A] time = 0.0804811, size = 223, normalized size = 0.72

$$-\frac{\sqrt{(a+bx)^2}\left(15a^2b^3e^2\left(10d^2ex+d^3+45de^2x^2+120e^3x^3\right)+35a^3b^2e^3\left(d^2+10dex+45e^2x^2\right)+70a^4be^4(d+10ex)+126a\right.}{1260e^6(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^11,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(126*a^5*e^5 + 70*a^4*b*e^4*(d + 10*e*x) + 35*a^3*b^2*e
^3*(d^2 + 10*d*e*x + 45*e^2*x^2) + 15*a^2*b^3*e^2*(d^3 + 10*d^2*e*x + 45*d*
e^2*x^2 + 120*e^3*x^3) + 5*a*b^4*e*(d^4 + 10*d^3*e*x + 45*d^2*e^2*x^2 + 120
*d*e^3*x^3 + 210*e^4*x^4) + b^5*(d^5 + 10*d^4*e*x + 45*d^3*e^2*x^2 + 120*d^
2*e^3*x^3 + 210*d*e^4*x^4 + 252*e^5*x^5)))/(1260*e^6*(a + b*x)*(d + e*x)^10
)
```

Maple [A] time = 0.159, size = 288, normalized size = 0.9

$$-\frac{252x^5b^5e^5+1050x^4ab^4e^5+210x^4b^5de^4+1800x^3a^2b^3e^5+600x^3ab^4de^4+120x^3b^5d^2e^3+1575x^2a^3b^2e^5+675x^2a^2b^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^11,x)
```

```
[Out] -1/1260/e^6*(252*b^5*e^5*x^5+1050*a*b^4*e^5*x^4+210*b^5*d*e^4*x^4+1800*a^2*
b^3*e^5*x^3+600*a*b^4*d*e^4*x^3+120*b^5*d^2*e^3*x^3+1575*a^3*b^2*e^5*x^2+67
5*a^2*b^3*d*e^4*x^2+225*a*b^4*d^2*e^3*x^2+45*b^5*d^3*e^2*x^2+700*a^4*b*e^5*
x+350*a^3*b^2*d*e^4*x+150*a^2*b^3*d^2*e^3*x+50*a*b^4*d^3*e^2*x+10*b^5*d^4*e
*x+126*a^5*e^5+70*a^4*b*d*e^4+35*a^3*b^2*d^2*e^3+15*a^2*b^3*d^3*e^2+5*a*b^4
*d^4*e+b^5*d^5)*((b*x+a)^2)^(5/2)/(e*x+d)^10/(b*x+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60735, size = 771, normalized size = 2.5

$$-\frac{252\,b^5e^5x^5 + b^5d^5 + 5\,ab^4d^4e + 15\,a^2b^3d^3e^2 + 35\,a^3b^2d^2e^3 + 70\,a^4bde^4 + 126\,a^5e^5 + 210\left(b^5de^4 + 5\,ab^4e^5\right)x^4 + 120\left(b^5\right.}{1260\left(e^{16}x^{10} + 10\,de^{15}x^9 + 45\,d^2e^{14}x^8 + 120\,d^3e^{13}x^7 +\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^11,x, algorithm="fricas")
```

```
[Out] -1/1260*(252*b^5*e^5*x^5 + b^5*d^5 + 5*a*b^4*d^4*e + 15*a^2*b^3*d^3*e^2 + 3
5*a^3*b^2*d^2*e^3 + 70*a^4*b*d*e^4 + 126*a^5*e^5 + 210*(b^5*d*e^4 + 5*a*b^4
*e^5)*x^4 + 120*(b^5*d^2*e^3 + 5*a*b^4*d*e^4 + 15*a^2*b^3*e^5)*x^3 + 45*(b^
5*d^3*e^2 + 5*a*b^4*d^2*e^3 + 15*a^2*b^3*d*e^4 + 35*a^3*b^2*e^5)*x^2 + 10*(
b^5*d^4*e + 5*a*b^4*d^3*e^2 + 15*a^2*b^3*d^2*e^3 + 35*a^3*b^2*d*e^4 + 70*a^
4*b*e^5)*x)/(e^16*x^10 + 10*d*e^15*x^9 + 45*d^2*e^14*x^8 + 120*d^3*e^13*x^7
 + 210*d^4*e^12*x^6 + 252*d^5*e^11*x^5 + 210*d^6*e^10*x^4 + 120*d^7*e^9*x^3
 + 45*d^8*e^8*x^2 + 10*d^9*e^7*x + d^10*e^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**11,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16376, size = 514, normalized size = 1.67

$$-\frac{\left(252\,b^5x^5e^5\mathrm{sgn}\left(bx + a\right) + 210\,b^5dx^4e^4\mathrm{sgn}\left(bx + a\right) + 120\,b^5d^2x^3e^3\mathrm{sgn}\left(bx + a\right) + 45\,b^5d^3x^2e^2\mathrm{sgn}\left(bx + a\right) + 10\,b^5d^4x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^11,x, algorithm="giac")
```

```
[Out] -1/1260*(252*b^5*x^5*e^5*sgn(b*x + a) + 210*b^5*d*x^4*e^4*sgn(b*x + a) + 12
0*b^5*d^2*x^3*e^3*sgn(b*x + a) + 45*b^5*d^3*x^2*e^2*sgn(b*x + a) + 10*b^5*d
^4*x*e*sgn(b*x + a) + b^5*d^5*sgn(b*x + a) + 1050*a*b^4*x^4*e^5*sgn(b*x + a
) + 600*a*b^4*d*x^3*e^4*sgn(b*x + a) + 225*a*b^4*d^2*x^2*e^3*sgn(b*x + a) +
 50*a*b^4*d^3*x*e^2*sgn(b*x + a) + 5*a*b^4*d^4*e*sgn(b*x + a) + 1800*a^2*b^
3*x^3*e^5*sgn(b*x + a) + 675*a^2*b^3*d*x^2*e^4*sgn(b*x + a) + 150*a^2*b^3*d
^2*x*e^3*sgn(b*x + a) + 15*a^2*b^3*d^3*e^2*sgn(b*x + a) + 1575*a^3*b^2*x^2*
e^5*sgn(b*x + a) + 350*a^3*b^2*d*x*e^4*sgn(b*x + a) + 35*a^3*b^2*d^2*e^3*sg
n(b*x + a) + 700*a^4*b*x*e^5*sgn(b*x + a) + 70*a^4*b*d*e^4*sgn(b*x + a) + 1
26*a^5*e^5*sgn(b*x + a))*e^(-6)/(x*e + d)^10
```

3.1586 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{12}}\,dx$

Optimal. Leaf size=308

$$-\frac{b^5\sqrt{a^2+2abx+b^2x^2}}{6e^6(a+bx)(d+ex)^6}+\frac{5b^4\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{7e^6(a+bx)(d+ex)^7}-\frac{5b^3\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{4e^6(a+bx)(d+ex)^8}+\frac{10b^2\sqrt{a^2+2abx+b^2x^2}(b}{9e^6(a+bx)(d+ex}$$

```
[Out] ((b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(11*e^6*(a + b*x)*(d + e*x)^1
1) - (b*(b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*e^6*(a + b*x)*(d +
e*x)^10) + (10*b^2*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(9*e^6*(a +
 b*x)*(d + e*x)^9) - (5*b^3*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4
*e^6*(a + b*x)*(d + e*x)^8) + (5*b^4*(b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x
^2])/(7*e^6*(a + b*x)*(d + e*x)^7) - (b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6
*e^6*(a + b*x)*(d + e*x)^6)
```

Rubi [A] time = 0.138008, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$-\frac{b^5\sqrt{a^2+2abx+b^2x^2}}{6e^6(a+bx)(d+ex)^6}+\frac{5b^4\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{7e^6(a+bx)(d+ex)^7}-\frac{5b^3\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{4e^6(a+bx)(d+ex)^8}+\frac{10b^2\sqrt{a^2+2abx+b^2x^2}(b}{9e^6(a+bx)(d+ex}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^12,x]
```


```
[Out] ((b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(11*e^6*(a + b*x)*(d + e*x)^1
1) - (b*(b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*e^6*(a + b*x)*(d +
e*x)^10) + (10*b^2*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(9*e^6*(a +
 b*x)*(d + e*x)^9) - (5*b^3*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4
*e^6*(a + b*x)*(d + e*x)^8) + (5*b^4*(b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x
^2])/(7*e^6*(a + b*x)*(d + e*x)^7) - (b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6
*e^6*(a + b*x)*(d + e*x)^6)
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{12}}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(ab+b^2x)^5}{(d+ex)^{12}}\,dx}{b^4(ab+b^2x)}$$

$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^5(bd-ae)^5}{e^5(d+ex)^{12}} + \frac{5b^6(bd-ae)^4}{e^5(d+ex)^{11}} - \frac{10b^7(bd-ae)^3}{e^5(d+ex)^{10}} + \frac{10b^8(bd-ae)^2}{e^5(d+ex)^9} - \frac{5b^9(bd-ae)}{e^5(d+ex)^8} + \dots\right.}{b^4(ab+b^2x)}$$

$$= \frac{(bd-ae)^5\sqrt{a^2+2abx+b^2x^2}}{11e^6(a+bx)(d+ex)^{11}} - \frac{b(bd-ae)^4\sqrt{a^2+2abx+b^2x^2}}{2e^6(a+bx)(d+ex)^{10}} + \frac{10b^2(bd-ae)^3\sqrt{a^2+\dots}}{9e^6(a+bx)(d+\dots}$$

Mathematica [A] time = 0.0794436, size = 223, normalized size = 0.72

$$-\frac{\sqrt{(a+bx)^2}\left(21a^2b^3e^2\left(11d^2ex+d^3+55de^2x^2+165e^3x^3\right)+56a^3b^2e^3\left(d^2+11dex+55e^2x^2\right)+126a^4be^4(d+11ex)+\dots\right.}{2772\dots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^12,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(252*a^5*e^5 + 126*a^4*b*e^4*(d + 11*e*x) + 56*a^3*b^2*
e^3*(d^2 + 11*d*e*x + 55*e^2*x^2) + 21*a^2*b^3*e^2*(d^3 + 11*d^2*e*x + 55*d
*e^2*x^2 + 165*e^3*x^3) + 6*a*b^4*e*(d^4 + 11*d^3*e*x + 55*d^2*e^2*x^2 + 16
5*d*e^3*x^3 + 330*e^4*x^4) + b^5*(d^5 + 11*d^4*e*x + 55*d^3*e^2*x^2 + 165*d
^2*e^3*x^3 + 330*d*e^4*x^4 + 462*e^5*x^5)))/(2772*e^6*(a + b*x)*(d + e*x)^1
1)
```

Maple [A] time = 0.156, size = 288, normalized size = 0.9

$$-\frac{462x^5b^5e^5+1980x^4ab^4e^5+330x^4b^5de^4+3465x^3a^2b^3e^5+990x^3ab^4de^4+165x^3b^5d^2e^3+3080x^2a^3b^2e^5+1155x^2\dots}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^12,x)
```

```
[Out] -1/2772/e^6*(462*b^5*e^5*x^5+1980*a*b^4*e^5*x^4+330*b^5*d*e^4*x^4+3465*a^2*
b^3*e^5*x^3+990*a*b^4*d*e^4*x^3+165*b^5*d^2*e^3*x^3+3080*a^3*b^2*e^5*x^2+11
55*a^2*b^3*d*e^4*x^2+330*a*b^4*d^2*e^3*x^2+55*b^5*d^3*e^2*x^2+1386*a^4*b*e^
5*x+616*a^3*b^2*d*e^4*x+231*a^2*b^3*d^2*e^3*x+66*a*b^4*d^3*e^2*x+11*b^5*d^4
*e*x+252*a^5*e^5+126*a^4*b*d*e^4+56*a^3*b^2*d^2*e^3+21*a^2*b^3*d^3*e^2+6*a*
b^4*d^4*e+b^5*d^5)*((b*x+a)^2)^(5/2)/(e*x+d)^11/(b*x+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^12,x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [A] time = 1.61667, size = 802, normalized size = 2.6

$$-\frac{462\,b^5e^5x^5+b^5d^5+6\,ab^4d^4e+21\,a^2b^3d^3e^2+56\,a^3b^2d^2e^3+126\,a^4bde^4+252\,a^5e^5+330\left(b^5de^4+6\,ab^4e^5\right)x^4+165\left(b^5d\right.}{2772\left(e^{17}x^{11}+11\,de^{16}x^{10}+55\,d^2e^{15}x^9+165\,d^3e^{14}x^8+330\,d^4e^1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^12,x, algorithm="fricas")
```

```
[Out] -1/2772*(462*b^5*e^5*x^5 + b^5*d^5 + 6*a*b^4*d^4*e + 21*a^2*b^3*d^3*e^2 + 5
6*a^3*b^2*d^2*e^3 + 126*a^4*b*d*e^4 + 252*a^5*e^5 + 330*(b^5*d*e^4 + 6*a*b^
4*e^5)*x^4 + 165*(b^5*d^2*e^3 + 6*a*b^4*d*e^4 + 21*a^2*b^3*e^5)*x^3 + 55*(b
^5*d^3*e^2 + 6*a*b^4*d^2*e^3 + 21*a^2*b^3*d*e^4 + 56*a^3*b^2*e^5)*x^2 + 11*
(b^5*d^4*e + 6*a*b^4*d^3*e^2 + 21*a^2*b^3*d^2*e^3 + 56*a^3*b^2*d*e^4 + 126*
a^4*b*e^5)*x)/(e^17*x^11 + 11*d*e^16*x^10 + 55*d^2*e^15*x^9 + 165*d^3*e^14*
x^8 + 330*d^4*e^13*x^7 + 462*d^5*e^12*x^6 + 462*d^6*e^11*x^5 + 330*d^7*e^10
*x^4 + 165*d^8*e^9*x^3 + 55*d^9*e^8*x^2 + 11*d^10*e^7*x + d^11*e^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**12,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17248, size = 514, normalized size = 1.67

$$-\frac{\left(462\,b^5x^5e^5\text{sgn}\left(bx+a\right)+330\,b^5dx^4e^4\text{sgn}\left(bx+a\right)+165\,b^5d^2x^3e^3\text{sgn}\left(bx+a\right)+55\,b^5d^3x^2e^2\text{sgn}\left(bx+a\right)+11\,b^5d^4xe\text{sg}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^12,x, algorithm="giac")
```

```
[Out] -1/2772*(462*b^5*x^5*e^5*sgn(b*x + a) + 330*b^5*d*x^4*e^4*sgn(b*x + a) + 16
5*b^5*d^2*x^3*e^3*sgn(b*x + a) + 55*b^5*d^3*x^2*e^2*sgn(b*x + a) + 11*b^5*d
^4*x*e*sgn(b*x + a) + b^5*d^5*sgn(b*x + a) + 1980*a*b^4*x^4*e^5*sgn(b*x + a
) + 990*a*b^4*d*x^3*e^4*sgn(b*x + a) + 330*a*b^4*d^2*x^2*e^3*sgn(b*x + a) +
 66*a*b^4*d^3*x*e^2*sgn(b*x + a) + 6*a*b^4*d^4*e*sgn(b*x + a) + 3465*a^2*b^
3*x^3*e^5*sgn(b*x + a) + 1155*a^2*b^3*d*x^2*e^4*sgn(b*x + a) + 231*a^2*b^3*
d^2*x*e^3*sgn(b*x + a) + 21*a^2*b^3*d^3*e^2*sgn(b*x + a) + 3080*a^3*b^2*x^2
*e^5*sgn(b*x + a) + 616*a^3*b^2*d*x*e^4*sgn(b*x + a) + 56*a^3*b^2*d^2*e^3*s
gn(b*x + a) + 1386*a^4*b*x*e^5*sgn(b*x + a) + 126*a^4*b*d*e^4*sgn(b*x + a)
+ 252*a^5*e^5*sgn(b*x + a))*e^(-6)/(x*e + d)^11
```

3.1587 $\int \frac{(d+ex)^4}{\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=222

$$\frac{ex(a+bx)(bd-ae)^3}{b^4\sqrt{a^2+2abx+b^2x^2}}+\frac{(a+bx)(d+ex)^2(bd-ae)^2}{2b^3\sqrt{a^2+2abx+b^2x^2}}+\frac{(a+bx)(d+ex)^3(bd-ae)}{3b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{(a+bx)(d+ex)^4}{4b\sqrt{a^2+2abx+b^2x^2}}+\frac{(a+bx)(\ldots}{b^5\sqrt{a\ldots}}$$

```
[Out] (e*(b*d - a*e)^3*x*(a + b*x))/(b^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + ((b*d -
 a*e)^2*(a + b*x)*(d + e*x)^2)/(2*b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + ((b*
d - a*e)*(a + b*x)*(d + e*x)^3)/(3*b^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + ((a
 + b*x)*(d + e*x)^4)/(4*b*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + ((b*d - a*e)^4*(
a + b*x)*Log[a + b*x])/(b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0803956, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$\frac{ex(a+bx)(bd-ae)^3}{b^4\sqrt{a^2+2abx+b^2x^2}}+\frac{(a+bx)(d+ex)^2(bd-ae)^2}{2b^3\sqrt{a^2+2abx+b^2x^2}}+\frac{(a+bx)(d+ex)^3(bd-ae)}{3b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{(a+bx)(d+ex)^4}{4b\sqrt{a^2+2abx+b^2x^2}}+\frac{(a+bx)(\ldots}{b^5\sqrt{a\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```


```
[Out] (e*(b*d - a*e)^3*x*(a + b*x))/(b^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + ((b*d -
 a*e)^2*(a + b*x)*(d + e*x)^2)/(2*b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + ((b*
d - a*e)*(a + b*x)*(d + e*x)^3)/(3*b^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + ((a
 + b*x)*(d + e*x)^4)/(4*b*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + ((b*d - a*e)^4*(
a + b*x)*Log[a + b*x])/(b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^4}{\sqrt{a^2+2abx+b^2x^2}}\,dx &= \frac{\left(ab+b^2x\right)\int \frac{(d+ex)^4}{ab+b^2x}\,dx}{\sqrt{a^2+2abx+b^2x^2}}\\ &= \frac{\left(ab+b^2x\right)\int \left(\frac{e(bd-ae)^3}{b^5}+\frac{(bd-ae)^4}{b^4\left(ab+b^2x\right)}+\frac{e(bd-ae)^2(d+ex)}{b^4}+\frac{e(bd-ae)(d+ex)^2}{b^3}+\frac{e(d+ex)^3}{b^2}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}\\ &= \frac{e(bd-ae)^3x(a+bx)}{b^4\sqrt{a^2+2abx+b^2x^2}}+\frac{(bd-ae)^2(a+bx)(d+ex)^2}{2b^3\sqrt{a^2+2abx+b^2x^2}}+\frac{(bd-ae)(a+bx)(d+ex)^3}{3b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{(a\ldots}{4b\sqrt{a\ldots}}\end{aligned}$$

Mathematica [A] time = 0.0806139, size = 130, normalized size = 0.59

$$\frac{(a+bx)\left(bex\left(6a^2be^2(8d+ex)-12a^3e^3-4ab^2e\left(18d^2+6dex+e^2x^2\right)+b^3\left(36d^2ex+48d^3+16de^2x^2+3e^3x^3\right)\right)+12(bd-\right.}{12b^5\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ((a + b*x)*(b*e*x*(-12*a^3*e^3 + 6*a^2*b*e^2*(8*d + e*x) - 4*a*b^2*e*(18*d^
2 + 6*d*e*x + e^2*x^2) + b^3*(48*d^3 + 36*d^2*e*x + 16*d*e^2*x^2 + 3*e^3*x^
3)) + 12*(b*d - a*e)^4*Log[a + b*x]))/(12*b^5*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.158, size = 223, normalized size = 1.

$$\frac{(bx+a)\left(3x^4b^4e^4-4x^3ab^3e^4+16x^3b^4de^3+6x^2a^2b^2e^4-24x^2ab^3de^3+36x^2b^4d^2e^2+12\ln(bx+a)a^4e^4-48\ln(bx+a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/((b*x+a)^2)^(1/2),x)
```

```
[Out] 1/12*(b*x+a)*(3*x^4*b^4*e^4-4*x^3*a*b^3*e^4+16*x^3*b^4*d*e^3+6*x^2*a^2*b^2*
e^4-24*x^2*a*b^3*d*e^3+36*x^2*b^4*d^2*e^2+12*ln(b*x+a)*a^4*e^4-48*ln(b*x+a)
*a^3*b*d*e^3+72*ln(b*x+a)*a^2*b^2*d^2*e^2-48*ln(b*x+a)*a*b^3*d^3*e+12*ln(b*
x+a)*b^4*d^4-12*x*a^3*b*e^4+48*x*a^2*b^2*d*e^3-72*x*a*b^3*d^2*e^2+48*x*b^4*
d^3*e)/((b*x+a)^2)^(1/2)/b^5
```

Maxima [B] time = 1.1538, size = 590, normalized size = 2.66

$$\frac{6a^2b^2d^2e^2\log\left(x+\frac{a}{b}\right)}{\left(b^2\right)^{\frac{5}{2}}}-\frac{20a^3bde^3\log\left(x+\frac{a}{b}\right)}{3\left(b^2\right)^{\frac{5}{2}}}+\frac{13a^4e^4\log\left(x+\frac{a}{b}\right)}{6\left(b^2\right)^{\frac{5}{2}}}-\frac{6abd^2e^2x}{\left(b^2\right)^{\frac{3}{2}}}+\frac{20a^2de^3x}{3\left(b^2\right)^{\frac{3}{2}}}-\frac{13a^3e^4x}{6\left(b^2\right)^{\frac{3}{2}}b}+\frac{3d^2e^2x^2}{\sqrt{b^2}}-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 6*a^2*b^2*d^2*e^2*log(x + a/b)/(b^2)^(5/2) - 20/3*a^3*b*d*e^3*log(x + a/b)/
(b^2)^(5/2) + 13/6*a^4*e^4*log(x + a/b)/(b^2)^(5/2) - 6*a*b*d^2*e^2*x/(b^2)
^(3/2) + 20/3*a^2*d*e^3*x/(b^2)^(3/2) - 13/6*a^3*e^4*x/((b^2)^(3/2)*b) + 3*
d^2*e^2*x^2/sqrt(b^2) - 10/3*a*d*e^3*x^2/(sqrt(b^2)*b) + 13/12*a^2*e^4*x^2/
(sqrt(b^2)*b^2) + 1/4*sqrt(b^2*x^2 + 2*a*b*x + a^2)*e^4*x^3/b^2 + sqrt(b^(-
2))*d^4*log(x + a/b) - 4*a*sqrt(b^(-2))*d^3*e*log(x + a/b)/b + 8/3*a^3*sqrt
(b^(-2))*d*e^3*log(x + a/b)/b^3 - 7/6*a^4*sqrt(b^(-2))*e^4*log(x + a/b)/b^4
 + 4/3*sqrt(b^2*x^2 + 2*a*b*x + a^2)*d*e^3*x^2/b^2 - 7/12*sqrt(b^2*x^2 + 2*
a*b*x + a^2)*a*e^4*x^2/b^3 + 4*sqrt(b^2*x^2 + 2*a*b*x + a^2)*d^3*e/b^2 - 8/
3*sqrt(b^2*x^2 + 2*a*b*x + a^2)*a^2*d*e^3/b^4 + 7/6*sqrt(b^2*x^2 + 2*a*b*x
+ a^2)*a^3*e^4/b^5
```

Fricas [A] time = 1.5845, size = 369, normalized size = 1.66

$$\frac{3\,b^4e^4x^4 + 4\left(4\,b^4de^3 - ab^3e^4\right)x^3 + 6\left(6\,b^4d^2e^2 - 4\,ab^3de^3 + a^2b^2e^4\right)x^2 + 12\left(4\,b^4d^3e - 6\,ab^3d^2e^2 + 4\,a^2b^2de^3 - a^3be^4\right)x}{12\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*e^4*x^4 + 4*(4*b^4*d*e^3 - a*b^3*e^4)*x^3 + 6*(6*b^4*d^2*e^2 -
4*a*b^3*d*e^3 + a^2*b^2*e^4)*x^2 + 12*(4*b^4*d^3*e - 6*a*b^3*d^2*e^2 + 4*a^
2*b^2*d*e^3 - a^3*b*e^4)*x + 12*(b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^
2 - 4*a^3*b*d*e^3 + a^4*e^4)*log(b*x + a))/b^5
```

Sympy [A] time = 0.647496, size = 134, normalized size = 0.6

$$\frac{e^4x^4}{4b} - \frac{x^3\left(ae^4 - 4bde^3\right)}{3b^2} + \frac{x^2\left(a^2e^4 - 4abde^3 + 6b^2d^2e^2\right)}{2b^3} - \frac{x\left(a^3e^4 - 4a^2bde^3 + 6ab^2d^2e^2 - 4b^3d^3e\right)}{b^4} + \frac{(ae - bd)^4\log(a}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/((b*x+a)**2)**(1/2),x)
```

```
[Out] e**4*x**4/(4*b) - x**3*(a*e**4 - 4*b*d*e**3)/(3*b**2) + x**2*(a**2*e**4 - 4
*a*b*d*e**3 + 6*b**2*d**2*e**2)/(2*b**3) - x*(a**3*e**4 - 4*a**2*b*d*e**3 +
 6*a*b**2*d**2*e**2 - 4*b**3*d**3*e)/b**4 + (a*e - b*d)**4*log(a + b*x)/b**
5
```

Giac [A] time = 1.22037, size = 356, normalized size = 1.6

$$\frac{3\,b^3x^4e^4\mathrm{sgn}\,(bx+a) + 16\,b^3dx^3e^3\mathrm{sgn}\,(bx+a) + 36\,b^3d^2x^2e^2\mathrm{sgn}\,(bx+a) + 48\,b^3d^3xe\mathrm{sgn}\,(bx+a) - 4\,ab^2x^3e^4\mathrm{sgn}\,(bx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(3*b^3*x^4*e^4*sgn(b*x + a) + 16*b^3*d*x^3*e^3*sgn(b*x + a) + 36*b^3*d
^2*x^2*e^2*sgn(b*x + a) + 48*b^3*d^3*x*e*sgn(b*x + a) - 4*a*b^2*x^3*e^4*sgn
(b*x + a) - 24*a*b^2*d*x^2*e^3*sgn(b*x + a) - 72*a*b^2*d^2*x*e^2*sgn(b*x +
a) + 6*a^2*b*x^2*e^4*sgn(b*x + a) + 48*a^2*b*d*x*e^3*sgn(b*x + a) - 12*a^3*
x*e^4*sgn(b*x + a))/b^4 + (b^4*d^4*sgn(b*x + a) - 4*a*b^3*d^3*e*sgn(b*x + a
) + 6*a^2*b^2*d^2*e^2*sgn(b*x + a) - 4*a^3*b*d*e^3*sgn(b*x + a) + a^4*e^4*s
gn(b*x + a))*log(abs(b*x + a))/b^5
```

3.1588 $\int \frac{(d+ex)^3}{\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=173

$$\frac{ex(a+bx)(bd-ae)^2}{b^3\sqrt{a^2+2abx+b^2x^2}}+\frac{(a+bx)(d+ex)^2(bd-ae)}{2b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{(a+bx)(d+ex)^3}{3b\sqrt{a^2+2abx+b^2x^2}}+\frac{(a+bx)(bd-ae)^3\log(a+bx)}{b^4\sqrt{a^2+2abx+b^2x^2}}$$

```
[Out] (e*(b*d - a*e)^2*x*(a + b*x))/(b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + ((b*d -
 a*e)*(a + b*x)*(d + e*x)^2)/(2*b^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + ((a +
b*x)*(d + e*x)^3)/(3*b*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + ((b*d - a*e)^3*(a +
 b*x)*Log[a + b*x])/(b^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0619336, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$\frac{ex(a+bx)(bd-ae)^2}{b^3\sqrt{a^2+2abx+b^2x^2}}+\frac{(a+bx)(d+ex)^2(bd-ae)}{2b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{(a+bx)(d+ex)^3}{3b\sqrt{a^2+2abx+b^2x^2}}+\frac{(a+bx)(bd-ae)^3\log(a+bx)}{b^4\sqrt{a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (e*(b*d - a*e)^2*x*(a + b*x))/(b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + ((b*d -
 a*e)*(a + b*x)*(d + e*x)^2)/(2*b^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + ((a +
b*x)*(d + e*x)^3)/(3*b*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + ((b*d - a*e)^3*(a +
 b*x)*Log[a + b*x])/(b^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^3}{\sqrt{a^2+2abx+b^2x^2}}\,dx &= \frac{\left(ab+b^2x\right)\int \frac{(d+ex)^3}{ab+b^2x}\,dx}{\sqrt{a^2+2abx+b^2x^2}}\\ &= \frac{\left(ab+b^2x\right)\int \left(\frac{e(bd-ae)^2}{b^4}+\frac{(bd-ae)^3}{b^3\left(ab+b^2x\right)}+\frac{e(bd-ae)(d+ex)}{b^3}+\frac{e(d+ex)^2}{b^2}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}\\ &= \frac{e(bd-ae)^2x(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}}+\frac{(bd-ae)(a+bx)(d+ex)^2}{2b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{(a+bx)(d+ex)^3}{3b\sqrt{a^2+2abx+b^2x^2}}+\frac{(bd-ae)^3(a+}{b^4\sqrt{a^2+2}}\end{aligned}$$

Mathematica [A] time = 0.0526176, size = 90, normalized size = 0.52

$$\frac{(a+bx)\left(bex\left(6a^2e^2-3abe(6d+ex)+b^2\left(18d^2+9dex+2e^2x^2\right)\right)+6(bd-ae)^3\log(a+bx)\right)}{6b^4\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ((a + b*x)*(b*e*x*(6*a^2*e^2 - 3*a*b*e*(6*d + e*x) + b^2*(18*d^2 + 9*d*e*x
+ 2*e^2*x^2)) + 6*(b*d - a*e)^3*Log[a + b*x]))/(6*b^4*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.154, size = 147, normalized size = 0.9

$$-\frac{(bx+a)\left(-2x^3b^3e^3+3x^2ab^2e^3-9x^2b^3de^2+6\ln(bx+a)a^3e^3-18\ln(bx+a)a^2bde^2+18\ln(bx+a)ab^2d^2e-6\ln\right.}{6b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/((b*x+a)^2)^(1/2),x)
```

```
[Out] -1/6*(b*x+a)*(-2*x^3*b^3*e^3+3*x^2*a*b^2*e^3-9*x^2*b^3*d*e^2+6*ln(b*x+a)*a^
3*e^3-18*ln(b*x+a)*a^2*b*d*e^2+18*ln(b*x+a)*a*b^2*d^2*e-6*ln(b*x+a)*b^3*d^3
-6*x*a^2*b*e^3+18*x*a*b^2*d*e^2-18*x*b^3*d^2*e)/((b*x+a)^2)^(1/2)/b^4
```

Maxima [B] time = 1.14639, size = 346, normalized size = 2.

$$\frac{3a^2b^2de^2\log\left(x+\frac{a}{b}\right)}{\left(b^2\right)^{\frac{5}{2}}}-\frac{5a^3be^3\log\left(x+\frac{a}{b}\right)}{3\left(b^2\right)^{\frac{5}{2}}}-\frac{3abde^2x}{\left(b^2\right)^{\frac{3}{2}}}+\frac{5a^2e^3x}{3\left(b^2\right)^{\frac{3}{2}}}+\frac{3de^2x^2}{2\sqrt{b^2}}-\frac{5ae^3x^2}{6\sqrt{b^2}b}+\sqrt{\frac{1}{b^2}}d^3\log\left(x+\frac{a}{b}\right)-\frac{3a\sqrt{\frac{1}{b^2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 3*a^2*b^2*d*e^2*log(x + a/b)/(b^2)^(5/2) - 5/3*a^3*b*e^3*log(x + a/b)/(b^2)
^(5/2) - 3*a*b*d*e^2*x/(b^2)^(3/2) + 5/3*a^2*e^3*x/(b^2)^(3/2) + 3/2*d*e^2*
x^2/sqrt(b^2) - 5/6*a*e^3*x^2/(sqrt(b^2)*b) + sqrt(b^(-2))*d^3*log(x + a/b)
 - 3*a*sqrt(b^(-2))*d^2*e*log(x + a/b)/b + 2/3*a^3*sqrt(b^(-2))*e^3*log(x +
 a/b)/b^3 + 1/3*sqrt(b^2*x^2 + 2*a*b*x + a^2)*e^3*x^2/b^2 + 3*sqrt(b^2*x^2
+ 2*a*b*x + a^2)*d^2*e/b^2 - 2/3*sqrt(b^2*x^2 + 2*a*b*x + a^2)*a^2*e^3/b^4
```

Fricas [A] time = 1.55755, size = 238, normalized size = 1.38

$$\frac{2b^3e^3x^3+3\left(3b^3de^2-ab^2e^3\right)x^2+6\left(3b^3d^2e-3ab^2de^2+a^2be^3\right)x+6\left(b^3d^3-3ab^2d^2e+3a^2bde^2-a^3e^3\right)\log(bx+a)}{6b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*e^3*x^3 + 3*(3*b^3*d*e^2 - a*b^2*e^3)*x^2 + 6*(3*b^3*d^2*e - 3*a
*b^2*d*e^2 + a^2*b*e^3)*x + 6*(b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^
3*e^3)*log(b*x + a))/b^4
```

Sympy [A] time = 0.549075, size = 82, normalized size = 0.47

$$\frac{e^3x^3}{3b} - \frac{x^2\left(ae^3 - 3bde^2\right)}{2b^2} + \frac{x\left(a^2e^3 - 3abde^2 + 3b^2d^2e\right)}{b^3} - \frac{(ae - bd)^3 \log{(a + bx)}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/((b*x+a)**2)**(1/2),x)
```

```
[Out] e**3*x**3/(3*b) - x**2*(a*e**3 - 3*b*d*e**2)/(2*b**2) + x*(a**2*e**3 - 3*a*
b*d*e**2 + 3*b**2*d**2*e)/b**3 - (a*e - b*d)**3*log(a + b*x)/b**4
```

Giac [A] time = 1.15909, size = 230, normalized size = 1.33

$$\frac{2\,b^2x^3e^3\mathrm{sgn}\left(bx+a\right) + 9\,b^2dx^2e^2\mathrm{sgn}\left(bx+a\right) + 18\,b^2d^2xe\mathrm{sgn}\left(bx+a\right) - 3\,abx^2e^3\mathrm{sgn}\left(bx+a\right) - 18\,abdxe^2\mathrm{sgn}\left(bx+a\right) + 6}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(2*b^2*x^3*e^3*sgn(b*x + a) + 9*b^2*d*x^2*e^2*sgn(b*x + a) + 18*b^2*d^2
*x*e*sgn(b*x + a) - 3*a*b*x^2*e^3*sgn(b*x + a) - 18*a*b*d*x*e^2*sgn(b*x + a
) + 6*a^2*x*e^3*sgn(b*x + a))/b^3 + (b^3*d^3*sgn(b*x + a) - 3*a*b^2*d^2*e*s
gn(b*x + a) + 3*a^2*b*d*e^2*sgn(b*x + a) - a^3*e^3*sgn(b*x + a))*log(abs(b*
x + a))/b^4
```

3.1589 $\int \frac{(d+ex)^2}{\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=124

$$\frac{ex(a+bx)(bd-ae)}{b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{(a+bx)(d+ex)^2}{2b\sqrt{a^2+2abx+b^2x^2}}+\frac{(a+bx)(bd-ae)^2\log(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}}$$

```
[Out] (e*(b*d - a*e)*x*(a + b*x))/(b^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + ((a + b*x
)*(d + e*x)^2)/(2*b*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + ((b*d - a*e)^2*(a + b*
x)*Log[a + b*x])/(b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0458448, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$\frac{ex(a+bx)(bd-ae)}{b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{(a+bx)(d+ex)^2}{2b\sqrt{a^2+2abx+b^2x^2}}+\frac{(a+bx)(bd-ae)^2\log(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (e*(b*d - a*e)*x*(a + b*x))/(b^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + ((a + b*x
)*(d + e*x)^2)/(2*b*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + ((b*d - a*e)^2*(a + b*
x)*Log[a + b*x])/(b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2}{\sqrt{a^2+2abx+b^2x^2}}\,dx &= \frac{\left(ab+b^2x\right)\int \frac{(d+ex)^2}{ab+b^2x}\,dx}{\sqrt{a^2+2abx+b^2x^2}}\\
&= \frac{\left(ab+b^2x\right)\int \left(\frac{e(bd-ae)}{b^3}+\frac{(bd-ae)^2}{b^2\left(ab+b^2x\right)}+\frac{e(d+ex)}{b^2}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}\\
&= \frac{e(bd-ae)x(a+bx)}{b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{(a+bx)(d+ex)^2}{2b\sqrt{a^2+2abx+b^2x^2}}+\frac{(bd-ae)^2(a+bx)\log(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.0337432, size = 59, normalized size = 0.48

$$\frac{(a+bx)\left(bex(-2ae+4bd+bex)+2(bd-ae)^2\log(a+bx)\right)}{2b^3\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ((a + b*x)*(b*e*x*(4*b*d - 2*a*e + b*e*x) + 2*(b*d - a*e)^2*Log[a + b*x]))/
(2*b^3*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.154, size = 87, normalized size = 0.7

$$\frac{(bx+a)\left(x^2b^2e^2+2\ln(bx+a)a^2e^2-4\ln(bx+a)abde+2\ln(bx+a)b^2d^2-2xabe^2+4xb^2de\right)}{2b^3}\frac{1}{\sqrt{(bx+a)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/((b*x+a)^2)^(1/2),x)
```

```
[Out] 1/2*(b*x+a)*(x^2*b^2*e^2+2*ln(b*x+a)*a^2*e^2-4*ln(b*x+a)*a*b*d*e+2*ln(b*x+a
)*b^2*d^2-2*x*a*b*e^2+4*x*b^2*d*e)/((b*x+a)^2)^(1/2)/b^3
```

Maxima [A] time = 1.06228, size = 153, normalized size = 1.23

$$\frac{a^2b^2e^2\log\left(x+\frac{a}{b}\right)}{\left(b^2\right)^{\frac{5}{2}}}-\frac{abe^2x}{\left(b^2\right)^{\frac{3}{2}}}+\frac{e^2x^2}{2\sqrt{b^2}}+\sqrt{\frac{1}{b^2}}d^2\log\left(x+\frac{a}{b}\right)-\frac{2a\sqrt{\frac{1}{b^2}}de\log\left(x+\frac{a}{b}\right)}{b}+\frac{2\sqrt{b^2x^2+2abx+a^2}de}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] a^2*b^2*e^2*log(x + a/b)/(b^2)^(5/2) - a*b*e^2*x/(b^2)^(3/2) + 1/2*e^2*x^2/
sqrt(b^2) + sqrt(b^(-2))*d^2*log(x + a/b) - 2*a*sqrt(b^(-2))*d*e*log(x + a/
b)/b + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*d*e/b^2
```

Fricas [A] time = 1.56328, size = 135, normalized size = 1.09

$$\frac{b^2e^2x^2+2\left(2b^2de-abe^2\right)x+2\left(b^2d^2-2abde+a^2e^2\right)\log(bx+a)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*e^2*x^2 + 2*(2*b^2*d*e - a*b*e^2)*x + 2*(b^2*d^2 - 2*a*b*d*e + a^2
*e^2)*log(b*x + a))/b^3
```

Sympy [A] time = 0.452613, size = 44, normalized size = 0.35

$$\frac{e^2x^2}{2b} - \frac{x\left(ae^2 - 2bde\right)}{b^2} + \frac{(ae - bd)^2\log\left(a + bx\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/((b*x+a)**2)**(1/2),x)
```

```
[Out] e**2*x**2/(2*b) - x*(a*e**2 - 2*b*d*e)/b**2 + (a*e - b*d)**2*log(a + b*x)/b
**3
```

Giac [A] time = 1.18255, size = 128, normalized size = 1.03

$$\frac{bx^2e^2\mathrm{sgn}\left(bx + a\right) + 4\,bdxe\mathrm{sgn}\left(bx + a\right) - 2\,axe^2\mathrm{sgn}\left(bx + a\right)}{2\,b^2} + \frac{\left(b^2d^2\mathrm{sgn}\left(bx + a\right) - 2\,abde\mathrm{sgn}\left(bx + a\right) + a^2e^2\mathrm{sgn}\left(bx\right.\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2*e^2*sgn(b*x + a) + 4*b*d*x*e*sgn(b*x + a) - 2*a*x*e^2*sgn(b*x +
a))/b^2 + (b^2*d^2*sgn(b*x + a) - 2*a*b*d*e*sgn(b*x + a) + a^2*e^2*sgn(b*x
+ a))*log(abs(b*x + a))/b^3
```

3.1590 $\int \frac{d+ex}{\sqrt{a^2+2abx+b^2x^2}} dx$

Optimal. Leaf size=69

$$\frac{(a+bx)(bd-ae)\log(a+bx)}{b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{e\sqrt{a^2+2abx+b^2x^2}}{b^2}$$

```
[Out] (e*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/b^2 + ((b*d - a*e)*(a + b*x)*Log[a + b*x]
)/(b^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0233859, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {640, 608, 31}

$$\frac{(a+bx)(bd-ae)\log(a+bx)}{b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{e\sqrt{a^2+2abx+b^2x^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (e*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/b^2 + ((b*d - a*e)*(a + b*x)*Log[a + b*x]
)/(b^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex}{\sqrt{a^2+2abx+b^2x^2}}\,dx &= \frac{e\sqrt{a^2+2abx+b^2x^2}}{b^2}+\frac{\left(2b^2d-2abe\right)\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx}{2b^2}\\
&= \frac{e\sqrt{a^2+2abx+b^2x^2}}{b^2}+\frac{\left(\left(2b^2d-2abe\right)\left(ab+b^2x\right)\right)\int \frac{1}{ab+b^2x}\,dx}{2b^2\sqrt{a^2+2abx+b^2x^2}}\\
&= \frac{e\sqrt{a^2+2abx+b^2x^2}}{b^2}+\frac{(bd-ae)(a+bx)\log(a+bx)}{b^2\sqrt{a^2+2abx+b^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.0172236, size = 40, normalized size = 0.58

$$\frac{(a+bx)((bd-ae)\log(a+bx)+bex)}{b^2\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ((a + b*x)*(b*e*x + (b*d - a*e)*Log[a + b*x]))/(b^2*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.164, size = 45, normalized size = 0.7

$$-\frac{(bx+a)(\ln(bx+a)ae-\ln(bx+a)bd-bxe)}{b^2}\frac{1}{\sqrt{(bx+a)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/((b*x+a)^2)^(1/2),x)
```

```
[Out] -(b*x+a)*(ln(b*x+a)*a*e-ln(b*x+a)*b*d-b*x*e)/((b*x+a)^2)^(1/2)/b^2
```

Maxima [A] time = 1.01973, size = 80, normalized size = 1.16

$$\sqrt{\frac{1}{b^2}}d\log\left(x+\frac{a}{b}\right)-\frac{a\sqrt{\frac{1}{b^2}}e\log\left(x+\frac{a}{b}\right)}{b}+\frac{\sqrt{b^2x^2+2\,abx+a^2}e}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(b^(-2))*d*log(x + a/b) - a*sqrt(b^(-2))*e*log(x + a/b)/b + sqrt(b^2*x^
2 + 2*a*b*x + a^2)*e/b^2
```

Fricas [A] time = 1.48752, size = 54, normalized size = 0.78

$$\frac{bex+(bd-ae)\log(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (b*e*x + (b*d - a*e)*log(b*x + a))/b^2
```

Sympy [A] time = 0.369018, size = 20, normalized size = 0.29

$$\frac{ex}{b}-\frac{(ae-bd)\log(a+bx)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/((b*x+a)**2)**(1/2),x)
```

```
[Out] e*x/b - (a*e - b*d)*log(a + b*x)/b**2
```

Giac [A] time = 1.13261, size = 62, normalized size = 0.9

$$\frac{xe\text{sgn}\,(bx+a)}{b}+\frac{(bd\text{sgn}\,(bx+a)-ae\text{sgn}\,(bx+a))\log\left(|bx+a|\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] x*e*sgn(b*x + a)/b + (b*d*sgn(b*x + a) - a*e*sgn(b*x + a))*log(abs(b*x + a)
)/b^2
```

3.1591 $\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=35

$$\frac{(a+bx)\log(a+bx)}{b\sqrt{a^2+2abx+b^2x^2}}$$

```
[Out] ((a + b*x)*Log[a + b*x])/(b*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0081635, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {608, 31}

$$\frac{(a+bx)\log(a+bx)}{b\sqrt{a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]

[Out] ((a + b*x)*Log[a + b*x])/(b*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx &= \frac{\left(ab+b^2x\right)\int \frac{1}{ab+b^2x}\,dx}{\sqrt{a^2+2abx+b^2x^2}}\\ &= \frac{(a+bx)\log(a+bx)}{b\sqrt{a^2+2abx+b^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0065614, size = 26, normalized size = 0.74

$$\frac{(a+bx)\log(a+bx)}{b\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]

[Out] ((a + b*x)*Log[a + b*x])/(b*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.043, size = 25, normalized size = 0.7

$$\frac{(bx+a)\ln(bx+a)}{b}\frac{1}{\sqrt{(bx+a)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^2)^(1/2),x)
```

```
[Out] (b*x+a)*ln(b*x+a)/b/((b*x+a)^2)^(1/2)
```

Maxima [A] time = 1.0589, size = 19, normalized size = 0.54

$$\sqrt{\frac{1}{b^2}}\log\left(x+\frac{a}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(b^(-2))*log(x + a/b)
```

Fricas [A] time = 1.65035, size = 22, normalized size = 0.63

$$\frac{\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] log(b*x + a)/b
```

Sympy [A] time = 0.087553, size = 7, normalized size = 0.2

$$\frac{\log(a+bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)**2)**(1/2),x)
```

```
[Out] log(a + b*x)/b
```

Giac [A] time = 1.15692, size = 23, normalized size = 0.66

$$\frac{\log(|bx+a|)\operatorname{sgn}(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))*sgn(b*x + a)/b
```

3.1592 $\int \frac{1}{(d+ex)\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=86

$$\frac{(a+bx)\log(a+bx)}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)}-\frac{(a+bx)\log(d+ex)}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)}$$

```
[Out] ((a + b*x)*Log[a + b*x])/((b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - ((a
+ b*x)*Log[d + e*x])/((b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0307776, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {646, 36, 31}

$$\frac{(a+bx)\log(a+bx)}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)}-\frac{(a+bx)\log(d+ex)}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

```
[Out] ((a + b*x)*Log[a + b*x])/((b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - ((a
+ b*x)*Log[d + e*x])/((b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)\sqrt{a^2+2abx+b^2x^2}}\,dx &= \frac{\left(ab+b^2x\right)\int \frac{1}{\left(ab+b^2x\right)(d+ex)}\,dx}{\sqrt{a^2+2abx+b^2x^2}}\\ &= \frac{\left(b\left(ab+b^2x\right)\right)\int \frac{1}{ab+b^2x}\,dx}{(bd-ae)\sqrt{a^2+2abx+b^2x^2}}-\frac{\left(e\left(ab+b^2x\right)\right)\int \frac{1}{d+ex}\,dx}{b(bd-ae)\sqrt{a^2+2abx+b^2x^2}}\\ &= \frac{(a+bx)\log(a+bx)}{(bd-ae)\sqrt{a^2+2abx+b^2x^2}}-\frac{(a+bx)\log(d+ex)}{(bd-ae)\sqrt{a^2+2abx+b^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0187443, size = 42, normalized size = 0.49

$$\frac{(a+bx)(\log(a+bx)-\log(d+ex))}{\sqrt{(a+bx)^2}(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

```
[Out] ((a + b*x)*(Log[a + b*x] - Log[d + e*x]))/((b*d - a*e)*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.157, size = 41, normalized size = 0.5

$$\frac{(bx+a)(\ln(ex+d)-\ln(bx+a))}{ae-bd}\frac{1}{\sqrt{(bx+a)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/((b*x+a)^2)^(1/2),x)
```

```
[Out] (b*x+a)*(ln(e*x+d)-ln(b*x+a))/((b*x+a)^2)^(1/2)/(a*e-b*d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59554, size = 58, normalized size = 0.67

$$\frac{\log(bx+a)-\log(ex+d)}{bd-ae}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (log(b*x + a) - log(e*x + d))/(b*d - a*e)
```

Sympy [B] time = 0.401514, size = 128, normalized size = 1.49

$$\frac{\log\left(x+\frac{-\frac{a^2e^2}{ae-bd}+\frac{2abde}{ae-bd}+ae-\frac{b^2d^2}{ae-bd}+bd}{2be}\right)}{ae-bd}-\frac{\log\left(x+\frac{\frac{a^2e^2}{ae-bd}-\frac{2abde}{ae-bd}+ae+\frac{b^2d^2}{ae-bd}+bd}{2be}\right)}{ae-bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/((b*x+a)**2)**(1/2),x)
```

```
[Out] log(x + (-a**2*e**2/(a*e - b*d) + 2*a*b*d*e/(a*e - b*d) + a*e - b**2*d**2/(
a*e - b*d) + b*d)/(2*b*e))/(a*e - b*d) - log(x + (a**2*e**2/(a*e - b*d) - 2
*a*b*d*e/(a*e - b*d) + a*e + b**2*d**2/(a*e - b*d) + b*d)/(2*b*e))/(a*e - b
*d)
```

Giac [A] time = 1.24142, size = 101, normalized size = 1.17

$$\frac{\log\left(\frac{|2\,bxe+bd+ae-|bd-ae||}{|2\,bxe+bd+ae+|bd-ae||}\right)\operatorname{sgn}(bx+a)}{|bd-ae|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] log(abs(2*b*x*e + b*d + a*e - abs(b*d - a*e))/abs(2*b*x*e + b*d + a*e + abs
(b*d - a*e)))*sgn(b*x + a)/abs(b*d - a*e)
```

3.1593 $\int \frac{1}{(d+ex)^2\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=131

$$\frac{a+bx}{\sqrt{a^2+2abx+b^2x^2}(d+ex)(bd-ae)}+\frac{b(a+bx)\log(a+bx)}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}-\frac{b(a+bx)\log(d+ex)}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}$$

```
[Out] (a + b*x)/((b*d - a*e)*(d + e*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (b*(a + b
*x)*Log[a + b*x])/((b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (b*(a + b
*x)*Log[d + e*x])/((b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0620819, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 44}

$$\frac{a+bx}{\sqrt{a^2+2abx+b^2x^2}(d+ex)(bd-ae)}+\frac{b(a+bx)\log(a+bx)}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}-\frac{b(a+bx)\log(d+ex)}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

```
[Out] (a + b*x)/((b*d - a*e)*(d + e*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (b*(a + b
*x)*Log[a + b*x])/((b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (b*(a + b
*x)*Log[d + e*x])/((b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)^2\sqrt{a^2+2abx+b^2x^2}}\,dx &= \frac{\left(ab+b^2x\right)\int \frac{1}{\left(ab+b^2x\right)(d+ex)^2}\,dx}{\sqrt{a^2+2abx+b^2x^2}}\\ &= \frac{\left(ab+b^2x\right)\int \left(\frac{b}{(bd-ae)^2(a+bx)}-\frac{e}{b(bd-ae)(d+ex)^2}-\frac{e}{(bd-ae)^2(d+ex)}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}\\ &= \frac{a+bx}{(bd-ae)(d+ex)\sqrt{a^2+2abx+b^2x^2}}+\frac{b(a+bx)\log(a+bx)}{(bd-ae)^2\sqrt{a^2+2abx+b^2x^2}}-\frac{b(a+}{(bd-ae)^2}\end{aligned}$$

Mathematica [A] time = 0.0439544, size = 69, normalized size = 0.53

$$\frac{(a+bx)(b(d+ex)\log(a+bx)-ae-b(d+ex)\log(d+ex)+bd)}{\sqrt{(a+bx)^2}(d+ex)(bd-ae)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

```
[Out] ((a + b*x)*(b*d - a*e + b*(d + e*x)*Log[a + b*x] - b*(d + e*x)*Log[d + e*x]
))/((b*d - a*e)^2*Sqrt[(a + b*x)^2]*(d + e*x))
```

Maple [A] time = 0.162, size = 82, normalized size = 0.6

$$-\frac{(bx+a)(\ln(ex+d)xbe-\ln(bx+a)xbe+\ln(ex+d)bd-\ln(bx+a)bd+ae-bd)}{(ae-bd)^2(ex+d)}\frac{1}{\sqrt{(bx+a)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/((b*x+a)^2)^(1/2),x)
```

```
[Out] -(b*x+a)*(ln(e*x+d)*x*b*e-ln(b*x+a)*x*b*e+ln(e*x+d)*b*d-ln(b*x+a)*b*d+a*e-b
*d)/((b*x+a)^2)^(1/2)/(a*e-b*d)^2/(e*x+d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59596, size = 198, normalized size = 1.51

$$\frac{bd-ae+(bex+bd)\log(bx+a)-(bex+bd)\log(ex+d)}{b^2d^3-2\,abd^2e+a^2de^2+\left(b^2d^2e-2\,abde^2+a^2e^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (b*d - a*e + (b*e*x + b*d)*log(b*x + a) - (b*e*x + b*d)*log(e*x + d))/(b^2*
d^3 - 2*a*b*d^2*e + a^2*d*e^2 + (b^2*d^2*e - 2*a*b*d*e^2 + a^2*e^3)*x)
```

Sympy [B] time = 0.95312, size = 233, normalized size = 1.78

$$-\frac{b\log\left(x+\frac{-\frac{a^3be^3}{(ae-bd)^2}+\frac{3a^2b^2de^2}{(ae-bd)^2}-\frac{3ab^3d^2e}{(ae-bd)^2}+abe+\frac{b^4d^3}{(ae-bd)^2}+b^2d}{2b^2e}\right)}{(ae-bd)^2}+\frac{b\log\left(x+\frac{\frac{a^3be^3}{(ae-bd)^2}-\frac{3a^2b^2de^2}{(ae-bd)^2}+\frac{3ab^3d^2e}{(ae-bd)^2}+abe-\frac{b^4d^3}{(ae-bd)^2}+b^2d}{2b^2e}\right)}{(ae-bd)^2}-\frac{1}{ade-bd^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/((b*x+a)**2)**(1/2),x)
```

```
[Out] -b*log(x + (-a**3*b*e**3/(a*e - b*d)**2 + 3*a**2*b**2*d*e**2/(a*e - b*d)**2
 - 3*a*b**3*d**2*e/(a*e - b*d)**2 + a*b*e + b**4*d**3/(a*e - b*d)**2 + b**2
*d)/(2*b**2*e))/(a*e - b*d)**2 + b*log(x + (a**3*b*e**3/(a*e - b*d)**2 - 3*
a**2*b**2*d*e**2/(a*e - b*d)**2 + 3*a*b**3*d**2*e/(a*e - b*d)**2 + a*b*e -
b**4*d**3/(a*e - b*d)**2 + b**2*d)/(2*b**2*e))/(a*e - b*d)**2 - 1/(a*d*e -
b*d**2 + x*(a*e**2 - b*d*e))
```

Giac [A] time = 1.19021, size = 139, normalized size = 1.06

$$\left(\frac{b^2\log\left(|bx+a|\right)}{b^3d^2-2\,ab^2de+a^2be^2}-\frac{be\log\left(|xe+d|\right)}{b^2d^2e-2\,abde^2+a^2e^3}+\frac{1}{(bd-ae)(xe+d)}\right)\mathrm{sgn}\left(bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] (b^2*log(abs(b*x + a))/(b^3*d^2 - 2*a*b^2*d*e + a^2*b*e^2) - b*e*log(abs(x*
e + d))/(b^2*d^2*e - 2*a*b*d*e^2 + a^2*e^3) + 1/((b*d - a*e)*(x*e + d)))*sg
n(b*x + a)
```

3.1594 $\int \frac{1}{(d+ex)^3\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=182

$$\frac{b(a+bx)}{\sqrt{a^2+2abx+b^2x^2}(d+ex)(bd-ae)^2}+\frac{a+bx}{2\sqrt{a^2+2abx+b^2x^2}(d+ex)^2(bd-ae)}+\frac{b^2(a+bx)\log(a+bx)}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3}-\frac{b^2(a}{\sqrt{a^2+2}}$$

```
[Out] (a + b*x)/(2*(b*d - a*e)*(d + e*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (b*(a
 + b*x))/((b*d - a*e)^2*(d + e*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (b^2*(a
+ b*x)*Log[a + b*x])/((b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (b^2*(
a + b*x)*Log[d + e*x])/((b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0813618, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 44}

$$\frac{b(a+bx)}{\sqrt{a^2+2abx+b^2x^2}(d+ex)(bd-ae)^2}+\frac{a+bx}{2\sqrt{a^2+2abx+b^2x^2}(d+ex)^2(bd-ae)}+\frac{b^2(a+bx)\log(a+bx)}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3}-\frac{b^2(a}{\sqrt{a^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

```
[Out] (a + b*x)/(2*(b*d - a*e)*(d + e*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (b*(a
 + b*x))/((b*d - a*e)^2*(d + e*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (b^2*(a
+ b*x)*Log[a + b*x])/((b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (b^2*(
a + b*x)*Log[d + e*x])/((b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)^3\sqrt{a^2+2abx+b^2x^2}}\,dx &= \frac{\left(ab+b^2x\right)\int \frac{1}{\left(ab+b^2x\right)(d+ex)^3}\,dx}{\sqrt{a^2+2abx+b^2x^2}}\\
&= \frac{\left(ab+b^2x\right)\int \left(\frac{b^2}{(bd-ae)^3(a+bx)}-\frac{e}{b(bd-ae)(d+ex)^3}-\frac{e}{(bd-ae)^2(d+ex)^2}-\frac{be}{(bd-ae)^3(d+ex)}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}\\
&= \frac{a+bx}{2(bd-ae)(d+ex)^2\sqrt{a^2+2abx+b^2x^2}}+\frac{b(a+bx)}{(bd-ae)^2(d+ex)\sqrt{a^2+2abx+b^2x^2}}+\frac{}{(b}
\end{aligned}$$

Mathematica [A] time = 0.0650985, size = 97, normalized size = 0.53

$$\frac{(a+bx)\left(2b^2(d+ex)^2\log(a+bx)+(bd-ae)(-ae+3bd+2bex)-2b^2(d+ex)^2\log(d+ex)\right)}{2\sqrt{(a+bx)^2}(d+ex)^2(bd-ae)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

```
[Out] ((a + b*x)*((b*d - a*e)*(3*b*d - a*e + 2*b*e*x) + 2*b^2*(d + e*x)^2*Log[a +
 b*x] - 2*b^2*(d + e*x)^2*Log[d + e*x]))/(2*(b*d - a*e)^3*Sqrt[(a + b*x)^2]
*(d + e*x)^2)
```

Maple [A] time = 0.165, size = 163, normalized size = 0.9

$$\frac{(bx+a)\,(2\,\ln(ex+d)\,x^2b^2e^2-2\,\ln(bx+a)\,x^2b^2e^2+4\,\ln(ex+d)\,xb^2de-4\,\ln(bx+a)\,xb^2de+2\,\ln(ex+d)\,b^2d^2-2}{2\,(ae-bd)^3\,(ex+d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/((b*x+a)^2)^(1/2),x)
```

```
[Out] 1/2*(b*x+a)*(2*ln(e*x+d)*x^2*b^2*e^2-2*ln(b*x+a)*x^2*b^2*e^2+4*ln(e*x+d)*x*
b^2*d*e-4*ln(b*x+a)*x*b^2*d*e+2*ln(e*x+d)*b^2*d^2-2*ln(b*x+a)*b^2*d^2+2*x*a
*b*e^2-2*x*b^2*d*e-a^2*e^2+4*a*b*d*e-3*b^2*d^2)/((b*x+a)^2)^(1/2)/(a*e-b*d)
^3/(e*x+d)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.61354, size = 490, normalized size = 2.69

$$\frac{3\,b^2d^2-4\,abde+a^2e^2+2\,(b^2de-abe^2)x+2\,(b^2e^2x^2+2\,b^2dex+b^2d^2)\log(bx+a)-2\,(b^2e^2x^2+2\,b^2dex+b^2d^2)\log}{2\,(b^3d^5-3\,ab^2d^4e+3\,a^2bd^3e^2-a^3d^2e^3+(b^3d^3e^2-3\,ab^2d^2e^3+3\,a^2bde^4-a^3e^5)x^2+2\,(b^3d^4e-3\,ab^2d^3e^2+3\,a^2bd^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(3*b^2*d^2 - 4*a*b*d*e + a^2*e^2 + 2*(b^2*d*e - a*b*e^2)*x + 2*(b^2*e^2
*x^2 + 2*b^2*d*e*x + b^2*d^2)*log(b*x + a) - 2*(b^2*e^2*x^2 + 2*b^2*d*e*x +
 b^2*d^2)*log(e*x + d))/(b^3*d^5 - 3*a*b^2*d^4*e + 3*a^2*b*d^3*e^2 - a^3*d^
2*e^3 + (b^3*d^3*e^2 - 3*a*b^2*d^2*e^3 + 3*a^2*b*d*e^4 - a^3*e^5)*x^2 + 2*(
```

```
b^3*d^4*e - 3*a*b^2*d^3*e^2 + 3*a^2*b*d^2*e^3 - a^3*d*e^4)*x)
```

Sympy [B] time = 1.47254, size = 381, normalized size = 2.09

$$\frac{b^2 \log\left(x + \frac{-\frac{a^4b^2e^4}{(ae-bd)^3} + \frac{4a^3b^3de^3}{(ae-bd)^3} - \frac{6a^2b^4d^2e^2}{(ae-bd)^3} + \frac{4ab^5d^3e}{(ae-bd)^3} + ab^2e - \frac{b^6d^4}{(ae-bd)^3} + b^3d}{2b^3e}\right)}{(ae-bd)^3} - \frac{b^2 \log\left(x + \frac{\frac{a^4b^2e^4}{(ae-bd)^3} - \frac{4a^3b^3de^3}{(ae-bd)^3} + \frac{6a^2b^4d^2e^2}{(ae-bd)^3} - \frac{4ab^5d^3e}{(ae-bd)^3} + ab^2e + \frac{b^6d^4}{(ae-bd)^3} + b^3d}{2b^3e}\right)}{(ae-bd)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/((b*x+a)**2)**(1/2),x)
```

```
[Out] b**2*log(x + (-a**4*b**2*e**4/(a*e - b*d)**3 + 4*a**3*b**3*d*e**3/(a*e - b*
d)**3 - 6*a**2*b**4*d**2*e**2/(a*e - b*d)**3 + 4*a*b**5*d**3*e/(a*e - b*d)*
*3 + a*b**2*e - b**6*d**4/(a*e - b*d)**3 + b**3*d)/(2*b**3*e))/(a*e - b*d)*
*3 - b**2*log(x + (a**4*b**2*e**4/(a*e - b*d)**3 - 4*a**3*b**3*d*e**3/(a*e
- b*d)**3 + 6*a**2*b**4*d**2*e**2/(a*e - b*d)**3 - 4*a*b**5*d**3*e/(a*e - b
*d)**3 + a*b**2*e + b**6*d**4/(a*e - b*d)**3 + b**3*d)/(2*b**3*e))/(a*e - b
*d)**3 + (-a*e + 3*b*d + 2*b*e*x)/(2*a**2*d**2*e**2 - 4*a*b*d**3*e + 2*b**2
*d**4 + x**2*(2*a**2*e**4 - 4*a*b*d*e**3 + 2*b**2*d**2*e**2) + x*(4*a**2*d*
e**3 - 8*a*b*d**2*e**2 + 4*b**2*d**3*e))
```

Giac [A] time = 1.16502, size = 235, normalized size = 1.29

$$\frac{1}{2}\left(\frac{2\,b^3\log\left(|bx+a|\right)}{b^4d^3 - 3\,ab^3d^2e + 3\,a^2b^2de^2 - a^3be^3} - \frac{2\,b^2e\log\left(|xe+d|\right)}{b^3d^3e - 3\,ab^2d^2e^2 + 3\,a^2bde^3 - a^3e^4} + \frac{3\,b^2d^2 - 4\,abde + a^2e^2 + 2\left(b^2de - abe^2\right)x}{(bd-ae)^3(xe+d)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*log(abs(b*x + a))/(b^4*d^3 - 3*a*b^3*d^2*e + 3*a^2*b^2*d*e^2 - a
^3*b*e^3) - 2*b^2*e*log(abs(x*e + d))/(b^3*d^3*e - 3*a*b^2*d^2*e^2 + 3*a^2*
b*d*e^3 - a^3*e^4) + (3*b^2*d^2 - 4*a*b*d*e + a^2*e^2 + 2*(b^2*d*e - a*b*e^
2)*x)/((b*d - a*e)^3*(x*e + d)^2))*sgn(b*x + a)
```

3.1595 $\int \frac{1}{(d+ex)^4\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=231

$$\frac{b^2(a+bx)}{\sqrt{a^2+2abx+b^2x^2}(d+ex)(bd-ae)^3}+\frac{b(a+bx)}{2\sqrt{a^2+2abx+b^2x^2}(d+ex)^2(bd-ae)^2}+\frac{a+bx}{3\sqrt{a^2+2abx+b^2x^2}(d+ex)^3(bd-\cdots}$$

```
[Out] (a + b*x)/(3*(b*d - a*e)*(d + e*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (b*(a
 + b*x))/(2*(b*d - a*e)^2*(d + e*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (b^2
*(a + b*x))/((b*d - a*e)^3*(d + e*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (b^3*
(a + b*x)*Log[a + b*x])/((b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (b^
3*(a + b*x)*Log[d + e*x])/((b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.102261, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 44}

$$\frac{b^2(a+bx)}{\sqrt{a^2+2abx+b^2x^2}(d+ex)(bd-ae)^3}+\frac{b(a+bx)}{2\sqrt{a^2+2abx+b^2x^2}(d+ex)^2(bd-ae)^2}+\frac{a+bx}{3\sqrt{a^2+2abx+b^2x^2}(d+ex)^3(bd-\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```


```
[Out] (a + b*x)/(3*(b*d - a*e)*(d + e*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (b*(a
 + b*x))/(2*(b*d - a*e)^2*(d + e*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (b^2
*(a + b*x))/((b*d - a*e)^3*(d + e*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (b^3*
(a + b*x)*Log[a + b*x])/((b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (b^
3*(a + b*x)*Log[d + e*x])/((b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)^4\sqrt{a^2+2abx+b^2x^2}}\,dx &= \frac{\left(ab+b^2x\right)\int \frac{1}{\left(ab+b^2x\right)(d+ex)^4}\,dx}{\sqrt{a^2+2abx+b^2x^2}}\\
&= \frac{\left(ab+b^2x\right)\int \left(\frac{b^3}{(bd-ae)^4(a+bx)}-\frac{e}{b(bd-ae)(d+ex)^4}-\frac{e}{(bd-ae)^2(d+ex)^3}-\frac{be}{(bd-ae)^3(d+ex)^2}-\frac{\cdots}{(bd-ae\cdots}\right.}{\sqrt{a^2+2abx+b^2x^2}}\\
&= \frac{a+bx}{3(bd-ae)(d+ex)^3\sqrt{a^2+2abx+b^2x^2}}+\frac{b(a+bx)}{2(bd-ae)^2(d+ex)^2\sqrt{a^2+2abx+b^2x^2}}\cdots
\end{aligned}$$

Mathematica [A] time = 0.0749858, size = 124, normalized size = 0.54

$$\frac{(a+bx)\left(6b^2(d+ex)^2(bd-ae)+6b^3(d+ex)^3\log(a+bx)+3b(d+ex)(bd-ae)^2+2(bd-ae)^3-6b^3(d+ex)^3\log(d+ex)\right.}{6\sqrt{(a+bx)^2}(d+ex)^3(bd-ae)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

```
[Out] ((a + b*x)*(2*(b*d - a*e)^3 + 3*b*(b*d - a*e)^2*(d + e*x) + 6*b^2*(b*d - a*
e)*(d + e*x)^2 + 6*b^3*(d + e*x)^3*Log[a + b*x] - 6*b^3*(d + e*x)^3*Log[d +
 e*x]))/(6*(b*d - a*e)^4*Sqrt[(a + b*x)^2]*(d + e*x)^3)
```

Maple [A] time = 0.165, size = 256, normalized size = 1.1

$$-\frac{(bx+a)\left(6\ln(ex+d)x^3b^3e^3-6\ln(bx+a)x^3b^3e^3+18\ln(ex+d)x^2b^3de^2-18\ln(bx+a)x^2b^3de^2+18\ln(ex+d)xb\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^4/((b*x+a)^2)^(1/2),x)
```

```
[Out] -1/6*(b*x+a)*(6*ln(e*x+d)*x^3*b^3*e^3-6*ln(b*x+a)*x^3*b^3*e^3+18*ln(e*x+d)*
x^2*b^3*d*e^2-18*ln(b*x+a)*x^2*b^3*d*e^2+18*ln(e*x+d)*x*b^3*d^2*e-18*ln(b*x
+a)*x*b^3*d^2*e+6*x^2*a*b^2*e^3-6*x^2*b^3*d*e^2+6*ln(e*x+d)*b^3*d^3-6*ln(b*
x+a)*b^3*d^3-3*x*a^2*b*e^3+18*x*a*b^2*d*e^2-15*x*b^3*d^2*e+2*a^3*e^3-9*d*e^
2*a^2*b+18*a*b^2*d^2*e-11*b^3*d^3)/((b*x+a)^2)^(1/2)/(a*e-b*d)^4/(e*x+d)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.62372, size = 855, normalized size = 3.7

$$\frac{11\,b^3d^3-18\,ab^2d^2e+9\,a^2bde^2-2\,a^3e^3+6\left(b^3de^2-ab^2e^3\right)x^2+3\left(5\,b^3d^2e-6\,ab^2de^2+a^2be^3\right)x+6\left(b^3e^3x^3+\right.}{6\left(b^4d^7-4\,ab^3d^6e+6\,a^2b^2d^5e^2-4\,a^3bd^4e^3+a^4d^3e^4+\left(b^4d^4e^3-4\,ab^3d^3e^4+6\,a^2b^2d^2e^5-4\,a^3bde^6+a^4e^7\right)x^3+3\left(b^4d^5e^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(11*b^3*d^3 - 18*a*b^2*d^2*e + 9*a^2*b*d*e^2 - 2*a^3*e^3 + 6*(b^3*d*e^2
 - a*b^2*e^3)*x^2 + 3*(5*b^3*d^2*e - 6*a*b^2*d*e^2 + a^2*b*e^3)*x + 6*(b^3*
e^3*x^3 + 3*b^3*d*e^2*x^2 + 3*b^3*d^2*e*x + b^3*d^3)*log(b*x + a) - 6*(b^3*
```

```
e^3*x^3 + 3*b^3*d*e^2*x^2 + 3*b^3*d^2*e*x + b^3*d^3)*log(e*x + d))/(b^4*d^7
 - 4*a*b^3*d^6*e + 6*a^2*b^2*d^5*e^2 - 4*a^3*b*d^4*e^3 + a^4*d^3*e^4 + (b^4
*d^4*e^3 - 4*a*b^3*d^3*e^4 + 6*a^2*b^2*d^2*e^5 - 4*a^3*b*d*e^6 + a^4*e^7)*x
^3 + 3*(b^4*d^5*e^2 - 4*a*b^3*d^4*e^3 + 6*a^2*b^2*d^3*e^4 - 4*a^3*b*d^2*e^5
 + a^4*d*e^6)*x^2 + 3*(b^4*d^6*e - 4*a*b^3*d^5*e^2 + 6*a^2*b^2*d^4*e^3 - 4*
a^3*b*d^3*e^4 + a^4*d^2*e^5)*x)
```

Sympy [B] time = 2.15661, size = 570, normalized size = 2.47

$$-\frac{b^3\log\left(x+\frac{-\frac{a^5b^3e^5}{(ae-bd)^4}+\frac{5a^4b^4de^4}{(ae-bd)^4}-\frac{10a^3b^5d^2e^3}{(ae-bd)^4}+\frac{10a^2b^6d^3e^2}{(ae-bd)^4}-\frac{5ab^7d^4e}{(ae-bd)^4}+ab^3e+\frac{b^8d^5}{(ae-bd)^4}+b^4d}{2b^4e}\right)}{(ae-bd)^4}+\frac{b^3\log\left(x+\frac{\frac{a^5b^3e^5}{(ae-bd)^4}-\frac{5a^4b^4de^4}{(ae-bd)^4}+\frac{10a^3b^5d^2e^3}{(ae-bd)^4}-\frac{10a^2b^6d^3e^2}{(ae-bd)^4}}{2b^4e}\right)}{(ae-bd)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**4/((b*x+a)**2)**(1/2),x)
```

```
[Out] -b**3*log(x + (-a**5*b**3*e**5/(a*e - b*d)**4 + 5*a**4*b**4*d*e**4/(a*e - b
*d)**4 - 10*a**3*b**5*d**2*e**3/(a*e - b*d)**4 + 10*a**2*b**6*d**3*e**2/(a*
e - b*d)**4 - 5*a*b**7*d**4*e/(a*e - b*d)**4 + a*b**3*e + b**8*d**5/(a*e -
b*d)**4 + b**4*d)/(2*b**4*e))/(a*e - b*d)**4 + b**3*log(x + (a**5*b**3*e**5
/(a*e - b*d)**4 - 5*a**4*b**4*d*e**4/(a*e - b*d)**4 + 10*a**3*b**5*d**2*e**
3/(a*e - b*d)**4 - 10*a**2*b**6*d**3*e**2/(a*e - b*d)**4 + 5*a*b**7*d**4*e/
(a*e - b*d)**4 + a*b**3*e - b**8*d**5/(a*e - b*d)**4 + b**4*d)/(2*b**4*e))/
(a*e - b*d)**4 - (2*a**2*e**2 - 7*a*b*d*e + 11*b**2*d**2 + 6*b**2*e**2*x**2
 + x*(-3*a*b*e**2 + 15*b**2*d*e))/(6*a**3*d**3*e**3 - 18*a**2*b*d**4*e**2 +
 18*a*b**2*d**5*e - 6*b**3*d**6 + x**3*(6*a**3*e**6 - 18*a**2*b*d*e**5 + 18
*a*b**2*d**2*e**4 - 6*b**3*d**3*e**3) + x**2*(18*a**3*d*e**5 - 54*a**2*b*d*
*2*e**4 + 54*a*b**2*d**3*e**3 - 18*b**3*d**4*e**2) + x*(18*a**3*d**2*e**4 -
 54*a**2*b*d**3*e**3 + 54*a*b**2*d**4*e**2 - 18*b**3*d**5*e))
```

Giac [A] time = 1.1684, size = 332, normalized size = 1.44

$$\frac{1}{6}\left(\frac{6\,b^4\log\left(|bx+a|\right)}{b^5d^4-4\,ab^4d^3e+6\,a^2b^3d^2e^2-4\,a^3b^2de^3+a^4be^4}-\frac{6\,b^3e\log\left(|xe+d|\right)}{b^4d^4e-4\,ab^3d^3e^2+6\,a^2b^2d^2e^3-4\,a^3bde^4+a^4e^5}+\frac{11\,b^3d^3-18}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(6*b^4*log(abs(b*x + a))/(b^5*d^4 - 4*a*b^4*d^3*e + 6*a^2*b^3*d^2*e^2 -
 4*a^3*b^2*d*e^3 + a^4*b*e^4) - 6*b^3*e*log(abs(x*e + d))/(b^4*d^4*e - 4*a*
b^3*d^3*e^2 + 6*a^2*b^2*d^2*e^3 - 4*a^3*b*d*e^4 + a^4*e^5) + (11*b^3*d^3 -
18*a*b^2*d^2*e + 9*a^2*b*d*e^2 - 2*a^3*e^3 + 6*(b^3*d*e^2 - a*b^2*e^3)*x^2
+ 3*(5*b^3*d^2*e - 6*a*b^2*d*e^2 + a^2*b*e^3)*x)/((b*d - a*e)^4*(x*e + d)^3
))*sgn(b*x + a)
```

3.1596 $\int \frac{(d+ex)^4}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=210

$$\frac{e^3x(a+bx)(4bd-3ae)}{b^4\sqrt{a^2+2abx+b^2x^2}}+\frac{6e^2(a+bx)(bd-ae)^2\log(a+bx)}{b^5\sqrt{a^2+2abx+b^2x^2}}-\frac{4e(bd-ae)^3}{b^5\sqrt{a^2+2abx+b^2x^2}}-\frac{(bd-ae)^4}{2b^5(a+bx)\sqrt{a^2+2abx+b^2x^2}}+$$

```
[Out] (-4*e*(b*d - a*e)^3)/(b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (b*d - a*e)^4/(2
*b^5*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (e^3*(4*b*d - 3*a*e)*x*(a +
 b*x))/(b^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (e^4*x^2*(a + b*x))/(2*b^3*Sqr
t[a^2 + 2*a*b*x + b^2*x^2]) + (6*e^2*(b*d - a*e)^2*(a + b*x)*Log[a + b*x])/
(b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.13473, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$\frac{e^3x(a+bx)(4bd-3ae)}{b^4\sqrt{a^2+2abx+b^2x^2}}+\frac{6e^2(a+bx)(bd-ae)^2\log(a+bx)}{b^5\sqrt{a^2+2abx+b^2x^2}}-\frac{4e(bd-ae)^3}{b^5\sqrt{a^2+2abx+b^2x^2}}-\frac{(bd-ae)^4}{2b^5(a+bx)\sqrt{a^2+2abx+b^2x^2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```


```
[Out] (-4*e*(b*d - a*e)^3)/(b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (b*d - a*e)^4/(2
*b^5*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (e^3*(4*b*d - 3*a*e)*x*(a +
 b*x))/(b^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (e^4*x^2*(a + b*x))/(2*b^3*Sqr
t[a^2 + 2*a*b*x + b^2*x^2]) + (6*e^2*(b*d - a*e)^2*(a + b*x)*Log[a + b*x])/
(b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(d+ex)^4}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x\right)\right)\int \frac{(d+ex)^4}{\left(ab+b^2x\right)^3}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= \frac{\left(b^2\left(ab+b^2x\right)\right)\int \left(\frac{e^3(4bd-3ae)}{b^7}+\frac{e^4x}{b^6}+\frac{(bd-ae)^4}{b^7(a+bx)^3}+\frac{4e(bd-ae)^3}{b^7(a+bx)^2}+\frac{6e^2(bd-ae)^2}{b^7(a+bx)}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{4e(bd-ae)^3}{b^5\sqrt{a^2+2abx+b^2x^2}} - \frac{(bd-ae)^4}{2b^5(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{e^3(4bd-3ae)x(a+bx)}{b^4\sqrt{a^2+2abx+b^2x^2}} + -$$

Mathematica [A] time = 0.0843384, size = 174, normalized size = 0.83

$$\frac{a^2b^2e^2\left(18d^2-16dex-11e^2x^2\right)+2a^3be^3(ex-10d)+7a^4e^4-4ab^3e\left(-6d^2ex+d^3-4de^2x^2+e^3x^3\right)+12e^2(a+bx)^2(bd}{2b^5(a+bx)\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (7*a^4*e^4 + 2*a^3*b*e^3*(-10*d + e*x) + a^2*b^2*e^2*(18*d^2 - 16*d*e*x - 1
1*e^2*x^2) - 4*a*b^3*e*(d^3 - 6*d^2*e*x - 4*d*e^2*x^2 + e^3*x^3) + b^4*(-d^
4 - 8*d^3*e*x + 8*d*e^3*x^3 + e^4*x^4) + 12*e^2*(b*d - a*e)^2*(a + b*x)^2*L
og[a + b*x])/(2*b^5*(a + b*x)*Sqrt[(a + b*x)^2])
```

Maple [B] time = 0.204, size = 341, normalized size = 1.6

$$\frac{\left(x^4b^4e^4+12\ln(bx+a)x^2a^2b^2e^4-24\ln(bx+a)x^2ab^3de^3+12\ln(bx+a)x^2b^4d^2e^2-4x^3ab^3e^4+8x^3b^4de^3+24\ln(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] 1/2*(x^4*b^4*e^4+12*ln(b*x+a)*x^2*a^2*b^2*e^4-24*ln(b*x+a)*x^2*a*b^3*d*e^3+
12*ln(b*x+a)*x^2*b^4*d^2*e^2-4*x^3*a*b^3*e^4+8*x^3*b^4*d*e^3+24*ln(b*x+a)*x
*a^3*b*e^4-48*ln(b*x+a)*x*a^2*b^2*d*e^3+24*ln(b*x+a)*x*a*b^3*d^2*e^2-11*x^2
*a^2*b^2*e^4+16*x^2*a*b^3*d*e^3+12*ln(b*x+a)*a^4*e^4-24*ln(b*x+a)*a^3*b*d*e
^3+12*ln(b*x+a)*a^2*b^2*d^2*e^2+2*x*a^3*b*e^4-16*x*a^2*b^2*d*e^3+24*x*a*b^3
*d^2*e^2-8*x*b^4*d^3*e+7*a^4*e^4-20*a^3*b*d*e^3+18*d^2*e^2*a^2*b^2-4*a*b^3*
d^3*e-b^4*d^4)*(b*x+a)/b^5/((b*x+a)^2)^(3/2)
```

Maxima [B] time = 1.08357, size = 657, normalized size = 3.13

$$\frac{e^4x^3}{2\sqrt{b^2x^2+2abx+a^2}b^2} + \frac{4de^3x^2}{\sqrt{b^2x^2+2abx+a^2}b^2} - \frac{5ae^4x^2}{2\sqrt{b^2x^2+2abx+a^2}b^3} + \frac{6d^2e^2\log\left(x+\frac{a}{b}\right)}{\left(b^2\right)^{\frac{3}{2}}} - \frac{12ade^3\log\left(x+\frac{a}{b}\right)}{\left(b^2\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*e^4*x^3/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^2) + 4*d*e^3*x^2/(sqrt(b^2*x^2
 + 2*a*b*x + a^2)*b^2) - 5/2*a*e^4*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^3)
+ 6*d^2*e^2*log(x + a/b)/(b^2)^(3/2) - 12*a*d*e^3*log(x + a/b)/((b^2)^(3/2)
*b) + 6*a^2*e^4*log(x + a/b)/((b^2)^(3/2)*b^2) + 9*a^2*b^2*d^2*e^2/((b^2)^(
7/2)*(x + a/b)^2) - 18*a^3*b*d*e^3/((b^2)^(7/2)*(x + a/b)^2) + 9*a^4*e^4/((
b^2)^(7/2)*(x + a/b)^2) + 12*a*b*d^2*e^2*x/((b^2)^(5/2)*(x + a/b)^2) - 24*a
^2*d*e^3*x/((b^2)^(5/2)*(x + a/b)^2) + 12*a^3*e^4*x/((b^2)^(5/2)*b*(x + a/b
)^2) - 4*d^3*e/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^2) + 8*a^2*d*e^3/(sqrt(b^2*
x^2 + 2*a*b*x + a^2)*b^4) - 5*a^3*e^4/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^5) -
 1/2*d^4/((b^2)^(3/2)*(x + a/b)^2) + 2*a*d^3*e/((b^2)^(3/2)*b*(x + a/b)^2)
- 4*a^3*d*e^3/((b^2)^(3/2)*b^3*(x + a/b)^2) + 5/2*a^4*e^4/((b^2)^(3/2)*b^4*
(x + a/b)^2)
```

Fricas [A] time = 1.58009, size = 586, normalized size = 2.79

$$\frac{b^4e^4x^4 - b^4d^4 - 4\,ab^3d^3e + 18\,a^2b^2d^2e^2 - 20\,a^3bde^3 + 7\,a^4e^4 + 4\left(2\,b^4de^3 - ab^3e^4\right)x^3 + \left(16\,ab^3de^3 - 11\,a^2b^2e^4\right)x^2 - 2\left(4\,b^4 \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*e^4*x^4 - b^4*d^4 - 4*a*b^3*d^3*e + 18*a^2*b^2*d^2*e^2 - 20*a^3*b*
d*e^3 + 7*a^4*e^4 + 4*(2*b^4*d*e^3 - a*b^3*e^4)*x^3 + (16*a*b^3*d*e^3 - 11*
a^2*b^2*e^4)*x^2 - 2*(4*b^4*d^3*e - 12*a*b^3*d^2*e^2 + 8*a^2*b^2*d*e^3 - a^
3*b*e^4)*x + 12*(a^2*b^2*d^2*e^2 - 2*a^3*b*d*e^3 + a^4*e^4 + (b^4*d^2*e^2 -
 2*a*b^3*d*e^3 + a^2*b^2*e^4)*x^2 + 2*(a*b^3*d^2*e^2 - 2*a^2*b^2*d*e^3 + a^
3*b*e^4)*x)*log(b*x + a))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^4}{\left((a+bx)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x)**4/((a + b*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.1597 $\int \frac{(d+ex)^3}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=161

$$\frac{3e^2(a+bx)(bd-ae)\log(a+bx)}{b^4\sqrt{a^2+2abx+b^2x^2}}-\frac{3e(bd-ae)^2}{b^4\sqrt{a^2+2abx+b^2x^2}}-\frac{(bd-ae)^3}{2b^4(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{e^3x(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}}$$

```
[Out] (-3*e*(b*d - a*e)^2)/(b^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (b*d - a*e)^3/(2
*b^4*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (e^3*x*(a + b*x))/(b^3*Sqrt
[a^2 + 2*a*b*x + b^2*x^2]) + (3*e^2*(b*d - a*e)*(a + b*x)*Log[a + b*x])/(b^
4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0940398, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$\frac{3e^2(a+bx)(bd-ae)\log(a+bx)}{b^4\sqrt{a^2+2abx+b^2x^2}}-\frac{3e(bd-ae)^2}{b^4\sqrt{a^2+2abx+b^2x^2}}-\frac{(bd-ae)^3}{2b^4(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{e^3x(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```


```
[Out] (-3*e*(b*d - a*e)^2)/(b^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (b*d - a*e)^3/(2
*b^4*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (e^3*x*(a + b*x))/(b^3*Sqrt
[a^2 + 2*a*b*x + b^2*x^2]) + (3*e^2*(b*d - a*e)*(a + b*x)*Log[a + b*x])/(b^
4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^3}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx &= \frac{\left(b^2\left(ab+b^2x\right)\right)\int \frac{(d+ex)^3}{\left(ab+b^2x\right)^3}\,dx}{\sqrt{a^2+2abx+b^2x^2}}\\ &= \frac{\left(b^2\left(ab+b^2x\right)\right)\int \left(\frac{e^3}{b^6}+\frac{(bd-ae)^3}{b^6(a+bx)^3}+\frac{3e(bd-ae)^2}{b^6(a+bx)^2}+\frac{3e^2(bd-ae)}{b^6(a+bx)}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}\\ &= -\frac{3e(bd-ae)^2}{b^4\sqrt{a^2+2abx+b^2x^2}}-\frac{(bd-ae)^3}{2b^4(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{e^3x(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}}+\frac{3e^2}{}\end{aligned}$$

Mathematica [A] time = 0.056599, size = 125, normalized size = 0.78

$$\frac{a^2be^2(9d-4ex)-5a^3e^3+ab^2e\left(-3d^2+12dex+4e^2x^2\right)-6e^2(a+bx)^2(ae-bd)\log(a+bx)+b^3\left(-\left(6d^2ex+d^3-2e^3x^3\right)\right)}{2b^4(a+bx)\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (-5*a^3*e^3 + a^2*b*e^2*(9*d - 4*e*x) + a*b^2*e*(-3*d^2 + 12*d*e*x + 4*e^2*
x^2) - b^3*(d^3 + 6*d^2*e*x - 2*e^3*x^3) - 6*e^2*(-(b*d) + a*e)*(a + b*x)^2
*Log[a + b*x])/(2*b^4*(a + b*x)*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.2, size = 209, normalized size = 1.3

$$-\frac{\left(6\ln(bx+a)x^2ab^2e^3-6\ln(bx+a)x^2b^3de^2-2x^3b^3e^3+12\ln(bx+a)xa^2be^3-12\ln(bx+a)xab^2de^2-4x^2ab^2e^3+\cdots\right.}{2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] -1/2*(6*ln(b*x+a)*x^2*a*b^2*e^3-6*ln(b*x+a)*x^2*b^3*d*e^2-2*x^3*b^3*e^3+12*
ln(b*x+a)*x*a^2*b*e^3-12*ln(b*x+a)*x*a*b^2*d*e^2-4*x^2*a*b^2*e^3+6*ln(b*x+a
)*a^3*e^3-6*ln(b*x+a)*a^2*b*d*e^2+4*x*a^2*b*e^3-12*x*a*b^2*d*e^2+6*x*b^3*d^
2*e+5*a^3*e^3-9*d*e^2*a^2*b+3*a*b^2*d^2*e+b^3*d^3)*(b*x+a)/b^4/((b*x+a)^2)^
(3/2)
```

Maxima [B] time = 1.05174, size = 390, normalized size = 2.42

$$\frac{e^3x^2}{\sqrt{b^2x^2+2\,abx+a^2b^2}}+\frac{3\,de^2\log\left(x+\frac{a}{b}\right)}{\left(b^2\right)^{\frac{3}{2}}}-\frac{3\,ae^3\log\left(x+\frac{a}{b}\right)}{\left(b^2\right)^{\frac{3}{2}}b}+\frac{9\,a^2b^2de^2}{2\left(b^2\right)^{\frac{7}{2}}\left(x+\frac{a}{b}\right)^2}-\frac{9\,a^3be^3}{2\left(b^2\right)^{\frac{7}{2}}\left(x+\frac{a}{b}\right)^2}+\frac{6\,abde^2x}{\left(b^2\right)^{\frac{5}{2}}\left(x+\frac{a}{b}\right)^2}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] e^3*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^2) + 3*d*e^2*log(x + a/b)/(b^2)^(3
/2) - 3*a*e^3*log(x + a/b)/((b^2)^(3/2)*b) + 9/2*a^2*b^2*d*e^2/((b^2)^(7/2)
*(x + a/b)^2) - 9/2*a^3*b*e^3/((b^2)^(7/2)*(x + a/b)^2) + 6*a*b*d*e^2*x/((b
^2)^(5/2)*(x + a/b)^2) - 6*a^2*e^3*x/((b^2)^(5/2)*(x + a/b)^2) - 3*d^2*e/(s
qrt(b^2*x^2 + 2*a*b*x + a^2)*b^2) + 2*a^2*e^3/(sqrt(b^2*x^2 + 2*a*b*x + a^2
)*b^4) - 1/2*d^3/((b^2)^(3/2)*(x + a/b)^2) + 3/2*a*d^2*e/((b^2)^(3/2)*b*(x
+ a/b)^2) - a^3*e^3/((b^2)^(3/2)*b^3*(x + a/b)^2)
```

Fricas [A] time = 1.6106, size = 375, normalized size = 2.33

$$\frac{2\,b^3e^3x^3+4\,ab^2e^3x^2-b^3d^3-3\,ab^2d^2e+9\,a^2bde^2-5\,a^3e^3-2\left(3\,b^3d^2e-6\,ab^2de^2+2\,a^2be^3\right)x+6\left(a^2bde^2-a^3e^3+\left(b^3de^2\cdots\right.\right.}{2\left(b^6x^2+2\,ab^5x+a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*e^3*x^3 + 4*a*b^2*e^3*x^2 - b^3*d^3 - 3*a*b^2*d^2*e + 9*a^2*b*d*
e^2 - 5*a^3*e^3 - 2*(3*b^3*d^2*e - 6*a*b^2*d*e^2 + 2*a^2*b*e^3)*x + 6*(a^2*
b*d*e^2 - a^3*e^3 + (b^3*d*e^2 - a*b^2*e^3)*x^2 + 2*(a*b^2*d*e^2 - a^2*b*e^
3)*x)*log(b*x + a))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^3}{\left((a+bx)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x)**3/((a + b*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.1598 $\int \frac{(d+ex)^2}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=117

$$-\frac{2e(bd-ae)}{b^3\sqrt{a^2+2abx+b^2x^2}}-\frac{(bd-ae)^2}{2b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{e^2(a+bx)\log(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}}$$

```
[Out] (-2*e*(b*d - a*e))/(b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (b*d - a*e)^2/(2*b
^3*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (e^2*(a + b*x)*Log[a + b*x])/
(b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0688655, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$-\frac{2e(bd-ae)}{b^3\sqrt{a^2+2abx+b^2x^2}}-\frac{(bd-ae)^2}{2b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{e^2(a+bx)\log(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (-2*e*(b*d - a*e))/(b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (b*d - a*e)^2/(2*b
^3*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (e^2*(a + b*x)*Log[a + b*x])/
(b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx &= \frac{\left(b^2\left(ab+b^2x\right)\right)\int \frac{(d+ex)^2}{\left(ab+b^2x\right)^3}\,dx}{\sqrt{a^2+2abx+b^2x^2}}\\
&= \frac{\left(b^2\left(ab+b^2x\right)\right)\int \left(\frac{(bd-ae)^2}{b^5(a+bx)^3}+\frac{2e(bd-ae)}{b^5(a+bx)^2}+\frac{e^2}{b^5(a+bx)}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}\\
&= -\frac{2e(bd-ae)}{b^3\sqrt{a^2+2abx+b^2x^2}}-\frac{(bd-ae)^2}{2b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{e^2(a+bx)\log(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.0324117, size = 67, normalized size = 0.57

$$\frac{2e^2(a+bx)^2\log(a+bx)-(bd-ae)(3ae+b(d+4ex))}{2b^3(a+bx)\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (-((b*d - a*e)*(3*a*e + b*(d + 4*e*x))) + 2*e^2*(a + b*x)^2*Log[a + b*x])/(
2*b^3*(a + b*x)*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.201, size = 104, normalized size = 0.9

$$\frac{\left(2\ln(bx+a)x^2b^2e^2+4\ln(bx+a)xabe^2+2\ln(bx+a)a^2e^2+4xabe^2-4xb^2de+3a^2e^2-2abde-b^2d^2\right)(bx+a)}{2b^3} ($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] 1/2*(2*ln(b*x+a)*x^2*b^2*e^2+4*ln(b*x+a)*x*a*b*e^2+2*ln(b*x+a)*a^2*e^2+4*x*
a*b*e^2-4*x*b^2*d*e+3*a^2*e^2-2*a*b*d*e-b^2*d^2)*(b*x+a)/b^3/((b*x+a)^2)^(3
/2)
```

Maxima [A] time = 1.089, size = 176, normalized size = 1.5

$$\frac{e^2\log\left(x+\frac{a}{b}\right)}{\left(b^2\right)^{\frac{3}{2}}}+\frac{3a^2b^2e^2}{2\left(b^2\right)^{\frac{7}{2}}\left(x+\frac{a}{b}\right)^2}+\frac{2abe^2x}{\left(b^2\right)^{\frac{5}{2}}\left(x+\frac{a}{b}\right)^2}-\frac{2de}{\sqrt{b^2x^2+2abx+a^2}b^2}-\frac{d^2}{2\left(b^2\right)^{\frac{3}{2}}\left(x+\frac{a}{b}\right)^2}+\frac{ade}{\left(b^2\right)^{\frac{3}{2}}b\left(x+\frac{a}{b}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] e^2*log(x + a/b)/(b^2)^(3/2) + 3/2*a^2*b^2*e^2/((b^2)^(7/2)*(x + a/b)^2) +
2*a*b*e^2*x/((b^2)^(5/2)*(x + a/b)^2) - 2*d*e/(sqrt(b^2*x^2 + 2*a*b*x + a^2
)*b^2) - 1/2*d^2/((b^2)^(3/2)*(x + a/b)^2) + a*d*e/((b^2)^(3/2)*b*(x + a/b)
^2)
```

Fricas [A] time = 1.53551, size = 207, normalized size = 1.77

$$-\frac{b^2d^2+2abde-3a^2e^2+4\left(b^2de-abe^2\right)x-2\left(b^2e^2x^2+2abe^2x+a^2e^2\right)\log(bx+a)}{2\left(b^5x^2+2ab^4x+a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*d^2 + 2*a*b*d*e - 3*a^2*e^2 + 4*(b^2*d*e - a*b*e^2)*x - 2*(b^2*e^
2*x^2 + 2*a*b*e^2*x + a^2*e^2)*log(b*x + a))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3
```

)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^2}{\left((a+bx)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x)**2/((a + b*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.1599 $\int \frac{d+ex}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=69

$$-\frac{bd-ae}{2b^2(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{e}{b^2\sqrt{a^2+2abx+b^2x^2}}$$

```
[Out] -(e/(b^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])) - (b*d - a*e)/(2*b^2*(a + b*x)*Sqr
t[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0218046, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {640, 607}

$$-\frac{bd-ae}{2b^2(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{e}{b^2\sqrt{a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] -(e/(b^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])) - (b*d - a*e)/(2*b^2*(a + b*x)*Sqr
t[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx &= -\frac{e}{b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{\left(2b^2d-2abe\right)\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx}{2b^2}\\ &= -\frac{e}{b^2\sqrt{a^2+2abx+b^2x^2}}-\frac{bd-ae}{2b^2(a+bx)\sqrt{a^2+2abx+b^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.014965, size = 39, normalized size = 0.57

$$\frac{-ae-b(d+2ex)}{2b^2(a+bx)\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (-(a*e) - b*(d + 2*e*x))/(2*b^2*(a + b*x)*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.154, size = 32, normalized size = 0.5

$$-\frac{(bx+a)(2bxe+ae+bd)}{2b^2}\left((bx+a)^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] -1/2*(b*x+a)*(2*b*e*x+a*e+b*d)/b^2/((b*x+a)^2)^(3/2)
```

Maxima [A] time = 1.01296, size = 85, normalized size = 1.23

$$-\frac{e}{\sqrt{b^2x^2+2abx+a^2}b^2}-\frac{d}{2\left(b^2\right)^{\frac{3}{2}}\left(x+\frac{a}{b}\right)^2}+\frac{ae}{2\left(b^2\right)^{\frac{3}{2}}b\left(x+\frac{a}{b}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -e/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^2) - 1/2*d/((b^2)^(3/2)*(x + a/b)^2) +
1/2*a*e/((b^2)^(3/2)*b*(x + a/b)^2)
```

Fricas [A] time = 1.53212, size = 81, normalized size = 1.17

$$-\frac{2bex+bd+ae}{2\left(b^4x^2+2ab^3x+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*e*x + b*d + a*e)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{d+ex}{\left((a+bx)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x)/((a + b*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.1600 $\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}}$$

```
[Out] -1/(2*b*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0047325, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {607}

$$-\frac{1}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(-3/2),x]
```

```
[Out] -1/(2*b*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=-\frac{1}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}}$$

Mathematica [A] time = 0.0087837, size = 23, normalized size = 0.68

$$-\frac{a+bx}{2b\left((a+bx)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(-3/2),x]
```

```
[Out] -(a + b*x)/(2*b*((a + b*x)^2)^(3/2))
```

Maple [A] time = 0.042, size = 20, normalized size = 0.6

$$-\frac{bx+a}{2\,b}\left((bx+a)^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] -1/2*(b*x+a)/b/((b*x+a)^2)^(3/2)
```

Maxima [A] time = 1.07083, size = 22, normalized size = 0.65

$$-\frac{1}{2\left(b^{2}\right)^{\frac{3}{2}}\left(x+\frac{a}{b}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2/((b^2)^(3/2)*(x + a/b)^2)
```

Fricas [A] time = 1.53499, size = 49, normalized size = 1.44

$$-\frac{1}{2\left(b^{3}x^{2}+2\,ab^{2}x+a^{2}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(a^{2}+2abx+b^{2}x^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x + b**2*x**2)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.1601 $\int \frac{1}{(d+ex)\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=165

$$\frac{e^2(a+bx)\log(a+bx)}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3}-\frac{e^2(a+bx)\log(d+ex)}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3}+\frac{e}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}-\frac{1}{2(a+bx)\sqrt{a^2+2abx+b^2x^2}}$$

```
[Out] e/((b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - 1/(2*(b*d - a*e)*(a + b*x
)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (e^2*(a + b*x)*Log[a + b*x])/((b*d - a*e
)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (e^2*(a + b*x)*Log[d + e*x])/((b*d - a
*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0969946, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 44}

$$\frac{e^2(a+bx)\log(a+bx)}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3}-\frac{e^2(a+bx)\log(d+ex)}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3}+\frac{e}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}-\frac{1}{2(a+bx)\sqrt{a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

```
[Out] e/((b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - 1/(2*(b*d - a*e)*(a + b*x
)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (e^2*(a + b*x)*Log[a + b*x])/((b*d - a*e
)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (e^2*(a + b*x)*Log[d + e*x])/((b*d - a
*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx&=\frac{\left(b^2\left(ab+b^2x\right)\right)\int \frac{1}{\left(ab+b^2x\right)^3(d+ex)}\,dx}{\sqrt{a^2+2abx+b^2x^2}}\\&=\frac{\left(b^2\left(ab+b^2x\right)\right)\int \left(\frac{1}{b^2(bd-ae)(a+bx)^3}-\frac{e}{b^2(bd-ae)^2(a+bx)^2}+\frac{e^2}{b^2(bd-ae)^3(a+bx)}-\frac{e^3}{b^3(bd-ae)^3(d+ex)}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}\\&=\frac{e}{(bd-ae)^2\sqrt{a^2+2abx+b^2x^2}}-\frac{1}{2(bd-ae)(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{e^2(a+}{(bd-ae)^3}\end{aligned}$$

Mathematica [A] time = 0.0550638, size = 92, normalized size = 0.56

$$\frac{-2e^2(a+bx)^2\log(d+ex)-(bd-ae)(b(d-2ex)-3ae)+2e^2(a+bx)^2\log(a+bx)}{2(a+bx)\sqrt{(a+bx)^2}(bd-ae)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

```
[Out] (-((b*d - a*e)*(-3*a*e + b*(d - 2*e*x))) + 2*e^2*(a + b*x)^2*Log[a + b*x] -
 2*e^2*(a + b*x)^2*Log[d + e*x])/(2*(b*d - a*e)^3*(a + b*x)*Sqrt[(a + b*x)^
2])
```

Maple [A] time = 0.203, size = 155, normalized size = 0.9

$$\frac{\left(2\ln(ex+d)x^2b^2e^2-2\ln(bx+a)x^2b^2e^2+4\ln(ex+d)xabe^2-4\ln(bx+a)xabe^2+2\ln(ex+d)a^2e^2-2\ln(bx+\right.}{2(ae-bd)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] 1/2*(2*ln(e*x+d)*x^2*b^2*e^2-2*ln(b*x+a)*x^2*b^2*e^2+4*ln(e*x+d)*x*a*b*e^2-
4*ln(b*x+a)*x*a*b*e^2+2*ln(e*x+d)*a^2*e^2-2*ln(b*x+a)*a^2*e^2+2*x*a*b*e^2-2
*x*b^2*d*e+3*a^2*e^2-4*a*b*d*e+b^2*d^2)*(b*x+a)/(a*e-b*d)^3/((b*x+a)^2)^(3/
2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.49254, size = 491, normalized size = 2.98

$$-\frac{b^2d^2-4abde+3a^2e^2-2\left(b^2de-abe^2\right)x-2\left(b^2e^2x^2+2abe^2x+a^2e^2\right)\log(bx+a)+2\left(b^2e^2x^2+2abe^2x+a^2e^2\right)\mathrm{lo}}{2\left(a^2b^3d^3-3a^3b^2d^2e+3a^4bde^2-a^5e^3+\left(b^5d^3-3ab^4d^2e+3a^2b^3de^2-a^3b^2e^3\right)x^2+2\left(ab^4d^3-3a^2b^3d^2e+3a^3b^2de\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*d^2 - 4*a*b*d*e + 3*a^2*e^2 - 2*(b^2*d*e - a*b*e^2)*x - 2*(b^2*e^
2*x^2 + 2*a*b*e^2*x + a^2*e^2)*log(b*x + a) + 2*(b^2*e^2*x^2 + 2*a*b*e^2*x
+ a^2*e^2)*log(e*x + d))/(a^2*b^3*d^3 - 3*a^3*b^2*d^2*e + 3*a^4*b*d*e^2 - a
^5*e^3 + (b^5*d^3 - 3*a*b^4*d^2*e + 3*a^2*b^3*d*e^2 - a^3*b^2*e^3)*x^2 + 2*
```

```
(a*b^4*d^3 - 3*a^2*b^3*d^2*e + 3*a^3*b^2*d*e^2 - a^4*b*e^3)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)\left((a+bx)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral(1/((d + e*x)*((a + b*x)**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.1602 $\int \frac{1}{(d+ex)^2\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=217

$$\frac{e^2(a+bx)}{\sqrt{a^2+2abx+b^2x^2}(d+ex)(bd-ae)^3}+\frac{3be^2(a+bx)\log(a+bx)}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^4}-\frac{3be^2(a+bx)\log(d+ex)}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^4}+\frac{2}{\sqrt{a^2+2abx+}}$$

```
[Out] (2*b*e)/((b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - b/(2*(b*d - a*e)^2*
(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (e^2*(a + b*x))/((b*d - a*e)^3*(
d + e*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (3*b*e^2*(a + b*x)*Log[a + b*x])/
((b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (3*b*e^2*(a + b*x)*Log[d +
e*x])/((b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.126441, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 44}

$$\frac{e^2(a+bx)}{\sqrt{a^2+2abx+b^2x^2}(d+ex)(bd-ae)^3}+\frac{3be^2(a+bx)\log(a+bx)}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^4}-\frac{3be^2(a+bx)\log(d+ex)}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^4}+\frac{2}{\sqrt{a^2+2abx+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```


```
[Out] (2*b*e)/((b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - b/(2*(b*d - a*e)^2*
(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (e^2*(a + b*x))/((b*d - a*e)^3*(
d + e*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (3*b*e^2*(a + b*x)*Log[a + b*x])/
((b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (3*b*e^2*(a + b*x)*Log[d +
e*x])/((b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x\right)\right)\int \frac{1}{\left(ab+b^2x\right)^3(d+ex)^2}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$
$$= \frac{\left(b^2\left(ab+b^2x\right)\right)\int \left(\frac{1}{b(bd-ae)^2(a+bx)^3} - \frac{2e}{b(bd-ae)^3(a+bx)^2} + \frac{3e^2}{b(bd-ae)^4(a+bx)} - \frac{e^3}{b^3(bd-ae)^3(d+ex)}\right.}{\sqrt{a^2+2abx+b^2x^2}}$$
$$= \frac{2be}{(bd-ae)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{b}{2(bd-ae)^2(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{}{(bd-a}$$

Mathematica [A] time = 0.0992049, size = 141, normalized size = 0.65

$$\frac{-(bd-ae)\left(-2a^2e^2-abe(5d+9ex)+b^2\left(d^2-3dex-6e^2x^2\right)\right)+6be^2(a+bx)^2(d+ex)\log(a+bx)-6be^2(a+bx)^2(d+ex)}{2(a+bx)\sqrt{(a+bx)^2}(d+ex)(bd-ae)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

```
[Out] (-((b*d - a*e)*(-2*a^2*e^2 - a*b*e*(5*d + 9*e*x) + b^2*(d^2 - 3*d*e*x - 6*e
^2*x^2))) + 6*b*e^2*(a + b*x)^2*(d + e*x)*Log[a + b*x] - 6*b*e^2*(a + b*x)^
2*(d + e*x)*Log[d + e*x])/(2*(b*d - a*e)^4*(a + b*x)*Sqrt[(a + b*x)^2]*(d +
 e*x))
```

Maple [B] time = 0.204, size = 329, normalized size = 1.5

$$-\frac{\left(6\ln(ex+d)x^3b^3e^3-6\ln(bx+a)x^3b^3e^3+12\ln(ex+d)x^2ab^2e^3+6\ln(ex+d)x^2b^3de^2-12\ln(bx+a)x^2ab^2e^3-6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] -1/2*(6*ln(e*x+d)*x^3*b^3*e^3-6*ln(b*x+a)*x^3*b^3*e^3+12*ln(e*x+d)*x^2*a*b^
2*e^3+6*ln(e*x+d)*x^2*b^3*d*e^2-12*ln(b*x+a)*x^2*a*b^2*e^3-6*ln(b*x+a)*x^2*
b^3*d*e^2+6*ln(e*x+d)*x*a^2*b*e^3+12*ln(e*x+d)*x*a*b^2*d*e^2-6*ln(b*x+a)*x*
a^2*b*e^3-12*ln(b*x+a)*x*a*b^2*d*e^2+6*x^2*a*b^2*e^3-6*x^2*b^3*d*e^2+6*ln(e
*x+d)*a^2*b*d*e^2-6*ln(b*x+a)*a^2*b*d*e^2+9*x*a^2*b*e^3-6*x*a*b^2*d*e^2-3*x
*b^3*d^2*e+2*a^3*e^3+3*d*e^2*a^2*b-6*a*b^2*d^2*e+b^3*d^3)*(b*x+a)/(e*x+d)/(
a*e-b*d)^4/((b*x+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.70269, size = 991, normalized size = 4.57

$$-\frac{b^3d^3 - 6\,ab^2d^2e + 3\,a^2bde^2 + 2\,a^3e^3 - 6\left(b^3de^2 - ab^2e^3\right)x^2 - 3\left(b^3d^2e + 2\,ab^2de^2 - 3\,a^2be^3\right)x - 6\left(b^3e^3x^3 + a^2bde^2 + \right.}{2\left(a^2b^4d^5 - 4\,a^3b^3d^4e + 6\,a^4b^2d^3e^2 - 4\,a^5bd^2e^3 + a^6de^4 + \left(b^6d^4e - 4\,ab^5d^3e^2 + 6\,a^2b^4d^2e^3 - 4\,a^3b^3de^4 + a^4b^2e^5\right)x^3 + \left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(b^3*d^3 - 6*a*b^2*d^2*e + 3*a^2*b*d*e^2 + 2*a^3*e^3 - 6*(b^3*d*e^2 -
a*b^2*e^3)*x^2 - 3*(b^3*d^2*e + 2*a*b^2*d*e^2 - 3*a^2*b*e^3)*x - 6*(b^3*e^3
*x^3 + a^2*b*d*e^2 + (b^3*d*e^2 + 2*a*b^2*e^3)*x^2 + (2*a*b^2*d*e^2 + a^2*b
*e^3)*x)*log(b*x + a) + 6*(b^3*e^3*x^3 + a^2*b*d*e^2 + (b^3*d*e^2 + 2*a*b^2
*e^3)*x^2 + (2*a*b^2*d*e^2 + a^2*b*e^3)*x)*log(e*x + d))/(a^2*b^4*d^5 - 4*a
^3*b^3*d^4*e + 6*a^4*b^2*d^3*e^2 - 4*a^5*b*d^2*e^3 + a^6*d*e^4 + (b^6*d^4*e
 - 4*a*b^5*d^3*e^2 + 6*a^2*b^4*d^2*e^3 - 4*a^3*b^3*d*e^4 + a^4*b^2*e^5)*x^3
 + (b^6*d^5 - 2*a*b^5*d^4*e - 2*a^2*b^4*d^3*e^2 + 8*a^3*b^3*d^2*e^3 - 7*a^4
*b^2*d*e^4 + 2*a^5*b*e^5)*x^2 + (2*a*b^5*d^5 - 7*a^2*b^4*d^4*e + 8*a^3*b^3*
d^3*e^2 - 2*a^4*b^2*d^2*e^3 - 2*a^5*b*d*e^4 + a^6*e^5)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)^2\left((a+bx)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral(1/((d + e*x)**2*((a + b*x)**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^2 + 2\,abx + a^2\right)^{\frac{3}{2}}(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*(e*x + d)^2), x)
```

3.1603 $\int \frac{1}{(d+ex)^3\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=276

$$\frac{3be^2(a+bx)}{\sqrt{a^2+2abx+b^2x^2}(d+ex)(bd-ae)^4}+\frac{e^2(a+bx)}{2\sqrt{a^2+2abx+b^2x^2}(d+ex)^2(bd-ae)^3}+\frac{6b^2e^2(a+bx)\log(a+bx)}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^5}-\frac{6b^2e^2}{\sqrt{a^2+}}$$

```
[Out] (3*b^2*e)/((b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - b^2/(2*(b*d - a*e
)^3*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (e^2*(a + b*x))/(2*(b*d - a*
e)^3*(d + e*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (3*b*e^2*(a + b*x))/((b*d
 - a*e)^4*(d + e*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (6*b^2*e^2*(a + b*x)*L
og[a + b*x])/((b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (6*b^2*e^2*(a
+ b*x)*Log[d + e*x])/((b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.151705, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 44}

$$\frac{3be^2(a+bx)}{\sqrt{a^2+2abx+b^2x^2}(d+ex)(bd-ae)^4}+\frac{e^2(a+bx)}{2\sqrt{a^2+2abx+b^2x^2}(d+ex)^2(bd-ae)^3}+\frac{6b^2e^2(a+bx)\log(a+bx)}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^5}-\frac{6b^2e^2}{\sqrt{a^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```


```
[Out] (3*b^2*e)/((b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - b^2/(2*(b*d - a*e
)^3*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (e^2*(a + b*x))/(2*(b*d - a*
e)^3*(d + e*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (3*b*e^2*(a + b*x))/((b*d
 - a*e)^4*(d + e*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (6*b^2*e^2*(a + b*x)*L
og[a + b*x])/((b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (6*b^2*e^2*(a
+ b*x)*Log[d + e*x])/((b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x\right)\right)\int \frac{1}{\left(ab+b^2x\right)^3(d+ex)^3}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$
$$= \frac{\left(b^2\left(ab+b^2x\right)\right)\int \left(\frac{1}{(bd-ae)^3(a+bx)^3} - \frac{3e}{(bd-ae)^4(a+bx)^2} + \frac{6e^2}{(bd-ae)^5(a+bx)} - \frac{e^3}{b^3(bd-ae)^3(d+ex}\right.}{\sqrt{a^2+2abx+b^2x^2}}$$
$$= \frac{3b^2e}{(bd-ae)^4\sqrt{a^2+2abx+b^2x^2}} - \frac{b^2}{2(bd-ae)^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{}{2(bd}$$

Mathematica [A] time = 0.0980738, size = 163, normalized size = 0.59

$$\frac{(a+bx)\left(-12b^2e^2(a+bx)^2\log(d+ex)+6b^2e(a+bx)(bd-ae)+b^2\left(-(bd-ae)^2\right)+12b^2e^2(a+bx)^2\log(a+bx)+\frac{6be^2}{}\right.}{2\left((a+bx)^2\right)^{3/2}(bd-ae)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

```
[Out] ((a + b*x)*(-(b^2*(b*d - a*e)^2) + 6*b^2*e*(b*d - a*e)*(a + b*x) + (e^2*(b*
d - a*e)^2*(a + b*x)^2)/(d + e*x)^2 + (6*b*e^2*(b*d - a*e)*(a + b*x)^2)/(d
+ e*x) + 12*b^2*e^2*(a + b*x)^2*Log[a + b*x] - 12*b^2*e^2*(a + b*x)^2*Log[d
 + e*x]))/(2*(b*d - a*e)^5*((a + b*x)^2)^(3/2))
```

Maple [B] time = 0.212, size = 508, normalized size = 1.8

$$\left(-24\ln(bx+a)xab^3d^2e^2+24\ln(ex+d)x^3ab^3e^4-8ab^3d^3e-48\ln(bx+a)x^2ab^3de^3-a^4e^4+b^4d^4-24\ln(bx+a)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] 1/2*(-24*ln(b*x+a)*x*a*b^3*d^2*e^2+24*ln(e*x+d)*x^3*a*b^3*e^4-8*a*b^3*d^3*e
-48*ln(b*x+a)*x^2*a*b^3*d*e^3-a^4*e^4+b^4*d^4-24*ln(b*x+a)*x^3*a*b^3*e^4-24
*ln(b*x+a)*x^3*b^4*d*e^3+12*ln(e*x+d)*x^2*a^2*b^2*e^4+12*ln(e*x+d)*x^2*b^4*
d^2*e^2+12*ln(e*x+d)*a^2*b^2*d^2*e^2+12*x^3*a*b^3*e^4+48*ln(e*x+d)*x^2*a*b^
3*d*e^3+24*ln(e*x+d)*x*a^2*b^2*d*e^3+24*ln(e*x+d)*x*a*b^3*d^2*e^2+12*ln(e*x
+d)*x^4*b^4*e^4-12*ln(b*x+a)*x^4*b^4*e^4-12*x^3*b^4*d*e^3+18*x^2*a^2*b^2*e^
4-18*x^2*b^4*d^2*e^2+4*x*a^3*b*e^4-4*x*b^4*d^3*e-12*ln(b*x+a)*a^2*b^2*d^2*e
^2+24*x*a^2*b^2*d*e^3-24*x*a*b^3*d^2*e^2-24*ln(b*x+a)*x*a^2*b^2*d*e^3-12*ln
(b*x+a)*x^2*b^4*d^2*e^2-12*ln(b*x+a)*x^2*a^2*b^2*e^4+24*ln(e*x+d)*x^3*b^4*d
*e^3+8*a^3*b*d*e^3)*(b*x+a)/(e*x+d)^2/(a*e-b*d)^5/((b*x+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.75502, size = 1488, normalized size = 5.39

$$-\frac{b^4d^4 - 8\,ab^3d^3e + 8\,a^3bde^3 - a^4e^4 - 12\left(b^4de^3 - ab^3e^4\right)x^3 - 18\left(b^4d^2e^2 - a^2b^2e^4\right)x^2 - 4\left(b^4d^3e + 6\,ab^3d^2e^2 - 6\,a^2b \cdots\right.}{2\left(a^2b^5d^7 - 5\,a^3b^4d^6e + 10\,a^4b^3d^5e^2 - 10\,a^5b^2d^4e^3 + 5\,a^6bd^3e^4 - a^7d^2e^5 + \left(b^7d^5e^2 - 5\,ab^6d^4e^3 + 10\,a^2b^5d^3e^4 - 10\,a^3b^4d^2e \cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(b^4*d^4 - 8*a*b^3*d^3*e + 8*a^3*b*d*e^3 - a^4*e^4 - 12*(b^4*d*e^3 - a
*b^3*e^4)*x^3 - 18*(b^4*d^2*e^2 - a^2*b^2*e^4)*x^2 - 4*(b^4*d^3*e + 6*a*b^3
*d^2*e^2 - 6*a^2*b^2*d*e^3 - a^3*b*e^4)*x - 12*(b^4*e^4*x^4 + a^2*b^2*d^2*e
^2 + 2*(b^4*d*e^3 + a*b^3*e^4)*x^3 + (b^4*d^2*e^2 + 4*a*b^3*d*e^3 + a^2*b^2
*e^4)*x^2 + 2*(a*b^3*d^2*e^2 + a^2*b^2*d*e^3)*x)*log(b*x + a) + 12*(b^4*e^4
*x^4 + a^2*b^2*d^2*e^2 + 2*(b^4*d*e^3 + a*b^3*e^4)*x^3 + (b^4*d^2*e^2 + 4*a
*b^3*d*e^3 + a^2*b^2*e^4)*x^2 + 2*(a*b^3*d^2*e^2 + a^2*b^2*d*e^3)*x)*log(e*
x + d))/(a^2*b^5*d^7 - 5*a^3*b^4*d^6*e + 10*a^4*b^3*d^5*e^2 - 10*a^5*b^2*d^
4*e^3 + 5*a^6*b*d^3*e^4 - a^7*d^2*e^5 + (b^7*d^5*e^2 - 5*a*b^6*d^4*e^3 + 10
*a^2*b^5*d^3*e^4 - 10*a^3*b^4*d^2*e^5 + 5*a^4*b^3*d*e^6 - a^5*b^2*e^7)*x^4
+ 2*(b^7*d^6*e - 4*a*b^6*d^5*e^2 + 5*a^2*b^5*d^4*e^3 - 5*a^4*b^3*d^2*e^5 +
4*a^5*b^2*d*e^6 - a^6*b*e^7)*x^3 + (b^7*d^7 - a*b^6*d^6*e - 9*a^2*b^5*d^5*e
^2 + 25*a^3*b^4*d^4*e^3 - 25*a^4*b^3*d^3*e^4 + 9*a^5*b^2*d^2*e^5 + a^6*b*d*
e^6 - a^7*e^7)*x^2 + 2*(a*b^6*d^7 - 4*a^2*b^5*d^6*e + 5*a^3*b^4*d^5*e^2 - 5
*a^5*b^2*d^3*e^4 + 4*a^6*b*d^2*e^5 - a^7*d*e^6)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)^3\left((a+bx)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral(1/((d + e*x)**3*((a + b*x)**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.1604 $\int \frac{(d+ex)^6}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=302

$$\frac{e^5x(a+bx)(6bd-5ae)}{b^6\sqrt{a^2+2abx+b^2x^2}}-\frac{20e^3(bd-ae)^3}{b^7\sqrt{a^2+2abx+b^2x^2}}-\frac{15e^2(bd-ae)^4}{2b^7(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{15e^4(a+bx)(bd-ae)^2\log(a+bx)}{b^7\sqrt{a^2+2abx+b^2x^2}}$$

```
[Out] (-20*e^3*(b*d - a*e)^3)/(b^7*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (b*d - a*e)^6
/(4*b^7*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (2*e*(b*d - a*e)^5)/(b
^7*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (15*e^2*(b*d - a*e)^4)/(2*b
^7*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (e^5*(6*b*d - 5*a*e)*x*(a + b
*x))/(b^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (e^6*x^2*(a + b*x))/(2*b^5*Sqrt[
a^2 + 2*a*b*x + b^2*x^2]) + (15*e^4*(b*d - a*e)^2*(a + b*x)*Log[a + b*x])/(
b^7*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.238804, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$\frac{e^5x(a+bx)(6bd-5ae)}{b^6\sqrt{a^2+2abx+b^2x^2}}-\frac{20e^3(bd-ae)^3}{b^7\sqrt{a^2+2abx+b^2x^2}}-\frac{15e^2(bd-ae)^4}{2b^7(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{15e^4(a+bx)(bd-ae)^2\log(a+bx)}{b^7\sqrt{a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^6/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```


```
[Out] (-20*e^3*(b*d - a*e)^3)/(b^7*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (b*d - a*e)^6
/(4*b^7*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (2*e*(b*d - a*e)^5)/(b
^7*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (15*e^2*(b*d - a*e)^4)/(2*b
^7*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (e^5*(6*b*d - 5*a*e)*x*(a + b
*x))/(b^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (e^6*x^2*(a + b*x))/(2*b^5*Sqrt[
a^2 + 2*a*b*x + b^2*x^2]) + (15*e^4*(b*d - a*e)^2*(a + b*x)*Log[a + b*x])/(
b^7*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(d+ex)^6}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x\right)\right)\int \frac{(d+ex)^6}{\left(ab+b^2x\right)^5}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$
$$= \frac{\left(b^4\left(ab+b^2x\right)\right)\int \left(\frac{e^5(6bd-5ae)}{b^{11}}+\frac{e^6x}{b^{10}}+\frac{(bd-ae)^6}{b^{11}(a+bx)^5}+\frac{6e(bd-ae)^5}{b^{11}(a+bx)^4}+\frac{15e^2(bd-ae)^4}{b^{11}(a+bx)^3}+\frac{20e^3(bd-ae)^3}{b^{11}(a+bx)^2}+\frac{15e^4}{b^{11}}\right.}{\sqrt{a^2+2abx+b^2x^2}}$$
$$= -\frac{20e^3(bd-ae)^3}{b^7\sqrt{a^2+2abx+b^2x^2}}-\frac{(bd-ae)^6}{4b^7(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}-\frac{2e(bd-ae)^5}{b^7(a+bx)^2\sqrt{a^2+2abx+b^2x^2}}$$

Mathematica [A] time = 0.147565, size = 313, normalized size = 1.04

$$\frac{-a^2b^4e^2\left(-540d^2e^2x^2+80d^3ex+5d^4+96de^3x^3+68e^4x^4\right)-4a^3b^3e^3\left(-110d^2ex+5d^3+126de^2x^2+8e^3x^3\right)+a^4b^2e^4\left(125\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^6/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (57*a^6*e^6 + 14*a^5*b*e^5*(-11*d + 12*e*x) + a^4*b^2*e^4*(125*d^2 - 496*d*
e*x + 132*e^2*x^2) - 4*a^3*b^3*e^3*(5*d^3 - 110*d^2*e*x + 126*d*e^2*x^2 + 8
*e^3*x^3) - a^2*b^4*e^2*(5*d^4 + 80*d^3*e*x - 540*d^2*e^2*x^2 + 96*d*e^3*x^
3 + 68*e^4*x^4) - 2*a*b^5*e*(d^5 + 10*d^4*e*x + 60*d^3*e^2*x^2 - 120*d^2*e^
3*x^3 - 48*d*e^4*x^4 + 6*e^5*x^5) - b^6*(d^6 + 8*d^5*e*x + 30*d^4*e^2*x^2 +
 80*d^3*e^3*x^3 - 24*d*e^5*x^5 - 2*e^6*x^6) + 60*e^4*(b*d - a*e)^2*(a + b*x
)^4*Log[a + b*x])/(4*b^7*(a + b*x)^3*Sqrt[(a + b*x)^2])
```

Maple [B] time = 0.205, size = 661, normalized size = 2.2

$$\frac{\left(-154\,a^5bde^5+57\,a^6e^6-30\,x^2b^6d^4e^2+168\,xa^5be^6-8\,xb^6d^5e-32\,x^3a^3b^3e^6-80\,x^3b^6d^3e^3+132\,x^2a^4b^2e^6-12\,x^5ab^5e^6+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^6/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] 1/4*(-154*a^5*b*d*e^5+57*a^6*e^6-30*x^2*b^6*d^4*e^2+168*x*a^5*b*e^6-8*x*b^6
*d^5*e-32*x^3*a^3*b^3*e^6-80*x^3*b^6*d^3*e^3+132*x^2*a^4*b^2*e^6-12*x^5*a*b
^5*e^6+24*x^5*b^6*d*e^5-68*x^4*a^2*b^4*e^6-120*ln(b*x+a)*x^4*a*b^5*d*e^5-2*
a*b^5*d^5*e+540*x^2*a^2*b^4*d^2*e^4-120*x^2*a*b^5*d^3*e^3+440*x*a^3*b^3*d^2
*e^4-80*x*a^2*b^4*d^3*e^3-20*x*a*b^5*d^4*e^2+240*x^3*a*b^5*d^2*e^4+60*ln(b*
x+a)*x^4*a^2*b^4*e^6+60*ln(b*x+a)*x^4*b^6*d^2*e^4+240*ln(b*x+a)*x^3*a^3*b^3
*e^6+360*ln(b*x+a)*x^2*a^4*b^2*e^6-d^6*b^6+2*x^6*b^6*e^6+240*ln(b*x+a)*x*a^
5*b*e^6-120*ln(b*x+a)*a^5*b*d*e^5+60*ln(b*x+a)*a^4*b^2*d^2*e^4-96*x^3*a^2*b
^4*d*e^5-504*x^2*a^3*b^3*d*e^5-496*x*a^4*b^2*d*e^5+96*x^4*a*b^5*d*e^5+60*ln
(b*x+a)*a^6*e^6-5*a^2*b^4*d^4*e^2+125*d^2*e^4*a^4*b^2-20*b^3*a^3*d^3*e^3-48
0*ln(b*x+a)*x*a^4*b^2*d*e^5+240*ln(b*x+a)*x*a^3*b^3*d^2*e^4-480*ln(b*x+a)*x
^3*a^2*b^4*d*e^5+240*ln(b*x+a)*x^3*a*b^5*d^2*e^4-720*ln(b*x+a)*x^2*a^3*b^3*
d*e^5+360*ln(b*x+a)*x^2*a^2*b^4*d^2*e^4)*(b*x+a)/b^7/((b*x+a)^2)^(5/2)
```

Maxima [B] time = 1.40594, size = 844, normalized size = 2.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*e^6*((2*b^6*x^6 - 12*a*b^5*x^5 - 68*a^2*b^4*x^4 - 32*a^3*b^3*x^3 + 132*
a^4*b^2*x^2 + 168*a^5*b*x + 57*a^6)/(b^11*x^4 + 4*a*b^10*x^3 + 6*a^2*b^9*x^
2 + 4*a^3*b^8*x + a^4*b^7) + 60*a^2*log(b*x + a)/b^7) + 1/2*d*e^5*((12*b^5*
x^5 + 48*a*b^4*x^4 - 48*a^2*b^3*x^3 - 252*a^3*b^2*x^2 - 248*a^4*b*x - 77*a^
5)/(b^10*x^4 + 4*a*b^9*x^3 + 6*a^2*b^8*x^2 + 4*a^3*b^7*x + a^4*b^6) - 60*a*
log(b*x + a)/b^6) + 5/4*d^2*e^4*((48*a*b^3*x^3 + 108*a^2*b^2*x^2 + 88*a^3*b
*x + 25*a^4)/(b^9*x^4 + 4*a*b^8*x^3 + 6*a^2*b^7*x^2 + 4*a^3*b^6*x + a^4*b^5
) + 12*log(b*x + a)/b^5) - 5/3*d^3*e^3*(12*x^2/((b^2*x^2 + 2*a*b*x + a^2)^(
3/2)*b^2) + 8*a^2/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^4) + 3*a^3*b/((b^2)^(9
/2)*(x + a/b)^4) - 8*a^2/((b^2)^(7/2)*(x + a/b)^3) + 6*a/((b^2)^(5/2)*b*(x
+ a/b)^2) - 6*a^3/((b^2)^(5/2)*b^3*(x + a/b)^4)) - 1/2*d^5*e*(4/((b^2*x^2 +
 2*a*b*x + a^2)^(3/2)*b^2) - 3*a/((b^2)^(5/2)*b*(x + a/b)^4)) - 5/4*d^4*e^2
*(3*a^2*b^2/((b^2)^(9/2)*(x + a/b)^4) - 8*a*b/((b^2)^(7/2)*(x + a/b)^3) + 6
/((b^2)^(5/2)*(x + a/b)^2)) - 1/4*d^6/((b^2)^(5/2)*(x + a/b)^4)
```

Fricas [B] time = 1.65025, size = 1158, normalized size = 3.83

$$\frac{2 b^6e^6x^6 - b^6d^6 - 2 ab^5d^5e - 5 a^2b^4d^4e^2 - 20 a^3b^3d^3e^3 + 125 a^4b^2d^2e^4 - 154 a^5bde^5 + 57 a^6e^6 + 12 \left(2 b^6de^5 - ab^5e^6\right)x^5 \cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^6*e^6*x^6 - b^6*d^6 - 2*a*b^5*d^5*e - 5*a^2*b^4*d^4*e^2 - 20*a^3*b
^3*d^3*e^3 + 125*a^4*b^2*d^2*e^4 - 154*a^5*b*d*e^5 + 57*a^6*e^6 + 12*(2*b^6
*d*e^5 - a*b^5*e^6)*x^5 + 4*(24*a*b^5*d*e^5 - 17*a^2*b^4*e^6)*x^4 - 16*(5*b
^6*d^3*e^3 - 15*a*b^5*d^2*e^4 + 6*a^2*b^4*d*e^5 + 2*a^3*b^3*e^6)*x^3 - 6*(5
*b^6*d^4*e^2 + 20*a*b^5*d^3*e^3 - 90*a^2*b^4*d^2*e^4 + 84*a^3*b^3*d*e^5 - 2
2*a^4*b^2*e^6)*x^2 - 4*(2*b^6*d^5*e + 5*a*b^5*d^4*e^2 + 20*a^2*b^4*d^3*e^3
- 110*a^3*b^3*d^2*e^4 + 124*a^4*b^2*d*e^5 - 42*a^5*b*e^6)*x + 60*(a^4*b^2*d
^2*e^4 - 2*a^5*b*d*e^5 + a^6*e^6 + (b^6*d^2*e^4 - 2*a*b^5*d*e^5 + a^2*b^4*e
^6)*x^4 + 4*(a*b^5*d^2*e^4 - 2*a^2*b^4*d*e^5 + a^3*b^3*e^6)*x^3 + 6*(a^2*b^
4*d^2*e^4 - 2*a^3*b^3*d*e^5 + a^4*b^2*e^6)*x^2 + 4*(a^3*b^3*d^2*e^4 - 2*a^4
*b^2*d*e^5 + a^5*b*e^6)*x)*log(b*x + a))/(b^11*x^4 + 4*a*b^10*x^3 + 6*a^2*b
^9*x^2 + 4*a^3*b^8*x + a^4*b^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^6}{\left((a + bx)^2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**6/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Integral((d + e*x)**6/((a + b*x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.1605 $\int \frac{(d+ex)^5}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=253

$$-\frac{10e^3(bd-ae)^2}{b^6\sqrt{a^2+2abx+b^2x^2}}-\frac{5e^2(bd-ae)^3}{b^6(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{5e^4(a+bx)(bd-ae)\log(a+bx)}{b^6\sqrt{a^2+2abx+b^2x^2}}-\frac{5e(bd-ae)^4}{3b^6(a+bx)^2\sqrt{a^2+2abx}}$$

```
[Out] (-10*e^3*(b*d - a*e)^2)/(b^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (b*d - a*e)^5
/(4*b^6*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (5*e*(b*d - a*e)^4)/(3
*b^6*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (5*e^2*(b*d - a*e)^3)/(b^
6*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (e^5*x*(a + b*x))/(b^5*Sqrt[a^
2 + 2*a*b*x + b^2*x^2]) + (5*e^4*(b*d - a*e)*(a + b*x)*Log[a + b*x])/(b^6*S
qrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.175031, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$-\frac{10e^3(bd-ae)^2}{b^6\sqrt{a^2+2abx+b^2x^2}}-\frac{5e^2(bd-ae)^3}{b^6(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{5e^4(a+bx)(bd-ae)\log(a+bx)}{b^6\sqrt{a^2+2abx+b^2x^2}}-\frac{5e(bd-ae)^4}{3b^6(a+bx)^2\sqrt{a^2+2abx}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```


```
[Out] (-10*e^3*(b*d - a*e)^2)/(b^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (b*d - a*e)^5
/(4*b^6*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (5*e*(b*d - a*e)^4)/(3
*b^6*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (5*e^2*(b*d - a*e)^3)/(b^
6*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (e^5*x*(a + b*x))/(b^5*Sqrt[a^
2 + 2*a*b*x + b^2*x^2]) + (5*e^4*(b*d - a*e)*(a + b*x)*Log[a + b*x])/(b^6*S
qrt[a^2 + 2*a*b*x + b^2*x^2])
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(d+ex)^5}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x\right)\right)\int \frac{(d+ex)^5}{\left(ab+b^2x\right)^5}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$
$$= \frac{\left(b^4\left(ab+b^2x\right)\right)\int \left(\frac{e^5}{b^{10}}+\frac{(bd-ae)^5}{b^{10}(a+bx)^5}+\frac{5e(bd-ae)^4}{b^{10}(a+bx)^4}+\frac{10e^2(bd-ae)^3}{b^{10}(a+bx)^3}+\frac{10e^3(bd-ae)^2}{b^{10}(a+bx)^2}+\frac{5e^4(bd-ae)}{b^{10}(a+bx)}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$
$$= -\frac{10e^3(bd-ae)^2}{b^6\sqrt{a^2+2abx+b^2x^2}} - \frac{(bd-ae)^5}{4b^6(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{5e(bd-ae)^4}{3b^6(a+bx)^2\sqrt{a^2+2abx+b^2}}$$

Mathematica [A] time = 0.129426, size = 242, normalized size = 0.96

$$\frac{-2a^2b^3e^2\left(60d^2ex+5d^3-270de^2x^2+24e^3x^3\right)-2a^3b^2e^3\left(15d^2-220dex+126e^2x^2\right)+a^4be^4(125d-248ex)-77a^5e^5+ab}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (-77*a^5*e^5 + a^4*b*e^4*(125*d - 248*e*x) - 2*a^3*b^2*e^3*(15*d^2 - 220*d*
e*x + 126*e^2*x^2) - 2*a^2*b^3*e^2*(5*d^3 + 60*d^2*e*x - 270*d*e^2*x^2 + 24
*e^3*x^3) + a*b^4*e*(-5*d^4 - 40*d^3*e*x - 180*d^2*e^2*x^2 + 240*d*e^3*x^3
+ 48*e^4*x^4) - b^5*(3*d^5 + 20*d^4*e*x + 60*d^3*e^2*x^2 + 120*d^2*e^3*x^3
- 12*e^5*x^5) - 60*e^4*(-(b*d) + a*e)*(a + b*x)^4*Log[a + b*x])/(12*b^6*(a
+ b*x)^3*Sqrt[(a + b*x)^2])
```

Maple [B] time = 0.204, size = 449, normalized size = 1.8

$$-\frac{\left(77\,a^5e^5+3\,b^5d^5-125\,de^4a^4b+20\,xb^5d^4e-48\,x^4ab^4e^5+48\,x^3a^2b^3e^5+120\,x^3b^5d^2e^3+252\,x^2a^3b^2e^5+60\,x^2b^5d^3e^2+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] -1/12*(77*a^5*e^5+3*b^5*d^5-125*d*e^4*a^4*b+20*x*b^5*d^4*e-48*x^4*a*b^4*e^5
+48*x^3*a^2*b^3*e^5+120*x^3*b^5*d^2*e^3+252*x^2*a^3*b^2*e^5+60*x^2*b^5*d^3*
e^2+248*x*a^4*b*e^5-240*ln(b*x+a)*x^3*a*b^4*d*e^4+180*x^2*a*b^4*d^2*e^3-540
*x^2*a^2*b^3*d*e^4-240*x^3*a*b^4*d*e^4+60*ln(b*x+a)*a^5*e^5+10*a^2*b^3*d^3*
e^2+30*a^3*b^2*d^2*e^3-12*x^5*b^5*e^5+40*x*a*b^4*d^3*e^2-440*x*a^3*b^2*d*e^
4+120*x*a^2*b^3*d^2*e^3+360*ln(b*x+a)*x^2*a^3*b^2*e^5+240*ln(b*x+a)*x*a^4*b
*e^5-60*ln(b*x+a)*a^4*b*d*e^4+60*ln(b*x+a)*x^4*a*b^4*e^5-60*ln(b*x+a)*x^4*b
^5*d*e^4+240*ln(b*x+a)*x^3*a^2*b^3*e^5-360*ln(b*x+a)*x^2*a^2*b^3*d*e^4-240*
ln(b*x+a)*x*a^3*b^2*d*e^4+5*a*b^4*d^4*e)*(b*x+a)/b^6/((b*x+a)^2)^(5/2)
```

Maxima [B] time = 1.18969, size = 664, normalized size = 2.62

$$\frac{1}{12}e^5\left(\frac{12\,b^5x^5+48\,ab^4x^4-48\,a^2b^3x^3-252\,a^3b^2x^2-248\,a^4bx-77\,a^5}{b^{10}x^4+4\,ab^9x^3+6\,a^2b^8x^2+4\,a^3b^7x+a^4b^6}-\frac{60\,a\log(bx+a)}{b^6}\right)+\frac{5}{12}de^4\left(\frac{48\,ab^3x^3+108\,a^2b}{b^9x^4+4\,ab^8x^3+6\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*e^5*((12*b^5*x^5 + 48*a*b^4*x^4 - 48*a^2*b^3*x^3 - 252*a^3*b^2*x^2 - 2
48*a^4*b*x - 77*a^5)/(b^10*x^4 + 4*a*b^9*x^3 + 6*a^2*b^8*x^2 + 4*a^3*b^7*x
+ a^4*b^6) - 60*a*log(b*x + a)/b^6) + 5/12*d*e^4*((48*a*b^3*x^3 + 108*a^2*b
^2*x^2 + 88*a^3*b*x + 25*a^4)/(b^9*x^4 + 4*a*b^8*x^3 + 6*a^2*b^7*x^2 + 4*a^
3*b^6*x + a^4*b^5) + 12*log(b*x + a)/b^5) - 5/6*d^2*e^3*(12*x^2/((b^2*x^2 +
 2*a*b*x + a^2)^(3/2)*b^2) + 8*a^2/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^4) +
3*a^3*b/((b^2)^(9/2)*(x + a/b)^4) - 8*a^2/((b^2)^(7/2)*(x + a/b)^3) + 6*a/(
(b^2)^(5/2)*b*(x + a/b)^2) - 6*a^3/((b^2)^(5/2)*b^3*(x + a/b)^4)) - 5/12*d^
4*e*(4/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^2) - 3*a/((b^2)^(5/2)*b*(x + a/b)
^4)) - 5/6*d^3*e^2*(3*a^2*b^2/((b^2)^(9/2)*(x + a/b)^4) - 8*a*b/((b^2)^(7/2
)*(x + a/b)^3) + 6/((b^2)^(5/2)*(x + a/b)^2)) - 1/4*d^5/((b^2)^(5/2)*(x + a
/b)^4)
```

Fricas [B] time = 1.63797, size = 838, normalized size = 3.31

$$\frac{12\,b^5e^5x^5 + 48\,ab^4e^5x^4 - 3\,b^5d^5 - 5\,ab^4d^4e - 10\,a^2b^3d^3e^2 - 30\,a^3b^2d^2e^3 + 125\,a^4bde^4 - 77\,a^5e^5 - 24\left(5\,b^5d^2e^3 - 10\,ab^4d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(12*b^5*e^5*x^5 + 48*a*b^4*e^5*x^4 - 3*b^5*d^5 - 5*a*b^4*d^4*e - 10*a^
2*b^3*d^3*e^2 - 30*a^3*b^2*d^2*e^3 + 125*a^4*b*d*e^4 - 77*a^5*e^5 - 24*(5*b
^5*d^2*e^3 - 10*a*b^4*d*e^4 + 2*a^2*b^3*e^5)*x^3 - 12*(5*b^5*d^3*e^2 + 15*a
*b^4*d^2*e^3 - 45*a^2*b^3*d*e^4 + 21*a^3*b^2*e^5)*x^2 - 4*(5*b^5*d^4*e + 10
*a*b^4*d^3*e^2 + 30*a^2*b^3*d^2*e^3 - 110*a^3*b^2*d*e^4 + 62*a^4*b*e^5)*x +
 60*(a^4*b*d*e^4 - a^5*e^5 + (b^5*d*e^4 - a*b^4*e^5)*x^4 + 4*(a*b^4*d*e^4 -
 a^2*b^3*e^5)*x^3 + 6*(a^2*b^3*d*e^4 - a^3*b^2*e^5)*x^2 + 4*(a^3*b^2*d*e^4
- a^4*b*e^5)*x)*log(b*x + a))/(b^10*x^4 + 4*a*b^9*x^3 + 6*a^2*b^8*x^2 + 4*a
^3*b^7*x + a^4*b^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^5}{\left((a+bx)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Integral((d + e*x)**5/((a + b*x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.1606 $\int \frac{(d+ex)^4}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=209

$$-\frac{4e^3(bd-ae)}{b^5\sqrt{a^2+2abx+b^2x^2}}-\frac{3e^2(bd-ae)^2}{b^5(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{4e(bd-ae)^3}{3b^5(a+bx)^2\sqrt{a^2+2abx+b^2x^2}}-\frac{(bd-ae)^4}{4b^5(a+bx)^3\sqrt{a^2+2abx}}$$

```
[Out] (-4*e^3*(b*d - a*e))/(b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (b*d - a*e)^4/(4
*b^5*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (4*e*(b*d - a*e)^3)/(3*b^
5*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (3*e^2*(b*d - a*e)^2)/(b^5*(
a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (e^4*(a + b*x)*Log[a + b*x])/(b^5
*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.126887, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$-\frac{4e^3(bd-ae)}{b^5\sqrt{a^2+2abx+b^2x^2}}-\frac{3e^2(bd-ae)^2}{b^5(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{4e(bd-ae)^3}{3b^5(a+bx)^2\sqrt{a^2+2abx+b^2x^2}}-\frac{(bd-ae)^4}{4b^5(a+bx)^3\sqrt{a^2+2abx}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```


```
[Out] (-4*e^3*(b*d - a*e))/(b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (b*d - a*e)^4/(4
*b^5*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (4*e*(b*d - a*e)^3)/(3*b^
5*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (3*e^2*(b*d - a*e)^2)/(b^5*(
a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (e^4*(a + b*x)*Log[a + b*x])/(b^5
*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(d+ex)^4}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x\right)\right)\int \frac{(d+ex)^4}{\left(ab+b^2x\right)^5}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$
$$= \frac{\left(b^4\left(ab+b^2x\right)\right)\int \left(\frac{(bd-ae)^4}{b^9(a+bx)^5}+\frac{4e(bd-ae)^3}{b^9(a+bx)^4}+\frac{6e^2(bd-ae)^2}{b^9(a+bx)^3}+\frac{4e^3(bd-ae)}{b^9(a+bx)^2}+\frac{e^4}{b^9(a+bx)}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$
$$= -\frac{4e^3(bd-ae)}{b^5\sqrt{a^2+2abx+b^2x^2}} - \frac{(bd-ae)^4}{4b^5(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{4e(bd-ae)^3}{3b^5(a+bx)^2\sqrt{a^2+2abx+b^2x^2}}$$

Mathematica [A] time = 0.0764801, size = 138, normalized size = 0.66

$$\frac{12e^4(a+bx)^4\log(a+bx)-(bd-ae)\left(a^2be^2(13d+88ex)+25a^3e^3+ab^2e\left(7d^2+40dex+108e^2x^2\right)+b^3\left(16d^2ex+3d^3+3\right.\right.}{12b^5(a+bx)^3\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (-((b*d - a*e)*(25*a^3*e^3 + a^2*b*e^2*(13*d + 88*e*x) + a*b^2*e*(7*d^2 + 4
0*d*e*x + 108*e^2*x^2) + b^3*(3*d^3 + 16*d^2*e*x + 36*d*e^2*x^2 + 48*e^3*x^
3))) + 12*e^4*(a + b*x)^4*Log[a + b*x])/(12*b^5*(a + b*x)^3*Sqrt[(a + b*x)^
2])
```

Maple [A] time = 0.198, size = 267, normalized size = 1.3

$$\frac{\left(12\ln(bx+a)x^4b^4e^4+48\ln(bx+a)x^3ab^3e^4+72\ln(bx+a)x^2a^2b^2e^4+48x^3ab^3e^4-48x^3b^4de^3+48\ln(bx+a)xa^3b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] 1/12*(12*ln(b*x+a)*x^4*b^4*e^4+48*ln(b*x+a)*x^3*a*b^3*e^4+72*ln(b*x+a)*x^2*
a^2*b^2*e^4+48*x^3*a*b^3*e^4-48*x^3*b^4*d*e^3+48*ln(b*x+a)*x*a^3*b*e^4+108*
x^2*a^2*b^2*e^4-72*x^2*a*b^3*d*e^3-36*x^2*b^4*d^2*e^2+12*ln(b*x+a)*a^4*e^4+
88*x*a^3*b*e^4-48*x*a^2*b^2*d*e^3-24*x*a*b^3*d^2*e^2-16*x*b^4*d^3*e+25*a^4*
e^4-12*a^3*b*d*e^3-6*d^2*e^2*a^2*b^2-4*a*b^3*d^3*e-3*b^4*d^4)*(b*x+a)/b^5/(
(b*x+a)^2)^(5/2)
```

Maxima [B] time = 1.14381, size = 502, normalized size = 2.4

$$\frac{1}{12}e^4\left(\frac{48ab^3x^3+108a^2b^2x^2+88a^3bx+25a^4}{b^9x^4+4ab^8x^3+6a^2b^7x^2+4a^3b^6x+a^4b^5}+\frac{12\log(bx+a)}{b^5}\right)-\frac{1}{3}de^3\left(\frac{12x^2}{\left(b^2x^2+2abx+a^2\right)^{\frac{3}{2}}b^2}+\frac{8a^2}{\left(b^2x^2+2abx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*e^4*((48*a*b^3*x^3 + 108*a^2*b^2*x^2 + 88*a^3*b*x + 25*a^4)/(b^9*x^4 +
 4*a*b^8*x^3 + 6*a^2*b^7*x^2 + 4*a^3*b^6*x + a^4*b^5) + 12*log(b*x + a)/b^5
) - 1/3*d*e^3*(12*x^2/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^2) + 8*a^2/((b^2*x
^2 + 2*a*b*x + a^2)^(3/2)*b^4) + 3*a^3*b/((b^2)^(9/2)*(x + a/b)^4) - 8*a^2/
((b^2)^(7/2)*(x + a/b)^3) + 6*a/((b^2)^(5/2)*b*(x + a/b)^2) - 6*a^3/((b^2)^
(5/2)*b^3*(x + a/b)^4)) - 1/3*d^3*e*(4/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^2
) - 3*a/((b^2)^(5/2)*b*(x + a/b)^4)) - 1/2*d^2*e^2*(3*a^2*b^2/((b^2)^(9/2)*
(x + a/b)^4) - 8*a*b/((b^2)^(7/2)*(x + a/b)^3) + 6/((b^2)^(5/2)*(x + a/b)^2
)) - 1/4*d^4/((b^2)^(5/2)*(x + a/b)^4)
```

Fricas [A] time = 1.51676, size = 545, normalized size = 2.61

$$-\frac{3\,b^4d^4+4\,ab^3d^3e+6\,a^2b^2d^2e^2+12\,a^3bde^3-25\,a^4e^4+48\left(b^4de^3-ab^3e^4\right)x^3+36\left(b^4d^2e^2+2\,ab^3de^3-3\,a^2b^2e^4\right)x^2+}{12\left(b^9x^4+4\,ab^8x^3+6\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*b^4*d^4 + 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 + 12*a^3*b*d*e^3 - 25*
a^4*e^4 + 48*(b^4*d*e^3 - a*b^3*e^4)*x^3 + 36*(b^4*d^2*e^2 + 2*a*b^3*d*e^3
- 3*a^2*b^2*e^4)*x^2 + 8*(2*b^4*d^3*e + 3*a*b^3*d^2*e^2 + 6*a^2*b^2*d*e^3 -
 11*a^3*b*e^4)*x - 12*(b^4*e^4*x^4 + 4*a*b^3*e^4*x^3 + 6*a^2*b^2*e^4*x^2 +
4*a^3*b*e^4*x + a^4*e^4)*log(b*x + a))/(b^9*x^4 + 4*a*b^8*x^3 + 6*a^2*b^7*x
^2 + 4*a^3*b^6*x + a^4*b^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^4}{\left((a+bx)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Integral((d + e*x)**4/((a + b*x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.1607 $\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=48

$$-\frac{(d+ex)^4}{4(a+bx)^3\sqrt{a^2+2abx+b^2x^2}(bd-ae)}$$

```
[Out] -(d + e*x)^4/(4*(b*d - a*e)*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0220864, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 37}

$$-\frac{(d+ex)^4}{4(a+bx)^3\sqrt{a^2+2abx+b^2x^2}(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] -(d + e*x)^4/(4*(b*d - a*e)*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^{5/2}}\,dx &= \frac{\left(b^4\left(ab+b^2x\right)\right)\int \frac{(d+ex)^3}{(ab+b^2x)^5}\,dx}{\sqrt{a^2+2abx+b^2x^2}}\\ &= -\frac{(d+ex)^4}{4(bd-ae)(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}\end{aligned}$$

Mathematica [B] time = 0.0431586, size = 106, normalized size = 2.21

$$\frac{-a^2be^2(d+4ex)-a^3e^3-ab^2e\left(d^2+4dex+6e^2x^2\right)+b^3\left(-\left(4d^2ex+d^3+6de^2x^2+4e^3x^3\right)\right)}{4b^4(a+bx)^3\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (-(a^3*e^3) - a^2*b*e^2*(d + 4*e*x) - a*b^2*e*(d^2 + 4*d*e*x + 6*e^2*x^2) -
 b^3*(d^3 + 4*d^2*e*x + 6*d*e^2*x^2 + 4*e^3*x^3))/(4*b^4*(a + b*x)^3*Sqrt[(
a + b*x)^2])
```

Maple [B] time = 0.155, size = 119, normalized size = 2.5

$$-\frac{(bx+a)\left(4x^3b^3e^3+6x^2ab^2e^3+6x^2b^3de^2+4xa^2be^3+4xab^2de^2+4xb^3d^2e+a^3e^3+de^2a^2b+ab^2d^2e+b^3d^3\right)}{4b^4}\left((bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] -1/4*(b*x+a)*(4*b^3*e^3*x^3+6*a*b^2*e^3*x^2+6*b^3*d*e^2*x^2+4*a^2*b*e^3*x+4
*a*b^2*d*e^2*x+4*b^3*d^2*e*x+a^3*e^3+a^2*b*d*e^2+a*b^2*d^2*e+b^3*d^3)/b^4/(
(b*x+a)^2)^(5/2)
```

Maxima [B] time = 1.1404, size = 392, normalized size = 8.17

$$-\frac{e^3x^2}{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}b^2}-\frac{d^2e}{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}b^2}-\frac{2\,a^2e^3}{3\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}b^4}-\frac{3\,a^2b^2de^2}{4\left(b^2\right)^{\frac{9}{2}}\left(x+\frac{a}{b}\right)^4}-\frac{a^3be^3}{4\left(b^2\right)^{\frac{9}{2}}\left(x+\frac{a}{b}\right)^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -e^3*x^2/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^2) - d^2*e/((b^2*x^2 + 2*a*b*x
+ a^2)^(3/2)*b^2) - 2/3*a^2*e^3/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^4) - 3/4
*a^2*b^2*d*e^2/((b^2)^(9/2)*(x + a/b)^4) - 1/4*a^3*b*e^3/((b^2)^(9/2)*(x +
a/b)^4) + 2*a*b*d*e^2/((b^2)^(7/2)*(x + a/b)^3) + 2/3*a^2*e^3/((b^2)^(7/2)*
(x + a/b)^3) - 3/2*d*e^2/((b^2)^(5/2)*(x + a/b)^2) - 1/2*a*e^3/((b^2)^(5/2)
*b*(x + a/b)^2) - 1/4*d^3/((b^2)^(5/2)*(x + a/b)^4) + 3/4*a*d^2*e/((b^2)^(5
/2)*b*(x + a/b)^4) + 1/2*a^3*e^3/((b^2)^(5/2)*b^3*(x + a/b)^4)
```

Fricas [B] time = 1.62099, size = 284, normalized size = 5.92

$$-\frac{4\,b^3e^3x^3+b^3d^3+ab^2d^2e+a^2bde^2+a^3e^3+6\left(b^3de^2+ab^2e^3\right)x^2+4\left(b^3d^2e+ab^2de^2+a^2be^3\right)x}{4\left(b^8x^4+4\,ab^7x^3+6\,a^2b^6x^2+4\,a^3b^5x+a^4b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^3*e^3*x^3 + b^3*d^3 + a*b^2*d^2*e + a^2*b*d*e^2 + a^3*e^3 + 6*(b^
3*d*e^2 + a*b^2*e^3)*x^2 + 4*(b^3*d^2*e + a*b^2*d*e^2 + a^2*b*e^3)*x)/(b^8*
x^4 + 4*a*b^7*x^3 + 6*a^2*b^6*x^2 + 4*a^3*b^5*x + a^4*b^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^3}{\left((a+bx)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Integral((d + e*x)**3/((a + b*x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.1608 $\int \frac{(d+ex)^2}{\left(a^2+2abx+b^2x^2\right)^{5/2}} dx$

Optimal. Leaf size=125

$$-\frac{2e(bd-ae)}{3b^3(a+bx)^2\sqrt{a^2+2abx+b^2x^2}}-\frac{(bd-ae)^2}{4b^3(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}-\frac{e^2}{2b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}$$

```
[Out] -(b*d - a*e)^2/(4*b^3*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (2*e*(b*
d - a*e))/(3*b^3*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - e^2/(2*b^3*(a
 + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0697736, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$-\frac{2e(bd-ae)}{3b^3(a+bx)^2\sqrt{a^2+2abx+b^2x^2}}-\frac{(bd-ae)^2}{4b^3(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}-\frac{e^2}{2b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] -(b*d - a*e)^2/(4*b^3*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (2*e*(b*
d - a*e))/(3*b^3*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - e^2/(2*b^3*(a
 + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2}{\left(a^2+2abx+b^2x^2\right)^{5/2}} dx &= \frac{\left(b^4\left(ab+b^2x\right)\right)\int \frac{(d+ex)^2}{\left(ab+b^2x\right)^5} dx}{\sqrt{a^2+2abx+b^2x^2}}\\
&= \frac{\left(b^4\left(ab+b^2x\right)\right)\int \left(\frac{(bd-ae)^2}{b^7(a+bx)^5}+\frac{2e(bd-ae)}{b^7(a+bx)^4}+\frac{e^2}{b^7(a+bx)^3}\right) dx}{\sqrt{a^2+2abx+b^2x^2}}\\
&= -\frac{(bd-ae)^2}{4b^3(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}-\frac{2e(bd-ae)}{3b^3(a+bx)^2\sqrt{a^2+2abx+b^2x^2}}-\frac{e^2}{2b^3(a+bx)\sqrt{a^2}}
\end{aligned}$$

Mathematica [A] time = 0.0281796, size = 69, normalized size = 0.55

$$\frac{-a^2e^2-2abe(d+2ex)+b^2\left(-\left(3d^2+8dex+6e^2x^2\right)\right)}{12b^3(a+bx)^3\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (-(a^2*e^2) - 2*a*b*e*(d + 2*e*x) - b^2*(3*d^2 + 8*d*e*x + 6*e^2*x^2))/(12*
b^3*(a + b*x)^3*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.154, size = 69, normalized size = 0.6

$$-\frac{(bx+a)\left(6e^2x^2b^2+4xabe^2+8xb^2de+a^2e^2+2abde+3b^2d^2\right)}{12b^3}\left((bx+a)^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] -1/12*(b*x+a)/b^3*(6*b^2*e^2*x^2+4*a*b*e^2*x+8*b^2*d*e*x+a^2*e^2+2*a*b*d*e+
3*b^2*d^2)/((b*x+a)^2)^(5/2)
```

Maxima [A] time = 1.14905, size = 178, normalized size = 1.42

$$-\frac{2de}{3\left(b^2x^2+2abx+a^2\right)^{\frac{3}{2}}b^2}-\frac{a^2b^2e^2}{4\left(b^2\right)^{\frac{9}{2}}\left(x+\frac{a}{b}\right)^4}+\frac{2abe^2}{3\left(b^2\right)^{\frac{7}{2}}\left(x+\frac{a}{b}\right)^3}-\frac{e^2}{2\left(b^2\right)^{\frac{5}{2}}\left(x+\frac{a}{b}\right)^2}-\frac{d^2}{4\left(b^2\right)^{\frac{5}{2}}\left(x+\frac{a}{b}\right)^4}+\frac{ade}{2\left(b^2\right)^{\frac{5}{2}}b\left(x+\frac{a}{b}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*d*e/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^2) - 1/4*a^2*b^2*e^2/((b^2)^(9/
2)*(x + a/b)^4) + 2/3*a*b*e^2/((b^2)^(7/2)*(x + a/b)^3) - 1/2*e^2/((b^2)^(5
/2)*(x + a/b)^2) - 1/4*d^2/((b^2)^(5/2)*(x + a/b)^4) + 1/2*a*d*e/((b^2)^(5/
2)*b*(x + a/b)^4)
```

Fricas [A] time = 1.51971, size = 201, normalized size = 1.61

$$-\frac{6b^2e^2x^2+3b^2d^2+2abde+a^2e^2+4\left(2b^2de+abe^2\right)x}{12\left(b^7x^4+4ab^6x^3+6a^2b^5x^2+4a^3b^4x+a^4b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^2*e^2*x^2 + 3*b^2*d^2 + 2*a*b*d*e + a^2*e^2 + 4*(2*b^2*d*e + a*b
*e^2)*x)/(b^7*x^4 + 4*a*b^6*x^3 + 6*a^2*b^5*x^2 + 4*a^3*b^4*x + a^4*b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^2}{\left((a+bx)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Integral((d + e*x)**2/((a + b*x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.1609 $\int \frac{d+ex}{(a^2+2abx+b^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=71

$$-\frac{bd-ae}{4b^2(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}-\frac{e}{3b^2\left(a^2+2abx+b^2x^2\right)^{3/2}}$$

```
[Out] -e/(3*b^2*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)) - (b*d - a*e)/(4*b^2*(a + b*x)*(
a^2 + 2*a*b*x + b^2*x^2)^(3/2))
```

Rubi [A] time = 0.0214001, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {640, 607}

$$-\frac{bd-ae}{4b^2(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}-\frac{e}{3b^2\left(a^2+2abx+b^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] -e/(3*b^2*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)) - (b*d - a*e)/(4*b^2*(a + b*x)*(
a^2 + 2*a*b*x + b^2*x^2)^(3/2))
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx &= -\frac{e}{3b^2\left(a^2+2abx+b^2x^2\right)^{3/2}}+\frac{\left(2b^2d-2abe\right)\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx}{2b^2}\\ &= -\frac{e}{3b^2\left(a^2+2abx+b^2x^2\right)^{3/2}}-\frac{bd-ae}{4b^2(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0159398, size = 39, normalized size = 0.55

$$\frac{-ae-3bd-4bex}{12b^2(a+bx)^3\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (-3*b*d - a*e - 4*b*e*x)/(12*b^2*(a + b*x)^3*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.15, size = 33, normalized size = 0.5

$$-\frac{(bx+a)(4\,bxe+ae+3\,bd)}{12\,b^2}\left((bx+a)^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] -1/12*(b*x+a)/b^2*(4*b*e*x+a*e+3*b*d)/((b*x+a)^2)^(5/2)
```

Maxima [A] time = 1.00733, size = 85, normalized size = 1.2

$$-\frac{e}{3\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}b^2}-\frac{d}{4\left(b^2\right)^{\frac{5}{2}}\left(x+\frac{a}{b}\right)^4}+\frac{ae}{4\left(b^2\right)^{\frac{5}{2}}b\left(x+\frac{a}{b}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*e/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^2) - 1/4*d/((b^2)^(5/2)*(x + a/b)
^4) + 1/4*a*e/((b^2)^(5/2)*b*(x + a/b)^4)
```

Fricas [A] time = 1.59836, size = 128, normalized size = 1.8

$$-\frac{4\,bex+3\,bd+ae}{12\left(b^6x^4+4\,ab^5x^3+6\,a^2b^4x^2+4\,a^3b^3x+a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(4*b*e*x + 3*b*d + a*e)/(b^6*x^4 + 4*a*b^5*x^3 + 6*a^2*b^4*x^2 + 4*a^
3*b^3*x + a^4*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{d+ex}{\left((a+bx)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Integral((d + e*x)/((a + b*x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.1610 $\int \frac{1}{(a^2+2abx+b^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{4b(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}$$

```
[Out] -1/(4*b*(a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))
```

Rubi [A] time = 0.0045317, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {607}

$$-\frac{1}{4b(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(-5/2),x]
```

```
[Out] -1/(4*b*(a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx=-\frac{1}{4b(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}$$

Mathematica [A] time = 0.0106671, size = 23, normalized size = 0.68

$$-\frac{a+bx}{4b\left((a+bx)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(-5/2),x]
```

```
[Out] -(a + b*x)/(4*b*((a + b*x)^2)^(5/2))
```

Maple [A] time = 0.041, size = 20, normalized size = 0.6

$$-\frac{bx+a}{4\,b}\left((bx+a)^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] -1/4*(b*x+a)/b/((b*x+a)^2)^(5/2)
```

Maxima [A] time = 1.03628, size = 22, normalized size = 0.65

$$-\frac{1}{4\left(b^2\right)^{\frac{5}{2}}\left(x+\frac{a}{b}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4/((b^2)^(5/2)*(x + a/b)^4)
```

Fricas [A] time = 1.55644, size = 92, normalized size = 2.71

$$-\frac{1}{4\left(b^5x^4+4\,ab^4x^3+6\,a^2b^3x^2+4\,a^3b^2x+a^4b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/4/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x + b**2*x**2)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.1611 $\int \frac{1}{(d+ex)\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=253

$$\frac{e^3}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^4}-\frac{e^2}{2(a+bx)\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3}+\frac{e^4(a+bx)\log(a+bx)}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^5}-\frac{e^4(a+bx}{\sqrt{a^2+2abx}}$$

```
[Out] e^3/((b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - 1/(4*(b*d - a*e)*(a + b
*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + e/(3*(b*d - a*e)^2*(a + b*x)^2*Sqrt[
a^2 + 2*a*b*x + b^2*x^2]) - e^2/(2*(b*d - a*e)^3*(a + b*x)*Sqrt[a^2 + 2*a*b
*x + b^2*x^2]) + (e^4*(a + b*x)*Log[a + b*x])/((b*d - a*e)^5*Sqrt[a^2 + 2*a
*b*x + b^2*x^2]) - (e^4*(a + b*x)*Log[d + e*x])/((b*d - a*e)^5*Sqrt[a^2 + 2
*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.157273, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 44}

$$\frac{e^3}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^4}-\frac{e^2}{2(a+bx)\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3}+\frac{e^4(a+bx)\log(a+bx)}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^5}-\frac{e^4(a+bx}{\sqrt{a^2+2abx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

```
[Out] e^3/((b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - 1/(4*(b*d - a*e)*(a + b
*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + e/(3*(b*d - a*e)^2*(a + b*x)^2*Sqrt[
a^2 + 2*a*b*x + b^2*x^2]) - e^2/(2*(b*d - a*e)^3*(a + b*x)*Sqrt[a^2 + 2*a*b
*x + b^2*x^2]) + (e^4*(a + b*x)*Log[a + b*x])/((b*d - a*e)^5*Sqrt[a^2 + 2*a
*b*x + b^2*x^2]) - (e^4*(a + b*x)*Log[d + e*x])/((b*d - a*e)^5*Sqrt[a^2 + 2
*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a^2+2abx+b^2x^2\right)^{5/2}}dx = \frac{\left(b^4\left(ab+b^2x\right)\right)\int \frac{1}{\left(ab+b^2x\right)^5(d+ex)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= \frac{\left(b^4\left(ab+b^2x\right)\right)\int \left(\frac{1}{b^4(bd-ae)(a+bx)^5} - \frac{e}{b^4(bd-ae)^2(a+bx)^4} + \frac{e^2}{b^4(bd-ae)^3(a+bx)^3} - \frac{e^3}{b^4(bd-ae)^4(a+b}\right.}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= \frac{e^3}{(bd-ae)^4\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{4(bd-ae)(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{}{3(bd-a}$$

Mathematica [A] time = 0.123977, size = 163, normalized size = 0.64

$$\frac{-(bd-ae)\left(a^2be^2(23d-52ex)-25a^3e^3+ab^2e\left(-13d^2+20dex-42e^2x^2\right)+b^3\left(-4d^2ex+3d^3+6de^2x^2-12e^3x^3\right)\right)-12e^4(}{12(a+bx)^3\sqrt{(a+bx)^2}(bd-ae)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

```
[Out] (-((b*d - a*e)*(-25*a^3*e^3 + a^2*b*e^2*(23*d - 52*e*x) + a*b^2*e*(-13*d^2
+ 20*d*e*x - 42*e^2*x^2) + b^3*(3*d^3 - 4*d^2*e*x + 6*d*e^2*x^2 - 12*e^3*x^
3))) + 12*e^4*(a + b*x)^4*Log[a + b*x] - 12*e^4*(a + b*x)^4*Log[d + e*x])/(
12*(b*d - a*e)^5*(a + b*x)^3*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.204, size = 359, normalized size = 1.4

$$\frac{\left(12\ln(ex+d)x^4b^4e^4 - 12\ln(bx+a)x^4b^4e^4 + 48\ln(ex+d)x^3ab^3e^4 - 48\ln(bx+a)x^3ab^3e^4 + 72\ln(ex+d)x^2a^2b^2e^4 -\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] 1/12*(12*ln(e*x+d)*x^4*b^4*e^4-12*ln(b*x+a)*x^4*b^4*e^4+48*ln(e*x+d)*x^3*a*
b^3*e^4-48*ln(b*x+a)*x^3*a*b^3*e^4+72*ln(e*x+d)*x^2*a^2*b^2*e^4-72*ln(b*x+a
)*x^2*a^2*b^2*e^4+12*x^3*a*b^3*e^4-12*x^3*b^4*d*e^3+48*ln(e*x+d)*x*a^3*b*e^
4-48*ln(b*x+a)*x*a^3*b*e^4+42*x^2*a^2*b^2*e^4-48*x^2*a*b^3*d*e^3+6*x^2*b^4*
d^2*e^2+12*ln(e*x+d)*a^4*e^4-12*ln(b*x+a)*a^4*e^4+52*x*a^3*b*e^4-72*x*a^2*b
^2*d*e^3+24*x*a*b^3*d^2*e^2-4*x*b^4*d^3*e+25*a^4*e^4-48*a^3*b*d*e^3+36*d^2*
e^2*a^2*b^2-16*a*b^3*d^3*e+3*b^4*d^4)*(b*x+a)/(a*e-b*d)^5/((b*x+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.69628, size = 1320, normalized size = 5.22

$$-\frac{3 b^4 d^4 - 16 a b^3 d^3 e + 36 a^2 b^2 d^2 e^2 - 48 a^3 b d e^3 + 25 a^4 e^4 - 12 (b^4 d e^3 - a b^3 e^4) x^3 + 6 (}{12 (a^4 b^5 d^5 - 5 a^5 b^4 d^4 e + 10 a^6 b^3 d^3 e^2 - 10 a^7 b^2 d^2 e^3 + 5 a^8 b d e^4 - a^9 e^5 + (b^9 d^5 - 5 a b^8 d^4 e + 10 a^2 b^7 d^3 e^2 - 10 a^3 b^6 d^2 e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*b^4*d^4 - 16*a*b^3*d^3*e + 36*a^2*b^2*d^2*e^2 - 48*a^3*b*d*e^3 + 2
5*a^4*e^4 - 12*(b^4*d*e^3 - a*b^3*e^4)*x^3 + 6*(b^4*d^2*e^2 - 8*a*b^3*d*e^3
 + 7*a^2*b^2*e^4)*x^2 - 4*(b^4*d^3*e - 6*a*b^3*d^2*e^2 + 18*a^2*b^2*d*e^3 -
 13*a^3*b*e^4)*x - 12*(b^4*e^4*x^4 + 4*a*b^3*e^4*x^3 + 6*a^2*b^2*e^4*x^2 +
4*a^3*b*e^4*x + a^4*e^4)*log(b*x + a) + 12*(b^4*e^4*x^4 + 4*a*b^3*e^4*x^3 +
 6*a^2*b^2*e^4*x^2 + 4*a^3*b*e^4*x + a^4*e^4)*log(e*x + d))/(a^4*b^5*d^5 -
5*a^5*b^4*d^4*e + 10*a^6*b^3*d^3*e^2 - 10*a^7*b^2*d^2*e^3 + 5*a^8*b*d*e^4 -
 a^9*e^5 + (b^9*d^5 - 5*a*b^8*d^4*e + 10*a^2*b^7*d^3*e^2 - 10*a^3*b^6*d^2*e
^3 + 5*a^4*b^5*d*e^4 - a^5*b^4*e^5)*x^4 + 4*(a*b^8*d^5 - 5*a^2*b^7*d^4*e +
10*a^3*b^6*d^3*e^2 - 10*a^4*b^5*d^2*e^3 + 5*a^5*b^4*d*e^4 - a^6*b^3*e^5)*x^
3 + 6*(a^2*b^7*d^5 - 5*a^3*b^6*d^4*e + 10*a^4*b^5*d^3*e^2 - 10*a^5*b^4*d^2*
e^3 + 5*a^6*b^3*d*e^4 - a^7*b^2*e^5)*x^2 + 4*(a^3*b^6*d^5 - 5*a^4*b^5*d^4*e
 + 10*a^5*b^4*d^3*e^2 - 10*a^6*b^3*d^2*e^3 + 5*a^7*b^2*d*e^4 - a^8*b*e^5)*x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex) \left((a + bx)^2\right)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Integral(1/((d + e*x)*((a + b*x)**2)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.1612 $\int \frac{1}{(d+ex)^2\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=307

$$\frac{e^4(a+bx)}{\sqrt{a^2+2abx+b^2x^2}(d+ex)(bd-ae)^5}+\frac{4be^3}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^5}-\frac{3be^2}{2(a+bx)\sqrt{a^2+2abx+b^2x^2}(bd-ae)^4}+\frac{5be^4(}{\sqrt{a^2+2}}$$

```
[Out] (4*b*e^3)/((b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - b/(4*(b*d - a*e)^
2*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (2*b*e)/(3*(b*d - a*e)^3*(a
+ b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (3*b*e^2)/(2*(b*d - a*e)^4*(a + b
*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (e^4*(a + b*x))/((b*d - a*e)^5*(d + e*
x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (5*b*e^4*(a + b*x)*Log[a + b*x])/((b*d
- a*e)^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (5*b*e^4*(a + b*x)*Log[d + e*x])/
((b*d - a*e)^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.210104, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 44}

$$\frac{e^4(a+bx)}{\sqrt{a^2+2abx+b^2x^2}(d+ex)(bd-ae)^5}+\frac{4be^3}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^5}-\frac{3be^2}{2(a+bx)\sqrt{a^2+2abx+b^2x^2}(bd-ae)^4}+\frac{5be^4(}{\sqrt{a^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

```
[Out] (4*b*e^3)/((b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - b/(4*(b*d - a*e)^
2*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (2*b*e)/(3*(b*d - a*e)^3*(a
+ b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (3*b*e^2)/(2*(b*d - a*e)^4*(a + b
*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (e^4*(a + b*x))/((b*d - a*e)^5*(d + e*
x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (5*b*e^4*(a + b*x)*Log[a + b*x])/((b*d
- a*e)^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (5*b*e^4*(a + b*x)*Log[d + e*x])/
((b*d - a*e)^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x\right)\right)\int \frac{1}{\left(ab+b^2x\right)^5(d+ex)^2}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= \frac{\left(b^4\left(ab+b^2x\right)\right)\int \left(\frac{1}{b^3(bd-ae)^2(a+bx)^5} - \frac{2e}{b^3(bd-ae)^3(a+bx)^4} + \frac{3e^2}{b^3(bd-ae)^4(a+bx)^3} - \frac{4}{b^3(bd-ae}\right.}{\sqrt{a^2+2abx+b^2}}$$

$$= \frac{4be^3}{(bd-ae)^5\sqrt{a^2+2abx+b^2x^2}} - \frac{b}{4(bd-ae)^2(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{}{3(b}$$

Mathematica [A] time = 0.119407, size = 167, normalized size = 0.54

$$\frac{\frac{12e^4(a+bx)^3(bd-ae)}{d+ex} + 48be^3(a+bx)^2(bd-ae) - 18be^2(a+bx)(bd-ae)^2 - 60be^4(a+bx)^3\log(d+ex) - \frac{3b(bd-ae)^4}{a+bx} + 8be(bd}{12\left((a+bx)^2\right)^{3/2}(bd-ae)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

```
[Out] (8*b*e*(b*d - a*e)^3 - (3*b*(b*d - a*e)^4)/(a + b*x) - 18*b*e^2*(b*d - a*e)
^2*(a + b*x) + 48*b*e^3*(b*d - a*e)*(a + b*x)^2 + (12*e^4*(b*d - a*e)*(a +
b*x)^3)/(d + e*x) + 60*b*e^4*(a + b*x)^3*Log[a + b*x] - 60*b*e^4*(a + b*x)^
3*Log[d + e*x])/(12*(b*d - a*e)^6*((a + b*x)^2)^(3/2))
```

Maple [B] time = 0.211, size = 651, normalized size = 2.1

$$-\frac{\left(240\ln(ex+d)xa^3b^2de^4 + 360\ln(ex+d)x^2a^2b^3de^4 + 240\ln(ex+d)x^2a^3b^2e^5 + 12a^5e^5 + 3b^5d^5 + 65de^4a^4b + 6(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] -1/12*(240*ln(e*x+d)*x*a^3*b^2*d*e^4+360*ln(e*x+d)*x^2*a^2*b^3*d*e^4+240*ln
(e*x+d)*x^2*a^3*b^2*e^5+12*a^5*e^5+3*b^5*d^5+65*d*e^4*a^4*b+60*ln(e*x+d)*x^
5*b^5*e^5-5*x*b^5*d^4*e+60*x^4*a*b^4*e^5-60*x^4*b^5*d*e^4+210*x^3*a^2*b^3*e
^5-30*x^3*b^5*d^2*e^3+260*x^2*a^3*b^2*e^5+10*x^2*b^5*d^3*e^2+125*x*a^4*b*e^
5-240*ln(b*x+a)*x^3*a*b^4*d*e^4+60*ln(e*x+d)*x*a^4*b*e^5-120*x^2*a*b^4*d^2*
e^3-150*x^2*a^2*b^3*d*e^4-180*x^3*a*b^4*d*e^4+60*ln(e*x+d)*a^4*b*d*e^4+60*a
^2*b^3*d^3*e^2-120*a^3*b^2*d^2*e^3+40*x*a*b^4*d^3*e^2+20*x*a^3*b^2*d*e^4-18
0*x*a^2*b^3*d^2*e^3-60*ln(b*x+a)*x^5*b^5*e^5+360*ln(e*x+d)*x^3*a^2*b^3*e^5-
240*ln(b*x+a)*x^2*a^3*b^2*e^5-60*ln(b*x+a)*x*a^4*b*e^5-60*ln(b*x+a)*a^4*b*d
*e^4+240*ln(e*x+d)*x^4*a*b^4*e^5+60*ln(e*x+d)*x^4*b^5*d*e^4-240*ln(b*x+a)*x
^4*a*b^4*e^5-60*ln(b*x+a)*x^4*b^5*d*e^4-360*ln(b*x+a)*x^3*a^2*b^3*e^5-360*l
n(b*x+a)*x^2*a^2*b^3*d*e^4+240*ln(e*x+d)*x^3*a*b^4*d*e^4-240*ln(b*x+a)*x*a^
3*b^2*d*e^4-20*a*b^4*d^4*e)*(b*x+a)/(e*x+d)/(a*e-b*d)^6/((b*x+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.91157, size = 2196, normalized size = 7.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*b^5*d^5 - 20*a*b^4*d^4*e + 60*a^2*b^3*d^3*e^2 - 120*a^3*b^2*d^2*e^
3 + 65*a^4*b*d*e^4 + 12*a^5*e^5 - 60*(b^5*d*e^4 - a*b^4*e^5)*x^4 - 30*(b^5*
d^2*e^3 + 6*a*b^4*d*e^4 - 7*a^2*b^3*e^5)*x^3 + 10*(b^5*d^3*e^2 - 12*a*b^4*d
^2*e^3 - 15*a^2*b^3*d*e^4 + 26*a^3*b^2*e^5)*x^2 - 5*(b^5*d^4*e - 8*a*b^4*d^
3*e^2 + 36*a^2*b^3*d^2*e^3 - 4*a^3*b^2*d*e^4 - 25*a^4*b*e^5)*x - 60*(b^5*e^
5*x^5 + a^4*b*d*e^4 + (b^5*d*e^4 + 4*a*b^4*e^5)*x^4 + 2*(2*a*b^4*d*e^4 + 3*
a^2*b^3*e^5)*x^3 + 2*(3*a^2*b^3*d*e^4 + 2*a^3*b^2*e^5)*x^2 + (4*a^3*b^2*d*e
^4 + a^4*b*e^5)*x)*log(b*x + a) + 60*(b^5*e^5*x^5 + a^4*b*d*e^4 + (b^5*d*e^
4 + 4*a*b^4*e^5)*x^4 + 2*(2*a*b^4*d*e^4 + 3*a^2*b^3*e^5)*x^3 + 2*(3*a^2*b^3
*d*e^4 + 2*a^3*b^2*e^5)*x^2 + (4*a^3*b^2*d*e^4 + a^4*b*e^5)*x)*log(e*x + d)
)/(a^4*b^6*d^7 - 6*a^5*b^5*d^6*e + 15*a^6*b^4*d^5*e^2 - 20*a^7*b^3*d^4*e^3
+ 15*a^8*b^2*d^3*e^4 - 6*a^9*b*d^2*e^5 + a^10*d*e^6 + (b^10*d^6*e - 6*a*b^9
*d^5*e^2 + 15*a^2*b^8*d^4*e^3 - 20*a^3*b^7*d^3*e^4 + 15*a^4*b^6*d^2*e^5 - 6
*a^5*b^5*d*e^6 + a^6*b^4*e^7)*x^5 + (b^10*d^7 - 2*a*b^9*d^6*e - 9*a^2*b^8*d
^5*e^2 + 40*a^3*b^7*d^4*e^3 - 65*a^4*b^6*d^3*e^4 + 54*a^5*b^5*d^2*e^5 - 23*
a^6*b^4*d*e^6 + 4*a^7*b^3*e^7)*x^4 + 2*(2*a*b^9*d^7 - 9*a^2*b^8*d^6*e + 12*
a^3*b^7*d^5*e^2 + 5*a^4*b^6*d^4*e^3 - 30*a^5*b^5*d^3*e^4 + 33*a^6*b^4*d^2*e
^5 - 16*a^7*b^3*d*e^6 + 3*a^8*b^2*e^7)*x^3 + 2*(3*a^2*b^8*d^7 - 16*a^3*b^7*
d^6*e + 33*a^4*b^6*d^5*e^2 - 30*a^5*b^5*d^4*e^3 + 5*a^6*b^4*d^3*e^4 + 12*a^
7*b^3*d^2*e^5 - 9*a^8*b^2*d*e^6 + 2*a^9*b*e^7)*x^2 + (4*a^3*b^7*d^7 - 23*a^
4*b^6*d^6*e + 54*a^5*b^5*d^5*e^2 - 65*a^6*b^4*d^4*e^3 + 40*a^7*b^3*d^3*e^4
- 9*a^8*b^2*d^2*e^5 - 2*a^9*b*d*e^6 + a^10*e^7)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^2 + 2\,abx + a^2\right)^{\frac{5}{2}}(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b^2*x^2 + 2*a*b*x + a^2)^(5/2)*(e*x + d)^2), x)
```

3.1613 $\int \frac{1}{(d+ex)^3\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=365

$$\frac{5be^4(a+bx)}{\sqrt{a^2+2abx+b^2x^2}(d+ex)(bd-ae)^6}+\frac{e^4(a+bx)}{2\sqrt{a^2+2abx+b^2x^2}(d+ex)^2(bd-ae)^5}+\frac{10b^2e^3}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^6}-\frac{}{(a+b}$$

```
[Out] (10*b^2*e^3)/((b*d - a*e)^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - b^2/(4*(b*d -
a*e)^3*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (b^2*e)/((b*d - a*e)^4*
(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (3*b^2*e^2)/((b*d - a*e)^5*(a
+ b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (e^4*(a + b*x))/(2*(b*d - a*e)^5*(d
 + e*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (5*b*e^4*(a + b*x))/((b*d - a*e)
^6*(d + e*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (15*b^2*e^4*(a + b*x)*Log[a +
 b*x])/((b*d - a*e)^7*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (15*b^2*e^4*(a + b*x
)*Log[d + e*x])/((b*d - a*e)^7*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.26331, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 44}

$$\frac{5be^4(a+bx)}{\sqrt{a^2+2abx+b^2x^2}(d+ex)(bd-ae)^6}+\frac{e^4(a+bx)}{2\sqrt{a^2+2abx+b^2x^2}(d+ex)^2(bd-ae)^5}+\frac{10b^2e^3}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^6}-\frac{}{(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

```
[Out] (10*b^2*e^3)/((b*d - a*e)^6*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - b^2/(4*(b*d -
a*e)^3*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (b^2*e)/((b*d - a*e)^4*
(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (3*b^2*e^2)/((b*d - a*e)^5*(a
+ b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (e^4*(a + b*x))/(2*(b*d - a*e)^5*(d
 + e*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (5*b*e^4*(a + b*x))/((b*d - a*e)
^6*(d + e*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (15*b^2*e^4*(a + b*x)*Log[a +
 b*x])/((b*d - a*e)^7*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (15*b^2*e^4*(a + b*x
)*Log[d + e*x])/((b*d - a*e)^7*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x\right)\right)\int \frac{1}{\left(ab+b^2x\right)^5(d+ex)^3}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= \frac{\left(b^4\left(ab+b^2x\right)\right)\int \left(\frac{1}{b^2(bd-ae)^3(a+bx)^5} - \frac{3e}{b^2(bd-ae)^4(a+bx)^4} + \frac{6e^2}{b^2(bd-ae)^5(a+bx)^3} - \frac{1\cdots}{b^2(bd-ae\cdots}\right.}{\sqrt{a^2+2\cdots}}$$

$$= \frac{10b^2e^3}{(bd-ae)^6\sqrt{a^2+2abx+b^2x^2}} - \frac{b^2}{4(bd-ae)^3(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{\cdots}{(bd\cdots}$$

Mathematica [A] time = 0.157949, size = 209, normalized size = 0.57

$$\frac{40b^2e^3(a+bx)^2(bd-ae) - 12b^2e^2(a+bx)(bd-ae)^2 - 60b^2e^4(a+bx)^3\log(d+ex) - \frac{b^2(bd-ae)^4}{a+bx} + 4b^2e(bd-ae)^3 + 60b^2\cdots}{4\left((a+bx)^2\right)^{3/2}(bd-ae)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

```
[Out] (4*b^2*e*(b*d - a*e)^3 - (b^2*(b*d - a*e)^4)/(a + b*x) - 12*b^2*e^2*(b*d -
a*e)^2*(a + b*x) + 40*b^2*e^3*(b*d - a*e)*(a + b*x)^2 + (2*e^4*(b*d - a*e)^
2*(a + b*x)^3)/(d + e*x)^2 + (20*b*e^4*(b*d - a*e)*(a + b*x)^3)/(d + e*x) +
 60*b^2*e^4*(a + b*x)^3*Log[a + b*x] - 60*b^2*e^4*(a + b*x)^3*Log[d + e*x])
/(4*(b*d - a*e)^7*((a + b*x)^2)^(3/2))
```

Maple [B] time = 0.235, size = 982, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] 1/4*(24*a^5*b*d*e^5-2*a^6*e^6-60*ln(b*x+a)*x^6*b^6*e^6+5*x^2*b^6*d^4*e^2+12
*x*a^5*b*e^6-2*x*b^6*d^5*e+260*x^3*a^3*b^3*e^6-20*x^3*b^6*d^3*e^3+125*x^2*a
^4*b^2*e^6+60*x^5*a*b^5*e^6-60*x^5*b^6*d*e^5+210*x^4*a^2*b^4*e^6-90*x^4*b^6
*d^2*e^4+60*ln(e*x+d)*x^6*b^6*e^6+240*ln(e*x+d)*x^5*a*b^5*e^6+120*ln(e*x+d)
*x^5*b^6*d*e^5-240*ln(b*x+a)*x^5*a*b^5*e^6-120*ln(b*x+a)*x^5*b^6*d*e^5+360*
ln(e*x+d)*x^4*a^2*b^4*e^6+60*ln(e*x+d)*x^4*b^6*d^2*e^4+240*ln(e*x+d)*x^3*a^
3*b^3*e^6-480*ln(b*x+a)*x^4*a*b^5*d*e^5-8*a*b^5*d^5*e-330*x^2*a^2*b^4*d^2*e
^4-80*x^2*a*b^5*d^3*e^3-100*x*a^3*b^3*d^2*e^4-120*x*a^2*b^4*d^3*e^3+20*x*a*
b^5*d^4*e^2-300*x^3*a*b^5*d^2*e^4-360*ln(b*x+a)*x^4*a^2*b^4*e^6-60*ln(b*x+a
)*x^4*b^6*d^2*e^4-240*ln(b*x+a)*x^3*a^3*b^3*e^6-60*ln(b*x+a)*x^2*a^4*b^2*e^
6+d^6*b^6+480*ln(e*x+d)*x^2*a^3*b^3*d*e^5+360*ln(e*x+d)*x^2*a^2*b^4*d^2*e^4
-60*ln(b*x+a)*a^4*b^2*d^2*e^4+60*x^3*a^2*b^4*d*e^5+280*x^2*a^3*b^3*d*e^5+19
0*x*a^4*b^2*d*e^5-120*x^4*a*b^5*d*e^5+30*a^2*b^4*d^4*e^2+480*ln(e*x+d)*x^4*
a*b^5*d*e^5+720*ln(e*x+d)*x^3*a^2*b^4*d*e^5+240*ln(e*x+d)*x^3*a*b^5*d^2*e^4
+35*d^2*e^4*a^4*b^2-80*b^3*a^3*d^3*e^3-120*ln(b*x+a)*x*a^4*b^2*d*e^5-240*ln
(b*x+a)*x*a^3*b^3*d^2*e^4-720*ln(b*x+a)*x^3*a^2*b^4*d*e^5-240*ln(b*x+a)*x^3
*a*b^5*d^2*e^4-480*ln(b*x+a)*x^2*a^3*b^3*d*e^5-360*ln(b*x+a)*x^2*a^2*b^4*d^
2*e^4+60*ln(e*x+d)*a^4*b^2*d^2*e^4+60*ln(e*x+d)*x^2*a^4*b^2*e^6+120*ln(e*x+
d)*x*a^4*b^2*d*e^5+240*ln(e*x+d)*x*a^3*b^3*d^2*e^4)*(b*x+a)/(e*x+d)^2/(a*e-
```

```
b*d)^7/((b*x+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.13331, size = 3136, normalized size = 8.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/4*(b^6*d^6 - 8*a*b^5*d^5*e + 30*a^2*b^4*d^4*e^2 - 80*a^3*b^3*d^3*e^3 + 3
5*a^4*b^2*d^2*e^4 + 24*a^5*b*d*e^5 - 2*a^6*e^6 - 60*(b^6*d*e^5 - a*b^5*e^6)
*x^5 - 30*(3*b^6*d^2*e^4 + 4*a*b^5*d*e^5 - 7*a^2*b^4*e^6)*x^4 - 20*(b^6*d^3
*e^3 + 15*a*b^5*d^2*e^4 - 3*a^2*b^4*d*e^5 - 13*a^3*b^3*e^6)*x^3 + 5*(b^6*d^
4*e^2 - 16*a*b^5*d^3*e^3 - 66*a^2*b^4*d^2*e^4 + 56*a^3*b^3*d*e^5 + 25*a^4*b
^2*e^6)*x^2 - 2*(b^6*d^5*e - 10*a*b^5*d^4*e^2 + 60*a^2*b^4*d^3*e^3 + 50*a^3
*b^3*d^2*e^4 - 95*a^4*b^2*d*e^5 - 6*a^5*b*e^6)*x - 60*(b^6*e^6*x^6 + a^4*b^
2*d^2*e^4 + 2*(b^6*d*e^5 + 2*a*b^5*e^6)*x^5 + (b^6*d^2*e^4 + 8*a*b^5*d*e^5
+ 6*a^2*b^4*e^6)*x^4 + 4*(a*b^5*d^2*e^4 + 3*a^2*b^4*d*e^5 + a^3*b^3*e^6)*x^
3 + (6*a^2*b^4*d^2*e^4 + 8*a^3*b^3*d*e^5 + a^4*b^2*e^6)*x^2 + 2*(2*a^3*b^3*
d^2*e^4 + a^4*b^2*d*e^5)*x)*log(b*x + a) + 60*(b^6*e^6*x^6 + a^4*b^2*d^2*e^
4 + 2*(b^6*d*e^5 + 2*a*b^5*e^6)*x^5 + (b^6*d^2*e^4 + 8*a*b^5*d*e^5 + 6*a^2*
b^4*e^6)*x^4 + 4*(a*b^5*d^2*e^4 + 3*a^2*b^4*d*e^5 + a^3*b^3*e^6)*x^3 + (6*a
^2*b^4*d^2*e^4 + 8*a^3*b^3*d*e^5 + a^4*b^2*e^6)*x^2 + 2*(2*a^3*b^3*d^2*e^4
+ a^4*b^2*d*e^5)*x)*log(e*x + d))/(a^4*b^7*d^9 - 7*a^5*b^6*d^8*e + 21*a^6*b
^5*d^7*e^2 - 35*a^7*b^4*d^6*e^3 + 35*a^8*b^3*d^5*e^4 - 21*a^9*b^2*d^4*e^5 +
 7*a^10*b*d^3*e^6 - a^11*d^2*e^7 + (b^11*d^7*e^2 - 7*a*b^10*d^6*e^3 + 21*a^
2*b^9*d^5*e^4 - 35*a^3*b^8*d^4*e^5 + 35*a^4*b^7*d^3*e^6 - 21*a^5*b^6*d^2*e^
7 + 7*a^6*b^5*d*e^8 - a^7*b^4*e^9)*x^6 + 2*(b^11*d^8*e - 5*a*b^10*d^7*e^2 +
 7*a^2*b^9*d^6*e^3 + 7*a^3*b^8*d^5*e^4 - 35*a^4*b^7*d^4*e^5 + 49*a^5*b^6*d^
3*e^6 - 35*a^6*b^5*d^2*e^7 + 13*a^7*b^4*d*e^8 - 2*a^8*b^3*e^9)*x^5 + (b^11*
d^9 + a*b^10*d^8*e - 29*a^2*b^9*d^7*e^2 + 91*a^3*b^8*d^6*e^3 - 119*a^4*b^7*
d^5*e^4 + 49*a^5*b^6*d^4*e^5 + 49*a^6*b^5*d^3*e^6 - 71*a^7*b^4*d^2*e^7 + 34
*a^8*b^3*d*e^8 - 6*a^9*b^2*e^9)*x^4 + 4*(a*b^10*d^9 - 4*a^2*b^9*d^8*e + a^3
*b^8*d^7*e^2 + 21*a^4*b^7*d^6*e^3 - 49*a^5*b^6*d^5*e^4 + 49*a^6*b^5*d^4*e^5
 - 21*a^7*b^4*d^3*e^6 - a^8*b^3*d^2*e^7 + 4*a^9*b^2*d*e^8 - a^10*b*e^9)*x^3
 + (6*a^2*b^9*d^9 - 34*a^3*b^8*d^8*e + 71*a^4*b^7*d^7*e^2 - 49*a^5*b^6*d^6*
e^3 - 49*a^6*b^5*d^5*e^4 + 119*a^7*b^4*d^4*e^5 - 91*a^8*b^3*d^3*e^6 + 29*a^
9*b^2*d^2*e^7 - a^10*b*d*e^8 - a^11*e^9)*x^2 + 2*(2*a^3*b^8*d^9 - 13*a^4*b^
7*d^8*e + 35*a^5*b^6*d^7*e^2 - 49*a^6*b^5*d^6*e^3 + 35*a^7*b^4*d^5*e^4 - 7*
a^8*b^3*d^4*e^5 - 7*a^9*b^2*d^3*e^6 + 5*a^10*b*d^2*e^7 - a^11*d*e^8)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.1614 $\int (d+ex) \left(9+12x+4x^2\right)^{5/2} \, dx$

Optimal. Leaf size=50

$$\frac{1}{24}(2x+3)\left(4x^2+12x+9\right)^{5/2}(2d-3e)+\frac{1}{28}e\left(4x^2+12x+9\right)^{7/2}$$

```
[Out] ((2*d - 3*e)*(3 + 2*x)*(9 + 12*x + 4*x^2)^(5/2))/24 + (e*(9 + 12*x + 4*x^2)
^(7/2))/28
```

Rubi [A] time = 0.0127645, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {640, 609}

$$\frac{1}{24}(2x+3)\left(4x^2+12x+9\right)^{5/2}(2d-3e)+\frac{1}{28}e\left(4x^2+12x+9\right)^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(9 + 12*x + 4*x^2)^(5/2),x]
```

```
[Out] ((2*d - 3*e)*(3 + 2*x)*(9 + 12*x + 4*x^2)^(5/2))/24 + (e*(9 + 12*x + 4*x^2)
^(7/2))/28
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned} \int (d+ex)\left(9+12x+4x^2\right)^{5/2}\,dx &= \frac{1}{28}e\left(9+12x+4x^2\right)^{7/2}+\frac{1}{2}(2d-3e)\int \left(9+12x+4x^2\right)^{5/2}\,dx\\ &= \frac{1}{24}(2d-3e)(3+2x)\left(9+12x+4x^2\right)^{5/2}+\frac{1}{28}e\left(9+12x+4x^2\right)^{7/2} \end{aligned}$$

Mathematica [A] time = 0.0311574, size = 81, normalized size = 1.62

$$\frac{x\sqrt{(2x+3)^2}\left(14d\left(16x^5+144x^4+540x^3+1080x^2+1215x+729\right)+3ex\left(64x^5+560x^4+2016x^3+3780x^2+3780x+1\right.\right.}{42(2x+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(9 + 12*x + 4*x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[(3 + 2*x)^2]*(14*d*(729 + 1215*x + 1080*x^2 + 540*x^3 + 144*x^4 + 1
6*x^5) + 3*e*x*(1701 + 3780*x + 3780*x^2 + 2016*x^3 + 560*x^4 + 64*x^5)))/(
42*(3 + 2*x))
```

Maple [B] time = 0.081, size = 86, normalized size = 1.7

$$\frac{x\left(192\,ex^{6}+224\,x^{5}d+1680\,x^{5}e+2016\,dx^{4}+6048\,ex^{4}+7560\,dx^{3}+11340\,x^{3}e+15120\,dx^{2}+11340\,ex^{2}+17010\,dx+\right.}{42\,\left(3+2\,x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(4*x^2+12*x+9)^(5/2),x)
```

```
[Out] 1/42*x*(192*e*x^6+224*d*x^5+1680*e*x^5+2016*d*x^4+6048*e*x^4+7560*d*x^3+113
40*e*x^3+15120*d*x^2+11340*e*x^2+17010*d*x+5103*e*x+10206*d)*((3+2*x)^2)^(5
/2)/(3+2*x)^5
```

Maxima [A] time = 1.60213, size = 105, normalized size = 2.1

$$\frac{1}{28}\left(4\,x^{2}+12\,x+9\right)^{\frac{7}{2}}e+\frac{1}{6}\left(4\,x^{2}+12\,x+9\right)^{\frac{5}{2}}dx-\frac{1}{4}\left(4\,x^{2}+12\,x+9\right)^{\frac{5}{2}}ex+\frac{1}{4}\left(4\,x^{2}+12\,x+9\right)^{\frac{5}{2}}d-\frac{3}{8}\left(4\,x^{2}+12\,x+9\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^2+12*x+9)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/28*(4*x^2 + 12*x + 9)^(7/2)*e + 1/6*(4*x^2 + 12*x + 9)^(5/2)*d*x - 1/4*(4
*x^2 + 12*x + 9)^(5/2)*e*x + 1/4*(4*x^2 + 12*x + 9)^(5/2)*d - 3/8*(4*x^2 +
12*x + 9)^(5/2)*e
```

Fricas [A] time = 1.47588, size = 176, normalized size = 3.52

$$\frac{32}{7}\,ex^{7}+\frac{8}{3}\,(2\,d+15\,e)x^{6}+48\,(d+3\,e)x^{5}+90\,(2\,d+3\,e)x^{4}+90\,(4\,d+3\,e)x^{3}+\frac{81}{2}\,(10\,d+3\,e)x^{2}+243\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^2+12*x+9)^(5/2),x, algorithm="fricas")
```

```
[Out] 32/7*e*x^7 + 8/3*(2*d + 15*e)*x^6 + 48*(d + 3*e)*x^5 + 90*(2*d + 3*e)*x^4 +
 90*(4*d + 3*e)*x^3 + 81/2*(10*d + 3*e)*x^2 + 243*d*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(d+ex)\left((2x+3)^{2}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x**2+12*x+9)**(5/2),x)
```

```
[Out] Integral((d + e*x)*((2*x + 3)**2)**(5/2), x)
```

Giac [B] time = 1.15301, size = 223, normalized size = 4.46

$$\frac{32}{7}\,x^7e\mathrm{sgn}\left(2\,x+3\right)+\frac{16}{3}\,dx^6\mathrm{sgn}\left(2\,x+3\right)+40\,x^6e\mathrm{sgn}\left(2\,x+3\right)+48\,dx^5\mathrm{sgn}\left(2\,x+3\right)+144\,x^5e\mathrm{sgn}\left(2\,x+3\right)+180\,dx^4\mathrm{sg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^2+12*x+9)^(5/2),x, algorithm="giac")
```

```
[Out] 32/7*x^7*e*sgn(2*x + 3) + 16/3*d*x^6*sgn(2*x + 3) + 40*x^6*e*sgn(2*x + 3) +
 48*d*x^5*sgn(2*x + 3) + 144*x^5*e*sgn(2*x + 3) + 180*d*x^4*sgn(2*x + 3) +
270*x^4*e*sgn(2*x + 3) + 360*d*x^3*sgn(2*x + 3) + 270*x^3*e*sgn(2*x + 3) +
405*d*x^2*sgn(2*x + 3) + 243/2*x^2*e*sgn(2*x + 3) + 243*d*x*sgn(2*x + 3) +
243/56*(14*d - 3*e)*sgn(2*x + 3)
```

3.1615 $\int (d+ex)\left(9+12x+4x^2\right)^{3/2}\,dx$

Optimal. Leaf size=50

$$\frac{1}{16}(2x+3)\left(4x^2+12x+9\right)^{3/2}(2d-3e)+\frac{1}{20}e\left(4x^2+12x+9\right)^{5/2}$$

```
[Out] ((2*d - 3*e)*(3 + 2*x)*(9 + 12*x + 4*x^2)^(3/2))/16 + (e*(9 + 12*x + 4*x^2)
^(5/2))/20
```

Rubi [A] time = 0.0121757, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {640, 609}

$$\frac{1}{16}(2x+3)\left(4x^2+12x+9\right)^{3/2}(2d-3e)+\frac{1}{20}e\left(4x^2+12x+9\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(9 + 12*x + 4*x^2)^(3/2),x]
```

```
[Out] ((2*d - 3*e)*(3 + 2*x)*(9 + 12*x + 4*x^2)^(3/2))/16 + (e*(9 + 12*x + 4*x^2)
^(5/2))/20
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(9+12x+4x^2\right)^{3/2}\,dx&=\frac{1}{20}e\left(9+12x+4x^2\right)^{5/2}+\frac{1}{2}(2d-3e)\int\left(9+12x+4x^2\right)^{3/2}\,dx\\&=\frac{1}{16}(2d-3e)(3+2x)\left(9+12x+4x^2\right)^{3/2}+\frac{1}{20}e\left(9+12x+4x^2\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.0218325, size = 57, normalized size = 1.14

$$\frac{x\sqrt{(2x+3)^2}\left(10d\left(2x^3+12x^2+27x+27\right)+ex\left(16x^3+90x^2+180x+135\right)\right)}{20x+30}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(9 + 12*x + 4*x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[(3 + 2*x)^2]*(10*d*(27 + 27*x + 12*x^2 + 2*x^3) + e*x*(135 + 180*x
+ 90*x^2 + 16*x^3)))/(30 + 20*x)
```

Maple [A] time = 0.08, size = 62, normalized size = 1.2

$$\frac{x\left(16\,ex^4+20\,dx^3+90\,x^3e+120\,dx^2+180\,ex^2+270\,dx+135\,ex+270\,d\right)}{10\,(3+2\,x)^3}\left((3+2\,x)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(4*x^2+12*x+9)^(3/2),x)
```

```
[Out] 1/10*x*(16*e*x^4+20*d*x^3+90*e*x^3+120*d*x^2+180*e*x^2+270*d*x+135*e*x+270*
d)*((3+2*x)^2)^(3/2)/(3+2*x)^3
```

Maxima [A] time = 1.5557, size = 105, normalized size = 2.1

$$\frac{1}{20}\left(4\,x^2+12\,x+9\right)^{\frac{5}{2}}e+\frac{1}{4}\left(4\,x^2+12\,x+9\right)^{\frac{3}{2}}dx-\frac{3}{8}\left(4\,x^2+12\,x+9\right)^{\frac{3}{2}}ex+\frac{3}{8}\left(4\,x^2+12\,x+9\right)^{\frac{3}{2}}d-\frac{9}{16}\left(4\,x^2+12\,x+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^2+12*x+9)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/20*(4*x^2 + 12*x + 9)^(5/2)*e + 1/4*(4*x^2 + 12*x + 9)^(3/2)*d*x - 3/8*(4
*x^2 + 12*x + 9)^(3/2)*e*x + 3/8*(4*x^2 + 12*x + 9)^(3/2)*d - 9/16*(4*x^2 +
 12*x + 9)^(3/2)*e
```

Fricas [A] time = 1.58034, size = 107, normalized size = 2.14

$$\frac{8}{5}\,ex^5+(2\,d+9\,e)x^4+6\,(2\,d+3\,e)x^3+\frac{27}{2}\,(2\,d+e)x^2+27\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^2+12*x+9)^(3/2),x, algorithm="fricas")
```

```
[Out] 8/5*e*x^5 + (2*d + 9*e)*x^4 + 6*(2*d + 3*e)*x^3 + 27/2*(2*d + e)*x^2 + 27*d
*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(d+ex)\left((2x+3)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x**2+12*x+9)**(3/2),x)
```

```
[Out] Integral((d + e*x)*((2*x + 3)**2)**(3/2), x)
```

Giac [B] time = 1.19265, size = 155, normalized size = 3.1

$$\frac{8}{5}\,x^5 e\,\mathrm{sgn}\left(2\,x+3\right)+2\,dx^4\mathrm{sgn}\left(2\,x+3\right)+9\,x^4 e\,\mathrm{sgn}\left(2\,x+3\right)+12\,dx^3\mathrm{sgn}\left(2\,x+3\right)+18\,x^3 e\,\mathrm{sgn}\left(2\,x+3\right)+27\,dx^2\mathrm{sgn}\left(2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^2+12*x+9)^(3/2),x, algorithm="giac")
```

```
[Out] 8/5*x^5*e*sgn(2*x + 3) + 2*d*x^4*sgn(2*x + 3) + 9*x^4*e*sgn(2*x + 3) + 12*d
*x^3*sgn(2*x + 3) + 18*x^3*e*sgn(2*x + 3) + 27*d*x^2*sgn(2*x + 3) + 27/2*x^
2*e*sgn(2*x + 3) + 27*d*x*sgn(2*x + 3) + 81/80*(10*d - 3*e)*sgn(2*x + 3)
```

3.1616 $\int (d+ex)\sqrt{9+12x+4x^2}\,dx$

Optimal. Leaf size=50

$$\frac{1}{8}(2x+3)\sqrt{4x^2+12x+9}(2d-3e)+\frac{1}{12}e\left(4x^2+12x+9\right)^{3/2}$$

```
[Out] ((2*d - 3*e)*(3 + 2*x)*Sqrt[9 + 12*x + 4*x^2])/8 + (e*(9 + 12*x + 4*x^2)^(3
/2))/12
```

Rubi [A] time = 0.0120345, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {640, 609}

$$\frac{1}{8}(2x+3)\sqrt{4x^2+12x+9}(2d-3e)+\frac{1}{12}e\left(4x^2+12x+9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*Sqrt[9 + 12*x + 4*x^2],x]
```

```
[Out] ((2*d - 3*e)*(3 + 2*x)*Sqrt[9 + 12*x + 4*x^2])/8 + (e*(9 + 12*x + 4*x^2)^(3
/2))/12
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int (d+ex)\sqrt{9+12x+4x^2}\,dx &= \frac{1}{12}e\left(9+12x+4x^2\right)^{3/2}+\frac{1}{2}(2d-3e)\int \sqrt{9+12x+4x^2}\,dx\\ &= \frac{1}{8}(2d-3e)(3+2x)\sqrt{9+12x+4x^2}+\frac{1}{12}e\left(9+12x+4x^2\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.0115699, size = 38, normalized size = 0.76

$$\frac{x\sqrt{(2x+3)^2}(6d(x+3)+ex(4x+9))}{6(2x+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Sqrt[9 + 12*x + 4*x^2],x]
```

```
[Out] (x*Sqrt[(3 + 2*x)^2]*(6*d*(3 + x) + e*x*(9 + 4*x)))/(6*(3 + 2*x))
```

Maple [A] time = 0.079, size = 38, normalized size = 0.8

$$\frac{x\left(4ex^{2}+6dx+9ex+18d\right)\sqrt{(3+2x)^{2}}}{18+12x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(4*x^2+12*x+9)^(1/2),x)
```

```
[Out] 1/6*x*(4*e*x^2+6*d*x+9*e*x+18*d)*((3+2*x)^2)^(1/2)/(3+2*x)
```

Maxima [A] time = 1.59159, size = 105, normalized size = 2.1

$$\frac{1}{12}\left(4x^{2}+12x+9\right)^{\frac{3}{2}}e+\frac{1}{2}\sqrt{4x^{2}+12x+9}dx-\frac{3}{4}\sqrt{4x^{2}+12x+9}ex+\frac{3}{4}\sqrt{4x^{2}+12x+9}d-\frac{9}{8}\sqrt{4x^{2}+12x+9}e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^2+12*x+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(4*x^2 + 12*x + 9)^(3/2)*e + 1/2*sqrt(4*x^2 + 12*x + 9)*d*x - 3/4*sqrt
(4*x^2 + 12*x + 9)*e*x + 3/4*sqrt(4*x^2 + 12*x + 9)*d - 9/8*sqrt(4*x^2 + 12
*x + 9)*e
```

Fricas [A] time = 1.55504, size = 55, normalized size = 1.1

$$\frac{2}{3}ex^{3}+\frac{1}{2}(2d+3e)x^{2}+3dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^2+12*x+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*e*x^3 + 1/2*(2*d + 3*e)*x^2 + 3*d*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(d+ex)\sqrt{(2x+3)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x**2+12*x+9)**(1/2),x)
```

```
[Out] Integral((d + e*x)*sqrt((2*x + 3)**2), x)
```

Giac [A] time = 1.18821, size = 86, normalized size = 1.72

$$\frac{2}{3}x^{3}e\mathrm{sgn}(2x+3)+dx^{2}\mathrm{sgn}(2x+3)+\frac{3}{2}x^{2}e\mathrm{sgn}(2x+3)+3dx\mathrm{sgn}(2x+3)+\frac{9}{8}(2d-e)\mathrm{sgn}(2x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^2+12*x+9)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*x^3*e*sgn(2*x + 3) + d*x^2*sgn(2*x + 3) + 3/2*x^2*e*sgn(2*x + 3) + 3*d*
x*sgn(2*x + 3) + 9/8*(2*d - e)*sgn(2*x + 3)
```

3.1617 $\int \frac{d+ex}{\sqrt{9+12x+4x^2}}\,dx$

Optimal. Leaf size=56

$$\frac{(2x+3)(2d-3e)\log(2x+3)}{4\sqrt{4x^2+12x+9}}+\frac{1}{4}e\sqrt{4x^2+12x+9}$$

```
[Out] (e*Sqrt[9 + 12*x + 4*x^2])/4 + ((2*d - 3*e)*(3 + 2*x)*Log[3 + 2*x])/(4*Sqrt
[9 + 12*x + 4*x^2])
```

Rubi [A] time = 0.0156218, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {640, 608, 31}

$$\frac{(2x+3)(2d-3e)\log(2x+3)}{4\sqrt{4x^2+12x+9}}+\frac{1}{4}e\sqrt{4x^2+12x+9}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/Sqrt[9 + 12*x + 4*x^2],x]
```

```
[Out] (e*Sqrt[9 + 12*x + 4*x^2])/4 + ((2*d - 3*e)*(3 + 2*x)*Log[3 + 2*x])/(4*Sqrt
[9 + 12*x + 4*x^2])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex}{\sqrt{9+12x+4x^2}}\,dx &= \frac{1}{4}e\sqrt{9+12x+4x^2}+\frac{1}{2}(2d-3e)\int \frac{1}{\sqrt{9+12x+4x^2}}\,dx\\ &= \frac{1}{4}e\sqrt{9+12x+4x^2}+\frac{((2d-3e)(6+4x))\int \frac{1}{6+4x}\,dx}{2\sqrt{9+12x+4x^2}}\\ &= \frac{1}{4}e\sqrt{9+12x+4x^2}+\frac{(2d-3e)(3+2x)\log(3+2x)}{4\sqrt{9+12x+4x^2}}\end{aligned}$$

Mathematica [A] time = 0.0153117, size = 42, normalized size = 0.75

$$\frac{(2x+3)((2d-3e)\log(2x+3)+e(2x+3))}{4\sqrt{(2x+3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/Sqrt[9 + 12*x + 4*x^2],x]
```

```
[Out] ((3 + 2*x)*(e*(3 + 2*x) + (2*d - 3*e)*Log[3 + 2*x]))/(4*Sqrt[(3 + 2*x)^2])
```

Maple [A] time = 0.12, size = 40, normalized size = 0.7

$$\frac{(3+2x)(2\ln(3+2x)d-3e\ln(3+2x)+2ex)}{4}\frac{1}{\sqrt{(3+2x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(4*x^2+12*x+9)^(1/2),x)
```

```
[Out] 1/4*(3+2*x)*(2*ln(3+2*x)*d-3*e*ln(3+2*x)+2*e*x)/((3+2*x)^2)^(1/2)
```

Maxima [A] time = 1.55833, size = 41, normalized size = 0.73

$$\frac{1}{2}d\log\left(x+\frac{3}{2}\right)-\frac{3}{4}e\log\left(x+\frac{3}{2}\right)+\frac{1}{4}\sqrt{4x^2+12x+9}e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(4*x^2+12*x+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*d*log(x + 3/2) - 3/4*e*log(x + 3/2) + 1/4*sqrt(4*x^2 + 12*x + 9)*e
```

Fricas [A] time = 1.60149, size = 54, normalized size = 0.96

$$\frac{1}{2}ex+\frac{1}{4}(2d-3e)\log(2x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(4*x^2+12*x+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*e*x + 1/4*(2*d - 3*e)*log(2*x + 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{d+ex}{\sqrt{(2x+3)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(4*x**2+12*x+9)**(1/2),x)
```

```
[Out] Integral((d + e*x)/sqrt((2*x + 3)**2), x)
```

Giac [A] time = 1.1582, size = 62, normalized size = 1.11

$$-\frac{1}{4}\,(2\,d-3\,e)\log\left(\left|-2\,x+\sqrt{4\,x^{2}+12\,x+9}-3\right|\right)+\frac{1}{4}\,\sqrt{4\,x^{2}+12\,x+9}e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(4*x^2+12*x+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(2*d - 3*e)*log(abs(-2*x + sqrt(4*x^2 + 12*x + 9) - 3)) + 1/4*sqrt(4*x
^2 + 12*x + 9)*e
```

3.1618 $\int \frac{d+ex}{(9+12x+4x^2)^{3/2}}\,dx$

Optimal. Leaf size=52

$$-\frac{2d-3e}{8(2x+3)\sqrt{4x^2+12x+9}}-\frac{e}{4\sqrt{4x^2+12x+9}}$$

```
[Out] -e/(4*Sqrt[9 + 12*x + 4*x^2]) - (2*d - 3*e)/(8*(3 + 2*x)*Sqrt[9 + 12*x + 4*
x^2])
```

Rubi [A] time = 0.0135592, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {640, 607}

$$-\frac{2d-3e}{8(2x+3)\sqrt{4x^2+12x+9}}-\frac{e}{4\sqrt{4x^2+12x+9}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(9 + 12*x + 4*x^2)^(3/2),x]
```

```
[Out] -e/(4*Sqrt[9 + 12*x + 4*x^2]) - (2*d - 3*e)/(8*(3 + 2*x)*Sqrt[9 + 12*x + 4*
x^2])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex}{(9+12x+4x^2)^{3/2}}\,dx &= -\frac{e}{4\sqrt{9+12x+4x^2}}+\frac{1}{2}(2d-3e)\int \frac{1}{(9+12x+4x^2)^{3/2}}\,dx\\ &= -\frac{e}{4\sqrt{9+12x+4x^2}}-\frac{2d-3e}{8(3+2x)\sqrt{9+12x+4x^2}}\end{aligned}$$

Mathematica [A] time = 0.0113364, size = 34, normalized size = 0.65

$$\frac{-2d-e(4x+3)}{8(2x+3)\sqrt{(2x+3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(9 + 12*x + 4*x^2)^(3/2),x]

[Out] (-2*d - e*(3 + 4*x))/(8*(3 + 2*x)*Sqrt[(3 + 2*x)^2])
```

Maple [A] time = 0.079, size = 28, normalized size = 0.5

$$-\frac{(3+2x)(4ex+2d+3e)}{8}\left((3+2x)^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(4*x^2+12*x+9)^(3/2),x)

[Out] -1/8*(3+2*x)*(4*e*x+2*d+3*e)/((3+2*x)^2)^(3/2)
```

Maxima [A] time = 1.56026, size = 49, normalized size = 0.94

$$-\frac{e}{4\sqrt{4x^2+12x+9}}-\frac{d}{4(2x+3)^2}+\frac{3e}{8(2x+3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(4*x^2+12*x+9)^(3/2),x, algorithm="maxima")

[Out] -1/4*e/sqrt(4*x^2 + 12*x + 9) - 1/4*d/(2*x + 3)^2 + 3/8*e/(2*x + 3)^2
```

Fricas [A] time = 1.54412, size = 61, normalized size = 1.17

$$-\frac{4ex+2d+3e}{8\left(4x^2+12x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(4*x^2+12*x+9)^(3/2),x, algorithm="fricas")

[Out] -1/8*(4*e*x + 2*d + 3*e)/(4*x^2 + 12*x + 9)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{d+ex}{\left((2x+3)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(4*x**2+12*x+9)**(3/2),x)

[Out] Integral((d + e*x)/((2*x + 3)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(4*x^2+12*x+9)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.1619 $\int \frac{d+ex}{(9+12x+4x^2)^{5/2}}\,dx$

Optimal. Leaf size=52

$$-\frac{2d-3e}{16(2x+3)\left(4x^2+12x+9\right)^{3/2}}-\frac{e}{12\left(4x^2+12x+9\right)^{3/2}}$$

```
[Out] -e/(12*(9 + 12*x + 4*x^2)^(3/2)) - (2*d - 3*e)/(16*(3 + 2*x)*(9 + 12*x + 4*
x^2)^(3/2))
```

Rubi [A] time = 0.0126159, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {640, 607}

$$-\frac{2d-3e}{16(2x+3)\left(4x^2+12x+9\right)^{3/2}}-\frac{e}{12\left(4x^2+12x+9\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(9 + 12*x + 4*x^2)^(5/2),x]
```

```
[Out] -e/(12*(9 + 12*x + 4*x^2)^(3/2)) - (2*d - 3*e)/(16*(3 + 2*x)*(9 + 12*x + 4*
x^2)^(3/2))
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex}{\left(9+12x+4x^2\right)^{5/2}}\,dx &= -\frac{e}{12\left(9+12x+4x^2\right)^{3/2}}+\frac{1}{2}(2d-3e)\int \frac{1}{\left(9+12x+4x^2\right)^{5/2}}\,dx\\ &= -\frac{e}{12\left(9+12x+4x^2\right)^{3/2}}-\frac{2d-3e}{16(3+2x)\left(9+12x+4x^2\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0140349, size = 34, normalized size = 0.65

$$\frac{-6d-e(8x+3)}{48(2x+3)^3\sqrt{(2x+3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(9 + 12*x + 4*x^2)^(5/2),x]
```

```
[Out] (-6*d - e*(3 + 8*x))/(48*(3 + 2*x)^3*Sqrt[(3 + 2*x)^2])
```

Maple [A] time = 0.078, size = 28, normalized size = 0.5

$$-\frac{(3+2x)(8ex+6d+3e)}{48}\left((3+2x)^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(4*x^2+12*x+9)^(5/2),x)
```

```
[Out] -1/48*(3+2*x)*(8*e*x+6*d+3*e)/((3+2*x)^2)^(5/2)
```

Maxima [A] time = 1.60159, size = 49, normalized size = 0.94

$$-\frac{e}{12\left(4x^2+12x+9\right)^{\frac{3}{2}}}-\frac{d}{8(2x+3)^4}+\frac{3e}{16(2x+3)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(4*x^2+12*x+9)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12*e/(4*x^2 + 12*x + 9)^(3/2) - 1/8*d/(2*x + 3)^4 + 3/16*e/(2*x + 3)^4
```

Fricas [A] time = 1.51083, size = 92, normalized size = 1.77

$$-\frac{8ex+6d+3e}{48\left(16x^4+96x^3+216x^2+216x+81\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(4*x^2+12*x+9)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/48*(8*e*x + 6*d + 3*e)/(16*x^4 + 96*x^3 + 216*x^2 + 216*x + 81)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{d+ex}{\left((2x+3)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(4*x**2+12*x+9)**(5/2),x)
```

```
[Out] Integral((d + e*x)/((2*x + 3)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(4*x^2+12*x+9)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.1620 $\int \frac{d+ex}{(9+12x+4x^2)^{7/2}}\,dx$

Optimal. Leaf size=52

$$-\frac{2d-3e}{24(2x+3)\left(4x^2+12x+9\right)^{5/2}}-\frac{e}{20\left(4x^2+12x+9\right)^{5/2}}$$

```
[Out] -e/(20*(9 + 12*x + 4*x^2)^(5/2)) - (2*d - 3*e)/(24*(3 + 2*x)*(9 + 12*x + 4*
x^2)^(5/2))
```

Rubi [A] time = 0.0141922, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {640, 607}

$$-\frac{2d-3e}{24(2x+3)\left(4x^2+12x+9\right)^{5/2}}-\frac{e}{20\left(4x^2+12x+9\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(9 + 12*x + 4*x^2)^(7/2),x]
```

```
[Out] -e/(20*(9 + 12*x + 4*x^2)^(5/2)) - (2*d - 3*e)/(24*(3 + 2*x)*(9 + 12*x + 4*
x^2)^(5/2))
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex}{\left(9+12x+4x^2\right)^{7/2}}\,dx &= -\frac{e}{20\left(9+12x+4x^2\right)^{5/2}}+\frac{1}{2}(2d-3e)\int \frac{1}{\left(9+12x+4x^2\right)^{7/2}}\,dx\\ &= -\frac{e}{20\left(9+12x+4x^2\right)^{5/2}}-\frac{2d-3e}{24(3+2x)\left(9+12x+4x^2\right)^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.0152839, size = 34, normalized size = 0.65

$$\frac{-10d-3(4ex+e)}{120(2x+3)^5\sqrt{(2x+3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(9 + 12*x + 4*x^2)^(7/2),x]

[Out] (-10*d - 3*(e + 4*e*x))/(120*(3 + 2*x)^5*Sqrt[(3 + 2*x)^2])
```

Maple [A] time = 0.08, size = 28, normalized size = 0.5

$$-\frac{(3+2x)(12ex+10d+3e)}{120}\left((3+2x)^2\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(4*x^2+12*x+9)^(7/2),x)

[Out] -1/120*(3+2*x)*(12*e*x+10*d+3*e)/((3+2*x)^2)^(7/2)
```

Maxima [A] time = 1.64083, size = 49, normalized size = 0.94

$$-\frac{e}{20\left(4x^2+12x+9\right)^{\frac{5}{2}}}-\frac{d}{12(2x+3)^6}+\frac{e}{8(2x+3)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(4*x^2+12*x+9)^(7/2),x, algorithm="maxima")

[Out] -1/20*e/(4*x^2 + 12*x + 9)^(5/2) - 1/12*d/(2*x + 3)^6 + 1/8*e/(2*x + 3)^6
```

Fricas [A] time = 1.48923, size = 131, normalized size = 2.52

$$-\frac{12ex+10d+3e}{120\left(64x^6+576x^5+2160x^4+4320x^3+4860x^2+2916x+729\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(4*x^2+12*x+9)^(7/2),x, algorithm="fricas")

[Out] -1/120*(12*e*x + 10*d + 3*e)/(64*x^6 + 576*x^5 + 2160*x^4 + 4320*x^3 + 4860
*x^2 + 2916*x + 729)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{d+ex}{\left((2x+3)^2\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(4*x**2+12*x+9)**(7/2),x)
```

```
[Out] Integral((d + e*x)/((2*x + 3)**2)**(7/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(4*x^2+12*x+9)^(7/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.1621 $\int (d+ex)^{7/2} \left(a^2+2abx+b^2x^2\right) dx$

Optimal. Leaf size=71

$$-\frac{4b(d+ex)^{11/2}(bd-ae)}{11e^3}+\frac{2(d+ex)^{9/2}(bd-ae)^2}{9e^3}+\frac{2b^2(d+ex)^{13/2}}{13e^3}$$

```
[Out] (2*(b*d - a*e)^2*(d + e*x)^(9/2))/(9*e^3) - (4*b*(b*d - a*e)*(d + e*x)^(11/
2))/(11*e^3) + (2*b^2*(d + e*x)^(13/2))/(13*e^3)
```

Rubi [A] time = 0.0283148, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$-\frac{4b(d+ex)^{11/2}(bd-ae)}{11e^3}+\frac{2(d+ex)^{9/2}(bd-ae)^2}{9e^3}+\frac{2b^2(d+ex)^{13/2}}{13e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)*(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] (2*(b*d - a*e)^2*(d + e*x)^(9/2))/(9*e^3) - (4*b*(b*d - a*e)*(d + e*x)^(11/
2))/(11*e^3) + (2*b^2*(d + e*x)^(13/2))/(13*e^3)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^{7/2}\left(a^2+2abx+b^2x^2\right)dx &= \int (a+bx)^2(d+ex)^{7/2}\,dx\\ &= \int \left(\frac{(-bd+ae)^2(d+ex)^{7/2}}{e^2}-\frac{2b(bd-ae)(d+ex)^{9/2}}{e^2}+\frac{b^2(d+ex)^{11/2}}{e^2}\right)dx\\ &= \frac{2(bd-ae)^2(d+ex)^{9/2}}{9e^3}-\frac{4b(bd-ae)(d+ex)^{11/2}}{11e^3}+\frac{2b^2(d+ex)^{13/2}}{13e^3}\end{aligned}$$

Mathematica [A] time = 0.0532309, size = 61, normalized size = 0.86

$$\frac{2(d+ex)^{9/2}\left(143a^2e^2+26abe(9ex-2d)+b^2\left(8d^2-36dex+99e^2x^2\right)\right)}{1287e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)*(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] (2*(d + e*x)^(9/2)*(143*a^2*e^2 + 26*a*b*e*(-2*d + 9*e*x) + b^2*(8*d^2 - 36
*d*e*x + 99*e^2*x^2)))/(1287*e^3)
```

Maple [A] time = 0.046, size = 63, normalized size = 0.9

$$\frac{198\,b^2x^2e^2+468\,xabe^2-72\,xb^2de+286\,a^2e^2-104\,abde+16\,b^2d^2}{1287\,e^3}(ex+d)^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)*(b^2*x^2+2*a*b*x+a^2),x)
```

```
[Out] 2/1287*(e*x+d)^(9/2)*(99*b^2*e^2*x^2+234*a*b*e^2*x-36*b^2*d*e*x+143*a^2*e^2
-52*a*b*d*e+8*b^2*d^2)/e^3
```

Maxima [A] time = 1.21825, size = 92, normalized size = 1.3

$$\frac{2\left(99\,(ex+d)^{\frac{13}{2}}b^2-234\,\left(b^2d-abe\right)(ex+d)^{\frac{11}{2}}+143\,\left(b^2d^2-2\,abde+a^2e^2\right)(ex+d)^{\frac{9}{2}}\right)}{1287\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

```
[Out] 2/1287*(99*(e*x + d)^(13/2)*b^2 - 234*(b^2*d - a*b*e)*(e*x + d)^(11/2) + 14
3*(b^2*d^2 - 2*a*b*d*e + a^2*e^2)*(e*x + d)^(9/2))/e^3
```

Fricas [B] time = 1.52034, size = 474, normalized size = 6.68

$$2\left(99\,b^2e^6x^6+8\,b^2d^6-52\,abd^5e+143\,a^2d^4e^2+18\left(20\,b^2de^5+13\,abe^6\right)x^5+\left(458\,b^2d^2e^4+884\,abde^5+143\,a^2e^6\right)x^4+4\,(5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

```
[Out] 2/1287*(99*b^2*e^6*x^6 + 8*b^2*d^6 - 52*a*b*d^5*e + 143*a^2*d^4*e^2 + 18*(2
0*b^2*d*e^5 + 13*a*b*e^6)*x^5 + (458*b^2*d^2*e^4 + 884*a*b*d*e^5 + 143*a^2*
e^6)*x^4 + 4*(53*b^2*d^3*e^3 + 299*a*b*d^2*e^4 + 143*a^2*d*e^5)*x^3 + 3*(b^
2*d^4*e^2 + 208*a*b*d^3*e^3 + 286*a^2*d^2*e^4)*x^2 - 2*(2*b^2*d^5*e - 13*a*
b*d^4*e^2 - 286*a^2*d^3*e^3)*x)*sqrt(e*x + d)/e^3
```

Sympy [A] time = 9.58377, size = 432, normalized size = 6.08

$$\begin{cases}\frac{2a^2d^4\sqrt{d+ex}}{9e}+\frac{8a^2d^3x\sqrt{d+ex}}{9}+\frac{4a^2d^2ex^2\sqrt{d+ex}}{3}+\frac{8a^2de^2x^3\sqrt{d+ex}}{9}+\frac{2a^2e^3x^4\sqrt{d+ex}}{9}-\frac{8abd^5\sqrt{d+ex}}{99e^2}+\frac{4abd^4x\sqrt{d+ex}}{99e}+\frac{32abd^3x^2\sqrt{d+ex}}{33}+\frac{184abd^2 \\ d^{\frac{7}{2}}\left(a^2x+abx^2+\frac{b^2x^3}{3}\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)*(b**2*x**2+2*a*b*x+a**2),x)
```

```
[Out] Piecewise((2*a**2*d**4*sqrt(d + e*x)/(9*e) + 8*a**2*d**3*x*sqrt(d + e*x)/9
+ 4*a**2*d**2*e*x**2*sqrt(d + e*x)/3 + 8*a**2*d*e**2*x**3*sqrt(d + e*x)/9 +
 2*a**2*e**3*x**4*sqrt(d + e*x)/9 - 8*a*b*d**5*sqrt(d + e*x)/(99*e**2) + 4*
a*b*d**4*x*sqrt(d + e*x)/(99*e) + 32*a*b*d**3*x**2*sqrt(d + e*x)/33 + 184*a
*b*d**2*e*x**3*sqrt(d + e*x)/99 + 136*a*b*d*e**2*x**4*sqrt(d + e*x)/99 + 4*
a*b*e**3*x**5*sqrt(d + e*x)/11 + 16*b**2*d**6*sqrt(d + e*x)/(1287*e**3) - 8
*b**2*d**5*x*sqrt(d + e*x)/(1287*e**2) + 2*b**2*d**4*x**2*sqrt(d + e*x)/(42
9*e) + 424*b**2*d**3*x**3*sqrt(d + e*x)/1287 + 916*b**2*d**2*e*x**4*sqrt(d
+ e*x)/1287 + 80*b**2*d*e**2*x**5*sqrt(d + e*x)/143 + 2*b**2*e**3*x**6*sqrt
(d + e*x)/13, Ne(e, 0)), (d**(7/2)*(a**2*x + a*b*x**2 + b**2*x**3/3), True)
)
```

Giac [B] time = 1.16192, size = 803, normalized size = 11.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

```
[Out] 2/45045*(6006*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a*b*d^3*e^(-1) + 42
9*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*b^2*
d^3*e^(-2) + 15015*(x*e + d)^(3/2)*a^2*d^3 + 2574*(15*(x*e + d)^(7/2) - 42*
(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*a*b*d^2*e^(-1) + 429*(35*(x*e +
 d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)
^(3/2)*d^3)*b^2*d^2*e^(-2) + 9009*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)
*a^2*d^2 + 858*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^
(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a*b*d*e^(-1) + 39*(315*(x*e + d)^(11/2
) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2
)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*b^2*d*e^(-2) + 1287*(15*(x*e + d)^(7/2) -
 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*a^2*d + 26*(315*(x*e + d)^(
11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^
(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*a*b*e^(-1) + 5*(693*(x*e + d)^(13/2)
- 4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/
2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*b^2*e^(-2) +
143*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 -
 105*(x*e + d)^(3/2)*d^3)*a^2)*e^(-1)
```

3.1622 $\int (d+ex)^{5/2} \left(a^2+2abx+b^2x^2\right) dx$

Optimal. Leaf size=71

$$-\frac{4b(d+ex)^{9/2}(bd-ae)}{9e^3}+\frac{2(d+ex)^{7/2}(bd-ae)^2}{7e^3}+\frac{2b^2(d+ex)^{11/2}}{11e^3}$$

```
[Out] (2*(b*d - a*e)^2*(d + e*x)^(7/2))/(7*e^3) - (4*b*(b*d - a*e)*(d + e*x)^(9/2
))/(9*e^3) + (2*b^2*(d + e*x)^(11/2))/(11*e^3)
```

Rubi [A] time = 0.0233756, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$-\frac{4b(d+ex)^{9/2}(bd-ae)}{9e^3}+\frac{2(d+ex)^{7/2}(bd-ae)^2}{7e^3}+\frac{2b^2(d+ex)^{11/2}}{11e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] (2*(b*d - a*e)^2*(d + e*x)^(7/2))/(7*e^3) - (4*b*(b*d - a*e)*(d + e*x)^(9/2
))/(9*e^3) + (2*b^2*(d + e*x)^(11/2))/(11*e^3)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^{5/2}\left(a^2+2abx+b^2x^2\right)dx &= \int (a+bx)^2(d+ex)^{5/2}\,dx\\
&= \int \left(\frac{(-bd+ae)^2(d+ex)^{5/2}}{e^2}-\frac{2b(bd-ae)(d+ex)^{7/2}}{e^2}+\frac{b^2(d+ex)^{9/2}}{e^2}\right)dx\\
&= \frac{2(bd-ae)^2(d+ex)^{7/2}}{7e^3}-\frac{4b(bd-ae)(d+ex)^{9/2}}{9e^3}+\frac{2b^2(d+ex)^{11/2}}{11e^3}
\end{aligned}$$

Mathematica [A] time = 0.0413262, size = 61, normalized size = 0.86

$$\frac{2(d+ex)^{7/2}\left(99a^2e^2+22abe(7ex-2d)+b^2\left(8d^2-28dex+63e^2x^2\right)\right)}{693e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] (2*(d + e*x)^(7/2)*(99*a^2*e^2 + 22*a*b*e*(-2*d + 7*e*x) + b^2*(8*d^2 - 28*
d*e*x + 63*e^2*x^2)))/(693*e^3)
```

Maple [A] time = 0.046, size = 63, normalized size = 0.9

$$\frac{126\,b^2x^2e^2+308\,xab e^2-56\,xb^2de+198\,a^2e^2-88\,abde+16\,b^2d^2}{693\,e^3}(ex+d)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2),x)
```

```
[Out] 2/693*(e*x+d)^(7/2)*(63*b^2*e^2*x^2+154*a*b*e^2*x-28*b^2*d*e*x+99*a^2*e^2-4
4*a*b*d*e+8*b^2*d^2)/e^3
```

Maxima [A] time = 1.06185, size = 92, normalized size = 1.3

$$\frac{2\left(63\,(ex+d)^{\frac{11}{2}}b^2-154\,\left(b^2d-abe\right)(ex+d)^{\frac{9}{2}}+99\,\left(b^2d^2-2\,abde+a^2e^2\right)(ex+d)^{\frac{7}{2}}\right)}{693\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

```
[Out] 2/693*(63*(e*x + d)^(11/2)*b^2 - 154*(b^2*d - a*b*e)*(e*x + d)^(9/2) + 99*(
b^2*d^2 - 2*a*b*d*e + a^2*e^2)*(e*x + d)^(7/2))/e^3
```

Fricas [B] time = 1.53275, size = 382, normalized size = 5.38

$$\frac{2\left(63\,b^2e^5x^5+8\,b^2d^5-44\,abd^4e+99\,a^2d^3e^2+7\left(23\,b^2de^4+22\,abe^5\right)x^4+\left(113\,b^2d^2e^3+418\,abde^4+99\,a^2e^5\right)x^3+3\,(b^2\right.}{693\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*b^2*e^5*x^5 + 8*b^2*d^5 - 44*a*b*d^4*e + 99*a^2*d^3*e^2 + 7*(23*b
^2*d*e^4 + 22*a*b*e^5)*x^4 + (113*b^2*d^2*e^3 + 418*a*b*d*e^4 + 99*a^2*e^5)
*x^3 + 3*(b^2*d^3*e^2 + 110*a*b*d^2*e^3 + 99*a^2*d*e^4)*x^2 - (4*b^2*d^4*e
- 22*a*b*d^3*e^2 - 297*a^2*d^2*e^3)*x)*sqrt(e*x + d)/e^3
```

Sympy [A] time = 3.87907, size = 355, normalized size = 5.

$$\begin{cases}\frac{2a^2d^3\sqrt{d+ex}}{7e}+\frac{6a^2d^2x\sqrt{d+ex}}{7}+\frac{6a^2dex^2\sqrt{d+ex}}{7}+\frac{2a^2e^2x^3\sqrt{d+ex}}{7}-\frac{8abd^4\sqrt{d+ex}}{63e^2}+\frac{4abd^3x\sqrt{d+ex}}{63e}+\frac{20abd^2x^2\sqrt{d+ex}}{21}+\frac{76abdex^3\sqrt{d+ex}}{63}+\frac{4abe^2}{} \\ d^{\frac{5}{2}}\left(a^2x+abx^2+\frac{b^2x^3}{3}\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(b**2*x**2+2*a*b*x+a**2),x)
```

```
[Out] Piecewise((2*a**2*d**3*sqrt(d + e*x)/(7*e) + 6*a**2*d**2*x*sqrt(d + e*x)/7
+ 6*a**2*d*e*x**2*sqrt(d + e*x)/7 + 2*a**2*e**2*x**3*sqrt(d + e*x)/7 - 8*a*
b*d**4*sqrt(d + e*x)/(63*e**2) + 4*a*b*d**3*x*sqrt(d + e*x)/(63*e) + 20*a*b
*d**2*x**2*sqrt(d + e*x)/21 + 76*a*b*d*e*x**3*sqrt(d + e*x)/63 + 4*a*b*e**2
*x**4*sqrt(d + e*x)/9 + 16*b**2*d**5*sqrt(d + e*x)/(693*e**3) - 8*b**2*d**4
*x*sqrt(d + e*x)/(693*e**2) + 2*b**2*d**3*x**2*sqrt(d + e*x)/(231*e) + 226*
b**2*d**2*x**3*sqrt(d + e*x)/693 + 46*b**2*d*e*x**4*sqrt(d + e*x)/99 + 2*b*
*2*e**2*x**5*sqrt(d + e*x)/11, Ne(e, 0)), (d**(5/2)*(a**2*x + a*b*x**2 + b*
*2*x**3/3), True))
```

Giac [B] time = 1.16392, size = 518, normalized size = 7.3

$$\frac{2}{3465}\left(462\left(3\,(xe+d)^{\frac{5}{2}}-5\,(xe+d)^{\frac{3}{2}}d\right)abd^2e^{(-1)}+33\left(15\,(xe+d)^{\frac{7}{2}}-42\,(xe+d)^{\frac{5}{2}}d+35\,(xe+d)^{\frac{3}{2}}d^2\right)b^2d^2e^{(-2)}+1155\,(xe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

```
[Out] 2/3465*(462*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a*b*d^2*e^(-1) + 33*(
15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*b^2*d^2
*e^(-2) + 1155*(x*e + d)^(3/2)*a^2*d^2 + 132*(15*(x*e + d)^(7/2) - 42*(x*e
+ d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*a*b*d*e^(-1) + 22*(35*(x*e + d)^(9/2
) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d
^3)*b^2*d*e^(-2) + 462*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^2*d + 22
*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 10
5*(x*e + d)^(3/2)*d^3)*a*b*e^(-1) + (315*(x*e + d)^(11/2) - 1540*(x*e + d)^
(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e +
 d)^(3/2)*d^4)*b^2*e^(-2) + 33*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d +
 35*(x*e + d)^(3/2)*d^2)*a^2)*e^(-1)
```

3.1623 $\int (d+ex)^{3/2} \left(a^2+2abx+b^2x^2\right) dx$

Optimal. Leaf size=71

$$-\frac{4b(d+ex)^{7/2}(bd-ae)}{7e^3}+\frac{2(d+ex)^{5/2}(bd-ae)^2}{5e^3}+\frac{2b^2(d+ex)^{9/2}}{9e^3}$$

```
[Out] (2*(b*d - a*e)^2*(d + e*x)^(5/2))/(5*e^3) - (4*b*(b*d - a*e)*(d + e*x)^(7/2
))/(7*e^3) + (2*b^2*(d + e*x)^(9/2))/(9*e^3)
```

Rubi [A] time = 0.0228057, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$-\frac{4b(d+ex)^{7/2}(bd-ae)}{7e^3}+\frac{2(d+ex)^{5/2}(bd-ae)^2}{5e^3}+\frac{2b^2(d+ex)^{9/2}}{9e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] (2*(b*d - a*e)^2*(d + e*x)^(5/2))/(5*e^3) - (4*b*(b*d - a*e)*(d + e*x)^(7/2
))/(7*e^3) + (2*b^2*(d + e*x)^(9/2))/(9*e^3)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^{3/2}\left(a^2+2abx+b^2x^2\right) dx &= \int (a+bx)^2(d+ex)^{3/2}\, dx\\ &= \int \left(\frac{(-bd+ae)^2(d+ex)^{3/2}}{e^2}-\frac{2b(bd-ae)(d+ex)^{5/2}}{e^2}+\frac{b^2(d+ex)^{7/2}}{e^2}\right) dx\\ &= \frac{2(bd-ae)^2(d+ex)^{5/2}}{5e^3}-\frac{4b(bd-ae)(d+ex)^{7/2}}{7e^3}+\frac{2b^2(d+ex)^{9/2}}{9e^3}\end{aligned}$$

Mathematica [A] time = 0.0368846, size = 61, normalized size = 0.86

$$\frac{2(d+ex)^{5/2}\left(63a^2e^2+18abe(5ex-2d)+b^2\left(8d^2-20dex+35e^2x^2\right)\right)}{315e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] (2*(d + e*x)^(5/2)*(63*a^2*e^2 + 18*a*b*e*(-2*d + 5*e*x) + b^2*(8*d^2 - 20*
d*e*x + 35*e^2*x^2)))/(315*e^3)
```

Maple [A] time = 0.046, size = 63, normalized size = 0.9

$$\frac{70\,b^2x^2e^2+180\,xabe^2-40\,xb^2de+126\,a^2e^2-72\,abde+16\,b^2d^2}{315\,e^3}(ex+d)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2),x)
```

```
[Out] 2/315*(e*x+d)^(5/2)*(35*b^2*e^2*x^2+90*a*b*e^2*x-20*b^2*d*e*x+63*a^2*e^2-36
*a*b*d*e+8*b^2*d^2)/e^3
```

Maxima [A] time = 1.03479, size = 92, normalized size = 1.3

$$\frac{2\left(35\,(ex+d)^{\frac{9}{2}}b^2-90\,\left(b^2d-abe\right)(ex+d)^{\frac{7}{2}}+63\,\left(b^2d^2-2\,abde+a^2e^2\right)(ex+d)^{\frac{5}{2}}\right)}{315\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*(e*x + d)^(9/2)*b^2 - 90*(b^2*d - a*b*e)*(e*x + d)^(7/2) + 63*(b^
2*d^2 - 2*a*b*d*e + a^2*e^2)*(e*x + d)^(5/2))/e^3
```

Fricas [B] time = 1.53832, size = 300, normalized size = 4.23

$$\frac{2\left(35\,b^2e^4x^4+8\,b^2d^4-36\,abd^3e+63\,a^2d^2e^2+10\left(5\,b^2de^3+9\,abe^4\right)x^3+3\left(b^2d^2e^2+48\,abde^3+21\,a^2e^4\right)x^2-2\left(2\,b^2d^3e-\right.\right.}{315\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^2*e^4*x^4 + 8*b^2*d^4 - 36*a*b*d^3*e + 63*a^2*d^2*e^2 + 10*(5*b
^2*d*e^3 + 9*a*b*e^4)*x^3 + 3*(b^2*d^2*e^2 + 48*a*b*d*e^3 + 21*a^2*e^4)*x^2
 - 2*(2*b^2*d^3*e - 9*a*b*d^2*e^2 - 63*a^2*d*e^3)*x)*sqrt(e*x + d)/e^3
```

Sympy [A] time = 9.39779, size = 240, normalized size = 3.38

$$a^2d\left(\begin{cases}\sqrt{d}x & \text{for } e=0\\ \frac{2(d+ex)^{\frac{3}{2}}}{3e} & \text{otherwise}\end{cases}\right)+\frac{2a^2\left(-\frac{d(d+ex)^{\frac{3}{2}}}{3}+\frac{(d+ex)^{\frac{5}{2}}}{5}\right)}{e}+\frac{4abd\left(-\frac{d(d+ex)^{\frac{3}{2}}}{3}+\frac{(d+ex)^{\frac{5}{2}}}{5}\right)}{e^2}+\frac{4ab\left(\frac{d^2(d+ex)^{\frac{3}{2}}}{3}-\frac{2d(d+ex)^{\frac{5}{2}}}{5}+\frac{(d+e}{7}\right.}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(b**2*x**2+2*a*b*x+a**2),x)
```

```
[Out] a**2*d*Piecewise((sqrt(d)*x, Eq(e, 0)), (2*(d + e*x)**(3/2)/(3*e), True)) +
 2*a**2*(-d*(d + e*x)**(3/2)/3 + (d + e*x)**(5/2)/5)/e + 4*a*b*d*(-d*(d + e
*x)**(3/2)/3 + (d + e*x)**(5/2)/5)/e**2 + 4*a*b*(d**2*(d + e*x)**(3/2)/3 -
2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e**2 + 2*b**2*d*(d**2*(d + e*x
)**(3/2)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e**3 + 2*b**2*(-d
**3*(d + e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d + e*x)**(7/2)/7
 + (d + e*x)**(9/2)/9)/e**3
```

Giac [B] time = 1.21849, size = 288, normalized size = 4.06

$$\frac{2}{315}\left(42\left(3\,(xe+d)^{\frac{5}{2}}-5\,(xe+d)^{\frac{3}{2}}d\right)abde^{(-1)}+3\left(15\,(xe+d)^{\frac{7}{2}}-42\,(xe+d)^{\frac{5}{2}}d+35\,(xe+d)^{\frac{3}{2}}d^2\right)b^2de^{(-2)}+105\,(xe+d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

```
[Out] 2/315*(42*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a*b*d*e^(-1) + 3*(15*(x
*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*b^2*d*e^(-2)
 + 105*(x*e + d)^(3/2)*a^2*d + 6*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d
 + 35*(x*e + d)^(3/2)*d^2)*a*b*e^(-1) + (35*(x*e + d)^(9/2) - 135*(x*e + d)
^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*b^2*e^(-2) +
21*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^2)*e^(-1)
```

3.1624 $\int \sqrt{d+ex}\left(a^2+2abx+b^2x^2\right)\,dx$

Optimal. Leaf size=71

$$-\frac{4b(d+ex)^{5/2}(bd-ae)}{5e^3}+\frac{2(d+ex)^{3/2}(bd-ae)^2}{3e^3}+\frac{2b^2(d+ex)^{7/2}}{7e^3}$$

```
[Out] (2*(b*d - a*e)^2*(d + e*x)^(3/2))/(3*e^3) - (4*b*(b*d - a*e)*(d + e*x)^(5/2
))/(5*e^3) + (2*b^2*(d + e*x)^(7/2))/(7*e^3)
```

Rubi [A] time = 0.0225009, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$-\frac{4b(d+ex)^{5/2}(bd-ae)}{5e^3}+\frac{2(d+ex)^{3/2}(bd-ae)^2}{3e^3}+\frac{2b^2(d+ex)^{7/2}}{7e^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] (2*(b*d - a*e)^2*(d + e*x)^(3/2))/(3*e^3) - (4*b*(b*d - a*e)*(d + e*x)^(5/2
))/(5*e^3) + (2*b^2*(d + e*x)^(7/2))/(7*e^3)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{d+ex}\left(a^2+2abx+b^2x^2\right)dx &= \int (a+bx)^2\sqrt{d+ex}\,dx\\ &= \int \left(\frac{(-bd+ae)^2\sqrt{d+ex}}{e^2}-\frac{2b(bd-ae)(d+ex)^{3/2}}{e^2}+\frac{b^2(d+ex)^{5/2}}{e^2}\right)dx\\ &= \frac{2(bd-ae)^2(d+ex)^{3/2}}{3e^3}-\frac{4b(bd-ae)(d+ex)^{5/2}}{5e^3}+\frac{2b^2(d+ex)^{7/2}}{7e^3}\end{aligned}$$

Mathematica [A] time = 0.0343695, size = 61, normalized size = 0.86

$$\frac{2(d+ex)^{3/2}\left(35a^2e^2+14abe(3ex-2d)+b^2\left(8d^2-12dex+15e^2x^2\right)\right)}{105e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(35*a^2*e^2 + 14*a*b*e*(-2*d + 3*e*x) + b^2*(8*d^2 - 12*
d*e*x + 15*e^2*x^2)))/(105*e^3)
```

Maple [A] time = 0.046, size = 63, normalized size = 0.9

$$\frac{30\,b^2x^2e^2+84\,xabe^2-24\,xb^2de+70\,a^2e^2-56\,abde+16\,b^2d^2}{105\,e^3}(ex+d)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)*(e*x+d)^(1/2),x)
```

```
[Out] 2/105*(e*x+d)^(3/2)*(15*b^2*e^2*x^2+42*a*b*e^2*x-12*b^2*d*e*x+35*a^2*e^2-28
*a*b*d*e+8*b^2*d^2)/e^3
```

Maxima [A] time = 1.08315, size = 92, normalized size = 1.3

$$\frac{2\left(15\,(ex+d)^{\frac{7}{2}}b^2-42\,\left(b^2d-abe\right)(ex+d)^{\frac{5}{2}}+35\,\left(b^2d^2-2\,abde+a^2e^2\right)(ex+d)^{\frac{3}{2}}\right)}{105\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*(e*x + d)^(7/2)*b^2 - 42*(b^2*d - a*b*e)*(e*x + d)^(5/2) + 35*(b^
2*d^2 - 2*a*b*d*e + a^2*e^2)*(e*x + d)^(3/2))/e^3
```

Fricas [A] time = 1.56796, size = 220, normalized size = 3.1

$$\frac{2\left(15\,b^2e^3x^3+8\,b^2d^3-28\,abd^2e+35\,a^2de^2+3\left(b^2de^2+14\,abe^3\right)x^2-\left(4\,b^2d^2e-14\,abde^2-35\,a^2e^3\right)x\right)\sqrt{ex+d}}{105\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^2*e^3*x^3 + 8*b^2*d^3 - 28*a*b*d^2*e + 35*a^2*d*e^2 + 3*(b^2*d*
e^2 + 14*a*b*e^3)*x^2 - (4*b^2*d^2*e - 14*a*b*d*e^2 - 35*a^2*e^3)*x)*sqrt(e
*x + d)/e^3
```

Sympy [A] time = 2.83358, size = 85, normalized size = 1.2

$$\frac{2\left(\frac{b^2(d+ex)^{\frac{7}{2}}}{7e^2}+\frac{(d+ex)^{\frac{5}{2}}\left(2abe-2b^2d\right)}{5e^2}+\frac{(d+ex)^{\frac{3}{2}}\left(a^2e^2-2abde+b^2d^2\right)}{3e^2}\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)*(e*x+d)**(1/2),x)
```

```
[Out] 2*(b**2*(d + e*x)**(7/2)/(7*e**2) + (d + e*x)**(5/2)*(2*a*b*e - 2*b**2*d)/(
5*e**2) + (d + e*x)**(3/2)*(a**2*e**2 - 2*a*b*d*e + b**2*d**2)/(3*e**2))/e
```

Giac [A] time = 1.18315, size = 117, normalized size = 1.65

$$\frac{2}{105}\left(14\left(3\,(xe+d)^{\frac{5}{2}}-5\,(xe+d)^{\frac{3}{2}}d\right)abe^{(-1)}+\left(15\,(xe+d)^{\frac{7}{2}}-42\,(xe+d)^{\frac{5}{2}}d+35\,(xe+d)^{\frac{3}{2}}d^2\right)b^2e^{(-2)}+35\,(xe+d)^{\frac{3}{2}}a^2\right)e^{(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(14*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a*b*e^(-1) + (15*(x*e +
 d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*b^2*e^(-2) + 35*
(x*e + d)^(3/2)*a^2)*e^(-1)
```

3.1625 $\int \frac{a^2+2abx+b^2x^2}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=69

$$-\frac{4b(d+ex)^{3/2}(bd-ae)}{3e^3}+\frac{2\sqrt{d+ex}(bd-ae)^2}{e^3}+\frac{2b^2(d+ex)^{5/2}}{5e^3}$$

```
[Out] (2*(b*d - a*e)^2*Sqrt[d + e*x])/e^3 - (4*b*(b*d - a*e)*(d + e*x)^(3/2))/(3*
e^3) + (2*b^2*(d + e*x)^(5/2))/(5*e^3)
```

Rubi [A] time = 0.022423, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$-\frac{4b(d+ex)^{3/2}(bd-ae)}{3e^3}+\frac{2\sqrt{d+ex}(bd-ae)^2}{e^3}+\frac{2b^2(d+ex)^{5/2}}{5e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)/Sqrt[d + e*x],x]
```

```
[Out] (2*(b*d - a*e)^2*Sqrt[d + e*x])/e^3 - (4*b*(b*d - a*e)*(d + e*x)^(3/2))/(3*
e^3) + (2*b^2*(d + e*x)^(5/2))/(5*e^3)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx+b^2x^2}{\sqrt{d+ex}}\,dx &= \int \frac{(a+bx)^2}{\sqrt{d+ex}}\,dx\\ &= \int \left(\frac{(-bd+ae)^2}{e^2\sqrt{d+ex}}-\frac{2b(bd-ae)\sqrt{d+ex}}{e^2}+\frac{b^2(d+ex)^{3/2}}{e^2}\right)dx\\ &= \frac{2(bd-ae)^2\sqrt{d+ex}}{e^3}-\frac{4b(bd-ae)(d+ex)^{3/2}}{3e^3}+\frac{2b^2(d+ex)^{5/2}}{5e^3}\end{aligned}$$

Mathematica [A] time = 0.0325503, size = 60, normalized size = 0.87

$$\frac{2\sqrt{d+ex}\left(15a^2e^2+10abe(ex-2d)+b^2\left(8d^2-4dex+3e^2x^2\right)\right)}{15e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[d + e*x]*(15*a^2*e^2 + 10*a*b*e*(-2*d + e*x) + b^2*(8*d^2 - 4*d*e*x
 + 3*e^2*x^2)))/(15*e^3)
```

Maple [A] time = 0.047, size = 63, normalized size = 0.9

$$\frac{6\,b^2x^2e^2+20\,xabe^2-8\,xb^2de+30\,a^2e^2-40\,abde+16\,b^2d^2}{15\,e^3}\sqrt{ex+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(1/2),x)
```

```
[Out] 2/15*(3*b^2*e^2*x^2+10*a*b*e^2*x-4*b^2*d*e*x+15*a^2*e^2-20*a*b*d*e+8*b^2*d^
2)*(e*x+d)^(1/2)/e^3
```

Maxima [A] time = 1.05642, size = 111, normalized size = 1.61

$$\frac{2\left(15\,\sqrt{ex+d}a^2+\frac{10\left((ex+d)^{\frac{3}{2}}-3\,\sqrt{ex+d}d\right)ab}{e}+\frac{\left(3\,(ex+d)^{\frac{5}{2}}-10\,(ex+d)^{\frac{3}{2}}d+15\,\sqrt{ex+d}d^2\right)b^2}{e^2}\right)}{15\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(e*x + d)*a^2 + 10*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*a*b/e
 + (3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*b^2/e^
2)/e
```

Fricas [A] time = 1.5299, size = 146, normalized size = 2.12

$$\frac{2\left(3\,b^2e^2x^2+8\,b^2d^2-20\,abde+15\,a^2e^2-2\left(2\,b^2de-5\,abe^2\right)x\right)\sqrt{ex+d}}{15\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*e^2*x^2 + 8*b^2*d^2 - 20*a*b*d*e + 15*a^2*e^2 - 2*(2*b^2*d*e -
5*a*b*e^2)*x)*sqrt(e*x + d)/e^3
```

Sympy [A] time = 9.33131, size = 236, normalized size = 3.42

$$\begin{cases} -\frac{\frac{2a^2d}{\sqrt{d+ex}}+2a^2\left(-\frac{d}{\sqrt{d+ex}}-\sqrt{d+ex}\right)+\frac{4abd\left(-\frac{d}{\sqrt{d+ex}}-\sqrt{d+ex}\right)}{e}+\frac{4ab\left(\frac{d^2}{\sqrt{d+ex}}+2d\sqrt{d+ex}-\frac{(d+ex)^{\frac{3}{2}}}{3}\right)}{e}+\frac{2b^2d\left(\frac{d^2}{\sqrt{d+ex}}+2d\sqrt{d+ex}-\frac{(d+ex)^{\frac{3}{2}}}{3}\right)}{e^2}+\frac{2b^2\left(-\frac{d^3}{\sqrt{d+ex}}-3d^2\sqrt{d+ex}+d(d+ex)^{\frac{3}{2}}-\frac{(d+ex)^{\frac{5}{2}}}{5}\right.}{e^2}}{e} \\ \frac{a^2x+abx^2+\frac{b^2x^3}{3}}{\sqrt{d}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)/(e*x+d)**(1/2),x)
```

```
[Out] Piecewise((-(2*a**2*d/sqrt(d + e*x) + 2*a**2*(-d/sqrt(d + e*x) - sqrt(d + e
*x)) + 4*a*b*d*(-d/sqrt(d + e*x) - sqrt(d + e*x))/e + 4*a*b*(d**2/sqrt(d +
e*x) + 2*d*sqrt(d + e*x) - (d + e*x)**(3/2)/3)/e + 2*b**2*d*(d**2/sqrt(d +
e*x) + 2*d*sqrt(d + e*x) - (d + e*x)**(3/2)/3)/e**2 + 2*b**2*(-d**3/sqrt(d
+ e*x) - 3*d**2*sqrt(d + e*x) + d*(d + e*x)**(3/2) - (d + e*x)**(5/2)/5)/e*
*2)/e, Ne(e, 0)), ((a**2*x + a*b*x**2 + b**2*x**3/3)/sqrt(d), True))
```

Giac [A] time = 1.15855, size = 115, normalized size = 1.67

$$\frac{2}{15}\left(10\left((xe+d)^{\frac{3}{2}}-3\sqrt{xe+d}d\right)abe^{(-1)}+\left(3(xe+d)^{\frac{5}{2}}-10(xe+d)^{\frac{3}{2}}d+15\sqrt{xe+d}d^{2}\right)b^{2}e^{(-2)}+15\sqrt{xe+d}a^{2}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(10*((x*e + d)^(3/2) - 3*sqrt(x*e + d)*d)*a*b*e^(-1) + (3*(x*e + d)^(5
/2) - 10*(x*e + d)^(3/2)*d + 15*sqrt(x*e + d)*d^2)*b^2*e^(-2) + 15*sqrt(x*e
 + d)*a^2)*e^(-1)
```

3.1626 $\int \frac{a^2+2abx+b^2x^2}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{4b\sqrt{d+ex}(bd-ae)}{e^3}-\frac{2(bd-ae)^2}{e^3\sqrt{d+ex}}+\frac{2b^2(d+ex)^{3/2}}{3e^3}$$

```
[Out] (-2*(b*d - a*e)^2)/(e^3*Sqrt[d + e*x]) - (4*b*(b*d - a*e)*Sqrt[d + e*x])/e^
3 + (2*b^2*(d + e*x)^(3/2))/(3*e^3)
```

Rubi [A] time = 0.0221356, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$-\frac{4b\sqrt{d+ex}(bd-ae)}{e^3}-\frac{2(bd-ae)^2}{e^3\sqrt{d+ex}}+\frac{2b^2(d+ex)^{3/2}}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^(3/2),x]
```

```
[Out] (-2*(b*d - a*e)^2)/(e^3*Sqrt[d + e*x]) - (4*b*(b*d - a*e)*Sqrt[d + e*x])/e^
3 + (2*b^2*(d + e*x)^(3/2))/(3*e^3)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a^2+2abx+b^2x^2}{(d+ex)^{3/2}}\,dx &= \int \frac{(a+bx)^2}{(d+ex)^{3/2}}\,dx\\
&= \int \left(\frac{(-bd+ae)^2}{e^2(d+ex)^{3/2}}-\frac{2b(bd-ae)}{e^2\sqrt{d+ex}}+\frac{b^2\sqrt{d+ex}}{e^2}\right)dx\\
&= -\frac{2(bd-ae)^2}{e^3\sqrt{d+ex}}-\frac{4b(bd-ae)\sqrt{d+ex}}{e^3}+\frac{2b^2(d+ex)^{3/2}}{3e^3}
\end{aligned}$$

Mathematica [A] time = 0.0305628, size = 59, normalized size = 0.88

$$\frac{2\left(-3a^2e^2+6abe(2d+ex)+b^2\left(-8d^2-4dex+e^2x^2\right)\right)}{3e^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^(3/2),x]
```

```
[Out] (2*(-3*a^2*e^2 + 6*a*b*e*(2*d + e*x) + b^2*(-8*d^2 - 4*d*e*x + e^2*x^2)))/(
3*e^3*Sqrt[d + e*x])
```

Maple [A] time = 0.045, size = 63, normalized size = 0.9

$$-\frac{-2\,b^2x^2e^2-12\,xabe^2+8\,xb^2de+6\,a^2e^2-24\,abde+16\,b^2d^2}{3\,e^3}\frac{1}{\sqrt{ex+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(3/2),x)
```

```
[Out] -2/3*(-b^2*e^2*x^2-6*a*b*e^2*x+4*b^2*d*e*x+3*a^2*e^2-12*a*b*d*e+8*b^2*d^2)/
(e*x+d)^(1/2)/e^3
```

Maxima [A] time = 1.10379, size = 101, normalized size = 1.51

$$\frac{2\left(\frac{(ex+d)^{\frac{3}{2}}b^2-6\left(b^2d-abe\right)\sqrt{ex+d}}{e^2}-\frac{3\left(b^2d^2-2\,abde+a^2e^2\right)}{\sqrt{ex+d}e^2}\right)}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(((e*x + d)^(3/2)*b^2 - 6*(b^2*d - a*b*e)*sqrt(e*x + d))/e^2 - 3*(b^2*d
^2 - 2*a*b*d*e + a^2*e^2)/(sqrt(e*x + d)*e^2))/e
```

Fricas [A] time = 1.57028, size = 157, normalized size = 2.34

$$\frac{2\left(b^2e^2x^2-8\,b^2d^2+12\,abde-3\,a^2e^2-2\left(2\,b^2de-3\,abe^2\right)x\right)\sqrt{ex+d}}{3\left(e^4x+de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^2*e^2*x^2 - 8*b^2*d^2 + 12*a*b*d*e - 3*a^2*e^2 - 2*(2*b^2*d*e - 3*a*
b*e^2)*x)*sqrt(e*x + d)/(e^4*x + d*e^3)
```

Sympy [A] time = 10.2983, size = 65, normalized size = 0.97

$$\frac{2b^2\,(d+ex)^{\frac{3}{2}}}{3e^3}+\frac{\sqrt{d+ex}\left(4abe-4b^2d\right)}{e^3}-\frac{2\,(ae-bd)^2}{e^3\sqrt{d+ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)/(e*x+d)**(3/2),x)
```

```
[Out] 2*b**2*(d + e*x)**(3/2)/(3*e**3) + sqrt(d + e*x)*(4*a*b*e - 4*b**2*d)/e**3
- 2*(a*e - b*d)**2/(e**3*sqrt(d + e*x))
```

Giac [A] time = 1.21209, size = 112, normalized size = 1.67

$$\frac{2}{3}\left((xe+d)^{\frac{3}{2}}b^2e^6 - 6\sqrt{xe+d}b^2de^6 + 6\sqrt{xe+d}abe^7\right)e^{(-9)} - \frac{2\left(b^2d^2 - 2\,abde + a^2e^2\right)e^{(-3)}}{\sqrt{xe+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*((x*e + d)^(3/2)*b^2*e^6 - 6*sqrt(x*e + d)*b^2*d*e^6 + 6*sqrt(x*e + d)*
a*b*e^7)*e^(-9) - 2*(b^2*d^2 - 2*a*b*d*e + a^2*e^2)*e^(-3)/sqrt(x*e + d)
```

3.1627 $\int \frac{a^2+2abx+b^2x^2}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=67

$$\frac{4b(bd-ae)}{e^3\sqrt{d+ex}}-\frac{2(bd-ae)^2}{3e^3(d+ex)^{3/2}}+\frac{2b^2\sqrt{d+ex}}{e^3}$$

```
[Out] (-2*(b*d - a*e)^2)/(3*e^3*(d + e*x)^(3/2)) + (4*b*(b*d - a*e))/(e^3*Sqrt[d
+ e*x]) + (2*b^2*Sqrt[d + e*x])/e^3
```

Rubi [A] time = 0.0222164, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$\frac{4b(bd-ae)}{e^3\sqrt{d+ex}}-\frac{2(bd-ae)^2}{3e^3(d+ex)^{3/2}}+\frac{2b^2\sqrt{d+ex}}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^(5/2),x]
```

```
[Out] (-2*(b*d - a*e)^2)/(3*e^3*(d + e*x)^(3/2)) + (4*b*(b*d - a*e))/(e^3*Sqrt[d
+ e*x]) + (2*b^2*Sqrt[d + e*x])/e^3
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx+b^2x^2}{(d+ex)^{5/2}}\,dx&=\int \frac{(a+bx)^2}{(d+ex)^{5/2}}\,dx\\&=\int \left(\frac{(-bd+ae)^2}{e^2(d+ex)^{5/2}}-\frac{2b(bd-ae)}{e^2(d+ex)^{3/2}}+\frac{b^2}{e^2\sqrt{d+ex}}\right)dx\\&=-\frac{2(bd-ae)^2}{3e^3(d+ex)^{3/2}}+\frac{4b(bd-ae)}{e^3\sqrt{d+ex}}+\frac{2b^2\sqrt{d+ex}}{e^3}\end{aligned}$$

Mathematica [A] time = 0.0360681, size = 62, normalized size = 0.93

$$\frac{-2a^2e^2-4abe(2d+3ex)+2b^2\left(8d^2+12dex+3e^2x^2\right)}{3e^3(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^(5/2),x]
```

```
[Out] (-2*a^2*e^2 - 4*a*b*e*(2*d + 3*e*x) + 2*b^2*(8*d^2 + 12*d*e*x + 3*e^2*x^2))
/(3*e^3*(d + e*x)^(3/2))
```

Maple [A] time = 0.046, size = 62, normalized size = 0.9

$$-\frac{-6\,b^2x^2e^2+12\,xabe^2-24\,xb^2de+2\,a^2e^2+8\,abde-16\,b^2d^2}{3\,e^3}(ex+d)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(5/2),x)
```

```
[Out] -2/3*(-3*b^2*e^2*x^2+6*a*b*e^2*x-12*b^2*d*e*x+a^2*e^2+4*a*b*d*e-8*b^2*d^2)/
(e*x+d)^(3/2)/e^3
```

Maxima [A] time = 1.0363, size = 97, normalized size = 1.45

$$\frac{2\left(\frac{3\sqrt{ex+d}b^2}{e^2}-\frac{b^2d^2-2\,abde+a^2e^2-6\left(b^2d-abe\right)(ex+d)}{(ex+d)^{\frac{3}{2}}e^2}\right)}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(3*sqrt(e*x + d)*b^2/e^2 - (b^2*d^2 - 2*a*b*d*e + a^2*e^2 - 6*(b^2*d -
a*b*e)*(e*x + d))/((e*x + d)^(3/2)*e^2))/e
```

Fricas [A] time = 1.44072, size = 174, normalized size = 2.6

$$\frac{2\left(3\,b^2e^2x^2+8\,b^2d^2-4\,abde-a^2e^2+6\left(2\,b^2de-abe^2\right)x\right)\sqrt{ex+d}}{3\left(e^5x^2+2\,de^4x+d^2e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*b^2*e^2*x^2 + 8*b^2*d^2 - 4*a*b*d*e - a^2*e^2 + 6*(2*b^2*d*e - a*b*e
^2)*x)*sqrt(e*x + d)/(e^5*x^2 + 2*d*e^4*x + d^2*e^3)
```

Sympy [A] time = 1.45511, size = 265, normalized size = 3.96

$$\begin{cases} -\frac{2a^2e^2}{3de^3\sqrt{d+ex}+3e^4x\sqrt{d+ex}}-\frac{8abde}{3de^3\sqrt{d+ex}+3e^4x\sqrt{d+ex}}-\frac{12abe^2x}{3de^3\sqrt{d+ex}+3e^4x\sqrt{d+ex}}+\frac{16b^2d^2}{3de^3\sqrt{d+ex}+3e^4x\sqrt{d+ex}}+\frac{24b^2dex}{3de^3\sqrt{d+ex}+3e^4x\sqrt{d+ex}}+\frac{6b^2e^2}{3de^3\sqrt{d+ex}+3} \\ \frac{a^2x+abx^2+\frac{b^2x^3}{3}}{d^{\frac{5}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)/(e*x+d)**(5/2),x)
```

```
[Out] Piecewise((-2*a**2*e**2/(3*d*e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)) -
 8*a*b*d*e/(3*d*e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)) - 12*a*b*e**2*
x/(3*d*e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)) + 16*b**2*d**2/(3*d*e**
3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)) + 24*b**2*d*e*x/(3*d*e**3*sqrt(d
+ e*x) + 3*e**4*x*sqrt(d + e*x)) + 6*b**2*e**2*x**2/(3*d*e**3*sqrt(d + e*x)
 + 3*e**4*x*sqrt(d + e*x)), Ne(e, 0)), ((a**2*x + a*b*x**2 + b**2*x**3/3)/d
**(5/2), True))
```

Giac [A] time = 1.24103, size = 101, normalized size = 1.51

$$2\sqrt{xe+d}b^2e^{(-3)} + \frac{2\left(6\,(xe+d)b^2d - b^2d^2 - 6\,(xe+d)abe + 2\,abde - a^2e^2\right)e^{(-3)}}{3\,(xe+d)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x*e + d)*b^2*e^(-3) + 2/3*(6*(x*e + d)*b^2*d - b^2*d^2 - 6*(x*e + d)
*a*b*e + 2*a*b*d*e - a^2*e^2)*e^(-3)/(x*e + d)^(3/2)
```

3.1628 $\int \frac{a^2+2abx+b^2x^2}{(d+ex)^{7/2}} dx$

Optimal. Leaf size=69

$$\frac{4b(bd-ae)}{3e^3(d+ex)^{3/2}}-\frac{2(bd-ae)^2}{5e^3(d+ex)^{5/2}}-\frac{2b^2}{e^3\sqrt{d+ex}}$$

```
[Out] (-2*(b*d - a*e)^2)/(5*e^3*(d + e*x)^(5/2)) + (4*b*(b*d - a*e))/(3*e^3*(d +
e*x)^(3/2)) - (2*b^2)/(e^3*Sqrt[d + e*x])
```

Rubi [A] time = 0.0260152, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$\frac{4b(bd-ae)}{3e^3(d+ex)^{3/2}}-\frac{2(bd-ae)^2}{5e^3(d+ex)^{5/2}}-\frac{2b^2}{e^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^(7/2),x]
```

```
[Out] (-2*(b*d - a*e)^2)/(5*e^3*(d + e*x)^(5/2)) + (4*b*(b*d - a*e))/(3*e^3*(d +
e*x)^(3/2)) - (2*b^2)/(e^3*Sqrt[d + e*x])
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx+b^2x^2}{(d+ex)^{7/2}}\,dx &= \int \frac{(a+bx)^2}{(d+ex)^{7/2}}\,dx\\ &= \int \left(\frac{(-bd+ae)^2}{e^2(d+ex)^{7/2}}-\frac{2b(bd-ae)}{e^2(d+ex)^{5/2}}+\frac{b^2}{e^2(d+ex)^{3/2}}\right)dx\\ &= -\frac{2(bd-ae)^2}{5e^3(d+ex)^{5/2}}+\frac{4b(bd-ae)}{3e^3(d+ex)^{3/2}}-\frac{2b^2}{e^3\sqrt{d+ex}}\end{aligned}$$

Mathematica [A] time = 0.0326638, size = 61, normalized size = 0.88

$$-\frac{2\left(3a^2e^2+2abe(2d+5ex)+b^2\left(8d^2+20dex+15e^2x^2\right)\right)}{15e^3(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^(7/2),x]
```

```
[Out] (-2*(3*a^2*e^2 + 2*a*b*e*(2*d + 5*e*x) + b^2*(8*d^2 + 20*d*e*x + 15*e^2*x^2
)))/(15*e^3*(d + e*x)^(5/2))
```

Maple [A] time = 0.044, size = 63, normalized size = 0.9

$$-\frac{30\,b^2x^2e^2+20\,xabe^2+40\,xb^2de+6\,a^2e^2+8\,abde+16\,b^2d^2}{15\,e^3}(ex+d)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(7/2),x)
```

```
[Out] -2/15*(15*b^2*e^2*x^2+10*a*b*e^2*x+20*b^2*d*e*x+3*a^2*e^2+4*a*b*d*e+8*b^2*d
^2)/(e*x+d)^(5/2)/e^3
```

Maxima [A] time = 1.03004, size = 88, normalized size = 1.28

$$-\frac{2\left(15\,(ex+d)^2b^2+3\,b^2d^2-6\,abde+3\,a^2e^2-10\left(b^2d-abe\right)(ex+d)\right)}{15\,(ex+d)^{\frac{5}{2}}e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/15*(15*(e*x + d)^2*b^2 + 3*b^2*d^2 - 6*a*b*d*e + 3*a^2*e^2 - 10*(b^2*d -
 a*b*e)*(e*x + d))/((e*x + d)^(5/2)*e^3)
```

Fricas [A] time = 1.47153, size = 204, normalized size = 2.96

$$-\frac{2\left(15\,b^2e^2x^2+8\,b^2d^2+4\,abde+3\,a^2e^2+10\left(2\,b^2de+abe^2\right)x\right)\sqrt{ex+d}}{15\left(e^6x^3+3\,de^5x^2+3\,d^2e^4x+d^3e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/15*(15*b^2*e^2*x^2 + 8*b^2*d^2 + 4*a*b*d*e + 3*a^2*e^2 + 10*(2*b^2*d*e +
 a*b*e^2)*x)*sqrt(e*x + d)/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3)
```

Sympy [A] time = 3.13579, size = 389, normalized size = 5.64

$$\begin{cases} -\frac{6a^2e^2}{15d^2e^3\sqrt{d+ex}+30de^4x\sqrt{d+ex}+15e^5x^2\sqrt{d+ex}}-\frac{8abde}{15d^2e^3\sqrt{d+ex}+30de^4x\sqrt{d+ex}+15e^5x^2\sqrt{d+ex}}-\frac{20abe^2x}{15d^2e^3\sqrt{d+ex}+30de^4x\sqrt{d+ex}+15e^5x^2\sqrt{d+ex}}-\frac{}{15d^2e^3\cdot} \\ \frac{a^2x+abx^2+\frac{b^2x^3}{3}}{d^{\frac{7}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)/(e*x+d)**(7/2),x)
```

```
[Out] Piecewise((-6*a**2*e**2/(15*d**2*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d +
e*x) + 15*e**5*x**2*sqrt(d + e*x)) - 8*a*b*d*e/(15*d**2*e**3*sqrt(d + e*x)
+ 30*d*e**4*x*sqrt(d + e*x) + 15*e**5*x**2*sqrt(d + e*x)) - 20*a*b*e**2*x/(
15*d**2*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d + e*x) + 15*e**5*x**2*sqrt(
d + e*x)) - 16*b**2*d**2/(15*d**2*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d +
 e*x) + 15*e**5*x**2*sqrt(d + e*x)) - 40*b**2*d*e*x/(15*d**2*e**3*sqrt(d +
e*x) + 30*d*e**4*x*sqrt(d + e*x) + 15*e**5*x**2*sqrt(d + e*x)) - 30*b**2*e*
*2*x**2/(15*d**2*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d + e*x) + 15*e**5*x
**2*sqrt(d + e*x)), Ne(e, 0)), ((a**2*x + a*b*x**2 + b**2*x**3/3)/d**(7/2),
 True))
```

Giac [A] time = 1.19207, size = 97, normalized size = 1.41

$$-\frac{2\left(15\,(xe+d)^{2}b^{2}-10\,(xe+d)b^{2}d+3\,b^{2}d^{2}+10\,(xe+d)abe-6\,abde+3\,a^{2}e^{2}\right)e^{(-3)}}{15\,(xe+d)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*(15*(x*e + d)^2*b^2 - 10*(x*e + d)*b^2*d + 3*b^2*d^2 + 10*(x*e + d)*a
*b*e - 6*a*b*d*e + 3*a^2*e^2)*e^(-3)/(x*e + d)^(5/2)
```

3.1629 $\int (d+ex)^{7/2}\left(a^2+2abx+b^2x^2\right)^2\,dx$

Optimal. Leaf size=129

$$-\frac{8b^3(d+ex)^{15/2}(bd-ae)}{15e^5}+\frac{12b^2(d+ex)^{13/2}(bd-ae)^2}{13e^5}-\frac{8b(d+ex)^{11/2}(bd-ae)^3}{11e^5}+\frac{2(d+ex)^{9/2}(bd-ae)^4}{9e^5}+\frac{2b^4(d+e}{17e^5}$$

```
[Out] (2*(b*d - a*e)^4*(d + e*x)^(9/2))/(9*e^5) - (8*b*(b*d - a*e)^3*(d + e*x)^(1
1/2))/(11*e^5) + (12*b^2*(b*d - a*e)^2*(d + e*x)^(13/2))/(13*e^5) - (8*b^3*
(b*d - a*e)*(d + e*x)^(15/2))/(15*e^5) + (2*b^4*(d + e*x)^(17/2))/(17*e^5)
```

Rubi [A] time = 0.0707597, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {27, 43}

$$-\frac{8b^3(d+ex)^{15/2}(bd-ae)}{15e^5}+\frac{12b^2(d+ex)^{13/2}(bd-ae)^2}{13e^5}-\frac{8b(d+ex)^{11/2}(bd-ae)^3}{11e^5}+\frac{2(d+ex)^{9/2}(bd-ae)^4}{9e^5}+\frac{2b^4(d+e}{17e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] (2*(b*d - a*e)^4*(d + e*x)^(9/2))/(9*e^5) - (8*b*(b*d - a*e)^3*(d + e*x)^(1
1/2))/(11*e^5) + (12*b^2*(b*d - a*e)^2*(d + e*x)^(13/2))/(13*e^5) - (8*b^3*
(b*d - a*e)*(d + e*x)^(15/2))/(15*e^5) + (2*b^4*(d + e*x)^(17/2))/(17*e^5)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^{7/2}\left(a^2+2abx+b^2x^2\right)^2\,dx &= \int (a+bx)^4(d+ex)^{7/2}\,dx\\ &= \int \left(\frac{(-bd+ae)^4(d+ex)^{7/2}}{e^4}-\frac{4b(bd-ae)^3(d+ex)^{9/2}}{e^4}+\frac{6b^2(bd-ae)^2(d+ex)^{11}}{e^4}\right.\\ &= \frac{2(bd-ae)^4(d+ex)^{9/2}}{9e^5}-\frac{8b(bd-ae)^3(d+ex)^{11/2}}{11e^5}+\frac{12b^2(bd-ae)^2(d+ex)^{13/2}}{13e^5}\end{aligned}$$

Mathematica [A] time = 0.117525, size = 101, normalized size = 0.78

$$\frac{2(d+ex)^{9/2}\left(50490b^2(d+ex)^2(bd-ae)^2-29172b^3(d+ex)^3(bd-ae)-39780b(d+ex)(bd-ae)^3+12155(bd-ae)^4+\right.}{109395e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] (2*(d + e*x)^(9/2)*(12155*(b*d - a*e)^4 - 39780*b*(b*d - a*e)^3*(d + e*x) +
 50490*b^2*(b*d - a*e)^2*(d + e*x)^2 - 29172*b^3*(b*d - a*e)*(d + e*x)^3 +
6435*b^4*(d + e*x)^4))/(109395*e^5)
```

Maple [A] time = 0.048, size = 186, normalized size = 1.4

$$\frac{12870\,x^4b^4e^4+58344\,x^3ab^3e^4-6864\,x^3b^4de^3+100980\,x^2a^2b^2e^4-26928\,x^2ab^3de^3+3168\,x^2b^4d^2e^2+79560\,xa^3be^4-36}{109395}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)*(b^2*x^2+2*a*b*x+a^2)^2,x)
```

```
[Out] 2/109395*(e*x+d)^(9/2)*(6435*b^4*e^4*x^4+29172*a*b^3*e^4*x^3-3432*b^4*d*e^3
*x^3+50490*a^2*b^2*e^4*x^2-13464*a*b^3*d*e^3*x^2+1584*b^4*d^2*e^2*x^2+39780
*a^3*b*e^4*x-18360*a^2*b^2*d*e^3*x+4896*a*b^3*d^2*e^2*x-576*b^4*d^3*e*x+121
55*a^4*e^4-8840*a^3*b*d*e^3+4080*a^2*b^2*d^2*e^2-1088*a*b^3*d^3*e+128*b^4*d
^4)/e^5
```

Maxima [A] time = 1.08627, size = 244, normalized size = 1.89

$$\frac{2\left(6435\,(ex+d)^{\frac{17}{2}}b^4-29172\left(b^4d-ab^3e\right)(ex+d)^{\frac{15}{2}}+50490\left(b^4d^2-2\,ab^3de+a^2b^2e^2\right)(ex+d)^{\frac{13}{2}}-39780\left(b^4d^3-3\,ab^3d^2\right.}{109395\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

```
[Out] 2/109395*(6435*(e*x + d)^(17/2)*b^4 - 29172*(b^4*d - a*b^3*e)*(e*x + d)^(15
/2) + 50490*(b^4*d^2 - 2*a*b^3*d*e + a^2*b^2*e^2)*(e*x + d)^(13/2) - 39780*
(b^4*d^3 - 3*a*b^3*d^2*e + 3*a^2*b^2*d*e^2 - a^3*b*e^3)*(e*x + d)^(11/2) +
12155*(b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^
4)*(e*x + d)^(9/2))/e^5
```

Fricas [B] time = 1.52538, size = 1022, normalized size = 7.92

$$\frac{2\left(6435\,b^4e^8x^8+128\,b^4d^8-1088\,ab^3d^7e+4080\,a^2b^2d^6e^2-8840\,a^3bd^5e^3+12155\,a^4d^4e^4+1716\left(13\,b^4de^7+17\,ab^3e^8\right)x^7\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

```
[Out] 2/109395*(6435*b^4*e^8*x^8 + 128*b^4*d^8 - 1088*a*b^3*d^7*e + 4080*a^2*b^2*
d^6*e^2 - 8840*a^3*b*d^5*e^3 + 12155*a^4*d^4*e^4 + 1716*(13*b^4*d*e^7 + 17*
a*b^3*e^8)*x^7 + 66*(401*b^4*d^2*e^6 + 1564*a*b^3*d*e^7 + 765*a^2*b^2*e^8)*
x^6 + 36*(303*b^4*d^3*e^5 + 3502*a*b^3*d^2*e^6 + 5100*a^2*b^2*d*e^7 + 1105*
a^3*b*e^8)*x^5 + 5*(7*b^4*d^4*e^4 + 10880*a*b^3*d^3*e^5 + 46716*a^2*b^2*d^2
*e^6 + 30056*a^3*b*d*e^7 + 2431*a^4*e^8)*x^4 - 20*(2*b^4*d^5*e^3 - 17*a*b^3
```

```
*d^4*e^4 - 5406*a^2*b^2*d^3*e^5 - 10166*a^3*b*d^2*e^6 - 2431*a^4*d*e^7)*x^3
 + 6*(8*b^4*d^6*e^2 - 68*a*b^3*d^5*e^3 + 255*a^2*b^2*d^4*e^4 + 17680*a^3*b*
d^3*e^5 + 12155*a^4*d^2*e^6)*x^2 - 4*(16*b^4*d^7*e - 136*a*b^3*d^6*e^2 + 51
0*a^2*b^2*d^5*e^3 - 1105*a^3*b*d^4*e^4 - 12155*a^4*d^3*e^5)*x)*sqrt(e*x + d
)/e^5
```

Sympy [A] time = 19.7384, size = 903, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)*(b**2*x**2+2*a*b*x+a**2)**2,x)
```

```
[Out] Piecewise((2*a**4*d**4*sqrt(d + e*x)/(9*e) + 8*a**4*d**3*x*sqrt(d + e*x)/9
+ 4*a**4*d**2*e*x**2*sqrt(d + e*x)/3 + 8*a**4*d*e**2*x**3*sqrt(d + e*x)/9 +
 2*a**4*e**3*x**4*sqrt(d + e*x)/9 - 16*a**3*b*d**5*sqrt(d + e*x)/(99*e**2)
+ 8*a**3*b*d**4*x*sqrt(d + e*x)/(99*e) + 64*a**3*b*d**3*x**2*sqrt(d + e*x)/
33 + 368*a**3*b*d**2*e*x**3*sqrt(d + e*x)/99 + 272*a**3*b*d*e**2*x**4*sqrt(
d + e*x)/99 + 8*a**3*b*e**3*x**5*sqrt(d + e*x)/11 + 32*a**2*b**2*d**6*sqrt(
d + e*x)/(429*e**3) - 16*a**2*b**2*d**5*x*sqrt(d + e*x)/(429*e**2) + 4*a**2
*b**2*d**4*x**2*sqrt(d + e*x)/(143*e) + 848*a**2*b**2*d**3*x**3*sqrt(d + e*
x)/429 + 1832*a**2*b**2*d**2*e*x**4*sqrt(d + e*x)/429 + 480*a**2*b**2*d*e**
2*x**5*sqrt(d + e*x)/143 + 12*a**2*b**2*e**3*x**6*sqrt(d + e*x)/13 - 128*a*
b**3*d**7*sqrt(d + e*x)/(6435*e**4) + 64*a*b**3*d**6*x*sqrt(d + e*x)/(6435*
e**3) - 16*a*b**3*d**5*x**2*sqrt(d + e*x)/(2145*e**2) + 8*a*b**3*d**4*x**3*
sqrt(d + e*x)/(1287*e) + 1280*a*b**3*d**3*x**4*sqrt(d + e*x)/1287 + 1648*a*
b**3*d**2*e*x**5*sqrt(d + e*x)/715 + 368*a*b**3*d*e**2*x**6*sqrt(d + e*x)/1
95 + 8*a*b**3*e**3*x**7*sqrt(d + e*x)/15 + 256*b**4*d**8*sqrt(d + e*x)/(109
395*e**5) - 128*b**4*d**7*x*sqrt(d + e*x)/(109395*e**4) + 32*b**4*d**6*x**2
*sqrt(d + e*x)/(36465*e**3) - 16*b**4*d**5*x**3*sqrt(d + e*x)/(21879*e**2)
+ 14*b**4*d**4*x**4*sqrt(d + e*x)/(21879*e) + 2424*b**4*d**3*x**5*sqrt(d +
e*x)/12155 + 1604*b**4*d**2*e*x**6*sqrt(d + e*x)/3315 + 104*b**4*d*e**2*x**
7*sqrt(d + e*x)/255 + 2*b**4*e**3*x**8*sqrt(d + e*x)/17, Ne(e, 0)), (d**(7/
2)*(a**4*x + 2*a**3*b*x**2 + 2*a**2*b**2*x**3 + a*b**3*x**4 + b**4*x**5/5),
 True))
```

Giac [B] time = 1.35548, size = 1729, normalized size = 13.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

```
[Out] 2/765765*(204204*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^3*b*d^3*e^(-1)
 + 43758*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^
2)*a^2*b^2*d^3*e^(-2) + 9724*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d +
189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a*b^3*d^3*e^(-3) + 221*(
315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 -
2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*b^4*d^3*e^(-4) + 25525
5*(x*e + d)^(3/2)*a^4*d^3 + 87516*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*
d + 35*(x*e + d)^(3/2)*d^2)*a^3*b*d^2*e^(-1) + 43758*(35*(x*e + d)^(9/2) -
135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*
```

a^2*b^2*d^2*e^(-2) + 2652*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*a*b^3*d^2*e^(-3) + 255*(693*(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*b^4*d^2*e^(-4) + 153153*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^4*d^2 + 29172*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a^3*b*d*e^(-1) + 3978*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*a^2*b^2*d*e^(-2) + 1020*(693*(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*a*b^3*d*e^(-3) + 51*(3003*(x*e + d)^(15/2) - 20790*(x*e + d)^(13/2)*d + 61425*(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525*(x*e + d)^(7/2)*d^4 - 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*b^4*d*e^(-4) + 21879*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*a^4*d + 884*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*a^3*b*e^(-1) + 510*(693*(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*a^2*b^2*e^(-2) + 68*(3003*(x*e + d)^(15/2) - 20790*(x*e + d)^(13/2)*d + 61425*(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525*(x*e + d)^(7/2)*d^4 - 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*a*b^3*e^(-3) + 7*(6435*(x*e + d)^(17/2) - 51051*(x*e + d)^(15/2)*d + 176715*(x*e + d)^(13/2)*d^2 - 348075*(x*e + d)^(11/2)*d^3 + 425425*(x*e + d)^(9/2)*d^4 - 328185*(x*e + d)^(7/2)*d^5 + 153153*(x*e + d)^(5/2)*d^6 - 36465*(x*e + d)^(3/2)*d^7)*b^4*e^(-4) + 2431*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a^4)*e^(-1)

3.1630 $\int (d+ex)^{5/2} \left(a^2+2abx+b^2x^2\right)^2 \, dx$

Optimal. Leaf size=129

$$-\frac{8b^3(d+ex)^{13/2}(bd-ae)}{13e^5}+\frac{12b^2(d+ex)^{11/2}(bd-ae)^2}{11e^5}-\frac{8b(d+ex)^{9/2}(bd-ae)^3}{9e^5}+\frac{2(d+ex)^{7/2}(bd-ae)^4}{7e^5}+\frac{2b^4(d+e}{15e^5}$$

```
[Out] (2*(b*d - a*e)^4*(d + e*x)^(7/2))/(7*e^5) - (8*b*(b*d - a*e)^3*(d + e*x)^(9
/2))/(9*e^5) + (12*b^2*(b*d - a*e)^2*(d + e*x)^(11/2))/(11*e^5) - (8*b^3*(b
*d - a*e)*(d + e*x)^(13/2))/(13*e^5) + (2*b^4*(d + e*x)^(15/2))/(15*e^5)
```

Rubi [A] time = 0.041644, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {27, 43}

$$-\frac{8b^3(d+ex)^{13/2}(bd-ae)}{13e^5}+\frac{12b^2(d+ex)^{11/2}(bd-ae)^2}{11e^5}-\frac{8b(d+ex)^{9/2}(bd-ae)^3}{9e^5}+\frac{2(d+ex)^{7/2}(bd-ae)^4}{7e^5}+\frac{2b^4(d+e}{15e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] (2*(b*d - a*e)^4*(d + e*x)^(7/2))/(7*e^5) - (8*b*(b*d - a*e)^3*(d + e*x)^(9
/2))/(9*e^5) + (12*b^2*(b*d - a*e)^2*(d + e*x)^(11/2))/(11*e^5) - (8*b^3*(b
*d - a*e)*(d + e*x)^(13/2))/(13*e^5) + (2*b^4*(d + e*x)^(15/2))/(15*e^5)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^{5/2}\left(a^2+2abx+b^2x^2\right)^2\,dx &= \int (a+bx)^4(d+ex)^{5/2}\,dx\\ &= \int \left(\frac{(-bd+ae)^4(d+ex)^{5/2}}{e^4}-\frac{4b(bd-ae)^3(d+ex)^{7/2}}{e^4}+\frac{6b^2(bd-ae)^2(d+ex)^{9/}}{e^4}\right.\\ &= \frac{2(bd-ae)^4(d+ex)^{7/2}}{7e^5}-\frac{8b(bd-ae)^3(d+ex)^{9/2}}{9e^5}+\frac{12b^2(bd-ae)^2(d+ex)^{11/2}}{11e^5}-\end{aligned}$$

Mathematica [A] time = 0.094288, size = 101, normalized size = 0.78

$$\frac{2(d+ex)^{7/2}\left(24570b^2(d+ex)^2(bd-ae)^2-13860b^3(d+ex)^3(bd-ae)-20020b(d+ex)(bd-ae)^3+6435(bd-ae)^4+\right.}{45045e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] (2*(d + e*x)^(7/2)*(6435*(b*d - a*e)^4 - 20020*b*(b*d - a*e)^3*(d + e*x) +
24570*b^2*(b*d - a*e)^2*(d + e*x)^2 - 13860*b^3*(b*d - a*e)*(d + e*x)^3 + 3
003*b^4*(d + e*x)^4))/(45045*e^5)
```

Maple [A] time = 0.046, size = 186, normalized size = 1.4

$$\frac{6006\,x^4b^4e^4 + 27720\,x^3ab^3e^4 - 3696\,x^3b^4de^3 + 49140\,x^2a^2b^2e^4 - 15120\,x^2ab^3de^3 + 2016\,x^2b^4d^2e^2 + 40040\,xa^3be^4 - 2184}{45045\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2)^2,x)
```

```
[Out] 2/45045*(e*x+d)^(7/2)*(3003*b^4*e^4*x^4+13860*a*b^3*e^4*x^3-1848*b^4*d*e^3*
x^3+24570*a^2*b^2*e^4*x^2-7560*a*b^3*d*e^3*x^2+1008*b^4*d^2*e^2*x^2+20020*a
^3*b*e^4*x-10920*a^2*b^2*d*e^3*x+3360*a*b^3*d^2*e^2*x-448*b^4*d^3*e*x+6435*
a^4*e^4-5720*a^3*b*d*e^3+3120*a^2*b^2*d^2*e^2-960*a*b^3*d^3*e+128*b^4*d^4)/
e^5
```

Maxima [A] time = 1.04471, size = 244, normalized size = 1.89

$$\frac{2\left(3003\,(ex+d)^{\frac{15}{2}}b^4 - 13860\left(b^4d - ab^3e\right)(ex+d)^{\frac{13}{2}} + 24570\left(b^4d^2 - 2\,ab^3de + a^2b^2e^2\right)(ex+d)^{\frac{11}{2}} - 20020\left(b^4d^3 - 3\,ab^3d^2\right.\right.}{45045\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

```
[Out] 2/45045*(3003*(e*x + d)^(15/2)*b^4 - 13860*(b^4*d - a*b^3*e)*(e*x + d)^(13/
2) + 24570*(b^4*d^2 - 2*a*b^3*d*e + a^2*b^2*e^2)*(e*x + d)^(11/2) - 20020*(
b^4*d^3 - 3*a*b^3*d^2*e + 3*a^2*b^2*d*e^2 - a^3*b*e^3)*(e*x + d)^(9/2) + 64
35*(b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4)*
(e*x + d)^(7/2))/e^5
```

Fricas [B] time = 1.54051, size = 855, normalized size = 6.63

$$\frac{2\left(3003\,b^4e^7x^7 + 128\,b^4d^7 - 960\,ab^3d^6e + 3120\,a^2b^2d^5e^2 - 5720\,a^3bd^4e^3 + 6435\,a^4d^3e^4 + 231\left(31\,b^4de^6 + 60\,ab^3e^7\right)x^6 + 63\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

```
[Out] 2/45045*(3003*b^4*e^7*x^7 + 128*b^4*d^7 - 960*a*b^3*d^6*e + 3120*a^2*b^2*d^
5*e^2 - 5720*a^3*b*d^4*e^3 + 6435*a^4*d^3*e^4 + 231*(31*b^4*d*e^6 + 60*a*b^
3*e^7)*x^6 + 63*(71*b^4*d^2*e^5 + 540*a*b^3*d*e^6 + 390*a^2*b^2*e^7)*x^5 +
35*(b^4*d^3*e^4 + 636*a*b^3*d^2*e^5 + 1794*a^2*b^2*d*e^6 + 572*a^3*b*e^7)*x
^4 - 5*(8*b^4*d^4*e^3 - 60*a*b^3*d^3*e^4 - 8814*a^2*b^2*d^2*e^5 - 10868*a^3
*b*d*e^6 - 1287*a^4*e^7)*x^3 + 3*(16*b^4*d^5*e^2 - 120*a*b^3*d^4*e^3 + 390*
```

```
a^2*b^2*d^3*e^4 + 14300*a^3*b*d^2*e^5 + 6435*a^4*d*e^6)*x^2 - (64*b^4*d^6*e
 - 480*a*b^3*d^5*e^2 + 1560*a^2*b^2*d^4*e^3 - 2860*a^3*b*d^3*e^4 - 19305*a^
4*d^2*e^5)*x)*sqrt(e*x + d)/e^5
```

Sympy [A] time = 33.5058, size = 960, normalized size = 7.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(b**2*x**2+2*a*b*x+a**2)**2,x)
```

```
[Out] a**4*d**2*Piecewise((sqrt(d)*x, Eq(e, 0)), (2*(d + e*x)**(3/2)/(3*e), True)
) + 4*a**4*d*(-d*(d + e*x)**(3/2)/3 + (d + e*x)**(5/2)/5)/e + 2*a**4*(d**2*
(d + e*x)**(3/2)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e + 8*a**
3*b*d**2*(-d*(d + e*x)**(3/2)/3 + (d + e*x)**(5/2)/5)/e**2 + 16*a**3*b*d*(d
**2*(d + e*x)**(3/2)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e**2
+ 8*a**3*b*(-d**3*(d + e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d +
 e*x)**(7/2)/7 + (d + e*x)**(9/2)/9)/e**2 + 12*a**2*b**2*d**2*(d**2*(d + e*
x)**(3/2)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e**3 + 24*a**2*b
**2*d*(-d**3*(d + e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d + e*x)
**(7/2)/7 + (d + e*x)**(9/2)/9)/e**3 + 12*a**2*b**2*(d**4*(d + e*x)**(3/2)/
3 - 4*d**3*(d + e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e*x)**
(9/2)/9 + (d + e*x)**(11/2)/11)/e**3 + 8*a*b**3*d**2*(-d**3*(d + e*x)**(3/2
)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d + e*x)**(7/2)/7 + (d + e*x)**(9/2)
/9)/e**4 + 16*a*b**3*d*(d**4*(d + e*x)**(3/2)/3 - 4*d**3*(d + e*x)**(5/2)/5
 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e*x)**(9/2)/9 + (d + e*x)**(11/2)/1
1)/e**4 + 8*a*b**3*(-d**5*(d + e*x)**(3/2)/3 + d**4*(d + e*x)**(5/2) - 10*d
**3*(d + e*x)**(7/2)/7 + 10*d**2*(d + e*x)**(9/2)/9 - 5*d*(d + e*x)**(11/2)
/11 + (d + e*x)**(13/2)/13)/e**4 + 2*b**4*d**2*(d**4*(d + e*x)**(3/2)/3 - 4
*d**3*(d + e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e*x)**(9/2)
/9 + (d + e*x)**(11/2)/11)/e**5 + 4*b**4*d*(-d**5*(d + e*x)**(3/2)/3 + d**4
*(d + e*x)**(5/2) - 10*d**3*(d + e*x)**(7/2)/7 + 10*d**2*(d + e*x)**(9/2)/9
 - 5*d*(d + e*x)**(11/2)/11 + (d + e*x)**(13/2)/13)/e**5 + 2*b**4*(d**6*(d
+ e*x)**(3/2)/3 - 6*d**5*(d + e*x)**(5/2)/5 + 15*d**4*(d + e*x)**(7/2)/7 -
20*d**3*(d + e*x)**(9/2)/9 + 15*d**2*(d + e*x)**(11/2)/11 - 6*d*(d + e*x)**
(13/2)/13 + (d + e*x)**(15/2)/15)/e**5
```

Giac [B] time = 1.25363, size = 1157, normalized size = 8.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

```
[Out] 2/45045*(12012*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^3*b*d^2*e^(-1) +
 2574*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*
a^2*b^2*d^2*e^(-2) + 572*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*
(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a*b^3*d^2*e^(-3) + 13*(315*(
x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*
(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*b^4*d^2*e^(-4) + 15015*(x*e
 + d)^(3/2)*a^4*d^2 + 3432*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*
(x*e + d)^(3/2)*d^2)*a^3*b*d*e^(-1) + 1716*(35*(x*e + d)^(9/2) - 135*(x*e +
```

d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a^2*b^2*d*
e^(-2) + 104*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d
)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*a*b^3*d*
e^(-3) + 10*(693*(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e +
d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*
(x*e + d)^(3/2)*d^5)*b^4*d*e^(-4) + 6006*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(
3/2)*d)*a^4*d + 572*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e
+ d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a^3*b*e^(-1) + 78*(315*(x*e + d)^
(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)
^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*a^2*b^2*e^(-2) + 20*(693*(x*e + d)^(
13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12870*(x*e +
d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*a*b^3*e
^(-3) + (3003*(x*e + d)^(15/2) - 20790*(x*e + d)^(13/2)*d + 61425*(x*e + d)
^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525*(x*e + d)^(7/2)*d^4 - 5405
4*(x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*b^4*e^(-4) + 429*(15*(x*
e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*a^4)*e^(-1)

3.1631 $\int (d+ex)^{3/2} \left(a^2+2abx+b^2x^2\right)^2 \, dx$

Optimal. Leaf size=129

$$-\frac{8b^3(d+ex)^{11/2}(bd-ae)}{11e^5}+\frac{4b^2(d+ex)^{9/2}(bd-ae)^2}{3e^5}-\frac{8b(d+ex)^{7/2}(bd-ae)^3}{7e^5}+\frac{2(d+ex)^{5/2}(bd-ae)^4}{5e^5}+\frac{2b^4(d+ex)^{\cdot}}{13e^5}$$

```
[Out] (2*(b*d - a*e)^4*(d + e*x)^(5/2))/(5*e^5) - (8*b*(b*d - a*e)^3*(d + e*x)^(7
/2))/(7*e^5) + (4*b^2*(b*d - a*e)^2*(d + e*x)^(9/2))/(3*e^5) - (8*b^3*(b*d
- a*e)*(d + e*x)^(11/2))/(11*e^5) + (2*b^4*(d + e*x)^(13/2))/(13*e^5)
```

Rubi [A] time = 0.0428612, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {27, 43}

$$-\frac{8b^3(d+ex)^{11/2}(bd-ae)}{11e^5}+\frac{4b^2(d+ex)^{9/2}(bd-ae)^2}{3e^5}-\frac{8b(d+ex)^{7/2}(bd-ae)^3}{7e^5}+\frac{2(d+ex)^{5/2}(bd-ae)^4}{5e^5}+\frac{2b^4(d+ex)^{\cdot}}{13e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] (2*(b*d - a*e)^4*(d + e*x)^(5/2))/(5*e^5) - (8*b*(b*d - a*e)^3*(d + e*x)^(7
/2))/(7*e^5) + (4*b^2*(b*d - a*e)^2*(d + e*x)^(9/2))/(3*e^5) - (8*b^3*(b*d
- a*e)*(d + e*x)^(11/2))/(11*e^5) + (2*b^4*(d + e*x)^(13/2))/(13*e^5)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^{3/2}\left(a^2+2abx+b^2x^2\right)^2\,dx &= \int (a+bx)^4(d+ex)^{3/2}\,dx\\ &= \int \left(\frac{(-bd+ae)^4(d+ex)^{3/2}}{e^4}-\frac{4b(bd-ae)^3(d+ex)^{5/2}}{e^4}+\frac{6b^2(bd-ae)^2(d+ex)^{7/\cdot}}{e^4}\right.\\ &= \frac{2(bd-ae)^4(d+ex)^{5/2}}{5e^5}-\frac{8b(bd-ae)^3(d+ex)^{7/2}}{7e^5}+\frac{4b^2(bd-ae)^2(d+ex)^{9/2}}{3e^5}-\end{aligned}$$

Mathematica [A] time = 0.0845709, size = 101, normalized size = 0.78

$$\frac{2(d+ex)^{5/2}\left(10010b^2(d+ex)^2(bd-ae)^2-5460b^3(d+ex)^3(bd-ae)-8580b(d+ex)(bd-ae)^3+3003(bd-ae)^4+115\right.}{15015e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] (2*(d + e*x)^(5/2)*(3003*(b*d - a*e)^4 - 8580*b*(b*d - a*e)^3*(d + e*x) + 1
0010*b^2*(b*d - a*e)^2*(d + e*x)^2 - 5460*b^3*(b*d - a*e)*(d + e*x)^3 + 115
5*b^4*(d + e*x)^4))/(15015*e^5)
```

Maple [A] time = 0.046, size = 186, normalized size = 1.4

$$\frac{2310\,x^4b^4e^4+10920\,x^3ab^3e^4-1680\,x^3b^4de^3+20020\,x^2a^2b^2e^4-7280\,x^2ab^3de^3+1120\,x^2b^4d^2e^2+17160\,xa^3be^4-11440\,}{15015\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^2,x)
```

```
[Out] 2/15015*(e*x+d)^(5/2)*(1155*b^4*e^4*x^4+5460*a*b^3*e^4*x^3-840*b^4*d*e^3*x^
3+10010*a^2*b^2*e^4*x^2-3640*a*b^3*d*e^3*x^2+560*b^4*d^2*e^2*x^2+8580*a^3*b
*e^4*x-5720*a^2*b^2*d*e^3*x+2080*a*b^3*d^2*e^2*x-320*b^4*d^3*e*x+3003*a^4*e
^4-3432*a^3*b*d*e^3+2288*a^2*b^2*d^2*e^2-832*a*b^3*d^3*e+128*b^4*d^4)/e^5
```

Maxima [A] time = 1.10382, size = 244, normalized size = 1.89

$$\frac{2\left(1155\,(ex+d)^{\frac{13}{2}}b^4-5460\left(b^4d-ab^3e\right)(ex+d)^{\frac{11}{2}}+10010\left(b^4d^2-2\,ab^3de+a^2b^2e^2\right)(ex+d)^{\frac{9}{2}}-8580\left(b^4d^3-3\,ab^3d^2e+\right.\right.}{15015\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

```
[Out] 2/15015*(1155*(e*x + d)^(13/2)*b^4 - 5460*(b^4*d - a*b^3*e)*(e*x + d)^(11/2
) + 10010*(b^4*d^2 - 2*a*b^3*d*e + a^2*b^2*e^2)*(e*x + d)^(9/2) - 8580*(b^4
*d^3 - 3*a*b^3*d^2*e + 3*a^2*b^2*d*e^2 - a^3*b*e^3)*(e*x + d)^(7/2) + 3003*
(b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4)*(e*
x + d)^(5/2))/e^5
```

Fricas [B] time = 1.49044, size = 702, normalized size = 5.44

$$\frac{2\left(1155\,b^4e^6x^6+128\,b^4d^6-832\,ab^3d^5e+2288\,a^2b^2d^4e^2-3432\,a^3bd^3e^3+3003\,a^4d^2e^4+210\left(7\,b^4de^5+26\,ab^3e^6\right)x^5+35\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

```
[Out] 2/15015*(1155*b^4*e^6*x^6 + 128*b^4*d^6 - 832*a*b^3*d^5*e + 2288*a^2*b^2*d^
4*e^2 - 3432*a^3*b*d^3*e^3 + 3003*a^4*d^2*e^4 + 210*(7*b^4*d*e^5 + 26*a*b^3
*e^6)*x^5 + 35*(b^4*d^2*e^4 + 208*a*b^3*d*e^5 + 286*a^2*b^2*e^6)*x^4 - 20*(
2*b^4*d^3*e^3 - 13*a*b^3*d^2*e^4 - 715*a^2*b^2*d*e^5 - 429*a^3*b*e^6)*x^3 +
 3*(16*b^4*d^4*e^2 - 104*a*b^3*d^3*e^3 + 286*a^2*b^2*d^2*e^4 + 4576*a^3*b*d
*e^5 + 1001*a^4*e^6)*x^2 - 2*(32*b^4*d^5*e - 208*a*b^3*d^4*e^2 + 572*a^2*b^
```

```
2*d^3*e^3 - 858*a^3*b*d^2*e^4 - 3003*a^4*d*e^5)*x)*sqrt(e*x + d)/e^5
```

Sympy [A] time = 18.9131, size = 559, normalized size = 4.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(b**2*x**2+2*a*b*x+a**2)**2,x)
```

```
[Out] a**4*d*Piecewise((sqrt(d)*x, Eq(e, 0)), (2*(d + e*x)**(3/2)/(3*e), True)) +
 2*a**4*(-d*(d + e*x)**(3/2)/3 + (d + e*x)**(5/2)/5)/e + 8*a**3*b*d*(-d*(d
+ e*x)**(3/2)/3 + (d + e*x)**(5/2)/5)/e**2 + 8*a**3*b*(d**2*(d + e*x)**(3/2
)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e**2 + 12*a**2*b**2*d*(d
**2*(d + e*x)**(3/2)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e**3
+ 12*a**2*b**2*(-d**3*(d + e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*
(d + e*x)**(7/2)/7 + (d + e*x)**(9/2)/9)/e**3 + 8*a*b**3*d*(-d**3*(d + e*x)
**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d + e*x)**(7/2)/7 + (d + e*x)*
*(9/2)/9)/e**4 + 8*a*b**3*(d**4*(d + e*x)**(3/2)/3 - 4*d**3*(d + e*x)**(5/2
)/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e*x)**(9/2)/9 + (d + e*x)**(11/2
)/11)/e**4 + 2*b**4*d*(d**4*(d + e*x)**(3/2)/3 - 4*d**3*(d + e*x)**(5/2)/5
+ 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e*x)**(9/2)/9 + (d + e*x)**(11/2)/11
)/e**5 + 2*b**4*(-d**5*(d + e*x)**(3/2)/3 + d**4*(d + e*x)**(5/2) - 10*d**3
*(d + e*x)**(7/2)/7 + 10*d**2*(d + e*x)**(9/2)/9 - 5*d*(d + e*x)**(11/2)/11
 + (d + e*x)**(13/2)/13)/e**5
```

Giac [B] time = 1.25953, size = 675, normalized size = 5.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

```
[Out] 2/45045*(12012*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^3*b*d*e^(-1) + 2
574*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*a^
2*b^2*d*e^(-2) + 572*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e
 + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a*b^3*d*e^(-3) + 13*(315*(x*e +
d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e +
 d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*b^4*d*e^(-4) + 15015*(x*e + d)^(3
/2)*a^4*d + 1716*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^
(3/2)*d^2)*a^3*b*e^(-1) + 858*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d +
 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a^2*b^2*e^(-2) + 52*(31
5*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 27
72*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*a*b^3*e^(-3) + 5*(693*(x
*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 1287
0*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5
)*b^4*e^(-4) + 3003*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^4)*e^(-1)
```

3.1632 $\int \sqrt{d+ex}\left(a^2+2abx+b^2x^2\right)^2\,dx$

Optimal. Leaf size=129

$$-\frac{8b^3(d+ex)^{9/2}(bd-ae)}{9e^5}+\frac{12b^2(d+ex)^{7/2}(bd-ae)^2}{7e^5}-\frac{8b(d+ex)^{5/2}(bd-ae)^3}{5e^5}+\frac{2(d+ex)^{3/2}(bd-ae)^4}{3e^5}+\frac{2b^4(d+ex)^{11/2}}{11e^5}$$

```
[Out] (2*(b*d - a*e)^4*(d + e*x)^(3/2))/(3*e^5) - (8*b*(b*d - a*e)^3*(d + e*x)^(5
/2))/(5*e^5) + (12*b^2*(b*d - a*e)^2*(d + e*x)^(7/2))/(7*e^5) - (8*b^3*(b*d
 - a*e)*(d + e*x)^(9/2))/(9*e^5) + (2*b^4*(d + e*x)^(11/2))/(11*e^5)
```

Rubi [A] time = 0.0540715, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {27, 43}

$$-\frac{8b^3(d+ex)^{9/2}(bd-ae)}{9e^5}+\frac{12b^2(d+ex)^{7/2}(bd-ae)^2}{7e^5}-\frac{8b(d+ex)^{5/2}(bd-ae)^3}{5e^5}+\frac{2(d+ex)^{3/2}(bd-ae)^4}{3e^5}+\frac{2b^4(d+ex)^{11/2}}{11e^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] (2*(b*d - a*e)^4*(d + e*x)^(3/2))/(3*e^5) - (8*b*(b*d - a*e)^3*(d + e*x)^(5
/2))/(5*e^5) + (12*b^2*(b*d - a*e)^2*(d + e*x)^(7/2))/(7*e^5) - (8*b^3*(b*d
 - a*e)*(d + e*x)^(9/2))/(9*e^5) + (2*b^4*(d + e*x)^(11/2))/(11*e^5)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{d+ex}\left(a^2+2abx+b^2x^2\right)^2\,dx&=\int (a+bx)^4\sqrt{d+ex}\,dx\\&=\int \left(\frac{(-bd+ae)^4\sqrt{d+ex}}{e^4}-\frac{4b(bd-ae)^3(d+ex)^{3/2}}{e^4}+\frac{6b^2(bd-ae)^2(d+ex)^{5/2}}{e^4}-\frac{4b^3\right.\\&=\frac{2(bd-ae)^4(d+ex)^{3/2}}{3e^5}-\frac{8b(bd-ae)^3(d+ex)^{5/2}}{5e^5}+\frac{12b^2(bd-ae)^2(d+ex)^{7/2}}{7e^5}-\frac{8b^3(\end{aligned}$$

Mathematica [A] time = 0.0845308, size = 101, normalized size = 0.78

$$\frac{2(d+ex)^{3/2}\left(2970b^2(d+ex)^2(bd-ae)^2-1540b^3(d+ex)^3(bd-ae)-2772b(d+ex)(bd-ae)^3+1155(bd-ae)^4+315b^4(d\right.}{3465e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] (2*(d + e*x)^(3/2)*(1155*(b*d - a*e)^4 - 2772*b*(b*d - a*e)^3*(d + e*x) + 2
970*b^2*(b*d - a*e)^2*(d + e*x)^2 - 1540*b^3*(b*d - a*e)*(d + e*x)^3 + 315*
b^4*(d + e*x)^4))/(3465*e^5)
```

Maple [A] time = 0.045, size = 186, normalized size = 1.4

$$\frac{630x^4b^4e^4 + 3080x^3ab^3e^4 - 560x^3b^4de^3 + 5940x^2a^2b^2e^4 - 2640x^2ab^3de^3 + 480x^2b^4d^2e^2 + 5544xa^3be^4 - 4752xa^2b}{3465e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^2*(e*x+d)^(1/2),x)
```

```
[Out] 2/3465*(e*x+d)^(3/2)*(315*b^4*e^4*x^4+1540*a*b^3*e^4*x^3-280*b^4*d*e^3*x^3+
2970*a^2*b^2*e^4*x^2-1320*a*b^3*d*e^3*x^2+240*b^4*d^2*e^2*x^2+2772*a^3*b*e^
4*x-2376*a^2*b^2*d*e^3*x+1056*a*b^3*d^2*e^2*x-192*b^4*d^3*e*x+1155*a^4*e^4-
1848*a^3*b*d*e^3+1584*a^2*b^2*d^2*e^2-704*a*b^3*d^3*e+128*b^4*d^4)/e^5
```

Maxima [A] time = 1.16986, size = 244, normalized size = 1.89

$$\frac{2\left(315(ex+d)^{\frac{11}{2}}b^4 - 1540\left(b^4d - ab^3e\right)(ex+d)^{\frac{9}{2}} + 2970\left(b^4d^2 - 2ab^3de + a^2b^2e^2\right)(ex+d)^{\frac{7}{2}} - 2772\left(b^4d^3 - 3ab^3d^2e +\right.\right.}{3465e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3465*(315*(e*x + d)^(11/2)*b^4 - 1540*(b^4*d - a*b^3*e)*(e*x + d)^(9/2) +
 2970*(b^4*d^2 - 2*a*b^3*d*e + a^2*b^2*e^2)*(e*x + d)^(7/2) - 2772*(b^4*d^3
 - 3*a*b^3*d^2*e + 3*a^2*b^2*d*e^2 - a^3*b*e^3)*(e*x + d)^(5/2) + 1155*(b^4
*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4)*(e*x +
d)^(3/2))/e^5
```

Fricas [B] time = 1.50096, size = 547, normalized size = 4.24

$$\frac{2\left(315b^4e^5x^5 + 128b^4d^5 - 704ab^3d^4e + 1584a^2b^2d^3e^2 - 1848a^3bd^2e^3 + 1155a^4de^4 + 35\left(b^4de^4 + 44ab^3e^5\right)x^4 - 10\left(4\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*b^4*e^5*x^5 + 128*b^4*d^5 - 704*a*b^3*d^4*e + 1584*a^2*b^2*d^3*
e^2 - 1848*a^3*b*d^2*e^3 + 1155*a^4*d*e^4 + 35*(b^4*d*e^4 + 44*a*b^3*e^5)*x
^4 - 10*(4*b^4*d^2*e^3 - 22*a*b^3*d*e^4 - 297*a^2*b^2*e^5)*x^3 + 6*(8*b^4*d
^3*e^2 - 44*a*b^3*d^2*e^3 + 99*a^2*b^2*d*e^4 + 462*a^3*b*e^5)*x^2 - (64*b^4
*d^4*e - 352*a*b^3*d^3*e^2 + 792*a^2*b^2*d^2*e^3 - 924*a^3*b*d*e^4 - 1155*a
^4*e^5)*x)*sqrt(e*x + d)/e^5
```

Sympy [A] time = 4.66578, size = 223, normalized size = 1.73

$$\frac{2\left(\frac{b^4(d+ex)^{\frac{11}{2}}}{11e^4}+\frac{(d+ex)^{\frac{9}{2}}\left(4ab^3e-4b^4d\right)}{9e^4}+\frac{(d+ex)^{\frac{7}{2}}\left(6a^2b^2e^2-12ab^3de+6b^4d^2\right)}{7e^4}+\frac{(d+ex)^{\frac{5}{2}}\left(4a^3be^3-12a^2b^2de^2+12ab^3d^2e-4b^4d^3\right)}{5e^4}+\frac{(d+ex)^{\frac{3}{2}}\left(a^4e^4-4a^3bde^3+\cdots\right.}{}\right.}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**2*(e*x+d)**(1/2),x)
```

```
[Out] 2*(b**4*(d + e*x)**(11/2)/(11*e**4) + (d + e*x)**(9/2)*(4*a*b**3*e - 4*b**4
*d)/(9*e**4) + (d + e*x)**(7/2)*(6*a**2*b**2*e**2 - 12*a*b**3*d*e + 6*b**4*
d**2)/(7*e**4) + (d + e*x)**(5/2)*(4*a**3*b*e**3 - 12*a**2*b**2*d*e**2 + 12
*a*b**3*d**2*e - 4*b**4*d**3)/(5*e**4) + (d + e*x)**(3/2)*(a**4*e**4 - 4*a*
*3*b*d*e**3 + 6*a**2*b**2*d**2*e**2 - 4*a*b**3*d**3*e + b**4*d**4)/(3*e**4)
)/e
```

Giac [A] time = 1.19623, size = 292, normalized size = 2.26

$$\frac{2}{3465}\left(924\left(3\,(xe+d)^{\frac{5}{2}}-5\,(xe+d)^{\frac{3}{2}}d\right)a^3be^{(-1)}+198\left(15\,(xe+d)^{\frac{7}{2}}-42\,(xe+d)^{\frac{5}{2}}d+35\,(xe+d)^{\frac{3}{2}}d^2\right)a^2b^2e^{(-2)}+44\left(35\,(x\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3465*(924*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^3*b*e^(-1) + 198*(1
5*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*a^2*b^2*
e^(-2) + 44*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/
2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a*b^3*e^(-3) + (315*(x*e + d)^(11/2) - 15
40*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3
+ 1155*(x*e + d)^(3/2)*d^4)*b^4*e^(-4) + 1155*(x*e + d)^(3/2)*a^4)*e^(-1)
```

3.1633 $\int \frac{(a^2+2abx+b^2x^2)^2}{\sqrt{d+ex}} dx$

Optimal. Leaf size=127

$$-\frac{8b^3(d+ex)^{7/2}(bd-ae)}{7e^5}+\frac{12b^2(d+ex)^{5/2}(bd-ae)^2}{5e^5}-\frac{8b(d+ex)^{3/2}(bd-ae)^3}{3e^5}+\frac{2\sqrt{d+ex}(bd-ae)^4}{e^5}+\frac{2b^4(d+ex)^{9/2}}{9e^5}$$

```
[Out] (2*(b*d - a*e)^4*Sqrt[d + e*x])/e^5 - (8*b*(b*d - a*e)^3*(d + e*x)^(3/2))/(
3*e^5) + (12*b^2*(b*d - a*e)^2*(d + e*x)^(5/2))/(5*e^5) - (8*b^3*(b*d - a*e
)*(d + e*x)^(7/2))/(7*e^5) + (2*b^4*(d + e*x)^(9/2))/(9*e^5)
```

Rubi [A] time = 0.0414936, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {27, 43}

$$-\frac{8b^3(d+ex)^{7/2}(bd-ae)}{7e^5}+\frac{12b^2(d+ex)^{5/2}(bd-ae)^2}{5e^5}-\frac{8b(d+ex)^{3/2}(bd-ae)^3}{3e^5}+\frac{2\sqrt{d+ex}(bd-ae)^4}{e^5}+\frac{2b^4(d+ex)^{9/2}}{9e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^2/Sqrt[d + e*x],x]
```

```
[Out] (2*(b*d - a*e)^4*Sqrt[d + e*x])/e^5 - (8*b*(b*d - a*e)^3*(d + e*x)^(3/2))/(
3*e^5) + (12*b^2*(b*d - a*e)^2*(d + e*x)^(5/2))/(5*e^5) - (8*b^3*(b*d - a*e
)*(d + e*x)^(7/2))/(7*e^5) + (2*b^4*(d + e*x)^(9/2))/(9*e^5)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx+b^2x^2)^2}{\sqrt{d+ex}} dx &= \int \frac{(a+bx)^4}{\sqrt{d+ex}} dx\\ &= \int \left(\frac{(-bd+ae)^4}{e^4\sqrt{d+ex}}-\frac{4b(bd-ae)^3\sqrt{d+ex}}{e^4}+\frac{6b^2(bd-ae)^2(d+ex)^{3/2}}{e^4}-\frac{4b^3(bd-ae)(d+ex}{e^4}\right.\\ &= \frac{2(bd-ae)^4\sqrt{d+ex}}{e^5}-\frac{8b(bd-ae)^3(d+ex)^{3/2}}{3e^5}+\frac{12b^2(bd-ae)^2(d+ex)^{5/2}}{5e^5}-\frac{8b^3(bd-ae}{7e}\end{aligned}$$

Mathematica [A] time = 0.0706069, size = 101, normalized size = 0.8

$$\frac{2\sqrt{d+ex}\left(378b^2(d+ex)^2(bd-ae)^2-180b^3(d+ex)^3(bd-ae)-420b(d+ex)(bd-ae)^3+315(bd-ae)^4+35b^4(d+ex\right.}{315e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[d + e*x]*(315*(b*d - a*e)^4 - 420*b*(b*d - a*e)^3*(d + e*x) + 378*b
^2*(b*d - a*e)^2*(d + e*x)^2 - 180*b^3*(b*d - a*e)*(d + e*x)^3 + 35*b^4*(d
+ e*x)^4))/(315*e^5)
```

Maple [A] time = 0.044, size = 186, normalized size = 1.5

$$\frac{70 x^4b^4e^4 + 360 x^3ab^3e^4 - 80 x^3b^4de^3 + 756 x^2a^2b^2e^4 - 432 x^2ab^3de^3 + 96 x^2b^4d^2e^2 + 840 xa^3be^4 - 1008 xa^2b^2de^3 + 576 x}{315 e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(1/2),x)
```

```
[Out] 2/315*(35*b^4*e^4*x^4+180*a*b^3*e^4*x^3-40*b^4*d*e^3*x^3+378*a^2*b^2*e^4*x^
2-216*a*b^3*d*e^3*x^2+48*b^4*d^2*e^2*x^2+420*a^3*b*e^4*x-504*a^2*b^2*d*e^3*
x+288*a*b^3*d^2*e^2*x-64*b^4*d^3*e*x+315*a^4*e^4-840*a^3*b*d*e^3+1008*a^2*b
^2*d^2*e^2-576*a*b^3*d^3*e+128*b^4*d^4)*(e*x+d)^(1/2)/e^5
```

Maxima [B] time = 1.17734, size = 333, normalized size = 2.62

$$\frac{2\left(315 \sqrt{ex+d}a^4 + 42\left(\frac{10\left((ex+d)^{\frac{3}{2}} - 3\sqrt{ex+d}d\right)ab}{e} + \frac{\left(3(ex+d)^{\frac{5}{2}} - 10(ex+d)^{\frac{3}{2}}d + 15\sqrt{ex+d}d^2\right)b^2}{e^2}\right)a^2 + \frac{84\left(3(ex+d)^{\frac{5}{2}} - 10(ex+d)^{\frac{3}{2}}d + 15\sqrt{ex+d}d^2\right)a^2b^2}{e^2}\right.}{315 e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(315*sqrt(e*x + d)*a^4 + 42*(10*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)
*a*b/e + (3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*
b^2/e^2)*a^2 + 84*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x +
 d)*d^2)*a^2*b^2/e^2 + 36*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e
*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a*b^3/e^3 + (35*(e*x + d)^(9/2) -
 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3
+ 315*sqrt(e*x + d)*d^4)*b^4/e^4)/e
```

Fricas [A] time = 1.49323, size = 405, normalized size = 3.19

$$\frac{2\left(35 b^4e^4x^4 + 128 b^4d^4 - 576 ab^3d^3e + 1008 a^2b^2d^2e^2 - 840 a^3bde^3 + 315 a^4e^4 - 20\left(2 b^4de^3 - 9 ab^3e^4\right)x^3 + 6\left(8 b^4d^2e^2 - \right.\right.}{315 e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*e^4*x^4 + 128*b^4*d^4 - 576*a*b^3*d^3*e + 1008*a^2*b^2*d^2*e^
2 - 840*a^3*b*d*e^3 + 315*a^4*e^4 - 20*(2*b^4*d*e^3 - 9*a*b^3*e^4)*x^3 + 6*
```

```
(8*b^4*d^2*e^2 - 36*a*b^3*d*e^3 + 63*a^2*b^2*e^4)*x^2 - 4*(16*b^4*d^3*e - 7
2*a*b^3*d^2*e^2 + 126*a^2*b^2*d*e^3 - 105*a^3*b*e^4)*x)*sqrt(e*x + d)/e^5
```

Sympy [A] time = 61.6239, size = 561, normalized size = 4.42

$$\begin{cases} -\frac{\frac{2a^4d}{\sqrt{d+ex}}+2a^4\left(-\frac{d}{\sqrt{d+ex}}-\sqrt{d+ex}\right)+\frac{8a^3bd\left(-\frac{d}{\sqrt{d+ex}}-\sqrt{d+ex}\right)}{e}+\frac{8a^3b\left(\frac{d^2}{\sqrt{d+ex}}+2d\sqrt{d+ex}-\frac{(d+ex)^{\frac{3}{2}}}{3}\right)}{e}+\frac{12a^2b^2d\left(\frac{d^2}{\sqrt{d+ex}}+2d\sqrt{d+ex}-\frac{(d+ex)^{\frac{3}{2}}}{3}\right)}{e^2}+\frac{12a^2b^2\left(-\frac{d^3}{\sqrt{d+ex}}-3d^2\sqrt{d+ex}+d(d+\cdots\right.}{e^2}}{\cdots} \\ \frac{a^4x+2a^3bx^2+2a^2b^2x^3+ab^3x^4+\frac{b^4x^5}{5}}{\sqrt{d}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**2/(e*x+d)**(1/2),x)
```

```
[Out] Piecewise((-(2*a**4*d/sqrt(d + e*x) + 2*a**4*(-d/sqrt(d + e*x) - sqrt(d + e
*x)) + 8*a**3*b*d*(-d/sqrt(d + e*x) - sqrt(d + e*x))/e + 8*a**3*b*(d**2/sqr
t(d + e*x) + 2*d*sqrt(d + e*x) - (d + e*x)**(3/2)/3)/e + 12*a**2*b**2*d*(d*
*2/sqrt(d + e*x) + 2*d*sqrt(d + e*x) - (d + e*x)**(3/2)/3)/e**2 + 12*a**2*b
**2*(-d**3/sqrt(d + e*x) - 3*d**2*sqrt(d + e*x) + d*(d + e*x)**(3/2) - (d +
 e*x)**(5/2)/5)/e**2 + 8*a*b**3*d*(-d**3/sqrt(d + e*x) - 3*d**2*sqrt(d + e*
x) + d*(d + e*x)**(3/2) - (d + e*x)**(5/2)/5)/e**3 + 8*a*b**3*(d**4/sqrt(d
+ e*x) + 4*d**3*sqrt(d + e*x) - 2*d**2*(d + e*x)**(3/2) + 4*d*(d + e*x)**(5
/2)/5 - (d + e*x)**(7/2)/7)/e**3 + 2*b**4*d*(d**4/sqrt(d + e*x) + 4*d**3*sq
rt(d + e*x) - 2*d**2*(d + e*x)**(3/2) + 4*d*(d + e*x)**(5/2)/5 - (d + e*x)*
*(7/2)/7)/e**4 + 2*b**4*(-d**5/sqrt(d + e*x) - 5*d**4*sqrt(d + e*x) + 10*d*
*3*(d + e*x)**(3/2)/3 - 2*d**2*(d + e*x)**(5/2) + 5*d*(d + e*x)**(7/2)/7 -
(d + e*x)**(9/2)/9)/e**4)/e, Ne(e, 0)), ((a**4*x + 2*a**3*b*x**2 + 2*a**2*b
**2*x**3 + a*b**3*x**4 + b**4*x**5/5)/sqrt(d), True))
```

Giac [A] time = 1.20947, size = 289, normalized size = 2.28

$$\frac{2}{315}\left(420\left((xe+d)^{\frac{3}{2}}-3\sqrt{xe+d}d\right)a^3be^{(-1)}+126\left(3(xe+d)^{\frac{5}{2}}-10(xe+d)^{\frac{3}{2}}d+15\sqrt{xe+d}d^2\right)a^2b^2e^{(-2)}+36\left(5(xe+\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(420*((x*e + d)^(3/2) - 3*sqrt(x*e + d)*d)*a^3*b*e^(-1) + 126*(3*(x*e
 + d)^(5/2) - 10*(x*e + d)^(3/2)*d + 15*sqrt(x*e + d)*d^2)*a^2*b^2*e^(-2) +
 36*(5*(x*e + d)^(7/2) - 21*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2 - 35
*sqrt(x*e + d)*d^3)*a*b^3*e^(-3) + (35*(x*e + d)^(9/2) - 180*(x*e + d)^(7/2
)*d + 378*(x*e + d)^(5/2)*d^2 - 420*(x*e + d)^(3/2)*d^3 + 315*sqrt(x*e + d)
*d^4)*b^4*e^(-4) + 315*sqrt(x*e + d)*a^4)*e^(-1)
```

3.1634 $\int \frac{(a^2+2abx+b^2x^2)^2}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{8b^3(d+ex)^{5/2}(bd-ae)}{5e^5}+\frac{4b^2(d+ex)^{3/2}(bd-ae)^2}{e^5}-\frac{8b\sqrt{d+ex}(bd-ae)^3}{e^5}-\frac{2(bd-ae)^4}{e^5\sqrt{d+ex}}+\frac{2b^4(d+ex)^{7/2}}{7e^5}$$

```
[Out] (-2*(b*d - a*e)^4)/(e^5*Sqrt[d + e*x]) - (8*b*(b*d - a*e)^3*Sqrt[d + e*x])/
e^5 + (4*b^2*(b*d - a*e)^2*(d + e*x)^(3/2))/e^5 - (8*b^3*(b*d - a*e)*(d + e
*x)^(5/2))/(5*e^5) + (2*b^4*(d + e*x)^(7/2))/(7*e^5)
```

Rubi [A] time = 0.0406202, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {27, 43}

$$-\frac{8b^3(d+ex)^{5/2}(bd-ae)}{5e^5}+\frac{4b^2(d+ex)^{3/2}(bd-ae)^2}{e^5}-\frac{8b\sqrt{d+ex}(bd-ae)^3}{e^5}-\frac{2(bd-ae)^4}{e^5\sqrt{d+ex}}+\frac{2b^4(d+ex)^{7/2}}{7e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^(3/2),x]
```

```
[Out] (-2*(b*d - a*e)^4)/(e^5*Sqrt[d + e*x]) - (8*b*(b*d - a*e)^3*Sqrt[d + e*x])/
e^5 + (4*b^2*(b*d - a*e)^2*(d + e*x)^(3/2))/e^5 - (8*b^3*(b*d - a*e)*(d + e
*x)^(5/2))/(5*e^5) + (2*b^4*(d + e*x)^(7/2))/(7*e^5)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx+b^2x^2)^2}{(d+ex)^{3/2}}\,dx &= \int \frac{(a+bx)^4}{(d+ex)^{3/2}}\,dx\\ &= \int \left(\frac{(-bd+ae)^4}{e^4(d+ex)^{3/2}}-\frac{4b(bd-ae)^3}{e^4\sqrt{d+ex}}+\frac{6b^2(bd-ae)^2\sqrt{d+ex}}{e^4}-\frac{4b^3(bd-ae)(d+ex)^{3/2}}{e^4}+\frac{b^4(d+\right.\\ &= -\frac{2(bd-ae)^4}{e^5\sqrt{d+ex}}-\frac{8b(bd-ae)^3\sqrt{d+ex}}{e^5}+\frac{4b^2(bd-ae)^2(d+ex)^{3/2}}{e^5}-\frac{8b^3(bd-ae)(d+ex)^{5/2}}{5e^5}+\end{aligned}$$

Mathematica [A] time = 0.069786, size = 101, normalized size = 0.82

$$\frac{2\left(70b^2(d+ex)^2(bd-ae)^2-28b^3(d+ex)^3(bd-ae)-140b(d+ex)(bd-ae)^3-35(bd-ae)^4+5b^4(d+ex)^4\right)}{35e^5\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^(3/2),x]
```

```
[Out] (2*(-35*(b*d - a*e)^4 - 140*b*(b*d - a*e)^3*(d + e*x) + 70*b^2*(b*d - a*e)^
2*(d + e*x)^2 - 28*b^3*(b*d - a*e)*(d + e*x)^3 + 5*b^4*(d + e*x)^4))/(35*e^
5*Sqrt[d + e*x])
```

Maple [A] time = 0.046, size = 186, normalized size = 1.5

$$-\frac{-10\,x^4b^4e^4-56\,x^3ab^3e^4+16\,x^3b^4de^3-140\,x^2a^2b^2e^4+112\,x^2ab^3de^3-32\,x^2b^4d^2e^2-280\,xa^3be^4+560\,xa^2b^2de^3-44}{35\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(3/2),x)
```

```
[Out] -2/35*(-5*b^4*e^4*x^4-28*a*b^3*e^4*x^3+8*b^4*d*e^3*x^3-70*a^2*b^2*e^4*x^2+5
6*a*b^3*d*e^3*x^2-16*b^4*d^2*e^2*x^2-140*a^3*b*e^4*x+280*a^2*b^2*d*e^3*x-22
4*a*b^3*d^2*e^2*x+64*b^4*d^3*e*x+35*a^4*e^4-280*a^3*b*d*e^3+560*a^2*b^2*d^2
*e^2-448*a*b^3*d^3*e+128*b^4*d^4)/(e*x+d)^(1/2)/e^5
```

Maxima [A] time = 1.04781, size = 255, normalized size = 2.07

$$\frac{2\left(\frac{5\,(ex+d)^{\frac{7}{2}}b^4-28\left(b^4d-ab^3e\right)(ex+d)^{\frac{5}{2}}+70\left(b^4d^2-2\,ab^3de+a^2b^2e^2\right)(ex+d)^{\frac{3}{2}}-140\left(b^4d^3-3\,ab^3d^2e+3\,a^2b^2de^2-a^3be^3\right)\sqrt{ex+d}}{e^4}-\frac{35\left(b^4d^4-4\,ab^3d^3e+6\,a^2b^2d^2e^2\right.}{\sqrt{ex+d}e^4}\right.}{35\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/35*((5*(e*x + d)^(7/2)*b^4 - 28*(b^4*d - a*b^3*e)*(e*x + d)^(5/2) + 70*(b
^4*d^2 - 2*a*b^3*d*e + a^2*b^2*e^2)*(e*x + d)^(3/2) - 140*(b^4*d^3 - 3*a*b^
3*d^2*e + 3*a^2*b^2*d*e^2 - a^3*b*e^3)*sqrt(e*x + d))/e^4 - 35*(b^4*d^4 - 4
*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4)/(sqrt(e*x + d)*
e^4))/e
```

Fricas [A] time = 1.50403, size = 412, normalized size = 3.35

$$\frac{2\left(5\,b^4e^4x^4-128\,b^4d^4+448\,ab^3d^3e-560\,a^2b^2d^2e^2+280\,a^3bde^3-35\,a^4e^4-4\left(2\,b^4de^3-7\,ab^3e^4\right)x^3+2\left(8\,b^4d^2e^2-28\right.\right.}{35\left(e^6x+de^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^4*e^4*x^4 - 128*b^4*d^4 + 448*a*b^3*d^3*e - 560*a^2*b^2*d^2*e^2 +
 280*a^3*b*d*e^3 - 35*a^4*e^4 - 4*(2*b^4*d*e^3 - 7*a*b^3*e^4)*x^3 + 2*(8*b^
4*d^2*e^2 - 28*a*b^3*d*e^3 + 35*a^2*b^2*e^4)*x^2 - 4*(16*b^4*d^3*e - 56*a*b
```

```
^3*d^2*e^2 + 70*a^2*b^2*d*e^3 - 35*a^3*b*e^4)*x)*sqrt(e*x + d)/(e^6*x + d*e
^5)
```

Sympy [A] time = 40.107, size = 168, normalized size = 1.37

$$\frac{2b^4(d+ex)^{\frac{7}{2}}}{7e^5}+\frac{(d+ex)^{\frac{5}{2}}\left(8ab^3e-8b^4d\right)}{5e^5}+\frac{(d+ex)^{\frac{3}{2}}\left(12a^2b^2e^2-24ab^3de+12b^4d^2\right)}{3e^5}+\frac{\sqrt{d+ex}\left(8a^3be^3-24a^2b^2de^2+2\right.}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**2/(e*x+d)**(3/2),x)
```

```
[Out] 2*b**4*(d + e*x)**(7/2)/(7*e**5) + (d + e*x)**(5/2)*(8*a*b**3*e - 8*b**4*d)
/(5*e**5) + (d + e*x)**(3/2)*(12*a**2*b**2*e**2 - 24*a*b**3*d*e + 12*b**4*d
**2)/(3*e**5) + sqrt(d + e*x)*(8*a**3*b*e**3 - 24*a**2*b**2*d*e**2 + 24*a*b
**3*d**2*e - 8*b**4*d**3)/e**5 - 2*(a*e - b*d)**4/(e**5*sqrt(d + e*x))
```

Giac [B] time = 1.18195, size = 320, normalized size = 2.6

$$\frac{2}{35}\left(5(xe+d)^{\frac{7}{2}}b^4e^{30}-28(xe+d)^{\frac{5}{2}}b^4de^{30}+70(xe+d)^{\frac{3}{2}}b^4d^2e^{30}-140\sqrt{xe+d}b^4d^3e^{30}+28(xe+d)^{\frac{5}{2}}ab^3e^{31}-140(xe+d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] 2/35*(5*(x*e + d)^(7/2)*b^4*e^30 - 28*(x*e + d)^(5/2)*b^4*d*e^30 + 70*(x*e
+ d)^(3/2)*b^4*d^2*e^30 - 140*sqrt(x*e + d)*b^4*d^3*e^30 + 28*(x*e + d)^(5/
2)*a*b^3*e^31 - 140*(x*e + d)^(3/2)*a*b^3*d*e^31 + 420*sqrt(x*e + d)*a*b^3*
d^2*e^31 + 70*(x*e + d)^(3/2)*a^2*b^2*e^32 - 420*sqrt(x*e + d)*a^2*b^2*d*e^
32 + 140*sqrt(x*e + d)*a^3*b*e^33)*e^(-35) - 2*(b^4*d^4 - 4*a*b^3*d^3*e + 6
*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4)*e^(-5)/sqrt(x*e + d)
```

3.1635 $\int \frac{\left(a^2+2abx+b^2x^2\right)^2}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=125

$$-\frac{8b^3(d+ex)^{3/2}(bd-ae)}{3e^5}+\frac{12b^2\sqrt{d+ex}(bd-ae)^2}{e^5}+\frac{8b(bd-ae)^3}{e^5\sqrt{d+ex}}-\frac{2(bd-ae)^4}{3e^5(d+ex)^{3/2}}+\frac{2b^4(d+ex)^{5/2}}{5e^5}$$

```
[Out] (-2*(b*d - a*e)^4)/(3*e^5*(d + e*x)^(3/2)) + (8*b*(b*d - a*e)^3)/(e^5*Sqrt[
d + e*x]) + (12*b^2*(b*d - a*e)^2*Sqrt[d + e*x])/e^5 - (8*b^3*(b*d - a*e)*(
d + e*x)^(3/2))/(3*e^5) + (2*b^4*(d + e*x)^(5/2))/(5*e^5)
```

Rubi [A] time = 0.0415078, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {27, 43}

$$-\frac{8b^3(d+ex)^{3/2}(bd-ae)}{3e^5}+\frac{12b^2\sqrt{d+ex}(bd-ae)^2}{e^5}+\frac{8b(bd-ae)^3}{e^5\sqrt{d+ex}}-\frac{2(bd-ae)^4}{3e^5(d+ex)^{3/2}}+\frac{2b^4(d+ex)^{5/2}}{5e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^(5/2),x]
```

```
[Out] (-2*(b*d - a*e)^4)/(3*e^5*(d + e*x)^(3/2)) + (8*b*(b*d - a*e)^3)/(e^5*Sqrt[
d + e*x]) + (12*b^2*(b*d - a*e)^2*Sqrt[d + e*x])/e^5 - (8*b^3*(b*d - a*e)*(
d + e*x)^(3/2))/(3*e^5) + (2*b^4*(d + e*x)^(5/2))/(5*e^5)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx+b^2x^2\right)^2}{(d+ex)^{5/2}}\,dx &= \int \frac{(a+bx)^4}{(d+ex)^{5/2}}\,dx\\
&= \int \left(\frac{(-bd+ae)^4}{e^4(d+ex)^{5/2}}-\frac{4b(bd-ae)^3}{e^4(d+ex)^{3/2}}+\frac{6b^2(bd-ae)^2}{e^4\sqrt{d+ex}}-\frac{4b^3(bd-ae)\sqrt{d+ex}}{e^4}+\frac{b^4(d+ex)^{3/2}}{e^4}\right.\\
&= -\frac{2(bd-ae)^4}{3e^5(d+ex)^{3/2}}+\frac{8b(bd-ae)^3}{e^5\sqrt{d+ex}}+\frac{12b^2(bd-ae)^2\sqrt{d+ex}}{e^5}-\frac{8b^3(bd-ae)(d+ex)^{3/2}}{3e^5}+\frac{2b}{}
\end{aligned}$$

Mathematica [A] time = 0.0716295, size = 101, normalized size = 0.81

$$\frac{2\left(90b^2(d+ex)^2(bd-ae)^2-20b^3(d+ex)^3(bd-ae)+60b(d+ex)(bd-ae)^3-5(bd-ae)^4+3b^4(d+ex)^4\right)}{15e^5(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^(5/2),x]
```

```
[Out] (2*(-5*(b*d - a*e)^4 + 60*b*(b*d - a*e)^3*(d + e*x) + 90*b^2*(b*d - a*e)^2*
(d + e*x)^2 - 20*b^3*(b*d - a*e)*(d + e*x)^3 + 3*b^4*(d + e*x)^4))/(15*e^5*
(d + e*x)^(3/2))
```

Maple [A] time = 0.049, size = 186, normalized size = 1.5

$$-\frac{-6x^4b^4e^4 - 40x^3ab^3e^4 + 16x^3b^4de^3 - 180x^2a^2b^2e^4 + 240x^2ab^3de^3 - 96x^2b^4d^2e^2 + 120xa^3be^4 - 720xa^2b^2de^3 + 960xa}{15e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(5/2),x)
```

```
[Out] -2/15*(-3*b^4*e^4*x^4-20*a*b^3*e^4*x^3+8*b^4*d*e^3*x^3-90*a^2*b^2*e^4*x^2+1
20*a*b^3*d*e^3*x^2-48*b^4*d^2*e^2*x^2+60*a^3*b*e^4*x-360*a^2*b^2*d*e^3*x+48
0*a*b^3*d^2*e^2*x-192*b^4*d^3*e*x+5*a^4*e^4+40*a^3*b*d*e^3-240*a^2*b^2*d^2*
e^2+320*a*b^3*d^3*e-128*b^4*d^4)/(e*x+d)^(3/2)/e^5
```

Maxima [A] time = 1.12195, size = 252, normalized size = 2.02

$$\frac{2\left(\frac{3(ex+d)^{\frac{5}{2}}b^4 - 20(b^4d - ab^3e)(ex+d)^{\frac{3}{2}} + 90(b^4d^2 - 2ab^3de + a^2b^2e^2)\sqrt{ex+d}}{e^4} - \frac{5(b^4d^4 - 4ab^3d^3e + 6a^2b^2d^2e^2 - 4a^3bde^3 + a^4e^4 - 12(b^4d^3 - 3ab^3d^2e + 3a^2b^2de^2 - a^3be^3}{(ex+d)^{\frac{3}{2}}e^4}\right.}{15e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*((3*(e*x + d)^(5/2)*b^4 - 20*(b^4*d - a*b^3*e)*(e*x + d)^(3/2) + 90*(b
^4*d^2 - 2*a*b^3*d*e + a^2*b^2*e^2)*sqrt(e*x + d))/e^4 - 5*(b^4*d^4 - 4*a*b
^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4 - 12*(b^4*d^3 - 3*a*
b^3*d^2*e + 3*a^2*b^2*d*e^2 - a^3*b*e^3)*(e*x + d))/((e*x + d)^(3/2)*e^4))/
e
```

Fricas [A] time = 1.61883, size = 431, normalized size = 3.45

$$\frac{2\left(3b^4e^4x^4 + 128b^4d^4 - 320ab^3d^3e + 240a^2b^2d^2e^2 - 40a^3bde^3 - 5a^4e^4 - 4\left(2b^4de^3 - 5ab^3e^4\right)x^3 + 6\left(8b^4d^2e^2 - 20ab^3de\right.\right.}{15\left(e^7x^2 + 2de^6x + d^2e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^4*e^4*x^4 + 128*b^4*d^4 - 320*a*b^3*d^3*e + 240*a^2*b^2*d^2*e^2 -
 40*a^3*b*d*e^3 - 5*a^4*e^4 - 4*(2*b^4*d*e^3 - 5*a*b^3*e^4)*x^3 + 6*(8*b^4*
d^2*e^2 - 20*a*b^3*d*e^3 + 15*a^2*b^2*e^4)*x^2 + 12*(16*b^4*d^3*e - 40*a*b^
3*d^2*e^2 + 30*a^2*b^2*d*e^3 - 5*a^3*b*e^4)*x)*sqrt(e*x + d)/(e^7*x^2 + 2*d
```

```
*e^6*x + d^2*e^5)
```

Sympy [A] time = 40.3786, size = 136, normalized size = 1.09

$$\frac{2b^4(d+ex)^{\frac{5}{2}}}{5e^5} - \frac{8b(ae-bd)^3}{e^5\sqrt{d+ex}} + \frac{(d+ex)^{\frac{3}{2}}\left(8ab^3e-8b^4d\right)}{3e^5} + \frac{\sqrt{d+ex}\left(12a^2b^2e^2-24ab^3de+12b^4d^2\right)}{e^5} - \frac{2(ae-bd)^4}{3e^5(d+ex)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**2/(e*x+d)**(5/2),x)
```

```
[Out] 2*b**4*(d + e*x)**(5/2)/(5*e**5) - 8*b*(a*e - b*d)**3/(e**5*sqrt(d + e*x))
+ (d + e*x)**(3/2)*(8*a*b**3*e - 8*b**4*d)/(3*e**5) + sqrt(d + e*x)*(12*a**
2*b**2*e**2 - 24*a*b**3*d*e + 12*b**4*d**2)/e**5 - 2*(a*e - b*d)**4/(3*e**5
*(d + e*x)**(3/2))
```

Giac [B] time = 1.15454, size = 309, normalized size = 2.47

$$\frac{2}{15}\left(3(xe+d)^{\frac{5}{2}}b^4e^{20} - 20(xe+d)^{\frac{3}{2}}b^4de^{20} + 90\sqrt{xe+d}b^4d^2e^{20} + 20(xe+d)^{\frac{3}{2}}ab^3e^{21} - 180\sqrt{xe+d}ab^3de^{21} + 90\sqrt{xe+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*(x*e + d)^(5/2)*b^4*e^20 - 20*(x*e + d)^(3/2)*b^4*d*e^20 + 90*sqrt(
x*e + d)*b^4*d^2*e^20 + 20*(x*e + d)^(3/2)*a*b^3*e^21 - 180*sqrt(x*e + d)*a
*b^3*d*e^21 + 90*sqrt(x*e + d)*a^2*b^2*e^22)*e^(-25) + 2/3*(12*(x*e + d)*b^
4*d^3 - b^4*d^4 - 36*(x*e + d)*a*b^3*d^2*e + 4*a*b^3*d^3*e + 36*(x*e + d)*a
^2*b^2*d*e^2 - 6*a^2*b^2*d^2*e^2 - 12*(x*e + d)*a^3*b*e^3 + 4*a^3*b*d*e^3 -
 a^4*e^4)*e^(-5)/(x*e + d)^(3/2)
```

3.1636 $\int \frac{(a^2+2abx+b^2x^2)^2}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=125

$$-\frac{8b^3\sqrt{d+ex}(bd-ae)}{e^5}-\frac{12b^2(bd-ae)^2}{e^5\sqrt{d+ex}}+\frac{8b(bd-ae)^3}{3e^5(d+ex)^{3/2}}-\frac{2(bd-ae)^4}{5e^5(d+ex)^{5/2}}+\frac{2b^4(d+ex)^{3/2}}{3e^5}$$

```
[Out] (-2*(b*d - a*e)^4)/(5*e^5*(d + e*x)^(5/2)) + (8*b*(b*d - a*e)^3)/(3*e^5*(d
+ e*x)^(3/2)) - (12*b^2*(b*d - a*e)^2)/(e^5*Sqrt[d + e*x]) - (8*b^3*(b*d -
a*e)*Sqrt[d + e*x])/e^5 + (2*b^4*(d + e*x)^(3/2))/(3*e^5)
```

Rubi [A] time = 0.0430044, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {27, 43}

$$-\frac{8b^3\sqrt{d+ex}(bd-ae)}{e^5}-\frac{12b^2(bd-ae)^2}{e^5\sqrt{d+ex}}+\frac{8b(bd-ae)^3}{3e^5(d+ex)^{3/2}}-\frac{2(bd-ae)^4}{5e^5(d+ex)^{5/2}}+\frac{2b^4(d+ex)^{3/2}}{3e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^(7/2),x]
```

```
[Out] (-2*(b*d - a*e)^4)/(5*e^5*(d + e*x)^(5/2)) + (8*b*(b*d - a*e)^3)/(3*e^5*(d
+ e*x)^(3/2)) - (12*b^2*(b*d - a*e)^2)/(e^5*Sqrt[d + e*x]) - (8*b^3*(b*d -
a*e)*Sqrt[d + e*x])/e^5 + (2*b^4*(d + e*x)^(3/2))/(3*e^5)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx+b^2x^2\right)^2}{(d+ex)^{7/2}}\,dx &= \int \frac{(a+bx)^4}{(d+ex)^{7/2}}\,dx\\
&= \int \left(\frac{(-bd+ae)^4}{e^4(d+ex)^{7/2}}-\frac{4b(bd-ae)^3}{e^4(d+ex)^{5/2}}+\frac{6b^2(bd-ae)^2}{e^4(d+ex)^{3/2}}-\frac{4b^3(bd-ae)}{e^4\sqrt{d+ex}}+\frac{b^4\sqrt{d+ex}}{e^4}\right)dx\\
&= -\frac{2(bd-ae)^4}{5e^5(d+ex)^{5/2}}+\frac{8b(bd-ae)^3}{3e^5(d+ex)^{3/2}}-\frac{12b^2(bd-ae)^2}{e^5\sqrt{d+ex}}-\frac{8b^3(bd-ae)\sqrt{d+ex}}{e^5}+\frac{2b^4(d+ex)^{3/2}}{3e^5}
\end{aligned}$$

Mathematica [A] time = 0.0715284, size = 101, normalized size = 0.81

$$\frac{2\left(-90b^2(d+ex)^2(bd-ae)^2-60b^3(d+ex)^3(bd-ae)+20b(d+ex)(bd-ae)^3-3(bd-ae)^4+5b^4(d+ex)^4\right)}{15e^5(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^(7/2),x]
```

```
[Out] (2*(-3*(b*d - a*e)^4 + 20*b*(b*d - a*e)^3*(d + e*x) - 90*b^2*(b*d - a*e)^2*
(d + e*x)^2 - 60*b^3*(b*d - a*e)*(d + e*x)^3 + 5*b^4*(d + e*x)^4))/(15*e^5*
(d + e*x)^(5/2))
```

Maple [A] time = 0.046, size = 186, normalized size = 1.5

$$-\frac{-10\,x^4b^4e^4-120\,x^3ab^3e^4+80\,x^3b^4de^3+180\,x^2a^2b^2e^4-720\,x^2ab^3de^3+480\,x^2b^4d^2e^2+40\,xa^3be^4+240\,xa^2b^2de^3-\cdots}{15\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(7/2),x)
```

```
[Out] -2/15*(-5*b^4*e^4*x^4-60*a*b^3*e^4*x^3+40*b^4*d*e^3*x^3+90*a^2*b^2*e^4*x^2-
360*a*b^3*d*e^3*x^2+240*b^4*d^2*e^2*x^2+20*a^3*b*e^4*x+120*a^2*b^2*d*e^3*x-
480*a*b^3*d^2*e^2*x+320*b^4*d^3*e*x+3*a^4*e^4+8*a^3*b*d*e^3+48*a^2*b^2*d^2*
e^2-192*a*b^3*d^3*e+128*b^4*d^4)/(e*x+d)^(5/2)/e^5
```

Maxima [A] time = 1.21617, size = 255, normalized size = 2.04

$$\frac{2\left(\frac{5\left((ex+d)^{\frac{3}{2}}b^4-12\left(b^4d-ab^3e\right)\sqrt{ex+d}\right)}{e^4}-\frac{3\,b^4d^4-12\,ab^3d^3e+18\,a^2b^2d^2e^2-12\,a^3bde^3+3\,a^4e^4+90\left(b^4d^2-2\,ab^3de+a^2b^2e^2\right)(ex+d)^2-20\left(b^4d^3-3\,ab^3d^2e+3\,a^2b^2\cdots\right.}{(ex+d)^{\frac{5}{2}}e^4}\right.}{15\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*(5*((e*x + d)^(3/2)*b^4 - 12*(b^4*d - a*b^3*e)*sqrt(e*x + d))/e^4 - (3
*b^4*d^4 - 12*a*b^3*d^3*e + 18*a^2*b^2*d^2*e^2 - 12*a^3*b*d*e^3 + 3*a^4*e^4
 + 90*(b^4*d^2 - 2*a*b^3*d*e + a^2*b^2*e^2)*(e*x + d)^2 - 20*(b^4*d^3 - 3*a
*b^3*d^2*e + 3*a^2*b^2*d*e^2 - a^3*b*e^3)*(e*x + d))/((e*x + d)^(5/2)*e^4))
/e
```

Fricas [A] time = 1.49603, size = 447, normalized size = 3.58

$$\frac{2\left(5\,b^4e^4x^4-128\,b^4d^4+192\,ab^3d^3e-48\,a^2b^2d^2e^2-8\,a^3bde^3-3\,a^4e^4-20\left(2\,b^4de^3-3\,ab^3e^4\right)x^3-30\left(8\,b^4d^2e^2-12\,ab\cdots\right.\right.}{15\left(e^8x^3+3\,de^7x^2+3\,d^2e^6x+d^3e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(5*b^4*e^4*x^4 - 128*b^4*d^4 + 192*a*b^3*d^3*e - 48*a^2*b^2*d^2*e^2 -
8*a^3*b*d*e^3 - 3*a^4*e^4 - 20*(2*b^4*d*e^3 - 3*a*b^3*e^4)*x^3 - 30*(8*b^4*
d^2*e^2 - 12*a*b^3*d*e^3 + 3*a^2*b^2*e^4)*x^2 - 20*(16*b^4*d^3*e - 24*a*b^3
```

```
*d^2*e^2 + 6*a^2*b^2*d*e^3 + a^3*b*e^4)*x)*sqrt(e*x + d)/(e^8*x^3 + 3*d*e^7
*x^2 + 3*d^2*e^6*x + d^3*e^5)
```

Sympy [A] time = 4.30085, size = 1008, normalized size = 8.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**2/(e*x+d)**(7/2),x)
```

```
[Out] Piecewise((-6*a**4*e**4/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d +
e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 16*a**3*b*d*e**3/(15*d**2*e**5*sqrt(d
+ e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 40*a**3*
b*e**4*x/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*
x**2*sqrt(d + e*x)) - 96*a**2*b**2*d**2*e**2/(15*d**2*e**5*sqrt(d + e*x) +
30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 240*a**2*b**2*d*e
**3*x/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**
2*sqrt(d + e*x)) - 180*a**2*b**2*e**4*x**2/(15*d**2*e**5*sqrt(d + e*x) + 30
*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) + 384*a*b**3*d**3*e/(
15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(
d + e*x)) + 960*a*b**3*d**2*e**2*x/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*
x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) + 720*a*b**3*d*e**3*x**2/(15*
d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d +
 e*x)) + 120*a*b**3*e**4*x**3/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqr
t(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 256*b**4*d**4/(15*d**2*e**5*sqrt
(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 640*b
**4*d**3*e*x/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e
**7*x**2*sqrt(d + e*x)) - 480*b**4*d**2*e**2*x**2/(15*d**2*e**5*sqrt(d + e*
x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 80*b**4*d*e*
*3*x**3/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x
**2*sqrt(d + e*x)) + 10*b**4*e**4*x**4/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e
**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)), Ne(e, 0)), ((a**4*x + 2*
a**3*b*x**2 + 2*a**2*b**2*x**3 + a*b**3*x**4 + b**4*x**5/5)/d**(7/2), True)
)
```

Giac [B] time = 1.24122, size = 305, normalized size = 2.44

$$\frac{2}{3}\left((xe+d)^{\frac{3}{2}}b^4e^{10} - 12\sqrt{xe+d}b^4de^{10} + 12\sqrt{xe+d}ab^3e^{11}\right)e^{(-15)} - \frac{2\left(90\,(xe+d)^2b^4d^2 - 20\,(xe+d)b^4d^3 + 3\,b^4d^4 - 180\,(x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] 2/3*((x*e + d)^(3/2)*b^4*e^10 - 12*sqrt(x*e + d)*b^4*d*e^10 + 12*sqrt(x*e +
 d)*a*b^3*e^11)*e^(-15) - 2/15*(90*(x*e + d)^2*b^4*d^2 - 20*(x*e + d)*b^4*d
^3 + 3*b^4*d^4 - 180*(x*e + d)^2*a*b^3*d*e + 60*(x*e + d)*a*b^3*d^2*e - 12*
a*b^3*d^3*e + 90*(x*e + d)^2*a^2*b^2*e^2 - 60*(x*e + d)*a^2*b^2*d*e^2 + 18*
a^2*b^2*d^2*e^2 + 20*(x*e + d)*a^3*b*e^3 - 12*a^3*b*d*e^3 + 3*a^4*e^4)*e^(-
5)/(x*e + d)^(5/2)
```

3.1637 $\int (d+ex)^{7/2} \left(a^2+2abx+b^2x^2\right)^3 \, dx$

Optimal. Leaf size=187

$$-\frac{12b^5(d+ex)^{19/2}(bd-ae)}{19e^7}+\frac{30b^4(d+ex)^{17/2}(bd-ae)^2}{17e^7}-\frac{8b^3(d+ex)^{15/2}(bd-ae)^3}{3e^7}+\frac{30b^2(d+ex)^{13/2}(bd-ae)^4}{13e^7}-12$$

```
[Out] (2*(b*d - a*e)^6*(d + e*x)^(9/2))/(9*e^7) - (12*b*(b*d - a*e)^5*(d + e*x)^(
11/2))/(11*e^7) + (30*b^2*(b*d - a*e)^4*(d + e*x)^(13/2))/(13*e^7) - (8*b^3
*(b*d - a*e)^3*(d + e*x)^(15/2))/(3*e^7) + (30*b^4*(b*d - a*e)^2*(d + e*x)^
(17/2))/(17*e^7) - (12*b^5*(b*d - a*e)*(d + e*x)^(19/2))/(19*e^7) + (2*b^6*
(d + e*x)^(21/2))/(21*e^7)
```

Rubi [A] time = 0.082821, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {27, 43}

$$-\frac{12b^5(d+ex)^{19/2}(bd-ae)}{19e^7}+\frac{30b^4(d+ex)^{17/2}(bd-ae)^2}{17e^7}-\frac{8b^3(d+ex)^{15/2}(bd-ae)^3}{3e^7}+\frac{30b^2(d+ex)^{13/2}(bd-ae)^4}{13e^7}-12$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```


```
[Out] (2*(b*d - a*e)^6*(d + e*x)^(9/2))/(9*e^7) - (12*b*(b*d - a*e)^5*(d + e*x)^(
11/2))/(11*e^7) + (30*b^2*(b*d - a*e)^4*(d + e*x)^(13/2))/(13*e^7) - (8*b^3
*(b*d - a*e)^3*(d + e*x)^(15/2))/(3*e^7) + (30*b^4*(b*d - a*e)^2*(d + e*x)^
(17/2))/(17*e^7) - (12*b^5*(b*d - a*e)*(d + e*x)^(19/2))/(19*e^7) + (2*b^6*
(d + e*x)^(21/2))/(21*e^7)
```


Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^{7/2}\left(a^2+2abx+b^2x^2\right)^3\,dx &= \int (a+bx)^6(d+ex)^{7/2}\,dx\\ &= \int \left(\frac{(-bd+ae)^6(d+ex)^{7/2}}{e^6}-\frac{6b(bd-ae)^5(d+ex)^{9/2}}{e^6}+\frac{15b^2(bd-ae)^4(d+ex)^1}{e^6}\right.\\ &= \frac{2(bd-ae)^6(d+ex)^{9/2}}{9e^7}-\frac{12b(bd-ae)^5(d+ex)^{11/2}}{11e^7}+\frac{30b^2(bd-ae)^4(d+ex)^{13/2}}{13e^7}\end{aligned}$$

Mathematica [A] time = 0.158601, size = 145, normalized size = 0.78

$$\frac{2(d+ex)^{9/2}\left(3357585b^2(d+ex)^2(bd-ae)^4-3879876b^3(d+ex)^3(bd-ae)^3+2567565b^4(d+ex)^4(bd-ae)^2-918918\right.}{2909907e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] (2*(d + e*x)^(9/2)*(323323*(b*d - a*e)^6 - 1587222*b*(b*d - a*e)^5*(d + e*x
) + 3357585*b^2*(b*d - a*e)^4*(d + e*x)^2 - 3879876*b^3*(b*d - a*e)^3*(d +
e*x)^3 + 2567565*b^4*(b*d - a*e)^2*(d + e*x)^4 - 918918*b^5*(b*d - a*e)*(d
+ e*x)^5 + 138567*b^6*(d + e*x)^6))/(2909907*e^7)
```

Maple [B] time = 0.049, size = 377, normalized size = 2.

$$\frac{277134\, b^6x^6e^6 + 1837836\, x^5ab^5e^6 - 175032\, x^5b^6de^5 + 5135130\, x^4a^2b^4e^6 - 1081080\, x^4ab^5de^5 + 102960\, x^4b^6d^2e^4 + 775975}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)*(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] 2/2909907*(e*x+d)^(9/2)*(138567*b^6*e^6*x^6+918918*a*b^5*e^6*x^5-87516*b^6*
d*e^5*x^5+2567565*a^2*b^4*e^6*x^4-540540*a*b^5*d*e^5*x^4+51480*b^6*d^2*e^4*
x^4+3879876*a^3*b^3*e^6*x^3-1369368*a^2*b^4*d*e^5*x^3+288288*a*b^5*d^2*e^4*
x^3-27456*b^6*d^3*e^3*x^3+3357585*a^4*b^2*e^6*x^2-1790712*a^3*b^3*d*e^5*x^2
+632016*a^2*b^4*d^2*e^4*x^2-133056*a*b^5*d^3*e^3*x^2+12672*b^6*d^4*e^2*x^2+
1587222*a^5*b*e^6*x-1220940*a^4*b^2*d*e^5*x+651168*a^3*b^3*d^2*e^4*x-229824
*a^2*b^4*d^3*e^3*x+48384*a*b^5*d^4*e^2*x-4608*b^6*d^5*e*x+323323*a^6*e^6-35
2716*a^5*b*d*e^5+271320*a^4*b^2*d^2*e^4-144704*a^3*b^3*d^3*e^3+51072*a^2*b^
4*d^4*e^2-10752*a*b^5*d^5*e+1024*b^6*d^6)/e^7
```

Maxima [B] time = 1.06954, size = 473, normalized size = 2.53

$$\frac{2\left(138567\,(ex+d)^{\frac{21}{2}}b^6 - 918918\left(b^6d - ab^5e\right)(ex+d)^{\frac{19}{2}} + 2567565\left(b^6d^2 - 2\,ab^5de + a^2b^4e^2\right)(ex+d)^{\frac{17}{2}} - 3879876\left(b^6d^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] 2/2909907*(138567*(e*x + d)^(21/2)*b^6 - 918918*(b^6*d - a*b^5*e)*(e*x + d)
^(19/2) + 2567565*(b^6*d^2 - 2*a*b^5*d*e + a^2*b^4*e^2)*(e*x + d)^(17/2) -
3879876*(b^6*d^3 - 3*a*b^5*d^2*e + 3*a^2*b^4*d*e^2 - a^3*b^3*e^3)*(e*x + d)
^(15/2) + 3357585*(b^6*d^4 - 4*a*b^5*d^3*e + 6*a^2*b^4*d^2*e^2 - 4*a^3*b^3*
d*e^3 + a^4*b^2*e^4)*(e*x + d)^(13/2) - 1587222*(b^6*d^5 - 5*a*b^5*d^4*e +
10*a^2*b^4*d^3*e^2 - 10*a^3*b^3*d^2*e^3 + 5*a^4*b^2*d*e^4 - a^5*b*e^5)*(e*x
 + d)^(11/2) + 323323*(b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^
3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 + a^6*e^6)*(e*x + d)^(9/
2))/e^7
```

Fricas [B] time = 1.60444, size = 1746, normalized size = 9.34

$$\frac{2\left(138567\, b^6e^{10}x^{10} + 1024\, b^6d^{10} - 10752\, ab^5d^9e + 51072\, a^2b^4d^8e^2 - 144704\, a^3b^3d^7e^3 + 271320\, a^4b^2d^6e^4 - 352716\, a^5bd^5e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] 2/2909907*(138567*b^6*e^10*x^10 + 1024*b^6*d^10 - 10752*a*b^5*d^9*e + 51072
*a^2*b^4*d^8*e^2 - 144704*a^3*b^3*d^7*e^3 + 271320*a^4*b^2*d^6*e^4 - 352716
*a^5*b*d^5*e^5 + 323323*a^6*d^4*e^6 + 14586*(32*b^6*d*e^9 + 63*a*b^5*e^10)*
x^9 + 3861*(138*b^6*d^2*e^8 + 812*a*b^5*d*e^9 + 665*a^2*b^4*e^10)*x^8 + 171
6*(121*b^6*d^3*e^7 + 2121*a*b^5*d^2*e^8 + 5187*a^2*b^4*d*e^9 + 2261*a^3*b^3
*e^10)*x^7 + 231*(b^6*d^4*e^6 + 6288*a*b^5*d^3*e^7 + 45714*a^2*b^4*d^2*e^8
+ 59432*a^3*b^3*d*e^9 + 14535*a^4*b^2*e^10)*x^6 - 126*(2*b^6*d^5*e^5 - 21*a
*b^5*d^4*e^6 - 34542*a^2*b^4*d^3*e^7 - 133076*a^3*b^3*d^2*e^8 - 96900*a^4*b
^2*d*e^9 - 12597*a^5*b*e^10)*x^5 + 7*(40*b^6*d^6*e^4 - 420*a*b^5*d^5*e^5 +
1995*a^2*b^4*d^4*e^6 + 1033600*a^3*b^3*d^3*e^7 + 2219010*a^4*b^2*d^2*e^8 +
856596*a^5*b*d*e^9 + 46189*a^6*e^10)*x^4 - 4*(80*b^6*d^7*e^3 - 840*a*b^5*d^
6*e^4 + 3990*a^2*b^4*d^5*e^5 - 11305*a^3*b^3*d^4*e^6 - 1797495*a^4*b^2*d^3*
e^7 - 2028117*a^5*b*d^2*e^8 - 323323*a^6*d*e^9)*x^3 + 3*(128*b^6*d^8*e^2 -
1344*a*b^5*d^7*e^3 + 6384*a^2*b^4*d^6*e^4 - 18088*a^3*b^3*d^5*e^5 + 33915*a
^4*b^2*d^4*e^6 + 1410864*a^5*b*d^3*e^7 + 646646*a^6*d^2*e^8)*x^2 - 2*(256*b
^6*d^9*e - 2688*a*b^5*d^8*e^2 + 12768*a^2*b^4*d^7*e^3 - 36176*a^3*b^3*d^6*e
^4 + 67830*a^4*b^2*d^5*e^5 - 88179*a^5*b*d^4*e^6 - 646646*a^6*d^3*e^7)*x)*s
qrt(e*x + d)/e^7
```

Sympy [A] time = 78.9065, size = 2450, normalized size = 13.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)*(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] a**6*d**3*Piecewise((sqrt(d)*x, Eq(e, 0)), (2*(d + e*x)**(3/2)/(3*e), True)
) + 6*a**6*d**2*(-d*(d + e*x)**(3/2)/3 + (d + e*x)**(5/2)/5)/e + 6*a**6*d*(
d**2*(d + e*x)**(3/2)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e +
2*a**6*(-d**3*(d + e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d + e*x
)**(7/2)/7 + (d + e*x)**(9/2)/9)/e + 12*a**5*b*d**3*(-d*(d + e*x)**(3/2)/3
+ (d + e*x)**(5/2)/5)/e**2 + 36*a**5*b*d**2*(d**2*(d + e*x)**(3/2)/3 - 2*d*
(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e**2 + 36*a**5*b*d*(-d**3*(d + e*x
)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d + e*x)**(7/2)/7 + (d + e*x)
**(9/2)/9)/e**2 + 12*a**5*b*(d**4*(d + e*x)**(3/2)/3 - 4*d**3*(d + e*x)**(5
/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e*x)**(9/2)/9 + (d + e*x)**(11
/2)/11)/e**2 + 30*a**4*b**2*d**3*(d**2*(d + e*x)**(3/2)/3 - 2*d*(d + e*x)**
(5/2)/5 + (d + e*x)**(7/2)/7)/e**3 + 90*a**4*b**2*d**2*(-d**3*(d + e*x)**(3
/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d + e*x)**(7/2)/7 + (d + e*x)**(9/
2)/9)/e**3 + 90*a**4*b**2*d*(d**4*(d + e*x)**(3/2)/3 - 4*d**3*(d + e*x)**(5
/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e*x)**(9/2)/9 + (d + e*x)**(11
/2)/11)/e**3 + 30*a**4*b**2*(-d**5*(d + e*x)**(3/2)/3 + d**4*(d + e*x)**(5/
2) - 10*d**3*(d + e*x)**(7/2)/7 + 10*d**2*(d + e*x)**(9/2)/9 - 5*d*(d + e*x
)**(11/2)/11 + (d + e*x)**(13/2)/13)/e**3 + 40*a**3*b**3*d**3*(-d**3*(d + e
*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d + e*x)**(7/2)/7 + (d + e*
x)**(9/2)/9)/e**4 + 120*a**3*b**3*d**2*(d**4*(d + e*x)**(3/2)/3 - 4*d**3*(d
 + e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e*x)**(9/2)/9 + (d
+ e*x)**(11/2)/11)/e**4 + 120*a**3*b**3*d*(-d**5*(d + e*x)**(3/2)/3 + d**4*
(d + e*x)**(5/2) - 10*d**3*(d + e*x)**(7/2)/7 + 10*d**2*(d + e*x)**(9/2)/9
- 5*d*(d + e*x)**(11/2)/11 + (d + e*x)**(13/2)/13)/e**4 + 40*a**3*b**3*(d**
6*(d + e*x)**(3/2)/3 - 6*d**5*(d + e*x)**(5/2)/5 + 15*d**4*(d + e*x)**(7/2)
/7 - 20*d**3*(d + e*x)**(9/2)/9 + 15*d**2*(d + e*x)**(11/2)/11 - 6*d*(d + e
```

```
*x)**(13/2)/13 + (d + e*x)**(15/2)/15)/e**4 + 30*a**2*b**4*d**3*(d**4*(d +
e*x)**(3/2)/3 - 4*d**3*(d + e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d
*(d + e*x)**(9/2)/9 + (d + e*x)**(11/2)/11)/e**5 + 90*a**2*b**4*d**2*(-d**5
*(d + e*x)**(3/2)/3 + d**4*(d + e*x)**(5/2) - 10*d**3*(d + e*x)**(7/2)/7 +
10*d**2*(d + e*x)**(9/2)/9 - 5*d*(d + e*x)**(11/2)/11 + (d + e*x)**(13/2)/1
3)/e**5 + 90*a**2*b**4*d*(d**6*(d + e*x)**(3/2)/3 - 6*d**5*(d + e*x)**(5/2)
/5 + 15*d**4*(d + e*x)**(7/2)/7 - 20*d**3*(d + e*x)**(9/2)/9 + 15*d**2*(d +
 e*x)**(11/2)/11 - 6*d*(d + e*x)**(13/2)/13 + (d + e*x)**(15/2)/15)/e**5 +
30*a**2*b**4*(-d**7*(d + e*x)**(3/2)/3 + 7*d**6*(d + e*x)**(5/2)/5 - 3*d**5
*(d + e*x)**(7/2) + 35*d**4*(d + e*x)**(9/2)/9 - 35*d**3*(d + e*x)**(11/2)/
11 + 21*d**2*(d + e*x)**(13/2)/13 - 7*d*(d + e*x)**(15/2)/15 + (d + e*x)**(
17/2)/17)/e**5 + 12*a*b**5*d**3*(-d**5*(d + e*x)**(3/2)/3 + d**4*(d + e*x)*
*(5/2) - 10*d**3*(d + e*x)**(7/2)/7 + 10*d**2*(d + e*x)**(9/2)/9 - 5*d*(d +
 e*x)**(11/2)/11 + (d + e*x)**(13/2)/13)/e**6 + 36*a*b**5*d**2*(d**6*(d + e
*x)**(3/2)/3 - 6*d**5*(d + e*x)**(5/2)/5 + 15*d**4*(d + e*x)**(7/2)/7 - 20*
d**3*(d + e*x)**(9/2)/9 + 15*d**2*(d + e*x)**(11/2)/11 - 6*d*(d + e*x)**(13
/2)/13 + (d + e*x)**(15/2)/15)/e**6 + 36*a*b**5*d*(-d**7*(d + e*x)**(3/2)/3
 + 7*d**6*(d + e*x)**(5/2)/5 - 3*d**5*(d + e*x)**(7/2) + 35*d**4*(d + e*x)*
*(9/2)/9 - 35*d**3*(d + e*x)**(11/2)/11 + 21*d**2*(d + e*x)**(13/2)/13 - 7*
d*(d + e*x)**(15/2)/15 + (d + e*x)**(17/2)/17)/e**6 + 12*a*b**5*(d**8*(d +
e*x)**(3/2)/3 - 8*d**7*(d + e*x)**(5/2)/5 + 4*d**6*(d + e*x)**(7/2) - 56*d*
*5*(d + e*x)**(9/2)/9 + 70*d**4*(d + e*x)**(11/2)/11 - 56*d**3*(d + e*x)**(
13/2)/13 + 28*d**2*(d + e*x)**(15/2)/15 - 8*d*(d + e*x)**(17/2)/17 + (d + e
*x)**(19/2)/19)/e**6 + 2*b**6*d**3*(d**6*(d + e*x)**(3/2)/3 - 6*d**5*(d + e
*x)**(5/2)/5 + 15*d**4*(d + e*x)**(7/2)/7 - 20*d**3*(d + e*x)**(9/2)/9 + 15
*d**2*(d + e*x)**(11/2)/11 - 6*d*(d + e*x)**(13/2)/13 + (d + e*x)**(15/2)/1
5)/e**7 + 6*b**6*d**2*(-d**7*(d + e*x)**(3/2)/3 + 7*d**6*(d + e*x)**(5/2)/5
 - 3*d**5*(d + e*x)**(7/2) + 35*d**4*(d + e*x)**(9/2)/9 - 35*d**3*(d + e*x)
**(11/2)/11 + 21*d**2*(d + e*x)**(13/2)/13 - 7*d*(d + e*x)**(15/2)/15 + (d
+ e*x)**(17/2)/17)/e**7 + 6*b**6*d*(d**8*(d + e*x)**(3/2)/3 - 8*d**7*(d + e
*x)**(5/2)/5 + 4*d**6*(d + e*x)**(7/2) - 56*d**5*(d + e*x)**(9/2)/9 + 70*d*
*4*(d + e*x)**(11/2)/11 - 56*d**3*(d + e*x)**(13/2)/13 + 28*d**2*(d + e*x)*
*(15/2)/15 - 8*d*(d + e*x)**(17/2)/17 + (d + e*x)**(19/2)/19)/e**7 + 2*b**6
*(-d**9*(d + e*x)**(3/2)/3 + 9*d**8*(d + e*x)**(5/2)/5 - 36*d**7*(d + e*x)*
*(7/2)/7 + 28*d**6*(d + e*x)**(9/2)/3 - 126*d**5*(d + e*x)**(11/2)/11 + 126
*d**4*(d + e*x)**(13/2)/13 - 28*d**3*(d + e*x)**(15/2)/5 + 36*d**2*(d + e*x
)**(17/2)/17 - 9*d*(d + e*x)**(19/2)/19 + (d + e*x)**(21/2)/21)/e**7
```

Giac [B] time = 1.36297, size = 2936, normalized size = 15.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] 2/14549535*(5819814*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^5*b*d^3*e^(
-1) + 2078505*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/
2)*d^2)*a^4*b^2*d^3*e^(-2) + 923780*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/
2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a^3*b^3*d^3*e^(-3
) + 62985*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(
7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*a^2*b^4*d^3
*e^(-4) + 9690*(693*(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e
 + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 30
03*(x*e + d)^(3/2)*d^5)*a*b^5*d^3*e^(-5) + 323*(3003*(x*e + d)^(15/2) - 207
90*(x*e + d)^(13/2)*d + 61425*(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)
*d^3 + 96525*(x*e + d)^(7/2)*d^4 - 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*e +
```

d)^(3/2)*d^6)*b^6*d^3*e^(-6) + 4849845*(x*e + d)^(3/2)*a^6*d^3 + 2494206*(
15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*a^5*b*d
^2*e^(-1) + 2078505*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e
+ d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a^4*b^2*d^2*e^(-2) + 251940*(315*
(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772
*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*a^3*b^3*d^2*e^(-3) + 72675
(693(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^
2 - 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(
3/2)*d^5)*a^2*b^4*d^2*e^(-4) + 5814*(3003*(x*e + d)^(15/2) - 20790*(x*e + d
)^(13/2)*d + 61425*(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 9652
5*(x*e + d)^(7/2)*d^4 - 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d
^6)*a*b^5*d^2*e^(-5) + 399*(6435*(x*e + d)^(17/2) - 51051*(x*e + d)^(15/2)*
d + 176715*(x*e + d)^(13/2)*d^2 - 348075*(x*e + d)^(11/2)*d^3 + 425425*(x*e
 + d)^(9/2)*d^4 - 328185*(x*e + d)^(7/2)*d^5 + 153153*(x*e + d)^(5/2)*d^6 -
 36465*(x*e + d)^(3/2)*d^7)*b^6*d^2*e^(-6) + 2909907*(3*(x*e + d)^(5/2) - 5
*(x*e + d)^(3/2)*d)*a^6*d^2 + 831402*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7
/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a^5*b*d*e^(-1) +
 188955*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/
2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*a^4*b^2*d*e^(
-2) + 96900*(693*(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e +
d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*
(x*e + d)^(3/2)*d^5)*a^3*b^3*d*e^(-3) + 14535*(3003*(x*e + d)^(15/2) - 2079
0*(x*e + d)^(13/2)*d + 61425*(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*
d^3 + 96525*(x*e + d)^(7/2)*d^4 - 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*e +
d)^(3/2)*d^6)*a^2*b^4*d*e^(-4) + 2394*(6435*(x*e + d)^(17/2) - 51051*(x*e +
 d)^(15/2)*d + 176715*(x*e + d)^(13/2)*d^2 - 348075*(x*e + d)^(11/2)*d^3 +
425425*(x*e + d)^(9/2)*d^4 - 328185*(x*e + d)^(7/2)*d^5 + 153153*(x*e + d)^
(5/2)*d^6 - 36465*(x*e + d)^(3/2)*d^7)*a*b^5*d*e^(-5) + 21*(109395*(x*e + d
)^(19/2) - 978120*(x*e + d)^(17/2)*d + 3879876*(x*e + d)^(15/2)*d^2 - 89535
60*(x*e + d)^(13/2)*d^3 + 13226850*(x*e + d)^(11/2)*d^4 - 12932920*(x*e + d
)^(9/2)*d^5 + 8314020*(x*e + d)^(7/2)*d^6 - 3325608*(x*e + d)^(5/2)*d^7 + 6
92835*(x*e + d)^(3/2)*d^8)*b^6*d*e^(-6) + 415701*(15*(x*e + d)^(7/2) - 42*(
x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*a^6*d + 25194*(315*(x*e + d)^(11
/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5
/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*a^5*b*e^(-1) + 24225*(693*(x*e + d)^(13
/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12870*(x*e + d)
^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*a^4*b^2*e
^(-2) + 6460*(3003*(x*e + d)^(15/2) - 20790*(x*e + d)^(13/2)*d + 61425*(x*e
 + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525*(x*e + d)^(7/2)*d^4 -
 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*a^3*b^3*e^(-3) + 19
95*(6435*(x*e + d)^(17/2) - 51051*(x*e + d)^(15/2)*d + 176715*(x*e + d)^(13
/2)*d^2 - 348075*(x*e + d)^(11/2)*d^3 + 425425*(x*e + d)^(9/2)*d^4 - 328185
*(x*e + d)^(7/2)*d^5 + 153153*(x*e + d)^(5/2)*d^6 - 36465*(x*e + d)^(3/2)*d
^7)*a^2*b^4*e^(-4) + 42*(109395*(x*e + d)^(19/2) - 978120*(x*e + d)^(17/2)*
d + 3879876*(x*e + d)^(15/2)*d^2 - 8953560*(x*e + d)^(13/2)*d^3 + 13226850*
(x*e + d)^(11/2)*d^4 - 12932920*(x*e + d)^(9/2)*d^5 + 8314020*(x*e + d)^(7/
2)*d^6 - 3325608*(x*e + d)^(5/2)*d^7 + 692835*(x*e + d)^(3/2)*d^8)*a*b^5*e^
(-5) + 3*(230945*(x*e + d)^(21/2) - 2297295*(x*e + d)^(19/2)*d + 10270260*(
x*e + d)^(17/2)*d^2 - 27159132*(x*e + d)^(15/2)*d^3 + 47006190*(x*e + d)^(1
3/2)*d^4 - 55552770*(x*e + d)^(11/2)*d^5 + 45265220*(x*e + d)^(9/2)*d^6 - 2
4942060*(x*e + d)^(7/2)*d^7 + 8729721*(x*e + d)^(5/2)*d^8 - 1616615*(x*e +
d)^(3/2)*d^9)*b^6*e^(-6) + 46189*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*
d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a^6)*e^(-1)

3.1638 $\int (d+ex)^{5/2}\left(a^2+2abx+b^2x^2\right)^3\,dx$

Optimal. Leaf size=185

$$-\frac{12b^5(d+ex)^{17/2}(bd-ae)}{17e^7}+\frac{2b^4(d+ex)^{15/2}(bd-ae)^2}{e^7}-\frac{40b^3(d+ex)^{13/2}(bd-ae)^3}{13e^7}+\frac{30b^2(d+ex)^{11/2}(bd-ae)^4}{11e^7}-\frac{4b(d\cdots}{}$$

```
[Out] (2*(b*d - a*e)^6*(d + e*x)^(7/2))/(7*e^7) - (4*b*(b*d - a*e)^5*(d + e*x)^(9
/2))/(3*e^7) + (30*b^2*(b*d - a*e)^4*(d + e*x)^(11/2))/(11*e^7) - (40*b^3*(
b*d - a*e)^3*(d + e*x)^(13/2))/(13*e^7) + (2*b^4*(b*d - a*e)^2*(d + e*x)^(1
5/2))/e^7 - (12*b^5*(b*d - a*e)*(d + e*x)^(17/2))/(17*e^7) + (2*b^6*(d + e*
x)^(19/2))/(19*e^7)
```

Rubi [A] time = 0.0608597, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {27, 43}

$$-\frac{12b^5(d+ex)^{17/2}(bd-ae)}{17e^7}+\frac{2b^4(d+ex)^{15/2}(bd-ae)^2}{e^7}-\frac{40b^3(d+ex)^{13/2}(bd-ae)^3}{13e^7}+\frac{30b^2(d+ex)^{11/2}(bd-ae)^4}{11e^7}-\frac{4b(d\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] (2*(b*d - a*e)^6*(d + e*x)^(7/2))/(7*e^7) - (4*b*(b*d - a*e)^5*(d + e*x)^(9
/2))/(3*e^7) + (30*b^2*(b*d - a*e)^4*(d + e*x)^(11/2))/(11*e^7) - (40*b^3*(
b*d - a*e)^3*(d + e*x)^(13/2))/(13*e^7) + (2*b^4*(b*d - a*e)^2*(d + e*x)^(1
5/2))/e^7 - (12*b^5*(b*d - a*e)*(d + e*x)^(17/2))/(17*e^7) + (2*b^6*(d + e*
x)^(19/2))/(19*e^7)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^{5/2}\left(a^2+2abx+b^2x^2\right)^3\,dx &= \int (a+bx)^6(d+ex)^{5/2}\,dx\\ &= \int \left(\frac{(-bd+ae)^6(d+ex)^{5/2}}{e^6}-\frac{6b(bd-ae)^5(d+ex)^{7/2}}{e^6}+\frac{15b^2(bd-ae)^4(d+ex)^{9/2}}{e^6}\cdots\right.\\ &= \frac{2(bd-ae)^6(d+ex)^{7/2}}{7e^7}-\frac{4b(bd-ae)^5(d+ex)^{9/2}}{3e^7}+\frac{30b^2(bd-ae)^4(d+ex)^{11/2}}{11e^7}-\cdots\end{aligned}$$

Mathematica [A] time = 0.112956, size = 145, normalized size = 0.78

$$\frac{2(d+ex)^{7/2}\left(1322685b^2(d+ex)^2(bd-ae)^4-1492260b^3(d+ex)^3(bd-ae)^3+969969b^4(d+ex)^4(bd-ae)^2-342342b^5(d\cdots\right.}{969969e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] (2*(d + e*x)^(7/2)*(138567*(b*d - a*e)^6 - 646646*b*(b*d - a*e)^5*(d + e*x)
 + 1322685*b^2*(b*d - a*e)^4*(d + e*x)^2 - 1492260*b^3*(b*d - a*e)^3*(d + e
*x)^3 + 969969*b^4*(b*d - a*e)^2*(d + e*x)^4 - 342342*b^5*(b*d - a*e)*(d +
e*x)^5 + 51051*b^6*(d + e*x)^6))/(969969*e^7)
```

Maple [B] time = 0.046, size = 377, normalized size = 2.

$$102102\,b^6x^6e^6 + 684684\,x^5ab^5e^6 - 72072\,x^5b^6de^5 + 1939938\,x^4a^2b^4e^6 - 456456\,x^4ab^5de^5 + 48048\,x^4b^6d^2e^4 + 298452$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] 2/969969*(e*x+d)^(7/2)*(51051*b^6*e^6*x^6+342342*a*b^5*e^6*x^5-36036*b^6*d*
e^5*x^5+969969*a^2*b^4*e^6*x^4-228228*a*b^5*d*e^5*x^4+24024*b^6*d^2*e^4*x^4
+1492260*a^3*b^3*e^6*x^3-596904*a^2*b^4*d*e^5*x^3+140448*a*b^5*d^2*e^4*x^3-
14784*b^6*d^3*e^3*x^3+1322685*a^4*b^2*e^6*x^2-813960*a^3*b^3*d*e^5*x^2+3255
84*a^2*b^4*d^2*e^4*x^2-76608*a*b^5*d^3*e^3*x^2+8064*b^6*d^4*e^2*x^2+646646*
a^5*b*e^6*x-587860*a^4*b^2*d*e^5*x+361760*a^3*b^3*d^2*e^4*x-144704*a^2*b^4*
d^3*e^3*x+34048*a*b^5*d^4*e^2*x-3584*b^6*d^5*e*x+138567*a^6*e^6-184756*a^5*
b*d*e^5+167960*a^4*b^2*d^2*e^4-103360*a^3*b^3*d^3*e^3+41344*a^2*b^4*d^4*e^2
-9728*a*b^5*d^5*e+1024*b^6*d^6)/e^7
```

Maxima [B] time = 1.06292, size = 473, normalized size = 2.56

$$2\left(51051\,(ex+d)^{\frac{19}{2}}b^6 - 342342\left(b^6d - ab^5e\right)(ex+d)^{\frac{17}{2}} + 969969\left(b^6d^2 - 2\,ab^5de + a^2b^4e^2\right)(ex+d)^{\frac{15}{2}} - 1492260\left(b^6d^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] 2/969969*(51051*(e*x + d)^(19/2)*b^6 - 342342*(b^6*d - a*b^5*e)*(e*x + d)^(
17/2) + 969969*(b^6*d^2 - 2*a*b^5*d*e + a^2*b^4*e^2)*(e*x + d)^(15/2) - 149
2260*(b^6*d^3 - 3*a*b^5*d^2*e + 3*a^2*b^4*d*e^2 - a^3*b^3*e^3)*(e*x + d)^(1
3/2) + 1322685*(b^6*d^4 - 4*a*b^5*d^3*e + 6*a^2*b^4*d^2*e^2 - 4*a^3*b^3*d*e
^3 + a^4*b^2*e^4)*(e*x + d)^(11/2) - 646646*(b^6*d^5 - 5*a*b^5*d^4*e + 10*a
^2*b^4*d^3*e^2 - 10*a^3*b^3*d^2*e^3 + 5*a^4*b^2*d*e^4 - a^5*b*e^5)*(e*x + d
)^(9/2) + 138567*(b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3
*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 + a^6*e^6)*(e*x + d)^(7/2))/e
^7
```

Fricas [B] time = 1.65116, size = 1497, normalized size = 8.09

$$2\left(51051\,b^6e^9x^9 + 1024\,b^6d^9 - 9728\,ab^5d^8e + 41344\,a^2b^4d^7e^2 - 103360\,a^3b^3d^6e^3 + 167960\,a^4b^2d^5e^4 - 184756\,a^5bd^4e^5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] 2/969969*(51051*b^6*e^9*x^9 + 1024*b^6*d^9 - 9728*a*b^5*d^8*e + 41344*a^2*b
^4*d^7*e^2 - 103360*a^3*b^3*d^6*e^3 + 167960*a^4*b^2*d^5*e^4 - 184756*a^5*b
*d^4*e^5 + 138567*a^6*d^3*e^6 + 9009*(13*b^6*d*e^8 + 38*a*b^5*e^9)*x^8 + 30
03*(23*b^6*d^2*e^7 + 266*a*b^5*d*e^8 + 323*a^2*b^4*e^9)*x^7 + 231*(b^6*d^3*
e^6 + 2090*a*b^5*d^2*e^7 + 10013*a^2*b^4*d*e^8 + 6460*a^3*b^3*e^9)*x^6 - 63
*(4*b^6*d^4*e^5 - 38*a*b^5*d^3*e^6 - 22933*a^2*b^4*d^2*e^7 - 58140*a^3*b^3*
d*e^8 - 20995*a^4*b^2*e^9)*x^5 + 7*(40*b^6*d^5*e^4 - 380*a*b^5*d^4*e^5 + 16
15*a^2*b^4*d^3*e^6 + 342380*a^3*b^3*d^2*e^7 + 482885*a^4*b^2*d*e^8 + 92378*
a^5*b*e^9)*x^4 - (320*b^6*d^6*e^3 - 3040*a*b^5*d^5*e^4 + 12920*a^2*b^4*d^4*
e^5 - 32300*a^3*b^3*d^3*e^6 - 2372435*a^4*b^2*d^2*e^7 - 1755182*a^5*b*d*e^8
 - 138567*a^6*e^9)*x^3 + 3*(128*b^6*d^7*e^2 - 1216*a*b^5*d^6*e^3 + 5168*a^2
*b^4*d^5*e^4 - 12920*a^3*b^3*d^4*e^5 + 20995*a^4*b^2*d^3*e^6 + 461890*a^5*b
*d^2*e^7 + 138567*a^6*d*e^8)*x^2 - (512*b^6*d^8*e - 4864*a*b^5*d^7*e^2 + 20
672*a^2*b^4*d^6*e^3 - 51680*a^3*b^3*d^5*e^4 + 83980*a^4*b^2*d^4*e^5 - 92378
*a^5*b*d^3*e^6 - 415701*a^6*d^2*e^7)*x)*sqrt(e*x + d)/e^7
```

Sympy [A] time = 51.2127, size = 1671, normalized size = 9.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] a**6*d**2*Piecewise((sqrt(d)*x, Eq(e, 0)), (2*(d + e*x)**(3/2)/(3*e), True)
) + 4*a**6*d*(-d*(d + e*x)**(3/2)/3 + (d + e*x)**(5/2)/5)/e + 2*a**6*(d**2*
(d + e*x)**(3/2)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e + 12*a*
*5*b*d**2*(-d*(d + e*x)**(3/2)/3 + (d + e*x)**(5/2)/5)/e**2 + 24*a**5*b*d*(
d**2*(d + e*x)**(3/2)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e**2
 + 12*a**5*b*(-d**3*(d + e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d
 + e*x)**(7/2)/7 + (d + e*x)**(9/2)/9)/e**2 + 30*a**4*b**2*d**2*(d**2*(d +
e*x)**(3/2)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e**3 + 60*a**4
*b**2*d*(-d**3*(d + e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d + e*
x)**(7/2)/7 + (d + e*x)**(9/2)/9)/e**3 + 30*a**4*b**2*(d**4*(d + e*x)**(3/2
)/3 - 4*d**3*(d + e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e*x)
**(9/2)/9 + (d + e*x)**(11/2)/11)/e**3 + 40*a**3*b**3*d**2*(-d**3*(d + e*x)
**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d + e*x)**(7/2)/7 + (d + e*x)*
*(9/2)/9)/e**4 + 80*a**3*b**3*d*(d**4*(d + e*x)**(3/2)/3 - 4*d**3*(d + e*x)
**(5/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e*x)**(9/2)/9 + (d + e*x)*
*(11/2)/11)/e**4 + 40*a**3*b**3*(-d**5*(d + e*x)**(3/2)/3 + d**4*(d + e*x)*
*(5/2) - 10*d**3*(d + e*x)**(7/2)/7 + 10*d**2*(d + e*x)**(9/2)/9 - 5*d*(d +
 e*x)**(11/2)/11 + (d + e*x)**(13/2)/13)/e**4 + 30*a**2*b**4*d**2*(d**4*(d
+ e*x)**(3/2)/3 - 4*d**3*(d + e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4
*d*(d + e*x)**(9/2)/9 + (d + e*x)**(11/2)/11)/e**5 + 60*a**2*b**4*d*(-d**5*
(d + e*x)**(3/2)/3 + d**4*(d + e*x)**(5/2) - 10*d**3*(d + e*x)**(7/2)/7 + 1
0*d**2*(d + e*x)**(9/2)/9 - 5*d*(d + e*x)**(11/2)/11 + (d + e*x)**(13/2)/13
)/e**5 + 30*a**2*b**4*(d**6*(d + e*x)**(3/2)/3 - 6*d**5*(d + e*x)**(5/2)/5
+ 15*d**4*(d + e*x)**(7/2)/7 - 20*d**3*(d + e*x)**(9/2)/9 + 15*d**2*(d + e*
x)**(11/2)/11 - 6*d*(d + e*x)**(13/2)/13 + (d + e*x)**(15/2)/15)/e**5 + 12*
a*b**5*d**2*(-d**5*(d + e*x)**(3/2)/3 + d**4*(d + e*x)**(5/2) - 10*d**3*(d
+ e*x)**(7/2)/7 + 10*d**2*(d + e*x)**(9/2)/9 - 5*d*(d + e*x)**(11/2)/11 + (
d + e*x)**(13/2)/13)/e**6 + 24*a*b**5*d*(d**6*(d + e*x)**(3/2)/3 - 6*d**5*(
d + e*x)**(5/2)/5 + 15*d**4*(d + e*x)**(7/2)/7 - 20*d**3*(d + e*x)**(9/2)/9
```

```
 + 15*d**2*(d + e*x)**(11/2)/11 - 6*d*(d + e*x)**(13/2)/13 + (d + e*x)**(15
/2)/15)/e**6 + 12*a*b**5*(-d**7*(d + e*x)**(3/2)/3 + 7*d**6*(d + e*x)**(5/2
)/5 - 3*d**5*(d + e*x)**(7/2) + 35*d**4*(d + e*x)**(9/2)/9 - 35*d**3*(d + e
*x)**(11/2)/11 + 21*d**2*(d + e*x)**(13/2)/13 - 7*d*(d + e*x)**(15/2)/15 +
(d + e*x)**(17/2)/17)/e**6 + 2*b**6*d**2*(d**6*(d + e*x)**(3/2)/3 - 6*d**5*
(d + e*x)**(5/2)/5 + 15*d**4*(d + e*x)**(7/2)/7 - 20*d**3*(d + e*x)**(9/2)/
9 + 15*d**2*(d + e*x)**(11/2)/11 - 6*d*(d + e*x)**(13/2)/13 + (d + e*x)**(1
5/2)/15)/e**7 + 4*b**6*d*(-d**7*(d + e*x)**(3/2)/3 + 7*d**6*(d + e*x)**(5/2
)/5 - 3*d**5*(d + e*x)**(7/2) + 35*d**4*(d + e*x)**(9/2)/9 - 35*d**3*(d + e
*x)**(11/2)/11 + 21*d**2*(d + e*x)**(13/2)/13 - 7*d*(d + e*x)**(15/2)/15 +
(d + e*x)**(17/2)/17)/e**7 + 2*b**6*(d**8*(d + e*x)**(3/2)/3 - 8*d**7*(d +
e*x)**(5/2)/5 + 4*d**6*(d + e*x)**(7/2) - 56*d**5*(d + e*x)**(9/2)/9 + 70*d
**4*(d + e*x)**(11/2)/11 - 56*d**3*(d + e*x)**(13/2)/13 + 28*d**2*(d + e*x)
**(15/2)/15 - 8*d*(d + e*x)**(17/2)/17 + (d + e*x)**(19/2)/19)/e**7
```

Giac [B] time = 1.3244, size = 2007, normalized size = 10.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] 2/14549535*(5819814*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^5*b*d^2*e^(
-1) + 2078505*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/
2)*d^2)*a^4*b^2*d^2*e^(-2) + 923780*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/
2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a^3*b^3*d^2*e^(-3
) + 62985*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(
7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*a^2*b^4*d^2
*e^(-4) + 9690*(693*(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e
 + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 30
03*(x*e + d)^(3/2)*d^5)*a*b^5*d^2*e^(-5) + 323*(3003*(x*e + d)^(15/2) - 207
90*(x*e + d)^(13/2)*d + 61425*(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)
*d^3 + 96525*(x*e + d)^(7/2)*d^4 - 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*e +
 d)^(3/2)*d^6)*b^6*d^2*e^(-6) + 4849845*(x*e + d)^(3/2)*a^6*d^2 + 1662804*(
15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*a^5*b*d
*e^(-1) + 1385670*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e +
d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a^4*b^2*d*e^(-2) + 167960*(315*(x*e
 + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*
e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*a^3*b^3*d*e^(-3) + 48450*(693*
(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12
870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d
^5)*a^2*b^4*d*e^(-4) + 3876*(3003*(x*e + d)^(15/2) - 20790*(x*e + d)^(13/2)
*d + 61425*(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525*(x*e +
 d)^(7/2)*d^4 - 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*a*b^
5*d*e^(-5) + 266*(6435*(x*e + d)^(17/2) - 51051*(x*e + d)^(15/2)*d + 176715
*(x*e + d)^(13/2)*d^2 - 348075*(x*e + d)^(11/2)*d^3 + 425425*(x*e + d)^(9/2
)*d^4 - 328185*(x*e + d)^(7/2)*d^5 + 153153*(x*e + d)^(5/2)*d^6 - 36465*(x*
e + d)^(3/2)*d^7)*b^6*d*e^(-6) + 1939938*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(
3/2)*d)*a^6*d + 277134*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x
*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a^5*b*e^(-1) + 62985*(315*(x*e
 + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*
e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*a^4*b^2*e^(-2) + 32300*(693*(x
*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 1287
0*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5
)*a^3*b^3*e^(-3) + 4845*(3003*(x*e + d)^(15/2) - 20790*(x*e + d)^(13/2)*d +
 61425*(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525*(x*e + d)^
```

(7/2)*d^4 - 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*a^2*b^4*
e^(-4) + 798*(6435*(x*e + d)^(17/2) - 51051*(x*e + d)^(15/2)*d + 176715*(x*
e + d)^(13/2)*d^2 - 348075*(x*e + d)^(11/2)*d^3 + 425425*(x*e + d)^(9/2)*d^
4 - 328185*(x*e + d)^(7/2)*d^5 + 153153*(x*e + d)^(5/2)*d^6 - 36465*(x*e +
d)^(3/2)*d^7)*a*b^5*e^(-5) + 7*(109395*(x*e + d)^(19/2) - 978120*(x*e + d)^
(17/2)*d + 3879876*(x*e + d)^(15/2)*d^2 - 8953560*(x*e + d)^(13/2)*d^3 + 13
226850*(x*e + d)^(11/2)*d^4 - 12932920*(x*e + d)^(9/2)*d^5 + 8314020*(x*e +
d)^(7/2)*d^6 - 3325608*(x*e + d)^(5/2)*d^7 + 692835*(x*e + d)^(3/2)*d^8)*b
^6*e^(-6) + 138567*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d
)^(3/2)*d^2)*a^6)*e^(-1)

3.1639 $\int (d+ex)^{3/2} \left(a^2+2abx+b^2x^2\right)^3 \, dx$

Optimal. Leaf size=187

$$-\frac{4b^5(d+ex)^{15/2}(bd-ae)}{5e^7}+\frac{30b^4(d+ex)^{13/2}(bd-ae)^2}{13e^7}-\frac{40b^3(d+ex)^{11/2}(bd-ae)^3}{11e^7}+\frac{10b^2(d+ex)^{9/2}(bd-ae)^4}{3e^7}-\frac{12b}{}$$

```
[Out] (2*(b*d - a*e)^6*(d + e*x)^(5/2))/(5*e^7) - (12*b*(b*d - a*e)^5*(d + e*x)^(
7/2))/(7*e^7) + (10*b^2*(b*d - a*e)^4*(d + e*x)^(9/2))/(3*e^7) - (40*b^3*(b
*d - a*e)^3*(d + e*x)^(11/2))/(11*e^7) + (30*b^4*(b*d - a*e)^2*(d + e*x)^(1
3/2))/(13*e^7) - (4*b^5*(b*d - a*e)*(d + e*x)^(15/2))/(5*e^7) + (2*b^6*(d +
 e*x)^(17/2))/(17*e^7)
```

Rubi [A] time = 0.0609793, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {27, 43}

$$-\frac{4b^5(d+ex)^{15/2}(bd-ae)}{5e^7}+\frac{30b^4(d+ex)^{13/2}(bd-ae)^2}{13e^7}-\frac{40b^3(d+ex)^{11/2}(bd-ae)^3}{11e^7}+\frac{10b^2(d+ex)^{9/2}(bd-ae)^4}{3e^7}-\frac{12b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```


```
[Out] (2*(b*d - a*e)^6*(d + e*x)^(5/2))/(5*e^7) - (12*b*(b*d - a*e)^5*(d + e*x)^(
7/2))/(7*e^7) + (10*b^2*(b*d - a*e)^4*(d + e*x)^(9/2))/(3*e^7) - (40*b^3*(b
*d - a*e)^3*(d + e*x)^(11/2))/(11*e^7) + (30*b^4*(b*d - a*e)^2*(d + e*x)^(1
3/2))/(13*e^7) - (4*b^5*(b*d - a*e)*(d + e*x)^(15/2))/(5*e^7) + (2*b^6*(d +
 e*x)^(17/2))/(17*e^7)
```


Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^{3/2}\left(a^2+2abx+b^2x^2\right)^3 \, dx &= \int (a+bx)^6(d+ex)^{3/2} \, dx\\ &= \int \left(\frac{(-bd+ae)^6(d+ex)^{3/2}}{e^6}-\frac{6b(bd-ae)^5(d+ex)^{5/2}}{e^6}+\frac{15b^2(bd-ae)^4(d+ex)^{7}}{e^6}\right.\\ &= \frac{2(bd-ae)^6(d+ex)^{5/2}}{5e^7}-\frac{12b(bd-ae)^5(d+ex)^{7/2}}{7e^7}+\frac{10b^2(bd-ae)^4(d+ex)^{9/2}}{3e^7}-\end{aligned}$$

Mathematica [A] time = 0.104006, size = 145, normalized size = 0.78

$$\frac{2(d+ex)^{5/2}\left(425425b^2(d+ex)^2(bd-ae)^4-464100b^3(d+ex)^3(bd-ae)^3+294525b^4(d+ex)^4(bd-ae)^2-102102b^5(d\right.}{255255e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] (2*(d + e*x)^(5/2)*(51051*(b*d - a*e)^6 - 218790*b*(b*d - a*e)^5*(d + e*x)
+ 425425*b^2*(b*d - a*e)^4*(d + e*x)^2 - 464100*b^3*(b*d - a*e)^3*(d + e*x)
^3 + 294525*b^4*(b*d - a*e)^2*(d + e*x)^4 - 102102*b^5*(b*d - a*e)*(d + e*x
)^5 + 15015*b^6*(d + e*x)^6))/(255255*e^7)
```

Maple [B] time = 0.047, size = 377, normalized size = 2.

$$30030\,b^6x^6e^6+204204\,x^5ab^5e^6-24024\,x^5b^6de^5+589050\,x^4a^2b^4e^6-157080\,x^4ab^5de^5+18480\,x^4b^6d^2e^4+928200\,x^3a^3b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] 2/255255*(e*x+d)^(5/2)*(15015*b^6*e^6*x^6+102102*a*b^5*e^6*x^5-12012*b^6*d*
e^5*x^5+294525*a^2*b^4*e^6*x^4-78540*a*b^5*d*e^5*x^4+9240*b^6*d^2*e^4*x^4+4
64100*a^3*b^3*e^6*x^3-214200*a^2*b^4*d*e^5*x^3+57120*a*b^5*d^2*e^4*x^3-6720
*b^6*d^3*e^3*x^3+425425*a^4*b^2*e^6*x^2-309400*a^3*b^3*d*e^5*x^2+142800*a^2
*b^4*d^2*e^4*x^2-38080*a*b^5*d^3*e^3*x^2+4480*b^6*d^4*e^2*x^2+218790*a^5*b*
e^6*x-243100*a^4*b^2*d*e^5*x+176800*a^3*b^3*d^2*e^4*x-81600*a^2*b^4*d^3*e^3
*x+21760*a*b^5*d^4*e^2*x-2560*b^6*d^5*e*x+51051*a^6*e^6-87516*a^5*b*d*e^5+9
7240*a^4*b^2*d^2*e^4-70720*a^3*b^3*d^3*e^3+32640*a^2*b^4*d^4*e^2-8704*a*b^5
*d^5*e+1024*b^6*d^6)/e^7
```

Maxima [B] time = 1.07083, size = 473, normalized size = 2.53

$$2\left(15015\,(ex+d)^{\frac{17}{2}}b^6-102102\left(b^6d-ab^5e\right)(ex+d)^{\frac{15}{2}}+294525\left(b^6d^2-2\,ab^5de+a^2b^4e^2\right)(ex+d)^{\frac{13}{2}}-464100\left(b^6d^3-3\,a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] 2/255255*(15015*(e*x + d)^(17/2)*b^6 - 102102*(b^6*d - a*b^5*e)*(e*x + d)^(
15/2) + 294525*(b^6*d^2 - 2*a*b^5*d*e + a^2*b^4*e^2)*(e*x + d)^(13/2) - 464
100*(b^6*d^3 - 3*a*b^5*d^2*e + 3*a^2*b^4*d*e^2 - a^3*b^3*e^3)*(e*x + d)^(11
/2) + 425425*(b^6*d^4 - 4*a*b^5*d^3*e + 6*a^2*b^4*d^2*e^2 - 4*a^3*b^3*d*e^3
 + a^4*b^2*e^4)*(e*x + d)^(9/2) - 218790*(b^6*d^5 - 5*a*b^5*d^4*e + 10*a^2*
b^4*d^3*e^2 - 10*a^3*b^3*d^2*e^3 + 5*a^4*b^2*d*e^4 - a^5*b*e^5)*(e*x + d)^(
7/2) + 51051*(b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3
*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 + a^6*e^6)*(e*x + d)^(5/2))/e^7
```

Fricas [B] time = 1.61811, size = 1256, normalized size = 6.72

$$2\left(15015\,b^6e^8x^8+1024\,b^6d^8-8704\,ab^5d^7e+32640\,a^2b^4d^6e^2-70720\,a^3b^3d^5e^3+97240\,a^4b^2d^4e^4-87516\,a^5bd^3e^5+5105\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] 2/255255*(15015*b^6*e^8*x^8 + 1024*b^6*d^8 - 8704*a*b^5*d^7*e + 32640*a^2*b
^4*d^6*e^2 - 70720*a^3*b^3*d^5*e^3 + 97240*a^4*b^2*d^4*e^4 - 87516*a^5*b*d^
3*e^5 + 51051*a^6*d^2*e^6 + 6006*(3*b^6*d*e^7 + 17*a*b^5*e^8)*x^7 + 231*(b^
6*d^2*e^6 + 544*a*b^5*d*e^7 + 1275*a^2*b^4*e^8)*x^6 - 42*(6*b^6*d^3*e^5 - 5
1*a*b^5*d^2*e^6 - 8925*a^2*b^4*d*e^7 - 11050*a^3*b^3*e^8)*x^5 + 35*(8*b^6*d
^4*e^4 - 68*a*b^5*d^3*e^5 + 255*a^2*b^4*d^2*e^6 + 17680*a^3*b^3*d*e^7 + 121
55*a^4*b^2*e^8)*x^4 - 10*(32*b^6*d^5*e^3 - 272*a*b^5*d^4*e^4 + 1020*a^2*b^4
*d^3*e^5 - 2210*a^3*b^3*d^2*e^6 - 60775*a^4*b^2*d*e^7 - 21879*a^5*b*e^8)*x^
3 + 3*(128*b^6*d^6*e^2 - 1088*a*b^5*d^5*e^3 + 4080*a^2*b^4*d^4*e^4 - 8840*a
^3*b^3*d^3*e^5 + 12155*a^4*b^2*d^2*e^6 + 116688*a^5*b*d*e^7 + 17017*a^6*e^8
)*x^2 - 2*(256*b^6*d^7*e - 2176*a*b^5*d^6*e^2 + 8160*a^2*b^4*d^5*e^3 - 1768
0*a^3*b^3*d^4*e^4 + 24310*a^4*b^2*d^3*e^5 - 21879*a^5*b*d^2*e^6 - 51051*a^6
*d*e^7)*x)*sqrt(e*x + d)/e^7
```

Sympy [A] time = 33.1206, size = 1000, normalized size = 5.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] a**6*d*Piecewise((sqrt(d)*x, Eq(e, 0)), (2*(d + e*x)**(3/2)/(3*e), True)) +
 2*a**6*(-d*(d + e*x)**(3/2)/3 + (d + e*x)**(5/2)/5)/e + 12*a**5*b*d*(-d*(d
 + e*x)**(3/2)/3 + (d + e*x)**(5/2)/5)/e**2 + 12*a**5*b*(d**2*(d + e*x)**(3
/2)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e**2 + 30*a**4*b**2*d*
(d**2*(d + e*x)**(3/2)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e**
3 + 30*a**4*b**2*(-d**3*(d + e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*
d*(d + e*x)**(7/2)/7 + (d + e*x)**(9/2)/9)/e**3 + 40*a**3*b**3*d*(-d**3*(d
+ e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d + e*x)**(7/2)/7 + (d +
 e*x)**(9/2)/9)/e**4 + 40*a**3*b**3*(d**4*(d + e*x)**(3/2)/3 - 4*d**3*(d +
e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e*x)**(9/2)/9 + (d + e
*x)**(11/2)/11)/e**4 + 30*a**2*b**4*d*(d**4*(d + e*x)**(3/2)/3 - 4*d**3*(d
+ e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e*x)**(9/2)/9 + (d +
 e*x)**(11/2)/11)/e**5 + 30*a**2*b**4*(-d**5*(d + e*x)**(3/2)/3 + d**4*(d +
 e*x)**(5/2) - 10*d**3*(d + e*x)**(7/2)/7 + 10*d**2*(d + e*x)**(9/2)/9 - 5*
d*(d + e*x)**(11/2)/11 + (d + e*x)**(13/2)/13)/e**5 + 12*a*b**5*d*(-d**5*(d
 + e*x)**(3/2)/3 + d**4*(d + e*x)**(5/2) - 10*d**3*(d + e*x)**(7/2)/7 + 10*
d**2*(d + e*x)**(9/2)/9 - 5*d*(d + e*x)**(11/2)/11 + (d + e*x)**(13/2)/13)/
e**6 + 12*a*b**5*(d**6*(d + e*x)**(3/2)/3 - 6*d**5*(d + e*x)**(5/2)/5 + 15*
d**4*(d + e*x)**(7/2)/7 - 20*d**3*(d + e*x)**(9/2)/9 + 15*d**2*(d + e*x)**(
11/2)/11 - 6*d*(d + e*x)**(13/2)/13 + (d + e*x)**(15/2)/15)/e**6 + 2*b**6*d
*(d**6*(d + e*x)**(3/2)/3 - 6*d**5*(d + e*x)**(5/2)/5 + 15*d**4*(d + e*x)**
(7/2)/7 - 20*d**3*(d + e*x)**(9/2)/9 + 15*d**2*(d + e*x)**(11/2)/11 - 6*d*(
d + e*x)**(13/2)/13 + (d + e*x)**(15/2)/15)/e**7 + 2*b**6*(-d**7*(d + e*x)*
*(3/2)/3 + 7*d**6*(d + e*x)**(5/2)/5 - 3*d**5*(d + e*x)**(7/2) + 35*d**4*(d
 + e*x)**(9/2)/9 - 35*d**3*(d + e*x)**(11/2)/11 + 21*d**2*(d + e*x)**(13/2)
/13 - 7*d*(d + e*x)**(15/2)/15 + (d + e*x)**(17/2)/17)/e**7
```

Giac [B] time = 1.24026, size = 1202, normalized size = 6.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] 2/765765*(306306*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^5*b*d*e^(-1) +
 109395*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2
)*a^4*b^2*d*e^(-2) + 48620*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 18
9*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a^3*b^3*d*e^(-3) + 3315*(3
15*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2
772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*a^2*b^4*d*e^(-4) + 510*
(693*(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2
 - 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3
/2)*d^5)*a*b^5*d*e^(-5) + 17*(3003*(x*e + d)^(15/2) - 20790*(x*e + d)^(13/2
)*d + 61425*(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525*(x*e
+ d)^(7/2)*d^4 - 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*b^6
*d*e^(-6) + 255255*(x*e + d)^(3/2)*a^6*d + 43758*(15*(x*e + d)^(7/2) - 42*(
x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*a^5*b*e^(-1) + 36465*(35*(x*e +
d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^
(3/2)*d^3)*a^4*b^2*e^(-2) + 4420*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/
2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)
^(3/2)*d^4)*a^3*b^3*e^(-3) + 1275*(693*(x*e + d)^(13/2) - 4095*(x*e + d)^(1
1/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e
+ d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*a^2*b^4*e^(-4) + 102*(3003*(x*e
+ d)^(15/2) - 20790*(x*e + d)^(13/2)*d + 61425*(x*e + d)^(11/2)*d^2 - 10010
0*(x*e + d)^(9/2)*d^3 + 96525*(x*e + d)^(7/2)*d^4 - 54054*(x*e + d)^(5/2)*d
^5 + 15015*(x*e + d)^(3/2)*d^6)*a*b^5*e^(-5) + 7*(6435*(x*e + d)^(17/2) - 5
1051*(x*e + d)^(15/2)*d + 176715*(x*e + d)^(13/2)*d^2 - 348075*(x*e + d)^(1
1/2)*d^3 + 425425*(x*e + d)^(9/2)*d^4 - 328185*(x*e + d)^(7/2)*d^5 + 153153
*(x*e + d)^(5/2)*d^6 - 36465*(x*e + d)^(3/2)*d^7)*b^6*e^(-6) + 51051*(3*(x*
e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^6)*e^(-1)
```

3.1640 $\int \sqrt{d+ex}\left(a^2+2abx+b^2x^2\right)^3\,dx$

Optimal. Leaf size=187

$$-\frac{12b^5(d+ex)^{13/2}(bd-ae)}{13e^7}+\frac{30b^4(d+ex)^{11/2}(bd-ae)^2}{11e^7}-\frac{40b^3(d+ex)^{9/2}(bd-ae)^3}{9e^7}+\frac{30b^2(d+ex)^{7/2}(bd-ae)^4}{7e^7}-\frac{12b}{}$$

```
[Out] (2*(b*d - a*e)^6*(d + e*x)^(3/2))/(3*e^7) - (12*b*(b*d - a*e)^5*(d + e*x)^(
5/2))/(5*e^7) + (30*b^2*(b*d - a*e)^4*(d + e*x)^(7/2))/(7*e^7) - (40*b^3*(b
*d - a*e)^3*(d + e*x)^(9/2))/(9*e^7) + (30*b^4*(b*d - a*e)^2*(d + e*x)^(11/
2))/(11*e^7) - (12*b^5*(b*d - a*e)*(d + e*x)^(13/2))/(13*e^7) + (2*b^6*(d +
 e*x)^(15/2))/(15*e^7)
```

Rubi [A] time = 0.0589647, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {27, 43}

$$-\frac{12b^5(d+ex)^{13/2}(bd-ae)}{13e^7}+\frac{30b^4(d+ex)^{11/2}(bd-ae)^2}{11e^7}-\frac{40b^3(d+ex)^{9/2}(bd-ae)^3}{9e^7}+\frac{30b^2(d+ex)^{7/2}(bd-ae)^4}{7e^7}-\frac{12b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```


```
[Out] (2*(b*d - a*e)^6*(d + e*x)^(3/2))/(3*e^7) - (12*b*(b*d - a*e)^5*(d + e*x)^(
5/2))/(5*e^7) + (30*b^2*(b*d - a*e)^4*(d + e*x)^(7/2))/(7*e^7) - (40*b^3*(b
*d - a*e)^3*(d + e*x)^(9/2))/(9*e^7) + (30*b^4*(b*d - a*e)^2*(d + e*x)^(11/
2))/(11*e^7) - (12*b^5*(b*d - a*e)*(d + e*x)^(13/2))/(13*e^7) + (2*b^6*(d +
 e*x)^(15/2))/(15*e^7)
```


Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{d+ex}\left(a^2+2abx+b^2x^2\right)^3\,dx &= \int (a+bx)^6\sqrt{d+ex}\,dx\\ &= \int \left(\frac{(-bd+ae)^6\sqrt{d+ex}}{e^6}-\frac{6b(bd-ae)^5(d+ex)^{3/2}}{e^6}+\frac{15b^2(bd-ae)^4(d+ex)^{5/2}}{e^6}-\right.\\ &= \frac{2(bd-ae)^6(d+ex)^{3/2}}{3e^7}-\frac{12b(bd-ae)^5(d+ex)^{5/2}}{5e^7}+\frac{30b^2(bd-ae)^4(d+ex)^{7/2}}{7e^7}-\end{aligned}$$

Mathematica [A] time = 0.101242, size = 145, normalized size = 0.78

$$\frac{2(d+ex)^{3/2}\left(96525b^2(d+ex)^2(bd-ae)^4-100100b^3(d+ex)^3(bd-ae)^3+61425b^4(d+ex)^4(bd-ae)^2-20790b^5(d+e\right.}{45045e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] (2*(d + e*x)^(3/2)*(15015*(b*d - a*e)^6 - 54054*b*(b*d - a*e)^5*(d + e*x) +
 96525*b^2*(b*d - a*e)^4*(d + e*x)^2 - 100100*b^3*(b*d - a*e)^3*(d + e*x)^3
 + 61425*b^4*(b*d - a*e)^2*(d + e*x)^4 - 20790*b^5*(b*d - a*e)*(d + e*x)^5
+ 3003*b^6*(d + e*x)^6))/(45045*e^7)
```

Maple [B] time = 0.047, size = 377, normalized size = 2.

$$\frac{6006\,b^6x^6e^6 + 41580\,x^5ab^5e^6 - 5544\,x^5b^6de^5 + 122850\,x^4a^2b^4e^6 - 37800\,x^4ab^5de^5 + 5040\,x^4b^6d^2e^4 + 200200\,x^3a^3b^3e^6 - \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^3*(e*x+d)^(1/2),x)
```

```
[Out] 2/45045*(e*x+d)^(3/2)*(3003*b^6*e^6*x^6+20790*a*b^5*e^6*x^5-2772*b^6*d*e^5*
x^5+61425*a^2*b^4*e^6*x^4-18900*a*b^5*d*e^5*x^4+2520*b^6*d^2*e^4*x^4+100100
*a^3*b^3*e^6*x^3-54600*a^2*b^4*d*e^5*x^3+16800*a*b^5*d^2*e^4*x^3-2240*b^6*d
^3*e^3*x^3+96525*a^4*b^2*e^6*x^2-85800*a^3*b^3*d*e^5*x^2+46800*a^2*b^4*d^2*
e^4*x^2-14400*a*b^5*d^3*e^3*x^2+1920*b^6*d^4*e^2*x^2+54054*a^5*b*e^6*x-7722
0*a^4*b^2*d*e^5*x+68640*a^3*b^3*d^2*e^4*x-37440*a^2*b^4*d^3*e^3*x+11520*a*b
^5*d^4*e^2*x-1536*b^6*d^5*e*x+15015*a^6*e^6-36036*a^5*b*d*e^5+51480*a^4*b^2
*d^2*e^4-45760*a^3*b^3*d^3*e^3+24960*a^2*b^4*d^4*e^2-7680*a*b^5*d^5*e+1024*
b^6*d^6)/e^7
```

Maxima [B] time = 1.07513, size = 473, normalized size = 2.53

$$\frac{2\left(3003\,(ex+d)^{\frac{15}{2}}b^6 - 20790\left(b^6d - ab^5e\right)(ex+d)^{\frac{13}{2}} + 61425\left(b^6d^2 - 2\,ab^5de + a^2b^4e^2\right)(ex+d)^{\frac{11}{2}} - 100100\left(b^6d^3 - 3\,ab^5 \cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(3003*(e*x + d)^(15/2)*b^6 - 20790*(b^6*d - a*b^5*e)*(e*x + d)^(13/
2) + 61425*(b^6*d^2 - 2*a*b^5*d*e + a^2*b^4*e^2)*(e*x + d)^(11/2) - 100100*
(b^6*d^3 - 3*a*b^5*d^2*e + 3*a^2*b^4*d*e^2 - a^3*b^3*e^3)*(e*x + d)^(9/2) +
 96525*(b^6*d^4 - 4*a*b^5*d^3*e + 6*a^2*b^4*d^2*e^2 - 4*a^3*b^3*d*e^3 + a^4
*b^2*e^4)*(e*x + d)^(7/2) - 54054*(b^6*d^5 - 5*a*b^5*d^4*e + 10*a^2*b^4*d^3
*e^2 - 10*a^3*b^3*d^2*e^3 + 5*a^4*b^2*d*e^4 - a^5*b*e^5)*(e*x + d)^(5/2) +
15015*(b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 +
15*a^4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 + a^6*e^6)*(e*x + d)^(3/2))/e^7
```

Fricas [B] time = 1.60004, size = 1027, normalized size = 5.49

$$\frac{2\left(3003\,b^6e^7x^7 + 1024\,b^6d^7 - 7680\,ab^5d^6e + 24960\,a^2b^4d^5e^2 - 45760\,a^3b^3d^4e^3 + 51480\,a^4b^2d^3e^4 - 36036\,a^5bd^2e^5 + 15015 \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3003*b^6*e^7*x^7 + 1024*b^6*d^7 - 7680*a*b^5*d^6*e + 24960*a^2*b^4
*d^5*e^2 - 45760*a^3*b^3*d^4*e^3 + 51480*a^4*b^2*d^3*e^4 - 36036*a^5*b*d^2*
e^5 + 15015*a^6*d*e^6 + 231*(b^6*d*e^6 + 90*a*b^5*e^7)*x^6 - 63*(4*b^6*d^2*
e^5 - 30*a*b^5*d*e^6 - 975*a^2*b^4*e^7)*x^5 + 35*(8*b^6*d^3*e^4 - 60*a*b^5*
d^2*e^5 + 195*a^2*b^4*d*e^6 + 2860*a^3*b^3*e^7)*x^4 - 5*(64*b^6*d^4*e^3 - 4
80*a*b^5*d^3*e^4 + 1560*a^2*b^4*d^2*e^5 - 2860*a^3*b^3*d*e^6 - 19305*a^4*b^
2*e^7)*x^3 + 3*(128*b^6*d^5*e^2 - 960*a*b^5*d^4*e^3 + 3120*a^2*b^4*d^3*e^4
- 5720*a^3*b^3*d^2*e^5 + 6435*a^4*b^2*d*e^6 + 18018*a^5*b*e^7)*x^2 - (512*b
^6*d^6*e - 3840*a*b^5*d^5*e^2 + 12480*a^2*b^4*d^4*e^3 - 22880*a^3*b^3*d^3*e
^4 + 25740*a^4*b^2*d^2*e^5 - 18018*a^5*b*d*e^6 - 15015*a^6*e^7)*x)*sqrt(e*x
 + d)/e^7
```

Sympy [B] time = 6.4708, size = 422, normalized size = 2.26

$$2\left(\frac{b^6(d+ex)^{\frac{15}{2}}}{15e^6}+\frac{(d+ex)^{\frac{13}{2}}\left(6ab^5e-6b^6d\right)}{13e^6}+\frac{(d+ex)^{\frac{11}{2}}\left(15a^2b^4e^2-30ab^5de+15b^6d^2\right)}{11e^6}+\frac{(d+ex)^{\frac{9}{2}}\left(20a^3b^3e^3-60a^2b^4de^2+60ab^5d^2e-20b^6d^3\right)}{9e^6}+\frac{(d+ex)^{\frac{7}{2}}(15\ldots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**3*(e*x+d)**(1/2),x)
```

```
[Out] 2*(b**6*(d + e*x)**(15/2)/(15*e**6) + (d + e*x)**(13/2)*(6*a*b**5*e - 6*b**
6*d)/(13*e**6) + (d + e*x)**(11/2)*(15*a**2*b**4*e**2 - 30*a*b**5*d*e + 15*
b**6*d**2)/(11*e**6) + (d + e*x)**(9/2)*(20*a**3*b**3*e**3 - 60*a**2*b**4*d
*e**2 + 60*a*b**5*d**2*e - 20*b**6*d**3)/(9*e**6) + (d + e*x)**(7/2)*(15*a*
*4*b**2*e**4 - 60*a**3*b**3*d*e**3 + 90*a**2*b**4*d**2*e**2 - 60*a*b**5*d**
3*e + 15*b**6*d**4)/(7*e**6) + (d + e*x)**(5/2)*(6*a**5*b*e**5 - 30*a**4*b*
*2*d*e**4 + 60*a**3*b**3*d**2*e**3 - 60*a**2*b**4*d**3*e**2 + 30*a*b**5*d**
4*e - 6*b**6*d**5)/(5*e**6) + (d + e*x)**(3/2)*(a**6*e**6 - 6*a**5*b*d*e**5
 + 15*a**4*b**2*d**2*e**4 - 20*a**3*b**3*d**3*e**3 + 15*a**2*b**4*d**4*e**2
 - 6*a*b**5*d**5*e + b**6*d**6)/(3*e**6))/e
```

Giac [B] time = 1.22188, size = 536, normalized size = 2.87

$$\frac{2}{45045}\left(18018\left(3\,(xe+d)^{\frac{5}{2}}-5\,(xe+d)^{\frac{3}{2}}d\right)a^5be^{(-1)}+6435\left(15\,(xe+d)^{\frac{7}{2}}-42\,(xe+d)^{\frac{5}{2}}d+35\,(xe+d)^{\frac{3}{2}}d^2\right)a^4b^2e^{(-2)}+2\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 2/45045*(18018*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^5*b*e^(-1) + 643
5*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*a^4*
b^2*e^(-2) + 2860*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e +
d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a^3*b^3*e^(-3) + 195*(315*(x*e + d)
^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d
)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*a^2*b^4*e^(-4) + 30*(693*(x*e + d)^
(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12870*(x*e +
 d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*a*b^5*
e^(-5) + (3003*(x*e + d)^(15/2) - 20790*(x*e + d)^(13/2)*d + 61425*(x*e + d
)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525*(x*e + d)^(7/2)*d^4 - 540
```

```
54*(x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*b^6*e^(-6) + 15015*(x*e
 + d)^(3/2)*a^6)*e^(-1)
```

3.1641 $\int \frac{\left(a^2+2abx+b^2x^2\right)^3}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=181

$$-\frac{12b^5(d+ex)^{11/2}(bd-ae)}{11e^7}+\frac{10b^4(d+ex)^{9/2}(bd-ae)^2}{3e^7}-\frac{40b^3(d+ex)^{7/2}(bd-ae)^3}{7e^7}+\frac{6b^2(d+ex)^{5/2}(bd-ae)^4}{e^7}-\frac{4b(d$$

```
[Out] (2*(b*d - a*e)^6*Sqrt[d + e*x])/e^7 - (4*b*(b*d - a*e)^5*(d + e*x)^(3/2))/e
^7 + (6*b^2*(b*d - a*e)^4*(d + e*x)^(5/2))/e^7 - (40*b^3*(b*d - a*e)^3*(d +
 e*x)^(7/2))/(7*e^7) + (10*b^4*(b*d - a*e)^2*(d + e*x)^(9/2))/(3*e^7) - (12
*b^5*(b*d - a*e)*(d + e*x)^(11/2))/(11*e^7) + (2*b^6*(d + e*x)^(13/2))/(13*
e^7)
```

Rubi [A] time = 0.0602919, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {27, 43}

$$-\frac{12b^5(d+ex)^{11/2}(bd-ae)}{11e^7}+\frac{10b^4(d+ex)^{9/2}(bd-ae)^2}{3e^7}-\frac{40b^3(d+ex)^{7/2}(bd-ae)^3}{7e^7}+\frac{6b^2(d+ex)^{5/2}(bd-ae)^4}{e^7}-\frac{4b(d$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^3/Sqrt[d + e*x],x]
```

```
[Out] (2*(b*d - a*e)^6*Sqrt[d + e*x])/e^7 - (4*b*(b*d - a*e)^5*(d + e*x)^(3/2))/e
^7 + (6*b^2*(b*d - a*e)^4*(d + e*x)^(5/2))/e^7 - (40*b^3*(b*d - a*e)^3*(d +
 e*x)^(7/2))/(7*e^7) + (10*b^4*(b*d - a*e)^2*(d + e*x)^(9/2))/(3*e^7) - (12
*b^5*(b*d - a*e)*(d + e*x)^(11/2))/(11*e^7) + (2*b^6*(d + e*x)^(13/2))/(13*
e^7)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx+b^2x^2\right)^3}{\sqrt{d+ex}}\,dx &= \int \frac{(a+bx)^6}{\sqrt{d+ex}}\,dx \\
&= \int \left(\frac{(-bd+ae)^6}{e^6\sqrt{d+ex}}-\frac{6b(bd-ae)^5\sqrt{d+ex}}{e^6}+\frac{15b^2(bd-ae)^4(d+ex)^{3/2}}{e^6}-\frac{20b^3(bd-ae)^3(d\\
&= \frac{2(bd-ae)^6\sqrt{d+ex}}{e^7}-\frac{4b(bd-ae)^5(d+ex)^{3/2}}{e^7}+\frac{6b^2(bd-ae)^4(d+ex)^{5/2}}{e^7}-\frac{40b^3(bd-ae}{7e}
\end{aligned}$$

Mathematica [A] time = 0.0812089, size = 145, normalized size = 0.8

$$\frac{2\sqrt{d+ex}\left(9009b^2(d+ex)^2(bd-ae)^4-8580b^3(d+ex)^3(bd-ae)^3+5005b^4(d+ex)^4(bd-ae)^2-1638b^5(d+ex)^5(bd-ae\right.}{3003e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[d + e*x]*(3003*(b*d - a*e)^6 - 6006*b*(b*d - a*e)^5*(d + e*x) + 900
9*b^2*(b*d - a*e)^4*(d + e*x)^2 - 8580*b^3*(b*d - a*e)^3*(d + e*x)^3 + 5005
*b^4*(b*d - a*e)^2*(d + e*x)^4 - 1638*b^5*(b*d - a*e)*(d + e*x)^5 + 231*b^6
*(d + e*x)^6))/(3003*e^7)
```

Maple [B] time = 0.048, size = 377, normalized size = 2.1

$$\frac{462\,b^6x^6e^6+3276\,x^5ab^5e^6-504\,x^5b^6de^5+10010\,x^4a^2b^4e^6-3640\,x^4ab^5de^5+560\,x^4b^6d^2e^4+17160\,x^3a^3b^3e^6-11440\,x^3a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(1/2),x)
```

```
[Out] 2/3003*(231*b^6*e^6*x^6+1638*a*b^5*e^6*x^5-252*b^6*d*e^5*x^5+5005*a^2*b^4*e
^6*x^4-1820*a*b^5*d*e^5*x^4+280*b^6*d^2*e^4*x^4+8580*a^3*b^3*e^6*x^3-5720*a
^2*b^4*d*e^5*x^3+2080*a*b^5*d^2*e^4*x^3-320*b^6*d^3*e^3*x^3+9009*a^4*b^2*e^
6*x^2-10296*a^3*b^3*d*e^5*x^2+6864*a^2*b^4*d^2*e^4*x^2-2496*a*b^5*d^3*e^3*x
^2+384*b^6*d^4*e^2*x^2+6006*a^5*b*e^6*x-12012*a^4*b^2*d*e^5*x+13728*a^3*b^3
*d^2*e^4*x-9152*a^2*b^4*d^3*e^3*x+3328*a*b^5*d^4*e^2*x-512*b^6*d^5*e*x+3003
*a^6*e^6-12012*a^5*b*d*e^5+24024*a^4*b^2*d^2*e^4-27456*a^3*b^3*d^3*e^3+1830
4*a^2*b^4*d^4*e^2-6656*a*b^5*d^5*e+1024*b^6*d^6)*(e*x+d)^(1/2)/e^7
```

Maxima [B] time = 1.03915, size = 729, normalized size = 4.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15015*(15015*sqrt(e*x + d)*a^6 + 3003*(10*((e*x + d)^(3/2) - 3*sqrt(e*x +
 d)*d)*a*b/e + (3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)
*d^2)*b^2/e^2)*a^4 + 3432*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e
*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^3*b^3/e^3 + 143*(84*(3*(e*x + d
)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a^2*b^2/e^2 + 36*(5*
(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e
*x + d)*d^3)*a*b^3/e^3 + (35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*
(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*b^4/
e^4)*a^2 + 572*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^
(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*a^2*b^4/e^4 +
130*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2
- 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d
^5)*a*b^5/e^5 + 5*(231*(e*x + d)^(13/2) - 1638*(e*x + d)^(11/2)*d + 5005*(e
*x + d)^(9/2)*d^2 - 8580*(e*x + d)^(7/2)*d^3 + 9009*(e*x + d)^(5/2)*d^4 - 6
```

```
006*(e*x + d)^(3/2)*d^5 + 3003*sqrt(e*x + d)*d^6)*b^6/e^6)/e
```

Fricas [B] time = 1.68577, size = 815, normalized size = 4.5

$$\frac{2\left(231\,b^6e^6x^6+1024\,b^6d^6-6656\,ab^5d^5e+18304\,a^2b^4d^4e^2-27456\,a^3b^3d^3e^3+24024\,a^4b^2d^2e^4-12012\,a^5bde^5+3003\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(231*b^6*e^6*x^6 + 1024*b^6*d^6 - 6656*a*b^5*d^5*e + 18304*a^2*b^4*d
^4*e^2 - 27456*a^3*b^3*d^3*e^3 + 24024*a^4*b^2*d^2*e^4 - 12012*a^5*b*d*e^5
+ 3003*a^6*e^6 - 126*(2*b^6*d*e^5 - 13*a*b^5*e^6)*x^5 + 35*(8*b^6*d^2*e^4 -
 52*a*b^5*d*e^5 + 143*a^2*b^4*e^6)*x^4 - 20*(16*b^6*d^3*e^3 - 104*a*b^5*d^2
*e^4 + 286*a^2*b^4*d*e^5 - 429*a^3*b^3*e^6)*x^3 + 3*(128*b^6*d^4*e^2 - 832*
a*b^5*d^3*e^3 + 2288*a^2*b^4*d^2*e^4 - 3432*a^3*b^3*d*e^5 + 3003*a^4*b^2*e^
6)*x^2 - 2*(256*b^6*d^5*e - 1664*a*b^5*d^4*e^2 + 4576*a^2*b^4*d^3*e^3 - 686
4*a^3*b^3*d^2*e^4 + 6006*a^4*b^2*d*e^5 - 3003*a^5*b*e^6)*x)*sqrt(e*x + d)/e
^7
```

Sympy [A] time = 98.0158, size = 1003, normalized size = 5.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**(1/2),x)
```

```
[Out] Piecewise((-(2*a**6*d/sqrt(d + e*x) + 2*a**6*(-d/sqrt(d + e*x) - sqrt(d + e
*x)) + 12*a**5*b*d*(-d/sqrt(d + e*x) - sqrt(d + e*x))/e + 12*a**5*b*(d**2/s
qrt(d + e*x) + 2*d*sqrt(d + e*x) - (d + e*x)**(3/2)/3)/e + 30*a**4*b**2*d*(
d**2/sqrt(d + e*x) + 2*d*sqrt(d + e*x) - (d + e*x)**(3/2)/3)/e**2 + 30*a**4
*b**2*(-d**3/sqrt(d + e*x) - 3*d**2*sqrt(d + e*x) + d*(d + e*x)**(3/2) - (d
 + e*x)**(5/2)/5)/e**2 + 40*a**3*b**3*d*(-d**3/sqrt(d + e*x) - 3*d**2*sqrt(
d + e*x) + d*(d + e*x)**(3/2) - (d + e*x)**(5/2)/5)/e**3 + 40*a**3*b**3*(d*
*4/sqrt(d + e*x) + 4*d**3*sqrt(d + e*x) - 2*d**2*(d + e*x)**(3/2) + 4*d*(d
+ e*x)**(5/2)/5 - (d + e*x)**(7/2)/7)/e**3 + 30*a**2*b**4*d*(d**4/sqrt(d +
e*x) + 4*d**3*sqrt(d + e*x) - 2*d**2*(d + e*x)**(3/2) + 4*d*(d + e*x)**(5/2
)/5 - (d + e*x)**(7/2)/7)/e**4 + 30*a**2*b**4*(-d**5/sqrt(d + e*x) - 5*d**4
*sqrt(d + e*x) + 10*d**3*(d + e*x)**(3/2)/3 - 2*d**2*(d + e*x)**(5/2) + 5*d
*(d + e*x)**(7/2)/7 - (d + e*x)**(9/2)/9)/e**4 + 12*a*b**5*d*(-d**5/sqrt(d
+ e*x) - 5*d**4*sqrt(d + e*x) + 10*d**3*(d + e*x)**(3/2)/3 - 2*d**2*(d + e*
x)**(5/2) + 5*d*(d + e*x)**(7/2)/7 - (d + e*x)**(9/2)/9)/e**5 + 12*a*b**5*(
d**6/sqrt(d + e*x) + 6*d**5*sqrt(d + e*x) - 5*d**4*(d + e*x)**(3/2) + 4*d**
3*(d + e*x)**(5/2) - 15*d**2*(d + e*x)**(7/2)/7 + 2*d*(d + e*x)**(9/2)/3 -
(d + e*x)**(11/2)/11)/e**5 + 2*b**6*d*(d**6/sqrt(d + e*x) + 6*d**5*sqrt(d +
 e*x) - 5*d**4*(d + e*x)**(3/2) + 4*d**3*(d + e*x)**(5/2) - 15*d**2*(d + e*
x)**(7/2)/7 + 2*d*(d + e*x)**(9/2)/3 - (d + e*x)**(11/2)/11)/e**6 + 2*b**6*
(-d**7/sqrt(d + e*x) - 7*d**6*sqrt(d + e*x) + 7*d**5*(d + e*x)**(3/2) - 7*d
**4*(d + e*x)**(5/2) + 5*d**3*(d + e*x)**(7/2) - 7*d**2*(d + e*x)**(9/2)/3
+ 7*d*(d + e*x)**(11/2)/11 - (d + e*x)**(13/2)/13)/e**6)/e, Ne(e, 0)), ((a*
*6*x + 3*a**5*b*x**2 + 5*a**4*b**2*x**3 + 5*a**3*b**3*x**4 + 3*a**2*b**4*x*
*5 + a*b**5*x**6 + b**6*x**7/7)/sqrt(d), True))
```

Giac [B] time = 1.18411, size = 533, normalized size = 2.94

$$\frac{2}{3003}\left(6006\left((xe+d)^{\frac{3}{2}}-3\sqrt{xe+d}d\right)a^5be^{(-1)}+3003\left(3(xe+d)^{\frac{5}{2}}-10(xe+d)^{\frac{3}{2}}d+15\sqrt{xe+d}d^2\right)a^4b^2e^{(-2)}+1716\left(5(xe\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3003*(6006*((x*e + d)^(3/2) - 3*sqrt(x*e + d)*d)*a^5*b*e^(-1) + 3003*(3*(
x*e + d)^(5/2) - 10*(x*e + d)^(3/2)*d + 15*sqrt(x*e + d)*d^2)*a^4*b^2*e^(-2
) + 1716*(5*(x*e + d)^(7/2) - 21*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2
 - 35*sqrt(x*e + d)*d^3)*a^3*b^3*e^(-3) + 143*(35*(x*e + d)^(9/2) - 180*(x*
e + d)^(7/2)*d + 378*(x*e + d)^(5/2)*d^2 - 420*(x*e + d)^(3/2)*d^3 + 315*sq
rt(x*e + d)*d^4)*a^2*b^4*e^(-4) + 26*(63*(x*e + d)^(11/2) - 385*(x*e + d)^(
9/2)*d + 990*(x*e + d)^(7/2)*d^2 - 1386*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d
)^(3/2)*d^4 - 693*sqrt(x*e + d)*d^5)*a*b^5*e^(-5) + (231*(x*e + d)^(13/2) -
 1638*(x*e + d)^(11/2)*d + 5005*(x*e + d)^(9/2)*d^2 - 8580*(x*e + d)^(7/2)*
d^3 + 9009*(x*e + d)^(5/2)*d^4 - 6006*(x*e + d)^(3/2)*d^5 + 3003*sqrt(x*e +
 d)*d^6)*b^6*e^(-6) + 3003*sqrt(x*e + d)*a^6)*e^(-1)
```

3.1642 $\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=179

$$-\frac{4b^5(d+ex)^{9/2}(bd-ae)}{3e^7}+\frac{30b^4(d+ex)^{7/2}(bd-ae)^2}{7e^7}-\frac{8b^3(d+ex)^{5/2}(bd-ae)^3}{e^7}+\frac{10b^2(d+ex)^{3/2}(bd-ae)^4}{e^7}-\frac{12b\sqrt{d}}{}$$

```
[Out] (-2*(b*d - a*e)^6)/(e^7*Sqrt[d + e*x]) - (12*b*(b*d - a*e)^5*Sqrt[d + e*x])
/e^7 + (10*b^2*(b*d - a*e)^4*(d + e*x)^(3/2))/e^7 - (8*b^3*(b*d - a*e)^3*(d
 + e*x)^(5/2))/e^7 + (30*b^4*(b*d - a*e)^2*(d + e*x)^(7/2))/(7*e^7) - (4*b^
5*(b*d - a*e)*(d + e*x)^(9/2))/(3*e^7) + (2*b^6*(d + e*x)^(11/2))/(11*e^7)
```

Rubi [A] time = 0.0590916, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {27, 43}

$$-\frac{4b^5(d+ex)^{9/2}(bd-ae)}{3e^7}+\frac{30b^4(d+ex)^{7/2}(bd-ae)^2}{7e^7}-\frac{8b^3(d+ex)^{5/2}(bd-ae)^3}{e^7}+\frac{10b^2(d+ex)^{3/2}(bd-ae)^4}{e^7}-\frac{12b\sqrt{d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^(3/2),x]
```

```
[Out] (-2*(b*d - a*e)^6)/(e^7*Sqrt[d + e*x]) - (12*b*(b*d - a*e)^5*Sqrt[d + e*x])
/e^7 + (10*b^2*(b*d - a*e)^4*(d + e*x)^(3/2))/e^7 - (8*b^3*(b*d - a*e)^3*(d
 + e*x)^(5/2))/e^7 + (30*b^4*(b*d - a*e)^2*(d + e*x)^(7/2))/(7*e^7) - (4*b^
5*(b*d - a*e)*(d + e*x)^(9/2))/(3*e^7) + (2*b^6*(d + e*x)^(11/2))/(11*e^7)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx+b^2x^2\right)^3}{(d+ex)^{3/2}}\,dx &= \int \frac{(a+bx)^6}{(d+ex)^{3/2}}\,dx \\
&= \int \left(\frac{(-bd+ae)^6}{e^6(d+ex)^{3/2}}-\frac{6b(bd-ae)^5}{e^6\sqrt{d+ex}}+\frac{15b^2(bd-ae)^4\sqrt{d+ex}}{e^6}-\frac{20b^3(bd-ae)^3(d+ex)^{3/2}}{e^6}+\right. \\
&= -\frac{2(bd-ae)^6}{e^7\sqrt{d+ex}}-\frac{12b(bd-ae)^5\sqrt{d+ex}}{e^7}+\frac{10b^2(bd-ae)^4(d+ex)^{3/2}}{e^7}-\frac{8b^3(bd-ae)^3(d+e}{e^7}
\end{aligned}$$

Mathematica [A] time = 0.0804628, size = 145, normalized size = 0.81

$$\frac{2\left(1155b^2(d+ex)^2(bd-ae)^4-924b^3(d+ex)^3(bd-ae)^3+495b^4(d+ex)^4(bd-ae)^2-154b^5(d+ex)^5(bd-ae)-1386b(d\right.}{231e^7\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^(3/2),x]
```

```
[Out] (2*(-231*(b*d - a*e)^6 - 1386*b*(b*d - a*e)^5*(d + e*x) + 1155*b^2*(b*d - a
*e)^4*(d + e*x)^2 - 924*b^3*(b*d - a*e)^3*(d + e*x)^3 + 495*b^4*(b*d - a*e)
^2*(d + e*x)^4 - 154*b^5*(b*d - a*e)*(d + e*x)^5 + 21*b^6*(d + e*x)^6))/(23
1*e^7*Sqrt[d + e*x])
```

Maple [B] time = 0.047, size = 377, normalized size = 2.1

$$-\frac{-42\,b^6x^6e^6-308\,x^5ab^5e^6+56\,x^5b^6de^5-990\,x^4a^2b^4e^6+440\,x^4ab^5de^5-80\,x^4b^6d^2e^4-1848\,x^3a^3b^3e^6+1584\,x^3a^2b^4de^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(3/2),x)
```

```
[Out] -2/231*(-21*b^6*e^6*x^6-154*a*b^5*e^6*x^5+28*b^6*d*e^5*x^5-495*a^2*b^4*e^6*
x^4+220*a*b^5*d*e^5*x^4-40*b^6*d^2*e^4*x^4-924*a^3*b^3*e^6*x^3+792*a^2*b^4*
d*e^5*x^3-352*a*b^5*d^2*e^4*x^3+64*b^6*d^3*e^3*x^3-1155*a^4*b^2*e^6*x^2+184
8*a^3*b^3*d*e^5*x^2-1584*a^2*b^4*d^2*e^4*x^2+704*a*b^5*d^3*e^3*x^2-128*b^6*
d^4*e^2*x^2-1386*a^5*b*e^6*x+4620*a^4*b^2*d*e^5*x-7392*a^3*b^3*d^2*e^4*x+63
36*a^2*b^4*d^3*e^3*x-2816*a*b^5*d^4*e^2*x+512*b^6*d^5*e*x+231*a^6*e^6-2772*
a^5*b*d*e^5+9240*a^4*b^2*d^2*e^4-14784*a^3*b^3*d^3*e^3+12672*a^2*b^4*d^4*e^
2-5632*a*b^5*d^5*e+1024*b^6*d^6)/(e*x+d)^(1/2)/e^7
```

Maxima [B] time = 1.01675, size = 483, normalized size = 2.7

$$2\left(\frac{21\,(ex+d)^{\frac{11}{2}}b^6-154\left(b^6d-ab^5e\right)(ex+d)^{\frac{9}{2}}+495\left(b^6d^2-2\,ab^5de+a^2b^4e^2\right)(ex+d)^{\frac{7}{2}}-924\left(b^6d^3-3\,ab^5d^2e+3\,a^2b^4de^2-a^3b^3e^3\right)(ex+d)^{\frac{5}{2}}+1155\left(b^6d^4-4\,ab^5d^3e+6\,a^2b^4d\right.}{e^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/231*((21*(e*x + d)^(11/2)*b^6 - 154*(b^6*d - a*b^5*e)*(e*x + d)^(9/2) + 4
95*(b^6*d^2 - 2*a*b^5*d*e + a^2*b^4*e^2)*(e*x + d)^(7/2) - 924*(b^6*d^3 - 3
*a*b^5*d^2*e + 3*a^2*b^4*d*e^2 - a^3*b^3*e^3)*(e*x + d)^(5/2) + 1155*(b^6*d
^4 - 4*a*b^5*d^3*e + 6*a^2*b^4*d^2*e^2 - 4*a^3*b^3*d*e^3 + a^4*b^2*e^4)*(e*
x + d)^(3/2) - 1386*(b^6*d^5 - 5*a*b^5*d^4*e + 10*a^2*b^4*d^3*e^2 - 10*a^3*
b^3*d^2*e^3 + 5*a^4*b^2*d*e^4 - a^5*b*e^5)*sqrt(e*x + d))/e^6 - 231*(b^6*d^
6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^
2*e^4 - 6*a^5*b*d*e^5 + a^6*e^6)/(sqrt(e*x + d)*e^6))/e
```

Fricas [B] time = 1.86338, size = 814, normalized size = 4.55

$$2\left(21\,b^6e^6x^6-1024\,b^6d^6+5632\,ab^5d^5e-12672\,a^2b^4d^4e^2+14784\,a^3b^3d^3e^3-9240\,a^4b^2d^2e^4+2772\,a^5bde^5-231\,a^6e^6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/231*(21*b^6*e^6*x^6 - 1024*b^6*d^6 + 5632*a*b^5*d^5*e - 12672*a^2*b^4*d^4
*e^2 + 14784*a^3*b^3*d^3*e^3 - 9240*a^4*b^2*d^2*e^4 + 2772*a^5*b*d*e^5 - 23
1*a^6*e^6 - 14*(2*b^6*d*e^5 - 11*a*b^5*e^6)*x^5 + 5*(8*b^6*d^2*e^4 - 44*a*b
^5*d*e^5 + 99*a^2*b^4*e^6)*x^4 - 4*(16*b^6*d^3*e^3 - 88*a*b^5*d^2*e^4 + 198
*a^2*b^4*d*e^5 - 231*a^3*b^3*e^6)*x^3 + (128*b^6*d^4*e^2 - 704*a*b^5*d^3*e^
3 + 1584*a^2*b^4*d^2*e^4 - 1848*a^3*b^3*d*e^5 + 1155*a^4*b^2*e^6)*x^2 - 2*(
256*b^6*d^5*e - 1408*a*b^5*d^4*e^2 + 3168*a^2*b^4*d^3*e^3 - 3696*a^3*b^3*d^
2*e^4 + 2310*a^4*b^2*d*e^5 - 693*a^5*b*e^6)*x)*sqrt(e*x + d)/(e^8*x + d*e^7
)
```

Sympy [A] time = 55.1659, size = 333, normalized size = 1.86

$$\frac{2b^6\left(d+ex\right)^{\frac{11}{2}}}{11e^7}+\frac{\left(d+ex\right)^{\frac{9}{2}}\left(12ab^5e-12b^6d\right)}{9e^7}+\frac{\left(d+ex\right)^{\frac{7}{2}}\left(30a^2b^4e^2-60ab^5de+30b^6d^2\right)}{7e^7}+\frac{\left(d+ex\right)^{\frac{5}{2}}\left(40a^3b^3e^3-120\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**(3/2),x)
```

```
[Out] 2*b**6*(d + e*x)**(11/2)/(11*e**7) + (d + e*x)**(9/2)*(12*a*b**5*e - 12*b**
6*d)/(9*e**7) + (d + e*x)**(7/2)*(30*a**2*b**4*e**2 - 60*a*b**5*d*e + 30*b*
*6*d**2)/(7*e**7) + (d + e*x)**(5/2)*(40*a**3*b**3*e**3 - 120*a**2*b**4*d*e
**2 + 120*a*b**5*d**2*e - 40*b**6*d**3)/(5*e**7) + (d + e*x)**(3/2)*(30*a**
4*b**2*e**4 - 120*a**3*b**3*d*e**3 + 180*a**2*b**4*d**2*e**2 - 120*a*b**5*d
**3*e + 30*b**6*d**4)/(3*e**7) + sqrt(d + e*x)*(12*a**5*b*e**5 - 60*a**4*b*
*2*d*e**4 + 120*a**3*b**3*d**2*e**3 - 120*a**2*b**4*d**3*e**2 + 60*a*b**5*d
**4*e - 12*b**6*d**5)/e**7 - 2*(a*e - b*d)**6/(e**7*sqrt(d + e*x))
```

Giac [B] time = 1.30341, size = 640, normalized size = 3.58

$$\frac{2}{231}\left(21\left(xe+d\right)^{\frac{11}{2}}b^6e^{70}-154\left(xe+d\right)^{\frac{9}{2}}b^6de^{70}+495\left(xe+d\right)^{\frac{7}{2}}b^6d^2e^{70}-924\left(xe+d\right)^{\frac{5}{2}}b^6d^3e^{70}+1155\left(xe+d\right)^{\frac{3}{2}}b^6d^4e^{70}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] 2/231*(21*(x*e + d)^(11/2)*b^6*e^70 - 154*(x*e + d)^(9/2)*b^6*d*e^70 + 495*
(x*e + d)^(7/2)*b^6*d^2*e^70 - 924*(x*e + d)^(5/2)*b^6*d^3*e^70 + 1155*(x*e
 + d)^(3/2)*b^6*d^4*e^70 - 1386*sqrt(x*e + d)*b^6*d^5*e^70 + 154*(x*e + d)^
(9/2)*a*b^5*e^71 - 990*(x*e + d)^(7/2)*a*b^5*d*e^71 + 2772*(x*e + d)^(5/2)*
a*b^5*d^2*e^71 - 4620*(x*e + d)^(3/2)*a*b^5*d^3*e^71 + 6930*sqrt(x*e + d)*a
*b^5*d^4*e^71 + 495*(x*e + d)^(7/2)*a^2*b^4*e^72 - 2772*(x*e + d)^(5/2)*a^2
```

```
*b^4*d*e^72 + 6930*(x*e + d)^(3/2)*a^2*b^4*d^2*e^72 - 13860*sqrt(x*e + d)*a
^2*b^4*d^3*e^72 + 924*(x*e + d)^(5/2)*a^3*b^3*e^73 - 4620*(x*e + d)^(3/2)*a
^3*b^3*d*e^73 + 13860*sqrt(x*e + d)*a^3*b^3*d^2*e^73 + 1155*(x*e + d)^(3/2)
*a^4*b^2*e^74 - 6930*sqrt(x*e + d)*a^4*b^2*d*e^74 + 1386*sqrt(x*e + d)*a^5*
b*e^75)*e^(-77) - 2*(b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*
b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 + a^6*e^6)*e^(-7)/sqrt(x*e
 + d)
```

3.1643 $\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=181

$$-\frac{12b^5(d+ex)^{7/2}(bd-ae)}{7e^7}+\frac{6b^4(d+ex)^{5/2}(bd-ae)^2}{e^7}-\frac{40b^3(d+ex)^{3/2}(bd-ae)^3}{3e^7}+\frac{30b^2\sqrt{d+ex}(bd-ae)^4}{e^7}+\frac{12b(bd-}{e^7\sqrt{d-}}$$

```
[Out] (-2*(b*d - a*e)^6)/(3*e^7*(d + e*x)^(3/2)) + (12*b*(b*d - a*e)^5)/(e^7*Sqrt
[d + e*x]) + (30*b^2*(b*d - a*e)^4*Sqrt[d + e*x])/e^7 - (40*b^3*(b*d - a*e)
^3*(d + e*x)^(3/2))/(3*e^7) + (6*b^4*(b*d - a*e)^2*(d + e*x)^(5/2))/e^7 - (
12*b^5*(b*d - a*e)*(d + e*x)^(7/2))/(7*e^7) + (2*b^6*(d + e*x)^(9/2))/(9*e^
7)
```

Rubi [A] time = 0.0592892, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {27, 43}

$$-\frac{12b^5(d+ex)^{7/2}(bd-ae)}{7e^7}+\frac{6b^4(d+ex)^{5/2}(bd-ae)^2}{e^7}-\frac{40b^3(d+ex)^{3/2}(bd-ae)^3}{3e^7}+\frac{30b^2\sqrt{d+ex}(bd-ae)^4}{e^7}+\frac{12b(bd-}{e^7\sqrt{d-}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^(5/2),x]
```

```
[Out] (-2*(b*d - a*e)^6)/(3*e^7*(d + e*x)^(3/2)) + (12*b*(b*d - a*e)^5)/(e^7*Sqrt
[d + e*x]) + (30*b^2*(b*d - a*e)^4*Sqrt[d + e*x])/e^7 - (40*b^3*(b*d - a*e)
^3*(d + e*x)^(3/2))/(3*e^7) + (6*b^4*(b*d - a*e)^2*(d + e*x)^(5/2))/e^7 - (
12*b^5*(b*d - a*e)*(d + e*x)^(7/2))/(7*e^7) + (2*b^6*(d + e*x)^(9/2))/(9*e^
7)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^{5/2}}\,dx=\int \frac{(a+bx)^6}{(d+ex)^{5/2}}\,dx$$

$$=\int \left(\frac{(-bd+ae)^6}{e^6(d+ex)^{5/2}}-\frac{6b(bd-ae)^5}{e^6(d+ex)^{3/2}}+\frac{15b^2(bd-ae)^4}{e^6\sqrt{d+ex}}-\frac{20b^3(bd-ae)^3\sqrt{d+ex}}{e^6}+15b^4(bd-\right.$$

$$=-\frac{2(bd-ae)^6}{3e^7(d+ex)^{3/2}}+\frac{12b(bd-ae)^5}{e^7\sqrt{d+ex}}+\frac{30b^2(bd-ae)^4\sqrt{d+ex}}{e^7}-\frac{40b^3(bd-ae)^3(d+ex)^{3/2}}{3e^7}+$$

Mathematica [A] time = 0.0834247, size = 145, normalized size = 0.8

$$\frac{2\left(945b^2(d+ex)^2(bd-ae)^4-420b^3(d+ex)^3(bd-ae)^3+189b^4(d+ex)^4(bd-ae)^2-54b^5(d+ex)^5(bd-ae)+378b(d+e\ldots}{63e^7(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^(5/2),x]
```

```
[Out] (2*(-21*(b*d - a*e)^6 + 378*b*(b*d - a*e)^5*(d + e*x) + 945*b^2*(b*d - a*e)
^4*(d + e*x)^2 - 420*b^3*(b*d - a*e)^3*(d + e*x)^3 + 189*b^4*(b*d - a*e)^2*
(d + e*x)^4 - 54*b^5*(b*d - a*e)*(d + e*x)^5 + 7*b^6*(d + e*x)^6))/(63*e^7*
(d + e*x)^(3/2))
```

Maple [B] time = 0.048, size = 377, normalized size = 2.1

$$-\frac{-14\,b^6x^6e^6-108\,x^5ab^5e^6+24\,x^5b^6de^5-378\,x^4a^2b^4e^6+216\,x^4ab^5de^5-48\,x^4b^6d^2e^4-840\,x^3a^3b^3e^6+1008\,x^3a^2b^4de^5-\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(5/2),x)
```

```
[Out] -2/63*(-7*b^6*e^6*x^6-54*a*b^5*e^6*x^5+12*b^6*d*e^5*x^5-189*a^2*b^4*e^6*x^4
+108*a*b^5*d*e^5*x^4-24*b^6*d^2*e^4*x^4-420*a^3*b^3*e^6*x^3+504*a^2*b^4*d*e
^5*x^3-288*a*b^5*d^2*e^4*x^3+64*b^6*d^3*e^3*x^3-945*a^4*b^2*e^6*x^2+2520*a^
3*b^3*d*e^5*x^2-3024*a^2*b^4*d^2*e^4*x^2+1728*a*b^5*d^3*e^3*x^2-384*b^6*d^4
*e^2*x^2+378*a^5*b*e^6*x-3780*a^4*b^2*d*e^5*x+10080*a^3*b^3*d^2*e^4*x-12096
*a^2*b^4*d^3*e^3*x+6912*a*b^5*d^4*e^2*x-1536*b^6*d^5*e*x+21*a^6*e^6+252*a^5
*b*d*e^5-2520*a^4*b^2*d^2*e^4+6720*a^3*b^3*d^3*e^3-8064*a^2*b^4*d^4*e^2+460
8*a*b^5*d^5*e-1024*b^6*d^6)/(e*x+d)^(3/2)/e^7
```

Maxima [B] time = 1.04811, size = 481, normalized size = 2.66

$$2\left(\frac{7\,(ex+d)^{\frac{9}{2}}b^6-54\left(b^6d-ab^5e\right)(ex+d)^{\frac{7}{2}}+189\left(b^6d^2-2\,ab^5de+a^2b^4e^2\right)(ex+d)^{\frac{5}{2}}-420\left(b^6d^3-3\,ab^5d^2e+3\,a^2b^4de^2-a^3b^3e^3\right)(ex+d)^{\frac{3}{2}}+945\left(b^6d^4-4\,ab^5d^3e+6\,a^2b^4d^2e^2-\ldots\right.}{e^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/63*((7*(e*x + d)^(9/2)*b^6 - 54*(b^6*d - a*b^5*e)*(e*x + d)^(7/2) + 189*(
b^6*d^2 - 2*a*b^5*d*e + a^2*b^4*e^2)*(e*x + d)^(5/2) - 420*(b^6*d^3 - 3*a*b
^5*d^2*e + 3*a^2*b^4*d*e^2 - a^3*b^3*e^3)*(e*x + d)^(3/2) + 945*(b^6*d^4 -
4*a*b^5*d^3*e + 6*a^2*b^4*d^2*e^2 - 4*a^3*b^3*d*e^3 + a^4*b^2*e^4)*sqrt(e*x
 + d))/e^6 - 21*(b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*
d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 + a^6*e^6 - 18*(b^6*d^5 - 5*a*
b^5*d^4*e + 10*a^2*b^4*d^3*e^2 - 10*a^3*b^3*d^2*e^3 + 5*a^4*b^2*d*e^4 - a^5
*b*e^5)*(e*x + d))/((e*x + d)^(3/2)*e^6))/e
```

Fricas [B] time = 1.81573, size = 822, normalized size = 4.54

$$\frac{2\left(7\,b^6e^6x^6+1024\,b^6d^6-4608\,ab^5d^5e+8064\,a^2b^4d^4e^2-6720\,a^3b^3d^3e^3+2520\,a^4b^2d^2e^4-252\,a^5bde^5-21\,a^6e^6-6\left(2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/63*(7*b^6*e^6*x^6 + 1024*b^6*d^6 - 4608*a*b^5*d^5*e + 8064*a^2*b^4*d^4*e^
2 - 6720*a^3*b^3*d^3*e^3 + 2520*a^4*b^2*d^2*e^4 - 252*a^5*b*d*e^5 - 21*a^6*
e^6 - 6*(2*b^6*d*e^5 - 9*a*b^5*e^6)*x^5 + 3*(8*b^6*d^2*e^4 - 36*a*b^5*d*e^5
 + 63*a^2*b^4*e^6)*x^4 - 4*(16*b^6*d^3*e^3 - 72*a*b^5*d^2*e^4 + 126*a^2*b^4
*d*e^5 - 105*a^3*b^3*e^6)*x^3 + 3*(128*b^6*d^4*e^2 - 576*a*b^5*d^3*e^3 + 10
08*a^2*b^4*d^2*e^4 - 840*a^3*b^3*d*e^5 + 315*a^4*b^2*e^6)*x^2 + 6*(256*b^6*
d^5*e - 1152*a*b^5*d^4*e^2 + 2016*a^2*b^4*d^3*e^3 - 1680*a^3*b^3*d^2*e^4 +
630*a^4*b^2*d*e^5 - 63*a^5*b*e^6)*x)*sqrt(e*x + d)/(e^9*x^2 + 2*d*e^8*x + d
^2*e^7)
```

Sympy [A] time = 66.3361, size = 270, normalized size = 1.49

$$\frac{2b^6\left(d+ex\right)^{\frac{9}{2}}}{9e^7}-\frac{12b\left(ae-bd\right)^5}{e^7\sqrt{d+ex}}+\frac{\left(d+ex\right)^{\frac{7}{2}}\left(12ab^5e-12b^6d\right)}{7e^7}+\frac{\left(d+ex\right)^{\frac{5}{2}}\left(30a^2b^4e^2-60ab^5de+30b^6d^2\right)}{5e^7}+\frac{\left(d+ex\right)^{\frac{3}{2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**(5/2),x)
```

```
[Out] 2*b**6*(d + e*x)**(9/2)/(9*e**7) - 12*b*(a*e - b*d)**5/(e**7*sqrt(d + e*x))
 + (d + e*x)**(7/2)*(12*a*b**5*e - 12*b**6*d)/(7*e**7) + (d + e*x)**(5/2)*(
30*a**2*b**4*e**2 - 60*a*b**5*d*e + 30*b**6*d**2)/(5*e**7) + (d + e*x)**(3/
2)*(40*a**3*b**3*e**3 - 120*a**2*b**4*d*e**2 + 120*a*b**5*d**2*e - 40*b**6*
d**3)/(3*e**7) + sqrt(d + e*x)*(30*a**4*b**2*e**4 - 120*a**3*b**3*d*e**3 +
180*a**2*b**4*d**2*e**2 - 120*a*b**5*d**3*e + 30*b**6*d**4)/e**7 - 2*(a*e -
 b*d)**6/(3*e**7*(d + e*x)**(3/2))
```

Giac [B] time = 1.22098, size = 624, normalized size = 3.45

$$\frac{2}{63}\left(7\left(xe+d\right)^{\frac{9}{2}}b^6e^{56}-54\left(xe+d\right)^{\frac{7}{2}}b^6de^{56}+189\left(xe+d\right)^{\frac{5}{2}}b^6d^2e^{56}-420\left(xe+d\right)^{\frac{3}{2}}b^6d^3e^{56}+945\sqrt{xe+d}b^6d^4e^{56}+54\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] 2/63*(7*(x*e + d)^(9/2)*b^6*e^56 - 54*(x*e + d)^(7/2)*b^6*d*e^56 + 189*(x*e
 + d)^(5/2)*b^6*d^2*e^56 - 420*(x*e + d)^(3/2)*b^6*d^3*e^56 + 945*sqrt(x*e
+ d)*b^6*d^4*e^56 + 54*(x*e + d)^(7/2)*a*b^5*e^57 - 378*(x*e + d)^(5/2)*a*b
^5*d*e^57 + 1260*(x*e + d)^(3/2)*a*b^5*d^2*e^57 - 3780*sqrt(x*e + d)*a*b^5*
d^3*e^57 + 189*(x*e + d)^(5/2)*a^2*b^4*e^58 - 1260*(x*e + d)^(3/2)*a^2*b^4*
d*e^58 + 5670*sqrt(x*e + d)*a^2*b^4*d^2*e^58 + 420*(x*e + d)^(3/2)*a^3*b^3*
e^59 - 3780*sqrt(x*e + d)*a^3*b^3*d*e^59 + 945*sqrt(x*e + d)*a^4*b^2*e^60)*
```

```
e^(-63) + 2/3*(18*(x*e + d)*b^6*d^5 - b^6*d^6 - 90*(x*e + d)*a*b^5*d^4*e +
6*a*b^5*d^5*e + 180*(x*e + d)*a^2*b^4*d^3*e^2 - 15*a^2*b^4*d^4*e^2 - 180*(x
*e + d)*a^3*b^3*d^2*e^3 + 20*a^3*b^3*d^3*e^3 + 90*(x*e + d)*a^4*b^2*d*e^4 -
 15*a^4*b^2*d^2*e^4 - 18*(x*e + d)*a^5*b*e^5 + 6*a^5*b*d*e^5 - a^6*e^6)*e^(
-7)/(x*e + d)^(3/2)
```

3.1644 $\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=179

$$-\frac{12b^5(d+ex)^{5/2}(bd-ae)}{5e^7}+\frac{10b^4(d+ex)^{3/2}(bd-ae)^2}{e^7}-\frac{40b^3\sqrt{d+ex}(bd-ae)^3}{e^7}-\frac{30b^2(bd-ae)^4}{e^7\sqrt{d+ex}}+\frac{4b(bd-ae)^5}{e^7(d+ex)^{3/2}}-\frac{2}{5e}$$

```
[Out] (-2*(b*d - a*e)^6)/(5*e^7*(d + e*x)^(5/2)) + (4*b*(b*d - a*e)^5)/(e^7*(d +
e*x)^(3/2)) - (30*b^2*(b*d - a*e)^4)/(e^7*Sqrt[d + e*x]) - (40*b^3*(b*d - a
*e)^3*Sqrt[d + e*x])/e^7 + (10*b^4*(b*d - a*e)^2*(d + e*x)^(3/2))/e^7 - (12
*b^5*(b*d - a*e)*(d + e*x)^(5/2))/(5*e^7) + (2*b^6*(d + e*x)^(7/2))/(7*e^7)
```

Rubi [A] time = 0.0591365, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {27, 43}

$$-\frac{12b^5(d+ex)^{5/2}(bd-ae)}{5e^7}+\frac{10b^4(d+ex)^{3/2}(bd-ae)^2}{e^7}-\frac{40b^3\sqrt{d+ex}(bd-ae)^3}{e^7}-\frac{30b^2(bd-ae)^4}{e^7\sqrt{d+ex}}+\frac{4b(bd-ae)^5}{e^7(d+ex)^{3/2}}-\frac{2}{5e}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^(7/2),x]
```

```
[Out] (-2*(b*d - a*e)^6)/(5*e^7*(d + e*x)^(5/2)) + (4*b*(b*d - a*e)^5)/(e^7*(d +
e*x)^(3/2)) - (30*b^2*(b*d - a*e)^4)/(e^7*Sqrt[d + e*x]) - (40*b^3*(b*d - a
*e)^3*Sqrt[d + e*x])/e^7 + (10*b^4*(b*d - a*e)^2*(d + e*x)^(3/2))/e^7 - (12
*b^5*(b*d - a*e)*(d + e*x)^(5/2))/(5*e^7) + (2*b^6*(d + e*x)^(7/2))/(7*e^7)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^{7/2}}\,dx &= \int \frac{(a+bx)^6}{(d+ex)^{7/2}}\,dx\\ &= \int \left(\frac{(-bd+ae)^6}{e^6(d+ex)^{7/2}}-\frac{6b(bd-ae)^5}{e^6(d+ex)^{5/2}}+\frac{15b^2(bd-ae)^4}{e^6(d+ex)^{3/2}}-\frac{20b^3(bd-ae)^3}{e^6\sqrt{d+ex}}+\frac{15b^4(bd-ae)^2\sqrt{d}}{e^6}\right.\\ &= -\frac{2(bd-ae)^6}{5e^7(d+ex)^{5/2}}+\frac{4b(bd-ae)^5}{e^7(d+ex)^{3/2}}-\frac{30b^2(bd-ae)^4}{e^7\sqrt{d+ex}}-\frac{40b^3(bd-ae)^3\sqrt{d+ex}}{e^7}+\frac{10b^4(bd-}{}\end{aligned}$$

Mathematica [A] time = 0.0814156, size = 145, normalized size = 0.81

$$\frac{2\left(-525b^2(d+ex)^2(bd-ae)^4-700b^3(d+ex)^3(bd-ae)^3+175b^4(d+ex)^4(bd-ae)^2-42b^5(d+ex)^5(bd-ae)+70b(d+e\right.}{35e^7(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^(7/2),x]
```

```
[Out] (2*(-7*(b*d - a*e)^6 + 70*b*(b*d - a*e)^5*(d + e*x) - 525*b^2*(b*d - a*e)^4
*(d + e*x)^2 - 700*b^3*(b*d - a*e)^3*(d + e*x)^3 + 175*b^4*(b*d - a*e)^2*(d
 + e*x)^4 - 42*b^5*(b*d - a*e)*(d + e*x)^5 + 5*b^6*(d + e*x)^6))/(35*e^7*(d
 + e*x)^(5/2))
```

Maple [B] time = 0.048, size = 377, normalized size = 2.1

$$-\frac{-10\,b^6x^6e^6-84\,x^5ab^5e^6+24\,x^5b^6de^5-350\,x^4a^2b^4e^6+280\,x^4ab^5de^5-80\,x^4b^6d^2e^4-1400\,x^3a^3b^3e^6+2800\,x^3a^2b^4de^5-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(7/2),x)
```

```
[Out] -2/35*(-5*b^6*e^6*x^6-42*a*b^5*e^6*x^5+12*b^6*d*e^5*x^5-175*a^2*b^4*e^6*x^4
+140*a*b^5*d*e^5*x^4-40*b^6*d^2*e^4*x^4-700*a^3*b^3*e^6*x^3+1400*a^2*b^4*d*
e^5*x^3-1120*a*b^5*d^2*e^4*x^3+320*b^6*d^3*e^3*x^3+525*a^4*b^2*e^6*x^2-4200
*a^3*b^3*d*e^5*x^2+8400*a^2*b^4*d^2*e^4*x^2-6720*a*b^5*d^3*e^3*x^2+1920*b^6
*d^4*e^2*x^2+70*a^5*b*e^6*x+700*a^4*b^2*d*e^5*x-5600*a^3*b^3*d^2*e^4*x+1120
0*a^2*b^4*d^3*e^3*x-8960*a*b^5*d^4*e^2*x+2560*b^6*d^5*e*x+7*a^6*e^6+28*a^5*
b*d*e^5+280*a^4*b^2*d^2*e^4-2240*a^3*b^3*d^3*e^3+4480*a^2*b^4*d^4*e^2-3584*
a*b^5*d^5*e+1024*b^6*d^6)/(e*x+d)^(5/2)/e^7
```

Maxima [B] time = 1.17603, size = 481, normalized size = 2.69

$$2\left(\frac{5\,(ex+d)^{\frac{7}{2}}b^6-42\left(b^6d-ab^5e\right)(ex+d)^{\frac{5}{2}}+175\left(b^6d^2-2\,ab^5de+a^2b^4e^2\right)(ex+d)^{\frac{3}{2}}-700\left(b^6d^3-3\,ab^5d^2e+3\,a^2b^4de^2-a^3b^3e^3\right)\sqrt{ex+d}}{e^6}-\frac{7\left(b^6d^6-6\,ab^5d^5e+15\,a^2b^4d^4e^2-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/35*((5*(e*x + d)^(7/2)*b^6 - 42*(b^6*d - a*b^5*e)*(e*x + d)^(5/2) + 175*(
b^6*d^2 - 2*a*b^5*d*e + a^2*b^4*e^2)*(e*x + d)^(3/2) - 700*(b^6*d^3 - 3*a*b
^5*d^2*e + 3*a^2*b^4*d*e^2 - a^3*b^3*e^3)*sqrt(e*x + d))/e^6 - 7*(b^6*d^6 -
 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e
^4 - 6*a^5*b*d*e^5 + a^6*e^6 + 75*(b^6*d^4 - 4*a*b^5*d^3*e + 6*a^2*b^4*d^2*
e^2 - 4*a^3*b^3*d*e^3 + a^4*b^2*e^4)*(e*x + d)^2 - 10*(b^6*d^5 - 5*a*b^5*d^
4*e + 10*a^2*b^4*d^3*e^2 - 10*a^3*b^3*d^2*e^3 + 5*a^4*b^2*d*e^4 - a^5*b*e^5
)*(e*x + d))/((e*x + d)^(5/2)*e^6))/e
```

Fricas [B] time = 1.84991, size = 834, normalized size = 4.66

$$\frac{2\left(5\,b^6e^6x^6-1024\,b^6d^6+3584\,ab^5d^5e-4480\,a^2b^4d^4e^2+2240\,a^3b^3d^3e^3-280\,a^4b^2d^2e^4-28\,a^5bde^5-7\,a^6e^6-6\left(2\,b^6d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^6*e^6*x^6 - 1024*b^6*d^6 + 3584*a*b^5*d^5*e - 4480*a^2*b^4*d^4*e^
2 + 2240*a^3*b^3*d^3*e^3 - 280*a^4*b^2*d^2*e^4 - 28*a^5*b*d*e^5 - 7*a^6*e^6
 - 6*(2*b^6*d*e^5 - 7*a*b^5*e^6)*x^5 + 5*(8*b^6*d^2*e^4 - 28*a*b^5*d*e^5 +
35*a^2*b^4*e^6)*x^4 - 20*(16*b^6*d^3*e^3 - 56*a*b^5*d^2*e^4 + 70*a^2*b^4*d*
e^5 - 35*a^3*b^3*e^6)*x^3 - 15*(128*b^6*d^4*e^2 - 448*a*b^5*d^3*e^3 + 560*a
^2*b^4*d^2*e^4 - 280*a^3*b^3*d*e^5 + 35*a^4*b^2*e^6)*x^2 - 10*(256*b^6*d^5*
e - 896*a*b^5*d^4*e^2 + 1120*a^2*b^4*d^3*e^3 - 560*a^3*b^3*d^2*e^4 + 70*a^4
*b^2*d*e^5 + 7*a^5*b*e^6)*x)*sqrt(e*x + d)/(e^10*x^3 + 3*d*e^9*x^2 + 3*d^2*
e^8*x + d^3*e^7)
```

Sympy [A] time = 123.569, size = 221, normalized size = 1.23

$$\frac{2b^6\left(d+ex\right)^{\frac{7}{2}}}{7e^7}-\frac{30b^2\left(ae-bd\right)^4}{e^7\sqrt{d+ex}}-\frac{4b\left(ae-bd\right)^5}{e^7\left(d+ex\right)^{\frac{3}{2}}}+\frac{\left(d+ex\right)^{\frac{5}{2}}\left(12ab^5e-12b^6d\right)}{5e^7}+\frac{\left(d+ex\right)^{\frac{3}{2}}\left(30a^2b^4e^2-60ab^5de+30b^6\right.}{3e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**(7/2),x)
```

```
[Out] 2*b**6*(d + e*x)**(7/2)/(7*e**7) - 30*b**2*(a*e - b*d)**4/(e**7*sqrt(d + e*
x)) - 4*b*(a*e - b*d)**5/(e**7*(d + e*x)**(3/2)) + (d + e*x)**(5/2)*(12*a*b
**5*e - 12*b**6*d)/(5*e**7) + (d + e*x)**(3/2)*(30*a**2*b**4*e**2 - 60*a*b*
*5*d*e + 30*b**6*d**2)/(3*e**7) + sqrt(d + e*x)*(40*a**3*b**3*e**3 - 120*a*
*2*b**4*d*e**2 + 120*a*b**5*d**2*e - 40*b**6*d**3)/e**7 - 2*(a*e - b*d)**6/
(5*e**7*(d + e*x)**(5/2))
```

Giac [B] time = 1.23942, size = 618, normalized size = 3.45

$$\frac{2}{35}\left(5\left(xe+d\right)^{\frac{7}{2}}b^6e^{42}-42\left(xe+d\right)^{\frac{5}{2}}b^6de^{42}+175\left(xe+d\right)^{\frac{3}{2}}b^6d^2e^{42}-700\sqrt{xe+d}b^6d^3e^{42}+42\left(xe+d\right)^{\frac{5}{2}}ab^5e^{43}-350\left(xe\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] 2/35*(5*(x*e + d)^(7/2)*b^6*e^42 - 42*(x*e + d)^(5/2)*b^6*d*e^42 + 175*(x*e
 + d)^(3/2)*b^6*d^2*e^42 - 700*sqrt(x*e + d)*b^6*d^3*e^42 + 42*(x*e + d)^(5
/2)*a*b^5*e^43 - 350*(x*e + d)^(3/2)*a*b^5*d*e^43 + 2100*sqrt(x*e + d)*a*b^
5*d^2*e^43 + 175*(x*e + d)^(3/2)*a^2*b^4*e^44 - 2100*sqrt(x*e + d)*a^2*b^4*
d*e^44 + 700*sqrt(x*e + d)*a^3*b^3*e^45)*e^(-49) - 2/5*(75*(x*e + d)^2*b^6*
d^4 - 10*(x*e + d)*b^6*d^5 + b^6*d^6 - 300*(x*e + d)^2*a*b^5*d^3*e + 50*(x*
e + d)*a*b^5*d^4*e - 6*a*b^5*d^5*e + 450*(x*e + d)^2*a^2*b^4*d^2*e^2 - 100*
(x*e + d)*a^2*b^4*d^3*e^2 + 15*a^2*b^4*d^4*e^2 - 300*(x*e + d)^2*a^3*b^3*d*
```

```
e^3 + 100*(x*e + d)*a^3*b^3*d^2*e^3 - 20*a^3*b^3*d^3*e^3 + 75*(x*e + d)^2*a
^4*b^2*e^4 - 50*(x*e + d)*a^4*b^2*d*e^4 + 15*a^4*b^2*d^2*e^4 + 10*(x*e + d)
*a^5*b*e^5 - 6*a^5*b*d*e^5 + a^6*e^6)*e^(-7)/(x*e + d)^(5/2)
```

3.1645 $\int \frac{(d+ex)^{9/2}}{a^2+2abx+b^2x^2} dx$

Optimal. Leaf size=162

$$\frac{9e(d+ex)^{5/2}(bd-ae)}{5b^3}+\frac{3e(d+ex)^{3/2}(bd-ae)^2}{b^4}+\frac{9e\sqrt{d+ex}(bd-ae)^3}{b^5}-\frac{9e(bd-ae)^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{11/2}}-\frac{(d+ex)^9}{b(a+bx}$$

```
[Out] (9*e*(b*d - a*e)^3*Sqrt[d + e*x])/b^5 + (3*e*(b*d - a*e)^2*(d + e*x)^(3/2))
/b^4 + (9*e*(b*d - a*e)*(d + e*x)^(5/2))/(5*b^3) + (9*e*(d + e*x)^(7/2))/(7
*b^2) - (d + e*x)^(9/2)/(b*(a + b*x)) - (9*e*(b*d - a*e)^(7/2)*ArcTanh[(Sqr
t[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/b^(11/2)
```

Rubi [A] time = 0.182752, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {27, 47, 50, 63, 208}

$$\frac{9e(d+ex)^{5/2}(bd-ae)}{5b^3}+\frac{3e(d+ex)^{3/2}(bd-ae)^2}{b^4}+\frac{9e\sqrt{d+ex}(bd-ae)^3}{b^5}-\frac{9e(bd-ae)^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{11/2}}-\frac{(d+ex)^9}{b(a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(9/2)/(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] (9*e*(b*d - a*e)^3*Sqrt[d + e*x])/b^5 + (3*e*(b*d - a*e)^2*(d + e*x)^(3/2))
/b^4 + (9*e*(b*d - a*e)*(d + e*x)^(5/2))/(5*b^3) + (9*e*(d + e*x)^(7/2))/(7
*b^2) - (d + e*x)^(9/2)/(b*(a + b*x)) - (9*e*(b*d - a*e)^(7/2)*ArcTanh[(Sqr
t[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/b^(11/2)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{9/2}}{a^2+2abx+b^2x^2}\,dx = \int \frac{(d+ex)^{9/2}}{(a+bx)^2}\,dx$$

$$= -\frac{(d+ex)^{9/2}}{b(a+bx)} + \frac{(9e)\int \frac{(d+ex)^{7/2}}{a+bx}\,dx}{2b}$$

$$= \frac{9e(d+ex)^{7/2}}{7b^2} - \frac{(d+ex)^{9/2}}{b(a+bx)} + \frac{(9e(bd-ae))\int \frac{(d+ex)^{5/2}}{a+bx}\,dx}{2b^2}$$

$$= \frac{9e(bd-ae)(d+ex)^{5/2}}{5b^3} + \frac{9e(d+ex)^{7/2}}{7b^2} - \frac{(d+ex)^{9/2}}{b(a+bx)} + \frac{\left(9e(bd-ae)^2\right)\int \frac{(d+ex)^{3/2}}{a+bx}\,dx}{2b^3}$$

$$= \frac{3e(bd-ae)^2(d+ex)^{3/2}}{b^4} + \frac{9e(bd-ae)(d+ex)^{5/2}}{5b^3} + \frac{9e(d+ex)^{7/2}}{7b^2} - \frac{(d+ex)^{9/2}}{b(a+bx)} + \frac{\left(9e(bd-ae)^3\right)}{2b^4}$$

$$= \frac{9e(bd-ae)^3\sqrt{d+ex}}{b^5} + \frac{3e(bd-ae)^2(d+ex)^{3/2}}{b^4} + \frac{9e(bd-ae)(d+ex)^{5/2}}{5b^3} + \frac{9e(d+ex)^{7/2}}{7b^2} - \frac{(d+}{b(a}$$

$$= \frac{9e(bd-ae)^3\sqrt{d+ex}}{b^5} + \frac{3e(bd-ae)^2(d+ex)^{3/2}}{b^4} + \frac{9e(bd-ae)(d+ex)^{5/2}}{5b^3} + \frac{9e(d+ex)^{7/2}}{7b^2} - \frac{(d+}{b(a}$$

$$= \frac{9e(bd-ae)^3\sqrt{d+ex}}{b^5} + \frac{3e(bd-ae)^2(d+ex)^{3/2}}{b^4} + \frac{9e(bd-ae)(d+ex)^{5/2}}{5b^3} + \frac{9e(d+ex)^{7/2}}{7b^2} - \frac{(d+}{b(a}$$

Mathematica [C] time = 0.024523, size = 50, normalized size = 0.31

$$\frac{2e(d+ex)^{11/2}\,{}_2F_1\left(2,\frac{11}{2};\frac{13}{2};-\frac{b(d+ex)}{ae-bd}\right)}{11(ae-bd)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(9/2)/(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] (2*e*(d + e*x)^(11/2)*Hypergeometric2F1[2, 11/2, 13/2, -((b*(d + e*x))/(-(b
*d) + a*e))])/(11*(-(b*d) + a*e)^2)
```

Maple [B] time = 0.213, size = 539, normalized size = 3.3

$$\frac{2e}{7b^2}(ex+d)^{\frac{7}{2}} - \frac{4ae^2}{5b^3}(ex+d)^{\frac{5}{2}} + \frac{4de}{5b^2}(ex+d)^{\frac{5}{2}} + 2\frac{(ex+d)^{3/2}a^2e^3}{b^4} - 4\frac{(ex+d)^{3/2}ade^2}{b^3} + 2\frac{e(ex+d)^{3/2}d^2}{b^2} - 8\frac{a^3e^4\sqrt{ex}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2),x)
```

```
[Out] 2/7*e*(e*x+d)^(7/2)/b^2-4/5/b^3*(e*x+d)^(5/2)*a*e^2+4/5*e/b^2*(e*x+d)^(5/2)
*d+2/b^4*(e*x+d)^(3/2)*a^2*e^3-4/b^3*(e*x+d)^(3/2)*a*d*e^2+2*e/b^2*(e*x+d)^
(3/2)*d^2-8/b^5*a^3*e^4*(e*x+d)^(1/2)+24/b^4*a^2*d*e^3*(e*x+d)^(1/2)-24/b^3
*a*d^2*e^2*(e*x+d)^(1/2)+8*e/b^2*d^3*(e*x+d)^(1/2)-1/b^5*(e*x+d)^(1/2)/(b*e
*x+a*e)*a^4*e^5+4/b^4*(e*x+d)^(1/2)/(b*e*x+a*e)*a^3*d*e^4-6/b^3*(e*x+d)^(1/
2)/(b*e*x+a*e)*d^2*e^3*a^2+4/b^2*(e*x+d)^(1/2)/(b*e*x+a*e)*a*d^3*e^2-e/b*(e
*x+d)^(1/2)/(b*e*x+a*e)*d^4+9/b^5/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2
)/((a*e-b*d)*b)^(1/2))*a^4*e^5-36/b^4/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^
(1/2)/((a*e-b*d)*b)^(1/2))*a^3*d*e^4+54/b^3/((a*e-b*d)*b)^(1/2)*arctan(b*(e
*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*d^2*e^3*a^2-36/b^2/((a*e-b*d)*b)^(1/2)*arc
tan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a*d^3*e^2+9*e/b/((a*e-b*d)*b)^(1/2
)*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*d^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.0296, size = 1450, normalized size = 8.95

$$\left[-\frac{315\left(ab^3d^3e - 3a^2b^2d^2e^2 + 3a^3bde^3 - a^4e^4 + \left(b^4d^3e - 3ab^3d^2e^2 + 3a^2b^2de^3 - a^3be^4\right)x\right)\sqrt{\frac{bd-ae}{b}}\log\left(\frac{bex+2bd-ae+2\sqrt{ex+}}{bx+a}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

```
[Out] [-1/70*(315*(a*b^3*d^3*e - 3*a^2*b^2*d^2*e^2 + 3*a^3*b*d*e^3 - a^4*e^4 + (b
^4*d^3*e - 3*a*b^3*d^2*e^2 + 3*a^2*b^2*d*e^3 - a^3*b*e^4)*x)*sqrt((b*d - a*
e)/b)*log((b*e*x + 2*b*d - a*e + 2*sqrt(e*x + d)*b*sqrt((b*d - a*e)/b))/(b*
x + a)) - 2*(10*b^4*e^4*x^4 - 35*b^4*d^4 + 528*a*b^3*d^3*e - 1218*a^2*b^2*d
^2*e^2 + 1050*a^3*b*d*e^3 - 315*a^4*e^4 + 2*(29*b^4*d*e^3 - 9*a*b^3*e^4)*x^
3 + 6*(26*b^4*d^2*e^2 - 23*a*b^3*d*e^3 + 7*a^2*b^2*e^4)*x^2 + 2*(194*b^4*d^
3*e - 426*a*b^3*d^2*e^2 + 357*a^2*b^2*d*e^3 - 105*a^3*b*e^4)*x)*sqrt(e*x +
d))/(b^6*x + a*b^5), -1/35*(315*(a*b^3*d^3*e - 3*a^2*b^2*d^2*e^2 + 3*a^3*b*
d*e^3 - a^4*e^4 + (b^4*d^3*e - 3*a*b^3*d^2*e^2 + 3*a^2*b^2*d*e^3 - a^3*b*e^
4)*x)*sqrt(-(b*d - a*e)/b)*arctan(-sqrt(e*x + d)*b*sqrt(-(b*d - a*e)/b)/(b*
d - a*e)) - (10*b^4*e^4*x^4 - 35*b^4*d^4 + 528*a*b^3*d^3*e - 1218*a^2*b^2*d
^2*e^2 + 1050*a^3*b*d*e^3 - 315*a^4*e^4 + 2*(29*b^4*d*e^3 - 9*a*b^3*e^4)*x^
3 + 6*(26*b^4*d^2*e^2 - 23*a*b^3*d*e^3 + 7*a^2*b^2*e^4)*x^2 + 2*(194*b^4*d^
3*e - 426*a*b^3*d^2*e^2 + 357*a^2*b^2*d*e^3 - 105*a^3*b*e^4)*x)*sqrt(e*x +
d))/(b^6*x + a*b^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(9/2)/(b**2*x**2+2*a*b*x+a**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26353, size = 522, normalized size = 3.22

$$\frac{9\left(b^4d^4e-4\,ab^3d^3e^2+6\,a^2b^2d^2e^3-4\,a^3bde^4+a^4e^5\right)\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)}{\sqrt{-b^2d+abe}b^5}-\frac{\sqrt{xe+d}b^4d^4e-4\sqrt{xe+d}ab^3d^3e^2+6\sqrt{xe+d}}{((xe+d)b-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

```
[Out] 9*(b^4*d^4*e - 4*a*b^3*d^3*e^2 + 6*a^2*b^2*d^2*e^3 - 4*a^3*b*d*e^4 + a^4*e^
5)*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^5)
- (sqrt(x*e + d)*b^4*d^4*e - 4*sqrt(x*e + d)*a*b^3*d^3*e^2 + 6*sqrt(x*e + d
)*a^2*b^2*d^2*e^3 - 4*sqrt(x*e + d)*a^3*b*d*e^4 + sqrt(x*e + d)*a^4*e^5)/((
(x*e + d)*b - b*d + a*e)*b^5) + 2/35*(5*(x*e + d)^(7/2)*b^12*e + 14*(x*e +
d)^(5/2)*b^12*d*e + 35*(x*e + d)^(3/2)*b^12*d^2*e + 140*sqrt(x*e + d)*b^12*
d^3*e - 14*(x*e + d)^(5/2)*a*b^11*e^2 - 70*(x*e + d)^(3/2)*a*b^11*d*e^2 - 4
20*sqrt(x*e + d)*a*b^11*d^2*e^2 + 35*(x*e + d)^(3/2)*a^2*b^10*e^3 + 420*sqr
t(x*e + d)*a^2*b^10*d*e^3 - 140*sqrt(x*e + d)*a^3*b^9*e^4)/b^14
```

3.1646 $\int \frac{(d+ex)^{7/2}}{a^2+2abx+b^2x^2} dx$

Optimal. Leaf size=137

$$\frac{7e(d+ex)^{3/2}(bd-ae)}{3b^3}+\frac{7e\sqrt{d+ex}(bd-ae)^2}{b^4}-\frac{7e(bd-ae)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{9/2}}-\frac{(d+ex)^{7/2}}{b(a+bx)}+\frac{7e(d+ex)^{5/2}}{5b^2}$$

```
[Out] (7*e*(b*d - a*e)^2*Sqrt[d + e*x])/b^4 + (7*e*(b*d - a*e)*(d + e*x)^(3/2))/(
3*b^3) + (7*e*(d + e*x)^(5/2))/(5*b^2) - (d + e*x)^(7/2)/(b*(a + b*x)) - (7
*e*(b*d - a*e)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/b^(9
/2)
```

Rubi [A] time = 0.0757445, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {27, 47, 50, 63, 208}

$$\frac{7e(d+ex)^{3/2}(bd-ae)}{3b^3}+\frac{7e\sqrt{d+ex}(bd-ae)^2}{b^4}-\frac{7e(bd-ae)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{9/2}}-\frac{(d+ex)^{7/2}}{b(a+bx)}+\frac{7e(d+ex)^{5/2}}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] (7*e*(b*d - a*e)^2*Sqrt[d + e*x])/b^4 + (7*e*(b*d - a*e)*(d + e*x)^(3/2))/(
3*b^3) + (7*e*(d + e*x)^(5/2))/(5*b^2) - (d + e*x)^(7/2)/(b*(a + b*x)) - (7
*e*(b*d - a*e)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/b^(9
/2)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{7/2}}{a^2+2abx+b^2x^2}\,dx = \int \frac{(d+ex)^{7/2}}{(a+bx)^2}\,dx$$
$$= -\frac{(d+ex)^{7/2}}{b(a+bx)} + \frac{(7e)\int \frac{(d+ex)^{5/2}}{a+bx}\,dx}{2b}$$
$$= \frac{7e(d+ex)^{5/2}}{5b^2} - \frac{(d+ex)^{7/2}}{b(a+bx)} + \frac{(7e(bd-ae))\int \frac{(d+ex)^{3/2}}{a+bx}\,dx}{2b^2}$$
$$= \frac{7e(bd-ae)(d+ex)^{3/2}}{3b^3} + \frac{7e(d+ex)^{5/2}}{5b^2} - \frac{(d+ex)^{7/2}}{b(a+bx)} + \frac{\left(7e(bd-ae)^2\right)\int \frac{\sqrt{d+ex}}{a+bx}\,dx}{2b^3}$$
$$= \frac{7e(bd-ae)^2\sqrt{d+ex}}{b^4} + \frac{7e(bd-ae)(d+ex)^{3/2}}{3b^3} + \frac{7e(d+ex)^{5/2}}{5b^2} - \frac{(d+ex)^{7/2}}{b(a+bx)} + \frac{\left(7e(bd-ae)^3\right)\int}{2b^4}$$
$$= \frac{7e(bd-ae)^2\sqrt{d+ex}}{b^4} + \frac{7e(bd-ae)(d+ex)^{3/2}}{3b^3} + \frac{7e(d+ex)^{5/2}}{5b^2} - \frac{(d+ex)^{7/2}}{b(a+bx)} + \frac{\left(7(bd-ae)^3\right)\mathrm{Su}}{}$$
$$= \frac{7e(bd-ae)^2\sqrt{d+ex}}{b^4} + \frac{7e(bd-ae)(d+ex)^{3/2}}{3b^3} + \frac{7e(d+ex)^{5/2}}{5b^2} - \frac{(d+ex)^{7/2}}{b(a+bx)} - \frac{7e(bd-ae)^{5/2}\tan}{b^{9/}}$$

Mathematica [C] time = 0.0173275, size = 50, normalized size = 0.36

$$\frac{2e(d+ex)^{9/2}\,{}_2F_1\left(2,\frac{9}{2};\frac{11}{2};-\frac{b(d+ex)}{ae-bd}\right)}{9(ae-bd)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)/(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] (2*e*(d + e*x)^(9/2)*Hypergeometric2F1[2, 9/2, 11/2, -((b*(d + e*x))/(-(b*d
) + a*e))])/(9*(-(b*d) + a*e)^2)
```

Maple [B] time = 0.202, size = 387, normalized size = 2.8

$$\frac{2e}{5b^2}(ex+d)^{\frac{5}{2}} - \frac{4ae^2}{3b^3}(ex+d)^{\frac{3}{2}} + \frac{4de}{3b^2}(ex+d)^{\frac{3}{2}} + 6\frac{a^2e^3\sqrt{ex+d}}{b^4} - 12\frac{ade^2\sqrt{ex+d}}{b^3} + 6\frac{ed^2\sqrt{ex+d}}{b^2} + \frac{a^3e^4}{b^4(bxe+ae)}\sqrt{ex}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2),x)
```

```
[Out] 2/5*e*(e*x+d)^(5/2)/b^2-4/3/b^3*(e*x+d)^(3/2)*a*e^2+4/3*e/b^2*(e*x+d)^(3/2)
*d+6/b^4*a^2*e^3*(e*x+d)^(1/2)-12/b^3*a*d*e^2*(e*x+d)^(1/2)+6*e/b^2*d^2*(e*
x+d)^(1/2)+1/b^4*(e*x+d)^(1/2)/(b*e*x+a*e)*a^3*e^4-3/b^3*(e*x+d)^(1/2)/(b*e
*x+a*e)*d*e^3*a^2+3/b^2*(e*x+d)^(1/2)/(b*e*x+a*e)*a*d^2*e^2-e/b*(e*x+d)^(1/
2)/(b*e*x+a*e)*d^3-7/b^4/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2)/((a*e-b
*d)*b)^(1/2))*a^3*e^4+21/b^3/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2)/((a
*e-b*d)*b)^(1/2))*d*e^3*a^2-21/b^2/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/
2)/((a*e-b*d)*b)^(1/2))*a*d^2*e^2+7*e/b/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d
)^(1/2)/((a*e-b*d)*b)^(1/2))*d^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.92009, size = 1049, normalized size = 7.66

$$\left[\frac{105\left(ab^2d^2e-2a^2bde^2+a^3e^3+\left(b^3d^2e-2ab^2de^2+a^2be^3\right)x\right)\sqrt{\frac{bd-ae}{b}}\log\left(\frac{bex+2bd-ae-2\sqrt{ex+d}b\sqrt{\frac{bd-ae}{b}}}{bx+a}\right)+2\left(6b^3e^3x^3-15\cdots\right.}{30\left(b^5x\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

```
[Out] [1/30*(105*(a*b^2*d^2*e - 2*a^2*b*d*e^2 + a^3*e^3 + (b^3*d^2*e - 2*a*b^2*d*
e^2 + a^2*b*e^3)*x)*sqrt((b*d - a*e)/b)*log((b*e*x + 2*b*d - a*e - 2*sqrt(e
*x + d)*b*sqrt((b*d - a*e)/b))/(b*x + a)) + 2*(6*b^3*e^3*x^3 - 15*b^3*d^3 +
 161*a*b^2*d^2*e - 245*a^2*b*d*e^2 + 105*a^3*e^3 + 2*(16*b^3*d*e^2 - 7*a*b^
2*e^3)*x^2 + 2*(58*b^3*d^2*e - 84*a*b^2*d*e^2 + 35*a^2*b*e^3)*x)*sqrt(e*x +
 d))/(b^5*x + a*b^4), -1/15*(105*(a*b^2*d^2*e - 2*a^2*b*d*e^2 + a^3*e^3 + (
b^3*d^2*e - 2*a*b^2*d*e^2 + a^2*b*e^3)*x)*sqrt(-(b*d - a*e)/b)*arctan(-sqrt
(e*x + d)*b*sqrt(-(b*d - a*e)/b)/(b*d - a*e)) - (6*b^3*e^3*x^3 - 15*b^3*d^3
 + 161*a*b^2*d^2*e - 245*a^2*b*d*e^2 + 105*a^3*e^3 + 2*(16*b^3*d*e^2 - 7*a*
b^2*e^3)*x^2 + 2*(58*b^3*d^2*e - 84*a*b^2*d*e^2 + 35*a^2*b*e^3)*x)*sqrt(e*x
 + d))/(b^5*x + a*b^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/(b**2*x**2+2*a*b*x+a**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2795, size = 379, normalized size = 2.77

$$\frac{7\left(b^3d^3e-3\,ab^2d^2e^2+3\,a^2bde^3-a^3e^4\right)\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)}{\sqrt{-b^2d+abe}b^4}-\frac{\sqrt{xe+d}b^3d^3e-3\,\sqrt{xe+d}ab^2d^2e^2+3\,\sqrt{xe+d}a^2bde^3-\sqrt{xe}}{((xe+d)b-bd+ae)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

```
[Out] 7*(b^3*d^3*e - 3*a*b^2*d^2*e^2 + 3*a^2*b*d*e^3 - a^3*e^4)*arctan(sqrt(x*e +
 d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^4) - (sqrt(x*e + d)*b^3
*d^3*e - 3*sqrt(x*e + d)*a*b^2*d^2*e^2 + 3*sqrt(x*e + d)*a^2*b*d*e^3 - sqrt
(x*e + d)*a^3*e^4)/(((x*e + d)*b - b*d + a*e)*b^4) + 2/15*(3*(x*e + d)^(5/2
)*b^8*e + 10*(x*e + d)^(3/2)*b^8*d*e + 45*sqrt(x*e + d)*b^8*d^2*e - 10*(x*e
 + d)^(3/2)*a*b^7*e^2 - 90*sqrt(x*e + d)*a*b^7*d*e^2 + 45*sqrt(x*e + d)*a^2
*b^6*e^3)/b^10
```

3.1647 $\int \frac{(d+ex)^{5/2}}{a^2+2abx+b^2x^2}\,dx$

Optimal. Leaf size=110

$$\frac{5e\sqrt{d+ex}(bd-ae)}{b^3}-\frac{5e(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{7/2}}-\frac{(d+ex)^{5/2}}{b(a+bx)}+\frac{5e(d+ex)^{3/2}}{3b^2}$$

```
[Out] (5*e*(b*d - a*e)*Sqrt[d + e*x])/b^3 + (5*e*(d + e*x)^(3/2))/(3*b^2) - (d +
e*x)^(5/2)/(b*(a + b*x)) - (5*e*(b*d - a*e)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[d +
 e*x])/Sqrt[b*d - a*e]])/b^(7/2)
```

Rubi [A] time = 0.0575072, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {27, 47, 50, 63, 208}

$$\frac{5e\sqrt{d+ex}(bd-ae)}{b^3}-\frac{5e(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{7/2}}-\frac{(d+ex)^{5/2}}{b(a+bx)}+\frac{5e(d+ex)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] (5*e*(b*d - a*e)*Sqrt[d + e*x])/b^3 + (5*e*(d + e*x)^(3/2))/(3*b^2) - (d +
e*x)^(5/2)/(b*(a + b*x)) - (5*e*(b*d - a*e)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[d +
 e*x])/Sqrt[b*d - a*e]])/b^(7/2)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^{5/2}}{a^2+2abx+b^2x^2}\,dx &= \int \frac{(d+ex)^{5/2}}{(a+bx)^2}\,dx\\
&= -\frac{(d+ex)^{5/2}}{b(a+bx)}+\frac{(5e)\int \frac{(d+ex)^{3/2}}{a+bx}\,dx}{2b}\\
&= \frac{5e(d+ex)^{3/2}}{3b^2}-\frac{(d+ex)^{5/2}}{b(a+bx)}+\frac{(5e(bd-ae))\int \frac{\sqrt{d+ex}}{a+bx}\,dx}{2b^2}\\
&= \frac{5e(bd-ae)\sqrt{d+ex}}{b^3}+\frac{5e(d+ex)^{3/2}}{3b^2}-\frac{(d+ex)^{5/2}}{b(a+bx)}+\frac{\left(5e(bd-ae)^2\right)\int \frac{1}{(a+bx)\sqrt{d+ex}}\,dx}{2b^3}\\
&= \frac{5e(bd-ae)\sqrt{d+ex}}{b^3}+\frac{5e(d+ex)^{3/2}}{3b^2}-\frac{(d+ex)^{5/2}}{b(a+bx)}+\frac{\left(5(bd-ae)^2\right)\mathrm{Subst}\left(\int \frac{1}{a-\frac{bd}{e}+\frac{bx^2}{e}}\,dx,x,\sqrt{d}\right.}{b^3}\\
&= \frac{5e(bd-ae)\sqrt{d+ex}}{b^3}+\frac{5e(d+ex)^{3/2}}{3b^2}-\frac{(d+ex)^{5/2}}{b(a+bx)}-\frac{5e(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.0166566, size = 50, normalized size = 0.45

$$\frac{2e(d+ex)^{7/2}\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};-\frac{b(d+ex)}{ae-bd}\right)}{7(ae-bd)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] (2*e*(d + e*x)^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, -((b*(d + e*x))/(-(b*d)
 + a*e))])/(7*(-(b*d) + a*e)^2)
```

Maple [B] time = 0.202, size = 258, normalized size = 2.4

$$\frac{2e}{3b^2}(ex+d)^{\frac{3}{2}}-4\frac{ae^2\sqrt{ex+d}}{b^3}+4\frac{e\sqrt{ex+d}d}{b^2}-\frac{a^2e^3}{b^3(bxe+ae)}\sqrt{ex+d}+2\frac{\sqrt{ex+d}ade^2}{b^2(bxe+ae)}-\frac{ed^2}{b(bxe+ae)}\sqrt{ex+d}+5\frac{a}{b^3\sqrt{(a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2),x)
```

```
[Out] 2/3*e*(e*x+d)^(3/2)/b^2-4/b^3*a*e^2*(e*x+d)^(1/2)+4*e/b^2*(e*x+d)^(1/2)*d-1
/b^3*(e*x+d)^(1/2)/(b*e*x+a*e)*a^2*e^3+2/b^2*(e*x+d)^(1/2)/(b*e*x+a*e)*a*d*
e^2-e/b*(e*x+d)^(1/2)/(b*e*x+a*e)*d^2+5/b^3/((a*e-b*d)*b)^(1/2)*arctan(b*(e
*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a^2*e^3-10/b^2/((a*e-b*d)*b)^(1/2)*arctan(
```

```
b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a*d*e^2+5*e/b/((a*e-b*d)*b)^(1/2)*arct
an(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.84785, size = 707, normalized size = 6.43

$$\left[-\frac{15\left(abde-a^2e^2+\left(b^2de-abe^2\right)x\right)\sqrt{\frac{bd-ae}{b}}\log\left(\frac{bex+2\,bd-ae+2\,\sqrt{ex+d}b\sqrt{\frac{bd-ae}{b}}}{bx+a}\right)-2\left(2\,b^2e^2x^2-3\,b^2d^2+20\,abde-15\,a^2e^2+\right.}{6\left(b^4x+ab^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

```
[Out] [-1/6*(15*(a*b*d*e - a^2*e^2 + (b^2*d*e - a*b*e^2)*x)*sqrt((b*d - a*e)/b)*l
og((b*e*x + 2*b*d - a*e + 2*sqrt(e*x + d)*b*sqrt((b*d - a*e)/b))/(b*x + a))
 - 2*(2*b^2*e^2*x^2 - 3*b^2*d^2 + 20*a*b*d*e - 15*a^2*e^2 + 2*(7*b^2*d*e -
5*a*b*e^2)*x)*sqrt(e*x + d))/(b^4*x + a*b^3), -1/3*(15*(a*b*d*e - a^2*e^2 +
 (b^2*d*e - a*b*e^2)*x)*sqrt(-(b*d - a*e)/b)*arctan(-sqrt(e*x + d)*b*sqrt(-
(b*d - a*e)/b)/(b*d - a*e)) - (2*b^2*e^2*x^2 - 3*b^2*d^2 + 20*a*b*d*e - 15*
a^2*e^2 + 2*(7*b^2*d*e - 5*a*b*e^2)*x)*sqrt(e*x + d))/(b^4*x + a*b^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(b**2*x**2+2*a*b*x+a**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18858, size = 258, normalized size = 2.35

$$\frac{5\left(b^2d^2e-2\,abde^2+a^2e^3\right)\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)}{\sqrt{-b^2d+abe}b^3}-\frac{\sqrt{xe+d}b^2d^2e-2\,\sqrt{xe+d}abde^2+\sqrt{xe+d}a^2e^3}{((xe+d)b-bd+ae)b^3}+\frac{2\left((xe+d)^{\frac{3}{2}}b^4e+6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

```
[Out] 5*(b^2*d^2*e - 2*a*b*d*e^2 + a^2*e^3)*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d +
a*b*e))/(sqrt(-b^2*d + a*b*e)*b^3) - (sqrt(x*e + d)*b^2*d^2*e - 2*sqrt(x*e
+ d)*a*b*d*e^2 + sqrt(x*e + d)*a^2*e^3)/(((x*e + d)*b - b*d + a*e)*b^3) + 2
/3*((x*e + d)^(3/2)*b^4*e + 6*sqrt(x*e + d)*b^4*d*e - 6*sqrt(x*e + d)*a*b^3
*e^2)/b^6
```

3.1648 $\int \frac{(d+ex)^{3/2}}{a^2+2abx+b^2x^2} dx$

Optimal. Leaf size=85

$$-\frac{3e\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{5/2}}-\frac{(d+ex)^{3/2}}{b(a+bx)}+\frac{3e\sqrt{d+ex}}{b^2}$$

```
[Out] (3*e*Sqrt[d + e*x])/b^2 - (d + e*x)^(3/2)/(b*(a + b*x)) - (3*e*Sqrt[b*d - a
*e]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/b^(5/2)
```

Rubi [A] time = 0.0400498, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {27, 47, 50, 63, 208}

$$-\frac{3e\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{5/2}}-\frac{(d+ex)^{3/2}}{b(a+bx)}+\frac{3e\sqrt{d+ex}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] (3*e*Sqrt[d + e*x])/b^2 - (d + e*x)^(3/2)/(b*(a + b*x)) - (3*e*Sqrt[b*d - a
*e]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/b^(5/2)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^{3/2}}{a^2+2abx+b^2x^2}\,dx &= \int \frac{(d+ex)^{3/2}}{(a+bx)^2}\,dx\\
&= -\frac{(d+ex)^{3/2}}{b(a+bx)}+\frac{(3e)\int \frac{\sqrt{d+ex}}{a+bx}\,dx}{2b}\\
&= \frac{3e\sqrt{d+ex}}{b^2}-\frac{(d+ex)^{3/2}}{b(a+bx)}+\frac{(3e(bd-ae))\int \frac{1}{(a+bx)\sqrt{d+ex}}\,dx}{2b^2}\\
&= \frac{3e\sqrt{d+ex}}{b^2}-\frac{(d+ex)^{3/2}}{b(a+bx)}+\frac{(3(bd-ae))\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{bd}{e}+\frac{bx^2}{e}}\,dx,x,\sqrt{d+ex}\right)}{b^2}\\
&= \frac{3e\sqrt{d+ex}}{b^2}-\frac{(d+ex)^{3/2}}{b(a+bx)}-\frac{3e\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [C] time = 0.0140168, size = 50, normalized size = 0.59

$$\frac{2e(d+ex)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};-\frac{b(d+ex)}{ae-bd}\right)}{5(ae-bd)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] (2*e*(d + e*x)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, -((b*(d + e*x))/(-(b*d)
 + a*e))])/(5*(-(b*d) + a*e)^2)
```

Maple [B] time = 0.2, size = 148, normalized size = 1.7

$$2\frac{e\sqrt{ex+d}}{b^2}+\frac{ae^2}{b^2(bxe+ae)}\sqrt{ex+d}-\frac{de}{b(bxe+ae)}\sqrt{ex+d}-3\frac{ae^2}{b^2\sqrt{(ae-bd)b}}\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)b}}\right)+3\frac{de}{b\sqrt{(ae-bd)b}}\,\mathrm{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2),x)
```

```
[Out] 2*e*(e*x+d)^(1/2)/b^2+1/b^2*(e*x+d)^(1/2)/(b*e*x+a*e)*a*e^2-e/b*(e*x+d)^(1/
2)/(b*e*x+a*e)*d-3/b^2/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2)/((a*e-b*d
)*b)^(1/2))*a*e^2+3*e/b/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2)/((a*e-b*
d)*b)^(1/2))*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89191, size = 455, normalized size = 5.35

$$\left[\frac{3\,(bex+ae)\sqrt{\frac{bd-ae}{b}}\log\left(\frac{bex+2\,bd-ae-2\,\sqrt{ex+d}b\sqrt{\frac{bd-ae}{b}}}{bx+a}\right)+2\,(2\,bex-bd+3\,ae)\sqrt{ex+d}}{2\,(b^3x+ab^2)}, -\frac{3\,(bex+ae)\sqrt{-\frac{bd-ae}{b}}\arctan\left(-\frac{\sqrt{e}}{}\right.}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*e*x + a*e)*sqrt((b*d - a*e)/b)*log((b*e*x + 2*b*d - a*e - 2*sqrt
(e*x + d)*b*sqrt((b*d - a*e)/b))/(b*x + a)) + 2*(2*b*e*x - b*d + 3*a*e)*sqr
t(e*x + d))/(b^3*x + a*b^2), -(3*(b*e*x + a*e)*sqrt(-(b*d - a*e)/b)*arctan(
-sqrt(e*x + d)*b*sqrt(-(b*d - a*e)/b)/(b*d - a*e)) - (2*b*e*x - b*d + 3*a*e
)*sqrt(e*x + d))/(b^3*x + a*b^2)]
```

Sympy [B] time = 108.324, size = 923, normalized size = 10.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(b**2*x**2+2*a*b*x+a**2),x)
```

```
[Out] 2*a**2*e**3*sqrt(d + e*x)/(2*a**2*b**2*e**2 - 2*a*b**3*d*e + 2*a*b**3*e**2*
x - 2*b**4*d*e*x) - a**2*e**3*sqrt(-1/(b*(a*e - b*d)**3))*log(-a**2*e**2*sq
rt(-1/(b*(a*e - b*d)**3)) + 2*a*b*d*e*sqrt(-1/(b*(a*e - b*d)**3)) - b**2*d*
*2*sqrt(-1/(b*(a*e - b*d)**3)) + sqrt(d + e*x))/(2*b**2) + a**2*e**3*sqrt(-
1/(b*(a*e - b*d)**3))*log(a**2*e**2*sqrt(-1/(b*(a*e - b*d)**3)) - 2*a*b*d*e
*sqrt(-1/(b*(a*e - b*d)**3)) + b**2*d**2*sqrt(-1/(b*(a*e - b*d)**3)) + sqrt
(d + e*x))/(2*b**2) - 4*a*d*e**2*sqrt(d + e*x)/(2*a**2*b*e**2 - 2*a*b**2*d*
e + 2*a*b**2*e**2*x - 2*b**3*d*e*x) + a*d*e**2*sqrt(-1/(b*(a*e - b*d)**3))*
log(-a**2*e**2*sqrt(-1/(b*(a*e - b*d)**3)) + 2*a*b*d*e*sqrt(-1/(b*(a*e - b*
d)**3)) - b**2*d**2*sqrt(-1/(b*(a*e - b*d)**3)) + sqrt(d + e*x))/b - a*d*e*
*2*sqrt(-1/(b*(a*e - b*d)**3))*log(a**2*e**2*sqrt(-1/(b*(a*e - b*d)**3)) -
2*a*b*d*e*sqrt(-1/(b*(a*e - b*d)**3)) + b**2*d**2*sqrt(-1/(b*(a*e - b*d)**3
)) + sqrt(d + e*x))/b - 4*a*e**2*atan(sqrt(d + e*x)/sqrt(a*e/b - d))/(b**3*
sqrt(a*e/b - d)) - d**2*e*sqrt(-1/(b*(a*e - b*d)**3))*log(-a**2*e**2*sqrt(-
1/(b*(a*e - b*d)**3)) + 2*a*b*d*e*sqrt(-1/(b*(a*e - b*d)**3)) - b**2*d**2*s
qrt(-1/(b*(a*e - b*d)**3)) + sqrt(d + e*x))/2 + d**2*e*sqrt(-1/(b*(a*e - b*
d)**3))*log(a**2*e**2*sqrt(-1/(b*(a*e - b*d)**3)) - 2*a*b*d*e*sqrt(-1/(b*(a
*e - b*d)**3)) + b**2*d**2*sqrt(-1/(b*(a*e - b*d)**3)) + sqrt(d + e*x))/2 +
```

```
 2*d**2*e*sqrt(d + e*x)/(2*a**2*e**2 - 2*a*b*d*e + 2*a*b*e**2*x - 2*b**2*d*
e*x) + 4*d*e*atan(sqrt(d + e*x)/sqrt(a*e/b - d))/(b**2*sqrt(a*e/b - d)) + 2
*e*sqrt(d + e*x)/b**2
```

Giac [A] time = 1.16355, size = 165, normalized size = 1.94

$$\frac{3\left(bde-ae^2\right)\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)}{\sqrt{-b^2d+abe}b^2}+\frac{2\sqrt{xe+d}e}{b^2}-\frac{\sqrt{xe+d}bde-\sqrt{xe+d}ae^2}{((xe+d)b-bd+ae)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

```
[Out] 3*(b*d*e - a*e^2)*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d
 + a*b*e)*b^2) + 2*sqrt(x*e + d)*e/b^2 - (sqrt(x*e + d)*b*d*e - sqrt(x*e +
d)*a*e^2)/(((x*e + d)*b - b*d + a*e)*b^2)
```

3.1649 $\int \frac{\sqrt{d+ex}}{a^2+2abx+b^2x^2}\,dx$

Optimal. Leaf size=70

$$-\frac{e\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}\sqrt{bd-ae}}-\frac{\sqrt{d+ex}}{b(a+bx)}$$

```
[Out] -(Sqrt[d + e*x]/(b*(a + b*x))) - (e*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*
d - a*e]])/(b^(3/2)*Sqrt[b*d - a*e])
```

Rubi [A] time = 0.0314011, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {27, 47, 63, 208}

$$-\frac{e\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}\sqrt{bd-ae}}-\frac{\sqrt{d+ex}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] -(Sqrt[d + e*x]/(b*(a + b*x))) - (e*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*
d - a*e]])/(b^(3/2)*Sqrt[b*d - a*e])
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{a^2+2abx+b^2x^2}\,dx = \int \frac{\sqrt{d+ex}}{(a+bx)^2}\,dx$$
$$= -\frac{\sqrt{d+ex}}{b(a+bx)} + \frac{e\int \frac{1}{(a+bx)\sqrt{d+ex}}\,dx}{2b}$$
$$= -\frac{\sqrt{d+ex}}{b(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bd}{e}+\frac{bx^2}{e}}\,dx, x, \sqrt{d+ex}\right)}{b}$$
$$= -\frac{\sqrt{d+ex}}{b(a+bx)} - \frac{e\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}\sqrt{bd-ae}}$$

Mathematica [A] time = 0.0753953, size = 69, normalized size = 0.99

$$\frac{e\tan^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{ae-bd}}\right)}{b^{3/2}\sqrt{ae-bd}} - \frac{\sqrt{d+ex}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] -(Sqrt[d + e*x]/(b*(a + b*x))) + (e*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b
*d) + a*e]])/(b^(3/2)*Sqrt[-(b*d) + a*e])
```

Maple [A] time = 0.199, size = 64, normalized size = 0.9

$$-\frac{e}{b(bxe+ae)}\sqrt{ex+d} + \frac{e}{b}\arctan\left(b\sqrt{ex+d}\frac{1}{\sqrt{(ae-bd)b}}\right)\frac{1}{\sqrt{(ae-bd)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2),x)
```

```
[Out] -e/b*(e*x+d)^(1/2)/(b*e*x+a*e)+e/b/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/
2)/((a*e-b*d)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.97895, size = 498, normalized size = 7.11

$$\left[\frac{\sqrt{b^2d-abe}(bex+ae)\log\left(\frac{bex+2bd-ae-2\sqrt{b^2d-abe}\sqrt{ex+d}}{bx+a}\right)-2\left(b^2d-abe\right)\sqrt{ex+d}}{2\left(ab^3d-a^2b^2e+\left(b^4d-ab^3e\right)x\right)},\frac{\sqrt{-b^2d+abe}(bex+ae)\arctan\left(\frac{\sqrt{-b^2d+}}{bex}\right.}{ab^3d-a^2b^2e+\left(b^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b^2*d - a*b*e)*(b*e*x + a*e)*log((b*e*x + 2*b*d - a*e - 2*sqrt(b
^2*d - a*b*e)*sqrt(e*x + d))/(b*x + a)) - 2*(b^2*d - a*b*e)*sqrt(e*x + d))/
(a*b^3*d - a^2*b^2*e + (b^4*d - a*b^3*e)*x), (sqrt(-b^2*d + a*b*e)*(b*e*x +
 a*e)*arctan(sqrt(-b^2*d + a*b*e)*sqrt(e*x + d)/(b*e*x + b*d)) - (b^2*d - a
*b*e)*sqrt(e*x + d))/(a*b^3*d - a^2*b^2*e + (b^4*d - a*b^3*e)*x)]
```

Sympy [B] time = 19.1932, size = 573, normalized size = 8.19

$$-\frac{2ae^2\sqrt{d+ex}}{2a^2be^2-2ab^2de+2ab^2e^2x-2b^3dex}+\frac{ae^2\sqrt{-\frac{1}{b(ae-bd)^3}}\log\left(-a^2e^2\sqrt{-\frac{1}{b(ae-bd)^3}}+2abde\sqrt{-\frac{1}{b(ae-bd)^3}}-b^2d^2\sqrt{-\frac{1}{b(ae-bd)^3}}\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(b**2*x**2+2*a*b*x+a**2),x)
```

```
[Out] -2*a*e**2*sqrt(d + e*x)/(2*a**2*b*e**2 - 2*a*b**2*d*e + 2*a*b**2*e**2*x - 2
*b**3*d*e*x) + a*e**2*sqrt(-1/(b*(a*e - b*d)**3))*log(-a**2*e**2*sqrt(-1/(b
*(a*e - b*d)**3)) + 2*a*b*d*e*sqrt(-1/(b*(a*e - b*d)**3)) - b**2*d**2*sqrt(
-1/(b*(a*e - b*d)**3)) + sqrt(d + e*x))/(2*b) - a*e**2*sqrt(-1/(b*(a*e - b*
d)**3))*log(a**2*e**2*sqrt(-1/(b*(a*e - b*d)**3)) - 2*a*b*d*e*sqrt(-1/(b*(a
*e - b*d)**3)) + b**2*d**2*sqrt(-1/(b*(a*e - b*d)**3)) + sqrt(d + e*x))/(2*
b) - d*e*sqrt(-1/(b*(a*e - b*d)**3))*log(-a**2*e**2*sqrt(-1/(b*(a*e - b*d)*
*3)) + 2*a*b*d*e*sqrt(-1/(b*(a*e - b*d)**3)) - b**2*d**2*sqrt(-1/(b*(a*e -
b*d)**3)) + sqrt(d + e*x))/2 + d*e*sqrt(-1/(b*(a*e - b*d)**3))*log(a**2*e**
2*sqrt(-1/(b*(a*e - b*d)**3)) - 2*a*b*d*e*sqrt(-1/(b*(a*e - b*d)**3)) + b**
2*d**2*sqrt(-1/(b*(a*e - b*d)**3)) + sqrt(d + e*x))/2 + 2*d*e*sqrt(d + e*x)
/(2*a**2*e**2 - 2*a*b*d*e + 2*a*b*e**2*x - 2*b**2*d*e*x) + 2*e*atan(sqrt(d
+ e*x)/sqrt(a*e/b - d))/(b**2*sqrt(a*e/b - d))
```

Giac [A] time = 1.22723, size = 108, normalized size = 1.54

$$\frac{\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)e}{\sqrt{-b^2d+abe}b}-\frac{\sqrt{xe+d}e}{((xe+d)b-bd+ae)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e/(sqrt(-b^2*d + a*b*e)*b) - s
qrt(x*e + d)*e/(((x*e + d)*b - b*d + a*e)*b)
```

3.1650 $\int \frac{1}{\sqrt{d+ex}\left(a^2+2abx+b^2x^2\right)}\,dx$

Optimal. Leaf size=76

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{\sqrt{b}(bd-ae)^{3/2}}-\frac{\sqrt{d+ex}}{(a+bx)(bd-ae)}$$

```
[Out] -(Sqrt[d + e*x]/((b*d - a*e)*(a + b*x))) + (e*ArcTanh[(Sqrt[b]*Sqrt[d + e*x
])/Sqrt[b*d - a*e]])/(Sqrt[b]*(b*d - a*e)^(3/2))
```

Rubi [A] time = 0.0444152, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {27, 51, 63, 208}

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{\sqrt{b}(bd-ae)^{3/2}}-\frac{\sqrt{d+ex}}{(a+bx)(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

```
[Out] -(Sqrt[d + e*x]/((b*d - a*e)*(a + b*x))) + (e*ArcTanh[(Sqrt[b]*Sqrt[d + e*x
])/Sqrt[b*d - a*e]])/(Sqrt[b]*(b*d - a*e)^(3/2))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\,(a^2+2abx+b^2x^2)}\,dx = \int \frac{1}{(a+bx)^2\sqrt{d+ex}}\,dx$$
$$= -\frac{\sqrt{d+ex}}{(bd-ae)(a+bx)} - \frac{e\int \frac{1}{(a+bx)\sqrt{d+ex}}\,dx}{2(bd-ae)}$$
$$= -\frac{\sqrt{d+ex}}{(bd-ae)(a+bx)} - \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bd}{e}+\frac{bx^2}{e}}\,dx, x, \sqrt{d+ex}\right)}{bd-ae}$$
$$= -\frac{\sqrt{d+ex}}{(bd-ae)(a+bx)} + \frac{e\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{\sqrt{b}(bd-ae)^{3/2}}$$

Mathematica [A] time = 0.0556035, size = 76, normalized size = 1.

$$\frac{e\tan^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{ae-bd}}\right)}{\sqrt{b}(ae-bd)^{3/2}} - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

```
[Out] -(Sqrt[d + e*x]/((b*d - a*e)*(a + b*x))) + (e*ArcTan[(Sqrt[b]*Sqrt[d + e*x]
)/Sqrt[-(b*d) + a*e]])/(Sqrt[b]*(-(b*d) + a*e)^(3/2))
```

Maple [A] time = 0.196, size = 77, normalized size = 1.

$$\frac{e}{(ae-bd)(bxe+ae)}\sqrt{ex+d} + \frac{e}{ae-bd}\arctan\left(b\sqrt{ex+d}\frac{1}{\sqrt{(ae-bd)\,b}}\right)\frac{1}{\sqrt{(ae-bd)\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2),x)
```

```
[Out] e*(e*x+d)^(1/2)/(a*e-b*d)/(b*e*x+a*e)+e/(a*e-b*d)/((a*e-b*d)*b)^(1/2)*arcta
n(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.99259, size = 603, normalized size = 7.93

$$\left[-\frac{\sqrt{b^2d-abe}(bex+ae)\log\left(\frac{bex+2\,bd-ae-2\,\sqrt{b^2d-abe}\sqrt{ex+d}}{bx+a}\right)+2\left(b^2d-abe\right)\sqrt{ex+d}}{2\left(ab^3d^2-2\,a^2b^2de+a^3be^2+\left(b^4d^2-2\,ab^3de+a^2b^2e^2\right)x\right)},-\frac{\sqrt{-b^2d+abe}(bex+ae)\arctan\left(\frac{\sqrt{-b^2d+\cdots}}{bex\cdots}\right)}{ab^3d^2-2\,a^2b^2de+a^3be^2+\left(b^4\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(b^2*d - a*b*e)*(b*e*x + a*e)*log((b*e*x + 2*b*d - a*e - 2*sqrt(
b^2*d - a*b*e)*sqrt(e*x + d))/(b*x + a)) + 2*(b^2*d - a*b*e)*sqrt(e*x + d))
/(a*b^3*d^2 - 2*a^2*b^2*d*e + a^3*b*e^2 + (b^4*d^2 - 2*a*b^3*d*e + a^2*b^2*
e^2)*x), -(sqrt(-b^2*d + a*b*e)*(b*e*x + a*e)*arctan(sqrt(-b^2*d + a*b*e)*s
qrt(e*x + d)/(b*e*x + b*d)) + (b^2*d - a*b*e)*sqrt(e*x + d))/(a*b^3*d^2 - 2
*a^2*b^2*d*e + a^3*b*e^2 + (b^4*d^2 - 2*a*b^3*d*e + a^2*b^2*e^2)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^2\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(b**2*x**2+2*a*b*x+a**2),x)
```

```
[Out] Integral(1/((a + b*x)**2*sqrt(d + e*x)), x)
```

Giac [A] time = 1.1706, size = 131, normalized size = 1.72

$$-\frac{\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)e}{\sqrt{-b^2d+abe}(bd-ae)}-\frac{\sqrt{xe+d}e}{((xe+d)b-bd+ae)(bd-ae)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

```
[Out] -arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e/(sqrt(-b^2*d + a*b*e)*(b*d
- a*e)) - sqrt(x*e + d)*e/(((x*e + d)*b - b*d + a*e)*(b*d - a*e))
```

3.1651 $\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)}\,dx$

Optimal. Leaf size=99

$$-\frac{3e}{\sqrt{d+ex}(bd-ae)^2}-\frac{1}{(a+bx)\sqrt{d+ex}(bd-ae)}+\frac{3\sqrt{b}e\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{5/2}}$$

```
[Out] (-3*e)/((b*d - a*e)^2*Sqrt[d + e*x]) - 1/((b*d - a*e)*(a + b*x)*Sqrt[d + e*
x]) + (3*Sqrt[b]*e*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(b*d -
 a*e)^(5/2)
```

Rubi [A] time = 0.0539409, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {27, 51, 63, 208}

$$-\frac{3e}{\sqrt{d+ex}(bd-ae)^2}-\frac{1}{(a+bx)\sqrt{d+ex}(bd-ae)}+\frac{3\sqrt{b}e\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

```
[Out] (-3*e)/((b*d - a*e)^2*Sqrt[d + e*x]) - 1/((b*d - a*e)*(a + b*x)*Sqrt[d + e*
x]) + (3*Sqrt[b]*e*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(b*d -
 a*e)^(5/2)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\left(a^2+2abx+b^2x^2\right)}\,dx = \int \frac{1}{(a+bx)^2(d+ex)^{3/2}}\,dx$$
$$= -\frac{1}{(bd-ae)(a+bx)\sqrt{d+ex}} - \frac{(3e)\int \frac{1}{(a+bx)(d+ex)^{3/2}}\,dx}{2(bd-ae)}$$
$$= -\frac{3e}{(bd-ae)^2\sqrt{d+ex}} - \frac{1}{(bd-ae)(a+bx)\sqrt{d+ex}} - \frac{(3be)\int \frac{1}{(a+bx)\sqrt{d+ex}}\,dx}{2(bd-ae)^2}$$
$$= -\frac{3e}{(bd-ae)^2\sqrt{d+ex}} - \frac{1}{(bd-ae)(a+bx)\sqrt{d+ex}} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{a-\frac{bd}{e}+\frac{bx^2}{e}}\,dx, x, \sqrt{}\right.}{(bd-ae)^2}$$
$$= -\frac{3e}{(bd-ae)^2\sqrt{d+ex}} - \frac{1}{(bd-ae)(a+bx)\sqrt{d+ex}} + \frac{3\sqrt{b}e\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{5/2}}$$

Mathematica [C] time = 0.0131378, size = 48, normalized size = 0.48

$$-\frac{2e\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};-\frac{b(d+ex)}{ae-bd}\right)}{\sqrt{d+ex}(ae-bd)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

```
[Out] (-2*e*Hypergeometric2F1[-1/2, 2, 1/2, -((b*(d + e*x))/(-(b*d) + a*e))])/((-
(b*d) + a*e)^2*Sqrt[d + e*x])
```

Maple [A] time = 0.207, size = 101, normalized size = 1.

$$-2\,{\frac {e}{ \left(ae-bd \right) ^{2}\sqrt {ex+d}}}-{\frac {be}{ \left(ae-bd \right) ^{2} \left(bxe+ae \right) }\sqrt {ex+d}}-3\,{\frac {be}{ \left(ae-bd \right) ^{2}\sqrt { \left(ae-bd \right) b}}\arctan \left({\frac {b\sqrt {ex+d}}{\sqrt { \left(ae-bd \right) b}}} \right) }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2),x)
```

```
[Out] -2*e/(a*e-b*d)^2/(e*x+d)^(1/2)-e*b/(a*e-b*d)^2*(e*x+d)^(1/2)/(b*e*x+a*e)-3*
e*b/(a*e-b*d)^2/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1
/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.01781, size = 883, normalized size = 8.92

$$\left[\frac{3\left(be^2x^2+ade+\left(bde+ae^2\right)x\right)\sqrt{\frac{b}{bd-ae}}\log\left(\frac{bex+2\,bd-ae+2\,(bd-ae)\sqrt{ex+d}\sqrt{\frac{b}{bd-ae}}}{bx+a}\right)-2\,(3\,bex+bd+2\,ae)\sqrt{ex+d}}{2\left(ab^2d^3-2\,a^2bd^2e+a^3de^2+\left(b^3d^2e-2\,ab^2de^2+a^2be^3\right)x^2+\left(b^3d^3-ab^2d^2e-a^2bde^2+a^3e^3\right)x\right)},\ \frac{3\left(be^2x^2+\right.}{ab^2d^3-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*e^2*x^2 + a*d*e + (b*d*e + a*e^2)*x)*sqrt(b/(b*d - a*e))*log((b*
e*x + 2*b*d - a*e + 2*(b*d - a*e)*sqrt(e*x + d)*sqrt(b/(b*d - a*e)))/(b*x +
 a)) - 2*(3*b*e*x + b*d + 2*a*e)*sqrt(e*x + d))/(a*b^2*d^3 - 2*a^2*b*d^2*e
+ a^3*d*e^2 + (b^3*d^2*e - 2*a*b^2*d*e^2 + a^2*b*e^3)*x^2 + (b^3*d^3 - a*b^
2*d^2*e - a^2*b*d*e^2 + a^3*e^3)*x), (3*(b*e^2*x^2 + a*d*e + (b*d*e + a*e^2
)*x)*sqrt(-b/(b*d - a*e))*arctan(-(b*d - a*e)*sqrt(e*x + d)*sqrt(-b/(b*d -
a*e))/(b*e*x + b*d)) - (3*b*e*x + b*d + 2*a*e)*sqrt(e*x + d))/(a*b^2*d^3 -
2*a^2*b*d^2*e + a^3*d*e^2 + (b^3*d^2*e - 2*a*b^2*d*e^2 + a^2*b*e^3)*x^2 + (
b^3*d^3 - a*b^2*d^2*e - a^2*b*d*e^2 + a^3*e^3)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^2\,(d+ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(b**2*x**2+2*a*b*x+a**2),x)
```

```
[Out] Integral(1/((a + b*x)**2*(d + e*x)**(3/2)), x)
```

Giac [A] time = 1.18709, size = 207, normalized size = 2.09

$$-\frac{3\,b\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)e}{\left(b^2d^2-2\,abde+a^2e^2\right)\sqrt{-b^2d+abe}}-\frac{3\,(xe+d)be-2\,bde+2\,ae^2}{\left(b^2d^2-2\,abde+a^2e^2\right)\left((xe+d)^{\frac{3}{2}}b-\sqrt{xe+d}bd+\sqrt{xe+d}ae\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

```
[Out] -3*b*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e/((b^2*d^2 - 2*a*b*d*e +
 a^2*e^2)*sqrt(-b^2*d + a*b*e)) - (3*(x*e + d)*b*e - 2*b*d*e + 2*a*e^2)/((b
^2*d^2 - 2*a*b*d*e + a^2*e^2)*((x*e + d)^(3/2)*b - sqrt(x*e + d)*b*d + sqrt
(x*e + d)*a*e))
```

3.1652 $\int \frac{1}{(d+ex)^{5/2}\left(a^2+2abx+b^2x^2\right)}\,dx$

Optimal. Leaf size=124

$$\frac{5b^{3/2}e\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{7/2}}-\frac{5be}{\sqrt{d+ex}(bd-ae)^3}-\frac{1}{(a+bx)(d+ex)^{3/2}(bd-ae)}-\frac{5e}{3(d+ex)^{3/2}(bd-ae)^2}$$

```
[Out] (-5*e)/(3*(b*d - a*e)^2*(d + e*x)^(3/2)) - 1/((b*d - a*e)*(a + b*x)*(d + e*
x)^(3/2)) - (5*b*e)/((b*d - a*e)^3*Sqrt[d + e*x]) + (5*b^(3/2)*e*ArcTanh[(S
qrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(b*d - a*e)^(7/2)
```

Rubi [A] time = 0.0641323, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {27, 51, 63, 208}

$$\frac{5b^{3/2}e\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{7/2}}-\frac{5be}{\sqrt{d+ex}(bd-ae)^3}-\frac{1}{(a+bx)(d+ex)^{3/2}(bd-ae)}-\frac{5e}{3(d+ex)^{3/2}(bd-ae)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

```
[Out] (-5*e)/(3*(b*d - a*e)^2*(d + e*x)^(3/2)) - 1/((b*d - a*e)*(a + b*x)*(d + e*
x)^(3/2)) - (5*b*e)/((b*d - a*e)^3*Sqrt[d + e*x]) + (5*b^(3/2)*e*ArcTanh[(S
qrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(b*d - a*e)^(7/2)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{5/2}\left(a^2+2abx+b^2x^2\right)}\,dx=\int \frac{1}{(a+bx)^2(d+ex)^{5/2}}\,dx$$

$$=-\frac{1}{(bd-ae)(a+bx)(d+ex)^{3/2}}-\frac{(5e)\int \frac{1}{(a+bx)(d+ex)^{5/2}}\,dx}{2(bd-ae)}$$

$$=-\frac{5e}{3(bd-ae)^2(d+ex)^{3/2}}-\frac{1}{(bd-ae)(a+bx)(d+ex)^{3/2}}-\frac{(5be)\int \frac{1}{(a+bx)(d+ex)^{3/2}}\,d}{2(bd-ae)^2}$$

$$=-\frac{5e}{3(bd-ae)^2(d+ex)^{3/2}}-\frac{1}{(bd-ae)(a+bx)(d+ex)^{3/2}}-\frac{5be}{(bd-ae)^3\sqrt{d+ex}}-$$

$$=-\frac{5e}{3(bd-ae)^2(d+ex)^{3/2}}-\frac{1}{(bd-ae)(a+bx)(d+ex)^{3/2}}-\frac{5be}{(bd-ae)^3\sqrt{d+ex}}-$$

$$=-\frac{5e}{3(bd-ae)^2(d+ex)^{3/2}}-\frac{1}{(bd-ae)(a+bx)(d+ex)^{3/2}}-\frac{5be}{(bd-ae)^3\sqrt{d+ex}}+$$

Mathematica [C] time = 0.0149508, size = 50, normalized size = 0.4

$$-\frac{2e\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};-\frac{b(d+ex)}{ae-bd}\right)}{3(d+ex)^{3/2}(ae-bd)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

```
[Out] (-2*e*Hypergeometric2F1[-3/2, 2, -1/2, -((b*(d + e*x))/(-(b*d) + a*e))])/(3
*(-(b*d) + a*e)^2*(d + e*x)^(3/2))
```

Maple [A] time = 0.204, size = 125, normalized size = 1.

$$-\frac{2e}{3\,(ae-bd)^2}(ex+d)^{-\frac{3}{2}}+4\frac{be}{(ae-bd)^3\sqrt{ex+d}}+\frac{b^2e}{(ae-bd)^3\,(bxe+ae)}\sqrt{ex+d}+5\frac{b^2e}{(ae-bd)^3\sqrt{(ae-bd)\,b}}\arctan\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2),x)
```

```
[Out] -2/3*e/(a*e-b*d)^2/(e*x+d)^(3/2)+4*e/(a*e-b*d)^3*b/(e*x+d)^(1/2)+e*b^2/(a*e
-b*d)^3*(e*x+d)^(1/2)/(b*e*x+a*e)+5*e*b^2/(a*e-b*d)^3/((a*e-b*d)*b)^(1/2)*a
rctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.02, size = 1592, normalized size = 12.84

$$\left[-\frac{15\left(b^2e^3x^3+abd^2e+\left(2\,b^2de^2+abe^3\right)x^2+\left(b^2d^2e+2\,abde^2\right)x\right)\sqrt{\frac{b}{bd-ae}}\log\left(\frac{bex+2\,bd-ae-2\,(bd-ae)\sqrt{ex+d}\sqrt{\frac{b}{bd-ae}}}{bx+a}\right)+2}{6\left(ab^3d^5-3\,a^2b^2d^4e+3\,a^3bd^3e^2-a^4d^2e^3+\left(b^4d^3e^2-3\,ab^3d^2e^3+3\,a^2b^2de^4-a^3be^5\right)x^3+\left(2\,b^4d^4e-5\,ab^3d^3e^2+3\,a^2b^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

```
[Out] [-1/6*(15*(b^2*e^3*x^3 + a*b*d^2*e + (2*b^2*d*e^2 + a*b*e^3)*x^2 + (b^2*d^2
*e + 2*a*b*d*e^2)*x)*sqrt(b/(b*d - a*e))*log((b*e*x + 2*b*d - a*e - 2*(b*d
- a*e)*sqrt(e*x + d)*sqrt(b/(b*d - a*e)))/(b*x + a)) + 2*(15*b^2*e^2*x^2 +
3*b^2*d^2 + 14*a*b*d*e - 2*a^2*e^2 + 10*(2*b^2*d*e + a*b*e^2)*x)*sqrt(e*x +
 d))/(a*b^3*d^5 - 3*a^2*b^2*d^4*e + 3*a^3*b*d^3*e^2 - a^4*d^2*e^3 + (b^4*d^
3*e^2 - 3*a*b^3*d^2*e^3 + 3*a^2*b^2*d*e^4 - a^3*b*e^5)*x^3 + (2*b^4*d^4*e -
 5*a*b^3*d^3*e^2 + 3*a^2*b^2*d^2*e^3 + a^3*b*d*e^4 - a^4*e^5)*x^2 + (b^4*d^
5 - a*b^3*d^4*e - 3*a^2*b^2*d^3*e^2 + 5*a^3*b*d^2*e^3 - 2*a^4*d*e^4)*x), 1/
3*(15*(b^2*e^3*x^3 + a*b*d^2*e + (2*b^2*d*e^2 + a*b*e^3)*x^2 + (b^2*d^2*e +
 2*a*b*d*e^2)*x)*sqrt(-b/(b*d - a*e))*arctan(-(b*d - a*e)*sqrt(e*x + d)*sqr
t(-b/(b*d - a*e))/(b*e*x + b*d)) - (15*b^2*e^2*x^2 + 3*b^2*d^2 + 14*a*b*d*e
 - 2*a^2*e^2 + 10*(2*b^2*d*e + a*b*e^2)*x)*sqrt(e*x + d))/(a*b^3*d^5 - 3*a^
2*b^2*d^4*e + 3*a^3*b*d^3*e^2 - a^4*d^2*e^3 + (b^4*d^3*e^2 - 3*a*b^3*d^2*e^
3 + 3*a^2*b^2*d*e^4 - a^3*b*e^5)*x^3 + (2*b^4*d^4*e - 5*a*b^3*d^3*e^2 + 3*a
^2*b^2*d^2*e^3 + a^3*b*d*e^4 - a^4*e^5)*x^2 + (b^4*d^5 - a*b^3*d^4*e - 3*a^
2*b^2*d^3*e^2 + 5*a^3*b*d^2*e^3 - 2*a^4*d*e^4)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^2\,(d+ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(5/2)/(b**2*x**2+2*a*b*x+a**2),x)
```

```
[Out] Integral(1/((a + b*x)**2*(d + e*x)**(5/2)), x)
```

Giac [B] time = 1.24007, size = 302, normalized size = 2.44

$$-\frac{5\,b^2\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)e}{\left(b^3d^3-3\,ab^2d^2e+3\,a^2bde^2-a^3e^3\right)\sqrt{-b^2d+abe}}-\frac{\sqrt{xe+d}b^2e}{\left(b^3d^3-3\,ab^2d^2e+3\,a^2bde^2-a^3e^3\right)\left((xe+d)b-bd+ae\right)}-\frac{}{3\left(b^3d^3-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

```
[Out] -5*b^2*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e/((b^3*d^3 - 3*a*b^2*d
^2*e + 3*a^2*b*d*e^2 - a^3*e^3)*sqrt(-b^2*d + a*b*e)) - sqrt(x*e + d)*b^2*e
/((b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3)*((x*e + d)*b - b*d +
a*e)) - 2/3*(6*(x*e + d)*b*e + b*d*e - a*e^2)/((b^3*d^3 - 3*a*b^2*d^2*e + 3
*a^2*b*d*e^2 - a^3*e^3)*(x*e + d)^(3/2))
```

3.1653 $\int \frac{1}{(d+ex)^{7/2}(a^2+2abx+b^2x^2)}\,dx$

Optimal. Leaf size=151

$$-\frac{7b^2e}{\sqrt{d+ex}(bd-ae)^4}+\frac{7b^{5/2}e\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{9/2}}-\frac{7be}{3(d+ex)^{3/2}(bd-ae)^3}-\frac{1}{(a+bx)(d+ex)^{5/2}(bd-ae)}-\frac{7e}{5(d+ex)^{5/2}(bd-$$

```
[Out] (-7*e)/(5*(b*d - a*e)^2*(d + e*x)^(5/2)) - 1/((b*d - a*e)*(a + b*x)*(d + e*
x)^(5/2)) - (7*b*e)/(3*(b*d - a*e)^3*(d + e*x)^(3/2)) - (7*b^2*e)/((b*d - a
*e)^4*Sqrt[d + e*x]) + (7*b^(5/2)*e*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*
d - a*e]])/(b*d - a*e)^(9/2)
```

Rubi [A] time = 0.0996385, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {27, 51, 63, 208}

$$-\frac{7b^2e}{\sqrt{d+ex}(bd-ae)^4}+\frac{7b^{5/2}e\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{9/2}}-\frac{7be}{3(d+ex)^{3/2}(bd-ae)^3}-\frac{1}{(a+bx)(d+ex)^{5/2}(bd-ae)}-\frac{7e}{5(d+ex)^{5/2}(bd-$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

```
[Out] (-7*e)/(5*(b*d - a*e)^2*(d + e*x)^(5/2)) - 1/((b*d - a*e)*(a + b*x)*(d + e*
x)^(5/2)) - (7*b*e)/(3*(b*d - a*e)^3*(d + e*x)^(3/2)) - (7*b^2*e)/((b*d - a
*e)^4*Sqrt[d + e*x]) + (7*b^(5/2)*e*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*
d - a*e]])/(b*d - a*e)^(9/2)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{7/2}\left(a^2+2abx+b^2x^2\right)}\,dx = \int \frac{1}{(a+bx)^2(d+ex)^{7/2}}\,dx$$

$$= -\frac{1}{(bd-ae)(a+bx)(d+ex)^{5/2}} - \frac{(7e)\int \frac{1}{(a+bx)(d+ex)^{7/2}}\,dx}{2(bd-ae)}$$

$$= -\frac{7e}{5(bd-ae)^2(d+ex)^{5/2}} - \frac{1}{(bd-ae)(a+bx)(d+ex)^{5/2}} - \frac{(7be)\int \frac{1}{(a+bx)(d+ex)^{5/2}}\,d}{2(bd-ae)^2}$$

$$= -\frac{7e}{5(bd-ae)^2(d+ex)^{5/2}} - \frac{1}{(bd-ae)(a+bx)(d+ex)^{5/2}} - \frac{7be}{3(bd-ae)^3(d+ex)^{3/2}}$$

$$= -\frac{7e}{5(bd-ae)^2(d+ex)^{5/2}} - \frac{1}{(bd-ae)(a+bx)(d+ex)^{5/2}} - \frac{7be}{3(bd-ae)^3(d+ex)^{3/2}}$$

$$= -\frac{7e}{5(bd-ae)^2(d+ex)^{5/2}} - \frac{1}{(bd-ae)(a+bx)(d+ex)^{5/2}} - \frac{7be}{3(bd-ae)^3(d+ex)^{3/2}}$$

$$= -\frac{7e}{5(bd-ae)^2(d+ex)^{5/2}} - \frac{1}{(bd-ae)(a+bx)(d+ex)^{5/2}} - \frac{7be}{3(bd-ae)^3(d+ex)^{3/2}}$$

Mathematica [C] time = 0.0171591, size = 50, normalized size = 0.33

$$-\frac{2e\,{}_2F_1\left(-\frac{5}{2},2;-\frac{3}{2};-\frac{b(d+ex)}{ae-bd}\right)}{5(d+ex)^{5/2}(ae-bd)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

```
[Out] (-2*e*Hypergeometric2F1[-5/2, 2, -3/2, -((b*(d + e*x))/(-(b*d) + a*e))])/(5
*(-(b*d) + a*e)^2*(d + e*x)^(5/2))
```

Maple [A] time = 0.207, size = 149, normalized size = 1.

$$-\frac{2e}{5(ae-bd)^2}(ex+d)^{-\frac{5}{2}} - 6\frac{b^2e}{(ae-bd)^4\sqrt{ex+d}} + \frac{4be}{3(ae-bd)^3}(ex+d)^{-\frac{3}{2}} - \frac{eb^3}{(ae-bd)^4(bxe+ae)}\sqrt{ex+d} - 7\frac{}{(ae-ba}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2),x)
```

```
[Out] -2/5*e/(a*e-b*d)^2/(e*x+d)^(5/2)-6*e/(a*e-b*d)^4*b^2/(e*x+d)^(1/2)+4/3*e/(a
*e-b*d)^3*b/(e*x+d)^(3/2)-e*b^3/(a*e-b*d)^4*(e*x+d)^(1/2)/(b*e*x+a*e)-7*e*b
^3/(a*e-b*d)^4/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/
2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.13549, size = 2471, normalized size = 16.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

```
[Out] [1/30*(105*(b^3*e^4*x^4 + a*b^2*d^3*e + (3*b^3*d*e^3 + a*b^2*e^4)*x^3 + 3*(
b^3*d^2*e^2 + a*b^2*d*e^3)*x^2 + (b^3*d^3*e + 3*a*b^2*d^2*e^2)*x)*sqrt(b/(b
*d - a*e))*log((b*e*x + 2*b*d - a*e + 2*(b*d - a*e)*sqrt(e*x + d)*sqrt(b/(b
*d - a*e)))/(b*x + a)) - 2*(105*b^3*e^3*x^3 + 15*b^3*d^3 + 116*a*b^2*d^2*e
- 32*a^2*b*d*e^2 + 6*a^3*e^3 + 35*(7*b^3*d*e^2 + 2*a*b^2*e^3)*x^2 + 7*(23*b
^3*d^2*e + 24*a*b^2*d*e^2 - 2*a^2*b*e^3)*x)*sqrt(e*x + d))/(a*b^4*d^7 - 4*a
^2*b^3*d^6*e + 6*a^3*b^2*d^5*e^2 - 4*a^4*b*d^4*e^3 + a^5*d^3*e^4 + (b^5*d^4
*e^3 - 4*a*b^4*d^3*e^4 + 6*a^2*b^3*d^2*e^5 - 4*a^3*b^2*d*e^6 + a^4*b*e^7)*x
^4 + (3*b^5*d^5*e^2 - 11*a*b^4*d^4*e^3 + 14*a^2*b^3*d^3*e^4 - 6*a^3*b^2*d^2
*e^5 - a^4*b*d*e^6 + a^5*e^7)*x^3 + 3*(b^5*d^6*e - 3*a*b^4*d^5*e^2 + 2*a^2*
b^3*d^4*e^3 + 2*a^3*b^2*d^3*e^4 - 3*a^4*b*d^2*e^5 + a^5*d*e^6)*x^2 + (b^5*d
^7 - a*b^4*d^6*e - 6*a^2*b^3*d^5*e^2 + 14*a^3*b^2*d^4*e^3 - 11*a^4*b*d^3*e^
4 + 3*a^5*d^2*e^5)*x), 1/15*(105*(b^3*e^4*x^4 + a*b^2*d^3*e + (3*b^3*d*e^3
+ a*b^2*e^4)*x^3 + 3*(b^3*d^2*e^2 + a*b^2*d*e^3)*x^2 + (b^3*d^3*e + 3*a*b^2
*d^2*e^2)*x)*sqrt(-b/(b*d - a*e))*arctan(-(b*d - a*e)*sqrt(e*x + d)*sqrt(-b
/(b*d - a*e))/(b*e*x + b*d)) - (105*b^3*e^3*x^3 + 15*b^3*d^3 + 116*a*b^2*d^
2*e - 32*a^2*b*d*e^2 + 6*a^3*e^3 + 35*(7*b^3*d*e^2 + 2*a*b^2*e^3)*x^2 + 7*(
23*b^3*d^2*e + 24*a*b^2*d*e^2 - 2*a^2*b*e^3)*x)*sqrt(e*x + d))/(a*b^4*d^7 -
 4*a^2*b^3*d^6*e + 6*a^3*b^2*d^5*e^2 - 4*a^4*b*d^4*e^3 + a^5*d^3*e^4 + (b^5
*d^4*e^3 - 4*a*b^4*d^3*e^4 + 6*a^2*b^3*d^2*e^5 - 4*a^3*b^2*d*e^6 + a^4*b*e^
7)*x^4 + (3*b^5*d^5*e^2 - 11*a*b^4*d^4*e^3 + 14*a^2*b^3*d^3*e^4 - 6*a^3*b^2
*d^2*e^5 - a^4*b*d*e^6 + a^5*e^7)*x^3 + 3*(b^5*d^6*e - 3*a*b^4*d^5*e^2 + 2*
a^2*b^3*d^4*e^3 + 2*a^3*b^2*d^3*e^4 - 3*a^4*b*d^2*e^5 + a^5*d*e^6)*x^2 + (b
^5*d^7 - a*b^4*d^6*e - 6*a^2*b^3*d^5*e^2 + 14*a^3*b^2*d^4*e^3 - 11*a^4*b*d^
3*e^4 + 3*a^5*d^2*e^5)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^2 (d+ex)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(7/2)/(b**2*x**2+2*a*b*x+a**2),x)
```

```
[Out] Integral(1/((a + b*x)**2*(d + e*x)**(7/2)), x)
```

Giac [B] time = 1.20258, size = 410, normalized size = 2.72

$$-\frac{7\,b^3\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)e}{\left(b^4d^4-4\,ab^3d^3e+6\,a^2b^2d^2e^2-4\,a^3bde^3+a^4e^4\right)\sqrt{-b^2d+abe}}-\frac{\sqrt{xe+d}b^3e}{\left(b^4d^4-4\,ab^3d^3e+6\,a^2b^2d^2e^2-4\,a^3bde^3+a^4e^4\right)((xe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

```
[Out] -7*b^3*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e/((b^4*d^4 - 4*a*b^3*d
^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4)*sqrt(-b^2*d + a*b*e)) -
 sqrt(x*e + d)*b^3*e/((b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*
b*d*e^3 + a^4*e^4)*((x*e + d)*b - b*d + a*e)) - 2/15*(45*(x*e + d)^2*b^2*e
+ 10*(x*e + d)*b^2*d*e + 3*b^2*d^2*e - 10*(x*e + d)*a*b*e^2 - 6*a*b*d*e^2 +
 3*a^2*e^3)/((b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 +
 a^4*e^4)*(x*e + d)^(5/2))
```

3.1654 $\int \frac{(d+ex)^{11/2}}{\left(a^2+2abx+b^2x^2\right)^2} dx$

Optimal. Leaf size=201

$$-\frac{33e^2(d+ex)^{7/2}}{8b^3(a+bx)}+\frac{77e^3(d+ex)^{3/2}(bd-ae)}{8b^5}+\frac{231e^3\sqrt{d+ex}(bd-ae)^2}{8b^6}-\frac{231e^3(bd-ae)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8b^{13/2}}-\frac{11e(d+e}{12b^2(a+}$$

```
[Out] (231*e^3*(b*d - a*e)^2*Sqrt[d + e*x])/(8*b^6) + (77*e^3*(b*d - a*e)*(d + e*
x)^(3/2))/(8*b^5) + (231*e^3*(d + e*x)^(5/2))/(40*b^4) - (33*e^2*(d + e*x)^
(7/2))/(8*b^3*(a + b*x)) - (11*e*(d + e*x)^(9/2))/(12*b^2*(a + b*x)^2) - (d
 + e*x)^(11/2)/(3*b*(a + b*x)^3) - (231*e^3*(b*d - a*e)^(5/2)*ArcTanh[(Sqrt
[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(8*b^(13/2))
```

Rubi [A] time = 0.130738, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {27, 47, 50, 63, 208}

$$-\frac{33e^2(d+ex)^{7/2}}{8b^3(a+bx)}+\frac{77e^3(d+ex)^{3/2}(bd-ae)}{8b^5}+\frac{231e^3\sqrt{d+ex}(bd-ae)^2}{8b^6}-\frac{231e^3(bd-ae)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8b^{13/2}}-\frac{11e(d+e}{12b^2(a+}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(11/2)/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] (231*e^3*(b*d - a*e)^2*Sqrt[d + e*x])/(8*b^6) + (77*e^3*(b*d - a*e)*(d + e*
x)^(3/2))/(8*b^5) + (231*e^3*(d + e*x)^(5/2))/(40*b^4) - (33*e^2*(d + e*x)^
(7/2))/(8*b^3*(a + b*x)) - (11*e*(d + e*x)^(9/2))/(12*b^2*(a + b*x)^2) - (d
 + e*x)^(11/2)/(3*b*(a + b*x)^3) - (231*e^3*(b*d - a*e)^(5/2)*ArcTanh[(Sqrt
[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(8*b^(13/2))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{11/2}}{\left(a^2+2abx+b^2x^2\right)^2}\,dx = \int \frac{(d+ex)^{11/2}}{(a+bx)^4}\,dx$$

$$= -\frac{(d+ex)^{11/2}}{3b(a+bx)^3} + \frac{(11e)\int \frac{(d+ex)^{9/2}}{(a+bx)^3}\,dx}{6b}$$

$$= -\frac{11e(d+ex)^{9/2}}{12b^2(a+bx)^2} - \frac{(d+ex)^{11/2}}{3b(a+bx)^3} + \frac{\left(33e^2\right)\int \frac{(d+ex)^{7/2}}{(a+bx)^2}\,dx}{8b^2}$$

$$= -\frac{33e^2(d+ex)^{7/2}}{8b^3(a+bx)} - \frac{11e(d+ex)^{9/2}}{12b^2(a+bx)^2} - \frac{(d+ex)^{11/2}}{3b(a+bx)^3} + \frac{\left(231e^3\right)\int \frac{(d+ex)^{5/2}}{a+bx}\,dx}{16b^3}$$

$$= \frac{231e^3(d+ex)^{5/2}}{40b^4} - \frac{33e^2(d+ex)^{7/2}}{8b^3(a+bx)} - \frac{11e(d+ex)^{9/2}}{12b^2(a+bx)^2} - \frac{(d+ex)^{11/2}}{3b(a+bx)^3} + \frac{\left(231e^3(bd-ae)\right)\int}{16b^4}$$

$$= \frac{77e^3(bd-ae)(d+ex)^{3/2}}{8b^5} + \frac{231e^3(d+ex)^{5/2}}{40b^4} - \frac{33e^2(d+ex)^{7/2}}{8b^3(a+bx)} - \frac{11e(d+ex)^{9/2}}{12b^2(a+bx)^2} - \frac{(d+ex}{3b(a+}$$

$$= \frac{231e^3(bd-ae)^2\sqrt{d+ex}}{8b^6} + \frac{77e^3(bd-ae)(d+ex)^{3/2}}{8b^5} + \frac{231e^3(d+ex)^{5/2}}{40b^4} - \frac{33e^2(d+ex)^{7/2}}{8b^3(a+bx)}$$

$$= \frac{231e^3(bd-ae)^2\sqrt{d+ex}}{8b^6} + \frac{77e^3(bd-ae)(d+ex)^{3/2}}{8b^5} + \frac{231e^3(d+ex)^{5/2}}{40b^4} - \frac{33e^2(d+ex)^{7/2}}{8b^3(a+bx)}$$

$$= \frac{231e^3(bd-ae)^2\sqrt{d+ex}}{8b^6} + \frac{77e^3(bd-ae)(d+ex)^{3/2}}{8b^5} + \frac{231e^3(d+ex)^{5/2}}{40b^4} - \frac{33e^2(d+ex)^{7/2}}{8b^3(a+bx)}$$

Mathematica [C] time = 0.0252909, size = 52, normalized size = 0.26

$$\frac{2e^3(d+ex)^{13/2}\,{}_2F_1\left(4,\frac{13}{2};\frac{15}{2};-\frac{b(d+ex)}{ae-bd}\right)}{13(ae-bd)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(11/2)/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] (2*e^3*(d + e*x)^(13/2)*Hypergeometric2F1[4, 13/2, 15/2, -((b*(d + e*x))/(-
(b*d) + a*e))])/(13*(-(b*d) + a*e)^4)
```

Maple [B] time = 0.209, size = 719, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(11/2)/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

```
[Out] -355/8*e^7/b^5/(b*e*x+a*e)^3*(e*x+d)^(1/2)*d*a^4-693/8*e^4/b^4/((a*e-b*d)*b
)^(1/2)*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a*d^2-267/8*e^5/b^3/(b*
e*x+a*e)^3*(e*x+d)^(5/2)*a^2*d+2/5*e^3*(e*x+d)^(5/2)/b^4+231/8*e^3/b^3/((a*
e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*d^3-40*e^4/b^5*
a*d*(e*x+d)^(1/2)+89/8*e^6/b^4/(b*e*x+a*e)^3*(e*x+d)^(5/2)*a^3+59/3*e^7/b^5
/(b*e*x+a*e)^3*(e*x+d)^(3/2)*a^4+71/8*e^8/b^6/(b*e*x+a*e)^3*(e*x+d)^(1/2)*a
^5-231/8*e^6/b^6/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(
1/2))*a^3-89/8*e^3/b/(b*e*x+a*e)^3*(e*x+d)^(5/2)*d^3+59/3*e^3/b/(b*e*x+a*e)
^3*(e*x+d)^(3/2)*d^4-8/3*e^4/b^5*(e*x+d)^(3/2)*a+20*e^5/b^6*a^2*(e*x+d)^(1/
2)+8/3*e^3/b^4*(e*x+d)^(3/2)*d+20*e^3/b^4*d^2*(e*x+d)^(1/2)+693/8*e^5/b^5/(
(a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*d*a^2-236/3*
e^6/b^4/(b*e*x+a*e)^3*(e*x+d)^(3/2)*a^3*d+118*e^5/b^3/(b*e*x+a*e)^3*(e*x+d)
^(3/2)*a^2*d^2-236/3*e^4/b^2/(b*e*x+a*e)^3*(e*x+d)^(3/2)*a*d^3-71/8*e^3/b/(
b*e*x+a*e)^3*(e*x+d)^(1/2)*d^5+267/8*e^4/b^2/(b*e*x+a*e)^3*(e*x+d)^(5/2)*a*
d^2+355/4*e^6/b^4/(b*e*x+a*e)^3*(e*x+d)^(1/2)*a^3*d^2-355/4*e^5/b^3/(b*e*x+
a*e)^3*(e*x+d)^(1/2)*a^2*d^3+355/8*e^4/b^2/(b*e*x+a*e)^3*(e*x+d)^(1/2)*a*d^
4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(11/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.0346, size = 2140, normalized size = 10.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(11/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/240*(3465*(a^3*b^2*d^2*e^3 - 2*a^4*b*d*e^4 + a^5*e^5 + (b^5*d^2*e^3 - 2*
a*b^4*d*e^4 + a^2*b^3*e^5)*x^3 + 3*(a*b^4*d^2*e^3 - 2*a^2*b^3*d*e^4 + a^3*b
^2*e^5)*x^2 + 3*(a^2*b^3*d^2*e^3 - 2*a^3*b^2*d*e^4 + a^4*b*e^5)*x)*sqrt((b*
d - a*e)/b)*log((b*e*x + 2*b*d - a*e - 2*sqrt(e*x + d)*b*sqrt((b*d - a*e)/b
))/(b*x + a)) + 2*(48*b^5*e^5*x^5 - 40*b^5*d^5 - 110*a*b^4*d^4*e - 495*a^2*
b^3*d^3*e^2 + 5313*a^3*b^2*d^2*e^3 - 8085*a^4*b*d*e^4 + 3465*a^5*e^5 + 16*(
26*b^5*d*e^4 - 11*a*b^4*e^5)*x^4 + 16*(173*b^5*d^2*e^3 - 242*a*b^4*d*e^4 +
99*a^2*b^3*e^5)*x^3 - 3*(445*b^5*d^3*e^2 - 4103*a*b^4*d^2*e^3 + 6039*a^2*b^
3*d*e^4 - 2541*a^3*b^2*e^5)*x^2 - 2*(155*b^5*d^4*e + 715*a*b^4*d^3*e^2 - 72
27*a^2*b^3*d^2*e^3 + 10857*a^3*b^2*d*e^4 - 4620*a^4*b*e^5)*x)*sqrt(e*x + d)
```

```
)/(b^9*x^3 + 3*a*b^8*x^2 + 3*a^2*b^7*x + a^3*b^6), -1/120*(3465*(a^3*b^2*d^
2*e^3 - 2*a^4*b*d*e^4 + a^5*e^5 + (b^5*d^2*e^3 - 2*a*b^4*d*e^4 + a^2*b^3*e^
5)*x^3 + 3*(a*b^4*d^2*e^3 - 2*a^2*b^3*d*e^4 + a^3*b^2*e^5)*x^2 + 3*(a^2*b^3
*d^2*e^3 - 2*a^3*b^2*d*e^4 + a^4*b*e^5)*x)*sqrt(-(b*d - a*e)/b)*arctan(-sqr
t(e*x + d)*b*sqrt(-(b*d - a*e)/b)/(b*d - a*e)) - (48*b^5*e^5*x^5 - 40*b^5*d
^5 - 110*a*b^4*d^4*e - 495*a^2*b^3*d^3*e^2 + 5313*a^3*b^2*d^2*e^3 - 8085*a^
4*b*d*e^4 + 3465*a^5*e^5 + 16*(26*b^5*d*e^4 - 11*a*b^4*e^5)*x^4 + 16*(173*b
^5*d^2*e^3 - 242*a*b^4*d*e^4 + 99*a^2*b^3*e^5)*x^3 - 3*(445*b^5*d^3*e^2 - 4
103*a*b^4*d^2*e^3 + 6039*a^2*b^3*d*e^4 - 2541*a^3*b^2*e^5)*x^2 - 2*(155*b^5
*d^4*e + 715*a*b^4*d^3*e^2 - 7227*a^2*b^3*d^2*e^3 + 10857*a^3*b^2*d*e^4 - 4
620*a^4*b*e^5)*x)*sqrt(e*x + d))/(b^9*x^3 + 3*a*b^8*x^2 + 3*a^2*b^7*x + a^3
*b^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(11/2)/(b**2*x**2+2*a*b*x+a**2)**2,x)

[Out] Timed out
```

Giac [B] time = 1.22319, size = 663, normalized size = 3.3

$$\frac{231\left(b^3d^3e^3 - 3\,ab^2d^2e^4 + 3\,a^2bde^5 - a^3e^6\right)\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)}{8\,\sqrt{-b^2d+abe}b^6} - \frac{267\,(xe+d)^{\frac{5}{2}}b^5d^3e^3 - 472\,(xe+d)^{\frac{3}{2}}b^5d^4e^3 + 213\,\sqrt{xe}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(11/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")

[Out] 231/8*(b^3*d^3*e^3 - 3*a*b^2*d^2*e^4 + 3*a^2*b*d*e^5 - a^3*e^6)*arctan(sqrt
(x*e + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^6) - 1/24*(267*(x
*e + d)^(5/2)*b^5*d^3*e^3 - 472*(x*e + d)^(3/2)*b^5*d^4*e^3 + 213*sqrt(x*e
+ d)*b^5*d^5*e^3 - 801*(x*e + d)^(5/2)*a*b^4*d^2*e^4 + 1888*(x*e + d)^(3/2)
*a*b^4*d^3*e^4 - 1065*sqrt(x*e + d)*a*b^4*d^4*e^4 + 801*(x*e + d)^(5/2)*a^2
*b^3*d*e^5 - 2832*(x*e + d)^(3/2)*a^2*b^3*d^2*e^5 + 2130*sqrt(x*e + d)*a^2*
b^3*d^3*e^5 - 267*(x*e + d)^(5/2)*a^3*b^2*e^6 + 1888*(x*e + d)^(3/2)*a^3*b^
2*d*e^6 - 2130*sqrt(x*e + d)*a^3*b^2*d^2*e^6 - 472*(x*e + d)^(3/2)*a^4*b*e^
7 + 1065*sqrt(x*e + d)*a^4*b*d*e^7 - 213*sqrt(x*e + d)*a^5*e^8)/(((x*e + d)
*b - b*d + a*e)^3*b^6) + 2/15*(3*(x*e + d)^(5/2)*b^16*e^3 + 20*(x*e + d)^(3
/2)*b^16*d*e^3 + 150*sqrt(x*e + d)*b^16*d^2*e^3 - 20*(x*e + d)^(3/2)*a*b^15
*e^4 - 300*sqrt(x*e + d)*a*b^15*d*e^4 + 150*sqrt(x*e + d)*a^2*b^14*e^5)/b^2
0
```

3.1655 $\int \frac{(d+ex)^{9/2}}{\left(a^2+2abx+b^2x^2\right)^2}\,dx$

Optimal. Leaf size=172

$$-\frac{21e^2(d+ex)^{5/2}}{8b^3(a+bx)}+\frac{105e^3\sqrt{d+ex}(bd-ae)}{8b^5}-\frac{105e^3(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8b^{11/2}}-\frac{3e(d+ex)^{7/2}}{4b^2(a+bx)^2}-\frac{(d+ex)^{9/2}}{3b(a+bx)^3}+\frac{35e^3(d}{8}$$

```
[Out] (105*e^3*(b*d - a*e)*Sqrt[d + e*x])/(8*b^5) + (35*e^3*(d + e*x)^(3/2))/(8*b
^4) - (21*e^2*(d + e*x)^(5/2))/(8*b^3*(a + b*x)) - (3*e*(d + e*x)^(7/2))/(4
*b^2*(a + b*x)^2) - (d + e*x)^(9/2)/(3*b*(a + b*x)^3) - (105*e^3*(b*d - a*e
)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(8*b^(11/2))
```

Rubi [A] time = 0.0973135, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {27, 47, 50, 63, 208}

$$-\frac{21e^2(d+ex)^{5/2}}{8b^3(a+bx)}+\frac{105e^3\sqrt{d+ex}(bd-ae)}{8b^5}-\frac{105e^3(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8b^{11/2}}-\frac{3e(d+ex)^{7/2}}{4b^2(a+bx)^2}-\frac{(d+ex)^{9/2}}{3b(a+bx)^3}+\frac{35e^3(d}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(9/2)/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```


```
[Out] (105*e^3*(b*d - a*e)*Sqrt[d + e*x])/(8*b^5) + (35*e^3*(d + e*x)^(3/2))/(8*b
^4) - (21*e^2*(d + e*x)^(5/2))/(8*b^3*(a + b*x)) - (3*e*(d + e*x)^(7/2))/(4
*b^2*(a + b*x)^2) - (d + e*x)^(9/2)/(3*b*(a + b*x)^3) - (105*e^3*(b*d - a*e
)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(8*b^(11/2))
```


Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{9/2}}{\left(a^2+2abx+b^2x^2\right)^2}\,dx=\int \frac{(d+ex)^{9/2}}{(a+bx)^4}\,dx$$

$$=-\frac{(d+ex)^{9/2}}{3b(a+bx)^3}+\frac{(3e)\int \frac{(d+ex)^{7/2}}{(a+bx)^3}\,dx}{2b}$$

$$=-\frac{3e(d+ex)^{7/2}}{4b^2(a+bx)^2}-\frac{(d+ex)^{9/2}}{3b(a+bx)^3}+\frac{\left(21e^2\right)\int \frac{(d+ex)^{5/2}}{(a+bx)^2}\,dx}{8b^2}$$

$$=-\frac{21e^2(d+ex)^{5/2}}{8b^3(a+bx)}-\frac{3e(d+ex)^{7/2}}{4b^2(a+bx)^2}-\frac{(d+ex)^{9/2}}{3b(a+bx)^3}+\frac{\left(105e^3\right)\int \frac{(d+ex)^{3/2}}{a+bx}\,dx}{16b^3}$$

$$=\frac{35e^3(d+ex)^{3/2}}{8b^4}-\frac{21e^2(d+ex)^{5/2}}{8b^3(a+bx)}-\frac{3e(d+ex)^{7/2}}{4b^2(a+bx)^2}-\frac{(d+ex)^{9/2}}{3b(a+bx)^3}+\frac{\left(105e^3(bd-ae)\right)\int \frac{\sqrt{d}}{a+}}{16b^4}$$

$$=\frac{105e^3(bd-ae)\sqrt{d+ex}}{8b^5}+\frac{35e^3(d+ex)^{3/2}}{8b^4}-\frac{21e^2(d+ex)^{5/2}}{8b^3(a+bx)}-\frac{3e(d+ex)^{7/2}}{4b^2(a+bx)^2}-\frac{(d+ex)^{9/2}}{3b(a+bx)}$$

$$=\frac{105e^3(bd-ae)\sqrt{d+ex}}{8b^5}+\frac{35e^3(d+ex)^{3/2}}{8b^4}-\frac{21e^2(d+ex)^{5/2}}{8b^3(a+bx)}-\frac{3e(d+ex)^{7/2}}{4b^2(a+bx)^2}-\frac{(d+ex)^{9/2}}{3b(a+bx)}$$

$$=\frac{105e^3(bd-ae)\sqrt{d+ex}}{8b^5}+\frac{35e^3(d+ex)^{3/2}}{8b^4}-\frac{21e^2(d+ex)^{5/2}}{8b^3(a+bx)}-\frac{3e(d+ex)^{7/2}}{4b^2(a+bx)^2}-\frac{(d+ex)^{9/2}}{3b(a+bx)}$$

Mathematica [C] time = 0.0197256, size = 52, normalized size = 0.3

$$\frac{2e^3(d+ex)^{11/2}\,{}_2F_1\left(4,\frac{11}{2};\frac{13}{2};-\frac{b(d+ex)}{ae-bd}\right)}{11(ae-bd)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(9/2)/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] (2*e^3*(d + e*x)^(11/2)*Hypergeometric2F1[4, 11/2, 13/2, -((b*(d + e*x))/(-
(b*d) + a*e))])/(11*(-(b*d) + a*e)^4)
```

Maple [B] time = 0.205, size = 525, normalized size = 3.1

$$\frac{2e^3}{3b^4}(ex+d)^{\frac{3}{2}}-8\frac{e^4a\sqrt{ex+d}}{b^5}+8\frac{e^3\sqrt{ex+d}d}{b^4}-\frac{55a^2e^5}{8b^3(bxe+ae)^3}(ex+d)^{\frac{5}{2}}+\frac{55e^4ad}{4b^2(bxe+ae)^3}(ex+d)^{\frac{5}{2}}-\frac{55e^3d^2}{8b(bxe+ae}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

```
[Out] 2/3*e^3*(e*x+d)^(3/2)/b^4-8*e^4/b^5*a*(e*x+d)^(1/2)+8*e^3/b^4*(e*x+d)^(1/2)
*d-55/8*e^5/b^3/(b*e*x+a*e)^3*(e*x+d)^(5/2)*a^2+55/4*e^4/b^2/(b*e*x+a*e)^3*
(e*x+d)^(5/2)*a*d-55/8*e^3/b/(b*e*x+a*e)^3*(e*x+d)^(5/2)*d^2-35/3*e^6/b^4/(
b*e*x+a*e)^3*(e*x+d)^(3/2)*a^3+35*e^5/b^3/(b*e*x+a*e)^3*(e*x+d)^(3/2)*a^2*d
-35*e^4/b^2/(b*e*x+a*e)^3*(e*x+d)^(3/2)*a*d^2+35/3*e^3/b/(b*e*x+a*e)^3*(e*x
+d)^(3/2)*d^3-41/8*e^7/b^5/(b*e*x+a*e)^3*(e*x+d)^(1/2)*a^4+41/2*e^6/b^4/(b*
e*x+a*e)^3*(e*x+d)^(1/2)*a^3*d-123/4*e^5/b^3/(b*e*x+a*e)^3*(e*x+d)^(1/2)*d^
2*a^2+41/2*e^4/b^2/(b*e*x+a*e)^3*(e*x+d)^(1/2)*a*d^3-41/8*e^3/b/(b*e*x+a*e)
^3*(e*x+d)^(1/2)*d^4+105/8*e^5/b^5/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/
2)/((a*e-b*d)*b)^(1/2))*a^2-105/4*e^4/b^4/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x
+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a*d+105/8*e^3/b^3/((a*e-b*d)*b)^(1/2)*arctan
(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.90122, size = 1528, normalized size = 8.88

$$\left[-\frac{315\left(a^3bde^3 - a^4e^4 + \left(b^4de^3 - ab^3e^4\right)x^3 + 3\left(ab^3de^3 - a^2b^2e^4\right)x^2 + 3\left(a^2b^2de^3 - a^3be^4\right)x\right)\sqrt{\frac{bd-ae}{b}}\log\left(\frac{bex+2\,bd-ae+2\,\sqrt{ex+d}b\cdots}{bx+a}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/48*(315*(a^3*b*d*e^3 - a^4*e^4 + (b^4*d*e^3 - a*b^3*e^4)*x^3 + 3*(a*b^3
*d*e^3 - a^2*b^2*e^4)*x^2 + 3*(a^2*b^2*d*e^3 - a^3*b*e^4)*x)*sqrt((b*d - a*
e)/b)*log((b*e*x + 2*b*d - a*e + 2*sqrt(e*x + d)*b*sqrt((b*d - a*e)/b))/(b*
x + a)) - 2*(16*b^4*e^4*x^4 - 8*b^4*d^4 - 18*a*b^3*d^3*e - 63*a^2*b^2*d^2*e
^2 + 420*a^3*b*d*e^3 - 315*a^4*e^4 + 16*(13*b^4*d*e^3 - 9*a*b^3*e^4)*x^3 -
3*(55*b^4*d^2*e^2 - 318*a*b^3*d*e^3 + 231*a^2*b^2*e^4)*x^2 - 2*(25*b^4*d^3*
e + 90*a*b^3*d^2*e^2 - 567*a^2*b^2*d*e^3 + 420*a^3*b*e^4)*x)*sqrt(e*x + d))
/(b^8*x^3 + 3*a*b^7*x^2 + 3*a^2*b^6*x + a^3*b^5), -1/24*(315*(a^3*b*d*e^3 -
 a^4*e^4 + (b^4*d*e^3 - a*b^3*e^4)*x^3 + 3*(a*b^3*d*e^3 - a^2*b^2*e^4)*x^2
+ 3*(a^2*b^2*d*e^3 - a^3*b*e^4)*x)*sqrt(-(b*d - a*e)/b)*arctan(-sqrt(e*x +
d)*b*sqrt(-(b*d - a*e)/b)/(b*d - a*e)) - (16*b^4*e^4*x^4 - 8*b^4*d^4 - 18*a
*b^3*d^3*e - 63*a^2*b^2*d^2*e^2 + 420*a^3*b*d*e^3 - 315*a^4*e^4 + 16*(13*b^
4*d*e^3 - 9*a*b^3*e^4)*x^3 - 3*(55*b^4*d^2*e^2 - 318*a*b^3*d*e^3 + 231*a^2*
b^2*e^4)*x^2 - 2*(25*b^4*d^3*e + 90*a*b^3*d^2*e^2 - 567*a^2*b^2*d*e^3 + 420
*a^3*b*e^4)*x)*sqrt(e*x + d))/(b^8*x^3 + 3*a*b^7*x^2 + 3*a^2*b^6*x + a^3*b^
```

```
5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(9/2)/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24183, size = 486, normalized size = 2.83

$$\frac{105\left(b^2d^2e^3 - 2\,abde^4 + a^2e^5\right)\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)}{8\sqrt{-b^2d + abe}b^5} - \frac{165\,(xe+d)^{\frac{5}{2}}b^4d^2e^3 - 280\,(xe+d)^{\frac{3}{2}}b^4d^3e^3 + 123\sqrt{xe+d}b^4d^4e^3 -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

```
[Out] 105/8*(b^2*d^2*e^3 - 2*a*b*d*e^4 + a^2*e^5)*arctan(sqrt(x*e + d)*b/sqrt(-b^
2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^5) - 1/24*(165*(x*e + d)^(5/2)*b^4*d^
2*e^3 - 280*(x*e + d)^(3/2)*b^4*d^3*e^3 + 123*sqrt(x*e + d)*b^4*d^4*e^3 - 3
30*(x*e + d)^(5/2)*a*b^3*d*e^4 + 840*(x*e + d)^(3/2)*a*b^3*d^2*e^4 - 492*sq
rt(x*e + d)*a*b^3*d^3*e^4 + 165*(x*e + d)^(5/2)*a^2*b^2*e^5 - 840*(x*e + d)
^(3/2)*a^2*b^2*d*e^5 + 738*sqrt(x*e + d)*a^2*b^2*d^2*e^5 + 280*(x*e + d)^(3
/2)*a^3*b*e^6 - 492*sqrt(x*e + d)*a^3*b*d*e^6 + 123*sqrt(x*e + d)*a^4*e^7)/
(((x*e + d)*b - b*d + a*e)^3*b^5) + 2/3*((x*e + d)^(3/2)*b^8*e^3 + 12*sqrt(
x*e + d)*b^8*d*e^3 - 12*sqrt(x*e + d)*a*b^7*e^4)/b^12
```

3.1656 $\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^2} dx$

Optimal. Leaf size=145

$$-\frac{35e^2(d+ex)^{3/2}}{24b^3(a+bx)}-\frac{35e^3\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8b^{9/2}}-\frac{7e(d+ex)^{5/2}}{12b^2(a+bx)^2}-\frac{(d+ex)^{7/2}}{3b(a+bx)^3}+\frac{35e^3\sqrt{d+ex}}{8b^4}$$

```
[Out] (35*e^3*Sqrt[d + e*x])/(8*b^4) - (35*e^2*(d + e*x)^(3/2))/(24*b^3*(a + b*x)
) - (7*e*(d + e*x)^(5/2))/(12*b^2*(a + b*x)^2) - (d + e*x)^(7/2)/(3*b*(a +
b*x)^3) - (35*e^3*Sqrt[b*d - a*e]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d
- a*e]])/(8*b^(9/2))
```

Rubi [A] time = 0.0692103, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {27, 47, 50, 63, 208}

$$-\frac{35e^2(d+ex)^{3/2}}{24b^3(a+bx)}-\frac{35e^3\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8b^{9/2}}-\frac{7e(d+ex)^{5/2}}{12b^2(a+bx)^2}-\frac{(d+ex)^{7/2}}{3b(a+bx)^3}+\frac{35e^3\sqrt{d+ex}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] (35*e^3*Sqrt[d + e*x])/(8*b^4) - (35*e^2*(d + e*x)^(3/2))/(24*b^3*(a + b*x)
) - (7*e*(d + e*x)^(5/2))/(12*b^2*(a + b*x)^2) - (d + e*x)^(7/2)/(3*b*(a +
b*x)^3) - (35*e^3*Sqrt[b*d - a*e]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d
- a*e]])/(8*b^(9/2))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{7/2}}{\left(a^2+2abx+b^2x^2\right)^2}\,dx = \int \frac{(d+ex)^{7/2}}{(a+bx)^4}\,dx$$

$$= -\frac{(d+ex)^{7/2}}{3b(a+bx)^3} + \frac{(7e)\int \frac{(d+ex)^{5/2}}{(a+bx)^3}\,dx}{6b}$$

$$= -\frac{7e(d+ex)^{5/2}}{12b^2(a+bx)^2} - \frac{(d+ex)^{7/2}}{3b(a+bx)^3} + \frac{\left(35e^2\right)\int \frac{(d+ex)^{3/2}}{(a+bx)^2}\,dx}{24b^2}$$

$$= -\frac{35e^2(d+ex)^{3/2}}{24b^3(a+bx)} - \frac{7e(d+ex)^{5/2}}{12b^2(a+bx)^2} - \frac{(d+ex)^{7/2}}{3b(a+bx)^3} + \frac{\left(35e^3\right)\int \frac{\sqrt{d+ex}}{a+bx}\,dx}{16b^3}$$

$$= \frac{35e^3\sqrt{d+ex}}{8b^4} - \frac{35e^2(d+ex)^{3/2}}{24b^3(a+bx)} - \frac{7e(d+ex)^{5/2}}{12b^2(a+bx)^2} - \frac{(d+ex)^{7/2}}{3b(a+bx)^3} + \frac{\left(35e^3(bd-ae)\right)\int \frac{}{(a+bx}}{16b^4}$$

$$= \frac{35e^3\sqrt{d+ex}}{8b^4} - \frac{35e^2(d+ex)^{3/2}}{24b^3(a+bx)} - \frac{7e(d+ex)^{5/2}}{12b^2(a+bx)^2} - \frac{(d+ex)^{7/2}}{3b(a+bx)^3} + \frac{\left(35e^2(bd-ae)\right)\text{Subst}}{}$$

$$= \frac{35e^3\sqrt{d+ex}}{8b^4} - \frac{35e^2(d+ex)^{3/2}}{24b^3(a+bx)} - \frac{7e(d+ex)^{5/2}}{12b^2(a+bx)^2} - \frac{(d+ex)^{7/2}}{3b(a+bx)^3} - \frac{35e^3\sqrt{bd-ae}\tanh^{-1}\left(\right.}{8b^{9/2}}$$

Mathematica [C] time = 0.0186881, size = 52, normalized size = 0.36

$$\frac{2e^3(d+ex)^{9/2}\,{}_2F_1\left(4,\frac{9}{2};\frac{11}{2};-\frac{b(d+ex)}{ae-bd}\right)}{9(ae-bd)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] (2*e^3*(d + e*x)^(9/2)*Hypergeometric2F1[4, 9/2, 11/2, -((b*(d + e*x))/(-(b
*d) + a*e))])/(9*(-(b*d) + a*e)^4)
```

Maple [B] time = 0.208, size = 352, normalized size = 2.4

$$2\,\frac{e^3\sqrt{ex+d}}{b^4} + \frac{29\,e^4a}{8\,b^2\,(bxe+ae)^3}\,(ex+d)^{\frac{5}{2}} - \frac{29\,e^3d}{8\,b\,(bxe+ae)^3}\,(ex+d)^{\frac{5}{2}} + \frac{17\,a^2e^5}{3\,b^3\,(bxe+ae)^3}\,(ex+d)^{\frac{3}{2}} - \frac{34\,e^4ad}{3\,b^2\,(bxe+ae)^3}\,(ex$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

```
[Out] 2*e^3*(e*x+d)^(1/2)/b^4+29/8*e^4/b^2/(b*e*x+a*e)^3*(e*x+d)^(5/2)*a-29/8*e^3
/b/(b*e*x+a*e)^3*(e*x+d)^(5/2)*d+17/3*e^5/b^3/(b*e*x+a*e)^3*(e*x+d)^(3/2)*a
^2-34/3*e^4/b^2/(b*e*x+a*e)^3*(e*x+d)^(3/2)*a*d+17/3*e^3/b/(b*e*x+a*e)^3*(e
*x+d)^(3/2)*d^2+19/8*e^6/b^4/(b*e*x+a*e)^3*(e*x+d)^(1/2)*a^3-57/8*e^5/b^3/(
b*e*x+a*e)^3*(e*x+d)^(1/2)*d*a^2+57/8*e^4/b^2/(b*e*x+a*e)^3*(e*x+d)^(1/2)*a
*d^2-19/8*e^3/b/(b*e*x+a*e)^3*(e*x+d)^(1/2)*d^3-35/8*e^4/b^4/((a*e-b*d)*b)^
(1/2)*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a+35/8*e^3/b^3/((a*e-b*d)
*b)^(1/2)*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.94319, size = 1065, normalized size = 7.34

$$\left[\frac{105\left(b^3e^3x^3+3\,ab^2e^3x^2+3\,a^2be^3x+a^3e^3\right)\sqrt{\frac{bd-ae}{b}}\log\left(\frac{bex+2\,bd-ae-2\,\sqrt{ex+d}b\sqrt{\frac{bd-ae}{b}}}{bx+a}\right)+2\left(48\,b^3e^3x^3-8\,b^3d^3-14\,ab^2d^2e-35\right.}{48\left(b^7x^3+3\,ab^6x^2+3\,a^2b^5x+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/48*(105*(b^3*e^3*x^3 + 3*a*b^2*e^3*x^2 + 3*a^2*b*e^3*x + a^3*e^3)*sqrt((
b*d - a*e)/b)*log((b*e*x + 2*b*d - a*e - 2*sqrt(e*x + d)*b*sqrt((b*d - a*e)
/b))/(b*x + a)) + 2*(48*b^3*e^3*x^3 - 8*b^3*d^3 - 14*a*b^2*d^2*e - 35*a^2*b
*d*e^2 + 105*a^3*e^3 - 3*(29*b^3*d*e^2 - 77*a*b^2*e^3)*x^2 - 2*(19*b^3*d^2*
e + 49*a*b^2*d*e^2 - 140*a^2*b*e^3)*x)*sqrt(e*x + d))/(b^7*x^3 + 3*a*b^6*x^
2 + 3*a^2*b^5*x + a^3*b^4), -1/24*(105*(b^3*e^3*x^3 + 3*a*b^2*e^3*x^2 + 3*a
^2*b*e^3*x + a^3*e^3)*sqrt(-(b*d - a*e)/b)*arctan(-sqrt(e*x + d)*b*sqrt(-(b
*d - a*e)/b)/(b*d - a*e)) - (48*b^3*e^3*x^3 - 8*b^3*d^3 - 14*a*b^2*d^2*e -
35*a^2*b*d*e^2 + 105*a^3*e^3 - 3*(29*b^3*d*e^2 - 77*a*b^2*e^3)*x^2 - 2*(19*
b^3*d^2*e + 49*a*b^2*d*e^2 - 140*a^2*b*e^3)*x)*sqrt(e*x + d))/(b^7*x^3 + 3*
a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

[Out] Timed out

Giac [B] time = 1.24008, size = 335, normalized size = 2.31

$$\frac{35\left(bde^3-ae^4\right)\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)}{8\sqrt{-b^2d+abe}b^4}+\frac{2\sqrt{xe+d}e^3}{b^4}-\frac{87\,(xe+d)^{\frac{5}{2}}b^3de^3-136\,(xe+d)^{\frac{3}{2}}b^3d^2e^3+57\sqrt{xe+d}b^3d^3e^3-87$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

```
[Out] 35/8*(b*d*e^3 - a*e^4)*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-
b^2*d + a*b*e)*b^4) + 2*sqrt(x*e + d)*e^3/b^4 - 1/24*(87*(x*e + d)^(5/2)*b^
3*d*e^3 - 136*(x*e + d)^(3/2)*b^3*d^2*e^3 + 57*sqrt(x*e + d)*b^3*d^3*e^3 -
87*(x*e + d)^(5/2)*a*b^2*e^4 + 272*(x*e + d)^(3/2)*a*b^2*d*e^4 - 171*sqrt(x
*e + d)*a*b^2*d^2*e^4 - 136*(x*e + d)^(3/2)*a^2*b*e^5 + 171*sqrt(x*e + d)*a
^2*b*d*e^5 - 57*sqrt(x*e + d)*a^3*e^6)/(((x*e + d)*b - b*d + a*e)^3*b^4)
```

3.1657 $\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^2}\,dx$

Optimal. Leaf size=126

$$-\frac{5e^2\sqrt{d+ex}}{8b^3(a+bx)}-\frac{5e^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8b^{7/2}\sqrt{bd-ae}}-\frac{5e(d+ex)^{3/2}}{12b^2(a+bx)^2}-\frac{(d+ex)^{5/2}}{3b(a+bx)^3}$$

```
[Out] (-5*e^2*Sqrt[d + e*x])/(8*b^3*(a + b*x)) - (5*e*(d + e*x)^(3/2))/(12*b^2*(a
 + b*x)^2) - (d + e*x)^(5/2)/(3*b*(a + b*x)^3) - (5*e^3*ArcTanh[(Sqrt[b]*Sq
rt[d + e*x])/Sqrt[b*d - a*e]])/(8*b^(7/2)*Sqrt[b*d - a*e])
```

Rubi [A] time = 0.0593683, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {27, 47, 63, 208}

$$-\frac{5e^2\sqrt{d+ex}}{8b^3(a+bx)}-\frac{5e^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8b^{7/2}\sqrt{bd-ae}}-\frac{5e(d+ex)^{3/2}}{12b^2(a+bx)^2}-\frac{(d+ex)^{5/2}}{3b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] (-5*e^2*Sqrt[d + e*x])/(8*b^3*(a + b*x)) - (5*e*(d + e*x)^(3/2))/(12*b^2*(a
 + b*x)^2) - (d + e*x)^(5/2)/(3*b*(a + b*x)^3) - (5*e^3*ArcTanh[(Sqrt[b]*Sq
rt[d + e*x])/Sqrt[b*d - a*e]])/(8*b^(7/2)*Sqrt[b*d - a*e])
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\left(a^2+2abx+b^2x^2\right)^2}\,dx = \int \frac{(d+ex)^{5/2}}{(a+bx)^4}\,dx$$

$$= -\frac{(d+ex)^{5/2}}{3b(a+bx)^3} + \frac{(5e)\int \frac{(d+ex)^{3/2}}{(a+bx)^3}\,dx}{6b}$$

$$= -\frac{5e(d+ex)^{3/2}}{12b^2(a+bx)^2} - \frac{(d+ex)^{5/2}}{3b(a+bx)^3} + \frac{\left(5e^2\right)\int \frac{\sqrt{d+ex}}{(a+bx)^2}\,dx}{8b^2}$$

$$= -\frac{5e^2\sqrt{d+ex}}{8b^3(a+bx)} - \frac{5e(d+ex)^{3/2}}{12b^2(a+bx)^2} - \frac{(d+ex)^{5/2}}{3b(a+bx)^3} + \frac{\left(5e^3\right)\int \frac{1}{(a+bx)\sqrt{d+ex}}\,dx}{16b^3}$$

$$= -\frac{5e^2\sqrt{d+ex}}{8b^3(a+bx)} - \frac{5e(d+ex)^{3/2}}{12b^2(a+bx)^2} - \frac{(d+ex)^{5/2}}{3b(a+bx)^3} + \frac{\left(5e^2\right)\text{Subst}\left(\int \frac{1}{a-\frac{bd}{e}+\frac{bx^2}{e}}\,dx, x, \sqrt{d+ex}\right)}{8b^3}$$

$$= -\frac{5e^2\sqrt{d+ex}}{8b^3(a+bx)} - \frac{5e(d+ex)^{3/2}}{12b^2(a+bx)^2} - \frac{(d+ex)^{5/2}}{3b(a+bx)^3} - \frac{5e^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8b^{7/2}\sqrt{bd-ae}}$$

Mathematica [A] time = 0.144229, size = 119, normalized size = 0.94

$$\frac{5e^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{ae-bd}}\right)}{8b^{7/2}\sqrt{ae-bd}} - \frac{\sqrt{d+ex}\left(15a^2e^2+10abe(d+4ex)+b^2\left(8d^2+26dex+33e^2x^2\right)\right)}{24b^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] -(Sqrt[d + e*x]*(15*a^2*e^2 + 10*a*b*e*(d + 4*e*x) + b^2*(8*d^2 + 26*d*e*x
+ 33*e^2*x^2)))/(24*b^3*(a + b*x)^3) + (5*e^3*ArcTan[(Sqrt[b]*Sqrt[d + e*x]
)/Sqrt[-(b*d) + a*e]])/(8*b^(7/2)*Sqrt[-(b*d) + a*e])
```

Maple [A] time = 0.2, size = 204, normalized size = 1.6

$$-\frac{11\,e^3}{8\,(bxe+ae)^3\,b}(ex+d)^{\frac{5}{2}} - \frac{5\,e^4a}{3\,(bxe+ae)^3\,b^2}(ex+d)^{\frac{3}{2}} + \frac{5\,e^3d}{3\,(bxe+ae)^3\,b}(ex+d)^{\frac{3}{2}} - \frac{5\,a^2e^5}{8\,(bxe+ae)^3\,b^3}\sqrt{ex+d} + \frac{5}{4\,(bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

```
[Out] -11/8*e^3/(b*e*x+a*e)^3/b*(e*x+d)^(5/2)-5/3*e^4/(b*e*x+a*e)^3/b^2*(e*x+d)^(
3/2)*a+5/3*e^3/(b*e*x+a*e)^3/b*(e*x+d)^(3/2)*d-5/8*e^5/(b*e*x+a*e)^3/b^3*(e
*x+d)^(1/2)*a^2+5/4*e^4/(b*e*x+a*e)^3/b^2*(e*x+d)^(1/2)*a*d-5/8*e^3/(b*e*x+
a*e)^3/b*(e*x+d)^(1/2)*d^2+5/8*e^3/b^3/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)
^(1/2)/((a*e-b*d)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.02755, size = 1162, normalized size = 9.22

$$\left[\frac{15\left(b^3e^3x^3+3\,ab^2e^3x^2+3\,a^2be^3x+a^3e^3\right)\sqrt{b^2d-abe}\log\left(\frac{bex+2\,bd-ae-2\,\sqrt{b^2d-abe}\sqrt{ex+d}}{bx+a}\right)-2\left(8\,b^4d^3+2\,ab^3d^2e+5\,a^2b^2de^2-\right.}{48\left(a^3b^5d-a^4b^4e+\left(b^8d-ab^7e\right)x^3+3\left(ab^7d-a^2b^6e\right)x^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/48*(15*(b^3*e^3*x^3 + 3*a*b^2*e^3*x^2 + 3*a^2*b*e^3*x + a^3*e^3)*sqrt(b^
2*d - a*b*e)*log((b*e*x + 2*b*d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d)
)/(b*x + a)) - 2*(8*b^4*d^3 + 2*a*b^3*d^2*e + 5*a^2*b^2*d*e^2 - 15*a^3*b*e^
3 + 33*(b^4*d*e^2 - a*b^3*e^3)*x^2 + 2*(13*b^4*d^2*e + 7*a*b^3*d*e^2 - 20*a
^2*b^2*e^3)*x)*sqrt(e*x + d))/(a^3*b^5*d - a^4*b^4*e + (b^8*d - a*b^7*e)*x^
3 + 3*(a*b^7*d - a^2*b^6*e)*x^2 + 3*(a^2*b^6*d - a^3*b^5*e)*x), 1/24*(15*(b
^3*e^3*x^3 + 3*a*b^2*e^3*x^2 + 3*a^2*b*e^3*x + a^3*e^3)*sqrt(-b^2*d + a*b*e
)*arctan(sqrt(-b^2*d + a*b*e)*sqrt(e*x + d)/(b*e*x + b*d)) - (8*b^4*d^3 + 2
*a*b^3*d^2*e + 5*a^2*b^2*d*e^2 - 15*a^3*b*e^3 + 33*(b^4*d*e^2 - a*b^3*e^3)*
x^2 + 2*(13*b^4*d^2*e + 7*a*b^3*d*e^2 - 20*a^2*b^2*e^3)*x)*sqrt(e*x + d))/(
a^3*b^5*d - a^4*b^4*e + (b^8*d - a*b^7*e)*x^3 + 3*(a*b^7*d - a^2*b^6*e)*x^2
 + 3*(a^2*b^6*d - a^3*b^5*e)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25548, size = 223, normalized size = 1.77

$$\frac{5\,\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)e^3}{8\,\sqrt{-b^2d+abe}b^3}-\frac{33\,(xe+d)^{\frac{5}{2}}b^2e^3-40\,(xe+d)^{\frac{3}{2}}b^2de^3+15\,\sqrt{xe+d}b^2d^2e^3+40\,(xe+d)^{\frac{3}{2}}abe^4-30\,\sqrt{xe+d}abde}{24\,((xe+d)b-bd+ae)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

```
[Out] 5/8*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e^3/(sqrt(-b^2*d + a*b*e)*
b^3) - 1/24*(33*(x*e + d)^(5/2)*b^2*e^3 - 40*(x*e + d)^(3/2)*b^2*d*e^3 + 15
```

```
*sqrt(x*e + d)*b^2*d^2*e^3 + 40*(x*e + d)^(3/2)*a*b*e^4 - 30*sqrt(x*e + d)*
a*b*d*e^4 + 15*sqrt(x*e + d)*a^2*e^5)/(((x*e + d)*b - b*d + a*e)^3*b^3)
```

3.1658 $\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^2}\,dx$

Optimal. Leaf size=136

$$-\frac{e^2\sqrt{d+ex}}{8b^2(a+bx)(bd-ae)}+\frac{e^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8b^{5/2}(bd-ae)^{3/2}}-\frac{e\sqrt{d+ex}}{4b^2(a+bx)^2}-\frac{(d+ex)^{3/2}}{3b(a+bx)^3}$$

```
[Out] -(e*Sqrt[d + e*x])/(4*b^2*(a + b*x)^2) - (e^2*Sqrt[d + e*x])/(8*b^2*(b*d -
a*e)*(a + b*x)) - (d + e*x)^(3/2)/(3*b*(a + b*x)^3) + (e^3*ArcTanh[(Sqrt[b]
*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(8*b^(5/2)*(b*d - a*e)^(3/2))
```

Rubi [A] time = 0.0719368, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {27, 47, 51, 63, 208}

$$-\frac{e^2\sqrt{d+ex}}{8b^2(a+bx)(bd-ae)}+\frac{e^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8b^{5/2}(bd-ae)^{3/2}}-\frac{e\sqrt{d+ex}}{4b^2(a+bx)^2}-\frac{(d+ex)^{3/2}}{3b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] -(e*Sqrt[d + e*x])/(4*b^2*(a + b*x)^2) - (e^2*Sqrt[d + e*x])/(8*b^2*(b*d -
a*e)*(a + b*x)) - (d + e*x)^(3/2)/(3*b*(a + b*x)^3) + (e^3*ArcTanh[(Sqrt[b]
*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(8*b^(5/2)*(b*d - a*e)^(3/2))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\left(a^2+2abx+b^2x^2\right)^2}\,dx = \int \frac{(d+ex)^{3/2}}{(a+bx)^4}\,dx$$

$$= -\frac{(d+ex)^{3/2}}{3b(a+bx)^3} + \frac{e\int \frac{\sqrt{d+ex}}{(a+bx)^3}\,dx}{2b}$$

$$= -\frac{e\sqrt{d+ex}}{4b^2(a+bx)^2} - \frac{(d+ex)^{3/2}}{3b(a+bx)^3} + \frac{e^2\int \frac{1}{(a+bx)^2\sqrt{d+ex}}\,dx}{8b^2}$$

$$= -\frac{e\sqrt{d+ex}}{4b^2(a+bx)^2} - \frac{e^2\sqrt{d+ex}}{8b^2(bd-ae)(a+bx)} - \frac{(d+ex)^{3/2}}{3b(a+bx)^3} - \frac{e^3\int \frac{1}{(a+bx)\sqrt{d+ex}}\,dx}{16b^2(bd-ae)}$$

$$= -\frac{e\sqrt{d+ex}}{4b^2(a+bx)^2} - \frac{e^2\sqrt{d+ex}}{8b^2(bd-ae)(a+bx)} - \frac{(d+ex)^{3/2}}{3b(a+bx)^3} - \frac{e^2\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{bd}{e}+\frac{bx^2}{e}}\,dx, x, \sqrt{d+\right.}{8b^2(bd-ae)}$$

$$= -\frac{e\sqrt{d+ex}}{4b^2(a+bx)^2} - \frac{e^2\sqrt{d+ex}}{8b^2(bd-ae)(a+bx)} - \frac{(d+ex)^{3/2}}{3b(a+bx)^3} + \frac{e^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8b^{5/2}(bd-ae)^{3/2}}$$

Mathematica [C] time = 0.0162283, size = 52, normalized size = 0.38

$$\frac{2e^3(d+ex)^{5/2}\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};-\frac{b(d+ex)}{ae-bd}\right)}{5(ae-bd)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] (2*e^3*(d + e*x)^(5/2)*Hypergeometric2F1[5/2, 4, 7/2, -((b*(d + e*x))/(-(b*
d) + a*e))])/(5*(-(b*d) + a*e)^4)
```

Maple [A] time = 0.207, size = 163, normalized size = 1.2

$$\frac{e^3}{8\,(bxe+ae)^3\,(ae-bd)}(ex+d)^{\frac{5}{2}} - \frac{e^3}{3\,(bxe+ae)^3\,b}(ex+d)^{\frac{3}{2}} - \frac{e^4a}{8\,(bxe+ae)^3\,b^2}\sqrt{ex+d} + \frac{e^3d}{8\,(bxe+ae)^3\,b}\sqrt{ex+d} + \frac{}{(8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

```
[Out] 1/8*e^3/(b*e*x+a*e)^3/(a*e-b*d)*(e*x+d)^(5/2)-1/3*e^3/(b*e*x+a*e)^3/b*(e*x+
d)^(3/2)-1/8*e^4/(b*e*x+a*e)^3/b^2*(e*x+d)^(1/2)*a+1/8*e^3/(b*e*x+a*e)^3/b*
```

```
(e*x+d)^(1/2)*d+1/8*e^3/(a*e-b*d)/b^2/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^
(1/2)/((a*e-b*d)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.858, size = 1368, normalized size = 10.06

$$\left[-\frac{3\left(b^3e^3x^3+3\,ab^2e^3x^2+3\,a^2be^3x+a^3e^3\right)\sqrt{b^2d-abe}\log\left(\frac{bex+2\,bd-ae-2\,\sqrt{b^2d-abe}\sqrt{ex+d}}{bx+a}\right)+2\left(8\,b^4d^3-10\,ab^3d^2e-a^2b^2de^2+\right.}{48\left(a^3b^5d^2-2\,a^4b^4de+a^5b^3e^2+\left(b^8d^2-2\,ab^7de+a^2b^6e^2\right)x^3+3\left(ab^7d^2-2\,a^2b^6de+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/48*(3*(b^3*e^3*x^3 + 3*a*b^2*e^3*x^2 + 3*a^2*b*e^3*x + a^3*e^3)*sqrt(b^
2*d - a*b*e)*log((b*e*x + 2*b*d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d)
)/(b*x + a)) + 2*(8*b^4*d^3 - 10*a*b^3*d^2*e - a^2*b^2*d*e^2 + 3*a^3*b*e^3
+ 3*(b^4*d*e^2 - a*b^3*e^3)*x^2 + 2*(7*b^4*d^2*e - 11*a*b^3*d*e^2 + 4*a^2*b
^2*e^3)*x)*sqrt(e*x + d))/(a^3*b^5*d^2 - 2*a^4*b^4*d*e + a^5*b^3*e^2 + (b^8
*d^2 - 2*a*b^7*d*e + a^2*b^6*e^2)*x^3 + 3*(a*b^7*d^2 - 2*a^2*b^6*d*e + a^3*
b^5*e^2)*x^2 + 3*(a^2*b^6*d^2 - 2*a^3*b^5*d*e + a^4*b^4*e^2)*x), -1/24*(3*(
b^3*e^3*x^3 + 3*a*b^2*e^3*x^2 + 3*a^2*b*e^3*x + a^3*e^3)*sqrt(-b^2*d + a*b*
e)*arctan(sqrt(-b^2*d + a*b*e)*sqrt(e*x + d)/(b*e*x + b*d)) + (8*b^4*d^3 -
10*a*b^3*d^2*e - a^2*b^2*d*e^2 + 3*a^3*b*e^3 + 3*(b^4*d*e^2 - a*b^3*e^3)*x^
2 + 2*(7*b^4*d^2*e - 11*a*b^3*d*e^2 + 4*a^2*b^2*e^3)*x)*sqrt(e*x + d))/(a^3
*b^5*d^2 - 2*a^4*b^4*d*e + a^5*b^3*e^2 + (b^8*d^2 - 2*a*b^7*d*e + a^2*b^6*e
^2)*x^3 + 3*(a*b^7*d^2 - 2*a^2*b^6*d*e + a^3*b^5*e^2)*x^2 + 3*(a^2*b^6*d^2
- 2*a^3*b^5*d*e + a^4*b^4*e^2)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19194, size = 258, normalized size = 1.9

$$-\frac{\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)e^3}{8\left(b^3d-ab^2e\right)\sqrt{-b^2d+abe}}-\frac{3(xe+d)^{\frac{5}{2}}b^2e^3+8(xe+d)^{\frac{3}{2}}b^2de^3-3\sqrt{xe+d}b^2d^2e^3-8(xe+d)^{\frac{3}{2}}abe^4+6\sqrt{xe+d}ab}{24\left(b^3d-ab^2e\right)((xe+d)b-bd+ae)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

```
[Out] -1/8*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e^3/((b^3*d - a*b^2*e)*sq
rt(-b^2*d + a*b*e)) - 1/24*(3*(x*e + d)^(5/2)*b^2*e^3 + 8*(x*e + d)^(3/2)*b
^2*d*e^3 - 3*sqrt(x*e + d)*b^2*d^2*e^3 - 8*(x*e + d)^(3/2)*a*b*e^4 + 6*sqrt
(x*e + d)*a*b*d*e^4 - 3*sqrt(x*e + d)*a^2*e^5)/((b^3*d - a*b^2*e)*((x*e + d
)*b - b*d + a*e)^3)
```

3.1659 $\int \frac{\sqrt{d+ex}}{\left(a^2+2abx+b^2x^2\right)^2}\,dx$

Optimal. Leaf size=146

$$-\frac{e^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8b^{3/2}(bd-ae)^{5/2}}+\frac{e^2\sqrt{d+ex}}{8b(a+bx)(bd-ae)^2}-\frac{e\sqrt{d+ex}}{12b(a+bx)^2(bd-ae)}-\frac{\sqrt{d+ex}}{3b(a+bx)^3}$$

```
[Out] -Sqrt[d + e*x]/(3*b*(a + b*x)^3) - (e*Sqrt[d + e*x])/(12*b*(b*d - a*e)*(a +
 b*x)^2) + (e^2*Sqrt[d + e*x])/(8*b*(b*d - a*e)^2*(a + b*x)) - (e^3*ArcTanh
[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(8*b^(3/2)*(b*d - a*e)^(5/2))
```

Rubi [A] time = 0.0687249, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {27, 47, 51, 63, 208}

$$-\frac{e^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8b^{3/2}(bd-ae)^{5/2}}+\frac{e^2\sqrt{d+ex}}{8b(a+bx)(bd-ae)^2}-\frac{e\sqrt{d+ex}}{12b(a+bx)^2(bd-ae)}-\frac{\sqrt{d+ex}}{3b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] -Sqrt[d + e*x]/(3*b*(a + b*x)^3) - (e*Sqrt[d + e*x])/(12*b*(b*d - a*e)*(a +
 b*x)^2) + (e^2*Sqrt[d + e*x])/(8*b*(b*d - a*e)^2*(a + b*x)) - (e^3*ArcTanh
[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(8*b^(3/2)*(b*d - a*e)^(5/2))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d+ex}}{\left(a^2+2abx+b^2x^2\right)^2}\,dx &= \int \frac{\sqrt{d+ex}}{(a+bx)^4}\,dx \\
&= -\frac{\sqrt{d+ex}}{3b(a+bx)^3}+\frac{e\int \frac{1}{(a+bx)^3\sqrt{d+ex}}\,dx}{6b} \\
&= -\frac{\sqrt{d+ex}}{3b(a+bx)^3}-\frac{e\sqrt{d+ex}}{12b(bd-ae)(a+bx)^2}-\frac{e^2\int \frac{1}{(a+bx)^2\sqrt{d+ex}}\,dx}{8b(bd-ae)} \\
&= -\frac{\sqrt{d+ex}}{3b(a+bx)^3}-\frac{e\sqrt{d+ex}}{12b(bd-ae)(a+bx)^2}+\frac{e^2\sqrt{d+ex}}{8b(bd-ae)^2(a+bx)}+\frac{e^3\int \frac{1}{(a+bx)\sqrt{d+ex}}\,dx}{16b(bd-ae)^2} \\
&= -\frac{\sqrt{d+ex}}{3b(a+bx)^3}-\frac{e\sqrt{d+ex}}{12b(bd-ae)(a+bx)^2}+\frac{e^2\sqrt{d+ex}}{8b(bd-ae)^2(a+bx)}+\frac{e^2\,\text{Subst}\left(\int \frac{1}{a-\frac{bd}{e}+\frac{bx^2}{e}}\,dx \ldots\right)}{8b(bd-ae)^2} \\
&= -\frac{\sqrt{d+ex}}{3b(a+bx)^3}-\frac{e\sqrt{d+ex}}{12b(bd-ae)(a+bx)^2}+\frac{e^2\sqrt{d+ex}}{8b(bd-ae)^2(a+bx)}-\frac{e^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8b^{3/2}(bd-ae)^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.0134836, size = 52, normalized size = 0.36

$$\frac{2e^3(d+ex)^{3/2}\,{}_2F_1\left(\frac{3}{2},4;\frac{5}{2};-\frac{b(d+ex)}{ae-bd}\right)}{3(ae-bd)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] (2*e^3*(d + e*x)^(3/2)*Hypergeometric2F1[3/2, 4, 5/2, -((b*(d + e*x))/(-(b*
d) + a*e))])/(3*(-(b*d) + a*e)^4)
```

Maple [A] time = 0.204, size = 170, normalized size = 1.2

$$\frac{e^3b}{8\,(bxe+ae)^3\left(a^2e^2-2\,abde+b^2d^2\right)}(ex+d)^{\frac{5}{2}}+\frac{e^3}{3\,(bxe+ae)^3\,(ae-bd)}(ex+d)^{\frac{3}{2}}-\frac{e^3}{8\,(bxe+ae)^3\,b}\sqrt{ex+d}+\frac{\ldots}{8\,b\left(a^2e \ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

```
[Out] 1/8*e^3/(b*e*x+a*e)^3*b/(a^2*e^2-2*a*b*d*e+b^2*d^2)*(e*x+d)^(5/2)+1/3*e^3/(
b*e*x+a*e)^3/(a*e-b*d)*(e*x+d)^(3/2)-1/8*e^3/(b*e*x+a*e)^3/b*(e*x+d)^(1/2)+
```

```
1/8*e^3/b/(a^2*e^2-2*a*b*d*e+b^2*d^2)/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^
(1/2)/((a*e-b*d)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.10518, size = 1581, normalized size = 10.83

$$\left[\frac{3\left(b^3e^3x^3+3\,ab^2e^3x^2+3\,a^2be^3x+a^3e^3\right)\sqrt{b^2d-abe}\log\left(\frac{bex+2\,bd-ae-2\,\sqrt{b^2d-abe}\sqrt{ex+d}}{bx+a}\right)-2\left(8\,b^4d^3-22\,ab^3d^2e+17\,a^2b^2de^2\right.}{48\left(a^3b^5d^3-3\,a^4b^4d^2e+3\,a^5b^3de^2-a^6b^2e^3+\left(b^8d^3-3\,ab^7d^2e+3\,a^2b^6de^2-a^3b^5e^3\right)x^3+3\left(ab^7d^3-3\,a^2b^6d^2e+3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(b^3*e^3*x^3 + 3*a*b^2*e^3*x^2 + 3*a^2*b*e^3*x + a^3*e^3)*sqrt(b^2
*d - a*b*e)*log((b*e*x + 2*b*d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d))
/(b*x + a)) - 2*(8*b^4*d^3 - 22*a*b^3*d^2*e + 17*a^2*b^2*d*e^2 - 3*a^3*b*e^
3 - 3*(b^4*d*e^2 - a*b^3*e^3)*x^2 + 2*(b^4*d^2*e - 5*a*b^3*d*e^2 + 4*a^2*b^
2*e^3)*x)*sqrt(e*x + d))/(a^3*b^5*d^3 - 3*a^4*b^4*d^2*e + 3*a^5*b^3*d*e^2 -
 a^6*b^2*e^3 + (b^8*d^3 - 3*a*b^7*d^2*e + 3*a^2*b^6*d*e^2 - a^3*b^5*e^3)*x^
3 + 3*(a*b^7*d^3 - 3*a^2*b^6*d^2*e + 3*a^3*b^5*d*e^2 - a^4*b^4*e^3)*x^2 + 3
*(a^2*b^6*d^3 - 3*a^3*b^5*d^2*e + 3*a^4*b^4*d*e^2 - a^5*b^3*e^3)*x), 1/24*(
3*(b^3*e^3*x^3 + 3*a*b^2*e^3*x^2 + 3*a^2*b*e^3*x + a^3*e^3)*sqrt(-b^2*d + a
*b*e)*arctan(sqrt(-b^2*d + a*b*e)*sqrt(e*x + d)/(b*e*x + b*d)) - (8*b^4*d^3
 - 22*a*b^3*d^2*e + 17*a^2*b^2*d*e^2 - 3*a^3*b*e^3 - 3*(b^4*d*e^2 - a*b^3*e
^3)*x^2 + 2*(b^4*d^2*e - 5*a*b^3*d*e^2 + 4*a^2*b^2*e^3)*x)*sqrt(e*x + d))/(
a^3*b^5*d^3 - 3*a^4*b^4*d^2*e + 3*a^5*b^3*d*e^2 - a^6*b^2*e^3 + (b^8*d^3 -
3*a*b^7*d^2*e + 3*a^2*b^6*d*e^2 - a^3*b^5*e^3)*x^3 + 3*(a*b^7*d^3 - 3*a^2*b
^6*d^2*e + 3*a^3*b^5*d*e^2 - a^4*b^4*e^3)*x^2 + 3*(a^2*b^6*d^3 - 3*a^3*b^5*
d^2*e + 3*a^4*b^4*d*e^2 - a^5*b^3*e^3)*x)]
```

Sympy [B] time = 108.858, size = 4592, normalized size = 31.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

```
[Out] -66*a**3*e**6*sqrt(d + e*x)/(48*a**6*b*e**6 - 144*a**5*b**2*d*e**5 + 144*a*
*5*b**2*e**6*x - 720*a**4*b**3*d*e**5*x + 144*a**4*b**3*e**4*(d + e*x)**2 +
 480*a**3*b**4*d**3*e**3 + 1440*a**3*b**4*d**2*e**4*x - 576*a**3*b**4*d*e**
```

3*(d + e*x)**2 + 48*a**3*b**4*e**3*(d + e*x)**3 - 720*a**2*b**5*d**4*e**2 - 1440*a**2*b**5*d**3*e**3*x + 864*a**2*b**5*d**2*e**2*(d + e*x)**2 - 144*a**2*b**5*d*e**2*(d + e*x)**3 + 432*a*b**6*d**5*e + 720*a*b**6*d**4*e**2*x - 576*a*b**6*d**3*e*(d + e*x)**2 + 144*a*b**6*d**2*e*(d + e*x)**3 - 96*b**7*d**6 - 144*b**7*d**5*e*x + 144*b**7*d**4*(d + e*x)**2 - 48*b**7*d**3*(d + e*x)**3) + 198*a**2*d*e**5*sqrt(d + e*x)/(48*a**6*e**6 - 144*a**5*b*d*e**5 + 144*a**5*b*e**6*x - 720*a**4*b**2*d*e**5*x + 144*a**4*b**2*e**4*(d + e*x)**2 + 480*a**3*b**3*d**3*e**3 + 1440*a**3*b**3*d**2*e**4*x - 576*a**3*b**3*d*e**3*(d + e*x)**2 + 48*a**3*b**3*e**3*(d + e*x)**3 - 720*a**2*b**4*d**4*e**2 - 1440*a**2*b**4*d**3*e**3*x + 864*a**2*b**4*d**2*e**2*(d + e*x)**2 - 144*a**2*b**4*d*e**2*(d + e*x)**3 + 432*a*b**5*d**5*e + 720*a*b**5*d**4*e**2*x - 576*a*b**5*d**3*e*(d + e*x)**2 + 144*a*b**5*d**2*e*(d + e*x)**3 - 96*b**6*d**6 - 144*b**6*d**5*e*x + 144*b**6*d**4*(d + e*x)**2 - 48*b**6*d**3*(d + e*x)**3) - 80*a**2*e**5*(d + e*x)**(3/2)/(48*a**6*e**6 - 144*a**5*b*d*e**5 + 144*a**5*b*e**6*x - 720*a**4*b**2*d*e**5*x + 144*a**4*b**2*e**4*(d + e*x)**2 + 480*a**3*b**3*d**3*e**3 + 1440*a**3*b**3*d**2*e**4*x - 576*a**3*b**3*d*e**3*(d + e*x)**2 + 48*a**3*b**3*e**3*(d + e*x)**3 - 720*a**2*b**4*d**4*e**2 - 1440*a**2*b**4*d**3*e**3*x + 864*a**2*b**4*d**2*e**2*(d + e*x)**2 - 144*a**2*b**4*d*e**2*(d + e*x)**3 + 432*a*b**5*d**5*e + 720*a*b**5*d**4*e**2*x - 576*a*b**5*d**3*e*(d + e*x)**2 + 144*a*b**5*d**2*e*(d + e*x)**3 - 96*b**6*d**6 - 144*b**6*d**5*e*x + 144*b**6*d**4*(d + e*x)**2 - 48*b**6*d**3*(d + e*x)**3) - 198*a*b*d**2*e**4*sqrt(d + e*x)/(48*a**6*e**6 - 144*a**5*b*d*e**5 + 144*a**5*b*e**6*x - 720*a**4*b**2*d*e**5*x + 144*a**4*b**2*e**4*(d + e*x)**2 + 480*a**3*b**3*d**3*e**3 + 1440*a**3*b**3*d**2*e**4*x - 576*a**3*b**3*d*e**3*(d + e*x)**2 + 48*a**3*b**3*e**3*(d + e*x)**3 - 720*a**2*b**4*d**4*e**2 - 1440*a**2*b**4*d**3*e**3*x + 864*a**2*b**4*d**2*e**2*(d + e*x)**2 - 144*a**2*b**4*d*e**2*(d + e*x)**3 + 432*a*b**5*d**5*e + 720*a*b**5*d**4*e**2*x - 576*a*b**5*d**3*e*(d + e*x)**2 + 144*a*b**5*d**2*e*(d + e*x)**3 - 96*b**6*d**6 - 144*b**6*d**5*e*x + 144*b**6*d**4*(d + e*x)**2 - 48*b**6*d**3*(d + e*x)**3) + 160*a*b*d*e**4*(d + e*x)**(3/2)/(48*a**6*e**6 - 144*a**5*b*d*e**5 + 144*a**5*b*e**6*x - 720*a**4*b**2*d*e**5*x + 144*a**4*b**2*e**4*(d + e*x)**2 + 480*a**3*b**3*d**3*e**3 + 1440*a**3*b**3*d**2*e**4*x - 576*a**3*b**3*d*e**3*(d + e*x)**2 + 48*a**3*b**3*e**3*(d + e*x)**3 - 720*a**2*b**4*d**4*e**2 - 1440*a**2*b**4*d**3*e**3*x + 864*a**2*b**4*d**2*e**2*(d + e*x)**2 - 144*a**2*b**4*d*e**2*(d + e*x)**3 + 432*a*b**5*d**5*e + 720*a*b**5*d**4*e**2*x - 576*a*b**5*d**3*e*(d + e*x)**2 + 144*a*b**5*d**2*e*(d + e*x)**3 - 96*b**6*d**6 - 144*b**6*d**5*e*x + 144*b**6*d**4*(d + e*x)**2 - 48*b**6*d**3*(d + e*x)**3) - 30*a*b*e**4*(d + e*x)**(5/2)/(48*a**6*e**6 - 144*a**5*b*d*e**5 + 144*a**5*b*e**6*x - 720*a**4*b**2*d*e**5*x + 144*a**4*b**2*e**4*(d + e*x)**2 + 480*a**3*b**3*d**3*e**3 + 1440*a**3*b**3*d**2*e**4*x - 576*a**3*b**3*d*e**3*(d + e*x)**2 + 48*a**3*b**3*e**3*(d + e*x)**3 - 720*a**2*b**4*d**4*e**2 - 1440*a**2*b**4*d**3*e**3*x + 864*a**2*b**4*d**2*e**2*(d + e*x)**2 - 144*a**2*b**4*d*e**2*(d + e*x)**3 + 432*a*b**5*d**5*e + 720*a*b**5*d**4*e**2*x - 576*a*b**5*d**3*e*(d + e*x)**2 + 144*a*b**5*d**2*e*(d + e*x)**3 - 96*b**6*d**6 - 144*b**6*d**5*e*x + 144*b**6*d**4*(d + e*x)**2 - 48*b**6*d**3*(d + e*x)**3) + 10*a*e**4*sqrt(d + e*x)/(8*a**4*b*e**4 - 16*a**3*b**2*d*e**3 + 16*a**3*b**2*e**4*x - 48*a**2*b**3*d*e**3*x + 8*a**2*b**3*e**2*(d + e*x)**2 + 16*a*b**4*d**3*e + 48*a*b**4*d**2*e**2*x - 16*a*b**4*d*e*(d + e*x)**2 - 8*b**5*d**4 - 16*b**5*d**3*e*x + 8*b**5*d**2*(d + e*x)**2) + 5*a*e**4*sqrt(-1/(b*(a*e - b*d)**7))*log(-a**4*e**4*sqrt(-1/(b*(a*e - b*d)**7)) + 4*a**3*b*d*e**3*sqrt(-1/(b*(a*e - b*d)**7)) - 6*a**2*b**2*d**2*e**2*sqrt(-1/(b*(a*e - b*d)**7)) + 4*a*b**3*d**3*e*sqrt(-1/(b*(a*e - b*d)**7)) - b**4*d**4*sqrt(-1/(b*(a*e - b*d)**7)) + sqrt(d + e*x))/(16*b) - 5*a*e**4*sqrt(-1/(b*(a*e - b*d)**7))*log(a**4*e**4*sqrt(-1/(b*(a*e - b*d)**7)) - 4*a**3*b*d*e**3*sqrt(-1/(b*(a*e - b*d)**7)) + 6*a**2*b**2*d**2*e**2*sqrt(-1/(b*(a*e - b*d)**7)) - 4*a*b**3*d**3*e*sqrt(-1/(b*(a*e - b*d)**7)) + b**4*d**4*sqrt(-1/(b*(a*e - b*d)**7)) + sqrt(d + e*x))/(16*b) + 66*b**2*d**3*e**3*sqrt(d + e*x)/(48*a**6*e**6 - 144*a**5*b*d*e**5 + 144*a**5*b*e**6*x - 720*a**4*b**2*d*e**5*x + 144*a**4*b**2*e**4*(d + e*x)**2 + 480*a**3*b**3*d**3*e**3

```
 + 1440*a**3*b**3*d**2*e**4*x - 576*a**3*b**3*d*e**3*(d + e*x)**2 + 48*a**3
*b**3*e**3*(d + e*x)**3 - 720*a**2*b**4*d**4*e**2 - 1440*a**2*b**4*d**3*e**
3*x + 864*a**2*b**4*d**2*e**2*(d + e*x)**2 - 144*a**2*b**4*d*e**2*(d + e*x)
**3 + 432*a*b**5*d**5*e + 720*a*b**5*d**4*e**2*x - 576*a*b**5*d**3*e*(d + e
*x)**2 + 144*a*b**5*d**2*e*(d + e*x)**3 - 96*b**6*d**6 - 144*b**6*d**5*e*x
+ 144*b**6*d**4*(d + e*x)**2 - 48*b**6*d**3*(d + e*x)**3) - 80*b**2*d**2*e*
*3*(d + e*x)**(3/2)/(48*a**6*e**6 - 144*a**5*b*d*e**5 + 144*a**5*b*e**6*x -
 720*a**4*b**2*d*e**5*x + 144*a**4*b**2*e**4*(d + e*x)**2 + 480*a**3*b**3*d
**3*e**3 + 1440*a**3*b**3*d**2*e**4*x - 576*a**3*b**3*d*e**3*(d + e*x)**2 +
 48*a**3*b**3*e**3*(d + e*x)**3 - 720*a**2*b**4*d**4*e**2 - 1440*a**2*b**4*
d**3*e**3*x + 864*a**2*b**4*d**2*e**2*(d + e*x)**2 - 144*a**2*b**4*d*e**2*(
d + e*x)**3 + 432*a*b**5*d**5*e + 720*a*b**5*d**4*e**2*x - 576*a*b**5*d**3*
e*(d + e*x)**2 + 144*a*b**5*d**2*e*(d + e*x)**3 - 96*b**6*d**6 - 144*b**6*d
**5*e*x + 144*b**6*d**4*(d + e*x)**2 - 48*b**6*d**3*(d + e*x)**3) + 30*b**2
*d*e**3*(d + e*x)**(5/2)/(48*a**6*e**6 - 144*a**5*b*d*e**5 + 144*a**5*b*e**
6*x - 720*a**4*b**2*d*e**5*x + 144*a**4*b**2*e**4*(d + e*x)**2 + 480*a**3*b
**3*d**3*e**3 + 1440*a**3*b**3*d**2*e**4*x - 576*a**3*b**3*d*e**3*(d + e*x)
**2 + 48*a**3*b**3*e**3*(d + e*x)**3 - 720*a**2*b**4*d**4*e**2 - 1440*a**2*
b**4*d**3*e**3*x + 864*a**2*b**4*d**2*e**2*(d + e*x)**2 - 144*a**2*b**4*d*e
**2*(d + e*x)**3 + 432*a*b**5*d**5*e + 720*a*b**5*d**4*e**2*x - 576*a*b**5*
d**3*e*(d + e*x)**2 + 144*a*b**5*d**2*e*(d + e*x)**3 - 96*b**6*d**6 - 144*b
**6*d**5*e*x + 144*b**6*d**4*(d + e*x)**2 - 48*b**6*d**3*(d + e*x)**3) - 5*
d*e**3*sqrt(-1/(b*(a*e - b*d)**7))*log(-a**4*e**4*sqrt(-1/(b*(a*e - b*d)**7
)) + 4*a**3*b*d*e**3*sqrt(-1/(b*(a*e - b*d)**7)) - 6*a**2*b**2*d**2*e**2*sq
rt(-1/(b*(a*e - b*d)**7)) + 4*a*b**3*d**3*e*sqrt(-1/(b*(a*e - b*d)**7)) - b
**4*d**4*sqrt(-1/(b*(a*e - b*d)**7)) + sqrt(d + e*x))/16 + 5*d*e**3*sqrt(-1
/(b*(a*e - b*d)**7))*log(a**4*e**4*sqrt(-1/(b*(a*e - b*d)**7)) - 4*a**3*b*d
*e**3*sqrt(-1/(b*(a*e - b*d)**7)) + 6*a**2*b**2*d**2*e**2*sqrt(-1/(b*(a*e -
 b*d)**7)) - 4*a*b**3*d**3*e*sqrt(-1/(b*(a*e - b*d)**7)) + b**4*d**4*sqrt(-
1/(b*(a*e - b*d)**7)) + sqrt(d + e*x))/16 - 10*d*e**3*sqrt(d + e*x)/(8*a**4
*e**4 - 16*a**3*b*d*e**3 + 16*a**3*b*e**4*x - 48*a**2*b**2*d*e**3*x + 8*a**
2*b**2*e**2*(d + e*x)**2 + 16*a*b**3*d**3*e + 48*a*b**3*d**2*e**2*x - 16*a*
b**3*d*e*(d + e*x)**2 - 8*b**4*d**4 - 16*b**4*d**3*e*x + 8*b**4*d**2*(d + e
*x)**2) + 6*e**3*(d + e*x)**(3/2)/(8*a**4*e**4 - 16*a**3*b*d*e**3 + 16*a**3
*b*e**4*x - 48*a**2*b**2*d*e**3*x + 8*a**2*b**2*e**2*(d + e*x)**2 + 16*a*b*
*3*d**3*e + 48*a*b**3*d**2*e**2*x - 16*a*b**3*d*e*(d + e*x)**2 - 8*b**4*d**
4 - 16*b**4*d**3*e*x + 8*b**4*d**2*(d + e*x)**2) - 3*e**3*sqrt(-1/(b*(a*e -
 b*d)**5))*log(-a**3*e**3*sqrt(-1/(b*(a*e - b*d)**5)) + 3*a**2*b*d*e**2*sqr
t(-1/(b*(a*e - b*d)**5)) - 3*a*b**2*d**2*e*sqrt(-1/(b*(a*e - b*d)**5)) + b*
*3*d**3*sqrt(-1/(b*(a*e - b*d)**5)) + sqrt(d + e*x))/(8*b) + 3*e**3*sqrt(-1
/(b*(a*e - b*d)**5))*log(a**3*e**3*sqrt(-1/(b*(a*e - b*d)**5)) - 3*a**2*b*d
*e**2*sqrt(-1/(b*(a*e - b*d)**5)) + 3*a*b**2*d**2*e*sqrt(-1/(b*(a*e - b*d)*
*5)) - b**3*d**3*sqrt(-1/(b*(a*e - b*d)**5)) + sqrt(d + e*x))/(8*b)
```

Giac [A] time = 1.25656, size = 285, normalized size = 1.95

$$\frac{\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)e^3}{8\left(b^3d^2-2\,ab^2de+a^2be^2\right)\sqrt{-b^2d+abe}}+\frac{3\,(xe+d)^{\frac{5}{2}}b^2e^3-8\,(xe+d)^{\frac{3}{2}}b^2de^3-3\,\sqrt{xe+d}b^2d^2e^3+8\,(xe+d)^{\frac{3}{2}}abe^4+6\,\sqrt{x}}{24\left(b^3d^2-2\,ab^2de+a^2be^2\right)((xe+d)b-bd+ae)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e^3/((b^3*d^2 - 2*a*b^2*d*
e + a^2*b*e^2)*sqrt(-b^2*d + a*b*e)) + 1/24*(3*(x*e + d)^(5/2)*b^2*e^3 - 8*
(x*e + d)^(3/2)*b^2*d*e^3 - 3*sqrt(x*e + d)*b^2*d^2*e^3 + 8*(x*e + d)^(3/2)
```

```
*a*b*e^4 + 6*sqrt(x*e + d)*a*b*d*e^4 - 3*sqrt(x*e + d)*a^2*e^5)/((b^3*d^2 -
 2*a*b^2*d*e + a^2*b*e^2)*((x*e + d)*b - b*d + a*e)^3)
```

3.1660 $\int \frac{1}{\sqrt{d+ex}\left(a^2+2abx+b^2x^2\right)^2}\,dx$

Optimal. Leaf size=147

$$-\frac{5e^2\sqrt{d+ex}}{8(a+bx)(bd-ae)^3}+\frac{5e^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8\sqrt{b}(bd-ae)^{7/2}}+\frac{5e\sqrt{d+ex}}{12(a+bx)^2(bd-ae)^2}-\frac{\sqrt{d+ex}}{3(a+bx)^3(bd-ae)}$$

```
[Out] -Sqrt[d + e*x]/(3*(b*d - a*e)*(a + b*x)^3) + (5*e*Sqrt[d + e*x])/(12*(b*d -
 a*e)^2*(a + b*x)^2) - (5*e^2*Sqrt[d + e*x])/(8*(b*d - a*e)^3*(a + b*x)) +
(5*e^3*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(8*Sqrt[b]*(b*d -
a*e)^(7/2))
```

Rubi [A] time = 0.0718597, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {27, 51, 63, 208}

$$-\frac{5e^2\sqrt{d+ex}}{8(a+bx)(bd-ae)^3}+\frac{5e^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8\sqrt{b}(bd-ae)^{7/2}}+\frac{5e\sqrt{d+ex}}{12(a+bx)^2(bd-ae)^2}-\frac{\sqrt{d+ex}}{3(a+bx)^3(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

```
[Out] -Sqrt[d + e*x]/(3*(b*d - a*e)*(a + b*x)^3) + (5*e*Sqrt[d + e*x])/(12*(b*d -
 a*e)^2*(a + b*x)^2) - (5*e^2*Sqrt[d + e*x])/(8*(b*d - a*e)^3*(a + b*x)) +
(5*e^3*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(8*Sqrt[b]*(b*d -
a*e)^(7/2))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\left(a^2+2abx+b^2x^2\right)^2}\,dx = \int \frac{1}{(a+bx)^4\sqrt{d+ex}}\,dx$$
$$= -\frac{\sqrt{d+ex}}{3(bd-ae)(a+bx)^3} - \frac{(5e)\int \frac{1}{(a+bx)^3\sqrt{d+ex}}\,dx}{6(bd-ae)}$$
$$= -\frac{\sqrt{d+ex}}{3(bd-ae)(a+bx)^3} + \frac{5e\sqrt{d+ex}}{12(bd-ae)^2(a+bx)^2} + \frac{\left(5e^2\right)\int \frac{1}{(a+bx)^2\sqrt{d+ex}}\,dx}{8(bd-ae)^2}$$
$$= -\frac{\sqrt{d+ex}}{3(bd-ae)(a+bx)^3} + \frac{5e\sqrt{d+ex}}{12(bd-ae)^2(a+bx)^2} - \frac{5e^2\sqrt{d+ex}}{8(bd-ae)^3(a+bx)} - \frac{\left(5e^3\right)\int -}{16(b}$$
$$= -\frac{\sqrt{d+ex}}{3(bd-ae)(a+bx)^3} + \frac{5e\sqrt{d+ex}}{12(bd-ae)^2(a+bx)^2} - \frac{5e^2\sqrt{d+ex}}{8(bd-ae)^3(a+bx)} - \frac{\left(5e^2\right)\mathrm{Su}}{}$$
$$= -\frac{\sqrt{d+ex}}{3(bd-ae)(a+bx)^3} + \frac{5e\sqrt{d+ex}}{12(bd-ae)^2(a+bx)^2} - \frac{5e^2\sqrt{d+ex}}{8(bd-ae)^3(a+bx)} + \frac{5e^3\tanh}{8\sqrt{b}(b}$$

Mathematica [C] time = 0.0114543, size = 50, normalized size = 0.34

$$\frac{2e^3\sqrt{d+ex}\,{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};-\frac{b(d+ex)}{ae-bd}\right)}{(ae-bd)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

```
[Out] (2*e^3*Sqrt[d + e*x]*Hypergeometric2F1[1/2, 4, 3/2, -((b*(d + e*x))/(-(b*d)
 + a*e))])/(-(b*d) + a*e)^4
```

Maple [A] time = 0.196, size = 147, normalized size = 1.

$$\frac{e^3}{(3ae-3bd)(bxe+ae)^3}\sqrt{ex+d} + \frac{5e^3}{12(ae-bd)^2(bxe+ae)^2}\sqrt{ex+d} + \frac{5e^3}{8(ae-bd)^3(bxe+ae)}\sqrt{ex+d} + \frac{5e^3}{8(ae-bd)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

```
[Out] 1/3*e^3*(e*x+d)^(1/2)/(a*e-b*d)/(b*e*x+a*e)^3+5/12*e^3/(a*e-b*d)^2*(e*x+d)^
(1/2)/(b*e*x+a*e)^2+5/8*e^3/(a*e-b*d)^3*(e*x+d)^(1/2)/(b*e*x+a*e)+5/8*e^3/(
a*e-b*d)^3/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.11195, size = 1805, normalized size = 12.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/48*(15*(b^3*e^3*x^3 + 3*a*b^2*e^3*x^2 + 3*a^2*b*e^3*x + a^3*e^3)*sqrt(b
^2*d - a*b*e)*log((b*e*x + 2*b*d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d
))/(b*x + a)) + 2*(8*b^4*d^3 - 34*a*b^3*d^2*e + 59*a^2*b^2*d*e^2 - 33*a^3*b
*e^3 + 15*(b^4*d*e^2 - a*b^3*e^3)*x^2 - 10*(b^4*d^2*e - 5*a*b^3*d*e^2 + 4*a
^2*b^2*e^3)*x)*sqrt(e*x + d))/(a^3*b^5*d^4 - 4*a^4*b^4*d^3*e + 6*a^5*b^3*d^
2*e^2 - 4*a^6*b^2*d*e^3 + a^7*b*e^4 + (b^8*d^4 - 4*a*b^7*d^3*e + 6*a^2*b^6*
d^2*e^2 - 4*a^3*b^5*d*e^3 + a^4*b^4*e^4)*x^3 + 3*(a*b^7*d^4 - 4*a^2*b^6*d^3
*e + 6*a^3*b^5*d^2*e^2 - 4*a^4*b^4*d*e^3 + a^5*b^3*e^4)*x^2 + 3*(a^2*b^6*d^
4 - 4*a^3*b^5*d^3*e + 6*a^4*b^4*d^2*e^2 - 4*a^5*b^3*d*e^3 + a^6*b^2*e^4)*x)
, -1/24*(15*(b^3*e^3*x^3 + 3*a*b^2*e^3*x^2 + 3*a^2*b*e^3*x + a^3*e^3)*sqrt(
-b^2*d + a*b*e)*arctan(sqrt(-b^2*d + a*b*e)*sqrt(e*x + d)/(b*e*x + b*d)) +
(8*b^4*d^3 - 34*a*b^3*d^2*e + 59*a^2*b^2*d*e^2 - 33*a^3*b*e^3 + 15*(b^4*d*e
^2 - a*b^3*e^3)*x^2 - 10*(b^4*d^2*e - 5*a*b^3*d*e^2 + 4*a^2*b^2*e^3)*x)*sqr
t(e*x + d))/(a^3*b^5*d^4 - 4*a^4*b^4*d^3*e + 6*a^5*b^3*d^2*e^2 - 4*a^6*b^2*
d*e^3 + a^7*b*e^4 + (b^8*d^4 - 4*a*b^7*d^3*e + 6*a^2*b^6*d^2*e^2 - 4*a^3*b^
5*d*e^3 + a^4*b^4*e^4)*x^3 + 3*(a*b^7*d^4 - 4*a^2*b^6*d^3*e + 6*a^3*b^5*d^2
*e^2 - 4*a^4*b^4*d*e^3 + a^5*b^3*e^4)*x^2 + 3*(a^2*b^6*d^4 - 4*a^3*b^5*d^3*
e + 6*a^4*b^4*d^2*e^2 - 4*a^5*b^3*d*e^3 + a^6*b^2*e^4)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^4\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

```
[Out] Integral(1/((a + b*x)**4*sqrt(d + e*x)), x)
```

Giac [A] time = 1.13792, size = 315, normalized size = 2.14

$$-\frac{5\arctan\left(\frac{\sqrt{xe+d}\,b}{\sqrt{-b^2d+abe}}\right)e^3}{8\left(b^3d^3-3\,ab^2d^2e+3\,a^2bde^2-a^3e^3\right)\sqrt{-b^2d+abe}}-\frac{15\,(xe+d)^{\frac{5}{2}}b^2e^3-40\,(xe+d)^{\frac{3}{2}}b^2de^3+33\,\sqrt{xe+d}\,b^2d^2e^3+40\,(xe\cdots}{24\left(b^3d^3-3\,ab^2d^2e+3\,a^2bde^2-a^3e^3\right)((\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

```
[Out] -5/8*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e^3/((b^3*d^3 - 3*a*b^2*d
^2*e + 3*a^2*b*d*e^2 - a^3*e^3)*sqrt(-b^2*d + a*b*e)) - 1/24*(15*(x*e + d)^
(5/2)*b^2*e^3 - 40*(x*e + d)^(3/2)*b^2*d*e^3 + 33*sqrt(x*e + d)*b^2*d^2*e^3
 + 40*(x*e + d)^(3/2)*a*b*e^4 - 66*sqrt(x*e + d)*a*b*d*e^4 + 33*sqrt(x*e +
d)*a^2*e^5)/((b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3)*((x*e + d)
*b - b*d + a*e)^3)
```

3.1661 $\int \frac{1}{(d+ex)^{3/2}\left(a^2+2abx+b^2x^2\right)^2}\,dx$

Optimal. Leaf size=173

$$-\frac{35e^3}{8\sqrt{d+ex}(bd-ae)^4}-\frac{35e^2}{24(a+bx)\sqrt{d+ex}(bd-ae)^3}+\frac{35\sqrt{b}e^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8(bd-ae)^{9/2}}+\frac{7e}{12(a+bx)^2\sqrt{d+ex}(bd-ae)^2}-\frac{}{3(a}$$

```
[Out] (-35*e^3)/(8*(b*d - a*e)^4*Sqrt[d + e*x]) - 1/(3*(b*d - a*e)*(a + b*x)^3*Sq
rt[d + e*x]) + (7*e)/(12*(b*d - a*e)^2*(a + b*x)^2*Sqrt[d + e*x]) - (35*e^2
)/(24*(b*d - a*e)^3*(a + b*x)*Sqrt[d + e*x]) + (35*Sqrt[b]*e^3*ArcTanh[(Sqr
t[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(8*(b*d - a*e)^(9/2))
```

Rubi [A] time = 0.0835521, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {27, 51, 63, 208}

$$-\frac{35e^3}{8\sqrt{d+ex}(bd-ae)^4}-\frac{35e^2}{24(a+bx)\sqrt{d+ex}(bd-ae)^3}+\frac{35\sqrt{b}e^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8(bd-ae)^{9/2}}+\frac{7e}{12(a+bx)^2\sqrt{d+ex}(bd-ae)^2}-\frac{}{3(a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

```
[Out] (-35*e^3)/(8*(b*d - a*e)^4*Sqrt[d + e*x]) - 1/(3*(b*d - a*e)*(a + b*x)^3*Sq
rt[d + e*x]) + (7*e)/(12*(b*d - a*e)^2*(a + b*x)^2*Sqrt[d + e*x]) - (35*e^2
)/(24*(b*d - a*e)^3*(a + b*x)*Sqrt[d + e*x]) + (35*Sqrt[b]*e^3*ArcTanh[(Sqr
t[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(8*(b*d - a*e)^(9/2))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\left(a^2+2abx+b^2x^2\right)^2}\,dx = \int \frac{1}{(a+bx)^4(d+ex)^{3/2}}\,dx$$

$$= -\frac{1}{3(bd-ae)(a+bx)^3\sqrt{d+ex}} - \frac{(7e)\int \frac{1}{(a+bx)^3(d+ex)^{3/2}}\,dx}{6(bd-ae)}$$

$$= -\frac{1}{3(bd-ae)(a+bx)^3\sqrt{d+ex}} + \frac{7e}{12(bd-ae)^2(a+bx)^2\sqrt{d+ex}} + \frac{\left(35e^2\right)\int \frac{}{(a+l}}{24(bd}$$

$$= -\frac{1}{3(bd-ae)(a+bx)^3\sqrt{d+ex}} + \frac{7e}{12(bd-ae)^2(a+bx)^2\sqrt{d+ex}} - \frac{}{24(bd-ae)^3}$$

$$= -\frac{35e^3}{8(bd-ae)^4\sqrt{d+ex}} - \frac{1}{3(bd-ae)(a+bx)^3\sqrt{d+ex}} + \frac{7e}{12(bd-ae)^2(a+bx)^2\sqrt{}}$$

$$= -\frac{35e^3}{8(bd-ae)^4\sqrt{d+ex}} - \frac{1}{3(bd-ae)(a+bx)^3\sqrt{d+ex}} + \frac{7e}{12(bd-ae)^2(a+bx)^2\sqrt{}}$$

$$= -\frac{35e^3}{8(bd-ae)^4\sqrt{d+ex}} - \frac{1}{3(bd-ae)(a+bx)^3\sqrt{d+ex}} + \frac{7e}{12(bd-ae)^2(a+bx)^2\sqrt{}}$$

Mathematica [C] time = 0.0147443, size = 50, normalized size = 0.29

$$-\frac{2e^3\,{}_2F_1\left(-\frac{1}{2},4;\frac{1}{2};-\frac{b(d+ex)}{ae-bd}\right)}{\sqrt{d+ex}(ae-bd)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

```
[Out] (-2*e^3*Hypergeometric2F1[-1/2, 4, 1/2, -((b*(d + e*x))/(-(b*d) + a*e))])/(
(-(b*d) + a*e)^4*Sqrt[d + e*x])
```

Maple [B] time = 0.21, size = 292, normalized size = 1.7

$$-2\frac{e^3}{(ae-bd)^4\sqrt{ex+d}} - \frac{19\,b^3e^3}{8\,(ae-bd)^4\,(bxe+ae)^3}(ex+d)^{\frac{5}{2}} - \frac{17\,e^4b^2a}{3\,(ae-bd)^4\,(bxe+ae)^3}(ex+d)^{\frac{3}{2}} + \frac{17\,b^3e^3d}{3\,(ae-bd)^4\,(bxe+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

```
[Out] -2*e^3/(a*e-b*d)^4/(e*x+d)^(1/2)-19/8*e^3*b^3/(a*e-b*d)^4/(b*e*x+a*e)^3*(e*
x+d)^(5/2)-17/3*e^4*b^2/(a*e-b*d)^4/(b*e*x+a*e)^3*(e*x+d)^(3/2)*a+17/3*e^3*
b^3/(a*e-b*d)^4/(b*e*x+a*e)^3*(e*x+d)^(3/2)*d-29/8*e^5*b/(a*e-b*d)^4/(b*e*x
+a*e)^3*(e*x+d)^(1/2)*a^2+29/4*e^4*b^2/(a*e-b*d)^4/(b*e*x+a*e)^3*(e*x+d)^(1
/2)*a*d-29/8*e^3*b^3/(a*e-b*d)^4/(b*e*x+a*e)^3*(e*x+d)^(1/2)*d^2-35/8*e^3*b
```

```
/(a*e-b*d)^4/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2)
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 2.46893, size = 2441, normalized size = 14.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")

[Out] [1/48*(105*(b^3*e^4*x^4 + a^3*d*e^3 + (b^3*d*e^3 + 3*a*b^2*e^4)*x^3 + 3*(a*
b^2*d*e^3 + a^2*b*e^4)*x^2 + (3*a^2*b*d*e^3 + a^3*e^4)*x)*sqrt(b/(b*d - a*e
))*log((b*e*x + 2*b*d - a*e + 2*(b*d - a*e)*sqrt(e*x + d)*sqrt(b/(b*d - a*e
)))/(b*x + a)) - 2*(105*b^3*e^3*x^3 + 8*b^3*d^3 - 38*a*b^2*d^2*e + 87*a^2*b
*d*e^2 + 48*a^3*e^3 + 35*(b^3*d*e^2 + 8*a*b^2*e^3)*x^2 - 7*(2*b^3*d^2*e - 1
4*a*b^2*d*e^2 - 33*a^2*b*e^3)*x)*sqrt(e*x + d))/(a^3*b^4*d^5 - 4*a^4*b^3*d^
4*e + 6*a^5*b^2*d^3*e^2 - 4*a^6*b*d^2*e^3 + a^7*d*e^4 + (b^7*d^4*e - 4*a*b^
6*d^3*e^2 + 6*a^2*b^5*d^2*e^3 - 4*a^3*b^4*d*e^4 + a^4*b^3*e^5)*x^4 + (b^7*d
^5 - a*b^6*d^4*e - 6*a^2*b^5*d^3*e^2 + 14*a^3*b^4*d^2*e^3 - 11*a^4*b^3*d*e^
4 + 3*a^5*b^2*e^5)*x^3 + 3*(a*b^6*d^5 - 3*a^2*b^5*d^4*e + 2*a^3*b^4*d^3*e^2
 + 2*a^4*b^3*d^2*e^3 - 3*a^5*b^2*d*e^4 + a^6*b*e^5)*x^2 + (3*a^2*b^5*d^5 -
11*a^3*b^4*d^4*e + 14*a^4*b^3*d^3*e^2 - 6*a^5*b^2*d^2*e^3 - a^6*b*d*e^4 + a
^7*e^5)*x), 1/24*(105*(b^3*e^4*x^4 + a^3*d*e^3 + (b^3*d*e^3 + 3*a*b^2*e^4)*
x^3 + 3*(a*b^2*d*e^3 + a^2*b*e^4)*x^2 + (3*a^2*b*d*e^3 + a^3*e^4)*x)*sqrt(-
b/(b*d - a*e))*arctan(-(b*d - a*e)*sqrt(e*x + d)*sqrt(-b/(b*d - a*e))/(b*e*
x + b*d)) - (105*b^3*e^3*x^3 + 8*b^3*d^3 - 38*a*b^2*d^2*e + 87*a^2*b*d*e^2
+ 48*a^3*e^3 + 35*(b^3*d*e^2 + 8*a*b^2*e^3)*x^2 - 7*(2*b^3*d^2*e - 14*a*b^2
*d*e^2 - 33*a^2*b*e^3)*x)*sqrt(e*x + d))/(a^3*b^4*d^5 - 4*a^4*b^3*d^4*e + 6
*a^5*b^2*d^3*e^2 - 4*a^6*b*d^2*e^3 + a^7*d*e^4 + (b^7*d^4*e - 4*a*b^6*d^3*e
^2 + 6*a^2*b^5*d^2*e^3 - 4*a^3*b^4*d*e^4 + a^4*b^3*e^5)*x^4 + (b^7*d^5 - a*
b^6*d^4*e - 6*a^2*b^5*d^3*e^2 + 14*a^3*b^4*d^2*e^3 - 11*a^4*b^3*d*e^4 + 3*a
^5*b^2*e^5)*x^3 + 3*(a*b^6*d^5 - 3*a^2*b^5*d^4*e + 2*a^3*b^4*d^3*e^2 + 2*a^
4*b^3*d^2*e^3 - 3*a^5*b^2*d*e^4 + a^6*b*e^5)*x^2 + (3*a^2*b^5*d^5 - 11*a^3*
b^4*d^4*e + 14*a^4*b^3*d^3*e^2 - 6*a^5*b^2*d^2*e^3 - a^6*b*d*e^4 + a^7*e^5)
*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22019, size = 437, normalized size = 2.53

$$-\frac{35\,b\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)e^3}{8\left(b^4d^4-4\,ab^3d^3e+6\,a^2b^2d^2e^2-4\,a^3bde^3+a^4e^4\right)\sqrt{-b^2d+abe}}-\frac{2\,e^3}{\left(b^4d^4-4\,ab^3d^3e+6\,a^2b^2d^2e^2-4\,a^3bde^3+a^4e^4\right)\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

```
[Out] -35/8*b*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e^3/((b^4*d^4 - 4*a*b^
3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4)*sqrt(-b^2*d + a*b*e)
) - 2*e^3/((b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a
^4*e^4)*sqrt(x*e + d)) - 1/24*(57*(x*e + d)^(5/2)*b^3*e^3 - 136*(x*e + d)^(
3/2)*b^3*d*e^3 + 87*sqrt(x*e + d)*b^3*d^2*e^3 + 136*(x*e + d)^(3/2)*a*b^2*e
^4 - 174*sqrt(x*e + d)*a*b^2*d*e^4 + 87*sqrt(x*e + d)*a^2*b*e^5)/((b^4*d^4
- 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4)*((x*e + d)*b
 - b*d + a*e)^3)
```

3.1662 $\int \frac{1}{(d+ex)^{5/2}\left(a^2+2abx+b^2x^2\right)^2}\,dx$

Optimal. Leaf size=200

$$\frac{105b^{3/2}e^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8(bd-ae)^{11/2}}-\frac{105be^3}{8\sqrt{d+ex}(bd-ae)^5}-\frac{35e^3}{8(d+ex)^{3/2}(bd-ae)^4}-\frac{21e^2}{8(a+bx)(d+ex)^{3/2}(bd-ae)^3}+\frac{}{4(a+bx)^2}$$

```
[Out] (-35*e^3)/(8*(b*d - a*e)^4*(d + e*x)^(3/2)) - 1/(3*(b*d - a*e)*(a + b*x)^3*
(d + e*x)^(3/2)) + (3*e)/(4*(b*d - a*e)^2*(a + b*x)^2*(d + e*x)^(3/2)) - (2
1*e^2)/(8*(b*d - a*e)^3*(a + b*x)*(d + e*x)^(3/2)) - (105*b*e^3)/(8*(b*d -
a*e)^5*Sqrt[d + e*x]) + (105*b^(3/2)*e^3*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sq
rt[b*d - a*e]])/(8*(b*d - a*e)^(11/2))
```

Rubi [A] time = 0.140279, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {27, 51, 63, 208}

$$\frac{105b^{3/2}e^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8(bd-ae)^{11/2}}-\frac{105be^3}{8\sqrt{d+ex}(bd-ae)^5}-\frac{35e^3}{8(d+ex)^{3/2}(bd-ae)^4}-\frac{21e^2}{8(a+bx)(d+ex)^{3/2}(bd-ae)^3}+\frac{}{4(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

```
[Out] (-35*e^3)/(8*(b*d - a*e)^4*(d + e*x)^(3/2)) - 1/(3*(b*d - a*e)*(a + b*x)^3*
(d + e*x)^(3/2)) + (3*e)/(4*(b*d - a*e)^2*(a + b*x)^2*(d + e*x)^(3/2)) - (2
1*e^2)/(8*(b*d - a*e)^3*(a + b*x)*(d + e*x)^(3/2)) - (105*b*e^3)/(8*(b*d -
a*e)^5*Sqrt[d + e*x]) + (105*b^(3/2)*e^3*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sq
rt[b*d - a*e]])/(8*(b*d - a*e)^(11/2))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{5/2}\left(a^2+2abx+b^2x^2\right)^2}\,dx = \int \frac{1}{(a+bx)^4(d+ex)^{5/2}}\,dx$$

$$= -\frac{1}{3(bd-ae)(a+bx)^3(d+ex)^{3/2}} - \frac{(3e)\int \frac{1}{(a+bx)^3(d+ex)^{5/2}}\,dx}{2(bd-ae)}$$

$$= -\frac{1}{3(bd-ae)(a+bx)^3(d+ex)^{3/2}} + \frac{3e}{4(bd-ae)^2(a+bx)^2(d+ex)^{3/2}} + \frac{\left(21e^2\right)\int}{8(}$$

$$= -\frac{1}{3(bd-ae)(a+bx)^3(d+ex)^{3/2}} + \frac{3e}{4(bd-ae)^2(a+bx)^2(d+ex)^{3/2}} - \frac{}{8(bd-ae}$$

$$= -\frac{35e^3}{8(bd-ae)^4(d+ex)^{3/2}} - \frac{1}{3(bd-ae)(a+bx)^3(d+ex)^{3/2}} + \frac{3e}{4(bd-ae)^2(a+bx}$$

$$= -\frac{35e^3}{8(bd-ae)^4(d+ex)^{3/2}} - \frac{1}{3(bd-ae)(a+bx)^3(d+ex)^{3/2}} + \frac{3e}{4(bd-ae)^2(a+bx}$$

$$= -\frac{35e^3}{8(bd-ae)^4(d+ex)^{3/2}} - \frac{1}{3(bd-ae)(a+bx)^3(d+ex)^{3/2}} + \frac{3e}{4(bd-ae)^2(a+bx}$$

$$= -\frac{35e^3}{8(bd-ae)^4(d+ex)^{3/2}} - \frac{1}{3(bd-ae)(a+bx)^3(d+ex)^{3/2}} + \frac{3e}{4(bd-ae)^2(a+bx}$$

Mathematica [C] time = 0.0173553, size = 52, normalized size = 0.26

$$-\frac{2e^3\,{}_2F_1\left(-\frac{3}{2},4;-\frac{1}{2};-\frac{b(d+ex)}{ae-bd}\right)}{3(d+ex)^{3/2}(ae-bd)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

```
[Out] (-2*e^3*Hypergeometric2F1[-3/2, 4, -1/2, -((b*(d + e*x))/(-(b*d) + a*e))])/
(3*(-(b*d) + a*e)^4*(d + e*x)^(3/2))
```

Maple [A] time = 0.212, size = 319, normalized size = 1.6

$$-\frac{2e^3}{3(ae-bd)^4}(ex+d)^{-\frac{3}{2}} + 8\frac{e^3b}{(ae-bd)^5\sqrt{ex+d}} + \frac{41e^3b^4}{8(ae-bd)^5(bxe+ae)^3}(ex+d)^{\frac{5}{2}} + \frac{35e^4b^3a}{3(ae-bd)^5(bxe+ae)^3}(ex+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

```
[Out] -2/3*e^3/(a*e-b*d)^4/(e*x+d)^(3/2)+8*e^3/(a*e-b*d)^5*b/(e*x+d)^(1/2)+41/8*e
^3*b^4/(a*e-b*d)^5/(b*e*x+a*e)^3*(e*x+d)^(5/2)+35/3*e^4*b^3/(a*e-b*d)^5/(b*
e*x+a*e)^3*(e*x+d)^(3/2)*a-35/3*e^3*b^4/(a*e-b*d)^5/(b*e*x+a*e)^3*(e*x+d)^(
```

```
3/2)*d+55/8*e^5*b^2/(a*e-b*d)^5/(b*e*x+a*e)^3*(e*x+d)^(1/2)*a^2-55/4*e^4*b^
3/(a*e-b*d)^5/(b*e*x+a*e)^3*(e*x+d)^(1/2)*a*d+55/8*e^3*b^4/(a*e-b*d)^5/(b*e
*x+a*e)^3*(e*x+d)^(1/2)*d^2+105/8*e^3*b^2/(a*e-b*d)^5/((a*e-b*d)*b)^(1/2)*a
rctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.56335, size = 3717, normalized size = 18.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/48*(315*(b^4*e^5*x^5 + a^3*b*d^2*e^3 + (2*b^4*d*e^4 + 3*a*b^3*e^5)*x^4
+ (b^4*d^2*e^3 + 6*a*b^3*d*e^4 + 3*a^2*b^2*e^5)*x^3 + (3*a*b^3*d^2*e^3 + 6*
a^2*b^2*d*e^4 + a^3*b*e^5)*x^2 + (3*a^2*b^2*d^2*e^3 + 2*a^3*b*d*e^4)*x)*sqr
t(b/(b*d - a*e))*log((b*e*x + 2*b*d - a*e - 2*(b*d - a*e)*sqrt(e*x + d)*sqr
t(b/(b*d - a*e)))/(b*x + a)) + 2*(315*b^4*e^4*x^4 + 8*b^4*d^4 - 50*a*b^3*d^
3*e + 165*a^2*b^2*d^2*e^2 + 208*a^3*b*d*e^3 - 16*a^4*e^4 + 420*(b^4*d*e^3 +
 2*a*b^3*e^4)*x^3 + 63*(b^4*d^2*e^2 + 18*a*b^3*d*e^3 + 11*a^2*b^2*e^4)*x^2
- 18*(b^4*d^3*e - 10*a*b^3*d^2*e^2 - 53*a^2*b^2*d*e^3 - 8*a^3*b*e^4)*x)*sqr
t(e*x + d))/(a^3*b^5*d^7 - 5*a^4*b^4*d^6*e + 10*a^5*b^3*d^5*e^2 - 10*a^6*b^
2*d^4*e^3 + 5*a^7*b*d^3*e^4 - a^8*d^2*e^5 + (b^8*d^5*e^2 - 5*a*b^7*d^4*e^3
+ 10*a^2*b^6*d^3*e^4 - 10*a^3*b^5*d^2*e^5 + 5*a^4*b^4*d*e^6 - a^5*b^3*e^7)*
x^5 + (2*b^8*d^6*e - 7*a*b^7*d^5*e^2 + 5*a^2*b^6*d^4*e^3 + 10*a^3*b^5*d^3*e
^4 - 20*a^4*b^4*d^2*e^5 + 13*a^5*b^3*d*e^6 - 3*a^6*b^2*e^7)*x^4 + (b^8*d^7
+ a*b^7*d^6*e - 17*a^2*b^6*d^5*e^2 + 35*a^3*b^5*d^4*e^3 - 25*a^4*b^4*d^3*e^
4 - a^5*b^3*d^2*e^5 + 9*a^6*b^2*d*e^6 - 3*a^7*b*e^7)*x^3 + (3*a*b^7*d^7 - 9
*a^2*b^6*d^6*e + a^3*b^5*d^5*e^2 + 25*a^4*b^4*d^4*e^3 - 35*a^5*b^3*d^3*e^4
+ 17*a^6*b^2*d^2*e^5 - a^7*b*d*e^6 - a^8*e^7)*x^2 + (3*a^2*b^6*d^7 - 13*a^3
*b^5*d^6*e + 20*a^4*b^4*d^5*e^2 - 10*a^5*b^3*d^4*e^3 - 5*a^6*b^2*d^3*e^4 +
7*a^7*b*d^2*e^5 - 2*a^8*d*e^6)*x), 1/24*(315*(b^4*e^5*x^5 + a^3*b*d^2*e^3 +
 (2*b^4*d*e^4 + 3*a*b^3*e^5)*x^4 + (b^4*d^2*e^3 + 6*a*b^3*d*e^4 + 3*a^2*b^2
*e^5)*x^3 + (3*a*b^3*d^2*e^3 + 6*a^2*b^2*d*e^4 + a^3*b*e^5)*x^2 + (3*a^2*b^
2*d^2*e^3 + 2*a^3*b*d*e^4)*x)*sqrt(-b/(b*d - a*e))*arctan(-(b*d - a*e)*sqrt
(e*x + d)*sqrt(-b/(b*d - a*e))/(b*e*x + b*d)) - (315*b^4*e^4*x^4 + 8*b^4*d^
4 - 50*a*b^3*d^3*e + 165*a^2*b^2*d^2*e^2 + 208*a^3*b*d*e^3 - 16*a^4*e^4 + 4
20*(b^4*d*e^3 + 2*a*b^3*e^4)*x^3 + 63*(b^4*d^2*e^2 + 18*a*b^3*d*e^3 + 11*a^
2*b^2*e^4)*x^2 - 18*(b^4*d^3*e - 10*a*b^3*d^2*e^2 - 53*a^2*b^2*d*e^3 - 8*a^
3*b*e^4)*x)*sqrt(e*x + d))/(a^3*b^5*d^7 - 5*a^4*b^4*d^6*e + 10*a^5*b^3*d^5*
e^2 - 10*a^6*b^2*d^4*e^3 + 5*a^7*b*d^3*e^4 - a^8*d^2*e^5 + (b^8*d^5*e^2 - 5
*a*b^7*d^4*e^3 + 10*a^2*b^6*d^3*e^4 - 10*a^3*b^5*d^2*e^5 + 5*a^4*b^4*d*e^6
- a^5*b^3*e^7)*x^5 + (2*b^8*d^6*e - 7*a*b^7*d^5*e^2 + 5*a^2*b^6*d^4*e^3 + 1
0*a^3*b^5*d^3*e^4 - 20*a^4*b^4*d^2*e^5 + 13*a^5*b^3*d*e^6 - 3*a^6*b^2*e^7)*
x^4 + (b^8*d^7 + a*b^7*d^6*e - 17*a^2*b^6*d^5*e^2 + 35*a^3*b^5*d^4*e^3 - 25
```

```
*a^4*b^4*d^3*e^4 - a^5*b^3*d^2*e^5 + 9*a^6*b^2*d*e^6 - 3*a^7*b*e^7)*x^3 + (
3*a*b^7*d^7 - 9*a^2*b^6*d^6*e + a^3*b^5*d^5*e^2 + 25*a^4*b^4*d^4*e^3 - 35*a
^5*b^3*d^3*e^4 + 17*a^6*b^2*d^2*e^5 - a^7*b*d*e^6 - a^8*e^7)*x^2 + (3*a^2*b
^6*d^7 - 13*a^3*b^5*d^6*e + 20*a^4*b^4*d^5*e^2 - 10*a^5*b^3*d^4*e^3 - 5*a^6
*b^2*d^3*e^4 + 7*a^7*b*d^2*e^5 - 2*a^8*d*e^6)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(5/2)/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22311, size = 576, normalized size = 2.88

$$-\frac{105\, b^2 \arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right) e^3}{8\left(b^5d^5 - 5\, ab^4d^4e + 10\, a^2b^3d^3e^2 - 10\, a^3b^2d^2e^3 + 5\, a^4bde^4 - a^5e^5\right)\sqrt{-b^2d + abe}} - \frac{315\,(xe+d)^4b^4e^3 - 840\,(xe+d)^3b^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

```
[Out] -105/8*b^2*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e^3/((b^5*d^5 - 5*a
*b^4*d^4*e + 10*a^2*b^3*d^3*e^2 - 10*a^3*b^2*d^2*e^3 + 5*a^4*b*d*e^4 - a^5*
e^5)*sqrt(-b^2*d + a*b*e)) - 1/24*(315*(x*e + d)^4*b^4*e^3 - 840*(x*e + d)^
3*b^4*d*e^3 + 693*(x*e + d)^2*b^4*d^2*e^3 - 144*(x*e + d)*b^4*d^3*e^3 - 16*
b^4*d^4*e^3 + 840*(x*e + d)^3*a*b^3*e^4 - 1386*(x*e + d)^2*a*b^3*d*e^4 + 43
2*(x*e + d)*a*b^3*d^2*e^4 + 64*a*b^3*d^3*e^4 + 693*(x*e + d)^2*a^2*b^2*e^5
- 432*(x*e + d)*a^2*b^2*d*e^5 - 96*a^2*b^2*d^2*e^5 + 144*(x*e + d)*a^3*b*e^
6 + 64*a^3*b*d*e^6 - 16*a^4*e^7)/((b^5*d^5 - 5*a*b^4*d^4*e + 10*a^2*b^3*d^3
*e^2 - 10*a^3*b^2*d^2*e^3 + 5*a^4*b*d*e^4 - a^5*e^5)*((x*e + d)^(3/2)*b - s
qrt(x*e + d)*b*d + sqrt(x*e + d)*a*e)^3)
```

3.1663 $\int \frac{1}{(d+ex)^{7/2}\left(a^2+2abx+b^2x^2\right)^2}\,dx$

Optimal. Leaf size=229

$$-\frac{231b^2e^3}{8\sqrt{d+ex}(bd-ae)^6}+\frac{231b^{5/2}e^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8(bd-ae)^{13/2}}-\frac{77be^3}{8(d+ex)^{3/2}(bd-ae)^5}-\frac{231e^3}{40(d+ex)^{5/2}(bd-ae)^4}-\frac{33e}{8(a+bx)(d+e}$$

```
[Out] (-231*e^3)/(40*(b*d - a*e)^4*(d + e*x)^(5/2)) - 1/(3*(b*d - a*e)*(a + b*x)^
3*(d + e*x)^(5/2)) + (11*e)/(12*(b*d - a*e)^2*(a + b*x)^2*(d + e*x)^(5/2))
- (33*e^2)/(8*(b*d - a*e)^3*(a + b*x)*(d + e*x)^(5/2)) - (77*b*e^3)/(8*(b*d
 - a*e)^5*(d + e*x)^(3/2)) - (231*b^2*e^3)/(8*(b*d - a*e)^6*Sqrt[d + e*x])
+ (231*b^(5/2)*e^3*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(8*(b*
d - a*e)^(13/2))
```

Rubi [A] time = 0.184573, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {27, 51, 63, 208}

$$-\frac{231b^2e^3}{8\sqrt{d+ex}(bd-ae)^6}+\frac{231b^{5/2}e^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8(bd-ae)^{13/2}}-\frac{77be^3}{8(d+ex)^{3/2}(bd-ae)^5}-\frac{231e^3}{40(d+ex)^{5/2}(bd-ae)^4}-\frac{33e}{8(a+bx)(d+e}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

```
[Out] (-231*e^3)/(40*(b*d - a*e)^4*(d + e*x)^(5/2)) - 1/(3*(b*d - a*e)*(a + b*x)^
3*(d + e*x)^(5/2)) + (11*e)/(12*(b*d - a*e)^2*(a + b*x)^2*(d + e*x)^(5/2))
- (33*e^2)/(8*(b*d - a*e)^3*(a + b*x)*(d + e*x)^(5/2)) - (77*b*e^3)/(8*(b*d
 - a*e)^5*(d + e*x)^(3/2)) - (231*b^2*e^3)/(8*(b*d - a*e)^6*Sqrt[d + e*x])
+ (231*b^(5/2)*e^3*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(8*(b*
d - a*e)^(13/2))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{7/2}\left(a^2+2abx+b^2x^2\right)^2}\,dx = \int \frac{1}{(a+bx)^4(d+ex)^{7/2}}\,dx$$

$$= -\frac{1}{3(bd-ae)(a+bx)^3(d+ex)^{5/2}} - \frac{(11e)\int \frac{1}{(a+bx)^3(d+ex)^{7/2}}\,dx}{6(bd-ae)}$$

$$= -\frac{1}{3(bd-ae)(a+bx)^3(d+ex)^{5/2}} + \frac{11e}{12(bd-ae)^2(a+bx)^2(d+ex)^{5/2}} + \frac{\left(33e^2\right)\int}{8}$$

$$= -\frac{1}{3(bd-ae)(a+bx)^3(d+ex)^{5/2}} + \frac{11e}{12(bd-ae)^2(a+bx)^2(d+ex)^{5/2}} - \frac{}{8(bd-a}$$

$$= -\frac{231e^3}{40(bd-ae)^4(d+ex)^{5/2}} - \frac{1}{3(bd-ae)(a+bx)^3(d+ex)^{5/2}} + \frac{11e}{12(bd-ae)^2(a+}$$

$$= -\frac{231e^3}{40(bd-ae)^4(d+ex)^{5/2}} - \frac{1}{3(bd-ae)(a+bx)^3(d+ex)^{5/2}} + \frac{11e}{12(bd-ae)^2(a+}$$

$$= -\frac{231e^3}{40(bd-ae)^4(d+ex)^{5/2}} - \frac{1}{3(bd-ae)(a+bx)^3(d+ex)^{5/2}} + \frac{11e}{12(bd-ae)^2(a+}$$

$$= -\frac{231e^3}{40(bd-ae)^4(d+ex)^{5/2}} - \frac{1}{3(bd-ae)(a+bx)^3(d+ex)^{5/2}} + \frac{11e}{12(bd-ae)^2(a+}$$

$$= -\frac{231e^3}{40(bd-ae)^4(d+ex)^{5/2}} - \frac{1}{3(bd-ae)(a+bx)^3(d+ex)^{5/2}} + \frac{11e}{12(bd-ae)^2(a+}$$

Mathematica [C] time = 0.021571, size = 52, normalized size = 0.23

$$-\frac{2e^3\,{}_2F_1\left(-\frac{5}{2},4;-\frac{3}{2};-\frac{b(d+ex)}{ae-bd}\right)}{5(d+ex)^{5/2}(ae-bd)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

```
[Out] (-2*e^3*Hypergeometric2F1[-5/2, 4, -3/2, -((b*(d + e*x))/(-(b*d) + a*e))])/
(5*(-(b*d) + a*e)^4*(d + e*x)^(5/2))
```

Maple [A] time = 0.212, size = 344, normalized size = 1.5

$$-\frac{2\,e^3}{5\,(ae-bd)^4}(ex+d)^{-\frac{5}{2}} - 20\,\frac{e^3b^2}{(ae-bd)^6\sqrt{ex+d}} + \frac{8\,e^3b}{3\,(ae-bd)^5}(ex+d)^{-\frac{3}{2}} - \frac{71\,e^3b^5}{8\,(ae-bd)^6\,(bxe+ae)^3}(ex+d)^{\frac{5}{2}} - \frac{}{3\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^2,x)

[Out] -2/5*e^3/(a*e-b*d)^4/(e*x+d)^(5/2)-20*e^3/(a*e-b*d)^6*b^2/(e*x+d)^(1/2)+8/3*e^3/(a*e-b*d)^5*b/(e*x+d)^(3/2)-71/8*e^3*b^5/(a*e-b*d)^6/(b*e*x+a*e)^3*(e*x+d)^(5/2)-59/3*e^4*b^4/(a*e-b*d)^6/(b*e*x+a*e)^3*(e*x+d)^(3/2)*a+59/3*e^3*b^5/(a*e-b*d)^6/(b*e*x+a*e)^3*(e*x+d)^(3/2)*d-89/8*e^5*b^3/(a*e-b*d)^6/(b*e*x+a*e)^3*(e*x+d)^(1/2)*a^2+89/4*e^4*b^4/(a*e-b*d)^6/(b*e*x+a*e)^3*(e*x+d)^(1/2)*a*d-89/8*e^3*b^5/(a*e-b*d)^6/(b*e*x+a*e)^3*(e*x+d)^(1/2)*d^2-231/8*e^3*b^3/(a*e-b*d)^6/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 2.77801, size = 5176, normalized size = 22.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")

[Out] [1/240*(3465*(b^5*e^6*x^6 + a^3*b^2*d^3*e^3 + 3*(b^5*d*e^5 + a*b^4*e^6)*x^5 + 3*(b^5*d^2*e^4 + 3*a*b^4*d*e^5 + a^2*b^3*e^6)*x^4 + (b^5*d^3*e^3 + 9*a*b^4*d^2*e^4 + 9*a^2*b^3*d*e^5 + a^3*b^2*e^6)*x^3 + 3*(a*b^4*d^3*e^3 + 3*a^2*b^3*d^2*e^4 + a^3*b^2*d*e^5)*x^2 + 3*(a^2*b^3*d^3*e^3 + a^3*b^2*d^2*e^4)*x)*sqrt(b/(b*d - a*e))*log((b*e*x + 2*b*d - a*e + 2*(b*d - a*e)*sqrt(e*x + d)*sqrt(b/(b*d - a*e)))/(b*x + a)) - 2*(3465*b^5*e^5*x^5 + 40*b^5*d^5 - 310*a*b^4*d^4*e + 1335*a^2*b^3*d^3*e^2 + 2768*a^3*b^2*d^2*e^3 - 416*a^4*b*d*e^4 + 48*a^5*e^5 + 1155*(7*b^5*d*e^4 + 8*a*b^4*e^5)*x^4 + 231*(23*b^5*d^2*e^3 + 94*a*b^4*d*e^4 + 33*a^2*b^3*e^5)*x^3 + 99*(5*b^5*d^3*e^2 + 146*a*b^4*d^2*e^3 + 183*a^2*b^3*d*e^4 + 16*a^3*b^2*e^5)*x^2 - 11*(10*b^5*d^4*e - 130*a*b^4*d^3*e^2 - 1119*a^2*b^3*d^2*e^3 - 352*a^3*b^2*d*e^4 + 16*a^4*b*e^5)*x)*sqrt(e*x + d))/(a^3*b^6*d^9 - 6*a^4*b^5*d^8*e + 15*a^5*b^4*d^7*e^2 - 20*a^6*b^3*d^6*e^3 + 15*a^7*b^2*d^5*e^4 - 6*a^8*b*d^4*e^5 + a^9*d^3*e^6 + (b^9*d^6*e^3 - 6*a*b^8*d^5*e^4 + 15*a^2*b^7*d^4*e^5 - 20*a^3*b^6*d^3*e^6 + 15*a^4*b^5*d^2*e^7 - 6*a^5*b^4*d*e^8 + a^6*b^3*e^9)*x^6 + 3*(b^9*d^7*e^2 - 5*a*b^8*d^6*e^3 + 9*a^2*b^7*d^5*e^4 - 5*a^3*b^6*d^4*e^5 - 5*a^4*b^5*d^3*e^6 + 9*a^5*b^4*d^2*e^7 - 5*a^6*b^3*d*e^8 + a^7*b^2*e^9)*x^5 + 3*(b^9*d^8*e - 3*a*b^8*d^7*e^2 - 2*a^2*b^7*d^6*e^3 + 19*a^3*b^6*d^5*e^4 - 30*a^4*b^5*d^4*e^5 + 19*a^5*b^4*d^3*e^6 - 2*a^6*b^3*d^2*e^7 - 3*a^7*b^2*d*e^8 + a^8*b*e^9)*x^4 + (b^9*d^9 + 3*a*b^8*d^8*e - 30*a^2*b^7*d^7*e^2 + 62*a^3*b^6*d^6*e^3 - 36*a^4*b^5*d^5*e^4 - 36*a^5*b^4*d^4*e^5 + 62*a^6*b^3*d^3*e^6 - 30*a^7*b^2*d^2*e^7 + 3*a^8*b*d*e^8 + a^9*e^9)*x^3 + 3*(a*b^8*d^9 - 3*a^2*b^7*d^8*e - 2*a^3*b^6*d^7*e^2 + 19*a^4*b^5*d^6*e^3 - 30*a^5*b^4*d^5*e^4 + 19*a^6*b^3*d^4*e^5 - 2*a^7*b^2*d^3*e^6 - 3*a^8*b*d^2*e^7 + a^9*d*e^8)*x^2 + 3*(a^2*b^7*d^9 - 5*a^3*b^6*d^8*e + 9*a^4*b^5*d^7*e^2 - 5*a^5*b^4*d^6*e^3 - 5*a^6*b^3*d^5*e^4 + 9*a^7

```
*b^2*d^4*e^5 - 5*a^8*b*d^3*e^6 + a^9*d^2*e^7)*x), 1/120*(3465*(b^5*e^6*x^6
+ a^3*b^2*d^3*e^3 + 3*(b^5*d*e^5 + a*b^4*e^6)*x^5 + 3*(b^5*d^2*e^4 + 3*a*b^
4*d*e^5 + a^2*b^3*e^6)*x^4 + (b^5*d^3*e^3 + 9*a*b^4*d^2*e^4 + 9*a^2*b^3*d*e
^5 + a^3*b^2*e^6)*x^3 + 3*(a*b^4*d^3*e^3 + 3*a^2*b^3*d^2*e^4 + a^3*b^2*d*e^
5)*x^2 + 3*(a^2*b^3*d^3*e^3 + a^3*b^2*d^2*e^4)*x)*sqrt(-b/(b*d - a*e))*arct
an(-(b*d - a*e)*sqrt(e*x + d)*sqrt(-b/(b*d - a*e))/(b*e*x + b*d)) - (3465*b
^5*e^5*x^5 + 40*b^5*d^5 - 310*a*b^4*d^4*e + 1335*a^2*b^3*d^3*e^2 + 2768*a^3
*b^2*d^2*e^3 - 416*a^4*b*d*e^4 + 48*a^5*e^5 + 1155*(7*b^5*d*e^4 + 8*a*b^4*e
^5)*x^4 + 231*(23*b^5*d^2*e^3 + 94*a*b^4*d*e^4 + 33*a^2*b^3*e^5)*x^3 + 99*(
5*b^5*d^3*e^2 + 146*a*b^4*d^2*e^3 + 183*a^2*b^3*d*e^4 + 16*a^3*b^2*e^5)*x^2
 - 11*(10*b^5*d^4*e - 130*a*b^4*d^3*e^2 - 1119*a^2*b^3*d^2*e^3 - 352*a^3*b^
2*d*e^4 + 16*a^4*b*e^5)*x)*sqrt(e*x + d))/(a^3*b^6*d^9 - 6*a^4*b^5*d^8*e +
15*a^5*b^4*d^7*e^2 - 20*a^6*b^3*d^6*e^3 + 15*a^7*b^2*d^5*e^4 - 6*a^8*b*d^4*
e^5 + a^9*d^3*e^6 + (b^9*d^6*e^3 - 6*a*b^8*d^5*e^4 + 15*a^2*b^7*d^4*e^5 - 2
0*a^3*b^6*d^3*e^6 + 15*a^4*b^5*d^2*e^7 - 6*a^5*b^4*d*e^8 + a^6*b^3*e^9)*x^6
 + 3*(b^9*d^7*e^2 - 5*a*b^8*d^6*e^3 + 9*a^2*b^7*d^5*e^4 - 5*a^3*b^6*d^4*e^5
 - 5*a^4*b^5*d^3*e^6 + 9*a^5*b^4*d^2*e^7 - 5*a^6*b^3*d*e^8 + a^7*b^2*e^9)*x
^5 + 3*(b^9*d^8*e - 3*a*b^8*d^7*e^2 - 2*a^2*b^7*d^6*e^3 + 19*a^3*b^6*d^5*e^
4 - 30*a^4*b^5*d^4*e^5 + 19*a^5*b^4*d^3*e^6 - 2*a^6*b^3*d^2*e^7 - 3*a^7*b^2
*d*e^8 + a^8*b*e^9)*x^4 + (b^9*d^9 + 3*a*b^8*d^8*e - 30*a^2*b^7*d^7*e^2 + 6
2*a^3*b^6*d^6*e^3 - 36*a^4*b^5*d^5*e^4 - 36*a^5*b^4*d^4*e^5 + 62*a^6*b^3*d^
3*e^6 - 30*a^7*b^2*d^2*e^7 + 3*a^8*b*d*e^8 + a^9*e^9)*x^3 + 3*(a*b^8*d^9 -
3*a^2*b^7*d^8*e - 2*a^3*b^6*d^7*e^2 + 19*a^4*b^5*d^6*e^3 - 30*a^5*b^4*d^5*e
^4 + 19*a^6*b^3*d^4*e^5 - 2*a^7*b^2*d^3*e^6 - 3*a^8*b*d^2*e^7 + a^9*d*e^8)*
x^2 + 3*(a^2*b^7*d^9 - 5*a^3*b^6*d^8*e + 9*a^4*b^5*d^7*e^2 - 5*a^5*b^4*d^6*
e^3 - 5*a^6*b^3*d^5*e^4 + 9*a^7*b^2*d^4*e^5 - 5*a^8*b*d^3*e^6 + a^9*d^2*e^7
)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(7/2)/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20805, size = 635, normalized size = 2.77

$$-\frac{231\, b^3 \arctan\left(\frac{\sqrt{xe+d}\,b}{\sqrt{-b^2d+abe}}\right)e^3}{8\left(b^6d^6 - 6\,ab^5d^5e + 15\,a^2b^4d^4e^2 - 20\,a^3b^3d^3e^3 + 15\,a^4b^2d^2e^4 - 6\,a^5bde^5 + a^6e^6\right)\sqrt{-b^2d+abe}} - \frac{2\left(150\,(xe+d)^2b^2e^3 \cdots\right.}{15\left(b^6d^6 - 6\,ab^5d^5e + \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

```
[Out] -231/8*b^3*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e^3/((b^6*d^6 - 6*a
*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 -
 6*a^5*b*d*e^5 + a^6*e^6)*sqrt(-b^2*d + a*b*e)) - 2/15*(150*(x*e + d)^2*b^2
*e^3 + 20*(x*e + d)*b^2*d*e^3 + 3*b^2*d^2*e^3 - 20*(x*e + d)*a*b*e^4 - 6*a*
b*d*e^4 + 3*a^2*e^5)/((b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^
3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 + a^6*e^6)*(x*e + d)^(5/
```

```
2)) - 1/24*(213*(x*e + d)^(5/2)*b^5*e^3 - 472*(x*e + d)^(3/2)*b^5*d*e^3 + 2
67*sqrt(x*e + d)*b^5*d^2*e^3 + 472*(x*e + d)^(3/2)*a*b^4*e^4 - 534*sqrt(x*e
 + d)*a*b^4*d*e^4 + 267*sqrt(x*e + d)*a^2*b^3*e^5)/((b^6*d^6 - 6*a*b^5*d^5*
e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a^5*b*
d*e^5 + a^6*e^6)*((x*e + d)*b - b*d + a*e)^3)
```

3.1664 $\int \frac{(d+ex)^{15/2}}{(a^2+2abx+b^2x^2)^3} dx$

Optimal. Leaf size=253

$$-\frac{13e^2(d+ex)^{11/2}}{16b^3(a+bx)^3}-\frac{143e^3(d+ex)^{9/2}}{64b^4(a+bx)^2}-\frac{1287e^4(d+ex)^{7/2}}{128b^5(a+bx)}+\frac{3003e^5(d+ex)^{3/2}(bd-ae)}{128b^7}+\frac{9009e^5\sqrt{d+ex}(bd-ae)^2}{128b^8}-$$

```
[Out] (9009*e^5*(b*d - a*e)^2*Sqrt[d + e*x])/(128*b^8) + (3003*e^5*(b*d - a*e)*(d
 + e*x)^(3/2))/(128*b^7) + (9009*e^5*(d + e*x)^(5/2))/(640*b^6) - (1287*e^4
*(d + e*x)^(7/2))/(128*b^5*(a + b*x)) - (143*e^3*(d + e*x)^(9/2))/(64*b^4*(
a + b*x)^2) - (13*e^2*(d + e*x)^(11/2))/(16*b^3*(a + b*x)^3) - (3*e*(d + e*
x)^(13/2))/(8*b^2*(a + b*x)^4) - (d + e*x)^(15/2)/(5*b*(a + b*x)^5) - (9009
*e^5*(b*d - a*e)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(1
28*b^(17/2))
```

Rubi [A] time = 0.183265, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {27, 47, 50, 63, 208}

$$-\frac{13e^2(d+ex)^{11/2}}{16b^3(a+bx)^3}-\frac{143e^3(d+ex)^{9/2}}{64b^4(a+bx)^2}-\frac{1287e^4(d+ex)^{7/2}}{128b^5(a+bx)}+\frac{3003e^5(d+ex)^{3/2}(bd-ae)}{128b^7}+\frac{9009e^5\sqrt{d+ex}(bd-ae)^2}{128b^8}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(15/2)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```


```
[Out] (9009*e^5*(b*d - a*e)^2*Sqrt[d + e*x])/(128*b^8) + (3003*e^5*(b*d - a*e)*(d
 + e*x)^(3/2))/(128*b^7) + (9009*e^5*(d + e*x)^(5/2))/(640*b^6) - (1287*e^4
*(d + e*x)^(7/2))/(128*b^5*(a + b*x)) - (143*e^3*(d + e*x)^(9/2))/(64*b^4*(
a + b*x)^2) - (13*e^2*(d + e*x)^(11/2))/(16*b^3*(a + b*x)^3) - (3*e*(d + e*
x)^(13/2))/(8*b^2*(a + b*x)^4) - (d + e*x)^(15/2)/(5*b*(a + b*x)^5) - (9009
*e^5*(b*d - a*e)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(1
28*b^(17/2))
```


Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{15/2}}{\left(a^2+2abx+b^2x^2\right)^3}\,dx = \int \frac{(d+ex)^{15/2}}{(a+bx)^6}\,dx$$

$$= -\frac{(d+ex)^{15/2}}{5b(a+bx)^5} + \frac{(3e)\int \frac{(d+ex)^{13/2}}{(a+bx)^5}\,dx}{2b}$$

$$= -\frac{3e(d+ex)^{13/2}}{8b^2(a+bx)^4} - \frac{(d+ex)^{15/2}}{5b(a+bx)^5} + \frac{\left(39e^2\right)\int \frac{(d+ex)^{11/2}}{(a+bx)^4}\,dx}{16b^2}$$

$$= -\frac{13e^2(d+ex)^{11/2}}{16b^3(a+bx)^3} - \frac{3e(d+ex)^{13/2}}{8b^2(a+bx)^4} - \frac{(d+ex)^{15/2}}{5b(a+bx)^5} + \frac{\left(143e^3\right)\int \frac{(d+ex)^{9/2}}{(a+bx)^3}\,dx}{32b^3}$$

$$= -\frac{143e^3(d+ex)^{9/2}}{64b^4(a+bx)^2} - \frac{13e^2(d+ex)^{11/2}}{16b^3(a+bx)^3} - \frac{3e(d+ex)^{13/2}}{8b^2(a+bx)^4} - \frac{(d+ex)^{15/2}}{5b(a+bx)^5} + \frac{\left(1287e^4\right)\int \frac{(d+ex)^{7/2}}{(a+bx)^2}\,d}{128b^4}$$

$$= -\frac{1287e^4(d+ex)^{7/2}}{128b^5(a+bx)} - \frac{143e^3(d+ex)^{9/2}}{64b^4(a+bx)^2} - \frac{13e^2(d+ex)^{11/2}}{16b^3(a+bx)^3} - \frac{3e(d+ex)^{13/2}}{8b^2(a+bx)^4} - \frac{(d+ex)^{15/2}}{5b(a+bx)^5} +$$

$$= \frac{9009e^5(d+ex)^{5/2}}{640b^6} - \frac{1287e^4(d+ex)^{7/2}}{128b^5(a+bx)} - \frac{143e^3(d+ex)^{9/2}}{64b^4(a+bx)^2} - \frac{13e^2(d+ex)^{11/2}}{16b^3(a+bx)^3} - \frac{3e(d+ex)^{13/}}{8b^2(a+bx)^4}$$

$$= \frac{3003e^5(bd-ae)(d+ex)^{3/2}}{128b^7} + \frac{9009e^5(d+ex)^{5/2}}{640b^6} - \frac{1287e^4(d+ex)^{7/2}}{128b^5(a+bx)} - \frac{143e^3(d+ex)^{9/2}}{64b^4(a+bx)^2} - \frac{13}{1}$$

$$= \frac{9009e^5(bd-ae)^2\sqrt{d+ex}}{128b^8} + \frac{3003e^5(bd-ae)(d+ex)^{3/2}}{128b^7} + \frac{9009e^5(d+ex)^{5/2}}{640b^6} - \frac{1287e^4(d+e}{128b^5(a+b}$$

$$= \frac{9009e^5(bd-ae)^2\sqrt{d+ex}}{128b^8} + \frac{3003e^5(bd-ae)(d+ex)^{3/2}}{128b^7} + \frac{9009e^5(d+ex)^{5/2}}{640b^6} - \frac{1287e^4(d+e}{128b^5(a+b}$$

$$= \frac{9009e^5(bd-ae)^2\sqrt{d+ex}}{128b^8} + \frac{3003e^5(bd-ae)(d+ex)^{3/2}}{128b^7} + \frac{9009e^5(d+ex)^{5/2}}{640b^6} - \frac{1287e^4(d+e}{128b^5(a+b}$$

Mathematica [C] time = 0.0270306, size = 52, normalized size = 0.21

$$\frac{2e^5(d+ex)^{17/2}\,{}_2F_1\left(6,\frac{17}{2};\frac{19}{2};-\frac{b(d+ex)}{ae-bd}\right)}{17(ae-bd)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(15/2)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] (2*e^5*(d + e*x)^(17/2)*Hypergeometric2F1[6, 17/2, 19/2, -((b*(d + e*x))/(-
(b*d) + a*e))])/(17*(-(b*d) + a*e)^6)
```

Maple [B] time = 0.244, size = 1164, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(15/2)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] 2/5*e^5*(e*x+d)^(5/2)/b^6-84*e^6/b^7*a*d*(e*x+d)^(1/2)+9443/64*e^9/b^5/(b*e
*x+a*e)^5*(e*x+d)^(7/2)*a^4+5327/128*e^8/b^4/(b*e*x+a*e)^5*(e*x+d)^(9/2)*a^
3+1001/5*e^10/b^6/(b*e*x+a*e)^5*(e*x+d)^(5/2)*a^5-5327/128*e^5/b/(b*e*x+a*e
)^5*(e*x+d)^(9/2)*d^3-1001/5*e^5/b/(b*e*x+a*e)^5*(e*x+d)^(5/2)*d^5+7837/64*
e^5/b/(b*e*x+a*e)^5*(e*x+d)^(3/2)*d^6-3633/128*e^5/b/(b*e*x+a*e)^5*(e*x+d)^
(1/2)*d^7+9443/64*e^5/b/(b*e*x+a*e)^5*(e*x+d)^(7/2)*d^4+9009/128*e^5/b^5/((
a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*d^3+3633/128*
e^12/b^8/(b*e*x+a*e)^5*(e*x+d)^(1/2)*a^7-9009/128*e^8/b^8/((a*e-b*d)*b)^(1/
2)*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a^3+7837/64*e^11/b^7/(b*e*x+
a*e)^5*(e*x+d)^(3/2)*a^6+117555/64*e^7/b^3/(b*e*x+a*e)^5*(e*x+d)^(3/2)*a^2*
d^4-23511/32*e^6/b^2/(b*e*x+a*e)^5*(e*x+d)^(3/2)*a*d^5-27027/128*e^6/b^6/((
a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a*d^2-25431/1
28*e^11/b^7/(b*e*x+a*e)^5*(e*x+d)^(1/2)*a^6*d+76293/128*e^10/b^6/(b*e*x+a*e
)^5*(e*x+d)^(1/2)*a^5*d^2+127155/128*e^8/b^4/(b*e*x+a*e)^5*(e*x+d)^(1/2)*a^
3*d^4-76293/128*e^7/b^3/(b*e*x+a*e)^5*(e*x+d)^(1/2)*a^2*d^5+4*e^5/b^6*(e*x+
d)^(3/2)*d+42*e^5/b^6*d^2*(e*x+d)^(1/2)-4*e^6/b^7*(e*x+d)^(3/2)*a+42*e^7/b^
8*a^2*(e*x+d)^(1/2)+25431/128*e^6/b^2/(b*e*x+a*e)^5*(e*x+d)^(1/2)*a*d^6-159
81/128*e^7/b^3/(b*e*x+a*e)^5*(e*x+d)^(9/2)*a^2*d+15981/128*e^6/b^2/(b*e*x+a
*e)^5*(e*x+d)^(9/2)*a*d^2-1001*e^9/b^5/(b*e*x+a*e)^5*(e*x+d)^(5/2)*a^4*d+20
02*e^8/b^4/(b*e*x+a*e)^5*(e*x+d)^(5/2)*a^3*d^2-2002*e^7/b^3/(b*e*x+a*e)^5*(
e*x+d)^(5/2)*a^2*d^3+1001*e^6/b^2/(b*e*x+a*e)^5*(e*x+d)^(5/2)*a*d^4-23511/3
2*e^10/b^6/(b*e*x+a*e)^5*(e*x+d)^(3/2)*a^5*d+117555/64*e^9/b^5/(b*e*x+a*e)^
5*(e*x+d)^(3/2)*a^4*d^2+27027/128*e^7/b^7/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x
+d)^(1/2)/((a*e-b*d)*b)^(1/2))*d*a^2-9443/16*e^6/b^2/(b*e*x+a*e)^5*(e*x+d)^
(7/2)*a*d^3+28329/32*e^7/b^3/(b*e*x+a*e)^5*(e*x+d)^(7/2)*a^2*d^2-39185/16*e
^8/b^4/(b*e*x+a*e)^5*(e*x+d)^(3/2)*a^3*d^3-127155/128*e^9/b^5/(b*e*x+a*e)^5
*(e*x+d)^(1/2)*a^4*d^3-9443/16*e^8/b^4/(b*e*x+a*e)^5*(e*x+d)^(7/2)*a^3*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(15/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.28154, size = 3553, normalized size = 14.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(15/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/1280*(45045*(a^5*b^2*d^2*e^5 - 2*a^6*b*d*e^6 + a^7*e^7 + (b^7*d^2*e^5 -
2*a*b^6*d*e^6 + a^2*b^5*e^7)*x^5 + 5*(a*b^6*d^2*e^5 - 2*a^2*b^5*d*e^6 + a^3
*b^4*e^7)*x^4 + 10*(a^2*b^5*d^2*e^5 - 2*a^3*b^4*d*e^6 + a^4*b^3*e^7)*x^3 +
10*(a^3*b^4*d^2*e^5 - 2*a^4*b^3*d*e^6 + a^5*b^2*e^7)*x^2 + 5*(a^4*b^3*d^2*e
^5 - 2*a^5*b^2*d*e^6 + a^6*b*e^7)*x)*sqrt((b*d - a*e)/b)*log((b*e*x + 2*b*d
 - a*e - 2*sqrt(e*x + d)*b*sqrt((b*d - a*e)/b))/(b*x + a)) + 2*(256*b^7*e^7
*x^7 - 128*b^7*d^7 - 240*a*b^6*d^6*e - 520*a^2*b^5*d^5*e^2 - 1430*a^3*b^4*d
^4*e^3 - 6435*a^4*b^3*d^3*e^4 + 69069*a^5*b^2*d^2*e^5 - 105105*a^6*b*d*e^6
+ 45045*a^7*e^7 + 256*(12*b^7*d*e^6 - 5*a*b^6*e^7)*x^6 + 256*(116*b^7*d^2*e
^5 - 160*a*b^6*d*e^6 + 65*a^2*b^5*e^7)*x^5 - 5*(5327*b^7*d^3*e^4 - 45677*a*
b^6*d^2*e^5 + 66157*a^2*b^5*d*e^6 - 27599*a^3*b^4*e^7)*x^4 - 10*(1211*b^7*d
^4*e^3 + 5810*a*b^6*d^3*e^4 - 54392*a^2*b^5*d^2*e^5 + 80366*a^3*b^4*d*e^6 -
 33891*a^4*b^3*e^7)*x^3 - 2*(2324*b^7*d^5*e^2 + 6545*a*b^6*d^4*e^3 + 30485*
a^2*b^5*d^3*e^4 - 302445*a^3*b^4*d^2*e^5 + 452595*a^4*b^3*d*e^6 - 192192*a^
5*b^2*e^7)*x^2 - 2*(568*b^7*d^6*e + 1240*a*b^6*d^5*e^2 + 3445*a^2*b^5*d^4*e
^3 + 15730*a^3*b^4*d^3*e^4 - 163020*a^4*b^3*d^2*e^5 + 246246*a^5*b^2*d*e^6
- 105105*a^6*b*e^7)*x)*sqrt(e*x + d))/(b^13*x^5 + 5*a*b^12*x^4 + 10*a^2*b^1
1*x^3 + 10*a^3*b^10*x^2 + 5*a^4*b^9*x + a^5*b^8), -1/640*(45045*(a^5*b^2*d^
2*e^5 - 2*a^6*b*d*e^6 + a^7*e^7 + (b^7*d^2*e^5 - 2*a*b^6*d*e^6 + a^2*b^5*e^
7)*x^5 + 5*(a*b^6*d^2*e^5 - 2*a^2*b^5*d*e^6 + a^3*b^4*e^7)*x^4 + 10*(a^2*b^
5*d^2*e^5 - 2*a^3*b^4*d*e^6 + a^4*b^3*e^7)*x^3 + 10*(a^3*b^4*d^2*e^5 - 2*a^
4*b^3*d*e^6 + a^5*b^2*e^7)*x^2 + 5*(a^4*b^3*d^2*e^5 - 2*a^5*b^2*d*e^6 + a^6
*b*e^7)*x)*sqrt(-(b*d - a*e)/b)*arctan(-sqrt(e*x + d)*b*sqrt(-(b*d - a*e)/b
)/(b*d - a*e)) - (256*b^7*e^7*x^7 - 128*b^7*d^7 - 240*a*b^6*d^6*e - 520*a^2
*b^5*d^5*e^2 - 1430*a^3*b^4*d^4*e^3 - 6435*a^4*b^3*d^3*e^4 + 69069*a^5*b^2*
d^2*e^5 - 105105*a^6*b*d*e^6 + 45045*a^7*e^7 + 256*(12*b^7*d*e^6 - 5*a*b^6*
e^7)*x^6 + 256*(116*b^7*d^2*e^5 - 160*a*b^6*d*e^6 + 65*a^2*b^5*e^7)*x^5 - 5
*(5327*b^7*d^3*e^4 - 45677*a*b^6*d^2*e^5 + 66157*a^2*b^5*d*e^6 - 27599*a^3*
b^4*e^7)*x^4 - 10*(1211*b^7*d^4*e^3 + 5810*a*b^6*d^3*e^4 - 54392*a^2*b^5*d^
2*e^5 + 80366*a^3*b^4*d*e^6 - 33891*a^4*b^3*e^7)*x^3 - 2*(2324*b^7*d^5*e^2
+ 6545*a*b^6*d^4*e^3 + 30485*a^2*b^5*d^3*e^4 - 302445*a^3*b^4*d^2*e^5 + 452
595*a^4*b^3*d*e^6 - 192192*a^5*b^2*e^7)*x^2 - 2*(568*b^7*d^6*e + 1240*a*b^6
*d^5*e^2 + 3445*a^2*b^5*d^4*e^3 + 15730*a^3*b^4*d^3*e^4 - 163020*a^4*b^3*d^
2*e^5 + 246246*a^5*b^2*d*e^6 - 105105*a^6*b*e^7)*x)*sqrt(e*x + d))/(b^13*x^
5 + 5*a*b^12*x^4 + 10*a^2*b^11*x^3 + 10*a^3*b^10*x^2 + 5*a^4*b^9*x + a^5*b^
8)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(15/2)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.38142, size = 1060, normalized size = 4.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(15/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] 9009/128*(b^3*d^3*e^5 - 3*a*b^2*d^2*e^6 + 3*a^2*b*d*e^7 - a^3*e^8)*arctan(s
qrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^8) - 1/640*(26
635*(x*e + d)^(9/2)*b^7*d^3*e^5 - 94430*(x*e + d)^(7/2)*b^7*d^4*e^5 + 12812
8*(x*e + d)^(5/2)*b^7*d^5*e^5 - 78370*(x*e + d)^(3/2)*b^7*d^6*e^5 + 18165*s
qrt(x*e + d)*b^7*d^7*e^5 - 79905*(x*e + d)^(9/2)*a*b^6*d^2*e^6 + 377720*(x*
e + d)^(7/2)*a*b^6*d^3*e^6 - 640640*(x*e + d)^(5/2)*a*b^6*d^4*e^6 + 470220*
(x*e + d)^(3/2)*a*b^6*d^5*e^6 - 127155*sqrt(x*e + d)*a*b^6*d^6*e^6 + 79905*
(x*e + d)^(9/2)*a^2*b^5*d*e^7 - 566580*(x*e + d)^(7/2)*a^2*b^5*d^2*e^7 + 12
81280*(x*e + d)^(5/2)*a^2*b^5*d^3*e^7 - 1175550*(x*e + d)^(3/2)*a^2*b^5*d^4
*e^7 + 381465*sqrt(x*e + d)*a^2*b^5*d^5*e^7 - 26635*(x*e + d)^(9/2)*a^3*b^4
*e^8 + 377720*(x*e + d)^(7/2)*a^3*b^4*d*e^8 - 1281280*(x*e + d)^(5/2)*a^3*b
^4*d^2*e^8 + 1567400*(x*e + d)^(3/2)*a^3*b^4*d^3*e^8 - 635775*sqrt(x*e + d)
*a^3*b^4*d^4*e^8 - 94430*(x*e + d)^(7/2)*a^4*b^3*e^9 + 640640*(x*e + d)^(5/
2)*a^4*b^3*d*e^9 - 1175550*(x*e + d)^(3/2)*a^4*b^3*d^2*e^9 + 635775*sqrt(x*
e + d)*a^4*b^3*d^3*e^9 - 128128*(x*e + d)^(5/2)*a^5*b^2*e^10 + 470220*(x*e
+ d)^(3/2)*a^5*b^2*d*e^10 - 381465*sqrt(x*e + d)*a^5*b^2*d^2*e^10 - 78370*(
x*e + d)^(3/2)*a^6*b*e^11 + 127155*sqrt(x*e + d)*a^6*b*d*e^11 - 18165*sqrt(
x*e + d)*a^7*e^12)/(((x*e + d)*b - b*d + a*e)^5*b^8) + 2/5*((x*e + d)^(5/2)
*b^24*e^5 + 10*(x*e + d)^(3/2)*b^24*d*e^5 + 105*sqrt(x*e + d)*b^24*d^2*e^5
- 10*(x*e + d)^(3/2)*a*b^23*e^6 - 210*sqrt(x*e + d)*a*b^23*d*e^6 + 105*sqrt
(x*e + d)*a^2*b^22*e^7)/b^30
```

3.1665 $\int \frac{(d+ex)^{13/2}}{(a^2+2abx+b^2x^2)^3} dx$

Optimal. Leaf size=224

$$-\frac{143e^2(d+ex)^{9/2}}{240b^3(a+bx)^3}-\frac{429e^3(d+ex)^{7/2}}{320b^4(a+bx)^2}-\frac{3003e^4(d+ex)^{5/2}}{640b^5(a+bx)}+\frac{3003e^5\sqrt{d+ex}(bd-ae)}{128b^7}-\frac{3003e^5(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+}}{\sqrt{bd-}}\right.}{128b^{15/2}}$$

```
[Out] (3003*e^5*(b*d - a*e)*Sqrt[d + e*x])/(128*b^7) + (1001*e^5*(d + e*x)^(3/2))
/(128*b^6) - (3003*e^4*(d + e*x)^(5/2))/(640*b^5*(a + b*x)) - (429*e^3*(d +
 e*x)^(7/2))/(320*b^4*(a + b*x)^2) - (143*e^2*(d + e*x)^(9/2))/(240*b^3*(a
+ b*x)^3) - (13*e*(d + e*x)^(11/2))/(40*b^2*(a + b*x)^4) - (d + e*x)^(13/2)
/(5*b*(a + b*x)^5) - (3003*e^5*(b*d - a*e)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[d +
e*x])/Sqrt[b*d - a*e]])/(128*b^(15/2))
```

Rubi [A] time = 0.139222, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {27, 47, 50, 63, 208}

$$-\frac{143e^2(d+ex)^{9/2}}{240b^3(a+bx)^3}-\frac{429e^3(d+ex)^{7/2}}{320b^4(a+bx)^2}-\frac{3003e^4(d+ex)^{5/2}}{640b^5(a+bx)}+\frac{3003e^5\sqrt{d+ex}(bd-ae)}{128b^7}-\frac{3003e^5(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+}}{\sqrt{bd-}}\right.}{128b^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(13/2)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```


```
[Out] (3003*e^5*(b*d - a*e)*Sqrt[d + e*x])/(128*b^7) + (1001*e^5*(d + e*x)^(3/2))
/(128*b^6) - (3003*e^4*(d + e*x)^(5/2))/(640*b^5*(a + b*x)) - (429*e^3*(d +
 e*x)^(7/2))/(320*b^4*(a + b*x)^2) - (143*e^2*(d + e*x)^(9/2))/(240*b^3*(a
+ b*x)^3) - (13*e*(d + e*x)^(11/2))/(40*b^2*(a + b*x)^4) - (d + e*x)^(13/2)
/(5*b*(a + b*x)^5) - (3003*e^5*(b*d - a*e)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[d +
e*x])/Sqrt[b*d - a*e]])/(128*b^(15/2))
```


Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{13/2}}{\left(a^2+2abx+b^2x^2\right)^3}\,dx = \int \frac{(d+ex)^{13/2}}{(a+bx)^6}\,dx$$

$$= -\frac{(d+ex)^{13/2}}{5b(a+bx)^5} + \frac{(13e)\int \frac{(d+ex)^{11/2}}{(a+bx)^5}\,dx}{10b}$$

$$= -\frac{13e(d+ex)^{11/2}}{40b^2(a+bx)^4} - \frac{(d+ex)^{13/2}}{5b(a+bx)^5} + \frac{\left(143e^2\right)\int \frac{(d+ex)^{9/2}}{(a+bx)^4}\,dx}{80b^2}$$

$$= -\frac{143e^2(d+ex)^{9/2}}{240b^3(a+bx)^3} - \frac{13e(d+ex)^{11/2}}{40b^2(a+bx)^4} - \frac{(d+ex)^{13/2}}{5b(a+bx)^5} + \frac{\left(429e^3\right)\int \frac{(d+ex)^{7/2}}{(a+bx)^3}\,dx}{160b^3}$$

$$= -\frac{429e^3(d+ex)^{7/2}}{320b^4(a+bx)^2} - \frac{143e^2(d+ex)^{9/2}}{240b^3(a+bx)^3} - \frac{13e(d+ex)^{11/2}}{40b^2(a+bx)^4} - \frac{(d+ex)^{13/2}}{5b(a+bx)^5} + \frac{\left(3003e^4\right)\int \frac{(d+ex}{(a+b}}{640b^4}$$

$$= -\frac{3003e^4(d+ex)^{5/2}}{640b^5(a+bx)} - \frac{429e^3(d+ex)^{7/2}}{320b^4(a+bx)^2} - \frac{143e^2(d+ex)^{9/2}}{240b^3(a+bx)^3} - \frac{13e(d+ex)^{11/2}}{40b^2(a+bx)^4} - \frac{(d+ex)^{13}}{5b(a+bx}$$

$$= \frac{1001e^5(d+ex)^{3/2}}{128b^6} - \frac{3003e^4(d+ex)^{5/2}}{640b^5(a+bx)} - \frac{429e^3(d+ex)^{7/2}}{320b^4(a+bx)^2} - \frac{143e^2(d+ex)^{9/2}}{240b^3(a+bx)^3} - \frac{13e(d+e}{40b^2(a+}$$

$$= \frac{3003e^5(bd-ae)\sqrt{d+ex}}{128b^7} + \frac{1001e^5(d+ex)^{3/2}}{128b^6} - \frac{3003e^4(d+ex)^{5/2}}{640b^5(a+bx)} - \frac{429e^3(d+ex)^{7/2}}{320b^4(a+bx)^2} - \frac{1}{2}$$

$$= \frac{3003e^5(bd-ae)\sqrt{d+ex}}{128b^7} + \frac{1001e^5(d+ex)^{3/2}}{128b^6} - \frac{3003e^4(d+ex)^{5/2}}{640b^5(a+bx)} - \frac{429e^3(d+ex)^{7/2}}{320b^4(a+bx)^2} - \frac{1}{2}$$

$$= \frac{3003e^5(bd-ae)\sqrt{d+ex}}{128b^7} + \frac{1001e^5(d+ex)^{3/2}}{128b^6} - \frac{3003e^4(d+ex)^{5/2}}{640b^5(a+bx)} - \frac{429e^3(d+ex)^{7/2}}{320b^4(a+bx)^2} - \frac{1}{2}$$

Mathematica [C] time = 0.0221142, size = 52, normalized size = 0.23

$$\frac{2e^5(d+ex)^{15/2}\,{}_2F_1\left(6,\frac{15}{2};\frac{17}{2};-\frac{b(d+ex)}{ae-bd}\right)}{15(ae-bd)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(13/2)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] (2*e^5*(d + e*x)^(15/2)*Hypergeometric2F1[6, 15/2, 17/2, -((b*(d + e*x))/(-
(b*d) + a*e))])/(15*(-(b*d) + a*e)^6)
```

Maple [B] time = 0.216, size = 908, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(13/2)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] 2/3*e^5*(e*x+d)^(3/2)/b^6+12*e^5/b^6*(e*x+d)^(1/2)*d-12*e^6/b^7*a*(e*x+d)^(
1/2)-48145/96*e^8/b^4/(b*e*x+a*e)^5*(e*x+d)^(3/2)*a^3*d^2+48145/96*e^7/b^3/
(b*e*x+a*e)^5*(e*x+d)^(3/2)*a^2*d^3-12131/64*e^6/b^2/(b*e*x+a*e)^5*(e*x+d)^
(7/2)*a*d^2+2373/64*e^6/b^2/(b*e*x+a*e)^5*(e*x+d)^(9/2)*a*d-3003/64*e^6/b^6
/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a*d-22005/
128*e^9/b^5/(b*e*x+a*e)^5*(e*x+d)^(1/2)*d^2*a^4+7335/32*e^8/b^4/(b*e*x+a*e)
^5*(e*x+d)^(1/2)*a^3*d^3-22005/128*e^7/b^3/(b*e*x+a*e)^5*(e*x+d)^(1/2)*a^2*
d^4+4401/64*e^6/b^2/(b*e*x+a*e)^5*(e*x+d)^(1/2)*a*d^5+12131/64*e^7/b^3/(b*e
*x+a*e)^5*(e*x+d)^(7/2)*a^2*d+5012/15*e^8/b^4/(b*e*x+a*e)^5*(e*x+d)^(5/2)*a
^3*d-2506/5*e^7/b^3/(b*e*x+a*e)^5*(e*x+d)^(5/2)*a^2*d^2+5012/15*e^6/b^2/(b*
e*x+a*e)^5*(e*x+d)^(5/2)*a*d^3-48145/192*e^6/b^2/(b*e*x+a*e)^5*(e*x+d)^(3/2
)*a*d^4+4401/64*e^10/b^6/(b*e*x+a*e)^5*(e*x+d)^(1/2)*a^5*d+48145/192*e^9/b^
5/(b*e*x+a*e)^5*(e*x+d)^(3/2)*a^4*d+3003/128*e^7/b^7/((a*e-b*d)*b)^(1/2)*ar
ctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a^2-1467/128*e^11/b^7/(b*e*x+a*e)
^5*(e*x+d)^(1/2)*a^6-2373/128*e^7/b^3/(b*e*x+a*e)^5*(e*x+d)^(9/2)*a^2-1253/
15*e^9/b^5/(b*e*x+a*e)^5*(e*x+d)^(5/2)*a^4-9629/192*e^10/b^6/(b*e*x+a*e)^5*
(e*x+d)^(3/2)*a^5-12131/192*e^8/b^4/(b*e*x+a*e)^5*(e*x+d)^(7/2)*a^3+3003/12
8*e^5/b^5/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*d
^2-2373/128*e^5/b/(b*e*x+a*e)^5*(e*x+d)^(9/2)*d^2-1253/15*e^5/b/(b*e*x+a*e)
^5*(e*x+d)^(5/2)*d^4+9629/192*e^5/b/(b*e*x+a*e)^5*(e*x+d)^(3/2)*d^5-1467/12
8*e^5/b/(b*e*x+a*e)^5*(e*x+d)^(1/2)*d^6+12131/192*e^5/b/(b*e*x+a*e)^5*(e*x+
d)^(7/2)*d^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(13/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.41751, size = 2743, normalized size = 12.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(13/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/3840*(45045*(a^5*b*d*e^5 - a^6*e^6 + (b^6*d*e^5 - a*b^5*e^6)*x^5 + 5*(a
*b^5*d*e^5 - a^2*b^4*e^6)*x^4 + 10*(a^2*b^4*d*e^5 - a^3*b^3*e^6)*x^3 + 10*(
a^3*b^3*d*e^5 - a^4*b^2*e^6)*x^2 + 5*(a^4*b^2*d*e^5 - a^5*b*e^6)*x)*sqrt((b
*d - a*e)/b)*log((b*e*x + 2*b*d - a*e + 2*sqrt(e*x + d)*b*sqrt((b*d - a*e)/
b))/(b*x + a)) - 2*(1280*b^6*e^6*x^6 - 384*b^6*d^6 - 624*a*b^5*d^5*e - 1144
*a^2*b^4*d^4*e^2 - 2574*a^3*b^3*d^3*e^3 - 9009*a^4*b^2*d^2*e^4 + 60060*a^5*
b*d*e^5 - 45045*a^6*e^6 + 1280*(19*b^6*d*e^5 - 13*a*b^5*e^6)*x^5 - 5*(7119*
b^6*d^2*e^4 - 38558*a*b^5*d*e^5 + 27599*a^2*b^4*e^6)*x^4 - 10*(2107*b^6*d^3
*e^3 + 7917*a*b^5*d^2*e^4 - 46475*a^2*b^4*d*e^5 + 33891*a^3*b^3*e^6)*x^3 -
2*(5012*b^6*d^4*e^2 + 11557*a*b^5*d^3*e^3 + 42042*a^2*b^4*d^2*e^4 - 260403*
a^3*b^3*d*e^5 + 192192*a^4*b^2*e^6)*x^2 - 2*(1464*b^6*d^5*e + 2704*a*b^5*d^
4*e^2 + 6149*a^2*b^4*d^3*e^3 + 21879*a^3*b^3*d^2*e^4 - 141141*a^4*b^2*d*e^5
 + 105105*a^5*b*e^6)*x)*sqrt(e*x + d))/(b^12*x^5 + 5*a*b^11*x^4 + 10*a^2*b^
10*x^3 + 10*a^3*b^9*x^2 + 5*a^4*b^8*x + a^5*b^7), -1/1920*(45045*(a^5*b*d*e
^5 - a^6*e^6 + (b^6*d*e^5 - a*b^5*e^6)*x^5 + 5*(a*b^5*d*e^5 - a^2*b^4*e^6)*
x^4 + 10*(a^2*b^4*d*e^5 - a^3*b^3*e^6)*x^3 + 10*(a^3*b^3*d*e^5 - a^4*b^2*e^
6)*x^2 + 5*(a^4*b^2*d*e^5 - a^5*b*e^6)*x)*sqrt(-(b*d - a*e)/b)*arctan(-sqrt
(e*x + d)*b*sqrt(-(b*d - a*e)/b)/(b*d - a*e)) - (1280*b^6*e^6*x^6 - 384*b^6
*d^6 - 624*a*b^5*d^5*e - 1144*a^2*b^4*d^4*e^2 - 2574*a^3*b^3*d^3*e^3 - 9009
*a^4*b^2*d^2*e^4 + 60060*a^5*b*d*e^5 - 45045*a^6*e^6 + 1280*(19*b^6*d*e^5 -
 13*a*b^5*e^6)*x^5 - 5*(7119*b^6*d^2*e^4 - 38558*a*b^5*d*e^5 + 27599*a^2*b^
4*e^6)*x^4 - 10*(2107*b^6*d^3*e^3 + 7917*a*b^5*d^2*e^4 - 46475*a^2*b^4*d*e^
5 + 33891*a^3*b^3*e^6)*x^3 - 2*(5012*b^6*d^4*e^2 + 11557*a*b^5*d^3*e^3 + 42
042*a^2*b^4*d^2*e^4 - 260403*a^3*b^3*d*e^5 + 192192*a^4*b^2*e^6)*x^2 - 2*(1
464*b^6*d^5*e + 2704*a*b^5*d^4*e^2 + 6149*a^2*b^4*d^3*e^3 + 21879*a^3*b^3*d
^2*e^4 - 141141*a^4*b^2*d*e^5 + 105105*a^5*b*e^6)*x)*sqrt(e*x + d))/(b^12*x
^5 + 5*a*b^11*x^4 + 10*a^2*b^10*x^3 + 10*a^3*b^9*x^2 + 5*a^4*b^8*x + a^5*b^
7)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(13/2)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32275, size = 828, normalized size = 3.7

$$\frac{3003\left(b^2d^2e^5 - 2\,abde^6 + a^2e^7\right)\arctan\left(\frac{\sqrt{xe+db}}{\sqrt{-b^2d+abe}}\right)}{128\sqrt{-b^2d+abe}b^7} - \frac{35595\,(xe+d)^{\frac{9}{2}}b^6d^2e^5 - 121310\,(xe+d)^{\frac{7}{2}}b^6d^3e^5 + 160384\,(xe+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(13/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] 3003/128*(b^2*d^2*e^5 - 2*a*b*d*e^6 + a^2*e^7)*arctan(sqrt(x*e + d)*b/sqrt(
-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^7) - 1/1920*(35595*(x*e + d)^(9/2)
*b^6*d^2*e^5 - 121310*(x*e + d)^(7/2)*b^6*d^3*e^5 + 160384*(x*e + d)^(5/2)*
b^6*d^4*e^5 - 96290*(x*e + d)^(3/2)*b^6*d^5*e^5 + 22005*sqrt(x*e + d)*b^6*d
^6*e^5 - 71190*(x*e + d)^(9/2)*a*b^5*d*e^6 + 363930*(x*e + d)^(7/2)*a*b^5*d
^2*e^6 - 641536*(x*e + d)^(5/2)*a*b^5*d^3*e^6 + 481450*(x*e + d)^(3/2)*a*b^
```

```
5*d^4*e^6 - 132030*sqrt(x*e + d)*a*b^5*d^5*e^6 + 35595*(x*e + d)^(9/2)*a^2*
b^4*e^7 - 363930*(x*e + d)^(7/2)*a^2*b^4*d*e^7 + 962304*(x*e + d)^(5/2)*a^2
*b^4*d^2*e^7 - 962900*(x*e + d)^(3/2)*a^2*b^4*d^3*e^7 + 330075*sqrt(x*e + d
)*a^2*b^4*d^4*e^7 + 121310*(x*e + d)^(7/2)*a^3*b^3*e^8 - 641536*(x*e + d)^(
5/2)*a^3*b^3*d*e^8 + 962900*(x*e + d)^(3/2)*a^3*b^3*d^2*e^8 - 440100*sqrt(x
*e + d)*a^3*b^3*d^3*e^8 + 160384*(x*e + d)^(5/2)*a^4*b^2*e^9 - 481450*(x*e
+ d)^(3/2)*a^4*b^2*d*e^9 + 330075*sqrt(x*e + d)*a^4*b^2*d^2*e^9 + 96290*(x*
e + d)^(3/2)*a^5*b*e^10 - 132030*sqrt(x*e + d)*a^5*b*d*e^10 + 22005*sqrt(x*
e + d)*a^6*e^11)/(((x*e + d)*b - b*d + a*e)^5*b^7) + 2/3*((x*e + d)^(3/2)*b
^12*e^5 + 18*sqrt(x*e + d)*b^12*d*e^5 - 18*sqrt(x*e + d)*a*b^11*e^6)/b^18
```

3.1666 $\int \frac{(d+ex)^{11/2}}{\left(a^2+2abx+b^2x^2\right)^3}\,dx$

Optimal. Leaf size=197

$$-\frac{33e^2(d+ex)^{7/2}}{80b^3(a+bx)^3}-\frac{231e^3(d+ex)^{5/2}}{320b^4(a+bx)^2}-\frac{231e^4(d+ex)^{3/2}}{128b^5(a+bx)}-\frac{693e^5\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{128b^{13/2}}-\frac{11e(d+ex)^{9/2}}{40b^2(a+bx)^4}-\frac{(d+ex}{5b(a+}$$

```
[Out] (693*e^5*Sqrt[d + e*x])/(128*b^6) - (231*e^4*(d + e*x)^(3/2))/(128*b^5*(a +
 b*x)) - (231*e^3*(d + e*x)^(5/2))/(320*b^4*(a + b*x)^2) - (33*e^2*(d + e*x
)^(7/2))/(80*b^3*(a + b*x)^3) - (11*e*(d + e*x)^(9/2))/(40*b^2*(a + b*x)^4)
 - (d + e*x)^(11/2)/(5*b*(a + b*x)^5) - (693*e^5*Sqrt[b*d - a*e]*ArcTanh[(S
qrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(128*b^(13/2))
```

Rubi [A] time = 0.10289, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {27, 47, 50, 63, 208}

$$-\frac{33e^2(d+ex)^{7/2}}{80b^3(a+bx)^3}-\frac{231e^3(d+ex)^{5/2}}{320b^4(a+bx)^2}-\frac{231e^4(d+ex)^{3/2}}{128b^5(a+bx)}-\frac{693e^5\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{128b^{13/2}}-\frac{11e(d+ex)^{9/2}}{40b^2(a+bx)^4}-\frac{(d+ex}{5b(a+}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(11/2)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] (693*e^5*Sqrt[d + e*x])/(128*b^6) - (231*e^4*(d + e*x)^(3/2))/(128*b^5*(a +
 b*x)) - (231*e^3*(d + e*x)^(5/2))/(320*b^4*(a + b*x)^2) - (33*e^2*(d + e*x
)^(7/2))/(80*b^3*(a + b*x)^3) - (11*e*(d + e*x)^(9/2))/(40*b^2*(a + b*x)^4)
 - (d + e*x)^(11/2)/(5*b*(a + b*x)^5) - (693*e^5*Sqrt[b*d - a*e]*ArcTanh[(S
qrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(128*b^(13/2))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{11/2}}{\left(a^2+2abx+b^2x^2\right)^3}\,dx = \int \frac{(d+ex)^{11/2}}{(a+bx)^6}\,dx$$

$$= -\frac{(d+ex)^{11/2}}{5b(a+bx)^5} + \frac{(11e)\int \frac{(d+ex)^{9/2}}{(a+bx)^5}\,dx}{10b}$$

$$= -\frac{11e(d+ex)^{9/2}}{40b^2(a+bx)^4} - \frac{(d+ex)^{11/2}}{5b(a+bx)^5} + \frac{\left(99e^2\right)\int \frac{(d+ex)^{7/2}}{(a+bx)^4}\,dx}{80b^2}$$

$$= -\frac{33e^2(d+ex)^{7/2}}{80b^3(a+bx)^3} - \frac{11e(d+ex)^{9/2}}{40b^2(a+bx)^4} - \frac{(d+ex)^{11/2}}{5b(a+bx)^5} + \frac{\left(231e^3\right)\int \frac{(d+ex)^{5/2}}{(a+bx)^3}\,dx}{160b^3}$$

$$= -\frac{231e^3(d+ex)^{5/2}}{320b^4(a+bx)^2} - \frac{33e^2(d+ex)^{7/2}}{80b^3(a+bx)^3} - \frac{11e(d+ex)^{9/2}}{40b^2(a+bx)^4} - \frac{(d+ex)^{11/2}}{5b(a+bx)^5} + \frac{\left(231e^4\right)\int \frac{(d+ex)^{3/2}}{(a+bx)^2}\,dx}{128b^4}$$

$$= -\frac{231e^4(d+ex)^{3/2}}{128b^5(a+bx)} - \frac{231e^3(d+ex)^{5/2}}{320b^4(a+bx)^2} - \frac{33e^2(d+ex)^{7/2}}{80b^3(a+bx)^3} - \frac{11e(d+ex)^{9/2}}{40b^2(a+bx)^4} - \frac{(d+ex)^{11/2}}{5b(a+bx)^5} + \frac{(6$$

$$= \frac{693e^5\sqrt{d+ex}}{128b^6} - \frac{231e^4(d+ex)^{3/2}}{128b^5(a+bx)} - \frac{231e^3(d+ex)^{5/2}}{320b^4(a+bx)^2} - \frac{33e^2(d+ex)^{7/2}}{80b^3(a+bx)^3} - \frac{11e(d+ex)^{9/2}}{40b^2(a+bx)^4} - \frac{(\ }{5$$

$$= \frac{693e^5\sqrt{d+ex}}{128b^6} - \frac{231e^4(d+ex)^{3/2}}{128b^5(a+bx)} - \frac{231e^3(d+ex)^{5/2}}{320b^4(a+bx)^2} - \frac{33e^2(d+ex)^{7/2}}{80b^3(a+bx)^3} - \frac{11e(d+ex)^{9/2}}{40b^2(a+bx)^4} - \frac{(\ }{5$$

$$= \frac{693e^5\sqrt{d+ex}}{128b^6} - \frac{231e^4(d+ex)^{3/2}}{128b^5(a+bx)} - \frac{231e^3(d+ex)^{5/2}}{320b^4(a+bx)^2} - \frac{33e^2(d+ex)^{7/2}}{80b^3(a+bx)^3} - \frac{11e(d+ex)^{9/2}}{40b^2(a+bx)^4} - \frac{(\ }{5$$

Mathematica [C] time = 0.022983, size = 52, normalized size = 0.26

$$\frac{2e^5(d+ex)^{13/2}\,{}_2F_1\left(6,\frac{13}{2};\frac{15}{2};-\frac{b(d+ex)}{ae-bd}\right)}{13(ae-bd)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(11/2)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] (2*e^5*(d + e*x)^(13/2)*Hypergeometric2F1[6, 13/2, 15/2, -((b*(d + e*x))/(-
(b*d) + a*e))])/(13*(-(b*d) + a*e)^6)
```

Maple [B] time = 0.212, size = 673, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(11/2)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] 2*e^5*(e*x+d)^(1/2)/b^6+843/128*e^6/b^2/(b*e*x+a*e)^5*(e*x+d)^(9/2)*a-843/1
28*e^5/b/(b*e*x+a*e)^5*(e*x+d)^(9/2)*d+1327/64*e^7/b^3/(b*e*x+a*e)^5*(e*x+d
)^(7/2)*a^2-1327/32*e^6/b^2/(b*e*x+a*e)^5*(e*x+d)^(7/2)*a*d+1327/64*e^5/b/(
b*e*x+a*e)^5*(e*x+d)^(7/2)*d^2+131/5*e^8/b^4/(b*e*x+a*e)^5*(e*x+d)^(5/2)*a^
3-393/5*e^7/b^3/(b*e*x+a*e)^5*(e*x+d)^(5/2)*a^2*d+393/5*e^6/b^2/(b*e*x+a*e)
^5*(e*x+d)^(5/2)*a*d^2-131/5*e^5/b/(b*e*x+a*e)^5*(e*x+d)^(5/2)*d^3+977/64*e
^9/b^5/(b*e*x+a*e)^5*(e*x+d)^(3/2)*a^4-977/16*e^8/b^4/(b*e*x+a*e)^5*(e*x+d)
^(3/2)*a^3*d+2931/32*e^7/b^3/(b*e*x+a*e)^5*(e*x+d)^(3/2)*a^2*d^2-977/16*e^6
/b^2/(b*e*x+a*e)^5*(e*x+d)^(3/2)*a*d^3+977/64*e^5/b/(b*e*x+a*e)^5*(e*x+d)^(
3/2)*d^4+437/128*e^10/b^6/(b*e*x+a*e)^5*(e*x+d)^(1/2)*a^5-2185/128*e^9/b^5/
(b*e*x+a*e)^5*(e*x+d)^(1/2)*d*a^4+2185/64*e^8/b^4/(b*e*x+a*e)^5*(e*x+d)^(1/
2)*a^3*d^2-2185/64*e^7/b^3/(b*e*x+a*e)^5*(e*x+d)^(1/2)*a^2*d^3+2185/128*e^6
/b^2/(b*e*x+a*e)^5*(e*x+d)^(1/2)*a*d^4-437/128*e^5/b/(b*e*x+a*e)^5*(e*x+d)^
(1/2)*d^5-693/128*e^6/b^6/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2)/((a*e-
b*d)*b)^(1/2))*a+693/128*e^5/b^5/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2)
/((a*e-b*d)*b)^(1/2))*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(11/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.22922, size = 1976, normalized size = 10.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(11/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/1280*(3465*(b^5*e^5*x^5 + 5*a*b^4*e^5*x^4 + 10*a^2*b^3*e^5*x^3 + 10*a^3*
b^2*e^5*x^2 + 5*a^4*b*e^5*x + a^5*e^5)*sqrt((b*d - a*e)/b)*log((b*e*x + 2*b
*d - a*e - 2*sqrt(e*x + d)*b*sqrt((b*d - a*e)/b))/(b*x + a)) + 2*(1280*b^5*
e^5*x^5 - 128*b^5*d^5 - 176*a*b^4*d^4*e - 264*a^2*b^3*d^3*e^2 - 462*a^3*b^2
*d^2*e^3 - 1155*a^4*b*d*e^4 + 3465*a^5*e^5 - 5*(843*b^5*d*e^4 - 2123*a*b^4*
e^5)*x^4 - 10*(359*b^5*d^2*e^3 + 968*a*b^4*d*e^4 - 2607*a^2*b^3*e^5)*x^3 -
2*(1124*b^5*d^3*e^2 + 2013*a*b^4*d^2*e^3 + 5247*a^2*b^3*d*e^4 - 14784*a^3*b
^2*e^5)*x^2 - 2*(408*b^5*d^4*e + 616*a*b^4*d^3*e^2 + 1089*a^2*b^3*d^2*e^3 +
 2772*a^3*b^2*d*e^4 - 8085*a^4*b*e^5)*x)*sqrt(e*x + d))/(b^11*x^5 + 5*a*b^1
0*x^4 + 10*a^2*b^9*x^3 + 10*a^3*b^8*x^2 + 5*a^4*b^7*x + a^5*b^6), -1/640*(3
465*(b^5*e^5*x^5 + 5*a*b^4*e^5*x^4 + 10*a^2*b^3*e^5*x^3 + 10*a^3*b^2*e^5*x^
```

```
2 + 5*a^4*b*e^5*x + a^5*e^5)*sqrt(-(b*d - a*e)/b)*arctan(-sqrt(e*x + d)*b*s
qrt(-(b*d - a*e)/b)/(b*d - a*e)) - (1280*b^5*e^5*x^5 - 128*b^5*d^5 - 176*a*
b^4*d^4*e - 264*a^2*b^3*d^3*e^2 - 462*a^3*b^2*d^2*e^3 - 1155*a^4*b*d*e^4 +
3465*a^5*e^5 - 5*(843*b^5*d*e^4 - 2123*a*b^4*e^5)*x^4 - 10*(359*b^5*d^2*e^3
 + 968*a*b^4*d*e^4 - 2607*a^2*b^3*e^5)*x^3 - 2*(1124*b^5*d^3*e^2 + 2013*a*b
^4*d^2*e^3 + 5247*a^2*b^3*d*e^4 - 14784*a^3*b^2*e^5)*x^2 - 2*(408*b^5*d^4*e
 + 616*a*b^4*d^3*e^2 + 1089*a^2*b^3*d^2*e^3 + 2772*a^3*b^2*d*e^4 - 8085*a^4
*b*e^5)*x)*sqrt(e*x + d))/(b^11*x^5 + 5*a*b^10*x^4 + 10*a^2*b^9*x^3 + 10*a^
3*b^8*x^2 + 5*a^4*b^7*x + a^5*b^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(11/2)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34365, size = 620, normalized size = 3.15

$$\frac{693\left(bde^5-ae^6\right)\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)}{128\sqrt{-b^2d+abe}b^6}+\frac{2\sqrt{xe+d}e^5}{b^6}-\frac{4215\,(xe+d)^{\frac{9}{2}}b^5de^5-13270\,(xe+d)^{\frac{7}{2}}b^5d^2e^5+16768\,(xe+d)^{\frac{5}{2}}b^5d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(11/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] 693/128*(b*d*e^5 - a*e^6)*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))/(sqr
t(-b^2*d + a*b*e)*b^6) + 2*sqrt(x*e + d)*e^5/b^6 - 1/640*(4215*(x*e + d)^(9
/2)*b^5*d*e^5 - 13270*(x*e + d)^(7/2)*b^5*d^2*e^5 + 16768*(x*e + d)^(5/2)*b
^5*d^3*e^5 - 9770*(x*e + d)^(3/2)*b^5*d^4*e^5 + 2185*sqrt(x*e + d)*b^5*d^5*
e^5 - 4215*(x*e + d)^(9/2)*a*b^4*e^6 + 26540*(x*e + d)^(7/2)*a*b^4*d*e^6 -
50304*(x*e + d)^(5/2)*a*b^4*d^2*e^6 + 39080*(x*e + d)^(3/2)*a*b^4*d^3*e^6 -
 10925*sqrt(x*e + d)*a*b^4*d^4*e^6 - 13270*(x*e + d)^(7/2)*a^2*b^3*e^7 + 50
304*(x*e + d)^(5/2)*a^2*b^3*d*e^7 - 58620*(x*e + d)^(3/2)*a^2*b^3*d^2*e^7 +
 21850*sqrt(x*e + d)*a^2*b^3*d^3*e^7 - 16768*(x*e + d)^(5/2)*a^3*b^2*e^8 +
39080*(x*e + d)^(3/2)*a^3*b^2*d*e^8 - 21850*sqrt(x*e + d)*a^3*b^2*d^2*e^8 -
 9770*(x*e + d)^(3/2)*a^4*b*e^9 + 10925*sqrt(x*e + d)*a^4*b*d*e^9 - 2185*sq
rt(x*e + d)*a^5*e^10)/(((x*e + d)*b - b*d + a*e)^5*b^6)
```

3.1667 $\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^3} dx$

Optimal. Leaf size=178

$$-\frac{63e^4\sqrt{d+ex}}{128b^5(a+bx)}-\frac{21e^3(d+ex)^{3/2}}{64b^4(a+bx)^2}-\frac{21e^2(d+ex)^{5/2}}{80b^3(a+bx)^3}-\frac{63e^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{128b^{11/2}\sqrt{bd-ae}}-\frac{9e(d+ex)^{7/2}}{40b^2(a+bx)^4}-\frac{(d+ex)^{9/2}}{5b(a+bx)^5}$$

```
[Out] (-63*e^4*Sqrt[d + e*x])/(128*b^5*(a + b*x)) - (21*e^3*(d + e*x)^(3/2))/(64*
b^4*(a + b*x)^2) - (21*e^2*(d + e*x)^(5/2))/(80*b^3*(a + b*x)^3) - (9*e*(d
+ e*x)^(7/2))/(40*b^2*(a + b*x)^4) - (d + e*x)^(9/2)/(5*b*(a + b*x)^5) - (6
3*e^5*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(128*b^(11/2)*Sqrt[
b*d - a*e])
```

Rubi [A] time = 0.0901336, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {27, 47, 63, 208}

$$-\frac{63e^4\sqrt{d+ex}}{128b^5(a+bx)}-\frac{21e^3(d+ex)^{3/2}}{64b^4(a+bx)^2}-\frac{21e^2(d+ex)^{5/2}}{80b^3(a+bx)^3}-\frac{63e^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{128b^{11/2}\sqrt{bd-ae}}-\frac{9e(d+ex)^{7/2}}{40b^2(a+bx)^4}-\frac{(d+ex)^{9/2}}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(9/2)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] (-63*e^4*Sqrt[d + e*x])/(128*b^5*(a + b*x)) - (21*e^3*(d + e*x)^(3/2))/(64*
b^4*(a + b*x)^2) - (21*e^2*(d + e*x)^(5/2))/(80*b^3*(a + b*x)^3) - (9*e*(d
+ e*x)^(7/2))/(40*b^2*(a + b*x)^4) - (d + e*x)^(9/2)/(5*b*(a + b*x)^5) - (6
3*e^5*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(128*b^(11/2)*Sqrt[
b*d - a*e])
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{9/2}}{\left(a^2+2abx+b^2x^2\right)^3}\,dx = \int \frac{(d+ex)^{9/2}}{(a+bx)^6}\,dx$$

$$= -\frac{(d+ex)^{9/2}}{5b(a+bx)^5} + \frac{(9e)\int \frac{(d+ex)^{7/2}}{(a+bx)^5}\,dx}{10b}$$

$$= -\frac{9e(d+ex)^{7/2}}{40b^2(a+bx)^4} - \frac{(d+ex)^{9/2}}{5b(a+bx)^5} + \frac{\left(63e^2\right)\int \frac{(d+ex)^{5/2}}{(a+bx)^4}\,dx}{80b^2}$$

$$= -\frac{21e^2(d+ex)^{5/2}}{80b^3(a+bx)^3} - \frac{9e(d+ex)^{7/2}}{40b^2(a+bx)^4} - \frac{(d+ex)^{9/2}}{5b(a+bx)^5} + \frac{\left(21e^3\right)\int \frac{(d+ex)^{3/2}}{(a+bx)^3}\,dx}{32b^3}$$

$$= -\frac{21e^3(d+ex)^{3/2}}{64b^4(a+bx)^2} - \frac{21e^2(d+ex)^{5/2}}{80b^3(a+bx)^3} - \frac{9e(d+ex)^{7/2}}{40b^2(a+bx)^4} - \frac{(d+ex)^{9/2}}{5b(a+bx)^5} + \frac{\left(63e^4\right)\int \frac{\sqrt{d+ex}}{(a+bx)^2}\,dx}{128b^4}$$

$$= -\frac{63e^4\sqrt{d+ex}}{128b^5(a+bx)} - \frac{21e^3(d+ex)^{3/2}}{64b^4(a+bx)^2} - \frac{21e^2(d+ex)^{5/2}}{80b^3(a+bx)^3} - \frac{9e(d+ex)^{7/2}}{40b^2(a+bx)^4} - \frac{(d+ex)^{9/2}}{5b(a+bx)^5} + \frac{\left(63e^5\right.}{}$$

$$= -\frac{63e^4\sqrt{d+ex}}{128b^5(a+bx)} - \frac{21e^3(d+ex)^{3/2}}{64b^4(a+bx)^2} - \frac{21e^2(d+ex)^{5/2}}{80b^3(a+bx)^3} - \frac{9e(d+ex)^{7/2}}{40b^2(a+bx)^4} - \frac{(d+ex)^{9/2}}{5b(a+bx)^5} + \frac{\left(63e^4\right.}{}$$

$$= -\frac{63e^4\sqrt{d+ex}}{128b^5(a+bx)} - \frac{21e^3(d+ex)^{3/2}}{64b^4(a+bx)^2} - \frac{21e^2(d+ex)^{5/2}}{80b^3(a+bx)^3} - \frac{9e(d+ex)^{7/2}}{40b^2(a+bx)^4} - \frac{(d+ex)^{9/2}}{5b(a+bx)^5} - \frac{63e^5\ \mathrm{t}}{128}$$

Mathematica [A] time = 0.314815, size = 178, normalized size = 1.

$$-\frac{63e^4\sqrt{d+ex}}{128b^5(a+bx)} - \frac{21e^3(d+ex)^{3/2}}{64b^4(a+bx)^2} - \frac{21e^2(d+ex)^{5/2}}{80b^3(a+bx)^3} + \frac{63e^5\tan^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{ae-bd}}\right)}{128b^{11/2}\sqrt{ae-bd}} - \frac{9e(d+ex)^{7/2}}{40b^2(a+bx)^4} - \frac{(d+ex)^{9/2}}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(9/2)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] (-63*e^4*Sqrt[d + e*x])/(128*b^5*(a + b*x)) - (21*e^3*(d + e*x)^(3/2))/(64*
b^4*(a + b*x)^2) - (21*e^2*(d + e*x)^(5/2))/(80*b^3*(a + b*x)^3) - (9*e*(d
+ e*x)^(7/2))/(40*b^2*(a + b*x)^4) - (d + e*x)^(9/2)/(5*b*(a + b*x)^5) + (6
3*e^5*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(128*b^(11/2)*Sqr
t[-(b*d) + a*e])
```

Maple [B] time = 0.209, size = 463, normalized size = 2.6

$$-\frac{193\,e^5}{128\,(bxe+ae)^5\,b}(ex+d)^{\frac{9}{2}} - \frac{237\,e^6a}{64\,(bxe+ae)^5\,b^2}(ex+d)^{\frac{7}{2}} + \frac{237\,e^5d}{64\,(bxe+ae)^5\,b}(ex+d)^{\frac{7}{2}} - \frac{21\,a^2e^7}{5\,(bxe+ae)^5\,b^3}(ex+d)^{\frac{5}{2}} + \frac{}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] -193/128*e^5/(b*e*x+a*e)^5/b*(e*x+d)^(9/2)-237/64*e^6/(b*e*x+a*e)^5/b^2*(e*
x+d)^(7/2)*a+237/64*e^5/(b*e*x+a*e)^5/b*(e*x+d)^(7/2)*d-21/5*e^7/(b*e*x+a*e
)^5/b^3*(e*x+d)^(5/2)*a^2+42/5*e^6/(b*e*x+a*e)^5/b^2*(e*x+d)^(5/2)*a*d-21/5
*e^5/(b*e*x+a*e)^5/b*(e*x+d)^(5/2)*d^2-147/64*e^8/(b*e*x+a*e)^5/b^4*(e*x+d)
^(3/2)*a^3+441/64*e^7/(b*e*x+a*e)^5/b^3*(e*x+d)^(3/2)*d*a^2-441/64*e^6/(b*e
*x+a*e)^5/b^2*(e*x+d)^(3/2)*a*d^2+147/64*e^5/(b*e*x+a*e)^5/b*(e*x+d)^(3/2)*
d^3-63/128*e^9/(b*e*x+a*e)^5/b^5*(e*x+d)^(1/2)*a^4+63/32*e^8/(b*e*x+a*e)^5/
b^4*(e*x+d)^(1/2)*a^3*d-189/64*e^7/(b*e*x+a*e)^5/b^3*(e*x+d)^(1/2)*d^2*a^2+
63/32*e^6/(b*e*x+a*e)^5/b^2*(e*x+d)^(1/2)*a*d^3-63/128*e^5/(b*e*x+a*e)^5/b*
(e*x+d)^(1/2)*d^4+63/128*e^5/b^5/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2)
/((a*e-b*d)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.14142, size = 2122, normalized size = 11.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/1280*(315*(b^5*e^5*x^5 + 5*a*b^4*e^5*x^4 + 10*a^2*b^3*e^5*x^3 + 10*a^3*b
^2*e^5*x^2 + 5*a^4*b*e^5*x + a^5*e^5)*sqrt(b^2*d - a*b*e)*log((b*e*x + 2*b*
d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d))/(b*x + a)) - 2*(128*b^6*d^5
+ 16*a*b^5*d^4*e + 24*a^2*b^4*d^3*e^2 + 42*a^3*b^3*d^2*e^3 + 105*a^4*b^2*d*
e^4 - 315*a^5*b*e^5 + 965*(b^6*d*e^4 - a*b^5*e^5)*x^4 + 10*(149*b^6*d^2*e^3
 + 88*a*b^5*d*e^4 - 237*a^2*b^4*e^5)*x^3 + 6*(228*b^6*d^3*e^2 + 61*a*b^5*d^
2*e^3 + 159*a^2*b^4*d*e^4 - 448*a^3*b^3*e^5)*x^2 + 2*(328*b^6*d^4*e + 56*a*
b^5*d^3*e^2 + 99*a^2*b^4*d^2*e^3 + 252*a^3*b^3*d*e^4 - 735*a^4*b^2*e^5)*x)*
sqrt(e*x + d))/(a^5*b^7*d - a^6*b^6*e + (b^12*d - a*b^11*e)*x^5 + 5*(a*b^11
*d - a^2*b^10*e)*x^4 + 10*(a^2*b^10*d - a^3*b^9*e)*x^3 + 10*(a^3*b^9*d - a^
4*b^8*e)*x^2 + 5*(a^4*b^8*d - a^5*b^7*e)*x), 1/640*(315*(b^5*e^5*x^5 + 5*a*
b^4*e^5*x^4 + 10*a^2*b^3*e^5*x^3 + 10*a^3*b^2*e^5*x^2 + 5*a^4*b*e^5*x + a^5
*e^5)*sqrt(-b^2*d + a*b*e)*arctan(sqrt(-b^2*d + a*b*e)*sqrt(e*x + d)/(b*e*x
 + b*d)) - (128*b^6*d^5 + 16*a*b^5*d^4*e + 24*a^2*b^4*d^3*e^2 + 42*a^3*b^3*
d^2*e^3 + 105*a^4*b^2*d*e^4 - 315*a^5*b*e^5 + 965*(b^6*d*e^4 - a*b^5*e^5)*x
^4 + 10*(149*b^6*d^2*e^3 + 88*a*b^5*d*e^4 - 237*a^2*b^4*e^5)*x^3 + 6*(228*b
^6*d^3*e^2 + 61*a*b^5*d^2*e^3 + 159*a^2*b^4*d*e^4 - 448*a^3*b^3*e^5)*x^2 +
2*(328*b^6*d^4*e + 56*a*b^5*d^3*e^2 + 99*a^2*b^4*d^2*e^3 + 252*a^3*b^3*d*e^
4 - 735*a^4*b^2*e^5)*x)*sqrt(e*x + d))/(a^5*b^7*d - a^6*b^6*e + (b^12*d - a
*b^11*e)*x^5 + 5*(a*b^11*d - a^2*b^10*e)*x^4 + 10*(a^2*b^10*d - a^3*b^9*e)*
x^3 + 10*(a^3*b^9*d - a^4*b^8*e)*x^2 + 5*(a^4*b^8*d - a^5*b^7*e)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(9/2)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26686, size = 451, normalized size = 2.53

$$\frac{63 \arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)e^5}{128\sqrt{-b^2d+abe}b^5} - \frac{965\,(xe+d)^{\frac{9}{2}}b^4e^5 - 2370\,(xe+d)^{\frac{7}{2}}b^4de^5 + 2688\,(xe+d)^{\frac{5}{2}}b^4d^2e^5 - 1470\,(xe+d)^{\frac{3}{2}}b^4d^3e^5 + 31}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] 63/128*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e^5/(sqrt(-b^2*d + a*b*
e)*b^5) - 1/640*(965*(x*e + d)^(9/2)*b^4*e^5 - 2370*(x*e + d)^(7/2)*b^4*d*e
^5 + 2688*(x*e + d)^(5/2)*b^4*d^2*e^5 - 1470*(x*e + d)^(3/2)*b^4*d^3*e^5 +
315*sqrt(x*e + d)*b^4*d^4*e^5 + 2370*(x*e + d)^(7/2)*a*b^3*e^6 - 5376*(x*e
+ d)^(5/2)*a*b^3*d*e^6 + 4410*(x*e + d)^(3/2)*a*b^3*d^2*e^6 - 1260*sqrt(x*e
 + d)*a*b^3*d^3*e^6 + 2688*(x*e + d)^(5/2)*a^2*b^2*e^7 - 4410*(x*e + d)^(3/
2)*a^2*b^2*d*e^7 + 1890*sqrt(x*e + d)*a^2*b^2*d^2*e^7 + 1470*(x*e + d)^(3/2
)*a^3*b*e^8 - 1260*sqrt(x*e + d)*a^3*b*d*e^8 + 315*sqrt(x*e + d)*a^4*e^9)/(
((x*e + d)*b - b*d + a*e)^5*b^5)
```

3.1668 $\int \frac{(d+ex)^{7/2}}{\left(a^2+2abx+b^2x^2\right)^3}\,dx$

Optimal. Leaf size=188

$$-\frac{7e^4\sqrt{d+ex}}{128b^4(a+bx)(bd-ae)}-\frac{7e^3\sqrt{d+ex}}{64b^4(a+bx)^2}-\frac{7e^2(d+ex)^{3/2}}{48b^3(a+bx)^3}+\frac{7e^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{128b^{9/2}(bd-ae)^{3/2}}-\frac{7e(d+ex)^{5/2}}{40b^2(a+bx)^4}-\frac{(d+ex)^{7/2}}{5b(a+bx)^5}$$

```
[Out] (-7*e^3*Sqrt[d + e*x])/(64*b^4*(a + b*x)^2) - (7*e^4*Sqrt[d + e*x])/(128*b^
4*(b*d - a*e)*(a + b*x)) - (7*e^2*(d + e*x)^(3/2))/(48*b^3*(a + b*x)^3) - (
7*e*(d + e*x)^(5/2))/(40*b^2*(a + b*x)^4) - (d + e*x)^(7/2)/(5*b*(a + b*x)^
5) + (7*e^5*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(128*b^(9/2)*
(b*d - a*e)^(3/2))
```

Rubi [A] time = 0.10614, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {27, 47, 51, 63, 208}

$$-\frac{7e^4\sqrt{d+ex}}{128b^4(a+bx)(bd-ae)}-\frac{7e^3\sqrt{d+ex}}{64b^4(a+bx)^2}-\frac{7e^2(d+ex)^{3/2}}{48b^3(a+bx)^3}+\frac{7e^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{128b^{9/2}(bd-ae)^{3/2}}-\frac{7e(d+ex)^{5/2}}{40b^2(a+bx)^4}-\frac{(d+ex)^{7/2}}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```


```
[Out] (-7*e^3*Sqrt[d + e*x])/(64*b^4*(a + b*x)^2) - (7*e^4*Sqrt[d + e*x])/(128*b^
4*(b*d - a*e)*(a + b*x)) - (7*e^2*(d + e*x)^(3/2))/(48*b^3*(a + b*x)^3) - (
7*e*(d + e*x)^(5/2))/(40*b^2*(a + b*x)^4) - (d + e*x)^(7/2)/(5*b*(a + b*x)^
5) + (7*e^5*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(128*b^(9/2)*
(b*d - a*e)^(3/2))
```


Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{7/2}}{\left(a^2+2abx+b^2x^2\right)^3}\,dx = \int \frac{(d+ex)^{7/2}}{(a+bx)^6}\,dx$$

$$= -\frac{(d+ex)^{7/2}}{5b(a+bx)^5} + \frac{(7e)\int \frac{(d+ex)^{5/2}}{(a+bx)^5}\,dx}{10b}$$

$$= -\frac{7e(d+ex)^{5/2}}{40b^2(a+bx)^4} - \frac{(d+ex)^{7/2}}{5b(a+bx)^5} + \frac{\left(7e^2\right)\int \frac{(d+ex)^{3/2}}{(a+bx)^4}\,dx}{16b^2}$$

$$= -\frac{7e^2(d+ex)^{3/2}}{48b^3(a+bx)^3} - \frac{7e(d+ex)^{5/2}}{40b^2(a+bx)^4} - \frac{(d+ex)^{7/2}}{5b(a+bx)^5} + \frac{\left(7e^3\right)\int \frac{\sqrt{d+ex}}{(a+bx)^3}\,dx}{32b^3}$$

$$= -\frac{7e^3\sqrt{d+ex}}{64b^4(a+bx)^2} - \frac{7e^2(d+ex)^{3/2}}{48b^3(a+bx)^3} - \frac{7e(d+ex)^{5/2}}{40b^2(a+bx)^4} - \frac{(d+ex)^{7/2}}{5b(a+bx)^5} + \frac{\left(7e^4\right)\int \frac{1}{(a+bx)^2\sqrt{d+ex}}\,dx}{128b^4}$$

$$= -\frac{7e^3\sqrt{d+ex}}{64b^4(a+bx)^2} - \frac{7e^4\sqrt{d+ex}}{128b^4(bd-ae)(a+bx)} - \frac{7e^2(d+ex)^{3/2}}{48b^3(a+bx)^3} - \frac{7e(d+ex)^{5/2}}{40b^2(a+bx)^4} - \frac{(d+ex)^{7/2}}{5b(a+bx)^5} -$$

$$= -\frac{7e^3\sqrt{d+ex}}{64b^4(a+bx)^2} - \frac{7e^4\sqrt{d+ex}}{128b^4(bd-ae)(a+bx)} - \frac{7e^2(d+ex)^{3/2}}{48b^3(a+bx)^3} - \frac{7e(d+ex)^{5/2}}{40b^2(a+bx)^4} - \frac{(d+ex)^{7/2}}{5b(a+bx)^5} -$$

$$= -\frac{7e^3\sqrt{d+ex}}{64b^4(a+bx)^2} - \frac{7e^4\sqrt{d+ex}}{128b^4(bd-ae)(a+bx)} - \frac{7e^2(d+ex)^{3/2}}{48b^3(a+bx)^3} - \frac{7e(d+ex)^{5/2}}{40b^2(a+bx)^4} - \frac{(d+ex)^{7/2}}{5b(a+bx)^5} +$$

Mathematica [C] time = 0.019642, size = 52, normalized size = 0.28

$$\frac{2e^5(d+ex)^{9/2}\,{}_2F_1\left(\frac{9}{2},6;\frac{11}{2};-\frac{b(d+ex)}{ae-bd}\right)}{9(ae-bd)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] (2*e^5*(d + e*x)^(9/2)*Hypergeometric2F1[9/2, 6, 11/2, -((b*(d + e*x))/(-(b
*d) + a*e))])/(9*(-(b*d) + a*e)^6)
```

Maple [B] time = 0.258, size = 360, normalized size = 1.9

$$\frac{7\,e^5}{128\,(bxe+ae)^5\,(ae-bd)}(ex+d)^{\frac{9}{2}} - \frac{79\,e^5}{192\,(bxe+ae)^5\,b}(ex+d)^{\frac{7}{2}} - \frac{7\,e^6a}{15\,(bxe+ae)^5\,b^2}(ex+d)^{\frac{5}{2}} + \frac{7\,e^5d}{15\,(bxe+ae)^5\,b}(ex+d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] 7/128*e^5/(b*e*x+a*e)^5/(a*e-b*d)*(e*x+d)^(9/2)-79/192*e^5/(b*e*x+a*e)^5/b*
(e*x+d)^(7/2)-7/15*e^6/(b*e*x+a*e)^5/b^2*(e*x+d)^(5/2)*a+7/15*e^5/(b*e*x+a*
e)^5/b*(e*x+d)^(5/2)*d-49/192*e^7/(b*e*x+a*e)^5/b^3*(e*x+d)^(3/2)*a^2+49/96
*e^6/(b*e*x+a*e)^5/b^2*(e*x+d)^(3/2)*a*d-49/192*e^5/(b*e*x+a*e)^5/b*(e*x+d)
^(3/2)*d^2-7/128*e^8/(b*e*x+a*e)^5/b^4*(e*x+d)^(1/2)*a^3+21/128*e^7/(b*e*x+
a*e)^5/b^3*(e*x+d)^(1/2)*d*a^2-21/128*e^6/(b*e*x+a*e)^5/b^2*(e*x+d)^(1/2)*a
*d^2+7/128*e^5/(b*e*x+a*e)^5/b*(e*x+d)^(1/2)*d^3+7/128*e^5/(a*e-b*d)/b^4/((
a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.898, size = 2458, normalized size = 13.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/3840*(105*(b^5*e^5*x^5 + 5*a*b^4*e^5*x^4 + 10*a^2*b^3*e^5*x^3 + 10*a^3*
b^2*e^5*x^2 + 5*a^4*b*e^5*x + a^5*e^5)*sqrt(b^2*d - a*b*e)*log((b*e*x + 2*b
*d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d))/(b*x + a)) + 2*(384*b^6*d^5
 - 432*a*b^5*d^4*e - 8*a^2*b^4*d^3*e^2 - 14*a^3*b^3*d^2*e^3 - 35*a^4*b^2*d*
e^4 + 105*a^5*b*e^5 + 105*(b^6*d*e^4 - a*b^5*e^5)*x^4 + 10*(121*b^6*d^2*e^3
 - 200*a*b^5*d*e^4 + 79*a^2*b^4*e^5)*x^3 + 2*(1052*b^6*d^3*e^2 - 1341*a*b^5
*d^2*e^3 - 159*a^2*b^4*d*e^4 + 448*a^3*b^3*e^5)*x^2 + 2*(744*b^6*d^4*e - 87
2*a*b^5*d^3*e^2 - 33*a^2*b^4*d^2*e^3 - 84*a^3*b^3*d*e^4 + 245*a^4*b^2*e^5)*
x)*sqrt(e*x + d))/(a^5*b^7*d^2 - 2*a^6*b^6*d*e + a^7*b^5*e^2 + (b^12*d^2 -
2*a*b^11*d*e + a^2*b^10*e^2)*x^5 + 5*(a*b^11*d^2 - 2*a^2*b^10*d*e + a^3*b^9
*e^2)*x^4 + 10*(a^2*b^10*d^2 - 2*a^3*b^9*d*e + a^4*b^8*e^2)*x^3 + 10*(a^3*b
^9*d^2 - 2*a^4*b^8*d*e + a^5*b^7*e^2)*x^2 + 5*(a^4*b^8*d^2 - 2*a^5*b^7*d*e
+ a^6*b^6*e^2)*x), -1/1920*(105*(b^5*e^5*x^5 + 5*a*b^4*e^5*x^4 + 10*a^2*b^3
*e^5*x^3 + 10*a^3*b^2*e^5*x^2 + 5*a^4*b*e^5*x + a^5*e^5)*sqrt(-b^2*d + a*b*
e)*arctan(sqrt(-b^2*d + a*b*e)*sqrt(e*x + d)/(b*e*x + b*d)) + (384*b^6*d^5
- 432*a*b^5*d^4*e - 8*a^2*b^4*d^3*e^2 - 14*a^3*b^3*d^2*e^3 - 35*a^4*b^2*d*e
^4 + 105*a^5*b*e^5 + 105*(b^6*d*e^4 - a*b^5*e^5)*x^4 + 10*(121*b^6*d^2*e^3
- 200*a*b^5*d*e^4 + 79*a^2*b^4*e^5)*x^3 + 2*(1052*b^6*d^3*e^2 - 1341*a*b^5*
d^2*e^3 - 159*a^2*b^4*d*e^4 + 448*a^3*b^3*e^5)*x^2 + 2*(744*b^6*d^4*e - 872
*a*b^5*d^3*e^2 - 33*a^2*b^4*d^2*e^3 - 84*a^3*b^3*d*e^4 + 245*a^4*b^2*e^5)*x
)*sqrt(e*x + d))/(a^5*b^7*d^2 - 2*a^6*b^6*d*e + a^7*b^5*e^2 + (b^12*d^2 - 2
*a*b^11*d*e + a^2*b^10*e^2)*x^5 + 5*(a*b^11*d^2 - 2*a^2*b^10*d*e + a^3*b^9*
e^2)*x^4 + 10*(a^2*b^10*d^2 - 2*a^3*b^9*d*e + a^4*b^8*e^2)*x^3 + 10*(a^3*b^
```

```
9*d^2 - 2*a^4*b^8*d*e + a^5*b^7*e^2)*x^2 + 5*(a^4*b^8*d^2 - 2*a^5*b^7*d*e +
 a^6*b^6*e^2)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26487, size = 486, normalized size = 2.59

$$-\frac{7\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)e^5}{128\left(b^5d-ab^4e\right)\sqrt{-b^2d+abe}}-\frac{105\,(xe+d)^{\frac{9}{2}}b^4e^5+790\,(xe+d)^{\frac{7}{2}}b^4de^5-896\,(xe+d)^{\frac{5}{2}}b^4d^2e^5+490\,(xe+d)^{\frac{3}{2}}b^4d^3e^5-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] -7/128*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e^5/((b^5*d - a*b^4*e)*
sqrt(-b^2*d + a*b*e)) - 1/1920*(105*(x*e + d)^(9/2)*b^4*e^5 + 790*(x*e + d)
^(7/2)*b^4*d*e^5 - 896*(x*e + d)^(5/2)*b^4*d^2*e^5 + 490*(x*e + d)^(3/2)*b^
4*d^3*e^5 - 105*sqrt(x*e + d)*b^4*d^4*e^5 - 790*(x*e + d)^(7/2)*a*b^3*e^6 +
 1792*(x*e + d)^(5/2)*a*b^3*d*e^6 - 1470*(x*e + d)^(3/2)*a*b^3*d^2*e^6 + 42
0*sqrt(x*e + d)*a*b^3*d^3*e^6 - 896*(x*e + d)^(5/2)*a^2*b^2*e^7 + 1470*(x*e
 + d)^(3/2)*a^2*b^2*d*e^7 - 630*sqrt(x*e + d)*a^2*b^2*d^2*e^7 - 490*(x*e +
d)^(3/2)*a^3*b*e^8 + 420*sqrt(x*e + d)*a^3*b*d*e^8 - 105*sqrt(x*e + d)*a^4*
e^9)/((b^5*d - a*b^4*e)*((x*e + d)*b - b*d + a*e)^5)
```

3.1669 $\int \frac{(d+ex)^{5/2}}{\left(a^2+2abx+b^2x^2\right)^3} \, dx$

Optimal. Leaf size=198

$$\frac{3e^4\sqrt{d+ex}}{128b^3(a+bx)(bd-ae)^2}-\frac{e^3\sqrt{d+ex}}{64b^3(a+bx)^2(bd-ae)}-\frac{e^2\sqrt{d+ex}}{16b^3(a+bx)^3}-\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{128b^{7/2}(bd-ae)^{5/2}}-\frac{e(d+ex)^{3/2}}{8b^2(a+bx)^4}-\frac{(d+ex)^{5/2}}{5b(a+bx)^5}$$

```
[Out] -(e^2*Sqrt[d + e*x])/(16*b^3*(a + b*x)^3) - (e^3*Sqrt[d + e*x])/(64*b^3*(b*
d - a*e)*(a + b*x)^2) + (3*e^4*Sqrt[d + e*x])/(128*b^3*(b*d - a*e)^2*(a + b
*x)) - (e*(d + e*x)^(3/2))/(8*b^2*(a + b*x)^4) - (d + e*x)^(5/2)/(5*b*(a +
b*x)^5) - (3*e^5*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(128*b^(
7/2)*(b*d - a*e)^(5/2))
```

Rubi [A] time = 0.102512, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {27, 47, 51, 63, 208}

$$\frac{3e^4\sqrt{d+ex}}{128b^3(a+bx)(bd-ae)^2}-\frac{e^3\sqrt{d+ex}}{64b^3(a+bx)^2(bd-ae)}-\frac{e^2\sqrt{d+ex}}{16b^3(a+bx)^3}-\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{128b^{7/2}(bd-ae)^{5/2}}-\frac{e(d+ex)^{3/2}}{8b^2(a+bx)^4}-\frac{(d+ex)^{5/2}}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] -(e^2*Sqrt[d + e*x])/(16*b^3*(a + b*x)^3) - (e^3*Sqrt[d + e*x])/(64*b^3*(b*
d - a*e)*(a + b*x)^2) + (3*e^4*Sqrt[d + e*x])/(128*b^3*(b*d - a*e)^2*(a + b
*x)) - (e*(d + e*x)^(3/2))/(8*b^2*(a + b*x)^4) - (d + e*x)^(5/2)/(5*b*(a +
b*x)^5) - (3*e^5*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(128*b^(
7/2)*(b*d - a*e)^(5/2))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\left(a^2+2abx+b^2x^2\right)^3}\,dx = \int \frac{(d+ex)^{5/2}}{(a+bx)^6}\,dx$$

$$= -\frac{(d+ex)^{5/2}}{5b(a+bx)^5} + \frac{e\int \frac{(d+ex)^{3/2}}{(a+bx)^5}\,dx}{2b}$$

$$= -\frac{e(d+ex)^{3/2}}{8b^2(a+bx)^4} - \frac{(d+ex)^{5/2}}{5b(a+bx)^5} + \frac{\left(3e^2\right)\int \frac{\sqrt{d+ex}}{(a+bx)^4}\,dx}{16b^2}$$

$$= -\frac{e^2\sqrt{d+ex}}{16b^3(a+bx)^3} - \frac{e(d+ex)^{3/2}}{8b^2(a+bx)^4} - \frac{(d+ex)^{5/2}}{5b(a+bx)^5} + \frac{e^3\int \frac{1}{(a+bx)^3\sqrt{d+ex}}\,dx}{32b^3}$$

$$= -\frac{e^2\sqrt{d+ex}}{16b^3(a+bx)^3} - \frac{e^3\sqrt{d+ex}}{64b^3(bd-ae)(a+bx)^2} - \frac{e(d+ex)^{3/2}}{8b^2(a+bx)^4} - \frac{(d+ex)^{5/2}}{5b(a+bx)^5} - \frac{\left(3e^4\right)\int \frac{1}{(a+bx)^2\sqrt{d+}}}{128b^3(bd-ae}$$

$$= -\frac{e^2\sqrt{d+ex}}{16b^3(a+bx)^3} - \frac{e^3\sqrt{d+ex}}{64b^3(bd-ae)(a+bx)^2} + \frac{3e^4\sqrt{d+ex}}{128b^3(bd-ae)^2(a+bx)} - \frac{e(d+ex)^{3/2}}{8b^2(a+bx)^4} - \frac{(d+}{5b(a}$$

$$= -\frac{e^2\sqrt{d+ex}}{16b^3(a+bx)^3} - \frac{e^3\sqrt{d+ex}}{64b^3(bd-ae)(a+bx)^2} + \frac{3e^4\sqrt{d+ex}}{128b^3(bd-ae)^2(a+bx)} - \frac{e(d+ex)^{3/2}}{8b^2(a+bx)^4} - \frac{(d+}{5b(a}$$

$$= -\frac{e^2\sqrt{d+ex}}{16b^3(a+bx)^3} - \frac{e^3\sqrt{d+ex}}{64b^3(bd-ae)(a+bx)^2} + \frac{3e^4\sqrt{d+ex}}{128b^3(bd-ae)^2(a+bx)} - \frac{e(d+ex)^{3/2}}{8b^2(a+bx)^4} - \frac{(d+}{5b(a}$$

Mathematica [C] time = 0.0184048, size = 52, normalized size = 0.26

$$\frac{2e^5(d+ex)^{7/2}\,{}_2F_1\left(\frac{7}{2},6;\frac{9}{2};-\frac{b(d+ex)}{ae-bd}\right)}{7(ae-bd)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] (2*e^5*(d + e*x)^(7/2)*Hypergeometric2F1[7/2, 6, 9/2, -((b*(d + e*x))/(-(b*
d) + a*e))])/(7*(-(b*d) + a*e)^6)
```

Maple [A] time = 0.205, size = 305, normalized size = 1.5

$$\frac{3\,e^5b}{128\,(bxe+ae)^5\left(a^2e^2-2\,abde+b^2d^2\right)}(ex+d)^{\frac{9}{2}} + \frac{7\,e^5}{64\,(bxe+ae)^5\,(ae-bd)}(ex+d)^{\frac{7}{2}} - \frac{e^5}{5\,(bxe+ae)^5\,b}(ex+d)^{\frac{5}{2}} - \frac{}{64\,(b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] 3/128*e^5/(b*e*x+a*e)^5*b/(a^2*e^2-2*a*b*d*e+b^2*d^2)*(e*x+d)^(9/2)+7/64*e^
5/(b*e*x+a*e)^5/(a*e-b*d)*(e*x+d)^(7/2)-1/5*e^5/(b*e*x+a*e)^5/b*(e*x+d)^(5/
2)-7/64*e^6/(b*e*x+a*e)^5/b^2*(e*x+d)^(3/2)*a+7/64*e^5/(b*e*x+a*e)^5/b*(e*x
+d)^(3/2)*d-3/128*e^7/(b*e*x+a*e)^5/b^3*(e*x+d)^(1/2)*a^2+3/64*e^6/(b*e*x+a
*e)^5/b^2*(e*x+d)^(1/2)*a*d-3/128*e^5/(b*e*x+a*e)^5/b*(e*x+d)^(1/2)*d^2+3/1
28*e^5/b^3/(a^2*e^2-2*a*b*d*e+b^2*d^2)/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)
^(1/2)/((a*e-b*d)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.91889, size = 2743, normalized size = 13.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/1280*(15*(b^5*e^5*x^5 + 5*a*b^4*e^5*x^4 + 10*a^2*b^3*e^5*x^3 + 10*a^3*b^
2*e^5*x^2 + 5*a^4*b*e^5*x + a^5*e^5)*sqrt(b^2*d - a*b*e)*log((b*e*x + 2*b*d
 - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d))/(b*x + a)) - 2*(128*b^6*d^5 -
 304*a*b^5*d^4*e + 184*a^2*b^4*d^3*e^2 + 2*a^3*b^3*d^2*e^3 + 5*a^4*b^2*d*e^
4 - 15*a^5*b*e^5 - 15*(b^6*d*e^4 - a*b^5*e^5)*x^4 + 10*(b^6*d^2*e^3 - 8*a*b
^5*d*e^4 + 7*a^2*b^4*e^5)*x^3 + 2*(124*b^6*d^3*e^2 - 357*a*b^5*d^2*e^3 + 29
7*a^2*b^4*d*e^4 - 64*a^3*b^3*e^5)*x^2 + 2*(168*b^6*d^4*e - 424*a*b^5*d^3*e^
2 + 279*a^2*b^4*d^2*e^3 + 12*a^3*b^3*d*e^4 - 35*a^4*b^2*e^5)*x)*sqrt(e*x +
d))/(a^5*b^7*d^3 - 3*a^6*b^6*d^2*e + 3*a^7*b^5*d*e^2 - a^8*b^4*e^3 + (b^12*
d^3 - 3*a*b^11*d^2*e + 3*a^2*b^10*d*e^2 - a^3*b^9*e^3)*x^5 + 5*(a*b^11*d^3
- 3*a^2*b^10*d^2*e + 3*a^3*b^9*d*e^2 - a^4*b^8*e^3)*x^4 + 10*(a^2*b^10*d^3
- 3*a^3*b^9*d^2*e + 3*a^4*b^8*d*e^2 - a^5*b^7*e^3)*x^3 + 10*(a^3*b^9*d^3 -
3*a^4*b^8*d^2*e + 3*a^5*b^7*d*e^2 - a^6*b^6*e^3)*x^2 + 5*(a^4*b^8*d^3 - 3*a
^5*b^7*d^2*e + 3*a^6*b^6*d*e^2 - a^7*b^5*e^3)*x), 1/640*(15*(b^5*e^5*x^5 +
5*a*b^4*e^5*x^4 + 10*a^2*b^3*e^5*x^3 + 10*a^3*b^2*e^5*x^2 + 5*a^4*b*e^5*x +
 a^5*e^5)*sqrt(-b^2*d + a*b*e)*arctan(sqrt(-b^2*d + a*b*e)*sqrt(e*x + d)/(b
*e*x + b*d)) - (128*b^6*d^5 - 304*a*b^5*d^4*e + 184*a^2*b^4*d^3*e^2 + 2*a^3
*b^3*d^2*e^3 + 5*a^4*b^2*d*e^4 - 15*a^5*b*e^5 - 15*(b^6*d*e^4 - a*b^5*e^5)*
x^4 + 10*(b^6*d^2*e^3 - 8*a*b^5*d*e^4 + 7*a^2*b^4*e^5)*x^3 + 2*(124*b^6*d^3
*e^2 - 357*a*b^5*d^2*e^3 + 297*a^2*b^4*d*e^4 - 64*a^3*b^3*e^5)*x^2 + 2*(168
*b^6*d^4*e - 424*a*b^5*d^3*e^2 + 279*a^2*b^4*d^2*e^3 + 12*a^3*b^3*d*e^4 - 3
5*a^4*b^2*e^5)*x)*sqrt(e*x + d))/(a^5*b^7*d^3 - 3*a^6*b^6*d^2*e + 3*a^7*b^5
*d*e^2 - a^8*b^4*e^3 + (b^12*d^3 - 3*a*b^11*d^2*e + 3*a^2*b^10*d*e^2 - a^3*
b^9*e^3)*x^5 + 5*(a*b^11*d^3 - 3*a^2*b^10*d^2*e + 3*a^3*b^9*d*e^2 - a^4*b^8
```

```
*e^3)*x^4 + 10*(a^2*b^10*d^3 - 3*a^3*b^9*d^2*e + 3*a^4*b^8*d*e^2 - a^5*b^7*
e^3)*x^3 + 10*(a^3*b^9*d^3 - 3*a^4*b^8*d^2*e + 3*a^5*b^7*d*e^2 - a^6*b^6*e^
3)*x^2 + 5*(a^4*b^8*d^3 - 3*a^5*b^7*d^2*e + 3*a^6*b^6*d*e^2 - a^7*b^5*e^3)*
x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24838, size = 518, normalized size = 2.62

$$\frac{3 \arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)e^5}{128\left(b^5d^2 - 2\,ab^4de + a^2b^3e^2\right)\sqrt{-b^2d+abe}} + \frac{15\,(xe+d)^{\frac{9}{2}}b^4e^5 - 70\,(xe+d)^{\frac{7}{2}}b^4de^5 - 128\,(xe+d)^{\frac{5}{2}}b^4d^2e^5 + 70\,(xe+d)^{\frac{3}{2}}b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] 3/128*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e^5/((b^5*d^2 - 2*a*b^4*
d*e + a^2*b^3*e^2)*sqrt(-b^2*d + a*b*e)) + 1/640*(15*(x*e + d)^(9/2)*b^4*e^
5 - 70*(x*e + d)^(7/2)*b^4*d*e^5 - 128*(x*e + d)^(5/2)*b^4*d^2*e^5 + 70*(x*
e + d)^(3/2)*b^4*d^3*e^5 - 15*sqrt(x*e + d)*b^4*d^4*e^5 + 70*(x*e + d)^(7/2
)*a*b^3*e^6 + 256*(x*e + d)^(5/2)*a*b^3*d*e^6 - 210*(x*e + d)^(3/2)*a*b^3*d
^2*e^6 + 60*sqrt(x*e + d)*a*b^3*d^3*e^6 - 128*(x*e + d)^(5/2)*a^2*b^2*e^7 +
 210*(x*e + d)^(3/2)*a^2*b^2*d*e^7 - 90*sqrt(x*e + d)*a^2*b^2*d^2*e^7 - 70*
(x*e + d)^(3/2)*a^3*b*e^8 + 60*sqrt(x*e + d)*a^3*b*d*e^8 - 15*sqrt(x*e + d)
*a^4*e^9)/((b^5*d^2 - 2*a*b^4*d*e + a^2*b^3*e^2)*((x*e + d)*b - b*d + a*e)^
5)
```

3.1670 $\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^3} dx$

Optimal. Leaf size=208

$$-\frac{3e^4\sqrt{d+ex}}{128b^2(a+bx)(bd-ae)^3}+\frac{e^3\sqrt{d+ex}}{64b^2(a+bx)^2(bd-ae)^2}-\frac{e^2\sqrt{d+ex}}{80b^2(a+bx)^3(bd-ae)}+\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{128b^{5/2}(bd-ae)^{7/2}}-\frac{3e\sqrt{d+ex}}{40b^2(a+bx}$$

```
[Out] (-3*e*Sqrt[d + e*x])/(40*b^2*(a + b*x)^4) - (e^2*Sqrt[d + e*x])/(80*b^2*(b*
d - a*e)*(a + b*x)^3) + (e^3*Sqrt[d + e*x])/(64*b^2*(b*d - a*e)^2*(a + b*x)
^2) - (3*e^4*Sqrt[d + e*x])/(128*b^2*(b*d - a*e)^3*(a + b*x)) - (d + e*x)^(
3/2)/(5*b*(a + b*x)^5) + (3*e^5*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d -
a*e]])/(128*b^(5/2)*(b*d - a*e)^(7/2))
```

Rubi [A] time = 0.123246, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {27, 47, 51, 63, 208}

$$-\frac{3e^4\sqrt{d+ex}}{128b^2(a+bx)(bd-ae)^3}+\frac{e^3\sqrt{d+ex}}{64b^2(a+bx)^2(bd-ae)^2}-\frac{e^2\sqrt{d+ex}}{80b^2(a+bx)^3(bd-ae)}+\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{128b^{5/2}(bd-ae)^{7/2}}-\frac{3e\sqrt{d+ex}}{40b^2(a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] (-3*e*Sqrt[d + e*x])/(40*b^2*(a + b*x)^4) - (e^2*Sqrt[d + e*x])/(80*b^2*(b*
d - a*e)*(a + b*x)^3) + (e^3*Sqrt[d + e*x])/(64*b^2*(b*d - a*e)^2*(a + b*x)
^2) - (3*e^4*Sqrt[d + e*x])/(128*b^2*(b*d - a*e)^3*(a + b*x)) - (d + e*x)^(
3/2)/(5*b*(a + b*x)^5) + (3*e^5*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d -
a*e]])/(128*b^(5/2)*(b*d - a*e)^(7/2))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\left(a^2+2abx+b^2x^2\right)^3}\,dx = \int \frac{(d+ex)^{3/2}}{(a+bx)^6}\,dx$$

$$= -\frac{(d+ex)^{3/2}}{5b(a+bx)^5} + \frac{(3e)\int \frac{\sqrt{d+ex}}{(a+bx)^5}\,dx}{10b}$$

$$= -\frac{3e\sqrt{d+ex}}{40b^2(a+bx)^4} - \frac{(d+ex)^{3/2}}{5b(a+bx)^5} + \frac{\left(3e^2\right)\int \frac{1}{(a+bx)^4\sqrt{d+ex}}\,dx}{80b^2}$$

$$= -\frac{3e\sqrt{d+ex}}{40b^2(a+bx)^4} - \frac{e^2\sqrt{d+ex}}{80b^2(bd-ae)(a+bx)^3} - \frac{(d+ex)^{3/2}}{5b(a+bx)^5} - \frac{e^3\int \frac{1}{(a+bx)^3\sqrt{d+ex}}\,dx}{32b^2(bd-ae)}$$

$$= -\frac{3e\sqrt{d+ex}}{40b^2(a+bx)^4} - \frac{e^2\sqrt{d+ex}}{80b^2(bd-ae)(a+bx)^3} + \frac{e^3\sqrt{d+ex}}{64b^2(bd-ae)^2(a+bx)^2} - \frac{(d+ex)^{3/2}}{5b(a+bx)^5} + \frac{\left(3e^4\right)}{12}$$

$$= -\frac{3e\sqrt{d+ex}}{40b^2(a+bx)^4} - \frac{e^2\sqrt{d+ex}}{80b^2(bd-ae)(a+bx)^3} + \frac{e^3\sqrt{d+ex}}{64b^2(bd-ae)^2(a+bx)^2} - \frac{3e^4\sqrt{d+ex}}{128b^2(bd-ae)^3(a+}$$

$$= -\frac{3e\sqrt{d+ex}}{40b^2(a+bx)^4} - \frac{e^2\sqrt{d+ex}}{80b^2(bd-ae)(a+bx)^3} + \frac{e^3\sqrt{d+ex}}{64b^2(bd-ae)^2(a+bx)^2} - \frac{3e^4\sqrt{d+ex}}{128b^2(bd-ae)^3(a+}$$

$$= -\frac{3e\sqrt{d+ex}}{40b^2(a+bx)^4} - \frac{e^2\sqrt{d+ex}}{80b^2(bd-ae)(a+bx)^3} + \frac{e^3\sqrt{d+ex}}{64b^2(bd-ae)^2(a+bx)^2} - \frac{3e^4\sqrt{d+ex}}{128b^2(bd-ae)^3(a+}$$

Mathematica [C] time = 0.0181231, size = 52, normalized size = 0.25

$$\frac{2e^5(d+ex)^{5/2}\,{}_2F_1\left(\frac{5}{2},6;\frac{7}{2};-\frac{b(d+ex)}{ae-bd}\right)}{5(ae-bd)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] (2*e^5*(d + e*x)^(5/2)*Hypergeometric2F1[5/2, 6, 7/2, -((b*(d + e*x))/(-(b*
d) + a*e))])/(5*(-(b*d) + a*e)^6)
```

Maple [A] time = 0.232, size = 300, normalized size = 1.4

$$\frac{3\,e^5b^2}{128\,(bxe+ae)^5\left(a^3e^3-3\,de^2a^2b+3\,ab^2d^2e-b^3d^3\right)}(ex+d)^{\frac{9}{2}} + \frac{7\,e^5b}{64\,(bxe+ae)^5\left(a^2e^2-2\,abde+b^2d^2\right)}(ex+d)^{\frac{7}{2}} + \frac{}{5\,(bxe}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] 3/128*e^5/(b*e*x+a*e)^5/(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d^2*e-b^3*d^3)*b^2*(
e*x+d)^(9/2)+7/64*e^5/(b*e*x+a*e)^5*b/(a^2*e^2-2*a*b*d*e+b^2*d^2)*(e*x+d)^(
7/2)+1/5*e^5/(b*e*x+a*e)^5/(a*e-b*d)*(e*x+d)^(5/2)-7/64*e^5/(b*e*x+a*e)^5/b
*(e*x+d)^(3/2)-3/128*e^6/(b*e*x+a*e)^5/b^2*(e*x+d)^(1/2)*a+3/128*e^5/(b*e*x
+a*e)^5/b*(e*x+d)^(1/2)*d+3/128*e^5/(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d^2*e-b^
3*d^3)/b^2/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.86769, size = 3065, normalized size = 14.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/1280*(15*(b^5*e^5*x^5 + 5*a*b^4*e^5*x^4 + 10*a^2*b^3*e^5*x^3 + 10*a^3*b
^2*e^5*x^2 + 5*a^4*b*e^5*x + a^5*e^5)*sqrt(b^2*d - a*b*e)*log((b*e*x + 2*b*
d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d))/(b*x + a)) + 2*(128*b^6*d^5
- 464*a*b^5*d^4*e + 584*a^2*b^4*d^3*e^2 - 258*a^3*b^3*d^2*e^3 - 5*a^4*b^2*d
*e^4 + 15*a^5*b*e^5 + 15*(b^6*d*e^4 - a*b^5*e^5)*x^4 - 10*(b^6*d^2*e^3 - 8*
a*b^5*d*e^4 + 7*a^2*b^4*e^5)*x^3 + 2*(4*b^6*d^3*e^2 - 27*a*b^5*d^2*e^3 + 87
*a^2*b^4*d*e^4 - 64*a^3*b^3*e^5)*x^2 + 2*(88*b^6*d^4*e - 344*a*b^5*d^3*e^2
+ 489*a^2*b^4*d^2*e^3 - 268*a^3*b^3*d*e^4 + 35*a^4*b^2*e^5)*x)*sqrt(e*x + d
))/(a^5*b^7*d^4 - 4*a^6*b^6*d^3*e + 6*a^7*b^5*d^2*e^2 - 4*a^8*b^4*d*e^3 + a
^9*b^3*e^4 + (b^12*d^4 - 4*a*b^11*d^3*e + 6*a^2*b^10*d^2*e^2 - 4*a^3*b^9*d*
e^3 + a^4*b^8*e^4)*x^5 + 5*(a*b^11*d^4 - 4*a^2*b^10*d^3*e + 6*a^3*b^9*d^2*e
^2 - 4*a^4*b^8*d*e^3 + a^5*b^7*e^4)*x^4 + 10*(a^2*b^10*d^4 - 4*a^3*b^9*d^3*
e + 6*a^4*b^8*d^2*e^2 - 4*a^5*b^7*d*e^3 + a^6*b^6*e^4)*x^3 + 10*(a^3*b^9*d^
4 - 4*a^4*b^8*d^3*e + 6*a^5*b^7*d^2*e^2 - 4*a^6*b^6*d*e^3 + a^7*b^5*e^4)*x^
2 + 5*(a^4*b^8*d^4 - 4*a^5*b^7*d^3*e + 6*a^6*b^6*d^2*e^2 - 4*a^7*b^5*d*e^3
+ a^8*b^4*e^4)*x), -1/640*(15*(b^5*e^5*x^5 + 5*a*b^4*e^5*x^4 + 10*a^2*b^3*e
^5*x^3 + 10*a^3*b^2*e^5*x^2 + 5*a^4*b*e^5*x + a^5*e^5)*sqrt(-b^2*d + a*b*e)
*arctan(sqrt(-b^2*d + a*b*e)*sqrt(e*x + d)/(b*e*x + b*d)) + (128*b^6*d^5 -
464*a*b^5*d^4*e + 584*a^2*b^4*d^3*e^2 - 258*a^3*b^3*d^2*e^3 - 5*a^4*b^2*d*e
^4 + 15*a^5*b*e^5 + 15*(b^6*d*e^4 - a*b^5*e^5)*x^4 - 10*(b^6*d^2*e^3 - 8*a*
b^5*d*e^4 + 7*a^2*b^4*e^5)*x^3 + 2*(4*b^6*d^3*e^2 - 27*a*b^5*d^2*e^3 + 87*a
^2*b^4*d*e^4 - 64*a^3*b^3*e^5)*x^2 + 2*(88*b^6*d^4*e - 344*a*b^5*d^3*e^2 +
489*a^2*b^4*d^2*e^3 - 268*a^3*b^3*d*e^4 + 35*a^4*b^2*e^5)*x)*sqrt(e*x + d))
/(a^5*b^7*d^4 - 4*a^6*b^6*d^3*e + 6*a^7*b^5*d^2*e^2 - 4*a^8*b^4*d*e^3 + a^9
*b^3*e^4 + (b^12*d^4 - 4*a*b^11*d^3*e + 6*a^2*b^10*d^2*e^2 - 4*a^3*b^9*d*e^
```

```
3 + a^4*b^8*e^4)*x^5 + 5*(a*b^11*d^4 - 4*a^2*b^10*d^3*e + 6*a^3*b^9*d^2*e^2
 - 4*a^4*b^8*d*e^3 + a^5*b^7*e^4)*x^4 + 10*(a^2*b^10*d^4 - 4*a^3*b^9*d^3*e
+ 6*a^4*b^8*d^2*e^2 - 4*a^5*b^7*d*e^3 + a^6*b^6*e^4)*x^3 + 10*(a^3*b^9*d^4
- 4*a^4*b^8*d^3*e + 6*a^5*b^7*d^2*e^2 - 4*a^6*b^6*d*e^3 + a^7*b^5*e^4)*x^2
+ 5*(a^4*b^8*d^4 - 4*a^5*b^7*d^3*e + 6*a^6*b^6*d^2*e^2 - 4*a^7*b^5*d*e^3 +
a^8*b^4*e^4)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24847, size = 556, normalized size = 2.67

$$-\frac{3 \arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)e^5}{128\left(b^5d^3-3\,ab^4d^2e+3\,a^2b^3de^2-a^3b^2e^3\right)\sqrt{-b^2d+abe}}-\frac{15\,(xe+d)^{\frac{9}{2}}b^4e^5-70\,(xe+d)^{\frac{7}{2}}b^4de^5+128\,(xe+d)^{\frac{5}{2}}b^4d^2e^5+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] -3/128*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e^5/((b^5*d^3 - 3*a*b^4
*d^2*e + 3*a^2*b^3*d*e^2 - a^3*b^2*e^3)*sqrt(-b^2*d + a*b*e)) - 1/640*(15*(
x*e + d)^(9/2)*b^4*e^5 - 70*(x*e + d)^(7/2)*b^4*d*e^5 + 128*(x*e + d)^(5/2)
*b^4*d^2*e^5 + 70*(x*e + d)^(3/2)*b^4*d^3*e^5 - 15*sqrt(x*e + d)*b^4*d^4*e^
5 + 70*(x*e + d)^(7/2)*a*b^3*e^6 - 256*(x*e + d)^(5/2)*a*b^3*d*e^6 - 210*(x
*e + d)^(3/2)*a*b^3*d^2*e^6 + 60*sqrt(x*e + d)*a*b^3*d^3*e^6 + 128*(x*e + d
)^(5/2)*a^2*b^2*e^7 + 210*(x*e + d)^(3/2)*a^2*b^2*d*e^7 - 90*sqrt(x*e + d)*
a^2*b^2*d^2*e^7 - 70*(x*e + d)^(3/2)*a^3*b*e^8 + 60*sqrt(x*e + d)*a^3*b*d*e
^8 - 15*sqrt(x*e + d)*a^4*e^9)/((b^5*d^3 - 3*a*b^4*d^2*e + 3*a^2*b^3*d*e^2
- a^3*b^2*e^3)*((x*e + d)*b - b*d + a*e)^5)
```

3.1671 $\int \frac{\sqrt{d+ex}}{\left(a^2+2abx+b^2x^2\right)^3}\,dx$

Optimal. Leaf size=218

$$-\frac{7e^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{128b^{3/2}(bd-ae)^{9/2}}+\frac{7e^4\sqrt{d+ex}}{128b(a+bx)(bd-ae)^4}-\frac{7e^3\sqrt{d+ex}}{192b(a+bx)^2(bd-ae)^3}+\frac{7e^2\sqrt{d+ex}}{240b(a+bx)^3(bd-ae)^2}-\frac{e\sqrt{d}}{40b(a+bx)}$$

```
[Out] -Sqrt[d + e*x]/(5*b*(a + b*x)^5) - (e*Sqrt[d + e*x])/(40*b*(b*d - a*e)*(a +
 b*x)^4) + (7*e^2*Sqrt[d + e*x])/(240*b*(b*d - a*e)^2*(a + b*x)^3) - (7*e^3
*Sqrt[d + e*x])/(192*b*(b*d - a*e)^3*(a + b*x)^2) + (7*e^4*Sqrt[d + e*x])/(
128*b*(b*d - a*e)^4*(a + b*x)) - (7*e^5*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqr
t[b*d - a*e]])/(128*b^(3/2)*(b*d - a*e)^(9/2))
```

Rubi [A] time = 0.150461, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {27, 47, 51, 63, 208}

$$-\frac{7e^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{128b^{3/2}(bd-ae)^{9/2}}+\frac{7e^4\sqrt{d+ex}}{128b(a+bx)(bd-ae)^4}-\frac{7e^3\sqrt{d+ex}}{192b(a+bx)^2(bd-ae)^3}+\frac{7e^2\sqrt{d+ex}}{240b(a+bx)^3(bd-ae)^2}-\frac{e\sqrt{d}}{40b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] -Sqrt[d + e*x]/(5*b*(a + b*x)^5) - (e*Sqrt[d + e*x])/(40*b*(b*d - a*e)*(a +
 b*x)^4) + (7*e^2*Sqrt[d + e*x])/(240*b*(b*d - a*e)^2*(a + b*x)^3) - (7*e^3
*Sqrt[d + e*x])/(192*b*(b*d - a*e)^3*(a + b*x)^2) + (7*e^4*Sqrt[d + e*x])/(
128*b*(b*d - a*e)^4*(a + b*x)) - (7*e^5*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqr
t[b*d - a*e]])/(128*b^(3/2)*(b*d - a*e)^(9/2))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\left(a^2+2abx+b^2x^2\right)^3}\,dx = \int \frac{\sqrt{d+ex}}{(a+bx)^6}\,dx$$

$$= -\frac{\sqrt{d+ex}}{5b(a+bx)^5} + \frac{e\int \frac{1}{(a+bx)^5\sqrt{d+ex}}\,dx}{10b}$$

$$= -\frac{\sqrt{d+ex}}{5b(a+bx)^5} - \frac{e\sqrt{d+ex}}{40b(bd-ae)(a+bx)^4} - \frac{\left(7e^2\right)\int \frac{1}{(a+bx)^4\sqrt{d+ex}}\,dx}{80b(bd-ae)}$$

$$= -\frac{\sqrt{d+ex}}{5b(a+bx)^5} - \frac{e\sqrt{d+ex}}{40b(bd-ae)(a+bx)^4} + \frac{7e^2\sqrt{d+ex}}{240b(bd-ae)^2(a+bx)^3} + \frac{\left(7e^3\right)\int \frac{1}{(a+bx)^3\sqrt{d+ex}}\,dx}{96b(bd-ae)^2}$$

$$= -\frac{\sqrt{d+ex}}{5b(a+bx)^5} - \frac{e\sqrt{d+ex}}{40b(bd-ae)(a+bx)^4} + \frac{7e^2\sqrt{d+ex}}{240b(bd-ae)^2(a+bx)^3} - \frac{7e^3\sqrt{d+ex}}{192b(bd-ae)^3(a+bx)^2}$$

$$= -\frac{\sqrt{d+ex}}{5b(a+bx)^5} - \frac{e\sqrt{d+ex}}{40b(bd-ae)(a+bx)^4} + \frac{7e^2\sqrt{d+ex}}{240b(bd-ae)^2(a+bx)^3} - \frac{7e^3\sqrt{d+ex}}{192b(bd-ae)^3(a+bx)^2}$$

$$= -\frac{\sqrt{d+ex}}{5b(a+bx)^5} - \frac{e\sqrt{d+ex}}{40b(bd-ae)(a+bx)^4} + \frac{7e^2\sqrt{d+ex}}{240b(bd-ae)^2(a+bx)^3} - \frac{7e^3\sqrt{d+ex}}{192b(bd-ae)^3(a+bx)^2}$$

$$= -\frac{\sqrt{d+ex}}{5b(a+bx)^5} - \frac{e\sqrt{d+ex}}{40b(bd-ae)(a+bx)^4} + \frac{7e^2\sqrt{d+ex}}{240b(bd-ae)^2(a+bx)^3} - \frac{7e^3\sqrt{d+ex}}{192b(bd-ae)^3(a+bx)^2}$$

Mathematica [C] time = 0.0141032, size = 52, normalized size = 0.24

$$\frac{2e^5(d+ex)^{3/2}\,{}_2F_1\left(\frac{3}{2},6;\frac{5}{2};-\frac{b(d+ex)}{ae-bd}\right)}{3(ae-bd)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] (2*e^5*(d + e*x)^(3/2)*Hypergeometric2F1[3/2, 6, 5/2, -((b*(d + e*x))/(-(b*
d) + a*e))])/(3*(-(b*d) + a*e)^6)
```

Maple [A] time = 0.204, size = 337, normalized size = 1.6

$$\frac{7\,e^5b^3}{128\,(bxe+ae)^5\left(a^4e^4-4\,a^3bde^3+6\,d^2e^2a^2b^2-4\,ab^3d^3e+b^4d^4\right)}(ex+d)^{\frac{9}{2}} + \frac{49\,e^5b^2}{192\,(bxe+ae)^5\left(a^3e^3-3\,de^2a^2b+3\,ab^2d^2e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] 7/128*e^5/(b*e*x+a*e)^5*b^3/(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-4*a*b^
3*d^3*e+b^4*d^4)*(e*x+d)^(9/2)+49/192*e^5/(b*e*x+a*e)^5/(a^3*e^3-3*a^2*b*d*
e^2+3*a*b^2*d^2*e-b^3*d^3)*b^2*(e*x+d)^(7/2)+7/15*e^5/(b*e*x+a*e)^5*b/(a^2*
e^2-2*a*b*d*e+b^2*d^2)*(e*x+d)^(5/2)+79/192*e^5/(b*e*x+a*e)^5/(a*e-b*d)*(e*
x+d)^(3/2)-7/128*e^5/(b*e*x+a*e)^5/b*(e*x+d)^(1/2)+7/128*e^5/b/(a^4*e^4-4*a
^3*b*d*e^3+6*a^2*b^2*d^2*e^2-4*a*b^3*d^3*e+b^4*d^4)/((a*e-b*d)*b)^(1/2)*arc
tan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.85345, size = 3452, normalized size = 15.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/3840*(105*(b^5*e^5*x^5 + 5*a*b^4*e^5*x^4 + 10*a^2*b^3*e^5*x^3 + 10*a^3*b
^2*e^5*x^2 + 5*a^4*b*e^5*x + a^5*e^5)*sqrt(b^2*d - a*b*e)*log((b*e*x + 2*b*
d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d))/(b*x + a)) - 2*(384*b^6*d^5
- 1872*a*b^5*d^4*e + 3592*a^2*b^4*d^3*e^2 - 3314*a^3*b^3*d^2*e^3 + 1315*a^4
*b^2*d*e^4 - 105*a^5*b*e^5 - 105*(b^6*d*e^4 - a*b^5*e^5)*x^4 + 70*(b^6*d^2*
e^3 - 8*a*b^5*d*e^4 + 7*a^2*b^4*e^5)*x^3 - 14*(4*b^6*d^3*e^2 - 27*a*b^5*d^2
*e^3 + 87*a^2*b^4*d*e^4 - 64*a^3*b^3*e^5)*x^2 + 2*(24*b^6*d^4*e - 152*a*b^5
*d^3*e^2 + 417*a^2*b^4*d^2*e^3 - 684*a^3*b^3*d*e^4 + 395*a^4*b^2*e^5)*x)*sq
rt(e*x + d))/(a^5*b^7*d^5 - 5*a^6*b^6*d^4*e + 10*a^7*b^5*d^3*e^2 - 10*a^8*b
^4*d^2*e^3 + 5*a^9*b^3*d*e^4 - a^10*b^2*e^5 + (b^12*d^5 - 5*a*b^11*d^4*e +
10*a^2*b^10*d^3*e^2 - 10*a^3*b^9*d^2*e^3 + 5*a^4*b^8*d*e^4 - a^5*b^7*e^5)*x
^5 + 5*(a*b^11*d^5 - 5*a^2*b^10*d^4*e + 10*a^3*b^9*d^3*e^2 - 10*a^4*b^8*d^2
*e^3 + 5*a^5*b^7*d*e^4 - a^6*b^6*e^5)*x^4 + 10*(a^2*b^10*d^5 - 5*a^3*b^9*d^
4*e + 10*a^4*b^8*d^3*e^2 - 10*a^5*b^7*d^2*e^3 + 5*a^6*b^6*d*e^4 - a^7*b^5*e
^5)*x^3 + 10*(a^3*b^9*d^5 - 5*a^4*b^8*d^4*e + 10*a^5*b^7*d^3*e^2 - 10*a^6*b
^6*d^2*e^3 + 5*a^7*b^5*d*e^4 - a^8*b^4*e^5)*x^2 + 5*(a^4*b^8*d^5 - 5*a^5*b^
7*d^4*e + 10*a^6*b^6*d^3*e^2 - 10*a^7*b^5*d^2*e^3 + 5*a^8*b^4*d*e^4 - a^9*b
^3*e^5)*x), 1/1920*(105*(b^5*e^5*x^5 + 5*a*b^4*e^5*x^4 + 10*a^2*b^3*e^5*x^3
 + 10*a^3*b^2*e^5*x^2 + 5*a^4*b*e^5*x + a^5*e^5)*sqrt(-b^2*d + a*b*e)*arcta
n(sqrt(-b^2*d + a*b*e)*sqrt(e*x + d)/(b*e*x + b*d)) - (384*b^6*d^5 - 1872*a
*b^5*d^4*e + 3592*a^2*b^4*d^3*e^2 - 3314*a^3*b^3*d^2*e^3 + 1315*a^4*b^2*d*e
^4 - 105*a^5*b*e^5 - 105*(b^6*d*e^4 - a*b^5*e^5)*x^4 + 70*(b^6*d^2*e^3 - 8*
a*b^5*d*e^4 + 7*a^2*b^4*e^5)*x^3 - 14*(4*b^6*d^3*e^2 - 27*a*b^5*d^2*e^3 + 8
7*a^2*b^4*d*e^4 - 64*a^3*b^3*e^5)*x^2 + 2*(24*b^6*d^4*e - 152*a*b^5*d^3*e^2
```

```
+ 417*a^2*b^4*d^2*e^3 - 684*a^3*b^3*d*e^4 + 395*a^4*b^2*e^5)*x)*sqrt(e*x +
 d))/(a^5*b^7*d^5 - 5*a^6*b^6*d^4*e + 10*a^7*b^5*d^3*e^2 - 10*a^8*b^4*d^2*e
^3 + 5*a^9*b^3*d*e^4 - a^10*b^2*e^5 + (b^12*d^5 - 5*a*b^11*d^4*e + 10*a^2*b
^10*d^3*e^2 - 10*a^3*b^9*d^2*e^3 + 5*a^4*b^8*d*e^4 - a^5*b^7*e^5)*x^5 + 5*(
a*b^11*d^5 - 5*a^2*b^10*d^4*e + 10*a^3*b^9*d^3*e^2 - 10*a^4*b^8*d^2*e^3 + 5
*a^5*b^7*d*e^4 - a^6*b^6*e^5)*x^4 + 10*(a^2*b^10*d^5 - 5*a^3*b^9*d^4*e + 10
*a^4*b^8*d^3*e^2 - 10*a^5*b^7*d^2*e^3 + 5*a^6*b^6*d*e^4 - a^7*b^5*e^5)*x^3
+ 10*(a^3*b^9*d^5 - 5*a^4*b^8*d^4*e + 10*a^5*b^7*d^3*e^2 - 10*a^6*b^6*d^2*e
^3 + 5*a^7*b^5*d*e^4 - a^8*b^4*e^5)*x^2 + 5*(a^4*b^8*d^5 - 5*a^5*b^7*d^4*e
+ 10*a^6*b^6*d^3*e^2 - 10*a^7*b^5*d^2*e^3 + 5*a^8*b^4*d*e^4 - a^9*b^3*e^5)*
x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18188, size = 583, normalized size = 2.67

$$\frac{7\,\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)e^5}{128\left(b^5d^4-4\,ab^4d^3e+6\,a^2b^3d^2e^2-4\,a^3b^2de^3+a^4be^4\right)\sqrt{-b^2d+abe}}+\frac{105\,(xe+d)^{\frac{9}{2}}b^4e^5-490\,(xe+d)^{\frac{7}{2}}b^4de^5+896\,(xe-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] 7/128*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e^5/((b^5*d^4 - 4*a*b^4*
d^3*e + 6*a^2*b^3*d^2*e^2 - 4*a^3*b^2*d*e^3 + a^4*b*e^4)*sqrt(-b^2*d + a*b*
e)) + 1/1920*(105*(x*e + d)^(9/2)*b^4*e^5 - 490*(x*e + d)^(7/2)*b^4*d*e^5 +
 896*(x*e + d)^(5/2)*b^4*d^2*e^5 - 790*(x*e + d)^(3/2)*b^4*d^3*e^5 - 105*sq
rt(x*e + d)*b^4*d^4*e^5 + 490*(x*e + d)^(7/2)*a*b^3*e^6 - 1792*(x*e + d)^(5
/2)*a*b^3*d*e^6 + 2370*(x*e + d)^(3/2)*a*b^3*d^2*e^6 + 420*sqrt(x*e + d)*a*
b^3*d^3*e^6 + 896*(x*e + d)^(5/2)*a^2*b^2*e^7 - 2370*(x*e + d)^(3/2)*a^2*b^
2*d*e^7 - 630*sqrt(x*e + d)*a^2*b^2*d^2*e^7 + 790*(x*e + d)^(3/2)*a^3*b*e^8
 + 420*sqrt(x*e + d)*a^3*b*d*e^8 - 105*sqrt(x*e + d)*a^4*e^9)/((b^5*d^4 - 4
*a*b^4*d^3*e + 6*a^2*b^3*d^2*e^2 - 4*a^3*b^2*d*e^3 + a^4*b*e^4)*((x*e + d)*
b - b*d + a*e)^5)
```

3.1672 $\int \frac{1}{\sqrt{d+ex}\left(a^2+2abx+b^2x^2\right)^3}\,dx$

Optimal. Leaf size=213

$$-\frac{63e^4\sqrt{d+ex}}{128(a+bx)(bd-ae)^5}+\frac{21e^3\sqrt{d+ex}}{64(a+bx)^2(bd-ae)^4}-\frac{21e^2\sqrt{d+ex}}{80(a+bx)^3(bd-ae)^3}+\frac{63e^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{128\sqrt{b}(bd-ae)^{11/2}}+\frac{9e\sqrt{d+ex}}{40(a+bx)^4(bd-}$$

```
[Out] -Sqrt[d + e*x]/(5*(b*d - a*e)*(a + b*x)^5) + (9*e*Sqrt[d + e*x])/(40*(b*d -
 a*e)^2*(a + b*x)^4) - (21*e^2*Sqrt[d + e*x])/(80*(b*d - a*e)^3*(a + b*x)^3
) + (21*e^3*Sqrt[d + e*x])/(64*(b*d - a*e)^4*(a + b*x)^2) - (63*e^4*Sqrt[d
+ e*x])/(128*(b*d - a*e)^5*(a + b*x)) + (63*e^5*ArcTanh[(Sqrt[b]*Sqrt[d + e
*x])/Sqrt[b*d - a*e]])/(128*Sqrt[b]*(b*d - a*e)^(11/2))
```

Rubi [A] time = 0.116132, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {27, 51, 63, 208}

$$-\frac{63e^4\sqrt{d+ex}}{128(a+bx)(bd-ae)^5}+\frac{21e^3\sqrt{d+ex}}{64(a+bx)^2(bd-ae)^4}-\frac{21e^2\sqrt{d+ex}}{80(a+bx)^3(bd-ae)^3}+\frac{63e^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{128\sqrt{b}(bd-ae)^{11/2}}+\frac{9e\sqrt{d+ex}}{40(a+bx)^4(bd-}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

```
[Out] -Sqrt[d + e*x]/(5*(b*d - a*e)*(a + b*x)^5) + (9*e*Sqrt[d + e*x])/(40*(b*d -
 a*e)^2*(a + b*x)^4) - (21*e^2*Sqrt[d + e*x])/(80*(b*d - a*e)^3*(a + b*x)^3
) + (21*e^3*Sqrt[d + e*x])/(64*(b*d - a*e)^4*(a + b*x)^2) - (63*e^4*Sqrt[d
+ e*x])/(128*(b*d - a*e)^5*(a + b*x)) + (63*e^5*ArcTanh[(Sqrt[b]*Sqrt[d + e
*x])/Sqrt[b*d - a*e]])/(128*Sqrt[b]*(b*d - a*e)^(11/2))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\left(a^2+2abx+b^2x^2\right)^3}dx = \int \frac{1}{(a+bx)^6\sqrt{d+ex}}dx$$

$$= -\frac{\sqrt{d+ex}}{5(bd-ae)(a+bx)^5} - \frac{(9e)\int \frac{1}{(a+bx)^5\sqrt{d+ex}}dx}{10(bd-ae)}$$

$$= -\frac{\sqrt{d+ex}}{5(bd-ae)(a+bx)^5} + \frac{9e\sqrt{d+ex}}{40(bd-ae)^2(a+bx)^4} + \frac{\left(63e^2\right)\int \frac{1}{(a+bx)^4\sqrt{d+ex}}dx}{80(bd-ae)^2}$$

$$= -\frac{\sqrt{d+ex}}{5(bd-ae)(a+bx)^5} + \frac{9e\sqrt{d+ex}}{40(bd-ae)^2(a+bx)^4} - \frac{21e^2\sqrt{d+ex}}{80(bd-ae)^3(a+bx)^3} - \frac{\left(21e^3\right)\int}{32(b}$$

$$= -\frac{\sqrt{d+ex}}{5(bd-ae)(a+bx)^5} + \frac{9e\sqrt{d+ex}}{40(bd-ae)^2(a+bx)^4} - \frac{21e^2\sqrt{d+ex}}{80(bd-ae)^3(a+bx)^3} + \frac{21e^3}{64(bd-a}$$

$$= -\frac{\sqrt{d+ex}}{5(bd-ae)(a+bx)^5} + \frac{9e\sqrt{d+ex}}{40(bd-ae)^2(a+bx)^4} - \frac{21e^2\sqrt{d+ex}}{80(bd-ae)^3(a+bx)^3} + \frac{21e^3}{64(bd-a}$$

$$= -\frac{\sqrt{d+ex}}{5(bd-ae)(a+bx)^5} + \frac{9e\sqrt{d+ex}}{40(bd-ae)^2(a+bx)^4} - \frac{21e^2\sqrt{d+ex}}{80(bd-ae)^3(a+bx)^3} + \frac{21e^3}{64(bd-a}$$

$$= -\frac{\sqrt{d+ex}}{5(bd-ae)(a+bx)^5} + \frac{9e\sqrt{d+ex}}{40(bd-ae)^2(a+bx)^4} - \frac{21e^2\sqrt{d+ex}}{80(bd-ae)^3(a+bx)^3} + \frac{21e^3}{64(bd-a}$$

Mathematica [C] time = 0.0132862, size = 50, normalized size = 0.23

$$\frac{2e^5\sqrt{d+ex}\,{}_2F_1\left(\frac{1}{2},6;\frac{3}{2};-\frac{b(d+ex)}{ae-bd}\right)}{(ae-bd)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

```
[Out] (2*e^5*Sqrt[d + e*x]*Hypergeometric2F1[1/2, 6, 3/2, -((b*(d + e*x))/(-(b*d)
 + a*e))])/(-(b*d) + a*e)^6
```

Maple [A] time = 0.198, size = 211, normalized size = 1.

$$\frac{e^5}{(5ae-5bd)(bxe+ae)^5}\sqrt{ex+d} + \frac{9e^5}{40(ae-bd)^2(bxe+ae)^4}\sqrt{ex+d} + \frac{21e^5}{80(ae-bd)^3(bxe+ae)^3}\sqrt{ex+d} + \frac{21e}{64(ae-bd)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] 1/5*e^5*(e*x+d)^(1/2)/(a*e-b*d)/(b*e*x+a*e)^5+9/40*e^5/(a*e-b*d)^2*(e*x+d)^
(1/2)/(b*e*x+a*e)^4+21/80*e^5/(a*e-b*d)^3*(e*x+d)^(1/2)/(b*e*x+a*e)^3+21/64
*e^5/(a*e-b*d)^4*(e*x+d)^(1/2)/(b*e*x+a*e)^2+63/128*e^5/(a*e-b*d)^5*(e*x+d)
```

```
^(1/2)/(b*e*x+a*e)+63/128*e^5/(a*e-b*d)^5/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x
+d)^(1/2)/((a*e-b*d)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 1.91918, size = 3794, normalized size = 17.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")

[Out] [-1/1280*(315*(b^5*e^5*x^5 + 5*a*b^4*e^5*x^4 + 10*a^2*b^3*e^5*x^3 + 10*a^3*
b^2*e^5*x^2 + 5*a^4*b*e^5*x + a^5*e^5)*sqrt(b^2*d - a*b*e)*log((b*e*x + 2*b
*d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d))/(b*x + a)) + 2*(128*b^6*d^5
 - 784*a*b^5*d^4*e + 2024*a^2*b^4*d^3*e^2 - 2858*a^3*b^3*d^2*e^3 + 2455*a^4
*b^2*d*e^4 - 965*a^5*b*e^5 + 315*(b^6*d*e^4 - a*b^5*e^5)*x^4 - 210*(b^6*d^2
*e^3 - 8*a*b^5*d*e^4 + 7*a^2*b^4*e^5)*x^3 + 42*(4*b^6*d^3*e^2 - 27*a*b^5*d^
2*e^3 + 87*a^2*b^4*d*e^4 - 64*a^3*b^3*e^5)*x^2 - 6*(24*b^6*d^4*e - 152*a*b^
5*d^3*e^2 + 417*a^2*b^4*d^2*e^3 - 684*a^3*b^3*d*e^4 + 395*a^4*b^2*e^5)*x)*s
qrt(e*x + d))/(a^5*b^7*d^6 - 6*a^6*b^6*d^5*e + 15*a^7*b^5*d^4*e^2 - 20*a^8*
b^4*d^3*e^3 + 15*a^9*b^3*d^2*e^4 - 6*a^10*b^2*d*e^5 + a^11*b*e^6 + (b^12*d^
6 - 6*a*b^11*d^5*e + 15*a^2*b^10*d^4*e^2 - 20*a^3*b^9*d^3*e^3 + 15*a^4*b^8*
d^2*e^4 - 6*a^5*b^7*d*e^5 + a^6*b^6*e^6)*x^5 + 5*(a*b^11*d^6 - 6*a^2*b^10*d
^5*e + 15*a^3*b^9*d^4*e^2 - 20*a^4*b^8*d^3*e^3 + 15*a^5*b^7*d^2*e^4 - 6*a^6
*b^6*d*e^5 + a^7*b^5*e^6)*x^4 + 10*(a^2*b^10*d^6 - 6*a^3*b^9*d^5*e + 15*a^4
*b^8*d^4*e^2 - 20*a^5*b^7*d^3*e^3 + 15*a^6*b^6*d^2*e^4 - 6*a^7*b^5*d*e^5 +
a^8*b^4*e^6)*x^3 + 10*(a^3*b^9*d^6 - 6*a^4*b^8*d^5*e + 15*a^5*b^7*d^4*e^2 -
 20*a^6*b^6*d^3*e^3 + 15*a^7*b^5*d^2*e^4 - 6*a^8*b^4*d*e^5 + a^9*b^3*e^6)*x
^2 + 5*(a^4*b^8*d^6 - 6*a^5*b^7*d^5*e + 15*a^6*b^6*d^4*e^2 - 20*a^7*b^5*d^3
*e^3 + 15*a^8*b^4*d^2*e^4 - 6*a^9*b^3*d*e^5 + a^10*b^2*e^6)*x), -1/640*(315
*(b^5*e^5*x^5 + 5*a*b^4*e^5*x^4 + 10*a^2*b^3*e^5*x^3 + 10*a^3*b^2*e^5*x^2 +
 5*a^4*b*e^5*x + a^5*e^5)*sqrt(-b^2*d + a*b*e)*arctan(sqrt(-b^2*d + a*b*e)*
sqrt(e*x + d)/(b*e*x + b*d)) + (128*b^6*d^5 - 784*a*b^5*d^4*e + 2024*a^2*b^
4*d^3*e^2 - 2858*a^3*b^3*d^2*e^3 + 2455*a^4*b^2*d*e^4 - 965*a^5*b*e^5 + 315
*(b^6*d*e^4 - a*b^5*e^5)*x^4 - 210*(b^6*d^2*e^3 - 8*a*b^5*d*e^4 + 7*a^2*b^4
*e^5)*x^3 + 42*(4*b^6*d^3*e^2 - 27*a*b^5*d^2*e^3 + 87*a^2*b^4*d*e^4 - 64*a^
3*b^3*e^5)*x^2 - 6*(24*b^6*d^4*e - 152*a*b^5*d^3*e^2 + 417*a^2*b^4*d^2*e^3
- 684*a^3*b^3*d*e^4 + 395*a^4*b^2*e^5)*x)*sqrt(e*x + d))/(a^5*b^7*d^6 - 6*a
^6*b^6*d^5*e + 15*a^7*b^5*d^4*e^2 - 20*a^8*b^4*d^3*e^3 + 15*a^9*b^3*d^2*e^4
 - 6*a^10*b^2*d*e^5 + a^11*b*e^6 + (b^12*d^6 - 6*a*b^11*d^5*e + 15*a^2*b^10
*d^4*e^2 - 20*a^3*b^9*d^3*e^3 + 15*a^4*b^8*d^2*e^4 - 6*a^5*b^7*d*e^5 + a^6*
b^6*e^6)*x^5 + 5*(a*b^11*d^6 - 6*a^2*b^10*d^5*e + 15*a^3*b^9*d^4*e^2 - 20*a
^4*b^8*d^3*e^3 + 15*a^5*b^7*d^2*e^4 - 6*a^6*b^6*d*e^5 + a^7*b^5*e^6)*x^4 +
10*(a^2*b^10*d^6 - 6*a^3*b^9*d^5*e + 15*a^4*b^8*d^4*e^2 - 20*a^5*b^7*d^3*e^
3 + 15*a^6*b^6*d^2*e^4 - 6*a^7*b^5*d*e^5 + a^8*b^4*e^6)*x^3 + 10*(a^3*b^9*d
```

```
^6 - 6*a^4*b^8*d^5*e + 15*a^5*b^7*d^4*e^2 - 20*a^6*b^6*d^3*e^3 + 15*a^7*b^5
*d^2*e^4 - 6*a^8*b^4*d*e^5 + a^9*b^3*e^6)*x^2 + 5*(a^4*b^8*d^6 - 6*a^5*b^7*
d^5*e + 15*a^6*b^6*d^4*e^2 - 20*a^7*b^5*d^3*e^3 + 15*a^8*b^4*d^2*e^4 - 6*a^
9*b^3*d*e^5 + a^10*b^2*e^6)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19648, size = 613, normalized size = 2.88

$$-\frac{63 \arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)e^5}{128\left(b^5d^5 - 5\,ab^4d^4e + 10\,a^2b^3d^3e^2 - 10\,a^3b^2d^2e^3 + 5\,a^4bde^4 - a^5e^5\right)\sqrt{-b^2d + abe}} - \frac{315\,(xe+d)^{\frac{9}{2}}b^4e^5 - 1470\,(xe+d)^{\frac{7}{2}}b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] -63/128*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e^5/((b^5*d^5 - 5*a*b^
4*d^4*e + 10*a^2*b^3*d^3*e^2 - 10*a^3*b^2*d^2*e^3 + 5*a^4*b*d*e^4 - a^5*e^5
)*sqrt(-b^2*d + a*b*e)) - 1/640*(315*(x*e + d)^(9/2)*b^4*e^5 - 1470*(x*e +
d)^(7/2)*b^4*d*e^5 + 2688*(x*e + d)^(5/2)*b^4*d^2*e^5 - 2370*(x*e + d)^(3/2
)*b^4*d^3*e^5 + 965*sqrt(x*e + d)*b^4*d^4*e^5 + 1470*(x*e + d)^(7/2)*a*b^3*
e^6 - 5376*(x*e + d)^(5/2)*a*b^3*d*e^6 + 7110*(x*e + d)^(3/2)*a*b^3*d^2*e^6
 - 3860*sqrt(x*e + d)*a*b^3*d^3*e^6 + 2688*(x*e + d)^(5/2)*a^2*b^2*e^7 - 71
10*(x*e + d)^(3/2)*a^2*b^2*d*e^7 + 5790*sqrt(x*e + d)*a^2*b^2*d^2*e^7 + 237
0*(x*e + d)^(3/2)*a^3*b*e^8 - 3860*sqrt(x*e + d)*a^3*b*d*e^8 + 965*sqrt(x*e
 + d)*a^4*e^9)/((b^5*d^5 - 5*a*b^4*d^4*e + 10*a^2*b^3*d^3*e^2 - 10*a^3*b^2*
d^2*e^3 + 5*a^4*b*d*e^4 - a^5*e^5)*((x*e + d)*b - b*d + a*e)^5)
```

3.1673 $\int \frac{1}{(d+ex)^{3/2}\left(a^2+2abx+b^2x^2\right)^3} dx$

Optimal. Leaf size=239

$$-\frac{693e^5}{128\sqrt{d+ex}(bd-ae)^6}-\frac{231e^4}{128(a+bx)\sqrt{d+ex}(bd-ae)^5}+\frac{231e^3}{320(a+bx)^2\sqrt{d+ex}(bd-ae)^4}-\frac{33e^2}{80(a+bx)^3\sqrt{d+ex}(bd-}$$

```
[Out] (-693*e^5)/(128*(b*d - a*e)^6*Sqrt[d + e*x]) - 1/(5*(b*d - a*e)*(a + b*x)^5
*Sqrt[d + e*x]) + (11*e)/(40*(b*d - a*e)^2*(a + b*x)^4*Sqrt[d + e*x]) - (33
*e^2)/(80*(b*d - a*e)^3*(a + b*x)^3*Sqrt[d + e*x]) + (231*e^3)/(320*(b*d -
a*e)^4*(a + b*x)^2*Sqrt[d + e*x]) - (231*e^4)/(128*(b*d - a*e)^5*(a + b*x)*
Sqrt[d + e*x]) + (693*Sqrt[b]*e^5*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d
- a*e]])/(128*(b*d - a*e)^(13/2))
```

Rubi [A] time = 0.16294, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {27, 51, 63, 208}

$$-\frac{693e^5}{128\sqrt{d+ex}(bd-ae)^6}-\frac{231e^4}{128(a+bx)\sqrt{d+ex}(bd-ae)^5}+\frac{231e^3}{320(a+bx)^2\sqrt{d+ex}(bd-ae)^4}-\frac{33e^2}{80(a+bx)^3\sqrt{d+ex}(bd-}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```


```
[Out] (-693*e^5)/(128*(b*d - a*e)^6*Sqrt[d + e*x]) - 1/(5*(b*d - a*e)*(a + b*x)^5
*Sqrt[d + e*x]) + (11*e)/(40*(b*d - a*e)^2*(a + b*x)^4*Sqrt[d + e*x]) - (33
*e^2)/(80*(b*d - a*e)^3*(a + b*x)^3*Sqrt[d + e*x]) + (231*e^3)/(320*(b*d -
a*e)^4*(a + b*x)^2*Sqrt[d + e*x]) - (231*e^4)/(128*(b*d - a*e)^5*(a + b*x)*
Sqrt[d + e*x]) + (693*Sqrt[b]*e^5*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d
- a*e]])/(128*(b*d - a*e)^(13/2))
```


Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\left(a^2+2abx+b^2x^2\right)^3}\,dx = \int \frac{1}{(a+bx)^6(d+ex)^{3/2}}\,dx$$

$$= -\frac{1}{5(bd-ae)(a+bx)^5\sqrt{d+ex}} - \frac{(11e)\int \frac{1}{(a+bx)^5(d+ex)^{3/2}}\,dx}{10(bd-ae)}$$

$$= -\frac{1}{5(bd-ae)(a+bx)^5\sqrt{d+ex}} + \frac{11e}{40(bd-ae)^2(a+bx)^4\sqrt{d+ex}} + \frac{\left(99e^2\right)\int \frac{1}{(a+bx)^4(\cdots}}{80(bd-\cdots}$$

$$= -\frac{1}{5(bd-ae)(a+bx)^5\sqrt{d+ex}} + \frac{11e}{40(bd-ae)^2(a+bx)^4\sqrt{d+ex}} - \frac{33e\cdots}{80(bd-ae)^3(a\cdots}$$

$$= -\frac{1}{5(bd-ae)(a+bx)^5\sqrt{d+ex}} + \frac{11e}{40(bd-ae)^2(a+bx)^4\sqrt{d+ex}} - \frac{33e\cdots}{80(bd-ae)^3(a\cdots}$$

$$= -\frac{1}{5(bd-ae)(a+bx)^5\sqrt{d+ex}} + \frac{11e}{40(bd-ae)^2(a+bx)^4\sqrt{d+ex}} - \frac{33e\cdots}{80(bd-ae)^3(a\cdots}$$

$$= -\frac{693e^5}{128(bd-ae)^6\sqrt{d+ex}} - \frac{1}{5(bd-ae)(a+bx)^5\sqrt{d+ex}} + \frac{11e}{40(bd-ae)^2(a+bx)^4\sqrt{\cdots}}$$

$$= -\frac{693e^5}{128(bd-ae)^6\sqrt{d+ex}} - \frac{1}{5(bd-ae)(a+bx)^5\sqrt{d+ex}} + \frac{11e}{40(bd-ae)^2(a+bx)^4\sqrt{\cdots}}$$

$$= -\frac{693e^5}{128(bd-ae)^6\sqrt{d+ex}} - \frac{1}{5(bd-ae)(a+bx)^5\sqrt{d+ex}} + \frac{11e}{40(bd-ae)^2(a+bx)^4\sqrt{\cdots}}$$

Mathematica [C] time = 0.0160358, size = 50, normalized size = 0.21

$$-\frac{2e^5\,{}_2F_1\left(-\frac{1}{2},6;\frac{1}{2};-\frac{b(d+ex)}{ae-bd}\right)}{\sqrt{d+ex}(ae-bd)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

```
[Out] (-2*e^5*Hypergeometric2F1[-1/2, 6, 1/2, -((b*(d + e*x))/(-(b*d) + a*e))])/(
(-(b*d) + a*e)^6*Sqrt[d + e*x])
```

Maple [B] time = 0.219, size = 641, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] -2*e^5/(a*e-b*d)^6/(e*x+d)^(1/2)-437/128*e^5*b^5/(a*e-b*d)^6/(b*e*x+a*e)^5*
(e*x+d)^(9/2)-977/64*e^6*b^4/(a*e-b*d)^6/(b*e*x+a*e)^5*(e*x+d)^(7/2)*a+977/
64*e^5*b^5/(a*e-b*d)^6/(b*e*x+a*e)^5*(e*x+d)^(7/2)*d-131/5*e^7*b^3/(a*e-b*d
)^6/(b*e*x+a*e)^5*(e*x+d)^(5/2)*a^2+262/5*e^6*b^4/(a*e-b*d)^6/(b*e*x+a*e)^5
*(e*x+d)^(5/2)*a*d-131/5*e^5*b^5/(a*e-b*d)^6/(b*e*x+a*e)^5*(e*x+d)^(5/2)*d^
2-1327/64*e^8*b^2/(a*e-b*d)^6/(b*e*x+a*e)^5*(e*x+d)^(3/2)*a^3+3981/64*e^7*b
^3/(a*e-b*d)^6/(b*e*x+a*e)^5*(e*x+d)^(3/2)*a^2*d-3981/64*e^6*b^4/(a*e-b*d)^
6/(b*e*x+a*e)^5*(e*x+d)^(3/2)*a*d^2+1327/64*e^5*b^5/(a*e-b*d)^6/(b*e*x+a*e)
^5*(e*x+d)^(3/2)*d^3-843/128*e^9*b/(a*e-b*d)^6/(b*e*x+a*e)^5*(e*x+d)^(1/2)*
a^4+843/32*e^8*b^2/(a*e-b*d)^6/(b*e*x+a*e)^5*(e*x+d)^(1/2)*a^3*d-2529/64*e^
7*b^3/(a*e-b*d)^6/(b*e*x+a*e)^5*(e*x+d)^(1/2)*d^2*a^2+843/32*e^6*b^4/(a*e-b
*d)^6/(b*e*x+a*e)^5*(e*x+d)^(1/2)*a*d^3-843/128*e^5*b^5/(a*e-b*d)^6/(b*e*x+
a*e)^5*(e*x+d)^(1/2)*d^4-693/128*e^5*b/(a*e-b*d)^6/((a*e-b*d)*b)^(1/2)*arct
an(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.33728, size = 4783, normalized size = 20.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/1280*(3465*(b^5*e^6*x^6 + a^5*d*e^5 + (b^5*d*e^5 + 5*a*b^4*e^6)*x^5 + 5*
(a*b^4*d*e^5 + 2*a^2*b^3*e^6)*x^4 + 10*(a^2*b^3*d*e^5 + a^3*b^2*e^6)*x^3 +
5*(2*a^3*b^2*d*e^5 + a^4*b*e^6)*x^2 + (5*a^4*b*d*e^5 + a^5*e^6)*x)*sqrt(b/(
b*d - a*e))*log((b*e*x + 2*b*d - a*e + 2*(b*d - a*e)*sqrt(e*x + d)*sqrt(b/(
b*d - a*e)))/(b*x + a)) - 2*(3465*b^5*e^5*x^5 + 128*b^5*d^5 - 816*a*b^4*d^4
*e + 2248*a^2*b^3*d^3*e^2 - 3590*a^3*b^2*d^2*e^3 + 4215*a^4*b*d*e^4 + 1280*
a^5*e^5 + 1155*(b^5*d*e^4 + 14*a*b^4*e^5)*x^4 - 462*(b^5*d^2*e^3 - 12*a*b^4
*d*e^4 - 64*a^2*b^3*e^5)*x^3 + 66*(4*b^5*d^3*e^2 - 33*a*b^4*d^2*e^3 + 159*a
^2*b^3*d*e^4 + 395*a^3*b^2*e^5)*x^2 - 11*(16*b^5*d^4*e - 112*a*b^4*d^3*e^2
+ 366*a^2*b^3*d^2*e^3 - 880*a^3*b^2*d*e^4 - 965*a^4*b*e^5)*x)*sqrt(e*x + d)
)/(a^5*b^6*d^7 - 6*a^6*b^5*d^6*e + 15*a^7*b^4*d^5*e^2 - 20*a^8*b^3*d^4*e^3
+ 15*a^9*b^2*d^3*e^4 - 6*a^10*b*d^2*e^5 + a^11*d*e^6 + (b^11*d^6*e - 6*a*b^
10*d^5*e^2 + 15*a^2*b^9*d^4*e^3 - 20*a^3*b^8*d^3*e^4 + 15*a^4*b^7*d^2*e^5 -
 6*a^5*b^6*d*e^6 + a^6*b^5*e^7)*x^6 + (b^11*d^7 - a*b^10*d^6*e - 15*a^2*b^9
*d^5*e^2 + 55*a^3*b^8*d^4*e^3 - 85*a^4*b^7*d^3*e^4 + 69*a^5*b^6*d^2*e^5 - 2
9*a^6*b^5*d*e^6 + 5*a^7*b^4*e^7)*x^5 + 5*(a*b^10*d^7 - 4*a^2*b^9*d^6*e + 3*
a^3*b^8*d^5*e^2 + 10*a^4*b^7*d^4*e^3 - 25*a^5*b^6*d^3*e^4 + 24*a^6*b^5*d^2*
e^5 - 11*a^7*b^4*d*e^6 + 2*a^8*b^3*e^7)*x^4 + 10*(a^2*b^9*d^7 - 5*a^3*b^8*d
^6*e + 9*a^4*b^7*d^5*e^2 - 5*a^5*b^6*d^4*e^3 - 5*a^6*b^5*d^3*e^4 + 9*a^7*b^
```

```
4*d^2*e^5 - 5*a^8*b^3*d*e^6 + a^9*b^2*e^7)*x^3 + 5*(2*a^3*b^8*d^7 - 11*a^4*
b^7*d^6*e + 24*a^5*b^6*d^5*e^2 - 25*a^6*b^5*d^4*e^3 + 10*a^7*b^4*d^3*e^4 +
3*a^8*b^3*d^2*e^5 - 4*a^9*b^2*d*e^6 + a^10*b*e^7)*x^2 + (5*a^4*b^7*d^7 - 29
*a^5*b^6*d^6*e + 69*a^6*b^5*d^5*e^2 - 85*a^7*b^4*d^4*e^3 + 55*a^8*b^3*d^3*e
^4 - 15*a^9*b^2*d^2*e^5 - a^10*b*d*e^6 + a^11*e^7)*x), 1/640*(3465*(b^5*e^6
*x^6 + a^5*d*e^5 + (b^5*d*e^5 + 5*a*b^4*e^6)*x^5 + 5*(a*b^4*d*e^5 + 2*a^2*b
^3*e^6)*x^4 + 10*(a^2*b^3*d*e^5 + a^3*b^2*e^6)*x^3 + 5*(2*a^3*b^2*d*e^5 + a
^4*b*e^6)*x^2 + (5*a^4*b*d*e^5 + a^5*e^6)*x)*sqrt(-b/(b*d - a*e))*arctan(-(
b*d - a*e)*sqrt(e*x + d)*sqrt(-b/(b*d - a*e))/(b*e*x + b*d)) - (3465*b^5*e^
5*x^5 + 128*b^5*d^5 - 816*a*b^4*d^4*e + 2248*a^2*b^3*d^3*e^2 - 3590*a^3*b^2
*d^2*e^3 + 4215*a^4*b*d*e^4 + 1280*a^5*e^5 + 1155*(b^5*d*e^4 + 14*a*b^4*e^5
)*x^4 - 462*(b^5*d^2*e^3 - 12*a*b^4*d*e^4 - 64*a^2*b^3*e^5)*x^3 + 66*(4*b^5
*d^3*e^2 - 33*a*b^4*d^2*e^3 + 159*a^2*b^3*d*e^4 + 395*a^3*b^2*e^5)*x^2 - 11
*(16*b^5*d^4*e - 112*a*b^4*d^3*e^2 + 366*a^2*b^3*d^2*e^3 - 880*a^3*b^2*d*e^
4 - 965*a^4*b*e^5)*x)*sqrt(e*x + d))/(a^5*b^6*d^7 - 6*a^6*b^5*d^6*e + 15*a^
7*b^4*d^5*e^2 - 20*a^8*b^3*d^4*e^3 + 15*a^9*b^2*d^3*e^4 - 6*a^10*b*d^2*e^5
+ a^11*d*e^6 + (b^11*d^6*e - 6*a*b^10*d^5*e^2 + 15*a^2*b^9*d^4*e^3 - 20*a^3
*b^8*d^3*e^4 + 15*a^4*b^7*d^2*e^5 - 6*a^5*b^6*d*e^6 + a^6*b^5*e^7)*x^6 + (b
^11*d^7 - a*b^10*d^6*e - 15*a^2*b^9*d^5*e^2 + 55*a^3*b^8*d^4*e^3 - 85*a^4*b
^7*d^3*e^4 + 69*a^5*b^6*d^2*e^5 - 29*a^6*b^5*d*e^6 + 5*a^7*b^4*e^7)*x^5 + 5
*(a*b^10*d^7 - 4*a^2*b^9*d^6*e + 3*a^3*b^8*d^5*e^2 + 10*a^4*b^7*d^4*e^3 - 2
5*a^5*b^6*d^3*e^4 + 24*a^6*b^5*d^2*e^5 - 11*a^7*b^4*d*e^6 + 2*a^8*b^3*e^7)*
x^4 + 10*(a^2*b^9*d^7 - 5*a^3*b^8*d^6*e + 9*a^4*b^7*d^5*e^2 - 5*a^5*b^6*d^4
*e^3 - 5*a^6*b^5*d^3*e^4 + 9*a^7*b^4*d^2*e^5 - 5*a^8*b^3*d*e^6 + a^9*b^2*e^
7)*x^3 + 5*(2*a^3*b^8*d^7 - 11*a^4*b^7*d^6*e + 24*a^5*b^6*d^5*e^2 - 25*a^6*
b^5*d^4*e^3 + 10*a^7*b^4*d^3*e^4 + 3*a^8*b^3*d^2*e^5 - 4*a^9*b^2*d*e^6 + a^
10*b*e^7)*x^2 + (5*a^4*b^7*d^7 - 29*a^5*b^6*d^6*e + 69*a^6*b^5*d^5*e^2 - 85
*a^7*b^4*d^4*e^3 + 55*a^8*b^3*d^3*e^4 - 15*a^9*b^2*d^2*e^5 - a^10*b*d*e^6 +
 a^11*e^7)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20611, size = 771, normalized size = 3.23

$$-\frac{693\, b \arctan\left(\frac{\sqrt{xe+d}\, b}{\sqrt{-b^2 d+abe}}\right) e^5}{128\left(b^6d^6 - 6\, ab^5d^5e + 15\, a^2b^4d^4e^2 - 20\, a^3b^3d^3e^3 + 15\, a^4b^2d^2e^4 - 6\, a^5bde^5 + a^6e^6\right)\sqrt{-b^2d+abe}} - \frac{\cdots}{\left(b^6d^6 - 6\, ab^5d^5e + 15 \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] -693/128*b*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e^5/((b^6*d^6 - 6*a
*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 -
 6*a^5*b*d*e^5 + a^6*e^6)*sqrt(-b^2*d + a*b*e)) - 2*e^5/((b^6*d^6 - 6*a*b^5
*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a
```

```
^5*b*d*e^5 + a^6*e^6)*sqrt(x*e + d)) - 1/640*(2185*(x*e + d)^(9/2)*b^5*e^5
- 9770*(x*e + d)^(7/2)*b^5*d*e^5 + 16768*(x*e + d)^(5/2)*b^5*d^2*e^5 - 1327
0*(x*e + d)^(3/2)*b^5*d^3*e^5 + 4215*sqrt(x*e + d)*b^5*d^4*e^5 + 9770*(x*e
+ d)^(7/2)*a*b^4*e^6 - 33536*(x*e + d)^(5/2)*a*b^4*d*e^6 + 39810*(x*e + d)^
(3/2)*a*b^4*d^2*e^6 - 16860*sqrt(x*e + d)*a*b^4*d^3*e^6 + 16768*(x*e + d)^(
5/2)*a^2*b^3*e^7 - 39810*(x*e + d)^(3/2)*a^2*b^3*d*e^7 + 25290*sqrt(x*e + d
)*a^2*b^3*d^2*e^7 + 13270*(x*e + d)^(3/2)*a^3*b^2*e^8 - 16860*sqrt(x*e + d)
*a^3*b^2*d*e^8 + 4215*sqrt(x*e + d)*a^4*b*e^9)/((b^6*d^6 - 6*a*b^5*d^5*e +
15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a^5*b*d*e^
5 + a^6*e^6)*((x*e + d)*b - b*d + a*e)^5)
```

3.1674 $\int \frac{1}{(d+ex)^{5/2}\left(a^2+2abx+b^2x^2\right)^3}\,dx$

Optimal. Leaf size=266

$$\frac{3003b^{3/2}e^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{128(bd-ae)^{15/2}}-\frac{3003be^5}{128\sqrt{d+ex}(bd-ae)^7}-\frac{1001e^5}{128(d+ex)^{3/2}(bd-ae)^6}-\frac{3003e^4}{640(a+bx)(d+ex)^{3/2}(bd-ae)^5}+\frac{3}{}$$

```
[Out] (-1001*e^5)/(128*(b*d - a*e)^6*(d + e*x)^(3/2)) - 1/(5*(b*d - a*e)*(a + b*x
)^5*(d + e*x)^(3/2)) + (13*e)/(40*(b*d - a*e)^2*(a + b*x)^4*(d + e*x)^(3/2)
) - (143*e^2)/(240*(b*d - a*e)^3*(a + b*x)^3*(d + e*x)^(3/2)) + (429*e^3)/(
320*(b*d - a*e)^4*(a + b*x)^2*(d + e*x)^(3/2)) - (3003*e^4)/(640*(b*d - a*e
)^5*(a + b*x)*(d + e*x)^(3/2)) - (3003*b*e^5)/(128*(b*d - a*e)^7*Sqrt[d + e
*x]) + (3003*b^(3/2)*e^5*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/
(128*(b*d - a*e)^(15/2))
```

Rubi [A] time = 0.243315, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {27, 51, 63, 208}

$$\frac{3003b^{3/2}e^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{128(bd-ae)^{15/2}}-\frac{3003be^5}{128\sqrt{d+ex}(bd-ae)^7}-\frac{1001e^5}{128(d+ex)^{3/2}(bd-ae)^6}-\frac{3003e^4}{640(a+bx)(d+ex)^{3/2}(bd-ae)^5}+\frac{3}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

```
[Out] (-1001*e^5)/(128*(b*d - a*e)^6*(d + e*x)^(3/2)) - 1/(5*(b*d - a*e)*(a + b*x
)^5*(d + e*x)^(3/2)) + (13*e)/(40*(b*d - a*e)^2*(a + b*x)^4*(d + e*x)^(3/2)
) - (143*e^2)/(240*(b*d - a*e)^3*(a + b*x)^3*(d + e*x)^(3/2)) + (429*e^3)/(
320*(b*d - a*e)^4*(a + b*x)^2*(d + e*x)^(3/2)) - (3003*e^4)/(640*(b*d - a*e
)^5*(a + b*x)*(d + e*x)^(3/2)) - (3003*b*e^5)/(128*(b*d - a*e)^7*Sqrt[d + e
*x]) + (3003*b^(3/2)*e^5*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/
(128*(b*d - a*e)^(15/2))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{5/2}\left(a^2+2abx+b^2x^2\right)^3}\,dx = \int \frac{1}{(a+bx)^6(d+ex)^{5/2}}\,dx$$

$$= -\frac{1}{5(bd-ae)(a+bx)^5(d+ex)^{3/2}} - \frac{(13e)\int \frac{1}{(a+bx)^5(d+ex)^{5/2}}\,dx}{10(bd-ae)}$$

$$= -\frac{1}{5(bd-ae)(a+bx)^5(d+ex)^{3/2}} + \frac{13e}{40(bd-ae)^2(a+bx)^4(d+ex)^{3/2}} + \frac{\left(143e^2\right)}{8}$$

$$= -\frac{1}{5(bd-ae)(a+bx)^5(d+ex)^{3/2}} + \frac{13e}{40(bd-ae)^2(a+bx)^4(d+ex)^{3/2}} - \frac{}{240(bd-}$$

$$= -\frac{1}{5(bd-ae)(a+bx)^5(d+ex)^{3/2}} + \frac{13e}{40(bd-ae)^2(a+bx)^4(d+ex)^{3/2}} - \frac{}{240(bd-}$$

$$= -\frac{1}{5(bd-ae)(a+bx)^5(d+ex)^{3/2}} + \frac{13e}{40(bd-ae)^2(a+bx)^4(d+ex)^{3/2}} - \frac{}{240(bd-}$$

$$= -\frac{1001e^5}{128(bd-ae)^6(d+ex)^{3/2}} - \frac{1}{5(bd-ae)(a+bx)^5(d+ex)^{3/2}} + \frac{13}{40(bd-ae)^2(a+}$$

$$= -\frac{1001e^5}{128(bd-ae)^6(d+ex)^{3/2}} - \frac{1}{5(bd-ae)(a+bx)^5(d+ex)^{3/2}} + \frac{13}{40(bd-ae)^2(a+}$$

$$= -\frac{1001e^5}{128(bd-ae)^6(d+ex)^{3/2}} - \frac{1}{5(bd-ae)(a+bx)^5(d+ex)^{3/2}} + \frac{13}{40(bd-ae)^2(a+}$$

$$= -\frac{1001e^5}{128(bd-ae)^6(d+ex)^{3/2}} - \frac{1}{5(bd-ae)(a+bx)^5(d+ex)^{3/2}} + \frac{13}{40(bd-ae)^2(a+}$$

Mathematica [C] time = 0.0207705, size = 52, normalized size = 0.2

$$-\frac{2e^5\,{}_2F_1\left(-\frac{3}{2},6;-\frac{1}{2};-\frac{b(d+ex)}{ae-bd}\right)}{3(d+ex)^{3/2}(ae-bd)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

```
[Out] (-2*e^5*Hypergeometric2F1[-3/2, 6, -1/2, -((b*(d + e*x))/(-(b*d) + a*e))])/
(3*(-(b*d) + a*e)^6*(d + e*x)^(3/2))
```

Maple [B] time = 0.217, size = 668, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] -2/3*e^5/(a*e-b*d)^6/(e*x+d)^(3/2)+12*e^5/(a*e-b*d)^7*b/(e*x+d)^(1/2)+1467/
128*e^5/(a*e-b*d)^7*b^6/(b*e*x+a*e)^5*(e*x+d)^(9/2)+9629/192*e^6/(a*e-b*d)^
7*b^5/(b*e*x+a*e)^5*(e*x+d)^(7/2)*a-9629/192*e^5/(a*e-b*d)^7*b^6/(b*e*x+a*e
)^5*(e*x+d)^(7/2)*d+1253/15*e^7/(a*e-b*d)^7*b^4/(b*e*x+a*e)^5*(e*x+d)^(5/2)
*a^2-2506/15*e^6/(a*e-b*d)^7*b^5/(b*e*x+a*e)^5*(e*x+d)^(5/2)*a*d+1253/15*e^
5/(a*e-b*d)^7*b^6/(b*e*x+a*e)^5*(e*x+d)^(5/2)*d^2+12131/192*e^8/(a*e-b*d)^7
*b^3/(b*e*x+a*e)^5*(e*x+d)^(3/2)*a^3-12131/64*e^7/(a*e-b*d)^7*b^4/(b*e*x+a*
e)^5*(e*x+d)^(3/2)*a^2*d+12131/64*e^6/(a*e-b*d)^7*b^5/(b*e*x+a*e)^5*(e*x+d)
^(3/2)*a*d^2-12131/192*e^5/(a*e-b*d)^7*b^6/(b*e*x+a*e)^5*(e*x+d)^(3/2)*d^3+
2373/128*e^9/(a*e-b*d)^7*b^2/(b*e*x+a*e)^5*(e*x+d)^(1/2)*a^4-2373/32*e^8/(a
*e-b*d)^7*b^3/(b*e*x+a*e)^5*(e*x+d)^(1/2)*a^3*d+7119/64*e^7/(a*e-b*d)^7*b^4
/(b*e*x+a*e)^5*(e*x+d)^(1/2)*d^2*a^2-2373/32*e^6/(a*e-b*d)^7*b^5/(b*e*x+a*e
)^5*(e*x+d)^(1/2)*a*d^3+2373/128*e^5/(a*e-b*d)^7*b^6/(b*e*x+a*e)^5*(e*x+d)^
(1/2)*d^4+3003/128*e^5/(a*e-b*d)^7*b^2/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)
^(1/2)/((a*e-b*d)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.50026, size = 6792, normalized size = 25.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/3840*(45045*(b^6*e^7*x^7 + a^5*b*d^2*e^5 + (2*b^6*d*e^6 + 5*a*b^5*e^7)*
x^6 + (b^6*d^2*e^5 + 10*a*b^5*d*e^6 + 10*a^2*b^4*e^7)*x^5 + 5*(a*b^5*d^2*e^
5 + 4*a^2*b^4*d*e^6 + 2*a^3*b^3*e^7)*x^4 + 5*(2*a^2*b^4*d^2*e^5 + 4*a^3*b^3
*d*e^6 + a^4*b^2*e^7)*x^3 + (10*a^3*b^3*d^2*e^5 + 10*a^4*b^2*d*e^6 + a^5*b*
e^7)*x^2 + (5*a^4*b^2*d^2*e^5 + 2*a^5*b*d*e^6)*x)*sqrt(b/(b*d - a*e))*log((
b*e*x + 2*b*d - a*e - 2*(b*d - a*e)*sqrt(e*x + d)*sqrt(b/(b*d - a*e)))/(b*x
 + a)) + 2*(45045*b^6*e^6*x^6 + 384*b^6*d^6 - 2928*a*b^5*d^5*e + 10024*a^2*
b^4*d^4*e^2 - 21070*a^3*b^3*d^3*e^3 + 35595*a^4*b^2*d^2*e^4 + 24320*a^5*b*d
*e^5 - 1280*a^6*e^6 + 30030*(2*b^6*d*e^5 + 7*a*b^5*e^6)*x^5 + 3003*(3*b^6*d
^2*e^4 + 94*a*b^5*d*e^5 + 128*a^2*b^4*e^6)*x^4 - 858*(3*b^6*d^3*e^3 - 51*a*
b^5*d^2*e^4 - 607*a^2*b^4*d*e^5 - 395*a^3*b^3*e^6)*x^3 + 143*(8*b^6*d^4*e^2
 - 86*a*b^5*d^3*e^3 + 588*a^2*b^4*d^2*e^4 + 3250*a^3*b^3*d*e^5 + 965*a^4*b^
2*e^6)*x^2 - 26*(24*b^6*d^5*e - 208*a*b^5*d^4*e^2 + 889*a^2*b^4*d^3*e^3 - 3
045*a^3*b^3*d^2*e^4 - 7415*a^4*b^2*d*e^5 - 640*a^5*b*e^6)*x)*sqrt(e*x + d))
/(a^5*b^7*d^9 - 7*a^6*b^6*d^8*e + 21*a^7*b^5*d^7*e^2 - 35*a^8*b^4*d^6*e^3 +
 35*a^9*b^3*d^5*e^4 - 21*a^10*b^2*d^4*e^5 + 7*a^11*b*d^3*e^6 - a^12*d^2*e^7
```

```
 + (b^12*d^7*e^2 - 7*a*b^11*d^6*e^3 + 21*a^2*b^10*d^5*e^4 - 35*a^3*b^9*d^4*
e^5 + 35*a^4*b^8*d^3*e^6 - 21*a^5*b^7*d^2*e^7 + 7*a^6*b^6*d*e^8 - a^7*b^5*e
^9)*x^7 + (2*b^12*d^8*e - 9*a*b^11*d^7*e^2 + 7*a^2*b^10*d^6*e^3 + 35*a^3*b^
9*d^5*e^4 - 105*a^4*b^8*d^4*e^5 + 133*a^5*b^7*d^3*e^6 - 91*a^6*b^6*d^2*e^7
+ 33*a^7*b^5*d*e^8 - 5*a^8*b^4*e^9)*x^6 + (b^12*d^9 + 3*a*b^11*d^8*e - 39*a
^2*b^10*d^7*e^2 + 105*a^3*b^9*d^6*e^3 - 105*a^4*b^8*d^5*e^4 - 21*a^5*b^7*d^
4*e^5 + 147*a^6*b^6*d^3*e^6 - 141*a^7*b^5*d^2*e^7 + 60*a^8*b^4*d*e^8 - 10*a
^9*b^3*e^9)*x^5 + 5*(a*b^11*d^9 - 3*a^2*b^10*d^8*e - 5*a^3*b^9*d^7*e^2 + 35
*a^4*b^8*d^6*e^3 - 63*a^5*b^7*d^5*e^4 + 49*a^6*b^6*d^4*e^5 - 7*a^7*b^5*d^3*
e^6 - 15*a^8*b^4*d^2*e^7 + 10*a^9*b^3*d*e^8 - 2*a^10*b^2*e^9)*x^4 + 5*(2*a^
2*b^10*d^9 - 10*a^3*b^9*d^8*e + 15*a^4*b^8*d^7*e^2 + 7*a^5*b^7*d^6*e^3 - 49
*a^6*b^6*d^5*e^4 + 63*a^7*b^5*d^4*e^5 - 35*a^8*b^4*d^3*e^6 + 5*a^9*b^3*d^2*
e^7 + 3*a^10*b^2*d*e^8 - a^11*b*e^9)*x^3 + (10*a^3*b^9*d^9 - 60*a^4*b^8*d^8
*e + 141*a^5*b^7*d^7*e^2 - 147*a^6*b^6*d^6*e^3 + 21*a^7*b^5*d^5*e^4 + 105*a
^8*b^4*d^4*e^5 - 105*a^9*b^3*d^3*e^6 + 39*a^10*b^2*d^2*e^7 - 3*a^11*b*d*e^8
 - a^12*e^9)*x^2 + (5*a^4*b^8*d^9 - 33*a^5*b^7*d^8*e + 91*a^6*b^6*d^7*e^2 -
 133*a^7*b^5*d^6*e^3 + 105*a^8*b^4*d^5*e^4 - 35*a^9*b^3*d^4*e^5 - 7*a^10*b^
2*d^3*e^6 + 9*a^11*b*d^2*e^7 - 2*a^12*d*e^8)*x), 1/1920*(45045*(b^6*e^7*x^7
 + a^5*b*d^2*e^5 + (2*b^6*d*e^6 + 5*a*b^5*e^7)*x^6 + (b^6*d^2*e^5 + 10*a*b^
5*d*e^6 + 10*a^2*b^4*e^7)*x^5 + 5*(a*b^5*d^2*e^5 + 4*a^2*b^4*d*e^6 + 2*a^3*
b^3*e^7)*x^4 + 5*(2*a^2*b^4*d^2*e^5 + 4*a^3*b^3*d*e^6 + a^4*b^2*e^7)*x^3 +
(10*a^3*b^3*d^2*e^5 + 10*a^4*b^2*d*e^6 + a^5*b*e^7)*x^2 + (5*a^4*b^2*d^2*e^
5 + 2*a^5*b*d*e^6)*x)*sqrt(-b/(b*d - a*e))*arctan(-(b*d - a*e)*sqrt(e*x + d
)*sqrt(-b/(b*d - a*e))/(b*e*x + b*d)) - (45045*b^6*e^6*x^6 + 384*b^6*d^6 -
2928*a*b^5*d^5*e + 10024*a^2*b^4*d^4*e^2 - 21070*a^3*b^3*d^3*e^3 + 35595*a^
4*b^2*d^2*e^4 + 24320*a^5*b*d*e^5 - 1280*a^6*e^6 + 30030*(2*b^6*d*e^5 + 7*a
*b^5*e^6)*x^5 + 3003*(3*b^6*d^2*e^4 + 94*a*b^5*d*e^5 + 128*a^2*b^4*e^6)*x^4
 - 858*(3*b^6*d^3*e^3 - 51*a*b^5*d^2*e^4 - 607*a^2*b^4*d*e^5 - 395*a^3*b^3*
e^6)*x^3 + 143*(8*b^6*d^4*e^2 - 86*a*b^5*d^3*e^3 + 588*a^2*b^4*d^2*e^4 + 32
50*a^3*b^3*d*e^5 + 965*a^4*b^2*e^6)*x^2 - 26*(24*b^6*d^5*e - 208*a*b^5*d^4*
e^2 + 889*a^2*b^4*d^3*e^3 - 3045*a^3*b^3*d^2*e^4 - 7415*a^4*b^2*d*e^5 - 640
*a^5*b*e^6)*x)*sqrt(e*x + d))/(a^5*b^7*d^9 - 7*a^6*b^6*d^8*e + 21*a^7*b^5*d
^7*e^2 - 35*a^8*b^4*d^6*e^3 + 35*a^9*b^3*d^5*e^4 - 21*a^10*b^2*d^4*e^5 + 7*
a^11*b*d^3*e^6 - a^12*d^2*e^7 + (b^12*d^7*e^2 - 7*a*b^11*d^6*e^3 + 21*a^2*b
^10*d^5*e^4 - 35*a^3*b^9*d^4*e^5 + 35*a^4*b^8*d^3*e^6 - 21*a^5*b^7*d^2*e^7
+ 7*a^6*b^6*d*e^8 - a^7*b^5*e^9)*x^7 + (2*b^12*d^8*e - 9*a*b^11*d^7*e^2 + 7
*a^2*b^10*d^6*e^3 + 35*a^3*b^9*d^5*e^4 - 105*a^4*b^8*d^4*e^5 + 133*a^5*b^7*
d^3*e^6 - 91*a^6*b^6*d^2*e^7 + 33*a^7*b^5*d*e^8 - 5*a^8*b^4*e^9)*x^6 + (b^1
2*d^9 + 3*a*b^11*d^8*e - 39*a^2*b^10*d^7*e^2 + 105*a^3*b^9*d^6*e^3 - 105*a^
4*b^8*d^5*e^4 - 21*a^5*b^7*d^4*e^5 + 147*a^6*b^6*d^3*e^6 - 141*a^7*b^5*d^2*
e^7 + 60*a^8*b^4*d*e^8 - 10*a^9*b^3*e^9)*x^5 + 5*(a*b^11*d^9 - 3*a^2*b^10*d
^8*e - 5*a^3*b^9*d^7*e^2 + 35*a^4*b^8*d^6*e^3 - 63*a^5*b^7*d^5*e^4 + 49*a^6
*b^6*d^4*e^5 - 7*a^7*b^5*d^3*e^6 - 15*a^8*b^4*d^2*e^7 + 10*a^9*b^3*d*e^8 -
2*a^10*b^2*e^9)*x^4 + 5*(2*a^2*b^10*d^9 - 10*a^3*b^9*d^8*e + 15*a^4*b^8*d^7
*e^2 + 7*a^5*b^7*d^6*e^3 - 49*a^6*b^6*d^5*e^4 + 63*a^7*b^5*d^4*e^5 - 35*a^8
*b^4*d^3*e^6 + 5*a^9*b^3*d^2*e^7 + 3*a^10*b^2*d*e^8 - a^11*b*e^9)*x^3 + (10
*a^3*b^9*d^9 - 60*a^4*b^8*d^8*e + 141*a^5*b^7*d^7*e^2 - 147*a^6*b^6*d^6*e^3
 + 21*a^7*b^5*d^5*e^4 + 105*a^8*b^4*d^4*e^5 - 105*a^9*b^3*d^3*e^6 + 39*a^10
*b^2*d^2*e^7 - 3*a^11*b*d*e^8 - a^12*e^9)*x^2 + (5*a^4*b^8*d^9 - 33*a^5*b^7
*d^8*e + 91*a^6*b^6*d^7*e^2 - 133*a^7*b^5*d^6*e^3 + 105*a^8*b^4*d^5*e^4 - 3
5*a^9*b^3*d^4*e^5 - 7*a^10*b^2*d^3*e^6 + 9*a^11*b*d^2*e^7 - 2*a^12*d*e^8)*x
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(5/2)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32092, size = 860, normalized size = 3.23

$$-\frac{3003\,b^2\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)e^5}{128\left(b^7d^7-7\,ab^6d^6e+21\,a^2b^5d^5e^2-35\,a^3b^4d^4e^3+35\,a^4b^3d^3e^4-21\,a^5b^2d^2e^5+7\,a^6bde^6-a^7e^7\right)\sqrt{-b^2d+abe}}-\frac{}{3\left(b^7d^7\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] -3003/128*b^2*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e^5/((b^7*d^7 -
7*a*b^6*d^6*e + 21*a^2*b^5*d^5*e^2 - 35*a^3*b^4*d^4*e^3 + 35*a^4*b^3*d^3*e^
4 - 21*a^5*b^2*d^2*e^5 + 7*a^6*b*d*e^6 - a^7*e^7)*sqrt(-b^2*d + a*b*e)) - 2
/3*(18*(x*e + d)*b*e^5 + b*d*e^5 - a*e^6)/((b^7*d^7 - 7*a*b^6*d^6*e + 21*a^
2*b^5*d^5*e^2 - 35*a^3*b^4*d^4*e^3 + 35*a^4*b^3*d^3*e^4 - 21*a^5*b^2*d^2*e^
5 + 7*a^6*b*d*e^6 - a^7*e^7)*(x*e + d)^(3/2)) - 1/1920*(22005*(x*e + d)^(9/
2)*b^6*e^5 - 96290*(x*e + d)^(7/2)*b^6*d*e^5 + 160384*(x*e + d)^(5/2)*b^6*d
^2*e^5 - 121310*(x*e + d)^(3/2)*b^6*d^3*e^5 + 35595*sqrt(x*e + d)*b^6*d^4*e
^5 + 96290*(x*e + d)^(7/2)*a*b^5*e^6 - 320768*(x*e + d)^(5/2)*a*b^5*d*e^6 +
 363930*(x*e + d)^(3/2)*a*b^5*d^2*e^6 - 142380*sqrt(x*e + d)*a*b^5*d^3*e^6
+ 160384*(x*e + d)^(5/2)*a^2*b^4*e^7 - 363930*(x*e + d)^(3/2)*a^2*b^4*d*e^7
 + 213570*sqrt(x*e + d)*a^2*b^4*d^2*e^7 + 121310*(x*e + d)^(3/2)*a^3*b^3*e^
8 - 142380*sqrt(x*e + d)*a^3*b^3*d*e^8 + 35595*sqrt(x*e + d)*a^4*b^2*e^9)/(
(b^7*d^7 - 7*a*b^6*d^6*e + 21*a^2*b^5*d^5*e^2 - 35*a^3*b^4*d^4*e^3 + 35*a^4
*b^3*d^3*e^4 - 21*a^5*b^2*d^2*e^5 + 7*a^6*b*d*e^6 - a^7*e^7)*((x*e + d)*b -
 b*d + a*e)^5)
```

3.1675 $\int \frac{1}{(d+ex)^{7/2}\left(a^2+2abx+b^2x^2\right)^3}\,dx$

Optimal. Leaf size=295

$$-\frac{9009b^2e^5}{128\sqrt{d+ex}(bd-ae)^8}+\frac{9009b^{5/2}e^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{128(bd-ae)^{17/2}}-\frac{3003be^5}{128(d+ex)^{3/2}(bd-ae)^7}-\frac{9009e^5}{640(d+ex)^{5/2}(bd-ae)^6}-\frac{}{128(a}$$

```
[Out] (-9009*e^5)/(640*(b*d - a*e)^6*(d + e*x)^(5/2)) - 1/(5*(b*d - a*e)*(a + b*x
)^5*(d + e*x)^(5/2)) + (3*e)/(8*(b*d - a*e)^2*(a + b*x)^4*(d + e*x)^(5/2))
- (13*e^2)/(16*(b*d - a*e)^3*(a + b*x)^3*(d + e*x)^(5/2)) + (143*e^3)/(64*(
b*d - a*e)^4*(a + b*x)^2*(d + e*x)^(5/2)) - (1287*e^4)/(128*(b*d - a*e)^5*(
a + b*x)*(d + e*x)^(5/2)) - (3003*b*e^5)/(128*(b*d - a*e)^7*(d + e*x)^(3/2)
) - (9009*b^2*e^5)/(128*(b*d - a*e)^8*Sqrt[d + e*x]) + (9009*b^(5/2)*e^5*Ar
cTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(128*(b*d - a*e)^(17/2))
```

Rubi [A] time = 0.270057, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {27, 51, 63, 208}

$$-\frac{9009b^2e^5}{128\sqrt{d+ex}(bd-ae)^8}+\frac{9009b^{5/2}e^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{128(bd-ae)^{17/2}}-\frac{3003be^5}{128(d+ex)^{3/2}(bd-ae)^7}-\frac{9009e^5}{640(d+ex)^{5/2}(bd-ae)^6}-\frac{}{128(a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```


```
[Out] (-9009*e^5)/(640*(b*d - a*e)^6*(d + e*x)^(5/2)) - 1/(5*(b*d - a*e)*(a + b*x
)^5*(d + e*x)^(5/2)) + (3*e)/(8*(b*d - a*e)^2*(a + b*x)^4*(d + e*x)^(5/2))
- (13*e^2)/(16*(b*d - a*e)^3*(a + b*x)^3*(d + e*x)^(5/2)) + (143*e^3)/(64*(
b*d - a*e)^4*(a + b*x)^2*(d + e*x)^(5/2)) - (1287*e^4)/(128*(b*d - a*e)^5*(
a + b*x)*(d + e*x)^(5/2)) - (3003*b*e^5)/(128*(b*d - a*e)^7*(d + e*x)^(3/2)
) - (9009*b^2*e^5)/(128*(b*d - a*e)^8*Sqrt[d + e*x]) + (9009*b^(5/2)*e^5*Ar
cTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(128*(b*d - a*e)^(17/2))
```


Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{7/2}\left(a^2+2abx+b^2x^2\right)^3}\,dx = \int \frac{1}{(a+bx)^6(d+ex)^{7/2}}\,dx$$

$$= -\frac{1}{5(bd-ae)(a+bx)^5(d+ex)^{5/2}} - \frac{(3e)\int \frac{1}{(a+bx)^5(d+ex)^{7/2}}\,dx}{2(bd-ae)}$$

$$= -\frac{1}{5(bd-ae)(a+bx)^5(d+ex)^{5/2}} + \frac{3e}{8(bd-ae)^2(a+bx)^4(d+ex)^{5/2}} + \frac{\left(39e^2\right)\int \frac{}{(a+b}}{16(bd}$$

$$= -\frac{1}{5(bd-ae)(a+bx)^5(d+ex)^{5/2}} + \frac{3e}{8(bd-ae)^2(a+bx)^4(d+ex)^{5/2}} - \frac{}{16(bd-ae)^3(}$$

$$= -\frac{1}{5(bd-ae)(a+bx)^5(d+ex)^{5/2}} + \frac{3e}{8(bd-ae)^2(a+bx)^4(d+ex)^{5/2}} - \frac{}{16(bd-ae)^3(}$$

$$= -\frac{1}{5(bd-ae)(a+bx)^5(d+ex)^{5/2}} + \frac{3e}{8(bd-ae)^2(a+bx)^4(d+ex)^{5/2}} - \frac{}{16(bd-ae)^3(}$$

$$= -\frac{9009e^5}{640(bd-ae)^6(d+ex)^{5/2}} - \frac{1}{5(bd-ae)(a+bx)^5(d+ex)^{5/2}} + \frac{3e}{8(bd-ae)^2(a+bx)}$$

$$= -\frac{9009e^5}{640(bd-ae)^6(d+ex)^{5/2}} - \frac{1}{5(bd-ae)(a+bx)^5(d+ex)^{5/2}} + \frac{3e}{8(bd-ae)^2(a+bx)}$$

$$= -\frac{9009e^5}{640(bd-ae)^6(d+ex)^{5/2}} - \frac{1}{5(bd-ae)(a+bx)^5(d+ex)^{5/2}} + \frac{3e}{8(bd-ae)^2(a+bx)}$$

$$= -\frac{9009e^5}{640(bd-ae)^6(d+ex)^{5/2}} - \frac{1}{5(bd-ae)(a+bx)^5(d+ex)^{5/2}} + \frac{3e}{8(bd-ae)^2(a+bx)}$$

$$= -\frac{9009e^5}{640(bd-ae)^6(d+ex)^{5/2}} - \frac{1}{5(bd-ae)(a+bx)^5(d+ex)^{5/2}} + \frac{3e}{8(bd-ae)^2(a+bx)}$$

Mathematica [C] time = 0.0263282, size = 52, normalized size = 0.18

$$-\frac{2e^5\,{}_2F_1\left(-\frac{5}{2},6;-\frac{3}{2};-\frac{b(d+ex)}{ae-bd}\right)}{5(d+ex)^{5/2}(ae-bd)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

```
[Out] (-2*e^5*Hypergeometric2F1[-5/2, 6, -3/2, -((b*(d + e*x))/(-(b*d) + a*e))])/
(5*(-(b*d) + a*e)^6*(d + e*x)^(5/2))
```

Maple [B] time = 0.218, size = 693, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] -2/5*e^5/(a*e-b*d)^6/(e*x+d)^(5/2)-42*e^5/(a*e-b*d)^8*b^2/(e*x+d)^(1/2)+4*e
^5/(a*e-b*d)^7*b/(e*x+d)^(3/2)-3633/128*e^5/(a*e-b*d)^8*b^7/(b*e*x+a*e)^5*(
e*x+d)^(9/2)-7837/64*e^6/(a*e-b*d)^8*b^6/(b*e*x+a*e)^5*(e*x+d)^(7/2)*a+7837
/64*e^5/(a*e-b*d)^8*b^7/(b*e*x+a*e)^5*(e*x+d)^(7/2)*d-1001/5*e^7/(a*e-b*d)^
8*b^5/(b*e*x+a*e)^5*(e*x+d)^(5/2)*a^2+2002/5*e^6/(a*e-b*d)^8*b^6/(b*e*x+a*e
)^5*(e*x+d)^(5/2)*a*d-1001/5*e^5/(a*e-b*d)^8*b^7/(b*e*x+a*e)^5*(e*x+d)^(5/2
)*d^2-9443/64*e^8/(a*e-b*d)^8*b^4/(b*e*x+a*e)^5*(e*x+d)^(3/2)*a^3+28329/64*
e^7/(a*e-b*d)^8*b^5/(b*e*x+a*e)^5*(e*x+d)^(3/2)*a^2*d-28329/64*e^6/(a*e-b*d
)^8*b^6/(b*e*x+a*e)^5*(e*x+d)^(3/2)*a*d^2+9443/64*e^5/(a*e-b*d)^8*b^7/(b*e*
x+a*e)^5*(e*x+d)^(3/2)*d^3-5327/128*e^9/(a*e-b*d)^8*b^3/(b*e*x+a*e)^5*(e*x+
d)^(1/2)*a^4+5327/32*e^8/(a*e-b*d)^8*b^4/(b*e*x+a*e)^5*(e*x+d)^(1/2)*a^3*d-
15981/64*e^7/(a*e-b*d)^8*b^5/(b*e*x+a*e)^5*(e*x+d)^(1/2)*d^2*a^2+5327/32*e^
6/(a*e-b*d)^8*b^6/(b*e*x+a*e)^5*(e*x+d)^(1/2)*a*d^3-5327/128*e^5/(a*e-b*d)^
8*b^7/(b*e*x+a*e)^5*(e*x+d)^(1/2)*d^4-9009/128*e^5/(a*e-b*d)^8*b^3/((a*e-b*
d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.74879, size = 9030, normalized size = 30.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/1280*(45045*(b^7*e^8*x^8 + a^5*b^2*d^3*e^5 + (3*b^7*d*e^7 + 5*a*b^6*e^8)
*x^7 + (3*b^7*d^2*e^6 + 15*a*b^6*d*e^7 + 10*a^2*b^5*e^8)*x^6 + (b^7*d^3*e^5
 + 15*a*b^6*d^2*e^6 + 30*a^2*b^5*d*e^7 + 10*a^3*b^4*e^8)*x^5 + 5*(a*b^6*d^3
*e^5 + 6*a^2*b^5*d^2*e^6 + 6*a^3*b^4*d*e^7 + a^4*b^3*e^8)*x^4 + (10*a^2*b^5
*d^3*e^5 + 30*a^3*b^4*d^2*e^6 + 15*a^4*b^3*d*e^7 + a^5*b^2*e^8)*x^3 + (10*a
^3*b^4*d^3*e^5 + 15*a^4*b^3*d^2*e^6 + 3*a^5*b^2*d*e^7)*x^2 + (5*a^4*b^3*d^3
*e^5 + 3*a^5*b^2*d^2*e^6)*x)*sqrt(b/(b*d - a*e))*log((b*e*x + 2*b*d - a*e +
 2*(b*d - a*e)*sqrt(e*x + d)*sqrt(b/(b*d - a*e)))/(b*x + a)) - 2*(45045*b^7
*e^7*x^7 + 128*b^7*d^7 - 1136*a*b^6*d^6*e + 4648*a^2*b^5*d^5*e^2 - 12110*a^
3*b^4*d^4*e^3 + 26635*a^4*b^3*d^3*e^4 + 29696*a^5*b^2*d^2*e^5 - 3072*a^6*b*
d*e^6 + 256*a^7*e^7 + 105105*(b^7*d*e^6 + 2*a*b^6*e^7)*x^6 + 3003*(23*b^7*d
^2*e^5 + 164*a*b^6*d*e^6 + 128*a^2*b^5*e^7)*x^5 + 2145*(3*b^7*d^3*e^4 + 152
```

```
*a*b^6*d^2*e^5 + 422*a^2*b^5*d*e^6 + 158*a^3*b^4*e^7)*x^4 - 715*(2*b^7*d^4*
e^3 - 44*a*b^6*d^3*e^4 - 846*a^2*b^5*d^2*e^5 - 1124*a^3*b^4*d*e^6 - 193*a^4
*b^3*e^7)*x^3 + 65*(8*b^7*d^5*e^2 - 106*a*b^6*d^4*e^3 + 938*a^2*b^5*d^3*e^4
 + 8368*a^3*b^4*d^2*e^5 + 5089*a^4*b^3*d*e^6 + 256*a^5*b^2*e^7)*x^2 - 5*(48
*b^7*d^6*e - 496*a*b^6*d^5*e^2 + 2618*a^2*b^5*d^4*e^3 - 11620*a^3*b^4*d^3*e
^4 - 45677*a^4*b^3*d^2*e^5 - 8192*a^5*b^2*d*e^6 + 256*a^6*b*e^7)*x)*sqrt(e*
x + d))/(a^5*b^8*d^11 - 8*a^6*b^7*d^10*e + 28*a^7*b^6*d^9*e^2 - 56*a^8*b^5*
d^8*e^3 + 70*a^9*b^4*d^7*e^4 - 56*a^10*b^3*d^6*e^5 + 28*a^11*b^2*d^5*e^6 -
8*a^12*b*d^4*e^7 + a^13*d^3*e^8 + (b^13*d^8*e^3 - 8*a*b^12*d^7*e^4 + 28*a^2
*b^11*d^6*e^5 - 56*a^3*b^10*d^5*e^6 + 70*a^4*b^9*d^4*e^7 - 56*a^5*b^8*d^3*e
^8 + 28*a^6*b^7*d^2*e^9 - 8*a^7*b^6*d*e^10 + a^8*b^5*e^11)*x^8 + (3*b^13*d^
9*e^2 - 19*a*b^12*d^8*e^3 + 44*a^2*b^11*d^7*e^4 - 28*a^3*b^10*d^6*e^5 - 70*
a^4*b^9*d^5*e^6 + 182*a^5*b^8*d^4*e^7 - 196*a^6*b^7*d^3*e^8 + 116*a^7*b^6*d
^2*e^9 - 37*a^8*b^5*d*e^10 + 5*a^9*b^4*e^11)*x^7 + (3*b^13*d^10*e - 9*a*b^1
2*d^9*e^2 - 26*a^2*b^11*d^8*e^3 + 172*a^3*b^10*d^7*e^4 - 350*a^4*b^9*d^6*e^
5 + 322*a^5*b^8*d^5*e^6 - 56*a^6*b^7*d^4*e^7 - 164*a^7*b^6*d^3*e^8 + 163*a^
8*b^5*d^2*e^9 - 65*a^9*b^4*d*e^10 + 10*a^10*b^3*e^11)*x^6 + (b^13*d^11 + 7*
a*b^12*d^10*e - 62*a^2*b^11*d^9*e^2 + 134*a^3*b^10*d^8*e^3 - 10*a^4*b^9*d^7
*e^4 - 406*a^5*b^8*d^6*e^5 + 728*a^6*b^7*d^5*e^6 - 568*a^7*b^6*d^4*e^7 + 16
1*a^8*b^5*d^3*e^8 + 55*a^9*b^4*d^2*e^9 - 50*a^10*b^3*d*e^10 + 10*a^11*b^2*e
^11)*x^5 + 5*(a*b^12*d^11 - 2*a^2*b^11*d^10*e - 14*a^3*b^10*d^9*e^2 + 65*a^
4*b^9*d^8*e^3 - 106*a^5*b^8*d^7*e^4 + 56*a^6*b^7*d^6*e^5 + 56*a^7*b^6*d^5*e
^6 - 106*a^8*b^5*d^4*e^7 + 65*a^9*b^4*d^3*e^8 - 14*a^10*b^3*d^2*e^9 - 2*a^1
1*b^2*d*e^10 + a^12*b*e^11)*x^4 + (10*a^2*b^11*d^11 - 50*a^3*b^10*d^10*e +
55*a^4*b^9*d^9*e^2 + 161*a^5*b^8*d^8*e^3 - 568*a^6*b^7*d^7*e^4 + 728*a^7*b^
6*d^6*e^5 - 406*a^8*b^5*d^5*e^6 - 10*a^9*b^4*d^4*e^7 + 134*a^10*b^3*d^3*e^8
 - 62*a^11*b^2*d^2*e^9 + 7*a^12*b*d*e^10 + a^13*e^11)*x^3 + (10*a^3*b^10*d^
11 - 65*a^4*b^9*d^10*e + 163*a^5*b^8*d^9*e^2 - 164*a^6*b^7*d^8*e^3 - 56*a^7
*b^6*d^7*e^4 + 322*a^8*b^5*d^6*e^5 - 350*a^9*b^4*d^5*e^6 + 172*a^10*b^3*d^4
*e^7 - 26*a^11*b^2*d^3*e^8 - 9*a^12*b*d^2*e^9 + 3*a^13*d*e^10)*x^2 + (5*a^4
*b^9*d^11 - 37*a^5*b^8*d^10*e + 116*a^6*b^7*d^9*e^2 - 196*a^7*b^6*d^8*e^3 +
 182*a^8*b^5*d^7*e^4 - 70*a^9*b^4*d^6*e^5 - 28*a^10*b^3*d^5*e^6 + 44*a^11*b
^2*d^4*e^7 - 19*a^12*b*d^3*e^8 + 3*a^13*d^2*e^9)*x), 1/640*(45045*(b^7*e^8*
x^8 + a^5*b^2*d^3*e^5 + (3*b^7*d*e^7 + 5*a*b^6*e^8)*x^7 + (3*b^7*d^2*e^6 +
15*a*b^6*d*e^7 + 10*a^2*b^5*e^8)*x^6 + (b^7*d^3*e^5 + 15*a*b^6*d^2*e^6 + 30
*a^2*b^5*d*e^7 + 10*a^3*b^4*e^8)*x^5 + 5*(a*b^6*d^3*e^5 + 6*a^2*b^5*d^2*e^6
 + 6*a^3*b^4*d*e^7 + a^4*b^3*e^8)*x^4 + (10*a^2*b^5*d^3*e^5 + 30*a^3*b^4*d^
2*e^6 + 15*a^4*b^3*d*e^7 + a^5*b^2*e^8)*x^3 + (10*a^3*b^4*d^3*e^5 + 15*a^4*
b^3*d^2*e^6 + 3*a^5*b^2*d*e^7)*x^2 + (5*a^4*b^3*d^3*e^5 + 3*a^5*b^2*d^2*e^6
)*x)*sqrt(-b/(b*d - a*e))*arctan(-(b*d - a*e)*sqrt(e*x + d)*sqrt(-b/(b*d -
a*e))/(b*e*x + b*d)) - (45045*b^7*e^7*x^7 + 128*b^7*d^7 - 1136*a*b^6*d^6*e
+ 4648*a^2*b^5*d^5*e^2 - 12110*a^3*b^4*d^4*e^3 + 26635*a^4*b^3*d^3*e^4 + 29
696*a^5*b^2*d^2*e^5 - 3072*a^6*b*d*e^6 + 256*a^7*e^7 + 105105*(b^7*d*e^6 +
2*a*b^6*e^7)*x^6 + 3003*(23*b^7*d^2*e^5 + 164*a*b^6*d*e^6 + 128*a^2*b^5*e^7
)*x^5 + 2145*(3*b^7*d^3*e^4 + 152*a*b^6*d^2*e^5 + 422*a^2*b^5*d*e^6 + 158*a
^3*b^4*e^7)*x^4 - 715*(2*b^7*d^4*e^3 - 44*a*b^6*d^3*e^4 - 846*a^2*b^5*d^2*e
^5 - 1124*a^3*b^4*d*e^6 - 193*a^4*b^3*e^7)*x^3 + 65*(8*b^7*d^5*e^2 - 106*a*
b^6*d^4*e^3 + 938*a^2*b^5*d^3*e^4 + 8368*a^3*b^4*d^2*e^5 + 5089*a^4*b^3*d*e
^6 + 256*a^5*b^2*e^7)*x^2 - 5*(48*b^7*d^6*e - 496*a*b^6*d^5*e^2 + 2618*a^2*
b^5*d^4*e^3 - 11620*a^3*b^4*d^3*e^4 - 45677*a^4*b^3*d^2*e^5 - 8192*a^5*b^2*
d*e^6 + 256*a^6*b*e^7)*x)*sqrt(e*x + d))/(a^5*b^8*d^11 - 8*a^6*b^7*d^10*e +
 28*a^7*b^6*d^9*e^2 - 56*a^8*b^5*d^8*e^3 + 70*a^9*b^4*d^7*e^4 - 56*a^10*b^3
*d^6*e^5 + 28*a^11*b^2*d^5*e^6 - 8*a^12*b*d^4*e^7 + a^13*d^3*e^8 + (b^13*d^
8*e^3 - 8*a*b^12*d^7*e^4 + 28*a^2*b^11*d^6*e^5 - 56*a^3*b^10*d^5*e^6 + 70*a
^4*b^9*d^4*e^7 - 56*a^5*b^8*d^3*e^8 + 28*a^6*b^7*d^2*e^9 - 8*a^7*b^6*d*e^10
 + a^8*b^5*e^11)*x^8 + (3*b^13*d^9*e^2 - 19*a*b^12*d^8*e^3 + 44*a^2*b^11*d^
7*e^4 - 28*a^3*b^10*d^6*e^5 - 70*a^4*b^9*d^5*e^6 + 182*a^5*b^8*d^4*e^7 - 19
6*a^6*b^7*d^3*e^8 + 116*a^7*b^6*d^2*e^9 - 37*a^8*b^5*d*e^10 + 5*a^9*b^4*e^1
1)*x^7 + (3*b^13*d^10*e - 9*a*b^12*d^9*e^2 - 26*a^2*b^11*d^8*e^3 + 172*a^3*
```

```
b^10*d^7*e^4 - 350*a^4*b^9*d^6*e^5 + 322*a^5*b^8*d^5*e^6 - 56*a^6*b^7*d^4*e
^7 - 164*a^7*b^6*d^3*e^8 + 163*a^8*b^5*d^2*e^9 - 65*a^9*b^4*d*e^10 + 10*a^1
0*b^3*e^11)*x^6 + (b^13*d^11 + 7*a*b^12*d^10*e - 62*a^2*b^11*d^9*e^2 + 134*
a^3*b^10*d^8*e^3 - 10*a^4*b^9*d^7*e^4 - 406*a^5*b^8*d^6*e^5 + 728*a^6*b^7*d
^5*e^6 - 568*a^7*b^6*d^4*e^7 + 161*a^8*b^5*d^3*e^8 + 55*a^9*b^4*d^2*e^9 - 5
0*a^10*b^3*d*e^10 + 10*a^11*b^2*e^11)*x^5 + 5*(a*b^12*d^11 - 2*a^2*b^11*d^1
0*e - 14*a^3*b^10*d^9*e^2 + 65*a^4*b^9*d^8*e^3 - 106*a^5*b^8*d^7*e^4 + 56*a
^6*b^7*d^6*e^5 + 56*a^7*b^6*d^5*e^6 - 106*a^8*b^5*d^4*e^7 + 65*a^9*b^4*d^3*
e^8 - 14*a^10*b^3*d^2*e^9 - 2*a^11*b^2*d*e^10 + a^12*b*e^11)*x^4 + (10*a^2*
b^11*d^11 - 50*a^3*b^10*d^10*e + 55*a^4*b^9*d^9*e^2 + 161*a^5*b^8*d^8*e^3 -
 568*a^6*b^7*d^7*e^4 + 728*a^7*b^6*d^6*e^5 - 406*a^8*b^5*d^5*e^6 - 10*a^9*b
^4*d^4*e^7 + 134*a^10*b^3*d^3*e^8 - 62*a^11*b^2*d^2*e^9 + 7*a^12*b*d*e^10 +
 a^13*e^11)*x^3 + (10*a^3*b^10*d^11 - 65*a^4*b^9*d^10*e + 163*a^5*b^8*d^9*e
^2 - 164*a^6*b^7*d^8*e^3 - 56*a^7*b^6*d^7*e^4 + 322*a^8*b^5*d^6*e^5 - 350*a
^9*b^4*d^5*e^6 + 172*a^10*b^3*d^4*e^7 - 26*a^11*b^2*d^3*e^8 - 9*a^12*b*d^2*
e^9 + 3*a^13*d*e^10)*x^2 + (5*a^4*b^9*d^11 - 37*a^5*b^8*d^10*e + 116*a^6*b^
7*d^9*e^2 - 196*a^7*b^6*d^8*e^3 + 182*a^8*b^5*d^7*e^4 - 70*a^9*b^4*d^6*e^5
- 28*a^10*b^3*d^5*e^6 + 44*a^11*b^2*d^4*e^7 - 19*a^12*b*d^3*e^8 + 3*a^13*d^
2*e^9)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(7/2)/(b**2*x**2+2*a*b*x+a**2)**3,x)

[Out] Timed out
```

Giac [B] time = 1.21175, size = 1193, normalized size = 4.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")

[Out] -9009/128*b^3*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e^5/((b^8*d^8 -
8*a*b^7*d^7*e + 28*a^2*b^6*d^6*e^2 - 56*a^3*b^5*d^5*e^3 + 70*a^4*b^4*d^4*e^
4 - 56*a^5*b^3*d^3*e^5 + 28*a^6*b^2*d^2*e^6 - 8*a^7*b*d*e^7 + a^8*e^8)*sqrt
(-b^2*d + a*b*e)) - 1/640*(45045*(x*e + d)^7*b^7*e^5 - 210210*(x*e + d)^6*b
^7*d*e^5 + 384384*(x*e + d)^5*b^7*d^2*e^5 - 338910*(x*e + d)^4*b^7*d^3*e^5
+ 137995*(x*e + d)^3*b^7*d^4*e^5 - 16640*(x*e + d)^2*b^7*d^5*e^5 - 1280*(x*
e + d)*b^7*d^6*e^5 - 256*b^7*d^7*e^5 + 210210*(x*e + d)^6*a*b^6*e^6 - 76876
8*(x*e + d)^5*a*b^6*d*e^6 + 1016730*(x*e + d)^4*a*b^6*d^2*e^6 - 551980*(x*e
 + d)^3*a*b^6*d^3*e^6 + 83200*(x*e + d)^2*a*b^6*d^4*e^6 + 7680*(x*e + d)*a*
b^6*d^5*e^6 + 1792*a*b^6*d^6*e^6 + 384384*(x*e + d)^5*a^2*b^5*e^7 - 1016730
*(x*e + d)^4*a^2*b^5*d*e^7 + 827970*(x*e + d)^3*a^2*b^5*d^2*e^7 - 166400*(x
*e + d)^2*a^2*b^5*d^3*e^7 - 19200*(x*e + d)*a^2*b^5*d^4*e^7 - 5376*a^2*b^5*
d^5*e^7 + 338910*(x*e + d)^4*a^3*b^4*e^8 - 551980*(x*e + d)^3*a^3*b^4*d*e^8
 + 166400*(x*e + d)^2*a^3*b^4*d^2*e^8 + 25600*(x*e + d)*a^3*b^4*d^3*e^8 + 8
960*a^3*b^4*d^4*e^8 + 137995*(x*e + d)^3*a^4*b^3*e^9 - 83200*(x*e + d)^2*a^
4*b^3*d*e^9 - 19200*(x*e + d)*a^4*b^3*d^2*e^9 - 8960*a^4*b^3*d^3*e^9 + 1664
```

```
0*(x*e + d)^2*a^5*b^2*e^10 + 7680*(x*e + d)*a^5*b^2*d*e^10 + 5376*a^5*b^2*d
^2*e^10 - 1280*(x*e + d)*a^6*b*e^11 - 1792*a^6*b*d*e^11 + 256*a^7*e^12)/((b
^8*d^8 - 8*a*b^7*d^7*e + 28*a^2*b^6*d^6*e^2 - 56*a^3*b^5*d^5*e^3 + 70*a^4*b
^4*d^4*e^4 - 56*a^5*b^3*d^3*e^5 + 28*a^6*b^2*d^2*e^6 - 8*a^7*b*d*e^7 + a^8*
e^8)*((x*e + d)^(3/2)*b - sqrt(x*e + d)*b*d + sqrt(x*e + d)*a*e)^5)
```

3.1676 $\int (d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}\,dx$

Optimal. Leaf size=96

$$\frac{2b\sqrt{a^2+2abx+b^2x^2}(d+ex)^{9/2}}{9e^2(a+bx)}-\frac{2\sqrt{a^2+2abx+b^2x^2}(d+ex)^{7/2}(bd-ae)}{7e^2(a+bx)}$$

```
[Out] (-2*(b*d - a*e)*(d + e*x)^(7/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*e^2*(a +
b*x)) + (2*b*(d + e*x)^(9/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(9*e^2*(a + b*x
))
```

Rubi [A] time = 0.039625, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {646, 43}

$$\frac{2b\sqrt{a^2+2abx+b^2x^2}(d+ex)^{9/2}}{9e^2(a+bx)}-\frac{2\sqrt{a^2+2abx+b^2x^2}(d+ex)^{7/2}(bd-ae)}{7e^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (-2*(b*d - a*e)*(d + e*x)^(7/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*e^2*(a +
b*x)) + (2*b*(d + e*x)^(9/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(9*e^2*(a + b*x
))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}\,dx&=\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(ab+b^2x\right)(d+ex)^{5/2}\,dx}{ab+b^2x}\\&=\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b(bd-ae)(d+ex)^{5/2}}{e}+\frac{b^2(d+ex)^{7/2}}{e}\right)dx}{ab+b^2x}\\&=-\frac{2(bd-ae)(d+ex)^{7/2}\sqrt{a^2+2abx+b^2x^2}}{7e^2(a+bx)}+\frac{2b(d+ex)^{9/2}\sqrt{a^2+2abx+b^2x^2}}{9e^2(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.0351934, size = 48, normalized size = 0.5

$$\frac{2\sqrt{(a+bx)^2}(d+ex)^{7/2}(9ae-2bd+7bex)}{63e^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (2*Sqrt[(a + b*x)^2]*(d + e*x)^(7/2)*(-2*b*d + 9*a*e + 7*b*e*x))/(63*e^2*(a
 + b*x))
```

Maple [A] time = 0.04, size = 43, normalized size = 0.5

$$\frac{14\,bxe + 18\,ae - 4\,bd}{63\,e^{2}\,(bx + a)}\,(ex + d)^{\frac{7}{2}}\,\sqrt{(bx + a)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*((b*x+a)^2)^(1/2),x)
```

```
[Out] 2/63*(e*x+d)^(7/2)*(7*b*e*x+9*a*e-2*b*d)*((b*x+a)^2)^(1/2)/e^2/(b*x+a)
```

Maxima [A] time = 1.13163, size = 126, normalized size = 1.31

$$\frac{2\left(7\,be^{4}x^{4} - 2\,bd^{4} + 9\,ad^{3}e + \left(19\,bde^{3} + 9\,ae^{4}\right)x^{3} + 3\left(5\,bd^{2}e^{2} + 9\,ade^{3}\right)x^{2} + \left(bd^{3}e + 27\,ad^{2}e^{2}\right)x\right)\sqrt{ex + d}}{63\,e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/63*(7*b*e^4*x^4 - 2*b*d^4 + 9*a*d^3*e + (19*b*d*e^3 + 9*a*e^4)*x^3 + 3*(5
*b*d^2*e^2 + 9*a*d*e^3)*x^2 + (b*d^3*e + 27*a*d^2*e^2)*x)*sqrt(e*x + d)/e^2
```

Fricas [A] time = 1.55586, size = 205, normalized size = 2.14

$$\frac{2\left(7\,be^{4}x^{4} - 2\,bd^{4} + 9\,ad^{3}e + \left(19\,bde^{3} + 9\,ae^{4}\right)x^{3} + 3\left(5\,bd^{2}e^{2} + 9\,ade^{3}\right)x^{2} + \left(bd^{3}e + 27\,ad^{2}e^{2}\right)x\right)\sqrt{ex + d}}{63\,e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/63*(7*b*e^4*x^4 - 2*b*d^4 + 9*a*d^3*e + (19*b*d*e^3 + 9*a*e^4)*x^3 + 3*(5
*b*d^2*e^2 + 9*a*d*e^3)*x^2 + (b*d^3*e + 27*a*d^2*e^2)*x)*sqrt(e*x + d)/e^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*((b*x+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18184, size = 327, normalized size = 3.41

$$\frac{2}{315}\left(21\left(3(xe+d)^{\frac{5}{2}}-5(xe+d)^{\frac{3}{2}}d\right)bd^2e^{(-1)}\mathrm{sgn}\,(bx+a)+105(xe+d)^{\frac{3}{2}}ad^2\mathrm{sgn}\,(bx+a)+6\left(15(xe+d)^{\frac{7}{2}}-42(xe+d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(21*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*b*d^2*e^(-1)*sgn(b*x +
a) + 105*(x*e + d)^(3/2)*a*d^2*sgn(b*x + a) + 6*(15*(x*e + d)^(7/2) - 42*(x
*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*b*d*e^(-1)*sgn(b*x + a) + 42*(3*(
x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a*d*sgn(b*x + a) + (35*(x*e + d)^(9/2
) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d
^3)*b*e^(-1)*sgn(b*x + a) + 3*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d +
35*(x*e + d)^(3/2)*d^2)*a*sgn(b*x + a))*e^(-1)
```

3.1677 $\int (d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}\,dx$

Optimal. Leaf size=96

$$\frac{2b\sqrt{a^2+2abx+b^2x^2}(d+ex)^{7/2}}{7e^2(a+bx)}-\frac{2\sqrt{a^2+2abx+b^2x^2}(d+ex)^{5/2}(bd-ae)}{5e^2(a+bx)}$$

```
[Out] (-2*(b*d - a*e)*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^2*(a +
b*x)) + (2*b*(d + e*x)^(7/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*e^2*(a + b*x
))
```

Rubi [A] time = 0.0380268, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {646, 43}

$$\frac{2b\sqrt{a^2+2abx+b^2x^2}(d+ex)^{7/2}}{7e^2(a+bx)}-\frac{2\sqrt{a^2+2abx+b^2x^2}(d+ex)^{5/2}(bd-ae)}{5e^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (-2*(b*d - a*e)*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^2*(a +
b*x)) + (2*b*(d + e*x)^(7/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*e^2*(a + b*x
))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}\,dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(ab+b^2x\right)(d+ex)^{3/2}\,dx}{ab+b^2x}\\ &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b(bd-ae)(d+ex)^{3/2}}{e}+\frac{b^2(d+ex)^{5/2}}{e}\right)dx}{ab+b^2x}\\ &= -\frac{2(bd-ae)(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}}{5e^2(a+bx)}+\frac{2b(d+ex)^{7/2}\sqrt{a^2+2abx+b^2x^2}}{7e^2(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.0256344, size = 48, normalized size = 0.5

$$\frac{2\sqrt{(a+bx)^2}(d+ex)^{5/2}(7ae-2bd+5bex)}{35e^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (2*Sqrt[(a + b*x)^2]*(d + e*x)^(5/2)*(-2*b*d + 7*a*e + 5*b*e*x))/(35*e^2*(a
 + b*x))
```

Maple [A] time = 0.041, size = 43, normalized size = 0.5

$$\frac{10\,bxe+14\,ae-4\,bd}{35\,e^2\,(bx+a)}(ex+d)^{\frac{5}{2}}\sqrt{(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*((b*x+a)^2)^(1/2),x)
```

```
[Out] 2/35*(e*x+d)^(5/2)*(5*b*e*x+7*a*e-2*b*d)*((b*x+a)^2)^(1/2)/e^2/(b*x+a)
```

Maxima [A] time = 1.15182, size = 93, normalized size = 0.97

$$\frac{2\left(5\,be^3x^3-2\,bd^3+7\,ad^2e+\left(8\,bde^2+7\,ae^3\right)x^2+\left(bd^2e+14\,ade^2\right)x\right)\sqrt{ex+d}}{35\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*b*e^3*x^3 - 2*b*d^3 + 7*a*d^2*e + (8*b*d*e^2 + 7*a*e^3)*x^2 + (b*d^
2*e + 14*a*d*e^2)*x)*sqrt(e*x + d)/e^2
```

Fricas [A] time = 1.49506, size = 155, normalized size = 1.61

$$\frac{2\left(5\,be^3x^3-2\,bd^3+7\,ad^2e+\left(8\,bde^2+7\,ae^3\right)x^2+\left(bd^2e+14\,ade^2\right)x\right)\sqrt{ex+d}}{35\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b*e^3*x^3 - 2*b*d^3 + 7*a*d^2*e + (8*b*d*e^2 + 7*a*e^3)*x^2 + (b*d^
2*e + 14*a*d*e^2)*x)*sqrt(e*x + d)/e^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*((b*x+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18572, size = 180, normalized size = 1.88

$$\frac{2}{105}\left(7\left(3\,(xe+d)^{\frac{5}{2}}-5\,(xe+d)^{\frac{3}{2}}d\right)bde^{(-1)}\mathrm{sgn}\,(bx+a)+35\,(xe+d)^{\frac{3}{2}}ad\mathrm{sgn}\,(bx+a)+\left(15\,(xe+d)^{\frac{7}{2}}-42\,(xe+d)^{\frac{5}{2}}d+35\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(7*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*b*d*e^(-1)*sgn(b*x + a)
+ 35*(x*e + d)^(3/2)*a*d*sgn(b*x + a) + (15*(x*e + d)^(7/2) - 42*(x*e + d)^
(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*b*e^(-1)*sgn(b*x + a) + 7*(3*(x*e + d)^(5
/2) - 5*(x*e + d)^(3/2)*d)*a*sgn(b*x + a))*e^(-1)
```

3.1678 $\int \sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}\,dx$

Optimal. Leaf size=96

$$\frac{2b\sqrt{a^2+2abx+b^2x^2}(d+ex)^{5/2}}{5e^2(a+bx)}-\frac{2\sqrt{a^2+2abx+b^2x^2}(d+ex)^{3/2}(bd-ae)}{3e^2(a+bx)}$$

```
[Out] (-2*(b*d - a*e)*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^2*(a +
b*x)) + (2*b*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^2*(a + b*x
))
```

Rubi [A] time = 0.0374104, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {646, 43}

$$\frac{2b\sqrt{a^2+2abx+b^2x^2}(d+ex)^{5/2}}{5e^2(a+bx)}-\frac{2\sqrt{a^2+2abx+b^2x^2}(d+ex)^{3/2}(bd-ae)}{3e^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (-2*(b*d - a*e)*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^2*(a +
b*x)) + (2*b*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^2*(a + b*x
))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}\,dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(ab+b^2x\right)\sqrt{d+ex}\,dx}{ab+b^2x}\\ &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b(bd-ae)\sqrt{d+ex}}{e}+\frac{b^2(d+ex)^{3/2}}{e}\right)dx}{ab+b^2x}\\ &= -\frac{2(bd-ae)(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}}{3e^2(a+bx)}+\frac{2b(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}}{5e^2(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.0226549, size = 48, normalized size = 0.5

$$\frac{2\sqrt{(a+bx)^2}(d+ex)^{3/2}(5ae-2bd+3bex)}{15e^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (2*Sqrt[(a + b*x)^2]*(d + e*x)^(3/2)*(-2*b*d + 5*a*e + 3*b*e*x))/(15*e^2*(a
 + b*x))
```

Maple [A] time = 0.04, size = 43, normalized size = 0.5

$$\frac{6\,bxe+10\,ae-4\,bd}{15\,e^2\,(bx+a)}\,(ex+d)^{\frac{3}{2}}\,\sqrt{(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*((b*x+a)^2)^(1/2),x)
```

```
[Out] 2/15*(e*x+d)^(3/2)*(3*b*e*x+5*a*e-2*b*d)*((b*x+a)^2)^(1/2)/e^2/(b*x+a)
```

Maxima [A] time = 1.10817, size = 62, normalized size = 0.65

$$\frac{2\left(3\,be^2x^2-2\,bd^2+5\,ade+\left(bde+5\,ae^2\right)x\right)\sqrt{ex+d}}{15\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*b*e^2*x^2 - 2*b*d^2 + 5*a*d*e + (b*d*e + 5*a*e^2)*x)*sqrt(e*x + d)/
e^2
```

Fricas [A] time = 1.43663, size = 108, normalized size = 1.12

$$\frac{2\left(3\,be^2x^2-2\,bd^2+5\,ade+\left(bde+5\,ae^2\right)x\right)\sqrt{ex+d}}{15\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b*e^2*x^2 - 2*b*d^2 + 5*a*d*e + (b*d*e + 5*a*e^2)*x)*sqrt(e*x + d)/
e^2
```

Sympy [A] time = 13.5209, size = 49, normalized size = 0.51

$$\frac{2a\,(d+ex)^{\frac{3}{2}}}{3e}-\frac{2bd\,(d+ex)^{\frac{3}{2}}}{3e^2}+\frac{2b\,(d+ex)^{\frac{5}{2}}}{5e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*((b*x+a)**2)**(1/2),x)
```

```
[Out] 2*a*(d + e*x)**(3/2)/(3*e) - 2*b*d*(d + e*x)**(3/2)/(3*e**2) + 2*b*(d + e*x
)**(5/2)/(5*e**2)
```

Giac [A] time = 1.12019, size = 73, normalized size = 0.76

$$\frac{2}{15}\left(\left(3(xe+d)^{\frac{5}{2}}-5(xe+d)^{\frac{3}{2}}d\right)be^{(-1)}\mathrm{sgn}\,(bx+a)+5(xe+d)^{\frac{3}{2}}a\mathrm{sgn}\,(bx+a)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*((3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*b*e^(-1)*sgn(b*x + a) + 5*(
x*e + d)^(3/2)*a*sgn(b*x + a))*e^(-1)
```

3.1679 $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=94

$$\frac{2b\sqrt{a^2+2abx+b^2x^2}(d+ex)^{3/2}}{3e^2(a+bx)}-\frac{2\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-ae)}{e^2(a+bx)}$$

```
[Out] (-2*(b*d - a*e)*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^2*(a + b*x)
) + (2*b*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^2*(a + b*x))
```

Rubi [A] time = 0.0359643, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {646, 43}

$$\frac{2b\sqrt{a^2+2abx+b^2x^2}(d+ex)^{3/2}}{3e^2(a+bx)}-\frac{2\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-ae)}{e^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/Sqrt[d + e*x],x]
```

```
[Out] (-2*(b*d - a*e)*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^2*(a + b*x)
) + (2*b*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^2*(a + b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx+b^2x^2}}{\sqrt{d+ex}}\,dx&=\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{ab+b^2x}{\sqrt{d+ex}}\,dx}{ab+b^2x}\\&=\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b(bd-ae)}{e\sqrt{d+ex}}+\frac{b^2\sqrt{d+ex}}{e}\right)dx}{ab+b^2x}\\&=-\frac{2(bd-ae)\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}}{e^2(a+bx)}+\frac{2b(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}}{3e^2(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.0275974, size = 47, normalized size = 0.5

$$\frac{2\sqrt{(a+bx)^2}\sqrt{d+ex}(3ae-2bd+bex)}{3e^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[(a + b*x)^2]*Sqrt[d + e*x]*(-2*b*d + 3*a*e + b*e*x))/(3*e^2*(a + b*
x))
```

Maple [A] time = 0.043, size = 42, normalized size = 0.5

$$\frac{2\,bxe + 6\,ae - 4\,bd}{3\,(bx + a)\,e^2}\sqrt{ex + d}\sqrt{(bx + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] 2/3*(e*x+d)^(1/2)*(b*e*x+3*a*e-2*b*d)*((b*x+a)^2)^(1/2)/e^2/(b*x+a)
```

Maxima [A] time = 1.07143, size = 62, normalized size = 0.66

$$\frac{2\left(be^2x^2 - 2\,bd^2 + 3\,ade - \left(bde - 3\,ae^2\right)x\right)}{3\,\sqrt{ex + d}e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*e^2*x^2 - 2*b*d^2 + 3*a*d*e - (b*d*e - 3*a*e^2)*x)/(sqrt(e*x + d)*e^
2)
```

Fricas [A] time = 1.55486, size = 63, normalized size = 0.67

$$\frac{2\,(bex - 2\,bd + 3\,ae)\sqrt{ex + d}}{3\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*e*x - 2*b*d + 3*a*e)*sqrt(e*x + d)/e^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(a + bx)^2}}{\sqrt{d + ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Integral(sqrt((a + b*x)**2)/sqrt(d + e*x), x)
```

Giac [A] time = 1.14869, size = 70, normalized size = 0.74

$$\frac{2}{3}\left(\left((xe+d)^{\frac{3}{2}}-3\sqrt{xe+d}d\right)be^{(-1)}\mathrm{sgn}\left(bx+a\right)+3\sqrt{xe+d}a\mathrm{sgn}\left(bx+a\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(((x*e + d)^(3/2) - 3*sqrt(x*e + d)*d)*b*e^(-1)*sgn(b*x + a) + 3*sqrt(x
*e + d)*a*sgn(b*x + a))*e^(-1)
```

3.1680 $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=92

$$\frac{2b\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}}{e^2(a+bx)}+\frac{2\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{e^2(a+bx)\sqrt{d+ex}}$$

```
[Out] (2*(b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^2*(a + b*x)*Sqrt[d + e*x])
 + (2*b*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^2*(a + b*x))
```

Rubi [A] time = 0.0357957, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {646, 43}

$$\frac{2b\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}}{e^2(a+bx)}+\frac{2\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{e^2(a+bx)\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^(3/2),x]
```

```
[Out] (2*(b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^2*(a + b*x)*Sqrt[d + e*x])
 + (2*b*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^2*(a + b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^{3/2}}\,dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{ab+b^2x}{(d+ex)^{3/2}}\,dx}{ab+b^2x}\\
&= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b(bd-ae)}{e(d+ex)^{3/2}}+\frac{b^2}{e\sqrt{d+ex}}\right)dx}{ab+b^2x}\\
&= \frac{2(bd-ae)\sqrt{a^2+2abx+b^2x^2}}{e^2(a+bx)\sqrt{d+ex}}+\frac{2b\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}}{e^2(a+bx)}
\end{aligned}$$

Mathematica [A] time = 0.0238114, size = 45, normalized size = 0.49

$$\frac{2\sqrt{(a+bx)^2}(-ae+2bd+bex)}{e^2(a+bx)\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^(3/2),x]
```

```
[Out] (2*Sqrt[(a + b*x)^2]*(2*b*d - a*e + b*e*x))/(e^2*(a + b*x)*Sqrt[d + e*x])
```

Maple [A] time = 0.041, size = 42, normalized size = 0.5

$$-2\frac{(-bxe + ae - 2\,bd)\sqrt{(bx + a)^2}}{\sqrt{ex + d}e^2\,(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)/(e*x+d)^(3/2),x)
```

```
[Out] -2/(e*x+d)^(1/2)*(-b*e*x+a*e-2*b*d)*((b*x+a)^2)^(1/2)/e^2/(b*x+a)
```

Maxima [A] time = 1.09141, size = 34, normalized size = 0.37

$$\frac{2\,(bex + 2\,bd - ae)}{\sqrt{ex + d}e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*(b*e*x + 2*b*d - a*e)/(sqrt(e*x + d)*e^2)
```

Fricas [A] time = 1.42545, size = 74, normalized size = 0.8

$$\frac{2\,(bex + 2\,bd - ae)\sqrt{ex + d}}{e^3x + de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(b*e*x + 2*b*d - a*e)*sqrt(e*x + d)/(e^3*x + d*e^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(a + bx)^2}}{(d + ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)/(e*x+d)**(3/2),x)
```

```
[Out] Integral(sqrt((a + b*x)**2)/(d + e*x)**(3/2), x)
```

Giac [A] time = 1.15545, size = 72, normalized size = 0.78

$$2\sqrt{xe+d}be^{(-2)}\mathrm{sgn}\,(bx+a)+\frac{2\,(bd\mathrm{sgn}\,(bx+a)-ae\mathrm{sgn}\,(bx+a))e^{(-2)}}{\sqrt{xe+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x*e + d)*b*e^(-2)*sgn(b*x + a) + 2*(b*d*sgn(b*x + a) - a*e*sgn(b*x +
 a))*e^(-2)/sqrt(x*e + d)
```

3.1681 $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=94

$$\frac{2\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{3e^2(a+bx)(d+ex)^{3/2}}-\frac{2b\sqrt{a^2+2abx+b^2x^2}}{e^2(a+bx)\sqrt{d+ex}}$$

```
[Out] (2*(b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^2*(a + b*x)*(d + e*x)^(3
/2)) - (2*b*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^2*(a + b*x)*Sqrt[d + e*x])
```

Rubi [A] time = 0.0365931, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {646, 43}

$$\frac{2\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{3e^2(a+bx)(d+ex)^{3/2}}-\frac{2b\sqrt{a^2+2abx+b^2x^2}}{e^2(a+bx)\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^(5/2),x]
```

```
[Out] (2*(b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^2*(a + b*x)*(d + e*x)^(3
/2)) - (2*b*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^2*(a + b*x)*Sqrt[d + e*x])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^{5/2}}\,dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{ab+b^2x}{(d+ex)^{5/2}}\,dx}{ab+b^2x}\\
&= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b(bd-ae)}{e(d+ex)^{5/2}}+\frac{b^2}{e(d+ex)^{3/2}}\right)dx}{ab+b^2x}\\
&= \frac{2(bd-ae)\sqrt{a^2+2abx+b^2x^2}}{3e^2(a+bx)(d+ex)^{3/2}}-\frac{2b\sqrt{a^2+2abx+b^2x^2}}{e^2(a+bx)\sqrt{d+ex}}
\end{aligned}$$

Mathematica [A] time = 0.0232539, size = 47, normalized size = 0.5

$$-\frac{2\sqrt{(a+bx)^2}(ae+2bd+3bex)}{3e^2(a+bx)(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[(a + b*x)^2]*(2*b*d + a*e + 3*b*e*x))/(3*e^2*(a + b*x)*(d + e*x)^(
3/2))
```

Maple [A] time = 0.043, size = 42, normalized size = 0.5

$$-\frac{6\,bxe+2\,ae+4\,bd}{3\,(bx+a)\,e^2}\sqrt{(bx+a)^2}\,(ex+d)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)/(e*x+d)^(5/2),x)
```

```
[Out] -2/3/(e*x+d)^(3/2)*(3*b*e*x+a*e+2*b*d)*((b*x+a)^2)^(1/2)/e^2/(b*x+a)
```

Maxima [A] time = 1.19077, size = 47, normalized size = 0.5

$$-\frac{2\,(3\,bex+2\,bd+ae)}{3\,(e^3x+de^2)\sqrt{ex+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(3*b*e*x + 2*b*d + a*e)/((e^3*x + d*e^2)*sqrt(e*x + d))
```

Fricas [A] time = 1.55682, size = 103, normalized size = 1.1

$$-\frac{2\,(3\,bex+2\,bd+ae)\sqrt{ex+d}}{3\,(e^4x^2+2\,de^3x+d^2e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*b*e*x + 2*b*d + a*e)*sqrt(e*x + d)/(e^4*x^2 + 2*d*e^3*x + d^2*e^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)/(e*x+d)**(5/2),x)
```

[Out] Timed out

Giac [A] time = 1.13638, size = 65, normalized size = 0.69

$$-\frac{2\left(3\left(xe+d\right)b\operatorname{sgn}\left(bx+a\right)-bd\operatorname{sgn}\left(bx+a\right)+ae\operatorname{sgn}\left(bx+a\right)\right)e^{(-2)}}{3\left(xe+d\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*(x*e + d)*b*sgn(b*x + a) - b*d*sgn(b*x + a) + a*e*sgn(b*x + a))*e^(
-2)/(x*e + d)^(3/2)
```

3.1682 $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=96

$$\frac{2\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{5e^2(a+bx)(d+ex)^{5/2}}-\frac{2b\sqrt{a^2+2abx+b^2x^2}}{3e^2(a+bx)(d+ex)^{3/2}}$$

```
[Out] (2*(b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^2*(a + b*x)*(d + e*x)^(5
/2)) - (2*b*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^2*(a + b*x)*(d + e*x)^(3/2)
)
```

Rubi [A] time = 0.0352242, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {646, 43}

$$\frac{2\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{5e^2(a+bx)(d+ex)^{5/2}}-\frac{2b\sqrt{a^2+2abx+b^2x^2}}{3e^2(a+bx)(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^(7/2),x]
```

```
[Out] (2*(b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^2*(a + b*x)*(d + e*x)^(5
/2)) - (2*b*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^2*(a + b*x)*(d + e*x)^(3/2)
)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^{7/2}}\,dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{ab+b^2x}{(d+ex)^{7/2}}\,dx}{ab+b^2x}\\
&= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b(bd-ae)}{e(d+ex)^{7/2}}+\frac{b^2}{e(d+ex)^{5/2}}\right)dx}{ab+b^2x}\\
&= \frac{2(bd-ae)\sqrt{a^2+2abx+b^2x^2}}{5e^2(a+bx)(d+ex)^{5/2}}-\frac{2b\sqrt{a^2+2abx+b^2x^2}}{3e^2(a+bx)(d+ex)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0244522, size = 48, normalized size = 0.5

$$-\frac{2\sqrt{(a+bx)^2}(3ae+2bd+5bex)}{15e^2(a+bx)(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^(7/2),x]
```

```
[Out] (-2*Sqrt[(a + b*x)^2]*(2*b*d + 3*a*e + 5*b*e*x))/(15*e^2*(a + b*x)*(d + e*x
)^(5/2))
```

Maple [A] time = 0.04, size = 43, normalized size = 0.5

$$-\frac{10\,bxe+6\,ae+4\,bd}{15\,(bx+a)\,e^2}\sqrt{(bx+a)^2}\,(ex+d)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)/(e*x+d)^(7/2),x)
```

```
[Out] -2/15/(e*x+d)^(5/2)*(5*b*e*x+3*a*e+2*b*d)*((b*x+a)^2)^(1/2)/e^2/(b*x+a)
```

Maxima [A] time = 1.07257, size = 63, normalized size = 0.66

$$-\frac{2\,(5\,bex+2\,bd+3\,ae)}{15\,\left(e^4x^2+2\,de^3x+d^2e^2\right)\sqrt{ex+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/15*(5*b*e*x + 2*b*d + 3*a*e)/((e^4*x^2 + 2*d*e^3*x + d^2*e^2)*sqrt(e*x +
 d))
```

Fricas [A] time = 1.53867, size = 128, normalized size = 1.33

$$-\frac{2\,(5\,bex+2\,bd+3\,ae)\sqrt{ex+d}}{15\,\left(e^5x^3+3\,de^4x^2+3\,d^2e^3x+d^3e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/15*(5*b*e*x + 2*b*d + 3*a*e)*sqrt(e*x + d)/(e^5*x^3 + 3*d*e^4*x^2 + 3*d^
2*e^3*x + d^3*e^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)/(e*x+d)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13031, size = 66, normalized size = 0.69

$$-\frac{2\,(5\,(xe+d)b\mathrm{sgn}\,(bx+a)-3\,bd\mathrm{sgn}\,(bx+a)+3\,ae\mathrm{sgn}\,(bx+a))e^{(-2)}}{15\,(xe+d)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*(5*(x*e + d)*b*sgn(b*x + a) - 3*b*d*sgn(b*x + a) + 3*a*e*sgn(b*x + a)
)*e^(-2)/(x*e + d)^(5/2)
```

3.1683 $\int (d+ex)^{5/2} \left(a^2+2abx+b^2x^2\right)^{3/2} \, dx$

Optimal. Leaf size=208

$$\frac{2b^3\sqrt{a^2+2abx+b^2x^2}(d+ex)^{13/2}}{13e^4(a+bx)}-\frac{6b^2\sqrt{a^2+2abx+b^2x^2}(d+ex)^{11/2}(bd-ae)}{11e^4(a+bx)}+\frac{2b\sqrt{a^2+2abx+b^2x^2}(d+ex)^{9/2}(bd-}{3e^4(a+bx)}$$

```
[Out] (-2*(b*d - a*e)^3*(d + e*x)^(7/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*e^4*(a
+ b*x)) + (2*b*(b*d - a*e)^2*(d + e*x)^(9/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
/(3*e^4*(a + b*x)) - (6*b^2*(b*d - a*e)*(d + e*x)^(11/2)*Sqrt[a^2 + 2*a*b*x
 + b^2*x^2])/(11*e^4*(a + b*x)) + (2*b^3*(d + e*x)^(13/2)*Sqrt[a^2 + 2*a*b*
x + b^2*x^2])/(13*e^4*(a + b*x))
```

Rubi [A] time = 0.07289, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {646, 43}

$$\frac{2b^3\sqrt{a^2+2abx+b^2x^2}(d+ex)^{13/2}}{13e^4(a+bx)}-\frac{6b^2\sqrt{a^2+2abx+b^2x^2}(d+ex)^{11/2}(bd-ae)}{11e^4(a+bx)}+\frac{2b\sqrt{a^2+2abx+b^2x^2}(d+ex)^{9/2}(bd-}{3e^4(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (-2*(b*d - a*e)^3*(d + e*x)^(7/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*e^4*(a
+ b*x)) + (2*b*(b*d - a*e)^2*(d + e*x)^(9/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
/(3*e^4*(a + b*x)) - (6*b^2*(b*d - a*e)*(d + e*x)^(11/2)*Sqrt[a^2 + 2*a*b*x
 + b^2*x^2])/(11*e^4*(a + b*x)) + (2*b^3*(d + e*x)^(13/2)*Sqrt[a^2 + 2*a*b*
x + b^2*x^2])/(13*e^4*(a + b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^{5/2}\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(ab+b^2x\right)^3(d+ex)^{5/2}\,dx}{b^2\left(ab+b^2x\right)}\\
&= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^3(bd-ae)^3(d+ex)^{5/2}}{e^3}+\frac{3b^4(bd-ae)^2(d+ex)^{7/2}}{e^3}-\frac{3b^5(bd-ae)(d+ex)^{9/2}}{e^3}+\right.}{b^2\left(ab+b^2x\right)}\\
&= -\frac{2(bd-ae)^3(d+ex)^{7/2}\sqrt{a^2+2abx+b^2x^2}}{7e^4(a+bx)}+\frac{2b(bd-ae)^2(d+ex)^{9/2}\sqrt{a^2+2abx}}{3e^4(a+bx)}
\end{aligned}$$

Mathematica [A] time = 0.0920237, size = 120, normalized size = 0.58

$$\frac{2\sqrt{(a+bx)^2}(d+ex)^{7/2}\left(143a^2be^2(7ex-2d)+429a^3e^3+13ab^2e\left(8d^2-28dex+63e^2x^2\right)+b^3\left(56d^2ex-16d^3-126de^2\cdots\right.\right.}{3003e^4(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[(a + b*x)^2]*(d + e*x)^(7/2)*(429*a^3*e^3 + 143*a^2*b*e^2*(-2*d + 7
*e*x) + 13*a*b^2*e*(8*d^2 - 28*d*e*x + 63*e^2*x^2) + b^3*(-16*d^3 + 56*d^2*
e*x - 126*d*e^2*x^2 + 231*e^3*x^3)))/(3003*e^4*(a + b*x))
```

Maple [A] time = 0.154, size = 132, normalized size = 0.6

$$\frac{462x^3b^3e^3+1638x^2ab^2e^3-252x^2b^3de^2+2002xa^2be^3-728xab^2de^2+112xb^3d^2e+858a^3e^3-572de^2a^2b+208ab^{\cdots}}{3003e^4(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] 2/3003*(e*x+d)^(7/2)*(231*b^3*e^3*x^3+819*a*b^2*e^3*x^2-126*b^3*d*e^2*x^2+1
001*a^2*b*e^3*x-364*a*b^2*d*e^2*x+56*b^3*d^2*e*x+429*a^3*e^3-286*a^2*b*d*e^
2+104*a*b^2*d^2*e-16*b^3*d^3)*((b*x+a)^2)^(3/2)/e^4/(b*x+a)^3
```

Maxima [A] time = 1.09352, size = 362, normalized size = 1.74

$$\frac{2\left(231b^3e^6x^6-16b^3d^6+104ab^2d^5e-286a^2bd^4e^2+429a^3d^3e^3+63\left(9b^3de^5+13ab^2e^6\right)x^5+7\left(53b^3d^2e^4+299ab^2\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3003*(231*b^3*e^6*x^6 - 16*b^3*d^6 + 104*a*b^2*d^5*e - 286*a^2*b*d^4*e^2
+ 429*a^3*d^3*e^3 + 63*(9*b^3*d*e^5 + 13*a*b^2*e^6)*x^5 + 7*(53*b^3*d^2*e^4
 + 299*a*b^2*d*e^5 + 143*a^2*b*e^6)*x^4 + (5*b^3*d^3*e^3 + 1469*a*b^2*d^2*e
^4 + 2717*a^2*b*d*e^5 + 429*a^3*e^6)*x^3 - 3*(2*b^3*d^4*e^2 - 13*a*b^2*d^3*
e^3 - 715*a^2*b*d^2*e^4 - 429*a^3*d*e^5)*x^2 + (8*b^3*d^5*e - 52*a*b^2*d^4*
e^2 + 143*a^2*b*d^3*e^3 + 1287*a^3*d^2*e^4)*x)*sqrt(e*x + d)/e^4
```

Fricas [A] time = 1.66259, size = 595, normalized size = 2.86

$$\frac{2\left(231b^3e^6x^6-16b^3d^6+104ab^2d^5e-286a^2bd^4e^2+429a^3d^3e^3+63\left(9b^3de^5+13ab^2e^6\right)x^5+7\left(53b^3d^2e^4+299ab^2\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(231*b^3*e^6*x^6 - 16*b^3*d^6 + 104*a*b^2*d^5*e - 286*a^2*b*d^4*e^2
+ 429*a^3*d^3*e^3 + 63*(9*b^3*d*e^5 + 13*a*b^2*e^6)*x^5 + 7*(53*b^3*d^2*e^4
 + 299*a*b^2*d*e^5 + 143*a^2*b*e^6)*x^4 + (5*b^3*d^3*e^3 + 1469*a*b^2*d^2*e
^4 + 2717*a^2*b*d*e^5 + 429*a^3*e^6)*x^3 - 3*(2*b^3*d^4*e^2 - 13*a*b^2*d^3*
e^3 - 715*a^2*b*d^2*e^4 - 429*a^3*d*e^5)*x^2 + (8*b^3*d^5*e - 52*a*b^2*d^4*
e^2 + 143*a^2*b*d^3*e^3 + 1287*a^3*d^2*e^4)*x)*sqrt(e*x + d)/e^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22581, size = 910, normalized size = 4.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/45045*(9009*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^2*b*d^2*e^(-1)*sg
n(b*x + a) + 1287*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)
^(3/2)*d^2)*a*b^2*d^2*e^(-2)*sgn(b*x + a) + 143*(35*(x*e + d)^(9/2) - 135*(
x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*b^3*d
^2*e^(-3)*sgn(b*x + a) + 15015*(x*e + d)^(3/2)*a^3*d^2*sgn(b*x + a) + 2574*
(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*a^2*b*
d*e^(-1)*sgn(b*x + a) + 858*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 1
89*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a*b^2*d*e^(-2)*sgn(b*x +
a) + 26*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/
2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*b^3*d*e^(-3)*
sgn(b*x + a) + 6006*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^3*d*sgn(b*x
 + a) + 429*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/
2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a^2*b*e^(-1)*sgn(b*x + a) + 39*(315*(x*e
+ d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e
 + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*a*b^2*e^(-2)*sgn(b*x + a) + 5*(
693*(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2
- 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/
2)*d^5)*b^3*e^(-3)*sgn(b*x + a) + 429*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5
/2)*d + 35*(x*e + d)^(3/2)*d^2)*a^3*sgn(b*x + a))*e^(-1)
```

3.1684 $\int (d+ex)^{3/2} \left(a^2+2abx+b^2x^2\right)^{3/2} dx$

Optimal. Leaf size=208

$$\frac{2b^3\sqrt{a^2+2abx+b^2x^2}(d+ex)^{11/2}}{11e^4(a+bx)}-\frac{2b^2\sqrt{a^2+2abx+b^2x^2}(d+ex)^{9/2}(bd-ae)}{3e^4(a+bx)}+\frac{6b\sqrt{a^2+2abx+b^2x^2}(d+ex)^{7/2}(bd-}{7e^4(a+bx)}$$

```
[Out] (-2*(b*d - a*e)^3*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^4*(a
+ b*x)) + (6*b*(b*d - a*e)^2*(d + e*x)^(7/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
/(7*e^4*(a + b*x)) - (2*b^2*(b*d - a*e)*(d + e*x)^(9/2)*Sqrt[a^2 + 2*a*b*x
+ b^2*x^2])/(3*e^4*(a + b*x)) + (2*b^3*(d + e*x)^(11/2)*Sqrt[a^2 + 2*a*b*x
+ b^2*x^2])/(11*e^4*(a + b*x))
```

Rubi [A] time = 0.0651006, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {646, 43}

$$\frac{2b^3\sqrt{a^2+2abx+b^2x^2}(d+ex)^{11/2}}{11e^4(a+bx)}-\frac{2b^2\sqrt{a^2+2abx+b^2x^2}(d+ex)^{9/2}(bd-ae)}{3e^4(a+bx)}+\frac{6b\sqrt{a^2+2abx+b^2x^2}(d+ex)^{7/2}(bd-}{7e^4(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```


```
[Out] (-2*(b*d - a*e)^3*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^4*(a
+ b*x)) + (6*b*(b*d - a*e)^2*(d + e*x)^(7/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
/(7*e^4*(a + b*x)) - (2*b^2*(b*d - a*e)*(d + e*x)^(9/2)*Sqrt[a^2 + 2*a*b*x
+ b^2*x^2])/(3*e^4*(a + b*x)) + (2*b^3*(d + e*x)^(11/2)*Sqrt[a^2 + 2*a*b*x
+ b^2*x^2])/(11*e^4*(a + b*x))
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (d+ex)^{3/2}\left(a^2+2abx+b^2x^2\right)^{3/2} dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(ab+b^2x\right)^3 (d+ex)^{3/2} dx}{b^2\left(ab+b^2x\right)}$$

$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^3(bd-ae)^3(d+ex)^{3/2}}{e^3}+\frac{3b^4(bd-ae)^2(d+ex)^{5/2}}{e^3}-\frac{3b^5(bd-ae)(d+ex)^{7/}}{e^3}\right.}{b^2\left(ab+b^2x\right)}$$

$$= -\frac{2(bd-ae)^3(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}}{5e^4(a+bx)}+\frac{6b(bd-ae)^2(d+ex)^{7/2}\sqrt{a^2+2a}}{7e^4(a+bx)}$$

Mathematica [A] time = 0.0729421, size = 120, normalized size = 0.58

$$\frac{2\sqrt{(a+bx)^2}(d+ex)^{5/2}\left(99a^2be^2(5ex-2d)+231a^3e^3+11ab^2e\left(8d^2-20dex+35e^2x^2\right)+b^3\left(40d^2ex-16d^3-70de^2x^2+1\right.\right.}{1155e^4(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[(a + b*x)^2]*(d + e*x)^(5/2)*(231*a^3*e^3 + 99*a^2*b*e^2*(-2*d + 5*
e*x) + 11*a*b^2*e*(8*d^2 - 20*d*e*x + 35*e^2*x^2) + b^3*(-16*d^3 + 40*d^2*e
*x - 70*d*e^2*x^2 + 105*e^3*x^3)))/(1155*e^4*(a + b*x))
```

Maple [A] time = 0.153, size = 132, normalized size = 0.6

$$\frac{210\,x^3b^3e^3+770\,x^2ab^2e^3-140\,x^2b^3de^2+990\,xa^2be^3-440\,xab^2de^2+80\,xb^3d^2e+462\,a^3e^3-396\,de^2a^2b+176\,ab^2d^2e-}{1155\,e^4\,(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] 2/1155*(e*x+d)^(5/2)*(105*b^3*e^3*x^3+385*a*b^2*e^3*x^2-70*b^3*d*e^2*x^2+49
5*a^2*b*e^3*x-220*a*b^2*d*e^2*x+40*b^3*d^2*e*x+231*a^3*e^3-198*a^2*b*d*e^2+
88*a*b^2*d^2*e-16*b^3*d^3)*((b*x+a)^2)^(3/2)/e^4/(b*x+a)^3
```

Maxima [A] time = 1.08715, size = 292, normalized size = 1.4

$$\frac{2\left(105\,b^3e^5x^5-16\,b^3d^5+88\,ab^2d^4e-198\,a^2bd^3e^2+231\,a^3d^2e^3+35\left(4\,b^3de^4+11\,ab^2e^5\right)x^4+5\left(b^3d^2e^3+110\,ab^2de^4+99\right.\right.}{115}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(105*b^3*e^5*x^5 - 16*b^3*d^5 + 88*a*b^2*d^4*e - 198*a^2*b*d^3*e^2 +
 231*a^3*d^2*e^3 + 35*(4*b^3*d*e^4 + 11*a*b^2*e^5)*x^4 + 5*(b^3*d^2*e^3 + 1
10*a*b^2*d*e^4 + 99*a^2*b*e^5)*x^3 - 3*(2*b^3*d^3*e^2 - 11*a*b^2*d^2*e^3 -
264*a^2*b*d*e^4 - 77*a^3*e^5)*x^2 + (8*b^3*d^4*e - 44*a*b^2*d^3*e^2 + 99*a^
2*b*d^2*e^3 + 462*a^3*d*e^4)*x)*sqrt(e*x + d)/e^4
```

Fricas [A] time = 1.54609, size = 474, normalized size = 2.28

$$\frac{2\left(105\,b^3e^5x^5-16\,b^3d^5+88\,ab^2d^4e-198\,a^2bd^3e^2+231\,a^3d^2e^3+35\left(4\,b^3de^4+11\,ab^2e^5\right)x^4+5\left(b^3d^2e^3+110\,ab^2de^4+99\right.\right.}{115}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*b^3*e^5*x^5 - 16*b^3*d^5 + 88*a*b^2*d^4*e - 198*a^2*b*d^3*e^2 +
 231*a^3*d^2*e^3 + 35*(4*b^3*d*e^4 + 11*a*b^2*e^5)*x^4 + 5*(b^3*d^2*e^3 + 1
```

```
10*a*b^2*d*e^4 + 99*a^2*b*e^5)*x^3 - 3*(2*b^3*d^3*e^2 - 11*a*b^2*d^2*e^3 -
264*a^2*b*d*e^4 - 77*a^3*e^5)*x^2 + (8*b^3*d^4*e - 44*a*b^2*d^3*e^2 + 99*a^
2*b*d^2*e^3 + 462*a^3*d*e^4)*x)*sqrt(e*x + d)/e^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d+ex)^{\frac{3}{2}}\left((a+bx)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)*((a + b*x)**2)**(3/2), x)
```

Giac [B] time = 1.1583, size = 528, normalized size = 2.54

$$\frac{2}{3465}\left(693\left(3\,(xe+d)^{\frac{5}{2}}-5\,(xe+d)^{\frac{3}{2}}d\right)a^2bde^{(-1)}\mathrm{sgn}\,(bx+a)+99\left(15\,(xe+d)^{\frac{7}{2}}-42\,(xe+d)^{\frac{5}{2}}d+35\,(xe+d)^{\frac{3}{2}}d^2\right)ab^2d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3465*(693*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^2*b*d*e^(-1)*sgn(b*
x + a) + 99*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)
*d^2)*a*b^2*d*e^(-2)*sgn(b*x + a) + 11*(35*(x*e + d)^(9/2) - 135*(x*e + d)^
(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*b^3*d*e^(-3)*s
gn(b*x + a) + 1155*(x*e + d)^(3/2)*a^3*d*sgn(b*x + a) + 99*(15*(x*e + d)^(7
/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*a^2*b*e^(-1)*sgn(b*x +
 a) + 33*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*
d^2 - 105*(x*e + d)^(3/2)*d^3)*a*b^2*e^(-2)*sgn(b*x + a) + (315*(x*e + d)^(
11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^
(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*b^3*e^(-3)*sgn(b*x + a) + 231*(3*(x*e
 + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^3*sgn(b*x + a))*e^(-1)
```

3.1685 $\int \sqrt{d+ex}\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=208

$$\frac{2b^3\sqrt{a^2+2abx+b^2x^2}(d+ex)^{9/2}}{9e^4(a+bx)}-\frac{6b^2\sqrt{a^2+2abx+b^2x^2}(d+ex)^{7/2}(bd-ae)}{7e^4(a+bx)}+\frac{6b\sqrt{a^2+2abx+b^2x^2}(d+ex)^{5/2}(bd-ae)}{5e^4(a+bx)}$$

```
[Out] (-2*(b*d - a*e)^3*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^4*(a
+ b*x)) + (6*b*(b*d - a*e)^2*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
/(5*e^4*(a + b*x)) - (6*b^2*(b*d - a*e)*(d + e*x)^(7/2)*Sqrt[a^2 + 2*a*b*x
+ b^2*x^2])/(7*e^4*(a + b*x)) + (2*b^3*(d + e*x)^(9/2)*Sqrt[a^2 + 2*a*b*x +
 b^2*x^2])/(9*e^4*(a + b*x))
```

Rubi [A] time = 0.0664567, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {646, 43}

$$\frac{2b^3\sqrt{a^2+2abx+b^2x^2}(d+ex)^{9/2}}{9e^4(a+bx)}-\frac{6b^2\sqrt{a^2+2abx+b^2x^2}(d+ex)^{7/2}(bd-ae)}{7e^4(a+bx)}+\frac{6b\sqrt{a^2+2abx+b^2x^2}(d+ex)^{5/2}(bd-ae)}{5e^4(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```


```
[Out] (-2*(b*d - a*e)^3*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^4*(a
+ b*x)) + (6*b*(b*d - a*e)^2*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
/(5*e^4*(a + b*x)) - (6*b^2*(b*d - a*e)*(d + e*x)^(7/2)*Sqrt[a^2 + 2*a*b*x
+ b^2*x^2])/(7*e^4*(a + b*x)) + (2*b^3*(d + e*x)^(9/2)*Sqrt[a^2 + 2*a*b*x +
 b^2*x^2])/(9*e^4*(a + b*x))
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{d+ex}\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(ab+b^2x\right)^3\sqrt{d+ex}\,dx}{b^2\left(ab+b^2x\right)}\\ &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^3(bd-ae)^3\sqrt{d+ex}}{e^3}+\frac{3b^4(bd-ae)^2(d+ex)^{3/2}}{e^3}-\frac{3b^5(bd-ae)(d+ex)^{5/2}}{e^3}+\frac{b^6(d+ex)^{7/2}}{e^3}\right)dx}{b^2\left(ab+b^2x\right)}\\ &= -\frac{2(bd-ae)^3(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}}{3e^4(a+bx)}+\frac{6b(bd-ae)^2(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}}{5e^4(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.0685035, size = 120, normalized size = 0.58

$$\frac{2\sqrt{(a+bx)^2}(d+ex)^{3/2}\left(63a^2be^2(3ex-2d)+105a^3e^3+9ab^2e\left(8d^2-12dex+15e^2x^2\right)+b^3\left(24d^2ex-16d^3-30de^2x^2+\right.\right.}{315e^4(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[(a + b*x)^2]*(d + e*x)^(3/2)*(105*a^3*e^3 + 63*a^2*b*e^2*(-2*d + 3*
e*x) + 9*a*b^2*e*(8*d^2 - 12*d*e*x + 15*e^2*x^2) + b^3*(-16*d^3 + 24*d^2*e*
x - 30*d*e^2*x^2 + 35*e^3*x^3)))/(315*e^4*(a + b*x))
```

Maple [A] time = 0.184, size = 132, normalized size = 0.6

$$\frac{70\,x^3b^3e^3+270\,x^2ab^2e^3-60\,x^2b^3de^2+378\,xa^2be^3-216\,xab^2de^2+48\,xb^3d^2e+210\,a^3e^3-252\,de^2a^2b+144\,ab^2d^2e-}{315\,e^4\,(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(3/2)*(e*x+d)^(1/2),x)
```

```
[Out] 2/315*(e*x+d)^(3/2)*(35*b^3*e^3*x^3+135*a*b^2*e^3*x^2-30*b^3*d*e^2*x^2+189*
a^2*b*e^3*x-108*a*b^2*d*e^2*x+24*b^3*d^2*e*x+105*a^3*e^3-126*a^2*b*d*e^2+72
*a*b^2*d^2*e-16*b^3*d^3)*((b*x+a)^2)^(3/2)/e^4/(b*x+a)^3
```

Maxima [A] time = 1.11669, size = 221, normalized size = 1.06

$$\frac{2\left(35\,b^3e^4x^4-16\,b^3d^4+72\,ab^2d^3e-126\,a^2bd^2e^2+105\,a^3de^3+5\left(b^3de^3+27\,ab^2e^4\right)x^3-3\left(2\,b^3d^2e^2-9\,ab^2de^3-63\,a^2\right.\right.}{315\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*b^3*e^4*x^4 - 16*b^3*d^4 + 72*a*b^2*d^3*e - 126*a^2*b*d^2*e^2 + 1
05*a^3*d*e^3 + 5*(b^3*d*e^3 + 27*a*b^2*e^4)*x^3 - 3*(2*b^3*d^2*e^2 - 9*a*b^
2*d*e^3 - 63*a^2*b*e^4)*x^2 + (8*b^3*d^3*e - 36*a*b^2*d^2*e^2 + 63*a^2*b*d*
e^3 + 105*a^3*e^4)*x)*sqrt(e*x + d)/e^4
```

Fricas [A] time = 1.55473, size = 359, normalized size = 1.73

$$\frac{2\left(35\,b^3e^4x^4-16\,b^3d^4+72\,ab^2d^3e-126\,a^2bd^2e^2+105\,a^3de^3+5\left(b^3de^3+27\,ab^2e^4\right)x^3-3\left(2\,b^3d^2e^2-9\,ab^2de^3-63\,a^2\right.\right.}{315\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^3*e^4*x^4 - 16*b^3*d^4 + 72*a*b^2*d^3*e - 126*a^2*b*d^2*e^2 + 1
05*a^3*d*e^3 + 5*(b^3*d*e^3 + 27*a*b^2*e^4)*x^3 - 3*(2*b^3*d^2*e^2 - 9*a*b^
```

```
2*d*e^3 - 63*a^2*b*e^4)*x^2 + (8*b^3*d^3*e - 36*a*b^2*d^2*e^2 + 63*a^2*b*d*
e^3 + 105*a^3*e^4)*x)*sqrt(e*x + d)/e^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d+ex}\left((a+bx)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)*(e*x+d)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*((a + b*x)**2)**(3/2), x)
```

Giac [A] time = 1.20514, size = 228, normalized size = 1.1

$$\frac{2}{315}\left(63\left(3\,(xe+d)^{\frac{5}{2}}-5\,(xe+d)^{\frac{3}{2}}d\right)a^2be^{(-1)}\mathrm{sgn}\,(bx+a)+9\left(15\,(xe+d)^{\frac{7}{2}}-42\,(xe+d)^{\frac{5}{2}}d+35\,(xe+d)^{\frac{3}{2}}d^2\right)ab^2e^{(-2)}\mathrm{sgn}\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(63*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^2*b*e^(-1)*sgn(b*x +
a) + 9*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)
*a*b^2*e^(-2)*sgn(b*x + a) + (35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d +
189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*b^3*e^(-3)*sgn(b*x + a)
+ 105*(x*e + d)^(3/2)*a^3*sgn(b*x + a))*e^(-1)
```

3.1686 $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=204

$$\frac{2b^3\sqrt{a^2+2abx+b^2x^2}(d+ex)^{7/2}}{7e^4(a+bx)}-\frac{6b^2\sqrt{a^2+2abx+b^2x^2}(d+ex)^{5/2}(bd-ae)}{5e^4(a+bx)}+\frac{2b\sqrt{a^2+2abx+b^2x^2}(d+ex)^{3/2}(bd-}{e^4(a+bx)}$$

```
[Out] (-2*(b*d - a*e)^3*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^4*(a + b*
x)) + (2*b*(b*d - a*e)^2*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^
4*(a + b*x)) - (6*b^2*(b*d - a*e)*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*
x^2])/(5*e^4*(a + b*x)) + (2*b^3*(d + e*x)^(7/2)*Sqrt[a^2 + 2*a*b*x + b^2*x
^2])/(7*e^4*(a + b*x))
```

Rubi [A] time = 0.0635751, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {646, 43}

$$\frac{2b^3\sqrt{a^2+2abx+b^2x^2}(d+ex)^{7/2}}{7e^4(a+bx)}-\frac{6b^2\sqrt{a^2+2abx+b^2x^2}(d+ex)^{5/2}(bd-ae)}{5e^4(a+bx)}+\frac{2b\sqrt{a^2+2abx+b^2x^2}(d+ex)^{3/2}(bd-}{e^4(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/Sqrt[d + e*x],x]
```

```
[Out] (-2*(b*d - a*e)^3*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^4*(a + b*
x)) + (2*b*(b*d - a*e)^2*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^
4*(a + b*x)) - (6*b^2*(b*d - a*e)*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*
x^2])/(5*e^4*(a + b*x)) + (2*b^3*(d + e*x)^(7/2)*Sqrt[a^2 + 2*a*b*x + b^2*x
^2])/(7*e^4*(a + b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{\sqrt{d+ex}}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^3}{\sqrt{d+ex}}\,dx}{b^2\left(ab+b^2x\right)}$$

$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^3(bd-ae)^3}{e^3\sqrt{d+ex}}+\frac{3b^4(bd-ae)^2\sqrt{d+ex}}{e^3}-\frac{3b^5(bd-ae)(d+ex)^{3/2}}{e^3}+\frac{b^6(d+ex)^{5/2}}{e^3}\right)dx}{b^2\left(ab+b^2x\right)}$$

$$= -\frac{2(bd-ae)^3\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}}{e^4(a+bx)}+\frac{2b(bd-ae)^2(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}}{e^4(a+bx)}-\frac{6b}{}$$

Mathematica [A] time = 0.0652632, size = 119, normalized size = 0.58

$$\frac{2\sqrt{(a+bx)^2}\sqrt{d+ex}\left(35a^2be^2(ex-2d)+35a^3e^3+7ab^2e\left(8d^2-4dex+3e^2x^2\right)+b^3\left(8d^2ex-16d^3-6de^2x^2+5e^3x^3\right)\right)}{35e^4(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[(a + b*x)^2]*Sqrt[d + e*x]*(35*a^3*e^3 + 35*a^2*b*e^2*(-2*d + e*x)
+ 7*a*b^2*e*(8*d^2 - 4*d*e*x + 3*e^2*x^2) + b^3*(-16*d^3 + 8*d^2*e*x - 6*d*
e^2*x^2 + 5*e^3*x^3)))/(35*e^4*(a + b*x))
```

Maple [A] time = 0.152, size = 132, normalized size = 0.7

$$\frac{10\,x^3b^3e^3+42\,x^2ab^2e^3-12\,x^2b^3de^2+70\,xa^2be^3-56\,xab^2de^2+16\,xb^3d^2e+70\,a^3e^3-140\,de^2a^2b+112\,ab^2d^2e-32\,b^3d^3}{35\,(bx+a)^3\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(1/2),x)
```

```
[Out] 2/35*(e*x+d)^(1/2)*(5*b^3*e^3*x^3+21*a*b^2*e^3*x^2-6*b^3*d*e^2*x^2+35*a^2*b
*e^3*x-28*a*b^2*d*e^2*x+8*b^3*d^2*e*x+35*a^3*e^3-70*a^2*b*d*e^2+56*a*b^2*d^
2*e-16*b^3*d^3)*((b*x+a)^2)^(3/2)/e^4/(b*x+a)^3
```

Maxima [A] time = 1.07642, size = 221, normalized size = 1.08

$$\frac{2\left(5\,b^3e^4x^4-16\,b^3d^4+56\,ab^2d^3e-70\,a^2bd^2e^2+35\,a^3de^3-\left(b^3de^3-21\,ab^2e^4\right)x^3+\left(2\,b^3d^2e^2-7\,ab^2de^3+35\,a^2be^4\right)x^2-\left(\right.}{35\sqrt{ex+d}e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*b^3*e^4*x^4 - 16*b^3*d^4 + 56*a*b^2*d^3*e - 70*a^2*b*d^2*e^2 + 35*a
^3*d*e^3 - (b^3*d*e^3 - 21*a*b^2*e^4)*x^3 + (2*b^3*d^2*e^2 - 7*a*b^2*d*e^3
+ 35*a^2*b*e^4)*x^2 - (8*b^3*d^3*e - 28*a*b^2*d^2*e^2 + 35*a^2*b*d*e^3 - 35
*a^3*e^4)*x)/(sqrt(e*x + d)*e^4)
```

Fricas [A] time = 1.51798, size = 251, normalized size = 1.23

$$\frac{2\left(5\,b^3e^3x^3-16\,b^3d^3+56\,ab^2d^2e-70\,a^2bde^2+35\,a^3e^3-3\left(2\,b^3de^2-7\,ab^2e^3\right)x^2+\left(8\,b^3d^2e-28\,ab^2de^2+35\,a^2be^3\right)x\right)}{35\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*e^3*x^3 - 16*b^3*d^3 + 56*a*b^2*d^2*e - 70*a^2*b*d*e^2 + 35*a^3
*e^3 - 3*(2*b^3*d*e^2 - 7*a*b^2*e^3)*x^2 + (8*b^3*d^2*e - 28*a*b^2*d*e^2 +
35*a^2*b*e^3)*x)*sqrt(e*x + d)/e^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22602, size = 225, normalized size = 1.1

$$\frac{2}{35}\left(35\left((xe+d)^{\frac{3}{2}}-3\,\sqrt{xe+d}d\right)a^2be^{(-1)}\mathrm{sgn}\,(bx+a)+7\left(3\,(xe+d)^{\frac{5}{2}}-10\,(xe+d)^{\frac{3}{2}}d+15\,\sqrt{xe+d}d^2\right)ab^2e^{(-2)}\mathrm{sgn}\,(bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 2/35*(35*((x*e + d)^(3/2) - 3*sqrt(x*e + d)*d)*a^2*b*e^(-1)*sgn(b*x + a) +
7*(3*(x*e + d)^(5/2) - 10*(x*e + d)^(3/2)*d + 15*sqrt(x*e + d)*d^2)*a*b^2*e
^(-2)*sgn(b*x + a) + (5*(x*e + d)^(7/2) - 21*(x*e + d)^(5/2)*d + 35*(x*e +
d)^(3/2)*d^2 - 35*sqrt(x*e + d)*d^3)*b^3*e^(-3)*sgn(b*x + a) + 35*sqrt(x*e
+ d)*a^3*sgn(b*x + a))*e^(-1)
```

3.1687 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=202

$$\frac{2b^3\sqrt{a^2+2abx+b^2x^2}(d+ex)^{5/2}}{5e^4(a+bx)}-\frac{2b^2\sqrt{a^2+2abx+b^2x^2}(d+ex)^{3/2}(bd-ae)}{e^4(a+bx)}+\frac{6b\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-ae)^2}{e^4(a+bx)}$$

```
[Out] (2*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^4*(a + b*x)*Sqrt[d + e*x
]) + (6*b*(b*d - a*e)^2*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^4*(
a + b*x)) - (2*b^2*(b*d - a*e)*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2
])/(e^4*(a + b*x)) + (2*b^3*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/
(5*e^4*(a + b*x))
```

Rubi [A] time = 0.0645532, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {646, 43}

$$\frac{2b^3\sqrt{a^2+2abx+b^2x^2}(d+ex)^{5/2}}{5e^4(a+bx)}-\frac{2b^2\sqrt{a^2+2abx+b^2x^2}(d+ex)^{3/2}(bd-ae)}{e^4(a+bx)}+\frac{6b\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-ae)^2}{e^4(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^(3/2),x]
```


```
[Out] (2*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^4*(a + b*x)*Sqrt[d + e*x
]) + (6*b*(b*d - a*e)^2*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^4*(
a + b*x)) - (2*b^2*(b*d - a*e)*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2
])/(e^4*(a + b*x)) + (2*b^3*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/
(5*e^4*(a + b*x))
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^3}{(d+ex)^{3/2}}\,dx}{b^2\left(ab+b^2x\right)}$$

$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^3(bd-ae)^3}{e^3(d+ex)^{3/2}}+\frac{3b^4(bd-ae)^2}{e^3\sqrt{d+ex}}-\frac{3b^5(bd-ae)\sqrt{d+ex}}{e^3}+\frac{b^6(d+ex)^{3/2}}{e^3}\right)dx}{b^2\left(ab+b^2x\right)}$$

$$= \frac{2(bd-ae)^3\sqrt{a^2+2abx+b^2x^2}}{e^4(a+bx)\sqrt{d+ex}}+\frac{6b(bd-ae)^2\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}}{e^4(a+bx)}-\frac{2b^2(bd-a}{}$$

Mathematica [A] time = 0.0595021, size = 119, normalized size = 0.59

$$-\frac{2\sqrt{(a+bx)^2}\left(-15a^2be^2(2d+ex)+5a^3e^3+5ab^2e\left(8d^2+4dex-e^2x^2\right)+b^3\left(-\left(8d^2ex+16d^3-2de^2x^2+e^3x^3\right)\right)\right)}{5e^4(a+bx)\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[(a + b*x)^2]*(5*a^3*e^3 - 15*a^2*b*e^2*(2*d + e*x) + 5*a*b^2*e*(8*
d^2 + 4*d*e*x - e^2*x^2) - b^3*(16*d^3 + 8*d^2*e*x - 2*d*e^2*x^2 + e^3*x^3)
))/(5*e^4*(a + b*x)*Sqrt[d + e*x])
```

Maple [A] time = 0.154, size = 132, normalized size = 0.7

$$-\frac{-2x^3b^3e^3-10x^2ab^2e^3+4x^2b^3de^2-30xa^2be^3+40xab^2de^2-16xb^3d^2e+10a^3e^3-60de^2a^2b+80ab^2d^2e-32b^3d^3}{5\,(bx+a)^3\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(3/2),x)
```

```
[Out] -2/5/(e*x+d)^(1/2)*(-b^3*e^3*x^3-5*a*b^2*e^3*x^2+2*b^3*d*e^2*x^2-15*a^2*b*e
^3*x+20*a*b^2*d*e^2*x-8*b^3*d^2*e*x+5*a^3*e^3-30*a^2*b*d*e^2+40*a*b^2*d^2*e
-16*b^3*d^3)*((b*x+a)^2)^(3/2)/e^4/(b*x+a)^3
```

Maxima [A] time = 1.16769, size = 154, normalized size = 0.76

$$\frac{2\left(b^3e^3x^3+16\,b^3d^3-40\,ab^2d^2e+30\,a^2bde^2-5\,a^3e^3-\left(2\,b^3de^2-5\,ab^2e^3\right)x^2+\left(8\,b^3d^2e-20\,ab^2de^2+15\,a^2be^3\right)x\right)}{5\,\sqrt{ex+d}e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b^3*e^3*x^3 + 16*b^3*d^3 - 40*a*b^2*d^2*e + 30*a^2*b*d*e^2 - 5*a^3*e^3
 - (2*b^3*d*e^2 - 5*a*b^2*e^3)*x^2 + (8*b^3*d^2*e - 20*a*b^2*d*e^2 + 15*a^2
*b*e^3)*x)/(sqrt(e*x + d)*e^4)
```

Fricas [A] time = 1.5908, size = 259, normalized size = 1.28

$$\frac{2\left(b^3e^3x^3+16\,b^3d^3-40\,ab^2d^2e+30\,a^2bde^2-5\,a^3e^3-\left(2\,b^3de^2-5\,ab^2e^3\right)x^2+\left(8\,b^3d^2e-20\,ab^2de^2+15\,a^2be^3\right)x\right)\sqrt{ex+d}}{5\left(e^5x+de^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^3*e^3*x^3 + 16*b^3*d^3 - 40*a*b^2*d^2*e + 30*a^2*b*d*e^2 - 5*a^3*e^3
 - (2*b^3*d*e^2 - 5*a*b^2*e^3)*x^2 + (8*b^3*d^2*e - 20*a*b^2*d*e^2 + 15*a^2
*b*e^3)*x)*sqrt(e*x + d)/(e^5*x + d*e^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((a+bx)^2\right)^{\frac{3}{2}}}{(d+ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/(e*x+d)**(3/2),x)
```

```
[Out] Integral(((a + b*x)**2)**(3/2)/(d + e*x)**(3/2), x)
```

Giac [A] time = 1.19567, size = 284, normalized size = 1.41

$$\frac{2}{5}\left((xe+d)^{\frac{5}{2}}b^3e^{16}\mathrm{sgn}\,(bx+a)-5\,(xe+d)^{\frac{3}{2}}b^3de^{16}\mathrm{sgn}\,(bx+a)+15\,\sqrt{xe+d}b^3d^2e^{16}\mathrm{sgn}\,(bx+a)+5\,(xe+d)^{\frac{3}{2}}ab^2e^{17}\mathrm{sgn}\,(b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] 2/5*((x*e + d)^(5/2)*b^3*e^16*sgn(b*x + a) - 5*(x*e + d)^(3/2)*b^3*d*e^16*s
gn(b*x + a) + 15*sqrt(x*e + d)*b^3*d^2*e^16*sgn(b*x + a) + 5*(x*e + d)^(3/2
)*a*b^2*e^17*sgn(b*x + a) - 30*sqrt(x*e + d)*a*b^2*d*e^17*sgn(b*x + a) + 15
*sqrt(x*e + d)*a^2*b*e^18*sgn(b*x + a))*e^(-20) + 2*(b^3*d^3*sgn(b*x + a) -
 3*a*b^2*d^2*e*sgn(b*x + a) + 3*a^2*b*d*e^2*sgn(b*x + a) - a^3*e^3*sgn(b*x
+ a))*e^(-4)/sqrt(x*e + d)
```

3.1688 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=204

$$\frac{2b^3\sqrt{a^2+2abx+b^2x^2}(d+ex)^{3/2}}{3e^4(a+bx)}-\frac{6b^2\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-ae)}{e^4(a+bx)}-\frac{6b\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{e^4(a+bx)\sqrt{d+ex}}+\frac{2\sqrt{a^2 \cdots}}{3 \cdots}$$

```
[Out] (2*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^4*(a + b*x)*(d + e*x)^
(3/2)) - (6*b*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^4*(a + b*x)*S
qrt[d + e*x]) - (6*b^2*(b*d - a*e)*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x
^2])/(e^4*(a + b*x)) + (2*b^3*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]
)/(3*e^4*(a + b*x))
```

Rubi [A] time = 0.0677803, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {646, 43}

$$\frac{2b^3\sqrt{a^2+2abx+b^2x^2}(d+ex)^{3/2}}{3e^4(a+bx)}-\frac{6b^2\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-ae)}{e^4(a+bx)}-\frac{6b\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{e^4(a+bx)\sqrt{d+ex}}+\frac{2\sqrt{a^2 \cdots}}{3 \cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^(5/2),x]
```

```
[Out] (2*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^4*(a + b*x)*(d + e*x)^
(3/2)) - (6*b*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^4*(a + b*x)*S
qrt[d + e*x]) - (6*b^2*(b*d - a*e)*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x
^2])/(e^4*(a + b*x)) + (2*b^3*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]
)/(3*e^4*(a + b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{(d+ex)^{5/2}}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^3}{(d+ex)^{5/2}}\,dx}{b^2\left(ab+b^2x\right)}$$

$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^3(bd-ae)^3}{e^3(d+ex)^{5/2}}+\frac{3b^4(bd-ae)^2}{e^3(d+ex)^{3/2}}-\frac{3b^5(bd-ae)}{e^3\sqrt{d+ex}}+\frac{b^6\sqrt{d+ex}}{e^3}\right)dx}{b^2\left(ab+b^2x\right)}$$

$$= \frac{2(bd-ae)^3\sqrt{a^2+2abx+b^2x^2}}{3e^4(a+bx)(d+ex)^{3/2}} - \frac{6b(bd-ae)^2\sqrt{a^2+2abx+b^2x^2}}{e^4(a+bx)\sqrt{d+ex}} - \frac{6b^2(bd-ae)\sqrt{d+ex}\sqrt{}}{e^4(a+b}$$

Mathematica [A] time = 0.0629554, size = 119, normalized size = 0.58

$$-\frac{2\sqrt{(a+bx)^2}\left(3a^2be^2(2d+3ex)+a^3e^3-3ab^2e\left(8d^2+12dex+3e^2x^2\right)+b^3\left(24d^2ex+16d^3+6de^2x^2-e^3x^3\right)\right)}{3e^4(a+bx)(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[(a + b*x)^2]*(a^3*e^3 + 3*a^2*b*e^2*(2*d + 3*e*x) - 3*a*b^2*e*(8*d
^2 + 12*d*e*x + 3*e^2*x^2) + b^3*(16*d^3 + 24*d^2*e*x + 6*d*e^2*x^2 - e^3*x
^3)))/(3*e^4*(a + b*x)*(d + e*x)^(3/2))
```

Maple [A] time = 0.153, size = 131, normalized size = 0.6

$$-\frac{-2\,x^3b^3e^3-18\,x^2ab^2e^3+12\,x^2b^3de^2+18\,xa^2be^3-72\,xab^2de^2+48\,xb^3d^2e+2\,a^3e^3+12\,de^2a^2b-48\,ab^2d^2e+32\,b^3d^3}{3\,(bx+a)^3\,e^4}\,((\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(5/2),x)
```

```
[Out] -2/3/(e*x+d)^(3/2)*(-b^3*e^3*x^3-9*a*b^2*e^3*x^2+6*b^3*d*e^2*x^2+9*a^2*b*e^
3*x-36*a*b^2*d*e^2*x+24*b^3*d^2*e*x+a^3*e^3+6*a^2*b*d*e^2-24*a*b^2*d^2*e+16
*b^3*d^3)*((b*x+a)^2)^(3/2)/e^4/(b*x+a)^3
```

Maxima [A] time = 1.10806, size = 169, normalized size = 0.83

$$\frac{2\left(b^3e^3x^3-16\,b^3d^3+24\,ab^2d^2e-6\,a^2bde^2-a^3e^3-3\left(2\,b^3de^2-3\,ab^2e^3\right)x^2-3\left(8\,b^3d^2e-12\,ab^2de^2+3\,a^2be^3\right)x\right)}{3\left(e^5x+de^4\right)\sqrt{ex+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b^3*e^3*x^3 - 16*b^3*d^3 + 24*a*b^2*d^2*e - 6*a^2*b*d*e^2 - a^3*e^3 -
3*(2*b^3*d*e^2 - 3*a*b^2*e^3)*x^2 - 3*(8*b^3*d^2*e - 12*a*b^2*d*e^2 + 3*a^2
*b*e^3)*x)/((e^5*x + d*e^4)*sqrt(e*x + d))
```

Fricas [A] time = 1.51978, size = 281, normalized size = 1.38

$$\frac{2\left(b^3e^3x^3-16\,b^3d^3+24\,ab^2d^2e-6\,a^2bde^2-a^3e^3-3\left(2\,b^3de^2-3\,ab^2e^3\right)x^2-3\left(8\,b^3d^2e-12\,ab^2de^2+3\,a^2be^3\right)x\right)\sqrt{ex+}}{3\left(e^6x^2+2\,de^5x+d^2e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^3*e^3*x^3 - 16*b^3*d^3 + 24*a*b^2*d^2*e - 6*a^2*b*d*e^2 - a^3*e^3 -
3*(2*b^3*d*e^2 - 3*a*b^2*e^3)*x^2 - 3*(8*b^3*d^2*e - 12*a*b^2*d*e^2 + 3*a^2
*b*e^3)*x)*sqrt(e*x + d)/(e^6*x^2 + 2*d*e^5*x + d^2*e^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((a+bx)^2\right)^{\frac{3}{2}}}{(d+ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/(e*x+d)**(5/2),x)
```

```
[Out] Integral(((a + b*x)**2)**(3/2)/(d + e*x)**(5/2), x)
```

Giac [A] time = 1.19761, size = 273, normalized size = 1.34

$$\frac{2}{3}\left((xe+d)^{\frac{3}{2}}b^3e^8\mathrm{sgn}\,(bx+a)-9\,\sqrt{xe+d}b^3de^8\mathrm{sgn}\,(bx+a)+9\,\sqrt{xe+d}ab^2e^9\mathrm{sgn}\,(bx+a)\right)e^{(-12)}-\frac{2\left(9\,(xe+d)b^3d^2\mathrm{sg}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*((x*e + d)^(3/2)*b^3*e^8*sgn(b*x + a) - 9*sqrt(x*e + d)*b^3*d*e^8*sgn(b
*x + a) + 9*sqrt(x*e + d)*a*b^2*e^9*sgn(b*x + a))*e^(-12) - 2/3*(9*(x*e + d
)*b^3*d^2*sgn(b*x + a) - b^3*d^3*sgn(b*x + a) - 18*(x*e + d)*a*b^2*d*e*sgn(
b*x + a) + 3*a*b^2*d^2*e*sgn(b*x + a) + 9*(x*e + d)*a^2*b*e^2*sgn(b*x + a)
- 3*a^2*b*d*e^2*sgn(b*x + a) + a^3*e^3*sgn(b*x + a))*e^(-4)/(x*e + d)^(3/2)
```

3.1689 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=202

$$\frac{2b^3\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}}{e^4(a+bx)}+\frac{6b^2\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{e^4(a+bx)\sqrt{d+ex}}-\frac{2b\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{e^4(a+bx)(d+ex)^{3/2}}+\frac{2\sqrt{a^2+2abx+b^2}}{5e^4(a+bx)(d}$$

```
[Out] (2*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^4*(a + b*x)*(d + e*x)^
(5/2)) - (2*b*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^4*(a + b*x)*(
d + e*x)^(3/2)) + (6*b^2*(b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^4*(a
 + b*x)*Sqrt[d + e*x]) + (2*b^3*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]
)/(e^4*(a + b*x))
```

Rubi [A] time = 0.066247, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {646, 43}

$$\frac{2b^3\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}}{e^4(a+bx)}+\frac{6b^2\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{e^4(a+bx)\sqrt{d+ex}}-\frac{2b\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{e^4(a+bx)(d+ex)^{3/2}}+\frac{2\sqrt{a^2+2abx+b^2}}{5e^4(a+bx)(d}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^(7/2),x]
```

```
[Out] (2*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^4*(a + b*x)*(d + e*x)^
(5/2)) - (2*b*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^4*(a + b*x)*(
d + e*x)^(3/2)) + (6*b^2*(b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^4*(a
 + b*x)*Sqrt[d + e*x]) + (2*b^3*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]
)/(e^4*(a + b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^3}{(d+ex)^{7/2}}\,dx}{b^2\left(ab+b^2x\right)}$$
$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^3(bd-ae)^3}{e^3(d+ex)^{7/2}}+\frac{3b^4(bd-ae)^2}{e^3(d+ex)^{5/2}}-\frac{3b^5(bd-ae)}{e^3(d+ex)^{3/2}}+\frac{b^6}{e^3\sqrt{d+ex}}\right)dx}{b^2\left(ab+b^2x\right)}$$
$$= \frac{2(bd-ae)^3\sqrt{a^2+2abx+b^2x^2}}{5e^4(a+bx)(d+ex)^{5/2}}-\frac{2b(bd-ae)^2\sqrt{a^2+2abx+b^2x^2}}{e^4(a+bx)(d+ex)^{3/2}}+\frac{6b^2(bd-ae)\sqrt{a^2+}}{e^4(a+bx)\sqrt{}}$$

Mathematica [A] time = 0.0647071, size = 118, normalized size = 0.58

$$-\frac{2\sqrt{(a+bx)^2}\left(a^2be^2(2d+5ex)+a^3e^3+ab^2e\left(8d^2+20dex+15e^2x^2\right)+b^3\left(-\left(40d^2ex+16d^3+30de^2x^2+5e^3x^3\right)\right)\right)}{5e^4(a+bx)(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^(7/2),x]
```

```
[Out] (-2*Sqrt[(a + b*x)^2]*(a^3*e^3 + a^2*b*e^2*(2*d + 5*e*x) + a*b^2*e*(8*d^2 +
 20*d*e*x + 15*e^2*x^2) - b^3*(16*d^3 + 40*d^2*e*x + 30*d*e^2*x^2 + 5*e^3*x
^3)))/(5*e^4*(a + b*x)*(d + e*x)^(5/2))
```

Maple [A] time = 0.153, size = 131, normalized size = 0.7

$$-\frac{-10\,x^3b^3e^3+30\,x^2ab^2e^3-60\,x^2b^3de^2+10\,xa^2be^3+40\,xab^2de^2-80\,xb^3d^2e+2\,a^3e^3+4\,de^2a^2b+16\,ab^2d^2e-32\,b^3d}{5\,(bx+a)^3\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(7/2),x)
```

```
[Out] -2/5/(e*x+d)^(5/2)*(-5*b^3*e^3*x^3+15*a*b^2*e^3*x^2-30*b^3*d*e^2*x^2+5*a^2*
b*e^3*x+20*a*b^2*d*e^2*x-40*b^3*d^2*e*x+a^3*e^3+2*a^2*b*d*e^2+8*a*b^2*d^2*e
-16*b^3*d^3)*((b*x+a)^2)^(3/2)/e^4/(b*x+a)^3
```

Maxima [A] time = 1.18217, size = 185, normalized size = 0.92

$$\frac{2\left(5\,b^3e^3x^3+16\,b^3d^3-8\,ab^2d^2e-2\,a^2bde^2-a^3e^3+15\left(2\,b^3de^2-ab^2e^3\right)x^2+5\left(8\,b^3d^2e-4\,ab^2de^2-a^2be^3\right)x\right)}{5\left(e^6x^2+2\,de^5x+d^2e^4\right)\sqrt{ex+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/5*(5*b^3*e^3*x^3 + 16*b^3*d^3 - 8*a*b^2*d^2*e - 2*a^2*b*d*e^2 - a^3*e^3 +
 15*(2*b^3*d*e^2 - a*b^2*e^3)*x^2 + 5*(8*b^3*d^2*e - 4*a*b^2*d*e^2 - a^2*b*
e^3)*x)/((e^6*x^2 + 2*d*e^5*x + d^2*e^4)*sqrt(e*x + d))
```

Fricas [A] time = 1.64484, size = 298, normalized size = 1.48

$$\frac{2\left(5\,b^3e^3x^3+16\,b^3d^3-8\,ab^2d^2e-2\,a^2bde^2-a^3e^3+15\left(2\,b^3de^2-ab^2e^3\right)x^2+5\left(8\,b^3d^2e-4\,ab^2de^2-a^2be^3\right)x\right)\sqrt{ex+d}}{5\left(e^7x^3+3\,de^6x^2+3\,d^2e^5x+d^3e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/5*(5*b^3*e^3*x^3 + 16*b^3*d^3 - 8*a*b^2*d^2*e - 2*a^2*b*d*e^2 - a^3*e^3 +
 15*(2*b^3*d*e^2 - a*b^2*e^3)*x^2 + 5*(8*b^3*d^2*e - 4*a*b^2*d*e^2 - a^2*b*
e^3)*x)*sqrt(e*x + d)/(e^7*x^3 + 3*d*e^6*x^2 + 3*d^2*e^5*x + d^3*e^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((a+bx)^2\right)^{\frac{3}{2}}}{(d+ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/(e*x+d)**(7/2),x)
```

```
[Out] Integral(((a + b*x)**2)**(3/2)/(d + e*x)**(7/2), x)
```

Giac [A] time = 1.21597, size = 265, normalized size = 1.31

$$2\sqrt{xe+d}b^3e^{(-4)}\mathrm{sgn}\left(bx+a\right)+\frac{2\left(15\,(xe+d)^2b^3d\mathrm{sgn}\left(bx+a\right)-5\,(xe+d)b^3d^2\mathrm{sgn}\left(bx+a\right)+b^3d^3\mathrm{sgn}\left(bx+a\right)-15\,(xe+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x*e + d)*b^3*e^(-4)*sgn(b*x + a) + 2/5*(15*(x*e + d)^2*b^3*d*sgn(b*x
 + a) - 5*(x*e + d)*b^3*d^2*sgn(b*x + a) + b^3*d^3*sgn(b*x + a) - 15*(x*e +
 d)^2*a*b^2*e*sgn(b*x + a) + 10*(x*e + d)*a*b^2*d*e*sgn(b*x + a) - 3*a*b^2*
d^2*e*sgn(b*x + a) - 5*(x*e + d)*a^2*b*e^2*sgn(b*x + a) + 3*a^2*b*d*e^2*sgn
(b*x + a) - a^3*e^3*sgn(b*x + a))*e^(-4)/(x*e + d)^(5/2)
```

3.1690 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{(d+ex)^{9/2}}\,dx$

Optimal. Leaf size=204

$$-\frac{2b^3\sqrt{a^2+2abx+b^2x^2}}{e^4(a+bx)\sqrt{d+ex}}+\frac{2b^2\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{e^4(a+bx)(d+ex)^{3/2}}-\frac{6b\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{5e^4(a+bx)(d+ex)^{5/2}}+\frac{2\sqrt{a^2+2abx+b^2x^2}(b}{7e^4(a+bx)(d+e}$$

```
[Out] (2*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*e^4*(a + b*x)*(d + e*x)^
(7/2)) - (6*b*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^4*(a + b*x)
*(d + e*x)^(5/2)) + (2*b^2*(b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^4*
(a + b*x)*(d + e*x)^(3/2)) - (2*b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^4*(a
+ b*x)*Sqrt[d + e*x])
```

Rubi [A] time = 0.0634645, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {646, 43}

$$-\frac{2b^3\sqrt{a^2+2abx+b^2x^2}}{e^4(a+bx)\sqrt{d+ex}}+\frac{2b^2\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{e^4(a+bx)(d+ex)^{3/2}}-\frac{6b\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{5e^4(a+bx)(d+ex)^{5/2}}+\frac{2\sqrt{a^2+2abx+b^2x^2}(b}{7e^4(a+bx)(d+e}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^(9/2),x]
```

```
[Out] (2*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*e^4*(a + b*x)*(d + e*x)^
(7/2)) - (6*b*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^4*(a + b*x)
*(d + e*x)^(5/2)) + (2*b^2*(b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^4*
(a + b*x)*(d + e*x)^(3/2)) - (2*b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^4*(a
+ b*x)*Sqrt[d + e*x])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{(d+ex)^{9/2}}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^3}{(d+ex)^{9/2}}\,dx}{b^2\left(ab+b^2x\right)}$$
$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^3(bd-ae)^3}{e^3(d+ex)^{9/2}}+\frac{3b^4(bd-ae)^2}{e^3(d+ex)^{7/2}}-\frac{3b^5(bd-ae)}{e^3(d+ex)^{5/2}}+\frac{b^6}{e^3(d+ex)^{3/2}}\right)dx}{b^2\left(ab+b^2x\right)}$$
$$= \frac{2(bd-ae)^3\sqrt{a^2+2abx+b^2x^2}}{7e^4(a+bx)(d+ex)^{7/2}} - \frac{6b(bd-ae)^2\sqrt{a^2+2abx+b^2x^2}}{5e^4(a+bx)(d+ex)^{5/2}} + \frac{2b^2(bd-ae)\sqrt{a^2+2al}}{e^4(a+bx)(d+ex}$$

Mathematica [A] time = 0.0632568, size = 119, normalized size = 0.58

$$-\frac{2\sqrt{(a+bx)^2}\left(3a^2be^2(2d+7ex)+5a^3e^3+ab^2e\left(8d^2+28dex+35e^2x^2\right)+b^3\left(56d^2ex+16d^3+70de^2x^2+35e^3x^3\right)\right)}{35e^4(a+bx)(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^(9/2),x]
```

```
[Out] (-2*Sqrt[(a + b*x)^2]*(5*a^3*e^3 + 3*a^2*b*e^2*(2*d + 7*e*x) + a*b^2*e*(8*d
^2 + 28*d*e*x + 35*e^2*x^2) + b^3*(16*d^3 + 56*d^2*e*x + 70*d*e^2*x^2 + 35*
e^3*x^3)))/(35*e^4*(a + b*x)*(d + e*x)^(7/2))
```

Maple [A] time = 0.184, size = 132, normalized size = 0.7

$$-\frac{70x^3b^3e^3+70x^2ab^2e^3+140x^2b^3de^2+42xa^2be^3+56xab^2de^2+112xb^3d^2e+10a^3e^3+12de^2a^2b+16ab^2d^2e+32b^3d}{35\,(bx+a)^3\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(9/2),x)
```

```
[Out] -2/35/(e*x+d)^(7/2)*(35*b^3*e^3*x^3+35*a*b^2*e^3*x^2+70*b^3*d*e^2*x^2+21*a^
2*b*e^3*x+28*a*b^2*d*e^2*x+56*b^3*d^2*e*x+5*a^3*e^3+6*a^2*b*d*e^2+8*a*b^2*d
^2*e+16*b^3*d^3)*((b*x+a)^2)^(3/2)/e^4/(b*x+a)^3
```

Maxima [A] time = 1.26868, size = 198, normalized size = 0.97

$$-\frac{2\left(35\,b^3e^3x^3+16\,b^3d^3+8\,ab^2d^2e+6\,a^2bde^2+5\,a^3e^3+35\left(2\,b^3de^2+ab^2e^3\right)x^2+7\left(8\,b^3d^2e+4\,ab^2de^2+3\,a^2be^3\right)x\right)}{35\left(e^7x^3+3\,de^6x^2+3\,d^2e^5x+d^3e^4\right)\sqrt{ex+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(9/2),x, algorithm="maxima")
```

```
[Out] -2/35*(35*b^3*e^3*x^3 + 16*b^3*d^3 + 8*a*b^2*d^2*e + 6*a^2*b*d*e^2 + 5*a^3*
e^3 + 35*(2*b^3*d*e^2 + a*b^2*e^3)*x^2 + 7*(8*b^3*d^2*e + 4*a*b^2*d*e^2 + 3
*a^2*b*e^3)*x)/((e^7*x^3 + 3*d*e^6*x^2 + 3*d^2*e^5*x + d^3*e^4)*sqrt(e*x +
d))
```

Fricas [A] time = 1.54083, size = 329, normalized size = 1.61

$$-\frac{2\left(35\,b^3e^3x^3+16\,b^3d^3+8\,ab^2d^2e+6\,a^2bde^2+5\,a^3e^3+35\left(2\,b^3de^2+ab^2e^3\right)x^2+7\left(8\,b^3d^2e+4\,ab^2de^2+3\,a^2be^3\right)x\right)\sqrt{}}{35\left(e^8x^4+4\,de^7x^3+6\,d^2e^6x^2+4\,d^3e^5x+d^4e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(9/2),x, algorithm="fricas")
```

```
[Out] -2/35*(35*b^3*e^3*x^3 + 16*b^3*d^3 + 8*a*b^2*d^2*e + 6*a^2*b*d*e^2 + 5*a^3*
e^3 + 35*(2*b^3*d*e^2 + a*b^2*e^3)*x^2 + 7*(8*b^3*d^2*e + 4*a*b^2*d*e^2 + 3
*a^2*b*e^3)*x)*sqrt(e*x + d)/(e^8*x^4 + 4*d*e^7*x^3 + 6*d^2*e^6*x^2 + 4*d^3
*e^5*x + d^4*e^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/(e*x+d)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22615, size = 262, normalized size = 1.28

$$-\frac{2\left(35\,(xe+d)^3b^3\mathrm{sgn}\,(bx+a)-35\,(xe+d)^2b^3d\mathrm{sgn}\,(bx+a)+21\,(xe+d)b^3d^2\mathrm{sgn}\,(bx+a)-5\,b^3d^3\mathrm{sgn}\,(bx+a)+35\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(9/2),x, algorithm="giac")
```

```
[Out] -2/35*(35*(x*e + d)^3*b^3*sgn(b*x + a) - 35*(x*e + d)^2*b^3*d*sgn(b*x + a)
+ 21*(x*e + d)*b^3*d^2*sgn(b*x + a) - 5*b^3*d^3*sgn(b*x + a) + 35*(x*e + d)
^2*a*b^2*e*sgn(b*x + a) - 42*(x*e + d)*a*b^2*d*e*sgn(b*x + a) + 15*a*b^2*d^
2*e*sgn(b*x + a) + 21*(x*e + d)*a^2*b*e^2*sgn(b*x + a) - 15*a^2*b*d*e^2*sgn
(b*x + a) + 5*a^3*e^3*sgn(b*x + a))*e^(-4)/(x*e + d)^(7/2)
```

3.1691 $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^{11/2}}\,dx$

Optimal. Leaf size=208

$$-\frac{2b^3\sqrt{a^2+2abx+b^2x^2}}{3e^4(a+bx)(d+ex)^{3/2}}+\frac{6b^2\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{5e^4(a+bx)(d+ex)^{5/2}}-\frac{6b\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{7e^4(a+bx)(d+ex)^{7/2}}+\frac{2\sqrt{a^2+2abx+b^2x^2}(bd-}{9e^4(a+bx)(d+ex)^{9/}}$$

```
[Out] (2*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(9*e^4*(a + b*x)*(d + e*x)^
(9/2)) - (6*b*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*e^4*(a + b*x)
*(d + e*x)^(7/2)) + (6*b^2*(b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^
4*(a + b*x)*(d + e*x)^(5/2)) - (2*b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^4
*(a + b*x)*(d + e*x)^(3/2))
```

Rubi [A] time = 0.0659555, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {646, 43}

$$-\frac{2b^3\sqrt{a^2+2abx+b^2x^2}}{3e^4(a+bx)(d+ex)^{3/2}}+\frac{6b^2\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{5e^4(a+bx)(d+ex)^{5/2}}-\frac{6b\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{7e^4(a+bx)(d+ex)^{7/2}}+\frac{2\sqrt{a^2+2abx+b^2x^2}(bd-}{9e^4(a+bx)(d+ex)^{9/}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^(11/2),x]
```

```
[Out] (2*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(9*e^4*(a + b*x)*(d + e*x)^
(9/2)) - (6*b*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*e^4*(a + b*x)
*(d + e*x)^(7/2)) + (6*b^2*(b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^
4*(a + b*x)*(d + e*x)^(5/2)) - (2*b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^4
*(a + b*x)*(d + e*x)^(3/2))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{(d+ex)^{11/2}}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^3}{(d+ex)^{11/2}}dx}{b^2\left(ab+b^2x\right)}$$

$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^3(bd-ae)^3}{e^3(d+ex)^{11/2}}+\frac{3b^4(bd-ae)^2}{e^3(d+ex)^{9/2}}-\frac{3b^5(bd-ae)}{e^3(d+ex)^{7/2}}+\frac{b^6}{e^3(d+ex)^{5/2}}\right)dx}{b^2\left(ab+b^2x\right)}$$

$$= \frac{2(bd-ae)^3\sqrt{a^2+2abx+b^2x^2}}{9e^4(a+bx)(d+ex)^{9/2}} - \frac{6b(bd-ae)^2\sqrt{a^2+2abx+b^2x^2}}{7e^4(a+bx)(d+ex)^{7/2}} + \frac{6b^2(bd-ae)\sqrt{a^2+}}{5e^4(a+bx)(d}$$

Mathematica [A] time = 0.0649713, size = 120, normalized size = 0.58

$$-\frac{2\sqrt{(a+bx)^2}\left(15a^2be^2(2d+9ex)+35a^3e^3+3ab^2e\left(8d^2+36dex+63e^2x^2\right)+b^3\left(72d^2ex+16d^3+126de^2x^2+105e^3x\right.\right.}{315e^4(a+bx)(d+ex)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^(11/2),x]
```

```
[Out] (-2*Sqrt[(a + b*x)^2]*(35*a^3*e^3 + 15*a^2*b*e^2*(2*d + 9*e*x) + 3*a*b^2*e*
(8*d^2 + 36*d*e*x + 63*e^2*x^2) + b^3*(16*d^3 + 72*d^2*e*x + 126*d*e^2*x^2
+ 105*e^3*x^3)))/(315*e^4*(a + b*x)*(d + e*x)^(9/2))
```

Maple [A] time = 0.153, size = 132, normalized size = 0.6

$$-\frac{210\,x^3b^3e^3+378\,x^2ab^2e^3+252\,x^2b^3de^2+270\,xa^2be^3+216\,xab^2de^2+144\,xb^3d^2e+70\,a^3e^3+60\,de^2a^2b+48\,ab^2d^2e}{315\,(bx+a)^3\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(11/2),x)
```

```
[Out] -2/315/(e*x+d)^(9/2)*(105*b^3*e^3*x^3+189*a*b^2*e^3*x^2+126*b^3*d*e^2*x^2+1
35*a^2*b*e^3*x+108*a*b^2*d*e^2*x+72*b^3*d^2*e*x+35*a^3*e^3+30*a^2*b*d*e^2+2
4*a*b^2*d^2*e+16*b^3*d^3)*((b*x+a)^2)^(3/2)/e^4/(b*x+a)^3
```

Maxima [A] time = 1.254, size = 215, normalized size = 1.03

$$-\frac{2\left(105\,b^3e^3x^3+16\,b^3d^3+24\,ab^2d^2e+30\,a^2bde^2+35\,a^3e^3+63\left(2\,b^3de^2+3\,ab^2e^3\right)x^2+9\left(8\,b^3d^2e+12\,ab^2de^2+15\,a^2\right.\right.}{315\left(e^8x^4+4\,de^7x^3+6\,d^2e^6x^2+4\,d^3e^5x+d^4e^4\right)\sqrt{ex+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(11/2),x, algorithm="maxima")
```

```
[Out] -2/315*(105*b^3*e^3*x^3 + 16*b^3*d^3 + 24*a*b^2*d^2*e + 30*a^2*b*d*e^2 + 35
*a^3*e^3 + 63*(2*b^3*d*e^2 + 3*a*b^2*e^3)*x^2 + 9*(8*b^3*d^2*e + 12*a*b^2*d
*e^2 + 15*a^2*b*e^3)*x)/((e^8*x^4 + 4*d*e^7*x^3 + 6*d^2*e^6*x^2 + 4*d^3*e^5
*x + d^4*e^4)*sqrt(e*x + d))
```

Fricas [A] time = 1.56084, size = 366, normalized size = 1.76

$$-\frac{2\left(105\,b^3e^3x^3+16\,b^3d^3+24\,ab^2d^2e+30\,a^2bde^2+35\,a^3e^3+63\left(2\,b^3de^2+3\,ab^2e^3\right)x^2+9\left(8\,b^3d^2e+12\,ab^2de^2+15\,a^2be^{3}\right.\right.}{315\left(e^9x^5+5\,de^8x^4+10\,d^2e^7x^3+10\,d^3e^6x^2+5\,d^4e^5x+d^5e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(11/2),x, algorithm="fricas")
```

```
[Out] -2/315*(105*b^3*e^3*x^3 + 16*b^3*d^3 + 24*a*b^2*d^2*e + 30*a^2*b*d*e^2 + 35
*a^3*e^3 + 63*(2*b^3*d*e^2 + 3*a*b^2*e^3)*x^2 + 9*(8*b^3*d^2*e + 12*a*b^2*d
*e^2 + 15*a^2*b*e^3)*x)*sqrt(e*x + d)/(e^9*x^5 + 5*d*e^8*x^4 + 10*d^2*e^7*x
^3 + 10*d^3*e^6*x^2 + 5*d^4*e^5*x + d^5*e^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/(e*x+d)**(11/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17838, size = 262, normalized size = 1.26

$$-\frac{2\left(105\,(xe+d)^3b^3\mathrm{sgn}\,(bx+a)-189\,(xe+d)^2b^3d\mathrm{sgn}\,(bx+a)+135\,(xe+d)b^3d^2\mathrm{sgn}\,(bx+a)-35\,b^3d^3\mathrm{sgn}\,(bx+a)+1\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(11/2),x, algorithm="giac")
```

```
[Out] -2/315*(105*(x*e + d)^3*b^3*sgn(b*x + a) - 189*(x*e + d)^2*b^3*d*sgn(b*x +
a) + 135*(x*e + d)*b^3*d^2*sgn(b*x + a) - 35*b^3*d^3*sgn(b*x + a) + 189*(x*
e + d)^2*a*b^2*e*sgn(b*x + a) - 270*(x*e + d)*a*b^2*d*e*sgn(b*x + a) + 105*
a*b^2*d^2*e*sgn(b*x + a) + 135*(x*e + d)*a^2*b*e^2*sgn(b*x + a) - 105*a^2*b
*d*e^2*sgn(b*x + a) + 35*a^3*e^3*sgn(b*x + a))*e^(-4)/(x*e + d)^(9/2)
```

3.1692 $\int (d+ex)^{5/2} \left(a^2+2abx+b^2x^2\right)^{5/2} dx$

Optimal. Leaf size=320

$$\frac{2b^5\sqrt{a^2+2abx+b^2x^2}(d+ex)^{17/2}}{17e^6(a+bx)}-\frac{2b^4\sqrt{a^2+2abx+b^2x^2}(d+ex)^{15/2}(bd-ae)}{3e^6(a+bx)}+\frac{20b^3\sqrt{a^2+2abx+b^2x^2}(d+ex)^{13/2}}{13e^6(a+bx)}$$

```
[Out] (-2*(b*d - a*e)^5*(d + e*x)^(7/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*e^6*(a
+ b*x)) + (10*b*(b*d - a*e)^4*(d + e*x)^(9/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]
)/(9*e^6*(a + b*x)) - (20*b^2*(b*d - a*e)^3*(d + e*x)^(11/2)*Sqrt[a^2 + 2*a
*b*x + b^2*x^2])/(11*e^6*(a + b*x)) + (20*b^3*(b*d - a*e)^2*(d + e*x)^(13/2
)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(13*e^6*(a + b*x)) - (2*b^4*(b*d - a*e)*(d
 + e*x)^(15/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^6*(a + b*x)) + (2*b^5*(d
 + e*x)^(17/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(17*e^6*(a + b*x))
```

Rubi [A] time = 0.116144, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {646, 43}

$$\frac{2b^5\sqrt{a^2+2abx+b^2x^2}(d+ex)^{17/2}}{17e^6(a+bx)}-\frac{2b^4\sqrt{a^2+2abx+b^2x^2}(d+ex)^{15/2}(bd-ae)}{3e^6(a+bx)}+\frac{20b^3\sqrt{a^2+2abx+b^2x^2}(d+ex)^{13/2}}{13e^6(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (-2*(b*d - a*e)^5*(d + e*x)^(7/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*e^6*(a
+ b*x)) + (10*b*(b*d - a*e)^4*(d + e*x)^(9/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]
)/(9*e^6*(a + b*x)) - (20*b^2*(b*d - a*e)^3*(d + e*x)^(11/2)*Sqrt[a^2 + 2*a
*b*x + b^2*x^2])/(11*e^6*(a + b*x)) + (20*b^3*(b*d - a*e)^2*(d + e*x)^(13/2
)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(13*e^6*(a + b*x)) - (2*b^4*(b*d - a*e)*(d
 + e*x)^(15/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^6*(a + b*x)) + (2*b^5*(d
 + e*x)^(17/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(17*e^6*(a + b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (d+ex)^{5/2}\left(a^2+2abx+b^2x^2\right)^{5/2}dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(ab+b^2x\right)^5 (d+ex)^{5/2}dx}{b^4\left(ab+b^2x\right)}$$

$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^5(bd-ae)^5(d+ex)^{5/2}}{e^5}+\frac{5b^6(bd-ae)^4(d+ex)^{7/2}}{e^5}-\frac{10b^7(bd-ae)^3(d+ex)^{9/2}}{e^5}\right.}{b^4\left(ab+b^2x\right)}$$

$$= -\frac{2(bd-ae)^5(d+ex)^{7/2}\sqrt{a^2+2abx+b^2x^2}}{7e^6(a+bx)}+\frac{10b(bd-ae)^4(d+ex)^{9/2}\sqrt{a^2+2ab}}{9e^6(a+bx)}$$

Mathematica [A] time = 0.182802, size = 141, normalized size = 0.44

$$\frac{2\left((a+bx)^2\right)^{5/2}(d+ex)^{7/2}\left(-139230b^2(d+ex)^2(bd-ae)^3+117810b^3(d+ex)^3(bd-ae)^2-51051b^4(d+ex)^4(bd-ae)+8\right.}{153153e^6(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (2*((a + b*x)^2)^(5/2)*(d + e*x)^(7/2)*(-21879*(b*d - a*e)^5 + 85085*b*(b*d
 - a*e)^4*(d + e*x) - 139230*b^2*(b*d - a*e)^3*(d + e*x)^2 + 117810*b^3*(b*
d - a*e)^2*(d + e*x)^3 - 51051*b^4*(b*d - a*e)*(d + e*x)^4 + 9009*b^5*(d +
e*x)^5))/(153153*e^6*(a + b*x)^5)
```

Maple [A] time = 0.155, size = 289, normalized size = 0.9

$$\frac{18018x^5b^5e^5+102102x^4ab^4e^5-12012x^4b^5de^4+235620x^3a^2b^3e^5-62832x^3ab^4de^4+7392x^3b^5d^2e^3+278460x^2a^3b^2e^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] 2/153153*(e*x+d)^(7/2)*(9009*b^5*e^5*x^5+51051*a*b^4*e^5*x^4-6006*b^5*d*e^4
*x^4+117810*a^2*b^3*e^5*x^3-31416*a*b^4*d*e^4*x^3+3696*b^5*d^2*e^3*x^3+1392
30*a^3*b^2*e^5*x^2-64260*a^2*b^3*d*e^4*x^2+17136*a*b^4*d^2*e^3*x^2-2016*b^5
*d^3*e^2*x^2+85085*a^4*b*e^5*x-61880*a^3*b^2*d*e^4*x+28560*a^2*b^3*d^2*e^3*
x-7616*a*b^4*d^3*e^2*x+896*b^5*d^4*e*x+21879*a^5*e^5-24310*a^4*b*d*e^4+1768
0*a^3*b^2*d^2*e^3-8160*a^2*b^3*d^3*e^2+2176*a*b^4*d^4*e-256*b^5*d^5)*((b*x+
a)^2)^(5/2)/e^6/(b*x+a)^5
```

Maxima [B] time = 1.1395, size = 671, normalized size = 2.1

$$\frac{2\left(9009b^5e^8x^8-256b^5d^8+2176ab^4d^7e-8160a^2b^3d^6e^2+17680a^3b^2d^5e^3-24310a^4bd^4e^4+21879a^5d^3e^5+3003\left(7b^5\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/153153*(9009*b^5*e^8*x^8 - 256*b^5*d^8 + 2176*a*b^4*d^7*e - 8160*a^2*b^3*
d^6*e^2 + 17680*a^3*b^2*d^5*e^3 - 24310*a^4*b*d^4*e^4 + 21879*a^5*d^3*e^5 +
```

```
 3003*(7*b^5*d*e^7 + 17*a*b^4*e^8)*x^7 + 231*(55*b^5*d^2*e^6 + 527*a*b^4*d*
e^7 + 510*a^2*b^3*e^8)*x^6 + 63*(b^5*d^3*e^5 + 1207*a*b^4*d^2*e^6 + 4590*a^
2*b^3*d*e^7 + 2210*a^3*b^2*e^8)*x^5 - 35*(2*b^5*d^4*e^4 - 17*a*b^4*d^3*e^5
- 5406*a^2*b^3*d^2*e^6 - 10166*a^3*b^2*d*e^7 - 2431*a^4*b*e^8)*x^4 + (80*b^
5*d^5*e^3 - 680*a*b^4*d^4*e^4 + 2550*a^2*b^3*d^3*e^5 + 249730*a^3*b^2*d^2*e
^6 + 230945*a^4*b*d*e^7 + 21879*a^5*e^8)*x^3 - 3*(32*b^5*d^6*e^2 - 272*a*b^
4*d^5*e^3 + 1020*a^2*b^3*d^4*e^4 - 2210*a^3*b^2*d^3*e^5 - 60775*a^4*b*d^2*e
^6 - 21879*a^5*d*e^7)*x^2 + (128*b^5*d^7*e - 1088*a*b^4*d^6*e^2 + 4080*a^2*
b^3*d^5*e^3 - 8840*a^3*b^2*d^4*e^4 + 12155*a^4*b*d^3*e^5 + 65637*a^5*d^2*e^
6)*x)*sqrt(e*x + d)/e^6
```

Fricas [B] time = 1.63599, size = 1148, normalized size = 3.59

$$\frac{2\left(9009\,b^5e^8x^8 - 256\,b^5d^8 + 2176\,ab^4d^7e - 8160\,a^2b^3d^6e^2 + 17680\,a^3b^2d^5e^3 - 24310\,a^4bd^4e^4 + 21879\,a^5d^3e^5 + 3003\,(7\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/153153*(9009*b^5*e^8*x^8 - 256*b^5*d^8 + 2176*a*b^4*d^7*e - 8160*a^2*b^3*
d^6*e^2 + 17680*a^3*b^2*d^5*e^3 - 24310*a^4*b*d^4*e^4 + 21879*a^5*d^3*e^5 +
 3003*(7*b^5*d*e^7 + 17*a*b^4*e^8)*x^7 + 231*(55*b^5*d^2*e^6 + 527*a*b^4*d*
e^7 + 510*a^2*b^3*e^8)*x^6 + 63*(b^5*d^3*e^5 + 1207*a*b^4*d^2*e^6 + 4590*a^
2*b^3*d*e^7 + 2210*a^3*b^2*e^8)*x^5 - 35*(2*b^5*d^4*e^4 - 17*a*b^4*d^3*e^5
- 5406*a^2*b^3*d^2*e^6 - 10166*a^3*b^2*d*e^7 - 2431*a^4*b*e^8)*x^4 + (80*b^
5*d^5*e^3 - 680*a*b^4*d^4*e^4 + 2550*a^2*b^3*d^3*e^5 + 249730*a^3*b^2*d^2*e
^6 + 230945*a^4*b*d*e^7 + 21879*a^5*e^8)*x^3 - 3*(32*b^5*d^6*e^2 - 272*a*b^
4*d^5*e^3 + 1020*a^2*b^3*d^4*e^4 - 2210*a^3*b^2*d^3*e^5 - 60775*a^4*b*d^2*e
^6 - 21879*a^5*d*e^7)*x^2 + (128*b^5*d^7*e - 1088*a*b^4*d^6*e^2 + 4080*a^2*
b^3*d^5*e^3 - 8840*a^3*b^2*d^4*e^4 + 12155*a^4*b*d^3*e^5 + 65637*a^5*d^2*e^
6)*x)*sqrt(e*x + d)/e^6
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28014, size = 1702, normalized size = 5.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

[Out] 2/765765*(255255*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^4*b*d^2*e^(-1)*sgn(b*x + a) + 72930*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*a^3*b^2*d^2*e^(-2)*sgn(b*x + a) + 24310*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a^2*b^3*d^2*e^(-3)*sgn(b*x + a) + 1105*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*a*b^4*d^2*e^(-4)*sgn(b*x + a) + 85*(693*(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*b^5*d^2*e^(-5)*sgn(b*x + a) + 255255*(x*e + d)^(3/2)*a^5*d^2*sgn(b*x + a) + 72930*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*a^4*b*d*e^(-1)*sgn(b*x + a) + 48620*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a^3*b^2*d*e^(-2)*sgn(b*x + a) + 4420*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*a^2*b^3*d*e^(-3)*sgn(b*x + a) + 850*(693*(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*a*b^4*d*e^(-4)*sgn(b*x + a) + 34*(3003*(x*e + d)^(15/2) - 20790*(x*e + d)^(13/2)*d + 61425*(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525*(x*e + d)^(7/2)*d^4 - 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*b^5*d*e^(-5)*sgn(b*x + a) + 102102*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^5*d*sgn(b*x + a) + 12155*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a^4*b*e^(-1)*sgn(b*x + a) + 2210*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*a^3*b^2*e^(-2)*sgn(b*x + a) + 850*(693*(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*a^2*b^3*e^(-3)*sgn(b*x + a) + 85*(3003*(x*e + d)^(15/2) - 20790*(x*e + d)^(13/2)*d + 61425*(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525*(x*e + d)^(7/2)*d^4 - 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*a*b^4*e^(-4)*sgn(b*x + a) + 7*(6435*(x*e + d)^(17/2) - 51051*(x*e + d)^(15/2)*d + 176715*(x*e + d)^(13/2)*d^2 - 348075*(x*e + d)^(11/2)*d^3 + 425425*(x*e + d)^(9/2)*d^4 - 328185*(x*e + d)^(7/2)*d^5 + 153153*(x*e + d)^(5/2)*d^6 - 36465*(x*e + d)^(3/2)*d^7)*b^5*e^(-5)*sgn(b*x + a) + 7293*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*a^5*sgn(b*x + a))*e^(-1)

3.1693 $\int (d+ex)^{3/2} \left(a^2+2abx+b^2x^2\right)^{5/2} dx$

Optimal. Leaf size=320

$$\frac{2b^5\sqrt{a^2+2abx+b^2x^2}(d+ex)^{15/2}}{15e^6(a+bx)}-\frac{10b^4\sqrt{a^2+2abx+b^2x^2}(d+ex)^{13/2}(bd-ae)}{13e^6(a+bx)}+\frac{20b^3\sqrt{a^2+2abx+b^2x^2}(d+ex)^{11/}}{11e^6(a+bx)}$$

```
[Out] (-2*(b*d - a*e)^5*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^6*(a
+ b*x)) + (10*b*(b*d - a*e)^4*(d + e*x)^(7/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]
)/(7*e^6*(a + b*x)) - (20*b^2*(b*d - a*e)^3*(d + e*x)^(9/2)*Sqrt[a^2 + 2*a*
b*x + b^2*x^2])/(9*e^6*(a + b*x)) + (20*b^3*(b*d - a*e)^2*(d + e*x)^(11/2)*
Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(11*e^6*(a + b*x)) - (10*b^4*(b*d - a*e)*(d
+ e*x)^(13/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(13*e^6*(a + b*x)) + (2*b^5*(d
 + e*x)^(15/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(15*e^6*(a + b*x))
```

Rubi [A] time = 0.0960453, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {646, 43}

$$\frac{2b^5\sqrt{a^2+2abx+b^2x^2}(d+ex)^{15/2}}{15e^6(a+bx)}-\frac{10b^4\sqrt{a^2+2abx+b^2x^2}(d+ex)^{13/2}(bd-ae)}{13e^6(a+bx)}+\frac{20b^3\sqrt{a^2+2abx+b^2x^2}(d+ex)^{11/}}{11e^6(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (-2*(b*d - a*e)^5*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^6*(a
+ b*x)) + (10*b*(b*d - a*e)^4*(d + e*x)^(7/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]
)/(7*e^6*(a + b*x)) - (20*b^2*(b*d - a*e)^3*(d + e*x)^(9/2)*Sqrt[a^2 + 2*a*
b*x + b^2*x^2])/(9*e^6*(a + b*x)) + (20*b^3*(b*d - a*e)^2*(d + e*x)^(11/2)*
Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(11*e^6*(a + b*x)) - (10*b^4*(b*d - a*e)*(d
+ e*x)^(13/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(13*e^6*(a + b*x)) + (2*b^5*(d
 + e*x)^(15/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(15*e^6*(a + b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (d+ex)^{3/2}\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(ab+b^2x\right)^5 (d+ex)^{3/2}\,dx}{b^4\left(ab+b^2x\right)}$$

$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^5(bd-ae)^5(d+ex)^{3/2}}{e^5}+\frac{5b^6(bd-ae)^4(d+ex)^{5/2}}{e^5}-\frac{10b^7(bd-ae)^3(d+ex)^{7/2}}{e^5}\right.}{b^4\left(ab+b^2x\right)}$$

$$= -\frac{2(bd-ae)^5(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}}{5e^6(a+bx)}+\frac{10b(bd-ae)^4(d+ex)^{7/2}\sqrt{a^2+2ab}}{7e^6(a+bx)}$$

Mathematica [A] time = 0.148892, size = 141, normalized size = 0.44

$$\frac{2\left((a+bx)^2\right)^{5/2}(d+ex)^{5/2}\left(-50050b^2(d+ex)^2(bd-ae)^3+40950b^3(d+ex)^3(bd-ae)^2-17325b^4(d+ex)^4(bd-ae)+321\right.}{45045e^6(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (2*((a + b*x)^2)^(5/2)*(d + e*x)^(5/2)*(-9009*(b*d - a*e)^5 + 32175*b*(b*d
- a*e)^4*(d + e*x) - 50050*b^2*(b*d - a*e)^3*(d + e*x)^2 + 40950*b^3*(b*d -
 a*e)^2*(d + e*x)^3 - 17325*b^4*(b*d - a*e)*(d + e*x)^4 + 3003*b^5*(d + e*x
)^5))/(45045*e^6*(a + b*x)^5)
```

Maple [A] time = 0.17, size = 289, normalized size = 0.9

$$\frac{6006\,x^5b^5e^5+34650\,x^4ab^4e^5-4620\,x^4b^5de^4+81900\,x^3a^2b^3e^5-25200\,x^3ab^4de^4+3360\,x^3b^5d^2e^3+100100\,x^2a^3b^2e^5-5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] 2/45045*(e*x+d)^(5/2)*(3003*b^5*e^5*x^5+17325*a*b^4*e^5*x^4-2310*b^5*d*e^4*
x^4+40950*a^2*b^3*e^5*x^3-12600*a*b^4*d*e^4*x^3+1680*b^5*d^2*e^3*x^3+50050*
a^3*b^2*e^5*x^2-27300*a^2*b^3*d*e^4*x^2+8400*a*b^4*d^2*e^3*x^2-1120*b^5*d^3
*e^2*x^2+32175*a^4*b*e^5*x-28600*a^3*b^2*d*e^4*x+15600*a^2*b^3*d^2*e^3*x-48
00*a*b^4*d^3*e^2*x+640*b^5*d^4*e*x+9009*a^5*e^5-12870*a^4*b*d*e^4+11440*a^3
*b^2*d^2*e^3-6240*a^2*b^3*d^3*e^2+1920*a*b^4*d^4*e-256*b^5*d^5)*((b*x+a)^2)
^(5/2)/e^6/(b*x+a)^5
```

Maxima [A] time = 1.10069, size = 564, normalized size = 1.76

$$\frac{2\left(3003\,b^5e^7x^7-256\,b^5d^7+1920\,ab^4d^6e-6240\,a^2b^3d^5e^2+11440\,a^3b^2d^4e^3-12870\,a^4bd^3e^4+9009\,a^5d^2e^5+231\left(16\,b^5de\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(3003*b^5*e^7*x^7 - 256*b^5*d^7 + 1920*a*b^4*d^6*e - 6240*a^2*b^3*d
^5*e^2 + 11440*a^3*b^2*d^4*e^3 - 12870*a^4*b*d^3*e^4 + 9009*a^5*d^2*e^5 + 2
```

```
31*(16*b^5*d*e^6 + 75*a*b^4*e^7)*x^6 + 63*(b^5*d^2*e^5 + 350*a*b^4*d*e^6 +
650*a^2*b^3*e^7)*x^5 - 35*(2*b^5*d^3*e^4 - 15*a*b^4*d^2*e^5 - 1560*a^2*b^3*
d*e^6 - 1430*a^3*b^2*e^7)*x^4 + 5*(16*b^5*d^4*e^3 - 120*a*b^4*d^3*e^4 + 390
*a^2*b^3*d^2*e^5 + 14300*a^3*b^2*d*e^6 + 6435*a^4*b*e^7)*x^3 - 3*(32*b^5*d^
5*e^2 - 240*a*b^4*d^4*e^3 + 780*a^2*b^3*d^3*e^4 - 1430*a^3*b^2*d^2*e^5 - 17
160*a^4*b*d*e^6 - 3003*a^5*e^7)*x^2 + (128*b^5*d^6*e - 960*a*b^4*d^5*e^2 +
3120*a^2*b^3*d^4*e^3 - 5720*a^3*b^2*d^3*e^4 + 6435*a^4*b*d^2*e^5 + 18018*a^
5*d*e^6)*x)*sqrt(e*x + d)/e^6
```

Fricas [A] time = 1.59294, size = 953, normalized size = 2.98

$$2\left(3003\, b^5e^7x^7 - 256\, b^5d^7 + 1920\, ab^4d^6e - 6240\, a^2b^3d^5e^2 + 11440\, a^3b^2d^4e^3 - 12870\, a^4bd^3e^4 + 9009\, a^5d^2e^5 + 231\left(16\, b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3003*b^5*e^7*x^7 - 256*b^5*d^7 + 1920*a*b^4*d^6*e - 6240*a^2*b^3*d
^5*e^2 + 11440*a^3*b^2*d^4*e^3 - 12870*a^4*b*d^3*e^4 + 9009*a^5*d^2*e^5 + 2
31*(16*b^5*d*e^6 + 75*a*b^4*e^7)*x^6 + 63*(b^5*d^2*e^5 + 350*a*b^4*d*e^6 +
650*a^2*b^3*e^7)*x^5 - 35*(2*b^5*d^3*e^4 - 15*a*b^4*d^2*e^5 - 1560*a^2*b^3*
d*e^6 - 1430*a^3*b^2*e^7)*x^4 + 5*(16*b^5*d^4*e^3 - 120*a*b^4*d^3*e^4 + 390
*a^2*b^3*d^2*e^5 + 14300*a^3*b^2*d*e^6 + 6435*a^4*b*e^7)*x^3 - 3*(32*b^5*d^
5*e^2 - 240*a*b^4*d^4*e^3 + 780*a^2*b^3*d^3*e^4 - 1430*a^3*b^2*d^2*e^5 - 17
160*a^4*b*d*e^6 - 3003*a^5*e^7)*x^2 + (128*b^5*d^6*e - 960*a*b^4*d^5*e^2 +
3120*a^2*b^3*d^4*e^3 - 5720*a^3*b^2*d^3*e^4 + 6435*a^4*b*d^2*e^5 + 18018*a^
5*d*e^6)*x)*sqrt(e*x + d)/e^6
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21951, size = 1017, normalized size = 3.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 2/45045*(15015*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^4*b*d*e^(-1)*sgn
(b*x + a) + 4290*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^
(3/2)*d^2)*a^3*b^2*d*e^(-2)*sgn(b*x + a) + 1430*(35*(x*e + d)^(9/2) - 135*(
x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a^2*b
^3*d*e^(-3)*sgn(b*x + a) + 65*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*
```

```
d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3
/2)*d^4)*a*b^4*d*e^(-4)*sgn(b*x + a) + 5*(693*(x*e + d)^(13/2) - 4095*(x*e
+ d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 + 900
9*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*b^5*d*e^(-5)*sgn(b*x + a)
 + 15015*(x*e + d)^(3/2)*a^5*d*sgn(b*x + a) + 2145*(15*(x*e + d)^(7/2) - 42
*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*a^4*b*e^(-1)*sgn(b*x + a) + 14
30*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 -
105*(x*e + d)^(3/2)*d^3)*a^3*b^2*e^(-2)*sgn(b*x + a) + 130*(315*(x*e + d)^(
11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^
(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*a^2*b^3*e^(-3)*sgn(b*x + a) + 25*(693
*(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 1
2870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*
d^5)*a*b^4*e^(-4)*sgn(b*x + a) + (3003*(x*e + d)^(15/2) - 20790*(x*e + d)^(
13/2)*d + 61425*(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525*(
x*e + d)^(7/2)*d^4 - 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)
*b^5*e^(-5)*sgn(b*x + a) + 3003*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a
^5*sgn(b*x + a))*e^(-1)
```

3.1694 $\int \sqrt{d+ex}\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=318

$$\frac{2b^5\sqrt{a^2+2abx+b^2x^2}(d+ex)^{13/2}}{13e^6(a+bx)}-\frac{10b^4\sqrt{a^2+2abx+b^2x^2}(d+ex)^{11/2}(bd-ae)}{11e^6(a+bx)}+\frac{20b^3\sqrt{a^2+2abx+b^2x^2}(d+ex)^{9/2}}{9e^6(a+bx)}$$

```
[Out] (-2*(b*d - a*e)^5*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^6*(a
+ b*x)) + (2*b*(b*d - a*e)^4*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
/(e^6*(a + b*x)) - (20*b^2*(b*d - a*e)^3*(d + e*x)^(7/2)*Sqrt[a^2 + 2*a*b*x
 + b^2*x^2])/(7*e^6*(a + b*x)) + (20*b^3*(b*d - a*e)^2*(d + e*x)^(9/2)*Sqrt
[a^2 + 2*a*b*x + b^2*x^2])/(9*e^6*(a + b*x)) - (10*b^4*(b*d - a*e)*(d + e*x
)^(11/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(11*e^6*(a + b*x)) + (2*b^5*(d + e*
x)^(13/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(13*e^6*(a + b*x))
```

Rubi [A] time = 0.0943637, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {646, 43}

$$\frac{2b^5\sqrt{a^2+2abx+b^2x^2}(d+ex)^{13/2}}{13e^6(a+bx)}-\frac{10b^4\sqrt{a^2+2abx+b^2x^2}(d+ex)^{11/2}(bd-ae)}{11e^6(a+bx)}+\frac{20b^3\sqrt{a^2+2abx+b^2x^2}(d+ex)^{9/2}}{9e^6(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```


```
[Out] (-2*(b*d - a*e)^5*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^6*(a
+ b*x)) + (2*b*(b*d - a*e)^4*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
/(e^6*(a + b*x)) - (20*b^2*(b*d - a*e)^3*(d + e*x)^(7/2)*Sqrt[a^2 + 2*a*b*x
 + b^2*x^2])/(7*e^6*(a + b*x)) + (20*b^3*(b*d - a*e)^2*(d + e*x)^(9/2)*Sqrt
[a^2 + 2*a*b*x + b^2*x^2])/(9*e^6*(a + b*x)) - (10*b^4*(b*d - a*e)*(d + e*x
)^(11/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(11*e^6*(a + b*x)) + (2*b^5*(d + e*
x)^(13/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(13*e^6*(a + b*x))
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \sqrt{d+ex}\left(a^2+2abx+b^2x^2\right)^{5/2} dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(ab+b^2x\right)^5 \sqrt{d+ex}\,dx}{b^4\left(ab+b^2x\right)}$$

$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^5(bd-ae)^5\sqrt{d+ex}}{e^5}+\frac{5b^6(bd-ae)^4(d+ex)^{3/2}}{e^5}-\frac{10b^7(bd-ae)^3(d+ex)^{5/2}}{e^5}+\ldots\right.}{b^4\left(ab+b^2x\right)}$$

$$= -\frac{2(bd-ae)^5(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}}{3e^6(a+bx)}+\frac{2b(bd-ae)^4(d+ex)^{5/2}\sqrt{a^2+2abx+\ldots}}{e^6(a+bx)}$$

Mathematica [A] time = 0.123743, size = 235, normalized size = 0.74

$$\frac{2\sqrt{(a+bx)^2}(d+ex)^{3/2}\left(286a^2b^3e^2\left(24d^2ex-16d^3-30de^2x^2+35e^3x^3\right)+858a^3b^2e^3\left(8d^2-12dex+15e^2x^2\right)+3003a^4be^4\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (2*Sqrt[(a + b*x)^2]*(d + e*x)^(3/2)*(3003*a^5*e^5 + 3003*a^4*b*e^4*(-2*d +
 3*e*x) + 858*a^3*b^2*e^3*(8*d^2 - 12*d*e*x + 15*e^2*x^2) + 286*a^2*b^3*e^2
*(-16*d^3 + 24*d^2*e*x - 30*d*e^2*x^2 + 35*e^3*x^3) + 13*a*b^4*e*(128*d^4 -
 192*d^3*e*x + 240*d^2*e^2*x^2 - 280*d*e^3*x^3 + 315*e^4*x^4) + b^5*(-256*d
^5 + 384*d^4*e*x - 480*d^3*e^2*x^2 + 560*d^2*e^3*x^3 - 630*d*e^4*x^4 + 693*
e^5*x^5)))/(9009*e^6*(a + b*x))
```

Maple [A] time = 0.155, size = 289, normalized size = 0.9

$$\frac{1386x^5b^5e^5+8190x^4ab^4e^5-1260x^4b^5de^4+20020x^3a^2b^3e^5-7280x^3ab^4de^4+1120x^3b^5d^2e^3+25740x^2a^3b^2e^5-17160\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)*(e*x+d)^(1/2),x)
```

```
[Out] 2/9009*(e*x+d)^(3/2)*(693*b^5*e^5*x^5+4095*a*b^4*e^5*x^4-630*b^5*d*e^4*x^4+
10010*a^2*b^3*e^5*x^3-3640*a*b^4*d*e^4*x^3+560*b^5*d^2*e^3*x^3+12870*a^3*b^
2*e^5*x^2-8580*a^2*b^3*d*e^4*x^2+3120*a*b^4*d^2*e^3*x^2-480*b^5*d^3*e^2*x^2
+9009*a^4*b*e^5*x-10296*a^3*b^2*d*e^4*x+6864*a^2*b^3*d^2*e^3*x-2496*a*b^4*d
^3*e^2*x+384*b^5*d^4*e*x+3003*a^5*e^5-6006*a^4*b*d*e^4+6864*a^3*b^2*d^2*e^3
-4576*a^2*b^3*d^3*e^2+1664*a*b^4*d^4*e-256*b^5*d^5)*((b*x+a)^2)^(5/2)/e^6/(
b*x+a)^5
```

Maxima [A] time = 1.12311, size = 456, normalized size = 1.43

$$\frac{2\left(693b^5e^6x^6-256b^5d^6+1664ab^4d^5e-4576a^2b^3d^4e^2+6864a^3b^2d^3e^3-6006a^4bd^2e^4+3003a^5de^5+63\left(b^5de^5+65a\ldots\right.\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9009*(693*b^5*e^6*x^6 - 256*b^5*d^6 + 1664*a*b^4*d^5*e - 4576*a^2*b^3*d^4
*e^2 + 6864*a^3*b^2*d^3*e^3 - 6006*a^4*b*d^2*e^4 + 3003*a^5*d*e^5 + 63*(b^5
*d*e^5 + 65*a*b^4*e^6)*x^5 - 35*(2*b^5*d^2*e^4 - 13*a*b^4*d*e^5 - 286*a^2*b
^3*e^6)*x^4 + 10*(8*b^5*d^3*e^3 - 52*a*b^4*d^2*e^4 + 143*a^2*b^3*d*e^5 + 12
87*a^3*b^2*e^6)*x^3 - 3*(32*b^5*d^4*e^2 - 208*a*b^4*d^3*e^3 + 572*a^2*b^3*d
^2*e^4 - 858*a^3*b^2*d*e^5 - 3003*a^4*b*e^6)*x^2 + (128*b^5*d^5*e - 832*a*b
^4*d^4*e^2 + 2288*a^2*b^3*d^3*e^3 - 3432*a^3*b^2*d^2*e^4 + 3003*a^4*b*d*e^5
 + 3003*a^5*e^6)*x)*sqrt(e*x + d)/e^6
```

Fricas [A] time = 1.6331, size = 761, normalized size = 2.39

$$\frac{2\left(693\, b^5 e^6 x^6 - 256\, b^5 d^6 + 1664\, a b^4 d^5 e - 4576\, a^2 b^3 d^4 e^2 + 6864\, a^3 b^2 d^3 e^3 - 6006\, a^4 b d^2 e^4 + 3003\, a^5 d e^5 + 63\left(b^5 d e^5 + 6\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```


```
[Out] 2/9009*(693*b^5*e^6*x^6 - 256*b^5*d^6 + 1664*a*b^4*d^5*e - 4576*a^2*b^3*d^4
*e^2 + 6864*a^3*b^2*d^3*e^3 - 6006*a^4*b*d^2*e^4 + 3003*a^5*d*e^5 + 63*(b^5
*d*e^5 + 65*a*b^4*e^6)*x^5 - 35*(2*b^5*d^2*e^4 - 13*a*b^4*d*e^5 - 286*a^2*b
^3*e^6)*x^4 + 10*(8*b^5*d^3*e^3 - 52*a*b^4*d^2*e^4 + 143*a^2*b^3*d*e^5 + 12
87*a^3*b^2*e^6)*x^3 - 3*(32*b^5*d^4*e^2 - 208*a*b^4*d^3*e^3 + 572*a^2*b^3*d
^2*e^4 - 858*a^3*b^2*d*e^5 - 3003*a^4*b*e^6)*x^2 + (128*b^5*d^5*e - 832*a*b
^4*d^4*e^2 + 2288*a^2*b^3*d^3*e^3 - 3432*a^3*b^2*d^2*e^4 + 3003*a^4*b*d*e^5
 + 3003*a^5*e^6)*x)*sqrt(e*x + d)/e^6
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d + ex}\left((a + bx)^2\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)*(e*x+d)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*((a + b*x)**2)**(5/2), x)
```

Giac [A] time = 1.19161, size = 454, normalized size = 1.43

$$\frac{2}{9009}\left(3003\left(3\,(xe+d)^{\frac{5}{2}} - 5\,(xe+d)^{\frac{3}{2}}d\right)a^4 b e^{(-1)}\mathrm{sgn}\,(bx+a) + 858\left(15\,(xe+d)^{\frac{7}{2}} - 42\,(xe+d)^{\frac{5}{2}}d + 35\,(xe+d)^{\frac{3}{2}}d^2\right)a^3 b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 2/9009*(3003*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^4*b*e^(-1)*sgn(b*x
 + a) + 858*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)
*d^2)*a^3*b^2*e^(-2)*sgn(b*x + a) + 286*(35*(x*e + d)^(9/2) - 135*(x*e + d)
^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a^2*b^3*e^(-3
)*sgn(b*x + a) + 13*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(
```

```
x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*a
*b^4*e^(-4)*sgn(b*x + a) + (693*(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d
+ 10010*(x*e + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5
/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*b^5*e^(-5)*sgn(b*x + a) + 3003*(x*e + d
)^(3/2)*a^5*sgn(b*x + a))*e^(-1)
```

3.1695 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=316

$$\frac{2b^5\sqrt{a^2+2abx+b^2x^2}(d+ex)^{11/2}}{11e^6(a+bx)}-\frac{10b^4\sqrt{a^2+2abx+b^2x^2}(d+ex)^{9/2}(bd-ae)}{9e^6(a+bx)}+\frac{20b^3\sqrt{a^2+2abx+b^2x^2}(d+ex)^{7/2}(}{7e^6(a+bx)}$$

```
[Out] (-2*(b*d - a*e)^5*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*
x)) + (10*b*(b*d - a*e)^4*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3
*e^6*(a + b*x)) - (4*b^2*(b*d - a*e)^3*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x +
 b^2*x^2])/(e^6*(a + b*x)) + (20*b^3*(b*d - a*e)^2*(d + e*x)^(7/2)*Sqrt[a^2
 + 2*a*b*x + b^2*x^2])/(7*e^6*(a + b*x)) - (10*b^4*(b*d - a*e)*(d + e*x)^(9
/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(9*e^6*(a + b*x)) + (2*b^5*(d + e*x)^(11
/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(11*e^6*(a + b*x))
```

Rubi [A] time = 0.0966293, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {646, 43}

$$\frac{2b^5\sqrt{a^2+2abx+b^2x^2}(d+ex)^{11/2}}{11e^6(a+bx)}-\frac{10b^4\sqrt{a^2+2abx+b^2x^2}(d+ex)^{9/2}(bd-ae)}{9e^6(a+bx)}+\frac{20b^3\sqrt{a^2+2abx+b^2x^2}(d+ex)^{7/2}(}{7e^6(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/Sqrt[d + e*x],x]
```



```
[Out] (-2*(b*d - a*e)^5*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*
x)) + (10*b*(b*d - a*e)^4*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3
*e^6*(a + b*x)) - (4*b^2*(b*d - a*e)^3*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x +
 b^2*x^2])/(e^6*(a + b*x)) + (20*b^3*(b*d - a*e)^2*(d + e*x)^(7/2)*Sqrt[a^2
 + 2*a*b*x + b^2*x^2])/(7*e^6*(a + b*x)) - (10*b^4*(b*d - a*e)*(d + e*x)^(9
/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(9*e^6*(a + b*x)) + (2*b^5*(d + e*x)^(11
/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(11*e^6*(a + b*x))
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{\sqrt{d+ex}}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^5}{\sqrt{d+ex}}\,dx}{b^4\left(ab+b^2x\right)}$$

$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^5(bd-ae)^5}{e^5\sqrt{d+ex}}+\frac{5b^6(bd-ae)^4\sqrt{d+ex}}{e^5}-\frac{10b^7(bd-ae)^3(d+ex)^{3/2}}{e^5}+\frac{10b^8(bd-ae)^2(d+ex)^{5/2}}{e^5}\right.}{b^4\left(ab+b^2x\right)}$$

$$= -\frac{2(bd-ae)^5\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}}{e^6(a+bx)}+\frac{10b(bd-ae)^4(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}}{3e^6(a+bx)}-$$

Mathematica [A] time = 0.12402, size = 234, normalized size = 0.74

$$\frac{2\sqrt{(a+bx)^2}\sqrt{d+ex}\left(198a^2b^3e^2\left(8d^2ex-16d^3-6de^2x^2+5e^3x^3\right)+462a^3b^2e^3\left(8d^2-4dex+3e^2x^2\right)+1155a^4be^4(ex-2d)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[(a + b*x)^2]*Sqrt[d + e*x]*(693*a^5*e^5 + 1155*a^4*b*e^4*(-2*d + e*
x) + 462*a^3*b^2*e^3*(8*d^2 - 4*d*e*x + 3*e^2*x^2) + 198*a^2*b^3*e^2*(-16*d
^3 + 8*d^2*e*x - 6*d*e^2*x^2 + 5*e^3*x^3) + 11*a*b^4*e*(128*d^4 - 64*d^3*e*
x + 48*d^2*e^2*x^2 - 40*d*e^3*x^3 + 35*e^4*x^4) + b^5*(-256*d^5 + 128*d^4*e
*x - 96*d^3*e^2*x^2 + 80*d^2*e^3*x^3 - 70*d*e^4*x^4 + 63*e^5*x^5)))/(693*e^
6*(a + b*x))
```

Maple [A] time = 0.155, size = 289, normalized size = 0.9

$$\frac{126x^5b^5e^5+770x^4ab^4e^5-140x^4b^5de^4+1980x^3a^2b^3e^5-880x^3ab^4de^4+160x^3b^5d^2e^3+2772x^2a^3b^2e^5-2376x^2a^2b^3de}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(1/2),x)
```

```
[Out] 2/693*(e*x+d)^(1/2)*(63*b^5*e^5*x^5+385*a*b^4*e^5*x^4-70*b^5*d*e^4*x^4+990*
a^2*b^3*e^5*x^3-440*a*b^4*d*e^4*x^3+80*b^5*d^2*e^3*x^3+1386*a^3*b^2*e^5*x^2
-1188*a^2*b^3*d*e^4*x^2+528*a*b^4*d^2*e^3*x^2-96*b^5*d^3*e^2*x^2+1155*a^4*b
*e^5*x-1848*a^3*b^2*d*e^4*x+1584*a^2*b^3*d^2*e^3*x-704*a*b^4*d^3*e^2*x+128*
b^5*d^4*e*x+693*a^5*e^5-2310*a^4*b*d*e^4+3696*a^3*b^2*d^2*e^3-3168*a^2*b^3*
d^3*e^2+1408*a*b^4*d^4*e-256*b^5*d^5)*((b*x+a)^2)^(5/2)/e^6/(b*x+a)^5
```

Maxima [A] time = 1.09507, size = 456, normalized size = 1.44

$$\frac{2\left(63b^5e^6x^6-256b^5d^6+1408ab^4d^5e-3168a^2b^3d^4e^2+3696a^3b^2d^3e^3-2310a^4bd^2e^4+693a^5de^5-7\left(b^5de^5-55ab^4e^6\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/693*(63*b^5*e^6*x^6 - 256*b^5*d^6 + 1408*a*b^4*d^5*e - 3168*a^2*b^3*d^4*e
^2 + 3696*a^3*b^2*d^3*e^3 - 2310*a^4*b*d^2*e^4 + 693*a^5*d*e^5 - 7*(b^5*d*e
^5 - 55*a*b^4*e^6)*x^5 + 5*(2*b^5*d^2*e^4 - 11*a*b^4*d*e^5 + 198*a^2*b^3*e^
6)*x^4 - 2*(8*b^5*d^3*e^3 - 44*a*b^4*d^2*e^4 + 99*a^2*b^3*d*e^5 - 693*a^3*b
^2*e^6)*x^3 + (32*b^5*d^4*e^2 - 176*a*b^4*d^3*e^3 + 396*a^2*b^3*d^2*e^4 - 4
62*a^3*b^2*d*e^5 + 1155*a^4*b*e^6)*x^2 - (128*b^5*d^5*e - 704*a*b^4*d^4*e^2
 + 1584*a^2*b^3*d^3*e^3 - 1848*a^3*b^2*d^2*e^4 + 1155*a^4*b*d*e^5 - 693*a^5
*e^6)*x)/(sqrt(e*x + d)*e^6)
```

Fricas [A] time = 1.60847, size = 586, normalized size = 1.85

$$2\left(63\,b^5e^5x^5 - 256\,b^5d^5 + 1408\,ab^4d^4e - 3168\,a^2b^3d^3e^2 + 3696\,a^3b^2d^2e^3 - 2310\,a^4bde^4 + 693\,a^5e^5 - 35\left(2\,b^5de^4 - 11\,ab\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*b^5*e^5*x^5 - 256*b^5*d^5 + 1408*a*b^4*d^4*e - 3168*a^2*b^3*d^3*e
^2 + 3696*a^3*b^2*d^2*e^3 - 2310*a^4*b*d*e^4 + 693*a^5*e^5 - 35*(2*b^5*d*e^
4 - 11*a*b^4*e^5)*x^4 + 10*(8*b^5*d^2*e^3 - 44*a*b^4*d*e^4 + 99*a^2*b^3*e^5
)*x^3 - 6*(16*b^5*d^3*e^2 - 88*a*b^4*d^2*e^3 + 198*a^2*b^3*d*e^4 - 231*a^3*
b^2*e^5)*x^2 + (128*b^5*d^4*e - 704*a*b^4*d^3*e^2 + 1584*a^2*b^3*d^2*e^3 -
1848*a^3*b^2*d*e^4 + 1155*a^4*b*e^5)*x)*sqrt(e*x + d)/e^6
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18321, size = 451, normalized size = 1.43

$$\frac{2}{693}\left(1155\left((xe+d)^{\frac{3}{2}} - 3\sqrt{xe+d}\,d\right)a^4be^{(-1)}\mathrm{sgn}\left(bx+a\right) + 462\left(3\,(xe+d)^{\frac{5}{2}} - 10\,(xe+d)^{\frac{3}{2}}d + 15\sqrt{xe+d}\,d^2\right)a^3b^2e^{(-2)}\mathrm{s}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 2/693*(1155*((x*e + d)^(3/2) - 3*sqrt(x*e + d)*d)*a^4*b*e^(-1)*sgn(b*x + a)
 + 462*(3*(x*e + d)^(5/2) - 10*(x*e + d)^(3/2)*d + 15*sqrt(x*e + d)*d^2)*a^
3*b^2*e^(-2)*sgn(b*x + a) + 198*(5*(x*e + d)^(7/2) - 21*(x*e + d)^(5/2)*d +
 35*(x*e + d)^(3/2)*d^2 - 35*sqrt(x*e + d)*d^3)*a^2*b^3*e^(-3)*sgn(b*x + a)
 + 11*(35*(x*e + d)^(9/2) - 180*(x*e + d)^(7/2)*d + 378*(x*e + d)^(5/2)*d^2
 - 420*(x*e + d)^(3/2)*d^3 + 315*sqrt(x*e + d)*d^4)*a*b^4*e^(-4)*sgn(b*x +
a) + (63*(x*e + d)^(11/2) - 385*(x*e + d)^(9/2)*d + 990*(x*e + d)^(7/2)*d^2
 - 1386*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4 - 693*sqrt(x*e + d)*
d^5)*b^5*e^(-5)*sgn(b*x + a) + 693*sqrt(x*e + d)*a^5*sgn(b*x + a))*e^(-1)
```

3.1696 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=314

$$\frac{2b^5\sqrt{a^2+2abx+b^2x^2}(d+ex)^{9/2}}{9e^6(a+bx)}-\frac{10b^4\sqrt{a^2+2abx+b^2x^2}(d+ex)^{7/2}(bd-ae)}{7e^6(a+bx)}+\frac{4b^3\sqrt{a^2+2abx+b^2x^2}(d+ex)^{5/2}(bd-a}{e^6(a+bx)}$$

```
[Out] (2*(b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*x)*Sqrt[d + e*x
]) + (10*b*(b*d - a*e)^4*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*
(a + b*x)) - (20*b^2*(b*d - a*e)^3*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2
*x^2])/(3*e^6*(a + b*x)) + (4*b^3*(b*d - a*e)^2*(d + e*x)^(5/2)*Sqrt[a^2 +
2*a*b*x + b^2*x^2])/(e^6*(a + b*x)) - (10*b^4*(b*d - a*e)*(d + e*x)^(7/2)*S
qrt[a^2 + 2*a*b*x + b^2*x^2])/(7*e^6*(a + b*x)) + (2*b^5*(d + e*x)^(9/2)*Sq
rt[a^2 + 2*a*b*x + b^2*x^2])/(9*e^6*(a + b*x))
```

Rubi [A] time = 0.0950377, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {646, 43}

$$\frac{2b^5\sqrt{a^2+2abx+b^2x^2}(d+ex)^{9/2}}{9e^6(a+bx)}-\frac{10b^4\sqrt{a^2+2abx+b^2x^2}(d+ex)^{7/2}(bd-ae)}{7e^6(a+bx)}+\frac{4b^3\sqrt{a^2+2abx+b^2x^2}(d+ex)^{5/2}(bd-a}{e^6(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^(3/2),x]
```


```
[Out] (2*(b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*x)*Sqrt[d + e*x
]) + (10*b*(b*d - a*e)^4*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*
(a + b*x)) - (20*b^2*(b*d - a*e)^3*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2
*x^2])/(3*e^6*(a + b*x)) + (4*b^3*(b*d - a*e)^2*(d + e*x)^(5/2)*Sqrt[a^2 +
2*a*b*x + b^2*x^2])/(e^6*(a + b*x)) - (10*b^4*(b*d - a*e)*(d + e*x)^(7/2)*S
qrt[a^2 + 2*a*b*x + b^2*x^2])/(7*e^6*(a + b*x)) + (2*b^5*(d + e*x)^(9/2)*Sq
rt[a^2 + 2*a*b*x + b^2*x^2])/(9*e^6*(a + b*x))
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{(d+ex)^{3/2}}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^5}{(d+ex)^{3/2}}\,dx}{b^4\left(ab+b^2x\right)}$$
$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^5(bd-ae)^5}{e^5(d+ex)^{3/2}}+\frac{5b^6(bd-ae)^4}{e^5\sqrt{d+ex}}-\frac{10b^7(bd-ae)^3\sqrt{d+ex}}{e^5}+\frac{10b^8(bd-ae)^2(d+ex)^{3/2}}{e^5}-\right.}{b^4\left(ab+b^2x\right)}$$
$$= \frac{2(bd-ae)^5\sqrt{a^2+2abx+b^2x^2}}{e^6(a+bx)\sqrt{d+ex}}+\frac{10b(bd-ae)^4\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}}{e^6(a+bx)}-\frac{20b^2(bd-}{}$$

Mathematica [A] time = 0.133589, size = 232, normalized size = 0.74

$$\frac{2\sqrt{(a+bx)^2}\left(126a^2b^3e^2\left(8d^2ex+16d^3-2de^2x^2+e^3x^3\right)+210a^3b^2e^3\left(-8d^2-4dex+e^2x^2\right)+315a^4be^4(2d+ex)-63a}{63e^6(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^(3/2),x]
```

```
[Out] (2*Sqrt[(a + b*x)^2]*(-63*a^5*e^5 + 315*a^4*b*e^4*(2*d + e*x) + 210*a^3*b^2
*e^3*(-8*d^2 - 4*d*e*x + e^2*x^2) + 126*a^2*b^3*e^2*(16*d^3 + 8*d^2*e*x - 2
*d*e^2*x^2 + e^3*x^3) + 9*a*b^4*e*(-128*d^4 - 64*d^3*e*x + 16*d^2*e^2*x^2 -
 8*d*e^3*x^3 + 5*e^4*x^4) + b^5*(256*d^5 + 128*d^4*e*x - 32*d^3*e^2*x^2 + 1
6*d^2*e^3*x^3 - 10*d*e^4*x^4 + 7*e^5*x^5)))/(63*e^6*(a + b*x)*Sqrt[d + e*x]
)
```

Maple [A] time = 0.157, size = 289, normalized size = 0.9

$$-\frac{-14\,x^5b^5e^5-90\,x^4ab^4e^5+20\,x^4b^5de^4-252\,x^3a^2b^3e^5+144\,x^3ab^4de^4-32\,x^3b^5d^2e^3-420\,x^2a^3b^2e^5+504\,x^2a^2b^3de^4-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(3/2),x)
```

```
[Out] -2/63/(e*x+d)^(1/2)*(-7*b^5*e^5*x^5-45*a*b^4*e^5*x^4+10*b^5*d*e^4*x^4-126*a
^2*b^3*e^5*x^3+72*a*b^4*d*e^4*x^3-16*b^5*d^2*e^3*x^3-210*a^3*b^2*e^5*x^2+25
2*a^2*b^3*d*e^4*x^2-144*a*b^4*d^2*e^3*x^2+32*b^5*d^3*e^2*x^2-315*a^4*b*e^5*
x+840*a^3*b^2*d*e^4*x-1008*a^2*b^3*d^2*e^3*x+576*a*b^4*d^3*e^2*x-128*b^5*d^
4*e*x+63*a^5*e^5-630*a^4*b*d*e^4+1680*a^3*b^2*d^2*e^3-2016*a^2*b^3*d^3*e^2+
1152*a*b^4*d^4*e-256*b^5*d^5)*((b*x+a)^2)^(5/2)/e^6/(b*x+a)^5
```

Maxima [A] time = 1.06593, size = 352, normalized size = 1.12

$$\frac{2\left(7\,b^5e^5x^5+256\,b^5d^5-1152\,ab^4d^4e+2016\,a^2b^3d^3e^2-1680\,a^3b^2d^2e^3+630\,a^4bde^4-63\,a^5e^5-5\left(2\,b^5de^4-9\,ab^4e^5\right)x^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/63*(7*b^5*e^5*x^5 + 256*b^5*d^5 - 1152*a*b^4*d^4*e + 2016*a^2*b^3*d^3*e^2
 - 1680*a^3*b^2*d^2*e^3 + 630*a^4*b*d*e^4 - 63*a^5*e^5 - 5*(2*b^5*d*e^4 - 9
*a*b^4*e^5)*x^4 + 2*(8*b^5*d^2*e^3 - 36*a*b^4*d*e^4 + 63*a^2*b^3*e^5)*x^3 -
 2*(16*b^5*d^3*e^2 - 72*a*b^4*d^2*e^3 + 126*a^2*b^3*d*e^4 - 105*a^3*b^2*e^5
)*x^2 + (128*b^5*d^4*e - 576*a*b^4*d^3*e^2 + 1008*a^2*b^3*d^2*e^3 - 840*a^3
*b^2*d*e^4 + 315*a^4*b*e^5)*x)/(sqrt(e*x + d)*e^6)
```

Fricas [A] time = 1.63708, size = 590, normalized size = 1.88

$$\frac{2\left(7\,b^5e^5x^5+256\,b^5d^5-1152\,ab^4d^4e+2016\,a^2b^3d^3e^2-1680\,a^3b^2d^2e^3+630\,a^4bde^4-63\,a^5e^5-5\left(2\,b^5de^4-9\,ab^4e^5\right)x^4+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/63*(7*b^5*e^5*x^5 + 256*b^5*d^5 - 1152*a*b^4*d^4*e + 2016*a^2*b^3*d^3*e^2
 - 1680*a^3*b^2*d^2*e^3 + 630*a^4*b*d*e^4 - 63*a^5*e^5 - 5*(2*b^5*d*e^4 - 9
*a*b^4*e^5)*x^4 + 2*(8*b^5*d^2*e^3 - 36*a*b^4*d*e^4 + 63*a^2*b^3*e^5)*x^3 -
 2*(16*b^5*d^3*e^2 - 72*a*b^4*d^2*e^3 + 126*a^2*b^3*d*e^4 - 105*a^3*b^2*e^5
)*x^2 + (128*b^5*d^4*e - 576*a*b^4*d^3*e^2 + 1008*a^2*b^3*d^2*e^3 - 840*a^3
*b^2*d*e^4 + 315*a^4*b*e^5)*x)*sqrt(e*x + d)/(e^7*x + d*e^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24668, size = 637, normalized size = 2.03

$$\frac{2}{63}\left(7\,(xe+d)^{\frac{9}{2}}b^5e^{48}\mathrm{sgn}\,(bx+a)-45\,(xe+d)^{\frac{7}{2}}b^5de^{48}\mathrm{sgn}\,(bx+a)+126\,(xe+d)^{\frac{5}{2}}b^5d^2e^{48}\mathrm{sgn}\,(bx+a)-210\,(xe+d)^{\frac{3}{2}}b^5d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] 2/63*(7*(x*e + d)^(9/2)*b^5*e^48*sgn(b*x + a) - 45*(x*e + d)^(7/2)*b^5*d*e^
48*sgn(b*x + a) + 126*(x*e + d)^(5/2)*b^5*d^2*e^48*sgn(b*x + a) - 210*(x*e
+ d)^(3/2)*b^5*d^3*e^48*sgn(b*x + a) + 315*sqrt(x*e + d)*b^5*d^4*e^48*sgn(b
*x + a) + 45*(x*e + d)^(7/2)*a*b^4*e^49*sgn(b*x + a) - 252*(x*e + d)^(5/2)*
a*b^4*d*e^49*sgn(b*x + a) + 630*(x*e + d)^(3/2)*a*b^4*d^2*e^49*sgn(b*x + a)
 - 1260*sqrt(x*e + d)*a*b^4*d^3*e^49*sgn(b*x + a) + 126*(x*e + d)^(5/2)*a^2
*b^3*e^50*sgn(b*x + a) - 630*(x*e + d)^(3/2)*a^2*b^3*d*e^50*sgn(b*x + a) +
1890*sqrt(x*e + d)*a^2*b^3*d^2*e^50*sgn(b*x + a) + 210*(x*e + d)^(3/2)*a^3*
b^2*e^51*sgn(b*x + a) - 1260*sqrt(x*e + d)*a^3*b^2*d*e^51*sgn(b*x + a) + 31
```

```
5*sqrt(x*e + d)*a^4*b*e^52*sgn(b*x + a))*e^(-54) + 2*(b^5*d^5*sgn(b*x + a)
- 5*a*b^4*d^4*e*sgn(b*x + a) + 10*a^2*b^3*d^3*e^2*sgn(b*x + a) - 10*a^3*b^2
*d^2*e^3*sgn(b*x + a) + 5*a^4*b*d*e^4*sgn(b*x + a) - a^5*e^5*sgn(b*x + a))*
e^(-6)/sqrt(x*e + d)
```

3.1697 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=314

$$\frac{2b^5\sqrt{a^2+2abx+b^2x^2}(d+ex)^{7/2}}{7e^6(a+bx)}-\frac{2b^4\sqrt{a^2+2abx+b^2x^2}(d+ex)^{5/2}(bd-ae)}{e^6(a+bx)}+\frac{20b^3\sqrt{a^2+2abx+b^2x^2}(d+ex)^{3/2}(bd-$$

```
[Out] (2*(b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^6*(a + b*x)*(d + e*x)^
(3/2)) - (10*b*(b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*x)*
Sqrt[d + e*x]) - (20*b^2*(b*d - a*e)^3*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b
^2*x^2])/(e^6*(a + b*x)) + (20*b^3*(b*d - a*e)^2*(d + e*x)^(3/2)*Sqrt[a^2 +
 2*a*b*x + b^2*x^2])/(3*e^6*(a + b*x)) - (2*b^4*(b*d - a*e)*(d + e*x)^(5/2)
*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*x)) + (2*b^5*(d + e*x)^(7/2)*Sq
rt[a^2 + 2*a*b*x + b^2*x^2])/(7*e^6*(a + b*x))
```

Rubi [A] time = 0.0969155, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {646, 43}

$$\frac{2b^5\sqrt{a^2+2abx+b^2x^2}(d+ex)^{7/2}}{7e^6(a+bx)}-\frac{2b^4\sqrt{a^2+2abx+b^2x^2}(d+ex)^{5/2}(bd-ae)}{e^6(a+bx)}+\frac{20b^3\sqrt{a^2+2abx+b^2x^2}(d+ex)^{3/2}(bd-$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^(5/2),x]
```

```
[Out] (2*(b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^6*(a + b*x)*(d + e*x)^
(3/2)) - (10*b*(b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*x)*
Sqrt[d + e*x]) - (20*b^2*(b*d - a*e)^3*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b
^2*x^2])/(e^6*(a + b*x)) + (20*b^3*(b*d - a*e)^2*(d + e*x)^(3/2)*Sqrt[a^2 +
 2*a*b*x + b^2*x^2])/(3*e^6*(a + b*x)) - (2*b^4*(b*d - a*e)*(d + e*x)^(5/2)
*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*x)) + (2*b^5*(d + e*x)^(7/2)*Sq
rt[a^2 + 2*a*b*x + b^2*x^2])/(7*e^6*(a + b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^5}{(d+ex)^{5/2}}\,dx}{b^4\left(ab+b^2x\right)}$$

$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^5(bd-ae)^5}{e^5(d+ex)^{5/2}}+\frac{5b^6(bd-ae)^4}{e^5(d+ex)^{3/2}}-\frac{10b^7(bd-ae)^3}{e^5\sqrt{d+ex}}+\frac{10b^8(bd-ae)^2\sqrt{d+ex}}{e^5}-\frac{5b^9(bd-a}{}\right.}{b^4\left(ab+b^2x\right)}$$

$$= \frac{2(bd-ae)^5\sqrt{a^2+2abx+b^2x^2}}{3e^6(a+bx)(d+ex)^{3/2}} - \frac{10b(bd-ae)^4\sqrt{a^2+2abx+b^2x^2}}{e^6(a+bx)\sqrt{d+ex}} - \frac{20b^2(bd-ae)^3\sqrt{d}}{e^6}$$

Mathematica [A] time = 0.116292, size = 235, normalized size = 0.75

$$-\frac{2\sqrt{(a+bx)^2}\left(70a^2b^3e^2\left(24d^2ex+16d^3+6de^2x^2-e^3x^3\right)-70a^3b^2e^3\left(8d^2+12dex+3e^2x^2\right)+35a^4be^4(2d+3ex)+7a}{21e^6(a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[(a + b*x)^2]*(7*a^5*e^5 + 35*a^4*b*e^4*(2*d + 3*e*x) - 70*a^3*b^2*
e^3*(8*d^2 + 12*d*e*x + 3*e^2*x^2) + 70*a^2*b^3*e^2*(16*d^3 + 24*d^2*e*x +
6*d*e^2*x^2 - e^3*x^3) - 7*a*b^4*e*(128*d^4 + 192*d^3*e*x + 48*d^2*e^2*x^2
- 8*d*e^3*x^3 + 3*e^4*x^4) + b^5*(256*d^5 + 384*d^4*e*x + 96*d^3*e^2*x^2 -
16*d^2*e^3*x^3 + 6*d*e^4*x^4 - 3*e^5*x^5)))/(21*e^6*(a + b*x)*(d + e*x)^(3/
2))
```

Maple [A] time = 0.153, size = 289, normalized size = 0.9

$$-\frac{-6x^5b^5e^5-42x^4ab^4e^5+12x^4b^5de^4-140x^3a^2b^3e^5+112x^3ab^4de^4-32x^3b^5d^2e^3-420x^2a^3b^2e^5+840x^2a^2b^3de^4-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(5/2),x)
```

```
[Out] -2/21/(e*x+d)^(3/2)*(-3*b^5*e^5*x^5-21*a*b^4*e^5*x^4+6*b^5*d*e^4*x^4-70*a^2
*b^3*e^5*x^3+56*a*b^4*d*e^4*x^3-16*b^5*d^2*e^3*x^3-210*a^3*b^2*e^5*x^2+420*
a^2*b^3*d*e^4*x^2-336*a*b^4*d^2*e^3*x^2+96*b^5*d^3*e^2*x^2+105*a^4*b*e^5*x-
840*a^3*b^2*d*e^4*x+1680*a^2*b^3*d^2*e^3*x-1344*a*b^4*d^3*e^2*x+384*b^5*d^4
*e*x+7*a^5*e^5+70*a^4*b*d*e^4-560*a^3*b^2*d^2*e^3+1120*a^2*b^3*d^3*e^2-896*
a*b^4*d^4*e+256*b^5*d^5)*((b*x+a)^2)^(5/2)/e^6/(b*x+a)^5
```

Maxima [A] time = 1.13879, size = 367, normalized size = 1.17

$$\frac{2\left(3b^5e^5x^5-256b^5d^5+896ab^4d^4e-1120a^2b^3d^3e^2+560a^3b^2d^2e^3-70a^4bde^4-7a^5e^5-3\left(2b^5de^4-7ab^4e^5\right)x^4+2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/21*(3*b^5*e^5*x^5 - 256*b^5*d^5 + 896*a*b^4*d^4*e - 1120*a^2*b^3*d^3*e^2
+ 560*a^3*b^2*d^2*e^3 - 70*a^4*b*d*e^4 - 7*a^5*e^5 - 3*(2*b^5*d*e^4 - 7*a*b
^4*e^5)*x^4 + 2*(8*b^5*d^2*e^3 - 28*a*b^4*d*e^4 + 35*a^2*b^3*e^5)*x^3 - 6*(
16*b^5*d^3*e^2 - 56*a*b^4*d^2*e^3 + 70*a^2*b^3*d*e^4 - 35*a^3*b^2*e^5)*x^2
- 3*(128*b^5*d^4*e - 448*a*b^4*d^3*e^2 + 560*a^2*b^3*d^2*e^3 - 280*a^3*b^2*
d*e^4 + 35*a^4*b*e^5)*x)/((e^7*x + d*e^6)*sqrt(e*x + d))
```

Fricas [A] time = 1.51022, size = 603, normalized size = 1.92

$$\frac{2\left(3\,b^5e^5x^5 - 256\,b^5d^5 + 896\,ab^4d^4e - 1120\,a^2b^3d^3e^2 + 560\,a^3b^2d^2e^3 - 70\,a^4bde^4 - 7\,a^5e^5 - 3\left(2\,b^5de^4 - 7\,ab^4e^5\right)x^4 + 2\left(8\,b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*b^5*e^5*x^5 - 256*b^5*d^5 + 896*a*b^4*d^4*e - 1120*a^2*b^3*d^3*e^2
+ 560*a^3*b^2*d^2*e^3 - 70*a^4*b*d*e^4 - 7*a^5*e^5 - 3*(2*b^5*d*e^4 - 7*a*b
^4*e^5)*x^4 + 2*(8*b^5*d^2*e^3 - 28*a*b^4*d*e^4 + 35*a^2*b^3*e^5)*x^3 - 6*(
16*b^5*d^3*e^2 - 56*a*b^4*d^2*e^3 + 70*a^2*b^3*d*e^4 - 35*a^3*b^2*e^5)*x^2
- 3*(128*b^5*d^4*e - 448*a*b^4*d^3*e^2 + 560*a^2*b^3*d^2*e^3 - 280*a^3*b^2*
d*e^4 + 35*a^4*b*e^5)*x)*sqrt(e*x + d)/(e^8*x^2 + 2*d*e^7*x + d^2*e^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21762, size = 621, normalized size = 1.98

$$\frac{2}{21}\left(3\,(xe+d)^{\frac{7}{2}}b^5e^{36}\mathrm{sgn}\,(bx+a) - 21\,(xe+d)^{\frac{5}{2}}b^5de^{36}\mathrm{sgn}\,(bx+a) + 70\,(xe+d)^{\frac{3}{2}}b^5d^2e^{36}\mathrm{sgn}\,(bx+a) - 210\,\sqrt{xe+d}b^5d^3e^{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] 2/21*(3*(x*e + d)^(7/2)*b^5*e^36*sgn(b*x + a) - 21*(x*e + d)^(5/2)*b^5*d*e^
36*sgn(b*x + a) + 70*(x*e + d)^(3/2)*b^5*d^2*e^36*sgn(b*x + a) - 210*sqrt(x
*e + d)*b^5*d^3*e^36*sgn(b*x + a) + 21*(x*e + d)^(5/2)*a*b^4*e^37*sgn(b*x +
 a) - 140*(x*e + d)^(3/2)*a*b^4*d*e^37*sgn(b*x + a) + 630*sqrt(x*e + d)*a*b
^4*d^2*e^37*sgn(b*x + a) + 70*(x*e + d)^(3/2)*a^2*b^3*e^38*sgn(b*x + a) - 6
30*sqrt(x*e + d)*a^2*b^3*d*e^38*sgn(b*x + a) + 210*sqrt(x*e + d)*a^3*b^2*e^
39*sgn(b*x + a))*e^(-42) - 2/3*(15*(x*e + d)*b^5*d^4*sgn(b*x + a) - b^5*d^5
*sgn(b*x + a) - 60*(x*e + d)*a*b^4*d^3*e*sgn(b*x + a) + 5*a*b^4*d^4*e*sgn(b
*x + a) + 90*(x*e + d)*a^2*b^3*d^2*e^2*sgn(b*x + a) - 10*a^2*b^3*d^3*e^2*sg
```

```
n(b*x + a) - 60*(x*e + d)*a^3*b^2*d*e^3*sgn(b*x + a) + 10*a^3*b^2*d^2*e^3*s
gn(b*x + a) + 15*(x*e + d)*a^4*b*e^4*sgn(b*x + a) - 5*a^4*b*d*e^4*sgn(b*x +
 a) + a^5*e^5*sgn(b*x + a))*e^(-6)/(x*e + d)^(3/2)
```

3.1698 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=316

$$\frac{2b^5\sqrt{a^2+2abx+b^2x^2}(d+ex)^{5/2}}{5e^6(a+bx)}-\frac{10b^4\sqrt{a^2+2abx+b^2x^2}(d+ex)^{3/2}(bd-ae)}{3e^6(a+bx)}+\frac{20b^3\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-a\ldots}{e^6(a+bx)}$$

```
[Out] (2*(b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^6*(a + b*x)*(d + e*x)^
(5/2)) - (10*b*(b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^6*(a + b*x
)*(d + e*x)^(3/2)) + (20*b^2*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(
e^6*(a + b*x)*Sqrt[d + e*x]) + (20*b^3*(b*d - a*e)^2*Sqrt[d + e*x]*Sqrt[a^2
 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*x)) - (10*b^4*(b*d - a*e)*(d + e*x)^(3/2
)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^6*(a + b*x)) + (2*b^5*(d + e*x)^(5/2)
*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^6*(a + b*x))
```

Rubi [A] time = 0.0972174, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {646, 43}

$$\frac{2b^5\sqrt{a^2+2abx+b^2x^2}(d+ex)^{5/2}}{5e^6(a+bx)}-\frac{10b^4\sqrt{a^2+2abx+b^2x^2}(d+ex)^{3/2}(bd-ae)}{3e^6(a+bx)}+\frac{20b^3\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-a\ldots}{e^6(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^(7/2),x]
```

```
[Out] (2*(b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^6*(a + b*x)*(d + e*x)^
(5/2)) - (10*b*(b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^6*(a + b*x
)*(d + e*x)^(3/2)) + (20*b^2*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(
e^6*(a + b*x)*Sqrt[d + e*x]) + (20*b^3*(b*d - a*e)^2*Sqrt[d + e*x]*Sqrt[a^2
 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*x)) - (10*b^4*(b*d - a*e)*(d + e*x)^(3/2
)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^6*(a + b*x)) + (2*b^5*(d + e*x)^(5/2)
*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(5*e^6*(a + b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{7/2}}dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(ab+b^2x)^5}{(d+ex)^{7/2}}dx}{b^4(ab+b^2x)}$$

$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^5(bd-ae)^5}{e^5(d+ex)^{7/2}}+\frac{5b^6(bd-ae)^4}{e^5(d+ex)^{5/2}}-\frac{10b^7(bd-ae)^3}{e^5(d+ex)^{3/2}}+\frac{10b^8(bd-ae)^2}{e^5\sqrt{d+ex}}-\frac{5b^9(bd-ae)\sqrt{d+}}{e^5}\right.}{b^4(ab+b^2x)}$$

$$= \frac{2(bd-ae)^5\sqrt{a^2+2abx+b^2x^2}}{5e^6(a+bx)(d+ex)^{5/2}} - \frac{10b(bd-ae)^4\sqrt{a^2+2abx+b^2x^2}}{3e^6(a+bx)(d+ex)^{3/2}} + \frac{20b^2(bd-ae)^3\sqrt{a}}{e^6(a+bx)}$$

Mathematica [A] time = 0.115685, size = 236, normalized size = 0.75

$$-\frac{2\sqrt{(a+bx)^2}\left(-30a^2b^3e^2\left(40d^2ex+16d^3+30de^2x^2+5e^3x^3\right)+10a^3b^2e^3\left(8d^2+20dex+15e^2x^2\right)+5a^4be^4(2d+5ex)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^(7/2),x]
```

```
[Out] (-2*Sqrt[(a + b*x)^2]*(3*a^5*e^5 + 5*a^4*b*e^4*(2*d + 5*e*x) + 10*a^3*b^2*e
^3*(8*d^2 + 20*d*e*x + 15*e^2*x^2) - 30*a^2*b^3*e^2*(16*d^3 + 40*d^2*e*x +
30*d*e^2*x^2 + 5*e^3*x^3) + 5*a*b^4*e*(128*d^4 + 320*d^3*e*x + 240*d^2*e^2*
x^2 + 40*d*e^3*x^3 - 5*e^4*x^4) - b^5*(256*d^5 + 640*d^4*e*x + 480*d^3*e^2*
x^2 + 80*d^2*e^3*x^3 - 10*d*e^4*x^4 + 3*e^5*x^5)))/(15*e^6*(a + b*x)*(d + e
*x)^(5/2))
```

Maple [A] time = 0.154, size = 289, normalized size = 0.9

$$-\frac{-6x^5b^5e^5-50x^4ab^4e^5+20x^4b^5de^4-300x^3a^2b^3e^5+400x^3ab^4de^4-160x^3b^5d^2e^3+300x^2a^3b^2e^5-1800x^2a^2b^3de^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(7/2),x)
```

```
[Out] -2/15/(e*x+d)^(5/2)*(-3*b^5*e^5*x^5-25*a*b^4*e^5*x^4+10*b^5*d*e^4*x^4-150*a
^2*b^3*e^5*x^3+200*a*b^4*d*e^4*x^3-80*b^5*d^2*e^3*x^3+150*a^3*b^2*e^5*x^2-9
00*a^2*b^3*d*e^4*x^2+1200*a*b^4*d^2*e^3*x^2-480*b^5*d^3*e^2*x^2+25*a^4*b*e^
5*x+200*a^3*b^2*d*e^4*x-1200*a^2*b^3*d^2*e^3*x+1600*a*b^4*d^3*e^2*x-640*b^5
*d^4*e*x+3*a^5*e^5+10*a^4*b*d*e^4+80*a^3*b^2*d^2*e^3-480*a^2*b^3*d^3*e^2+64
0*a*b^4*d^4*e-256*b^5*d^5)*((b*x+a)^2)^(5/2)/e^6/(b*x+a)^5
```

Maxima [A] time = 1.09525, size = 382, normalized size = 1.21

$$\frac{2\left(3b^5e^5x^5+256b^5d^5-640ab^4d^4e+480a^2b^3d^3e^2-80a^3b^2d^2e^3-10a^4bde^4-3a^5e^5-5\left(2b^5de^4-5ab^4e^5\right)x^4+10\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*b^5*e^5*x^5 + 256*b^5*d^5 - 640*a*b^4*d^4*e + 480*a^2*b^3*d^3*e^2 -
 80*a^3*b^2*d^2*e^3 - 10*a^4*b*d*e^4 - 3*a^5*e^5 - 5*(2*b^5*d*e^4 - 5*a*b^4
*e^5)*x^4 + 10*(8*b^5*d^2*e^3 - 20*a*b^4*d*e^4 + 15*a^2*b^3*e^5)*x^3 + 30*(
16*b^5*d^3*e^2 - 40*a*b^4*d^2*e^3 + 30*a^2*b^3*d*e^4 - 5*a^3*b^2*e^5)*x^2 +
 5*(128*b^5*d^4*e - 320*a*b^4*d^3*e^2 + 240*a^2*b^3*d^2*e^3 - 40*a^3*b^2*d*
e^4 - 5*a^4*b*e^5)*x)/((e^8*x^2 + 2*d*e^7*x + d^2*e^6)*sqrt(e*x + d))
```

Fricas [A] time = 1.57548, size = 621, normalized size = 1.97

$$\frac{2\left(3\,b^5e^5x^5+256\,b^5d^5-640\,ab^4d^4e+480\,a^2b^3d^3e^2-80\,a^3b^2d^2e^3-10\,a^4bde^4-3\,a^5e^5-5\left(2\,b^5de^4-5\,ab^4e^5\right)x^4+10\left(8\,b^5\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^5*e^5*x^5 + 256*b^5*d^5 - 640*a*b^4*d^4*e + 480*a^2*b^3*d^3*e^2 -
 80*a^3*b^2*d^2*e^3 - 10*a^4*b*d*e^4 - 3*a^5*e^5 - 5*(2*b^5*d*e^4 - 5*a*b^4
*e^5)*x^4 + 10*(8*b^5*d^2*e^3 - 20*a*b^4*d*e^4 + 15*a^2*b^3*e^5)*x^3 + 30*(
16*b^5*d^3*e^2 - 40*a*b^4*d^2*e^3 + 30*a^2*b^3*d*e^4 - 5*a^3*b^2*e^5)*x^2 +
 5*(128*b^5*d^4*e - 320*a*b^4*d^3*e^2 + 240*a^2*b^3*d^2*e^3 - 40*a^3*b^2*d*
e^4 - 5*a^4*b*e^5)*x)*sqrt(e*x + d)/(e^9*x^3 + 3*d*e^8*x^2 + 3*d^2*e^7*x +
d^3*e^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21552, size = 620, normalized size = 1.96

$$\frac{2}{15}\left(3\,(xe+d)^{\frac{5}{2}}b^5e^{24}\mathrm{sgn}\,(bx+a)-25\,(xe+d)^{\frac{3}{2}}b^5de^{24}\mathrm{sgn}\,(bx+a)+150\,\sqrt{xe+d}b^5d^2e^{24}\mathrm{sgn}\,(bx+a)+25\,(xe+d)^{\frac{3}{2}}ab^4e^{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*(x*e + d)^(5/2)*b^5*e^24*sgn(b*x + a) - 25*(x*e + d)^(3/2)*b^5*d*e^
24*sgn(b*x + a) + 150*sqrt(x*e + d)*b^5*d^2*e^24*sgn(b*x + a) + 25*(x*e + d
)^(3/2)*a*b^4*e^25*sgn(b*x + a) - 300*sqrt(x*e + d)*a*b^4*d*e^25*sgn(b*x +
a) + 150*sqrt(x*e + d)*a^2*b^3*e^26*sgn(b*x + a))*e^(-30) + 2/15*(150*(x*e
+ d)^2*b^5*d^3*sgn(b*x + a) - 25*(x*e + d)*b^5*d^4*sgn(b*x + a) + 3*b^5*d^5
*sgn(b*x + a) - 450*(x*e + d)^2*a*b^4*d^2*e*sgn(b*x + a) + 100*(x*e + d)*a*
b^4*d^3*e*sgn(b*x + a) - 15*a*b^4*d^4*e*sgn(b*x + a) + 450*(x*e + d)^2*a^2*
b^3*d*e^2*sgn(b*x + a) - 150*(x*e + d)*a^2*b^3*d^2*e^2*sgn(b*x + a) + 30*a^
```

```
2*b^3*d^3*e^2*sgn(b*x + a) - 150*(x*e + d)^2*a^3*b^2*e^3*sgn(b*x + a) + 100
*(x*e + d)*a^3*b^2*d*e^3*sgn(b*x + a) - 30*a^3*b^2*d^2*e^3*sgn(b*x + a) - 2
5*(x*e + d)*a^4*b*e^4*sgn(b*x + a) + 15*a^4*b*d*e^4*sgn(b*x + a) - 3*a^5*e^
5*sgn(b*x + a))*e^(-6)/(x*e + d)^(5/2)
```

3.1699 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{9/2}}\,dx$

Optimal. Leaf size=314

$$\frac{2b^5\sqrt{a^2+2abx+b^2x^2}(d+ex)^{3/2}}{3e^6(a+bx)}-\frac{10b^4\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-ae)}{e^6(a+bx)}-\frac{20b^3\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{e^6(a+bx)\sqrt{d+ex}}+\frac{20b^2}{}$$

```
[Out] (2*(b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*e^6*(a + b*x)*(d + e*x)^
(7/2)) - (2*b*(b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*x)*(
d + e*x)^(5/2)) + (20*b^2*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e
^6*(a + b*x)*(d + e*x)^(3/2)) - (20*b^3*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x +
b^2*x^2])/(e^6*(a + b*x)*Sqrt[d + e*x]) - (10*b^4*(b*d - a*e)*Sqrt[d + e*x]
*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*x)) + (2*b^5*(d + e*x)^(3/2)*Sq
rt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^6*(a + b*x))
```

Rubi [A] time = 0.0942204, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {646, 43}

$$\frac{2b^5\sqrt{a^2+2abx+b^2x^2}(d+ex)^{3/2}}{3e^6(a+bx)}-\frac{10b^4\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-ae)}{e^6(a+bx)}-\frac{20b^3\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{e^6(a+bx)\sqrt{d+ex}}+\frac{20b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^(9/2),x]
```


```
[Out] (2*(b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*e^6*(a + b*x)*(d + e*x)^
(7/2)) - (2*b*(b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*x)*(
d + e*x)^(5/2)) + (20*b^2*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(3*e
^6*(a + b*x)*(d + e*x)^(3/2)) - (20*b^3*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x +
b^2*x^2])/(e^6*(a + b*x)*Sqrt[d + e*x]) - (10*b^4*(b*d - a*e)*Sqrt[d + e*x]
*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*x)) + (2*b^5*(d + e*x)^(3/2)*Sq
rt[a^2 + 2*a*b*x + b^2*x^2])/(3*e^6*(a + b*x))
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{(d+ex)^{9/2}}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^5}{(d+ex)^{9/2}}\,dx}{b^4\left(ab+b^2x\right)}$$

$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^5(bd-ae)^5}{e^5(d+ex)^{9/2}}+\frac{5b^6(bd-ae)^4}{e^5(d+ex)^{7/2}}-\frac{10b^7(bd-ae)^3}{e^5(d+ex)^{5/2}}+\frac{10b^8(bd-ae)^2}{e^5(d+ex)^{3/2}}-\frac{5b^9(bd-ae)}{e^5\sqrt{d+ex}}+ \cdots}{b^4\left(ab+b^2x\right)}$$

$$= \frac{2(bd-ae)^5\sqrt{a^2+2abx+b^2x^2}}{7e^6(a+bx)(d+ex)^{7/2}} - \frac{2b(bd-ae)^4\sqrt{a^2+2abx+b^2x^2}}{e^6(a+bx)(d+ex)^{5/2}} + \frac{20b^2(bd-ae)^3\sqrt{a^2 \cdots}}{3e^6(a+bx)(\cdots}$$

Mathematica [A] time = 0.115255, size = 235, normalized size = 0.75

$$-\frac{2\sqrt{(a+bx)^2}\left(6a^2b^3e^2\left(56d^2ex+16d^3+70de^2x^2+35e^3x^3\right)+2a^3b^2e^3\left(8d^2+28dex+35e^2x^2\right)+3a^4be^4(2d+7ex)+\cdots\right)}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^(9/2),x]
```

```
[Out] (-2*Sqrt[(a + b*x)^2]*(3*a^5*e^5 + 3*a^4*b*e^4*(2*d + 7*e*x) + 2*a^3*b^2*e^
3*(8*d^2 + 28*d*e*x + 35*e^2*x^2) + 6*a^2*b^3*e^2*(16*d^3 + 56*d^2*e*x + 70
*d*e^2*x^2 + 35*e^3*x^3) - 3*a*b^4*e*(128*d^4 + 448*d^3*e*x + 560*d^2*e^2*x
^2 + 280*d*e^3*x^3 + 35*e^4*x^4) + b^5*(256*d^5 + 896*d^4*e*x + 1120*d^3*e^
2*x^2 + 560*d^2*e^3*x^3 + 70*d*e^4*x^4 - 7*e^5*x^5)))/(21*e^6*(a + b*x)*(d
+ e*x)^(7/2))
```

Maple [A] time = 0.157, size = 289, normalized size = 0.9

$$-\frac{-14x^5b^5e^5-210x^4ab^4e^5+140x^4b^5de^4+420x^3a^2b^3e^5-1680x^3ab^4de^4+1120x^3b^5d^2e^3+140x^2a^3b^2e^5+840x^2a^2b\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(9/2),x)
```

```
[Out] -2/21/(e*x+d)^(7/2)*(-7*b^5*e^5*x^5-105*a*b^4*e^5*x^4+70*b^5*d*e^4*x^4+210*
a^2*b^3*e^5*x^3-840*a*b^4*d*e^4*x^3+560*b^5*d^2*e^3*x^3+70*a^3*b^2*e^5*x^2+
420*a^2*b^3*d*e^4*x^2-1680*a*b^4*d^2*e^3*x^2+1120*b^5*d^3*e^2*x^2+21*a^4*b*
e^5*x+56*a^3*b^2*d*e^4*x+336*a^2*b^3*d^2*e^3*x-1344*a*b^4*d^3*e^2*x+896*b^5
*d^4*e*x+3*a^5*e^5+6*a^4*b*d*e^4+16*a^3*b^2*d^2*e^3+96*a^2*b^3*d^3*e^2-384*
a*b^4*d^4*e+256*b^5*d^5)*((b*x+a)^2)^(5/2)/e^6/(b*x+a)^5
```

Maxima [A] time = 1.16313, size = 396, normalized size = 1.26

$$\frac{2\left(7b^5e^5x^5-256b^5d^5+384ab^4d^4e-96a^2b^3d^3e^2-16a^3b^2d^2e^3-6a^4bde^4-3a^5e^5-35\left(2b^5de^4-3ab^4e^5\right)x^4-70\left(8\cdots\right.\right.}{21\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/21*(7*b^5*e^5*x^5 - 256*b^5*d^5 + 384*a*b^4*d^4*e - 96*a^2*b^3*d^3*e^2 -
16*a^3*b^2*d^2*e^3 - 6*a^4*b*d*e^4 - 3*a^5*e^5 - 35*(2*b^5*d*e^4 - 3*a*b^4*
e^5)*x^4 - 70*(8*b^5*d^2*e^3 - 12*a*b^4*d*e^4 + 3*a^2*b^3*e^5)*x^3 - 70*(16
*b^5*d^3*e^2 - 24*a*b^4*d^2*e^3 + 6*a^2*b^3*d*e^4 + a^3*b^2*e^5)*x^2 - 7*(1
28*b^5*d^4*e - 192*a*b^4*d^3*e^2 + 48*a^2*b^3*d^2*e^3 + 8*a^3*b^2*d*e^4 + 3
*a^4*b*e^5)*x)/((e^9*x^3 + 3*d*e^8*x^2 + 3*d^2*e^7*x + d^3*e^6)*sqrt(e*x +
d))
```

Fricas [A] time = 1.62108, size = 635, normalized size = 2.02

$$\frac{2\left(7\,b^5e^5x^5-256\,b^5d^5+384\,ab^4d^4e-96\,a^2b^3d^3e^2-16\,a^3b^2d^2e^3-6\,a^4bde^4-3\,a^5e^5-35\left(2\,b^5de^4-3\,ab^4e^5\right)x^4-70\left(8\,b^5d\right.\right.}{21\left(e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/21*(7*b^5*e^5*x^5 - 256*b^5*d^5 + 384*a*b^4*d^4*e - 96*a^2*b^3*d^3*e^2 -
16*a^3*b^2*d^2*e^3 - 6*a^4*b*d*e^4 - 3*a^5*e^5 - 35*(2*b^5*d*e^4 - 3*a*b^4*
e^5)*x^4 - 70*(8*b^5*d^2*e^3 - 12*a*b^4*d*e^4 + 3*a^2*b^3*e^5)*x^3 - 70*(16
*b^5*d^3*e^2 - 24*a*b^4*d^2*e^3 + 6*a^2*b^3*d*e^4 + a^3*b^2*e^5)*x^2 - 7*(1
28*b^5*d^4*e - 192*a*b^4*d^3*e^2 + 48*a^2*b^3*d^2*e^3 + 8*a^3*b^2*d*e^4 + 3
*a^4*b*e^5)*x)*sqrt(e*x + d)/(e^10*x^4 + 4*d*e^9*x^3 + 6*d^2*e^8*x^2 + 4*d^
3*e^7*x + d^4*e^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24065, size = 616, normalized size = 1.96

$$\frac{2}{3}\left((xe+d)^{\frac{3}{2}}b^5e^{12}\mathrm{sgn}\left(bx+a\right)-15\,\sqrt{xe+d}b^5de^{12}\mathrm{sgn}\left(bx+a\right)+15\,\sqrt{xe+d}ab^4e^{13}\mathrm{sgn}\left(bx+a\right)\right)e^{(-18)}-\frac{2\left(210\,(xe+d)^3b^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(9/2),x, algorithm="giac")
```

```
[Out] 2/3*((x*e + d)^(3/2)*b^5*e^12*sgn(b*x + a) - 15*sqrt(x*e + d)*b^5*d*e^12*sg
n(b*x + a) + 15*sqrt(x*e + d)*a*b^4*e^13*sgn(b*x + a))*e^(-18) - 2/21*(210*
(x*e + d)^3*b^5*d^2*sgn(b*x + a) - 70*(x*e + d)^2*b^5*d^3*sgn(b*x + a) + 21
*(x*e + d)*b^5*d^4*sgn(b*x + a) - 3*b^5*d^5*sgn(b*x + a) - 420*(x*e + d)^3*
a*b^4*d*e*sgn(b*x + a) + 210*(x*e + d)^2*a*b^4*d^2*e*sgn(b*x + a) - 84*(x*e
 + d)*a*b^4*d^3*e*sgn(b*x + a) + 15*a*b^4*d^4*e*sgn(b*x + a) + 210*(x*e + d
)^3*a^2*b^3*e^2*sgn(b*x + a) - 210*(x*e + d)^2*a^2*b^3*d*e^2*sgn(b*x + a) +
```

```
 126*(x*e + d)*a^2*b^3*d^2*e^2*sgn(b*x + a) - 30*a^2*b^3*d^3*e^2*sgn(b*x +
a) + 70*(x*e + d)^2*a^3*b^2*e^3*sgn(b*x + a) - 84*(x*e + d)*a^3*b^2*d*e^3*s
gn(b*x + a) + 30*a^3*b^2*d^2*e^3*sgn(b*x + a) + 21*(x*e + d)*a^4*b*e^4*sgn(
b*x + a) - 15*a^4*b*d*e^4*sgn(b*x + a) + 3*a^5*e^5*sgn(b*x + a))*e^(-6)/(x*
e + d)^(7/2)
```

3.1700 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{11/2}}\,dx$

Optimal. Leaf size=314

$$\frac{2b^5\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}}{e^6(a+bx)}+\frac{10b^4\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{e^6(a+bx)\sqrt{d+ex}}-\frac{20b^3\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{3e^6(a+bx)(d+ex)^{3/2}}+\frac{4b^2\sqrt{a^2+2abx}}{e^6(a+bx}$$

```
[Out] (2*(b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(9*e^6*(a + b*x)*(d + e*x)^
(9/2)) - (10*b*(b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*e^6*(a + b*x
)*(d + e*x)^(7/2)) + (4*b^2*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e
^6*(a + b*x)*(d + e*x)^(5/2)) - (20*b^3*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x +
b^2*x^2])/(3*e^6*(a + b*x)*(d + e*x)^(3/2)) + (10*b^4*(b*d - a*e)*Sqrt[a^2
+ 2*a*b*x + b^2*x^2])/(e^6*(a + b*x)*Sqrt[d + e*x]) + (2*b^5*Sqrt[d + e*x]*
Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*x))
```

Rubi [A] time = 0.0963662, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {646, 43}

$$\frac{2b^5\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}}{e^6(a+bx)}+\frac{10b^4\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{e^6(a+bx)\sqrt{d+ex}}-\frac{20b^3\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{3e^6(a+bx)(d+ex)^{3/2}}+\frac{4b^2\sqrt{a^2+2abx}}{e^6(a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^(11/2),x]
```

```
[Out] (2*(b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(9*e^6*(a + b*x)*(d + e*x)^
(9/2)) - (10*b*(b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(7*e^6*(a + b*x
)*(d + e*x)^(7/2)) + (4*b^2*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e
^6*(a + b*x)*(d + e*x)^(5/2)) - (20*b^3*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x +
b^2*x^2])/(3*e^6*(a + b*x)*(d + e*x)^(3/2)) + (10*b^4*(b*d - a*e)*Sqrt[a^2
+ 2*a*b*x + b^2*x^2])/(e^6*(a + b*x)*Sqrt[d + e*x]) + (2*b^5*Sqrt[d + e*x]*
Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{(d+ex)^{11/2}}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^5}{(d+ex)^{11/2}}\,dx}{b^4\left(ab+b^2x\right)}$$

$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^5(bd-ae)^5}{e^5(d+ex)^{11/2}}+\frac{5b^6(bd-ae)^4}{e^5(d+ex)^{9/2}}-\frac{10b^7(bd-ae)^3}{e^5(d+ex)^{7/2}}+\frac{10b^8(bd-ae)^2}{e^5(d+ex)^{5/2}}-\frac{5b^9(bd-ae)}{e^5(d+ex)^{3/2}}+\right.}{b^4\left(ab+b^2x\right)}$$

$$= \frac{2(bd-ae)^5\sqrt{a^2+2abx+b^2x^2}}{9e^6(a+bx)(d+ex)^{9/2}} - \frac{10b(bd-ae)^4\sqrt{a^2+2abx+b^2x^2}}{7e^6(a+bx)(d+ex)^{7/2}} + \frac{4b^2(bd-ae)^3\sqrt{a^2}}{e^6(a+bx)(d}$$

Mathematica [A] time = 0.119564, size = 235, normalized size = 0.75

$$-\frac{2\sqrt{(a+bx)^2}\left(2a^2b^3e^2\left(72d^2ex+16d^3+126de^2x^2+105e^3x^3\right)+2a^3b^2e^3\left(8d^2+36dex+63e^2x^2\right)+5a^4be^4(2d+9ex)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^(11/2),x]
```

```
[Out] (-2*Sqrt[(a + b*x)^2]*(7*a^5*e^5 + 5*a^4*b*e^4*(2*d + 9*e*x) + 2*a^3*b^2*e^
3*(8*d^2 + 36*d*e*x + 63*e^2*x^2) + 2*a^2*b^3*e^2*(16*d^3 + 72*d^2*e*x + 12
6*d*e^2*x^2 + 105*e^3*x^3) + a*b^4*e*(128*d^4 + 576*d^3*e*x + 1008*d^2*e^2*
x^2 + 840*d*e^3*x^3 + 315*e^4*x^4) - b^5*(256*d^5 + 1152*d^4*e*x + 2016*d^3
*e^2*x^2 + 1680*d^2*e^3*x^3 + 630*d*e^4*x^4 + 63*e^5*x^5)))/(63*e^6*(a + b*
x)*(d + e*x)^(9/2))
```

Maple [A] time = 0.154, size = 289, normalized size = 0.9

$$-\frac{-126\,x^5b^5e^5+630\,x^4ab^4e^5-1260\,x^4b^5de^4+420\,x^3a^2b^3e^5+1680\,x^3ab^4de^4-3360\,x^3b^5d^2e^3+252\,x^2a^3b^2e^5+504\,x^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(11/2),x)
```

```
[Out] -2/63/(e*x+d)^(9/2)*(-63*b^5*e^5*x^5+315*a*b^4*e^5*x^4-630*b^5*d*e^4*x^4+21
0*a^2*b^3*e^5*x^3+840*a*b^4*d*e^4*x^3-1680*b^5*d^2*e^3*x^3+126*a^3*b^2*e^5*
x^2+252*a^2*b^3*d*e^4*x^2+1008*a*b^4*d^2*e^3*x^2-2016*b^5*d^3*e^2*x^2+45*a^
4*b*e^5*x+72*a^3*b^2*d*e^4*x+144*a^2*b^3*d^2*e^3*x+576*a*b^4*d^3*e^2*x-1152
*b^5*d^4*e*x+7*a^5*e^5+10*a^4*b*d*e^4+16*a^3*b^2*d^2*e^3+32*a^2*b^3*d^3*e^2
+128*a*b^4*d^4*e-256*b^5*d^5)*((b*x+a)^2)^(5/2)/e^6/(b*x+a)^5
```

Maxima [A] time = 1.13801, size = 412, normalized size = 1.31

$$\frac{2\left(63\,b^5e^5x^5+256\,b^5d^5-128\,ab^4d^4e-32\,a^2b^3d^3e^2-16\,a^3b^2d^2e^3-10\,a^4bde^4-7\,a^5e^5+315\left(2\,b^5de^4-ab^4e^5\right)x^4+210\right.}{63\left(e^{10}x^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(11/2),x, algorithm="maxima")
```

```
[Out] 2/63*(63*b^5*e^5*x^5 + 256*b^5*d^5 - 128*a*b^4*d^4*e - 32*a^2*b^3*d^3*e^2 -
 16*a^3*b^2*d^2*e^3 - 10*a^4*b*d*e^4 - 7*a^5*e^5 + 315*(2*b^5*d*e^4 - a*b^4
*e^5)*x^4 + 210*(8*b^5*d^2*e^3 - 4*a*b^4*d*e^4 - a^2*b^3*e^5)*x^3 + 126*(16
*b^5*d^3*e^2 - 8*a*b^4*d^2*e^3 - 2*a^2*b^3*d*e^4 - a^3*b^2*e^5)*x^2 + 9*(12
8*b^5*d^4*e - 64*a*b^4*d^3*e^2 - 16*a^2*b^3*d^2*e^3 - 8*a^3*b^2*d*e^4 - 5*a
^4*b*e^5)*x)/((e^10*x^4 + 4*d*e^9*x^3 + 6*d^2*e^8*x^2 + 4*d^3*e^7*x + d^4*e
^6)*sqrt(e*x + d))
```

Fricas [A] time = 1.65967, size = 657, normalized size = 2.09

$$\frac{2\left(63\,b^5e^5x^5 + 256\,b^5d^5 - 128\,ab^4d^4e - 32\,a^2b^3d^3e^2 - 16\,a^3b^2d^2e^3 - 10\,a^4bde^4 - 7\,a^5e^5 + 315\left(2\,b^5de^4 - ab^4e^5\right)x^4 + 210\left(8\right.\right.}{63\left(e^{11}x^5 + 5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(11/2),x, algorithm="fricas")
```

```
[Out] 2/63*(63*b^5*e^5*x^5 + 256*b^5*d^5 - 128*a*b^4*d^4*e - 32*a^2*b^3*d^3*e^2 -
 16*a^3*b^2*d^2*e^3 - 10*a^4*b*d*e^4 - 7*a^5*e^5 + 315*(2*b^5*d*e^4 - a*b^4
*e^5)*x^4 + 210*(8*b^5*d^2*e^3 - 4*a*b^4*d*e^4 - a^2*b^3*e^5)*x^3 + 126*(16
*b^5*d^3*e^2 - 8*a*b^4*d^2*e^3 - 2*a^2*b^3*d*e^4 - a^3*b^2*e^5)*x^2 + 9*(12
8*b^5*d^4*e - 64*a*b^4*d^3*e^2 - 16*a^2*b^3*d^2*e^3 - 8*a^3*b^2*d*e^4 - 5*a
^4*b*e^5)*x)*sqrt(e*x + d)/(e^11*x^5 + 5*d*e^10*x^4 + 10*d^2*e^9*x^3 + 10*d
^3*e^8*x^2 + 5*d^4*e^7*x + d^5*e^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**(11/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18664, size = 608, normalized size = 1.94

$$2\,\sqrt{xe+d}b^5e^{(-6)}\mathrm{sgn}\left(bx+a\right) + \frac{2\left(315\,(xe+d)^4b^5d\mathrm{sgn}\left(bx+a\right) - 210\,(xe+d)^3b^5d^2\mathrm{sgn}\left(bx+a\right) + 126\,(xe+d)^2b^5d^3\mathrm{sgn}\left(b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(11/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x*e + d)*b^5*e^(-6)*sgn(b*x + a) + 2/63*(315*(x*e + d)^4*b^5*d*sgn(b
*x + a) - 210*(x*e + d)^3*b^5*d^2*sgn(b*x + a) + 126*(x*e + d)^2*b^5*d^3*sg
n(b*x + a) - 45*(x*e + d)*b^5*d^4*sgn(b*x + a) + 7*b^5*d^5*sgn(b*x + a) - 3
15*(x*e + d)^4*a*b^4*e*sgn(b*x + a) + 420*(x*e + d)^3*a*b^4*d*e*sgn(b*x + a
) - 378*(x*e + d)^2*a*b^4*d^2*e*sgn(b*x + a) + 180*(x*e + d)*a*b^4*d^3*e*sg
n(b*x + a) - 35*a*b^4*d^4*e*sgn(b*x + a) - 210*(x*e + d)^3*a^2*b^3*e^2*sgn(
b*x + a) + 378*(x*e + d)^2*a^2*b^3*d*e^2*sgn(b*x + a) - 270*(x*e + d)*a^2*b
```

```
^3*d^2*e^2*sgn(b*x + a) + 70*a^2*b^3*d^3*e^2*sgn(b*x + a) - 126*(x*e + d)^2
*a^3*b^2*e^3*sgn(b*x + a) + 180*(x*e + d)*a^3*b^2*d*e^3*sgn(b*x + a) - 70*a
^3*b^2*d^2*e^3*sgn(b*x + a) - 45*(x*e + d)*a^4*b*e^4*sgn(b*x + a) + 35*a^4*
b*d*e^4*sgn(b*x + a) - 7*a^5*e^5*sgn(b*x + a))*e^(-6)/(x*e + d)^(9/2)
```

3.1701 $\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{(d+ex)^{13/2}}\,dx$

Optimal. Leaf size=316

$$-\frac{2b^5\sqrt{a^2+2abx+b^2x^2}}{e^6(a+bx)\sqrt{d+ex}}+\frac{10b^4\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{3e^6(a+bx)(d+ex)^{3/2}}-\frac{4b^3\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{e^6(a+bx)(d+ex)^{5/2}}+\frac{20b^2\sqrt{a^2+2abx+b^2x}}{7e^6(a+bx)(d+e}$$

```
[Out] (2*(b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(11*e^6*(a + b*x)*(d + e*x)
^(11/2)) - (10*b*(b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(9*e^6*(a + b
*x)*(d + e*x)^(9/2)) + (20*b^2*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
/(7*e^6*(a + b*x)*(d + e*x)^(7/2)) - (4*b^3*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*
x + b^2*x^2])/(e^6*(a + b*x)*(d + e*x)^(5/2)) + (10*b^4*(b*d - a*e)*Sqrt[a^
2 + 2*a*b*x + b^2*x^2])/(3*e^6*(a + b*x)*(d + e*x)^(3/2)) - (2*b^5*Sqrt[a^2
 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*x)*Sqrt[d + e*x])
```

Rubi [A] time = 0.0957733, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {646, 43}

$$-\frac{2b^5\sqrt{a^2+2abx+b^2x^2}}{e^6(a+bx)\sqrt{d+ex}}+\frac{10b^4\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{3e^6(a+bx)(d+ex)^{3/2}}-\frac{4b^3\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{e^6(a+bx)(d+ex)^{5/2}}+\frac{20b^2\sqrt{a^2+2abx+b^2x}}{7e^6(a+bx)(d+e}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^(13/2),x]
```


```
[Out] (2*(b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(11*e^6*(a + b*x)*(d + e*x)
^(11/2)) - (10*b*(b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(9*e^6*(a + b
*x)*(d + e*x)^(9/2)) + (20*b^2*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
/(7*e^6*(a + b*x)*(d + e*x)^(7/2)) - (4*b^3*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*
x + b^2*x^2])/(e^6*(a + b*x)*(d + e*x)^(5/2)) + (10*b^4*(b*d - a*e)*Sqrt[a^
2 + 2*a*b*x + b^2*x^2])/(3*e^6*(a + b*x)*(d + e*x)^(3/2)) - (2*b^5*Sqrt[a^2
 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*x)*Sqrt[d + e*x])
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{(d+ex)^{13/2}}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^5}{(d+ex)^{13/2}}dx}{b^4\left(ab+b^2x\right)}$$

$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^5(bd-ae)^5}{e^5(d+ex)^{13/2}}+\frac{5b^6(bd-ae)^4}{e^5(d+ex)^{11/2}}-\frac{10b^7(bd-ae)^3}{e^5(d+ex)^{9/2}}+\frac{10b^8(bd-ae)^2}{e^5(d+ex)^{7/2}}-\frac{5b^9(bd-ae)}{e^5(d+ex)^{5/2}}+\right.}{b^4\left(ab+b^2x\right)}$$

$$= \frac{2(bd-ae)^5\sqrt{a^2+2abx+b^2x^2}}{11e^6(a+bx)(d+ex)^{11/2}}-\frac{10b(bd-ae)^4\sqrt{a^2+2abx+b^2x^2}}{9e^6(a+bx)(d+ex)^{9/2}}+\frac{20b^2(bd-ae)^3\sqrt{a}}{7e^6(a+bx)(}$$

Mathematica [A] time = 0.117079, size = 234, normalized size = 0.74

$$-\frac{2\sqrt{(a+bx)^2}\left(6a^2b^3e^2\left(88d^2ex+16d^3+198de^2x^2+231e^3x^3\right)+10a^3b^2e^3\left(8d^2+44dex+99e^2x^2\right)+35a^4be^4(2d+11e\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^(13/2),x]
```

```
[Out] (-2*Sqrt[(a + b*x)^2]*(63*a^5*e^5 + 35*a^4*b*e^4*(2*d + 11*e*x) + 10*a^3*b^
2*e^3*(8*d^2 + 44*d*e*x + 99*e^2*x^2) + 6*a^2*b^3*e^2*(16*d^3 + 88*d^2*e*x
+ 198*d*e^2*x^2 + 231*e^3*x^3) + a*b^4*e*(128*d^4 + 704*d^3*e*x + 1584*d^2*
e^2*x^2 + 1848*d*e^3*x^3 + 1155*e^4*x^4) + b^5*(256*d^5 + 1408*d^4*e*x + 31
68*d^3*e^2*x^2 + 3696*d^2*e^3*x^3 + 2310*d*e^4*x^4 + 693*e^5*x^5)))/(693*e^
6*(a + b*x)*(d + e*x)^(11/2))
```

Maple [A] time = 0.155, size = 289, normalized size = 0.9

$$-\frac{1386\,x^5b^5e^5+2310\,x^4ab^4e^5+4620\,x^4b^5de^4+2772\,x^3a^2b^3e^5+3696\,x^3ab^4de^4+7392\,x^3b^5d^2e^3+1980\,x^2a^3b^2e^5+23}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(13/2),x)
```

```
[Out] -2/693/(e*x+d)^(11/2)*(693*b^5*e^5*x^5+1155*a*b^4*e^5*x^4+2310*b^5*d*e^4*x^
4+1386*a^2*b^3*e^5*x^3+1848*a*b^4*d*e^4*x^3+3696*b^5*d^2*e^3*x^3+990*a^3*b^
2*e^5*x^2+1188*a^2*b^3*d*e^4*x^2+1584*a*b^4*d^2*e^3*x^2+3168*b^5*d^3*e^2*x^
2+385*a^4*b*e^5*x+440*a^3*b^2*d*e^4*x+528*a^2*b^3*d^2*e^3*x+704*a*b^4*d^3*e
^2*x+1408*b^5*d^4*e*x+63*a^5*e^5+70*a^4*b*d*e^4+80*a^3*b^2*d^2*e^3+96*a^2*b
^3*d^3*e^2+128*a*b^4*d^4*e+256*b^5*d^5)*((b*x+a)^2)^(5/2)/e^6/(b*x+a)^5
```

Maxima [A] time = 1.13069, size = 425, normalized size = 1.34

$$-\frac{2\left(693\,b^5e^5x^5+256\,b^5d^5+128\,ab^4d^4e+96\,a^2b^3d^3e^2+80\,a^3b^2d^2e^3+70\,a^4bde^4+63\,a^5e^5+1155\left(2\,b^5de^4+ab^4e^5\right)x^4\right.}{693\left(e^{11}x^5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(13/2),x, algorithm="maxima")
```

```
[Out] -2/693*(693*b^5*e^5*x^5 + 256*b^5*d^5 + 128*a*b^4*d^4*e + 96*a^2*b^3*d^3*e^
2 + 80*a^3*b^2*d^2*e^3 + 70*a^4*b*d*e^4 + 63*a^5*e^5 + 1155*(2*b^5*d*e^4 +
a*b^4*e^5)*x^4 + 462*(8*b^5*d^2*e^3 + 4*a*b^4*d*e^4 + 3*a^2*b^3*e^5)*x^3 +
198*(16*b^5*d^3*e^2 + 8*a*b^4*d^2*e^3 + 6*a^2*b^3*d*e^4 + 5*a^3*b^2*e^5)*x^
2 + 11*(128*b^5*d^4*e + 64*a*b^4*d^3*e^2 + 48*a^2*b^3*d^2*e^3 + 40*a^3*b^2*
d*e^4 + 35*a^4*b*e^5)*x)/((e^11*x^5 + 5*d*e^10*x^4 + 10*d^2*e^9*x^3 + 10*d^
3*e^8*x^2 + 5*d^4*e^7*x + d^5*e^6)*sqrt(e*x + d))
```

Fricas [A] time = 1.57252, size = 698, normalized size = 2.21

$$-\frac{2\left(693\,b^5e^5x^5+256\,b^5d^5+128\,ab^4d^4e+96\,a^2b^3d^3e^2+80\,a^3b^2d^2e^3+70\,a^4bde^4+63\,a^5e^5+1155\left(2\,b^5de^4+ab^4e^5\right)x^4+\cdots\right.}{693\left(e^{12}x^6+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(13/2),x, algorithm="fricas")
```

```
[Out] -2/693*(693*b^5*e^5*x^5 + 256*b^5*d^5 + 128*a*b^4*d^4*e + 96*a^2*b^3*d^3*e^
2 + 80*a^3*b^2*d^2*e^3 + 70*a^4*b*d*e^4 + 63*a^5*e^5 + 1155*(2*b^5*d*e^4 +
a*b^4*e^5)*x^4 + 462*(8*b^5*d^2*e^3 + 4*a*b^4*d*e^4 + 3*a^2*b^3*e^5)*x^3 +
198*(16*b^5*d^3*e^2 + 8*a*b^4*d^2*e^3 + 6*a^2*b^3*d*e^4 + 5*a^3*b^2*e^5)*x^
2 + 11*(128*b^5*d^4*e + 64*a*b^4*d^3*e^2 + 48*a^2*b^3*d^2*e^3 + 40*a^3*b^2*
d*e^4 + 35*a^4*b*e^5)*x)*sqrt(e*x + d)/(e^12*x^6 + 6*d*e^11*x^5 + 15*d^2*e^
10*x^4 + 20*d^3*e^9*x^3 + 15*d^4*e^8*x^2 + 6*d^5*e^7*x + d^6*e^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**(13/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21552, size = 603, normalized size = 1.91

$$-\frac{2\left(693\,(xe+d)^5b^5\mathrm{sgn}\,(bx+a)-1155\,(xe+d)^4b^5d\mathrm{sgn}\,(bx+a)+1386\,(xe+d)^3b^5d^2\mathrm{sgn}\,(bx+a)-990\,(xe+d)^2b^5d^3\mathrm{sgn}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(13/2),x, algorithm="giac")
```

```
[Out] -2/693*(693*(x*e + d)^5*b^5*sgn(b*x + a) - 1155*(x*e + d)^4*b^5*d*sgn(b*x +
 a) + 1386*(x*e + d)^3*b^5*d^2*sgn(b*x + a) - 990*(x*e + d)^2*b^5*d^3*sgn(b
*x + a) + 385*(x*e + d)*b^5*d^4*sgn(b*x + a) - 63*b^5*d^5*sgn(b*x + a) + 11
55*(x*e + d)^4*a*b^4*e*sgn(b*x + a) - 2772*(x*e + d)^3*a*b^4*d*e*sgn(b*x +
a) + 2970*(x*e + d)^2*a*b^4*d^2*e*sgn(b*x + a) - 1540*(x*e + d)*a*b^4*d^3*e
*sgn(b*x + a) + 315*a*b^4*d^4*e*sgn(b*x + a) + 1386*(x*e + d)^3*a^2*b^3*e^2
*sgn(b*x + a) - 2970*(x*e + d)^2*a^2*b^3*d*e^2*sgn(b*x + a) + 2310*(x*e + d
```

```
)*a^2*b^3*d^2*e^2*sgn(b*x + a) - 630*a^2*b^3*d^3*e^2*sgn(b*x + a) + 990*(x*
e + d)^2*a^3*b^2*e^3*sgn(b*x + a) - 1540*(x*e + d)*a^3*b^2*d*e^3*sgn(b*x +
a) + 630*a^3*b^2*d^2*e^3*sgn(b*x + a) + 385*(x*e + d)*a^4*b*e^4*sgn(b*x + a
) - 315*a^4*b*d*e^4*sgn(b*x + a) + 63*a^5*e^5*sgn(b*x + a))*e^(-6)/(x*e + d
)^(11/2)
```

3.1702 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{15/2}}\,dx$

Optimal. Leaf size=318

$$-\frac{2b^5\sqrt{a^2+2abx+b^2x^2}}{3e^6(a+bx)(d+ex)^{3/2}}+\frac{2b^4\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{e^6(a+bx)(d+ex)^{5/2}}-\frac{20b^3\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{7e^6(a+bx)(d+ex)^{7/2}}+\frac{20b^2\sqrt{a^2+2abx+b^2x}}{9e^6(a+bx)(d+e}$$

```
[Out] (2*(b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(13*e^6*(a + b*x)*(d + e*x)
^(13/2)) - (10*b*(b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(11*e^6*(a +
b*x)*(d + e*x)^(11/2)) + (20*b^2*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2
])/(9*e^6*(a + b*x)*(d + e*x)^(9/2)) - (20*b^3*(b*d - a*e)^2*Sqrt[a^2 + 2*a
*b*x + b^2*x^2])/(7*e^6*(a + b*x)*(d + e*x)^(7/2)) + (2*b^4*(b*d - a*e)*Sqr
t[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*x)*(d + e*x)^(5/2)) - (2*b^5*Sqrt[a
^2 + 2*a*b*x + b^2*x^2])/(3*e^6*(a + b*x)*(d + e*x)^(3/2))
```

Rubi [A] time = 0.09774, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {646, 43}

$$-\frac{2b^5\sqrt{a^2+2abx+b^2x^2}}{3e^6(a+bx)(d+ex)^{3/2}}+\frac{2b^4\sqrt{a^2+2abx+b^2x^2}(bd-ae)}{e^6(a+bx)(d+ex)^{5/2}}-\frac{20b^3\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}{7e^6(a+bx)(d+ex)^{7/2}}+\frac{20b^2\sqrt{a^2+2abx+b^2x}}{9e^6(a+bx)(d+e}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^(15/2),x]
```


```
[Out] (2*(b*d - a*e)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(13*e^6*(a + b*x)*(d + e*x)
^(13/2)) - (10*b*(b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(11*e^6*(a +
b*x)*(d + e*x)^(11/2)) + (20*b^2*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2
])/(9*e^6*(a + b*x)*(d + e*x)^(9/2)) - (20*b^3*(b*d - a*e)^2*Sqrt[a^2 + 2*a
*b*x + b^2*x^2])/(7*e^6*(a + b*x)*(d + e*x)^(7/2)) + (2*b^4*(b*d - a*e)*Sqr
t[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(a + b*x)*(d + e*x)^(5/2)) - (2*b^5*Sqrt[a
^2 + 2*a*b*x + b^2*x^2])/(3*e^6*(a + b*x)*(d + e*x)^(3/2))
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{(d+ex)^{15/2}}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(ab+b^2x\right)^5}{(d+ex)^{15/2}}\,dx}{b^4\left(ab+b^2x\right)}$$
$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^5(bd-ae)^5}{e^5(d+ex)^{15/2}}+\frac{5b^6(bd-ae)^4}{e^5(d+ex)^{13/2}}-\frac{10b^7(bd-ae)^3}{e^5(d+ex)^{11/2}}+\frac{10b^8(bd-ae)^2}{e^5(d+ex)^{9/2}}-\frac{5b^9(bd-ae)}{e^5(d+ex)^{7/2}}+\right.}{b^4\left(ab+b^2x\right)}$$
$$= \frac{2(bd-ae)^5\sqrt{a^2+2abx+b^2x^2}}{13e^6(a+bx)(d+ex)^{13/2}} - \frac{10b(bd-ae)^4\sqrt{a^2+2abx+b^2x^2}}{11e^6(a+bx)(d+ex)^{11/2}} + \frac{20b^2(bd-ae)^3\sqrt{a}}{9e^6(a+bx)(}$$

Mathematica [A] time = 0.120595, size = 235, normalized size = 0.74

$$-\frac{2\sqrt{(a+bx)^2}\left(30a^2b^3e^2\left(104d^2ex+16d^3+286de^2x^2+429e^3x^3\right)+70a^3b^2e^3\left(8d^2+52dex+143e^2x^2\right)+315a^4be^4(2d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^(15/2),x]
```

```
[Out] (-2*Sqrt[(a + b*x)^2]*(693*a^5*e^5 + 315*a^4*b*e^4*(2*d + 13*e*x) + 70*a^3*
b^2*e^3*(8*d^2 + 52*d*e*x + 143*e^2*x^2) + 30*a^2*b^3*e^2*(16*d^3 + 104*d^2
*e*x + 286*d*e^2*x^2 + 429*e^3*x^3) + 3*a*b^4*e*(128*d^4 + 832*d^3*e*x + 22
88*d^2*e^2*x^2 + 3432*d*e^3*x^3 + 3003*e^4*x^4) + b^5*(256*d^5 + 1664*d^4*e
*x + 4576*d^3*e^2*x^2 + 6864*d^2*e^3*x^3 + 6006*d*e^4*x^4 + 3003*e^5*x^5)))
/(9009*e^6*(a + b*x)*(d + e*x)^(13/2))
```

Maple [A] time = 0.155, size = 289, normalized size = 0.9

$$-\frac{6006x^5b^5e^5+18018x^4ab^4e^5+12012x^4b^5de^4+25740x^3a^2b^3e^5+20592x^3ab^4de^4+13728x^3b^5d^2e^3+20020x^2a^3b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(15/2),x)
```

```
[Out] -2/9009/(e*x+d)^(13/2)*(3003*b^5*e^5*x^5+9009*a*b^4*e^5*x^4+6006*b^5*d*e^4*
x^4+12870*a^2*b^3*e^5*x^3+10296*a*b^4*d*e^4*x^3+6864*b^5*d^2*e^3*x^3+10010*
a^3*b^2*e^5*x^2+8580*a^2*b^3*d*e^4*x^2+6864*a*b^4*d^2*e^3*x^2+4576*b^5*d^3*
e^2*x^2+4095*a^4*b*e^5*x+3640*a^3*b^2*d*e^4*x+3120*a^2*b^3*d^2*e^3*x+2496*a
*b^4*d^3*e^2*x+1664*b^5*d^4*e*x+693*a^5*e^5+630*a^4*b*d*e^4+560*a^3*b^2*d^2
*e^3+480*a^2*b^3*d^3*e^2+384*a*b^4*d^4*e+256*b^5*d^5)*((b*x+a)^2)^(5/2)/e^6
/(b*x+a)^5
```

Maxima [A] time = 1.20604, size = 441, normalized size = 1.39

$$-\frac{2\left(3003b^5e^5x^5+256b^5d^5+384ab^4d^4e+480a^2b^3d^3e^2+560a^3b^2d^2e^3+630a^4bde^4+693a^5e^5+3003\left(2b^5de^4+3\right.\right.}{9009\left(e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(15/2),x, algorithm="maxima")
```

```
[Out] -2/9009*(3003*b^5*e^5*x^5 + 256*b^5*d^5 + 384*a*b^4*d^4*e + 480*a^2*b^3*d^3
*e^2 + 560*a^3*b^2*d^2*e^3 + 630*a^4*b*d*e^4 + 693*a^5*e^5 + 3003*(2*b^5*d*
e^4 + 3*a*b^4*e^5)*x^4 + 858*(8*b^5*d^2*e^3 + 12*a*b^4*d*e^4 + 15*a^2*b^3*e
^5)*x^3 + 286*(16*b^5*d^3*e^2 + 24*a*b^4*d^2*e^3 + 30*a^2*b^3*d*e^4 + 35*a^
3*b^2*e^5)*x^2 + 13*(128*b^5*d^4*e + 192*a*b^4*d^3*e^2 + 240*a^2*b^3*d^2*e^
3 + 280*a^3*b^2*d*e^4 + 315*a^4*b*e^5)*x)/((e^12*x^6 + 6*d*e^11*x^5 + 15*d^
2*e^10*x^4 + 20*d^3*e^9*x^3 + 15*d^4*e^8*x^2 + 6*d^5*e^7*x + d^6*e^6)*sqrt(
e*x + d))
```

Fricas [A] time = 1.63259, size = 745, normalized size = 2.34

$$-\frac{2\left(3003\,b^5e^5x^5+256\,b^5d^5+384\,ab^4d^4e+480\,a^2b^3d^3e^2+560\,a^3b^2d^2e^3+630\,a^4bde^4+693\,a^5e^5+3003\left(2\,b^5de^4+3\,ab^4\right.\right.}{9009\left(e^{13}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(15/2),x, algorithm="fricas")
```

```
[Out] -2/9009*(3003*b^5*e^5*x^5 + 256*b^5*d^5 + 384*a*b^4*d^4*e + 480*a^2*b^3*d^3
*e^2 + 560*a^3*b^2*d^2*e^3 + 630*a^4*b*d*e^4 + 693*a^5*e^5 + 3003*(2*b^5*d*
e^4 + 3*a*b^4*e^5)*x^4 + 858*(8*b^5*d^2*e^3 + 12*a*b^4*d*e^4 + 15*a^2*b^3*e
^5)*x^3 + 286*(16*b^5*d^3*e^2 + 24*a*b^4*d^2*e^3 + 30*a^2*b^3*d*e^4 + 35*a^
3*b^2*e^5)*x^2 + 13*(128*b^5*d^4*e + 192*a*b^4*d^3*e^2 + 240*a^2*b^3*d^2*e^
3 + 280*a^3*b^2*d*e^4 + 315*a^4*b*e^5)*x)*sqrt(e*x + d)/(e^13*x^7 + 7*d*e^1
2*x^6 + 21*d^2*e^11*x^5 + 35*d^3*e^10*x^4 + 35*d^4*e^9*x^3 + 21*d^5*e^8*x^2
 + 7*d^6*e^7*x + d^7*e^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**(15/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18664, size = 603, normalized size = 1.9

$$-\frac{2\left(3003\left(xe+d\right)^5b^5\mathrm{sgn}\left(bx+a\right)-9009\left(xe+d\right)^4b^5d\mathrm{sgn}\left(bx+a\right)+12870\left(xe+d\right)^3b^5d^2\mathrm{sgn}\left(bx+a\right)-10010\left(xe+d\right)^2b^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(15/2),x, algorithm="giac")
```

```
[Out] -2/9009*(3003*(x*e + d)^5*b^5*sgn(b*x + a) - 9009*(x*e + d)^4*b^5*d*sgn(b*x
 + a) + 12870*(x*e + d)^3*b^5*d^2*sgn(b*x + a) - 10010*(x*e + d)^2*b^5*d^3*
sgn(b*x + a) + 4095*(x*e + d)*b^5*d^4*sgn(b*x + a) - 693*b^5*d^5*sgn(b*x +
a) + 9009*(x*e + d)^4*a*b^4*e*sgn(b*x + a) - 25740*(x*e + d)^3*a*b^4*d*e*sg
n(b*x + a) + 30030*(x*e + d)^2*a*b^4*d^2*e*sgn(b*x + a) - 16380*(x*e + d)*a
```

```
*b^4*d^3*e*sgn(b*x + a) + 3465*a*b^4*d^4*e*sgn(b*x + a) + 12870*(x*e + d)^3
*a^2*b^3*e^2*sgn(b*x + a) - 30030*(x*e + d)^2*a^2*b^3*d*e^2*sgn(b*x + a) +
24570*(x*e + d)*a^2*b^3*d^2*e^2*sgn(b*x + a) - 6930*a^2*b^3*d^3*e^2*sgn(b*x
 + a) + 10010*(x*e + d)^2*a^3*b^2*e^3*sgn(b*x + a) - 16380*(x*e + d)*a^3*b^
2*d*e^3*sgn(b*x + a) + 6930*a^3*b^2*d^2*e^3*sgn(b*x + a) + 4095*(x*e + d)*a
^4*b*e^4*sgn(b*x + a) - 3465*a^4*b*d*e^4*sgn(b*x + a) + 693*a^5*e^5*sgn(b*x
 + a))*e^(-6)/(x*e + d)^(13/2)
```

3.1703 $\int \frac{(d+ex)^{7/2}}{\sqrt{a^2+2abx+b^2x^2}} dx$

Optimal. Leaf size=263

$$\frac{2(a+bx)\sqrt{d+ex}(bd-ae)^3}{b^4\sqrt{a^2+2abx+b^2x^2}}+\frac{2(a+bx)(d+ex)^{3/2}(bd-ae)^2}{3b^3\sqrt{a^2+2abx+b^2x^2}}+\frac{2(a+bx)(d+ex)^{5/2}(bd-ae)}{5b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{2(a+bx)(d+ex)^{7/2}}{7b\sqrt{a^2+2abx+b^2x^2}}-$$

```
[Out] (2*(b*d - a*e)^3*(a + b*x)*Sqrt[d + e*x])/(b^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2
]) + (2*(b*d - a*e)^2*(a + b*x)*(d + e*x)^(3/2))/(3*b^3*Sqrt[a^2 + 2*a*b*x
+ b^2*x^2]) + (2*(b*d - a*e)*(a + b*x)*(d + e*x)^(5/2))/(5*b^2*Sqrt[a^2 + 2
*a*b*x + b^2*x^2]) + (2*(a + b*x)*(d + e*x)^(7/2))/(7*b*Sqrt[a^2 + 2*a*b*x
+ b^2*x^2]) - (2*(b*d - a*e)^(7/2)*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x]
)/Sqrt[b*d - a*e]])/(b^(9/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.16931, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {646, 50, 63, 208}

$$\frac{2(a+bx)\sqrt{d+ex}(bd-ae)^3}{b^4\sqrt{a^2+2abx+b^2x^2}}+\frac{2(a+bx)(d+ex)^{3/2}(bd-ae)^2}{3b^3\sqrt{a^2+2abx+b^2x^2}}+\frac{2(a+bx)(d+ex)^{5/2}(bd-ae)}{5b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{2(a+bx)(d+ex)^{7/2}}{7b\sqrt{a^2+2abx+b^2x^2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (2*(b*d - a*e)^3*(a + b*x)*Sqrt[d + e*x])/(b^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2
]) + (2*(b*d - a*e)^2*(a + b*x)*(d + e*x)^(3/2))/(3*b^3*Sqrt[a^2 + 2*a*b*x
+ b^2*x^2]) + (2*(b*d - a*e)*(a + b*x)*(d + e*x)^(5/2))/(5*b^2*Sqrt[a^2 + 2
*a*b*x + b^2*x^2]) + (2*(a + b*x)*(d + e*x)^(7/2))/(7*b*Sqrt[a^2 + 2*a*b*x
+ b^2*x^2]) - (2*(b*d - a*e)^(7/2)*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x]
)/Sqrt[b*d - a*e]])/(b^(9/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{7/2}}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{\left(ab+b^2x\right)\int \frac{(d+ex)^{7/2}}{ab+b^2x}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= \frac{2(a+bx)(d+ex)^{7/2}}{7b\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(\left(b^2d-abe\right)\left(ab+b^2x\right)\right)\int \frac{(d+ex)^{5/2}}{ab+b^2x}\,dx}{b^2\sqrt{a^2+2abx+b^2x^2}}$$

$$= \frac{2(bd-ae)(a+bx)(d+ex)^{5/2}}{5b^2\sqrt{a^2+2abx+b^2x^2}} + \frac{2(a+bx)(d+ex)^{7/2}}{7b\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(\left(b^2d-abe\right)^2\left(ab+b^2x\right)\right)\int \frac{(d+ex}{ab+b}}{b^4\sqrt{a^2+2abx+b^2x^2}}$$

$$= \frac{2(bd-ae)^2(a+bx)(d+ex)^{3/2}}{3b^3\sqrt{a^2+2abx+b^2x^2}} + \frac{2(bd-ae)(a+bx)(d+ex)^{5/2}}{5b^2\sqrt{a^2+2abx+b^2x^2}} + \frac{2(a+bx)(d+ex)^{7/2}}{7b\sqrt{a^2+2abx+b^2x^2}} +$$

$$= \frac{2(bd-ae)^3(a+bx)\sqrt{d+ex}}{b^4\sqrt{a^2+2abx+b^2x^2}} + \frac{2(bd-ae)^2(a+bx)(d+ex)^{3/2}}{3b^3\sqrt{a^2+2abx+b^2x^2}} + \frac{2(bd-ae)(a+bx)(d+ex)}{5b^2\sqrt{a^2+2abx+b^2x^2}}$$

$$= \frac{2(bd-ae)^3(a+bx)\sqrt{d+ex}}{b^4\sqrt{a^2+2abx+b^2x^2}} + \frac{2(bd-ae)^2(a+bx)(d+ex)^{3/2}}{3b^3\sqrt{a^2+2abx+b^2x^2}} + \frac{2(bd-ae)(a+bx)(d+ex)}{5b^2\sqrt{a^2+2abx+b^2x^2}}$$

$$= \frac{2(bd-ae)^3(a+bx)\sqrt{d+ex}}{b^4\sqrt{a^2+2abx+b^2x^2}} + \frac{2(bd-ae)^2(a+bx)(d+ex)^{3/2}}{3b^3\sqrt{a^2+2abx+b^2x^2}} + \frac{2(bd-ae)(a+bx)(d+ex)}{5b^2\sqrt{a^2+2abx+b^2x^2}}$$

Mathematica [A] time = 0.255444, size = 150, normalized size = 0.57

$$\frac{(a+bx)\left(\frac{14(bd-ae)\left(5(bd-ae)\left(\sqrt{b}\sqrt{d+ex}(-3ae+4bd+bex)-3(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\right)+3b^{5/2}(d+ex)^{5/2}\right)}{15b^{7/2}}+2(d+ex)^{7/2}\right)}{7b\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ((a + b*x)*(2*(d + e*x)^(7/2) + (14*(b*d - a*e)*(3*b^(5/2)*(d + e*x)^(5/2)
+ 5*(b*d - a*e)*(Sqrt[b]*Sqrt[d + e*x]*(4*b*d - 3*a*e + b*e*x) - 3*(b*d - a
*e)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])))/(15*b^(7/2)))
)/(7*b*Sqrt[(a + b*x)^2])
```

Maple [B] time = 0.232, size = 462, normalized size = 1.8

$$\frac{2\,bx+2\,a}{105\,b^4}\left(15\,\sqrt{(ae-bd)\,b}\,(ex+d)^{7/2}\,b^3 - 21\,\sqrt{(ae-bd)\,b}\,(ex+d)^{5/2}\,ab^2e + 21\,\sqrt{(ae-bd)\,b}\,(ex+d)^{5/2}\,b^3d + 105\,\mathrm{arct}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/((b*x+a)^2)^(1/2),x)
```

```
[Out] 2/105*(b*x+a)*(15*((a*e-b*d)*b)^(1/2)*(e*x+d)^(7/2)*b^3-21*((a*e-b*d)*b)^(1
/2)*(e*x+d)^(5/2)*a*b^2*e+21*((a*e-b*d)*b)^(1/2)*(e*x+d)^(5/2)*b^3*d+105*ar
ctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a^4*e^4-420*arctan(b*(e*x+d)^(1/2
)/((a*e-b*d)*b)^(1/2))*a^3*b*d*e^3+630*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)
^(1/2))*a^2*b^2*d^2*e^2-420*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a*b
^3*d^3*e+105*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*b^4*d^4+35*((a*e-b
*d)*b)^(1/2)*(e*x+d)^(3/2)*a^2*b*e^2-70*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*a
*b^2*d*e+35*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*b^3*d^2-105*((a*e-b*d)*b)^(1/
2)*(e*x+d)^(1/2)*a^3*e^3+315*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*a^2*b*d*e^2-
315*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*a*b^2*d^2*e+105*((a*e-b*d)*b)^(1/2)*(
e*x+d)^(1/2)*b^3*d^3)/((b*x+a)^2)^(1/2)/b^4/((a*e-b*d)*b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{7}{2}}}{\sqrt{(bx+a)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(7/2)/sqrt((b*x + a)^2), x)
```

Fricas [A] time = 1.70972, size = 940, normalized size = 3.57

$$\left[-\frac{105\left(b^3d^3 - 3\,ab^2d^2e + 3\,a^2bde^2 - a^3e^3\right)\sqrt{\frac{bd-ae}{b}}\log\left(\frac{bex+2\,bd-ae+2\,\sqrt{ex+d}b\sqrt{\frac{bd-ae}{b}}}{bx+a}\right) - 2\left(15\,b^3e^3x^3 + 176\,b^3d^3 - 406\,ab^2d^2e + \right.}{105\,b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/105*(105*(b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3)*sqrt((b*d
 - a*e)/b)*log((b*e*x + 2*b*d - a*e + 2*sqrt(e*x + d)*b*sqrt((b*d - a*e)/b))
/(b*x + a)) - 2*(15*b^3*e^3*x^3 + 176*b^3*d^3 - 406*a*b^2*d^2*e + 350*a^2*b
*d*e^2 - 105*a^3*e^3 + 3*(22*b^3*d*e^2 - 7*a*b^2*e^3)*x^2 + (122*b^3*d^2*e
 - 112*a*b^2*d*e^2 + 35*a^2*b*e^3)*x)*sqrt(e*x + d))/b^4, -2/105*(105*(b^3*d
^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3)*sqrt(-(b*d - a*e)/b)*arctan(-
sqrt(e*x + d)*b*sqrt(-(b*d - a*e)/b)/(b*d - a*e)) - (15*b^3*e^3*x^3 + 176*b
^3*d^3 - 406*a*b^2*d^2*e + 350*a^2*b*d*e^2 - 105*a^3*e^3 + 3*(22*b^3*d*e^2
 - 7*a*b^2*e^3)*x^2 + (122*b^3*d^2*e - 112*a*b^2*d*e^2 + 35*a^2*b*e^3)*x)*sq
rt(e*x + d))/b^4]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/((b*x+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17778, size = 478, normalized size = 1.82

$$\frac{2\left(b^4d^4\mathrm{sgn}\left(bx+a\right)-4\,ab^3d^3e\mathrm{sgn}\left(bx+a\right)+6\,a^2b^2d^2e^2\mathrm{sgn}\left(bx+a\right)-4\,a^3bde^3\mathrm{sgn}\left(bx+a\right)+a^4e^4\mathrm{sgn}\left(bx+a\right)\right)\mathrm{arcta}}{\sqrt{-b^2d+abe}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*(b^4*d^4*sgn(b*x + a) - 4*a*b^3*d^3*e*sgn(b*x + a) + 6*a^2*b^2*d^2*e^2*sg
n(b*x + a) - 4*a^3*b*d*e^3*sgn(b*x + a) + a^4*e^4*sgn(b*x + a))*arctan(sqrt
(x*e + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^4) + 2/105*(15*(x
*e + d)^(7/2)*b^6*sgn(b*x + a) + 21*(x*e + d)^(5/2)*b^6*d*sgn(b*x + a) + 35
*(x*e + d)^(3/2)*b^6*d^2*sgn(b*x + a) + 105*sqrt(x*e + d)*b^6*d^3*sgn(b*x +
 a) - 21*(x*e + d)^(5/2)*a*b^5*e*sgn(b*x + a) - 70*(x*e + d)^(3/2)*a*b^5*d*
e*sgn(b*x + a) - 315*sqrt(x*e + d)*a*b^5*d^2*e*sgn(b*x + a) + 35*(x*e + d)^
(3/2)*a^2*b^4*e^2*sgn(b*x + a) + 315*sqrt(x*e + d)*a^2*b^4*d*e^2*sgn(b*x +
a) - 105*sqrt(x*e + d)*a^3*b^3*e^3*sgn(b*x + a))/b^7
```

3.1704 $\int \frac{(d+ex)^{5/2}}{\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=212

$$\frac{2(a+bx)\sqrt{d+ex}(bd-ae)^2}{b^3\sqrt{a^2+2abx+b^2x^2}}+\frac{2(a+bx)(d+ex)^{3/2}(bd-ae)}{3b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{2(a+bx)(d+ex)^{5/2}}{5b\sqrt{a^2+2abx+b^2x^2}}-\frac{2(a+bx)(bd-ae)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{7/2}\sqrt{a^2+2abx+b^2x^2}}$$

```
[Out] (2*(b*d - a*e)^2*(a + b*x)*Sqrt[d + e*x])/(b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2
]) + (2*(b*d - a*e)*(a + b*x)*(d + e*x)^(3/2))/(3*b^2*Sqrt[a^2 + 2*a*b*x +
b^2*x^2]) + (2*(a + b*x)*(d + e*x)^(5/2))/(5*b*Sqrt[a^2 + 2*a*b*x + b^2*x^2
]) - (2*(b*d - a*e)^(5/2)*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*
d - a*e]])/(b^(7/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.109214, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {646, 50, 63, 208}

$$\frac{2(a+bx)\sqrt{d+ex}(bd-ae)^2}{b^3\sqrt{a^2+2abx+b^2x^2}}+\frac{2(a+bx)(d+ex)^{3/2}(bd-ae)}{3b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{2(a+bx)(d+ex)^{5/2}}{5b\sqrt{a^2+2abx+b^2x^2}}-\frac{2(a+bx)(bd-ae)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{7/2}\sqrt{a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (2*(b*d - a*e)^2*(a + b*x)*Sqrt[d + e*x])/(b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2
]) + (2*(b*d - a*e)*(a + b*x)*(d + e*x)^(3/2))/(3*b^2*Sqrt[a^2 + 2*a*b*x +
b^2*x^2]) + (2*(a + b*x)*(d + e*x)^(5/2))/(5*b*Sqrt[a^2 + 2*a*b*x + b^2*x^2
]) - (2*(b*d - a*e)^(5/2)*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*
d - a*e]])/(b^(7/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{\left(ab+b^2x\right)\int \frac{(d+ex)^{5/2}}{ab+b^2x}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= \frac{2(a+bx)(d+ex)^{5/2}}{5b\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(\left(b^2d-abe\right)\left(ab+b^2x\right)\right)\int \frac{(d+ex)^{3/2}}{ab+b^2x}\,dx}{b^2\sqrt{a^2+2abx+b^2x^2}}$$

$$= \frac{2(bd-ae)(a+bx)(d+ex)^{3/2}}{3b^2\sqrt{a^2+2abx+b^2x^2}} + \frac{2(a+bx)(d+ex)^{5/2}}{5b\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(\left(b^2d-abe\right)^2\left(ab+b^2x\right)\right)\int \frac{\sqrt{d+e}}{ab+b^2}}{b^4\sqrt{a^2+2abx+b^2x^2}}$$

$$= \frac{2(bd-ae)^2(a+bx)\sqrt{d+ex}}{b^3\sqrt{a^2+2abx+b^2x^2}} + \frac{2(bd-ae)(a+bx)(d+ex)^{3/2}}{3b^2\sqrt{a^2+2abx+b^2x^2}} + \frac{2(a+bx)(d+ex)^{5/2}}{5b\sqrt{a^2+2abx+b^2x^2}} + \cdots$$

$$= \frac{2(bd-ae)^2(a+bx)\sqrt{d+ex}}{b^3\sqrt{a^2+2abx+b^2x^2}} + \frac{2(bd-ae)(a+bx)(d+ex)^{3/2}}{3b^2\sqrt{a^2+2abx+b^2x^2}} + \frac{2(a+bx)(d+ex)^{5/2}}{5b\sqrt{a^2+2abx+b^2x^2}} + \cdots$$

$$= \frac{2(bd-ae)^2(a+bx)\sqrt{d+ex}}{b^3\sqrt{a^2+2abx+b^2x^2}} + \frac{2(bd-ae)(a+bx)(d+ex)^{3/2}}{3b^2\sqrt{a^2+2abx+b^2x^2}} + \frac{2(a+bx)(d+ex)^{5/2}}{5b\sqrt{a^2+2abx+b^2x^2}} - \cdots$$

Mathematica [A] time = 0.105951, size = 127, normalized size = 0.6

$$\frac{2(a+bx)\left(\sqrt{b}\sqrt{d+ex}\left(15a^2e^2-5abe(7d+ex)+b^2\left(23d^2+11dex+3e^2x^2\right)\right)-15(bd-ae)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\right)}{15b^{7/2}\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (2*(a + b*x)*(Sqrt[b]*Sqrt[d + e*x]*(15*a^2*e^2 - 5*a*b*e*(7*d + e*x) + b^2
*(23*d^2 + 11*d*e*x + 3*e^2*x^2)) - 15*(b*d - a*e)^(5/2)*ArcTanh[(Sqrt[b]*S
qrt[d + e*x])/Sqrt[b*d - a*e]]))/(15*b^(7/2)*Sqrt[(a + b*x)^2])
```

Maple [B] time = 0.232, size = 309, normalized size = 1.5

$$\frac{2bx+2a}{15b^3}\left(3\sqrt{(ae-bd)b}\,(ex+d)^{5/2}b^2-5\sqrt{(ae-bd)b}\,(ex+d)^{3/2}abe+5\sqrt{(ae-bd)b}\,(ex+d)^{3/2}b^2d-15\arctan\left(\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/((b*x+a)^2)^(1/2),x)
```

```
[Out] 2/15*(b*x+a)*(3*((a*e-b*d)*b)^(1/2)*(e*x+d)^(5/2)*b^2-5*((a*e-b*d)*b)^(1/2)
*(e*x+d)^(3/2)*a*b*e+5*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*b^2*d-15*arctan(b*
(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a^3*e^3+45*arctan(b*(e*x+d)^(1/2)/((a*e-
b*d)*b)^(1/2))*a^2*b*d*e^2-45*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a
```

```
*b^2*d^2*e+15*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*b^3*d^3+15*((a*e-
b*d)*b)^(1/2)*(e*x+d)^(1/2)*a^2*e^2-30*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*a*
b*d*e+15*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*b^2*d^2)/((b*x+a)^2)^(1/2)/b^3/(
(a*e-b*d)*b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\sqrt{(bx+a)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/sqrt((b*x + a)^2), x)
```

Fricas [A] time = 1.71745, size = 644, normalized size = 3.04

$$\left[\frac{15\left(b^2d^2-2\,abde+a^2e^2\right)\sqrt{\frac{bd-ae}{b}}\log\left(\frac{bex+2\,bd-ae-2\,\sqrt{ex+d}b\sqrt{\frac{bd-ae}{b}}}{bx+a}\right)+2\left(3\,b^2e^2x^2+23\,b^2d^2-35\,abde+15\,a^2e^2+\left(11\,b^2de-5\right.\right.}{15\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/15*(15*(b^2*d^2 - 2*a*b*d*e + a^2*e^2)*sqrt((b*d - a*e)/b)*log((b*e*x +
2*b*d - a*e - 2*sqrt(e*x + d)*b*sqrt((b*d - a*e)/b))/(b*x + a)) + 2*(3*b^2*
e^2*x^2 + 23*b^2*d^2 - 35*a*b*d*e + 15*a^2*e^2 + (11*b^2*d*e - 5*a*b*e^2)*x
)*sqrt(e*x + d))/b^3, -2/15*(15*(b^2*d^2 - 2*a*b*d*e + a^2*e^2)*sqrt(-(b*d
- a*e)/b)*arctan(-sqrt(e*x + d)*b*sqrt(-(b*d - a*e)/b)/(b*d - a*e)) - (3*b^
2*e^2*x^2 + 23*b^2*d^2 - 35*a*b*d*e + 15*a^2*e^2 + (11*b^2*d*e - 5*a*b*e^2)
*x)*sqrt(e*x + d))/b^3]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/((b*x+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19106, size = 324, normalized size = 1.53

$$\frac{2\left(b^3d^3\mathrm{sgn}\left(bx+a\right)-3\,ab^2d^2e\mathrm{sgn}\left(bx+a\right)+3\,a^2bde^2\mathrm{sgn}\left(bx+a\right)-a^3e^3\mathrm{sgn}\left(bx+a\right)\right)\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)}{\sqrt{-b^2d+abe}b^3}+\frac{2\left(3\,(xe+d)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*(b^3*d^3*sgn(b*x + a) - 3*a*b^2*d^2*e*sgn(b*x + a) + 3*a^2*b*d*e^2*sgn(b*
x + a) - a^3*e^3*sgn(b*x + a))*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))
/(sqrt(-b^2*d + a*b*e)*b^3) + 2/15*(3*(x*e + d)^(5/2)*b^4*sgn(b*x + a) + 5*
(x*e + d)^(3/2)*b^4*d*sgn(b*x + a) + 15*sqrt(x*e + d)*b^4*d^2*sgn(b*x + a)
- 5*(x*e + d)^(3/2)*a*b^3*e*sgn(b*x + a) - 30*sqrt(x*e + d)*a*b^3*d*e*sgn(b
*x + a) + 15*sqrt(x*e + d)*a^2*b^2*e^2*sgn(b*x + a))/b^5
```

3.1705 $\int \frac{(d+ex)^{3/2}}{\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=161

$$\frac{2(a+bx)\sqrt{d+ex}(bd-ae)}{b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{2(a+bx)(d+ex)^{3/2}}{3b\sqrt{a^2+2abx+b^2x^2}}-\frac{2(a+bx)(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{5/2}\sqrt{a^2+2abx+b^2x^2}}$$

```
[Out] (2*(b*d - a*e)*(a + b*x)*Sqrt[d + e*x])/(b^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
 + (2*(a + b*x)*(d + e*x)^(3/2))/(3*b*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (2*(
b*d - a*e)^(3/2)*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]]
)/(b^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0780594, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {646, 50, 63, 208}

$$\frac{2(a+bx)\sqrt{d+ex}(bd-ae)}{b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{2(a+bx)(d+ex)^{3/2}}{3b\sqrt{a^2+2abx+b^2x^2}}-\frac{2(a+bx)(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{5/2}\sqrt{a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (2*(b*d - a*e)*(a + b*x)*Sqrt[d + e*x])/(b^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
 + (2*(a + b*x)*(d + e*x)^(3/2))/(3*b*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (2*(
b*d - a*e)^(3/2)*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]]
)/(b^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{\left(ab+b^2x\right)\int \frac{(d+ex)^{3/2}}{ab+b^2x}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$
$$= \frac{2(a+bx)(d+ex)^{3/2}}{3b\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(\left(b^2d-abe\right)\left(ab+b^2x\right)\right)\int \frac{\sqrt{d+ex}}{ab+b^2x}\,dx}{b^2\sqrt{a^2+2abx+b^2x^2}}$$
$$= \frac{2(bd-ae)(a+bx)\sqrt{d+ex}}{b^2\sqrt{a^2+2abx+b^2x^2}} + \frac{2(a+bx)(d+ex)^{3/2}}{3b\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(\left(b^2d-abe\right)^2\left(ab+b^2x\right)\right)\int \frac{1}{(ab+b^2x)}}{b^4\sqrt{a^2+2abx+b^2x^2}}$$
$$= \frac{2(bd-ae)(a+bx)\sqrt{d+ex}}{b^2\sqrt{a^2+2abx+b^2x^2}} + \frac{2(a+bx)(d+ex)^{3/2}}{3b\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(2\left(b^2d-abe\right)^2\left(ab+b^2x\right)\right)\text{Subst}}{b^4e\sqrt{a^2+2ab}}$$
$$= \frac{2(bd-ae)(a+bx)\sqrt{d+ex}}{b^2\sqrt{a^2+2abx+b^2x^2}} + \frac{2(a+bx)(d+ex)^{3/2}}{3b\sqrt{a^2+2abx+b^2x^2}} - \frac{2(bd-ae)^{3/2}(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d-}}{\sqrt{bd-}}\right.}{b^{5/2}\sqrt{a^2+2abx+b^2x^2}}$$

Mathematica [A] time = 0.0552392, size = 96, normalized size = 0.6

$$\frac{2(a+bx)\left(\sqrt{b}\sqrt{d+ex}(-3ae+4bd+bex)-3(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\right)}{3b^{5/2}\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (2*(a + b*x)*(Sqrt[b]*Sqrt[d + e*x]*(4*b*d - 3*a*e + b*e*x) - 3*(b*d - a*e)
^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]]))/(3*b^(5/2)*Sqrt[(
a + b*x)^2])
```

Maple [A] time = 0.233, size = 188, normalized size = 1.2

$$\frac{2\,bx+2\,a}{3\,b^2}\left(\sqrt{(ae-bd)\,b}\,(ex+d)^{\frac{3}{2}}b+3\,\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)\,b}}\right)a^2e^2-6\,\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)\,b}}\right)abde+3\,\arctan\left(\frac{b\sqrt{}}{\sqrt{(a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/((b*x+a)^2)^(1/2),x)
```

```
[Out] 2/3*(b*x+a)*(((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*b+3*arctan(b*(e*x+d)^(1/2)/(
(a*e-b*d)*b)^(1/2))*a^2*e^2-6*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a
*b*d*e+3*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*b^2*d^2-3*((a*e-b*d)*b
)^(1/2)*(e*x+d)^(1/2)*a*e+3*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*b*d)/((b*x+a)
^2)^(1/2)/b^2/((a*e-b*d)*b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{(bx+a)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/sqrt((b*x + a)^2), x)
```

Fricas [A] time = 1.68801, size = 424, normalized size = 2.63

$$\left[-\frac{3\,(bd-ae)\sqrt{\frac{bd-ae}{b}}\log\left(\frac{bex+2\,bd-ae+2\,\sqrt{ex+d}b\sqrt{\frac{bd-ae}{b}}}{bx+a}\right)-2\,(bex+4\,bd-3\,ae)\sqrt{ex+d}}{3\,b^2}, -\frac{2\left(3\,(bd-ae)\sqrt{-\frac{bd-ae}{b}}\arctan\left(-\frac{\sqrt{ex+}}{}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(b*d - a*e)*sqrt((b*d - a*e)/b)*log((b*e*x + 2*b*d - a*e + 2*sqrt(
e*x + d)*b*sqrt((b*d - a*e)/b))/(b*x + a)) - 2*(b*e*x + 4*b*d - 3*a*e)*sqrt
(e*x + d))/b^2, -2/3*(3*(b*d - a*e)*sqrt(-(b*d - a*e)/b)*arctan(-sqrt(e*x +
 d)*b*sqrt(-(b*d - a*e)/b)/(b*d - a*e)) - (b*e*x + 4*b*d - 3*a*e)*sqrt(e*x
+ d))/b^2]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/((b*x+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19885, size = 200, normalized size = 1.24

$$\frac{2\left(b^2d^2\mathrm{sgn}\,(bx+a)-2\,abde\mathrm{sgn}\,(bx+a)+a^2e^2\mathrm{sgn}\,(bx+a)\right)\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)}{\sqrt{-b^2d+abe}b^2}+\frac{2\left((xe+d)^{\frac{3}{2}}b^2\mathrm{sgn}\,(bx+a)+3\,\sqrt{xe+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*(b^2*d^2*sgn(b*x + a) - 2*a*b*d*e*sgn(b*x + a) + a^2*e^2*sgn(b*x + a))*ar
ctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^2) + 2/3
*((x*e + d)^(3/2)*b^2*sgn(b*x + a) + 3*sqrt(x*e + d)*b^2*d*sgn(b*x + a) - 3
*sqrt(x*e + d)*a*b*e*sgn(b*x + a))/b^3
```

3.1706 $\int \frac{\sqrt{d+ex}}{\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=112

$$\frac{2(a+bx)\sqrt{d+ex}}{b\sqrt{a^2+2abx+b^2x^2}}-\frac{2(a+bx)\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}\sqrt{a^2+2abx+b^2x^2}}$$

```
[Out] (2*(a + b*x)*Sqrt[d + e*x])/(b*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (2*Sqrt[b*d
 - a*e]*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(b^(3/2
)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0532999, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {646, 50, 63, 208}

$$\frac{2(a+bx)\sqrt{d+ex}}{b\sqrt{a^2+2abx+b^2x^2}}-\frac{2(a+bx)\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}\sqrt{a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (2*(a + b*x)*Sqrt[d + e*x])/(b*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (2*Sqrt[b*d
 - a*e]*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(b^(3/2
)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d+ex}}{\sqrt{a^2+2abx+b^2x^2}}\,dx &= \frac{\left(ab+b^2x\right)\int \frac{\sqrt{d+ex}}{ab+b^2x}\,dx}{\sqrt{a^2+2abx+b^2x^2}}\\
&= \frac{2(a+bx)\sqrt{d+ex}}{b\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(\left(b^2d-abe\right)\left(ab+b^2x\right)\right)\int \frac{1}{\left(ab+b^2x\right)\sqrt{d+ex}}\,dx}{b^2\sqrt{a^2+2abx+b^2x^2}}\\
&= \frac{2(a+bx)\sqrt{d+ex}}{b\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(2\left(b^2d-abe\right)\left(ab+b^2x\right)\right)\text{Subst}\left(\int \frac{1}{ab-\frac{b^2d}{e}+\frac{b^2x^2}{e}}\,dx, x, \sqrt{d+ex}\right)}{b^2e\sqrt{a^2+2abx+b^2x^2}}\\
&= \frac{2(a+bx)\sqrt{d+ex}}{b\sqrt{a^2+2abx+b^2x^2}} - \frac{2\sqrt{bd-ae}(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}\sqrt{a^2+2abx+b^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.0209021, size = 81, normalized size = 0.72

$$\frac{2(a+bx)\left(\sqrt{b}\sqrt{d+ex}-\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)\right)}{b^{3/2}\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (2*(a + b*x)*(Sqrt[b]*Sqrt[d + e*x] - Sqrt[b*d - a*e]*ArcTanh[(Sqrt[b]*Sqrt
[d + e*x])/Sqrt[b*d - a*e]]))/(b^(3/2)*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.23, size = 104, normalized size = 0.9

$$2\frac{bx+a}{\sqrt{(bx+a)^2}b\sqrt{(ae-bd)b}}\left(-\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)b}}\right)ae+\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)b}}\right)bd+\sqrt{ex+d}\sqrt{(ae-bd)b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/((b*x+a)^2)^(1/2),x)
```

```
[Out] 2*(b*x+a)*(-arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a*e+arctan(b*(e*x+d
)^(1/2)/((a*e-b*d)*b)^(1/2))*b*d+(e*x+d)^(1/2)*((a*e-b*d)*b)^(1/2))/((b*x+a
)^2)^(1/2)/b/((a*e-b*d)*b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{(bx+a)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/sqrt((b*x + a)^2), x)
```

Fricas [A] time = 1.65198, size = 306, normalized size = 2.73

$$\left[\frac{\sqrt{\frac{bd-ae}{b}}\log\left(\frac{bex+2\,bd-ae-2\,\sqrt{ex+d}b\sqrt{\frac{bd-ae}{b}}}{bx+a}\right)+2\,\sqrt{ex+d}}{b}, -\frac{2\left(\sqrt{-\frac{bd-ae}{b}}\arctan\left(-\frac{\sqrt{ex+d}b\sqrt{-\frac{bd-ae}{b}}}{bd-ae}\right)-\sqrt{ex+d}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt((b*d - a*e)/b)*log((b*e*x + 2*b*d - a*e - 2*sqrt(e*x + d)*b*sqrt((b*
d - a*e)/b))/(b*x + a)) + 2*sqrt(e*x + d))/b, -2*(sqrt(-(b*d - a*e)/b)*arct
an(-sqrt(e*x + d)*b*sqrt(-(b*d - a*e)/b)/(b*d - a*e)) - sqrt(e*x + d))/b]
```

Sympy [A] time = 9.13563, size = 95, normalized size = 0.85

$$\sqrt{-\frac{ae}{b^3}+\frac{d}{b^2}}\log\left(-b\sqrt{-\frac{ae}{b^3}+\frac{d}{b^2}}+\sqrt{d+ex}\right)-\sqrt{-\frac{ae}{b^3}+\frac{d}{b^2}}\log\left(b\sqrt{-\frac{ae}{b^3}+\frac{d}{b^2}}+\sqrt{d+ex}\right)+\frac{2\sqrt{d+ex}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/((b*x+a)**2)**(1/2),x)
```

```
[Out] sqrt(-a*e/b**3 + d/b**2)*log(-b*sqrt(-a*e/b**3 + d/b**2) + sqrt(d + e*x)) -
 sqrt(-a*e/b**3 + d/b**2)*log(b*sqrt(-a*e/b**3 + d/b**2) + sqrt(d + e*x)) +
 2*sqrt(d + e*x)/b
```

Giac [A] time = 1.14444, size = 99, normalized size = 0.88

$$2\left(\frac{(bd-ae)\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)}{\sqrt{-b^2d+abeb}}+\frac{\sqrt{xe+d}}{b}\right)\mathrm{sgn}\left(bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*((b*d - a*e)*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d +
a*b*e)*b) + sqrt(x*e + d)/b)*sgn(b*x + a)
```

3.1707 $\int \frac{1}{\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=72

$$-\frac{2(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{\sqrt{b}\sqrt{a^2+2abx+b^2x^2}\sqrt{bd-ae}}$$

```
[Out] (-2*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(Sqrt[b]*Sq
rt[b*d - a*e]*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0393768, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {646, 63, 208}

$$-\frac{2(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{\sqrt{b}\sqrt{a^2+2abx+b^2x^2}\sqrt{bd-ae}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```


```
[Out] (-2*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(Sqrt[b]*Sq
rt[b*d - a*e]*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{\left(ab+b^2x\right)\int \frac{1}{\left(ab+b^2x\right)\sqrt{d+ex}}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$
$$= \frac{\left(2\left(ab+b^2x\right)\right)\text{Subst}\left(\int \frac{1}{ab-\frac{b^2d}{e}+\frac{b^2x^2}{e}}\,dx, x, \sqrt{d+ex}\right)}{e\sqrt{a^2+2abx+b^2x^2}}$$
$$= -\frac{2(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{\sqrt{b}\sqrt{bd-ae}\sqrt{a^2+2abx+b^2x^2}}$$

Mathematica [A] time = 0.0194446, size = 63, normalized size = 0.88

$$-\frac{2(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{\sqrt{b}\sqrt{(a+bx)^2}\sqrt{bd-ae}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

```
[Out] (-2*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(Sqrt[b]*Sq
rt[b*d - a*e]*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.23, size = 51, normalized size = 0.7

$$2\,\frac{bx+a}{\sqrt{(bx+a)^2}\sqrt{(ae-bd)\,b}}\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)\,b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/((b*x+a)^2)^(1/2),x)
```

```
[Out] 2/((b*x+a)^2)^(1/2)*(b*x+a)/((a*e-b*d)*b)^(1/2)*arctan(b*(e*x+d)^(1/2)/((a*
e-b*d)*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(bx+a)^2}\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((b*x + a)^2)*sqrt(e*x + d)), x)
```

Fricas [A] time = 1.70336, size = 266, normalized size = 3.69

$$\left[\frac{\log\left(\frac{bex+2\,bd-ae-2\,\sqrt{b^2d-abe}\sqrt{ex+d}}{bx+a}\right)}{\sqrt{b^2d-abe}}, \frac{2\,\sqrt{-b^2d+abe}\arctan\left(\frac{\sqrt{-b^2d+abe}\sqrt{ex+d}}{bex+bd}\right)}{b^2d-abe}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*e*x + 2*b*d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d))/(b*x + a))
/sqrt(b^2*d - a*b*e), 2*sqrt(-b^2*d + a*b*e)*arctan(sqrt(-b^2*d + a*b*e)*sq
rt(e*x + d)/(b*e*x + b*d))/(b^2*d - a*b*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d+ex}\sqrt{(a+bx)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/((b*x+a)**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d + e*x)*sqrt((a + b*x)**2)), x)
```

Giac [A] time = 1.14415, size = 63, normalized size = 0.88

$$\frac{2\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)\operatorname{sgn}(bx+a)}{\sqrt{-b^2d+abe}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*sgn(b*x + a)/sqrt(-b^2*d + a
*b*e)
```

3.1708 $\int \frac{1}{(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=119

$$\frac{2(a+bx)}{\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-ae)}-\frac{2\sqrt{b}(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^{3/2}}$$

```
[Out] (2*(a + b*x))/((b*d - a*e)*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (
2*Sqrt[b]*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/((b*d
 - a*e)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0666665, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {646, 51, 63, 208}

$$\frac{2(a+bx)}{\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-ae)}-\frac{2\sqrt{b}(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

```
[Out] (2*(a + b*x))/((b*d - a*e)*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (
2*Sqrt[b]*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/((b*d
 - a*e)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{(ab+b^2x)\int \frac{1}{(ab+b^2x)(d+ex)^{3/2}}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$
$$= \frac{2(a+bx)}{(bd-ae)\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(b\left(ab+b^2x\right)\right)\int \frac{1}{(ab+b^2x)\sqrt{d+ex}}\,dx}{(bd-ae)\sqrt{a^2+2abx+b^2x^2}}$$
$$= \frac{2(a+bx)}{(bd-ae)\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(2b\left(ab+b^2x\right)\right)\text{Subst}\left(\int \frac{1}{ab-\frac{b^2d}{e}+\frac{b^2x^2}{e}}\,dx, x,\right.}{e(bd-ae)\sqrt{a^2+2abx+b^2x^2}}$$
$$= \frac{2(a+bx)}{(bd-ae)\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} - \frac{2\sqrt{b}(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{3/2}\sqrt{a^2+2abx+b^2x^2}}$$

Mathematica [C] time = 0.0165321, size = 62, normalized size = 0.52

$$-\frac{2(a+bx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b(d+ex)}{bd-ae}\right)}{\sqrt{(a+bx)^2}\sqrt{d+ex}(ae-bd)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

```
[Out] (-2*(a + b*x)*Hypergeometric2F1[-1/2, 1, 1/2, (b*(d + e*x))/(b*d - a*e)])/(
(-(b*d) + a*e)*Sqrt[(a + b*x)^2]*Sqrt[d + e*x])
```

Maple [A] time = 0.233, size = 90, normalized size = 0.8

$$-2\,\frac{bx+a}{\sqrt{(bx+a)^2}\,(ae-bd)\,\sqrt{(ae-bd)\,b}\sqrt{ex+d}}\left(b\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)\,b}}\right)\sqrt{ex+d}+\sqrt{(ae-bd)\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/((b*x+a)^2)^(1/2),x)
```

```
[Out] -2*(b*x+a)*(b*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*(e*x+d)^(1/2)+((a
*e-b*d)*b)^(1/2))/((b*x+a)^2)^(1/2)/(a*e-b*d)/((a*e-b*d)*b)^(1/2)/(e*x+d)^(
1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(bx+a)^2}(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((b*x + a)^2)*(e*x + d)^(3/2)), x)
```

Fricas [A] time = 1.65611, size = 456, normalized size = 3.83

$$\left[-\frac{(ex+d)\sqrt{\frac{b}{bd-ae}}\log\left(\frac{bex+2\,bd-ae+2\,(bd-ae)\sqrt{ex+d}\sqrt{\frac{b}{bd-ae}}}{bx+a}\right)-2\,\sqrt{ex+d}}{bd^2-ade+\left(bde-ae^2\right)x}, -\frac{2\left((ex+d)\sqrt{-\frac{b}{bd-ae}}\arctan\left(-\frac{(bd-ae)\sqrt{ex+d}\sqrt{-\frac{b}{bd-ae}}}{bex+bd}\right)\right.}{bd^2-ade+\left(bde-ae^2\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-((e*x + d)*sqrt(b/(b*d - a*e))*log((b*e*x + 2*b*d - a*e + 2*(b*d - a*e)*s
qrt(e*x + d)*sqrt(b/(b*d - a*e)))/(b*x + a)) - 2*sqrt(e*x + d))/(b*d^2 - a*
d*e + (b*d*e - a*e^2)*x), -2*((e*x + d)*sqrt(-b/(b*d - a*e))*arctan(-(b*d -
 a*e)*sqrt(e*x + d)*sqrt(-b/(b*d - a*e))/(b*e*x + b*d)) - sqrt(e*x + d))/(b
*d^2 - a*d*e + (b*d*e - a*e^2)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)^{\frac{3}{2}}\sqrt{(a+bx)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/((b*x+a)**2)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x)**(3/2)*sqrt((a + b*x)**2)), x)
```

Giac [A] time = 1.17445, size = 109, normalized size = 0.92

$$2\left(\frac{b\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)}{\sqrt{-b^2d+abe}(bd-ae)}+\frac{1}{(bd-ae)\sqrt{xe+d}}\right)\mathrm{sgn}\,(bx+a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*(b*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*(b*
d - a*e)) + 1/((b*d - a*e)*sqrt(x*e + d)))*sgn(b*x + a)
```

3.1709 $\int \frac{1}{(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=168

$$\frac{2b(a+bx)}{\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-ae)^2}+\frac{2(a+bx)}{3\sqrt{a^2+2abx+b^2x^2}(d+ex)^{3/2}(bd-ae)}-\frac{2b^{3/2}(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^{5/2}}$$

```
[Out] (2*(a + b*x))/(3*(b*d - a*e)*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
 + (2*b*(a + b*x))/((b*d - a*e)^2*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^
2]) - (2*b^(3/2)*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]]
)/((b*d - a*e)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0708907, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {646, 51, 63, 208}

$$\frac{2b(a+bx)}{\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-ae)^2}+\frac{2(a+bx)}{3\sqrt{a^2+2abx+b^2x^2}(d+ex)^{3/2}(bd-ae)}-\frac{2b^{3/2}(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

```
[Out] (2*(a + b*x))/(3*(b*d - a*e)*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
 + (2*b*(a + b*x))/((b*d - a*e)^2*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^
2]) - (2*b^(3/2)*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]]
)/((b*d - a*e)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{\left(ab+b^2x\right)\int \frac{1}{\left(ab+b^2x\right)(d+ex)^{5/2}}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= \frac{2(a+bx)}{3(bd-ae)(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(b\left(ab+b^2x\right)\right)\int \frac{1}{\left(ab+b^2x\right)(d+ex)^{3/2}}\,dx}{(bd-ae)\sqrt{a^2+2abx+b^2x^2}}$$

$$= \frac{2(a+bx)}{3(bd-ae)(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}} + \frac{2b(a+bx)}{(bd-ae)^2\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}}$$

$$= \frac{2(a+bx)}{3(bd-ae)(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}} + \frac{2b(a+bx)}{(bd-ae)^2\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}}$$

$$= \frac{2(a+bx)}{3(bd-ae)(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}} + \frac{2b(a+bx)}{(bd-ae)^2\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}}$$

Mathematica [C] time = 0.0173199, size = 64, normalized size = 0.38

$$\frac{2(a+bx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b(d+ex)}{bd-ae}\right)}{3\sqrt{(a+bx)^2}(d+ex)^{3/2}(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

```
[Out] (2*(a + b*x)*Hypergeometric2F1[-3/2, 1, -1/2, (b*(d + e*x))/(b*d - a*e)])/(
3*(b*d - a*e)*Sqrt[(a + b*x)^2]*(d + e*x)^(3/2))
```

Maple [A] time = 0.249, size = 130, normalized size = 0.8

$$\frac{2\,bx+2\,a}{3\,(ae-bd)^2}\left(3\,b^2\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)\,b}}\right)(ex+d)^{3/2}+3\,\sqrt{(ae-bd)\,b}xbe-\sqrt{(ae-bd)\,b}ae+4\,\sqrt{(ae-bd)\,b}bd\right)(ex+d)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(5/2)/((b*x+a)^2)^(1/2),x)
```

```
[Out] 2/3*(b*x+a)*(3*b^2*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*(e*x+d)^(3/2
)+3*((a*e-b*d)*b)^(1/2)*x*b*e-((a*e-b*d)*b)^(1/2)*a*e+4*((a*e-b*d)*b)^(1/2)
*b*d)/((b*x+a)^2)^(1/2)/(a*e-b*d)^2/(e*x+d)^(3/2)/((a*e-b*d)*b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(bx+a)^2}(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((b*x + a)^2)*(e*x + d)^(5/2)), x)
```

Fricas [A] time = 1.59893, size = 841, normalized size = 5.01

$$\left[\frac{3\left(be^2x^2+2\,bdex+bd^2\right)\sqrt{\frac{b}{bd-ae}}\log\left(\frac{bex+2\,bd-ae-2\,(bd-ae)\sqrt{ex+d}\sqrt{\frac{b}{bd-ae}}}{bx+a}\right)+2\,(3\,bex+4\,bd-ae)\sqrt{ex+d}}{3\left(b^2d^4-2\,abd^3e+a^2d^2e^2+\left(b^2d^2e^2-2\,abde^3+a^2e^4\right)x^2+2\left(b^2d^3e-2\,abd^2e^2+a^2de^3\right)x\right)},-\frac{2\left(3\left(be^2x^2+2\,bdex\right.\right.}{3\left(b^2d^4-2\,abd^3e\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b*e^2*x^2 + 2*b*d*e*x + b*d^2)*sqrt(b/(b*d - a*e))*log((b*e*x + 2*
b*d - a*e - 2*(b*d - a*e)*sqrt(e*x + d)*sqrt(b/(b*d - a*e)))/(b*x + a)) + 2
*(3*b*e*x + 4*b*d - a*e)*sqrt(e*x + d))/(b^2*d^4 - 2*a*b*d^3*e + a^2*d^2*e^
2 + (b^2*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4)*x^2 + 2*(b^2*d^3*e - 2*a*b*d^2*e^
2 + a^2*d*e^3)*x), -2/3*(3*(b*e^2*x^2 + 2*b*d*e*x + b*d^2)*sqrt(-b/(b*d - a
*e))*arctan(-(b*d - a*e)*sqrt(e*x + d)*sqrt(-b/(b*d - a*e))/(b*e*x + b*d))
- (3*b*e*x + 4*b*d - a*e)*sqrt(e*x + d))/(b^2*d^4 - 2*a*b*d^3*e + a^2*d^2*e
^2 + (b^2*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4)*x^2 + 2*(b^2*d^3*e - 2*a*b*d^2*e
^2 + a^2*d*e^3)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(5/2)/((b*x+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22782, size = 170, normalized size = 1.01

$$\frac{2}{3}\left(\frac{3\,b^2\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)}{\left(b^2d^2-2\,abde+a^2e^2\right)\sqrt{-b^2d+abe}}+\frac{3\,(xe+d)b+bd-ae}{\left(b^2d^2-2\,abde+a^2e^2\right)(xe+d)^{\frac{3}{2}}}\right)\mathrm{sgn}\,(bx+a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*b^2*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))/((b^2*d^2 - 2*a*b*d
*e + a^2*e^2)*sqrt(-b^2*d + a*b*e)) + (3*(x*e + d)*b + b*d - a*e)/((b^2*d^2
 - 2*a*b*d*e + a^2*e^2)*(x*e + d)^(3/2)))*sgn(b*x + a)
```

3.1710 $\int \frac{1}{(d+ex)^{7/2}\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=219

$$\frac{2b^2(a+bx)}{\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-ae)^3}+\frac{2b(a+bx)}{3\sqrt{a^2+2abx+b^2x^2}(d+ex)^{3/2}(bd-ae)^2}+\frac{2(a+bx)}{5\sqrt{a^2+2abx+b^2x^2}(d+ex)^{5/2}(bd}$$

```
[Out] (2*(a + b*x))/(5*(b*d - a*e)*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
 + (2*b*(a + b*x))/(3*(b*d - a*e)^2*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^
2*x^2]) + (2*b^2*(a + b*x))/((b*d - a*e)^3*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x
 + b^2*x^2]) - (2*b^(5/2)*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*
d - a*e]])/((b*d - a*e)^(7/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0914409, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {646, 51, 63, 208}

$$\frac{2b^2(a+bx)}{\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-ae)^3}+\frac{2b(a+bx)}{3\sqrt{a^2+2abx+b^2x^2}(d+ex)^{3/2}(bd-ae)^2}+\frac{2(a+bx)}{5\sqrt{a^2+2abx+b^2x^2}(d+ex)^{5/2}(bd}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(7/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

```
[Out] (2*(a + b*x))/(5*(b*d - a*e)*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
 + (2*b*(a + b*x))/(3*(b*d - a*e)^2*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^
2*x^2]) + (2*b^2*(a + b*x))/((b*d - a*e)^3*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x
 + b^2*x^2]) - (2*b^(5/2)*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*
d - a*e]])/((b*d - a*e)^(7/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{7/2}\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{\left(ab+b^2x\right)\int \frac{1}{\left(ab+b^2x\right)(d+ex)^{7/2}}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= \frac{2(a+bx)}{5(bd-ae)(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(b\left(ab+b^2x\right)\right)\int \frac{1}{\left(ab+b^2x\right)(d+ex)^{5/2}}\,dx}{(bd-ae)\sqrt{a^2+2abx+b^2x^2}}$$

$$= \frac{2(a+bx)}{5(bd-ae)(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}} + \frac{2b(a+bx)}{3(bd-ae)^2(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x}}$$

$$= \frac{2(a+bx)}{5(bd-ae)(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}} + \frac{2b(a+bx)}{3(bd-ae)^2(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x}}$$

$$= \frac{2(a+bx)}{5(bd-ae)(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}} + \frac{2b(a+bx)}{3(bd-ae)^2(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x}}$$

$$= \frac{2(a+bx)}{5(bd-ae)(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}} + \frac{2b(a+bx)}{3(bd-ae)^2(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x}}$$

Mathematica [C] time = 0.0202933, size = 64, normalized size = 0.29

$$\frac{2(a+bx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\frac{b(d+ex)}{bd-ae}\right)}{5\sqrt{(a+bx)^2}(d+ex)^{5/2}(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(7/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

```
[Out] (2*(a + b*x)*Hypergeometric2F1[-5/2, 1, -3/2, (b*(d + e*x))/(b*d - a*e)])/(
5*(b*d - a*e)*Sqrt[(a + b*x)^2]*(d + e*x)^(5/2))
```

Maple [A] time = 0.266, size = 202, normalized size = 0.9

$$-\frac{2\,bx+2\,a}{15\,(ae-bd)^3}\left(15\,b^3\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)\,b}}\right)(ex+d)^{5/2}+15\,\sqrt{(ae-bd)\,b}x^2b^2e^2-5\,\sqrt{(ae-bd)\,b}xabe^2+35\,\sqrt{(ae-bd)\,b}x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(7/2)/((b*x+a)^2)^(1/2),x)
```

```
[Out] -2/15*(b*x+a)*(15*b^3*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*(e*x+d)^(
5/2)+15*((a*e-b*d)*b)^(1/2)*x^2*b^2*e^2-5*((a*e-b*d)*b)^(1/2)*x*a*b*e^2+35*
((a*e-b*d)*b)^(1/2)*x*b^2*d*e+3*((a*e-b*d)*b)^(1/2)*a^2*e^2-11*((a*e-b*d)*b
)^(1/2)*a*b*d*e+23*((a*e-b*d)*b)^(1/2)*b^2*d^2)/((b*x+a)^2)^(1/2)/(a*e-b*d)
```

```
^3/(e*x+d)^(5/2)/((a*e-b*d)*b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(bx+a)^2}(ex+d)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((b*x + a)^2)*(e*x + d)^(7/2)), x)
```

Fricas [B] time = 1.7733, size = 1434, normalized size = 6.55

$$\left[-\frac{15\left(b^2e^3x^3+3\,b^2de^2x^2+3\,b^2d^2ex+b^2d^3\right)\sqrt{\frac{b}{bd-ae}}\log\left(\frac{bex+2\,bd-ae+2\,(bd-ae)\sqrt{ex+d}\sqrt{\frac{b}{bd-ae}}}{bx+a}\right)-2\left(15\,b^2e^2x^2+23\,b\right.}{15\left(b^3d^6-3\,ab^2d^5e+3\,a^2bd^4e^2-a^3d^3e^3+\left(b^3d^3e^3-3\,ab^2d^2e^4+3\,a^2bde^5-a^3e^6\right)x^3+3\left(b^3d^4e^2-3\,ab^2d^3e^3+3\,a^2b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/15*(15*(b^2*e^3*x^3 + 3*b^2*d*e^2*x^2 + 3*b^2*d^2*e*x + b^2*d^3)*sqrt(b
/(b*d - a*e))*log((b*e*x + 2*b*d - a*e + 2*(b*d - a*e)*sqrt(e*x + d)*sqrt(b
/(b*d - a*e)))/(b*x + a)) - 2*(15*b^2*e^2*x^2 + 23*b^2*d^2 - 11*a*b*d*e + 3
*a^2*e^2 + 5*(7*b^2*d*e - a*b*e^2)*x)*sqrt(e*x + d))/(b^3*d^6 - 3*a*b^2*d^5
*e + 3*a^2*b*d^4*e^2 - a^3*d^3*e^3 + (b^3*d^3*e^3 - 3*a*b^2*d^2*e^4 + 3*a^2
*b*d*e^5 - a^3*e^6)*x^3 + 3*(b^3*d^4*e^2 - 3*a*b^2*d^3*e^3 + 3*a^2*b*d^2*e^
4 - a^3*d*e^5)*x^2 + 3*(b^3*d^5*e - 3*a*b^2*d^4*e^2 + 3*a^2*b*d^3*e^3 - a^3
*d^2*e^4)*x), -2/15*(15*(b^2*e^3*x^3 + 3*b^2*d*e^2*x^2 + 3*b^2*d^2*e*x + b^
2*d^3)*sqrt(-b/(b*d - a*e))*arctan(-(b*d - a*e)*sqrt(e*x + d)*sqrt(-b/(b*d
- a*e))/(b*e*x + b*d)) - (15*b^2*e^2*x^2 + 23*b^2*d^2 - 11*a*b*d*e + 3*a^2*
e^2 + 5*(7*b^2*d*e - a*b*e^2)*x)*sqrt(e*x + d))/(b^3*d^6 - 3*a*b^2*d^5*e +
3*a^2*b*d^4*e^2 - a^3*d^3*e^3 + (b^3*d^3*e^3 - 3*a*b^2*d^2*e^4 + 3*a^2*b*d*
e^5 - a^3*e^6)*x^3 + 3*(b^3*d^4*e^2 - 3*a*b^2*d^3*e^3 + 3*a^2*b*d^2*e^4 - a
^3*d*e^5)*x^2 + 3*(b^3*d^5*e - 3*a*b^2*d^4*e^2 + 3*a^2*b*d^3*e^3 - a^3*d^2*
e^4)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(7/2)/((b*x+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14513, size = 265, normalized size = 1.21

$$\frac{2}{15}\left(\frac{15\,b^3\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)}{\left(b^3d^3-3\,ab^2d^2e+3\,a^2bde^2-a^3e^3\right)\sqrt{-b^2d+abe}}+\frac{15\,(xe+d)^2b^2+5\,(xe+d)b^2d+3\,b^2d^2-5\,(xe+d)abe-6\,abde+}{\left(b^3d^3-3\,ab^2d^2e+3\,a^2bde^2-a^3e^3\right)(xe+d)^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*b^3*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))/((b^3*d^3 - 3*a*b
^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3)*sqrt(-b^2*d + a*b*e)) + (15*(x*e + d)^2
*b^2 + 5*(x*e + d)*b^2*d + 3*b^2*d^2 - 5*(x*e + d)*a*b*e - 6*a*b*d*e + 3*a^
2*e^2)/((b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3)*(x*e + d)^(5/2)
))*sgn(b*x + a)
```

3.1711 $\int \frac{(d+ex)^{9/2}}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=308

$$\frac{63e^2(a+bx)(d+ex)^{5/2}}{20b^3\sqrt{a^2+2abx+b^2x^2}}+\frac{21e^2(a+bx)(d+ex)^{3/2}(bd-ae)}{4b^4\sqrt{a^2+2abx+b^2x^2}}+\frac{63e^2(a+bx)\sqrt{d+ex}(bd-ae)^2}{4b^5\sqrt{a^2+2abx+b^2x^2}}-\frac{63e^2(a+bx)(bd-ae)^5}{4b^{11/2}\sqrt{a^2+2}}$$

```
[Out] (63*e^2*(b*d - a*e)^2*(a + b*x)*Sqrt[d + e*x])/(4*b^5*Sqrt[a^2 + 2*a*b*x +
b^2*x^2]) + (21*e^2*(b*d - a*e)*(a + b*x)*(d + e*x)^(3/2))/(4*b^4*Sqrt[a^2
+ 2*a*b*x + b^2*x^2]) + (63*e^2*(a + b*x)*(d + e*x)^(5/2))/(20*b^3*Sqrt[a^2
 + 2*a*b*x + b^2*x^2]) - (9*e*(d + e*x)^(7/2))/(4*b^2*Sqrt[a^2 + 2*a*b*x +
b^2*x^2]) - (d + e*x)^(9/2)/(2*b*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) -
 (63*e^2*(b*d - a*e)^(5/2)*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b
*d - a*e]])/(4*b^(11/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.174307, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {646, 47, 50, 63, 208}

$$\frac{63e^2(a+bx)(d+ex)^{5/2}}{20b^3\sqrt{a^2+2abx+b^2x^2}}+\frac{21e^2(a+bx)(d+ex)^{3/2}(bd-ae)}{4b^4\sqrt{a^2+2abx+b^2x^2}}+\frac{63e^2(a+bx)\sqrt{d+ex}(bd-ae)^2}{4b^5\sqrt{a^2+2abx+b^2x^2}}-\frac{63e^2(a+bx)(bd-ae)^5}{4b^{11/2}\sqrt{a^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(9/2)/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```


```
[Out] (63*e^2*(b*d - a*e)^2*(a + b*x)*Sqrt[d + e*x])/(4*b^5*Sqrt[a^2 + 2*a*b*x +
b^2*x^2]) + (21*e^2*(b*d - a*e)*(a + b*x)*(d + e*x)^(3/2))/(4*b^4*Sqrt[a^2
+ 2*a*b*x + b^2*x^2]) + (63*e^2*(a + b*x)*(d + e*x)^(5/2))/(20*b^3*Sqrt[a^2
 + 2*a*b*x + b^2*x^2]) - (9*e*(d + e*x)^(7/2))/(4*b^2*Sqrt[a^2 + 2*a*b*x +
b^2*x^2]) - (d + e*x)^(9/2)/(2*b*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) -
 (63*e^2*(b*d - a*e)^(5/2)*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b
*d - a*e]])/(4*b^(11/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{9/2}}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x\right)\right)\int \frac{(d+ex)^{9/2}}{\left(ab+b^2x\right)^3}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{(d+ex)^{9/2}}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(9e\left(ab+b^2x\right)\right)\int \frac{(d+ex)^{7/2}}{\left(ab+b^2x\right)^2}\,dx}{4\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{9e(d+ex)^{7/2}}{4b^2\sqrt{a^2+2abx+b^2x^2}} - \frac{(d+ex)^{9/2}}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(63e^2\left(ab+b^2x\right)\right)\int \frac{(d+ex)^{5/2}}{ab+b^2x}\,dx}{8b^2\sqrt{a^2+2abx+b^2x^2}}$$

$$= \frac{63e^2(a+bx)(d+ex)^{5/2}}{20b^3\sqrt{a^2+2abx+b^2x^2}} - \frac{9e(d+ex)^{7/2}}{4b^2\sqrt{a^2+2abx+b^2x^2}} - \frac{(d+ex)^{9/2}}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{(63e}{}$$

$$= \frac{21e^2(bd-ae)(a+bx)(d+ex)^{3/2}}{4b^4\sqrt{a^2+2abx+b^2x^2}} + \frac{63e^2(a+bx)(d+ex)^{5/2}}{20b^3\sqrt{a^2+2abx+b^2x^2}} - \frac{9e(d+ex)^{7/2}}{4b^2\sqrt{a^2+2abx+b^2x^2}} - \frac{}{2b}$$

$$= \frac{63e^2(bd-ae)^2(a+bx)\sqrt{d+ex}}{4b^5\sqrt{a^2+2abx+b^2x^2}} + \frac{21e^2(bd-ae)(a+bx)(d+ex)^{3/2}}{4b^4\sqrt{a^2+2abx+b^2x^2}} + \frac{63e^2(a+bx)(d+ex)^{}}{20b^3\sqrt{a^2+2abx+b^2}}$$

$$= \frac{63e^2(bd-ae)^2(a+bx)\sqrt{d+ex}}{4b^5\sqrt{a^2+2abx+b^2x^2}} + \frac{21e^2(bd-ae)(a+bx)(d+ex)^{3/2}}{4b^4\sqrt{a^2+2abx+b^2x^2}} + \frac{63e^2(a+bx)(d+ex)^{}}{20b^3\sqrt{a^2+2abx+b^2}}$$

$$= \frac{63e^2(bd-ae)^2(a+bx)\sqrt{d+ex}}{4b^5\sqrt{a^2+2abx+b^2x^2}} + \frac{21e^2(bd-ae)(a+bx)(d+ex)^{3/2}}{4b^4\sqrt{a^2+2abx+b^2x^2}} + \frac{63e^2(a+bx)(d+ex)^{}}{20b^3\sqrt{a^2+2abx+b^2}}$$

Mathematica [C] time = 0.0461776, size = 67, normalized size = 0.22

$$-\frac{2e^2(a+bx)(d+ex)^{11/2}\,{}_2F_1\left(3,\frac{11}{2};\frac{13}{2};\frac{b(d+ex)}{bd-ae}\right)}{11\sqrt{(a+bx)^2}(bd-ae)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(9/2)/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (-2*e^2*(a + b*x)*(d + e*x)^(11/2)*Hypergeometric2F1[3, 11/2, 13/2, (b*(d +
 e*x))/(b*d - a*e)])/(11*(b*d - a*e)^3*Sqrt[(a + b*x)^2])
```

Maple [B] time = 0.282, size = 1115, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] 1/20*(-480*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*x^2*a*b^3*d*e^3-960*((a*e-b*d)
*b)^(1/2)*(e*x+d)^(1/2)*x*a^2*b^2*d*e^3+480*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/
2)*x*a*b^3*d^2*e^2+80*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*x*a*b^3*d*e^2+1890*
arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x*a^3*b^2*d*e^4-1890*arctan(b*(
e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x*a^2*b^3*d^2*e^3+630*arctan(b*(e*x+d)^(1
/2)/((a*e-b*d)*b)^(1/2))*x*a*b^4*d^3*e^2-215*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3
/2)*a^2*b^2*d*e^2+255*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*a*b^3*d^2*e+16*((a*
e-b*d)*b)^(1/2)*(e*x+d)^(5/2)*x*a*b^3*e^2-40*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3
/2)*x^2*a*b^3*e^3+315*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*a^4*e^4+75*((a*e-b*
d)*b)^(1/2)*(e*x+d)^(1/2)*b^4*d^4-85*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*b^4*
d^3+240*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*x^2*a^2*b^2*e^4+240*((a*e-b*d)*b)
^(1/2)*(e*x+d)^(1/2)*x^2*b^4*d^2*e^2-315*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*
b)^(1/2))*a^5*e^5-630*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x*a^4*b*e
^5+45*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*a^3*b*e^3+945*arctan(b*(e*x+d)^(1/2
)/((a*e-b*d)*b)^(1/2))*a^4*b*d*e^4-945*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)
^(1/2))*a^3*b^2*d^2*e^3+315*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a^2
*b^3*d^3*e^2-315*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^2*a^3*b^2*e^
5+315*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^2*b^5*d^3*e^2+8*((a*e-b
*d)*b)^(1/2)*(e*x+d)^(5/2)*a^2*b^2*e^2+8*((a*e-b*d)*b)^(1/2)*(e*x+d)^(5/2)*
x^2*b^4*e^2+40*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*x^2*b^4*d*e^2+945*arctan(b
*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^2*a^2*b^3*d*e^4-945*arctan(b*(e*x+d)^
(1/2)/((a*e-b*d)*b)^(1/2))*x^2*a*b^4*d^2*e^3-80*((a*e-b*d)*b)^(1/2)*(e*x+d)
^(3/2)*x*a^2*b^2*e^3+480*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*x*a^3*b*e^4-780*
((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*a^3*b*d*e^3+690*((a*e-b*d)*b)^(1/2)*(e*x+
d)^(1/2)*a^2*b^2*d^2*e^2-300*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*a*b^3*d^3*e)
*(b*x+a)/((a*e-b*d)*b)^(1/2)/b^5/((b*x+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{9}{2}}}{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(9/2)/(b^2*x^2 + 2*a*b*x + a^2)^(3/2), x)
```

Fricas [A] time = 1.69455, size = 1535, normalized size = 4.98

$$\left[\frac{315\left(a^2b^2d^2e^2-2\,a^3bde^3+a^4e^4+\left(b^4d^2e^2-2\,ab^3de^3+a^2b^2e^4\right)x^2+2\left(ab^3d^2e^2-2\,a^2b^2de^3+a^3be^4\right)x\right)\sqrt{\frac{bd-ae}{b}}\log\left(\frac{bex+2\,b\cdots}{\cdots}\right)}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/40*(315*(a^2*b^2*d^2*e^2 - 2*a^3*b*d*e^3 + a^4*e^4 + (b^4*d^2*e^2 - 2*a*
b^3*d*e^3 + a^2*b^2*e^4)*x^2 + 2*(a*b^3*d^2*e^2 - 2*a^2*b^2*d*e^3 + a^3*b*e
^4)*x)*sqrt((b*d - a*e)/b)*log((b*e*x + 2*b*d - a*e - 2*sqrt(e*x + d)*b*sqr
t((b*d - a*e)/b))/(b*x + a)) + 2*(8*b^4*e^4*x^4 - 10*b^4*d^4 - 45*a*b^3*d^3
*e + 483*a^2*b^2*d^2*e^2 - 735*a^3*b*d*e^3 + 315*a^4*e^4 + 8*(7*b^4*d*e^3 -
 3*a*b^3*e^4)*x^3 + 24*(12*b^4*d^2*e^2 - 17*a*b^3*d*e^3 + 7*a^2*b^2*e^4)*x^
2 - (85*b^4*d^3*e - 831*a*b^3*d^2*e^2 + 1239*a^2*b^2*d*e^3 - 525*a^3*b*e^4)
*x)*sqrt(e*x + d))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5), -1/20*(315*(a^2*b^2*d^2
*e^2 - 2*a^3*b*d*e^3 + a^4*e^4 + (b^4*d^2*e^2 - 2*a*b^3*d*e^3 + a^2*b^2*e^4
)*x^2 + 2*(a*b^3*d^2*e^2 - 2*a^2*b^2*d*e^3 + a^3*b*e^4)*x)*sqrt(-(b*d - a*e
)/b)*arctan(-sqrt(e*x + d)*b*sqrt(-(b*d - a*e)/b)/(b*d - a*e)) - (8*b^4*e^4
*x^4 - 10*b^4*d^4 - 45*a*b^3*d^3*e + 483*a^2*b^2*d^2*e^2 - 735*a^3*b*d*e^3
+ 315*a^4*e^4 + 8*(7*b^4*d*e^3 - 3*a*b^3*e^4)*x^3 + 24*(12*b^4*d^2*e^2 - 17
*a*b^3*d*e^3 + 7*a^2*b^2*e^4)*x^2 - (85*b^4*d^3*e - 831*a*b^3*d^2*e^2 + 123
9*a^2*b^2*d*e^3 - 525*a^3*b*e^4)*x)*sqrt(e*x + d))/(b^7*x^2 + 2*a*b^6*x + a
^2*b^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(9/2)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.288, size = 602, normalized size = 1.95

$$\frac{63\left(b^3d^3e^2-3\,ab^2d^2e^3+3\,a^2bde^4-a^3e^5\right)\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)}{4\sqrt{-b^2d+abe}b^5\mathrm{sgn}\left((xe+d)be-bde+ae^2\right)}-\frac{17\,(xe+d)^{\frac{3}{2}}b^4d^3e^2-15\,\sqrt{xe+d}b^4d^4e^2-51\,(xe+d)^{\frac{3}{2}}ab^3\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 63/4*(b^3*d^3*e^2 - 3*a*b^2*d^2*e^3 + 3*a^2*b*d*e^4 - a^3*e^5)*arctan(sqrt(
x*e + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^5*sgn((x*e + d)*b*
e - b*d*e + a*e^2)) - 1/4*(17*(x*e + d)^(3/2)*b^4*d^3*e^2 - 15*sqrt(x*e + d
)*b^4*d^4*e^2 - 51*(x*e + d)^(3/2)*a*b^3*d^2*e^3 + 60*sqrt(x*e + d)*a*b^3*d
^3*e^3 + 51*(x*e + d)^(3/2)*a^2*b^2*d*e^4 - 90*sqrt(x*e + d)*a^2*b^2*d^2*e^
```

```
4 - 17*(x*e + d)^(3/2)*a^3*b*e^5 + 60*sqrt(x*e + d)*a^3*b*d*e^5 - 15*sqrt(x
*e + d)*a^4*e^6)/(((x*e + d)*b - b*d + a*e)^2*b^5*sgn((x*e + d)*b*e - b*d*e
 + a*e^2)) + 2/5*((x*e + d)^(5/2)*b^12*e^2 + 5*(x*e + d)^(3/2)*b^12*d*e^2 +
 30*sqrt(x*e + d)*b^12*d^2*e^2 - 5*(x*e + d)^(3/2)*a*b^11*e^3 - 60*sqrt(x*e
 + d)*a*b^11*d*e^3 + 30*sqrt(x*e + d)*a^2*b^10*e^4)/(b^15*sgn((x*e + d)*b*e
 - b*d*e + a*e^2))
```

3.1712 $\int \frac{(d+ex)^{7/2}}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=254

$$\frac{35e^2(a+bx)(d+ex)^{3/2}}{12b^3\sqrt{a^2+2abx+b^2x^2}}+\frac{35e^2(a+bx)\sqrt{d+ex}(bd-ae)}{4b^4\sqrt{a^2+2abx+b^2x^2}}-\frac{35e^2(a+bx)(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{4b^{9/2}\sqrt{a^2+2abx+b^2x^2}}-\frac{(d+ex}{2b(a+bx)\sqrt{a^2+}}$$

```
[Out] (35*e^2*(b*d - a*e)*(a + b*x)*Sqrt[d + e*x])/(4*b^4*Sqrt[a^2 + 2*a*b*x + b^
2*x^2]) + (35*e^2*(a + b*x)*(d + e*x)^(3/2))/(12*b^3*Sqrt[a^2 + 2*a*b*x + b
^2*x^2]) - (7*e*(d + e*x)^(5/2))/(4*b^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (d
 + e*x)^(7/2)/(2*b*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (35*e^2*(b*d
- a*e)^(3/2)*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(4
*b^(9/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.128755, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {646, 47, 50, 63, 208}

$$\frac{35e^2(a+bx)(d+ex)^{3/2}}{12b^3\sqrt{a^2+2abx+b^2x^2}}+\frac{35e^2(a+bx)\sqrt{d+ex}(bd-ae)}{4b^4\sqrt{a^2+2abx+b^2x^2}}-\frac{35e^2(a+bx)(bd-ae)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{4b^{9/2}\sqrt{a^2+2abx+b^2x^2}}-\frac{(d+ex}{2b(a+bx)\sqrt{a^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (35*e^2*(b*d - a*e)*(a + b*x)*Sqrt[d + e*x])/(4*b^4*Sqrt[a^2 + 2*a*b*x + b^
2*x^2]) + (35*e^2*(a + b*x)*(d + e*x)^(3/2))/(12*b^3*Sqrt[a^2 + 2*a*b*x + b
^2*x^2]) - (7*e*(d + e*x)^(5/2))/(4*b^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (d
 + e*x)^(7/2)/(2*b*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (35*e^2*(b*d
- a*e)^(3/2)*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(4
*b^(9/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{7/2}}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x\right)\right)\int \frac{(d+ex)^{7/2}}{\left(ab+b^2x\right)^3}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{(d+ex)^{7/2}}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(7e\left(ab+b^2x\right)\right)\int \frac{(d+ex)^{5/2}}{\left(ab+b^2x\right)^2}\,dx}{4\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{7e(d+ex)^{5/2}}{4b^2\sqrt{a^2+2abx+b^2x^2}} - \frac{(d+ex)^{7/2}}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(35e^2\left(ab+b^2x\right)\right)\int \frac{(d+ex)^{3/2}}{ab+b^2x}}{8b^2\sqrt{a^2+2abx+b^2x^2}}$$

$$= \frac{35e^2(a+bx)(d+ex)^{3/2}}{12b^3\sqrt{a^2+2abx+b^2x^2}} - \frac{7e(d+ex)^{5/2}}{4b^2\sqrt{a^2+2abx+b^2x^2}} - \frac{(d+ex)^{7/2}}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{(3}{}$$

$$= \frac{35e^2(bd-ae)(a+bx)\sqrt{d+ex}}{4b^4\sqrt{a^2+2abx+b^2x^2}} + \frac{35e^2(a+bx)(d+ex)^{3/2}}{12b^3\sqrt{a^2+2abx+b^2x^2}} - \frac{7e(d+ex)^{5/2}}{4b^2\sqrt{a^2+2abx+b^2x^2}} - \frac{}{2b}$$

$$= \frac{35e^2(bd-ae)(a+bx)\sqrt{d+ex}}{4b^4\sqrt{a^2+2abx+b^2x^2}} + \frac{35e^2(a+bx)(d+ex)^{3/2}}{12b^3\sqrt{a^2+2abx+b^2x^2}} - \frac{7e(d+ex)^{5/2}}{4b^2\sqrt{a^2+2abx+b^2x^2}} - \frac{}{2b}$$

$$= \frac{35e^2(bd-ae)(a+bx)\sqrt{d+ex}}{4b^4\sqrt{a^2+2abx+b^2x^2}} + \frac{35e^2(a+bx)(d+ex)^{3/2}}{12b^3\sqrt{a^2+2abx+b^2x^2}} - \frac{7e(d+ex)^{5/2}}{4b^2\sqrt{a^2+2abx+b^2x^2}} - \frac{}{2b}$$

Mathematica [C] time = 0.035838, size = 67, normalized size = 0.26

$$-\frac{2e^2(a+bx)(d+ex)^{9/2}\,{}_2F_1\left(3,\frac{9}{2};\frac{11}{2};\frac{b(d+ex)}{bd-ae}\right)}{9\sqrt{(a+bx)^2}(bd-ae)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (-2*e^2*(a + b*x)*(d + e*x)^(9/2)*Hypergeometric2F1[3, 9/2, 11/2, (b*(d + e
*x))/(b*d - a*e)])/(9*(b*d - a*e)^3*Sqrt[(a + b*x)^2])
```

Maple [B] time = 0.279, size = 714, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] 1/12*(8*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*x^2*b^3*e^2+105*arctan(b*(e*x+d)^
(1/2)/((a*e-b*d)*b)^(1/2))*x^2*a^2*b^2*e^4-210*arctan(b*(e*x+d)^(1/2)/((a*e
-b*d)*b)^(1/2))*x^2*a*b^3*d*e^3+105*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1
/2))*x^2*b^4*d^2*e^2+16*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*x*a*b^2*e^2-72*((
a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*x^2*a*b^2*e^3+72*((a*e-b*d)*b)^(1/2)*(e*x+d
)^(1/2)*x^2*b^3*d*e^2+210*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x*a^3
*b*e^4-420*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x*a^2*b^2*d*e^3+210*
arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x*a*b^3*d^2*e^2-31*((a*e-b*d)*b
)^(1/2)*(e*x+d)^(3/2)*a^2*b*e^2+78*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*a*b^2*
d*e-39*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*b^3*d^2-144*((a*e-b*d)*b)^(1/2)*(e
*x+d)^(1/2)*x*a^2*b*e^3+144*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*x*a*b^2*d*e^2
+105*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a^4*e^4-210*arctan(b*(e*x+
d)^(1/2)/((a*e-b*d)*b)^(1/2))*a^3*b*d*e^3+105*arctan(b*(e*x+d)^(1/2)/((a*e-
b*d)*b)^(1/2))*a^2*b^2*d^2*e^2-105*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*a^3*e^
3+171*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*a^2*b*d*e^2-99*((a*e-b*d)*b)^(1/2)*
(e*x+d)^(1/2)*a*b^2*d^2*e+33*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*b^3*d^3)*(b*
x+a)/((a*e-b*d)*b)^(1/2)/b^4/((b*x+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{7}{2}}}{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(7/2)/(b^2*x^2 + 2*a*b*x + a^2)^(3/2), x)
```

Fricas [A] time = 1.75002, size = 1094, normalized size = 4.31

$$\left[-\frac{105\left(a^2bde^2 - a^3e^3 + \left(b^3de^2 - ab^2e^3\right)x^2 + 2\left(ab^2de^2 - a^2be^3\right)x\right)\sqrt{\frac{bd-ae}{b}}\log\left(\frac{bex+2\,bd-ae+2\,\sqrt{ex+d}b\sqrt{\frac{bd-ae}{b}}}{bx+a}\right) - 2\left(8\,b^3e^3x^3 - 6\,b\right.}{24\left(b^6x^2 + 2\,al\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(105*(a^2*b*d*e^2 - a^3*e^3 + (b^3*d*e^2 - a*b^2*e^3)*x^2 + 2*(a*b^2
*d*e^2 - a^2*b*e^3)*x)*sqrt((b*d - a*e)/b)*log((b*e*x + 2*b*d - a*e + 2*sqr
```

```
t(e*x + d)*b*sqrt((b*d - a*e)/b))/(b*x + a)) - 2*(8*b^3*e^3*x^3 - 6*b^3*d^3
 - 21*a*b^2*d^2*e + 140*a^2*b*d*e^2 - 105*a^3*e^3 + 8*(10*b^3*d*e^2 - 7*a*b
^2*e^3)*x^2 - (39*b^3*d^2*e - 238*a*b^2*d*e^2 + 175*a^2*b*e^3)*x)*sqrt(e*x
+ d))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4), -1/12*(105*(a^2*b*d*e^2 - a^3*e^3 +
(b^3*d*e^2 - a*b^2*e^3)*x^2 + 2*(a*b^2*d*e^2 - a^2*b*e^3)*x)*sqrt(-(b*d - a
*e)/b)*arctan(-sqrt(e*x + d)*b*sqrt(-(b*d - a*e)/b)/(b*d - a*e)) - (8*b^3*e
^3*x^3 - 6*b^3*d^3 - 21*a*b^2*d^2*e + 140*a^2*b*d*e^2 - 105*a^3*e^3 + 8*(10
*b^3*d*e^2 - 7*a*b^2*e^3)*x^2 - (39*b^3*d^2*e - 238*a*b^2*d*e^2 + 175*a^2*b
*e^3)*x)*sqrt(e*x + d))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22808, size = 455, normalized size = 1.79

$$\frac{35\left(b^2d^2e^2-2\,abde^3+a^2e^4\right)\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)}{4\,\sqrt{-b^2d+abe}b^4\mathrm{sgn}\left((xe+d)be-bde+ae^2\right)}-\frac{13\,(xe+d)^{\frac{3}{2}}b^3d^2e^2-11\,\sqrt{xe+d}b^3d^3e^2-26\,(xe+d)^{\frac{3}{2}}ab^2de^3+33\,\sqrt{}}{4\left((xe+d)b-bd+ae\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 35/4*(b^2*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4)*arctan(sqrt(x*e + d)*b/sqrt(-b^2
*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^4*sgn((x*e + d)*b*e - b*d*e + a*e^2))
- 1/4*(13*(x*e + d)^(3/2)*b^3*d^2*e^2 - 11*sqrt(x*e + d)*b^3*d^3*e^2 - 26*(
x*e + d)^(3/2)*a*b^2*d*e^3 + 33*sqrt(x*e + d)*a*b^2*d^2*e^3 + 13*(x*e + d)^
(3/2)*a^2*b*e^4 - 33*sqrt(x*e + d)*a^2*b*d*e^4 + 11*sqrt(x*e + d)*a^3*e^5)/
(((x*e + d)*b - b*d + a*e)^2*b^4*sgn((x*e + d)*b*e - b*d*e + a*e^2)) + 2/3*
((x*e + d)^(3/2)*b^6*e^2 + 9*sqrt(x*e + d)*b^6*d*e^2 - 9*sqrt(x*e + d)*a*b^
5*e^3)/(b^9*sgn((x*e + d)*b*e - b*d*e + a*e^2))
```

3.1713 $\int \frac{(d+ex)^{5/2}}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=202

$$\frac{15e^2(a+bx)\sqrt{d+ex}}{4b^3\sqrt{a^2+2abx+b^2x^2}}-\frac{15e^2(a+bx)\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{4b^{7/2}\sqrt{a^2+2abx+b^2x^2}}-\frac{(d+ex)^{5/2}}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{5e(d+ex)^{3/2}}{4b^2\sqrt{a^2+2abx+b^2}}$$

```
[Out] (15*e^2*(a + b*x)*Sqrt[d + e*x])/(4*b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (5
*e*(d + e*x)^(3/2))/(4*b^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (d + e*x)^(5/2)
/(2*b*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (15*e^2*Sqrt[b*d - a*e]*(a
 + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(4*b^(7/2)*Sqrt[a
^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.094029, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {646, 47, 50, 63, 208}

$$\frac{15e^2(a+bx)\sqrt{d+ex}}{4b^3\sqrt{a^2+2abx+b^2x^2}}-\frac{15e^2(a+bx)\sqrt{bd-ae}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{4b^{7/2}\sqrt{a^2+2abx+b^2x^2}}-\frac{(d+ex)^{5/2}}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{5e(d+ex)^{3/2}}{4b^2\sqrt{a^2+2abx+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (15*e^2*(a + b*x)*Sqrt[d + e*x])/(4*b^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (5
*e*(d + e*x)^(3/2))/(4*b^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (d + e*x)^(5/2)
/(2*b*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (15*e^2*Sqrt[b*d - a*e]*(a
 + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(4*b^(7/2)*Sqrt[a
^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x\right)\right)\int \frac{(d+ex)^{5/2}}{\left(ab+b^2x\right)^3}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{(d+ex)^{5/2}}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(5e\left(ab+b^2x\right)\right)\int \frac{(d+ex)^{3/2}}{\left(ab+b^2x\right)^2}\,dx}{4\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{5e(d+ex)^{3/2}}{4b^2\sqrt{a^2+2abx+b^2x^2}} - \frac{(d+ex)^{5/2}}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(15e^2\left(ab+b^2x\right)\right)\int \frac{\sqrt{d+ex}}{ab+b^2x}\,d}{8b^2\sqrt{a^2+2abx+b^2x^2}}$$

$$= \frac{15e^2(a+bx)\sqrt{d+ex}}{4b^3\sqrt{a^2+2abx+b^2x^2}} - \frac{5e(d+ex)^{3/2}}{4b^2\sqrt{a^2+2abx+b^2x^2}} - \frac{(d+ex)^{5/2}}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{(15}{}$$

$$= \frac{15e^2(a+bx)\sqrt{d+ex}}{4b^3\sqrt{a^2+2abx+b^2x^2}} - \frac{5e(d+ex)^{3/2}}{4b^2\sqrt{a^2+2abx+b^2x^2}} - \frac{(d+ex)^{5/2}}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{(15}{}$$

$$= \frac{15e^2(a+bx)\sqrt{d+ex}}{4b^3\sqrt{a^2+2abx+b^2x^2}} - \frac{5e(d+ex)^{3/2}}{4b^2\sqrt{a^2+2abx+b^2x^2}} - \frac{(d+ex)^{5/2}}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{15e}{}$$

Mathematica [C] time = 0.038997, size = 67, normalized size = 0.33

$$-\frac{2e^2(a+bx)(d+ex)^{7/2}\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};\frac{b(d+ex)}{bd-ae}\right)}{7\sqrt{(a+bx)^2}(bd-ae)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (-2*e^2*(a + b*x)*(d + e*x)^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, (b*(d + e*
x))/(b*d - a*e)])/(7*(b*d - a*e)^3*Sqrt[(a + b*x)^2])
```

Maple [B] time = 0.275, size = 413, normalized size = 2.

$$\frac{bx+a}{4\,b^3}\left(-15\,\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)\,b}}\right)x^2ab^2e^3+15\,\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)\,b}}\right)x^2b^3de^2+8\,\sqrt{(ae-bd)\,b}\sqrt{ex+d}x^2b^2e^2-30\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] 1/4*(-15*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^2*a*b^2*e^3+15*arcta
n(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^2*b^3*d*e^2+8*((a*e-b*d)*b)^(1/2)*
(e*x+d)^(1/2)*x^2*b^2*e^2-30*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x*
a^2*b*e^3+30*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x*a*b^2*d*e^2+9*((
a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*a*b*e-9*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*b
^2*d+16*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*x*a*b*e^2-15*arctan(b*(e*x+d)^(1/
2)/((a*e-b*d)*b)^(1/2))*a^3*e^3+15*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/
2))*a^2*b*d*e^2+15*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*a^2*e^2-14*((a*e-b*d)*
b)^(1/2)*(e*x+d)^(1/2)*a*b*d*e+7*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*b^2*d^2)
*(b*x+a)/((a*e-b*d)*b)^(1/2)/b^3/((b*x+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/(b^2*x^2 + 2*a*b*x + a^2)^(3/2), x)
```

Fricas [A] time = 1.64884, size = 728, normalized size = 3.6

$$\left[\frac{15\left(b^2e^2x^2+2\,abe^2x+a^2e^2\right)\sqrt{\frac{bd-ae}{b}}\log\left(\frac{bex+2\,bd-ae-2\,\sqrt{ex+d}b\sqrt{\frac{bd-ae}{b}}}{bx+a}\right)+2\left(8\,b^2e^2x^2-2\,b^2d^2-5\,abde+15\,a^2e^2-\left(9\,b^2de-2\right.\right.}{8\left(b^5x^2+2\,ab^4x+a^2b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*e^2*x^2 + 2*a*b*e^2*x + a^2*e^2)*sqrt((b*d - a*e)/b)*log((b*e
*x + 2*b*d - a*e - 2*sqrt(e*x + d)*b*sqrt((b*d - a*e)/b))/(b*x + a)) + 2*(8
*b^2*e^2*x^2 - 2*b^2*d^2 - 5*a*b*d*e + 15*a^2*e^2 - (9*b^2*d*e - 25*a*b*e^2
)*x)*sqrt(e*x + d))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3), -1/4*(15*(b^2*e^2*x^2
+ 2*a*b*e^2*x + a^2*e^2)*sqrt(-(b*d - a*e)/b)*arctan(-sqrt(e*x + d)*b*sqrt(
-(b*d - a*e)/b)/(b*d - a*e)) - (8*b^2*e^2*x^2 - 2*b^2*d^2 - 5*a*b*d*e + 15*
a^2*e^2 - (9*b^2*d*e - 25*a*b*e^2)*x)*sqrt(e*x + d))/(b^5*x^2 + 2*a*b^4*x +
 a^2*b^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22455, size = 332, normalized size = 1.64

$$\frac{15\left(bde^2 - ae^3\right)\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)}{4\sqrt{-b^2d+abe}b^3\mathrm{sgn}\left((xe+d)be - bde + ae^2\right)} + \frac{2\sqrt{xe+d}e^2}{b^3\mathrm{sgn}\left((xe+d)be - bde + ae^2\right)} - \frac{9(xe+d)^{\frac{3}{2}}b^2de^2 - 7\sqrt{xe+d}b^2d^2e^2}{4((xe+d)b - bd +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 15/4*(b*d*e^2 - a*e^3)*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-
b^2*d + a*b*e)*b^3*sgn((x*e + d)*b*e - b*d*e + a*e^2)) + 2*sqrt(x*e + d)*e^
2/(b^3*sgn((x*e + d)*b*e - b*d*e + a*e^2)) - 1/4*(9*(x*e + d)^(3/2)*b^2*d*e
^2 - 7*sqrt(x*e + d)*b^2*d^2*e^2 - 9*(x*e + d)^(3/2)*a*b*e^3 + 14*sqrt(x*e
+ d)*a*b*d*e^3 - 7*sqrt(x*e + d)*a^2*e^4)/(((x*e + d)*b - b*d + a*e)^2*b^3*
sgn((x*e + d)*b*e - b*d*e + a*e^2))
```

3.1714 $\int \frac{(d+ex)^{3/2}}{\left(a^2+2abx+b^2x^2\right)^{3/2}} dx$

Optimal. Leaf size=158

$$-\frac{3e^2(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{4b^{5/2}\sqrt{a^2+2abx+b^2x^2}\sqrt{bd-ae}}-\frac{3e\sqrt{d+ex}}{4b^2\sqrt{a^2+2abx+b^2x^2}}-\frac{(d+ex)^{3/2}}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}}$$

```
[Out] (-3*e*Sqrt[d + e*x])/(4*b^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (d + e*x)^(3/2
)/(2*b*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (3*e^2*(a + b*x)*ArcTanh[
(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(4*b^(5/2)*Sqrt[b*d - a*e]*Sqrt[a
^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.075453, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {646, 47, 63, 208}

$$-\frac{3e^2(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{4b^{5/2}\sqrt{a^2+2abx+b^2x^2}\sqrt{bd-ae}}-\frac{3e\sqrt{d+ex}}{4b^2\sqrt{a^2+2abx+b^2x^2}}-\frac{(d+ex)^{3/2}}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```


```
[Out] (-3*e*Sqrt[d + e*x])/(4*b^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (d + e*x)^(3/2
)/(2*b*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (3*e^2*(a + b*x)*ArcTanh[
(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(4*b^(5/2)*Sqrt[b*d - a*e]*Sqrt[a
^2 + 2*a*b*x + b^2*x^2])
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x\right)\right)\int \frac{(d+ex)^{3/2}}{\left(ab+b^2x\right)^3}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{(d+ex)^{3/2}}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(3e\left(ab+b^2x\right)\right)\int \frac{\sqrt{d+ex}}{\left(ab+b^2x\right)^2}\,dx}{4\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{3e\sqrt{d+ex}}{4b^2\sqrt{a^2+2abx+b^2x^2}} - \frac{(d+ex)^{3/2}}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(3e^2\left(ab+b^2x\right)\right)\int \frac{1}{\left(ab+b^2x\right)\sqrt{}}}{8b^2\sqrt{a^2+2abx+b^2x^{}}}$$

$$= -\frac{3e\sqrt{d+ex}}{4b^2\sqrt{a^2+2abx+b^2x^2}} - \frac{(d+ex)^{3/2}}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(3e\left(ab+b^2x\right)\right)\text{Subst}\left(\int -\right.}{4b^2\sqrt{a^2+2a}}$$

$$= -\frac{3e\sqrt{d+ex}}{4b^2\sqrt{a^2+2abx+b^2x^2}} - \frac{(d+ex)^{3/2}}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{3e^2(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d}}{\sqrt{bd-}}\right.}{4b^{5/2}\sqrt{bd-ae}\sqrt{a^2+2abx+}}$$

Mathematica [A] time = 0.108416, size = 110, normalized size = 0.7

$$\frac{\frac{3e^2(a+bx)^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{ae-bd}}\right)}{\sqrt{ae-bd}} - \sqrt{b}\sqrt{d+ex}(3ae+2bd+5bex)}{4b^{5/2}(a+bx)\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (-(Sqrt[b]*Sqrt[d + e*x]*(2*b*d + 3*a*e + 5*b*e*x)) + (3*e^2*(a + b*x)^2*Ar
cTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/Sqrt[-(b*d) + a*e])/(4*b^
(5/2)*(a + b*x)*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.271, size = 194, normalized size = 1.2

$$-\frac{bx+a}{4\,b^2}\left(-3\,\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)\,b}}\right)x^2b^2e^2 - 6\,\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)\,b}}\right)xabe^2 + 5\,\sqrt{(ae-bd)\,b}\,(ex+d)^{3/2}\,b - 3\,\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] -1/4*(-3*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^2*b^2*e^2-6*arctan(b
*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x*a*b*e^2+5*((a*e-b*d)*b)^(1/2)*(e*x+d)
^(3/2)*b-3*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a^2*e^2+3*((a*e-b*d)
*b)^(1/2)*(e*x+d)^(1/2)*a*e-3*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*b*d)*(b*x+a
```

```
)/((a*e-b*d)*b)^(1/2)/b^2/((b*x+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(b^2*x^2 + 2*a*b*x + a^2)^(3/2), x)
```

Fricas [A] time = 1.6693, size = 795, normalized size = 5.03

$$\left[\frac{3\left(b^2e^2x^2+2\,abe^2x+a^2e^2\right)\sqrt{b^2d-abe}\log\left(\frac{bex+2\,bd-ae-2\,\sqrt{b^2d-abe}\sqrt{ex+d}}{bx+a}\right)-2\left(2\,b^3d^2+ab^2de-3\,a^2be^2+5\left(b^3de-ab^2e^2\right)x\right)}{8\left(a^2b^4d-a^3b^3e+\left(b^6d-ab^5e\right)x^2+2\left(ab^5d-a^2b^4e\right)x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b^2*e^2*x^2 + 2*a*b*e^2*x + a^2*e^2)*sqrt(b^2*d - a*b*e)*log((b*e*
x + 2*b*d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d))/(b*x + a)) - 2*(2*b^
3*d^2 + a*b^2*d*e - 3*a^2*b*e^2 + 5*(b^3*d*e - a*b^2*e^2)*x)*sqrt(e*x + d))
/(a^2*b^4*d - a^3*b^3*e + (b^6*d - a*b^5*e)*x^2 + 2*(a*b^5*d - a^2*b^4*e)*x
), 1/4*(3*(b^2*e^2*x^2 + 2*a*b*e^2*x + a^2*e^2)*sqrt(-b^2*d + a*b*e)*arctan
(sqrt(-b^2*d + a*b*e)*sqrt(e*x + d)/(b*e*x + b*d)) - (2*b^3*d^2 + a*b^2*d*e
 - 3*a^2*b*e^2 + 5*(b^3*d*e - a*b^2*e^2)*x)*sqrt(e*x + d))/(a^2*b^4*d - a^3
*b^3*e + (b^6*d - a*b^5*e)*x^2 + 2*(a*b^5*d - a^2*b^4*e)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1912, size = 216, normalized size = 1.37

$$\frac{3\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)e^2}{4\,\sqrt{-b^2d+abe}b^2\mathrm{sgn}\left((xe+d)be-bde+ae^2\right)}-\frac{5\,(xe+d)^{\frac{3}{2}}be^2-3\,\sqrt{xe+d}bde^2+3\,\sqrt{xe+d}ae^3}{4\,((xe+d)b-bd+ae)^2b^2\mathrm{sgn}\left((xe+d)be-bde+ae^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 3/4*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e^2/(sqrt(-b^2*d + a*b*e)*
b^2*sgn((x*e + d)*b*e - b*d*e + a*e^2)) - 1/4*(5*(x*e + d)^(3/2)*b*e^2 - 3*
sqrt(x*e + d)*b*d*e^2 + 3*sqrt(x*e + d)*a*e^3)/(((x*e + d)*b - b*d + a*e)^2
*b^2*sgn((x*e + d)*b*e - b*d*e + a*e^2))
```

3.1715 $\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=168

$$\frac{e^2(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{4b^{3/2}\sqrt{a^2+2abx+b^2x^2}(bd-ae)^{3/2}}-\frac{e\sqrt{d+ex}}{4b\sqrt{a^2+2abx+b^2x^2}(bd-ae)}-\frac{\sqrt{d+ex}}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}}$$

```
[Out] -(e*Sqrt[d + e*x])/(4*b*(b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - Sqrt[d
 + e*x]/(2*b*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (e^2*(a + b*x)*ArcT
anh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(4*b^(3/2)*(b*d - a*e)^(3/2)*
Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0875864, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {646, 47, 51, 63, 208}

$$\frac{e^2(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{4b^{3/2}\sqrt{a^2+2abx+b^2x^2}(bd-ae)^{3/2}}-\frac{e\sqrt{d+ex}}{4b\sqrt{a^2+2abx+b^2x^2}(bd-ae)}-\frac{\sqrt{d+ex}}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] -(e*Sqrt[d + e*x])/(4*b*(b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - Sqrt[d
 + e*x]/(2*b*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (e^2*(a + b*x)*ArcT
anh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(4*b^(3/2)*(b*d - a*e)^(3/2)*
Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x\right)\right)\int \frac{\sqrt{d+ex}}{\left(ab+b^2x\right)^3}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{\sqrt{d+ex}}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{\left(e\left(ab+b^2x\right)\right)\int \frac{1}{\left(ab+b^2x\right)^2\sqrt{d+ex}}\,dx}{4\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{e\sqrt{d+ex}}{4b(bd-ae)\sqrt{a^2+2abx+b^2x^2}}-\frac{\sqrt{d+ex}}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{\left(e^2\left(ab+b^2x\right)\right)\int \frac{}{(a}}{8b(bd-ae)\sqrt{a^2+}}$$

$$= -\frac{e\sqrt{d+ex}}{4b(bd-ae)\sqrt{a^2+2abx+b^2x^2}}-\frac{\sqrt{d+ex}}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{\left(e\left(ab+b^2x\right)\right)\text{Subs}}{4b(bd-a}$$

$$= -\frac{e\sqrt{d+ex}}{4b(bd-ae)\sqrt{a^2+2abx+b^2x^2}}-\frac{\sqrt{d+ex}}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{e^2(a+bx)\tanh}{4b^{3/2}(bd-ae)^{3/2}\sqrt{a}}$$

Mathematica [C] time = 0.0253277, size = 67, normalized size = 0.4

$$-\frac{2e^2(a+bx)(d+ex)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{b(d+ex)}{bd-ae}\right)}{3\sqrt{(a+bx)^2}(bd-ae)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (-2*e^2*(a + b*x)*(d + e*x)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, (b*(d + e*
x))/(b*d - a*e)])/(3*(b*d - a*e)^3*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.271, size = 200, normalized size = 1.2

$$\frac{bx+a}{4\,(ae-bd)\,b}\left(\arctan\left(b\sqrt{ex+d}\frac{1}{\sqrt{(ae-bd)\,b}}\right)x^2b^2e^2+2\,\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)\,b}}\right)xabe^2+\sqrt{(ae-bd)\,b}\,(ex+d)^{\frac{3}{2}}\,b+\text{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] 1/4*(arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^2*b^2*e^2+2*arctan(b*(e*
x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x*a*b*e^2+((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)
*b+arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a^2*e^2-((a*e-b*d)*b)^(1/2)*
(e*x+d)^(1/2)*a*e+((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*b*d)*(b*x+a)/((a*e-b*d)
*b)^(1/2)/b/(a*e-b*d)/((b*x+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(b^2*x^2 + 2*a*b*x + a^2)^(3/2), x)
```

Fricas [A] time = 1.70585, size = 949, normalized size = 5.65

$$\left[-\frac{\left(b^2e^2x^2+2\,abe^2x+a^2e^2\right)\sqrt{b^2d-abe}\log\left(\frac{bex+2\,bd-ae-2\,\sqrt{b^2d-abe}\sqrt{ex+d}}{bx+a}\right)+2\left(2\,b^3d^2-3\,ab^2de+a^2be^2+\left(b^3de-ab^2e^2\right)x\right)\sqrt{}}{8\left(a^2b^4d^2-2\,a^3b^3de+a^4b^2e^2+\left(b^6d^2-2\,ab^5de+a^2b^4e^2\right)x^2+2\left(ab^5d^2-2\,a^2b^4de+a^3b^3e^2\right)x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((b^2*e^2*x^2 + 2*a*b*e^2*x + a^2*e^2)*sqrt(b^2*d - a*b*e)*log((b*e*x
 + 2*b*d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d))/(b*x + a)) + 2*(2*b^3
*d^2 - 3*a*b^2*d*e + a^2*b*e^2 + (b^3*d*e - a*b^2*e^2)*x)*sqrt(e*x + d))/(a
^2*b^4*d^2 - 2*a^3*b^3*d*e + a^4*b^2*e^2 + (b^6*d^2 - 2*a*b^5*d*e + a^2*b^4
*e^2)*x^2 + 2*(a*b^5*d^2 - 2*a^2*b^4*d*e + a^3*b^3*e^2)*x), -1/4*((b^2*e^2*
x^2 + 2*a*b*e^2*x + a^2*e^2)*sqrt(-b^2*d + a*b*e)*arctan(sqrt(-b^2*d + a*b*
e)*sqrt(e*x + d)/(b*e*x + b*d)) + (2*b^3*d^2 - 3*a*b^2*d*e + a^2*b*e^2 + (b
^3*d*e - a*b^2*e^2)*x)*sqrt(e*x + d))/(a^2*b^4*d^2 - 2*a^3*b^3*d*e + a^4*b^
2*e^2 + (b^6*d^2 - 2*a*b^5*d*e + a^2*b^4*e^2)*x^2 + 2*(a*b^5*d^2 - 2*a^2*b^
4*d*e + a^3*b^3*e^2)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}}{\left(\left(a+bx\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/((a + b*x)**2)**(3/2), x)
```

Giac [A] time = 1.21077, size = 297, normalized size = 1.77

$$-\frac{\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)e^2}{4\left(b^2d\operatorname{sgn}\left((xe+d)be-bde+ae^2\right)-abe\operatorname{sgn}\left((xe+d)be-bde+ae^2\right)\right)\sqrt{-b^2d+abe}}-\frac{(xe+}{4\left(b^2d\operatorname{sgn}\left((xe+d)be-bde+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e^2/((b^2*d*sgn((x*e + d)
*b*e - b*d*e + a*e^2) - a*b*e*sgn((x*e + d)*b*e - b*d*e + a*e^2))*sqrt(-b^2
*d + a*b*e)) - 1/4*((x*e + d)^(3/2)*b*e^2 + sqrt(x*e + d)*b*d*e^2 - sqrt(x*
e + d)*a*e^3)/((b^2*d*sgn((x*e + d)*b*e - b*d*e + a*e^2) - a*b*e*sgn((x*e +
 d)*b*e - b*d*e + a*e^2))*((x*e + d)*b - b*d + a*e)^2)
```

3.1716 $\int \frac{1}{\sqrt{d+ex}\left(a^2+2abx+b^2x^2\right)^{3/2}} \, dx$

Optimal. Leaf size=172

$$-\frac{3e^2(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{4\sqrt{b}\sqrt{a^2+2abx+b^2x^2}(bd-ae)^{5/2}}+\frac{3e\sqrt{d+ex}}{4\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}-\frac{\sqrt{d+ex}}{2(a+bx)\sqrt{a^2+2abx+b^2x^2}(bd-ae)}$$

```
[Out] (3*e*Sqrt[d + e*x])/(4*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - Sqrt[
d + e*x]/(2*(b*d - a*e)*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (3*e^2*(
a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(4*Sqrt[b]*(b*d
- a*e)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0871563, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {646, 51, 63, 208}

$$-\frac{3e^2(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{4\sqrt{b}\sqrt{a^2+2abx+b^2x^2}(bd-ae)^{5/2}}+\frac{3e\sqrt{d+ex}}{4\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}-\frac{\sqrt{d+ex}}{2(a+bx)\sqrt{a^2+2abx+b^2x^2}(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

```
[Out] (3*e*Sqrt[d + e*x])/(4*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - Sqrt[
d + e*x]/(2*(b*d - a*e)*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (3*e^2*(
a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(4*Sqrt[b]*(b*d
- a*e)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x\right)\right)\int \frac{1}{\left(ab+b^2x\right)^3\sqrt{d+ex}}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{\sqrt{d+ex}}{2(bd-ae)(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{\left(3be\left(ab+b^2x\right)\right)\int \frac{1}{\left(ab+b^2x\right)^2\sqrt{d+ex}}\,dx}{4(bd-ae)\sqrt{a^2+2abx+b^2x^2}}$$

$$= \frac{3e\sqrt{d+ex}}{4(bd-ae)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{\sqrt{d+ex}}{2(bd-ae)(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(3e^2\right.}{8(}$$

$$= \frac{3e\sqrt{d+ex}}{4(bd-ae)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{\sqrt{d+ex}}{2(bd-ae)(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(3e\left(\right.\right.}{}$$

$$= \frac{3e\sqrt{d+ex}}{4(bd-ae)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{\sqrt{d+ex}}{2(bd-ae)(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{3e}{4\sqrt{b}}$$

Mathematica [C] time = 0.0204898, size = 65, normalized size = 0.38

$$-\frac{2e^2(a+bx)\sqrt{d+ex}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{b(d+ex)}{bd-ae}\right)}{\sqrt{(a+bx)^2}(bd-ae)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

```
[Out] (-2*e^2*(a + b*x)*Sqrt[d + e*x]*Hypergeometric2F1[1/2, 3, 3/2, (b*(d + e*x)
)/(b*d - a*e)])/((b*d - a*e)^3*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.27, size = 203, normalized size = 1.2

$$\frac{bx+a}{4\,(ae-bd)^2}\left(3\,\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)\,b}}\right)x^2b^2e^2+6\,\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)\,b}}\right)xabe^2+3\,\sqrt{(ae-bd)\,b}\sqrt{ex+d}xbe+3\,\mathrm{arcta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(1/2),x)
```

```
[Out] 1/4*(3*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^2*b^2*e^2+6*arctan(b*(
e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x*a*b*e^2+3*((a*e-b*d)*b)^(1/2)*(e*x+d)^(
1/2)*x*b*e+3*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a^2*e^2+5*((a*e-b*
d)*b)^(1/2)*(e*x+d)^(1/2)*a*e-2*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*b*d)*(b*x
+a)/((a*e-b*d)*b)^(1/2)/(a*e-b*d)^2/((b*x+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^2 + 2\,abx + a^2\right)^{\frac{3}{2}}\sqrt{ex + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*sqrt(e*x + d)), x)
```

Fricas [B] time = 1.76714, size = 1119, normalized size = 6.51

$$\left[\frac{3\left(b^2e^2x^2 + 2\,abe^2x + a^2e^2\right)\sqrt{b^2d - abe}\log\left(\frac{bex+2\,bd-ae-2\,\sqrt{b^2d-abe}\sqrt{ex+d}}{bx+a}\right) - 2\left(2\,b^3d^2 - 7\,ab^2de + 5\,a^2be^2 - 3\left(b^3de - ab^2e^2\right)\right.}{8\left(a^2b^4d^3 - 3\,a^3b^3d^2e + 3\,a^4b^2de^2 - a^5be^3 + \left(b^6d^3 - 3\,ab^5d^2e + 3\,a^2b^4de^2 - a^3b^3e^3\right)x^2 + 2\left(ab^5d^3 - 3\,a^2b^4d^2e + 3\,a^3b^3de^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/8*(3*(b^2*e^2*x^2 + 2*a*b*e^2*x + a^2*e^2)*sqrt(b^2*d - a*b*e)*log((b*e*
x + 2*b*d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d))/(b*x + a)) - 2*(2*b^
3*d^2 - 7*a*b^2*d*e + 5*a^2*b*e^2 - 3*(b^3*d*e - a*b^2*e^2)*x)*sqrt(e*x + d
))/(a^2*b^4*d^3 - 3*a^3*b^3*d^2*e + 3*a^4*b^2*d*e^2 - a^5*b*e^3 + (b^6*d^3
- 3*a*b^5*d^2*e + 3*a^2*b^4*d*e^2 - a^3*b^3*e^3)*x^2 + 2*(a*b^5*d^3 - 3*a^2
*b^4*d^2*e + 3*a^3*b^3*d*e^2 - a^4*b^2*e^3)*x), 1/4*(3*(b^2*e^2*x^2 + 2*a*b
*e^2*x + a^2*e^2)*sqrt(-b^2*d + a*b*e)*arctan(sqrt(-b^2*d + a*b*e)*sqrt(e*x
 + d)/(b*e*x + b*d)) - (2*b^3*d^2 - 7*a*b^2*d*e + 5*a^2*b*e^2 - 3*(b^3*d*e
- a*b^2*e^2)*x)*sqrt(e*x + d))/(a^2*b^4*d^3 - 3*a^3*b^3*d^2*e + 3*a^4*b^2*d
*e^2 - a^5*b*e^3 + (b^6*d^3 - 3*a*b^5*d^2*e + 3*a^2*b^4*d*e^2 - a^3*b^3*e^3
)*x^2 + 2*(a*b^5*d^3 - 3*a^2*b^4*d^2*e + 3*a^3*b^3*d*e^2 - a^4*b^2*e^3)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d + ex}\left((a + bx)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**2+2*a*b*x+a**2)**(3/2)/(e*x+d)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d + e*x)*((a + b*x)**2)**(3/2)), x)
```

Giac [B] time = 1.17725, size = 383, normalized size = 2.23

$$\frac{3\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)e^2}{4\left(b^2d^2\mathrm{sgn}\left((xe + d)be - bde + ae^2\right) - 2\,abde\mathrm{sgn}\left((xe + d)be - bde + ae^2\right) + a^2e^2\mathrm{sgn}\left((xe + d)be - bde + ae^2\right)\right)\sqrt{-b^2d + abe}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 3/4*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e^2/((b^2*d^2*sgn((x*e + d
)*b*e - b*d*e + a*e^2) - 2*a*b*d*e*sgn((x*e + d)*b*e - b*d*e + a*e^2) + a^2
*e^2*sgn((x*e + d)*b*e - b*d*e + a*e^2))*sqrt(-b^2*d + a*b*e)) + 1/4*(3*(x*
e + d)^(3/2)*b*e^2 - 5*sqrt(x*e + d)*b*d*e^2 + 5*sqrt(x*e + d)*a*e^3)/((b^2
*d^2*sgn((x*e + d)*b*e - b*d*e + a*e^2) - 2*a*b*d*e*sgn((x*e + d)*b*e - b*d
*e + a*e^2) + a^2*e^2*sgn((x*e + d)*b*e - b*d*e + a*e^2))*((x*e + d)*b - b*
d + a*e)^2)
```

3.1717 $\int \frac{1}{(d+ex)^{3/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=223

$$\frac{15e^2(a+bx)}{4\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-ae)^3}-\frac{15\sqrt{b}e^2(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{4\sqrt{a^2+2abx+b^2x^2}(bd-ae)^{7/2}}+\frac{5e}{4\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-ae)^2}-2$$

```
[Out] (5*e)/(4*(b*d - a*e)^2*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - 1/(2*
(b*d - a*e)*(a + b*x)*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (15*e^
2*(a + b*x))/(4*(b*d - a*e)^3*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
- (15*Sqrt[b]*e^2*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]
])/(4*(b*d - a*e)^(7/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.106826, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {646, 51, 63, 208}

$$\frac{15e^2(a+bx)}{4\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-ae)^3}-\frac{15\sqrt{b}e^2(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{4\sqrt{a^2+2abx+b^2x^2}(bd-ae)^{7/2}}+\frac{5e}{4\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-ae)^2}-2$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

```
[Out] (5*e)/(4*(b*d - a*e)^2*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - 1/(2*
(b*d - a*e)*(a + b*x)*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (15*e^
2*(a + b*x))/(4*(b*d - a*e)^3*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
- (15*Sqrt[b]*e^2*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]
])/(4*(b*d - a*e)^(7/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x\right)\right)\int \frac{1}{\left(ab+b^2x\right)^3(d+ex)^{3/2}}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{1}{2(bd-ae)(a+bx)\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} - \frac{\left(5be\left(ab+b^2x\right)\right)\int \frac{}{\left(ab+b^2x\right)}}{4(bd-ae)\sqrt{a^2+2abx}}$$

$$= \frac{5e}{4(bd-ae)^2\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{2(bd-ae)(a+bx)\sqrt{d+ex}\sqrt{a^2+2}}$$

$$= \frac{5e}{4(bd-ae)^2\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{2(bd-ae)(a+bx)\sqrt{d+ex}\sqrt{a^2+2}}$$

$$= \frac{5e}{4(bd-ae)^2\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{2(bd-ae)(a+bx)\sqrt{d+ex}\sqrt{a^2+2}}$$

$$= \frac{5e}{4(bd-ae)^2\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{2(bd-ae)(a+bx)\sqrt{d+ex}\sqrt{a^2+2}}$$

Mathematica [C] time = 0.0215676, size = 65, normalized size = 0.29

$$\frac{2e^2(a+bx)\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\frac{b(d+ex)}{bd-ae}\right)}{\sqrt{(a+bx)^2}\sqrt{d+ex}(bd-ae)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

```
[Out] (2*e^2*(a + b*x)*Hypergeometric2F1[-1/2, 3, 1/2, (b*(d + e*x))/(b*d - a*e)]
)/((b*d - a*e)^3*Sqrt[(a + b*x)^2]*Sqrt[d + e*x])
```

Maple [A] time = 0.283, size = 285, normalized size = 1.3

$$-\frac{bx+a}{4\,(ae-bd)^3}\left(15\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)\,b}}\right)\sqrt{ex+d}x^2b^3e^2+30\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)\,b}}\right)\sqrt{ex+d}xab^2e^2+15\sqrt{(ae-bd)\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] -1/4*(15*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*(e*x+d)^(1/2)*x^2*b^3*
e^2+30*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*(e*x+d)^(1/2)*x*a*b^2*e^
```

```
2+15*((a*e-b*d)*b)^(1/2)*x^2*b^2*e^2+15*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b
)^(1/2))*(e*x+d)^(1/2)*a^2*b*e^2+25*((a*e-b*d)*b)^(1/2)*x*a*b*e^2+5*((a*e-b
*d)*b)^(1/2)*x*b^2*d*e+8*((a*e-b*d)*b)^(1/2)*a^2*e^2+9*((a*e-b*d)*b)^(1/2)*
a*b*d*e-2*((a*e-b*d)*b)^(1/2)*b^2*d^2)*(b*x+a)/((a*e-b*d)*b)^(1/2)/(e*x+d)^
(1/2)/(a*e-b*d)^3/((b*x+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^2 + 2\,abx + a^2\right)^{\frac{3}{2}}(ex + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*(e*x + d)^(3/2)), x)
```

Fricas [B] time = 1.63644, size = 1582, normalized size = 7.09

$$\left[-\frac{15\left(b^2e^3x^3 + a^2de^2 + \left(b^2de^2 + 2\,abe^3\right)x^2 + \left(2\,abde^2 + a^2e^3\right)x\right)\sqrt{\frac{b}{bd-ae}}\log\left(\frac{bex+2\,bd-ae+2\,(bd-ae)\sqrt{ex+d}\sqrt{\frac{b}{bd-ae}}}{bx+a}\right) - 2}{8\left(a^2b^3d^4 - 3\,a^3b^2d^3e + 3\,a^4bd^2e^2 - a^5de^3 + \left(b^5d^3e - 3\,ab^4d^2e^2 + 3\,a^2b^3de^3 - a^3b^2e^4\right)x^3 + \left(b^5d^4 - ab^4d^3e - 3\,a^2b^3d^2e^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [-1/8*(15*(b^2*e^3*x^3 + a^2*d*e^2 + (b^2*d*e^2 + 2*a*b*e^3)*x^2 + (2*a*b*d
*e^2 + a^2*e^3)*x)*sqrt(b/(b*d - a*e))*log((b*e*x + 2*b*d - a*e + 2*(b*d -
a*e)*sqrt(e*x + d)*sqrt(b/(b*d - a*e)))/(b*x + a)) - 2*(15*b^2*e^2*x^2 - 2*
b^2*d^2 + 9*a*b*d*e + 8*a^2*e^2 + 5*(b^2*d*e + 5*a*b*e^2)*x)*sqrt(e*x + d))
/(a^2*b^3*d^4 - 3*a^3*b^2*d^3*e + 3*a^4*b*d^2*e^2 - a^5*d*e^3 + (b^5*d^3*e
- 3*a*b^4*d^2*e^2 + 3*a^2*b^3*d*e^3 - a^3*b^2*e^4)*x^3 + (b^5*d^4 - a*b^4*d
^3*e - 3*a^2*b^3*d^2*e^2 + 5*a^3*b^2*d*e^3 - 2*a^4*b*e^4)*x^2 + (2*a*b^4*d^
4 - 5*a^2*b^3*d^3*e + 3*a^3*b^2*d^2*e^2 + a^4*b*d*e^3 - a^5*e^4)*x), -1/4*(
15*(b^2*e^3*x^3 + a^2*d*e^2 + (b^2*d*e^2 + 2*a*b*e^3)*x^2 + (2*a*b*d*e^2 +
a^2*e^3)*x)*sqrt(-b/(b*d - a*e))*arctan(-(b*d - a*e)*sqrt(e*x + d)*sqrt(-b/
(b*d - a*e))/(b*e*x + b*d)) - (15*b^2*e^2*x^2 - 2*b^2*d^2 + 9*a*b*d*e + 8*a
^2*e^2 + 5*(b^2*d*e + 5*a*b*e^2)*x)*sqrt(e*x + d))/(a^2*b^3*d^4 - 3*a^3*b^2
*d^3*e + 3*a^4*b*d^2*e^2 - a^5*d*e^3 + (b^5*d^3*e - 3*a*b^4*d^2*e^2 + 3*a^2
*b^3*d*e^3 - a^3*b^2*e^4)*x^3 + (b^5*d^4 - a*b^4*d^3*e - 3*a^2*b^3*d^2*e^2
+ 5*a^3*b^2*d*e^3 - 2*a^4*b*e^4)*x^2 + (2*a*b^4*d^4 - 5*a^2*b^3*d^3*e + 3*a
^3*b^2*d^2*e^2 + a^4*b*d*e^3 - a^5*e^4)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex)^{\frac{3}{2}}\left((a + bx)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral(1/((d + e*x)**(3/2)*((a + b*x)**2)**(3/2)), x)
```

Giac [B] time = 1.25694, size = 674, normalized size = 3.02

$$\frac{15\, b \arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)e^2}{4\left(b^3d^3\mathrm{sgn}\left((xe+d)be-bde+ae^2\right)-3\,ab^2d^2e\mathrm{sgn}\left((xe+d)be-bde+ae^2\right)+3\,a^2bde^2\mathrm{sgn}\left((xe+d)be-bde+ae^2\right)-a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 15/4*b*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e^2/((b^3*d^3*sgn((x*e
+ d)*b*e - b*d*e + a*e^2) - 3*a*b^2*d^2*e*sgn((x*e + d)*b*e - b*d*e + a*e^2
) + 3*a^2*b*d*e^2*sgn((x*e + d)*b*e - b*d*e + a*e^2) - a^3*e^3*sgn((x*e + d
)*b*e - b*d*e + a*e^2))*sqrt(-b^2*d + a*b*e)) + 2*e^2/((b^3*d^3*sgn((x*e +
d)*b*e - b*d*e + a*e^2) - 3*a*b^2*d^2*e*sgn((x*e + d)*b*e - b*d*e + a*e^2)
+ 3*a^2*b*d*e^2*sgn((x*e + d)*b*e - b*d*e + a*e^2) - a^3*e^3*sgn((x*e + d)*
b*e - b*d*e + a*e^2))*sqrt(x*e + d)) + 1/4*(7*(x*e + d)^(3/2)*b^2*e^2 - 9*s
qrt(x*e + d)*b^2*d*e^2 + 9*sqrt(x*e + d)*a*b*e^3)/((b^3*d^3*sgn((x*e + d)*b
*e - b*d*e + a*e^2) - 3*a*b^2*d^2*e*sgn((x*e + d)*b*e - b*d*e + a*e^2) + 3*
a^2*b*d*e^2*sgn((x*e + d)*b*e - b*d*e + a*e^2) - a^3*e^3*sgn((x*e + d)*b*e
- b*d*e + a*e^2))*((x*e + d)*b - b*d + a*e)^2)
```

3.1718 $\int \frac{1}{(d+ex)^{5/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=275

$$\frac{35be^2(a+bx)}{4\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-ae)^4}+\frac{35e^2(a+bx)}{12\sqrt{a^2+2abx+b^2x^2}(d+ex)^{3/2}(bd-ae)^3}-\frac{35b^{3/2}e^2(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{4\sqrt{a^2+2abx+b^2x^2}(bd-ae)^{9/2}}$$

```
[Out] (7*e)/(4*(b*d - a*e)^2*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - 1/(
2*(b*d - a*e)*(a + b*x)*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (3
5*e^2*(a + b*x))/(12*(b*d - a*e)^3*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2
*x^2]) + (35*b*e^2*(a + b*x))/(4*(b*d - a*e)^4*Sqrt[d + e*x]*Sqrt[a^2 + 2*a
*b*x + b^2*x^2]) - (35*b^(3/2)*e^2*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x]
)/Sqrt[b*d - a*e]])/(4*(b*d - a*e)^(9/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.124641, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {646, 51, 63, 208}

$$\frac{35be^2(a+bx)}{4\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-ae)^4}+\frac{35e^2(a+bx)}{12\sqrt{a^2+2abx+b^2x^2}(d+ex)^{3/2}(bd-ae)^3}-\frac{35b^{3/2}e^2(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{4\sqrt{a^2+2abx+b^2x^2}(bd-ae)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```


```
[Out] (7*e)/(4*(b*d - a*e)^2*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - 1/(
2*(b*d - a*e)*(a + b*x)*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (3
5*e^2*(a + b*x))/(12*(b*d - a*e)^3*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2
*x^2]) + (35*b*e^2*(a + b*x))/(4*(b*d - a*e)^4*Sqrt[d + e*x]*Sqrt[a^2 + 2*a
*b*x + b^2*x^2]) - (35*b^(3/2)*e^2*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x]
)/Sqrt[b*d - a*e]])/(4*(b*d - a*e)^(9/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{5/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}dx = \frac{\left(b^2\left(ab+b^2x\right)\right)\int \frac{1}{\left(ab+b^2x\right)^3(d+ex)^{5/2}}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{1}{2(bd-ae)(a+bx)(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}} - \frac{\left(7be\left(ab+b^2x\right)\right)\int \frac{}{\left(ab+b\right.}}{4(bd-ae)\sqrt{a^2+2ai}}$$

$$= \frac{7e}{4(bd-ae)^2(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{2(bd-ae)(a+bx)(d+ex)^{3/2}\sqrt{a^2}}$$

$$= \frac{7e}{4(bd-ae)^2(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{2(bd-ae)(a+bx)(d+ex)^{3/2}\sqrt{a^2}}$$

$$= \frac{7e}{4(bd-ae)^2(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{2(bd-ae)(a+bx)(d+ex)^{3/2}\sqrt{a^2}}$$

$$= \frac{7e}{4(bd-ae)^2(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{2(bd-ae)(a+bx)(d+ex)^{3/2}\sqrt{a^2}}$$

$$= \frac{7e}{4(bd-ae)^2(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{2(bd-ae)(a+bx)(d+ex)^{3/2}\sqrt{a^2}}$$

Mathematica [C] time = 0.0240417, size = 67, normalized size = 0.24

$$\frac{2e^2(a+bx)\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};\frac{b(d+ex)}{bd-ae}\right)}{3\sqrt{(a+bx)^2}(d+ex)^{3/2}(bd-ae)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

```
[Out] (2*e^2*(a + b*x)*Hypergeometric2F1[-3/2, 3, -1/2, (b*(d + e*x))/(b*d - a*e)
])/(3*(b*d - a*e)^3*Sqrt[(a + b*x)^2]*(d + e*x)^(3/2))
```

Maple [B] time = 0.279, size = 388, normalized size = 1.4

$$\frac{bx+a}{12\,(ae-bd)^4}\left(105\,\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)\,b}}\right)(ex+d)^{3/2}x^2b^4e^2+210\,\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)\,b}}\right)(ex+d)^{3/2}xab^3e^2+105\sqrt{(a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] 1/12*(105*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*(e*x+d)^(3/2)*x^2*b^4
*e^2+210*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*(e*x+d)^(3/2)*x*a*b^3*
e^2+105*((a*e-b*d)*b)^(1/2)*x^3*b^3*e^3+105*arctan(b*(e*x+d)^(1/2)/((a*e-b*
d)*b)^(1/2))*(e*x+d)^(3/2)*a^2*b^2*e^2+175*((a*e-b*d)*b)^(1/2)*x^2*a*b^2*e^
3+140*((a*e-b*d)*b)^(1/2)*x^2*b^3*d*e^2+56*((a*e-b*d)*b)^(1/2)*x*a^2*b*e^3+
238*((a*e-b*d)*b)^(1/2)*x*a*b^2*d*e^2+21*((a*e-b*d)*b)^(1/2)*x*b^3*d^2*e-8*
((a*e-b*d)*b)^(1/2)*a^3*e^3+80*((a*e-b*d)*b)^(1/2)*a^2*b*d*e^2+39*((a*e-b*d
)*b)^(1/2)*a*b^2*d^2*e-6*((a*e-b*d)*b)^(1/2)*b^3*d^3)*(b*x+a)/((a*e-b*d)*b)
^(1/2)/(e*x+d)^(3/2)/(a*e-b*d)^4/((b*x+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*(e*x + d)^(5/2)), x)
```

Fricas [B] time = 1.80953, size = 2472, normalized size = 8.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/24*(105*(b^3*e^4*x^4 + a^2*b*d^2*e^2 + 2*(b^3*d*e^3 + a*b^2*e^4)*x^3 + (
b^3*d^2*e^2 + 4*a*b^2*d*e^3 + a^2*b*e^4)*x^2 + 2*(a*b^2*d^2*e^2 + a^2*b*d*e
^3)*x)*sqrt(b/(b*d - a*e))*log((b*e*x + 2*b*d - a*e - 2*(b*d - a*e)*sqrt(e*
x + d)*sqrt(b/(b*d - a*e)))/(b*x + a)) + 2*(105*b^3*e^3*x^3 - 6*b^3*d^3 + 3
9*a*b^2*d^2*e + 80*a^2*b*d*e^2 - 8*a^3*e^3 + 35*(4*b^3*d*e^2 + 5*a*b^2*e^3)
*x^2 + 7*(3*b^3*d^2*e + 34*a*b^2*d*e^2 + 8*a^2*b*e^3)*x)*sqrt(e*x + d))/(a^
2*b^4*d^6 - 4*a^3*b^3*d^5*e + 6*a^4*b^2*d^4*e^2 - 4*a^5*b*d^3*e^3 + a^6*d^2
*e^4 + (b^6*d^4*e^2 - 4*a*b^5*d^3*e^3 + 6*a^2*b^4*d^2*e^4 - 4*a^3*b^3*d*e^5
 + a^4*b^2*e^6)*x^4 + 2*(b^6*d^5*e - 3*a*b^5*d^4*e^2 + 2*a^2*b^4*d^3*e^3 +
2*a^3*b^3*d^2*e^4 - 3*a^4*b^2*d*e^5 + a^5*b*e^6)*x^3 + (b^6*d^6 - 9*a^2*b^4
*d^4*e^2 + 16*a^3*b^3*d^3*e^3 - 9*a^4*b^2*d^2*e^4 + a^6*e^6)*x^2 + 2*(a*b^5
*d^6 - 3*a^2*b^4*d^5*e + 2*a^3*b^3*d^4*e^2 + 2*a^4*b^2*d^3*e^3 - 3*a^5*b*d^
2*e^4 + a^6*d*e^5)*x), -1/12*(105*(b^3*e^4*x^4 + a^2*b*d^2*e^2 + 2*(b^3*d*e
^3 + a*b^2*e^4)*x^3 + (b^3*d^2*e^2 + 4*a*b^2*d*e^3 + a^2*b*e^4)*x^2 + 2*(a*
b^2*d^2*e^2 + a^2*b*d*e^3)*x)*sqrt(-b/(b*d - a*e))*arctan(-(b*d - a*e)*sqrt
(e*x + d)*sqrt(-b/(b*d - a*e))/(b*e*x + b*d)) - (105*b^3*e^3*x^3 - 6*b^3*d^
3 + 39*a*b^2*d^2*e + 80*a^2*b*d*e^2 - 8*a^3*e^3 + 35*(4*b^3*d*e^2 + 5*a*b^2
*e^3)*x^2 + 7*(3*b^3*d^2*e + 34*a*b^2*d*e^2 + 8*a^2*b*e^3)*x)*sqrt(e*x + d)
)/(a^2*b^4*d^6 - 4*a^3*b^3*d^5*e + 6*a^4*b^2*d^4*e^2 - 4*a^5*b*d^3*e^3 + a^
6*d^2*e^4 + (b^6*d^4*e^2 - 4*a*b^5*d^3*e^3 + 6*a^2*b^4*d^2*e^4 - 4*a^3*b^3*
d*e^5 + a^4*b^2*e^6)*x^4 + 2*(b^6*d^5*e - 3*a*b^5*d^4*e^2 + 2*a^2*b^4*d^3*e
```

```
^3 + 2*a^3*b^3*d^2*e^4 - 3*a^4*b^2*d*e^5 + a^5*b*e^6)*x^3 + (b^6*d^6 - 9*a^
2*b^4*d^4*e^2 + 16*a^3*b^3*d^3*e^3 - 9*a^4*b^2*d^2*e^4 + a^6*e^6)*x^2 + 2*(
a*b^5*d^6 - 3*a^2*b^4*d^5*e + 2*a^3*b^3*d^4*e^2 + 2*a^4*b^2*d^3*e^3 - 3*a^5
*b*d^2*e^4 + a^6*d*e^5)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)^{\frac{5}{2}}\left((a+bx)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(5/2)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral(1/((d + e*x)**(5/2)*((a + b*x)**2)**(3/2)), x)
```

Giac [B] time = 1.27909, size = 844, normalized size = 3.07

$$\frac{35\,b^2\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)e^2}{4\left(b^4d^4\mathrm{sgn}\left((xe+d)be-bde+ae^2\right)-4\,ab^3d^3e\mathrm{sgn}\left((xe+d)be-bde+ae^2\right)+6\,a^2b^2d^2e^2\mathrm{sgn}\left((xe+d)be-bde+ae^2\right)-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 35/4*b^2*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e^2/((b^4*d^4*sgn((x*
e + d)*b*e - b*d*e + a*e^2) - 4*a*b^3*d^3*e*sgn((x*e + d)*b*e - b*d*e + a*e
^2) + 6*a^2*b^2*d^2*e^2*sgn((x*e + d)*b*e - b*d*e + a*e^2) - 4*a^3*b*d*e^3*
sgn((x*e + d)*b*e - b*d*e + a*e^2) + a^4*e^4*sgn((x*e + d)*b*e - b*d*e + a*
e^2))*sqrt(-b^2*d + a*b*e)) + 2/3*(9*(x*e + d)*b*e^2 + b*d*e^2 - a*e^3)/((b
^4*d^4*sgn((x*e + d)*b*e - b*d*e + a*e^2) - 4*a*b^3*d^3*e*sgn((x*e + d)*b*e
 - b*d*e + a*e^2) + 6*a^2*b^2*d^2*e^2*sgn((x*e + d)*b*e - b*d*e + a*e^2) -
4*a^3*b*d*e^3*sgn((x*e + d)*b*e - b*d*e + a*e^2) + a^4*e^4*sgn((x*e + d)*b*
e - b*d*e + a*e^2))*(x*e + d)^(3/2)) + 1/4*(11*(x*e + d)^(3/2)*b^3*e^2 - 13
*sqrt(x*e + d)*b^3*d*e^2 + 13*sqrt(x*e + d)*a*b^2*e^3)/((b^4*d^4*sgn((x*e +
 d)*b*e - b*d*e + a*e^2) - 4*a*b^3*d^3*e*sgn((x*e + d)*b*e - b*d*e + a*e^2)
 + 6*a^2*b^2*d^2*e^2*sgn((x*e + d)*b*e - b*d*e + a*e^2) - 4*a^3*b*d*e^3*sgn
((x*e + d)*b*e - b*d*e + a*e^2) + a^4*e^4*sgn((x*e + d)*b*e - b*d*e + a*e^2
))*((x*e + d)*b - b*d + a*e)^2)
```

3.1719 $\int \frac{1}{(d+ex)^{7/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=329

$$\frac{63b^2e^2(a+bx)}{4\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-ae)^5}+\frac{21be^2(a+bx)}{4\sqrt{a^2+2abx+b^2x^2}(d+ex)^{3/2}(bd-ae)^4}+\frac{63e^2(a+bx)}{20\sqrt{a^2+2abx+b^2x^2}(d+ex)^{5/2}(bd$$

```
[Out] (9*e)/(4*(b*d - a*e)^2*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - 1/(
2*(b*d - a*e)*(a + b*x)*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (6
3*e^2*(a + b*x))/(20*(b*d - a*e)^3*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2
*x^2]) + (21*b*e^2*(a + b*x))/(4*(b*d - a*e)^4*(d + e*x)^(3/2)*Sqrt[a^2 + 2
*a*b*x + b^2*x^2]) + (63*b^2*e^2*(a + b*x))/(4*(b*d - a*e)^5*Sqrt[d + e*x]*
Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (63*b^(5/2)*e^2*(a + b*x)*ArcTanh[(Sqrt[b]
*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(4*(b*d - a*e)^(11/2)*Sqrt[a^2 + 2*a*b*x
+ b^2*x^2])
```

Rubi [A] time = 0.160639, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {646, 51, 63, 208}

$$\frac{63b^2e^2(a+bx)}{4\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-ae)^5}+\frac{21be^2(a+bx)}{4\sqrt{a^2+2abx+b^2x^2}(d+ex)^{3/2}(bd-ae)^4}+\frac{63e^2(a+bx)}{20\sqrt{a^2+2abx+b^2x^2}(d+ex)^{5/2}(bd$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```


```
[Out] (9*e)/(4*(b*d - a*e)^2*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - 1/(
2*(b*d - a*e)*(a + b*x)*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (6
3*e^2*(a + b*x))/(20*(b*d - a*e)^3*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2
*x^2]) + (21*b*e^2*(a + b*x))/(4*(b*d - a*e)^4*(d + e*x)^(3/2)*Sqrt[a^2 + 2
*a*b*x + b^2*x^2]) + (63*b^2*e^2*(a + b*x))/(4*(b*d - a*e)^5*Sqrt[d + e*x]*
Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (63*b^(5/2)*e^2*(a + b*x)*ArcTanh[(Sqrt[b]
*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(4*(b*d - a*e)^(11/2)*Sqrt[a^2 + 2*a*b*x
+ b^2*x^2])
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{7/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x\right)\right)\int \frac{1}{\left(ab+b^2x\right)^3(d+ex)^{7/2}}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{1}{2(bd-ae)(a+bx)(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}} - \frac{\left(9be\left(ab+b^2x\right)\right)\int \frac{}{\left(ab+b\right.}}{4(bd-ae)\sqrt{a^2+2a}}$$

$$= \frac{9e}{4(bd-ae)^2(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{2(bd-ae)(a+bx)(d+ex)^{5/2}\sqrt{a^2}}$$

$$= \frac{9e}{4(bd-ae)^2(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{2(bd-ae)(a+bx)(d+ex)^{5/2}\sqrt{a^2}}$$

$$= \frac{9e}{4(bd-ae)^2(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{2(bd-ae)(a+bx)(d+ex)^{5/2}\sqrt{a^2}}$$

$$= \frac{9e}{4(bd-ae)^2(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{2(bd-ae)(a+bx)(d+ex)^{5/2}\sqrt{a^2}}$$

$$= \frac{9e}{4(bd-ae)^2(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{2(bd-ae)(a+bx)(d+ex)^{5/2}\sqrt{a^2}}$$

$$= \frac{9e}{4(bd-ae)^2(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{2(bd-ae)(a+bx)(d+ex)^{5/2}\sqrt{a^2}}$$

Mathematica [C] time = 0.0273192, size = 67, normalized size = 0.2

$$\frac{2e^2(a+bx)\,{}_2F_1\left(-\frac{5}{2},3;-\frac{3}{2};\frac{b(d+ex)}{bd-ae}\right)}{5\sqrt{(a+bx)^2}(d+ex)^{5/2}(bd-ae)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

```
[Out] (2*e^2*(a + b*x)*Hypergeometric2F1[-5/2, 3, -3/2, (b*(d + e*x))/(b*d - a*e)
])/(5*(b*d - a*e)^3*Sqrt[(a + b*x)^2]*(d + e*x)^(5/2))
```

Maple [B] time = 0.307, size = 518, normalized size = 1.6

$$-\frac{bx+a}{20\left(ae-bd\right)^{5}}\left(315\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{\left(ae-bd\right)b}}\right)\left(ex+d\right)^{5/2}x^{2}b^{5}e^{2}+630\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{\left(ae-bd\right)b}}\right)\left(ex+d\right)^{5/2}xab^{4}e^{2}+315\sqrt{(ae}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] -1/20*(315*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*(e*x+d)^(5/2)*x^2*b^
5*e^2+630*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*(e*x+d)^(5/2)*x*a*b^4
*e^2+315*((a*e-b*d)*b)^(1/2)*x^4*b^4*e^4+315*arctan(b*(e*x+d)^(1/2)/((a*e-b
*d)*b)^(1/2))*(e*x+d)^(5/2)*a^2*b^3*e^2+525*((a*e-b*d)*b)^(1/2)*x^3*a*b^3*e
^4+735*((a*e-b*d)*b)^(1/2)*x^3*b^4*d*e^3+168*((a*e-b*d)*b)^(1/2)*x^2*a^2*b^
2*e^4+1239*((a*e-b*d)*b)^(1/2)*x^2*a*b^3*d*e^3+483*((a*e-b*d)*b)^(1/2)*x^2*
b^4*d^2*e^2-24*((a*e-b*d)*b)^(1/2)*x*a^3*b*e^4+408*((a*e-b*d)*b)^(1/2)*x*a^
2*b^2*d*e^3+831*((a*e-b*d)*b)^(1/2)*x*a*b^3*d^2*e^2+45*((a*e-b*d)*b)^(1/2)*
x*b^4*d^3*e+8*((a*e-b*d)*b)^(1/2)*a^4*e^4-56*((a*e-b*d)*b)^(1/2)*a^3*b*d*e^
3+288*((a*e-b*d)*b)^(1/2)*a^2*b^2*d^2*e^2+85*((a*e-b*d)*b)^(1/2)*a*b^3*d^3*
e-10*((a*e-b*d)*b)^(1/2)*b^4*d^4)*(b*x+a)/((a*e-b*d)*b)^(1/2)/(e*x+d)^(5/2)
/(a*e-b*d)^5/((b*x+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^{2}x^{2}+2\,abx+a^{2}\right)^{\frac{3}{2}}\left(ex+d\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*(e*x + d)^(7/2)), x)
```

Fricas [B] time = 1.91255, size = 3753, normalized size = 11.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [-1/40*(315*(b^4*e^5*x^5 + a^2*b^2*d^3*e^2 + (3*b^4*d*e^4 + 2*a*b^3*e^5)*x^
4 + (3*b^4*d^2*e^3 + 6*a*b^3*d*e^4 + a^2*b^2*e^5)*x^3 + (b^4*d^3*e^2 + 6*a*
b^3*d^2*e^3 + 3*a^2*b^2*d*e^4)*x^2 + (2*a*b^3*d^3*e^2 + 3*a^2*b^2*d^2*e^3)*
x)*sqrt(b/(b*d - a*e))*log((b*e*x + 2*b*d - a*e + 2*(b*d - a*e)*sqrt(e*x +
d)*sqrt(b/(b*d - a*e)))/(b*x + a)) - 2*(315*b^4*e^4*x^4 - 10*b^4*d^4 + 85*a
*b^3*d^3*e + 288*a^2*b^2*d^2*e^2 - 56*a^3*b*d*e^3 + 8*a^4*e^4 + 105*(7*b^4*
d*e^3 + 5*a*b^3*e^4)*x^3 + 21*(23*b^4*d^2*e^2 + 59*a*b^3*d*e^3 + 8*a^2*b^2*
e^4)*x^2 + 3*(15*b^4*d^3*e + 277*a*b^3*d^2*e^2 + 136*a^2*b^2*d*e^3 - 8*a^3*
b*e^4)*x)*sqrt(e*x + d))/(a^2*b^5*d^8 - 5*a^3*b^4*d^7*e + 10*a^4*b^3*d^6*e^
2 - 10*a^5*b^2*d^5*e^3 + 5*a^6*b*d^4*e^4 - a^7*d^3*e^5 + (b^7*d^5*e^3 - 5*a
```

```
*b^6*d^4*e^4 + 10*a^2*b^5*d^3*e^5 - 10*a^3*b^4*d^2*e^6 + 5*a^4*b^3*d*e^7 -
a^5*b^2*e^8)*x^5 + (3*b^7*d^6*e^2 - 13*a*b^6*d^5*e^3 + 20*a^2*b^5*d^4*e^4 -
 10*a^3*b^4*d^3*e^5 - 5*a^4*b^3*d^2*e^6 + 7*a^5*b^2*d*e^7 - 2*a^6*b*e^8)*x^
4 + (3*b^7*d^7*e - 9*a*b^6*d^6*e^2 + a^2*b^5*d^5*e^3 + 25*a^3*b^4*d^4*e^4 -
 35*a^4*b^3*d^3*e^5 + 17*a^5*b^2*d^2*e^6 - a^6*b*d*e^7 - a^7*e^8)*x^3 + (b^
7*d^8 + a*b^6*d^7*e - 17*a^2*b^5*d^6*e^2 + 35*a^3*b^4*d^5*e^3 - 25*a^4*b^3*
d^4*e^4 - a^5*b^2*d^3*e^5 + 9*a^6*b*d^2*e^6 - 3*a^7*d*e^7)*x^2 + (2*a*b^6*d
^8 - 7*a^2*b^5*d^7*e + 5*a^3*b^4*d^6*e^2 + 10*a^4*b^3*d^5*e^3 - 20*a^5*b^2*
d^4*e^4 + 13*a^6*b*d^3*e^5 - 3*a^7*d^2*e^6)*x), -1/20*(315*(b^4*e^5*x^5 + a
^2*b^2*d^3*e^2 + (3*b^4*d*e^4 + 2*a*b^3*e^5)*x^4 + (3*b^4*d^2*e^3 + 6*a*b^3
*d*e^4 + a^2*b^2*e^5)*x^3 + (b^4*d^3*e^2 + 6*a*b^3*d^2*e^3 + 3*a^2*b^2*d*e^
4)*x^2 + (2*a*b^3*d^3*e^2 + 3*a^2*b^2*d^2*e^3)*x)*sqrt(-b/(b*d - a*e))*arct
an(-(b*d - a*e)*sqrt(e*x + d)*sqrt(-b/(b*d - a*e))/(b*e*x + b*d)) - (315*b^
4*e^4*x^4 - 10*b^4*d^4 + 85*a*b^3*d^3*e + 288*a^2*b^2*d^2*e^2 - 56*a^3*b*d*
e^3 + 8*a^4*e^4 + 105*(7*b^4*d*e^3 + 5*a*b^3*e^4)*x^3 + 21*(23*b^4*d^2*e^2
+ 59*a*b^3*d*e^3 + 8*a^2*b^2*e^4)*x^2 + 3*(15*b^4*d^3*e + 277*a*b^3*d^2*e^2
 + 136*a^2*b^2*d*e^3 - 8*a^3*b*e^4)*x)*sqrt(e*x + d))/(a^2*b^5*d^8 - 5*a^3*
b^4*d^7*e + 10*a^4*b^3*d^6*e^2 - 10*a^5*b^2*d^5*e^3 + 5*a^6*b*d^4*e^4 - a^7
*d^3*e^5 + (b^7*d^5*e^3 - 5*a*b^6*d^4*e^4 + 10*a^2*b^5*d^3*e^5 - 10*a^3*b^4
*d^2*e^6 + 5*a^4*b^3*d*e^7 - a^5*b^2*e^8)*x^5 + (3*b^7*d^6*e^2 - 13*a*b^6*d
^5*e^3 + 20*a^2*b^5*d^4*e^4 - 10*a^3*b^4*d^3*e^5 - 5*a^4*b^3*d^2*e^6 + 7*a^
5*b^2*d*e^7 - 2*a^6*b*e^8)*x^4 + (3*b^7*d^7*e - 9*a*b^6*d^6*e^2 + a^2*b^5*d
^5*e^3 + 25*a^3*b^4*d^4*e^4 - 35*a^4*b^3*d^3*e^5 + 17*a^5*b^2*d^2*e^6 - a^6
*b*d*e^7 - a^7*e^8)*x^3 + (b^7*d^8 + a*b^6*d^7*e - 17*a^2*b^5*d^6*e^2 + 35*
a^3*b^4*d^5*e^3 - 25*a^4*b^3*d^4*e^4 - a^5*b^2*d^3*e^5 + 9*a^6*b*d^2*e^6 -
3*a^7*d*e^7)*x^2 + (2*a*b^6*d^8 - 7*a^2*b^5*d^7*e + 5*a^3*b^4*d^6*e^2 + 10*
a^4*b^3*d^5*e^3 - 20*a^5*b^2*d^4*e^4 + 13*a^6*b*d^3*e^5 - 3*a^7*d^2*e^6)*x)
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(7/2)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.29158, size = 1046, normalized size = 3.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 63/4*b^3*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e^2/((b^5*d^5*sgn((x*
e + d)*b*e - b*d*e + a*e^2) - 5*a*b^4*d^4*e*sgn((x*e + d)*b*e - b*d*e + a*e
^2) + 10*a^2*b^3*d^3*e^2*sgn((x*e + d)*b*e - b*d*e + a*e^2) - 10*a^3*b^2*d^
2*e^3*sgn((x*e + d)*b*e - b*d*e + a*e^2) + 5*a^4*b*d*e^4*sgn((x*e + d)*b*e
- b*d*e + a*e^2) - a^5*e^5*sgn((x*e + d)*b*e - b*d*e + a*e^2))*sqrt(-b^2*d
+ a*b*e)) + 1/4*(15*(x*e + d)^(3/2)*b^4*e^2 - 17*sqrt(x*e + d)*b^4*d*e^2 +
17*sqrt(x*e + d)*a*b^3*e^3)/((b^5*d^5*sgn((x*e + d)*b*e - b*d*e + a*e^2) -
```

```
5*a*b^4*d^4*e*sgn((x*e + d)*b*e - b*d*e + a*e^2) + 10*a^2*b^3*d^3*e^2*sgn((
x*e + d)*b*e - b*d*e + a*e^2) - 10*a^3*b^2*d^2*e^3*sgn((x*e + d)*b*e - b*d*
e + a*e^2) + 5*a^4*b*d*e^4*sgn((x*e + d)*b*e - b*d*e + a*e^2) - a^5*e^5*sgn
((x*e + d)*b*e - b*d*e + a*e^2))*((x*e + d)*b - b*d + a*e)^2) + 2/5*(30*(x*
e + d)^2*b^2*e^2 + 5*(x*e + d)*b^2*d*e^2 + b^2*d^2*e^2 - 5*(x*e + d)*a*b*e^
3 - 2*a*b*d*e^3 + a^2*e^4)/((b^5*d^5*sgn((x*e + d)*b*e - b*d*e + a*e^2) - 5
*a*b^4*d^4*e*sgn((x*e + d)*b*e - b*d*e + a*e^2) + 10*a^2*b^3*d^3*e^2*sgn((x
*e + d)*b*e - b*d*e + a*e^2) - 10*a^3*b^2*d^2*e^3*sgn((x*e + d)*b*e - b*d*e
 + a*e^2) + 5*a^4*b*d*e^4*sgn((x*e + d)*b*e - b*d*e + a*e^2) - a^5*e^5*sgn(
(x*e + d)*b*e - b*d*e + a*e^2))*(x*e + d)^(5/2))
```

3.1720 $\int \frac{(d+ex)^{13/2}}{\left(a^2+2abx+b^2x^2\right)^{5/2}} dx$

Optimal. Leaf size=400

$$-\frac{143e^2(d+ex)^{9/2}}{96b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{429e^3(d+ex)^{7/2}}{64b^4\sqrt{a^2+2abx+b^2x^2}}+\frac{3003e^4(a+bx)(d+ex)^{5/2}}{320b^5\sqrt{a^2+2abx+b^2x^2}}+\frac{1001e^4(a+bx)(d+ex)^{3/2}(}{64b^6\sqrt{a^2+2abx+b}}$$

```
[Out] (3003*e^4*(b*d - a*e)^2*(a + b*x)*Sqrt[d + e*x])/(64*b^7*Sqrt[a^2 + 2*a*b*x
 + b^2*x^2]) + (1001*e^4*(b*d - a*e)*(a + b*x)*(d + e*x)^(3/2))/(64*b^6*Sqr
t[a^2 + 2*a*b*x + b^2*x^2]) + (3003*e^4*(a + b*x)*(d + e*x)^(5/2))/(320*b^5
*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (429*e^3*(d + e*x)^(7/2))/(64*b^4*Sqrt[a^
2 + 2*a*b*x + b^2*x^2]) - (143*e^2*(d + e*x)^(9/2))/(96*b^3*(a + b*x)*Sqrt[
a^2 + 2*a*b*x + b^2*x^2]) - (13*e*(d + e*x)^(11/2))/(24*b^2*(a + b*x)^2*Sqr
t[a^2 + 2*a*b*x + b^2*x^2]) - (d + e*x)^(13/2)/(4*b*(a + b*x)^3*Sqrt[a^2 +
2*a*b*x + b^2*x^2]) - (3003*e^4*(b*d - a*e)^(5/2)*(a + b*x)*ArcTanh[(Sqrt[b
]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(64*b^(15/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^
2])
```

Rubi [A] time = 0.237824, antiderivative size = 400, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {646, 47, 50, 63, 208}

$$-\frac{143e^2(d+ex)^{9/2}}{96b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{429e^3(d+ex)^{7/2}}{64b^4\sqrt{a^2+2abx+b^2x^2}}+\frac{3003e^4(a+bx)(d+ex)^{5/2}}{320b^5\sqrt{a^2+2abx+b^2x^2}}+\frac{1001e^4(a+bx)(d+ex)^{3/2}(}{64b^6\sqrt{a^2+2abx+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(13/2)/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```


```
[Out] (3003*e^4*(b*d - a*e)^2*(a + b*x)*Sqrt[d + e*x])/(64*b^7*Sqrt[a^2 + 2*a*b*x
 + b^2*x^2]) + (1001*e^4*(b*d - a*e)*(a + b*x)*(d + e*x)^(3/2))/(64*b^6*Sqr
t[a^2 + 2*a*b*x + b^2*x^2]) + (3003*e^4*(a + b*x)*(d + e*x)^(5/2))/(320*b^5
*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (429*e^3*(d + e*x)^(7/2))/(64*b^4*Sqrt[a^
2 + 2*a*b*x + b^2*x^2]) - (143*e^2*(d + e*x)^(9/2))/(96*b^3*(a + b*x)*Sqrt[
a^2 + 2*a*b*x + b^2*x^2]) - (13*e*(d + e*x)^(11/2))/(24*b^2*(a + b*x)^2*Sqr
t[a^2 + 2*a*b*x + b^2*x^2]) - (d + e*x)^(13/2)/(4*b*(a + b*x)^3*Sqrt[a^2 +
2*a*b*x + b^2*x^2]) - (3003*e^4*(b*d - a*e)^(5/2)*(a + b*x)*ArcTanh[(Sqrt[b
]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(64*b^(15/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^
2])
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
```

```
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{13/2}}{\left(a^2+2abx+b^2x^2\right)^{5/2}}dx = \frac{\left(b^4\left(ab+b^2x\right)\right)\int \frac{(d+ex)^{13/2}}{\left(ab+b^2x\right)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{(d+ex)^{13/2}}{4b(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(13b^2e\left(ab+b^2x\right)\right)\int \frac{(d+ex)^{11/2}}{\left(ab+b^2x\right)^4}dx}{8\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{13e(d+ex)^{11/2}}{24b^2(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{(d+ex)^{13/2}}{4b(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(143e^2\left(ab+b^2\right.\right.}{48\sqrt{a^2+2}}$$

$$= -\frac{143e^2(d+ex)^{9/2}}{96b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{13e(d+ex)^{11/2}}{24b^2(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{(d+}{4b(a+bx)^3\sqrt{a}}$$

$$= -\frac{429e^3(d+ex)^{7/2}}{64b^4\sqrt{a^2+2abx+b^2x^2}} - \frac{143e^2(d+ex)^{9/2}}{96b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{13e(d+ex)^{11/2}}{24b^2(a+bx)^2\sqrt{a^2+2al}}$$

$$= \frac{3003e^4(a+bx)(d+ex)^{5/2}}{320b^5\sqrt{a^2+2abx+b^2x^2}} - \frac{429e^3(d+ex)^{7/2}}{64b^4\sqrt{a^2+2abx+b^2x^2}} - \frac{143e^2(d+ex)^{9/2}}{96b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}$$

$$= \frac{1001e^4(bd-ae)(a+bx)(d+ex)^{3/2}}{64b^6\sqrt{a^2+2abx+b^2x^2}} + \frac{3003e^4(a+bx)(d+ex)^{5/2}}{320b^5\sqrt{a^2+2abx+b^2x^2}} - \frac{429e^3(d+ex)^{7/2}}{64b^4\sqrt{a^2+2abx+b^2}}$$

$$= \frac{3003e^4(bd-ae)^2(a+bx)\sqrt{d+ex}}{64b^7\sqrt{a^2+2abx+b^2x^2}} + \frac{1001e^4(bd-ae)(a+bx)(d+ex)^{3/2}}{64b^6\sqrt{a^2+2abx+b^2x^2}} + \frac{3003e^4(a+b}{320b^5\sqrt{a^2+}}$$

$$= \frac{3003e^4(bd-ae)^2(a+bx)\sqrt{d+ex}}{64b^7\sqrt{a^2+2abx+b^2x^2}} + \frac{1001e^4(bd-ae)(a+bx)(d+ex)^{3/2}}{64b^6\sqrt{a^2+2abx+b^2x^2}} + \frac{3003e^4(a+b}{320b^5\sqrt{a^2+}}$$

$$= \frac{3003e^4(bd-ae)^2(a+bx)\sqrt{d+ex}}{64b^7\sqrt{a^2+2abx+b^2x^2}} + \frac{1001e^4(bd-ae)(a+bx)(d+ex)^{3/2}}{64b^6\sqrt{a^2+2abx+b^2x^2}} + \frac{3003e^4(a+b}{320b^5\sqrt{a^2+}}$$

Mathematica [C] time = 0.0550612, size = 67, normalized size = 0.17

$$-\frac{2e^4(a+bx)(d+ex)^{15/2}\,{}_2F_1\left(5,\frac{15}{2};\frac{17}{2};\frac{b(d+ex)}{bd-ae}\right)}{15\sqrt{(a+bx)^2}(bd-ae)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(13/2)/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (-2*e^4*(a + b*x)*(d + e*x)^(15/2)*Hypergeometric2F1[5, 15/2, 17/2, (b*(d +
 e*x))/(b*d - a*e)])/(15*(b*d - a*e)^5*Sqrt[(a + b*x)^2])
```

Maple [B] time = 0.284, size = 2192, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(13/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] 1/960*(59219*((a*e-b*d)*b)^(1/2)*(e*x+d)^(5/2)*a^4*b^2*e^4-180180*arctan(b*
(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x*a^6*b*e^7+49965*((a*e-b*d)*b)^(1/2)*(e
*x+d)^(3/2)*a^5*b*e^5+135135*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a^
6*b*d*e^6-135135*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a^5*b^2*d^2*e^
5+45045*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a^4*b^3*d^3*e^4+384*((a
*e-b*d)*b)^(1/2)*(e*x+d)^(5/2)*x^4*b^6*e^4-45045*arctan(b*(e*x+d)^(1/2)/((a
*e-b*d)*b)^(1/2))*x^4*a^3*b^4*e^7+45045*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b
)^(1/2))*x^4*b^7*d^3*e^4-180180*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))
*x^3*a^4*b^3*e^7+22155*((a*e-b*d)*b)^(1/2)*(e*x+d)^(7/2)*a^3*b^3*e^3-270270
*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^2*a^5*b^2*e^7-22155*((a*e-b*
d)*b)^(1/2)*(e*x+d)^(7/2)*b^6*d^3+58835*((a*e-b*d)*b)^(1/2)*(e*x+d)^(5/2)*b
^6*d^4-53165*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*b^6*d^5+45045*((a*e-b*d)*b)^
(1/2)*(e*x+d)^(1/2)*a^6*e^6+16245*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*b^6*d^6
-45045*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a^7*e^7+12800*((a*e-b*d)
*b)^(1/2)*(e*x+d)^(3/2)*x^3*a*b^5*d*e^4-57600*((a*e-b*d)*b)^(1/2)*(e*x+d)^(
1/2)*x^4*a*b^5*d*e^5+19200*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*x^2*a^2*b^4*d*
e^4-230400*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*x^3*a^2*b^4*d*e^5+115200*((a*e
-b*d)*b)^(1/2)*(e*x+d)^(1/2)*x^3*a*b^5*d^2*e^4+12800*((a*e-b*d)*b)^(1/2)*(e
*x+d)^(3/2)*x*a^3*b^3*d*e^4-345600*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*x^2*a^
3*b^3*d*e^5+172800*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*x^2*a^2*b^4*d^2*e^4-23
0400*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*x*a^4*b^2*d*e^5+115200*((a*e-b*d)*b)
^(1/2)*(e*x+d)^(1/2)*x*a^3*b^3*d^2*e^4+115200*((a*e-b*d)*b)^(1/2)*(e*x+d)^(
1/2)*x^3*a^3*b^3*e^6+810810*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^2
*a^4*b^3*d*e^6-810810*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^2*a^3*b
^4*d^2*e^5+270270*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^2*a^2*b^5*d
^3*e^4-235340*((a*e-b*d)*b)^(1/2)*(e*x+d)^(5/2)*a^3*b^3*d*e^3+353010*((a*e-
b*d)*b)^(1/2)*(e*x+d)^(5/2)*a^2*b^4*d^2*e^2-235340*((a*e-b*d)*b)^(1/2)*(e*x
+d)^(5/2)*a*b^5*d^3*e-12800*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*x*a^4*b^2*e^5
+172800*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*x^2*a^4*b^2*e^6+540540*arctan(b*(
e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x*a^5*b^2*d*e^6-540540*arctan(b*(e*x+d)^(
1/2)/((a*e-b*d)*b)^(1/2))*x*a^4*b^3*d^2*e^5+180180*arctan(b*(e*x+d)^(1/2)/(
(a*e-b*d)*b)^(1/2))*x*a^3*b^4*d^3*e^4-262625*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3
/2)*a^4*b^2*d*e^4+531650*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*a^3*b^3*d^2*e^3-
531650*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*a^2*b^4*d^3*e^2+265825*((a*e-b*d)*
b)^(1/2)*(e*x+d)^(3/2)*a*b^5*d^4*e+115200*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)
*x*a^5*b*e^6-155070*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*a^5*b*d*e^5+272475*((
a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*a^4*b^2*d^2*e^4+1536*((a*e-b*d)*b)^(1/2)*(e
*x+d)^(5/2)*x^3*a*b^5*e^4-3200*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*x^4*a*b^5*
e^5+3200*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*x^4*b^6*d*e^4+135135*arctan(b*(e
*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^4*a^2*b^5*d*e^6-135135*arctan(b*(e*x+d)^
(1/2)/((a*e-b*d)*b)^(1/2))*x^4*a*b^6*d^2*e^5+2304*((a*e-b*d)*b)^(1/2)*(e*x+
d)^(5/2)*x^2*a^2*b^4*e^4-12800*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*x^3*a^2*b^
4*e^5+28800*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*x^4*a^2*b^4*e^6+28800*((a*e-b
*d)*b)^(1/2)*(e*x+d)^(1/2)*x^4*b^6*d^2*e^4+540540*arctan(b*(e*x+d)^(1/2)/((
a*e-b*d)*b)^(1/2))*x^3*a^3*b^4*d*e^6-540540*arctan(b*(e*x+d)^(1/2)/((a*e-b*
d)*b)^(1/2))*x^3*a^2*b^5*d^2*e^5-324900*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*a
^3*b^3*d^3*e^3+243675*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*a^2*b^4*d^4*e^2-974
70*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*a*b^5*d^5*e+180180*arctan(b*(e*x+d)^(1
/2)/((a*e-b*d)*b)^(1/2))*x^3*a*b^6*d^3*e^4-66465*((a*e-b*d)*b)^(1/2)*(e*x+d
)^(7/2)*a^2*b^4*d*e^2+66465*((a*e-b*d)*b)^(1/2)*(e*x+d)^(7/2)*a*b^5*d^2*e+1
536*((a*e-b*d)*b)^(1/2)*(e*x+d)^(5/2)*x*a^3*b^3*e^4-19200*((a*e-b*d)*b)^(1/
2)*(e*x+d)^(3/2)*x^2*a^3*b^3*e^5)*(b*x+a)/((a*e-b*d)*b)^(1/2)/b^7/((b*x+a)^
2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{13}{2}}}{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(13/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(13/2)/(b^2*x^2 + 2*a*b*x + a^2)^(5/2), x)
```

Fricas [B] time = 1.81771, size = 2846, normalized size = 7.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(13/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/1920*(45045*(a^4*b^2*d^2*e^4 - 2*a^5*b*d*e^5 + a^6*e^6 + (b^6*d^2*e^4 -
2*a*b^5*d*e^5 + a^2*b^4*e^6)*x^4 + 4*(a*b^5*d^2*e^4 - 2*a^2*b^4*d*e^5 + a^3
*b^3*e^6)*x^3 + 6*(a^2*b^4*d^2*e^4 - 2*a^3*b^3*d*e^5 + a^4*b^2*e^6)*x^2 + 4
*(a^3*b^3*d^2*e^4 - 2*a^4*b^2*d*e^5 + a^5*b*e^6)*x)*sqrt((b*d - a*e)/b)*log
((b*e*x + 2*b*d - a*e - 2*sqrt(e*x + d)*b*sqrt((b*d - a*e)/b))/(b*x + a)) +
 2*(384*b^6*e^6*x^6 - 240*b^6*d^6 - 520*a*b^5*d^5*e - 1430*a^2*b^4*d^4*e^2
- 6435*a^3*b^3*d^3*e^3 + 69069*a^4*b^2*d^2*e^4 - 105105*a^5*b*d*e^5 + 45045
*a^6*e^6 + 128*(31*b^6*d*e^5 - 13*a*b^5*e^6)*x^5 + 128*(253*b^6*d^2*e^4 - 3
51*a*b^5*d*e^5 + 143*a^2*b^4*e^6)*x^4 - (22155*b^6*d^3*e^3 - 196001*a*b^5*d
^2*e^4 + 285857*a^2*b^4*d*e^5 - 119691*a^3*b^3*e^6)*x^3 - (7630*b^6*d^4*e^2
 + 35945*a*b^5*d^3*e^3 - 347919*a^2*b^4*d^2*e^4 + 517803*a^3*b^3*d*e^5 - 21
9219*a^4*b^2*e^6)*x^2 - (1960*b^6*d^5*e + 5460*a*b^5*d^4*e^2 + 25025*a^2*b^
4*d^3*e^3 - 256971*a^3*b^3*d^2*e^4 + 387387*a^4*b^2*d*e^5 - 165165*a^5*b*e^
6)*x)*sqrt(e*x + d))/(b^11*x^4 + 4*a*b^10*x^3 + 6*a^2*b^9*x^2 + 4*a^3*b^8*x
 + a^4*b^7), -1/960*(45045*(a^4*b^2*d^2*e^4 - 2*a^5*b*d*e^5 + a^6*e^6 + (b^
6*d^2*e^4 - 2*a*b^5*d*e^5 + a^2*b^4*e^6)*x^4 + 4*(a*b^5*d^2*e^4 - 2*a^2*b^4
*d*e^5 + a^3*b^3*e^6)*x^3 + 6*(a^2*b^4*d^2*e^4 - 2*a^3*b^3*d*e^5 + a^4*b^2*
e^6)*x^2 + 4*(a^3*b^3*d^2*e^4 - 2*a^4*b^2*d*e^5 + a^5*b*e^6)*x)*sqrt(-(b*d
- a*e)/b)*arctan(-sqrt(e*x + d)*b*sqrt(-(b*d - a*e)/b)/(b*d - a*e)) - (384*
b^6*e^6*x^6 - 240*b^6*d^6 - 520*a*b^5*d^5*e - 1430*a^2*b^4*d^4*e^2 - 6435*a
^3*b^3*d^3*e^3 + 69069*a^4*b^2*d^2*e^4 - 105105*a^5*b*d*e^5 + 45045*a^6*e^6
 + 128*(31*b^6*d*e^5 - 13*a*b^5*e^6)*x^5 + 128*(253*b^6*d^2*e^4 - 351*a*b^5
*d*e^5 + 143*a^2*b^4*e^6)*x^4 - (22155*b^6*d^3*e^3 - 196001*a*b^5*d^2*e^4 +
 285857*a^2*b^4*d*e^5 - 119691*a^3*b^3*e^6)*x^3 - (7630*b^6*d^4*e^2 + 35945
*a*b^5*d^3*e^3 - 347919*a^2*b^4*d^2*e^4 + 517803*a^3*b^3*d*e^5 - 219219*a^4
*b^2*e^6)*x^2 - (1960*b^6*d^5*e + 5460*a*b^5*d^4*e^2 + 25025*a^2*b^4*d^3*e^
3 - 256971*a^3*b^3*d^2*e^4 + 387387*a^4*b^2*d*e^5 - 165165*a^5*b*e^6)*x)*sq
rt(e*x + d))/(b^11*x^4 + 4*a*b^10*x^3 + 6*a^2*b^9*x^2 + 4*a^3*b^8*x + a^4*b
^7)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(13/2)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.38168, size = 945, normalized size = 2.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(13/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 3003/64*(b^3*d^3*e^4 - 3*a*b^2*d^2*e^5 + 3*a^2*b*d*e^6 - a^3*e^7)*arctan(sq
rt(x*e + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^7*sgn((x*e + d)
*b*e - b*d*e + a*e^2)) - 1/192*(4431*(x*e + d)^(7/2)*b^6*d^3*e^4 - 11767*(x
*e + d)^(5/2)*b^6*d^4*e^4 + 10633*(x*e + d)^(3/2)*b^6*d^5*e^4 - 3249*sqrt(x
*e + d)*b^6*d^6*e^4 - 13293*(x*e + d)^(7/2)*a*b^5*d^2*e^5 + 47068*(x*e + d)
^(5/2)*a*b^5*d^3*e^5 - 53165*(x*e + d)^(3/2)*a*b^5*d^4*e^5 + 19494*sqrt(x*e
 + d)*a*b^5*d^5*e^5 + 13293*(x*e + d)^(7/2)*a^2*b^4*d*e^6 - 70602*(x*e + d)
^(5/2)*a^2*b^4*d^2*e^6 + 106330*(x*e + d)^(3/2)*a^2*b^4*d^3*e^6 - 48735*sqr
t(x*e + d)*a^2*b^4*d^4*e^6 - 4431*(x*e + d)^(7/2)*a^3*b^3*e^7 + 47068*(x*e
+ d)^(5/2)*a^3*b^3*d*e^7 - 106330*(x*e + d)^(3/2)*a^3*b^3*d^2*e^7 + 64980*s
qrt(x*e + d)*a^3*b^3*d^3*e^7 - 11767*(x*e + d)^(5/2)*a^4*b^2*e^8 + 53165*(x
*e + d)^(3/2)*a^4*b^2*d*e^8 - 48735*sqrt(x*e + d)*a^4*b^2*d^2*e^8 - 10633*(
x*e + d)^(3/2)*a^5*b*e^9 + 19494*sqrt(x*e + d)*a^5*b*d*e^9 - 3249*sqrt(x*e
+ d)*a^6*e^10)/(((x*e + d)*b - b*d + a*e)^4*b^7*sgn((x*e + d)*b*e - b*d*e +
 a*e^2)) + 2/15*(3*(x*e + d)^(5/2)*b^20*e^4 + 25*(x*e + d)^(3/2)*b^20*d*e^4
 + 225*sqrt(x*e + d)*b^20*d^2*e^4 - 25*(x*e + d)^(3/2)*a*b^19*e^5 - 450*sqr
t(x*e + d)*a*b^19*d*e^5 + 225*sqrt(x*e + d)*a^2*b^18*e^6)/(b^25*sgn((x*e +
d)*b*e - b*d*e + a*e^2))
```

3.1721 $\int \frac{(d+ex)^{11/2}}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=346

$$-\frac{33e^2(d+ex)^{7/2}}{32b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{231e^3(d+ex)^{5/2}}{64b^4\sqrt{a^2+2abx+b^2x^2}}+\frac{385e^4(a+bx)(d+ex)^{3/2}}{64b^5\sqrt{a^2+2abx+b^2x^2}}+\frac{1155e^4(a+bx)\sqrt{d+ex}(bd}{64b^6\sqrt{a^2+2abx+b^2x^2}}$$

```
[Out] (1155*e^4*(b*d - a*e)*(a + b*x)*Sqrt[d + e*x])/(64*b^6*Sqrt[a^2 + 2*a*b*x +
 b^2*x^2]) + (385*e^4*(a + b*x)*(d + e*x)^(3/2))/(64*b^5*Sqrt[a^2 + 2*a*b*x
 + b^2*x^2]) - (231*e^3*(d + e*x)^(5/2))/(64*b^4*Sqrt[a^2 + 2*a*b*x + b^2*x
^2]) - (33*e^2*(d + e*x)^(7/2))/(32*b^3*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*
x^2]) - (11*e*(d + e*x)^(9/2))/(24*b^2*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2
*x^2]) - (d + e*x)^(11/2)/(4*b*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) -
 (1155*e^4*(b*d - a*e)^(3/2)*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt
[b*d - a*e]])/(64*b^(13/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.188314, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {646, 47, 50, 63, 208}

$$-\frac{33e^2(d+ex)^{7/2}}{32b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{231e^3(d+ex)^{5/2}}{64b^4\sqrt{a^2+2abx+b^2x^2}}+\frac{385e^4(a+bx)(d+ex)^{3/2}}{64b^5\sqrt{a^2+2abx+b^2x^2}}+\frac{1155e^4(a+bx)\sqrt{d+ex}(bd}{64b^6\sqrt{a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(11/2)/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```


```
[Out] (1155*e^4*(b*d - a*e)*(a + b*x)*Sqrt[d + e*x])/(64*b^6*Sqrt[a^2 + 2*a*b*x +
 b^2*x^2]) + (385*e^4*(a + b*x)*(d + e*x)^(3/2))/(64*b^5*Sqrt[a^2 + 2*a*b*x
 + b^2*x^2]) - (231*e^3*(d + e*x)^(5/2))/(64*b^4*Sqrt[a^2 + 2*a*b*x + b^2*x
^2]) - (33*e^2*(d + e*x)^(7/2))/(32*b^3*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*
x^2]) - (11*e*(d + e*x)^(9/2))/(24*b^2*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2
*x^2]) - (d + e*x)^(11/2)/(4*b*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) -
 (1155*e^4*(b*d - a*e)^(3/2)*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt
[b*d - a*e]])/(64*b^(13/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{11/2}}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x\right)\right)\int \frac{(d+ex)^{11/2}}{\left(ab+b^2x\right)^5}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{(d+ex)^{11/2}}{4b(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(11b^2e\left(ab+b^2x\right)\right)\int \frac{(d+ex)^{9/2}}{\left(ab+b^2x\right)^4}\,dx}{8\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{11e(d+ex)^{9/2}}{24b^2(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{(d+ex)^{11/2}}{4b(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(33e^2\left(ab+b^2x\right)\right)\int}{16\sqrt{a^2+2abx}}$$

$$= -\frac{33e^2(d+ex)^{7/2}}{32b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{11e(d+ex)^{9/2}}{24b^2(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{(d+ex)}{4b(a+bx)^3\sqrt{a^2+}}$$

$$= -\frac{231e^3(d+ex)^{5/2}}{64b^4\sqrt{a^2+2abx+b^2x^2}} - \frac{33e^2(d+ex)^{7/2}}{32b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{11e(d+ex)^{9/2}}{24b^2(a+bx)^2\sqrt{a^2+2abx+}}$$

$$= \frac{385e^4(a+bx)(d+ex)^{3/2}}{64b^5\sqrt{a^2+2abx+b^2x^2}} - \frac{231e^3(d+ex)^{5/2}}{64b^4\sqrt{a^2+2abx+b^2x^2}} - \frac{33e^2(d+ex)^{7/2}}{32b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{}{2}$$

$$= \frac{1155e^4(bd-ae)(a+bx)\sqrt{d+ex}}{64b^6\sqrt{a^2+2abx+b^2x^2}} + \frac{385e^4(a+bx)(d+ex)^{3/2}}{64b^5\sqrt{a^2+2abx+b^2x^2}} - \frac{231e^3(d+ex)^{5/2}}{64b^4\sqrt{a^2+2abx+b^2x^2}} - \frac{}{3}$$

$$= \frac{1155e^4(bd-ae)(a+bx)\sqrt{d+ex}}{64b^6\sqrt{a^2+2abx+b^2x^2}} + \frac{385e^4(a+bx)(d+ex)^{3/2}}{64b^5\sqrt{a^2+2abx+b^2x^2}} - \frac{231e^3(d+ex)^{5/2}}{64b^4\sqrt{a^2+2abx+b^2x^2}} - \frac{}{3}$$

$$= \frac{1155e^4(bd-ae)(a+bx)\sqrt{d+ex}}{64b^6\sqrt{a^2+2abx+b^2x^2}} + \frac{385e^4(a+bx)(d+ex)^{3/2}}{64b^5\sqrt{a^2+2abx+b^2x^2}} - \frac{231e^3(d+ex)^{5/2}}{64b^4\sqrt{a^2+2abx+b^2x^2}} - \frac{}{3}$$

Mathematica [C] time = 0.0491772, size = 67, normalized size = 0.19

$$-\frac{2e^4(a+bx)(d+ex)^{13/2}\,{}_2F_1\left(5,\frac{13}{2};\frac{15}{2};\frac{b(d+ex)}{bd-ae}\right)}{13\sqrt{(a+bx)^2}(bd-ae)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(11/2)/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (-2*e^4*(a + b*x)*(d + e*x)^(13/2)*Hypergeometric2F1[5, 13/2, 15/2, (b*(d +
 e*x))/(b*d - a*e)])/(13*(b*d - a*e)^5*Sqrt[(a + b*x)^2])
```

Maple [B] time = 0.288, size = 1471, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(11/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] -1/192*(-3465*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^4*a^2*b^4*e^6-3
465*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^4*b^6*d^2*e^4-13860*arcta
n(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^3*a^3*b^3*e^6+2295*((a*e-b*d)*b)^(
1/2)*(e*x+d)^(7/2)*a^2*b^3*e^2-20790*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(
1/2))*x^2*a^4*b^2*e^6+5855*((a*e-b*d)*b)^(1/2)*(e*x+d)^(5/2)*a^3*b^2*e^3-13
860*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x*a^5*b*e^6+5025*((a*e-b*d)
*b)^(1/2)*(e*x+d)^(3/2)*a^4*b*e^4+6930*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)
^(1/2))*a^5*b*d*e^5-3465*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a^4*b^
2*d^2*e^4-128*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*x^4*b^5*e^4+2295*((a*e-b*d)
*b)^(1/2)*(e*x+d)^(7/2)*b^5*d^2-5855*((a*e-b*d)*b)^(1/2)*(e*x+d)^(5/2)*b^5*
d^3+5153*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*b^5*d^4+3465*((a*e-b*d)*b)^(1/2)
*(e*x+d)^(1/2)*a^5*e^5-1545*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*b^5*d^5-3465*
arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a^6*e^6-7680*((a*e-b*d)*b)^(1/2
)*(e*x+d)^(1/2)*x^3*a*b^4*d*e^4-11520*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*x^2
*a^2*b^3*d*e^4-7680*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*x*a^3*b^2*d*e^4-4590*
((a*e-b*d)*b)^(1/2)*(e*x+d)^(7/2)*a*b^4*d*e-768*((a*e-b*d)*b)^(1/2)*(e*x+d)
^(3/2)*x^2*a^2*b^3*e^4+7680*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*x^3*a^2*b^3*e
^5+41580*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^2*a^3*b^3*d*e^5-2079
0*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^2*a^2*b^4*d^2*e^4-17565*((a
*e-b*d)*b)^(1/2)*(e*x+d)^(5/2)*a^2*b^3*d*e^2+17565*((a*e-b*d)*b)^(1/2)*(e*x
+d)^(5/2)*a*b^4*d^2*e-512*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*x*a^3*b^2*e^4+6
930*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^4*a*b^5*d*e^5-512*((a*e-b
*d)*b)^(1/2)*(e*x+d)^(3/2)*x^3*a*b^4*e^4+1920*((a*e-b*d)*b)^(1/2)*(e*x+d)^(
1/2)*x^4*a*b^4*e^5+11520*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*x^2*a^3*b^2*e^5+
27720*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x*a^4*b^2*d*e^5-13860*arc
tan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x*a^3*b^3*d^2*e^4-20612*((a*e-b*d)
*b)^(1/2)*(e*x+d)^(3/2)*a^3*b^2*d*e^3+30918*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/
2)*a^2*b^3*d^2*e^2-20612*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*a*b^4*d^3*e+7680
*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*x*a^4*b*e^5-9645*((a*e-b*d)*b)^(1/2)*(e*
x+d)^(1/2)*a^4*b*d*e^4+15450*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*a^3*b^2*d^2*
e^3-15450*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*a^2*b^3*d^3*e^2+7725*((a*e-b*d)
*b)^(1/2)*(e*x+d)^(1/2)*a*b^4*d^4*e-1920*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*
x^4*b^5*d*e^4+27720*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^3*a^2*b^4
*d*e^5-13860*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^3*a*b^5*d^2*e^4)
*(b*x+a)/((a*e-b*d)*b)^(1/2)/b^6/((b*x+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{11}{2}}}{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(11/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")

[Out] integrate((e*x + d)^(11/2)/(b^2*x^2 + 2*a*b*x + a^2)^(5/2), x)
```

Fricas [B] time = 1.70008, size = 2079, normalized size = 6.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(11/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")

[Out] [-1/384*(3465*(a^4*b*d*e^4 - a^5*e^5 + (b^5*d*e^4 - a*b^4*e^5)*x^4 + 4*(a*b
^4*d*e^4 - a^2*b^3*e^5)*x^3 + 6*(a^2*b^3*d*e^4 - a^3*b^2*e^5)*x^2 + 4*(a^3*
b^2*d*e^4 - a^4*b*e^5)*x)*sqrt((b*d - a*e)/b)*log((b*e*x + 2*b*d - a*e + 2*
sqrt(e*x + d)*b*sqrt((b*d - a*e)/b))/(b*x + a)) - 2*(128*b^5*e^5*x^5 - 48*b
^5*d^5 - 88*a*b^4*d^4*e - 198*a^2*b^3*d^3*e^2 - 693*a^3*b^2*d^2*e^3 + 4620*
a^4*b*d*e^4 - 3465*a^5*e^5 + 128*(16*b^5*d*e^4 - 11*a*b^4*e^5)*x^4 - (2295*
b^5*d^2*e^3 - 12782*a*b^4*d*e^4 + 9207*a^2*b^3*e^5)*x^3 - (1030*b^5*d^3*e^2
 + 3795*a*b^4*d^2*e^3 - 22968*a^2*b^3*d*e^4 + 16863*a^3*b^2*e^5)*x^2 - (328
*b^5*d^4*e + 748*a*b^4*d^3*e^2 + 2673*a^2*b^3*d^2*e^3 - 17094*a^3*b^2*d*e^4
 + 12705*a^4*b*e^5)*x)*sqrt(e*x + d))/(b^10*x^4 + 4*a*b^9*x^3 + 6*a^2*b^8*x
^2 + 4*a^3*b^7*x + a^4*b^6), -1/192*(3465*(a^4*b*d*e^4 - a^5*e^5 + (b^5*d*e
^4 - a*b^4*e^5)*x^4 + 4*(a*b^4*d*e^4 - a^2*b^3*e^5)*x^3 + 6*(a^2*b^3*d*e^4
- a^3*b^2*e^5)*x^2 + 4*(a^3*b^2*d*e^4 - a^4*b*e^5)*x)*sqrt(-(b*d - a*e)/b)*
arctan(-sqrt(e*x + d)*b*sqrt(-(b*d - a*e)/b)/(b*d - a*e)) - (128*b^5*e^5*x^
5 - 48*b^5*d^5 - 88*a*b^4*d^4*e - 198*a^2*b^3*d^3*e^2 - 693*a^3*b^2*d^2*e^3
 + 4620*a^4*b*d*e^4 - 3465*a^5*e^5 + 128*(16*b^5*d*e^4 - 11*a*b^4*e^5)*x^4
- (2295*b^5*d^2*e^3 - 12782*a*b^4*d*e^4 + 9207*a^2*b^3*e^5)*x^3 - (1030*b^5
*d^3*e^2 + 3795*a*b^4*d^2*e^3 - 22968*a^2*b^3*d*e^4 + 16863*a^3*b^2*e^5)*x^
2 - (328*b^5*d^4*e + 748*a*b^4*d^3*e^2 + 2673*a^2*b^3*d^2*e^3 - 17094*a^3*b
^2*d*e^4 + 12705*a^4*b*e^5)*x)*sqrt(e*x + d))/(b^10*x^4 + 4*a*b^9*x^3 + 6*a
^2*b^8*x^2 + 4*a^3*b^7*x + a^4*b^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(11/2)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)

[Out] Timed out
```

Giac [B] time = 1.33739, size = 740, normalized size = 2.14

$$\frac{1155\left(b^{2}d^{2}e^{4}-2\,abde^{5}+a^{2}e^{6}\right)\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^{2}d+abe}}\right)}{64\sqrt{-b^{2}d+abe}b^{6}\mathrm{sgn}\left((xe+d)be-bde+ae^{2}\right)}-\frac{2295\,(xe+d)^{\frac{7}{2}}b^{5}d^{2}e^{4}-5855\,(xe+d)^{\frac{5}{2}}b^{5}d^{3}e^{4}+5153\,(xe+d)^{\frac{3}{2}}b^{5}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(11/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1155/64*(b^2*d^2*e^4 - 2*a*b*d*e^5 + a^2*e^6)*arctan(sqrt(x*e + d)*b/sqrt(-
b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^6*sgn((x*e + d)*b*e - b*d*e + a*e^2
)) - 1/192*(2295*(x*e + d)^(7/2)*b^5*d^2*e^4 - 5855*(x*e + d)^(5/2)*b^5*d^3
*e^4 + 5153*(x*e + d)^(3/2)*b^5*d^4*e^4 - 1545*sqrt(x*e + d)*b^5*d^5*e^4 -
4590*(x*e + d)^(7/2)*a*b^4*d*e^5 + 17565*(x*e + d)^(5/2)*a*b^4*d^2*e^5 - 20
612*(x*e + d)^(3/2)*a*b^4*d^3*e^5 + 7725*sqrt(x*e + d)*a*b^4*d^4*e^5 + 2295
*(x*e + d)^(7/2)*a^2*b^3*e^6 - 17565*(x*e + d)^(5/2)*a^2*b^3*d*e^6 + 30918*
(x*e + d)^(3/2)*a^2*b^3*d^2*e^6 - 15450*sqrt(x*e + d)*a^2*b^3*d^3*e^6 + 585
5*(x*e + d)^(5/2)*a^3*b^2*e^7 - 20612*(x*e + d)^(3/2)*a^3*b^2*d*e^7 + 15450
*sqrt(x*e + d)*a^3*b^2*d^2*e^7 + 5153*(x*e + d)^(3/2)*a^4*b*e^8 - 7725*sqrt
(x*e + d)*a^4*b*d*e^8 + 1545*sqrt(x*e + d)*a^5*e^9)/(((x*e + d)*b - b*d + a
*e)^4*b^6*sgn((x*e + d)*b*e - b*d*e + a*e^2)) + 2/3*((x*e + d)^(3/2)*b^10*e
^4 + 15*sqrt(x*e + d)*b^10*d*e^4 - 15*sqrt(x*e + d)*a*b^9*e^5)/(b^15*sgn((x
*e + d)*b*e - b*d*e + a*e^2))
```

3.1722 $\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=294

$$-\frac{21e^2(d+ex)^{5/2}}{32b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{105e^3(d+ex)^{3/2}}{64b^4\sqrt{a^2+2abx+b^2x^2}}+\frac{315e^4(a+bx)\sqrt{d+ex}}{64b^5\sqrt{a^2+2abx+b^2x^2}}-\frac{315e^4(a+bx)\sqrt{bd-ae}\tanh^{-1}}{64b^{11/2}\sqrt{a^2+2abx+b^2}}$$

```
[Out] (315*e^4*(a + b*x)*Sqrt[d + e*x])/(64*b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) -
(105*e^3*(d + e*x)^(3/2))/(64*b^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (21*e^2*
(d + e*x)^(5/2))/(32*b^3*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (3*e*(d
 + e*x)^(7/2))/(8*b^2*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (d + e*x
)^(9/2)/(4*b*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (315*e^4*Sqrt[b*d
 - a*e]*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(64*b^(
11/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.14133, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {646, 47, 50, 63, 208}

$$-\frac{21e^2(d+ex)^{5/2}}{32b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{105e^3(d+ex)^{3/2}}{64b^4\sqrt{a^2+2abx+b^2x^2}}+\frac{315e^4(a+bx)\sqrt{d+ex}}{64b^5\sqrt{a^2+2abx+b^2x^2}}-\frac{315e^4(a+bx)\sqrt{bd-ae}\tanh^{-1}}{64b^{11/2}\sqrt{a^2+2abx+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(9/2)/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```


```
[Out] (315*e^4*(a + b*x)*Sqrt[d + e*x])/(64*b^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) -
(105*e^3*(d + e*x)^(3/2))/(64*b^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (21*e^2*
(d + e*x)^(5/2))/(32*b^3*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (3*e*(d
 + e*x)^(7/2))/(8*b^2*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (d + e*x
)^(9/2)/(4*b*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (315*e^4*Sqrt[b*d
 - a*e]*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(64*b^(
11/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{9/2}}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x\right)\right)\int \frac{(d+ex)^{9/2}}{\left(ab+b^2x\right)^5}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{(d+ex)^{9/2}}{4b(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(9b^2e\left(ab+b^2x\right)\right)\int \frac{(d+ex)^{7/2}}{\left(ab+b^2x\right)^4}\,dx}{8\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{3e(d+ex)^{7/2}}{8b^2(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{(d+ex)^{9/2}}{4b(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(21e^2\left(ab+b^2x\right)\right)}{16\sqrt{a^2+2a}}$$

$$= -\frac{21e^2(d+ex)^{5/2}}{32b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{3e(d+ex)^{7/2}}{8b^2(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{(d+e}{4b(a+bx)^3\sqrt{a^2}}$$

$$= -\frac{105e^3(d+ex)^{3/2}}{64b^4\sqrt{a^2+2abx+b^2x^2}} - \frac{21e^2(d+ex)^{5/2}}{32b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{3e(d+ex)^{7/2}}{8b^2(a+bx)^2\sqrt{a^2+2ab}}$$

$$= \frac{315e^4(a+bx)\sqrt{d+ex}}{64b^5\sqrt{a^2+2abx+b^2x^2}} - \frac{105e^3(d+ex)^{3/2}}{64b^4\sqrt{a^2+2abx+b^2x^2}} - \frac{21e^2(d+ex)^{5/2}}{32b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} -$$

$$= \frac{315e^4(a+bx)\sqrt{d+ex}}{64b^5\sqrt{a^2+2abx+b^2x^2}} - \frac{105e^3(d+ex)^{3/2}}{64b^4\sqrt{a^2+2abx+b^2x^2}} - \frac{21e^2(d+ex)^{5/2}}{32b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} -$$

$$= \frac{315e^4(a+bx)\sqrt{d+ex}}{64b^5\sqrt{a^2+2abx+b^2x^2}} - \frac{105e^3(d+ex)^{3/2}}{64b^4\sqrt{a^2+2abx+b^2x^2}} - \frac{21e^2(d+ex)^{5/2}}{32b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} -$$

Mathematica [C] time = 0.0450814, size = 67, normalized size = 0.23

$$-\frac{2e^4(a+bx)(d+ex)^{11/2}\,{}_2F_1\left(5,\frac{11}{2};\frac{13}{2};\frac{b(d+ex)}{bd-ae}\right)}{11\sqrt{(a+bx)^2}(bd-ae)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(9/2)/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (-2*e^4*(a + b*x)*(d + e*x)^(11/2)*Hypergeometric2F1[5, 11/2, 13/2, (b*(d +
 e*x))/(b*d - a*e)])/(11*(b*d - a*e)^5*Sqrt[(a + b*x)^2])
```

Maple [B] time = 0.282, size = 892, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] 1/64*(-315*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^4*a*b^4*e^5+315*ar
ctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^4*b^5*d*e^4+128*((a*e-b*d)*b)^(
1/2)*(e*x+d)^(1/2)*x^4*b^4*e^4-1260*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1
/2))*x^3*a^2*b^3*e^5+325*((a*e-b*d)*b)^(1/2)*(e*x+d)^(7/2)*a*b^3*e+1260*arc
tan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x*a^3*b^2*d*e^4-1929*((a*e-b*d)*b)
^(1/2)*(e*x+d)^(3/2)*a^2*b^2*d*e^2+1929*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*a
*b^3*d^2*e+315*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*a^4*e^4+187*((a*e-b*d)*b)^
(1/2)*(e*x+d)^(1/2)*b^4*d^4-643*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*b^4*d^3-3
25*((a*e-b*d)*b)^(1/2)*(e*x+d)^(7/2)*b^4*d+765*((a*e-b*d)*b)^(1/2)*(e*x+d)^
(5/2)*b^4*d^2+768*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*x^2*a^2*b^2*e^4-315*arc
tan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a^5*e^5-1260*arctan(b*(e*x+d)^(1/2
)/((a*e-b*d)*b)^(1/2))*x*a^4*b*e^5+643*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*a^
3*b*e^3+315*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a^4*b*d*e^4-1890*ar
ctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^2*a^3*b^2*e^5+765*((a*e-b*d)*b)
^(1/2)*(e*x+d)^(5/2)*a^2*b^2*e^2+1890*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^
(1/2))*x^2*a^2*b^3*d*e^4+512*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*x*a^3*b*e^4-
748*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*a^3*b*d*e^3+1260*arctan(b*(e*x+d)^(1/
2)/((a*e-b*d)*b)^(1/2))*x^3*a*b^4*d*e^4+512*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/
2)*x^3*a*b^3*e^4-1530*((a*e-b*d)*b)^(1/2)*(e*x+d)^(5/2)*a*b^3*d*e+1122*((a*
e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*a^2*b^2*d^2*e^2-748*((a*e-b*d)*b)^(1/2)*(e*x+
d)^(1/2)*a*b^3*d^3*e)*(b*x+a)/((a*e-b*d)*b)^(1/2)/b^5/((b*x+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{9}{2}}}{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(9/2)/(b^2*x^2 + 2*a*b*x + a^2)^(5/2), x)
```

Fricas [A] time = 1.67546, size = 1453, normalized size = 4.94

$$\left[\frac{315\left(b^4e^4x^4+4\,ab^3e^4x^3+6\,a^2b^2e^4x^2+4\,a^3be^4x+a^4e^4\right)\sqrt{\frac{bd-ae}{b}}\log\left(\frac{bex+2\,bd-ae-2\,\sqrt{ex+d}b\sqrt{\frac{bd-ae}{b}}}{bx+a}\right)+2\left(128\,b^4e^4x^4-16\,b^4d^4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/128*(315*(b^4*e^4*x^4 + 4*a*b^3*e^4*x^3 + 6*a^2*b^2*e^4*x^2 + 4*a^3*b*e^
4*x + a^4*e^4)*sqrt((b*d - a*e)/b)*log((b*e*x + 2*b*d - a*e - 2*sqrt(e*x +
d)*b*sqrt((b*d - a*e)/b))/(b*x + a)) + 2*(128*b^4*e^4*x^4 - 16*b^4*d^4 - 24
*a*b^3*d^3*e - 42*a^2*b^2*d^2*e^2 - 105*a^3*b*d*e^3 + 315*a^4*e^4 - (325*b^
4*d*e^3 - 837*a*b^3*e^4)*x^3 - 3*(70*b^4*d^2*e^2 + 185*a*b^3*d*e^3 - 511*a^
2*b^2*e^4)*x^2 - (88*b^4*d^3*e + 156*a*b^3*d^2*e^2 + 399*a^2*b^2*d*e^3 - 11
55*a^3*b*e^4)*x)*sqrt(e*x + d))/(b^9*x^4 + 4*a*b^8*x^3 + 6*a^2*b^7*x^2 + 4*
a^3*b^6*x + a^4*b^5), -1/64*(315*(b^4*e^4*x^4 + 4*a*b^3*e^4*x^3 + 6*a^2*b^2
*e^4*x^2 + 4*a^3*b*e^4*x + a^4*e^4)*sqrt(-(b*d - a*e)/b)*arctan(-sqrt(e*x +
 d)*b*sqrt(-(b*d - a*e)/b)/(b*d - a*e)) - (128*b^4*e^4*x^4 - 16*b^4*d^4 - 2
4*a*b^3*d^3*e - 42*a^2*b^2*d^2*e^2 - 105*a^3*b*d*e^3 + 315*a^4*e^4 - (325*b
^4*d*e^3 - 837*a*b^3*e^4)*x^3 - 3*(70*b^4*d^2*e^2 + 185*a*b^3*d*e^3 - 511*a
^2*b^2*e^4)*x^2 - (88*b^4*d^3*e + 156*a*b^3*d^2*e^2 + 399*a^2*b^2*d*e^3 - 1
155*a^3*b*e^4)*x)*sqrt(e*x + d))/(b^9*x^4 + 4*a*b^8*x^3 + 6*a^2*b^7*x^2 + 4
*a^3*b^6*x + a^4*b^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(9/2)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31808, size = 560, normalized size = 1.9

$$\frac{315\left(bde^4-ae^5\right)\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)}{64\sqrt{-b^2d+abe}b^5\operatorname{sgn}\left((xe+d)be-bde+ae^2\right)}+\frac{2\sqrt{xe+d}e^4}{b^5\operatorname{sgn}\left((xe+d)be-bde+ae^2\right)}-\frac{325(xe+d)^{\frac{7}{2}}b^4de^4-765(xe+d)^{\frac{5}{2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 315/64*(b*d*e^4 - a*e^5)*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt
(-b^2*d + a*b*e)*b^5*sgn((x*e + d)*b*e - b*d*e + a*e^2)) + 2*sqrt(x*e + d)*
e^4/(b^5*sgn((x*e + d)*b*e - b*d*e + a*e^2)) - 1/64*(325*(x*e + d)^(7/2)*b^
4*d*e^4 - 765*(x*e + d)^(5/2)*b^4*d^2*e^4 + 643*(x*e + d)^(3/2)*b^4*d^3*e^4
 - 187*sqrt(x*e + d)*b^4*d^4*e^4 - 325*(x*e + d)^(7/2)*a*b^3*e^5 + 1530*(x*
e + d)^(5/2)*a*b^3*d*e^5 - 1929*(x*e + d)^(3/2)*a*b^3*d^2*e^5 + 748*sqrt(x*
e + d)*a*b^3*d^3*e^5 - 765*(x*e + d)^(5/2)*a^2*b^2*e^6 + 1929*(x*e + d)^(3/
2)*a^2*b^2*d*e^6 - 1122*sqrt(x*e + d)*a^2*b^2*d^2*e^6 - 643*(x*e + d)^(3/2)
*a^3*b*e^7 + 748*sqrt(x*e + d)*a^3*b*d*e^7 - 187*sqrt(x*e + d)*a^4*e^8)/(((
x*e + d)*b - b*d + a*e)^4*b^5*sgn((x*e + d)*b*e - b*d*e + a*e^2))
```

3.1723 $\int \frac{(d+ex)^{7/2}}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=250

$$-\frac{35e^3\sqrt{d+ex}}{64b^4\sqrt{a^2+2abx+b^2x^2}}-\frac{35e^2(d+ex)^{3/2}}{96b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{35e^4(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{64b^{9/2}\sqrt{a^2+2abx+b^2x^2}\sqrt{bd-ae}}-\frac{7e(d+ex)^{5/2}}{24b^2(a+bx)^2\sqrt{a^2+\cdots}}$$

```
[Out] (-35*e^3*Sqrt[d + e*x])/(64*b^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (35*e^2*(d
 + e*x)^(3/2))/(96*b^3*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (7*e*(d +
e*x)^(5/2))/(24*b^2*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (d + e*x)
^(7/2)/(4*b*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (35*e^4*(a + b*x)*
ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(64*b^(9/2)*Sqrt[b*d - a*
e]*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.11792, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {646, 47, 63, 208}

$$-\frac{35e^3\sqrt{d+ex}}{64b^4\sqrt{a^2+2abx+b^2x^2}}-\frac{35e^2(d+ex)^{3/2}}{96b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{35e^4(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{64b^{9/2}\sqrt{a^2+2abx+b^2x^2}\sqrt{bd-ae}}-\frac{7e(d+ex)^{5/2}}{24b^2(a+bx)^2\sqrt{a^2+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```


```
[Out] (-35*e^3*Sqrt[d + e*x])/(64*b^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (35*e^2*(d
 + e*x)^(3/2))/(96*b^3*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (7*e*(d +
e*x)^(5/2))/(24*b^2*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (d + e*x)
^(7/2)/(4*b*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (35*e^4*(a + b*x)*
ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(64*b^(9/2)*Sqrt[b*d - a*
e]*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{7/2}}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x\right)\right)\int \frac{(d+ex)^{7/2}}{\left(ab+b^2x\right)^5}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{(d+ex)^{7/2}}{4b(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(7b^2e\left(ab+b^2x\right)\right)\int \frac{(d+ex)^{5/2}}{\left(ab+b^2x\right)^4}\,dx}{8\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{7e(d+ex)^{5/2}}{24b^2(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{(d+ex)^{7/2}}{4b(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(35e^2\left(ab+b^2x\right.\right.}{48\sqrt{a^2+2\ldots}}$$

$$= -\frac{35e^2(d+ex)^{3/2}}{96b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{7e(d+ex)^{5/2}}{24b^2(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{(d+\ldots}{4b(a+bx)^3\sqrt{a\ldots}}$$

$$= -\frac{35e^3\sqrt{d+ex}}{64b^4\sqrt{a^2+2abx+b^2x^2}} - \frac{35e^2(d+ex)^{3/2}}{96b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{7e(d+ex)^{5/2}}{24b^2(a+bx)^2\sqrt{a^2+2a\ldots}}$$

$$= -\frac{35e^3\sqrt{d+ex}}{64b^4\sqrt{a^2+2abx+b^2x^2}} - \frac{35e^2(d+ex)^{3/2}}{96b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{7e(d+ex)^{5/2}}{24b^2(a+bx)^2\sqrt{a^2+2a\ldots}}$$

$$= -\frac{35e^3\sqrt{d+ex}}{64b^4\sqrt{a^2+2abx+b^2x^2}} - \frac{35e^2(d+ex)^{3/2}}{96b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{7e(d+ex)^{5/2}}{24b^2(a+bx)^2\sqrt{a^2+2a\ldots}}$$

Mathematica [A] time = 0.223078, size = 182, normalized size = 0.73

$$\frac{\frac{105e^4(a+bx)^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{ae-bd}}\right)}{\sqrt{ae-bd}} - \sqrt{b}\sqrt{d+ex}\left(35a^2be^2(2d+11ex)+105a^3e^3+7ab^2e\left(8d^2+36dex+73e^2x^2\right)+b^3\left(200d^2ex+\ldots\right.\right.}{192b^{9/2}(a+bx)^3\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (-(Sqrt[b]*Sqrt[d + e*x]*(105*a^3*e^3 + 35*a^2*b*e^2*(2*d + 11*e*x) + 7*a*b
^2*e*(8*d^2 + 36*d*e*x + 73*e^2*x^2) + b^3*(48*d^3 + 200*d^2*e*x + 326*d*e^
2*x^2 + 279*e^3*x^3))) + (105*e^4*(a + b*x)^4*ArcTan[(Sqrt[b]*Sqrt[d + e*x]
)/Sqrt[-(b*d) + a*e]])/Sqrt[-(b*d) + a*e])/(192*b^(9/2)*(a + b*x)^3*Sqrt[(a
 + b*x)^2])
```

Maple [B] time = 0.277, size = 467, normalized size = 1.9

$$-\frac{bx+a}{192\,b^4}\left(-105\,\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)\,b}}\right)x^4b^4e^4 - 420\,\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)\,b}}\right)x^3ab^3e^4 + 279\,\sqrt{(ae-bd)\,b}\,(ex+d)^{7/2}\,b^3 - \ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] -1/192*(-105*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^4*b^4*e^4-420*ar
ctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^3*a*b^3*e^4+279*((a*e-b*d)*b)^(
1/2)*(e*x+d)^(7/2)*b^3-630*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^2*
a^2*b^2*e^4+511*((a*e-b*d)*b)^(1/2)*(e*x+d)^(5/2)*a*b^2*e-511*((a*e-b*d)*b)
^(1/2)*(e*x+d)^(5/2)*b^3*d-420*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*
x*a^3*b*e^4+385*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*a^2*b*e^2-770*((a*e-b*d)*
b)^(1/2)*(e*x+d)^(3/2)*a*b^2*d*e+385*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*b^3*
d^2-105*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a^4*e^4+105*((a*e-b*d)*
b)^(1/2)*(e*x+d)^(1/2)*a^3*e^3-315*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*a^2*b*
d*e^2+315*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*a*b^2*d^2*e-105*((a*e-b*d)*b)^(
1/2)*(e*x+d)^(1/2)*b^3*d^3)*(b*x+a)/((a*e-b*d)*b)^(1/2)/b^4/((b*x+a)^2)^(5/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{7}{2}}}{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(7/2)/(b^2*x^2 + 2*a*b*x + a^2)^(5/2), x)
```

Fricas [B] time = 1.70959, size = 1602, normalized size = 6.41

$$\left[\frac{105\left(b^4e^4x^4+4\,ab^3e^4x^3+6\,a^2b^2e^4x^2+4\,a^3be^4x+a^4e^4\right)\sqrt{b^2d-abe}\log\left(\frac{bex+2\,bd-ae-2\,\sqrt{b^2d-abe}\sqrt{ex+d}}{bx+a}\right)-2\left(48\,b^5d^4+8\,ab^4d\right.}{384\left(a^4b^6d-a^5b^5e+\left(b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/384*(105*(b^4*e^4*x^4 + 4*a*b^3*e^4*x^3 + 6*a^2*b^2*e^4*x^2 + 4*a^3*b*e^
4*x + a^4*e^4)*sqrt(b^2*d - a*b*e)*log((b*e*x + 2*b*d - a*e - 2*sqrt(b^2*d
 - a*b*e)*sqrt(e*x + d))/(b*x + a)) - 2*(48*b^5*d^4 + 8*a*b^4*d^3*e + 14*a^2
*b^3*d^2*e^2 + 35*a^3*b^2*d*e^3 - 105*a^4*b*e^4 + 279*(b^5*d*e^3 - a*b^4*e^
4)*x^3 + (326*b^5*d^2*e^2 + 185*a*b^4*d*e^3 - 511*a^2*b^3*e^4)*x^2 + (200*b
^5*d^3*e + 52*a*b^4*d^2*e^2 + 133*a^2*b^3*d*e^3 - 385*a^3*b^2*e^4)*x)*sqrt(
e*x + d))/(a^4*b^6*d - a^5*b^5*e + (b^10*d - a*b^9*e)*x^4 + 4*(a*b^9*d - a^
2*b^8*e)*x^3 + 6*(a^2*b^8*d - a^3*b^7*e)*x^2 + 4*(a^3*b^7*d - a^4*b^6*e)*x)
, 1/192*(105*(b^4*e^4*x^4 + 4*a*b^3*e^4*x^3 + 6*a^2*b^2*e^4*x^2 + 4*a^3*b*e
^4*x + a^4*e^4)*sqrt(-b^2*d + a*b*e)*arctan(sqrt(-b^2*d + a*b*e)*sqrt(e*x +
 d)/(b*e*x + b*d)) - (48*b^5*d^4 + 8*a*b^4*d^3*e + 14*a^2*b^3*d^2*e^2 + 35*
a^3*b^2*d*e^3 - 105*a^4*b*e^4 + 279*(b^5*d*e^3 - a*b^4*e^4)*x^3 + (326*b^5*
d^2*e^2 + 185*a*b^4*d*e^3 - 511*a^2*b^3*e^4)*x^2 + (200*b^5*d^3*e + 52*a*b^
4*d^2*e^2 + 133*a^2*b^3*d*e^3 - 385*a^3*b^2*e^4)*x)*sqrt(e*x + d))/(a^4*b^6
```

```
*d - a^5*b^5*e + (b^10*d - a*b^9*e)*x^4 + 4*(a*b^9*d - a^2*b^8*e)*x^3 + 6*(
a^2*b^8*d - a^3*b^7*e)*x^2 + 4*(a^3*b^7*d - a^4*b^6*e)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20095, size = 387, normalized size = 1.55

$$\frac{35 \arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)e^4}{64\sqrt{-b^2d+abe}b^4\mathrm{sgn}\left((xe+d)be-bde+ae^2\right)} - \frac{279\,(xe+d)^{\frac{7}{2}}b^3e^4 - 511\,(xe+d)^{\frac{5}{2}}b^3de^4 + 385\,(xe+d)^{\frac{3}{2}}b^3d^2e^4 - 105}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 35/64*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e^4/(sqrt(-b^2*d + a*b*e
)*b^4*sgn((x*e + d)*b*e - b*d*e + a*e^2)) - 1/192*(279*(x*e + d)^(7/2)*b^3*
e^4 - 511*(x*e + d)^(5/2)*b^3*d*e^4 + 385*(x*e + d)^(3/2)*b^3*d^2*e^4 - 105
*sqrt(x*e + d)*b^3*d^3*e^4 + 511*(x*e + d)^(5/2)*a*b^2*e^5 - 770*(x*e + d)^
(3/2)*a*b^2*d*e^5 + 315*sqrt(x*e + d)*a*b^2*d^2*e^5 + 385*(x*e + d)^(3/2)*a
^2*b*e^6 - 315*sqrt(x*e + d)*a^2*b*d*e^6 + 105*sqrt(x*e + d)*a^3*e^7)/(((x*
e + d)*b - b*d + a*e)^4*b^4*sgn((x*e + d)*b*e - b*d*e + a*e^2))
```

3.1724 $\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=260

$$-\frac{5e^3\sqrt{d+ex}}{64b^3\sqrt{a^2+2abx+b^2x^2}(bd-ae)}-\frac{5e^2\sqrt{d+ex}}{32b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{5e^4(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{64b^{7/2}\sqrt{a^2+2abx+b^2x^2}(bd-ae)^{3/2}}-\frac{}{24b^2(a+}$$

```
[Out] (-5*e^3*Sqrt[d + e*x])/(64*b^3*(b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) -
 (5*e^2*Sqrt[d + e*x])/(32*b^3*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (
5*e*(d + e*x)^(3/2))/(24*b^2*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (
d + e*x)^(5/2)/(4*b*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (5*e^4*(a
+ b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(64*b^(7/2)*(b*d -
 a*e)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.139203, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {646, 47, 51, 63, 208}

$$-\frac{5e^3\sqrt{d+ex}}{64b^3\sqrt{a^2+2abx+b^2x^2}(bd-ae)}-\frac{5e^2\sqrt{d+ex}}{32b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{5e^4(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{64b^{7/2}\sqrt{a^2+2abx+b^2x^2}(bd-ae)^{3/2}}-\frac{}{24b^2(a+}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (-5*e^3*Sqrt[d + e*x])/(64*b^3*(b*d - a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) -
 (5*e^2*Sqrt[d + e*x])/(32*b^3*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (
5*e*(d + e*x)^(3/2))/(24*b^2*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (
d + e*x)^(5/2)/(4*b*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (5*e^4*(a
+ b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(64*b^(7/2)*(b*d -
 a*e)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x\right)\right)\int \frac{(d+ex)^{5/2}}{\left(ab+b^2x\right)^5}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{(d+ex)^{5/2}}{4b(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(5b^2e\left(ab+b^2x\right)\right)\int \frac{(d+ex)^{3/2}}{\left(ab+b^2x\right)^4}\,dx}{8\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{5e(d+ex)^{3/2}}{24b^2(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{(d+ex)^{5/2}}{4b(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(5e^2\left(ab+b^2x\right)\right.}{16\sqrt{a^2+2a}}$$

$$= -\frac{5e^2\sqrt{d+ex}}{32b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{5e(d+ex)^{3/2}}{24b^2(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{(d+}{4b(a+bx)^3\sqrt{a}}$$

$$= -\frac{5e^3\sqrt{d+ex}}{64b^3(bd-ae)\sqrt{a^2+2abx+b^2x^2}} - \frac{5e^2\sqrt{d+ex}}{32b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{5e(d}{24b^2(a+bx)^2}$$

$$= -\frac{5e^3\sqrt{d+ex}}{64b^3(bd-ae)\sqrt{a^2+2abx+b^2x^2}} - \frac{5e^2\sqrt{d+ex}}{32b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{5e(d}{24b^2(a+bx)^2}$$

$$= -\frac{5e^3\sqrt{d+ex}}{64b^3(bd-ae)\sqrt{a^2+2abx+b^2x^2}} - \frac{5e^2\sqrt{d+ex}}{32b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{5e(d}{24b^2(a+bx)^2}$$

Mathematica [C] time = 0.0402505, size = 67, normalized size = 0.26

$$-\frac{2e^4(a+bx)(d+ex)^{7/2}\,{}_2F_1\left(\frac{7}{2},5;\frac{9}{2};\frac{b(d+ex)}{bd-ae}\right)}{7\sqrt{(a+bx)^2}(bd-ae)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (-2*e^4*(a + b*x)*(d + e*x)^(7/2)*Hypergeometric2F1[7/2, 5, 9/2, (b*(d + e*
x))/(b*d - a*e)])/(7*(b*d - a*e)^5*Sqrt[(a + b*x)^2])
```

Maple [B] time = 0.275, size = 477, normalized size = 1.8

$$\frac{bx+a}{192\,(ae-bd)\,b^3}\left(15\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)\,b}}\right)x^4b^4e^4+60\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)\,b}}\right)x^3ab^3e^4+15\sqrt{(ae-bd)\,b}\,(ex+d)^{7/2}\,b^3+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] 1/192*(15*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^4*b^4*e^4+60*arctan
(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^3*a*b^3*e^4+15*((a*e-b*d)*b)^(1/2)*
(e*x+d)^(7/2)*b^3+90*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^2*a^2*b^
2*e^4-73*((a*e-b*d)*b)^(1/2)*(e*x+d)^(5/2)*a*b^2*e+73*((a*e-b*d)*b)^(1/2)*(
e*x+d)^(5/2)*b^3*d+60*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x*a^3*b*e
^4-55*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*a^2*b*e^2+110*((a*e-b*d)*b)^(1/2)*(
e*x+d)^(3/2)*a*b^2*d*e-55*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*b^3*d^2+15*arct
an(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a^4*e^4-15*((a*e-b*d)*b)^(1/2)*(e*x
+d)^(1/2)*a^3*e^3+45*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*a^2*b*d*e^2-45*((a*e
-b*d)*b)^(1/2)*(e*x+d)^(1/2)*a*b^2*d^2*e+15*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/
2)*b^3*d^3)*(b*x+a)/((a*e-b*d)*b)^(1/2)/b^3/(a*e-b*d)/((b*x+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/(b^2*x^2 + 2*a*b*x + a^2)^(5/2), x)
```

Fricas [B] time = 1.73771, size = 1859, normalized size = 7.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(15*(b^4*e^4*x^4 + 4*a*b^3*e^4*x^3 + 6*a^2*b^2*e^4*x^2 + 4*a^3*b*e^
4*x + a^4*e^4)*sqrt(b^2*d - a*b*e)*log((b*e*x + 2*b*d - a*e - 2*sqrt(b^2*d
- a*b*e)*sqrt(e*x + d))/(b*x + a)) + 2*(48*b^5*d^4 - 56*a*b^4*d^3*e - 2*a^2
*b^3*d^2*e^2 - 5*a^3*b^2*d*e^3 + 15*a^4*b*e^4 + 15*(b^5*d*e^3 - a*b^4*e^4)*
x^3 + (118*b^5*d^2*e^2 - 191*a*b^4*d*e^3 + 73*a^2*b^3*e^4)*x^2 + (136*b^5*d
^3*e - 172*a*b^4*d^2*e^2 - 19*a^2*b^3*d*e^3 + 55*a^3*b^2*e^4)*x)*sqrt(e*x +
 d))/(a^4*b^6*d^2 - 2*a^5*b^5*d*e + a^6*b^4*e^2 + (b^10*d^2 - 2*a*b^9*d*e +
 a^2*b^8*e^2)*x^4 + 4*(a*b^9*d^2 - 2*a^2*b^8*d*e + a^3*b^7*e^2)*x^3 + 6*(a^
2*b^8*d^2 - 2*a^3*b^7*d*e + a^4*b^6*e^2)*x^2 + 4*(a^3*b^7*d^2 - 2*a^4*b^6*d
*e + a^5*b^5*e^2)*x), -1/192*(15*(b^4*e^4*x^4 + 4*a*b^3*e^4*x^3 + 6*a^2*b^2
*e^4*x^2 + 4*a^3*b*e^4*x + a^4*e^4)*sqrt(-b^2*d + a*b*e)*arctan(sqrt(-b^2*d
```

```
 + a*b*e)*sqrt(e*x + d)/(b*e*x + b*d)) + (48*b^5*d^4 - 56*a*b^4*d^3*e - 2*a
^2*b^3*d^2*e^2 - 5*a^3*b^2*d*e^3 + 15*a^4*b*e^4 + 15*(b^5*d*e^3 - a*b^4*e^4
)*x^3 + (118*b^5*d^2*e^2 - 191*a*b^4*d*e^3 + 73*a^2*b^3*e^4)*x^2 + (136*b^5
*d^3*e - 172*a*b^4*d^2*e^2 - 19*a^2*b^3*d*e^3 + 55*a^3*b^2*e^4)*x)*sqrt(e*x
 + d))/(a^4*b^6*d^2 - 2*a^5*b^5*d*e + a^6*b^4*e^2 + (b^10*d^2 - 2*a*b^9*d*e
 + a^2*b^8*e^2)*x^4 + 4*(a*b^9*d^2 - 2*a^2*b^8*d*e + a^3*b^7*e^2)*x^3 + 6*(
a^2*b^8*d^2 - 2*a^3*b^7*d*e + a^4*b^6*e^2)*x^2 + 4*(a^3*b^7*d^2 - 2*a^4*b^6
*d*e + a^5*b^5*e^2)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23455, size = 477, normalized size = 1.83

$$-\frac{5\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)e^4}{64\left(b^4d\mathrm{sgn}\left((xe+d)be-bde+ae^2\right)-ab^3e\mathrm{sgn}\left((xe+d)be-bde+ae^2\right)\right)\sqrt{-b^2d+abe}}-\frac{15\,(xe+d)^{\frac{7}{2}}b^3e^4+73\,(xe+d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -5/64*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e^4/((b^4*d*sgn((x*e + d
)*b*e - b*d*e + a*e^2) - a*b^3*e*sgn((x*e + d)*b*e - b*d*e + a*e^2))*sqrt(-
b^2*d + a*b*e)) - 1/192*(15*(x*e + d)^(7/2)*b^3*e^4 + 73*(x*e + d)^(5/2)*b^
3*d*e^4 - 55*(x*e + d)^(3/2)*b^3*d^2*e^4 + 15*sqrt(x*e + d)*b^3*d^3*e^4 - 7
3*(x*e + d)^(5/2)*a*b^2*e^5 + 110*(x*e + d)^(3/2)*a*b^2*d*e^5 - 45*sqrt(x*e
 + d)*a*b^2*d^2*e^5 - 55*(x*e + d)^(3/2)*a^2*b*e^6 + 45*sqrt(x*e + d)*a^2*b
*d*e^6 - 15*sqrt(x*e + d)*a^3*e^7)/((b^4*d*sgn((x*e + d)*b*e - b*d*e + a*e^
2) - a*b^3*e*sgn((x*e + d)*b*e - b*d*e + a*e^2))*((x*e + d)*b - b*d + a*e)^
4)
```

3.1725 $\int \frac{(d+ex)^{3/2}}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=270

$$\frac{3e^3\sqrt{d+ex}}{64b^2\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}-\frac{e^2\sqrt{d+ex}}{32b^2(a+bx)\sqrt{a^2+2abx+b^2x^2}(bd-ae)}-\frac{3e^4(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{64b^{5/2}\sqrt{a^2+2abx+b^2x^2}(bd-ae)^{5/2}}-$$

```
[Out] (3*e^3*Sqrt[d + e*x])/(64*b^2*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
- (e*Sqrt[d + e*x])/(8*b^2*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (e^
2*Sqrt[d + e*x])/(32*b^2*(b*d - a*e)*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2
]) - (d + e*x)^(3/2)/(4*b*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (3*e
^4*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(64*b^(5/2)*
(b*d - a*e)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.139302, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {646, 47, 51, 63, 208}

$$\frac{3e^3\sqrt{d+ex}}{64b^2\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}-\frac{e^2\sqrt{d+ex}}{32b^2(a+bx)\sqrt{a^2+2abx+b^2x^2}(bd-ae)}-\frac{3e^4(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{64b^{5/2}\sqrt{a^2+2abx+b^2x^2}(bd-ae)^{5/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```


```
[Out] (3*e^3*Sqrt[d + e*x])/(64*b^2*(b*d - a*e)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
- (e*Sqrt[d + e*x])/(8*b^2*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (e^
2*Sqrt[d + e*x])/(32*b^2*(b*d - a*e)*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2
]) - (d + e*x)^(3/2)/(4*b*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (3*e
^4*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(64*b^(5/2)*
(b*d - a*e)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x\right)\right)\int \frac{(d+ex)^{3/2}}{\left(ab+b^2x\right)^5}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{(d+ex)^{3/2}}{4b(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(3b^2e\left(ab+b^2x\right)\right)\int \frac{\sqrt{d+ex}}{\left(ab+b^2x\right)^4}\,dx}{8\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{e\sqrt{d+ex}}{8b^2(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{(d+ex)^{3/2}}{4b(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(e^2\left(ab+b^2x\right)\right)\int}{16\sqrt{a^2+2}}$$

$$= -\frac{e\sqrt{d+ex}}{8b^2(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{e^2\sqrt{d+ex}}{32b^2(bd-ae)(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{}{4b(a+}$$

$$= \frac{3e^3\sqrt{d+ex}}{64b^2(bd-ae)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{e\sqrt{d+ex}}{8b^2(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{e}{32b^2(bd-ae)(a}$$

$$= \frac{3e^3\sqrt{d+ex}}{64b^2(bd-ae)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{e\sqrt{d+ex}}{8b^2(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{e}{32b^2(bd-ae)(a}$$

$$= \frac{3e^3\sqrt{d+ex}}{64b^2(bd-ae)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{e\sqrt{d+ex}}{8b^2(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{e}{32b^2(bd-ae)(a}$$

Mathematica [C] time = 0.0343844, size = 67, normalized size = 0.25

$$-\frac{2e^4(a+bx)(d+ex)^{5/2}\,{}_2F_1\left(\frac{5}{2},5;\frac{7}{2};\frac{b(d+ex)}{bd-ae}\right)}{5\sqrt{(a+bx)^2}(bd-ae)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (-2*e^4*(a + b*x)*(d + e*x)^(5/2)*Hypergeometric2F1[5/2, 5, 7/2, (b*(d + e*
x))/(b*d - a*e)])/(5*(b*d - a*e)^5*Sqrt[(a + b*x)^2])
```

Maple [B] time = 0.279, size = 477, normalized size = 1.8

$$\frac{bx+a}{64\,(ae-bd)^2\,b^2}\left(3\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)\,b}}\right)x^4b^4e^4+12\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)\,b}}\right)x^3ab^3e^4+3\sqrt{(ae-bd)\,b}\,(ex+d)^{7/2}\,b^3+18$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] 1/64*(b*x+a)*(3*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^4*b^4*e^4+12*
arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^3*a*b^3*e^4+3*((a*e-b*d)*b)^(
1/2)*(e*x+d)^(7/2)*b^3+18*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^2*a
^2*b^2*e^4+11*((a*e-b*d)*b)^(1/2)*(e*x+d)^(5/2)*a*b^2*e-11*((a*e-b*d)*b)^(1
/2)*(e*x+d)^(5/2)*b^3*d+12*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x*a^
3*b*e^4-11*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*a^2*b*e^2+22*((a*e-b*d)*b)^(1/
2)*(e*x+d)^(3/2)*a*b^2*d*e-11*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*b^3*d^2+3*a
rctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a^4*e^4-3*((a*e-b*d)*b)^(1/2)*(e
*x+d)^(1/2)*a^3*e^3+9*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*a^2*b*d*e^2-9*((a*e
-b*d)*b)^(1/2)*(e*x+d)^(1/2)*a*b^2*d^2*e+3*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2
)*b^3*d^3)/((a*e-b*d)*b)^(1/2)/b^2/(a*e-b*d)^2/((b*x+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(b^2*x^2 + 2*a*b*x + a^2)^(5/2), x)
```

Fricas [B] time = 1.85406, size = 2101, normalized size = 7.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/128*(3*(b^4*e^4*x^4 + 4*a*b^3*e^4*x^3 + 6*a^2*b^2*e^4*x^2 + 4*a^3*b*e^4*
x + a^4*e^4)*sqrt(b^2*d - a*b*e)*log((b*e*x + 2*b*d - a*e - 2*sqrt(b^2*d -
a*b*e)*sqrt(e*x + d))/(b*x + a)) - 2*(16*b^5*d^4 - 40*a*b^4*d^3*e + 26*a^2*
b^3*d^2*e^2 + a^3*b^2*d*e^3 - 3*a^4*b*e^4 - 3*(b^5*d*e^3 - a*b^4*e^4)*x^3 +
 (2*b^5*d^2*e^2 - 13*a*b^4*d*e^3 + 11*a^2*b^3*e^4)*x^2 + (24*b^5*d^3*e - 68
*a*b^4*d^2*e^2 + 55*a^2*b^3*d*e^3 - 11*a^3*b^2*e^4)*x)*sqrt(e*x + d))/(a^4*
b^6*d^3 - 3*a^5*b^5*d^2*e + 3*a^6*b^4*d*e^2 - a^7*b^3*e^3 + (b^10*d^3 - 3*a
*b^9*d^2*e + 3*a^2*b^8*d*e^2 - a^3*b^7*e^3)*x^4 + 4*(a*b^9*d^3 - 3*a^2*b^8*
d^2*e + 3*a^3*b^7*d*e^2 - a^4*b^6*e^3)*x^3 + 6*(a^2*b^8*d^3 - 3*a^3*b^7*d^2
*e + 3*a^4*b^6*d*e^2 - a^5*b^5*e^3)*x^2 + 4*(a^3*b^7*d^3 - 3*a^4*b^6*d^2*e
+ 3*a^5*b^5*d*e^2 - a^6*b^4*e^3)*x), 1/64*(3*(b^4*e^4*x^4 + 4*a*b^3*e^4*x^3
```

```
 + 6*a^2*b^2*e^4*x^2 + 4*a^3*b*e^4*x + a^4*e^4)*sqrt(-b^2*d + a*b*e)*arctan
(sqrt(-b^2*d + a*b*e)*sqrt(e*x + d)/(b*e*x + b*d)) - (16*b^5*d^4 - 40*a*b^4
*d^3*e + 26*a^2*b^3*d^2*e^2 + a^3*b^2*d*e^3 - 3*a^4*b*e^4 - 3*(b^5*d*e^3 -
a*b^4*e^4)*x^3 + (2*b^5*d^2*e^2 - 13*a*b^4*d*e^3 + 11*a^2*b^3*e^4)*x^2 + (2
4*b^5*d^3*e - 68*a*b^4*d^2*e^2 + 55*a^2*b^3*d*e^3 - 11*a^3*b^2*e^4)*x)*sqrt
(e*x + d))/(a^4*b^6*d^3 - 3*a^5*b^5*d^2*e + 3*a^6*b^4*d*e^2 - a^7*b^3*e^3 +
 (b^10*d^3 - 3*a*b^9*d^2*e + 3*a^2*b^8*d*e^2 - a^3*b^7*e^3)*x^4 + 4*(a*b^9*
d^3 - 3*a^2*b^8*d^2*e + 3*a^3*b^7*d*e^2 - a^4*b^6*e^3)*x^3 + 6*(a^2*b^8*d^3
 - 3*a^3*b^7*d^2*e + 3*a^4*b^6*d*e^2 - a^5*b^5*e^3)*x^2 + 4*(a^3*b^7*d^3 -
3*a^4*b^6*d^2*e + 3*a^5*b^5*d*e^2 - a^6*b^4*e^3)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24221, size = 568, normalized size = 2.1

$$\frac{3\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)e^4}{64\left(b^4d^2\mathrm{sgn}\left((xe+d)be-bde+ae^2\right)-2\,ab^3de\mathrm{sgn}\left((xe+d)be-bde+ae^2\right)+a^2b^2e^2\mathrm{sgn}\left((xe+d)be-bde+ae^2\right)\right)\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 3/64*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e^4/((b^4*d^2*sgn((x*e +
d)*b*e - b*d*e + a*e^2) - 2*a*b^3*d*e*sgn((x*e + d)*b*e - b*d*e + a*e^2) +
a^2*b^2*e^2*sgn((x*e + d)*b*e - b*d*e + a*e^2))*sqrt(-b^2*d + a*b*e)) + 1/6
4*(3*(x*e + d)^(7/2)*b^3*e^4 - 11*(x*e + d)^(5/2)*b^3*d*e^4 - 11*(x*e + d)^
(3/2)*b^3*d^2*e^4 + 3*sqrt(x*e + d)*b^3*d^3*e^4 + 11*(x*e + d)^(5/2)*a*b^2*
e^5 + 22*(x*e + d)^(3/2)*a*b^2*d*e^5 - 9*sqrt(x*e + d)*a*b^2*d^2*e^5 - 11*(
x*e + d)^(3/2)*a^2*b*e^6 + 9*sqrt(x*e + d)*a^2*b*d*e^6 - 3*sqrt(x*e + d)*a^
3*e^7)/((b^4*d^2*sgn((x*e + d)*b*e - b*d*e + a*e^2) - 2*a*b^3*d*e*sgn((x*e
+ d)*b*e - b*d*e + a*e^2) + a^2*b^2*e^2*sgn((x*e + d)*b*e - b*d*e + a*e^2))
*((x*e + d)*b - b*d + a*e)^4)
```

3.1726 $\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=280

$$-\frac{5e^3\sqrt{d+ex}}{64b\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3}+\frac{5e^2\sqrt{d+ex}}{96b(a+bx)\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}+\frac{5e^4(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{64b^{3/2}\sqrt{a^2+2abx+b^2x^2}(bd-ae)^{7/2}}-$$

```
[Out] (-5*e^3*Sqrt[d + e*x])/(64*b*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) -
 Sqrt[d + e*x]/(4*b*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (e*Sqrt[d
+ e*x])/(24*b*(b*d - a*e)*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (5*e
^2*Sqrt[d + e*x])/(96*b*(b*d - a*e)^2*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^
2]) + (5*e^4*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(6
4*b^(3/2)*(b*d - a*e)^(7/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.151276, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {646, 47, 51, 63, 208}

$$-\frac{5e^3\sqrt{d+ex}}{64b\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3}+\frac{5e^2\sqrt{d+ex}}{96b(a+bx)\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2}+\frac{5e^4(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{64b^{3/2}\sqrt{a^2+2abx+b^2x^2}(bd-ae)^{7/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (-5*e^3*Sqrt[d + e*x])/(64*b*(b*d - a*e)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) -
 Sqrt[d + e*x]/(4*b*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (e*Sqrt[d
+ e*x])/(24*b*(b*d - a*e)*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (5*e
^2*Sqrt[d + e*x])/(96*b*(b*d - a*e)^2*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^
2]) + (5*e^4*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(6
4*b^(3/2)*(b*d - a*e)^(7/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
```

```
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x\right)\right)\int \frac{\sqrt{d+ex}}{\left(ab+b^2x\right)^5}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{\sqrt{d+ex}}{4b(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{\left(b^2e\left(ab+b^2x\right)\right)\int \frac{1}{\left(ab+b^2x\right)^4\sqrt{d+ex}}\,dx}{8\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{\sqrt{d+ex}}{4b(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{e\sqrt{d+ex}}{24b(bd-ae)(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{\left(5be^2\left(a \ldots\right.\right.}{48(b\ldots}$$

$$= -\frac{\sqrt{d+ex}}{4b(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{e\sqrt{d+ex}}{24b(bd-ae)(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} + \frac{\ldots}{96b(bd\ldots}$$

$$= -\frac{5e^3\sqrt{d+ex}}{64b(bd-ae)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{\sqrt{d+ex}}{4b(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{\ldots}{24b(bd-ae)(a+\ldots}$$

$$= -\frac{5e^3\sqrt{d+ex}}{64b(bd-ae)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{\sqrt{d+ex}}{4b(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{\ldots}{24b(bd-ae)(a+\ldots}$$

$$= -\frac{5e^3\sqrt{d+ex}}{64b(bd-ae)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{\sqrt{d+ex}}{4b(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{\ldots}{24b(bd-ae)(a+\ldots}$$

Mathematica [C] time = 0.0276875, size = 67, normalized size = 0.24

$$-\frac{2e^4(a+bx)(d+ex)^{3/2}\,{}_2F_1\left(\frac{3}{2},5;\frac{5}{2};\frac{b(d+ex)}{bd-ae}\right)}{3\sqrt{(a+bx)^2}(bd-ae)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (-2*e^4*(a + b*x)*(d + e*x)^(3/2)*Hypergeometric2F1[3/2, 5, 5/2, (b*(d + e*
x))/(b*d - a*e)])/(3*(b*d - a*e)^5*Sqrt[(a + b*x)^2])
```

Maple [B] time = 0.276, size = 500, normalized size = 1.8

$$\frac{bx+a}{192\,(ae-bd)\,b\left(a^{2}e^{2}-2\,abde+b^{2}d^{2}\right)}\left(15\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)\,b}}\right)x^{4}b^{4}e^{4}+60\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)\,b}}\right)x^{3}ab^{3}e^{4}+15\sqrt{(ae-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] 1/192*(b*x+a)*(15*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^4*b^4*e^4+6
0*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^3*a*b^3*e^4+15*((a*e-b*d)*b
)^(1/2)*(e*x+d)^(7/2)*b^3+90*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^
2*a^2*b^2*e^4+55*((a*e-b*d)*b)^(1/2)*(e*x+d)^(5/2)*a*b^2*e-55*((a*e-b*d)*b)
^(1/2)*(e*x+d)^(5/2)*b^3*d+60*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x
*a^3*b*e^4+73*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*a^2*b*e^2-146*((a*e-b*d)*b)
^(1/2)*(e*x+d)^(3/2)*a*b^2*d*e+73*((a*e-b*d)*b)^(1/2)*(e*x+d)^(3/2)*b^3*d^2
+15*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*a^4*e^4-15*((a*e-b*d)*b)^(1
/2)*(e*x+d)^(1/2)*a^3*e^3+45*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*a^2*b*d*e^2-
45*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*a*b^2*d^2*e+15*((a*e-b*d)*b)^(1/2)*(e*
x+d)^(1/2)*b^3*d^3)/((a*e-b*d)*b)^(1/2)/b/(a*e-b*d)/(a^2*e^2-2*a*b*d*e+b^2*
d^2)/((b*x+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\left(b^{2}x^{2}+2\,abx+a^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(b^2*x^2 + 2*a*b*x + a^2)^(5/2), x)
```

Fricas [B] time = 1.83105, size = 2404, normalized size = 8.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(15*(b^4*e^4*x^4 + 4*a*b^3*e^4*x^3 + 6*a^2*b^2*e^4*x^2 + 4*a^3*b*e^
4*x + a^4*e^4)*sqrt(b^2*d - a*b*e)*log((b*e*x + 2*b*d - a*e - 2*sqrt(b^2*d
- a*b*e)*sqrt(e*x + d))/(b*x + a)) + 2*(48*b^5*d^4 - 184*a*b^4*d^3*e + 254*
a^2*b^3*d^2*e^2 - 133*a^3*b^2*d*e^3 + 15*a^4*b*e^4 + 15*(b^5*d*e^3 - a*b^4*
e^4)*x^3 - 5*(2*b^5*d^2*e^2 - 13*a*b^4*d*e^3 + 11*a^2*b^3*e^4)*x^2 + (8*b^5
*d^3*e - 44*a*b^4*d^2*e^2 + 109*a^2*b^3*d*e^3 - 73*a^3*b^2*e^4)*x)*sqrt(e*x
 + d))/(a^4*b^6*d^4 - 4*a^5*b^5*d^3*e + 6*a^6*b^4*d^2*e^2 - 4*a^7*b^3*d*e^3
 + a^8*b^2*e^4 + (b^10*d^4 - 4*a*b^9*d^3*e + 6*a^2*b^8*d^2*e^2 - 4*a^3*b^7*
d*e^3 + a^4*b^6*e^4)*x^4 + 4*(a*b^9*d^4 - 4*a^2*b^8*d^3*e + 6*a^3*b^7*d^2*e
^2 - 4*a^4*b^6*d*e^3 + a^5*b^5*e^4)*x^3 + 6*(a^2*b^8*d^4 - 4*a^3*b^7*d^3*e
```

```
+ 6*a^4*b^6*d^2*e^2 - 4*a^5*b^5*d*e^3 + a^6*b^4*e^4)*x^2 + 4*(a^3*b^7*d^4 -
 4*a^4*b^6*d^3*e + 6*a^5*b^5*d^2*e^2 - 4*a^6*b^4*d*e^3 + a^7*b^3*e^4)*x), -
1/192*(15*(b^4*e^4*x^4 + 4*a*b^3*e^4*x^3 + 6*a^2*b^2*e^4*x^2 + 4*a^3*b*e^4*
x + a^4*e^4)*sqrt(-b^2*d + a*b*e)*arctan(sqrt(-b^2*d + a*b*e)*sqrt(e*x + d)
/(b*e*x + b*d)) + (48*b^5*d^4 - 184*a*b^4*d^3*e + 254*a^2*b^3*d^2*e^2 - 133
*a^3*b^2*d*e^3 + 15*a^4*b*e^4 + 15*(b^5*d*e^3 - a*b^4*e^4)*x^3 - 5*(2*b^5*d
^2*e^2 - 13*a*b^4*d*e^3 + 11*a^2*b^3*e^4)*x^2 + (8*b^5*d^3*e - 44*a*b^4*d^2
*e^2 + 109*a^2*b^3*d*e^3 - 73*a^3*b^2*e^4)*x)*sqrt(e*x + d))/(a^4*b^6*d^4 -
 4*a^5*b^5*d^3*e + 6*a^6*b^4*d^2*e^2 - 4*a^7*b^3*d*e^3 + a^8*b^2*e^4 + (b^1
0*d^4 - 4*a*b^9*d^3*e + 6*a^2*b^8*d^2*e^2 - 4*a^3*b^7*d*e^3 + a^4*b^6*e^4)*
x^4 + 4*(a*b^9*d^4 - 4*a^2*b^8*d^3*e + 6*a^3*b^7*d^2*e^2 - 4*a^4*b^6*d*e^3
+ a^5*b^5*e^4)*x^3 + 6*(a^2*b^8*d^4 - 4*a^3*b^7*d^3*e + 6*a^4*b^6*d^2*e^2 -
 4*a^5*b^5*d*e^3 + a^6*b^4*e^4)*x^2 + 4*(a^3*b^7*d^4 - 4*a^4*b^6*d^3*e + 6*
a^5*b^5*d^2*e^2 - 4*a^6*b^4*d*e^3 + a^7*b^3*e^4)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28878, size = 660, normalized size = 2.36

$$-\frac{5\arctan\left(\frac{\sqrt{xe+d}\,b}{\sqrt{-b^2d+abe}}\right)e^4}{64\left(b^4d^3\mathrm{sgn}\left((xe+d)be-bde+ae^2\right)-3\,ab^3d^2e\,\mathrm{sgn}\left((xe+d)be-bde+ae^2\right)+3\,a^2b^2de^2\mathrm{sgn}\left((xe+d)be-bde+ae^2\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -5/64*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e^4/((b^4*d^3*sgn((x*e +
 d)*b*e - b*d*e + a*e^2) - 3*a*b^3*d^2*e*sgn((x*e + d)*b*e - b*d*e + a*e^2)
 + 3*a^2*b^2*d*e^2*sgn((x*e + d)*b*e - b*d*e + a*e^2) - a^3*b*e^3*sgn((x*e
+ d)*b*e - b*d*e + a*e^2))*sqrt(-b^2*d + a*b*e)) - 1/192*(15*(x*e + d)^(7/2
)*b^3*e^4 - 55*(x*e + d)^(5/2)*b^3*d*e^4 + 73*(x*e + d)^(3/2)*b^3*d^2*e^4 +
 15*sqrt(x*e + d)*b^3*d^3*e^4 + 55*(x*e + d)^(5/2)*a*b^2*e^5 - 146*(x*e + d
)^(3/2)*a*b^2*d*e^5 - 45*sqrt(x*e + d)*a*b^2*d^2*e^5 + 73*(x*e + d)^(3/2)*a
^2*b*e^6 + 45*sqrt(x*e + d)*a^2*b*d*e^6 - 15*sqrt(x*e + d)*a^3*e^7)/((b^4*d
^3*sgn((x*e + d)*b*e - b*d*e + a*e^2) - 3*a*b^3*d^2*e*sgn((x*e + d)*b*e - b
*d*e + a*e^2) + 3*a^2*b^2*d*e^2*sgn((x*e + d)*b*e - b*d*e + a*e^2) - a^3*b*
e^3*sgn((x*e + d)*b*e - b*d*e + a*e^2))*((x*e + d)*b - b*d + a*e)^4)
```

3.1727 $\int \frac{1}{\sqrt{d+ex}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=278

$$\frac{35e^3\sqrt{d+ex}}{64\sqrt{a^2+2abx+b^2x^2}(bd-ae)^4}-\frac{35e^2\sqrt{d+ex}}{96(a+bx)\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3}-\frac{35e^4(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{64\sqrt{b}\sqrt{a^2+2abx+b^2x^2}(bd-ae)^{9/2}}+\frac{}{24(a}$$

```
[Out] (35*e^3*Sqrt[d + e*x])/(64*(b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - S
qrt[d + e*x]/(4*(b*d - a*e)*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (7
*e*Sqrt[d + e*x])/(24*(b*d - a*e)^2*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^
2]) - (35*e^2*Sqrt[d + e*x])/(96*(b*d - a*e)^3*(a + b*x)*Sqrt[a^2 + 2*a*b*x
 + b^2*x^2]) - (35*e^4*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d -
 a*e]])/(64*Sqrt[b]*(b*d - a*e)^(9/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.142837, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {646, 51, 63, 208}

$$\frac{35e^3\sqrt{d+ex}}{64\sqrt{a^2+2abx+b^2x^2}(bd-ae)^4}-\frac{35e^2\sqrt{d+ex}}{96(a+bx)\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3}-\frac{35e^4(a+bx)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{64\sqrt{b}\sqrt{a^2+2abx+b^2x^2}(bd-ae)^{9/2}}+\frac{}{24(a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```


```
[Out] (35*e^3*Sqrt[d + e*x])/(64*(b*d - a*e)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - S
qrt[d + e*x]/(4*(b*d - a*e)*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (7
*e*Sqrt[d + e*x])/(24*(b*d - a*e)^2*(a + b*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^
2]) - (35*e^2*Sqrt[d + e*x])/(96*(b*d - a*e)^3*(a + b*x)*Sqrt[a^2 + 2*a*b*x
 + b^2*x^2]) - (35*e^4*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d -
 a*e]])/(64*Sqrt[b]*(b*d - a*e)^(9/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\left(a^2+2abx+b^2x^2\right)^{5/2}}dx = \frac{\left(b^4\left(ab+b^2x\right)\right)\int \frac{1}{\left(ab+b^2x\right)^5\sqrt{d+ex}}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{\sqrt{d+ex}}{4(bd-ae)(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{\left(7b^3e\left(ab+b^2x\right)\right)\int \frac{1}{\left(ab+b^2x\right)^4\sqrt{d+ex}}d}{8(bd-ae)\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{\sqrt{d+ex}}{4(bd-ae)(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{7e\sqrt{d+ex}}{24(bd-ae)^2(a+bx)^2\sqrt{a^2+2abx+}}$$

$$= -\frac{\sqrt{d+ex}}{4(bd-ae)(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{7e\sqrt{d+ex}}{24(bd-ae)^2(a+bx)^2\sqrt{a^2+2abx+}}$$

$$= \frac{35e^3\sqrt{d+ex}}{64(bd-ae)^4\sqrt{a^2+2abx+b^2x^2}} - \frac{\sqrt{d+ex}}{4(bd-ae)(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{}{24}$$

$$= \frac{35e^3\sqrt{d+ex}}{64(bd-ae)^4\sqrt{a^2+2abx+b^2x^2}} - \frac{\sqrt{d+ex}}{4(bd-ae)(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{}{24}$$

$$= \frac{35e^3\sqrt{d+ex}}{64(bd-ae)^4\sqrt{a^2+2abx+b^2x^2}} - \frac{\sqrt{d+ex}}{4(bd-ae)(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{}{24}$$

Mathematica [C] time = 0.0240033, size = 65, normalized size = 0.23

$$-\frac{2e^4(a+bx)\sqrt{d+ex}\,{}_2F_1\left(\frac{1}{2},5;\frac{3}{2};\frac{b(d+ex)}{bd-ae}\right)}{\sqrt{(a+bx)^2}(bd-ae)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

```
[Out] (-2*e^4*(a + b*x)*Sqrt[d + e*x]*Hypergeometric2F1[1/2, 5, 3/2, (b*(d + e*x)
)/(b*d - a*e)])/((b*d - a*e)^5*Sqrt[(a + b*x)^2])
```

Maple [B] time = 0.268, size = 497, normalized size = 1.8

$$\frac{bx+a}{192\,(ae-bd)^4}\left(105\,\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)\,b}}\right)x^4b^4e^4+420\,\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)\,b}}\right)x^3ab^3e^4+105\,\sqrt{(ae-bd)\,b}\sqrt{ex+d}x^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(1/2),x)
```

```
[Out] 1/192*(105*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^4*b^4*e^4+420*arct
an(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*x^3*a*b^3*e^4+105*((a*e-b*d)*b)^(1/
2)*(e*x+d)^(1/2)*x^3*b^3*e^3+630*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2)
)*x^2*a^2*b^2*e^4+385*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*x^2*a*b^2*e^3-70*((
a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*x^2*b^3*d*e^2+420*arctan(b*(e*x+d)^(1/2)/((
a*e-b*d)*b)^(1/2))*x*a^3*b*e^4+511*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*x*a^2*
b*e^3-252*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*x*a*b^2*d*e^2+56*((a*e-b*d)*b)^
(1/2)*(e*x+d)^(1/2)*x*b^3*d^2*e+105*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1
/2))*a^4*e^4+279*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*a^3*e^3-326*((a*e-b*d)*b
)^(1/2)*(e*x+d)^(1/2)*a^2*b*d*e^2+200*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*a*b
^2*d^2*e-48*((a*e-b*d)*b)^(1/2)*(e*x+d)^(1/2)*b^3*d^3)*(b*x+a)/((a*e-b*d)*b
)^(1/2)/(a*e-b*d)^4/((b*x+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^2 + 2\,abx + a^2\right)^{\frac{5}{2}}\sqrt{ex + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b^2*x^2 + 2*a*b*x + a^2)^(5/2)*sqrt(e*x + d)), x)
```

Fricas [B] time = 1.73517, size = 2709, normalized size = 9.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/384*(105*(b^4*e^4*x^4 + 4*a*b^3*e^4*x^3 + 6*a^2*b^2*e^4*x^2 + 4*a^3*b*e^
4*x + a^4*e^4)*sqrt(b^2*d - a*b*e)*log((b*e*x + 2*b*d - a*e - 2*sqrt(b^2*d
- a*b*e)*sqrt(e*x + d))/(b*x + a)) - 2*(48*b^5*d^4 - 248*a*b^4*d^3*e + 526*
a^2*b^3*d^2*e^2 - 605*a^3*b^2*d*e^3 + 279*a^4*b*e^4 - 105*(b^5*d*e^3 - a*b^
4*e^4)*x^3 + 35*(2*b^5*d^2*e^2 - 13*a*b^4*d*e^3 + 11*a^2*b^3*e^4)*x^2 - 7*(
8*b^5*d^3*e - 44*a*b^4*d^2*e^2 + 109*a^2*b^3*d*e^3 - 73*a^3*b^2*e^4)*x)*sqr
t(e*x + d))/(a^4*b^6*d^5 - 5*a^5*b^5*d^4*e + 10*a^6*b^4*d^3*e^2 - 10*a^7*b^
3*d^2*e^3 + 5*a^8*b^2*d*e^4 - a^9*b*e^5 + (b^10*d^5 - 5*a*b^9*d^4*e + 10*a^
2*b^8*d^3*e^2 - 10*a^3*b^7*d^2*e^3 + 5*a^4*b^6*d*e^4 - a^5*b^5*e^5)*x^4 + 4
*(a*b^9*d^5 - 5*a^2*b^8*d^4*e + 10*a^3*b^7*d^3*e^2 - 10*a^4*b^6*d^2*e^3 + 5
*a^5*b^5*d*e^4 - a^6*b^4*e^5)*x^3 + 6*(a^2*b^8*d^5 - 5*a^3*b^7*d^4*e + 10*a
^4*b^6*d^3*e^2 - 10*a^5*b^5*d^2*e^3 + 5*a^6*b^4*d*e^4 - a^7*b^3*e^5)*x^2 +
4*(a^3*b^7*d^5 - 5*a^4*b^6*d^4*e + 10*a^5*b^5*d^3*e^2 - 10*a^6*b^4*d^2*e^3
+ 5*a^7*b^3*d*e^4 - a^8*b^2*e^5)*x), 1/192*(105*(b^4*e^4*x^4 + 4*a*b^3*e^4*
x^3 + 6*a^2*b^2*e^4*x^2 + 4*a^3*b*e^4*x + a^4*e^4)*sqrt(-b^2*d + a*b*e)*arc
tan(sqrt(-b^2*d + a*b*e)*sqrt(e*x + d)/(b*e*x + b*d)) - (48*b^5*d^4 - 248*a
*b^4*d^3*e + 526*a^2*b^3*d^2*e^2 - 605*a^3*b^2*d*e^3 + 279*a^4*b*e^4 - 105*
(b^5*d*e^3 - a*b^4*e^4)*x^3 + 35*(2*b^5*d^2*e^2 - 13*a*b^4*d*e^3 + 11*a^2*b
```

```
^3*e^4)*x^2 - 7*(8*b^5*d^3*e - 44*a*b^4*d^2*e^2 + 109*a^2*b^3*d*e^3 - 73*a^
3*b^2*e^4)*x)*sqrt(e*x + d))/(a^4*b^6*d^5 - 5*a^5*b^5*d^4*e + 10*a^6*b^4*d^
3*e^2 - 10*a^7*b^3*d^2*e^3 + 5*a^8*b^2*d*e^4 - a^9*b*e^5 + (b^10*d^5 - 5*a*
b^9*d^4*e + 10*a^2*b^8*d^3*e^2 - 10*a^3*b^7*d^2*e^3 + 5*a^4*b^6*d*e^4 - a^5
*b^5*e^5)*x^4 + 4*(a*b^9*d^5 - 5*a^2*b^8*d^4*e + 10*a^3*b^7*d^3*e^2 - 10*a^
4*b^6*d^2*e^3 + 5*a^5*b^5*d*e^4 - a^6*b^4*e^5)*x^3 + 6*(a^2*b^8*d^5 - 5*a^3
*b^7*d^4*e + 10*a^4*b^6*d^3*e^2 - 10*a^5*b^5*d^2*e^3 + 5*a^6*b^4*d*e^4 - a^
7*b^3*e^5)*x^2 + 4*(a^3*b^7*d^5 - 5*a^4*b^6*d^4*e + 10*a^5*b^5*d^3*e^2 - 10
*a^6*b^4*d^2*e^3 + 5*a^7*b^3*d*e^4 - a^8*b^2*e^5)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26337, size = 744, normalized size = 2.68

$$\frac{35\arctan\left(\frac{\sqrt{xe+d}b}{\sqrt{-b^2d+abe}}\right)e^4}{64\left(b^4d^4\mathrm{sgn}\left((xe+d)be-bde+ae^2\right)-4\,ab^3d^3e\mathrm{sgn}\left((xe+d)be-bde+ae^2\right)+6\,a^2b^2d^2e^2\mathrm{sgn}\left((xe+d)be-bde+ae^2\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 35/64*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e^4/((b^4*d^4*sgn((x*e +
 d)*b*e - b*d*e + a*e^2) - 4*a*b^3*d^3*e*sgn((x*e + d)*b*e - b*d*e + a*e^2)
 + 6*a^2*b^2*d^2*e^2*sgn((x*e + d)*b*e - b*d*e + a*e^2) - 4*a^3*b*d*e^3*sgn
((x*e + d)*b*e - b*d*e + a*e^2) + a^4*e^4*sgn((x*e + d)*b*e - b*d*e + a*e^2
))*sqrt(-b^2*d + a*b*e)) + 1/192*(105*(x*e + d)^(7/2)*b^3*e^4 - 385*(x*e +
d)^(5/2)*b^3*d*e^4 + 511*(x*e + d)^(3/2)*b^3*d^2*e^4 - 279*sqrt(x*e + d)*b^
3*d^3*e^4 + 385*(x*e + d)^(5/2)*a*b^2*e^5 - 1022*(x*e + d)^(3/2)*a*b^2*d*e^
5 + 837*sqrt(x*e + d)*a*b^2*d^2*e^5 + 511*(x*e + d)^(3/2)*a^2*b*e^6 - 837*s
qrt(x*e + d)*a^2*b*d*e^6 + 279*sqrt(x*e + d)*a^3*e^7)/((b^4*d^4*sgn((x*e +
d)*b*e - b*d*e + a*e^2) - 4*a*b^3*d^3*e*sgn((x*e + d)*b*e - b*d*e + a*e^2)
+ 6*a^2*b^2*d^2*e^2*sgn((x*e + d)*b*e - b*d*e + a*e^2) - 4*a^3*b*d*e^3*sgn(
(x*e + d)*b*e - b*d*e + a*e^2) + a^4*e^4*sgn((x*e + d)*b*e - b*d*e + a*e^2)
)*((x*e + d)*b - b*d + a*e)^4)
```

3.1728 $\int \frac{1}{(d+ex)^{3/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=329

$$\frac{315e^4(a+bx)}{64\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-ae)^5}+\frac{105e^3}{64\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-ae)^4}-\frac{21e^2}{32(a+bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{d+}}$$

```
[Out] (105*e^3)/(64*(b*d - a*e)^4*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) -
1/(4*(b*d - a*e)*(a + b*x)^3*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) +
 (3*e)/(8*(b*d - a*e)^2*(a + b*x)^2*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*
x^2]) - (21*e^2)/(32*(b*d - a*e)^3*(a + b*x)*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b
*x + b^2*x^2]) + (315*e^4*(a + b*x))/(64*(b*d - a*e)^5*Sqrt[d + e*x]*Sqrt[a
^2 + 2*a*b*x + b^2*x^2]) - (315*Sqrt[b]*e^4*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt
[d + e*x])/Sqrt[b*d - a*e]])/(64*(b*d - a*e)^(11/2)*Sqrt[a^2 + 2*a*b*x + b^
2*x^2])
```

Rubi [A] time = 0.181807, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {646, 51, 63, 208}

$$\frac{315e^4(a+bx)}{64\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-ae)^5}+\frac{105e^3}{64\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-ae)^4}-\frac{21e^2}{32(a+bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{d+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

```
[Out] (105*e^3)/(64*(b*d - a*e)^4*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) -
1/(4*(b*d - a*e)*(a + b*x)^3*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) +
 (3*e)/(8*(b*d - a*e)^2*(a + b*x)^2*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*
x^2]) - (21*e^2)/(32*(b*d - a*e)^3*(a + b*x)*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b
*x + b^2*x^2]) + (315*e^4*(a + b*x))/(64*(b*d - a*e)^5*Sqrt[d + e*x]*Sqrt[a
^2 + 2*a*b*x + b^2*x^2]) - (315*Sqrt[b]*e^4*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt
[d + e*x])/Sqrt[b*d - a*e]])/(64*(b*d - a*e)^(11/2)*Sqrt[a^2 + 2*a*b*x + b^
2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x\right)\right)\int \frac{1}{\left(ab+b^2x\right)^5(d+ex)^{3/2}}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{1}{4(bd-ae)(a+bx)^3\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} - \frac{\left(9b^3e\left(ab+b^2x\right)\right)\int \frac{}{\left(ab+b\right.}}{8(bd-ae)\sqrt{a^2+2ab}}$$

$$= -\frac{1}{4(bd-ae)(a+bx)^3\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} + \frac{3e}{8(bd-ae)^2(a+bx)^2\sqrt{d+}}$$

$$= -\frac{1}{4(bd-ae)(a+bx)^3\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} + \frac{3e}{8(bd-ae)^2(a+bx)^2\sqrt{d+}}$$

$$= \frac{105e^3}{64(bd-ae)^4\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{4(bd-ae)(a+bx)^3\sqrt{d+ex}\sqrt{a^2+}}$$

$$= \frac{105e^3}{64(bd-ae)^4\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{4(bd-ae)(a+bx)^3\sqrt{d+ex}\sqrt{a^2+}}$$

$$= \frac{105e^3}{64(bd-ae)^4\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{4(bd-ae)(a+bx)^3\sqrt{d+ex}\sqrt{a^2+}}$$

$$= \frac{105e^3}{64(bd-ae)^4\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{4(bd-ae)(a+bx)^3\sqrt{d+ex}\sqrt{a^2+}}$$

Mathematica [C] time = 0.0252979, size = 65, normalized size = 0.2

$$\frac{2e^4(a+bx)\,{}_2F_1\left(-\frac{1}{2},5;\frac{1}{2};\frac{b(d+ex)}{bd-ae}\right)}{\sqrt{(a+bx)^2}\sqrt{d+ex}(bd-ae)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

```
[Out] (2*e^4*(a + b*x)*Hypergeometric2F1[-1/2, 5, 1/2, (b*(d + e*x))/(b*d - a*e)]
)/((b*d - a*e)^5*Sqrt[(a + b*x)^2]*Sqrt[d + e*x])
```

Maple [B] time = 0.285, size = 602, normalized size = 1.8

$$-\frac{bx+a}{64\,(ae-bd)^5}\left(315\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)\,b}}\right)\sqrt{ex+d}\,x^4b^5e^4+1260\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{(ae-bd)\,b}}\right)\sqrt{ex+d}\,x^3ab^4e^4+315\sqrt{(ae-b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] -1/64*(315*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*(e*x+d)^(1/2)*x^4*b^
5*e^4+1260*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*(e*x+d)^(1/2)*x^3*a*
b^4*e^4+315*((a*e-b*d)*b)^(1/2)*x^4*b^4*e^4+1890*arctan(b*(e*x+d)^(1/2)/((a
*e-b*d)*b)^(1/2))*(e*x+d)^(1/2)*x^2*a^2*b^3*e^4+1155*((a*e-b*d)*b)^(1/2)*x^
3*a*b^3*e^4+105*((a*e-b*d)*b)^(1/2)*x^3*b^4*d*e^3+1260*arctan(b*(e*x+d)^(1/
2)/((a*e-b*d)*b)^(1/2))*(e*x+d)^(1/2)*x*a^3*b^2*e^4+1533*((a*e-b*d)*b)^(1/2
)*x^2*a^2*b^2*e^4+399*((a*e-b*d)*b)^(1/2)*x^2*a*b^3*d*e^3-42*((a*e-b*d)*b)^
(1/2)*x^2*b^4*d^2*e^2+315*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*(e*x+
d)^(1/2)*a^4*b*e^4+837*((a*e-b*d)*b)^(1/2)*x*a^3*b*e^4+555*((a*e-b*d)*b)^(1
/2)*x*a^2*b^2*d*e^3-156*((a*e-b*d)*b)^(1/2)*x*a*b^3*d^2*e^2+24*((a*e-b*d)*b
)^(1/2)*x*b^4*d^3*e+128*((a*e-b*d)*b)^(1/2)*a^4*e^4+325*((a*e-b*d)*b)^(1/2)
*a^3*b*d*e^3-210*((a*e-b*d)*b)^(1/2)*a^2*b^2*d^2*e^2+88*((a*e-b*d)*b)^(1/2)
*a*b^3*d^3*e-16*((a*e-b*d)*b)^(1/2)*b^4*d^4)*(b*x+a)/((a*e-b*d)*b)^(1/2)/(e
*x+d)^(1/2)/(a*e-b*d)^5/((b*x+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b^2*x^2 + 2*a*b*x + a^2)^(5/2)*(e*x + d)^(3/2)), x)
```

Fricas [B] time = 2.01033, size = 3549, normalized size = 10.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas"
)
```

```
[Out] [-1/128*(315*(b^4*e^5*x^5 + a^4*d*e^4 + (b^4*d*e^4 + 4*a*b^3*e^5)*x^4 + 2*(
2*a*b^3*d*e^4 + 3*a^2*b^2*e^5)*x^3 + 2*(3*a^2*b^2*d*e^4 + 2*a^3*b*e^5)*x^2
+ (4*a^3*b*d*e^4 + a^4*e^5)*x)*sqrt(b/(b*d - a*e))*log((b*e*x + 2*b*d - a*e
 + 2*(b*d - a*e)*sqrt(e*x + d)*sqrt(b/(b*d - a*e)))/(b*x + a)) - 2*(315*b^4
*e^4*x^4 - 16*b^4*d^4 + 88*a*b^3*d^3*e - 210*a^2*b^2*d^2*e^2 + 325*a^3*b*d*
e^3 + 128*a^4*e^4 + 105*(b^4*d*e^3 + 11*a*b^3*e^4)*x^3 - 21*(2*b^4*d^2*e^2
- 19*a*b^3*d*e^3 - 73*a^2*b^2*e^4)*x^2 + 3*(8*b^4*d^3*e - 52*a*b^3*d^2*e^2
+ 185*a^2*b^2*d*e^3 + 279*a^3*b*e^4)*x)*sqrt(e*x + d))/(a^4*b^5*d^6 - 5*a^5
```

```
*b^4*d^5*e + 10*a^6*b^3*d^4*e^2 - 10*a^7*b^2*d^3*e^3 + 5*a^8*b*d^2*e^4 - a^
9*d*e^5 + (b^9*d^5*e - 5*a*b^8*d^4*e^2 + 10*a^2*b^7*d^3*e^3 - 10*a^3*b^6*d^
2*e^4 + 5*a^4*b^5*d*e^5 - a^5*b^4*e^6)*x^5 + (b^9*d^6 - a*b^8*d^5*e - 10*a^
2*b^7*d^4*e^2 + 30*a^3*b^6*d^3*e^3 - 35*a^4*b^5*d^2*e^4 + 19*a^5*b^4*d*e^5
- 4*a^6*b^3*e^6)*x^4 + 2*(2*a*b^8*d^6 - 7*a^2*b^7*d^5*e + 5*a^3*b^6*d^4*e^2
 + 10*a^4*b^5*d^3*e^3 - 20*a^5*b^4*d^2*e^4 + 13*a^6*b^3*d*e^5 - 3*a^7*b^2*e
^6)*x^3 + 2*(3*a^2*b^7*d^6 - 13*a^3*b^6*d^5*e + 20*a^4*b^5*d^4*e^2 - 10*a^5
*b^4*d^3*e^3 - 5*a^6*b^3*d^2*e^4 + 7*a^7*b^2*d*e^5 - 2*a^8*b*e^6)*x^2 + (4*
a^3*b^6*d^6 - 19*a^4*b^5*d^5*e + 35*a^5*b^4*d^4*e^2 - 30*a^6*b^3*d^3*e^3 +
10*a^7*b^2*d^2*e^4 + a^8*b*d*e^5 - a^9*e^6)*x), -1/64*(315*(b^4*e^5*x^5 + a
^4*d*e^4 + (b^4*d*e^4 + 4*a*b^3*e^5)*x^4 + 2*(2*a*b^3*d*e^4 + 3*a^2*b^2*e^5
)*x^3 + 2*(3*a^2*b^2*d*e^4 + 2*a^3*b*e^5)*x^2 + (4*a^3*b*d*e^4 + a^4*e^5)*x
)*sqrt(-b/(b*d - a*e))*arctan(-(b*d - a*e)*sqrt(e*x + d)*sqrt(-b/(b*d - a*e
))/(b*e*x + b*d)) - (315*b^4*e^4*x^4 - 16*b^4*d^4 + 88*a*b^3*d^3*e - 210*a^
2*b^2*d^2*e^2 + 325*a^3*b*d*e^3 + 128*a^4*e^4 + 105*(b^4*d*e^3 + 11*a*b^3*e
^4)*x^3 - 21*(2*b^4*d^2*e^2 - 19*a*b^3*d*e^3 - 73*a^2*b^2*e^4)*x^2 + 3*(8*b
^4*d^3*e - 52*a*b^3*d^2*e^2 + 185*a^2*b^2*d*e^3 + 279*a^3*b*e^4)*x)*sqrt(e*
x + d))/(a^4*b^5*d^6 - 5*a^5*b^4*d^5*e + 10*a^6*b^3*d^4*e^2 - 10*a^7*b^2*d^
3*e^3 + 5*a^8*b*d^2*e^4 - a^9*d*e^5 + (b^9*d^5*e - 5*a*b^8*d^4*e^2 + 10*a^2
*b^7*d^3*e^3 - 10*a^3*b^6*d^2*e^4 + 5*a^4*b^5*d*e^5 - a^5*b^4*e^6)*x^5 + (b
^9*d^6 - a*b^8*d^5*e - 10*a^2*b^7*d^4*e^2 + 30*a^3*b^6*d^3*e^3 - 35*a^4*b^5
*d^2*e^4 + 19*a^5*b^4*d*e^5 - 4*a^6*b^3*e^6)*x^4 + 2*(2*a*b^8*d^6 - 7*a^2*b
^7*d^5*e + 5*a^3*b^6*d^4*e^2 + 10*a^4*b^5*d^3*e^3 - 20*a^5*b^4*d^2*e^4 + 13
*a^6*b^3*d*e^5 - 3*a^7*b^2*e^6)*x^3 + 2*(3*a^2*b^7*d^6 - 13*a^3*b^6*d^5*e +
 20*a^4*b^5*d^4*e^2 - 10*a^5*b^4*d^3*e^3 - 5*a^6*b^3*d^2*e^4 + 7*a^7*b^2*d*
e^5 - 2*a^8*b*e^6)*x^2 + (4*a^3*b^6*d^6 - 19*a^4*b^5*d^5*e + 35*a^5*b^4*d^4
*e^2 - 30*a^6*b^3*d^3*e^3 + 10*a^7*b^2*d^2*e^4 + a^8*b*d*e^5 - a^9*e^6)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)^{\frac{3}{2}}\left((a+bx)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Integral(1/((d + e*x)**(3/2)*((a + b*x)**2)**(5/2)), x)
```

Giac [B] time = 1.34138, size = 1129, normalized size = 3.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 315/64*b*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e^4/((b^5*d^5*sgn((x*
e + d)*b*e - b*d*e + a*e^2) - 5*a*b^4*d^4*e*sgn((x*e + d)*b*e - b*d*e + a*e
^2) + 10*a^2*b^3*d^3*e^2*sgn((x*e + d)*b*e - b*d*e + a*e^2) - 10*a^3*b^2*d^
2*e^3*sgn((x*e + d)*b*e - b*d*e + a*e^2) + 5*a^4*b*d*e^4*sgn((x*e + d)*b*e
- b*d*e + a*e^2) - a^5*e^5*sgn((x*e + d)*b*e - b*d*e + a*e^2))*sqrt(-b^2*d
+ a*b*e)) + 2*e^4/((b^5*d^5*sgn((x*e + d)*b*e - b*d*e + a*e^2) - 5*a*b^4*d^
4*e*sgn((x*e + d)*b*e - b*d*e + a*e^2) + 10*a^2*b^3*d^3*e^2*sgn((x*e + d)*b
```

```
*e - b*d*e + a*e^2) - 10*a^3*b^2*d^2*e^3*sgn((x*e + d)*b*e - b*d*e + a*e^2)
 + 5*a^4*b*d*e^4*sgn((x*e + d)*b*e - b*d*e + a*e^2) - a^5*e^5*sgn((x*e + d)
*b*e - b*d*e + a*e^2))*sqrt(x*e + d)) + 1/64*(187*(x*e + d)^(7/2)*b^4*e^4 -
 643*(x*e + d)^(5/2)*b^4*d*e^4 + 765*(x*e + d)^(3/2)*b^4*d^2*e^4 - 325*sqrt
(x*e + d)*b^4*d^3*e^4 + 643*(x*e + d)^(5/2)*a*b^3*e^5 - 1530*(x*e + d)^(3/2
)*a*b^3*d*e^5 + 975*sqrt(x*e + d)*a*b^3*d^2*e^5 + 765*(x*e + d)^(3/2)*a^2*b
^2*e^6 - 975*sqrt(x*e + d)*a^2*b^2*d*e^6 + 325*sqrt(x*e + d)*a^3*b*e^7)/((b
^5*d^5*sgn((x*e + d)*b*e - b*d*e + a*e^2) - 5*a*b^4*d^4*e*sgn((x*e + d)*b*e
 - b*d*e + a*e^2) + 10*a^2*b^3*d^3*e^2*sgn((x*e + d)*b*e - b*d*e + a*e^2) -
 10*a^3*b^2*d^2*e^3*sgn((x*e + d)*b*e - b*d*e + a*e^2) + 5*a^4*b*d*e^4*sgn(
(x*e + d)*b*e - b*d*e + a*e^2) - a^5*e^5*sgn((x*e + d)*b*e - b*d*e + a*e^2)
)*((x*e + d)*b - b*d + a*e)^4)
```

3.1729 $\int \frac{1}{(d+ex)^{5/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=381

$$\frac{1155be^4(a+bx)}{64\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-ae)^6}+\frac{385e^4(a+bx)}{64\sqrt{a^2+2abx+b^2x^2}(d+ex)^{3/2}(bd-ae)^5}+\frac{231e^3}{64\sqrt{a^2+2abx+b^2x^2}(d+ex)^{3}}$$

```
[Out] (231*e^3)/(64*(b*d - a*e)^4*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
- 1/(4*(b*d - a*e)*(a + b*x)^3*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2
]) + (11*e)/(24*(b*d - a*e)^2*(a + b*x)^2*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*
x + b^2*x^2]) - (33*e^2)/(32*(b*d - a*e)^3*(a + b*x)*(d + e*x)^(3/2)*Sqrt[a
^2 + 2*a*b*x + b^2*x^2]) + (385*e^4*(a + b*x))/(64*(b*d - a*e)^5*(d + e*x)^
(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (1155*b*e^4*(a + b*x))/(64*(b*d - a*
e)^6*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (1155*b^(3/2)*e^4*(a +
b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(64*(b*d - a*e)^(13/
2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.237396, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {646, 51, 63, 208}

$$\frac{1155be^4(a+bx)}{64\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-ae)^6}+\frac{385e^4(a+bx)}{64\sqrt{a^2+2abx+b^2x^2}(d+ex)^{3/2}(bd-ae)^5}+\frac{231e^3}{64\sqrt{a^2+2abx+b^2x^2}(d+ex)^{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```


```
[Out] (231*e^3)/(64*(b*d - a*e)^4*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
- 1/(4*(b*d - a*e)*(a + b*x)^3*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2
]) + (11*e)/(24*(b*d - a*e)^2*(a + b*x)^2*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*
x + b^2*x^2]) - (33*e^2)/(32*(b*d - a*e)^3*(a + b*x)*(d + e*x)^(3/2)*Sqrt[a
^2 + 2*a*b*x + b^2*x^2]) + (385*e^4*(a + b*x))/(64*(b*d - a*e)^5*(d + e*x)^
(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (1155*b*e^4*(a + b*x))/(64*(b*d - a*
e)^6*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (1155*b^(3/2)*e^4*(a +
b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(64*(b*d - a*e)^(13/
2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{5/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x\right)\right)\int \frac{1}{\left(ab+b^2x\right)^5(d+ex)^{5/2}}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{1}{4(bd-ae)(a+bx)^3(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}} - \frac{\left(11b^3e\left(ab+b^2x\right)\right)\int \frac{}{\left(ab+b\right.}}{8(bd-ae)\sqrt{a^2+2ab}}$$

$$= -\frac{1}{4(bd-ae)(a+bx)^3(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}} + \frac{11}{24(bd-ae)^2(a+bx)^2(d+}$$

$$= -\frac{1}{4(bd-ae)(a+bx)^3(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}} + \frac{11}{24(bd-ae)^2(a+bx)^2(d+}$$

$$= \frac{231e^3}{64(bd-ae)^4(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{4(bd-ae)(a+bx)^3(d+ex)^{3/2}\sqrt{a^2}}$$

$$= \frac{231e^3}{64(bd-ae)^4(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{4(bd-ae)(a+bx)^3(d+ex)^{3/2}\sqrt{a^2}}$$

$$= \frac{231e^3}{64(bd-ae)^4(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{4(bd-ae)(a+bx)^3(d+ex)^{3/2}\sqrt{a^2}}$$

$$= \frac{231e^3}{64(bd-ae)^4(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{4(bd-ae)(a+bx)^3(d+ex)^{3/2}\sqrt{a^2}}$$

$$= \frac{231e^3}{64(bd-ae)^4(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{4(bd-ae)(a+bx)^3(d+ex)^{3/2}\sqrt{a^2}}$$

Mathematica [C] time = 0.0281996, size = 67, normalized size = 0.18

$$\frac{2e^4(a+bx)\,{}_2F_1\left(-\frac{3}{2},5;-\frac{1}{2};\frac{b(d+ex)}{bd-ae}\right)}{3\sqrt{(a+bx)^2}(d+ex)^{3/2}(bd-ae)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

```
[Out] (2*e^4*(a + b*x)*Hypergeometric2F1[-3/2, 5, -1/2, (b*(d + e*x))/(b*d - a*e)
])/(3*(b*d - a*e)^5*Sqrt[(a + b*x)^2]*(d + e*x)^(3/2))
```

Maple [B] time = 0.286, size = 763, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] 1/192*(3465*((a*e-b*d)*b)^(1/2)*x^5*b^5*e^5+20790*arctan(b*(e*x+d)^(1/2)/((
a*e-b*d)*b)^(1/2))*(e*x+d)^(3/2)*x^2*a^2*b^4*e^4+9207*((a*e-b*d)*b)^(1/2)*x
^2*a^3*b^2*e^5+693*((a*e-b*d)*b)^(1/2)*x^3*b^5*d^2*e^3-198*((a*e-b*d)*b)^(1
/2)*x^2*b^5*d^3*e^2+88*((a*e-b*d)*b)^(1/2)*x*b^5*d^4*e+3465*arctan(b*(e*x+d
)^(1/2)/((a*e-b*d)*b)^(1/2))*(e*x+d)^(3/2)*x^4*b^6*e^4+1408*((a*e-b*d)*b)^(
1/2)*x*a^4*b*e^5+12705*((a*e-b*d)*b)^(1/2)*x^4*a*b^4*e^5+4620*((a*e-b*d)*b)
^(1/2)*x^4*b^5*d*e^4+16863*((a*e-b*d)*b)^(1/2)*x^3*a^2*b^3*e^5+3465*arctan(
b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*(e*x+d)^(3/2)*a^4*b^2*e^4+2295*((a*e-b
*d)*b)^(1/2)*a^3*b^2*d^2*e^3-1030*((a*e-b*d)*b)^(1/2)*a^2*b^3*d^3*e^2+328*(
(a*e-b*d)*b)^(1/2)*a*b^4*d^4*e+2048*((a*e-b*d)*b)^(1/2)*a^4*b*d*e^4-48*((a*
e-b*d)*b)^(1/2)*b^5*d^5-128*((a*e-b*d)*b)^(1/2)*a^5*e^5+13860*arctan(b*(e*x
+d)^(1/2)/((a*e-b*d)*b)^(1/2))*(e*x+d)^(3/2)*x*a^3*b^3*e^4+17094*((a*e-b*d)
*b)^(1/2)*x^3*a*b^4*d*e^4+2673*((a*e-b*d)*b)^(1/2)*x^2*a*b^4*d^2*e^3+3795*(
(a*e-b*d)*b)^(1/2)*x*a^2*b^3*d^2*e^3-748*((a*e-b*d)*b)^(1/2)*x*a*b^4*d^3*e^
2+13860*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*(e*x+d)^(3/2)*x^3*a*b^5
*e^4+22968*((a*e-b*d)*b)^(1/2)*x^2*a^2*b^3*d*e^4+12782*((a*e-b*d)*b)^(1/2)*
x*a^3*b^2*d*e^4)*(b*x+a)/((a*e-b*d)*b)^(1/2)/(e*x+d)^(3/2)/(a*e-b*d)^6/((b*
x+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^2 + 2\,abx + a^2\right)^{\frac{5}{2}}(ex + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b^2*x^2 + 2*a*b*x + a^2)^(5/2)*(e*x + d)^(5/2)), x)
```

Fricas [B] time = 2.14567, size = 5126, normalized size = 13.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas"
)
```

```
[Out] [1/384*(3465*(b^5*e^6*x^6 + a^4*b*d^2*e^4 + 2*(b^5*d*e^5 + 2*a*b^4*e^6)*x^5
 + (b^5*d^2*e^4 + 8*a*b^4*d*e^5 + 6*a^2*b^3*e^6)*x^4 + 4*(a*b^4*d^2*e^4 + 3
*a^2*b^3*d*e^5 + a^3*b^2*e^6)*x^3 + (6*a^2*b^3*d^2*e^4 + 8*a^3*b^2*d*e^5 +
a^4*b*e^6)*x^2 + 2*(2*a^3*b^2*d^2*e^4 + a^4*b*d*e^5)*x)*sqrt(b/(b*d - a*e))
*log((b*e*x + 2*b*d - a*e - 2*(b*d - a*e)*sqrt(e*x + d)*sqrt(b/(b*d - a*e))
)/(b*x + a)) + 2*(3465*b^5*e^5*x^5 - 48*b^5*d^5 + 328*a*b^4*d^4*e - 1030*a^
2*b^3*d^3*e^2 + 2295*a^3*b^2*d^2*e^3 + 2048*a^4*b*d*e^4 - 128*a^5*e^5 + 115
5*(4*b^5*d*e^4 + 11*a*b^4*e^5)*x^4 + 231*(3*b^5*d^2*e^3 + 74*a*b^4*d*e^4 +
73*a^2*b^3*e^5)*x^3 - 99*(2*b^5*d^3*e^2 - 27*a*b^4*d^2*e^3 - 232*a^2*b^3*d*
e^4 - 93*a^3*b^2*e^5)*x^2 + 11*(8*b^5*d^4*e - 68*a*b^4*d^3*e^2 + 345*a^2*b^
3*d^2*e^3 + 1162*a^3*b^2*d*e^4 + 128*a^4*b*e^5)*x)*sqrt(e*x + d))/(a^4*b^6*
d^8 - 6*a^5*b^5*d^7*e + 15*a^6*b^4*d^6*e^2 - 20*a^7*b^3*d^5*e^3 + 15*a^8*b^
2*d^4*e^4 - 6*a^9*b*d^3*e^5 + a^10*d^2*e^6 + (b^10*d^6*e^2 - 6*a*b^9*d^5*e^
3 + 15*a^2*b^8*d^4*e^4 - 20*a^3*b^7*d^3*e^5 + 15*a^4*b^6*d^2*e^6 - 6*a^5*b^
5*d*e^7 + a^6*b^4*e^8)*x^6 + 2*(b^10*d^7*e - 4*a*b^9*d^6*e^2 + 3*a^2*b^8*d^
5*e^3 + 10*a^3*b^7*d^4*e^4 - 25*a^4*b^6*d^3*e^5 + 24*a^5*b^5*d^2*e^6 - 11*a
^6*b^4*d*e^7 + 2*a^7*b^3*e^8)*x^5 + (b^10*d^8 + 2*a*b^9*d^7*e - 27*a^2*b^8*
d^6*e^2 + 64*a^3*b^7*d^5*e^3 - 55*a^4*b^6*d^4*e^4 - 6*a^5*b^5*d^3*e^5 + 43*
a^6*b^4*d^2*e^6 - 28*a^7*b^3*d*e^7 + 6*a^8*b^2*e^8)*x^4 + 4*(a*b^9*d^8 - 3*
a^2*b^8*d^7*e - 2*a^3*b^7*d^6*e^2 + 19*a^4*b^6*d^5*e^3 - 30*a^5*b^5*d^4*e^4
 + 19*a^6*b^4*d^3*e^5 - 2*a^7*b^3*d^2*e^6 - 3*a^8*b^2*d*e^7 + a^9*b*e^8)*x^
3 + (6*a^2*b^8*d^8 - 28*a^3*b^7*d^7*e + 43*a^4*b^6*d^6*e^2 - 6*a^5*b^5*d^5*
e^3 - 55*a^6*b^4*d^4*e^4 + 64*a^7*b^3*d^3*e^5 - 27*a^8*b^2*d^2*e^6 + 2*a^9*
b*d*e^7 + a^10*e^8)*x^2 + 2*(2*a^3*b^7*d^8 - 11*a^4*b^6*d^7*e + 24*a^5*b^5*
d^6*e^2 - 25*a^6*b^4*d^5*e^3 + 10*a^7*b^3*d^4*e^4 + 3*a^8*b^2*d^3*e^5 - 4*a
^9*b*d^2*e^6 + a^10*d*e^7)*x), -1/192*(3465*(b^5*e^6*x^6 + a^4*b*d^2*e^4 +
2*(b^5*d*e^5 + 2*a*b^4*e^6)*x^5 + (b^5*d^2*e^4 + 8*a*b^4*d*e^5 + 6*a^2*b^3*
e^6)*x^4 + 4*(a*b^4*d^2*e^4 + 3*a^2*b^3*d*e^5 + a^3*b^2*e^6)*x^3 + (6*a^2*b
^3*d^2*e^4 + 8*a^3*b^2*d*e^5 + a^4*b*e^6)*x^2 + 2*(2*a^3*b^2*d^2*e^4 + a^4*
b*d*e^5)*x)*sqrt(-b/(b*d - a*e))*arctan(-(b*d - a*e)*sqrt(e*x + d)*sqrt(-b/
(b*d - a*e))/(b*e*x + b*d)) - (3465*b^5*e^5*x^5 - 48*b^5*d^5 + 328*a*b^4*d^
4*e - 1030*a^2*b^3*d^3*e^2 + 2295*a^3*b^2*d^2*e^3 + 2048*a^4*b*d*e^4 - 128*
a^5*e^5 + 1155*(4*b^5*d*e^4 + 11*a*b^4*e^5)*x^4 + 231*(3*b^5*d^2*e^3 + 74*a
*b^4*d*e^4 + 73*a^2*b^3*e^5)*x^3 - 99*(2*b^5*d^3*e^2 - 27*a*b^4*d^2*e^3 - 2
32*a^2*b^3*d*e^4 - 93*a^3*b^2*e^5)*x^2 + 11*(8*b^5*d^4*e - 68*a*b^4*d^3*e^2
 + 345*a^2*b^3*d^2*e^3 + 1162*a^3*b^2*d*e^4 + 128*a^4*b*e^5)*x)*sqrt(e*x +
d))/(a^4*b^6*d^8 - 6*a^5*b^5*d^7*e + 15*a^6*b^4*d^6*e^2 - 20*a^7*b^3*d^5*e^
3 + 15*a^8*b^2*d^4*e^4 - 6*a^9*b*d^3*e^5 + a^10*d^2*e^6 + (b^10*d^6*e^2 - 6
*a*b^9*d^5*e^3 + 15*a^2*b^8*d^4*e^4 - 20*a^3*b^7*d^3*e^5 + 15*a^4*b^6*d^2*e
^6 - 6*a^5*b^5*d*e^7 + a^6*b^4*e^8)*x^6 + 2*(b^10*d^7*e - 4*a*b^9*d^6*e^2 +
 3*a^2*b^8*d^5*e^3 + 10*a^3*b^7*d^4*e^4 - 25*a^4*b^6*d^3*e^5 + 24*a^5*b^5*d
^2*e^6 - 11*a^6*b^4*d*e^7 + 2*a^7*b^3*e^8)*x^5 + (b^10*d^8 + 2*a*b^9*d^7*e
- 27*a^2*b^8*d^6*e^2 + 64*a^3*b^7*d^5*e^3 - 55*a^4*b^6*d^4*e^4 - 6*a^5*b^5*
d^3*e^5 + 43*a^6*b^4*d^2*e^6 - 28*a^7*b^3*d*e^7 + 6*a^8*b^2*e^8)*x^4 + 4*(a
*b^9*d^8 - 3*a^2*b^8*d^7*e - 2*a^3*b^7*d^6*e^2 + 19*a^4*b^6*d^5*e^3 - 30*a^
5*b^5*d^4*e^4 + 19*a^6*b^4*d^3*e^5 - 2*a^7*b^3*d^2*e^6 - 3*a^8*b^2*d*e^7 +
a^9*b*e^8)*x^3 + (6*a^2*b^8*d^8 - 28*a^3*b^7*d^7*e + 43*a^4*b^6*d^6*e^2 - 6
*a^5*b^5*d^5*e^3 - 55*a^6*b^4*d^4*e^4 + 64*a^7*b^3*d^3*e^5 - 27*a^8*b^2*d^2
*e^6 + 2*a^9*b*d*e^7 + a^10*e^8)*x^2 + 2*(2*a^3*b^7*d^8 - 11*a^4*b^6*d^7*e
+ 24*a^5*b^5*d^6*e^2 - 25*a^6*b^4*d^5*e^3 + 10*a^7*b^3*d^4*e^4 + 3*a^8*b^2*
d^3*e^5 - 4*a^9*b*d^2*e^6 + a^10*d*e^7)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(5/2)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32493, size = 1299, normalized size = 3.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1155/64*b^2*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e^4/((b^6*d^6*sgn(
(x*e + d)*b*e - b*d*e + a*e^2) - 6*a*b^5*d^5*e*sgn((x*e + d)*b*e - b*d*e +
a*e^2) + 15*a^2*b^4*d^4*e^2*sgn((x*e + d)*b*e - b*d*e + a*e^2) - 20*a^3*b^3
*d^3*e^3*sgn((x*e + d)*b*e - b*d*e + a*e^2) + 15*a^4*b^2*d^2*e^4*sgn((x*e +
 d)*b*e - b*d*e + a*e^2) - 6*a^5*b*d*e^5*sgn((x*e + d)*b*e - b*d*e + a*e^2)
 + a^6*e^6*sgn((x*e + d)*b*e - b*d*e + a*e^2))*sqrt(-b^2*d + a*b*e)) + 2/3*
(15*(x*e + d)*b*e^4 + b*d*e^4 - a*e^5)/((b^6*d^6*sgn((x*e + d)*b*e - b*d*e
+ a*e^2) - 6*a*b^5*d^5*e*sgn((x*e + d)*b*e - b*d*e + a*e^2) + 15*a^2*b^4*d^
4*e^2*sgn((x*e + d)*b*e - b*d*e + a*e^2) - 20*a^3*b^3*d^3*e^3*sgn((x*e + d)
*b*e - b*d*e + a*e^2) + 15*a^4*b^2*d^2*e^4*sgn((x*e + d)*b*e - b*d*e + a*e^
2) - 6*a^5*b*d*e^5*sgn((x*e + d)*b*e - b*d*e + a*e^2) + a^6*e^6*sgn((x*e +
d)*b*e - b*d*e + a*e^2))*(x*e + d)^(3/2)) + 1/192*(1545*(x*e + d)^(7/2)*b^5
*e^4 - 5153*(x*e + d)^(5/2)*b^5*d*e^4 + 5855*(x*e + d)^(3/2)*b^5*d^2*e^4 -
2295*sqrt(x*e + d)*b^5*d^3*e^4 + 5153*(x*e + d)^(5/2)*a*b^4*e^5 - 11710*(x*
e + d)^(3/2)*a*b^4*d*e^5 + 6885*sqrt(x*e + d)*a*b^4*d^2*e^5 + 5855*(x*e + d
)^(3/2)*a^2*b^3*e^6 - 6885*sqrt(x*e + d)*a^2*b^3*d*e^6 + 2295*sqrt(x*e + d)
*a^3*b^2*e^7)/((b^6*d^6*sgn((x*e + d)*b*e - b*d*e + a*e^2) - 6*a*b^5*d^5*e*
sgn((x*e + d)*b*e - b*d*e + a*e^2) + 15*a^2*b^4*d^4*e^2*sgn((x*e + d)*b*e -
 b*d*e + a*e^2) - 20*a^3*b^3*d^3*e^3*sgn((x*e + d)*b*e - b*d*e + a*e^2) + 1
5*a^4*b^2*d^2*e^4*sgn((x*e + d)*b*e - b*d*e + a*e^2) - 6*a^5*b*d*e^5*sgn((x
*e + d)*b*e - b*d*e + a*e^2) + a^6*e^6*sgn((x*e + d)*b*e - b*d*e + a*e^2))*
((x*e + d)*b - b*d + a*e)^4)
```

3.1730 $\int \frac{1}{(d+ex)^{7/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=435

$$\frac{3003b^2e^4(a+bx)}{64\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-ae)^7}+\frac{1001be^4(a+bx)}{64\sqrt{a^2+2abx+b^2x^2}(d+ex)^{3/2}(bd-ae)^6}+\frac{3003e^4(a+bx)}{320\sqrt{a^2+2abx+b^2x^2}(d+ex)^{5/2}}$$

```
[Out] (429*e^3)/(64*(b*d - a*e)^4*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
- 1/(4*(b*d - a*e)*(a + b*x)^3*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2
]) + (13*e)/(24*(b*d - a*e)^2*(a + b*x)^2*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*
x + b^2*x^2]) - (143*e^2)/(96*(b*d - a*e)^3*(a + b*x)*(d + e*x)^(5/2)*Sqrt[
a^2 + 2*a*b*x + b^2*x^2]) + (3003*e^4*(a + b*x))/(320*(b*d - a*e)^5*(d + e*
x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (1001*b*e^4*(a + b*x))/(64*(b*d -
 a*e)^6*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (3003*b^2*e^4*(a +
 b*x))/(64*(b*d - a*e)^7*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (30
03*b^(5/2)*e^4*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/
(64*(b*d - a*e)^(15/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.255969, antiderivative size = 435, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {646, 51, 63, 208}

$$\frac{3003b^2e^4(a+bx)}{64\sqrt{a^2+2abx+b^2x^2}\sqrt{d+ex}(bd-ae)^7}+\frac{1001be^4(a+bx)}{64\sqrt{a^2+2abx+b^2x^2}(d+ex)^{3/2}(bd-ae)^6}+\frac{3003e^4(a+bx)}{320\sqrt{a^2+2abx+b^2x^2}(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

```
[Out] (429*e^3)/(64*(b*d - a*e)^4*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
- 1/(4*(b*d - a*e)*(a + b*x)^3*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2
]) + (13*e)/(24*(b*d - a*e)^2*(a + b*x)^2*(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*
x + b^2*x^2]) - (143*e^2)/(96*(b*d - a*e)^3*(a + b*x)*(d + e*x)^(5/2)*Sqrt[
a^2 + 2*a*b*x + b^2*x^2]) + (3003*e^4*(a + b*x))/(320*(b*d - a*e)^5*(d + e*
x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (1001*b*e^4*(a + b*x))/(64*(b*d -
 a*e)^6*(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) + (3003*b^2*e^4*(a +
 b*x))/(64*(b*d - a*e)^7*Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - (30
03*b^(5/2)*e^4*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/
(64*(b*d - a*e)^(15/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
```

```
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{7/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x\right)\right)\int \frac{1}{\left(ab+b^2x\right)^5(d+ex)^{7/2}}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$= -\frac{1}{4(bd-ae)(a+bx)^3(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}} - \frac{\left(13b^3e\left(ab+b^2x\right)\right)\int \cdots}{8(bd-ae)\sqrt{a^2+2\cdots}}$$

$$= -\frac{1}{4(bd-ae)(a+bx)^3(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}} + \frac{\cdots}{24(bd-ae)^2(a+bx)^2\cdots}$$

$$= -\frac{1}{4(bd-ae)(a+bx)^3(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}} + \frac{\cdots}{24(bd-ae)^2(a+bx)^2\cdots}$$

$$= \frac{429e^3}{64(bd-ae)^4(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{4(bd-ae)(a+bx)^3(d+ex)^{5/2}\cdots}$$

$$= \frac{429e^3}{64(bd-ae)^4(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{4(bd-ae)(a+bx)^3(d+ex)^{5/2}\cdots}$$

$$= \frac{429e^3}{64(bd-ae)^4(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{4(bd-ae)(a+bx)^3(d+ex)^{5/2}\cdots}$$

$$= \frac{429e^3}{64(bd-ae)^4(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{4(bd-ae)(a+bx)^3(d+ex)^{5/2}\cdots}$$

$$= \frac{429e^3}{64(bd-ae)^4(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{4(bd-ae)(a+bx)^3(d+ex)^{5/2}\cdots}$$

$$= \frac{429e^3}{64(bd-ae)^4(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{4(bd-ae)(a+bx)^3(d+ex)^{5/2}\cdots}$$

Mathematica [C] time = 0.0351494, size = 67, normalized size = 0.15

$$\frac{2e^4(a+bx)\,{}_2F_1\left(-\frac{5}{2},5;-\frac{3}{2};\frac{b(d+ex)}{bd-ae}\right)}{5\sqrt{(a+bx)^2}(d+ex)^{5/2}(bd-ae)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

```
[Out] (2*e^4*(a + b*x)*Hypergeometric2F1[-5/2, 5, -3/2, (b*(d + e*x))/(b*d - a*e)
])/(5*(b*d - a*e)^5*Sqrt[(a + b*x)^2]*(d + e*x)^(5/2))
```

Maple [B] time = 0.291, size = 951, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] -1/960*(22155*((a*e-b*d)*b)^(1/2)*a^3*b^3*d^3*e^3-7630*((a*e-b*d)*b)^(1/2)*
a^2*b^4*d^4*e^2+1960*((a*e-b*d)*b)^(1/2)*a*b^5*d^5*e+180180*arctan(b*(e*x+d
)^(1/2)/((a*e-b*d)*b)^(1/2))*(e*x+d)^(5/2)*x*a^3*b^4*e^4+45045*((a*e-b*d)*b
)^(1/2)*x^6*b^6*e^6+6435*((a*e-b*d)*b)^(1/2)*x^3*b^6*d^3*e^3-1430*((a*e-b*d
)*b)^(1/2)*x^2*b^6*d^4*e^2+520*((a*e-b*d)*b)^(1/2)*x*b^6*d^5*e+165165*((a*e
-b*d)*b)^(1/2)*x^5*a*b^5*e^6+105105*((a*e-b*d)*b)^(1/2)*x^5*b^6*d*e^5+45045
*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*(e*x+d)^(5/2)*x^4*b^7*e^4+2192
19*((a*e-b*d)*b)^(1/2)*x^4*a^2*b^4*e^6+69069*((a*e-b*d)*b)^(1/2)*x^4*b^6*d^
2*e^4+45045*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*(e*x+d)^(5/2)*a^4*b
^3*e^4+119691*((a*e-b*d)*b)^(1/2)*x^3*a^3*b^3*e^6+18304*((a*e-b*d)*b)^(1/2)
*x^2*a^4*b^2*e^6-1664*((a*e-b*d)*b)^(1/2)*x*a^5*b*e^6-3968*((a*e-b*d)*b)^(1
/2)*a^5*b*d*e^5+32384*((a*e-b*d)*b)^(1/2)*a^4*b^2*d^2*e^4-240*((a*e-b*d)*b)
^(1/2)*b^6*d^6+384*((a*e-b*d)*b)^(1/2)*a^6*e^6+387387*((a*e-b*d)*b)^(1/2)*x
^4*a*b^5*d*e^5+180180*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*(e*x+d)^(
5/2)*x^3*a*b^6*e^4+270270*arctan(b*(e*x+d)^(1/2)/((a*e-b*d)*b)^(1/2))*(e*x+
d)^(5/2)*x^2*a^2*b^5*e^4+517803*((a*e-b*d)*b)^(1/2)*x^3*a^2*b^4*d*e^5+25697
1*((a*e-b*d)*b)^(1/2)*x^3*a*b^5*d^2*e^4+285857*((a*e-b*d)*b)^(1/2)*x^2*a^3*
b^3*d*e^5+347919*((a*e-b*d)*b)^(1/2)*x^2*a^2*b^4*d^2*e^4+44928*((a*e-b*d)*b
)^(1/2)*x*a^4*b^2*d*e^5+196001*((a*e-b*d)*b)^(1/2)*x*a^3*b^3*d^2*e^4+25025*
((a*e-b*d)*b)^(1/2)*x^2*a*b^5*d^3*e^3+35945*((a*e-b*d)*b)^(1/2)*x*a^2*b^4*d
^3*e^3-5460*((a*e-b*d)*b)^(1/2)*x*a*b^5*d^4*e^2)*(b*x+a)/((a*e-b*d)*b)^(1/2
)/(e*x+d)^(5/2)/(a*e-b*d)^7/((b*x+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^2 + 2\,abx + a^2\right)^{\frac{5}{2}}(ex + d)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b^2*x^2 + 2*a*b*x + a^2)^(5/2)*(e*x + d)^(7/2)), x)
```

Fricas [B] time = 2.32661, size = 7048, normalized size = 16.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas"
)
```

```
[Out] [-1/1920*(45045*(b^6*e^7*x^7 + a^4*b^2*d^3*e^4 + (3*b^6*d*e^6 + 4*a*b^5*e^7
)*x^6 + 3*(b^6*d^2*e^5 + 4*a*b^5*d*e^6 + 2*a^2*b^4*e^7)*x^5 + (b^6*d^3*e^4
+ 12*a*b^5*d^2*e^5 + 18*a^2*b^4*d*e^6 + 4*a^3*b^3*e^7)*x^4 + (4*a*b^5*d^3*e
^4 + 18*a^2*b^4*d^2*e^5 + 12*a^3*b^3*d*e^6 + a^4*b^2*e^7)*x^3 + 3*(2*a^2*b^
4*d^3*e^4 + 4*a^3*b^3*d^2*e^5 + a^4*b^2*d*e^6)*x^2 + (4*a^3*b^3*d^3*e^4 + 3
*a^4*b^2*d^2*e^5)*x)*sqrt(b/(b*d - a*e))*log((b*e*x + 2*b*d - a*e + 2*(b*d
- a*e)*sqrt(e*x + d)*sqrt(b/(b*d - a*e)))/(b*x + a)) - 2*(45045*b^6*e^6*x^6
 - 240*b^6*d^6 + 1960*a*b^5*d^5*e - 7630*a^2*b^4*d^4*e^2 + 22155*a^3*b^3*d^
3*e^3 + 32384*a^4*b^2*d^2*e^4 - 3968*a^5*b*d*e^5 + 384*a^6*e^6 + 15015*(7*b
^6*d*e^5 + 11*a*b^5*e^6)*x^5 + 3003*(23*b^6*d^2*e^4 + 129*a*b^5*d*e^5 + 73*
a^2*b^4*e^6)*x^4 + 429*(15*b^6*d^3*e^3 + 599*a*b^5*d^2*e^4 + 1207*a^2*b^4*d
*e^5 + 279*a^3*b^3*e^6)*x^3 - 143*(10*b^6*d^4*e^2 - 175*a*b^5*d^3*e^3 - 243
3*a^2*b^4*d^2*e^4 - 1999*a^3*b^3*d*e^5 - 128*a^4*b^2*e^6)*x^2 + 13*(40*b^6*
d^5*e - 420*a*b^5*d^4*e^2 + 2765*a^2*b^4*d^3*e^3 + 15077*a^3*b^3*d^2*e^4 +
3456*a^4*b^2*d*e^5 - 128*a^5*b*e^6)*x)*sqrt(e*x + d))/(a^4*b^7*d^10 - 7*a^5
*b^6*d^9*e + 21*a^6*b^5*d^8*e^2 - 35*a^7*b^4*d^7*e^3 + 35*a^8*b^3*d^6*e^4 -
 21*a^9*b^2*d^5*e^5 + 7*a^10*b*d^4*e^6 - a^11*d^3*e^7 + (b^11*d^7*e^3 - 7*a
*b^10*d^6*e^4 + 21*a^2*b^9*d^5*e^5 - 35*a^3*b^8*d^4*e^6 + 35*a^4*b^7*d^3*e^
7 - 21*a^5*b^6*d^2*e^8 + 7*a^6*b^5*d*e^9 - a^7*b^4*e^10)*x^7 + (3*b^11*d^8*
e^2 - 17*a*b^10*d^7*e^3 + 35*a^2*b^9*d^6*e^4 - 21*a^3*b^8*d^5*e^5 - 35*a^4*
b^7*d^4*e^6 + 77*a^5*b^6*d^3*e^7 - 63*a^6*b^5*d^2*e^8 + 25*a^7*b^4*d*e^9 -
4*a^8*b^3*e^10)*x^6 + 3*(b^11*d^9*e - 3*a*b^10*d^8*e^2 - 5*a^2*b^9*d^7*e^3
+ 35*a^3*b^8*d^6*e^4 - 63*a^4*b^7*d^5*e^5 + 49*a^5*b^6*d^4*e^6 - 7*a^6*b^5*
d^3*e^7 - 15*a^7*b^4*d^2*e^8 + 10*a^8*b^3*d*e^9 - 2*a^9*b^2*e^10)*x^5 + (b^
11*d^10 + 5*a*b^10*d^9*e - 45*a^2*b^9*d^8*e^2 + 95*a^3*b^8*d^7*e^3 - 35*a^4
*b^7*d^6*e^4 - 147*a^5*b^6*d^5*e^5 + 245*a^6*b^5*d^4*e^6 - 155*a^7*b^4*d^3*
e^7 + 30*a^8*b^3*d^2*e^8 + 10*a^9*b^2*d*e^9 - 4*a^10*b*e^10)*x^4 + (4*a*b^1
0*d^10 - 10*a^2*b^9*d^9*e - 30*a^3*b^8*d^8*e^2 + 155*a^4*b^7*d^7*e^3 - 245*
a^5*b^6*d^6*e^4 + 147*a^6*b^5*d^5*e^5 + 35*a^7*b^4*d^4*e^6 - 95*a^8*b^3*d^3
*e^7 + 45*a^9*b^2*d^2*e^8 - 5*a^10*b*d*e^9 - a^11*e^10)*x^3 + 3*(2*a^2*b^9*
d^10 - 10*a^3*b^8*d^9*e + 15*a^4*b^7*d^8*e^2 + 7*a^5*b^6*d^7*e^3 - 49*a^6*b
^5*d^6*e^4 + 63*a^7*b^4*d^5*e^5 - 35*a^8*b^3*d^4*e^6 + 5*a^9*b^2*d^3*e^7 +
3*a^10*b*d^2*e^8 - a^11*d*e^9)*x^2 + (4*a^3*b^8*d^10 - 25*a^4*b^7*d^9*e + 6
3*a^5*b^6*d^8*e^2 - 77*a^6*b^5*d^7*e^3 + 35*a^7*b^4*d^6*e^4 + 21*a^8*b^3*d^
5*e^5 - 35*a^9*b^2*d^4*e^6 + 17*a^10*b*d^3*e^7 - 3*a^11*d^2*e^8)*x), -1/960
*(45045*(b^6*e^7*x^7 + a^4*b^2*d^3*e^4 + (3*b^6*d*e^6 + 4*a*b^5*e^7)*x^6 +
3*(b^6*d^2*e^5 + 4*a*b^5*d*e^6 + 2*a^2*b^4*e^7)*x^5 + (b^6*d^3*e^4 + 12*a*b
^5*d^2*e^5 + 18*a^2*b^4*d*e^6 + 4*a^3*b^3*e^7)*x^4 + (4*a*b^5*d^3*e^4 + 18*
a^2*b^4*d^2*e^5 + 12*a^3*b^3*d*e^6 + a^4*b^2*e^7)*x^3 + 3*(2*a^2*b^4*d^3*e^
4 + 4*a^3*b^3*d^2*e^5 + a^4*b^2*d*e^6)*x^2 + (4*a^3*b^3*d^3*e^4 + 3*a^4*b^2
*d^2*e^5)*x)*sqrt(-b/(b*d - a*e))*arctan(-(b*d - a*e)*sqrt(e*x + d)*sqrt(-b
/(b*d - a*e))/(b*e*x + b*d)) - (45045*b^6*e^6*x^6 - 240*b^6*d^6 + 1960*a*b^
5*d^5*e - 7630*a^2*b^4*d^4*e^2 + 22155*a^3*b^3*d^3*e^3 + 32384*a^4*b^2*d^2*
e^4 - 3968*a^5*b*d*e^5 + 384*a^6*e^6 + 15015*(7*b^6*d*e^5 + 11*a*b^5*e^6)*x
^5 + 3003*(23*b^6*d^2*e^4 + 129*a*b^5*d*e^5 + 73*a^2*b^4*e^6)*x^4 + 429*(15
*b^6*d^3*e^3 + 599*a*b^5*d^2*e^4 + 1207*a^2*b^4*d*e^5 + 279*a^3*b^3*e^6)*x^
3 - 143*(10*b^6*d^4*e^2 - 175*a*b^5*d^3*e^3 - 2433*a^2*b^4*d^2*e^4 - 1999*a
^3*b^3*d*e^5 - 128*a^4*b^2*e^6)*x^2 + 13*(40*b^6*d^5*e - 420*a*b^5*d^4*e^2
+ 2765*a^2*b^4*d^3*e^3 + 15077*a^3*b^3*d^2*e^4 + 3456*a^4*b^2*d*e^5 - 128*a
^5*b*e^6)*x)*sqrt(e*x + d))/(a^4*b^7*d^10 - 7*a^5*b^6*d^9*e + 21*a^6*b^5*d^
```

```
8*e^2 - 35*a^7*b^4*d^7*e^3 + 35*a^8*b^3*d^6*e^4 - 21*a^9*b^2*d^5*e^5 + 7*a^
10*b*d^4*e^6 - a^11*d^3*e^7 + (b^11*d^7*e^3 - 7*a*b^10*d^6*e^4 + 21*a^2*b^9
*d^5*e^5 - 35*a^3*b^8*d^4*e^6 + 35*a^4*b^7*d^3*e^7 - 21*a^5*b^6*d^2*e^8 + 7
*a^6*b^5*d*e^9 - a^7*b^4*e^10)*x^7 + (3*b^11*d^8*e^2 - 17*a*b^10*d^7*e^3 +
35*a^2*b^9*d^6*e^4 - 21*a^3*b^8*d^5*e^5 - 35*a^4*b^7*d^4*e^6 + 77*a^5*b^6*d
^3*e^7 - 63*a^6*b^5*d^2*e^8 + 25*a^7*b^4*d*e^9 - 4*a^8*b^3*e^10)*x^6 + 3*(b
^11*d^9*e - 3*a*b^10*d^8*e^2 - 5*a^2*b^9*d^7*e^3 + 35*a^3*b^8*d^6*e^4 - 63*
a^4*b^7*d^5*e^5 + 49*a^5*b^6*d^4*e^6 - 7*a^6*b^5*d^3*e^7 - 15*a^7*b^4*d^2*e
^8 + 10*a^8*b^3*d*e^9 - 2*a^9*b^2*e^10)*x^5 + (b^11*d^10 + 5*a*b^10*d^9*e -
 45*a^2*b^9*d^8*e^2 + 95*a^3*b^8*d^7*e^3 - 35*a^4*b^7*d^6*e^4 - 147*a^5*b^6
*d^5*e^5 + 245*a^6*b^5*d^4*e^6 - 155*a^7*b^4*d^3*e^7 + 30*a^8*b^3*d^2*e^8 +
 10*a^9*b^2*d*e^9 - 4*a^10*b*e^10)*x^4 + (4*a*b^10*d^10 - 10*a^2*b^9*d^9*e
- 30*a^3*b^8*d^8*e^2 + 155*a^4*b^7*d^7*e^3 - 245*a^5*b^6*d^6*e^4 + 147*a^6*
b^5*d^5*e^5 + 35*a^7*b^4*d^4*e^6 - 95*a^8*b^3*d^3*e^7 + 45*a^9*b^2*d^2*e^8
- 5*a^10*b*d*e^9 - a^11*e^10)*x^3 + 3*(2*a^2*b^9*d^10 - 10*a^3*b^8*d^9*e +
15*a^4*b^7*d^8*e^2 + 7*a^5*b^6*d^7*e^3 - 49*a^6*b^5*d^6*e^4 + 63*a^7*b^4*d^
5*e^5 - 35*a^8*b^3*d^4*e^6 + 5*a^9*b^2*d^3*e^7 + 3*a^10*b*d^2*e^8 - a^11*d*
e^9)*x^2 + (4*a^3*b^8*d^10 - 25*a^4*b^7*d^9*e + 63*a^5*b^6*d^8*e^2 - 77*a^6
*b^5*d^7*e^3 + 35*a^7*b^4*d^6*e^4 + 21*a^8*b^3*d^5*e^5 - 35*a^9*b^2*d^4*e^6
 + 17*a^10*b*d^3*e^7 - 3*a^11*d^2*e^8)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(7/2)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)

[Out] Timed out
```

Giac [B] time = 1.42333, size = 1504, normalized size = 3.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")

[Out] 3003/64*b^3*arctan(sqrt(x*e + d)*b/sqrt(-b^2*d + a*b*e))*e^4/((b^7*d^7*sgn(
(x*e + d)*b*e - b*d*e + a*e^2) - 7*a*b^6*d^6*e*sgn((x*e + d)*b*e - b*d*e +
a*e^2) + 21*a^2*b^5*d^5*e^2*sgn((x*e + d)*b*e - b*d*e + a*e^2) - 35*a^3*b^4
*d^4*e^3*sgn((x*e + d)*b*e - b*d*e + a*e^2) + 35*a^4*b^3*d^3*e^4*sgn((x*e +
 d)*b*e - b*d*e + a*e^2) - 21*a^5*b^2*d^2*e^5*sgn((x*e + d)*b*e - b*d*e + a
*e^2) + 7*a^6*b*d*e^6*sgn((x*e + d)*b*e - b*d*e + a*e^2) - a^7*e^7*sgn((x*e
 + d)*b*e - b*d*e + a*e^2))*sqrt(-b^2*d + a*b*e)) + 2/15*(225*(x*e + d)^2*b
^2*e^4 + 25*(x*e + d)*b^2*d*e^4 + 3*b^2*d^2*e^4 - 25*(x*e + d)*a*b*e^5 - 6*
a*b*d*e^5 + 3*a^2*e^6)/((b^7*d^7*sgn((x*e + d)*b*e - b*d*e + a*e^2) - 7*a*b
^6*d^6*e*sgn((x*e + d)*b*e - b*d*e + a*e^2) + 21*a^2*b^5*d^5*e^2*sgn((x*e +
 d)*b*e - b*d*e + a*e^2) - 35*a^3*b^4*d^4*e^3*sgn((x*e + d)*b*e - b*d*e + a
*e^2) + 35*a^4*b^3*d^3*e^4*sgn((x*e + d)*b*e - b*d*e + a*e^2) - 21*a^5*b^2*
d^2*e^5*sgn((x*e + d)*b*e - b*d*e + a*e^2) + 7*a^6*b*d*e^6*sgn((x*e + d)*b*
e - b*d*e + a*e^2) - a^7*e^7*sgn((x*e + d)*b*e - b*d*e + a*e^2))*(x*e + d)^
(5/2)) + 1/192*(3249*(x*e + d)^(7/2)*b^6*e^4 - 10633*(x*e + d)^(5/2)*b^6*d*
```

```
e^4 + 11767*(x*e + d)^(3/2)*b^6*d^2*e^4 - 4431*sqrt(x*e + d)*b^6*d^3*e^4 +
10633*(x*e + d)^(5/2)*a*b^5*e^5 - 23534*(x*e + d)^(3/2)*a*b^5*d*e^5 + 13293
*sqrt(x*e + d)*a*b^5*d^2*e^5 + 11767*(x*e + d)^(3/2)*a^2*b^4*e^6 - 13293*sq
rt(x*e + d)*a^2*b^4*d*e^6 + 4431*sqrt(x*e + d)*a^3*b^3*e^7)/((b^7*d^7*sgn((
x*e + d)*b*e - b*d*e + a*e^2) - 7*a*b^6*d^6*e*sgn((x*e + d)*b*e - b*d*e + a
*e^2) + 21*a^2*b^5*d^5*e^2*sgn((x*e + d)*b*e - b*d*e + a*e^2) - 35*a^3*b^4*
d^4*e^3*sgn((x*e + d)*b*e - b*d*e + a*e^2) + 35*a^4*b^3*d^3*e^4*sgn((x*e +
d)*b*e - b*d*e + a*e^2) - 21*a^5*b^2*d^2*e^5*sgn((x*e + d)*b*e - b*d*e + a*
e^2) + 7*a^6*b*d*e^6*sgn((x*e + d)*b*e - b*d*e + a*e^2) - a^7*e^7*sgn((x*e
+ d)*b*e - b*d*e + a*e^2))*((x*e + d)*b - b*d + a*e)^4)
```

3.1731 $\int (d+ex)^m \left(a^2+2abx+b^2x^2\right)^3 \, dx$

Optimal. Leaf size=206

$$\frac{15b^2(bd-ae)^4(d+ex)^{m+3}}{e^7(m+3)}-\frac{20b^3(bd-ae)^3(d+ex)^{m+4}}{e^7(m+4)}+\frac{15b^4(bd-ae)^2(d+ex)^{m+5}}{e^7(m+5)}-\frac{6b^5(bd-ae)(d+ex)^{m+6}}{e^7(m+6)}+\frac{(bd-\cdots}{\cdots}$$

```
[Out] ((b*d - a*e)^6*(d + e*x)^(1 + m))/(e^7*(1 + m)) - (6*b*(b*d - a*e)^5*(d + e
*x)^(2 + m))/(e^7*(2 + m)) + (15*b^2*(b*d - a*e)^4*(d + e*x)^(3 + m))/(e^7*
(3 + m)) - (20*b^3*(b*d - a*e)^3*(d + e*x)^(4 + m))/(e^7*(4 + m)) + (15*b^4
*(b*d - a*e)^2*(d + e*x)^(5 + m))/(e^7*(5 + m)) - (6*b^5*(b*d - a*e)*(d + e
*x)^(6 + m))/(e^7*(6 + m)) + (b^6*(d + e*x)^(7 + m))/(e^7*(7 + m))
```

Rubi [A] time = 0.109974, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$\frac{15b^2(bd-ae)^4(d+ex)^{m+3}}{e^7(m+3)}-\frac{20b^3(bd-ae)^3(d+ex)^{m+4}}{e^7(m+4)}+\frac{15b^4(bd-ae)^2(d+ex)^{m+5}}{e^7(m+5)}-\frac{6b^5(bd-ae)(d+ex)^{m+6}}{e^7(m+6)}+\frac{(bd-\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] ((b*d - a*e)^6*(d + e*x)^(1 + m))/(e^7*(1 + m)) - (6*b*(b*d - a*e)^5*(d + e
*x)^(2 + m))/(e^7*(2 + m)) + (15*b^2*(b*d - a*e)^4*(d + e*x)^(3 + m))/(e^7*
(3 + m)) - (20*b^3*(b*d - a*e)^3*(d + e*x)^(4 + m))/(e^7*(4 + m)) + (15*b^4
*(b*d - a*e)^2*(d + e*x)^(5 + m))/(e^7*(5 + m)) - (6*b^5*(b*d - a*e)*(d + e
*x)^(6 + m))/(e^7*(6 + m)) + (b^6*(d + e*x)^(7 + m))/(e^7*(7 + m))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^m \left(a^2+2abx+b^2x^2\right)^3 \, dx &= \int (a+bx)^6(d+ex)^m \, dx\\ &= \int \left(\frac{(-bd+ae)^6(d+ex)^m}{e^6}-\frac{6b(bd-ae)^5(d+ex)^{1+m}}{e^6}+\frac{15b^2(bd-ae)^4(d+ex)^{2+m}}{e^6} \cdots\right.\\ &= \frac{(bd-ae)^6(d+ex)^{1+m}}{e^7(1+m)}-\frac{6b(bd-ae)^5(d+ex)^{2+m}}{e^7(2+m)}+\frac{15b^2(bd-ae)^4(d+ex)^{3+m}}{e^7(3+m)}-2\cdots\end{aligned}$$

Mathematica [A] time = 0.179302, size = 175, normalized size = 0.85

$$\frac{(d+ex)^{m+1}\left(\frac{15b^2(d+ex)^2(bd-ae)^4}{m+3}-\frac{20b^3(d+ex)^3(bd-ae)^3}{m+4}+\frac{15b^4(d+ex)^4(bd-ae)^2}{m+5}-\frac{6b^5(d+ex)^5(bd-ae)}{m+6}-\frac{6b(d+ex)(bd-ae)^5}{m+2}+\frac{(bd-ae)^6}{m+1}+\frac{b^6(d+ex)^6}{m+7}\right)}{e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] ((d + e*x)^(1 + m)*((b*d - a*e)^6/(1 + m) - (6*b*(b*d - a*e)^5*(d + e*x))/(
2 + m) + (15*b^2*(b*d - a*e)^4*(d + e*x)^2)/(3 + m) - (20*b^3*(b*d - a*e)^3
*(d + e*x)^3)/(4 + m) + (15*b^4*(b*d - a*e)^2*(d + e*x)^4)/(5 + m) - (6*b^5
*(b*d - a*e)*(d + e*x)^5)/(6 + m) + (b^6*(d + e*x)^6)/(7 + m)))/e^7
```

Maple [B] time = 0.053, size = 2157, normalized size = 10.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] (e*x+d)^(1+m)*(b^6*e^6*m^6*x^6+6*a*b^5*e^6*m^6*x^5+21*b^6*e^6*m^5*x^6+15*a^
2*b^4*e^6*m^6*x^4+132*a*b^5*e^6*m^5*x^5-6*b^6*d*e^5*m^5*x^5+175*b^6*e^6*m^4
*x^6+20*a^3*b^3*e^6*m^6*x^3+345*a^2*b^4*e^6*m^5*x^4-30*a*b^5*d*e^5*m^5*x^4+
1140*a*b^5*e^6*m^4*x^5-90*b^6*d*e^5*m^4*x^5+735*b^6*e^6*m^3*x^6+15*a^4*b^2*
e^6*m^6*x^2+480*a^3*b^3*e^6*m^5*x^3-60*a^2*b^4*d*e^5*m^5*x^3+3105*a^2*b^4*e
^6*m^4*x^4-510*a*b^5*d*e^5*m^4*x^4+4920*a*b^5*e^6*m^3*x^5+30*b^6*d^2*e^4*m^
4*x^4-510*b^6*d*e^5*m^3*x^5+1624*b^6*e^6*m^2*x^6+6*a^5*b*e^6*m^6*x+375*a^4*
b^2*e^6*m^5*x^2-60*a^3*b^3*d*e^5*m^5*x^2+4520*a^3*b^3*e^6*m^4*x^3-1140*a^2*
b^4*d*e^5*m^4*x^3+13875*a^2*b^4*e^6*m^3*x^4+120*a*b^5*d^2*e^4*m^4*x^3-3150*
a*b^5*d*e^5*m^3*x^4+11094*a*b^5*e^6*m^2*x^5+300*b^6*d^2*e^4*m^3*x^4-1350*b^
6*d*e^5*m^2*x^5+1764*b^6*e^6*m*x^6+a^6*e^6*m^6+156*a^5*b*e^6*m^5*x-30*a^4*b
^2*d*e^5*m^5*x+3705*a^4*b^2*e^6*m^4*x^2-1260*a^3*b^3*d*e^5*m^4*x^2+21120*a^
3*b^3*e^6*m^3*x^3+180*a^2*b^4*d^2*e^4*m^4*x^2-7860*a^2*b^4*d*e^5*m^3*x^3+32
160*a^2*b^4*e^6*m^2*x^4+1560*a*b^5*d^2*e^4*m^3*x^3-8850*a*b^5*d*e^5*m^2*x^4
+12228*a*b^5*e^6*m*x^5-120*b^6*d^3*e^3*m^3*x^3+1050*b^6*d^2*e^4*m^2*x^4-164
4*b^6*d*e^5*m*x^5+720*b^6*e^6*x^6+27*a^6*e^6*m^5-6*a^5*b*d*e^5*m^5+1620*a^5
*b*e^6*m^4*x-690*a^4*b^2*d*e^5*m^4*x+18285*a^4*b^2*e^6*m^3*x^2+120*a^3*b^3*
d^2*e^4*m^4*x-9780*a^3*b^3*d*e^5*m^3*x^2+50900*a^3*b^3*e^6*m^2*x^3+2880*a^2
*b^4*d^2*e^4*m^3*x^2-24060*a^2*b^4*d*e^5*m^2*x^3+36180*a^2*b^4*e^6*m*x^4-36
0*a*b^5*d^3*e^3*m^3*x^2+6360*a*b^5*d^2*e^4*m^2*x^3-11220*a*b^5*d*e^5*m*x^4+
5040*a*b^5*e^6*x^5-720*b^6*d^3*e^3*m^2*x^3+1500*b^6*d^2*e^4*m*x^4-720*b^6*d
*e^5*x^5+295*a^6*e^6*m^4-150*a^5*b*d*e^5*m^4+8520*a^5*b*e^6*m^3*x+30*a^4*b^
2*d^2*e^4*m^4-6030*a^4*b^2*d*e^5*m^3*x+46680*a^4*b^2*e^6*m^2*x^2+2280*a^3*b
^3*d^2*e^4*m^3*x-34020*a^3*b^3*d*e^5*m^2*x^2+59040*a^3*b^3*e^6*m*x^3-360*a^
2*b^4*d^3*e^3*m^3*x+14940*a^2*b^4*d^2*e^4*m^2*x^2-32400*a^2*b^4*d*e^5*m*x^3
+15120*a^2*b^4*e^6*x^4-3600*a*b^5*d^3*e^3*m^2*x^2+9960*a*b^5*d^2*e^4*m*x^3-
5040*a*b^5*d*e^5*x^4+360*b^6*d^4*e^2*m^2*x^2-1320*b^6*d^3*e^3*m*x^3+720*b^6
*d^2*e^4*x^4+1665*a^6*e^6*m^3-1470*a^5*b*d*e^5*m^3+23574*a^5*b*e^6*m^2*x+66
0*a^4*b^2*d^2*e^4*m^3-24510*a^4*b^2*d*e^5*m^2*x+56940*a^4*b^2*e^6*m*x^2-120
*a^3*b^3*d^3*e^3*m^3+15000*a^3*b^3*d^2*e^4*m^2*x-50640*a^3*b^3*d*e^5*m*x^2+
25200*a^3*b^3*e^6*x^3-5040*a^2*b^4*d^3*e^3*m^2*x+27360*a^2*b^4*d^2*e^4*m*x^
2-15120*a^2*b^4*d*e^5*x^3+720*a*b^5*d^4*e^2*m^2*x-8280*a*b^5*d^3*e^3*m*x^2+
5040*a*b^5*d^2*e^4*x^3+1080*b^6*d^4*e^2*m*x^2-720*b^6*d^3*e^3*x^3+5104*a^6*
e^6*m^2-7050*a^5*b*d*e^5*m^2+31644*a^5*b*e^6*m*x+5370*a^4*b^2*d^2*e^4*m^2-4
```

```
4340*a^4*b^2*d*e^5*m*x+25200*a^4*b^2*e^6*x^2-2160*a^3*b^3*d^3*e^3*m^2+38040
*a^3*b^3*d^2*e^4*m*x-25200*a^3*b^3*d*e^5*x^2+360*a^2*b^4*d^4*e^2*m^2-19800*
a^2*b^4*d^3*e^3*m*x+15120*a^2*b^4*d^2*e^4*x^2+5760*a*b^5*d^4*e^2*m*x-5040*a
*b^5*d^3*e^3*x^2-720*b^6*d^5*e*m*x+720*b^6*d^4*e^2*x^2+8028*a^6*e^6*m-16524
*a^5*b*d*e^5*m+15120*a^5*b*e^6*x+19140*a^4*b^2*d^2*e^4*m-25200*a^4*b^2*d*e^
5*x-12840*a^3*b^3*d^3*e^3*m+25200*a^3*b^3*d^2*e^4*x+4680*a^2*b^4*d^4*e^2*m-
15120*a^2*b^4*d^3*e^3*x-720*a*b^5*d^5*e*m+5040*a*b^5*d^4*e^2*x-720*b^6*d^5*
e*x+5040*a^6*e^6-15120*a^5*b*d*e^5+25200*a^4*b^2*d^2*e^4-25200*a^3*b^3*d^3*
e^3+15120*a^2*b^4*d^4*e^2-5040*a*b^5*d^5*e+720*b^6*d^6)/e^7/(m^7+28*m^6+322
*m^5+1960*m^4+6769*m^3+13132*m^2+13068*m+5040)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.89748, size = 4810, normalized size = 23.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] (a^6*d*e^6*m^6 + 720*b^6*d^7 - 5040*a*b^5*d^6*e + 15120*a^2*b^4*d^5*e^2 - 2
5200*a^3*b^3*d^4*e^3 + 25200*a^4*b^2*d^3*e^4 - 15120*a^5*b*d^2*e^5 + 5040*a
^6*d*e^6 + (b^6*e^7*m^6 + 21*b^6*e^7*m^5 + 175*b^6*e^7*m^4 + 735*b^6*e^7*m^
3 + 1624*b^6*e^7*m^2 + 1764*b^6*e^7*m + 720*b^6*e^7)*x^7 + (5040*a*b^5*e^7
+ (b^6*d*e^6 + 6*a*b^5*e^7)*m^6 + 3*(5*b^6*d*e^6 + 44*a*b^5*e^7)*m^5 + 5*(1
7*b^6*d*e^6 + 228*a*b^5*e^7)*m^4 + 15*(15*b^6*d*e^6 + 328*a*b^5*e^7)*m^3 +
2*(137*b^6*d*e^6 + 5547*a*b^5*e^7)*m^2 + 12*(10*b^6*d*e^6 + 1019*a*b^5*e^7)
*m)*x^6 - 3*(2*a^5*b*d^2*e^5 - 9*a^6*d*e^6)*m^5 + 3*(5040*a^2*b^4*e^7 + (2*
a*b^5*d*e^6 + 5*a^2*b^4*e^7)*m^6 - (2*b^6*d^2*e^5 - 34*a*b^5*d*e^6 - 115*a^
2*b^4*e^7)*m^5 - 5*(4*b^6*d^2*e^5 - 42*a*b^5*d*e^6 - 207*a^2*b^4*e^7)*m^4 -
 5*(14*b^6*d^2*e^5 - 118*a*b^5*d*e^6 - 925*a^2*b^4*e^7)*m^3 - 4*(25*b^6*d^2
*e^5 - 187*a*b^5*d*e^6 - 2680*a^2*b^4*e^7)*m^2 - 12*(4*b^6*d^2*e^5 - 28*a*b
^5*d*e^6 - 1005*a^2*b^4*e^7)*m)*x^5 + 5*(6*a^4*b^2*d^3*e^4 - 30*a^5*b*d^2*e
^5 + 59*a^6*d*e^6)*m^4 + 5*(5040*a^3*b^3*e^7 + (3*a^2*b^4*d*e^6 + 4*a^3*b^3
*e^7)*m^6 - 3*(2*a*b^5*d^2*e^5 - 19*a^2*b^4*d*e^6 - 32*a^3*b^3*e^7)*m^5 + (
6*b^6*d^3*e^4 - 78*a*b^5*d^2*e^5 + 393*a^2*b^4*d*e^6 + 904*a^3*b^3*e^7)*m^4
 + 3*(12*b^6*d^3*e^4 - 106*a*b^5*d^2*e^5 + 401*a^2*b^4*d*e^6 + 1408*a^3*b^3
*e^7)*m^3 + 2*(33*b^6*d^3*e^4 - 249*a*b^5*d^2*e^5 + 810*a^2*b^4*d*e^6 + 509
0*a^3*b^3*e^7)*m^2 + 36*(b^6*d^3*e^4 - 7*a*b^5*d^2*e^5 + 21*a^2*b^4*d*e^6 +
 328*a^3*b^3*e^7)*m)*x^4 - 15*(8*a^3*b^3*d^4*e^3 - 44*a^4*b^2*d^3*e^4 + 98*
a^5*b*d^2*e^5 - 111*a^6*d*e^6)*m^3 + 5*(5040*a^4*b^2*e^7 + (4*a^3*b^3*d*e^6
 + 3*a^4*b^2*e^7)*m^6 - 3*(4*a^2*b^4*d^2*e^5 - 28*a^3*b^3*d*e^6 - 25*a^4*b^
2*e^7)*m^5 + (24*a*b^5*d^3*e^4 - 192*a^2*b^4*d^2*e^5 + 652*a^3*b^3*d*e^6 +
741*a^4*b^2*e^7)*m^4 - 3*(8*b^6*d^4*e^3 - 80*a*b^5*d^3*e^4 + 332*a^2*b^4*d^
2*e^5 - 756*a^3*b^3*d*e^6 - 1219*a^4*b^2*e^7)*m^3 - 8*(9*b^6*d^4*e^3 - 69*a
*b^5*d^3*e^4 + 228*a^2*b^4*d^2*e^5 - 422*a^3*b^3*d*e^6 - 1167*a^4*b^2*e^7)*
```

```
m^2 - 12*(4*b^6*d^4*e^3 - 28*a*b^5*d^3*e^4 + 84*a^2*b^4*d^2*e^5 - 140*a^3*b
^3*d*e^6 - 949*a^4*b^2*e^7)*m)*x^3 + 2*(180*a^2*b^4*d^5*e^2 - 1080*a^3*b^3*
d^4*e^3 + 2685*a^4*b^2*d^3*e^4 - 3525*a^5*b*d^2*e^5 + 2552*a^6*d*e^6)*m^2 +
 3*(5040*a^5*b*e^7 + (5*a^4*b^2*d*e^6 + 2*a^5*b*e^7)*m^6 - (20*a^3*b^3*d^2*
e^5 - 115*a^4*b^2*d*e^6 - 52*a^5*b*e^7)*m^5 + 5*(12*a^2*b^4*d^3*e^4 - 76*a^
3*b^3*d^2*e^5 + 201*a^4*b^2*d*e^6 + 108*a^5*b*e^7)*m^4 - 5*(24*a*b^5*d^4*e^
3 - 168*a^2*b^4*d^3*e^4 + 500*a^3*b^3*d^2*e^5 - 817*a^4*b^2*d*e^6 - 568*a^5
*b*e^7)*m^3 + 2*(60*b^6*d^5*e^2 - 480*a*b^5*d^4*e^3 + 1650*a^2*b^4*d^3*e^4
- 3170*a^3*b^3*d^2*e^5 + 3695*a^4*b^2*d*e^6 + 3929*a^5*b*e^7)*m^2 + 12*(10*
b^6*d^5*e^2 - 70*a*b^5*d^4*e^3 + 210*a^2*b^4*d^3*e^4 - 350*a^3*b^3*d^2*e^5
+ 350*a^4*b^2*d*e^6 + 879*a^5*b*e^7)*m)*x^2 - 12*(60*a*b^5*d^6*e - 390*a^2*
b^4*d^5*e^2 + 1070*a^3*b^3*d^4*e^3 - 1595*a^4*b^2*d^3*e^4 + 1377*a^5*b*d^2*
e^5 - 669*a^6*d*e^6)*m + (5040*a^6*e^7 + (6*a^5*b*d*e^6 + a^6*e^7)*m^6 - 3*
(10*a^4*b^2*d^2*e^5 - 50*a^5*b*d*e^6 - 9*a^6*e^7)*m^5 + 5*(24*a^3*b^3*d^3*e
^4 - 132*a^4*b^2*d^2*e^5 + 294*a^5*b*d*e^6 + 59*a^6*e^7)*m^4 - 15*(24*a^2*b
^4*d^4*e^3 - 144*a^3*b^3*d^3*e^4 + 358*a^4*b^2*d^2*e^5 - 470*a^5*b*d*e^6 -
111*a^6*e^7)*m^3 + 4*(180*a*b^5*d^5*e^2 - 1170*a^2*b^4*d^4*e^3 + 3210*a^3*b
^3*d^3*e^4 - 4785*a^4*b^2*d^2*e^5 + 4131*a^5*b*d*e^6 + 1276*a^6*e^7)*m^2 -
36*(20*b^6*d^6*e - 140*a*b^5*d^5*e^2 + 420*a^2*b^4*d^4*e^3 - 700*a^3*b^3*d^
3*e^4 + 700*a^4*b^2*d^2*e^5 - 420*a^5*b*d*e^6 - 223*a^6*e^7)*m)*x)*(e*x + d
)^m/(e^7*m^7 + 28*e^7*m^6 + 322*e^7*m^5 + 1960*e^7*m^4 + 6769*e^7*m^3 + 131
32*e^7*m^2 + 13068*e^7*m + 5040*e^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(b**2*x**2+2*a*b*x+a**2)**3,x)

[Out] Timed out
```

Giac [B] time = 1.23667, size = 5234, normalized size = 25.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")

[Out] ((x*e + d)^m*b^6*m^6*x^7*e^7 + (x*e + d)^m*b^6*d*m^6*x^6*e^6 + 6*(x*e + d)^
m*a*b^5*m^6*x^6*e^7 + 21*(x*e + d)^m*b^6*m^5*x^7*e^7 + 6*(x*e + d)^m*a*b^5*
d*m^6*x^5*e^6 + 15*(x*e + d)^m*b^6*d*m^5*x^6*e^6 - 6*(x*e + d)^m*b^6*d^2*m^
5*x^5*e^5 + 15*(x*e + d)^m*a^2*b^4*m^6*x^5*e^7 + 132*(x*e + d)^m*a*b^5*m^5*
x^6*e^7 + 175*(x*e + d)^m*b^6*m^4*x^7*e^7 + 15*(x*e + d)^m*a^2*b^4*d*m^6*x^
4*e^6 + 102*(x*e + d)^m*a*b^5*d*m^5*x^5*e^6 + 85*(x*e + d)^m*b^6*d*m^4*x^6*
e^6 - 30*(x*e + d)^m*a*b^5*d^2*m^5*x^4*e^5 - 60*(x*e + d)^m*b^6*d^2*m^4*x^5
*e^5 + 30*(x*e + d)^m*b^6*d^3*m^4*x^4*e^4 + 20*(x*e + d)^m*a^3*b^3*m^6*x^4*
e^7 + 345*(x*e + d)^m*a^2*b^4*m^5*x^5*e^7 + 1140*(x*e + d)^m*a*b^5*m^4*x^6*
e^7 + 735*(x*e + d)^m*b^6*m^3*x^7*e^7 + 20*(x*e + d)^m*a^3*b^3*d*m^6*x^3*e^
6 + 285*(x*e + d)^m*a^2*b^4*d*m^5*x^4*e^6 + 630*(x*e + d)^m*a*b^5*d*m^4*x^5
*e^6 + 225*(x*e + d)^m*b^6*d*m^3*x^6*e^6 - 60*(x*e + d)^m*a^2*b^4*d^2*m^5*x
^3*e^5 - 390*(x*e + d)^m*a*b^5*d^2*m^4*x^4*e^5 - 210*(x*e + d)^m*b^6*d^2*m^
```

3*x^5*e^5 + 120*(x*e + d)^m*a*b^5*d^3*m^4*x^3*e^4 + 180*(x*e + d)^m*b^6*d^3*m^3*x^4*e^4 - 120*(x*e + d)^m*b^6*d^4*m^3*x^3*e^3 + 15*(x*e + d)^m*a^4*b^2*m^6*x^3*e^7 + 480*(x*e + d)^m*a^3*b^3*m^5*x^4*e^7 + 3105*(x*e + d)^m*a^2*b^4*m^4*x^5*e^7 + 4920*(x*e + d)^m*a*b^5*m^3*x^6*e^7 + 1624*(x*e + d)^m*b^6*m^2*x^7*e^7 + 15*(x*e + d)^m*a^4*b^2*d*m^6*x^2*e^6 + 420*(x*e + d)^m*a^3*b^3*d*m^5*x^3*e^6 + 1965*(x*e + d)^m*a^2*b^4*d*m^4*x^4*e^6 + 1770*(x*e + d)^m*a*b^5*d*m^3*x^5*e^6 + 274*(x*e + d)^m*b^6*d*m^2*x^6*e^6 - 60*(x*e + d)^m*a^3*b^3*d^2*m^5*x^2*e^5 - 960*(x*e + d)^m*a^2*b^4*d^2*m^4*x^3*e^5 - 1590*(x*e + d)^m*a*b^5*d^2*m^3*x^4*e^5 - 300*(x*e + d)^m*b^6*d^2*m^2*x^5*e^5 + 180*(x*e + d)^m*a^2*b^4*d^3*m^4*x^2*e^4 + 1200*(x*e + d)^m*a*b^5*d^3*m^3*x^3*e^4 + 330*(x*e + d)^m*b^6*d^3*m^2*x^4*e^4 - 360*(x*e + d)^m*a*b^5*d^4*m^3*x^2*e^3 - 360*(x*e + d)^m*b^6*d^4*m^2*x^3*e^3 + 360*(x*e + d)^m*b^6*d^5*m^2*x^2*e^2 + 6*(x*e + d)^m*a^5*b*m^6*x^2*e^7 + 375*(x*e + d)^m*a^4*b^2*m^5*x^3*e^7 + 4520*(x*e + d)^m*a^3*b^3*m^4*x^4*e^7 + 13875*(x*e + d)^m*a^2*b^4*m^3*x^5*e^7 + 11094*(x*e + d)^m*a*b^5*m^2*x^6*e^7 + 1764*(x*e + d)^m*b^6*m*x^7*e^7 + 6*(x*e + d)^m*a^5*b*d*m^6*x*e^6 + 345*(x*e + d)^m*a^4*b^2*d*m^5*x^2*e^6 + 3260*(x*e + d)^m*a^3*b^3*d*m^4*x^3*e^6 + 6015*(x*e + d)^m*a^2*b^4*d*m^3*x^4*e^6 + 2244*(x*e + d)^m*a*b^5*d*m^2*x^5*e^6 + 120*(x*e + d)^m*b^6*d*m*x^6*e^6 - 30*(x*e + d)^m*a^4*b^2*d^2*m^5*x*e^5 - 1140*(x*e + d)^m*a^3*b^3*d^2*m^4*x^2*e^5 - 4980*(x*e + d)^m*a^2*b^4*d^2*m^3*x^3*e^5 - 2490*(x*e + d)^m*a*b^5*d^2*m^2*x^4*e^5 - 144*(x*e + d)^m*b^6*d^2*m*x^5*e^5 + 120*(x*e + d)^m*a^3*b^3*d^3*m^4*x*e^4 + 2520*(x*e + d)^m*a^2*b^4*d^3*m^3*x^2*e^4 + 2760*(x*e + d)^m*a*b^5*d^3*m^2*x^3*e^4 + 180*(x*e + d)^m*b^6*d^3*m*x^4*e^4 - 360*(x*e + d)^m*a^2*b^4*d^4*m^3*x*e^3 - 2880*(x*e + d)^m*a*b^5*d^4*m^2*x^2*e^3 - 240*(x*e + d)^m*b^6*d^4*m*x^3*e^3 + 720*(x*e + d)^m*a*b^5*d^5*m^2*x*e^2 + 360*(x*e + d)^m*b^6*d^5*m*x^2*e^2 - 720*(x*e + d)^m*b^6*d^6*m*x*e + (x*e + d)^m*a^6*m^6*x*e^7 + 156*(x*e + d)^m*a^5*b*m^5*x^2*e^7 + 3705*(x*e + d)^m*a^4*b^2*m^4*x^3*e^7 + 21120*(x*e + d)^m*a^3*b^3*m^3*x^4*e^7 + 32160*(x*e + d)^m*a^2*b^4*m^2*x^5*e^7 + 12228*(x*e + d)^m*a*b^5*m*x^6*e^7 + 720*(x*e + d)^m*b^6*x^7*e^7 + (x*e + d)^m*a^6*d*m^6*e^6 + 150*(x*e + d)^m*a^5*b*d*m^5*x*e^6 + 3015*(x*e + d)^m*a^4*b^2*d*m^4*x^2*e^6 + 11340*(x*e + d)^m*a^3*b^3*d*m^3*x^3*e^6 + 8100*(x*e + d)^m*a^2*b^4*d*m^2*x^4*e^6 + 1008*(x*e + d)^m*a*b^5*d*m*x^5*e^6 - 6*(x*e + d)^m*a^5*b*d^2*m^5*e^5 - 660*(x*e + d)^m*a^4*b^2*d^2*m^4*x*e^5 - 7500*(x*e + d)^m*a^3*b^3*d^2*m^3*x^2*e^5 - 9120*(x*e + d)^m*a^2*b^4*d^2*m^2*x^3*e^5 - 1260*(x*e + d)^m*a*b^5*d^2*m*x^4*e^5 + 30*(x*e + d)^m*a^4*b^2*d^3*m^4*e^4 + 2160*(x*e + d)^m*a^3*b^3*d^3*m^3*x*e^4 + 9900*(x*e + d)^m*a^2*b^4*d^3*m^2*x^2*e^4 + 1680*(x*e + d)^m*a*b^5*d^3*m*x^3*e^4 - 120*(x*e + d)^m*a^3*b^3*d^4*m^3*e^3 - 4680*(x*e + d)^m*a^2*b^4*d^4*m^2*x*e^3 - 2520*(x*e + d)^m*a*b^5*d^4*m*x^2*e^3 + 360*(x*e + d)^m*a^2*b^4*d^5*m^2*e^2 + 5040*(x*e + d)^m*a*b^5*d^5*m*x*e^2 - 720*(x*e + d)^m*a*b^5*d^6*m*e + 720*(x*e + d)^m*b^6*d^7 + 27*(x*e + d)^m*a^6*m^5*x*e^7 + 1620*(x*e + d)^m*a^5*b*m^4*x^2*e^7 + 18285*(x*e + d)^m*a^4*b^2*m^3*x^3*e^7 + 50900*(x*e + d)^m*a^3*b^3*m^2*x^4*e^7 + 36180*(x*e + d)^m*a^2*b^4*m*x^5*e^7 + 5040*(x*e + d)^m*a*b^5*x^6*e^7 + 27*(x*e + d)^m*a^6*d*m^5*e^6 + 1470*(x*e + d)^m*a^5*b*d*m^4*x*e^6 + 12255*(x*e + d)^m*a^4*b^2*d*m^3*x^2*e^6 + 16880*(x*e + d)^m*a^3*b^3*d*m^2*x^3*e^6 + 3780*(x*e + d)^m*a^2*b^4*d*m*x^4*e^6 - 150*(x*e + d)^m*a^5*b*d^2*m^4*e^5 - 5370*(x*e + d)^m*a^4*b^2*d^2*m^3*x*e^5 - 19020*(x*e + d)^m*a^3*b^3*d^2*m^2*x^2*e^5 - 5040*(x*e + d)^m*a^2*b^4*d^2*m*x^3*e^5 + 660*(x*e + d)^m*a^4*b^2*d^3*m^3*e^4 + 12840*(x*e + d)^m*a^3*b^3*d^3*m^2*x*e^4 + 7560*(x*e + d)^m*a^2*b^4*d^3*m*x^2*e^4 - 2160*(x*e + d)^m*a^3*b^3*d^4*m^2*e^3 - 15120*(x*e + d)^m*a^2*b^4*d^4*m*x*e^3 + 4680*(x*e + d)^m*a^2*b^4*d^5*m*e^2 - 5040*(x*e + d)^m*a*b^5*d^6*e + 295*(x*e + d)^m*a^6*m^4*x*e^7 + 8520*(x*e + d)^m*a^5*b*m^3*x^2*e^7 + 46680*(x*e + d)^m*a^4*b^2*m^2*x^3*e^7 + 59040*(x*e + d)^m*a^3*b^3*m*x^4*e^7 + 15120*(x*e + d)^m*a^2*b^4*x^5*e^7 + 295*(x*e + d)^m*a^6*d*m^4*e^6 + 7050*(x*e + d)^m*a^5*b*d*m^3*x*e^6 + 22170*(x*e + d)^m*a^4*b^2*d*m^2*x^2*e^6 + 8400*(x*e + d)^m*a^3*b^3*d*m*x^3*e^6 - 1470*(x*e + d)^m*a^5*b*d^2*m^3*e^5 - 19140*(x*e + d)^m*a^4*b^2*d^2*m^2*x*e^5 - 12600*(x*e + d)^m*a^3*b^3*d^2*m*x^2*e^5 + 5370*(x*e + d)^m*a^4*b^2*d^3*m^2*e^4 + 25200*(x*e + d)^m*a^3*b^3*d^3*m*x*e^4 - 12840*(x*e + d)^m*a^3*b^

```
3*d^4*m*e^3 + 15120*(x*e + d)^m*a^2*b^4*d^5*e^2 + 1665*(x*e + d)^m*a^6*m^3*
x*e^7 + 23574*(x*e + d)^m*a^5*b*m^2*x^2*e^7 + 56940*(x*e + d)^m*a^4*b^2*m*x
^3*e^7 + 25200*(x*e + d)^m*a^3*b^3*x^4*e^7 + 1665*(x*e + d)^m*a^6*d*m^3*e^6
 + 16524*(x*e + d)^m*a^5*b*d*m^2*x*e^6 + 12600*(x*e + d)^m*a^4*b^2*d*m*x^2*
e^6 - 7050*(x*e + d)^m*a^5*b*d^2*m^2*e^5 - 25200*(x*e + d)^m*a^4*b^2*d^2*m*
x*e^5 + 19140*(x*e + d)^m*a^4*b^2*d^3*m*e^4 - 25200*(x*e + d)^m*a^3*b^3*d^4
*e^3 + 5104*(x*e + d)^m*a^6*m^2*x*e^7 + 31644*(x*e + d)^m*a^5*b*m*x^2*e^7 +
 25200*(x*e + d)^m*a^4*b^2*x^3*e^7 + 5104*(x*e + d)^m*a^6*d*m^2*e^6 + 15120
*(x*e + d)^m*a^5*b*d*m*x*e^6 - 16524*(x*e + d)^m*a^5*b*d^2*m*e^5 + 25200*(x
*e + d)^m*a^4*b^2*d^3*e^4 + 8028*(x*e + d)^m*a^6*m*x*e^7 + 15120*(x*e + d)^
m*a^5*b*x^2*e^7 + 8028*(x*e + d)^m*a^6*d*m*e^6 - 15120*(x*e + d)^m*a^5*b*d^
2*e^5 + 5040*(x*e + d)^m*a^6*x*e^7 + 5040*(x*e + d)^m*a^6*d*e^6)/(m^7*e^7 +
 28*m^6*e^7 + 322*m^5*e^7 + 1960*m^4*e^7 + 6769*m^3*e^7 + 13132*m^2*e^7 + 1
3068*m*e^7 + 5040*e^7)
```

3.1732 $\int (d+ex)^m \left(a^2+2abx+b^2x^2\right)^2 \, dx$

Optimal. Leaf size=142

$$\frac{6b^2(bd-ae)^2(d+ex)^{m+3}}{e^5(m+3)}-\frac{4b^3(bd-ae)(d+ex)^{m+4}}{e^5(m+4)}+\frac{(bd-ae)^4(d+ex)^{m+1}}{e^5(m+1)}-\frac{4b(bd-ae)^3(d+ex)^{m+2}}{e^5(m+2)}+\frac{b^4(d+ex)^{m+5}}{e^5(m+5)}$$

```
[Out] ((b*d - a*e)^4*(d + e*x)^(1 + m))/(e^5*(1 + m)) - (4*b*(b*d - a*e)^3*(d + e
*x)^(2 + m))/(e^5*(2 + m)) + (6*b^2*(b*d - a*e)^2*(d + e*x)^(3 + m))/(e^5*(
3 + m)) - (4*b^3*(b*d - a*e)*(d + e*x)^(4 + m))/(e^5*(4 + m)) + (b^4*(d + e
*x)^(5 + m))/(e^5*(5 + m))
```

Rubi [A] time = 0.0673952, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 43}

$$\frac{6b^2(bd-ae)^2(d+ex)^{m+3}}{e^5(m+3)}-\frac{4b^3(bd-ae)(d+ex)^{m+4}}{e^5(m+4)}+\frac{(bd-ae)^4(d+ex)^{m+1}}{e^5(m+1)}-\frac{4b(bd-ae)^3(d+ex)^{m+2}}{e^5(m+2)}+\frac{b^4(d+ex)^{m+5}}{e^5(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] ((b*d - a*e)^4*(d + e*x)^(1 + m))/(e^5*(1 + m)) - (4*b*(b*d - a*e)^3*(d + e
*x)^(2 + m))/(e^5*(2 + m)) + (6*b^2*(b*d - a*e)^2*(d + e*x)^(3 + m))/(e^5*(
3 + m)) - (4*b^3*(b*d - a*e)*(d + e*x)^(4 + m))/(e^5*(4 + m)) + (b^4*(d + e
*x)^(5 + m))/(e^5*(5 + m))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^m \left(a^2+2abx+b^2x^2\right)^2 \, dx &= \int (a+bx)^4(d+ex)^m \, dx\\ &= \int \left(\frac{(-bd+ae)^4(d+ex)^m}{e^4}-\frac{4b(bd-ae)^3(d+ex)^{1+m}}{e^4}+\frac{6b^2(bd-ae)^2(d+ex)^{2+m}}{e^4}-\right.\\ &= \frac{(bd-ae)^4(d+ex)^{1+m}}{e^5(1+m)}-\frac{4b(bd-ae)^3(d+ex)^{2+m}}{e^5(2+m)}+\frac{6b^2(bd-ae)^2(d+ex)^{3+m}}{e^5(3+m)}-4b\end{aligned}$$

Mathematica [A] time = 0.104427, size = 121, normalized size = 0.85

$$\frac{(d+ex)^{m+1}\left(\frac{6b^2(d+ex)^2(bd-ae)^2}{m+3}-\frac{4b^3(d+ex)^3(bd-ae)}{m+4}-\frac{4b(d+ex)(bd-ae)^3}{m+2}+\frac{(bd-ae)^4}{m+1}+\frac{b^4(d+ex)^4}{m+5}\right)}{e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + m)*((b*d - a*e)^4/(1 + m) - (4*b*(b*d - a*e)^3*(d + e*x))/(
2 + m) + (6*b^2*(b*d - a*e)^2*(d + e*x)^2)/(3 + m) - (4*b^3*(b*d - a*e)*(d
+ e*x)^3)/(4 + m) + (b^4*(d + e*x)^4)/(5 + m)))/e^5
```

Maple [B] time = 0.051, size = 768, normalized size = 5.4

$$(ex+d)^{1+m}\left(b^4e^4m^4x^4+4\,ab^3e^4m^4x^3+10\,b^4e^4m^3x^4+6\,a^2b^2e^4m^4x^2+44\,ab^3e^4m^3x^3-4\,b^4de^3m^3x^3+35\,b^4e^4m^2x^4+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^2,x)
```

```
[Out] (e*x+d)^(1+m)*(b^4*e^4*m^4*x^4+4*a*b^3*e^4*m^4*x^3+10*b^4*e^4*m^3*x^4+6*a^2
*b^2*e^4*m^4*x^2+44*a*b^3*e^4*m^3*x^3-4*b^4*d*e^3*m^3*x^3+35*b^4*e^4*m^2*x^
4+4*a^3*b*e^4*m^4*x+72*a^2*b^2*e^4*m^3*x^2-12*a*b^3*d*e^3*m^3*x^2+164*a*b^3
*e^4*m^2*x^3-24*b^4*d*e^3*m^2*x^3+50*b^4*e^4*m*x^4+a^4*e^4*m^4+52*a^3*b*e^4
*m^3*x-12*a^2*b^2*d*e^3*m^3*x+294*a^2*b^2*e^4*m^2*x^2-96*a*b^3*d*e^3*m^2*x^
2+244*a*b^3*e^4*m*x^3+12*b^4*d^2*e^2*m^2*x^2-44*b^4*d*e^3*m*x^3+24*b^4*e^4*
x^4+14*a^4*e^4*m^3-4*a^3*b*d*e^3*m^3+236*a^3*b*e^4*m^2*x-120*a^2*b^2*d*e^3*
m^2*x+468*a^2*b^2*e^4*m*x^2+24*a*b^3*d^2*e^2*m^2*x-204*a*b^3*d*e^3*m*x^2+12
0*a*b^3*e^4*x^3+36*b^4*d^2*e^2*m*x^2-24*b^4*d*e^3*x^3+71*a^4*e^4*m^2-48*a^3
*b*d*e^3*m^2+428*a^3*b*e^4*m*x+12*a^2*b^2*d^2*e^2*m^2-348*a^2*b^2*d*e^3*m*x
+240*a^2*b^2*e^4*x^2+144*a*b^3*d^2*e^2*m*x-120*a*b^3*d*e^3*x^2-24*b^4*d^3*e
*m*x+24*b^4*d^2*e^2*x^2+154*a^4*e^4*m-188*a^3*b*d*e^3*m+240*a^3*b*e^4*x+108
*a^2*b^2*d^2*e^2*m-240*a^2*b^2*d*e^3*x-24*a*b^3*d^3*e*m+120*a*b^3*d^2*e^2*x
-24*b^4*d^3*e*x+120*a^4*e^4-240*a^3*b*d*e^3+240*a^2*b^2*d^2*e^2-120*a*b^3*d
^3*e+24*b^4*d^4)/e^5/(m^5+15*m^4+85*m^3+225*m^2+274*m+120)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.7684, size = 1890, normalized size = 13.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

```
[Out] (a^4*d*e^4*m^4 + 24*b^4*d^5 - 120*a*b^3*d^4*e + 240*a^2*b^2*d^3*e^2 - 240*a
^3*b*d^2*e^3 + 120*a^4*d*e^4 + (b^4*e^5*m^4 + 10*b^4*e^5*m^3 + 35*b^4*e^5*m
^2 + 50*b^4*e^5*m + 24*b^4*e^5)*x^5 + (120*a*b^3*e^5 + (b^4*d*e^4 + 4*a*b^3
*e^5)*m^4 + 2*(3*b^4*d*e^4 + 22*a*b^3*e^5)*m^3 + (11*b^4*d*e^4 + 164*a*b^3*
e^5)*m^2 + 2*(3*b^4*d*e^4 + 122*a*b^3*e^5)*m)*x^4 - 2*(2*a^3*b*d^2*e^3 - 7*
a^4*d*e^4)*m^3 + 2*(120*a^2*b^2*e^5 + (2*a*b^3*d*e^4 + 3*a^2*b^2*e^5)*m^4 -
 2*(b^4*d^2*e^3 - 8*a*b^3*d*e^4 - 18*a^2*b^2*e^5)*m^3 - (6*b^4*d^2*e^3 - 34
*a*b^3*d*e^4 - 147*a^2*b^2*e^5)*m^2 - 2*(2*b^4*d^2*e^3 - 10*a*b^3*d*e^4 - 1
17*a^2*b^2*e^5)*m)*x^3 + (12*a^2*b^2*d^3*e^2 - 48*a^3*b*d^2*e^3 + 71*a^4*d*
e^4)*m^2 + 2*(120*a^3*b*e^5 + (3*a^2*b^2*d*e^4 + 2*a^3*b*e^5)*m^4 - 2*(3*a*
b^3*d^2*e^3 - 15*a^2*b^2*d*e^4 - 13*a^3*b*e^5)*m^3 + (6*b^4*d^3*e^2 - 36*a*
b^3*d^2*e^3 + 87*a^2*b^2*d*e^4 + 118*a^3*b*e^5)*m^2 + 2*(3*b^4*d^3*e^2 - 15
*a*b^3*d^2*e^3 + 30*a^2*b^2*d*e^4 + 107*a^3*b*e^5)*m)*x^2 - 2*(12*a*b^3*d^4
*e - 54*a^2*b^2*d^3*e^2 + 94*a^3*b*d^2*e^3 - 77*a^4*d*e^4)*m + (120*a^4*e^5
 + (4*a^3*b*d*e^4 + a^4*e^5)*m^4 - 2*(6*a^2*b^2*d^2*e^3 - 24*a^3*b*d*e^4 -
7*a^4*e^5)*m^3 + (24*a*b^3*d^3*e^2 - 108*a^2*b^2*d^2*e^3 + 188*a^3*b*d*e^4
+ 71*a^4*e^5)*m^2 - 2*(12*b^4*d^4*e - 60*a*b^3*d^3*e^2 + 120*a^2*b^2*d^2*e^
3 - 120*a^3*b*d*e^4 - 77*a^4*e^5)*m)*x)*(e*x + d)^m/(e^5*m^5 + 15*e^5*m^4 +
 85*e^5*m^3 + 225*e^5*m^2 + 274*e^5*m + 120*e^5)
```

Sympy [A] time = 9.75118, size = 8531, normalized size = 60.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(b**2*x**2+2*a*b*x+a**2)**2,x)
```

```
[Out] Piecewise((d**m*(a**4*x + 2*a**3*b*x**2 + 2*a**2*b**2*x**3 + a*b**3*x**4 +
b**4*x**5/5), Eq(e, 0)), (-3*a**4*d**2*e**4/(12*d**6*e**5 + 48*d**5*e**6*x
+ 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4) - 4*a**3*b*d**
3*e**3/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x*
*3 + 12*d**2*e**9*x**4) - 16*a**3*b*d**2*e**4*x/(12*d**6*e**5 + 48*d**5*e**
6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4) + 24*a**2*
b**2*d*e**5*x**3/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d*
*3*e**8*x**3 + 12*d**2*e**9*x**4) + 6*a**2*b**2*e**6*x**4/(12*d**6*e**5 + 4
8*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4)
+ 12*a*b**3*d*e**5*x**4/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2
+ 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4) + 12*b**4*d**6*log(d/e + x)/(12*d*
*6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*
e**9*x**4) + 7*b**4*d**6/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2
 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4) + 48*b**4*d**5*e*x*log(d/e + x)/(
12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*
d**2*e**9*x**4) + 16*b**4*d**5*e*x/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4
*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4) + 72*b**4*d**4*e**2*x**
2*log(d/e + x)/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3
*e**8*x**3 + 12*d**2*e**9*x**4) + 48*b**4*d**3*e**3*x**3*log(d/e + x)/(12*d
**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2
*e**9*x**4) - 24*b**4*d**3*e**3*x**3/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d*
*4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4) + 12*b**4*d**2*e**4*x
**4*log(d/e + x)/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d*
*3*e**8*x**3 + 12*d**2*e**9*x**4) - 18*b**4*d**2*e**4*x**4/(12*d**6*e**5 +
48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4)
, Eq(m, -5)), (-a**4*d*e**4/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2
 + 3*d*e**8*x**3) - 2*a**3*b*d**2*e**3/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**
2*e**7*x**2 + 3*d*e**8*x**3) - 6*a**3*b*d*e**4*x/(3*d**4*e**5 + 9*d**3*e**6
*x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3) + 6*a**2*b**2*e**5*x**3/(3*d**4*e**5
```

```
 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3) + 12*a*b**3*d**4*e*log
(d/e + x)/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3)
+ 10*a*b**3*d**4*e/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e*
*8*x**3) + 36*a*b**3*d**3*e**2*x*log(d/e + x)/(3*d**4*e**5 + 9*d**3*e**6*x
+ 9*d**2*e**7*x**2 + 3*d*e**8*x**3) + 18*a*b**3*d**3*e**2*x/(3*d**4*e**5 +
9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3) + 36*a*b**3*d**2*e**3*x**
2*log(d/e + x)/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x
**3) + 12*a*b**3*d*e**4*x**3*log(d/e + x)/(3*d**4*e**5 + 9*d**3*e**6*x + 9*
d**2*e**7*x**2 + 3*d*e**8*x**3) - 12*a*b**3*d*e**4*x**3/(3*d**4*e**5 + 9*d*
*3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3) - 12*b**4*d**5*log(d/e + x)/(
3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3) - 10*b**4*d
**5/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3) - 36*b
**4*d**4*e*x*log(d/e + x)/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 +
 3*d*e**8*x**3) - 18*b**4*d**4*e*x/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e*
*7*x**2 + 3*d*e**8*x**3) - 36*b**4*d**3*e**2*x**2*log(d/e + x)/(3*d**4*e**5
 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3) - 12*b**4*d**2*e**3*x*
*3*log(d/e + x)/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*
x**3) + 12*b**4*d**2*e**3*x**3/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x
**2 + 3*d*e**8*x**3) + 3*b**4*d*e**4*x**4/(3*d**4*e**5 + 9*d**3*e**6*x + 9*
d**2*e**7*x**2 + 3*d*e**8*x**3), Eq(m, -4)), (-a**4*e**4/(2*d**2*e**5 + 4*d
*e**6*x + 2*e**7*x**2) - 4*a**3*b*d*e**3/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7
*x**2) - 8*a**3*b*e**4*x/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 12*a**2
*b**2*d**2*e**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 18*
a**2*b**2*d**2*e**2/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 24*a**2*b**2
*d*e**3*x*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 24*a**2*b
**2*d*e**3*x/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 12*a**2*b**2*e**4*x
**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 24*a*b**3*d**3*
e*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 36*a*b**3*d**3*e/
(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 48*a*b**3*d**2*e**2*x*log(d/e +
x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 48*a*b**3*d**2*e**2*x/(2*d**2
*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 24*a*b**3*d*e**3*x**2*log(d/e + x)/(2*d
**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 8*a*b**3*e**4*x**3/(2*d**2*e**5 + 4*
d*e**6*x + 2*e**7*x**2) + 12*b**4*d**4*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6
*x + 2*e**7*x**2) + 18*b**4*d**4/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) +
 24*b**4*d**3*e*x*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 2
4*b**4*d**3*e*x/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 12*b**4*d**2*e**
2*x**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 4*b**4*d*e**
3*x**3/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + b**4*e**4*x**4/(2*d**2*e*
*5 + 4*d*e**6*x + 2*e**7*x**2), Eq(m, -3)), (-3*a**4*e**4/(3*d*e**5 + 3*e**
6*x) + 12*a**3*b*d*e**3*log(d/e + x)/(3*d*e**5 + 3*e**6*x) + 12*a**3*b*d*e*
*3/(3*d*e**5 + 3*e**6*x) + 12*a**3*b*e**4*x*log(d/e + x)/(3*d*e**5 + 3*e**6
*x) - 36*a**2*b**2*d**2*e**2*log(d/e + x)/(3*d*e**5 + 3*e**6*x) - 36*a**2*b
**2*d**2*e**2/(3*d*e**5 + 3*e**6*x) - 36*a**2*b**2*d*e**3*x*log(d/e + x)/(3
*d*e**5 + 3*e**6*x) + 18*a**2*b**2*e**4*x**2/(3*d*e**5 + 3*e**6*x) + 36*a*b
**3*d**3*e*log(d/e + x)/(3*d*e**5 + 3*e**6*x) + 36*a*b**3*d**3*e/(3*d*e**5
+ 3*e**6*x) + 36*a*b**3*d**2*e**2*x*log(d/e + x)/(3*d*e**5 + 3*e**6*x) - 18
*a*b**3*d*e**3*x**2/(3*d*e**5 + 3*e**6*x) + 6*a*b**3*e**4*x**3/(3*d*e**5 +
3*e**6*x) - 12*b**4*d**4*log(d/e + x)/(3*d*e**5 + 3*e**6*x) - 12*b**4*d**4/
(3*d*e**5 + 3*e**6*x) - 12*b**4*d**3*e*x*log(d/e + x)/(3*d*e**5 + 3*e**6*x)
 + 6*b**4*d**2*e**2*x**2/(3*d*e**5 + 3*e**6*x) - 2*b**4*d*e**3*x**3/(3*d*e*
*5 + 3*e**6*x) + b**4*e**4*x**4/(3*d*e**5 + 3*e**6*x), Eq(m, -2)), (a**4*lo
g(d/e + x)/e - 4*a**3*b*d*log(d/e + x)/e**2 + 4*a**3*b*x/e + 6*a**2*b**2*d*
*2*log(d/e + x)/e**3 - 6*a**2*b**2*d*x/e**2 + 3*a**2*b**2*x**2/e - 4*a*b**3
*d**3*log(d/e + x)/e**4 + 4*a*b**3*d**2*x/e**3 - 2*a*b**3*d*x**2/e**2 + 4*a
*b**3*x**3/(3*e) + b**4*d**4*log(d/e + x)/e**5 - b**4*d**3*x/e**4 + b**4*d*
*2*x**2/(2*e**3) - b**4*d*x**3/(3*e**2) + b**4*x**4/(4*e), Eq(m, -1)), (a**
4*d*e**4*m**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e
**5*m**2 + 274*e**5*m + 120*e**5) + 14*a**4*d*e**4*m**3*(d + e*x)**m/(e**5*
m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5)
```

+ 71*a**4*d*e**4*m**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 154*a**4*d*e**4*m*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 120*a**4*d*e**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + a**4*e**5*m**4*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 14*a**4*e**5*m**3*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 71*a**4*e**5*m**2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 154*a**4*e**5*m*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 120*a**4*e**5*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 4*a**3*b*d**2*e**3*m**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 48*a**3*b*d**2*e**3*m**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 188*a**3*b*d**2*e**3*m*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 240*a**3*b*d**2*e**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 4*a**3*b*d*e**4*m**4*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 48*a**3*b*d*e**4*m**3*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 188*a**3*b*d*e**4*m**2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 240*a**3*b*d*e**4*m*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 4*a**3*b*e**5*m**4*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 52*a**3*b*e**5*m**3*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 236*a**3*b*e**5*m**2*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 428*a**3*b*e**5*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 240*a**3*b*e**5*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 12*a**2*b**2*d**3*e**2*m**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 108*a**2*b**2*d**3*e**2*m*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 240*a**2*b**2*d**3*e**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 12*a**2*b**2*d**2*e**3*m**3*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 108*a**2*b**2*d**2*e**3*m**2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 240*a**2*b**2*d**2*e**3*m*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 6*a**2*b**2*d*e**4*m**4*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 60*a**2*b**2*d*e**4*m**3*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 174*a**2*b**2*d*e**4*m**2*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 120*a**2*b**2*d*e**4*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 6*a**2*b**2*e**5*m**4*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 72*a**2*b**2*e**5*m**3*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 294*a**2*b**2*e**5*m**2*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 468*a**2*b**2*e**5*m*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 240*a**2*b**2*e**5*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 24*a*b**3*d**4*e*m*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 +

```
85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 120*a*b**3*d**4*e*(
d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*
e**5*m + 120*e**5) + 24*a*b**3*d**3*e**2*m**2*x*(d + e*x)**m/(e**5*m**5 + 1
5*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 120*a
*b**3*d**3*e**2*m*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 +
 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 12*a*b**3*d**2*e**3*m**3*x**2*(d
+ e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e*
*5*m + 120*e**5) - 72*a*b**3*d**2*e**3*m**2*x**2*(d + e*x)**m/(e**5*m**5 +
15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 60*a
*b**3*d**2*e**3*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**
3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 4*a*b**3*d*e**4*m**4*x**3*(d +
 e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**
5*m + 120*e**5) + 32*a*b**3*d*e**4*m**3*x**3*(d + e*x)**m/(e**5*m**5 + 15*e
**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 68*a*b**
3*d*e**4*m**2*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 +
225*e**5*m**2 + 274*e**5*m + 120*e**5) + 40*a*b**3*d*e**4*m*x**3*(d + e*x)*
*m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m +
120*e**5) + 4*a*b**3*e**5*m**4*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4
+ 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 44*a*b**3*e**5*m*
*3*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m*
*2 + 274*e**5*m + 120*e**5) + 164*a*b**3*e**5*m**2*x**4*(d + e*x)**m/(e**5*
m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5)
 + 244*a*b**3*e**5*m*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*
m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 120*a*b**3*e**5*x**4*(d + e
*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*
m + 120*e**5) + 24*b**4*d**5*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e*
*5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 24*b**4*d**4*e*m*x*(d +
e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5
*m + 120*e**5) + 12*b**4*d**3*e**2*m**2*x**2*(d + e*x)**m/(e**5*m**5 + 15*e
**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 12*b**4*
d**3*e**2*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 22
5*e**5*m**2 + 274*e**5*m + 120*e**5) - 4*b**4*d**2*e**3*m**3*x**3*(d + e*x)
**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m +
 120*e**5) - 12*b**4*d**2*e**3*m**2*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*
m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 8*b**4*d**2*
e**3*m*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**
5*m**2 + 274*e**5*m + 120*e**5) + b**4*d*e**4*m**4*x**4*(d + e*x)**m/(e**5*
m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5)
 + 6*b**4*d*e**4*m**3*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5
*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 11*b**4*d*e**4*m**2*x**4*(
d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*
e**5*m + 120*e**5) + 6*b**4*d*e**4*m*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5
*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + b**4*e**5*m
**4*x**5*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m
**2 + 274*e**5*m + 120*e**5) + 10*b**4*e**5*m**3*x**5*(d + e*x)**m/(e**5*m*
*5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) +
 35*b**4*e**5*m**2*x**5*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m*
*3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 50*b**4*e**5*m*x**5*(d + e*x)
**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m +
 120*e**5) + 24*b**4*e**5*x**5*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*
e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5), True))
```

Giac [B] time = 1.1804, size = 2064, normalized size = 14.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*b^4*m^4*x^5*e^5 + (x*e + d)^m*b^4*d*m^4*x^4*e^4 + 4*(x*e + d)^
m*a*b^3*m^4*x^4*e^5 + 10*(x*e + d)^m*b^4*m^3*x^5*e^5 + 4*(x*e + d)^m*a*b^3*
d*m^4*x^3*e^4 + 6*(x*e + d)^m*b^4*d*m^3*x^4*e^4 - 4*(x*e + d)^m*b^4*d^2*m^3
*x^3*e^3 + 6*(x*e + d)^m*a^2*b^2*m^4*x^3*e^5 + 44*(x*e + d)^m*a*b^3*m^3*x^4
*e^5 + 35*(x*e + d)^m*b^4*m^2*x^5*e^5 + 6*(x*e + d)^m*a^2*b^2*d*m^4*x^2*e^4
 + 32*(x*e + d)^m*a*b^3*d*m^3*x^3*e^4 + 11*(x*e + d)^m*b^4*d*m^2*x^4*e^4 -
12*(x*e + d)^m*a*b^3*d^2*m^3*x^2*e^3 - 12*(x*e + d)^m*b^4*d^2*m^2*x^3*e^3 +
 12*(x*e + d)^m*b^4*d^3*m^2*x^2*e^2 + 4*(x*e + d)^m*a^3*b*m^4*x^2*e^5 + 72*
(x*e + d)^m*a^2*b^2*m^3*x^3*e^5 + 164*(x*e + d)^m*a*b^3*m^2*x^4*e^5 + 50*(x
*e + d)^m*b^4*m*x^5*e^5 + 4*(x*e + d)^m*a^3*b*d*m^4*x*e^4 + 60*(x*e + d)^m*
a^2*b^2*d*m^3*x^2*e^4 + 68*(x*e + d)^m*a*b^3*d*m^2*x^3*e^4 + 6*(x*e + d)^m*
b^4*d*m*x^4*e^4 - 12*(x*e + d)^m*a^2*b^2*d^2*m^3*x*e^3 - 72*(x*e + d)^m*a*b
^3*d^2*m^2*x^2*e^3 - 8*(x*e + d)^m*b^4*d^2*m*x^3*e^3 + 24*(x*e + d)^m*a*b^3
*d^3*m^2*x*e^2 + 12*(x*e + d)^m*b^4*d^3*m*x^2*e^2 - 24*(x*e + d)^m*b^4*d^4*
m*x*e + (x*e + d)^m*a^4*m^4*x*e^5 + 52*(x*e + d)^m*a^3*b*m^3*x^2*e^5 + 294*
(x*e + d)^m*a^2*b^2*m^2*x^3*e^5 + 244*(x*e + d)^m*a*b^3*m*x^4*e^5 + 24*(x*e
 + d)^m*b^4*x^5*e^5 + (x*e + d)^m*a^4*d*m^4*e^4 + 48*(x*e + d)^m*a^3*b*d*m^
3*x*e^4 + 174*(x*e + d)^m*a^2*b^2*d*m^2*x^2*e^4 + 40*(x*e + d)^m*a*b^3*d*m*
x^3*e^4 - 4*(x*e + d)^m*a^3*b*d^2*m^3*e^3 - 108*(x*e + d)^m*a^2*b^2*d^2*m^2
*x*e^3 - 60*(x*e + d)^m*a*b^3*d^2*m*x^2*e^3 + 12*(x*e + d)^m*a^2*b^2*d^3*m^
2*e^2 + 120*(x*e + d)^m*a*b^3*d^3*m*x*e^2 - 24*(x*e + d)^m*a*b^3*d^4*m*e +
24*(x*e + d)^m*b^4*d^5 + 14*(x*e + d)^m*a^4*m^3*x*e^5 + 236*(x*e + d)^m*a^3
*b*m^2*x^2*e^5 + 468*(x*e + d)^m*a^2*b^2*m*x^3*e^5 + 120*(x*e + d)^m*a*b^3*
x^4*e^5 + 14*(x*e + d)^m*a^4*d*m^3*e^4 + 188*(x*e + d)^m*a^3*b*d*m^2*x*e^4
+ 120*(x*e + d)^m*a^2*b^2*d*m*x^2*e^4 - 48*(x*e + d)^m*a^3*b*d^2*m^2*e^3 -
240*(x*e + d)^m*a^2*b^2*d^2*m*x*e^3 + 108*(x*e + d)^m*a^2*b^2*d^3*m*e^2 - 1
20*(x*e + d)^m*a*b^3*d^4*e + 71*(x*e + d)^m*a^4*m^2*x*e^5 + 428*(x*e + d)^m
*a^3*b*m*x^2*e^5 + 240*(x*e + d)^m*a^2*b^2*x^3*e^5 + 71*(x*e + d)^m*a^4*d*m
^2*e^4 + 240*(x*e + d)^m*a^3*b*d*m*x*e^4 - 188*(x*e + d)^m*a^3*b*d^2*m*e^3
+ 240*(x*e + d)^m*a^2*b^2*d^3*e^2 + 154*(x*e + d)^m*a^4*m*x*e^5 + 240*(x*e
+ d)^m*a^3*b*x^2*e^5 + 154*(x*e + d)^m*a^4*d*m*e^4 - 240*(x*e + d)^m*a^3*b*
d^2*e^3 + 120*(x*e + d)^m*a^4*x*e^5 + 120*(x*e + d)^m*a^4*d*e^4)/(m^5*e^5 +
 15*m^4*e^5 + 85*m^3*e^5 + 225*m^2*e^5 + 274*m*e^5 + 120*e^5)
```

3.1733 $\int (d+ex)^m \left(a^2+2abx+b^2x^2\right)\,dx$

Optimal. Leaf size=78

$$\frac{(bd-ae)^2(d+ex)^{m+1}}{e^3(m+1)}-\frac{2b(bd-ae)(d+ex)^{m+2}}{e^3(m+2)}+\frac{b^2(d+ex)^{m+3}}{e^3(m+3)}$$

```
[Out] ((b*d - a*e)^2*(d + e*x)^(1 + m))/(e^3*(1 + m)) - (2*b*(b*d - a*e)*(d + e*x
)^(2 + m))/(e^3*(2 + m)) + (b^2*(d + e*x)^(3 + m))/(e^3*(3 + m))
```

Rubi [A] time = 0.0331921, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {27, 43}

$$\frac{(bd-ae)^2(d+ex)^{m+1}}{e^3(m+1)}-\frac{2b(bd-ae)(d+ex)^{m+2}}{e^3(m+2)}+\frac{b^2(d+ex)^{m+3}}{e^3(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] ((b*d - a*e)^2*(d + e*x)^(1 + m))/(e^3*(1 + m)) - (2*b*(b*d - a*e)*(d + e*x
)^(2 + m))/(e^3*(2 + m)) + (b^2*(d + e*x)^(3 + m))/(e^3*(3 + m))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^m \left(a^2+2abx+b^2x^2\right)\,dx &= \int (a+bx)^2(d+ex)^m\,dx\\ &= \int \left(\frac{(-bd+ae)^2(d+ex)^m}{e^2}-\frac{2b(bd-ae)(d+ex)^{1+m}}{e^2}+\frac{b^2(d+ex)^{2+m}}{e^2}\right)dx\\ &= \frac{(bd-ae)^2(d+ex)^{1+m}}{e^3(1+m)}-\frac{2b(bd-ae)(d+ex)^{2+m}}{e^3(2+m)}+\frac{b^2(d+ex)^{3+m}}{e^3(3+m)}\end{aligned}$$

Mathematica [A] time = 0.0708185, size = 67, normalized size = 0.86

$$\frac{(d+ex)^{m+1}\left(-\frac{2b(d+ex)(bd-ae)}{m+2}+\frac{(bd-ae)^2}{m+1}+\frac{b^2(d+ex)^2}{m+3}\right)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] ((d + e*x)^(1 + m)*((b*d - a*e)^2/(1 + m) - (2*b*(b*d - a*e)*(d + e*x))/(2
+ m) + (b^2*(d + e*x)^2)/(3 + m)))/e^3
```

Maple [B] time = 0.047, size = 159, normalized size = 2.

$$\frac{(ex+d)^{1+m}\left(b^2e^2m^2x^2 + 2\,abe^2m^2x + 3\,b^2e^2mx^2 + a^2e^2m^2 + 8\,abe^2mx - 2\,b^2demx + 2\,b^2x^2e^2 + 5\,a^2e^2m - 2\,abdem + 6\,ab\right.}{e^3\left(m^3 + 6\,m^2 + 11\,m + 6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2),x)
```

```
[Out] (e*x+d)^(1+m)*(b^2*e^2*m^2*x^2+2*a*b*e^2*m^2*x+3*b^2*e^2*m*x^2+a^2*e^2*m^2+
8*a*b*e^2*m*x-2*b^2*d*e*m*x+2*b^2*e^2*x^2+5*a^2*e^2*m-2*a*b*d*e*m+6*a*b*e^2
*x-2*b^2*d*e*x+6*a^2*e^2-6*a*b*d*e+2*b^2*d^2)/e^3/(m^3+6*m^2+11*m+6)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.75875, size = 478, normalized size = 6.13

$$\frac{\left(a^2de^2m^2 + 2\,b^2d^3 - 6\,abd^2e + 6\,a^2de^2 + \left(b^2e^3m^2 + 3\,b^2e^3m + 2\,b^2e^3\right)x^3 + \left(6\,abe^3 + \left(b^2de^2 + 2\,abe^3\right)m^2 + \left(b^2de^2 + 8\,abe^3\right.\right.\right.}{e^3m^3 + 6\,e^3m^2 + 11\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

```
[Out] (a^2*d*e^2*m^2 + 2*b^2*d^3 - 6*a*b*d^2*e + 6*a^2*d*e^2 + (b^2*e^3*m^2 + 3*b
^2*e^3*m + 2*b^2*e^3)*x^3 + (6*a*b*e^3 + (b^2*d*e^2 + 2*a*b*e^3)*m^2 + (b^2
*d*e^2 + 8*a*b*e^3)*m)*x^2 - (2*a*b*d^2*e - 5*a^2*d*e^2)*m + (6*a^2*e^3 + (
2*a*b*d*e^2 + a^2*e^3)*m^2 - (2*b^2*d^2*e - 6*a*b*d*e^2 - 5*a^2*e^3)*m)*x)*
(e*x + d)^m/(e^3*m^3 + 6*e^3*m^2 + 11*e^3*m + 6*e^3)
```

Sympy [A] time = 2.07865, size = 1506, normalized size = 19.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(b**2*x**2+2*a*b*x+a**2),x)
```

```
[Out] Piecewise((d**m*(a**2*x + a*b*x**2 + b**2*x**3/3), Eq(e, 0)), (-a**2*e**2/(
2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2) - 2*a*b*d*e/(2*d**2*e**3 + 4*d*e**4
*x + 2*e**5*x**2) - 4*a*b*e**2*x/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2) +
 2*b**2*d**2*log(d/e + x)/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2) + 3*b**2
*d**2/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2) + 4*b**2*d*e*x*log(d/e + x)/
(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2) + 4*b**2*d*e*x/(2*d**2*e**3 + 4*d*
e**4*x + 2*e**5*x**2) + 2*b**2*e**2*x**2*log(d/e + x)/(2*d**2*e**3 + 4*d*e*
*4*x + 2*e**5*x**2), Eq(m, -3)), (-a**2*e**2/(d*e**3 + e**4*x) + 2*a*b*d*e*
log(d/e + x)/(d*e**3 + e**4*x) + 2*a*b*d*e/(d*e**3 + e**4*x) + 2*a*b*e**2*x
*log(d/e + x)/(d*e**3 + e**4*x) - 2*b**2*d**2*log(d/e + x)/(d*e**3 + e**4*x
) - 2*b**2*d**2/(d*e**3 + e**4*x) - 2*b**2*d*e*x*log(d/e + x)/(d*e**3 + e**
4*x) + b**2*e**2*x**2/(d*e**3 + e**4*x), Eq(m, -2)), (a**2*log(d/e + x)/e -
 2*a*b*d*log(d/e + x)/e**2 + 2*a*b*x/e + b**2*d**2*log(d/e + x)/e**3 - b**2
*d*x/e**2 + b**2*x**2/(2*e), Eq(m, -1)), (a**2*d*e**2*m**2*(d + e*x)**m/(e*
*3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + 5*a**2*d*e**2*m*(d + e*x)**m/
(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + 6*a**2*d*e**2*(d + e*x)**m
/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + a**2*e**3*m**2*x*(d + e*x
)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + 5*a**2*e**3*m*x*(d +
e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + 6*a**2*e**3*x*(d +
 e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) - 2*a*b*d**2*e*m*(d
 + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) - 6*a*b*d**2*e*(d
 + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + 2*a*b*d*e**2*m*
*2*x*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + 6*a*b*d*
e**2*m*x*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + 2*a*
b*e**3*m**2*x**2*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3
) + 8*a*b*e**3*m*x**2*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6
*e**3) + 6*a*b*e**3*x**2*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m
+ 6*e**3) + 2*b**2*d**3*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m +
 6*e**3) - 2*b**2*d**2*e*m*x*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**
3*m + 6*e**3) + b**2*d*e**2*m**2*x**2*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2
 + 11*e**3*m + 6*e**3) + b**2*d*e**2*m*x**2*(d + e*x)**m/(e**3*m**3 + 6*e**
3*m**2 + 11*e**3*m + 6*e**3) + b**2*e**3*m**2*x**3*(d + e*x)**m/(e**3*m**3
+ 6*e**3*m**2 + 11*e**3*m + 6*e**3) + 3*b**2*e**3*m*x**3*(d + e*x)**m/(e**3
*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + 2*b**2*e**3*x**3*(d + e*x)**m/(
e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3), True))
```

Giac [B] time = 1.20745, size = 524, normalized size = 6.72

$$\frac{(xe+d)^m b^2 m^2 x^3 e^3 + (xe+d)^m b^2 d m^2 x^2 e^2 + 2\,(xe+d)^m a b m^2 x^2 e^3 + 3\,(xe+d)^m b^2 m x^3 e^3 + 2\,(xe+d)^m a b d m^2 x e^2 + (xe \cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*b^2*m^2*x^3*e^3 + (x*e + d)^m*b^2*d*m^2*x^2*e^2 + 2*(x*e + d)^
m*a*b*m^2*x^2*e^3 + 3*(x*e + d)^m*b^2*m*x^3*e^3 + 2*(x*e + d)^m*a*b*d*m^2*x
*e^2 + (x*e + d)^m*b^2*d*m*x^2*e^2 - 2*(x*e + d)^m*b^2*d^2*m*x*e + (x*e + d
)^m*a^2*m^2*x*e^3 + 8*(x*e + d)^m*a*b*m*x^2*e^3 + 2*(x*e + d)^m*b^2*x^3*e^3
 + (x*e + d)^m*a^2*d*m^2*e^2 + 6*(x*e + d)^m*a*b*d*m*x*e^2 - 2*(x*e + d)^m*
a*b*d^2*m*e + 2*(x*e + d)^m*b^2*d^3 + 5*(x*e + d)^m*a^2*m*x*e^3 + 6*(x*e +
d)^m*a*b*x^2*e^3 + 5*(x*e + d)^m*a^2*d*m*e^2 - 6*(x*e + d)^m*a*b*d^2*e + 6*
(x*e + d)^m*a^2*x*e^3 + 6*(x*e + d)^m*a^2*d*e^2)/(m^3*e^3 + 6*m^2*e^3 + 11*
m*e^3 + 6*e^3)
```

3.1734 $\int \frac{(d+ex)^m}{a^2+2abx+b^2x^2} dx$

Optimal. Leaf size=51

$$\frac{e(d+ex)^{m+1} {}_2F_1\left(2,m+1;m+2;\frac{b(d+ex)}{bd-ae}\right)}{(m+1)(bd-ae)^2}$$

```
[Out] (e*(d + e*x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, (b*(d + e*x))/(b*d
- a*e)])/((b*d - a*e)^2*(1 + m))
```

Rubi [A] time = 0.0176339, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 68}

$$\frac{e(d+ex)^{m+1} {}_2F_1\left(2,m+1;m+2;\frac{b(d+ex)}{bd-ae}\right)}{(m+1)(bd-ae)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(a^2 + 2*a*b*x + b^2*x^2),x]
```


```
[Out] (e*(d + e*x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, (b*(d + e*x))/(b*d
- a*e)])/((b*d - a*e)^2*(1 + m))
```


Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^m}{a^2+2abx+b^2x^2} dx &= \int \frac{(d+ex)^m}{(a+bx)^2} dx\\ &= \frac{e(d+ex)^{1+m} {}_2F_1\left(2,1+m;2+m;\frac{b(d+ex)}{bd-ae}\right)}{(bd-ae)^2(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0139702, size = 52, normalized size = 1.02

$$\frac{e(d+ex)^{m+1} {}_2F_1\left(2,m+1;m+2;-\frac{b(d+ex)}{ae-bd}\right)}{(m+1)(ae-bd)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(a^2 + 2*a*b*x + b^2*x^2),x]
```

```
[Out] (e*(d + e*x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((b*(d + e*x))/(-(
b*d) + a*e))])/((-(b*d) + a*e)^2*(1 + m))
```

Maple [F] time = 1.005, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{b^2x^2+2\,abx+a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2),x)
```

```
[Out] int((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{b^2x^2+2\,abx+a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(ex+d)^m}{b^2x^2+2\,abx+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(b**2*x**2+2*a*b*x+a**2),x)
```

```
[Out] Integral((d + e*x)**m/(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{b^2x^2+2\,abx+a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(b^2*x^2 + 2*a*b*x + a^2), x)
```

3.1735 $\int \frac{(d+ex)^m}{(a^2+2abx+b^2x^2)^2}\,dx$

Optimal. Leaf size=53

$$\frac{e^3(d+ex)^{m+1}\,{}_2F_1\left(4,m+1;m+2;\frac{b(d+ex)}{bd-ae}\right)}{(m+1)(bd-ae)^4}$$

```
[Out] (e^3*(d + e*x)^(1 + m)*Hypergeometric2F1[4, 1 + m, 2 + m, (b*(d + e*x))/(b*
d - a*e)])/((b*d - a*e)^4*(1 + m))
```

Rubi [A] time = 0.0152125, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 68}

$$\frac{e^3(d+ex)^{m+1}\,{}_2F_1\left(4,m+1;m+2;\frac{b(d+ex)}{bd-ae}\right)}{(m+1)(bd-ae)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] (e^3*(d + e*x)^(1 + m)*Hypergeometric2F1[4, 1 + m, 2 + m, (b*(d + e*x))/(b*
d - a*e)])/((b*d - a*e)^4*(1 + m))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^m}{\left(a^2+2abx+b^2x^2\right)^2}\,dx &= \int \frac{(d+ex)^m}{(a+bx)^4}\,dx\\ &= \frac{e^3(d+ex)^{1+m}\,{}_2F_1\left(4,1+m;2+m;\frac{b(d+ex)}{bd-ae}\right)}{(bd-ae)^4(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0151031, size = 54, normalized size = 1.02

$$\frac{e^3(d+ex)^{m+1}\,{}_2F_1\left(4,m+1;m+2;-\frac{b(d+ex)}{ae-bd}\right)}{(m+1)(ae-bd)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

```
[Out] (e^3*(d + e*x)^(1 + m)*Hypergeometric2F1[4, 1 + m, 2 + m, -((b*(d + e*x))/(
-(b*d) + a*e))])/((-(b*d) + a*e)^4*(1 + m))
```

Maple [F] time = 1.092, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(b^2x^2+2\,abx+a^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

```
[Out] int((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(b^2x^2+2\,abx+a^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/(b^2*x^2 + 2*a*b*x + a^2)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^m}{b^4x^4+4\,ab^3x^3+6\,a^2b^2x^2+4\,a^3bx+a^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m/(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a
^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

```
[Out] Timed out
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{(b^2x^2+2\,abx+a^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(b^2*x^2 + 2*a*b*x + a^2)^2, x)
```

3.1736 $\int \frac{(d+ex)^m}{(a^2+2abx+b^2x^2)^3} dx$

Optimal. Leaf size=53

$$\frac{e^5(d+ex)^{m+1} {}_2F_1\left(6,m+1;m+2;\frac{b(d+ex)}{bd-ae}\right)}{(m+1)(bd-ae)^6}$$

```
[Out] (e^5*(d + e*x)^(1 + m)*Hypergeometric2F1[6, 1 + m, 2 + m, (b*(d + e*x))/(b*
d - a*e)])/((b*d - a*e)^6*(1 + m))
```

Rubi [A] time = 0.0168016, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {27, 68}

$$\frac{e^5(d+ex)^{m+1} {}_2F_1\left(6,m+1;m+2;\frac{b(d+ex)}{bd-ae}\right)}{(m+1)(bd-ae)^6}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] (e^5*(d + e*x)^(1 + m)*Hypergeometric2F1[6, 1 + m, 2 + m, (b*(d + e*x))/(b*
d - a*e)])/((b*d - a*e)^6*(1 + m))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^m}{(a^2+2abx+b^2x^2)^3} dx &= \int \frac{(d+ex)^m}{(a+bx)^6} dx\\ &= \frac{e^5(d+ex)^{1+m} {}_2F_1\left(6,1+m;2+m;\frac{b(d+ex)}{bd-ae}\right)}{(bd-ae)^6(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0158343, size = 54, normalized size = 1.02

$$\frac{e^5(d+ex)^{m+1} {}_2F_1\left(6,m+1;m+2;-\frac{b(d+ex)}{ae-bd}\right)}{(m+1)(ae-bd)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
[Out] (e^5*(d + e*x)^(1 + m)*Hypergeometric2F1[6, 1 + m, 2 + m, -((b*(d + e*x))/(
-(b*d) + a*e))])/((-(b*d) + a*e)^6*(1 + m))
```

Maple [F] time = 1.235, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{(b^2x^2+2\,abx+a^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

```
[Out] int((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{(b^2x^2+2\,abx+a^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/(b^2*x^2 + 2*a*b*x + a^2)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^m}{b^6x^6+6\,ab^5x^5+15\,a^2b^4x^4+20\,a^3b^3x^3+15\,a^4b^2x^2+6\,a^5bx+a^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m/(b^6*x^6 + 6*a*b^5*x^5 + 15*a^2*b^4*x^4 + 20*a^3*b^3*x
^3 + 15*a^4*b^2*x^2 + 6*a^5*b*x + a^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{(b^2x^2+2\,abx+a^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(b^2*x^2 + 2*a*b*x + a^2)^3, x)
```

3.1737 $\int (d+ex)^m \left(a^2+2abx+b^2x^2\right)^{5/2} \, dx$

Optimal. Leaf size=337

$$-\frac{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^5(d+ex)^{m+1}}{e^6(m+1)(a+bx)}+\frac{5b\sqrt{a^2+2abx+b^2x^2}(bd-ae)^4(d+ex)^{m+2}}{e^6(m+2)(a+bx)}-\frac{10b^2\sqrt{a^2+2abx+b^2x^2}(bd}{e^6(m+3)(a}$$

```
[Out] -(((b*d - a*e)^5*(d + e*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(1 +
 m)*(a + b*x))) + (5*b*(b*d - a*e)^4*(d + e*x)^(2 + m)*Sqrt[a^2 + 2*a*b*x +
 b^2*x^2])/(e^6*(2 + m)*(a + b*x)) - (10*b^2*(b*d - a*e)^3*(d + e*x)^(3 + m
)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(3 + m)*(a + b*x)) + (10*b^3*(b*d - a
*e)^2*(d + e*x)^(4 + m)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(4 + m)*(a + b*
x)) - (5*b^4*(b*d - a*e)*(d + e*x)^(5 + m)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(
e^6*(5 + m)*(a + b*x)) + (b^5*(d + e*x)^(6 + m)*Sqrt[a^2 + 2*a*b*x + b^2*x^
2])/(e^6*(6 + m)*(a + b*x))
```

Rubi [A] time = 0.141864, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$-\frac{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^5(d+ex)^{m+1}}{e^6(m+1)(a+bx)}+\frac{5b\sqrt{a^2+2abx+b^2x^2}(bd-ae)^4(d+ex)^{m+2}}{e^6(m+2)(a+bx)}-\frac{10b^2\sqrt{a^2+2abx+b^2x^2}(bd}{e^6(m+3)(a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] -(((b*d - a*e)^5*(d + e*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(1 +
 m)*(a + b*x))) + (5*b*(b*d - a*e)^4*(d + e*x)^(2 + m)*Sqrt[a^2 + 2*a*b*x +
 b^2*x^2])/(e^6*(2 + m)*(a + b*x)) - (10*b^2*(b*d - a*e)^3*(d + e*x)^(3 + m
)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(3 + m)*(a + b*x)) + (10*b^3*(b*d - a
*e)^2*(d + e*x)^(4 + m)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^6*(4 + m)*(a + b*
x)) - (5*b^4*(b*d - a*e)*(d + e*x)^(5 + m)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(
e^6*(5 + m)*(a + b*x)) + (b^5*(d + e*x)^(6 + m)*Sqrt[a^2 + 2*a*b*x + b^2*x^
2])/(e^6*(6 + m)*(a + b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (d+ex)^m \left(a^2+2abx+b^2x^2\right)^{5/2} dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(ab+b^2x\right)^5 (d+ex)^m dx}{b^4\left(ab+b^2x\right)}$$
$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^5(bd-ae)^5(d+ex)^m}{e^5}+\frac{5b^6(bd-ae)^4(d+ex)^{1+m}}{e^5}-\frac{10b^7(bd-ae)^3(d+ex)^{2+m}}{e^5}\right.}{b^4\left(ab+b^2x\right)}$$
$$= -\frac{(bd-ae)^5(d+ex)^{1+m}\sqrt{a^2+2abx+b^2x^2}}{e^6(1+m)(a+bx)}+\frac{5b(bd-ae)^4(d+ex)^{2+m}\sqrt{a^2+2abx}}{e^6(2+m)(a+bx)}$$

Mathematica [A] time = 0.264481, size = 167, normalized size = 0.5

$$\frac{\left((a+bx)^2\right)^{5/2}(d+ex)^{m+1}\left(-\frac{10b^2(d+ex)^2(bd-ae)^3}{m+3}+\frac{10b^3(d+ex)^3(bd-ae)^2}{m+4}-\frac{5b^4(d+ex)^4(bd-ae)}{m+5}+\frac{5b(d+ex)(bd-ae)^4}{m+2}-\frac{(bd-ae)^5}{m+1}+\frac{b^5(d+ex)^5}{m+6}\right)}{e^6(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (((a + b*x)^2)^(5/2)*(d + e*x)^(1 + m)*(-((b*d - a*e)^5/(1 + m)) + (5*b*(b*
d - a*e)^4*(d + e*x))/(2 + m) - (10*b^2*(b*d - a*e)^3*(d + e*x)^2)/(3 + m)
+ (10*b^3*(b*d - a*e)^2*(d + e*x)^3)/(4 + m) - (5*b^4*(b*d - a*e)*(d + e*x)
^4)/(5 + m) + (b^5*(d + e*x)^5)/(6 + m)))/(e^6*(a + b*x)^5)
```

Maple [B] time = 0.16, size = 1361, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] ((b*x+a)^2)^(5/2)*(e*x+d)^(1+m)*(b^5*e^5*m^5*x^5+5*a*b^4*e^5*m^5*x^4+15*b^5
*e^5*m^4*x^5+10*a^2*b^3*e^5*m^5*x^3+80*a*b^4*e^5*m^4*x^4-5*b^5*d*e^4*m^4*x^
4+85*b^5*e^5*m^3*x^5+10*a^3*b^2*e^5*m^5*x^2+170*a^2*b^3*e^5*m^4*x^3-20*a*b^
4*d*e^4*m^4*x^3+475*a*b^4*e^5*m^3*x^4-50*b^5*d*e^4*m^3*x^4+225*b^5*e^5*m^2*
x^5+5*a^4*b*e^5*m^5*x+180*a^3*b^2*e^5*m^4*x^2-30*a^2*b^3*d*e^4*m^4*x^2+1070
*a^2*b^3*e^5*m^3*x^3-240*a*b^4*d*e^4*m^3*x^3+1300*a*b^4*e^5*m^2*x^4+20*b^5*
d^2*e^3*m^3*x^3-175*b^5*d*e^4*m^2*x^4+274*b^5*e^5*m*x^5+a^5*e^5*m^5+95*a^4*
b*e^5*m^4*x-20*a^3*b^2*d*e^4*m^4*x+1210*a^3*b^2*e^5*m^3*x^2-420*a^2*b^3*d*e
^4*m^3*x^2+3070*a^2*b^3*e^5*m^2*x^3+60*a*b^4*d^2*e^3*m^3*x^2-940*a*b^4*d*e^
4*m^2*x^3+1620*a*b^4*e^5*m*x^4+120*b^5*d^2*e^3*m^2*x^3-250*b^5*d*e^4*m*x^4+
120*b^5*e^5*x^5+20*a^5*e^5*m^4-5*a^4*b*d*e^4*m^4+685*a^4*b*e^5*m^3*x-320*a^
3*b^2*d*e^4*m^3*x+3720*a^3*b^2*e^5*m^2*x^2+60*a^2*b^3*d^2*e^3*m^3*x-1950*a^
2*b^3*d*e^4*m^2*x^2+3960*a^2*b^3*e^5*m*x^3+540*a*b^4*d^2*e^3*m^2*x^2-1440*a
*b^4*d*e^4*m*x^3+720*a*b^4*e^5*x^4-60*b^5*d^3*e^2*m^2*x^2+220*b^5*d^2*e^3*m
*x^3-120*b^5*d*e^4*x^4+155*a^5*e^5*m^3-90*a^4*b*d*e^4*m^3+2305*a^4*b*e^5*m^
2*x+20*a^3*b^2*d^2*e^3*m^3-1780*a^3*b^2*d*e^4*m^2*x+5080*a^3*b^2*e^5*m*x^2+
720*a^2*b^3*d^2*e^3*m^2*x-3360*a^2*b^3*d*e^4*m*x^2+1800*a^2*b^3*e^5*x^3-120
*a*b^4*d^3*e^2*m^2*x+1200*a*b^4*d^2*e^3*m*x^2-720*a*b^4*d*e^4*x^3-180*b^5*d
^3*e^2*m*x^2+120*b^5*d^2*e^3*x^3+580*a^5*e^5*m^2-595*a^4*b*d*e^4*m^2+3510*a
^4*b*e^5*m*x+300*a^3*b^2*d^2*e^3*m^2-3880*a^3*b^2*d*e^4*m*x+2400*a^3*b^2*e^
5*x^2-60*a^2*b^3*d^3*e^2*m^2+2460*a^2*b^3*d^2*e^3*m*x-1800*a^2*b^3*d*e^4*x^
```

```
2-840*a*b^4*d^3*e^2*m*x+720*a*b^4*d^2*e^3*x^2+120*b^5*d^4*e*m*x-120*b^5*d^3
*e^2*x^2+1044*a^5*e^5*m-1710*a^4*b*d*e^4*m+1800*a^4*b*e^5*x+1480*a^3*b^2*d^
2*e^3*m-2400*a^3*b^2*d*e^4*x-660*a^2*b^3*d^3*e^2*m+1800*a^2*b^3*d^2*e^3*x+1
20*a*b^4*d^4*e*m-720*a*b^4*d^3*e^2*x+120*b^5*d^4*e*x+720*a^5*e^5-1800*a^4*b
*d*e^4+2400*a^3*b^2*d^2*e^3-1800*a^2*b^3*d^3*e^2+720*a*b^4*d^4*e-120*b^5*d^
5)/(b*x+a)^5/e^6/(m^6+21*m^5+175*m^4+735*m^3+1624*m^2+1764*m+720)
```

Maxima [B] time = 1.24332, size = 1069, normalized size = 3.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] ((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*b^5*e^6*x^6 - 60*(m^2 + 11
*m + 30)*a^2*b^3*d^4*e^2 + 20*(m^3 + 15*m^2 + 74*m + 120)*a^3*b^2*d^3*e^3 -
 5*(m^4 + 18*m^3 + 119*m^2 + 342*m + 360)*a^4*b*d^2*e^4 + (m^5 + 20*m^4 + 1
55*m^3 + 580*m^2 + 1044*m + 720)*a^5*d*e^5 + 120*a*b^4*d^5*e*(m + 6) - 120*
b^5*d^6 + ((m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*b^5*d*e^5 + 5*(m^5 + 16*
m^4 + 95*m^3 + 260*m^2 + 324*m + 144)*a*b^4*e^6)*x^5 - 5*((m^4 + 6*m^3 + 11
*m^2 + 6*m)*b^5*d^2*e^4 - (m^5 + 12*m^4 + 47*m^3 + 72*m^2 + 36*m)*a*b^4*d*e
^5 - 2*(m^5 + 17*m^4 + 107*m^3 + 307*m^2 + 396*m + 180)*a^2*b^3*e^6)*x^4 +
10*(2*(m^3 + 3*m^2 + 2*m)*b^5*d^3*e^3 - 2*(m^4 + 9*m^3 + 20*m^2 + 12*m)*a*b
^4*d^2*e^4 + (m^5 + 14*m^4 + 65*m^3 + 112*m^2 + 60*m)*a^2*b^3*d*e^5 + (m^5
+ 18*m^4 + 121*m^3 + 372*m^2 + 508*m + 240)*a^3*b^2*e^6)*x^3 - 5*(12*(m^2 +
 m)*b^5*d^4*e^2 - 12*(m^3 + 7*m^2 + 6*m)*a*b^4*d^3*e^3 + 6*(m^4 + 12*m^3 +
41*m^2 + 30*m)*a^2*b^3*d^2*e^4 - 2*(m^5 + 16*m^4 + 89*m^3 + 194*m^2 + 120*m
)*a^3*b^2*d*e^5 - (m^5 + 19*m^4 + 137*m^3 + 461*m^2 + 702*m + 360)*a^4*b*e^
6)*x^2 - (120*(m^2 + 6*m)*a*b^4*d^4*e^2 - 60*(m^3 + 11*m^2 + 30*m)*a^2*b^3*
d^3*e^3 + 20*(m^4 + 15*m^3 + 74*m^2 + 120*m)*a^3*b^2*d^2*e^4 - 5*(m^5 + 18*
m^4 + 119*m^3 + 342*m^2 + 360*m)*a^4*b*d*e^5 - (m^5 + 20*m^4 + 155*m^3 + 58
0*m^2 + 1044*m + 720)*a^5*e^6 - 120*b^5*d^5*e*m)*x)*(e*x + d)^m/((m^6 + 21*
m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^6)
```

Fricas [B] time = 1.84347, size = 3060, normalized size = 9.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] (a^5*d*e^5*m^5 - 120*b^5*d^6 + 720*a*b^4*d^5*e - 1800*a^2*b^3*d^4*e^2 + 240
0*a^3*b^2*d^3*e^3 - 1800*a^4*b*d^2*e^4 + 720*a^5*d*e^5 + (b^5*e^6*m^5 + 15*
b^5*e^6*m^4 + 85*b^5*e^6*m^3 + 225*b^5*e^6*m^2 + 274*b^5*e^6*m + 120*b^5*e^
6)*x^6 + (720*a*b^4*e^6 + (b^5*d*e^5 + 5*a*b^4*e^6)*m^5 + 10*(b^5*d*e^5 + 8
*a*b^4*e^6)*m^4 + 5*(7*b^5*d*e^5 + 95*a*b^4*e^6)*m^3 + 50*(b^5*d*e^5 + 26*a
*b^4*e^6)*m^2 + 12*(2*b^5*d*e^5 + 135*a*b^4*e^6)*m)*x^5 - 5*(a^4*b*d^2*e^4
- 4*a^5*d*e^5)*m^4 + 5*(360*a^2*b^3*e^6 + (a*b^4*d*e^5 + 2*a^2*b^3*e^6)*m^5
 - (b^5*d^2*e^4 - 12*a*b^4*d*e^5 - 34*a^2*b^3*e^6)*m^4 - (6*b^5*d^2*e^4 - 4
7*a*b^4*d*e^5 - 214*a^2*b^3*e^6)*m^3 - (11*b^5*d^2*e^4 - 72*a*b^4*d*e^5 - 6
14*a^2*b^3*e^6)*m^2 - 6*(b^5*d^2*e^4 - 6*a*b^4*d*e^5 - 132*a^2*b^3*e^6)*m)*
x^4 + 5*(4*a^3*b^2*d^3*e^3 - 18*a^4*b*d^2*e^4 + 31*a^5*d*e^5)*m^3 + 10*(240
*a^3*b^2*e^6 + (a^2*b^3*d*e^5 + a^3*b^2*e^6)*m^5 - 2*(a*b^4*d^2*e^4 - 7*a^2
```

```
*b^3*d*e^5 - 9*a^3*b^2*e^6)*m^4 + (2*b^5*d^3*e^3 - 18*a*b^4*d^2*e^4 + 65*a^
2*b^3*d*e^5 + 121*a^3*b^2*e^6)*m^3 + 2*(3*b^5*d^3*e^3 - 20*a*b^4*d^2*e^4 +
56*a^2*b^3*d*e^5 + 186*a^3*b^2*e^6)*m^2 + 4*(b^5*d^3*e^3 - 6*a*b^4*d^2*e^4
+ 15*a^2*b^3*d*e^5 + 127*a^3*b^2*e^6)*m)*x^3 - 5*(12*a^2*b^3*d^4*e^2 - 60*a
^3*b^2*d^3*e^3 + 119*a^4*b*d^2*e^4 - 116*a^5*d*e^5)*m^2 + 5*(360*a^4*b*e^6
+ (2*a^3*b^2*d*e^5 + a^4*b*e^6)*m^5 - (6*a^2*b^3*d^2*e^4 - 32*a^3*b^2*d*e^5
 - 19*a^4*b*e^6)*m^4 + (12*a*b^4*d^3*e^3 - 72*a^2*b^3*d^2*e^4 + 178*a^3*b^2
*d*e^5 + 137*a^4*b*e^6)*m^3 - (12*b^5*d^4*e^2 - 84*a*b^4*d^3*e^3 + 246*a^2*
b^3*d^2*e^4 - 388*a^3*b^2*d*e^5 - 461*a^4*b*e^6)*m^2 - 6*(2*b^5*d^4*e^2 - 1
2*a*b^4*d^3*e^3 + 30*a^2*b^3*d^2*e^4 - 40*a^3*b^2*d*e^5 - 117*a^4*b*e^6)*m)
*x^2 + 2*(60*a*b^4*d^5*e - 330*a^2*b^3*d^4*e^2 + 740*a^3*b^2*d^3*e^3 - 855*
a^4*b*d^2*e^4 + 522*a^5*d*e^5)*m + (720*a^5*e^6 + (5*a^4*b*d*e^5 + a^5*e^6)
*m^5 - 10*(2*a^3*b^2*d^2*e^4 - 9*a^4*b*d*e^5 - 2*a^5*e^6)*m^4 + 5*(12*a^2*b
^3*d^3*e^3 - 60*a^3*b^2*d^2*e^4 + 119*a^4*b*d*e^5 + 31*a^5*e^6)*m^3 - 10*(1
2*a*b^4*d^4*e^2 - 66*a^2*b^3*d^3*e^3 + 148*a^3*b^2*d^2*e^4 - 171*a^4*b*d*e^
5 - 58*a^5*e^6)*m^2 + 12*(10*b^5*d^5*e - 60*a*b^4*d^4*e^2 + 150*a^2*b^3*d^3
*e^3 - 200*a^3*b^2*d^2*e^4 + 150*a^4*b*d*e^5 + 87*a^5*e^6)*m)*x)*(e*x + d)^
m/(e^6*m^6 + 21*e^6*m^5 + 175*e^6*m^4 + 735*e^6*m^3 + 1624*e^6*m^2 + 1764*e
^6*m + 720*e^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.38734, size = 4316, normalized size = 12.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*b^5*m^5*x^6*e^6*sgn(b*x + a) + (x*e + d)^m*b^5*d*m^5*x^5*e^5*s
gn(b*x + a) + 5*(x*e + d)^m*a*b^4*m^5*x^5*e^6*sgn(b*x + a) + 15*(x*e + d)^m
*b^5*m^4*x^6*e^6*sgn(b*x + a) + 5*(x*e + d)^m*a*b^4*d*m^5*x^4*e^5*sgn(b*x +
 a) + 10*(x*e + d)^m*b^5*d*m^4*x^5*e^5*sgn(b*x + a) - 5*(x*e + d)^m*b^5*d^2
*m^4*x^4*e^4*sgn(b*x + a) + 10*(x*e + d)^m*a^2*b^3*m^5*x^4*e^6*sgn(b*x + a)
 + 80*(x*e + d)^m*a*b^4*m^4*x^5*e^6*sgn(b*x + a) + 85*(x*e + d)^m*b^5*m^3*x
^6*e^6*sgn(b*x + a) + 10*(x*e + d)^m*a^2*b^3*d*m^5*x^3*e^5*sgn(b*x + a) + 6
0*(x*e + d)^m*a*b^4*d*m^4*x^4*e^5*sgn(b*x + a) + 35*(x*e + d)^m*b^5*d*m^3*x
^5*e^5*sgn(b*x + a) - 20*(x*e + d)^m*a*b^4*d^2*m^4*x^3*e^4*sgn(b*x + a) - 3
0*(x*e + d)^m*b^5*d^2*m^3*x^4*e^4*sgn(b*x + a) + 20*(x*e + d)^m*b^5*d^3*m^3
*x^3*e^3*sgn(b*x + a) + 10*(x*e + d)^m*a^3*b^2*m^5*x^3*e^6*sgn(b*x + a) + 1
70*(x*e + d)^m*a^2*b^3*m^4*x^4*e^6*sgn(b*x + a) + 475*(x*e + d)^m*a*b^4*m^3
*x^5*e^6*sgn(b*x + a) + 225*(x*e + d)^m*b^5*m^2*x^6*e^6*sgn(b*x + a) + 10*(
x*e + d)^m*a^3*b^2*d*m^5*x^2*e^5*sgn(b*x + a) + 140*(x*e + d)^m*a^2*b^3*d*m
^4*x^3*e^5*sgn(b*x + a) + 235*(x*e + d)^m*a*b^4*d*m^3*x^4*e^5*sgn(b*x + a)
+ 50*(x*e + d)^m*b^5*d*m^2*x^5*e^5*sgn(b*x + a) - 30*(x*e + d)^m*a^2*b^3*d^
```

2*m^4*x^2*e^4*sgn(b*x + a) - 180*(x*e + d)^m*a*b^4*d^2*m^3*x^3*e^4*sgn(b*x + a) - 55*(x*e + d)^m*b^5*d^2*m^2*x^4*e^4*sgn(b*x + a) + 60*(x*e + d)^m*a*b^4*d^3*m^3*x^2*e^3*sgn(b*x + a) + 60*(x*e + d)^m*b^5*d^3*m^2*x^3*e^3*sgn(b*x + a) - 60*(x*e + d)^m*b^5*d^4*m^2*x^2*e^2*sgn(b*x + a) + 5*(x*e + d)^m*a^4*b*m^5*x^2*e^6*sgn(b*x + a) + 180*(x*e + d)^m*a^3*b^2*m^4*x^3*e^6*sgn(b*x + a) + 1070*(x*e + d)^m*a^2*b^3*m^3*x^4*e^6*sgn(b*x + a) + 1300*(x*e + d)^m*a*b^4*m^2*x^5*e^6*sgn(b*x + a) + 274*(x*e + d)^m*b^5*m*x^6*e^6*sgn(b*x + a) + 5*(x*e + d)^m*a^4*b*d*m^5*x*e^5*sgn(b*x + a) + 160*(x*e + d)^m*a^3*b^2*d*m^4*x^2*e^5*sgn(b*x + a) + 650*(x*e + d)^m*a^2*b^3*d*m^3*x^3*e^5*sgn(b*x + a) + 360*(x*e + d)^m*a*b^4*d*m^2*x^4*e^5*sgn(b*x + a) + 24*(x*e + d)^m*b^5*d*m*x^5*e^5*sgn(b*x + a) - 20*(x*e + d)^m*a^3*b^2*d^2*m^4*x*e^4*sgn(b*x + a) - 360*(x*e + d)^m*a^2*b^3*d^2*m^3*x^2*e^4*sgn(b*x + a) - 400*(x*e + d)^m*a*b^4*d^2*m^2*x^3*e^4*sgn(b*x + a) - 30*(x*e + d)^m*b^5*d^2*m*x^4*e^4*sgn(b*x + a) + 60*(x*e + d)^m*a^2*b^3*d^3*m^3*x*e^3*sgn(b*x + a) + 420*(x*e + d)^m*a*b^4*d^3*m^2*x^2*e^3*sgn(b*x + a) + 40*(x*e + d)^m*b^5*d^3*m*x^3*e^3*sgn(b*x + a) - 120*(x*e + d)^m*a*b^4*d^4*m^2*x*e^2*sgn(b*x + a) - 60*(x*e + d)^m*b^5*d^4*m*x^2*e^2*sgn(b*x + a) + 120*(x*e + d)^m*b^5*d^5*m*x*e*sgn(b*x + a) + (x*e + d)^m*a^5*m^5*x*e^6*sgn(b*x + a) + 95*(x*e + d)^m*a^4*b*m^4*x^2*e^6*sgn(b*x + a) + 1210*(x*e + d)^m*a^3*b^2*m^3*x^3*e^6*sgn(b*x + a) + 3070*(x*e + d)^m*a^2*b^3*m^2*x^4*e^6*sgn(b*x + a) + 1620*(x*e + d)^m*a*b^4*m*x^5*e^6*sgn(b*x + a) + 120*(x*e + d)^m*b^5*x^6*e^6*sgn(b*x + a) + (x*e + d)^m*a^5*d*m^5*e^5*sgn(b*x + a) + 90*(x*e + d)^m*a^4*b*d*m^4*x*e^5*sgn(b*x + a) + 890*(x*e + d)^m*a^3*b^2*d*m^3*x^2*e^5*sgn(b*x + a) + 1120*(x*e + d)^m*a^2*b^3*d*m^2*x^3*e^5*sgn(b*x + a) + 180*(x*e + d)^m*a*b^4*d*m*x^4*e^5*sgn(b*x + a) - 5*(x*e + d)^m*a^4*b*d^2*m^4*e^4*sgn(b*x + a) - 300*(x*e + d)^m*a^3*b^2*d^2*m^3*x*e^4*sgn(b*x + a) - 1230*(x*e + d)^m*a^2*b^3*d^2*m^2*x^2*e^4*sgn(b*x + a) - 240*(x*e + d)^m*a*b^4*d^2*m*x^3*e^4*sgn(b*x + a) + 20*(x*e + d)^m*a^3*b^2*d^3*m^3*e^3*sgn(b*x + a) + 660*(x*e + d)^m*a^2*b^3*d^3*m^2*x*e^3*sgn(b*x + a) + 360*(x*e + d)^m*a*b^4*d^3*m*x^2*e^3*sgn(b*x + a) - 60*(x*e + d)^m*a^2*b^3*d^4*m^2*e^2*sgn(b*x + a) - 720*(x*e + d)^m*a*b^4*d^4*m*x*e^2*sgn(b*x + a) + 120*(x*e + d)^m*a*b^4*d^5*m*e*sgn(b*x + a) - 120*(x*e + d)^m*b^5*d^6*sgn(b*x + a) + 20*(x*e + d)^m*a^5*m^4*x*e^6*sgn(b*x + a) + 685*(x*e + d)^m*a^4*b*m^3*x^2*e^6*sgn(b*x + a) + 3720*(x*e + d)^m*a^3*b^2*m^2*x^3*e^6*sgn(b*x + a) + 3960*(x*e + d)^m*a^2*b^3*m*x^4*e^6*sgn(b*x + a) + 720*(x*e + d)^m*a*b^4*x^5*e^6*sgn(b*x + a) + 20*(x*e + d)^m*a^5*d*m^4*e^5*sgn(b*x + a) + 595*(x*e + d)^m*a^4*b*d*m^3*x*e^5*sgn(b*x + a) + 1940*(x*e + d)^m*a^3*b^2*d*m^2*x^2*e^5*sgn(b*x + a) + 600*(x*e + d)^m*a^2*b^3*d*m*x^3*e^5*sgn(b*x + a) - 90*(x*e + d)^m*a^4*b*d^2*m^3*e^4*sgn(b*x + a) - 1480*(x*e + d)^m*a^3*b^2*d^2*m^2*x*e^4*sgn(b*x + a) - 900*(x*e + d)^m*a^2*b^3*d^2*m*x^2*e^4*sgn(b*x + a) + 300*(x*e + d)^m*a^3*b^2*d^3*m^2*e^3*sgn(b*x + a) + 1800*(x*e + d)^m*a^2*b^3*d^3*m*x*e^3*sgn(b*x + a) - 660*(x*e + d)^m*a^2*b^3*d^4*m*e^2*sgn(b*x + a) + 720*(x*e + d)^m*a*b^4*d^5*e*sgn(b*x + a) + 155*(x*e + d)^m*a^5*m^3*x*e^6*sgn(b*x + a) + 2305*(x*e + d)^m*a^4*b*m^2*x^2*e^6*sgn(b*x + a) + 5080*(x*e + d)^m*a^3*b^2*m*x^3*e^6*sgn(b*x + a) + 1800*(x*e + d)^m*a^2*b^3*x^4*e^6*sgn(b*x + a) + 155*(x*e + d)^m*a^5*d*m^3*e^5*sgn(b*x + a) + 1710*(x*e + d)^m*a^4*b*d*m^2*x*e^5*sgn(b*x + a) + 1200*(x*e + d)^m*a^3*b^2*d*m*x^2*e^5*sgn(b*x + a) - 595*(x*e + d)^m*a^4*b*d^2*m^2*e^4*sgn(b*x + a) - 2400*(x*e + d)^m*a^3*b^2*d^2*m*x*e^4*sgn(b*x + a) + 1480*(x*e + d)^m*a^3*b^2*d^3*m*e^3*sgn(b*x + a) - 1800*(x*e + d)^m*a^2*b^3*d^4*e^2*sgn(b*x + a) + 580*(x*e + d)^m*a^5*m^2*x*e^6*sgn(b*x + a) + 3510*(x*e + d)^m*a^4*b*m*x^2*e^6*sgn(b*x + a) + 2400*(x*e + d)^m*a^3*b^2*x^3*e^6*sgn(b*x + a) + 580*(x*e + d)^m*a^5*d*m^2*e^5*sgn(b*x + a) + 1800*(x*e + d)^m*a^4*b*d*m*x*e^5*sgn(b*x + a) - 1710*(x*e + d)^m*a^4*b*d^2*m*e^4*sgn(b*x + a) + 2400*(x*e + d)^m*a^3*b^2*d^3*e^3*sgn(b*x + a) + 1044*(x*e + d)^m*a^5*m*x*e^6*sgn(b*x + a) + 1800*(x*e + d)^m*a^4*b*x^2*e^6*sgn(b*x + a) + 1044*(x*e + d)^m*a^5*d*m*e^5*sgn(b*x + a) - 1800*(x*e + d)^m*a^4*b*d^2*e^4*sgn(b*x + a) + 720*(x*e + d)^m*a^5*x*e^6*sgn(b*x + a) + 720*(x*e + d)^m*a^5*d*e^5*sgn(b*x + a))/(m^6*e^6 + 21*m^5*e^6 + 175*m^4*e^6 + 735*m^3*e^6 + 1624*m^2*e^6 + 1764*m*e^6 + 720*e^6)

3.1738 $\int (d+ex)^m \left(a^2+2abx+b^2x^2\right)^{3/2} \, dx$

Optimal. Leaf size=219

$$-\frac{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3(d+ex)^{m+1}}{e^4(m+1)(a+bx)}+\frac{3b\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2(d+ex)^{m+2}}{e^4(m+2)(a+bx)}-\frac{3b^2\sqrt{a^2+2abx+b^2x^2}(bd\cdots}{e^4(m+3)(a+\cdots}$$

```
[Out] -(((b*d - a*e)^3*(d + e*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^4*(1 +
 m)*(a + b*x))) + (3*b*(b*d - a*e)^2*(d + e*x)^(2 + m)*Sqrt[a^2 + 2*a*b*x +
 b^2*x^2])/(e^4*(2 + m)*(a + b*x)) - (3*b^2*(b*d - a*e)*(d + e*x)^(3 + m)*S
qrt[a^2 + 2*a*b*x + b^2*x^2])/(e^4*(3 + m)*(a + b*x)) + (b^3*(d + e*x)^(4 +
 m)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^4*(4 + m)*(a + b*x))
```

Rubi [A] time = 0.0876123, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$-\frac{\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3(d+ex)^{m+1}}{e^4(m+1)(a+bx)}+\frac{3b\sqrt{a^2+2abx+b^2x^2}(bd-ae)^2(d+ex)^{m+2}}{e^4(m+2)(a+bx)}-\frac{3b^2\sqrt{a^2+2abx+b^2x^2}(bd\cdots}{e^4(m+3)(a+\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```


```
[Out] -(((b*d - a*e)^3*(d + e*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^4*(1 +
 m)*(a + b*x))) + (3*b*(b*d - a*e)^2*(d + e*x)^(2 + m)*Sqrt[a^2 + 2*a*b*x +
 b^2*x^2])/(e^4*(2 + m)*(a + b*x)) - (3*b^2*(b*d - a*e)*(d + e*x)^(3 + m)*S
qrt[a^2 + 2*a*b*x + b^2*x^2])/(e^4*(3 + m)*(a + b*x)) + (b^3*(d + e*x)^(4 +
 m)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^4*(4 + m)*(a + b*x))
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (d+ex)^m \left(a^2+2abx+b^2x^2\right)^{3/2} dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(ab+b^2x\right)^3 (d+ex)^m \, dx}{b^2\left(ab+b^2x\right)}$$

$$= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b^3(bd-ae)^3(d+ex)^m}{e^3}+\frac{3b^4(bd-ae)^2(d+ex)^{1+m}}{e^3}-\frac{3b^5(bd-ae)(d+ex)^{2+\cdots}}{e^3}\cdots\right.}{b^2\left(ab+b^2x\right)}$$

$$= -\frac{(bd-ae)^3(d+ex)^{1+m}\sqrt{a^2+2abx+b^2x^2}}{e^4(1+m)(a+bx)}+\frac{3b(bd-ae)^2(d+ex)^{2+m}\sqrt{a^2+2\cdots}}{e^4(2+m)(a+bx)}$$

Mathematica [A] time = 0.118283, size = 113, normalized size = 0.52

$$\frac{\left((a+bx)^2\right)^{3/2}(d+ex)^{m+1}\left(-\frac{3b^2(d+ex)^2(bd-ae)}{m+3}+\frac{3b(d+ex)(bd-ae)^2}{m+2}-\frac{(bd-ae)^3}{m+1}+\frac{b^3(d+ex)^3}{m+4}\right)}{e^4(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (((a + b*x)^2)^(3/2)*(d + e*x)^(1 + m)*(-((b*d - a*e)^3/(1 + m)) + (3*b*(b*
d - a*e)^2*(d + e*x))/(2 + m) - (3*b^2*(b*d - a*e)*(d + e*x)^2)/(3 + m) + (
b^3*(d + e*x)^3)/(4 + m)))/(e^4*(a + b*x)^3)
```

Maple [B] time = 0.155, size = 402, normalized size = 1.8

$$(ex+d)^{1+m}\left(b^3e^3m^3x^3+3ab^2e^3m^3x^2+6b^3e^3m^2x^3+3a^2be^3m^3x+21ab^2e^3m^2x^2-3b^3de^2m^2x^2+11b^3e^3mx^3+a^3e^3m^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] ((b*x+a)^2)^(3/2)*(e*x+d)^(1+m)*(b^3*e^3*m^3*x^3+3*a*b^2*e^3*m^3*x^2+6*b^3*
e^3*m^2*x^3+3*a^2*b*e^3*m^3*x+21*a*b^2*e^3*m^2*x^2-3*b^3*d*e^2*m^2*x^2+11*b
^3*e^3*m*x^3+a^3*e^3*m^3+24*a^2*b*e^3*m^2*x-6*a*b^2*d*e^2*m^2*x+42*a*b^2*e^
3*m*x^2-9*b^3*d*e^2*m*x^2+6*b^3*e^3*x^3+9*a^3*e^3*m^2-3*a^2*b*d*e^2*m^2+57*
a^2*b*e^3*m*x-30*a*b^2*d*e^2*m*x+24*a*b^2*e^3*x^2+6*b^3*d^2*e*m*x-6*b^3*d*e
^2*x^2+26*a^3*e^3*m-21*a^2*b*d*e^2*m+36*a^2*b*e^3*x+6*a*b^2*d^2*e*m-24*a*b^
2*d*e^2*x+6*b^3*d^2*e*x+24*a^3*e^3-36*a^2*b*d*e^2+24*a*b^2*d^2*e-6*b^3*d^3)
/(b*x+a)^3/e^4/(m^4+10*m^3+35*m^2+50*m+24)
```

Maxima [A] time = 1.18128, size = 412, normalized size = 1.88

$$\left(\left(m^3+6m^2+11m+6\right)b^3e^4x^4-3\left(m^2+7m+12\right)a^2bd^2e^2+\left(m^3+9m^2+26m+24\right)a^3de^3+6ab^2d^3e(m+4)-6b^3d^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] ((m^3 + 6*m^2 + 11*m + 6)*b^3*e^4*x^4 - 3*(m^2 + 7*m + 12)*a^2*b*d^2*e^2 +
(m^3 + 9*m^2 + 26*m + 24)*a^3*d*e^3 + 6*a*b^2*d^3*e*(m + 4) - 6*b^3*d^4 + (
(m^3 + 3*m^2 + 2*m)*b^3*d*e^3 + 3*(m^3 + 7*m^2 + 14*m + 8)*a*b^2*e^4)*x^3 -
 3*((m^2 + m)*b^3*d^2*e^2 - (m^3 + 5*m^2 + 4*m)*a*b^2*d*e^3 - (m^3 + 8*m^2
+ 19*m + 12)*a^2*b*e^4)*x^2 - (6*(m^2 + 4*m)*a*b^2*d^2*e^2 - 3*(m^3 + 7*m^2
 + 12*m)*a^2*b*d*e^3 - (m^3 + 9*m^2 + 26*m + 24)*a^3*e^4 - 6*b^3*d^3*e*m)*x
)*(e*x + d)^m/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4)
```

Fricas [B] time = 1.67178, size = 1011, normalized size = 4.62

$$\left(a^3de^3m^3-6b^3d^4+24ab^2d^3e-36a^2bd^2e^2+24a^3de^3+\left(b^3e^4m^3+6b^3e^4m^2+11b^3e^4m+6b^3e^4\right)x^4+\left(24ab^2e^4+\left(b^3de\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (a^3*d*e^3*m^3 - 6*b^3*d^4 + 24*a*b^2*d^3*e - 36*a^2*b*d^2*e^2 + 24*a^3*d*e
^3 + (b^3*e^4*m^3 + 6*b^3*e^4*m^2 + 11*b^3*e^4*m + 6*b^3*e^4)*x^4 + (24*a*b
^2*e^4 + (b^3*d*e^3 + 3*a*b^2*e^4)*m^3 + 3*(b^3*d*e^3 + 7*a*b^2*e^4)*m^2 +
2*(b^3*d*e^3 + 21*a*b^2*e^4)*m)*x^3 - 3*(a^2*b*d^2*e^2 - 3*a^3*d*e^3)*m^2 +
 3*(12*a^2*b*e^4 + (a*b^2*d*e^3 + a^2*b*e^4)*m^3 - (b^3*d^2*e^2 - 5*a*b^2*d
*e^3 - 8*a^2*b*e^4)*m^2 - (b^3*d^2*e^2 - 4*a*b^2*d*e^3 - 19*a^2*b*e^4)*m)*x
^2 + (6*a*b^2*d^3*e - 21*a^2*b*d^2*e^2 + 26*a^3*d*e^3)*m + (24*a^3*e^4 + (3
*a^2*b*d*e^3 + a^3*e^4)*m^3 - 3*(2*a*b^2*d^2*e^2 - 7*a^2*b*d*e^3 - 3*a^3*e^
4)*m^2 + 2*(3*b^3*d^3*e - 12*a*b^2*d^2*e^2 + 18*a^2*b*d*e^3 + 13*a^3*e^4)*m
)*x)*(e*x + d)^m/(e^4*m^4 + 10*e^4*m^3 + 35*e^4*m^2 + 50*e^4*m + 24*e^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24507, size = 1451, normalized size = 6.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*b^3*m^3*x^4*e^4*sgn(b*x + a) + (x*e + d)^m*b^3*d*m^3*x^3*e^3*s
gn(b*x + a) + 3*(x*e + d)^m*a*b^2*m^3*x^3*e^4*sgn(b*x + a) + 6*(x*e + d)^m*
b^3*m^2*x^4*e^4*sgn(b*x + a) + 3*(x*e + d)^m*a*b^2*d*m^3*x^2*e^3*sgn(b*x +
a) + 3*(x*e + d)^m*b^3*d*m^2*x^3*e^3*sgn(b*x + a) - 3*(x*e + d)^m*b^3*d^2*m
^2*x^2*e^2*sgn(b*x + a) + 3*(x*e + d)^m*a^2*b*m^3*x^2*e^4*sgn(b*x + a) + 21
*(x*e + d)^m*a*b^2*m^2*x^3*e^4*sgn(b*x + a) + 11*(x*e + d)^m*b^3*m*x^4*e^4*
sgn(b*x + a) + 3*(x*e + d)^m*a^2*b*d*m^3*x*e^3*sgn(b*x + a) + 15*(x*e + d)^
m*a*b^2*d*m^2*x^2*e^3*sgn(b*x + a) + 2*(x*e + d)^m*b^3*d*m*x^3*e^3*sgn(b*x
+ a) - 6*(x*e + d)^m*a*b^2*d^2*m^2*x*e^2*sgn(b*x + a) - 3*(x*e + d)^m*b^3*d
^2*m*x^2*e^2*sgn(b*x + a) + 6*(x*e + d)^m*b^3*d^3*m*x*e*sgn(b*x + a) + (x*e
 + d)^m*a^3*m^3*x*e^4*sgn(b*x + a) + 24*(x*e + d)^m*a^2*b*m^2*x^2*e^4*sgn(b
*x + a) + 42*(x*e + d)^m*a*b^2*m*x^3*e^4*sgn(b*x + a) + 6*(x*e + d)^m*b^3*x
^4*e^4*sgn(b*x + a) + (x*e + d)^m*a^3*d*m^3*e^3*sgn(b*x + a) + 21*(x*e + d)
^m*a^2*b*d*m^2*x*e^3*sgn(b*x + a) + 12*(x*e + d)^m*a*b^2*d*m*x^2*e^3*sgn(b*
x + a) - 3*(x*e + d)^m*a^2*b*d^2*m^2*e^2*sgn(b*x + a) - 24*(x*e + d)^m*a*b^
2*d^2*m*x*e^2*sgn(b*x + a) + 6*(x*e + d)^m*a*b^2*d^3*m*e*sgn(b*x + a) - 6*(
x*e + d)^m*b^3*d^4*sgn(b*x + a) + 9*(x*e + d)^m*a^3*m^2*x*e^4*sgn(b*x + a)
+ 57*(x*e + d)^m*a^2*b*m*x^2*e^4*sgn(b*x + a) + 24*(x*e + d)^m*a*b^2*x^3*e^
4*sgn(b*x + a) + 9*(x*e + d)^m*a^3*d*m^2*e^3*sgn(b*x + a) + 36*(x*e + d)^m*
a^2*b*d*m*x*e^3*sgn(b*x + a) - 21*(x*e + d)^m*a^2*b*d^2*m*e^2*sgn(b*x + a)
+ 24*(x*e + d)^m*a*b^2*d^3*e*sgn(b*x + a) + 26*(x*e + d)^m*a^3*m*x*e^4*sgn(
```

```
b*x + a) + 36*(x*e + d)^m*a^2*b*x^2*e^4*sgn(b*x + a) + 26*(x*e + d)^m*a^3*d
*m*e^3*sgn(b*x + a) - 36*(x*e + d)^m*a^2*b*d^2*e^2*sgn(b*x + a) + 24*(x*e +
 d)^m*a^3*x*e^4*sgn(b*x + a) + 24*(x*e + d)^m*a^3*d*e^3*sgn(b*x + a))/(m^4*
e^4 + 10*m^3*e^4 + 35*m^2*e^4 + 50*m*e^4 + 24*e^4)
```

3.1739 $\int (d+ex)^m \sqrt{a^2+2abx+b^2x^2}\,dx$

Optimal. Leaf size=101

$$\frac{b\sqrt{a^2+2abx+b^2x^2}(d+ex)^{m+2}}{e^2(m+2)(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}(bd-ae)(d+ex)^{m+1}}{e^2(m+1)(a+bx)}$$

```
[Out] -(((b*d - a*e)*(d + e*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^2*(1 + m
)*(a + b*x))) + (b*(d + e*x)^(2 + m)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^2*(2
 + m)*(a + b*x))
```

Rubi [A] time = 0.0446243, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 43}

$$\frac{b\sqrt{a^2+2abx+b^2x^2}(d+ex)^{m+2}}{e^2(m+2)(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}(bd-ae)(d+ex)^{m+1}}{e^2(m+1)(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] -(((b*d - a*e)*(d + e*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^2*(1 + m
)*(a + b*x))) + (b*(d + e*x)^(2 + m)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(e^2*(2
 + m)*(a + b*x))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^m\sqrt{a^2+2abx+b^2x^2}\,dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(ab+b^2x\right)(d+ex)^m\,dx}{ab+b^2x}\\
&= \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(-\frac{b(bd-ae)(d+ex)^m}{e}+\frac{b^2(d+ex)^{1+m}}{e}\right)dx}{ab+b^2x}\\
&= -\frac{(bd-ae)(d+ex)^{1+m}\sqrt{a^2+2abx+b^2x^2}}{e^2(1+m)(a+bx)}+\frac{b(d+ex)^{2+m}\sqrt{a^2+2abx+b^2x^2}}{e^2(2+m)(a+bx)}
\end{aligned}$$

Mathematica [A] time = 0.0437715, size = 59, normalized size = 0.58

$$\frac{\sqrt{(a+bx)^2}(d+ex)^{m+1}(ae(m+2)-bd+be(m+1)x)}{e^2(m+1)(m+2)(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(d + e*x)^(1 + m)*(-(b*d) + a*e*(2 + m) + b*e*(1 + m)*x)
)/(e^2*(1 + m)*(2 + m)*(a + b*x))
```

Maple [A] time = 0.152, size = 62, normalized size = 0.6

$$\frac{(ex+d)^{1+m}(bemx+aem+bxe+2\,ae-bd)}{(bx+a)\,e^2\left(m^2+3\,m+2\right)}\sqrt{(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^(1/2),x)
```

```
[Out] ((b*x+a)^2)^(1/2)*(e*x+d)^(1+m)*(b*e*m*x+a*e*m+b*e*x+2*a*e-b*d)/(b*x+a)/e^2
/(m^2+3*m+2)
```

Maxima [A] time = 1.07785, size = 84, normalized size = 0.83

$$\frac{\left(be^2(m+1)x^2+ade(m+2)-bd^2+\left(ae^2(m+2)+bdem\right)x\right)(ex+d)^m}{\left(m^2+3\,m+2\right)e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (b*e^2*(m + 1)*x^2 + a*d*e*(m + 2) - b*d^2 + (a*e^2*(m + 2) + b*d*e*m)*x)*(
e*x + d)^m/((m^2 + 3*m + 2)*e^2)
```

Fricas [A] time = 1.63927, size = 171, normalized size = 1.69

$$\frac{\left(adem-bd^2+2\,ade+\left(be^2m+be^2\right)x^2+\left(2\,ae^2+\left(bde+ae^2\right)m\right)x\right)(ex+d)^m}{e^2m^2+3\,e^2m+2\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (a*d*e*m - b*d^2 + 2*a*d*e + (b*e^2*m + b*e^2)*x^2 + (2*a*e^2 + (b*d*e + a*
e^2)*m)*x)*(e*x + d)^m/(e^2*m^2 + 3*e^2*m + 2*e^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d+ex)^m\sqrt{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(b**2*x**2+2*a*b*x+a**2)**(1/2),x)
```

```
[Out] Integral((d + e*x)**m*sqrt((a + b*x)**2), x)
```

Giac [B] time = 1.16174, size = 248, normalized size = 2.46

$$\frac{(xe+d)^m bmx^2e^2\mathrm{sgn}(bx+a) + (xe+d)^m bdmxe\mathrm{sgn}(bx+a) + (xe+d)^m amxe^2\mathrm{sgn}(bx+a) + (xe+d)^m bx^2e^2\mathrm{sgn}(bx+}{m^2e^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*b*m*x^2*e^2*sgn(b*x + a) + (x*e + d)^m*b*d*m*x*e*sgn(b*x + a)
+ (x*e + d)^m*a*m*x*e^2*sgn(b*x + a) + (x*e + d)^m*b*x^2*e^2*sgn(b*x + a) +
 (x*e + d)^m*a*d*m*e*sgn(b*x + a) - (x*e + d)^m*b*d^2*sgn(b*x + a) + 2*(x*e
 + d)^m*a*x*e^2*sgn(b*x + a) + 2*(x*e + d)^m*a*d*e*sgn(b*x + a))/(m^2*e^2 +
 3*m*e^2 + 2*e^2)
```

3.1740 $\int \frac{(d+ex)^m}{\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=76

$$-\frac{(a+bx)(d+ex)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{b(d+ex)}{bd-ae}\right)}{(m+1)\sqrt{a^2+2abx+b^2x^2}(bd-ae)}$$

```
[Out] -(((a + b*x)*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (b*(d + e
*x))/(b*d - a*e)])/((b*d - a*e)*(1 + m)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]))
```

Rubi [A] time = 0.0374753, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 68}

$$-\frac{(a+bx)(d+ex)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{b(d+ex)}{bd-ae}\right)}{(m+1)\sqrt{a^2+2abx+b^2x^2}(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] -(((a + b*x)*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (b*(d + e
*x))/(b*d - a*e)])/((b*d - a*e)*(1 + m)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^m}{\sqrt{a^2+2abx+b^2x^2}}\,dx &= \frac{\left(ab+b^2x\right)\int \frac{(d+ex)^m}{ab+b^2x}\,dx}{\sqrt{a^2+2abx+b^2x^2}}\\ &= -\frac{(a+bx)(d+ex)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{b(d+ex)}{bd-ae}\right)}{(bd-ae)(1+m)\sqrt{a^2+2abx+b^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0239368, size = 67, normalized size = 0.88

$$-\frac{(a+bx)(d+ex)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{b(d+ex)}{bd-ae}\right)}{(m+1)\sqrt{(a+bx)^2}(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] -(((a + b*x)*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (b*(d + e
*x))/(b*d - a*e)])/((b*d - a*e)*(1 + m)*Sqrt[(a + b*x)^2]))
```

Maple [F] time = 1.032, size = 0, normalized size = 0.

$$\int (ex+d)^m \frac{1}{\sqrt{b^2x^2+2\,abx+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^(1/2),x)
```

```
[Out] int((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\sqrt{b^2x^2+2\,abx+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/sqrt(b^2*x^2 + 2*a*b*x + a^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^m}{\sqrt{b^2x^2+2\,abx+a^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m/sqrt(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m}{\sqrt{(a+bx)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(b**2*x**2+2*a*b*x+a**2)**(1/2),x)
```

```
[Out] Integral((d + e*x)**m/sqrt((a + b*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\sqrt{b^2x^2+2\,abx+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/sqrt(b^2*x^2 + 2*a*b*x + a^2), x)
```

3.1741 $\int \frac{(d+ex)^m}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=79

$$-\frac{e^2(a+bx)(d+ex)^{m+1}\,{}_2F_1\left(3,m+1;m+2;\frac{b(d+ex)}{bd-ae}\right)}{(m+1)\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3}$$

```
[Out] -((e^2*(a + b*x)*(d + e*x)^(1 + m)*Hypergeometric2F1[3, 1 + m, 2 + m, (b*(d
 + e*x))/(b*d - a*e)])/((b*d - a*e)^3*(1 + m)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]
))
```

Rubi [A] time = 0.0467206, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 68}

$$-\frac{e^2(a+bx)(d+ex)^{m+1}\,{}_2F_1\left(3,m+1;m+2;\frac{b(d+ex)}{bd-ae}\right)}{(m+1)\sqrt{a^2+2abx+b^2x^2}(bd-ae)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] -((e^2*(a + b*x)*(d + e*x)^(1 + m)*Hypergeometric2F1[3, 1 + m, 2 + m, (b*(d
 + e*x))/(b*d - a*e)])/((b*d - a*e)^3*(1 + m)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]
))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^m}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx &= \frac{\left(b^2\left(ab+b^2x\right)\right)\int \frac{(d+ex)^m}{\left(ab+b^2x\right)^3}\,dx}{\sqrt{a^2+2abx+b^2x^2}}\\ &= -\frac{e^2(a+bx)(d+ex)^{1+m}\,{}_2F_1\left(3,1+m;2+m;\frac{b(d+ex)}{bd-ae}\right)}{(bd-ae)^3(1+m)\sqrt{a^2+2abx+b^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.030895, size = 72, normalized size = 0.91

$$\frac{e^2(a+bx)^3(d+ex)^{m+1}\,{}_2F_1\left(3,m+1;m+2;-\frac{b(d+ex)}{ae-bd}\right)}{(m+1)\left((a+bx)^2\right)^{3/2}(ae-bd)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (e^2*(a + b*x)^3*(d + e*x)^(1 + m)*Hypergeometric2F1[3, 1 + m, 2 + m, -((b*
(d + e*x))/(-(b*d) + a*e))])/((-(b*d) + a*e)^3*(1 + m)*((a + b*x)^2)^(3/2))
```

Maple [F] time = 1.041, size = 0, normalized size = 0.

$$\int (ex+d)^m \left(b^2x^2+2abx+a^2\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

```
[Out] int((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(b^2x^2+2abx+a^2\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/(b^2*x^2 + 2*a*b*x + a^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b^2x^2+2abx+a^2}(ex+d)^m}{b^4x^4+4ab^3x^3+6a^2b^2x^2+4a^3bx+a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^2 + 2*a*b*x + a^2)*(e*x + d)^m/(b^4*x^4 + 4*a*b^3*x^3 +
 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m}{\left((a+bx)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x)**m/((a + b*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(b^2*x^2 + 2*a*b*x + a^2)^(3/2), x)
```

3.1742 $\int \frac{(d+ex)^m}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=79

$$-\frac{e^4(a+bx)(d+ex)^{m+1}\,{}_2F_1\left(5,m+1;m+2;\frac{b(d+ex)}{bd-ae}\right)}{(m+1)\sqrt{a^2+2abx+b^2x^2}(bd-ae)^5}$$

```
[Out] -((e^4*(a + b*x)*(d + e*x)^(1 + m)*Hypergeometric2F1[5, 1 + m, 2 + m, (b*(d
 + e*x))/(b*d - a*e)])/((b*d - a*e)^5*(1 + m)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]
))
```

Rubi [A] time = 0.0501979, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {646, 68}

$$-\frac{e^4(a+bx)(d+ex)^{m+1}\,{}_2F_1\left(5,m+1;m+2;\frac{b(d+ex)}{bd-ae}\right)}{(m+1)\sqrt{a^2+2abx+b^2x^2}(bd-ae)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] -((e^4*(a + b*x)*(d + e*x)^(1 + m)*Hypergeometric2F1[5, 1 + m, 2 + m, (b*(d
 + e*x))/(b*d - a*e)])/((b*d - a*e)^5*(1 + m)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]
))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^m}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx &= \frac{\left(b^4\left(ab+b^2x\right)\right)\int \frac{(d+ex)^m}{\left(ab+b^2x\right)^5}\,dx}{\sqrt{a^2+2abx+b^2x^2}}\\ &= -\frac{e^4(a+bx)(d+ex)^{1+m}\,{}_2F_1\left(5,1+m;2+m;\frac{b(d+ex)}{bd-ae}\right)}{(bd-ae)^5(1+m)\sqrt{a^2+2abx+b^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0339767, size = 72, normalized size = 0.91

$$\frac{e^4(a+bx)^5(d+ex)^{m+1}\,{}_2F_1\left(5,m+1;m+2;-\frac{b(d+ex)}{ae-bd}\right)}{(m+1)\left((a+bx)^2\right)^{5/2}(ae-bd)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
[Out] (e^4*(a + b*x)^5*(d + e*x)^(1 + m)*Hypergeometric2F1[5, 1 + m, 2 + m, -((b*
(d + e*x))/(-(b*d) + a*e))])/((-(b*d) + a*e)^5*(1 + m)*((a + b*x)^2)^(5/2))
```

Maple [F] time = 1.085, size = 0, normalized size = 0.

$$\int (ex+d)^m\left(b^2x^2+2abx+a^2\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

```
[Out] int((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(b^2x^2+2abx+a^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/(b^2*x^2 + 2*a*b*x + a^2)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b^2x^2+2abx+a^2}(ex+d)^m}{b^6x^6+6ab^5x^5+15a^2b^4x^4+20a^3b^3x^3+15a^4b^2x^2+6a^5bx+a^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^2 + 2*a*b*x + a^2)*(e*x + d)^m/(b^6*x^6 + 6*a*b^5*x^5 +
 15*a^2*b^4*x^4 + 20*a^3*b^3*x^3 + 15*a^4*b^2*x^2 + 6*a^5*b*x + a^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(b^2*x^2 + 2*a*b*x + a^2)^(5/2), x)
```

3.1743 $\int (d+ex)^m \left(a^2+2abx+b^2x^2\right)^p\,dx$

Optimal. Leaf size=85

$$\frac{\left(a^2+2abx+b^2x^2\right)^p (d+ex)^{m+1} \left(-\frac{e(a+bx)}{bd-ae}\right)^{-2p} {}_2F_1\left(m+1,-2p;m+2;\frac{b(d+ex)}{bd-ae}\right)}{e(m+1)}$$

```
[Out] ((d + e*x)^(1 + m)*(a^2 + 2*a*b*x + b^2*x^2)^p*Hypergeometric2F1[1 + m, -2*
p, 2 + m, (b*(d + e*x))/(b*d - a*e)])/(e*(1 + m)*(-((e*(a + b*x))/(b*d - a*
e)))^(2*p))
```

Rubi [A] time = 0.0543152, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {646, 70, 69}

$$\frac{\left(a^2+2abx+b^2x^2\right)^p (d+ex)^{m+1} \left(-\frac{e(a+bx)}{bd-ae}\right)^{-2p} {}_2F_1\left(m+1,-2p;m+2;\frac{b(d+ex)}{bd-ae}\right)}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a^2 + 2*a*b*x + b^2*x^2)^p,x]
```

```
[Out] ((d + e*x)^(1 + m)*(a^2 + 2*a*b*x + b^2*x^2)^p*Hypergeometric2F1[1 + m, -2*
p, 2 + m, (b*(d + e*x))/(b*d - a*e)])/(e*(1 + m)*(-((e*(a + b*x))/(b*d - a*
e)))^(2*p))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (d+ex)^m \left(a^2+2abx+b^2x^2\right)^p dx = \left(\left(ab+b^2x\right)^{-2p}\left(a^2+2abx+b^2x^2\right)^p\right)\int \left(ab+b^2x\right)^{2p}(d+ex)^m dx$$
$$= \left(\left(\frac{e\left(ab+b^2x\right)}{-b^2d+abe}\right)^{-2p}\left(a^2+2abx+b^2x^2\right)^p\right)\int (d+ex)^m\left(-\frac{ae}{bd-ae}-\frac{bex}{bd-ae}\right)^{2p} dx$$
$$= \frac{\left(-\frac{e(a+bx)}{bd-ae}\right)^{-2p}(d+ex)^{1+m}\left(a^2+2abx+b^2x^2\right)^p {}_2F_1\left(1+m,-2p;2+m;\frac{b(d+ex)}{bd-ae}\right)}{e(1+m)}$$

Mathematica [A] time = 0.0350256, size = 75, normalized size = 0.88

$$\frac{\left((a+bx)^2\right)^p(d+ex)^{m+1}\left(\frac{e(a+bx)}{ae-bd}\right)^{-2p} {}_2F_1\left(m+1,-2p;m+2;\frac{b(d+ex)}{bd-ae}\right)}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(a^2 + 2*a*b*x + b^2*x^2)^p,x]
```

```
[Out] (((a + b*x)^2)^p*(d + e*x)^(1 + m)*Hypergeometric2F1[1 + m, -2*p, 2 + m, (b
*(d + e*x))/(b*d - a*e)])/(e*(1 + m)*((e*(a + b*x))/(-(b*d) + a*e))^(2*p))
```

Maple [F] time = 1.316, size = 0, normalized size = 0.

$$\int (ex+d)^m \left(b^2x^2+2\,abx+a^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^p,x)
```

```
[Out] int((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^2+2\,abx+a^2\right)^p(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2)^p*(e*x + d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2x^2+2\,abx+a^2\right)^p(ex+d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)^p*(e*x + d)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(b**2*x**2+2*a*b*x+a**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^2 + 2\,abx + a^2\right)^p (ex + d)^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2)^p*(e*x + d)^m, x)
```

3.1744 $\int (d+ex)^3 \left(a^2+2abx+b^2x^2\right)^p \, dx$

Optimal. Leaf size=181

$$\frac{3e^2(a+bx)^3(bd-ae)\left(a^2+2abx+b^2x^2\right)^p}{b^4(2p+3)}+\frac{3e(a+bx)^2(bd-ae)^2\left(a^2+2abx+b^2x^2\right)^p}{2b^4(p+1)}+\frac{(a+bx)(bd-ae)^3\left(a^2+2abx+\right.}{b^4(2p+1)}$$

```
[Out] ((b*d - a*e)^3*(a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^p)/(b^4*(1 + 2*p)) + (3*
e*(b*d - a*e)^2*(a + b*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^p)/(2*b^4*(1 + p)) +
(3*e^2*(b*d - a*e)*(a + b*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^p)/(b^4*(3 + 2*p))
 + (e^3*(a + b*x)^4*(a^2 + 2*a*b*x + b^2*x^2)^p)/(2*b^4*(2 + p))
```

Rubi [A] time = 0.0923914, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {646, 43}

$$\frac{3e^2(a+bx)^3(bd-ae)\left(a^2+2abx+b^2x^2\right)^p}{b^4(2p+3)}+\frac{3e(a+bx)^2(bd-ae)^2\left(a^2+2abx+b^2x^2\right)^p}{2b^4(p+1)}+\frac{(a+bx)(bd-ae)^3\left(a^2+2abx+\right.}{b^4(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^p,x]
```


```
[Out] ((b*d - a*e)^3*(a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^p)/(b^4*(1 + 2*p)) + (3*
e*(b*d - a*e)^2*(a + b*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^p)/(2*b^4*(1 + p)) +
(3*e^2*(b*d - a*e)*(a + b*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^p)/(b^4*(3 + 2*p))
 + (e^3*(a + b*x)^4*(a^2 + 2*a*b*x + b^2*x^2)^p)/(2*b^4*(2 + p))
```


Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^3\left(a^2+2abx+b^2x^2\right)^p\,dx &= \left(\left(ab+b^2x\right)^{-2p}\left(a^2+2abx+b^2x^2\right)^p\right)\int \left(ab+b^2x\right)^{2p}(d+ex)^3\,dx\\ &= \left(\left(ab+b^2x\right)^{-2p}\left(a^2+2abx+b^2x^2\right)^p\right)\int\left(\frac{(bd-ae)^3\left(ab+b^2x\right)^{2p}}{b^3}+\frac{3e(bd-ae)^2\left(ab\right.}{b^4}\right.\\ &= \frac{(bd-ae)^3(a+bx)\left(a^2+2abx+b^2x^2\right)^p}{b^4(1+2p)}+\frac{3e(bd-ae)^2(a+bx)^2\left(a^2+2abx+b^2x^2\right)^{\cdot}}{2b^4(1+p)}\end{aligned}$$

Mathematica [A] time = 0.0857858, size = 107, normalized size = 0.59

$$\frac{(a+bx)\left((a+bx)^2\right)^p\left(\frac{6e^2(a+bx)^2(bd-ae)}{2p+3}+\frac{3e(a+bx)(bd-ae)^2}{p+1}+\frac{2(bd-ae)^3}{2p+1}+\frac{e^3(a+bx)^3}{p+2}\right)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^p,x]
```

```
[Out] ((a + b*x)*((a + b*x)^2)^p*((2*(b*d - a*e)^3)/(1 + 2*p) + (3*e*(b*d - a*e)^
2*(a + b*x))/(1 + p) + (6*e^2*(b*d - a*e)*(a + b*x)^2)/(3 + 2*p) + (e^3*(a
+ b*x)^3)/(2 + p)))/(2*b^4)
```

Maple [B] time = 0.049, size = 405, normalized size = 2.2

$$-\frac{\left(b^2x^2+2\,abx+a^2\right)^p\left(-4\,b^3e^3p^3x^3-12\,b^3de^2p^3x^2-12\,b^3e^3p^2x^3+6\,ab^2e^3p^2x^2-12\,b^3d^2ep^3x-42\,b^3de^2p^2x^2-11\,b^3e^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^p,x)
```

```
[Out] -1/2*(b^2*x^2+2*a*b*x+a^2)^p*(-4*b^3*e^3*p^3*x^3-12*b^3*d*e^2*p^3*x^2-12*b^
3*e^3*p^2*x^3+6*a*b^2*e^3*p^2*x^2-12*b^3*d^2*e*p^3*x-42*b^3*d*e^2*p^2*x^2-1
1*b^3*e^3*p*x^3+12*a*b^2*d*e^2*p^2*x+9*a*b^2*e^3*p*x^2-4*b^3*d^3*p^3-48*b^3
*d^2*e*p^2*x-42*b^3*d*e^2*p*x^2-3*b^3*e^3*x^3-6*a^2*b*e^3*p*x+6*a*b^2*d^2*e
*p^2+30*a*b^2*d*e^2*p*x+3*a*b^2*e^3*x^2-18*b^3*d^3*p^2-57*b^3*d^2*e*p*x-12*
b^3*d*e^2*x^2-6*a^2*b*d*e^2*p-3*a^2*b*e^3*x+21*a*b^2*d^2*e*p+12*a*b^2*d*e^2
*x-26*b^3*d^3*p-18*b^3*d^2*e*x+3*a^3*e^3-12*a^2*b*d*e^2+18*a*b^2*d^2*e-12*b
^3*d^3)*(b*x+a)/b^4/(4*p^4+20*p^3+35*p^2+25*p+6)
```

Maxima [A] time = 1.12158, size = 373, normalized size = 2.06

$$\frac{(bx+a)(bx+a)^{2p}d^3}{b(2p+1)}+\frac{3\left(b^2(2p+1)x^2+2\,abpx-a^2\right)(bx+a)^{2p}d^2e}{2\left(2p^2+3p+1\right)b^2}+\frac{3\left(\left(2p^2+3p+1\right)b^3x^3+\left(2p^2+p\right)ab^2x^2-2\,a\right.}{\left(4p^3+12p^2+11p+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="maxima")
```

```
[Out] (b*x + a)*(b*x + a)^(2*p)*d^3/(b*(2*p + 1)) + 3/2*(b^2*(2*p + 1)*x^2 + 2*a*
b*p*x - a^2)*(b*x + a)^(2*p)*d^2*e/((2*p^2 + 3*p + 1)*b^2) + 3*((2*p^2 + 3*
p + 1)*b^3*x^3 + (2*p^2 + p)*a*b^2*x^2 - 2*a^2*b*p*x + a^3)*(b*x + a)^(2*p)
*d*e^2/((4*p^3 + 12*p^2 + 11*p + 3)*b^3) + 1/2*((4*p^3 + 12*p^2 + 11*p + 3)
*b^4*x^4 + 2*(2*p^3 + 3*p^2 + p)*a*b^3*x^3 - 3*(2*p^2 + p)*a^2*b^2*x^2 + 6*
a^3*b*p*x - 3*a^4)*(b*x + a)^(2*p)*e^3/((4*p^4 + 20*p^3 + 35*p^2 + 25*p + 6
)*b^4)
```

Fricas [B] time = 1.72555, size = 1053, normalized size = 5.82

$$\frac{\left(4\,ab^3d^3p^3+12\,ab^3d^3-18\,a^2b^2d^2e+12\,a^3bde^2-3\,a^4e^3+\left(4\,b^4e^3p^3+12\,b^4e^3p^2+11\,b^4e^3p+3\,b^4e^3\right)x^4+2\left(6\,b^4de^2+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*b^3*d^3*p^3 + 12*a*b^3*d^3 - 18*a^2*b^2*d^2*e + 12*a^3*b*d*e^2 - 3
*a^4*e^3 + (4*b^4*e^3*p^3 + 12*b^4*e^3*p^2 + 11*b^4*e^3*p + 3*b^4*e^3)*x^4
+ 2*(6*b^4*d*e^2 + 2*(3*b^4*d*e^2 + a*b^3*e^3)*p^3 + 3*(7*b^4*d*e^2 + a*b^3
*e^3)*p^2 + (21*b^4*d*e^2 + a*b^3*e^3)*p)*x^3 + 6*(3*a*b^3*d^3 - a^2*b^2*d^
2*e)*p^2 + 3*(6*b^4*d^2*e + 4*(b^4*d^2*e + a*b^3*d*e^2)*p^3 + 2*(8*b^4*d^2*
e + 5*a*b^3*d*e^2 - a^2*b^2*e^3)*p^2 + (19*b^4*d^2*e + 4*a*b^3*d*e^2 - a^2*
b^2*e^3)*p)*x^2 + (26*a*b^3*d^3 - 21*a^2*b^2*d^2*e + 6*a^3*b*d*e^2)*p + 2*(
6*b^4*d^3 + 2*(b^4*d^3 + 3*a*b^3*d^2*e)*p^3 + 3*(3*b^4*d^3 + 7*a*b^3*d^2*e
- 2*a^2*b^2*d*e^2)*p^2 + (13*b^4*d^3 + 18*a*b^3*d^2*e - 12*a^2*b^2*d*e^2 +
3*a^3*b*e^3)*p)*x)*(b^2*x^2 + 2*a*b*x + a^2)^p/(4*b^4*p^4 + 20*b^4*p^3 + 35
*b^4*p^2 + 25*b^4*p + 6*b^4)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(b**2*x**2+2*a*b*x+a**2)**p,x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.23718, size = 1710, normalized size = 9.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/2*(4*(b^2*x^2 + 2*a*b*x + a^2)^p*b^4*p^3*x^4*e^3 + 12*(b^2*x^2 + 2*a*b*x
+ a^2)^p*b^4*d*p^3*x^3*e^2 + 12*(b^2*x^2 + 2*a*b*x + a^2)^p*b^4*d^2*p^3*x^2
*e + 4*(b^2*x^2 + 2*a*b*x + a^2)^p*b^4*d^3*p^3*x + 4*(b^2*x^2 + 2*a*b*x + a
^2)^p*a*b^3*p^3*x^3*e^3 + 12*(b^2*x^2 + 2*a*b*x + a^2)^p*b^4*p^2*x^4*e^3 +
12*(b^2*x^2 + 2*a*b*x + a^2)^p*a*b^3*d*p^3*x^2*e^2 + 42*(b^2*x^2 + 2*a*b*x
+ a^2)^p*b^4*d*p^2*x^3*e^2 + 12*(b^2*x^2 + 2*a*b*x + a^2)^p*a*b^3*d^2*p^3*x
*e + 48*(b^2*x^2 + 2*a*b*x + a^2)^p*b^4*d^2*p^2*x^2*e + 4*(b^2*x^2 + 2*a*b*
x + a^2)^p*a*b^3*d^3*p^3 + 18*(b^2*x^2 + 2*a*b*x + a^2)^p*b^4*d^3*p^2*x + 6
*(b^2*x^2 + 2*a*b*x + a^2)^p*a*b^3*p^2*x^3*e^3 + 11*(b^2*x^2 + 2*a*b*x + a^
2)^p*b^4*p*x^4*e^3 + 30*(b^2*x^2 + 2*a*b*x + a^2)^p*a*b^3*d*p^2*x^2*e^2 + 4
2*(b^2*x^2 + 2*a*b*x + a^2)^p*b^4*d*p*x^3*e^2 + 42*(b^2*x^2 + 2*a*b*x + a^2
)^p*a*b^3*d^2*p^2*x*e + 57*(b^2*x^2 + 2*a*b*x + a^2)^p*b^4*d^2*p*x^2*e + 18
*(b^2*x^2 + 2*a*b*x + a^2)^p*a*b^3*d^3*p^2 + 26*(b^2*x^2 + 2*a*b*x + a^2)^p
*b^4*d^3*p*x - 6*(b^2*x^2 + 2*a*b*x + a^2)^p*a^2*b^2*p^2*x^2*e^3 + 2*(b^2*x
^2 + 2*a*b*x + a^2)^p*a*b^3*p*x^3*e^3 + 3*(b^2*x^2 + 2*a*b*x + a^2)^p*b^4*x
^4*e^3 - 12*(b^2*x^2 + 2*a*b*x + a^2)^p*a^2*b^2*d*p^2*x*e^2 + 12*(b^2*x^2 +
 2*a*b*x + a^2)^p*a*b^3*d*p*x^2*e^2 + 12*(b^2*x^2 + 2*a*b*x + a^2)^p*b^4*d*
x^3*e^2 - 6*(b^2*x^2 + 2*a*b*x + a^2)^p*a^2*b^2*d^2*p^2*e + 36*(b^2*x^2 + 2
*a*b*x + a^2)^p*a*b^3*d^2*p*x*e + 18*(b^2*x^2 + 2*a*b*x + a^2)^p*b^4*d^2*x^
2*e + 26*(b^2*x^2 + 2*a*b*x + a^2)^p*a*b^3*d^3*p + 12*(b^2*x^2 + 2*a*b*x +
```

```
a^2)^p*b^4*d^3*x - 3*(b^2*x^2 + 2*a*b*x + a^2)^p*a^2*b^2*p*x^2*e^3 - 24*(b^
2*x^2 + 2*a*b*x + a^2)^p*a^2*b^2*d*p*x*e^2 - 21*(b^2*x^2 + 2*a*b*x + a^2)^p
*a^2*b^2*d^2*p*e + 12*(b^2*x^2 + 2*a*b*x + a^2)^p*a*b^3*d^3 + 6*(b^2*x^2 +
2*a*b*x + a^2)^p*a^3*b*p*x*e^3 + 6*(b^2*x^2 + 2*a*b*x + a^2)^p*a^3*b*d*p*e^
2 - 18*(b^2*x^2 + 2*a*b*x + a^2)^p*a^2*b^2*d^2*e + 12*(b^2*x^2 + 2*a*b*x +
a^2)^p*a^3*b*d*e^2 - 3*(b^2*x^2 + 2*a*b*x + a^2)^p*a^4*e^3)/(4*b^4*p^4 + 20
*b^4*p^3 + 35*b^4*p^2 + 25*b^4*p + 6*b^4)
```

3.1745 $\int (d+ex)^2 \left(a^2+2abx+b^2x^2\right)^p \, dx$

Optimal. Leaf size=127

$$\frac{e(a+bx)^2(bd-ae)\left(a^2+2abx+b^2x^2\right)^p}{b^3(p+1)}+\frac{(a+bx)(bd-ae)^2\left(a^2+2abx+b^2x^2\right)^p}{b^3(2p+1)}+\frac{e^2(a+bx)^3\left(a^2+2abx+b^2x^2\right)^p}{b^3(2p+3)}$$

```
[Out] ((b*d - a*e)^2*(a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^p)/(b^3*(1 + 2*p)) + (e*
(b*d - a*e)*(a + b*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^p)/(b^3*(1 + p)) + (e^2*(
a + b*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^p)/(b^3*(3 + 2*p))
```

Rubi [A] time = 0.0611493, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {646, 43}

$$\frac{e(a+bx)^2(bd-ae)\left(a^2+2abx+b^2x^2\right)^p}{b^3(p+1)}+\frac{(a+bx)(bd-ae)^2\left(a^2+2abx+b^2x^2\right)^p}{b^3(2p+1)}+\frac{e^2(a+bx)^3\left(a^2+2abx+b^2x^2\right)^p}{b^3(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^p,x]
```

```
[Out] ((b*d - a*e)^2*(a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^p)/(b^3*(1 + 2*p)) + (e*
(b*d - a*e)*(a + b*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^p)/(b^3*(1 + p)) + (e^2*(
a + b*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^p)/(b^3*(3 + 2*p))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^2\left(a^2+2abx+b^2x^2\right)^p\,dx &= \left(\left(ab+b^2x\right)^{-2p}\left(a^2+2abx+b^2x^2\right)^p\right)\int\left(ab+b^2x\right)^{2p}(d+ex)^2\,dx\\ &= \left(\left(ab+b^2x\right)^{-2p}\left(a^2+2abx+b^2x^2\right)^p\right)\int\left(\frac{(bd-ae)^2\left(ab+b^2x\right)^{2p}}{b^2}+\frac{2e(bd-ae)\left(ab\right.}{b^3}\right.\\ &= \frac{(bd-ae)^2(a+bx)\left(a^2+2abx+b^2x^2\right)^p}{b^3(1+2p)}+\frac{e(bd-ae)(a+bx)^2\left(a^2+2abx+b^2x^2\right)^p}{b^3(1+p)}\end{aligned}$$

Mathematica [A] time = 0.0782135, size = 75, normalized size = 0.59

$$\frac{(a+bx)\left((a+bx)^2\right)^p\left(\frac{e(a+bx)(bd-ae)}{p+1}+\frac{(bd-ae)^2}{2p+1}+\frac{e^2(a+bx)^2}{2p+3}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^p,x]
```

```
[Out] ((a + b*x)*((a + b*x)^2)^p*((b*d - a*e)^2/(1 + 2*p) + (e*(b*d - a*e)*(a + b
*x))/(1 + p) + (e^2*(a + b*x)^2)/(3 + 2*p)))/b^3
```

Maple [A] time = 0.046, size = 175, normalized size = 1.4

$$\frac{\left(2b^2e^2p^2x^2+4b^2dep^2x+3b^2e^2px^2-2abe^2px+2b^2d^2p^2+8b^2depx+e^2x^2b^2-2abdep-abe^2x+5b^2d^2p+3xb^2de\right.}{b^3\left(4p^3+12p^2+11p+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^p,x)
```

```
[Out] (b*x+a)*(2*b^2*e^2*p^2*x^2+4*b^2*d*e*p^2*x+3*b^2*e^2*p*x^2-2*a*b*e^2*p*x+2*
b^2*d^2*p^2+8*b^2*d*e*p*x+b^2*e^2*x^2-2*a*b*d*e*p-a*b*e^2*x+5*b^2*d^2*p+3*b
^2*d*e*x+a^2*e^2-3*a*b*d*e+3*b^2*d^2)*(b^2*x^2+2*a*b*x+a^2)^p/b^3/(4*p^3+12
*p^2+11*p+3)
```

Maxima [A] time = 1.12736, size = 212, normalized size = 1.67

$$\frac{(bx+a)(bx+a)^{2p}d^2}{b(2p+1)}+\frac{\left(b^2(2p+1)x^2+2abpx-a^2\right)(bx+a)^{2p}de}{\left(2p^2+3p+1\right)b^2}+\frac{\left(\left(2p^2+3p+1\right)b^3x^3+\left(2p^2+p\right)ab^2x^2-2a^2bp\right.}{\left(4p^3+12p^2+11p+3\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="maxima")
```

```
[Out] (b*x + a)*(b*x + a)^(2*p)*d^2/(b*(2*p + 1)) + (b^2*(2*p + 1)*x^2 + 2*a*b*p*
x - a^2)*(b*x + a)^(2*p)*d*e/((2*p^2 + 3*p + 1)*b^2) + ((2*p^2 + 3*p + 1)*b
^3*x^3 + (2*p^2 + p)*a*b^2*x^2 - 2*a^2*b*p*x + a^3)*(b*x + a)^(2*p)*e^2/((4
*p^3 + 12*p^2 + 11*p + 3)*b^3)
```

Fricas [A] time = 1.65259, size = 509, normalized size = 4.01

$$\frac{\left(2ab^2d^2p^2+3ab^2d^2-3a^2bde+a^3e^2+\left(2b^3e^2p^2+3b^3e^2p+b^3e^2\right)x^3+\left(3b^3de+2\left(2b^3de+ab^2e^2\right)p^2+\left(8b^3de+ab^2\right.\right.\right.}{4b^3p^3+12b^3p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="fricas")
```

```
[Out] (2*a*b^2*d^2*p^2 + 3*a*b^2*d^2 - 3*a^2*b*d*e + a^3*e^2 + (2*b^3*e^2*p^2 + 3
*b^3*e^2*p + b^3*e^2)*x^3 + (3*b^3*d*e + 2*(2*b^3*d*e + a*b^2*e^2)*p^2 + (8
*b^3*d*e + a*b^2*e^2)*p)*x^2 + (5*a*b^2*d^2 - 2*a^2*b*d*e)*p + (3*b^3*d^2 +
 2*(b^3*d^2 + 2*a*b^2*d*e)*p^2 + (5*b^3*d^2 + 6*a*b^2*d*e - 2*a^2*b*e^2)*p)
*x)*(b^2*x^2 + 2*a*b*x + a^2)^p/(4*b^3*p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(b**2*x**2+2*a*b*x+a**2)**p,x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.14645, size = 821, normalized size = 6.46

$$\frac{2\left(b^2x^2+2\,abx+a^2\right)^p b^3p^2x^3e^2+4\left(b^2x^2+2\,abx+a^2\right)^p b^3dp^2x^2e+2\left(b^2x^2+2\,abx+a^2\right)^p b^3d^2p^2x+2\left(b^2x^2+2\,abx+a^2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="giac")
```

```
[Out] (2*(b^2*x^2 + 2*a*b*x + a^2)^p*b^3*p^2*x^3*e^2 + 4*(b^2*x^2 + 2*a*b*x + a^2
)^p*b^3*d*p^2*x^2*e + 2*(b^2*x^2 + 2*a*b*x + a^2)^p*b^3*d^2*p^2*x + 2*(b^2*
x^2 + 2*a*b*x + a^2)^p*a*b^2*p^2*x^2*e^2 + 3*(b^2*x^2 + 2*a*b*x + a^2)^p*b^
3*p*x^3*e^2 + 4*(b^2*x^2 + 2*a*b*x + a^2)^p*a*b^2*d*p^2*x*e + 8*(b^2*x^2 +
2*a*b*x + a^2)^p*b^3*d*p*x^2*e + 2*(b^2*x^2 + 2*a*b*x + a^2)^p*a*b^2*d^2*p^
2 + 5*(b^2*x^2 + 2*a*b*x + a^2)^p*b^3*d^2*p*x + (b^2*x^2 + 2*a*b*x + a^2)^p
*a*b^2*p*x^2*e^2 + (b^2*x^2 + 2*a*b*x + a^2)^p*b^3*x^3*e^2 + 6*(b^2*x^2 + 2
*a*b*x + a^2)^p*a*b^2*d*p*x*e + 3*(b^2*x^2 + 2*a*b*x + a^2)^p*b^3*d*x^2*e +
 5*(b^2*x^2 + 2*a*b*x + a^2)^p*a*b^2*d^2*p + 3*(b^2*x^2 + 2*a*b*x + a^2)^p*
b^3*d^2*x - 2*(b^2*x^2 + 2*a*b*x + a^2)^p*a^2*b*p*x*e^2 - 2*(b^2*x^2 + 2*a*
b*x + a^2)^p*a^2*b*d*p*e + 3*(b^2*x^2 + 2*a*b*x + a^2)^p*a*b^2*d^2 - 3*(b^2
*x^2 + 2*a*b*x + a^2)^p*a^2*b*d*e + (b^2*x^2 + 2*a*b*x + a^2)^p*a^3*e^2)/(4
*b^3*p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)
```

3.1746 $\int (d+ex)\left(a^2+2abx+b^2x^2\right)^p\,dx$

Optimal. Leaf size=76

$$\frac{(a+bx)(bd-ae)\left(a^2+2abx+b^2x^2\right)^p}{b^2(2p+1)}+\frac{e\left(a^2+2abx+b^2x^2\right)^{p+1}}{2b^2(p+1)}$$

```
[Out] ((b*d - a*e)*(a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^p)/(b^2*(1 + 2*p)) + (e*(a
^2 + 2*a*b*x + b^2*x^2)^(1 + p))/(2*b^2*(1 + p))
```

Rubi [A] time = 0.0238942, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {640, 609}

$$\frac{(a+bx)(bd-ae)\left(a^2+2abx+b^2x^2\right)^p}{b^2(2p+1)}+\frac{e\left(a^2+2abx+b^2x^2\right)^{p+1}}{2b^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^p,x]
```

```
[Out] ((b*d - a*e)*(a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^p)/(b^2*(1 + 2*p)) + (e*(a
^2 + 2*a*b*x + b^2*x^2)^(1 + p))/(2*b^2*(1 + p))
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(a^2+2abx+b^2x^2\right)^p\,dx&=\frac{e\left(a^2+2abx+b^2x^2\right)^{1+p}}{2b^2(1+p)}+\frac{\left(2b^2d-2abe\right)\int\left(a^2+2abx+b^2x^2\right)^p\,dx}{2b^2}\\&=\frac{(bd-ae)(a+bx)\left(a^2+2abx+b^2x^2\right)^p}{b^2(1+2p)}+\frac{e\left(a^2+2abx+b^2x^2\right)^{1+p}}{2b^2(1+p)}\end{aligned}$$

Mathematica [A] time = 0.0340394, size = 54, normalized size = 0.71

$$\frac{(a+bx)\left((a+bx)^2\right)^p(-ae+2bd(p+1)+be(2p+1)x)}{2b^2(p+1)(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^p,x]
```

```
[Out] ((a + b*x)*((a + b*x)^2)^p*(-(a*e) + 2*b*d*(1 + p) + b*e*(1 + 2*p)*x))/(2*b
^2*(1 + p)*(1 + 2*p))
```

Maple [A] time = 0.041, size = 65, normalized size = 0.9

$$-\frac{\left(b^{2}x^{2}+2\,abx+a^{2}\right)^{p}\left(-2\,bepx-2\,bdp-bxe+ae-2\,bd\right)(bx+a)}{2\,b^{2}\left(2\,p^{2}+3\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^p,x)
```

```
[Out] -1/2*(b^2*x^2+2*a*b*x+a^2)^p*(-2*b*e*p*x-2*b*d*p-b*e*x+a*e-2*b*d)*(b*x+a)/b
^2/(2*p^2+3*p+1)
```

Maxima [A] time = 1.08004, size = 105, normalized size = 1.38

$$\frac{(bx+a)(bx+a)^{2\,p}d}{b\left(2\,p+1\right)}+\frac{\left(b^{2}\left(2\,p+1\right)x^{2}+2\,abpx-a^{2}\right)(bx+a)^{2\,p}e}{2\left(2\,p^{2}+3\,p+1\right)b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="maxima")
```

```
[Out] (b*x + a)*(b*x + a)^(2*p)*d/(b*(2*p + 1)) + 1/2*(b^2*(2*p + 1)*x^2 + 2*a*b*
p*x - a^2)*(b*x + a)^(2*p)*e/((2*p^2 + 3*p + 1)*b^2)
```

Fricas [A] time = 1.68205, size = 204, normalized size = 2.68

$$\frac{\left(2\,abdp+2\,abd-a^{2}e+\left(2\,b^{2}ep+b^{2}e\right)x^{2}+2\left(b^{2}d+\left(b^{2}d+abe\right)p\right)x\right)\left(b^{2}x^{2}+2\,abx+a^{2}\right)^{p}}{2\left(2\,b^{2}p^{2}+3\,b^{2}p+b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*d*p + 2*a*b*d - a^2*e + (2*b^2*e*p + b^2*e)*x^2 + 2*(b^2*d + (b^
2*d + a*b*e)*p)*x)*(b^2*x^2 + 2*a*b*x + a^2)^p/(2*b^2*p^2 + 3*b^2*p + b^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b**2*x**2+2*a*b*x+a**2)**p,x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.15629, size = 308, normalized size = 4.05

$$\frac{2\left(b^2x^2+2\,abx+a^2\right)^p b^2px^2e+2\left(b^2x^2+2\,abx+a^2\right)^p b^2dpx+2\left(b^2x^2+2\,abx+a^2\right)^p abpxe+\left(b^2x^2+2\,abx+a^2\right)^p b^2x^2}{2\left(2\,b^2p^2+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/2*(2*(b^2*x^2 + 2*a*b*x + a^2)^p*b^2*p*x^2*e + 2*(b^2*x^2 + 2*a*b*x + a^2
)^p*b^2*d*p*x + 2*(b^2*x^2 + 2*a*b*x + a^2)^p*a*b*p*x*e + (b^2*x^2 + 2*a*b*
x + a^2)^p*b^2*x^2*e + 2*(b^2*x^2 + 2*a*b*x + a^2)^p*a*b*d*p + 2*(b^2*x^2 +
 2*a*b*x + a^2)^p*b^2*d*x + 2*(b^2*x^2 + 2*a*b*x + a^2)^p*a*b*d - (b^2*x^2
+ 2*a*b*x + a^2)^p*a^2*e)/(2*b^2*p^2 + 3*b^2*p + b^2)
```

3.1747 $\int \left(a^2+2abx+b^2x^2\right)^p\,dx$

Optimal. Leaf size=34

$$\frac{(a+bx)\left(a^2+2abx+b^2x^2\right)^p}{b(2p+1)}$$

```
[Out] ((a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^p)/(b*(1 + 2*p))
```

Rubi [A] time = 0.0056333, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {609}

$$\frac{(a+bx)\left(a^2+2abx+b^2x^2\right)^p}{b(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^p,x]
```

```
[Out] ((a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^p)/(b*(1 + 2*p))
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \left(a^2+2abx+b^2x^2\right)^p\,dx=\frac{(a+bx)\left(a^2+2abx+b^2x^2\right)^p}{b(1+2p)}$$

Mathematica [A] time = 0.0111934, size = 23, normalized size = 0.68

$$\frac{(a+bx)\left((a+bx)^2\right)^p}{2bp+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^p,x]
```

```
[Out] ((a + b*x)*((a + b*x)^2)^p)/(b + 2*b*p)
```

Maple [A] time = 0.041, size = 35, normalized size = 1.

$$\frac{(bx+a)\left(b^2x^2+2\,abx+a^2\right)^p}{b\left(1+2\,p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^p,x)
```

```
[Out] (b*x+a)*(b^2*x^2+2*a*b*x+a^2)^p/b/(1+2*p)
```

Maxima [A] time = 1.11127, size = 34, normalized size = 1.

$$\frac{(bx+a)(bx+a)^{2p}}{b(2p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="maxima")
```

```
[Out] (b*x + a)*(b*x + a)^(2*p)/(b*(2*p + 1))
```

Fricas [A] time = 1.60567, size = 69, normalized size = 2.03

$$\frac{(bx+a)\left(b^2x^2+2\,abx+a^2\right)^p}{2\,bp+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="fricas")
```

```
[Out] (b*x + a)*(b^2*x^2 + 2*a*b*x + a^2)^p/(2*b*p + b)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**p,x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.16335, size = 69, normalized size = 2.03

$$\frac{\left(b^2x^2+2\,abx+a^2\right)^p bx+\left(b^2x^2+2\,abx+a^2\right)^p a}{2\,bp+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="giac")
```

```
[Out] ((b^2*x^2 + 2*a*b*x + a^2)^p*b*x + (b^2*x^2 + 2*a*b*x + a^2)^p*a)/(2*b*p +
b)
```

3.1748 $\int \frac{\left(a^2+2abx+b^2x^2\right)^p}{d+ex} dx$

Optimal. Leaf size=71

$$\frac{(a+bx)\left(a^2+2abx+b^2x^2\right)^p {}_2F_1\left(1,2p+1;2(p+1);-\frac{e(a+bx)}{bd-ae}\right)}{(2p+1)(bd-ae)}$$

```
[Out] ((a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^p*Hypergeometric2F1[1, 1 + 2*p, 2*(1 +
 p), -((e*(a + b*x))/(b*d - a*e))])/((b*d - a*e)*(1 + 2*p))
```

Rubi [A] time = 0.0323544, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {646, 68}

$$\frac{(a+bx)\left(a^2+2abx+b^2x^2\right)^p {}_2F_1\left(1,2p+1;2(p+1);-\frac{e(a+bx)}{bd-ae}\right)}{(2p+1)(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^p/(d + e*x),x]
```

```
[Out] ((a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^p*Hypergeometric2F1[1, 1 + 2*p, 2*(1 +
 p), -((e*(a + b*x))/(b*d - a*e))])/((b*d - a*e)*(1 + 2*p))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx+b^2x^2\right)^p}{d+ex} dx &= \left(\left(ab+b^2x\right)^{-2p}\left(a^2+2abx+b^2x^2\right)^p\right)\int \frac{\left(ab+b^2x\right)^{2p}}{d+ex} dx\\ &= \frac{(a+bx)\left(a^2+2abx+b^2x^2\right)^p {}_2F_1\left(1,1+2p;2(1+p);-\frac{e(a+bx)}{bd-ae}\right)}{(bd-ae)(1+2p)}\end{aligned}$$

Mathematica [A] time = 0.0169132, size = 62, normalized size = 0.87

$$-\frac{(a+bx)\left((a+bx)^2\right)^p {}_2F_1\left(1,2p+1;2p+2;\frac{e(a+bx)}{ae-bd}\right)}{(2p+1)(ae-bd)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^p/(d + e*x),x]
```

```
[Out] -(((a + b*x)*((a + b*x)^2)^p*Hypergeometric2F1[1, 1 + 2*p, 2 + 2*p, (e*(a +
 b*x))/(-(b*d) + a*e)])/((-(b*d) + a*e)*(1 + 2*p)))
```

Maple [F] time = 1.215, size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^2+2\,abx+a^2\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d),x)
```

```
[Out] int((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^2+2\,abx+a^2\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2)^p/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2x^2+2\,abx+a^2\right)^p}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)^p/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((a+bx)^2\right)^p}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**p/(e*x+d),x)
```

```
[Out] Integral(((a + b*x)**2)**p/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^2 + 2\,abx + a^2\right)^p}{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2)^p/(e*x + d), x)
```

3.1749 $\int \frac{(a^2+2abx+b^2x^2)^p}{(d+ex)^2} dx$

Optimal. Leaf size=72

$$\frac{b(a+bx)\left(a^2+2abx+b^2x^2\right)^p {}_2F_1\left(2,2p+1;2(p+1);-\frac{e(a+bx)}{bd-ae}\right)}{(2p+1)(bd-ae)^2}$$

```
[Out] (b*(a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^p*Hypergeometric2F1[2, 1 + 2*p, 2*(1
 + p), -((e*(a + b*x))/(b*d - a*e))])/((b*d - a*e)^2*(1 + 2*p))
```

Rubi [A] time = 0.0304151, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {646, 68}

$$\frac{b(a+bx)\left(a^2+2abx+b^2x^2\right)^p {}_2F_1\left(2,2p+1;2(p+1);-\frac{e(a+bx)}{bd-ae}\right)}{(2p+1)(bd-ae)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^p/(d + e*x)^2,x]
```

```
[Out] (b*(a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^p*Hypergeometric2F1[2, 1 + 2*p, 2*(1
 + p), -((e*(a + b*x))/(b*d - a*e))])/((b*d - a*e)^2*(1 + 2*p))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx+b^2x^2\right)^p}{(d+ex)^2} dx &= \left(\left(ab+b^2x\right)^{-2p}\left(a^2+2abx+b^2x^2\right)^p\right)\int \frac{\left(ab+b^2x\right)^{2p}}{(d+ex)^2} dx\\ &= \frac{b(a+bx)\left(a^2+2abx+b^2x^2\right)^p {}_2F_1\left(2,1+2p;2(1+p);-\frac{e(a+bx)}{bd-ae}\right)}{(bd-ae)^2(1+2p)}\end{aligned}$$

Mathematica [A] time = 0.0160252, size = 63, normalized size = 0.88

$$\frac{b(a+bx)\left((a+bx)^2\right)^p {}_2F_1\left(2,2p+1;2p+2;-\frac{e(a+bx)}{bd-ae}\right)}{(2p+1)(bd-ae)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^p/(d + e*x)^2,x]
```

```
[Out] (b*(a + b*x)*((a + b*x)^2)^p*Hypergeometric2F1[2, 1 + 2*p, 2 + 2*p, -((e*(a
 + b*x))/(b*d - a*e))])/((b*d - a*e)^2*(1 + 2*p))
```

Maple [F] time = 1.188, size = 0, normalized size = 0.

$$\int \frac{\left(b^{2} x^{2}+2 a b x +a^{2}\right)^{p}}{\left(e x +d \right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^2,x)
```

```
[Out] int((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^{2} x^{2}+2 a b x +a^{2}\right)^{p}}{\left(e x +d \right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2)^p/(e*x + d)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^{2} x^{2}+2 \, a b x +a^{2}\right)^{p}}{e^{2} x^{2}+2 \, d e x +d^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)^p/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx\right)^{2}\right)^{p}}{\left(d+ex\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**p/(e*x+d)**2,x)
```

```
[Out] Integral(((a + b*x)**2)**p/(d + e*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^2 + 2\,abx + a^2\right)^p}{\left(ex + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2)^p/(e*x + d)^2, x)
```

3.1750 $\int \frac{\left(a^2+2abx+b^2x^2\right)^p}{(d+ex)^3}\,dx$

Optimal. Leaf size=74

$$\frac{b^2(a+bx)\left(a^2+2abx+b^2x^2\right)^p\,{}_2F_1\left(3,2p+1;2(p+1);-\frac{e(a+bx)}{bd-ae}\right)}{(2p+1)(bd-ae)^3}$$

```
[Out] (b^2*(a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^p*Hypergeometric2F1[3, 1 + 2*p, 2*
(1 + p), -((e*(a + b*x))/(b*d - a*e))])/((b*d - a*e)^3*(1 + 2*p))
```

Rubi [A] time = 0.0296786, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {646, 68}

$$\frac{b^2(a+bx)\left(a^2+2abx+b^2x^2\right)^p\,{}_2F_1\left(3,2p+1;2(p+1);-\frac{e(a+bx)}{bd-ae}\right)}{(2p+1)(bd-ae)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^p/(d + e*x)^3,x]
```

```
[Out] (b^2*(a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^p*Hypergeometric2F1[3, 1 + 2*p, 2*
(1 + p), -((e*(a + b*x))/(b*d - a*e))])/((b*d - a*e)^3*(1 + 2*p))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx+b^2x^2\right)^p}{(d+ex)^3}\,dx &= \left(\left(ab+b^2x\right)^{-2p}\left(a^2+2abx+b^2x^2\right)^p\right)\int \frac{\left(ab+b^2x\right)^{2p}}{(d+ex)^3}\,dx\\ &= \frac{b^2(a+bx)\left(a^2+2abx+b^2x^2\right)^p\,{}_2F_1\left(3,1+2p;2(1+p);-\frac{e(a+bx)}{bd-ae}\right)}{(bd-ae)^3(1+2p)}\end{aligned}$$

Mathematica [A] time = 0.0151866, size = 65, normalized size = 0.88

$$\frac{b^2(a+bx)\left((a+bx)^2\right)^p\,{}_2F_1\left(3,2p+1;2p+2;-\frac{e(a+bx)}{bd-ae}\right)}{(2p+1)(bd-ae)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^p/(d + e*x)^3,x]
```

```
[Out] (b^2*(a + b*x)*((a + b*x)^2)^p*Hypergeometric2F1[3, 1 + 2*p, 2 + 2*p, -((e*
(a + b*x))/(b*d - a*e))])/((b*d - a*e)^3*(1 + 2*p))
```

Maple [F] time = 1.209, size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^2 + 2\,abx + a^2\right)^p}{\left(ex + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^3,x)
```

```
[Out] int((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^2 + 2\,abx + a^2\right)^p}{\left(ex + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2)^p/(e*x + d)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^2 + 2\,abx + a^2\right)^p}{e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)^p/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d
^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**p/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^2 + 2\,abx + a^2\right)^p}{(ex + d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2)^p/(e*x + d)^3, x)
```

3.1751 $\int (d+ex)^{3/2} \left(a^2+2abx+b^2x^2\right)^p \, dx$

Optimal. Leaf size=83

$$\frac{2(d+ex)^{5/2} \left(a^2+2abx+b^2x^2\right)^p \left(-\frac{e(a+bx)}{bd-ae}\right)^{-2p} {}_2F_1\left(\frac{5}{2},-2p;\frac{7}{2};\frac{b(d+ex)}{bd-ae}\right)}{5e}$$

```
[Out] (2*(d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^p*Hypergeometric2F1[5/2, -2*p,
 7/2, (b*(d + e*x))/(b*d - a*e)])/(5*e*(-((e*(a + b*x))/(b*d - a*e)))^(2*p)
)
```

Rubi [A] time = 0.0409771, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {646, 70, 69}

$$\frac{2(d+ex)^{5/2} \left(a^2+2abx+b^2x^2\right)^p \left(-\frac{e(a+bx)}{bd-ae}\right)^{-2p} {}_2F_1\left(\frac{5}{2},-2p;\frac{7}{2};\frac{b(d+ex)}{bd-ae}\right)}{5e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^p,x]
```

```
[Out] (2*(d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^p*Hypergeometric2F1[5/2, -2*p,
 7/2, (b*(d + e*x))/(b*d - a*e)])/(5*e*(-((e*(a + b*x))/(b*d - a*e)))^(2*p)
)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (d+ex)^{3/2} \left(a^2+2abx+b^2x^2\right)^p \, dx = \left(\left(ab+b^2x\right)^{-2p} \left(a^2+2abx+b^2x^2\right)^p\right) \int \left(ab+b^2x\right)^{2p} (d+ex)^{3/2} \, dx$$
$$= \left(\left(\frac{e\left(ab+b^2x\right)}{-b^2d+abe}\right)^{-2p} \left(a^2+2abx+b^2x^2\right)^p\right) \int (d+ex)^{3/2} \left(-\frac{ae}{bd-ae}-\frac{bex}{bd-ae}\right)^{2p} dx$$
$$= \frac{2\left(-\frac{e(a+bx)}{bd-ae}\right)^{-2p} (d+ex)^{5/2} \left(a^2+2abx+b^2x^2\right)^p {}_2F_1\left(\frac{5}{2},-2p;\frac{7}{2};\frac{b(d+ex)}{bd-ae}\right)}{5e}$$

Mathematica [A] time = 0.0295678, size = 73, normalized size = 0.88

$$\frac{2(d+ex)^{5/2} \left((a+bx)^2\right)^p \left(\frac{e(a+bx)}{ae-bd}\right)^{-2p} {}_2F_1\left(\frac{5}{2},-2p;\frac{7}{2};\frac{b(d+ex)}{bd-ae}\right)}{5e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^p,x]
```

```
[Out] (2*((a + b*x)^2)^p*(d + e*x)^(5/2)*Hypergeometric2F1[5/2, -2*p, 7/2, (b*(d
+ e*x))/(b*d - a*e)])/(5*e*((e*(a + b*x))/(-(b*d) + a*e))^(2*p))
```

Maple [F] time = 1.185, size = 0, normalized size = 0.

$$\int (ex+d)^{\frac{3}{2}} \left(b^2x^2+2\,abx+a^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^p,x)
```

```
[Out] int((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^{\frac{3}{2}} \left(b^2x^2+2\,abx+a^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*(b^2*x^2 + 2*a*b*x + a^2)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex+d)^{\frac{3}{2}} \left(b^2x^2+2\,abx+a^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^(3/2)*(b^2*x^2 + 2*a*b*x + a^2)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(b**2*x**2+2*a*b*x+a**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^{\frac{3}{2}} \left(b^2x^2 + 2\,abx + a^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)*(b^2*x^2 + 2*a*b*x + a^2)^p, x)
```

3.1752 $\int \sqrt{d+ex}\left(a^2+2abx+b^2x^2\right)^p\,dx$

Optimal. Leaf size=83

$$\frac{2(d+ex)^{3/2}\left(a^2+2abx+b^2x^2\right)^p\left(-\frac{e(a+bx)}{bd-ae}\right)^{-2p}{}_2F_1\left(\frac{3}{2},-2p;\frac{5}{2};\frac{b(d+ex)}{bd-ae}\right)}{3e}$$

```
[Out] (2*(d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^p*Hypergeometric2F1[3/2, -2*p,
 5/2, (b*(d + e*x))/(b*d - a*e)])/(3*e*(-((e*(a + b*x))/(b*d - a*e)))^(2*p)
)
```

Rubi [A] time = 0.038224, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {646, 70, 69}

$$\frac{2(d+ex)^{3/2}\left(a^2+2abx+b^2x^2\right)^p\left(-\frac{e(a+bx)}{bd-ae}\right)^{-2p}{}_2F_1\left(\frac{3}{2},-2p;\frac{5}{2};\frac{b(d+ex)}{bd-ae}\right)}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^p,x]
```

```
[Out] (2*(d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^p*Hypergeometric2F1[3/2, -2*p,
 5/2, (b*(d + e*x))/(b*d - a*e)])/(3*e*(-((e*(a + b*x))/(b*d - a*e)))^(2*p)
)
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \sqrt{d+ex}\left(a^2+2abx+b^2x^2\right)^p\,dx = \left(\left(ab+b^2x\right)^{-2p}\left(a^2+2abx+b^2x^2\right)^p\right)\int \left(ab+b^2x\right)^{2p}\sqrt{d+ex}\,dx$$
$$= \left(\left(\frac{e\left(ab+b^2x\right)}{-b^2d+abe}\right)^{-2p}\left(a^2+2abx+b^2x^2\right)^p\right)\int \sqrt{d+ex}\left(-\frac{ae}{bd-ae}-\frac{bex}{bd-ae}\right)^{2p}dx$$
$$= \frac{2\left(-\frac{e(a+bx)}{bd-ae}\right)^{-2p}(d+ex)^{3/2}\left(a^2+2abx+b^2x^2\right)^p {}_2F_1\left(\frac{3}{2},-2p;\frac{5}{2};\frac{b(d+ex)}{bd-ae}\right)}{3e}$$

Mathematica [A] time = 0.0224397, size = 73, normalized size = 0.88

$$\frac{2(d+ex)^{3/2}\left((a+bx)^2\right)^p\left(\frac{e(a+bx)}{ae-bd}\right)^{-2p} {}_2F_1\left(\frac{3}{2},-2p;\frac{5}{2};\frac{b(d+ex)}{bd-ae}\right)}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^p,x]
```

```
[Out] (2*((a + b*x)^2)^p*(d + e*x)^(3/2)*Hypergeometric2F1[3/2, -2*p, 5/2, (b*(d
+ e*x))/(b*d - a*e)])/(3*e*((e*(a + b*x))/(-(b*d) + a*e))^(2*p))
```

Maple [F] time = 1.162, size = 0, normalized size = 0.

$$\int \sqrt{ex+d}\left(b^2x^2+2\,abx+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2)^p,x)
```

```
[Out] int((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex+d}\left(b^2x^2+2\,abx+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*(b^2*x^2 + 2*a*b*x + a^2)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{ex+d}\left(b^2x^2+2\,abx+a^2\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*(b^2*x^2 + 2*a*b*x + a^2)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(b**2*x**2+2*a*b*x+a**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex+d}\left(b^2x^2+2\,abx+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(b^2*x^2 + 2*a*b*x + a^2)^p, x)
```

3.1753 $\int \frac{(a^2+2abx+b^2x^2)^p}{\sqrt{d+ex}} dx$

Optimal. Leaf size=81

$$\frac{2\sqrt{d+ex}\left(a^2+2abx+b^2x^2\right)^p\left(-\frac{e(a+bx)}{bd-ae}\right)^{-2p}{}_2F_1\left(\frac{1}{2},-2p;\frac{3}{2};\frac{b(d+ex)}{bd-ae}\right)}{e}$$

```
[Out] (2*Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^p*Hypergeometric2F1[1/2, -2*p, 3
/2, (b*(d + e*x))/(b*d - a*e)])/(e*(-((e*(a + b*x))/(b*d - a*e)))^(2*p))
```

Rubi [A] time = 0.0379554, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {646, 70, 69}

$$\frac{2\sqrt{d+ex}\left(a^2+2abx+b^2x^2\right)^p\left(-\frac{e(a+bx)}{bd-ae}\right)^{-2p}{}_2F_1\left(\frac{1}{2},-2p;\frac{3}{2};\frac{b(d+ex)}{bd-ae}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^p/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^p*Hypergeometric2F1[1/2, -2*p, 3
/2, (b*(d + e*x))/(b*d - a*e)])/(e*(-((e*(a + b*x))/(b*d - a*e)))^(2*p))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^p}{\sqrt{d+ex}}\,dx = \left(\left(ab+b^2x\right)^{-2p}\left(a^2+2abx+b^2x^2\right)^p\right)\int \frac{\left(ab+b^2x\right)^{2p}}{\sqrt{d+ex}}\,dx$$
$$= \left(\left(\frac{e\left(ab+b^2x\right)}{-b^2d+abe}\right)^{-2p}\left(a^2+2abx+b^2x^2\right)^p\right)\int \frac{\left(-\frac{ae}{bd-ae}-\frac{bex}{bd-ae}\right)^{2p}}{\sqrt{d+ex}}\,dx$$
$$= \frac{2\left(-\frac{e(a+bx)}{bd-ae}\right)^{-2p}\sqrt{d+ex}\left(a^2+2abx+b^2x^2\right)^p\,{}_2F_1\left(\frac{1}{2},-2p;\frac{3}{2};\frac{b(d+ex)}{bd-ae}\right)}{e}$$

Mathematica [A] time = 0.016849, size = 71, normalized size = 0.88

$$\frac{2\sqrt{d+ex}\left((a+bx)^2\right)^p\left(\frac{e(a+bx)}{ae-bd}\right)^{-2p}\,{}_2F_1\left(\frac{1}{2},-2p;\frac{3}{2};\frac{b(d+ex)}{bd-ae}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^p/Sqrt[d + e*x],x]
```

```
[Out] (2*((a + b*x)^2)^p*Sqrt[d + e*x]*Hypergeometric2F1[1/2, -2*p, 3/2, (b*(d +
e*x))/(b*d - a*e)])/(e*((e*(a + b*x))/(-(b*d) + a*e))^(2*p))
```

Maple [F] time = 1.143, size = 0, normalized size = 0.

$$\int \left(b^2x^2+2\,abx+a^2\right)^p\frac{1}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(1/2),x)
```

```
[Out] int((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^2+2\,abx+a^2\right)^p}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2)^p/sqrt(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^2+2\,abx+a^2\right)^p}{\sqrt{ex+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)^p/sqrt(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((a+bx)^2\right)^p}{\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**p/(e*x+d)**(1/2),x)
```

```
[Out] Integral(((a + b*x)**2)**p/sqrt(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^2+2\,abx+a^2\right)^p}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2)^p/sqrt(e*x + d), x)
```

3.1754 $\int \frac{\left(a^2+2abx+b^2x^2\right)^p}{(d+ex)^{3/2}} dx$

Optimal. Leaf size=81

$$-\frac{2\left(a^2+2abx+b^2x^2\right)^p\left(-\frac{e(a+bx)}{bd-ae}\right)^{-2p}{}_2F_1\left(-\frac{1}{2},-2p;\frac{1}{2};\frac{b(d+ex)}{bd-ae}\right)}{e\sqrt{d+ex}}$$

```
[Out] (-2*(a^2 + 2*a*b*x + b^2*x^2)^p*Hypergeometric2F1[-1/2, -2*p, 1/2, (b*(d +
e*x))/(b*d - a*e)])/(e*(-((e*(a + b*x))/(b*d - a*e)))^(2*p)*Sqrt[d + e*x])
```

Rubi [A] time = 0.0390484, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {646, 70, 69}

$$-\frac{2\left(a^2+2abx+b^2x^2\right)^p\left(-\frac{e(a+bx)}{bd-ae}\right)^{-2p}{}_2F_1\left(-\frac{1}{2},-2p;\frac{1}{2};\frac{b(d+ex)}{bd-ae}\right)}{e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^p/(d + e*x)^(3/2),x]
```

```
[Out] (-2*(a^2 + 2*a*b*x + b^2*x^2)^p*Hypergeometric2F1[-1/2, -2*p, 1/2, (b*(d +
e*x))/(b*d - a*e)])/(e*(-((e*(a + b*x))/(b*d - a*e)))^(2*p)*Sqrt[d + e*x])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(a^2+2abx+b^2x^2)^p}{(d+ex)^{3/2}}\,dx = \left((ab+b^2x)^{-2p}(a^2+2abx+b^2x^2)^p\right)\int \frac{(ab+b^2x)^{2p}}{(d+ex)^{3/2}}\,dx$$
$$= \left(\left(\frac{e(ab+b^2x)}{-b^2d+abe}\right)^{-2p}(a^2+2abx+b^2x^2)^p\right)\int \frac{\left(-\frac{ae}{bd-ae}-\frac{bex}{bd-ae}\right)^{2p}}{(d+ex)^{3/2}}\,dx$$
$$= -\frac{2\left(-\frac{e(a+bx)}{bd-ae}\right)^{-2p}(a^2+2abx+b^2x^2)^p\,{}_2F_1\left(-\frac{1}{2},-2p;\frac{1}{2};\frac{b(d+ex)}{bd-ae}\right)}{e\sqrt{d+ex}}$$

Mathematica [A] time = 0.0183813, size = 71, normalized size = 0.88

$$-\frac{2\left((a+bx)^2\right)^p\left(\frac{e(a+bx)}{ae-bd}\right)^{-2p}{}_2F_1\left(-\frac{1}{2},-2p;\frac{1}{2};\frac{b(d+ex)}{bd-ae}\right)}{e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^p/(d + e*x)^(3/2),x]

[Out] (-2*((a + b*x)^2)^p*Hypergeometric2F1[-1/2, -2*p, 1/2, (b*(d + e*x))/(b*d -
 a*e)])/(e*((e*(a + b*x))/(-(b*d) + a*e))^(2*p)*Sqrt[d + e*x])
```

Maple [F] time = 1.161, size = 0, normalized size = 0.

$$\int \left(b^2x^2+2\,abx+a^2\right)^p (ex+d)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(3/2),x)

[Out] int((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^2+2\,abx+a^2\right)^p}{(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(3/2),x, algorithm="maxima")

[Out] integrate((b^2*x^2 + 2*a*b*x + a^2)^p/(e*x + d)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex+d}\left(b^2x^2+2\,abx+a^2\right)^p}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*(b^2*x^2 + 2*a*b*x + a^2)^p/(e^2*x^2 + 2*d*e*x + d^2
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((a+bx)^2\right)^p}{(d+ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**p/(e*x+d)**(3/2),x)
```

```
[Out] Integral(((a + b*x)**2)**p/(d + e*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^2+2\,abx+a^2\right)^p}{(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2)^p/(e*x + d)^(3/2), x)
```

3.1755 $\int \frac{(a^2+2abx+b^2x^2)^p}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{2\left(a^2+2abx+b^2x^2\right)^p\left(-\frac{e(a+bx)}{bd-ae}\right)^{-2p}\,{}_2F_1\left(-\frac{3}{2},-2p;-\frac{1}{2};\frac{b(d+ex)}{bd-ae}\right)}{3e(d+ex)^{3/2}}$$

```
[Out] (-2*(a^2 + 2*a*b*x + b^2*x^2)^p*Hypergeometric2F1[-3/2, -2*p, -1/2, (b*(d +
 e*x))/(b*d - a*e)])/(3*e*(-((e*(a + b*x))/(b*d - a*e)))^(2*p)*(d + e*x)^(3
/2))
```

Rubi [A] time = 0.0400015, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {646, 70, 69}

$$-\frac{2\left(a^2+2abx+b^2x^2\right)^p\left(-\frac{e(a+bx)}{bd-ae}\right)^{-2p}\,{}_2F_1\left(-\frac{3}{2},-2p;-\frac{1}{2};\frac{b(d+ex)}{bd-ae}\right)}{3e(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x + b^2*x^2)^p/(d + e*x)^(5/2),x]
```

```
[Out] (-2*(a^2 + 2*a*b*x + b^2*x^2)^p*Hypergeometric2F1[-3/2, -2*p, -1/2, (b*(d +
 e*x))/(b*d - a*e)])/(3*e*(-((e*(a + b*x))/(b*d - a*e)))^(2*p)*(d + e*x)^(3
/2))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^p}{(d+ex)^{5/2}}\,dx = \left(\left(ab+b^2x\right)^{-2p}\left(a^2+2abx+b^2x^2\right)^p\right)\int \frac{\left(ab+b^2x\right)^{2p}}{(d+ex)^{5/2}}\,dx$$

$$= \left(\left(\frac{e\left(ab+b^2x\right)}{-b^2d+abe}\right)^{-2p}\left(a^2+2abx+b^2x^2\right)^p\right)\int \frac{\left(-\frac{ae}{bd-ae}-\frac{bex}{bd-ae}\right)^{2p}}{(d+ex)^{5/2}}\,dx$$

$$= -\frac{2\left(-\frac{e(a+bx)}{bd-ae}\right)^{-2p}\left(a^2+2abx+b^2x^2\right)^p\,{}_2F_1\left(-\frac{3}{2},-2p;-\frac{1}{2};\frac{b(d+ex)}{bd-ae}\right)}{3e(d+ex)^{3/2}}$$

Mathematica [A] time = 0.0206526, size = 73, normalized size = 0.88

$$-\frac{2\left((a+bx)^2\right)^p\left(\frac{e(a+bx)}{ae-bd}\right)^{-2p}\,{}_2F_1\left(-\frac{3}{2},-2p;-\frac{1}{2};\frac{b(d+ex)}{bd-ae}\right)}{3e(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x + b^2*x^2)^p/(d + e*x)^(5/2),x]
```

```
[Out] (-2*((a + b*x)^2)^p*Hypergeometric2F1[-3/2, -2*p, -1/2, (b*(d + e*x))/(b*d
- a*e)])/(3*e*((e*(a + b*x))/(-(b*d) + a*e))^(2*p)*(d + e*x)^(3/2))
```

Maple [F] time = 1.18, size = 0, normalized size = 0.

$$\int \left(b^2x^2+2\,abx+a^2\right)^p (ex+d)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(5/2),x)
```

```
[Out] int((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^2+2\,abx+a^2\right)^p}{(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2)^p/(e*x + d)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex+d}\left(b^2x^2+2\,abx+a^2\right)^p}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*(b^2*x^2 + 2*a*b*x + a^2)^p/(e^3*x^3 + 3*d*e^2*x^2 +
 3*d^2*e*x + d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((a+bx)^2\right)^p}{(d+ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2)**p/(e*x+d)**(5/2),x)
```

```
[Out] Integral(((a + b*x)**2)**p/(d + e*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^2+2\,abx+a^2\right)^p}{(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2)^p/(e*x + d)^(5/2), x)
```

3.1756 $\int (d+ex)^m \left(a^2+2abx+b^2x^2\right)^{5+p}\,dx$

Optimal. Leaf size=97

$$\frac{(bd-ae)^{10}\left(a^2+2abx+b^2x^2\right)^p(d+ex)^{m+1}\left(-\frac{e(a+bx)}{bd-ae}\right)^{-2p}{}_2F_1\left(m+1,-2(p+5);m+2;\frac{b(d+ex)}{bd-ae}\right)}{e^{11}(m+1)}$$

```
[Out] ((b*d - a*e)^10*(d + e*x)^(1 + m)*(a^2 + 2*a*b*x + b^2*x^2)^p*Hypergeometri
c2F1[1 + m, -2*(5 + p), 2 + m, (b*(d + e*x))/(b*d - a*e)])/(e^11*(1 + m)*(-
((e*(a + b*x))/(b*d - a*e)))^(2*p))
```

Rubi [A] time = 0.0632328, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {646, 70, 69}

$$\frac{(bd-ae)^{10}\left(a^2+2abx+b^2x^2\right)^p(d+ex)^{m+1}\left(-\frac{e(a+bx)}{bd-ae}\right)^{-2p}{}_2F_1\left(m+1,-2(p+5);m+2;\frac{b(d+ex)}{bd-ae}\right)}{e^{11}(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a^2 + 2*a*b*x + b^2*x^2)^(5 + p),x]
```

```
[Out] ((b*d - a*e)^10*(d + e*x)^(1 + m)*(a^2 + 2*a*b*x + b^2*x^2)^p*Hypergeometri
c2F1[1 + m, -2*(5 + p), 2 + m, (b*(d + e*x))/(b*d - a*e)])/(e^11*(1 + m)*(-
((e*(a + b*x))/(b*d - a*e)))^(2*p))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (d+ex)^m \left(a^2+2abx+b^2x^2\right)^{5+p} dx = \frac{\left(\left(ab+b^2x\right)^{-2p}\left(a^2+2abx+b^2x^2\right)^p\right)\int \left(ab+b^2x\right)^{2(5+p)}(d+ex)^m dx}{b^{10}}$$

$$= \frac{\left(\left(-b^2d+abe\right)^{10}\left(\frac{e\left(ab+b^2x\right)}{-b^2d+abe}\right)^{-2p}\left(a^2+2abx+b^2x^2\right)^p\right)\int (d+ex)^m \left(-\frac{ae}{bd-ae}-\frac{be}{bd-}\right.}{b^{10}e^{10}}$$

$$= \frac{(bd-ae)^{10}\left(-\frac{e(a+bx)}{bd-ae}\right)^{-2p}(d+ex)^{1+m}\left(a^2+2abx+b^2x^2\right)^p {}_2F_1\left(1+m,-2(5+\right.}{e^{11}(1+m)}$$

Mathematica [A] time = 0.0579863, size = 87, normalized size = 0.9

$$\frac{(bd-ae)^{10}\left((a+bx)^2\right)^p(d+ex)^{m+1}\left(\frac{e(a+bx)}{ae-bd}\right)^{-2p} {}_2F_1\left(m+1,-2(p+5);m+2;\frac{b(d+ex)}{bd-ae}\right)}{e^{11}(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(a^2 + 2*a*b*x + b^2*x^2)^(5 + p),x]
```

```
[Out] ((b*d - a*e)^10*((a + b*x)^2)^p*(d + e*x)^(1 + m)*Hypergeometric2F1[1 + m,
-2*(5 + p), 2 + m, (b*(d + e*x))/(b*d - a*e)])/(e^11*(1 + m)*((e*(a + b*x))
/(-(b*d) + a*e))^(2*p))
```

Maple [F] time = 1.21, size = 0, normalized size = 0.

$$\int (ex+d)^m \left(b^2x^2+2\,abx+a^2\right)^{5+p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^(5+p),x)
```

```
[Out] int((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^(5+p),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^2+2\,abx+a^2\right)^{p+5}(ex+d)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^(5+p),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2)^(p + 5)*(e*x + d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2x^2+2\,abx+a^2\right)^{p+5}(ex+d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^(5+p),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)^(p + 5)*(e*x + d)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(b**2*x**2+2*a*b*x+a**2)**(5+p),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^2 + 2\,abx + a^2\right)^{p+5} (ex + d)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^(5+p),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2)^(p + 5)*(e*x + d)^m, x)
```

3.1757 $\int (d+ex)^{-3-2p} \left(a^2+2abx+b^2x^2\right)^p \, dx$

Optimal. Leaf size=115

$$\frac{b(a+bx)\left(a^2+2abx+b^2x^2\right)^p (d+ex)^{-2p-1}}{2(p+1)(2p+1)(bd-ae)^2}+\frac{(a+bx)\left(a^2+2abx+b^2x^2\right)^p (d+ex)^{-2(p+1)}}{2(p+1)(bd-ae)}$$

```
[Out] (b*(a + b*x)*(d + e*x)^(-1 - 2*p)*(a^2 + 2*a*b*x + b^2*x^2)^p)/(2*(b*d - a*
e)^2*(1 + p)*(1 + 2*p)) + ((a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^p)/(2*(b*d -
 a*e)*(1 + p)*(d + e*x)^(2*(1 + p)))
```

Rubi [A] time = 0.0482377, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {646, 45, 37}

$$\frac{b(a+bx)\left(a^2+2abx+b^2x^2\right)^p (d+ex)^{-2p-1}}{2(p+1)(2p+1)(bd-ae)^2}+\frac{(a+bx)\left(a^2+2abx+b^2x^2\right)^p (d+ex)^{-2(p+1)}}{2(p+1)(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(-3 - 2*p)*(a^2 + 2*a*b*x + b^2*x^2)^p,x]
```

```
[Out] (b*(a + b*x)*(d + e*x)^(-1 - 2*p)*(a^2 + 2*a*b*x + b^2*x^2)^p)/(2*(b*d - a*
e)^2*(1 + p)*(1 + 2*p)) + ((a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^p)/(2*(b*d -
 a*e)*(1 + p)*(d + e*x)^(2*(1 + p)))
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int (d+ex)^{-3-2p}\left(a^2+2abx+b^2x^2\right)^p dx = \left(\left(ab+b^2x\right)^{-2p}\left(a^2+2abx+b^2x^2\right)^p\right)\int \left(ab+b^2x\right)^{2p}(d+ex)^{-3-2p}dx$$

$$= \frac{(a+bx)(d+ex)^{-2(1+p)}\left(a^2+2abx+b^2x^2\right)^p}{2(bd-ae)(1+p)} + \frac{\left(b\left(ab+b^2x\right)^{-2p}\left(a^2+2abx+b^2\right.\right.}{2(bd-\right.}$$

$$= \frac{b(a+bx)(d+ex)^{-1-2p}\left(a^2+2abx+b^2x^2\right)^p}{2(bd-ae)^2(1+p)(1+2p)} + \frac{(a+bx)(d+ex)^{-2(1+p)}\left(a^2+2ab\right.}{2(bd-ae)(1+p)}$$

Mathematica [A] time = 0.0399814, size = 72, normalized size = 0.63

$$\frac{(a+bx)\left((a+bx)^2\right)^p(d+ex)^{-2(p+1)}(-ae(2p+1)+2bd(p+1)+bex)}{2(p+1)(2p+1)(bd-ae)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(-3 - 2*p)*(a^2 + 2*a*b*x + b^2*x^2)^p,x]
```

```
[Out] ((a + b*x)*((a + b*x)^2)^p*(2*b*d*(1 + p) - a*e*(1 + 2*p) + b*e*x))/(2*(b*d
 - a*e)^2*(1 + p)*(1 + 2*p)*(d + e*x)^(2*(1 + p)))
```

Maple [A] time = 0.044, size = 139, normalized size = 1.2

$$-\frac{(ex+d)^{-2-2p}(bx+a)\left(2aep-2bdp-bxe+ae-2bd\right)\left(b^2x^2+2abx+a^2\right)^p}{4a^2e^2p^2-8abdep^2+4b^2d^2p^2+6a^2e^2p-12abdep+6b^2d^2p+2a^2e^2-4abde+2b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(-3-2*p)*(b^2*x^2+2*a*b*x+a^2)^p,x)
```

```
[Out] -1/2*(e*x+d)^(-2-2*p)*(b*x+a)*(2*a*e*p-2*b*d*p-b*e*x+a*e-2*b*d)*(b^2*x^2+2*
a*b*x+a^2)^p/(2*a^2*e^2*p^2-4*a*b*d*e*p^2+2*b^2*d^2*p^2+3*a^2*e^2*p-6*a*b*d
*e*p+3*b^2*d^2*p+a^2*e^2-2*a*b*d*e+b^2*d^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^2+2abx+a^2\right)^p(ex+d)^{-2p-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-3-2*p)*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2)^p*(e*x + d)^(-2*p - 3), x)
```

Fricas [A] time = 1.92459, size = 451, normalized size = 3.92

$$\frac{\left(b^2e^2x^3+2abd^2-a^2de+\left(3b^2de+2\left(b^2de-abe^2\right)p\right)x^2+2\left(abd^2-a^2de\right)p+\left(2b^2d^2+2abde-a^2e^2+2\left(b^2d^2-a^2e^2\right)p\right)\right.}{2\left(b^2d^2-2abde+a^2e^2+2\left(b^2d^2-2abde+a^2e^2\right)p^2+3\left(b^2d^2-2abde+a^2e^2\right)p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-3-2*p)*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*e^2*x^3 + 2*a*b*d^2 - a^2*d*e + (3*b^2*d*e + 2*(b^2*d*e - a*b*e^2)
*p)*x^2 + 2*(a*b*d^2 - a^2*d*e)*p + (2*b^2*d^2 + 2*a*b*d*e - a^2*e^2 + 2*(b
^2*d^2 - a^2*e^2)*p)*x)*(b^2*x^2 + 2*a*b*x + a^2)^p*(e*x + d)^(-2*p - 3)/(b
^2*d^2 - 2*a*b*d*e + a^2*e^2 + 2*(b^2*d^2 - 2*a*b*d*e + a^2*e^2)*p^2 + 3*(b
^2*d^2 - 2*a*b*d*e + a^2*e^2)*p)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(-3-2*p)*(b**2*x**2+2*a*b*x+a**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^2 + 2\,abx + a^2\right)^p (ex + d)^{-2\,p-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-3-2*p)*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2)^p*(e*x + d)^(-2*p - 3), x)
```

3.1758 $\int (d+ex)\left(9+12x+4x^2\right)^p\,dx$

Optimal. Leaf size=60

$$\frac{(2x+3)(2d-3e)\left(4x^2+12x+9\right)^p}{4(2p+1)}+\frac{e\left(4x^2+12x+9\right)^{p+1}}{8(p+1)}$$

```
[Out] ((2*d - 3*e)*(3 + 2*x)*(9 + 12*x + 4*x^2)^p)/(4*(1 + 2*p)) + (e*(9 + 12*x +
 4*x^2)^(1 + p))/(8*(1 + p))
```

Rubi [A] time = 0.0149501, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {640, 609}

$$\frac{(2x+3)(2d-3e)\left(4x^2+12x+9\right)^p}{4(2p+1)}+\frac{e\left(4x^2+12x+9\right)^{p+1}}{8(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(9 + 12*x + 4*x^2)^p,x]
```

```
[Out] ((2*d - 3*e)*(3 + 2*x)*(9 + 12*x + 4*x^2)^p)/(4*(1 + 2*p)) + (e*(9 + 12*x +
 4*x^2)^(1 + p))/(8*(1 + p))
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(9+12x+4x^2\right)^p\,dx &= \frac{e\left(9+12x+4x^2\right)^{1+p}}{8(1+p)}+\frac{1}{2}(2d-3e)\int \left(9+12x+4x^2\right)^p\,dx\\ &= \frac{(2d-3e)(3+2x)\left(9+12x+4x^2\right)^p}{4(1+2p)}+\frac{e\left(9+12x+4x^2\right)^{1+p}}{8(1+p)}\end{aligned}$$

Mathematica [A] time = 0.0246458, size = 48, normalized size = 0.8

$$\frac{(2x+3)\left((2x+3)^2\right)^p(4d(p+1)+e((4p+2)x-3))}{8(p+1)(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(9 + 12*x + 4*x^2)^p,x]
```

```
[Out] ((3 + 2*x)*((3 + 2*x)^2)^p*(4*d*(1 + p) + e*(-3 + (2 + 4*p)*x)))/(8*(1 + p)
*(1 + 2*p))
```

Maple [A] time = 0.042, size = 52, normalized size = 0.9

$$\frac{\left(4x^{2}+12x+9\right)^{p}\left(4epx+4dp+2ex+4d-3e\right)\left(3+2x\right)}{16p^{2}+24p+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(4*x^2+12*x+9)^p,x)
```

```
[Out] 1/8*(4*x^2+12*x+9)^p*(4*e*p*x+4*d*p+2*e*x+4*d-3*e)*(3+2*x)/(2*p^2+3*p+1)
```

Maxima [A] time = 1.07015, size = 88, normalized size = 1.47

$$\frac{\left(4\left(2p+1\right)x^{2}+12px-9\right)e(2x+3)^{2p}}{8\left(2p^{2}+3p+1\right)}+\frac{d(2x+3)^{2p}(2x+3)}{2\left(2p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^2+12*x+9)^p,x, algorithm="maxima")
```

```
[Out] 1/8*(4*(2*p + 1)*x^2 + 12*p*x - 9)*e*(2*x + 3)^(2*p)/(2*p^2 + 3*p + 1) + 1/
2*d*(2*x + 3)^(2*p)*(2*x + 3)/(2*p + 1)
```

Fricas [A] time = 1.8246, size = 154, normalized size = 2.57

$$\frac{\left(4\left(2ep+e\right)x^{2}+12dp+4\left((2d+3e)p+2d\right)x+12d-9e\right)\left(4x^{2}+12x+9\right)^{p}}{8\left(2p^{2}+3p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^2+12*x+9)^p,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(2*e*p + e)*x^2 + 12*d*p + 4*((2*d + 3*e)*p + 2*d)*x + 12*d - 9*e)*(
4*x^2 + 12*x + 9)^p/(2*p^2 + 3*p + 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x**2+12*x+9)**p,x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.16227, size = 205, normalized size = 3.42

$$\frac{8\left(4x^{2}+12x+9\right)^{p}px^{2}e+8\left(4x^{2}+12x+9\right)^{p}dpx+12\left(4x^{2}+12x+9\right)^{p}pxe+4\left(4x^{2}+12x+9\right)^{p}x^{2}e+12\left(4x^{2}+12x+\right.}{8\left(2p^{2}+3p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^2+12*x+9)^p,x, algorithm="giac")
```

```
[Out] 1/8*(8*(4*x^2 + 12*x + 9)^p*p*x^2*e + 8*(4*x^2 + 12*x + 9)^p*d*p*x + 12*(4*
x^2 + 12*x + 9)^p*p*x*e + 4*(4*x^2 + 12*x + 9)^p*x^2*e + 12*(4*x^2 + 12*x +
 9)^p*d*p + 8*(4*x^2 + 12*x + 9)^p*d*x + 12*(4*x^2 + 12*x + 9)^p*d - 9*(4*x
^2 + 12*x + 9)^p*e)/(2*p^2 + 3*p + 1)
```

3.1759 $\int (a+bx)^3 \left(ac+(bc+ad)x+bdx^2\right) \, dx$

Optimal. Leaf size=38

$$\frac{(a+bx)^5(bc-ad)}{5b^2}+\frac{d(a+bx)^6}{6b^2}$$

```
[Out] ((b*c - a*d)*(a + b*x)^5)/(5*b^2) + (d*(a + b*x)^6)/(6*b^2)
```

Rubi [A] time = 0.0178423, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {626, 43}

$$\frac{(a+bx)^5(bc-ad)}{5b^2}+\frac{d(a+bx)^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^5)/(5*b^2) + (d*(a + b*x)^6)/(6*b^2)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (a+bx)^3 \left(ac+(bc+ad)x+bdx^2\right) \, dx &= \int (a+bx)^4(c+dx) \, dx\\
&= \int \left(\frac{(bc-ad)(a+bx)^4}{b}+\frac{d(a+bx)^5}{b}\right) dx\\
&= \frac{(bc-ad)(a+bx)^5}{5b^2}+\frac{d(a+bx)^6}{6b^2}
\end{aligned}$$

Mathematica [B] time = 0.0180026, size = 84, normalized size = 2.21

$$\frac{1}{30}x\left(15a^2b^2x^2(4c+3dx)+20a^3bx(3c+2dx)+15a^4(2c+dx)+6ab^3x^3(5c+4dx)+b^4x^4(6c+5dx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

```
[Out] (x*(15*a^4*(2*c + d*x) + 20*a^3*b*x*(3*c + 2*d*x) + 15*a^2*b^2*x^2*(4*c + 3
*d*x) + 6*a*b^3*x^3*(5*c + 4*d*x) + b^4*x^4*(6*c + 5*d*x)))/30
```

Maple [B] time = 0.039, size = 133, normalized size = 3.5

$$\frac{b^4dx^6}{6}+\frac{\left(3\,adb^3+b^3\,(ad+bc)\right)x^5}{5}+\frac{\left(3\,b^2a^2d+3\,b^2a\,(ad+bc)+ab^3c\right)x^4}{4}+\frac{\left(a^3bd+3\,ba^2\,(ad+bc)+3\,b^2a^2c\right)x^3}{3}+\frac{\left(a^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(a*c+(a*d+b*c)*x+b*d*x^2),x)
```

```
[Out] 1/6*b^4*d*x^6+1/5*(3*a*d*b^3+b^3*(a*d+b*c))*x^5+1/4*(3*b^2*a^2*d+3*b^2*a*(a
*d+b*c)+a*b^3*c)*x^4+1/3*(a^3*b*d+3*b*a^2*(a*d+b*c)+3*b^2*a^2*c)*x^3+1/2*(a
^3*(a*d+b*c)+3*b*a^3*c)*x^2+a^4*c*x
```

Maxima [B] time = 1.09863, size = 130, normalized size = 3.42

$$\frac{1}{6}\,b^4dx^6+a^4cx+\frac{1}{5}\left(b^4c+4\,ab^3d\right)x^5+\frac{1}{2}\left(2\,ab^3c+3\,a^2b^2d\right)x^4+\frac{2}{3}\left(3\,a^2b^2c+2\,a^3bd\right)x^3+\frac{1}{2}\left(4\,a^3bc+a^4d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="maxima")
```

```
[Out] 1/6*b^4*d*x^6 + a^4*c*x + 1/5*(b^4*c + 4*a*b^3*d)*x^5 + 1/2*(2*a*b^3*c + 3*
a^2*b^2*d)*x^4 + 2/3*(3*a^2*b^2*c + 2*a^3*b*d)*x^3 + 1/2*(4*a^3*b*c + a^4*d
)*x^2
```

Fricas [B] time = 1.53655, size = 217, normalized size = 5.71

$$\frac{1}{6}x^6db^4+\frac{1}{5}x^5cb^4+\frac{4}{5}x^5db^3a+x^4cb^3a+\frac{3}{2}x^4db^2a^2+2x^3cb^2a^2+\frac{4}{3}x^3dba^3+2x^2cba^3+\frac{1}{2}x^2da^4+xca^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*d*b^4 + 1/5*x^5*c*b^4 + 4/5*x^5*d*b^3*a + x^4*c*b^3*a + 3/2*x^4*d*b
^2*a^2 + 2*x^3*c*b^2*a^2 + 4/3*x^3*d*b*a^3 + 2*x^2*c*b*a^3 + 1/2*x^2*d*a^4
+ x*c*a^4
```

Sympy [B] time = 0.140041, size = 100, normalized size = 2.63

$$a^4cx+\frac{b^4dx^6}{6}+x^5\left(\frac{4ab^3d}{5}+\frac{b^4c}{5}\right)+x^4\left(\frac{3a^2b^2d}{2}+ab^3c\right)+x^3\left(\frac{4a^3bd}{3}+2a^2b^2c\right)+x^2\left(\frac{a^4d}{2}+2a^3bc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

```
[Out] a**4*c*x + b**4*d*x**6/6 + x**5*(4*a*b**3*d/5 + b**4*c/5) + x**4*(3*a**2*b*
*2*d/2 + a*b**3*c) + x**3*(4*a**3*b*d/3 + 2*a**2*b**2*c) + x**2*(a**4*d/2 +
 2*a**3*b*c)
```

Giac [B] time = 1.16292, size = 131, normalized size = 3.45

$$\frac{1}{6}b^4dx^6 + \frac{1}{5}b^4cx^5 + \frac{4}{5}ab^3dx^5 + ab^3cx^4 + \frac{3}{2}a^2b^2dx^4 + 2a^2b^2cx^3 + \frac{4}{3}a^3bdx^3 + 2a^3bcx^2 + \frac{1}{2}a^4dx^2 + a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="giac")
```

```
[Out] 1/6*b^4*d*x^6 + 1/5*b^4*c*x^5 + 4/5*a*b^3*d*x^5 + a*b^3*c*x^4 + 3/2*a^2*b^2
*d*x^4 + 2*a^2*b^2*c*x^3 + 4/3*a^3*b*d*x^3 + 2*a^3*b*c*x^2 + 1/2*a^4*d*x^2
+ a^4*c*x
```

3.1760 $\int (a+bx)^2 \left(ac+(bc+ad)x+bdx^2\right) dx$

Optimal. Leaf size=38

$$\frac{(a+bx)^4(bc-ad)}{4b^2}+\frac{d(a+bx)^5}{5b^2}$$

```
[Out] ((b*c - a*d)*(a + b*x)^4)/(4*b^2) + (d*(a + b*x)^5)/(5*b^2)
```

Rubi [A] time = 0.0156864, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {626, 43}

$$\frac{(a+bx)^4(bc-ad)}{4b^2}+\frac{d(a+bx)^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^4)/(4*b^2) + (d*(a + b*x)^5)/(5*b^2)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2\left(ac+(bc+ad)x+bdx^2\right)dx &= \int (a+bx)^3(c+dx)\,dx\\ &= \int \left(\frac{(bc-ad)(a+bx)^3}{b}+\frac{d(a+bx)^4}{b}\right)dx\\ &= \frac{(bc-ad)(a+bx)^4}{4b^2}+\frac{d(a+bx)^5}{5b^2}\end{aligned}$$

Mathematica [A] time = 0.010389, size = 67, normalized size = 1.76

$$\frac{1}{2}a^2x^2(ad+3bc)+a^3cx+\frac{1}{4}b^2x^4(3ad+bc)+abx^3(ad+bc)+\frac{1}{5}b^3dx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

```
[Out] a^3*c*x + (a^2*(3*b*c + a*d)*x^2)/2 + a*b*(b*c + a*d)*x^3 + (b^2*(b*c + 3*a
*d)*x^4)/4 + (b^3*d*x^5)/5
```

Maple [B] time = 0.041, size = 94, normalized size = 2.5

$$\frac{b^3dx^5}{5}+\frac{\left(2\,ab^2d+b^2\,(ad+bc)\right)x^4}{4}+\frac{\left(a^2bd+2\,ab\,(ad+bc)+acb^2\right)x^3}{3}+\frac{\left(a^2\,(ad+bc)+2\,a^2bc\right)x^2}{2}+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(a*c+(a*d+b*c)*x+b*d*x^2),x)
```

```
[Out] 1/5*b^3*d*x^5+1/4*(2*a*b^2*d+b^2*(a*d+b*c))*x^4+1/3*(a^2*b*d+2*a*b*(a*d+b*c
)+a*c*b^2)*x^3+1/2*(a^2*(a*d+b*c)+2*a^2*b*c)*x^2+a^3*c*x
```

Maxima [B] time = 1.16033, size = 93, normalized size = 2.45

$$\frac{1}{5}\,b^3dx^5+a^3cx+\frac{1}{4}\left(b^3c+3\,ab^2d\right)x^4+\left(ab^2c+a^2bd\right)x^3+\frac{1}{2}\left(3\,a^2bc+a^3d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="maxima")
```

```
[Out] 1/5*b^3*d*x^5 + a^3*c*x + 1/4*(b^3*c + 3*a*b^2*d)*x^4 + (a*b^2*c + a^2*b*d)
*x^3 + 1/2*(3*a^2*b*c + a^3*d)*x^2
```

Fricas [B] time = 1.6618, size = 163, normalized size = 4.29

$$\frac{1}{5}x^5db^3+\frac{1}{4}x^4cb^3+\frac{3}{4}x^4db^2a+x^3cb^2a+x^3dba^2+\frac{3}{2}x^2cba^2+\frac{1}{2}x^2da^3+xca^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="fricas")
```

```
[Out] 1/5*x^5*d*b^3 + 1/4*x^4*c*b^3 + 3/4*x^4*d*b^2*a + x^3*c*b^2*a + x^3*d*b*a^2
 + 3/2*x^2*c*b*a^2 + 1/2*x^2*d*a^3 + x*c*a^3
```

Sympy [B] time = 0.084755, size = 73, normalized size = 1.92

$$a^3cx+\frac{b^3dx^5}{5}+x^4\left(\frac{3ab^2d}{4}+\frac{b^3c}{4}\right)+x^3\left(a^2bd+ab^2c\right)+x^2\left(\frac{a^3d}{2}+\frac{3a^2bc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

```
[Out] a**3*c*x + b**3*d*x**5/5 + x**4*(3*a*b**2*d/4 + b**3*c/4) + x**3*(a**2*b*d
+ a*b**2*c) + x**2*(a**3*d/2 + 3*a**2*b*c/2)
```

Giac [B] time = 1.17151, size = 97, normalized size = 2.55

$$\frac{1}{5}b^3dx^5 + \frac{1}{4}b^3cx^4 + \frac{3}{4}ab^2dx^4 + ab^2cx^3 + a^2bdx^3 + \frac{3}{2}a^2bcx^2 + \frac{1}{2}a^3dx^2 + a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="giac")
```

```
[Out] 1/5*b^3*d*x^5 + 1/4*b^3*c*x^4 + 3/4*a*b^2*d*x^4 + a*b^2*c*x^3 + a^2*b*d*x^3
 + 3/2*a^2*b*c*x^2 + 1/2*a^3*d*x^2 + a^3*c*x
```

3.1761 $\int (a+bx)\left(ac+(bc+ad)x+bdx^2\right)\,dx$

Optimal. Leaf size=38

$$\frac{(a+bx)^3(bc-ad)}{3b^2}+\frac{d(a+bx)^4}{4b^2}$$

```
[Out] ((b*c - a*d)*(a + b*x)^3)/(3*b^2) + (d*(a + b*x)^4)/(4*b^2)
```

Rubi [A] time = 0.0284002, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {626, 43}

$$\frac{(a+bx)^3(bc-ad)}{3b^2}+\frac{d(a+bx)^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^3)/(3*b^2) + (d*(a + b*x)^4)/(4*b^2)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)\left(ac+(bc+ad)x+bdx^2\right)\,dx &= \int (a+bx)^2(c+dx)\,dx\\ &= \int \left(\frac{(bc-ad)(a+bx)^2}{b}+\frac{d(a+bx)^3}{b}\right)dx\\ &= \frac{(bc-ad)(a+bx)^3}{3b^2}+\frac{d(a+bx)^4}{4b^2}\end{aligned}$$

Mathematica [A] time = 0.0074121, size = 46, normalized size = 1.21

$$\frac{1}{12}x\left(6a^2(2c+dx)+4abx(3c+2dx)+b^2x^2(4c+3dx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

```
[Out] (x*(6*a^2*(2*c + d*x) + 4*a*b*x*(3*c + 2*d*x) + b^2*x^2*(4*c + 3*d*x)))/12
```

Maple [A] time = 0.04, size = 55, normalized size = 1.5

$$\frac{b^2dx^4}{4}+\frac{(abd+b(ad+bc))x^3}{3}+\frac{(a(ad+bc)+abc)x^2}{2}+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x^2),x)
```

```
[Out] 1/4*b^2*d*x^4+1/3*(a*b*d+b*(a*d+b*c))*x^3+1/2*(a*(a*d+b*c)+a*b*c)*x^2+a^2*c
*x
```

Maxima [A] time = 1.06322, size = 65, normalized size = 1.71

$$\frac{1}{4}b^2dx^4+a^2cx+\frac{1}{3}\left(b^2c+2\,abd\right)x^3+\frac{1}{2}\left(2\,abc+a^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*d*x^4 + a^2*c*x + 1/3*(b^2*c + 2*a*b*d)*x^3 + 1/2*(2*a*b*c + a^2*d)
*x^2
```

Fricas [A] time = 1.58811, size = 115, normalized size = 3.03

$$\frac{1}{4}x^4db^2+\frac{1}{3}x^3cb^2+\frac{2}{3}x^3dba+x^2cba+\frac{1}{2}x^2da^2+xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*d*b^2 + 1/3*x^3*c*b^2 + 2/3*x^3*d*b*a + x^2*c*b*a + 1/2*x^2*d*a^2 +
 x*c*a^2
```

Sympy [A] time = 0.07156, size = 49, normalized size = 1.29

$$a^2cx+\frac{b^2dx^4}{4}+x^3\left(\frac{2abd}{3}+\frac{b^2c}{3}\right)+x^2\left(\frac{a^2d}{2}+abc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

```
[Out] a**2*c*x + b**2*d*x**4/4 + x**3*(2*a*b*d/3 + b**2*c/3) + x**2*(a**2*d/2 + a
*b*c)
```

Giac [A] time = 1.20138, size = 66, normalized size = 1.74

$$\frac{1}{4}b^2dx^4 + \frac{1}{3}b^2cx^3 + \frac{2}{3}abdx^3 + abcx^2 + \frac{1}{2}a^2dx^2 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="giac")
```

```
[Out] 1/4*b^2*d*x^4 + 1/3*b^2*c*x^3 + 2/3*a*b*d*x^3 + a*b*c*x^2 + 1/2*a^2*d*x^2 +
 a^2*c*x
```

3.1762 $\int (ac + (bc + ad)x + bdx^2)\, dx$

Optimal. Leaf size=28

$$\frac{1}{2}x^2(ad + bc) + acx + \frac{1}{3}bdx^3$$

```
[Out] a*c*x + ((b*c + a*d)*x^2)/2 + (b*d*x^3)/3
```

Rubi [A] time = 0.0077028, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{1}{2}x^2(ad + bc) + acx + \frac{1}{3}bdx^3$$

Antiderivative was successfully verified.

```
[In] Int[a*c + (b*c + a*d)*x + b*d*x^2,x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^2)/2 + (b*d*x^3)/3
```

Rubi steps

$$\int (ac + (bc + ad)x + bdx^2)\, dx = acx + \frac{1}{2}(bc + ad)x^2 + \frac{1}{3}bdx^3$$

Mathematica [A] time = 0.0000459, size = 32, normalized size = 1.14

$$acx + \frac{1}{2}adx^2 + \frac{1}{2}bcx^2 + \frac{1}{3}bdx^3$$

Antiderivative was successfully verified.

```
[In] Integrate[a*c + (b*c + a*d)*x + b*d*x^2,x]
```

```
[Out] a*c*x + (b*c*x^2)/2 + (a*d*x^2)/2 + (b*d*x^3)/3
```

Maple [A] time = 0.039, size = 25, normalized size = 0.9

$$acx + \frac{(ad + bc)\,x^2}{2} + \frac{bdx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*c+(a*d+b*c)*x+b*d*x^2,x)
```

```
[Out] a*c*x+1/2*(a*d+b*c)*x^2+1/3*b*d*x^3
```

Maxima [A] time = 1.04278, size = 32, normalized size = 1.14

$$\frac{1}{3}\,bdx^3 + acx + \frac{1}{2}\,(bc + ad)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c+(a*d+b*c)*x+b*d*x^2,x, algorithm="maxima")
```

```
[Out] 1/3*b*d*x^3 + a*c*x + 1/2*(b*c + a*d)*x^2
```

Fricas [A] time = 1.60225, size = 66, normalized size = 2.36

$$\frac{1}{3}x^3db + \frac{1}{2}x^2cb + \frac{1}{2}x^2da + xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c+(a*d+b*c)*x+b*d*x^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3*d*b + 1/2*x^2*c*b + 1/2*x^2*d*a + x*c*a
```

Sympy [A] time = 0.122718, size = 26, normalized size = 0.93

$$acx + \frac{bdx^3}{3} + x^2\left(\frac{ad}{2} + \frac{bc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c+(a*d+b*c)*x+b*d*x**2,x)
```

```
[Out] a*c*x + b*d*x**3/3 + x**2*(a*d/2 + b*c/2)
```

Giac [A] time = 1.24618, size = 32, normalized size = 1.14

$$\frac{1}{3}\,bdx^3 + acx + \frac{1}{2}\,(bc + ad)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c+(a*d+b*c)*x+b*d*x^2,x, algorithm="giac")
```

```
[Out] 1/3*b*d*x^3 + a*c*x + 1/2*(b*c + a*d)*x^2
```

3.1763 $\int \frac{ac+(bc+ad)x+bdx^2}{a+bx}\,dx$

Optimal. Leaf size=12

$$cx+\frac{dx^2}{2}$$

[Out] c*x + (d*x^2)/2

Rubi [A] time = 0.0065882, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {24}

$$cx+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)/(a + b*x),x]

[Out] c*x + (d*x^2)/2

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{ac+(bc+ad)x+bdx^2}{a+bx}\,dx&=\frac{\int \left(b^2c+b^2dx\right)\,dx}{b^2}\\&=cx+\frac{dx^2}{2}\end{aligned}$$

Mathematica [A] time = 0.0006399, size = 12, normalized size = 1.

$$cx+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)/(a + b*x),x]

[Out] c*x + (d*x^2)/2

Maple [A] time = 0.038, size = 11, normalized size = 0.9

$$cx+\frac{dx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a),x)
```

```
[Out] c*x+1/2*d*x^2
```

Maxima [A] time = 1.1512, size = 14, normalized size = 1.17

$$\frac{1}{2}dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*d*x^2 + c*x
```

Fricas [A] time = 1.77442, size = 23, normalized size = 1.92

$$\frac{1}{2}dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*d*x^2 + c*x
```

Sympy [A] time = 0.110716, size = 8, normalized size = 0.67

$$cx + \frac{dx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x**2)/(b*x+a),x)
```

```
[Out] c*x + d*x**2/2
```

Giac [A] time = 1.21701, size = 14, normalized size = 1.17

$$\frac{1}{2}dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*d*x^2 + c*x
```

3.1764 $\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^2}\,dx$

Optimal. Leaf size=25

$$\frac{(bc-ad)\log(a+bx)}{b^2}+\frac{dx}{b}$$

```
[Out] (d*x)/b + ((b*c - a*d)*Log[a + b*x])/b^2
```

Rubi [A] time = 0.0207689, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {24, 43}

$$\frac{(bc-ad)\log(a+bx)}{b^2}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)/(a + b*x)^2,x]
```

```
[Out] (d*x)/b + ((b*c - a*d)*Log[a + b*x])/b^2
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^2}\,dx &= \frac{\int \frac{b^2c+b^2dx}{a+bx}\,dx}{b^2}\\
&= \frac{\int \left(bd+\frac{b(bc-ad)}{a+bx}\right)dx}{b^2}\\
&= \frac{dx}{b}+\frac{(bc-ad)\log(a+bx)}{b^2}
\end{aligned}$$

Mathematica [A] time = 0.0071398, size = 25, normalized size = 1.

$$\frac{(bc-ad)\log(a+bx)}{b^2}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)/(a + b*x)^2,x]
```

```
[Out] (d*x)/b + ((b*c - a*d)*Log[a + b*x])/b^2
```

Maple [A] time = 0.041, size = 32, normalized size = 1.3

$$\frac{dx}{b} - \frac{\ln(bx+a)\,ad}{b^2} + \frac{\ln(bx+a)\,c}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^2,x)
```

```
[Out] d*x/b-1/b^2*ln(b*x+a)*a*d+1/b*ln(b*x+a)*c
```

Maxima [A] time = 1.0755, size = 34, normalized size = 1.36

$$\frac{dx}{b} + \frac{(bc-ad)\log(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] d*x/b + (b*c - a*d)*log(b*x + a)/b^2
```

Fricas [A] time = 1.71667, size = 54, normalized size = 2.16

$$\frac{bdx + (bc-ad)\log(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b*d*x + (b*c - a*d)*log(b*x + a))/b^2
```

Sympy [A] time = 0.578064, size = 20, normalized size = 0.8

$$\frac{dx}{b} - \frac{(ad-bc)\log(a+bx)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x**2)/(b*x+a)**2,x)
```

```
[Out] d*x/b - (a*d - b*c)*log(a + b*x)/b**2
```

Giac [B] time = 1.1685, size = 158, normalized size = 6.32

$$bd\left(\frac{2\,a\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^3}+\frac{bx+a}{b^3}-\frac{a^2}{(bx+a)b^3}\right)-\frac{(bc+ad)\left(\frac{\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b}-\frac{a}{(bx+a)b}\right)}{b}-\frac{ac}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] b*d*(2*a*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^3 + (b*x + a)/b^3 - a^2/(
(b*x + a)*b^3)) - (b*c + a*d)*(log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b - a
/((b*x + a)*b))/b - a*c/((b*x + a)*b)
```

3.1765 $\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^3} dx$

Optimal. Leaf size=32

$$\frac{d\log(a+bx)}{b^2}-\frac{bc-ad}{b^2(a+bx)}$$

```
[Out] -((b*c - a*d)/(b^2*(a + b*x))) + (d*Log[a + b*x])/b^2
```

Rubi [A] time = 0.024534, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {24, 43}

$$\frac{d\log(a+bx)}{b^2}-\frac{bc-ad}{b^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)/(a + b*x)^3,x]
```

```
[Out] -((b*c - a*d)/(b^2*(a + b*x))) + (d*Log[a + b*x])/b^2
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_Symbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^3}\,dx &= \frac{\int \frac{b^2c+b^2dx}{(a+bx)^2}\,dx}{b^2}\\ &= \frac{\int \left(\frac{b(bc-ad)}{(a+bx)^2}+\frac{bd}{a+bx}\right)dx}{b^2}\\ &= -\frac{bc-ad}{b^2(a+bx)}+\frac{d\log(a+bx)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.0101296, size = 31, normalized size = 0.97

$$\frac{ad-bc}{b^2(a+bx)}+\frac{d\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)/(a + b*x)^3,x]
```

```
[Out] (-(b*c) + a*d)/(b^2*(a + b*x)) + (d*Log[a + b*x])/b^2
```

Maple [A] time = 0.045, size = 39, normalized size = 1.2

$$\frac{d \ln (bx+a)}{b^2}+\frac{ad}{b^2 (bx+a)}-\frac{c}{b (bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^3,x)
```

```
[Out] d*ln(b*x+a)/b^2+1/b^2/(b*x+a)*a*d-1/b/(b*x+a)*c
```

Maxima [A] time = 1.0242, size = 47, normalized size = 1.47

$$-\frac{bc-ad}{b^3x+ab^2}+\frac{d \log (bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -(b*c - a*d)/(b^3*x + a*b^2) + d*log(b*x + a)/b^2
```

Fricas [A] time = 1.82422, size = 80, normalized size = 2.5

$$-\frac{bc-ad-(bdx+ad) \log (bx+a)}{b^3x+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(b*c - a*d - (b*d*x + a*d)*log(b*x + a))/(b^3*x + a*b^2)
```

Sympy [A] time = 0.510869, size = 27, normalized size = 0.84

$$\frac{ad-bc}{ab^2+b^3x}+\frac{d \log (a+bx)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x**2)/(b*x+a)**3,x)
```

```
[Out] (a*d - b*c)/(a*b**2 + b**3*x) + d*log(a + b*x)/b**2
```

Giac [A] time = 1.23979, size = 45, normalized size = 1.41

$$\frac{d\log\left(|bx+a|\right)}{b^2}-\frac{bc-ad}{(bx+a)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] d*log(abs(b*x + a))/b^2 - (b*c - a*d)/((b*x + a)*b^2)
```

3.1766 $\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^4} dx$

Optimal. Leaf size=28

$$-\frac{(c+dx)^2}{2(a+bx)^2(bc-ad)}$$

```
[Out] -(c + d*x)^2/(2*(b*c - a*d)*(a + b*x)^2)
```

Rubi [A] time = 0.0109377, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {24, 37}

$$-\frac{(c+dx)^2}{2(a+bx)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)/(a + b*x)^4,x]
```

```
[Out] -(c + d*x)^2/(2*(b*c - a*d)*(a + b*x)^2)
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^4} dx &= \frac{\int \frac{b^2c+b^2dx}{(a+bx)^3} dx}{b^2}\\ &= -\frac{(c+dx)^2}{2(bc-ad)(a+bx)^2}\end{aligned}$$

Mathematica [A] time = 0.009448, size = 26, normalized size = 0.93

$$-\frac{ad+b(c+2dx)}{2b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)/(a + b*x)^4,x]
```

[Out] -(a*d + b*(c + 2*d*x))/(2*b^2*(a + b*x)^2)

Maple [A] time = 0.045, size = 35, normalized size = 1.3

$$-\frac{-ad+bc}{2\,b^2\,(bx+a)^2}-\frac{d}{b^2\,(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^4,x)

[Out] -1/2*(-a*d+b*c)/b^2/(b*x+a)^2-d/b^2/(b*x+a)

Maxima [A] time = 1.06197, size = 51, normalized size = 1.82

$$-\frac{2\,bdx+bc+ad}{2\,\left(b^4x^2+2\,ab^3x+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^4,x, algorithm="maxima")

[Out] -1/2*(2*b*d*x + b*c + a*d)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)

Fricas [A] time = 1.6709, size = 81, normalized size = 2.89

$$-\frac{2\,bdx+bc+ad}{2\,\left(b^4x^2+2\,ab^3x+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^4,x, algorithm="fricas")

[Out] -1/2*(2*b*d*x + b*c + a*d)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)

Sympy [A] time = 0.524705, size = 39, normalized size = 1.39

$$-\frac{ad+bc+2bdx}{2a^2b^2+4ab^3x+2b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*c+(a*d+b*c)*x+b*d*x**2)/(b*x+a)**4,x)

[Out] -(a*d + b*c + 2*b*d*x)/(2*a**2*b**2 + 4*a*b**3*x + 2*b**4*x**2)

Giac [A] time = 1.17529, size = 32, normalized size = 1.14

$$-\frac{2\,bdx + bc + ad}{2\,(bx + a)^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*d*x + b*c + a*d)/((b*x + a)^2*b^2)
```

3.1767 $\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^5} dx$

Optimal. Leaf size=38

$$-\frac{bc-ad}{3b^2(a+bx)^3}-\frac{d}{2b^2(a+bx)^2}$$

```
[Out] -(b*c - a*d)/(3*b^2*(a + b*x)^3) - d/(2*b^2*(a + b*x)^2)
```

Rubi [A] time = 0.0242644, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {24, 43}

$$-\frac{bc-ad}{3b^2(a+bx)^3}-\frac{d}{2b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)/(a + b*x)^5,x]
```

```
[Out] -(b*c - a*d)/(3*b^2*(a + b*x)^3) - d/(2*b^2*(a + b*x)^2)
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^5}\,dx &= \frac{\int \frac{b^2c+b^2dx}{(a+bx)^4}\,dx}{b^2}\\
&= \frac{\int \left(\frac{b(bc-ad)}{(a+bx)^4}+\frac{bd}{(a+bx)^3}\right)dx}{b^2}\\
&= -\frac{bc-ad}{3b^2(a+bx)^3}-\frac{d}{2b^2(a+bx)^2}
\end{aligned}$$

Mathematica [A] time = 0.0087894, size = 27, normalized size = 0.71

$$-\frac{ad+2bc+3bdx}{6b^2(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)/(a + b*x)^5,x]
```

```
[Out] -(2*b*c + a*d + 3*b*d*x)/(6*b^2*(a + b*x)^3)
```

Maple [A] time = 0.044, size = 35, normalized size = 0.9

$$-\frac{d}{2\,b^2\left(bx+a\right)^2}-\frac{-ad+bc}{3\,b^2\left(bx+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^5,x)
```

```
[Out] -1/2*d/b^2/(b*x+a)^2-1/3*(-a*d+b*c)/b^2/(b*x+a)^3
```

Maxima [A] time = 1.08868, size = 68, normalized size = 1.79

$$-\frac{3\,bdx+2\,bc+ad}{6\left(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/6*(3*b*d*x + 2*b*c + a*d)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2
)
```

Fricas [A] time = 1.78756, size = 105, normalized size = 2.76

$$-\frac{3\,bdx+2\,bc+ad}{6\left(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b*d*x + 2*b*c + a*d)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2
)
```

Sympy [A] time = 0.57405, size = 53, normalized size = 1.39

$$-\frac{ad+2bc+3bdx}{6a^3b^2+18a^2b^3x+18ab^4x^2+6b^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x**2)/(b*x+a)**5,x)
```

```
[Out] -(a*d + 2*b*c + 3*b*d*x)/(6*a**3*b**2 + 18*a**2*b**3*x + 18*a*b**4*x**2 + 6
*b**5*x**3)
```

Giac [A] time = 1.14854, size = 55, normalized size = 1.45

$$-\frac{c}{3(bx+a)^3b}-\frac{d}{2(bx+a)^2b^2}+\frac{ad}{3(bx+a)^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/3*c/((b*x + a)^3*b) - 1/2*d/((b*x + a)^2*b^2) + 1/3*a*d/((b*x + a)^3*b^2
)
```

3.1768 $\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^6}\,dx$

Optimal. Leaf size=38

$$-\frac{bc-ad}{4b^2(a+bx)^4}-\frac{d}{3b^2(a+bx)^3}$$

```
[Out] -(b*c - a*d)/(4*b^2*(a + b*x)^4) - d/(3*b^2*(a + b*x)^3)
```

Rubi [A] time = 0.0260423, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {24, 43}

$$-\frac{bc-ad}{4b^2(a+bx)^4}-\frac{d}{3b^2(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)/(a + b*x)^6,x]
```

```
[Out] -(b*c - a*d)/(4*b^2*(a + b*x)^4) - d/(3*b^2*(a + b*x)^3)
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^6}\,dx &= \frac{\int \frac{b^2c+b^2dx}{(a+bx)^5}\,dx}{b^2}\\ &= \frac{\int \left(\frac{b(bc-ad)}{(a+bx)^5}+\frac{bd}{(a+bx)^4}\right)dx}{b^2}\\ &= -\frac{bc-ad}{4b^2(a+bx)^4}-\frac{d}{3b^2(a+bx)^3}\end{aligned}$$

Mathematica [A] time = 0.0089455, size = 27, normalized size = 0.71

$$-\frac{ad+3bc+4bdx}{12b^2(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)/(a + b*x)^6,x]
```

```
[Out] -(3*b*c + a*d + 4*b*d*x)/(12*b^2*(a + b*x)^4)
```

Maple [A] time = 0.044, size = 35, normalized size = 0.9

$$-\frac{-ad+bc}{4\,b^2\,(bx+a)^4}-\frac{d}{3\,b^2\,(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^6,x)
```

```
[Out] -1/4*(-a*d+b*c)/b^2/(b*x+a)^4-1/3*d/b^2/(b*x+a)^3
```

Maxima [A] time = 1.04554, size = 82, normalized size = 2.16

$$-\frac{4\,bdx+3\,bc+ad}{12\,\left(b^6x^4+4\,ab^5x^3+6\,a^2b^4x^2+4\,a^3b^3x+a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/12*(4*b*d*x + 3*b*c + a*d)/(b^6*x^4 + 4*a*b^5*x^3 + 6*a^2*b^4*x^2 + 4*a^
3*b^3*x + a^4*b^2)
```

Fricas [A] time = 1.77898, size = 128, normalized size = 3.37

$$-\frac{4\,bdx+3\,bc+ad}{12\,\left(b^6x^4+4\,ab^5x^3+6\,a^2b^4x^2+4\,a^3b^3x+a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] -1/12*(4*b*d*x + 3*b*c + a*d)/(b^6*x^4 + 4*a*b^5*x^3 + 6*a^2*b^4*x^2 + 4*a^
3*b^3*x + a^4*b^2)
```

Sympy [B] time = 0.738193, size = 65, normalized size = 1.71

$$-\frac{ad+3bc+4bdx}{12a^4b^2+48a^3b^3x+72a^2b^4x^2+48ab^5x^3+12b^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x**2)/(b*x+a)**6,x)
```

```
[Out] -(a*d + 3*b*c + 4*b*d*x)/(12*a**4*b**2 + 48*a**3*b**3*x + 72*a**2*b**4*x**2
 + 48*a*b**5*x**3 + 12*b**6*x**4)
```

Giac [A] time = 1.17497, size = 34, normalized size = 0.89

$$-\frac{4\,bdx+3\,bc+ad}{12\,(bx+a)^4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -1/12*(4*b*d*x + 3*b*c + a*d)/((b*x + a)^4*b^2)
```

3.1769 $\int (a+bx)^3 \left(ac+(bc+ad)x+bdx^2\right)^2 \, dx$

Optimal. Leaf size=65

$$\frac{2d(a+bx)^7(bc-ad)}{7b^3}+\frac{(a+bx)^6(bc-ad)^2}{6b^3}+\frac{d^2(a+bx)^8}{8b^3}$$

```
[Out] ((b*c - a*d)^2*(a + b*x)^6)/(6*b^3) + (2*d*(b*c - a*d)*(a + b*x)^7)/(7*b^3)
 + (d^2*(a + b*x)^8)/(8*b^3)
```

Rubi [A] time = 0.129975, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$\frac{2d(a+bx)^7(bc-ad)}{7b^3}+\frac{(a+bx)^6(bc-ad)^2}{6b^3}+\frac{d^2(a+bx)^8}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x)^6)/(6*b^3) + (2*d*(b*c - a*d)*(a + b*x)^7)/(7*b^3)
 + (d^2*(a + b*x)^8)/(8*b^3)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3 \left(ac+(bc+ad)x+bdx^2\right)^2 \, dx &= \int (a+bx)^5(c+dx)^2 \, dx\\ &= \int \left(\frac{(bc-ad)^2(a+bx)^5}{b^2}+\frac{2d(bc-ad)(a+bx)^6}{b^2}+\frac{d^2(a+bx)^7}{b^2}\right) dx\\ &= \frac{(bc-ad)^2(a+bx)^6}{6b^3}+\frac{2d(bc-ad)(a+bx)^7}{7b^3}+\frac{d^2(a+bx)^8}{8b^3}\end{aligned}$$

Mathematica [B] time = 0.0313715, size = 189, normalized size = 2.91

$$\frac{1}{6}b^3x^6\left(10a^2d^2+10abcd+b^2c^2\right)+ab^2x^5\left(2a^2d^2+4abcd+b^2c^2\right)+\frac{5}{4}a^2bx^4\left(a^2d^2+4abcd+2b^2c^2\right)+\frac{1}{3}a^3x^3\left(a^2d^2+10\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

```
[Out] a^5*c^2*x + (a^4*c*(5*b*c + 2*a*d)*x^2)/2 + (a^3*(10*b^2*c^2 + 10*a*b*c*d +
 a^2*d^2)*x^3)/3 + (5*a^2*b*(2*b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^4)/4 + a*b^
2*(b^2*c^2 + 4*a*b*c*d + 2*a^2*d^2)*x^5 + (b^3*(b^2*c^2 + 10*a*b*c*d + 10*a
^2*d^2)*x^6)/6 + (b^4*d*(2*b*c + 5*a*d)*x^7)/7 + (b^5*d^2*x^8)/8
```

Maple [B] time = 0.039, size = 315, normalized size = 4.9

$$\frac{b^5d^2x^8}{8} + \frac{\left(3\,b^4ad^2 + 2\,b^4\,(ad+bc)\,d\right)x^7}{7} + \frac{\left(3\,b^3a^2d^2 + 6\,b^3a\,(ad+bc)\,d + b^3\left(2\,cabd + (ad+bc)^2\right)\right)x^6}{6} + \frac{\left(a^3b^2d^2 + 6\,b^2a^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(a*c+(a*d+b*c)*x+b*d*x^2)^2,x)
```

```
[Out] 1/8*b^5*d^2*x^8+1/7*(3*b^4*a*d^2+2*b^4*(a*d+b*c)*d)*x^7+1/6*(3*b^3*a^2*d^2+
6*b^3*a*(a*d+b*c)*d+b^3*(2*c*a*b*d+(a*d+b*c)^2))*x^6+1/5*(a^3*b^2*d^2+6*b^2
*a^2*(a*d+b*c)*d+3*b^2*a*(2*c*a*b*d+(a*d+b*c)^2)+2*b^3*a*c*(a*d+b*c))*x^5+1
/4*(2*a^3*(a*d+b*c)*b*d+3*b*a^2*(2*c*a*b*d+(a*d+b*c)^2)+6*b^2*a^2*c*(a*d+b*
c)+a^2*b^3*c^2)*x^4+1/3*(a^3*(2*c*a*b*d+(a*d+b*c)^2)+6*b*a^3*c*(a*d+b*c)+3*
b^2*a^3*c^2)*x^3+1/2*(2*a^4*c*(a*d+b*c)+3*b*a^4*c^2)*x^2+a^5*c^2*x
```

Maxima [B] time = 1.0531, size = 266, normalized size = 4.09

$$\frac{1}{8}\,b^5d^2x^8 + a^5c^2x + \frac{1}{7}\left(2\,b^5cd + 5\,ab^4d^2\right)x^7 + \frac{1}{6}\left(b^5c^2 + 10\,ab^4cd + 10\,a^2b^3d^2\right)x^6 + \left(ab^4c^2 + 4\,a^2b^3cd + 2\,a^3b^2d^2\right)x^5 + \frac{5}{4}\left(2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^5*d^2*x^8 + a^5*c^2*x + 1/7*(2*b^5*c*d + 5*a*b^4*d^2)*x^7 + 1/6*(b^5*
c^2 + 10*a*b^4*c*d + 10*a^2*b^3*d^2)*x^6 + (a*b^4*c^2 + 4*a^2*b^3*c*d + 2*a
^3*b^2*d^2)*x^5 + 5/4*(2*a^2*b^3*c^2 + 4*a^3*b^2*c*d + a^4*b*d^2)*x^4 + 1/3
*(10*a^3*b^2*c^2 + 10*a^4*b*c*d + a^5*d^2)*x^3 + 1/2*(5*a^4*b*c^2 + 2*a^5*c
*d)*x^2
```

Fricas [B] time = 1.57015, size = 460, normalized size = 7.08

$$\frac{1}{8}x^8d^2b^5 + \frac{2}{7}x^7dcb^5 + \frac{5}{7}x^7d^2b^4a + \frac{1}{6}x^6c^2b^5 + \frac{5}{3}x^6dcb^4a + \frac{5}{3}x^6d^2b^3a^2 + x^5c^2b^4a + 4x^5dcb^3a^2 + 2x^5d^2b^2a^3 + \frac{5}{2}x^4c^2b^3a^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*d^2*b^5 + 2/7*x^7*d*c*b^5 + 5/7*x^7*d^2*b^4*a + 1/6*x^6*c^2*b^5 + 5
/3*x^6*d*c*b^4*a + 5/3*x^6*d^2*b^3*a^2 + x^5*c^2*b^4*a + 4*x^5*d*c*b^3*a^2
+ 2*x^5*d^2*b^2*a^3 + 5/2*x^4*c^2*b^3*a^2 + 5*x^4*d*c*b^2*a^3 + 5/4*x^4*d^2
*b*a^4 + 10/3*x^3*c^2*b^2*a^3 + 10/3*x^3*d*c*b*a^4 + 1/3*x^3*d^2*a^5 + 5/2*
```

```
x^2*c^2*b*a^4 + x^2*d*c*a^5 + x*c^2*a^5
```

Sympy [B] time = 0.219368, size = 218, normalized size = 3.35

$$a^5c^2x + \frac{b^5d^2x^8}{8} + x^7\left(\frac{5ab^4d^2}{7} + \frac{2b^5cd}{7}\right) + x^6\left(\frac{5a^2b^3d^2}{3} + \frac{5ab^4cd}{3} + \frac{b^5c^2}{6}\right) + x^5\left(2a^3b^2d^2 + 4a^2b^3cd + ab^4c^2\right) + x^4\left(\frac{5a^4}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(a*c+(a*d+b*c)*x+b*d*x**2)**2,x)
```

```
[Out] a**5*c**2*x + b**5*d**2*x**8/8 + x**7*(5*a*b**4*d**2/7 + 2*b**5*c*d/7) + x*
*6*(5*a**2*b**3*d**2/3 + 5*a*b**4*c*d/3 + b**5*c**2/6) + x**5*(2*a**3*b**2*
d**2 + 4*a**2*b**3*c*d + a*b**4*c**2) + x**4*(5*a**4*b*d**2/4 + 5*a**3*b**2
*c*d + 5*a**2*b**3*c**2/2) + x**3*(a**5*d**2/3 + 10*a**4*b*c*d/3 + 10*a**3*
b**2*c**2/3) + x**2*(a**5*c*d + 5*a**4*b*c**2/2)
```

Giac [B] time = 1.15711, size = 286, normalized size = 4.4

$$\frac{1}{8}b^5d^2x^8 + \frac{2}{7}b^5cdx^7 + \frac{5}{7}ab^4d^2x^7 + \frac{1}{6}b^5c^2x^6 + \frac{5}{3}ab^4cdx^6 + \frac{5}{3}a^2b^3d^2x^6 + ab^4c^2x^5 + 4a^2b^3cdx^5 + 2a^3b^2d^2x^5 + \frac{5}{2}a^2b^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*b^5*d^2*x^8 + 2/7*b^5*c*d*x^7 + 5/7*a*b^4*d^2*x^7 + 1/6*b^5*c^2*x^6 + 5
/3*a*b^4*c*d*x^6 + 5/3*a^2*b^3*d^2*x^6 + a*b^4*c^2*x^5 + 4*a^2*b^3*c*d*x^5
+ 2*a^3*b^2*d^2*x^5 + 5/2*a^2*b^3*c^2*x^4 + 5*a^3*b^2*c*d*x^4 + 5/4*a^4*b*d
^2*x^4 + 10/3*a^3*b^2*c^2*x^3 + 10/3*a^4*b*c*d*x^3 + 1/3*a^5*d^2*x^3 + 5/2*
a^4*b*c^2*x^2 + a^5*c*d*x^2 + a^5*c^2*x
```

3.1770 $\int (a+bx)^2 \left(ac+(bc+ad)x+bdx^2\right)^2 \, dx$

Optimal. Leaf size=65

$$\frac{d(a+bx)^6(bc-ad)}{3b^3}+\frac{(a+bx)^5(bc-ad)^2}{5b^3}+\frac{d^2(a+bx)^7}{7b^3}$$

```
[Out] ((b*c - a*d)^2*(a + b*x)^5)/(5*b^3) + (d*(b*c - a*d)*(a + b*x)^6)/(3*b^3) +
 (d^2*(a + b*x)^7)/(7*b^3)
```

Rubi [A] time = 0.0897019, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$\frac{d(a+bx)^6(bc-ad)}{3b^3}+\frac{(a+bx)^5(bc-ad)^2}{5b^3}+\frac{d^2(a+bx)^7}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x)^5)/(5*b^3) + (d*(b*c - a*d)*(a + b*x)^6)/(3*b^3) +
 (d^2*(a + b*x)^7)/(7*b^3)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (a+bx)^2 \left(ac+(bc+ad)x+bdx^2\right)^2 \, dx &= \int (a+bx)^4(c+dx)^2 \, dx \\
&= \int \left(\frac{(bc-ad)^2(a+bx)^4}{b^2}+\frac{2d(bc-ad)(a+bx)^5}{b^2}+\frac{d^2(a+bx)^6}{b^2}\right) dx \\
&= \frac{(bc-ad)^2(a+bx)^5}{5b^3}+\frac{d(bc-ad)(a+bx)^6}{3b^3}+\frac{d^2(a+bx)^7}{7b^3}
\end{aligned}$$

Mathematica [B] time = 0.0200078, size = 148, normalized size = 2.28

$$\frac{1}{5}b^2x^5\left(6a^2d^2+8abcd+b^2c^2\right)+abx^4\left(a^2d^2+3abcd+b^2c^2\right)+\frac{1}{3}a^2x^3\left(a^2d^2+8abcd+6b^2c^2\right)+a^3cx^2(ad+2bc)+a^4c^2x+$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

```
[Out] a^4*c^2*x + a^3*c*(2*b*c + a*d)*x^2 + (a^2*(6*b^2*c^2 + 8*a*b*c*d + a^2*d^2
)*x^3)/3 + a*b*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^4 + (b^2*(b^2*c^2 + 8*a*b*
c*d + 6*a^2*d^2)*x^5)/5 + (b^3*d*(b*c + 2*a*d)*x^6)/3 + (b^4*d^2*x^7)/7
```

Maple [B] time = 0.039, size = 231, normalized size = 3.6

$$\frac{b^4d^2x^7}{7} + \frac{\left(2\,d^2ab^3 + 2\,b^3\,(ad+bc)\,d\right)x^6}{6} + \frac{\left(a^2b^2d^2 + 4\,ab^2\,(ad+bc)\,d + b^2\left(2\,cabd + (ad+bc)^2\right)\right)x^5}{5} + \frac{\left(2\,a^2\,(ad+bc)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(a*c+(a*d+b*c)*x+b*d*x^2)^2,x)
```

```
[Out] 1/7*b^4*d^2*x^7+1/6*(2*d^2*a*b^3+2*b^3*(a*d+b*c)*d)*x^6+1/5*(a^2*b^2*d^2+4*
a*b^2*(a*d+b*c)*d+b^2*(2*c*a*b*d+(a*d+b*c)^2))*x^5+1/4*(2*a^2*(a*d+b*c)*b*d
+2*a*b*(2*c*a*b*d+(a*d+b*c)^2)+2*b^2*a*c*(a*d+b*c))*x^4+1/3*(a^2*(2*c*a*b*d
+(a*d+b*c)^2)+4*a^2*b*c*(a*d+b*c)+a^2*b^2*c^2)*x^3+1/2*(2*a^3*c*(a*d+b*c)+2
*a^3*b*c^2)*x^2+a^4*c^2*x
```

Maxima [B] time = 1.04526, size = 211, normalized size = 3.25

$$\frac{1}{7}\,b^4d^2x^7 + a^4c^2x + \frac{1}{3}\left(b^4cd + 2\,ab^3d^2\right)x^6 + \frac{1}{5}\left(b^4c^2 + 8\,ab^3cd + 6\,a^2b^2d^2\right)x^5 + \left(ab^3c^2 + 3\,a^2b^2cd + a^3bd^2\right)x^4 + \frac{1}{3}\left(6\,a^2b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^4*d^2*x^7 + a^4*c^2*x + 1/3*(b^4*c*d + 2*a*b^3*d^2)*x^6 + 1/5*(b^4*c^
2 + 8*a*b^3*c*d + 6*a^2*b^2*d^2)*x^5 + (a*b^3*c^2 + 3*a^2*b^2*c*d + a^3*b*d
^2)*x^4 + 1/3*(6*a^2*b^2*c^2 + 8*a^3*b*c*d + a^4*d^2)*x^3 + (2*a^3*b*c^2 +
a^4*c*d)*x^2
```

Fricas [B] time = 1.68828, size = 363, normalized size = 5.58

$$\frac{1}{7}x^7d^2b^4 + \frac{1}{3}x^6dcb^4 + \frac{2}{3}x^6d^2b^3a + \frac{1}{5}x^5c^2b^4 + \frac{8}{5}x^5dcb^3a + \frac{6}{5}x^5d^2b^2a^2 + x^4c^2b^3a + 3x^4dcb^2a^2 + x^4d^2ba^3 + 2x^3c^2b^2a^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*d^2*b^4 + 1/3*x^6*d*c*b^4 + 2/3*x^6*d^2*b^3*a + 1/5*x^5*c^2*b^4 + 8
/5*x^5*d*c*b^3*a + 6/5*x^5*d^2*b^2*a^2 + x^4*c^2*b^3*a + 3*x^4*d*c*b^2*a^2
+ x^4*d^2*b*a^3 + 2*x^3*c^2*b^2*a^2 + 8/3*x^3*d*c*b*a^3 + 1/3*x^3*d^2*a^4 +
 2*x^2*c^2*b*a^3 + x^2*d*c*a^4 + x*c^2*a^4
```

Sympy [B] time = 0.126889, size = 168, normalized size = 2.58

$$a^4c^2x + \frac{b^4d^2x^7}{7} + x^6\left(\frac{2ab^3d^2}{3} + \frac{b^4cd}{3}\right) + x^5\left(\frac{6a^2b^2d^2}{5} + \frac{8ab^3cd}{5} + \frac{b^4c^2}{5}\right) + x^4\left(a^3bd^2 + 3a^2b^2cd + ab^3c^2\right) + x^3\left(\frac{a^4d^2}{3} + \frac{8a}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(a*c+(a*d+b*c)*x+b*d*x**2)**2,x)
```

```
[Out] a**4*c**2*x + b**4*d**2*x**7/7 + x**6*(2*a*b**3*d**2/3 + b**4*c*d/3) + x**5
*(6*a**2*b**2*d**2/5 + 8*a*b**3*c*d/5 + b**4*c**2/5) + x**4*(a**3*b*d**2 +
3*a**2*b**2*c*d + a*b**3*c**2) + x**3*(a**4*d**2/3 + 8*a**3*b*c*d/3 + 2*a**
2*b**2*c**2) + x**2*(a**4*c*d + 2*a**3*b*c**2)
```

Giac [B] time = 1.191, size = 230, normalized size = 3.54

$$\frac{1}{7}b^4d^2x^7 + \frac{1}{3}b^4cdx^6 + \frac{2}{3}ab^3d^2x^6 + \frac{1}{5}b^4c^2x^5 + \frac{8}{5}ab^3cdx^5 + \frac{6}{5}a^2b^2d^2x^5 + ab^3c^2x^4 + 3a^2b^2cdx^4 + a^3bd^2x^4 + 2a^2b^2c^2x^3 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/7*b^4*d^2*x^7 + 1/3*b^4*c*d*x^6 + 2/3*a*b^3*d^2*x^6 + 1/5*b^4*c^2*x^5 + 8
/5*a*b^3*c*d*x^5 + 6/5*a^2*b^2*d^2*x^5 + a*b^3*c^2*x^4 + 3*a^2*b^2*c*d*x^4
+ a^3*b*d^2*x^4 + 2*a^2*b^2*c^2*x^3 + 8/3*a^3*b*c*d*x^3 + 1/3*a^4*d^2*x^3 +
 2*a^3*b*c^2*x^2 + a^4*c*d*x^2 + a^4*c^2*x
```

3.1771 $\int (a+bx)\left(ac+(bc+ad)x+bdx^2\right)^2\,dx$

Optimal. Leaf size=65

$$\frac{2d(a+bx)^5(bc-ad)}{5b^3}+\frac{(a+bx)^4(bc-ad)^2}{4b^3}+\frac{d^2(a+bx)^6}{6b^3}$$

```
[Out] ((b*c - a*d)^2*(a + b*x)^4)/(4*b^3) + (2*d*(b*c - a*d)*(a + b*x)^5)/(5*b^3)
 + (d^2*(a + b*x)^6)/(6*b^3)
```

Rubi [A] time = 0.0657123, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {626, 43}

$$\frac{2d(a+bx)^5(bc-ad)}{5b^3}+\frac{(a+bx)^4(bc-ad)^2}{4b^3}+\frac{d^2(a+bx)^6}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x)^4)/(4*b^3) + (2*d*(b*c - a*d)*(a + b*x)^5)/(5*b^3)
 + (d^2*(a + b*x)^6)/(6*b^3)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)\left(ac+(bc+ad)x+bdx^2\right)^2\,dx &= \int (a+bx)^3(c+dx)^2\,dx\\ &= \int \left(\frac{(bc-ad)^2(a+bx)^3}{b^2}+\frac{2d(bc-ad)(a+bx)^4}{b^2}+\frac{d^2(a+bx)^5}{b^2}\right)dx\\ &= \frac{(bc-ad)^2(a+bx)^4}{4b^3}+\frac{2d(bc-ad)(a+bx)^5}{5b^3}+\frac{d^2(a+bx)^6}{6b^3}\end{aligned}$$

Mathematica [A] time = 0.0144083, size = 122, normalized size = 1.88

$$\frac{1}{4}bx^4\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{3}ax^3\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{2}a^2cx^2(2ad+3bc)+a^3c^2x+\frac{1}{5}b^2dx^5(3ad+2bc)+\frac{1}{6}b$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

```
[Out] a^3*c^2*x + (a^2*c*(3*b*c + 2*a*d)*x^2)/2 + (a*(3*b^2*c^2 + 6*a*b*c*d + a^2
*d^2)*x^3)/3 + (b*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^4)/4 + (b^2*d*(2*b*c
+ 3*a*d)*x^5)/5 + (b^3*d^2*x^6)/6
```

Maple [B] time = 0.038, size = 147, normalized size = 2.3

$$\frac{b^3d^2x^6}{6}+\frac{\left(ab^2d^2+2\,b^2\left(ad+bc\right)d\right)x^5}{5}+\frac{\left(2\,a\left(ad+bc\right)bd+b\left(2\,cabd+\left(ad+bc\right)^2\right)\right)x^4}{4}+\frac{\left(a\left(2\,cabd+\left(ad+bc\right)^2\right)+2\,ba\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x^2)^2,x)
```

```
[Out] 1/6*b^3*d^2*x^6+1/5*(a*b^2*d^2+2*b^2*(a*d+b*c)*d)*x^5+1/4*(2*a*(a*d+b*c)*b*
d+b*(2*c*a*b*d+(a*d+b*c)^2))*x^4+1/3*(a*(2*c*a*b*d+(a*d+b*c)^2)+2*b*a*c*(a*
d+b*c))*x^3+1/2*(2*a^2*c*(a*d+b*c)+a^2*b*c^2)*x^2+a^3*c^2*x
```

Maxima [B] time = 1.10535, size = 167, normalized size = 2.57

$$\frac{1}{6}\,b^3d^2x^6+a^3c^2x+\frac{1}{5}\left(2\,b^3cd+3\,ab^2d^2\right)x^5+\frac{1}{4}\left(b^3c^2+6\,ab^2cd+3\,a^2bd^2\right)x^4+\frac{1}{3}\left(3\,ab^2c^2+6\,a^2bcd+a^3d^2\right)x^3+\frac{1}{2}\left(3\,a^2b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^3*d^2*x^6 + a^3*c^2*x + 1/5*(2*b^3*c*d + 3*a*b^2*d^2)*x^5 + 1/4*(b^3*
c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^4 + 1/3*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3
*d^2)*x^3 + 1/2*(3*a^2*b*c^2 + 2*a^3*c*d)*x^2
```

Fricas [B] time = 1.58249, size = 285, normalized size = 4.38

$$\frac{1}{6}x^6d^2b^3+\frac{2}{5}x^5dcb^3+\frac{3}{5}x^5d^2b^2a+\frac{1}{4}x^4c^2b^3+\frac{3}{2}x^4dcb^2a+\frac{3}{4}x^4d^2ba^2+x^3c^2b^2a+2x^3dcba^2+\frac{1}{3}x^3d^2a^3+\frac{3}{2}x^2c^2ba^2+x^2dc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*x^6*d^2*b^3 + 2/5*x^5*d*c*b^3 + 3/5*x^5*d^2*b^2*a + 1/4*x^4*c^2*b^3 + 3
/2*x^4*d*c*b^2*a + 3/4*x^4*d^2*b*a^2 + x^3*c^2*b^2*a + 2*x^3*d*c*b*a^2 + 1/
3*x^3*d^2*a^3 + 3/2*x^2*c^2*b*a^2 + x^2*d*c*a^3 + x*c^2*a^3
```

Sympy [B] time = 0.102693, size = 133, normalized size = 2.05

$$a^3c^2x+\frac{b^3d^2x^6}{6}+x^5\left(\frac{3ab^2d^2}{5}+\frac{2b^3cd}{5}\right)+x^4\left(\frac{3a^2bd^2}{4}+\frac{3ab^2cd}{2}+\frac{b^3c^2}{4}\right)+x^3\left(\frac{a^3d^2}{3}+2a^2bcd+ab^2c^2\right)+x^2\left(a^3cd+\frac{3a^2b}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x**2)**2,x)
```

```
[Out] a**3*c**2*x + b**3*d**2*x**6/6 + x**5*(3*a*b**2*d**2/5 + 2*b**3*c*d/5) + x*
*4*(3*a**2*b*d**2/4 + 3*a*b**2*c*d/2 + b**3*c**2/4) + x**3*(a**3*d**2/3 + 2
*a**2*b*c*d + a*b**2*c**2) + x**2*(a**3*c*d + 3*a**2*b*c**2/2)
```

Giac [B] time = 1.2061, size = 176, normalized size = 2.71

$$\frac{1}{6}\,b^3d^2x^6+\frac{2}{5}\,b^3cdx^5+\frac{3}{5}\,ab^2d^2x^5+\frac{1}{4}\,b^3c^2x^4+\frac{3}{2}\,ab^2cdx^4+\frac{3}{4}\,a^2bd^2x^4+ab^2c^2x^3+2\,a^2bcdx^3+\frac{1}{3}\,a^3d^2x^3+\frac{3}{2}\,a^2bc^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*b^3*d^2*x^6 + 2/5*b^3*c*d*x^5 + 3/5*a*b^2*d^2*x^5 + 1/4*b^3*c^2*x^4 + 3
/2*a*b^2*c*d*x^4 + 3/4*a^2*b*d^2*x^4 + a*b^2*c^2*x^3 + 2*a^2*b*c*d*x^3 + 1/
3*a^3*d^2*x^3 + 3/2*a^2*b*c^2*x^2 + a^3*c*d*x^2 + a^3*c^2*x
```

3.1772 $\int \left(ac+(bc+ad)x+bdx^2\right)^2\,dx$

Optimal. Leaf size=65

$$-\frac{b(c+dx)^4(bc-ad)}{2d^3}+\frac{(c+dx)^3(bc-ad)^2}{3d^3}+\frac{b^2(c+dx)^5}{5d^3}$$

```
[Out] ((b*c - a*d)^2*(c + d*x)^3)/(3*d^3) - (b*(b*c - a*d)*(c + d*x)^4)/(2*d^3) +
 (b^2*(c + d*x)^5)/(5*d^3)
```

Rubi [A] time = 0.0692441, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {610, 43}

$$-\frac{b(c+dx)^4(bc-ad)}{2d^3}+\frac{(c+dx)^3(bc-ad)^2}{3d^3}+\frac{b^2(c+dx)^5}{5d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x)^3)/(3*d^3) - (b*(b*c - a*d)*(c + d*x)^4)/(2*d^3) +
 (b^2*(c + d*x)^5)/(5*d^3)
```

Rule 610

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Dist[1/c^p, Int[Simp[b/2 - q/2 + c*x, x]^p*Simp[b/2 + q/2 + c
*x, x]^p, x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(ac+(bc+ad)x+bdx^2\right)^2\,dx &= \frac{\int (bc+bdx)^2(ad+bdx)^2\,dx}{b^2d^2}\\ &= \frac{\int \left((bc-ad)^2(bc+bdx)^2-2(bc-ad)(bc+bdx)^3+(bc+bdx)^4\right)dx}{b^2d^2}\\ &= \frac{(bc-ad)^2(c+dx)^3}{3d^3}-\frac{b(bc-ad)(c+dx)^4}{2d^3}+\frac{b^2(c+dx)^5}{5d^3}\end{aligned}$$

Mathematica [A] time = 0.0099373, size = 79, normalized size = 1.22

$$\frac{1}{3}x^3\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{1}{2}bdx^4(ad+bc)+acx^2(ad+bc)+\frac{1}{5}b^2d^2x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

```
[Out] a^2*c^2*x + a*c*(b*c + a*d)*x^2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^3)/3 +
 (b*d*(b*c + a*d)*x^4)/2 + (b^2*d^2*x^5)/5
```

Maple [A] time = 0.039, size = 69, normalized size = 1.1

$$\frac{b^2d^2x^5}{5}+\frac{(ad+bc)\,bdx^4}{2}+\frac{\left(2\,cabd+(ad+bc)^2\right)x^3}{3}+ac\,(ad+bc)\,x^2+a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+(a*d+b*c)*x+b*d*x^2)^2,x)
```

```
[Out] 1/5*b^2*d^2*x^5+1/2*(a*d+b*c)*b*d*x^4+1/3*(2*c*a*b*d+(a*d+b*c)^2)*x^3+a*c*(
a*d+b*c)*x^2+a^2*c^2*x
```

Maxima [A] time = 1.04471, size = 97, normalized size = 1.49

$$\frac{1}{5}\,b^2d^2x^5+\frac{1}{2}\,(bc+ad)bdx^4+a^2c^2x+\frac{1}{3}\,(bc+ad)^2x^3+\frac{1}{3}\left(2\,bdx^3+3\,(bc+ad)x^2\right)ac$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*d^2*x^5 + 1/2*(b*c + a*d)*b*d*x^4 + a^2*c^2*x + 1/3*(b*c + a*d)^2*x
^3 + 1/3*(2*b*d*x^3 + 3*(b*c + a*d)*x^2)*a*c
```

Fricas [A] time = 1.4657, size = 198, normalized size = 3.05

$$\frac{1}{5}x^5d^2b^2+\frac{1}{2}x^4dcb^2+\frac{1}{2}x^4d^2ba+\frac{1}{3}x^3c^2b^2+\frac{4}{3}x^3dcba+\frac{1}{3}x^3d^2a^2+x^2c^2ba+x^2dca^2+xc^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*d^2*b^2 + 1/2*x^4*d*c*b^2 + 1/2*x^4*d^2*b*a + 1/3*x^3*c^2*b^2 + 4/3
*x^3*d*c*b*a + 1/3*x^3*d^2*a^2 + x^2*c^2*b*a + x^2*d*c*a^2 + x*c^2*a^2
```

Sympy [A] time = 0.086148, size = 87, normalized size = 1.34

$$a^2c^2x+\frac{b^2d^2x^5}{5}+x^4\left(\frac{abd^2}{2}+\frac{b^2cd}{2}\right)+x^3\left(\frac{a^2d^2}{3}+\frac{4abcd}{3}+\frac{b^2c^2}{3}\right)+x^2\left(a^2cd+abc^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x**2)**2,x)
```

```
[Out] a**2*c**2*x + b**2*d**2*x**5/5 + x**4*(a*b*d**2/2 + b**2*c*d/2) + x**3*(a**
2*d**2/3 + 4*a*b*c*d/3 + b**2*c**2/3) + x**2*(a**2*c*d + a*b*c**2)
```

Giac [A] time = 1.2152, size = 120, normalized size = 1.85

$$\frac{1}{5}\,b^2d^2x^5+\frac{1}{2}\,b^2cdx^4+\frac{1}{2}\,abd^2x^4+\frac{1}{3}\,b^2c^2x^3+\frac{4}{3}\,abcdx^3+\frac{1}{3}\,a^2d^2x^3+abc^2x^2+a^2cdx^2+a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/5*b^2*d^2*x^5 + 1/2*b^2*c*d*x^4 + 1/2*a*b*d^2*x^4 + 1/3*b^2*c^2*x^3 + 4/3
*a*b*c*d*x^3 + 1/3*a^2*d^2*x^3 + a*b*c^2*x^2 + a^2*c*d*x^2 + a^2*c^2*x
```

3.1773 $\int \frac{(ac+(bc+ad)x+bdx^2)^2}{a+bx} dx$

Optimal. Leaf size=38

$$\frac{b(c+dx)^4}{4d^2}-\frac{(c+dx)^3(bc-ad)}{3d^2}$$

```
[Out] -((b*c - a*d)*(c + d*x)^3)/(3*d^2) + (b*(c + d*x)^4)/(4*d^2)
```

Rubi [A] time = 0.034688, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$\frac{b(c+dx)^4}{4d^2}-\frac{(c+dx)^3(bc-ad)}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x),x]
```

```
[Out] -((b*c - a*d)*(c + d*x)^3)/(3*d^2) + (b*(c + d*x)^4)/(4*d^2)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ac+(bc+ad)x+bdx^2)^2}{a+bx}\,dx &= \int (a+bx)(c+dx)^2\,dx\\ &= \int \left(\frac{(-bc+ad)(c+dx)^2}{d}+\frac{b(c+dx)^3}{d}\right)dx\\ &= -\frac{(bc-ad)(c+dx)^3}{3d^2}+\frac{b(c+dx)^4}{4d^2}\end{aligned}$$

Mathematica [A] time = 0.009309, size = 47, normalized size = 1.24

$$\frac{1}{12}x\left(4dx^2(ad+2bc)+6cx(2ad+bc)+12ac^2+3bd^2x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x),x]
```

```
[Out] (x*(12*a*c^2 + 6*c*(b*c + 2*a*d)*x + 4*d*(2*b*c + a*d)*x^2 + 3*b*d^2*x^3))/
12
```

Maple [A] time = 0.038, size = 55, normalized size = 1.5

$$\frac{bd^2x^4}{4} + \frac{(bcd + d(ad + bc))x^3}{3} + \frac{(c(ad + bc) + acd)x^2}{2} + xac^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a),x)
```

```
[Out] 1/4*b*d^2*x^4+1/3*(b*c*d+d*(a*d+b*c))*x^3+1/2*(c*(a*d+b*c)+a*c*d)*x^2+x*a*c
^2
```

Maxima [A] time = 1.05459, size = 65, normalized size = 1.71

$$\frac{1}{4}bd^2x^4 + ac^2x + \frac{1}{3}\left(2bcd + ad^2\right)x^3 + \frac{1}{2}\left(bc^2 + 2acd\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*b*d^2*x^4 + a*c^2*x + 1/3*(2*b*c*d + a*d^2)*x^3 + 1/2*(b*c^2 + 2*a*c*d)
*x^2
```

Fricas [A] time = 1.79481, size = 109, normalized size = 2.87

$$\frac{1}{4}bd^2x^4 + ac^2x + \frac{1}{3}\left(2bcd + ad^2\right)x^3 + \frac{1}{2}\left(bc^2 + 2acd\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*b*d^2*x^4 + a*c^2*x + 1/3*(2*b*c*d + a*d^2)*x^3 + 1/2*(b*c^2 + 2*a*c*d)
*x^2
```

Sympy [A] time = 0.128249, size = 49, normalized size = 1.29

$$ac^2x + \frac{bd^2x^4}{4} + x^3\left(\frac{ad^2}{3} + \frac{2bcd}{3}\right) + x^2\left(acd + \frac{bc^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x**2)**2/(b*x+a),x)
```

```
[Out] a*c**2*x + b*d**2*x**4/4 + x**3*(a*d**2/3 + 2*b*c*d/3) + x**2*(a*c*d + b*c*
*2/2)
```

Giac [A] time = 1.20942, size = 66, normalized size = 1.74

$$\frac{1}{4}bd^2x^4 + \frac{2}{3}bcdx^3 + \frac{1}{3}ad^2x^3 + \frac{1}{2}bc^2x^2 + acdx^2 + ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*b*d^2*x^4 + 2/3*b*c*d*x^3 + 1/3*a*d^2*x^3 + 1/2*b*c^2*x^2 + a*c*d*x^2 +
 a*c^2*x
```

3.1774 $\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^2}{(a+bx)^2} dx$

Optimal. Leaf size=14

$$\frac{(c+dx)^3}{3d}$$

```
[Out] (c + d*x)^3/(3*d)
```

Rubi [A] time = 0.0099224, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 32}

$$\frac{(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^2,x]
```

```
[Out] (c + d*x)^3/(3*d)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^2}{(a+bx)^2} dx &= \int (c+dx)^2 dx \\
&= \frac{(c+dx)^3}{3d}
\end{aligned}$$

Mathematica [A] time = 0.0011442, size = 14, normalized size = 1.

$$\frac{(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^2,x]
```

```
[Out] (c + d*x)^3/(3*d)
```

Maple [A] time = 0.038, size = 13, normalized size = 0.9

$$\frac{(dx+c)^3}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^2,x)
```

```
[Out] 1/3*(d*x+c)^3/d
```

Maxima [A] time = 1.04203, size = 27, normalized size = 1.93

$$\frac{1}{3}d^2x^3 + cdx^2 + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*d^2*x^3 + c*d*x^2 + c^2*x
```

Fricas [A] time = 1.66728, size = 42, normalized size = 3.

$$\frac{1}{3}d^2x^3 + cdx^2 + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*d^2*x^3 + c*d*x^2 + c^2*x
```

Sympy [B] time = 0.189729, size = 19, normalized size = 1.36

$$c^2x + cdx^2 + \frac{d^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x**2)**2/(b*x+a)**2,x)
```

```
[Out] c**2*x + c*d*x**2 + d**2*x**3/3
```

Giac [B] time = 1.22177, size = 113, normalized size = 8.07

$$\frac{\left(\frac{3b^2c^2}{(bx+a)^2} + \frac{3bcd}{bx+a} - \frac{6abcd}{(bx+a)^2} - \frac{3ad^2}{bx+a} + \frac{3a^2d^2}{(bx+a)^2} + d^2\right)(bx+a)^3}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*b^2*c^2/(b*x + a)^2 + 3*b*c*d/(b*x + a) - 6*a*b*c*d/(b*x + a)^2 - 3*
a*d^2/(b*x + a) + 3*a^2*d^2/(b*x + a)^2 + d^2)*(b*x + a)^3/b^3
```

3.1775 $\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^3} dx$

Optimal. Leaf size=49

$$\frac{dx(bc-ad)}{b^2}+\frac{(bc-ad)^2\log(a+bx)}{b^3}+\frac{(c+dx)^2}{2b}$$

```
[Out] (d*(b*c - a*d)*x)/b^2 + (c + d*x)^2/(2*b) + ((b*c - a*d)^2*Log[a + b*x])/b^
3
```

Rubi [A] time = 0.0281078, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$\frac{dx(bc-ad)}{b^2}+\frac{(bc-ad)^2\log(a+bx)}{b^3}+\frac{(c+dx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^3,x]
```

```
[Out] (d*(b*c - a*d)*x)/b^2 + (c + d*x)^2/(2*b) + ((b*c - a*d)^2*Log[a + b*x])/b^
3
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^3}\,dx &= \int \frac{(c+dx)^2}{a+bx}\,dx\\
&= \int \left(\frac{d(bc-ad)}{b^2}+\frac{(bc-ad)^2}{b^2(a+bx)}+\frac{d(c+dx)}{b}\right)dx\\
&= \frac{d(bc-ad)x}{b^2}+\frac{(c+dx)^2}{2b}+\frac{(bc-ad)^2\log(a+bx)}{b^3}
\end{aligned}$$

Mathematica [A] time = 0.0153433, size = 43, normalized size = 0.88

$$\frac{bdx(-2ad+4bc+bdx)+2(bc-ad)^2\log(a+bx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^3,x]
```

```
[Out] (b*d*x*(4*b*c - 2*a*d + b*d*x) + 2*(b*c - a*d)^2*Log[a + b*x])/(2*b^3)
```

Maple [A] time = 0.04, size = 74, normalized size = 1.5

$$\frac{d^2x^2}{2\,b}-\frac{ad^2x}{b^2}+2\,\frac{cdx}{b}+\frac{\ln\left(bx+a\right)a^2d^2}{b^3}-2\,\frac{\ln\left(bx+a\right)cad}{b^2}+\frac{\ln\left(bx+a\right)c^2}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^3,x)
```

```
[Out] 1/2*d^2/b*x^2-d^2/b^2*a*x+2*d/b*x*c+1/b^3*ln(b*x+a)*a^2*d^2-2/b^2*ln(b*x+a)
*c*a*d+1/b*ln(b*x+a)*c^2
```

Maxima [A] time = 1.09436, size = 82, normalized size = 1.67

$$\frac{bd^2x^2+2\left(2\,bcd-ad^2\right)x}{2\,b^2}+\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(bx+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b*d^2*x^2 + 2*(2*b*c*d - a*d^2)*x)/b^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*log(b*x + a)/b^3
```

Fricas [A] time = 1.4725, size = 135, normalized size = 2.76

$$\frac{b^2d^2x^2+2\left(2\,b^2cd-abd^2\right)x+2\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(bx+a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^2 + 2*(2*b^2*c*d - a*b*d^2)*x + 2*(b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*log(b*x + a))/b^3
```

Sympy [A] time = 0.832072, size = 44, normalized size = 0.9

$$\frac{d^2x^2}{2b}-\frac{x\left(ad^2-2bcd\right)}{b^2}+\frac{\left(ad-bc\right)^2\log\left(a+bx\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x**2)**2/(b*x+a)**3,x)
```

```
[Out] d**2*x**2/(2*b) - x*(a*d**2 - 2*b*c*d)/b**2 + (a*d - b*c)**2*log(a + b*x)/b
**3
```

Giac [A] time = 1.20167, size = 81, normalized size = 1.65

$$\frac{bd^2x^2 + 4\,bcdx - 2\,ad^2x}{2\,b^2} + \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(|bx + a|\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*(b*d^2*x^2 + 4*b*c*d*x - 2*a*d^2*x)/b^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*log(abs(b*x + a))/b^3
```

3.1776 $\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^4} dx$

Optimal. Leaf size=51

$$-\frac{(bc-ad)^2}{b^3(a+bx)}+\frac{2d(bc-ad)\log(a+bx)}{b^3}+\frac{d^2x}{b^2}$$

```
[Out] (d^2*x)/b^2 - (b*c - a*d)^2/(b^3*(a + b*x)) + (2*d*(b*c - a*d)*Log[a + b*x]
)/b^3
```

Rubi [A] time = 0.0442666, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$-\frac{(bc-ad)^2}{b^3(a+bx)}+\frac{2d(bc-ad)\log(a+bx)}{b^3}+\frac{d^2x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^4,x]
```

```
[Out] (d^2*x)/b^2 - (b*c - a*d)^2/(b^3*(a + b*x)) + (2*d*(b*c - a*d)*Log[a + b*x]
)/b^3
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^4} dx &= \int \frac{(c+dx)^2}{(a+bx)^2} dx \\
&= \int \left(\frac{d^2}{b^2}+\frac{(bc-ad)^2}{b^2(a+bx)^2}+\frac{2d(bc-ad)}{b^2(a+bx)}\right) dx \\
&= \frac{d^2x}{b^2}-\frac{(bc-ad)^2}{b^3(a+bx)}+\frac{2d(bc-ad)\log(a+bx)}{b^3}
\end{aligned}$$

Mathematica [A] time = 0.0364032, size = 47, normalized size = 0.92

$$\frac{-\frac{(bc-ad)^2}{a+bx}+2d(bc-ad)\log(a+bx)+bd^2x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^4,x]
```

```
[Out] (b*d^2*x - (b*c - a*d)^2/(a + b*x) + 2*d*(b*c - a*d)*Log[a + b*x])/b^3
```

Maple [A] time = 0.045, size = 86, normalized size = 1.7

$$\frac{d^2x}{b^2} - 2\,\frac{d^2\ln(bx+a)\,a}{b^3} + 2\,\frac{d\ln(bx+a)\,c}{b^2} - \frac{a^2d^2}{b^3(bx+a)} + 2\,\frac{acd}{b^2(bx+a)} - \frac{c^2}{b(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^4,x)
```

```
[Out] d^2*x/b^2-2*d^2/b^3*ln(b*x+a)*a+2*d/b^2*ln(b*x+a)*c-1/b^3/(b*x+a)*a^2*d^2+2
/b^2/(b*x+a)*c*a*d-1/b/(b*x+a)*c^2
```

Maxima [A] time = 1.0366, size = 90, normalized size = 1.76

$$\frac{d^2x}{b^2} - \frac{b^2c^2 - 2\,abcd + a^2d^2}{b^4x + ab^3} + \frac{2\left(bcd - ad^2\right)\log(bx+a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] d^2*x/b^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(b^4*x + a*b^3) + 2*(b*c*d - a*
d^2)*log(b*x + a)/b^3
```

Fricas [A] time = 1.52461, size = 184, normalized size = 3.61

$$\frac{b^2d^2x^2 + abd^2x - b^2c^2 + 2\,abcd - a^2d^2 + 2\left(abcd - a^2d^2 + \left(b^2cd - abd^2\right)x\right)\log(bx+a)}{b^4x + ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^2 + a*b*d^2*x - b^2*c^2 + 2*a*b*c*d - a^2*d^2 + 2*(a*b*c*d - a^2
*d^2 + (b^2*c*d - a*b*d^2)*x)*log(b*x + a))/(b^4*x + a*b^3)
```

Sympy [A] time = 0.765751, size = 60, normalized size = 1.18

$$-\frac{a^2d^2 - 2abcd + b^2c^2}{ab^3 + b^4x} + \frac{d^2x}{b^2} - \frac{2d(ad - bc)\log(a + bx)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x**2)**2/(b*x+a)**4,x)
```

```
[Out] -(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(a*b**3 + b**4*x) + d**2*x/b**2 - 2*d*
(a*d - b*c)*log(a + b*x)/b**3
```

Giac [A] time = 1.22063, size = 88, normalized size = 1.73

$$\frac{d^2x}{b^2} + \frac{2\left(bcd - ad^2\right)\log\left(|bx + a|\right)}{b^3} - \frac{b^2c^2 - 2\,abcd + a^2d^2}{(bx + a)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] d^2*x/b^2 + 2*(b*c*d - a*d^2)*log(abs(b*x + a))/b^3 - (b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)/((b*x + a)*b^3)
```

3.1777 $\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^2}{(a+bx)^5}\,dx$

Optimal. Leaf size=59

$$-\frac{2d(bc-ad)}{b^3(a+bx)}-\frac{(bc-ad)^2}{2b^3(a+bx)^2}+\frac{d^2\log(a+bx)}{b^3}$$

```
[Out] -(b*c - a*d)^2/(2*b^3*(a + b*x)^2) - (2*d*(b*c - a*d))/(b^3*(a + b*x)) + (d
^2*Log[a + b*x])/b^3
```

Rubi [A] time = 0.0431223, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$-\frac{2d(bc-ad)}{b^3(a+bx)}-\frac{(bc-ad)^2}{2b^3(a+bx)^2}+\frac{d^2\log(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^5,x]
```

```
[Out] -(b*c - a*d)^2/(2*b^3*(a + b*x)^2) - (2*d*(b*c - a*d))/(b^3*(a + b*x)) + (d
^2*Log[a + b*x])/b^3
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^2}{(a+bx)^5}\,dx &= \int \frac{(c+dx)^2}{(a+bx)^3}\,dx \\
&= \int \left(\frac{(bc-ad)^2}{b^2(a+bx)^3}+\frac{2d(bc-ad)}{b^2(a+bx)^2}+\frac{d^2}{b^2(a+bx)}\right)dx \\
&= -\frac{(bc-ad)^2}{2b^3(a+bx)^2}-\frac{2d(bc-ad)}{b^3(a+bx)}+\frac{d^2\log(a+bx)}{b^3}
\end{aligned}$$

Mathematica [A] time = 0.0241648, size = 49, normalized size = 0.83

$$\frac{2d^2\log(a+bx)-\frac{(bc-ad)(3ad+b(c+4dx))}{(a+bx)^2}}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^5,x]
```

```
[Out] (-(((b*c - a*d)*(3*a*d + b*(c + 4*d*x)))/(a + b*x)^2) + 2*d^2*Log[a + b*x])
/(2*b^3)
```

Maple [A] time = 0.045, size = 92, normalized size = 1.6

$$-\frac{a^2d^2}{2\,b^3\,(bx+a)^2}+\frac{acd}{b^2\,(bx+a)^2}-\frac{c^2}{2\,b\,(bx+a)^2}+\frac{d^2\ln(bx+a)}{b^3}+2\,\frac{ad^2}{b^3\,(bx+a)}-2\,\frac{cd}{b^2\,(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^5,x)
```

```
[Out] -1/2/b^3/(b*x+a)^2*a^2*d^2+1/b^2/(b*x+a)^2*c*a*d-1/2/b/(b*x+a)^2*c^2+d^2*ln
(b*x+a)/b^3+2*d^2/b^3/(b*x+a)*a-2*d/b^2/(b*x+a)*c
```

Maxima [A] time = 0.994966, size = 107, normalized size = 1.81

$$-\frac{b^2c^2+2\,abcd-3\,a^2d^2+4\left(b^2cd-abd^2\right)x}{2\left(b^5x^2+2\,ab^4x+a^2b^3\right)}+\frac{d^2\log(bx+a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x)/(b^5*x^2 +
 2*a*b^4*x + a^2*b^3) + d^2*log(b*x + a)/b^3
```

Fricas [A] time = 1.60843, size = 207, normalized size = 3.51

$$-\frac{b^2c^2+2\,abcd-3\,a^2d^2+4\left(b^2cd-abd^2\right)x-2\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)\log(bx+a)}{2\left(b^5x^2+2\,ab^4x+a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x - 2*(b^2*d^
2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3
)
```

Sympy [A] time = 0.853165, size = 80, normalized size = 1.36

$$\frac{3a^2d^2-2abcd-b^2c^2+x\left(4abd^2-4b^2cd\right)}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{d^2\log(a+bx)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x**2)**2/(b*x+a)**5,x)
```

```
[Out] (3*a**2*d**2 - 2*a*b*c*d - b**2*c**2 + x*(4*a*b*d**2 - 4*b**2*c*d))/(2*a**2
*b**3 + 4*a*b**4*x + 2*b**5*x**2) + d**2*log(a + b*x)/b**3
```

Giac [A] time = 1.19446, size = 149, normalized size = 2.53

$$-\frac{d^2\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^3}-\frac{\frac{b^5c^2}{(bx+a)^2}+\frac{4\,b^4cd}{bx+a}-\frac{2\,ab^4cd}{(bx+a)^2}-\frac{4\,ab^3d^2}{bx+a}+\frac{a^2b^3d^2}{(bx+a)^2}}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -d^2*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^3 - 1/2*(b^5*c^2/(b*x + a)^2
+ 4*b^4*c*d/(b*x + a) - 2*a*b^4*c*d/(b*x + a)^2 - 4*a*b^3*d^2/(b*x + a) + a
^2*b^3*d^2/(b*x + a)^2)/b^6
```

3.1778 $\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^2}{(a+bx)^6} dx$

Optimal. Leaf size=28

$$-\frac{(c+dx)^3}{3(a+bx)^3(bc-ad)}$$

```
[Out] -(c + d*x)^3/(3*(b*c - a*d)*(a + b*x)^3)
```

Rubi [A] time = 0.0112407, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 37}

$$-\frac{(c+dx)^3}{3(a+bx)^3(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^6,x]
```

```
[Out] -(c + d*x)^3/(3*(b*c - a*d)*(a + b*x)^3)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^2}{(a+bx)^6} dx &= \int \frac{(c+dx)^2}{(a+bx)^4} dx\\ &= -\frac{(c+dx)^3}{3(bc-ad)(a+bx)^3}\end{aligned}$$

Mathematica [A] time = 0.022367, size = 53, normalized size = 1.89

$$-\frac{a^2d^2+abd(c+3dx)+b^2\left(c^2+3cdx+3d^2x^2\right)}{3b^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^6,x]
```

```
[Out] -(a^2*d^2 + a*b*d*(c + 3*d*x) + b^2*(c^2 + 3*c*d*x + 3*d^2*x^2))/(3*b^3*(a
+ b*x)^3)
```

Maple [B] time = 0.043, size = 70, normalized size = 2.5

$$\frac{(ad-bc)\,d}{b^3\,(bx+a)^2}-\frac{a^2d^2-2\,cabd+b^2c^2}{3\,b^3\,(bx+a)^3}-\frac{d^2}{b^3\,(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^6,x)
```

```
[Out] (a*d-b*c)*d/b^3/(b*x+a)^2-1/3*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/(b*x+a)^3-d^2
/b^3/(b*x+a)
```

Maxima [B] time = 1.18399, size = 113, normalized size = 4.04

$$-\frac{3\,b^2d^2x^2+b^2c^2+abcd+a^2d^2+3\,\left(b^2cd+abd^2\right)x}{3\,\left(b^6x^3+3\,ab^5x^2+3\,a^2b^4x+a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^2*d^2*x^2 + b^2*c^2 + a*b*c*d + a^2*d^2 + 3*(b^2*c*d + a*b*d^2)*x
)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3*b^3)
```

Fricas [B] time = 1.58046, size = 170, normalized size = 6.07

$$-\frac{3\,b^2d^2x^2+b^2c^2+abcd+a^2d^2+3\,\left(b^2cd+abd^2\right)x}{3\,\left(b^6x^3+3\,ab^5x^2+3\,a^2b^4x+a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*d^2*x^2 + b^2*c^2 + a*b*c*d + a^2*d^2 + 3*(b^2*c*d + a*b*d^2)*x
)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3*b^3)
```

Sympy [B] time = 1.26452, size = 88, normalized size = 3.14

$$-\frac{a^2d^2+abcd+b^2c^2+3b^2d^2x^2+x\left(3abd^2+3b^2cd\right)}{3a^3b^3+9a^2b^4x+9ab^5x^2+3b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x**2)**2/(b*x+a)**6,x)
```

```
[Out] -(a**2*d**2 + a*b*c*d + b**2*c**2 + 3*b**2*d**2*x**2 + x*(3*a*b*d**2 + 3*b*
*2*c*d))/(3*a**3*b**3 + 9*a**2*b**4*x + 9*a*b**5*x**2 + 3*b**6*x**3)
```

Giac [B] time = 1.18049, size = 80, normalized size = 2.86

$$-\frac{3\,b^2d^2x^2 + 3\,b^2cdx + 3\,abd^2x + b^2c^2 + abcd + a^2d^2}{3\,(bx+a)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*d^2*x^2 + 3*b^2*c*d*x + 3*a*b*d^2*x + b^2*c^2 + a*b*c*d + a^2*d
^2)/((b*x + a)^3*b^3)
```

3.1779 $\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^2}{(a+bx)^7}\,dx$

Optimal. Leaf size=65

$$-\frac{2d(bc-ad)}{3b^3(a+bx)^3}-\frac{(bc-ad)^2}{4b^3(a+bx)^4}-\frac{d^2}{2b^3(a+bx)^2}$$

```
[Out] -(b*c - a*d)^2/(4*b^3*(a + b*x)^4) - (2*d*(b*c - a*d))/(3*b^3*(a + b*x)^3)
- d^2/(2*b^3*(a + b*x)^2)
```

Rubi [A] time = 0.0435792, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$-\frac{2d(bc-ad)}{3b^3(a+bx)^3}-\frac{(bc-ad)^2}{4b^3(a+bx)^4}-\frac{d^2}{2b^3(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^7,x]
```

```
[Out] -(b*c - a*d)^2/(4*b^3*(a + b*x)^4) - (2*d*(b*c - a*d))/(3*b^3*(a + b*x)^3)
- d^2/(2*b^3*(a + b*x)^2)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^2}{(a+bx)^7}\,dx &= \int \frac{(c+dx)^2}{(a+bx)^5}\,dx \\
&= \int \left(\frac{(bc-ad)^2}{b^2(a+bx)^5}+\frac{2d(bc-ad)}{b^2(a+bx)^4}+\frac{d^2}{b^2(a+bx)^3}\right)dx \\
&= -\frac{(bc-ad)^2}{4b^3(a+bx)^4}-\frac{2d(bc-ad)}{3b^3(a+bx)^3}-\frac{d^2}{2b^3(a+bx)^2}
\end{aligned}$$

Mathematica [A] time = 0.0189296, size = 56, normalized size = 0.86

$$-\frac{a^2d^2+2abd(c+2dx)+b^2\left(3c^2+8cdx+6d^2x^2\right)}{12b^3(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^7,x]
```

```
[Out] -(a^2*d^2 + 2*a*b*d*(c + 2*d*x) + b^2*(3*c^2 + 8*c*d*x + 6*d^2*x^2))/(12*b^
3*(a + b*x)^4)
```

Maple [A] time = 0.044, size = 71, normalized size = 1.1

$$-\frac{d^2}{2\,b^3\,(bx+a)^2}-\frac{a^2d^2-2\,cabd+b^2c^2}{4\,b^3\,(bx+a)^4}+\frac{(2\,ad-2\,bc)\,d}{3\,b^3\,(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^7,x)
```

```
[Out] -1/2*d^2/b^3/(b*x+a)^2-1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/(b*x+a)^4+2/3*(a
*d-b*c)*d/b^3/(b*x+a)^3
```

Maxima [A] time = 1.17405, size = 132, normalized size = 2.03

$$-\frac{6\,b^2d^2x^2+3\,b^2c^2+2\,abcd+a^2d^2+4\,\left(2\,b^2cd+abd^2\right)x}{12\,\left(b^7x^4+4\,ab^6x^3+6\,a^2b^5x^2+4\,a^3b^4x+a^4b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/12*(6*b^2*d^2*x^2 + 3*b^2*c^2 + 2*a*b*c*d + a^2*d^2 + 4*(2*b^2*c*d + a*b
*d^2)*x)/(b^7*x^4 + 4*a*b^6*x^3 + 6*a^2*b^5*x^2 + 4*a^3*b^4*x + a^4*b^3)
```

Fricas [A] time = 1.62276, size = 201, normalized size = 3.09

$$-\frac{6\,b^2d^2x^2+3\,b^2c^2+2\,abcd+a^2d^2+4\,\left(2\,b^2cd+abd^2\right)x}{12\,\left(b^7x^4+4\,ab^6x^3+6\,a^2b^5x^2+4\,a^3b^4x+a^4b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^2*d^2*x^2 + 3*b^2*c^2 + 2*a*b*c*d + a^2*d^2 + 4*(2*b^2*c*d + a*b
*d^2)*x)/(b^7*x^4 + 4*a*b^6*x^3 + 6*a^2*b^5*x^2 + 4*a^3*b^4*x + a^4*b^3)
```

Sympy [A] time = 1.37642, size = 104, normalized size = 1.6

$$-\frac{a^2d^2+2abcd+3b^2c^2+6b^2d^2x^2+x\left(4abd^2+8b^2cd\right)}{12a^4b^3+48a^3b^4x+72a^2b^5x^2+48ab^6x^3+12b^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x**2)**2/(b*x+a)**7,x)
```

```
[Out] -(a**2*d**2 + 2*a*b*c*d + 3*b**2*c**2 + 6*b**2*d**2*x**2 + x*(4*a*b*d**2 +
8*b**2*c*d))/(12*a**4*b**3 + 48*a**3*b**4*x + 72*a**2*b**5*x**2 + 48*a*b**6
*x**3 + 12*b**7*x**4)
```

Giac [A] time = 1.24807, size = 82, normalized size = 1.26

$$-\frac{6\,b^{2}d^{2}x^{2}+8\,b^{2}cdx+4\,abd^{2}x+3\,b^{2}c^{2}+2\,abcd+a^{2}d^{2}}{12\,(bx+a)^{4}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/12*(6*b^2*d^2*x^2 + 8*b^2*c*d*x + 4*a*b*d^2*x + 3*b^2*c^2 + 2*a*b*c*d +
a^2*d^2)/((b*x + a)^4*b^3)
```

3.1780 $\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^2}{(a+bx)^8}\,dx$

Optimal. Leaf size=65

$$-\frac{d(bc-ad)}{2b^3(a+bx)^4}-\frac{(bc-ad)^2}{5b^3(a+bx)^5}-\frac{d^2}{3b^3(a+bx)^3}$$

```
[Out] -(b*c - a*d)^2/(5*b^3*(a + b*x)^5) - (d*(b*c - a*d))/(2*b^3*(a + b*x)^4) -
d^2/(3*b^3*(a + b*x)^3)
```

Rubi [A] time = 0.0435076, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$-\frac{d(bc-ad)}{2b^3(a+bx)^4}-\frac{(bc-ad)^2}{5b^3(a+bx)^5}-\frac{d^2}{3b^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^8,x]
```

```
[Out] -(b*c - a*d)^2/(5*b^3*(a + b*x)^5) - (d*(b*c - a*d))/(2*b^3*(a + b*x)^4) -
d^2/(3*b^3*(a + b*x)^3)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^2}{(a+bx)^8}\,dx &= \int \frac{(c+dx)^2}{(a+bx)^6}\,dx\\
&= \int \left(\frac{(bc-ad)^2}{b^2(a+bx)^6}+\frac{2d(bc-ad)}{b^2(a+bx)^5}+\frac{d^2}{b^2(a+bx)^4}\right)dx\\
&= -\frac{(bc-ad)^2}{5b^3(a+bx)^5}-\frac{d(bc-ad)}{2b^3(a+bx)^4}-\frac{d^2}{3b^3(a+bx)^3}
\end{aligned}$$

Mathematica [A] time = 0.0241188, size = 57, normalized size = 0.88

$$-\frac{a^2d^2+abd(3c+5dx)+b^2\left(6c^2+15cdx+10d^2x^2\right)}{30b^3(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^8,x]
```

```
[Out] -(a^2*d^2 + a*b*d*(3*c + 5*d*x) + b^2*(6*c^2 + 15*c*d*x + 10*d^2*x^2))/(30*
b^3*(a + b*x)^5)
```

Maple [A] time = 0.044, size = 71, normalized size = 1.1

$$-\frac{a^2d^2-2\,cabd+b^2c^2}{5\,b^3\,(bx+a)^5}+\frac{(ad-bc)\,d}{2\,b^3\,(bx+a)^4}-\frac{d^2}{3\,b^3\,(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^8,x)
```

```
[Out] -1/5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/(b*x+a)^5+1/2*(a*d-b*c)*d/b^3/(b*x+a)^
4-1/3*d^2/b^3/(b*x+a)^3
```

Maxima [A] time = 1.0295, size = 147, normalized size = 2.26

$$-\frac{10\,b^2d^2x^2+6\,b^2c^2+3\,abcd+a^2d^2+5\left(3\,b^2cd+abd^2\right)x}{30\left(b^8x^5+5\,ab^7x^4+10\,a^2b^6x^3+10\,a^3b^5x^2+5\,a^4b^4x+a^5b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^8,x, algorithm="maxima")
```

```
[Out] -1/30*(10*b^2*d^2*x^2 + 6*b^2*c^2 + 3*a*b*c*d + a^2*d^2 + 5*(3*b^2*c*d + a*
b*d^2)*x)/(b^8*x^5 + 5*a*b^7*x^4 + 10*a^2*b^6*x^3 + 10*a^3*b^5*x^2 + 5*a^4*
b^4*x + a^5*b^3)
```

Fricas [A] time = 1.42936, size = 227, normalized size = 3.49

$$-\frac{10\,b^2d^2x^2+6\,b^2c^2+3\,abcd+a^2d^2+5\left(3\,b^2cd+abd^2\right)x}{30\left(b^8x^5+5\,ab^7x^4+10\,a^2b^6x^3+10\,a^3b^5x^2+5\,a^4b^4x+a^5b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^8,x, algorithm="fricas")
```

```
[Out] -1/30*(10*b^2*d^2*x^2 + 6*b^2*c^2 + 3*a*b*c*d + a^2*d^2 + 5*(3*b^2*c*d + a*
b*d^2)*x)/(b^8*x^5 + 5*a*b^7*x^4 + 10*a^2*b^6*x^3 + 10*a^3*b^5*x^2 + 5*a^4*
b^4*x + a^5*b^3)
```

Sympy [B] time = 1.6324, size = 116, normalized size = 1.78

$$-\frac{a^2d^2+3abcd+6b^2c^2+10b^2d^2x^2+x\left(5abd^2+15b^2cd\right)}{30a^5b^3+150a^4b^4x+300a^3b^5x^2+300a^2b^6x^3+150ab^7x^4+30b^8x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x**2)**2/(b*x+a)**8,x)
```

```
[Out] -(a**2*d**2 + 3*a*b*c*d + 6*b**2*c**2 + 10*b**2*d**2*x**2 + x*(5*a*b*d**2 +
 15*b**2*c*d))/(30*a**5*b**3 + 150*a**4*b**4*x + 300*a**3*b**5*x**2 + 300*a
**2*b**6*x**3 + 150*a*b**7*x**4 + 30*b**8*x**5)
```

Giac [A] time = 1.20867, size = 82, normalized size = 1.26

$$-\frac{10\,b^2d^2x^2+15\,b^2cdx+5\,abd^2x+6\,b^2c^2+3\,abcd+a^2d^2}{30\,(bx+a)^5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^8,x, algorithm="giac")
```

```
[Out] -1/30*(10*b^2*d^2*x^2 + 15*b^2*c*d*x + 5*a*b*d^2*x + 6*b^2*c^2 + 3*a*b*c*d
+ a^2*d^2)/((b*x + a)^5*b^3)
```

3.1781 $\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^2}{(a+bx)^9}\,dx$

Optimal. Leaf size=65

$$-\frac{2d(bc-ad)}{5b^3(a+bx)^5}-\frac{(bc-ad)^2}{6b^3(a+bx)^6}-\frac{d^2}{4b^3(a+bx)^4}$$

```
[Out] -(b*c - a*d)^2/(6*b^3*(a + b*x)^6) - (2*d*(b*c - a*d))/(5*b^3*(a + b*x)^5)
- d^2/(4*b^3*(a + b*x)^4)
```

Rubi [A] time = 0.0411441, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$-\frac{2d(bc-ad)}{5b^3(a+bx)^5}-\frac{(bc-ad)^2}{6b^3(a+bx)^6}-\frac{d^2}{4b^3(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^9,x]
```

```
[Out] -(b*c - a*d)^2/(6*b^3*(a + b*x)^6) - (2*d*(b*c - a*d))/(5*b^3*(a + b*x)^5)
- d^2/(4*b^3*(a + b*x)^4)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^2}{(a+bx)^9}\,dx &= \int \frac{(c+dx)^2}{(a+bx)^7}\,dx\\ &= \int \left(\frac{(bc-ad)^2}{b^2(a+bx)^7}+\frac{2d(bc-ad)}{b^2(a+bx)^6}+\frac{d^2}{b^2(a+bx)^5}\right)dx\\ &= -\frac{(bc-ad)^2}{6b^3(a+bx)^6}-\frac{2d(bc-ad)}{5b^3(a+bx)^5}-\frac{d^2}{4b^3(a+bx)^4}\end{aligned}$$

Mathematica [A] time = 0.0201183, size = 58, normalized size = 0.89

$$-\frac{a^2d^2+2abd(2c+3dx)+b^2\left(10c^2+24cdx+15d^2x^2\right)}{60b^3(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^9,x]
```

```
[Out] -(a^2*d^2 + 2*a*b*d*(2*c + 3*d*x) + b^2*(10*c^2 + 24*c*d*x + 15*d^2*x^2))/(
60*b^3*(a + b*x)^6)
```

Maple [A] time = 0.044, size = 71, normalized size = 1.1

$$\frac{(2\,ad-2\,bc)\,d}{5\,b^3\,(bx+a)^5}-\frac{d^2}{4\,b^3\,(bx+a)^4}-\frac{a^2d^2-2\,cabd+b^2c^2}{6\,b^3\,(bx+a)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^9,x)
```

```
[Out] 2/5*(a*d-b*c)*d/b^3/(b*x+a)^5-1/4*d^2/b^3/(b*x+a)^4-1/6*(a^2*d^2-2*a*b*c*d+
b^2*c^2)/b^3/(b*x+a)^6
```

Maxima [B] time = 1.03486, size = 162, normalized size = 2.49

$$-\frac{15\,b^2d^2x^2+10\,b^2c^2+4\,abcd+a^2d^2+6\left(4\,b^2cd+abd^2\right)x}{60\left(b^9x^6+6\,ab^8x^5+15\,a^2b^7x^4+20\,a^3b^6x^3+15\,a^4b^5x^2+6\,a^5b^4x+a^6b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^9,x, algorithm="maxima")
```

```
[Out] -1/60*(15*b^2*d^2*x^2 + 10*b^2*c^2 + 4*a*b*c*d + a^2*d^2 + 6*(4*b^2*c*d + a
*b*d^2)*x)/(b^9*x^6 + 6*a*b^8*x^5 + 15*a^2*b^7*x^4 + 20*a^3*b^6*x^3 + 15*a^
4*b^5*x^2 + 6*a^5*b^4*x + a^6*b^3)
```

Fricas [B] time = 1.53198, size = 251, normalized size = 3.86

$$-\frac{15\,b^2d^2x^2+10\,b^2c^2+4\,abcd+a^2d^2+6\left(4\,b^2cd+abd^2\right)x}{60\left(b^9x^6+6\,ab^8x^5+15\,a^2b^7x^4+20\,a^3b^6x^3+15\,a^4b^5x^2+6\,a^5b^4x+a^6b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^9,x, algorithm="fricas")
```

```
[Out] -1/60*(15*b^2*d^2*x^2 + 10*b^2*c^2 + 4*a*b*c*d + a^2*d^2 + 6*(4*b^2*c*d + a
*b*d^2)*x)/(b^9*x^6 + 6*a*b^8*x^5 + 15*a^2*b^7*x^4 + 20*a^3*b^6*x^3 + 15*a^
4*b^5*x^2 + 6*a^5*b^4*x + a^6*b^3)
```

Sympy [B] time = 1.9433, size = 128, normalized size = 1.97

$$-\frac{a^2d^2+4abcd+10b^2c^2+15b^2d^2x^2+x\left(6abd^2+24b^2cd\right)}{60a^6b^3+360a^5b^4x+900a^4b^5x^2+1200a^3b^6x^3+900a^2b^7x^4+360ab^8x^5+60b^9x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x**2)**2/(b*x+a)**9,x)
```

```
[Out] -(a**2*d**2 + 4*a*b*c*d + 10*b**2*c**2 + 15*b**2*d**2*x**2 + x*(6*a*b*d**2
+ 24*b**2*c*d))/(60*a**6*b**3 + 360*a**5*b**4*x + 900*a**4*b**5*x**2 + 1200
*a**3*b**6*x**3 + 900*a**2*b**7*x**4 + 360*a*b**8*x**5 + 60*b**9*x**6)
```

Giac [A] time = 1.23679, size = 82, normalized size = 1.26

$$-\frac{15\,b^2d^2x^2+24\,b^2cdx+6\,abd^2x+10\,b^2c^2+4\,abcd+a^2d^2}{60\,(bx+a)^6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^9,x, algorithm="giac")
```

```
[Out] -1/60*(15*b^2*d^2*x^2 + 24*b^2*c*d*x + 6*a*b*d^2*x + 10*b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)/((b*x + a)^6*b^3)
```

3.1782 $\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^2}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=65

$$-\frac{d(bc-ad)}{3b^3(a+bx)^6}-\frac{(bc-ad)^2}{7b^3(a+bx)^7}-\frac{d^2}{5b^3(a+bx)^5}$$

```
[Out] -(b*c - a*d)^2/(7*b^3*(a + b*x)^7) - (d*(b*c - a*d))/(3*b^3*(a + b*x)^6) -
d^2/(5*b^3*(a + b*x)^5)
```

Rubi [A] time = 0.0413255, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$-\frac{d(bc-ad)}{3b^3(a+bx)^6}-\frac{(bc-ad)^2}{7b^3(a+bx)^7}-\frac{d^2}{5b^3(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^10,x]
```

```
[Out] -(b*c - a*d)^2/(7*b^3*(a + b*x)^7) - (d*(b*c - a*d))/(3*b^3*(a + b*x)^6) -
d^2/(5*b^3*(a + b*x)^5)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^2}{(a+bx)^{10}}\,dx &= \int \frac{(c+dx)^2}{(a+bx)^8}\,dx\\ &= \int \left(\frac{(bc-ad)^2}{b^2(a+bx)^8}+\frac{2d(bc-ad)}{b^2(a+bx)^7}+\frac{d^2}{b^2(a+bx)^6}\right)dx\\ &= -\frac{(bc-ad)^2}{7b^3(a+bx)^7}-\frac{d(bc-ad)}{3b^3(a+bx)^6}-\frac{d^2}{5b^3(a+bx)^5}\end{aligned}$$

Mathematica [A] time = 0.0234481, size = 57, normalized size = 0.88

$$-\frac{a^2d^2+abd(5c+7dx)+b^2\left(15c^2+35cdx+21d^2x^2\right)}{105b^3(a+bx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^10,x]
```

```
[Out] -(a^2*d^2 + a*b*d*(5*c + 7*d*x) + b^2*(15*c^2 + 35*c*d*x + 21*d^2*x^2))/(10
5*b^3*(a + b*x)^7)
```

Maple [A] time = 0.049, size = 71, normalized size = 1.1

$$-\frac{d^2}{5\,b^3\,(bx+a)^5}-\frac{a^2d^2-2\,cabd+b^2c^2}{7\,b^3\,(bx+a)^7}+\frac{(ad-bc)\,d}{3\,b^3\,(bx+a)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^10,x)
```

```
[Out] -1/5*d^2/b^3/(b*x+a)^5-1/7*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/(b*x+a)^7+1/3*(a
*d-b*c)*d/b^3/(b*x+a)^6
```

Maxima [B] time = 1.05412, size = 177, normalized size = 2.72

$$-\frac{21\,b^2d^2x^2+15\,b^2c^2+5\,abcd+a^2d^2+7\left(5\,b^2cd+abd^2\right)x}{105\left(b^{10}x^7+7\,ab^9x^6+21\,a^2b^8x^5+35\,a^3b^7x^4+35\,a^4b^6x^3+21\,a^5b^5x^2+7\,a^6b^4x+a^7b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/105*(21*b^2*d^2*x^2 + 15*b^2*c^2 + 5*a*b*c*d + a^2*d^2 + 7*(5*b^2*c*d +
a*b*d^2)*x)/(b^10*x^7 + 7*a*b^9*x^6 + 21*a^2*b^8*x^5 + 35*a^3*b^7*x^4 + 35*
a^4*b^6*x^3 + 21*a^5*b^5*x^2 + 7*a^6*b^4*x + a^7*b^3)
```

Fricas [B] time = 1.49137, size = 277, normalized size = 4.26

$$-\frac{21\,b^2d^2x^2+15\,b^2c^2+5\,abcd+a^2d^2+7\left(5\,b^2cd+abd^2\right)x}{105\left(b^{10}x^7+7\,ab^9x^6+21\,a^2b^8x^5+35\,a^3b^7x^4+35\,a^4b^6x^3+21\,a^5b^5x^2+7\,a^6b^4x+a^7b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/105*(21*b^2*d^2*x^2 + 15*b^2*c^2 + 5*a*b*c*d + a^2*d^2 + 7*(5*b^2*c*d +
a*b*d^2)*x)/(b^10*x^7 + 7*a*b^9*x^6 + 21*a^2*b^8*x^5 + 35*a^3*b^7*x^4 + 35*
a^4*b^6*x^3 + 21*a^5*b^5*x^2 + 7*a^6*b^4*x + a^7*b^3)
```

Sympy [B] time = 2.81169, size = 139, normalized size = 2.14

$$-\frac{a^2d^2+5abcd+15b^2c^2+21b^2d^2x^2+x\left(7abd^2+35b^2cd\right)}{105a^7b^3+735a^6b^4x+2205a^5b^5x^2+3675a^4b^6x^3+3675a^3b^7x^4+2205a^2b^8x^5+735ab^9x^6+105b^{10}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x**2)**2/(b*x+a)**10,x)
```

```
[Out] -(a**2*d**2 + 5*a*b*c*d + 15*b**2*c**2 + 21*b**2*d**2*x**2 + x*(7*a*b*d**2
+ 35*b**2*c*d))/(105*a**7*b**3 + 735*a**6*b**4*x + 2205*a**5*b**5*x**2 + 36
75*a**4*b**6*x**3 + 3675*a**3*b**7*x**4 + 2205*a**2*b**8*x**5 + 735*a*b**9*
x**6 + 105*b**10*x**7)
```

Giac [A] time = 1.20101, size = 82, normalized size = 1.26

$$-\frac{21\,b^2d^2x^2+35\,b^2cdx+7\,abd^2x+15\,b^2c^2+5\,abcd+a^2d^2}{105\,(bx+a)^7b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/105*(21*b^2*d^2*x^2 + 35*b^2*c*d*x + 7*a*b*d^2*x + 15*b^2*c^2 + 5*a*b*c*
d + a^2*d^2)/((b*x + a)^7*b^3)
```

3.1783 $\int (a+bx)^3 \left(ac+(bc+ad)x+bdx^2\right)^3 \, dx$

Optimal. Leaf size=92

$$\frac{d^2(a+bx)^9(bc-ad)}{3b^4}+\frac{3d(a+bx)^8(bc-ad)^2}{8b^4}+\frac{(a+bx)^7(bc-ad)^3}{7b^4}+\frac{d^3(a+bx)^{10}}{10b^4}$$

```
[Out] ((b*c - a*d)^3*(a + b*x)^7)/(7*b^4) + (3*d*(b*c - a*d)^2*(a + b*x)^8)/(8*b^
4) + (d^2*(b*c - a*d)*(a + b*x)^9)/(3*b^4) + (d^3*(a + b*x)^10)/(10*b^4)
```

Rubi [A] time = 0.219694, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$\frac{d^2(a+bx)^9(bc-ad)}{3b^4}+\frac{3d(a+bx)^8(bc-ad)^2}{8b^4}+\frac{(a+bx)^7(bc-ad)^3}{7b^4}+\frac{d^3(a+bx)^{10}}{10b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

```
[Out] ((b*c - a*d)^3*(a + b*x)^7)/(7*b^4) + (3*d*(b*c - a*d)^2*(a + b*x)^8)/(8*b^
4) + (d^2*(b*c - a*d)*(a + b*x)^9)/(3*b^4) + (d^3*(a + b*x)^10)/(10*b^4)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3\left(ac+(bc+ad)x+bdx^2\right)^3\,dx&=\int (a+bx)^6(c+dx)^3\,dx\\&=\int\left(\frac{(bc-ad)^3(a+bx)^6}{b^3}+\frac{3d(bc-ad)^2(a+bx)^7}{b^3}+\frac{3d^2(bc-ad)(a+bx)^8}{b^3}\right.\\&=\frac{(bc-ad)^3(a+bx)^7}{7b^4}+\frac{3d(bc-ad)^2(a+bx)^8}{8b^4}+\frac{d^2(bc-ad)(a+bx)^9}{3b^4}+\frac{d^3(\end{aligned}$$

Mathematica [B] time = 0.0824648, size = 276, normalized size = 3.

$$\frac{1}{840}x\left(210a^4b^2x^2\left(45c^2dx+20c^3+36cd^2x^2+10d^3x^3\right)+120a^3b^3x^3\left(84c^2dx+35c^3+70cd^2x^2+20d^3x^3\right)+45a^2b^4x^4\right($$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

```
[Out] (x*(210*a^6*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 252*a^5*b*x*(10*c
^3 + 20*c^2*d*x + 15*c*d^2*x^2 + 4*d^3*x^3) + 210*a^4*b^2*x^2*(20*c^3 + 45*
c^2*d*x + 36*c*d^2*x^2 + 10*d^3*x^3) + 120*a^3*b^3*x^3*(35*c^3 + 84*c^2*d*x
 + 70*c*d^2*x^2 + 20*d^3*x^3) + 45*a^2*b^4*x^4*(56*c^3 + 140*c^2*d*x + 120*
c*d^2*x^2 + 35*d^3*x^3) + 10*a*b^5*x^5*(84*c^3 + 216*c^2*d*x + 189*c*d^2*x^
2 + 56*d^3*x^3) + b^6*x^6*(120*c^3 + 315*c^2*d*x + 280*c*d^2*x^2 + 84*d^3*x
^3)))/840
```

Maple [B] time = 0.04, size = 811, normalized size = 8.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(a*c+(a*d+b*c)*x+b*d*x^2)^3,x)
```

```
[Out] 1/10*b^6*d^3*x^10+1/9*(3*b^5*a*d^3+3*b^5*(a*d+b*c)*d^2)*x^9+1/8*(3*b^4*a^2*
d^3+9*b^4*a*(a*d+b*c)*d^2+b^3*(a*b^2*c*d^2+2*(a*d+b*c)^2*b*d+b*d*(2*c*a*b*d
+(a*d+b*c)^2)))*x^8+1/7*(a^3*b^3*d^3+9*b^3*a^2*(a*d+b*c)*d^2+3*b^2*a*(a*b^2
*c*d^2+2*(a*d+b*c)^2*b*d+b*d*(2*c*a*b*d+(a*d+b*c)^2))+b^3*(4*a*c*(a*d+b*c)*
b*d+(a*d+b*c)*(2*c*a*b*d+(a*d+b*c)^2)))*x^7+1/6*(3*a^3*(a*d+b*c)*b^2*d^2+3*
b*a^2*(a*b^2*c*d^2+2*(a*d+b*c)^2*b*d+b*d*(2*c*a*b*d+(a*d+b*c)^2))+3*b^2*a*(
4*a*c*(a*d+b*c)*b*d+(a*d+b*c)*(2*c*a*b*d+(a*d+b*c)^2))+b^3*(a*c*(2*c*a*b*d+
(a*d+b*c)^2)+2*(a*d+b*c)^2*a*c+b*d*a^2*c^2))*x^6+1/5*(a^3*(a*b^2*c*d^2+2*(a
*d+b*c)^2*b*d+b*d*(2*c*a*b*d+(a*d+b*c)^2))+3*b*a^2*(4*a*c*(a*d+b*c)*b*d+(a*
d+b*c)*(2*c*a*b*d+(a*d+b*c)^2))+3*b^2*a*(a*c*(2*c*a*b*d+(a*d+b*c)^2)+2*(a*d
+b*c)^2*a*c+b*d*a^2*c^2)+3*b^3*a^2*c^2*(a*d+b*c))*x^5+1/4*(a^3*(4*a*c*(a*d+
b*c)*b*d+(a*d+b*c)*(2*c*a*b*d+(a*d+b*c)^2))+3*b*a^2*(a*c*(2*c*a*b*d+(a*d+b*
c)^2)+2*(a*d+b*c)^2*a*c+b*d*a^2*c^2)+9*b^2*a^3*c^2*(a*d+b*c)+a^3*b^3*c^3)*x
^4+1/3*(a^3*(a*c*(2*c*a*b*d+(a*d+b*c)^2)+2*(a*d+b*c)^2*a*c+b*d*a^2*c^2)+9*b
*a^4*c^2*(a*d+b*c)+3*b^2*a^4*c^3)*x^3+1/2*(3*a^5*c^2*(a*d+b*c)+3*b*a^5*c^3)
*x^2+a^6*c^3*x
```

Maxima [B] time = 1.04754, size = 441, normalized size = 4.79

$$\frac{1}{10} b^6 d^3 x^{10} + a^6 c^3 x + \frac{1}{3} \left(b^6 c d^2 + 2\, a b^5 d^3\right) x^9 + \frac{3}{8} \left(b^6 c^2 d + 6\, a b^5 c d^2 + 5\, a^2 b^4 d^3\right) x^8 + \frac{1}{7} \left(b^6 c^3 + 18\, a b^5 c^2 d + 45\, a^2 b^4 c d^2 + 2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/10*b^6*d^3*x^10 + a^6*c^3*x + 1/3*(b^6*c*d^2 + 2*a*b^5*d^3)*x^9 + 3/8*(b^
6*c^2*d + 6*a*b^5*c*d^2 + 5*a^2*b^4*d^3)*x^8 + 1/7*(b^6*c^3 + 18*a*b^5*c^2*
d + 45*a^2*b^4*c*d^2 + 20*a^3*b^3*d^3)*x^7 + 1/2*(2*a*b^5*c^3 + 15*a^2*b^4*
c^2*d + 20*a^3*b^3*c*d^2 + 5*a^4*b^2*d^3)*x^6 + 3/5*(5*a^2*b^4*c^3 + 20*a^3
*b^3*c^2*d + 15*a^4*b^2*c*d^2 + 2*a^5*b*d^3)*x^5 + 1/4*(20*a^3*b^3*c^3 + 45
*a^4*b^2*c^2*d + 18*a^5*b*c*d^2 + a^6*d^3)*x^4 + (5*a^4*b^2*c^3 + 6*a^5*b*c
^2*d + a^6*c*d^2)*x^3 + 3/2*(2*a^5*b*c^3 + a^6*c^2*d)*x^2
```

Fricas [B] time = 1.28594, size = 776, normalized size = 8.43

$$\frac{1}{10}x^{10}d^3b^6 + \frac{1}{3}x^9d^2cb^6 + \frac{2}{3}x^9d^3b^5a + \frac{3}{8}x^8dc^2b^6 + \frac{9}{4}x^8d^2cb^5a + \frac{15}{8}x^8d^3b^4a^2 + \frac{1}{7}x^7c^3b^6 + \frac{18}{7}x^7dc^2b^5a + \frac{45}{7}x^7d^2cb^4a^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*d^3*b^6 + 1/3*x^9*d^2*c*b^6 + 2/3*x^9*d^3*b^5*a + 3/8*x^8*d*c^2*b
^6 + 9/4*x^8*d^2*c*b^5*a + 15/8*x^8*d^3*b^4*a^2 + 1/7*x^7*c^3*b^6 + 18/7*x^
7*d*c^2*b^5*a + 45/7*x^7*d^2*c*b^4*a^2 + 20/7*x^7*d^3*b^3*a^3 + x^6*c^3*b^5
*a + 15/2*x^6*d*c^2*b^4*a^2 + 10*x^6*d^2*c*b^3*a^3 + 5/2*x^6*d^3*b^2*a^4 +
3*x^5*c^3*b^4*a^2 + 12*x^5*d*c^2*b^3*a^3 + 9*x^5*d^2*c*b^2*a^4 + 6/5*x^5*d^
3*b*a^5 + 5*x^4*c^3*b^3*a^3 + 45/4*x^4*d*c^2*b^2*a^4 + 9/2*x^4*d^2*c*b*a^5
+ 1/4*x^4*d^3*a^6 + 5*x^3*c^3*b^2*a^4 + 6*x^3*d*c^2*b*a^5 + x^3*d^2*c*a^6 +
 3*x^2*c^3*b*a^5 + 3/2*x^2*d*c^2*a^6 + x*c^3*a^6
```

Sympy [B] time = 0.245507, size = 364, normalized size = 3.96

$$a^6c^3x + \frac{b^6d^3x^{10}}{10} + x^9\left(\frac{2ab^5d^3}{3} + \frac{b^6cd^2}{3}\right) + x^8\left(\frac{15a^2b^4d^3}{8} + \frac{9ab^5cd^2}{4} + \frac{3b^6c^2d}{8}\right) + x^7\left(\frac{20a^3b^3d^3}{7} + \frac{45a^2b^4cd^2}{7} + \frac{18ab^5c}{7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(a*c+(a*d+b*c)*x+b*d*x**2)**3,x)
```

```
[Out] a**6*c**3*x + b**6*d**3*x**10/10 + x**9*(2*a*b**5*d**3/3 + b**6*c*d**2/3) +
 x**8*(15*a**2*b**4*d**3/8 + 9*a*b**5*c*d**2/4 + 3*b**6*c**2*d/8) + x**7*(2
0*a**3*b**3*d**3/7 + 45*a**2*b**4*c*d**2/7 + 18*a*b**5*c**2*d/7 + b**6*c**3
/7) + x**6*(5*a**4*b**2*d**3/2 + 10*a**3*b**3*c*d**2 + 15*a**2*b**4*c**2*d/
2 + a*b**5*c**3) + x**5*(6*a**5*b*d**3/5 + 9*a**4*b**2*c*d**2 + 12*a**3*b**
3*c**2*d + 3*a**2*b**4*c**3) + x**4*(a**6*d**3/4 + 9*a**5*b*c*d**2/2 + 45*a
**4*b**2*c**2*d/4 + 5*a**3*b**3*c**3) + x**3*(a**6*c*d**2 + 6*a**5*b*c**2*d
 + 5*a**4*b**2*c**3) + x**2*(3*a**6*c**2*d/2 + 3*a**5*b*c**3)
```

Giac [B] time = 1.21896, size = 489, normalized size = 5.32

$$\frac{1}{10}b^6d^3x^{10} + \frac{1}{3}b^6cd^2x^9 + \frac{2}{3}ab^5d^3x^9 + \frac{3}{8}b^6c^2dx^8 + \frac{9}{4}ab^5cd^2x^8 + \frac{15}{8}a^2b^4d^3x^8 + \frac{1}{7}b^6c^3x^7 + \frac{18}{7}ab^5c^2dx^7 + \frac{45}{7}a^2b^4cd^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/10*b^6*d^3*x^10 + 1/3*b^6*c*d^2*x^9 + 2/3*a*b^5*d^3*x^9 + 3/8*b^6*c^2*d*x
^8 + 9/4*a*b^5*c*d^2*x^8 + 15/8*a^2*b^4*d^3*x^8 + 1/7*b^6*c^3*x^7 + 18/7*a*
b^5*c^2*d*x^7 + 45/7*a^2*b^4*c*d^2*x^7 + 20/7*a^3*b^3*d^3*x^7 + a*b^5*c^3*x
^6 + 15/2*a^2*b^4*c^2*d*x^6 + 10*a^3*b^3*c*d^2*x^6 + 5/2*a^4*b^2*d^3*x^6 +
3*a^2*b^4*c^3*x^5 + 12*a^3*b^3*c^2*d*x^5 + 9*a^4*b^2*c*d^2*x^5 + 6/5*a^5*b*
d^3*x^5 + 5*a^3*b^3*c^3*x^4 + 45/4*a^4*b^2*c^2*d*x^4 + 9/2*a^5*b*c*d^2*x^4
+ 1/4*a^6*d^3*x^4 + 5*a^4*b^2*c^3*x^3 + 6*a^5*b*c^2*d*x^3 + a^6*c*d^2*x^3 +
 3*a^5*b*c^3*x^2 + 3/2*a^6*c^2*d*x^2 + a^6*c^3*x
```

3.1784 $\int (a+bx)^2 \left(ac+(bc+ad)x+bdx^2\right)^3 \, dx$

Optimal. Leaf size=92

$$\frac{3d^2(a+bx)^8(bc-ad)}{8b^4}+\frac{3d(a+bx)^7(bc-ad)^2}{7b^4}+\frac{(a+bx)^6(bc-ad)^3}{6b^4}+\frac{d^3(a+bx)^9}{9b^4}$$

```
[Out] ((b*c - a*d)^3*(a + b*x)^6)/(6*b^4) + (3*d*(b*c - a*d)^2*(a + b*x)^7)/(7*b^
4) + (3*d^2*(b*c - a*d)*(a + b*x)^8)/(8*b^4) + (d^3*(a + b*x)^9)/(9*b^4)
```

Rubi [A] time = 0.15726, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$\frac{3d^2(a+bx)^8(bc-ad)}{8b^4}+\frac{3d(a+bx)^7(bc-ad)^2}{7b^4}+\frac{(a+bx)^6(bc-ad)^3}{6b^4}+\frac{d^3(a+bx)^9}{9b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

```
[Out] ((b*c - a*d)^3*(a + b*x)^6)/(6*b^4) + (3*d*(b*c - a*d)^2*(a + b*x)^7)/(7*b^
4) + (3*d^2*(b*c - a*d)*(a + b*x)^8)/(8*b^4) + (d^3*(a + b*x)^9)/(9*b^4)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2\left(ac+(bc+ad)x+bdx^2\right)^3 dx &= \int (a+bx)^5(c+dx)^3 dx\\ &= \int \left(\frac{(bc-ad)^3(a+bx)^5}{b^3}+\frac{3d(bc-ad)^2(a+bx)^6}{b^3}+\frac{3d^2(bc-ad)(a+bx)^7}{b^3}+\ldots\right.\\ &= \frac{(bc-ad)^3(a+bx)^6}{6b^4}+\frac{3d(bc-ad)^2(a+bx)^7}{7b^4}+\frac{3d^2(bc-ad)(a+bx)^8}{8b^4}+\ldots\end{aligned}$$

Mathematica [B] time = 0.058977, size = 235, normalized size = 2.55

$$\frac{1}{504}x\left(84a^3b^2x^2\left(45c^2dx+20c^3+36cd^2x^2+10d^3x^3\right)+36a^2b^3x^3\left(84c^2dx+35c^3+70cd^2x^2+20d^3x^3\right)+126a^4bx\left(20c^2d\ldots\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

```
[Out] (x*(126*a^5*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 126*a^4*b*x*(10*c
^3 + 20*c^2*d*x + 15*c*d^2*x^2 + 4*d^3*x^3) + 84*a^3*b^2*x^2*(20*c^3 + 45*c
^2*d*x + 36*c*d^2*x^2 + 10*d^3*x^3) + 36*a^2*b^3*x^3*(35*c^3 + 84*c^2*d*x +
 70*c*d^2*x^2 + 20*d^3*x^3) + 9*a*b^4*x^4*(56*c^3 + 140*c^2*d*x + 120*c*d^2
*x^2 + 35*d^3*x^3) + b^5*x^5*(84*c^3 + 216*c^2*d*x + 189*c*d^2*x^2 + 56*d^3
*x^3)))/504
```

Maple [B] time = 0.04, size = 601, normalized size = 6.5

$$\frac{b^5d^3x^9}{9}+\frac{\left(2\,ab^4d^3+3\,b^4\,(ad+bc)\,d^2\right)x^8}{8}+\frac{\left(a^2b^3d^3+6\,ab^3\,(ad+bc)\,d^2+b^2\left(ab^2cd^2+2\,(ad+bc)^2\,bd+bd\left(2\,cabd+\right.\right.}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(a*c+(a*d+b*c)*x+b*d*x^2)^3,x)
```

```
[Out] 1/9*b^5*d^3*x^9+1/8*(2*a*b^4*d^3+3*b^4*(a*d+b*c)*d^2)*x^8+1/7*(a^2*b^3*d^3+
6*a*b^3*(a*d+b*c)*d^2+b^2*(a*b^2*c*d^2+2*(a*d+b*c)^2*b*d+b*d*(2*c*a*b*d+(a*
d+b*c)^2)))*x^7+1/6*(3*a^2*(a*d+b*c)*b^2*d^2+2*a*b*(a*b^2*c*d^2+2*(a*d+b*c)
^2*b*d+b*d*(2*c*a*b*d+(a*d+b*c)^2))+b^2*(4*a*c*(a*d+b*c)*b*d+(a*d+b*c)*(2*c
*a*b*d+(a*d+b*c)^2)))*x^6+1/5*(a^2*(a*b^2*c*d^2+2*(a*d+b*c)^2*b*d+b*d*(2*c*
a*b*d+(a*d+b*c)^2))+2*a*b*(4*a*c*(a*d+b*c)*b*d+(a*d+b*c)*(2*c*a*b*d+(a*d+b*
c)^2))+b^2*(a*c*(2*c*a*b*d+(a*d+b*c)^2)+2*(a*d+b*c)^2*a*c+b*d*a^2*c^2))*x^5
+1/4*(a^2*(4*a*c*(a*d+b*c)*b*d+(a*d+b*c)*(2*c*a*b*d+(a*d+b*c)^2))+2*a*b*(a*
c*(2*c*a*b*d+(a*d+b*c)^2)+2*(a*d+b*c)^2*a*c+b*d*a^2*c^2)+3*b^2*a^2*c^2*(a*d
+b*c))*x^4+1/3*(a^2*(a*c*(2*c*a*b*d+(a*d+b*c)^2)+2*(a*d+b*c)^2*a*c+b*d*a^2*
c^2)+6*a^3*b*c^2*(a*d+b*c)+a^3*b^2*c^3)*x^3+1/2*(3*a^4*c^2*(a*d+b*c)+2*a^4*
b*c^3)*x^2+a^5*c^3*x
```

Maxima [B] time = 1.03591, size = 374, normalized size = 4.07

$$\frac{1}{9}\,b^5d^3x^9+a^5c^3x+\frac{1}{8}\left(3\,b^5cd^2+5\,ab^4d^3\right)x^8+\frac{1}{7}\left(3\,b^5c^2d+15\,ab^4cd^2+10\,a^2b^3d^3\right)x^7+\frac{1}{6}\left(b^5c^3+15\,ab^4c^2d+30\,a^2b^3c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/9*b^5*d^3*x^9 + a^5*c^3*x + 1/8*(3*b^5*c*d^2 + 5*a*b^4*d^3)*x^8 + 1/7*(3*
b^5*c^2*d + 15*a*b^4*c*d^2 + 10*a^2*b^3*d^3)*x^7 + 1/6*(b^5*c^3 + 15*a*b^4*
c^2*d + 30*a^2*b^3*c*d^2 + 10*a^3*b^2*d^3)*x^6 + (a*b^4*c^3 + 6*a^2*b^3*c^2
*d + 6*a^3*b^2*c*d^2 + a^4*b*d^3)*x^5 + 1/4*(10*a^2*b^3*c^3 + 30*a^3*b^2*c^
2*d + 15*a^4*b*c*d^2 + a^5*d^3)*x^4 + 1/3*(10*a^3*b^2*c^3 + 15*a^4*b*c^2*d
+ 3*a^5*c*d^2)*x^3 + 1/2*(5*a^4*b*c^3 + 3*a^5*c^2*d)*x^2
```

Fricas [B] time = 1.38934, size = 651, normalized size = 7.08

$$\frac{1}{9}x^9d^3b^5+\frac{3}{8}x^8d^2cb^5+\frac{5}{8}x^8d^3b^4a+\frac{3}{7}x^7dc^2b^5+\frac{15}{7}x^7d^2cb^4a+\frac{10}{7}x^7d^3b^3a^2+\frac{1}{6}x^6c^3b^5+\frac{5}{2}x^6dc^2b^4a+5x^6d^2cb^3a^2+\frac{5}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*d^3*b^5 + 3/8*x^8*d^2*c*b^5 + 5/8*x^8*d^3*b^4*a + 3/7*x^7*d*c^2*b^5
 + 15/7*x^7*d^2*c*b^4*a + 10/7*x^7*d^3*b^3*a^2 + 1/6*x^6*c^3*b^5 + 5/2*x^6*
d*c^2*b^4*a + 5*x^6*d^2*c*b^3*a^2 + 5/3*x^6*d^3*b^2*a^3 + x^5*c^3*b^4*a + 6
*x^5*d*c^2*b^3*a^2 + 6*x^5*d^2*c*b^2*a^3 + x^5*d^3*b*a^4 + 5/2*x^4*c^3*b^3*
a^2 + 15/2*x^4*d*c^2*b^2*a^3 + 15/4*x^4*d^2*c*b*a^4 + 1/4*x^4*d^3*a^5 + 10/
3*x^3*c^3*b^2*a^3 + 5*x^3*d*c^2*b*a^4 + x^3*d^2*c*a^5 + 5/2*x^2*c^3*b*a^4 +
 3/2*x^2*d*c^2*a^5 + x*c^3*a^5
```

Sympy [B] time = 0.190995, size = 308, normalized size = 3.35

$$a^5c^3x + \frac{b^5d^3x^9}{9} + x^8\left(\frac{5ab^4d^3}{8} + \frac{3b^5cd^2}{8}\right) + x^7\left(\frac{10a^2b^3d^3}{7} + \frac{15ab^4cd^2}{7} + \frac{3b^5c^2d}{7}\right) + x^6\left(\frac{5a^3b^2d^3}{3} + 5a^2b^3cd^2 + \frac{5ab^4c^2d}{2} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(a*c+(a*d+b*c)*x+b*d*x**2)**3,x)
```

```
[Out] a**5*c**3*x + b**5*d**3*x**9/9 + x**8*(5*a*b**4*d**3/8 + 3*b**5*c*d**2/8) +
 x**7*(10*a**2*b**3*d**3/7 + 15*a*b**4*c*d**2/7 + 3*b**5*c**2*d/7) + x**6*(
5*a**3*b**2*d**3/3 + 5*a**2*b**3*c*d**2 + 5*a*b**4*c**2*d/2 + b**5*c**3/6)
+ x**5*(a**4*b*d**3 + 6*a**3*b**2*c*d**2 + 6*a**2*b**3*c**2*d + a*b**4*c**3
) + x**4*(a**5*d**3/4 + 15*a**4*b*c*d**2/4 + 15*a**3*b**2*c**2*d/2 + 5*a**2
*b**3*c**3/2) + x**3*(a**5*c*d**2 + 5*a**4*b*c**2*d + 10*a**3*b**2*c**3/3)
+ x**2*(3*a**5*c**2*d/2 + 5*a**4*b*c**3/2)
```

Giac [B] time = 1.22973, size = 409, normalized size = 4.45

$$\frac{1}{9}b^5d^3x^9 + \frac{3}{8}b^5cd^2x^8 + \frac{5}{8}ab^4d^3x^8 + \frac{3}{7}b^5c^2dx^7 + \frac{15}{7}ab^4cd^2x^7 + \frac{10}{7}a^2b^3d^3x^7 + \frac{1}{6}b^5c^3x^6 + \frac{5}{2}ab^4c^2dx^6 + 5a^2b^3cd^2x^6 + \frac{5}{3}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/9*b^5*d^3*x^9 + 3/8*b^5*c*d^2*x^8 + 5/8*a*b^4*d^3*x^8 + 3/7*b^5*c^2*d*x^7
 + 15/7*a*b^4*c*d^2*x^7 + 10/7*a^2*b^3*d^3*x^7 + 1/6*b^5*c^3*x^6 + 5/2*a*b^
4*c^2*d*x^6 + 5*a^2*b^3*c*d^2*x^6 + 5/3*a^3*b^2*d^3*x^6 + a*b^4*c^3*x^5 + 6
*a^2*b^3*c^2*d*x^5 + 6*a^3*b^2*c*d^2*x^5 + a^4*b*d^3*x^5 + 5/2*a^2*b^3*c^3*
x^4 + 15/2*a^3*b^2*c^2*d*x^4 + 15/4*a^4*b*c*d^2*x^4 + 1/4*a^5*d^3*x^4 + 10/
3*a^3*b^2*c^3*x^3 + 5*a^4*b*c^2*d*x^3 + a^5*c*d^2*x^3 + 5/2*a^4*b*c^3*x^2 +
 3/2*a^5*c^2*d*x^2 + a^5*c^3*x
```

3.1785 $\int (a+bx)\left(ac+(bc+ad)x+bdx^2\right)^3\,dx$

Optimal. Leaf size=92

$$\frac{3d^2(a+bx)^7(bc-ad)}{7b^4}+\frac{d(a+bx)^6(bc-ad)^2}{2b^4}+\frac{(a+bx)^5(bc-ad)^3}{5b^4}+\frac{d^3(a+bx)^8}{8b^4}$$

```
[Out] ((b*c - a*d)^3*(a + b*x)^5)/(5*b^4) + (d*(b*c - a*d)^2*(a + b*x)^6)/(2*b^4)
 + (3*d^2*(b*c - a*d)*(a + b*x)^7)/(7*b^4) + (d^3*(a + b*x)^8)/(8*b^4)
```

Rubi [A] time = 0.120538, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {626, 43}

$$\frac{3d^2(a+bx)^7(bc-ad)}{7b^4}+\frac{d(a+bx)^6(bc-ad)^2}{2b^4}+\frac{(a+bx)^5(bc-ad)^3}{5b^4}+\frac{d^3(a+bx)^8}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

```
[Out] ((b*c - a*d)^3*(a + b*x)^5)/(5*b^4) + (d*(b*c - a*d)^2*(a + b*x)^6)/(2*b^4)
 + (3*d^2*(b*c - a*d)*(a + b*x)^7)/(7*b^4) + (d^3*(a + b*x)^8)/(8*b^4)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)\left(ac+(bc+ad)x+bdx^2\right)^3\,dx &= \int (a+bx)^4(c+dx)^3\,dx\\ &= \int \left(\frac{(bc-ad)^3(a+bx)^4}{b^3}+\frac{3d(bc-ad)^2(a+bx)^5}{b^3}+\frac{3d^2(bc-ad)(a+bx)^6}{b^3}+\right.\\ &= \frac{(bc-ad)^3(a+bx)^5}{5b^4}+\frac{d(bc-ad)^2(a+bx)^6}{2b^4}+\frac{3d^2(bc-ad)(a+bx)^7}{7b^4}+\frac{d^3(a\end{aligned}$$

Mathematica [B] time = 0.0247937, size = 217, normalized size = 2.36

$$\frac{1}{2}b^2dx^6\left(2a^2d^2+4abcd+b^2c^2\right)+\frac{1}{5}bx^5\left(18a^2bcd^2+4a^3d^3+12ab^2c^2d+b^3c^3\right)+\frac{1}{4}ax^4\left(12a^2bcd^2+a^3d^3+18ab^2c^2d+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

```
[Out] a^4*c^3*x + (a^3*c^2*(4*b*c + 3*a*d)*x^2)/2 + a^2*c*(2*b^2*c^2 + 4*a*b*c*d
+ a^2*d^2)*x^3 + (a*(4*b^3*c^3 + 18*a*b^2*c^2*d + 12*a^2*b*c*d^2 + a^3*d^3)
*x^4)/4 + (b*(b^3*c^3 + 12*a*b^2*c^2*d + 18*a^2*b*c*d^2 + 4*a^3*d^3)*x^5)/5
 + (b^2*d*(b^2*c^2 + 4*a*b*c*d + 2*a^2*d^2)*x^6)/2 + (b^3*d^2*(3*b*c + 4*a*
d)*x^7)/7 + (b^4*d^3*x^8)/8
```

Maple [B] time = 0.04, size = 391, normalized size = 4.3

$$\frac{b^4d^3x^8}{8}+\frac{\left(ab^3d^3+3\,b^3\,(ad+bc)\,d^2\right)x^7}{7}+\frac{\left(3\,a\,(ad+bc)\,b^2d^2+b\left(ab^2cd^2+2\,(ad+bc)^2\,bd+bd\left(2\,cabd+(ad+bc)^2\right)\right)\right)x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x^2)^3,x)
```

```
[Out] 1/8*b^4*d^3*x^8+1/7*(a*b^3*d^3+3*b^3*(a*d+b*c)*d^2)*x^7+1/6*(3*a*(a*d+b*c)*
b^2*d^2+b*(a*b^2*c*d^2+2*(a*d+b*c)^2*b*d+b*d*(2*c*a*b*d+(a*d+b*c)^2)))*x^6+
1/5*(a*(a*b^2*c*d^2+2*(a*d+b*c)^2*b*d+b*d*(2*c*a*b*d+(a*d+b*c)^2))+b*(4*a*c
*(a*d+b*c)*b*d+(a*d+b*c)*(2*c*a*b*d+(a*d+b*c)^2)))*x^5+1/4*(a*(4*a*c*(a*d+b
*c)*b*d+(a*d+b*c)*(2*c*a*b*d+(a*d+b*c)^2))+b*(a*c*(2*c*a*b*d+(a*d+b*c)^2)+2
*(a*d+b*c)^2*a*c+b*d*a^2*c^2))*x^4+1/3*(a*(a*c*(2*c*a*b*d+(a*d+b*c)^2)+2*(a
*d+b*c)^2*a*c+b*d*a^2*c^2)+3*b*a^2*c^2*(a*d+b*c))*x^3+1/2*(3*a^3*c^2*(a*d+b
*c)+a^3*b*c^3)*x^2+a^4*c^3*x
```

Maxima [B] time = 1.04555, size = 304, normalized size = 3.3

$$\frac{1}{8}\,b^4d^3x^8+a^4c^3x+\frac{1}{7}\left(3\,b^4cd^2+4\,ab^3d^3\right)x^7+\frac{1}{2}\left(b^4c^2d+4\,ab^3cd^2+2\,a^2b^2d^3\right)x^6+\frac{1}{5}\left(b^4c^3+12\,ab^3c^2d+18\,a^2b^2cd^2+4\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*b^4*d^3*x^8 + a^4*c^3*x + 1/7*(3*b^4*c*d^2 + 4*a*b^3*d^3)*x^7 + 1/2*(b^
4*c^2*d + 4*a*b^3*c*d^2 + 2*a^2*b^2*d^3)*x^6 + 1/5*(b^4*c^3 + 12*a*b^3*c^2*
d + 18*a^2*b^2*c*d^2 + 4*a^3*b*d^3)*x^5 + 1/4*(4*a*b^3*c^3 + 18*a^2*b^2*c^2
*d + 12*a^3*b*c*d^2 + a^4*d^3)*x^4 + (2*a^2*b^2*c^3 + 4*a^3*b*c^2*d + a^4*c
*d^2)*x^3 + 1/2*(4*a^3*b*c^3 + 3*a^4*c^2*d)*x^2
```

Fricas [B] time = 1.39332, size = 520, normalized size = 5.65

$$\frac{1}{8}x^8d^3b^4+\frac{3}{7}x^7d^2cb^4+\frac{4}{7}x^7d^3b^3a+\frac{1}{2}x^6dc^2b^4+2x^6d^2cb^3a+x^6d^3b^2a^2+\frac{1}{5}x^5c^3b^4+\frac{12}{5}x^5dc^2b^3a+\frac{18}{5}x^5d^2cb^2a^2+\frac{4}{5}x^5d^3b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*d^3*b^4 + 3/7*x^7*d^2*c*b^4 + 4/7*x^7*d^3*b^3*a + 1/2*x^6*d*c^2*b^4
 + 2*x^6*d^2*c*b^3*a + x^6*d^3*b^2*a^2 + 1/5*x^5*c^3*b^4 + 12/5*x^5*d*c^2*b
```

```
^3*a + 18/5*x^5*d^2*c*b^2*a^2 + 4/5*x^5*d^3*b*a^3 + x^4*c^3*b^3*a + 9/2*x^4
*d*c^2*b^2*a^2 + 3*x^4*d^2*c*b*a^3 + 1/4*x^4*d^3*a^4 + 2*x^3*c^3*b^2*a^2 +
4*x^3*d*c^2*b*a^3 + x^3*d^2*c*a^4 + 2*x^2*c^3*b*a^3 + 3/2*x^2*d*c^2*a^4 + x
*c^3*a^4
```

Sympy [B] time = 0.279936, size = 243, normalized size = 2.64

$$a^4c^3x + \frac{b^4d^3x^8}{8} + x^7\left(\frac{4ab^3d^3}{7} + \frac{3b^4cd^2}{7}\right) + x^6\left(a^2b^2d^3 + 2ab^3cd^2 + \frac{b^4c^2d}{2}\right) + x^5\left(\frac{4a^3bd^3}{5} + \frac{18a^2b^2cd^2}{5} + \frac{12ab^3c^2d}{5} + \frac{b}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x**2)**3,x)
```

```
[Out] a**4*c**3*x + b**4*d**3*x**8/8 + x**7*(4*a*b**3*d**3/7 + 3*b**4*c*d**2/7) +
 x**6*(a**2*b**2*d**3 + 2*a*b**3*c*d**2 + b**4*c**2*d/2) + x**5*(4*a**3*b*d
**3/5 + 18*a**2*b**2*c*d**2/5 + 12*a*b**3*c**2*d/5 + b**4*c**3/5) + x**4*(a
**4*d**3/4 + 3*a**3*b*c*d**2 + 9*a**2*b**2*c**2*d/2 + a*b**3*c**3) + x**3*(
a**4*c*d**2 + 4*a**3*b*c**2*d + 2*a**2*b**2*c**3) + x**2*(3*a**4*c**2*d/2 +
 2*a**3*b*c**3)
```

Giac [B] time = 1.18625, size = 331, normalized size = 3.6

$$\frac{1}{8}b^4d^3x^8 + \frac{3}{7}b^4cd^2x^7 + \frac{4}{7}ab^3d^3x^7 + \frac{1}{2}b^4c^2dx^6 + 2\,ab^3cd^2x^6 + a^2b^2d^3x^6 + \frac{1}{5}b^4c^3x^5 + \frac{12}{5}ab^3c^2dx^5 + \frac{18}{5}a^2b^2cd^2x^5 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*b^4*d^3*x^8 + 3/7*b^4*c*d^2*x^7 + 4/7*a*b^3*d^3*x^7 + 1/2*b^4*c^2*d*x^6
 + 2*a*b^3*c*d^2*x^6 + a^2*b^2*d^3*x^6 + 1/5*b^4*c^3*x^5 + 12/5*a*b^3*c^2*d
*x^5 + 18/5*a^2*b^2*c*d^2*x^5 + 4/5*a^3*b*d^3*x^5 + a*b^3*c^3*x^4 + 9/2*a^2
*b^2*c^2*d*x^4 + 3*a^3*b*c*d^2*x^4 + 1/4*a^4*d^3*x^4 + 2*a^2*b^2*c^3*x^3 +
4*a^3*b*c^2*d*x^3 + a^4*c*d^2*x^3 + 2*a^3*b*c^3*x^2 + 3/2*a^4*c^2*d*x^2 + a
^4*c^3*x
```

3.1786 $\int \left(ac + (bc+ad)x + bdx^2\right)^3 dx$

Optimal. Leaf size=92

$$-\frac{b^2(c+dx)^6(bc-ad)}{2d^4}+\frac{3b(c+dx)^5(bc-ad)^2}{5d^4}-\frac{(c+dx)^4(bc-ad)^3}{4d^4}+\frac{b^3(c+dx)^7}{7d^4}$$

```
[Out] -((b*c - a*d)^3*(c + d*x)^4)/(4*d^4) + (3*b*(b*c - a*d)^2*(c + d*x)^5)/(5*d
^4) - (b^2*(b*c - a*d)*(c + d*x)^6)/(2*d^4) + (b^3*(c + d*x)^7)/(7*d^4)
```

Rubi [A] time = 0.117105, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {610, 43}

$$-\frac{b^2(c+dx)^6(bc-ad)}{2d^4}+\frac{3b(c+dx)^5(bc-ad)^2}{5d^4}-\frac{(c+dx)^4(bc-ad)^3}{4d^4}+\frac{b^3(c+dx)^7}{7d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

```
[Out] -((b*c - a*d)^3*(c + d*x)^4)/(4*d^4) + (3*b*(b*c - a*d)^2*(c + d*x)^5)/(5*d
^4) - (b^2*(b*c - a*d)*(c + d*x)^6)/(2*d^4) + (b^3*(c + d*x)^7)/(7*d^4)
```

Rule 610

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Dist[1/c^p, Int[Simp[b/2 - q/2 + c*x, x]^p*Simp[b/2 + q/2 + c
*x, x]^p, x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(ac+(bc+ad)x+bdx^2\right)^3 dx &= \frac{\int (bc+bdx)^3(ad+bdx)^3\,dx}{b^3d^3}\\ &= \frac{\int \left(-(bc-ad)^3(bc+bdx)^3+3(bc-ad)^2(bc+bdx)^4-3(bc-ad)(bc+bdx)^5+(bc+bd\right.}{b^3d^3}\\ &= -\frac{(bc-ad)^3(c+dx)^4}{4d^4}+\frac{3b(bc-ad)^2(c+dx)^5}{5d^4}-\frac{b^2(bc-ad)(c+dx)^6}{2d^4}+\frac{b^3(c+dx)^7}{7d^4}\end{aligned}$$

Mathematica [A] time = 0.0179502, size = 161, normalized size = 1.75

$$\frac{3}{5}bdx^5\left(a^2d^2+3abcd+b^2c^2\right)+\frac{1}{4}x^4\left(9a^2bcd^2+a^3d^3+9ab^2c^2d+b^3c^3\right)+acx^3\left(a^2d^2+3abcd+b^2c^2\right)+\frac{3}{2}a^2c^2x^2(ad+bc)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

```
[Out] a^3*c^3*x + (3*a^2*c^2*(b*c + a*d)*x^2)/2 + a*c*(b^2*c^2 + 3*a*b*c*d + a^2*
d^2)*x^3 + ((b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*x^4)/4 + (3
*b*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^5)/5 + (b^2*d^2*(b*c + a*d)*x^6)/2 +
 (b^3*d^3*x^7)/7
```

Maple [B] time = 0.041, size = 194, normalized size = 2.1

$$\frac{b^3d^3x^7}{7} + \frac{(ad+bc)\,b^2d^2x^6}{2} + \frac{\left(ab^2cd^2 + 2\,(ad+bc)^2\,bd + bd\left(2\,cabd + (ad+bc)^2\right)\right)x^5}{5} + \frac{\left(4\,ac\,(ad+bc)\,bd + (ad+bc)\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+(a*d+b*c)*x+b*d*x^2)^3,x)
```

```
[Out] 1/7*b^3*d^3*x^7+1/2*(a*d+b*c)*b^2*d^2*x^6+1/5*(a*b^2*c*d^2+2*(a*d+b*c)^2*b*
d+b*d*(2*c*a*b*d+(a*d+b*c)^2))*x^5+1/4*(4*a*c*(a*d+b*c)*b*d+(a*d+b*c)*(2*c*
a*b*d+(a*d+b*c)^2))*x^4+1/3*(a*c*(2*c*a*b*d+(a*d+b*c)^2)+2*(a*d+b*c)^2*a*c+
b*d*a^2*c^2)*x^3+3/2*a^2*c^2*(a*d+b*c)*x^2+a^3*c^3*x
```

Maxima [A] time = 1.0822, size = 189, normalized size = 2.05

$$\frac{1}{7}\,b^3d^3x^7 + \frac{1}{2}\,(bc+ad)b^2d^2x^6 + \frac{3}{5}\,(bc+ad)^2bdx^5 + a^3c^3x + \frac{1}{4}\,(bc+ad)^3x^4 + \frac{1}{2}\left(2\,bdx^3 + 3\,(bc+ad)x^2\right)a^2c^2 + \frac{1}{10}\left(6\,b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/7*b^3*d^3*x^7 + 1/2*(b*c + a*d)*b^2*d^2*x^6 + 3/5*(b*c + a*d)^2*b*d*x^5 +
 a^3*c^3*x + 1/4*(b*c + a*d)^3*x^4 + 1/2*(2*b*d*x^3 + 3*(b*c + a*d)*x^2)*a^
2*c^2 + 1/10*(6*b^2*d^2*x^5 + 15*(b*c + a*d)*b*d*x^4 + 10*(b*c + a*d)^2*x^3
)*a*c
```

Fricas [B] time = 1.39292, size = 409, normalized size = 4.45

$$\frac{1}{7}x^7d^3b^3 + \frac{1}{2}x^6d^2cb^3 + \frac{1}{2}x^6d^3b^2a + \frac{3}{5}x^5dc^2b^3 + \frac{9}{5}x^5d^2cb^2a + \frac{3}{5}x^5d^3ba^2 + \frac{1}{4}x^4c^3b^3 + \frac{9}{4}x^4dc^2b^2a + \frac{9}{4}x^4d^2cba^2 + \frac{1}{4}x^4d^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*d^3*b^3 + 1/2*x^6*d^2*c*b^3 + 1/2*x^6*d^3*b^2*a + 3/5*x^5*d*c^2*b^3
 + 9/5*x^5*d^2*c*b^2*a + 3/5*x^5*d^3*b*a^2 + 1/4*x^4*c^3*b^3 + 9/4*x^4*d*c^
2*b^2*a + 9/4*x^4*d^2*c*b*a^2 + 1/4*x^4*d^3*a^3 + x^3*c^3*b^2*a + 3*x^3*d*c
^2*b*a^2 + x^3*d^2*c*a^3 + 3/2*x^2*c^3*b*a^2 + 3/2*x^2*d*c^2*a^3 + x*c^3*a^
3
```

Sympy [B] time = 0.219996, size = 190, normalized size = 2.07

$$a^3c^3x + \frac{b^3d^3x^7}{7} + x^6\left(\frac{ab^2d^3}{2} + \frac{b^3cd^2}{2}\right) + x^5\left(\frac{3a^2bd^3}{5} + \frac{9ab^2cd^2}{5} + \frac{3b^3c^2d}{5}\right) + x^4\left(\frac{a^3d^3}{4} + \frac{9a^2bcd^2}{4} + \frac{9ab^2c^2d}{4} + \frac{b^3c^3}{4}\right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x**2)**3,x)
```

```
[Out] a**3*c**3*x + b**3*d**3*x**7/7 + x**6*(a*b**2*d**3/2 + b**3*c*d**2/2) + x**
5*(3*a**2*b*d**3/5 + 9*a*b**2*c*d**2/5 + 3*b**3*c**2*d/5) + x**4*(a**3*d**3
/4 + 9*a**2*b*c*d**2/4 + 9*a*b**2*c**2*d/4 + b**3*c**3/4) + x**3*(a**3*c*d*
*2 + 3*a**2*b*c**2*d + a*b**2*c**3) + x**2*(3*a**3*c**2*d/2 + 3*a**2*b*c**3
/2)
```

Giac [B] time = 1.19604, size = 254, normalized size = 2.76

$$\frac{1}{7}b^3d^3x^7 + \frac{1}{2}b^3cd^2x^6 + \frac{1}{2}ab^2d^3x^6 + \frac{3}{5}b^3c^2dx^5 + \frac{9}{5}ab^2cd^2x^5 + \frac{3}{5}a^2bd^3x^5 + \frac{1}{4}b^3c^3x^4 + \frac{9}{4}ab^2c^2dx^4 + \frac{9}{4}a^2bcd^2x^4 + \frac{1}{4}a^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/7*b^3*d^3*x^7 + 1/2*b^3*c*d^2*x^6 + 1/2*a*b^2*d^3*x^6 + 3/5*b^3*c^2*d*x^5
 + 9/5*a*b^2*c*d^2*x^5 + 3/5*a^2*b*d^3*x^5 + 1/4*b^3*c^3*x^4 + 9/4*a*b^2*c^
2*d*x^4 + 9/4*a^2*b*c*d^2*x^4 + 1/4*a^3*d^3*x^4 + a*b^2*c^3*x^3 + 3*a^2*b*c
^2*d*x^3 + a^3*c*d^2*x^3 + 3/2*a^2*b*c^3*x^2 + 3/2*a^3*c^2*d*x^2 + a^3*c^3*
x
```

3.1787 $\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^3}{a+bx} \, dx$

Optimal. Leaf size=65

$$-\frac{2b(c+dx)^5(bc-ad)}{5d^3}+\frac{(c+dx)^4(bc-ad)^2}{4d^3}+\frac{b^2(c+dx)^6}{6d^3}$$

```
[Out] ((b*c - a*d)^2*(c + d*x)^4)/(4*d^3) - (2*b*(b*c - a*d)*(c + d*x)^5)/(5*d^3)
 + (b^2*(c + d*x)^6)/(6*d^3)
```

Rubi [A] time = 0.0718319, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$-\frac{2b(c+dx)^5(bc-ad)}{5d^3}+\frac{(c+dx)^4(bc-ad)^2}{4d^3}+\frac{b^2(c+dx)^6}{6d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x),x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x)^4)/(4*d^3) - (2*b*(b*c - a*d)*(c + d*x)^5)/(5*d^3)
 + (b^2*(c + d*x)^6)/(6*d^3)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\left(ac+(bc+ad)x+bdx^2\right)^3}{a+bx} \, dx &= \int (a+bx)^2(c+dx)^3 \, dx \\ &= \int \left(\frac{(-bc+ad)^2(c+dx)^3}{d^2}-\frac{2b(bc-ad)(c+dx)^4}{d^2}+\frac{b^2(c+dx)^5}{d^2}\right) dx \\ &= \frac{(bc-ad)^2(c+dx)^4}{4d^3}-\frac{2b(bc-ad)(c+dx)^5}{5d^3}+\frac{b^2(c+dx)^6}{6d^3} \end{aligned}$$

Mathematica [A] time = 0.0128684, size = 122, normalized size = 1.88

$$\frac{1}{4}dx^4\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{3}cx^3\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{2}ac^2x^2(3ad+2bc)+\frac{1}{5}bd^2x^5(2ad+3bc)+\frac{1}{6}b$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x),x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^2)/2 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^3)/3 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4)/4 + (b*d^2*(3*b*c
+ 2*a*d)*x^5)/5 + (b^2*d^3*x^6)/6
```

Maple [B] time = 0.04, size = 147, normalized size = 2.3

$$\frac{b^2d^3x^6}{6}+\frac{\left(cb^2d^2+2\,d^2\,(ad+bc)\,b\right)x^5}{5}+\frac{\left(2\,c\,(ad+bc)\,bd+d\left(2\,cabd+(ad+bc)^2\right)\right)x^4}{4}+\frac{\left(c\left(2\,cabd+(ad+bc)^2\right)+2\,dac\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a),x)
```

```
[Out] 1/6*b^2*d^3*x^6+1/5*(c*b^2*d^2+2*d^2*(a*d+b*c)*b)*x^5+1/4*(2*c*(a*d+b*c)*b*
d+d*(2*c*a*b*d+(a*d+b*c)^2))*x^4+1/3*(c*(2*c*a*b*d+(a*d+b*c)^2)+2*d*a*c*(a*
d+b*c))*x^3+1/2*(2*c^2*a*(a*d+b*c)+d*a^2*c^2)*x^2+x*a^2*c^3
```

Maxima [B] time = 1.02757, size = 167, normalized size = 2.57

$$\frac{1}{6}\,b^2d^3x^6+a^2c^3x+\frac{1}{5}\left(3\,b^2cd^2+2\,abd^3\right)x^5+\frac{1}{4}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^4+\frac{1}{3}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^3+\frac{1}{2}\left(2\,abc\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/6*b^2*d^3*x^6 + a^2*c^3*x + 1/5*(3*b^2*c*d^2 + 2*a*b*d^3)*x^5 + 1/4*(3*b^
2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^4 + 1/3*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c
*d^2)*x^3 + 1/2*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2
```

Fricas [B] time = 1.55957, size = 266, normalized size = 4.09

$$\frac{1}{6}\,b^2d^3x^6+a^2c^3x+\frac{1}{5}\left(3\,b^2cd^2+2\,abd^3\right)x^5+\frac{1}{4}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^4+\frac{1}{3}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^3+\frac{1}{2}\left(2\,abc\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*b^2*d^3*x^6 + a^2*c^3*x + 1/5*(3*b^2*c*d^2 + 2*a*b*d^3)*x^5 + 1/4*(3*b^
2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^4 + 1/3*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c
*d^2)*x^3 + 1/2*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2
```

Sympy [B] time = 0.256044, size = 133, normalized size = 2.05

$$a^2c^3x+\frac{b^2d^3x^6}{6}+x^5\left(\frac{2abd^3}{5}+\frac{3b^2cd^2}{5}\right)+x^4\left(\frac{a^2d^3}{4}+\frac{3abcd^2}{2}+\frac{3b^2c^2d}{4}\right)+x^3\left(a^2cd^2+2abc^2d+\frac{b^2c^3}{3}\right)+x^2\left(\frac{3a^2c^2d}{2}+ab\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x**2)**3/(b*x+a),x)
```

```
[Out] a**2*c**3*x + b**2*d**3*x**6/6 + x**5*(2*a*b*d**3/5 + 3*b**2*c*d**2/5) + x*
*4*(a**2*d**3/4 + 3*a*b*c*d**2/2 + 3*b**2*c**2*d/4) + x**3*(a**2*c*d**2 + 2
*a*b*c**2*d + b**2*c**3/3) + x**2*(3*a**2*c**2*d/2 + a*b*c**3)
```

Giac [B] time = 1.21458, size = 176, normalized size = 2.71

$$\frac{1}{6}\,b^2d^3x^6 + \frac{3}{5}\,b^2cd^2x^5 + \frac{2}{5}\,abd^3x^5 + \frac{3}{4}\,b^2c^2dx^4 + \frac{3}{2}\,abcd^2x^4 + \frac{1}{4}\,a^2d^3x^4 + \frac{1}{3}\,b^2c^3x^3 + 2\,abc^2dx^3 + a^2cd^2x^3 + abc^3x^2 + \frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/6*b^2*d^3*x^6 + 3/5*b^2*c*d^2*x^5 + 2/5*a*b*d^3*x^5 + 3/4*b^2*c^2*d*x^4 +
 3/2*a*b*c*d^2*x^4 + 1/4*a^2*d^3*x^4 + 1/3*b^2*c^3*x^3 + 2*a*b*c^2*d*x^3 +
a^2*c*d^2*x^3 + a*b*c^3*x^2 + 3/2*a^2*c^2*d*x^2 + a^2*c^3*x
```

3.1788 $\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^3}{(a+bx)^2}\,dx$

Optimal. Leaf size=38

$$\frac{b(c+dx)^5}{5d^2}-\frac{(c+dx)^4(bc-ad)}{4d^2}$$

```
[Out] -((b*c - a*d)*(c + d*x)^4)/(4*d^2) + (b*(c + d*x)^5)/(5*d^2)
```

Rubi [A] time = 0.0228092, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$\frac{b(c+dx)^5}{5d^2}-\frac{(c+dx)^4(bc-ad)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^2,x]
```

```
[Out] -((b*c - a*d)*(c + d*x)^4)/(4*d^2) + (b*(c + d*x)^5)/(5*d^2)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^3}{(a+bx)^2}\,dx &= \int (a+bx)(c+dx)^3\,dx\\ &= \int \left(\frac{(-bc+ad)(c+dx)^3}{d}+\frac{b(c+dx)^4}{d}\right)dx\\ &= -\frac{(bc-ad)(c+dx)^4}{4d^2}+\frac{b(c+dx)^5}{5d^2}\end{aligned}$$

Mathematica [A] time = 0.0077657, size = 67, normalized size = 1.76

$$\frac{1}{2}c^2x^2(3ad+bc)+\frac{1}{4}d^2x^4(ad+3bc)+cdx^3(ad+bc)+ac^3x+\frac{1}{5}bd^3x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^2,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^2)/2 + c*d*(b*c + a*d)*x^3 + (d^2*(3*b*c + a
*d)*x^4)/4 + (b*d^3*x^5)/5
```

Maple [B] time = 0.041, size = 94, normalized size = 2.5

$$\frac{d^3bx^5}{5}+\frac{\left(2\,cd^2b+d^2\,(ad+bc)\right)x^4}{4}+\frac{\left(c^2bd+2\,cd\,(ad+bc)+acd^2\right)x^3}{3}+\frac{\left(c^2\,(ad+bc)+2\,ac^2d\right)x^2}{2}+ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^2,x)
```

```
[Out] 1/5*d^3*b*x^5+1/4*(2*c*d^2*b+d^2*(a*d+b*c))*x^4+1/3*(c^2*b*d+2*c*d*(a*d+b*c
)+a*c*d^2)*x^3+1/2*(c^2*(a*d+b*c)+2*a*c^2*d)*x^2+a*c^3*x
```

Maxima [B] time = 1.01124, size = 93, normalized size = 2.45

$$\frac{1}{5}\,bd^3x^5+ac^3x+\frac{1}{4}\left(3\,bcd^2+ad^3\right)x^4+\left(bc^2d+acd^2\right)x^3+\frac{1}{2}\left(bc^3+3\,ac^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/5*b*d^3*x^5 + a*c^3*x + 1/4*(3*b*c*d^2 + a*d^3)*x^4 + (b*c^2*d + a*c*d^2)
*x^3 + 1/2*(b*c^3 + 3*a*c^2*d)*x^2
```

Fricas [B] time = 1.58697, size = 150, normalized size = 3.95

$$\frac{1}{5}\,bd^3x^5+ac^3x+\frac{1}{4}\left(3\,bcd^2+ad^3\right)x^4+\left(bc^2d+acd^2\right)x^3+\frac{1}{2}\left(bc^3+3\,ac^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/5*b*d^3*x^5 + a*c^3*x + 1/4*(3*b*c*d^2 + a*d^3)*x^4 + (b*c^2*d + a*c*d^2)
*x^3 + 1/2*(b*c^3 + 3*a*c^2*d)*x^2
```

Sympy [B] time = 0.240515, size = 73, normalized size = 1.92

$$ac^3x+\frac{bd^3x^5}{5}+x^4\left(\frac{ad^3}{4}+\frac{3bcd^2}{4}\right)+x^3\left(acd^2+bc^2d\right)+x^2\left(\frac{3ac^2d}{2}+\frac{bc^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x**2)**3/(b*x+a)**2,x)
```

```
[Out] a*c**3*x + b*d**3*x**5/5 + x**4*(a*d**3/4 + 3*b*c*d**2/4) + x**3*(a*c*d**2
+ b*c**2*d) + x**2*(3*a*c**2*d/2 + b*c**3/2)
```

Giac [B] time = 1.16105, size = 209, normalized size = 5.5

$$\frac{\left(\frac{10\,b^3c^3}{(bx+a)^3}+\frac{20\,b^2c^2d}{(bx+a)^2}-\frac{30\,ab^2c^2d}{(bx+a)^3}+\frac{15\,bcd^2}{bx+a}-\frac{40\,abcd^2}{(bx+a)^2}+\frac{30\,a^2bcd^2}{(bx+a)^3}-\frac{15\,ad^3}{bx+a}+\frac{20\,a^2d^3}{(bx+a)^2}-\frac{10\,a^3d^3}{(bx+a)^3}+4\,d^3\right)(bx+a)^5}{20\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/20*(10*b^3*c^3/(b*x + a)^3 + 20*b^2*c^2*d/(b*x + a)^2 - 30*a*b^2*c^2*d/(b
*x + a)^3 + 15*b*c*d^2/(b*x + a) - 40*a*b*c*d^2/(b*x + a)^2 + 30*a^2*b*c*d^
2/(b*x + a)^3 - 15*a*d^3/(b*x + a) + 20*a^2*d^3/(b*x + a)^2 - 10*a^3*d^3/(b
*x + a)^3 + 4*d^3)*(b*x + a)^5/b^4
```

3.1789 $\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^3}{(a+bx)^3}\,dx$

Optimal. Leaf size=14

$$\frac{(c+dx)^4}{4d}$$

```
[Out] (c + d*x)^4/(4*d)
```

Rubi [A] time = 0.0099483, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 32}

$$\frac{(c+dx)^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^3,x]
```

```
[Out] (c + d*x)^4/(4*d)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^3}{(a+bx)^3}\,dx &= \int (c+dx)^3\,dx\\ &= \frac{(c+dx)^4}{4d}\end{aligned}$$

Mathematica [A] time = 0.0015153, size = 14, normalized size = 1.

$$\frac{(c+dx)^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^3,x]
```

```
[Out] (c + d*x)^4/(4*d)
```

Maple [A] time = 0.04, size = 13, normalized size = 0.9

$$\frac{(dx + c)^4}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^3,x)
```

```
[Out] 1/4*(d*x+c)^4/d
```

Maxima [B] time = 1.03494, size = 42, normalized size = 3.

$$\frac{1}{4}d^3x^4 + cd^2x^3 + \frac{3}{2}c^2dx^2 + c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*d^3*x^4 + c*d^2*x^3 + 3/2*c^2*d*x^2 + c^3*x
```

Fricas [B] time = 1.4753, size = 66, normalized size = 4.71

$$\frac{1}{4}d^3x^4 + cd^2x^3 + \frac{3}{2}c^2dx^2 + c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*d^3*x^4 + c*d^2*x^3 + 3/2*c^2*d*x^2 + c^3*x
```

Sympy [B] time = 0.145323, size = 32, normalized size = 2.29

$$c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x**2)**3/(b*x+a)**3,x)
```

```
[Out] c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4
```

Giac [B] time = 1.22382, size = 42, normalized size = 3.

$$\frac{1}{4}d^3x^4 + cd^2x^3 + \frac{3}{2}c^2dx^2 + c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*d^3*x^4 + c*d^2*x^3 + 3/2*c^2*d*x^2 + c^3*x
```

3.1790 $\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^3}{(a+bx)^4}\,dx$

Optimal. Leaf size=73

$$\frac{dx(bc-ad)^2}{b^3}+\frac{(c+dx)^2(bc-ad)}{2b^2}+\frac{(bc-ad)^3\log(a+bx)}{b^4}+\frac{(c+dx)^3}{3b}$$

```
[Out] (d*(b*c - a*d)^2*x)/b^3 + ((b*c - a*d)*(c + d*x)^2)/(2*b^2) + (c + d*x)^3/(
3*b) + ((b*c - a*d)^3*Log[a + b*x])/b^4
```

Rubi [A] time = 0.0371784, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$\frac{dx(bc-ad)^2}{b^3}+\frac{(c+dx)^2(bc-ad)}{2b^2}+\frac{(bc-ad)^3\log(a+bx)}{b^4}+\frac{(c+dx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^4,x]
```

```
[Out] (d*(b*c - a*d)^2*x)/b^3 + ((b*c - a*d)*(c + d*x)^2)/(2*b^2) + (c + d*x)^3/(
3*b) + ((b*c - a*d)^3*Log[a + b*x])/b^4
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^3}{(a+bx)^4}\,dx &= \int \frac{(c+dx)^3}{a+bx}\,dx\\ &= \int \left(\frac{d(bc-ad)^2}{b^3}+\frac{(bc-ad)^3}{b^3(a+bx)}+\frac{d(bc-ad)(c+dx)}{b^2}+\frac{d(c+dx)^2}{b}\right)dx\\ &= \frac{d(bc-ad)^2x}{b^3}+\frac{(bc-ad)(c+dx)^2}{2b^2}+\frac{(c+dx)^3}{3b}+\frac{(bc-ad)^3\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.0287097, size = 74, normalized size = 1.01

$$\frac{bdx\left(6a^2d^2-3abd(6c+dx)+b^2\left(18c^2+9cdx+2d^2x^2\right)\right)+6(bc-ad)^3\log(a+bx)}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^4,x]
```

```
[Out] (b*d*x*(6*a^2*d^2 - 3*a*b*d*(6*c + d*x) + b^2*(18*c^2 + 9*c*d*x + 2*d^2*x^2
)) + 6*(b*c - a*d)^3*Log[a + b*x])/(6*b^4)
```

Maple [A] time = 0.042, size = 133, normalized size = 1.8

$$\frac{d^3x^3}{3\,b}-\frac{d^3x^2a}{2\,b^2}+\frac{3\,d^2x^2c}{2\,b}+\frac{d^3a^2x}{b^3}-3\,\frac{acd^2x}{b^2}+3\,\frac{c^2dx}{b}-\frac{\ln(bx+a)\,a^3d^3}{b^4}+3\,\frac{\ln(bx+a)\,ca^2d^2}{b^3}-3\,\frac{\ln(bx+a)\,ac^2d}{b^2}+\dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^4,x)
```

```
[Out] 1/3*d^3/b*x^3-1/2*d^3/b^2*x^2*a+3/2*d^2/b*x^2*c+d^3/b^3*a^2*x-3*d^2/b^2*c*a
*x+3*d/b*c^2*x-1/b^4*ln(b*x+a)*a^3*d^3+3/b^3*ln(b*x+a)*c*a^2*d^2-3/b^2*ln(b
*x+a)*a*c^2*d+1/b*ln(b*x+a)*c^3
```

Maxima [A] time = 1.03305, size = 154, normalized size = 2.11

$$\frac{2\,b^2d^3x^3+3\left(3\,b^2cd^2-abd^3\right)x^2+6\left(3\,b^2c^2d-3\,abcd^2+a^2d^3\right)x}{6\,b^3}+\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log(bx+a)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^2*d^3*x^3 + 3*(3*b^2*c*d^2 - a*b*d^3)*x^2 + 6*(3*b^2*c^2*d - 3*a*b
*c*d^2 + a^2*d^3)*x)/b^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*log(b*x + a)/b^4
```

Fricas [A] time = 1.52819, size = 238, normalized size = 3.26

$$\frac{2\,b^3d^3x^3+3\left(3\,b^3cd^2-ab^2d^3\right)x^2+6\left(3\,b^3c^2d-3\,ab^2cd^2+a^2bd^3\right)x+6\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log(bx+a}{6\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*d^3*x^3 + 3*(3*b^3*c*d^2 - a*b^2*d^3)*x^2 + 6*(3*b^3*c^2*d - 3*a
*b^2*c*d^2 + a^2*b*d^3)*x + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*log(b*x + a))/b^4
```

Sympy [A] time = 0.799061, size = 82, normalized size = 1.12

$$\frac{d^3x^3}{3b}-\frac{x^2\left(ad^3-3bcd^2\right)}{2b^2}+\frac{x\left(a^2d^3-3abcd^2+3b^2c^2d\right)}{b^3}-\frac{(ad-bc)^3\log(a+bx)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x**2)**3/(b*x+a)**4,x)
```

```
[Out] d**3*x**3/(3*b) - x**2*(a*d**3 - 3*b*c*d**2)/(2*b**2) + x*(a**2*d**3 - 3*a*
b*c*d**2 + 3*b**2*c**2*d)/b**3 - (a*d - b*c)**3*log(a + b*x)/b**4
```

Giac [A] time = 1.26836, size = 155, normalized size = 2.12

$$\frac{2\,b^2d^3x^3 + 9\,b^2cd^2x^2 - 3\,abd^3x^2 + 18\,b^2c^2dx - 18\,abcd^2x + 6\,a^2d^3x}{6\,b^3} + \frac{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\log\left(|bx + a|\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/6*(2*b^2*d^3*x^3 + 9*b^2*c*d^2*x^2 - 3*a*b*d^3*x^2 + 18*b^2*c^2*d*x - 18*
a*b*c*d^2*x + 6*a^2*d^3*x)/b^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*log(abs(b*x + a))/b^4
```

3.1791 $\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^3}{(a+bx)^5}\,dx$

Optimal. Leaf size=75

$$\frac{d^2x(3bc-2ad)}{b^3}-\frac{(bc-ad)^3}{b^4(a+bx)}+\frac{3d(bc-ad)^2\log(a+bx)}{b^4}+\frac{d^3x^2}{2b^2}$$

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^2)/(2*b^2) - (b*c - a*d)^3/(b^4*(a + b
*x)) + (3*d*(b*c - a*d)^2*Log[a + b*x])/b^4
```

Rubi [A] time = 0.0670233, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$\frac{d^2x(3bc-2ad)}{b^3}-\frac{(bc-ad)^3}{b^4(a+bx)}+\frac{3d(bc-ad)^2\log(a+bx)}{b^4}+\frac{d^3x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^5,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^2)/(2*b^2) - (b*c - a*d)^3/(b^4*(a + b
*x)) + (3*d*(b*c - a*d)^2*Log[a + b*x])/b^4
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^3}{(a+bx)^5}\,dx &= \int \frac{(c+dx)^3}{(a+bx)^2}\,dx\\ &= \int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{d^3x}{b^2}+\frac{(bc-ad)^3}{b^3(a+bx)^2}+\frac{3d(bc-ad)^2}{b^3(a+bx)}\right)dx\\ &= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^2}{2b^2}-\frac{(bc-ad)^3}{b^4(a+bx)}+\frac{3d(bc-ad)^2\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.0494445, size = 72, normalized size = 0.96

$$\frac{2bd^2x(3bc-2ad)-\frac{2(bc-ad)^3}{a+bx}+6d(bc-ad)^2\log(a+bx)+b^2d^3x^2}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^5,x]
```

```
[Out] (2*b*d^2*(3*b*c - 2*a*d)*x + b^2*d^3*x^2 - (2*(b*c - a*d)^3)/(a + b*x) + 6*
d*(b*c - a*d)^2*Log[a + b*x])/(2*b^4)
```

Maple [B] time = 0.046, size = 149, normalized size = 2.

$$\frac{d^3x^2}{2\,b^2}-2\,\frac{ad^3x}{b^3}+3\,\frac{cd^2x}{b^2}+3\,\frac{d^3\ln\left(bx+a\right)a^2}{b^4}-6\,\frac{d^2\ln\left(bx+a\right)ca}{b^3}+3\,\frac{d\ln\left(bx+a\right)c^2}{b^2}+\frac{a^3d^3}{b^4\left(bx+a\right)}-3\,\frac{a^2cd^2}{b^3\left(bx+a\right)}+3\,\frac{}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^5,x)
```

```
[Out] 1/2*d^3*x^2/b^2-2*d^3/b^3*a*x+3*d^2/b^2*x*c+3/b^4*d^3*ln(b*x+a)*a^2-6/b^3*d
^2*ln(b*x+a)*c*a+3/b^2*d*ln(b*x+a)*c^2+1/b^4/(b*x+a)*a^3*d^3-3/b^3/(b*x+a)*
c*a^2*d^2+3/b^2/(b*x+a)*a*c^2*d-1/b/(b*x+a)*c^3
```

Maxima [A] time = 1.03238, size = 159, normalized size = 2.12

$$-\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{b^5x+ab^4}+\frac{bd^3x^2+2\left(3\,bcd^2-2\,ad^3\right)x}{2\,b^3}+\frac{3\left(b^2c^2d-2\,abcd^2+a^2d^3\right)\log\left(bx+a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(b^5*x + a*b^4) + 1/2*
(b*d^3*x^2 + 2*(3*b*c*d^2 - 2*a*d^3)*x)/b^3 + 3*(b^2*c^2*d - 2*a*b*c*d^2 +
a^2*d^3)*log(b*x + a)/b^4
```

Fricas [B] time = 1.51793, size = 354, normalized size = 4.72

$$\frac{b^3d^3x^3-2\,b^3c^3+6\,ab^2c^2d-6\,a^2bcd^2+2\,a^3d^3+3\left(2\,b^3cd^2-ab^2d^3\right)x^2+2\left(3\,ab^2cd^2-2\,a^2bd^3\right)x+6\left(ab^2c^2d-2\,a^2bcd^2+\right.}{2\left(b^5x+ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*d^3*x^3 - 2*b^3*c^3 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + 2*a^3*d^3 +
3*(2*b^3*c*d^2 - a*b^2*d^3)*x^2 + 2*(3*a*b^2*c*d^2 - 2*a^2*b*d^3)*x + 6*(a*
b^2*c^2*d - 2*a^2*b*c*d^2 + a^3*d^3 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^
3)*x)*log(b*x + a))/(b^5*x + a*b^4)
```

Sympy [A] time = 1.14946, size = 100, normalized size = 1.33

$$\frac{a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - b^3c^3}{ab^4 + b^5x} + \frac{d^3x^2}{2b^2} - \frac{x\left(2ad^3 - 3bcd^2\right)}{b^3} + \frac{3d\left(ad - bc\right)^2\log\left(a + bx\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x**2)**3/(b*x+a)**5,x)
```

```
[Out] (a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3)/(a*b**4 + b**5*
x) + d**3*x**2/(2*b**2) - x*(2*a*d**3 - 3*b*c*d**2)/b**3 + 3*d*(a*d - b*c)*
*2*log(a + b*x)/b**4
```

Giac [B] time = 1.23487, size = 225, normalized size = 3.

$$\frac{\left(d^3 + \frac{6\left(b^2cd^2 - abd^3\right)}{(bx+a)b}\right)(bx+a)^2}{2\,b^4} - \frac{3\left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^4} - \frac{\frac{b^5c^3}{bx+a} - \frac{3\,ab^4c^2d}{bx+a} + \frac{3\,a^2b^3cd^2}{bx+a} - \frac{a^3b^2d^3}{bx+a}}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/2*(d^3 + 6*(b^2*c*d^2 - a*b*d^3)/((b*x + a)*b))*(b*x + a)^2/b^4 - 3*(b^2*
c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^4 -
 (b^5*c^3/(b*x + a) - 3*a*b^4*c^2*d/(b*x + a) + 3*a^2*b^3*c*d^2/(b*x + a) -
 a^3*b^2*d^3/(b*x + a))/b^6
```

3.1792 $\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^3}{(a+bx)^6} \, dx$

Optimal. Leaf size=78

$$\frac{3d^2(bc-ad)\log(a+bx)}{b^4}-\frac{3d(bc-ad)^2}{b^4(a+bx)}-\frac{(bc-ad)^3}{2b^4(a+bx)^2}+\frac{d^3x}{b^3}$$

```
[Out] (d^3*x)/b^3 - (b*c - a*d)^3/(2*b^4*(a + b*x)^2) - (3*d*(b*c - a*d)^2)/(b^4*
(a + b*x)) + (3*d^2*(b*c - a*d)*Log[a + b*x])/b^4
```

Rubi [A] time = 0.0606103, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$\frac{3d^2(bc-ad)\log(a+bx)}{b^4}-\frac{3d(bc-ad)^2}{b^4(a+bx)}-\frac{(bc-ad)^3}{2b^4(a+bx)^2}+\frac{d^3x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^6,x]
```

```
[Out] (d^3*x)/b^3 - (b*c - a*d)^3/(2*b^4*(a + b*x)^2) - (3*d*(b*c - a*d)^2)/(b^4*
(a + b*x)) + (3*d^2*(b*c - a*d)*Log[a + b*x])/b^4
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^3}{(a+bx)^6} \, dx &= \int \frac{(c+dx)^3}{(a+bx)^3} \, dx \\
&= \int \left(\frac{d^3}{b^3}+\frac{(bc-ad)^3}{b^3(a+bx)^3}+\frac{3d(bc-ad)^2}{b^3(a+bx)^2}+\frac{3d^2(bc-ad)}{b^3(a+bx)}\right) dx \\
&= \frac{d^3x}{b^3}-\frac{(bc-ad)^3}{2b^4(a+bx)^2}-\frac{3d(bc-ad)^2}{b^4(a+bx)}+\frac{3d^2(bc-ad)\log(a+bx)}{b^4}
\end{aligned}$$

Mathematica [A] time = 0.0403758, size = 114, normalized size = 1.46

$$\frac{a^2bd^2(9c-4dx)-5a^3d^3+ab^2d\left(-3c^2+12cdx+4d^2x^2\right)-6d^2(a+bx)^2(ad-bc)\log(a+bx)+b^3\left(-\left(6c^2dx+c^3-2d^3x^3\right)\right)}{2b^4(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^6,x]
```

```
[Out] (-5*a^3*d^3 + a^2*b*d^2*(9*c - 4*d*x) + a*b^2*d*(-3*c^2 + 12*c*d*x + 4*d^2*
x^2) - b^3*(c^3 + 6*c^2*d*x - 2*d^3*x^3) - 6*d^2*(-(b*c) + a*d)*(a + b*x)^2
*Log[a + b*x])/(2*b^4*(a + b*x)^2)
```

Maple [B] time = 0.048, size = 160, normalized size = 2.1

$$\frac{d^3x}{b^3}+\frac{a^3d^3}{2\,b^4\,(bx+a)^2}-\frac{3\,a^2cd^2}{2\,b^3\,(bx+a)^2}+\frac{3\,ac^2d}{2\,b^2\,(bx+a)^2}-\frac{c^3}{2\,b\,(bx+a)^2}-3\,\frac{d^3\ln(bx+a)\,a}{b^4}+3\,\frac{d^2\ln(bx+a)\,c}{b^3}-3\,\frac{a^2}{b^4\,(b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^6,x)
```

```
[Out] d^3*x/b^3+1/2/b^4/(b*x+a)^2*a^3*d^3-3/2/b^3/(b*x+a)^2*c*a^2*d^2+3/2/b^2/(b*
x+a)^2*a*c^2*d-1/2/b/(b*x+a)^2*c^3-3/b^4*d^3*ln(b*x+a)*a+3/b^3*d^2*ln(b*x+a
)*c-3/b^4*d^3/(b*x+a)*a^2+6/b^3*d^2/(b*x+a)*c*a-3/b^2*d/(b*x+a)*c^2
```

Maxima [A] time = 1.04744, size = 169, normalized size = 2.17

$$\frac{d^3x}{b^3}-\frac{b^3c^3+3\,ab^2c^2d-9\,a^2bcd^2+5\,a^3d^3+6\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x}{2\left(b^6x^2+2\,ab^5x+a^2b^4\right)}+\frac{3\left(bcd^2-ad^3\right)\log(bx+a)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] d^3*x/b^3 - 1/2*(b^3*c^3 + 3*a*b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3 + 6*(b
^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x)/(b^6*x^2 + 2*a*b^5*x + a^2*b^4) +
3*(b*c*d^2 - a*d^3)*log(b*x + a)/b^4
```

Fricas [B] time = 1.6434, size = 375, normalized size = 4.81

$$\frac{2\,b^3d^3x^3+4\,ab^2d^3x^2-b^3c^3-3\,ab^2c^2d+9\,a^2bcd^2-5\,a^3d^3-2\left(3\,b^3c^2d-6\,ab^2cd^2+2\,a^2bd^3\right)x+6\left(a^2bcd^2-a^3d^3+(b\right.}{2\left(b^6x^2+2\,ab^5x+a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*d^3*x^3 + 4*a*b^2*d^3*x^2 - b^3*c^3 - 3*a*b^2*c^2*d + 9*a^2*b*c*
d^2 - 5*a^3*d^3 - 2*(3*b^3*c^2*d - 6*a*b^2*c*d^2 + 2*a^2*b*d^3)*x + 6*(a^2*
b*c*d^2 - a^3*d^3 + (b^3*c*d^2 - a*b^2*d^3)*x^2 + 2*(a*b^2*c*d^2 - a^2*b*d^
3)*x)*log(b*x + a))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

Sympy [A] time = 1.57843, size = 128, normalized size = 1.64

$$-\frac{5a^3d^3-9a^2bcd^2+3ab^2c^2d+b^3c^3+x\left(6a^2bd^3-12ab^2cd^2+6b^3c^2d\right)}{2a^2b^4+4ab^5x+2b^6x^2}+\frac{d^3x}{b^3}-\frac{3d^2\left(ad-bc\right)\log\left(a+bx\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x**2)**3/(b*x+a)**6,x)
```

```
[Out] -(5*a**3*d**3 - 9*a**2*b*c*d**2 + 3*a*b**2*c**2*d + b**3*c**3 + x*(6*a**2*b
*d**3 - 12*a*b**2*c*d**2 + 6*b**3*c**2*d))/(2*a**2*b**4 + 4*a*b**5*x + 2*b*
*6*x**2) + d**3*x/b**3 - 3*d**2*(a*d - b*c)*log(a + b*x)/b**4
```

Giac [A] time = 1.20084, size = 151, normalized size = 1.94

$$\frac{d^3x}{b^3}+\frac{3\left(bcd^2-ad^3\right)\log\left(|bx+a|\right)}{b^4}-\frac{b^3c^3+3\,ab^2c^2d-9\,a^2bcd^2+5\,a^3d^3+6\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x}{2\left(bx+a\right)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] d^3*x/b^3 + 3*(b*c*d^2 - a*d^3)*log(abs(b*x + a))/b^4 - 1/2*(b^3*c^3 + 3*a*
b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3 + 6*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*
b*d^3)*x)/((b*x + a)^2*b^4)
```

3.1793 $\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^3}{(a+bx)^7} dx$

Optimal. Leaf size=86

$$-\frac{3d^2(bc-ad)}{b^4(a+bx)}-\frac{3d(bc-ad)^2}{2b^4(a+bx)^2}-\frac{(bc-ad)^3}{3b^4(a+bx)^3}+\frac{d^3\log(a+bx)}{b^4}$$

```
[Out] -(b*c - a*d)^3/(3*b^4*(a + b*x)^3) - (3*d*(b*c - a*d)^2)/(2*b^4*(a + b*x)^2
) - (3*d^2*(b*c - a*d))/(b^4*(a + b*x)) + (d^3*Log[a + b*x])/b^4
```

Rubi [A] time = 0.0600761, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$-\frac{3d^2(bc-ad)}{b^4(a+bx)}-\frac{3d(bc-ad)^2}{2b^4(a+bx)^2}-\frac{(bc-ad)^3}{3b^4(a+bx)^3}+\frac{d^3\log(a+bx)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^7,x]
```

```
[Out] -(b*c - a*d)^3/(3*b^4*(a + b*x)^3) - (3*d*(b*c - a*d)^2)/(2*b^4*(a + b*x)^2
) - (3*d^2*(b*c - a*d))/(b^4*(a + b*x)) + (d^3*Log[a + b*x])/b^4
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^3}{(a+bx)^7}\,dx &= \int \frac{(c+dx)^3}{(a+bx)^4}\,dx\\
&= \int \left(\frac{(bc-ad)^3}{b^3(a+bx)^4}+\frac{3d(bc-ad)^2}{b^3(a+bx)^3}+\frac{3d^2(bc-ad)}{b^3(a+bx)^2}+\frac{d^3}{b^3(a+bx)}\right)dx\\
&= -\frac{(bc-ad)^3}{3b^4(a+bx)^3}-\frac{3d(bc-ad)^2}{2b^4(a+bx)^2}-\frac{3d^2(bc-ad)}{b^4(a+bx)}+\frac{d^3\log(a+bx)}{b^4}
\end{aligned}$$

Mathematica [A] time = 0.0410163, size = 80, normalized size = 0.93

$$\frac{6d^3\log(a+bx)-\frac{(bc-ad)\left(11a^2d^2+abd(5c+27dx)+b^2\left(2c^2+9cdx+18d^2x^2\right)\right)}{(a+bx)^3}}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^7,x]
```

```
[Out] (-(((b*c - a*d)*(11*a^2*d^2 + a*b*d*(5*c + 27*d*x) + b^2*(2*c^2 + 9*c*d*x +
 18*d^2*x^2)))/(a + b*x)^3) + 6*d^3*Log[a + b*x])/(6*b^4)
```

Maple [B] time = 0.045, size = 166, normalized size = 1.9

$$-\frac{3\,a^2d^3}{2\,b^4\,(bx+a)^2}+3\,\frac{acd^2}{b^3\,(bx+a)^2}-\frac{3\,c^2d}{2\,b^2\,(bx+a)^2}+\frac{a^3d^3}{3\,b^4\,(bx+a)^3}-\frac{a^2cd^2}{b^3\,(bx+a)^3}+\frac{ac^2d}{b^2\,(bx+a)^3}-\frac{c^3}{3\,b\,(bx+a)^3}+\frac{d^3\ln(bx}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^7,x)
```

```
[Out] -3/2*d^3/b^4/(b*x+a)^2*a^2+3*d^2/b^3/(b*x+a)^2*c*a-3/2*d/b^2/(b*x+a)^2*c^2+
1/3/b^4/(b*x+a)^3*a^3*d^3-1/b^3/(b*x+a)^3*c*a^2*d^2+1/b^2/(b*x+a)^3*a*c^2*d
-1/3/b/(b*x+a)^3*c^3+d^3*ln(b*x+a)/b^4+3/b^4*d^3/(b*x+a)*a-3/b^3*d^2/(b*x+a
)*c
```

Maxima [A] time = 1.14136, size = 192, normalized size = 2.23

$$-\frac{2\,b^3c^3+3\,ab^2c^2d+6\,a^2bcd^2-11\,a^3d^3+18\left(b^3cd^2-ab^2d^3\right)x^2+9\left(b^3c^2d+2\,ab^2cd^2-3\,a^2bd^3\right)x}{6\left(b^7x^3+3\,ab^6x^2+3\,a^2b^5x+a^3b^4\right)}+\frac{d^3\log(bx+a)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/6*(2*b^3*c^3 + 3*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 11*a^3*d^3 + 18*(b^3*c*d^
2 - a*b^2*d^3)*x^2 + 9*(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*x)/(b^7*x^
3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4) + d^3*log(b*x + a)/b^4
```

Fricas [B] time = 1.55612, size = 360, normalized size = 4.19

$$-\frac{2\,b^3c^3+3\,ab^2c^2d+6\,a^2bcd^2-11\,a^3d^3+18\left(b^3cd^2-ab^2d^3\right)x^2+9\left(b^3c^2d+2\,ab^2cd^2-3\,a^2bd^3\right)x-6\left(b^3d^3x^3+3\,ab^2d^3x^2\right.}{6\left(b^7x^3+3\,ab^6x^2+3\,a^2b^5x+a^3b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b^3*c^3 + 3*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 11*a^3*d^3 + 18*(b^3*c*d^
2 - a*b^2*d^3)*x^2 + 9*(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*x - 6*(b^3
*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))/(b^7*x^
3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4)
```

Sympy [A] time = 2.22226, size = 148, normalized size = 1.72

$$\frac{11a^3d^3 - 6a^2bcd^2 - 3ab^2c^2d - 2b^3c^3 + x^2\left(18ab^2d^3 - 18b^3cd^2\right) + x\left(27a^2bd^3 - 18ab^2cd^2 - 9b^3c^2d\right)}{6a^3b^4 + 18a^2b^5x + 18ab^6x^2 + 6b^7x^3} + \frac{d^3\log\left(a + bx\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x**2)**3/(b*x+a)**7,x)
```

```
[Out] (11*a**3*d**3 - 6*a**2*b*c*d**2 - 3*a*b**2*c**2*d - 2*b**3*c**3 + x**2*(18*
a*b**2*d**3 - 18*b**3*c*d**2) + x*(27*a**2*b*d**3 - 18*a*b**2*c*d**2 - 9*b*
*3*c**2*d))/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) +
 d**3*log(a + b*x)/b**4
```

Giac [A] time = 1.20789, size = 159, normalized size = 1.85

$$\frac{d^3\log\left(|bx + a|\right)}{b^4} - \frac{18\left(b^2cd^2 - abd^3\right)x^2 + 9\left(b^2c^2d + 2\,abcd^2 - 3\,a^2d^3\right)x + \frac{2\,b^3c^3 + 3\,ab^2c^2d + 6\,a^2bcd^2 - 11\,a^3d^3}{b}}{6\,(bx + a)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] d^3*log(abs(b*x + a))/b^4 - 1/6*(18*(b^2*c*d^2 - a*b*d^3)*x^2 + 9*(b^2*c^2*
d + 2*a*b*c*d^2 - 3*a^2*d^3)*x + (2*b^3*c^3 + 3*a*b^2*c^2*d + 6*a^2*b*c*d^2
 - 11*a^3*d^3)/b)/((b*x + a)^3*b^3)
```

3.1794 $\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^3}{(a+bx)^8}\,dx$

Optimal. Leaf size=28

$$-\frac{(c+dx)^4}{4(a+bx)^4(bc-ad)}$$

```
[Out] -(c + d*x)^4/(4*(b*c - a*d)*(a + b*x)^4)
```

Rubi [A] time = 0.0116204, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 37}

$$-\frac{(c+dx)^4}{4(a+bx)^4(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^8,x]
```

```
[Out] -(c + d*x)^4/(4*(b*c - a*d)*(a + b*x)^4)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^3}{(a+bx)^8}\,dx &= \int \frac{(c+dx)^3}{(a+bx)^5}\,dx\\ &= -\frac{(c+dx)^4}{4(bc-ad)(a+bx)^4}\end{aligned}$$

Mathematica [B] time = 0.0294395, size = 91, normalized size = 3.25

$$-\frac{a^2bd^2(c+4dx)+a^3d^3+ab^2d\left(c^2+4cdx+6d^2x^2\right)+b^3\left(4c^2dx+c^3+6cd^2x^2+4d^3x^3\right)}{4b^4(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^8,x]
```

```
[Out] -(a^3*d^3 + a^2*b*d^2*(c + 4*d*x) + a*b^2*d*(c^2 + 4*c*d*x + 6*d^2*x^2) + b
^3*(c^3 + 4*c^2*d*x + 6*c*d^2*x^2 + 4*d^3*x^3))/(4*b^4*(a + b*x)^4)
```

Maple [B] time = 0.045, size = 122, normalized size = 4.4

$$\frac{3\,d^2\,(ad-bc)}{2\,b^4\,(bx+a)^2}-\frac{-a^3d^3+3\,cba^2d^2-3\,ac^2db^2+c^3b^3}{4\,b^4\,(bx+a)^4}-\frac{d\left(a^2d^2-2\,cabd+b^2c^2\right)}{b^4\,(bx+a)^3}-\frac{d^3}{b^4\,(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^8,x)
```

```
[Out] 3/2*d^2*(a*d-b*c)/b^4/(b*x+a)^2-1/4*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b
^3*c^3)/b^4/(b*x+a)^4-d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^4/(b*x+a)^3-d^3/b^4/(
b*x+a)
```

Maxima [B] time = 1.23457, size = 193, normalized size = 6.89

$$-\frac{4\,b^3d^3x^3+b^3c^3+ab^2c^2d+a^2bcd^2+a^3d^3+6\left(b^3cd^2+ab^2d^3\right)x^2+4\left(b^3c^2d+ab^2cd^2+a^2bd^3\right)x}{4\left(b^8x^4+4\,ab^7x^3+6\,a^2b^6x^2+4\,a^3b^5x+a^4b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^8,x, algorithm="maxima")
```

```
[Out] -1/4*(4*b^3*d^3*x^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3 + 6*(b^
3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + a*b^2*c*d^2 + a^2*b*d^3)*x)/(b^8*
x^4 + 4*a*b^7*x^3 + 6*a^2*b^6*x^2 + 4*a^3*b^5*x + a^4*b^4)
```

Fricas [B] time = 1.46081, size = 284, normalized size = 10.14

$$-\frac{4\,b^3d^3x^3+b^3c^3+ab^2c^2d+a^2bcd^2+a^3d^3+6\left(b^3cd^2+ab^2d^3\right)x^2+4\left(b^3c^2d+ab^2cd^2+a^2bd^3\right)x}{4\left(b^8x^4+4\,ab^7x^3+6\,a^2b^6x^2+4\,a^3b^5x+a^4b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^8,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^3*d^3*x^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3 + 6*(b^
3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + a*b^2*c*d^2 + a^2*b*d^3)*x)/(b^8*
x^4 + 4*a*b^7*x^3 + 6*a^2*b^6*x^2 + 4*a^3*b^5*x + a^4*b^4)
```

Sympy [B] time = 2.90295, size = 153, normalized size = 5.46

$$-\frac{a^3d^3+a^2bcd^2+ab^2c^2d+b^3c^3+4b^3d^3x^3+x^2\left(6ab^2d^3+6b^3cd^2\right)+x\left(4a^2bd^3+4ab^2cd^2+4b^3c^2d\right)}{4a^4b^4+16a^3b^5x+24a^2b^6x^2+16ab^7x^3+4b^8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x**2)**3/(b*x+a)**8,x)
```

```
[Out] -(a**3*d**3 + a**2*b*c*d**2 + a*b**2*c**2*d + b**3*c**3 + 4*b**3*d**3*x**3
+ x**2*(6*a*b**2*d**3 + 6*b**3*c*d**2) + x*(4*a**2*b*d**3 + 4*a*b**2*c*d**2
 + 4*b**3*c**2*d))/(4*a**4*b**4 + 16*a**3*b**5*x + 24*a**2*b**6*x**2 + 16*a
*b**7*x**3 + 4*b**8*x**4)
```

Giac [B] time = 1.16293, size = 150, normalized size = 5.36

$$-\frac{4\,b^3d^3x^3+6\,b^3cd^2x^2+6\,ab^2d^3x^2+4\,b^3c^2dx+4\,ab^2cd^2x+4\,a^2bd^3x+b^3c^3+ab^2c^2d+a^2bcd^2+a^3d^3}{4\,(bx+a)^4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^8,x, algorithm="giac")
```

```
[Out] -1/4*(4*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*a*b^2*d^3*x^2 + 4*b^3*c^2*d*x + 4
*a*b^2*c*d^2*x + 4*a^2*b*d^3*x + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2 + a^3*
d^3)/((b*x + a)^4*b^4)
```

3.1795 $\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^9}\,dx$

Optimal. Leaf size=58

$$\frac{d(c+dx)^4}{20(a+bx)^4(bc-ad)^2}-\frac{(c+dx)^4}{5(a+bx)^5(bc-ad)}$$

```
[Out] -(c + d*x)^4/(5*(b*c - a*d)*(a + b*x)^5) + (d*(c + d*x)^4)/(20*(b*c - a*d)^
2*(a + b*x)^4)
```

Rubi [A] time = 0.0189528, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {626, 45, 37}

$$\frac{d(c+dx)^4}{20(a+bx)^4(bc-ad)^2}-\frac{(c+dx)^4}{5(a+bx)^5(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^9,x]
```

```
[Out] -(c + d*x)^4/(5*(b*c - a*d)*(a + b*x)^5) + (d*(c + d*x)^4)/(20*(b*c - a*d)^
2*(a + b*x)^4)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^3}{(a+bx)^9}\,dx = \int \frac{(c+dx)^3}{(a+bx)^6}\,dx$$
$$= -\frac{(c+dx)^4}{5(bc-ad)(a+bx)^5} - \frac{d\int \frac{(c+dx)^3}{(a+bx)^5}\,dx}{5(bc-ad)}$$
$$= -\frac{(c+dx)^4}{5(bc-ad)(a+bx)^5} + \frac{d(c+dx)^4}{20(bc-ad)^2(a+bx)^4}$$

Mathematica [A] time = 0.033927, size = 97, normalized size = 1.67

$$-\frac{a^2bd^2(2c+5dx)+a^3d^3+ab^2d\left(3c^2+10cdx+10d^2x^2\right)+b^3\left(15c^2dx+4c^3+20cd^2x^2+10d^3x^3\right)}{20b^4(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^9,x]
```

```
[Out] -(a^3*d^3 + a^2*b*d^2*(2*c + 5*d*x) + a*b^2*d*(3*c^2 + 10*c*d*x + 10*d^2*x^
2) + b^3*(4*c^3 + 15*c^2*d*x + 20*c*d^2*x^2 + 10*d^3*x^3))/(20*b^4*(a + b*x
)^5)
```

Maple [B] time = 0.044, size = 121, normalized size = 2.1

$$-\frac{d^3}{2\,b^4\,(bx+a)^2}-\frac{-a^3d^3+3\,cba^2d^2-3\,ac^2db^2+c^3b^3}{5\,b^4\,(bx+a)^5}-\frac{3\,d\left(a^2d^2-2\,cabd+b^2c^2\right)}{4\,b^4\,(bx+a)^4}+\frac{d^2\,(ad-bc)}{b^4\,(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^9,x)
```

```
[Out] -1/2*d^3/b^4/(b*x+a)^2-1/5*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b
^4/(b*x+a)^5-3/4*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^4/(b*x+a)^4+d^2*(a*d-b*c)/
b^4/(b*x+a)^3
```

Maxima [B] time = 1.00864, size = 216, normalized size = 3.72

$$-\frac{10\,b^3d^3x^3+4\,b^3c^3+3\,ab^2c^2d+2\,a^2bcd^2+a^3d^3+10\left(2\,b^3cd^2+ab^2d^3\right)x^2+5\left(3\,b^3c^2d+2\,ab^2cd^2+a^2bd^3\right)x}{20\left(b^9x^5+5\,ab^8x^4+10\,a^2b^7x^3+10\,a^3b^6x^2+5\,a^4b^5x+a^5b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^9,x, algorithm="maxima")
```

```
[Out] -1/20*(10*b^3*d^3*x^3 + 4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*c*d^2 + a^3*d^3
 + 10*(2*b^3*c*d^2 + a*b^2*d^3)*x^2 + 5*(3*b^3*c^2*d + 2*a*b^2*c*d^2 + a^2*
b*d^3)*x)/(b^9*x^5 + 5*a*b^8*x^4 + 10*a^2*b^7*x^3 + 10*a^3*b^6*x^2 + 5*a^4*
b^5*x + a^5*b^4)
```

Fricas [B] time = 1.55489, size = 328, normalized size = 5.66

$$-\frac{10\,b^3d^3x^3+4\,b^3c^3+3\,ab^2c^2d+2\,a^2bcd^2+a^3d^3+10\,\left(2\,b^3cd^2+ab^2d^3\right)x^2+5\,\left(3\,b^3c^2d+2\,ab^2cd^2+a^2bd^3\right)x}{20\,\left(b^9x^5+5\,ab^8x^4+10\,a^2b^7x^3+10\,a^3b^6x^2+5\,a^4b^5x+a^5b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^9,x, algorithm="fricas")
```

```
[Out] -1/20*(10*b^3*d^3*x^3 + 4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*c*d^2 + a^3*d^3
 + 10*(2*b^3*c*d^2 + a*b^2*d^3)*x^2 + 5*(3*b^3*c^2*d + 2*a*b^2*c*d^2 + a^2*
b*d^3)*x)/(b^9*x^5 + 5*a*b^8*x^4 + 10*a^2*b^7*x^3 + 10*a^3*b^6*x^2 + 5*a^4*
b^5*x + a^5*b^4)
```

Sympy [B] time = 5.88685, size = 170, normalized size = 2.93

$$-\frac{a^3d^3+2a^2bcd^2+3ab^2c^2d+4b^3c^3+10b^3d^3x^3+x^2\left(10ab^2d^3+20b^3cd^2\right)+x\left(5a^2bd^3+10ab^2cd^2+15b^3c^2d\right)}{20a^5b^4+100a^4b^5x+200a^3b^6x^2+200a^2b^7x^3+100ab^8x^4+20b^9x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x**2)**3/(b*x+a)**9,x)
```

```
[Out] -(a**3*d**3 + 2*a**2*b*c*d**2 + 3*a*b**2*c**2*d + 4*b**3*c**3 + 10*b**3*d**
3*x**3 + x**2*(10*a*b**2*d**3 + 20*b**3*c*d**2) + x*(5*a**2*b*d**3 + 10*a*b
**2*c*d**2 + 15*b**3*c**2*d))/(20*a**5*b**4 + 100*a**4*b**5*x + 200*a**3*b*
*6*x**2 + 200*a**2*b**7*x**3 + 100*a*b**8*x**4 + 20*b**9*x**5)
```

Giac [B] time = 1.2378, size = 154, normalized size = 2.66

$$-\frac{10\,b^3d^3x^3+20\,b^3cd^2x^2+10\,ab^2d^3x^2+15\,b^3c^2dx+10\,ab^2cd^2x+5\,a^2bd^3x+4\,b^3c^3+3\,ab^2c^2d+2\,a^2bcd^2+a^3d^3}{20\,(bx+a)^5b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^9,x, algorithm="giac")
```

```
[Out] -1/20*(10*b^3*d^3*x^3 + 20*b^3*c*d^2*x^2 + 10*a*b^2*d^3*x^2 + 15*b^3*c^2*d*
x + 10*a*b^2*c*d^2*x + 5*a^2*b*d^3*x + 4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*
c*d^2 + a^3*d^3)/((b*x + a)^5*b^4)
```

3.1796 $\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^3}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=92

$$-\frac{3d^2(bc-ad)}{4b^4(a+bx)^4}-\frac{3d(bc-ad)^2}{5b^4(a+bx)^5}-\frac{(bc-ad)^3}{6b^4(a+bx)^6}-\frac{d^3}{3b^4(a+bx)^3}$$

```
[Out] -(b*c - a*d)^3/(6*b^4*(a + b*x)^6) - (3*d*(b*c - a*d)^2)/(5*b^4*(a + b*x)^5
) - (3*d^2*(b*c - a*d))/(4*b^4*(a + b*x)^4) - d^3/(3*b^4*(a + b*x)^3)
```

Rubi [A] time = 0.0611387, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$-\frac{3d^2(bc-ad)}{4b^4(a+bx)^4}-\frac{3d(bc-ad)^2}{5b^4(a+bx)^5}-\frac{(bc-ad)^3}{6b^4(a+bx)^6}-\frac{d^3}{3b^4(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^10,x]
```

```
[Out] -(b*c - a*d)^3/(6*b^4*(a + b*x)^6) - (3*d*(b*c - a*d)^2)/(5*b^4*(a + b*x)^5
) - (3*d^2*(b*c - a*d))/(4*b^4*(a + b*x)^4) - d^3/(3*b^4*(a + b*x)^3)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^3}{(a+bx)^{10}}\,dx &= \int \frac{(c+dx)^3}{(a+bx)^7}\,dx\\ &= \int \left(\frac{(bc-ad)^3}{b^3(a+bx)^7}+\frac{3d(bc-ad)^2}{b^3(a+bx)^6}+\frac{3d^2(bc-ad)}{b^3(a+bx)^5}+\frac{d^3}{b^3(a+bx)^4}\right)dx\\ &= -\frac{(bc-ad)^3}{6b^4(a+bx)^6}-\frac{3d(bc-ad)^2}{5b^4(a+bx)^5}-\frac{3d^2(bc-ad)}{4b^4(a+bx)^4}-\frac{d^3}{3b^4(a+bx)^3}\end{aligned}$$

Mathematica [A] time = 0.0299068, size = 97, normalized size = 1.05

$$-\frac{3a^2bd^2(c+2dx)+a^3d^3+3ab^2d\left(2c^2+6cdx+5d^2x^2\right)+b^3\left(36c^2dx+10c^3+45cd^2x^2+20d^3x^3\right)}{60b^4(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^10,x]
```

```
[Out] -(a^3*d^3 + 3*a^2*b*d^2*(c + 2*d*x) + 3*a*b^2*d*(2*c^2 + 6*c*d*x + 5*d^2*x^
2) + b^3*(10*c^3 + 36*c^2*d*x + 45*c*d^2*x^2 + 20*d^3*x^3))/(60*b^4*(a + b*
x)^6)
```

Maple [A] time = 0.044, size = 122, normalized size = 1.3

$$-\frac{3\,d\left(a^2d^2-2\,cabd+b^2c^2\right)}{5\,b^4\,(bx+a)^5}+\frac{3\,d^2\,(ad-bc)}{4\,b^4\,(bx+a)^4}-\frac{d^3}{3\,b^4\,(bx+a)^3}-\frac{-a^3d^3+3\,cba^2d^2-3\,ac^2db^2+c^3b^3}{6\,b^4\,(bx+a)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^10,x)
```

```
[Out] -3/5*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^4/(b*x+a)^5+3/4*d^2*(a*d-b*c)/b^4/(b*x
+a)^4-1/3*d^3/b^4/(b*x+a)^3-1/6*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c
^3)/b^4/(b*x+a)^6
```

Maxima [B] time = 1.13016, size = 231, normalized size = 2.51

$$-\frac{20\,b^3d^3x^3+10\,b^3c^3+6\,ab^2c^2d+3\,a^2bcd^2+a^3d^3+15\left(3\,b^3cd^2+ab^2d^3\right)x^2+6\left(6\,b^3c^2d+3\,ab^2cd^2+a^2bd^3\right)x}{60\left(b^{10}x^6+6\,ab^9x^5+15\,a^2b^8x^4+20\,a^3b^7x^3+15\,a^4b^6x^2+6\,a^5b^5x+a^6b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/60*(20*b^3*d^3*x^3 + 10*b^3*c^3 + 6*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^
3 + 15*(3*b^3*c*d^2 + a*b^2*d^3)*x^2 + 6*(6*b^3*c^2*d + 3*a*b^2*c*d^2 + a^2
*b*d^3)*x)/(b^10*x^6 + 6*a*b^9*x^5 + 15*a^2*b^8*x^4 + 20*a^3*b^7*x^3 + 15*a
^4*b^6*x^2 + 6*a^5*b^5*x + a^6*b^4)
```

Fricas [B] time = 1.52496, size = 354, normalized size = 3.85

$$-\frac{20\,b^3d^3x^3+10\,b^3c^3+6\,ab^2c^2d+3\,a^2bcd^2+a^3d^3+15\left(3\,b^3cd^2+ab^2d^3\right)x^2+6\left(6\,b^3c^2d+3\,ab^2cd^2+a^2bd^3\right)x}{60\left(b^{10}x^6+6\,ab^9x^5+15\,a^2b^8x^4+20\,a^3b^7x^3+15\,a^4b^6x^2+6\,a^5b^5x+a^6b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/60*(20*b^3*d^3*x^3 + 10*b^3*c^3 + 6*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^
3 + 15*(3*b^3*c*d^2 + a*b^2*d^3)*x^2 + 6*(6*b^3*c^2*d + 3*a*b^2*c*d^2 + a^2
*b*d^3)*x)/(b^10*x^6 + 6*a*b^9*x^5 + 15*a^2*b^8*x^4 + 20*a^3*b^7*x^3 + 15*a
^4*b^6*x^2 + 6*a^5*b^5*x + a^6*b^4)
```

Sympy [B] time = 8.09449, size = 182, normalized size = 1.98

$$-\frac{a^3d^3 + 3a^2bcd^2 + 6ab^2c^2d + 10b^3c^3 + 20b^3d^3x^3 + x^2\left(15ab^2d^3 + 45b^3cd^2\right) + x\left(6a^2bd^3 + 18ab^2cd^2 + 36b^3c^2d\right)}{60a^6b^4 + 360a^5b^5x + 900a^4b^6x^2 + 1200a^3b^7x^3 + 900a^2b^8x^4 + 360ab^9x^5 + 60b^{10}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x**2)**3/(b*x+a)**10,x)
```

```
[Out] -(a**3*d**3 + 3*a**2*b*c*d**2 + 6*a*b**2*c**2*d + 10*b**3*c**3 + 20*b**3*d*
*3*x**3 + x**2*(15*a*b**2*d**3 + 45*b**3*c*d**2) + x*(6*a**2*b*d**3 + 18*a*
b**2*c*d**2 + 36*b**3*c**2*d))/(60*a**6*b**4 + 360*a**5*b**5*x + 900*a**4*b
**6*x**2 + 1200*a**3*b**7*x**3 + 900*a**2*b**8*x**4 + 360*a*b**9*x**5 + 60*
b**10*x**6)
```

Giac [A] time = 1.22086, size = 154, normalized size = 1.67

$$-\frac{20\,b^3d^3x^3 + 45\,b^3cd^2x^2 + 15\,ab^2d^3x^2 + 36\,b^3c^2dx + 18\,ab^2cd^2x + 6\,a^2bd^3x + 10\,b^3c^3 + 6\,ab^2c^2d + 3\,a^2bcd^2 + a^3d^3}{60\,(bx + a)^6b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/60*(20*b^3*d^3*x^3 + 45*b^3*c*d^2*x^2 + 15*a*b^2*d^3*x^2 + 36*b^3*c^2*d*
x + 18*a*b^2*c*d^2*x + 6*a^2*b*d^3*x + 10*b^3*c^3 + 6*a*b^2*c^2*d + 3*a^2*b
*c*d^2 + a^3*d^3)/((b*x + a)^6*b^4)
```

3.1797 $\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^3}{(a+bx)^{11}}\,dx$

Optimal. Leaf size=92

$$-\frac{3d^2(bc-ad)}{5b^4(a+bx)^5}-\frac{d(bc-ad)^2}{2b^4(a+bx)^6}-\frac{(bc-ad)^3}{7b^4(a+bx)^7}-\frac{d^3}{4b^4(a+bx)^4}$$

```
[Out] -(b*c - a*d)^3/(7*b^4*(a + b*x)^7) - (d*(b*c - a*d)^2)/(2*b^4*(a + b*x)^6)
- (3*d^2*(b*c - a*d))/(5*b^4*(a + b*x)^5) - d^3/(4*b^4*(a + b*x)^4)
```

Rubi [A] time = 0.0576906, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$-\frac{3d^2(bc-ad)}{5b^4(a+bx)^5}-\frac{d(bc-ad)^2}{2b^4(a+bx)^6}-\frac{(bc-ad)^3}{7b^4(a+bx)^7}-\frac{d^3}{4b^4(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^11,x]
```

```
[Out] -(b*c - a*d)^3/(7*b^4*(a + b*x)^7) - (d*(b*c - a*d)^2)/(2*b^4*(a + b*x)^6)
- (3*d^2*(b*c - a*d))/(5*b^4*(a + b*x)^5) - d^3/(4*b^4*(a + b*x)^4)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^3}{(a+bx)^{11}}\,dx &= \int \frac{(c+dx)^3}{(a+bx)^8}\,dx\\ &= \int \left(\frac{(bc-ad)^3}{b^3(a+bx)^8}+\frac{3d(bc-ad)^2}{b^3(a+bx)^7}+\frac{3d^2(bc-ad)}{b^3(a+bx)^6}+\frac{d^3}{b^3(a+bx)^5}\right)dx\\ &= -\frac{(bc-ad)^3}{7b^4(a+bx)^7}-\frac{d(bc-ad)^2}{2b^4(a+bx)^6}-\frac{3d^2(bc-ad)}{5b^4(a+bx)^5}-\frac{d^3}{4b^4(a+bx)^4}\end{aligned}$$

Mathematica [A] time = 0.03055, size = 97, normalized size = 1.05

$$-\frac{a^2bd^2(4c+7dx)+a^3d^3+ab^2d\left(10c^2+28cdx+21d^2x^2\right)+b^3\left(70c^2dx+20c^3+84cd^2x^2+35d^3x^3\right)}{140b^4(a+bx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^11,x]
```

```
[Out] -(a^3*d^3 + a^2*b*d^2*(4*c + 7*d*x) + a*b^2*d*(10*c^2 + 28*c*d*x + 21*d^2*x
^2) + b^3*(20*c^3 + 70*c^2*d*x + 84*c*d^2*x^2 + 35*d^3*x^3))/(140*b^4*(a +
b*x)^7)
```

Maple [A] time = 0.045, size = 122, normalized size = 1.3

$$\frac{3\,d^2\,(ad-bc)}{5\,b^4\,(bx+a)^5}-\frac{d^3}{4\,b^4\,(bx+a)^4}-\frac{-a^3d^3+3\,cba^2d^2-3\,ac^2db^2+c^3b^3}{7\,b^4\,(bx+a)^7}-\frac{d\left(a^2d^2-2\,cabd+b^2c^2\right)}{2\,b^4\,(bx+a)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^11,x)
```

```
[Out] 3/5*d^2*(a*d-b*c)/b^4/(b*x+a)^5-1/4*d^3/b^4/(b*x+a)^4-1/7*(-a^3*d^3+3*a^2*b
*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^4/(b*x+a)^7-1/2*d*(a^2*d^2-2*a*b*c*d+b^2*c^
2)/b^4/(b*x+a)^6
```

Maxima [B] time = 1.11019, size = 246, normalized size = 2.67

$$-\frac{35\,b^3d^3x^3+20\,b^3c^3+10\,ab^2c^2d+4\,a^2bcd^2+a^3d^3+21\left(4\,b^3cd^2+ab^2d^3\right)x^2+7\left(10\,b^3c^2d+4\,ab^2cd^2+a^2bd^3\right)x}{140\left(b^{11}x^7+7\,ab^{10}x^6+21\,a^2b^9x^5+35\,a^3b^8x^4+35\,a^4b^7x^3+21\,a^5b^6x^2+7\,a^6b^5x+a^7b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^11,x, algorithm="maxima")
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + 20*b^3*c^3 + 10*a*b^2*c^2*d + 4*a^2*b*c*d^2 + a^3*
d^3 + 21*(4*b^3*c*d^2 + a*b^2*d^3)*x^2 + 7*(10*b^3*c^2*d + 4*a*b^2*c*d^2 +
a^2*b*d^3)*x)/(b^11*x^7 + 7*a*b^10*x^6 + 21*a^2*b^9*x^5 + 35*a^3*b^8*x^4 +
35*a^4*b^7*x^3 + 21*a^5*b^6*x^2 + 7*a^6*b^5*x + a^7*b^4)
```

Fricas [B] time = 1.52746, size = 382, normalized size = 4.15

$$-\frac{35\,b^3d^3x^3+20\,b^3c^3+10\,ab^2c^2d+4\,a^2bcd^2+a^3d^3+21\left(4\,b^3cd^2+ab^2d^3\right)x^2+7\left(10\,b^3c^2d+4\,ab^2cd^2+a^2bd^3\right)x}{140\left(b^{11}x^7+7\,ab^{10}x^6+21\,a^2b^9x^5+35\,a^3b^8x^4+35\,a^4b^7x^3+21\,a^5b^6x^2+7\,a^6b^5x+a^7b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^11,x, algorithm="fricas")
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + 20*b^3*c^3 + 10*a*b^2*c^2*d + 4*a^2*b*c*d^2 + a^3*
d^3 + 21*(4*b^3*c*d^2 + a*b^2*d^3)*x^2 + 7*(10*b^3*c^2*d + 4*a*b^2*c*d^2 +
a^2*b*d^3)*x)/(b^11*x^7 + 7*a*b^10*x^6 + 21*a^2*b^9*x^5 + 35*a^3*b^8*x^4 +
35*a^4*b^7*x^3 + 21*a^5*b^6*x^2 + 7*a^6*b^5*x + a^7*b^4)
```

Sympy [B] time = 13.7032, size = 194, normalized size = 2.11

$$-\frac{a^3d^3 + 4a^2bcd^2 + 10ab^2c^2d + 20b^3c^3 + 35b^3d^3x^3 + x^2\left(21ab^2d^3 + 84b^3cd^2\right) + x\left(7a^2bd^3 + 28ab^2cd^2 + 70b^3c^2d\right)}{140a^7b^4 + 980a^6b^5x + 2940a^5b^6x^2 + 4900a^4b^7x^3 + 4900a^3b^8x^4 + 2940a^2b^9x^5 + 980ab^{10}x^6 + 140b^{11}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x**2)**3/(b*x+a)**11,x)
```

```
[Out] -(a**3*d**3 + 4*a**2*b*c*d**2 + 10*a*b**2*c**2*d + 20*b**3*c**3 + 35*b**3*d
**3*x**3 + x**2*(21*a*b**2*d**3 + 84*b**3*c*d**2) + x*(7*a**2*b*d**3 + 28*a
*b**2*c*d**2 + 70*b**3*c**2*d))/(140*a**7*b**4 + 980*a**6*b**5*x + 2940*a**
5*b**6*x**2 + 4900*a**4*b**7*x**3 + 4900*a**3*b**8*x**4 + 2940*a**2*b**9*x*
*5 + 980*a*b**10*x**6 + 140*b**11*x**7)
```

Giac [A] time = 1.26302, size = 154, normalized size = 1.67

$$-\frac{35\,b^3d^3x^3 + 84\,b^3cd^2x^2 + 21\,ab^2d^3x^2 + 70\,b^3c^2dx + 28\,ab^2cd^2x + 7\,a^2bd^3x + 20\,b^3c^3 + 10\,ab^2c^2d + 4\,a^2bcd^2 + a^3d^3}{140\,(bx + a)^7b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^11,x, algorithm="giac")
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + 84*b^3*c*d^2*x^2 + 21*a*b^2*d^3*x^2 + 70*b^3*c^2*d
*x + 28*a*b^2*c*d^2*x + 7*a^2*b*d^3*x + 20*b^3*c^3 + 10*a*b^2*c^2*d + 4*a^2
*b*c*d^2 + a^3*d^3)/((b*x + a)^7*b^4)
```

3.1798 $\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^3}{(a+bx)^{12}}\,dx$

Optimal. Leaf size=92

$$-\frac{d^2(bc-ad)}{2b^4(a+bx)^6}-\frac{3d(bc-ad)^2}{7b^4(a+bx)^7}-\frac{(bc-ad)^3}{8b^4(a+bx)^8}-\frac{d^3}{5b^4(a+bx)^5}$$

```
[Out] -(b*c - a*d)^3/(8*b^4*(a + b*x)^8) - (3*d*(b*c - a*d)^2)/(7*b^4*(a + b*x)^7
) - (d^2*(b*c - a*d))/(2*b^4*(a + b*x)^6) - d^3/(5*b^4*(a + b*x)^5)
```

Rubi [A] time = 0.0551401, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$-\frac{d^2(bc-ad)}{2b^4(a+bx)^6}-\frac{3d(bc-ad)^2}{7b^4(a+bx)^7}-\frac{(bc-ad)^3}{8b^4(a+bx)^8}-\frac{d^3}{5b^4(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^12,x]
```

```
[Out] -(b*c - a*d)^3/(8*b^4*(a + b*x)^8) - (3*d*(b*c - a*d)^2)/(7*b^4*(a + b*x)^7
) - (d^2*(b*c - a*d))/(2*b^4*(a + b*x)^6) - d^3/(5*b^4*(a + b*x)^5)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^3}{(a+bx)^{12}}\,dx &= \int \frac{(c+dx)^3}{(a+bx)^9}\,dx\\
&= \int \left(\frac{(bc-ad)^3}{b^3(a+bx)^9}+\frac{3d(bc-ad)^2}{b^3(a+bx)^8}+\frac{3d^2(bc-ad)}{b^3(a+bx)^7}+\frac{d^3}{b^3(a+bx)^6}\right)dx\\
&= -\frac{(bc-ad)^3}{8b^4(a+bx)^8}-\frac{3d(bc-ad)^2}{7b^4(a+bx)^7}-\frac{d^2(bc-ad)}{2b^4(a+bx)^6}-\frac{d^3}{5b^4(a+bx)^5}
\end{aligned}$$

Mathematica [A] time = 0.0354081, size = 97, normalized size = 1.05

$$-\frac{a^2bd^2(5c+8dx)+a^3d^3+ab^2d\left(15c^2+40cdx+28d^2x^2\right)+b^3\left(120c^2dx+35c^3+140cd^2x^2+56d^3x^3\right)}{280b^4(a+bx)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^12,x]
```

```
[Out] -(a^3*d^3 + a^2*b*d^2*(5*c + 8*d*x) + a*b^2*d*(15*c^2 + 40*c*d*x + 28*d^2*x
^2) + b^3*(35*c^3 + 120*c^2*d*x + 140*c*d^2*x^2 + 56*d^3*x^3))/(280*b^4*(a
+ b*x)^8)
```

Maple [A] time = 0.044, size = 122, normalized size = 1.3

$$-\frac{d^3}{5\,b^4\,(bx+a)^5}-\frac{3\,d\left(a^2d^2-2\,cabd+b^2c^2\right)}{7\,b^4\,(bx+a)^7}-\frac{-a^3d^3+3\,cba^2d^2-3\,ac^2db^2+c^3b^3}{8\,b^4\,(bx+a)^8}+\frac{d^2\,(ad-bc)}{2\,b^4\,(bx+a)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^12,x)
```

```
[Out] -1/5*d^3/b^4/(b*x+a)^5-3/7*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^4/(b*x+a)^7-1/8*
(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^4/(b*x+a)^8+1/2*d^2*(a*d-b
*c)/b^4/(b*x+a)^6
```

Maxima [B] time = 1.0898, size = 261, normalized size = 2.84

$$-\frac{56\,b^3d^3x^3+35\,b^3c^3+15\,ab^2c^2d+5\,a^2bcd^2+a^3d^3+28\left(5\,b^3cd^2+ab^2d^3\right)x^2+8\left(15\,b^3c^2d+5\,ab^2cd^2+a^2bd^3\right)x}{280\left(b^{12}x^8+8\,ab^{11}x^7+28\,a^2b^{10}x^6+56\,a^3b^9x^5+70\,a^4b^8x^4+56\,a^5b^7x^3+28\,a^6b^6x^2+8\,a^7b^5x+a^8b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^12,x, algorithm="maxima")
```

```
[Out] -1/280*(56*b^3*d^3*x^3 + 35*b^3*c^3 + 15*a*b^2*c^2*d + 5*a^2*b*c*d^2 + a^3*
d^3 + 28*(5*b^3*c*d^2 + a*b^2*d^3)*x^2 + 8*(15*b^3*c^2*d + 5*a*b^2*c*d^2 +
a^2*b*d^3)*x)/(b^12*x^8 + 8*a*b^11*x^7 + 28*a^2*b^10*x^6 + 56*a^3*b^9*x^5 +
 70*a^4*b^8*x^4 + 56*a^5*b^7*x^3 + 28*a^6*b^6*x^2 + 8*a^7*b^5*x + a^8*b^4)
```

Fricas [B] time = 1.62724, size = 406, normalized size = 4.41

$$-\frac{56\,b^3d^3x^3+35\,b^3c^3+15\,ab^2c^2d+5\,a^2bcd^2+a^3d^3+28\left(5\,b^3cd^2+ab^2d^3\right)x^2+8\left(15\,b^3c^2d+5\,ab^2cd^2+a^2bd^3\right)x}{280\left(b^{12}x^8+8\,ab^{11}x^7+28\,a^2b^{10}x^6+56\,a^3b^9x^5+70\,a^4b^8x^4+56\,a^5b^7x^3+28\,a^6b^6x^2+8\,a^7b^5x+a^8b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^12,x, algorithm="fricas")
```

```
[Out] -1/280*(56*b^3*d^3*x^3 + 35*b^3*c^3 + 15*a*b^2*c^2*d + 5*a^2*b*c*d^2 + a^3*
d^3 + 28*(5*b^3*c*d^2 + a*b^2*d^3)*x^2 + 8*(15*b^3*c^2*d + 5*a*b^2*c*d^2 +
a^2*b*d^3)*x)/(b^12*x^8 + 8*a*b^11*x^7 + 28*a^2*b^10*x^6 + 56*a^3*b^9*x^5 +
 70*a^4*b^8*x^4 + 56*a^5*b^7*x^3 + 28*a^6*b^6*x^2 + 8*a^7*b^5*x + a^8*b^4)
```

Sympy [B] time = 25.5739, size = 206, normalized size = 2.24

$$-\frac{a^3d^3 + 5a^2bcd^2 + 15ab^2c^2d + 35b^3c^3 + 56b^3d^3x^3 + x^2\left(28ab^2d^3 + 140b^3cd^2\right) + x\left(8a^2bd^3 + 40ab^2cd^2 + 120b^3c^2d\right)}{280a^8b^4 + 2240a^7b^5x + 7840a^6b^6x^2 + 15680a^5b^7x^3 + 19600a^4b^8x^4 + 15680a^3b^9x^5 + 7840a^2b^{10}x^6 + 2240ab^{11}x^7 + 280b^{12}x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x**2)**3/(b*x+a)**12,x)
```

```
[Out] -(a**3*d**3 + 5*a**2*b*c*d**2 + 15*a*b**2*c**2*d + 35*b**3*c**3 + 56*b**3*d
**3*x**3 + x**2*(28*a*b**2*d**3 + 140*b**3*c*d**2) + x*(8*a**2*b*d**3 + 40*
a*b**2*c*d**2 + 120*b**3*c**2*d))/(280*a**8*b**4 + 2240*a**7*b**5*x + 7840*
a**6*b**6*x**2 + 15680*a**5*b**7*x**3 + 19600*a**4*b**8*x**4 + 15680*a**3*b
**9*x**5 + 7840*a**2*b**10*x**6 + 2240*a*b**11*x**7 + 280*b**12*x**8)
```

Giac [A] time = 1.29392, size = 154, normalized size = 1.67

$$-\frac{56\,b^3d^3x^3 + 140\,b^3cd^2x^2 + 28\,ab^2d^3x^2 + 120\,b^3c^2dx + 40\,ab^2cd^2x + 8\,a^2bd^3x + 35\,b^3c^3 + 15\,ab^2c^2d + 5\,a^2bcd^2 + a^3d^3}{280\,(bx+a)^8b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^12,x, algorithm="giac")
```

```
[Out] -1/280*(56*b^3*d^3*x^3 + 140*b^3*c*d^2*x^2 + 28*a*b^2*d^3*x^2 + 120*b^3*c^2
*d*x + 40*a*b^2*c*d^2*x + 8*a^2*b*d^3*x + 35*b^3*c^3 + 15*a*b^2*c^2*d + 5*a
^2*b*c*d^2 + a^3*d^3)/((b*x + a)^8*b^4)
```

3.1799 $\int \frac{(a+bx)^6}{ac+(bc+ad)x+bdx^2}\,dx$

Optimal. Leaf size=122

$$\frac{bx(bc-ad)^4}{d^5}-\frac{(a+bx)^2(bc-ad)^3}{2d^4}+\frac{(a+bx)^3(bc-ad)^2}{3d^3}-\frac{(a+bx)^4(bc-ad)}{4d^2}-\frac{(bc-ad)^5\log(c+dx)}{d^6}+\frac{(a+bx)^5}{5d}$$

```
[Out] (b*(b*c - a*d)^4*x)/d^5 - ((b*c - a*d)^3*(a + b*x)^2)/(2*d^4) + ((b*c - a*d
)^2*(a + b*x)^3)/(3*d^3) - ((b*c - a*d)*(a + b*x)^4)/(4*d^2) + (a + b*x)^5/
(5*d) - ((b*c - a*d)^5*Log[c + d*x])/d^6
```

Rubi [A] time = 0.0626873, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$\frac{bx(bc-ad)^4}{d^5}-\frac{(a+bx)^2(bc-ad)^3}{2d^4}+\frac{(a+bx)^3(bc-ad)^2}{3d^3}-\frac{(a+bx)^4(bc-ad)}{4d^2}-\frac{(bc-ad)^5\log(c+dx)}{d^6}+\frac{(a+bx)^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^6/(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

```
[Out] (b*(b*c - a*d)^4*x)/d^5 - ((b*c - a*d)^3*(a + b*x)^2)/(2*d^4) + ((b*c - a*d
)^2*(a + b*x)^3)/(3*d^3) - ((b*c - a*d)*(a + b*x)^4)/(4*d^2) + (a + b*x)^5/
(5*d) - ((b*c - a*d)^5*Log[c + d*x])/d^6
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^6}{ac+(bc+ad)x+bdx^2}\,dx &= \int \frac{(a+bx)^5}{c+dx}\,dx\\ &= \int \left(\frac{b(bc-ad)^4}{d^5}-\frac{b(bc-ad)^3(a+bx)}{d^4}+\frac{b(bc-ad)^2(a+bx)^2}{d^3}-\frac{b(bc-ad)(a+bx)^3}{d^2}+\right.\\ &= \frac{b(bc-ad)^4x}{d^5}-\frac{(bc-ad)^3(a+bx)^2}{2d^4}+\frac{(bc-ad)^2(a+bx)^3}{3d^3}-\frac{(bc-ad)(a+bx)^4}{4d^2}+\frac{(a+}{5}\end{aligned}$$

Mathematica [A] time = 0.0641482, size = 167, normalized size = 1.37

$$\frac{bdx\left(100a^2b^2d^2\left(6c^2-3cdx+2d^2x^2\right)+300a^3bd^3(dx-2c)+300a^4d^4+25ab^3d\left(6c^2dx-12c^3-4cd^2x^2+3d^3x^3\right)+b^4\right.}{60d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^6/(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

```
[Out] (b*d*x*(300*a^4*d^4 + 300*a^3*b*d^3*(-2*c + d*x) + 100*a^2*b^2*d^2*(6*c^2 -
 3*c*d*x + 2*d^2*x^2) + 25*a*b^3*d*(-12*c^3 + 6*c^2*d*x - 4*c*d^2*x^2 + 3*d
^3*x^3) + b^4*(60*c^4 - 30*c^3*d*x + 20*c^2*d^2*x^2 - 15*c*d^3*x^3 + 12*d^4
*x^4)) - 60*(b*c - a*d)^5*Log[c + d*x])/(60*d^6)
```

Maple [B] time = 0.043, size = 302, normalized size = 2.5

$$\frac{b^5x^5}{5d}+\frac{5ab^4x^4}{4d}-\frac{b^5x^4c}{4d^2}+\frac{10a^2b^3x^3}{3d}-\frac{5b^4x^3ac}{3d^2}+\frac{b^5x^3c^2}{3d^3}+5\frac{a^3b^2x^2}{d}-5\frac{b^3x^2a^2c}{d^2}+\frac{5b^4x^2ac^2}{2d^3}-\frac{b^5x^2c^3}{2d^4}+5\frac{ba^4x}{d}-10$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^6/(a*c+(a*d+b*c)*x+b*d*x^2),x)
```

```
[Out] 1/5*b^5/d*x^5+5/4*b^4/d*x^4*a-1/4*b^5/d^2*x^4*c+10/3*b^3/d*x^3*a^2-5/3*b^4/
d^2*x^3*a*c+1/3*b^5/d^3*x^3*c^2+5*b^2/d*x^2*a^3-5*b^3/d^2*x^2*a^2*c+5/2*b^4
/d^3*x^2*a*c^2-1/2*b^5/d^4*x^2*c^3+5*b/d*a^4*x-10*b^2/d^2*a^3*c*x+10*b^3/d^
3*a^2*c^2*x-5*b^4/d^4*a*c^3*x+b^5/d^5*c^4*x+1/d*ln(d*x+c)*a^5-5/d^2*ln(d*x+
c)*a^4*b*c+10/d^3*ln(d*x+c)*a^3*b^2*c^2-10/d^4*ln(d*x+c)*a^2*b^3*c^3+5/d^5*
ln(d*x+c)*a*b^4*c^4-1/d^6*ln(d*x+c)*b^5*c^5
```

Maxima [B] time = 1.04921, size = 348, normalized size = 2.85

$$\frac{12\,b^5d^4x^5-15\left(b^5cd^3-5\,ab^4d^4\right)x^4+20\left(b^5c^2d^2-5\,ab^4cd^3+10\,a^2b^3d^4\right)x^3-30\left(b^5c^3d-5\,ab^4c^2d^2+10\,a^2b^3cd^3-10\,a^3b^2\right.}{60\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="maxima")
```

```
[Out] 1/60*(12*b^5*d^4*x^5 - 15*(b^5*c*d^3 - 5*a*b^4*d^4)*x^4 + 20*(b^5*c^2*d^2 -
 5*a*b^4*c*d^3 + 10*a^2*b^3*d^4)*x^3 - 30*(b^5*c^3*d - 5*a*b^4*c^2*d^2 + 10
*a^2*b^3*c*d^3 - 10*a^3*b^2*d^4)*x^2 + 60*(b^5*c^4 - 5*a*b^4*c^3*d + 10*a^2
*b^3*c^2*d^2 - 10*a^3*b^2*c*d^3 + 5*a^4*b*d^4)*x)/d^5 - (b^5*c^5 - 5*a*b^4*
c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*
log(d*x + c)/d^6
```

Fricas [B] time = 1.61941, size = 537, normalized size = 4.4

$$\frac{12\,b^5d^5x^5-15\left(b^5cd^4-5\,ab^4d^5\right)x^4+20\left(b^5c^2d^3-5\,ab^4cd^4+10\,a^2b^3d^5\right)x^3-30\left(b^5c^3d^2-5\,ab^4c^2d^3+10\,a^2b^3cd^4-10\,a^3b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="fricas")
```

```
[Out] 1/60*(12*b^5*d^5*x^5 - 15*(b^5*c*d^4 - 5*a*b^4*d^5)*x^4 + 20*(b^5*c^2*d^3 -
 5*a*b^4*c*d^4 + 10*a^2*b^3*d^5)*x^3 - 30*(b^5*c^3*d^2 - 5*a*b^4*c^2*d^3 +
```

```
10*a^2*b^3*c*d^4 - 10*a^3*b^2*d^5)*x^2 + 60*(b^5*c^4*d - 5*a*b^4*c^3*d^2 +
10*a^2*b^3*c^2*d^3 - 10*a^3*b^2*c*d^4 + 5*a^4*b*d^5)*x - 60*(b^5*c^5 - 5*a*
b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d
^5)*log(d*x + c))/d^6
```

Sympy [A] time = 1.05147, size = 202, normalized size = 1.66

$$\frac{b^5x^5}{5d} + \frac{x^4\left(5ab^4d - b^5c\right)}{4d^2} + \frac{x^3\left(10a^2b^3d^2 - 5ab^4cd + b^5c^2\right)}{3d^3} + \frac{x^2\left(10a^3b^2d^3 - 10a^2b^3cd^2 + 5ab^4c^2d - b^5c^3\right)}{2d^4} + \frac{x\left(5a^4bd^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**6/(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

```
[Out] b**5*x**5/(5*d) + x**4*(5*a*b**4*d - b**5*c)/(4*d**2) + x**3*(10*a**2*b**3*
d**2 - 5*a*b**4*c*d + b**5*c**2)/(3*d**3) + x**2*(10*a**3*b**2*d**3 - 10*a*
*2*b**3*c*d**2 + 5*a*b**4*c**2*d - b**5*c**3)/(2*d**4) + x*(5*a**4*b*d**4 -
 10*a**3*b**2*c*d**3 + 10*a**2*b**3*c**2*d**2 - 5*a*b**4*c**3*d + b**5*c**4
)/d**5 + (a*d - b*c)**5*log(c + d*x)/d**6
```

Giac [B] time = 1.25377, size = 369, normalized size = 3.02

$$\frac{12\,b^5d^4x^5 - 15\,b^5cd^3x^4 + 75\,ab^4d^4x^4 + 20\,b^5c^2d^2x^3 - 100\,ab^4cd^3x^3 + 200\,a^2b^3d^4x^3 - 30\,b^5c^3dx^2 + 150\,ab^4c^2d^2x^2 - 30}{60\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="giac")
```

```
[Out] 1/60*(12*b^5*d^4*x^5 - 15*b^5*c*d^3*x^4 + 75*a*b^4*d^4*x^4 + 20*b^5*c^2*d^2
*x^3 - 100*a*b^4*c*d^3*x^3 + 200*a^2*b^3*d^4*x^3 - 30*b^5*c^3*d*x^2 + 150*a
*b^4*c^2*d^2*x^2 - 300*a^2*b^3*c*d^3*x^2 + 300*a^3*b^2*d^4*x^2 + 60*b^5*c^4
*x - 300*a*b^4*c^3*d*x + 600*a^2*b^3*c^2*d^2*x - 600*a^3*b^2*c*d^3*x + 300*
a^4*b*d^4*x)/d^5 - (b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b
^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*log(abs(d*x + c))/d^6
```

3.1800 $\int \frac{(a+bx)^5}{ac+(bc+ad)x+bdx^2}\,dx$

Optimal. Leaf size=98

$$-\frac{bx(bc-ad)^3}{d^4}+\frac{(a+bx)^2(bc-ad)^2}{2d^3}-\frac{(a+bx)^3(bc-ad)}{3d^2}+\frac{(bc-ad)^4\log(c+dx)}{d^5}+\frac{(a+bx)^4}{4d}$$

```
[Out] -((b*(b*c - a*d)^3*x)/d^4) + ((b*c - a*d)^2*(a + b*x)^2)/(2*d^3) - ((b*c -
a*d)*(a + b*x)^3)/(3*d^2) + (a + b*x)^4/(4*d) + ((b*c - a*d)^4*Log[c + d*x]
)/d^5
```

Rubi [A] time = 0.046089, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$-\frac{bx(bc-ad)^3}{d^4}+\frac{(a+bx)^2(bc-ad)^2}{2d^3}-\frac{(a+bx)^3(bc-ad)}{3d^2}+\frac{(bc-ad)^4\log(c+dx)}{d^5}+\frac{(a+bx)^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

```
[Out] -((b*(b*c - a*d)^3*x)/d^4) + ((b*c - a*d)^2*(a + b*x)^2)/(2*d^3) - ((b*c -
a*d)*(a + b*x)^3)/(3*d^2) + (a + b*x)^4/(4*d) + ((b*c - a*d)^4*Log[c + d*x]
)/d^5
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{ac+(bc+ad)x+bdx^2}\,dx &= \int \frac{(a+bx)^4}{c+dx}\,dx\\ &= \int \left(-\frac{b(bc-ad)^3}{d^4}+\frac{b(bc-ad)^2(a+bx)}{d^3}-\frac{b(bc-ad)(a+bx)^2}{d^2}+\frac{b(a+bx)^3}{d}+\frac{(-bc+ad)}{d^4(c+dx)}\right.\\ &= -\frac{b(bc-ad)^3x}{d^4}+\frac{(bc-ad)^2(a+bx)^2}{2d^3}-\frac{(bc-ad)(a+bx)^3}{3d^2}+\frac{(a+bx)^4}{4d}+\frac{(bc-ad)^4\log(c}{d^5}\end{aligned}$$

Mathematica [A] time = 0.0403413, size = 115, normalized size = 1.17

$$\frac{bdx\left(36a^2bd^2(dx-2c)+48a^3d^3+8ab^2d\left(6c^2-3cdx+2d^2x^2\right)+b^3\left(6c^2dx-12c^3-4cd^2x^2+3d^3x^3\right)\right)+12(bc-ad)^4\log(}{12d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

```
[Out] (b*d*x*(48*a^3*d^3 + 36*a^2*b*d^2*(-2*c + d*x) + 8*a*b^2*d*(6*c^2 - 3*c*d*x
 + 2*d^2*x^2) + b^3*(-12*c^3 + 6*c^2*d*x - 4*c*d^2*x^2 + 3*d^3*x^3)) + 12*(
b*c - a*d)^4*Log[c + d*x])/(12*d^5)
```

Maple [B] time = 0.041, size = 209, normalized size = 2.1

$$\frac{b^4x^4}{4d}+\frac{4\,ab^3x^3}{3\,d}-\frac{b^4x^3c}{3\,d^2}+3\,\frac{b^2x^2a^2}{d}-2\,\frac{b^3x^2ac}{d^2}+\frac{x^2b^4c^2}{2\,d^3}+4\,\frac{xa^3b}{d}-6\,\frac{b^2ca^2x}{d^2}+4\,\frac{xab^3c^2}{d^3}-\frac{b^4c^3x}{d^4}+\frac{\ln(dx+c)\,a^4}{d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(a*c+(a*d+b*c)*x+b*d*x^2),x)
```

```
[Out] 1/4*b^4/d*x^4+4/3*b^3/d*x^3*a-1/3*b^4/d^2*x^3*c+3*b^2/d*x^2*a^2-2*b^3/d^2*x
^2*a*c+1/2*b^4/d^3*x^2*c^2+4*b/d*a^3*x-6*b^2/d^2*c*a^2*x+4*b^3/d^3*a*c^2*x-
b^4/d^4*c^3*x+1/d*ln(d*x+c)*a^4-4/d^2*ln(d*x+c)*a^3*b*c+6/d^3*ln(d*x+c)*a^2
*b^2*c^2-4/d^4*ln(d*x+c)*a*b^3*c^3+1/d^5*ln(d*x+c)*b^4*c^4
```

Maxima [A] time = 1.09025, size = 239, normalized size = 2.44

$$\frac{3\,b^4d^3x^4-4\left(b^4cd^2-4\,ab^3d^3\right)x^3+6\left(b^4c^2d-4\,ab^3cd^2+6\,a^2b^2d^3\right)x^2-12\left(b^4c^3-4\,ab^3c^2d+6\,a^2b^2cd^2-4\,a^3bd^3\right)x}{12\,d^4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*b^4*d^3*x^4 - 4*(b^4*c*d^2 - 4*a*b^3*d^3)*x^3 + 6*(b^4*c^2*d - 4*a*
b^3*c*d^2 + 6*a^2*b^2*d^3)*x^2 - 12*(b^4*c^3 - 4*a*b^3*c^2*d + 6*a^2*b^2*c*
d^2 - 4*a^3*b*d^3)*x)/d^4 + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
4*a^3*b*c*d^3 + a^4*d^4)*log(d*x + c)/d^5
```

Fricas [A] time = 1.6222, size = 369, normalized size = 3.77

$$\frac{3\,b^4d^4x^4-4\left(b^4cd^3-4\,ab^3d^4\right)x^3+6\left(b^4c^2d^2-4\,ab^3cd^3+6\,a^2b^2d^4\right)x^2-12\left(b^4c^3d-4\,ab^3c^2d^2+6\,a^2b^2cd^3-4\,a^3bd^4\right)x}{12\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*d^4*x^4 - 4*(b^4*c*d^3 - 4*a*b^3*d^4)*x^3 + 6*(b^4*c^2*d^2 - 4*
a*b^3*c*d^3 + 6*a^2*b^2*d^4)*x^2 - 12*(b^4*c^3*d - 4*a*b^3*c^2*d^2 + 6*a^2*
b^2*c*d^3 - 4*a^3*b*d^4)*x + 12*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^
2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(d*x + c))/d^5
```

Sympy [A] time = 0.891777, size = 134, normalized size = 1.37

$$\frac{b^4x^4}{4d} + \frac{x^3\left(4ab^3d - b^4c\right)}{3d^2} + \frac{x^2\left(6a^2b^2d^2 - 4ab^3cd + b^4c^2\right)}{2d^3} + \frac{x\left(4a^3bd^3 - 6a^2b^2cd^2 + 4ab^3c^2d - b^4c^3\right)}{d^4} + \frac{(ad - bc)^4\log(c +}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

```
[Out] b**4*x**4/(4*d) + x**3*(4*a*b**3*d - b**4*c)/(3*d**2) + x**2*(6*a**2*b**2*d
**2 - 4*a*b**3*c*d + b**4*c**2)/(2*d**3) + x*(4*a**3*b*d**3 - 6*a**2*b**2*c
*d**2 + 4*a*b**3*c**2*d - b**4*c**3)/d**4 + (a*d - b*c)**4*log(c + d*x)/d**
5
```

Giac [A] time = 1.26526, size = 248, normalized size = 2.53

$$\frac{3\,b^4d^3x^4 - 4\,b^4cd^2x^3 + 16\,ab^3d^3x^3 + 6\,b^4c^2dx^2 - 24\,ab^3cd^2x^2 + 36\,a^2b^2d^3x^2 - 12\,b^4c^3x + 48\,ab^3c^2dx - 72\,a^2b^2cd^2x + 48\,a}{12\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="giac")
```

```
[Out] 1/12*(3*b^4*d^3*x^4 - 4*b^4*c*d^2*x^3 + 16*a*b^3*d^3*x^3 + 6*b^4*c^2*d*x^2
- 24*a*b^3*c*d^2*x^2 + 36*a^2*b^2*d^3*x^2 - 12*b^4*c^3*x + 48*a*b^3*c^2*d*x
 - 72*a^2*b^2*c*d^2*x + 48*a^3*b*d^3*x)/d^4 + (b^4*c^4 - 4*a*b^3*c^3*d + 6*
a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(abs(d*x + c))/d^5
```

3.1801 $\int \frac{(a+bx)^4}{ac+(bc+ad)x+bdx^2}\,dx$

Optimal. Leaf size=74

$$\frac{bx(bc-ad)^2}{d^3}-\frac{(a+bx)^2(bc-ad)}{2d^2}-\frac{(bc-ad)^3\log(c+dx)}{d^4}+\frac{(a+bx)^3}{3d}$$

```
[Out] (b*(b*c - a*d)^2*x)/d^3 - ((b*c - a*d)*(a + b*x)^2)/(2*d^2) + (a + b*x)^3/(
3*d) - ((b*c - a*d)^3*Log[c + d*x])/d^4
```

Rubi [A] time = 0.0361576, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$\frac{bx(bc-ad)^2}{d^3}-\frac{(a+bx)^2(bc-ad)}{2d^2}-\frac{(bc-ad)^3\log(c+dx)}{d^4}+\frac{(a+bx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

```
[Out] (b*(b*c - a*d)^2*x)/d^3 - ((b*c - a*d)*(a + b*x)^2)/(2*d^2) + (a + b*x)^3/(
3*d) - ((b*c - a*d)^3*Log[c + d*x])/d^4
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{ac+(bc+ad)x+bdx^2}\,dx &= \int \frac{(a+bx)^3}{c+dx}\,dx\\ &= \int \left(\frac{b(bc-ad)^2}{d^3}-\frac{b(bc-ad)(a+bx)}{d^2}+\frac{b(a+bx)^2}{d}+\frac{(-bc+ad)^3}{d^3(c+dx)}\right)dx\\ &= \frac{b(bc-ad)^2x}{d^3}-\frac{(bc-ad)(a+bx)^2}{2d^2}+\frac{(a+bx)^3}{3d}-\frac{(bc-ad)^3\log(c+dx)}{d^4}\end{aligned}$$

Mathematica [A] time = 0.026245, size = 74, normalized size = 1.

$$\frac{bdx\left(18a^2d^2+9abd(dx-2c)+b^2\left(6c^2-3cdx+2d^2x^2\right)\right)-6(bc-ad)^3\log(c+dx)}{6d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

```
[Out] (b*d*x*(18*a^2*d^2 + 9*a*b*d*(-2*c + d*x) + b^2*(6*c^2 - 3*c*d*x + 2*d^2*x^
2)) - 6*(b*c - a*d)^3*Log[c + d*x])/(6*d^4)
```

Maple [A] time = 0.04, size = 133, normalized size = 1.8

$$\frac{b^3x^3}{3d}+\frac{3ab^2x^2}{2d}-\frac{b^3x^2c}{2d^2}+3\frac{ba^2x}{d}-3\frac{acb^2x}{d^2}+\frac{b^3c^2x}{d^3}+\frac{\ln(dx+c)a^3}{d}-3\frac{\ln(dx+c)cba^2}{d^2}+3\frac{\ln(dx+c)ac^2b^2}{d^3}-\frac{\ln(dx+}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2),x)
```

```
[Out] 1/3*b^3/d*x^3+3/2*b^2/d*x^2*a-1/2*b^3/d^2*x^2*c+3*b/d*a^2*x-3*b^2/d^2*c*a*x
+b^3/d^3*c^2*x+1/d*ln(d*x+c)*a^3-3/d^2*ln(d*x+c)*c*b*a^2+3/d^3*ln(d*x+c)*a*
c^2*b^2-1/d^4*ln(d*x+c)*b^3*c^3
```

Maxima [A] time = 1.06319, size = 154, normalized size = 2.08

$$\frac{2\,b^3d^2x^3-3\left(b^3cd-3\,ab^2d^2\right)x^2+6\left(b^3c^2-3\,ab^2cd+3\,a^2bd^2\right)x}{6\,d^3}-\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(dx+c\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^3*d^2*x^3 - 3*(b^3*c*d - 3*a*b^2*d^2)*x^2 + 6*(b^3*c^2 - 3*a*b^2*c
*d + 3*a^2*b*d^2)*x)/d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*log(d*x + c)/d^4
```

Fricas [A] time = 1.57452, size = 238, normalized size = 3.22

$$\frac{2\,b^3d^3x^3-3\left(b^3cd^2-3\,ab^2d^3\right)x^2+6\left(b^3c^2d-3\,ab^2cd^2+3\,a^2bd^3\right)x-6\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(dx+c\right)}{6\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*d^3*x^3 - 3*(b^3*c*d^2 - 3*a*b^2*d^3)*x^2 + 6*(b^3*c^2*d - 3*a*b
^2*c*d^2 + 3*a^2*b*d^3)*x - 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*log(d*x + c))/d^4
```

Sympy [A] time = 0.536191, size = 82, normalized size = 1.11

$$\frac{b^3x^3}{3d}+\frac{x^2\left(3ab^2d-b^3c\right)}{2d^2}+\frac{x\left(3a^2bd^2-3ab^2cd+b^3c^2\right)}{d^3}+\frac{\left(ad-bc\right)^3\log\left(c+dx\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

```
[Out] b**3*x**3/(3*d) + x**2*(3*a*b**2*d - b**3*c)/(2*d**2) + x*(3*a**2*b*d**2 -
3*a*b**2*c*d + b**3*c**2)/d**3 + (a*d - b*c)**3*log(c + d*x)/d**4
```

Giac [A] time = 1.15981, size = 157, normalized size = 2.12

$$\frac{2\,b^3d^2x^3 - 3\,b^3cdx^2 + 9\,ab^2d^2x^2 + 6\,b^3c^2x - 18\,ab^2cdx + 18\,a^2bd^2x}{6\,d^3} - \frac{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\log\left(|dx + c|\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="giac")
```

```
[Out] 1/6*(2*b^3*d^2*x^3 - 3*b^3*c*d*x^2 + 9*a*b^2*d^2*x^2 + 6*b^3*c^2*x - 18*a*b
^2*c*d*x + 18*a^2*b*d^2*x)/d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*log(abs(d*x + c))/d^4
```

3.1802 $\int \frac{(a+bx)^3}{ac+(bc+ad)x+bdx^2}\,dx$

Optimal. Leaf size=50

$$-\frac{bx(bc-ad)}{d^2}+\frac{(bc-ad)^2\log(c+dx)}{d^3}+\frac{(a+bx)^2}{2d}$$

```
[Out] -((b*(b*c - a*d)*x)/d^2) + (a + b*x)^2/(2*d) + ((b*c - a*d)^2*Log[c + d*x])
/d^3
```

Rubi [A] time = 0.0254921, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$-\frac{bx(bc-ad)}{d^2}+\frac{(bc-ad)^2\log(c+dx)}{d^3}+\frac{(a+bx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

```
[Out] -((b*(b*c - a*d)*x)/d^2) + (a + b*x)^2/(2*d) + ((b*c - a*d)^2*Log[c + d*x])
/d^3
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{ac+(bc+ad)x+bdx^2}\,dx &= \int \frac{(a+bx)^2}{c+dx}\,dx\\ &= \int \left(-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}+\frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx\\ &= -\frac{b(bc-ad)x}{d^2}+\frac{(a+bx)^2}{2d}+\frac{(bc-ad)^2\log(c+dx)}{d^3}\end{aligned}$$

Mathematica [A] time = 0.0158795, size = 43, normalized size = 0.86

$$\frac{bdx(4ad-2bc+bdx)+2(bc-ad)^2\log(c+dx)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

```
[Out] (b*d*x*(-2*b*c + 4*a*d + b*d*x) + 2*(b*c - a*d)^2*Log[c + d*x])/(2*d^3)
```

Maple [A] time = 0.041, size = 74, normalized size = 1.5

$$\frac{b^2x^2}{2\,d}+2\,\frac{abx}{d}-\frac{b^2xc}{d^2}+\frac{\ln\left(dx+c\right)a^2}{d}-2\,\frac{\ln\left(dx+c\right)cab}{d^2}+\frac{\ln\left(dx+c\right)b^2c^2}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2),x)
```

```
[Out] 1/2*b^2/d*x^2+2*b/d*a*x-b^2/d^2*x*c+1/d*ln(d*x+c)*a^2-2/d^2*ln(d*x+c)*c*a*b
+1/d^3*ln(d*x+c)*b^2*c^2
```

Maxima [A] time = 1.04225, size = 81, normalized size = 1.62

$$\frac{b^2dx^2-2\left(b^2c-2\,abd\right)x}{2\,d^2}+\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(dx+c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*d*x^2 - 2*(b^2*c - 2*a*b*d)*x)/d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*log(d*x + c)/d^3
```

Fricas [A] time = 1.5661, size = 135, normalized size = 2.7

$$\frac{b^2d^2x^2-2\left(b^2cd-2\,abd^2\right)x+2\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(dx+c\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^2 - 2*(b^2*c*d - 2*a*b*d^2)*x + 2*(b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*log(d*x + c))/d^3
```

Sympy [A] time = 0.602858, size = 44, normalized size = 0.88

$$\frac{b^2x^2}{2d}+\frac{x\left(2abd-b^2c\right)}{d^2}+\frac{(ad-bc)^2\log\left(c+dx\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

```
[Out] b**2*x**2/(2*d) + x*(2*a*b*d - b**2*c)/d**2 + (a*d - b*c)**2*log(c + d*x)/d
**3
```

Giac [A] time = 1.19037, size = 81, normalized size = 1.62

$$\frac{b^2dx^2 - 2\,b^2cx + 4\,abdx}{2\,d^2} + \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(|dx + c|\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d*x^2 - 2*b^2*c*x + 4*a*b*d*x)/d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*log(abs(d*x + c))/d^3
```

3.1803 $\int \frac{(a+bx)^2}{ac+(bc+ad)x+bdx^2}\,dx$

Optimal. Leaf size=26

$$\frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}$$

```
[Out] (b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2
```

Rubi [A] time = 0.0220228, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$\frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

```
[Out] (b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{ac+(bc+ad)x+bdx^2}\,dx &= \int \frac{a+bx}{c+dx}\,dx\\ &= \int \left(\frac{b}{d}+\frac{-bc+ad}{d(c+dx)}\right)dx\\ &= \frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}\end{aligned}$$

Mathematica [A] time = 0.0070554, size = 25, normalized size = 0.96

$$\frac{(ad-bc)\log(c+dx)}{d^2}+\frac{bx}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

```
[Out] (b*x)/d + ((-(b*c) + a*d)*Log[c + d*x])/d^2
```

Maple [A] time = 0.04, size = 32, normalized size = 1.2

$$\frac{bx}{d}+\frac{\ln\left(dx+c\right)a}{d}-\frac{\ln\left(dx+c\right)bc}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2),x)
```

```
[Out] b*x/d+1/d*ln(d*x+c)*a-1/d^2*ln(d*x+c)*b*c
```

Maxima [A] time = 1.06554, size = 35, normalized size = 1.35

$$\frac{bx}{d}-\frac{(bc-ad)\log\left(dx+c\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="maxima")
```

```
[Out] b*x/d - (b*c - a*d)*log(d*x + c)/d^2
```

Fricas [A] time = 1.50057, size = 54, normalized size = 2.08

$$\frac{bdx-(bc-ad)\log\left(dx+c\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="fricas")
```

```
[Out] (b*d*x - (b*c - a*d)*log(d*x + c))/d^2
```

Sympy [A] time = 0.506721, size = 20, normalized size = 0.77

$$\frac{bx}{d}+\frac{(ad-bc)\log\left(c+dx\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

```
[Out] b*x/d + (a*d - b*c)*log(c + d*x)/d**2
```

Giac [A] time = 1.1909, size = 36, normalized size = 1.38

$$\frac{bx}{d}-\frac{(bc-ad)\log\left(|dx+c|\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="giac")
```

```
[Out] b*x/d - (b*c - a*d)*log(abs(d*x + c))/d^2
```

3.1804 $\int \frac{a+bx}{ac+(bc+ad)x+bdx^2}\,dx$

Optimal. Leaf size=10

$$\frac{\log(c+dx)}{d}$$

```
[Out] Log[c + d*x]/d
```

Rubi [A] time = 0.0050334, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {626, 31}

$$\frac{\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a*c + (b*c + a*d)*x + b*d*x^2),x]

[Out] Log[c + d*x]/d
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx}{ac+(bc+ad)x+bdx^2}\,dx &= \int \frac{1}{c+dx}\,dx \\ &= \frac{\log(c+dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.0011979, size = 10, normalized size = 1.

$$\frac{\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a*c + (b*c + a*d)*x + b*d*x^2),x]

[Out] Log[c + d*x]/d
```

Maple [A] time = 0.039, size = 11, normalized size = 1.1

$$\frac{\ln\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2),x)
```

```
[Out] ln(d*x+c)/d
```

Maxima [A] time = 1.04392, size = 14, normalized size = 1.4

$$\frac{\log\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="maxima")
```

```
[Out] log(d*x + c)/d
```

Fricas [A] time = 1.40938, size = 22, normalized size = 2.2

$$\frac{\log\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="fricas")
```

```
[Out] log(d*x + c)/d
```

Sympy [A] time = 0.139582, size = 7, normalized size = 0.7

$$\frac{\log\left(c+dx\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

```
[Out] log(c + d*x)/d
```

Giac [A] time = 1.18707, size = 15, normalized size = 1.5

$$\frac{\log\left(|dx+c|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="giac")

[Out] log(abs(d*x + c))/d
```

3.1805 $\int \frac{1}{ac+(bc+ad)x+bdx^2}\,dx$

Optimal. Leaf size=36

$$\frac{\log(a+bx)}{bc-ad}-\frac{\log(c+dx)}{bc-ad}$$

```
[Out] Log[a + b*x]/(b*c - a*d) - Log[c + d*x]/(b*c - a*d)
```

Rubi [A] time = 0.0122863, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {616, 31}

$$\frac{\log(a+bx)}{bc-ad}-\frac{\log(c+dx)}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)^(-1),x]
```

```
[Out] Log[a + b*x]/(b*c - a*d) - Log[c + d*x]/(b*c - a*d)
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ac+(bc+ad)x+bdx^2}\,dx &= -\frac{(bd)\int \frac{1}{bc+bdx}\,dx}{bc-ad}+\frac{(bd)\int \frac{1}{ad+bdx}\,dx}{bc-ad}\\ &= \frac{\log(a+bx)}{bc-ad}-\frac{\log(c+dx)}{bc-ad}\end{aligned}$$

Mathematica [A] time = 0.0109429, size = 26, normalized size = 0.72

$$\frac{\log(a+bx)-\log(c+dx)}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^(-1),x]
```

```
[Out] (Log[a + b*x] - Log[c + d*x])/(b*c - a*d)
```

Maple [A] time = 0.043, size = 37, normalized size = 1.

$$\frac{\ln(dx+c)}{ad-bc}-\frac{\ln(bx+a)}{ad-bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c+(a*d+b*c)*x+b*d*x^2),x)
```

```
[Out] 1/(a*d-b*c)*ln(d*x+c)-1/(a*d-b*c)*ln(b*x+a)
```

Maxima [A] time = 1.00709, size = 49, normalized size = 1.36

$$\frac{\log(bx+a)}{bc-ad}-\frac{\log(dx+c)}{bc-ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="maxima")
```

```
[Out] log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d)
```

Fricas [A] time = 1.56742, size = 58, normalized size = 1.61

$$\frac{\log(bx+a)-\log(dx+c)}{bc-ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="fricas")
```

```
[Out] (log(b*x + a) - log(d*x + c))/(b*c - a*d)
```

Sympy [B] time = 0.410633, size = 128, normalized size = 3.56

$$\frac{\log\left(x+\frac{-\frac{a^2d^2}{ad-bc}+\frac{2abcd}{ad-bc}+ad-\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{ad-bc}-\frac{\log\left(x+\frac{\frac{a^2d^2}{ad-bc}-\frac{2abcd}{ad-bc}+ad+\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{ad-bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

```
[Out] log(x + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*c**2/(
a*d - b*c) + b*c)/(2*b*d))/(a*d - b*c) - log(x + (a**2*d**2/(a*d - b*c) - 2
*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b*d))/(a*d - b
*c)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1806 $\int \frac{1}{(a+bx)(ac+(bc+ad)x+bdx^2)}\,dx$

Optimal. Leaf size=57

$$-\frac{1}{(a+bx)(bc-ad)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}$$

```
[Out] -(1/((b*c - a*d)*(a + b*x))) - (d*Log[a + b*x])/(b*c - a*d)^2 + (d*Log[c +
d*x])/(b*c - a*d)^2
```

Rubi [A] time = 0.0355253, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 44}

$$-\frac{1}{(a+bx)(bc-ad)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(a*c + (b*c + a*d)*x + b*d*x^2)),x]
```

```
[Out] -(1/((b*c - a*d)*(a + b*x))) - (d*Log[a + b*x])/(b*c - a*d)^2 + (d*Log[c +
d*x])/(b*c - a*d)^2
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)\left(ac+(bc+ad)x+bdx^2\right)}\,dx &= \int \frac{1}{(a+bx)^2(c+dx)}\,dx\\ &= \int \left(\frac{b}{(bc-ad)(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)}\right)dx\\ &= -\frac{1}{(bc-ad)(a+bx)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}\end{aligned}$$

Mathematica [A] time = 0.0238165, size = 53, normalized size = 0.93

$$\frac{d(a+bx)\log(c+dx)-d(a+bx)\log(a+bx)+ad-bc}{(a+bx)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(a*c + (b*c + a*d)*x + b*d*x^2)),x]
```

```
[Out] (-(b*c) + a*d - d*(a + b*x)*Log[a + b*x] + d*(a + b*x)*Log[c + d*x])/((b*c
- a*d)^2*(a + b*x))
```

Maple [A] time = 0.05, size = 57, normalized size = 1.

$$\frac{d\ln(dx+c)}{(ad-bc)^2}+\frac{1}{(ad-bc)(bx+a)}-\frac{d\ln(bx+a)}{(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2),x)
```

```
[Out] d/(a*d-b*c)^2*ln(d*x+c)+1/(a*d-b*c)/(b*x+a)-d/(a*d-b*c)^2*ln(b*x+a)
```

Maxima [A] time = 1.0461, size = 124, normalized size = 2.18

$$-\frac{d\log(bx+a)}{b^2c^2-2\,abcd+a^2d^2}+\frac{d\log(dx+c)}{b^2c^2-2\,abcd+a^2d^2}-\frac{1}{abc-a^2d+\left(b^2c-abd\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="maxima")
```

```
[Out] -d*log(b*x + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + d*log(d*x + c)/(b^2*c^2 -
 2*a*b*c*d + a^2*d^2) - 1/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

Fricas [A] time = 1.67954, size = 200, normalized size = 3.51

$$-\frac{bc-ad+(bdx+ad)\log(bx+a)-(bdx+ad)\log(dx+c)}{ab^2c^2-2\,a^2bcd+a^3d^2+\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="fricas")
```

```
[Out] -(b*c - a*d + (b*d*x + a*d)*log(b*x + a) - (b*d*x + a*d)*log(d*x + c))/(a*b
^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x)
```

Sympy [B] time = 1.17334, size = 233, normalized size = 4.09

$$\frac{d\log\left(x+\frac{-\frac{a^3d^4}{(ad-bc)^2}+\frac{3a^2bcd^3}{(ad-bc)^2}-\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2+\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{(ad-bc)^2}-\frac{d\log\left(x+\frac{\frac{a^3d^4}{(ad-bc)^2}-\frac{3a^2bcd^3}{(ad-bc)^2}+\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2-\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{(ad-bc)^2}+\frac{1}{a^2d-abc+x\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

```
[Out] d*log(x + (-a**3*d**4/(a*d - b*c)**2 + 3*a**2*b*c*d**3/(a*d - b*c)**2 - 3*a
*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 + b**3*c**3*d/(a*d - b*c)**2 + b*c*
d)/(2*b*d**2))/(a*d - b*c)**2 - d*log(x + (a**3*d**4/(a*d - b*c)**2 - 3*a**
2*b*c*d**3/(a*d - b*c)**2 + 3*a*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 - b*
*3*c**3*d/(a*d - b*c)**2 + b*c*d)/(2*b*d**2))/(a*d - b*c)**2 + 1/(a**2*d -
a*b*c + x*(a*b*d - b**2*c))
```

Giac [A] time = 1.21447, size = 127, normalized size = 2.23

$$-\frac{bd\log\left(|bx+a|\right)}{b^3c^2-2\,ab^2cd+a^2bd^2}+\frac{d^2\log\left(|dx+c|\right)}{b^2c^2d-2\,abcd^2+a^2d^3}-\frac{1}{(bc-ad)(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="giac")
```

```
[Out] -b*d*log(abs(b*x + a))/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + d^2*log(abs(d*
x + c))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3) - 1/((b*c - a*d)*(b*x + a))
```

3.1807 $\int \frac{1}{(a+bx)^2\left(ac+(bc+ad)x+bdx^2\right)}\,dx$

Optimal. Leaf size=82

$$\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}+\frac{d}{(a+bx)(bc-ad)^2}-\frac{1}{2(a+bx)^2(bc-ad)}$$

```
[Out] -1/(2*(b*c - a*d)*(a + b*x)^2) + d/((b*c - a*d)^2*(a + b*x)) + (d^2*Log[a +
 b*x])/(b*c - a*d)^3 - (d^2*Log[c + d*x])/(b*c - a*d)^3
```

Rubi [A] time = 0.0563078, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 44}

$$\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}+\frac{d}{(a+bx)(bc-ad)^2}-\frac{1}{2(a+bx)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(a*c + (b*c + a*d)*x + b*d*x^2)),x]
```

```
[Out] -1/(2*(b*c - a*d)*(a + b*x)^2) + d/((b*c - a*d)^2*(a + b*x)) + (d^2*Log[a +
 b*x])/(b*c - a*d)^3 - (d^2*Log[c + d*x])/(b*c - a*d)^3
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx)^2\left(ac+(bc+ad)x+bdx^2\right)}\,dx=\int \frac{1}{(a+bx)^3(c+dx)}\,dx$$

$$=\int \left(\frac{b}{(bc-ad)(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{bd^2}{(bc-ad)^3(a+bx)}-(bc-$$

$$=-\frac{1}{2(bc-ad)(a+bx)^2}+\frac{d}{(bc-ad)^2(a+bx)}+\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+}{(bc-ad)^3}$$

Mathematica [A] time = 0.0628019, size = 67, normalized size = 0.82

$$\frac{\frac{(bc-ad)(3ad-bc+2bdx)}{(a+bx)^2}+2d^2\log(a+bx)-2d^2\log(c+dx)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(a*c + (b*c + a*d)*x + b*d*x^2)),x]
```

```
[Out] (((b*c - a*d)*(-(b*c) + 3*a*d + 2*b*d*x))/(a + b*x)^2 + 2*d^2*Log[a + b*x]
- 2*d^2*Log[c + d*x])/(2*(b*c - a*d)^3)
```

Maple [A] time = 0.051, size = 81, normalized size = 1.

$$\frac{d^2\ln(dx+c)}{(ad-bc)^3}+\frac{1}{(2ad-2bc)(bx+a)^2}+\frac{d}{(ad-bc)^2(bx+a)}-\frac{d^2\ln(bx+a)}{(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2),x)
```

```
[Out] d^2/(a*d-b*c)^3*ln(d*x+c)+1/2/(a*d-b*c)/(b*x+a)^2+d/(a*d-b*c)^2/(b*x+a)-d^2
/(a*d-b*c)^3*ln(b*x+a)
```

Maxima [B] time = 1.04429, size = 273, normalized size = 3.33

$$\frac{d^2\log(bx+a)}{b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3}-\frac{d^2\log(dx+c)}{b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3}+\frac{2bdx-bc\cdots}{2\left(a^2b^2c^2-2a^3bcd+a^4d^2+\left(b^4c^2-2ab^3cd+a\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="maxima")
```

```
[Out] d^2*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - d^2*
log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 1/2*(2*b
*d*x - b*c + 3*a*d)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b
^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x)
```

Fricas [B] time = 1.65853, size = 491, normalized size = 5.99

$$-\frac{b^2c^2-4abcd+3a^2d^2-2\left(b^2cd-abd^2\right)x-2\left(b^2d^2x^2+2abd^2x+a^2d^2\right)\log(bx+a)+2\left(b^2d^2x^2+2abd^2x+a^2d^2\right)\log\cdots}{2\left(a^2b^3c^3-3a^3b^2c^2d+3a^4bcd^2-a^5d^3+\left(b^5c^3-3ab^4c^2d+3a^2b^3cd^2-a^3b^2d^3\right)x^2+2\left(ab^4c^3-3a^2b^3c^2d+3a^3b^2cd^2-\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x - 2*(b^2*d^
2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x
+ a^2*d^2)*log(d*x + c))/(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a
^5*d^3 + (b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*x^2 + 2*
(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - a^4*b*d^3)*x)
```

Sympy [B] time = 1.92071, size = 381, normalized size = 4.65

$$\frac{d^2\log\left(x+\frac{-\frac{a^4d^6}{(ad-bc)^3}+\frac{4a^3bcd^5}{(ad-bc)^3}-\frac{6a^2b^2c^2d^4}{(ad-bc)^3}+\frac{4ab^3c^3d^3}{(ad-bc)^3}+ad^3-\frac{b^4c^4d^2}{(ad-bc)^3}+bcd^2}{2bd^3}\right)}{(ad-bc)^3}-\frac{d^2\log\left(x+\frac{\frac{a^4d^6}{(ad-bc)^3}-\frac{4a^3bcd^5}{(ad-bc)^3}+\frac{6a^2b^2c^2d^4}{(ad-bc)^3}-\frac{4ab^3c^3d^3}{(ad-bc)^3}+ad^3+\frac{b^4c^4d^2}{(ad-bc)^3}+bcd^2}{2bd^3}\right)}{(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

```
[Out] d**2*log(x + (-a**4*d**6/(a*d - b*c)**3 + 4*a**3*b*c*d**5/(a*d - b*c)**3 -
6*a**2*b**2*c**2*d**4/(a*d - b*c)**3 + 4*a*b**3*c**3*d**3/(a*d - b*c)**3 +
a*d**3 - b**4*c**4*d**2/(a*d - b*c)**3 + b*c*d**2)/(2*b*d**3))/(a*d - b*c)*
*3 - d**2*log(x + (a**4*d**6/(a*d - b*c)**3 - 4*a**3*b*c*d**5/(a*d - b*c)**
3 + 6*a**2*b**2*c**2*d**4/(a*d - b*c)**3 - 4*a*b**3*c**3*d**3/(a*d - b*c)**
3 + a*d**3 + b**4*c**4*d**2/(a*d - b*c)**3 + b*c*d**2)/(2*b*d**3))/(a*d - b
*c)**3 + (3*a*d - b*c + 2*b*d*x)/(2*a**4*d**2 - 4*a**3*b*c*d + 2*a**2*b**2*
c**2 + x**2*(2*a**2*b**2*d**2 - 4*a*b**3*c*d + 2*b**4*c**2) + x*(4*a**3*b*d
**2 - 8*a**2*b**2*c*d + 4*a*b**3*c**2))
```

Giac [A] time = 1.19967, size = 196, normalized size = 2.39

$$-\frac{bd^2\log\left(\left|-\frac{bc}{bx+a}+\frac{ad}{bx+a}-d\right|\right)}{b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3}-\frac{\frac{b^3c}{(bx+a)^2}-\frac{2\,b^2d}{bx+a}-\frac{ab^2d}{(bx+a)^2}}{2\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="giac")
```

```
[Out] -b*d^2*log(abs(-b*c/(b*x + a) + a*d/(b*x + a) - d))/(b^4*c^3 - 3*a*b^3*c^2*
d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) - 1/2*(b^3*c/(b*x + a)^2 - 2*b^2*d/(b*x +
a) - a*b^2*d/(b*x + a)^2)/(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)
```

3.1808 $\int \frac{1}{(a+bx)^3\left(ac+(bc+ad)x+bdx^2\right)}\,dx$

Optimal. Leaf size=107

$$-\frac{d^2}{(a+bx)(bc-ad)^3}-\frac{d^3\log(a+bx)}{(bc-ad)^4}+\frac{d^3\log(c+dx)}{(bc-ad)^4}+\frac{d}{2(a+bx)^2(bc-ad)^2}-\frac{1}{3(a+bx)^3(bc-ad)}$$

```
[Out] -1/(3*(b*c - a*d)*(a + b*x)^3) + d/(2*(b*c - a*d)^2*(a + b*x)^2) - d^2/((b*
c - a*d)^3*(a + b*x)) - (d^3*Log[a + b*x])/(b*c - a*d)^4 + (d^3*Log[c + d*x
])/(b*c - a*d)^4
```

Rubi [A] time = 0.0712196, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 44}

$$-\frac{d^2}{(a+bx)(bc-ad)^3}-\frac{d^3\log(a+bx)}{(bc-ad)^4}+\frac{d^3\log(c+dx)}{(bc-ad)^4}+\frac{d}{2(a+bx)^2(bc-ad)^2}-\frac{1}{3(a+bx)^3(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(a*c + (b*c + a*d)*x + b*d*x^2)),x]
```

```
[Out] -1/(3*(b*c - a*d)*(a + b*x)^3) + d/(2*(b*c - a*d)^2*(a + b*x)^2) - d^2/((b*
c - a*d)^3*(a + b*x)) - (d^3*Log[a + b*x])/(b*c - a*d)^4 + (d^3*Log[c + d*x
])/(b*c - a*d)^4
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx)^3\left(ac+(bc+ad)x+bdx^2\right)}\,dx = \int \frac{1}{(a+bx)^4(c+dx)}\,dx$$

$$= \int \left(\frac{b}{(bc-ad)(a+bx)^4}-\frac{bd}{(bc-ad)^2(a+bx)^3}+\frac{bd^2}{(bc-ad)^3(a+bx)^2}-\frac{b}{(bc-ad}\right.$$

$$= -\frac{1}{3(bc-ad)(a+bx)^3}+\frac{d}{2(bc-ad)^2(a+bx)^2}-\frac{d^2}{(bc-ad)^3(a+bx)}-\frac{d^3\log(a}{(bc-a}$$

Mathematica [A] time = 0.042251, size = 107, normalized size = 1.

$$-\frac{d^2}{(a+bx)(bc-ad)^3}-\frac{d^3\log(a+bx)}{(bc-ad)^4}+\frac{d^3\log(c+dx)}{(bc-ad)^4}+\frac{d}{2(a+bx)^2(bc-ad)^2}+\frac{1}{3(a+bx)^3(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(a*c + (b*c + a*d)*x + b*d*x^2)),x]
```

```
[Out] 1/(3*(-(b*c) + a*d)*(a + b*x)^3) + d/(2*(b*c - a*d)^2*(a + b*x)^2) - d^2/((
b*c - a*d)^3*(a + b*x)) - (d^3*Log[a + b*x])/(b*c - a*d)^4 + (d^3*Log[c + d
*x])/(b*c - a*d)^4
```

Maple [A] time = 0.051, size = 103, normalized size = 1.

$$\frac{d^3 \ln(dx+c)}{(ad-bc)^4} + \frac{1}{(3\,ad-3\,bc)\,(bx+a)^3} + \frac{d}{2\,(ad-bc)^2\,(bx+a)^2} - \frac{d^3 \ln(bx+a)}{(ad-bc)^4} + \frac{d^2}{(ad-bc)^3\,(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2),x)
```

```
[Out] d^3/(a*d-b*c)^4*ln(d*x+c)+1/3/(a*d-b*c)/(b*x+a)^3+1/2*d/(a*d-b*c)^2/(b*x+a)
^2-d^3/(a*d-b*c)^4*ln(b*x+a)+d^2/(a*d-b*c)^3/(b*x+a)
```

Maxima [B] time = 1.24775, size = 487, normalized size = 4.55

$$-\frac{d^3 \log(bx+a)}{b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4} + \frac{d^3 \log(dx+c)}{b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4} - \frac{}{6\left(a^3b^3c^3 - 3\,a^4b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="maxima")
```

```
[Out] -d^3*log(b*x + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*
d^3 + a^4*d^4) + d^3*log(d*x + c)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*
d^2 - 4*a^3*b*c*d^3 + a^4*d^4) - 1/6*(6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d
 + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*d +
 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3
*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*
d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3)*
x)
```

Fricas [B] time = 1.60064, size = 856, normalized size = 8.

$$-\frac{2\,b^3c^3 - 9\,ab^2c^2d + 18\,a^2bcd^2 - 11\,a^3d^3 + 6\left(b^3cd^2 - ab^2d^3\right)x^2 - 3\left(b^3c^2d - 6\,ab^2cd^2 + 5\,a^2bd^3\right)x + 6\left(b^3d^3\right.}{6\left(a^3b^4c^4 - 4\,a^4b^3c^3d + 6\,a^5b^2c^2d^2 - 4\,a^6bcd^3 + a^7d^4 + \left(b^7c^4 - 4\,ab^6c^3d + 6\,a^2b^5c^2d^2 - 4\,a^3b^4cd^3 + a^4b^3d^4\right)x^3 + 3\left(al\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="fricas")
```

```
[Out] -1/6*(2*b^3*c^3 - 9*a*b^2*c^2*d + 18*a^2*b*c*d^2 - 11*a^3*d^3 + 6*(b^3*c*d^
2 - a*b^2*d^3)*x^2 - 3*(b^3*c^2*d - 6*a*b^2*c*d^2 + 5*a^2*b*d^3)*x + 6*(b^3
*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(b^3
```

```
*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c))/(a^3*b^
4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4 + (b^
7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4)*
x^3 + 3*(a*b^6*c^4 - 4*a^2*b^5*c^3*d + 6*a^3*b^4*c^2*d^2 - 4*a^4*b^3*c*d^3
+ a^5*b^2*d^4)*x^2 + 3*(a^2*b^5*c^4 - 4*a^3*b^4*c^3*d + 6*a^4*b^3*c^2*d^2 -
 4*a^5*b^2*c*d^3 + a^6*b*d^4)*x)
```

Sympy [B] time = 2.87301, size = 570, normalized size = 5.33

$$\frac{d^3\log\left(x+\frac{-\frac{a^5d^8}{(ad-bc)^4}+\frac{5a^4bcd^7}{(ad-bc)^4}-\frac{10a^3b^2c^2d^6}{(ad-bc)^4}+\frac{10a^2b^3c^3d^5}{(ad-bc)^4}-\frac{5ab^4c^4d^4}{(ad-bc)^4}+ad^4+\frac{b^5c^5d^3}{(ad-bc)^4}+bcd^3}{2bd^4}\right)}{(ad-bc)^4}-\frac{d^3\log\left(x+\frac{\frac{a^5d^8}{(ad-bc)^4}-\frac{5a^4bcd^7}{(ad-bc)^4}+\frac{10a^3b^2c^2d^6}{(ad-bc)^4}-\frac{10a^2b^3c^3d^5}{(ad-bc)^4}+\frac{5ab^4c\cdots}{(ad-b\cdots}}{2bd^4}\cdots\right)}{(ad-bc)^4}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

```
[Out] d**3*log(x + (-a**5*d**8/(a*d - b*c)**4 + 5*a**4*b*c*d**7/(a*d - b*c)**4 -
10*a**3*b**2*c**2*d**6/(a*d - b*c)**4 + 10*a**2*b**3*c**3*d**5/(a*d - b*c)*
*4 - 5*a*b**4*c**4*d**4/(a*d - b*c)**4 + a*d**4 + b**5*c**5*d**3/(a*d - b*c
)**4 + b*c*d**3)/(2*b*d**4))/(a*d - b*c)**4 - d**3*log(x + (a**5*d**8/(a*d
- b*c)**4 - 5*a**4*b*c*d**7/(a*d - b*c)**4 + 10*a**3*b**2*c**2*d**6/(a*d -
b*c)**4 - 10*a**2*b**3*c**3*d**5/(a*d - b*c)**4 + 5*a*b**4*c**4*d**4/(a*d -
 b*c)**4 + a*d**4 - b**5*c**5*d**3/(a*d - b*c)**4 + b*c*d**3)/(2*b*d**4))/(
a*d - b*c)**4 + (11*a**2*d**2 - 7*a*b*c*d + 2*b**2*c**2 + 6*b**2*d**2*x**2
+ x*(15*a*b*d**2 - 3*b**2*c*d))/(6*a**6*d**3 - 18*a**5*b*c*d**2 + 18*a**4*b
**2*c**2*d - 6*a**3*b**3*c**3 + x**3*(6*a**3*b**3*d**3 - 18*a**2*b**4*c*d**
2 + 18*a*b**5*c**2*d - 6*b**6*c**3) + x**2*(18*a**4*b**2*d**3 - 54*a**3*b**
3*c*d**2 + 54*a**2*b**4*c**2*d - 18*a*b**5*c**3) + x*(18*a**5*b*d**3 - 54*a
**4*b**2*c*d**2 + 54*a**3*b**3*c**2*d - 18*a**2*b**4*c**3))
```

Giac [B] time = 1.2146, size = 328, normalized size = 3.07

$$-\frac{bd^3\log\left(|bx+a|\right)}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4}+\frac{d^4\log\left(|dx+c|\right)}{b^4c^4d-4\,ab^3c^3d^2+6\,a^2b^2c^2d^3-4\,a^3bcd^4+a^4d^5}-\frac{2\,b^3c^3-9\,ab^2c^2\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="giac")
```

```
[Out] -b*d^3*log(abs(b*x + a))/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a
^3*b^2*c*d^3 + a^4*b*d^4) + d^4*log(abs(d*x + c))/(b^4*c^4*d - 4*a*b^3*c^3*
d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5) - 1/6*(2*b^3*c^3 - 9*a*b
^2*c^2*d + 18*a^2*b*c*d^2 - 11*a^3*d^3 + 6*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 3*
(b^3*c^2*d - 6*a*b^2*c*d^2 + 5*a^2*b*d^3)*x)/((b*c - a*d)^4*(b*x + a)^3)
```

3.1809 $\int \frac{1}{(a+bx)^4\left(ac+(bc+ad)x+bdx^2\right)}\,dx$

Optimal. Leaf size=130

$$\frac{d^3}{(a+bx)(bc-ad)^4}-\frac{d^2}{2(a+bx)^2(bc-ad)^3}+\frac{d^4\log(a+bx)}{(bc-ad)^5}-\frac{d^4\log(c+dx)}{(bc-ad)^5}+\frac{d}{3(a+bx)^3(bc-ad)^2}-\frac{1}{4(a+bx)^4(bc-}$$

```
[Out] -1/(4*(b*c - a*d)*(a + b*x)^4) + d/(3*(b*c - a*d)^2*(a + b*x)^3) - d^2/(2*(
b*c - a*d)^3*(a + b*x)^2) + d^3/((b*c - a*d)^4*(a + b*x)) + (d^4*Log[a + b*
x])/(b*c - a*d)^5 - (d^4*Log[c + d*x])/(b*c - a*d)^5
```

Rubi [A] time = 0.0922863, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 44}

$$\frac{d^3}{(a+bx)(bc-ad)^4}-\frac{d^2}{2(a+bx)^2(bc-ad)^3}+\frac{d^4\log(a+bx)}{(bc-ad)^5}-\frac{d^4\log(c+dx)}{(bc-ad)^5}+\frac{d}{3(a+bx)^3(bc-ad)^2}-\frac{1}{4(a+bx)^4(bc-}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^4*(a*c + (b*c + a*d)*x + b*d*x^2)),x]
```

```
[Out] -1/(4*(b*c - a*d)*(a + b*x)^4) + d/(3*(b*c - a*d)^2*(a + b*x)^3) - d^2/(2*(
b*c - a*d)^3*(a + b*x)^2) + d^3/((b*c - a*d)^4*(a + b*x)) + (d^4*Log[a + b*
x])/(b*c - a*d)^5 - (d^4*Log[c + d*x])/(b*c - a*d)^5
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^4\left(ac+(bc+ad)x+bdx^2\right)}\,dx &= \int \frac{1}{(a+bx)^5(c+dx)}\,dx\\ &= \int \left(\frac{b}{(bc-ad)(a+bx)^5}-\frac{bd}{(bc-ad)^2(a+bx)^4}+\frac{bd^2}{(bc-ad)^3(a+bx)^3}-\frac{}{(bc-}\right.\\ &= -\frac{1}{4(bc-ad)(a+bx)^4}+\frac{d}{3(bc-ad)^2(a+bx)^3}-\frac{d^2}{2(bc-ad)^3(a+bx)^2}+\frac{}{(bc}\end{aligned}$$

Mathematica [A] time = 0.0507717, size = 130, normalized size = 1.

$$\frac{d^3}{(a+bx)(bc-ad)^4}-\frac{d^2}{2(a+bx)^2(bc-ad)^3}+\frac{d^4\log(a+bx)}{(bc-ad)^5}-\frac{d^4\log(c+dx)}{(bc-ad)^5}+\frac{d}{3(a+bx)^3(bc-ad)^2}+\frac{1}{4(a+bx)^4(ad-}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^4*(a*c + (b*c + a*d)*x + b*d*x^2)),x]
```

```
[Out] 1/(4*(-(b*c) + a*d)*(a + b*x)^4) + d/(3*(b*c - a*d)^2*(a + b*x)^3) - d^2/(2
*(b*c - a*d)^3*(a + b*x)^2) + d^3/((b*c - a*d)^4*(a + b*x)) + (d^4*Log[a +
b*x])/(b*c - a*d)^5 - (d^4*Log[c + d*x])/(b*c - a*d)^5
```

Maple [A] time = 0.052, size = 125, normalized size = 1.

$$\frac{d^4\ln(dx+c)}{(ad-bc)^5}+\frac{1}{(4ad-4bc)(bx+a)^4}+\frac{d}{3(ad-bc)^2(bx+a)^3}+\frac{d^3}{(ad-bc)^4(bx+a)}-\frac{d^4\ln(bx+a)}{(ad-bc)^5}+\frac{d^2}{2(ad-bc)^3(bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2),x)
```

```
[Out] d^4/(a*d-b*c)^5*ln(d*x+c)+1/4/(a*d-b*c)/(b*x+a)^4+1/3*d/(a*d-b*c)^2/(b*x+a)
^3+d^3/(a*d-b*c)^4/(b*x+a)-d^4/(a*d-b*c)^5*ln(b*x+a)+1/2*d^2/(a*d-b*c)^3/(b
*x+a)^2
```

Maxima [B] time = 1.27895, size = 753, normalized size = 5.79

$$\frac{d^4\log(bx+a)}{b^5c^5-5ab^4c^4d+10a^2b^3c^3d^2-10a^3b^2c^2d^3+5a^4bcd^4-a^5d^5}-\frac{d^4\log(dx+c)}{b^5c^5-5ab^4c^4d+10a^2b^3c^3d^2-10a^3b^2c^2d^3+5a^4bcd^4-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="maxima")
```

```
[Out] d^4*log(b*x + a)/(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2
*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5) - d^4*log(d*x + c)/(b^5*c^5 - 5*a*b^4*c
^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5) +
 1/12*(12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^
3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13
*a^2*b*d^3)*x)/(a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b
*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5
*c*d^3 + a^4*b^4*d^4)*x^4 + 4*(a*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*
d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4)*x^3 + 6*(a^2*b^6*c^4 - 4*a^3*b^5*c^3*d
 + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*d^3 + a^6*b^2*d^4)*x^2 + 4*(a^3*b^5*c^4
- 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*x)
```

Fricas [B] time = 1.73601, size = 1320, normalized size = 10.15

$$-\frac{3b^4c^4-16ab^3c^3d+36a^2b^2c^2d^2-48a^3bcd^3+25a^4d^4-12\left(b^4cd^3-ab^3d^4\right)x^3+6\left(b^4c^2\right.}{12\left(a^4b^5c^5-5a^5b^4c^4d+10a^6b^3c^3d^2-10a^7b^2c^2d^3+5a^8bcd^4-a^9d^5+\left(b^9c^5-5ab^8c^4d+10a^2b^7c^3d^2-10a^3b^6c^2d^3+5\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*b^4*c^4 - 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 - 48*a^3*b*c*d^3 + 2
5*a^4*d^4 - 12*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 6*(b^4*c^2*d^2 - 8*a*b^3*c*d^3
 + 7*a^2*b^2*d^4)*x^2 - 4*(b^4*c^3*d - 6*a*b^3*c^2*d^2 + 18*a^2*b^2*c*d^3 -
 13*a^3*b*d^4)*x - 12*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 +
4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 12*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 +
 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(d*x + c))/(a^4*b^5*c^5 -
5*a^5*b^4*c^4*d + 10*a^6*b^3*c^3*d^2 - 10*a^7*b^2*c^2*d^3 + 5*a^8*b*c*d^4 -
 a^9*d^5 + (b^9*c^5 - 5*a*b^8*c^4*d + 10*a^2*b^7*c^3*d^2 - 10*a^3*b^6*c^2*d
^3 + 5*a^4*b^5*c*d^4 - a^5*b^4*d^5)*x^4 + 4*(a*b^8*c^5 - 5*a^2*b^7*c^4*d +
10*a^3*b^6*c^3*d^2 - 10*a^4*b^5*c^2*d^3 + 5*a^5*b^4*c*d^4 - a^6*b^3*d^5)*x^
3 + 6*(a^2*b^7*c^5 - 5*a^3*b^6*c^4*d + 10*a^4*b^5*c^3*d^2 - 10*a^5*b^4*c^2*
d^3 + 5*a^6*b^3*c*d^4 - a^7*b^2*d^5)*x^2 + 4*(a^3*b^6*c^5 - 5*a^4*b^5*c^4*d
 + 10*a^5*b^4*c^3*d^2 - 10*a^6*b^3*c^2*d^3 + 5*a^7*b^2*c*d^4 - a^8*b*d^5)*x
)
```

Sympy [B] time = 4.33716, size = 802, normalized size = 6.17

$$\frac{d^4 \log\left(x + \frac{-\frac{a^6d^{10}}{(ad-bc)^5} + \frac{6a^5bcd^9}{(ad-bc)^5} - \frac{15a^4b^2c^2d^8}{(ad-bc)^5} + \frac{20a^3b^3c^3d^7}{(ad-bc)^5} - \frac{15a^2b^4c^4d^6}{(ad-bc)^5} + \frac{6ab^5c^5d^5}{(ad-bc)^5} + ad^5 - \frac{b^6c^6d^4}{(ad-bc)^5} + bcd^4}{2bd^5}\right)}{(ad-bc)^5} - \frac{d^4 \log\left(x + \frac{\frac{a^6d^{10}}{(ad-bc)^5} - \frac{6a^5bcd^9}{(ad-bc)^5} + \frac{15a^4b^2c^2d^8}{(ad-bc)^5} - 2}{}\right)}{(ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**4/(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

```
[Out] d**4*log(x + (-a**6*d**10/(a*d - b*c)**5 + 6*a**5*b*c*d**9/(a*d - b*c)**5 -
 15*a**4*b**2*c**2*d**8/(a*d - b*c)**5 + 20*a**3*b**3*c**3*d**7/(a*d - b*c)
**5 - 15*a**2*b**4*c**4*d**6/(a*d - b*c)**5 + 6*a*b**5*c**5*d**5/(a*d - b*c
)**5 + a*d**5 - b**6*c**6*d**4/(a*d - b*c)**5 + b*c*d**4)/(2*b*d**5))/(a*d
- b*c)**5 - d**4*log(x + (a**6*d**10/(a*d - b*c)**5 - 6*a**5*b*c*d**9/(a*d
- b*c)**5 + 15*a**4*b**2*c**2*d**8/(a*d - b*c)**5 - 20*a**3*b**3*c**3*d**7/
(a*d - b*c)**5 + 15*a**2*b**4*c**4*d**6/(a*d - b*c)**5 - 6*a*b**5*c**5*d**5
/(a*d - b*c)**5 + a*d**5 + b**6*c**6*d**4/(a*d - b*c)**5 + b*c*d**4)/(2*b*d
**5))/(a*d - b*c)**5 + (25*a**3*d**3 - 23*a**2*b*c*d**2 + 13*a*b**2*c**2*d
- 3*b**3*c**3 + 12*b**3*d**3*x**3 + x**2*(42*a*b**2*d**3 - 6*b**3*c*d**2) +
 x*(52*a**2*b*d**3 - 20*a*b**2*c*d**2 + 4*b**3*c**2*d))/(12*a**8*d**4 - 48*
a**7*b*c*d**3 + 72*a**6*b**2*c**2*d**2 - 48*a**5*b**3*c**3*d + 12*a**4*b**4
*c**4 + x**4*(12*a**4*b**4*d**4 - 48*a**3*b**5*c*d**3 + 72*a**2*b**6*c**2*d
**2 - 48*a*b**7*c**3*d + 12*b**8*c**4) + x**3*(48*a**5*b**3*d**4 - 192*a**4
*b**4*c*d**3 + 288*a**3*b**5*c**2*d**2 - 192*a**2*b**6*c**3*d + 48*a*b**7*c
**4) + x**2*(72*a**6*b**2*d**4 - 288*a**5*b**3*c*d**3 + 432*a**4*b**4*c**2*
d**2 - 288*a**3*b**5*c**3*d + 72*a**2*b**6*c**4) + x*(48*a**7*b*d**4 - 192*
a**6*b**2*c*d**3 + 288*a**5*b**3*c**2*d**2 - 192*a**4*b**4*c**3*d + 48*a**3
*b**5*c**4))
```

Giac [B] time = 1.19591, size = 456, normalized size = 3.51

$$\frac{bd^4\log\left(|bx+a|\right)}{b^6c^5 - 5\,ab^5c^4d + 10\,a^2b^4c^3d^2 - 10\,a^3b^3c^2d^3 + 5\,a^4b^2cd^4 - a^5bd^5} - \frac{d^5\log\left(|dx+c|\right)}{b^5c^5d - 5\,ab^4c^4d^2 + 10\,a^2b^3c^3d^3 - 10\,a^3b^2c^2d^4 + 5\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="giac")
```

```
[Out] b*d^4*log(abs(b*x + a))/(b^6*c^5 - 5*a*b^5*c^4*d + 10*a^2*b^4*c^3*d^2 - 10*
a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^5*b*d^5) - d^5*log(abs(d*x + c))/(b^5
*c^5*d - 5*a*b^4*c^4*d^2 + 10*a^2*b^3*c^3*d^3 - 10*a^3*b^2*c^2*d^4 + 5*a^4*
b*c*d^5 - a^5*d^6) - 1/12*(3*b^4*c^4 - 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2
- 48*a^3*b*c*d^3 + 25*a^4*d^4 - 12*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 6*(b^4*c^2
*d^2 - 8*a*b^3*c*d^3 + 7*a^2*b^2*d^4)*x^2 - 4*(b^4*c^3*d - 6*a*b^3*c^2*d^2
+ 18*a^2*b^2*c*d^3 - 13*a^3*b*d^4)*x)/((b*c - a*d)^5*(b*x + a)^4)
```

3.1810 $\int \frac{(a+bx)^6}{\left(ac+(bc+ad)x+bdx^2\right)^2}\,dx$

Optimal. Leaf size=104

$$-\frac{2b^3(c+dx)^2(bc-ad)}{d^5}+\frac{6b^2x(bc-ad)^2}{d^4}-\frac{(bc-ad)^4}{d^5(c+dx)}-\frac{4b(bc-ad)^3\log(c+dx)}{d^5}+\frac{b^4(c+dx)^3}{3d^5}$$

```
[Out] (6*b^2*(b*c - a*d)^2*x)/d^4 - (b*c - a*d)^4/(d^5*(c + d*x)) - (2*b^3*(b*c -
 a*d)*(c + d*x)^2)/d^5 + (b^4*(c + d*x)^3)/(3*d^5) - (4*b*(b*c - a*d)^3*Log
[c + d*x])/d^5
```

Rubi [A] time = 0.104787, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$-\frac{2b^3(c+dx)^2(bc-ad)}{d^5}+\frac{6b^2x(bc-ad)^2}{d^4}-\frac{(bc-ad)^4}{d^5(c+dx)}-\frac{4b(bc-ad)^3\log(c+dx)}{d^5}+\frac{b^4(c+dx)^3}{3d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^6/(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

```
[Out] (6*b^2*(b*c - a*d)^2*x)/d^4 - (b*c - a*d)^4/(d^5*(c + d*x)) - (2*b^3*(b*c -
 a*d)*(c + d*x)^2)/d^5 + (b^4*(c + d*x)^3)/(3*d^5) - (4*b*(b*c - a*d)^3*Log
[c + d*x])/d^5
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^6}{\left(ac+(bc+ad)x+bdx^2\right)^2}\,dx &= \int \frac{(a+bx)^4}{(c+dx)^2}\,dx\\ &= \int \left(\frac{6b^2(bc-ad)^2}{d^4}+\frac{(-bc+ad)^4}{d^4(c+dx)^2}-\frac{4b(bc-ad)^3}{d^4(c+dx)}-\frac{4b^3(bc-ad)(c+dx)}{d^4}+\frac{b^4(c+d\\ &= \frac{6b^2(bc-ad)^2x}{d^4}-\frac{(bc-ad)^4}{d^5(c+dx)}-\frac{2b^3(bc-ad)(c+dx)^2}{d^5}+\frac{b^4(c+dx)^3}{3d^5}-\frac{4b(bc-ad)^3}{d^5}\end{aligned}$$

Mathematica [A] time = 0.0602713, size = 165, normalized size = 1.59

$$\frac{18a^2b^2d^2\left(-c^2+cdx+d^2x^2\right)+12a^3bcd^3-3a^4d^4+6ab^3d\left(-4c^2dx+2c^3-3cd^2x^2+d^3x^3\right)-12b(c+dx)(bc-ad)^3\log}{3d^5(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^6/(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

```
[Out] (12*a^3*b*c*d^3 - 3*a^4*d^4 + 18*a^2*b^2*d^2*(-c^2 + c*d*x + d^2*x^2) + 6*a
*b^3*d*(2*c^3 - 4*c^2*d*x - 3*c*d^2*x^2 + d^3*x^3) + b^4*(-3*c^4 + 9*c^3*d*
x + 6*c^2*d^2*x^2 - 2*c*d^3*x^3 + d^4*x^4) - 12*b*(b*c - a*d)^3*(c + d*x)*L
og[c + d*x])/(3*d^5*(c + d*x))
```

Maple [B] time = 0.048, size = 230, normalized size = 2.2

$$\frac{b^4x^3}{3\,d^2}+2\,\frac{b^3x^2a}{d^2}-\frac{b^4x^2c}{d^3}+6\,\frac{a^2b^2x}{d^2}-8\,\frac{ab^3cx}{d^3}+3\,\frac{b^4c^2x}{d^4}+4\,\frac{b\ln(dx+c)\,a^3}{d^2}-12\,\frac{b^2\ln(dx+c)\,ca^2}{d^3}+12\,\frac{b^3\ln(dx+c)\,ac^2}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^6/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x)
```

```
[Out] 1/3*b^4/d^2*x^3+2*b^3/d^2*x^2*a-b^4/d^3*x^2*c+6*b^2/d^2*a^2*x-8*b^3/d^3*c*a
*x+3*b^4/d^4*c^2*x+4*b/d^2*ln(d*x+c)*a^3-12*b^2/d^3*ln(d*x+c)*c*a^2+12*b^3/
d^4*ln(d*x+c)*a*c^2-4*b^4/d^5*ln(d*x+c)*c^3-1/d/(d*x+c)*a^4+4/d^2/(d*x+c)*c
*a^3*b-6/d^3/(d*x+c)*a^2*b^2*c^2+4/d^4/(d*x+c)*a*b^3*c^3-1/d^5/(d*x+c)*b^4*
c^4
```

Maxima [A] time = 1.02825, size = 247, normalized size = 2.38

$$-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{d^6x+cd^5}+\frac{b^4d^2x^3-3\left(b^4cd-2\,ab^3d^2\right)x^2+3\left(3\,b^4c^2-8\,ab^3cd+6\,a^2b^2d^2\right)x}{3\,d^4}-\frac{4(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="maxima")
```

```
[Out] -(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(d
^6*x + c*d^5) + 1/3*(b^4*d^2*x^3 - 3*(b^4*c*d - 2*a*b^3*d^2)*x^2 + 3*(3*b^4
*c^2 - 8*a*b^3*c*d + 6*a^2*b^2*d^2)*x)/d^4 - 4*(b^4*c^3 - 3*a*b^3*c^2*d + 3
*a^2*b^2*c*d^2 - a^3*b*d^3)*log(d*x + c)/d^5
```

Fricas [B] time = 1.53139, size = 540, normalized size = 5.19

$$\frac{b^4d^4x^4-3\,b^4c^4+12\,ab^3c^3d-18\,a^2b^2c^2d^2+12\,a^3bcd^3-3\,a^4d^4-2\left(b^4cd^3-3\,ab^3d^4\right)x^3+6\left(b^4c^2d^2-3\,ab^3cd^3+3\,a^2b^2d^4\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*d^4*x^4 - 3*b^4*c^4 + 12*a*b^3*c^3*d - 18*a^2*b^2*c^2*d^2 + 12*a^3
*b*c*d^3 - 3*a^4*d^4 - 2*(b^4*c*d^3 - 3*a*b^3*d^4)*x^3 + 6*(b^4*c^2*d^2 - 3
*a*b^3*c*d^3 + 3*a^2*b^2*d^4)*x^2 + 3*(3*b^4*c^3*d - 8*a*b^3*c^2*d^2 + 6*a^
2*b^2*c*d^3)*x - 12*(b^4*c^4 - 3*a*b^3*c^3*d + 3*a^2*b^2*c^2*d^2 - a^3*b*c*
```

```
d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x)*log(d*
x + c))/(d^6*x + c*d^5)
```

Sympy [A] time = 1.27619, size = 151, normalized size = 1.45

$$\frac{b^4x^3}{3d^2}+\frac{4b(ad-bc)^3\log(c+dx)}{d^5}-\frac{a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4}{cd^5+d^6x}+\frac{x^2\left(2ab^3d-b^4c\right)}{d^3}+\frac{x\left(6a^2b^2d^2-8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**6/(a*c+(a*d+b*c)*x+b*d*x**2)**2,x)
```

```
[Out] b**4*x**3/(3*d**2) + 4*b*(a*d - b*c)**3*log(c + d*x)/d**5 - (a**4*d**4 - 4*
a**3*b*c*d**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4)/(c*d**
5 + d**6*x) + x**2*(2*a*b**3*d - b**4*c)/d**3 + x*(6*a**2*b**2*d**2 - 8*a*b
**3*c*d + 3*b**4*c**2)/d**4
```

Giac [A] time = 1.20685, size = 254, normalized size = 2.44

$$-\frac{4\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)\log(|dx+c|)}{d^5}+\frac{b^4d^4x^3-3\,b^4cd^3x^2+6\,ab^3d^4x^2+9\,b^4c^2d^2x-24\,ab^3cd^3x+1}{3\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="giac")
```

```
[Out] -4*(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*log(abs(d*x + c)
)/d^5 + 1/3*(b^4*d^4*x^3 - 3*b^4*c*d^3*x^2 + 6*a*b^3*d^4*x^2 + 9*b^4*c^2*d^
2*x - 24*a*b^3*c*d^3*x + 18*a^2*b^2*d^4*x)/d^6 - (b^4*c^4 - 4*a*b^3*c^3*d +
 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/((d*x + c)*d^5)
```

3.1811 $\int \frac{(a+bx)^5}{\left(ac+(bc+ad)x+bdx^2\right)^2}\,dx$

Optimal. Leaf size=75

$$-\frac{b^2x(2bc-3ad)}{d^3}+\frac{(bc-ad)^3}{d^4(c+dx)}+\frac{3b(bc-ad)^2\log(c+dx)}{d^4}+\frac{b^3x^2}{2d^2}$$

```
[Out] -((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^2)/(2*d^2) + (b*c - a*d)^3/(d^4*(c
+ d*x)) + (3*b*(b*c - a*d)^2*Log[c + d*x])/d^4
```

Rubi [A] time = 0.0664855, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$-\frac{b^2x(2bc-3ad)}{d^3}+\frac{(bc-ad)^3}{d^4(c+dx)}+\frac{3b(bc-ad)^2\log(c+dx)}{d^4}+\frac{b^3x^2}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

```
[Out] -((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^2)/(2*d^2) + (b*c - a*d)^3/(d^4*(c
+ d*x)) + (3*b*(b*c - a*d)^2*Log[c + d*x])/d^4
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^5}{\left(ac+(bc+ad)x+bdx^2\right)^2}\,dx &= \int \frac{(a+bx)^3}{(c+dx)^2}\,dx \\
&= \int \left(-\frac{b^2(2bc-3ad)}{d^3}+\frac{b^3x}{d^2}+\frac{(-bc+ad)^3}{d^3(c+dx)^2}+\frac{3b(bc-ad)^2}{d^3(c+dx)}\right)dx \\
&= -\frac{b^2(2bc-3ad)x}{d^3}+\frac{b^3x^2}{2d^2}+\frac{(bc-ad)^3}{d^4(c+dx)}+\frac{3b(bc-ad)^2\log(c+dx)}{d^4}
\end{aligned}$$

Mathematica [A] time = 0.0341325, size = 114, normalized size = 1.52

$$\frac{3a^2bcd^2-a^3d^3-3ab^2c^2d+b^3c^3}{d^4(c+dx)}+\frac{3\left(a^2bd^2-2ab^2cd+b^3c^2\right)\log(c+dx)}{d^4}-\frac{b^2x(2bc-3ad)}{d^3}+\frac{b^3x^2}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

```
[Out] -((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^2)/(2*d^2) + (b^3*c^3 - 3*a*b^2*c^2
*d + 3*a^2*b*c*d^2 - a^3*d^3)/(d^4*(c + d*x)) + (3*(b^3*c^2 - 2*a*b^2*c*d +
 a^2*b*d^2)*Log[c + d*x])/d^4
```

Maple [B] time = 0.046, size = 149, normalized size = 2.

$$\frac{b^3x^2}{2\,d^2}+3\,\frac{b^2ax}{d^2}-2\,\frac{b^3cx}{d^3}+3\,\frac{b\ln(dx+c)\,a^2}{d^2}-6\,\frac{b^2\ln(dx+c)\,ca}{d^3}+3\,\frac{b^3\ln(dx+c)\,c^2}{d^4}-\frac{a^3}{d\,(dx+c)}+3\,\frac{bca^2}{d^2\,(dx+c)}-3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x)
```

```
[Out] 1/2*b^3*x^2/d^2+3*b^2/d^2*a*x-2*b^3/d^3*x*c+3*b/d^2*ln(d*x+c)*a^2-6*b^2/d^3
*ln(d*x+c)*c*a+3*b^3/d^4*ln(d*x+c)*c^2-1/d/(d*x+c)*a^3+3/d^2/(d*x+c)*c*b*a^
2-3/d^3/(d*x+c)*a*c^2*b^2+1/d^4/(d*x+c)*b^3*c^3
```

Maxima [A] time = 1.05338, size = 158, normalized size = 2.11

$$\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{d^5x+cd^4}+\frac{b^3dx^2-2\left(2\,b^3c-3\,ab^2d\right)x}{2\,d^3}+\frac{3\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)\log(dx+c)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="maxima")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(d^5*x + c*d^4) + 1/2*(
b^3*d*x^2 - 2*(2*b^3*c - 3*a*b^2*d)*x)/d^3 + 3*(b^3*c^2 - 2*a*b^2*c*d + a^2
*b*d^2)*log(d*x + c)/d^4
```

Fricas [B] time = 1.52327, size = 354, normalized size = 4.72

$$\frac{b^3d^3x^3+2\,b^3c^3-6\,ab^2c^2d+6\,a^2bcd^2-2\,a^3d^3-3\left(b^3cd^2-2\,ab^2d^3\right)x^2-2\left(2\,b^3c^2d-3\,ab^2cd^2\right)x+6\left(b^3c^3-2\,ab^2c^2d+\right.}{2\left(d^5x+cd^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*d^3*x^3 + 2*b^3*c^3 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 2*a^3*d^3 -
3*(b^3*c*d^2 - 2*a*b^2*d^3)*x^2 - 2*(2*b^3*c^2*d - 3*a*b^2*c*d^2)*x + 6*(b^
3*c^3 - 2*a*b^2*c^2*d + a^2*b*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^
3)*x)*log(d*x + c))/(d^5*x + c*d^4)
```

Sympy [A] time = 1.13319, size = 100, normalized size = 1.33

$$\frac{b^3x^2}{2d^2} + \frac{3b(ad-bc)^2\log(c+dx)}{d^4} - \frac{a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - b^3c^3}{cd^4 + d^5x} + \frac{x(3ab^2d - 2b^3c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(a*c+(a*d+b*c)*x+b*d*x**2)**2,x)
```

```
[Out] b**3*x**2/(2*d**2) + 3*b*(a*d - b*c)**2*log(c + d*x)/d**4 - (a**3*d**3 - 3*
a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3)/(c*d**4 + d**5*x) + x*(3*a*b**
2*d - 2*b**3*c)/d**3
```

Giac [A] time = 1.17548, size = 159, normalized size = 2.12

$$\frac{3\left(b^3c^2 - 2\,ab^2cd + a^2bd^2\right)\log(|dx+c|)}{d^4} + \frac{b^3d^2x^2 - 4\,b^3cdx + 6\,ab^2d^2x}{2\,d^4} + \frac{b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3}{(dx+c)d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="giac")
```

```
[Out] 3*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*log(abs(d*x + c))/d^4 + 1/2*(b^3*d^2*
x^2 - 4*b^3*c*d*x + 6*a*b^2*d^2*x)/d^4 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b
*c*d^2 - a^3*d^3)/((d*x + c)*d^4)
```

3.1812 $\int \frac{(a+bx)^4}{\left(ac+(bc+ad)x+bdx^2\right)^2} dx$

Optimal. Leaf size=51

$$-\frac{(bc-ad)^2}{d^3(c+dx)}-\frac{2b(bc-ad)\log(c+dx)}{d^3}+\frac{b^2x}{d^2}$$

```
[Out] (b^2*x)/d^2 - (b*c - a*d)^2/(d^3*(c + d*x)) - (2*b*(b*c - a*d)*Log[c + d*x]
)/d^3
```

Rubi [A] time = 0.0434578, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$-\frac{(bc-ad)^2}{d^3(c+dx)}-\frac{2b(bc-ad)\log(c+dx)}{d^3}+\frac{b^2x}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

```
[Out] (b^2*x)/d^2 - (b*c - a*d)^2/(d^3*(c + d*x)) - (2*b*(b*c - a*d)*Log[c + d*x]
)/d^3
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{\left(ac+(bc+ad)x+bdx^2\right)^2} dx &= \int \frac{(a+bx)^2}{(c+dx)^2} dx \\ &= \int \left(\frac{b^2}{d^2}+\frac{(-bc+ad)^2}{d^2(c+dx)^2}-\frac{2b(bc-ad)}{d^2(c+dx)}\right) dx \\ &= \frac{b^2x}{d^2}-\frac{(bc-ad)^2}{d^3(c+dx)}-\frac{2b(bc-ad)\log(c+dx)}{d^3}\end{aligned}$$

Mathematica [A] time = 0.0345981, size = 47, normalized size = 0.92

$$\frac{-\frac{(bc-ad)^2}{c+dx}+2b(ad-bc)\log(c+dx)+b^2dx}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

```
[Out] (b^2*d*x - (b*c - a*d)^2/(c + d*x) + 2*b*(-(b*c) + a*d)*Log[c + d*x])/d^3
```

Maple [A] time = 0.044, size = 86, normalized size = 1.7

$$\frac{b^2x}{d^2}+2\,\frac{b\ln(dx+c)\,a}{d^2}-2\,\frac{b^2\ln(dx+c)\,c}{d^3}-\frac{a^2}{d\,(dx+c)}+2\,\frac{abc}{d^2\,(dx+c)}-\frac{b^2c^2}{d^3\,(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x)
```

```
[Out] b^2*x/d^2+2*b/d^2*ln(d*x+c)*a-2*b^2/d^3*ln(d*x+c)*c-1/d/(d*x+c)*a^2+2/d^2/(
d*x+c)*c*a*b-1/d^3/(d*x+c)*b^2*c^2
```

Maxima [A] time = 1.12773, size = 90, normalized size = 1.76

$$\frac{b^2x}{d^2}-\frac{b^2c^2-2\,abcd+a^2d^2}{d^4x+cd^3}-\frac{2\left(b^2c-abd\right)\log(dx+c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="maxima")
```

```
[Out] b^2*x/d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(d^4*x + c*d^3) - 2*(b^2*c - a*
b*d)*log(d*x + c)/d^3
```

Fricas [A] time = 1.47149, size = 184, normalized size = 3.61

$$\frac{b^2d^2x^2+b^2cdx-b^2c^2+2\,abcd-a^2d^2-2\left(b^2c^2-abcd+\left(b^2cd-abd^2\right)x\right)\log(dx+c)}{d^4x+cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^2 + b^2*c*d*x - b^2*c^2 + 2*a*b*c*d - a^2*d^2 - 2*(b^2*c^2 - a*b
*c*d + (b^2*c*d - a*b*d^2)*x)*log(d*x + c))/(d^4*x + c*d^3)
```

Sympy [A] time = 0.744323, size = 60, normalized size = 1.18

$$\frac{b^2x}{d^2}+\frac{2b\,(ad-bc)\log(c+dx)}{d^3}-\frac{a^2d^2-2abcd+b^2c^2}{cd^3+d^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(a*c+(a*d+b*c)*x+b*d*x**2)**2,x)
```

```
[Out] b**2*x/d**2 + 2*b*(a*d - b*c)*log(c + d*x)/d**3 - (a**2*d**2 - 2*a*b*c*d +
b**2*c**2)/(c*d**3 + d**4*x)
```

Giac [A] time = 1.26112, size = 88, normalized size = 1.73

$$\frac{b^2x}{d^2} - \frac{2\left(b^2c - abd\right)\log\left(|dx + c|\right)}{d^3} - \frac{b^2c^2 - 2\,abcd + a^2d^2}{(dx + c)d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="giac")
```

```
[Out] b^2*x/d^2 - 2*(b^2*c - a*b*d)*log(abs(d*x + c))/d^3 - (b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)/((d*x + c)*d^3)
```

3.1813 $\int \frac{(a+bx)^3}{\left(ac+(bc+ad)x+bdx^2\right)^2}\,dx$

Optimal. Leaf size=31

$$\frac{bc-ad}{d^2(c+dx)}+\frac{b\log(c+dx)}{d^2}$$

```
[Out] (b*c - a*d)/(d^2*(c + d*x)) + (b*Log[c + d*x])/d^2
```

Rubi [A] time = 0.0254543, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$\frac{bc-ad}{d^2(c+dx)}+\frac{b\log(c+dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

```
[Out] (b*c - a*d)/(d^2*(c + d*x)) + (b*Log[c + d*x])/d^2
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{\left(ac+(bc+ad)x+bdx^2\right)^2}\,dx &= \int \frac{a+bx}{(c+dx)^2}\,dx\\ &= \int \left(\frac{-bc+ad}{d(c+dx)^2}+\frac{b}{d(c+dx)}\right)dx\\ &= \frac{bc-ad}{d^2(c+dx)}+\frac{b\log(c+dx)}{d^2}\end{aligned}$$

Mathematica [A] time = 0.0101975, size = 31, normalized size = 1.

$$\frac{bc-ad}{d^2(c+dx)}+\frac{b\log(c+dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

```
[Out] (b*c - a*d)/(d^2*(c + d*x)) + (b*Log[c + d*x])/d^2
```

Maple [A] time = 0.044, size = 39, normalized size = 1.3

$$\frac{b\ln(dx+c)}{d^2} - \frac{a}{d(dx+c)} + \frac{bc}{d^2(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x)
```

```
[Out] b*ln(d*x+c)/d^2-1/d/(d*x+c)*a+1/d^2/(d*x+c)*b*c
```

Maxima [A] time = 1.04856, size = 46, normalized size = 1.48

$$\frac{bc-ad}{d^3x+cd^2} + \frac{b\log(dx+c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="maxima")
```

```
[Out] (b*c - a*d)/(d^3*x + c*d^2) + b*log(d*x + c)/d^2
```

Fricas [A] time = 1.58518, size = 78, normalized size = 2.52

$$\frac{bc-ad+(bdx+bc)\log(dx+c)}{d^3x+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="fricas")
```

```
[Out] (b*c - a*d + (b*d*x + b*c)*log(d*x + c))/(d^3*x + c*d^2)
```

Sympy [A] time = 0.44771, size = 27, normalized size = 0.87

$$\frac{b\log(c+dx)}{d^2} - \frac{ad-bc}{cd^2+d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(a*c+(a*d+b*c)*x+b*d*x**2)**2,x)
```

```
[Out] b*log(c + d*x)/d**2 - (a*d - b*c)/(c*d**2 + d**3*x)
```

Giac [A] time = 1.19807, size = 43, normalized size = 1.39

$$\frac{b\log(|dx+c|)}{d^2}+\frac{bc-ad}{(dx+c)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="giac")
```

```
[Out] b*log(abs(d*x + c))/d^2 + (b*c - a*d)/((d*x + c)*d^2)
```

3.1814 $\int \frac{(a+bx)^2}{\left(ac+(bc+ad)x+bdx^2\right)^2} dx$

Optimal. Leaf size=12

$$-\frac{1}{d(c+dx)}$$

```
[Out] -(1/(d*(c + d*x)))
```

Rubi [A] time = 0.0081565, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 32}

$$-\frac{1}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

```
[Out] -(1/(d*(c + d*x)))
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{\left(ac+(bc+ad)x+bdx^2\right)^2} dx &= \int \frac{1}{(c+dx)^2} dx\\ &= -\frac{1}{d(c+dx)}\end{aligned}$$

Mathematica [A] time = 0.0029981, size = 12, normalized size = 1.

$$-\frac{1}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

```
[Out] -(1/(d*(c + d*x)))
```

Maple [A] time = 0.037, size = 13, normalized size = 1.1

$$-\frac{1}{d\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x)
```

```
[Out] -1/d/(d*x+c)
```

Maxima [A] time = 1.02133, size = 18, normalized size = 1.5

$$-\frac{1}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/(d^2*x + c*d)
```

Fricas [A] time = 1.51142, size = 24, normalized size = 2.

$$-\frac{1}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/(d^2*x + c*d)
```

Sympy [A] time = 0.325532, size = 10, normalized size = 0.83

$$-\frac{1}{cd+d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(a*c+(a*d+b*c)*x+b*d*x**2)**2,x)
```

```
[Out] -1/(c*d + d**2*x)
```

Giac [A] time = 1.17501, size = 16, normalized size = 1.33

$$-\frac{1}{(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="giac")

[Out] -1/((d*x + c)*d)
```

3.1815 $\int \frac{a+bx}{\left(ac+(bc+ad)x+bdx^2\right)^2}\,dx$

Optimal. Leaf size=56

$$\frac{1}{(c+dx)(bc-ad)}+\frac{b\log(a+bx)}{(bc-ad)^2}-\frac{b\log(c+dx)}{(bc-ad)^2}$$

```
[Out] 1/((b*c - a*d)*(c + d*x)) + (b*Log[a + b*x])/(b*c - a*d)^2 - (b*Log[c + d*x
])/(b*c - a*d)^2
```

Rubi [A] time = 0.0316406, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {626, 44}

$$\frac{1}{(c+dx)(bc-ad)}+\frac{b\log(a+bx)}{(bc-ad)^2}-\frac{b\log(c+dx)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

```
[Out] 1/((b*c - a*d)*(c + d*x)) + (b*Log[a + b*x])/(b*c - a*d)^2 - (b*Log[c + d*x
])/(b*c - a*d)^2
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{\left(ac+(bc+ad)x+bdx^2\right)^2}\,dx &= \int \frac{1}{(a+bx)(c+dx)^2}\,dx\\ &= \int \left(\frac{b^2}{(bc-ad)^2(a+bx)}-\frac{d}{(bc-ad)(c+dx)^2}-\frac{bd}{(bc-ad)^2(c+dx)}\right)dx\\ &= \frac{1}{(bc-ad)(c+dx)}+\frac{b\log(a+bx)}{(bc-ad)^2}-\frac{b\log(c+dx)}{(bc-ad)^2}\end{aligned}$$

Mathematica [A] time = 0.0237145, size = 53, normalized size = 0.95

$$\frac{b(c+dx)\log(a+bx)-ad-b(c+dx)\log(c+dx)+bc}{(c+dx)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

```
[Out] (b*c - a*d + b*(c + d*x)*Log[a + b*x] - b*(c + d*x)*Log[c + d*x])/((b*c - a
*d)^2*(c + d*x))
```

Maple [A] time = 0.055, size = 58, normalized size = 1.

$$-\frac{1}{(ad-bc)(dx+c)}-\frac{b\ln(dx+c)}{(ad-bc)^2}+\frac{b\ln(bx+a)}{(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x)
```

```
[Out] -1/(a*d-b*c)/(d*x+c)-b/(a*d-b*c)^2*ln(d*x+c)+b/(a*d-b*c)^2*ln(b*x+a)
```

Maxima [A] time = 1.14036, size = 122, normalized size = 2.18

$$\frac{b\log(bx+a)}{b^2c^2-2\,abcd+a^2d^2}-\frac{b\log(dx+c)}{b^2c^2-2\,abcd+a^2d^2}+\frac{1}{bc^2-acd+\left(bcd-ad^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="maxima")
```

```
[Out] b*log(b*x + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - b*log(d*x + c)/(b^2*c^2 -
2*a*b*c*d + a^2*d^2) + 1/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)
```

Fricas [A] time = 1.65618, size = 198, normalized size = 3.54

$$\frac{bc-ad+(bdx+bc)\log(bx+a)-(bdx+bc)\log(dx+c)}{b^2c^3-2\,abc^2d+a^2cd^2+\left(b^2c^2d-2\,abcd^2+a^2d^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="fricas")
```

```
[Out] (b*c - a*d + (b*d*x + b*c)*log(b*x + a) - (b*d*x + b*c)*log(d*x + c))/(b^2*
c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x)
```

Sympy [B] time = 1.19887, size = 233, normalized size = 4.16

$$-\frac{b\log\left(x+\frac{-\frac{a^3bd^3}{(ad-bc)^2}+\frac{3a^2b^2cd^2}{(ad-bc)^2}-\frac{3ab^3c^2d}{(ad-bc)^2}+abd+\frac{b^4c^3}{(ad-bc)^2}+b^2c}{2b^2d}\right)}{(ad-bc)^2}+\frac{b\log\left(x+\frac{\frac{a^3bd^3}{(ad-bc)^2}-\frac{3a^2b^2cd^2}{(ad-bc)^2}+\frac{3ab^3c^2d}{(ad-bc)^2}+abd-\frac{b^4c^3}{(ad-bc)^2}+b^2c}{2b^2d}\right)}{(ad-bc)^2}-\frac{1}{acd-bc^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x**2)**2,x)
```

```
[Out] -b*log(x + (-a**3*b*d**3/(a*d - b*c)**2 + 3*a**2*b**2*c*d**2/(a*d - b*c)**2
 - 3*a*b**3*c**2*d/(a*d - b*c)**2 + a*b*d + b**4*c**3/(a*d - b*c)**2 + b**2
*c)/(2*b**2*d))/(a*d - b*c)**2 + b*log(x + (a**3*b*d**3/(a*d - b*c)**2 - 3*
a**2*b**2*c*d**2/(a*d - b*c)**2 + 3*a*b**3*c**2*d/(a*d - b*c)**2 + a*b*d -
b**4*c**3/(a*d - b*c)**2 + b**2*c)/(2*b**2*d))/(a*d - b*c)**2 - 1/(a*c*d -
b*c**2 + x*(a*d**2 - b*c*d))
```

Giac [A] time = 1.18335, size = 126, normalized size = 2.25

$$\frac{b^2\log(|bx+a|)}{b^3c^2-2\,ab^2cd+a^2bd^2}-\frac{bd\log(|dx+c|)}{b^2c^2d-2\,abcd^2+a^2d^3}+\frac{1}{(bc-ad)(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="giac")
```

```
[Out] b^2*log(abs(b*x + a))/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - b*d*log(abs(d*x
 + c))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3) + 1/((b*c - a*d)*(d*x + c))
```

3.1816 $\int \frac{1}{(ac+(bc+ad)x+bdx^2)^2}\,dx$

Optimal. Leaf size=86

$$-\frac{ad+bc+2bdx}{(bc-ad)^2\left(x(ad+bc)+ac+bdx^2\right)}-\frac{2bd\log(a+bx)}{(bc-ad)^3}+\frac{2bd\log(c+dx)}{(bc-ad)^3}$$

```
[Out] -((b*c + a*d + 2*b*d*x)/((b*c - a*d)^2*(a*c + (b*c + a*d)*x + b*d*x^2))) -
(2*b*d*Log[a + b*x])/(b*c - a*d)^3 + (2*b*d*Log[c + d*x])/(b*c - a*d)^3
```

Rubi [A] time = 0.0261368, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {614, 616, 31}

$$-\frac{ad+bc+2bdx}{(bc-ad)^2\left(x(ad+bc)+ac+bdx^2\right)}-\frac{2bd\log(a+bx)}{(bc-ad)^3}+\frac{2bd\log(c+dx)}{(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)^(-2),x]
```

```
[Out] -((b*c + a*d + 2*b*d*x)/((b*c - a*d)^2*(a*c + (b*c + a*d)*x + b*d*x^2))) -
(2*b*d*Log[a + b*x])/(b*c - a*d)^3 + (2*b*d*Log[c + d*x])/(b*c - a*d)^3
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(ac+(bc+ad)x+bdx^2\right)^2}\,dx &= -\frac{bc+ad+2bdx}{(bc-ad)^2\left(ac+(bc+ad)x+bdx^2\right)}-\frac{(2bd)\int \frac{1}{ac+(bc+ad)x+bdx^2}\,dx}{(bc-ad)^2}\\ &= -\frac{bc+ad+2bdx}{(bc-ad)^2\left(ac+(bc+ad)x+bdx^2\right)}+\frac{\left(2b^2d^2\right)\int \frac{1}{bc+bdx}\,dx}{(bc-ad)^3}-\frac{\left(2b^2d^2\right)\int \frac{1}{ad+bdx}\,dx}{(bc-ad)^3}\\ &= -\frac{bc+ad+2bdx}{(bc-ad)^2\left(ac+(bc+ad)x+bdx^2\right)}-\frac{2bd\log(a+bx)}{(bc-ad)^3}+\frac{2bd\log(c+dx)}{(bc-ad)^3}\end{aligned}$$

Mathematica [A] time = 0.0663112, size = 66, normalized size = 0.77

$$\frac{\frac{b(ad-bc)}{a+bx}+\frac{d(ad-bc)}{c+dx}-2bd\log(a+bx)+2bd\log(c+dx)}{(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^(-2),x]
```

```
[Out] ((b*(-(b*c) + a*d))/(a + b*x) + (d*(-(b*c) + a*d))/(c + d*x) - 2*b*d*Log[a
+ b*x] + 2*b*d*Log[c + d*x])/(b*c - a*d)^3
```

Maple [A] time = 0.051, size = 82, normalized size = 1.

$$-\frac{d}{(ad-bc)^2(dx+c)}-2\frac{bd\ln(dx+c)}{(ad-bc)^3}-\frac{b}{(ad-bc)^2(bx+a)}+2\frac{bd\ln(bx+a)}{(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x)
```

```
[Out] -d/(a*d-b*c)^2/(d*x+c)-2*d/(a*d-b*c)^3*b*ln(d*x+c)-b/(a*d-b*c)^2/(b*x+a)+2*
d/(a*d-b*c)^3*b*ln(b*x+a)
```

Maxima [B] time = 1.1048, size = 281, normalized size = 3.27

$$-\frac{2\,bd\log(bx+a)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}+\frac{2\,bd\log(dx+c)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}-\frac{2\,bdx+}{ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(b^3c^2d-2\,ab^2cd^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="maxima")
```

```
[Out] -2*b*d*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 2
*b*d*log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - (2*
b*d*x + b*c + a*d)/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*
a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d
^3)*x)
```

Fricas [B] time = 1.61919, size = 486, normalized size = 5.65

$$-\frac{b^2c^2-a^2d^2+2\left(b^2cd-abd^2\right)x+2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right)x\right)\log(bx+a)-2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right)x\right)}{ab^3c^4-3\,a^2b^2c^3d+3\,a^3bc^2d^2-a^4cd^3+\left(b^4c^3d-3\,ab^3c^2d^2+3\,a^2b^2cd^3-a^3bd^4\right)x^2+\left(b^4c^4-2\,ab^3c^3d+2\,a^3bcd^3-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="fricas")
```

```
[Out] -(b^2*c^2 - a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x + 2*(b^2*d^2*x^2 + a*b*c*d +
(b^2*c*d + a*b*d^2)*x)*log(b*x + a) - 2*(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d +
```

```
a*b*d^2)*x)*log(d*x + c))/(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 -
a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^
2 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*x)
```

Sympy [B] time = 1.93193, size = 405, normalized size = 4.71

$$-\frac{2bd\log\left(x+\frac{-\frac{2a^4bd^5}{(ad-bc)^3}+\frac{8a^3b^2cd^4}{(ad-bc)^3}-\frac{12a^2b^3c^2d^3}{(ad-bc)^3}+\frac{8ab^4c^3d^2}{(ad-bc)^3}+2abd^2-\frac{2b^5c^4d}{(ad-bc)^3}+2b^2cd}{4b^2d^2}\right)}{(ad-bc)^3}+\frac{2bd\log\left(x+\frac{\frac{2a^4bd^5}{(ad-bc)^3}-\frac{8a^3b^2cd^4}{(ad-bc)^3}+\frac{12a^2b^3c^2d^3}{(ad-bc)^3}-\frac{8ab^4c^3d^2}{(ad-bc)^3}+2abd}{4b^2d^2}\right.}{(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+(a*d+b*c)*x+b*d*x**2)**2,x)
```

```
[Out] -2*b*d*log(x + (-2*a**4*b*d**5/(a*d - b*c)**3 + 8*a**3*b**2*c*d**4/(a*d - b
*c)**3 - 12*a**2*b**3*c**2*d**3/(a*d - b*c)**3 + 8*a*b**4*c**3*d**2/(a*d -
b*c)**3 + 2*a*b*d**2 - 2*b**5*c**4*d/(a*d - b*c)**3 + 2*b**2*c*d)/(4*b**2*d
**2))/(a*d - b*c)**3 + 2*b*d*log(x + (2*a**4*b*d**5/(a*d - b*c)**3 - 8*a**3
*b**2*c*d**4/(a*d - b*c)**3 + 12*a**2*b**3*c**2*d**3/(a*d - b*c)**3 - 8*a*b
**4*c**3*d**2/(a*d - b*c)**3 + 2*a*b*d**2 + 2*b**5*c**4*d/(a*d - b*c)**3 +
2*b**2*c*d)/(4*b**2*d**2))/(a*d - b*c)**3 - (a*d + b*c + 2*b*d*x)/(a**3*c*d
**2 - 2*a**2*b*c**2*d + a*b**2*c**3 + x**2*(a**2*b*d**3 - 2*a*b**2*c*d**2 +
b**3*c**2*d) + x*(a**3*d**3 - a**2*b*c*d**2 - a*b**2*c**2*d + b**3*c**3))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1817 $\int \frac{1}{(a+bx)\left(ac+(bc+ad)x+bdx^2\right)^2}\,dx$

Optimal. Leaf size=109

$$\frac{d^2}{(c+dx)(bc-ad)^3}+\frac{3bd^2\log(a+bx)}{(bc-ad)^4}-\frac{3bd^2\log(c+dx)}{(bc-ad)^4}+\frac{2bd}{(a+bx)(bc-ad)^3}-\frac{b}{2(a+bx)^2(bc-ad)^2}$$

```
[Out] -b/(2*(b*c - a*d)^2*(a + b*x)^2) + (2*b*d)/((b*c - a*d)^3*(a + b*x)) + d^2/
((b*c - a*d)^3*(c + d*x)) + (3*b*d^2*Log[a + b*x])/(b*c - a*d)^4 - (3*b*d^2
*Log[c + d*x])/(b*c - a*d)^4
```

Rubi [A] time = 0.0778673, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 44}

$$\frac{d^2}{(c+dx)(bc-ad)^3}+\frac{3bd^2\log(a+bx)}{(bc-ad)^4}-\frac{3bd^2\log(c+dx)}{(bc-ad)^4}+\frac{2bd}{(a+bx)(bc-ad)^3}-\frac{b}{2(a+bx)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(a*c + (b*c + a*d)*x + b*d*x^2)^2),x]
```

```
[Out] -b/(2*(b*c - a*d)^2*(a + b*x)^2) + (2*b*d)/((b*c - a*d)^3*(a + b*x)) + d^2/
((b*c - a*d)^3*(c + d*x)) + (3*b*d^2*Log[a + b*x])/(b*c - a*d)^4 - (3*b*d^2
*Log[c + d*x])/(b*c - a*d)^4
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx)\left(ac+(bc+ad)x+bdx^2\right)^2}\,dx=\int \frac{1}{(a+bx)^3(c+dx)^2}\,dx$$
$$=\int \left(\frac{b^2}{(bc-ad)^2(a+bx)^3}-\frac{2b^2d}{(bc-ad)^3(a+bx)^2}+\frac{3b^2d^2}{(bc-ad)^4(a+bx)}-\frac{\cdots}{(bc-ad\cdots}\right.$$
$$=-\frac{b}{2(bc-ad)^2(a+bx)^2}+\frac{2bd}{(bc-ad)^3(a+bx)}+\frac{d^2}{(bc-ad)^3(c+dx)}+\frac{3bd^2\log(\cdots}{(bc-\cdots}$$

Mathematica [A] time = 0.0723044, size = 98, normalized size = 0.9

$$\frac{\frac{2d^2(bc-ad)}{c+dx}+\frac{4bd(bc-ad)}{a+bx}-\frac{b(bc-ad)^2}{(a+bx)^2}+6bd^2\log(a+bx)-6bd^2\log(c+dx)}{2(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(a*c + (b*c + a*d)*x + b*d*x^2)^2),x]
```

```
[Out] (-((b*(b*c - a*d)^2)/(a + b*x)^2) + (4*b*d*(b*c - a*d))/(a + b*x) + (2*d^2*
(b*c - a*d))/(c + d*x) + 6*b*d^2*Log[a + b*x] - 6*b*d^2*Log[c + d*x])/(2*(b
*c - a*d)^4)
```

Maple [A] time = 0.052, size = 109, normalized size = 1.

$$-\frac{d^2}{(ad-bc)^3(dx+c)}-3\frac{d^2b\ln(dx+c)}{(ad-bc)^4}-\frac{b}{2(ad-bc)^2(bx+a)^2}+3\frac{d^2b\ln(bx+a)}{(ad-bc)^4}-2\frac{bd}{(ad-bc)^3(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x)
```

```
[Out] -d^2/(a*d-b*c)^3/(d*x+c)-3*d^2/(a*d-b*c)^4*b*ln(d*x+c)-1/2*b/(a*d-b*c)^2/(b
*x+a)^2+3*d^2/(a*d-b*c)^4*b*ln(b*x+a)-2*b/(a*d-b*c)^3*d/(b*x+a)
```

Maxima [B] time = 1.1036, size = 521, normalized size = 4.78

$$\frac{3\,bd^2\log(bx+a)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}-\frac{3\,bd^2\log(dx+c)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}+\frac{}{2\left(a^2b^3c^4-3\,a^3b^2c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="maxima")
```

```
[Out] 3*b*d^2*log(b*x + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b
*c*d^3 + a^4*d^4) - 3*b*d^2*log(d*x + c)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b
^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) + 1/2*(6*b^2*d^2*x^2 - b^2*c^2 + 5*a*
b*c*d + 2*a^2*d^2 + 3*(b^2*c*d + 3*a*b*d^2)*x)/(a^2*b^3*c^4 - 3*a^3*b^2*c^3
*d + 3*a^4*b*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3
*c*d^3 - a^3*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*
a^3*b^2*c*d^3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b
^2*c^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*x)
```

Fricas [B] time = 1.61282, size = 991, normalized size = 9.09

$$-\frac{b^3c^3-6\,ab^2c^2d+3\,a^2bcd^2+2\,a^3d^3-6\left(b^3cd^2-ab^2d^3\right)x^2-3\left(b^3c^2d+2\,ab^2cd^2-3\,a^2bd^3\right)x-6\left(b^3d^3x^3+a^2bcd^2+\right.}{2\left(a^2b^4c^5-4\,a^3b^3c^4d+6\,a^4b^2c^3d^2-4\,a^5bc^2d^3+a^6cd^4+\left(b^6c^4d-4\,ab^5c^3d^2+6\,a^2b^4c^2d^3-4\,a^3b^3cd^4+a^4b^2d^5\right)x^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^3*c^3 - 6*a*b^2*c^2*d + 3*a^2*b*c*d^2 + 2*a^3*d^3 - 6*(b^3*c*d^2 -
a*b^2*d^3)*x^2 - 3*(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*x - 6*(b^3*d^3
*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b
```

```
*d^3)*x)*log(b*x + a) + 6*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2
*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(d*x + c))/(a^2*b^4*c^5 - 4*a
^3*b^3*c^4*d + 6*a^4*b^2*c^3*d^2 - 4*a^5*b*c^2*d^3 + a^6*c*d^4 + (b^6*c^4*d
 - 4*a*b^5*c^3*d^2 + 6*a^2*b^4*c^2*d^3 - 4*a^3*b^3*c*d^4 + a^4*b^2*d^5)*x^3
 + (b^6*c^5 - 2*a*b^5*c^4*d - 2*a^2*b^4*c^3*d^2 + 8*a^3*b^3*c^2*d^3 - 7*a^4
*b^2*c*d^4 + 2*a^5*b*d^5)*x^2 + (2*a*b^5*c^5 - 7*a^2*b^4*c^4*d + 8*a^3*b^3*
c^3*d^2 - 2*a^4*b^2*c^2*d^3 - 2*a^5*b*c*d^4 + a^6*d^5)*x)
```

Sympy [B] time = 3.87319, size = 632, normalized size = 5.8

$$-\frac{3bd^2\log\left(x+\frac{-\frac{3a^5bd^7}{(ad-bc)^4}+\frac{15a^4b^2cd^6}{(ad-bc)^4}-\frac{30a^3b^3c^2d^5}{(ad-bc)^4}+\frac{30a^2b^4c^3d^4}{(ad-bc)^4}-\frac{15ab^5c^4d^3}{(ad-bc)^4}+3abd^3+\frac{3b^6c^5d^2}{(ad-bc)^4}+3b^2cd^2}{6b^2d^3}\right)}{(ad-bc)^4}+\frac{3bd^2\log\left(x+\frac{\frac{3a^5bd^7}{(ad-bc)^4}-\frac{15a^4b^2cd^6}{(ad-bc)^4}+\frac{30a^3b^3c^2d^5}{(ad-bc)^4}-\cdots}{\cdots}\right)}{(ad-\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x**2)**2,x)
```

```
[Out] -3*b*d**2*log(x + (-3*a**5*b*d**7/(a*d - b*c)**4 + 15*a**4*b**2*c*d**6/(a*d
 - b*c)**4 - 30*a**3*b**3*c**2*d**5/(a*d - b*c)**4 + 30*a**2*b**4*c**3*d**4
/(a*d - b*c)**4 - 15*a*b**5*c**4*d**3/(a*d - b*c)**4 + 3*a*b*d**3 + 3*b**6*
c**5*d**2/(a*d - b*c)**4 + 3*b**2*c*d**2)/(6*b**2*d**3))/(a*d - b*c)**4 + 3
*b*d**2*log(x + (3*a**5*b*d**7/(a*d - b*c)**4 - 15*a**4*b**2*c*d**6/(a*d -
b*c)**4 + 30*a**3*b**3*c**2*d**5/(a*d - b*c)**4 - 30*a**2*b**4*c**3*d**4/(a
*d - b*c)**4 + 15*a*b**5*c**4*d**3/(a*d - b*c)**4 + 3*a*b*d**3 - 3*b**6*c**
5*d**2/(a*d - b*c)**4 + 3*b**2*c*d**2)/(6*b**2*d**3))/(a*d - b*c)**4 - (2*a
**2*d**2 + 5*a*b*c*d - b**2*c**2 + 6*b**2*d**2*x**2 + x*(9*a*b*d**2 + 3*b**
2*c*d))/(2*a**5*c*d**3 - 6*a**4*b*c**2*d**2 + 6*a**3*b**2*c**3*d - 2*a**2*b
**3*c**4 + x**3*(2*a**3*b**2*d**4 - 6*a**2*b**3*c*d**3 + 6*a*b**4*c**2*d**2
 - 2*b**5*c**3*d) + x**2*(4*a**4*b*d**4 - 10*a**3*b**2*c*d**3 + 6*a**2*b**3
*c**2*d**2 + 2*a*b**4*c**3*d - 2*b**5*c**4) + x*(2*a**5*d**4 - 2*a**4*b*c*d
**3 - 6*a**3*b**2*c**2*d**2 + 10*a**2*b**3*c**3*d - 4*a*b**4*c**4))
```

Giac [B] time = 1.21765, size = 342, normalized size = 3.14

$$\frac{3\,b^2d^2\log\left(|bx+a|\right)}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4}-\frac{3\,bd^3\log\left(|dx+c|\right)}{b^4c^4d-4\,ab^3c^3d^2+6\,a^2b^2c^2d^3-4\,a^3bcd^4+a^4d^5}-\frac{b^3c^3-6\,ab^2c^2d+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="giac")
```

```
[Out] 3*b^2*d^2*log(abs(b*x + a))/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 -
4*a^3*b^2*c*d^3 + a^4*b*d^4) - 3*b*d^3*log(abs(d*x + c))/(b^4*c^4*d - 4*a*b
^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5) - 1/2*(b^3*c^3 -
6*a*b^2*c^2*d + 3*a^2*b*c*d^2 + 2*a^3*d^3 - 6*(b^3*c*d^2 - a*b^2*d^3)*x^2 -
 3*(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*x)/((b*c - a*d)^4*(b*x + a)^2*
(d*x + c))
```

3.1818 $\int \frac{(a+bx)^8}{\left(ac+(bc+ad)x+bdx^2\right)^3}\,dx$

Optimal. Leaf size=133

$$-\frac{5b^4(c+dx)^2(bc-ad)}{2d^6}+\frac{10b^3x(bc-ad)^2}{d^5}-\frac{10b^2(bc-ad)^3\log(c+dx)}{d^6}-\frac{5b(bc-ad)^4}{d^6(c+dx)}+\frac{(bc-ad)^5}{2d^6(c+dx)^2}+\frac{b^5(c+dx)^3}{3d^6}$$

```
[Out] (10*b^3*(b*c - a*d)^2*x)/d^5 + (b*c - a*d)^5/(2*d^6*(c + d*x)^2) - (5*b*(b*
c - a*d)^4)/(d^6*(c + d*x)) - (5*b^4*(b*c - a*d)*(c + d*x)^2)/(2*d^6) + (b^
5*(c + d*x)^3)/(3*d^6) - (10*b^2*(b*c - a*d)^3*Log[c + d*x])/d^6
```

Rubi [A] time = 0.135829, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$-\frac{5b^4(c+dx)^2(bc-ad)}{2d^6}+\frac{10b^3x(bc-ad)^2}{d^5}-\frac{10b^2(bc-ad)^3\log(c+dx)}{d^6}-\frac{5b(bc-ad)^4}{d^6(c+dx)}+\frac{(bc-ad)^5}{2d^6(c+dx)^2}+\frac{b^5(c+dx)^3}{3d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^8/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

```
[Out] (10*b^3*(b*c - a*d)^2*x)/d^5 + (b*c - a*d)^5/(2*d^6*(c + d*x)^2) - (5*b*(b*
c - a*d)^4)/(d^6*(c + d*x)) - (5*b^4*(b*c - a*d)*(c + d*x)^2)/(2*d^6) + (b^
5*(c + d*x)^3)/(3*d^6) - (10*b^2*(b*c - a*d)^3*Log[c + d*x])/d^6
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^8}{\left(ac+(bc+ad)x+bdx^2\right)^3}\,dx &= \int \frac{(a+bx)^5}{(c+dx)^3}\,dx\\ &= \int \left(\frac{10b^3(bc-ad)^2}{d^5}+\frac{(-bc+ad)^5}{d^5(c+dx)^3}+\frac{5b(bc-ad)^4}{d^5(c+dx)^2}-\frac{10b^2(bc-ad)^3}{d^5(c+dx)}-\frac{5b^4(bc-ad)(\right.\\ &= \frac{10b^3(bc-ad)^2x}{d^5}+\frac{(bc-ad)^5}{2d^6(c+dx)^2}-\frac{5b(bc-ad)^4}{d^6(c+dx)}-\frac{5b^4(bc-ad)(c+dx)^2}{2d^6}+\frac{b^5(c+d}{3d^6}\end{aligned}$$

Mathematica [A] time = 0.0803342, size = 230, normalized size = 1.73

$$30a^2b^3d^2\left(-4c^2dx-5c^3+4cd^2x^2+2d^3x^3\right)+30a^3b^2cd^3(3c+4dx)-15a^4bd^4(c+2dx)-3a^5d^5+15ab^4d\left(-11c^2d^2x^2+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^8/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

```
[Out] (-3*a^5*d^5 - 15*a^4*b*d^4*(c + 2*d*x) + 30*a^3*b^2*c*d^3*(3*c + 4*d*x) + 3
0*a^2*b^3*d^2*(-5*c^3 - 4*c^2*d*x + 4*c*d^2*x^2 + 2*d^3*x^3) + 15*a*b^4*d*(
7*c^4 + 2*c^3*d*x - 11*c^2*d^2*x^2 - 4*c*d^3*x^3 + d^4*x^4) + b^5*(-27*c^5
+ 6*c^4*d*x + 63*c^3*d^2*x^2 + 20*c^2*d^3*x^3 - 5*c*d^4*x^4 + 2*d^5*x^5) -
60*b^2*(b*c - a*d)^3*(c + d*x)^2*Log[c + d*x])/(6*d^6*(c + d*x)^2)
```

Maple [B] time = 0.051, size = 346, normalized size = 2.6

$$\frac{b^5x^3}{3d^3}+\frac{5b^4x^2a}{2d^3}-\frac{3b^5x^2c}{2d^4}+10\frac{a^2b^3x}{d^3}-15\frac{ab^4cx}{d^4}+6\frac{b^5c^2x}{d^5}+10\frac{b^2\ln(dx+c)a^3}{d^3}-30\frac{b^3\ln(dx+c)ca^2}{d^4}+30\frac{b^4\ln(dx+}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^8/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x)
```

```
[Out] 1/3*b^5/d^3*x^3+5/2*b^4/d^3*x^2*a-3/2*b^5/d^4*x^2*c+10*b^3/d^3*a^2*x-15*b^4
/d^4*c*a*x+6*b^5/d^5*c^2*x+10*b^2/d^3*ln(d*x+c)*a^3-30*b^3/d^4*ln(d*x+c)*c*
a^2+30*b^4/d^5*ln(d*x+c)*a*c^2-10*b^5/d^6*ln(d*x+c)*c^3-1/2/d/(d*x+c)^2*a^5
+5/2/d^2/(d*x+c)^2*c*a^4*b-5/d^3/(d*x+c)^2*c^2*a^3*b^2+5/d^4/(d*x+c)^2*a^2*
b^3*c^3-5/2/d^5/(d*x+c)^2*a*b^4*c^4+1/2/d^6/(d*x+c)^2*b^5*c^5-5*b/d^2/(d*x+
c)*a^4+20*b^2/d^3/(d*x+c)*a^3*c-30*b^3/d^4/(d*x+c)*a^2*c^2+20*b^4/d^5/(d*x+
c)*a*c^3-5*b^5/d^6/(d*x+c)*c^4
```

Maxima [B] time = 1.11074, size = 366, normalized size = 2.75

$$-\frac{9b^5c^5-35ab^4c^4d+50a^2b^3c^3d^2-30a^3b^2c^2d^3+5a^4bcd^4+a^5d^5+10\left(b^5c^4d-4ab^4c^3d^2+6a^2b^3c^2d^3-4a^3b^2cd^4+a^4b}{2\left(d^8x^2+2cd^7x+c^2d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(9*b^5*c^5 - 35*a*b^4*c^4*d + 50*a^2*b^3*c^3*d^2 - 30*a^3*b^2*c^2*d^3
+ 5*a^4*b*c*d^4 + a^5*d^5 + 10*(b^5*c^4*d - 4*a*b^4*c^3*d^2 + 6*a^2*b^3*c^2
*d^3 - 4*a^3*b^2*c*d^4 + a^4*b*d^5)*x)/(d^8*x^2 + 2*c*d^7*x + c^2*d^6) + 1/
6*(2*b^5*d^2*x^3 - 3*(3*b^5*c*d - 5*a*b^4*d^2)*x^2 + 6*(6*b^5*c^2 - 15*a*b^
4*c*d + 10*a^2*b^3*d^2)*x)/d^5 - 10*(b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*
d^2 - a^3*b^2*d^3)*log(d*x + c)/d^6
```

Fricas [B] time = 1.59612, size = 840, normalized size = 6.32

$$\frac{2b^5d^5x^5-27b^5c^5+105ab^4c^4d-150a^2b^3c^3d^2+90a^3b^2c^2d^3-15a^4bcd^4-3a^5d^5-5\left(b^5cd^4-3ab^4d^5\right)x^4+20\left(b^5c^2d^3-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^5*d^5*x^5 - 27*b^5*c^5 + 105*a*b^4*c^4*d - 150*a^2*b^3*c^3*d^2 + 9
0*a^3*b^2*c^2*d^3 - 15*a^4*b*c*d^4 - 3*a^5*d^5 - 5*(b^5*c*d^4 - 3*a*b^4*d^5
)*x^4 + 20*(b^5*c^2*d^3 - 3*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + 3*(21*b^5*c^
3*d^2 - 55*a*b^4*c^2*d^3 + 40*a^2*b^3*c*d^4)*x^2 + 6*(b^5*c^4*d + 5*a*b^4*c
^3*d^2 - 20*a^2*b^3*c^2*d^3 + 20*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x - 60*(b^5*c
^5 - 3*a*b^4*c^4*d + 3*a^2*b^3*c^3*d^2 - a^3*b^2*c^2*d^3 + (b^5*c^3*d^2 - 3
*a*b^4*c^2*d^3 + 3*a^2*b^3*c*d^4 - a^3*b^2*d^5)*x^2 + 2*(b^5*c^4*d - 3*a*b^
4*c^3*d^2 + 3*a^2*b^3*c^2*d^3 - a^3*b^2*c*d^4)*x)*log(d*x + c))/(d^8*x^2 +
2*c*d^7*x + c^2*d^6)
```

Sympy [B] time = 3.39264, size = 253, normalized size = 1.9

$$\frac{b^5x^3}{3d^3} + \frac{10b^2\left(ad-bc\right)^3\log\left(c+dx\right)}{d^6} - \frac{a^5d^5+5a^4bcd^4-30a^3b^2c^2d^3+50a^2b^3c^3d^2-35ab^4c^4d+9b^5c^5+x\left(10a^4bd^5-4\right.}{2c^2d^6+4cd^7x+2d^8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**8/(a*c+(a*d+b*c)*x+b*d*x**2)**3,x)
```

```
[Out] b**5*x**3/(3*d**3) + 10*b**2*(a*d - b*c)**3*log(c + d*x)/d**6 - (a**5*d**5
+ 5*a**4*b*c*d**4 - 30*a**3*b**2*c**2*d**3 + 50*a**2*b**3*c**3*d**2 - 35*a*
b**4*c**4*d + 9*b**5*c**5 + x*(10*a**4*b*d**5 - 40*a**3*b**2*c*d**4 + 60*a*
*2*b**3*c**2*d**3 - 40*a*b**4*c**3*d**2 + 10*b**5*c**4*d))/(2*c**2*d**6 + 4
*c*d**7*x + 2*d**8*x**2) + x**2*(5*a*b**4*d - 3*b**5*c)/(2*d**4) + x*(10*a*
*2*b**3*d**2 - 15*a*b**4*c*d + 6*b**5*c**2)/d**5
```

Giac [B] time = 1.25914, size = 356, normalized size = 2.68

$$-\frac{10\left(b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2-a^3b^2d^3\right)\log\left(|dx+c|\right)}{d^6} - \frac{9\,b^5c^5-35\,ab^4c^4d+50\,a^2b^3c^3d^2-30\,a^3b^2c^2d^3+5\,a^4bcd^4}{2\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="giac")
```

```
[Out] -10*(b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*log(abs(d*x +
 c))/d^6 - 1/2*(9*b^5*c^5 - 35*a*b^4*c^4*d + 50*a^2*b^3*c^3*d^2 - 30*a^3*b^
2*c^2*d^3 + 5*a^4*b*c*d^4 + a^5*d^5 + 10*(b^5*c^4*d - 4*a*b^4*c^3*d^2 + 6*a
^2*b^3*c^2*d^3 - 4*a^3*b^2*c*d^4 + a^4*b*d^5)*x)/((d*x + c)^2*d^6) + 1/6*(2
*b^5*d^6*x^3 - 9*b^5*c*d^5*x^2 + 15*a*b^4*d^6*x^2 + 36*b^5*c^2*d^4*x - 90*a
*b^4*c*d^5*x + 60*a^2*b^3*d^6*x)/d^9
```

3.1819 $\int \frac{(a+bx)^7}{\left(ac+(bc+ad)x+bdx^2\right)^3}\,dx$

Optimal. Leaf size=103

$$-\frac{b^3x(3bc-4ad)}{d^4}+\frac{6b^2(bc-ad)^2\log(c+dx)}{d^5}+\frac{4b(bc-ad)^3}{d^5(c+dx)}-\frac{(bc-ad)^4}{2d^5(c+dx)^2}+\frac{b^4x^2}{2d^3}$$

```
[Out] -((b^3*(3*b*c - 4*a*d)*x)/d^4) + (b^4*x^2)/(2*d^3) - (b*c - a*d)^4/(2*d^5*(
c + d*x)^2) + (4*b*(b*c - a*d)^3)/(d^5*(c + d*x)) + (6*b^2*(b*c - a*d)^2*Lo
g[c + d*x])/d^5
```

Rubi [A] time = 0.0881793, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$-\frac{b^3x(3bc-4ad)}{d^4}+\frac{6b^2(bc-ad)^2\log(c+dx)}{d^5}+\frac{4b(bc-ad)^3}{d^5(c+dx)}-\frac{(bc-ad)^4}{2d^5(c+dx)^2}+\frac{b^4x^2}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

```
[Out] -((b^3*(3*b*c - 4*a*d)*x)/d^4) + (b^4*x^2)/(2*d^3) - (b*c - a*d)^4/(2*d^5*(
c + d*x)^2) + (4*b*(b*c - a*d)^3)/(d^5*(c + d*x)) + (6*b^2*(b*c - a*d)^2*Lo
g[c + d*x])/d^5
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^7}{\left(ac+(bc+ad)x+bdx^2\right)^3}\,dx &= \int \frac{(a+bx)^4}{(c+dx)^3}\,dx\\
&= \int \left(-\frac{b^3(3bc-4ad)}{d^4}+\frac{b^4x}{d^3}+\frac{(-bc+ad)^4}{d^4(c+dx)^3}-\frac{4b(bc-ad)^3}{d^4(c+dx)^2}+\frac{6b^2(bc-ad)^2}{d^4(c+dx)}\right)dx\\
&= -\frac{b^3(3bc-4ad)x}{d^4}+\frac{b^4x^2}{2d^3}-\frac{(bc-ad)^4}{2d^5(c+dx)^2}+\frac{4b(bc-ad)^3}{d^5(c+dx)}+\frac{6b^2(bc-ad)^2\log(c+dx)}{d^5}
\end{aligned}$$

Mathematica [A] time = 0.0540216, size = 167, normalized size = 1.62

$$\frac{6a^2b^2cd^2(3c+4dx)-4a^3bd^3(c+2dx)-a^4d^4+4ab^3d\left(-4c^2dx-5c^3+4cd^2x^2+2d^3x^3\right)+12b^2(c+dx)^2(bc-ad)^2\log(c+}{2d^5(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

```
[Out] (-(a^4*d^4) - 4*a^3*b*d^3*(c + 2*d*x) + 6*a^2*b^2*c*d^2*(3*c + 4*d*x) + 4*a
*b^3*d*(-5*c^3 - 4*c^2*d*x + 4*c*d^2*x^2 + 2*d^3*x^3) + b^4*(7*c^4 + 2*c^3*
d*x - 11*c^2*d^2*x^2 - 4*c*d^3*x^3 + d^4*x^4) + 12*b^2*(b*c - a*d)^2*(c + d
*x)^2*Log[c + d*x])/(2*d^5*(c + d*x)^2)
```

Maple [B] time = 0.048, size = 245, normalized size = 2.4

$$\frac{b^4x^2}{2d^3}+4\frac{ab^3x}{d^3}-3\frac{b^4xc}{d^4}+6\frac{b^2\ln(dx+c)a^2}{d^3}-12\frac{b^3\ln(dx+c)ca}{d^4}+6\frac{b^4\ln(dx+c)c^2}{d^5}-\frac{a^4}{2d(dx+c)^2}+2\frac{ca^3b}{d^2(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x)
```

```
[Out] 1/2*b^4*x^2/d^3+4*b^3/d^3*a*x-3*b^4/d^4*x*c+6*b^2/d^3*ln(d*x+c)*a^2-12*b^3/
d^4*ln(d*x+c)*c*a+6*b^4/d^5*ln(d*x+c)*c^2-1/2/d/(d*x+c)^2*a^4+2/d^2/(d*x+c)
^2*c*a^3*b-3/d^3/(d*x+c)^2*a^2*b^2*c^2+2/d^4/(d*x+c)^2*a*b^3*c^3-1/2/d^5/(d
*x+c)^2*b^4*c^4-4*b/d^2/(d*x+c)*a^3+12*b^2/d^3/(d*x+c)*c*a^2-12*b^3/d^4/(d*
x+c)*a*c^2+4*b^4/d^5/(d*x+c)*c^3
```

Maxima [A] time = 1.08637, size = 258, normalized size = 2.5

$$\frac{7b^4c^4-20ab^3c^3d+18a^2b^2c^2d^2-4a^3bcd^3-a^4d^4+8\left(b^4c^3d-3ab^3c^2d^2+3a^2b^2cd^3-a^3bd^4\right)x}{2\left(d^7x^2+2cd^6x+c^2d^5\right)}+\frac{b^4dx^2-2\left(3b^4c-\right.}{2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*
d^4 + 8*(b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x)/(d^7
*x^2 + 2*c*d^6*x + c^2*d^5) + 1/2*(b^4*d*x^2 - 2*(3*b^4*c - 4*a*b^3*d)*x)/d
^4 + 6*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*log(d*x + c)/d^5
```

Fricas [B] time = 1.49897, size = 586, normalized size = 5.69

$$\frac{b^4d^4x^4+7b^4c^4-20ab^3c^3d+18a^2b^2c^2d^2-4a^3bcd^3-a^4d^4-4\left(b^4cd^3-2ab^3d^4\right)x^3-\left(11b^4c^2d^2-16ab^3cd^3\right)x^2+2\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*d^4*x^4 + 7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 - a^4*d^4 - 4*(b^4*c*d^3 - 2*a*b^3*d^4)*x^3 - (11*b^4*c^2*d^2 - 16*
a*b^3*c*d^3)*x^2 + 2*(b^4*c^3*d - 8*a*b^3*c^2*d^2 + 12*a^2*b^2*c*d^3 - 4*a^
```

```
3*b*d^4)*x + 12*(b^4*c^4 - 2*a*b^3*c^3*d + a^2*b^2*c^2*d^2 + (b^4*c^2*d^2 -
 2*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(b^4*c^3*d - 2*a*b^3*c^2*d^2 + a^2*b^
2*c*d^3)*x)*log(d*x + c))/(d^7*x^2 + 2*c*d^6*x + c^2*d^5)
```

Sympy [A] time = 1.80211, size = 184, normalized size = 1.79

$$\frac{b^4x^2}{2d^3} + \frac{6b^2(ad-bc)^2\log(c+dx)}{d^5} - \frac{a^4d^4 + 4a^3bcd^3 - 18a^2b^2c^2d^2 + 20ab^3c^3d - 7b^4c^4 + x\left(8a^3bd^4 - 24a^2b^2cd^3 + 24ab^3c^2\right.}{2c^2d^5 + 4cd^6x + 2d^7x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/(a*c+(a*d+b*c)*x+b*d*x**2)**3,x)
```

```
[Out] b**4*x**2/(2*d**3) + 6*b**2*(a*d - b*c)**2*log(c + d*x)/d**5 - (a**4*d**4 +
 4*a**3*b*c*d**3 - 18*a**2*b**2*c**2*d**2 + 20*a*b**3*c**3*d - 7*b**4*c**4
+ x*(8*a**3*b*d**4 - 24*a**2*b**2*c*d**3 + 24*a*b**3*c**2*d**2 - 8*b**4*c**
3*d))/(2*c**2*d**5 + 4*c*d**6*x + 2*d**7*x**2) + x*(4*a*b**3*d - 3*b**4*c)/
d**4
```

Giac [A] time = 1.26366, size = 247, normalized size = 2.4

$$\frac{6\left(b^4c^2 - 2\,ab^3cd + a^2b^2d^2\right)\log(|dx+c|)}{d^5} + \frac{b^4d^3x^2 - 6\,b^4cd^2x + 8\,ab^3d^3x}{2\,d^6} + \frac{7\,b^4c^4 - 20\,ab^3c^3d + 18\,a^2b^2c^2d^2 - 4\,a^3bcd^3 -}{2\,(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="giac")
```

```
[Out] 6*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*log(abs(d*x + c))/d^5 + 1/2*(b^4*d^
3*x^2 - 6*b^4*c*d^2*x + 8*a*b^3*d^3*x)/d^6 + 1/2*(7*b^4*c^4 - 20*a*b^3*c^3*
d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*d^4 + 8*(b^4*c^3*d - 3*a*b^3*c
^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x)/((d*x + c)^2*d^5)
```

3.1820 $\int \frac{(a+bx)^6}{\left(ac+(bc+ad)x+bdx^2\right)^3}\,dx$

Optimal. Leaf size=78

$$-\frac{3b^2(bc-ad)\log(c+dx)}{d^4}-\frac{3b(bc-ad)^2}{d^4(c+dx)}+\frac{(bc-ad)^3}{2d^4(c+dx)^2}+\frac{b^3x}{d^3}$$

```
[Out] (b^3*x)/d^3 + (b*c - a*d)^3/(2*d^4*(c + d*x)^2) - (3*b*(b*c - a*d)^2)/(d^4*
(c + d*x)) - (3*b^2*(b*c - a*d)*Log[c + d*x])/d^4
```

Rubi [A] time = 0.0613338, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$-\frac{3b^2(bc-ad)\log(c+dx)}{d^4}-\frac{3b(bc-ad)^2}{d^4(c+dx)}+\frac{(bc-ad)^3}{2d^4(c+dx)^2}+\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^6/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

```
[Out] (b^3*x)/d^3 + (b*c - a*d)^3/(2*d^4*(c + d*x)^2) - (3*b*(b*c - a*d)^2)/(d^4*
(c + d*x)) - (3*b^2*(b*c - a*d)*Log[c + d*x])/d^4
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^6}{\left(ac+(bc+ad)x+bdx^2\right)^3}\,dx &= \int \frac{(a+bx)^3}{(c+dx)^3}\,dx\\
&= \int \left(\frac{b^3}{d^3}+\frac{(-bc+ad)^3}{d^3(c+dx)^3}+\frac{3b(bc-ad)^2}{d^3(c+dx)^2}-\frac{3b^2(bc-ad)}{d^3(c+dx)}\right)dx\\
&= \frac{b^3x}{d^3}+\frac{(bc-ad)^3}{2d^4(c+dx)^2}-\frac{3b(bc-ad)^2}{d^4(c+dx)}-\frac{3b^2(bc-ad)\log(c+dx)}{d^4}
\end{aligned}$$

Mathematica [A] time = 0.038989, size = 114, normalized size = 1.46

$$\frac{-3a^2bd^2(c+2dx)-a^3d^3+3ab^2cd(3c+4dx)-6b^2(c+dx)^2(bc-ad)\log(c+dx)+b^3\left(-4c^2dx-5c^3+4cd^2x^2+2d^3x^3\right)}{2d^4(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^6/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

```
[Out] (-(a^3*d^3) - 3*a^2*b*d^2*(c + 2*d*x) + 3*a*b^2*c*d*(3*c + 4*d*x) + b^3*(-5
*c^3 - 4*c^2*d*x + 4*c*d^2*x^2 + 2*d^3*x^3) - 6*b^2*(b*c - a*d)*(c + d*x)^2
*Log[c + d*x])/(2*d^4*(c + d*x)^2)
```

Maple [B] time = 0.046, size = 160, normalized size = 2.1

$$\frac{b^3x}{d^3}+3\frac{b^2\ln(dx+c)\,a}{d^3}-3\frac{b^3\ln(dx+c)\,c}{d^4}-\frac{a^3}{2d\,(dx+c)^2}+\frac{3\,bca^2}{2d^2\,(dx+c)^2}-\frac{3\,ab^2c^2}{2d^3\,(dx+c)^2}+\frac{b^3c^3}{2d^4\,(dx+c)^2}-3\frac{ba^2}{d^2\,(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^6/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x)
```

```
[Out] b^3*x/d^3+3*b^2/d^3*ln(d*x+c)*a-3*b^3/d^4*ln(d*x+c)*c-1/2/d/(d*x+c)^2*a^3+3
/2/d^2/(d*x+c)^2*c*b*a^2-3/2/d^3/(d*x+c)^2*a*c^2*b^2+1/2/d^4/(d*x+c)^2*b^3*
c^3-3*b/d^2/(d*x+c)*a^2+6*b^2/d^3/(d*x+c)*c*a-3*b^3/d^4/(d*x+c)*c^2
```

Maxima [A] time = 1.04148, size = 169, normalized size = 2.17

$$\frac{b^3x}{d^3}-\frac{5\,b^3c^3-9\,ab^2c^2d+3\,a^2bcd^2+a^3d^3+6\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x}{2\left(d^6x^2+2\,cd^5x+c^2d^4\right)}-\frac{3\left(b^3c-ab^2d\right)\log(dx+c)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="maxima")
```

```
[Out] b^3*x/d^3 - 1/2*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3 + 6*(b
^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x)/(d^6*x^2 + 2*c*d^5*x + c^2*d^4) -
3*(b^3*c - a*b^2*d)*log(d*x + c)/d^4
```

Fricas [B] time = 1.62815, size = 375, normalized size = 4.81

$$\frac{2\,b^3d^3x^3+4\,b^3cd^2x^2-5\,b^3c^3+9\,ab^2c^2d-3\,a^2bcd^2-a^3d^3-2\left(2\,b^3c^2d-6\,ab^2cd^2+3\,a^2bd^3\right)x-6\left(b^3c^3-ab^2c^2d+\left(b^3cd\right.\right.}{2\left(d^6x^2+2\,cd^5x+c^2d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*d^3*x^3 + 4*b^3*c*d^2*x^2 - 5*b^3*c^3 + 9*a*b^2*c^2*d - 3*a^2*b*
c*d^2 - a^3*d^3 - 2*(2*b^3*c^2*d - 6*a*b^2*c*d^2 + 3*a^2*b*d^3)*x - 6*(b^3*
c^3 - a*b^2*c^2*d + (b^3*c*d^2 - a*b^2*d^3)*x^2 + 2*(b^3*c^2*d - a*b^2*c*d^
2)*x)*log(d*x + c))/(d^6*x^2 + 2*c*d^5*x + c^2*d^4)
```

Sympy [A] time = 1.58782, size = 128, normalized size = 1.64

$$\frac{b^3x}{d^3} + \frac{3b^2(ad - bc)\log(c + dx)}{d^4} - \frac{a^3d^3 + 3a^2bcd^2 - 9ab^2c^2d + 5b^3c^3 + x(6a^2bd^3 - 12ab^2cd^2 + 6b^3c^2d)}{2c^2d^4 + 4cd^5x + 2d^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**6/(a*c+(a*d+b*c)*x+b*d*x**2)**3,x)
```

```
[Out] b**3*x/d**3 + 3*b**2*(a*d - b*c)*log(c + d*x)/d**4 - (a**3*d**3 + 3*a**2*b*
c*d**2 - 9*a*b**2*c**2*d + 5*b**3*c**3 + x*(6*a**2*b*d**3 - 12*a*b**2*c*d**
2 + 6*b**3*c**2*d))/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2)
```

Giac [A] time = 1.13169, size = 151, normalized size = 1.94

$$\frac{b^3x}{d^3} - \frac{3(b^3c - ab^2d)\log(|dx + c|)}{d^4} - \frac{5b^3c^3 - 9ab^2c^2d + 3a^2bcd^2 + a^3d^3 + 6(b^3c^2d - 2ab^2cd^2 + a^2bd^3)x}{2(dx + c)^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="giac")
```

```
[Out] b^3*x/d^3 - 3*(b^3*c - a*b^2*d)*log(abs(d*x + c))/d^4 - 1/2*(5*b^3*c^3 - 9*
a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3 + 6*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*
b*d^3)*x)/((d*x + c)^2*d^4)
```

3.1821 $\int \frac{(a+bx)^5}{\left(ac+(bc+ad)x+bdx^2\right)^3} dx$

Optimal. Leaf size=59

$$\frac{2b(bc-ad)}{d^3(c+dx)}-\frac{(bc-ad)^2}{2d^3(c+dx)^2}+\frac{b^2\log(c+dx)}{d^3}$$

```
[Out] -(b*c - a*d)^2/(2*d^3*(c + d*x)^2) + (2*b*(b*c - a*d))/(d^3*(c + d*x)) + (b
^2*Log[c + d*x])/d^3
```

Rubi [A] time = 0.0432568, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 43}

$$\frac{2b(bc-ad)}{d^3(c+dx)}-\frac{(bc-ad)^2}{2d^3(c+dx)^2}+\frac{b^2\log(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

```
[Out] -(b*c - a*d)^2/(2*d^3*(c + d*x)^2) + (2*b*(b*c - a*d))/(d^3*(c + d*x)) + (b
^2*Log[c + d*x])/d^3
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^5}{\left(ac+(bc+ad)x+bdx^2\right)^3}\,dx &= \int \frac{(a+bx)^2}{(c+dx)^3}\,dx\\
&= \int \left(\frac{(-bc+ad)^2}{d^2(c+dx)^3}-\frac{2b(bc-ad)}{d^2(c+dx)^2}+\frac{b^2}{d^2(c+dx)}\right)dx\\
&= -\frac{(bc-ad)^2}{2d^3(c+dx)^2}+\frac{2b(bc-ad)}{d^3(c+dx)}+\frac{b^2\log(c+dx)}{d^3}
\end{aligned}$$

Mathematica [A] time = 0.0236377, size = 48, normalized size = 0.81

$$\frac{\frac{(bc-ad)(ad+3bc+4bdx)}{(c+dx)^2}+2b^2\log(c+dx)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

```
[Out] (((b*c - a*d)*(3*b*c + a*d + 4*b*d*x))/(c + d*x)^2 + 2*b^2*Log[c + d*x])/(2
*d^3)
```

Maple [A] time = 0.046, size = 92, normalized size = 1.6

$$\frac{b^2\ln(dx+c)}{d^3}-\frac{a^2}{2d(dx+c)^2}+\frac{abc}{d^2(dx+c)^2}-\frac{b^2c^2}{2d^3(dx+c)^2}-2\frac{ab}{d^2(dx+c)}+2\frac{b^2c}{d^3(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x)
```

```
[Out] b^2*ln(d*x+c)/d^3-1/2/d/(d*x+c)^2*a^2+1/d^2/(d*x+c)^2*c*a*b-1/2/d^3/(d*x+c)
^2*b^2*c^2-2*b/d^2/(d*x+c)*a+2*b^2/d^3/(d*x+c)*c
```

Maxima [A] time = 1.15721, size = 108, normalized size = 1.83

$$\frac{3b^2c^2-2abcd-a^2d^2+4\left(b^2cd-abd^2\right)x}{2\left(d^5x^2+2cd^4x+c^2d^3\right)}+\frac{b^2\log(dx+c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x)/(d^5*x^2 +
2*c*d^4*x + c^2*d^3) + b^2*log(d*x + c)/d^3
```

Fricas [A] time = 1.60057, size = 205, normalized size = 3.47

$$\frac{3b^2c^2-2abcd-a^2d^2+4\left(b^2cd-abd^2\right)x+2\left(b^2d^2x^2+2b^2cdx+b^2c^2\right)\log(dx+c)}{2\left(d^5x^2+2cd^4x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x + 2*(b^2*d^2
*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d*x + c))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [A] time = 1.15639, size = 80, normalized size = 1.36

$$\frac{b^2\log(c+dx)}{d^3}-\frac{a^2d^2+2abcd-3b^2c^2+x\left(4abd^2-4b^2cd\right)}{2c^2d^3+4cd^4x+2d^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(a*c+(a*d+b*c)*x+b*d*x**2)**3,x)
```

```
[Out] b**2*log(c + d*x)/d**3 - (a**2*d**2 + 2*a*b*c*d - 3*b**2*c**2 + x*(4*a*b*d*
*2 - 4*b**2*c*d))/(2*c**2*d**3 + 4*c*d**4*x + 2*d**5*x**2)
```

Giac [A] time = 1.23623, size = 93, normalized size = 1.58

$$\frac{b^2 \log(|dx + c|)}{d^3} + \frac{4\left(b^2c - abd\right)x + \frac{3\,b^2c^2 - 2\,abcd - a^2d^2}{d}}{2\,(dx + c)^2 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="giac")
```

```
[Out] b^2*log(abs(d*x + c))/d^3 + 1/2*(4*(b^2*c - a*b*d)*x + (3*b^2*c^2 - 2*a*b*c
*d - a^2*d^2)/d)/((d*x + c)^2*d^2)
```

3.1822 $\int \frac{(a+bx)^4}{\left(ac+(bc+ad)x+bdx^2\right)^3}\,dx$

Optimal. Leaf size=28

$$\frac{(a+bx)^2}{2(c+dx)^2(bc-ad)}$$

```
[Out] (a + b*x)^2/(2*(b*c - a*d)*(c + d*x)^2)
```

Rubi [A] time = 0.0093834, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 37}

$$\frac{(a+bx)^2}{2(c+dx)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

```
[Out] (a + b*x)^2/(2*(b*c - a*d)*(c + d*x)^2)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{\left(ac+(bc+ad)x+bdx^2\right)^3}\,dx &= \int \frac{a+bx}{(c+dx)^3}\,dx\\ &= \frac{(a+bx)^2}{2(bc-ad)(c+dx)^2}\end{aligned}$$

Mathematica [A] time = 0.0087586, size = 26, normalized size = 0.93

$$-\frac{ad+b(c+2dx)}{2d^2(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

```
[Out] -(a*d + b*(c + 2*d*x))/(2*d^2*(c + d*x)^2)
```

Maple [A] time = 0.043, size = 35, normalized size = 1.3

$$-\frac{ad-bc}{2\,d^{2}\left(dx+c\right)^{2}}-\frac{b}{d^{2}\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x)
```

```
[Out] -1/2*(a*d-b*c)/d^2/(d*x+c)^2-1/d^2*b/(d*x+c)
```

Maxima [A] time = 1.08441, size = 51, normalized size = 1.82

$$-\frac{2\,bdx+bc+ad}{2\left(d^{4}x^{2}+2\,cd^{3}x+c^{2}d^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*d*x + b*c + a*d)/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

Fricas [A] time = 1.62033, size = 81, normalized size = 2.89

$$-\frac{2\,bdx+bc+ad}{2\left(d^{4}x^{2}+2\,cd^{3}x+c^{2}d^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x + b*c + a*d)/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

Sympy [A] time = 0.567457, size = 39, normalized size = 1.39

$$-\frac{ad+bc+2bdx}{2c^{2}d^{2}+4cd^{3}x+2d^{4}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(a*c+(a*d+b*c)*x+b*d*x**2)**3,x)
```

```
[Out] -(a*d + b*c + 2*b*d*x)/(2*c**2*d**2 + 4*c*d**3*x + 2*d**4*x**2)
```

Giac [A] time = 1.17273, size = 32, normalized size = 1.14

$$-\frac{2\,bdx + bc + ad}{2\,(dx + c)^2 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*d*x + b*c + a*d)/((d*x + c)^2*d^2)
```

3.1823 $\int \frac{(a+bx)^3}{\left(ac+(bc+ad)x+bdx^2\right)^3}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{2d(c+dx)^2}$$

[Out] -1/(2*d*(c + d*x)^2)

Rubi [A] time = 0.0081064, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 32}

$$-\frac{1}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^3/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]

[Out] -1/(2*d*(c + d*x)^2)

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{\left(ac+(bc+ad)x+bdx^2\right)^3}\,dx &= \int \frac{1}{(c+dx)^3}\,dx\\ &= -\frac{1}{2d(c+dx)^2}\end{aligned}$$

Mathematica [A] time = 0.002949, size = 14, normalized size = 1.

$$-\frac{1}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^3/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]

[Out] -1/(2*d*(c + d*x)^2)

Maple [A] time = 0.039, size = 13, normalized size = 0.9

$$-\frac{1}{2d(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x)
```

```
[Out] -1/2/d/(d*x+c)^2
```

Maxima [A] time = 1.11179, size = 32, normalized size = 2.29

$$-\frac{1}{2\left(d^3x^2+2\,cd^2x+c^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/2/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Fricas [A] time = 1.59199, size = 49, normalized size = 3.5

$$-\frac{1}{2\left(d^3x^2+2\,cd^2x+c^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Sympy [B] time = 0.45007, size = 26, normalized size = 1.86

$$-\frac{1}{2c^2d+4cd^2x+2d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(a*c+(a*d+b*c)*x+b*d*x**2)**3,x)
```

```
[Out] -1/(2*c**2*d + 4*c*d**2*x + 2*d**3*x**2)
```

Giac [A] time = 1.27555, size = 16, normalized size = 1.14

$$-\frac{1}{2(dx+c)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="giac")
```

```
[Out] -1/2/((d*x + c)^2*d)
```

3.1824 $\int \frac{(a+bx)^2}{\left(ac+(bc+ad)x+bdx^2\right)^3} dx$

Optimal. Leaf size=82

$$\frac{b^2\log(a+bx)}{(bc-ad)^3}-\frac{b^2\log(c+dx)}{(bc-ad)^3}+\frac{b}{(c+dx)(bc-ad)^2}+\frac{1}{2(c+dx)^2(bc-ad)}$$

```
[Out] 1/(2*(b*c - a*d)*(c + d*x)^2) + b/((b*c - a*d)^2*(c + d*x)) + (b^2*Log[a +
b*x])/(b*c - a*d)^3 - (b^2*Log[c + d*x])/(b*c - a*d)^3
```

Rubi [A] time = 0.0507484, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 44}

$$\frac{b^2\log(a+bx)}{(bc-ad)^3}-\frac{b^2\log(c+dx)}{(bc-ad)^3}+\frac{b}{(c+dx)(bc-ad)^2}+\frac{1}{2(c+dx)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

```
[Out] 1/(2*(b*c - a*d)*(c + d*x)^2) + b/((b*c - a*d)^2*(c + d*x)) + (b^2*Log[a +
b*x])/(b*c - a*d)^3 - (b^2*Log[c + d*x])/(b*c - a*d)^3
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2}{\left(ac+(bc+ad)x+bdx^2\right)^3} dx &= \int \frac{1}{(a+bx)(c+dx)^3} dx \\
&= \int \left(\frac{b^3}{(bc-ad)^3(a+bx)}-\frac{d}{(bc-ad)(c+dx)^3}-\frac{bd}{(bc-ad)^2(c+dx)^2}-\frac{b^2d}{(bc-ad)^3(c+
\end{aligned}$$
$$= \frac{1}{2(bc-ad)(c+dx)^2}+\frac{b}{(bc-ad)^2(c+dx)}+\frac{b^2\log(a+bx)}{(bc-ad)^3}-\frac{b^2\log(c+dx)}{(bc-ad)^3}$$

Mathematica [A] time = 0.047041, size = 67, normalized size = 0.82

$$\frac{2b^2\log(a+bx)+\frac{(bc-ad)(-ad+3bc+2bdx)}{(c+dx)^2}-2b^2\log(c+dx)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

```
[Out] (((b*c - a*d)*(3*b*c - a*d + 2*b*d*x))/(c + d*x)^2 + 2*b^2*Log[a + b*x] - 2
*b^2*Log[c + d*x])/(2*(b*c - a*d)^3)
```

Maple [A] time = 0.05, size = 81, normalized size = 1.

$$-\frac{1}{(2ad-2bc)(dx+c)^2}+\frac{b^2\ln(dx+c)}{(ad-bc)^3}+\frac{b}{(ad-bc)^2(dx+c)}-\frac{b^2\ln(bx+a)}{(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x)
```

```
[Out] -1/2/(a*d-b*c)/(d*x+c)^2+b^2/(a*d-b*c)^3*ln(d*x+c)+b/(a*d-b*c)^2/(d*x+c)-b^
2/(a*d-b*c)^3*ln(b*x+a)
```

Maxima [B] time = 1.17323, size = 273, normalized size = 3.33

$$\frac{b^2\log(bx+a)}{b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3}-\frac{b^2\log(dx+c)}{b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3}+\frac{2bdx+3b\cdots}{2\left(b^2c^4-2abc^3d+a^2c^2d^2+\left(b^2c^2d^2-2abcd^3+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="maxima")
```

```
[Out] b^2*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - b^2*
log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 1/2*(2*b
*d*x + 3*b*c - a*d)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2
*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)
```

Fricas [B] time = 1.62624, size = 490, normalized size = 5.98

$$\frac{3b^2c^2-4abcd+a^2d^2+2\left(b^2cd-abd^2\right)x+2\left(b^2d^2x^2+2b^2cdx+b^2c^2\right)\log(bx+a)-2\left(b^2d^2x^2+2b^2cdx+b^2c^2\right)\log(\cdots}{2\left(b^3c^5-3ab^2c^4d+3a^2bc^3d^2-a^3c^2d^3+\left(b^3c^3d^2-3ab^2c^2d^3+3a^2bcd^4-a^3d^5\right)x^2+2\left(b^3c^4d-3ab^2c^3d^2+3a^2bc^2d^3-\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x + 2*(b^2*d^2
*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2)*log(d*x + c))/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^
2*d^3 + (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*x^2 + 2*(
b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*x)
```

Sympy [B] time = 2.38015, size = 381, normalized size = 4.65

$$\frac{b^2\log\left(x+\frac{-\frac{a^4b^2d^4}{(ad-bc)^3}+\frac{4a^3b^3cd^3}{(ad-bc)^3}-\frac{6a^2b^4c^2d^2}{(ad-bc)^3}+\frac{4ab^5c^3d}{(ad-bc)^3}+ab^2d-\frac{b^6c^4}{(ad-bc)^3}+b^3c}{2b^3d}\right)}{(ad-bc)^3}-\frac{b^2\log\left(x+\frac{\frac{a^4b^2d^4}{(ad-bc)^3}-\frac{4a^3b^3cd^3}{(ad-bc)^3}+\frac{6a^2b^4c^2d^2}{(ad-bc)^3}-\frac{4ab^5c^3d}{(ad-bc)^3}+ab^2d+\frac{b^6c^4}{(ad-bc)^3}+b^3}{2b^3d}\right.}{(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(a*c+(a*d+b*c)*x+b*d*x**2)**3,x)
```

```
[Out] b**2*log(x + (-a**4*b**2*d**4/(a*d - b*c)**3 + 4*a**3*b**3*c*d**3/(a*d - b*
c)**3 - 6*a**2*b**4*c**2*d**2/(a*d - b*c)**3 + 4*a*b**5*c**3*d/(a*d - b*c)*
*3 + a*b**2*d - b**6*c**4/(a*d - b*c)**3 + b**3*c)/(2*b**3*d))/(a*d - b*c)*
*3 - b**2*log(x + (a**4*b**2*d**4/(a*d - b*c)**3 - 4*a**3*b**3*c*d**3/(a*d
- b*c)**3 + 6*a**2*b**4*c**2*d**2/(a*d - b*c)**3 - 4*a*b**5*c**3*d/(a*d - b
*c)**3 + a*b**2*d + b**6*c**4/(a*d - b*c)**3 + b**3*c)/(2*b**3*d))/(a*d - b
*c)**3 + (-a*d + 3*b*c + 2*b*d*x)/(2*a**2*c**2*d**2 - 4*a*b*c**3*d + 2*b**2
*c**4 + x**2*(2*a**2*d**4 - 4*a*b*c*d**3 + 2*b**2*c**2*d**2) + x*(4*a**2*c*
d**3 - 8*a*b*c**2*d**2 + 4*b**2*c**3*d))
```

Giac [B] time = 1.17193, size = 223, normalized size = 2.72

$$\frac{b^3\log\left(|bx+a|\right)}{b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3}-\frac{b^2d\log\left(|dx+c|\right)}{b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3-a^3d^4}+\frac{3\,b^2c^2-4\,abcd+a^2d^2+2\left(b^2cd-abd^2\right)}{2\,(bc-ad)^3(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="giac")
```

```
[Out] b^3*log(abs(b*x + a))/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^
3) - b^2*d*log(abs(d*x + c))/(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 -
 a^3*d^4) + 1/2*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x)
/((b*c - a*d)^3*(d*x + c)^2)
```

3.1825 $\int \frac{a+bx}{\left(ac+(bc+ad)x+bdx^2\right)^3}\,dx$

Optimal. Leaf size=110

$$-\frac{b^2}{(a+bx)(bc-ad)^3}-\frac{3b^2d\log(a+bx)}{(bc-ad)^4}+\frac{3b^2d\log(c+dx)}{(bc-ad)^4}-\frac{2bd}{(c+dx)(bc-ad)^3}-\frac{d}{2(c+dx)^2(bc-ad)^2}$$

```
[Out] -(b^2/((b*c - a*d)^3*(a + b*x))) - d/(2*(b*c - a*d)^2*(c + d*x)^2) - (2*b*d
)/((b*c - a*d)^3*(c + d*x)) - (3*b^2*d*Log[a + b*x])/(b*c - a*d)^4 + (3*b^2
*d*Log[c + d*x])/(b*c - a*d)^4
```

Rubi [A] time = 0.0764191, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {626, 44}

$$-\frac{b^2}{(a+bx)(bc-ad)^3}-\frac{3b^2d\log(a+bx)}{(bc-ad)^4}+\frac{3b^2d\log(c+dx)}{(bc-ad)^4}-\frac{2bd}{(c+dx)(bc-ad)^3}-\frac{d}{2(c+dx)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

```
[Out] -(b^2/((b*c - a*d)^3*(a + b*x))) - d/(2*(b*c - a*d)^2*(c + d*x)^2) - (2*b*d
)/((b*c - a*d)^3*(c + d*x)) - (3*b^2*d*Log[a + b*x])/(b*c - a*d)^4 + (3*b^2
*d*Log[c + d*x])/(b*c - a*d)^4
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{\left(ac+(bc+ad)x+bdx^2\right)^3}\,dx &= \int \frac{1}{(a+bx)^2(c+dx)^3}\,dx\\ &= \int \left(\frac{b^3}{(bc-ad)^3(a+bx)^2}-\frac{3b^3d}{(bc-ad)^4(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)^3}+\frac{2bd^2}{(bc-ad)^3(c+d\right.\\ &= -\frac{b^2}{(bc-ad)^3(a+bx)}-\frac{d}{2(bc-ad)^2(c+dx)^2}-\frac{2bd}{(bc-ad)^3(c+dx)}-\frac{3b^2d\log(a+bx)}{(bc-ad)^4}+\end{aligned}$$

Mathematica [A] time = 0.0950963, size = 97, normalized size = 0.88

$$-\frac{\frac{2b^2(bc-ad)}{a+bx}+6b^2d\log(a+bx)+\frac{4bd(bc-ad)}{c+dx}+\frac{d(bc-ad)^2}{(c+dx)^2}-6b^2d\log(c+dx)}{2(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

```
[Out] -((2*b^2*(b*c - a*d))/(a + b*x) + (d*(b*c - a*d)^2)/(c + d*x)^2 + (4*b*d*(b
*c - a*d))/(c + d*x) + 6*b^2*d*Log[a + b*x] - 6*b^2*d*Log[c + d*x])/(2*(b*c
 - a*d)^4)
```

Maple [A] time = 0.053, size = 108, normalized size = 1.

$$-\frac{d}{2\,(ad-bc)^{2}\,(dx+c)^{2}}+3\,\frac{b^{2}d\ln(dx+c)}{(ad-bc)^{4}}+2\,\frac{bd}{(ad-bc)^{3}\,(dx+c)}+\frac{b^{2}}{(ad-bc)^{3}\,(bx+a)}-3\,\frac{b^{2}d\ln(bx+a)}{(ad-bc)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x)
```

```
[Out] -1/2*d/(a*d-b*c)^2/(d*x+c)^2+3*d/(a*d-b*c)^4*b^2*ln(d*x+c)+2*d/(a*d-b*c)^3*
b/(d*x+c)+b^2/(a*d-b*c)^3/(b*x+a)-3*d/(a*d-b*c)^4*b^2*ln(b*x+a)
```

Maxima [B] time = 1.30531, size = 521, normalized size = 4.74

$$-\frac{3\,b^{2}d\log(bx+a)}{b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}}+\frac{3\,b^{2}d\log(dx+c)}{b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}}-\frac{}{2\left(ab^{3}c^{5}-3\,a^{2}b^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="maxima")
```

```
[Out] -3*b^2*d*log(b*x + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4) + 3*b^2*d*log(d*x + c)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*
b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) - 1/2*(6*b^2*d^2*x^2 + 2*b^2*c^2 + 5
*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d + a*b*d^2)*x)/(a*b^3*c^5 - 3*a^2*b^2*c^4*
d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*
b^2*c*d^4 - a^3*b*d^5)*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2
*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*
d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x)
```

Fricas [B] time = 1.65349, size = 991, normalized size = 9.01

$$-\frac{2\,b^{3}c^{3}+3\,ab^{2}c^{2}d-6\,a^{2}bcd^{2}+a^{3}d^{3}+6\left(b^{3}cd^{2}-ab^{2}d^{3}\right)x^{2}+3\left(3\,b^{3}c^{2}d-2\,ab^{2}cd^{2}-a^{2}bd^{3}\right)x+6\left(b^{3}d^{3}x^{3}+ab^{2}c^{2}d+\right.}{2\left(ab^{4}c^{6}-4\,a^{2}b^{3}c^{5}d+6\,a^{3}b^{2}c^{4}d^{2}-4\,a^{4}bc^{3}d^{3}+a^{5}c^{2}d^{4}+\left(b^{5}c^{4}d^{2}-4\,ab^{4}c^{3}d^{3}+6\,a^{2}b^{3}c^{2}d^{4}-4\,a^{3}b^{2}cd^{5}+a^{4}bd^{6}\right)x^{3}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*c^3 + 3*a*b^2*c^2*d - 6*a^2*b*c*d^2 + a^3*d^3 + 6*(b^3*c*d^2 -
a*b^2*d^3)*x^2 + 3*(3*b^3*c^2*d - 2*a*b^2*c*d^2 - a^2*b*d^3)*x + 6*(b^3*d^3
*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c
```

```
*d^2)*x)*log(b*x + a) - 6*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2
*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(d*x + c))/(a*b^4*c^6 - 4*a^2
*b^3*c^5*d + 6*a^3*b^2*c^4*d^2 - 4*a^4*b*c^3*d^3 + a^5*c^2*d^4 + (b^5*c^4*d
^2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^5 + a^4*b*d^6)*x^3
 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^3*c^3*d^3 - 2*a^3*b^2*c^2*d^4 -
 2*a^4*b*c*d^5 + a^5*d^6)*x^2 + (b^5*c^6 - 2*a*b^4*c^5*d - 2*a^2*b^3*c^4*d^
2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*x)
```

Sympy [B] time = 3.53382, size = 632, normalized size = 5.75

$$\frac{3b^2d\log\left(x+\frac{-\frac{3a^5b^2d^6}{(ad-bc)^4}+\frac{15a^4b^3cd^5}{(ad-bc)^4}-\frac{30a^3b^4c^2d^4}{(ad-bc)^4}+\frac{30a^2b^5c^3d^3}{(ad-bc)^4}-\frac{15ab^6c^4d^2}{(ad-bc)^4}+3ab^2d^2+\frac{3b^7c^5d}{(ad-bc)^4}+3b^3cd}{6b^3d^2}\right)}{(ad-bc)^4}-\frac{3b^2d\log\left(x+\frac{\frac{3a^5b^2d^6}{(ad-bc)^4}-\frac{15a^4b^3cd^5}{(ad-bc)^4}+\frac{30a^3b^4c^2d^4}{(ad-bc)^4}-\cdots\right.}{(ad-bc\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x**2)**3,x)
```

```
[Out] 3*b**2*d*log(x + (-3*a**5*b**2*d**6/(a*d - b*c)**4 + 15*a**4*b**3*c*d**5/(a
*d - b*c)**4 - 30*a**3*b**4*c**2*d**4/(a*d - b*c)**4 + 30*a**2*b**5*c**3*d*
*3/(a*d - b*c)**4 - 15*a*b**6*c**4*d**2/(a*d - b*c)**4 + 3*a*b**2*d**2 + 3*
b**7*c**5*d/(a*d - b*c)**4 + 3*b**3*c*d)/(6*b**3*d**2))/(a*d - b*c)**4 - 3*
b**2*d*log(x + (3*a**5*b**2*d**6/(a*d - b*c)**4 - 15*a**4*b**3*c*d**5/(a*d
- b*c)**4 + 30*a**3*b**4*c**2*d**4/(a*d - b*c)**4 - 30*a**2*b**5*c**3*d**3/
(a*d - b*c)**4 + 15*a*b**6*c**4*d**2/(a*d - b*c)**4 + 3*a*b**2*d**2 - 3*b**
7*c**5*d/(a*d - b*c)**4 + 3*b**3*c*d)/(6*b**3*d**2))/(a*d - b*c)**4 + (-a**
2*d**2 + 5*a*b*c*d + 2*b**2*c**2 + 6*b**2*d**2*x**2 + x*(3*a*b*d**2 + 9*b**
2*c*d))/(2*a**4*c**2*d**3 - 6*a**3*b*c**3*d**2 + 6*a**2*b**2*c**4*d - 2*a*b
**3*c**5 + x**3*(2*a**3*b*d**5 - 6*a**2*b**2*c*d**4 + 6*a*b**3*c**2*d**3 -
2*b**4*c**3*d**2) + x**2*(2*a**4*d**5 - 2*a**3*b*c*d**4 - 6*a**2*b**2*c**2*
d**3 + 10*a*b**3*c**3*d**2 - 4*b**4*c**4*d) + x*(4*a**4*c*d**4 - 10*a**3*b*
c**2*d**3 + 6*a**2*b**2*c**3*d**2 + 2*a*b**3*c**4*d - 2*b**4*c**5))
```

Giac [B] time = 1.2279, size = 343, normalized size = 3.12

$$-\frac{3\,b^3d\log\left(|bx+a|\right)}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4}+\frac{3\,b^2d^2\log\left(|dx+c|\right)}{b^4c^4d-4\,ab^3c^3d^2+6\,a^2b^2c^2d^3-4\,a^3bcd^4+a^4d^5}-\frac{2\,b^3c^3+3\,ab^2c^2\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="giac")
```

```
[Out] -3*b^3*d*log(abs(b*x + a))/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4
*a^3*b^2*c*d^3 + a^4*b*d^4) + 3*b^2*d^2*log(abs(d*x + c))/(b^4*c^4*d - 4*a*
b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5) - 1/2*(2*b^3*c^3
 + 3*a*b^2*c^2*d - 6*a^2*b*c*d^2 + a^3*d^3 + 6*(b^3*c*d^2 - a*b^2*d^3)*x^2
+ 3*(3*b^3*c^2*d - 2*a*b^2*c*d^2 - a^2*b*d^3)*x)/((b*c - a*d)^4*(b*x + a)*(
d*x + c)^2)
```

3.1826 $\int \frac{1}{\left(ac+(bc+ad)x+bdx^2\right)^3}\,dx$

Optimal. Leaf size=143

$$\frac{6b^2d^2\log(a+bx)}{(bc-ad)^5}-\frac{6b^2d^2\log(c+dx)}{(bc-ad)^5}+\frac{3bd(ad+bc+2bdx)}{(bc-ad)^4\left(x(ad+bc)+ac+bdx^2\right)}-\frac{ad+bc+2bdx}{2(bc-ad)^2\left(x(ad+bc)+ac+bdx^2\right)^2}$$

```
[Out] -(b*c + a*d + 2*b*d*x)/(2*(b*c - a*d)^2*(a*c + (b*c + a*d)*x + b*d*x^2)^2)
+ (3*b*d*(b*c + a*d + 2*b*d*x))/((b*c - a*d)^4*(a*c + (b*c + a*d)*x + b*d*x
^2)) + (6*b^2*d^2*Log[a + b*x])/(b*c - a*d)^5 - (6*b^2*d^2*Log[c + d*x])/(b
*c - a*d)^5
```

Rubi [A] time = 0.0486094, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {614, 616, 31}

$$\frac{6b^2d^2\log(a+bx)}{(bc-ad)^5}-\frac{6b^2d^2\log(c+dx)}{(bc-ad)^5}+\frac{3bd(ad+bc+2bdx)}{(bc-ad)^4\left(x(ad+bc)+ac+bdx^2\right)}-\frac{ad+bc+2bdx}{2(bc-ad)^2\left(x(ad+bc)+ac+bdx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + (b*c + a*d)*x + b*d*x^2)^(-3),x]
```

```
[Out] -(b*c + a*d + 2*b*d*x)/(2*(b*c - a*d)^2*(a*c + (b*c + a*d)*x + b*d*x^2)^2)
+ (3*b*d*(b*c + a*d + 2*b*d*x))/((b*c - a*d)^4*(a*c + (b*c + a*d)*x + b*d*x
^2)) + (6*b^2*d^2*Log[a + b*x])/(b*c - a*d)^5 - (6*b^2*d^2*Log[c + d*x])/(b
*c - a*d)^5
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\left(ac+(bc+ad)x+bdx^2\right)^3}\,dx = -\frac{bc+ad+2bdx}{2(bc-ad)^2\left(ac+(bc+ad)x+bdx^2\right)^2} - \frac{(3bd)\int \frac{1}{\left(ac+(bc+ad)x+bdx^2\right)^2}\,dx}{(bc-ad)^2}$$

$$= -\frac{bc+ad+2bdx}{2(bc-ad)^2\left(ac+(bc+ad)x+bdx^2\right)^2} + \frac{3bd(bc+ad+2bdx)}{(bc-ad)^4\left(ac+(bc+ad)x+bdx^2\right)} + \frac{\left(6b^2c\right.}{}$$

$$= -\frac{bc+ad+2bdx}{2(bc-ad)^2\left(ac+(bc+ad)x+bdx^2\right)^2} + \frac{3bd(bc+ad+2bdx)}{(bc-ad)^4\left(ac+(bc+ad)x+bdx^2\right)} - \frac{\left(6b^3a\right.}{(}$$

$$= -\frac{bc+ad+2bdx}{2(bc-ad)^2\left(ac+(bc+ad)x+bdx^2\right)^2} + \frac{3bd(bc+ad+2bdx)}{(bc-ad)^4\left(ac+(bc+ad)x+bdx^2\right)} + \frac{6b^2d^2}{(b}$$

Mathematica [A] time = 0.109183, size = 128, normalized size = 0.9

$$\frac{\frac{6b^2d(bc-ad)}{a+bx} - \frac{b^2(bc-ad)^2}{(a+bx)^2} + 12b^2d^2\log(a+bx) + \frac{6bd^2(bc-ad)}{c+dx} + \frac{d^2(bc-ad)^2}{(c+dx)^2} - 12b^2d^2\log(c+dx)}{2(bc-ad)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^(-3),x]
```

```
[Out] (-((b^2*(b*c - a*d)^2)/(a + b*x)^2) + (6*b^2*d*(b*c - a*d))/(a + b*x) + (d^
2*(b*c - a*d)^2)/(c + d*x)^2 + (6*b*d^2*(b*c - a*d))/(c + d*x) + 12*b^2*d^2
*Log[a + b*x] - 12*b^2*d^2*Log[c + d*x])/(2*(b*c - a*d)^5)
```

Maple [A] time = 0.053, size = 140, normalized size = 1.

$$-\frac{d^2}{2\,(ad-bc)^3\,(dx+c)^2} + 6\,\frac{b^2d^2\ln(dx+c)}{(ad-bc)^5} + 3\,\frac{bd^2}{(ad-bc)^4\,(dx+c)} + \frac{b^2}{2\,(ad-bc)^3\,(bx+a)^2} - 6\,\frac{b^2d^2\ln(bx+a)}{(ad-bc)^5} + 3\,\frac{}{(ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x)
```

```
[Out] -1/2*d^2/(a*d-b*c)^3/(d*x+c)^2+6*d^2/(a*d-b*c)^5*b^2*ln(d*x+c)+3*d^2/(a*d-b
*c)^4*b/(d*x+c)+1/2*b^2/(a*d-b*c)^3/(b*x+a)^2-6*d^2/(a*d-b*c)^5*b^2*ln(b*x+
a)+3*b^2/(a*d-b*c)^4*d/(b*x+a)
```

Maxima [B] time = 1.15268, size = 802, normalized size = 5.61

$$\frac{6\,b^2d^2\log(bx+a)}{b^5c^5 - 5\,ab^4c^4d + 10\,a^2b^3c^3d^2 - 10\,a^3b^2c^2d^3 + 5\,a^4bcd^4 - a^5d^5} - \frac{6\,b^2d^2\log(dx+c)}{b^5c^5 - 5\,ab^4c^4d + 10\,a^2b^3c^3d^2 - 10\,a^3b^2c^2d^3 + 5\,a^4bcd^4 -}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="maxima")
```

```
[Out] 6*b^2*d^2*log(b*x + a)/(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a
^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5) - 6*b^2*d^2*log(d*x + c)/(b^5*c^5
```

```
 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4
- a^5*d^5) + 1/2*(12*b^3*d^3*x^3 - b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*d^2
- a^3*d^3 + 18*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c*d^2 +
 a^2*b*d^3)*x)/(a^2*b^4*c^6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b
*c^3*d^3 + a^6*c^2*d^4 + (b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4
 - 4*a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*
a^2*b^4*c^3*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*x^3 + (b
^6*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 + a^6*d
^6)*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 + 2*a^4*b^2*c^
3*d^3 - 3*a^5*b*c^2*d^4 + a^6*c*d^5)*x)
```

Fricas [B] time = 1.75451, size = 1488, normalized size = 10.41

$$-\frac{b^4c^4 - 8\,ab^3c^3d + 8\,a^3bcd^3 - a^4d^4 - 12\left(b^4cd^3 - ab^3d^4\right)x^3 - 18\left(b^4c^2d^2 - a^2b^2d^4\right)x^2 - 4\left(b^4c^3d + 6\,ab^3c^2d^2 - 6\right.}{2\left(a^2b^5c^7 - 5\,a^3b^4c^6d + 10\,a^4b^3c^5d^2 - 10\,a^5b^2c^4d^3 + 5\,a^6bc^3d^4 - a^7c^2d^5 + \left(b^7c^5d^2 - 5\,ab^6c^4d^3 + 10\,a^2b^5c^3d^4 - 10\,a^3b^4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b^4*c^4 - 8*a*b^3*c^3*d + 8*a^3*b*c*d^3 - a^4*d^4 - 12*(b^4*c*d^3 - a
*b^3*d^4)*x^3 - 18*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^2 - 4*(b^4*c^3*d + 6*a*b^3
*c^2*d^2 - 6*a^2*b^2*c*d^3 - a^3*b*d^4)*x - 12*(b^4*d^4*x^4 + a^2*b^2*c^2*d
^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a*b^3*c*d^3 + a^2*b^2
*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(b*x + a) + 12*(b^4*d^4
*x^4 + a^2*b^2*c^2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a
*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(d*
x + c))/(a^2*b^5*c^7 - 5*a^3*b^4*c^6*d + 10*a^4*b^3*c^5*d^2 - 10*a^5*b^2*c^
4*d^3 + 5*a^6*b*c^3*d^4 - a^7*c^2*d^5 + (b^7*c^5*d^2 - 5*a*b^6*c^4*d^3 + 10
*a^2*b^5*c^3*d^4 - 10*a^3*b^4*c^2*d^5 + 5*a^4*b^3*c*d^6 - a^5*b^2*d^7)*x^4
+ 2*(b^7*c^6*d - 4*a*b^6*c^5*d^2 + 5*a^2*b^5*c^4*d^3 - 5*a^4*b^3*c^2*d^5 +
4*a^5*b^2*c*d^6 - a^6*b*d^7)*x^3 + (b^7*c^7 - a*b^6*c^6*d - 9*a^2*b^5*c^5*d
^2 + 25*a^3*b^4*c^4*d^3 - 25*a^4*b^3*c^3*d^4 + 9*a^5*b^2*c^2*d^5 + a^6*b*c*
d^6 - a^7*d^7)*x^2 + 2*(a*b^6*c^7 - 4*a^2*b^5*c^6*d + 5*a^3*b^4*c^5*d^2 - 5
*a^5*b^2*c^3*d^4 + 4*a^6*b*c^2*d^5 - a^7*c*d^6)*x)
```

Sympy [B] time = 3.79196, size = 881, normalized size = 6.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+(a*d+b*c)*x+b*d*x**2)**3,x)
```

```
[Out] 6*b**2*d**2*log(x + (-6*a**6*b**2*d**8/(a*d - b*c)**5 + 36*a**5*b**3*c*d**7
/(a*d - b*c)**5 - 90*a**4*b**4*c**2*d**6/(a*d - b*c)**5 + 120*a**3*b**5*c**
3*d**5/(a*d - b*c)**5 - 90*a**2*b**6*c**4*d**4/(a*d - b*c)**5 + 36*a*b**7*c
**5*d**3/(a*d - b*c)**5 + 6*a*b**2*d**3 - 6*b**8*c**6*d**2/(a*d - b*c)**5 +
 6*b**3*c*d**2)/(12*b**3*d**3))/(a*d - b*c)**5 - 6*b**2*d**2*log(x + (6*a**
6*b**2*d**8/(a*d - b*c)**5 - 36*a**5*b**3*c*d**7/(a*d - b*c)**5 + 90*a**4*b
**4*c**2*d**6/(a*d - b*c)**5 - 120*a**3*b**5*c**3*d**5/(a*d - b*c)**5 + 90*
a**2*b**6*c**4*d**4/(a*d - b*c)**5 - 36*a*b**7*c**5*d**3/(a*d - b*c)**5 + 6
*a*b**2*d**3 + 6*b**8*c**6*d**2/(a*d - b*c)**5 + 6*b**3*c*d**2)/(12*b**3*d*
*3))/(a*d - b*c)**5 + (-a**3*d**3 + 7*a**2*b*c*d**2 + 7*a*b**2*c**2*d - b**
```

```
3*c**3 + 12*b**3*d**3*x**3 + x**2*(18*a*b**2*d**3 + 18*b**3*c*d**2) + x*(4*
a**2*b*d**3 + 28*a*b**2*c*d**2 + 4*b**3*c**2*d))/(2*a**6*c**2*d**4 - 8*a**5
*b*c**3*d**3 + 12*a**4*b**2*c**4*d**2 - 8*a**3*b**3*c**5*d + 2*a**2*b**4*c*
*6 + x**4*(2*a**4*b**2*d**6 - 8*a**3*b**3*c*d**5 + 12*a**2*b**4*c**2*d**4 -
 8*a*b**5*c**3*d**3 + 2*b**6*c**4*d**2) + x**3*(4*a**5*b*d**6 - 12*a**4*b**
2*c*d**5 + 8*a**3*b**3*c**2*d**4 + 8*a**2*b**4*c**3*d**3 - 12*a*b**5*c**4*d
**2 + 4*b**6*c**5*d) + x**2*(2*a**6*d**6 - 18*a**4*b**2*c**2*d**4 + 32*a**3
*b**3*c**3*d**3 - 18*a**2*b**4*c**4*d**2 + 2*b**6*c**6) + x*(4*a**6*c*d**5
- 12*a**5*b*c**2*d**4 + 8*a**4*b**2*c**3*d**3 + 8*a**3*b**3*c**4*d**2 - 12*
a**2*b**4*c**5*d + 4*a*b**5*c**6))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1827 $\int \frac{1}{(a+bx)\left(ac+(bc+ad)x+bdx^2\right)^3}\,dx$

Optimal. Leaf size=170

$$-\frac{6b^2d^2}{(a+bx)(bc-ad)^5}-\frac{10b^2d^3\log(a+bx)}{(bc-ad)^6}+\frac{10b^2d^3\log(c+dx)}{(bc-ad)^6}+\frac{3b^2d}{2(a+bx)^2(bc-ad)^4}-\frac{b^2}{3(a+bx)^3(bc-ad)^3}-\frac{(c+}{}$$

```
[Out] -b^2/(3*(b*c - a*d)^3*(a + b*x)^3) + (3*b^2*d)/(2*(b*c - a*d)^4*(a + b*x)^2
) - (6*b^2*d^2)/((b*c - a*d)^5*(a + b*x)) - d^3/(2*(b*c - a*d)^4*(c + d*x)^
2) - (4*b*d^3)/((b*c - a*d)^5*(c + d*x)) - (10*b^2*d^3*Log[a + b*x])/(b*c -
 a*d)^6 + (10*b^2*d^3*Log[c + d*x])/(b*c - a*d)^6
```

Rubi [A] time = 0.159918, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {626, 44}

$$-\frac{6b^2d^2}{(a+bx)(bc-ad)^5}-\frac{10b^2d^3\log(a+bx)}{(bc-ad)^6}+\frac{10b^2d^3\log(c+dx)}{(bc-ad)^6}+\frac{3b^2d}{2(a+bx)^2(bc-ad)^4}-\frac{b^2}{3(a+bx)^3(bc-ad)^3}-\frac{(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(a*c + (b*c + a*d)*x + b*d*x^2)^3),x]
```

```
[Out] -b^2/(3*(b*c - a*d)^3*(a + b*x)^3) + (3*b^2*d)/(2*(b*c - a*d)^4*(a + b*x)^2
) - (6*b^2*d^2)/((b*c - a*d)^5*(a + b*x)) - d^3/(2*(b*c - a*d)^4*(c + d*x)^
2) - (4*b*d^3)/((b*c - a*d)^5*(c + d*x)) - (10*b^2*d^3*Log[a + b*x])/(b*c -
 a*d)^6 + (10*b^2*d^3*Log[c + d*x])/(b*c - a*d)^6
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx)\left(ac+(bc+ad)x+bdx^2\right)^3}\,dx=\int \frac{1}{(a+bx)^4(c+dx)^3}\,dx$$
$$=\int \left(\frac{b^3}{(bc-ad)^3(a+bx)^4}-\frac{3b^3d}{(bc-ad)^4(a+bx)^3}+\frac{6b^3d^2}{(bc-ad)^5(a+bx)^2}-\frac{}{(bc}\right.$$
$$=-\frac{b^2}{3(bc-ad)^3(a+bx)^3}+\frac{3b^2d}{2(bc-ad)^4(a+bx)^2}-\frac{6b^2d^2}{(bc-ad)^5(a+bx)}-\frac{}{2(bc}$$

Mathematica [A] time = 0.211589, size = 154, normalized size = 0.91

$$-\frac{\frac{36b^2d^2(bc-ad)}{a+bx}-\frac{9b^2d(bc-ad)^2}{(a+bx)^2}+\frac{2b^2(bc-ad)^3}{(a+bx)^3}+60b^2d^3\log(a+bx)+\frac{24bd^3(bc-ad)}{c+dx}+\frac{3d^3(bc-ad)^2}{(c+dx)^2}-60b^2d^3\log(c+dx)}{6(bc-ad)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(a*c + (b*c + a*d)*x + b*d*x^2)^3),x]
```

```
[Out] -((2*b^2*(b*c - a*d)^3)/(a + b*x)^3 - (9*b^2*d*(b*c - a*d)^2)/(a + b*x)^2 +
 (36*b^2*d^2*(b*c - a*d))/(a + b*x) + (3*d^3*(b*c - a*d)^2)/(c + d*x)^2 + (
24*b*d^3*(b*c - a*d))/(c + d*x) + 60*b^2*d^3*Log[a + b*x] - 60*b^2*d^3*Log[
c + d*x])/(6*(b*c - a*d)^6)
```

Maple [A] time = 0.054, size = 165, normalized size = 1.

$$-\frac{d^3}{2\,(ad-bc)^4\,(dx+c)^2}+10\,\frac{d^3b^2\ln(dx+c)}{(ad-bc)^6}+4\,\frac{d^3b}{(ad-bc)^5\,(dx+c)}+\frac{b^2}{3\,(ad-bc)^3\,(bx+a)^3}-10\,\frac{d^3b^2\ln(bx+a)}{(ad-bc)^6}+6\,\frac{}{(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x)
```

```
[Out] -1/2*d^3/(a*d-b*c)^4/(d*x+c)^2+10*d^3/(a*d-b*c)^6*b^2*ln(d*x+c)+4*d^3/(a*d-
b*c)^5*b/(d*x+c)+1/3*b^2/(a*d-b*c)^3/(b*x+a)^3-10*d^3/(a*d-b*c)^6*b^2*ln(b*
x+a)+6*b^2/(a*d-b*c)^5*d^2/(b*x+a)+3/2*b^2/(a*d-b*c)^4*d/(b*x+a)^2
```

Maxima [B] time = 1.35181, size = 1200, normalized size = 7.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="maxima")
```

```
[Out] -10*b^2*d^3*log(b*x + a)/(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20
*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6) + 10*b^2*d
^3*log(d*x + c)/(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*
c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6) - 1/6*(60*b^4*d^4*x
^4 + 2*b^4*c^4 - 13*a*b^3*c^3*d + 47*a^2*b^2*c^2*d^2 + 27*a^3*b*c*d^3 - 3*a
^4*d^4 + 30*(3*b^4*c*d^3 + 5*a*b^3*d^4)*x^3 + 10*(2*b^4*c^2*d^2 + 23*a*b^3*
c*d^3 + 11*a^2*b^2*d^4)*x^2 - 5*(b^4*c^3*d - 11*a*b^3*c^2*d^2 - 35*a^2*b^2*
c*d^3 - 3*a^3*b*d^4)*x)/(a^3*b^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*d^2
 - 10*a^6*b^2*c^4*d^3 + 5*a^7*b*c^3*d^4 - a^8*c^2*d^5 + (b^8*c^5*d^2 - 5*a*
b^7*c^4*d^3 + 10*a^2*b^6*c^3*d^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6 - a
^5*b^3*d^7)*x^5 + (2*b^8*c^6*d - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4*d^3 + 10*a
^3*b^5*c^3*d^4 - 20*a^4*b^4*c^2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*b^2*d^7)*x^4
 + (b^8*c^7 + a*b^7*c^6*d - 17*a^2*b^6*c^5*d^2 + 35*a^3*b^5*c^4*d^3 - 25*a^
4*b^4*c^3*d^4 - a^5*b^3*c^2*d^5 + 9*a^6*b^2*c*d^6 - 3*a^7*b*d^7)*x^3 + (3*a
*b^7*c^7 - 9*a^2*b^6*c^6*d + a^3*b^5*c^5*d^2 + 25*a^4*b^4*c^4*d^3 - 35*a^5*
b^3*c^3*d^4 + 17*a^6*b^2*c^2*d^5 - a^7*b*c*d^6 - a^8*d^7)*x^2 + (3*a^2*b^6*
c^7 - 13*a^3*b^5*c^6*d + 20*a^4*b^4*c^5*d^2 - 10*a^5*b^3*c^4*d^3 - 5*a^6*b^
2*c^3*d^4 + 7*a^7*b*c^2*d^5 - 2*a^8*c*d^6)*x)
```

Fricas [B] time = 1.72821, size = 2313, normalized size = 13.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b^5*c^5 - 15*a*b^4*c^4*d + 60*a^2*b^3*c^3*d^2 - 20*a^3*b^2*c^2*d^3
- 30*a^4*b*c*d^4 + 3*a^5*d^5 + 60*(b^5*c*d^4 - a*b^4*d^5)*x^4 + 30*(3*b^5*c
^2*d^3 + 2*a*b^4*c*d^4 - 5*a^2*b^3*d^5)*x^3 + 10*(2*b^5*c^3*d^2 + 21*a*b^4*
c^2*d^3 - 12*a^2*b^3*c*d^4 - 11*a^3*b^2*d^5)*x^2 - 5*(b^5*c^4*d - 12*a*b^4*
c^3*d^2 - 24*a^2*b^3*c^2*d^3 + 32*a^3*b^2*c*d^4 + 3*a^4*b*d^5)*x + 60*(b^5*
d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3
+ 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 +
 a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x)*log(b*x + a) -
 60*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5
*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^
3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x)*log(d
*x + c))/(a^3*b^6*c^8 - 6*a^4*b^5*c^7*d + 15*a^5*b^4*c^6*d^2 - 20*a^6*b^3*c
^5*d^3 + 15*a^7*b^2*c^4*d^4 - 6*a^8*b*c^3*d^5 + a^9*c^2*d^6 + (b^9*c^6*d^2
- 6*a*b^8*c^5*d^3 + 15*a^2*b^7*c^4*d^4 - 20*a^3*b^6*c^3*d^5 + 15*a^4*b^5*c^
2*d^6 - 6*a^5*b^4*c*d^7 + a^6*b^3*d^8)*x^5 + (2*b^9*c^7*d - 9*a*b^8*c^6*d^2
 + 12*a^2*b^7*c^5*d^3 + 5*a^3*b^6*c^4*d^4 - 30*a^4*b^5*c^3*d^5 + 33*a^5*b^4
*c^2*d^6 - 16*a^6*b^3*c*d^7 + 3*a^7*b^2*d^8)*x^4 + (b^9*c^8 - 18*a^2*b^7*c^
6*d^2 + 52*a^3*b^6*c^5*d^3 - 60*a^4*b^5*c^4*d^4 + 24*a^5*b^4*c^3*d^5 + 10*a
^6*b^3*c^2*d^6 - 12*a^7*b^2*c*d^7 + 3*a^8*b*d^8)*x^3 + (3*a*b^8*c^8 - 12*a^
2*b^7*c^7*d + 10*a^3*b^6*c^6*d^2 + 24*a^4*b^5*c^5*d^3 - 60*a^5*b^4*c^4*d^4
+ 52*a^6*b^3*c^3*d^5 - 18*a^7*b^2*c^2*d^6 + a^9*d^8)*x^2 + (3*a^2*b^7*c^8 -
 16*a^3*b^6*c^7*d + 33*a^4*b^5*c^6*d^2 - 30*a^5*b^4*c^5*d^3 + 5*a^6*b^3*c^4
*d^4 + 12*a^7*b^2*c^3*d^5 - 9*a^8*b*c^2*d^6 + 2*a^9*c*d^7)*x)
```

Sympy [B] time = 7.18453, size = 1217, normalized size = 7.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x**2)**3,x)
```

```
[Out] 10*b**2*d**3*log(x + (-10*a**7*b**2*d**10/(a*d - b*c)**6 + 70*a**6*b**3*c*d
**9/(a*d - b*c)**6 - 210*a**5*b**4*c**2*d**8/(a*d - b*c)**6 + 350*a**4*b**5
*c**3*d**7/(a*d - b*c)**6 - 350*a**3*b**6*c**4*d**6/(a*d - b*c)**6 + 210*a*
*2*b**7*c**5*d**5/(a*d - b*c)**6 - 70*a*b**8*c**6*d**4/(a*d - b*c)**6 + 10*
a*b**2*d**4 + 10*b**9*c**7*d**3/(a*d - b*c)**6 + 10*b**3*c*d**3)/(20*b**3*d
**4))/(a*d - b*c)**6 - 10*b**2*d**3*log(x + (10*a**7*b**2*d**10/(a*d - b*c)
**6 - 70*a**6*b**3*c*d**9/(a*d - b*c)**6 + 210*a**5*b**4*c**2*d**8/(a*d - b
*c)**6 - 350*a**4*b**5*c**3*d**7/(a*d - b*c)**6 + 350*a**3*b**6*c**4*d**6/(
a*d - b*c)**6 - 210*a**2*b**7*c**5*d**5/(a*d - b*c)**6 + 70*a*b**8*c**6*d**
4/(a*d - b*c)**6 + 10*a*b**2*d**4 - 10*b**9*c**7*d**3/(a*d - b*c)**6 + 10*b
**3*c*d**3)/(20*b**3*d**4))/(a*d - b*c)**6 + (-3*a**4*d**4 + 27*a**3*b*c*d*
*3 + 47*a**2*b**2*c**2*d**2 - 13*a*b**3*c**3*d + 2*b**4*c**4 + 60*b**4*d**4
*x**4 + x**3*(150*a*b**3*d**4 + 90*b**4*c*d**3) + x**2*(110*a**2*b**2*d**4
+ 230*a*b**3*c*d**3 + 20*b**4*c**2*d**2) + x*(15*a**3*b*d**4 + 175*a**2*b**
2*c*d**3 + 55*a*b**3*c**2*d**2 - 5*b**4*c**3*d))/(6*a**8*c**2*d**5 - 30*a**
```

```
7*b*c**3*d**4 + 60*a**6*b**2*c**4*d**3 - 60*a**5*b**3*c**5*d**2 + 30*a**4*b
**4*c**6*d - 6*a**3*b**5*c**7 + x**5*(6*a**5*b**3*d**7 - 30*a**4*b**4*c*d**
6 + 60*a**3*b**5*c**2*d**5 - 60*a**2*b**6*c**3*d**4 + 30*a*b**7*c**4*d**3 -
 6*b**8*c**5*d**2) + x**4*(18*a**6*b**2*d**7 - 78*a**5*b**3*c*d**6 + 120*a*
*4*b**4*c**2*d**5 - 60*a**3*b**5*c**3*d**4 - 30*a**2*b**6*c**4*d**3 + 42*a*
b**7*c**5*d**2 - 12*b**8*c**6*d) + x**3*(18*a**7*b*d**7 - 54*a**6*b**2*c*d*
*6 + 6*a**5*b**3*c**2*d**5 + 150*a**4*b**4*c**3*d**4 - 210*a**3*b**5*c**4*d
**3 + 102*a**2*b**6*c**5*d**2 - 6*a*b**7*c**6*d - 6*b**8*c**7) + x**2*(6*a*
*8*d**7 + 6*a**7*b*c*d**6 - 102*a**6*b**2*c**2*d**5 + 210*a**5*b**3*c**3*d*
*4 - 150*a**4*b**4*c**4*d**3 - 6*a**3*b**5*c**5*d**2 + 54*a**2*b**6*c**6*d
- 18*a*b**7*c**7) + x*(12*a**8*c*d**6 - 42*a**7*b*c**2*d**5 + 30*a**6*b**2*
c**3*d**4 + 60*a**5*b**3*c**4*d**3 - 120*a**4*b**4*c**5*d**2 + 78*a**3*b**5
*c**6*d - 18*a**2*b**6*c**7))
```

Giac [B] time = 1.2078, size = 618, normalized size = 3.64

$$-\frac{10\,b^3d^3\log\left(|bx+a|\right)}{b^7c^6-6\,ab^6c^5d+15\,a^2b^5c^4d^2-20\,a^3b^4c^3d^3+15\,a^4b^3c^2d^4-6\,a^5b^2cd^5+a^6bd^6}+\frac{10\,b^2d^4\,\mathrm{lo}}{b^6c^6d-6\,ab^5c^5d^2+15\,a^2b^4c^4d^3-20\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="giac")
```

```
[Out] -10*b^3*d^3*log(abs(b*x + a))/(b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2
 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6) +
 10*b^2*d^4*log(abs(d*x + c))/(b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4
*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7) -
 1/6*(2*b^5*c^5 - 15*a*b^4*c^4*d + 60*a^2*b^3*c^3*d^2 - 20*a^3*b^2*c^2*d^3
- 30*a^4*b*c*d^4 + 3*a^5*d^5 + 60*(b^5*c*d^4 - a*b^4*d^5)*x^4 + 30*(3*b^5*c
^2*d^3 + 2*a*b^4*c*d^4 - 5*a^2*b^3*d^5)*x^3 + 10*(2*b^5*c^3*d^2 + 21*a*b^4*
c^2*d^3 - 12*a^2*b^3*c*d^4 - 11*a^3*b^2*d^5)*x^2 - 5*(b^5*c^4*d - 12*a*b^4*
c^3*d^2 - 24*a^2*b^3*c^2*d^3 + 32*a^3*b^2*c*d^4 + 3*a^4*b*d^5)*x)/((b*c - a
*d)^6*(b*x + a)^3*(d*x + c)^2)
```

3.1828 $\int (d+ex)^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right) dx$

Optimal. Leaf size=39

$$\frac{1}{6}(d+ex)^6\left(a-\frac{cd^2}{e^2}\right)+\frac{cd(d+ex)^7}{7e^2}$$

```
[Out] ((a - (c*d^2)/e^2)*(d + e*x)^6)/6 + (c*d*(d + e*x)^7)/(7*e^2)
```

Rubi [A] time = 0.0226517, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {626, 43}

$$\frac{1}{6}(d+ex)^6\left(a-\frac{cd^2}{e^2}\right)+\frac{cd(d+ex)^7}{7e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] ((a - (c*d^2)/e^2)*(d + e*x)^6)/6 + (c*d*(d + e*x)^7)/(7*e^2)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)dx &= \int (ae+cdx)(d+ex)^5\,dx\\ &= \int \left(\frac{\left(-cd^2+ae^2\right)(d+ex)^5}{e}+\frac{cd(d+ex)^6}{e}\right)dx\\ &= \frac{1}{6}\left(a-\frac{cd^2}{e^2}\right)(d+ex)^6+\frac{cd(d+ex)^7}{7e^2}\end{aligned}$$

Mathematica [B] time = 0.0310917, size = 117, normalized size = 3.

$$\frac{1}{42}x\left(7ae\left(20d^3e^2x^2+15d^2e^3x^3+15d^4ex+6d^5+6de^4x^4+e^5x^5\right)+cdx\left(105d^3e^2x^2+84d^2e^3x^3+70d^4ex+21d^5+35d\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] (x*(7*a*e*(6*d^5 + 15*d^4*e*x + 20*d^3*e^2*x^2 + 15*d^2*e^3*x^3 + 6*d*e^4*x
^4 + e^5*x^5) + c*d*x*(21*d^5 + 70*d^4*e*x + 105*d^3*e^2*x^2 + 84*d^2*e^3*x
^3 + 35*d*e^4*x^4 + 6*e^5*x^5)))/42
```

Maple [B] time = 0.041, size = 198, normalized size = 5.1

$$\frac{e^5dcx^7}{7}+\frac{\left(4\,d^2e^4c+e^4\left(ae^2+cd^2\right)\right)x^6}{6}+\frac{\left(6\,d^3e^3c+4\,de^3\left(ae^2+cd^2\right)+e^5ad\right)x^5}{5}+\frac{\left(4\,d^4e^2c+6\,d^2e^2\left(ae^2+cd^2\right)+4\,d^2e^4a\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

```
[Out] 1/7*e^5*d*c*x^7+1/6*(4*d^2*e^4*c+e^4*(a*e^2+c*d^2))*x^6+1/5*(6*d^3*e^3*c+4*
d*e^3*(a*e^2+c*d^2)+e^5*a*d)*x^5+1/4*(4*d^4*e^2*c+6*d^2*e^2*(a*e^2+c*d^2)+4
*d^2*e^4*a)*x^4+1/3*(d^5*e*c+4*d^3*e*(a*e^2+c*d^2)+6*d^3*e^3*a)*x^3+1/2*(d^
4*(a*e^2+c*d^2)+4*d^4*e^2*a)*x^2+d^5*a*e*x
```

Maxima [B] time = 1.08482, size = 163, normalized size = 4.18

$$\frac{1}{7}\,cde^5x^7+ad^5ex+\frac{1}{6}\left(5\,cd^2e^4+ae^6\right)x^6+\left(2\,cd^3e^3+ade^5\right)x^5+\frac{5}{2}\left(cd^4e^2+ad^2e^4\right)x^4+\frac{5}{3}\left(cd^5e+2\,ad^3e^3\right)x^3+\frac{1}{2}\left(cd^6+5\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="maxima"
)
```

```
[Out] 1/7*c*d*e^5*x^7 + a*d^5*e*x + 1/6*(5*c*d^2*e^4 + a*e^6)*x^6 + (2*c*d^3*e^3
+ a*d*e^5)*x^5 + 5/2*(c*d^4*e^2 + a*d^2*e^4)*x^4 + 5/3*(c*d^5*e + 2*a*d^3*e
^3)*x^3 + 1/2*(c*d^6 + 5*a*d^4*e^2)*x^2
```

Fricas [B] time = 1.34521, size = 286, normalized size = 7.33

$$\frac{1}{7}x^7e^5dc+\frac{5}{6}x^6e^4d^2c+\frac{1}{6}x^6e^6a+2x^5e^3d^3c+x^5e^5da+\frac{5}{2}x^4e^2d^4c+\frac{5}{2}x^4e^4d^2a+\frac{5}{3}x^3ed^5c+\frac{10}{3}x^3e^3d^3a+\frac{1}{2}x^2d^6c+\frac{5}{2}x^2e^2d^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fricas"
)
```

```
[Out] 1/7*x^7*e^5*d*c + 5/6*x^6*e^4*d^2*c + 1/6*x^6*e^6*a + 2*x^5*e^3*d^3*c + x^5
*e^5*d*a + 5/2*x^4*e^2*d^4*c + 5/2*x^4*e^4*d^2*a + 5/3*x^3*e*d^5*c + 10/3*x
^3*e^3*d^3*a + 1/2*x^2*d^6*c + 5/2*x^2*e^2*d^4*a + x*e*d^5*a
```

Sympy [B] time = 0.106722, size = 136, normalized size = 3.49

$$ad^5ex+\frac{cde^5x^7}{7}+x^6\left(\frac{ae^6}{6}+\frac{5cd^2e^4}{6}\right)+x^5\left(ade^5+2cd^3e^3\right)+x^4\left(\frac{5ad^2e^4}{2}+\frac{5cd^4e^2}{2}\right)+x^3\left(\frac{10ad^3e^3}{3}+\frac{5cd^5e}{3}\right)+x^2\left(\frac{5ad^4e^2}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

```
[Out] a*d**5*e*x + c*d*e**5*x**7/7 + x**6*(a*e**6/6 + 5*c*d**2*e**4/6) + x**5*(a*
d*e**5 + 2*c*d**3*e**3) + x**4*(5*a*d**2*e**4/2 + 5*c*d**4*e**2/2) + x**3*(
10*a*d**3*e**3/3 + 5*c*d**5*e/3) + x**2*(5*a*d**4*e**2/2 + c*d**6/2)
```

Giac [B] time = 1.26842, size = 162, normalized size = 4.15

$$\frac{1}{7}\,cdx^7e^5 + \frac{5}{6}\,cd^2x^6e^4 + 2\,cd^3x^5e^3 + \frac{5}{2}\,cd^4x^4e^2 + \frac{5}{3}\,cd^5x^3e + \frac{1}{2}\,cd^6x^2 + \frac{1}{6}\,ax^6e^6 + adx^5e^5 + \frac{5}{2}\,ad^2x^4e^4 + \frac{10}{3}\,ad^3x^3e^3 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")
```

```
[Out] 1/7*c*d*x^7*e^5 + 5/6*c*d^2*x^6*e^4 + 2*c*d^3*x^5*e^3 + 5/2*c*d^4*x^4*e^2 +
 5/3*c*d^5*x^3*e + 1/2*c*d^6*x^2 + 1/6*a*x^6*e^6 + a*d*x^5*e^5 + 5/2*a*d^2*
x^4*e^4 + 10/3*a*d^3*x^3*e^3 + 5/2*a*d^4*x^2*e^2 + a*d^5*x*e
```

3.1829 $\int (d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right) dx$

Optimal. Leaf size=39

$$\frac{1}{5}(d+ex)^5\left(a-\frac{cd^2}{e^2}\right)+\frac{cd(d+ex)^6}{6e^2}$$

```
[Out] ((a - (c*d^2)/e^2)*(d + e*x)^5)/5 + (c*d*(d + e*x)^6)/(6*e^2)
```

Rubi [A] time = 0.0180514, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {626, 43}

$$\frac{1}{5}(d+ex)^5\left(a-\frac{cd^2}{e^2}\right)+\frac{cd(d+ex)^6}{6e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] ((a - (c*d^2)/e^2)*(d + e*x)^5)/5 + (c*d*(d + e*x)^6)/(6*e^2)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)dx &= \int (ae+cdx)(d+ex)^4\,dx\\ &= \int \left(\frac{\left(-cd^2+ae^2\right)(d+ex)^4}{e}+\frac{cd(d+ex)^5}{e}\right)dx\\ &= \frac{1}{5}\left(a-\frac{cd^2}{e^2}\right)(d+ex)^5+\frac{cd(d+ex)^6}{6e^2}\end{aligned}$$

Mathematica [B] time = 0.0190197, size = 95, normalized size = 2.44

$$\frac{1}{30}x\left(6ae\left(10d^2e^2x^2+10d^3ex+5d^4+5de^3x^3+e^4x^4\right)+cdx\left(45d^2e^2x^2+40d^3ex+15d^4+24de^3x^3+5e^4x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] (x*(6*a*e*(5*d^4 + 10*d^3*e*x + 10*d^2*e^2*x^2 + 5*d*e^3*x^3 + e^4*x^4) + c
*d*x*(15*d^4 + 40*d^3*e*x + 45*d^2*e^2*x^2 + 24*d*e^3*x^3 + 5*e^4*x^4)))/30
```

Maple [B] time = 0.04, size = 155, normalized size = 4.

$$\frac{e^4dcx^6}{6}+\frac{\left(3\,d^2e^3c+e^3\left(ae^2+cd^2\right)\right)x^5}{5}+\frac{\left(3\,d^3e^2c+3\,de^2\left(ae^2+cd^2\right)+e^4ad\right)x^4}{4}+\frac{\left(d^4ec+3\,d^2e\left(ae^2+cd^2\right)+3\,d^2e^3a\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

```
[Out] 1/6*e^4*d*c*x^6+1/5*(3*d^2*e^3*c+e^3*(a*e^2+c*d^2))*x^5+1/4*(3*d^3*e^2*c+3*
d*e^2*(a*e^2+c*d^2)+e^4*a*d)*x^4+1/3*(d^4*e*c+3*d^2*e*(a*e^2+c*d^2)+3*d^2*e
^3*a)*x^3+1/2*(d^3*(a*e^2+c*d^2)+3*d^3*e^2*a)*x^2+d^4*a*e*x
```

Maxima [B] time = 1.01701, size = 138, normalized size = 3.54

$$\frac{1}{6}\,cde^4x^6+ad^4ex+\frac{1}{5}\left(4\,cd^2e^3+ae^5\right)x^5+\frac{1}{2}\left(3\,cd^3e^2+2\,ade^4\right)x^4+\frac{2}{3}\left(2\,cd^4e+3\,ad^2e^3\right)x^3+\frac{1}{2}\left(cd^5+4\,ad^3e^2\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="maxima"
)
```

```
[Out] 1/6*c*d*e^4*x^6 + a*d^4*e*x + 1/5*(4*c*d^2*e^3 + a*e^5)*x^5 + 1/2*(3*c*d^3*
e^2 + 2*a*d*e^4)*x^4 + 2/3*(2*c*d^4*e + 3*a*d^2*e^3)*x^3 + 1/2*(c*d^5 + 4*a
*d^3*e^2)*x^2
```

Fricas [B] time = 1.27457, size = 228, normalized size = 5.85

$$\frac{1}{6}x^6e^4dc+\frac{4}{5}x^5e^3d^2c+\frac{1}{5}x^5e^5a+\frac{3}{2}x^4e^2d^3c+x^4e^4da+\frac{4}{3}x^3ed^4c+2x^3e^3d^2a+\frac{1}{2}x^2d^5c+2x^2e^2d^3a+xed^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fricas"
)
```

```
[Out] 1/6*x^6*e^4*d*c + 4/5*x^5*e^3*d^2*c + 1/5*x^5*e^5*a + 3/2*x^4*e^2*d^3*c + x
^4*e^4*d*a + 4/3*x^3*e*d^4*c + 2*x^3*e^3*d^2*a + 1/2*x^2*d^5*c + 2*x^2*e^2*
d^3*a + x*e*d^4*a
```

Sympy [B] time = 0.092746, size = 107, normalized size = 2.74

$$ad^4ex+\frac{cde^4x^6}{6}+x^5\left(\frac{ae^5}{5}+\frac{4cd^2e^3}{5}\right)+x^4\left(ade^4+\frac{3cd^3e^2}{2}\right)+x^3\left(2ad^2e^3+\frac{4cd^4e}{3}\right)+x^2\left(2ad^3e^2+\frac{cd^5}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

```
[Out] a*d**4*e*x + c*d*e**4*x**6/6 + x**5*(a*e**5/5 + 4*c*d**2*e**3/5) + x**4*(a*
d*e**4 + 3*c*d**3*e**2/2) + x**3*(2*a*d**2*e**3 + 4*c*d**4*e/3) + x**2*(2*a
*d**3*e**2 + c*d**5/2)
```

Giac [B] time = 1.23442, size = 132, normalized size = 3.38

$$\frac{1}{6}\,cdx^6e^4 + \frac{4}{5}\,cd^2x^5e^3 + \frac{3}{2}\,cd^3x^4e^2 + \frac{4}{3}\,cd^4x^3e + \frac{1}{2}\,cd^5x^2 + \frac{1}{5}\,ax^5e^5 + adx^4e^4 + 2\,ad^2x^3e^3 + 2\,ad^3x^2e^2 + ad^4xe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")
```

```
[Out] 1/6*c*d*x^6*e^4 + 4/5*c*d^2*x^5*e^3 + 3/2*c*d^3*x^4*e^2 + 4/3*c*d^4*x^3*e +
 1/2*c*d^5*x^2 + 1/5*a*x^5*e^5 + a*d*x^4*e^4 + 2*a*d^2*x^3*e^3 + 2*a*d^3*x^
2*e^2 + a*d^4*x*e
```

3.1830 $\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right) dx$

Optimal. Leaf size=39

$$\frac{1}{4}(d+ex)^4\left(a-\frac{cd^2}{e^2}\right)+\frac{cd(d+ex)^5}{5e^2}$$

```
[Out] ((a - (c*d^2)/e^2)*(d + e*x)^4)/4 + (c*d*(d + e*x)^5)/(5*e^2)
```

Rubi [A] time = 0.0171029, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {626, 43}

$$\frac{1}{4}(d+ex)^4\left(a-\frac{cd^2}{e^2}\right)+\frac{cd(d+ex)^5}{5e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] ((a - (c*d^2)/e^2)*(d + e*x)^4)/4 + (c*d*(d + e*x)^5)/(5*e^2)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)dx &= \int (ae+cdx)(d+ex)^3\,dx\\
&= \int \left(\frac{\left(-cd^2+ae^2\right)(d+ex)^3}{e}+\frac{cd(d+ex)^4}{e}\right)dx\\
&= \frac{1}{4}\left(a-\frac{cd^2}{e^2}\right)(d+ex)^4+\frac{cd(d+ex)^5}{5e^2}
\end{aligned}$$

Mathematica [A] time = 0.0195682, size = 73, normalized size = 1.87

$$\frac{1}{20}x\left(5ae\left(6d^2ex+4d^3+4de^2x^2+e^3x^3\right)+cdx\left(20d^2ex+10d^3+15de^2x^2+4e^3x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] (x*(5*a*e*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) + c*d*x*(10*d^3 + 20*
d^2*e*x + 15*d*e^2*x^2 + 4*e^3*x^3)))/20
```

Maple [B] time = 0.04, size = 112, normalized size = 2.9

$$\frac{e^{3} d c x^{5}}{5}+\frac{\left(2 d^{2} e^{2} c+e^{2} \left(a e^{2}+c d^{2}\right)\right) x^{4}}{4}+\frac{\left(c d^{3} e+2 d e \left(a e^{2}+c d^{2}\right)+a d e^{3}\right) x^{3}}{3}+\frac{\left(d^{2} \left(a e^{2}+c d^{2}\right)+2 a d^{2} e^{2}\right) x^{2}}{2}+d^{3} a e x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

```
[Out] 1/5*e^3*d*c*x^5+1/4*(2*d^2*e^2*c+e^2*(a*e^2+c*d^2))*x^4+1/3*(c*d^3*e+2*d*e*
(a*e^2+c*d^2)+a*d*e^3)*x^3+1/2*(d^2*(a*e^2+c*d^2)+2*a*d^2*e^2)*x^2+d^3*a*e*
x
```

Maxima [B] time = 1.1614, size = 101, normalized size = 2.59

$$\frac{1}{5} c d e^{3} x^{5} + a d^{3} e x + \frac{1}{4} \left(3 c d^{2} e^{2} + a e^{4}\right) x^{4} + \left(c d^{3} e + a d e^{3}\right) x^{3} + \frac{1}{2} \left(c d^{4} + 3 a d^{2} e^{2}\right) x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="maxima"
)
```

```
[Out] 1/5*c*d*e^3*x^5 + a*d^3*e*x + 1/4*(3*c*d^2*e^2 + a*e^4)*x^4 + (c*d^3*e + a*
d*e^3)*x^3 + 1/2*(c*d^4 + 3*a*d^2*e^2)*x^2
```

Fricas [B] time = 1.32734, size = 174, normalized size = 4.46

$$\frac{1}{5} x^{5} e^{3} d c + \frac{3}{4} x^{4} e^{2} d^{2} c + \frac{1}{4} x^{4} e^{4} a + x^{3} e d^{3} c + x^{3} e^{3} d a + \frac{1}{2} x^{2} d^{4} c + \frac{3}{2} x^{2} e^{2} d^{2} a + x e d^{3} a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fricas"
)
```

```
[Out] 1/5*x^5*e^3*d*c + 3/4*x^4*e^2*d^2*c + 1/4*x^4*e^4*a + x^3*e*d^3*c + x^3*e^3
*d*a + 1/2*x^2*d^4*c + 3/2*x^2*e^2*d^2*a + x*e*d^3*a
```

Sympy [B] time = 0.115237, size = 80, normalized size = 2.05

$$a d^{3} e x + \frac{c d e^{3} x^{5}}{5} + x^{4} \left(\frac{a e^{4}}{4} + \frac{3 c d^{2} e^{2}}{4}\right) + x^{3} \left(a d e^{3} + c d^{3} e\right) + x^{2} \left(\frac{3 a d^{2} e^{2}}{2} + \frac{c d^{4}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

```
[Out] a*d**3*e*x + c*d*e**3*x**5/5 + x**4*(a*e**4/4 + 3*c*d**2*e**2/4) + x**3*(a*
d*e**3 + c*d**3*e) + x**2*(3*a*d**2*e**2/2 + c*d**4/2)
```

Giac [B] time = 1.24991, size = 101, normalized size = 2.59

$$\frac{1}{5}\,cdx^5e^3+\frac{3}{4}\,cd^2x^4e^2+cd^3x^3e+\frac{1}{2}\,cd^4x^2+\frac{1}{4}\,ax^4e^4+adx^3e^3+\frac{3}{2}\,ad^2x^2e^2+ad^3xe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")
```

```
[Out] 1/5*c*d*x^5*e^3 + 3/4*c*d^2*x^4*e^2 + c*d^3*x^3*e + 1/2*c*d^4*x^2 + 1/4*a*x
^4*e^4 + a*d*x^3*e^3 + 3/2*a*d^2*x^2*e^2 + a*d^3*x*e
```

3.1831 $\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx$

Optimal. Leaf size=39

$$\frac{1}{3}(d+ex)^3\left(a-\frac{cd^2}{e^2}\right)+\frac{cd(d+ex)^4}{4e^2}$$

```
[Out] ((a - (c*d^2)/e^2)*(d + e*x)^3)/3 + (c*d*(d + e*x)^4)/(4*e^2)
```

Rubi [A] time = 0.033846, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {626, 43}

$$\frac{1}{3}(d+ex)^3\left(a-\frac{cd^2}{e^2}\right)+\frac{cd(d+ex)^4}{4e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] ((a - (c*d^2)/e^2)*(d + e*x)^3)/3 + (c*d*(d + e*x)^4)/(4*e^2)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx &= \int (ae+cdx)(d+ex)^2\,dx\\ &= \int \left(\frac{\left(-cd^2+ae^2\right)(d+ex)^2}{e}+\frac{cd(d+ex)^3}{e}\right)dx\\ &= \frac{1}{3}\left(a-\frac{cd^2}{e^2}\right)(d+ex)^3+\frac{cd(d+ex)^4}{4e^2}\end{aligned}$$

Mathematica [A] time = 0.011793, size = 51, normalized size = 1.31

$$\frac{1}{12}x\left(4ae\left(3d^2+3dex+e^2x^2\right)+cdx\left(6d^2+8dex+3e^2x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] (x*(4*a*e*(3*d^2 + 3*d*e*x + e^2*x^2) + c*d*x*(6*d^2 + 8*d*e*x + 3*e^2*x^2)
))/12
```

Maple [A] time = 0.039, size = 69, normalized size = 1.8

$$\frac{de^2cx^4}{4}+\frac{\left(d^2ec+e\left(ae^2+cd^2\right)\right)x^3}{3}+\frac{\left(d\left(ae^2+cd^2\right)+ade^2\right)x^2}{2}+ad^2ex$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

```
[Out] 1/4*d*e^2*c*x^4+1/3*(d^2*e*c+e*(a*e^2+c*d^2))*x^3+1/2*(d*(a*e^2+c*d^2)+a*d*
e^2)*x^2+a*d^2*e*x
```

Maxima [A] time = 1.02497, size = 73, normalized size = 1.87

$$\frac{1}{4}\,cde^2x^4+ad^2ex+\frac{1}{3}\left(2\,cd^2e+ae^3\right)x^3+\frac{1}{2}\left(cd^3+2\,ade^2\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*c*d*e^2*x^4 + a*d^2*e*x + 1/3*(2*c*d^2*e + a*e^3)*x^3 + 1/2*(c*d^3 + 2*
a*d*e^2)*x^2
```

Fricas [A] time = 1.3799, size = 126, normalized size = 3.23

$$\frac{1}{4}x^4e^2dc+\frac{2}{3}x^3ed^2c+\frac{1}{3}x^3e^3a+\frac{1}{2}x^2d^3c+x^2e^2da+xed^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*e^2*d*c + 2/3*x^3*e*d^2*c + 1/3*x^3*e^3*a + 1/2*x^2*d^3*c + x^2*e^2
*d*a + x*e*d^2*a
```

Sympy [A] time = 0.1642, size = 56, normalized size = 1.44

$$ad^2ex+\frac{cde^2x^4}{4}+x^3\left(\frac{ae^3}{3}+\frac{2cd^2e}{3}\right)+x^2\left(ade^2+\frac{cd^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

```
[Out] a*d**2*e*x + c*d*e**2*x**4/4 + x**3*(a*e**3/3 + 2*c*d**2*e/3) + x**2*(a*d*e
**2 + c*d**3/2)
```

Giac [A] time = 1.16421, size = 73, normalized size = 1.87

$$\frac{1}{4}cdx^4e^2 + \frac{2}{3}cd^2x^3e + \frac{1}{2}cd^3x^2 + \frac{1}{3}ax^3e^3 + adx^2e^2 + ad^2xe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")
```

```
[Out] 1/4*c*d*x^4*e^2 + 2/3*c*d^2*x^3*e + 1/2*c*d^3*x^2 + 1/3*a*x^3*e^3 + a*d*x^2
*e^2 + a*d^2*x*e
```

3.1832 $\int \left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right) dx$

Optimal. Leaf size=34

$$\frac{1}{2}x^2\left(ae^2 + cd^2\right) + adex + \frac{1}{3}cdex^3$$

```
[Out] a*d*e*x + ((c*d^2 + a*e^2)*x^2)/2 + (c*d*e*x^3)/3
```

Rubi [A] time = 0.0089247, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{1}{2}x^2\left(ae^2 + cd^2\right) + adex + \frac{1}{3}cdex^3$$

Antiderivative was successfully verified.

```
[In] Int[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2,x]
```

```
[Out] a*d*e*x + ((c*d^2 + a*e^2)*x^2)/2 + (c*d*e*x^3)/3
```

Rubi steps

$$\int \left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right) dx = adex + \frac{1}{2}\left(cd^2 + ae^2\right)x^2 + \frac{1}{3}cdex^3$$

Mathematica [A] time = 0.0000464, size = 38, normalized size = 1.12

$$adex + \frac{1}{2}ae^2x^2 + \frac{1}{2}cd^2x^2 + \frac{1}{3}cdex^3$$

Antiderivative was successfully verified.

```
[In] Integrate[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2,x]
```

```
[Out] a*d*e*x + (c*d^2*x^2)/2 + (a*e^2*x^2)/2 + (c*d*e*x^3)/3
```

Maple [A] time = 0.039, size = 31, normalized size = 0.9

$$adex + \frac{\left(ae^2 + cd^2\right)x^2}{2} + \frac{cdex^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2,x)
```

```
[Out] a*d*e*x+1/2*(a*e^2+c*d^2)*x^2+1/3*c*d*e*x^3
```

Maxima [A] time = 1.0985, size = 41, normalized size = 1.21

$$\frac{1}{3}\,cdex^3 + adex + \frac{1}{2}\left(cd^2 + ae^2\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2,x, algorithm="maxima")
```

```
[Out] 1/3*c*d*e*x^3 + a*d*e*x + 1/2*(c*d^2 + a*e^2)*x^2
```

Fricas [A] time = 1.28492, size = 77, normalized size = 2.26

$$\frac{1}{3}x^3edc + \frac{1}{2}x^2d^2c + \frac{1}{2}x^2e^2a + xeda$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3*e*d*c + 1/2*x^2*d^2*c + 1/2*x^2*e^2*a + x*e*d*a
```

Sympy [A] time = 0.188778, size = 32, normalized size = 0.94

$$adex + \frac{cdex^3}{3} + x^2\left(\frac{ae^2}{2} + \frac{cd^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2,x)
```

```
[Out] a*d*e*x + c*d*e*x**3/3 + x**2*(a*e**2/2 + c*d**2/2)
```

Giac [A] time = 1.15327, size = 42, normalized size = 1.24

$$\frac{1}{3}\,cdx^3e + adxe + \frac{1}{2}\left(cd^2 + ae^2\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2,x, algorithm="giac")
```

```
[Out] 1/3*c*d*x^3*e + a*d*x*e + 1/2*(c*d^2 + a*e^2)*x^2
```

3.1833 $\int \frac{ade+(cd^2+ae^2)x+cdex^2}{d+ex} dx$

Optimal. Leaf size=14

$$aex + \frac{1}{2}cdx^2$$

```
[Out] a*e*x + (c*d*x^2)/2
```

Rubi [A] time = 0.0079364, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.03, Rules used = {24}

$$aex + \frac{1}{2}cdx^2$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x),x]
```

```
[Out] a*e*x + (c*d*x^2)/2
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{ade + (cd^2 + ae^2)x + cdex^2}{d + ex} dx &= \frac{\int (ae^3 + cde^2x) dx}{e^2} \\ &= aex + \frac{1}{2}cdx^2\end{aligned}$$

Mathematica [A] time = 0.0008907, size = 14, normalized size = 1.

$$aex + \frac{1}{2}cdx^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x),x]
```

```
[Out] a*e*x + (c*d*x^2)/2
```

Maple [A] time = 0.038, size = 13, normalized size = 0.9

$$aex + \frac{cdx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d),x)
```

```
[Out] a*e*x+1/2*c*d*x^2
```

Maxima [A] time = 1.03206, size = 16, normalized size = 1.14

$$\frac{1}{2}\,cdx^2 + aex$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*c*d*x^2 + a*e*x
```

Fricas [A] time = 1.59621, size = 28, normalized size = 2.

$$\frac{1}{2}\,cdx^2 + aex$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/2*c*d*x^2 + a*e*x
```

Sympy [A] time = 0.128321, size = 12, normalized size = 0.86

$$aex + \frac{cdx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)/(e*x+d),x)
```

```
[Out] a*e*x + c*d*x**2/2
```

Giac [A] time = 1.15146, size = 26, normalized size = 1.86

$$\frac{1}{2}\left(cdx^2e^2 + 2\,axe^3\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d),x, algorithm="giac")
```

```
[Out] 1/2*(c*d*x^2*e^2 + 2*a*x*e^3)*e^(-2)
```

3.1834 $\int \frac{ade+\left(cd^2+ae^2\right)x+cdex^2}{(d+ex)^2} dx$

Optimal. Leaf size=26

$$\left(a-\frac{cd^2}{e^2}\right)\log(d+ex)+\frac{cdx}{e}$$

```
[Out] (c*d*x)/e + (a - (c*d^2)/e^2)*Log[d + e*x]
```

Rubi [A] time = 0.0281776, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {24, 43}

$$\left(a-\frac{cd^2}{e^2}\right)\log(d+ex)+\frac{cdx}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^2,x]
```

```
[Out] (c*d*x)/e + (a - (c*d^2)/e^2)*Log[d + e*x]
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{ade+\left(cd^2+ae^2\right)x+cdex^2}{(d+ex)^2}\,dx &= \frac{\int \frac{ae^3+cde^2x}{d+ex}\,dx}{e^2}\\
&= \frac{\int \left(cde+\frac{-cd^2e+ae^3}{d+ex}\right)dx}{e^2}\\
&= \frac{cdx}{e}+\left(a-\frac{cd^2}{e^2}\right)\log(d+ex)
\end{aligned}$$

Mathematica [A] time = 0.0091383, size = 30, normalized size = 1.15

$$\frac{\left(ae^2-cd^2\right)\log(d+ex)}{e^2}+\frac{cdx}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^2,x]
```

```
[Out] (c*d*x)/e + ((-(c*d^2) + a*e^2)*Log[d + e*x])/e^2
```

Maple [A] time = 0.04, size = 32, normalized size = 1.2

$$\frac{cdx}{e} + \ln(ex + d)\,a - \frac{\ln(ex + d)\,cd^2}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^2,x)
```

```
[Out] c*d*x/e+ln(e*x+d)*a-1/e^2*ln(e*x+d)*c*d^2
```

Maxima [A] time = 1.01912, size = 42, normalized size = 1.62

$$\frac{cdx}{e} - \frac{\left(cd^2 - ae^2\right)\log(ex + d)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^2,x, algorithm="maxima"
)
```

```
[Out] c*d*x/e - (c*d^2 - a*e^2)*log(e*x + d)/e^2
```

Fricas [A] time = 1.56012, size = 62, normalized size = 2.38

$$\frac{cdex - \left(cd^2 - ae^2\right)\log(ex + d)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^2,x, algorithm="fricas"
)
```

```
[Out] (c*d*e*x - (c*d^2 - a*e^2)*log(e*x + d))/e^2
```

Sympy [A] time = 0.392545, size = 26, normalized size = 1.

$$\frac{cdx}{e} + \frac{\left(ae^2 - cd^2\right)\log(d + ex)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)/(e*x+d)**2,x)
```

```
[Out] c*d*x/e + (a*e**2 - c*d**2)*log(d + e*x)/e**2
```

Giac [B] time = 1.24215, size = 158, normalized size = 6.08

$$\left(2\,de^{(-3)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^2}\right)+(xe+d)e^{(-3)}-\frac{d^2e^{(-3)}}{xe+d}\right)cde-\left(cd^2+ae^2\right)\left(e^{(-1)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^2}\right)-\frac{de^{(-1)}}{xe+d}\right)e^{(-1)}-\frac{ad}{xe+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] (2*d*e^(-3)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2) + (x*e + d)*e^(-3) - d^2*e
^(-3)/(x*e + d))*c*d*e - (c*d^2 + a*e^2)*(e^(-1)*log(abs(x*e + d)*e^(-1)/(x
*e + d)^2) - d*e^(-1)/(x*e + d))*e^(-1) - a*d/(x*e + d)
```

3.1835 $\int \frac{ade+(cd^2+ae^2)x+cdex^2}{(d+ex)^3}\,dx$

Optimal. Leaf size=33

$$\frac{cd\log(d+ex)}{e^2}-\frac{a-\frac{cd^2}{e^2}}{d+ex}$$

```
[Out] -((a - (c*d^2)/e^2)/(d + e*x)) + (c*d*Log[d + e*x])/e^2
```

Rubi [A] time = 0.0282534, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {24, 43}

$$\frac{cd\log(d+ex)}{e^2}-\frac{a-\frac{cd^2}{e^2}}{d+ex}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^3,x]
```

```
[Out] -((a - (c*d^2)/e^2)/(d + e*x)) + (c*d*Log[d + e*x])/e^2
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_Symbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{ade+\left(cd^2+ae^2\right)x+cdex^2}{(d+ex)^3}\,dx&=\frac{\int \frac{ae^3+cde^2x}{(d+ex)^2}\,dx}{e^2}\\&=\frac{\int \left(\frac{-cd^2e+ae^3}{(d+ex)^2}+\frac{cde}{d+ex}\right)dx}{e^2}\\&=-\frac{a-\frac{cd^2}{e^2}}{d+ex}+\frac{cd\log(d+ex)}{e^2}\end{aligned}$$

Mathematica [A] time = 0.0127439, size = 36, normalized size = 1.09

$$\frac{cd^2-ae^2}{e^2(d+ex)}+\frac{cd\log(d+ex)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^3,x]
```

```
[Out] (c*d^2 - a*e^2)/(e^2*(d + e*x)) + (c*d*Log[d + e*x])/e^2
```

Maple [A] time = 0.046, size = 39, normalized size = 1.2

$$\frac{cd\ln(ex+d)}{e^2}-\frac{a}{ex+d}+\frac{cd^2}{e^2(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^3,x)
```

```
[Out] c*d*ln(e*x+d)/e^2-1/(e*x+d)*a+1/e^2/(e*x+d)*c*d^2
```

Maxima [A] time = 1.1247, size = 53, normalized size = 1.61

$$\frac{cd\log(ex+d)}{e^2}+\frac{cd^2-ae^2}{e^3x+de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^3,x, algorithm="maxima"
)
```

```
[Out] c*d*log(e*x + d)/e^2 + (c*d^2 - a*e^2)/(e^3*x + d*e^2)
```

Fricas [A] time = 1.52604, size = 89, normalized size = 2.7

$$\frac{cd^2-ae^2+\left(cdex+cd^2\right)\log(ex+d)}{e^3x+de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^3,x, algorithm="fricas"
)
```

```
[Out] (c*d^2 - a*e^2 + (c*d*e*x + c*d^2)*log(e*x + d))/(e^3*x + d*e^2)
```

Sympy [A] time = 0.538249, size = 32, normalized size = 0.97

$$\frac{cd\log(d+ex)}{e^2}-\frac{ae^2-cd^2}{de^2+e^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)/(e*x+d)**3,x)
```

```
[Out] c*d*log(d + e*x)/e**2 - (a*e**2 - c*d**2)/(d*e**2 + e**3*x)
```

Giac [A] time = 1.2081, size = 70, normalized size = 2.12

$$cde^{(-2)}\log\left(|xe+d|\right)+\frac{\left(cd^3-ade^2+\left(cd^2e-ae^3\right)x\right)e^{(-2)}}{(xe+d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] c*d*e^(-2)*log(abs(x*e + d)) + (c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)*e^(-
2)/(x*e + d)^2
```

3.1836 $\int \frac{ade+\left(cd^2+ae^2\right)x+cdex^2}{(d+ex)^4}\,dx$

Optimal. Leaf size=35

$$\frac{(ae+cdx)^2}{2(d+ex)^2\left(cd^2-ae^2\right)}$$

```
[Out] (a*e + c*d*x)^2/(2*(c*d^2 - a*e^2)*(d + e*x)^2)
```

Rubi [A] time = 0.0137279, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {24, 37}

$$\frac{(ae+cdx)^2}{2(d+ex)^2\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^4,x]
```

```
[Out] (a*e + c*d*x)^2/(2*(c*d^2 - a*e^2)*(d + e*x)^2)
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{ade+\left(cd^2+ae^2\right)x+cdex^2}{(d+ex)^4}\,dx &= \frac{\int \frac{ae^3+cde^2x}{(d+ex)^3}\,dx}{e^2}\\ &= \frac{(ae+cdx)^2}{2\left(cd^2-ae^2\right)(d+ex)^2}\end{aligned}$$

Mathematica [A] time = 0.0109906, size = 29, normalized size = 0.83

$$-\frac{ae^2+cd(d+2ex)}{2e^2(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^4,x]
```

```
[Out] -(a*e^2 + c*d*(d + 2*e*x))/(2*e^2*(d + e*x)^2)
```

Maple [A] time = 0.043, size = 40, normalized size = 1.1

$$-\frac{ae^2 - cd^2}{2e^2(ex+d)^2} - \frac{cd}{e^2(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^4,x)
```

```
[Out] -1/2*(a*e^2-c*d^2)/e^2/(e*x+d)^2-c*d/e^2/(e*x+d)
```

Maxima [A] time = 1.05566, size = 58, normalized size = 1.66

$$-\frac{2\,cdex + cd^2 + ae^2}{2\left(e^4x^2 + 2\,de^3x + d^2e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^4,x, algorithm="maxima"
)
```

```
[Out] -1/2*(2*c*d*e*x + c*d^2 + a*e^2)/(e^4*x^2 + 2*d*e^3*x + d^2*e^2)
```

Fricas [A] time = 1.59144, size = 89, normalized size = 2.54

$$-\frac{2\,cdex + cd^2 + ae^2}{2\left(e^4x^2 + 2\,de^3x + d^2e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^4,x, algorithm="fricas"
)
```

```
[Out] -1/2*(2*c*d*e*x + c*d^2 + a*e^2)/(e^4*x^2 + 2*d*e^3*x + d^2*e^2)
```

Sympy [A] time = 0.830981, size = 44, normalized size = 1.26

$$-\frac{ae^2 + cd^2 + 2cdex}{2d^2e^2 + 4de^3x + 2e^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)/(e*x+d)**4,x)
```

```
[Out] -(a*e**2 + c*d**2 + 2*c*d*e*x)/(2*d**2*e**2 + 4*d*e**3*x + 2*e**4*x**2)
```

Giac [A] time = 1.17246, size = 62, normalized size = 1.77

$$-\frac{\left(2\,cdx^2e^2+3\,cd^2xe+cd^3+axe^3+ade^2\right)e^{(-2)}}{2\,(xe+d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -1/2*(2*c*d*x^2*e^2 + 3*c*d^2*x*e + c*d^3 + a*x*e^3 + a*d*e^2)*e^(-2)/(x*e
+ d)^3
```

3.1837 $\int \frac{ade+(cd^2+ae^2)x+cdex^2}{(d+ex)^5}\,dx$

Optimal. Leaf size=39

$$-\frac{a-\frac{cd^2}{e^2}}{3(d+ex)^3}-\frac{cd}{2e^2(d+ex)^2}$$

```
[Out] -(a - (c*d^2)/e^2)/(3*(d + e*x)^3) - (c*d)/(2*e^2*(d + e*x)^2)
```

Rubi [A] time = 0.0276398, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {24, 43}

$$-\frac{a-\frac{cd^2}{e^2}}{3(d+ex)^3}-\frac{cd}{2e^2(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^5,x]
```

```
[Out] -(a - (c*d^2)/e^2)/(3*(d + e*x)^3) - (c*d)/(2*e^2*(d + e*x)^2)
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{ade+\left(cd^2+ae^2\right)x+cdex^2}{(d+ex)^5}\,dx &= \frac{\int \frac{ae^3+cde^2x}{(d+ex)^4}\,dx}{e^2}\\
&= \frac{\int \left(\frac{-cd^2e+ae^3}{(d+ex)^4}+\frac{cde}{(d+ex)^3}\right)dx}{e^2}\\
&= -\frac{a-\frac{cd^2}{e^2}}{3(d+ex)^3}-\frac{cd}{2e^2(d+ex)^2}
\end{aligned}$$

Mathematica [A] time = 0.0118837, size = 30, normalized size = 0.77

$$-\frac{2ae^2+cd(d+3ex)}{6e^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^5,x]
```

```
[Out] -(2*a*e^2 + c*d*(d + 3*e*x))/(6*e^2*(d + e*x)^3)
```

Maple [A] time = 0.044, size = 40, normalized size = 1.

$$-\frac{cd}{2\,e^2\,(ex+d)^2}-\frac{ae^2-cd^2}{3\,e^2\,(ex+d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^5,x)
```

```
[Out] -1/2*c*d/e^2/(e*x+d)^2-1/3*(a*e^2-c*d^2)/e^2/(e*x+d)^3
```

Maxima [A] time = 1.12517, size = 74, normalized size = 1.9

$$-\frac{3\,cdex+cd^2+2\,ae^2}{6\left(e^5x^3+3\,de^4x^2+3\,d^2e^3x+d^3e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^5,x, algorithm="maxima"
)
```

```
[Out] -1/6*(3*c*d*e*x + c*d^2 + 2*a*e^2)/(e^5*x^3 + 3*d*e^4*x^2 + 3*d^2*e^3*x + d
^3*e^2)
```

Fricas [A] time = 1.56125, size = 113, normalized size = 2.9

$$-\frac{3\,cdex+cd^2+2\,ae^2}{6\left(e^5x^3+3\,de^4x^2+3\,d^2e^3x+d^3e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^5,x, algorithm="fricas"
)
```

```
[Out] -1/6*(3*c*d*e*x + c*d^2 + 2*a*e^2)/(e^5*x^3 + 3*d*e^4*x^2 + 3*d^2*e^3*x + d
^3*e^2)
```

Sympy [A] time = 0.85635, size = 58, normalized size = 1.49

$$-\frac{2ae^2+cd^2+3cdex}{6d^3e^2+18d^2e^3x+18de^4x^2+6e^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)/(e*x+d)**5,x)
```

```
[Out] -(2*a*e**2 + c*d**2 + 3*c*d*e*x)/(6*d**3*e**2 + 18*d**2*e**3*x + 18*d*e**4*
x**2 + 6*e**5*x**3)
```

Giac [A] time = 1.27447, size = 57, normalized size = 1.46

$$-\frac{cde^{(-2)}}{2\,(xe+d)^2}+\frac{cd^2e^{(-2)}}{3\,(xe+d)^3}-\frac{a}{3\,(xe+d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] -1/2*c*d*e^(-2)/(x*e + d)^2 + 1/3*c*d^2*e^(-2)/(x*e + d)^3 - 1/3*a/(x*e + d
)^3
```

3.1838 $\int \frac{ade+(cd^2+ae^2)x+cdex^2}{(d+ex)^6} dx$

Optimal. Leaf size=39

$$-\frac{a-\frac{cd^2}{e^2}}{4(d+ex)^4}-\frac{cd}{3e^2(d+ex)^3}$$

```
[Out] -(a - (c*d^2)/e^2)/(4*(d + e*x)^4) - (c*d)/(3*e^2*(d + e*x)^3)
```

Rubi [A] time = 0.0278199, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {24, 43}

$$-\frac{a-\frac{cd^2}{e^2}}{4(d+ex)^4}-\frac{cd}{3e^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^6,x]
```

```
[Out] -(a - (c*d^2)/e^2)/(4*(d + e*x)^4) - (c*d)/(3*e^2*(d + e*x)^3)
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{ade+(cd^2+ae^2)x+cdex^2}{(d+ex)^6}dx &= \frac{\int \frac{ae^3+cde^2x}{(d+ex)^5}dx}{e^2}\\ &= \frac{\int \left(\frac{-cd^2e+ae^3}{(d+ex)^5}+\frac{cde}{(d+ex)^4}\right)dx}{e^2}\\ &= -\frac{a-\frac{cd^2}{e^2}}{4(d+ex)^4}-\frac{cd}{3e^2(d+ex)^3}\end{aligned}$$

Mathematica [A] time = 0.012282, size = 30, normalized size = 0.77

$$-\frac{3ae^2+cd(d+4ex)}{12e^2(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^6,x]
```

```
[Out] -(3*a*e^2 + c*d*(d + 4*e*x))/(12*e^2*(d + e*x)^4)
```

Maple [A] time = 0.043, size = 40, normalized size = 1.

$$-\frac{ae^2-cd^2}{4\,e^2\,(ex+d)^4}-\frac{cd}{3\,e^2\,(ex+d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^6,x)
```

```
[Out] -1/4*(a*e^2-c*d^2)/e^2/(e*x+d)^4-1/3*c*d/e^2/(e*x+d)^3
```

Maxima [A] time = 1.17402, size = 89, normalized size = 2.28

$$-\frac{4\,cdex+cd^2+3\,ae^2}{12\,\left(e^6x^4+4\,de^5x^3+6\,d^2e^4x^2+4\,d^3e^3x+d^4e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^6,x, algorithm="maxima"
)
```

```
[Out] -1/12*(4*c*d*e*x + c*d^2 + 3*a*e^2)/(e^6*x^4 + 4*d*e^5*x^3 + 6*d^2*e^4*x^2
+ 4*d^3*e^3*x + d^4*e^2)
```

Fricas [A] time = 1.44876, size = 136, normalized size = 3.49

$$-\frac{4\,cdex+cd^2+3\,ae^2}{12\,\left(e^6x^4+4\,de^5x^3+6\,d^2e^4x^2+4\,d^3e^3x+d^4e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^6,x, algorithm="fricas"
)
```

```
[Out] -1/12*(4*c*d*e*x + c*d^2 + 3*a*e^2)/(e^6*x^4 + 4*d*e^5*x^3 + 6*d^2*e^4*x^2
+ 4*d^3*e^3*x + d^4*e^2)
```

Sympy [B] time = 0.842324, size = 70, normalized size = 1.79

$$-\frac{3ae^2+cd^2+4cdex}{12d^4e^2+48d^3e^3x+72d^2e^4x^2+48de^5x^3+12e^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)/(e*x+d)**6,x)
```

```
[Out] -(3*a*e**2 + c*d**2 + 4*c*d*e*x)/(12*d**4*e**2 + 48*d**3*e**3*x + 72*d**2*e
**4*x**2 + 48*d*e**5*x**3 + 12*e**6*x**4)
```

Giac [A] time = 1.22475, size = 65, normalized size = 1.67

$$-\frac{\left(4\,cdx^2e^2 + 5\,cd^2xe + cd^3 + 3\,axe^3 + 3\,ade^2\right)e^{(-2)}}{12\,(xe + d)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^6,x, algorithm="giac")
```

```
[Out] -1/12*(4*c*d*x^2*e^2 + 5*c*d^2*x*e + c*d^3 + 3*a*x*e^3 + 3*a*d*e^2)*e^(-2)/
(x*e + d)^5
```

3.1839 $\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2 \, dx$

Optimal. Leaf size=77

$$-\frac{cd(d+ex)^6\left(cd^2-ae^2\right)}{3e^3}+\frac{(d+ex)^5\left(cd^2-ae^2\right)^2}{5e^3}+\frac{c^2d^2(d+ex)^7}{7e^3}$$

```
[Out] ((c*d^2 - a*e^2)^2*(d + e*x)^5)/(5*e^3) - (c*d*(c*d^2 - a*e^2)*(d + e*x)^6)
/(3*e^3) + (c^2*d^2*(d + e*x)^7)/(7*e^3)
```

Rubi [A] time = 0.131867, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$-\frac{cd(d+ex)^6\left(cd^2-ae^2\right)}{3e^3}+\frac{(d+ex)^5\left(cd^2-ae^2\right)^2}{5e^3}+\frac{c^2d^2(d+ex)^7}{7e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] ((c*d^2 - a*e^2)^2*(d + e*x)^5)/(5*e^3) - (c*d*(c*d^2 - a*e^2)*(d + e*x)^6)
/(3*e^3) + (c^2*d^2*(d + e*x)^7)/(7*e^3)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2\,dx &= \int (ae+cdx)^2(d+ex)^4\,dx\\ &= \int \left(\frac{\left(-cd^2+ae^2\right)^2(d+ex)^4}{e^2}-\frac{2cd\left(cd^2-ae^2\right)(d+ex)^5}{e^2}+\frac{c^2d^2(d+ex)^6}{e^2}\right)dx\\ &= \frac{\left(cd^2-ae^2\right)^2(d+ex)^5}{5e^3}-\frac{cd\left(cd^2-ae^2\right)(d+ex)^6}{3e^3}+\frac{c^2d^2(d+ex)^7}{7e^3}\end{aligned}$$

Mathematica [B] time = 0.0315438, size = 160, normalized size = 2.08

$$a^2\left(2d^2e^4x^3+2d^3e^3x^2+d^4e^2x+de^5x^4+\frac{e^6x^5}{5}\right)+\frac{1}{15}acdex^2\left(45d^2e^2x^2+40d^3ex+15d^4+24de^3x^3+5e^4x^4\right)+\frac{1}{105}c^2d^2x^3$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] (a*c*d*e*x^2*(15*d^4 + 40*d^3*e*x + 45*d^2*e^2*x^2 + 24*d*e^3*x^3 + 5*e^4*x
^4))/15 + (c^2*d^2*x^3*(35*d^4 + 105*d^3*e*x + 126*d^2*e^2*x^2 + 70*d*e^3*x
^3 + 15*e^4*x^4))/105 + a^2*(d^4*e^2*x + 2*d^3*e^3*x^2 + 2*d^2*e^4*x^3 + d*
e^5*x^4 + (e^6*x^5)/5)
```

Maple [B] time = 0.041, size = 295, normalized size = 3.8

$$\frac{e^4d^2c^2x^7}{7}+\frac{\left(2\,d^3e^3c^2+2\,e^3\left(ae^2+cd^2\right)dc\right)x^6}{6}+\frac{\left(d^4e^2c^2+4\,d^2e^2\left(ae^2+cd^2\right)c+e^2\left(2\,acd^2e^2+\left(ae^2+cd^2\right)^2\right)\right)x^5}{5}+\frac{(2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

```
[Out] 1/7*e^4*d^2*c^2*x^7+1/6*(2*d^3*e^3*c^2+2*e^3*(a*e^2+c*d^2)*d*c)*x^6+1/5*(d^
4*e^2*c^2+4*d^2*e^2*(a*e^2+c*d^2)*c+e^2*(2*a*c*d^2*e^2+(a*e^2+c*d^2)^2))*x^
5+1/4*(2*d^3*(a*e^2+c*d^2)*e*c+2*d*e*(2*a*c*d^2*e^2+(a*e^2+c*d^2)^2)+2*e^3*
a*d*(a*e^2+c*d^2))*x^4+1/3*(d^2*(2*a*c*d^2*e^2+(a*e^2+c*d^2)^2)+4*d^2*e^2*a
*(a*e^2+c*d^2)+e^4*a^2*d^2)*x^3+1/2*(2*d^3*a*e*(a*e^2+c*d^2)+2*d^3*e^3*a^2)
*x^2+d^4*a^2*e^2*x
```

Maxima [B] time = 1.13418, size = 232, normalized size = 3.01

$$\frac{1}{7}\,c^2d^2e^4x^7+a^2d^4e^2x+\frac{1}{3}\left(2\,c^2d^3e^3+acde^5\right)x^6+\frac{1}{5}\left(6\,c^2d^4e^2+8\,acd^2e^4+a^2e^6\right)x^5+\left(c^2d^5e+3\,acd^3e^3+a^2de^5\right)x^4+\frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="maxim
a")
```

```
[Out] 1/7*c^2*d^2*e^4*x^7 + a^2*d^4*e^2*x + 1/3*(2*c^2*d^3*e^3 + a*c*d*e^5)*x^6 +
 1/5*(6*c^2*d^4*e^2 + 8*a*c*d^2*e^4 + a^2*e^6)*x^5 + (c^2*d^5*e + 3*a*c*d^3
*e^3 + a^2*d*e^5)*x^4 + 1/3*(c^2*d^6 + 8*a*c*d^4*e^2 + 6*a^2*d^2*e^4)*x^3 +
 (a*c*d^5*e + 2*a^2*d^3*e^3)*x^2
```

Fricas [B] time = 1.38587, size = 390, normalized size = 5.06

$$\frac{1}{7}x^7e^4d^2c^2+\frac{2}{3}x^6e^3d^3c^2+\frac{1}{3}x^6e^5dca+\frac{6}{5}x^5e^2d^4c^2+\frac{8}{5}x^5e^4d^2ca+\frac{1}{5}x^5e^6a^2+x^4ed^5c^2+3x^4e^3d^3ca+x^4e^5da^2+\frac{1}{3}x^3d^6c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="frica
s")
```

```
[Out] 1/7*x^7*e^4*d^2*c^2 + 2/3*x^6*e^3*d^3*c^2 + 1/3*x^6*e^5*d*c*a + 6/5*x^5*e^2
*d^4*c^2 + 8/5*x^5*e^4*d^2*c*a + 1/5*x^5*e^6*a^2 + x^4*e*d^5*c^2 + 3*x^4*e^
```

```
3*d^3*c*a + x^4*e^5*d*a^2 + 1/3*x^3*d^6*c^2 + 8/3*x^3*e^2*d^4*c*a + 2*x^3*e
^4*d^2*a^2 + x^2*e*d^5*c*a + 2*x^2*e^3*d^3*a^2 + x*e^2*d^4*a^2
```

Sympy [B] time = 0.135621, size = 185, normalized size = 2.4

$$a^2d^4e^2x + \frac{c^2d^2e^4x^7}{7} + x^6\left(\frac{acde^5}{3} + \frac{2c^2d^3e^3}{3}\right) + x^5\left(\frac{a^2e^6}{5} + \frac{8acd^2e^4}{5} + \frac{6c^2d^4e^2}{5}\right) + x^4\left(a^2de^5 + 3acd^3e^3 + c^2d^5e\right) + x^3\left(2a^2d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

```
[Out] a**2*d**4*e**2*x + c**2*d**2*e**4*x**7/7 + x**6*(a*c*d*e**5/3 + 2*c**2*d**3
*e**3/3) + x**5*(a**2*e**6/5 + 8*a*c*d**2*e**4/5 + 6*c**2*d**4*e**2/5) + x*
*4*(a**2*d*e**5 + 3*a*c*d**3*e**3 + c**2*d**5*e) + x**3*(2*a**2*d**2*e**4 +
 8*a*c*d**4*e**2/3 + c**2*d**6/3) + x**2*(2*a**2*d**3*e**3 + a*c*d**5*e)
```

Giac [B] time = 1.20042, size = 238, normalized size = 3.09

$$\frac{1}{7}c^2d^2x^7e^4 + \frac{2}{3}c^2d^3x^6e^3 + \frac{6}{5}c^2d^4x^5e^2 + c^2d^5x^4e + \frac{1}{3}c^2d^6x^3 + \frac{1}{3}acdx^6e^5 + \frac{8}{5}acd^2x^5e^4 + 3\,acd^3x^4e^3 + \frac{8}{3}acd^4x^3e^2 + acd^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac"
)
```

```
[Out] 1/7*c^2*d^2*x^7*e^4 + 2/3*c^2*d^3*x^6*e^3 + 6/5*c^2*d^4*x^5*e^2 + c^2*d^5*x
^4*e + 1/3*c^2*d^6*x^3 + 1/3*a*c*d*x^6*e^5 + 8/5*a*c*d^2*x^5*e^4 + 3*a*c*d^
3*x^4*e^3 + 8/3*a*c*d^4*x^3*e^2 + a*c*d^5*x^2*e + 1/5*a^2*x^5*e^6 + a^2*d*x
^4*e^5 + 2*a^2*d^2*x^3*e^4 + 2*a^2*d^3*x^2*e^3 + a^2*d^4*x*e^2
```

3.1840 $\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2\,dx$

Optimal. Leaf size=77

$$-\frac{2cd(d+ex)^5\left(cd^2-ae^2\right)}{5e^3}+\frac{(d+ex)^4\left(cd^2-ae^2\right)^2}{4e^3}+\frac{c^2d^2(d+ex)^6}{6e^3}$$

```
[Out] ((c*d^2 - a*e^2)^2*(d + e*x)^4)/(4*e^3) - (2*c*d*(c*d^2 - a*e^2)*(d + e*x)^
5)/(5*e^3) + (c^2*d^2*(d + e*x)^6)/(6*e^3)
```

Rubi [A] time = 0.0792925, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {626, 43}

$$-\frac{2cd(d+ex)^5\left(cd^2-ae^2\right)}{5e^3}+\frac{(d+ex)^4\left(cd^2-ae^2\right)^2}{4e^3}+\frac{c^2d^2(d+ex)^6}{6e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] ((c*d^2 - a*e^2)^2*(d + e*x)^4)/(4*e^3) - (2*c*d*(c*d^2 - a*e^2)*(d + e*x)^
5)/(5*e^3) + (c^2*d^2*(d + e*x)^6)/(6*e^3)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2\,dx &= \int (ae+cdx)^2(d+ex)^3\,dx\\
&= \int \left(\frac{\left(-cd^2+ae^2\right)^2(d+ex)^3}{e^2}-\frac{2cd\left(cd^2-ae^2\right)(d+ex)^4}{e^2}+\frac{c^2d^2(d+ex)}{e^2}\right.\\
&= \frac{\left(cd^2-ae^2\right)^2(d+ex)^4}{4e^3}-\frac{2cd\left(cd^2-ae^2\right)(d+ex)^5}{5e^3}+\frac{c^2d^2(d+ex)^6}{6e^3}
\end{aligned}$$

Mathematica [A] time = 0.0323462, size = 120, normalized size = 1.56

$$\frac{1}{60}x\left(15a^2e^2\left(6d^2ex+4d^3+4de^2x^2+e^3x^3\right)+6acdex\left(20d^2ex+10d^3+15de^2x^2+4e^3x^3\right)+c^2d^2x^2\left(45d^2ex+20d^3+3\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] (x*(15*a^2*e^2*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) + 6*a*c*d*e*x*(1
0*d^3 + 20*d^2*e*x + 15*d*e^2*x^2 + 4*e^3*x^3) + c^2*d^2*x^2*(20*d^3 + 45*d
^2*e*x + 36*d*e^2*x^2 + 10*e^3*x^3)))/60
```

Maple [B] time = 0.039, size = 195, normalized size = 2.5

$$\frac{d^2e^3c^2x^6}{6}+\frac{\left(d^3e^2c^2+2\,e^2\left(ae^2+cd^2\right)dc\right)x^5}{5}+\frac{\left(2\,d^2\left(ae^2+cd^2\right)ec+e\left(2\,acd^2e^2+\left(ae^2+cd^2\right)^2\right)\right)x^4}{4}+\frac{\left(d\left(2\,acd^2e^2+\left(ae^2\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

```
[Out] 1/6*d^2*e^3*c^2*x^6+1/5*(d^3*e^2*c^2+2*e^2*(a*e^2+c*d^2)*d*c)*x^5+1/4*(2*d^
2*(a*e^2+c*d^2)*e*c+e*(2*a*c*d^2*e^2+(a*e^2+c*d^2)^2))*x^4+1/3*(d*(2*a*c*d^
2*e^2+(a*e^2+c*d^2)^2)+2*e^2*a*d*(a*e^2+c*d^2))*x^3+1/2*(2*d^2*a*e*(a*e^2+c
*d^2)+a^2*d^2*e^3)*x^2+a^2*d^3*e^2*x
```

Maxima [A] time = 1.63677, size = 189, normalized size = 2.45

$$\frac{1}{6}\,c^2d^2e^3x^6+a^2d^3e^2x+\frac{1}{5}\left(3\,c^2d^3e^2+2\,acde^4\right)x^5+\frac{1}{4}\left(3\,c^2d^4e+6\,acd^2e^3+a^2e^5\right)x^4+\frac{1}{3}\left(c^2d^5+6\,acd^3e^2+3\,a^2de^4\right)x^3+\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="maxima"
)
```

```
[Out] 1/6*c^2*d^2*e^3*x^6 + a^2*d^3*e^2*x + 1/5*(3*c^2*d^3*e^2 + 2*a*c*d*e^4)*x^5
 + 1/4*(3*c^2*d^4*e + 6*a*c*d^2*e^3 + a^2*e^5)*x^4 + 1/3*(c^2*d^5 + 6*a*c*d
^3*e^2 + 3*a^2*d*e^4)*x^3 + 1/2*(2*a*c*d^4*e + 3*a^2*d^2*e^3)*x^2
```

Fricas [B] time = 1.42418, size = 312, normalized size = 4.05

$$\frac{1}{6}x^6e^3d^2c^2+\frac{3}{5}x^5e^2d^3c^2+\frac{2}{5}x^5e^4dca+\frac{3}{4}x^4ed^4c^2+\frac{3}{2}x^4e^3d^2ca+\frac{1}{4}x^4e^5a^2+\frac{1}{3}x^3d^5c^2+2x^3e^2d^3ca+x^3e^4da^2+x^2ed^4ca+\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="fricas"
)
```

```
[Out] 1/6*x^6*e^3*d^2*c^2 + 3/5*x^5*e^2*d^3*c^2 + 2/5*x^5*e^4*d*c*a + 3/4*x^4*e*d
^4*c^2 + 3/2*x^4*e^3*d^2*c*a + 1/4*x^4*e^5*a^2 + 1/3*x^3*d^5*c^2 + 2*x^3*e^
2*d^3*c*a + x^3*e^4*d*a^2 + x^2*e*d^4*c*a + 3/2*x^2*e^3*d^2*a^2 + x*e^2*d^3
*a^2
```

Sympy [B] time = 0.107471, size = 150, normalized size = 1.95

$$a^2d^3e^2x + \frac{c^2d^2e^3x^6}{6} + x^5\left(\frac{2acde^4}{5} + \frac{3c^2d^3e^2}{5}\right) + x^4\left(\frac{a^2e^5}{4} + \frac{3acd^2e^3}{2} + \frac{3c^2d^4e}{4}\right) + x^3\left(a^2de^4 + 2acd^3e^2 + \frac{c^2d^5}{3}\right) + x^2\left(\frac{3}{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

```
[Out] a**2*d**3*e**2*x + c**2*d**2*e**3*x**6/6 + x**5*(2*a*c*d*e**4/5 + 3*c**2*d*
*3*e**2/5) + x**4*(a**2*e**5/4 + 3*a*c*d**2*e**3/2 + 3*c**2*d**4*e/4) + x**
3*(a**2*d*e**4 + 2*a*c*d**3*e**2 + c**2*d**5/3) + x**2*(3*a**2*d**2*e**3/2
+ a*c*d**4*e)
```

Giac [A] time = 1.15863, size = 188, normalized size = 2.44

$$\frac{1}{6}c^2d^2x^6e^3 + \frac{3}{5}c^2d^3x^5e^2 + \frac{3}{4}c^2d^4x^4e + \frac{1}{3}c^2d^5x^3 + \frac{2}{5}acdx^5e^4 + \frac{3}{2}acd^2x^4e^3 + 2\,acd^3x^3e^2 + acd^4x^2e + \frac{1}{4}a^2x^4e^5 + a^2dx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*c^2*d^2*x^6*e^3 + 3/5*c^2*d^3*x^5*e^2 + 3/4*c^2*d^4*x^4*e + 1/3*c^2*d^5
*x^3 + 2/5*a*c*d*x^5*e^4 + 3/2*a*c*d^2*x^4*e^3 + 2*a*c*d^3*x^3*e^2 + a*c*d^
4*x^2*e + 1/4*a^2*x^4*e^5 + a^2*d*x^3*e^4 + 3/2*a^2*d^2*x^2*e^3 + a^2*d^3*x
*e^2
```

3.1841 $\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2 dx$

Optimal. Leaf size=77

$$-\frac{cd(d+ex)^4\left(cd^2-ae^2\right)}{2e^3}+\frac{(d+ex)^3\left(cd^2-ae^2\right)^2}{3e^3}+\frac{c^2d^2(d+ex)^5}{5e^3}$$

```
[Out] ((c*d^2 - a*e^2)^2*(d + e*x)^3)/(3*e^3) - (c*d*(c*d^2 - a*e^2)*(d + e*x)^4)
/(2*e^3) + (c^2*d^2*(d + e*x)^5)/(5*e^3)
```

Rubi [A] time = 0.0882099, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {610, 43}

$$-\frac{cd(d+ex)^4\left(cd^2-ae^2\right)}{2e^3}+\frac{(d+ex)^3\left(cd^2-ae^2\right)^2}{3e^3}+\frac{c^2d^2(d+ex)^5}{5e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] ((c*d^2 - a*e^2)^2*(d + e*x)^3)/(3*e^3) - (c*d*(c*d^2 - a*e^2)*(d + e*x)^4)
/(2*e^3) + (c^2*d^2*(d + e*x)^5)/(5*e^3)
```

Rule 610

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Dist[1/c^p, Int[Simp[b/2 - q/2 + c*x, x]^p*Simp[b/2 + q/2 + c
*x, x]^p, x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2 dx &= \frac{\int \left(cd^2+cdex\right)^2\left(ae^2+cdex\right)^2 dx}{c^2d^2e^2}\\
&= \frac{\int \left(\left(cd^2-ae^2\right)^2\left(cd^2+cdex\right)^2-2\left(cd^2-ae^2\right)\left(cd^2+cdex\right)^3+\left(cd^2+cdex\right)^4\right)dx}{c^2d^2e^2}\\
&= \frac{\left(cd^2-ae^2\right)^2(d+ex)^3}{3e^3}-\frac{cd\left(cd^2-ae^2\right)(d+ex)^4}{2e^3}+\frac{c^2d^2(d+ex)^5}{5e^3}
\end{aligned}$$

Mathematica [A] time = 0.0242541, size = 87, normalized size = 1.13

$$\frac{1}{30}x\left(10a^2e^2\left(3d^2+3dex+e^2x^2\right)+5acdex\left(6d^2+8dex+3e^2x^2\right)+c^2d^2x^2\left(10d^2+15dex+6e^2x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] (x*(10*a^2*e^2*(3*d^2 + 3*d*e*x + e^2*x^2) + 5*a*c*d*e*x*(6*d^2 + 8*d*e*x +
 3*e^2*x^2) + c^2*d^2*x^2*(10*d^2 + 15*d*e*x + 6*e^2*x^2)))/30
```

Maple [A] time = 0.044, size = 93, normalized size = 1.2

$$\frac{d^2e^2c^2x^5}{5}+\frac{\left(ae^2+cd^2\right)decx^4}{2}+\frac{\left(2\,acd^2e^2+\left(ae^2+cd^2\right)^2\right)x^3}{3}+ade\left(ae^2+cd^2\right)x^2+a^2d^2e^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

```
[Out] 1/5*d^2*e^2*c^2*x^5+1/2*(a*e^2+c*d^2)*d*e*c*x^4+1/3*(2*a*c*d^2*e^2+(a*e^2+c
*d^2)^2)*x^3+a*d*e*(a*e^2+c*d^2)*x^2+a^2*d^2*e^2*x
```

Maxima [A] time = 1.0831, size = 126, normalized size = 1.64

$$\frac{1}{5}\,c^2d^2e^2x^5+\frac{1}{2}\left(cd^2+ae^2\right)cdex^4+a^2d^2e^2x+\frac{1}{3}\left(cd^2+ae^2\right)^2x^3+\frac{1}{3}\left(2\,cdex^3+3\left(cd^2+ae^2\right)x^2\right)ade$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/5*c^2*d^2*e^2*x^5 + 1/2*(c*d^2 + a*e^2)*c*d*e*x^4 + a^2*d^2*e^2*x + 1/3*(
c*d^2 + a*e^2)^2*x^3 + 1/3*(2*c*d*e*x^3 + 3*(c*d^2 + a*e^2)*x^2)*a*d*e
```

Fricas [A] time = 1.37704, size = 225, normalized size = 2.92

$$\frac{1}{5}x^5e^2d^2c^2+\frac{1}{2}x^4ed^3c^2+\frac{1}{2}x^4e^3dca+\frac{1}{3}x^3d^4c^2+\frac{4}{3}x^3e^2d^2ca+\frac{1}{3}x^3e^4a^2+x^2ed^3ca+x^2e^3da^2+xe^2d^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*e^2*d^2*c^2 + 1/2*x^4*e*d^3*c^2 + 1/2*x^4*e^3*d*c*a + 1/3*x^3*d^4*c
^2 + 4/3*x^3*e^2*d^2*c*a + 1/3*x^3*e^4*a^2 + x^2*e*d^3*c*a + x^2*e^3*d*a^2
+ x*e^2*d^2*a^2
```

Sympy [A] time = 0.119938, size = 104, normalized size = 1.35

$$a^2d^2e^2x+\frac{c^2d^2e^2x^5}{5}+x^4\left(\frac{acde^3}{2}+\frac{c^2d^3e}{2}\right)+x^3\left(\frac{a^2e^4}{3}+\frac{4acd^2e^2}{3}+\frac{c^2d^4}{3}\right)+x^2\left(a^2de^3+acd^3e\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

```
[Out] a**2*d**2*e**2*x + c**2*d**2*e**2*x**5/5 + x**4*(a*c*d*e**3/2 + c**2*d**3*e
/2) + x**3*(a**2*e**4/3 + 4*a*c*d**2*e**2/3 + c**2*d**4/3) + x**2*(a**2*d*e
**3 + a*c*d**3*e)
```

Giac [A] time = 1.18371, size = 136, normalized size = 1.77

$$\frac{1}{5}c^2d^2x^5e^2 + \frac{1}{2}c^2d^3x^4e + \frac{1}{3}c^2d^4x^3 + \frac{1}{2}acdx^4e^3 + \frac{4}{3}acd^2x^3e^2 + acd^3x^2e + \frac{1}{3}a^2x^3e^4 + a^2dx^2e^3 + a^2d^2xe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/5*c^2*d^2*x^5*e^2 + 1/2*c^2*d^3*x^4*e + 1/3*c^2*d^4*x^3 + 1/2*a*c*d*x^4*e
^3 + 4/3*a*c*d^2*x^3*e^2 + a*c*d^3*x^2*e + 1/3*a^2*x^3*e^4 + a^2*d*x^2*e^3
+ a^2*d^2*x*e^2
```

3.1842 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{d+ex}\,dx$

Optimal. Leaf size=54

$$\frac{\left(cd^2-ae^2\right)(ae+cdx)^3}{3c^2d^2}+\frac{e(ae+cdx)^4}{4c^2d^2}$$

```
[Out] ((c*d^2 - a*e^2)*(a*e + c*d*x)^3)/(3*c^2*d^2) + (e*(a*e + c*d*x)^4)/(4*c^2*
d^2)
```

Rubi [A] time = 0.0490563, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$\frac{\left(cd^2-ae^2\right)(ae+cdx)^3}{3c^2d^2}+\frac{e(ae+cdx)^4}{4c^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x),x]
```

```
[Out] ((c*d^2 - a*e^2)*(a*e + c*d*x)^3)/(3*c^2*d^2) + (e*(a*e + c*d*x)^4)/(4*c^2*
d^2)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{d+ex}\,dx &= \int (ae+cdx)^2(d+ex)\,dx\\
&= \int \left(\frac{\left(cd^2-ae^2\right)(ae+cdx)^2}{cd}+\frac{e(ae+cdx)^3}{cd}\right)dx\\
&= \frac{\left(cd^2-ae^2\right)(ae+cdx)^3}{3c^2d^2}+\frac{e(ae+cdx)^4}{4c^2d^2}
\end{aligned}$$

Mathematica [A] time = 0.0125745, size = 54, normalized size = 1.

$$\frac{1}{12}x\left(6a^2e^2(2d+ex)+4acdex(3d+2ex)+c^2d^2x^2(4d+3ex)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x),x]
```

```
[Out] (x*(6*a^2*e^2*(2*d + e*x) + 4*a*c*d*e*x*(3*d + 2*e*x) + c^2*d^2*x^2*(4*d +
3*e*x)))/12
```

Maple [A] time = 0.038, size = 77, normalized size = 1.4

$$\frac{c^2d^2ex^4}{4}+\frac{\left(ade^2c+cd\left(ae^2+cd^2\right)\right)x^3}{3}+\frac{\left(ae\left(ae^2+cd^2\right)+cd^2ae\right)x^2}{2}+a^2e^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d),x)
```

```
[Out] 1/4*c^2*d^2*e*x^4+1/3*(a*d*e^2*c+c*d*(a*e^2+c*d^2))*x^3+1/2*(a*e*(a*e^2+c*d
^2)+c*d^2*a*e)*x^2+a^2*e^2*d*x
```

Maxima [A] time = 1.10627, size = 86, normalized size = 1.59

$$\frac{1}{4}c^2d^2ex^4+a^2de^2x+\frac{1}{3}\left(c^2d^3+2\,acde^2\right)x^3+\frac{1}{2}\left(2\,acd^2e+a^2e^3\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d),x, algorithm="maxima"
)
```

```
[Out] 1/4*c^2*d^2*e*x^4 + a^2*d*e^2*x + 1/3*(c^2*d^3 + 2*a*c*d*e^2)*x^3 + 1/2*(2*
a*c*d^2*e + a^2*e^3)*x^2
```

Fricas [A] time = 1.52867, size = 136, normalized size = 2.52

$$\frac{1}{4}c^2d^2ex^4+a^2de^2x+\frac{1}{3}\left(c^2d^3+2\,acde^2\right)x^3+\frac{1}{2}\left(2\,acd^2e+a^2e^3\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d),x, algorithm="fricas"
)
```

```
[Out] 1/4*c^2*d^2*e*x^4 + a^2*d*e^2*x + 1/3*(c^2*d^3 + 2*a*c*d*e^2)*x^3 + 1/2*(2*
a*c*d^2*e + a^2*e^3)*x^2
```

Sympy [A] time = 0.168092, size = 66, normalized size = 1.22

$$a^2de^2x+\frac{c^2d^2ex^4}{4}+x^3\left(\frac{2acde^2}{3}+\frac{c^2d^3}{3}\right)+x^2\left(\frac{a^2e^3}{2}+acd^2e\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d),x)
```

```
[Out] a**2*d*e**2*x + c**2*d**2*e*x**4/4 + x**3*(2*a*c*d*e**2/3 + c**2*d**3/3) +
x**2*(a**2*e**3/2 + a*c*d**2*e)
```

Giac [A] time = 1.1643, size = 97, normalized size = 1.8

$$\frac{1}{12}\left(3\,c^2d^2x^4e^5+4\,c^2d^3x^3e^4+8\,acdx^3e^6+12\,acd^2x^2e^5+6\,a^2x^2e^7+12\,a^2dxe^6\right)e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d),x, algorithm="giac")
```

```
[Out] 1/12*(3*c^2*d^2*x^4*e^5 + 4*c^2*d^3*x^3*e^4 + 8*a*c*d*x^3*e^6 + 12*a*c*d^2*
x^2*e^5 + 6*a^2*x^2*e^7 + 12*a^2*d*x*e^6)*e^(-4)
```

3.1843 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^2} \, dx$

Optimal. Leaf size=20

$$\frac{(ae+cdx)^3}{3cd}$$

```
[Out] (a*e + c*d*x)^3/(3*c*d)
```

Rubi [A] time = 0.012909, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 32}

$$\frac{(ae+cdx)^3}{3cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^2,x]
```

```
[Out] (a*e + c*d*x)^3/(3*c*d)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^2} \, dx &= \int (ae+cdx)^2 \, dx \\
&= \frac{(ae+cdx)^3}{3cd}
\end{aligned}$$

Mathematica [A] time = 0.0019549, size = 20, normalized size = 1.

$$\frac{(ae+cdx)^3}{3cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^2,x]
```

```
[Out] (a*e + c*d*x)^3/(3*c*d)
```

Maple [A] time = 0.039, size = 19, normalized size = 1.

$$\frac{(cdx + ae)^3}{3\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^2,x)
```

```
[Out] 1/3*(c*d*x+a*e)^3/c/d
```

Maxima [A] time = 0.988521, size = 38, normalized size = 1.9

$$\frac{1}{3}\,c^2d^2x^3 + acdex^2 + a^2e^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^2,x, algorithm="maxim
a")
```

```
[Out] 1/3*c^2*d^2*x^3 + a*c*d*e*x^2 + a^2*e^2*x
```

Fricas [A] time = 1.51214, size = 58, normalized size = 2.9

$$\frac{1}{3}\,c^2d^2x^3 + acdex^2 + a^2e^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^2,x, algorithm="frica
s")
```

```
[Out] 1/3*c^2*d^2*x^3 + a*c*d*e*x^2 + a^2*e^2*x
```

Sympy [B] time = 0.14883, size = 29, normalized size = 1.45

$$a^2e^2x + acdex^2 + \frac{c^2d^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d)**2,x)
```

```
[Out] a**2*e**2*x + a*c*d*e*x**2 + c**2*d**2*x**3/3
```

Giac [B] time = 1.19405, size = 116, normalized size = 5.8

$$\frac{1}{3}\left(c^2d^2 - \frac{3\left(c^2d^3e - acde^3\right)e^{(-1)}}{xe + d} + \frac{3\left(c^2d^4e^2 - 2\,acd^2e^4 + a^2e^6\right)e^{(-2)}}{(xe + d)^2}\right)(xe + d)^3e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^2,x, algorithm="giac"
)
```

```
[Out] 1/3*(c^2*d^2 - 3*(c^2*d^3*e - a*c*d*e^3)*e^(-1)/(x*e + d) + 3*(c^2*d^4*e^2
- 2*a*c*d^2*e^4 + a^2*e^6)*e^(-2)/(x*e + d)^2)*(x*e + d)^3*e^(-3)
```

3.1844 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^3}\,dx$

Optimal. Leaf size=62

$$-\frac{cdx\left(cd^2-ae^2\right)}{e^2}+\frac{\left(cd^2-ae^2\right)^2\log(d+ex)}{e^3}+\frac{(ae+cdx)^2}{2e}$$

```
[Out] -((c*d*(c*d^2 - a*e^2)*x)/e^2) + (a*e + c*d*x)^2/(2*e) + ((c*d^2 - a*e^2)^2
*Log[d + e*x])/e^3
```

Rubi [A] time = 0.0391704, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$-\frac{cdx\left(cd^2-ae^2\right)}{e^2}+\frac{\left(cd^2-ae^2\right)^2\log(d+ex)}{e^3}+\frac{(ae+cdx)^2}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^3,x]
```

```
[Out] -((c*d*(c*d^2 - a*e^2)*x)/e^2) + (a*e + c*d*x)^2/(2*e) + ((c*d^2 - a*e^2)^2
*Log[d + e*x])/e^3
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^3}\,dx &= \int \frac{(ae+cdx)^2}{d+ex}\,dx\\ &= \int \left(-\frac{cd\left(cd^2-ae^2\right)}{e^2}+\frac{cd(ae+cdx)}{e}+\frac{\left(-cd^2+ae^2\right)^2}{e^2(d+ex)}\right)dx\\ &= -\frac{cd\left(cd^2-ae^2\right)x}{e^2}+\frac{(ae+cdx)^2}{2e}+\frac{\left(cd^2-ae^2\right)^2\log(d+ex)}{e^3}\end{aligned}$$

Mathematica [A] time = 0.0215863, size = 52, normalized size = 0.84

$$\frac{2\left(cd^2-ae^2\right)^2\log(d+ex)+cdex\left(4ae^2+cd(ex-2d)\right)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^3,x]
```

```
[Out] (c*d*e*x*(4*a*e^2 + c*d*(-2*d + e*x)) + 2*(c*d^2 - a*e^2)^2*Log[d + e*x])/(
2*e^3)
```

Maple [A] time = 0.042, size = 77, normalized size = 1.2

$$\frac{c^2d^2x^2}{2e}+2\,cdax-\frac{c^2d^3x}{e^2}+e\ln(ex+d)\,a^2-2\,\frac{\ln(ex+d)\,acd^2}{e}+\frac{\ln(ex+d)\,c^2d^4}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^3,x)
```

```
[Out] 1/2*c^2*d^2/e*x^2+2*c*d*a*x-c^2*d^3/e^2*x+e*ln(e*x+d)*a^2-2/e*ln(e*x+d)*a*c
*d^2+1/e^3*ln(e*x+d)*c^2*d^4
```

Maxima [A] time = 1.11431, size = 97, normalized size = 1.56

$$\frac{c^2d^2ex^2-2\left(c^2d^3-2\,acde^2\right)x}{2\,e^2}+\frac{\left(c^2d^4-2\,acd^2e^2+a^2e^4\right)\log(ex+d)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^3,x, algorithm="maxim
a")
```

```
[Out] 1/2*(c^2*d^2*e*x^2 - 2*(c^2*d^3 - 2*a*c*d*e^2)*x)/e^2 + (c^2*d^4 - 2*a*c*d^
2*e^2 + a^2*e^4)*log(e*x + d)/e^3
```

Fricas [A] time = 1.58243, size = 151, normalized size = 2.44

$$\frac{c^2d^2e^2x^2-2\left(c^2d^3e-2\,acde^3\right)x+2\left(c^2d^4-2\,acd^2e^2+a^2e^4\right)\log(ex+d)}{2\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^3,x, algorithm="frica
s")
```

```
[Out] 1/2*(c^2*d^2*e^2*x^2 - 2*(c^2*d^3*e - 2*a*c*d*e^3)*x + 2*(c^2*d^4 - 2*a*c*d
^2*e^2 + a^2*e^4)*log(e*x + d))/e^3
```

Sympy [A] time = 0.538764, size = 56, normalized size = 0.9

$$\frac{c^2d^2x^2}{2e}+\frac{x\left(2acde^2-c^2d^3\right)}{e^2}+\frac{\left(ae^2-cd^2\right)^2\log(d+ex)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d)**3,x)
```

```
[Out] c**2*d**2*x**2/(2*e) + x*(2*a*c*d*e**2 - c**2*d**3)/e**2 + (a*e**2 - c*d**2
)**2*log(d + e*x)/e**3
```

Giac [A] time = 1.25246, size = 96, normalized size = 1.55

$$\left(c^2d^4-2\,acd^2e^2+a^2e^4\right)e^{(-3)}\log\left(|xe+d|\right)+\frac{1}{2}\left(c^2d^2x^2e^5-2\,c^2d^3xe^4+4\,acdxe^6\right)e^{(-6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^3,x, algorithm="giac"
)
```

```
[Out] (c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*e^(-3)*log(abs(x*e + d)) + 1/2*(c^2*d^2
*x^2*e^5 - 2*c^2*d^3*x*e^4 + 4*a*c*d*x*e^6)*e^(-6)
```

3.1845 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^4} dx$

Optimal. Leaf size=63

$$-\frac{\left(cd^2-ae^2\right)^2}{e^3(d+ex)}-\frac{2cd\left(cd^2-ae^2\right)\log(d+ex)}{e^3}+\frac{c^2d^2x}{e^2}$$

```
[Out] (c^2*d^2*x)/e^2 - (c*d^2 - a*e^2)^2/(e^3*(d + e*x)) - (2*c*d*(c*d^2 - a*e^2
)*Log[d + e*x])/e^3
```

Rubi [A] time = 0.0520952, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$-\frac{\left(cd^2-ae^2\right)^2}{e^3(d+ex)}-\frac{2cd\left(cd^2-ae^2\right)\log(d+ex)}{e^3}+\frac{c^2d^2x}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^4,x]
```

```
[Out] (c^2*d^2*x)/e^2 - (c*d^2 - a*e^2)^2/(e^3*(d + e*x)) - (2*c*d*(c*d^2 - a*e^2
)*Log[d + e*x])/e^3
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^4} dx &= \int \frac{(ae+cdx)^2}{(d+ex)^2} dx\\
&= \int \left(\frac{c^2d^2}{e^2}+\frac{\left(-cd^2+ae^2\right)^2}{e^2(d+ex)^2}-\frac{2cd\left(cd^2-ae^2\right)}{e^2(d+ex)}\right) dx\\
&= \frac{c^2d^2x}{e^2}-\frac{\left(cd^2-ae^2\right)^2}{e^3(d+ex)}-\frac{2cd\left(cd^2-ae^2\right)\log(d+ex)}{e^3}
\end{aligned}$$

Mathematica [A] time = 0.0409001, size = 59, normalized size = 0.94

$$\frac{-\frac{\left(cd^2-ae^2\right)^2}{d+ex}+2cd\left(ae^2-cd^2\right)\log(d+ex)+c^2d^2ex}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^4,x]
```

```
[Out] (c^2*d^2*e*x - (c*d^2 - a*e^2)^2/(d + e*x) + 2*c*d*(-(c*d^2) + a*e^2)*Log[d
 + e*x])/e^3
```

Maple [A] time = 0.046, size = 92, normalized size = 1.5

$$\frac{c^2d^2x}{e^2}+2\,\frac{cd\ln(ex+d)\,a}{e}-2\,\frac{c^2d^3\ln(ex+d)}{e^3}-\frac{a^2e}{ex+d}+2\,\frac{acd^2}{e\,(ex+d)}-\frac{c^2d^4}{e^3\,(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^4,x)
```

```
[Out] c^2*d^2*x/e^2+2*d/e*c*ln(e*x+d)*a-2*d^3/e^3*c^2*ln(e*x+d)-e/(e*x+d)*a^2+2/e
/(e*x+d)*a*c*d^2-1/e^3/(e*x+d)*c^2*d^4
```

Maxima [A] time = 1.01256, size = 107, normalized size = 1.7

$$\frac{c^2d^2x}{e^2}-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{e^4x+de^3}-\frac{2\left(c^2d^3-acde^2\right)\log(ex+d)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^4,x, algorithm="maxim
a")
```

```
[Out] c^2*d^2*x/e^2 - (c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(e^4*x + d*e^3) - 2*(c^
2*d^3 - a*c*d*e^2)*log(e*x + d)/e^3
```

Fricas [A] time = 1.50685, size = 208, normalized size = 3.3

$$\frac{c^2d^2e^2x^2+c^2d^3ex-c^2d^4+2\,acd^2e^2-a^2e^4-2\left(c^2d^4-acd^2e^2+\left(c^2d^3e-acde^3\right)x\right)\log(ex+d)}{e^4x+de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^4,x, algorithm="frica
s")
```

```
[Out] (c^2*d^2*e^2*x^2 + c^2*d^3*e*x - c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4 - 2*(c^2
*d^4 - a*c*d^2*e^2 + (c^2*d^3*e - a*c*d*e^3)*x)*log(e*x + d))/(e^4*x + d*e^
3)
```

Sympy [A] time = 0.647083, size = 71, normalized size = 1.13

$$\frac{c^2d^2x}{e^2}+\frac{2cd\left(ae^2-cd^2\right)\log(d+ex)}{e^3}-\frac{a^2e^4-2acd^2e^2+c^2d^4}{de^3+e^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d)**4,x)
```

```
[Out] c**2*d**2*x/e**2 + 2*c*d*(a*e**2 - c*d**2)*log(d + e*x)/e**3 - (a**2*e**4 -
 2*a*c*d**2*e**2 + c**2*d**4)/(d*e**3 + e**4*x)
```

Giac [B] time = 1.2321, size = 181, normalized size = 2.87

$$c^2d^2xe^{(-2)} - 2\left(c^2d^3 - acde^2\right)e^{(-3)}\log\left(|xe+d|\right) - \frac{\left(c^2d^6 - 2\,acd^4e^2 + a^2d^2e^4 + \left(c^2d^4e^2 - 2\,acd^2e^4 + a^2e^6\right)x^2 + 2\left(c^2d^5e - 2\,a\right.\right.}{(xe+d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^4,x, algorithm="giac"
)
```

```
[Out] c^2*d^2*x*e^(-2) - 2*(c^2*d^3 - a*c*d*e^2)*e^(-3)*log(abs(x*e + d)) - (c^2*
d^6 - 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^4*e^2 - 2*a*c*d^2*e^4 + a^2*e^6)
*x^2 + 2*(c^2*d^5*e - 2*a*c*d^3*e^3 + a^2*d*e^5)*x)*e^(-3)/(x*e + d)^3
```

3.1846 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^5}\,dx$

Optimal. Leaf size=71

$$\frac{2cd\left(cd^2-ae^2\right)}{e^3(d+ex)}-\frac{\left(cd^2-ae^2\right)^2}{2e^3(d+ex)^2}+\frac{c^2d^2\log(d+ex)}{e^3}$$

```
[Out] -(c*d^2 - a*e^2)^2/(2*e^3*(d + e*x)^2) + (2*c*d*(c*d^2 - a*e^2))/(e^3*(d +
e*x)) + (c^2*d^2*Log[d + e*x])/e^3
```

Rubi [A] time = 0.0519226, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$\frac{2cd\left(cd^2-ae^2\right)}{e^3(d+ex)}-\frac{\left(cd^2-ae^2\right)^2}{2e^3(d+ex)^2}+\frac{c^2d^2\log(d+ex)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^5,x]
```

```
[Out] -(c*d^2 - a*e^2)^2/(2*e^3*(d + e*x)^2) + (2*c*d*(c*d^2 - a*e^2))/(e^3*(d +
e*x)) + (c^2*d^2*Log[d + e*x])/e^3
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^5}\,dx &= \int \frac{(ae+cdx)^2}{(d+ex)^3}\,dx\\
&= \int \left(\frac{\left(-cd^2+ae^2\right)^2}{e^2(d+ex)^3}-\frac{2cd\left(cd^2-ae^2\right)}{e^2(d+ex)^2}+\frac{c^2d^2}{e^2(d+ex)}\right)dx\\
&= -\frac{\left(cd^2-ae^2\right)^2}{2e^3(d+ex)^2}+\frac{2cd\left(cd^2-ae^2\right)}{e^3(d+ex)}+\frac{c^2d^2\log(d+ex)}{e^3}
\end{aligned}$$

Mathematica [A] time = 0.0314704, size = 59, normalized size = 0.83

$$\frac{\frac{\left(cd^2-ae^2\right)\left(ae^2+cd(3d+4ex)\right)}{(d+ex)^2}+2c^2d^2\log(d+ex)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^5,x]
```

```
[Out] (((c*d^2 - a*e^2)*(a*e^2 + c*d*(3*d + 4*e*x)))/(d + e*x)^2 + 2*c^2*d^2*Log[
d + e*x])/(2*e^3)
```

Maple [A] time = 0.046, size = 98, normalized size = 1.4

$$-\frac{a^2e}{2\,(ex+d)^2}+\frac{acd^2}{e\,(ex+d)^2}-\frac{c^2d^4}{2\,e^3\,(ex+d)^2}+\frac{c^2d^2\ln(ex+d)}{e^3}-2\,\frac{acd}{e\,(ex+d)}+2\,\frac{c^2d^3}{e^3\,(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^5,x)
```

```
[Out] -1/2*e/(e*x+d)^2*a^2+1/e/(e*x+d)^2*a*c*d^2-1/2/e^3/(e*x+d)^2*c^2*d^4+c^2*d^
2*ln(e*x+d)/e^3-2*d/e*c/(e*x+d)*a+2*d^3/e^3*c^2/(e*x+d)
```

Maxima [A] time = 1.07287, size = 122, normalized size = 1.72

$$\frac{c^2d^2\log(ex+d)}{e^3}+\frac{3\,c^2d^4-2\,acd^2e^2-a^2e^4+4\left(c^2d^3e-acde^3\right)x}{2\left(e^5x^2+2\,de^4x+d^2e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^5,x, algorithm="maxim
a")
```

```
[Out] c^2*d^2*log(e*x + d)/e^3 + 1/2*(3*c^2*d^4 - 2*a*c*d^2*e^2 - a^2*e^4 + 4*(c^
2*d^3*e - a*c*d*e^3)*x)/(e^5*x^2 + 2*d*e^4*x + d^2*e^3)
```

Fricas [A] time = 1.65813, size = 224, normalized size = 3.15

$$\frac{3\,c^2d^4-2\,acd^2e^2-a^2e^4+4\left(c^2d^3e-acde^3\right)x+2\left(c^2d^2e^2x^2+2\,c^2d^3ex+c^2d^4\right)\log(ex+d)}{2\left(e^5x^2+2\,de^4x+d^2e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^5,x, algorithm="frica
s")
```

```
[Out] 1/2*(3*c^2*d^4 - 2*a*c*d^2*e^2 - a^2*e^4 + 4*(c^2*d^3*e - a*c*d*e^3)*x + 2*
(c^2*d^2*e^2*x^2 + 2*c^2*d^3*e*x + c^2*d^4)*log(e*x + d))/(e^5*x^2 + 2*d*e^
4*x + d^2*e^3)
```

Sympy [A] time = 0.966675, size = 90, normalized size = 1.27

$$\frac{c^2d^2\log(d+ex)}{e^3}-\frac{a^2e^4+2acd^2e^2-3c^2d^4+x\left(4acde^3-4c^2d^3e\right)}{2d^2e^3+4de^4x+2e^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d)**5,x)
```

```
[Out] c**2*d**2*log(d + e*x)/e**3 - (a**2*e**4 + 2*a*c*d**2*e**2 - 3*c**2*d**4 +
x*(4*a*c*d*e**3 - 4*c**2*d**3*e))/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2)
```

Giac [A] time = 1.22589, size = 159, normalized size = 2.24

$$-c^2d^2e^{(-3)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^2}\right)+\frac{1}{2}\left(\frac{4\,c^2d^3e^9}{xe+d}-\frac{c^2d^4e^9}{(xe+d)^2}-\frac{4\,acde^{11}}{xe+d}+\frac{2\,acd^2e^{11}}{(xe+d)^2}-\frac{a^2e^{13}}{(xe+d)^2}\right)e^{(-12)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^5,x, algorithm="giac"
)
```

```
[Out] -c^2*d^2*e^(-3)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2) + 1/2*(4*c^2*d^3*e^9/(
x*e + d) - c^2*d^4*e^9/(x*e + d)^2 - 4*a*c*d*e^11/(x*e + d) + 2*a*c*d^2*e^1
1/(x*e + d)^2 - a^2*e^13/(x*e + d)^2)*e^(-12)
```

3.1847 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^6}\,dx$

Optimal. Leaf size=35

$$\frac{(ae+cdx)^3}{3(d+ex)^3\left(cd^2-ae^2\right)}$$

```
[Out] (a*e + c*d*x)^3/(3*(c*d^2 - a*e^2)*(d + e*x)^3)
```

Rubi [A] time = 0.0141029, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 37}

$$\frac{(ae+cdx)^3}{3(d+ex)^3\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^6,x]
```

```
[Out] (a*e + c*d*x)^3/(3*(c*d^2 - a*e^2)*(d + e*x)^3)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^6}\,dx &= \int \frac{(ae+cdx)^2}{(d+ex)^4}\,dx\\ &= \frac{(ae+cdx)^3}{3\left(cd^2-ae^2\right)(d+ex)^3}\end{aligned}$$

Mathematica [A] time = 0.0277713, size = 59, normalized size = 1.69

$$-\frac{a^2e^4+acde^2(d+3ex)+c^2d^2\left(d^2+3dex+3e^2x^2\right)}{3e^3(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^6,x]
```

```
[Out] -(a^2*e^4 + a*c*d*e^2*(d + 3*e*x) + c^2*d^2*(d^2 + 3*d*e*x + 3*e^2*x^2))/(3
*e^3*(d + e*x)^3)
```

Maple [B] time = 0.043, size = 83, normalized size = 2.4

$$-\frac{cd\left(ae^{2}-cd^{2}\right)}{e^{3}\left(ex+d\right)^{2}}-\frac{c^{2}d^{2}}{e^{3}\left(ex+d\right)}-\frac{a^{2}e^{4}-2\,acd^{2}e^{2}+c^{2}d^{4}}{3\,e^{3}\left(ex+d\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^6,x)
```

```
[Out] -c*d*(a*e^2-c*d^2)/e^3/(e*x+d)^2-d^2/e^3*c^2/(e*x+d)-1/3*(a^2*e^4-2*a*c*d^2
*e^2+c^2*d^4)/e^3/(e*x+d)^3
```

Maxima [B] time = 1.0719, size = 127, normalized size = 3.63

$$-\frac{3\,c^{2}d^{2}e^{2}x^{2}+c^{2}d^{4}+acd^{2}e^{2}+a^{2}e^{4}+3\left(c^{2}d^{3}e+acde^{3}\right)x}{3\left(e^{6}x^{3}+3\,de^{5}x^{2}+3\,d^{2}e^{4}x+d^{3}e^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^6,x, algorithm="maxim
a")
```

```
[Out] -1/3*(3*c^2*d^2*e^2*x^2 + c^2*d^4 + a*c*d^2*e^2 + a^2*e^4 + 3*(c^2*d^3*e +
a*c*d*e^3)*x)/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3)
```

Fricas [B] time = 1.66687, size = 186, normalized size = 5.31

$$-\frac{3\,c^{2}d^{2}e^{2}x^{2}+c^{2}d^{4}+acd^{2}e^{2}+a^{2}e^{4}+3\left(c^{2}d^{3}e+acde^{3}\right)x}{3\left(e^{6}x^{3}+3\,de^{5}x^{2}+3\,d^{2}e^{4}x+d^{3}e^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^6,x, algorithm="frica
s")
```

```
[Out] -1/3*(3*c^2*d^2*e^2*x^2 + c^2*d^4 + a*c*d^2*e^2 + a^2*e^4 + 3*(c^2*d^3*e +
a*c*d*e^3)*x)/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3)
```

Sympy [B] time = 1.54047, size = 99, normalized size = 2.83

$$-\frac{a^{2}e^{4}+acd^{2}e^{2}+c^{2}d^{4}+3c^{2}d^{2}e^{2}x^{2}+x\left(3acde^{3}+3c^{2}d^{3}e\right)}{3d^{3}e^{3}+9d^{2}e^{4}x+9de^{5}x^{2}+3e^{6}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d)**6,x)
```

```
[Out] -(a**2*e**4 + a*c*d**2*e**2 + c**2*d**4 + 3*c**2*d**2*e**2*x**2 + x*(3*a*c*
d*e**3 + 3*c**2*d**3*e))/(3*d**3*e**3 + 9*d**2*e**4*x + 9*d*e**5*x**2 + 3*e
**6*x**3)
```

Giac [B] time = 1.22456, size = 185, normalized size = 5.29

$$-\frac{\left(3\,c^2d^2x^4e^4+9\,c^2d^3x^3e^3+10\,c^2d^4x^2e^2+5\,c^2d^5xe+c^2d^6+3\,acdx^3e^5+7\,acd^2x^2e^4+5\,acd^3xe^3+acd^4e^2+a^2x^2e^6+2\,a^2d}{3\,(xe+d)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^6,x, algorithm="giac"
)
```

```
[Out] -1/3*(3*c^2*d^2*x^4*e^4 + 9*c^2*d^3*x^3*e^3 + 10*c^2*d^4*x^2*e^2 + 5*c^2*d^
5*x*e + c^2*d^6 + 3*a*c*d*x^3*e^5 + 7*a*c*d^2*x^2*e^4 + 5*a*c*d^3*x*e^3 + a
*c*d^4*e^2 + a^2*x^2*e^6 + 2*a^2*d*x*e^5 + a^2*d^2*e^4)*e^(-3)/(x*e + d)^5
```

3.1848 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^7} dx$

Optimal. Leaf size=77

$$\frac{2cd\left(cd^2-ae^2\right)}{3e^3(d+ex)^3}-\frac{\left(cd^2-ae^2\right)^2}{4e^3(d+ex)^4}-\frac{c^2d^2}{2e^3(d+ex)^2}$$

```
[Out] -(c*d^2 - a*e^2)^2/(4*e^3*(d + e*x)^4) + (2*c*d*(c*d^2 - a*e^2))/(3*e^3*(d
+ e*x)^3) - (c^2*d^2)/(2*e^3*(d + e*x)^2)
```

Rubi [A] time = 0.0498161, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$\frac{2cd\left(cd^2-ae^2\right)}{3e^3(d+ex)^3}-\frac{\left(cd^2-ae^2\right)^2}{4e^3(d+ex)^4}-\frac{c^2d^2}{2e^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^7,x]
```

```
[Out] -(c*d^2 - a*e^2)^2/(4*e^3*(d + e*x)^4) + (2*c*d*(c*d^2 - a*e^2))/(3*e^3*(d
+ e*x)^3) - (c^2*d^2)/(2*e^3*(d + e*x)^2)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^7} dx &= \int \frac{(ae+cdx)^2}{(d+ex)^5} dx \\
&= \int \left(\frac{\left(-cd^2+ae^2\right)^2}{e^2(d+ex)^5}-\frac{2cd\left(cd^2-ae^2\right)}{e^2(d+ex)^4}+\frac{c^2d^2}{e^2(d+ex)^3}\right) dx \\
&= -\frac{\left(cd^2-ae^2\right)^2}{4e^3(d+ex)^4}+\frac{2cd\left(cd^2-ae^2\right)}{3e^3(d+ex)^3}-\frac{c^2d^2}{2e^3(d+ex)^2}
\end{aligned}$$

Mathematica [A] time = 0.0227091, size = 61, normalized size = 0.79

$$-\frac{3a^2e^4+2acde^2(d+4ex)+c^2d^2\left(d^2+4dex+6e^2x^2\right)}{12e^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^7,x]
```

```
[Out] -(3*a^2*e^4 + 2*a*c*d*e^2*(d + 4*e*x) + c^2*d^2*(d^2 + 4*d*e*x + 6*e^2*x^2)
)/(12*e^3*(d + e*x)^4)
```

Maple [A] time = 0.043, size = 83, normalized size = 1.1

$$-\frac{c^2d^2}{2\,e^3\,(ex+d)^2}-\frac{a^2e^4-2\,acd^2e^2+c^2d^4}{4\,e^3\,(ex+d)^4}-\frac{2\,cd\left(ae^2-cd^2\right)}{3\,e^3\,(ex+d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^7,x)
```

```
[Out] -1/2*c^2*d^2/e^3/(e*x+d)^2-1/4*(a^2*e^4-2*a*c*d^2*e^2+c^2*d^4)/e^3/(e*x+d)^
4-2/3*c*d*(a*e^2-c*d^2)/e^3/(e*x+d)^3
```

Maxima [A] time = 1.04598, size = 146, normalized size = 1.9

$$-\frac{6\,c^2d^2e^2x^2+c^2d^4+2\,acd^2e^2+3\,a^2e^4+4\left(c^2d^3e+2\,acde^3\right)x}{12\left(e^7x^4+4\,de^6x^3+6\,d^2e^5x^2+4\,d^3e^4x+d^4e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^7,x, algorithm="maxim
a")
```

```
[Out] -1/12*(6*c^2*d^2*e^2*x^2 + c^2*d^4 + 2*a*c*d^2*e^2 + 3*a^2*e^4 + 4*(c^2*d^3
*e + 2*a*c*d*e^3)*x)/(e^7*x^4 + 4*d*e^6*x^3 + 6*d^2*e^5*x^2 + 4*d^3*e^4*x +
 d^4*e^3)
```

Fricas [A] time = 1.51108, size = 217, normalized size = 2.82

$$-\frac{6\,c^2d^2e^2x^2+c^2d^4+2\,acd^2e^2+3\,a^2e^4+4\left(c^2d^3e+2\,acde^3\right)x}{12\left(e^7x^4+4\,de^6x^3+6\,d^2e^5x^2+4\,d^3e^4x+d^4e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^7,x, algorithm="frica
s")
```

```
[Out] -1/12*(6*c^2*d^2*e^2*x^2 + c^2*d^4 + 2*a*c*d^2*e^2 + 3*a^2*e^4 + 4*(c^2*d^3
*e + 2*a*c*d*e^3)*x)/(e^7*x^4 + 4*d*e^6*x^3 + 6*d^2*e^5*x^2 + 4*d^3*e^4*x +
 d^4*e^3)
```

Sympy [A] time = 1.47222, size = 114, normalized size = 1.48

$$-\frac{3a^2e^4 + 2acd^2e^2 + c^2d^4 + 6c^2d^2e^2x^2 + x\left(8acde^3 + 4c^2d^3e\right)}{12d^4e^3 + 48d^3e^4x + 72d^2e^5x^2 + 48de^6x^3 + 12e^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d)**7,x)
```

```
[Out] -(3*a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4 + 6*c**2*d**2*e**2*x**2 + x*(8*
a*c*d*e**3 + 4*c**2*d**3*e))/(12*d**4*e**3 + 48*d**3*e**4*x + 72*d**2*e**5*
x**2 + 48*d*e**6*x**3 + 12*e**7*x**4)
```

Giac [A] time = 1.23884, size = 189, normalized size = 2.45

$$-\frac{\left(6\,c^2d^2x^4e^4 + 16\,c^2d^3x^3e^3 + 15\,c^2d^4x^2e^2 + 6\,c^2d^5xe + c^2d^6 + 8\,acdx^3e^5 + 18\,acd^2x^2e^4 + 12\,acd^3xe^3 + 2\,acd^4e^2 + 3\,a^2x^2 \cdots\right.}{12\,(xe+d)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^7,x, algorithm="giac"
)
```

```
[Out] -1/12*(6*c^2*d^2*x^4*e^4 + 16*c^2*d^3*x^3*e^3 + 15*c^2*d^4*x^2*e^2 + 6*c^2*
d^5*x*e + c^2*d^6 + 8*a*c*d*x^3*e^5 + 18*a*c*d^2*x^2*e^4 + 12*a*c*d^3*x*e^3
 + 2*a*c*d^4*e^2 + 3*a^2*x^2*e^6 + 6*a^2*d*x*e^5 + 3*a^2*d^2*e^4)*e^(-3)/(x
*e + d)^6
```

3.1849 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^8}\,dx$

Optimal. Leaf size=77

$$\frac{cd\left(cd^2-ae^2\right)}{2e^3(d+ex)^4}-\frac{\left(cd^2-ae^2\right)^2}{5e^3(d+ex)^5}-\frac{c^2d^2}{3e^3(d+ex)^3}$$

```
[Out] -(c*d^2 - a*e^2)^2/(5*e^3*(d + e*x)^5) + (c*d*(c*d^2 - a*e^2))/(2*e^3*(d +
e*x)^4) - (c^2*d^2)/(3*e^3*(d + e*x)^3)
```

Rubi [A] time = 0.04866, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$\frac{cd\left(cd^2-ae^2\right)}{2e^3(d+ex)^4}-\frac{\left(cd^2-ae^2\right)^2}{5e^3(d+ex)^5}-\frac{c^2d^2}{3e^3(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^8,x]
```

```
[Out] -(c*d^2 - a*e^2)^2/(5*e^3*(d + e*x)^5) + (c*d*(c*d^2 - a*e^2))/(2*e^3*(d +
e*x)^4) - (c^2*d^2)/(3*e^3*(d + e*x)^3)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^8}\,dx &= \int \frac{(ae+cdx)^2}{(d+ex)^6}\,dx\\
&= \int \left(\frac{\left(-cd^2+ae^2\right)^2}{e^2(d+ex)^6}-\frac{2cd\left(cd^2-ae^2\right)}{e^2(d+ex)^5}+\frac{c^2d^2}{e^2(d+ex)^4}\right)dx\\
&= -\frac{\left(cd^2-ae^2\right)^2}{5e^3(d+ex)^5}+\frac{cd\left(cd^2-ae^2\right)}{2e^3(d+ex)^4}-\frac{c^2d^2}{3e^3(d+ex)^3}
\end{aligned}$$

Mathematica [A] time = 0.0297077, size = 61, normalized size = 0.79

$$-\frac{6a^2e^4+3acde^2(d+5ex)+c^2d^2\left(d^2+5dex+10e^2x^2\right)}{30e^3(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^8,x]
```

```
[Out] -(6*a^2*e^4 + 3*a*c*d*e^2*(d + 5*e*x) + c^2*d^2*(d^2 + 5*d*e*x + 10*e^2*x^2
))/(30*e^3*(d + e*x)^5)
```

Maple [A] time = 0.044, size = 83, normalized size = 1.1

$$-\frac{cd\left(ae^{2}-cd^{2}\right)}{2\,e^{3}\left(ex+d\right)^{4}}-\frac{c^{2}d^{2}}{3\,e^{3}\left(ex+d\right)^{3}}-\frac{a^{2}e^{4}-2\,acd^{2}e^{2}+c^{2}d^{4}}{5\,e^{3}\left(ex+d\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^8,x)
```

```
[Out] -1/2*c*d*(a*e^2-c*d^2)/e^3/(e*x+d)^4-1/3*c^2*d^2/e^3/(e*x+d)^3-1/5*(a^2*e^4
-2*a*c*d^2*e^2+c^2*d^4)/e^3/(e*x+d)^5
```

Maxima [A] time = 1.058, size = 161, normalized size = 2.09

$$-\frac{10\,c^{2}d^{2}e^{2}x^{2}+c^{2}d^{4}+3\,acd^{2}e^{2}+6\,a^{2}e^{4}+5\left(c^{2}d^{3}e+3\,acde^{3}\right)x}{30\left(e^{8}x^{5}+5\,de^{7}x^{4}+10\,d^{2}e^{6}x^{3}+10\,d^{3}e^{5}x^{2}+5\,d^{4}e^{4}x+d^{5}e^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^8,x, algorithm="maxim
a")
```

```
[Out] -1/30*(10*c^2*d^2*e^2*x^2 + c^2*d^4 + 3*a*c*d^2*e^2 + 6*a^2*e^4 + 5*(c^2*d^
3*e + 3*a*c*d*e^3)*x)/(e^8*x^5 + 5*d*e^7*x^4 + 10*d^2*e^6*x^3 + 10*d^3*e^5*
x^2 + 5*d^4*e^4*x + d^5*e^3)
```

Fricas [A] time = 1.6301, size = 243, normalized size = 3.16

$$-\frac{10\,c^{2}d^{2}e^{2}x^{2}+c^{2}d^{4}+3\,acd^{2}e^{2}+6\,a^{2}e^{4}+5\left(c^{2}d^{3}e+3\,acde^{3}\right)x}{30\left(e^{8}x^{5}+5\,de^{7}x^{4}+10\,d^{2}e^{6}x^{3}+10\,d^{3}e^{5}x^{2}+5\,d^{4}e^{4}x+d^{5}e^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^8,x, algorithm="frica
s")
```

```
[Out] -1/30*(10*c^2*d^2*e^2*x^2 + c^2*d^4 + 3*a*c*d^2*e^2 + 6*a^2*e^4 + 5*(c^2*d^
3*e + 3*a*c*d*e^3)*x)/(e^8*x^5 + 5*d*e^7*x^4 + 10*d^2*e^6*x^3 + 10*d^3*e^5*
x^2 + 5*d^4*e^4*x + d^5*e^3)
```

Sympy [A] time = 2.08551, size = 126, normalized size = 1.64

$$-\frac{6a^2e^4 + 3acd^2e^2 + c^2d^4 + 10c^2d^2e^2x^2 + x\left(15acde^3 + 5c^2d^3e\right)}{30d^5e^3 + 150d^4e^4x + 300d^3e^5x^2 + 300d^2e^6x^3 + 150de^7x^4 + 30e^8x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d)**8,x)
```

```
[Out] -(6*a**2*e**4 + 3*a*c*d**2*e**2 + c**2*d**4 + 10*c**2*d**2*e**2*x**2 + x*(1
5*a*c*d*e**3 + 5*c**2*d**3*e))/(30*d**5*e**3 + 150*d**4*e**4*x + 300*d**3*e
**5*x**2 + 300*d**2*e**6*x**3 + 150*d*e**7*x**4 + 30*e**8*x**5)
```

Giac [A] time = 1.22419, size = 189, normalized size = 2.45

$$-\frac{\left(10\,c^2d^2x^4e^4 + 25\,c^2d^3x^3e^3 + 21\,c^2d^4x^2e^2 + 7\,c^2d^5xe + c^2d^6 + 15\,acdx^3e^5 + 33\,acd^2x^2e^4 + 21\,acd^3xe^3 + 3\,acd^4e^2 + 6\,a^2x^2\right.}{30\,(xe + d)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^8,x, algorithm="giac"
)
```

```
[Out] -1/30*(10*c^2*d^2*x^4*e^4 + 25*c^2*d^3*x^3*e^3 + 21*c^2*d^4*x^2*e^2 + 7*c^2
*d^5*x*e + c^2*d^6 + 15*a*c*d*x^3*e^5 + 33*a*c*d^2*x^2*e^4 + 21*a*c*d^3*x*e
^3 + 3*a*c*d^4*e^2 + 6*a^2*x^2*e^6 + 12*a^2*d*x*e^5 + 6*a^2*d^2*e^4)*e^(-3)
/(x*e + d)^7
```

3.1850 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^9}\,dx$

Optimal. Leaf size=77

$$\frac{2cd\left(cd^2-ae^2\right)}{5e^3(d+ex)^5}-\frac{\left(cd^2-ae^2\right)^2}{6e^3(d+ex)^6}-\frac{c^2d^2}{4e^3(d+ex)^4}$$

```
[Out] -(c*d^2 - a*e^2)^2/(6*e^3*(d + e*x)^6) + (2*c*d*(c*d^2 - a*e^2))/(5*e^3*(d
+ e*x)^5) - (c^2*d^2)/(4*e^3*(d + e*x)^4)
```

Rubi [A] time = 0.0491455, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$\frac{2cd\left(cd^2-ae^2\right)}{5e^3(d+ex)^5}-\frac{\left(cd^2-ae^2\right)^2}{6e^3(d+ex)^6}-\frac{c^2d^2}{4e^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^9,x]
```

```
[Out] -(c*d^2 - a*e^2)^2/(6*e^3*(d + e*x)^6) + (2*c*d*(c*d^2 - a*e^2))/(5*e^3*(d
+ e*x)^5) - (c^2*d^2)/(4*e^3*(d + e*x)^4)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^9}\,dx &= \int \frac{(ae+cdx)^2}{(d+ex)^7}\,dx\\
&= \int \left(\frac{\left(-cd^2+ae^2\right)^2}{e^2(d+ex)^7}-\frac{2cd\left(cd^2-ae^2\right)}{e^2(d+ex)^6}+\frac{c^2d^2}{e^2(d+ex)^5}\right)dx\\
&= -\frac{\left(cd^2-ae^2\right)^2}{6e^3(d+ex)^6}+\frac{2cd\left(cd^2-ae^2\right)}{5e^3(d+ex)^5}-\frac{c^2d^2}{4e^3(d+ex)^4}
\end{aligned}$$

Mathematica [A] time = 0.0250267, size = 61, normalized size = 0.79

$$-\frac{10a^2e^4+4acde^2(d+6ex)+c^2d^2\left(d^2+6dex+15e^2x^2\right)}{60e^3(d+ex)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^9,x]
```

```
[Out] -(10*a^2*e^4 + 4*a*c*d*e^2*(d + 6*e*x) + c^2*d^2*(d^2 + 6*d*e*x + 15*e^2*x^
2))/(60*e^3*(d + e*x)^6)
```

Maple [A] time = 0.045, size = 83, normalized size = 1.1

$$-\frac{c^2d^2}{4\,e^3\,(ex+d)^4}-\frac{a^2e^4-2\,acd^2e^2+c^2d^4}{6\,e^3\,(ex+d)^6}-\frac{2\,cd\left(ae^2-cd^2\right)}{5\,e^3\,(ex+d)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^9,x)
```

```
[Out] -1/4*c^2*d^2/e^3/(e*x+d)^4-1/6*(a^2*e^4-2*a*c*d^2*e^2+c^2*d^4)/e^3/(e*x+d)^
6-2/5*c*d*(a*e^2-c*d^2)/e^3/(e*x+d)^5
```

Maxima [A] time = 1.05644, size = 176, normalized size = 2.29

$$-\frac{15\,c^2d^2e^2x^2+c^2d^4+4\,acd^2e^2+10\,a^2e^4+6\left(c^2d^3e+4\,acde^3\right)x}{60\left(e^9x^6+6\,de^8x^5+15\,d^2e^7x^4+20\,d^3e^6x^3+15\,d^4e^5x^2+6\,d^5e^4x+d^6e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^9,x, algorithm="maxim
a")
```

```
[Out] -1/60*(15*c^2*d^2*e^2*x^2 + c^2*d^4 + 4*a*c*d^2*e^2 + 10*a^2*e^4 + 6*(c^2*d
^3*e + 4*a*c*d*e^3)*x)/(e^9*x^6 + 6*d*e^8*x^5 + 15*d^2*e^7*x^4 + 20*d^3*e^6
*x^3 + 15*d^4*e^5*x^2 + 6*d^5*e^4*x + d^6*e^3)
```

Fricas [A] time = 1.58881, size = 267, normalized size = 3.47

$$-\frac{15\,c^2d^2e^2x^2+c^2d^4+4\,acd^2e^2+10\,a^2e^4+6\left(c^2d^3e+4\,acde^3\right)x}{60\left(e^9x^6+6\,de^8x^5+15\,d^2e^7x^4+20\,d^3e^6x^3+15\,d^4e^5x^2+6\,d^5e^4x+d^6e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^9,x, algorithm="frica
s")
```

```
[Out] -1/60*(15*c^2*d^2*e^2*x^2 + c^2*d^4 + 4*a*c*d^2*e^2 + 10*a^2*e^4 + 6*(c^2*d
^3*e + 4*a*c*d*e^3)*x)/(e^9*x^6 + 6*d*e^8*x^5 + 15*d^2*e^7*x^4 + 20*d^3*e^6
*x^3 + 15*d^4*e^5*x^2 + 6*d^5*e^4*x + d^6*e^3)
```

Sympy [B] time = 3.18706, size = 138, normalized size = 1.79

$$-\frac{10a^2e^4 + 4acd^2e^2 + c^2d^4 + 15c^2d^2e^2x^2 + x\left(24acde^3 + 6c^2d^3e\right)}{60d^6e^3 + 360d^5e^4x + 900d^4e^5x^2 + 1200d^3e^6x^3 + 900d^2e^7x^4 + 360de^8x^5 + 60e^9x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d)**9,x)
```

```
[Out] -(10*a**2*e**4 + 4*a*c*d**2*e**2 + c**2*d**4 + 15*c**2*d**2*e**2*x**2 + x*(
24*a*c*d*e**3 + 6*c**2*d**3*e))/(60*d**6*e**3 + 360*d**5*e**4*x + 900*d**4*
e**5*x**2 + 1200*d**3*e**6*x**3 + 900*d**2*e**7*x**4 + 360*d*e**8*x**5 + 60
*e**9*x**6)
```

Giac [A] time = 1.20518, size = 189, normalized size = 2.45

$$-\frac{\left(15\,c^2d^2x^4e^4 + 36\,c^2d^3x^3e^3 + 28\,c^2d^4x^2e^2 + 8\,c^2d^5xe + c^2d^6 + 24\,acdx^3e^5 + 52\,acd^2x^2e^4 + 32\,acd^3xe^3 + 4\,acd^4e^2 + 10\right.}{60\,(xe+d)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^9,x, algorithm="giac"
)
```

```
[Out] -1/60*(15*c^2*d^2*x^4*e^4 + 36*c^2*d^3*x^3*e^3 + 28*c^2*d^4*x^2*e^2 + 8*c^2
*d^5*x*e + c^2*d^6 + 24*a*c*d*x^3*e^5 + 52*a*c*d^2*x^2*e^4 + 32*a*c*d^3*x*e
^3 + 4*a*c*d^4*e^2 + 10*a^2*x^2*e^6 + 20*a^2*d*x*e^5 + 10*a^2*d^2*e^4)*e^(-
3)/(x*e + d)^8
```

3.1851 $\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3 dx$

Optimal. Leaf size=111

$$-\frac{3c^2d^2(d+ex)^8\left(cd^2-ae^2\right)}{8e^4}+\frac{3cd(d+ex)^7\left(cd^2-ae^2\right)^2}{7e^4}-\frac{(d+ex)^6\left(cd^2-ae^2\right)^3}{6e^4}+\frac{c^3d^3(d+ex)^9}{9e^4}$$

```
[Out] -((c*d^2 - a*e^2)^3*(d + e*x)^6)/(6*e^4) + (3*c*d*(c*d^2 - a*e^2)^2*(d + e*
x)^7)/(7*e^4) - (3*c^2*d^2*(c*d^2 - a*e^2)*(d + e*x)^8)/(8*e^4) + (c^3*d^3*
(d + e*x)^9)/(9*e^4)
```

Rubi [A] time = 0.233257, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$-\frac{3c^2d^2(d+ex)^8\left(cd^2-ae^2\right)}{8e^4}+\frac{3cd(d+ex)^7\left(cd^2-ae^2\right)^2}{7e^4}-\frac{(d+ex)^6\left(cd^2-ae^2\right)^3}{6e^4}+\frac{c^3d^3(d+ex)^9}{9e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] -((c*d^2 - a*e^2)^3*(d + e*x)^6)/(6*e^4) + (3*c*d*(c*d^2 - a*e^2)^2*(d + e*
x)^7)/(7*e^4) - (3*c^2*d^2*(c*d^2 - a*e^2)*(d + e*x)^8)/(8*e^4) + (c^3*d^3*
(d + e*x)^9)/(9*e^4)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3 dx &= \int (ae+cdx)^3(d+ex)^5 dx\\ &= \int \left(\frac{\left(-cd^2+ae^2\right)^3(d+ex)^5}{e^3}+\frac{3cd\left(cd^2-ae^2\right)^2(d+ex)^6}{e^3}-\frac{3c^2d^2\left(cd^2-a\right.}{e}\right.\\ &= -\frac{\left(cd^2-ae^2\right)^3(d+ex)^6}{6e^4}+\frac{3cd\left(cd^2-ae^2\right)^2(d+ex)^7}{7e^4}-\frac{3c^2d^2\left(cd^2-ae^2\right)}{8e^4}\end{aligned}$$

Mathematica [B] time = 0.0729191, size = 255, normalized size = 2.3

$$\frac{1}{504}x\left(36a^2cde^2x\left(105d^3e^2x^2+84d^2e^3x^3+70d^4ex+21d^5+35de^4x^4+6e^5x^5\right)+84a^3e^3\left(20d^3e^2x^2+15d^2e^3x^3+15d^4ex+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] (x*(84*a^3*e^3*(6*d^5 + 15*d^4*e*x + 20*d^3*e^2*x^2 + 15*d^2*e^3*x^3 + 6*d*
e^4*x^4 + e^5*x^5) + 36*a^2*c*d*e^2*x*(21*d^5 + 70*d^4*e*x + 105*d^3*e^2*x^
2 + 84*d^2*e^3*x^3 + 35*d*e^4*x^4 + 6*e^5*x^5) + 9*a*c^2*d^2*e*x^2*(56*d^5
+ 210*d^4*e*x + 336*d^3*e^2*x^2 + 280*d^2*e^3*x^3 + 120*d*e^4*x^4 + 21*e^5*
x^5) + c^3*d^3*x^3*(126*d^5 + 504*d^4*e*x + 840*d^3*e^2*x^2 + 720*d^2*e^3*x
^3 + 315*d*e^4*x^4 + 56*e^5*x^5)))/504
```

Maple [B] time = 0.04, size = 801, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

```
[Out] 1/9*e^5*d^3*c^3*x^9+1/8*(2*d^4*e^4*c^3+3*e^4*(a*e^2+c*d^2)*d^2*c^2)*x^8+1/7
*(d^5*e^3*c^3+6*d^3*e^3*(a*e^2+c*d^2)*c^2+e^2*(a*d^3*e^3*c^2+2*(a*e^2+c*d^2
)^2*d*e*c+d*e*c*(2*a*c*d^2*e^2+(a*e^2+c*d^2)^2)))*x^7+1/6*(3*d^4*(a*e^2+c*d
^2)*e^2*c^2+2*d*e*(a*d^3*e^3*c^2+2*(a*e^2+c*d^2)^2*d*e*c+d*e*c*(2*a*c*d^2*e
^2+(a*e^2+c*d^2)^2))+e^2*(4*a*d^2*e^2*(a*e^2+c*d^2)*c+(a*e^2+c*d^2)*(2*a*c*
d^2*e^2+(a*e^2+c*d^2)^2)))*x^6+1/5*(d^2*(a*d^3*e^3*c^2+2*(a*e^2+c*d^2)^2*d*
e*c+d*e*c*(2*a*c*d^2*e^2+(a*e^2+c*d^2)^2))+2*d*e*(4*a*d^2*e^2*(a*e^2+c*d^2)
*c+(a*e^2+c*d^2)*(2*a*c*d^2*e^2+(a*e^2+c*d^2)^2))+e^2*(a*d*e*(2*a*c*d^2*e^2
+(a*e^2+c*d^2)^2)+2*(a*e^2+c*d^2)^2*a*d*e+d^3*e^3*c*a^2))*x^5+1/4*(d^2*(4*a
*d^2*e^2*(a*e^2+c*d^2)*c+(a*e^2+c*d^2)*(2*a*c*d^2*e^2+(a*e^2+c*d^2)^2))+2*d
*e*(a*d*e*(2*a*c*d^2*e^2+(a*e^2+c*d^2)^2)+2*(a*e^2+c*d^2)^2*a*d*e+d^3*e^3*c
*a^2)+3*e^4*a^2*d^2*(a*e^2+c*d^2))*x^4+1/3*(d^2*(a*d*e*(2*a*c*d^2*e^2+(a*e^
2+c*d^2)^2)+2*(a*e^2+c*d^2)^2*a*d*e+d^3*e^3*c*a^2)+6*d^3*e^3*a^2*(a*e^2+c*d
^2)+e^5*a^3*d^3)*x^3+1/2*(3*d^4*a^2*e^2*(a*e^2+c*d^2)+2*d^4*e^4*a^3)*x^2+d^
5*a^3*e^3*x
```

Maxima [B] time = 0.999495, size = 409, normalized size = 3.68

$$\frac{1}{9}\,c^3d^3e^5x^9 + a^3d^5e^3x + \frac{1}{8}\left(5\,c^3d^4e^4 + 3\,ac^2d^2e^6\right)x^8 + \frac{1}{7}\left(10\,c^3d^5e^3 + 15\,ac^2d^3e^5 + 3\,a^2cde^7\right)x^7 + \frac{1}{6}\left(10\,c^3d^6e^2 + 30\,ac^2d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxim
a")
```

```
[Out] 1/9*c^3*d^3*e^5*x^9 + a^3*d^5*e^3*x + 1/8*(5*c^3*d^4*e^4 + 3*a*c^2*d^2*e^6)
*x^8 + 1/7*(10*c^3*d^5*e^3 + 15*a*c^2*d^3*e^5 + 3*a^2*c*d*e^7)*x^7 + 1/6*(1
0*c^3*d^6*e^2 + 30*a*c^2*d^4*e^4 + 15*a^2*c*d^2*e^6 + a^3*e^8)*x^6 + (c^3*d
^7*e + 6*a*c^2*d^5*e^3 + 6*a^2*c*d^3*e^5 + a^3*d*e^7)*x^5 + 1/4*(c^3*d^8 +
15*a*c^2*d^6*e^2 + 30*a^2*c*d^4*e^4 + 10*a^3*d^2*e^6)*x^4 + 1/3*(3*a*c^2*d^
7*e + 15*a^2*c*d^5*e^3 + 10*a^3*d^3*e^5)*x^3 + 1/2*(3*a^2*c*d^6*e^2 + 5*a^3
*d^4*e^4)*x^2
```

Fricas [B] time = 1.37211, size = 694, normalized size = 6.25

$$\frac{1}{9}x^9e^5d^3c^3 + \frac{5}{8}x^8e^4d^4c^3 + \frac{3}{8}x^8e^6d^2c^2a + \frac{10}{7}x^7e^3d^5c^3 + \frac{15}{7}x^7e^5d^3c^2a + \frac{3}{7}x^7e^7dca^2 + \frac{5}{3}x^6e^2d^6c^3 + 5x^6e^4d^4c^2a + \frac{5}{2}x^6e^6d^2ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="frica
s")
```

```
[Out] 1/9*x^9*e^5*d^3*c^3 + 5/8*x^8*e^4*d^4*c^3 + 3/8*x^8*e^6*d^2*c^2*a + 10/7*x^
7*e^3*d^5*c^3 + 15/7*x^7*e^5*d^3*c^2*a + 3/7*x^7*e^7*d*c*a^2 + 5/3*x^6*e^2*
d^6*c^3 + 5*x^6*e^4*d^4*c^2*a + 5/2*x^6*e^6*d^2*c*a^2 + 1/6*x^6*e^8*a^3 + x
^5*e*d^7*c^3 + 6*x^5*e^3*d^5*c^2*a + 6*x^5*e^5*d^3*c*a^2 + x^5*e^7*d*a^3 +
1/4*x^4*d^8*c^3 + 15/4*x^4*e^2*d^6*c^2*a + 15/2*x^4*e^4*d^4*c*a^2 + 5/2*x^4
*e^6*d^2*a^3 + x^3*e*d^7*c^2*a + 5*x^3*e^3*d^5*c*a^2 + 10/3*x^3*e^5*d^3*a^3
 + 3/2*x^2*e^2*d^6*c*a^2 + 5/2*x^2*e^4*d^4*a^3 + x*e^3*d^5*a^3
```

Sympy [B] time = 0.197032, size = 335, normalized size = 3.02

$$a^3d^5e^3x + \frac{c^3d^3e^5x^9}{9} + x^8\left(\frac{3ac^2d^2e^6}{8} + \frac{5c^3d^4e^4}{8}\right) + x^7\left(\frac{3a^2cde^7}{7} + \frac{15ac^2d^3e^5}{7} + \frac{10c^3d^5e^3}{7}\right) + x^6\left(\frac{a^3e^8}{6} + \frac{5a^2cd^2e^6}{2} + 5ac^2d^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

```
[Out] a**3*d**5*e**3*x + c**3*d**3*e**5*x**9/9 + x**8*(3*a*c**2*d**2*e**6/8 + 5*c
**3*d**4*e**4/8) + x**7*(3*a**2*c*d*e**7/7 + 15*a*c**2*d**3*e**5/7 + 10*c**
3*d**5*e**3/7) + x**6*(a**3*e**8/6 + 5*a**2*c*d**2*e**6/2 + 5*a*c**2*d**4*e
**4 + 5*c**3*d**6*e**2/3) + x**5*(a**3*d*e**7 + 6*a**2*c*d**3*e**5 + 6*a*c*
*2*d**5*e**3 + c**3*d**7*e) + x**4*(5*a**3*d**2*e**6/2 + 15*a**2*c*d**4*e**
4/2 + 15*a*c**2*d**6*e**2/4 + c**3*d**8/4) + x**3*(10*a**3*d**3*e**5/3 + 5*
a**2*c*d**5*e**3 + a*c**2*d**7*e) + x**2*(5*a**3*d**4*e**4/2 + 3*a**2*c*d**
6*e**2/2)
```

Giac [B] time = 1.14721, size = 419, normalized size = 3.77

$$\frac{1}{9}c^3d^3x^9e^5 + \frac{5}{8}c^3d^4x^8e^4 + \frac{10}{7}c^3d^5x^7e^3 + \frac{5}{3}c^3d^6x^6e^2 + c^3d^7x^5e + \frac{1}{4}c^3d^8x^4 + \frac{3}{8}ac^2d^2x^8e^6 + \frac{15}{7}ac^2d^3x^7e^5 + 5ac^2d^4x^6e^4 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac"
)
```

```
[Out] 1/9*c^3*d^3*x^9*e^5 + 5/8*c^3*d^4*x^8*e^4 + 10/7*c^3*d^5*x^7*e^3 + 5/3*c^3*
d^6*x^6*e^2 + c^3*d^7*x^5*e + 1/4*c^3*d^8*x^4 + 3/8*a*c^2*d^2*x^8*e^6 + 15/
7*a*c^2*d^3*x^7*e^5 + 5*a*c^2*d^4*x^6*e^4 + 6*a*c^2*d^5*x^5*e^3 + 15/4*a*c^
2*d^6*x^4*e^2 + a*c^2*d^7*x^3*e + 3/7*a^2*c*d*x^7*e^7 + 5/2*a^2*c*d^2*x^6*e
^6 + 6*a^2*c*d^3*x^5*e^5 + 15/2*a^2*c*d^4*x^4*e^4 + 5*a^2*c*d^5*x^3*e^3 + 3
/2*a^2*c*d^6*x^2*e^2 + 1/6*a^3*x^6*e^8 + a^3*d*x^5*e^7 + 5/2*a^3*d^2*x^4*e^
6 + 10/3*a^3*d^3*x^3*e^5 + 5/2*a^3*d^4*x^2*e^4 + a^3*d^5*x*e^3
```

3.1852 $\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3\,dx$

Optimal. Leaf size=111

$$-\frac{3c^2d^2(d+ex)^7\left(cd^2-ae^2\right)}{7e^4}+\frac{cd(d+ex)^6\left(cd^2-ae^2\right)^2}{2e^4}-\frac{(d+ex)^5\left(cd^2-ae^2\right)^3}{5e^4}+\frac{c^3d^3(d+ex)^8}{8e^4}$$

```
[Out] -((c*d^2 - a*e^2)^3*(d + e*x)^5)/(5*e^4) + (c*d*(c*d^2 - a*e^2)^2*(d + e*x)
^6)/(2*e^4) - (3*c^2*d^2*(c*d^2 - a*e^2)*(d + e*x)^7)/(7*e^4) + (c^3*d^3*(d
 + e*x)^8)/(8*e^4)
```

Rubi [A] time = 0.160326, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {626, 43}

$$-\frac{3c^2d^2(d+ex)^7\left(cd^2-ae^2\right)}{7e^4}+\frac{cd(d+ex)^6\left(cd^2-ae^2\right)^2}{2e^4}-\frac{(d+ex)^5\left(cd^2-ae^2\right)^3}{5e^4}+\frac{c^3d^3(d+ex)^8}{8e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] -((c*d^2 - a*e^2)^3*(d + e*x)^5)/(5*e^4) + (c*d*(c*d^2 - a*e^2)^2*(d + e*x)
^6)/(2*e^4) - (3*c^2*d^2*(c*d^2 - a*e^2)*(d + e*x)^7)/(7*e^4) + (c^3*d^3*(d
 + e*x)^8)/(8*e^4)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3\,dx &= \int (ae+cdx)^3(d+ex)^4\,dx\\ &= \int \left(\frac{\left(-cd^2+ae^2\right)^3(d+ex)^4}{e^3}+\frac{3cd\left(cd^2-ae^2\right)^2(d+ex)^5}{e^3}-\frac{3c^2d^2\left(cd^2\right.}{}\right.\\ &= -\frac{\left(cd^2-ae^2\right)^3(d+ex)^5}{5e^4}+\frac{cd\left(cd^2-ae^2\right)^2(d+ex)^6}{2e^4}-\frac{3c^2d^2\left(cd^2-ae^2\right.}{7e^4}\end{aligned}$$

Mathematica [A] time = 0.0633384, size = 211, normalized size = 1.9

$$\frac{1}{280}x\left(28a^2cde^2x\left(45d^2e^2x^2+40d^3ex+15d^4+24de^3x^3+5e^4x^4\right)+56a^3e^3\left(10d^2e^2x^2+10d^3ex+5d^4+5de^3x^3+e^4x^4\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] (x*(56*a^3*e^3*(5*d^4 + 10*d^3*e*x + 10*d^2*e^2*x^2 + 5*d*e^3*x^3 + e^4*x^4
) + 28*a^2*c*d*e^2*x*(15*d^4 + 40*d^3*e*x + 45*d^2*e^2*x^2 + 24*d*e^3*x^3 +
 5*e^4*x^4) + 8*a*c^2*d^2*e*x^2*(35*d^4 + 105*d^3*e*x + 126*d^2*e^2*x^2 + 7
0*d*e^3*x^3 + 15*e^4*x^4) + c^3*d^3*x^3*(70*d^4 + 224*d^3*e*x + 280*d^2*e^2
*x^2 + 160*d*e^3*x^3 + 35*e^4*x^4)))/280
```

Maple [B] time = 0.04, size = 531, normalized size = 4.8

$$\frac{d^3e^4c^3x^8}{8}+\frac{\left(d^4e^3c^3+3\,e^3\left(ae^2+cd^2\right)d^2c^2\right)x^7}{7}+\frac{\left(3\,d^3\left(ae^2+cd^2\right)e^2c^2+e\left(ad^3e^3c^2+2\left(ae^2+cd^2\right)^2dec+dec\left(2\,acd^2e^2+\right.\right.\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

```
[Out] 1/8*d^3*e^4*c^3*x^8+1/7*(d^4*e^3*c^3+3*e^3*(a*e^2+c*d^2)*d^2*c^2)*x^7+1/6*(
3*d^3*(a*e^2+c*d^2)*e^2*c^2+e*(a*d^3*e^3*c^2+2*(a*e^2+c*d^2)^2*d*e*c+d*e*c*
(2*a*c*d^2*e^2+(a*e^2+c*d^2)^2)))*x^6+1/5*(d*(a*d^3*e^3*c^2+2*(a*e^2+c*d^2)
^2*d*e*c+d*e*c*(2*a*c*d^2*e^2+(a*e^2+c*d^2)^2))+e*(4*a*d^2*e^2*(a*e^2+c*d^2
)*c+(a*e^2+c*d^2)*(2*a*c*d^2*e^2+(a*e^2+c*d^2)^2)))*x^5+1/4*(d*(4*a*d^2*e^2
*(a*e^2+c*d^2)*c+(a*e^2+c*d^2)*(2*a*c*d^2*e^2+(a*e^2+c*d^2)^2))+e*(a*d*e*(2
*a*c*d^2*e^2+(a*e^2+c*d^2)^2)+2*(a*e^2+c*d^2)^2*a*d*e+d^3*e^3*c*a^2))*x^4+1
/3*(d*(a*d*e*(2*a*c*d^2*e^2+(a*e^2+c*d^2)^2)+2*(a*e^2+c*d^2)^2*a*d*e+d^3*e^
3*c*a^2)+3*e^3*a^2*d^2*(a*e^2+c*d^2))*x^3+1/2*(3*d^3*a^2*e^2*(a*e^2+c*d^2)+
a^3*d^3*e^4)*x^2+a^3*d^4*e^3*x
```

Maxima [B] time = 1.01722, size = 339, normalized size = 3.05

$$\frac{1}{8}\,c^3d^3e^4x^8+a^3d^4e^3x+\frac{1}{7}\left(4\,c^3d^4e^3+3\,ac^2d^2e^5\right)x^7+\frac{1}{2}\left(2\,c^3d^5e^2+4\,ac^2d^3e^4+a^2cde^6\right)x^6+\frac{1}{5}\left(4\,c^3d^6e+18\,ac^2d^4e^3+12\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxima"
)
```

```
[Out] 1/8*c^3*d^3*e^4*x^8 + a^3*d^4*e^3*x + 1/7*(4*c^3*d^4*e^3 + 3*a*c^2*d^2*e^5)
*x^7 + 1/2*(2*c^3*d^5*e^2 + 4*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x^6 + 1/5*(4*c^3
*d^6*e + 18*a*c^2*d^4*e^3 + 12*a^2*c*d^2*e^5 + a^3*e^7)*x^5 + 1/4*(c^3*d^7
+ 12*a*c^2*d^5*e^2 + 18*a^2*c*d^3*e^4 + 4*a^3*d*e^6)*x^4 + (a*c^2*d^6*e + 4
*a^2*c*d^4*e^3 + 2*a^3*d^2*e^5)*x^3 + 1/2*(3*a^2*c*d^5*e^2 + 4*a^3*d^3*e^4)
*x^2
```

Fricas [B] time = 1.30077, size = 563, normalized size = 5.07

$$\frac{1}{8}x^8e^4d^3c^3+\frac{4}{7}x^7e^3d^4c^3+\frac{3}{7}x^7e^5d^2c^2a+x^6e^2d^5c^3+2x^6e^4d^3c^2a+\frac{1}{2}x^6e^6dca^2+\frac{4}{5}x^5ed^6c^3+\frac{18}{5}x^5e^3d^4c^2a+\frac{12}{5}x^5e^5d^2ca^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="fricas"
)
```

```
[Out] 1/8*x^8*e^4*d^3*c^3 + 4/7*x^7*e^3*d^4*c^3 + 3/7*x^7*e^5*d^2*c^2*a + x^6*e^2
*d^5*c^3 + 2*x^6*e^4*d^3*c^2*a + 1/2*x^6*e^6*d*c*a^2 + 4/5*x^5*e*d^6*c^3 +
18/5*x^5*e^3*d^4*c^2*a + 12/5*x^5*e^5*d^2*c*a^2 + 1/5*x^5*e^7*a^3 + 1/4*x^4
*d^7*c^3 + 3*x^4*e^2*d^5*c^2*a + 9/2*x^4*e^4*d^3*c*a^2 + x^4*e^6*d*a^3 + x^
3*e*d^6*c^2*a + 4*x^3*e^3*d^4*c*a^2 + 2*x^3*e^5*d^2*a^3 + 3/2*x^2*e^2*d^5*c
*a^2 + 2*x^2*e^4*d^3*a^3 + x*e^3*d^4*a^3
```

Sympy [B] time = 0.179508, size = 270, normalized size = 2.43

$$a^3d^4e^3x + \frac{c^3d^3e^4x^8}{8} + x^7\left(\frac{3ac^2d^2e^5}{7} + \frac{4c^3d^4e^3}{7}\right) + x^6\left(\frac{a^2cde^6}{2} + 2ac^2d^3e^4 + c^3d^5e^2\right) + x^5\left(\frac{a^3e^7}{5} + \frac{12a^2cd^2e^5}{5} + \frac{18ac^2d^4}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

```
[Out] a**3*d**4*e**3*x + c**3*d**3*e**4*x**8/8 + x**7*(3*a*c**2*d**2*e**5/7 + 4*c
**3*d**4*e**3/7) + x**6*(a**2*c*d*e**6/2 + 2*a*c**2*d**3*e**4 + c**3*d**5*e
**2) + x**5*(a**3*e**7/5 + 12*a**2*c*d**2*e**5/5 + 18*a*c**2*d**4*e**3/5 +
4*c**3*d**6*e/5) + x**4*(a**3*d*e**6 + 9*a**2*c*d**3*e**4/2 + 3*a*c**2*d**5
*e**2 + c**3*d**7/4) + x**3*(2*a**3*d**2*e**5 + 4*a**2*c*d**4*e**3 + a*c**2
*d**6*e) + x**2*(2*a**3*d**3*e**4 + 3*a**2*c*d**5*e**2/2)
```

Giac [B] time = 1.21402, size = 346, normalized size = 3.12

$$\frac{1}{8}c^3d^3x^8e^4 + \frac{4}{7}c^3d^4x^7e^3 + c^3d^5x^6e^2 + \frac{4}{5}c^3d^6x^5e + \frac{1}{4}c^3d^7x^4 + \frac{3}{7}ac^2d^2x^7e^5 + 2ac^2d^3x^6e^4 + \frac{18}{5}ac^2d^4x^5e^3 + 3ac^2d^5x^4e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*c^3*d^3*x^8*e^4 + 4/7*c^3*d^4*x^7*e^3 + c^3*d^5*x^6*e^2 + 4/5*c^3*d^6*x
^5*e + 1/4*c^3*d^7*x^4 + 3/7*a*c^2*d^2*x^7*e^5 + 2*a*c^2*d^3*x^6*e^4 + 18/5
*a*c^2*d^4*x^5*e^3 + 3*a*c^2*d^5*x^4*e^2 + a*c^2*d^6*x^3*e + 1/2*a^2*c*d*x^
6*e^6 + 12/5*a^2*c*d^2*x^5*e^5 + 9/2*a^2*c*d^3*x^4*e^4 + 4*a^2*c*d^4*x^3*e^
3 + 3/2*a^2*c*d^5*x^2*e^2 + 1/5*a^3*x^5*e^7 + a^3*d*x^4*e^6 + 2*a^3*d^2*x^3
*e^5 + 2*a^3*d^3*x^2*e^4 + a^3*d^4*x*e^3
```

3.1853 $\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3\,dx$

Optimal. Leaf size=111

$$-\frac{c^2d^2(d+ex)^6\left(cd^2-ae^2\right)}{2e^4}+\frac{3cd(d+ex)^5\left(cd^2-ae^2\right)^2}{5e^4}-\frac{(d+ex)^4\left(cd^2-ae^2\right)^3}{4e^4}+\frac{c^3d^3(d+ex)^7}{7e^4}$$

```
[Out] -((c*d^2 - a*e^2)^3*(d + e*x)^4)/(4*e^4) + (3*c*d*(c*d^2 - a*e^2)^2*(d + e*
x)^5)/(5*e^4) - (c^2*d^2*(c*d^2 - a*e^2)*(d + e*x)^6)/(2*e^4) + (c^3*d^3*(d
 + e*x)^7)/(7*e^4)
```

Rubi [A] time = 0.148613, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {610, 43}

$$-\frac{c^2d^2(d+ex)^6\left(cd^2-ae^2\right)}{2e^4}+\frac{3cd(d+ex)^5\left(cd^2-ae^2\right)^2}{5e^4}-\frac{(d+ex)^4\left(cd^2-ae^2\right)^3}{4e^4}+\frac{c^3d^3(d+ex)^7}{7e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] -((c*d^2 - a*e^2)^3*(d + e*x)^4)/(4*e^4) + (3*c*d*(c*d^2 - a*e^2)^2*(d + e*
x)^5)/(5*e^4) - (c^2*d^2*(c*d^2 - a*e^2)*(d + e*x)^6)/(2*e^4) + (c^3*d^3*(d
 + e*x)^7)/(7*e^4)
```

Rule 610

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[1/c^p, Int[Simp[b/2 - q/2 + c*x, x]^p*Simp[b/2 + q/2 + c
*x, x]^p, x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3\,dx &= \frac{\int \left(cd^2+cdex\right)^3\left(ae^2+cdex\right)^3\,dx}{c^3d^3e^3}\\ &= \frac{\int \left(-\left(cd^2-ae^2\right)^3\left(cd^2+cdex\right)^3+3\left(cd^2-ae^2\right)^2\left(cd^2+cdex\right)^4-3\left(cd^2-ae^2\right)\left(cd^2\right.\right.}{c^3d^3e^3}\\ &= -\frac{\left(cd^2-ae^2\right)^3(d+ex)^4}{4e^4}+\frac{3cd\left(cd^2-ae^2\right)^2(d+ex)^5}{5e^4}-\frac{c^2d^2\left(cd^2-ae^2\right)(d+ex)^6}{2e^4}\end{aligned}$$

Mathematica [A] time = 0.0485335, size = 167, normalized size = 1.5

$$\frac{1}{140}x\left(21a^2cde^2x\left(20d^2ex+10d^3+15de^2x^2+4e^3x^3\right)+35a^3e^3\left(6d^2ex+4d^3+4de^2x^2+e^3x^3\right)+7ac^2d^2ex^2\left(45d^2ex+20d^3\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] (x*(35*a^3*e^3*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) + 21*a^2*c*d*e^2
*x*(10*d^3 + 20*d^2*e*x + 15*d*e^2*x^2 + 4*e^3*x^3) + 7*a*c^2*d^2*e*x^2*(20
*d^3 + 45*d^2*e*x + 36*d*e^2*x^2 + 10*e^3*x^3) + c^3*d^3*x^3*(35*d^3 + 84*d
^2*e*x + 70*d*e^2*x^2 + 20*e^3*x^3)))/140
```

Maple [B] time = 0.038, size = 266, normalized size = 2.4

$$\frac{d^3e^3c^3x^7}{7} + \frac{\left(ae^2 + cd^2\right)d^2e^2c^2x^6}{2} + \frac{\left(ad^3e^3c^2 + 2\left(ae^2 + cd^2\right)^2 dec + dec\left(2\,acd^2e^2 + \left(ae^2 + cd^2\right)^2\right)\right)x^5}{5} + \frac{\left(4\,ad^2e^2\left(ae^2 +\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

```
[Out] 1/7*d^3*e^3*c^3*x^7+1/2*(a*e^2+c*d^2)*d^2*e^2*c^2*x^6+1/5*(a*d^3*e^3*c^2+2*
(a*e^2+c*d^2)^2*d*e*c+d*e*c*(2*a*c*d^2*e^2+(a*e^2+c*d^2)^2))*x^5+1/4*(4*a*d
^2*e^2*(a*e^2+c*d^2)*c+(a*e^2+c*d^2)*(2*a*c*d^2*e^2+(a*e^2+c*d^2)^2))*x^4+1
/3*(a*d*e*(2*a*c*d^2*e^2+(a*e^2+c*d^2)^2)+2*(a*e^2+c*d^2)^2*a*d*e+d^3*e^3*c
*a^2)*x^3+3/2*a^2*d^2*e^2*(a*e^2+c*d^2)*x^2+a^3*d^3*e^3*x
```

Maxima [A] time = 1.10195, size = 247, normalized size = 2.23

$$\frac{1}{7}\,c^3d^3e^3x^7 + \frac{1}{2}\left(cd^2 + ae^2\right)c^2d^2e^2x^6 + a^3d^3e^3x + \frac{3}{5}\left(cd^2 + ae^2\right)^2 cdex^5 + \frac{1}{2}\left(2\,cdex^3 + 3\left(cd^2 + ae^2\right)x^2\right)a^2d^2e^2 + \frac{1}{4}\left(cd^2 +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/7*c^3*d^3*e^3*x^7 + 1/2*(c*d^2 + a*e^2)*c^2*d^2*e^2*x^6 + a^3*d^3*e^3*x +
 3/5*(c*d^2 + a*e^2)^2*c*d*e*x^5 + 1/2*(2*c*d*e*x^3 + 3*(c*d^2 + a*e^2)*x^2
)*a^2*d^2*e^2 + 1/4*(c*d^2 + a*e^2)^3*x^4 + 1/10*(6*c^2*d^2*e^2*x^5 + 15*(c
*d^2 + a*e^2)*c*d*e*x^4 + 10*(c*d^2 + a*e^2)^2*x^3)*a*d*e
```

Fricas [B] time = 1.37993, size = 452, normalized size = 4.07

$$\frac{1}{7}x^7e^3d^3c^3 + \frac{1}{2}x^6e^2d^4c^3 + \frac{1}{2}x^6e^4d^2c^2a + \frac{3}{5}x^5ed^5c^3 + \frac{9}{5}x^5e^3d^3c^2a + \frac{3}{5}x^5e^5dca^2 + \frac{1}{4}x^4d^6c^3 + \frac{9}{4}x^4e^2d^4c^2a + \frac{9}{4}x^4e^4d^2ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*e^3*d^3*c^3 + 1/2*x^6*e^2*d^4*c^3 + 1/2*x^6*e^4*d^2*c^2*a + 3/5*x^5
*e*d^5*c^3 + 9/5*x^5*e^3*d^3*c^2*a + 3/5*x^5*e^5*d*c*a^2 + 1/4*x^4*d^6*c^3
+ 9/4*x^4*e^2*d^4*c^2*a + 9/4*x^4*e^4*d^2*c*a^2 + 1/4*x^4*e^6*a^3 + x^3*e*d
^5*c^2*a + 3*x^3*e^3*d^3*c*a^2 + x^3*e^5*d*a^3 + 3/2*x^2*e^2*d^4*c*a^2 + 3/
```

```
2*x^2*e^4*d^2*a^3 + x*e^3*d^3*a^3
```

Sympy [B] time = 0.145143, size = 218, normalized size = 1.96

$$a^3d^3e^3x + \frac{c^3d^3e^3x^7}{7} + x^6\left(\frac{ac^2d^2e^4}{2} + \frac{c^3d^4e^2}{2}\right) + x^5\left(\frac{3a^2cde^5}{5} + \frac{9ac^2d^3e^3}{5} + \frac{3c^3d^5e}{5}\right) + x^4\left(\frac{a^3e^6}{4} + \frac{9a^2cd^2e^4}{4} + \frac{9ac^2d^4e^2}{4} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

```
[Out] a**3*d**3*e**3*x + c**3*d**3*e**3*x**7/7 + x**6*(a*c**2*d**2*e**4/2 + c**3*
d**4*e**2/2) + x**5*(3*a**2*c*d*e**5/5 + 9*a*c**2*d**3*e**3/5 + 3*c**3*d**5
*e/5) + x**4*(a**3*e**6/4 + 9*a**2*c*d**2*e**4/4 + 9*a*c**2*d**4*e**2/4 + c
**3*d**6/4) + x**3*(a**3*d*e**5 + 3*a**2*c*d**3*e**3 + a*c**2*d**5*e) + x**
2*(3*a**3*d**2*e**4/2 + 3*a**2*c*d**4*e**2/2)
```

Giac [A] time = 1.16336, size = 274, normalized size = 2.47

$$\frac{1}{7}c^3d^3x^7e^3 + \frac{1}{2}c^3d^4x^6e^2 + \frac{3}{5}c^3d^5x^5e + \frac{1}{4}c^3d^6x^4 + \frac{1}{2}ac^2d^2x^6e^4 + \frac{9}{5}ac^2d^3x^5e^3 + \frac{9}{4}ac^2d^4x^4e^2 + ac^2d^5x^3e + \frac{3}{5}a^2cdx^5e^5 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/7*c^3*d^3*x^7*e^3 + 1/2*c^3*d^4*x^6*e^2 + 3/5*c^3*d^5*x^5*e + 1/4*c^3*d^6
*x^4 + 1/2*a*c^2*d^2*x^6*e^4 + 9/5*a*c^2*d^3*x^5*e^3 + 9/4*a*c^2*d^4*x^4*e^
2 + a*c^2*d^5*x^3*e + 3/5*a^2*c*d*x^5*e^5 + 9/4*a^2*c*d^2*x^4*e^4 + 3*a^2*c
*d^3*x^3*e^3 + 3/2*a^2*c*d^4*x^2*e^2 + 1/4*a^3*x^4*e^6 + a^3*d*x^3*e^5 + 3/
2*a^3*d^2*x^2*e^4 + a^3*d^3*x*e^3
```

3.1854 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{d+ex}\,dx$

Optimal. Leaf size=91

$$\frac{e^2(ae+cdx)^6}{6c^3d^3}+\frac{2e\left(cd^2-ae^2\right)(ae+cdx)^5}{5c^3d^3}+\frac{\left(cd^2-ae^2\right)^2(ae+cdx)^4}{4c^3d^3}$$

```
[Out] ((c*d^2 - a*e^2)^2*(a*e + c*d*x)^4)/(4*c^3*d^3) + (2*e*(c*d^2 - a*e^2)*(a*e
 + c*d*x)^5)/(5*c^3*d^3) + (e^2*(a*e + c*d*x)^6)/(6*c^3*d^3)
```

Rubi [A] time = 0.0992741, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$\frac{e^2(ae+cdx)^6}{6c^3d^3}+\frac{2e\left(cd^2-ae^2\right)(ae+cdx)^5}{5c^3d^3}+\frac{\left(cd^2-ae^2\right)^2(ae+cdx)^4}{4c^3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x),x]
```

```
[Out] ((c*d^2 - a*e^2)^2*(a*e + c*d*x)^4)/(4*c^3*d^3) + (2*e*(c*d^2 - a*e^2)*(a*e
 + c*d*x)^5)/(5*c^3*d^3) + (e^2*(a*e + c*d*x)^6)/(6*c^3*d^3)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{d+ex}\,dx &= \int (ae+cdx)^3(d+ex)^2\,dx\\ &= \int \left(\frac{\left(cd^2-ae^2\right)^2(ae+cdx)^3}{c^2d^2}+\frac{2e\left(cd^2-ae^2\right)(ae+cdx)^4}{c^2d^2}+\frac{e^2(ae+cdx)^5}{c^2d^2}\right)dx\\ &= \frac{\left(cd^2-ae^2\right)^2(ae+cdx)^4}{4c^3d^3}+\frac{2e\left(cd^2-ae^2\right)(ae+cdx)^5}{5c^3d^3}+\frac{e^2(ae+cdx)^6}{6c^3d^3}\end{aligned}$$

Mathematica [A] time = 0.0391323, size = 123, normalized size = 1.35

$$\frac{1}{60}x\left(15a^2cde^2x\left(6d^2+8dex+3e^2x^2\right)+20a^3e^3\left(3d^2+3dex+e^2x^2\right)+6ac^2d^2ex^2\left(10d^2+15dex+6e^2x^2\right)+c^3d^3x^3\left(15a\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x),x]
```

```
[Out] (x*(20*a^3*e^3*(3*d^2 + 3*d*e*x + e^2*x^2) + 15*a^2*c*d*e^2*x*(6*d^2 + 8*d*
e*x + 3*e^2*x^2) + 6*a*c^2*d^2*e*x^2*(10*d^2 + 15*d*e*x + 6*e^2*x^2) + c^3*
d^3*x^3*(15*d^2 + 24*d*e*x + 10*e^2*x^2)))/60
```

Maple [B] time = 0.038, size = 205, normalized size = 2.3

$$\frac{c^3d^3e^2x^6}{6}+\frac{\left(ae^3d^2c^2+2\,c^2d^2\left(ae^2+cd^2\right)e\right)x^5}{5}+\frac{\left(2\,ae^2\left(ae^2+cd^2\right)dc+cd\left(2\,acd^2e^2+\left(ae^2+cd^2\right)^2\right)\right)x^4}{4}+\frac{\left(ae\left(2\,acd^2e^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d),x)
```

```
[Out] 1/6*c^3*d^3*e^2*x^6+1/5*(a*e^3*d^2*c^2+2*c^2*d^2*(a*e^2+c*d^2)*e)*x^5+1/4*(
2*a*e^2*(a*e^2+c*d^2)*d*c+c*d*(2*a*c*d^2*e^2+(a*e^2+c*d^2)^2))*x^4+1/3*(a*e
*(2*a*c*d^2*e^2+(a*e^2+c*d^2)^2)+2*c*d^2*a*e*(a*e^2+c*d^2))*x^3+1/2*(2*a^2*
e^2*d*(a*e^2+c*d^2)+c*d^3*a^2*e^2)*x^2+a^3*e^3*d^2*x
```

Maxima [A] time = 1.0788, size = 203, normalized size = 2.23

$$\frac{1}{6}\,c^3d^3e^2x^6+a^3d^2e^3x+\frac{1}{5}\left(2\,c^3d^4e+3\,ac^2d^2e^3\right)x^5+\frac{1}{4}\left(c^3d^5+6\,ac^2d^3e^2+3\,a^2cde^4\right)x^4+\frac{1}{3}\left(3\,ac^2d^4e+6\,a^2cd^2e^3+a^3e^5\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d),x, algorithm="maxima"
)
```

```
[Out] 1/6*c^3*d^3*e^2*x^6 + a^3*d^2*e^3*x + 1/5*(2*c^3*d^4*e + 3*a*c^2*d^2*e^3)*x
^5 + 1/4*(c^3*d^5 + 6*a*c^2*d^3*e^2 + 3*a^2*c*d*e^4)*x^4 + 1/3*(3*a*c^2*d^4
*e + 6*a^2*c*d^2*e^3 + a^3*e^5)*x^3 + 1/2*(3*a^2*c*d^3*e^2 + 2*a^3*d*e^4)*x
^2
```

Fricas [A] time = 1.52703, size = 309, normalized size = 3.4

$$\frac{1}{6}\,c^3d^3e^2x^6+a^3d^2e^3x+\frac{1}{5}\left(2\,c^3d^4e+3\,ac^2d^2e^3\right)x^5+\frac{1}{4}\left(c^3d^5+6\,ac^2d^3e^2+3\,a^2cde^4\right)x^4+\frac{1}{3}\left(3\,ac^2d^4e+6\,a^2cd^2e^3+a^3e^5\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d),x, algorithm="fricas"
)
```

```
[Out] 1/6*c^3*d^3*e^2*x^6 + a^3*d^2*e^3*x + 1/5*(2*c^3*d^4*e + 3*a*c^2*d^2*e^3)*x
^5 + 1/4*(c^3*d^5 + 6*a*c^2*d^3*e^2 + 3*a^2*c*d*e^4)*x^4 + 1/3*(3*a*c^2*d^4
*e + 6*a^2*c*d^2*e^3 + a^3*e^5)*x^3 + 1/2*(3*a^2*c*d^3*e^2 + 2*a^3*d*e^4)*x
^2
```

Sympy [A] time = 0.152798, size = 160, normalized size = 1.76

$$a^3d^2e^3x + \frac{c^3d^3e^2x^6}{6} + x^5\left(\frac{3ac^2d^2e^3}{5} + \frac{2c^3d^4e}{5}\right) + x^4\left(\frac{3a^2cde^4}{4} + \frac{3ac^2d^3e^2}{2} + \frac{c^3d^5}{4}\right) + x^3\left(\frac{a^3e^5}{3} + 2a^2cd^2e^3 + ac^2d^4e\right) + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d),x)
```

```
[Out] a**3*d**2*e**3*x + c**3*d**3*e**2*x**6/6 + x**5*(3*a*c**2*d**2*e**3/5 + 2*c
**3*d**4*e/5) + x**4*(3*a**2*c*d*e**4/4 + 3*a*c**2*d**3*e**2/2 + c**3*d**5/
4) + x**3*(a**3*e**5/3 + 2*a**2*c*d**2*e**3 + a*c**2*d**4*e) + x**2*(a**3*d
*e**4 + 3*a**2*c*d**3*e**2/2)
```

Giac [A] time = 1.24299, size = 213, normalized size = 2.34

$$\frac{1}{60}\left(10\,c^3d^3x^6e^8 + 24\,c^3d^4x^5e^7 + 15\,c^3d^5x^4e^6 + 36\,ac^2d^2x^5e^9 + 90\,ac^2d^3x^4e^8 + 60\,ac^2d^4x^3e^7 + 45\,a^2cdx^4e^{10} + 120\,a^2cd^2 \dots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d),x, algorithm="giac")
```

```
[Out] 1/60*(10*c^3*d^3*x^6*e^8 + 24*c^3*d^4*x^5*e^7 + 15*c^3*d^5*x^4*e^6 + 36*a*c
^2*d^2*x^5*e^9 + 90*a*c^2*d^3*x^4*e^8 + 60*a*c^2*d^4*x^3*e^7 + 45*a^2*c*d*x
^4*e^10 + 120*a^2*c*d^2*x^3*e^9 + 90*a^2*c*d^3*x^2*e^8 + 20*a^3*x^3*e^11 +
60*a^3*d*x^2*e^10 + 60*a^3*d^2*x*e^9)*e^(-6)
```

3.1855 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^2}\,dx$

Optimal. Leaf size=54

$$\frac{\left(cd^2-ae^2\right)(ae+cdx)^4}{4c^2d^2}+\frac{e(ae+cdx)^5}{5c^2d^2}$$

```
[Out] ((c*d^2 - a*e^2)*(a*e + c*d*x)^4)/(4*c^2*d^2) + (e*(a*e + c*d*x)^5)/(5*c^2*
d^2)
```

Rubi [A] time = 0.0312005, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$\frac{\left(cd^2-ae^2\right)(ae+cdx)^4}{4c^2d^2}+\frac{e(ae+cdx)^5}{5c^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^2,x]
```

```
[Out] ((c*d^2 - a*e^2)*(a*e + c*d*x)^4)/(4*c^2*d^2) + (e*(a*e + c*d*x)^5)/(5*c^2*
d^2)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^2}\,dx &= \int (ae+cdx)^3(d+ex)\,dx\\
&= \int \left(\frac{\left(cd^2-ae^2\right)(ae+cdx)^3}{cd}+\frac{e(ae+cdx)^4}{cd}\right)dx\\
&= \frac{\left(cd^2-ae^2\right)(ae+cdx)^4}{4c^2d^2}+\frac{e(ae+cdx)^5}{5c^2d^2}
\end{aligned}$$

Mathematica [A] time = 0.0259652, size = 79, normalized size = 1.46

$$\frac{1}{20}x\left(10a^2cde^2x(3d+2ex)+10a^3e^3(2d+ex)+5ac^2d^2ex^2(4d+3ex)+c^3d^3x^3(5d+4ex)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^2,x]
```

```
[Out] (x*(10*a^3*e^3*(2*d + e*x) + 10*a^2*c*d*e^2*x*(3*d + 2*e*x) + 5*a*c^2*d^2*e
*x^2*(4*d + 3*e*x) + c^3*d^3*x^3*(5*d + 4*e*x)))/20
```

Maple [B] time = 0.04, size = 136, normalized size = 2.5

$$\frac{c^3d^3ex^5}{5}+\frac{\left(2\,ac^2d^2e^2+c^2d^2\left(ae^2+cd^2\right)\right)x^4}{4}+\frac{\left(a^2e^3dc+2\,acde\left(ae^2+cd^2\right)+c^2d^3ae\right)x^3}{3}+\frac{\left(a^2e^2\left(ae^2+cd^2\right)+2\,a^2cd^2\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^2,x)
```

```
[Out] 1/5*c^3*d^3*e*x^5+1/4*(2*a*c^2*d^2*e^2+c^2*d^2*(a*e^2+c*d^2))*x^4+1/3*(a^2*
e^3*d*c+2*a*c*d*e*(a*e^2+c*d^2)+c^2*d^3*a*e)*x^3+1/2*(a^2*e^2*(a*e^2+c*d^2)
+2*a^2*c*d^2*e^2)*x^2+a^3*e^3*d*x
```

Maxima [A] time = 1.08454, size = 128, normalized size = 2.37

$$\frac{1}{5}\,c^3d^3ex^5+a^3de^3x+\frac{1}{4}\left(c^3d^4+3\,ac^2d^2e^2\right)x^4+\left(ac^2d^3e+a^2cde^3\right)x^3+\frac{1}{2}\left(3\,a^2cd^2e^2+a^3e^4\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^2,x, algorithm="maxim
a")
```

```
[Out] 1/5*c^3*d^3*e*x^5 + a^3*d*e^3*x + 1/4*(c^3*d^4 + 3*a*c^2*d^2*e^2)*x^4 + (a*
c^2*d^3*e + a^2*c*d*e^3)*x^3 + 1/2*(3*a^2*c*d^2*e^2 + a^3*e^4)*x^2
```

Fricas [A] time = 1.37772, size = 193, normalized size = 3.57

$$\frac{1}{5}\,c^3d^3ex^5+a^3de^3x+\frac{1}{4}\left(c^3d^4+3\,ac^2d^2e^2\right)x^4+\left(ac^2d^3e+a^2cde^3\right)x^3+\frac{1}{2}\left(3\,a^2cd^2e^2+a^3e^4\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^2,x, algorithm="frica
s")
```

```
[Out] 1/5*c^3*d^3*e*x^5 + a^3*d*e^3*x + 1/4*(c^3*d^4 + 3*a*c^2*d^2*e^2)*x^4 + (a*
c^2*d^3*e + a^2*c*d*e^3)*x^3 + 1/2*(3*a^2*c*d^2*e^2 + a^3*e^4)*x^2
```

Sympy [B] time = 0.144167, size = 100, normalized size = 1.85

$$a^3de^3x+\frac{c^3d^3ex^5}{5}+x^4\left(\frac{3ac^2d^2e^2}{4}+\frac{c^3d^4}{4}\right)+x^3\left(a^2cde^3+ac^2d^3e\right)+x^2\left(\frac{a^3e^4}{2}+\frac{3a^2cd^2e^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**2,x)
```

```
[Out] a**3*d*e**3*x + c**3*d**3*e*x**5/5 + x**4*(3*a*c**2*d**2*e**2/4 + c**3*d**4
/4) + x**3*(a**2*c*d*e**3 + a*c**2*d**3*e) + x**2*(a**3*e**4/2 + 3*a**2*c*d
**2*e**2/2)
```

Giac [B] time = 1.19977, size = 197, normalized size = 3.65

$$\frac{1}{20}\left(4\,c^3d^3 - \frac{15\left(c^3d^4e - ac^2d^2e^3\right)e^{(-1)}}{xe+d} + \frac{20\left(c^3d^5e^2 - 2\,ac^2d^3e^4 + a^2cde^6\right)e^{(-2)}}{(xe+d)^2} - \frac{10\left(c^3d^6e^3 - 3\,ac^2d^4e^5 + 3\,a^2cd^2e^7 - a^3e^9\right.}{(xe+d)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^2,x, algorithm="giac"
)
```

```
[Out] 1/20*(4*c^3*d^3 - 15*(c^3*d^4*e - a*c^2*d^2*e^3)*e^(-1)/(x*e + d) + 20*(c^3
*d^5*e^2 - 2*a*c^2*d^3*e^4 + a^2*c*d*e^6)*e^(-2)/(x*e + d)^2 - 10*(c^3*d^6*
e^3 - 3*a*c^2*d^4*e^5 + 3*a^2*c*d^2*e^7 - a^3*e^9)*e^(-3)/(x*e + d)^3)*(x*e
 + d)^5*e^(-4)
```

3.1856 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^3}\,dx$

Optimal. Leaf size=20

$$\frac{(ae+cdx)^4}{4cd}$$

```
[Out] (a*e + c*d*x)^4/(4*c*d)
```

Rubi [A] time = 0.0133856, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 32}

$$\frac{(ae+cdx)^4}{4cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^3,x]
```

```
[Out] (a*e + c*d*x)^4/(4*c*d)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^3}\,dx &= \int (ae+cdx)^3\,dx\\ &= \frac{(ae+cdx)^4}{4cd}\end{aligned}$$

Mathematica [A] time = 0.0025022, size = 20, normalized size = 1.

$$\frac{(ae+cdx)^4}{4cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^3,x]
```

```
[Out] (a*e + c*d*x)^4/(4*c*d)
```

Maple [A] time = 0.04, size = 19, normalized size = 1.

$$\frac{(cdx + ae)^4}{4\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^3,x)
```

```
[Out] 1/4*(c*d*x+a*e)^4/c/d
```

Maxima [B] time = 1.08492, size = 61, normalized size = 3.05

$$\frac{1}{4}\,c^3d^3x^4 + ac^2d^2ex^3 + \frac{3}{2}\,a^2cde^2x^2 + a^3e^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^3,x, algorithm="maxim
a")
```

```
[Out] 1/4*c^3*d^3*x^4 + a*c^2*d^2*e*x^3 + 3/2*a^2*c*d*e^2*x^2 + a^3*e^3*x
```

Fricas [B] time = 1.52203, size = 93, normalized size = 4.65

$$\frac{1}{4}\,c^3d^3x^4 + ac^2d^2ex^3 + \frac{3}{2}\,a^2cde^2x^2 + a^3e^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^3,x, algorithm="frica
s")
```

```
[Out] 1/4*c^3*d^3*x^4 + a*c^2*d^2*e*x^3 + 3/2*a^2*c*d*e^2*x^2 + a^3*e^3*x
```

Sympy [B] time = 0.137744, size = 49, normalized size = 2.45

$$a^3e^3x + \frac{3a^2cde^2x^2}{2} + ac^2d^2ex^3 + \frac{c^3d^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**3,x)
```

```
[Out] a**3*e**3*x + 3*a**2*c*d*e**2*x**2/2 + a*c**2*d**2*e*x**3 + c**3*d**3*x**4/
4
```

Giac [B] time = 1.23946, size = 69, normalized size = 3.45

$$\frac{1}{4}\left(c^3d^3x^4e^{12}+4\,ac^2d^2x^3e^{13}+6\,a^2cdx^2e^{14}+4\,a^3xe^{15}\right)e^{(-12)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^3,x, algorithm="giac"
)
```

```
[Out] 1/4*(c^3*d^3*x^4*e^12 + 4*a*c^2*d^2*x^3*e^13 + 6*a^2*c*d*x^2*e^14 + 4*a^3*x
*e^15)*e^(-12)
```

3.1857 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^4}\,dx$

Optimal. Leaf size=89

$$\frac{cdx\left(cd^2-ae^2\right)^2}{e^3}+\frac{1}{2}\left(a-\frac{cd^2}{e^2}\right)(ae+cdx)^2-\frac{\left(cd^2-ae^2\right)^3\log(d+ex)}{e^4}+\frac{(ae+cdx)^3}{3e}$$

```
[Out] (c*d*(c*d^2 - a*e^2)^2*x)/e^3 + ((a - (c*d^2)/e^2)*(a*e + c*d*x)^2)/2 + (a*
e + c*d*x)^3/(3*e) - ((c*d^2 - a*e^2)^3*Log[d + e*x])/e^4
```

Rubi [A] time = 0.0486573, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$\frac{cdx\left(cd^2-ae^2\right)^2}{e^3}+\frac{1}{2}\left(a-\frac{cd^2}{e^2}\right)(ae+cdx)^2-\frac{\left(cd^2-ae^2\right)^3\log(d+ex)}{e^4}+\frac{(ae+cdx)^3}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^4,x]
```

```
[Out] (c*d*(c*d^2 - a*e^2)^2*x)/e^3 + ((a - (c*d^2)/e^2)*(a*e + c*d*x)^2)/2 + (a*
e + c*d*x)^3/(3*e) - ((c*d^2 - a*e^2)^3*Log[d + e*x])/e^4
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^4}\,dx &= \int \frac{(ae+cdx)^3}{d+ex}\,dx\\ &= \int \left(\frac{cd\left(cd^2-ae^2\right)^2}{e^3}-\frac{cd\left(cd^2-ae^2\right)(ae+cdx)}{e^2}+\frac{cd(ae+cdx)^2}{e}+\frac{\left(-cd^2+ae^2\right)^3}{e^3(d+ex)}\right)\\ &= \frac{cd\left(cd^2-ae^2\right)^2x}{e^3}+\frac{1}{2}\left(a-\frac{cd^2}{e^2}\right)(ae+cdx)^2+\frac{(ae+cdx)^3}{3e}-\frac{\left(cd^2-ae^2\right)^3\log(d+ex)}{e^4}\end{aligned}$$

Mathematica [A] time = 0.0358542, size = 85, normalized size = 0.96

$$\frac{cdex\left(18a^2e^4+9acde^2(ex-2d)+c^2d^2\left(6d^2-3dex+2e^2x^2\right)\right)-6\left(cd^2-ae^2\right)^3\log(d+ex)}{6e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^4,x]
```

```
[Out] (c*d*e*x*(18*a^2*e^4 + 9*a*c*d*e^2*(-2*d + e*x) + c^2*d^2*(6*d^2 - 3*d*e*x
+ 2*e^2*x^2)) - 6*(c*d^2 - a*e^2)^3*Log[d + e*x])/(6*e^4)
```

Maple [A] time = 0.043, size = 138, normalized size = 1.6

$$\frac{x^3c^3d^3}{3\,e}+\frac{3\,c^2d^2x^2a}{2}-\frac{c^3d^4x^2}{2\,e^2}+3\,cdea^2x-3\,\frac{c^2d^3ax}{e}+\frac{c^3d^5x}{e^3}+e^2\ln\left(ex+d\right)a^3-3\,\ln\left(ex+d\right)a^2cd^2+3\,\frac{\ln\left(ex+d\right)ac}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^4,x)
```

```
[Out] 1/3*c^3*d^3/e*x^3+3/2*c^2*d^2*x^2*a-1/2*c^3*d^4/e^2*x^2+3*c*d*e*a^2*x-3*c^2
*d^3/e*a*x+c^3*d^5/e^3*x+e^2*ln(e*x+d)*a^3-3*ln(e*x+d)*a^2*c*d^2+3/e^2*ln(e
*x+d)*a*c^2*d^4-1/e^4*ln(e*x+d)*c^3*d^6
```

Maxima [A] time = 1.12567, size = 177, normalized size = 1.99

$$\frac{2\,c^3d^3e^2x^3-3\left(c^3d^4e-3\,ac^2d^2e^3\right)x^2+6\left(c^3d^5-3\,ac^2d^3e^2+3\,a^2cde^4\right)x}{6\,e^3}-\frac{\left(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6\right)\log\left(ex\right.}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^4,x, algorithm="maxim
a")
```

```
[Out] 1/6*(2*c^3*d^3*e^2*x^3 - 3*(c^3*d^4*e - 3*a*c^2*d^2*e^3)*x^2 + 6*(c^3*d^5 -
 3*a*c^2*d^3*e^2 + 3*a^2*c*d*e^4)*x)/e^3 - (c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a
^2*c*d^2*e^4 - a^3*e^6)*log(e*x + d)/e^4
```

Fricas [A] time = 1.67437, size = 262, normalized size = 2.94

$$\frac{2\,c^3d^3e^3x^3-3\left(c^3d^4e^2-3\,ac^2d^2e^4\right)x^2+6\left(c^3d^5e-3\,ac^2d^3e^3+3\,a^2cde^5\right)x-6\left(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6\right)\log}{6\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^4,x, algorithm="frica
s")
```

```
[Out] 1/6*(2*c^3*d^3*e^3*x^3 - 3*(c^3*d^4*e^2 - 3*a*c^2*d^2*e^4)*x^2 + 6*(c^3*d^5
*e - 3*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*x - 6*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*
a^2*c*d^2*e^4 - a^3*e^6)*log(e*x + d))/e^4
```

Sympy [A] time = 0.651007, size = 100, normalized size = 1.12

$$\frac{c^3d^3x^3}{3e} + \frac{x^2\left(3ac^2d^2e^2 - c^3d^4\right)}{2e^2} + \frac{x\left(3a^2cde^4 - 3ac^2d^3e^2 + c^3d^5\right)}{e^3} + \frac{\left(ae^2 - cd^2\right)^3\log\left(d + ex\right)}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**4,x)
```

```
[Out] c**3*d**3*x**3/(3*e) + x**2*(3*a*c**2*d**2*e**2 - c**3*d**4)/(2*e**2) + x*(
3*a**2*c*d*e**4 - 3*a*c**2*d**3*e**2 + c**3*d**5)/e**3 + (a*e**2 - c*d**2)*
*3*log(d + e*x)/e**4
```

Giac [A] time = 1.34463, size = 173, normalized size = 1.94

$$-\left(c^3d^6 - 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 - a^3e^6\right)e^{(-4)}\log\left(|xe + d|\right) + \frac{1}{6}\left(2\,c^3d^3x^3e^{11} - 3\,c^3d^4x^2e^{10} + 6\,c^3d^5xe^9 + 9\,ac^2d^2x^2e^{12} - 18\,ac\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^4,x, algorithm="giac"
)
```

```
[Out] -(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*e^(-4)*log(abs(x*e
 + d)) + 1/6*(2*c^3*d^3*x^3*e^11 - 3*c^3*d^4*x^2*e^10 + 6*c^3*d^5*x*e^9 + 9
*a*c^2*d^2*x^2*e^12 - 18*a*c^2*d^3*x*e^11 + 18*a^2*c*d*x*e^13)*e^(-12)
```

3.1858 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^5}\,dx$

Optimal. Leaf size=94

$$-\frac{c^2d^2x\left(2cd^2-3ae^2\right)}{e^3}+\frac{\left(cd^2-ae^2\right)^3}{e^4(d+ex)}+\frac{3cd\left(cd^2-ae^2\right)^2\log(d+ex)}{e^4}+\frac{c^3d^3x^2}{2e^2}$$

```
[Out] -((c^2*d^2*(2*c*d^2 - 3*a*e^2)*x)/e^3) + (c^3*d^3*x^2)/(2*e^2) + (c*d^2 - a
*e^2)^3/(e^4*(d + e*x)) + (3*c*d*(c*d^2 - a*e^2)^2*Log[d + e*x])/e^4
```

Rubi [A] time = 0.0814272, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$-\frac{c^2d^2x\left(2cd^2-3ae^2\right)}{e^3}+\frac{\left(cd^2-ae^2\right)^3}{e^4(d+ex)}+\frac{3cd\left(cd^2-ae^2\right)^2\log(d+ex)}{e^4}+\frac{c^3d^3x^2}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^5,x]
```

```
[Out] -((c^2*d^2*(2*c*d^2 - 3*a*e^2)*x)/e^3) + (c^3*d^3*x^2)/(2*e^2) + (c*d^2 - a
*e^2)^3/(e^4*(d + e*x)) + (3*c*d*(c*d^2 - a*e^2)^2*Log[d + e*x])/e^4
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^5}\,dx &= \int \frac{(ae+cdx)^3}{(d+ex)^2}\,dx\\ &= \int \left(-\frac{c^2d^2\left(2cd^2-3ae^2\right)}{e^3}+\frac{c^3d^3x}{e^2}+\frac{\left(-cd^2+ae^2\right)^3}{e^3(d+ex)^2}+\frac{3cd\left(cd^2-ae^2\right)^2}{e^3(d+ex)}\right)dx\\ &= -\frac{c^2d^2\left(2cd^2-3ae^2\right)x}{e^3}+\frac{c^3d^3x^2}{2e^2}+\frac{\left(cd^2-ae^2\right)^3}{e^4(d+ex)}+\frac{3cd\left(cd^2-ae^2\right)^2\log(d+ex)}{e^4}\end{aligned}$$

Mathematica [A] time = 0.0412388, size = 129, normalized size = 1.37

$$\frac{6a^2cd^2e^4-2a^3e^6+6ac^2d^2e^2\left(-d^2+dex+e^2x^2\right)+6cd(d+ex)\left(cd^2-ae^2\right)^2\log(d+ex)+c^3d^3\left(-4d^2ex+2d^3-3de^2x^2\right.}{2e^4(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^5,x]
```

```
[Out] (6*a^2*c*d^2*e^4 - 2*a^3*e^6 + 6*a*c^2*d^2*e^2*(-d^2 + d*e*x + e^2*x^2) + c
^3*d^3*(2*d^3 - 4*d^2*e*x - 3*d*e^2*x^2 + e^3*x^3) + 6*c*d*(c*d^2 - a*e^2)^
2*(d + e*x)*Log[d + e*x])/(2*e^4*(d + e*x))
```

Maple [A] time = 0.046, size = 156, normalized size = 1.7

$$\frac{c^3d^3x^2}{2\,e^2}+3\,\frac{ac^2d^2x}{e}-2\,\frac{c^3d^4x}{e^3}+3\,dc\ln\left(ex+d\right)a^2-6\,\frac{c^2d^3\ln\left(ex+d\right)a}{e^2}+3\,\frac{c^3d^5\ln\left(ex+d\right)}{e^4}-\frac{e^2a^3}{ex+d}+3\,\frac{a^2cd^2}{ex+d}-3\,\frac{ac}{e^2\,(e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^5,x)
```

```
[Out] 1/2*c^3*d^3*x^2/e^2+3*c^2*d^2/e*a*x-2*c^3*d^4/e^3*x+3*d*c*ln(e*x+d)*a^2-6*d
^3/e^2*c^2*ln(e*x+d)*a+3*d^5/e^4*c^3*ln(e*x+d)-e^2/(e*x+d)*a^3+3/(e*x+d)*a^
2*c*d^2-3/e^2/(e*x+d)*a*c^2*d^4+1/e^4/(e*x+d)*c^3*d^6
```

Maxima [A] time = 1.06743, size = 184, normalized size = 1.96

$$\frac{c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6}{e^5x+de^4}+\frac{c^3d^3ex^2-2\left(2\,c^3d^4-3\,ac^2d^2e^2\right)x}{2\,e^3}+\frac{3\left(c^3d^5-2\,ac^2d^3e^2+a^2cde^4\right)\log\left(ex+d\right)}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^5,x, algorithm="maxim
a")
```

```
[Out] (c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)/(e^5*x + d*e^4) + 1
/2*(c^3*d^3*e*x^2 - 2*(2*c^3*d^4 - 3*a*c^2*d^2*e^2)*x)/e^3 + 3*(c^3*d^5 - 2
*a*c^2*d^3*e^2 + a^2*c*d*e^4)*log(e*x + d)/e^4
```

Fricas [B] time = 1.62843, size = 386, normalized size = 4.11

$$\frac{c^3d^3e^3x^3+2\,c^3d^6-6\,ac^2d^4e^2+6\,a^2cd^2e^4-2\,a^3e^6-3\left(c^3d^4e^2-2\,ac^2d^2e^4\right)x^2-2\left(2\,c^3d^5e-3\,ac^2d^3e^3\right)x+6\left(c^3d^6-2\,ac^2d\right.}{2\left(e^5x+de^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^5,x, algorithm="frica
s")
```

```
[Out] 1/2*(c^3*d^3*e^3*x^3 + 2*c^3*d^6 - 6*a*c^2*d^4*e^2 + 6*a^2*c*d^2*e^4 - 2*a^
3*e^6 - 3*(c^3*d^4*e^2 - 2*a*c^2*d^2*e^4)*x^2 - 2*(2*c^3*d^5*e - 3*a*c^2*d^
3*e^3)*x + 6*(c^3*d^6 - 2*a*c^2*d^4*e^2 + a^2*c*d^2*e^4 + (c^3*d^5*e - 2*a*
c^2*d^3*e^3 + a^2*c*d*e^5)*x)*log(e*x + d))/(e^5*x + d*e^4)
```

Sympy [A] time = 1.09834, size = 119, normalized size = 1.27

$$\frac{c^3d^3x^2}{2e^2}+\frac{3cd\left(ae^2-cd^2\right)^2\log\left(d+ex\right)}{e^4}-\frac{a^3e^6-3a^2cd^2e^4+3ac^2d^4e^2-c^3d^6}{de^4+e^5x}+\frac{x\left(3ac^2d^2e^2-2c^3d^4\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**5,x)
```

```
[Out] c**3*d**3*x**2/(2*e**2) + 3*c*d*(a*e**2 - c*d**2)**2*log(d + e*x)/e**4 - (a
**3*e**6 - 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 - c**3*d**6)/(d*e**4 + e
**5*x) + x*(3*a*c**2*d**2*e**2 - 2*c**3*d**4)/e**3
```

Giac [A] time = 1.2441, size = 240, normalized size = 2.55

$$\frac{1}{2}\left(c^3d^3-\frac{6\left(c^3d^4e-ac^2d^2e^3\right)e^{(-1)}}{xe+d}\right)(xe+d)^2e^{(-4)}-3\left(c^3d^5-2\,ac^2d^3e^2+a^2cde^4\right)e^{(-4)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^2}\right)+\left(\frac{c^3d^6e^{20}}{xe+d}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^5,x, algorithm="giac"
)
```

```
[Out] 1/2*(c^3*d^3 - 6*(c^3*d^4*e - a*c^2*d^2*e^3)*e^(-1)/(x*e + d))*(x*e + d)^2*
e^(-4) - 3*(c^3*d^5 - 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*e^(-4)*log(abs(x*e + d
)*e^(-1)/(x*e + d)^2) + (c^3*d^6*e^20/(x*e + d) - 3*a*c^2*d^4*e^22/(x*e + d
) + 3*a^2*c*d^2*e^24/(x*e + d) - a^3*e^26/(x*e + d))*e^(-24)
```

3.1859 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^6}\,dx$

Optimal. Leaf size=97

$$-\frac{3c^2d^2\left(cd^2-ae^2\right)\log(d+ex)}{e^4}-\frac{3cd\left(cd^2-ae^2\right)^2}{e^4(d+ex)}+\frac{\left(cd^2-ae^2\right)^3}{2e^4(d+ex)^2}+\frac{c^3d^3x}{e^3}$$

```
[Out] (c^3*d^3*x)/e^3 + (c*d^2 - a*e^2)^3/(2*e^4*(d + e*x)^2) - (3*c*d*(c*d^2 - a
*e^2)^2)/(e^4*(d + e*x)) - (3*c^2*d^2*(c*d^2 - a*e^2)*Log[d + e*x])/e^4
```

Rubi [A] time = 0.0737391, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$-\frac{3c^2d^2\left(cd^2-ae^2\right)\log(d+ex)}{e^4}-\frac{3cd\left(cd^2-ae^2\right)^2}{e^4(d+ex)}+\frac{\left(cd^2-ae^2\right)^3}{2e^4(d+ex)^2}+\frac{c^3d^3x}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^6,x]
```

```
[Out] (c^3*d^3*x)/e^3 + (c*d^2 - a*e^2)^3/(2*e^4*(d + e*x)^2) - (3*c*d*(c*d^2 - a
*e^2)^2)/(e^4*(d + e*x)) - (3*c^2*d^2*(c*d^2 - a*e^2)*Log[d + e*x])/e^4
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^6}\,dx &= \int \frac{(ae+cdx)^3}{(d+ex)^3}\,dx\\
&= \int \left(\frac{c^3d^3}{e^3}+\frac{\left(-cd^2+ae^2\right)^3}{e^3(d+ex)^3}+\frac{3cd\left(cd^2-ae^2\right)^2}{e^3(d+ex)^2}-\frac{3c^2d^2\left(cd^2-ae^2\right)}{e^3(d+ex)}\right)dx\\
&= \frac{c^3d^3x}{e^3}+\frac{\left(cd^2-ae^2\right)^3}{2e^4(d+ex)^2}-\frac{3cd\left(cd^2-ae^2\right)^2}{e^4(d+ex)}-\frac{3c^2d^2\left(cd^2-ae^2\right)\log(d+ex)}{e^4}
\end{aligned}$$

Mathematica [A] time = 0.0454638, size = 129, normalized size = 1.33

$$\frac{-3a^2cde^4(d+2ex)-a^3e^6+3ac^2d^3e^2(3d+4ex)-6c^2d^2(d+ex)^2\left(cd^2-ae^2\right)\log(d+ex)+c^3d^3\left(-4d^2ex-5d^3+4de^2x^2+\right.}{2e^4(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^6,x]
```

```
[Out] (-(a^3*e^6) - 3*a^2*c*d*e^4*(d + 2*e*x) + 3*a*c^2*d^3*e^2*(3*d + 4*e*x) + c
^3*d^3*(-5*d^3 - 4*d^2*e*x + 4*d*e^2*x^2 + 2*e^3*x^3) - 6*c^2*d^2*(c*d^2 -
a*e^2)*(d + e*x)^2*Log[d + e*x])/(2*e^4*(d + e*x)^2)
```

Maple [A] time = 0.048, size = 167, normalized size = 1.7

$$\frac{c^3d^3x}{e^3}-\frac{e^2a^3}{2\,(ex+d)^2}+\frac{3\,a^2cd^2}{2\,(ex+d)^2}-\frac{3\,ac^2d^4}{2\,e^2\,(ex+d)^2}+\frac{c^3d^6}{2\,e^4\,(ex+d)^2}+3\,\frac{c^2d^2\ln(ex+d)\,a}{e^2}-3\,\frac{c^3d^4\ln(ex+d)}{e^4}-3\,\frac{cda}{ex+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^6,x)
```

```
[Out] c^3*d^3*x/e^3-1/2*e^2/(e*x+d)^2*a^3+3/2/(e*x+d)^2*a^2*c*d^2-3/2/e^2/(e*x+d)
^2*a*c^2*d^4+1/2/e^4/(e*x+d)^2*c^3*d^6+3*c^2*d^2/e^2*ln(e*x+d)*a-3*c^3*d^4/
e^4*ln(e*x+d)-3*d*c/(e*x+d)*a^2+6*d^3/e^2*c^2/(e*x+d)*a-3*d^5/e^4*c^3/(e*x+
d)
```

Maxima [A] time = 1.09244, size = 192, normalized size = 1.98

$$\frac{c^3d^3x}{e^3}-\frac{5\,c^3d^6-9\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6+6\left(c^3d^5e-2\,ac^2d^3e^3+a^2cde^5\right)x}{2\left(e^6x^2+2\,de^5x+d^2e^4\right)}-\frac{3\left(c^3d^4-ac^2d^2e^2\right)\log(ex+d)}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^6,x, algorithm="maxim
a")
```

```
[Out] c^3*d^3*x/e^3 - 1/2*(5*c^3*d^6 - 9*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^
6 + 6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x)/(e^6*x^2 + 2*d*e^5*x +
 d^2*e^4) - 3*(c^3*d^4 - a*c^2*d^2*e^2)*log(e*x + d)/e^4
```

Fricas [B] time = 1.5291, size = 408, normalized size = 4.21

$$\frac{2\,c^3d^3e^3x^3+4\,c^3d^4e^2x^2-5\,c^3d^6+9\,ac^2d^4e^2-3\,a^2cd^2e^4-a^3e^6-2\left(2\,c^3d^5e-6\,ac^2d^3e^3+3\,a^2cde^5\right)x-6\left(c^3d^6-ac^2d^4e\right.}{2\left(e^6x^2+2\,de^5x+d^2e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^6,x, algorithm="frica
s")
```

```
[Out] 1/2*(2*c^3*d^3*e^3*x^3 + 4*c^3*d^4*e^2*x^2 - 5*c^3*d^6 + 9*a*c^2*d^4*e^2 -
3*a^2*c*d^2*e^4 - a^3*e^6 - 2*(2*c^3*d^5*e - 6*a*c^2*d^3*e^3 + 3*a^2*c*d*e^
5)*x - 6*(c^3*d^6 - a*c^2*d^4*e^2 + (c^3*d^4*e^2 - a*c^2*d^2*e^4)*x^2 + 2*(
c^3*d^5*e - a*c^2*d^3*e^3)*x)*log(e*x + d))/(e^6*x^2 + 2*d*e^5*x + d^2*e^4)
```

Sympy [A] time = 1.92822, size = 144, normalized size = 1.48

$$\frac{c^3d^3x}{e^3} + \frac{3c^2d^2\left(ae^2 - cd^2\right)\log\left(d + ex\right)}{e^4} - \frac{a^3e^6 + 3a^2cd^2e^4 - 9ac^2d^4e^2 + 5c^3d^6 + x\left(6a^2cde^5 - 12ac^2d^3e^3 + 6c^3d^5e\right)}{2d^2e^4 + 4de^5x + 2e^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**6,x)
```

```
[Out] c**3*d**3*x/e**3 + 3*c**2*d**2*(a*e**2 - c*d**2)*log(d + e*x)/e**4 - (a**3*
e**6 + 3*a**2*c*d**2*e**4 - 9*a*c**2*d**4*e**2 + 5*c**3*d**6 + x*(6*a**2*c*
d*e**5 - 12*a*c**2*d**3*e**3 + 6*c**3*d**5*e))/(2*d**2*e**4 + 4*d*e**5*x +
2*e**6*x**2)
```

Giac [B] time = 1.20684, size = 352, normalized size = 3.63

$$c^3d^3xe^{(-3)} - 3\left(c^3d^4 - ac^2d^2e^2\right)e^{(-4)}\log\left(|xe + d|\right) - \frac{\left(5\,c^3d^9 - 9\,ac^2d^7e^2 + 3\,a^2cd^5e^4 + a^3d^3e^6 + 6\left(c^3d^5e^4 - 2\,ac^2d^3e^6 + a^2cd\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^6,x, algorithm="giac"
)
```

```
[Out] c^3*d^3*x*e^(-3) - 3*(c^3*d^4 - a*c^2*d^2*e^2)*e^(-4)*log(abs(x*e + d)) - 1
/2*(5*c^3*d^9 - 9*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4 + a^3*d^3*e^6 + 6*(c^3*d^
5*e^4 - 2*a*c^2*d^3*e^6 + a^2*c*d*e^8)*x^4 + (23*c^3*d^6*e^3 - 45*a*c^2*d^4
*e^5 + 21*a^2*c*d^2*e^7 + a^3*e^9)*x^3 + 3*(11*c^3*d^7*e^2 - 21*a*c^2*d^5*e
^4 + 9*a^2*c*d^3*e^6 + a^3*d*e^8)*x^2 + 3*(7*c^3*d^8*e - 13*a*c^2*d^6*e^3 +
 5*a^2*c*d^4*e^5 + a^3*d^2*e^7)*x)*e^(-4)/(x*e + d)^5
```

3.1860 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^7} dx$

Optimal. Leaf size=105

$$\frac{3c^2d^2\left(cd^2-ae^2\right)}{e^4(d+ex)}-\frac{3cd\left(cd^2-ae^2\right)^2}{2e^4(d+ex)^2}+\frac{\left(cd^2-ae^2\right)^3}{3e^4(d+ex)^3}+\frac{c^3d^3\log(d+ex)}{e^4}$$

```
[Out] (c*d^2 - a*e^2)^3/(3*e^4*(d + e*x)^3) - (3*c*d*(c*d^2 - a*e^2)^2)/(2*e^4*(d
 + e*x)^2) + (3*c^2*d^2*(c*d^2 - a*e^2))/(e^4*(d + e*x)) + (c^3*d^3*Log[d +
 e*x])/e^4
```

Rubi [A] time = 0.0750259, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$\frac{3c^2d^2\left(cd^2-ae^2\right)}{e^4(d+ex)}-\frac{3cd\left(cd^2-ae^2\right)^2}{2e^4(d+ex)^2}+\frac{\left(cd^2-ae^2\right)^3}{3e^4(d+ex)^3}+\frac{c^3d^3\log(d+ex)}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^7,x]
```

```
[Out] (c*d^2 - a*e^2)^3/(3*e^4*(d + e*x)^3) - (3*c*d*(c*d^2 - a*e^2)^2)/(2*e^4*(d
 + e*x)^2) + (3*c^2*d^2*(c*d^2 - a*e^2))/(e^4*(d + e*x)) + (c^3*d^3*Log[d +
 e*x])/e^4
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^7} dx &= \int \frac{(ae+cdx)^3}{(d+ex)^4} dx\\
&= \int \left(\frac{\left(-cd^2+ae^2\right)^3}{e^3(d+ex)^4}+\frac{3cd\left(cd^2-ae^2\right)^2}{e^3(d+ex)^3}-\frac{3c^2d^2\left(cd^2-ae^2\right)}{e^3(d+ex)^2}+\frac{c^3d^3}{e^3(d+ex)}\right) dx\\
&= \frac{\left(cd^2-ae^2\right)^3}{3e^4(d+ex)^3}-\frac{3cd\left(cd^2-ae^2\right)^2}{2e^4(d+ex)^2}+\frac{3c^2d^2\left(cd^2-ae^2\right)}{e^4(d+ex)}+\frac{c^3d^3\log(d+ex)}{e^4}
\end{aligned}$$

Mathematica [A] time = 0.0474225, size = 92, normalized size = 0.88

$$\frac{\frac{(cd^2-ae^2)(2a^2e^4+acde^2(5d+9ex)+c^2d^2(11d^2+27dex+18e^2x^2))}{(d+ex)^3}+6c^3d^3\log(d+ex)}{6e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^7,x]
```

```
[Out] (((c*d^2 - a*e^2)*(2*a^2*e^4 + a*c*d*e^2*(5*d + 9*e*x) + c^2*d^2*(11*d^2 +
27*d*e*x + 18*e^2*x^2)))/(d + e*x)^3 + 6*c^3*d^3*Log[d + e*x])/(6*e^4)
```

Maple [A] time = 0.044, size = 173, normalized size = 1.7

$$-\frac{3\,cda^2}{2\,(ex+d)^2}+3\,\frac{ac^2d^3}{e^2\,(ex+d)^2}-\frac{3\,c^3d^5}{2\,e^4\,(ex+d)^2}+\frac{c^3d^3\ln(ex+d)}{e^4}-3\,\frac{ac^2d^2}{e^2\,(ex+d)}+3\,\frac{c^3d^4}{e^4\,(ex+d)}-\frac{e^2a^3}{3\,(ex+d)^3}+\frac{a^2cd^2}{(ex+d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^7,x)
```

```
[Out] -3/2*c*d/(e*x+d)^2*a^2+3*c^2*d^3/e^2/(e*x+d)^2*a-3/2*c^3*d^5/e^4/(e*x+d)^2+
c^3*d^3*ln(e*x+d)/e^4-3*c^2*d^2/e^2/(e*x+d)*a+3*c^3*d^4/e^4/(e*x+d)-1/3*e^2
/(e*x+d)^3*a^3+1/(e*x+d)^3*a^2*c*d^2-1/e^2/(e*x+d)^3*a*c^2*d^4+1/3/e^4/(e*x
+d)^3*c^3*d^6
```

Maxima [A] time = 1.07755, size = 213, normalized size = 2.03

$$\frac{c^3d^3\log(ex+d)}{e^4}+\frac{11\,c^3d^6-6\,ac^2d^4e^2-3\,a^2cd^2e^4-2\,a^3e^6+18\left(c^3d^4e^2-ac^2d^2e^4\right)x^2+9\left(3\,c^3d^5e-2\,ac^2d^3e^3-a^2cde^5\right)x}{6\left(e^7x^3+3\,de^6x^2+3\,d^2e^5x+d^3e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^7,x, algorithm="maxim
a")
```

```
[Out] c^3*d^3*log(e*x + d)/e^4 + 1/6*(11*c^3*d^6 - 6*a*c^2*d^4*e^2 - 3*a^2*c*d^2*
e^4 - 2*a^3*e^6 + 18*(c^3*d^4*e^2 - a*c^2*d^2*e^4)*x^2 + 9*(3*c^3*d^5*e - 2
*a*c^2*d^3*e^3 - a^2*c*d*e^5)*x)/(e^7*x^3 + 3*d*e^6*x^2 + 3*d^2*e^5*x + d^3
*e^4)
```

Fricas [A] time = 1.60749, size = 386, normalized size = 3.68

$$\frac{11\,c^3d^6-6\,ac^2d^4e^2-3\,a^2cd^2e^4-2\,a^3e^6+18\left(c^3d^4e^2-ac^2d^2e^4\right)x^2+9\left(3\,c^3d^5e-2\,ac^2d^3e^3-a^2cde^5\right)x+6\left(c^3d^3e^3x^3+3\,c^3\right.}{6\left(e^7x^3+3\,de^6x^2+3\,d^2e^5x+d^3e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^7,x, algorithm="frica
s")
```

```
[Out] 1/6*(11*c^3*d^6 - 6*a*c^2*d^4*e^2 - 3*a^2*c*d^2*e^4 - 2*a^3*e^6 + 18*(c^3*d
^4*e^2 - a*c^2*d^2*e^4)*x^2 + 9*(3*c^3*d^5*e - 2*a*c^2*d^3*e^3 - a^2*c*d*e^
5)*x + 6*(c^3*d^3*e^3*x^3 + 3*c^3*d^4*e^2*x^2 + 3*c^3*d^5*e*x + c^3*d^6)*lo
g(e*x + d))/(e^7*x^3 + 3*d*e^6*x^2 + 3*d^2*e^5*x + d^3*e^4)
```

Sympy [A] time = 3.2611, size = 163, normalized size = 1.55

$$\frac{c^3d^3\log(d+ex)}{e^4} - \frac{2a^3e^6 + 3a^2cd^2e^4 + 6ac^2d^4e^2 - 11c^3d^6 + x^2\left(18ac^2d^2e^4 - 18c^3d^4e^2\right) + x\left(9a^2cde^5 + 18ac^2d^3e^3 - 27c\right.}{6d^3e^4 + 18d^2e^5x + 18de^6x^2 + 6e^7x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**7,x)
```

```
[Out] c**3*d**3*log(d + e*x)/e**4 - (2*a**3*e**6 + 3*a**2*c*d**2*e**4 + 6*a*c**2*
d**4*e**2 - 11*c**3*d**6 + x**2*(18*a*c**2*d**2*e**4 - 18*c**3*d**4*e**2) +
 x*(9*a**2*c*d*e**5 + 18*a*c**2*d**3*e**3 - 27*c**3*d**5*e))/(6*d**3*e**4 +
 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3)
```

Giac [B] time = 1.22466, size = 365, normalized size = 3.48

$$c^3d^3e^{(-4)}\log(|xe+d|) + \frac{\left(11\,c^3d^9 - 6\,ac^2d^7e^2 - 3\,a^2cd^5e^4 - 2\,a^3d^3e^6 + 18\left(c^3d^4e^5 - ac^2d^2e^7\right)x^5 + 9\left(9\,c^3d^5e^4 - 8\,ac^2d^3e^6\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^7,x, algorithm="giac"
)
```

```
[Out] c^3*d^3*e^(-4)*log(abs(x*e + d)) + 1/6*(11*c^3*d^9 - 6*a*c^2*d^7*e^2 - 3*a^
2*c*d^5*e^4 - 2*a^3*d^3*e^6 + 18*(c^3*d^4*e^5 - a*c^2*d^2*e^7)*x^5 + 9*(9*c
^3*d^5*e^4 - 8*a*c^2*d^3*e^6 - a^2*c*d*e^8)*x^4 + 2*(73*c^3*d^6*e^3 - 57*a*
c^2*d^4*e^5 - 15*a^2*c*d^2*e^7 - a^3*e^9)*x^3 + 6*(22*c^3*d^7*e^2 - 15*a*c^
2*d^5*e^4 - 6*a^2*c*d^3*e^6 - a^3*d*e^8)*x^2 + 6*(10*c^3*d^8*e - 6*a*c^2*d^
6*e^3 - 3*a^2*c*d^4*e^5 - a^3*d^2*e^7)*x)*e^(-4)/(x*e + d)^6
```

3.1861 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^8}\,dx$

Optimal. Leaf size=35

$$\frac{(ae+cdx)^4}{4(d+ex)^4\left(cd^2-ae^2\right)}$$

```
[Out] (a*e + c*d*x)^4/(4*(c*d^2 - a*e^2)*(d + e*x)^4)
```

Rubi [A] time = 0.0152227, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 37}

$$\frac{(ae+cdx)^4}{4(d+ex)^4\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^8,x]
```

```
[Out] (a*e + c*d*x)^4/(4*(c*d^2 - a*e^2)*(d + e*x)^4)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^8}\,dx &= \int \frac{(ae+cdx)^3}{(d+ex)^5}\,dx\\ &= \frac{(ae+cdx)^4}{4\left(cd^2-ae^2\right)(d+ex)^4}\end{aligned}$$

Mathematica [B] time = 0.0397116, size = 100, normalized size = 2.86

$$-\frac{a^2cde^4(d+4ex)+a^3e^6+ac^2d^2e^2\left(d^2+4dex+6e^2x^2\right)+c^3d^3\left(4d^2ex+d^3+6de^2x^2+4e^3x^3\right)}{4e^4(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^8,x]
```

```
[Out] -(a^3*e^6 + a^2*c*d*e^4*(d + 4*e*x) + a*c^2*d^2*e^2*(d^2 + 4*d*e*x + 6*e^2*
x^2) + c^3*d^3*(d^3 + 4*d^2*e*x + 6*d*e^2*x^2 + 4*e^3*x^3))/(4*e^4*(d + e*x
)^4)
```

Maple [B] time = 0.045, size = 141, normalized size = 4.

$$-\frac{3 c^2 d^2 \left(a e^2-c d^2\right)}{2 e^4 (e x+d)^2}-\frac{a^3 e^6-3 a^2 c d^2 e^4+3 a c^2 d^4 e^2-c^3 d^6}{4 e^4 (e x+d)^4}-\frac{c^3 d^3}{e^4 (e x+d)}-\frac{c d \left(a^2 e^4-2 a c d^2 e^2+c^2 d^4\right)}{e^4 (e x+d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^8,x)
```

```
[Out] -3/2*c^2*d^2*(a*e^2-c*d^2)/e^4/(e*x+d)^2-1/4*(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c
^2*d^4*e^2-c^3*d^6)/e^4/(e*x+d)^4-d^3/e^4*c^3/(e*x+d)-c*d*(a^2*e^4-2*a*c*d^
2*e^2+c^2*d^4)/e^4/(e*x+d)^3
```

Maxima [B] time = 1.13931, size = 213, normalized size = 6.09

$$-\frac{4 c^3 d^3 e^3 x^3+c^3 d^6+a c^2 d^4 e^2+a^2 c d^2 e^4+a^3 e^6+6 \left(c^3 d^4 e^2+a c^2 d^2 e^4\right) x^2+4 \left(c^3 d^5 e+a c^2 d^3 e^3+a^2 c d e^5\right) x}{4 \left(e^8 x^4+4 d e^7 x^3+6 d^2 e^6 x^2+4 d^3 e^5 x+d^4 e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^8,x, algorithm="maxim
a")
```

```
[Out] -1/4*(4*c^3*d^3*e^3*x^3 + c^3*d^6 + a*c^2*d^4*e^2 + a^2*c*d^2*e^4 + a^3*e^6
 + 6*(c^3*d^4*e^2 + a*c^2*d^2*e^4)*x^2 + 4*(c^3*d^5*e + a*c^2*d^3*e^3 + a^2
*c*d*e^5)*x)/(e^8*x^4 + 4*d*e^7*x^3 + 6*d^2*e^6*x^2 + 4*d^3*e^5*x + d^4*e^4
)
```

Fricas [B] time = 1.52173, size = 308, normalized size = 8.8

$$-\frac{4 c^3 d^3 e^3 x^3+c^3 d^6+a c^2 d^4 e^2+a^2 c d^2 e^4+a^3 e^6+6 \left(c^3 d^4 e^2+a c^2 d^2 e^4\right) x^2+4 \left(c^3 d^5 e+a c^2 d^3 e^3+a^2 c d e^5\right) x}{4 \left(e^8 x^4+4 d e^7 x^3+6 d^2 e^6 x^2+4 d^3 e^5 x+d^4 e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^8,x, algorithm="frica
s")
```

```
[Out] -1/4*(4*c^3*d^3*e^3*x^3 + c^3*d^6 + a*c^2*d^4*e^2 + a^2*c*d^2*e^4 + a^3*e^6
 + 6*(c^3*d^4*e^2 + a*c^2*d^2*e^4)*x^2 + 4*(c^3*d^5*e + a*c^2*d^3*e^3 + a^2
*c*d*e^5)*x)/(e^8*x^4 + 4*d*e^7*x^3 + 6*d^2*e^6*x^2 + 4*d^3*e^5*x + d^4*e^4
)
```

Sympy [B] time = 4.7249, size = 168, normalized size = 4.8

$$-\frac{a^3e^6 + a^2cd^2e^4 + ac^2d^4e^2 + c^3d^6 + 4c^3d^3e^3x^3 + x^2\left(6ac^2d^2e^4 + 6c^3d^4e^2\right) + x\left(4a^2cde^5 + 4ac^2d^3e^3 + 4c^3d^5e\right)}{4d^4e^4 + 16d^3e^5x + 24d^2e^6x^2 + 16de^7x^3 + 4e^8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**8,x)
```

```
[Out] -(a**3*e**6 + a**2*c*d**2*e**4 + a*c**2*d**4*e**2 + c**3*d**6 + 4*c**3*d**3
*e**3*x**3 + x**2*(6*a*c**2*d**2*e**4 + 6*c**3*d**4*e**2) + x*(4*a**2*c*d*e
**5 + 4*a*c**2*d**3*e**3 + 4*c**3*d**5*e))/(4*d**4*e**4 + 16*d**3*e**5*x +
24*d**2*e**6*x**2 + 16*d*e**7*x**3 + 4*e**8*x**4)
```

Giac [B] time = 1.18985, size = 373, normalized size = 10.66

$$-\frac{\left(4\,c^3d^3x^6e^6 + 18\,c^3d^4x^5e^5 + 34\,c^3d^5x^4e^4 + 35\,c^3d^6x^3e^3 + 21\,c^3d^7x^2e^2 + 7\,c^3d^8xe + c^3d^9 + 6\,ac^2d^2x^5e^7 + 22\,ac^2d^3x^4e^6 + 3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^8,x, algorithm="giac"
)
```

```
[Out] -1/4*(4*c^3*d^3*x^6*e^6 + 18*c^3*d^4*x^5*e^5 + 34*c^3*d^5*x^4*e^4 + 35*c^3*
d^6*x^3*e^3 + 21*c^3*d^7*x^2*e^2 + 7*c^3*d^8*x*e + c^3*d^9 + 6*a*c^2*d^2*x^
5*e^7 + 22*a*c^2*d^3*x^4*e^6 + 31*a*c^2*d^4*x^3*e^5 + 21*a*c^2*d^5*x^2*e^4
+ 7*a*c^2*d^6*x*e^3 + a*c^2*d^7*e^2 + 4*a^2*c*d*x^4*e^8 + 13*a^2*c*d^2*x^3*
e^7 + 15*a^2*c*d^3*x^2*e^6 + 7*a^2*c*d^4*x*e^5 + a^2*c*d^5*e^4 + a^3*x^3*e^
9 + 3*a^3*d*x^2*e^8 + 3*a^3*d^2*x*e^7 + a^3*d^3*e^6)*e^(-4)/(x*e + d)^7
```

3.1862 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^9}\,dx$

Optimal. Leaf size=73

$$\frac{cd(ae+cdx)^4}{20(d+ex)^4\left(cd^2-ae^2\right)^2}+\frac{(ae+cdx)^4}{5(d+ex)^5\left(cd^2-ae^2\right)}$$

```
[Out] (a*e + c*d*x)^4/(5*(c*d^2 - a*e^2)*(d + e*x)^5) + (c*d*(a*e + c*d*x)^4)/(20
*(c*d^2 - a*e^2)^2*(d + e*x)^4)
```

Rubi [A] time = 0.0262825, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {626, 45, 37}

$$\frac{cd(ae+cdx)^4}{20(d+ex)^4\left(cd^2-ae^2\right)^2}+\frac{(ae+cdx)^4}{5(d+ex)^5\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^9,x]
```

```
[Out] (a*e + c*d*x)^4/(5*(c*d^2 - a*e^2)*(d + e*x)^5) + (c*d*(a*e + c*d*x)^4)/(20
*(c*d^2 - a*e^2)^2*(d + e*x)^4)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^9}\,dx=\int \frac{(ae+cdx)^3}{(d+ex)^6}\,dx$$
$$=\frac{(ae+cdx)^4}{5\left(cd^2-ae^2\right)(d+ex)^5}+\frac{(cd)\int \frac{(ae+cdx)^3}{(d+ex)^5}\,dx}{5\left(cd^2-ae^2\right)}$$
$$=\frac{(ae+cdx)^4}{5\left(cd^2-ae^2\right)(d+ex)^5}+\frac{cd(ae+cdx)^4}{20\left(cd^2-ae^2\right)^2(d+ex)^4}$$

Mathematica [A] time = 0.0406616, size = 103, normalized size = 1.41

$$-\frac{3a^2cde^4(d+5ex)+4a^3e^6+2ac^2d^2e^2\left(d^2+5dex+10e^2x^2\right)+c^3d^3\left(5d^2ex+d^3+10de^2x^2+10e^3x^3\right)}{20e^4(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^9,x]
```

```
[Out] -(4*a^3*e^6 + 3*a^2*c*d*e^4*(d + 5*e*x) + 2*a*c^2*d^2*e^2*(d^2 + 5*d*e*x +
10*e^2*x^2) + c^3*d^3*(d^3 + 5*d^2*e*x + 10*d*e^2*x^2 + 10*e^3*x^3))/(20*e^
4*(d + e*x)^5)
```

Maple [B] time = 0.046, size = 141, normalized size = 1.9

$$-\frac{c^3d^3}{2\,e^4\,(ex+d)^2}-\frac{3\,cd\left(a^2e^4-2\,acd^2e^2+c^2d^4\right)}{4\,e^4\,(ex+d)^4}-\frac{c^2d^2\left(ae^2-cd^2\right)}{e^4\,(ex+d)^3}-\frac{a^3e^6-3\,a^2cd^2e^4+3\,ac^2d^4e^2-c^3d^6}{5\,e^4\,(ex+d)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^9,x)
```

```
[Out] -1/2*d^3/e^4*c^3/(e*x+d)^2-3/4*c*d*(a^2*e^4-2*a*c*d^2*e^2+c^2*d^4)/e^4/(e*x
+d)^4-c^2*d^2*(a*e^2-c*d^2)/e^4/(e*x+d)^3-1/5*(a^3*e^6-3*a^2*c*d^2*e^4+3*a*
c^2*d^4*e^2-c^3*d^6)/e^4/(e*x+d)^5
```

Maxima [B] time = 1.12977, size = 236, normalized size = 3.23

$$-\frac{10\,c^3d^3e^3x^3+c^3d^6+2\,ac^2d^4e^2+3\,a^2cd^2e^4+4\,a^3e^6+10\left(c^3d^4e^2+2\,ac^2d^2e^4\right)x^2+5\left(c^3d^5e+2\,ac^2d^3e^3+3\,a^2cde^5\right)x}{20\left(e^9x^5+5\,de^8x^4+10\,d^2e^7x^3+10\,d^3e^6x^2+5\,d^4e^5x+d^5e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^9,x, algorithm="maxim
a")
```

```
[Out] -1/20*(10*c^3*d^3*e^3*x^3 + c^3*d^6 + 2*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 4
*a^3*e^6 + 10*(c^3*d^4*e^2 + 2*a*c^2*d^2*e^4)*x^2 + 5*(c^3*d^5*e + 2*a*c^2*
d^3*e^3 + 3*a^2*c*d*e^5)*x)/(e^9*x^5 + 5*d*e^8*x^4 + 10*d^2*e^7*x^3 + 10*d^
3*e^6*x^2 + 5*d^4*e^5*x + d^5*e^4)
```

Fricas [B] time = 1.50647, size = 352, normalized size = 4.82

$$-\frac{10\,c^{3}d^{3}e^{3}x^{3}+c^{3}d^{6}+2\,ac^{2}d^{4}e^{2}+3\,a^{2}cd^{2}e^{4}+4\,a^{3}e^{6}+10\left(c^{3}d^{4}e^{2}+2\,ac^{2}d^{2}e^{4}\right)x^{2}+5\left(c^{3}d^{5}e+2\,ac^{2}d^{3}e^{3}+3\,a^{2}cde^{5}\right)x}{20\left(e^{9}x^{5}+5\,de^{8}x^{4}+10\,d^{2}e^{7}x^{3}+10\,d^{3}e^{6}x^{2}+5\,d^{4}e^{5}x+d^{5}e^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^9,x, algorithm="frica
s")
```

```
[Out] -1/20*(10*c^3*d^3*e^3*x^3 + c^3*d^6 + 2*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 4
*a^3*e^6 + 10*(c^3*d^4*e^2 + 2*a*c^2*d^2*e^4)*x^2 + 5*(c^3*d^5*e + 2*a*c^2*
d^3*e^3 + 3*a^2*c*d*e^5)*x)/(e^9*x^5 + 5*d*e^8*x^4 + 10*d^2*e^7*x^3 + 10*d^
3*e^6*x^2 + 5*d^4*e^5*x + d^5*e^4)
```

Sympy [B] time = 18.4928, size = 185, normalized size = 2.53

$$-\frac{4a^{3}e^{6}+3a^{2}cd^{2}e^{4}+2ac^{2}d^{4}e^{2}+c^{3}d^{6}+10c^{3}d^{3}e^{3}x^{3}+x^{2}\left(20ac^{2}d^{2}e^{4}+10c^{3}d^{4}e^{2}\right)+x\left(15a^{2}cde^{5}+10ac^{2}d^{3}e^{3}+5c^{3}d^{5}e\right)}{20d^{5}e^{4}+100d^{4}e^{5}x+200d^{3}e^{6}x^{2}+200d^{2}e^{7}x^{3}+100de^{8}x^{4}+20e^{9}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**9,x)
```

```
[Out] -(4*a**3*e**6 + 3*a**2*c*d**2*e**4 + 2*a*c**2*d**4*e**2 + c**3*d**6 + 10*c*
*3*d**3*e**3*x**3 + x**2*(20*a*c**2*d**2*e**4 + 10*c**3*d**4*e**2) + x*(15*
a**2*c*d*e**5 + 10*a*c**2*d**3*e**3 + 5*c**3*d**5*e))/(20*d**5*e**4 + 100*d
**4*e**5*x + 200*d**3*e**6*x**2 + 200*d**2*e**7*x**3 + 100*d*e**8*x**4 + 20
*e**9*x**5)
```

Giac [B] time = 1.21506, size = 378, normalized size = 5.18

$$-\frac{\left(10\,c^{3}d^{3}x^{6}e^{6}+40\,c^{3}d^{4}x^{5}e^{5}+65\,c^{3}d^{5}x^{4}e^{4}+56\,c^{3}d^{6}x^{3}e^{3}+28\,c^{3}d^{7}x^{2}e^{2}+8\,c^{3}d^{8}xe+c^{3}d^{9}+20\,ac^{2}d^{2}x^{5}e^{7}+70\,ac^{2}d^{3}x^{4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^9,x, algorithm="giac"
)
```

```
[Out] -1/20*(10*c^3*d^3*x^6*e^6 + 40*c^3*d^4*x^5*e^5 + 65*c^3*d^5*x^4*e^4 + 56*c^
3*d^6*x^3*e^3 + 28*c^3*d^7*x^2*e^2 + 8*c^3*d^8*x*e + c^3*d^9 + 20*a*c^2*d^2
*x^5*e^7 + 70*a*c^2*d^3*x^4*e^6 + 92*a*c^2*d^4*x^3*e^5 + 56*a*c^2*d^5*x^2*e
^4 + 16*a*c^2*d^6*x*e^3 + 2*a*c^2*d^7*e^2 + 15*a^2*c*d*x^4*e^8 + 48*a^2*c*d
^2*x^3*e^7 + 54*a^2*c*d^3*x^2*e^6 + 24*a^2*c*d^4*x*e^5 + 3*a^2*c*d^5*e^4 +
4*a^3*x^3*e^9 + 12*a^3*d*x^2*e^8 + 12*a^3*d^2*x*e^7 + 4*a^3*d^3*e^6)*e^(-4)
/(x*e + d)^8
```

3.1863 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{10}}\,dx$

Optimal. Leaf size=111

$$\frac{3c^2d^2\left(cd^2-ae^2\right)}{4e^4(d+ex)^4}-\frac{3cd\left(cd^2-ae^2\right)^2}{5e^4(d+ex)^5}+\frac{\left(cd^2-ae^2\right)^3}{6e^4(d+ex)^6}-\frac{c^3d^3}{3e^4(d+ex)^3}$$

```
[Out] (c*d^2 - a*e^2)^3/(6*e^4*(d + e*x)^6) - (3*c*d*(c*d^2 - a*e^2)^2)/(5*e^4*(d
 + e*x)^5) + (3*c^2*d^2*(c*d^2 - a*e^2))/(4*e^4*(d + e*x)^4) - (c^3*d^3)/(3
*e^4*(d + e*x)^3)
```

Rubi [A] time = 0.071321, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$\frac{3c^2d^2\left(cd^2-ae^2\right)}{4e^4(d+ex)^4}-\frac{3cd\left(cd^2-ae^2\right)^2}{5e^4(d+ex)^5}+\frac{\left(cd^2-ae^2\right)^3}{6e^4(d+ex)^6}-\frac{c^3d^3}{3e^4(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^10,x]
```

```
[Out] (c*d^2 - a*e^2)^3/(6*e^4*(d + e*x)^6) - (3*c*d*(c*d^2 - a*e^2)^2)/(5*e^4*(d
 + e*x)^5) + (3*c^2*d^2*(c*d^2 - a*e^2))/(4*e^4*(d + e*x)^4) - (c^3*d^3)/(3
*e^4*(d + e*x)^3)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{10}}\,dx &= \int \frac{(ae+cdx)^3}{(d+ex)^7}\,dx\\
&= \int \left(\frac{\left(-cd^2+ae^2\right)^3}{e^3(d+ex)^7}+\frac{3cd\left(cd^2-ae^2\right)^2}{e^3(d+ex)^6}-\frac{3c^2d^2\left(cd^2-ae^2\right)}{e^3(d+ex)^5}+\frac{c^3d^3}{e^3(d+ex)^4}\right)dx\\
&= \frac{\left(cd^2-ae^2\right)^3}{6e^4(d+ex)^6}-\frac{3cd\left(cd^2-ae^2\right)^2}{5e^4(d+ex)^5}+\frac{3c^2d^2\left(cd^2-ae^2\right)}{4e^4(d+ex)^4}-\frac{c^3d^3}{3e^4(d+ex)^3}
\end{aligned}$$

Mathematica [A] time = 0.0349013, size = 103, normalized size = 0.93

$$-\frac{6a^2cde^4(d+6ex)+10a^3e^6+3ac^2d^2e^2\left(d^2+6dex+15e^2x^2\right)+c^3d^3\left(6d^2ex+d^3+15de^2x^2+20e^3x^3\right)}{60e^4(d+ex)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^10,x]
```

```
[Out] -(10*a^3*e^6 + 6*a^2*c*d*e^4*(d + 6*e*x) + 3*a*c^2*d^2*e^2*(d^2 + 6*d*e*x +
 15*e^2*x^2) + c^3*d^3*(d^3 + 6*d^2*e*x + 15*d*e^2*x^2 + 20*e^3*x^3))/(60*e
^4*(d + e*x)^6)
```

Maple [A] time = 0.045, size = 141, normalized size = 1.3

$$-\frac{3c^2d^2\left(ae^2-cd^2\right)}{4e^4\left(ex+d\right)^4}-\frac{a^3e^6-3a^2cd^2e^4+3ac^2d^4e^2-c^3d^6}{6e^4\left(ex+d\right)^6}-\frac{c^3d^3}{3e^4\left(ex+d\right)^3}-\frac{3cd\left(a^2e^4-2acd^2e^2+c^2d^4\right)}{5e^4\left(ex+d\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^10,x)
```

```
[Out] -3/4*c^2*d^2*(a*e^2-c*d^2)/e^4/(e*x+d)^4-1/6*(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c
^2*d^4*e^2-c^3*d^6)/e^4/(e*x+d)^6-1/3*c^3*d^3/e^4/(e*x+d)^3-3/5*c*d*(a^2*e^
4-2*a*c*d^2*e^2+c^2*d^4)/e^4/(e*x+d)^5
```

Maxima [A] time = 1.15409, size = 251, normalized size = 2.26

$$-\frac{20c^3d^3e^3x^3+c^3d^6+3ac^2d^4e^2+6a^2cd^2e^4+10a^3e^6+15\left(c^3d^4e^2+3ac^2d^2e^4\right)x^2+6\left(c^3d^5e+3ac^2d^3e^3+6a^2cde^5\right)x}{60\left(e^{10}x^6+6de^9x^5+15d^2e^8x^4+20d^3e^7x^3+15d^4e^6x^2+6d^5e^5x+d^6e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^10,x, algorithm="maxi
ma")
```

```
[Out] -1/60*(20*c^3*d^3*e^3*x^3 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 6*a^2*c*d^2*e^4 + 1
0*a^3*e^6 + 15*(c^3*d^4*e^2 + 3*a*c^2*d^2*e^4)*x^2 + 6*(c^3*d^5*e + 3*a*c^2
*d^3*e^3 + 6*a^2*c*d*e^5)*x)/(e^10*x^6 + 6*d*e^9*x^5 + 15*d^2*e^8*x^4 + 20*
d^3*e^7*x^3 + 15*d^4*e^6*x^2 + 6*d^5*e^5*x + d^6*e^4)
```

Fricas [A] time = 1.53929, size = 378, normalized size = 3.41

$$-\frac{20c^3d^3e^3x^3+c^3d^6+3ac^2d^4e^2+6a^2cd^2e^4+10a^3e^6+15\left(c^3d^4e^2+3ac^2d^2e^4\right)x^2+6\left(c^3d^5e+3ac^2d^3e^3+6a^2cde^5\right)x}{60\left(e^{10}x^6+6de^9x^5+15d^2e^8x^4+20d^3e^7x^3+15d^4e^6x^2+6d^5e^5x+d^6e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^10,x, algorithm="fric
as")
```

```
[Out] -1/60*(20*c^3*d^3*e^3*x^3 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 6*a^2*c*d^2*e^4 + 1
0*a^3*e^6 + 15*(c^3*d^4*e^2 + 3*a*c^2*d^2*e^4)*x^2 + 6*(c^3*d^5*e + 3*a*c^2
*d^3*e^3 + 6*a^2*c*d*e^5)*x)/(e^10*x^6 + 6*d*e^9*x^5 + 15*d^2*e^8*x^4 + 20*
d^3*e^7*x^3 + 15*d^4*e^6*x^2 + 6*d^5*e^5*x + d^6*e^4)
```

Sympy [A] time = 24.9106, size = 197, normalized size = 1.77

$$-\frac{10a^3e^6 + 6a^2cd^2e^4 + 3ac^2d^4e^2 + c^3d^6 + 20c^3d^3e^3x^3 + x^2\left(45ac^2d^2e^4 + 15c^3d^4e^2\right) + x\left(36a^2cde^5 + 18ac^2d^3e^3 + 6c^3d^5e\right)}{60d^6e^4 + 360d^5e^5x + 900d^4e^6x^2 + 1200d^3e^7x^3 + 900d^2e^8x^4 + 360de^9x^5 + 60e^{10}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**10,x)
```

```
[Out] -(10*a**3*e**6 + 6*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6 + 20*c
**3*d**3*e**3*x**3 + x**2*(45*a*c**2*d**2*e**4 + 15*c**3*d**4*e**2) + x*(36
*a**2*c*d*e**5 + 18*a*c**2*d**3*e**3 + 6*c**3*d**5*e))/(60*d**6*e**4 + 360*
d**5*e**5*x + 900*d**4*e**6*x**2 + 1200*d**3*e**7*x**3 + 900*d**2*e**8*x**4
 + 360*d*e**9*x**5 + 60*e**10*x**6)
```

Giac [B] time = 1.20137, size = 378, normalized size = 3.41

$$-\frac{\left(20\,c^3d^3x^6e^6 + 75\,c^3d^4x^5e^5 + 111\,c^3d^5x^4e^4 + 84\,c^3d^6x^3e^3 + 36\,c^3d^7x^2e^2 + 9\,c^3d^8xe + c^3d^9 + 45\,ac^2d^2x^5e^7 + 153\,ac^2d^3x^4e^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^10,x, algorithm="giac
")
```

```
[Out] -1/60*(20*c^3*d^3*x^6*e^6 + 75*c^3*d^4*x^5*e^5 + 111*c^3*d^5*x^4*e^4 + 84*c
^3*d^6*x^3*e^3 + 36*c^3*d^7*x^2*e^2 + 9*c^3*d^8*x*e + c^3*d^9 + 45*a*c^2*d^
2*x^5*e^7 + 153*a*c^2*d^3*x^4*e^6 + 192*a*c^2*d^4*x^3*e^5 + 108*a*c^2*d^5*x
^2*e^4 + 27*a*c^2*d^6*x*e^3 + 3*a*c^2*d^7*e^2 + 36*a^2*c*d*x^4*e^8 + 114*a^
2*c*d^2*x^3*e^7 + 126*a^2*c*d^3*x^2*e^6 + 54*a^2*c*d^4*x*e^5 + 6*a^2*c*d^5*
e^4 + 10*a^3*x^3*e^9 + 30*a^3*d*x^2*e^8 + 30*a^3*d^2*x*e^7 + 10*a^3*d^3*e^6
)*e^(-4)/(x*e + d)^9
```

3.1864 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{11}} dx$

Optimal. Leaf size=111

$$\frac{3c^2d^2\left(cd^2-ae^2\right)}{5e^4(d+ex)^5}-\frac{cd\left(cd^2-ae^2\right)^2}{2e^4(d+ex)^6}+\frac{\left(cd^2-ae^2\right)^3}{7e^4(d+ex)^7}-\frac{c^3d^3}{4e^4(d+ex)^4}$$

```
[Out] (c*d^2 - a*e^2)^3/(7*e^4*(d + e*x)^7) - (c*d*(c*d^2 - a*e^2)^2)/(2*e^4*(d +
 e*x)^6) + (3*c^2*d^2*(c*d^2 - a*e^2))/(5*e^4*(d + e*x)^5) - (c^3*d^3)/(4*e
^4*(d + e*x)^4)
```

Rubi [A] time = 0.0682159, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$\frac{3c^2d^2\left(cd^2-ae^2\right)}{5e^4(d+ex)^5}-\frac{cd\left(cd^2-ae^2\right)^2}{2e^4(d+ex)^6}+\frac{\left(cd^2-ae^2\right)^3}{7e^4(d+ex)^7}-\frac{c^3d^3}{4e^4(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^11,x]
```


```
[Out] (c*d^2 - a*e^2)^3/(7*e^4*(d + e*x)^7) - (c*d*(c*d^2 - a*e^2)^2)/(2*e^4*(d +
 e*x)^6) + (3*c^2*d^2*(c*d^2 - a*e^2))/(5*e^4*(d + e*x)^5) - (c^3*d^3)/(4*e
^4*(d + e*x)^4)
```


Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{11}}\,dx &= \int \frac{(ae+cdx)^3}{(d+ex)^8}\,dx\\
&= \int \left(\frac{\left(-cd^2+ae^2\right)^3}{e^3(d+ex)^8}+\frac{3cd\left(cd^2-ae^2\right)^2}{e^3(d+ex)^7}-\frac{3c^2d^2\left(cd^2-ae^2\right)}{e^3(d+ex)^6}+\frac{c^3d^3}{e^3(d+ex)^5}\right)dx\\
&= \frac{\left(cd^2-ae^2\right)^3}{7e^4(d+ex)^7}-\frac{cd\left(cd^2-ae^2\right)^2}{2e^4(d+ex)^6}+\frac{3c^2d^2\left(cd^2-ae^2\right)}{5e^4(d+ex)^5}-\frac{c^3d^3}{4e^4(d+ex)^4}
\end{aligned}$$

Mathematica [A] time = 0.03442, size = 103, normalized size = 0.93

$$-\frac{10a^2cde^4(d+7ex)+20a^3e^6+4ac^2d^2e^2\left(d^2+7dex+21e^2x^2\right)+c^3d^3\left(7d^2ex+d^3+21de^2x^2+35e^3x^3\right)}{140e^4(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^11,x]
```

```
[Out] -(20*a^3*e^6 + 10*a^2*c*d*e^4*(d + 7*e*x) + 4*a*c^2*d^2*e^2*(d^2 + 7*d*e*x
+ 21*e^2*x^2) + c^3*d^3*(d^3 + 7*d^2*e*x + 21*d*e^2*x^2 + 35*e^3*x^3))/(140
*e^4*(d + e*x)^7)
```

Maple [A] time = 0.047, size = 141, normalized size = 1.3

$$-\frac{a^3e^6-3\,a^2cd^2e^4+3\,ac^2d^4e^2-c^3d^6}{7\,e^4\,(ex+d)^7}-\frac{c^3d^3}{4\,e^4\,(ex+d)^4}-\frac{cd\left(a^2e^4-2\,acd^2e^2+c^2d^4\right)}{2\,e^4\,(ex+d)^6}-\frac{3\,c^2d^2\left(ae^2-cd^2\right)}{5\,e^4\,(ex+d)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^11,x)
```

```
[Out] -1/7*(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2-c^3*d^6)/e^4/(e*x+d)^7-1/4*c^
3*d^3/e^4/(e*x+d)^4-1/2*c*d*(a^2*e^4-2*a*c*d^2*e^2+c^2*d^4)/e^4/(e*x+d)^6-3
/5*c^2*d^2*(a*e^2-c*d^2)/e^4/(e*x+d)^5
```

Maxima [A] time = 1.06908, size = 266, normalized size = 2.4

$$-\frac{35\,c^3d^3e^3x^3+c^3d^6+4\,ac^2d^4e^2+10\,a^2cd^2e^4+20\,a^3e^6+21\left(c^3d^4e^2+4\,ac^2d^2e^4\right)x^2+7\left(c^3d^5e+4\,ac^2d^3e^3+10\,a^2cde^5\right)x}{140\left(e^{11}x^7+7\,de^{10}x^6+21\,d^2e^9x^5+35\,d^3e^8x^4+35\,d^4e^7x^3+21\,d^5e^6x^2+7\,d^6e^5x+d^7e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^11,x, algorithm="maxi
ma")
```

```
[Out] -1/140*(35*c^3*d^3*e^3*x^3 + c^3*d^6 + 4*a*c^2*d^4*e^2 + 10*a^2*c*d^2*e^4 +
 20*a^3*e^6 + 21*(c^3*d^4*e^2 + 4*a*c^2*d^2*e^4)*x^2 + 7*(c^3*d^5*e + 4*a*c
^2*d^3*e^3 + 10*a^2*c*d*e^5)*x)/(e^11*x^7 + 7*d*e^10*x^6 + 21*d^2*e^9*x^5 +
 35*d^3*e^8*x^4 + 35*d^4*e^7*x^3 + 21*d^5*e^6*x^2 + 7*d^6*e^5*x + d^7*e^4)
```

Fricas [A] time = 1.52223, size = 406, normalized size = 3.66

$$-\frac{35\,c^3d^3e^3x^3+c^3d^6+4\,ac^2d^4e^2+10\,a^2cd^2e^4+20\,a^3e^6+21\left(c^3d^4e^2+4\,ac^2d^2e^4\right)x^2+7\left(c^3d^5e+4\,ac^2d^3e^3+10\,a^2cde^5\right)x}{140\left(e^{11}x^7+7\,de^{10}x^6+21\,d^2e^9x^5+35\,d^3e^8x^4+35\,d^4e^7x^3+21\,d^5e^6x^2+7\,d^6e^5x+d^7e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^11,x, algorithm="fric
as")
```

```
[Out] -1/140*(35*c^3*d^3*e^3*x^3 + c^3*d^6 + 4*a*c^2*d^4*e^2 + 10*a^2*c*d^2*e^4 +
 20*a^3*e^6 + 21*(c^3*d^4*e^2 + 4*a*c^2*d^2*e^4)*x^2 + 7*(c^3*d^5*e + 4*a*c
^2*d^3*e^3 + 10*a^2*c*d*e^5)*x)/(e^11*x^7 + 7*d*e^10*x^6 + 21*d^2*e^9*x^5 +
 35*d^3*e^8*x^4 + 35*d^4*e^7*x^3 + 21*d^5*e^6*x^2 + 7*d^6*e^5*x + d^7*e^4)
```

Sympy [B] time = 100.358, size = 209, normalized size = 1.88

$$-\frac{20a^3e^6 + 10a^2cd^2e^4 + 4ac^2d^4e^2 + c^3d^6 + 35c^3d^3e^3x^3 + x^2\left(84ac^2d^2e^4 + 21c^3d^4e^2\right) + x\left(70a^2cde^5 + 28ac^2d^3e^3 + 7c^3d^5\right.}{140d^7e^4 + 980d^6e^5x + 2940d^5e^6x^2 + 4900d^4e^7x^3 + 4900d^3e^8x^4 + 2940d^2e^9x^5 + 980de^{10}x^6 + 140e^{11}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**11,x)
```

```
[Out] -(20*a**3*e**6 + 10*a**2*c*d**2*e**4 + 4*a*c**2*d**4*e**2 + c**3*d**6 + 35*
c**3*d**3*e**3*x**3 + x**2*(84*a*c**2*d**2*e**4 + 21*c**3*d**4*e**2) + x*(7
0*a**2*c*d*e**5 + 28*a*c**2*d**3*e**3 + 7*c**3*d**5*e))/(140*d**7*e**4 + 98
0*d**6*e**5*x + 2940*d**5*e**6*x**2 + 4900*d**4*e**7*x**3 + 4900*d**3*e**8*
x**4 + 2940*d**2*e**9*x**5 + 980*d*e**10*x**6 + 140*e**11*x**7)
```

Giac [B] time = 1.20712, size = 378, normalized size = 3.41

$$-\frac{\left(35\,c^3d^3x^6e^6 + 126\,c^3d^4x^5e^5 + 175\,c^3d^5x^4e^4 + 120\,c^3d^6x^3e^3 + 45\,c^3d^7x^2e^2 + 10\,c^3d^8xe + c^3d^9 + 84\,ac^2d^2x^5e^7 + 280\,ac\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^11,x, algorithm="giac
")
```

```
[Out] -1/140*(35*c^3*d^3*x^6*e^6 + 126*c^3*d^4*x^5*e^5 + 175*c^3*d^5*x^4*e^4 + 12
0*c^3*d^6*x^3*e^3 + 45*c^3*d^7*x^2*e^2 + 10*c^3*d^8*x*e + c^3*d^9 + 84*a*c^
2*d^2*x^5*e^7 + 280*a*c^2*d^3*x^4*e^6 + 340*a*c^2*d^4*x^3*e^5 + 180*a*c^2*d
^5*x^2*e^4 + 40*a*c^2*d^6*x*e^3 + 4*a*c^2*d^7*e^2 + 70*a^2*c*d*x^4*e^8 + 22
0*a^2*c*d^2*x^3*e^7 + 240*a^2*c*d^3*x^2*e^6 + 100*a^2*c*d^4*x*e^5 + 10*a^2*
c*d^5*e^4 + 20*a^3*x^3*e^9 + 60*a^3*d*x^2*e^8 + 60*a^3*d^2*x*e^7 + 20*a^3*d
^3*e^6)*e^(-4)/(x*e + d)^10
```

3.1865 $\int \frac{(d+ex)^5}{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx$

Optimal. Leaf size=131

$$\frac{ex\left(cd^2-ae^2\right)^3}{c^4d^4}+\frac{(d+ex)^2\left(cd^2-ae^2\right)^2}{2c^3d^3}+\frac{(d+ex)^3\left(cd^2-ae^2\right)}{3c^2d^2}+\frac{\left(cd^2-ae^2\right)^4\log(ae+cdx)}{c^5d^5}+\frac{(d+ex)^4}{4cd}$$

```
[Out] (e*(c*d^2 - a*e^2)^3*x)/(c^4*d^4) + ((c*d^2 - a*e^2)^2*(d + e*x)^2)/(2*c^3*
d^3) + ((c*d^2 - a*e^2)*(d + e*x)^3)/(3*c^2*d^2) + (d + e*x)^4/(4*c*d) + ((
c*d^2 - a*e^2)^4*Log[a*e + c*d*x])/(c^5*d^5)
```

Rubi [A] time = 0.0674149, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$\frac{ex\left(cd^2-ae^2\right)^3}{c^4d^4}+\frac{(d+ex)^2\left(cd^2-ae^2\right)^2}{2c^3d^3}+\frac{(d+ex)^3\left(cd^2-ae^2\right)}{3c^2d^2}+\frac{\left(cd^2-ae^2\right)^4\log(ae+cdx)}{c^5d^5}+\frac{(d+ex)^4}{4cd}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] (e*(c*d^2 - a*e^2)^3*x)/(c^4*d^4) + ((c*d^2 - a*e^2)^2*(d + e*x)^2)/(2*c^3*
d^3) + ((c*d^2 - a*e^2)*(d + e*x)^3)/(3*c^2*d^2) + (d + e*x)^4/(4*c*d) + ((
c*d^2 - a*e^2)^4*Log[a*e + c*d*x])/(c^5*d^5)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(d+ex)^5}{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx=\int \frac{(d+ex)^4}{ae+cdx}\,dx$$

$$=\int \left(\frac{e\left(cd^2-ae^2\right)^3}{c^4d^4}+\frac{\left(cd^2-ae^2\right)^4}{c^4d^4(ae+cdx)}+\frac{e\left(cd^2-ae^2\right)^2(d+ex)}{c^3d^3}+\frac{e\left(cd^2-ae^2\right)(d+ex)}{c^2d^2}\right.$$

$$=\frac{e\left(cd^2-ae^2\right)^3x}{c^4d^4}+\frac{\left(cd^2-ae^2\right)^2(d+ex)^2}{2c^3d^3}+\frac{\left(cd^2-ae^2\right)(d+ex)^3}{3c^2d^2}+\frac{(d+ex)^4}{4cd}+\frac{(cd^2}{}$$

Mathematica [A] time = 0.0529092, size = 134, normalized size = 1.02

$$\frac{cdex\left(6a^2cde^4(8d+ex)-12a^3e^6-4ac^2d^2e^2\left(18d^2+6dex+e^2x^2\right)+c^3d^3\left(36d^2ex+48d^3+16de^2x^2+3e^3x^3\right)\right)+12\left(cd\right.}{12c^5d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] (c*d*e*x*(-12*a^3*e^6 + 6*a^2*c*d*e^4*(8*d + e*x) - 4*a*c^2*d^2*e^2*(18*d^2
 + 6*d*e*x + e^2*x^2) + c^3*d^3*(48*d^3 + 36*d^2*e*x + 16*d*e^2*x^2 + 3*e^3
*x^3)) + 12*(c*d^2 - a*e^2)^4*Log[a*e + c*d*x])/(12*c^5*d^5)
```

Maple [A] time = 0.043, size = 239, normalized size = 1.8

$$\frac{e^4x^4}{4cd}-\frac{e^5x^3a}{3c^2d^2}+\frac{4e^3x^3}{3c}+\frac{e^6x^2a^2}{2c^3d^3}-2\frac{e^4x^2a}{c^2d}+3\frac{de^2x^2}{c}-\frac{e^7a^3x}{c^4d^4}+4\frac{a^2e^5x}{c^3d^2}-6\frac{ae^3x}{c^2}+4\frac{ed^2x}{c}+\frac{\ln(cdx+ae)a^4e^8}{c^5d^5}-4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

```
[Out] 1/4*e^4/c/d*x^4-1/3*e^5/c^2/d^2*x^3*a+4/3*e^3/c*x^3+1/2*e^6/c^3/d^3*x^2*a^2
-2*e^4/c^2/d*x^2*a+3*e^2/c*d*x^2-e^7/c^4/d^4*a^3*x+4*e^5/c^3/d^2*a^2*x-6*e^
3/c^2*a*x+4*e/c*d^2*x+1/c^5/d^5*ln(c*d*x+a*e)*a^4*e^8-4/c^4/d^3*ln(c*d*x+a*
e)*a^3*e^6+6/c^3/d*ln(c*d*x+a*e)*a^2*e^4-4/c^2*d*ln(c*d*x+a*e)*a*e^2+1/c*d^
3*ln(c*d*x+a*e)
```

Maxima [A] time = 1.0299, size = 277, normalized size = 2.11

$$\frac{3c^3d^3e^4x^4+4\left(4c^3d^4e^3-ac^2d^2e^5\right)x^3+6\left(6c^3d^5e^2-4ac^2d^3e^4+a^2cde^6\right)x^2+12\left(4c^3d^6e-6ac^2d^4e^3+4a^2cd^2e^5-a^3e\right.}{12c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="maxima"
)
```

```
[Out] 1/12*(3*c^3*d^3*e^4*x^4 + 4*(4*c^3*d^4*e^3 - a*c^2*d^2*e^5)*x^3 + 6*(6*c^3*
d^5*e^2 - 4*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x^2 + 12*(4*c^3*d^6*e - 6*a*c^2*d^
4*e^3 + 4*a^2*c*d^2*e^5 - a^3*e^7)*x)/(c^4*d^4) + (c^4*d^8 - 4*a*c^3*d^6*e^
2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*log(c*d*x + a*e)/(c^5*d^
5)
```

Fricas [A] time = 1.6241, size = 414, normalized size = 3.16

$$\frac{3c^4d^4e^4x^4+4\left(4c^4d^5e^3-ac^3d^3e^5\right)x^3+6\left(6c^4d^6e^2-4ac^3d^4e^4+a^2c^2d^2e^6\right)x^2+12\left(4c^4d^7e-6ac^3d^5e^3+4a^2c^2d^3e^5-\right.}{12c^5d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fricas"
)
```

```
[Out] 1/12*(3*c^4*d^4*e^4*x^4 + 4*(4*c^4*d^5*e^3 - a*c^3*d^3*e^5)*x^3 + 6*(6*c^4*
d^6*e^2 - 4*a*c^3*d^4*e^4 + a^2*c^2*d^2*e^6)*x^2 + 12*(4*c^4*d^7*e - 6*a*c^
3*d^5*e^3 + 4*a^2*c^2*d^3*e^5 - a^3*c*d*e^7)*x + 12*(c^4*d^8 - 4*a*c^3*d^6*
e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*log(c*d*x + a*e))/(c^5
*d^5)
```

Sympy [A] time = 0.671662, size = 162, normalized size = 1.24

$$\frac{e^4x^4}{4cd} - \frac{x^3\left(ae^5 - 4cd^2e^3\right)}{3c^2d^2} + \frac{x^2\left(a^2e^6 - 4acd^2e^4 + 6c^2d^4e^2\right)}{2c^3d^3} - \frac{x\left(a^3e^7 - 4a^2cd^2e^5 + 6ac^2d^4e^3 - 4c^3d^6e\right)}{c^4d^4} + \frac{\left(ae^2 - cd^2\right)^4 \log (}{c^5d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

```
[Out] e**4*x**4/(4*c*d) - x**3*(a*e**5 - 4*c*d**2*e**3)/(3*c**2*d**2) + x**2*(a**
2*e**6 - 4*a*c*d**2*e**4 + 6*c**2*d**4*e**2)/(2*c**3*d**3) - x*(a**3*e**7 -
 4*a**2*c*d**2*e**5 + 6*a*c**2*d**4*e**3 - 4*c**3*d**6*e)/(c**4*d**4) + (a*
e**2 - c*d**2)**4*log(a*e + c*d*x)/(c**5*d**5)
```

Giac [B] time = 1.27892, size = 486, normalized size = 3.71

$$\frac{\left(3\,c^3d^3x^4e^8 + 16\,c^3d^4x^3e^7 + 36\,c^3d^5x^2e^6 + 48\,c^3d^6xe^5 - 4\,ac^2d^2x^3e^9 - 24\,ac^2d^3x^2e^8 - 72\,ac^2d^4xe^7 + 6\,a^2cdx^2e^{10} + 48\,a^2cd^2}{12\,c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")
```

```
[Out] 1/12*(3*c^3*d^3*x^4*e^8 + 16*c^3*d^4*x^3*e^7 + 36*c^3*d^5*x^2*e^6 + 48*c^3*
d^6*x*e^5 - 4*a*c^2*d^2*x^3*e^9 - 24*a*c^2*d^3*x^2*e^8 - 72*a*c^2*d^4*x*e^7
 + 6*a^2*c*d*x^2*e^10 + 48*a^2*c*d^2*x*e^9 - 12*a^3*x*e^11)*e^(-4)/(c^4*d^4
) + 1/2*(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 +
a^4*e^8)*log(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)/(c^5*d^5) + (c^5*d^10 -
 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^
8 - a^5*e^10)*arctan((2*c*d*x*e + c*d^2 + a*e^2)/sqrt(-c^2*d^4 + 2*a*c*d^2*
e^2 - a^2*e^4))/(sqrt(-c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4)*c^5*d^5)
```

3.1866 $\int \frac{(d+ex)^4}{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal. Leaf size=100

$$\frac{ex\left(cd^2-ae^2\right)^2}{c^3d^3}+\frac{(d+ex)^2\left(cd^2-ae^2\right)}{2c^2d^2}+\frac{\left(cd^2-ae^2\right)^3\log(ae+cdx)}{c^4d^4}+\frac{(d+ex)^3}{3cd}$$

```
[Out] (e*(c*d^2 - a*e^2)^2*x)/(c^3*d^3) + ((c*d^2 - a*e^2)*(d + e*x)^2)/(2*c^2*d^
2) + (d + e*x)^3/(3*c*d) + ((c*d^2 - a*e^2)^3*Log[a*e + c*d*x])/(c^4*d^4)
```

Rubi [A] time = 0.0452864, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$\frac{ex\left(cd^2-ae^2\right)^2}{c^3d^3}+\frac{(d+ex)^2\left(cd^2-ae^2\right)}{2c^2d^2}+\frac{\left(cd^2-ae^2\right)^3\log(ae+cdx)}{c^4d^4}+\frac{(d+ex)^3}{3cd}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] (e*(c*d^2 - a*e^2)^2*x)/(c^3*d^3) + ((c*d^2 - a*e^2)*(d + e*x)^2)/(2*c^2*d^
2) + (d + e*x)^3/(3*c*d) + ((c*d^2 - a*e^2)^3*Log[a*e + c*d*x])/(c^4*d^4)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^4}{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx&=\int \frac{(d+ex)^3}{ae+cdx}\,dx\\&=\int \left(\frac{e\left(cd^2-ae^2\right)^2}{c^3d^3}+\frac{\left(cd^2-ae^2\right)^3}{c^3d^3(ae+cdx)}+\frac{e\left(cd^2-ae^2\right)(d+ex)}{c^2d^2}+\frac{e(d+ex)^2}{cd}\right)dx\\&=\frac{e\left(cd^2-ae^2\right)^2x}{c^3d^3}+\frac{\left(cd^2-ae^2\right)(d+ex)^2}{2c^2d^2}+\frac{(d+ex)^3}{3cd}+\frac{\left(cd^2-ae^2\right)^3\log(ae+cdx)}{c^4d^4}\end{aligned}$$

Mathematica [A] time = 0.0342907, size = 91, normalized size = 0.91

$$\frac{cdex\left(6a^2e^4-3acde^2(6d+ex)+c^2d^2\left(18d^2+9dex+2e^2x^2\right)\right)+6\left(cd^2-ae^2\right)^3\log(ae+cdx)}{6c^4d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] (c*d*e*x*(6*a^2*e^4 - 3*a*c*d*e^2*(6*d + e*x) + c^2*d^2*(18*d^2 + 9*d*e*x +
 2*e^2*x^2)) + 6*(c*d^2 - a*e^2)^3*Log[a*e + c*d*x])/(6*c^4*d^4)
```

Maple [A] time = 0.042, size = 157, normalized size = 1.6

$$\frac{e^3x^3}{3cd}-\frac{e^4x^2a}{2c^2d^2}+\frac{3e^2x^2}{2c}+\frac{a^2e^5x}{c^3d^3}-3\frac{ae^3x}{c^2d}+3\frac{dex}{c}-\frac{\ln(cdx+ae)a^3e^6}{c^4d^4}+3\frac{\ln(cdx+ae)a^2e^4}{c^3d^2}-3\frac{\ln(cdx+ae)ae^2}{c^2}+\frac{d^2\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

```
[Out] 1/3*e^3/c/d*x^3-1/2*e^4/c^2/d^2*x^2*a+3/2*e^2/c*x^2+e^5/c^3/d^3*a^2*x-3*e^3
/c^2/d*a*x+3*e/c*d*x-1/c^4/d^4*ln(c*d*x+a*e)*a^3*e^6+3/c^3/d^2*ln(c*d*x+a*e
)*a^2*e^4-3/c^2*ln(c*d*x+a*e)*a*e^2+1/c*d^2*ln(c*d*x+a*e)
```

Maxima [A] time = 1.04188, size = 182, normalized size = 1.82

$$\frac{2c^2d^2e^3x^3+3\left(3c^2d^3e^2-acde^4\right)x^2+6\left(3c^2d^4e-3acd^2e^3+a^2e^5\right)x}{6c^3d^3}+\frac{\left(c^3d^6-3ac^2d^4e^2+3a^2cd^2e^4-a^3e^6\right)\log(cdx+ae}{c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="maxima"
)
```

```
[Out] 1/6*(2*c^2*d^2*e^3*x^3 + 3*(3*c^2*d^3*e^2 - a*c*d*e^4)*x^2 + 6*(3*c^2*d^4*e
 - 3*a*c*d^2*e^3 + a^2*e^5)*x)/(c^3*d^3) + (c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a
^2*c*d^2*e^4 - a^3*e^6)*log(c*d*x + a*e)/(c^4*d^4)
```

Fricas [A] time = 1.49823, size = 275, normalized size = 2.75

$$\frac{2c^3d^3e^3x^3+3\left(3c^3d^4e^2-ac^2d^2e^4\right)x^2+6\left(3c^3d^5e-3ac^2d^3e^3+a^2cde^5\right)x+6\left(c^3d^6-3ac^2d^4e^2+3a^2cd^2e^4-a^3e^6\right)\log(cd}{6c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fricas"
)
```

```
[Out] 1/6*(2*c^3*d^3*e^3*x^3 + 3*(3*c^3*d^4*e^2 - a*c^2*d^2*e^4)*x^2 + 6*(3*c^3*d
^5*e - 3*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x + 6*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*
a^2*c*d^2*e^4 - a^3*e^6)*log(c*d*x + a*e))/(c^4*d^4)
```

Sympy [A] time = 0.606648, size = 104, normalized size = 1.04

$$\frac{e^3x^3}{3cd} - \frac{x^2\left(ae^4 - 3cd^2e^2\right)}{2c^2d^2} + \frac{x\left(a^2e^5 - 3acd^2e^3 + 3c^2d^4e\right)}{c^3d^3} - \frac{\left(ae^2 - cd^2\right)^3\log\left(ae + cdx\right)}{c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

```
[Out] e**3*x**3/(3*c*d) - x**2*(a*e**4 - 3*c*d**2*e**2)/(2*c**2*d**2) + x*(a**2*e
**5 - 3*a*c*d**2*e**3 + 3*c**2*d**4*e)/(c**3*d**3) - (a*e**2 - c*d**2)**3*l
og(a*e + c*d*x)/(c**4*d**4)
```

Giac [B] time = 1.21262, size = 374, normalized size = 3.74

$$\frac{\left(2\,c^2d^2x^3e^6 + 9\,c^2d^3x^2e^5 + 18\,c^2d^4xe^4 - 3\,acdx^2e^7 - 18\,acd^2xe^6 + 6\,a^2xe^8\right)e^{(-3)}}{6\,c^3d^3} + \frac{\left(c^3d^6 - 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 - a^3e^6\right)}{2\,c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")
```

```
[Out] 1/6*(2*c^2*d^2*x^3*e^6 + 9*c^2*d^3*x^2*e^5 + 18*c^2*d^4*x*e^4 - 3*a*c*d*x^2
*e^7 - 18*a*c*d^2*x*e^6 + 6*a^2*x*e^8)*e^(-3)/(c^3*d^3) + 1/2*(c^3*d^6 - 3*
a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*log(c*d*x^2*e + c*d^2*x + a*x*e^
2 + a*d*e)/(c^4*d^4) + (c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a
^3*c*d^2*e^6 + a^4*e^8)*arctan((2*c*d*x*e + c*d^2 + a*e^2)/sqrt(-c^2*d^4 +
2*a*c*d^2*e^2 - a^2*e^4))/(sqrt(-c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4)*c^4*d^4
)
```

3.1867 $\int \frac{(d+ex)^3}{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal. Leaf size=69

$$\frac{ex\left(cd^2-ae^2\right)}{c^2d^2}+\frac{\left(cd^2-ae^2\right)^2\log(ae+cdx)}{c^3d^3}+\frac{(d+ex)^2}{2cd}$$

```
[Out] (e*(c*d^2 - a*e^2)*x)/(c^2*d^2) + (d + e*x)^2/(2*c*d) + ((c*d^2 - a*e^2)^2*
Log[a*e + c*d*x])/(c^3*d^3)
```

Rubi [A] time = 0.0331473, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$\frac{ex\left(cd^2-ae^2\right)}{c^2d^2}+\frac{\left(cd^2-ae^2\right)^2\log(ae+cdx)}{c^3d^3}+\frac{(d+ex)^2}{2cd}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] (e*(c*d^2 - a*e^2)*x)/(c^2*d^2) + (d + e*x)^2/(2*c*d) + ((c*d^2 - a*e^2)^2*
Log[a*e + c*d*x])/(c^3*d^3)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^3}{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx &= \int \frac{(d+ex)^2}{ae+cdx}\,dx\\ &= \int \left(\frac{e\left(cd^2-ae^2\right)}{c^2d^2}+\frac{\left(cd^2-ae^2\right)^2}{c^2d^2(ae+cdx)}+\frac{e(d+ex)}{cd}\right)dx\\ &= \frac{e\left(cd^2-ae^2\right)x}{c^2d^2}+\frac{(d+ex)^2}{2cd}+\frac{\left(cd^2-ae^2\right)^2\log(ae+cdx)}{c^3d^3}\end{aligned}$$

Mathematica [A] time = 0.020929, size = 58, normalized size = 0.84

$$\frac{2\left(cd^2-ae^2\right)^2\log(ae+cdx)+cdex\left(cd(4d+ex)-2ae^2\right)}{2c^3d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] (c*d*e*x*(-2*a*e^2 + c*d*(4*d + e*x)) + 2*(c*d^2 - a*e^2)^2*Log[a*e + c*d*x
])/(2*c^3*d^3)
```

Maple [A] time = 0.041, size = 93, normalized size = 1.4

$$\frac{e^2x^2}{2\,cd}-\frac{ae^3x}{c^2d^2}+2\,\frac{ex}{c}+\frac{\ln\left(cdx+ae\right)a^2e^4}{c^3d^3}-2\,\frac{\ln\left(cdx+ae\right)ae^2}{c^2d}+\frac{d\ln\left(cdx+ae\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

```
[Out] 1/2*e^2/c/d*x^2-e^3/c^2/d^2*a*x+2*e/c*x+1/c^3/d^3*ln(c*d*x+a*e)*a^2*e^4-2/c
^2/d*ln(c*d*x+a*e)*a*e^2+1/c*d*ln(c*d*x+a*e)
```

Maxima [A] time = 1.05332, size = 104, normalized size = 1.51

$$\frac{cde^2x^2+2\left(2\,cd^2e-ae^3\right)x}{2\,c^2d^2}+\frac{\left(c^2d^4-2\,acd^2e^2+a^2e^4\right)\log\left(cdx+ae\right)}{c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="maxima"
)
```

```
[Out] 1/2*(c*d*e^2*x^2 + 2*(2*c*d^2*e - a*e^3)*x)/(c^2*d^2) + (c^2*d^4 - 2*a*c*d^
2*e^2 + a^2*e^4)*log(c*d*x + a*e)/(c^3*d^3)
```

Fricas [A] time = 1.57952, size = 165, normalized size = 2.39

$$\frac{c^2d^2e^2x^2+2\left(2\,c^2d^3e-acde^3\right)x+2\left(c^2d^4-2\,acd^2e^2+a^2e^4\right)\log\left(cdx+ae\right)}{2\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fricas"
)
```

```
[Out] 1/2*(c^2*d^2*e^2*x^2 + 2*(2*c^2*d^3*e - a*c*d*e^3)*x + 2*(c^2*d^4 - 2*a*c*d
^2*e^2 + a^2*e^4)*log(c*d*x + a*e))/(c^3*d^3)
```

Sympy [A] time = 0.439244, size = 61, normalized size = 0.88

$$\frac{e^2x^2}{2cd}-\frac{x\left(ae^3-2cd^2e\right)}{c^2d^2}+\frac{\left(ae^2-cd^2\right)^2\log\left(ae+cdx\right)}{c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

```
[Out] e**2*x**2/(2*c*d) - x*(a*e**3 - 2*c*d**2*e)/(c**2*d**2) + (a*e**2 - c*d**2)
**2*log(a*e + c*d*x)/(c**3*d**3)
```

Giac [B] time = 1.27223, size = 282, normalized size = 4.09

$$\frac{\left(cdx^2e^4 + 4\,cd^2xe^3 - 2\,axe^5\right)e^{(-2)}}{2\,c^2d^2} + \frac{\left(c^2d^4 - 2\,acd^2e^2 + a^2e^4\right)\log\left(cdx^2e + cd^2x + axe^2 + ade\right)}{2\,c^3d^3} + \frac{\left(c^3d^6 - 3\,ac^2d^4e^2 + 3\,a^2cd\right.}{\sqrt{-c^2d^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(c*d*x^2*e^4 + 4*c*d^2*x*e^3 - 2*a*x*e^5)*e^(-2)/(c^2*d^2) + 1/2*(c^2*d
^4 - 2*a*c*d^2*e^2 + a^2*e^4)*log(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)/(c
^3*d^3) + (c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*arctan((2
*c*d*x*e + c*d^2 + a*e^2)/sqrt(-c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4))/(sqrt(-
c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4)*c^3*d^3)
```

3.1868 $\int \frac{(d+ex)^2}{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx$

Optimal. Leaf size=38

$$\frac{\left(cd^2-ae^2\right)\log(ae+cdx)}{c^2d^2}+\frac{ex}{cd}$$

```
[Out] (e*x)/(c*d) + ((c*d^2 - a*e^2)*Log[a*e + c*d*x])/(c^2*d^2)
```

Rubi [A] time = 0.0323342, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$\frac{\left(cd^2-ae^2\right)\log(ae+cdx)}{c^2d^2}+\frac{ex}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] (e*x)/(c*d) + ((c*d^2 - a*e^2)*Log[a*e + c*d*x])/(c^2*d^2)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2}{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx &= \int \frac{d+ex}{ae+cdx}\,dx\\
&= \int \left(\frac{e}{cd}+\frac{cd^2-ae^2}{cd(ae+cdx)}\right)dx\\
&= \frac{ex}{cd}+\frac{\left(cd^2-ae^2\right)\log(ae+cdx)}{c^2d^2}
\end{aligned}$$

Mathematica [A] time = 0.0089051, size = 35, normalized size = 0.92

$$\frac{\left(cd^2-ae^2\right)\log(ae+cdx)+cdex}{c^2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] (c*d*e*x + (c*d^2 - a*e^2)*Log[a*e + c*d*x])/(c^2*d^2)
```

Maple [A] time = 0.041, size = 45, normalized size = 1.2

$$\frac{ex}{cd} - \frac{\ln(cdx + ae)\,ae^2}{c^2d^2} + \frac{\ln(cdx + ae)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

```
[Out] e*x/c/d-1/c^2/d^2*ln(c*d*x+a*e)*a*e^2+1/c*ln(c*d*x+a*e)
```

Maxima [A] time = 1.0063, size = 51, normalized size = 1.34

$$\frac{ex}{cd} + \frac{\left(cd^2 - ae^2\right)\log(cdx + ae)}{c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="maxima"
)
```

```
[Out] e*x/(c*d) + (c*d^2 - a*e^2)*log(c*d*x + a*e)/(c^2*d^2)
```

Fricas [A] time = 1.55843, size = 76, normalized size = 2.

$$\frac{cdex + \left(cd^2 - ae^2\right)\log(cdx + ae)}{c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fricas"
)
```

```
[Out] (c*d*e*x + (c*d^2 - a*e^2)*log(c*d*x + a*e))/(c^2*d^2)
```

Sympy [A] time = 0.376055, size = 32, normalized size = 0.84

$$\frac{ex}{cd} - \frac{\left(ae^2 - cd^2\right)\log(ae + cdx)}{c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

```
[Out] e*x/(c*d) - (a*e**2 - c*d**2)*log(a*e + c*d*x)/(c**2*d**2)
```

Giac [B] time = 1.25951, size = 215, normalized size = 5.66

$$\frac{xe}{cd} + \frac{\left(cd^2 - ae^2\right)\log\left(cdx^2e + cd^2x + axe^2 + ade\right)}{2\,c^2d^2} + \frac{\left(c^2d^4 - 2\,acd^2e^2 + a^2e^4\right)\arctan\left(\frac{2\,cdxe+cd^2+ae^2}{\sqrt{-c^2d^4+2\,acd^2e^2-a^2e^4}}\right)}{\sqrt{-c^2d^4 + 2\,acd^2e^2 - a^2e^4}c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")
```

```
[Out] x*e/(c*d) + 1/2*(c*d^2 - a*e^2)*log(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)/
(c^2*d^2) + (c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*arctan((2*c*d*x*e + c*d^2 +
 a*e^2)/sqrt(-c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4))/(sqrt(-c^2*d^4 + 2*a*c*d^
2*e^2 - a^2*e^4)*c^2*d^2)
```

3.1869 $\int \frac{d+ex}{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal. Leaf size=16

$$\frac{\log(ae+cdx)}{cd}$$

```
[Out] Log[a*e + c*d*x]/(c*d)
```

Rubi [A] time = 0.0061629, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {626, 31}

$$\frac{\log(ae+cdx)}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] Log[a*e + c*d*x]/(c*d)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex}{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx &= \int \frac{1}{ae+cdx}\,dx\\ &= \frac{\log(ae+cdx)}{cd}\end{aligned}$$

Mathematica [A] time = 0.0015131, size = 16, normalized size = 1.

$$\frac{\log(ae+cdx)}{cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] Log[a*e + c*d*x]/(c*d)
```

Maple [A] time = 0.039, size = 17, normalized size = 1.1

$$\frac{\ln\left(cdx+ae\right)}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

```
[Out] ln(c*d*x+a*e)/c/d
```

Maxima [A] time = 1.0236, size = 22, normalized size = 1.38

$$\frac{\log\left(cdx+ae\right)}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="maxima")
```

```
[Out] log(c*d*x + a*e)/(c*d)
```

Fricas [A] time = 1.62955, size = 32, normalized size = 2.

$$\frac{\log\left(cdx+ae\right)}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fricas")
```

```
[Out] log(c*d*x + a*e)/(c*d)
```

Sympy [A] time = 0.083825, size = 12, normalized size = 0.75

$$\frac{\log\left(ae+cdx\right)}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

```
[Out] log(a*e + c*d*x)/(c*d)
```

Giac [B] time = 1.22298, size = 170, normalized size = 10.62

$$\frac{\left(cd^2-ae^2\right)\arctan\left(\frac{2\,cdxe+cd^2+ae^2}{\sqrt{-c^2d^4+2\,acd^2e^2-a^2e^4}}\right)}{\sqrt{-c^2d^4+2\,acd^2e^2-a^2e^4}cd}+\frac{\log\left(cdx^2e+cd^2x+axe^2+ade\right)}{2\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")
```

```
[Out] (c*d^2 - a*e^2)*arctan((2*c*d*x*e + c*d^2 + a*e^2)/sqrt(-c^2*d^4 + 2*a*c*d^
2*e^2 - a^2*e^4))/(sqrt(-c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4)*c*d) + 1/2*log(
c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)/(c*d)
```

3.1870 $\int \frac{1}{ade+(cd^2+ae^2)x+cdex^2} dx$

Optimal. Leaf size=47

$$\frac{\log(ae+cdx)}{cd^2-ae^2} - \frac{\log(d+ex)}{cd^2-ae^2}$$

```
[Out] Log[a*e + c*d*x]/(c*d^2 - a*e^2) - Log[d + e*x]/(c*d^2 - a*e^2)
```

Rubi [A] time = 0.0154168, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {616, 31}

$$\frac{\log(ae+cdx)}{cd^2-ae^2} - \frac{\log(d+ex)}{cd^2-ae^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-1),x]
```

```
[Out] Log[a*e + c*d*x]/(c*d^2 - a*e^2) - Log[d + e*x]/(c*d^2 - a*e^2)
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ade+(cd^2+ae^2)x+cdex^2} dx &= -\frac{(cde)\int \frac{1}{cd^2+cdex} dx}{cd^2-ae^2} + \frac{(cde)\int \frac{1}{ae^2+cdex} dx}{cd^2-ae^2}\\ &= \frac{\log(ae+cdx)}{cd^2-ae^2} - \frac{\log(d+ex)}{cd^2-ae^2}\end{aligned}$$

Mathematica [A] time = 0.0137618, size = 33, normalized size = 0.7

$$\frac{\log(ae+cdx)-\log(d+ex)}{cd^2-ae^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-1),x]
```

```
[Out] (Log[a*e + c*d*x] - Log[d + e*x])/(c*d^2 - a*e^2)
```

Maple [A] time = 0.045, size = 48, normalized size = 1.

$$\frac{\ln(ex+d)}{ae^2-cd^2}-\frac{\ln(cdx+ae)}{ae^2-cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

```
[Out] 1/(a*e^2-c*d^2)*ln(e*x+d)-1/(a*e^2-c*d^2)*ln(c*d*x+a*e)
```

Maxima [A] time = 1.06451, size = 63, normalized size = 1.34

$$\frac{\log(cdx+ae)}{cd^2-ae^2}-\frac{\log(ex+d)}{cd^2-ae^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="maxima")
```

```
[Out] log(c*d*x + a*e)/(c*d^2 - a*e^2) - log(e*x + d)/(c*d^2 - a*e^2)
```

Fricas [A] time = 1.54686, size = 69, normalized size = 1.47

$$\frac{\log(cdx+ae)-\log(ex+d)}{cd^2-ae^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fricas")
```

```
[Out] (log(c*d*x + a*e) - log(e*x + d))/(c*d^2 - a*e^2)
```

Sympy [B] time = 0.361001, size = 172, normalized size = 3.66

$$\frac{\log\left(x+\frac{-\frac{a^2e^4}{ae^2-cd^2}+\frac{2acd^2e^2}{ae^2-cd^2}+ae^2-\frac{c^2d^4}{ae^2-cd^2}+cd^2}{2cde}\right)}{ae^2-cd^2}-\frac{\log\left(x+\frac{\frac{a^2e^4}{ae^2-cd^2}-\frac{2acd^2e^2}{ae^2-cd^2}+ae^2+\frac{c^2d^4}{ae^2-cd^2}+cd^2}{2cde}\right)}{ae^2-cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

```
[Out] log(x + (-a**2*e**4/(a*e**2 - c*d**2) + 2*a*c*d**2*e**2/(a*e**2 - c*d**2) +
 a*e**2 - c**2*d**4/(a*e**2 - c*d**2) + c*d**2)/(2*c*d*e))/(a*e**2 - c*d**2
) - log(x + (a**2*e**4/(a*e**2 - c*d**2) - 2*a*c*d**2*e**2/(a*e**2 - c*d**2
) + a*e**2 + c**2*d**4/(a*e**2 - c*d**2) + c*d**2)/(2*c*d*e))/(a*e**2 - c*d
**2)
```

Giac [A] time = 1.28223, size = 101, normalized size = 2.15

$$\frac{2\,\arctan\left(\frac{2\,cdxe+cd^2+ae^2}{\sqrt{-c^2d^4+2\,acd^2e^2-a^2e^4}}\right)}{\sqrt{-c^2d^4+2\,acd^2e^2-a^2e^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")
```

```
[Out] 2*arctan((2*c*d*x*e + c*d^2 + a*e^2)/sqrt(-c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^
4))/sqrt(-c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4)
```

3.1871 $\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}\,dx$

Optimal. Leaf size=73

$$\frac{1}{(d+ex)\left(cd^2-ae^2\right)}+\frac{cd\log(ae+cdx)}{\left(cd^2-ae^2\right)^2}-\frac{cd\log(d+ex)}{\left(cd^2-ae^2\right)^2}$$

```
[Out] 1/((c*d^2 - a*e^2)*(d + e*x)) + (c*d*Log[a*e + c*d*x])/(c*d^2 - a*e^2)^2 -
(c*d*Log[d + e*x])/(c*d^2 - a*e^2)^2
```

Rubi [A] time = 0.0505175, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 44}

$$\frac{1}{(d+ex)\left(cd^2-ae^2\right)}+\frac{cd\log(ae+cdx)}{\left(cd^2-ae^2\right)^2}-\frac{cd\log(d+ex)}{\left(cd^2-ae^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)),x]
```

```
[Out] 1/((c*d^2 - a*e^2)*(d + e*x)) + (c*d*Log[a*e + c*d*x])/(c*d^2 - a*e^2)^2 -
(c*d*Log[d + e*x])/(c*d^2 - a*e^2)^2
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}\,dx &= \int \frac{1}{(ae+cdx)(d+ex)^2}\,dx\\
&= \int \left(\frac{c^2d^2}{\left(cd^2-ae^2\right)^2(ae+cdx)}-\frac{e}{\left(cd^2-ae^2\right)(d+ex)^2}-\frac{cde}{\left(cd^2-ae^2\right)^2(d+ex)}\right)dx\\
&= \frac{1}{\left(cd^2-ae^2\right)(d+ex)}+\frac{cd\log(ae+cdx)}{\left(cd^2-ae^2\right)^2}-\frac{cd\log(d+ex)}{\left(cd^2-ae^2\right)^2}
\end{aligned}$$

Mathematica [A] time = 0.0292618, size = 66, normalized size = 0.9

$$\frac{cd(d+ex)\log(ae+cdx)-ae^2+cd^2-cd(d+ex)\log(d+ex)}{(d+ex)\left(cd^2-ae^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)),x]
```

```
[Out] (c*d^2 - a*e^2 + c*d*(d + e*x)*Log[a*e + c*d*x] - c*d*(d + e*x)*Log[d + e*x
])/((c*d^2 - a*e^2)^2*(d + e*x))
```

Maple [A] time = 0.076, size = 75, normalized size = 1.

$$-\frac{1}{\left(ae^{2}-cd^{2}\right)\left(ex+d\right)}-\frac{cd\ln\left(ex+d\right)}{\left(ae^{2}-cd^{2}\right)^{2}}+\frac{cd\ln\left(cdx+ae\right)}{\left(ae^{2}-cd^{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

```
[Out] -1/(a*e^2-c*d^2)/(e*x+d)-c*d/(a*e^2-c*d^2)^2*ln(e*x+d)+c*d/(a*e^2-c*d^2)^2*
ln(c*d*x+a*e)
```

Maxima [A] time = 0.998684, size = 144, normalized size = 1.97

$$\frac{cd\log\left(cdx+ae\right)}{c^{2}d^{4}-2\,acd^{2}e^{2}+a^{2}e^{4}}-\frac{cd\log\left(ex+d\right)}{c^{2}d^{4}-2\,acd^{2}e^{2}+a^{2}e^{4}}+\frac{1}{cd^{3}-ade^{2}+\left(cd^{2}e-ae^{3}\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="maxima"
)
```

```
[Out] c*d*log(c*d*x + a*e)/(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4) - c*d*log(e*x + d)
/(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4) + 1/(c*d^3 - a*d*e^2 + (c*d^2*e - a*e^
3)*x)
```

Fricas [A] time = 1.91676, size = 225, normalized size = 3.08

$$\frac{cd^{2}-ae^{2}+\left(cdex+cd^{2}\right)\log\left(cdx+ae\right)-\left(cdex+cd^{2}\right)\log\left(ex+d\right)}{c^{2}d^{5}-2\,acd^{3}e^{2}+a^{2}de^{4}+\left(c^{2}d^{4}e-2\,acd^{2}e^{3}+a^{2}e^{5}\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fricas"
)
```

```
[Out] (c*d^2 - a*e^2 + (c*d*e*x + c*d^2)*log(c*d*x + a*e) - (c*d*e*x + c*d^2)*log
(e*x + d))/(c^2*d^5 - 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e - 2*a*c*d^2*e^
3 + a^2*e^5)*x)
```

Sympy [B] time = 1.00351, size = 301, normalized size = 4.12

$$-\frac{cd\log\left(x+\frac{-\frac{a^3cde^6}{(ae^2-cd^2)^2}+\frac{3a^2c^2d^3e^4}{(ae^2-cd^2)^2}-\frac{3ac^3d^5e^2}{(ae^2-cd^2)^2}+acde^2+\frac{c^4d^7}{(ae^2-cd^2)^2}+c^2d^3}{2c^2d^2e}\right)}{(ae^2-cd^2)^2}+\frac{cd\log\left(x+\frac{\frac{a^3cde^6}{(ae^2-cd^2)^2}-\frac{3a^2c^2d^3e^4}{(ae^2-cd^2)^2}+\frac{3ac^3d^5e^2}{(ae^2-cd^2)^2}+acde^2-\frac{c^4d^7}{(ae^2-cd^2)^2}+c^2d^3}{2c^2d^2e}\right)}{(ae^2-cd^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

```
[Out] -c*d*log(x + (-a**3*c*d*e**6/(a*e**2 - c*d**2)**2 + 3*a**2*c**2*d**3*e**4/(
a*e**2 - c*d**2)**2 - 3*a*c**3*d**5*e**2/(a*e**2 - c*d**2)**2 + a*c*d*e**2
+ c**4*d**7/(a*e**2 - c*d**2)**2 + c**2*d**3)/(2*c**2*d**2*e))/(a*e**2 - c*
d**2)**2 + c*d*log(x + (a**3*c*d*e**6/(a*e**2 - c*d**2)**2 - 3*a**2*c**2*d*
*3*e**4/(a*e**2 - c*d**2)**2 + 3*a*c**3*d**5*e**2/(a*e**2 - c*d**2)**2 + a*
c*d*e**2 - c**4*d**7/(a*e**2 - c*d**2)**2 + c**2*d**3)/(2*c**2*d**2*e))/(a*
e**2 - c*d**2)**2 - 1/(a*d*e**2 - c*d**3 + x*(a*e**3 - c*d**2*e))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1872 $\int \frac{1}{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}\,dx$

Optimal. Leaf size=108

$$\frac{c^2d^2\log(ae+cdx)}{\left(cd^2-ae^2\right)^3}-\frac{c^2d^2\log(d+ex)}{\left(cd^2-ae^2\right)^3}+\frac{cd}{(d+ex)\left(cd^2-ae^2\right)^2}+\frac{1}{2(d+ex)^2\left(cd^2-ae^2\right)}$$

```
[Out] 1/(2*(c*d^2 - a*e^2)*(d + e*x)^2) + (c*d)/((c*d^2 - a*e^2)^2*(d + e*x)) + (
c^2*d^2*Log[a*e + c*d*x])/(c*d^2 - a*e^2)^3 - (c^2*d^2*Log[d + e*x])/(c*d^2
 - a*e^2)^3
```

Rubi [A] time = 0.0720214, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 44}

$$\frac{c^2d^2\log(ae+cdx)}{\left(cd^2-ae^2\right)^3}-\frac{c^2d^2\log(d+ex)}{\left(cd^2-ae^2\right)^3}+\frac{cd}{(d+ex)\left(cd^2-ae^2\right)^2}+\frac{1}{2(d+ex)^2\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)),x]
```

```
[Out] 1/(2*(c*d^2 - a*e^2)*(d + e*x)^2) + (c*d)/((c*d^2 - a*e^2)^2*(d + e*x)) + (
c^2*d^2*Log[a*e + c*d*x])/(c*d^2 - a*e^2)^3 - (c^2*d^2*Log[d + e*x])/(c*d^2
 - a*e^2)^3
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}\,dx=\int \frac{1}{(ae+cdx)(d+ex)^3}\,dx$$

$$=\int \left(\frac{c^3d^3}{\left(cd^2-ae^2\right)^3(ae+cdx)}-\frac{e}{\left(cd^2-ae^2\right)(d+ex)^3}-\frac{cde}{\left(cd^2-ae^2\right)^2(d+\right.$$

$$=\frac{1}{2\left(cd^2-ae^2\right)(d+ex)^2}+\frac{cd}{\left(cd^2-ae^2\right)^2(d+ex)}+\frac{c^2d^2\log(ae+cdx)}{\left(cd^2-ae^2\right)^3}-$$

Mathematica [A] time = 0.0476664, size = 102, normalized size = 0.94

$$\frac{2c^2d^2(d+ex)^2\log(ae+cdx)+\left(cd^2-ae^2\right)\left(cd(3d+2ex)-ae^2\right)-2c^2d^2(d+ex)^2\log(d+ex)}{2(d+ex)^2\left(cd^2-ae^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)),x]
```

```
[Out] ((c*d^2 - a*e^2)*(-(a*e^2) + c*d*(3*d + 2*e*x)) + 2*c^2*d^2*(d + e*x)^2*Log
[a*e + c*d*x] - 2*c^2*d^2*(d + e*x)^2*Log[d + e*x])/(2*(c*d^2 - a*e^2)^3*(d
 + e*x)^2)
```

Maple [A] time = 0.05, size = 107, normalized size = 1.

$$-\frac{1}{\left(2\,ae^2-2\,cd^2\right)(ex+d)^2}+\frac{c^2d^2\ln(ex+d)}{\left(ae^2-cd^2\right)^3}+\frac{cd}{\left(ae^2-cd^2\right)^2(ex+d)}-\frac{c^2d^2\ln(cdx+ae)}{\left(ae^2-cd^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

```
[Out] -1/2/(a*e^2-c*d^2)/(e*x+d)^2+c^2*d^2/(a*e^2-c*d^2)^3*ln(e*x+d)+c*d/(a*e^2-c
*d^2)^2/(e*x+d)-c^2*d^2/(a*e^2-c*d^2)^3*ln(c*d*x+a*e)
```

Maxima [B] time = 1.14348, size = 308, normalized size = 2.85

$$\frac{c^2d^2\log(cdx+ae)}{c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6}-\frac{c^2d^2\log(ex+d)}{c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6}+\frac{2\,cdex+3}{2\left(c^2d^6-2\,acd^4e^2+a^2d^2e^4+\left(c^2d^4e^2-2\,acd\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="maxim
a")
```

```
[Out] c^2*d^2*log(c*d*x + a*e)/(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3
*e^6) - c^2*d^2*log(e*x + d)/(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 -
 a^3*e^6) + 1/2*(2*c*d*e*x + 3*c*d^2 - a*e^2)/(c^2*d^6 - 2*a*c*d^4*e^2 + a^
2*d^2*e^4 + (c^2*d^4*e^2 - 2*a*c*d^2*e^4 + a^2*e^6)*x^2 + 2*(c^2*d^5*e - 2*
a*c*d^3*e^3 + a^2*d*e^5)*x)
```

Fricas [B] time = 1.94723, size = 528, normalized size = 4.89

$$\frac{3\,c^2d^4-4\,acd^2e^2+a^2e^4+2\left(c^2d^3e-acde^3\right)x+2\left(c^2d^2e^2x^2+2\,c^2d^3ex+c^2d^4\right)\log(cdx+ae)-2\left(c^2d^2e^2x^2+2\,c^2d^3ex+c\right.}{2\left(c^3d^8-3\,ac^2d^6e^2+3\,a^2cd^4e^4-a^3d^2e^6+\left(c^3d^6e^2-3\,ac^2d^4e^4+3\,a^2cd^2e^6-a^3e^8\right)x^2+2\left(c^3d^7e-3\,ac^2d^5e^3+3\,a^2cd^3e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="frica
s")
```

```
[Out] 1/2*(3*c^2*d^4 - 4*a*c*d^2*e^2 + a^2*e^4 + 2*(c^2*d^3*e - a*c*d*e^3)*x + 2*
(c^2*d^2*e^2*x^2 + 2*c^2*d^3*e*x + c^2*d^4)*log(c*d*x + a*e) - 2*(c^2*d^2*e
^2*x^2 + 2*c^2*d^3*e*x + c^2*d^4)*log(e*x + d))/(c^3*d^8 - 3*a*c^2*d^6*e^2
+ 3*a^2*c*d^4*e^4 - a^3*d^2*e^6 + (c^3*d^6*e^2 - 3*a*c^2*d^4*e^4 + 3*a^2*c*
d^2*e^6 - a^3*e^8)*x^2 + 2*(c^3*d^7*e - 3*a*c^2*d^5*e^3 + 3*a^2*c*d^3*e^5 -
 a^3*d*e^7)*x)
```

Sympy [B] time = 1.52632, size = 471, normalized size = 4.36

$$\frac{c^2d^2\log\left(x+\frac{-\frac{a^4c^2d^2e^8}{(ae^2-cd^2)^3}+\frac{4a^3c^3d^4e^6}{(ae^2-cd^2)^3}-\frac{6a^2c^4d^6e^4}{(ae^2-cd^2)^3}+\frac{4ac^5d^8e^2}{(ae^2-cd^2)^3}+ac^2d^2e^2-\frac{c^6d^{10}}{(ae^2-cd^2)^3}+c^3d^4}{2c^3d^3e}\right)}{(ae^2-cd^2)^3}-\frac{c^2d^2\log\left(x+\frac{\frac{a^4c^2d^2e^8}{(ae^2-cd^2)^3}-\frac{4a^3c^3d^4e^6}{(ae^2-cd^2)^3}+\frac{6a^2c^4d^6e^4}{(ae^2-cd^2)^3}-\frac{4ac}{(ae^2}}{2c^3d^3e}\right.}{(ae^2-cd^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

```
[Out] c**2*d**2*log(x + (-a**4*c**2*d**2*e**8/(a*e**2 - c*d**2)**3 + 4*a**3*c**3*
d**4*e**6/(a*e**2 - c*d**2)**3 - 6*a**2*c**4*d**6*e**4/(a*e**2 - c*d**2)**3
 + 4*a*c**5*d**8*e**2/(a*e**2 - c*d**2)**3 + a*c**2*d**2*e**2 - c**6*d**10/
(a*e**2 - c*d**2)**3 + c**3*d**4)/(2*c**3*d**3*e))/(a*e**2 - c*d**2)**3 - c
**2*d**2*log(x + (a**4*c**2*d**2*e**8/(a*e**2 - c*d**2)**3 - 4*a**3*c**3*d*
*4*e**6/(a*e**2 - c*d**2)**3 + 6*a**2*c**4*d**6*e**4/(a*e**2 - c*d**2)**3 -
 4*a*c**5*d**8*e**2/(a*e**2 - c*d**2)**3 + a*c**2*d**2*e**2 + c**6*d**10/(a
*e**2 - c*d**2)**3 + c**3*d**4)/(2*c**3*d**3*e))/(a*e**2 - c*d**2)**3 + (-a
*e**2 + 3*c*d**2 + 2*c*d*e*x)/(2*a**2*d**2*e**4 - 4*a*c*d**4*e**2 + 2*c**2*
d**6 + x**2*(2*a**2*e**6 - 4*a*c*d**2*e**4 + 2*c**2*d**4*e**2) + x*(4*a**2*
d*e**5 - 8*a*c*d**3*e**3 + 4*c**2*d**5*e))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError
```

3.1873 $\int \frac{1}{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}\,dx$

Optimal. Leaf size=139

$$\frac{c^2d^2}{(d+ex)\left(cd^2-ae^2\right)^3}+\frac{c^3d^3\log(ae+cdx)}{\left(cd^2-ae^2\right)^4}-\frac{c^3d^3\log(d+ex)}{\left(cd^2-ae^2\right)^4}+\frac{cd}{2(d+ex)^2\left(cd^2-ae^2\right)^2}+\frac{1}{3(d+ex)^3\left(cd^2-ae^2\right)}$$

```
[Out] 1/(3*(c*d^2 - a*e^2)*(d + e*x)^3) + (c*d)/(2*(c*d^2 - a*e^2)^2*(d + e*x)^2)
 + (c^2*d^2)/((c*d^2 - a*e^2)^3*(d + e*x)) + (c^3*d^3*Log[a*e + c*d*x])/(c*
d^2 - a*e^2)^4 - (c^3*d^3*Log[d + e*x])/(c*d^2 - a*e^2)^4
```

Rubi [A] time = 0.0989909, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 44}

$$\frac{c^2d^2}{(d+ex)\left(cd^2-ae^2\right)^3}+\frac{c^3d^3\log(ae+cdx)}{\left(cd^2-ae^2\right)^4}-\frac{c^3d^3\log(d+ex)}{\left(cd^2-ae^2\right)^4}+\frac{cd}{2(d+ex)^2\left(cd^2-ae^2\right)^2}+\frac{1}{3(d+ex)^3\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)),x]
```

```
[Out] 1/(3*(c*d^2 - a*e^2)*(d + e*x)^3) + (c*d)/(2*(c*d^2 - a*e^2)^2*(d + e*x)^2)
 + (c^2*d^2)/((c*d^2 - a*e^2)^3*(d + e*x)) + (c^3*d^3*Log[a*e + c*d*x])/(c*
d^2 - a*e^2)^4 - (c^3*d^3*Log[d + e*x])/(c*d^2 - a*e^2)^4
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}\,dx &= \int \frac{1}{(ae+cdx)(d+ex)^4}\,dx\\ &= \int \left(\frac{c^4d^4}{\left(cd^2-ae^2\right)^4(ae+cdx)}-\frac{e}{\left(cd^2-ae^2\right)(d+ex)^4}-\frac{cde}{\left(cd^2-ae^2\right)^2(d+ex\right.\\ &= \frac{1}{3\left(cd^2-ae^2\right)(d+ex)^3}+\frac{cd}{2\left(cd^2-ae^2\right)^2(d+ex)^2}+\frac{c^2d^2}{\left(cd^2-ae^2\right)^3(d+ex)}\end{aligned}$$

Mathematica [A] time = 0.0684324, size = 135, normalized size = 0.97

$$\frac{\left(cd^2-ae^2\right)\left(2a^2e^4-acde^2(7d+3ex)+c^2d^2\left(11d^2+15dex+6e^2x^2\right)\right)+6c^3d^3(d+ex)^3\log(ae+cdx)-6c^3d^3(d+ex)^3\,\mathrm{l}}{6(d+ex)^3\left(cd^2-ae^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)),x]
```

```
[Out] ((c*d^2 - a*e^2)*(2*a^2*e^4 - a*c*d*e^2*(7*d + 3*e*x) + c^2*d^2*(11*d^2 + 1
5*d*e*x + 6*e^2*x^2)) + 6*c^3*d^3*(d + e*x)^3*Log[a*e + c*d*x] - 6*c^3*d^3*
(d + e*x)^3*Log[d + e*x])/(6*(c*d^2 - a*e^2)^4*(d + e*x)^3)
```

Maple [A] time = 0.053, size = 137, normalized size = 1.

$$-\frac{1}{\left(3\,ae^2-3\,cd^2\right)(ex+d)^3}-\frac{c^2d^2}{\left(ae^2-cd^2\right)^3(ex+d)}+\frac{cd}{2\left(ae^2-cd^2\right)^2(ex+d)^2}-\frac{c^3d^3\ln(ex+d)}{\left(ae^2-cd^2\right)^4}+\frac{c^3d^3\ln(cdx+ae)}{\left(ae^2-cd^2\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

```
[Out] -1/3/(a*e^2-c*d^2)/(e*x+d)^3-c^2*d^2/(a*e^2-c*d^2)^3/(e*x+d)+1/2*c*d/(a*e^2
-c*d^2)^2/(e*x+d)^2-c^3*d^3/(a*e^2-c*d^2)^4*ln(e*x+d)+c^3*d^3/(a*e^2-c*d^2)
^4*ln(c*d*x+a*e)
```

Maxima [B] time = 1.15512, size = 531, normalized size = 3.82

$$\frac{c^3d^3\log(cdx+ae)}{c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8}-\frac{c^3d^3\log(ex+d)}{c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8}+\frac{}{6\left(c^3d^9-3\,ac^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="maxim
a")
```

```
[Out] c^3*d^3*log(c*d*x + a*e)/(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4
*a^3*c*d^2*e^6 + a^4*e^8) - c^3*d^3*log(e*x + d)/(c^4*d^8 - 4*a*c^3*d^6*e^2
 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8) + 1/6*(6*c^2*d^2*e^2*x^2
+ 11*c^2*d^4 - 7*a*c*d^2*e^2 + 2*a^2*e^4 + 3*(5*c^2*d^3*e - a*c*d*e^3)*x)/(
c^3*d^9 - 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4 - a^3*d^3*e^6 + (c^3*d^6*e^3 -
3*a*c^2*d^4*e^5 + 3*a^2*c*d^2*e^7 - a^3*e^9)*x^3 + 3*(c^3*d^7*e^2 - 3*a*c^2
*d^5*e^4 + 3*a^2*c*d^3*e^6 - a^3*d*e^8)*x^2 + 3*(c^3*d^8*e - 3*a*c^2*d^6*e^
3 + 3*a^2*c*d^4*e^5 - a^3*d^2*e^7)*x)
```

Fricas [B] time = 1.86856, size = 906, normalized size = 6.52

$$\frac{11\,c^3d^6-18\,ac^2d^4e^2+9\,a^2cd^2e^4-2\,a^3e^6+6\left(c^3d^4e^2-ac^2d^2e^4\right)x^2+3\left(5\,c^3d^5e-6\,ac^2d^3e^3+a^2cde^5\right)x+6\left(c^3d^3\right.}{6\left(c^4d^{11}-4\,ac^3d^9e^2+6\,a^2c^2d^7e^4-4\,a^3cd^5e^6+a^4d^3e^8+\left(c^4d^8e^3-4\,ac^3d^6e^5+6\,a^2c^2d^4e^7-4\,a^3cd^2e^9+a^4e^{11}\right)x^3+3\left(c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="frica
s")
```

```
[Out] 1/6*(11*c^3*d^6 - 18*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 2*a^3*e^6 + 6*(c^3*d
^4*e^2 - a*c^2*d^2*e^4)*x^2 + 3*(5*c^3*d^5*e - 6*a*c^2*d^3*e^3 + a^2*c*d*e^
5)*x + 6*(c^3*d^3*e^3*x^3 + 3*c^3*d^4*e^2*x^2 + 3*c^3*d^5*e*x + c^3*d^6)*lo
g(c*d*x + a*e) - 6*(c^3*d^3*e^3*x^3 + 3*c^3*d^4*e^2*x^2 + 3*c^3*d^5*e*x + c
^3*d^6)*log(e*x + d))/(c^4*d^11 - 4*a*c^3*d^9*e^2 + 6*a^2*c^2*d^7*e^4 - 4*a
^3*c*d^5*e^6 + a^4*d^3*e^8 + (c^4*d^8*e^3 - 4*a*c^3*d^6*e^5 + 6*a^2*c^2*d^4
*e^7 - 4*a^3*c*d^2*e^9 + a^4*e^11)*x^3 + 3*(c^4*d^9*e^2 - 4*a*c^3*d^7*e^4 +
 6*a^2*c^2*d^5*e^6 - 4*a^3*c*d^3*e^8 + a^4*d*e^10)*x^2 + 3*(c^4*d^10*e - 4*
a*c^3*d^8*e^3 + 6*a^2*c^2*d^6*e^5 - 4*a^3*c*d^4*e^7 + a^4*d^2*e^9)*x)
```

Sympy [B] time = 2.40193, size = 672, normalized size = 4.83

$$-\frac{c^3 d^3 \log\left(x + \frac{-\frac{a^5c^3d^3e^{10}}{\left(ae^2-cd^2\right)^4} + \frac{5a^4c^4d^5e^8}{\left(ae^2-cd^2\right)^4} - \frac{10a^3c^5d^7e^6}{\left(ae^2-cd^2\right)^4} + \frac{10a^2c^6d^9e^4}{\left(ae^2-cd^2\right)^4} - \frac{5ac^7d^{11}e^2}{\left(ae^2-cd^2\right)^4} + ac^3d^3e^2 + \frac{c^8d^{13}}{\left(ae^2-cd^2\right)^4} + c^4d^5}{2c^4d^4e}\right)}{\left(ae^2-cd^2\right)^4} + \frac{c^3d^3\log\left(x + \frac{\frac{a^5c^3d^3e^{10}}{\left(ae^2-cd^2\right)^4} - \frac{5a^4c^4d^5e^8}{\left(ae^2-cd^2\right)^4} + \frac{10a^3c}{\left(ae^2-\right.}}{}\right.}{\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

```
[Out] -c**3*d**3*log(x + (-a**5*c**3*d**3*e**10/(a*e**2 - c*d**2)**4 + 5*a**4*c**
4*d**5*e**8/(a*e**2 - c*d**2)**4 - 10*a**3*c**5*d**7*e**6/(a*e**2 - c*d**2)
**4 + 10*a**2*c**6*d**9*e**4/(a*e**2 - c*d**2)**4 - 5*a*c**7*d**11*e**2/(a*
e**2 - c*d**2)**4 + a*c**3*d**3*e**2 + c**8*d**13/(a*e**2 - c*d**2)**4 + c*
*4*d**5)/(2*c**4*d**4*e))/(a*e**2 - c*d**2)**4 + c**3*d**3*log(x + (a**5*c*
*3*d**3*e**10/(a*e**2 - c*d**2)**4 - 5*a**4*c**4*d**5*e**8/(a*e**2 - c*d**2
)**4 + 10*a**3*c**5*d**7*e**6/(a*e**2 - c*d**2)**4 - 10*a**2*c**6*d**9*e**4
/(a*e**2 - c*d**2)**4 + 5*a*c**7*d**11*e**2/(a*e**2 - c*d**2)**4 + a*c**3*d
**3*e**2 - c**8*d**13/(a*e**2 - c*d**2)**4 + c**4*d**5)/(2*c**4*d**4*e))/(a
*e**2 - c*d**2)**4 - (2*a**2*e**4 - 7*a*c*d**2*e**2 + 11*c**2*d**4 + 6*c**2
*d**2*e**2*x**2 + x*(-3*a*c*d*e**3 + 15*c**2*d**3*e))/(6*a**3*d**3*e**6 - 1
8*a**2*c*d**5*e**4 + 18*a*c**2*d**7*e**2 - 6*c**3*d**9 + x**3*(6*a**3*e**9
- 18*a**2*c*d**2*e**7 + 18*a*c**2*d**4*e**5 - 6*c**3*d**6*e**3) + x**2*(18*
a**3*d*e**8 - 54*a**2*c*d**3*e**6 + 54*a*c**2*d**5*e**4 - 18*c**3*d**7*e**2
) + x*(18*a**3*d**2*e**7 - 54*a**2*c*d**4*e**5 + 54*a*c**2*d**6*e**3 - 18*c
**3*d**8*e))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError
```

3.1874 $\int \frac{(d+ex)^8}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$

Optimal. Leaf size=219

$$\frac{e^6(ae+cdx)^5}{5c^7d^7}+\frac{3e^5\left(cd^2-ae^2\right)(ae+cdx)^4}{2c^7d^7}+\frac{5e^4\left(cd^2-ae^2\right)^2(ae+cdx)^3}{c^7d^7}+\frac{10e^3\left(cd^2-ae^2\right)^3(ae+cdx)^2}{c^7d^7}+\frac{15e^2x\left(cd\right.}{c^6\ldots}$$

```
[Out] (15*e^2*(c*d^2 - a*e^2)^4*x)/(c^6*d^6) - (c*d^2 - a*e^2)^6/(c^7*d^7*(a*e +
c*d*x)) + (10*e^3*(c*d^2 - a*e^2)^3*(a*e + c*d*x)^2)/(c^7*d^7) + (5*e^4*(c*
d^2 - a*e^2)^2*(a*e + c*d*x)^3)/(c^7*d^7) + (3*e^5*(c*d^2 - a*e^2)*(a*e + c
*d*x)^4)/(2*c^7*d^7) + (e^6*(a*e + c*d*x)^5)/(5*c^7*d^7) + (6*e*(c*d^2 - a*
e^2)^5*Log[a*e + c*d*x])/(c^7*d^7)
```

Rubi [A] time = 0.27523, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$\frac{e^6(ae+cdx)^5}{5c^7d^7}+\frac{3e^5\left(cd^2-ae^2\right)(ae+cdx)^4}{2c^7d^7}+\frac{5e^4\left(cd^2-ae^2\right)^2(ae+cdx)^3}{c^7d^7}+\frac{10e^3\left(cd^2-ae^2\right)^3(ae+cdx)^2}{c^7d^7}+\frac{15e^2x\left(cd\right.}{c^6\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^8/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```


```
[Out] (15*e^2*(c*d^2 - a*e^2)^4*x)/(c^6*d^6) - (c*d^2 - a*e^2)^6/(c^7*d^7*(a*e +
c*d*x)) + (10*e^3*(c*d^2 - a*e^2)^3*(a*e + c*d*x)^2)/(c^7*d^7) + (5*e^4*(c*
d^2 - a*e^2)^2*(a*e + c*d*x)^3)/(c^7*d^7) + (3*e^5*(c*d^2 - a*e^2)*(a*e + c
*d*x)^4)/(2*c^7*d^7) + (e^6*(a*e + c*d*x)^5)/(5*c^7*d^7) + (6*e*(c*d^2 - a*
e^2)^5*Log[a*e + c*d*x])/(c^7*d^7)
```


Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(d+ex)^8}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}dx = \int \frac{(d+ex)^6}{(ae+cdx)^2}dx$$

$$= \int \left(\frac{15e^2\left(cd^2-ae^2\right)^4}{c^6d^6} + \frac{\left(cd^2-ae^2\right)^6}{c^6d^6(ae+cdx)^2} + \frac{6e\left(cd^2-ae^2\right)^5}{c^6d^6(ae+cdx)} + \frac{20\left(cd^2e-ae^3\right)^3(ae}{c^6d^6}\right.$$

$$= \frac{15e^2\left(cd^2-ae^2\right)^4x}{c^6d^6} - \frac{\left(cd^2-ae^2\right)^6}{c^7d^7(ae+cdx)} + \frac{10e^3\left(cd^2-ae^2\right)^3(ae+cdx)^2}{c^7d^7} + \frac{5e^4\left(cd^2-\right.}{}$$

Mathematica [A] time = 0.124329, size = 339, normalized size = 1.55

$$\frac{-30a^4c^2d^2e^8\left(5d^2+8dex-e^2x^2\right)+10a^3c^3d^3e^6\left(45d^2ex+20d^3-15de^2x^2-e^3x^3\right)-5a^2c^4d^4e^4\left(-60d^2e^2x^2+80d^3ex+30d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^8/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] (-10*a^6*e^12 + 10*a^5*c*d*e^10*(6*d + 5*e*x) - 30*a^4*c^2*d^2*e^8*(5*d^2 +
 8*d*e*x - e^2*x^2) + 10*a^3*c^3*d^3*e^6*(20*d^3 + 45*d^2*e*x - 15*d*e^2*x^
2 - e^3*x^3) - 5*a^2*c^4*d^4*e^4*(30*d^4 + 80*d^3*e*x - 60*d^2*e^2*x^2 - 10
*d*e^3*x^3 - e^4*x^4) + a*c^5*d^5*e^2*(60*d^5 + 150*d^4*e*x - 300*d^3*e^2*x
^2 - 100*d^2*e^3*x^3 - 25*d*e^4*x^4 - 3*e^5*x^5) + c^6*d^6*(-10*d^6 + 150*d
^4*e^2*x^2 + 100*d^3*e^3*x^3 + 50*d^2*e^4*x^4 + 15*d*e^5*x^5 + 2*e^6*x^6) -
 60*e*(-(c*d^2) + a*e^2)^5*(a*e + c*d*x)*Log[a*e + c*d*x])/(10*c^7*d^7*(a*e
 + c*d*x))
```

Maple [B] time = 0.052, size = 502, normalized size = 2.3

$$-\frac{a^6e^{12}}{c^7d^7(cdx+ae)} + 6\frac{a^5e^{10}}{c^6d^5(cdx+ae)} - 4\frac{e^6x^3a}{c^3d^2} - 15\frac{a^4e^8}{c^5d^3(cdx+ae)} - 2\frac{e^9x^2a^3}{c^5d^5} - 40\frac{ae^4x}{c^3} + \frac{e^6x^5}{5c^2d^2} + \frac{3e^5x^4}{2c^2d} + 10\frac{e^3dx^2}{c^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^8/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

```
[Out] -1/c^7/d^7/(c*d*x+a*e)*a^6*e^12+6/c^6/d^5/(c*d*x+a*e)*a^5*e^10-4*e^6/c^3/d^
2*x^3*a-15/c^5/d^3/(c*d*x+a*e)*a^4*e^8-2*e^9/c^5/d^5*x^2*a^3-40*e^4/c^3*a*x
+1/5*e^6/c^2/d^2*x^5+3/2*e^5/c^2/d*x^4+10*e^3/c^2*d*x^2+e^8/c^4/d^4*x^3*a^2
-1/2*e^7/c^3/d^3*x^4*a+15*e^2/c^2*d^2*x+6*d^3*e/c^2*ln(c*d*x+a*e)+9*e^7/c^4
/d^3*x^2*a^2-15*e^5/c^3/d*x^2*a+5*e^10/c^6/d^6*a^4*x-24*e^8/c^5/d^4*a^3*x+2
0/c^4/d/(c*d*x+a*e)*a^3*e^6-15/c^3*d/(c*d*x+a*e)*a^2*e^4-60/d^3*e^7/c^5*ln(
c*d*x+a*e)*a^3+60/d*e^5/c^4*ln(c*d*x+a*e)*a^2-30*d*e^3/c^3*ln(c*d*x+a*e)*a+
5*e^4/c^2*x^3-1/c*d^5/(c*d*x+a*e)+45*e^6/c^4/d^2*a^2*x+6/c^2*d^3/(c*d*x+a*e
)*a*e^2-6/d^7*e^11/c^7*ln(c*d*x+a*e)*a^5+30/d^5*e^9/c^6*ln(c*d*x+a*e)*a^4
```

Maxima [A] time = 1.0221, size = 537, normalized size = 2.45

$$-\frac{c^6d^{12}-6ac^5d^{10}e^2+15a^2c^4d^8e^4-20a^3c^3d^6e^6+15a^4c^2d^4e^8-6a^5cd^2e^{10}+a^6e^{12}}{c^8d^8x+ac^7d^7e} + \frac{2c^4d^4e^6x^5+5\left(3c^4d^5e^5-ac^3d^3e^7\right)x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^8/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="maxim
a")
```

```
[Out] -(c^6*d^12 - 6*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 + 1
5*a^4*c^2*d^4*e^8 - 6*a^5*c*d^2*e^10 + a^6*e^12)/(c^8*d^8*x + a*c^7*d^7*e)
+ 1/10*(2*c^4*d^4*e^6*x^5 + 5*(3*c^4*d^5*e^5 - a*c^3*d^3*e^7)*x^4 + 10*(5*c
^4*d^6*e^4 - 4*a*c^3*d^4*e^6 + a^2*c^2*d^2*e^8)*x^3 + 10*(10*c^4*d^7*e^3 -
15*a*c^3*d^5*e^5 + 9*a^2*c^2*d^3*e^7 - 2*a^3*c*d*e^9)*x^2 + 10*(15*c^4*d^8*
e^2 - 40*a*c^3*d^6*e^4 + 45*a^2*c^2*d^4*e^6 - 24*a^3*c*d^2*e^8 + 5*a^4*e^10
)*x)/(c^6*d^6) + 6*(c^5*d^10*e - 5*a*c^4*d^8*e^3 + 10*a^2*c^3*d^6*e^5 - 10*
a^3*c^2*d^4*e^7 + 5*a^4*c*d^2*e^9 - a^5*e^11)*log(c*d*x + a*e)/(c^7*d^7)
```

Fricas [B] time = 1.80114, size = 1112, normalized size = 5.08

$$\frac{2\,c^6d^6e^6x^6 - 10\,c^6d^{12} + 60\,ac^5d^{10}e^2 - 150\,a^2c^4d^8e^4 + 200\,a^3c^3d^6e^6 - 150\,a^4c^2d^4e^8 + 60\,a^5cd^2e^{10} - 10\,a^6e^{12} + 3\,\big(5\,c^6d^7e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^8/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="frica
s")
```

```
[Out] 1/10*(2*c^6*d^6*e^6*x^6 - 10*c^6*d^12 + 60*a*c^5*d^10*e^2 - 150*a^2*c^4*d^8
*e^4 + 200*a^3*c^3*d^6*e^6 - 150*a^4*c^2*d^4*e^8 + 60*a^5*c*d^2*e^10 - 10*a
^6*e^12 + 3*(5*c^6*d^7*e^5 - a*c^5*d^5*e^7)*x^5 + 5*(10*c^6*d^8*e^4 - 5*a*c
^5*d^6*e^6 + a^2*c^4*d^4*e^8)*x^4 + 10*(10*c^6*d^9*e^3 - 10*a*c^5*d^7*e^5 +
 5*a^2*c^4*d^5*e^7 - a^3*c^3*d^3*e^9)*x^3 + 30*(5*c^6*d^10*e^2 - 10*a*c^5*d
^8*e^4 + 10*a^2*c^4*d^6*e^6 - 5*a^3*c^3*d^4*e^8 + a^4*c^2*d^2*e^10)*x^2 + 1
0*(15*a*c^5*d^9*e^3 - 40*a^2*c^4*d^7*e^5 + 45*a^3*c^3*d^5*e^7 - 24*a^4*c^2*
d^3*e^9 + 5*a^5*c*d*e^11)*x + 60*(a*c^5*d^10*e^2 - 5*a^2*c^4*d^8*e^4 + 10*a
^3*c^3*d^6*e^6 - 10*a^4*c^2*d^4*e^8 + 5*a^5*c*d^2*e^10 - a^6*e^12 + (c^6*d^
11*e - 5*a*c^5*d^9*e^3 + 10*a^2*c^4*d^7*e^5 - 10*a^3*c^3*d^5*e^7 + 5*a^4*c^
2*d^3*e^9 - a^5*c*d*e^11)*x)*log(c*d*x + a*e))/(c^8*d^8*x + a*c^7*d^7*e)
```

Sympy [A] time = 2.33021, size = 348, normalized size = 1.59

$$-\frac{a^6e^{12} - 6a^5cd^2e^{10} + 15a^4c^2d^4e^8 - 20a^3c^3d^6e^6 + 15a^2c^4d^8e^4 - 6ac^5d^{10}e^2 + c^6d^{12}}{ac^7d^7e + c^8d^8x} + \frac{e^6x^5}{5c^2d^2} - \frac{x^4\left(ae^7 - 3cd^2e^5\right)}{2c^3d^3} + \frac{x^3\left(a^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**8/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

```
[Out] -(a**6*e**12 - 6*a**5*c*d**2*e**10 + 15*a**4*c**2*d**4*e**8 - 20*a**3*c**3*
d**6*e**6 + 15*a**2*c**4*d**8*e**4 - 6*a*c**5*d**10*e**2 + c**6*d**12)/(a*c
**7*d**7*e + c**8*d**8*x) + e**6*x**5/(5*c**2*d**2) - x**4*(a*e**7 - 3*c*d*
*2*e**5)/(2*c**3*d**3) + x**3*(a**2*e**8 - 4*a*c*d**2*e**6 + 5*c**2*d**4*e*
*4)/(c**4*d**4) - x**2*(2*a**3*e**9 - 9*a**2*c*d**2*e**7 + 15*a*c**2*d**4*e
**5 - 10*c**3*d**6*e**3)/(c**5*d**5) + x*(5*a**4*e**10 - 24*a**3*c*d**2*e**
8 + 45*a**2*c**2*d**4*e**6 - 40*a*c**3*d**6*e**4 + 15*c**4*d**8*e**2)/(c**6
*d**6) - 6*e*(a*e**2 - c*d**2)**5*log(a*e + c*d*x)/(c**7*d**7)
```

Giac [B] time = 1.54775, size = 1075, normalized size = 4.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^8/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac"
)
```

```
[Out] 6*(c^8*d^16*e - 8*a*c^7*d^14*e^3 + 28*a^2*c^6*d^12*e^5 - 56*a^3*c^5*d^10*e^
7 + 70*a^4*c^4*d^8*e^9 - 56*a^5*c^3*d^6*e^11 + 28*a^6*c^2*d^4*e^13 - 8*a^7*
c*d^2*e^15 + a^8*e^17)*arctan((2*c*d*x*e + c*d^2 + a*e^2)/sqrt(-c^2*d^4 + 2
*a*c*d^2*e^2 - a^2*e^4))/((c^9*d^11 - 2*a*c^8*d^9*e^2 + a^2*c^7*d^7*e^4)*sq
rt(-c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4)) + 3*(c^5*d^10*e - 5*a*c^4*d^8*e^3 +
 10*a^2*c^3*d^6*e^5 - 10*a^3*c^2*d^4*e^7 + 5*a^4*c*d^2*e^9 - a^5*e^11)*log(
c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)/(c^7*d^7) - (c^8*d^17 - 8*a*c^7*d^15
*e^2 + 28*a^2*c^6*d^13*e^4 - 56*a^3*c^5*d^11*e^6 + 70*a^4*c^4*d^9*e^8 - 56*
a^5*c^3*d^7*e^10 + 28*a^6*c^2*d^5*e^12 - 8*a^7*c*d^3*e^14 + a^8*d*e^16 + (c
^8*d^16*e - 8*a*c^7*d^14*e^3 + 28*a^2*c^6*d^12*e^5 - 56*a^3*c^5*d^10*e^7 +
70*a^4*c^4*d^8*e^9 - 56*a^5*c^3*d^6*e^11 + 28*a^6*c^2*d^4*e^13 - 8*a^7*c*d^
2*e^15 + a^8*e^17)*x)/((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*(c*d*x^2*e + c*d
^2*x + a*x*e^2 + a*d*e)*c^7*d^7) + 1/10*(2*c^8*d^8*x^5*e^16 + 15*c^8*d^9*x^
4*e^15 + 50*c^8*d^10*x^3*e^14 + 100*c^8*d^11*x^2*e^13 + 150*c^8*d^12*x*e^12
 - 5*a*c^7*d^7*x^4*e^17 - 40*a*c^7*d^8*x^3*e^16 - 150*a*c^7*d^9*x^2*e^15 -
400*a*c^7*d^10*x*e^14 + 10*a^2*c^6*d^6*x^3*e^18 + 90*a^2*c^6*d^7*x^2*e^17 +
 450*a^2*c^6*d^8*x*e^16 - 20*a^3*c^5*d^5*x^2*e^19 - 240*a^3*c^5*d^6*x*e^18
+ 50*a^4*c^4*d^4*x*e^20)*e^(-10)/(c^10*d^10)
```

3.1875 $\int \frac{(d+ex)^7}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2} dx$

Optimal. Leaf size=184

$$\frac{e^5(ae+cdx)^4}{4c^6d^6}+\frac{5e^4\left(cd^2-ae^2\right)(ae+cdx)^3}{3c^6d^6}+\frac{5e^3\left(cd^2-ae^2\right)^2(ae+cdx)^2}{c^6d^6}+\frac{10e^2x\left(cd^2-ae^2\right)^3}{c^5d^5}-\frac{\left(cd^2-ae^2\right)^5}{c^6d^6(ae+cdx)}+\frac{5e}{}$$

```
[Out] (10*e^2*(c*d^2 - a*e^2)^3*x)/(c^5*d^5) - (c*d^2 - a*e^2)^5/(c^6*d^6*(a*e +
c*d*x)) + (5*e^3*(c*d^2 - a*e^2)^2*(a*e + c*d*x)^2)/(c^6*d^6) + (5*e^4*(c*d
^2 - a*e^2)*(a*e + c*d*x)^3)/(3*c^6*d^6) + (e^5*(a*e + c*d*x)^4)/(4*c^6*d^6
) + (5*e*(c*d^2 - a*e^2)^4*Log[a*e + c*d*x])/(c^6*d^6)
```

Rubi [A] time = 0.197296, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$\frac{e^5(ae+cdx)^4}{4c^6d^6}+\frac{5e^4\left(cd^2-ae^2\right)(ae+cdx)^3}{3c^6d^6}+\frac{5e^3\left(cd^2-ae^2\right)^2(ae+cdx)^2}{c^6d^6}+\frac{10e^2x\left(cd^2-ae^2\right)^3}{c^5d^5}-\frac{\left(cd^2-ae^2\right)^5}{c^6d^6(ae+cdx)}+\frac{5e}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^7/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] (10*e^2*(c*d^2 - a*e^2)^3*x)/(c^5*d^5) - (c*d^2 - a*e^2)^5/(c^6*d^6*(a*e +
c*d*x)) + (5*e^3*(c*d^2 - a*e^2)^2*(a*e + c*d*x)^2)/(c^6*d^6) + (5*e^4*(c*d
^2 - a*e^2)*(a*e + c*d*x)^3)/(3*c^6*d^6) + (e^5*(a*e + c*d*x)^4)/(4*c^6*d^6
) + (5*e*(c*d^2 - a*e^2)^4*Log[a*e + c*d*x])/(c^6*d^6)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(d+ex)^7}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2} dx = \int \frac{(d+ex)^5}{(ae+cdx)^2} dx$$

$$= \int \left(\frac{10e^2\left(cd^2-ae^2\right)^3}{c^5d^5}+\frac{\left(cd^2-ae^2\right)^5}{c^5d^5(ae+cdx)^2}+\frac{5e\left(cd^2-ae^2\right)^4}{c^5d^5(ae+cdx)}+\frac{10e^3\left(cd^2-ae^2\right)^2}{c^5d^5}\right.$$

$$= \frac{10e^2\left(cd^2-ae^2\right)^3x}{c^5d^5}-\frac{\left(cd^2-ae^2\right)^5}{c^6d^6(ae+cdx)}+\frac{5e^3\left(cd^2-ae^2\right)^2(ae+cdx)^2}{c^6d^6}+\frac{5e^4\left(cd^2\right.}{}$$

Mathematica [A] time = 0.0858901, size = 263, normalized size = 1.43

$$\frac{30a^3c^2d^2e^6\left(4d^2+6dex-e^2x^2\right)-10a^2c^3d^3e^4\left(24d^2ex+12d^3-12de^2x^2-e^3x^3\right)-12a^4cde^8(5d+4ex)+12a^5e^{10}+5ac^4d^4e\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^7/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] (12*a^5*e^10 - 12*a^4*c*d*e^8*(5*d + 4*e*x) + 30*a^3*c^2*d^2*e^6*(4*d^2 + 6
*d*e*x - e^2*x^2) - 10*a^2*c^3*d^3*e^4*(12*d^3 + 24*d^2*e*x - 12*d*e^2*x^2
- e^3*x^3) + 5*a*c^4*d^4*e^2*(12*d^4 + 24*d^3*e*x - 36*d^2*e^2*x^2 - 8*d*e^
3*x^3 - e^4*x^4) + c^5*d^5*(-12*d^5 + 120*d^3*e^2*x^2 + 60*d^2*e^3*x^3 + 20
*d*e^4*x^4 + 3*e^5*x^5) + 60*e*(c*d^2 - a*e^2)^4*(a*e + c*d*x)*Log[a*e + c*
d*x])/(12*c^6*d^6*(a*e + c*d*x))
```

Maple [B] time = 0.049, size = 378, normalized size = 2.1

$$\frac{e^5x^4}{4c^2d^2}-\frac{2e^6x^3a}{3c^3d^3}+\frac{5e^4x^3}{3c^2d}+\frac{3e^7x^2a^2}{2c^4d^4}-5\frac{e^5x^2a}{c^3d^2}+5\frac{e^3x^2}{c^2}-4\frac{a^3e^8x}{c^5d^5}+15\frac{a^2e^6x}{c^4d^3}-20\frac{ae^4x}{c^3d}+10\frac{de^2x}{c^2}+\frac{a^5e^{10}}{c^6d^6(cdx+ae)}-5\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

```
[Out] 1/4*e^5/c^2/d^2*x^4-2/3*e^6/c^3/d^3*x^3*a+5/3*e^4/c^2/d*x^3+3/2*e^7/c^4/d^4
*x^2*a^2-5*e^5/c^3/d^2*x^2*a+5*e^3/c^2*x^2-4*e^8/c^5/d^5*a^3*x+15*e^6/c^4/d
^3*a^2*x-20*e^4/c^3/d*a*x+10*e^2/c^2*d*x+1/c^6/d^6/(c*d*x+a*e)*a^5*e^10-5/c
^5/d^4/(c*d*x+a*e)*a^4*e^8+10/c^4/d^2/(c*d*x+a*e)*a^3*e^6-10/c^3/(c*d*x+a*e
)*a^2*e^4+5/c^2*d^2/(c*d*x+a*e)*a*e^2-1/c*d^4/(c*d*x+a*e)+5/d^6*e^9/c^6*ln(
c*d*x+a*e)*a^4-20/d^4*e^7/c^5*ln(c*d*x+a*e)*a^3+30/d^2*e^5/c^4*ln(c*d*x+a*e
)*a^2-20*e^3/c^3*ln(c*d*x+a*e)*a+5*d^2*e/c^2*ln(c*d*x+a*e)
```

Maxima [A] time = 1.10539, size = 405, normalized size = 2.2

$$-\frac{c^5d^{10}-5ac^4d^8e^2+10a^2c^3d^6e^4-10a^3c^2d^4e^6+5a^4cd^2e^8-a^5e^{10}}{c^7d^7x+ac^6d^6e}+\frac{3c^3d^3e^5x^4+4\left(5c^3d^4e^4-2ac^2d^2e^6\right)x^3+6\left(10c^3d^5e\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="maxim
a")
```

```
[Out] -(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4*e^6 + 5*
a^4*c*d^2*e^8 - a^5*e^10)/(c^7*d^7*x + a*c^6*d^6*e) + 1/12*(3*c^3*d^3*e^5*x
^4 + 4*(5*c^3*d^4*e^4 - 2*a*c^2*d^2*e^6)*x^3 + 6*(10*c^3*d^5*e^3 - 10*a*c^2
*d^3*e^5 + 3*a^2*c*d*e^7)*x^2 + 12*(10*c^3*d^6*e^2 - 20*a*c^2*d^4*e^4 + 15*
a^2*c*d^2*e^6 - 4*a^3*e^8)*x)/(c^5*d^5) + 5*(c^4*d^8*e - 4*a*c^3*d^6*e^3 +
6*a^2*c^2*d^4*e^5 - 4*a^3*c*d^2*e^7 + a^4*e^9)*log(c*d*x + a*e)/(c^6*d^6)
```

Fricas [B] time = 1.96588, size = 841, normalized size = 4.57

$$\frac{3\,c^{5}d^{5}e^{5}x^{5} - 12\,c^{5}d^{10} + 60\,ac^{4}d^{8}e^{2} - 120\,a^{2}c^{3}d^{6}e^{4} + 120\,a^{3}c^{2}d^{4}e^{6} - 60\,a^{4}cd^{2}e^{8} + 12\,a^{5}e^{10} + 5\left(4\,c^{5}d^{6}e^{4} - ac^{4}d^{4}e^{6}\right)x^{4} + \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="frica
s")
```

```
[Out] 1/12*(3*c^5*d^5*e^5*x^5 - 12*c^5*d^10 + 60*a*c^4*d^8*e^2 - 120*a^2*c^3*d^6*
e^4 + 120*a^3*c^2*d^4*e^6 - 60*a^4*c*d^2*e^8 + 12*a^5*e^10 + 5*(4*c^5*d^6*e
^4 - a*c^4*d^4*e^6)*x^4 + 10*(6*c^5*d^7*e^3 - 4*a*c^4*d^5*e^5 + a^2*c^3*d^3
*e^7)*x^3 + 30*(4*c^5*d^8*e^2 - 6*a*c^4*d^6*e^4 + 4*a^2*c^3*d^4*e^6 - a^3*c
^2*d^2*e^8)*x^2 + 12*(10*a*c^4*d^7*e^3 - 20*a^2*c^3*d^5*e^5 + 15*a^3*c^2*d^
3*e^7 - 4*a^4*c*d*e^9)*x + 60*(a*c^4*d^8*e^2 - 4*a^2*c^3*d^6*e^4 + 6*a^3*c^
2*d^4*e^6 - 4*a^4*c*d^2*e^8 + a^5*e^10 + (c^5*d^9*e - 4*a*c^4*d^7*e^3 + 6*a
^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*c*d*e^9)*x)*log(c*d*x + a*e))/(c^7
*d^7*x + a*c^6*d^6*e)
```

Sympy [A] time = 1.58569, size = 265, normalized size = 1.44

$$\frac{a^{5}e^{10} - 5a^{4}cd^{2}e^{8} + 10a^{3}c^{2}d^{4}e^{6} - 10a^{2}c^{3}d^{6}e^{4} + 5ac^{4}d^{8}e^{2} - c^{5}d^{10}}{ac^{6}d^{6}e + c^{7}d^{7}x} + \frac{e^{5}x^{4}}{4c^{2}d^{2}} - \frac{x^{3}\left(2ae^{6} - 5cd^{2}e^{4}\right)}{3c^{3}d^{3}} + \frac{x^{2}\left(3a^{2}e^{7} - 10acd^{2}e^{5} \cdots\right)}{2c^{4}d^{4}} \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**7/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

```
[Out] (a**5*e**10 - 5*a**4*c*d**2*e**8 + 10*a**3*c**2*d**4*e**6 - 10*a**2*c**3*d*
*6*e**4 + 5*a*c**4*d**8*e**2 - c**5*d**10)/(a*c**6*d**6*e + c**7*d**7*x) +
e**5*x**4/(4*c**2*d**2) - x**3*(2*a*e**6 - 5*c*d**2*e**4)/(3*c**3*d**3) + x
**2*(3*a**2*e**7 - 10*a*c*d**2*e**5 + 10*c**2*d**4*e**3)/(2*c**4*d**4) - x*
(4*a**3*e**8 - 15*a**2*c*d**2*e**6 + 20*a*c**2*d**4*e**4 - 10*c**3*d**6*e**
2)/(c**5*d**5) + 5*e*(a*e**2 - c*d**2)**4*log(a*e + c*d*x)/(c**6*d**6)
```

Giac [B] time = 1.46234, size = 909, normalized size = 4.94

$$\frac{5\left(c^{7}d^{14}e - 7\,ac^{6}d^{12}e^{3} + 21\,a^{2}c^{5}d^{10}e^{5} - 35\,a^{3}c^{4}d^{8}e^{7} + 35\,a^{4}c^{3}d^{6}e^{9} - 21\,a^{5}c^{2}d^{4}e^{11} + 7\,a^{6}cd^{2}e^{13} - a^{7}e^{15}\right)\arctan\left(\frac{2\,cdxe \cdots}{\sqrt{-c^{2}d^{4} + \cdots}}\right)}{\left(c^{8}d^{10} - 2\,ac^{7}d^{8}e^{2} + a^{2}c^{6}d^{6}e^{4}\right)\sqrt{-c^{2}d^{4} + 2\,acd^{2}e^{2} - a^{2}e^{4}}} \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac"
)
```

```
[Out] 5*(c^7*d^14*e - 7*a*c^6*d^12*e^3 + 21*a^2*c^5*d^10*e^5 - 35*a^3*c^4*d^8*e^7
 + 35*a^4*c^3*d^6*e^9 - 21*a^5*c^2*d^4*e^11 + 7*a^6*c*d^2*e^13 - a^7*e^15)*
arctan((2*c*d*x*e + c*d^2 + a*e^2)/sqrt(-c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4)
)/((c^8*d^10 - 2*a*c^7*d^8*e^2 + a^2*c^6*d^6*e^4)*sqrt(-c^2*d^4 + 2*a*c*d^2
*e^2 - a^2*e^4)) + 5/2*(c^4*d^8*e - 4*a*c^3*d^6*e^3 + 6*a^2*c^2*d^4*e^5 - 4
*a^3*c*d^2*e^7 + a^4*e^9)*log(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)/(c^6*d
```

```
^6) - (c^7*d^15 - 7*a*c^6*d^13*e^2 + 21*a^2*c^5*d^11*e^4 - 35*a^3*c^4*d^9*e
^6 + 35*a^4*c^3*d^7*e^8 - 21*a^5*c^2*d^5*e^10 + 7*a^6*c*d^3*e^12 - a^7*d*e^
14 + (c^7*d^14*e - 7*a*c^6*d^12*e^3 + 21*a^2*c^5*d^10*e^5 - 35*a^3*c^4*d^8*
e^7 + 35*a^4*c^3*d^6*e^9 - 21*a^5*c^2*d^4*e^11 + 7*a^6*c*d^2*e^13 - a^7*e^1
5)*x)/((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*(c*d*x^2*e + c*d^2*x + a*x*e^2 +
 a*d*e)*c^6*d^6) + 1/12*(3*c^6*d^6*x^4*e^13 + 20*c^6*d^7*x^3*e^12 + 60*c^6*
d^8*x^2*e^11 + 120*c^6*d^9*x*e^10 - 8*a*c^5*d^5*x^3*e^14 - 60*a*c^5*d^6*x^2
*e^13 - 240*a*c^5*d^7*x*e^12 + 18*a^2*c^4*d^4*x^2*e^15 + 180*a^2*c^4*d^5*x*
e^14 - 48*a^3*c^3*d^3*x*e^16)*e^(-8)/(c^8*d^8)
```

3.1876 $\int \frac{(d+ex)^6}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2} dx$

Optimal. Leaf size=145

$$\frac{e^2x\left(3a^2e^4-8acd^2e^2+6c^2d^4\right)}{c^4d^4}+\frac{e^3x^2\left(2cd^2-ae^2\right)}{c^3d^3}-\frac{\left(cd^2-ae^2\right)^4}{c^5d^5(ae+cdx)}+\frac{4e\left(cd^2-ae^2\right)^3\log(ae+cdx)}{c^5d^5}+\frac{e^4x^3}{3c^2d^2}$$

```
[Out] (e^2*(6*c^2*d^4 - 8*a*c*d^2*e^2 + 3*a^2*e^4)*x)/(c^4*d^4) + (e^3*(2*c*d^2 -
 a*e^2)*x^2)/(c^3*d^3) + (e^4*x^3)/(3*c^2*d^2) - (c*d^2 - a*e^2)^4/(c^5*d^5
*(a*e + c*d*x)) + (4*e*(c*d^2 - a*e^2)^3*Log[a*e + c*d*x])/(c^5*d^5)
```

Rubi [A] time = 0.151851, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$\frac{e^2x\left(3a^2e^4-8acd^2e^2+6c^2d^4\right)}{c^4d^4}+\frac{e^3x^2\left(2cd^2-ae^2\right)}{c^3d^3}-\frac{\left(cd^2-ae^2\right)^4}{c^5d^5(ae+cdx)}+\frac{4e\left(cd^2-ae^2\right)^3\log(ae+cdx)}{c^5d^5}+\frac{e^4x^3}{3c^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^6/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] (e^2*(6*c^2*d^4 - 8*a*c*d^2*e^2 + 3*a^2*e^4)*x)/(c^4*d^4) + (e^3*(2*c*d^2 -
 a*e^2)*x^2)/(c^3*d^3) + (e^4*x^3)/(3*c^2*d^2) - (c*d^2 - a*e^2)^4/(c^5*d^5
*(a*e + c*d*x)) + (4*e*(c*d^2 - a*e^2)^3*Log[a*e + c*d*x])/(c^5*d^5)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^6}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2} dx &= \int \frac{(d+ex)^4}{(ae+cdx)^2} dx\\
&= \int \left(\frac{6c^2d^4e^2-8acd^2e^4+3a^2e^6}{c^4d^4}+\frac{2e^3\left(2cd^2-ae^2\right)x}{c^3d^3}+\frac{e^4x^2}{c^2d^2}+\frac{\left(cd^2-ae^2\right)^4}{c^4d^4(ae+cdx)^2}\right.\\
&= \frac{e^2\left(6c^2d^4-8acd^2e^2+3a^2e^4\right)x}{c^4d^4}+\frac{e^3\left(2cd^2-ae^2\right)x^2}{c^3d^3}+\frac{e^4x^3}{3c^2d^2}-\frac{\left(cd^2-ae^2\right)^4}{c^5d^5(ae+cdx)}
\end{aligned}$$

Mathematica [A] time = 0.0756049, size = 196, normalized size = 1.35

$$\frac{-6a^2c^2d^2e^4\left(3d^2+4dex-e^2x^2\right)+3a^3cde^6(4d+3ex)-3a^4e^8+2ac^3d^3e^2\left(9d^2ex+6d^3-9de^2x^2-e^3x^3\right)-12e\left(ae^2-cd^2\right)^3}{3c^5d^5(ae+cdx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^6/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] (-3*a^4*e^8 + 3*a^3*c*d*e^6*(4*d + 3*e*x) - 6*a^2*c^2*d^2*e^4*(3*d^2 + 4*d*
e*x - e^2*x^2) + 2*a*c^3*d^3*e^2*(6*d^3 + 9*d^2*e*x - 9*d*e^2*x^2 - e^3*x^3
) + c^4*d^4*(-3*d^4 + 18*d^2*e^2*x^2 + 6*d*e^3*x^3 + e^4*x^4) - 12*e*(-(c*d
^2) + a*e^2)^3*(a*e + c*d*x)*Log[a*e + c*d*x])/(3*c^5*d^5*(a*e + c*d*x))
```

Maple [A] time = 0.048, size = 275, normalized size = 1.9

$$\frac{e^4x^3}{3\,c^2d^2}-\frac{e^5x^2a}{c^3d^3}+2\,\frac{e^3x^2}{c^2d}+3\,\frac{a^2e^6x}{c^4d^4}-8\,\frac{ae^4x}{c^3d^2}+6\,\frac{e^2x}{c^2}-\frac{a^4e^8}{c^5d^5\,(cdx+ae)}+4\,\frac{a^3e^6}{c^4d^3\,(cdx+ae)}-6\,\frac{a^2e^4}{c^3d\,(cdx+ae)}+4\,\frac{ade^2}{c^2\,(cdx+ae)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

```
[Out] 1/3*e^4*x^3/c^2/d^2-e^5/c^3/d^3*x^2*a+2*e^3/c^2/d*x^2+3*e^6/c^4/d^4*a^2*x-8
*e^4/c^3/d^2*a*x+6*e^2/c^2*x-1/c^5/d^5/(c*d*x+a*e)*a^4*e^8+4/c^4/d^3/(c*d*x
+a*e)*a^3*e^6-6/c^3/d/(c*d*x+a*e)*a^2*e^4+4/c^2*d/(c*d*x+a*e)*a*e^2-1/c*d^3
/(c*d*x+a*e)-4/d^5*e^7/c^5*ln(c*d*x+a*e)*a^3+12/d^3*e^5/c^4*ln(c*d*x+a*e)*a
^2-12/d*e^3/c^3*ln(c*d*x+a*e)*a+4*d*e/c^2*ln(c*d*x+a*e)
```

Maxima [A] time = 1.032, size = 289, normalized size = 1.99

$$-\frac{c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8}{c^6d^6x+ac^5d^5e}+\frac{c^2d^2e^4x^3+3\left(2\,c^2d^3e^3-acde^5\right)x^2+3\left(6\,c^2d^4e^2-8\,acd^2e^4+3\,a^2e^6\right)x}{3\,c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="maxim
a")
```

```
[Out] -(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8
)/(c^6*d^6*x + a*c^5*d^5*e) + 1/3*(c^2*d^2*e^4*x^3 + 3*(2*c^2*d^3*e^3 - a*c
*d*e^5)*x^2 + 3*(6*c^2*d^4*e^2 - 8*a*c*d^2*e^4 + 3*a^2*e^6)*x)/(c^4*d^4) +
4*(c^3*d^6*e - 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 - a^3*e^7)*log(c*d*x + a*e
)/(c^5*d^5)
```

Fricas [B] time = 1.94051, size = 602, normalized size = 4.15

$$c^4d^4e^4x^4-3\,c^4d^8+12\,ac^3d^6e^2-18\,a^2c^2d^4e^4+12\,a^3cd^2e^6-3\,a^4e^8+2\left(3\,c^4d^5e^3-ac^3d^3e^5\right)x^3+6\left(3\,c^4d^6e^2-3\,ac^3d^4e^4+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="frica
s")
```

```
[Out] 1/3*(c^4*d^4*e^4*x^4 - 3*c^4*d^8 + 12*a*c^3*d^6*e^2 - 18*a^2*c^2*d^4*e^4 +
12*a^3*c*d^2*e^6 - 3*a^4*e^8 + 2*(3*c^4*d^5*e^3 - a*c^3*d^3*e^5)*x^3 + 6*(3
*c^4*d^6*e^2 - 3*a*c^3*d^4*e^4 + a^2*c^2*d^2*e^6)*x^2 + 3*(6*a*c^3*d^5*e^3
- 8*a^2*c^2*d^3*e^5 + 3*a^3*c*d*e^7)*x + 12*(a*c^3*d^6*e^2 - 3*a^2*c^2*d^4*
e^4 + 3*a^3*c*d^2*e^6 - a^4*e^8 + (c^4*d^7*e - 3*a*c^3*d^5*e^3 + 3*a^2*c^2*
d^3*e^5 - a^3*c*d*e^7)*x)*log(c*d*x + a*e))/(c^6*d^6*x + a*c^5*d^5*e)
```

Sympy [A] time = 1.31122, size = 187, normalized size = 1.29

$$-\frac{a^4e^8 - 4a^3cd^2e^6 + 6a^2c^2d^4e^4 - 4ac^3d^6e^2 + c^4d^8}{ac^5d^5e + c^6d^6x} + \frac{e^4x^3}{3c^2d^2} - \frac{x^2\left(ae^5 - 2cd^2e^3\right)}{c^3d^3} + \frac{x\left(3a^2e^6 - 8acd^2e^4 + 6c^2d^4e^2\right)}{c^4d^4} - \frac{4e\left(a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**6/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

```
[Out] -(a**4*e**8 - 4*a**3*c*d**2*e**6 + 6*a**2*c**2*d**4*e**4 - 4*a*c**3*d**6*e*
*2 + c**4*d**8)/(a*c**5*d**5*e + c**6*d**6*x) + e**4*x**3/(3*c**2*d**2) - x
**2*(a*e**5 - 2*c*d**2*e**3)/(c**3*d**3) + x*(3*a**2*e**6 - 8*a*c*d**2*e**4
 + 6*c**2*d**4*e**2)/(c**4*d**4) - 4*e*(a*e**2 - c*d**2)**3*log(a*e + c*d*x
)/(c**5*d**5)
```

Giac [B] time = 1.47024, size = 757, normalized size = 5.22

$$\frac{4\left(c^6d^{12}e - 6\,ac^5d^{10}e^3 + 15\,a^2c^4d^8e^5 - 20\,a^3c^3d^6e^7 + 15\,a^4c^2d^4e^9 - 6\,a^5cd^2e^{11} + a^6e^{13}\right)\arctan\left(\frac{2\,cdxe + cd^2 + ae^2}{\sqrt{-c^2d^4 + 2\,acd^2e^2 - a^2e^4}}\right)}{\left(c^7d^9 - 2\,ac^6d^7e^2 + a^2c^5d^5e^4\right)\sqrt{-c^2d^4 + 2\,acd^2e^2 - a^2e^4}} + \frac{2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac"
)
```

```
[Out] 4*(c^6*d^12*e - 6*a*c^5*d^10*e^3 + 15*a^2*c^4*d^8*e^5 - 20*a^3*c^3*d^6*e^7
+ 15*a^4*c^2*d^4*e^9 - 6*a^5*c*d^2*e^11 + a^6*e^13)*arctan((2*c*d*x*e + c*d
^2 + a*e^2)/sqrt(-c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4))/((c^7*d^9 - 2*a*c^6*d
^7*e^2 + a^2*c^5*d^5*e^4)*sqrt(-c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4)) + 2*(c^
3*d^6*e - 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 - a^3*e^7)*log(c*d*x^2*e + c*d^
2*x + a*x*e^2 + a*d*e)/(c^5*d^5) + 1/3*(c^4*d^4*x^3*e^10 + 6*c^4*d^5*x^2*e^
9 + 18*c^4*d^6*x*e^8 - 3*a*c^3*d^3*x^2*e^11 - 24*a*c^3*d^4*x*e^10 + 9*a^2*c
^2*d^2*x*e^12)*e^(-6)/(c^6*d^6) - (c^6*d^13 - 6*a*c^5*d^11*e^2 + 15*a^2*c^4
*d^9*e^4 - 20*a^3*c^3*d^7*e^6 + 15*a^4*c^2*d^5*e^8 - 6*a^5*c*d^3*e^10 + a^6
*d*e^12 + (c^6*d^12*e - 6*a*c^5*d^10*e^3 + 15*a^2*c^4*d^8*e^5 - 20*a^3*c^3*
d^6*e^7 + 15*a^4*c^2*d^4*e^9 - 6*a^5*c*d^2*e^11 + a^6*e^13)*x)/((c^2*d^4 -
2*a*c*d^2*e^2 + a^2*e^4)*(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*c^5*d^5)
```

3.1877 $\int \frac{(d+ex)^5}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$

Optimal. Leaf size=105

$$\frac{e^2x\left(3cd^2-2ae^2\right)}{c^3d^3}-\frac{\left(cd^2-ae^2\right)^3}{c^4d^4(ae+cdx)}+\frac{3e\left(cd^2-ae^2\right)^2\log(ae+cdx)}{c^4d^4}+\frac{e^3x^2}{2c^2d^2}$$

```
[Out] (e^2*(3*c*d^2 - 2*a*e^2)*x)/(c^3*d^3) + (e^3*x^2)/(2*c^2*d^2) - (c*d^2 - a*
e^2)^3/(c^4*d^4*(a*e + c*d*x)) + (3*e*(c*d^2 - a*e^2)^2*Log[a*e + c*d*x])/(
c^4*d^4)
```

Rubi [A] time = 0.0928664, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$\frac{e^2x\left(3cd^2-2ae^2\right)}{c^3d^3}-\frac{\left(cd^2-ae^2\right)^3}{c^4d^4(ae+cdx)}+\frac{3e\left(cd^2-ae^2\right)^2\log(ae+cdx)}{c^4d^4}+\frac{e^3x^2}{2c^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] (e^2*(3*c*d^2 - 2*a*e^2)*x)/(c^3*d^3) + (e^3*x^2)/(2*c^2*d^2) - (c*d^2 - a*
e^2)^3/(c^4*d^4*(a*e + c*d*x)) + (3*e*(c*d^2 - a*e^2)^2*Log[a*e + c*d*x])/(
c^4*d^4)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^5}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx &= \int \frac{(d+ex)^3}{(ae+cdx)^2}\,dx\\
&= \int \left(\frac{3cd^2e^2-2ae^4}{c^3d^3}+\frac{e^3x}{c^2d^2}+\frac{\left(cd^2-ae^2\right)^3}{c^3d^3(ae+cdx)^2}+\frac{3e\left(cd^2-ae^2\right)^2}{c^3d^3(ae+cdx)}\right)dx\\
&= \frac{e^2\left(3cd^2-2ae^2\right)x}{c^3d^3}+\frac{e^3x^2}{2c^2d^2}-\frac{\left(cd^2-ae^2\right)^3}{c^4d^4(ae+cdx)}+\frac{3e\left(cd^2-ae^2\right)^2\log(ae+cdx)}{c^4d^4}
\end{aligned}$$

Mathematica [A] time = 0.0475453, size = 142, normalized size = 1.35

$$\frac{-3a^2cd^2e^4+a^3e^6+3ac^2d^4e^2-c^3d^6}{c^4d^4(ae+cdx)}+\frac{3\left(a^2e^5-2acd^2e^3+c^2d^4e\right)\log(ae+cdx)}{c^4d^4}-\frac{e^2x\left(2ae^2-3cd^2\right)}{c^3d^3}+\frac{e^3x^2}{2c^2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] -((e^2*(-3*c*d^2 + 2*a*e^2)*x)/(c^3*d^3)) + (e^3*x^2)/(2*c^2*d^2) + (-(c^3*
d^6) + 3*a*c^2*d^4*e^2 - 3*a^2*c*d^2*e^4 + a^3*e^6)/(c^4*d^4*(a*e + c*d*x))
 + (3*(c^2*d^4*e - 2*a*c*d^2*e^3 + a^2*e^5)*Log[a*e + c*d*x])/(c^4*d^4)
```

Maple [A] time = 0.047, size = 184, normalized size = 1.8

$$\frac{e^3x^2}{2c^2d^2}-2\frac{e^4ax}{c^3d^3}+3\frac{e^2x}{c^2d}+\frac{a^3e^6}{c^4d^4(cdx+ae)}-3\frac{a^2e^4}{c^3d^2(cdx+ae)}+3\frac{ae^2}{c^2(cdx+ae)}-\frac{d^2}{c(cdx+ae)}+3\frac{e^5\ln(cdx+ae)a^2}{c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

```
[Out] 1/2*e^3*x^2/c^2/d^2-2*e^4/c^3/d^3*a*x+3*e^2/c^2/d*x+1/c^4/d^4/(c*d*x+a*e)*a
^3*e^6-3/c^3/d^2/(c*d*x+a*e)*a^2*e^4+3/c^2/(c*d*x+a*e)*a*e^2-1/c*d^2/(c*d*x
+a*e)+3/d^4*e^5/c^4*ln(c*d*x+a*e)*a^2-6/d^2*e^3/c^3*ln(c*d*x+a*e)*a+3*e/c^2
*ln(c*d*x+a*e)
```

Maxima [A] time = 1.033, size = 193, normalized size = 1.84

$$-\frac{c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6}{c^5d^5x+ac^4d^4e}+\frac{cde^3x^2+2\left(3\,cd^2e^2-2\,ae^4\right)x}{2\,c^3d^3}+\frac{3\left(c^2d^4e-2\,acd^2e^3+a^2e^5\right)\log(cdx+ae)}{c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="maxim
a")
```

```
[Out] -(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)/(c^5*d^5*x + a*c^4
*d^4*e) + 1/2*(c*d*e^3*x^2 + 2*(3*c*d^2*e^2 - 2*a*e^4)*x)/(c^3*d^3) + 3*(c^
2*d^4*e - 2*a*c*d^2*e^3 + a^2*e^5)*log(c*d*x + a*e)/(c^4*d^4)
```

Fricas [A] time = 1.82145, size = 408, normalized size = 3.89

$$\frac{c^3d^3e^3x^3-2\,c^3d^6+6\,ac^2d^4e^2-6\,a^2cd^2e^4+2\,a^3e^6+3\left(2\,c^3d^4e^2-ac^2d^2e^4\right)x^2+2\left(3\,ac^2d^3e^3-2\,a^2cde^5\right)x+6\left(ac^2d^4e^2\right.}{2\left(c^5d^5x+ac^4d^4e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="frica
s")
```

```
[Out] 1/2*(c^3*d^3*e^3*x^3 - 2*c^3*d^6 + 6*a*c^2*d^4*e^2 - 6*a^2*c*d^2*e^4 + 2*a^
3*e^6 + 3*(2*c^3*d^4*e^2 - a*c^2*d^2*e^4)*x^2 + 2*(3*a*c^2*d^3*e^3 - 2*a^2*
c*d*e^5)*x + 6*(a*c^2*d^4*e^2 - 2*a^2*c*d^2*e^4 + a^3*e^6 + (c^3*d^5*e - 2*
a*c^2*d^3*e^3 + a^2*c*d*e^5)*x)*log(c*d*x + a*e))/(c^5*d^5*x + a*c^4*d^4*e)
```

Sympy [A] time = 1.08642, size = 131, normalized size = 1.25

$$\frac{a^3e^6 - 3a^2cd^2e^4 + 3ac^2d^4e^2 - c^3d^6}{ac^4d^4e + c^5d^5x} + \frac{e^3x^2}{2c^2d^2} - \frac{x\left(2ae^4 - 3cd^2e^2\right)}{c^3d^3} + \frac{3e\left(ae^2 - cd^2\right)^2\log\left(ae + cdx\right)}{c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

```
[Out] (a**3*e**6 - 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 - c**3*d**6)/(a*c**4*d
**4*e + c**5*d**5*x) + e**3*x**2/(2*c**2*d**2) - x*(2*a*e**4 - 3*c*d**2*e**
2)/(c**3*d**3) + 3*e*(a*e**2 - c*d**2)**2*log(a*e + c*d*x)/(c**4*d**4)
```

Giac [B] time = 1.37179, size = 629, normalized size = 5.99

$$\frac{3\left(c^5d^{10}e - 5\,ac^4d^8e^3 + 10\,a^2c^3d^6e^5 - 10\,a^3c^2d^4e^7 + 5\,a^4cd^2e^9 - a^5e^{11}\right)\arctan\left(\frac{2\,cdxe + cd^2 + ae^2}{\sqrt{-c^2d^4 + 2\,acd^2e^2 - a^2e^4}}\right)}{\left(c^6d^8 - 2\,ac^5d^6e^2 + a^2c^4d^4e^4\right)\sqrt{-c^2d^4 + 2\,acd^2e^2 - a^2e^4}} + \frac{\left(c^2d^2x^2e^7 + 6\,c^2d^3xe\right.}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac"
)
```

```
[Out] 3*(c^5*d^10*e - 5*a*c^4*d^8*e^3 + 10*a^2*c^3*d^6*e^5 - 10*a^3*c^2*d^4*e^7 +
 5*a^4*c*d^2*e^9 - a^5*e^11)*arctan((2*c*d*x*e + c*d^2 + a*e^2)/sqrt(-c^2*d
^4 + 2*a*c*d^2*e^2 - a^2*e^4))/((c^6*d^8 - 2*a*c^5*d^6*e^2 + a^2*c^4*d^4*e^
4)*sqrt(-c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4)) + 1/2*(c^2*d^2*x^2*e^7 + 6*c^2
*d^3*x*e^6 - 4*a*c*d*x*e^8)*e^(-4)/(c^4*d^4) + 3/2*(c^2*d^4*e - 2*a*c*d^2*e
^3 + a^2*e^5)*log(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)/(c^4*d^4) - (c^5*d
^11 - 5*a*c^4*d^9*e^2 + 10*a^2*c^3*d^7*e^4 - 10*a^3*c^2*d^5*e^6 + 5*a^4*c*d
^3*e^8 - a^5*d*e^10 + (c^5*d^10*e - 5*a*c^4*d^8*e^3 + 10*a^2*c^3*d^6*e^5 -
10*a^3*c^2*d^4*e^7 + 5*a^4*c*d^2*e^9 - a^5*e^11)*x)/((c^2*d^4 - 2*a*c*d^2*e
^2 + a^2*e^4)*(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*c^4*d^4)
```

3.1878 $\int \frac{(d+ex)^4}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$

Optimal. Leaf size=74

$$-\frac{\left(cd^2-ae^2\right)^2}{c^3d^3(ae+cdx)}+\frac{2e\left(cd^2-ae^2\right)\log(ae+cdx)}{c^3d^3}+\frac{e^2x}{c^2d^2}$$

```
[Out] (e^2*x)/(c^2*d^2) - (c*d^2 - a*e^2)^2/(c^3*d^3*(a*e + c*d*x)) + (2*e*(c*d^2
 - a*e^2)*Log[a*e + c*d*x])/(c^3*d^3)
```

Rubi [A] time = 0.0591841, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$-\frac{\left(cd^2-ae^2\right)^2}{c^3d^3(ae+cdx)}+\frac{2e\left(cd^2-ae^2\right)\log(ae+cdx)}{c^3d^3}+\frac{e^2x}{c^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] (e^2*x)/(c^2*d^2) - (c*d^2 - a*e^2)^2/(c^3*d^3*(a*e + c*d*x)) + (2*e*(c*d^2
 - a*e^2)*Log[a*e + c*d*x])/(c^3*d^3)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^4}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx &= \int \frac{(d+ex)^2}{(ae+cdx)^2}\,dx\\
&= \int \left(\frac{e^2}{c^2d^2}+\frac{\left(cd^2-ae^2\right)^2}{c^2d^2(ae+cdx)^2}+\frac{2\left(cd^2e-ae^3\right)}{c^2d^2(ae+cdx)}\right)dx\\
&= \frac{e^2x}{c^2d^2}-\frac{\left(cd^2-ae^2\right)^2}{c^3d^3(ae+cdx)}+\frac{2e\left(cd^2-ae^2\right)\log(ae+cdx)}{c^3d^3}
\end{aligned}$$

Mathematica [A] time = 0.0461451, size = 65, normalized size = 0.88

$$\frac{-\frac{\left(cd^2-ae^2\right)^2}{ae+cdx}+2\left(cd^2e-ae^3\right)\log(ae+cdx)+cde^2x}{c^3d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] (c*d*e^2*x - (c*d^2 - a*e^2)^2/(a*e + c*d*x) + 2*(c*d^2*e - a*e^3)*Log[a*e
+ c*d*x])/(c^3*d^3)
```

Maple [A] time = 0.053, size = 114, normalized size = 1.5

$$\frac{e^2x}{c^2d^2} - \frac{a^2e^4}{c^3d^3\left(cdx+ae\right)} + 2\frac{ae^2}{c^2d\left(cdx+ae\right)} - \frac{d}{c\left(cdx+ae\right)} - 2\frac{e^3\ln\left(cdx+ae\right)a}{c^3d^3} + 2\frac{e\ln\left(cdx+ae\right)}{c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

```
[Out] e^2*x/c^2/d^2-1/c^3/d^3/(c*d*x+a*e)*a^2*e^4+2/c^2/d/(c*d*x+a*e)*a*e^2-1/c*d
/(c*d*x+a*e)-2/d^3*e^3/c^3*ln(c*d*x+a*e)*a+2/d*e/c^2*ln(c*d*x+a*e)
```

Maxima [A] time = 1.02031, size = 120, normalized size = 1.62

$$-\frac{c^2d^4 - 2\,acd^2e^2 + a^2e^4}{c^4d^4x + ac^3d^3e} + \frac{e^2x}{c^2d^2} + \frac{2\left(cd^2e - ae^3\right)\log\left(cdx+ae\right)}{c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="maxim
a")
```

```
[Out] -(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(c^4*d^4*x + a*c^3*d^3*e) + e^2*x/(c^2
*d^2) + 2*(c*d^2*e - a*e^3)*log(c*d*x + a*e)/(c^3*d^3)
```

Fricas [A] time = 1.92026, size = 227, normalized size = 3.07

$$\frac{c^2d^2e^2x^2 + acde^3x - c^2d^4 + 2\,acd^2e^2 - a^2e^4 + 2\left(acd^2e^2 - a^2e^4 + \left(c^2d^3e - acde^3\right)x\right)\log\left(cdx+ae\right)}{c^4d^4x + ac^3d^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="frica
s")
```

```
[Out] (c^2*d^2*e^2*x^2 + a*c*d*e^3*x - c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4 + 2*(a*c
*d^2*e^2 - a^2*e^4 + (c^2*d^3*e - a*c*d*e^3)*x)*log(c*d*x + a*e))/(c^4*d^4*
x + a*c^3*d^3*e)
```

Sympy [A] time = 0.68862, size = 85, normalized size = 1.15

$$-\frac{a^2e^4 - 2acd^2e^2 + c^2d^4}{ac^3d^3e + c^4d^4x} + \frac{e^2x}{c^2d^2} - \frac{2e\left(ae^2 - cd^2\right)\log\left(ae+cdx\right)}{c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

```
[Out] -(a**2*e**4 - 2*a*c*d**2*e**2 + c**2*d**4)/(a*c**3*d**3*e + c**4*d**4*x) +
e**2*x/(c**2*d**2) - 2*e*(a*e**2 - c*d**2)*log(a*e + c*d*x)/(c**3*d**3)
```

Giac [B] time = 1.29919, size = 525, normalized size = 7.09

$$\frac{2\left(c^4d^8e-4\,ac^3d^6e^3+6\,a^2c^2d^4e^5-4\,a^3cd^2e^7+a^4e^9\right)\arctan\left(\frac{2\,cdxe+cd^2+ae^2}{\sqrt{-c^2d^4+2\,acd^2e^2-a^2e^4}}\right)}{\left(c^5d^7-2\,ac^4d^5e^2+a^2c^3d^3e^4\right)\sqrt{-c^2d^4+2\,acd^2e^2-a^2e^4}}+\frac{xe^2}{c^2d^2}+\frac{\left(cd^2e-ae^3\right)\log\left(cdx^2e+c\right.}{c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac"
)
```

```
[Out] 2*(c^4*d^8*e - 4*a*c^3*d^6*e^3 + 6*a^2*c^2*d^4*e^5 - 4*a^3*c*d^2*e^7 + a^4*
e^9)*arctan((2*c*d*x*e + c*d^2 + a*e^2)/sqrt(-c^2*d^4 + 2*a*c*d^2*e^2 - a^2
*e^4))/((c^5*d^7 - 2*a*c^4*d^5*e^2 + a^2*c^3*d^3*e^4)*sqrt(-c^2*d^4 + 2*a*c
*d^2*e^2 - a^2*e^4)) + x*e^2/(c^2*d^2) + (c*d^2*e - a*e^3)*log(c*d*x^2*e +
c*d^2*x + a*x*e^2 + a*d*e)/(c^3*d^3) - ((c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*
c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)/c + (c^4*d^8*e - 4*a*c^3*d^6*e^3 +
 6*a^2*c^2*d^4*e^5 - 4*a^3*c*d^2*e^7 + a^4*e^9)*x/(c*d))/((c^2*d^4 - 2*a*c*
d^2*e^2 + a^2*e^4)*(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*c^2*d^2)
```

3.1879 $\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$

Optimal. Leaf size=48

$$\frac{e\log(ae+cdx)}{c^2d^2}-\frac{cd^2-ae^2}{c^2d^2(ae+cdx)}$$

```
[Out] -((c*d^2 - a*e^2)/(c^2*d^2*(a*e + c*d*x))) + (e*Log[a*e + c*d*x])/(c^2*d^2)
```

Rubi [A] time = 0.0363662, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$\frac{e\log(ae+cdx)}{c^2d^2}-\frac{cd^2-ae^2}{c^2d^2(ae+cdx)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] -((c*d^2 - a*e^2)/(c^2*d^2*(a*e + c*d*x))) + (e*Log[a*e + c*d*x])/(c^2*d^2)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx &= \int \frac{d+ex}{(ae+cdx)^2}\,dx\\
&= \int \left(\frac{cd^2-ae^2}{cd(ae+cdx)^2}+\frac{e}{cd(ae+cdx)}\right)dx\\
&= -\frac{cd^2-ae^2}{c^2d^2(ae+cdx)}+\frac{e\log(ae+cdx)}{c^2d^2}
\end{aligned}$$

Mathematica [A] time = 0.012727, size = 47, normalized size = 0.98

$$\frac{ae^2-cd^2}{c^2d^2(ae+cdx)}+\frac{e\log(ae+cdx)}{c^2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] (-(c*d^2) + a*e^2)/(c^2*d^2*(a*e + c*d*x)) + (e*Log[a*e + c*d*x])/(c^2*d^2)
```

Maple [A] time = 0.045, size = 55, normalized size = 1.2

$$\frac{ae^2}{c^2d^2\left(cdx+ae\right)}-\frac{1}{c\left(cdx+ae\right)}+\frac{e\ln\left(cdx+ae\right)}{c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

```
[Out] 1/c^2/d^2/(c*d*x+a*e)*a*e^2-1/c/(c*d*x+a*e)+e*ln(c*d*x+a*e)/c^2/d^2
```

Maxima [A] time = 1.02123, size = 70, normalized size = 1.46

$$-\frac{cd^2-ae^2}{c^3d^3x+ac^2d^2e}+\frac{e\log\left(cdx+ae\right)}{c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="maxim
a")
```

```
[Out] -(c*d^2 - a*e^2)/(c^3*d^3*x + a*c^2*d^2*e) + e*log(c*d*x + a*e)/(c^2*d^2)
```

Fricas [A] time = 1.88916, size = 109, normalized size = 2.27

$$-\frac{cd^2-ae^2-\left(cdex+ae^2\right)\log\left(cdx+ae\right)}{c^3d^3x+ac^2d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="frica
s")
```

```
[Out] -(c*d^2 - a*e^2 - (c*d*e*x + a*e^2)*log(c*d*x + a*e))/(c^3*d^3*x + a*c^2*d^
2*e)
```

Sympy [A] time = 0.444943, size = 46, normalized size = 0.96

$$\frac{ae^2-cd^2}{ac^2d^2e+c^3d^3x}+\frac{e\log\left(ae+cdx\right)}{c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

```
[Out] (a*e**2 - c*d**2)/(a*c**2*d**2*e + c**3*d**3*x) + e*log(a*e + c*d*x)/(c**2*
d**2)
```

Giac [B] time = 1.20888, size = 435, normalized size = 9.06

$$\frac{\left(c^3d^6e-3\,ac^2d^4e^3+3\,a^2cd^2e^5-a^3e^7\right)\arctan\left(\frac{2\,cdxe+cd^2+ae^2}{\sqrt{-c^2d^4+2\,acd^2e^2-a^2e^4}}\right)}{\left(c^4d^6-2\,ac^3d^4e^2+a^2c^2d^2e^4\right)\sqrt{-c^2d^4+2\,acd^2e^2-a^2e^4}}+\frac{e\log\left(cdx^2e+cd^2x+axe^2+ade\right)}{2\,c^2d^2}-\frac{c^3d^7-3\,ac^2d^5e}{\left(c^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac"
)
```

```
[Out] (c^3*d^6*e - 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 - a^3*e^7)*arctan((2*c*d*x*e
 + c*d^2 + a*e^2)/sqrt(-c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4))/((c^4*d^6 - 2*a
*c^3*d^4*e^2 + a^2*c^2*d^2*e^4)*sqrt(-c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4)) +
 1/2*e*log(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)/(c^2*d^2) - (c^3*d^7 - 3*
a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 - a^3*d*e^6 + (c^3*d^6*e - 3*a*c^2*d^4*e^3
+ 3*a^2*c*d^2*e^5 - a^3*e^7)*x)/((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*(c*d*x
^2*e + c*d^2*x + a*x*e^2 + a*d*e)*c^2*d^2)
```

3.1880 $\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2} dx$

Optimal. Leaf size=18

$$-\frac{1}{cd(ae+cdx)}$$

[Out] -(1/(c*d*(a*e + c*d*x)))

Rubi [A] time = 0.0098234, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 32}

$$-\frac{1}{cd(ae+cdx)}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]

[Out] -(1/(c*d*(a*e + c*d*x)))

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2} \, dx &= \int \frac{1}{(ae+cdx)^2} \, dx \\ &= -\frac{1}{cd(ae+cdx)} \end{aligned}$$

Mathematica [A] time = 0.0038684, size = 18, normalized size = 1.

$$-\frac{1}{cd(ae+cdx)}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]

[Out] -(1/(c*d*(a*e + c*d*x)))

Maple [A] time = 0.039, size = 19, normalized size = 1.1

$$-\frac{1}{cd\,(cdx+ae)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

```
[Out] -1/c/d/(c*d*x+a*e)
```

Maxima [A] time = 1.01625, size = 24, normalized size = 1.33

$$-\frac{1}{c^2d^2x+acde}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="maxim
a")
```

```
[Out] -1/(c^2*d^2*x + a*c*d*e)
```

Fricas [A] time = 1.73763, size = 35, normalized size = 1.94

$$-\frac{1}{c^2d^2x+acde}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="frica
s")
```

```
[Out] -1/(c^2*d^2*x + a*c*d*e)
```

Sympy [A] time = 0.372318, size = 17, normalized size = 0.94

$$-\frac{1}{acde+c^2d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

```
[Out] -1/(a*c*d*e + c**2*d**2*x)
```

Giac [B] time = 1.19251, size = 147, normalized size = 8.17

$$-\frac{c^2d^4xe+c^2d^5-2\,acd^2xe^3-2\,acd^3e^2+a^2xe^5+a^2de^4}{\left(c^3d^5-2\,ac^2d^3e^2+a^2cde^4\right)\left(cdx^2e+cd^2x+axe^2+ade\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac"
)
```

```
[Out] -(c^2*d^4*x*e + c^2*d^5 - 2*a*c*d^2*x*e^3 - 2*a*c*d^3*e^2 + a^2*x*e^5 + a^2
*d*e^4)/((c^3*d^5 - 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*(c*d*x^2*e + c*d^2*x + a
*x*e^2 + a*d*e))
```

3.1881 $\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$

Optimal. Leaf size=75

$$-\frac{1}{\left(cd^2-ae^2\right)(ae+cdx)}-\frac{e\log(ae+cdx)}{\left(cd^2-ae^2\right)^2}+\frac{e\log(d+ex)}{\left(cd^2-ae^2\right)^2}$$

```
[Out] -(1/((c*d^2 - a*e^2)*(a*e + c*d*x))) - (e*Log[a*e + c*d*x])/(c*d^2 - a*e^2)
^2 + (e*Log[d + e*x])/(c*d^2 - a*e^2)^2
```

Rubi [A] time = 0.0446562, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {626, 44}

$$-\frac{1}{\left(cd^2-ae^2\right)(ae+cdx)}-\frac{e\log(ae+cdx)}{\left(cd^2-ae^2\right)^2}+\frac{e\log(d+ex)}{\left(cd^2-ae^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] -(1/((c*d^2 - a*e^2)*(a*e + c*d*x))) - (e*Log[a*e + c*d*x])/(c*d^2 - a*e^2)
^2 + (e*Log[d + e*x])/(c*d^2 - a*e^2)^2
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx &= \int \frac{1}{(ae+cdx)^2(d+ex)}\,dx\\
&= \int \left(\frac{cd}{\left(cd^2-ae^2\right)(ae+cdx)^2}-\frac{cde}{\left(cd^2-ae^2\right)^2(ae+cdx)}+\frac{e^2}{\left(cd^2-ae^2\right)^2(d+ex)}\right)dx\\
&= -\frac{1}{\left(cd^2-ae^2\right)(ae+cdx)}-\frac{e\log(ae+cdx)}{\left(cd^2-ae^2\right)^2}+\frac{e\log(d+ex)}{\left(cd^2-ae^2\right)^2}
\end{aligned}$$

Mathematica [A] time = 0.0297919, size = 74, normalized size = 0.99

$$\frac{1}{\left(ae^2-cd^2\right)(ae+cdx)}-\frac{e\log(ae+cdx)}{\left(ae^2-cd^2\right)^2}+\frac{e\log(d+ex)}{\left(ae^2-cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] 1/((-(c*d^2) + a*e^2)*(a*e + c*d*x)) - (e*Log[a*e + c*d*x])/(-(c*d^2) + a*e
^2)^2 + (e*Log[d + e*x])/(-(c*d^2) + a*e^2)^2
```

Maple [A] time = 0.074, size = 75, normalized size = 1.

$$\frac{e\ln(ex+d)}{(ae^2-cd^2)^2}+\frac{1}{(ae^2-cd^2)(cdx+ae)}-\frac{e\ln(cdx+ae)}{(ae^2-cd^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

```
[Out] e/(a*e^2-c*d^2)^2*ln(e*x+d)+1/(a*e^2-c*d^2)/(c*d*x+a*e)-e/(a*e^2-c*d^2)^2*l
n(c*d*x+a*e)
```

Maxima [A] time = 1.02519, size = 153, normalized size = 2.04

$$-\frac{e\log(cdx+ae)}{c^2d^4-2\,acd^2e^2+a^2e^4}+\frac{e\log(ex+d)}{c^2d^4-2\,acd^2e^2+a^2e^4}-\frac{1}{acd^2e-a^2e^3+(c^2d^3-acde^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="maxima"
)
```

```
[Out] -e*log(c*d*x + a*e)/(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4) + e*log(e*x + d)/(c
^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4) - 1/(a*c*d^2*e - a^2*e^3 + (c^2*d^3 - a*c
*d*e^2)*x)
```

Fricas [A] time = 1.90353, size = 238, normalized size = 3.17

$$-\frac{cd^2-ae^2+(cdex+ae^2)\log(cdx+ae)-(cdex+ae^2)\log(ex+d)}{ac^2d^4e-2\,a^2cd^2e^3+a^3e^5+(c^3d^5-2\,ac^2d^3e^2+a^2cde^4)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="fricas"
)
```

```
[Out] -(c*d^2 - a*e^2 + (c*d*e*x + a*e^2)*log(c*d*x + a*e) - (c*d*e*x + a*e^2)*lo
g(e*x + d))/(a*c^2*d^4*e - 2*a^2*c*d^2*e^3 + a^3*e^5 + (c^3*d^5 - 2*a*c^2*d
^3*e^2 + a^2*c*d*e^4)*x)
```

Sympy [B] time = 1.05649, size = 287, normalized size = 3.83

$$\frac{e\log\left(x+\frac{-\frac{a^3e^7}{(ae^2-cd^2)^2}+\frac{3a^2cd^2e^5}{(ae^2-cd^2)^2}-\frac{3ac^2d^4e^3}{(ae^2-cd^2)^2}+ae^3+\frac{c^3d^6e}{(ae^2-cd^2)^2}+cd^2e}{2cde^2}\right)}{(ae^2-cd^2)^2}-\frac{e\log\left(x+\frac{\frac{a^3e^7}{(ae^2-cd^2)^2}-\frac{3a^2cd^2e^5}{(ae^2-cd^2)^2}+\frac{3ac^2d^4e^3}{(ae^2-cd^2)^2}+ae^3-\frac{c^3d^6e}{(ae^2-cd^2)^2}+cd^2e}{2cde^2}\right)}{(ae^2-cd^2)^2}+\frac{}{a^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

```
[Out] e*log(x + (-a**3*e**7/(a*e**2 - c*d**2)**2 + 3*a**2*c*d**2*e**5/(a*e**2 - c
*d**2)**2 - 3*a*c**2*d**4*e**3/(a*e**2 - c*d**2)**2 + a*e**3 + c**3*d**6*e/
(a*e**2 - c*d**2)**2 + c*d**2*e)/(2*c*d*e**2))/(a*e**2 - c*d**2)**2 - e*log
(x + (a**3*e**7/(a*e**2 - c*d**2)**2 - 3*a**2*c*d**2*e**5/(a*e**2 - c*d**2)
**2 + 3*a*c**2*d**4*e**3/(a*e**2 - c*d**2)**2 + a*e**3 - c**3*d**6*e/(a*e**
2 - c*d**2)**2 + c*d**2*e)/(2*c*d*e**2))/(a*e**2 - c*d**2)**2 + 1/(a**2*e**
3 - a*c*d**2*e + x*(a*c*d*e**2 - c**2*d**3))
```

Giac [B] time = 1.17913, size = 262, normalized size = 3.49

$$-\frac{2\left(cd^2e-ae^3\right)\arctan\left(\frac{2\,cdxe+cd^2+ae^2}{\sqrt{-c^2d^4+2\,acd^2e^2-a^2e^4}}\right)}{\left(c^2d^4-2\,acd^2e^2+a^2e^4\right)\sqrt{-c^2d^4+2\,acd^2e^2-a^2e^4}}-\frac{cd^2xe+cd^3-axe^3-ade^2}{\left(c^2d^4-2\,acd^2e^2+a^2e^4\right)\left(cdx^2e+cd^2x+axe^2+ade\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac")
```

```
[Out] -2*(c*d^2*e - a*e^3)*arctan((2*c*d*x*e + c*d^2 + a*e^2)/sqrt(-c^2*d^4 + 2*a
*c*d^2*e^2 - a^2*e^4))/((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-c^2*d^4 +
 2*a*c*d^2*e^2 - a^2*e^4)) - (c*d^2*x*e + c*d^3 - a*x*e^3 - a*d*e^2)/((c^2*
d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))
```

3.1882 $\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$

Optimal. Leaf size=114

$$-\frac{ae^2+cd^2+2cdex}{\left(cd^2-ae^2\right)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)}-\frac{2cde\log(ae+cdx)}{\left(cd^2-ae^2\right)^3}+\frac{2cde\log(d+ex)}{\left(cd^2-ae^2\right)^3}$$

```
[Out] -((c*d^2 + a*e^2 + 2*c*d*e*x)/((c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2))) - (2*c*d*e*Log[a*e + c*d*x])/(c*d^2 - a*e^2)^3 + (2*c*d*e*L
og[d + e*x])/(c*d^2 - a*e^2)^3
```

Rubi [A] time = 0.0331131, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {614, 616, 31}

$$-\frac{ae^2+cd^2+2cdex}{\left(cd^2-ae^2\right)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)}-\frac{2cde\log(ae+cdx)}{\left(cd^2-ae^2\right)^3}+\frac{2cde\log(d+ex)}{\left(cd^2-ae^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-2),x]
```

```
[Out] -((c*d^2 + a*e^2 + 2*c*d*e*x)/((c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2))) - (2*c*d*e*Log[a*e + c*d*x])/(c*d^2 - a*e^2)^3 + (2*c*d*e*L
og[d + e*x])/(c*d^2 - a*e^2)^3
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx = -\frac{cd^2+ae^2+2cdex}{\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)} - \frac{(2cde)\int \frac{1}{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx}{\left(cd^2-ae^2\right)^2}$$

$$= -\frac{cd^2+ae^2+2cdex}{\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)} + \frac{\left(2c^2d^2e^2\right)\int \frac{1}{cd^2+cdex}\,dx}{\left(cd^2-ae^2\right)^3} - \frac{\left(2c^2d^2e^2\right.}{\left(cd\right.}$$

$$= -\frac{cd^2+ae^2+2cdex}{\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)} - \frac{2cde\log(ae+cdx)}{\left(cd^2-ae^2\right)^3} + \frac{2cde\log(d+}{\left(cd^2-ae^2\right)}$$

Mathematica [A] time = 0.094808, size = 86, normalized size = 0.75

$$\frac{\frac{\left(cd^2-ae^2\right)\left(ae^2+cd(d+2ex)\right)}{(d+ex)(ae+cdx)}+2cde\log(ae+cdx)-2cde\log(d+ex)}{\left(ae^2-cd^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-2),x]
```

```
[Out] (((c*d^2 - a*e^2)*(a*e^2 + c*d*(d + 2*e*x)))/((a*e + c*d*x)*(d + e*x)) + 2*
c*d*e*Log[a*e + c*d*x] - 2*c*d*e*Log[d + e*x])/(-(c*d^2) + a*e^2)^3
```

Maple [A] time = 0.052, size = 107, normalized size = 0.9

$$-\frac{e}{\left(ae^2-cd^2\right)^2(ex+d)} - 2\frac{dec\ln(ex+d)}{\left(ae^2-cd^2\right)^3} - \frac{cd}{\left(ae^2-cd^2\right)^2(cdx+ae)} + 2\frac{dec\ln(cdx+ae)}{\left(ae^2-cd^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

```
[Out] -e/(a*e^2-c*d^2)^2/(e*x+d)-2*e/(a*e^2-c*d^2)^3*c*d*ln(e*x+d)-c*d/(a*e^2-c*d
^2)^2/(c*d*x+a*e)+2*e/(a*e^2-c*d^2)^3*c*d*ln(c*d*x+a*e)
```

Maxima [B] time = 1.06506, size = 319, normalized size = 2.8

$$-\frac{2\,cde\log(cdx+ae)}{c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6} + \frac{2\,cde\log(ex+d)}{c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6} - \frac{2\,cd}{ac^2d^5e-2\,a^2cd^3e^3+a^3de^5+\left(c^3d^5e-2\,ac^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="maxima")
```

```
[Out] -2*c*d*e*log(c*d*x + a*e)/(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^
3*e^6) + 2*c*d*e*log(e*x + d)/(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4
- a^3*e^6) - (2*c*d*e*x + c*d^2 + a*e^2)/(a*c^2*d^5*e - 2*a^2*c*d^3*e^3 + a
^3*d*e^5 + (c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x^2 + (c^3*d^6 - a*c
^2*d^4*e^2 - a^2*c*d^2*e^4 + a^3*e^6)*x)
```

Fricas [B] time = 1.92847, size = 545, normalized size = 4.78

$$-\frac{c^2d^4 - a^2e^4 + 2\left(c^2d^3e - acde^3\right)x + 2\left(c^2d^2e^2x^2 + acd^2e^2 + \left(c^2d^3e + acde^3\right)x\right)\log\left(cdx + ae\right) - 2\left(c^2d^2e^2x^2 + acd^2e^2 + \right.}{ac^3d^7e - 3\,a^2c^2d^5e^3 + 3\,a^3cd^3e^5 - a^4de^7 + \left(c^4d^7e - 3\,ac^3d^5e^3 + 3\,a^2c^2d^3e^5 - a^3cde^7\right)x^2 + \left(c^4d^8 - 2\,ac^3d^6e^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="fricas")
```

```
[Out] -(c^2*d^4 - a^2*e^4 + 2*(c^2*d^3*e - a*c*d*e^3)*x + 2*(c^2*d^2*e^2*x^2 + a*
c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)*log(c*d*x + a*e) - 2*(c^2*d^2*e^2*x^
2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)*log(e*x + d))/(a*c^3*d^7*e - 3
*a^2*c^2*d^5*e^3 + 3*a^3*c*d^3*e^5 - a^4*d*e^7 + (c^4*d^7*e - 3*a*c^3*d^5*e
^3 + 3*a^2*c^2*d^3*e^5 - a^3*c*d*e^7)*x^2 + (c^4*d^8 - 2*a*c^3*d^6*e^2 + 2*
a^3*c*d^2*e^6 - a^4*e^8)*x)
```

Sympy [B] time = 1.53199, size = 484, normalized size = 4.25

$$-\frac{2cde\log\left(x+\frac{-\frac{2a^4cde^9}{\left(ae^2-cd^2\right)^3}+\frac{8a^3c^2d^3e^7}{\left(ae^2-cd^2\right)^3}-\frac{12a^2c^3d^5e^5}{\left(ae^2-cd^2\right)^3}+\frac{8ac^4d^7e^3}{\left(ae^2-cd^2\right)^3}+2acde^3-\frac{2c^5d^9e}{\left(ae^2-cd^2\right)^3}+2c^2d^3e}{4c^2d^2e^2}\right)}{\left(ae^2-cd^2\right)^3}+\frac{2cde\log\left(x+\frac{\frac{2a^4cde^9}{\left(ae^2-cd^2\right)^3}-\frac{8a^3c^2d^3e^7}{\left(ae^2-cd^2\right)^3}+\frac{12a^2c^3d^5e^5}{\left(ae^2-cd^2\right)^3}-\cdots}{4c\cdots}\right.}{\left(ae^2-cd^2\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

```
[Out] -2*c*d*e*log(x + (-2*a**4*c*d*e**9/(a*e**2 - c*d**2)**3 + 8*a**3*c**2*d**3*
e**7/(a*e**2 - c*d**2)**3 - 12*a**2*c**3*d**5*e**5/(a*e**2 - c*d**2)**3 + 8
*a*c**4*d**7*e**3/(a*e**2 - c*d**2)**3 + 2*a*c*d*e**3 - 2*c**5*d**9*e/(a*e*
*2 - c*d**2)**3 + 2*c**2*d**3*e)/(4*c**2*d**2*e**2))/(a*e**2 - c*d**2)**3 +
 2*c*d*e*log(x + (2*a**4*c*d*e**9/(a*e**2 - c*d**2)**3 - 8*a**3*c**2*d**3*e
**7/(a*e**2 - c*d**2)**3 + 12*a**2*c**3*d**5*e**5/(a*e**2 - c*d**2)**3 - 8*
a*c**4*d**7*e**3/(a*e**2 - c*d**2)**3 + 2*a*c*d*e**3 + 2*c**5*d**9*e/(a*e**
2 - c*d**2)**3 + 2*c**2*d**3*e)/(4*c**2*d**2*e**2))/(a*e**2 - c*d**2)**3 -
(a*e**2 + c*d**2 + 2*c*d*e*x)/(a**3*d*e**5 - 2*a**2*c*d**3*e**3 + a*c**2*d*
*5*e + x**2*(a**2*c*d*e**5 - 2*a*c**2*d**3*e**3 + c**3*d**5*e) + x*(a**3*e*
*6 - a**2*c*d**2*e**4 - a*c**2*d**4*e**2 + c**3*d**6))
```

Giac [A] time = 1.31586, size = 238, normalized size = 2.09

$$-\frac{4\,cd\arctan\left(\frac{2\,cdxe+cd^2+ae^2}{\sqrt{-c^2d^4+2\,acd^2e^2-a^2e^4}}\right)e}{\left(c^2d^4-2\,acd^2e^2+a^2e^4\right)\sqrt{-c^2d^4+2\,acd^2e^2-a^2e^4}}-\frac{2\,cdxe+cd^2+ae^2}{\left(c^2d^4-2\,acd^2e^2+a^2e^4\right)\left(cdx^2e+cd^2x+axe^2+ade\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac")
```

```
[Out] -4*c*d*arctan((2*c*d*x*e + c*d^2 + a*e^2)/sqrt(-c^2*d^4 + 2*a*c*d^2*e^2 - a
^2*e^4))*e/((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-c^2*d^4 + 2*a*c*d^2*e
```

```
^2 - a^2*e^4)) - (2*c*d*x*e + c*d^2 + a*e^2)/((c^2*d^4 - 2*a*c*d^2*e^2 + a^
2*e^4)*(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))
```

3.1883 $\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$

Optimal. Leaf size=146

$$-\frac{c^2d^2}{\left(cd^2-ae^2\right)^3(ae+cdx)}-\frac{3c^2d^2e\log(ae+cdx)}{\left(cd^2-ae^2\right)^4}+\frac{3c^2d^2e\log(d+ex)}{\left(cd^2-ae^2\right)^4}-\frac{2cde}{(d+ex)\left(cd^2-ae^2\right)^3}-\frac{e}{2(d+ex)^2\left(cd^2-ae^2\right)^2}$$

```
[Out] -((c^2*d^2)/((c*d^2 - a*e^2)^3*(a*e + c*d*x))) - e/(2*(c*d^2 - a*e^2)^2*(d
+ e*x)^2) - (2*c*d*e)/((c*d^2 - a*e^2)^3*(d + e*x)) - (3*c^2*d^2*e*Log[a*e
+ c*d*x])/(c*d^2 - a*e^2)^4 + (3*c^2*d^2*e*Log[d + e*x])/(c*d^2 - a*e^2)^4
```

Rubi [A] time = 0.109691, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 44}

$$-\frac{c^2d^2}{\left(cd^2-ae^2\right)^3(ae+cdx)}-\frac{3c^2d^2e\log(ae+cdx)}{\left(cd^2-ae^2\right)^4}+\frac{3c^2d^2e\log(d+ex)}{\left(cd^2-ae^2\right)^4}-\frac{2cde}{(d+ex)\left(cd^2-ae^2\right)^3}-\frac{e}{2(d+ex)^2\left(cd^2-ae^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2),x]
```

```
[Out] -((c^2*d^2)/((c*d^2 - a*e^2)^3*(a*e + c*d*x))) - e/(2*(c*d^2 - a*e^2)^2*(d
+ e*x)^2) - (2*c*d*e)/((c*d^2 - a*e^2)^3*(d + e*x)) - (3*c^2*d^2*e*Log[a*e
+ c*d*x])/(c*d^2 - a*e^2)^4 + (3*c^2*d^2*e*Log[d + e*x])/(c*d^2 - a*e^2)^4
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx=\int \frac{1}{(ae+cdx)^2(d+ex)^3}\,dx$$

$$=\int \left(\frac{c^3d^3}{\left(cd^2-ae^2\right)^3(ae+cdx)^2}-\frac{3c^3d^3e}{\left(cd^2-ae^2\right)^4(ae+cdx)}+\frac{e^2}{\left(cd^2-ae^2\right)^2(}\right.$$

$$=-\frac{c^2d^2}{\left(cd^2-ae^2\right)^3(ae+cdx)}-\frac{e}{2\left(cd^2-ae^2\right)^2(d+ex)^2}-\frac{2cde}{\left(cd^2-ae^2\right)^3(d+}$$

Mathematica [A] time = 0.0953845, size = 130, normalized size = 0.89

$$\frac{\frac{2c^2d^2\left(ae^2-cd^2\right)}{ae+cdx}-6c^2d^2e\log(ae+cdx)+\frac{4cde\left(ae^2-cd^2\right)}{d+ex}-\frac{e\left(cd^2-ae^2\right)^2}{(d+ex)^2}+6c^2d^2e\log(d+ex)}{2\left(cd^2-ae^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2),x]
```

```
[Out] ((2*c^2*d^2*(-(c*d^2) + a*e^2))/(a*e + c*d*x) - (e*(c*d^2 - a*e^2)^2)/(d +
e*x)^2 + (4*c*d*e*(-(c*d^2) + a*e^2))/(d + e*x) - 6*c^2*d^2*e*Log[a*e + c*d
*x] + 6*c^2*d^2*e*Log[d + e*x])/(2*(c*d^2 - a*e^2)^4)
```

Maple [A] time = 0.054, size = 144, normalized size = 1.

$$-\frac{e}{2\left(ae^2-cd^2\right)^2(ex+d)^2}+3\frac{c^2d^2e\ln(ex+d)}{\left(ae^2-cd^2\right)^4}+2\frac{dec}{\left(ae^2-cd^2\right)^3(ex+d)}+\frac{c^2d^2}{\left(ae^2-cd^2\right)^3(cdx+ae)}-3\frac{c^2d^2e\ln(cdx+ae)}{\left(ae^2-cd^2\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

```
[Out] -1/2*e/(a*e^2-c*d^2)^2/(e*x+d)^2+3*e/(a*e^2-c*d^2)^4*c^2*d^2*ln(e*x+d)+2*e/
(a*e^2-c*d^2)^3*c*d/(e*x+d)+c^2*d^2/(a*e^2-c*d^2)^3/(c*d*x+a*e)-3*e/(a*e^2-
c*d^2)^4*c^2*d^2*ln(c*d*x+a*e)
```

Maxima [B] time = 1.12091, size = 571, normalized size = 3.91

$$-\frac{3c^2d^2e\log(cdx+ae)}{c^4d^8-4ac^3d^6e^2+6a^2c^2d^4e^4-4a^3cd^2e^6+a^4e^8}+\frac{3c^2d^2e\log(ex+d)}{c^4d^8-4ac^3d^6e^2+6a^2c^2d^4e^4-4a^3cd^2e^6+a^4e^8}-\frac{}{2\left(ac^3d^8e-3a^2c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="maxim
a")
```

```
[Out] -3*c^2*d^2*e*log(c*d*x + a*e)/(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^
4 - 4*a^3*c*d^2*e^6 + a^4*e^8) + 3*c^2*d^2*e*log(e*x + d)/(c^4*d^8 - 4*a*c^
3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8) - 1/2*(6*c^2*d^2
*e^2*x^2 + 2*c^2*d^4 + 5*a*c*d^2*e^2 - a^2*e^4 + 3*(3*c^2*d^3*e + a*c*d*e^3
)*x)/(a*c^3*d^8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^
4*d^7*e^2 - 3*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4
*d^8*e - 5*a*c^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2
 + (c^4*d^9 - a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d
*e^8)*x)
```

Fricas [B] time = 2.00916, size = 1077, normalized size = 7.38

$$-\frac{2c^3d^6+3ac^2d^4e^2-6a^2cd^2e^4+a^3e^6+6\left(c^3d^4e^2-ac^2d^2e^4\right)x^2+3\left(3c^3d^5e-2ac^2d^3e^3-a^2cde^5\right)x+6\left(c^3d^3e^3x^3+ac^2d^4\right.}{2\left(ac^4d^{10}e-4a^2c^3d^8e^3+6a^3c^2d^6e^5-4a^4cd^4e^7+a^5d^2e^9+\left(c^5d^9e^2-4ac^4d^7e^4+6a^2c^3d^5e^6-4a^3c^2d^3e^8+a^4cde^{10}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="frica
s")
```

```
[Out] -1/2*(2*c^3*d^6 + 3*a*c^2*d^4*e^2 - 6*a^2*c*d^2*e^4 + a^3*e^6 + 6*(c^3*d^4*
e^2 - a*c^2*d^2*e^4)*x^2 + 3*(3*c^3*d^5*e - 2*a*c^2*d^3*e^3 - a^2*c*d*e^5)*
x + 6*(c^3*d^3*e^3*x^3 + a*c^2*d^4*e^2 + (2*c^3*d^4*e^2 + a*c^2*d^2*e^4)*x^
2 + (c^3*d^5*e + 2*a*c^2*d^3*e^3)*x)*log(c*d*x + a*e) - 6*(c^3*d^3*e^3*x^3
+ a*c^2*d^4*e^2 + (2*c^3*d^4*e^2 + a*c^2*d^2*e^4)*x^2 + (c^3*d^5*e + 2*a*c^
2*d^3*e^3)*x)*log(e*x + d))/(a*c^4*d^10*e - 4*a^2*c^3*d^8*e^3 + 6*a^3*c^2*d
^6*e^5 - 4*a^4*c*d^4*e^7 + a^5*d^2*e^9 + (c^5*d^9*e^2 - 4*a*c^4*d^7*e^4 + 6
*a^2*c^3*d^5*e^6 - 4*a^3*c^2*d^3*e^8 + a^4*c*d*e^10)*x^3 + (2*c^5*d^10*e -
7*a*c^4*d^8*e^3 + 8*a^2*c^3*d^6*e^5 - 2*a^3*c^2*d^4*e^7 - 2*a^4*c*d^2*e^9 +
 a^5*e^11)*x^2 + (c^5*d^11 - 2*a*c^4*d^9*e^2 - 2*a^2*c^3*d^7*e^4 + 8*a^3*c^
2*d^5*e^6 - 7*a^4*c*d^3*e^8 + 2*a^5*d*e^10)*x)
```

Sympy [B] time = 2.54374, size = 734, normalized size = 5.03

$$\frac{3c^2d^2e\log\left(x+\frac{-\frac{3a^5c^2d^2e^{11}}{(ae^2-cd^2)^4}+\frac{15a^4c^3d^4e^9}{(ae^2-cd^2)^4}-\frac{30a^3c^4d^6e^7}{(ae^2-cd^2)^4}+\frac{30a^2c^5d^8e^5}{(ae^2-cd^2)^4}-\frac{15ac^6d^{10}e^3}{(ae^2-cd^2)^4}+3ac^2d^2e^3+\frac{3c^7d^{12}e}{(ae^2-cd^2)^4}+3c^3d^4e}{6c^3d^3e^2}\right)}{(ae^2-cd^2)^4}-\frac{3c^2d^2e\log\left(x+\frac{\frac{3a^5c^2d^2e^{11}}{(ae^2-cd^2)^4}-\frac{15a^4c}{(ae^2-}\cdots\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

```
[Out] 3*c**2*d**2*e*log(x + (-3*a**5*c**2*d**2*e**11/(a*e**2 - c*d**2)**4 + 15*a*
*4*c**3*d**4*e**9/(a*e**2 - c*d**2)**4 - 30*a**3*c**4*d**6*e**7/(a*e**2 - c
*d**2)**4 + 30*a**2*c**5*d**8*e**5/(a*e**2 - c*d**2)**4 - 15*a*c**6*d**10*e
**3/(a*e**2 - c*d**2)**4 + 3*a*c**2*d**2*e**3 + 3*c**7*d**12*e/(a*e**2 - c*
d**2)**4 + 3*c**3*d**4*e)/(6*c**3*d**3*e**2))/(a*e**2 - c*d**2)**4 - 3*c**2
*d**2*e*log(x + (3*a**5*c**2*d**2*e**11/(a*e**2 - c*d**2)**4 - 15*a**4*c**3
*d**4*e**9/(a*e**2 - c*d**2)**4 + 30*a**3*c**4*d**6*e**7/(a*e**2 - c*d**2)*
*4 - 30*a**2*c**5*d**8*e**5/(a*e**2 - c*d**2)**4 + 15*a*c**6*d**10*e**3/(a*
e**2 - c*d**2)**4 + 3*a*c**2*d**2*e**3 - 3*c**7*d**12*e/(a*e**2 - c*d**2)**
4 + 3*c**3*d**4*e)/(6*c**3*d**3*e**2))/(a*e**2 - c*d**2)**4 + (-a**2*e**4 +
 5*a*c*d**2*e**2 + 2*c**2*d**4 + 6*c**2*d**2*e**2*x**2 + x*(3*a*c*d*e**3 +
9*c**2*d**3*e))/(2*a**4*d**2*e**7 - 6*a**3*c*d**4*e**5 + 6*a**2*c**2*d**6*e
**3 - 2*a*c**3*d**8*e + x**3*(2*a**3*c*d*e**8 - 6*a**2*c**2*d**3*e**6 + 6*a
*c**3*d**5*e**4 - 2*c**4*d**7*e**2) + x**2*(2*a**4*e**9 - 2*a**3*c*d**2*e**
7 - 6*a**2*c**2*d**4*e**5 + 10*a*c**3*d**6*e**3 - 4*c**4*d**8*e) + x*(4*a**
4*d*e**8 - 10*a**3*c*d**3*e**6 + 6*a**2*c**2*d**5*e**4 + 2*a*c**3*d**7*e**2
 - 2*c**4*d**9))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError
```

3.1884 $\int \frac{1}{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$

Optimal. Leaf size=176

$$-\frac{c^3d^3}{\left(cd^2-ae^2\right)^4(ae+cdx)}-\frac{3c^2d^2e}{(d+ex)\left(cd^2-ae^2\right)^4}-\frac{4c^3d^3e\log(ae+cdx)}{\left(cd^2-ae^2\right)^5}+\frac{4c^3d^3e\log(d+ex)}{\left(cd^2-ae^2\right)^5}-\frac{cde}{(d+ex)^2\left(cd^2-ae^2\right)^3}$$

```
[Out] -((c^3*d^3)/((c*d^2 - a*e^2)^4*(a*e + c*d*x))) - e/(3*(c*d^2 - a*e^2)^2*(d
+ e*x)^3) - (c*d*e)/((c*d^2 - a*e^2)^3*(d + e*x)^2) - (3*c^2*d^2*e)/((c*d^2
 - a*e^2)^4*(d + e*x)) - (4*c^3*d^3*e*Log[a*e + c*d*x])/(c*d^2 - a*e^2)^5 +
 (4*c^3*d^3*e*Log[d + e*x])/(c*d^2 - a*e^2)^5
```

Rubi [A] time = 0.153476, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 44}

$$-\frac{c^3d^3}{\left(cd^2-ae^2\right)^4(ae+cdx)}-\frac{3c^2d^2e}{(d+ex)\left(cd^2-ae^2\right)^4}-\frac{4c^3d^3e\log(ae+cdx)}{\left(cd^2-ae^2\right)^5}+\frac{4c^3d^3e\log(d+ex)}{\left(cd^2-ae^2\right)^5}-\frac{cde}{(d+ex)^2\left(cd^2-ae^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2),x]
```

```
[Out] -((c^3*d^3)/((c*d^2 - a*e^2)^4*(a*e + c*d*x))) - e/(3*(c*d^2 - a*e^2)^2*(d
+ e*x)^3) - (c*d*e)/((c*d^2 - a*e^2)^3*(d + e*x)^2) - (3*c^2*d^2*e)/((c*d^2
 - a*e^2)^4*(d + e*x)) - (4*c^3*d^3*e*Log[a*e + c*d*x])/(c*d^2 - a*e^2)^5 +
 (4*c^3*d^3*e*Log[d + e*x])/(c*d^2 - a*e^2)^5
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx=\int \frac{1}{(ae+cdx)^2(d+ex)^4}\,dx$$

$$=\int \left(\frac{c^4d^4}{\left(cd^2-ae^2\right)^4(ae+cdx)^2}-\frac{4c^4d^4e}{\left(cd^2-ae^2\right)^5(ae+cdx)}+\frac{e^2}{\left(cd^2-ae^2\right)^2}\right.$$

$$=-\frac{c^3d^3}{\left(cd^2-ae^2\right)^4(ae+cdx)}-\frac{e}{3\left(cd^2-ae^2\right)^2(d+ex)^3}-\frac{cde}{\left(cd^2-ae^2\right)^3(d\right.}$$

Mathematica [A] time = 0.141354, size = 160, normalized size = 0.91

$$\frac{\frac{3c^3d^3(cd^2-ae^2)}{ae+cdx}+\frac{9c^2d^2e(cd^2-ae^2)}{d+ex}+12c^3d^3e\log(ae+cdx)+\frac{3cde(cd^2-ae^2)^2}{(d+ex)^2}-\frac{e(ae^2-cd^2)^3}{(d+ex)^3}-12c^3d^3e\log(d+ex)}{3\left(ae^2-cd^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2),x]
```

```
[Out] ((3*c^3*d^3*(c*d^2 - a*e^2))/(a*e + c*d*x) - (e*(-(c*d^2) + a*e^2)^3)/(d +
e*x)^3 + (3*c*d*e*(c*d^2 - a*e^2)^2)/(d + e*x)^2 + (9*c^2*d^2*e*(c*d^2 - a*
e^2))/(d + e*x) + 12*c^3*d^3*e*Log[a*e + c*d*x] - 12*c^3*d^3*e*Log[d + e*x]
)/(3*(-(c*d^2) + a*e^2)^5)
```

Maple [A] time = 0.056, size = 174, normalized size = 1.

$$-\frac{e}{3\left(ae^2-cd^2\right)^2(ex+d)^3}-4\frac{c^3ed^3\ln(ex+d)}{\left(ae^2-cd^2\right)^5}-3\frac{c^2d^2e}{\left(ae^2-cd^2\right)^4(ex+d)}+\frac{dec}{\left(ae^2-cd^2\right)^3(ex+d)^2}-\frac{c^3d^3}{\left(ae^2-cd^2\right)^4(cdx+ae}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

```
[Out] -1/3*e/(a*e^2-c*d^2)^2/(e*x+d)^3-4*e/(a*e^2-c*d^2)^5*c^3*d^3*ln(e*x+d)-3*e/
(a*e^2-c*d^2)^4*c^2*d^2/(e*x+d)+e/(a*e^2-c*d^2)^3*c*d/(e*x+d)^2-c^3*d^3/(a*
e^2-c*d^2)^4/(c*d*x+a*e)+4*e/(a*e^2-c*d^2)^5*c^3*d^3*ln(c*d*x+a*e)
```

Maxima [B] time = 1.14173, size = 865, normalized size = 4.91

$$-\frac{4c^3d^3e\log(cdx+ae)}{c^5d^{10}-5ac^4d^8e^2+10a^2c^3d^6e^4-10a^3c^2d^4e^6+5a^4cd^2e^8-a^5e^{10}}+\frac{4c^3d^3e\log(ex+d)}{c^5d^{10}-5ac^4d^8e^2+10a^2c^3d^6e^4-10a^3c^2d^4e^6+5a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="max
ima")
```

```
[Out] -4*c^3*d^3*e*log(c*d*x + a*e)/(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*
e^4 - 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10) + 4*c^3*d^3*e*log(e*
x + d)/(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4*e^
6 + 5*a^4*c*d^2*e^8 - a^5*e^10) - 1/3*(12*c^3*d^3*e^3*x^3 + 3*c^3*d^6 + 13*
a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4 + a^3*e^6 + 6*(5*c^3*d^4*e^2 + a*c^2*d^2*e^
4)*x^2 + 2*(11*c^3*d^5*e + 8*a*c^2*d^3*e^3 - a^2*c*d*e^5)*x)/(a*c^4*d^11*e
- 4*a^2*c^3*d^9*e^3 + 6*a^3*c^2*d^7*e^5 - 4*a^4*c*d^5*e^7 + a^5*d^3*e^9 + (
c^5*d^9*e^3 - 4*a*c^4*d^7*e^5 + 6*a^2*c^3*d^5*e^7 - 4*a^3*c^2*d^3*e^9 + a^4
*c*d*e^11)*x^4 + (3*c^5*d^10*e^2 - 11*a*c^4*d^8*e^4 + 14*a^2*c^3*d^6*e^6 -
6*a^3*c^2*d^4*e^8 - a^4*c*d^2*e^10 + a^5*e^12)*x^3 + 3*(c^5*d^11*e - 3*a*c^
4*d^9*e^3 + 2*a^2*c^3*d^7*e^5 + 2*a^3*c^2*d^5*e^7 - 3*a^4*c*d^3*e^9 + a^5*d
*e^11)*x^2 + (c^5*d^12 - a*c^4*d^10*e^2 - 6*a^2*c^3*d^8*e^4 + 14*a^3*c^2*d^
6*e^6 - 11*a^4*c*d^4*e^8 + 3*a^5*d^2*e^10)*x)
```

Fricas [B] time = 2.06131, size = 1615, normalized size = 9.18

$$-\frac{3\,c^4d^8+10\,ac^3d^6e^2-18\,a^2c^2d^4e^4+6\,a^3cd^2e^6-a^4e^8+12\left(c^4d^5e^3-ac^3d^3e^5\right)x^3+6\left(5\,c^4d^6e^2-4\,ac^3d^4e^4-a^2\cdots\right.}{3\left(ac^5d^{13}e-5\,a^2c^4d^{11}e^3+10\,a^3c^3d^9e^5-10\,a^4c^2d^7e^7+5\,a^5cd^5e^9-a^6d^3e^{11}+\left(c^6d^{11}e^3-5\,ac^5d^9e^5+10\,a^2c^4d^7e^7-10\,\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="fri
cas")
```

```
[Out] -1/3*(3*c^4*d^8 + 10*a*c^3*d^6*e^2 - 18*a^2*c^2*d^4*e^4 + 6*a^3*c*d^2*e^6 -
 a^4*e^8 + 12*(c^4*d^5*e^3 - a*c^3*d^3*e^5)*x^3 + 6*(5*c^4*d^6*e^2 - 4*a*c^
3*d^4*e^4 - a^2*c^2*d^2*e^6)*x^2 + 2*(11*c^4*d^7*e - 3*a*c^3*d^5*e^3 - 9*a^
2*c^2*d^3*e^5 + a^3*c*d*e^7)*x + 12*(c^4*d^4*e^4*x^4 + a*c^3*d^6*e^2 + (3*c
^4*d^5*e^3 + a*c^3*d^3*e^5)*x^3 + 3*(c^4*d^6*e^2 + a*c^3*d^4*e^4)*x^2 + (c^
4*d^7*e + 3*a*c^3*d^5*e^3)*x)*log(c*d*x + a*e) - 12*(c^4*d^4*e^4*x^4 + a*c^
3*d^6*e^2 + (3*c^4*d^5*e^3 + a*c^3*d^3*e^5)*x^3 + 3*(c^4*d^6*e^2 + a*c^3*d^
4*e^4)*x^2 + (c^4*d^7*e + 3*a*c^3*d^5*e^3)*x)*log(e*x + d))/(a*c^5*d^13*e -
 5*a^2*c^4*d^11*e^3 + 10*a^3*c^3*d^9*e^5 - 10*a^4*c^2*d^7*e^7 + 5*a^5*c*d^5
*e^9 - a^6*d^3*e^11 + (c^6*d^11*e^3 - 5*a*c^5*d^9*e^5 + 10*a^2*c^4*d^7*e^7
- 10*a^3*c^3*d^5*e^9 + 5*a^4*c^2*d^3*e^11 - a^5*c*d*e^13)*x^4 + (3*c^6*d^12
*e^2 - 14*a*c^5*d^10*e^4 + 25*a^2*c^4*d^8*e^6 - 20*a^3*c^3*d^6*e^8 + 5*a^4*
c^2*d^4*e^10 + 2*a^5*c*d^2*e^12 - a^6*e^14)*x^3 + 3*(c^6*d^13*e - 4*a*c^5*d
^11*e^3 + 5*a^2*c^4*d^9*e^5 - 5*a^4*c^2*d^5*e^9 + 4*a^5*c*d^3*e^11 - a^6*d*
e^13)*x^2 + (c^6*d^14 - 2*a*c^5*d^12*e^2 - 5*a^2*c^4*d^10*e^4 + 20*a^3*c^3*
d^8*e^6 - 25*a^4*c^2*d^6*e^8 + 14*a^5*c*d^4*e^10 - 3*a^6*d^2*e^12)*x)
```

Sympy [B] time = 3.9785, size = 994, normalized size = 5.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

```
[Out] -4*c**3*d**3*e*log(x + (-4*a**6*c**3*d**3*e**13/(a*e**2 - c*d**2)**5 + 24*a
**5*c**4*d**5*e**11/(a*e**2 - c*d**2)**5 - 60*a**4*c**5*d**7*e**9/(a*e**2 -
 c*d**2)**5 + 80*a**3*c**6*d**9*e**7/(a*e**2 - c*d**2)**5 - 60*a**2*c**7*d*
*11*e**5/(a*e**2 - c*d**2)**5 + 24*a*c**8*d**13*e**3/(a*e**2 - c*d**2)**5 +
 4*a*c**3*d**3*e**3 - 4*c**9*d**15*e/(a*e**2 - c*d**2)**5 + 4*c**4*d**5*e)/
(8*c**4*d**4*e**2))/(a*e**2 - c*d**2)**5 + 4*c**3*d**3*e*log(x + (4*a**6*c*
*3*d**3*e**13/(a*e**2 - c*d**2)**5 - 24*a**5*c**4*d**5*e**11/(a*e**2 - c*d*
*2)**5 + 60*a**4*c**5*d**7*e**9/(a*e**2 - c*d**2)**5 - 80*a**3*c**6*d**9*e*
*7/(a*e**2 - c*d**2)**5 + 60*a**2*c**7*d**11*e**5/(a*e**2 - c*d**2)**5 - 24
*a*c**8*d**13*e**3/(a*e**2 - c*d**2)**5 + 4*a*c**3*d**3*e**3 + 4*c**9*d**15
*e/(a*e**2 - c*d**2)**5 + 4*c**4*d**5*e)/(8*c**4*d**4*e**2))/(a*e**2 - c*d*
*2)**5 - (a**3*e**6 - 5*a**2*c*d**2*e**4 + 13*a*c**2*d**4*e**2 + 3*c**3*d**
6 + 12*c**3*d**3*e**3*x**3 + x**2*(6*a*c**2*d**2*e**4 + 30*c**3*d**4*e**2)
+ x*(-2*a**2*c*d*e**5 + 16*a*c**2*d**3*e**3 + 22*c**3*d**5*e))/(3*a**5*d**3
*e**9 - 12*a**4*c*d**5*e**7 + 18*a**3*c**2*d**7*e**5 - 12*a**2*c**3*d**9*e*
*3 + 3*a*c**4*d**11*e + x**4*(3*a**4*c*d*e**11 - 12*a**3*c**2*d**3*e**9 + 1
8*a**2*c**3*d**5*e**7 - 12*a*c**4*d**7*e**5 + 3*c**5*d**9*e**3) + x**3*(3*a
**5*e**12 - 3*a**4*c*d**2*e**10 - 18*a**3*c**2*d**4*e**8 + 42*a**2*c**3*d**
6*e**6 - 33*a*c**4*d**8*e**4 + 9*c**5*d**10*e**2) + x**2*(9*a**5*d*e**11 -
```

```
27*a**4*c*d**3*e**9 + 18*a**3*c**2*d**5*e**7 + 18*a**2*c**3*d**7*e**5 - 27*
a*c**4*d**9*e**3 + 9*c**5*d**11*e) + x*(9*a**5*d**2*e**10 - 33*a**4*c*d**4*
e**8 + 42*a**3*c**2*d**6*e**6 - 18*a**2*c**3*d**8*e**4 - 3*a*c**4*d**10*e**
2 + 3*c**5*d**12))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="gia
c")
```

```
[Out] Exception raised: NotImplementedError
```

3.1885 $\int \frac{(d+ex)^9}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal. Leaf size=221

$$\frac{e^6(ae+cdx)^4}{4c^7d^7}+\frac{2e^5\left(cd^2-ae^2\right)(ae+cdx)^3}{c^7d^7}+\frac{15e^4\left(cd^2-ae^2\right)^2(ae+cdx)^2}{2c^7d^7}+\frac{20e^3x\left(cd^2-ae^2\right)^3}{c^6d^6}-\frac{6e\left(cd^2-ae^2\right)^5}{c^7d^7(ae+cdx)}-$$

```
[Out] (20*e^3*(c*d^2 - a*e^2)^3*x)/(c^6*d^6) - (c*d^2 - a*e^2)^6/(2*c^7*d^7*(a*e
+ c*d*x)^2) - (6*e*(c*d^2 - a*e^2)^5)/(c^7*d^7*(a*e + c*d*x)) + (15*e^4*(c*
d^2 - a*e^2)^2*(a*e + c*d*x)^2)/(2*c^7*d^7) + (2*e^5*(c*d^2 - a*e^2)*(a*e +
 c*d*x)^3)/(c^7*d^7) + (e^6*(a*e + c*d*x)^4)/(4*c^7*d^7) + (15*e^2*(c*d^2 -
 a*e^2)^4*Log[a*e + c*d*x])/(c^7*d^7)
```

Rubi [A] time = 0.261405, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$\frac{e^6(ae+cdx)^4}{4c^7d^7}+\frac{2e^5\left(cd^2-ae^2\right)(ae+cdx)^3}{c^7d^7}+\frac{15e^4\left(cd^2-ae^2\right)^2(ae+cdx)^2}{2c^7d^7}+\frac{20e^3x\left(cd^2-ae^2\right)^3}{c^6d^6}-\frac{6e\left(cd^2-ae^2\right)^5}{c^7d^7(ae+cdx)}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^9/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] (20*e^3*(c*d^2 - a*e^2)^3*x)/(c^6*d^6) - (c*d^2 - a*e^2)^6/(2*c^7*d^7*(a*e
+ c*d*x)^2) - (6*e*(c*d^2 - a*e^2)^5)/(c^7*d^7*(a*e + c*d*x)) + (15*e^4*(c*
d^2 - a*e^2)^2*(a*e + c*d*x)^2)/(2*c^7*d^7) + (2*e^5*(c*d^2 - a*e^2)*(a*e +
 c*d*x)^3)/(c^7*d^7) + (e^6*(a*e + c*d*x)^4)/(4*c^7*d^7) + (15*e^2*(c*d^2 -
 a*e^2)^4*Log[a*e + c*d*x])/(c^7*d^7)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(d+ex)^9}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx = \int \frac{(d+ex)^6}{(ae+cdx)^3}\,dx$$
$$= \int \left(\frac{20\left(cd^2e-ae^3\right)^3}{c^6d^6} + \frac{\left(cd^2-ae^2\right)^6}{c^6d^6(ae+cdx)^3} + \frac{6e\left(cd^2-ae^2\right)^5}{c^6d^6(ae+cdx)^2} + \frac{15e^2\left(cd^2-ae^2\right)^4}{c^6d^6(ae+cdx)} + \right.$$
$$= \frac{20e^3\left(cd^2-ae^2\right)^3x}{c^6d^6} - \frac{\left(cd^2-ae^2\right)^6}{2c^7d^7(ae+cdx)^2} - \frac{6e\left(cd^2-ae^2\right)^5}{c^7d^7(ae+cdx)} + \frac{15e^4\left(cd^2-ae^2\right)^2(ae}{2c^7d^7}$$

Mathematica [A] time = 0.129065, size = 337, normalized size = 1.52

$$\frac{2a^4c^2d^2e^8\left(105d^2+12dex-34e^2x^2\right)-4a^3c^3d^3e^6\left(-15d^2ex+50d^3-63de^2x^2+5e^3x^3\right)+5a^2c^4d^4e^4\left(-66d^2e^2x^2-32d^3ex\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^9/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] (22*a^6*e^12 - 4*a^5*c*d*e^10*(27*d + 4*e*x) + 2*a^4*c^2*d^2*e^8*(105*d^2 +
 12*d*e*x - 34*e^2*x^2) - 4*a^3*c^3*d^3*e^6*(50*d^3 - 15*d^2*e*x - 63*d*e^2
*x^2 + 5*e^3*x^3) + 5*a^2*c^4*d^4*e^4*(18*d^4 - 32*d^3*e*x - 66*d^2*e^2*x^2
 + 16*d*e^3*x^3 + e^4*x^4) - 2*a*c^5*d^5*e^2*(6*d^5 - 60*d^4*e*x - 80*d^3*e
^2*x^2 + 60*d^2*e^3*x^3 + 10*d*e^4*x^4 + e^5*x^5) + c^6*d^6*(-2*d^6 - 24*d^
5*e*x + 80*d^3*e^3*x^3 + 30*d^2*e^4*x^4 + 8*d*e^5*x^5 + e^6*x^6) + 60*e^2*(
c*d^2 - a*e^2)^4*(a*e + c*d*x)^2*Log[a*e + c*d*x])/(4*c^7*d^7*(a*e + c*d*x)
^2)
```

Maple [B] time = 0.051, size = 544, normalized size = 2.5

$$-\frac{15\,a^4e^8}{2\,c^5d^3\,(cdx+ae)^2} + 15\,\frac{e^{10}\ln(cdx+ae)\,a^4}{c^7d^7} - 60\,\frac{e^8\ln(cdx+ae)\,a^3}{c^6d^5} + 3\,\frac{e^8x^2a^2}{c^5d^5} + 30\,\frac{ade^3}{c^3\,(cdx+ae)} - \frac{a^6e^{12}}{2\,c^7d^7\,(cdx+ae)^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^9/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

```
[Out] -15/2/c^5/d^3/(c*d*x+a*e)^2*a^4*e^8+15/c^7/d^7*e^10*ln(c*d*x+a*e)*a^4-60/c^
6/d^5*e^8*ln(c*d*x+a*e)*a^3+3*e^8/c^5/d^5*x^2*a^2+30*d*e^3/c^3/(c*d*x+a*e)*
a-1/2/c^7/d^7/(c*d*x+a*e)^2*a^6*e^12+3/c^6/d^5/(c*d*x+a*e)^2*a^5*e^10+1/4*e
^6/c^3/d^3*x^4+2*e^5/c^3/d^2*x^3+15/2*e^4/c^3/d*x^2-6*d^3*e/c^2/(c*d*x+a*e)
+15/c^3*d*e^2*ln(c*d*x+a*e)+90/c^5/d^3*e^6*ln(c*d*x+a*e)*a^2-60/c^4/d*e^4*l
n(c*d*x+a*e)*a-e^7/c^4/d^4*x^3*a-9*e^6/c^4/d^3*x^2*a-10*e^9/c^6/d^6*a^3*x+3
6*e^7/c^5/d^4*a^2*x-45*e^5/c^4/d^2*a*x+20*e^3/c^3*x-1/2/c*d^5/(c*d*x+a*e)^2
-60/d*e^5/c^4/(c*d*x+a*e)*a^2+60/d^3*e^7/c^5/(c*d*x+a*e)*a^3+3/c^2*d^3/(c*d
*x+a*e)^2*a*e^2+6/d^7*e^11/c^7/(c*d*x+a*e)*a^5-30/d^5*e^9/c^6/(c*d*x+a*e)*a
^4-15/2/c^3*d/(c*d*x+a*e)^2*a^2*e^4+10/c^4/d/(c*d*x+a*e)^2*a^3*e^6
```

Maxima [A] time = 1.07531, size = 551, normalized size = 2.49

$$-\frac{c^6d^{12}+6\,ac^5d^{10}e^2-45\,a^2c^4d^8e^4+100\,a^3c^3d^6e^6-105\,a^4c^2d^4e^8+54\,a^5cd^2e^{10}-11\,a^6e^{12}+12\left(c^6d^{11}e-5\,ac^5d^9e^3+10\,a^2\right.}{2\left(c^9d^9x^2+2\,ac^8d^8ex+a^2c^7d^7e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^9/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxim
a")
```

```
[Out] -1/2*(c^6*d^12 + 6*a*c^5*d^10*e^2 - 45*a^2*c^4*d^8*e^4 + 100*a^3*c^3*d^6*e^
6 - 105*a^4*c^2*d^4*e^8 + 54*a^5*c*d^2*e^10 - 11*a^6*e^12 + 12*(c^6*d^11*e
- 5*a*c^5*d^9*e^3 + 10*a^2*c^4*d^7*e^5 - 10*a^3*c^3*d^5*e^7 + 5*a^4*c^2*d^3
*e^9 - a^5*c*d*e^11)*x)/(c^9*d^9*x^2 + 2*a*c^8*d^8*e*x + a^2*c^7*d^7*e^2) +
 1/4*(c^3*d^3*e^6*x^4 + 4*(2*c^3*d^4*e^5 - a*c^2*d^2*e^7)*x^3 + 6*(5*c^3*d^
5*e^4 - 6*a*c^2*d^3*e^6 + 2*a^2*c*d*e^8)*x^2 + 4*(20*c^3*d^6*e^3 - 45*a*c^2
*d^4*e^5 + 36*a^2*c*d^2*e^7 - 10*a^3*e^9)*x)/(c^6*d^6) + 15*(c^4*d^8*e^2 -
4*a*c^3*d^6*e^4 + 6*a^2*c^2*d^4*e^6 - 4*a^3*c*d^2*e^8 + a^4*e^10)*log(c*d*x
 + a*e)/(c^7*d^7)
```

Fricas [B] time = 1.8488, size = 1220, normalized size = 5.52

$$\frac{c^6d^6e^6x^6 - 2\,c^6d^{12} - 12\,ac^5d^{10}e^2 + 90\,a^2c^4d^8e^4 - 200\,a^3c^3d^6e^6 + 210\,a^4c^2d^4e^8 - 108\,a^5cd^2e^{10} + 22\,a^6e^{12} + 2\left(4\,c^6d^7e^5 - \dots\right.}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^9/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="frica
s")
```

```
[Out] 1/4*(c^6*d^6*e^6*x^6 - 2*c^6*d^12 - 12*a*c^5*d^10*e^2 + 90*a^2*c^4*d^8*e^4
- 200*a^3*c^3*d^6*e^6 + 210*a^4*c^2*d^4*e^8 - 108*a^5*c*d^2*e^10 + 22*a^6*e
^12 + 2*(4*c^6*d^7*e^5 - a*c^5*d^5*e^7)*x^5 + 5*(6*c^6*d^8*e^4 - 4*a*c^5*d^
6*e^6 + a^2*c^4*d^4*e^8)*x^4 + 20*(4*c^6*d^9*e^3 - 6*a*c^5*d^7*e^5 + 4*a^2*
c^4*d^5*e^7 - a^3*c^3*d^3*e^9)*x^3 + 2*(80*a*c^5*d^8*e^4 - 165*a^2*c^4*d^6*
e^6 + 126*a^3*c^3*d^4*e^8 - 34*a^4*c^2*d^2*e^10)*x^2 - 4*(6*c^6*d^11*e - 30
*a*c^5*d^9*e^3 + 40*a^2*c^4*d^7*e^5 - 15*a^3*c^3*d^5*e^7 - 6*a^4*c^2*d^3*e^
9 + 4*a^5*c*d*e^11)*x + 60*(a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 + 6*a^4*c^2
*d^4*e^8 - 4*a^5*c*d^2*e^10 + a^6*e^12 + (c^6*d^10*e^2 - 4*a*c^5*d^8*e^4 +
6*a^2*c^4*d^6*e^6 - 4*a^3*c^3*d^4*e^8 + a^4*c^2*d^2*e^10)*x^2 + 2*(a*c^5*d^
9*e^3 - 4*a^2*c^4*d^7*e^5 + 6*a^3*c^3*d^5*e^7 - 4*a^4*c^2*d^3*e^9 + a^5*c*d
*e^11)*x)*log(c*d*x + a*e))/(c^9*d^9*x^2 + 2*a*c^8*d^8*e*x + a^2*c^7*d^7*e^
2)
```

Sympy [A] time = 16.1664, size = 386, normalized size = 1.75

$$\frac{11a^6e^{12} - 54a^5cd^2e^{10} + 105a^4c^2d^4e^8 - 100a^3c^3d^6e^6 + 45a^2c^4d^8e^4 - 6ac^5d^{10}e^2 - c^6d^{12} + x\left(12a^5cde^{11} - 60a^4c^2d^3e^9 + 12\dots\right.}{2a^2c^7d^7e^2 + 4ac^8d^8ex + 2c^9d^9x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**9/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

```
[Out] (11*a**6*e**12 - 54*a**5*c*d**2*e**10 + 105*a**4*c**2*d**4*e**8 - 100*a**3*
c**3*d**6*e**6 + 45*a**2*c**4*d**8*e**4 - 6*a*c**5*d**10*e**2 - c**6*d**12
+ x*(12*a**5*c*d*e**11 - 60*a**4*c**2*d**3*e**9 + 120*a**3*c**3*d**5*e**7 -
 120*a**2*c**4*d**7*e**5 + 60*a*c**5*d**9*e**3 - 12*c**6*d**11*e))/(2*a**2*
c**7*d**7*e**2 + 4*a*c**8*d**8*e*x + 2*c**9*d**9*x**2) + e**6*x**4/(4*c**3*
```

```
d**3) - x**3*(a*e**7 - 2*c*d**2*e**5)/(c**4*d**4) + x**2*(6*a**2*e**8 - 18*
a*c*d**2*e**6 + 15*c**2*d**4*e**4)/(2*c**5*d**5) - x*(10*a**3*e**9 - 36*a**
2*c*d**2*e**7 + 45*a*c**2*d**4*e**5 - 20*c**3*d**6*e**3)/(c**6*d**6) + 15*e
**2*(a*e**2 - c*d**2)**4*log(a*e + c*d*x)/(c**7*d**7)
```

Giac [B] time = 18.4971, size = 1442, normalized size = 6.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^9/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac"
)
```

```
[Out] 15*(c^9*d^18*e^2 - 9*a*c^8*d^16*e^4 + 36*a^2*c^7*d^14*e^6 - 84*a^3*c^6*d^12
*e^8 + 126*a^4*c^5*d^10*e^10 - 126*a^5*c^4*d^8*e^12 + 84*a^6*c^3*d^6*e^14 -
 36*a^7*c^2*d^4*e^16 + 9*a^8*c*d^2*e^18 - a^9*e^20)*arctan((2*c*d*x*e + c*d
^2 + a*e^2)/sqrt(-c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4))/((c^11*d^15 - 4*a*c^1
0*d^13*e^2 + 6*a^2*c^9*d^11*e^4 - 4*a^3*c^8*d^9*e^6 + a^4*c^7*d^7*e^8)*sqrt
(-c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4)) + 15/2*(c^4*d^8*e^2 - 4*a*c^3*d^6*e^4
 + 6*a^2*c^2*d^4*e^6 - 4*a^3*c*d^2*e^8 + a^4*e^10)*log(c*d*x^2*e + c*d^2*x
+ a*x*e^2 + a*d*e)/(c^7*d^7) - 1/2*(c^10*d^22 + 2*a*c^9*d^20*e^2 - 63*a^2*c
^8*d^18*e^4 + 312*a^3*c^7*d^16*e^6 - 798*a^4*c^6*d^14*e^8 + 1260*a^5*c^5*d^
12*e^10 - 1302*a^6*c^4*d^10*e^12 + 888*a^7*c^3*d^8*e^14 - 387*a^8*c^2*d^6*e
^16 + 98*a^9*c*d^4*e^18 - 11*a^10*d^2*e^20 + 12*(c^10*d^19*e^3 - 9*a*c^9*d^
17*e^5 + 36*a^2*c^8*d^15*e^7 - 84*a^3*c^7*d^13*e^9 + 126*a^4*c^6*d^11*e^11
- 126*a^5*c^5*d^9*e^13 + 84*a^6*c^4*d^7*e^15 - 36*a^7*c^3*d^5*e^17 + 9*a^8*
c^2*d^3*e^19 - a^9*c*d*e^21)*x^3 + (25*c^10*d^20*e^2 - 214*a*c^9*d^18*e^4 +
 801*a^2*c^8*d^16*e^6 - 1704*a^3*c^7*d^14*e^8 + 2226*a^4*c^6*d^12*e^10 - 17
64*a^5*c^5*d^10*e^12 + 714*a^6*c^4*d^8*e^14 + 24*a^7*c^3*d^6*e^16 - 171*a^8
*c^2*d^4*e^18 + 74*a^9*c*d^2*e^20 - 11*a^10*e^22)*x^2 + 2*(7*c^10*d^21*e -
52*a*c^9*d^19*e^3 + 153*a^2*c^8*d^17*e^5 - 192*a^3*c^7*d^15*e^7 - 42*a^4*c^
6*d^13*e^9 + 504*a^5*c^5*d^11*e^11 - 798*a^6*c^4*d^9*e^13 + 672*a^7*c^3*d^7
*e^15 - 333*a^8*c^2*d^5*e^17 + 92*a^9*c*d^3*e^19 - 11*a^10*d*e^21)*x)/((c^2
*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)^2*(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)^2
*c^7*d^7) + 1/4*(c^9*d^9*x^4*e^18 + 8*c^9*d^10*x^3*e^17 + 30*c^9*d^11*x^2*e
^16 + 80*c^9*d^12*x*e^15 - 4*a*c^8*d^8*x^3*e^19 - 36*a*c^8*d^9*x^2*e^18 - 1
80*a*c^8*d^10*x*e^17 + 12*a^2*c^7*d^7*x^2*e^20 + 144*a^2*c^7*d^8*x*e^19 - 4
0*a^3*c^6*d^6*x*e^21)*e^(-12)/(c^12*d^12)
```

3.1886 $\int \frac{(d+ex)^8}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal. Leaf size=185

$$\frac{e^3x\left(6a^2e^4-15acd^2e^2+10c^2d^4\right)}{c^5d^5}+\frac{e^4x^2\left(5cd^2-3ae^2\right)}{2c^4d^4}-\frac{5e\left(cd^2-ae^2\right)^4}{c^6d^6(ae+cdx)}-\frac{\left(cd^2-ae^2\right)^5}{2c^6d^6(ae+cdx)^2}+\frac{10e^2\left(cd^2-ae^2\right)^3\log(a\dots}{c^6d^6}$$

```
[Out] (e^3*(10*c^2*d^4 - 15*a*c*d^2*e^2 + 6*a^2*e^4)*x)/(c^5*d^5) + (e^4*(5*c*d^2
 - 3*a*e^2)*x^2)/(2*c^4*d^4) + (e^5*x^3)/(3*c^3*d^3) - (c*d^2 - a*e^2)^5/(2
*c^6*d^6*(a*e + c*d*x)^2) - (5*e*(c*d^2 - a*e^2)^4)/(c^6*d^6*(a*e + c*d*x))
 + (10*e^2*(c*d^2 - a*e^2)^3*Log[a*e + c*d*x])/(c^6*d^6)
```

Rubi [A] time = 0.184028, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$\frac{e^3x\left(6a^2e^4-15acd^2e^2+10c^2d^4\right)}{c^5d^5}+\frac{e^4x^2\left(5cd^2-3ae^2\right)}{2c^4d^4}-\frac{5e\left(cd^2-ae^2\right)^4}{c^6d^6(ae+cdx)}-\frac{\left(cd^2-ae^2\right)^5}{2c^6d^6(ae+cdx)^2}+\frac{10e^2\left(cd^2-ae^2\right)^3\log(a\dots}{c^6d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^8/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] (e^3*(10*c^2*d^4 - 15*a*c*d^2*e^2 + 6*a^2*e^4)*x)/(c^5*d^5) + (e^4*(5*c*d^2
 - 3*a*e^2)*x^2)/(2*c^4*d^4) + (e^5*x^3)/(3*c^3*d^3) - (c*d^2 - a*e^2)^5/(2
*c^6*d^6*(a*e + c*d*x)^2) - (5*e*(c*d^2 - a*e^2)^4)/(c^6*d^6*(a*e + c*d*x))
 + (10*e^2*(c*d^2 - a*e^2)^3*Log[a*e + c*d*x])/(c^6*d^6)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(d+ex)^8}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx=\int \frac{(d+ex)^5}{(ae+cdx)^3}\,dx$$

$$=\int \left(\frac{10c^2d^4e^3-15acd^2e^5+6a^2e^7}{c^5d^5}+\frac{e^4\left(5cd^2-3ae^2\right)x}{c^4d^4}+\frac{e^5x^2}{c^3d^3}+\frac{\left(cd^2-ae^2\right)^5}{c^5d^5(ae+cdx\dots}\right.$$

$$=\frac{e^3\left(10c^2d^4-15acd^2e^2+6a^2e^4\right)x}{c^5d^5}+\frac{e^4\left(5cd^2-3ae^2\right)x^2}{2c^4d^4}+\frac{e^5x^3}{3c^3d^3}-\frac{\left(cd^2-ae\dots\right.}{2c^6d^6(ae+\dots}$$

Mathematica [A] time = 0.0904686, size = 262, normalized size = 1.42

$$\frac{3a^3c^2d^2e^6\left(-50d^2+10dex+21e^2x^2\right)+5a^2c^3d^3e^4\left(-24d^2ex+18d^3-33de^2x^2+4e^3x^3\right)+3a^4cde^8(35d+2ex)-27a^5e^{10}-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^8/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] (-27*a^5*e^10 + 3*a^4*c*d*e^8*(35*d + 2*e*x) + 3*a^3*c^2*d^2*e^6*(-50*d^2 +
 10*d*e*x + 21*e^2*x^2) + 5*a^2*c^3*d^3*e^4*(18*d^3 - 24*d^2*e*x - 33*d*e^2
*x^2 + 4*e^3*x^3) - 5*a*c^4*d^4*e^2*(3*d^4 - 24*d^3*e*x - 24*d^2*e^2*x^2 +
12*d*e^3*x^3 + e^4*x^4) + c^5*d^5*(-3*d^5 - 30*d^4*e*x + 60*d^2*e^3*x^3 + 1
5*d*e^4*x^4 + 2*e^5*x^5) - 60*e^2*(-(c*d^2) + a*e^2)^3*(a*e + c*d*x)^2*Log[
a*e + c*d*x])/(6*c^6*d^6*(a*e + c*d*x)^2)
```

Maple [B] time = 0.051, size = 412, normalized size = 2.2

$$\frac{e^5x^3}{3c^3d^3}-\frac{3e^6x^2a}{2c^4d^4}+\frac{5e^4x^2}{2c^3d^2}+6\frac{a^2e^7x}{c^5d^5}-15\frac{ae^5x}{c^4d^3}+10\frac{e^3x}{c^3d}+\frac{a^5e^{10}}{2c^6d^6(cdx+ae)^2}-\frac{5a^4e^8}{2c^5d^4(cdx+ae)^2}+5\frac{a^3e^6}{c^4d^2(cdx+ae)^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^8/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

```
[Out] 1/3*e^5*x^3/c^3/d^3-3/2*e^6/c^4/d^4*x^2*a+5/2*e^4/c^3/d^2*x^2+6*e^7/c^5/d^5
*a^2*x-15*e^5/c^4/d^3*a*x+10*e^3/c^3/d*x+1/2/c^6/d^6/(c*d*x+a*e)^2*a^5*e^10
-5/2/c^5/d^4/(c*d*x+a*e)^2*a^4*e^8+5/c^4/d^2/(c*d*x+a*e)^2*a^3*e^6-5/c^3/(c
*d*x+a*e)^2*a^2*e^4+5/2/c^2*d^2/(c*d*x+a*e)^2*a*e^2-1/2/c*d^4/(c*d*x+a*e)^2
-5/d^6*e^9/c^6/(c*d*x+a*e)*a^4+20/d^4*e^7/c^5/(c*d*x+a*e)*a^3-30/d^2*e^5/c^
4/(c*d*x+a*e)*a^2+20*e^3/c^3/(c*d*x+a*e)*a-5*d^2*e/c^2/(c*d*x+a*e)-10/c^6/d
^6*e^8*ln(c*d*x+a*e)*a^3+30/c^5/d^4*e^6*ln(c*d*x+a*e)*a^2-30/c^4/d^2*e^4*ln
(c*d*x+a*e)*a+10/c^3*e^2*ln(c*d*x+a*e)
```

Maxima [A] time = 1.06387, size = 419, normalized size = 2.26

$$-\frac{c^5d^{10}+5ac^4d^8e^2-30a^2c^3d^6e^4+50a^3c^2d^4e^6-35a^4cd^2e^8+9a^5e^{10}+10\left(c^5d^9e-4ac^4d^7e^3+6a^2c^3d^5e^5-4a^3c^2d^3e^7+\right.}{2\left(c^8d^8x^2+2ac^7d^7ex+a^2c^6d^6e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^8/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxim
a")
```

```
[Out] -1/2*(c^5*d^10 + 5*a*c^4*d^8*e^2 - 30*a^2*c^3*d^6*e^4 + 50*a^3*c^2*d^4*e^6
- 35*a^4*c*d^2*e^8 + 9*a^5*e^10 + 10*(c^5*d^9*e - 4*a*c^4*d^7*e^3 + 6*a^2*c
^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*c*d*e^9)*x)/(c^8*d^8*x^2 + 2*a*c^7*d^7
*e*x + a^2*c^6*d^6*e^2) + 1/6*(2*c^2*d^2*e^5*x^3 + 3*(5*c^2*d^3*e^4 - 3*a*c
*d*e^6)*x^2 + 6*(10*c^2*d^4*e^3 - 15*a*c*d^2*e^5 + 6*a^2*e^7)*x)/(c^5*d^5)
+ 10*(c^3*d^6*e^2 - 3*a*c^2*d^4*e^4 + 3*a^2*c*d^2*e^6 - a^3*e^8)*log(c*d*x
+ a*e)/(c^6*d^6)
```

Fricas [B] time = 1.89907, size = 930, normalized size = 5.03

$$\frac{2\,c^5d^5e^5x^5 - 3\,c^5d^{10} - 15\,ac^4d^8e^2 + 90\,a^2c^3d^6e^4 - 150\,a^3c^2d^4e^6 + 105\,a^4cd^2e^8 - 27\,a^5e^{10} + 5\left(3\,c^5d^6e^4 - ac^4d^4e^6\right)x^4 + 2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^8/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="frica
s")
```

```
[Out] 1/6*(2*c^5*d^5*e^5*x^5 - 3*c^5*d^10 - 15*a*c^4*d^8*e^2 + 90*a^2*c^3*d^6*e^4
 - 150*a^3*c^2*d^4*e^6 + 105*a^4*c*d^2*e^8 - 27*a^5*e^10 + 5*(3*c^5*d^6*e^4
 - a*c^4*d^4*e^6)*x^4 + 20*(3*c^5*d^7*e^3 - 3*a*c^4*d^5*e^5 + a^2*c^3*d^3*e
^7)*x^3 + 3*(40*a*c^4*d^6*e^4 - 55*a^2*c^3*d^4*e^6 + 21*a^3*c^2*d^2*e^8)*x^
2 - 6*(5*c^5*d^9*e - 20*a*c^4*d^7*e^3 + 20*a^2*c^3*d^5*e^5 - 5*a^3*c^2*d^3*
e^7 - a^4*c*d*e^9)*x + 60*(a^2*c^3*d^6*e^4 - 3*a^3*c^2*d^4*e^6 + 3*a^4*c*d^
2*e^8 - a^5*e^10 + (c^5*d^8*e^2 - 3*a*c^4*d^6*e^4 + 3*a^2*c^3*d^4*e^6 - a^3
*c^2*d^2*e^8)*x^2 + 2*(a*c^4*d^7*e^3 - 3*a^2*c^3*d^5*e^5 + 3*a^3*c^2*d^3*e^
7 - a^4*c*d*e^9)*x)*log(c*d*x + a*e))/(c^8*d^8*x^2 + 2*a*c^7*d^7*e*x + a^2*
c^6*d^6*e^2)
```

Sympy [A] time = 4.57694, size = 299, normalized size = 1.62

$$-\frac{9a^5e^{10} - 35a^4cd^2e^8 + 50a^3c^2d^4e^6 - 30a^2c^3d^6e^4 + 5ac^4d^8e^2 + c^5d^{10} + x\left(10a^4cde^9 - 40a^3c^2d^3e^7 + 60a^2c^3d^5e^5 - 40ac^4\right.}{2a^2c^6d^6e^2 + 4ac^7d^7ex + 2c^8d^8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**8/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

```
[Out] -(9*a**5*e**10 - 35*a**4*c*d**2*e**8 + 50*a**3*c**2*d**4*e**6 - 30*a**2*c**
3*d**6*e**4 + 5*a*c**4*d**8*e**2 + c**5*d**10 + x*(10*a**4*c*d*e**9 - 40*a*
*3*c**2*d**3*e**7 + 60*a**2*c**3*d**5*e**5 - 40*a*c**4*d**7*e**3 + 10*c**5*
d**9*e))/(2*a**2*c**6*d**6*e**2 + 4*a*c**7*d**7*e*x + 2*c**8*d**8*x**2) + e
**5*x**3/(3*c**3*d**3) - x**2*(3*a*e**6 - 5*c*d**2*e**4)/(2*c**4*d**4) + x*
(6*a**2*e**7 - 15*a*c*d**2*e**5 + 10*c**2*d**4*e**3)/(c**5*d**5) - 10*e**2*
(a*e**2 - c*d**2)**3*log(a*e + c*d*x)/(c**6*d**6)
```

Giac [B] time = 15.0715, size = 1260, normalized size = 6.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^8/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac"
)
```

```
[Out] 10*(c^8*d^16*e^2 - 8*a*c^7*d^14*e^4 + 28*a^2*c^6*d^12*e^6 - 56*a^3*c^5*d^10
*e^8 + 70*a^4*c^4*d^8*e^10 - 56*a^5*c^3*d^6*e^12 + 28*a^6*c^2*d^4*e^14 - 8*
a^7*c*d^2*e^16 + a^8*e^18)*arctan((2*c*d*x*e + c*d^2 + a*e^2)/sqrt(-c^2*d^4
 + 2*a*c*d^2*e^2 - a^2*e^4))/((c^10*d^14 - 4*a*c^9*d^12*e^2 + 6*a^2*c^8*d^1
0*e^4 - 4*a^3*c^7*d^8*e^6 + a^4*c^6*d^6*e^8)*sqrt(-c^2*d^4 + 2*a*c*d^2*e^2
- a^2*e^4)) + 5*(c^3*d^6*e^2 - 3*a*c^2*d^4*e^4 + 3*a^2*c*d^2*e^6 - a^3*e^8)
```

```
*log(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)/(c^6*d^6) - 1/2*(c^9*d^20 + a*c
^8*d^18*e^2 - 44*a^2*c^7*d^16*e^4 + 196*a^3*c^6*d^14*e^6 - 434*a^4*c^5*d^12
*e^8 + 574*a^5*c^4*d^10*e^10 - 476*a^6*c^3*d^8*e^12 + 244*a^7*c^2*d^6*e^14
- 71*a^8*c*d^4*e^16 + 9*a^9*d^2*e^18 + 10*(c^9*d^17*e^3 - 8*a*c^8*d^15*e^5
+ 28*a^2*c^7*d^13*e^7 - 56*a^3*c^6*d^11*e^9 + 70*a^4*c^5*d^9*e^11 - 56*a^5*
c^4*d^7*e^13 + 28*a^6*c^3*d^5*e^15 - 8*a^7*c^2*d^3*e^17 + a^8*c*d*e^19)*x^3
 + 3*(7*c^9*d^18*e^2 - 53*a*c^8*d^16*e^4 + 172*a^2*c^7*d^14*e^6 - 308*a^3*c
^6*d^12*e^8 + 322*a^4*c^5*d^10*e^10 - 182*a^5*c^4*d^8*e^12 + 28*a^6*c^3*d^6
*e^14 + 28*a^7*c^2*d^4*e^16 - 17*a^8*c*d^2*e^18 + 3*a^9*e^20)*x^2 + 6*(2*c^
9*d^19*e - 13*a*c^8*d^17*e^3 + 32*a^2*c^7*d^15*e^5 - 28*a^3*c^6*d^13*e^7 -
28*a^4*c^5*d^11*e^9 + 98*a^5*c^4*d^9*e^11 - 112*a^6*c^3*d^7*e^13 + 68*a^7*c
^2*d^5*e^15 - 22*a^8*c*d^3*e^17 + 3*a^9*d*e^19)*x)/((c^2*d^4 - 2*a*c*d^2*e^
2 + a^2*e^4)^2*(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)^2*c^6*d^6) + 1/6*(2*
c^6*d^6*x^3*e^14 + 15*c^6*d^7*x^2*e^13 + 60*c^6*d^8*x*e^12 - 9*a*c^5*d^5*x^
2*e^15 - 90*a*c^5*d^6*x*e^14 + 36*a^2*c^4*d^4*x*e^16)*e^(-9)/(c^9*d^9)
```

3.1887 $\int \frac{(d+ex)^7}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal. Leaf size=142

$$\frac{e^3x\left(4cd^2-3ae^2\right)}{c^4d^4}-\frac{4e\left(cd^2-ae^2\right)^3}{c^5d^5(ae+cdx)}-\frac{\left(cd^2-ae^2\right)^4}{2c^5d^5(ae+cdx)^2}+\frac{6e^2\left(cd^2-ae^2\right)^2\log(ae+cdx)}{c^5d^5}+\frac{e^4x^2}{2c^3d^3}$$

```
[Out] (e^3*(4*c*d^2 - 3*a*e^2)*x)/(c^4*d^4) + (e^4*x^2)/(2*c^3*d^3) - (c*d^2 - a*
e^2)^4/(2*c^5*d^5*(a*e + c*d*x)^2) - (4*e*(c*d^2 - a*e^2)^3)/(c^5*d^5*(a*e
+ c*d*x)) + (6*e^2*(c*d^2 - a*e^2)^2*Log[a*e + c*d*x])/(c^5*d^5)
```

Rubi [A] time = 0.135345, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$\frac{e^3x\left(4cd^2-3ae^2\right)}{c^4d^4}-\frac{4e\left(cd^2-ae^2\right)^3}{c^5d^5(ae+cdx)}-\frac{\left(cd^2-ae^2\right)^4}{2c^5d^5(ae+cdx)^2}+\frac{6e^2\left(cd^2-ae^2\right)^2\log(ae+cdx)}{c^5d^5}+\frac{e^4x^2}{2c^3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^7/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] (e^3*(4*c*d^2 - 3*a*e^2)*x)/(c^4*d^4) + (e^4*x^2)/(2*c^3*d^3) - (c*d^2 - a*
e^2)^4/(2*c^5*d^5*(a*e + c*d*x)^2) - (4*e*(c*d^2 - a*e^2)^3)/(c^5*d^5*(a*e
+ c*d*x)) + (6*e^2*(c*d^2 - a*e^2)^2*Log[a*e + c*d*x])/(c^5*d^5)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(d+ex)^7}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx=\int \frac{(d+ex)^4}{(ae+cdx)^3}\,dx$$

$$=\int \left(\frac{4cd^2e^3-3ae^5}{c^4d^4}+\frac{e^4x}{c^3d^3}+\frac{\left(cd^2-ae^2\right)^4}{c^4d^4(ae+cdx)^3}+\frac{4e\left(cd^2-ae^2\right)^3}{c^4d^4(ae+cdx)^2}+\frac{6\left(cd^2e-ae\right.}{c^4d^4(ae+cd}\right.$$

$$=\frac{e^3\left(4cd^2-3ae^2\right)x}{c^4d^4}+\frac{e^4x^2}{2c^3d^3}-\frac{\left(cd^2-ae^2\right)^4}{2c^5d^5(ae+cdx)^2}-\frac{4e\left(cd^2-ae^2\right)^3}{c^5d^5(ae+cdx)}+\frac{6e^2\left(cd^2-\right.}{}$$

Mathematica [A] time = 0.0702236, size = 191, normalized size = 1.35

$$\frac{a^2c^2d^2e^4\left(18d^2-16dex-11e^2x^2\right)+2a^3cde^6(ex-10d)+7a^4e^8-4ac^3d^3e^2\left(-6d^2ex+d^3-4de^2x^2+e^3x^3\right)+12e^2\left(cd^2-ae\right.}{2c^5d^5(ae+cdx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^7/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] (7*a^4*e^8 + 2*a^3*c*d*e^6*(-10*d + e*x) + a^2*c^2*d^2*e^4*(18*d^2 - 16*d*e
*x - 11*e^2*x^2) - 4*a*c^3*d^3*e^2*(d^3 - 6*d^2*e*x - 4*d*e^2*x^2 + e^3*x^3
) + c^4*d^4*(-d^4 - 8*d^3*e*x + 8*d*e^3*x^3 + e^4*x^4) + 12*e^2*(c*d^2 - a*
e^2)^2*(a*e + c*d*x)^2*Log[a*e + c*d*x])/(2*c^5*d^5*(a*e + c*d*x)^2)
```

Maple [B] time = 0.05, size = 302, normalized size = 2.1

$$\frac{e^4x^2}{2\,c^3d^3}-3\,\frac{ae^5x}{c^4d^4}+4\,\frac{e^3x}{c^3d^2}-\frac{a^4e^8}{2\,c^5d^5\,(cdx+ae)^2}+2\,\frac{a^3e^6}{c^4d^3\,(cdx+ae)^2}-3\,\frac{a^2e^4}{c^3d\,(cdx+ae)^2}+2\,\frac{ade^2}{c^2\,(cdx+ae)^2}-\frac{d^3}{2\,c\,(cdx+ae)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

```
[Out] 1/2*e^4*x^2/c^3/d^3-3*e^5/c^4/d^4*a*x+4*e^3/c^3/d^2*x-1/2/c^5/d^5/(c*d*x+a*
e)^2*a^4*e^8+2/c^4/d^3/(c*d*x+a*e)^2*a^3*e^6-3/c^3/d/(c*d*x+a*e)^2*a^2*e^4+
2/c^2*d/(c*d*x+a*e)^2*a*e^2-1/2/c*d^3/(c*d*x+a*e)^2+4/d^5*e^7/c^5/(c*d*x+a*
e)*a^3-12/d^3*e^5/c^4/(c*d*x+a*e)*a^2+12/d*e^3/c^3/(c*d*x+a*e)*a-4*d*e/c^2/
(c*d*x+a*e)+6/c^5/d^5*e^6*ln(c*d*x+a*e)*a^2-12/c^4/d^3*e^4*ln(c*d*x+a*e)*a+
6/c^3/d*e^2*ln(c*d*x+a*e)
```

Maxima [A] time = 1.07075, size = 302, normalized size = 2.13

$$-\frac{c^4d^8+4\,ac^3d^6e^2-18\,a^2c^2d^4e^4+20\,a^3cd^2e^6-7\,a^4e^8+8\left(c^4d^7e-3\,ac^3d^5e^3+3\,a^2c^2d^3e^5-a^3cde^7\right)x}{2\left(c^7d^7x^2+2\,ac^6d^6ex+a^2c^5d^5e^2\right)}+\frac{cde^4x^2+2\left(4\,cd^2e\right.}{2\,c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxim
a")
```

```
[Out] -1/2*(c^4*d^8 + 4*a*c^3*d^6*e^2 - 18*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 7
*a^4*e^8 + 8*(c^4*d^7*e - 3*a*c^3*d^5*e^3 + 3*a^2*c^2*d^3*e^5 - a^3*c*d*e^7
)*x)/(c^7*d^7*x^2 + 2*a*c^6*d^6*e*x + a^2*c^5*d^5*e^2) + 1/2*(c*d*e^4*x^2 +
 2*(4*c*d^2*e^3 - 3*a*e^5)*x)/(c^4*d^4) + 6*(c^2*d^4*e^2 - 2*a*c*d^2*e^4 +
a^2*e^6)*log(c*d*x + a*e)/(c^5*d^5)
```

Fricas [B] time = 1.89035, size = 664, normalized size = 4.68

$$\frac{c^4d^4e^4x^4-c^4d^8-4\,ac^3d^6e^2+18\,a^2c^2d^4e^4-20\,a^3cd^2e^6+7\,a^4e^8+4\left(2\,c^4d^5e^3-ac^3d^3e^5\right)x^3+\left(16\,ac^3d^4e^4-11\,a^2c^2d^2e^6\right)x^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="frica
s")
```

```
[Out] 1/2*(c^4*d^4*e^4*x^4 - c^4*d^8 - 4*a*c^3*d^6*e^2 + 18*a^2*c^2*d^4*e^4 - 20*
a^3*c*d^2*e^6 + 7*a^4*e^8 + 4*(2*c^4*d^5*e^3 - a*c^3*d^3*e^5)*x^3 + (16*a*c
^3*d^4*e^4 - 11*a^2*c^2*d^2*e^6)*x^2 - 2*(4*c^4*d^7*e - 12*a*c^3*d^5*e^3 +
8*a^2*c^2*d^3*e^5 - a^3*c*d*e^7)*x + 12*(a^2*c^2*d^4*e^4 - 2*a^3*c*d^2*e^6
+ a^4*e^8 + (c^4*d^6*e^2 - 2*a*c^3*d^4*e^4 + a^2*c^2*d^2*e^6)*x^2 + 2*(a*c^
3*d^5*e^3 - 2*a^2*c^2*d^3*e^5 + a^3*c*d*e^7)*x)*log(c*d*x + a*e))/(c^7*d^7*
x^2 + 2*a*c^6*d^6*e*x + a^2*c^5*d^5*e^2)
```

Sympy [A] time = 2.65236, size = 224, normalized size = 1.58

$$\frac{7a^4e^8 - 20a^3cd^2e^6 + 18a^2c^2d^4e^4 - 4ac^3d^6e^2 - c^4d^8 + x\left(8a^3cde^7 - 24a^2c^2d^3e^5 + 24ac^3d^5e^3 - 8c^4d^7e\right)}{2a^2c^5d^5e^2 + 4ac^6d^6ex + 2c^7d^7x^2} + \frac{e^4x^2}{2c^3d^3} - \frac{x\left(3ae\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**7/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

```
[Out] (7*a**4*e**8 - 20*a**3*c*d**2*e**6 + 18*a**2*c**2*d**4*e**4 - 4*a*c**3*d**6
*e**2 - c**4*d**8 + x*(8*a**3*c*d*e**7 - 24*a**2*c**2*d**3*e**5 + 24*a*c**3
*d**5*e**3 - 8*c**4*d**7*e))/(2*a**2*c**5*d**5*e**2 + 4*a*c**6*d**6*e*x + 2
*c**7*d**7*x**2) + e**4*x**2/(2*c**3*d**3) - x*(3*a*e**5 - 4*c*d**2*e**3)/(
c**4*d**4) + 6*e**2*(a*e**2 - c*d**2)**2*log(a*e + c*d*x)/(c**5*d**5)
```

Giac [B] time = 14.7952, size = 1084, normalized size = 7.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac"
)
```

```
[Out] 6*(c^7*d^14*e^2 - 7*a*c^6*d^12*e^4 + 21*a^2*c^5*d^10*e^6 - 35*a^3*c^4*d^8*e
^8 + 35*a^4*c^3*d^6*e^10 - 21*a^5*c^2*d^4*e^12 + 7*a^6*c*d^2*e^14 - a^7*e^1
6)*arctan((2*c*d*x*e + c*d^2 + a*e^2)/sqrt(-c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e
^4))/((c^9*d^13 - 4*a*c^8*d^11*e^2 + 6*a^2*c^7*d^9*e^4 - 4*a^3*c^6*d^7*e^6
+ a^4*c^5*d^5*e^8)*sqrt(-c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4)) + 3*(c^2*d^4*e
^2 - 2*a*c*d^2*e^4 + a^2*e^6)*log(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)/(c
^5*d^5) + 1/2*(c^3*d^3*x^2*e^10 + 8*c^3*d^4*x*e^9 - 6*a*c^2*d^2*x*e^11)*e^(
-6)/(c^6*d^6) - 1/2*(c^8*d^18 - 28*a^2*c^6*d^14*e^4 + 112*a^3*c^5*d^12*e^6
- 210*a^4*c^4*d^10*e^8 + 224*a^5*c^3*d^8*e^10 - 140*a^6*c^2*d^6*e^12 + 48*a
^7*c*d^4*e^14 - 7*a^8*d^2*e^16 + 8*(c^8*d^15*e^3 - 7*a*c^7*d^13*e^5 + 21*a^
2*c^6*d^11*e^7 - 35*a^3*c^5*d^9*e^9 + 35*a^4*c^4*d^7*e^11 - 21*a^5*c^3*d^5*
e^13 + 7*a^6*c^2*d^3*e^15 - a^7*c*d*e^17)*x^3 + (17*c^8*d^16*e^2 - 112*a*c^
7*d^14*e^4 + 308*a^2*c^6*d^12*e^6 - 448*a^3*c^5*d^10*e^8 + 350*a^4*c^4*d^8*
e^10 - 112*a^5*c^3*d^6*e^12 - 28*a^6*c^2*d^4*e^14 + 32*a^7*c*d^2*e^16 - 7*a
^8*e^18)*x^2 + 2*(5*c^8*d^17*e - 28*a*c^7*d^15*e^3 + 56*a^2*c^6*d^13*e^5 -
28*a^3*c^5*d^11*e^7 - 70*a^4*c^4*d^9*e^9 + 140*a^5*c^3*d^7*e^11 - 112*a^6*c
^2*d^5*e^13 + 44*a^7*c*d^3*e^15 - 7*a^8*d*e^17)*x)/((c^2*d^4 - 2*a*c*d^2*e^
```

```
2 + a^2*e^4)^2*(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)^2*c^5*d^5)
```

3.1888 $\int \frac{(d+ex)^6}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3} dx$

Optimal. Leaf size=111

$$-\frac{3e\left(cd^2-ae^2\right)^2}{c^4d^4(ae+cdx)}-\frac{\left(cd^2-ae^2\right)^3}{2c^4d^4(ae+cdx)^2}+\frac{3e^2\left(cd^2-ae^2\right)\log(ae+cdx)}{c^4d^4}+\frac{e^3x}{c^3d^3}$$

```
[Out] (e^3*x)/(c^3*d^3) - (c*d^2 - a*e^2)^3/(2*c^4*d^4*(a*e + c*d*x)^2) - (3*e*(c
*d^2 - a*e^2)^2)/(c^4*d^4*(a*e + c*d*x)) + (3*e^2*(c*d^2 - a*e^2)*Log[a*e +
 c*d*x])/(c^4*d^4)
```

Rubi [A] time = 0.0920715, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$-\frac{3e\left(cd^2-ae^2\right)^2}{c^4d^4(ae+cdx)}-\frac{\left(cd^2-ae^2\right)^3}{2c^4d^4(ae+cdx)^2}+\frac{3e^2\left(cd^2-ae^2\right)\log(ae+cdx)}{c^4d^4}+\frac{e^3x}{c^3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^6/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```


```
[Out] (e^3*x)/(c^3*d^3) - (c*d^2 - a*e^2)^3/(2*c^4*d^4*(a*e + c*d*x)^2) - (3*e*(c
*d^2 - a*e^2)^2)/(c^4*d^4*(a*e + c*d*x)) + (3*e^2*(c*d^2 - a*e^2)*Log[a*e +
 c*d*x])/(c^4*d^4)
```


Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^6}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3} dx &= \int \frac{(d+ex)^3}{(ae+cdx)^3} dx\\
&= \int \left(\frac{e^3}{c^3d^3}+\frac{\left(cd^2-ae^2\right)^3}{c^3d^3(ae+cdx)^3}+\frac{3e\left(cd^2-ae^2\right)^2}{c^3d^3(ae+cdx)^2}+\frac{3\left(cd^2e^2-ae^4\right)}{c^3d^3(ae+cdx)}\right) dx\\
&= \frac{e^3x}{c^3d^3}-\frac{\left(cd^2-ae^2\right)^3}{2c^4d^4(ae+cdx)^2}-\frac{3e\left(cd^2-ae^2\right)^2}{c^4d^4(ae+cdx)}+\frac{3e^2\left(cd^2-ae^2\right)\log(ae+cdx)}{c^4d^4}
\end{aligned}$$

Mathematica [A] time = 0.0531893, size = 139, normalized size = 1.25

$$\frac{a^2cde^4(9d-4ex)-5a^3e^6+ac^2d^2e^2\left(-3d^2+12dex+4e^2x^2\right)-6e^2\left(ae^2-cd^2\right)(ae+cdx)^2\log(ae+cdx)-c^3\left(-2d^3e^3x^3+6\right.}{2c^4d^4(ae+cdx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^6/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] (-5*a^3*e^6 + a^2*c*d*e^4*(9*d - 4*e*x) + a*c^2*d^2*e^2*(-3*d^2 + 12*d*e*x
+ 4*e^2*x^2) - c^3*(d^6 + 6*d^5*e*x - 2*d^3*e^3*x^3) - 6*e^2*(-(c*d^2) + a*
e^2)*(a*e + c*d*x)^2*Log[a*e + c*d*x])/(2*c^4*d^4*(a*e + c*d*x)^2)
```

Maple [A] time = 0.048, size = 201, normalized size = 1.8

$$\frac{e^3x}{c^3d^3}+\frac{a^3e^6}{2\,c^4d^4\,(cdx+ae)^2}-\frac{3\,a^2e^4}{2\,c^3d^2\,(cdx+ae)^2}+\frac{3\,ae^2}{2\,c^2\,(cdx+ae)^2}-\frac{d^2}{2\,c\,(cdx+ae)^2}-3\,\frac{a^2e^5}{c^4d^4\,(cdx+ae)}+6\,\frac{ae^3}{c^3d^2\,(cdx+ae)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

```
[Out] e^3*x/c^3/d^3+1/2/c^4/d^4/(c*d*x+a*e)^2*a^3*e^6-3/2/c^3/d^2/(c*d*x+a*e)^2*a
^2*e^4+3/2/c^2/(c*d*x+a*e)^2*a*e^2-1/2/c*d^2/(c*d*x+a*e)^2-3/d^4*e^5/c^4/(c
*d*x+a*e)*a^2+6/d^2*e^3/c^3/(c*d*x+a*e)*a-3*e/c^2/(c*d*x+a*e)-3/c^4/d^4*e^4
*ln(c*d*x+a*e)*a+3/c^3/d^2*e^2*ln(c*d*x+a*e)
```

Maxima [A] time = 1.0967, size = 211, normalized size = 1.9

$$-\frac{c^3d^6+3\,ac^2d^4e^2-9\,a^2cd^2e^4+5\,a^3e^6+6\left(c^3d^5e-2\,ac^2d^3e^3+a^2cde^5\right)x}{2\left(c^6d^6x^2+2\,ac^5d^5ex+a^2c^4d^4e^2\right)}+\frac{e^3x}{c^3d^3}+\frac{3\left(cd^2e^2-ae^4\right)\log\left(cdx+ae\right)}{c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxim
a")
```

```
[Out] -1/2*(c^3*d^6 + 3*a*c^2*d^4*e^2 - 9*a^2*c*d^2*e^4 + 5*a^3*e^6 + 6*(c^3*d^5*
e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x)/(c^6*d^6*x^2 + 2*a*c^5*d^5*e*x + a^2*
c^4*d^4*e^2) + e^3*x/(c^3*d^3) + 3*(c*d^2*e^2 - a*e^4)*log(c*d*x + a*e)/(c^
4*d^4)
```

Fricas [B] time = 1.85677, size = 443, normalized size = 3.99

$$\frac{2\,c^3d^3e^3x^3+4\,ac^2d^2e^4x^2-c^3d^6-3\,ac^2d^4e^2+9\,a^2cd^2e^4-5\,a^3e^6-2\left(3\,c^3d^5e-6\,ac^2d^3e^3+2\,a^2cde^5\right)x+6\left(a^2cd^2e^4-a^3e^6\right.}{2\left(c^6d^6x^2+2\,ac^5d^5ex+a^2c^4d^4e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="frica
s")
```

```
[Out] 1/2*(2*c^3*d^3*e^3*x^3 + 4*a*c^2*d^2*e^4*x^2 - c^3*d^6 - 3*a*c^2*d^4*e^2 +
9*a^2*c*d^2*e^4 - 5*a^3*e^6 - 2*(3*c^3*d^5*e - 6*a*c^2*d^3*e^3 + 2*a^2*c*d*
e^5)*x + 6*(a^2*c*d^2*e^4 - a^3*e^6 + (c^3*d^4*e^2 - a*c^2*d^2*e^4)*x^2 + 2
*(a*c^2*d^3*e^3 - a^2*c*d*e^5)*x)*log(c*d*x + a*e))/(c^6*d^6*x^2 + 2*a*c^5*
d^5*e*x + a^2*c^4*d^4*e^2)
```

Sympy [A] time = 1.49333, size = 163, normalized size = 1.47

$$-\frac{5a^3e^6-9a^2cd^2e^4+3ac^2d^4e^2+c^3d^6+x\left(6a^2cde^5-12ac^2d^3e^3+6c^3d^5e\right)}{2a^2c^4d^4e^2+4ac^5d^5ex+2c^6d^6x^2}+\frac{e^3x}{c^3d^3}-\frac{3e^2\left(ae^2-cd^2\right)\log\left(ae+cdx\right)}{c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**6/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

```
[Out] -(5*a**3*e**6 - 9*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6 + x*(6*
a**2*c*d*e**5 - 12*a*c**2*d**3*e**3 + 6*c**3*d**5*e))/(2*a**2*c**4*d**4*e**
2 + 4*a*c**5*d**5*e*x + 2*c**6*d**6*x**2) + e**3*x/(c**3*d**3) - 3*e**2*(a*
e**2 - c*d**2)*log(a*e + c*d*x)/(c**4*d**4)
```

Giac [B] time = 1.34645, size = 944, normalized size = 8.5

$$\frac{3\left(c^6d^{12}e^2-6\,ac^5d^{10}e^4+15\,a^2c^4d^8e^6-20\,a^3c^3d^6e^8+15\,a^4c^2d^4e^{10}-6\,a^5cd^2e^{12}+a^6e^{14}\right)\arctan\left(\frac{2\,cdxe+cd^2+ae^2}{\sqrt{-c^2d^4+2\,acd^2e^2-a^2e^4}}\right)}{\left(c^8d^{12}-4\,ac^7d^{10}e^2+6\,a^2c^6d^8e^4-4\,a^3c^5d^6e^6+a^4c^4d^4e^8\right)\sqrt{-c^2d^4+2\,acd^2e^2-a^2e^4}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac"
)
```

```
[Out] 3*(c^6*d^12*e^2 - 6*a*c^5*d^10*e^4 + 15*a^2*c^4*d^8*e^6 - 20*a^3*c^3*d^6*e^
8 + 15*a^4*c^2*d^4*e^10 - 6*a^5*c*d^2*e^12 + a^6*e^14)*arctan((2*c*d*x*e +
c*d^2 + a*e^2)/sqrt(-c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4))/((c^8*d^12 - 4*a*c
^7*d^10*e^2 + 6*a^2*c^6*d^8*e^4 - 4*a^3*c^5*d^6*e^6 + a^4*c^4*d^4*e^8)*sqrt
(-c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4)) + x*e^3/(c^3*d^3) + 3/2*(c*d^2*e^2 -
a*e^4)*log(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)/(c^4*d^4) - 1/2*(c^7*d^16
 - a*c^6*d^14*e^2 - 15*a^2*c^5*d^12*e^4 + 55*a^3*c^4*d^10*e^6 - 85*a^4*c^3*
d^8*e^8 + 69*a^5*c^2*d^6*e^10 - 29*a^6*c*d^4*e^12 + 5*a^7*d^2*e^14 + 6*(c^7
*d^13*e^3 - 6*a*c^6*d^11*e^5 + 15*a^2*c^5*d^9*e^7 - 20*a^3*c^4*d^7*e^9 + 15
*a^4*c^3*d^5*e^11 - 6*a^5*c^2*d^3*e^13 + a^6*c*d*e^15)*x^3 + (13*c^7*d^14*e
^2 - 73*a*c^6*d^12*e^4 + 165*a^2*c^5*d^10*e^6 - 185*a^3*c^4*d^8*e^8 + 95*a^
4*c^3*d^6*e^10 - 3*a^5*c^2*d^4*e^12 - 17*a^6*c*d^2*e^14 + 5*a^7*e^16)*x^2 +
 2*(4*c^7*d^15*e - 19*a*c^6*d^13*e^3 + 30*a^2*c^5*d^11*e^5 - 5*a^3*c^4*d^9*
e^7 - 40*a^4*c^3*d^7*e^9 + 51*a^5*c^2*d^5*e^11 - 26*a^6*c*d^3*e^13 + 5*a^7*
d*e^15)*x)/((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)^2*(c*d*x^2*e + c*d^2*x + a*
x*e^2 + a*d*e)^2*c^4*d^4)
```

3.1889 $\int \frac{(d+ex)^5}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal. Leaf size=85

$$-\frac{2e\left(cd^2-ae^2\right)}{c^3d^3(ae+cdx)}-\frac{\left(cd^2-ae^2\right)^2}{2c^3d^3(ae+cdx)^2}+\frac{e^2\log(ae+cdx)}{c^3d^3}$$

```
[Out] -(c*d^2 - a*e^2)^2/(2*c^3*d^3*(a*e + c*d*x)^2) - (2*e*(c*d^2 - a*e^2))/(c^3
*d^3*(a*e + c*d*x)) + (e^2*Log[a*e + c*d*x])/(c^3*d^3)
```

Rubi [A] time = 0.0614312, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$-\frac{2e\left(cd^2-ae^2\right)}{c^3d^3(ae+cdx)}-\frac{\left(cd^2-ae^2\right)^2}{2c^3d^3(ae+cdx)^2}+\frac{e^2\log(ae+cdx)}{c^3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] -(c*d^2 - a*e^2)^2/(2*c^3*d^3*(a*e + c*d*x)^2) - (2*e*(c*d^2 - a*e^2))/(c^3
*d^3*(a*e + c*d*x)) + (e^2*Log[a*e + c*d*x])/(c^3*d^3)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^5}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx &= \int \frac{(d+ex)^2}{(ae+cdx)^3}\,dx\\
&= \int \left(\frac{\left(cd^2-ae^2\right)^2}{c^2d^2(ae+cdx)^3}+\frac{2\left(cd^2e-ae^3\right)}{c^2d^2(ae+cdx)^2}+\frac{e^2}{c^2d^2(ae+cdx)}\right)dx\\
&= -\frac{\left(cd^2-ae^2\right)^2}{2c^3d^3(ae+cdx)^2}-\frac{2e\left(cd^2-ae^2\right)}{c^3d^3(ae+cdx)}+\frac{e^2\log(ae+cdx)}{c^3d^3}
\end{aligned}$$

Mathematica [A] time = 0.0336488, size = 65, normalized size = 0.76

$$\frac{2e^2\log(ae+cdx)-\frac{\left(cd^2-ae^2\right)\left(3ae^2+cd(d+4ex)\right)}{(ae+cdx)^2}}{2c^3d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] (-(((c*d^2 - a*e^2)*(3*a*e^2 + c*d*(d + 4*e*x)))/(a*e + c*d*x)^2) + 2*e^2*L
og[a*e + c*d*x])/(2*c^3*d^3)
```

Maple [A] time = 0.046, size = 123, normalized size = 1.5

$$-\frac{a^2e^4}{2\,c^3d^3\,(cdx+ae)^2}+\frac{ae^2}{c^2d\,(cdx+ae)^2}-\frac{d}{2\,c\,(cdx+ae)^2}+2\,\frac{ae^3}{c^3d^3\,(cdx+ae)}-2\,\frac{e}{c^2d\,(cdx+ae)}+\frac{e^2\ln(cdx+ae)}{c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

```
[Out] -1/2/c^3/d^3/(c*d*x+a*e)^2*a^2*e^4+1/c^2/d/(c*d*x+a*e)^2*a*e^2-1/2/c*d/(c*d
*x+a*e)^2+2/d^3*e^3/c^3/(c*d*x+a*e)*a-2/d*e/c^2/(c*d*x+a*e)+e^2*ln(c*d*x+a*
e)/c^3/d^3
```

Maxima [A] time = 1.07021, size = 142, normalized size = 1.67

$$-\frac{c^2d^4+2\,acd^2e^2-3\,a^2e^4+4\left(c^2d^3e-acde^3\right)x}{2\left(c^5d^5x^2+2\,ac^4d^4ex+a^2c^3d^3e^2\right)}+\frac{e^2\log(cdx+ae)}{c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxim
a")
```

```
[Out] -1/2*(c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4 + 4*(c^2*d^3*e - a*c*d*e^3)*x)/(c
^5*d^5*x^2 + 2*a*c^4*d^4*e*x + a^2*c^3*d^3*e^2) + e^2*log(c*d*x + a*e)/(c^3
*d^3)
```

Fricas [A] time = 1.98946, size = 255, normalized size = 3.

$$-\frac{c^2d^4+2\,acd^2e^2-3\,a^2e^4+4\left(c^2d^3e-acde^3\right)x-2\left(c^2d^2e^2x^2+2\,acde^3x+a^2e^4\right)\log(cdx+ae)}{2\left(c^5d^5x^2+2\,ac^4d^4ex+a^2c^3d^3e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="frica
s")
```

```
[Out] -1/2*(c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4 + 4*(c^2*d^3*e - a*c*d*e^3)*x - 2
*(c^2*d^2*e^2*x^2 + 2*a*c*d*e^3*x + a^2*e^4)*log(c*d*x + a*e))/(c^5*d^5*x^2
 + 2*a*c^4*d^4*e*x + a^2*c^3*d^3*e^2)
```

Sympy [A] time = 0.852945, size = 109, normalized size = 1.28

$$\frac{3a^2e^4-2acd^2e^2-c^2d^4+x\left(4acde^3-4c^2d^3e\right)}{2a^2c^3d^3e^2+4ac^4d^4ex+2c^5d^5x^2}+\frac{e^2\log\left(ae+cdx\right)}{c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

```
[Out] (3*a**2*e**4 - 2*a*c*d**2*e**2 - c**2*d**4 + x*(4*a*c*d*e**3 - 4*c**2*d**3*
e))/(2*a**2*c**3*d**3*e**2 + 4*a*c**4*d**4*e*x + 2*c**5*d**5*x**2) + e**2*l
og(a*e + c*d*x)/(c**3*d**3)
```

Giac [B] time = 1.33168, size = 811, normalized size = 9.54

$$\frac{\left(c^5d^{10}e^2-5\,ac^4d^8e^4+10\,a^2c^3d^6e^6-10\,a^3c^2d^4e^8+5\,a^4cd^2e^{10}-a^5e^{12}\right)\arctan\left(\frac{2\,cdxe+cd^2+ae^2}{\sqrt{-c^2d^4+2\,acd^2e^2-a^2e^4}}\right)}{\left(c^7d^{11}-4\,ac^6d^9e^2+6\,a^2c^5d^7e^4-4\,a^3c^4d^5e^6+a^4c^3d^3e^8\right)\sqrt{-c^2d^4+2\,acd^2e^2-a^2e^4}}+\frac{e^2\log\left(cdx^2e+cd^2x\right.}{2\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac"
)
```

```
[Out] (c^5*d^10*e^2 - 5*a*c^4*d^8*e^4 + 10*a^2*c^3*d^6*e^6 - 10*a^3*c^2*d^4*e^8 +
 5*a^4*c*d^2*e^10 - a^5*e^12)*arctan((2*c*d*x*e + c*d^2 + a*e^2)/sqrt(-c^2*
d^4 + 2*a*c*d^2*e^2 - a^2*e^4))/((c^7*d^11 - 4*a*c^6*d^9*e^2 + 6*a^2*c^5*d^
7*e^4 - 4*a^3*c^4*d^5*e^6 + a^4*c^3*d^3*e^8)*sqrt(-c^2*d^4 + 2*a*c*d^2*e^2
- a^2*e^4)) + 1/2*e^2*log(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)/(c^3*d^3)
- 1/2*(c^6*d^14 - 2*a*c^5*d^12*e^2 - 5*a^2*c^4*d^10*e^4 + 20*a^3*c^3*d^8*e^
6 - 25*a^4*c^2*d^6*e^8 + 14*a^5*c*d^4*e^10 - 3*a^6*d^2*e^12 + 4*(c^6*d^11*e
^3 - 5*a*c^5*d^9*e^5 + 10*a^2*c^4*d^7*e^7 - 10*a^3*c^3*d^5*e^9 + 5*a^4*c^2*
d^3*e^11 - a^5*c*d*e^13)*x^3 + 3*(3*c^6*d^12*e^2 - 14*a*c^5*d^10*e^4 + 25*a
^2*c^4*d^8*e^6 - 20*a^3*c^3*d^6*e^8 + 5*a^4*c^2*d^4*e^10 + 2*a^5*c*d^2*e^12
 - a^6*e^14)*x^2 + 6*(c^6*d^13*e - 4*a*c^5*d^11*e^3 + 5*a^2*c^4*d^9*e^5 - 5
*a^4*c^2*d^5*e^9 + 4*a^5*c*d^3*e^11 - a^6*d*e^13)*x)/((c^2*d^4 - 2*a*c*d^2*
e^2 + a^2*e^4)^2*(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)^2*c^3*d^3)
```

3.1890 $\int \frac{(d+ex)^4}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal. Leaf size=35

$$-\frac{(d+ex)^2}{2\left(cd^2-ae^2\right)(ae+cdx)^2}$$

```
[Out] -(d + e*x)^2/(2*(c*d^2 - a*e^2)*(a*e + c*d*x)^2)
```

Rubi [A] time = 0.0131445, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 37}

$$-\frac{(d+ex)^2}{2\left(cd^2-ae^2\right)(ae+cdx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] -(d + e*x)^2/(2*(c*d^2 - a*e^2)*(a*e + c*d*x)^2)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^4}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx &= \int \frac{d+ex}{(ae+cdx)^3}\,dx\\ &= -\frac{(d+ex)^2}{2\left(cd^2-ae^2\right)(ae+cdx)^2}\end{aligned}$$

Mathematica [A] time = 0.0132388, size = 35, normalized size = 1.

$$-\frac{ae^2+cd(d+2ex)}{2c^2d^2(ae+cdx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] -(a*e^2 + c*d*(d + 2*e*x))/(2*c^2*d^2*(a*e + c*d*x)^2)
```

Maple [A] time = 0.042, size = 51, normalized size = 1.5

$$-\frac{-ae^2+cd^2}{2\,c^2d^2\,(cdx+ae)^2}-\frac{e}{c^2d^2\,(cdx+ae)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

```
[Out] -1/2*(-a*e^2+c*d^2)/c^2/d^2/(c*d*x+a*e)^2-1/d^2/c^2*e/(c*d*x+a*e)
```

Maxima [A] time = 1.08134, size = 76, normalized size = 2.17

$$-\frac{2\,cdex+cd^2+ae^2}{2\left(c^4d^4x^2+2\,ac^3d^3ex+a^2c^2d^2e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxim
a")
```

```
[Out] -1/2*(2*c*d*e*x + c*d^2 + a*e^2)/(c^4*d^4*x^2 + 2*a*c^3*d^3*e*x + a^2*c^2*d
^2*e^2)
```

Fricas [A] time = 1.83694, size = 113, normalized size = 3.23

$$-\frac{2\,cdex+cd^2+ae^2}{2\left(c^4d^4x^2+2\,ac^3d^3ex+a^2c^2d^2e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="frica
s")
```

```
[Out] -1/2*(2*c*d*e*x + c*d^2 + a*e^2)/(c^4*d^4*x^2 + 2*a*c^3*d^3*e*x + a^2*c^2*d
^2*e^2)
```

Sympy [B] time = 0.61767, size = 60, normalized size = 1.71

$$-\frac{ae^2+cd^2+2cdex}{2a^2c^2d^2e^2+4ac^3d^3ex+2c^4d^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

```
[Out] -(a*e**2 + c*d**2 + 2*c*d*e*x)/(2*a**2*c**2*d**2*e**2 + 4*a*c**3*d**3*e*x +
 2*c**4*d**4*x**2)
```

Giac [B] time = 3.10052, size = 509, normalized size = 14.54

$$-\frac{2\,c^5d^9x^3e^3+5\,c^5d^{10}x^2e^2+4\,c^5d^{11}xe+c^5d^{12}-8\,ac^4d^7x^3e^5-19\,ac^4d^8x^2e^4-14\,ac^4d^9xe^3-3\,ac^4d^{10}e^2+12\,a^2c^3d^5x^3e^7}{2\,\left(c^6d^{10}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac"
)
```

```
[Out] -1/2*(2*c^5*d^9*x^3*e^3 + 5*c^5*d^10*x^2*e^2 + 4*c^5*d^11*x*e + c^5*d^12 -
8*a*c^4*d^7*x^3*e^5 - 19*a*c^4*d^8*x^2*e^4 - 14*a*c^4*d^9*x*e^3 - 3*a*c^4*d
^10*e^2 + 12*a^2*c^3*d^5*x^3*e^7 + 26*a^2*c^3*d^6*x^2*e^6 + 16*a^2*c^3*d^7*
x*e^5 + 2*a^2*c^3*d^8*e^4 - 8*a^3*c^2*d^3*x^3*e^9 - 14*a^3*c^2*d^4*x^2*e^8
- 4*a^3*c^2*d^5*x*e^7 + 2*a^3*c^2*d^6*e^6 + 2*a^4*c*d*x^3*e^11 + a^4*c*d^2*
x^2*e^10 - 4*a^4*c*d^3*x*e^9 - 3*a^4*c*d^4*e^8 + a^5*x^2*e^12 + 2*a^5*d*x*e
^11 + a^5*d^2*e^10)/((c^6*d^10 - 4*a*c^5*d^8*e^2 + 6*a^2*c^4*d^6*e^4 - 4*a^
3*c^3*d^4*e^6 + a^4*c^2*d^2*e^8)*(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)^2)
```

3.1891 $\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal. Leaf size=20

$$-\frac{1}{2cd(ae+cdx)^2}$$

```
[Out] -1/(2*c*d*(a*e + c*d*x)^2)
```

Rubi [A] time = 0.0102619, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 32}

$$-\frac{1}{2cd(ae+cdx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]

[Out] -1/(2*c*d*(a*e + c*d*x)^2)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx &= \int \frac{1}{(ae+cdx)^3}\,dx\\ &= -\frac{1}{2cd(ae+cdx)^2}\end{aligned}$$

Mathematica [A] time = 0.0035411, size = 20, normalized size = 1.

$$-\frac{1}{2cd(ae+cdx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]

[Out] -1/(2*c*d*(a*e + c*d*x)^2)
```

Maple [A] time = 0.039, size = 19, normalized size = 1.

$$-\frac{1}{2\,cd\,(cdx+ae)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

```
[Out] -1/2/c/d/(c*d*x+a*e)^2
```

Maxima [A] time = 1.0685, size = 47, normalized size = 2.35

$$-\frac{1}{2\left(c^{3}d^{3}x^{2}+2\,ac^{2}d^{2}ex+a^{2}cde^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxim
a")
```

```
[Out] -1/2/(c^3*d^3*x^2 + 2*a*c^2*d^2*e*x + a^2*c*d*e^2)
```

Fricas [A] time = 1.94773, size = 70, normalized size = 3.5

$$-\frac{1}{2\left(c^{3}d^{3}x^{2}+2\,ac^{2}d^{2}ex+a^{2}cde^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="frica
s")
```

```
[Out] -1/2/(c^3*d^3*x^2 + 2*a*c^2*d^2*e*x + a^2*c*d*e^2)
```

Sympy [B] time = 0.450763, size = 39, normalized size = 1.95

$$-\frac{1}{2a^{2}cde^{2}+4ac^{2}d^{2}ex+2c^{3}d^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

```
[Out] -1/(2*a**2*c*d*e**2 + 4*a*c**2*d**2*e*x + 2*c**3*d**3*x**2)
```

Giac [B] time = 1.37128, size = 350, normalized size = 17.5

$$-\frac{c^4d^8x^2e^2 + 2\,c^4d^9xe + c^4d^{10} - 4\,ac^3d^6x^2e^4 - 8\,ac^3d^7xe^3 - 4\,ac^3d^8e^2 + 6\,a^2c^2d^4x^2e^6 + 12\,a^2c^2d^5xe^5 + 6\,a^2c^2d^6e^4 - 4\,a^3cd^2}{2\left(c^5d^9 - 4\,ac^4d^7e^2 + 6\,a^2c^3d^5e^4 - 4\,a^3c^2d^3e^6 + a^4cde^8\right)\left(cdx^2e + cd^2x + a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac"
)
```

```
[Out] -1/2*(c^4*d^8*x^2*e^2 + 2*c^4*d^9*x*e + c^4*d^10 - 4*a*c^3*d^6*x^2*e^4 - 8*
a*c^3*d^7*x*e^3 - 4*a*c^3*d^8*e^2 + 6*a^2*c^2*d^4*x^2*e^6 + 12*a^2*c^2*d^5*
x*e^5 + 6*a^2*c^2*d^6*e^4 - 4*a^3*c*d^2*x^2*e^8 - 8*a^3*c*d^3*x*e^7 - 4*a^3
*c*d^4*e^6 + a^4*x^2*e^10 + 2*a^4*d*x*e^9 + a^4*d^2*e^8)/((c^5*d^9 - 4*a*c^
4*d^7*e^2 + 6*a^2*c^3*d^5*e^4 - 4*a^3*c^2*d^3*e^6 + a^4*c*d*e^8)*(c*d*x^2*e
 + c*d^2*x + a*x*e^2 + a*d*e)^2)
```

3.1892 $\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal. Leaf size=107

$$\frac{e}{\left(cd^2-ae^2\right)^2(ae+cdx)}-\frac{1}{2\left(cd^2-ae^2\right)(ae+cdx)^2}+\frac{e^2\log(ae+cdx)}{\left(cd^2-ae^2\right)^3}-\frac{e^2\log(d+ex)}{\left(cd^2-ae^2\right)^3}$$

```
[Out] -1/(2*(c*d^2 - a*e^2)*(a*e + c*d*x)^2) + e/((c*d^2 - a*e^2)^2*(a*e + c*d*x)
) + (e^2*Log[a*e + c*d*x])/(c*d^2 - a*e^2)^3 - (e^2*Log[d + e*x])/(c*d^2 -
a*e^2)^3
```

Rubi [A] time = 0.0710984, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 44}

$$\frac{e}{\left(cd^2-ae^2\right)^2(ae+cdx)}-\frac{1}{2\left(cd^2-ae^2\right)(ae+cdx)^2}+\frac{e^2\log(ae+cdx)}{\left(cd^2-ae^2\right)^3}-\frac{e^2\log(d+ex)}{\left(cd^2-ae^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] -1/(2*(c*d^2 - a*e^2)*(a*e + c*d*x)^2) + e/((c*d^2 - a*e^2)^2*(a*e + c*d*x)
) + (e^2*Log[a*e + c*d*x])/(c*d^2 - a*e^2)^3 - (e^2*Log[d + e*x])/(c*d^2 -
a*e^2)^3
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx &= \int \frac{1}{(ae+cdx)^3(d+ex)}\,dx\\ &= \int \left(\frac{cd}{\left(cd^2-ae^2\right)(ae+cdx)^3}-\frac{cde}{\left(cd^2-ae^2\right)^2(ae+cdx)^2}+\frac{cde^2}{\left(cd^2-ae^2\right)^3(ae+cd\right.\\ &= -\frac{1}{2\left(cd^2-ae^2\right)(ae+cdx)^2}+\frac{e}{\left(cd^2-ae^2\right)^2(ae+cdx)}+\frac{e^2\log(ae+cdx)}{\left(cd^2-ae^2\right)^3}-\frac{e^2\log}{(cd}\end{aligned}$$

Mathematica [A] time = 0.0770763, size = 83, normalized size = 0.78

$$-\frac{\frac{(cd^2-ae^2)(cd(d-2ex)-3ae^2)}{(ae+cdx)^2}-2e^2\log(ae+cdx)+2e^2\log(d+ex)}{2\left(cd^2-ae^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] -((((c*d^2 - a*e^2)*(-3*a*e^2 + c*d*(d - 2*e*x)))/(a*e + c*d*x)^2 - 2*e^2*Lo
g[a*e + c*d*x] + 2*e^2*Log[d + e*x])/(2*(c*d^2 - a*e^2)^3)
```

Maple [A] time = 0.05, size = 106, normalized size = 1.

$$\frac{e^2\ln(ex+d)}{\left(ae^2-cd^2\right)^3}+\frac{1}{\left(2ae^2-2cd^2\right)(cdx+ae)^2}+\frac{e}{\left(ae^2-cd^2\right)^2(cdx+ae)}-\frac{e^2\ln(cdx+ae)}{\left(ae^2-cd^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

```
[Out] e^2/(a*e^2-c*d^2)^3*ln(e*x+d)+1/2/(a*e^2-c*d^2)/(c*d*x+a*e)^2+e/(a*e^2-c*d^
2)^2/(c*d*x+a*e)-e^2/(a*e^2-c*d^2)^3*ln(c*d*x+a*e)
```

Maxima [B] time = 1.09754, size = 321, normalized size = 3.

$$\frac{e^2\log(cdx+ae)}{c^3d^6-3ac^2d^4e^2+3a^2cd^2e^4-a^3e^6}-\frac{e^2\log(ex+d)}{c^3d^6-3ac^2d^4e^2+3a^2cd^2e^4-a^3e^6}+\frac{2cd}{2\left(a^2c^2d^4e^2-2a^3cd^2e^4+a^4e^6+\left(c^4d^6-2ac^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxim
a")
```

```
[Out] e^2*log(c*d*x + a*e)/(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6
) - e^2*log(e*x + d)/(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6
) + 1/2*(2*c*d*e*x - c*d^2 + 3*a*e^2)/(a^2*c^2*d^4*e^2 - 2*a^3*c*d^2*e^4 +
a^4*e^6 + (c^4*d^6 - 2*a*c^3*d^4*e^2 + a^2*c^2*d^2*e^4)*x^2 + 2*(a*c^3*d^5*
e - 2*a^2*c^2*d^3*e^3 + a^3*c*d*e^5)*x)
```

Fricas [B] time = 2.00002, size = 556, normalized size = 5.2

$$-\frac{c^2d^4-4acd^2e^2+3a^2e^4-2\left(c^2d^3e-acde^3\right)x-2\left(c^2d^2e^2x^2+2acde^3x+a^2e^4\right)\log(cdx+ae)+2\left(c^2d^2e^2x^2+2acde^3x\right.}{2\left(a^2c^3d^6e^2-3a^3c^2d^4e^4+3a^4cd^2e^6-a^5e^8+\left(c^5d^8-3ac^4d^6e^2+3a^2c^3d^4e^4-a^3c^2d^2e^6\right)x^2+2\left(ac^4d^7e-3a^2c^3d^5e^3+3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="frica
s")
```

```
[Out] -1/2*(c^2*d^4 - 4*a*c*d^2*e^2 + 3*a^2*e^4 - 2*(c^2*d^3*e - a*c*d*e^3)*x - 2
*(c^2*d^2*e^2*x^2 + 2*a*c*d*e^3*x + a^2*e^4)*log(c*d*x + a*e) + 2*(c^2*d^2*
e^2*x^2 + 2*a*c*d*e^3*x + a^2*e^4)*log(e*x + d))/(a^2*c^3*d^6*e^2 - 3*a^3*c
^2*d^4*e^4 + 3*a^4*c*d^2*e^6 - a^5*e^8 + (c^5*d^8 - 3*a*c^4*d^6*e^2 + 3*a^2
*c^3*d^4*e^4 - a^3*c^2*d^2*e^6)*x^2 + 2*(a*c^4*d^7*e - 3*a^2*c^3*d^5*e^3 +
3*a^3*c^2*d^3*e^5 - a^4*c*d*e^7)*x)
```

Sympy [B] time = 1.61083, size = 457, normalized size = 4.27

$$\frac{e^2\log\left(x+\frac{-\frac{a^4e^{10}}{(ae^2-cd^2)^3}+\frac{4a^3cd^2e^8}{(ae^2-cd^2)^3}-\frac{6a^2c^2d^4e^6}{(ae^2-cd^2)^3}+\frac{4ac^3d^6e^4}{(ae^2-cd^2)^3}+ae^4-\frac{c^4d^8e^2}{(ae^2-cd^2)^3}+cd^2e^2}{2cde^3}\right)}{\left(ae^2-cd^2\right)^3}-\frac{e^2\log\left(x+\frac{\frac{a^4e^{10}}{(ae^2-cd^2)^3}-\frac{4a^3cd^2e^8}{(ae^2-cd^2)^3}+\frac{6a^2c^2d^4e^6}{(ae^2-cd^2)^3}-\frac{4ac^3d^6e^4}{(ae^2-cd^2)^3}+ae}{2cde^3}\right)}{\left(ae^2-cd^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

```
[Out] e**2*log(x + (-a**4*e**10/(a*e**2 - c*d**2)**3 + 4*a**3*c*d**2*e**8/(a*e**2
 - c*d**2)**3 - 6*a**2*c**2*d**4*e**6/(a*e**2 - c*d**2)**3 + 4*a*c**3*d**6*
e**4/(a*e**2 - c*d**2)**3 + a*e**4 - c**4*d**8*e**2/(a*e**2 - c*d**2)**3 +
c*d**2*e**2)/(2*c*d*e**3))/(a*e**2 - c*d**2)**3 - e**2*log(x + (a**4*e**10/
(a*e**2 - c*d**2)**3 - 4*a**3*c*d**2*e**8/(a*e**2 - c*d**2)**3 + 6*a**2*c**
2*d**4*e**6/(a*e**2 - c*d**2)**3 - 4*a*c**3*d**6*e**4/(a*e**2 - c*d**2)**3
+ a*e**4 + c**4*d**8*e**2/(a*e**2 - c*d**2)**3 + c*d**2*e**2)/(2*c*d*e**3))
/(a*e**2 - c*d**2)**3 + (3*a*e**2 - c*d**2 + 2*c*d*e*x)/(2*a**4*e**6 - 4*a*
*3*c*d**2*e**4 + 2*a**2*c**2*d**4*e**2 + x**2*(2*a**2*c**2*d**2*e**4 - 4*a*
c**3*d**4*e**2 + 2*c**4*d**6) + x*(4*a**3*c*d*e**5 - 8*a**2*c**2*d**3*e**3
+ 4*a*c**3*d**5*e))
```

Giac [B] time = 1.24947, size = 535, normalized size = 5.

$$\frac{2\left(c^2d^4e^2-2\,acd^2e^4+a^2e^6\right)\arctan\left(\frac{2\,cdxe+cd^2+ae^2}{\sqrt{-c^2d^4+2\,acd^2e^2-a^2e^4}}\right)}{\left(c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8\right)\sqrt{-c^2d^4+2\,acd^2e^2-a^2e^4}}+\frac{2\,c^3d^5x^3e^3+3\,c^3d^6x^2e^2-c^3d^8-4\,ac^2d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac"
)
```

```
[Out] 2*(c^2*d^4*e^2 - 2*a*c*d^2*e^4 + a^2*e^6)*arctan((2*c*d*x*e + c*d^2 + a*e^2
)/sqrt(-c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4))/((c^4*d^8 - 4*a*c^3*d^6*e^2 + 6
*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*sqrt(-c^2*d^4 + 2*a*c*d^2*e^2
 - a^2*e^4)) + 1/2*(2*c^3*d^5*x^3*e^3 + 3*c^3*d^6*x^2*e^2 - c^3*d^8 - 4*a*c
^2*d^3*x^3*e^5 - 3*a*c^2*d^4*x^2*e^4 + 6*a*c^2*d^5*x*e^3 + 5*a*c^2*d^6*e^2
+ 2*a^2*c*d*x^3*e^7 - 3*a^2*c*d^2*x^2*e^6 - 12*a^2*c*d^3*x*e^5 - 7*a^2*c*d^
4*e^4 + 3*a^3*x^2*e^8 + 6*a^3*d*x*e^7 + 3*a^3*d^2*e^6)/((c^4*d^8 - 4*a*c^3*
d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*(c*d*x^2*e + c*d^2
*x + a*x*e^2 + a*d*e)^2)
```

3.1893 $\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal. Leaf size=142

$$\frac{e^2}{(d+ex)\left(cd^2-ae^2\right)^3}+\frac{2cde}{\left(cd^2-ae^2\right)^3(ae+cdx)}-\frac{cd}{2\left(cd^2-ae^2\right)^2(ae+cdx)^2}+\frac{3cde^2\log(ae+cdx)}{\left(cd^2-ae^2\right)^4}-\frac{3cde^2\log(d+ex)}{\left(cd^2-ae^2\right)^4}$$

```
[Out] -(c*d)/(2*(c*d^2 - a*e^2)^2*(a*e + c*d*x)^2) + (2*c*d*e)/((c*d^2 - a*e^2)^3
*(a*e + c*d*x)) + e^2/((c*d^2 - a*e^2)^3*(d + e*x)) + (3*c*d*e^2*Log[a*e +
c*d*x])/(c*d^2 - a*e^2)^4 - (3*c*d*e^2*Log[d + e*x])/(c*d^2 - a*e^2)^4
```

Rubi [A] time = 0.112844, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {626, 44}

$$\frac{e^2}{(d+ex)\left(cd^2-ae^2\right)^3}+\frac{2cde}{\left(cd^2-ae^2\right)^3(ae+cdx)}-\frac{cd}{2\left(cd^2-ae^2\right)^2(ae+cdx)^2}+\frac{3cde^2\log(ae+cdx)}{\left(cd^2-ae^2\right)^4}-\frac{3cde^2\log(d+ex)}{\left(cd^2-ae^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] -(c*d)/(2*(c*d^2 - a*e^2)^2*(a*e + c*d*x)^2) + (2*c*d*e)/((c*d^2 - a*e^2)^3
*(a*e + c*d*x)) + e^2/((c*d^2 - a*e^2)^3*(d + e*x)) + (3*c*d*e^2*Log[a*e +
c*d*x])/(c*d^2 - a*e^2)^4 - (3*c*d*e^2*Log[d + e*x])/(c*d^2 - a*e^2)^4
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx &= \int \frac{1}{(ae+cdx)^3(d+ex)^2}\,dx\\ &= \int \left(\frac{c^2d^2}{\left(cd^2-ae^2\right)^2(ae+cdx)^3}-\frac{2c^2d^2e}{\left(cd^2-ae^2\right)^3(ae+cdx)^2}+\frac{3c^2d^2e^2}{\left(cd^2-ae^2\right)^4(ae+cdx)}\right.\\ &= -\frac{cd}{2\left(cd^2-ae^2\right)^2(ae+cdx)^2}+\frac{2cde}{\left(cd^2-ae^2\right)^3(ae+cdx)}+\frac{e^2}{\left(cd^2-ae^2\right)^3(d+ex)}+\frac{3}{}\end{aligned}$$

Mathematica [A] time = 0.102824, size = 127, normalized size = 0.89

$$\frac{\frac{4cde(cd^2-ae^2)}{ae+cdx}+\frac{2cd^2e^2-2ae^4}{d+ex}-\frac{cd(cd^2-ae^2)^2}{(ae+cdx)^2}+6cde^2\log(ae+cdx)-6cde^2\log(d+ex)}{2\left(cd^2-ae^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] (-((c*d*(c*d^2 - a*e^2)^2)/(a*e + c*d*x)^2) + (4*c*d*e*(c*d^2 - a*e^2))/(a*
e + c*d*x) + (2*c*d^2*e^2 - 2*a*e^4)/(d + e*x) + 6*c*d*e^2*Log[a*e + c*d*x]
 - 6*c*d*e^2*Log[d + e*x])/(2*(c*d^2 - a*e^2)^4)
```

Maple [A] time = 0.055, size = 142, normalized size = 1.

$$-\frac{e^2}{\left(ae^2-cd^2\right)^3(ex+d)}-3\frac{e^2cd\ln(ex+d)}{\left(ae^2-cd^2\right)^4}-\frac{cd}{2\left(ae^2-cd^2\right)^2(cdx+ae)^2}+3\frac{e^2cd\ln(cdx+ae)}{\left(ae^2-cd^2\right)^4}-2\frac{dec}{\left(ae^2-cd^2\right)^3(cdx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

```
[Out] -e^2/(a*e^2-c*d^2)^3/(e*x+d)-3*e^2/(a*e^2-c*d^2)^4*c*d*ln(e*x+d)-1/2*c*d/(a
*e^2-c*d^2)^2/(c*d*x+a*e)^2+3*e^2/(a*e^2-c*d^2)^4*c*d*ln(c*d*x+a*e)-2*c*d/(
a*e^2-c*d^2)^3*e/(c*d*x+a*e)
```

Maxima [B] time = 1.22685, size = 579, normalized size = 4.08

$$\frac{3\,cde^2\log(cdx+ae)}{c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8}-\frac{3\,cde^2\log(ex+d)}{c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8}+\frac{}{2\left(a^2c^3d^7e^2-3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxima"
)
```

```
[Out] 3*c*d*e^2*log(c*d*x + a*e)/(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 -
 4*a^3*c*d^2*e^6 + a^4*e^8) - 3*c*d*e^2*log(e*x + d)/(c^4*d^8 - 4*a*c^3*d^6
*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8) + 1/2*(6*c^2*d^2*e^2*
x^2 - c^2*d^4 + 5*a*c*d^2*e^2 + 2*a^2*e^4 + 3*(c^2*d^3*e + 3*a*c*d*e^3)*x)/
(a^2*c^3*d^7*e^2 - 3*a^3*c^2*d^5*e^4 + 3*a^4*c*d^3*e^6 - a^5*d*e^8 + (c^5*d
^8*e - 3*a*c^4*d^6*e^3 + 3*a^2*c^3*d^4*e^5 - a^3*c^2*d^2*e^7)*x^3 + (c^5*d^
9 - a*c^4*d^7*e^2 - 3*a^2*c^3*d^5*e^4 + 5*a^3*c^2*d^3*e^6 - 2*a^4*c*d*e^8)*
x^2 + (2*a*c^4*d^8*e - 5*a^2*c^3*d^6*e^3 + 3*a^3*c^2*d^4*e^5 + a^4*c*d^2*e^
7 - a^5*e^9)*x)
```

Fricas [B] time = 2.03729, size = 1099, normalized size = 7.74

$$-\frac{c^3d^6-6\,ac^2d^4e^2+3\,a^2cd^2e^4+2\,a^3e^6-6\left(c^3d^4e^2-ac^2d^2e^4\right)x^2-3\left(c^3d^5e+2\,ac^2d^3e^3-3\,a^2cde^5\right)x-6\left(c^3d^3e^3x^3+a^2\right.}{2\left(a^2c^4d^9e^2-4\,a^3c^3d^7e^4+6\,a^4c^2d^5e^6-4\,a^5cd^3e^8+a^6de^{10}+\left(c^6d^{10}e-4\,ac^5d^8e^3+6\,a^2c^4d^6e^5-4\,a^3c^3d^4e^7+a^4c^2d\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="fricas"
)
```

```
[Out] -1/2*(c^3*d^6 - 6*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 2*a^3*e^6 - 6*(c^3*d^4*
e^2 - a*c^2*d^2*e^4)*x^2 - 3*(c^3*d^5*e + 2*a*c^2*d^3*e^3 - 3*a^2*c*d*e^5)*
x - 6*(c^3*d^3*e^3*x^3 + a^2*c*d^2*e^4 + (c^3*d^4*e^2 + 2*a*c^2*d^2*e^4)*x^
2 + (2*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x)*log(c*d*x + a*e) + 6*(c^3*d^3*e^3*x^
3 + a^2*c*d^2*e^4 + (c^3*d^4*e^2 + 2*a*c^2*d^2*e^4)*x^2 + (2*a*c^2*d^3*e^3
+ a^2*c*d*e^5)*x)*log(e*x + d))/(a^2*c^4*d^9*e^2 - 4*a^3*c^3*d^7*e^4 + 6*a^
4*c^2*d^5*e^6 - 4*a^5*c*d^3*e^8 + a^6*d*e^10 + (c^6*d^10*e - 4*a*c^5*d^8*e^
3 + 6*a^2*c^4*d^6*e^5 - 4*a^3*c^3*d^4*e^7 + a^4*c^2*d^2*e^9)*x^3 + (c^6*d^1
1 - 2*a*c^5*d^9*e^2 - 2*a^2*c^4*d^7*e^4 + 8*a^3*c^3*d^5*e^6 - 7*a^4*c^2*d^3
*e^8 + 2*a^5*c*d*e^10)*x^2 + (2*a*c^5*d^10*e - 7*a^2*c^4*d^8*e^3 + 8*a^3*c^
3*d^6*e^5 - 2*a^4*c^2*d^4*e^7 - 2*a^5*c*d^2*e^9 + a^6*e^11)*x)
```

Sympy [B] time = 2.62892, size = 734, normalized size = 5.17

$$-\frac{3cde^2\log\left(x+\frac{-\frac{3a^5cde^{12}}{(ae^2-cd^2)^4}+\frac{15a^4c^2d^3e^{10}}{(ae^2-cd^2)^4}-\frac{30a^3c^3d^5e^8}{(ae^2-cd^2)^4}+\frac{30a^2c^4d^7e^6}{(ae^2-cd^2)^4}-\frac{15ac^5d^9e^4}{(ae^2-cd^2)^4}+3acde^4+\frac{3c^6d^{11}e^2}{(ae^2-cd^2)^4}+3c^2d^3e^2}{6c^2d^2e^3}\right)}{(ae^2-cd^2)^4}+\frac{3cde^2\log\left(x+\frac{\frac{3a^5cde^{12}}{(ae^2-cd^2)^4}-\frac{15a^4c^2d^3e^{10}}{(ae^2-cd^2)^4}\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

```
[Out] -3*c*d*e**2*log(x + (-3*a**5*c*d*e**12/(a*e**2 - c*d**2)**4 + 15*a**4*c**2*
d**3*e**10/(a*e**2 - c*d**2)**4 - 30*a**3*c**3*d**5*e**8/(a*e**2 - c*d**2)*
*4 + 30*a**2*c**4*d**7*e**6/(a*e**2 - c*d**2)**4 - 15*a*c**5*d**9*e**4/(a*e
**2 - c*d**2)**4 + 3*a*c*d*e**4 + 3*c**6*d**11*e**2/(a*e**2 - c*d**2)**4 +
3*c**2*d**3*e**2)/(6*c**2*d**2*e**3))/(a*e**2 - c*d**2)**4 + 3*c*d*e**2*log
(x + (3*a**5*c*d*e**12/(a*e**2 - c*d**2)**4 - 15*a**4*c**2*d**3*e**10/(a*e*
*2 - c*d**2)**4 + 30*a**3*c**3*d**5*e**8/(a*e**2 - c*d**2)**4 - 30*a**2*c**
4*d**7*e**6/(a*e**2 - c*d**2)**4 + 15*a*c**5*d**9*e**4/(a*e**2 - c*d**2)**4
 + 3*a*c*d*e**4 - 3*c**6*d**11*e**2/(a*e**2 - c*d**2)**4 + 3*c**2*d**3*e**2
)/(6*c**2*d**2*e**3))/(a*e**2 - c*d**2)**4 - (2*a**2*e**4 + 5*a*c*d**2*e**2
 - c**2*d**4 + 6*c**2*d**2*e**2*x**2 + x*(9*a*c*d*e**3 + 3*c**2*d**3*e))/(2
*a**5*d*e**8 - 6*a**4*c*d**3*e**6 + 6*a**3*c**2*d**5*e**4 - 2*a**2*c**3*d**
7*e**2 + x**3*(2*a**3*c**2*d**2*e**7 - 6*a**2*c**3*d**4*e**5 + 6*a*c**4*d**
6*e**3 - 2*c**5*d**8*e) + x**2*(4*a**4*c*d*e**8 - 10*a**3*c**2*d**3*e**6 +
6*a**2*c**3*d**5*e**4 + 2*a*c**4*d**7*e**2 - 2*c**5*d**9) + x*(2*a**5*e**9
- 2*a**4*c*d**2*e**7 - 6*a**3*c**2*d**4*e**5 + 10*a**2*c**3*d**6*e**3 - 4*a
*c**4*d**8*e))
```

Giac [B] time = 1.25553, size = 483, normalized size = 3.4

$$\frac{6\left(c^2d^3e^2-acde^4\right)\arctan\left(\frac{2\,cdxe+cd^2+ae^2}{\sqrt{-c^2d^4+2\,acd^2e^2-a^2e^4}}\right)}{\left(c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8\right)\sqrt{-c^2d^4+2\,acd^2e^2-a^2e^4}}+\frac{6\,c^3d^4x^3e^3+9\,c^3d^5x^2e^2+2\,c^3d^6xe-c^3d^7\cdots}{2\left(c^4d^8-4\,ac\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac")
```

```
[Out] 6*(c^2*d^3*e^2 - a*c*d*e^4)*arctan((2*c*d*x*e + c*d^2 + a*e^2)/sqrt(-c^2*d^
4 + 2*a*c*d^2*e^2 - a^2*e^4))/((c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e
^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*sqrt(-c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4)) +
 1/2*(6*c^3*d^4*x^3*e^3 + 9*c^3*d^5*x^2*e^2 + 2*c^3*d^6*x*e - c^3*d^7 - 6*a
*c^2*d^2*x^3*e^5 + 12*a*c^2*d^4*x*e^3 + 6*a*c^2*d^5*e^2 - 9*a^2*c*d*x^2*e^6
 - 12*a^2*c*d^2*x*e^5 - 3*a^2*c*d^3*e^4 - 2*a^3*x*e^7 - 2*a^3*d*e^6)/((c^4*
d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*(c*d
*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)^2)
```

3.1894 $\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal. Leaf size=191

$$\frac{6c^2d^2e^2\log(ae+cdx)}{\left(cd^2-ae^2\right)^5}-\frac{6c^2d^2e^2\log(d+ex)}{\left(cd^2-ae^2\right)^5}+\frac{3cde\left(ae^2+cd^2+2cdex\right)}{\left(cd^2-ae^2\right)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)}-\frac{ae^2+cd^2+2cd}{2\left(cd^2-ae^2\right)^2\left(x\left(ae^2+cd^2\right)\right.}$$

```
[Out] -(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^2) + (3*c*d*e*(c*d^2 + a*e^2 + 2*c*d*e*x))/((c*d^2 - a*e^2)^
4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)) + (6*c^2*d^2*e^2*Log[a*e + c*d*x
])/(c*d^2 - a*e^2)^5 - (6*c^2*d^2*e^2*Log[d + e*x])/(c*d^2 - a*e^2)^5
```

Rubi [A] time = 0.0648698, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {614, 616, 31}

$$\frac{6c^2d^2e^2\log(ae+cdx)}{\left(cd^2-ae^2\right)^5}-\frac{6c^2d^2e^2\log(d+ex)}{\left(cd^2-ae^2\right)^5}+\frac{3cde\left(ae^2+cd^2+2cdex\right)}{\left(cd^2-ae^2\right)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)}-\frac{ae^2+cd^2+2cd}{2\left(cd^2-ae^2\right)^2\left(x\left(ae^2+cd^2\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-3),x]
```

```
[Out] -(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^2) + (3*c*d*e*(c*d^2 + a*e^2 + 2*c*d*e*x))/((c*d^2 - a*e^2)^
4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)) + (6*c^2*d^2*e^2*Log[a*e + c*d*x
])/(c*d^2 - a*e^2)^5 - (6*c^2*d^2*e^2*Log[d + e*x])/(c*d^2 - a*e^2)^5
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx=-\frac{cd^2+ae^2+2cdex}{2\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}-\frac{(3cde)\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}}{\left(cd^2-ae^2\right)^2}$$

$$=-\frac{cd^2+ae^2+2cdex}{2\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}+\frac{3cde\left(cd^2+ae^2+2c\right.}{\left(cd^2-ae^2\right)^4\left(ade+\left(cd^2+ae\right.\right.}$$

$$=-\frac{cd^2+ae^2+2cdex}{2\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}+\frac{3cde\left(cd^2+ae^2+2c\right.}{\left(cd^2-ae^2\right)^4\left(ade+\left(cd^2+ae\right.\right.}$$

$$=-\frac{cd^2+ae^2+2cdex}{2\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}+\frac{3cde\left(cd^2+ae^2+2c\right.}{\left(cd^2-ae^2\right)^4\left(ade+\left(cd^2+ae\right.\right.}$$

Mathematica [A] time = 0.151466, size = 168, normalized size = 0.88

$$\frac{\frac{6c^2d^2e\left(ae^2-cd^2\right)}{ae+cdx}+\frac{c^2d^2\left(cd^2-ae^2\right)^2}{(ae+cdx)^2}-12c^2d^2e^2\log(ae+cdx)+\frac{6cde^2\left(ae^2-cd^2\right)}{d+ex}-\frac{\left(cd^2e-ae^3\right)^2}{(d+ex)^2}+12c^2d^2e^2\log(d+ex)}{2\left(ae^2-cd^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-3),x]
```

```
[Out] ((c^2*d^2*(c*d^2 - a*e^2)^2)/(a*e + c*d*x)^2 + (6*c^2*d^2*e*(-(c*d^2) + a*e
^2))/(a*e + c*d*x) - (c*d^2*e - a*e^3)^2/(d + e*x)^2 + (6*c*d*e^2*(-(c*d^2)
 + a*e^2))/(d + e*x) - 12*c^2*d^2*e^2*Log[a*e + c*d*x] + 12*c^2*d^2*e^2*Log
[d + e*x])/(2*(-(c*d^2) + a*e^2)^5)
```

Maple [A] time = 0.055, size = 186, normalized size = 1.

$$-\frac{e^2}{2\left(ae^2-cd^2\right)^3(ex+d)^2}+6\frac{e^2c^2d^2\ln(ex+d)}{\left(ae^2-cd^2\right)^5}+3\frac{cde^2}{\left(ae^2-cd^2\right)^4(ex+d)}+\frac{c^2d^2}{2\left(ae^2-cd^2\right)^3(cdx+ae)^2}-6\frac{e^2c^2d^2\ln(c}{\left(ae^2-c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

```
[Out] -1/2*e^2/(a*e^2-c*d^2)^3/(e*x+d)^2+6*e^2/(a*e^2-c*d^2)^5*c^2*d^2*ln(e*x+d)+
3*e^2/(a*e^2-c*d^2)^4*c*d/(e*x+d)+1/2*c^2*d^2/(a*e^2-c*d^2)^3/(c*d*x+a*e)^2
-6*e^2/(a*e^2-c*d^2)^5*c^2*d^2*ln(c*d*x+a*e)+3*c^2*d^2/(a*e^2-c*d^2)^4*e/(c
*d*x+a*e)
```

Maxima [B] time = 1.16293, size = 867, normalized size = 4.54

$$\frac{6\,c^2d^2e^2\log(cdx+ae)}{c^5d^{10}-5\,ac^4d^8e^2+10\,a^2c^3d^6e^4-10\,a^3c^2d^4e^6+5\,a^4cd^2e^8-a^5e^{10}}-\frac{6\,c^2d^2e^2\log(ex+d)}{c^5d^{10}-5\,ac^4d^8e^2+10\,a^2c^3d^6e^4-10\,a^3c^2d^4e^6+5\,a\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxima")
```

```
[Out] 6*c^2*d^2*e^2*log(c*d*x + a*e)/(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6
*e^4 - 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10) - 6*c^2*d^2*e^2*log
(e*x + d)/(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4
*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10) + 1/2*(12*c^3*d^3*e^3*x^3 - c^3*d^6 + 7*
a*c^2*d^4*e^2 + 7*a^2*c*d^2*e^4 - a^3*e^6 + 18*(c^3*d^4*e^2 + a*c^2*d^2*e^4
)*x^2 + 4*(c^3*d^5*e + 7*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x)/(a^2*c^4*d^10*e^2
- 4*a^3*c^3*d^8*e^4 + 6*a^4*c^2*d^6*e^6 - 4*a^5*c*d^4*e^8 + a^6*d^2*e^10 +
(c^6*d^10*e^2 - 4*a*c^5*d^8*e^4 + 6*a^2*c^4*d^6*e^6 - 4*a^3*c^3*d^4*e^8 + a
^4*c^2*d^2*e^10)*x^4 + 2*(c^6*d^11*e - 3*a*c^5*d^9*e^3 + 2*a^2*c^4*d^7*e^5
+ 2*a^3*c^3*d^5*e^7 - 3*a^4*c^2*d^3*e^9 + a^5*c*d*e^11)*x^3 + (c^6*d^12 - 9
*a^2*c^4*d^8*e^4 + 16*a^3*c^3*d^6*e^6 - 9*a^4*c^2*d^4*e^8 + a^6*e^12)*x^2 +
 2*(a*c^5*d^11*e - 3*a^2*c^4*d^9*e^3 + 2*a^3*c^3*d^7*e^5 + 2*a^4*c^2*d^5*e^
7 - 3*a^5*c*d^3*e^9 + a^6*d*e^11)*x)
```

Fricas [B] time = 2.09057, size = 1628, normalized size = 8.52

$$-\frac{c^4d^8 - 8\,ac^3d^6e^2 + 8\,a^3cd^2e^6 - a^4e^8 - 12\left(c^4d^5e^3 - ac^3d^3e^5\right)x^3 - 18\left(c^4d^6e^2 - a^2c^2d^2e^6\right)x^2 - 4\left(c^4d^7e + 6\,ac^3d^5e^3 \cdots\right.}{2\left(a^2c^5d^{12}e^2 - 5\,a^3c^4d^{10}e^4 + 10\,a^4c^3d^8e^6 - 10\,a^5c^2d^6e^8 + 5\,a^6cd^4e^{10} - a^7d^2e^{12} + \left(c^7d^{12}e^2 - 5\,ac^6d^{10}e^4 + 10\,a^2c^5d^8e^6 - 10 \cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(c^4*d^8 - 8*a*c^3*d^6*e^2 + 8*a^3*c*d^2*e^6 - a^4*e^8 - 12*(c^4*d^5*e
^3 - a*c^3*d^3*e^5)*x^3 - 18*(c^4*d^6*e^2 - a^2*c^2*d^2*e^6)*x^2 - 4*(c^4*d
^7*e + 6*a*c^3*d^5*e^3 - 6*a^2*c^2*d^3*e^5 - a^3*c*d*e^7)*x - 12*(c^4*d^4*e
^4*x^4 + a^2*c^2*d^4*e^4 + 2*(c^4*d^5*e^3 + a*c^3*d^3*e^5)*x^3 + (c^4*d^6*e
^2 + 4*a*c^3*d^4*e^4 + a^2*c^2*d^2*e^6)*x^2 + 2*(a*c^3*d^5*e^3 + a^2*c^2*d^
3*e^5)*x)*log(c*d*x + a*e) + 12*(c^4*d^4*e^4*x^4 + a^2*c^2*d^4*e^4 + 2*(c^4
*d^5*e^3 + a*c^3*d^3*e^5)*x^3 + (c^4*d^6*e^2 + 4*a*c^3*d^4*e^4 + a^2*c^2*d^
2*e^6)*x^2 + 2*(a*c^3*d^5*e^3 + a^2*c^2*d^3*e^5)*x)*log(e*x + d))/(a^2*c^5*
d^12*e^2 - 5*a^3*c^4*d^10*e^4 + 10*a^4*c^3*d^8*e^6 - 10*a^5*c^2*d^6*e^8 + 5
*a^6*c*d^4*e^10 - a^7*d^2*e^12 + (c^7*d^12*e^2 - 5*a*c^6*d^10*e^4 + 10*a^2*
c^5*d^8*e^6 - 10*a^3*c^4*d^6*e^8 + 5*a^4*c^3*d^4*e^10 - a^5*c^2*d^2*e^12)*x
^4 + 2*(c^7*d^13*e - 4*a*c^6*d^11*e^3 + 5*a^2*c^5*d^9*e^5 - 5*a^4*c^3*d^5*e
^9 + 4*a^5*c^2*d^3*e^11 - a^6*c*d*e^13)*x^3 + (c^7*d^14 - a*c^6*d^12*e^2 -
9*a^2*c^5*d^10*e^4 + 25*a^3*c^4*d^8*e^6 - 25*a^4*c^3*d^6*e^8 + 9*a^5*c^2*d^
4*e^10 + a^6*c*d^2*e^12 - a^7*e^14)*x^2 + 2*(a*c^6*d^13*e - 4*a^2*c^5*d^11*
e^3 + 5*a^3*c^4*d^9*e^5 - 5*a^5*c^2*d^5*e^9 + 4*a^6*c*d^3*e^11 - a^7*d*e^13
)*x)
```

Sympy [B] time = 3.74097, size = 1001, normalized size = 5.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

```
[Out] 6*c**2*d**2*e**2*log(x + (-6*a**6*c**2*d**2*e**14/(a*e**2 - c*d**2)**5 + 36
*a**5*c**3*d**4*e**12/(a*e**2 - c*d**2)**5 - 90*a**4*c**4*d**6*e**10/(a*e**
```

```
2 - c*d**2)**5 + 120*a**3*c**5*d**8*e**8/(a*e**2 - c*d**2)**5 - 90*a**2*c**
6*d**10*e**6/(a*e**2 - c*d**2)**5 + 36*a*c**7*d**12*e**4/(a*e**2 - c*d**2)*
*5 + 6*a*c**2*d**2*e**4 - 6*c**8*d**14*e**2/(a*e**2 - c*d**2)**5 + 6*c**3*d
**4*e**2)/(12*c**3*d**3*e**3))/(a*e**2 - c*d**2)**5 - 6*c**2*d**2*e**2*log(
x + (6*a**6*c**2*d**2*e**14/(a*e**2 - c*d**2)**5 - 36*a**5*c**3*d**4*e**12/
(a*e**2 - c*d**2)**5 + 90*a**4*c**4*d**6*e**10/(a*e**2 - c*d**2)**5 - 120*a
**3*c**5*d**8*e**8/(a*e**2 - c*d**2)**5 + 90*a**2*c**6*d**10*e**6/(a*e**2 -
 c*d**2)**5 - 36*a*c**7*d**12*e**4/(a*e**2 - c*d**2)**5 + 6*a*c**2*d**2*e**
4 + 6*c**8*d**14*e**2/(a*e**2 - c*d**2)**5 + 6*c**3*d**4*e**2)/(12*c**3*d**
3*e**3))/(a*e**2 - c*d**2)**5 + (-a**3*e**6 + 7*a**2*c*d**2*e**4 + 7*a*c**2
*d**4*e**2 - c**3*d**6 + 12*c**3*d**3*e**3*x**3 + x**2*(18*a*c**2*d**2*e**4
 + 18*c**3*d**4*e**2) + x*(4*a**2*c*d*e**5 + 28*a*c**2*d**3*e**3 + 4*c**3*d
**5*e))/(2*a**6*d**2*e**10 - 8*a**5*c*d**4*e**8 + 12*a**4*c**2*d**6*e**6 -
8*a**3*c**3*d**8*e**4 + 2*a**2*c**4*d**10*e**2 + x**4*(2*a**4*c**2*d**2*e**
10 - 8*a**3*c**3*d**4*e**8 + 12*a**2*c**4*d**6*e**6 - 8*a*c**5*d**8*e**4 +
2*c**6*d**10*e**2) + x**3*(4*a**5*c*d*e**11 - 12*a**4*c**2*d**3*e**9 + 8*a*
*3*c**3*d**5*e**7 + 8*a**2*c**4*d**7*e**5 - 12*a*c**5*d**9*e**3 + 4*c**6*d*
*11*e) + x**2*(2*a**6*e**12 - 18*a**4*c**2*d**4*e**8 + 32*a**3*c**3*d**6*e*
*6 - 18*a**2*c**4*d**8*e**4 + 2*c**6*d**12) + x*(4*a**6*d*e**11 - 12*a**5*c
*d**3*e**9 + 8*a**4*c**2*d**5*e**7 + 8*a**3*c**3*d**7*e**5 - 12*a**2*c**4*d
**9*e**3 + 4*a*c**5*d**11*e))
```

Giac [A] time = 1.22823, size = 440, normalized size = 2.3

$$\frac{12\,c^2d^2\arctan\left(\frac{2\,cdxe+cd^2+ae^2}{\sqrt{-c^2d^4+2\,acd^2e^2-a^2e^4}}\right)e^2}{\left(c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8\right)\sqrt{-c^2d^4+2\,acd^2e^2-a^2e^4}}+\frac{12\,c^3d^3x^3e^3+18\,c^3d^4x^2e^2+4\,c^3d^5xe-\cdots}{2\left(c^4d^8-4\,ac^3d^6e^2+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac")
```

```
[Out] 12*c^2*d^2*arctan((2*c*d*x*e + c*d^2 + a*e^2)/sqrt(-c^2*d^4 + 2*a*c*d^2*e^2
 - a^2*e^4))*e^2/((c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*
d^2*e^6 + a^4*e^8)*sqrt(-c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4)) + 1/2*(12*c^3*
d^3*x^3*e^3 + 18*c^3*d^4*x^2*e^2 + 4*c^3*d^5*x*e - c^3*d^6 + 18*a*c^2*d^2*x
^2*e^4 + 28*a*c^2*d^3*x*e^3 + 7*a*c^2*d^4*e^2 + 4*a^2*c*d*x*e^5 + 7*a^2*c*d
^2*e^4 - a^3*e^6)/((c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c
*d^2*e^6 + a^4*e^8)*(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)^2)
```

3.1895 $\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal. Leaf size=223

$$\frac{4c^3d^3e}{\left(cd^2-ae^2\right)^5(ae+cdx)}-\frac{c^3d^3}{2\left(cd^2-ae^2\right)^4(ae+cdx)^2}+\frac{6c^2d^2e^2}{(d+ex)\left(cd^2-ae^2\right)^5}+\frac{10c^3d^3e^2\log(ae+cdx)}{\left(cd^2-ae^2\right)^6}-\frac{10c^3d^3e^2\log(d+}{\left(cd^2-ae^2\right)^6}$$

```
[Out] -(c^3*d^3)/(2*(c*d^2 - a*e^2)^4*(a*e + c*d*x)^2) + (4*c^3*d^3*e)/((c*d^2 -
a*e^2)^5*(a*e + c*d*x)) + e^2/(3*(c*d^2 - a*e^2)^3*(d + e*x)^3) + (3*c*d*e^
2)/(2*(c*d^2 - a*e^2)^4*(d + e*x)^2) + (6*c^2*d^2*e^2)/((c*d^2 - a*e^2)^5*(
d + e*x)) + (10*c^3*d^3*e^2*Log[a*e + c*d*x])/(c*d^2 - a*e^2)^6 - (10*c^3*d
^3*e^2*Log[d + e*x])/(c*d^2 - a*e^2)^6
```

Rubi [A] time = 0.223859, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 44}

$$\frac{4c^3d^3e}{\left(cd^2-ae^2\right)^5(ae+cdx)}-\frac{c^3d^3}{2\left(cd^2-ae^2\right)^4(ae+cdx)^2}+\frac{6c^2d^2e^2}{(d+ex)\left(cd^2-ae^2\right)^5}+\frac{10c^3d^3e^2\log(ae+cdx)}{\left(cd^2-ae^2\right)^6}-\frac{10c^3d^3e^2\log(d+}{\left(cd^2-ae^2\right)^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3),x]
```

```
[Out] -(c^3*d^3)/(2*(c*d^2 - a*e^2)^4*(a*e + c*d*x)^2) + (4*c^3*d^3*e)/((c*d^2 -
a*e^2)^5*(a*e + c*d*x)) + e^2/(3*(c*d^2 - a*e^2)^3*(d + e*x)^3) + (3*c*d*e^
2)/(2*(c*d^2 - a*e^2)^4*(d + e*x)^2) + (6*c^2*d^2*e^2)/((c*d^2 - a*e^2)^5*(
d + e*x)) + (10*c^3*d^3*e^2*Log[a*e + c*d*x])/(c*d^2 - a*e^2)^6 - (10*c^3*d
^3*e^2*Log[d + e*x])/(c*d^2 - a*e^2)^6
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx=\int \frac{1}{(ae+cdx)^3(d+ex)^4}\,dx$$
$$=\int\left(\frac{c^4d^4}{\left(cd^2-ae^2\right)^4(ae+cdx)^3}-\frac{4c^4d^4e}{\left(cd^2-ae^2\right)^5(ae+cdx)^2}+\frac{10c^4}{\left(cd^2-ae^2\right)^6}\right.$$
$$=-\frac{c^3d^3}{2\left(cd^2-ae^2\right)^4(ae+cdx)^2}+\frac{4c^3d^3e}{\left(cd^2-ae^2\right)^5(ae+cdx)}+\frac{e^2}{3\left(cd^2-ae^2\right)^3}$$

Mathematica [A] time = 0.194587, size = 201, normalized size = 0.9

$$\frac{\frac{24c^3d^3e\left(cd^2-ae^2\right)}{ae+cdx}-\frac{3c^3d^3\left(cd^2-ae^2\right)^2}{(ae+cdx)^2}+\frac{36c^2d^2e^2\left(cd^2-ae^2\right)}{d+ex}+60c^3d^3e^2\log(ae+cdx)+\frac{9cd\left(cd^2e-ae^3\right)^2}{(d+ex)^2}-\frac{2e^2\left(ae^2-cd^2\right)^3}{(d+ex)^3}-60c^3d^3e^2\log(}{6\left(cd^2-ae^2\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3),x]
```

```
[Out] ((-3*c^3*d^3*(c*d^2 - a*e^2)^2)/(a*e + c*d*x)^2 + (24*c^3*d^3*e*(c*d^2 - a*
e^2))/(a*e + c*d*x) - (2*e^2*(-(c*d^2) + a*e^2)^3)/(d + e*x)^3 + (9*c*d*(c*
d^2*e - a*e^3)^2)/(d + e*x)^2 + (36*c^2*d^2*e^2*(c*d^2 - a*e^2))/(d + e*x)
+ 60*c^3*d^3*e^2*Log[a*e + c*d*x] - 60*c^3*d^3*e^2*Log[d + e*x])/(6*(c*d^2
- a*e^2)^6)
```

Maple [A] time = 0.053, size = 218, normalized size = 1.

$$-\frac{e^2}{3\left(ae^2-cd^2\right)^3(ex+d)^3}-10\frac{c^3d^3e^2\ln(ex+d)}{\left(ae^2-cd^2\right)^6}-6\frac{c^2e^2d^2}{\left(ae^2-cd^2\right)^5(ex+d)}+\frac{3cde^2}{2\left(ae^2-cd^2\right)^4(ex+d)^2}-\frac{c^3d}{2\left(ae^2-cd^2\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

```
[Out] -1/3*e^2/(a*e^2-c*d^2)^3/(e*x+d)^3-10*e^2/(a*e^2-c*d^2)^6*c^3*d^3*ln(e*x+d)
-6*e^2/(a*e^2-c*d^2)^5*c^2*d^2/(e*x+d)+3/2*e^2/(a*e^2-c*d^2)^4*c*d/(e*x+d)^
2-1/2*c^3*d^3/(a*e^2-c*d^2)^4/(c*d*x+a*e)^2+10*e^2/(a*e^2-c*d^2)^6*c^3*d^3*
ln(c*d*x+a*e)-4*c^3*d^3/(a*e^2-c*d^2)^5*e/(c*d*x+a*e)
```

Maxima [B] time = 1.27441, size = 1278, normalized size = 5.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxim
a")
```

```
[Out] 10*c^3*d^3*e^2*log(c*d*x + a*e)/(c^6*d^12 - 6*a*c^5*d^10*e^2 + 15*a^2*c^4*d
^8*e^4 - 20*a^3*c^3*d^6*e^6 + 15*a^4*c^2*d^4*e^8 - 6*a^5*c*d^2*e^10 + a^6*e
```

```
^12) - 10*c^3*d^3*e^2*log(e*x + d)/(c^6*d^12 - 6*a*c^5*d^10*e^2 + 15*a^2*c^
4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 + 15*a^4*c^2*d^4*e^8 - 6*a^5*c*d^2*e^10 + a^
6*e^12) + 1/6*(60*c^4*d^4*e^4*x^4 - 3*c^4*d^8 + 27*a*c^3*d^6*e^2 + 47*a^2*c
^2*d^4*e^4 - 13*a^3*c*d^2*e^6 + 2*a^4*e^8 + 30*(5*c^4*d^5*e^3 + 3*a*c^3*d^3
*e^5)*x^3 + 10*(11*c^4*d^6*e^2 + 23*a*c^3*d^4*e^4 + 2*a^2*c^2*d^2*e^6)*x^2
+ 5*(3*c^4*d^7*e + 35*a*c^3*d^5*e^3 + 11*a^2*c^2*d^3*e^5 - a^3*c*d*e^7)*x)/
(a^2*c^5*d^13*e^2 - 5*a^3*c^4*d^11*e^4 + 10*a^4*c^3*d^9*e^6 - 10*a^5*c^2*d^
7*e^8 + 5*a^6*c*d^5*e^10 - a^7*d^3*e^12 + (c^7*d^12*e^3 - 5*a*c^6*d^10*e^5
+ 10*a^2*c^5*d^8*e^7 - 10*a^3*c^4*d^6*e^9 + 5*a^4*c^3*d^4*e^11 - a^5*c^2*d^
2*e^13)*x^5 + (3*c^7*d^13*e^2 - 13*a*c^6*d^11*e^4 + 20*a^2*c^5*d^9*e^6 - 10
*a^3*c^4*d^7*e^8 - 5*a^4*c^3*d^5*e^10 + 7*a^5*c^2*d^3*e^12 - 2*a^6*c*d*e^14
)*x^4 + (3*c^7*d^14*e - 9*a*c^6*d^12*e^3 + a^2*c^5*d^10*e^5 + 25*a^3*c^4*d^
8*e^7 - 35*a^4*c^3*d^6*e^9 + 17*a^5*c^2*d^4*e^11 - a^6*c*d^2*e^13 - a^7*e^1
5)*x^3 + (c^7*d^15 + a*c^6*d^13*e^2 - 17*a^2*c^5*d^11*e^4 + 35*a^3*c^4*d^9*
e^6 - 25*a^4*c^3*d^7*e^8 - a^5*c^2*d^5*e^10 + 9*a^6*c*d^3*e^12 - 3*a^7*d*e^
14)*x^2 + (2*a*c^6*d^14*e - 7*a^2*c^5*d^12*e^3 + 5*a^3*c^4*d^10*e^5 + 10*a^
4*c^3*d^8*e^7 - 20*a^5*c^2*d^6*e^9 + 13*a^6*c*d^4*e^11 - 3*a^7*d^2*e^13)*x)
```

Fricas [B] time = 2.41755, size = 2485, normalized size = 11.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="frica
s")
```

```
[Out] -1/6*(3*c^5*d^10 - 30*a*c^4*d^8*e^2 - 20*a^2*c^3*d^6*e^4 + 60*a^3*c^2*d^4*e
^6 - 15*a^4*c*d^2*e^8 + 2*a^5*e^10 - 60*(c^5*d^6*e^4 - a*c^4*d^4*e^6)*x^4 -
 30*(5*c^5*d^7*e^3 - 2*a*c^4*d^5*e^5 - 3*a^2*c^3*d^3*e^7)*x^3 - 10*(11*c^5*
d^8*e^2 + 12*a*c^4*d^6*e^4 - 21*a^2*c^3*d^4*e^6 - 2*a^3*c^2*d^2*e^8)*x^2 -
5*(3*c^5*d^9*e + 32*a*c^4*d^7*e^3 - 24*a^2*c^3*d^5*e^5 - 12*a^3*c^2*d^3*e^7
 + a^4*c*d*e^9)*x - 60*(c^5*d^5*e^5*x^5 + a^2*c^3*d^6*e^4 + (3*c^5*d^6*e^4
+ 2*a*c^4*d^4*e^6)*x^4 + (3*c^5*d^7*e^3 + 6*a*c^4*d^5*e^5 + a^2*c^3*d^3*e^7
)*x^3 + (c^5*d^8*e^2 + 6*a*c^4*d^6*e^4 + 3*a^2*c^3*d^4*e^6)*x^2 + (2*a*c^4*
d^7*e^3 + 3*a^2*c^3*d^5*e^5)*x)*log(c*d*x + a*e) + 60*(c^5*d^5*e^5*x^5 + a^
2*c^3*d^6*e^4 + (3*c^5*d^6*e^4 + 2*a*c^4*d^4*e^6)*x^4 + (3*c^5*d^7*e^3 + 6*
a*c^4*d^5*e^5 + a^2*c^3*d^3*e^7)*x^3 + (c^5*d^8*e^2 + 6*a*c^4*d^6*e^4 + 3*a
^2*c^3*d^4*e^6)*x^2 + (2*a*c^4*d^7*e^3 + 3*a^2*c^3*d^5*e^5)*x)*log(e*x + d)
)/(a^2*c^6*d^15*e^2 - 6*a^3*c^5*d^13*e^4 + 15*a^4*c^4*d^11*e^6 - 20*a^5*c^3
*d^9*e^8 + 15*a^6*c^2*d^7*e^10 - 6*a^7*c*d^5*e^12 + a^8*d^3*e^14 + (c^8*d^1
4*e^3 - 6*a*c^7*d^12*e^5 + 15*a^2*c^6*d^10*e^7 - 20*a^3*c^5*d^8*e^9 + 15*a^
4*c^4*d^6*e^11 - 6*a^5*c^3*d^4*e^13 + a^6*c^2*d^2*e^15)*x^5 + (3*c^8*d^15*e
^2 - 16*a*c^7*d^13*e^4 + 33*a^2*c^6*d^11*e^6 - 30*a^3*c^5*d^9*e^8 + 5*a^4*c
^4*d^7*e^10 + 12*a^5*c^3*d^5*e^12 - 9*a^6*c^2*d^3*e^14 + 2*a^7*c*d*e^16)*x^
4 + (3*c^8*d^16*e - 12*a*c^7*d^14*e^3 + 10*a^2*c^6*d^12*e^5 + 24*a^3*c^5*d^
10*e^7 - 60*a^4*c^4*d^8*e^9 + 52*a^5*c^3*d^6*e^11 - 18*a^6*c^2*d^4*e^13 + a
^8*e^17)*x^3 + (c^8*d^17 - 18*a^2*c^6*d^13*e^4 + 52*a^3*c^5*d^11*e^6 - 60*a
^4*c^4*d^9*e^8 + 24*a^5*c^3*d^7*e^10 + 10*a^6*c^2*d^5*e^12 - 12*a^7*c*d^3*e
^14 + 3*a^8*d*e^16)*x^2 + (2*a*c^7*d^16*e - 9*a^2*c^6*d^14*e^3 + 12*a^3*c^5
*d^12*e^5 + 5*a^4*c^4*d^10*e^7 - 30*a^5*c^3*d^8*e^9 + 33*a^6*c^2*d^6*e^11 -
 16*a^7*c*d^4*e^13 + 3*a^8*d^2*e^15)*x)
```

Sympy [B] time = 6.329, size = 1353, normalized size = 6.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

```
[Out] -10*c**3*d**3*e**2*log(x + (-10*a**7*c**3*d**3*e**16/(a*e**2 - c*d**2)**6 +
 70*a**6*c**4*d**5*e**14/(a*e**2 - c*d**2)**6 - 210*a**5*c**5*d**7*e**12/(a
*e**2 - c*d**2)**6 + 350*a**4*c**6*d**9*e**10/(a*e**2 - c*d**2)**6 - 350*a*
*3*c**7*d**11*e**8/(a*e**2 - c*d**2)**6 + 210*a**2*c**8*d**13*e**6/(a*e**2
- c*d**2)**6 - 70*a*c**9*d**15*e**4/(a*e**2 - c*d**2)**6 + 10*a*c**3*d**3*e
**4 + 10*c**10*d**17*e**2/(a*e**2 - c*d**2)**6 + 10*c**4*d**5*e**2)/(20*c**
4*d**4*e**3))/(a*e**2 - c*d**2)**6 + 10*c**3*d**3*e**2*log(x + (10*a**7*c**
3*d**3*e**16/(a*e**2 - c*d**2)**6 - 70*a**6*c**4*d**5*e**14/(a*e**2 - c*d**
2)**6 + 210*a**5*c**5*d**7*e**12/(a*e**2 - c*d**2)**6 - 350*a**4*c**6*d**9*
e**10/(a*e**2 - c*d**2)**6 + 350*a**3*c**7*d**11*e**8/(a*e**2 - c*d**2)**6
- 210*a**2*c**8*d**13*e**6/(a*e**2 - c*d**2)**6 + 70*a*c**9*d**15*e**4/(a*e
**2 - c*d**2)**6 + 10*a*c**3*d**3*e**4 - 10*c**10*d**17*e**2/(a*e**2 - c*d*
*2)**6 + 10*c**4*d**5*e**2)/(20*c**4*d**4*e**3))/(a*e**2 - c*d**2)**6 - (2*
a**4*e**8 - 13*a**3*c*d**2*e**6 + 47*a**2*c**2*d**4*e**4 + 27*a*c**3*d**6*e
**2 - 3*c**4*d**8 + 60*c**4*d**4*e**4*x**4 + x**3*(90*a*c**3*d**3*e**5 + 15
0*c**4*d**5*e**3) + x**2*(20*a**2*c**2*d**2*e**6 + 230*a*c**3*d**4*e**4 + 1
10*c**4*d**6*e**2) + x*(-5*a**3*c*d*e**7 + 55*a**2*c**2*d**3*e**5 + 175*a*c
**3*d**5*e**3 + 15*c**4*d**7*e))/(6*a**7*d**3*e**12 - 30*a**6*c*d**5*e**10
+ 60*a**5*c**2*d**7*e**8 - 60*a**4*c**3*d**9*e**6 + 30*a**3*c**4*d**11*e**4
 - 6*a**2*c**5*d**13*e**2 + x**5*(6*a**5*c**2*d**2*e**13 - 30*a**4*c**3*d**
4*e**11 + 60*a**3*c**4*d**6*e**9 - 60*a**2*c**5*d**8*e**7 + 30*a*c**6*d**10
*e**5 - 6*c**7*d**12*e**3) + x**4*(12*a**6*c*d*e**14 - 42*a**5*c**2*d**3*e*
*12 + 30*a**4*c**3*d**5*e**10 + 60*a**3*c**4*d**7*e**8 - 120*a**2*c**5*d**9
*e**6 + 78*a*c**6*d**11*e**4 - 18*c**7*d**13*e**2) + x**3*(6*a**7*e**15 + 6
*a**6*c*d**2*e**13 - 102*a**5*c**2*d**4*e**11 + 210*a**4*c**3*d**6*e**9 - 1
50*a**3*c**4*d**8*e**7 - 6*a**2*c**5*d**10*e**5 + 54*a*c**6*d**12*e**3 - 18
*c**7*d**14*e) + x**2*(18*a**7*d*e**14 - 54*a**6*c*d**3*e**12 + 6*a**5*c**2
*d**5*e**10 + 150*a**4*c**3*d**7*e**8 - 210*a**3*c**4*d**9*e**6 + 102*a**2*
c**5*d**11*e**4 - 6*a*c**6*d**13*e**2 - 6*c**7*d**15) + x*(18*a**7*d**2*e**
13 - 78*a**6*c*d**4*e**11 + 120*a**5*c**2*d**6*e**9 - 60*a**4*c**3*d**8*e**
7 - 30*a**3*c**4*d**10*e**5 + 42*a**2*c**5*d**12*e**3 - 12*a*c**6*d**14*e))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError
```

3.1896 $\int \frac{(d+ex)^{10}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx$

Optimal. Leaf size=217

$$\frac{e^4x\left(10a^2e^4-24acd^2e^2+15c^2d^4\right)}{c^6d^6}+\frac{e^5x^2\left(3cd^2-2ae^2\right)}{c^5d^5}-\frac{15e^2\left(cd^2-ae^2\right)^4}{c^7d^7(ae+cdx)}-\frac{3e\left(cd^2-ae^2\right)^5}{c^7d^7(ae+cdx)^2}-\frac{\left(cd^2-ae^2\right)^6}{3c^7d^7(ae+cdx)^3}+\frac{20e^3}{}$$

```
[Out] (e^4*(15*c^2*d^4 - 24*a*c*d^2*e^2 + 10*a^2*e^4)*x)/(c^6*d^6) + (e^5*(3*c*d^
2 - 2*a*e^2)*x^2)/(c^5*d^5) + (e^6*x^3)/(3*c^4*d^4) - (c*d^2 - a*e^2)^6/(3*
c^7*d^7*(a*e + c*d*x)^3) - (3*e*(c*d^2 - a*e^2)^5)/(c^7*d^7*(a*e + c*d*x)^2
) - (15*e^2*(c*d^2 - a*e^2)^4)/(c^7*d^7*(a*e + c*d*x)) + (20*e^3*(c*d^2 - a
*e^2)^3*Log[a*e + c*d*x])/(c^7*d^7)
```

Rubi [A] time = 0.271207, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$\frac{e^4x\left(10a^2e^4-24acd^2e^2+15c^2d^4\right)}{c^6d^6}+\frac{e^5x^2\left(3cd^2-2ae^2\right)}{c^5d^5}-\frac{15e^2\left(cd^2-ae^2\right)^4}{c^7d^7(ae+cdx)}-\frac{3e\left(cd^2-ae^2\right)^5}{c^7d^7(ae+cdx)^2}-\frac{\left(cd^2-ae^2\right)^6}{3c^7d^7(ae+cdx)^3}+\frac{20e^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^10/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```


```
[Out] (e^4*(15*c^2*d^4 - 24*a*c*d^2*e^2 + 10*a^2*e^4)*x)/(c^6*d^6) + (e^5*(3*c*d^
2 - 2*a*e^2)*x^2)/(c^5*d^5) + (e^6*x^3)/(3*c^4*d^4) - (c*d^2 - a*e^2)^6/(3*
c^7*d^7*(a*e + c*d*x)^3) - (3*e*(c*d^2 - a*e^2)^5)/(c^7*d^7*(a*e + c*d*x)^2
) - (15*e^2*(c*d^2 - a*e^2)^4)/(c^7*d^7*(a*e + c*d*x)) + (20*e^3*(c*d^2 - a
*e^2)^3*Log[a*e + c*d*x])/(c^7*d^7)
```


Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(d+ex)^{10}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx = \int \frac{(d+ex)^6}{(ae+cdx)^4}\,dx$$

$$= \int \left(\frac{15c^2d^4e^4-24acd^2e^6+10a^2e^8}{c^6d^6}+\frac{2e^5\left(3cd^2-2ae^2\right)x}{c^5d^5}+\frac{e^6x^2}{c^4d^4}+\frac{\left(cd^2-ae^2\right.}{c^6d^6(ae+c}\right.$$

$$= \frac{e^4\left(15c^2d^4-24acd^2e^2+10a^2e^4\right)x}{c^6d^6}+\frac{e^5\left(3cd^2-2ae^2\right)x^2}{c^5d^5}+\frac{e^6x^3}{3c^4d^4}-\frac{\left(cd^2-a\right.}{3c^7d^7(ae+}$$

Mathematica [A] time = 0.141924, size = 335, normalized size = 1.54

$$\frac{3a^4c^2d^2e^8\left(-65d^2+81dex+13e^2x^2\right)+a^3c^3d^3e^6\left(-405d^2ex+110d^3-27de^2x^2+73e^3x^3\right)-3a^2c^4d^4e^4\left(45d^2e^2x^2-90d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^10/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

```
[Out] (-37*a^6*e^12 + 3*a^5*c*d*e^10*(47*d - 17*e*x) + 3*a^4*c^2*d^2*e^8*(-65*d^2
 + 81*d*e*x + 13*e^2*x^2) + a^3*c^3*d^3*e^6*(110*d^3 - 405*d^2*e*x - 27*d*e
^2*x^2 + 73*e^3*x^3) - 3*a^2*c^4*d^4*e^4*(5*d^4 - 90*d^3*e*x + 45*d^2*e^2*x
^2 + 63*d*e^3*x^3 - 5*e^4*x^4) - 3*a*c^5*d^5*e^2*(d^5 + 15*d^4*e*x - 60*d^3
*e^2*x^2 - 45*d^2*e^3*x^3 + 15*d*e^4*x^4 + e^5*x^5) + c^6*d^6*(-d^6 - 9*d^5
*e*x - 45*d^4*e^2*x^2 + 45*d^2*e^4*x^4 + 9*d*e^5*x^5 + e^6*x^6) - 60*e^3*(-
(c*d^2) + a*e^2)^3*(a*e + c*d*x)^3*Log[a*e + c*d*x])/(3*c^7*d^7*(a*e + c*d*
x)^3)
```

Maple [B] time = 0.054, size = 578, normalized size = 2.7

$$-5\,\frac{a^4e^8}{c^5d^3\,(cdx+ae)^3}+\frac{20\,a^3e^6}{3\,c^4d\,(cdx+ae)^3}-5\,\frac{a^2de^4}{c^3\,(cdx+ae)^3}+2\,\frac{d^3ae^2}{c^2\,(cdx+ae)^3}+3\,\frac{a^5e^{11}}{c^7d^7\,(cdx+ae)^2}-15\,\frac{e^9a^4}{d^5c^6\,(cdx+ae)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^10/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x)
```

```
[Out] -5/c^5/d^3/(c*d*x+a*e)^3*a^4*e^8+20/3/c^4/d/(c*d*x+a*e)^3*a^3*e^6-5/c^3*d/(
c*d*x+a*e)^3*a^2*e^4+2/c^2*d^3/(c*d*x+a*e)^3*a*e^2+3/d^7*e^11/c^7/(c*d*x+a*
e)^2*a^5-15/d^5*e^9/c^6/(c*d*x+a*e)^2*a^4+30/d^3*e^7/c^5/(c*d*x+a*e)^2*a^3-
30/d*e^5/c^4/(c*d*x+a*e)^2*a^2+15*d*e^3/c^3/(c*d*x+a*e)^2*a-15/c^7/d^7*e^10
/(c*d*x+a*e)*a^4+60/c^6/d^5*e^8/(c*d*x+a*e)*a^3-90/c^5/d^3*e^6/(c*d*x+a*e)*
a^2+60/c^4/d*e^4/(c*d*x+a*e)*a-20/c^7/d^7*e^9*ln(c*d*x+a*e)*a^3+60/c^6/d^5*
e^7*ln(c*d*x+a*e)*a^2-60/c^5/d^3*e^5*ln(c*d*x+a*e)*a+10*e^8/c^6/d^6*a^2*x-2
*e^7/c^5/d^5*x^2*a-24*e^6/c^5/d^4*a*x+3*e^5/c^4/d^3*x^2+15*e^4/c^4/d^2*x+20
/c^4/d*e^3*ln(c*d*x+a*e)-3*d^3*e/c^2/(c*d*x+a*e)^2-15/c^3*d*e^2/(c*d*x+a*e)
-1/3/c*d^5/(c*d*x+a*e)^3-1/3/c^7/d^7/(c*d*x+a*e)^3*a^6*e^12+2/c^6/d^5/(c*d*
x+a*e)^3*a^5*e^10+1/3*e^6*x^3/c^4/d^4
```

Maxima [A] time = 1.0903, size = 572, normalized size = 2.64

$$-\frac{c^6d^{12}+3\,ac^5d^{10}e^2+15\,a^2c^4d^8e^4-110\,a^3c^3d^6e^6+195\,a^4c^2d^4e^8-141\,a^5cd^2e^{10}+37\,a^6e^{12}+45\left(c^6d^{10}e^2-4\,ac^5d^8e^4+\right.}{3\left(c^{10}d^{10}x^3+3\,ac^9d^9ex^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^10/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="maxi
ma")
```

```
[Out] -1/3*(c^6*d^12 + 3*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 - 110*a^3*c^3*d^6*e^
6 + 195*a^4*c^2*d^4*e^8 - 141*a^5*c*d^2*e^10 + 37*a^6*e^12 + 45*(c^6*d^10*e
^2 - 4*a*c^5*d^8*e^4 + 6*a^2*c^4*d^6*e^6 - 4*a^3*c^3*d^4*e^8 + a^4*c^2*d^2*
e^10)*x^2 + 9*(c^6*d^11*e + 5*a*c^5*d^9*e^3 - 30*a^2*c^4*d^7*e^5 + 50*a^3*c
^3*d^5*e^7 - 35*a^4*c^2*d^3*e^9 + 9*a^5*c*d*e^11)*x)/(c^10*d^10*x^3 + 3*a*c
^9*d^9*e*x^2 + 3*a^2*c^8*d^8*e^2*x + a^3*c^7*d^7*e^3) + 1/3*(c^2*d^2*e^6*x^
3 + 3*(3*c^2*d^3*e^5 - 2*a*c*d*e^7)*x^2 + 3*(15*c^2*d^4*e^4 - 24*a*c*d^2*e^
6 + 10*a^2*e^8)*x)/(c^6*d^6) + 20*(c^3*d^6*e^3 - 3*a*c^2*d^4*e^5 + 3*a^2*c*
d^2*e^7 - a^3*e^9)*log(c*d*x + a*e)/(c^7*d^7)
```

Fricas [B] time = 1.95182, size = 1292, normalized size = 5.95

$$\frac{c^6d^6e^6x^6 - c^6d^{12} - 3\,ac^5d^{10}e^2 - 15\,a^2c^4d^8e^4 + 110\,a^3c^3d^6e^6 - 195\,a^4c^2d^4e^8 + 141\,a^5cd^2e^{10} - 37\,a^6e^{12} + 3\left(3\,c^6d^7e^5 - ac^5d^5e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^10/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="fric
as")
```

```
[Out] 1/3*(c^6*d^6*e^6*x^6 - c^6*d^12 - 3*a*c^5*d^10*e^2 - 15*a^2*c^4*d^8*e^4 + 1
10*a^3*c^3*d^6*e^6 - 195*a^4*c^2*d^4*e^8 + 141*a^5*c*d^2*e^10 - 37*a^6*e^12
 + 3*(3*c^6*d^7*e^5 - a*c^5*d^5*e^7)*x^5 + 15*(3*c^6*d^8*e^4 - 3*a*c^5*d^6*
e^6 + a^2*c^4*d^4*e^8)*x^4 + (135*a*c^5*d^7*e^5 - 189*a^2*c^4*d^5*e^7 + 73*
a^3*c^3*d^3*e^9)*x^3 - 3*(15*c^6*d^10*e^2 - 60*a*c^5*d^8*e^4 + 45*a^2*c^4*d
^6*e^6 + 9*a^3*c^3*d^4*e^8 - 13*a^4*c^2*d^2*e^10)*x^2 - 3*(3*c^6*d^11*e + 1
5*a*c^5*d^9*e^3 - 90*a^2*c^4*d^7*e^5 + 135*a^3*c^3*d^5*e^7 - 81*a^4*c^2*d^3
*e^9 + 17*a^5*c*d*e^11)*x + 60*(a^3*c^3*d^6*e^6 - 3*a^4*c^2*d^4*e^8 + 3*a^5
*c*d^2*e^10 - a^6*e^12 + (c^6*d^9*e^3 - 3*a*c^5*d^7*e^5 + 3*a^2*c^4*d^5*e^7
 - a^3*c^3*d^3*e^9)*x^3 + 3*(a*c^5*d^8*e^4 - 3*a^2*c^4*d^6*e^6 + 3*a^3*c^3*
d^4*e^8 - a^4*c^2*d^2*e^10)*x^2 + 3*(a^2*c^4*d^7*e^5 - 3*a^3*c^3*d^5*e^7 +
3*a^4*c^2*d^3*e^9 - a^5*c*d*e^11)*x)*log(c*d*x + a*e))/(c^10*d^10*x^3 + 3*a
*c^9*d^9*e*x^2 + 3*a^2*c^8*d^8*e^2*x + a^3*c^7*d^7*e^3)
```

Sympy [A] time = 134.566, size = 422, normalized size = 1.94

$$-\frac{37a^6e^{12} - 141a^5cd^2e^{10} + 195a^4c^2d^4e^8 - 110a^3c^3d^6e^6 + 15a^2c^4d^8e^4 + 3ac^5d^{10}e^2 + c^6d^{12} + x^2\left(45a^4c^2d^2e^{10} - 180a^3c^3d^4e^8\right.}{3a^3c^7d^7e^3 + 9a^2c^8d^8e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**10/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**4,x)
```

```
[Out] -(37*a**6*e**12 - 141*a**5*c*d**2*e**10 + 195*a**4*c**2*d**4*e**8 - 110*a**
3*c**3*d**6*e**6 + 15*a**2*c**4*d**8*e**4 + 3*a*c**5*d**10*e**2 + c**6*d**1
2 + x**2*(45*a**4*c**2*d**2*e**10 - 180*a**3*c**3*d**4*e**8 + 270*a**2*c**4
*d**6*e**6 - 180*a*c**5*d**8*e**4 + 45*c**6*d**10*e**2) + x*(81*a**5*c*d*e*
*11 - 315*a**4*c**2*d**3*e**9 + 450*a**3*c**3*d**5*e**7 - 270*a**2*c**4*d**
```

```
7*e**5 + 45*a*c**5*d**9*e**3 + 9*c**6*d**11*e))/(3*a**3*c**7*d**7*e**3 + 9*
a**2*c**8*d**8*e**2*x + 9*a*c**9*d**9*e*x**2 + 3*c**10*d**10*x**3) + e**6*x
**3/(3*c**4*d**4) - x**2*(2*a*e**7 - 3*c*d**2*e**5)/(c**5*d**5) + x*(10*a**
2*e**8 - 24*a*c*d**2*e**6 + 15*c**2*d**4*e**4)/(c**6*d**6) - 20*e**3*(a*e**
2 - c*d**2)**3*log(a*e + c*d*x)/(c**7*d**7)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^10/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="giac
")
```

```
[Out] sage0*x
```

3.1897 $\int \frac{(d+ex)^9}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx$

Optimal. Leaf size=179

$$\frac{e^4x\left(5cd^2-4ae^2\right)}{c^5d^5}-\frac{10e^2\left(cd^2-ae^2\right)^3}{c^6d^6(ae+cdx)}-\frac{5e\left(cd^2-ae^2\right)^4}{2c^6d^6(ae+cdx)^2}-\frac{\left(cd^2-ae^2\right)^5}{3c^6d^6(ae+cdx)^3}+\frac{10e^3\left(cd^2-ae^2\right)^2\log(ae+cdx)}{c^6d^6}+\frac{e^5x^2}{2c^4d^4}$$

```
[Out] (e^4*(5*c*d^2 - 4*a*e^2)*x)/(c^5*d^5) + (e^5*x^2)/(2*c^4*d^4) - (c*d^2 - a*
e^2)^5/(3*c^6*d^6*(a*e + c*d*x)^3) - (5*e*(c*d^2 - a*e^2)^4)/(2*c^6*d^6*(a*
e + c*d*x)^2) - (10*e^2*(c*d^2 - a*e^2)^3)/(c^6*d^6*(a*e + c*d*x)) + (10*e^
3*(c*d^2 - a*e^2)^2*Log[a*e + c*d*x])/(c^6*d^6)
```

Rubi [A] time = 0.173534, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$\frac{e^4x\left(5cd^2-4ae^2\right)}{c^5d^5}-\frac{10e^2\left(cd^2-ae^2\right)^3}{c^6d^6(ae+cdx)}-\frac{5e\left(cd^2-ae^2\right)^4}{2c^6d^6(ae+cdx)^2}-\frac{\left(cd^2-ae^2\right)^5}{3c^6d^6(ae+cdx)^3}+\frac{10e^3\left(cd^2-ae^2\right)^2\log(ae+cdx)}{c^6d^6}+\frac{e^5x^2}{2c^4d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^9/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

```
[Out] (e^4*(5*c*d^2 - 4*a*e^2)*x)/(c^5*d^5) + (e^5*x^2)/(2*c^4*d^4) - (c*d^2 - a*
e^2)^5/(3*c^6*d^6*(a*e + c*d*x)^3) - (5*e*(c*d^2 - a*e^2)^4)/(2*c^6*d^6*(a*
e + c*d*x)^2) - (10*e^2*(c*d^2 - a*e^2)^3)/(c^6*d^6*(a*e + c*d*x)) + (10*e^
3*(c*d^2 - a*e^2)^2*Log[a*e + c*d*x])/(c^6*d^6)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(d+ex)^9}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx=\int \frac{(d+ex)^5}{(ae+cdx)^4}\,dx$$

$$=\int \left(\frac{5cd^2e^4-4ae^6}{c^5d^5}+\frac{e^5x}{c^4d^4}+\frac{\left(cd^2-ae^2\right)^5}{c^5d^5(ae+cdx)^4}+\frac{5e\left(cd^2-ae^2\right)^4}{c^5d^5(ae+cdx)^3}+\frac{10e^2\left(cd^2-ae^2\right.}{c^5d^5(ae+cdx)}\right.$$

$$=\frac{e^4\left(5cd^2-4ae^2\right)x}{c^5d^5}+\frac{e^5x^2}{2c^4d^4}-\frac{\left(cd^2-ae^2\right)^5}{3c^6d^6(ae+cdx)^3}-\frac{5e\left(cd^2-ae^2\right)^4}{2c^6d^6(ae+cdx)^2}-\frac{10e^2\left(cd^2-\right.}{c^6d^6(ae+\right.}$$

Mathematica [A] time = 0.0941166, size = 259, normalized size = 1.45

$$\frac{a^3c^2d^2e^6\left(110d^2-270dex-9e^2x^2\right)-a^2c^3d^3e^4\left(-270d^2ex+20d^3+90de^2x^2+63e^3x^3\right)+a^4cde^8(81ex-130d)+47a^5e^{1}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^9/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

```
[Out] (47*a^5*e^10 + a^4*c*d*e^8*(-130*d + 81*e*x) + a^3*c^2*d^2*e^6*(110*d^2 - 2
70*d*e*x - 9*e^2*x^2) - a^2*c^3*d^3*e^4*(20*d^3 - 270*d^2*e*x + 90*d*e^2*x^
2 + 63*e^3*x^3) - 5*a*c^4*d^4*e^2*(d^4 + 12*d^3*e*x - 36*d^2*e^2*x^2 - 18*d
*e^3*x^3 + 3*e^4*x^4) + c^5*d^5*(-2*d^5 - 15*d^4*e*x - 60*d^3*e^2*x^2 + 30*
d*e^4*x^4 + 3*e^5*x^5) + 60*e^3*(c*d^2 - a*e^2)^2*(a*e + c*d*x)^3*Log[a*e +
 c*d*x])/(6*c^6*d^6*(a*e + c*d*x)^3)
```

Maple [B] time = 0.049, size = 436, normalized size = 2.4

$$\frac{e^5x^2}{2\,c^4d^4}-4\,\frac{ae^6x}{c^5d^5}+5\,\frac{e^4x}{c^4d^3}+\frac{a^5e^{10}}{3\,c^6d^6\,(cdx+ae)^3}-\frac{5\,a^4e^8}{3\,c^5d^4\,(cdx+ae)^3}+\frac{10\,a^3e^6}{3\,c^4d^2\,(cdx+ae)^3}-\frac{10\,a^2e^4}{3\,c^3\,(cdx+ae)^3}+\frac{5\,ad^2}{3\,c^2\,(cdx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^9/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x)
```

```
[Out] 1/2*e^5*x^2/c^4/d^4-4*e^6/c^5/d^5*a*x+5*e^4/c^4/d^3*x+1/3/c^6/d^6/(c*d*x+a*
e)^3*a^5*e^10-5/3/c^5/d^4/(c*d*x+a*e)^3*a^4*e^8+10/3/c^4/d^2/(c*d*x+a*e)^3*
a^3*e^6-10/3/c^3/(c*d*x+a*e)^3*a^2*e^4+5/3/c^2*d^2/(c*d*x+a*e)^3*a*e^2-1/3/
c*d^4/(c*d*x+a*e)^3-5/2/d^6*e^9/c^6/(c*d*x+a*e)^2*a^4+10/d^4*e^7/c^5/(c*d*x
+a*e)^2*a^3-15/d^2*e^5/c^4/(c*d*x+a*e)^2*a^2+10*e^3/c^3/(c*d*x+a*e)^2*a-5/2
*d^2*e/c^2/(c*d*x+a*e)^2+10/c^6/d^6*e^8/(c*d*x+a*e)*a^3-30/c^5/d^4*e^6/(c*d
*x+a*e)*a^2+30/c^4/d^2*e^4/(c*d*x+a*e)*a-10/c^3*e^2/(c*d*x+a*e)+10/c^6/d^6*
e^7*ln(c*d*x+a*e)*a^2-20/c^5/d^4*e^5*ln(c*d*x+a*e)*a+10/c^4/d^2*e^3*ln(c*d*
x+a*e)
```

Maxima [A] time = 1.09504, size = 440, normalized size = 2.46

$$-\frac{2\,c^5d^{10}+5\,ac^4d^8e^2+20\,a^2c^3d^6e^4-110\,a^3c^2d^4e^6+130\,a^4cd^2e^8-47\,a^5e^{10}+60\left(c^5d^8e^2-3\,ac^4d^6e^4+3\,a^2c^3d^4e^6-a^3c\right.}{6\left(c^9d^9x^3+3\,ac^8d^8ex^2+3\,a^2c^7d^7e^2x+a^3c^6d^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^9/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="maxim
a")
```

```
[Out] -1/6*(2*c^5*d^10 + 5*a*c^4*d^8*e^2 + 20*a^2*c^3*d^6*e^4 - 110*a^3*c^2*d^4*e
^6 + 130*a^4*c*d^2*e^8 - 47*a^5*e^10 + 60*(c^5*d^8*e^2 - 3*a*c^4*d^6*e^4 +
3*a^2*c^3*d^4*e^6 - a^3*c^2*d^2*e^8)*x^2 + 15*(c^5*d^9*e + 4*a*c^4*d^7*e^3
- 18*a^2*c^3*d^5*e^5 + 20*a^3*c^2*d^3*e^7 - 7*a^4*c*d*e^9)*x)/(c^9*d^9*x^3
+ 3*a*c^8*d^8*e*x^2 + 3*a^2*c^7*d^7*e^2*x + a^3*c^6*d^6*e^3) + 1/2*(c*d*e^5
*x^2 + 2*(5*c*d^2*e^4 - 4*a*e^6)*x)/(c^5*d^5) + 10*(c^2*d^4*e^3 - 2*a*c*d^2
*e^5 + a^2*e^7)*log(c*d*x + a*e)/(c^6*d^6)
```

Fricas [B] time = 1.93881, size = 976, normalized size = 5.45

$$\frac{3\,c^5d^5e^5x^5 - 2\,c^5d^{10} - 5\,ac^4d^8e^2 - 20\,a^2c^3d^6e^4 + 110\,a^3c^2d^4e^6 - 130\,a^4cd^2e^8 + 47\,a^5e^{10} + 15\left(2\,c^5d^6e^4 - ac^4d^4e^6\right)x^4 + 9\,(10}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^9/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="frica
s")
```

```
[Out] 1/6*(3*c^5*d^5*e^5*x^5 - 2*c^5*d^10 - 5*a*c^4*d^8*e^2 - 20*a^2*c^3*d^6*e^4
+ 110*a^3*c^2*d^4*e^6 - 130*a^4*c*d^2*e^8 + 47*a^5*e^10 + 15*(2*c^5*d^6*e^4
 - a*c^4*d^4*e^6)*x^4 + 9*(10*a*c^4*d^5*e^5 - 7*a^2*c^3*d^3*e^7)*x^3 - 3*(2
0*c^5*d^8*e^2 - 60*a*c^4*d^6*e^4 + 30*a^2*c^3*d^4*e^6 + 3*a^3*c^2*d^2*e^8)*
x^2 - 3*(5*c^5*d^9*e + 20*a*c^4*d^7*e^3 - 90*a^2*c^3*d^5*e^5 + 90*a^3*c^2*d
^3*e^7 - 27*a^4*c*d*e^9)*x + 60*(a^3*c^2*d^4*e^6 - 2*a^4*c*d^2*e^8 + a^5*e^
10 + (c^5*d^7*e^3 - 2*a*c^4*d^5*e^5 + a^2*c^3*d^3*e^7)*x^3 + 3*(a*c^4*d^6*e
^4 - 2*a^2*c^3*d^4*e^6 + a^3*c^2*d^2*e^8)*x^2 + 3*(a^2*c^3*d^5*e^5 - 2*a^3*
c^2*d^3*e^7 + a^4*c*d*e^9)*x)*log(c*d*x + a*e))/(c^9*d^9*x^3 + 3*a*c^8*d^8*
e*x^2 + 3*a^2*c^7*d^7*e^2*x + a^3*c^6*d^6*e^3)
```

Sympy [A] time = 42.7509, size = 335, normalized size = 1.87

$$\frac{47a^5e^{10} - 130a^4cd^2e^8 + 110a^3c^2d^4e^6 - 20a^2c^3d^6e^4 - 5ac^4d^8e^2 - 2c^5d^{10} + x^2\left(60a^3c^2d^2e^8 - 180a^2c^3d^4e^6 + 180ac^4d^6e^4 - 60\right.}{6a^3c^6d^6e^3 + 18a^2c^7d^7e^2x + 18ac^8d^8ex^2 + 6c^9d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**9/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**4,x)
```

```
[Out] (47*a**5*e**10 - 130*a**4*c*d**2*e**8 + 110*a**3*c**2*d**4*e**6 - 20*a**2*c
**3*d**6*e**4 - 5*a*c**4*d**8*e**2 - 2*c**5*d**10 + x**2*(60*a**3*c**2*d**2
*e**8 - 180*a**2*c**3*d**4*e**6 + 180*a*c**4*d**6*e**4 - 60*c**5*d**8*e**2)
 + x*(105*a**4*c*d*e**9 - 300*a**3*c**2*d**3*e**7 + 270*a**2*c**3*d**5*e**5
 - 60*a*c**4*d**7*e**3 - 15*c**5*d**9*e))/(6*a**3*c**6*d**6*e**3 + 18*a**2*
c**7*d**7*e**2*x + 18*a*c**8*d**8*e*x**2 + 6*c**9*d**9*x**3) + e**5*x**2/(2
*c**4*d**4) - x*(4*a*e**6 - 5*c*d**2*e**4)/(c**5*d**5) + 10*e**3*(a*e**2 -
c*d**2)**2*log(a*e + c*d*x)/(c**6*d**6)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^9/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.1898 $\int \frac{(d+ex)^8}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx$

Optimal. Leaf size=146

$$-\frac{6e^2\left(cd^2-ae^2\right)^2}{c^5d^5(ae+cdx)}-\frac{2e\left(cd^2-ae^2\right)^3}{c^5d^5(ae+cdx)^2}-\frac{\left(cd^2-ae^2\right)^4}{3c^5d^5(ae+cdx)^3}+\frac{4e^3\left(cd^2-ae^2\right)\log(ae+cdx)}{c^5d^5}+\frac{e^4x}{c^4d^4}$$

```
[Out] (e^4*x)/(c^4*d^4) - (c*d^2 - a*e^2)^4/(3*c^5*d^5*(a*e + c*d*x)^3) - (2*e*(c
*d^2 - a*e^2)^3)/(c^5*d^5*(a*e + c*d*x)^2) - (6*e^2*(c*d^2 - a*e^2)^2)/(c^5
*d^5*(a*e + c*d*x)) + (4*e^3*(c*d^2 - a*e^2)*Log[a*e + c*d*x])/(c^5*d^5)
```

Rubi [A] time = 0.133414, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$-\frac{6e^2\left(cd^2-ae^2\right)^2}{c^5d^5(ae+cdx)}-\frac{2e\left(cd^2-ae^2\right)^3}{c^5d^5(ae+cdx)^2}-\frac{\left(cd^2-ae^2\right)^4}{3c^5d^5(ae+cdx)^3}+\frac{4e^3\left(cd^2-ae^2\right)\log(ae+cdx)}{c^5d^5}+\frac{e^4x}{c^4d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^8/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

```
[Out] (e^4*x)/(c^4*d^4) - (c*d^2 - a*e^2)^4/(3*c^5*d^5*(a*e + c*d*x)^3) - (2*e*(c
*d^2 - a*e^2)^3)/(c^5*d^5*(a*e + c*d*x)^2) - (6*e^2*(c*d^2 - a*e^2)^2)/(c^5
*d^5*(a*e + c*d*x)) + (4*e^3*(c*d^2 - a*e^2)*Log[a*e + c*d*x])/(c^5*d^5)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(d+ex)^8}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx=\int \frac{(d+ex)^4}{(ae+cdx)^4}\,dx$$

$$=\int \left(\frac{e^4}{c^4d^4}+\frac{\left(cd^2-ae^2\right)^4}{c^4d^4(ae+cdx)^4}+\frac{4e\left(cd^2-ae^2\right)^3}{c^4d^4(ae+cdx)^3}+\frac{6\left(cd^2e-ae^3\right)^2}{c^4d^4(ae+cdx)^2}+\frac{4\left(cd^2e^3-\right.}{c^4d^4(ae+}\right.$$

$$=\frac{e^4x}{c^4d^4}-\frac{\left(cd^2-ae^2\right)^4}{3c^5d^5(ae+cdx)^3}-\frac{2e\left(cd^2-ae^2\right)^3}{c^5d^5(ae+cdx)^2}-\frac{6e^2\left(cd^2-ae^2\right)^2}{c^5d^5(ae+cdx)}+\frac{4e^3\left(cd^2-ae\right.}{c}$$

Mathematica [A] time = 0.0741037, size = 194, normalized size = 1.33

$$\frac{-3a^2c^2d^2e^4\left(2d^2-18dex+3e^2x^2\right)+a^3cde^6(22d-27ex)-13a^4e^8+ac^3d^3e^2\left(-18d^2ex-2d^3+36de^2x^2+9e^3x^3\right)-12e^3\left(a\right.}{3c^5d^5(ae+cdx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^8/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

```
[Out] (-13*a^4*e^8 + a^3*c*d*e^6*(22*d - 27*e*x) - 3*a^2*c^2*d^2*e^4*(2*d^2 - 18*
d*e*x + 3*e^2*x^2) + a*c^3*d^3*e^2*(-2*d^3 - 18*d^2*e*x + 36*d*e^2*x^2 + 9*
e^3*x^3) - c^4*(d^8 + 6*d^7*e*x + 18*d^6*e^2*x^2 - 3*d^4*e^4*x^4) - 12*e^3*
(-(c*d^2) + a*e^2)*(a*e + c*d*x)^3*Log[a*e + c*d*x])/(3*c^5*d^5*(a*e + c*d*
x)^3)
```

Maple [B] time = 0.048, size = 318, normalized size = 2.2

$$\frac{e^4x}{c^4d^4}-\frac{a^4e^8}{3\,c^5d^5\,(cdx+ae)^3}+\frac{4\,a^3e^6}{3\,c^4d^3\,(cdx+ae)^3}-2\,\frac{a^2e^4}{c^3d\,(cdx+ae)^3}+\frac{4\,ade^2}{3\,c^2\,(cdx+ae)^3}-\frac{d^3}{3\,c\,(cdx+ae)^3}+2\,\frac{a^3e^7}{c^5d^5\,(cdx+ae)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^8/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x)
```

```
[Out] e^4*x/c^4/d^4-1/3/c^5/d^5/(c*d*x+a*e)^3*a^4*e^8+4/3/c^4/d^3/(c*d*x+a*e)^3*a
^3*e^6-2/c^3/d/(c*d*x+a*e)^3*a^2*e^4+4/3/c^2*d/(c*d*x+a*e)^3*a*e^2-1/3/c*d^
3/(c*d*x+a*e)^3+2/d^5*e^7/c^5/(c*d*x+a*e)^2*a^3-6/d^3*e^5/c^4/(c*d*x+a*e)^2
*a^2+6/d*e^3/c^3/(c*d*x+a*e)^2*a-2*d*e/c^2/(c*d*x+a*e)^2-6/c^5/d^5*e^6/(c*d
*x+a*e)*a^2+12/c^4/d^3*e^4/(c*d*x+a*e)*a-6/c^3/d*e^2/(c*d*x+a*e)-4/c^5/d^5*
e^5*ln(c*d*x+a*e)*a+4/c^4/d^3*e^3*ln(c*d*x+a*e)
```

Maxima [A] time = 1.14443, size = 328, normalized size = 2.25

$$-\frac{c^4d^8+2\,ac^3d^6e^2+6\,a^2c^2d^4e^4-22\,a^3cd^2e^6+13\,a^4e^8+18\left(c^4d^6e^2-2\,ac^3d^4e^4+a^2c^2d^2e^6\right)x^2+6\left(c^4d^7e+3\,ac^3d^5e^3-9\,a^2\right.}{3\left(c^8d^8x^3+3\,ac^7d^7ex^2+3\,a^2c^6d^6e^2x+a^3c^5d^5e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^8/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="maxim
a")
```

```
[Out] -1/3*(c^4*d^8 + 2*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 22*a^3*c*d^2*e^6 + 13
*a^4*e^8 + 18*(c^4*d^6*e^2 - 2*a*c^3*d^4*e^4 + a^2*c^2*d^2*e^6)*x^2 + 6*(c^
4*d^7*e + 3*a*c^3*d^5*e^3 - 9*a^2*c^2*d^3*e^5 + 5*a^3*c*d*e^7)*x)/(c^8*d^8*
x^3 + 3*a*c^7*d^7*e*x^2 + 3*a^2*c^6*d^6*e^2*x + a^3*c^5*d^5*e^3) + e^4*x/(c
^4*d^4) + 4*(c*d^2*e^3 - a*e^5)*log(c*d*x + a*e)/(c^5*d^5)
```

Fricas [B] time = 1.85276, size = 675, normalized size = 4.62

$$\frac{3\,c^4d^4e^4x^4+9\,ac^3d^3e^5x^3-c^4d^8-2\,ac^3d^6e^2-6\,a^2c^2d^4e^4+22\,a^3cd^2e^6-13\,a^4e^8-9\left(2\,c^4d^6e^2-4\,ac^3d^4e^4+a^2c^2d^2e^6\right)x^2-}{3\left(c^8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^8/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="frica
s")
```

```
[Out] 1/3*(3*c^4*d^4*e^4*x^4 + 9*a*c^3*d^3*e^5*x^3 - c^4*d^8 - 2*a*c^3*d^6*e^2 -
6*a^2*c^2*d^4*e^4 + 22*a^3*c*d^2*e^6 - 13*a^4*e^8 - 9*(2*c^4*d^6*e^2 - 4*a*
c^3*d^4*e^4 + a^2*c^2*d^2*e^6)*x^2 - 3*(2*c^4*d^7*e + 6*a*c^3*d^5*e^3 - 18*
a^2*c^2*d^3*e^5 + 9*a^3*c*d*e^7)*x + 12*(a^3*c*d^2*e^6 - a^4*e^8 + (c^4*d^5
*e^3 - a*c^3*d^3*e^5)*x^3 + 3*(a*c^3*d^4*e^4 - a^2*c^2*d^2*e^6)*x^2 + 3*(a^
2*c^2*d^3*e^5 - a^3*c*d*e^7)*x)*log(c*d*x + a*e))/(c^8*d^8*x^3 + 3*a*c^7*d^
7*e*x^2 + 3*a^2*c^6*d^6*e^2*x + a^3*c^5*d^5*e^3)
```

Sympy [A] time = 11.7701, size = 257, normalized size = 1.76

$$-\frac{13a^4e^8 - 22a^3cd^2e^6 + 6a^2c^2d^4e^4 + 2ac^3d^6e^2 + c^4d^8 + x^2\left(18a^2c^2d^2e^6 - 36ac^3d^4e^4 + 18c^4d^6e^2\right) + x\left(30a^3cde^7 - 54a^2c^2\right.}{3a^3c^5d^5e^3 + 9a^2c^6d^6e^2x + 9ac^7d^7ex^2 + 3c^8d^8x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**8/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**4,x)
```

```
[Out] -(13*a**4*e**8 - 22*a**3*c*d**2*e**6 + 6*a**2*c**2*d**4*e**4 + 2*a*c**3*d**
6*e**2 + c**4*d**8 + x**2*(18*a**2*c**2*d**2*e**6 - 36*a*c**3*d**4*e**4 + 1
8*c**4*d**6*e**2) + x*(30*a**3*c*d*e**7 - 54*a**2*c**2*d**3*e**5 + 18*a*c**
3*d**5*e**3 + 6*c**4*d**7*e))/(3*a**3*c**5*d**5*e**3 + 9*a**2*c**6*d**6*e**
2*x + 9*a*c**7*d**7*e*x**2 + 3*c**8*d**8*x**3) + e**4*x/(c**4*d**4) - 4*e**
3*(a*e**2 - c*d**2)*log(a*e + c*d*x)/(c**5*d**5)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^8/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="giac"
)
```

```
[Out] sage0*x
```

3.1899 $\int \frac{(d+ex)^7}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx$

Optimal. Leaf size=122

$$-\frac{3e^2\left(cd^2-ae^2\right)}{c^4d^4(ae+cdx)}-\frac{3e\left(cd^2-ae^2\right)^2}{2c^4d^4(ae+cdx)^2}-\frac{\left(cd^2-ae^2\right)^3}{3c^4d^4(ae+cdx)^3}+\frac{e^3\log(ae+cdx)}{c^4d^4}$$

```
[Out] -(c*d^2 - a*e^2)^3/(3*c^4*d^4*(a*e + c*d*x)^3) - (3*e*(c*d^2 - a*e^2)^2)/(2
*c^4*d^4*(a*e + c*d*x)^2) - (3*e^2*(c*d^2 - a*e^2))/(c^4*d^4*(a*e + c*d*x))
 + (e^3*Log[a*e + c*d*x])/(c^4*d^4)
```

Rubi [A] time = 0.0882848, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$-\frac{3e^2\left(cd^2-ae^2\right)}{c^4d^4(ae+cdx)}-\frac{3e\left(cd^2-ae^2\right)^2}{2c^4d^4(ae+cdx)^2}-\frac{\left(cd^2-ae^2\right)^3}{3c^4d^4(ae+cdx)^3}+\frac{e^3\log(ae+cdx)}{c^4d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^7/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

```
[Out] -(c*d^2 - a*e^2)^3/(3*c^4*d^4*(a*e + c*d*x)^3) - (3*e*(c*d^2 - a*e^2)^2)/(2
*c^4*d^4*(a*e + c*d*x)^2) - (3*e^2*(c*d^2 - a*e^2))/(c^4*d^4*(a*e + c*d*x))
 + (e^3*Log[a*e + c*d*x])/(c^4*d^4)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^7}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx &= \int \frac{(d+ex)^3}{(ae+cdx)^4}\,dx\\
&= \int \left(\frac{\left(cd^2-ae^2\right)^3}{c^3d^3(ae+cdx)^4}+\frac{3e\left(cd^2-ae^2\right)^2}{c^3d^3(ae+cdx)^3}+\frac{3\left(cd^2e^2-ae^4\right)}{c^3d^3(ae+cdx)^2}+\frac{e^3}{c^3d^3(ae+cdx)}\right)dx\\
&= -\frac{\left(cd^2-ae^2\right)^3}{3c^4d^4(ae+cdx)^3}-\frac{3e\left(cd^2-ae^2\right)^2}{2c^4d^4(ae+cdx)^2}-\frac{3e^2\left(cd^2-ae^2\right)}{c^4d^4(ae+cdx)}+\frac{e^3\log(ae+cdx)}{c^4d^4}
\end{aligned}$$

Mathematica [A] time = 0.0516119, size = 99, normalized size = 0.81

$$\frac{6e^3\log(ae+cdx)-\frac{(cd^2-ae^2)(11a^2e^4+acde^2(5d+27ex)+c^2d^2(2d^2+9dex+18e^2x^2))}{(ae+cdx)^3}}{6c^4d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^7/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

```
[Out] (-(((c*d^2 - a*e^2)*(11*a^2*e^4 + a*c*d*e^2*(5*d + 27*e*x) + c^2*d^2*(2*d^2
 + 9*d*e*x + 18*e^2*x^2)))/(a*e + c*d*x)^3) + 6*e^3*Log[a*e + c*d*x])/(6*c^
4*d^4)
```

Maple [A] time = 0.045, size = 210, normalized size = 1.7

$$\frac{a^3e^6}{3\,c^4d^4\,(cdx+ae)^3}-\frac{a^2e^4}{c^3d^2\,(cdx+ae)^3}+\frac{ae^2}{c^2\,(cdx+ae)^3}-\frac{d^2}{3\,c\,(cdx+ae)^3}-\frac{3\,a^2e^5}{2\,c^4d^4\,(cdx+ae)^2}+3\,\frac{ae^3}{c^3d^2\,(cdx+ae)^2}-\frac{}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x)
```

```
[Out] 1/3/c^4/d^4/(c*d*x+a*e)^3*a^3*e^6-1/c^3/d^2/(c*d*x+a*e)^3*a^2*e^4+1/c^2/(c*
d*x+a*e)^3*a*e^2-1/3/c*d^2/(c*d*x+a*e)^3-3/2*e^5/c^4/d^4/(c*d*x+a*e)^2*a^2+
3*e^3/c^3/d^2/(c*d*x+a*e)^2*a-3/2*e/c^2/(c*d*x+a*e)^2+3/c^4/d^4*e^4/(c*d*x+
a*e)*a-3/c^3/d^2*e^2/(c*d*x+a*e)+e^3*ln(c*d*x+a*e)/c^4/d^4
```

Maxima [A] time = 1.17454, size = 242, normalized size = 1.98

$$-\frac{2\,c^3d^6+3\,ac^2d^4e^2+6\,a^2cd^2e^4-11\,a^3e^6+18\left(c^3d^4e^2-ac^2d^2e^4\right)x^2+9\left(c^3d^5e+2\,ac^2d^3e^3-3\,a^2cde^5\right)x}{6\left(c^7d^7x^3+3\,ac^6d^6ex^2+3\,a^2c^5d^5e^2x+a^3c^4d^4e^3\right)}+\frac{e^3\log(cdx}{c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="maxim
a")
```

```
[Out] -1/6*(2*c^3*d^6 + 3*a*c^2*d^4*e^2 + 6*a^2*c*d^2*e^4 - 11*a^3*e^6 + 18*(c^3*
d^4*e^2 - a*c^2*d^2*e^4)*x^2 + 9*(c^3*d^5*e + 2*a*c^2*d^3*e^3 - 3*a^2*c*d*e
^5)*x)/(c^7*d^7*x^3 + 3*a*c^6*d^6*e*x^2 + 3*a^2*c^5*d^5*e^2*x + a^3*c^4*d^4
*e^3) + e^3*log(c*d*x + a*e)/(c^4*d^4)
```

Fricas [A] time = 1.80898, size = 433, normalized size = 3.55

$$-\frac{2\,c^3d^6+3\,ac^2d^4e^2+6\,a^2cd^2e^4-11\,a^3e^6+18\left(c^3d^4e^2-ac^2d^2e^4\right)x^2+9\left(c^3d^5e+2\,ac^2d^3e^3-3\,a^2cde^5\right)x-6\left(c^3d^3e^3x^3\right.}{6\left(c^7d^7x^3+3\,ac^6d^6ex^2+3\,a^2c^5d^5e^2x+a^3c^4d^4e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="frica
s")
```

```
[Out] -1/6*(2*c^3*d^6 + 3*a*c^2*d^4*e^2 + 6*a^2*c*d^2*e^4 - 11*a^3*e^6 + 18*(c^3*
d^4*e^2 - a*c^2*d^2*e^4)*x^2 + 9*(c^3*d^5*e + 2*a*c^2*d^3*e^3 - 3*a^2*c*d*e
^5)*x - 6*(c^3*d^3*e^3*x^3 + 3*a*c^2*d^2*e^4*x^2 + 3*a^2*c*d*e^5*x + a^3*e^
6)*log(c*d*x + a*e))/(c^7*d^7*x^3 + 3*a*c^6*d^6*e*x^2 + 3*a^2*c^5*d^5*e^2*x
 + a^3*c^4*d^4*e^3)
```

Sympy [A] time = 3.17158, size = 189, normalized size = 1.55

$$\frac{11a^3e^6 - 6a^2cd^2e^4 - 3ac^2d^4e^2 - 2c^3d^6 + x^2\left(18ac^2d^2e^4 - 18c^3d^4e^2\right) + x\left(27a^2cde^5 - 18ac^2d^3e^3 - 9c^3d^5e\right)}{6a^3c^4d^4e^3 + 18a^2c^5d^5e^2x + 18ac^6d^6ex^2 + 6c^7d^7x^3} + \frac{e^3\log\left(ae + cdx\right)}{c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**7/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**4,x)
```

```
[Out] (11*a**3*e**6 - 6*a**2*c*d**2*e**4 - 3*a*c**2*d**4*e**2 - 2*c**3*d**6 + x**
2*(18*a*c**2*d**2*e**4 - 18*c**3*d**4*e**2) + x*(27*a**2*c*d*e**5 - 18*a*c*
*2*d**3*e**3 - 9*c**3*d**5*e))/(6*a**3*c**4*d**4*e**3 + 18*a**2*c**5*d**5*e
**2*x + 18*a*c**6*d**6*e*x**2 + 6*c**7*d**7*x**3) + e**3*log(a*e + c*d*x)/(
c**4*d**4)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.1900 $\int \frac{(d+ex)^6}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx$

Optimal. Leaf size=35

$$-\frac{(d+ex)^3}{3\left(cd^2-ae^2\right)(ae+cdx)^3}$$

```
[Out] -(d + e*x)^3/(3*(c*d^2 - a*e^2)*(a*e + c*d*x)^3)
```

Rubi [A] time = 0.0125113, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 37}

$$-\frac{(d+ex)^3}{3\left(cd^2-ae^2\right)(ae+cdx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^6/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

```
[Out] -(d + e*x)^3/(3*(c*d^2 - a*e^2)*(a*e + c*d*x)^3)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^6}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx &= \int \frac{(d+ex)^2}{(ae+cdx)^4}\,dx\\ &= -\frac{(d+ex)^3}{3\left(cd^2-ae^2\right)(ae+cdx)^3}\end{aligned}$$

Mathematica [A] time = 0.0315722, size = 65, normalized size = 1.86

$$-\frac{a^2e^4+acde^2(d+3ex)+c^2d^2\left(d^2+3dex+3e^2x^2\right)}{3c^3d^3(ae+cdx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^6/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

```
[Out] -(a^2*e^4 + a*c*d*e^2*(d + 3*e*x) + c^2*d^2*(d^2 + 3*d*e*x + 3*e^2*x^2))/(3
*c^3*d^3*(a*e + c*d*x)^3)
```

Maple [B] time = 0.044, size = 96, normalized size = 2.7

$$-\frac{a^2e^4-2\,acd^2e^2+c^2d^4}{3\,c^3d^3\,(cdx+ae)^3}+\frac{e\left(ae^2-cd^2\right)}{c^3d^3\,(cdx+ae)^2}-\frac{e^2}{c^3d^3\,(cdx+ae)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x)
```

```
[Out] -1/3*(a^2*e^4-2*a*c*d^2*e^2+c^2*d^4)/c^3/d^3/(c*d*x+a*e)^3+e*(a*e^2-c*d^2)/
c^3/d^3/(c*d*x+a*e)^2-e^2/c^3/d^3/(c*d*x+a*e)
```

Maxima [B] time = 1.09496, size = 153, normalized size = 4.37

$$-\frac{3\,c^2d^2e^2x^2+c^2d^4+acd^2e^2+a^2e^4+3\left(c^2d^3e+acde^3\right)x}{3\left(c^6d^6x^3+3\,ac^5d^5ex^2+3\,a^2c^4d^4e^2x+a^3c^3d^3e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="maxim
a")
```

```
[Out] -1/3*(3*c^2*d^2*e^2*x^2 + c^2*d^4 + a*c*d^2*e^2 + a^2*e^4 + 3*(c^2*d^3*e +
a*c*d*e^3)*x)/(c^6*d^6*x^3 + 3*a*c^5*d^5*e*x^2 + 3*a^2*c^4*d^4*e^2*x + a^3*
c^3*d^3*e^3)
```

Fricas [B] time = 1.81493, size = 221, normalized size = 6.31

$$-\frac{3\,c^2d^2e^2x^2+c^2d^4+acd^2e^2+a^2e^4+3\left(c^2d^3e+acde^3\right)x}{3\left(c^6d^6x^3+3\,ac^5d^5ex^2+3\,a^2c^4d^4e^2x+a^3c^3d^3e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="frica
s")
```

```
[Out] -1/3*(3*c^2*d^2*e^2*x^2 + c^2*d^4 + a*c*d^2*e^2 + a^2*e^4 + 3*(c^2*d^3*e +
a*c*d*e^3)*x)/(c^6*d^6*x^3 + 3*a*c^5*d^5*e*x^2 + 3*a^2*c^4*d^4*e^2*x + a^3*
c^3*d^3*e^3)
```

Sympy [B] time = 1.31524, size = 121, normalized size = 3.46

$$-\frac{a^2e^4+acd^2e^2+c^2d^4+3c^2d^2e^2x^2+x\left(3acde^3+3c^2d^3e\right)}{3a^3c^3d^3e^3+9a^2c^4d^4e^2x+9ac^5d^5ex^2+3c^6d^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**6/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**4,x)
```

```
[Out] -(a**2*e**4 + a*c*d**2*e**2 + c**2*d**4 + 3*c**2*d**2*e**2*x**2 + x*(3*a*c*
d*e**3 + 3*c**2*d**3*e))/(3*a**3*c**3*d**3*e**3 + 9*a**2*c**4*d**4*e**2*x +
 9*a*c**5*d**5*e*x**2 + 3*c**6*d**6*x**3)
```

Giac [B] time = 90.6134, size = 1110, normalized size = 31.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="giac"
)
```

```
[Out] -1/3*(3*c^8*d^14*x^5*e^5 + 12*c^8*d^15*x^4*e^4 + 19*c^8*d^16*x^3*e^3 + 15*c
^8*d^17*x^2*e^2 + 6*c^8*d^18*x*e + c^8*d^19 - 18*a*c^7*d^12*x^5*e^7 - 69*a*
c^7*d^13*x^4*e^6 - 104*a*c^7*d^14*x^3*e^5 - 78*a*c^7*d^15*x^2*e^4 - 30*a*c^
7*d^16*x*e^3 - 5*a*c^7*d^17*e^2 + 45*a^2*c^6*d^10*x^5*e^9 + 162*a^2*c^6*d^1
1*x^4*e^8 + 226*a^2*c^6*d^12*x^3*e^7 + 156*a^2*c^6*d^13*x^2*e^6 + 57*a^2*c^
6*d^14*x*e^5 + 10*a^2*c^6*d^15*e^4 - 60*a^3*c^5*d^8*x^5*e^11 - 195*a^3*c^5*
d^9*x^4*e^10 - 236*a^3*c^5*d^10*x^3*e^9 - 138*a^3*c^5*d^11*x^2*e^8 - 48*a^3
*c^5*d^12*x*e^7 - 11*a^3*c^5*d^13*e^6 + 45*a^4*c^4*d^6*x^5*e^13 + 120*a^4*c
^4*d^7*x^4*e^12 + 100*a^4*c^4*d^8*x^3*e^11 + 30*a^4*c^4*d^9*x^2*e^10 + 15*a
^4*c^4*d^10*x*e^9 + 10*a^4*c^4*d^11*e^8 - 18*a^5*c^3*d^4*x^5*e^15 - 27*a^5*
c^3*d^5*x^4*e^14 + 16*a^5*c^3*d^6*x^3*e^13 + 30*a^5*c^3*d^7*x^2*e^12 - 6*a^
5*c^3*d^8*x*e^11 - 11*a^5*c^3*d^9*e^10 + 3*a^6*c^2*d^2*x^5*e^17 - 6*a^6*c^2
*d^3*x^4*e^16 - 26*a^6*c^2*d^4*x^3*e^15 - 12*a^6*c^2*d^5*x^2*e^14 + 15*a^6*
c^2*d^6*x*e^13 + 10*a^6*c^2*d^7*e^12 + 3*a^7*c*d*x^4*e^18 + 4*a^7*c*d^2*x^3
*e^17 - 6*a^7*c*d^3*x^2*e^16 - 12*a^7*c*d^4*x*e^15 - 5*a^7*c*d^5*e^14 + a^8
*x^3*e^19 + 3*a^8*d*x^2*e^18 + 3*a^8*d^2*x*e^17 + a^8*d^3*e^16)/((c^9*d^15
- 6*a*c^8*d^13*e^2 + 15*a^2*c^7*d^11*e^4 - 20*a^3*c^6*d^9*e^6 + 15*a^4*c^5*
d^7*e^8 - 6*a^5*c^4*d^5*e^10 + a^6*c^3*d^3*e^12)*(c*d*x^2*e + c*d^2*x + a*x
*e^2 + a*d*e)^3)
```

3.1901 $\int \frac{(d+ex)^5}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx$

Optimal. Leaf size=54

$$-\frac{cd^2-ae^2}{3c^2d^2(ae+cdx)^3}-\frac{e}{2c^2d^2(ae+cdx)^2}$$

```
[Out] -(c*d^2 - a*e^2)/(3*c^2*d^2*(a*e + c*d*x)^3) - e/(2*c^2*d^2*(a*e + c*d*x)^2
)
```

Rubi [A] time = 0.0390207, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$-\frac{cd^2-ae^2}{3c^2d^2(ae+cdx)^3}-\frac{e}{2c^2d^2(ae+cdx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

```
[Out] -(c*d^2 - a*e^2)/(3*c^2*d^2*(a*e + c*d*x)^3) - e/(2*c^2*d^2*(a*e + c*d*x)^2
)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^5}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx &= \int \frac{d+ex}{(ae+cdx)^4}\,dx\\
&= \int \left(\frac{cd^2-ae^2}{cd(ae+cdx)^4}+\frac{e}{cd(ae+cdx)^3}\right)dx\\
&= -\frac{cd^2-ae^2}{3c^2d^2(ae+cdx)^3}-\frac{e}{2c^2d^2(ae+cdx)^2}
\end{aligned}$$

Mathematica [A] time = 0.0169026, size = 37, normalized size = 0.69

$$-\frac{ae^2+cd(2d+3ex)}{6c^2d^2(ae+cdx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

```
[Out] -(a*e^2 + c*d*(2*d + 3*e*x))/(6*c^2*d^2*(a*e + c*d*x)^3)
```

Maple [A] time = 0.043, size = 51, normalized size = 0.9

$$-\frac{-ae^2+cd^2}{3\,c^2d^2\,(cdx+ae)^3}-\frac{e}{2\,c^2d^2\,(cdx+ae)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x)
```

```
[Out] -1/3*(-a*e^2+c*d^2)/c^2/d^2/(c*d*x+a*e)^3-1/2*e/c^2/d^2/(c*d*x+a*e)^2
```

Maxima [A] time = 1.0737, size = 100, normalized size = 1.85

$$-\frac{3\,cdex+2\,cd^2+ae^2}{6\,\left(c^5d^5x^3+3\,ac^4d^4ex^2+3\,a^2c^3d^3e^2x+a^3c^2d^2e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="maxim
a")
```

```
[Out] -1/6*(3*c*d*e*x + 2*c*d^2 + a*e^2)/(c^5*d^5*x^3 + 3*a*c^4*d^4*e*x^2 + 3*a^2
*c^3*d^3*e^2*x + a^3*c^2*d^2*e^3)
```

Fricas [A] time = 1.98335, size = 149, normalized size = 2.76

$$-\frac{3\,cdex+2\,cd^2+ae^2}{6\,\left(c^5d^5x^3+3\,ac^4d^4ex^2+3\,a^2c^3d^3e^2x+a^3c^2d^2e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="frica
s")
```

```
[Out] -1/6*(3*c*d*e*x + 2*c*d^2 + a*e^2)/(c^5*d^5*x^3 + 3*a*c^4*d^4*e*x^2 + 3*a^2
*c^3*d^3*e^2*x + a^3*c^2*d^2*e^3)
```

Sympy [A] time = 0.851355, size = 80, normalized size = 1.48

$$-\frac{ae^2+2cd^2+3cdex}{6a^3c^2d^2e^3+18a^2c^3d^3e^2x+18ac^4d^4ex^2+6c^5d^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**4,x)
```

```
[Out] -(a*e**2 + 2*c*d**2 + 3*c*d*e*x)/(6*a**3*c**2*d**2*e**3 + 18*a**2*c**3*d**3
*e**2*x + 18*a*c**4*d**4*e*x**2 + 6*c**5*d**5*x**3)
```

Giac [B] time = 1.5247, size = 865, normalized size = 16.02

$$-\frac{3\,c^7d^{13}x^4e^4 + 11\,c^7d^{14}x^3e^3 + 15\,c^7d^{15}x^2e^2 + 9\,c^7d^{16}xe + 2\,c^7d^{17} - 18\,ac^6d^{11}x^4e^6 - 65\,ac^6d^{12}x^3e^5 - 87\,ac^6d^{13}x^2e^4 - 51\,ac^6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="giac"
)
```

```
[Out] -1/6*(3*c^7*d^13*x^4*e^4 + 11*c^7*d^14*x^3*e^3 + 15*c^7*d^15*x^2*e^2 + 9*c^
7*d^16*x*e + 2*c^7*d^17 - 18*a*c^6*d^11*x^4*e^6 - 65*a*c^6*d^12*x^3*e^5 - 8
7*a*c^6*d^13*x^2*e^4 - 51*a*c^6*d^14*x*e^3 - 11*a*c^6*d^15*e^2 + 45*a^2*c^5
*d^9*x^4*e^8 + 159*a^2*c^5*d^10*x^3*e^7 + 207*a^2*c^5*d^11*x^2*e^6 + 117*a^
2*c^5*d^12*x*e^5 + 24*a^2*c^5*d^13*e^4 - 60*a^3*c^4*d^7*x^4*e^10 - 205*a^3*
c^4*d^8*x^3*e^9 - 255*a^3*c^4*d^9*x^2*e^8 - 135*a^3*c^4*d^10*x*e^7 - 25*a^3
*c^4*d^11*e^6 + 45*a^4*c^3*d^5*x^4*e^12 + 145*a^4*c^3*d^6*x^3*e^11 + 165*a^
4*c^3*d^7*x^2*e^10 + 75*a^4*c^3*d^8*x*e^9 + 10*a^4*c^3*d^9*e^8 - 18*a^5*c^2
*d^3*x^4*e^14 - 51*a^5*c^2*d^4*x^3*e^13 - 45*a^5*c^2*d^5*x^2*e^12 - 9*a^5*c
^2*d^6*x*e^11 + 3*a^5*c^2*d^7*e^10 + 3*a^6*c*d*x^4*e^16 + 5*a^6*c*d^2*x^3*e
^15 - 3*a^6*c*d^3*x^2*e^14 - 9*a^6*c*d^4*x*e^13 - 4*a^6*c*d^5*e^12 + a^7*x^
3*e^17 + 3*a^7*d*x^2*e^16 + 3*a^7*d^2*x*e^15 + a^7*d^3*e^14)/((c^8*d^14 - 6
*a*c^7*d^12*e^2 + 15*a^2*c^6*d^10*e^4 - 20*a^3*c^5*d^8*e^6 + 15*a^4*c^4*d^6
*e^8 - 6*a^5*c^3*d^4*e^10 + a^6*c^2*d^2*e^12)*(c*d*x^2*e + c*d^2*x + a*x*e^
2 + a*d*e)^3)
```

3.1902 $\int \frac{(d+ex)^4}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4} dx$

Optimal. Leaf size=20

$$-\frac{1}{3cd(ae+cdx)^3}$$

[Out] -1/(3*c*d*(a*e + c*d*x)^3)

Rubi [A] time = 0.0106371, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 32}

$$-\frac{1}{3cd(ae+cdx)^3}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^4/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]

[Out] -1/(3*c*d*(a*e + c*d*x)^3)

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(d+ex)^4}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4} dx &= \int \frac{1}{(ae+cdx)^4} dx \\ &= -\frac{1}{3cd(ae+cdx)^3} \end{aligned}$$

Mathematica [A] time = 0.0037334, size = 20, normalized size = 1.

$$-\frac{1}{3cd(ae+cdx)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^4/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]

[Out] -1/(3*c*d*(a*e + c*d*x)^3)

Maple [A] time = 0.038, size = 19, normalized size = 1.

$$-\frac{1}{3\,cd\,(cdx+ae)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x)
```

```
[Out] -1/3/c/d/(c*d*x+a*e)^3
```

Maxima [B] time = 1.11815, size = 70, normalized size = 3.5

$$-\frac{1}{3\left(c^4d^4x^3+3\,ac^3d^3ex^2+3\,a^2c^2d^2e^2x+a^3cde^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="maxim
a")
```

```
[Out] -1/3/(c^4*d^4*x^3 + 3*a*c^3*d^3*e*x^2 + 3*a^2*c^2*d^2*e^2*x + a^3*c*d*e^3)
```

Fricas [B] time = 1.7866, size = 103, normalized size = 5.15

$$-\frac{1}{3\left(c^4d^4x^3+3\,ac^3d^3ex^2+3\,a^2c^2d^2e^2x+a^3cde^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="frica
s")
```

```
[Out] -1/3/(c^4*d^4*x^3 + 3*a*c^3*d^3*e*x^2 + 3*a^2*c^2*d^2*e^2*x + a^3*c*d*e^3)
```

Sympy [B] time = 0.672494, size = 58, normalized size = 2.9

$$-\frac{1}{3a^3cde^3+9a^2c^2d^2e^2x+9ac^3d^3ex^2+3c^4d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**4,x)
```

```
[Out] -1/(3*a**3*c*d*e**3 + 9*a**2*c**2*d**2*e**2*x + 9*a*c**3*d**3*e*x**2 + 3*c*
*4*d**4*x**3)
```

Giac [B] time = 3.41515, size = 639, normalized size = 31.95

$$-\frac{c^6d^{12}x^3e^3 + 3\,c^6d^{13}x^2e^2 + 3\,c^6d^{14}xe + c^6d^{15} - 6\,ac^5d^{10}x^3e^5 - 18\,ac^5d^{11}x^2e^4 - 18\,ac^5d^{12}xe^3 - 6\,ac^5d^{13}e^2 + 15\,a^2c^4d^8x^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="giac"
)
```

```
[Out] -1/3*(c^6*d^12*x^3*e^3 + 3*c^6*d^13*x^2*e^2 + 3*c^6*d^14*x*e + c^6*d^15 - 6
*a*c^5*d^10*x^3*e^5 - 18*a*c^5*d^11*x^2*e^4 - 18*a*c^5*d^12*x*e^3 - 6*a*c^5
*d^13*e^2 + 15*a^2*c^4*d^8*x^3*e^7 + 45*a^2*c^4*d^9*x^2*e^6 + 45*a^2*c^4*d^
10*x*e^5 + 15*a^2*c^4*d^11*e^4 - 20*a^3*c^3*d^6*x^3*e^9 - 60*a^3*c^3*d^7*x^
2*e^8 - 60*a^3*c^3*d^8*x*e^7 - 20*a^3*c^3*d^9*e^6 + 15*a^4*c^2*d^4*x^3*e^11
 + 45*a^4*c^2*d^5*x^2*e^10 + 45*a^4*c^2*d^6*x*e^9 + 15*a^4*c^2*d^7*e^8 - 6*
a^5*c*d^2*x^3*e^13 - 18*a^5*c*d^3*x^2*e^12 - 18*a^5*c*d^4*x*e^11 - 6*a^5*c*
d^5*e^10 + a^6*x^3*e^15 + 3*a^6*d*x^2*e^14 + 3*a^6*d^2*x*e^13 + a^6*d^3*e^1
2)/((c^7*d^13 - 6*a*c^6*d^11*e^2 + 15*a^2*c^5*d^9*e^4 - 20*a^3*c^4*d^7*e^6
+ 15*a^4*c^3*d^5*e^8 - 6*a^5*c^2*d^3*e^10 + a^6*c*d*e^12)*(c*d*x^2*e + c*d^
2*x + a*x*e^2 + a*d*e)^3)
```

3.1903 $\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx$

Optimal. Leaf size=139

$$-\frac{e^2}{\left(cd^2-ae^2\right)^3(ae+cdx)}+\frac{e}{2\left(cd^2-ae^2\right)^2(ae+cdx)^2}-\frac{1}{3\left(cd^2-ae^2\right)(ae+cdx)^3}-\frac{e^3\log(ae+cdx)}{\left(cd^2-ae^2\right)^4}+\frac{e^3\log(d+ex)}{\left(cd^2-ae^2\right)^4}$$

```
[Out] -1/(3*(c*d^2 - a*e^2)*(a*e + c*d*x)^3) + e/(2*(c*d^2 - a*e^2)^2*(a*e + c*d*
x)^2) - e^2/((c*d^2 - a*e^2)^3*(a*e + c*d*x)) - (e^3*Log[a*e + c*d*x])/(c*d
^2 - a*e^2)^4 + (e^3*Log[d + e*x])/(c*d^2 - a*e^2)^4
```

Rubi [A] time = 0.0984611, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 44}

$$-\frac{e^2}{\left(cd^2-ae^2\right)^3(ae+cdx)}+\frac{e}{2\left(cd^2-ae^2\right)^2(ae+cdx)^2}-\frac{1}{3\left(cd^2-ae^2\right)(ae+cdx)^3}-\frac{e^3\log(ae+cdx)}{\left(cd^2-ae^2\right)^4}+\frac{e^3\log(d+ex)}{\left(cd^2-ae^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

```
[Out] -1/(3*(c*d^2 - a*e^2)*(a*e + c*d*x)^3) + e/(2*(c*d^2 - a*e^2)^2*(a*e + c*d*
x)^2) - e^2/((c*d^2 - a*e^2)^3*(a*e + c*d*x)) - (e^3*Log[a*e + c*d*x])/(c*d
^2 - a*e^2)^4 + (e^3*Log[d + e*x])/(c*d^2 - a*e^2)^4
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx &= \int \frac{1}{(ae+cdx)^4(d+ex)}\,dx\\
&= \int \left(\frac{cd}{\left(cd^2-ae^2\right)(ae+cdx)^4}-\frac{cde}{\left(cd^2-ae^2\right)^2(ae+cdx)^3}+\frac{cde^2}{\left(cd^2-ae^2\right)^3(ae+cdx)^2}\right.\\
&= -\frac{1}{3\left(cd^2-ae^2\right)(ae+cdx)^3}+\frac{e}{2\left(cd^2-ae^2\right)^2(ae+cdx)^2}-\frac{e^2}{\left(cd^2-ae^2\right)^3(ae+cdx)}
\end{aligned}$$

Mathematica [A] time = 0.100535, size = 117, normalized size = 0.84

$$-\frac{\frac{(cd^2-ae^2)\left(11a^2e^4+acde^2(15ex-7d)+c^2d^2\left(2d^2-3dex+6e^2x^2\right)\right)}{(ae+cdx)^3}+6e^3\log(ae+cdx)-6e^3\log(d+ex)}{6\left(cd^2-ae^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

```
[Out] -((((c*d^2 - a*e^2)*(11*a^2*e^4 + a*c*d*e^2*(-7*d + 15*e*x) + c^2*d^2*(2*d^2
 - 3*d*e*x + 6*e^2*x^2)))/(a*e + c*d*x)^3 + 6*e^3*Log[a*e + c*d*x] - 6*e^3*
Log[d + e*x])/(6*(c*d^2 - a*e^2)^4)
```

Maple [A] time = 0.052, size = 135, normalized size = 1.

$$\frac{e^3\ln(ex+d)}{\left(ae^2-cd^2\right)^4}+\frac{1}{\left(3\,ae^2-3\,cd^2\right)(cdx+ae)^3}+\frac{e}{2\left(ae^2-cd^2\right)^2(cdx+ae)^2}+\frac{e^2}{\left(ae^2-cd^2\right)^3(cdx+ae)}-\frac{e^3\ln(cdx+ae)}{\left(ae^2-cd^2\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x)
```

```
[Out] e^3/(a*e^2-c*d^2)^4*ln(e*x+d)+1/3/(a*e^2-c*d^2)/(c*d*x+a*e)^3+1/2*e/(a*e^2-
c*d^2)^2/(c*d*x+a*e)^2+e^2/(a*e^2-c*d^2)^3/(c*d*x+a*e)-e^3/(a*e^2-c*d^2)^4*
ln(c*d*x+a*e)
```

Maxima [B] time = 1.07751, size = 556, normalized size = 4.

$$-\frac{e^3\log(cdx+ae)}{c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8}+\frac{e^3\log(ex+d)}{c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8}-\frac{}{6\left(a^3c^3d^6e^3-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="maxim
a")
```

```
[Out] -e^3*log(c*d*x + a*e)/(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^
3*c*d^2*e^6 + a^4*e^8) + e^3*log(e*x + d)/(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^
2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8) - 1/6*(6*c^2*d^2*e^2*x^2 + 2*c^2
*d^4 - 7*a*c*d^2*e^2 + 11*a^2*e^4 - 3*(c^2*d^3*e - 5*a*c*d*e^3)*x)/(a^3*c^3
*d^6*e^3 - 3*a^4*c^2*d^4*e^5 + 3*a^5*c*d^2*e^7 - a^6*e^9 + (c^6*d^9 - 3*a*c
^5*d^7*e^2 + 3*a^2*c^4*d^5*e^4 - a^3*c^3*d^3*e^6)*x^3 + 3*(a*c^5*d^8*e - 3*
a^2*c^4*d^6*e^3 + 3*a^3*c^3*d^4*e^5 - a^4*c^2*d^2*e^7)*x^2 + 3*(a^2*c^4*d^7
*e^2 - 3*a^3*c^3*d^5*e^4 + 3*a^4*c^2*d^3*e^6 - a^5*c*d*e^8)*x)
```

Fricas [B] time = 1.99822, size = 961, normalized size = 6.91

$$-\frac{2\,c^3d^6-9\,ac^2d^4e^2+18\,a^2cd^2e^4-11\,a^3e^6+6\left(c^3d^4e^2-ac^2d^2e^4\right)x^2-3\left(c^3d^5e-6\,ac^2d^3e^3+5\,a^2cde^5\right)x+}{6\left(a^3c^4d^8e^3-4\,a^4c^3d^6e^5+6\,a^5c^2d^4e^7-4\,a^6cd^2e^9+a^7e^{11}+\left(c^7d^{11}-4\,ac^6d^9e^2+6\,a^2c^5d^7e^4-4\,a^3c^4d^5e^6+a^4c^3d^3e^8\right)x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="frica
s")
```

```
[Out] -1/6*(2*c^3*d^6 - 9*a*c^2*d^4*e^2 + 18*a^2*c*d^2*e^4 - 11*a^3*e^6 + 6*(c^3*
d^4*e^2 - a*c^2*d^2*e^4)*x^2 - 3*(c^3*d^5*e - 6*a*c^2*d^3*e^3 + 5*a^2*c*d*e
^5)*x + 6*(c^3*d^3*e^3*x^3 + 3*a*c^2*d^2*e^4*x^2 + 3*a^2*c*d*e^5*x + a^3*e^
6)*log(c*d*x + a*e) - 6*(c^3*d^3*e^3*x^3 + 3*a*c^2*d^2*e^4*x^2 + 3*a^2*c*d*
e^5*x + a^3*e^6)*log(e*x + d))/(a^3*c^4*d^8*e^3 - 4*a^4*c^3*d^6*e^5 + 6*a^5
*c^2*d^4*e^7 - 4*a^6*c*d^2*e^9 + a^7*e^11 + (c^7*d^11 - 4*a*c^6*d^9*e^2 + 6
*a^2*c^5*d^7*e^4 - 4*a^3*c^4*d^5*e^6 + a^4*c^3*d^3*e^8)*x^3 + 3*(a*c^6*d^10
*e - 4*a^2*c^5*d^8*e^3 + 6*a^3*c^4*d^6*e^5 - 4*a^4*c^3*d^4*e^7 + a^5*c^2*d^
2*e^9)*x^2 + 3*(a^2*c^5*d^9*e^2 - 4*a^3*c^4*d^7*e^4 + 6*a^4*c^3*d^5*e^6 - 4
*a^5*c^2*d^3*e^8 + a^6*c*d*e^10)*x)
```

Sympy [B] time = 2.6889, size = 668, normalized size = 4.81

$$\frac{e^3\log\left(x+\frac{-\frac{a^5e^{13}}{(ae^2-cd^2)^4}+\frac{5a^4cd^2e^{11}}{(ae^2-cd^2)^4}-\frac{10a^3c^2d^4e^9}{(ae^2-cd^2)^4}+\frac{10a^2c^3d^6e^7}{(ae^2-cd^2)^4}-\frac{5ac^4d^8e^5}{(ae^2-cd^2)^4}+ae^5+\frac{c^5d^{10}e^3}{(ae^2-cd^2)^4}+cd^2e^3}{2cde^4}\right)}{\left(ae^2-cd^2\right)^4}-\frac{e^3\log\left(x+\frac{\frac{a^5e^{13}}{(ae^2-cd^2)^4}-\frac{5a^4cd^2e^{11}}{(ae^2-cd^2)^4}+\frac{10a^3c^2d^4e^9}{(ae^2-cd^2)^4}-\frac{10a^2c\ldots}{(ae^2-\ldots}}{2c\ldots}\right.}{\left(ae^2-cd^2\right)\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**4,x)
```

```
[Out] e**3*log(x + (-a**5*e**13/(a*e**2 - c*d**2)**4 + 5*a**4*c*d**2*e**11/(a*e**
2 - c*d**2)**4 - 10*a**3*c**2*d**4*e**9/(a*e**2 - c*d**2)**4 + 10*a**2*c**3
*d**6*e**7/(a*e**2 - c*d**2)**4 - 5*a*c**4*d**8*e**5/(a*e**2 - c*d**2)**4 +
 a*e**5 + c**5*d**10*e**3/(a*e**2 - c*d**2)**4 + c*d**2*e**3)/(2*c*d*e**4))
/(a*e**2 - c*d**2)**4 - e**3*log(x + (a**5*e**13/(a*e**2 - c*d**2)**4 - 5*a
**4*c*d**2*e**11/(a*e**2 - c*d**2)**4 + 10*a**3*c**2*d**4*e**9/(a*e**2 - c*
d**2)**4 - 10*a**2*c**3*d**6*e**7/(a*e**2 - c*d**2)**4 + 5*a*c**4*d**8*e**5
/(a*e**2 - c*d**2)**4 + a*e**5 - c**5*d**10*e**3/(a*e**2 - c*d**2)**4 + c*d
**2*e**3)/(2*c*d*e**4))/(a*e**2 - c*d**2)**4 + (11*a**2*e**4 - 7*a*c*d**2*e
**2 + 2*c**2*d**4 + 6*c**2*d**2*e**2*x**2 + x*(15*a*c*d*e**3 - 3*c**2*d**3*
e))/(6*a**6*e**9 - 18*a**5*c*d**2*e**7 + 18*a**4*c**2*d**4*e**5 - 6*a**3*c*
*3*d**6*e**3 + x**3*(6*a**3*c**3*d**3*e**6 - 18*a**2*c**4*d**5*e**4 + 18*a*
c**5*d**7*e**2 - 6*c**6*d**9) + x**2*(18*a**4*c**2*d**2*e**7 - 54*a**3*c**3
*d**4*e**5 + 54*a**2*c**4*d**6*e**3 - 18*a*c**5*d**8*e) + x*(18*a**5*c*d*e*
*8 - 54*a**4*c**2*d**3*e**6 + 54*a**3*c**3*d**5*e**4 - 18*a**2*c**4*d**7*e*
*2))
```

Giac [B] time = 1.34534, size = 973, normalized size = 7.

$$\frac{2\left(c^3d^6e^3-3\,ac^2d^4e^5+3\,a^2cd^2e^7-a^3e^9\right)\arctan\left(-\frac{2\,cdxe+cd^2+ae^2}{\sqrt{-c^2d^4+2\,acd^2e^2-a^2e^4}}\right)}{\left(c^6d^{12}-6\,ac^5d^{10}e^2+15\,a^2c^4d^8e^4-20\,a^3c^3d^6e^6+15\,a^4c^2d^4e^8-6\,a^5cd^2e^{10}+a^6e^{12}\right)\sqrt{-c^2d^4+2\,acd^2e^2-a^2e^4}}-\frac{6\,c^5d^8x^5e^5\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="giac"
)
```

```
[Out] 2*(c^3*d^6*e^3 - 3*a*c^2*d^4*e^5 + 3*a^2*c*d^2*e^7 - a^3*e^9)*arctan(-(2*c*
d*x*e + c*d^2 + a*e^2)/sqrt(-c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4))/((c^6*d^12
 - 6*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 + 15*a^4*c^2*
d^4*e^8 - 6*a^5*c*d^2*e^10 + a^6*e^12)*sqrt(-c^2*d^4 + 2*a*c*d^2*e^2 - a^2*
e^4)) - 1/6*(6*c^5*d^8*x^5*e^5 + 15*c^5*d^9*x^4*e^4 + 11*c^5*d^10*x^3*e^3 +
 3*c^5*d^11*x^2*e^2 + 3*c^5*d^12*x*e + 2*c^5*d^13 - 18*a*c^4*d^6*x^5*e^7 -
30*a*c^4*d^7*x^4*e^6 + 5*a*c^4*d^8*x^3*e^5 + 15*a*c^4*d^9*x^2*e^4 - 15*a*c^
4*d^10*x*e^3 - 13*a*c^4*d^11*e^2 + 18*a^2*c^3*d^4*x^5*e^9 - 70*a^2*c^3*d^6*
x^3*e^7 - 30*a^2*c^3*d^7*x^2*e^6 + 60*a^2*c^3*d^8*x*e^5 + 38*a^2*c^3*d^9*e^
4 - 6*a^3*c^2*d^2*x^5*e^11 + 30*a^3*c^2*d^3*x^4*e^10 + 70*a^3*c^2*d^4*x^3*e
^9 - 30*a^3*c^2*d^5*x^2*e^8 - 120*a^3*c^2*d^6*x*e^7 - 56*a^3*c^2*d^7*e^6 -
15*a^4*c*d*x^4*e^12 - 5*a^4*c*d^2*x^3*e^11 + 75*a^4*c*d^3*x^2*e^10 + 105*a^
4*c*d^4*x*e^9 + 40*a^4*c*d^5*e^8 - 11*a^5*x^3*e^13 - 33*a^5*d*x^2*e^12 - 33
*a^5*d^2*x*e^11 - 11*a^5*d^3*e^10)/((c^6*d^12 - 6*a*c^5*d^10*e^2 + 15*a^2*c
^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 + 15*a^4*c^2*d^4*e^8 - 6*a^5*c*d^2*e^10 + a
^6*e^12)*(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)^3)
```

3.1904 $\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4} dx$

Optimal. Leaf size=173

$$-\frac{e^3}{(d+ex)\left(cd^2-ae^2\right)^4}-\frac{3cde^2}{\left(cd^2-ae^2\right)^4(ae+cdx)}+\frac{cde}{\left(cd^2-ae^2\right)^3(ae+cdx)^2}-\frac{cd}{3\left(cd^2-ae^2\right)^2(ae+cdx)^3}-\frac{4cde^3\log(ae+}{\left(cd^2-ae^2\right)}$$

```
[Out] -(c*d)/(3*(c*d^2 - a*e^2)^2*(a*e + c*d*x)^3) + (c*d*e)/((c*d^2 - a*e^2)^3*(
a*e + c*d*x)^2) - (3*c*d*e^2)/((c*d^2 - a*e^2)^4*(a*e + c*d*x)) - e^3/((c*d
^2 - a*e^2)^4*(d + e*x)) - (4*c*d*e^3*Log[a*e + c*d*x])/(c*d^2 - a*e^2)^5 +
 (4*c*d*e^3*Log[d + e*x])/(c*d^2 - a*e^2)^5
```

Rubi [A] time = 0.152527, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 44}

$$-\frac{e^3}{(d+ex)\left(cd^2-ae^2\right)^4}-\frac{3cde^2}{\left(cd^2-ae^2\right)^4(ae+cdx)}+\frac{cde}{\left(cd^2-ae^2\right)^3(ae+cdx)^2}-\frac{cd}{3\left(cd^2-ae^2\right)^2(ae+cdx)^3}-\frac{4cde^3\log(ae+}{\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```


```
[Out] -(c*d)/(3*(c*d^2 - a*e^2)^2*(a*e + c*d*x)^3) + (c*d*e)/((c*d^2 - a*e^2)^3*(
a*e + c*d*x)^2) - (3*c*d*e^2)/((c*d^2 - a*e^2)^4*(a*e + c*d*x)) - e^3/((c*d
^2 - a*e^2)^4*(d + e*x)) - (4*c*d*e^3*Log[a*e + c*d*x])/(c*d^2 - a*e^2)^5 +
 (4*c*d*e^3*Log[d + e*x])/(c*d^2 - a*e^2)^5
```


Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}dx &= \int \frac{1}{(ae+cdx)^4(d+ex)^2}dx\\ &= \int \left(\frac{c^2d^2}{\left(cd^2-ae^2\right)^2(ae+cdx)^4}-\frac{2c^2d^2e}{\left(cd^2-ae^2\right)^3(ae+cdx)^3}+\frac{3c^2d^2e^2}{\left(cd^2-ae^2\right)^4(ae+cdx)^2}\right.\\ &= -\frac{cd}{3\left(cd^2-ae^2\right)^2(ae+cdx)^3}+\frac{cde}{\left(cd^2-ae^2\right)^3(ae+cdx)^2}-\frac{3cde^2}{\left(cd^2-ae^2\right)^4(ae+cdx)}\end{aligned}$$

Mathematica [A] time = 0.149459, size = 157, normalized size = 0.91

$$\frac{\frac{9cde^2(cd^2-ae^2)}{ae+cdx}-\frac{3cde(cd^2-ae^2)^2}{(ae+cdx)^2}+\frac{3cd^2e^3-3ae^5}{d+ex}+\frac{cd(cd^2-ae^2)^3}{(ae+cdx)^3}+12cde^3\log(ae+cdx)-12cde^3\log(d+ex)}{3\left(ae^2-cd^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

```
[Out] ((c*d*(c*d^2 - a*e^2)^3)/(a*e + c*d*x)^3 - (3*c*d*e*(c*d^2 - a*e^2)^2)/(a*e
 + c*d*x)^2 + (9*c*d*e^2*(c*d^2 - a*e^2))/(a*e + c*d*x) + (3*c*d^2*e^3 - 3*
a*e^5)/(d + e*x) + 12*c*d*e^3*Log[a*e + c*d*x] - 12*c*d*e^3*Log[d + e*x])/(
3*(-(c*d^2) + a*e^2)^5)
```

Maple [A] time = 0.056, size = 173, normalized size = 1.

$$-\frac{e^3}{\left(ae^2-cd^2\right)^4(ex+d)}-4\frac{e^3cd\ln(ex+d)}{\left(ae^2-cd^2\right)^5}-\frac{cd}{3\left(ae^2-cd^2\right)^2(cdx+ae)^3}+4\frac{e^3cd\ln(cdx+ae)}{\left(ae^2-cd^2\right)^5}-3\frac{e^2cd}{\left(ae^2-cd^2\right)^4(cdx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x)
```

```
[Out] -e^3/(a*e^2-c*d^2)^4/(e*x+d)-4*e^3/(a*e^2-c*d^2)^5*c*d*ln(e*x+d)-1/3*c*d/(a
*e^2-c*d^2)^2/(c*d*x+a*e)^3+4*e^3/(a*e^2-c*d^2)^5*c*d*ln(c*d*x+a*e)-3*c*d/(
a*e^2-c*d^2)^4*e^2/(c*d*x+a*e)-c*d/(a*e^2-c*d^2)^3*e/(c*d*x+a*e)^2
```

Maxima [B] time = 1.23983, size = 886, normalized size = 5.12

$$-\frac{4\,cde^3\log(cdx+ae)}{c^5d^{10}-5\,ac^4d^8e^2+10\,a^2c^3d^6e^4-10\,a^3c^2d^4e^6+5\,a^4cd^2e^8-a^5e^{10}}+\frac{4\,cde^3\log(ex+d)}{c^5d^{10}-5\,ac^4d^8e^2+10\,a^2c^3d^6e^4-10\,a^3c^2d^4e^6+5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="maxim
a")
```

```
[Out] -4*c*d*e^3*log(c*d*x + a*e)/(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^
4 - 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10) + 4*c*d*e^3*log(e*x +
d)/(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4*e^6 +
5*a^4*c*d^2*e^8 - a^5*e^10) - 1/3*(12*c^3*d^3*e^3*x^3 + c^3*d^6 - 5*a*c^2*d
^4*e^2 + 13*a^2*c*d^2*e^4 + 3*a^3*e^6 + 6*(c^3*d^4*e^2 + 5*a*c^2*d^2*e^4)*x
^2 - 2*(c^3*d^5*e - 8*a*c^2*d^3*e^3 - 11*a^2*c*d*e^5)*x)/(a^3*c^4*d^9*e^3 -
 4*a^4*c^3*d^7*e^5 + 6*a^5*c^2*d^5*e^7 - 4*a^6*c*d^3*e^9 + a^7*d*e^11 + (c^
7*d^11*e - 4*a*c^6*d^9*e^3 + 6*a^2*c^5*d^7*e^5 - 4*a^3*c^4*d^5*e^7 + a^4*c^
3*d^3*e^9)*x^4 + (c^7*d^12 - a*c^6*d^10*e^2 - 6*a^2*c^5*d^8*e^4 + 14*a^3*c^
4*d^6*e^6 - 11*a^4*c^3*d^4*e^8 + 3*a^5*c^2*d^2*e^10)*x^3 + 3*(a*c^6*d^11*e
- 3*a^2*c^5*d^9*e^3 + 2*a^3*c^4*d^7*e^5 + 2*a^4*c^3*d^5*e^7 - 3*a^5*c^2*d^3
*e^9 + a^6*c*d*e^11)*x^2 + (3*a^2*c^5*d^10*e^2 - 11*a^3*c^4*d^8*e^4 + 14*a^
4*c^3*d^6*e^6 - 6*a^5*c^2*d^4*e^8 - a^6*c*d^2*e^10 + a^7*e^12)*x)
```

Fricas [B] time = 2.2465, size = 1669, normalized size = 9.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="frica
s")
```

```
[Out] -1/3*(c^4*d^8 - 6*a*c^3*d^6*e^2 + 18*a^2*c^2*d^4*e^4 - 10*a^3*c*d^2*e^6 - 3
*a^4*e^8 + 12*(c^4*d^5*e^3 - a*c^3*d^3*e^5)*x^3 + 6*(c^4*d^6*e^2 + 4*a*c^3*
d^4*e^4 - 5*a^2*c^2*d^2*e^6)*x^2 - 2*(c^4*d^7*e - 9*a*c^3*d^5*e^3 - 3*a^2*c
^2*d^3*e^5 + 11*a^3*c*d*e^7)*x + 12*(c^4*d^4*e^4*x^4 + a^3*c*d^2*e^6 + (c^4
*d^5*e^3 + 3*a*c^3*d^3*e^5)*x^3 + 3*(a*c^3*d^4*e^4 + a^2*c^2*d^2*e^6)*x^2 +
 (3*a^2*c^2*d^3*e^5 + a^3*c*d*e^7)*x)*log(c*d*x + a*e) - 12*(c^4*d^4*e^4*x^
4 + a^3*c*d^2*e^6 + (c^4*d^5*e^3 + 3*a*c^3*d^3*e^5)*x^3 + 3*(a*c^3*d^4*e^4
+ a^2*c^2*d^2*e^6)*x^2 + (3*a^2*c^2*d^3*e^5 + a^3*c*d*e^7)*x)*log(e*x + d))
/(a^3*c^5*d^11*e^3 - 5*a^4*c^4*d^9*e^5 + 10*a^5*c^3*d^7*e^7 - 10*a^6*c^2*d^
5*e^9 + 5*a^7*c*d^3*e^11 - a^8*d*e^13 + (c^8*d^13*e - 5*a*c^7*d^11*e^3 + 10
*a^2*c^6*d^9*e^5 - 10*a^3*c^5*d^7*e^7 + 5*a^4*c^4*d^5*e^9 - a^5*c^3*d^3*e^1
1)*x^4 + (c^8*d^14 - 2*a*c^7*d^12*e^2 - 5*a^2*c^6*d^10*e^4 + 20*a^3*c^5*d^8
*e^6 - 25*a^4*c^4*d^6*e^8 + 14*a^5*c^3*d^4*e^10 - 3*a^6*c^2*d^2*e^12)*x^3 +
 3*(a*c^7*d^13*e - 4*a^2*c^6*d^11*e^3 + 5*a^3*c^5*d^9*e^5 - 5*a^5*c^3*d^5*e
^9 + 4*a^6*c^2*d^3*e^11 - a^7*c*d*e^13)*x^2 + (3*a^2*c^6*d^12*e^2 - 14*a^3*
c^5*d^10*e^4 + 25*a^4*c^4*d^8*e^6 - 20*a^5*c^3*d^6*e^8 + 5*a^6*c^2*d^4*e^10
 + 2*a^7*c*d^2*e^12 - a^8*e^14)*x)
```

Sympy [B] time = 4.49505, size = 1005, normalized size = 5.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**4,x)
```

```
[Out] -4*c*d*e**3*log(x + (-4*a**6*c*d*e**15/(a*e**2 - c*d**2)**5 + 24*a**5*c**2*
d**3*e**13/(a*e**2 - c*d**2)**5 - 60*a**4*c**3*d**5*e**11/(a*e**2 - c*d**2)
**5 + 80*a**3*c**4*d**7*e**9/(a*e**2 - c*d**2)**5 - 60*a**2*c**5*d**9*e**7/
(a*e**2 - c*d**2)**5 + 24*a*c**6*d**11*e**5/(a*e**2 - c*d**2)**5 + 4*a*c*d*
e**5 - 4*c**7*d**13*e**3/(a*e**2 - c*d**2)**5 + 4*c**2*d**3*e**3)/(8*c**2*d
**2*e**4))/(a*e**2 - c*d**2)**5 + 4*c*d*e**3*log(x + (4*a**6*c*d*e**15/(a*e
**2 - c*d**2)**5 - 24*a**5*c**2*d**3*e**13/(a*e**2 - c*d**2)**5 + 60*a**4*c
**3*d**5*e**11/(a*e**2 - c*d**2)**5 - 80*a**3*c**4*d**7*e**9/(a*e**2 - c*d*
*2)**5 + 60*a**2*c**5*d**9*e**7/(a*e**2 - c*d**2)**5 - 24*a*c**6*d**11*e**5
/(a*e**2 - c*d**2)**5 + 4*a*c*d*e**5 + 4*c**7*d**13*e**3/(a*e**2 - c*d**2)*
*5 + 4*c**2*d**3*e**3)/(8*c**2*d**2*e**4))/(a*e**2 - c*d**2)**5 - (3*a**3*e
**6 + 13*a**2*c*d**2*e**4 - 5*a*c**2*d**4*e**2 + c**3*d**6 + 12*c**3*d**3*e
**3*x**3 + x**2*(30*a*c**2*d**2*e**4 + 6*c**3*d**4*e**2) + x*(22*a**2*c*d*e
**5 + 16*a*c**2*d**3*e**3 - 2*c**3*d**5*e))/(3*a**7*d*e**11 - 12*a**6*c*d**
3*e**9 + 18*a**5*c**2*d**5*e**7 - 12*a**4*c**3*d**7*e**5 + 3*a**3*c**4*d**9
*e**3 + x**4*(3*a**4*c**3*d**3*e**9 - 12*a**3*c**4*d**5*e**7 + 18*a**2*c**5
*d**7*e**5 - 12*a*c**6*d**9*e**3 + 3*c**7*d**11*e) + x**3*(9*a**5*c**2*d**2
*e**10 - 33*a**4*c**3*d**4*e**8 + 42*a**3*c**4*d**6*e**6 - 18*a**2*c**5*d**
8*e**4 - 3*a*c**6*d**10*e**2 + 3*c**7*d**12) + x**2*(9*a**6*c*d*e**11 - 27*
a**5*c**2*d**3*e**9 + 18*a**4*c**3*d**5*e**7 + 18*a**3*c**4*d**7*e**5 - 27*
```

```
a**2*c**5*d**9*e**3 + 9*a*c**6*d**11*e) + x*(3*a**7*e**12 - 3*a**6*c*d**2*e
**10 - 18*a**5*c**2*d**4*e**8 + 42*a**4*c**3*d**6*e**6 - 33*a**3*c**4*d**8*
e**4 + 9*a**2*c**5*d**10*e**2))
```

Giac [B] time = 1.28201, size = 907, normalized size = 5.24

$$\frac{8\left(c^{3}d^{5}e^{3}-2\,ac^{2}d^{3}e^{5}+a^{2}cde^{7}\right)\arctan\left(-\frac{2\,cdxe+cd^{2}+ae^{2}}{\sqrt{-c^{2}d^{4}+2\,acd^{2}e^{2}-a^{2}e^{4}}}\right)}{\left(c^{6}d^{12}-6\,ac^{5}d^{10}e^{2}+15\,a^{2}c^{4}d^{8}e^{4}-20\,a^{3}c^{3}d^{6}e^{6}+15\,a^{4}c^{2}d^{4}e^{8}-6\,a^{5}cd^{2}e^{10}+a^{6}e^{12}\right)\sqrt{-c^{2}d^{4}+2\,acd^{2}e^{2}-a^{2}e^{4}}}-\frac{12\,c^{5}d^{7}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="giac"
)
```

```
[Out] 8*(c^3*d^5*e^3 - 2*a*c^2*d^3*e^5 + a^2*c*d*e^7)*arctan(-(2*c*d*x*e + c*d^2
+ a*e^2)/sqrt(-c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4))/((c^6*d^12 - 6*a*c^5*d^1
0*e^2 + 15*a^2*c^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 + 15*a^4*c^2*d^4*e^8 - 6*a^
5*c*d^2*e^10 + a^6*e^12)*sqrt(-c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4)) - 1/3*(1
2*c^5*d^7*x^5*e^5 + 30*c^5*d^8*x^4*e^4 + 22*c^5*d^9*x^3*e^3 + 3*c^5*d^10*x^
2*e^2 + c^5*d^12 - 24*a*c^4*d^5*x^5*e^7 - 30*a*c^4*d^6*x^4*e^6 + 32*a*c^4*d
^7*x^3*e^5 + 51*a*c^4*d^8*x^2*e^4 + 6*a*c^4*d^9*x*e^3 - 7*a*c^4*d^10*e^2 +
12*a^2*c^3*d^3*x^5*e^9 - 30*a^2*c^3*d^4*x^4*e^8 - 108*a^2*c^3*d^5*x^3*e^7 -
 54*a^2*c^3*d^6*x^2*e^6 + 36*a^2*c^3*d^7*x*e^5 + 24*a^2*c^3*d^8*e^4 + 30*a^
3*c^2*d^2*x^4*e^10 + 32*a^3*c^2*d^3*x^3*e^9 - 54*a^3*c^2*d^4*x^2*e^8 - 84*a
^3*c^2*d^5*x*e^7 - 28*a^3*c^2*d^6*e^6 + 22*a^4*c*d*x^3*e^11 + 51*a^4*c*d^2*
x^2*e^10 + 36*a^4*c*d^3*x*e^9 + 7*a^4*c*d^4*e^8 + 3*a^5*x^2*e^12 + 6*a^5*d*
x*e^11 + 3*a^5*d^2*e^10)/((c^6*d^12 - 6*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4
 - 20*a^3*c^3*d^6*e^6 + 15*a^4*c^2*d^4*e^8 - 6*a^5*c*d^2*e^10 + a^6*e^12)*(
c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)^3)
```

3.1905 $\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx$

Optimal. Leaf size=226

$$-\frac{6c^2d^2e^2}{\left(cd^2-ae^2\right)^5(ae+cdx)}+\frac{3c^2d^2e}{2\left(cd^2-ae^2\right)^4(ae+cdx)^2}-\frac{c^2d^2}{3\left(cd^2-ae^2\right)^3(ae+cdx)^3}-\frac{10c^2d^2e^3\log(ae+cdx)}{\left(cd^2-ae^2\right)^6}+\frac{10c^2d^2e^3\log}{\left(cd^2-\right.}$$

```
[Out] -(c^2*d^2)/(3*(c*d^2 - a*e^2)^3*(a*e + c*d*x)^3) + (3*c^2*d^2*e)/(2*(c*d^2
- a*e^2)^4*(a*e + c*d*x)^2) - (6*c^2*d^2*e^2)/((c*d^2 - a*e^2)^5*(a*e + c*d
*x)) - e^3/(2*(c*d^2 - a*e^2)^4*(d + e*x)^2) - (4*c*d*e^3)/((c*d^2 - a*e^2)
^5*(d + e*x)) - (10*c^2*d^2*e^3*Log[a*e + c*d*x])/(c*d^2 - a*e^2)^6 + (10*c
^2*d^2*e^3*Log[d + e*x])/(c*d^2 - a*e^2)^6
```

Rubi [A] time = 0.207764, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {626, 44}

$$-\frac{6c^2d^2e^2}{\left(cd^2-ae^2\right)^5(ae+cdx)}+\frac{3c^2d^2e}{2\left(cd^2-ae^2\right)^4(ae+cdx)^2}-\frac{c^2d^2}{3\left(cd^2-ae^2\right)^3(ae+cdx)^3}-\frac{10c^2d^2e^3\log(ae+cdx)}{\left(cd^2-ae^2\right)^6}+\frac{10c^2d^2e^3\log}{\left(cd^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```


```
[Out] -(c^2*d^2)/(3*(c*d^2 - a*e^2)^3*(a*e + c*d*x)^3) + (3*c^2*d^2*e)/(2*(c*d^2
- a*e^2)^4*(a*e + c*d*x)^2) - (6*c^2*d^2*e^2)/((c*d^2 - a*e^2)^5*(a*e + c*d
*x)) - e^3/(2*(c*d^2 - a*e^2)^4*(d + e*x)^2) - (4*c*d*e^3)/((c*d^2 - a*e^2)
^5*(d + e*x)) - (10*c^2*d^2*e^3*Log[a*e + c*d*x])/(c*d^2 - a*e^2)^6 + (10*c
^2*d^2*e^3*Log[d + e*x])/(c*d^2 - a*e^2)^6
```


Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx=\int \frac{1}{(ae+cdx)^4(d+ex)^3}\,dx$$
$$=\int \left(\frac{c^3d^3}{\left(cd^2-ae^2\right)^3(ae+cdx)^4}-\frac{3c^3d^3e}{\left(cd^2-ae^2\right)^4(ae+cdx)^3}+\frac{6c^3d^3e^2}{\left(cd^2-ae^2\right)^5(ae+cd\right.$$
$$=-\frac{c^2d^2}{3\left(cd^2-ae^2\right)^3(ae+cdx)^3}+\frac{3c^2d^2e}{2\left(cd^2-ae^2\right)^4(ae+cdx)^2}-\frac{6c^2d^2e^2}{\left(cd^2-ae^2\right)^5(ae+c}$$

Mathematica [A] time = 0.193569, size = 206, normalized size = 0.91

$$\frac{\frac{36c^2d^2e^2\left(ae^2-cd^2\right)}{ae+cdx}+\frac{9c^2d^2e\left(cd^2-ae^2\right)^2}{(ae+cdx)^2}+\frac{2c^2d^2\left(ae^2-cd^2\right)^3}{(ae+cdx)^3}-60c^2d^2e^3\log(ae+cdx)+\frac{24cde^3\left(ae^2-cd^2\right)}{d+ex}-\frac{3e^3\left(cd^2-ae^2\right)^2}{(d+ex)^2}+60c^2d^2e^3\log(}{6\left(cd^2-ae^2\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

```
[Out] ((2*c^2*d^2*(-(c*d^2) + a*e^2)^3)/(a*e + c*d*x)^3 + (9*c^2*d^2*e*(c*d^2 - a
*e^2)^2)/(a*e + c*d*x)^2 + (36*c^2*d^2*e^2*(-(c*d^2) + a*e^2))/(a*e + c*d*x
) - (3*e^3*(c*d^2 - a*e^2)^2)/(d + e*x)^2 + (24*c*d*e^3*(-(c*d^2) + a*e^2))
/(d + e*x) - 60*c^2*d^2*e^3*Log[a*e + c*d*x] + 60*c^2*d^2*e^3*Log[d + e*x])
/(6*(c*d^2 - a*e^2)^6)
```

Maple [A] time = 0.056, size = 221, normalized size = 1.

$$-\frac{e^3}{2\left(ae^2-cd^2\right)^4(ex+d)^2}+10\frac{e^3c^2d^2\ln(ex+d)}{\left(ae^2-cd^2\right)^6}+4\frac{e^3cd}{\left(ae^2-cd^2\right)^5(ex+d)}+\frac{c^2d^2}{3\left(ae^2-cd^2\right)^3(cdx+ae)^3}-10\frac{e^3c^2d^2\ln}{\left(ae^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x)
```

```
[Out] -1/2*e^3/(a*e^2-c*d^2)^4/(e*x+d)^2+10*e^3/(a*e^2-c*d^2)^6*c^2*d^2*ln(e*x+d)
+4*e^3/(a*e^2-c*d^2)^5*c*d/(e*x+d)+1/3*c^2*d^2/(a*e^2-c*d^2)^3/(c*d*x+a*e)^
3-10*e^3/(a*e^2-c*d^2)^6*c^2*d^2*ln(c*d*x+a*e)+6*c^2*d^2/(a*e^2-c*d^2)^5*e^
2/(c*d*x+a*e)+3/2*c^2*d^2/(a*e^2-c*d^2)^4*e/(c*d*x+a*e)^2
```

Maxima [B] time = 1.28939, size = 1291, normalized size = 5.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="maxima"
)
```

```
[Out] -10*c^2*d^2*e^3*log(c*d*x + a*e)/(c^6*d^12 - 6*a*c^5*d^10*e^2 + 15*a^2*c^4*
d^8*e^4 - 20*a^3*c^3*d^6*e^6 + 15*a^4*c^2*d^4*e^8 - 6*a^5*c*d^2*e^10 + a^6*
```

```
e^12) + 10*c^2*d^2*e^3*log(e*x + d)/(c^6*d^12 - 6*a*c^5*d^10*e^2 + 15*a^2*c
^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 + 15*a^4*c^2*d^4*e^8 - 6*a^5*c*d^2*e^10 + a
^6*e^12) - 1/6*(60*c^4*d^4*e^4*x^4 + 2*c^4*d^8 - 13*a*c^3*d^6*e^2 + 47*a^2*
c^2*d^4*e^4 + 27*a^3*c*d^2*e^6 - 3*a^4*e^8 + 30*(3*c^4*d^5*e^3 + 5*a*c^3*d^
3*e^5)*x^3 + 10*(2*c^4*d^6*e^2 + 23*a*c^3*d^4*e^4 + 11*a^2*c^2*d^2*e^6)*x^2
 - 5*(c^4*d^7*e - 11*a*c^3*d^5*e^3 - 35*a^2*c^2*d^3*e^5 - 3*a^3*c*d*e^7)*x)
/(a^3*c^5*d^12*e^3 - 5*a^4*c^4*d^10*e^5 + 10*a^5*c^3*d^8*e^7 - 10*a^6*c^2*d
^6*e^9 + 5*a^7*c*d^4*e^11 - a^8*d^2*e^13 + (c^8*d^13*e^2 - 5*a*c^7*d^11*e^4
 + 10*a^2*c^6*d^9*e^6 - 10*a^3*c^5*d^7*e^8 + 5*a^4*c^4*d^5*e^10 - a^5*c^3*d
^3*e^12)*x^5 + (2*c^8*d^14*e - 7*a*c^7*d^12*e^3 + 5*a^2*c^6*d^10*e^5 + 10*a
^3*c^5*d^8*e^7 - 20*a^4*c^4*d^6*e^9 + 13*a^5*c^3*d^4*e^11 - 3*a^6*c^2*d^2*e
^13)*x^4 + (c^8*d^15 + a*c^7*d^13*e^2 - 17*a^2*c^6*d^11*e^4 + 35*a^3*c^5*d^
9*e^6 - 25*a^4*c^4*d^7*e^8 - a^5*c^3*d^5*e^10 + 9*a^6*c^2*d^3*e^12 - 3*a^7*
c*d*e^14)*x^3 + (3*a*c^7*d^14*e - 9*a^2*c^6*d^12*e^3 + a^3*c^5*d^10*e^5 + 2
5*a^4*c^4*d^8*e^7 - 35*a^5*c^3*d^6*e^9 + 17*a^6*c^2*d^4*e^11 - a^7*c*d^2*e^
13 - a^8*e^15)*x^2 + (3*a^2*c^6*d^13*e^2 - 13*a^3*c^5*d^11*e^4 + 20*a^4*c^4
*d^9*e^6 - 10*a^5*c^3*d^7*e^8 - 5*a^6*c^2*d^5*e^10 + 7*a^7*c*d^3*e^12 - 2*a
^8*d*e^14)*x)
```

Fricas [B] time = 2.37401, size = 2518, normalized size = 11.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="fricas"
)
```

```
[Out] -1/6*(2*c^5*d^10 - 15*a*c^4*d^8*e^2 + 60*a^2*c^3*d^6*e^4 - 20*a^3*c^2*d^4*e
^6 - 30*a^4*c*d^2*e^8 + 3*a^5*e^10 + 60*(c^5*d^6*e^4 - a*c^4*d^4*e^6)*x^4 +
 30*(3*c^5*d^7*e^3 + 2*a*c^4*d^5*e^5 - 5*a^2*c^3*d^3*e^7)*x^3 + 10*(2*c^5*d
^8*e^2 + 21*a*c^4*d^6*e^4 - 12*a^2*c^3*d^4*e^6 - 11*a^3*c^2*d^2*e^8)*x^2 -
5*(c^5*d^9*e - 12*a*c^4*d^7*e^3 - 24*a^2*c^3*d^5*e^5 + 32*a^3*c^2*d^3*e^7 +
 3*a^4*c*d*e^9)*x + 60*(c^5*d^5*e^5*x^5 + a^3*c^2*d^4*e^6 + (2*c^5*d^6*e^4
+ 3*a*c^4*d^4*e^6)*x^4 + (c^5*d^7*e^3 + 6*a*c^4*d^5*e^5 + 3*a^2*c^3*d^3*e^7
)*x^3 + (3*a*c^4*d^6*e^4 + 6*a^2*c^3*d^4*e^6 + a^3*c^2*d^2*e^8)*x^2 + (3*a^
2*c^3*d^5*e^5 + 2*a^3*c^2*d^3*e^7)*x)*log(c*d*x + a*e) - 60*(c^5*d^5*e^5*x^
5 + a^3*c^2*d^4*e^6 + (2*c^5*d^6*e^4 + 3*a*c^4*d^4*e^6)*x^4 + (c^5*d^7*e^3
+ 6*a*c^4*d^5*e^5 + 3*a^2*c^3*d^3*e^7)*x^3 + (3*a*c^4*d^6*e^4 + 6*a^2*c^3*d
^4*e^6 + a^3*c^2*d^2*e^8)*x^2 + (3*a^2*c^3*d^5*e^5 + 2*a^3*c^2*d^3*e^7)*x)*
log(e*x + d))/(a^3*c^6*d^14*e^3 - 6*a^4*c^5*d^12*e^5 + 15*a^5*c^4*d^10*e^7
- 20*a^6*c^3*d^8*e^9 + 15*a^7*c^2*d^6*e^11 - 6*a^8*c*d^4*e^13 + a^9*d^2*e^1
5 + (c^9*d^15*e^2 - 6*a*c^8*d^13*e^4 + 15*a^2*c^7*d^11*e^6 - 20*a^3*c^6*d^9
*e^8 + 15*a^4*c^5*d^7*e^10 - 6*a^5*c^4*d^5*e^12 + a^6*c^3*d^3*e^14)*x^5 + (
2*c^9*d^16*e - 9*a*c^8*d^14*e^3 + 12*a^2*c^7*d^12*e^5 + 5*a^3*c^6*d^10*e^7
- 30*a^4*c^5*d^8*e^9 + 33*a^5*c^4*d^6*e^11 - 16*a^6*c^3*d^4*e^13 + 3*a^7*c^
2*d^2*e^15)*x^4 + (c^9*d^17 - 18*a^2*c^7*d^13*e^4 + 52*a^3*c^6*d^11*e^6 - 6
0*a^4*c^5*d^9*e^8 + 24*a^5*c^4*d^7*e^10 + 10*a^6*c^3*d^5*e^12 - 12*a^7*c^2*
d^3*e^14 + 3*a^8*c*d*e^16)*x^3 + (3*a*c^8*d^16*e - 12*a^2*c^7*d^14*e^3 + 10
*a^3*c^6*d^12*e^5 + 24*a^4*c^5*d^10*e^7 - 60*a^5*c^4*d^8*e^9 + 52*a^6*c^3*d
^6*e^11 - 18*a^7*c^2*d^4*e^13 + a^9*e^17)*x^2 + (3*a^2*c^7*d^15*e^2 - 16*a^
3*c^6*d^13*e^4 + 33*a^4*c^5*d^11*e^6 - 30*a^5*c^4*d^9*e^8 + 5*a^6*c^3*d^7*e
^10 + 12*a^7*c^2*d^5*e^12 - 9*a^8*c*d^3*e^14 + 2*a^9*d*e^16)*x)
```

Sympy [B] time = 7.05294, size = 1363, normalized size = 6.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**4,x)
```

```
[Out] 10*c**2*d**2*e**3*log(x + (-10*a**7*c**2*d**2*e**17/(a*e**2 - c*d**2)**6 +
70*a**6*c**3*d**4*e**15/(a*e**2 - c*d**2)**6 - 210*a**5*c**4*d**6*e**13/(a*
e**2 - c*d**2)**6 + 350*a**4*c**5*d**8*e**11/(a*e**2 - c*d**2)**6 - 350*a**
3*c**6*d**10*e**9/(a*e**2 - c*d**2)**6 + 210*a**2*c**7*d**12*e**7/(a*e**2 -
 c*d**2)**6 - 70*a*c**8*d**14*e**5/(a*e**2 - c*d**2)**6 + 10*a*c**2*d**2*e*
*5 + 10*c**9*d**16*e**3/(a*e**2 - c*d**2)**6 + 10*c**3*d**4*e**3)/(20*c**3*
d**3*e**4))/(a*e**2 - c*d**2)**6 - 10*c**2*d**2*e**3*log(x + (10*a**7*c**2*
d**2*e**17/(a*e**2 - c*d**2)**6 - 70*a**6*c**3*d**4*e**15/(a*e**2 - c*d**2)
**6 + 210*a**5*c**4*d**6*e**13/(a*e**2 - c*d**2)**6 - 350*a**4*c**5*d**8*e*
*11/(a*e**2 - c*d**2)**6 + 350*a**3*c**6*d**10*e**9/(a*e**2 - c*d**2)**6 -
210*a**2*c**7*d**12*e**7/(a*e**2 - c*d**2)**6 + 70*a*c**8*d**14*e**5/(a*e**
2 - c*d**2)**6 + 10*a*c**2*d**2*e**5 - 10*c**9*d**16*e**3/(a*e**2 - c*d**2)
**6 + 10*c**3*d**4*e**3)/(20*c**3*d**3*e**4))/(a*e**2 - c*d**2)**6 + (-3*a*
*4*e**8 + 27*a**3*c*d**2*e**6 + 47*a**2*c**2*d**4*e**4 - 13*a*c**3*d**6*e**
2 + 2*c**4*d**8 + 60*c**4*d**4*e**4*x**4 + x**3*(150*a*c**3*d**3*e**5 + 90*
c**4*d**5*e**3) + x**2*(110*a**2*c**2*d**2*e**6 + 230*a*c**3*d**4*e**4 + 20
*c**4*d**6*e**2) + x*(15*a**3*c*d*e**7 + 175*a**2*c**2*d**3*e**5 + 55*a*c**
3*d**5*e**3 - 5*c**4*d**7*e))/(6*a**8*d**2*e**13 - 30*a**7*c*d**4*e**11 + 6
0*a**6*c**2*d**6*e**9 - 60*a**5*c**3*d**8*e**7 + 30*a**4*c**4*d**10*e**5 -
6*a**3*c**5*d**12*e**3 + x**5*(6*a**5*c**3*d**3*e**12 - 30*a**4*c**4*d**5*e
**10 + 60*a**3*c**5*d**7*e**8 - 60*a**2*c**6*d**9*e**6 + 30*a*c**7*d**11*e*
*4 - 6*c**8*d**13*e**2) + x**4*(18*a**6*c**2*d**2*e**13 - 78*a**5*c**3*d**4
*e**11 + 120*a**4*c**4*d**6*e**9 - 60*a**3*c**5*d**8*e**7 - 30*a**2*c**6*d*
*10*e**5 + 42*a*c**7*d**12*e**3 - 12*c**8*d**14*e) + x**3*(18*a**7*c*d*e**1
4 - 54*a**6*c**2*d**3*e**12 + 6*a**5*c**3*d**5*e**10 + 150*a**4*c**4*d**7*e
**8 - 210*a**3*c**5*d**9*e**6 + 102*a**2*c**6*d**11*e**4 - 6*a*c**7*d**13*e
**2 - 6*c**8*d**15) + x**2*(6*a**8*e**15 + 6*a**7*c*d**2*e**13 - 102*a**6*c
**2*d**4*e**11 + 210*a**5*c**3*d**6*e**9 - 150*a**4*c**4*d**8*e**7 - 6*a**3
*c**5*d**10*e**5 + 54*a**2*c**6*d**12*e**3 - 18*a*c**7*d**14*e) + x*(12*a**
8*d*e**14 - 42*a**7*c*d**3*e**12 + 30*a**6*c**2*d**5*e**10 + 60*a**5*c**3*d
**7*e**8 - 120*a**4*c**4*d**9*e**6 + 78*a**3*c**5*d**11*e**4 - 18*a**2*c**6
*d**13*e**2))
```

Giac [B] time = 1.23536, size = 791, normalized size = 3.5

$$-\frac{20\left(c^3d^4e^3 - ac^2d^2e^5\right)\arctan\left(\frac{2\,cdxe+cd^2+ae^2}{\sqrt{-c^2d^4+2\,acd^2e^2-a^2e^4}}\right)}{\left(c^6d^{12} - 6\,ac^5d^{10}e^2 + 15\,a^2c^4d^8e^4 - 20\,a^3c^3d^6e^6 + 15\,a^4c^2d^4e^8 - 6\,a^5cd^2e^{10} + a^6e^{12}\right)\sqrt{-c^2d^4+2\,acd^2e^2-a^2e^4}} - 60\,c^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="giac")
```

```
[Out] -20*(c^3*d^4*e^3 - a*c^2*d^2*e^5)*arctan((2*c*d*x*e + c*d^2 + a*e^2)/sqrt(-
c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4))/((c^6*d^12 - 6*a*c^5*d^10*e^2 + 15*a^2*
c^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 + 15*a^4*c^2*d^4*e^8 - 6*a^5*c*d^2*e^10 +
a^6*e^12)*sqrt(-c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4)) - 1/6*(60*c^5*d^6*x^5*e
^5 + 150*c^5*d^7*x^4*e^4 + 110*c^5*d^8*x^3*e^3 + 15*c^5*d^9*x^2*e^2 - 3*c^5
```

```
*d^10*x*e + 2*c^5*d^11 - 60*a*c^4*d^4*x^5*e^7 + 270*a*c^4*d^6*x^3*e^5 + 270
*a*c^4*d^7*x^2*e^4 + 45*a*c^4*d^8*x*e^3 - 15*a*c^4*d^9*e^2 - 150*a^2*c^3*d^
3*x^4*e^8 - 270*a^2*c^3*d^4*x^3*e^7 + 180*a^2*c^3*d^6*x*e^5 + 60*a^2*c^3*d^
7*e^4 - 110*a^3*c^2*d^2*x^3*e^9 - 270*a^3*c^2*d^3*x^2*e^8 - 180*a^3*c^2*d^4
*x*e^7 - 20*a^3*c^2*d^5*e^6 - 15*a^4*c*d*x^2*e^10 - 45*a^4*c*d^2*x*e^9 - 30
*a^4*c*d^3*e^8 + 3*a^5*x*e^11 + 3*a^5*d*e^10)/((c^6*d^12 - 6*a*c^5*d^10*e^2
 + 15*a^2*c^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 + 15*a^4*c^2*d^4*e^8 - 6*a^5*c*d
^2*e^10 + a^6*e^12)*(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)^3)
```

3.1906 $\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx$

Optimal. Leaf size=262

$$-\frac{10c^2d^2e^2\left(ae^2+cd^2+2cdex\right)}{\left(cd^2-ae^2\right)^6\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)}-\frac{20c^3d^3e^3\log(ae+cdx)}{\left(cd^2-ae^2\right)^7}+\frac{20c^3d^3e^3\log(d+ex)}{\left(cd^2-ae^2\right)^7}+\frac{5cde\left(ae^2+\right.}{3\left(cd^2-ae^2\right)^4\left(x\left(ae^2\right.\right.}$$

```
[Out] -(c*d^2 + a*e^2 + 2*c*d*e*x)/(3*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^3) + (5*c*d*e*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^2 - a*e^2
)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2) - (10*c^2*d^2*e^2*(c*d^2 + a
*e^2 + 2*c*d*e*x))/((c*d^2 - a*e^2)^6*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)) - (20*c^3*d^3*e^3*Log[a*e + c*d*x])/(c*d^2 - a*e^2)^7 + (20*c^3*d^3*e^3
*Log[d + e*x])/(c*d^2 - a*e^2)^7
```

Rubi [A] time = 0.103507, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {614, 616, 31}

$$-\frac{10c^2d^2e^2\left(ae^2+cd^2+2cdex\right)}{\left(cd^2-ae^2\right)^6\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)}-\frac{20c^3d^3e^3\log(ae+cdx)}{\left(cd^2-ae^2\right)^7}+\frac{20c^3d^3e^3\log(d+ex)}{\left(cd^2-ae^2\right)^7}+\frac{5cde\left(ae^2+\right.}{3\left(cd^2-ae^2\right)^4\left(x\left(ae^2\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-4),x]
```


```
[Out] -(c*d^2 + a*e^2 + 2*c*d*e*x)/(3*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^3) + (5*c*d*e*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^2 - a*e^2
)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2) - (10*c^2*d^2*e^2*(c*d^2 + a
*e^2 + 2*c*d*e*x))/((c*d^2 - a*e^2)^6*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)) - (20*c^3*d^3*e^3*Log[a*e + c*d*x])/(c*d^2 - a*e^2)^7 + (20*c^3*d^3*e^3
*Log[d + e*x])/(c*d^2 - a*e^2)^7
```


Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx = -\frac{cd^2+ae^2+2cdex}{3\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3} - \frac{(10cde)\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,d}{3\left(cd^2-ae^2\right)^2}$$

$$= -\frac{cd^2+ae^2+2cdex}{3\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3} + \frac{5cde\left(cd^2+ae^2+2cd\right.}{3\left(cd^2-ae^2\right)^4\left(ade+\left(cd^2+ae^2\right.\right.}$$

$$= -\frac{cd^2+ae^2+2cdex}{3\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3} + \frac{5cde\left(cd^2+ae^2+2cd\right.}{3\left(cd^2-ae^2\right)^4\left(ade+\left(cd^2+ae^2\right.\right.}$$

$$= -\frac{cd^2+ae^2+2cdex}{3\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3} + \frac{5cde\left(cd^2+ae^2+2cd\right.}{3\left(cd^2-ae^2\right)^4\left(ade+\left(cd^2+ae^2\right.\right.}$$

$$= -\frac{cd^2+ae^2+2cdex}{3\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3} + \frac{5cde\left(cd^2+ae^2+2cd\right.}{3\left(cd^2-ae^2\right)^4\left(ade+\left(cd^2+ae^2\right.\right.}$$

Mathematica [A] time = 0.261452, size = 234, normalized size = 0.89

$$\frac{\frac{30c^3d^3e^2\left(cd^2-ae^2\right)}{ae+cdx} - \frac{6c^3d^3e\left(cd^2-ae^2\right)^2}{(ae+cdx)^2} + \frac{c^3d^3\left(cd^2-ae^2\right)^3}{(ae+cdx)^3} + \frac{30c^2d^2e^3\left(cd^2-ae^2\right)}{d+ex} + 60c^3d^3e^3\log(ae+cdx) + \frac{6cde^3\left(cd^2-ae^2\right)^2}{(d+ex)^2} + \frac{\left(cd^2e-ae^3\right)^3}{(d+ex)^3} - 6}{3\left(ae^2-cd^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-4),x]
```

```
[Out] ((c^3*d^3*(c*d^2 - a*e^2)^3)/(a*e + c*d*x)^3 - (6*c^3*d^3*e*(c*d^2 - a*e^2)
^2)/(a*e + c*d*x)^2 + (30*c^3*d^3*e^2*(c*d^2 - a*e^2))/(a*e + c*d*x) + (c*d
^2*e - a*e^3)^3/(d + e*x)^3 + (6*c*d*e^3*(c*d^2 - a*e^2)^2)/(d + e*x)^2 + (
30*c^2*d^2*e^3*(c*d^2 - a*e^2))/(d + e*x) + 60*c^3*d^3*e^3*Log[a*e + c*d*x]
 - 60*c^3*d^3*e^3*Log[d + e*x])/(3*(-(c*d^2) + a*e^2)^7)
```

Maple [A] time = 0.055, size = 253, normalized size = 1.

$$-\frac{e^3}{3\left(ae^2-cd^2\right)^4(ex+d)^3} - 20\frac{e^3c^3d^3\ln(ex+d)}{\left(ae^2-cd^2\right)^7} - 10\frac{c^2d^2e^3}{\left(ae^2-cd^2\right)^6(ex+d)} + 2\frac{e^3cd}{\left(ae^2-cd^2\right)^5(ex+d)^2} - \frac{c^3d^3}{3\left(ae^2-cd^2\right)^4(c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x)
```

```
[Out] -1/3*e^3/(a*e^2-c*d^2)^4/(e*x+d)^3-20*e^3/(a*e^2-c*d^2)^7*c^3*d^3*ln(e*x+d)
-10*e^3/(a*e^2-c*d^2)^6*c^2*d^2/(e*x+d)+2*e^3/(a*e^2-c*d^2)^5*c*d/(e*x+d)^2
-1/3*c^3*d^3/(a*e^2-c*d^2)^4/(c*d*x+a*e)^3+20*e^3/(a*e^2-c*d^2)^7*c^3*d^3*l
n(c*d*x+a*e)-10*c^3*d^3/(a*e^2-c*d^2)^6*e^2/(c*d*x+a*e)-2*c^3*d^3/(a*e^2-c*
d^2)^5*e/(c*d*x+a*e)^2
```

Maxima [B] time = 1.23206, size = 1725, normalized size = 6.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="maxima")
```

```
[Out] -20*c^3*d^3*e^3*log(c*d*x + a*e)/(c^7*d^14 - 7*a*c^6*d^12*e^2 + 21*a^2*c^5*
d^10*e^4 - 35*a^3*c^4*d^8*e^6 + 35*a^4*c^3*d^6*e^8 - 21*a^5*c^2*d^4*e^10 +
7*a^6*c*d^2*e^12 - a^7*e^14) + 20*c^3*d^3*e^3*log(e*x + d)/(c^7*d^14 - 7*a*
c^6*d^12*e^2 + 21*a^2*c^5*d^10*e^4 - 35*a^3*c^4*d^8*e^6 + 35*a^4*c^3*d^6*e^
8 - 21*a^5*c^2*d^4*e^10 + 7*a^6*c*d^2*e^12 - a^7*e^14) - 1/3*(60*c^5*d^5*e^
5*x^5 + c^5*d^10 - 8*a*c^4*d^8*e^2 + 37*a^2*c^3*d^6*e^4 + 37*a^3*c^2*d^4*e^
6 - 8*a^4*c*d^2*e^8 + a^5*e^10 + 150*(c^5*d^6*e^4 + a*c^4*d^4*e^6)*x^4 + 10
*(11*c^5*d^7*e^3 + 38*a*c^4*d^5*e^5 + 11*a^2*c^3*d^3*e^7)*x^3 + 15*(c^5*d^8
*e^2 + 19*a*c^4*d^6*e^4 + 19*a^2*c^3*d^4*e^6 + a^3*c^2*d^2*e^8)*x^2 - 3*(c^
5*d^9*e - 14*a*c^4*d^7*e^3 - 74*a^2*c^3*d^5*e^5 - 14*a^3*c^2*d^3*e^7 + a^4*
c*d*e^9)*x)/(a^3*c^6*d^15*e^3 - 6*a^4*c^5*d^13*e^5 + 15*a^5*c^4*d^11*e^7 -
20*a^6*c^3*d^9*e^9 + 15*a^7*c^2*d^7*e^11 - 6*a^8*c*d^5*e^13 + a^9*d^3*e^15
+ (c^9*d^15*e^3 - 6*a*c^8*d^13*e^5 + 15*a^2*c^7*d^11*e^7 - 20*a^3*c^6*d^9*e
^9 + 15*a^4*c^5*d^7*e^11 - 6*a^5*c^4*d^5*e^13 + a^6*c^3*d^3*e^15)*x^6 + 3*(
c^9*d^16*e^2 - 5*a*c^8*d^14*e^4 + 9*a^2*c^7*d^12*e^6 - 5*a^3*c^6*d^10*e^8 -
 5*a^4*c^5*d^8*e^10 + 9*a^5*c^4*d^6*e^12 - 5*a^6*c^3*d^4*e^14 + a^7*c^2*d^2
*e^16)*x^5 + 3*(c^9*d^17*e - 3*a*c^8*d^15*e^3 - 2*a^2*c^7*d^13*e^5 + 19*a^3
*c^6*d^11*e^7 - 30*a^4*c^5*d^9*e^9 + 19*a^5*c^4*d^7*e^11 - 2*a^6*c^3*d^5*e^
13 - 3*a^7*c^2*d^3*e^15 + a^8*c*d*e^17)*x^4 + (c^9*d^18 + 3*a*c^8*d^16*e^2
- 30*a^2*c^7*d^14*e^4 + 62*a^3*c^6*d^12*e^6 - 36*a^4*c^5*d^10*e^8 - 36*a^5*
c^4*d^8*e^10 + 62*a^6*c^3*d^6*e^12 - 30*a^7*c^2*d^4*e^14 + 3*a^8*c*d^2*e^16
 + a^9*e^18)*x^3 + 3*(a*c^8*d^17*e - 3*a^2*c^7*d^15*e^3 - 2*a^3*c^6*d^13*e^
5 + 19*a^4*c^5*d^11*e^7 - 30*a^5*c^4*d^9*e^9 + 19*a^6*c^3*d^7*e^11 - 2*a^7*
c^2*d^5*e^13 - 3*a^8*c*d^3*e^15 + a^9*d*e^17)*x^2 + 3*(a^2*c^7*d^16*e^2 - 5
*a^3*c^6*d^14*e^4 + 9*a^4*c^5*d^12*e^6 - 5*a^5*c^4*d^10*e^8 - 5*a^6*c^3*d^8
*e^10 + 9*a^7*c^2*d^6*e^12 - 5*a^8*c*d^4*e^14 + a^9*d^2*e^16)*x)
```

Fricas [B] time = 2.2726, size = 3263, normalized size = 12.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(c^6*d^12 - 9*a*c^5*d^10*e^2 + 45*a^2*c^4*d^8*e^4 - 45*a^4*c^2*d^4*e^8
 + 9*a^5*c*d^2*e^10 - a^6*e^12 + 60*(c^6*d^7*e^5 - a*c^5*d^5*e^7)*x^5 + 150
*(c^6*d^8*e^4 - a^2*c^4*d^4*e^8)*x^4 + 10*(11*c^6*d^9*e^3 + 27*a*c^5*d^7*e^
5 - 27*a^2*c^4*d^5*e^7 - 11*a^3*c^3*d^3*e^9)*x^3 + 15*(c^6*d^10*e^2 + 18*a*
c^5*d^8*e^4 - 18*a^3*c^3*d^4*e^8 - a^4*c^2*d^2*e^10)*x^2 - 3*(c^6*d^11*e -
15*a*c^5*d^9*e^3 - 60*a^2*c^4*d^7*e^5 + 60*a^3*c^3*d^5*e^7 + 15*a^4*c^2*d^3
*e^9 - a^5*c*d*e^11)*x + 60*(c^6*d^6*e^6*x^6 + a^3*c^3*d^6*e^6 + 3*(c^6*d^7
*e^5 + a*c^5*d^5*e^7)*x^5 + 3*(c^6*d^8*e^4 + 3*a*c^5*d^6*e^6 + a^2*c^4*d^4*
e^8)*x^4 + (c^6*d^9*e^3 + 9*a*c^5*d^7*e^5 + 9*a^2*c^4*d^5*e^7 + a^3*c^3*d^3
*e^9)*x^3 + 3*(a*c^5*d^8*e^4 + 3*a^2*c^4*d^6*e^6 + a^3*c^3*d^4*e^8)*x^2 + 3
*(a^2*c^4*d^7*e^5 + a^3*c^3*d^5*e^7)*x)*log(c*d*x + a*e) - 60*(c^6*d^6*e^6*
x^6 + a^3*c^3*d^6*e^6 + 3*(c^6*d^7*e^5 + a*c^5*d^5*e^7)*x^5 + 3*(c^6*d^8*e^
4 + 3*a*c^5*d^6*e^6 + a^2*c^4*d^4*e^8)*x^4 + (c^6*d^9*e^3 + 9*a*c^5*d^7*e^5
 + 9*a^2*c^4*d^5*e^7 + a^3*c^3*d^3*e^9)*x^3 + 3*(a*c^5*d^8*e^4 + 3*a^2*c^4*
d^6*e^6 + a^3*c^3*d^4*e^8)*x^2 + 3*(a^2*c^4*d^7*e^5 + a^3*c^3*d^5*e^7)*x)*l
og(e*x + d))/(a^3*c^7*d^17*e^3 - 7*a^4*c^6*d^15*e^5 + 21*a^5*c^5*d^13*e^7 -
 35*a^6*c^4*d^11*e^9 + 35*a^7*c^3*d^9*e^11 - 21*a^8*c^2*d^7*e^13 + 7*a^9*c*
d^5*e^15 - a^10*d^3*e^17 + (c^10*d^17*e^3 - 7*a*c^9*d^15*e^5 + 21*a^2*c^8*d
```

```
^13*e^7 - 35*a^3*c^7*d^11*e^9 + 35*a^4*c^6*d^9*e^11 - 21*a^5*c^5*d^7*e^13 +
 7*a^6*c^4*d^5*e^15 - a^7*c^3*d^3*e^17)*x^6 + 3*(c^10*d^18*e^2 - 6*a*c^9*d^
16*e^4 + 14*a^2*c^8*d^14*e^6 - 14*a^3*c^7*d^12*e^8 + 14*a^5*c^5*d^8*e^12 -
14*a^6*c^4*d^6*e^14 + 6*a^7*c^3*d^4*e^16 - a^8*c^2*d^2*e^18)*x^5 + 3*(c^10*
d^19*e - 4*a*c^9*d^17*e^3 + a^2*c^8*d^15*e^5 + 21*a^3*c^7*d^13*e^7 - 49*a^4
*c^6*d^11*e^9 + 49*a^5*c^5*d^9*e^11 - 21*a^6*c^4*d^7*e^13 - a^7*c^3*d^5*e^1
5 + 4*a^8*c^2*d^3*e^17 - a^9*c*d*e^19)*x^4 + (c^10*d^20 + 2*a*c^9*d^18*e^2
- 33*a^2*c^8*d^16*e^4 + 92*a^3*c^7*d^14*e^6 - 98*a^4*c^6*d^12*e^8 + 98*a^6*
c^4*d^8*e^12 - 92*a^7*c^3*d^6*e^14 + 33*a^8*c^2*d^4*e^16 - 2*a^9*c*d^2*e^18
 - a^10*e^20)*x^3 + 3*(a*c^9*d^19*e - 4*a^2*c^8*d^17*e^3 + a^3*c^7*d^15*e^5
 + 21*a^4*c^6*d^13*e^7 - 49*a^5*c^5*d^11*e^9 + 49*a^6*c^4*d^9*e^11 - 21*a^7
*c^3*d^7*e^13 - a^8*c^2*d^5*e^15 + 4*a^9*c*d^3*e^17 - a^10*d*e^19)*x^2 + 3*
(a^2*c^8*d^18*e^2 - 6*a^3*c^7*d^16*e^4 + 14*a^4*c^6*d^14*e^6 - 14*a^5*c^5*d
^12*e^8 + 14*a^7*c^3*d^8*e^12 - 14*a^8*c^2*d^6*e^14 + 6*a^9*c*d^4*e^16 - a^
10*d^2*e^18)*x)
```

Sympy [B] time = 11.443, size = 1742, normalized size = 6.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**4,x)
```

```
[Out] -20*c**3*d**3*e**3*log(x + (-20*a**8*c**3*d**3*e**19/(a*e**2 - c*d**2)**7 +
 160*a**7*c**4*d**5*e**17/(a*e**2 - c*d**2)**7 - 560*a**6*c**5*d**7*e**15/(
a*e**2 - c*d**2)**7 + 1120*a**5*c**6*d**9*e**13/(a*e**2 - c*d**2)**7 - 1400
*a**4*c**7*d**11*e**11/(a*e**2 - c*d**2)**7 + 1120*a**3*c**8*d**13*e**9/(a*
e**2 - c*d**2)**7 - 560*a**2*c**9*d**15*e**7/(a*e**2 - c*d**2)**7 + 160*a*c
**10*d**17*e**5/(a*e**2 - c*d**2)**7 + 20*a*c**3*d**3*e**5 - 20*c**11*d**19
*e**3/(a*e**2 - c*d**2)**7 + 20*c**4*d**5*e**3)/(40*c**4*d**4*e**4))/(a*e**
2 - c*d**2)**7 + 20*c**3*d**3*e**3*log(x + (20*a**8*c**3*d**3*e**19/(a*e**2
 - c*d**2)**7 - 160*a**7*c**4*d**5*e**17/(a*e**2 - c*d**2)**7 + 560*a**6*c*
*5*d**7*e**15/(a*e**2 - c*d**2)**7 - 1120*a**5*c**6*d**9*e**13/(a*e**2 - c*
d**2)**7 + 1400*a**4*c**7*d**11*e**11/(a*e**2 - c*d**2)**7 - 1120*a**3*c**8
*d**13*e**9/(a*e**2 - c*d**2)**7 + 560*a**2*c**9*d**15*e**7/(a*e**2 - c*d**
2)**7 - 160*a*c**10*d**17*e**5/(a*e**2 - c*d**2)**7 + 20*a*c**3*d**3*e**5 +
 20*c**11*d**19*e**3/(a*e**2 - c*d**2)**7 + 20*c**4*d**5*e**3)/(40*c**4*d**
4*e**4))/(a*e**2 - c*d**2)**7 - (a**5*e**10 - 8*a**4*c*d**2*e**8 + 37*a**3*
c**2*d**4*e**6 + 37*a**2*c**3*d**6*e**4 - 8*a*c**4*d**8*e**2 + c**5*d**10 +
 60*c**5*d**5*e**5*x**5 + x**4*(150*a*c**4*d**4*e**6 + 150*c**5*d**6*e**4)
+ x**3*(110*a**2*c**3*d**3*e**7 + 380*a*c**4*d**5*e**5 + 110*c**5*d**7*e**3
) + x**2*(15*a**3*c**2*d**2*e**8 + 285*a**2*c**3*d**4*e**6 + 285*a*c**4*d**
6*e**4 + 15*c**5*d**8*e**2) + x*(-3*a**4*c*d*e**9 + 42*a**3*c**2*d**3*e**7
+ 222*a**2*c**3*d**5*e**5 + 42*a*c**4*d**7*e**3 - 3*c**5*d**9*e))/(3*a**9*d
**3*e**15 - 18*a**8*c*d**5*e**13 + 45*a**7*c**2*d**7*e**11 - 60*a**6*c**3*d
**9*e**9 + 45*a**5*c**4*d**11*e**7 - 18*a**4*c**5*d**13*e**5 + 3*a**3*c**6*
d**15*e**3 + x**6*(3*a**6*c**3*d**3*e**15 - 18*a**5*c**4*d**5*e**13 + 45*a*
*4*c**5*d**7*e**11 - 60*a**3*c**6*d**9*e**9 + 45*a**2*c**7*d**11*e**7 - 18*
a*c**8*d**13*e**5 + 3*c**9*d**15*e**3) + x**5*(9*a**7*c**2*d**2*e**16 - 45*
a**6*c**3*d**4*e**14 + 81*a**5*c**4*d**6*e**12 - 45*a**4*c**5*d**8*e**10 -
45*a**3*c**6*d**10*e**8 + 81*a**2*c**7*d**12*e**6 - 45*a*c**8*d**14*e**4 +
9*c**9*d**16*e**2) + x**4*(9*a**8*c*d*e**17 - 27*a**7*c**2*d**3*e**15 - 18*
a**6*c**3*d**5*e**13 + 171*a**5*c**4*d**7*e**11 - 270*a**4*c**5*d**9*e**9 +
 171*a**3*c**6*d**11*e**7 - 18*a**2*c**7*d**13*e**5 - 27*a*c**8*d**15*e**3
+ 9*c**9*d**17*e) + x**3*(3*a**9*e**18 + 9*a**8*c*d**2*e**16 - 90*a**7*c**2
*d**4*e**14 + 186*a**6*c**3*d**6*e**12 - 108*a**5*c**4*d**8*e**10 - 108*a**
```

```
4*c**5*d**10*e**8 + 186*a**3*c**6*d**12*e**6 - 90*a**2*c**7*d**14*e**4 + 9*
a*c**8*d**16*e**2 + 3*c**9*d**18) + x**2*(9*a**9*d*e**17 - 27*a**8*c*d**3*e
**15 - 18*a**7*c**2*d**5*e**13 + 171*a**6*c**3*d**7*e**11 - 270*a**5*c**4*d
**9*e**9 + 171*a**4*c**5*d**11*e**7 - 18*a**3*c**6*d**13*e**5 - 27*a**2*c**
7*d**15*e**3 + 9*a*c**8*d**17*e) + x*(9*a**9*d**2*e**16 - 45*a**8*c*d**4*e*
*14 + 81*a**7*c**2*d**6*e**12 - 45*a**6*c**3*d**8*e**10 - 45*a**5*c**4*d**1
0*e**8 + 81*a**4*c**5*d**12*e**6 - 45*a**3*c**6*d**14*e**4 + 9*a**2*c**7*d*
*16*e**2))
```

Giac [B] time = 1.21765, size = 718, normalized size = 2.74

$$-\frac{40\,c^3 d^3 \arctan\left(\frac{2\,cdxe+cd^2+ae^2}{\sqrt{-c^2d^4+2\,acd^2e^2-a^2e^4}}\right)e^3}{\left(c^6d^{12}-6\,ac^5d^{10}e^2+15\,a^2c^4d^8e^4-20\,a^3c^3d^6e^6+15\,a^4c^2d^4e^8-6\,a^5cd^2e^{10}+a^6e^{12}\right)\sqrt{-c^2d^4+2\,acd^2e^2-a^2e^4}}-\frac{60\,c^5 \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="giac")
```

```
[Out] -40*c^3*d^3*arctan((2*c*d*x*e + c*d^2 + a*e^2)/sqrt(-c^2*d^4 + 2*a*c*d^2*e^
2 - a^2*e^4))*e^3/((c^6*d^12 - 6*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 - 20*a
^3*c^3*d^6*e^6 + 15*a^4*c^2*d^4*e^8 - 6*a^5*c*d^2*e^10 + a^6*e^12)*sqrt(-c^
2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4)) - 1/3*(60*c^5*d^5*x^5*e^5 + 150*c^5*d^6*x
^4*e^4 + 110*c^5*d^7*x^3*e^3 + 15*c^5*d^8*x^2*e^2 - 3*c^5*d^9*x*e + c^5*d^1
0 + 150*a*c^4*d^4*x^4*e^6 + 380*a*c^4*d^5*x^3*e^5 + 285*a*c^4*d^6*x^2*e^4 +
 42*a*c^4*d^7*x*e^3 - 8*a*c^4*d^8*e^2 + 110*a^2*c^3*d^3*x^3*e^7 + 285*a^2*c
^3*d^4*x^2*e^6 + 222*a^2*c^3*d^5*x*e^5 + 37*a^2*c^3*d^6*e^4 + 15*a^3*c^2*d^
2*x^2*e^8 + 42*a^3*c^2*d^3*x*e^7 + 37*a^3*c^2*d^4*e^6 - 3*a^4*c*d*x*e^9 - 8
*a^4*c*d^2*e^8 + a^5*e^10)/((c^6*d^12 - 6*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e
^4 - 20*a^3*c^3*d^6*e^6 + 15*a^4*c^2*d^4*e^8 - 6*a^5*c*d^2*e^10 + a^6*e^12)
*(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)^3)
```

3.1907 $\int (d+ex)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal. Leaf size=388

$$\frac{21\left(cd^2-ae^2\right)^4\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{512c^5d^5e}+\frac{7\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{64c^4d^4}+21($$

```
[Out] (21*(c*d^2 - a*e^2)^4*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])/(512*c^5*d^5*e) + (7*(c*d^2 - a*e^2)^3*(a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2)^(3/2))/(64*c^4*d^4) + (21*(c*d^2 - a*e^2)^2*(d + e*
x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(160*c^3*d^3) + (3*(c*d^2
 - a*e^2)*(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(20*c^
2*d^2) + ((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(6*c*d
) - (21*(c*d^2 - a*e^2)^6*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sq
rt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(1024*c^(11/2)
*d^(11/2)*e^(3/2))
```

Rubi [A] time = 0.441113, antiderivative size = 388, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {670, 640, 612, 621, 206}

$$\frac{21\left(cd^2-ae^2\right)^4\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{512c^5d^5e}+\frac{7\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{64c^4d^4}+21($$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (21*(c*d^2 - a*e^2)^4*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])/(512*c^5*d^5*e) + (7*(c*d^2 - a*e^2)^3*(a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2)^(3/2))/(64*c^4*d^4) + (21*(c*d^2 - a*e^2)^2*(d + e*
x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(160*c^3*d^3) + (3*(c*d^2
 - a*e^2)*(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(20*c^
2*d^2) + ((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(6*c*d
) - (21*(c*d^2 - a*e^2)^6*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sq
rt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(1024*c^(11/2)
*d^(11/2)*e^(3/2))
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx = \frac{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6cd} + \frac{\left(3\left(d^2-\frac{ae^2}{c}\right)\right)\int (d+ex)^3\sqrt{}}{}$$

$$= \frac{3\left(cd^2-ae^2\right)(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{20c^2d^2} + \frac{(d+ex)^3\left(ad\right.}{}$$

$$= \frac{21\left(cd^2-ae^2\right)^2(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{160c^3d^3} + \frac{3\left(cd^2-ae^2\right.}{}$$

$$= \frac{7\left(cd^2-ae^2\right)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{64c^4d^4} + \frac{21\left(cd^2-ae^2\right)^2(d+e}{}$$

$$= \frac{21\left(cd^2-ae^2\right)^4\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{512c^5d^5e} + \frac{7(}{}$$

$$= \frac{21\left(cd^2-ae^2\right)^4\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{512c^5d^5e} + \frac{7(}{}$$

$$= \frac{21\left(cd^2-ae^2\right)^4\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{512c^5d^5e} + \frac{7(}{}$$

Mathematica [A] time = 2.46123, size = 320, normalized size = 0.82

$$\frac{(ae+cdx)\sqrt{(d+ex)(ae+cdx)}\left(1152c^7d^7(d+ex)^3\left(cd^2-ae^2\right)+1008c^6d^6(d+ex)^2\left(cd^2-ae^2\right)^2+840c^5d^5(d+ex)\left(cd^2\right.\right.}{7680c^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] ((a*e + c*d*x)*Sqrt[(a*e + c*d*x)*(d + e*x)]*(630*(c^2*d^3 - a*c*d*e^2)^4 +
 (315*c^4*d^4*(c*d^2 - a*e^2)^5)/(e*(a*e + c*d*x)) + 840*c^5*d^5*(c*d^2 - a
```

```
*e^2)^3*(d + e*x) + 1008*c^6*d^6*(c*d^2 - a*e^2)^2*(d + e*x)^2 + 1152*c^7*d
^7*(c*d^2 - a*e^2)*(d + e*x)^3 + 1280*c^8*d^8*(d + e*x)^4 - (315*c^(7/2)*d^
(7/2)*Sqrt[c*d]*(c*d^2 - a*e^2)^(11/2)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqr
t[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(e^(3/2)*(a*e + c*d*x)^(3
/2)*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])))/(7680*c^9*d^9)
```

Maple [B] time = 0.102, size = 1327, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] -63/512*e^7/d^3/c^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^4+149/160*e*d
/c*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-21/1024/e*d^7*c*ln((1/2*a*e^2+
1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(
d*e*c)^(1/2)+63/128*e^4/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^2+2
1/256*e^3*d/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2-63/512*e*d^3/c*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a+1/6*e^3*x^3*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(3/2)/d/c+21/256*e^5/d/c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)*a^3+21/512*e^9/d^5/c^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^5+63/
512*e*d^5*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a+147/320*e^4/d^2/c^3*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(3/2)*a^2-7/64*e^6/d^4/c^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(3/2)*a^3+21/512/e*d^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+21/256*d
^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+107/192*d^2/c*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(3/2)-237/320*e^2/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
3/2)*a+13/20*e^2/c*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-315/1024*e^3
*d^3/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^2-21/64*e^2*d^2/c*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)*x*a-3/20*e^4/d^2/c^2*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(3/2)*a+21/256*e^8/d^4/c^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^4
+105/256*e^5*d/c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^3+21/160*e^5/d^3/c^3*x*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^2-21/64*e^6/d^2/c^3*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)*x*a^3-21/1024*e^11/d^5/c^5*ln((1/2*a*e^2+1/2*c*d^2+c
*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2
)*a^6+63/512*e^9/d^3/c^4*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^5-9/16*e^3/d/c^2*x*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a-315/1024*e^7/d/c^3*ln((1/2*a*e^2+1/
2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*
e*c)^(1/2)*a^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.1066, size = 2280, normalized size = 5.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/30720*(315*(c^6*d^12 - 6*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 - 20*a^3*c^
3*d^6*e^6 + 15*a^4*c^2*d^4*e^8 - 6*a^5*c*d^2*e^10 + a^6*e^12)*sqrt(c*d*e)*l
og(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(
c^2*d^3*e + a*c*d*e^3)*x) + 4*(1280*c^6*d^6*e^6*x^5 + 315*c^6*d^11*e + 3335
*a*c^5*d^9*e^3 - 5058*a^2*c^4*d^7*e^5 + 4158*a^3*c^3*d^5*e^7 - 1785*a^4*c^2
*d^3*e^9 + 315*a^5*c*d*e^11 + 128*(49*c^6*d^7*e^5 + a*c^5*d^5*e^7)*x^4 + 16
*(759*c^6*d^8*e^4 + 50*a*c^5*d^6*e^6 - 9*a^2*c^4*d^4*e^8)*x^3 + 8*(1429*c^6
*d^9*e^3 + 267*a*c^5*d^7*e^5 - 117*a^2*c^4*d^5*e^7 + 21*a^3*c^3*d^3*e^9)*x^
2 + 2*(2455*c^6*d^10*e^2 + 1612*a*c^5*d^8*e^4 - 1350*a^2*c^4*d^6*e^6 + 588*
a^3*c^3*d^4*e^8 - 105*a^4*c^2*d^2*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x))/(c^6*d^6*e^2), 1/15360*(315*(c^6*d^12 - 6*a*c^5*d^10*e^2 + 15*
a^2*c^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 + 15*a^4*c^2*d^4*e^8 - 6*a^5*c*d^2*e^1
0 + a^6*e^12)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*
e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(1280*c^6*d^6*e^6*x^5 + 315*c^6*d^11*
e + 3335*a*c^5*d^9*e^3 - 5058*a^2*c^4*d^7*e^5 + 4158*a^3*c^3*d^5*e^7 - 1785
*a^4*c^2*d^3*e^9 + 315*a^5*c*d*e^11 + 128*(49*c^6*d^7*e^5 + a*c^5*d^5*e^7)*
x^4 + 16*(759*c^6*d^8*e^4 + 50*a*c^5*d^6*e^6 - 9*a^2*c^4*d^4*e^8)*x^3 + 8*(
1429*c^6*d^9*e^3 + 267*a*c^5*d^7*e^5 - 117*a^2*c^4*d^5*e^7 + 21*a^3*c^3*d^3
*e^9)*x^2 + 2*(2455*c^6*d^10*e^2 + 1612*a*c^5*d^8*e^4 - 1350*a^2*c^4*d^6*e^
6 + 588*a^3*c^3*d^4*e^8 - 105*a^4*c^2*d^2*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e +
 (c*d^2 + a*e^2)*x))/(c^6*d^6*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26479, size = 655, normalized size = 1.69

$$\frac{1}{7680}\sqrt{cdx^2e + cd^2x + axe^2 + ade}\Bigg(2\Bigg(4\Bigg(2\Bigg(8\Bigg(10\,xe^4 + \frac{\left(49\,c^5d^6e^8 + ac^4d^4e^{10}\right)e^{(-5)}}{c^5d^5}\Bigg)x + \frac{\left(759\,c^5d^7e^7 + 50\,ac^4d^5e^9 - 9\,a^2\right.}{c^5d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] 1/7680*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*(2*(4*(2*(8*(10*x*e^4 +
(49*c^5*d^6*e^8 + a*c^4*d^4*e^10)*e^(-5)/(c^5*d^5))*x + (759*c^5*d^7*e^7 +
50*a*c^4*d^5*e^9 - 9*a^2*c^3*d^3*e^11)*e^(-5)/(c^5*d^5))*x + (1429*c^5*d^8*
e^6 + 267*a*c^4*d^6*e^8 - 117*a^2*c^3*d^4*e^10 + 21*a^3*c^2*d^2*e^12)*e^(-5
)/(c^5*d^5))*x + (2455*c^5*d^9*e^5 + 1612*a*c^4*d^7*e^7 - 1350*a^2*c^3*d^5*
e^9 + 588*a^3*c^2*d^3*e^11 - 105*a^4*c*d*e^13)*e^(-5)/(c^5*d^5))*x + (315*c
^5*d^10*e^4 + 3335*a*c^4*d^8*e^6 - 5058*a^2*c^3*d^6*e^8 + 4158*a^3*c^2*d^4*
e^10 - 1785*a^4*c*d^2*e^12 + 315*a^5*e^14)*e^(-5)/(c^5*d^5)) + 21/1024*(c^6
*d^12 - 6*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 + 15*a^4
*c^2*d^4*e^8 - 6*a^5*c*d^2*e^10 + a^6*e^12)*sqrt(c*d)*e^(-3/2)*log(abs(-sqr
t(c*d)*c*d^2*e^(1/2) - 2*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x +
a*x*e^2 + a*d*e))*c*d*e - sqrt(c*d)*a*e^(5/2)))/(c^6*d^6)
```

3.1908 $\int (d+ex)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal. Leaf size=328

$$\frac{7\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128c^4d^4e}+\frac{7\left(cd^2-ae^2\right)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{48c^3d^3}+$$

```
[Out] (7*(c*d^2 - a*e^2)^3*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(128*c^4*d^4*e) + (7*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(3/2))/(48*c^3*d^3) + (7*(c*d^2 - a*e^2)*(d + e*x)*(
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(40*c^2*d^2) + ((d + e*x)^2*(
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*c*d) - (7*(c*d^2 - a*e^2)^
5*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(256*c^(9/2)*d^(9/2)*e^(3/2))
```

Rubi [A] time = 0.275629, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {670, 640, 612, 621, 206}

$$\frac{7\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128c^4d^4e}+\frac{7\left(cd^2-ae^2\right)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{48c^3d^3}+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```


```
[Out] (7*(c*d^2 - a*e^2)^3*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(128*c^4*d^4*e) + (7*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(3/2))/(48*c^3*d^3) + (7*(c*d^2 - a*e^2)*(d + e*x)*(
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(40*c^2*d^2) + ((d + e*x)^2*(
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*c*d) - (7*(c*d^2 - a*e^2)^
5*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(256*c^(9/2)*d^(9/2)*e^(3/2))
```


Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
```

```
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx = \frac{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5cd}+\frac{\left(7\left(d^2-\frac{ae^2}{c}\right)\right)\int (d+ex)^2\sqrt{ad\cdots}}{10\cdots}$$

$$=\frac{7\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{40c^2d^2}+\frac{(d+ex)^2\left(ade+\cdots\right.}{\cdots}$$

$$=\frac{7\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{48c^3d^3}+\frac{7\left(cd^2-ae^2\right)(d+ex)\left(ad\cdots\right.}{4\cdots}$$

$$=\frac{7\left(cd^2-ae^2\right)^3\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^4d^4e}+\frac{7\left(cd^2\cdots\right.}{\cdots}$$

$$=\frac{7\left(cd^2-ae^2\right)^3\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^4d^4e}+\frac{7\left(cd^2\cdots\right.}{\cdots}$$

$$=\frac{7\left(cd^2-ae^2\right)^3\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^4d^4e}+\frac{7\left(cd^2\cdots\right.}{\cdots}$$

Mathematica [A] time = 1.91636, size = 291, normalized size = 0.89

$$\frac{(ae+cdx)\sqrt{(d+ex)(ae+cdx)}\left(336c^5d^5(d+ex)^2\left(cd^2-ae^2\right)+280c^4d^4(d+ex)\left(cd^2-ae^2\right)^2+\frac{105c^3d^3\left(cd^2-ae^2\right)^4}{e(ae+cdx)}-\frac{105c^{5/2}d^{5/2}\cdots}{\cdots}\right.}{1920c^7d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] ((a*e + c*d*x)*Sqrt[(a*e + c*d*x)*(d + e*x)]*(210*(c^2*d^3 - a*c*d*e^2)^3 +
 (105*c^3*d^3*(c*d^2 - a*e^2)^4)/(e*(a*e + c*d*x)) + 280*c^4*d^4*(c*d^2 - a
*e^2)^2*(d + e*x) + 336*c^5*d^5*(c*d^2 - a*e^2)*(d + e*x)^2 + 384*c^6*d^6*(
d + e*x)^3 - (105*c^(5/2)*d^(5/2)*Sqrt[c*d]*(c*d^2 - a*e^2)^(9/2)*ArcSinh[(
Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])]
)/(e^(3/2)*(a*e + c*d*x)^(3/2)*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])))/(19
20*c^7*d^7)
```

Maple [B] time = 0.053, size = 968, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] -7/40*e^3/d^2/c^2*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a-7/64*e^6/d^3/
c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^3+21/64*e^4/d/c^2*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^2+7/256*e^9/d^4/c^4*ln((1/2*a*e^2+1/2*c*
d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)
^(1/2)*a^5-35/256*e^7/d^2/c^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2
)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^4-21/64*e^2*d/c*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a-35/128*e^3*d^2/c*ln((1/2*a*e^2+
1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(
d*e*c)^(1/2)*a^2+23/40*e/c*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+25/48*
d/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+7/128/e*d^4*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)+7/64*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x-7/6
4*e*d^2/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a-7/256/e*d^6*c*ln((1/2*a
*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2))/(d*e*c)^(1/2)+35/128*e^5/c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(
1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^3+1/5*e^2*x^2
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/d/c+35/256*e*d^4*ln((1/2*a*e^2+1/2
*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e
*c)^(1/2)*a-7/15*e^2/d/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a+7/48*e
^4/d^3/c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^2-7/128*e^7/d^4/c^4*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^4+7/64*e^5/d^2/c^3*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2)*a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.09149, size = 1813, normalized size = 5.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/7680*(105*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*
d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c
```

```
^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x
) + 4*(384*c^5*d^5*e^5*x^4 + 105*c^5*d^9*e + 790*a*c^4*d^7*e^3 - 896*a^2*c^
3*d^5*e^5 + 490*a^3*c^2*d^3*e^7 - 105*a^4*c*d*e^9 + 48*(31*c^5*d^6*e^4 + a*
c^4*d^4*e^6)*x^3 + 8*(263*c^5*d^7*e^3 + 32*a*c^4*d^5*e^5 - 7*a^2*c^3*d^3*e^
7)*x^2 + 2*(605*c^5*d^8*e^2 + 289*a*c^4*d^6*e^4 - 161*a^2*c^3*d^4*e^6 + 35*
a^3*c^2*d^2*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5*d^5*e
^2), 1/3840*(105*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*
c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)
/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(384*c^5*
d^5*e^5*x^4 + 105*c^5*d^9*e + 790*a*c^4*d^7*e^3 - 896*a^2*c^3*d^5*e^5 + 490
*a^3*c^2*d^3*e^7 - 105*a^4*c*d*e^9 + 48*(31*c^5*d^6*e^4 + a*c^4*d^4*e^6)*x^
3 + 8*(263*c^5*d^7*e^3 + 32*a*c^4*d^5*e^5 - 7*a^2*c^3*d^3*e^7)*x^2 + 2*(605
*c^5*d^8*e^2 + 289*a*c^4*d^6*e^4 - 161*a^2*c^3*d^4*e^6 + 35*a^3*c^2*d^2*e^8
)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5*d^5*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2647, size = 521, normalized size = 1.59

$$\frac{1}{1920}\sqrt{cdx^2e+cd^2x+axe^2+ade}\left(2\left(4\left(6\left(8\,xe^3+\frac{\left(31\,c^4d^5e^6+ac^3d^3e^8\right)e^{(-4)}}{c^4d^4}\right)x+\frac{\left(263\,c^4d^6e^5+32\,ac^3d^4e^7-7\,a^2c^2d^2e^9\right)e}{c^4d^4}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] 1/1920*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*(2*(4*(6*(8*x*e^3 + (31*
c^4*d^5*e^6 + a*c^3*d^3*e^8)*e^(-4)/(c^4*d^4))*x + (263*c^4*d^6*e^5 + 32*a*
c^3*d^4*e^7 - 7*a^2*c^2*d^2*e^9)*e^(-4)/(c^4*d^4))*x + (605*c^4*d^7*e^4 + 2
89*a*c^3*d^5*e^6 - 161*a^2*c^2*d^3*e^8 + 35*a^3*c*d*e^10)*e^(-4)/(c^4*d^4))
*x + (105*c^4*d^8*e^3 + 790*a*c^3*d^6*e^5 - 896*a^2*c^2*d^4*e^7 + 490*a^3*c
*d^2*e^9 - 105*a^4*e^11)*e^(-4)/(c^4*d^4)) + 7/256*(c^5*d^10 - 5*a*c^4*d^8*
e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)
*sqrt(c*d)*e^(-3/2)*log(abs(-sqrt(c*d)*c*d^2*e^(1/2) - 2*(sqrt(c*d)*x*e^(1/
2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*c*d*e - sqrt(c*d)*a*e^(5/
2)))/(c^5*d^5)
```

3.1909 $\int (d+ex)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$

Optimal. Leaf size=268

$$\frac{5\left(cd^2-ae^2\right)^2\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64c^3d^3e}+\frac{5\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24c^2d^2}-\cdots 5($$

```
[Out] (5*(c*d^2 - a*e^2)^2*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(64*c^3*d^3*e) + (5*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2)^(3/2))/(24*c^2*d^2) + ((d + e*x)*(a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2)^(3/2))/(4*c*d) - (5*(c*d^2 - a*e^2)^4*ArcTanh[(c*d^2 + a*
e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2])])/(128*c^(7/2)*d^(7/2)*e^(3/2))
```

Rubi [A] time = 0.18869, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {670, 640, 612, 621, 206}

$$\frac{5\left(cd^2-ae^2\right)^2\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64c^3d^3e}+\frac{5\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24c^2d^2}-\cdots 5($$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```


```
[Out] (5*(c*d^2 - a*e^2)^2*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(64*c^3*d^3*e) + (5*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2)^(3/2))/(24*c^2*d^2) + ((d + e*x)*(a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2)^(3/2))/(4*c*d) - (5*(c*d^2 - a*e^2)^4*ArcTanh[(c*d^2 + a*
e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2])])/(128*c^(7/2)*d^(7/2)*e^(3/2))
```


Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx = \frac{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4cd} + \frac{\left(5\left(d^2-\frac{ae^2}{c}\right)\right)\int (d+ex)\sqrt{ade \cdots}}{8d}$$

$$= \frac{5\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24c^2d^2} + \frac{(d+ex)\left(ade+\left(cd^2+ae^2 \cdots\right.\right.}{4cd}$$

$$= \frac{5\left(cd^2-ae^2\right)^2\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^3d^3e} + \frac{5\left(cd^2 \cdots\right.}{\cdots}$$

$$= \frac{5\left(cd^2-ae^2\right)^2\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^3d^3e} + \frac{5\left(cd^2 \cdots\right.}{\cdots}$$

$$= \frac{5\left(cd^2-ae^2\right)^2\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^3d^3e} + \frac{5\left(cd^2 \cdots\right.}{\cdots}$$

Mathematica [A] time = 0.950161, size = 316, normalized size = 1.18

$$\frac{\sqrt{cd}\left(\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{cd}(d+ex)\left(a^2c^2d^2e^3\left(73d^2-19dex-2e^2x^2\right)+5a^3cde^5(ex-11d)+15a^4e^7+ac^3d^3e\left(191d^2ex+15d^3+17\cdots\right.\right.\right.}{192c^{9/2}d^{9/2}e^3\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (Sqrt[c*d]*(Sqrt[c]*Sqrt[d]*Sqrt[c*d]*Sqrt[e]*(d + e*x)*(15*a^4*e^7 + 5*a^3
*c*d*e^5*(-11*d + e*x) + a^2*c^2*d^2*e^3*(73*d^2 - 19*d*e*x - 2*e^2*x^2) +
c^4*d^4*x*(15*d^3 + 118*d^2*e*x + 136*d*e^2*x^2 + 48*e^3*x^3) + a*c^3*d^3*e
*(15*d^3 + 191*d^2*e*x + 172*d*e^2*x^2 + 56*e^3*x^3)) - 15*(c*d^2 - a*e^2)^
(9/2)*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt
[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])]))/(
192*c^(9/2)*d^(9/2)*e^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [B] time = 0.05, size = 730, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] 11/24/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-5/16*e^2/c*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2)*x*a+5/64*e^5/d^3/c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)*a^3-5/64*e^3/d/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2-5/
64*e*d/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a-5/128/e*d^5*c*ln((1/2*a*
e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
))/(d*e*c)^(1/2)+5/32*e*a*d^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2
)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+1/4*e*x*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/d/c-5/24*e^2/d^2/c^2*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(3/2)*a+5/32*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+5/64/e
*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5/32*e^4/d^2/c^2*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^2-5/128*e^7/d^3/c^3*ln((1/2*a*e^2+1/2*c*d^2+
c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/
2)*a^4-15/64*e^3*d/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^2+5/32*e^5*a^3/c^2/d*ln((
1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2))/(d*e*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.84461, size = 1424, normalized size = 5.31

$$\left[\frac{15\left(c^4d^8 - 4\,ac^3d^6e^2 + 6\,a^2c^2d^4e^4 - 4\,a^3cd^2e^6 + a^4e^8\right)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2 + c^2d^4 + 6\,acd^2e^2 + a^2e^4 - 4\,\sqrt{cdex^2 + ade}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/768*(15*(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6
 + a^4*e^8)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a
^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 +
 a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(48*c^4*d^4*e^4*x^3
+ 15*c^4*d^7*e + 73*a*c^3*d^5*e^3 - 55*a^2*c^2*d^3*e^5 + 15*a^3*c*d*e^7 + 8
*(17*c^4*d^5*e^3 + a*c^3*d^3*e^5)*x^2 + 2*(59*c^4*d^6*e^2 + 18*a*c^3*d^4*e^
4 - 5*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4
*d^4*e^2), 1/384*(15*(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3
*c*d^2*e^6 + a^4*e^8)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d
^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 +
a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(48*c^4*d^4*e^4*x^3 + 15*c^4*
```

```
d^7*e + 73*a*c^3*d^5*e^3 - 55*a^2*c^2*d^3*e^5 + 15*a^3*c*d*e^7 + 8*(17*c^4*
d^5*e^3 + a*c^3*d^3*e^5)*x^2 + 2*(59*c^4*d^6*e^2 + 18*a*c^3*d^4*e^4 - 5*a^2
*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^4*e^2)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{(d+ex)(ae+cdx)}(d+ex)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)**2, x)
```

Giac [A] time = 1.24141, size = 402, normalized size = 1.5

$$\frac{1}{192}\sqrt{cdx^2e+cd^2x+axe^2+ade}\left(2\left(4\left(6\,xe^2+\frac{\left(17\,c^3d^4e^4+ac^2d^2e^6\right)e^{(-3)}}{c^3d^3}\right)x+\frac{\left(59\,c^3d^5e^3+18\,ac^2d^3e^5-5\,a^2cde^7\right)e^{(-3)}}{c^3d^3}\right)x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] 1/192*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*(2*(4*(6*x*e^2 + (17*c^3*
d^4*e^4 + a*c^2*d^2*e^6)*e^(-3)/(c^3*d^3))*x + (59*c^3*d^5*e^3 + 18*a*c^2*d
^3*e^5 - 5*a^2*c*d*e^7)*e^(-3)/(c^3*d^3))*x + (15*c^3*d^6*e^2 + 73*a*c^2*d^
4*e^4 - 55*a^2*c*d^2*e^6 + 15*a^3*e^8)*e^(-3)/(c^3*d^3)) + 5/128*(c^4*d^8 -
 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*sqrt(c*d)
*e^(-3/2)*log(abs(-sqrt(c*d)*c*d^2*e^(1/2) - 2*(sqrt(c*d)*x*e^(1/2) - sqrt(
c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*c*d*e - sqrt(c*d)*a*e^(5/2)))/(c^4*
d^4)
```

3.1910 $\int (d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal. Leaf size=210

$$\frac{\left(cd^2-ae^2\right)\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8c^2d^2e}-\frac{\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16c^{5/2}d^{5/2}e^{3/2}}+$$

```
[Out] ((c*d^2 - a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2])/(8*c^2*d^2*e) + (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2
)/(3*c*d) - ((c*d^2 - a*e^2)^3*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[
c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*c^(5/
2)*d^(5/2)*e^(3/2))
```

Rubi [A] time = 0.104133, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {640, 612, 621, 206}

$$\frac{\left(cd^2-ae^2\right)\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8c^2d^2e}-\frac{\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16c^{5/2}d^{5/2}e^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] ((c*d^2 - a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2])/(8*c^2*d^2*e) + (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2
)/(3*c*d) - ((c*d^2 - a*e^2)^3*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[
c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*c^(5/
2)*d^(5/2)*e^(3/2))
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\int (d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx = \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3cd}+\frac{\left(d^2-\frac{ae^2}{c}\right)\int\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}{2d}$$

$$=\frac{\left(cd^2-ae^2\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^2d^2e}+\frac{\left(ade+\left(cd\right.\right.}{}$$

$$=\frac{\left(cd^2-ae^2\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^2d^2e}+\frac{\left(ade+\left(cd\right.\right.}{}$$

$$=\frac{\left(cd^2-ae^2\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^2d^2e}+\frac{\left(ade+\left(cd\right.\right.}{}$$

Mathematica [A] time = 0.701341, size = 262, normalized size = 1.25

$$\frac{\sqrt{cd}\left(\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{cd}(d+ex)\left(a^2cde^3(8d-ex)-3a^3e^5+ac^2d^2e\left(3d^2+22dex+10e^2x^2\right)+c^3d^3x\left(3d^2+14dex+8e^2x^2\right)\right)-3\right.}{24c^{7/2}d^{7/2}e^{3/2}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (Sqrt[c*d]*(Sqrt[c]*Sqrt[d]*Sqrt[c*d]*Sqrt[e]*(d + e*x)*(-3*a^3*e^5 + a^2*c
*d*e^3*(8*d - e*x) + c^3*d^3*x*(3*d^2 + 14*d*e*x + 8*e^2*x^2) + a*c^2*d^2*e
*(3*d^2 + 22*d*e*x + 10*e^2*x^2)) - 3*(c*d^2 - a*e^2)^(7/2)*Sqrt[a*e + c*d*
x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*S
qrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])]))/(24*c^(7/2)*d^(7/2)*e^
(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [B] time = 0.05, size = 465, normalized size = 2.2

$$\frac{1}{3\,cd}\left(ade+\left(ae^2+cd^2\right)x+cdex^2\right)^{\frac{3}{2}}-\frac{ae^2x}{4\,cd}\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2}+\frac{dx}{4}\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2}-\frac{a^2e^3}{8\,c^2d^2}\sqrt{a\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] 1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d-1/4/d*e^2/c*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)*x*a+1/4*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*
x-1/8/d^2*e^3/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2+1/8*d^2/e*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/16/d^2*e^5/c^2*ln((1/2*a*e^2+1/2*c*d
^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^
(1/2)*a^3-3/16*e^3/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^2+3/16*e*d^2*ln((1/2*a*e^
2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))
/(d*e*c)^(1/2)*a-1/16*d^4/e*c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2
```

```
)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72906, size = 1123, normalized size = 5.35

$$\left[-\frac{3\left(c^3d^6 - 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 - a^3e^6\right)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2 + c^2d^4 + 6\,acd^2e^2 + a^2e^4 + 4\,\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right.}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="fri
cas")
```

```
[Out] [-1/96*(3*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(c*d*
e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) +
 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(8*c^3*d^3*e^3*x^2 + 3*c^3*d^5*e + 8*a*c^
2*d^3*e^3 - 3*a^2*c*d*e^5 + 2*(7*c^3*d^4*e^2 + a*c^2*d^2*e^4)*x)*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^2), 1/48*(3*(c^3*d^6 - 3*a*c^
2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c
^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(8*c^3*d^3*e
^3*x^2 + 3*c^3*d^5*e + 8*a*c^2*d^3*e^3 - 3*a^2*c*d*e^5 + 2*(7*c^3*d^4*e^2 +
 a*c^2*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^
2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{(d + ex)\,(ae + cdx)}\,(d + ex)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x), x)
```

Giac [A] time = 1.26302, size = 304, normalized size = 1.45

$$\frac{1}{24}\sqrt{cdx^2e+cd^2x+axe^2+ade}\left(2\left(4\,xe+\frac{\left(7\,c^2d^3e^2+acde^4\right)e^{(-2)}}{c^2d^2}\right)x+\frac{\left(3\,c^2d^4e+8\,acd^2e^3-3\,a^2e^5\right)e^{(-2)}}{c^2d^2}\right)+\frac{\left(c^3d^6-3\,ac^2d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="gia
c")
```

```
[Out] 1/24*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*(2*(4*x*e + (7*c^2*d^3*e^2
 + a*c*d*e^4)*e^(-2)/(c^2*d^2))*x + (3*c^2*d^4*e + 8*a*c*d^2*e^3 - 3*a^2*e^
5)*e^(-2)/(c^2*d^2)) + 1/16*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 -
a^3*e^6)*sqrt(c*d)*e^(-3/2)*log(abs(-sqrt(c*d)*c*d^2*e^(1/2) - 2*(sqrt(c*d)
*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*c*d*e - sqrt(c*d)
*a*e^(5/2)))/(c^3*d^3)
```

3.1911 $\int \sqrt{ade + (cd^2 + ae^2)x + cdex^2}\, dx$

Optimal. Leaf size=159

$$\frac{(ae^2 + cd^2 + 2cdex)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{4cde} - \frac{(cd^2 - ae^2)^2 \tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{3/2}}$$

```
[Out] ((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(
4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c
]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(3/2)
*d^(3/2)*e^(3/2))
```

Rubi [A] time = 0.0502793, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {612, 621, 206}

$$\frac{(ae^2 + cd^2 + 2cdex)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{4cde} - \frac{(cd^2 - ae^2)^2 \tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] ((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(
4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c
]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(3/2)
*d^(3/2)*e^(3/2))
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx = \frac{\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4cde} - \frac{\left(cd^2-ae^2\right)^2 \int \frac{1}{\sqrt{ade+\left(cd^2+ae^2\right)x+}}}{8cde}$$
$$= \frac{\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4cde} - \frac{\left(cd^2-ae^2\right)^2 \mathrm{Subst}\left(\int \frac{1}{4cde-x^2}\right.}{4cd}$$
$$= \frac{\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4cde} - \frac{\left(cd^2-ae^2\right)^2 \tanh^{-1}\left(\frac{c}{2\sqrt{c}\sqrt{d}\sqrt{e}}\right.}{8c^{3/2}d^{3/2}e^{3/}}$$

Mathematica [A] time = 0.487997, size = 214, normalized size = 1.35

$$\frac{\sqrt{c}\sqrt{d}\left(\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{cd}(d+ex)\left(a^2e^3+acde(d+3ex)+c^2d^2x(d+2ex)\right)-\left(cd^2-ae^2\right)^{5/2}\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}}{\sqrt{cd}\sqrt{c}}\right.\right.}{4e^{3/2}(cd)^{5/2}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[d]*(Sqrt[c]*Sqrt[d]*Sqrt[c*d]*Sqrt[e]*(d + e*x)*(a^2*e^3 + c^
2*d^2*x*(d + 2*e*x) + a*c*d*e*(d + 3*e*x)) - (c*d^2 - a*e^2)^(5/2)*Sqrt[a*e
 + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sq
rt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])]))/(4*(c*d)^(5/2)*
e^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] time = 0.044, size = 265, normalized size = 1.7

$$\frac{2\,cdex+ae^2+cd^2}{4\,dec}\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2}-\frac{a^2e^3}{8\,cd}\ln\left(\left(\frac{ae^2}{2}+\frac{cd^2}{2}+cdex\right)\frac{1}{\sqrt{dec}}+\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] 1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/e-1
/8/d*e^3/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^2+1/4*d*e*ln((1/2*a*e^2+1/2*c*d^2+c
*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2
)*a-1/8*d^3/e*c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.79373, size = 888, normalized size = 5.58

$$\left[\frac{\left(c^2d^4 - 2\,acd^2e^2 + a^2e^4\right)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2 + c^2d^4 + 6\,acd^2e^2 + a^2e^4 - 4\,\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\left(2\,cdex + cd^2\right.\right.}{16\,c^2d^2e^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^
2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e
^3)*x) + 4*(2*c^2*d^2*e^2*x + c^2*d^3*e + a*c*d*e^3)*sqrt(c*d*e*x^2 + a*d*e
 + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^2), 1/8*((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e
^4)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*
c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2
*d^3*e + a*c*d*e^3)*x)) + 2*(2*c^2*d^2*e^2*x + c^2*d^3*e + a*c*d*e^3)*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ade + cdex^2 + x\left(ae^2 + cd^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)), x)
```

Giac [A] time = 1.35217, size = 219, normalized size = 1.38

$$\frac{1}{4}\sqrt{cdx^2e + cd^2x + axe^2 + ade}\left(2\,x + \frac{\left(cd^2 + ae^2\right)e^{(-1)}}{cd}\right) + \frac{\left(c^2d^4 - 2\,acd^2e^2 + a^2e^4\right)\sqrt{cd}e^{\left(-\frac{3}{2}\right)}\log\left(\left|-\sqrt{cd}cd^2e^{\frac{1}{2}} - 2\left(\sqrt{cdx}\right.\right.\right.}{8\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*(2*x + (c*d^2 + a*e^2)*e^(-
1)/(c*d)) + 1/8*(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(c*d)*e^(-3/2)*log(
abs(-sqrt(c*d)*c*d^2*e^(1/2) - 2*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*
d^2*x + a*x*e^2 + a*d*e))*c*d*e - sqrt(c*d)*a*e^(5/2)))/(c^2*d^2)
```

3.1912 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx$

Optimal. Leaf size=131

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e}-\frac{\left(cd^2-ae^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}e^{3/2}}$$

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/e - ((c*d^2 - a*e^2)*ArcTanh[(c
*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqrt[d]*e^(3/2))
```

Rubi [A] time = 0.0672677, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {664, 621, 206}

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e}-\frac{\left(cd^2-ae^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x),x]
```


```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/e - ((c*d^2 - a*e^2)*ArcTanh[(c
*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqrt[d]*e^(3/2))
```


Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}dx = \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{e} - \frac{(2cd^2e-e(cd^2+ae^2))\int \frac{1}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}d}{2e^2}$$
$$= \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{e} - \frac{(2cd^2e-e(cd^2+ae^2))\,\text{Subst}\left(\int \frac{1}{4cde-x^2}dx, x,\right.}{e^2}$$
$$= \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{e} - \frac{(cd^2-ae^2)\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex}}\right.}{2\sqrt{c}\sqrt{d}e^{3/2}}$$

Mathematica [A] time = 0.786715, size = 155, normalized size = 1.18

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\sqrt{e}-\frac{c^{3/2}d^{3/2}\sqrt{cd^2-ae^2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{(cd)^{3/2}\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}\right)}{e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[e] - (c^(3/2)*d^(3/2)*Sqrt[c*d^2 - a*e
^2]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]])/(Sqrt[c*d]*Sqrt[c*d
^2 - a*e^2]])]/((c*d)^(3/2)*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 -
 a*e^2)])))/e^(3/2)
```

Maple [A] time = 0.045, size = 205, normalized size = 1.6

$$\frac{1}{e}\sqrt{cde\left(\frac{d}{e}+x\right)^2+(ae^2-cd^2)\left(\frac{d}{e}+x\right)}+\frac{ae}{2}\ln\left(\left(\frac{ae^2}{2}-\frac{cd^2}{2}+\left(\frac{d}{e}+x\right)cde\right)\frac{1}{\sqrt{dec}}+\sqrt{cde\left(\frac{d}{e}+x\right)^2+(ae^2-cd^2)\left(\frac{d}{e}+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x)
```

```
[Out] 1/e*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)+1/2*e*ln((1/2*a*e^2-1/2*c
*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(
1/2))/(d*e*c)^(1/2)*a-1/2/e*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^
(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)*c*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70254, size = 725, normalized size = 5.53

$$\left[\frac{4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}cde-\left(cd^2-ae^2\right)\sqrt{cde}\log\left(8c^2d^2e^2x^2+c^2d^4+6acd^2e^2+a^2e^4+4\sqrt{cdex^2+ade+\left(cd^2\right.}\right.}{4cde^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="fri
cas")
```

```
[Out] [1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*c*d*e - (c*d^2 - a*e^2)
*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqr
t(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x))/(c*d*e^2), 1/2*(2*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*c*d*e + (c*d^2 - a*e^2)*sqrt(-c*d*e)*arctan(1/2
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sq
rt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)))/(c
*d*e^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{(d+ex)\,(ae+cdx)}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError
```

3.1913 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=124

$$\frac{\sqrt{c}\sqrt{d}\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{e^{3/2}}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)}$$

```
[Out] (-2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e*(d + e*x)) + (Sqrt[c]*S
qrt[d]*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/e^(3/2)
```

Rubi [A] time = 0.054264, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {662, 621, 206}

$$\frac{\sqrt{c}\sqrt{d}\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{e^{3/2}}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^2,x]
```

```
[Out] (-2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e*(d + e*x)) + (Sqrt[c]*S
qrt[d]*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/e^(3/2)
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(d+ex)^2}\,dx = -\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e(d+ex)} + \frac{(cd)\int \frac{1}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{e}$$

$$= -\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e(d+ex)} + \frac{(2cd)\,\text{Subst}\left(\int \frac{1}{4cde-x^2}\,dx, x, \frac{cd^2+ae^2+2cdex}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{e}$$

$$= -\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e(d+ex)} + \frac{\sqrt{c}\sqrt{d}\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{e^{3/2}}$$

Mathematica [A] time = 0.692493, size = 164, normalized size = 1.32

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{cd}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{\sqrt{cd^2-ae^2}\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}-\frac{\sqrt{e}}{d+ex}\right)}{e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^2,x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(Sqrt[e]/(d + e*x)) + (Sqrt[c]*Sqrt[d]*S
qrt[c*d]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqr
t[c*d^2 - a*e^2])])/(Sqrt[c*d^2 - a*e^2]*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e
*x))/(c*d^2 - a*e^2)])))/e^(3/2)
```

Maple [B] time = 0.048, size = 317, normalized size = 2.6

$$-2\,\frac{1}{e^2\left(ae^2-cd^2\right)}\left(cde\left(\frac{d}{e}+x\right)^2+\left(ae^2-cd^2\right)\left(\frac{d}{e}+x\right)\right)^{3/2}\left(\frac{d}{e}+x\right)^{-2}+2\,\frac{cd}{e\left(ae^2-cd^2\right)}\sqrt{cde\left(\frac{d}{e}+x\right)^2+\left(ae^2-cd^2\right)\left(\frac{d}{e}+x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^2,x)
```

```
[Out] -2/e^2/(a*e^2-c*d^2)/(d/e+x)^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2
)+2/e*d*c/(a*e^2-c*d^2)*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)+e*d*c
/(a*e^2-c*d^2)*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*
(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)*a-1/e*d^3*c^2/(a*e^2-
c*d^2)*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^
2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^2,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.18697, size = 697, normalized size = 5.62

$$\left[\frac{(ex+d)\sqrt{\frac{cd}{e}}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4+4\left(2\,cde^2x+cd^2e+ae^3\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{\frac{cd}{e}}+8\right.}{2\left(e^2x+de\right)} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^2,x, algorithm="f
ricas")
```

```
[Out] [1/2*((e*x + d)*sqrt(c*d/e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2
 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x))/(e^2*x + d*e), -((e*x + d)*sqrt(-c*d/e)*ar
ctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a
*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) +
 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(e^2*x + d*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}}{(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**2,x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^2,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError
```

3.1914 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^3}\,dx$

Optimal. Leaf size=54

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3(d+ex)^3\left(cd^2-ae^2\right)}$$

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*(c*d^2 - a*e^2)*(d + e
*x)^3)
```

Rubi [A] time = 0.020352, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {650}

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3(d+ex)^3\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^3,x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*(c*d^2 - a*e^2)*(d + e
*x)^3)
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(d+ex)^3}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3\left(cd^2-ae^2\right)(d+ex)^3}$$

Mathematica [A] time = 0.0207341, size = 43, normalized size = 0.8

$$\frac{2((d+ex)(ae+cdx))^{3/2}}{3(d+ex)^3\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^3,x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2))/(3*(c*d^2 - a*e^2)*(d + e*x)^3)
```

Maple [A] time = 0.044, size = 58, normalized size = 1.1

$$-\frac{2\,cdx+2\,ae}{3\,\left(ex+d\right)^{2}\left(ae^{2}-cd^{2}\right)}\sqrt{cdex^{2}+ae^{2}x+cd^{2}x+ade}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^3,x)
```

```
[Out] -2/3*(c*d*x+a*e)/(e*x+d)^2/(a*e^2-c*d^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^
(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^3,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.57038, size = 182, normalized size = 3.37

$$\frac{2\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}(cdx+ae)}{3\left(cd^{4}-ad^{2}e^{2}+\left(cd^{2}e^{2}-ae^{4}\right)x^{2}+2\left(cd^{3}e-ade^{3}\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^3,x, algorithm="f
ricas")
```

```
[Out] 2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*x + a*e)/(c*d^4 - a*d^
2*e^2 + (c*d^2*e^2 - a*e^4)*x^2 + 2*(c*d^3*e - a*d*e^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^3,x, algorithm="g
iac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1915 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=111

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{15(d+ex)^3\left(cd^2-ae^2\right)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5(d+ex)^4\left(cd^2-ae^2\right)}$$

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*(c*d^2 - a*e^2)*(d + e
*x)^4) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(15*(c*d^2 -
 a*e^2)^2*(d + e*x)^3)
```

Rubi [A] time = 0.0469014, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {658, 650}

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{15(d+ex)^3\left(cd^2-ae^2\right)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5(d+ex)^4\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^4,x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*(c*d^2 - a*e^2)*(d + e
*x)^4) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(15*(c*d^2 -
 a*e^2)^2*(d + e*x)^3)
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(d+ex)^4}\,dx &= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5\left(cd^2-ae^2\right)(d+ex)^4}+\frac{(2cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(d+ex)^3}\,dx}{5\left(cd^2-ae^2\right)}\\ &= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5\left(cd^2-ae^2\right)(d+ex)^4}+\frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{15\left(cd^2-ae^2\right)^2(d+ex)^3}\end{aligned}$$

Mathematica [A] time = 0.0311292, size = 61, normalized size = 0.55

$$\frac{2((d+ex)(ae+cdx))^{3/2}\left(cd(5d+2ex)-3ae^2\right)}{15(d+ex)^4\left(cd^2-ae^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^4,x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-3*a*e^2 + c*d*(5*d + 2*e*x)))/(15*(c*d
^2 - a*e^2)^2*(d + e*x)^4)
```

Maple [A] time = 0.043, size = 90, normalized size = 0.8

$$-\frac{(2\,cdx+2\,ae)\left(-2\,cdex+3\,ae^2-5\,cd^2\right)}{15\,(ex+d)^3\left(a^2e^4-2\,acd^2e^2+c^2d^4\right)}\sqrt{cdex^2+ae^2x+cd^2x+ade}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^4,x)
```

```
[Out] -2/15*(c*d*x+a*e)*(-2*c*d*e*x+3*a*e^2-5*c*d^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a
*d*e)^(1/2)/(e*x+d)^3/(a^2*e^4-2*a*c*d^2*e^2+c^2*d^4)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^4,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.94177, size = 413, normalized size = 3.72

$$\frac{2\left(2\,c^2d^2ex^2+5\,acd^2e-3\,a^2e^3+\left(5\,c^2d^3-acde^2\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{15\left(c^2d^7-2\,acd^5e^2+a^2d^3e^4+\left(c^2d^4e^3-2\,acd^2e^5+a^2e^7\right)x^3+3\left(c^2d^5e^2-2\,acd^3e^4+a^2de^6\right)x^2+3\left(c^2d^6e-2\,acd^4e^3+a^2d\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^4,x, algorithm="f
ricas")
```

```
[Out] 2/15*(2*c^2*d^2*e*x^2 + 5*a*c*d^2*e - 3*a^2*e^3 + (5*c^2*d^3 - a*c*d*e^2)*x
)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(c^2*d^7 - 2*a*c*d^5*e^2 + a^
2*d^3*e^4 + (c^2*d^4*e^3 - 2*a*c*d^2*e^5 + a^2*e^7)*x^3 + 3*(c^2*d^5*e^2 -
2*a*c*d^3*e^4 + a^2*d*e^6)*x^2 + 3*(c^2*d^6*e - 2*a*c*d^4*e^3 + a^2*d^2*e^5
```

```
)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^4,x, algorithm="g
iac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1916 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(d+ex)^5}\,dx$

Optimal. Leaf size=171

$$\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{105(d+ex)^3\left(cd^2-ae^2\right)^3}+\frac{8cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{35(d+ex)^4\left(cd^2-ae^2\right)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7(d+ex)^5\left(cd^2-ae^2\right)}$$

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(7*(c*d^2 - a*e^2)*(d + e
*x)^5) + (8*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(35*(c*d^2 -
 a*e^2)^2*(d + e*x)^4) + (16*c^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2))/(105*(c*d^2 - a*e^2)^3*(d + e*x)^3)
```

Rubi [A] time = 0.080947, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {658, 650}

$$\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{105(d+ex)^3\left(cd^2-ae^2\right)^3}+\frac{8cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{35(d+ex)^4\left(cd^2-ae^2\right)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7(d+ex)^5\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^5,x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(7*(c*d^2 - a*e^2)*(d + e
*x)^5) + (8*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(35*(c*d^2 -
 a*e^2)^2*(d + e*x)^4) + (16*c^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2))/(105*(c*d^2 - a*e^2)^3*(d + e*x)^3)
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^5}dx = \frac{2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{7(cd^2-ae^2)(d+ex)^5} + \frac{(4cd)\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^4}dx}{7(cd^2-ae^2)}$$

$$= \frac{2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{7(cd^2-ae^2)(d+ex)^5} + \frac{8cd\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{35(cd^2-ae^2)^2(d+ex)^4} + \frac{(8c}{}$$

$$= \frac{2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{7(cd^2-ae^2)(d+ex)^5} + \frac{8cd\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{35(cd^2-ae^2)^2(d+ex)^4} + \frac{16c}{}$$

Mathematica [A] time = 0.0441684, size = 124, normalized size = 0.73

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(3a^2cde^3(ex-14d)+15a^3e^5+ac^2d^2e\left(35d^2-14dex-4e^2x^2\right)+c^3d^3x\left(35d^2+28dex+8e^2x^2\right)\right)}{105(d+ex)^4\left(cd^2-ae^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^5,x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(15*a^3*e^5 + 3*a^2*c*d*e^3*(-14*d + e*x)
+ a*c^2*d^2*e*(35*d^2 - 14*d*e*x - 4*e^2*x^2) + c^3*d^3*x*(35*d^2 + 28*d*e*
x + 8*e^2*x^2)))/(105*(c*d^2 - a*e^2)^3*(d + e*x)^4)
```

Maple [A] time = 0.045, size = 146, normalized size = 0.9

$$-\frac{(2\,cdx+2\,ae)\left(8\,c^2d^2e^2x^2-12\,acde^3x+28\,c^2d^3ex+15\,a^2e^4-42\,acd^2e^2+35\,c^2d^4\right)}{105\,(ex+d)^4\left(a^3e^6-3\,a^2cd^2e^4+3\,ac^2d^4e^2-c^3d^6\right)}\sqrt{cdex^2+ae^2x+cd^2x+ade}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^5,x)
```

```
[Out] -2/105*(c*d*x+a*e)*(8*c^2*d^2*e^2*x^2-12*a*c*d*e^3*x+28*c^2*d^3*e*x+15*a^2*
e^4-42*a*c*d^2*e^2+35*c^2*d^4)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/(e*x
+d)^4/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2-c^3*d^6)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^5,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 17.0962, size = 740, normalized size = 4.33

$$\frac{2\left(8\,c^3d^3e^2x^3+35\,ac^2d^4e-42\,a^2cd^2e^3+15\,a^3e^5+4\left(7\,c^3d^4e-ac^2d^2e^3\right)x^2+\left(\cdots\right.\right.}{105\left(c^3d^{10}-3\,ac^2d^8e^2+3\,a^2cd^6e^4-a^3d^4e^6+\left(c^3d^6e^4-3\,ac^2d^4e^6+3\,a^2cd^2e^8-a^3e^{10}\right)x^4+4\left(c^3d^7e^3-3\,ac^2d^5e^5+3\,a^2cd^3\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^5,x, algorithm="f
ricas")
```

```
[Out] 2/105*(8*c^3*d^3*e^2*x^3 + 35*a*c^2*d^4*e - 42*a^2*c*d^2*e^3 + 15*a^3*e^5 +
 4*(7*c^3*d^4*e - a*c^2*d^2*e^3)*x^2 + (35*c^3*d^5 - 14*a*c^2*d^3*e^2 + 3*a
^2*c*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(c^3*d^10 - 3*a*
c^2*d^8*e^2 + 3*a^2*c*d^6*e^4 - a^3*d^4*e^6 + (c^3*d^6*e^4 - 3*a*c^2*d^4*e^
6 + 3*a^2*c*d^2*e^8 - a^3*e^10)*x^4 + 4*(c^3*d^7*e^3 - 3*a*c^2*d^5*e^5 + 3*
a^2*c*d^3*e^7 - a^3*d*e^9)*x^3 + 6*(c^3*d^8*e^2 - 3*a*c^2*d^6*e^4 + 3*a^2*c
*d^4*e^6 - a^3*d^2*e^8)*x^2 + 4*(c^3*d^9*e - 3*a*c^2*d^7*e^3 + 3*a^2*c*d^5*
e^5 - a^3*d^3*e^7)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**5,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^5,x, algorithm="g
iac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1917 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(d+ex)^6}\,dx$

Optimal. Leaf size=231

$$\frac{32c^3d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{315(d+ex)^3\left(cd^2-ae^2\right)^4}+\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{105(d+ex)^4\left(cd^2-ae^2\right)^3}+\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)}{21(d+ex)^5\left(cd^2-ae^2\right)^2}$$

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(9*(c*d^2 - a*e^2)*(d + e
*x)^6) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(21*(c*d^2 -
 a*e^2)^2*(d + e*x)^5) + (16*c^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2))/(105*(c*d^2 - a*e^2)^3*(d + e*x)^4) + (32*c^3*d^3*(a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2)^(3/2))/(315*(c*d^2 - a*e^2)^4*(d + e*x)^3)
```

Rubi [A] time = 0.115105, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {658, 650}

$$\frac{32c^3d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{315(d+ex)^3\left(cd^2-ae^2\right)^4}+\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{105(d+ex)^4\left(cd^2-ae^2\right)^3}+\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)}{21(d+ex)^5\left(cd^2-ae^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^6,x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(9*(c*d^2 - a*e^2)*(d + e
*x)^6) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(21*(c*d^2 -
 a*e^2)^2*(d + e*x)^5) + (16*c^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2))/(105*(c*d^2 - a*e^2)^3*(d + e*x)^4) + (32*c^3*d^3*(a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2)^(3/2))/(315*(c*d^2 - a*e^2)^4*(d + e*x)^3)
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(d+ex)^6}dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{9\left(cd^2-ae^2\right)(d+ex)^6} + \frac{(2cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(d+ex)^5}dx}{3\left(cd^2-ae^2\right)}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{9\left(cd^2-ae^2\right)(d+ex)^6} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{21\left(cd^2-ae^2\right)^2(d+ex)^5} + \frac{\left(8c^2d^2\right.}{}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{9\left(cd^2-ae^2\right)(d+ex)^6} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{21\left(cd^2-ae^2\right)^2(d+ex)^5} + \frac{16c^2d^2}{}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{9\left(cd^2-ae^2\right)(d+ex)^6} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{21\left(cd^2-ae^2\right)^2(d+ex)^5} + \frac{16c^2d^2}{}$$

Mathematica [A] time = 0.0637766, size = 180, normalized size = 0.78

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(3a^2c^2d^2e^3\left(-63d^2+9dex+2e^2x^2\right)+5a^3cde^5(27d-ex)-35a^4e^7+ac^3d^3e\left(-63d^2ex+105d^3-36d\right.\right.}{315(d+ex)^5\left(cd^2-ae^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^6,x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-35*a^4*e^7 + 5*a^3*c*d*e^5*(27*d - e*x)
+ 3*a^2*c^2*d^2*e^3*(-63*d^2 + 9*d*e*x + 2*e^2*x^2) + a*c^3*d^3*e*(105*d^3
- 63*d^2*e*x - 36*d*e^2*x^2 - 8*e^3*x^3) + c^4*d^4*x*(105*d^3 + 126*d^2*e*x
 + 72*d*e^2*x^2 + 16*e^3*x^3)))/(315*(c*d^2 - a*e^2)^4*(d + e*x)^5)
```

Maple [A] time = 0.047, size = 217, normalized size = 0.9

$$-\frac{(2\,cdx+2\,ae)\left(-16\,c^3d^3e^3x^3+24\,ac^2d^2e^4x^2-72\,c^3d^4e^2x^2-30\,a^2cde^5x+108\,ac^2d^3e^3x-126\,c^3d^5ex+35\,a^3e^6-135\,a^2c\right.}{315\,(ex+d)^5\left(a^4e^8-4\,a^3cd^2e^6+6\,a^2c^2d^4e^4-4\,ac^3d^6e^2+c^4d^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^6,x)
```

```
[Out] -2/315*(c*d*x+a*e)*(-16*c^3*d^3*e^3*x^3+24*a*c^2*d^2*e^4*x^2-72*c^3*d^4*e^2
*x^2-30*a^2*c*d*e^5*x+108*a*c^2*d^3*e^3*x-126*c^3*d^5*e*x+35*a^3*e^6-135*a^
2*c*d^2*e^4+189*a*c^2*d^4*e^2-105*c^3*d^6)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e
)^(1/2)/(e*x+d)^5/(a^4*e^8-4*a^3*c*d^2*e^6+6*a^2*c^2*d^4*e^4-4*a*c^3*d^6*e^
2+c^4*d^8)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^6,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 68.5094, size = 1161, normalized size = 5.03

$$\frac{2\left(16\,c^4d^4e^3x^4+105\,ac^3d^6e-189\,a^2c^2d^4e^3\right.}{315\left(c^4d^{13}-4\,ac^3d^{11}e^2+6\,a^2c^2d^9e^4-4\,a^3cd^7e^6+a^4d^5e^8+\left(c^4d^8e^5-4\,ac^3d^6e^7+6\,a^2c^2d^4e^9-4\,a^3cd^2e^{11}+a^4e^{13}\right)x^5+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^6,x, algorithm="f
ricas")
```

```
[Out] 2/315*(16*c^4*d^4*e^3*x^4 + 105*a*c^3*d^6*e - 189*a^2*c^2*d^4*e^3 + 135*a^3
*c*d^2*e^5 - 35*a^4*e^7 + 8*(9*c^4*d^5*e^2 - a*c^3*d^3*e^4)*x^3 + 6*(21*c^4
*d^6*e - 6*a*c^3*d^4*e^3 + a^2*c^2*d^2*e^5)*x^2 + (105*c^4*d^7 - 63*a*c^3*d
^5*e^2 + 27*a^2*c^2*d^3*e^4 - 5*a^3*c*d*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x)/(c^4*d^13 - 4*a*c^3*d^11*e^2 + 6*a^2*c^2*d^9*e^4 - 4*a^3*c
*d^7*e^6 + a^4*d^5*e^8 + (c^4*d^8*e^5 - 4*a*c^3*d^6*e^7 + 6*a^2*c^2*d^4*e^9
 - 4*a^3*c*d^2*e^11 + a^4*e^13)*x^5 + 5*(c^4*d^9*e^4 - 4*a*c^3*d^7*e^6 + 6*
a^2*c^2*d^5*e^8 - 4*a^3*c*d^3*e^10 + a^4*d*e^12)*x^4 + 10*(c^4*d^10*e^3 - 4
*a*c^3*d^8*e^5 + 6*a^2*c^2*d^6*e^7 - 4*a^3*c*d^4*e^9 + a^4*d^2*e^11)*x^3 +
10*(c^4*d^11*e^2 - 4*a*c^3*d^9*e^4 + 6*a^2*c^2*d^7*e^6 - 4*a^3*c*d^5*e^8 +
a^4*d^3*e^10)*x^2 + 5*(c^4*d^12*e - 4*a*c^3*d^10*e^3 + 6*a^2*c^2*d^8*e^5 -
4*a^3*c*d^6*e^7 + a^4*d^4*e^9)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**6,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^6,x, algorithm="g
iac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1918 $\int (d+ex)^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} dx$

Optimal. Leaf size=461

$$-\frac{99\left(cd^2-ae^2\right)^6\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{16384c^6d^6e^2}+\frac{33\left(cd^2-ae^2\right)^4\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+\right.}{2048c^5d^5e}$$

```
[Out] (-99*(c*d^2 - a*e^2)^6*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])/(16384*c^6*d^6*e^2) + (33*(c*d^2 - a*e^2)^4*(c*d^2 + a
*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(2048*c^5*
d^5*e) + (33*(c*d^2 - a*e^2)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2
))/(640*c^4*d^4) + (33*(c*d^2 - a*e^2)^2*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(5/2))/(448*c^3*d^3) + (11*(c*d^2 - a*e^2)*(d + e*x)^2*(a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(112*c^2*d^2) + ((d + e*x)^3*(a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(8*c*d) + (99*(c*d^2 - a*e^2)^8
*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(32768*c^(13/2)*d^(13/2)*e^(5/2))
```

Rubi [A] time = 0.548807, antiderivative size = 461, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {670, 640, 612, 621, 206}

$$-\frac{99\left(cd^2-ae^2\right)^6\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{16384c^6d^6e^2}+\frac{33\left(cd^2-ae^2\right)^4\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+\right.}{2048c^5d^5e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```


```
[Out] (-99*(c*d^2 - a*e^2)^6*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])/(16384*c^6*d^6*e^2) + (33*(c*d^2 - a*e^2)^4*(c*d^2 + a
*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(2048*c^5*
d^5*e) + (33*(c*d^2 - a*e^2)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2
))/(640*c^4*d^4) + (33*(c*d^2 - a*e^2)^2*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(5/2))/(448*c^3*d^3) + (11*(c*d^2 - a*e^2)*(d + e*x)^2*(a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(112*c^2*d^2) + ((d + e*x)^3*(a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(8*c*d) + (99*(c*d^2 - a*e^2)^8
*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(32768*c^(13/2)*d^(13/2)*e^(5/2))
```


Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx=\frac{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8cd}+\frac{\left(11\left(d^2-\frac{ae^2}{c}\right)\right)\int(d+e\cdots}{\cdots}$$

$$=\frac{11\left(cd^2-ae^2\right)(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{112c^2d^2}+\frac{(d+ex)^3\cdots}{\cdots}$$

$$=\frac{33\left(cd^2-ae^2\right)^2(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{448c^3d^3}+\frac{11\left(cd^2-\cdots\right.}{\cdots}$$

$$=\frac{33\left(cd^2-ae^2\right)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{640c^4d^4}+\frac{33\left(cd^2-ae^2\right)^2(\cdots}{\cdots}$$

$$=\frac{33\left(cd^2-ae^2\right)^4\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2048c^5d^5e}\cdots$$

$$=-\frac{99\left(cd^2-ae^2\right)^6\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16384c^6d^6e^2}\cdots$$

$$=-\frac{99\left(cd^2-ae^2\right)^6\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16384c^6d^6e^2}\cdots$$

$$=-\frac{99\left(cd^2-ae^2\right)^6\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16384c^6d^6e^2}\cdots$$

Mathematica [B] time = 6.40913, size = 1276, normalized size = 2.77

$$\frac{2\left(cd^2-ae^2\right)^5(ae+cdx)((ae+cdx)(d+ex))^{3/2}\left(\frac{cde(ae+cdx)}{\left(cd^2-ae^2\right)\left(\frac{c^2d^3}{cd^2-ae^2}-\frac{acde^2}{cd^2-ae^2}\right)}+1\right)^{13/2}\left(\frac{5}{16}\left(\frac{1}{\frac{cde(ae+cdx)}{\left(cd^2-ae^2\right)\left(\frac{c^2d^3}{cd^2-ae^2}-\frac{acde^2}{cd^2-ae^2}\right)}+1}+\frac{}{14\left(\frac{cde}{\left(cd^2-ae^2\right)\left(\cdots\right)}\cdots\right.}\cdots\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (2*(c*d^2 - a*e^2)^5*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^(3/2)*(1 + (c*
d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2
)/(c*d^2 - a*e^2))))^(13/2)*((5*(33/(256*(1 + (c*d*e*(a*e + c*d*x))/((c*d^2
 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))))^6) +
33/(128*(1 + (c*d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e
^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))))^5) + 33/(80*(1 + (c*d*e*(a*e + c*d*x))
/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))
))^4) + 33/(56*(1 + (c*d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^
2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))))^3) + 11/(14*(1 + (c*d*e*(a*e +
c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 -
a*e^2))))^2) + (1 + (c*d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^
2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))))^(-1)))/16 - (495*(c*d^2 - a*e^2
)^3*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))^3*((2*c*d*e*(
a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*
d^2 - a*e^2))) - (4*c^2*d^2*e^2*(a*e + c*d*x)^2)/(3*(c*d^2 - a*e^2)^2*((c^2
*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))^2) - (2*Sqrt[c]*Sqrt[d
]*Sqrt[e]*Sqrt[a*e + c*d*x]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d
*x])/(Sqrt[c*d^2 - a*e^2]*Sqrt[(c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d
^2 - a*e^2)])])/(Sqrt[c*d^2 - a*e^2]*Sqrt[(c^2*d^3)/(c*d^2 - a*e^2) - (a*c*
d*e^2)/(c*d^2 - a*e^2)]*Sqrt[1 + (c*d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c
^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2)))])))/(65536*c^3*d^3*
e^3*(a*e + c*d*x)^3*(1 + (c*d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/
(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))))^6)))/(5*c^6*d^6*((c*d)/((c
^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2)))^(11/2)*(d + e*x)*Sq
rt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])
```

Maple [B] time = 0.061, size = 2065, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] 495/16384*d^6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a+33/1024*d^4*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x+223/640*d^2/c*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(5/2)+33/2048/e*d^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-99/2048
*e^7/d^3/c^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^4+33/1024*e^5/d/c^3*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^3+53/112*e^2/c*x^2*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(5/2)-1793/4480*e^2/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(5/2)*a+495/16384*e^6/c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^4-99
/16384/e^2*d^8*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-33/640*e^6/d^4/c^4
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*a^3+1/8*e^3*x^3*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(5/2)/d/c-99/8192/e*d^7*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)*x-99/16384*e^12/d^6/c^6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^7+
495/16384*e^10/d^4/c^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^6+297/4096
*e*d^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a+1023/4480*e^4/d^2/c^3*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*a^2+33/1024*e^3*d/c^2*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(3/2)*a^2-99/2048*e*d^3/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(3/2)*a+495/16384*e^4*d^2/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a
^3-891/16384*e^2*d^4/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2+33/2048*
e^9/d^5/c^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^5-99/4096*d^8*c*ln((1
/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2))/(d*e*c)^(1/2)*a-891/16384*e^8/d^2/c^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)*a^5+99/32768/e^2*d^10*c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*
c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+289/448*e*d
/c*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+693/8192*e^2*d^6*ln((1/2*a*e^2
+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/
(d*e*c)^(1/2)*a^2+99/512*e^4/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*
a^2-693/4096*e^8/c^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^5-693/4096*e^4*d^4/c*ln((
1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2))/(d*e*c)^(1/2)*a^3+297/4096*e^9/d^3/c^4*a^5*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2)*x-1485/8192*e^7/d/c^3*a^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)*x+33/1024*e^8/d^4/c^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*a^4
-33/256*e^6/d^2/c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*a^3-99/8192*e
^11/d^5/c^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^6+99/32768*e^14/d^6
/c^6*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^8-11/112*e^4/d^2/c^2*x^2*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(5/2)*a+495/2048*e^5*d/c^2*a^3*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)*x-99/4096*e^12/d^4/c^5*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*
c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^7+693/819
2*e^10/d^2/c^4*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^6+3465/16384*e^6*d^2/c^2*ln((1/
2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2))/(d*e*c)^(1/2)*a^4+33/448*e^5/d^3/c^3*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(5/2)*a^2-33/256*e^2*d^2/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*a
-11/32*e^3/d/c^2*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*a-1485/8192*e^3*
d^3/c*a^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.69937, size = 3494, normalized size = 7.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="f
ricas")
```

```
[Out] [1/2293760*(3465*(c^8*d^16 - 8*a*c^7*d^14*e^2 + 28*a^2*c^6*d^12*e^4 - 56*a^
3*c^5*d^10*e^6 + 70*a^4*c^4*d^8*e^8 - 56*a^5*c^3*d^6*e^10 + 28*a^6*c^2*d^4*
e^12 - 8*a^7*c*d^2*e^14 + a^8*e^16)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2
*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x)
+ 4*(71680*c^8*d^8*e^8*x^7 - 3465*c^8*d^15*e + 26565*a*c^7*d^13*e^3 + 14090
3*a^2*c^6*d^11*e^5 - 193699*a^3*c^5*d^9*e^7 + 166749*a^4*c^4*d^7*e^9 - 8847
3*a^5*c^3*d^5*e^11 + 26565*a^6*c^2*d^3*e^13 - 3465*a^7*c*d*e^15 + 5120*(81*
c^8*d^9*e^7 + 17*a*c^7*d^7*e^9)*x^6 + 1280*(769*c^8*d^10*e^6 + 406*a*c^7*d^
8*e^8 + a^2*c^6*d^6*e^10)*x^5 + 128*(9461*c^8*d^11*e^5 + 10067*a*c^7*d^9*e^
7 + 83*a^2*c^6*d^7*e^9 - 11*a^3*c^5*d^5*e^11)*x^4 + 16*(49251*c^8*d^12*e^4
+ 105748*a*c^7*d^10*e^6 + 2450*a^2*c^6*d^8*e^8 - 748*a^3*c^5*d^6*e^10 + 99*
a^4*c^4*d^4*e^12)*x^3 + 8*(28441*c^8*d^13*e^3 + 153301*a*c^7*d^11*e^5 + 106
42*a^2*c^6*d^9*e^7 - 5742*a^3*c^5*d^7*e^9 + 1749*a^4*c^4*d^5*e^11 - 231*a^5
*c^3*d^3*e^13)*x^2 + 2*(1155*c^8*d^14*e^2 + 220598*a*c^7*d^12*e^4 + 61709*a
^2*c^6*d^10*e^6 - 53900*a^3*c^5*d^8*e^8 + 28941*a^4*c^4*d^6*e^10 - 8778*a^5
*c^3*d^4*e^12 + 1155*a^6*c^2*d^2*e^14)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x))/(c^7*d^7*e^3), -1/1146880*(3465*(c^8*d^16 - 8*a*c^7*d^14*e^2 +
28*a^2*c^6*d^12*e^4 - 56*a^3*c^5*d^10*e^6 + 70*a^4*c^4*d^8*e^8 - 56*a^5*c^3
*d^6*e^10 + 28*a^6*c^2*d^4*e^12 - 8*a^7*c*d^2*e^14 + a^8*e^16)*sqrt(-c*d*e)
*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2
+ a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e
^3)*x)) - 2*(71680*c^8*d^8*e^8*x^7 - 3465*c^8*d^15*e + 26565*a*c^7*d^13*e^3
 + 140903*a^2*c^6*d^11*e^5 - 193699*a^3*c^5*d^9*e^7 + 166749*a^4*c^4*d^7*e^
9 - 88473*a^5*c^3*d^5*e^11 + 26565*a^6*c^2*d^3*e^13 - 3465*a^7*c*d*e^15 + 5
120*(81*c^8*d^9*e^7 + 17*a*c^7*d^7*e^9)*x^6 + 1280*(769*c^8*d^10*e^6 + 406*
a*c^7*d^8*e^8 + a^2*c^6*d^6*e^10)*x^5 + 128*(9461*c^8*d^11*e^5 + 10067*a*c^
7*d^9*e^7 + 83*a^2*c^6*d^7*e^9 - 11*a^3*c^5*d^5*e^11)*x^4 + 16*(49251*c^8*d
^12*e^4 + 105748*a*c^7*d^10*e^6 + 2450*a^2*c^6*d^8*e^8 - 748*a^3*c^5*d^6*e^
10 + 99*a^4*c^4*d^4*e^12)*x^3 + 8*(28441*c^8*d^13*e^3 + 153301*a*c^7*d^11*e
^5 + 10642*a^2*c^6*d^9*e^7 - 5742*a^3*c^5*d^7*e^9 + 1749*a^4*c^4*d^5*e^11 -
 231*a^5*c^3*d^3*e^13)*x^2 + 2*(1155*c^8*d^14*e^2 + 220598*a*c^7*d^12*e^4 +
 61709*a^2*c^6*d^10*e^6 - 53900*a^3*c^5*d^8*e^8 + 28941*a^4*c^4*d^6*e^10 -
8778*a^5*c^3*d^4*e^12 + 1155*a^6*c^2*d^2*e^14)*x)*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x))/(c^7*d^7*e^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37158, size = 992, normalized size = 2.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="g
iac")

[Out] 1/573440*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*(2*(4*(2*(8*(10*(4*(14
*c*d*x*e^5 + (81*c^8*d^9*e^11 + 17*a*c^7*d^7*e^13)*e^(-7)/(c^7*d^7))*x + (7
69*c^8*d^10*e^10 + 406*a*c^7*d^8*e^12 + a^2*c^6*d^6*e^14)*e^(-7)/(c^7*d^7))
*x + (9461*c^8*d^11*e^9 + 10067*a*c^7*d^9*e^11 + 83*a^2*c^6*d^7*e^13 - 11*a
^3*c^5*d^5*e^15)*e^(-7)/(c^7*d^7))*x + (49251*c^8*d^12*e^8 + 105748*a*c^7*d
^10*e^10 + 2450*a^2*c^6*d^8*e^12 - 748*a^3*c^5*d^6*e^14 + 99*a^4*c^4*d^4*e^
16)*e^(-7)/(c^7*d^7))*x + (28441*c^8*d^13*e^7 + 153301*a*c^7*d^11*e^9 + 106
42*a^2*c^6*d^9*e^11 - 5742*a^3*c^5*d^7*e^13 + 1749*a^4*c^4*d^5*e^15 - 231*a
^5*c^3*d^3*e^17)*e^(-7)/(c^7*d^7))*x + (1155*c^8*d^14*e^6 + 220598*a*c^7*d^
12*e^8 + 61709*a^2*c^6*d^10*e^10 - 53900*a^3*c^5*d^8*e^12 + 28941*a^4*c^4*d
^6*e^14 - 8778*a^5*c^3*d^4*e^16 + 1155*a^6*c^2*d^2*e^18)*e^(-7)/(c^7*d^7))*
x - (3465*c^8*d^15*e^5 - 26565*a*c^7*d^13*e^7 - 140903*a^2*c^6*d^11*e^9 + 1
93699*a^3*c^5*d^9*e^11 - 166749*a^4*c^4*d^7*e^13 + 88473*a^5*c^3*d^5*e^15 -
 26565*a^6*c^2*d^3*e^17 + 3465*a^7*c*d*e^19)*e^(-7)/(c^7*d^7)) - 99/32768*(
c^8*d^16 - 8*a*c^7*d^14*e^2 + 28*a^2*c^6*d^12*e^4 - 56*a^3*c^5*d^10*e^6 + 7
0*a^4*c^4*d^8*e^8 - 56*a^5*c^3*d^6*e^10 + 28*a^6*c^2*d^4*e^12 - 8*a^7*c*d^2
*e^14 + a^8*e^16)*sqrt(c*d)*e^(-5/2)*log(abs(-sqrt(c*d)*c*d^2*e^(1/2) - 2*(
sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*c*d*e -
sqrt(c*d)*a*e^(5/2)))/(c^7*d^7)
```

3.1919 $\int (d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} dx$

Optimal. Leaf size=401

$$-\frac{9\left(cd^2-ae^2\right)^5\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{1024c^5d^5e^2}+\frac{3\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+a\right.}{128c^4d^4e}$$

```
[Out] (-9*(c*d^2 - a*e^2)^5*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])/(1024*c^5*d^5*e^2) + (3*(c*d^2 - a*e^2)^3*(c*d^2 + a*e^
2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(128*c^4*d^4*
e) + (3*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(4
0*c^3*d^3) + (3*(c*d^2 - a*e^2)*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(5/2))/(28*c^2*d^2) + ((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(5/2))/(7*c*d) + (9*(c*d^2 - a*e^2)^7*ArcTanh[(c*d^2 + a*e^2 + 2*c*d
*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])])/(2048*c^(11/2)*d^(11/2)*e^(5/2))
```

Rubi [A] time = 0.372587, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {670, 640, 612, 621, 206}

$$-\frac{9\left(cd^2-ae^2\right)^5\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{1024c^5d^5e^2}+\frac{3\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+a\right.}{128c^4d^4e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```


```
[Out] (-9*(c*d^2 - a*e^2)^5*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])/(1024*c^5*d^5*e^2) + (3*(c*d^2 - a*e^2)^3*(c*d^2 + a*e^
2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(128*c^4*d^4*
e) + (3*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(4
0*c^3*d^3) + (3*(c*d^2 - a*e^2)*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(5/2))/(28*c^2*d^2) + ((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(5/2))/(7*c*d) + (9*(c*d^2 - a*e^2)^7*ArcTanh[(c*d^2 + a*e^2 + 2*c*d
*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])])/(2048*c^(11/2)*d^(11/2)*e^(5/2))
```


Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}dx=\frac{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7cd}+\frac{\left(9\left(d^2-\frac{ae^2}{c}\right)\right)\int(d+ex$$

$$=\frac{3\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{28c^2d^2}+\frac{(d+ex)^2\,(\ldots}{}$$

$$=\frac{3\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{40c^3d^3}+\frac{3\left(cd^2-ae^2\right)(d+\ldots}{}$$

$$=\frac{3\left(cd^2-ae^2\right)^3\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{128c^4d^4e}\ldots$$

$$=-\frac{9\left(cd^2-ae^2\right)^5\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{1024c^5d^5e^2}+\ldots$$

$$=-\frac{9\left(cd^2-ae^2\right)^5\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{1024c^5d^5e^2}+\ldots$$

$$=-\frac{9\left(cd^2-ae^2\right)^5\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{1024c^5d^5e^2}+\ldots$$

Mathematica [B] time = 6.25364, size = 1196, normalized size = 2.98

$$\frac{2\left(cd^2-ae^2\right)^4(ae+cdx)((ae+cdx)(d+ex))^{3/2}\left(\frac{cde(ae+cdx)}{\left(cd^2-ae^2\right)\left(\frac{c^2d^3}{cd^2-ae^2}-\frac{acde^2}{cd^2-ae^2}\right)}+1\right)^{11/2}\left(\frac{5}{14}\left(\frac{1}{\frac{cde(ae+cdx)}{\left(cd^2-ae^2\right)\left(\frac{c^2d^3}{cd^2-ae^2}-\frac{acde^2}{cd^2-ae^2}\right)}+1}+\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (2*(c*d^2 - a*e^2)^4*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^(3/2)*(1 + (c*
d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2
)/(c*d^2 - a*e^2))))^(11/2)*((5*(21/(128*(1 + (c*d*e*(a*e + c*d*x))/((c*d^2
 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))))^5) +
21/(64*(1 + (c*d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^
2) - (a*c*d*e^2)/(c*d^2 - a*e^2))))^4) + 21/(40*(1 + (c*d*e*(a*e + c*d*x))/
((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2)))
)^3) + 3/(4*(1 + (c*d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 -
 a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))))^2) + (1 + (c*d*e*(a*e + c*d*x))/((
c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))))^
(-1)))/14 - (45*(c*d^2 - a*e^2)^3*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/
(c*d^2 - a*e^2))^3*((2*c*d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*
d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))) - (4*c^2*d^2*e^2*(a*e + c*d*x)
^2)/(3*(c*d^2 - a*e^2)^2*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 -
a*e^2))^2) - (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*ArcSinh[(Sqrt[c]*
Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d^2 - a*e^2]*Sqrt[(c^2*d^3)/(c*d
^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2)])])/(Sqrt[c*d^2 - a*e^2]*Sqrt[(c^
2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2)]*Sqrt[1 + (c*d*e*(a*e
+ c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2
- a*e^2)))])))/(4096*c^3*d^3*e^3*(a*e + c*d*x)^3*(1 + (c*d*e*(a*e + c*d*x))
/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))
)^5)))/(5*c^5*d^5*((c*d)/((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 -
 a*e^2)))^(9/2)*(d + e*x)*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])
```

Maple [B] time = 0.057, size = 1586, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] 3/64*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x+3/128/e*d^4*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(3/2)+13/40*d/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
```

```
2)+9/256*d^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a+9/2048/e^2*d^9*c^2*l
n((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2))/(d*e*c)^(1/2)-9/35*e^2/d/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(5/2)*a-3/64*e*d^2/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a+45/256*e^5/c
^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^3+189/2048*e^2*d^5*ln((1/2*a
*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2))/(d*e*c)^(1/2)*a^2+45/512*e*d^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*
x*a-63/2048*d^7*c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a-9/1024/e^2*d^7*c*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)+11/28*e/c*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(5/2)-3/128*e^7/d^4/c^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^4-9/256*e
^8/d^3/c^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^5+9/1024*e^10/d^5/c^5*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^6+1/7*e^2*x^2*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(5/2)/d/c-45/1024*e^2*d^3/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)*a^2+45/1024*e^6/d/c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^4+3
/40*e^4/d^3/c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*a^2-9/512/e*d^6*c*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+3/64*e^5/d^2/c^3*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(3/2)*a^3-3/64*e^6/d^3/c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(3/2)*x*a^3+9/64*e^4/d/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*a^2
-189/2048*e^8/d/c^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^5+315/2048*e^6*d/c^2*ln((1
/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2))/(d*e*c)^(1/2)*a^4-3/28*e^3/d^2/c^2*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(5/2)*a+9/512*e^9/d^4/c^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^5
-45/512*e^7/d^2/c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^4-9/64*e^2*
d/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*a-315/2048*e^4*d^3/c*ln((1/2*
a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2))/(d*e*c)^(1/2)*a^3-9/2048*e^12/d^5/c^5*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)
/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^7+6
3/2048*e^10/d^3/c^4*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^6-45/256*e^3*d^2/c*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.4543, size = 2804, normalized size = 6.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="f
ricas")
```

```
[Out] [1/143360*(315*(c^7*d^14 - 7*a*c^6*d^12*e^2 + 21*a^2*c^5*d^10*e^4 - 35*a^3*
c^4*d^8*e^6 + 35*a^4*c^3*d^6*e^8 - 21*a^5*c^2*d^4*e^10 + 7*a^6*c*d^2*e^12 -
```

```
a^7*e^14)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^
2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 +
a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(5120*c^7*d^7*e^7*x^6
 - 315*c^7*d^13*e + 2100*a*c^6*d^11*e^3 + 8393*a^2*c^5*d^9*e^5 - 9216*a^3*c
^4*d^7*e^7 + 5943*a^4*c^3*d^5*e^9 - 2100*a^5*c^2*d^3*e^11 + 315*a^6*c*d*e^1
3 + 1280*(19*c^7*d^8*e^6 + 5*a*c^6*d^6*e^8)*x^5 + 128*(351*c^7*d^9*e^5 + 24
8*a*c^6*d^7*e^7 + a^2*c^5*d^5*e^9)*x^4 + 16*(2441*c^7*d^10*e^4 + 3909*a*c^6
*d^8*e^6 + 59*a^2*c^5*d^6*e^8 - 9*a^3*c^4*d^4*e^10)*x^3 + 8*(1771*c^7*d^11*
e^3 + 7562*a*c^6*d^9*e^5 + 384*a^2*c^5*d^7*e^7 - 138*a^3*c^4*d^5*e^9 + 21*a
^4*c^3*d^3*e^11)*x^2 + 2*(105*c^7*d^12*e^2 + 13643*a*c^6*d^10*e^4 + 2962*a^
2*c^5*d^8*e^6 - 1938*a^3*c^4*d^6*e^8 + 693*a^4*c^3*d^4*e^10 - 105*a^5*c^2*d
^2*e^12)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^6*d^6*e^3), -1/
71680*(315*(c^7*d^14 - 7*a*c^6*d^12*e^2 + 21*a^2*c^5*d^10*e^4 - 35*a^3*c^4*
d^8*e^6 + 35*a^4*c^3*d^6*e^8 - 21*a^5*c^2*d^4*e^10 + 7*a^6*c*d^2*e^12 - a^7
*e^14)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (
c^2*d^3*e + a*c*d*e^3)*x)) - 2*(5120*c^7*d^7*e^7*x^6 - 315*c^7*d^13*e + 210
0*a*c^6*d^11*e^3 + 8393*a^2*c^5*d^9*e^5 - 9216*a^3*c^4*d^7*e^7 + 5943*a^4*c
^3*d^5*e^9 - 2100*a^5*c^2*d^3*e^11 + 315*a^6*c*d*e^13 + 1280*(19*c^7*d^8*e^
6 + 5*a*c^6*d^6*e^8)*x^5 + 128*(351*c^7*d^9*e^5 + 248*a*c^6*d^7*e^7 + a^2*c
^5*d^5*e^9)*x^4 + 16*(2441*c^7*d^10*e^4 + 3909*a*c^6*d^8*e^6 + 59*a^2*c^5*d
^6*e^8 - 9*a^3*c^4*d^4*e^10)*x^3 + 8*(1771*c^7*d^11*e^3 + 7562*a*c^6*d^9*e^
5 + 384*a^2*c^5*d^7*e^7 - 138*a^3*c^4*d^5*e^9 + 21*a^4*c^3*d^3*e^11)*x^2 +
2*(105*c^7*d^12*e^2 + 13643*a*c^6*d^10*e^4 + 2962*a^2*c^5*d^8*e^6 - 1938*a^
3*c^4*d^6*e^8 + 693*a^4*c^3*d^4*e^10 - 105*a^5*c^2*d^2*e^12)*x)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^6*d^6*e^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)

[Out] Timed out
```

Giac [A] time = 1.31157, size = 824, normalized size = 2.05

$$\frac{1}{35840}\sqrt{cdx^2e+cd^2x+axe^2+ade}\left(2\left(4\left(2\left(8\left(10\left(4\,cdxe^4+\frac{\left(19\,c^7d^8e^9+5\,ac^6d^6e^{11}\right)e^{(-6)}}{c^6d^6}\right)x+\frac{\left(351\,c^7d^9e^8+248\,ac^6d^7e^{10}\right.}{c^6d^6}\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="g
iac")

[Out] 1/35840*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*(2*(4*(2*(8*(10*(4*c*d*
x*e^4 + (19*c^7*d^8*e^9 + 5*a*c^6*d^6*e^11)*e^(-6)/(c^6*d^6))*x + (351*c^7*
d^9*e^8 + 248*a*c^6*d^7*e^10 + a^2*c^5*d^5*e^12)*e^(-6)/(c^6*d^6))*x + (244
1*c^7*d^10*e^7 + 3909*a*c^6*d^8*e^9 + 59*a^2*c^5*d^6*e^11 - 9*a^3*c^4*d^4*e
^13)*e^(-6)/(c^6*d^6))*x + (1771*c^7*d^11*e^6 + 7562*a*c^6*d^9*e^8 + 384*a^
2*c^5*d^7*e^10 - 138*a^3*c^4*d^5*e^12 + 21*a^4*c^3*d^3*e^14)*e^(-6)/(c^6*d^
```

```
6))*x + (105*c^7*d^12*e^5 + 13643*a*c^6*d^10*e^7 + 2962*a^2*c^5*d^8*e^9 - 1
938*a^3*c^4*d^6*e^11 + 693*a^4*c^3*d^4*e^13 - 105*a^5*c^2*d^2*e^15)*e^(-6)/
(c^6*d^6))*x - (315*c^7*d^13*e^4 - 2100*a*c^6*d^11*e^6 - 8393*a^2*c^5*d^9*e
^8 + 9216*a^3*c^4*d^7*e^10 - 5943*a^4*c^3*d^5*e^12 + 2100*a^5*c^2*d^3*e^14
- 315*a^6*c*d*e^16)*e^(-6)/(c^6*d^6)) - 9/2048*(c^7*d^14 - 7*a*c^6*d^12*e^2
 + 21*a^2*c^5*d^10*e^4 - 35*a^3*c^4*d^8*e^6 + 35*a^4*c^3*d^6*e^8 - 21*a^5*c
^2*d^4*e^10 + 7*a^6*c*d^2*e^12 - a^7*e^14)*sqrt(c*d)*e^(-5/2)*log(abs(-sqrt
(c*d)*c*d^2*e^(1/2) - 2*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a
*x*e^2 + a*d*e))*c*d*e - sqrt(c*d)*a*e^(5/2)))/(c^6*d^6)
```

3.1920 $\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} dx$

Optimal. Leaf size=341

$$-\frac{7\left(cd^2-ae^2\right)^4\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{512c^4d^4e^2}+\frac{7\left(cd^2-ae^2\right)^2\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+a\right.}{192c^3d^3e}$$

```
[Out] (-7*(c*d^2 - a*e^2)^4*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2]])/(512*c^4*d^4*e^2) + (7*(c*d^2 - a*e^2)^2*(c*d^2 + a*e^2
 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(192*c^3*d^3*e
) + (7*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(60*c
^2*d^2) + ((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(6*c*d)
 + (7*(c*d^2 - a*e^2)^6*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt
[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]])/(1024*c^(9/2)*d^
(9/2)*e^(5/2))
```

Rubi [A] time = 0.263849, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {670, 640, 612, 621, 206}

$$-\frac{7\left(cd^2-ae^2\right)^4\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{512c^4d^4e^2}+\frac{7\left(cd^2-ae^2\right)^2\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+a\right.}{192c^3d^3e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```



```
[Out] (-7*(c*d^2 - a*e^2)^4*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2]])/(512*c^4*d^4*e^2) + (7*(c*d^2 - a*e^2)^2*(c*d^2 + a*e^2
 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(192*c^3*d^3*e
) + (7*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(60*c
^2*d^2) + ((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(6*c*d)
 + (7*(c*d^2 - a*e^2)^6*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt
[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]])/(1024*c^(9/2)*d^
(9/2)*e^(5/2))
```


Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}dx=\frac{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{6cd}+\frac{\left(7\left(d^2-\frac{ae^2}{c}\right)\right)\int (d+ex)$$

$$=\frac{7\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{60c^2d^2}+\frac{(d+ex)\left(ade+\left(cd^2\right.\right.}{6}$$

$$=\frac{7\left(cd^2-ae^2\right)^2\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{192c^3d^3e}-$$

$$=-\frac{7\left(cd^2-ae^2\right)^4\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{512c^4d^4e^2}+$$

$$=-\frac{7\left(cd^2-ae^2\right)^4\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{512c^4d^4e^2}+$$

$$=-\frac{7\left(cd^2-ae^2\right)^4\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{512c^4d^4e^2}+$$

Mathematica [A] time = 3.22449, size = 328, normalized size = 0.96

$$\frac{(ae+cdx)^2\sqrt{(d+ex)(ae+cdx)}\left(896c^5d^5(d+ex)^2\left(cd^2-ae^2\right)+560c^4d^4(d+ex)\left(cd^2-ae^2\right)^2+\frac{70c^3d^3\left(cd^2-ae^2\right)^4}{e(ae+cdx)}-\frac{105c^3d^3}{e^2(a}\right.}{7680c^7d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] ((a*e + c*d*x)^2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(280*(c^2*d^3 - a*c*d*e^2)^3
 - (105*c^3*d^3*(c*d^2 - a*e^2)^5)/(e^2*(a*e + c*d*x)^2) + (70*c^3*d^3*(c*d
^2 - a*e^2)^4)/(e*(a*e + c*d*x)) + 560*c^4*d^4*(c*d^2 - a*e^2)^2*(d + e*x)
+ 896*c^5*d^5*(c*d^2 - a*e^2)*(d + e*x)^2 + 1280*c^6*d^6*(d + e*x)^3 + (105
*c^(5/2)*d^(5/2)*Sqrt[c*d]*(c*d^2 - a*e^2)^(11/2)*ArcSinh[(Sqrt[c]*Sqrt[d]*
Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(e^(5/2)*(a*e
```

```
+ c*d*x)^(5/2)*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])))/(7680*c^7*d^7)
```

Maple [B] time = 0.053, size = 1302, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)

[Out] -21/512*d^6*c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a-7/192*e^3/d/c^2*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(3/2)*a^2-7/48*e^2/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2)*x*a+1/6*e*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/d/c+7/64*e*a*d^3*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+105/1024*e^6/c^2*ln((1/2*a*e^2+1/2*
c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*
c)^(1/2)*a^4-7/512/e^2*d^6*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-7/256*
e^4/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^3+21/512*e^6/d^2/c^3*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^4-7/256*e^2*d^2/c*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)*a^2-7/192*e*d/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2
)*a-7/60*e^2/d^2/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*a+7/192*e^5/d^
3/c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^3+7/1024/e^2*d^8*c^2*ln((1/
2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2))/(d*e*c)^(1/2)+105/1024*e^2*d^4*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e
*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^2+7/96*d
^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x+21/512*d^4*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)*a+7/192/e*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-
7/512*e^8/d^4/c^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^5-7/256/e*d^5*c
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+17/60/c*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(5/2)+7/96*e^4/d^2/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*
a^2-7/256*e^7/d^3/c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^4-21/128*
e^3*d/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^2+7/1024*e^10/d^4/c^4*l
n((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2))/(d*e*c)^(1/2)*a^6-21/512*e^8/d^2/c^3*ln((1/2*a*e^2+1/2*c*d^2+c*
d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)
*a^5-35/256*e^4*d^2/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^3+7/64*e^5*a^3/c^2/d*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="m
axima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 2.18231, size = 2271, normalized size = 6.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="f
ricas")
```

```
[Out] [1/30720*(105*(c^6*d^12 - 6*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 - 20*a^3*c^
3*d^6*e^6 + 15*a^4*c^2*d^4*e^8 - 6*a^5*c*d^2*e^10 + a^6*e^12)*sqrt(c*d*e)*l
og(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(
c^2*d^3*e + a*c*d*e^3)*x) + 4*(1280*c^6*d^6*e^6*x^5 - 105*c^6*d^11*e + 595*
a*c^5*d^9*e^3 + 1686*a^2*c^4*d^7*e^5 - 1386*a^3*c^3*d^5*e^7 + 595*a^4*c^2*d
^3*e^9 - 105*a^5*c*d*e^11 + 128*(37*c^6*d^7*e^5 + 13*a*c^5*d^5*e^7)*x^4 + 1
6*(387*c^6*d^8*e^4 + 410*a*c^5*d^6*e^6 + 3*a^2*c^4*d^4*e^8)*x^3 + 8*(377*c^
6*d^9*e^3 + 1191*a*c^5*d^7*e^5 + 39*a^2*c^4*d^5*e^7 - 7*a^3*c^3*d^3*e^9)*x^
2 + 2*(35*c^6*d^10*e^2 + 2876*a*c^5*d^8*e^4 + 450*a^2*c^4*d^6*e^6 - 196*a^3
*c^3*d^4*e^8 + 35*a^4*c^2*d^2*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x))/(c^5*d^5*e^3), -1/15360*(105*(c^6*d^12 - 6*a*c^5*d^10*e^2 + 15*a^2
*c^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 + 15*a^4*c^2*d^4*e^8 - 6*a^5*c*d^2*e^10 +
 a^6*e^12)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2
 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(1280*c^6*d^6*e^6*x^5 - 105*c^6*d^11*e +
 595*a*c^5*d^9*e^3 + 1686*a^2*c^4*d^7*e^5 - 1386*a^3*c^3*d^5*e^7 + 595*a^4*
c^2*d^3*e^9 - 105*a^5*c*d*e^11 + 128*(37*c^6*d^7*e^5 + 13*a*c^5*d^5*e^7)*x^
4 + 16*(387*c^6*d^8*e^4 + 410*a*c^5*d^6*e^6 + 3*a^2*c^4*d^4*e^8)*x^3 + 8*(3
77*c^6*d^9*e^3 + 1191*a*c^5*d^7*e^5 + 39*a^2*c^4*d^5*e^7 - 7*a^3*c^3*d^3*e^
9)*x^2 + 2*(35*c^6*d^10*e^2 + 2876*a*c^5*d^8*e^4 + 450*a^2*c^4*d^6*e^6 - 19
6*a^3*c^3*d^4*e^8 + 35*a^4*c^2*d^2*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x))/(c^5*d^5*e^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26031, size = 671, normalized size = 1.97

$$\frac{1}{7680}\sqrt{cdx^2e + cd^2x + axe^2 + ade}\left(2\left(4\left(2\left(8\left(10\,cdxe^3 + \frac{\left(37\,c^6d^7e^7 + 13\,ac^5d^5e^9\right)e^{(-5)}}{c^5d^5}\right)x + \frac{\left(387\,c^6d^8e^6 + 410\,ac^5d^6e^8\right.}{c^5d^5}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="g
iac")
```

```
[Out] 1/7680*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*(2*(4*(2*(8*(10*c*d*x*e^
3 + (37*c^6*d^7*e^7 + 13*a*c^5*d^5*e^9)*e^(-5)/(c^5*d^5))*x + (387*c^6*d^8*
e^6 + 410*a*c^5*d^6*e^8 + 3*a^2*c^4*d^4*e^10)*e^(-5)/(c^5*d^5))*x + (377*c^
6*d^9*e^5 + 1191*a*c^5*d^7*e^7 + 39*a^2*c^4*d^5*e^9 - 7*a^3*c^3*d^3*e^11)*e
```

```
^(-5)/(c^5*d^5))*x + (35*c^6*d^10*e^4 + 2876*a*c^5*d^8*e^6 + 450*a^2*c^4*d^
6*e^8 - 196*a^3*c^3*d^4*e^10 + 35*a^4*c^2*d^2*e^12)*e^(-5)/(c^5*d^5))*x - (
105*c^6*d^11*e^3 - 595*a*c^5*d^9*e^5 - 1686*a^2*c^4*d^7*e^7 + 1386*a^3*c^3*
d^5*e^9 - 595*a^4*c^2*d^3*e^11 + 105*a^5*c*d*e^13)*e^(-5)/(c^5*d^5)) - 7/10
24*(c^6*d^12 - 6*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 +
 15*a^4*c^2*d^4*e^8 - 6*a^5*c*d^2*e^10 + a^6*e^12)*sqrt(c*d)*e^(-5/2)*log(a
bs(-sqrt(c*d)*c*d^2*e^(1/2) - 2*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d
^2*x + a*x*e^2 + a*d*e))*c*d*e - sqrt(c*d)*a*e^(5/2)))/(c^5*d^5)
```

3.1921 $\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx$

Optimal. Leaf size=283

$$-\frac{3\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128c^3d^3e^2}+\frac{\left(cd^2-ae^2\right)\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+a\right.}{16c^2d^2e}$$

```
[Out] (-3*(c*d^2 - a*e^2)^3*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])/(128*c^3*d^3*e^2) + ((c*d^2 - a*e^2)*(c*d^2 + a*e^2 + 2
*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(16*c^2*d^2*e) + (
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(5*c*d) + (3*(c*d^2 - a*e^2)^5
*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(256*c^(7/2)*d^(7/2)*e^(5/2))
```

Rubi [A] time = 0.161692, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {640, 612, 621, 206}

$$-\frac{3\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128c^3d^3e^2}+\frac{\left(cd^2-ae^2\right)\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+a\right.}{16c^2d^2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (-3*(c*d^2 - a*e^2)^3*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])/(128*c^3*d^3*e^2) + ((c*d^2 - a*e^2)*(c*d^2 + a*e^2 + 2
*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(16*c^2*d^2*e) + (
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(5*c*d) + (3*(c*d^2 - a*e^2)^5
*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(256*c^(7/2)*d^(7/2)*e^(5/2))
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}dx = \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5cd} + \frac{\left(d^2-\frac{ae^2}{c}\right)\int\left(ade+\left(cd^2+ae^2\right)x+c\right.}{2d}$$

$$= \frac{\left(cd^2-ae^2\right)\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{16c^2d^2e} + \frac{\left(ade\right.}{}$$

$$= -\frac{3\left(cd^2-ae^2\right)^3\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^3d^3e^2} + \frac{\left(cd\right.}{}$$

$$= -\frac{3\left(cd^2-ae^2\right)^3\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^3d^3e^2} + \frac{\left(cd\right.}{}$$

$$= -\frac{3\left(cd^2-ae^2\right)^3\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^3d^3e^2} + \frac{\left(cd\right.}{}$$

Mathematica [A] time = 2.23107, size = 299, normalized size = 1.06

$$\frac{(ae+cdx)^2\sqrt{(d+ex)(ae+cdx)}\left(-\frac{15c^2d^2\left(cd^2-ae^2\right)^4}{e^2(ae+cdx)^2}+\frac{10c^2d^2\left(cd^2-ae^2\right)^3}{e(ae+cdx)}+80c^3d^3(d+ex)\left(cd^2-ae^2\right)+\frac{15c^{3/2}d^{3/2}\sqrt{cd}\left(cd^2-ae^2\right)^{9/2}\sinh^{\cdots}}{e^{5/2}(ae+cdx)^{5/2}\sqrt{\frac{c\cdots}{c\cdots}}}\right.}{640c^5d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] ((a*e + c*d*x)^2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(40*(c^2*d^3 - a*c*d*e^2)^2
- (15*c^2*d^2*(c*d^2 - a*e^2)^4)/(e^2*(a*e + c*d*x)^2) + (10*c^2*d^2*(c*d^2
 - a*e^2)^3)/(e*(a*e + c*d*x)) + 80*c^3*d^3*(c*d^2 - a*e^2)*(d + e*x) + 128
*c^4*d^4*(d + e*x)^2 + (15*c^(3/2)*d^(3/2)*Sqrt[c*d]*(c*d^2 - a*e^2)^(9/2)*
ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 -
 a*e^2])])/(e^(5/2)*(a*e + c*d*x)^(5/2)*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2
)])))/(640*c^5*d^5)
```

Maple [B] time = 0.051, size = 917, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] -3/64/d*e^4/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^3+9/64*e*d^2*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a-3/128*d^5/e^2*c*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)+15/128*d^3*e^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c
)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^2+3/256*d^
```

```
7/e^2*c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)-1/16/d^2*e^3/c^2*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(3/2)*a^2-15/256*d^5*c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c
)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a-3/64*d^4/e
*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+3/128/d^3*e^6/c^3*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^4-9/64*e^3/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)*x*a^2+3/64*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a+1/5*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d+1/8*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(3/2)*x+1/16*d^2/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-1/8/d*e^2/c*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*a+3/64/d^2*e^5/c^2*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)*x*a^3-3/256/d^3*e^8/c^3*ln((1/2*a*e^2+1/2*c*d^2+c*
d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)
*a^5+15/256/d*e^6/c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^4-15/128*d*e^4/c*ln((1/2
*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2))/(d*e*c)^(1/2)*a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.02347, size = 1790, normalized size = 6.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="fri
cas")
```

```
[Out] [1/2560*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d
^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^
2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x)
 + 4*(128*c^5*d^5*e^5*x^4 - 15*c^5*d^9*e + 70*a*c^4*d^7*e^3 + 128*a^2*c^3*d
^5*e^5 - 70*a^3*c^2*d^3*e^7 + 15*a^4*c*d*e^9 + 16*(21*c^5*d^6*e^4 + 11*a*c^
4*d^4*e^6)*x^3 + 8*(31*c^5*d^7*e^3 + 64*a*c^4*d^5*e^5 + a^2*c^3*d^3*e^7)*x^
2 + 2*(5*c^5*d^8*e^2 + 233*a*c^4*d^6*e^4 + 23*a^2*c^3*d^4*e^6 - 5*a^3*c^2*d
^2*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^4*e^3), -1/1
280*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4*e
^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2
*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(128*c^5*d^5*e^5*x
^4 - 15*c^5*d^9*e + 70*a*c^4*d^7*e^3 + 128*a^2*c^3*d^5*e^5 - 70*a^3*c^2*d^3
*e^7 + 15*a^4*c*d*e^9 + 16*(21*c^5*d^6*e^4 + 11*a*c^4*d^4*e^6)*x^3 + 8*(31*
c^5*d^7*e^3 + 64*a*c^4*d^5*e^5 + a^2*c^3*d^3*e^7)*x^2 + 2*(5*c^5*d^8*e^2 +
233*a*c^4*d^6*e^4 + 23*a^2*c^3*d^4*e^6 - 5*a^3*c^2*d^2*e^8)*x)*sqrt(c*d*e*x
```

```
^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^4*e^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31569, size = 536, normalized size = 1.89

$$\frac{1}{640}\sqrt{cdx^2e + cd^2x + axe^2 + ade}\left(2\left(4\left(2\left(8\,cdxe^2 + \frac{\left(21\,c^5d^6e^5 + 11\,ac^4d^4e^7\right)e^{(-4)}}{c^4d^4}\right)x + \frac{\left(31\,c^5d^7e^4 + 64\,ac^4d^5e^6 + a^2c^3d^3e^8\right)}{c^4d^4}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="gia
c")
```

```
[Out] 1/640*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*(2*(4*(2*(8*c*d*x*e^2 + (
21*c^5*d^6*e^5 + 11*a*c^4*d^4*e^7)*e^(-4)/(c^4*d^4))*x + (31*c^5*d^7*e^4 +
64*a*c^4*d^5*e^6 + a^2*c^3*d^3*e^8)*e^(-4)/(c^4*d^4))*x + (5*c^5*d^8*e^3 +
233*a*c^4*d^6*e^5 + 23*a^2*c^3*d^4*e^7 - 5*a^3*c^2*d^2*e^9)*e^(-4)/(c^4*d^4
))*x - (15*c^5*d^9*e^2 - 70*a*c^4*d^7*e^4 - 128*a^2*c^3*d^5*e^6 + 70*a^3*c^
2*d^3*e^8 - 15*a^4*c*d*e^10)*e^(-4)/(c^4*d^4)) - 3/256*(c^5*d^10 - 5*a*c^4*
d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e
^10)*sqrt(c*d)*e^(-5/2)*log(abs(-sqrt(c*d)*c*d^2*e^(1/2) - 2*(sqrt(c*d)*x*e
^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*c*d*e - sqrt(c*d)*a*e
^(5/2)))/(c^4*d^4)
```

3.1922 $\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx$

Optimal. Leaf size=232

$$-\frac{3\left(cd^2-ae^2\right)^2\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64c^2d^2e^2}+\frac{3\left(cd^2-ae^2\right)^4\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex}}\right)}{128c^{5/2}d^{5/2}e^{5/2}}$$

```
[Out] (-3*(c*d^2 - a*e^2)^2*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])/(64*c^2*d^2*e^2) + ((c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(8*c*d*e) + (3*(c*d^2 - a*e^2)^4*Ar
cTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2])])/(128*c^(5/2)*d^(5/2)*e^(5/2))
```

Rubi [A] time = 0.0885656, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {612, 621, 206}

$$-\frac{3\left(cd^2-ae^2\right)^2\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64c^2d^2e^2}+\frac{3\left(cd^2-ae^2\right)^4\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex}}\right)}{128c^{5/2}d^{5/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```


```
[Out] (-3*(c*d^2 - a*e^2)^2*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])/(64*c^2*d^2*e^2) + ((c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(8*c*d*e) + (3*(c*d^2 - a*e^2)^4*Ar
cTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2])])/(128*c^(5/2)*d^(5/2)*e^(5/2))
```


Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2} dx = \frac{\left(cd^2 + ae^2 + 2cdex\right)\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}{8cde} - \frac{\left(3\left(cd^2 - ae^2\right)^2\right)\int \sqrt{ade}}{16}$$

$$= -\frac{3\left(cd^2 - ae^2\right)^2\left(cd^2 + ae^2 + 2cdex\right)\sqrt{ade + \left(cd^2 + ae^2\right)x + cdex^2}}{64c^2d^2e^2} + \frac{\left(cd^2 + ae^2\right.}{}$$

$$= -\frac{3\left(cd^2 - ae^2\right)^2\left(cd^2 + ae^2 + 2cdex\right)\sqrt{ade + \left(cd^2 + ae^2\right)x + cdex^2}}{64c^2d^2e^2} + \frac{\left(cd^2 + ae^2\right.}{}$$

$$= -\frac{3\left(cd^2 - ae^2\right)^2\left(cd^2 + ae^2 + 2cdex\right)\sqrt{ade + \left(cd^2 + ae^2\right)x + cdex^2}}{64c^2d^2e^2} + \frac{\left(cd^2 + ae^2\right.}{}$$

Mathematica [A] time = 0.954585, size = 317, normalized size = 1.37

$$\frac{\sqrt{cd}\left(3\left(cd^2 - ae^2\right)^{9/2}\sqrt{ae + cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right) - \sqrt{c}\sqrt{d}\sqrt{e}\sqrt{cd}(d + ex)\left(-a^2c^2d^2e^3\left(11d^2 + 55dex + 26e^2\right.\right.\right.}{64c^{7/2}d^{7/2}e^{5/2}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (Sqrt[c*d]*(-(Sqrt[c]*Sqrt[d]*Sqrt[c*d]*Sqrt[e]*(d + e*x)*(3*a^4*e^7 + a^3*
c*d*e^5*(-11*d + e*x) - a^2*c^2*d^2*e^3*(11*d^2 + 55*d*e*x + 26*e^2*x^2) +
a*c^3*d^3*e*(3*d^3 - 13*d^2*e*x - 68*d*e^2*x^2 - 40*e^3*x^3) + c^4*d^4*x*(3
*d^3 - 2*d^2*e*x - 24*d*e^2*x^2 - 16*e^3*x^3))) + 3*(c*d^2 - a*e^2)^(9/2)*S
qrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqr
t[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])]))/(64*c^(7
/2)*d^(7/2)*e^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [B] time = 0.043, size = 671, normalized size = 2.9

$$\frac{2\,cdex + ae^2 + cd^2}{8\,dec}\left(ade + \left(ae^2 + cd^2\right)x + cdex^2\right)^{\frac{3}{2}} - \frac{3\,e^3xa^2}{32\,cd}\sqrt{ade + \left(ae^2 + cd^2\right)x + cdex^2} + \frac{3\,adex}{16}\sqrt{ade + \left(ae^2 + cd^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] 1/8*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/e-3
/32/d*e^3/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^2+3/16*d*e*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a-3/32*d^3/e*c*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)*x-3/64/d^2*e^4/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^
3+3/64*e^2/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2+3/64*d^2*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a-3/64*d^4/e^2*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)+3/128/d^2*e^6/c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/
2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^4-3/32*e^4/c*ln
((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2))/(d*e*c)^(1/2)*a^3+9/64*d^2*e^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/
(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^2-3/
32*d^4*c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a+3/128*d^6/e^2*c^2*ln((1/2*a*e^2+1/2*c
```

```
*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c
)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.90906, size = 1399, normalized size = 6.03

$$\left[\frac{3\left(c^4d^8 - 4\,ac^3d^6e^2 + 6\,a^2c^2d^4e^4 - 4\,a^3cd^2e^6 + a^4e^8\right)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2 + c^2d^4 + 6\,acd^2e^2 + a^2e^4 + 4\,\sqrt{cdex^2 + ade +}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/256*(3*(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6
+ a^4*e^8)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^
2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 +
a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(16*c^4*d^4*e^4*x^3 -
 3*c^4*d^7*e + 11*a*c^3*d^5*e^3 + 11*a^2*c^2*d^3*e^5 - 3*a^3*c*d*e^7 + 24*(
c^4*d^5*e^3 + a*c^3*d^3*e^5)*x^2 + 2*(c^4*d^6*e^2 + 22*a*c^3*d^4*e^4 + a^2*
c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^3),
 -1/128*(3*(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6
 + a^4*e^8)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^
2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(16*c^4*d^4*e^4*x^3 - 3*c^4*d^7*e + 11*
a*c^3*d^5*e^3 + 11*a^2*c^2*d^3*e^5 - 3*a^3*c*d*e^7 + 24*(c^4*d^5*e^3 + a*c^
3*d^3*e^5)*x^2 + 2*(c^4*d^6*e^2 + 22*a*c^3*d^4*e^4 + a^2*c^2*d^2*e^6)*x)*sq
rt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33639, size = 414, normalized size = 1.78

$$\frac{1}{64}\sqrt{cdx^2e+cd^2x+axe^2+ade}\left(2\left(4\left(2\,cdxe+\frac{3\left(c^4d^5e^3+ac^3d^3e^5\right)e^{(-3)}}{c^3d^3}\right)x+\frac{\left(c^4d^6e^2+22\,ac^3d^4e^4+a^2c^2d^2e^6\right)e^{(-3)}}{c^3d^3}\right)x-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/64*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*(2*(4*(2*c*d*x*e + 3*(c^4*
d^5*e^3 + a*c^3*d^3*e^5)*e^(-3)/(c^3*d^3))*x + (c^4*d^6*e^2 + 22*a*c^3*d^4*
e^4 + a^2*c^2*d^2*e^6)*e^(-3)/(c^3*d^3))*x - (3*c^4*d^7*e - 11*a*c^3*d^5*e^
3 - 11*a^2*c^2*d^3*e^5 + 3*a^3*c*d*e^7)*e^(-3)/(c^3*d^3)) - 3/128*(c^4*d^8
- 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*sqrt(c*d
)*e^(-5/2)*log(abs(-sqrt(c*d)*c*d^2*e^(1/2) - 2*(sqrt(c*d)*x*e^(1/2) - sqrt
(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*c*d*e - sqrt(c*d)*a*e^(5/2)))/(c^3
*d^3)
```

3.1923 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=201

$$\frac{\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16c^{3/2}d^{3/2}e^{5/2}}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}+\frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(ae^2+cd^2+2cde\right.$$

```
[Out] ((a/(c*d) - d/e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2])/8 + (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(3*e) +
((c*d^2 - a*e^2)^3*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*S
qrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*c^(3/2)*d^(3/2)*e
^(5/2))
```

Rubi [A] time = 0.110995, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {664, 612, 621, 206}

$$\frac{\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16c^{3/2}d^{3/2}e^{5/2}}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}+\frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(ae^2+cd^2+2cde\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] ((a/(c*d) - d/e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2])/8 + (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(3*e) +
((c*d^2 - a*e^2)^3*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*S
qrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*c^(3/2)*d^(3/2)*e
^(5/2))
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx = \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e} - \frac{\left(2cd^2e-e\left(cd^2+ae^2\right)\right)\int\sqrt{ade+\left(cd^2+ae^2\right)}}{2e^2}$$

$$= \frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}+\frac{\left(ade+\left(cd^2+\right.\right.}{}$$

$$= \frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}+\frac{\left(ade+\left(cd^2+\right.\right.}{}$$

$$= \frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}+\frac{\left(ade+\left(cd^2+\right.\right.}{}$$

Mathematica [A] time = 0.681475, size = 264, normalized size = 1.31

$$\frac{\sqrt{c}\sqrt{d}\left(3\left(cd^2-ae^2\right)^{7/2}\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)-\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{cd}(d+ex)\left(-a^2cde^3(8d+17ex)-3a^3e^5+\right.\right.}{24e^{5/2}(cd)^{5/2}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] (Sqrt[c]*Sqrt[d]*(-(Sqrt[c]*Sqrt[d]*Sqrt[c*d]*Sqrt[e]*(d + e*x)*(-3*a^3*e^5
 - a^2*c*d*e^3*(8*d + 17*e*x) + a*c^2*d^2*e*(3*d^2 - 10*d*e*x - 22*e^2*x^2)
 + c^3*d^3*x*(3*d^2 - 2*d*e*x - 8*e^2*x^2))) + 3*(c*d^2 - a*e^2)^(7/2)*Sqrt
[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d
]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])]))/(24*(c*d)^(
5/2)*e^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [B] time = 0.047, size = 566, normalized size = 2.8

$$\frac{1}{3e}\left(cde\left(\frac{d}{e}+x\right)^2+\left(ae^2-cd^2\right)\left(\frac{d}{e}+x\right)\right)^{\frac{3}{2}}+\frac{aex}{4}\sqrt{cde\left(\frac{d}{e}+x\right)^2+\left(ae^2-cd^2\right)\left(\frac{d}{e}+x\right)}+\frac{a^2e^2}{8\,cd}\sqrt{cde\left(\frac{d}{e}+x\right)^2+\left(ae^2-cd^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x)
```

```
[Out] 1/3/e*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)+1/4*e*a*(c*d*e*(d/e+x)^
2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x+1/8*e^2*a^2/d/c*(c*d*e*(d/e+x)^2+(a*e^2-c*
d^2)*(d/e+x))^(1/2)-1/16*e^4*a^3/d/c*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)
/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)
+3/16*e^2*a^2*d*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e
```

```
*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-3/16*a*d^3*c*ln((1/2
*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2
)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-1/4/e*c*d^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*
(d/e+x))^(1/2)*x-1/8/e^2*c*d^3*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2
)+1/16/e^2*c^2*d^5*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*
d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.76744, size = 1125, normalized size = 5.6

$$\left[-\frac{3\left(c^3d^6 - 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 - a^3e^6\right)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2 + c^2d^4 + 6\,acd^2e^2 + a^2e^4 - 4\,\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right.}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="fri
cas")
```

```
[Out] [-1/96*(3*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(c*d*
e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) +
 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(8*c^3*d^3*e^3*x^2 - 3*c^3*d^5*e + 8*a*c^
2*d^3*e^3 + 3*a^2*c*d*e^5 + 2*(c^3*d^4*e^2 + 7*a*c^2*d^2*e^4)*x)*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^3), -1/48*(3*(c^3*d^6 - 3*a*c
^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(
c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(8*c^3*d^3*
e^3*x^2 - 3*c^3*d^5*e + 8*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5 + 2*(c^3*d^4*e^2 +
7*a*c^2*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e
^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
```

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError
```

3.1924 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=187

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2e(d+ex)}+\frac{3}{4}\left(a-\frac{cd^2}{e^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}+\frac{3\left(cd^2-ae^2\right)^2\tanh^{-1}\left(\frac{ae^2+cd^2+}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+\right.}}\right.}{8\sqrt{c}\sqrt{d}e^{5/2}}$$

```
[Out] (3*(a - (c*d^2)/e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/4 + (a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(2*e*(d + e*x)) + (3*(c*d^2 - a*e^
2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*Sqrt[c]*Sqrt[d]*e^(5/2))
```

Rubi [A] time = 0.126264, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {664, 621, 206}

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2e(d+ex)}+\frac{3}{4}\left(a-\frac{cd^2}{e^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}+\frac{3\left(cd^2-ae^2\right)^2\tanh^{-1}\left(\frac{ae^2+cd^2+}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+\right.}}\right.}{8\sqrt{c}\sqrt{d}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^2,x]
```


```
[Out] (3*(a - (c*d^2)/e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/4 + (a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(2*e*(d + e*x)) + (3*(c*d^2 - a*e^
2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*Sqrt[c]*Sqrt[d]*e^(5/2))
```


Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^2}dx = \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2e(d+ex)} - \frac{\left(3\left(2cd^2e-e\left(cd^2+ae^2\right)\right)\right)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+c}}{d+ex}}{4e^2}$$

$$= \frac{3}{4}\left(a-\frac{cd^2}{e^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2} + \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2e(d+ex)} +$$

$$= \frac{3}{4}\left(a-\frac{cd^2}{e^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2} + \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2e(d+ex)} +$$

$$= \frac{3}{4}\left(a-\frac{cd^2}{e^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2} + \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2e(d+ex)} +$$

Mathematica [A] time = 0.379775, size = 209, normalized size = 1.12

$$\frac{\sqrt{e}(cd)^{3/2}(d+ex)\left(5a^2e^3+acde(7ex-3d)+c^2d^2x(2ex-3d)\right)+3\sqrt{c}\sqrt{d}\left(cd^2-ae^2\right)^{5/2}\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}}{\sqrt{cd}\sqrt{c}}\right.}{4e^{5/2}(cd)^{3/2}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^2,x]
```

```
[Out] ((c*d)^(3/2)*Sqrt[e]*(d + e*x)*(5*a^2*e^3 + c^2*d^2*x*(-3*d + 2*e*x) + a*c*
d*e*(-3*d + 7*e*x)) + 3*Sqrt[c]*Sqrt[d]*(c*d^2 - a*e^2)^(5/2)*Sqrt[a*e + c*
d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]
*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(4*(c*d)^(3/2)*e^(5/2
)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [B] time = 0.049, size = 757, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^2,x)
```

```
[Out] 2/e^2/(a*e^2-c*d^2)/(d/e+x)^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(5/2)
-2/e*d*c/(a*e^2-c*d^2)*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)-3/2*e*
d*c/(a*e^2-c*d^2)*a*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-3/4*e^2
/(a*e^2-c*d^2)*a^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)+3/8*e^4/(a
*e^2-c*d^2)*a^3*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e
*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-9/8*e^2*d^2*c/(a*e^2
-c*d^2)*a^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/
e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+9/8*d^4*c^2/(a*e^2-c*d^2
)*a*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(
a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+3/2/e*d^3*c^2/(a*e^2-c*d^2)*(c*d
*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x+3/4/e^2*d^4*c^2/(a*e^2-c*d^2)*(
c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-3/8/e^2*d^6*c^3/(a*e^2-c*d^2)*
ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^
2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^2,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70366, size = 895, normalized size = 4.79

$$\left[\frac{3\left(c^2d^4-2\,acd^2e^2+a^2e^4\right)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4+4\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2\,cdex+cd\right.\right.}{16\,cde^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^2,x, algorithm="f
ricas")
```

```
[Out] [1/16*(3*(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*
x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d
*e^3)*x) + 4*(2*c^2*d^2*e^2*x - 3*c^2*d^3*e + 5*a*c*d*e^3)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x))/(c*d*e^3), -1/8*(3*(c^2*d^4 - 2*a*c*d^2*e^2 +
a^2*e^4)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 +
 (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(2*c^2*d^2*e^2*x - 3*c^2*d^3*e + 5*a*c*d*e
^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c*d*e^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^2,x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.1925 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^3}\,dx$

Optimal. Leaf size=175

$$-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{e(d+ex)^2}+\frac{3cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^2}-\frac{3\sqrt{c}\sqrt{d}\left(cd^2-ae^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+c}}\right.}{2e^{5/2}}$$

```
[Out] (3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/e^2 - (2*(a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(e*(d + e*x)^2) - (3*Sqrt[c]*Sqrt[d]*(c*d^
2 - a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*e^(5/2))
```

Rubi [A] time = 0.104034, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {662, 664, 621, 206}

$$-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{e(d+ex)^2}+\frac{3cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^2}-\frac{3\sqrt{c}\sqrt{d}\left(cd^2-ae^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+c}}\right.}{2e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^3,x]
```

```
[Out] (3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/e^2 - (2*(a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(e*(d + e*x)^2) - (3*Sqrt[c]*Sqrt[d]*(c*d^
2 - a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*e^(5/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^3}dx = -\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{e(d+ex)^2}+\frac{(3cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}dx}{e}$$

$$=\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^2}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{e(d+ex)^2}-\frac{\left(3cd\left(c\right.\right.}{}$$

$$=\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^2}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{e(d+ex)^2}-\frac{\left(3cd\left(c\right.\right.}{}$$

$$=\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^2}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{e(d+ex)^2}-\frac{3\sqrt{c}\sqrt{c}}{}$$

Mathematica [C] time = 0.0760555, size = 108, normalized size = 0.62

$$\frac{2cd(ae+cdx)^2\sqrt{(d+ex)(ae+cdx)}\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{5\left(cd^2-ae^2\right)^2\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^3,x]
```

```
[Out] (2*c*d*(a*e + c*d*x)^2*Sqrt[(a*e + c*d*x)*(d + e*x)]*Hypergeometric2F1[3/2,
 5/2, 7/2, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(5*(c*d^2 - a*e^2)^2*Sqrt
[(c*d*(d + e*x))/(c*d^2 - a*e^2)])
```

Maple [B] time = 0.052, size = 837, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^3,x)
```

```
[Out] -2/e^3/(a*e^2-c*d^2)/(d/e+x)^3*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(5/2
)+8/e^2*d*c/(a*e^2-c*d^2)^2/(d/e+x)^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x
))^(5/2)-8/e*d^2*c^2/(a*e^2-c*d^2)^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x)
)^(3/2)-6*e*d^2*c^2/(a*e^2-c*d^2)^2*a*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x
))^(1/2)*x-3*e^2*d*c/(a*e^2-c*d^2)^2*a^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/
e+x))^(1/2)+3/2*e^4*d*c/(a*e^2-c*d^2)^2*a^3*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)
*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c
)^(1/2)-9/2*e^2*d^3*c^2/(a*e^2-c*d^2)^2*a^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)
*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c
```

```
)^(1/2)+9/2*d^5*c^3/(a*e^2-c*d^2)^2*a*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e
)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2
)+6/e*d^4*c^3/(a*e^2-c*d^2)^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)
*x+3/e^2*d^5*c^3/(a*e^2-c*d^2)^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1
/2)-3/2/e^2*d^7*c^4/(a*e^2-c*d^2)^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/
(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^3,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.53509, size = 873, normalized size = 4.99

$$\left[\frac{3\left(cd^3 - ade^2 + \left(cd^2e - ae^3\right)x\right)\sqrt{\frac{cd}{e}}\log\left(8\,c^2d^2e^2x^2 + c^2d^4 + 6\,acd^2e^2 + a^2e^4 - 4\left(2\,cde^2x + cd^2e + ae^3\right)\sqrt{cdex^2 + ade +}\right.}{4\left(e^3x + de^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^3,x, algorithm="f
ricas")
```

```
[Out] [1/4*(3*(c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)*sqrt(c*d/e)*log(8*c^2*d^2*e
^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*(2*c*d*e^2*x + c*d^2*e + a*e
^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e
+ a*c*d*e^3)*x) + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*e*x +
3*c*d^2 - 2*a*e^2))/(e^3*x + d*e^2), 1/2*(3*(c*d^3 - a*d*e^2 + (c*d^2*e - a
*e^3)*x)*sqrt(-c*d/e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 + a*c*d^2*e + (c^
2*d^3 + a*c*d*e^2)*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d
*e*x + 3*c*d^2 - 2*a*e^2))/(e^3*x + d*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^3,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError
```

3.1926 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^4} dx$

Optimal. Leaf size=169

$$\frac{c^{3/2}d^{3/2}\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{e^{5/2}}-\frac{2cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^2(d+ex)}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^3}$$

```
[Out] (-2*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e^2*(d + e*x)) - (2*(
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*e*(d + e*x)^3) + (c^(3/2)*
d^(3/2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])])/e^(5/2)
```

Rubi [A] time = 0.0915419, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {662, 621, 206}

$$\frac{c^{3/2}d^{3/2}\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{e^{5/2}}-\frac{2cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^2(d+ex)}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^4,x]
```

```
[Out] (-2*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e^2*(d + e*x)) - (2*(
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*e*(d + e*x)^3) + (c^(3/2)*
d^(3/2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])])/e^(5/2)
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^4}dx = -\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e(d+ex)^3} + \frac{(cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(d+ex)^2}dx}{e}$$

$$= -\frac{2cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^2(d+ex)} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e(d+ex)^3} + \frac{\left(c^2d^2\right.}{}$$

$$= -\frac{2cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^2(d+ex)} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e(d+ex)^3} + \frac{\left(2c^2d\right.}{}$$

$$= -\frac{2cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^2(d+ex)} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e(d+ex)^3} + \frac{c^{3/2}d^3}{}$$

Mathematica [A] time = 0.707244, size = 184, normalized size = 1.09

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(\frac{3c^{3/2}d^{3/2}\sqrt{cd}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{\sqrt{cd^2-ae^2}\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}} - \frac{\sqrt{e}\left(ae^2+cd(3d+4ex)\right)}{(d+ex)^2}\right)}{3e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^4,x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[e]*(a*e^2 + c*d*(3*d + 4*e*x)))/(
d + e*x)^2) + (3*c^(3/2)*d^(3/2)*Sqrt[c*d]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]
*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(Sqrt[c*d^2 - a*e^2]*
Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])))/(3*e^(5/2))
```

Maple [B] time = 0.049, size = 914, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^4,x)
```

```
[Out] -2/3/e^4/(a*e^2-c*d^2)/(d/e+x)^4*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(5
/2)-4/3/e^3*d*c/(a*e^2-c*d^2)^2/(d/e+x)^3*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d
/e+x))^(5/2)+16/3/e^2*d^2*c^2/(a*e^2-c*d^2)^3/(d/e+x)^2*(c*d*e*(d/e+x)^2+(a
*e^2-c*d^2)*(d/e+x))^(5/2)-16/3/e*d^3*c^3/(a*e^2-c*d^2)^3*(c*d*e*(d/e+x)^2+
(a*e^2-c*d^2)*(d/e+x))^(3/2)-4*e*d^3*c^3/(a*e^2-c*d^2)^3*a*(c*d*e*(d/e+x)^2
+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-2*e^2*d^2*c^2/(a*e^2-c*d^2)^3*a^2*(c*d*e*(d
/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)+e^4*d^2*c^2/(a*e^2-c*d^2)^3*a^3*ln((1/
2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^
2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-3*e^2*d^4*c^3/(a*e^2-c*d^2)^3*a^2*ln((1/2*
a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)
*(d/e+x))^(1/2))/(d*e*c)^(1/2)+3*d^6*c^4/(a*e^2-c*d^2)^3*a*ln((1/2*a*e^2-1/
2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x)
)^(1/2))/(d*e*c)^(1/2)+4/e*d^5*c^4/(a*e^2-c*d^2)^3*(c*d*e*(d/e+x)^2+(a*e^2-
c*d^2)*(d/e+x))^(1/2)*x+2/e^2*d^6*c^4/(a*e^2-c*d^2)^3*(c*d*e*(d/e+x)^2+(a*e
^2-c*d^2)*(d/e+x))^(1/2)-1/e^2*d^8*c^5/(a*e^2-c*d^2)^3*ln((1/2*a*e^2-1/2*c*
```

```
d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1
/2))/(d*e*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^4,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.38952, size = 907, normalized size = 5.37

$$\left[\frac{3\left(cde^2x^2+2\,cd^2ex+cd^3\right)\sqrt{\frac{cd}{e}}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4+4\left(2\,cde^2x+cd^2e+ae^3\right)\sqrt{cdex^2+ade+\left(cd\right.}\right.}{6\left(e^4x^2+2\,de^3x+d^2e^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^4,x, algorithm="f
ricas")
```

```
[Out] [1/6*(3*(c*d*e^2*x^2 + 2*c*d^2*e*x + c*d^3)*sqrt(c*d/e)*log(8*c^2*d^2*e^2*x
^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*
c*d*e^3)*x) - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(4*c*d*e*x + 3*
c*d^2 + a*e^2))/(e^4*x^2 + 2*d*e^3*x + d^2*e^2), -1/3*(3*(c*d*e^2*x^2 + 2*c
*d^2*e*x + c*d^3)*sqrt(-c*d/e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 + a*c*d^
2*e + (c^2*d^3 + a*c*d*e^2)*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*(4*c*d*e*x + 3*c*d^2 + a*e^2))/(e^4*x^2 + 2*d*e^3*x + d^2*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**4,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^4,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError
```

3.1927 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^5}\,dx$

Optimal. Leaf size=54

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5(d+ex)^5\left(cd^2-ae^2\right)}$$

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*(c*d^2 - a*e^2)*(d + e
*x)^5)
```

Rubi [A] time = 0.0205957, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {650}

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5(d+ex)^5\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^5,x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*(c*d^2 - a*e^2)*(d + e
*x)^5)
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^5}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5\left(cd^2-ae^2\right)(d+ex)^5}$$

Mathematica [A] time = 0.0321824, size = 43, normalized size = 0.8

$$\frac{2((d+ex)(ae+cdx))^{5/2}}{5(d+ex)^5\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^5,x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2))/(5*(c*d^2 - a*e^2)*(d + e*x)^5)
```

Maple [A] time = 0.043, size = 58, normalized size = 1.1

$$-\frac{2\,cdx+2\,ae}{5\,\left(ex+d\right)^{4}\left(ae^{2}-cd^{2}\right)}\left(cdex^{2}+ae^{2}x+cd^{2}x+ade\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^5,x)
```

```
[Out] -2/5*(c*d*x+a*e)/(e*x+d)^4/(a*e^2-c*d^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^
(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^5,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 7.51395, size = 258, normalized size = 4.78

$$\frac{2\left(c^{2}d^{2}x^{2}+2\,acdex+a^{2}e^{2}\right)\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}}{5\left(cd^{5}-ad^{3}e^{2}+\left(cd^{2}e^{3}-ae^{5}\right)x^{3}+3\left(cd^{3}e^{2}-ade^{4}\right)x^{2}+3\left(cd^{4}e-ad^{2}e^{3}\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^5,x, algorithm="f
ricas")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)/(c*d^5 - a*d^3*e^2 + (c*d^2*e^3 - a*e^5)*x^3 + 3*(c*d^3*e^2 - a*
d*e^4)*x^2 + 3*(c*d^4*e - a*d^2*e^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**5,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^5,x, algorithm="g
iac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1928 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^6}\,dx$

Optimal. Leaf size=111

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{35(d+ex)^5\left(cd^2-ae^2\right)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7(d+ex)^6\left(cd^2-ae^2\right)}$$

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(7*(c*d^2 - a*e^2)*(d + e
*x)^6) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(35*(c*d^2 -
 a*e^2)^2*(d + e*x)^5)
```

Rubi [A] time = 0.0479102, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {658, 650}

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{35(d+ex)^5\left(cd^2-ae^2\right)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7(d+ex)^6\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^6,x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(7*(c*d^2 - a*e^2)*(d + e
*x)^6) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(35*(c*d^2 -
 a*e^2)^2*(d + e*x)^5)
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^6}\,dx &= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7\left(cd^2-ae^2\right)(d+ex)^6}+\frac{(2cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^5}\,dx}{7\left(cd^2-ae^2\right)}\\ &= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7\left(cd^2-ae^2\right)(d+ex)^6}+\frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{35\left(cd^2-ae^2\right)^2(d+ex)^5}\end{aligned}$$

Mathematica [A] time = 0.047925, size = 61, normalized size = 0.55

$$\frac{2((d+ex)(ae+cdx))^{5/2}\left(cd(7d+2ex)-5ae^2\right)}{35(d+ex)^6\left(cd^2-ae^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^6,x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-5*a*e^2 + c*d*(7*d + 2*e*x)))/(35*(c*d
^2 - a*e^2)^2*(d + e*x)^6)
```

Maple [A] time = 0.045, size = 90, normalized size = 0.8

$$-\frac{(2\,cdx+2\,ae)\left(-2\,cdex+5\,ae^2-7\,cd^2\right)}{35\,(ex+d)^5\left(a^2e^4-2\,acd^2e^2+c^2d^4\right)}\left(cdex^2+ae^2x+cd^2x+ade\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^6,x)
```

```
[Out] -2/35*(c*d*x+a*e)*(-2*c*d*e*x+5*a*e^2-7*c*d^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a
*d*e)^(3/2)/(e*x+d)^5/(a^2*e^4-2*a*c*d^2*e^2+c^2*d^4)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^6,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 22.4999, size = 548, normalized size = 4.94

$$\frac{2\left(2\,c^3d^3ex^3+7\,a^2cd^2e^2-5\,a^3e^4+\left(7\,c^3d^4-ac^2d^2e^2\right)x^2+2\left(7\,ac^2d^3e-4\,a^2cde^3\right)x\right)\sqrt{cdex^2+}}{35\left(c^2d^8-2\,acd^6e^2+a^2d^4e^4+\left(c^2d^4e^4-2\,acd^2e^6+a^2e^8\right)x^4+4\left(c^2d^5e^3-2\,acd^3e^5+a^2de^7\right)x^3+6\left(c^2d^6e^2-2\,acd^4e^4+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^6,x, algorithm="f
ricas")
```

```
[Out] 2/35*(2*c^3*d^3*e*x^3 + 7*a^2*c*d^2*e^2 - 5*a^3*e^4 + (7*c^3*d^4 - a*c^2*d^
2*e^2)*x^2 + 2*(7*a*c^2*d^3*e - 4*a^2*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x)/(c^2*d^8 - 2*a*c*d^6*e^2 + a^2*d^4*e^4 + (c^2*d^4*e^4 -
2*a*c*d^2*e^6 + a^2*e^8)*x^4 + 4*(c^2*d^5*e^3 - 2*a*c*d^3*e^5 + a^2*d*e^7)*
```

```
x^3 + 6*(c^2*d^6*e^2 - 2*a*c*d^4*e^4 + a^2*d^2*e^6)*x^2 + 4*(c^2*d^7*e - 2*
a*c*d^5*e^3 + a^2*d^3*e^5)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**6,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^6,x, algorithm="g
iac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1929 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^7}\,dx$

Optimal. Leaf size=171

$$\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{315(d+ex)^5\left(cd^2-ae^2\right)^3}+\frac{8cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{63(d+ex)^6\left(cd^2-ae^2\right)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9(d+ex)^7\left(cd^2-ae^2\right)}$$

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(9*(c*d^2 - a*e^2)*(d + e
*x)^7) + (8*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(63*(c*d^2 -
 a*e^2)^2*(d + e*x)^6) + (16*c^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2))/(315*(c*d^2 - a*e^2)^3*(d + e*x)^5)
```

Rubi [A] time = 0.0780033, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {658, 650}

$$\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{315(d+ex)^5\left(cd^2-ae^2\right)^3}+\frac{8cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{63(d+ex)^6\left(cd^2-ae^2\right)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9(d+ex)^7\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^7,x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(9*(c*d^2 - a*e^2)*(d + e
*x)^7) + (8*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(63*(c*d^2 -
 a*e^2)^2*(d + e*x)^6) + (16*c^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2))/(315*(c*d^2 - a*e^2)^3*(d + e*x)^5)
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^7}\,dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{9\left(cd^2-ae^2\right)(d+ex)^7} + \frac{(4cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^6}dx}{9\left(cd^2-ae^2\right)}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{9\left(cd^2-ae^2\right)(d+ex)^7} + \frac{8cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{63\left(cd^2-ae^2\right)^2(d+ex)^6} + \frac{(8c^2}{}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{9\left(cd^2-ae^2\right)(d+ex)^7} + \frac{8cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{63\left(cd^2-ae^2\right)^2(d+ex)^6} + \frac{16c^2}{}$$

Mathematica [A] time = 0.0703643, size = 94, normalized size = 0.55

$$\frac{2((d+ex)(ae+cdx))^{5/2}\left(35a^2e^4-10acde^2(9d+2ex)+c^2d^2\left(63d^2+36dex+8e^2x^2\right)\right)}{315(d+ex)^7\left(cd^2-ae^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^7,x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(35*a^2*e^4 - 10*a*c*d*e^2*(9*d + 2*e*x)
 + c^2*d^2*(63*d^2 + 36*d*e*x + 8*e^2*x^2)))/(315*(c*d^2 - a*e^2)^3*(d + e*
x)^7)
```

Maple [A] time = 0.047, size = 146, normalized size = 0.9

$$-\frac{(2\,cdx+2\,ae)\left(8\,c^2d^2e^2x^2-20\,acde^3x+36\,c^2d^3ex+35\,a^2e^4-90\,acd^2e^2+63\,c^2d^4\right)}{315\,(ex+d)^6\left(a^3e^6-3\,a^2cd^2e^4+3\,ac^2d^4e^2-c^3d^6\right)}\left(cdex^2+ae^2x+cd^2x+ade\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^7,x)
```

```
[Out] -2/315*(c*d*x+a*e)*(8*c^2*d^2*e^2*x^2-20*a*c*d*e^3*x+36*c^2*d^3*e*x+35*a^2*
e^4-90*a*c*d^2*e^2+63*c^2*d^4)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)/(e*x
+d)^6/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2-c^3*d^6)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^7,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 89.4596, size = 937, normalized size = 5.48

$$\frac{2\left(8c^4d^4e^2x^4+63a^2c^2d^4e^2-90a^3cd^2e^4+35a^4e^6+4\left(9c^4d^5e-ac^3d^3e\right.\right.}{315\left(c^3d^{11}-3ac^2d^9e^2+3a^2cd^7e^4-a^3d^5e^6+\left(c^3d^6e^5-3ac^2d^4e^7+3a^2cd^2e^9-a^3e^{11}\right)x^5+5\left(c^3d^7e^4-3ac^2d^5e^6+3a^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^7,x, algorithm="f
ricas")
```

```
[Out] 2/315*(8*c^4*d^4*e^2*x^4 + 63*a^2*c^2*d^4*e^2 - 90*a^3*c*d^2*e^4 + 35*a^4*e
^6 + 4*(9*c^4*d^5*e - a*c^3*d^3*e^3)*x^3 + 3*(21*c^4*d^6 - 6*a*c^3*d^4*e^2
+ a^2*c^2*d^2*e^4)*x^2 + 2*(63*a*c^3*d^5*e - 72*a^2*c^2*d^3*e^3 + 25*a^3*c*
d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(c^3*d^11 - 3*a*c^2*d
^9*e^2 + 3*a^2*c*d^7*e^4 - a^3*d^5*e^6 + (c^3*d^6*e^5 - 3*a*c^2*d^4*e^7 + 3
*a^2*c*d^2*e^9 - a^3*e^11)*x^5 + 5*(c^3*d^7*e^4 - 3*a*c^2*d^5*e^6 + 3*a^2*c
*d^3*e^8 - a^3*d*e^10)*x^4 + 10*(c^3*d^8*e^3 - 3*a*c^2*d^6*e^5 + 3*a^2*c*d^
4*e^7 - a^3*d^2*e^9)*x^3 + 10*(c^3*d^9*e^2 - 3*a*c^2*d^7*e^4 + 3*a^2*c*d^5*
e^6 - a^3*d^3*e^8)*x^2 + 5*(c^3*d^10*e - 3*a*c^2*d^8*e^3 + 3*a^2*c*d^6*e^5
- a^3*d^4*e^7)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**7,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^7,x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.1930 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^8} dx$

Optimal. Leaf size=231

$$\frac{32c^3d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{1155(d+ex)^5\left(cd^2-ae^2\right)^4}+\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{231(d+ex)^6\left(cd^2-ae^2\right)^3}+\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{33(d+ex)^7\left(cd^2-ae^2\right)^2}$$

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(11*(c*d^2 - a*e^2)*(d +
e*x)^8) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(33*(c*d^2
- a*e^2)^2*(d + e*x)^7) + (16*c^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2))/(231*(c*d^2 - a*e^2)^3*(d + e*x)^6) + (32*c^3*d^3*(a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2)^(5/2))/(1155*(c*d^2 - a*e^2)^4*(d + e*x)^5)
```

Rubi [A] time = 0.115717, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {658, 650}

$$\frac{32c^3d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{1155(d+ex)^5\left(cd^2-ae^2\right)^4}+\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{231(d+ex)^6\left(cd^2-ae^2\right)^3}+\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{33(d+ex)^7\left(cd^2-ae^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^8,x]
```


```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(11*(c*d^2 - a*e^2)*(d +
e*x)^8) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(33*(c*d^2
- a*e^2)^2*(d + e*x)^7) + (16*c^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2))/(231*(c*d^2 - a*e^2)^3*(d + e*x)^6) + (32*c^3*d^3*(a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2)^(5/2))/(1155*(c*d^2 - a*e^2)^4*(d + e*x)^5)
```


Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^8}dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11\left(cd^2-ae^2\right)(d+ex)^8} + \frac{(6cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^7}dx}{11\left(cd^2-ae^2\right)}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11\left(cd^2-ae^2\right)(d+ex)^8} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{33\left(cd^2-ae^2\right)^2(d+ex)^7} + \cdots$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11\left(cd^2-ae^2\right)(d+ex)^8} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{33\left(cd^2-ae^2\right)^2(d+ex)^7} + \cdots$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11\left(cd^2-ae^2\right)(d+ex)^8} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{33\left(cd^2-ae^2\right)^2(d+ex)^7} + \cdots$$

Mathematica [A] time = 0.0909509, size = 138, normalized size = 0.6

$$\frac{2((d+ex)(ae+cdx))^{5/2}\left(35a^2cde^4(11d+2ex)-105a^3e^6-5ac^2d^2e^2\left(99d^2+44dex+8e^2x^2\right)+c^3d^3\left(198d^2ex+231d^3 \cdots\right.\right.}{1155(d+ex)^8\left(cd^2-ae^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^8,x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-105*a^3*e^6 + 35*a^2*c*d*e^4*(11*d + 2
*e*x) - 5*a*c^2*d^2*e^2*(99*d^2 + 44*d*e*x + 8*e^2*x^2) + c^3*d^3*(231*d^3
+ 198*d^2*e*x + 88*d*e^2*x^2 + 16*e^3*x^3)))/(1155*(c*d^2 - a*e^2)^4*(d + e
*x)^8)
```

Maple [A] time = 0.049, size = 217, normalized size = 0.9

$$-\frac{(2\,cdx+2\,ae)\left(-16\,c^3d^3e^3x^3+40\,ac^2d^2e^4x^2-88\,c^3d^4e^2x^2-70\,a^2cde^5x+220\,ac^2d^3e^3x-198\,c^3d^5ex+105\,a^3e^6-385 \cdots\right.}{1155\,(ex+d)^7\left(a^4e^8-4\,a^3cd^2e^6+6\,a^2c^2d^4e^4-4\,ac^3d^6e^2+c^4d^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^8,x)
```

```
[Out] -2/1155*(c*d*x+a*e)*(-16*c^3*d^3*e^3*x^3+40*a*c^2*d^2*e^4*x^2-88*c^3*d^4*e^
2*x^2-70*a^2*c*d*e^5*x+220*a*c^2*d^3*e^3*x-198*c^3*d^5*e*x+105*a^3*e^6-385*
a^2*c*d^2*e^4+495*a*c^2*d^4*e^2-231*c^3*d^6)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d
*e)^(3/2)/(e*x+d)^7/(a^4*e^8-4*a^3*c*d^2*e^6+6*a^2*c^2*d^4*e^4-4*a*c^3*d^6*
e^2+c^4*d^8)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^8,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^8,x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**8,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^8,x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.1931 $\int (d+ex)^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx$

Optimal. Leaf size=534

$$\frac{143\left(cd^2-ae^2\right)^8\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{131072c^7d^7e^3}-\frac{143\left(cd^2-ae^2\right)^6\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd\right.\right.}{49152c^6d^6e^2}$$

```
[Out] (143*(c*d^2 - a*e^2)^8*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])/(131072*c^7*d^7*e^3) - (143*(c*d^2 - a*e^2)^6*(c*d^2 +
 a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(49152*c
^6*d^6*e^2) + (143*(c*d^2 - a*e^2)^4*(c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(15360*c^5*d^5*e) + (143*(c*d^2 - a*e^
2)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(4480*c^4*d^4) + (143*(
c*d^2 - a*e^2)^2*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(
2880*c^3*d^3) + (13*(c*d^2 - a*e^2)*(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(7/2))/(180*c^2*d^2) + ((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2)^(7/2))/(10*c*d) - (143*(c*d^2 - a*e^2)^10*ArcTanh[(c*d^2 + a*
e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2])])/(262144*c^(15/2)*d^(15/2)*e^(7/2))
```

Rubi [A] time = 0.676372, antiderivative size = 534, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {670, 640, 612, 621, 206}

$$\frac{143\left(cd^2-ae^2\right)^8\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{131072c^7d^7e^3}-\frac{143\left(cd^2-ae^2\right)^6\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd\right.\right.}{49152c^6d^6e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```


```
[Out] (143*(c*d^2 - a*e^2)^8*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])/(131072*c^7*d^7*e^3) - (143*(c*d^2 - a*e^2)^6*(c*d^2 +
 a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(49152*c
^6*d^6*e^2) + (143*(c*d^2 - a*e^2)^4*(c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(15360*c^5*d^5*e) + (143*(c*d^2 - a*e^
2)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(4480*c^4*d^4) + (143*(
c*d^2 - a*e^2)^2*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(
2880*c^3*d^3) + (13*(c*d^2 - a*e^2)*(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(7/2))/(180*c^2*d^2) + ((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2)^(7/2))/(10*c*d) - (143*(c*d^2 - a*e^2)^10*ArcTanh[(c*d^2 + a*
e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2])])/(262144*c^(15/2)*d^(15/2)*e^(7/2))
```


Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}dx = \frac{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{10cd}+\frac{\left(13\left(d^2-\frac{ae^2}{c}\right)\right)\int(d+ex)^3}{}$$

$$=\frac{13\left(cd^2-ae^2\right)(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{180c^2d^2}+\frac{(d+ex)^3\left(a\right.}{}$$

$$=\frac{143\left(cd^2-ae^2\right)^2(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{2880c^3d^3}+\frac{13\left(cd^2-a\right.}{}$$

$$=\frac{143\left(cd^2-ae^2\right)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{4480c^4d^4}+\frac{143\left(cd^2-ae^2\right)^2(d}{}$$

$$=\frac{143\left(cd^2-ae^2\right)^4\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{15360c^5d^5e}+$$

$$=-\frac{143\left(cd^2-ae^2\right)^6\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{49152c^6d^6e^2}$$

$$=\frac{143\left(cd^2-ae^2\right)^8\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{131072c^7d^7e^3}-\frac{14}{}$$

$$=\frac{143\left(cd^2-ae^2\right)^8\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{131072c^7d^7e^3}-\frac{14}{}$$

$$=\frac{143\left(cd^2-ae^2\right)^8\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{131072c^7d^7e^3}-\frac{14}{}$$

Mathematica [B] time = 6.8119, size = 1439, normalized size = 2.69

$$\frac{2\left(cd^2-ae^2\right)^6 (ae+cdx)((ae+cdx)(d+ex))^{5/2}\left(\frac{cde(ae+cdx)}{(cd^2-ae^2)\left(\frac{c^2d^3}{cd^2-ae^2}-\frac{acde^2}{cd^2-ae^2}\right)}+1\right)^{15/2}\left(\frac{1001(cd^2-ae^2)^4\left(\frac{16c^3d^3e^3(ae+cdx)^3}{15(cd^2-ae^2)^3\left(\frac{c^2d^3}{cd^2-ae^2}-\frac{acde^2}{cd^2-ae^2}\right)^3}-\cdots\right.}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (2*(c*d^2 - a*e^2)^6*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^(5/2)*(1 + (c*
d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2
)/(c*d^2 - a*e^2))))^(15/2)*((7*(143/(4096*(1 + (c*d*e*(a*e + c*d*x))/((c*d
^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))))^7)
+ 143/(1536*(1 + (c*d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 -
 a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))))^6) + 143/(768*(1 + (c*d*e*(a*e + c
*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a
*e^2))))^5) + 143/(448*(1 + (c*d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^
3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))))^4) + 143/(288*(1 + (c*d
*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)
/(c*d^2 - a*e^2))))^3) + 13/(18*(1 + (c*d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)
*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))))^2) + (1 + (c*d
*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)
/(c*d^2 - a*e^2))))^(-1)))/20 + (1001*(c*d^2 - a*e^2)^4*((c^2*d^3)/(c*d^2 -
 a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))^4*((2*c*d*e*(a*e + c*d*x))/((c*d^2 -
 a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))) - (4*c^2
*d^2*e^2*(a*e + c*d*x)^2)/(3*(c*d^2 - a*e^2)^2*((c^2*d^3)/(c*d^2 - a*e^2) -
 (a*c*d*e^2)/(c*d^2 - a*e^2))^2) + (16*c^3*d^3*e^3*(a*e + c*d*x)^3)/(15*(c*
d^2 - a*e^2)^3*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))^3)
 - (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sq
rt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d^2 - a*e^2]*Sqrt[(c^2*d^3)/(c*d^2 - a*e^2
) - (a*c*d*e^2)/(c*d^2 - a*e^2)])])/(Sqrt[c*d^2 - a*e^2]*Sqrt[(c^2*d^3)/(c*
d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2)]*Sqrt[1 + (c*d*e*(a*e + c*d*x))/
((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2)))
])))/(524288*c^4*d^4*e^4*(a*e + c*d*x)^4*(1 + (c*d*e*(a*e + c*d*x))/((c*d^2
 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))))^7)))/
(7*c^7*d^7*((c*d)/((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))
)^(13/2)*(d + e*x)^2*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])
```

Maple [B] time = 0.065, size = 2973, normalized size = 5.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)

[Out] -143/4480*e^6/d^4/c^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)*a^3-1001/131072/e*d^9*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a+1423/2880*e*d/c*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)+1001/65536*e^5*d^3/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^4+1/10*e^3*x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/d/c+1001/65536*e^7*d/c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^5-429/16384*e^8/d^2/c^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^5+143/65536/e^2*d^10*c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+143/131072*e^15/d^7/c^7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^9-1001/131072*e^13/d^5/c^6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^8+715/32768*e^11/d^3/c^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^7-143/49152/e^2*d^8*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+143/131072/e^3*d^11*c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+715/49152*e^6/c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^4+67/180*e^2/c*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)-10777/40320*e^2/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)*a+715/32768*e*d^7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2+143/15360/e*d^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+143/7680*d^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*x+715/49152*d^6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a+1137/4480*d^2/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)-143/5120*e*d^3/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*a+5863/40320*e^4/d^2/c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)*a^2-143/262144/e^3*d^13*c^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)-143/24576/e*d^7*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x+715/49152*e^4*d^2/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^3-429/16384*e^2*d^4/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^2+143/15360*e^9/d^5/c^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*a^5-143/5120*e^7/d^3/c^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*a^4-143/49152*e^12/d^6/c^6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^7+715/49152*e^10/d^4/c^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^6+143/7680*e^5/d/c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*a^3+143/7680*e^3*d/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*a^2+143/4096*e*d^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*a+143/1280*e^4/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*x*a^2-1001/8192*e^8/c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^5+1001/16384*e^2*d^6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^2+2145/32768*e^3*d^7*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^3-1001/32768*e^3*d^5/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^3-1001/32768*e^9/d/c^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^6-143/8192*d^8*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a-39/160*e^3/d/c^2*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)*a-715/8192*e^3*d^3/c*a^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x-143/1920*e^2*d^2/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*x*a+2145/32768*e^11/d/c^4*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^7-15015/131072*e^9*d/c^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^6+143/65536*e^14/d^6/c^6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^8-143/8192*e^12/d^4/c^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^7+1001/16384*e^10/d^2/c^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^6+715/131072/e*d^11*c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a-13/180*e^4/d^2/c^2*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)*a+9009/65536*e^7*d^3/c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^5-15015/131072*e^5*d^5/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^4+715/6144*e^5*d/c^2*a^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x+5005/32768*e^6*d^2/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^4-1001/8192*e^4*d^4/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^3+143/2880*e^5/d^3/c^3*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)*a^2-6435/262144*e*d^9*c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^2+143/4096*e^9/d^3/c^4*a^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x-715/8192*e^7/d/c^

```
3*a^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x+143/7680*e^8/d^4/c^4*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*x*a^4-143/1920*e^6/d^2/c^3*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(5/2)*x*a^3-143/24576*e^11/d^5/c^5*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(3/2)*x*a^6-143/262144*e^17/d^7/c^7*ln((1/2*a*e^2+1/2*c*d^2+c
*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2
)*a^10+715/131072*e^15/d^5/c^6*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/
2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^9-6435/262144*e
^13/d^3/c^5*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^8
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="m
axima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 4.02886, size = 5203, normalized size = 9.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="f
ricas")

[Out] [1/165150720*(45045*(c^10*d^20 - 10*a*c^9*d^18*e^2 + 45*a^2*c^8*d^16*e^4 -
120*a^3*c^7*d^14*e^6 + 210*a^4*c^6*d^12*e^8 - 252*a^5*c^5*d^10*e^10 + 210*a
^6*c^4*d^8*e^12 - 120*a^7*c^3*d^6*e^14 + 45*a^8*c^2*d^4*e^16 - 10*a^9*c*d^2
*e^18 + a^10*e^20)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*
e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x +
c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(4128768*c^10
*d^10*e^10*x^9 + 45045*c^10*d^19*e - 435435*a*c^9*d^17*e^3 + 1885884*a^2*c^
8*d^15*e^5 + 6983100*a^3*c^7*d^13*e^7 - 9035650*a^4*c^6*d^11*e^9 + 8003710*
a^5*c^5*d^9*e^11 - 4813380*a^6*c^4*d^7*e^13 + 1885884*a^7*c^3*d^5*e^15 - 43
5435*a^8*c^2*d^3*e^17 + 45045*a^9*c*d*e^19 + 229376*(121*c^10*d^11*e^9 + 41
*a*c^9*d^9*e^11)*x^8 + 14336*(5503*c^10*d^12*e^8 + 4482*a*c^9*d^10*e^10 + 3
83*a^2*c^8*d^8*e^12)*x^7 + 1024*(119055*c^10*d^13*e^7 + 182129*a*c^9*d^11*e
^9 + 37489*a^2*c^8*d^9*e^11 + 15*a^3*c^7*d^7*e^13)*x^6 + 256*(424895*c^10*d
^14*e^6 + 1157740*a*c^9*d^12*e^8 + 448938*a^2*c^8*d^10*e^10 + 620*a^3*c^7*d
^8*e^12 - 65*a^4*c^6*d^6*e^14)*x^5 + 128*(419983*c^10*d^15*e^5 + 2149035*a*
c^9*d^13*e^7 + 1490630*a^2*c^8*d^11*e^9 + 5830*a^3*c^7*d^9*e^11 - 1365*a^4*
c^6*d^7*e^13 + 143*a^5*c^5*d^5*e^15)*x^4 + 16*(735993*c^10*d^16*e^4 + 90234
98*a*c^9*d^14*e^6 + 11825815*a^2*c^8*d^12*e^8 + 132300*a^3*c^7*d^10*e^10 -
52585*a^4*c^6*d^8*e^12 + 12298*a^5*c^5*d^6*e^14 - 1287*a^6*c^4*d^4*e^16)*x^
3 + 8*(3003*c^10*d^17*e^3 + 4394937*a*c^9*d^15*e^5 + 13885683*a^2*c^8*d^13*
e^7 + 508825*a^3*c^7*d^11*e^9 - 310375*a^4*c^6*d^9*e^11 + 123123*a^5*c^5*d^
7*e^13 - 28743*a^6*c^4*d^5*e^15 + 3003*a^7*c^3*d^3*e^17)*x^2 - 2*(15015*c^1
0*d^18*e^2 - 144144*a*c^9*d^16*e^4 - 17075244*a^2*c^8*d^14*e^6 - 2878000*a^
3*c^7*d^12*e^8 + 2579850*a^4*c^6*d^10*e^10 - 1567280*a^5*c^5*d^8*e^12 + 619
476*a^6*c^4*d^6*e^14 - 144144*a^7*c^3*d^4*e^16 + 15015*a^8*c^2*d^2*e^18)*x)
```

```
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^8*d^8*e^4), 1/82575360*(45
045*(c^10*d^20 - 10*a*c^9*d^18*e^2 + 45*a^2*c^8*d^16*e^4 - 120*a^3*c^7*d^14
*e^6 + 210*a^4*c^6*d^12*e^8 - 252*a^5*c^5*d^10*e^10 + 210*a^6*c^4*d^8*e^12
- 120*a^7*c^3*d^6*e^14 + 45*a^8*c^2*d^4*e^16 - 10*a^9*c*d^2*e^18 + a^10*e^2
0)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c
*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*
d^3*e + a*c*d*e^3)*x)) + 2*(4128768*c^10*d^10*e^10*x^9 + 45045*c^10*d^19*e
- 435435*a*c^9*d^17*e^3 + 1885884*a^2*c^8*d^15*e^5 + 6983100*a^3*c^7*d^13*e
^7 - 9035650*a^4*c^6*d^11*e^9 + 8003710*a^5*c^5*d^9*e^11 - 4813380*a^6*c^4*
d^7*e^13 + 1885884*a^7*c^3*d^5*e^15 - 435435*a^8*c^2*d^3*e^17 + 45045*a^9*c
*d*e^19 + 229376*(121*c^10*d^11*e^9 + 41*a*c^9*d^9*e^11)*x^8 + 14336*(5503*
c^10*d^12*e^8 + 4482*a*c^9*d^10*e^10 + 383*a^2*c^8*d^8*e^12)*x^7 + 1024*(11
9055*c^10*d^13*e^7 + 182129*a*c^9*d^11*e^9 + 37489*a^2*c^8*d^9*e^11 + 15*a^
3*c^7*d^7*e^13)*x^6 + 256*(424895*c^10*d^14*e^6 + 1157740*a*c^9*d^12*e^8 +
448938*a^2*c^8*d^10*e^10 + 620*a^3*c^7*d^8*e^12 - 65*a^4*c^6*d^6*e^14)*x^5
+ 128*(419983*c^10*d^15*e^5 + 2149035*a*c^9*d^13*e^7 + 1490630*a^2*c^8*d^11
*e^9 + 5830*a^3*c^7*d^9*e^11 - 1365*a^4*c^6*d^7*e^13 + 143*a^5*c^5*d^5*e^15
)*x^4 + 16*(735993*c^10*d^16*e^4 + 9023498*a*c^9*d^14*e^6 + 11825815*a^2*c^
8*d^12*e^8 + 132300*a^3*c^7*d^10*e^10 - 52585*a^4*c^6*d^8*e^12 + 12298*a^5*
c^5*d^6*e^14 - 1287*a^6*c^4*d^4*e^16)*x^3 + 8*(3003*c^10*d^17*e^3 + 4394937
*a*c^9*d^15*e^5 + 13885683*a^2*c^8*d^13*e^7 + 508825*a^3*c^7*d^11*e^9 - 310
375*a^4*c^6*d^9*e^11 + 123123*a^5*c^5*d^7*e^13 - 28743*a^6*c^4*d^5*e^15 + 3
003*a^7*c^3*d^3*e^17)*x^2 - 2*(15015*c^10*d^18*e^2 - 144144*a*c^9*d^16*e^4
- 17075244*a^2*c^8*d^14*e^6 - 2878000*a^3*c^7*d^12*e^8 + 2579850*a^4*c^6*d^
10*e^10 - 1567280*a^5*c^5*d^8*e^12 + 619476*a^6*c^4*d^6*e^14 - 144144*a^7*c
^3*d^4*e^16 + 15015*a^8*c^2*d^2*e^18)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x))/(c^8*d^8*e^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)

[Out] Timed out
```

Giac [B] time = 2.01313, size = 1397, normalized size = 2.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="g
iac")

[Out] 1/41287680*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*(2*(4*(2*(8*(2*(4*(1
4*(16*(18*c^2*d^2*x*e^6 + (121*c^11*d^12*e^14 + 41*a*c^10*d^10*e^16)*e^(-9)
/(c^9*d^9))*x + (5503*c^11*d^13*e^13 + 4482*a*c^10*d^11*e^15 + 383*a^2*c^9*
d^9*e^17)*e^(-9)/(c^9*d^9))*x + (119055*c^11*d^14*e^12 + 182129*a*c^10*d^12
*e^14 + 37489*a^2*c^9*d^10*e^16 + 15*a^3*c^8*d^8*e^18)*e^(-9)/(c^9*d^9))*x
+ (424895*c^11*d^15*e^11 + 1157740*a*c^10*d^13*e^13 + 448938*a^2*c^9*d^11*e
^15 + 620*a^3*c^8*d^9*e^17 - 65*a^4*c^7*d^7*e^19)*e^(-9)/(c^9*d^9))*x + (41
```

9983*c^11*d^16*e^10 + 2149035*a*c^10*d^14*e^12 + 1490630*a^2*c^9*d^12*e^14 + 5830*a^3*c^8*d^10*e^16 - 1365*a^4*c^7*d^8*e^18 + 143*a^5*c^6*d^6*e^20)*e^(-9)/(c^9*d^9))*x + (735993*c^11*d^17*e^9 + 9023498*a*c^10*d^15*e^11 + 11825815*a^2*c^9*d^13*e^13 + 132300*a^3*c^8*d^11*e^15 - 52585*a^4*c^7*d^9*e^17 + 12298*a^5*c^6*d^7*e^19 - 1287*a^6*c^5*d^5*e^21)*e^(-9)/(c^9*d^9))*x + (3003*c^11*d^18*e^8 + 4394937*a*c^10*d^16*e^10 + 13885683*a^2*c^9*d^14*e^12 + 508825*a^3*c^8*d^12*e^14 - 310375*a^4*c^7*d^10*e^16 + 123123*a^5*c^6*d^8*e^18 - 28743*a^6*c^5*d^6*e^20 + 3003*a^7*c^4*d^4*e^22)*e^(-9)/(c^9*d^9))*x - (15015*c^11*d^19*e^7 - 144144*a*c^10*d^17*e^9 - 17075244*a^2*c^9*d^15*e^11 - 2878000*a^3*c^8*d^13*e^13 + 2579850*a^4*c^7*d^11*e^15 - 1567280*a^5*c^6*d^9*e^17 + 619476*a^6*c^5*d^7*e^19 - 144144*a^7*c^4*d^5*e^21 + 15015*a^8*c^3*d^3*e^23)*e^(-9)/(c^9*d^9))*x + (45045*c^11*d^20*e^6 - 435435*a*c^10*d^18*e^8 + 1885884*a^2*c^9*d^16*e^10 + 6983100*a^3*c^8*d^14*e^12 - 9035650*a^4*c^7*d^12*e^14 + 8003710*a^5*c^6*d^10*e^16 - 4813380*a^6*c^5*d^8*e^18 + 1885884*a^7*c^4*d^6*e^20 - 435435*a^8*c^3*d^4*e^22 + 45045*a^9*c^2*d^2*e^24)*e^(-9)/(c^9*d^9)) + 143/262144*(c^10*d^20 - 10*a*c^9*d^18*e^2 + 45*a^2*c^8*d^16*e^4 - 120*a^3*c^7*d^14*e^6 + 210*a^4*c^6*d^12*e^8 - 252*a^5*c^5*d^10*e^10 + 210*a^6*c^4*d^8*e^12 - 120*a^7*c^3*d^6*e^14 + 45*a^8*c^2*d^4*e^16 - 10*a^9*c*d^2*e^18 + a^10*e^20)*sqrt(c*d)*e^(-7/2)*log(abs(-sqrt(c*d)*c*d^2*e^(1/2) - 2*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*c*d*e - sqrt(c*d)*a*e^(5/2)))/(c^8*d^8)

3.1932 $\int (d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2} dx$

Optimal. Leaf size=474

$$\frac{55\left(cd^2-ae^2\right)^7\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{32768c^6d^6e^3}-\frac{55\left(cd^2-ae^2\right)^5\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+\right.}{12288c^5d^5e^2}$$

```
[Out] (55*(c*d^2 - a*e^2)^7*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])/(32768*c^6*d^6*e^3) - (55*(c*d^2 - a*e^2)^5*(c*d^2 + a*
e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(12288*c^5*
d^5*e^2) + (11*(c*d^2 - a*e^2)^3*(c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(768*c^4*d^4*e) + (11*(c*d^2 - a*e^2)^2*(a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(224*c^3*d^3) + (11*(c*d^2 - a
*e^2)*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(144*c^2*d^2
) + ((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(9*c*d) - (
55*(c*d^2 - a*e^2)^9*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]
*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(65536*c^(13/2)*d^(
13/2)*e^(7/2))
```

Rubi [A] time = 0.492717, antiderivative size = 474, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {670, 640, 612, 621, 206}

$$\frac{55\left(cd^2-ae^2\right)^7\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{32768c^6d^6e^3}-\frac{55\left(cd^2-ae^2\right)^5\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+\right.}{12288c^5d^5e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (55*(c*d^2 - a*e^2)^7*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])/(32768*c^6*d^6*e^3) - (55*(c*d^2 - a*e^2)^5*(c*d^2 + a*
e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(12288*c^5*
d^5*e^2) + (11*(c*d^2 - a*e^2)^3*(c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(768*c^4*d^4*e) + (11*(c*d^2 - a*e^2)^2*(a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(224*c^3*d^3) + (11*(c*d^2 - a
*e^2)*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(144*c^2*d^2
) + ((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(9*c*d) - (
55*(c*d^2 - a*e^2)^9*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]
*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(65536*c^(13/2)*d^(
13/2)*e^(7/2))
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx=\frac{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{9cd}+\frac{\left(11\left(d^2-\frac{ae^2}{c}\right)\right)\int (d+e\ldots}{\ldots}$$

$$=\frac{11\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{144c^2d^2}+\frac{(d+ex)^2\left(\ldots\right.}{\ldots}$$

$$=\frac{11\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{224c^3d^3}+\frac{11\left(cd^2-ae^2\right)(d\ldots}{\ldots}$$

$$=\frac{11\left(cd^2-ae^2\right)^3\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{768c^4d^4e}\ldots$$

$$=-\frac{55\left(cd^2-ae^2\right)^5\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/\ldots}}{12288c^5d^5e^2}\ldots$$

$$=\frac{55\left(cd^2-ae^2\right)^7\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32768c^6d^6e^3}-\ldots$$

$$=\frac{55\left(cd^2-ae^2\right)^7\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32768c^6d^6e^3}-\ldots$$

$$=\frac{55\left(cd^2-ae^2\right)^7\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32768c^6d^6e^3}-\ldots$$

Mathematica [B] time = 6.51024, size = 1359, normalized size = 2.87

$$\frac{2\left(cd^2-ae^2\right)^5 (ae+cdx)((ae+cdx)(d+ex))^{5/2}\left(\frac{cde(ae+cdx)}{\left(cd^2-ae^2\right)\left(\frac{c^2d^3}{cd^2-ae^2}-\frac{acde^2}{cd^2-ae^2}\right)}+1\right)^{13/2}\left(\frac{385\left(cd^2-ae^2\right)^4\left(\frac{16c^3d^3e^3(ae+cdx)^3}{15\left(cd^2-ae^2\right)^3\left(\frac{c^2d^3}{cd^2-ae^2}-\frac{acde^2}{cd^2-ae^2}\right)^3}-\frac{\cdots}{3(cd^2-\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (2*(c*d^2 - a*e^2)^5*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^(5/2)*(1 + (c*
d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2
)/(c*d^2 - a*e^2))))^(13/2)*((7*(99/(2048*(1 + (c*d*e*(a*e + c*d*x))/((c*d^
2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))))^6) +
 33/(256*(1 + (c*d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*
e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))))^5) + 33/(128*(1 + (c*d*e*(a*e + c*d*x
))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2
))))^4) + 99/(224*(1 + (c*d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c
*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))))^3) + 11/(16*(1 + (c*d*e*(a*e
 + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2
 - a*e^2))))^2) + (1 + (c*d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c
*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))))^(-1)))/18 + (385*(c*d^2 - a*
e^2)^4*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))^4*((2*c*d*
e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/
(c*d^2 - a*e^2))) - (4*c^2*d^2*e^2*(a*e + c*d*x)^2)/(3*(c*d^2 - a*e^2)^2*((
c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))^2) + (16*c^3*d^3*e^
3*(a*e + c*d*x)^3)/(15*(c*d^2 - a*e^2)^3*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*
d*e^2)/(c*d^2 - a*e^2))^3) - (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*A
rcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d^2 - a*e^2]*Sqr
t[(c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2)])])/(Sqrt[c*d^2 -
 a*e^2]*Sqrt[(c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2)]*Sqrt[
1 + (c*d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*
c*d*e^2)/(c*d^2 - a*e^2)))])))/(131072*c^4*d^4*e^4*(a*e + c*d*x)^4*(1 + (c*
d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2
)/(c*d^2 - a*e^2))))^6)))/(7*c^6*d^6*((c*d)/((c^2*d^3)/(c*d^2 - a*e^2) - (a
*c*d*e^2)/(c*d^2 - a*e^2)))^(11/2)*(d + e*x)^2*Sqrt[(c*d*(d + e*x))/(c*d^2
- a*e^2)])
```

Maple [B] time = 0.063, size = 2368, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] -11/768*e^7/d^4/c^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*a^4+11/384*e^5/
d^2/c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*a^3+55/16384/e^2*d^9*c^2*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x-165/16384/e*d^8*c*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)*a-385/16384*d^7*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2)*x*a+1/9*e^2*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/d/c-1155/1638
4*e^9/c^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^6+275/6144*e*d^4*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(3/2)*x*a+275/3072*e^5/c^2*a^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(3/2)*x+1155/16384*e^3*d^6*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1
/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^3+1155/16384*e
^2*d^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^2-55/32768*e^13/d^6/c^6*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^8+165/16384*e^11/d^4/c^5*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^7-275/12288*e^2*d^3/c*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(3/2)*a^2-11/63*e^2/d/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(7/2)*a-385/16384*e^3*d^4/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^3+55/
12288*e^10/d^5/c^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^6-55/3072*e^8/
d^3/c^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^5+275/12288*e^6/d/c^3*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^4-55/6144/e*d^6*c*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(3/2)*x-55/65536/e^3*d^12*c^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e
*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+11
/224*e^4/d^3/c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)*a^2-11/384*e*d^2/c
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*a-385/16384*e^9/d^2/c^4*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^6+53/224*d/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(7/2)+55/3072*d^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a+11/768/e*d^
4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+11/384*d^3*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(5/2)*x-55/12288/e^2*d^7*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2)+55/32768/e^3*d^10*c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+43/144*e/
c*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)+385/16384*e^7/c^3*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)*a^5+385/16384*e*d^6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)*a^2+495/65536/e*d^10*c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*
c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a-11/144*e^
3/d^2/c^2*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)*a-275/6144*e^7/d^2/c^3*
a^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x-495/16384*e*d^8*c*ln((1/2*a*e
^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
)/(d*e*c)^(1/2)*a^2-275/3072*e^3*d^2/c*a^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(3/2)*x+11/128*e^4/d/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*x*a^2-19
25/16384*e^4*d^3/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^3-11/128*e^2
*d/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*x*a+55/65536*e^15/d^6/c^6*ln((
1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2))/(d*e*c)^(1/2)*a^9-495/65536*e^13/d^4/c^5*ln((1/2*a*e^2+1/2*c*d^2+c
*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2
)*a^8+385/16384*e^10/d^3/c^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^6-
1155/16384*e^8/d/c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^5+1925/163
84*e^6*d/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^4-11/384*e^6/d^3/c
^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*x*a^3+495/16384*e^11/d^2/c^4*ln(
(1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2))/(d*e*c)^(1/2)*a^7+3465/32768*e^7*d^2/c^2*ln((1/2*a*e^2+1/2*c*d^2+
c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/
2)*a^5-3465/32768*e^5*d^4/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^4+55/6144*e^9/d^4/
c^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*a^5-55/16384*e^12/d^5/c^5*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^7
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.30871, size = 4213, normalized size = 8.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="f
ricas")
```

```
[Out] [1/8257536*(3465*(c^9*d^18 - 9*a*c^8*d^16*e^2 + 36*a^2*c^7*d^14*e^4 - 84*a^
3*c^6*d^12*e^6 + 126*a^4*c^5*d^10*e^8 - 126*a^5*c^4*d^8*e^10 + 84*a^6*c^3*d
^6*e^12 - 36*a^7*c^2*d^4*e^14 + 9*a^8*c*d^2*e^16 - a^9*e^18)*sqrt(c*d*e)*lo
g(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c
^2*d^3*e + a*c*d*e^3)*x) + 4*(229376*c^9*d^9*e^9*x^8 + 3465*c^9*d^17*e - 30
030*a*c^8*d^15*e^3 + 115038*a^2*c^7*d^13*e^5 + 334602*a^3*c^6*d^11*e^7 - 36
0448*a^4*c^5*d^9*e^9 + 255222*a^5*c^4*d^7*e^11 - 115038*a^6*c^3*d^5*e^13 +
30030*a^7*c^2*d^3*e^15 - 3465*a^8*c*d*e^17 + 14336*(91*c^9*d^10*e^8 + 37*a*
c^8*d^8*e^10)*x^7 + 1024*(2955*c^9*d^11*e^7 + 3008*a*c^8*d^9*e^9 + 309*a^2*
c^7*d^7*e^11)*x^6 + 256*(14075*c^9*d^12*e^6 + 28695*a*c^8*d^10*e^8 + 7401*a
^2*c^7*d^8*e^10 + 5*a^3*c^6*d^6*e^12)*x^5 + 128*(17419*c^9*d^13*e^5 + 71074
*a*c^8*d^11*e^7 + 36864*a^2*c^7*d^9*e^9 + 94*a^3*c^6*d^7*e^11 - 11*a^4*c^5*
d^5*e^13)*x^4 + 16*(36765*c^9*d^14*e^4 + 373583*a*c^8*d^12*e^6 + 390018*a^2
*c^7*d^10*e^8 + 3198*a^3*c^6*d^8*e^10 - 847*a^4*c^5*d^6*e^12 + 99*a^5*c^4*d
^4*e^14)*x^3 + 8*(231*c^9*d^15*e^3 + 219204*a*c^8*d^13*e^5 + 572739*a^2*c^7
*d^11*e^7 + 16384*a^3*c^6*d^9*e^9 - 7491*a^4*c^5*d^7*e^11 + 1980*a^5*c^4*d^
5*e^13 - 231*a^6*c^3*d^3*e^15)*x^2 - 2*(1155*c^9*d^16*e^2 - 9933*a*c^8*d^14
*e^4 - 847017*a^2*c^7*d^12*e^6 - 115609*a^3*c^6*d^10*e^8 + 82841*a^4*c^5*d^
8*e^10 - 37719*a^5*c^4*d^6*e^12 + 9933*a^6*c^3*d^4*e^14 - 1155*a^7*c^2*d^2*
e^16)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^7*d^7*e^4), 1/4128
768*(3465*(c^9*d^18 - 9*a*c^8*d^16*e^2 + 36*a^2*c^7*d^14*e^4 - 84*a^3*c^6*d
^12*e^6 + 126*a^4*c^5*d^10*e^8 - 126*a^5*c^4*d^8*e^10 + 84*a^6*c^3*d^6*e^12
 - 36*a^7*c^2*d^4*e^14 + 9*a^8*c*d^2*e^16 - a^9*e^18)*sqrt(-c*d*e)*arctan(1
/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*
sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) +
 2*(229376*c^9*d^9*e^9*x^8 + 3465*c^9*d^17*e - 30030*a*c^8*d^15*e^3 + 11503
8*a^2*c^7*d^13*e^5 + 334602*a^3*c^6*d^11*e^7 - 360448*a^4*c^5*d^9*e^9 + 255
222*a^5*c^4*d^7*e^11 - 115038*a^6*c^3*d^5*e^13 + 30030*a^7*c^2*d^3*e^15 - 3
465*a^8*c*d*e^17 + 14336*(91*c^9*d^10*e^8 + 37*a*c^8*d^8*e^10)*x^7 + 1024*(
2955*c^9*d^11*e^7 + 3008*a*c^8*d^9*e^9 + 309*a^2*c^7*d^7*e^11)*x^6 + 256*(1
4075*c^9*d^12*e^6 + 28695*a*c^8*d^10*e^8 + 7401*a^2*c^7*d^8*e^10 + 5*a^3*c^
6*d^6*e^12)*x^5 + 128*(17419*c^9*d^13*e^5 + 71074*a*c^8*d^11*e^7 + 36864*a^
2*c^7*d^9*e^9 + 94*a^3*c^6*d^7*e^11 - 11*a^4*c^5*d^5*e^13)*x^4 + 16*(36765*
c^9*d^14*e^4 + 373583*a*c^8*d^12*e^6 + 390018*a^2*c^7*d^10*e^8 + 3198*a^3*c
^6*d^8*e^10 - 847*a^4*c^5*d^6*e^12 + 99*a^5*c^4*d^4*e^14)*x^3 + 8*(231*c^9*
d^15*e^3 + 219204*a*c^8*d^13*e^5 + 572739*a^2*c^7*d^11*e^7 + 16384*a^3*c^6*
d^9*e^9 - 7491*a^4*c^5*d^7*e^11 + 1980*a^5*c^4*d^5*e^13 - 231*a^6*c^3*d^3*e
^15)*x^2 - 2*(1155*c^9*d^16*e^2 - 9933*a*c^8*d^14*e^4 - 847017*a^2*c^7*d^12
*e^6 - 115609*a^3*c^6*d^10*e^8 + 82841*a^4*c^5*d^8*e^10 - 37719*a^5*c^4*d^6
*e^12 + 9933*a^6*c^3*d^4*e^14 - 1155*a^7*c^2*d^2*e^16)*x)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x))/(c^7*d^7*e^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.74539, size = 1193, normalized size = 2.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="g
iac")
```

```
[Out] 1/2064384*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*(2*(4*(2*(8*(2*(4*(14
*(16*c^2*d^2*x*e^5 + (91*c^10*d^11*e^12 + 37*a*c^9*d^9*e^14)*e^(-8)/(c^8*d^
8))*x + (2955*c^10*d^12*e^11 + 3008*a*c^9*d^10*e^13 + 309*a^2*c^8*d^8*e^15)
*e^(-8)/(c^8*d^8))*x + (14075*c^10*d^13*e^10 + 28695*a*c^9*d^11*e^12 + 7401
*a^2*c^8*d^9*e^14 + 5*a^3*c^7*d^7*e^16)*e^(-8)/(c^8*d^8))*x + (17419*c^10*d
^14*e^9 + 71074*a*c^9*d^12*e^11 + 36864*a^2*c^8*d^10*e^13 + 94*a^3*c^7*d^8*
e^15 - 11*a^4*c^6*d^6*e^17)*e^(-8)/(c^8*d^8))*x + (36765*c^10*d^15*e^8 + 37
3583*a*c^9*d^13*e^10 + 390018*a^2*c^8*d^11*e^12 + 3198*a^3*c^7*d^9*e^14 - 8
47*a^4*c^6*d^7*e^16 + 99*a^5*c^5*d^5*e^18)*e^(-8)/(c^8*d^8))*x + (231*c^10*
d^16*e^7 + 219204*a*c^9*d^14*e^9 + 572739*a^2*c^8*d^12*e^11 + 16384*a^3*c^7
*d^10*e^13 - 7491*a^4*c^6*d^8*e^15 + 1980*a^5*c^5*d^6*e^17 - 231*a^6*c^4*d^
4*e^19)*e^(-8)/(c^8*d^8))*x - (1155*c^10*d^17*e^6 - 9933*a*c^9*d^15*e^8 - 8
47017*a^2*c^8*d^13*e^10 - 115609*a^3*c^7*d^11*e^12 + 82841*a^4*c^6*d^9*e^14
 - 37719*a^5*c^5*d^7*e^16 + 9933*a^6*c^4*d^5*e^18 - 1155*a^7*c^3*d^3*e^20)*
e^(-8)/(c^8*d^8))*x + (3465*c^10*d^18*e^5 - 30030*a*c^9*d^16*e^7 + 115038*a
^2*c^8*d^14*e^9 + 334602*a^3*c^7*d^12*e^11 - 360448*a^4*c^6*d^10*e^13 + 255
222*a^5*c^5*d^8*e^15 - 115038*a^6*c^4*d^6*e^17 + 30030*a^7*c^3*d^4*e^19 - 3
465*a^8*c^2*d^2*e^21)*e^(-8)/(c^8*d^8)) + 55/65536*(c^9*d^18 - 9*a*c^8*d^16
*e^2 + 36*a^2*c^7*d^14*e^4 - 84*a^3*c^6*d^12*e^6 + 126*a^4*c^5*d^10*e^8 - 1
26*a^5*c^4*d^8*e^10 + 84*a^6*c^3*d^6*e^12 - 36*a^7*c^2*d^4*e^14 + 9*a^8*c*d
^2*e^16 - a^9*e^18)*sqrt(c*d)*e^(-7/2)*log(abs(-sqrt(c*d)*c*d^2*e^(1/2) - 2
*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*c*d*e
- sqrt(c*d)*a*e^(5/2)))/(c^7*d^7)
```

3.1933 $\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx$

Optimal. Leaf size=414

$$\frac{45\left(cd^2-ae^2\right)^6\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{16384c^5d^5e^3}-\frac{15\left(cd^2-ae^2\right)^4\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+\cdots\right.}{2048c^4d^4e^2}$$

```
[Out] (45*(c*d^2 - a*e^2)^6*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2]])/(16384*c^5*d^5*e^3) - (15*(c*d^2 - a*e^2)^4*(c*d^2 + a*
e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(2048*c^4*d
^4*e^2) + (3*(c*d^2 - a*e^2)^2*(c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2)^(5/2))/(128*c^3*d^3*e) + (9*(c*d^2 - a*e^2)*(a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(112*c^2*d^2) + ((d + e*x)*(a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(8*c*d) - (45*(c*d^2 - a*e^2)^8*ArcT
anh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2])])/(32768*c^(11/2)*d^(11/2)*e^(7/2))
```

Rubi [A] time = 0.35434, antiderivative size = 414, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {670, 640, 612, 621, 206}

$$\frac{45\left(cd^2-ae^2\right)^6\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{16384c^5d^5e^3}-\frac{15\left(cd^2-ae^2\right)^4\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+\cdots\right.}{2048c^4d^4e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (45*(c*d^2 - a*e^2)^6*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2]])/(16384*c^5*d^5*e^3) - (15*(c*d^2 - a*e^2)^4*(c*d^2 + a*
e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(2048*c^4*d
^4*e^2) + (3*(c*d^2 - a*e^2)^2*(c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2)^(5/2))/(128*c^3*d^3*e) + (9*(c*d^2 - a*e^2)*(a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(112*c^2*d^2) + ((d + e*x)*(a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(8*c*d) - (45*(c*d^2 - a*e^2)^8*ArcT
anh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2])])/(32768*c^(11/2)*d^(11/2)*e^(7/2))
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2} dx = \frac{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{8cd} + \frac{\left(9\left(d^2-\frac{ae^2}{c}\right)\right)\int (d+ex)$$

$$= \frac{9\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{112c^2d^2} + \frac{(d+ex)\left(ade+\left(cd^2\right.\right.}{8}$$

$$= \frac{3\left(cd^2-ae^2\right)^2\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{128c^3d^3e}$$

$$= -\frac{15\left(cd^2-ae^2\right)^4\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/}}{2048c^4d^4e^2}$$

$$= \frac{45\left(cd^2-ae^2\right)^6\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16384c^5d^5e^3} -$$

$$= \frac{45\left(cd^2-ae^2\right)^6\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16384c^5d^5e^3} -$$

$$= \frac{45\left(cd^2-ae^2\right)^6\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16384c^5d^5e^3} -$$

Mathematica [B] time = 6.31834, size = 1279, normalized size = 3.09

$$2\left(cd^2-ae^2\right)^4(ae+cdx)((ae+cdx)(d+ex))^{5/2}\left(\frac{cde(ae+cdx)}{\left(cd^2-ae^2\right)\left(\frac{c^2d^3}{cd^2-ae^2}-\frac{acde^2}{cd^2-ae^2}\right)}+1\right)^{11/2}\left(\frac{315\left(cd^2-ae^2\right)^4\left(\frac{16c^3d^3e^3(ae+cdx)^3}{15\left(cd^2-ae^2\right)^3\left(\frac{c^2d^3}{cd^2-ae^2}-\frac{acde^2}{cd^2-ae^2}\right)^3}-\cdots\right.}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (2*(c*d^2 - a*e^2)^4*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^(5/2)*(1 + (c*
d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2
)/(c*d^2 - a*e^2))))^(11/2)*((7*(9/(128*(1 + (c*d*e*(a*e + c*d*x))/((c*d^2
- a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))))^5) + 3
/(16*(1 + (c*d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2)
 - (a*c*d*e^2)/(c*d^2 - a*e^2))))^4) + 3/(8*(1 + (c*d*e*(a*e + c*d*x))/((c*
d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))))^3)
 + 9/(14*(1 + (c*d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*
e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))))^2) + (1 + (c*d*e*(a*e + c*d*x))/((c*d
^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))))^(-1
)))/16 + (315*(c*d^2 - a*e^2)^4*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c
*d^2 - a*e^2))^4*((2*c*d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^
2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))) - (4*c^2*d^2*e^2*(a*e + c*d*x)^2
)/(3*(c*d^2 - a*e^2)^2*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*
e^2))^2) + (16*c^3*d^3*e^3*(a*e + c*d*x)^3)/(15*(c*d^2 - a*e^2)^3*((c^2*d^3
)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))^3) - (2*Sqrt[c]*Sqrt[d]*Sq
rt[e]*Sqrt[a*e + c*d*x]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])
/(Sqrt[c*d^2 - a*e^2]*Sqrt[(c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 -
 a*e^2)])])/(Sqrt[c*d^2 - a*e^2]*Sqrt[(c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^
2)/(c*d^2 - a*e^2)]*Sqrt[1 + (c*d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d
^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2)))])))/(65536*c^4*d^4*e^4*
(a*e + c*d*x)^4*(1 + (c*d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d
^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))))^5)))/(7*c^5*d^5*((c*d)/(((c^2*d
^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2)))^(9/2)*(d + e*x)^2*Sqrt[
(c*d*(d + e*x))/(c*d^2 - a*e^2)])
```

Maple [B] time = 0.054, size = 2044, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] -9/112*e^2/d^2/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)*a-225/16384/e*d^
7*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a+3/128*e^5/d^3/c^3*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(5/2)*a^3-135/4096*d^6*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)*x*a-3/128*e^3/d/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*a^
2-15/1024*e^2*d^2/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^2+1/8*e*x*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/d/c+315/4096*e^3*d^5*ln((1/2*a*e^2+1/2
*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e
*c)^(1/2)*a^3+675/8192*e^6/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^
4+15/256*e*a*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x-15/1024/e*d^5*c*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x+675/8192*e^2*d^4*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2)*x*a^2-3/32*e^2/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(5/2)*x*a-225/16384*e^5*d/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^4+4
5/8192/e^2*d^8*c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+45/16384*e^11/
d^5/c^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^7-225/16384*e^9/d^3/c^4*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^6-225/16384*e^3*d^3/c*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^3-45/32768/e^3*d^11*c^3*ln((1/2*a*e^2+1/2*c*d
^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^
(1/2)-15/2048*e^8/d^4/c^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^5+45/20
48*e^6/d^2/c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^4+3/64*d^2*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*x+45/2048*d^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(3/2)*a+3/128/e*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)-3/128*e*d
/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*a+405/16384*e^7/d/c^3*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^5+23/112/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(7/2)+405/16384*e*d^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2-15/1024
*e^4/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^3-15/2048/e^2*d^6*c*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+45/16384/e^3*d^9*c^2*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2)-315/8192*e*d^7*c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*
e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^2+45/40
96/e*d^9*c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a-45/32768*e^13/d^5/c^5*ln((1/2*a*e
^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
)/(d*e*c)^(1/2)*a^8+45/4096*e^11/d^3/c^4*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(
d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^7-315
/8192*e^9/d/c^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^6+315/4096*e^7*d/c^2*ln((1/2*a
*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2))/(d*e*c)^(1/2)*a^5+45/8192*e^10/d^4/c^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)*x*a^6-135/4096*e^8/d^2/c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*
x*a^5+15/256*e^5*a^3/c^2/d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x-225/20
48*e^4*d^2/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^3-15/1024*e^7/d^3/
c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*a^4+3/64*e^4/d^2/c^2*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*x*a^2-45/512*e^3*d/c*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(3/2)*x*a^2-1575/16384*e^5*d^3/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*
x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.04319, size = 3398, normalized size = 8.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="f
ricas")
```

```
[Out] [1/458752*(315*(c^8*d^16 - 8*a*c^7*d^14*e^2 + 28*a^2*c^6*d^12*e^4 - 56*a^3*
c^5*d^10*e^6 + 70*a^4*c^4*d^8*e^8 - 56*a^5*c^3*d^6*e^10 + 28*a^6*c^2*d^4*e^
12 - 8*a^7*c*d^2*e^14 + a^8*e^16)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d
^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) +
4*(14336*c^8*d^8*e^8*x^7 + 315*c^8*d^15*e - 2415*a*c^7*d^13*e^3 + 8043*a^2*
c^6*d^11*e^5 + 17609*a^3*c^5*d^9*e^7 - 15159*a^4*c^4*d^7*e^9 + 8043*a^5*c^3
*d^5*e^11 - 2415*a^6*c^2*d^3*e^13 + 315*a^7*c*d*e^15 + 1024*(65*c^8*d^9*e^7
 + 33*a*c^7*d^7*e^9)*x^6 + 768*(155*c^8*d^10*e^6 + 210*a*c^7*d^8*e^8 + 27*a
^2*c^6*d^6*e^10)*x^5 + 128*(769*c^8*d^11*e^5 + 2343*a*c^7*d^9*e^7 + 807*a^2
*c^6*d^7*e^9 + a^3*c^5*d^5*e^11)*x^4 + 16*(2039*c^8*d^12*e^4 + 16452*a*c^7*
d^10*e^6 + 12810*a^2*c^6*d^8*e^8 + 68*a^3*c^5*d^6*e^10 - 9*a^4*c^4*d^4*e^12
)*x^3 + 24*(7*c^8*d^13*e^3 + 4043*a*c^7*d^11*e^5 + 8366*a^2*c^6*d^9*e^7 + 1
74*a^3*c^5*d^7*e^9 - 53*a^4*c^4*d^5*e^11 + 7*a^5*c^3*d^3*e^13)*x^2 - 2*(105
*c^8*d^14*e^2 - 798*a*c^7*d^12*e^4 - 46521*a^2*c^6*d^10*e^6 - 4900*a^3*c^5*
d^8*e^8 + 2631*a^4*c^4*d^6*e^10 - 798*a^5*c^3*d^4*e^12 + 105*a^6*c^2*d^2*e^
14)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^6*d^6*e^4), 1/229376
*(315*(c^8*d^16 - 8*a*c^7*d^14*e^2 + 28*a^2*c^6*d^12*e^4 - 56*a^3*c^5*d^10*
e^6 + 70*a^4*c^4*d^8*e^8 - 56*a^5*c^3*d^6*e^10 + 28*a^6*c^2*d^4*e^12 - 8*a^
7*c*d^2*e^14 + a^8*e^16)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2
 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(14336*c^8*d^8*e^8*x^7 + 3
15*c^8*d^15*e - 2415*a*c^7*d^13*e^3 + 8043*a^2*c^6*d^11*e^5 + 17609*a^3*c^5
*d^9*e^7 - 15159*a^4*c^4*d^7*e^9 + 8043*a^5*c^3*d^5*e^11 - 2415*a^6*c^2*d^3
*e^13 + 315*a^7*c*d*e^15 + 1024*(65*c^8*d^9*e^7 + 33*a*c^7*d^7*e^9)*x^6 + 7
68*(155*c^8*d^10*e^6 + 210*a*c^7*d^8*e^8 + 27*a^2*c^6*d^6*e^10)*x^5 + 128*(
769*c^8*d^11*e^5 + 2343*a*c^7*d^9*e^7 + 807*a^2*c^6*d^7*e^9 + a^3*c^5*d^5*e
^11)*x^4 + 16*(2039*c^8*d^12*e^4 + 16452*a*c^7*d^10*e^6 + 12810*a^2*c^6*d^8
*e^8 + 68*a^3*c^5*d^6*e^10 - 9*a^4*c^4*d^4*e^12)*x^3 + 24*(7*c^8*d^13*e^3 +
 4043*a*c^7*d^11*e^5 + 8366*a^2*c^6*d^9*e^7 + 174*a^3*c^5*d^7*e^9 - 53*a^4*
c^4*d^5*e^11 + 7*a^5*c^3*d^3*e^13)*x^2 - 2*(105*c^8*d^14*e^2 - 798*a*c^7*d^
12*e^4 - 46521*a^2*c^6*d^10*e^6 - 4900*a^3*c^5*d^8*e^8 + 2631*a^4*c^4*d^6*e
^10 - 798*a^5*c^3*d^4*e^12 + 105*a^6*c^2*d^2*e^14)*x)*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x))/(c^6*d^6*e^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.84902, size = 1006, normalized size = 2.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="g
iac")
```

```
[Out] 1/114688*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*(2*(4*(2*(8*(2*(4*(14*
c^2*d^2*x*e^4 + (65*c^9*d^10*e^10 + 33*a*c^8*d^8*e^12)*e^(-7)/(c^7*d^7))*x
+ 3*(155*c^9*d^11*e^9 + 210*a*c^8*d^9*e^11 + 27*a^2*c^7*d^7*e^13)*e^(-7)/(c
^7*d^7))*x + (769*c^9*d^12*e^8 + 2343*a*c^8*d^10*e^10 + 807*a^2*c^7*d^8*e^1
2 + a^3*c^6*d^6*e^14)*e^(-7)/(c^7*d^7))*x + (2039*c^9*d^13*e^7 + 16452*a*c^
8*d^11*e^9 + 12810*a^2*c^7*d^9*e^11 + 68*a^3*c^6*d^7*e^13 - 9*a^4*c^5*d^5*e
^15)*e^(-7)/(c^7*d^7))*x + 3*(7*c^9*d^14*e^6 + 4043*a*c^8*d^12*e^8 + 8366*a
^2*c^7*d^10*e^10 + 174*a^3*c^6*d^8*e^12 - 53*a^4*c^5*d^6*e^14 + 7*a^5*c^4*d
^4*e^16)*e^(-7)/(c^7*d^7))*x - (105*c^9*d^15*e^5 - 798*a*c^8*d^13*e^7 - 465
21*a^2*c^7*d^11*e^9 - 4900*a^3*c^6*d^9*e^11 + 2631*a^4*c^5*d^7*e^13 - 798*a
^5*c^4*d^5*e^15 + 105*a^6*c^3*d^3*e^17)*e^(-7)/(c^7*d^7))*x + (315*c^9*d^16
*e^4 - 2415*a*c^8*d^14*e^6 + 8043*a^2*c^7*d^12*e^8 + 17609*a^3*c^6*d^10*e^1
0 - 15159*a^4*c^5*d^8*e^12 + 8043*a^5*c^4*d^6*e^14 - 2415*a^6*c^3*d^4*e^16
+ 315*a^7*c^2*d^2*e^18)*e^(-7)/(c^7*d^7)) + 45/32768*(c^8*d^16 - 8*a*c^7*d^
14*e^2 + 28*a^2*c^6*d^12*e^4 - 56*a^3*c^5*d^10*e^6 + 70*a^4*c^4*d^8*e^8 - 5
6*a^5*c^3*d^6*e^10 + 28*a^6*c^2*d^4*e^12 - 8*a^7*c*d^2*e^14 + a^8*e^16)*sqr
t(c*d)*e^(-7/2)*log(abs(-sqrt(c*d)*c*d^2*e^(1/2) - 2*(sqrt(c*d)*x*e^(1/2) -
 sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*c*d*e - sqrt(c*d)*a*e^(5/2)))
/(c^6*d^6)
```

3.1934 $\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}dx$

Optimal. Leaf size=356

$$\frac{5\left(cd^2-ae^2\right)^5\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{1024c^4d^4e^3}-\frac{5\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade\right.}{384c^3d^3e^2}$$

```
[Out] (5*(c*d^2 - a*e^2)^5*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(1024*c^4*d^4*e^3) - (5*(c*d^2 - a*e^2)^3*(c*d^2 + a*e^2
 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(384*c^3*d^3*e
^2) + ((c*d^2 - a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(5/2))/(24*c^2*d^2*e) + (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(7/2)/(7*c*d) - (5*(c*d^2 - a*e^2)^7*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*
x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]
)/(2048*c^(9/2)*d^(9/2)*e^(7/2))
```

Rubi [A] time = 0.238534, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {640, 612, 621, 206}

$$\frac{5\left(cd^2-ae^2\right)^5\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{1024c^4d^4e^3}-\frac{5\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade\right.}{384c^3d^3e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```


```
[Out] (5*(c*d^2 - a*e^2)^5*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(1024*c^4*d^4*e^3) - (5*(c*d^2 - a*e^2)^3*(c*d^2 + a*e^2
 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(384*c^3*d^3*e
^2) + ((c*d^2 - a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(5/2))/(24*c^2*d^2*e) + (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(7/2)/(7*c*d) - (5*(c*d^2 - a*e^2)^7*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*
x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]
)/(2048*c^(9/2)*d^(9/2)*e^(7/2))
```


Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx = \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7cd}+\frac{\left(d^2-\frac{ae^2}{c}\right)\int\left(ade+\left(cd^2+ae^2\right)x\right.}{2d}$$

$$= \frac{\left(cd^2-ae^2\right)\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{24c^2d^2e}+\frac{(a}{}$$

$$= -\frac{5\left(cd^2-ae^2\right)^3\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{384c^3d^3e^2}$$

$$= \frac{5\left(cd^2-ae^2\right)^5\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{1024c^4d^4e^3}-\frac{5}{}$$

$$= \frac{5\left(cd^2-ae^2\right)^5\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{1024c^4d^4e^3}-\frac{5}{}$$

$$= \frac{5\left(cd^2-ae^2\right)^5\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{1024c^4d^4e^3}-\frac{5}{}$$

Mathematica [B] time = 6.20005, size = 1199, normalized size = 3.37

$$\frac{2\left(cd^2-ae^2\right)^3(ae+cdx)((ae+cdx)(d+ex))^{5/2}\left(\frac{cde(ae+cdx)}{\left(cd^2-ae^2\right)\left(\frac{c^2d^3}{cd^2-ae^2}-\frac{acde^2}{cd^2-ae^2}\right)}+1\right)^{9/2}\left(35\left(cd^2-ae^2\right)^4\left(\frac{16c^3d^3e^3(ae+cdx)^3}{15\left(cd^2-ae^2\right)^3\left(\frac{c^2d^3}{cd^2-ae^2}-\frac{acde^2}{cd^2-ae^2}\right)^3}-\frac{}{3(ca}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (2*(c*d^2 - a*e^2)^3*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^(5/2)*(1 + (c*
d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2
)/(c*d^2 - a*e^2))))^(9/2)*((7/(64*(1 + (c*d*e*(a*e + c*d*x))/((c*d^2 - a*e
^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))))^4) + 7/(24*
(1 + (c*d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a
```

```
*c*d*e^2)/(c*d^2 - a*e^2))))^3) + 7/(12*(1 + (c*d*e*(a*e + c*d*x))/((c*d^2
- a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))))^2) + (
1 + (c*d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*
c*d*e^2)/(c*d^2 - a*e^2))))^(-1))/2 + (35*(c*d^2 - a*e^2)^4*((c^2*d^3)/(c*d
^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))^4*((2*c*d*e*(a*e + c*d*x))/((c*d
^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))) - (4
*c^2*d^2*e^2*(a*e + c*d*x)^2)/(3*(c*d^2 - a*e^2)^2*((c^2*d^3)/(c*d^2 - a*e^
2) - (a*c*d*e^2)/(c*d^2 - a*e^2))^2) + (16*c^3*d^3*e^3*(a*e + c*d*x)^3)/(15
*(c*d^2 - a*e^2)^3*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2)
)^3) - (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*ArcSinh[(Sqrt[c]*Sqrt[d
]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d^2 - a*e^2]*Sqrt[(c^2*d^3)/(c*d^2 - a
*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2)])])/(Sqrt[c*d^2 - a*e^2]*Sqrt[(c^2*d^3)
/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2)]*Sqrt[1 + (c*d*e*(a*e + c*d*
x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^
2)))])))/(4096*c^4*d^4*e^4*(a*e + c*d*x)^4*(1 + (c*d*e*(a*e + c*d*x))/((c*d
^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))))^4))
)/(7*c^4*d^4*((c*d)/((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2
)))^(7/2)*(d + e*x)^2*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])
```

Maple [B] time = 0.05, size = 1533, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] 5/2048/d^4*e^11/c^4*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^7-35/2048/d^2*e^9/c^3*ln((
1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2))/(d*e*c)^(1/2)*a^6+35/2048*d^8/e*c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*
x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a+2
5/512/d*e^6/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^4+5/192/d^2*e^5
/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*a^3-25/256*d*e^4/c*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^3-175/2048*d^2*e^5/c*ln((1/2*a*e^2+1/2*c
*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c
)^(1/2)*a^4-1/12/d*e^2/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*x*a-5/512/
d^3*e^8/c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^5-105/2048*e*d^6*c*
ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2))/(d*e*c)^(1/2)*a^2+5/1024*d^8/e^3*c^2*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)-5/384*d^5/e^2*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+25/1
024*e*d^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2-25/1024*e^5/c^2*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^4-5/192*d^4/e*c*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(3/2)*x+5/64*e*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*a+
25/256*d^3*e^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^2+175/2048*d^4*e
^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^3-5/64*e^3/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(3/2)*x*a^2-1/24/d^2*e^3/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*a^
2+5/256/d^2*e^7/c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^5+105/2048*e^
7/c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^5-25/512*d^5*c*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)*x*a-5/192/d*e^4/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*
a^3-5/2048*d^10/e^3*c^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+5/512*d^7/e^2*c^2*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x-5/1024/d^4*e^9/c^4*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2)*a^6+1/24*d^2/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+
1/12*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*x+5/192*d^3*(a*d*e+(a*e^2+c*
```

```
d^2)*x+c*d*e*x^2)^(3/2)*a-5/256*d^6/e*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)*a+5/384/d^3*e^6/c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^4+1/7*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.69538, size = 2782, normalized size = 7.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="fri
cas")
```

```
[Out] [1/86016*(105*(c^7*d^14 - 7*a*c^6*d^12*e^2 + 21*a^2*c^5*d^10*e^4 - 35*a^3*c
^4*d^8*e^6 + 35*a^4*c^3*d^6*e^8 - 21*a^5*c^2*d^4*e^10 + 7*a^6*c*d^2*e^12 -
a^7*e^14)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2
*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a
*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(3072*c^7*d^7*e^7*x^6
+ 105*c^7*d^13*e - 700*a*c^6*d^11*e^3 + 1981*a^2*c^5*d^9*e^5 + 3072*a^3*c^4
*d^7*e^7 - 1981*a^4*c^3*d^5*e^9 + 700*a^5*c^2*d^3*e^11 - 105*a^6*c*d*e^13 +
 256*(43*c^7*d^8*e^6 + 29*a*c^6*d^6*e^8)*x^5 + 128*(107*c^7*d^9*e^5 + 216*a
*c^6*d^7*e^7 + 37*a^2*c^5*d^5*e^9)*x^4 + 16*(381*c^7*d^10*e^4 + 2281*a*c^6*
d^8*e^6 + 1175*a^2*c^5*d^6*e^8 + 3*a^3*c^4*d^4*e^10)*x^3 + 8*(7*c^7*d^11*e^
3 + 2258*a*c^6*d^9*e^5 + 3456*a^2*c^5*d^7*e^7 + 46*a^3*c^4*d^5*e^9 - 7*a^4*
c^3*d^3*e^11)*x^2 - 2*(35*c^7*d^12*e^2 - 231*a*c^6*d^10*e^4 - 8570*a^2*c^5*
d^8*e^6 - 646*a^3*c^4*d^6*e^8 + 231*a^4*c^3*d^4*e^10 - 35*a^5*c^2*d^2*e^12)
*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5*d^5*e^4), 1/43008*(10
5*(c^7*d^14 - 7*a*c^6*d^12*e^2 + 21*a^2*c^5*d^10*e^4 - 35*a^3*c^4*d^8*e^6 +
 35*a^4*c^3*d^6*e^8 - 21*a^5*c^2*d^4*e^10 + 7*a^6*c*d^2*e^12 - a^7*e^14)*sq
rt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*
x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e
 + a*c*d*e^3)*x)) + 2*(3072*c^7*d^7*e^7*x^6 + 105*c^7*d^13*e - 700*a*c^6*d^
11*e^3 + 1981*a^2*c^5*d^9*e^5 + 3072*a^3*c^4*d^7*e^7 - 1981*a^4*c^3*d^5*e^9
 + 700*a^5*c^2*d^3*e^11 - 105*a^6*c*d*e^13 + 256*(43*c^7*d^8*e^6 + 29*a*c^6
*d^6*e^8)*x^5 + 128*(107*c^7*d^9*e^5 + 216*a*c^6*d^7*e^7 + 37*a^2*c^5*d^5*e
^9)*x^4 + 16*(381*c^7*d^10*e^4 + 2281*a*c^6*d^8*e^6 + 1175*a^2*c^5*d^6*e^8
+ 3*a^3*c^4*d^4*e^10)*x^3 + 8*(7*c^7*d^11*e^3 + 2258*a*c^6*d^9*e^5 + 3456*a
^2*c^5*d^7*e^7 + 46*a^3*c^4*d^5*e^9 - 7*a^4*c^3*d^3*e^11)*x^2 - 2*(35*c^7*d
^12*e^2 - 231*a*c^6*d^10*e^4 - 8570*a^2*c^5*d^8*e^6 - 646*a^3*c^4*d^6*e^8 +
 231*a^4*c^3*d^4*e^10 - 35*a^5*c^2*d^2*e^12)*x)*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x))/(c^5*d^5*e^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.7552, size = 836, normalized size = 2.35

$$\frac{1}{21504}\sqrt{cdx^2e + cd^2x + axe^2 + ade}\left(2\left(4\left(2\left(8\left(2\left(12\,c^2d^2xe^3 + \frac{\left(43\,c^8d^9e^8 + 29\,ac^7d^7e^{10}\right)e^{(-6)}}{c^6d^6}\right)x + \frac{\left(107\,c^8d^{10}e^7 + 216\,ac^7d\right.}{c^6}\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="gia
c")
```

```
[Out] 1/21504*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*(2*(4*(2*(8*(2*(12*c^2*
d^2*x*e^3 + (43*c^8*d^9*e^8 + 29*a*c^7*d^7*e^10)*e^(-6)/(c^6*d^6))*x + (107
*c^8*d^10*e^7 + 216*a*c^7*d^8*e^9 + 37*a^2*c^6*d^6*e^11)*e^(-6)/(c^6*d^6))*
x + (381*c^8*d^11*e^6 + 2281*a*c^7*d^9*e^8 + 1175*a^2*c^6*d^7*e^10 + 3*a^3*
c^5*d^5*e^12)*e^(-6)/(c^6*d^6))*x + (7*c^8*d^12*e^5 + 2258*a*c^7*d^10*e^7 +
 3456*a^2*c^6*d^8*e^9 + 46*a^3*c^5*d^6*e^11 - 7*a^4*c^4*d^4*e^13)*e^(-6)/(c
^6*d^6))*x - (35*c^8*d^13*e^4 - 231*a*c^7*d^11*e^6 - 8570*a^2*c^6*d^9*e^8 -
 646*a^3*c^5*d^7*e^10 + 231*a^4*c^4*d^5*e^12 - 35*a^5*c^3*d^3*e^14)*e^(-6)/
(c^6*d^6))*x + (105*c^8*d^14*e^3 - 700*a*c^7*d^12*e^5 + 1981*a^2*c^6*d^10*e
^7 + 3072*a^3*c^5*d^8*e^9 - 1981*a^4*c^4*d^6*e^11 + 700*a^5*c^3*d^4*e^13 -
105*a^6*c^2*d^2*e^15)*e^(-6)/(c^6*d^6)) + 5/2048*(c^7*d^14 - 7*a*c^6*d^12*e
^2 + 21*a^2*c^5*d^10*e^4 - 35*a^3*c^4*d^8*e^6 + 35*a^4*c^3*d^6*e^8 - 21*a^5
*c^2*d^4*e^10 + 7*a^6*c*d^2*e^12 - a^7*e^14)*sqrt(c*d)*e^(-7/2)*log(abs(-sq
rt(c*d)*c*d^2*e^(1/2) - 2*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x +
 a*x*e^2 + a*d*e))*c*d*e - sqrt(c*d)*a*e^(5/2)))/(c^5*d^5)
```

3.1935 $\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2} dx$

Optimal. Leaf size=305

$$\frac{5\left(cd^2-ae^2\right)^4\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{512c^3d^3e^3}-\frac{5\left(cd^2-ae^2\right)^2\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+\right.}{192c^2d^2e^2}$$

```
[Out] (5*(c*d^2 - a*e^2)^4*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(512*c^3*d^3*e^3) - (5*(c*d^2 - a*e^2)^2*(c*d^2 + a*e^2
+ 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(192*c^2*d^2*e^
2) + ((c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2))/(12*c*d*e) - (5*(c*d^2 - a*e^2)^6*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)
/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/
(1024*c^(7/2)*d^(7/2)*e^(7/2))
```

Rubi [A] time = 0.147549, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {612, 621, 206}

$$\frac{5\left(cd^2-ae^2\right)^4\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{512c^3d^3e^3}-\frac{5\left(cd^2-ae^2\right)^2\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+\right.}{192c^2d^2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```


```
[Out] (5*(c*d^2 - a*e^2)^4*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(512*c^3*d^3*e^3) - (5*(c*d^2 - a*e^2)^2*(c*d^2 + a*e^2
+ 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(192*c^2*d^2*e^
2) + ((c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2))/(12*c*d*e) - (5*(c*d^2 - a*e^2)^6*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)
/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/
(1024*c^(7/2)*d^(7/2)*e^(7/2))
```


Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx=\frac{\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{12cde}-\frac{\left(5\left(cd^2-ae^2\right)^2\right)\int\left(ade+\right.}{2}$$

$$=-\frac{5\left(cd^2-ae^2\right)^2\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{192c^2d^2e^2}+\frac{\left(cd^2+ae\right.}{}$$

$$=\frac{5\left(cd^2-ae^2\right)^4\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{512c^3d^3e^3}-\frac{5\left(cd^2-ae^2\right)}{}$$

$$=\frac{5\left(cd^2-ae^2\right)^4\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{512c^3d^3e^3}-\frac{5\left(cd^2-ae^2\right)}{}$$

$$=\frac{5\left(cd^2-ae^2\right)^4\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{512c^3d^3e^3}-\frac{5\left(cd^2-ae^2\right)}{}$$

Mathematica [B] time = 6.11832, size = 1119, normalized size = 3.67

$$\frac{2\left(cd^2-ae^2\right)^2(ae+cdx)((ae+cdx)(d+ex))^{5/2}\left(\frac{cde(ae+cdx)}{\left(cd^2-ae^2\right)\left(\frac{c^2d^3}{cd^2-ae^2}-\frac{acde^2}{cd^2-ae^2}\right)}+1\right)^{7/2}\left(\frac{35\left(cd^2-ae^2\right)^4\left(\frac{16c^3d^3e^3(ae+cdx)^3}{15\left(cd^2-ae^2\right)^3\left(\frac{c^2d^3}{cd^2-ae^2}-\frac{acde^2}{cd^2-ae^2}\right)^3}-\frac{4}{3\left(cd^2-ae\right.}\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (2*(c*d^2 - a*e^2)^2*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^(5/2)*(1 + (c*
d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2
)/(c*d^2 - a*e^2))))^(7/2)*((7*(3/(16*(1 + (c*d*e*(a*e + c*d*x))/((c*d^2 -
a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))))^3) + 1/(
2*(1 + (c*d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) -
(a*c*d*e^2)/(c*d^2 - a*e^2))))^2) + (1 + (c*d*e*(a*e + c*d*x))/((c*d^2 - a*
e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))))^(-1)))/12
+ (35*(c*d^2 - a*e^2)^4*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a
*e^2))^4*((2*c*d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^
2) - (a*c*d*e^2)/(c*d^2 - a*e^2))) - (4*c^2*d^2*e^2*(a*e + c*d*x)^2)/(3*(c*
d^2 - a*e^2)^2*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))^2)
 + (16*c^3*d^3*e^3*(a*e + c*d*x)^3)/(15*(c*d^2 - a*e^2)^3*((c^2*d^3)/(c*d^2
 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))^3) - (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sq
rt[a*e + c*d*x]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c
*d^2 - a*e^2]*Sqrt[(c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2)]
)])/(Sqrt[c*d^2 - a*e^2]*Sqrt[(c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^
```

```
2 - a*e^2)]*Sqrt[1 + (c*d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d
^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2)))]])))/(2048*c^4*d^4*e^4*(a*e + c*
d*x)^4*(1 + (c*d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^
2) - (a*c*d*e^2)/(c*d^2 - a*e^2))))^3)))/(7*c^3*d^3*((c*d)/((c^2*d^3)/(c*d^
2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2)))^(5/2)*(d + e*x)^2*Sqrt[(c*d*(d +
 e*x))/(c*d^2 - a*e^2)])
```

Maple [B] time = 0.045, size = 1247, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] 15/512*d^7/e*c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a-5/1024/d^3*e^9/c^3*ln((1/2*a*
e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
))/(d*e*c)^(1/2)*a^6+5/512*d^7/e^3*c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)+5/256*d^3*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2+5/192*e^2/c*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^2-5/192*d^4/e^2*c*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(3/2)+5/192*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a+1
5/512/d*e^7/c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^5-75/1024*d*e^5/c*ln((1/2*a*e^
2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))
/(d*e*c)^(1/2)*a^4-75/1024*d^5*e*c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)
^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^2-5/96/d*e^
3/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*a^2+5/256/d^2*e^6/c^2*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^4+5/256*d^6/e^2*c^2*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2)*x+5/48*d*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*
a-5/192/d^2*e^4/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^3-5/96*d^3/e*
c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x-15/512*d^5/e*c*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2)*a-5/64*d^4*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)*x*a-5/64*e^4/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^3+15/128*d^2*e
^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^2+25/256*d^3*e^3*ln((1/2*a*e
^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
)/(d*e*c)^(1/2)*a^3+5/512/d^3*e^7/c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)*a^5-15/512/d*e^5/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^4+5/256*d
*e^3/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^3-5/1024*d^9/e^3*c^3*ln((1
/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2))/(d*e*c)^(1/2)+1/12*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(5/2)/c/d/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.1914, size = 2199, normalized size = 7.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6144*(15*(c^6*d^12 - 6*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 - 20*a^3*c^3*
d^6*e^6 + 15*a^4*c^2*d^4*e^8 - 6*a^5*c*d^2*e^10 + a^6*e^12)*sqrt(c*d*e)*log
(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^
2*d^3*e + a*c*d*e^3)*x) + 4*(256*c^6*d^6*e^6*x^5 + 15*c^6*d^11*e - 85*a*c^5
*d^9*e^3 + 198*a^2*c^4*d^7*e^5 + 198*a^3*c^3*d^5*e^7 - 85*a^4*c^2*d^3*e^9 +
 15*a^5*c*d*e^11 + 640*(c^6*d^7*e^5 + a*c^5*d^5*e^7)*x^4 + 16*(27*c^6*d^8*e
^4 + 106*a*c^5*d^6*e^6 + 27*a^2*c^4*d^4*e^8)*x^3 + 8*(c^6*d^9*e^3 + 159*a*c
^5*d^7*e^5 + 159*a^2*c^4*d^5*e^7 + a^3*c^3*d^3*e^9)*x^2 - 2*(5*c^6*d^10*e^2
 - 28*a*c^5*d^8*e^4 - 594*a^2*c^4*d^6*e^6 - 28*a^3*c^3*d^4*e^8 + 5*a^4*c^2*
d^2*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^4*e^4), 1/
3072*(15*(c^6*d^12 - 6*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 - 20*a^3*c^3*d^6
*e^6 + 15*a^4*c^2*d^4*e^8 - 6*a^5*c*d^2*e^10 + a^6*e^12)*sqrt(-c*d*e)*arcta
n(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^
2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)
) + 2*(256*c^6*d^6*e^6*x^5 + 15*c^6*d^11*e - 85*a*c^5*d^9*e^3 + 198*a^2*c^4
*d^7*e^5 + 198*a^3*c^3*d^5*e^7 - 85*a^4*c^2*d^3*e^9 + 15*a^5*c*d*e^11 + 640
*(c^6*d^7*e^5 + a*c^5*d^5*e^7)*x^4 + 16*(27*c^6*d^8*e^4 + 106*a*c^5*d^6*e^6
 + 27*a^2*c^4*d^4*e^8)*x^3 + 8*(c^6*d^9*e^3 + 159*a*c^5*d^7*e^5 + 159*a^2*c
^4*d^5*e^7 + a^3*c^3*d^3*e^9)*x^2 - 2*(5*c^6*d^10*e^2 - 28*a*c^5*d^8*e^4 -
594*a^2*c^4*d^6*e^6 - 28*a^3*c^3*d^4*e^8 + 5*a^4*c^2*d^2*e^10)*x)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^4*e^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29178, size = 678, normalized size = 2.22

$$\frac{1}{1536}\sqrt{cdx^2e + cd^2x + axe^2 + ade}\left(2\left(4\left(2\left(8\left(2\,c^2d^2xe^2 + \frac{5\left(c^7d^8e^6 + ac^6d^6e^8\right)e^{(-5)}}{c^5d^5}\right)x + \frac{\left(27\,c^7d^9e^5 + 106\,ac^6d^7e^7 + 27\,a^2c\right.}{c^5d^5}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/1536*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*(2*(4*(2*(8*(2*c^2*d^2*x
*e^2 + 5*(c^7*d^8*e^6 + a*c^6*d^6*e^8)*e^(-5)/(c^5*d^5))*x + (27*c^7*d^9*e^
5 + 106*a*c^6*d^7*e^7 + 27*a^2*c^5*d^5*e^9)*e^(-5)/(c^5*d^5))*x + (c^7*d^10
```

```
*e^4 + 159*a*c^6*d^8*e^6 + 159*a^2*c^5*d^6*e^8 + a^3*c^4*d^4*e^10)*e^(-5)/(
c^5*d^5))*x - (5*c^7*d^11*e^3 - 28*a*c^6*d^9*e^5 - 594*a^2*c^5*d^7*e^7 - 28
*a^3*c^4*d^5*e^9 + 5*a^4*c^3*d^3*e^11)*e^(-5)/(c^5*d^5))*x + (15*c^7*d^12*e
^2 - 85*a*c^6*d^10*e^4 + 198*a^2*c^5*d^8*e^6 + 198*a^3*c^4*d^6*e^8 - 85*a^4
*c^3*d^4*e^10 + 15*a^5*c^2*d^2*e^12)*e^(-5)/(c^5*d^5)) + 5/1024*(c^6*d^12 -
 6*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 + 15*a^4*c^2*d^
4*e^8 - 6*a^5*c*d^2*e^10 + a^6*e^12)*sqrt(c*d)*e^(-7/2)*log(abs(-sqrt(c*d)*
c*d^2*e^(1/2) - 2*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2
 + a*d*e))*c*d*e - sqrt(c*d)*a*e^(5/2)))/(c^4*d^4)
```

3.1936 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=274

$$\frac{3\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128c^2d^2e^3}-\frac{3\left(cd^2-ae^2\right)^5\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{256c^{5/2}d^{5/2}e^{7/2}}+$$

```
[Out] (3*(c*d^2 - a*e^2)^3*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(128*c^2*d^2*e^3) + ((a/(c*d) - d/e^2)*(c*d^2 + a*e^2 +
2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/16 + (a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(5*e) - (3*(c*d^2 - a*e^2)^5*ArcTanh[(c*d
^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2]])])/(256*c^(5/2)*d^(5/2)*e^(7/2))
```

Rubi [A] time = 0.178594, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {664, 612, 621, 206}

$$\frac{3\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128c^2d^2e^3}-\frac{3\left(cd^2-ae^2\right)^5\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{256c^{5/2}d^{5/2}e^{7/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x),x]
```

```
[Out] (3*(c*d^2 - a*e^2)^3*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(128*c^2*d^2*e^3) + ((a/(c*d) - d/e^2)*(c*d^2 + a*e^2 +
2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/16 + (a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(5*e) - (3*(c*d^2 - a*e^2)^5*ArcTanh[(c*d
^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2]])])/(256*c^(5/2)*d^(5/2)*e^(7/2))
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}dx = \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5e} - \frac{\left(2cd^2e-e\left(cd^2+ae^2\right)\right)\int\left(ade+\left(cd^2+ae\right.\right.}{2e^2}$$

$$= \frac{1}{16}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} + \frac{\left(ade+\left(\right.\right.}{}$$

$$= \frac{3\left(cd^2-ae^2\right)^3\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2e^3} + \frac{1}{16}\left(\frac{a}{cd}-\right.$$

$$= \frac{3\left(cd^2-ae^2\right)^3\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2e^3} + \frac{1}{16}\left(\frac{a}{cd}-\right.$$

$$= \frac{3\left(cd^2-ae^2\right)^3\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2e^3} + \frac{1}{16}\left(\frac{a}{cd}-\right.$$

Mathematica [A] time = 1.24583, size = 384, normalized size = 1.4

$$\frac{\sqrt{cd}\left(\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{cd}(d+ex)\left(2a^3c^2d^2e^5\left(64d^2+268dex+129e^2x^2\right)+2a^2c^3d^3e^3\left(87d^2ex-35d^3+489de^2x^2+292e^3x^3\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x),x]
```

```
[Out] (Sqrt[c*d]*(Sqrt[c]*Sqrt[d]*Sqrt[c*d]*Sqrt[e]*(d + e*x)*(-15*a^5*e^9 + 5*a^
4*c*d*e^7*(14*d - e*x) + 2*a^3*c^2*d^2*e^5*(64*d^2 + 268*d*e*x + 129*e^2*x^
2) + 2*a^2*c^3*d^3*e^3*(-35*d^3 + 87*d^2*e*x + 489*d*e^2*x^2 + 292*e^3*x^3)
 + c^5*d^5*x*(15*d^4 - 10*d^3*e*x + 8*d^2*e^2*x^2 + 176*d*e^3*x^3 + 128*e^4
*x^4) + a*c^4*d^4*e*(15*d^4 - 80*d^3*e*x + 54*d^2*e^2*x^2 + 688*d*e^3*x^3 +
 464*e^4*x^4)) - 15*(c*d^2 - a*e^2)^(11/2)*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d +
 e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])
/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])]))/(640*c^(7/2)*d^(7/2)*e^(7/2)*Sqrt[(a*e
+ c*d*x)*(d + e*x)])
```

Maple [B] time = 0.049, size = 1123, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x)
```

```
[Out] 1/5/e*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(5/2)-1/16/e^2*c*d^3*(c*d*e*(
d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)+3/128/e^3*c^2*d^6*(c*d*e*(d/e+x)^2+(a
*e^2-c*d^2)*(d/e+x))^(1/2)+3/64*e^3*a^3/c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d
/e+x))^(1/2)-1/8/e*c*d^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)*x-3/
64*e^4*a^3/d/c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-9/64*a*d^3*c
*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x+3/256*e^7*a^5/d^2/c^2*ln((
1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*
d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-15/128*e*a^2*d^4*c*ln((1/2*a*e^2-1/2*c*d
^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/
2))/(d*e*c)^(1/2)+15/256/e*a*d^6*c^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)
/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)
-15/256*e^5*a^4/c*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d
*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+15/128*e^3*a^3*d^2
*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e
^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+3/64/e^2*c^2*d^5*(c*d*e*(d/e+x)^2+(
a*e^2-c*d^2)*(d/e+x))^(1/2)*x+1/16*e^2*a^2/d/c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^
2)*(d/e+x))^(3/2)+9/64*e^2*a^2*d*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1
/2)*x-3/128*e^5*a^4/d^2/c^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-3
/64/e*a*d^4*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-3/256/e^3*c^3*d
^8*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a
*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+1/8*e*a*(c*d*e*(d/e+x)^2+(a*e^2-c
*d^2)*(d/e+x))^(3/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.91429, size = 1789, normalized size = 6.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="fri
cas")
```

```
[Out] [1/2560*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d
^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^
2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x)
 + 4*(128*c^5*d^5*e^5*x^4 + 15*c^5*d^9*e - 70*a*c^4*d^7*e^3 + 128*a^2*c^3*d
^5*e^5 + 70*a^3*c^2*d^3*e^7 - 15*a^4*c*d*e^9 + 16*(11*c^5*d^6*e^4 + 21*a*c^
4*d^4*e^6)*x^3 + 8*(c^5*d^7*e^3 + 64*a*c^4*d^5*e^5 + 31*a^2*c^3*d^3*e^7)*x^
2 - 2*(5*c^5*d^8*e^2 - 23*a*c^4*d^6*e^4 - 233*a^2*c^3*d^4*e^6 - 5*a^3*c^2*d
^2*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^4), 1/12
80*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4*e^
6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*
```

```
e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(128*c^5*d^5*e^5*x^
4 + 15*c^5*d^9*e - 70*a*c^4*d^7*e^3 + 128*a^2*c^3*d^5*e^5 + 70*a^3*c^2*d^3*
e^7 - 15*a^4*c*d*e^9 + 16*(11*c^5*d^6*e^4 + 21*a*c^4*d^4*e^6)*x^3 + 8*(c^5*
d^7*e^3 + 64*a*c^4*d^5*e^5 + 31*a^2*c^3*d^3*e^7)*x^2 - 2*(5*c^5*d^8*e^2 - 2
3*a*c^4*d^6*e^4 - 233*a^2*c^3*d^4*e^6 - 5*a^3*c^2*d^2*e^8)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError
```

3.1937 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=261

$$-\frac{5\left(cd^2-ae^2\right)^4\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{128c^{3/2}d^{3/2}e^{7/2}}+\frac{5\left(cd^2-ae^2\right)^2\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64cde^3}$$

```
[Out] (5*(c*d^2 - a*e^2)^2*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(64*c*d*e^3) + (5*(a - (c*d^2)/e^2)*(a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2)^(3/2))/24 + ((a*e + c*d*x)*(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2)^(3/2))/(4*e) - (5*(c*d^2 - a*e^2)^4*ArcTanh[(c*d^2 + a*e^2 + 2*
c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])])/(128*c^(3/2)*d^(3/2)*e^(7/2))
```

Rubi [A] time = 0.220456, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {654, 670, 640, 612, 621, 206}

$$-\frac{5\left(cd^2-ae^2\right)^4\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{128c^{3/2}d^{3/2}e^{7/2}}+\frac{5\left(cd^2-ae^2\right)^2\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64cde^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^2,x]
```

```
[Out] (5*(c*d^2 - a*e^2)^2*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(64*c*d*e^3) + (5*(a - (c*d^2)/e^2)*(a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2)^(3/2))/24 + ((a*e + c*d*x)*(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2)^(3/2))/(4*e) - (5*(c*d^2 - a*e^2)^4*ArcTanh[(c*d^2 + a*e^2 + 2*
c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])])/(128*c^(3/2)*d^(3/2)*e^(7/2))
```

Rule 654

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Int[(a + b*x + c*x^2)^(m + p)/(a/d + (c*x)/e)^m, x] /; FreeQ[{a, b
, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
!IntegerQ[p] && IntegerQ[m] && RationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m,
 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^2}\,dx = \int (ae+cdx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx$$

$$= \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4e} + \frac{\left(5\left(2acde^2-cd\left(cd^2+ae^2\right)\right)\right)\int$$

$$= \frac{5}{24}\left(a-\frac{cd^2}{e^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} + \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right.\right.}{4e}$$

$$= \frac{5\left(cd^2-ae^2\right)^2\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cde^3} + \frac{5}{24}\left(a-\right.$$

$$= \frac{5\left(cd^2-ae^2\right)^2\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cde^3} + \frac{5}{24}\left(a-\right.$$

$$= \frac{5\left(cd^2-ae^2\right)^2\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cde^3} + \frac{5}{24}\left(a-\right.$$

Mathematica [A] time = 0.899307, size = 316, normalized size = 1.21

$$\frac{\sqrt{c}\sqrt{d}\left(\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{cd}(d+ex)\left(a^2c^2d^2e^3\left(-55d^2+109dex+254e^2x^2\right)+a^3cde^5(73d+133ex)+15a^4e^7+ac^3d^3e\left(-65d^2\right.\right.\right.}{192e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^2,x]
```

```
[Out] (Sqrt[c]*Sqrt[d]*(Sqrt[c]*Sqrt[d]*Sqrt[c*d]*Sqrt[e]*(d + e*x)*(15*a^4*e^7 +
 a^3*c*d*e^5*(73*d + 133*e*x) + a^2*c^2*d^2*e^3*(-55*d^2 + 109*d*e*x + 254*
```

```
e^2*x^2) + c^4*d^4*x*(15*d^3 - 10*d^2*e*x + 8*d*e^2*x^2 + 48*e^3*x^3) + a*c
^3*d^3*e*(15*d^3 - 65*d^2*e*x + 44*d*e^2*x^2 + 184*e^3*x^3)) - 15*(c*d^2 -
a*e^2)^(9/2)*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSin
h[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2
])]))/(192*(c*d)^(5/2)*e^(7/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [B] time = 0.049, size = 1455, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^2,x)
```

```
[Out] 2/3/e^2/(a*e^2-c*d^2)/(d/e+x)^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(7/
2)-2/3/e*d*c/(a*e^2-c*d^2)*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(5/2)-15
/32*e^2*d^2*c/(a*e^2-c*d^2)*a^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/
2)*x+5/64*e^5/d/c/(a*e^2-c*d^2)*a^4*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))
^(1/2)+5/32/e*d^5*c^2/(a*e^2-c*d^2)*a*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x
))^(1/2)+25/128*e^5*d/(a*e^2-c*d^2)*a^4*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d
*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1
/2)-25/64*e^3*d^3*c/(a*e^2-c*d^2)*a^3*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e
)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2
)-5/32/e^2*d^6*c^3/(a*e^2-c*d^2)*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1
/2)*x-5/24*e^2/(a*e^2-c*d^2)*a^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3
/2)-5/12*e*d*c/(a*e^2-c*d^2)*a*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2
)*x-5/32*e^3*d/(a*e^2-c*d^2)*a^3*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1
/2)+5/12/e*d^3*c^2/(a*e^2-c*d^2)*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3
/2)*x+5/24/e^2*d^4*c^2/(a*e^2-c*d^2)*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x)
)^(3/2)-5/64/e^3*d^7*c^3/(a*e^2-c*d^2)*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+
x))^(1/2)+5/32*e^4/(a*e^2-c*d^2)*a^3*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x)
)^(1/2)*x+15/32*d^4*c^2/(a*e^2-c*d^2)*a*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e
+x))^(1/2)*x-5/128*e^7/d/c/(a*e^2-c*d^2)*a^5*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x
)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*
c)^(1/2)+25/64*e*d^5*c^2/(a*e^2-c*d^2)*a^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*
c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)
^(1/2)-25/128/e*d^7*c^3/(a*e^2-c*d^2)*a*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d
*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1
/2)+5/128/e^3*d^9*c^4/(a*e^2-c*d^2)*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/
(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^2,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.9126, size = 1424, normalized size = 5.46

$$\left[\frac{15\left(c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8\right)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4-4\,\sqrt{cdex^2+ade}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^2,x, algorithm="f
ricas")
```

```
[Out] [1/768*(15*(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6
 + a^4*e^8)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a
^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 +
 a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(48*c^4*d^4*e^4*x^3
+ 15*c^4*d^7*e - 55*a*c^3*d^5*e^3 + 73*a^2*c^2*d^3*e^5 + 15*a^3*c*d*e^7 + 8
*(c^4*d^5*e^3 + 17*a*c^3*d^3*e^5)*x^2 - 2*(5*c^4*d^6*e^2 - 18*a*c^3*d^4*e^4
 - 59*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2
*d^2*e^4), 1/384*(15*(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3
*c*d^2*e^6 + a^4*e^8)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d
^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 +
a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(48*c^4*d^4*e^4*x^3 + 15*c^4*
d^7*e - 55*a*c^3*d^5*e^3 + 73*a^2*c^2*d^3*e^5 + 15*a^3*c*d*e^7 + 8*(c^4*d^5
*e^3 + 17*a*c^3*d^3*e^5)*x^2 - 2*(5*c^4*d^6*e^2 - 18*a*c^3*d^4*e^4 - 59*a^2
*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^4)
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^2,x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.1938 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^3}\,dx$

Optimal. Leaf size=244

$$\frac{5\left(cd^2-ae^2\right)\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8e^3}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{e(d+ex)^2}-\frac{5cd\left(x\left(ae^2+cd^2\right)\right.}{3e}$$

```
[Out] (5*(c*d^2 - a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2])/(8*e^3) - (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2))/(3*e^2) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(e*(d + e
*x)^2) - (5*(c*d^2 - a*e^2)^3*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c
]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*Sqrt[c
]*Sqrt[d]*e^(7/2))
```

Rubi [A] time = 0.20238, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {662, 664, 612, 621, 206}

$$\frac{5\left(cd^2-ae^2\right)\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8e^3}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{e(d+ex)^2}-\frac{5cd\left(x\left(ae^2+cd^2\right)\right.}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^3,x]
```


```
[Out] (5*(c*d^2 - a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2])/(8*e^3) - (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2))/(3*e^2) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(e*(d + e
*x)^2) - (5*(c*d^2 - a*e^2)^3*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c
]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*Sqrt[c
]*Sqrt[d]*e^(7/2))
```


Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^3}\,dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{e(d+ex)^2} - \frac{(5cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx}{e}$$

$$= -\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e^2} + \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{e(d+ex)^2} +$$

$$= \frac{5\left(cd^2-ae^2\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8e^3} - \frac{5cd\left(ade+\right.}{}$$

$$= \frac{5\left(cd^2-ae^2\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8e^3} - \frac{5cd\left(ade+\right.}{}$$

$$= \frac{5\left(cd^2-ae^2\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8e^3} - \frac{5cd\left(ade+\right.}{}$$

Mathematica [A] time = 0.524665, size = 252, normalized size = 1.03

$$\frac{\sqrt{e}(cd)^{3/2}(d+ex)\left(a^2cde^3(59ex-40d)+33a^3e^5+ac^2d^2e\left(15d^2-50dex+34e^2x^2\right)+c^3d^3x\left(15d^2-10dex+8e^2x^2\right)\right)-}{24e^{7/2}(cd)^{3/2}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^3,x]
```

```
[Out] ((c*d)^(3/2)*Sqrt[e]*(d + e*x)*(33*a^3*e^5 + a^2*c*d*e^3*(-40*d + 59*e*x) +
 c^3*d^3*x*(15*d^2 - 10*d*e*x + 8*e^2*x^2) + a*c^2*d^2*e*(15*d^2 - 50*d*e*x
 + 34*e^2*x^2)) - 15*Sqrt[c]*Sqrt[d]*(c*d^2 - a*e^2)^(7/2)*Sqrt[a*e + c*d*x
]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sq
rt[a*e + c*d*x]])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(24*(c*d)^(3/2)*e^(7/2)*
Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [B] time = 0.051, size = 1531, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^3,x)
```

```
[Out] 2/e^3/(a*e^2-c*d^2)/(d/e+x)^3*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(7/2)
-16/3/e^2*d*c/(a*e^2-c*d^2)^2/(d/e+x)^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e
+x))^(7/2)+16/3/e*d^2*c^2/(a*e^2-c*d^2)^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d
/e+x))^(5/2)+5/4*e^3*d^2*c/(a*e^2-c*d^2)^2*a^3*(c*d*e*(d/e+x)^2+(a*e^2-c*d^
2)*(d/e+x))^(1/2)-10/3/e*d^4*c^3/(a*e^2-c*d^2)^2*(c*d*e*(d/e+x)^2+(a*e^2-c*
d^2)*(d/e+x))^(3/2)*x-5/3/e^2*d^5*c^3/(a*e^2-c*d^2)^2*(c*d*e*(d/e+x)^2+(a*e
^2-c*d^2)*(d/e+x))^(3/2)+5/8/e^3*d^8*c^4/(a*e^2-c*d^2)^2*(c*d*e*(d/e+x)^2+(
a*e^2-c*d^2)*(d/e+x))^(1/2)+10/3*e*d^2*c^2/(a*e^2-c*d^2)^2*a*(c*d*e*(d/e+x)
^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)*x-5/4*e^4*d*c/(a*e^2-c*d^2)^2*a^3*(c*d*e*(d
/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-15/4*d^5*c^3/(a*e^2-c*d^2)^2*a*(c*d*
e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-5/16/e^3*d^10*c^5/(a*e^2-c*d^2)^
2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*
e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+25/8*e^3*d^4*c^2/(a*e^2-c*d^2)^2*a
^3*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a
*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+5/4/e^2*d^7*c^4/(a*e^2-c*d^2)^2*(
c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x+15/4*e^2*d^3*c^2/(a*e^2-c*d^
2)^2*a^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-5/8*e^5/(a*e^2-c*d
^2)^2*a^4*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-5/4/e*d^6*c^3/(a*e^
2-c*d^2)^2*a*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-25/16*e^5*d^2*c/
(a*e^2-c*d^2)^2*a^4*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c
*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+5/3*e^2*d*c/(a*e
^2-c*d^2)^2*a^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)+5/16*e^7/(a*e
^2-c*d^2)^2*a^5*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e
*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-25/8*e*d^6*c^3/(a*e^
2-c*d^2)^2*a^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*
(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+25/16/e*d^8*c^4/(a*e^
2-c*d^2)^2*a*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d
/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^3,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82404, size = 1130, normalized size = 4.63

$$\left[\frac{15\left(c^3d^6 - 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 - a^3e^6\right)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2 + c^2d^4 + 6\,acd^2e^2 + a^2e^4 - 4\,\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^3,x, algorithm="f
ricas")
```

```
[Out] [1/96*(15*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(c*d*
e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) +
 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(8*c^3*d^3*e^3*x^2 + 15*c^3*d^5*e - 40*a*
c^2*d^3*e^3 + 33*a^2*c*d*e^5 - 2*(5*c^3*d^4*e^2 - 13*a*c^2*d^2*e^4)*x)*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c*d*e^4), 1/48*(15*(c^3*d^6 - 3*a
*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)
/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(8*c^3*d^
3*e^3*x^2 + 15*c^3*d^5*e - 40*a*c^2*d^3*e^3 + 33*a^2*c*d*e^5 - 2*(5*c^3*d^4
*e^2 - 13*a*c^2*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c
*d*e^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^3,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError
```

3.1939 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=235

$$-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{e(d+ex)^3}+\frac{5cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2e^2(d+ex)}+\frac{15cd\left(a-\frac{cd^2}{e^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4e}$$

```
[Out] (15*c*d*(a - (c*d^2)/e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*e
) + (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(2*e^2*(d + e*x))
 - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(e*(d + e*x)^3) + (15*
Sqrt[c]*Sqrt[d]*(c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sq
rt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*e^(
7/2))
```

Rubi [A] time = 0.211624, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {662, 664, 621, 206}

$$-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{e(d+ex)^3}+\frac{5cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2e^2(d+ex)}+\frac{15cd\left(a-\frac{cd^2}{e^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^4,x]
```

```
[Out] (15*c*d*(a - (c*d^2)/e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*e
) + (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(2*e^2*(d + e*x))
 - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(e*(d + e*x)^3) + (15*
Sqrt[c]*Sqrt[d]*(c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sq
rt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*e^(
7/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^4}\,dx = -\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{e(d+ex)^3}+\frac{(5cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^2}\,dx}{e}$$

$$= \frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2e^2(d+ex)} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{e(d+ex)^3} - $$

$$= -\frac{15cd\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e^3} + \frac{5cd\left(ade+\left(cd^2+ae^2\right)x+\right.}{2e^2(d+ex)}$$

$$= -\frac{15cd\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e^3} + \frac{5cd\left(ade+\left(cd^2+ae^2\right)x+\right.}{2e^2(d+ex)}$$

$$= -\frac{15cd\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e^3} + \frac{5cd\left(ade+\left(cd^2+ae^2\right)x+\right.}{2e^2(d+ex)}$$

Mathematica [C] time = 0.0779383, size = 108, normalized size = 0.46

$$\frac{2cd(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\,{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{7\left(cd^2-ae^2\right)^2\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^4,x]
```

```
[Out] (2*c*d*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*Hypergeometric2F1[3/2,
 7/2, 9/2, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(7*(c*d^2 - a*e^2)^2*Sqrt
[(c*d*(d + e*x))/(c*d^2 - a*e^2)])
```

Maple [B] time = 0.05, size = 1617, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^4,x)
```

```
[Out] -2/e^4/(a*e^2-c*d^2)/(d/e+x)^4*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(7/2
)+12/e^3*d*c/(a*e^2-c*d^2)^2/(d/e+x)^3*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+
```

```
x))^(7/2)-32/e^2*d^2*c^2/(a*e^2-c*d^2)^3/(d/e+x)^2*(c*d*e*(d/e+x)^2+(a*e^2-
c*d^2)*(d/e+x))^(7/2)+32/e*d^3*c^3/(a*e^2-c*d^2)^3*(c*d*e*(d/e+x)^2+(a*e^2-
c*d^2)*(d/e+x))^(5/2)+15/2*e^3*d^3*c^2/(a*e^2-c*d^2)^3*a^3*(c*d*e*(d/e+x)^2
+(a*e^2-c*d^2)*(d/e+x))^(1/2)-20/e*d^5*c^4/(a*e^2-c*d^2)^3*(c*d*e*(d/e+x)^2
+(a*e^2-c*d^2)*(d/e+x))^(3/2)*x-10/e^2*d^6*c^4/(a*e^2-c*d^2)^3*(c*d*e*(d/e+
x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)+15/4/e^3*d^9*c^5/(a*e^2-c*d^2)^3*(c*d*e*(
d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-15/2*e^4*d^2*c^2/(a*e^2-c*d^2)^3*a^3*
(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x+20*e*d^3*c^3/(a*e^2-c*d^2)^
3*a*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)*x-15/8/e^3*d^11*c^6/(a*e^
2-c*d^2)^3*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e
+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-75/8*e^5*d^3*c^2/(a*e^2-c
*d^2)^3*a^4*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/
e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+75/4*e^3*d^5*c^3/(a*e^2-
c*d^2)^3*a^3*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d
/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+15/2/e^2*d^8*c^5/(a*e^2
-c*d^2)^3*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x+45/2*e^2*d^4*c^3/
(a*e^2-c*d^2)^3*a^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-15/4*e^
5*d*c/(a*e^2-c*d^2)^3*a^4*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-15/
2/e*d^7*c^4/(a*e^2-c*d^2)^3*a*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)
+10*e^2*d^2*c^2/(a*e^2-c*d^2)^3*a^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))
^(3/2)-45/2*d^6*c^4/(a*e^2-c*d^2)^3*a*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x
))^(1/2)*x+15/8*e^7*d*c/(a*e^2-c*d^2)^3*a^5*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)
*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c
)^(1/2)-75/4*e*d^7*c^4/(a*e^2-c*d^2)^3*a^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*
c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)
^(1/2)+75/8/e*d^9*c^5/(a*e^2-c*d^2)^3*a*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d
*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1
/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^4,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.8389, size = 1160, normalized size = 4.94

$$\left[\frac{15\left(c^2d^5-2\,acd^3e^2+a^2de^4+\left(c^2d^4e-2\,acd^2e^3+a^2e^5\right)x\right)\sqrt{\frac{cd}{e}}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4+4\left(2\,cde^2x+cd^2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^4,x, algorithm="f
ricas")
```

```
[Out] [1/16*(15*(c^2*d^5 - 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e - 2*a*c*d^2*e^3
 + a^2*e^5)*x)*sqrt(c*d/e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2
```

```
+ a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d
^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(2*c^2*d^2*e^
2*x^2 - 15*c^2*d^4 + 25*a*c*d^2*e^2 - 8*a^2*e^4 - (5*c^2*d^3*e - 9*a*c*d*e^
3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(e^4*x + d*e^3), -1/8*(1
5*(c^2*d^5 - 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e - 2*a*c*d^2*e^3 + a^2*e
^5)*x)*sqrt(-c*d/e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 + a*c*d^2*e + (c^2*
d^3 + a*c*d*e^2)*x)) - 2*(2*c^2*d^2*e^2*x^2 - 15*c^2*d^4 + 25*a*c*d^2*e^2 -
 8*a^2*e^4 - (5*c^2*d^3*e - 9*a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x))/(e^4*x + d*e^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**4,x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^4,x, algorithm="g
iac")

[Out] Exception raised: TypeError
```

3.1940 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^5}\,dx$

Optimal. Leaf size=226

$$\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^3}-\frac{5c^{3/2}d^{3/2}\left(cd^2-ae^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2e^{7/2}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade\right.}{3e(d+ex)^4}$$

```
[Out] (5*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/e^3 - (10*c*d*(a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*e^2*(d + e*x)^2) - (2*(a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(3*e*(d + e*x)^4) - (5*c^(3/2)*d^(3/
2)*(c*d^2 - a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*S
qrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*e^(7/2))
```

Rubi [A] time = 0.147749, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {662, 664, 621, 206}

$$\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^3}-\frac{5c^{3/2}d^{3/2}\left(cd^2-ae^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2e^{7/2}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade\right.}{3e(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^5,x]
```

```
[Out] (5*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/e^3 - (10*c*d*(a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*e^2*(d + e*x)^2) - (2*(a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(3*e*(d + e*x)^4) - (5*c^(3/2)*d^(3/
2)*(c*d^2 - a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*S
qrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*e^(7/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^5}dx = -\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3e(d+ex)^4}+\frac{(5cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^3}dx}{3e}$$

$$= -\frac{10cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e^2(d+ex)^2}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3e(d+ex)^4}$$

$$= \frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^3}-\frac{10cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e^2(d+ex)^2}$$

$$= \frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^3}-\frac{10cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e^2(d+ex)^2}$$

$$= \frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^3}-\frac{10cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e^2(d+ex)^2}$$

Mathematica [C] time = 0.0742471, size = 112, normalized size = 0.5

$$\frac{2c^2d^2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\,{}_2F_1\left(\frac{5}{2},\frac{7}{2};\frac{9}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{7\left(cd^2-ae^2\right)^3\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^5,x]
```

```
[Out] (2*c^2*d^2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*Hypergeometric2F1[
5/2, 7/2, 9/2, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(7*(c*d^2 - a*e^2)^3*
Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])
```

Maple [B] time = 0.051, size = 1695, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^5,x)
```

```
[Out] -2/3/e^5/(a*e^2-c*d^2)/(d/e+x)^5*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(7
/2)-8/3/e^4*d*c/(a*e^2-c*d^2)^2/(d/e+x)^4*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d
/e+x))^(7/2)+16/e^3*d^2*c^2/(a*e^2-c*d^2)^3/(d/e+x)^3*(c*d*e*(d/e+x)^2+(a*e
```

```
^2-c*d^2)*(d/e+x))^(7/2)-128/3/e^2*d^3*c^3/(a*e^2-c*d^2)^4/(d/e+x)^2*(c*d*e
*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(7/2)+128/3/e*d^4*c^4/(a*e^2-c*d^2)^4*(c*
d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(5/2)-80/3/e*d^6*c^5/(a*e^2-c*d^2)^4*(
c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)*x-40/3/e^2*d^7*c^5/(a*e^2-c*d^
2)^4*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)+5/e^3*d^10*c^6/(a*e^2-c*
d^2)^4*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-10*e^4*d^3*c^3/(a*e^2-
c*d^2)^4*a^3*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-30*d^7*c^5/(a*
e^2-c*d^2)^4*a*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x+5/2*e^7*d^2*
c^2/(a*e^2-c*d^2)^4*a^5*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2
)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-25*e*d^8*c^5
/(a*e^2-c*d^2)^4*a^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(
c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+80/3*e*d^4*c^4/
(a*e^2-c*d^2)^4*a*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)*x+10*e^3*d^
4*c^3/(a*e^2-c*d^2)^4*a^3*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)+10/
e^2*d^9*c^6/(a*e^2-c*d^2)^4*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x
-5/2/e^3*d^12*c^7/(a*e^2-c*d^2)^4*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d
*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-10
/e*d^8*c^5/(a*e^2-c*d^2)^4*a*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-
25/2*e^5*d^4*c^3/(a*e^2-c*d^2)^4*a^4*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)
/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)
+25*e^3*d^6*c^4/(a*e^2-c*d^2)^4*a^3*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/
(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+
40/3*e^2*d^3*c^3/(a*e^2-c*d^2)^4*a^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x)
)^(3/2)+30*e^2*d^5*c^4/(a*e^2-c*d^2)^4*a^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(
d/e+x))^(1/2)*x-5*e^5*d^2*c^2/(a*e^2-c*d^2)^4*a^4*(c*d*e*(d/e+x)^2+(a*e^2-c
*d^2)*(d/e+x))^(1/2)+25/2/e*d^10*c^6/(a*e^2-c*d^2)^4*a*ln((1/2*a*e^2-1/2*c*
d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1
/2))/(d*e*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^5,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.64561, size = 1231, normalized size = 5.45

$$\left[\frac{15\left(c^2d^5 - acd^3e^2 + \left(c^2d^3e^2 - acde^4\right)x^2 + 2\left(c^2d^4e - acd^2e^3\right)x\right)\sqrt{\frac{cd}{e}}\log\left(8c^2d^2e^2x^2 + c^2d^4 + 6acd^2e^2 + a^2e^4 - 4\left(2cde^2x\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^5,x, algorithm="f
ricas")
```

```
[Out] [1/12*(15*(c^2*d^5 - a*c*d^3*e^2 + (c^2*d^3*e^2 - a*c*d*e^4)*x^2 + 2*(c^2*d
^4*e - a*c*d^2*e^3)*x)*sqrt(c*d/e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*
```

```
d^2*e^2 + a^2*e^4 - 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(3*c^
2*d^2*e^2*x^2 + 15*c^2*d^4 - 10*a*c*d^2*e^2 - 2*a^2*e^4 + 2*(10*c^2*d^3*e -
 7*a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(e^5*x^2 + 2*
d*e^4*x + d^2*e^3), 1/6*(15*(c^2*d^5 - a*c*d^3*e^2 + (c^2*d^3*e^2 - a*c*d*e
^4)*x^2 + 2*(c^2*d^4*e - a*c*d^2*e^3)*x)*sqrt(-c*d/e)*arctan(1/2*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/
(c^2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) + 2*(3*c^2*d^2*e^2*x
^2 + 15*c^2*d^4 - 10*a*c*d^2*e^2 - 2*a^2*e^4 + 2*(10*c^2*d^3*e - 7*a*c*d*e^
3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(e^5*x^2 + 2*d*e^4*x + d
^2*e^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**5,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^5,x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.1941 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^6}\,dx$

Optimal. Leaf size=218

$$-\frac{2c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^3(d+ex)}+\frac{c^{5/2}d^{5/2}\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{e^{7/2}}-\frac{2cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)}{3e^2(d+ex)^3}$$

```
[Out] (-2*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e^3*(d + e*x)) -
(2*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*e^2*(d + e*x)^3) -
 (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*e*(d + e*x)^5) + (c^(
5/2)*d^(5/2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/e^(7/2)
```

Rubi [A] time = 0.132549, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {662, 621, 206}

$$-\frac{2c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^3(d+ex)}+\frac{c^{5/2}d^{5/2}\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{e^{7/2}}-\frac{2cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)}{3e^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^6,x]
```

```
[Out] (-2*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e^3*(d + e*x)) -
(2*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*e^2*(d + e*x)^3) -
 (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*e*(d + e*x)^5) + (c^(
5/2)*d^(5/2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/e^(7/2)
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^6}\,dx = -\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5e(d+ex)^5}+\frac{(cd)\int\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^4}dx}{e}$$

$$= -\frac{2cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e^2(d+ex)^3}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5e(d+ex)^5}+$$

$$= -\frac{2c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^3(d+ex)}-\frac{2cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e^2(d+ex)^3}$$

$$= -\frac{2c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^3(d+ex)}-\frac{2cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e^2(d+ex)^3}$$

$$= -\frac{2c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^3(d+ex)}-\frac{2cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e^2(d+ex)^3}$$

Mathematica [A] time = 1.11802, size = 217, normalized size = 1.

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(\frac{15c^{5/2}d^{5/2}\sqrt{cd}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{\sqrt{cd^2-ae^2}\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}-\frac{\sqrt{e}\left(3a^2e^4+acde^2(5d+11ex)+c^2d^2\left(15d^2+35dex+23e^2x^2\right)\right)}{(d+ex)^3}\right)}{15e^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^6,x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[e]*(3*a^2*e^4 + a*c*d*e^2*(5*d +
11*e*x) + c^2*d^2*(15*d^2 + 35*d*e*x + 23*e^2*x^2)))/(d + e*x)^3) + (15*c^(
5/2)*d^(5/2)*Sqrt[c*d]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/
(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(Sqrt[c*d^2 - a*e^2]*Sqrt[a*e + c*d*x]*Sq
rt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])))/(15*e^(7/2))
```

Maple [B] time = 0.052, size = 1764, normalized size = 8.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^6,x)
```

```
[Out] -2/5/e^6/(a*e^2-c*d^2)/(d/e+x)^6*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(7
/2)-4/15/e^5*d*c/(a*e^2-c*d^2)^2/(d/e+x)^5*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(
d/e+x))^(7/2)-16/15/e^4*d^2*c^2/(a*e^2-c*d^2)^3/(d/e+x)^4*(c*d*e*(d/e+x)^2+
(a*e^2-c*d^2)*(d/e+x))^(7/2)+32/5/e^3*d^3*c^3/(a*e^2-c*d^2)^4/(d/e+x)^3*(c*
d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(7/2)-256/15/e^2*d^4*c^4/(a*e^2-c*d^2)
^5/(d/e+x)^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(7/2)+256/15/e*d^5*c^5
/(a*e^2-c*d^2)^5*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(5/2)+32/3*e*d^5*c
^5/(a*e^2-c*d^2)^5*a*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)*x+4*e^3*
d^5*c^4/(a*e^2-c*d^2)^5*a^3*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-1
2*d^8*c^6/(a*e^2-c*d^2)^5*a*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x
+e^7*d^3*c^3/(a*e^2-c*d^2)^5*a^5*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*
```

```
e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-10*
e*d^9*c^6/(a*e^2-c*d^2)^5*a^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c
)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+5/e*d^
11*c^7/(a*e^2-c*d^2)^5*a*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/
2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-32/3/e*d^7*
c^6/(a*e^2-c*d^2)^5*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)*x-16/3/e^
2*d^8*c^6/(a*e^2-c*d^2)^5*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)+2/e
^3*d^11*c^7/(a*e^2-c*d^2)^5*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)+4
/e^2*d^10*c^7/(a*e^2-c*d^2)^5*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)
*x-1/e^3*d^13*c^8/(a*e^2-c*d^2)^5*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d
*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+16
/3*e^2*d^4*c^4/(a*e^2-c*d^2)^5*a^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^
(3/2)+12*e^2*d^6*c^5/(a*e^2-c*d^2)^5*a^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/
e+x))^(1/2)*x-2*e^5*d^3*c^3/(a*e^2-c*d^2)^5*a^4*(c*d*e*(d/e+x)^2+(a*e^2-c*d
^2)*(d/e+x))^(1/2)-4/e*d^9*c^6/(a*e^2-c*d^2)^5*a*(c*d*e*(d/e+x)^2+(a*e^2-c*
d^2)*(d/e+x))^(1/2)-5*e^5*d^5*c^4/(a*e^2-c*d^2)^5*a^4*ln((1/2*a*e^2-1/2*c*d
^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/
2))/(d*e*c)^(1/2)+10*e^3*d^7*c^5/(a*e^2-c*d^2)^5*a^3*ln((1/2*a*e^2-1/2*c*d^
2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2
))/(d*e*c)^(1/2)-4*e^4*d^4*c^4/(a*e^2-c*d^2)^5*a^3*(c*d*e*(d/e+x)^2+(a*e^2-
c*d^2)*(d/e+x))^(1/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^6,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 13.5571, size = 1210, normalized size = 5.55

$$\left[\frac{15\left(c^2d^2e^3x^3+3c^2d^3e^2x^2+3c^2d^4ex+c^2d^5\right)\sqrt{\frac{cd}{e}}\log\left(8c^2d^2e^2x^2+c^2d^4+6acd^2e^2+a^2e^4+4\left(2cde^2x+cd^2e+ae^3\right)\sqrt{cd\cdots}\right.}{30\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^6,x, algorithm="f
ricas")
```

```
[Out] [1/30*(15*(c^2*d^2*e^3*x^3 + 3*c^2*d^3*e^2*x^2 + 3*c^2*d^4*e*x + c^2*d^5)*s
qrt(c*d/e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*(2
*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*s
qrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(23*c^2*d^2*e^2*x^2 + 15*c^2*
d^4 + 5*a*c*d^2*e^2 + 3*a^2*e^4 + (35*c^2*d^3*e + 11*a*c*d*e^3)*x)*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x +
 d^3*e^3), -1/15*(15*(c^2*d^2*e^3*x^3 + 3*c^2*d^3*e^2*x^2 + 3*c^2*d^4*e*x +
 c^2*d^5)*sqrt(-c*d/e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 + a*c*d^2*e + (c
```

```
^2*d^3 + a*c*d*e^2)*x)) + 2*(23*c^2*d^2*e^2*x^2 + 15*c^2*d^4 + 5*a*c*d^2*e^
2 + 3*a^2*e^4 + (35*c^2*d^3*e + 11*a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x))/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**6,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^6,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError
```

3.1942 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^7}\,dx$

Optimal. Leaf size=54

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7(d+ex)^7\left(cd^2-ae^2\right)}$$

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(7*(c*d^2 - a*e^2)*(d + e
*x)^7)
```

Rubi [A] time = 0.020905, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {650}

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7(d+ex)^7\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^7,x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(7*(c*d^2 - a*e^2)*(d + e
*x)^7)
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^7}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7\left(cd^2-ae^2\right)(d+ex)^7}$$

Mathematica [A] time = 0.0458011, size = 43, normalized size = 0.8

$$\frac{2((d+ex)(ae+cdx))^{7/2}}{7(d+ex)^7\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^7,x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(7/2))/(7*(c*d^2 - a*e^2)*(d + e*x)^7)
```

Maple [A] time = 0.046, size = 58, normalized size = 1.1

$$-\frac{2\,cdx+2\,ae}{7\,(ex+d)^{6}\left(ae^{2}-cd^{2}\right)}\left(cdex^{2}+ae^{2}x+cd^{2}x+ade\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^7,x)
```

```
[Out] -2/7*(c*d*x+a*e)/(e*x+d)^6/(a*e^2-c*d^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^
(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^7,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 26.5348, size = 333, normalized size = 6.17

$$\frac{2\left(c^{3}d^{3}x^{3}+3\,ac^{2}d^{2}ex^{2}+3\,a^{2}cde^{2}x+a^{3}e^{3}\right)\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}}{7\left(cd^{6}-ad^{4}e^{2}+\left(cd^{2}e^{4}-ae^{6}\right)x^{4}+4\left(cd^{3}e^{3}-ade^{5}\right)x^{3}+6\left(cd^{4}e^{2}-ad^{2}e^{4}\right)x^{2}+4\left(cd^{5}e-ad^{3}e^{3}\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^7,x, algorithm="f
ricas")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(c*d^6 - a*d^4*e^2 + (c*d^2*e^4 - a*e^6)
*x^4 + 4*(c*d^3*e^3 - a*d*e^5)*x^3 + 6*(c*d^4*e^2 - a*d^2*e^4)*x^2 + 4*(c*d
^5*e - a*d^3*e^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**7,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^7,x, algorithm="g
iac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1943 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^8}\,dx$

Optimal. Leaf size=111

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{63(d+ex)^7\left(cd^2-ae^2\right)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9(d+ex)^8\left(cd^2-ae^2\right)}$$

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(9*(c*d^2 - a*e^2)*(d + e
*x)^8) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(63*(c*d^2 -
 a*e^2)^2*(d + e*x)^7)
```

Rubi [A] time = 0.0488611, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {658, 650}

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{63(d+ex)^7\left(cd^2-ae^2\right)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9(d+ex)^8\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^8,x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(9*(c*d^2 - a*e^2)*(d + e
*x)^8) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(63*(c*d^2 -
 a*e^2)^2*(d + e*x)^7)
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^8}\,dx &= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{9\left(cd^2-ae^2\right)(d+ex)^8}+\frac{(2cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^7}\,dx}{9\left(cd^2-ae^2\right)}\\ &= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{9\left(cd^2-ae^2\right)(d+ex)^8}+\frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{63\left(cd^2-ae^2\right)^2(d+ex)^7}\end{aligned}$$

Mathematica [A] time = 0.0408023, size = 71, normalized size = 0.64

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\left(cd(9d+2ex)-7ae^2\right)}{63(d+ex)^5\left(cd^2-ae^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^8,x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-7*a*e^2 + c*d*(9*d + 2*e
*x)))/(63*(c*d^2 - a*e^2)^2*(d + e*x)^5)
```

Maple [A] time = 0.045, size = 90, normalized size = 0.8

$$-\frac{(2\,cdx+2\,ae)\left(-2\,cdex+7\,ae^2-9\,cd^2\right)}{63\,(ex+d)^7\left(a^2e^4-2\,acd^2e^2+c^2d^4\right)}\left(cdex^2+ae^2x+cd^2x+ade\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^8,x)
```

```
[Out] -2/63*(c*d*x+a*e)*(-2*c*d*e*x+7*a*e^2-9*c*d^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a
*d*e)^(5/2)/(e*x+d)^7/(a^2*e^4-2*a*c*d^2*e^2+c^2*d^4)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^8,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 96.0911, size = 686, normalized size = 6.18

$$\frac{2\left(2\,c^4d^4ex^4+9\,a^3cd^2e^3-7\,a^4e^5+\left(9\,c^4d^5-ac^3d^3e^2\right)x^3+3\left(9\,ac^3d^4e-5\,a^2c^2d^2e^3\right)x^2+\left(27\,a^2\right.\right.}{63\left(c^2d^9-2\,acd^7e^2+a^2d^5e^4+\left(c^2d^4e^5-2\,acd^2e^7+a^2e^9\right)x^5+5\left(c^2d^5e^4-2\,acd^3e^6+a^2de^8\right)x^4+10\left(c^2d^6e^3-2\,acd^4e^5+a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^8,x, algorithm="f
ricas")
```

```
[Out] 2/63*(2*c^4*d^4*e*x^4 + 9*a^3*c*d^2*e^3 - 7*a^4*e^5 + (9*c^4*d^5 - a*c^3*d^
3*e^2)*x^3 + 3*(9*a*c^3*d^4*e - 5*a^2*c^2*d^2*e^3)*x^2 + (27*a^2*c^2*d^3*e^
2 - 19*a^3*c*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(c^2*d^9
 - 2*a*c*d^7*e^2 + a^2*d^5*e^4 + (c^2*d^4*e^5 - 2*a*c*d^2*e^7 + a^2*e^9)*x^
```

```
5 + 5*(c^2*d^5*e^4 - 2*a*c*d^3*e^6 + a^2*d*e^8)*x^4 + 10*(c^2*d^6*e^3 - 2*a
*c*d^4*e^5 + a^2*d^2*e^7)*x^3 + 10*(c^2*d^7*e^2 - 2*a*c*d^5*e^4 + a^2*d^3*e
^6)*x^2 + 5*(c^2*d^8*e - 2*a*c*d^6*e^3 + a^2*d^4*e^5)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**8,x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^8,x, algorithm="g
iac")

[Out] Timed out
```

3.1944 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^9}\,dx$

Optimal. Leaf size=171

$$\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{693(d+ex)^7\left(cd^2-ae^2\right)^3}+\frac{8cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{99(d+ex)^8\left(cd^2-ae^2\right)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11(d+ex)^9\left(cd^2-ae^2\right)}$$

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(11*(c*d^2 - a*e^2)*(d +
e*x)^9) + (8*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(99*(c*d^2
- a*e^2)^2*(d + e*x)^8) + (16*c^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(7/2))/(693*(c*d^2 - a*e^2)^3*(d + e*x)^7)
```

Rubi [A] time = 0.0804385, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {658, 650}

$$\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{693(d+ex)^7\left(cd^2-ae^2\right)^3}+\frac{8cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{99(d+ex)^8\left(cd^2-ae^2\right)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11(d+ex)^9\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^9,x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(11*(c*d^2 - a*e^2)*(d +
e*x)^9) + (8*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(99*(c*d^2
- a*e^2)^2*(d + e*x)^8) + (16*c^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(7/2))/(693*(c*d^2 - a*e^2)^3*(d + e*x)^7)
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^9}dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{11\left(cd^2-ae^2\right)(d+ex)^9} + \frac{(4cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^8}dx}{11\left(cd^2-ae^2\right)}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{11\left(cd^2-ae^2\right)(d+ex)^9} + \frac{8cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{99\left(cd^2-ae^2\right)^2(d+ex)^8} + \cdots$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{11\left(cd^2-ae^2\right)(d+ex)^9} + \frac{8cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{99\left(cd^2-ae^2\right)^2(d+ex)^8} + \cdots$$

Mathematica [A] time = 0.0606825, size = 104, normalized size = 0.61

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\left(63a^2e^4-14acde^2(11d+2ex)+c^2d^2\left(99d^2+44dex+8e^2x^2\right)\right)}{693(d+ex)^6\left(cd^2-ae^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^9,x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(63*a^2*e^4 - 14*a*c*d*e^2
*(11*d + 2*e*x) + c^2*d^2*(99*d^2 + 44*d*e*x + 8*e^2*x^2)))/(693*(c*d^2 - a
*e^2)^3*(d + e*x)^6)
```

Maple [A] time = 0.046, size = 146, normalized size = 0.9

$$-\frac{(2\,cdx+2\,ae)\left(8\,c^2d^2e^2x^2-28\,acde^3x+44\,c^2d^3ex+63\,a^2e^4-154\,acd^2e^2+99\,c^2d^4\right)}{693\,(ex+d)^8\left(a^3e^6-3\,a^2cd^2e^4+3\,ac^2d^4e^2-c^3d^6\right)}\left(cdex^2+ae^2x+cd^2x+ade\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^9,x)
```

```
[Out] -2/693*(c*d*x+a*e)*(8*c^2*d^2*e^2*x^2-28*a*c*d*e^3*x+44*c^2*d^3*e*x+63*a^2*
e^4-154*a*c*d^2*e^2+99*c^2*d^4)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)/(e*
x+d)^8/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2-c^3*d^6)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^9,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^9,x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**9,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^9,x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.1945 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{10}}\,dx$

Optimal. Leaf size=231

$$\frac{32c^3d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{3003(d+ex)^7\left(cd^2-ae^2\right)^4}+\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{429(d+ex)^8\left(cd^2-ae^2\right)^3}+\frac{12cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right.}{143(d+ex)^9\left(cd^2-ae^2\right)^2}$$

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(13*(c*d^2 - a*e^2)*(d +
e*x)^10) + (12*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(143*(c*d
^2 - a*e^2)^2*(d + e*x)^9) + (16*c^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(7/2))/(429*(c*d^2 - a*e^2)^3*(d + e*x)^8) + (32*c^3*d^3*(a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(3003*(c*d^2 - a*e^2)^4*(d + e*x)^7)
```

Rubi [A] time = 0.12055, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {658, 650}

$$\frac{32c^3d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{3003(d+ex)^7\left(cd^2-ae^2\right)^4}+\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{429(d+ex)^8\left(cd^2-ae^2\right)^3}+\frac{12cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right.}{143(d+ex)^9\left(cd^2-ae^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^10,x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(13*(c*d^2 - a*e^2)*(d +
e*x)^10) + (12*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(143*(c*d
^2 - a*e^2)^2*(d + e*x)^9) + (16*c^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(7/2))/(429*(c*d^2 - a*e^2)^3*(d + e*x)^8) + (32*c^3*d^3*(a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(3003*(c*d^2 - a*e^2)^4*(d + e*x)^7)
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{10}}\,dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{13\left(cd^2-ae^2\right)(d+ex)^{10}} + \frac{(6cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^9}dx}{13\left(cd^2-ae^2\right)}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{13\left(cd^2-ae^2\right)(d+ex)^{10}} + \frac{12cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{143\left(cd^2-ae^2\right)^2(d+ex)^9} + \frac{(2\cdots}{\cdots}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{13\left(cd^2-ae^2\right)(d+ex)^{10}} + \frac{12cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{143\left(cd^2-ae^2\right)^2(d+ex)^9} + \frac{16\cdots}{\cdots}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{13\left(cd^2-ae^2\right)(d+ex)^{10}} + \frac{12cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{143\left(cd^2-ae^2\right)^2(d+ex)^9} + \frac{16\cdots}{\cdots}$$

Mathematica [A] time = 0.0797922, size = 148, normalized size = 0.64

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\left(63a^2cde^4(13d+2ex)-231a^3e^6-7ac^2d^2e^2\left(143d^2+52dex+8e^2x^2\right)+c^3d^3\left(286d^2ex+\cdots\right.\right.}{3003(d+ex)^7\left(cd^2-ae^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^10,x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-231*a^3*e^6 + 63*a^2*c*d
*e^4*(13*d + 2*e*x) - 7*a*c^2*d^2*e^2*(143*d^2 + 52*d*e*x + 8*e^2*x^2) + c^
3*d^3*(429*d^3 + 286*d^2*e*x + 104*d*e^2*x^2 + 16*e^3*x^3)))/(3003*(c*d^2 -
 a*e^2)^4*(d + e*x)^7)
```

Maple [A] time = 0.05, size = 217, normalized size = 0.9

$$-\frac{(2\,cdx+2\,ae)\left(-16\,c^3d^3e^3x^3+56\,ac^2d^2e^4x^2-104\,c^3d^4e^2x^2-126\,a^2cde^5x+364\,ac^2d^3e^3x-286\,c^3d^5ex+231\,a^3e^6-819\cdots\right.}{3003\,(ex+d)^9\left(a^4e^8-4\,a^3cd^2e^6+6\,a^2c^2d^4e^4-4\,ac^3d^6e^2+c^4d^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^10,x)
```

```
[Out] -2/3003*(c*d*x+a*e)*(-16*c^3*d^3*e^3*x^3+56*a*c^2*d^2*e^4*x^2-104*c^3*d^4*e
^2*x^2-126*a^2*c*d*e^5*x+364*a*c^2*d^3*e^3*x-286*c^3*d^5*e*x+231*a^3*e^6-81
9*a^2*c*d^2*e^4+1001*a*c^2*d^4*e^2-429*c^3*d^6)*(c*d*e*x^2+a*e^2*x+c*d^2*x+
a*d*e)^(5/2)/(e*x+d)^9/(a^4*e^8-4*a^3*c*d^2*e^6+6*a^2*c^2*d^4*e^4-4*a*c^3*d
^6*e^2+c^4*d^8)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^10,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^10,x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**10,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^10,x, algorithm="
giac")
```

```
[Out] Timed out
```

3.1946 $\int \frac{(d+ex)^3}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$

Optimal. Leaf size=255

$$\frac{5\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8c^3d^3}+\frac{5(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{12c^2d^2}+\frac{5\left(cd^2-ae^2\right)^3\tanh}{1}$$

```
[Out] (5*(c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*c^3*d^
3) + (5*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(12*c^2*d^2) + ((d + e*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(3*c*d) + (5*(c*d^2 - a*e^2)^3*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqr
t[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*c^(
7/2)*d^(7/2)*Sqrt[e])
```

Rubi [A] time = 0.196533, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {670, 640, 621, 206}

$$\frac{5\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8c^3d^3}+\frac{5(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{12c^2d^2}+\frac{5\left(cd^2-ae^2\right)^3\tanh}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```


```
[Out] (5*(c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*c^3*d^
3) + (5*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(12*c^2*d^2) + ((d + e*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(3*c*d) + (5*(c*d^2 - a*e^2)^3*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqr
t[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*c^(
7/2)*d^(7/2)*Sqrt[e])
```


Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^3}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{(d+ex)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3cd} + \frac{\left(5\left(d^2-\frac{ae^2}{c}\right)\right)\int \frac{(d+ex)^2}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,d}{6d}$$

$$= \frac{5\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12c^2d^2} + \frac{(d+ex)^2\sqrt{ade+\left(cd^2+\right.}}{3cd}$$

$$= \frac{5\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^3d^3} + \frac{5\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(c\right.}}{12c^2d^2}$$

$$= \frac{5\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^3d^3} + \frac{5\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(c\right.}}{12c^2d^2}$$

$$= \frac{5\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^3d^3} + \frac{5\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(c\right.}}{12c^2d^2}$$

Mathematica [A] time = 0.585563, size = 214, normalized size = 0.84

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\sqrt{c}\sqrt{d}\left(15a^2e^4-10acde^2(4d+ex)+c^2d^2\left(33d^2+26dex+8e^2x^2\right)\right)+\frac{15\sqrt{cd}\left(cd^2-ae^2\right)^{5/2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{}}{\sqrt{cd}\sqrt{cd^2}}\right.}{\sqrt{e}\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}\right.}{24c^{7/2}d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*(15*a^2*e^4 - 10*a*c*d*e^2*
(4*d + e*x) + c^2*d^2*(33*d^2 + 26*d*e*x + 8*e^2*x^2)) + (15*Sqrt[c*d]*(c*d
^2 - a*e^2)^(5/2)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt
[c*d]*Sqrt[c*d^2 - a*e^2])])/(Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x)
)/(c*d^2 - a*e^2)])))/(24*c^(7/2)*d^(7/2))
```

Maple [B] time = 0.056, size = 513, normalized size = 2.

$$\frac{e^2x^2}{3\,cd}\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2} - \frac{5\,ae^3x}{12\,c^2d^2}\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2} + \frac{13\,ex}{12\,c}\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2} + \frac{5\,a}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] 1/3*e^2*x^2/d/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-5/12*e^3/d^2/c^2*x*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a+13/12*e/c*x*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)+5/8*e^4/d^3/c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*
a^2-5/3*e^2/d/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a+11/8*d/c*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-5/16*e^6/d^3/c^3*ln((1/2*a*e^2+1/2*c*d^2+
c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/
2)*a^3+15/16*e^4/d/c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^2-15/16*e^2*d/c*ln((1/2
*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2))/(d*e*c)^(1/2)*a+5/16*d^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/
2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.54175, size = 1137, normalized size = 4.46

$$\left[\frac{15\left(c^3d^6 - 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 - a^3e^6\right)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2 + c^2d^4 + 6\,acd^2e^2 + a^2e^4 + 4\,\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/96*(15*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(c*d*
e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) +
 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(8*c^3*d^3*e^3*x^2 + 33*c^3*d^5*e - 40*a*
c^2*d^3*e^3 + 15*a^2*c*d*e^5 + 2*(13*c^3*d^4*e^2 - 5*a*c^2*d^2*e^4)*x)*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^4*e), -1/48*(15*(c^3*d^6 -
3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-c*d*e)*arctan(1/2*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d
*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(8*c^3
*d^3*e^3*x^2 + 33*c^3*d^5*e - 40*a*c^2*d^3*e^3 + 15*a^2*c*d*e^5 + 2*(13*c^3
*d^4*e^2 - 5*a*c^2*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))
/(c^4*d^4*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^3}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral((d + e*x)**3/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Giac [A] time = 1.39729, size = 316, normalized size = 1.24

$$\frac{1}{24}\sqrt{cdx^2e + ade + \left(cd^2 + ae^2\right)x}\left(2x\left(2x\left(\frac{4xe^2}{cd} + \frac{\left(13c^2d^3e^3 - 5acde^5\right)e^{(-2)}}{c^3d^3}\right) + \frac{\left(33c^2d^4e^2 - 40acd^2e^4 + 15a^2e^6\right)e^{(-2)}}{c^3d^3}\right) - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] 1/24*sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)*(2*x*(4*x*e^2/(c*d) + (13*
c^2*d^3*e^3 - 5*a*c*d*e^5)*e^(-2)/(c^3*d^3)) + (33*c^2*d^4*e^2 - 40*a*c*d^2
*e^4 + 15*a^2*e^6)*e^(-2)/(c^3*d^3)) - 5/16*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*
a^2*c*d^2*e^4 - a^3*e^6)*sqrt(c*d)*e^(-1/2)*log(abs(-sqrt(c*d)*c*d^2*e^(1/2
) - 2*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x))*c
*d*e - sqrt(c*d)*a*e^(5/2)))/(c^4*d^4)
```

3.1947 $\int \frac{(d+ex)^2}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=195

$$\frac{3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4c^2d^2}+\frac{3\left(cd^2-ae^2\right)^2\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{5/2}d^{5/2}\sqrt{e}}+\frac{(d+ex)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2cd}$$

```
[Out] (3*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*c^2*d^2)
 + ((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*c*d) + (3*(c*
d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[
e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(5/2)*d^(5/2)*Sqrt[e
])
```

Rubi [A] time = 0.115318, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {670, 640, 621, 206}

$$\frac{3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4c^2d^2}+\frac{3\left(cd^2-ae^2\right)^2\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{5/2}d^{5/2}\sqrt{e}}+\frac{(d+ex)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2cd}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (3*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*c^2*d^2)
 + ((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*c*d) + (3*(c*
d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[
e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(5/2)*d^(5/2)*Sqrt[e
])
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^2}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2cd}+\frac{\left(3\left(d^2-\frac{ae^2}{c}\right)\right)\int\frac{d+ex}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,d}{4d}$$

$$=\frac{3\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4c^2d^2}+\frac{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+c}}{2cd}$$

$$=\frac{3\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4c^2d^2}+\frac{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+c}}{2cd}$$

$$=\frac{3\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4c^2d^2}+\frac{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+c}}{2cd}$$

Mathematica [A] time = 0.546682, size = 182, normalized size = 0.93

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{3\sqrt{cd}\left(cd^2-ae^2\right)^{3/2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{\sqrt{e}\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}+\sqrt{c}\sqrt{d}\left(cd(5d+2ex)-3ae^2\right)\right)}{4c^{5/2}d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*(-3*a*e^2 + c*d*(5*d + 2*e*
x)) + (3*Sqrt[c*d]*(c*d^2 - a*e^2)^(3/2)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*S
qrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(Sqrt[e]*Sqrt[a*e + c*d
*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])))/(4*c^(5/2)*d^(5/2))
```

Maple [A] time = 0.051, size = 318, normalized size = 1.6

$$\frac{ex}{2\,cd}\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2}-\frac{3\,ae^2}{4\,c^2d^2}\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2}+\frac{5}{4\,c}\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2}+\frac{3\,a^2e^4}{8\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] 1/2*e*x/d/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/4*e^2/d^2/c^2*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a+5/4/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2)+3/8*e^4/d^2/c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^2-3/4*e^2/c*ln((1/2*a*e^
2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))
```

```
/(d*e*c)^(1/2)*a+3/8*d^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.1435, size = 900, normalized size = 4.62

$$\left[\frac{3\left(c^2d^4 - 2\,acd^2e^2 + a^2e^4\right)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2 + c^2d^4 + 6\,acd^2e^2 + a^2e^4 + 4\,\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\left(2\,cdex + cd^2 + \right.\right.}{16\,c^3d^3e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/16*(3*(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*
x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d
*e^3)*x) + 4*(2*c^2*d^2*e^2*x + 5*c^2*d^3*e - 3*a*c*d*e^3)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e), -1/8*(3*(c^2*d^4 - 2*a*c*d^2*e^2
+ a^2*e^4)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2
 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(2*c^2*d^2*e^2*x + 5*c^2*d^3*e - 3*a*c*d
*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^2}{\sqrt{(d + ex)\,(ae + cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral((d + e*x)**2/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Giac [A] time = 1.37125, size = 238, normalized size = 1.22

$$\frac{1}{4}\sqrt{cdx^2e + ade + \left(cd^2 + ae^2\right)x}\left(\frac{2\,xe}{cd} + \frac{\left(5\,cd^2e - 3\,ae^3\right)e^{(-1)}}{c^2d^2}\right) - \frac{3\left(c^2d^4 - 2\,acd^2e^2 + a^2e^4\right)\sqrt{cd}e^{\left(-\frac{1}{2}\right)}\log\left(\left|-\sqrt{cd}cd^2e^{\frac{1}{2}} - \right.\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] 1/4*sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)*(2*x*e/(c*d) + (5*c*d^2*e -
 3*a*e^3)*e^(-1)/(c^2*d^2)) - 3/8*(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(
c*d)*e^(-1/2)*log(abs(-sqrt(c*d)*c*d^2*e^(1/2) - 2*(sqrt(c*d)*x*e^(1/2) - s
qrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x))*c*d*e - sqrt(c*d)*a*e^(5/2)))/(
c^3*d^3)
```

3.1948 $\int \frac{d+ex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=134

$$\frac{(cd^2-ae^2)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2c^{3/2}d^{3/2}\sqrt{e}}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd}$$

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(c*d) + ((c*d^2 - a*e^2)*ArcTan
h[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2])])/(2*c^(3/2)*d^(3/2)*Sqrt[e])
```

Rubi [A] time = 0.055865, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {640, 621, 206}

$$\frac{(cd^2-ae^2)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2c^{3/2}d^{3/2}\sqrt{e}}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```



```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(c*d) + ((c*d^2 - a*e^2)*ArcTan
h[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2])])/(2*c^(3/2)*d^(3/2)*Sqrt[e])
```


Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}dx = \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{cd} + \frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}dx}{2d}$$

$$= \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{cd} + \frac{\left(d^2-\frac{ae^2}{c}\right)\text{Subst}\left(\int \frac{1}{4cde-x^2}dx, x, \frac{cd^2+ae^2+2c}{\sqrt{ade+(cd^2+ae^2}}\right.}{d}$$

$$= \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{cd} + \frac{(cd^2-ae^2)\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex}}\right.}{2c^{3/2}d^{3/2}\sqrt{e}}$$

Mathematica [A] time = 0.203789, size = 203, normalized size = 1.51

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}+\sqrt{cd}\sqrt{cd^2-ae^2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)\right)}{c^{3/2}d^{3/2}\sqrt{e}\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*S
qrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)] + Sqrt[c*d]*Sqrt[c*d^2 - a*e^2]*ArcSin
h[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2
])]))/(c^(3/2)*d^(3/2)*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^
2 - a*e^2)])
```

Maple [A] time = 0.049, size = 171, normalized size = 1.3

$$\frac{1}{cd}\sqrt{ade+(ae^2+cd^2)x+cdex^2} - \frac{ae^2}{2cd}\ln\left(\left(\frac{ae^2}{2}+\frac{cd^2}{2}+cdex\right)\frac{1}{\sqrt{dec}}+\sqrt{ade+(ae^2+cd^2)x+cdex^2}\right)\frac{1}{\sqrt{dec}}+\frac{d}{2}\ln\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] (a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d-1/2/d*e^2/c*ln((1/2*a*e^2+1/2*c
*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c
)^(1/2)*a+1/2*d*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.96632, size = 730, normalized size = 5.45

$$\left[\frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}cde-(cd^2-ae^2)\sqrt{cde}\log\left(8c^2d^2e^2x^2+c^2d^4+6acd^2e^2+a^2e^4-4\sqrt{cdex^2+ade+(cd^2+\cdots}\right.}{4c^2d^2e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="fri
cas")
```

```
[Out] [1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*c*d*e - (c*d^2 - a*e^2)
*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqr
t(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x))/(c^2*d^2*e), 1/2*(2*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*c*d*e - (c*d^2 - a*e^2)*sqrt(-c*d*e)*arctan(1
/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*
sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)))/
(c^2*d^2*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d+ex}{\sqrt{(d+ex)(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral((d + e*x)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Giac [A] time = 1.30856, size = 181, normalized size = 1.35

$$-\frac{(cd^2-ae^2)\sqrt{cd}e^{\left(-\frac{1}{2}\right)}\log\left(\left|-\sqrt{cd}cd^2e^{\frac{1}{2}}-2\left(\sqrt{cd}xe^{\frac{1}{2}}-\sqrt{cdx^2e+ade+(cd^2+ae^2)x}\right)cde-\sqrt{cd}ae^{\frac{5}{2}}\right|\right)}{2c^2d^2}+\frac{\sqrt{cdx^2e+ade+(\cdots}}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="gia
c")
```

```
[Out] -1/2*(c*d^2 - a*e^2)*sqrt(c*d)*e^(-1/2)*log(abs(-sqrt(c*d)*c*d^2*e^(1/2) -
2*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x))*c*d*e
 - sqrt(c*d)*a*e^(5/2)))/(c^2*d^2) + sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^
2)*x)/(c*d)
```

3.1949 $\int \frac{1}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=82

$$\frac{\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}\sqrt{e}}$$

```
[Out] ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(Sqrt[c]*Sqrt[d]*Sqrt[e])
```

Rubi [A] time = 0.0250971, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {621, 206}

$$\frac{\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(Sqrt[c]*Sqrt[d]*Sqrt[e])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{4cde-x^2}\,dx, x, \frac{cd^2+ae^2+2cdex}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{\sqrt{c}\sqrt{d}\sqrt{e}}\end{aligned}$$

Mathematica [A] time = 0.0744948, size = 148, normalized size = 1.8

$$\frac{2\sqrt{cd}\sqrt{cd^2-ae^2}\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{c^{3/2}d^{3/2}\sqrt{e}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (2*Sqrt[c*d]*Sqrt[c*d^2 - a*e^2]*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*
d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d
]*Sqrt[c*d^2 - a*e^2])])/(c^(3/2)*d^(3/2)*Sqrt[e]*Sqrt[(a*e + c*d*x)*(d + e
*x)])
```

Maple [A] time = 0.046, size = 62, normalized size = 0.8

$$\ln\left(\left(\frac{ae^2}{2}+\frac{cd^2}{2}+cdex\right)\frac{1}{\sqrt{dec}}+\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2}\right)\frac{1}{\sqrt{dec}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2))/(d*e*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.91133, size = 521, normalized size = 6.35

$$\left[\frac{\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4+4\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2\,cdex+cd^2+ae^2\right)\sqrt{cde}+8\left(c^2d^3e+ac\right.\right.}{2\,cde}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4
+ 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)
*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x)/(c*d*e), -sqrt(-c*d*e)*arctan(1
/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*
sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x))/(
c*d*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ade + cdex^2 + x\left(ae^2 + cd^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)), x)
```

Giac [A] time = 1.31971, size = 116, normalized size = 1.41

$$-\frac{\sqrt{cd}e^{\left(-\frac{1}{2}\right)}\log\left(\left|-\sqrt{cd}cd^2e^{\frac{1}{2}} - 2\left(\sqrt{cd}xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)cde - \sqrt{cd}ae^{\frac{5}{2}}\right|\right)}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(c*d)*e^(-1/2)*log(abs(-sqrt(c*d)*c*d^2*e^(1/2) - 2*(sqrt(c*d)*x*e^(1/
2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*c*d*e - sqrt(c*d)*a*e^(5/
2)))/(c*d)
```

3.1950 $\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=52

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)\left(cd^2-ae^2\right)}$$

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d^2 - a*e^2)*(d + e*x))
```

Rubi [A] time = 0.0198817, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {650}

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d^2 - a*e^2)*(d + e*x))
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx=\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\left(cd^2-ae^2\right)(d+ex)}$$

Mathematica [A] time = 0.0166788, size = 42, normalized size = 0.81

$$\frac{2(ae+cdx)}{\left(cd^2-ae^2\right)\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (2*(a*e + c*d*x))/((c*d^2 - a*e^2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] time = 0.045, size = 51, normalized size = 1.

$$-2\frac{cdx+ae}{\left(ae^{2}-cd^{2}\right)\sqrt{cde\,x^{2}+ae^{2}x+cd^{2}x+ade}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] -2*(c*d*x+a*e)/(a*e^2-c*d^2)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.19918, size = 117, normalized size = 2.25

$$\frac{2\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}}{cd^{3}-ade^{2}+\left(cd^{2}e-ae^{3}\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(c*d^3 - a*d*e^2 + (c*d^2*e -
 a*e^3)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1951 $\int \frac{1}{(d+ex)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=111

$$\frac{4cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3(d+ex)\left(cd^2-ae^2\right)^2}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3(d+ex)^2\left(cd^2-ae^2\right)}$$

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d^2 - a*e^2)*(d + e*x
)^2) + (4*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d^2 - a*e^
2)^2*(d + e*x))
```

Rubi [A] time = 0.0470202, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {658, 650}

$$\frac{4cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3(d+ex)\left(cd^2-ae^2\right)^2}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3(d+ex)^2\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d^2 - a*e^2)*(d + e*x
)^2) + (4*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d^2 - a*e^
2)^2*(d + e*x))
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx &= \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3\left(cd^2-ae^2\right)(d+ex)^2}+\frac{(2cd)\int \frac{1}{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{3\left(cd^2-ae^2\right)}\\ &= \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3\left(cd^2-ae^2\right)(d+ex)^2}+\frac{4cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3\left(cd^2-ae^2\right)^2(d+ex)}\end{aligned}$$

Mathematica [A] time = 0.0405676, size = 61, normalized size = 0.55

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(cd(3d+2ex)-ae^2\right)}{3(d+ex)^2\left(cd^2-ae^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(a*e^2) + c*d*(3*d + 2*e*x)))/(3*(c*d^2
- a*e^2)^2*(d + e*x)^2)
```

Maple [A] time = 0.044, size = 89, normalized size = 0.8

$$-\frac{(2\,cdx+2\,ae)\left(-2\,cdex+ae^2-3\,cd^2\right)}{(3\,ex+3\,d)\left(a^2e^4-2\,acd^2e^2+c^2d^4\right)}\frac{1}{\sqrt{cdex^2+ae^2x+cd^2x+ade}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(-2*c*d*e*x+a*e^2-3*c*d^2)/(e*x+d)/(a^2*e^4-2*a*c*d^2*e^2+
c^2*d^4)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.73859, size = 285, normalized size = 2.57

$$\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2\,cdex+3\,cd^2-ae^2\right)}{3\left(c^2d^6-2\,acd^4e^2+a^2d^2e^4+\left(c^2d^4e^2-2\,acd^2e^4+a^2e^6\right)x^2+2\left(c^2d^5e-2\,acd^3e^3+a^2de^5\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"fricas")
```

```
[Out] 2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + 3*c*d^2 - a*e^
2)/(c^2*d^6 - 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^4*e^2 - 2*a*c*d^2*e^4 +
a^2*e^6)*x^2 + 2*(c^2*d^5*e - 2*a*c*d^3*e^3 + a^2*d*e^5)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError
```

3.1952 $\int \frac{1}{(d+ex)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=171

$$\frac{16c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{15(d+ex)\left(cd^2-ae^2\right)^3}+\frac{8cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{15(d+ex)^2\left(cd^2-ae^2\right)^2}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5(d+ex)^3\left(cd^2-ae^2\right)}$$

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*(c*d^2 - a*e^2)*(d + e*x
)^3) + (8*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(15*(c*d^2 - a*e
^2)^2*(d + e*x)^2) + (16*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(15*(c*d^2 - a*e^2)^3*(d + e*x))
```

Rubi [A] time = 0.078461, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {658, 650}

$$\frac{16c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{15(d+ex)\left(cd^2-ae^2\right)^3}+\frac{8cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{15(d+ex)^2\left(cd^2-ae^2\right)^2}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5(d+ex)^3\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```


```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*(c*d^2 - a*e^2)*(d + e*x
)^3) + (8*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(15*(c*d^2 - a*e
^2)^2*(d + e*x)^2) + (16*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(15*(c*d^2 - a*e^2)^3*(d + e*x))
```


Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5\left(cd^2-ae^2\right)(d+ex)^3} + \frac{(4cd)\int \frac{1}{(d+ex)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{5\left(cd^2-ae^2\right)}$$

$$= \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5\left(cd^2-ae^2\right)(d+ex)^3} + \frac{8cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{15\left(cd^2-ae^2\right)^2(d+ex)^2} +$$

$$= \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5\left(cd^2-ae^2\right)(d+ex)^3} + \frac{8cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{15\left(cd^2-ae^2\right)^2(d+ex)^2} +$$

Mathematica [A] time = 0.0484204, size = 94, normalized size = 0.55

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(3a^2e^4-2acde^2(5d+2ex)+c^2d^2\left(15d^2+20dex+8e^2x^2\right)\right)}{15(d+ex)^3\left(cd^2-ae^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(3*a^2*e^4 - 2*a*c*d*e^2*(5*d + 2*e*x) + c
^2*d^2*(15*d^2 + 20*d*e*x + 8*e^2*x^2)))/(15*(c*d^2 - a*e^2)^3*(d + e*x)^3)
```

Maple [A] time = 0.046, size = 146, normalized size = 0.9

$$-\frac{(2\,cdx+2\,ae)\left(8\,c^2d^2e^2x^2-4\,acde^3x+20\,c^2d^3ex+3\,a^2e^4-10\,acd^2e^2+15\,c^2d^4\right)}{15\,\left(a^3e^6-3\,a^2cd^2e^4+3\,ac^2d^4e^2-c^3d^6\right)(ex+d)^2}\frac{1}{\sqrt{cdex^2+ae^2x+cd^2x+ade}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] -2/15*(c*d*x+a*e)*(8*c^2*d^2*e^2*x^2-4*a*c*d*e^3*x+20*c^2*d^3*e*x+3*a^2*e^4
-10*a*c*d^2*e^2+15*c^2*d^4)/(e*x+d)^2/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*
e^2-c^3*d^6)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 9.18276, size = 551, normalized size = 3.22

$$\frac{2\left(8c^2d^2e^2x^2+15c^2d^4-10acd^2e^2+3a^2e^4+4\left(5c^2d^3e-acde^3\right)x\right)\sqrt{cdex^2+ad}}{15\left(c^3d^9-3ac^2d^7e^2+3a^2cd^5e^4-a^3d^3e^6+\left(c^3d^6e^3-3ac^2d^4e^5+3a^2cd^2e^7-a^3e^9\right)x^3+3\left(c^3d^7e^2-3ac^2d^5e^4+3a^2cd^3e^6\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"fricas")
```

```
[Out] 2/15*(8*c^2*d^2*e^2*x^2 + 15*c^2*d^4 - 10*a*c*d^2*e^2 + 3*a^2*e^4 + 4*(5*c^
2*d^3*e - a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(c^3*d^
9 - 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4 - a^3*d^3*e^6 + (c^3*d^6*e^3 - 3*a*c^
2*d^4*e^5 + 3*a^2*c*d^2*e^7 - a^3*e^9)*x^3 + 3*(c^3*d^7*e^2 - 3*a*c^2*d^5*e
^4 + 3*a^2*c*d^3*e^6 - a^3*d*e^8)*x^2 + 3*(c^3*d^8*e - 3*a*c^2*d^6*e^3 + 3*
a^2*c*d^4*e^5 - a^3*d^2*e^7)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1953 $\int \frac{1}{(d+ex)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=231

$$\frac{32c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{35(d+ex)\left(cd^2-ae^2\right)^4}+\frac{16c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{35(d+ex)^2\left(cd^2-ae^2\right)^3}+\frac{12cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{35(d+ex)^3\left(cd^2-ae^2\right)^2}+\cdots$$

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(7*(c*d^2 - a*e^2)*(d + e*x
)^4) + (12*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(35*(c*d^2 - a*
e^2)^2*(d + e*x)^3) + (16*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(35*(c*d^2 - a*e^2)^3*(d + e*x)^2) + (32*c^3*d^3*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2])/(35*(c*d^2 - a*e^2)^4*(d + e*x))
```

Rubi [A] time = 0.115357, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {658, 650}

$$\frac{32c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{35(d+ex)\left(cd^2-ae^2\right)^4}+\frac{16c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{35(d+ex)^2\left(cd^2-ae^2\right)^3}+\frac{12cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{35(d+ex)^3\left(cd^2-ae^2\right)^2}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(7*(c*d^2 - a*e^2)*(d + e*x
)^4) + (12*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(35*(c*d^2 - a*
e^2)^2*(d + e*x)^3) + (16*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(35*(c*d^2 - a*e^2)^3*(d + e*x)^2) + (32*c^3*d^3*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2])/(35*(c*d^2 - a*e^2)^4*(d + e*x))
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7\left(cd^2-ae^2\right)(d+ex)^4} + \frac{(6cd)\int \frac{1}{(d+ex)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}dx}{7\left(cd^2-ae^2\right)}$$

$$= \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7\left(cd^2-ae^2\right)(d+ex)^4} + \frac{12cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35\left(cd^2-ae^2\right)^2(d+ex)^3} + \frac{(2}{}$$

$$= \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7\left(cd^2-ae^2\right)(d+ex)^4} + \frac{12cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35\left(cd^2-ae^2\right)^2(d+ex)^3} + \frac{1}{}$$

$$= \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7\left(cd^2-ae^2\right)(d+ex)^4} + \frac{12cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35\left(cd^2-ae^2\right)^2(d+ex)^3} + \frac{1}{}$$

Mathematica [A] time = 0.0690381, size = 138, normalized size = 0.6

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(3a^2cde^4(7d+2ex)-5a^3e^6-ac^2d^2e^2\left(35d^2+28dex+8e^2x^2\right)+c^3d^3\left(70d^2ex+35d^3+56de^2x^2+1\right.\right.}{35(d+ex)^4\left(cd^2-ae^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-5*a^3*e^6 + 3*a^2*c*d*e^4*(7*d + 2*e*x)
- a*c^2*d^2*e^2*(35*d^2 + 28*d*e*x + 8*e^2*x^2) + c^3*d^3*(35*d^3 + 70*d^2*
e*x + 56*d*e^2*x^2 + 16*e^3*x^3)))/(35*(c*d^2 - a*e^2)^4*(d + e*x)^4)
```

Maple [A] time = 0.049, size = 217, normalized size = 0.9

$$-\frac{(2\,cdx+2\,ae)\left(-16\,c^3d^3e^3x^3+8\,ac^2d^2e^4x^2-56\,c^3d^4e^2x^2-6\,a^2cde^5x+28\,ac^2d^3e^3x-70\,c^3d^5ex+5\,a^3e^6-21\,a^2cd^2e^4+3\right.}{35\left(a^4e^8-4\,a^3cd^2e^6+6\,a^2c^2d^4e^4-4\,ac^3d^6e^2+c^4d^8\right)(ex+d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] -2/35*(c*d*x+a*e)*(-16*c^3*d^3*e^3*x^3+8*a*c^2*d^2*e^4*x^2-56*c^3*d^4*e^2*x
^2-6*a^2*c*d*e^5*x+28*a*c^2*d^3*e^3*x-70*c^3*d^5*e*x+5*a^3*e^6-21*a^2*c*d^2
*e^4+35*a*c^2*d^4*e^2-35*c^3*d^6)/(e*x+d)^3/(a^4*e^8-4*a^3*c*d^2*e^6+6*a^2*
c^2*d^4*e^4-4*a*c^3*d^6*e^2+c^4*d^8)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 34.0082, size = 910, normalized size = 3.94

$$\frac{2\left(16\,c^3d^3e^3x^3+35\,c^3d^6-35\,ac^2d^4e^2+21\,a^2c\cdots\right.}{35\left(c^4d^{12}-4\,ac^3d^{10}e^2+6\,a^2c^2d^8e^4-4\,a^3cd^6e^6+a^4d^4e^8+\left(c^4d^8e^4-4\,ac^3d^6e^6+6\,a^2c^2d^4e^8-4\,a^3cd^2e^{10}+a^4e^{12}\right)x^4+4\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"fricas")
```

```
[Out] 2/35*(16*c^3*d^3*e^3*x^3 + 35*c^3*d^6 - 35*a*c^2*d^4*e^2 + 21*a^2*c*d^2*e^4
 - 5*a^3*e^6 + 8*(7*c^3*d^4*e^2 - a*c^2*d^2*e^4)*x^2 + 2*(35*c^3*d^5*e - 14
*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)/(c^4*d^12 - 4*a*c^3*d^10*e^2 + 6*a^2*c^2*d^8*e^4 - 4*a^3*c*d^6*e^6 + a^
4*d^4*e^8 + (c^4*d^8*e^4 - 4*a*c^3*d^6*e^6 + 6*a^2*c^2*d^4*e^8 - 4*a^3*c*d^
2*e^10 + a^4*e^12)*x^4 + 4*(c^4*d^9*e^3 - 4*a*c^3*d^7*e^5 + 6*a^2*c^2*d^5*e
^7 - 4*a^3*c*d^3*e^9 + a^4*d*e^11)*x^3 + 6*(c^4*d^10*e^2 - 4*a*c^3*d^8*e^4
+ 6*a^2*c^2*d^6*e^6 - 4*a^3*c*d^4*e^8 + a^4*d^2*e^10)*x^2 + 4*(c^4*d^11*e -
 4*a*c^3*d^9*e^3 + 6*a^2*c^2*d^7*e^5 - 4*a^3*c*d^5*e^7 + a^4*d^3*e^9)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1954 $\int \frac{(d+ex)^5}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=302

$$\frac{7e(d+ex)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3c^2d^2}+\frac{35e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{12c^3d^3}+\frac{35e\left(cd^2-ae^2\right)^2\sqrt{x(\cdots}}{8\cdots}$$

```
[Out] (-2*(d + e*x)^4)/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (35*e*
(c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*c^4*d^4)
+ (35*e*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(12*c^3*d^3) + (7*e*(d + e*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(3*c^2*d^2) + (35*Sqrt[e]*(c*d^2 - a*e^2)^3*ArcTanh[(c*d^2 + a*e^2 +
2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])])/(16*c^(9/2)*d^(9/2))
```

Rubi [A] time = 0.24369, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {668, 670, 640, 621, 206}

$$\frac{7e(d+ex)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3c^2d^2}+\frac{35e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{12c^3d^3}+\frac{35e\left(cd^2-ae^2\right)^2\sqrt{x(\cdots}}{8\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (-2*(d + e*x)^4)/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (35*e*
(c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*c^4*d^4)
+ (35*e*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(12*c^3*d^3) + (7*e*(d + e*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(3*c^2*d^2) + (35*Sqrt[e]*(c*d^2 - a*e^2)^3*ArcTanh[(c*d^2 + a*e^2 +
2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])])/(16*c^(9/2)*d^(9/2))
```

Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^5}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2(d+ex)^4}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{(7e)\int \frac{(d+ex)^3}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{cd}$$

$$= -\frac{2(d+ex)^4}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{7e(d+ex)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{3c^2d^2}$$

$$= -\frac{2(d+ex)^4}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{35e\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+a\right.}}{12c^3d^3}$$

$$= -\frac{2(d+ex)^4}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{35e\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)x+}}{8c^4d^4}$$

$$= -\frac{2(d+ex)^4}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{35e\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)x+}}{8c^4d^4}$$

$$= -\frac{2(d+ex)^4}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{35e\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)x+}}{8c^4d^4}$$

Mathematica [C] time = 0.0713392, size = 100, normalized size = 0.33

$$-\frac{2\left(cd^2-ae^2\right)^4\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}\,{}_2F_1\left(-\frac{7}{2},-\frac{1}{2};\frac{1}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{c^5d^5\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (-2*(c*d^2 - a*e^2)^4*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*Hypergeometric2
F1[-7/2, -1/2, 1/2, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(c^5*d^5*Sqrt[(a
*e + c*d*x)*(d + e*x)])
```

Maple [B] time = 0.065, size = 1850, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] 2*d^5*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)-125/32*d^7*c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+23/12*e^3/c*x^3/(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)+1/3*e^4*x^4/d/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+
35/16*e*d^2/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)-125/32*d^3/c/(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)-35/32*e^8/d^5/c^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)*a^4+245/48*e^6/d^3/c^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^3-35/16*
e*d^2/c*x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+395/48*e^2*d/c^2/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a-28/3*e^4/d/c^3/(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)*a^2+125/24*e^2*d/c*x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)-105/16*e^3/c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a+5/12*e^2*d^5/(-a^2*e^4+2*a*c*
d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a+105/16*e^3/c^2*x
/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a+385/48*e^9/d^2/c^3/(-a^2*e^4+2*a
*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^4-53/24*e^5
*d^2/c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)*x*a^2-35/16*e^11/d^4/c^4/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^5+35/12*e^10/d^3/c^4/(-a^2*e^4+2*a*c*d^2*e^
2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^5-35/16*e^7/d^4/c^4*ln
((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2))/(d*e*c)^(1/2)*a^3+35/16*e^7/d^4/c^4*x/(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)*a^3-7/32*e^8/d/c^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^4-203/24*e^7/c^2/(-a^2*e^4+2*a*c*d^2*e^2-c
^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^3+415/48*e^3*d^4/(-a^2*
e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a+103/
32*e^4*d^3/c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)*a^2-14/3*e^4/d/c^2*x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a
-16/3*e^6*d/c^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)*a^3+35/24*e^6/d^3/c^3*x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)*a^2-7/12*e^5/d^2/c^2*x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a+105
/16*e^5/d^2/c^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^2-105/16*e^5/d^2/c^3*x/(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2-125/16*e*d^6*c/(-a^2*e^4+2*a*c*d^2*e^2
-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x-35/32*e^12/d^5/c^5/(-a^
2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 10.0756, size = 1574, normalized size = 5.21

$$\left[\frac{105\left(ac^3d^6e-3a^2c^2d^4e^3+3a^3cd^2e^5-a^4e^7+\left(c^4d^7-3ac^3d^5e^2+3a^2c^2d^3e^4-a^3cde^6\right)x\right)\sqrt{\frac{e}{cd}}\log\left(8c^2d^2e^2x^2+c^2d^4+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="f
ricas")
```

```
[Out] [1/96*(105*(a*c^3*d^6*e - 3*a^2*c^2*d^4*e^3 + 3*a^3*c*d^2*e^5 - a^4*e^7 + (
c^4*d^7 - 3*a*c^3*d^5*e^2 + 3*a^2*c^2*d^3*e^4 - a^3*c*d*e^6)*x)*sqrt(e/(c*d
))*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 8*(c^2*d^3*e
 + a*c*d*e^3)*x + 4*(2*c^2*d^2*e*x + c^2*d^3 + a*c*d*e^2)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e/(c*d))) + 4*(8*c^3*d^3*e^3*x^3 - 48*c^3*d
^6 + 231*a*c^2*d^4*e^2 - 280*a^2*c*d^2*e^4 + 105*a^3*e^6 + 2*(19*c^3*d^4*e^
2 - 7*a*c^2*d^2*e^4)*x^2 + (87*c^3*d^5*e - 98*a*c^2*d^3*e^3 + 35*a^2*c*d*e^
5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5*d^5*x + a*c^4*d^4*e
), -1/48*(105*(a*c^3*d^6*e - 3*a^2*c^2*d^4*e^3 + 3*a^3*c*d^2*e^5 - a^4*e^7
+ (c^4*d^7 - 3*a*c^3*d^5*e^2 + 3*a^2*c^2*d^3*e^4 - a^3*c*d*e^6)*x)*sqrt(-e/
(c*d))*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x +
c*d^2 + a*e^2)*sqrt(-e/(c*d))/(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x)
) - 2*(8*c^3*d^3*e^3*x^3 - 48*c^3*d^6 + 231*a*c^2*d^4*e^2 - 280*a^2*c*d^2*e
^4 + 105*a^3*e^6 + 2*(19*c^3*d^4*e^2 - 7*a*c^2*d^2*e^4)*x^2 + (87*c^3*d^5*e
 - 98*a*c^2*d^3*e^3 + 35*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x))/(c^5*d^5*x + a*c^4*d^4*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^5}{((d+ex)(ae+cdx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral((d + e*x)**5/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Giac [B] time = 1.34287, size = 840, normalized size = 2.78

$$\frac{\left(\left(2\left(\frac{4\left(c^5d^7e^7-2ac^4d^5e^9+a^2c^3d^3e^{11}\right)x}{c^6d^8e^3-2ac^5d^6e^5+a^2c^4d^4e^7}+\frac{23c^5d^8e^6-53ac^4d^6e^8+37a^2c^3d^4e^{10}-7a^3c^2d^2e^{12}}{c^6d^8e^3-2ac^5d^6e^5+a^2c^4d^4e^7}\right)x+\frac{125c^5d^9e^5-362ac^4d^7e^7+384a^2c^3d^5e^9-182a^3c^2d^3e^{11}+3}{c^6d^8e^3-2ac^5d^6e^5+a^2c^4d^4e^7}\right.\right.}{24\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="g
iac")
```

```
[Out] 1/24*(((2*(4*(c^5*d^7*e^7 - 2*a*c^4*d^5*e^9 + a^2*c^3*d^3*e^11)*x/(c^6*d^8*
e^3 - 2*a*c^5*d^6*e^5 + a^2*c^4*d^4*e^7) + (23*c^5*d^8*e^6 - 53*a*c^4*d^6*e
^8 + 37*a^2*c^3*d^4*e^10 - 7*a^3*c^2*d^2*e^12)/(c^6*d^8*e^3 - 2*a*c^5*d^6*e
^5 + a^2*c^4*d^4*e^7))*x + (125*c^5*d^9*e^5 - 362*a*c^4*d^7*e^7 + 384*a^2*c
^3*d^5*e^9 - 182*a^3*c^2*d^3*e^11 + 35*a^4*c*d*e^13)/(c^6*d^8*e^3 - 2*a*c^5
*d^6*e^5 + a^2*c^4*d^4*e^7))*x + (39*c^5*d^10*e^4 + 55*a*c^4*d^8*e^6 - 472*
a^2*c^3*d^6*e^8 + 728*a^3*c^2*d^4*e^10 - 455*a^4*c*d^2*e^12 + 105*a^5*e^14)
/(c^6*d^8*e^3 - 2*a*c^5*d^6*e^5 + a^2*c^4*d^4*e^7))*x - (48*c^5*d^11*e^3 -
327*a*c^4*d^9*e^5 + 790*a^2*c^3*d^7*e^7 - 896*a^3*c^2*d^5*e^9 + 490*a^4*c*d
^3*e^11 - 105*a^5*d*e^13)/(c^6*d^8*e^3 - 2*a*c^5*d^6*e^5 + a^2*c^4*d^4*e^7)
)/sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x) - 35/16*(c^3*d^6*e - 3*a*c^2*
d^4*e^3 + 3*a^2*c*d^2*e^5 - a^3*e^7)*sqrt(c*d)*e^(-1/2)*log(abs(-sqrt(c*d)*
c*d^2*e^(1/2) - 2*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + a*d*e + (c*d^2 +
a*e^2)*x))*c*d*e - sqrt(c*d)*a*e^(5/2)))/(c^5*d^5)
```

3.1955 $\int \frac{(d+ex)^4}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=241

$$\frac{5e(d+ex)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2c^2d^2}+\frac{15e\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4c^3d^3}+\frac{15\sqrt{e}\left(cd^2-ae^2\right)^2\tanh^{-1}\left(\cdots\right.}{8c^{7\cdots}}$$

```
[Out] (-2*(d + e*x)^3)/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (15*e*
(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*c^3*d^3) +
(5*e*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*c^2*d^2) + (
15*Sqrt[e]*(c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]
*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(7/2)*
d^(7/2))
```

Rubi [A] time = 0.161916, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {668, 670, 640, 621, 206}

$$\frac{5e(d+ex)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2c^2d^2}+\frac{15e\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4c^3d^3}+\frac{15\sqrt{e}\left(cd^2-ae^2\right)^2\tanh^{-1}\left(\cdots\right.}{8c^{7\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```


```
[Out] (-2*(d + e*x)^3)/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (15*e*
(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*c^3*d^3) +
(5*e*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*c^2*d^2) + (
15*Sqrt[e]*(c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]
*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(7/2)*
d^(7/2))
```


Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^4}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2(d+ex)^3}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{(5e)\int \frac{(d+ex)^2}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{cd}$$

$$= -\frac{2(d+ex)^3}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{5e(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2c^2d^2}+$$

$$= -\frac{2(d+ex)^3}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{15e\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}{4c^3d^3}$$

$$= -\frac{2(d+ex)^3}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{15e\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}{4c^3d^3}$$

$$= -\frac{2(d+ex)^3}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{15e\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}{4c^3d^3}$$

Mathematica [C] time = 0.0575939, size = 100, normalized size = 0.41

$$-\frac{2\left(cd^2-ae^2\right)^3\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{c^4d^4\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (-2*(c*d^2 - a*e^2)^3*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*Hypergeometric2
F1[-5/2, -1/2, 1/2, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(c^4*d^4*Sqrt[(a
*e + c*d*x)*(d + e*x)])
```

Maple [B] time = 0.059, size = 1428, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] -49/16*d^2/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+15/8*e*d/c*ln((1/2*a*e
^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
)/(d*e*c)^(1/2)-25/16*e^2*d^4/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)*a-11/8*e^6/c^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)
/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^3+1/2*e^3*x^3/d/c/(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)-55/16*e^4/d^2/c^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2)*a^2+15/8*e^9/d^3/c^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^4-5*e^7/d/c^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d
^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^3+9/4*e^5*d/c/(-a^2*e^4+2*a
*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^2-49/16*d^6
*c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
+73/16*e^2/c^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a+11/4*e^2/c*x^2/(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+2*d^4*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d
^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+15/16*e^6/d
^4/c^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^3-15/8*e*d/c*x/(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)+21/8*e^4*d^2/c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4
)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2+15/8*e^5/d^3/c^3*ln((1/2*a*e^
2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))
/(d*e*c)^(1/2)*a^2-49/8*e*d^5*c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+15/16*e^10/d^4/c^4/(-a^2*e^4+2*a*c*d^2*e^2-
c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^5-25/16*e^8/d^2/c^3/(-a^
2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^4+3*
e^3*d^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2)*x*a+15/4*e^3/d/c^2*x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a-15/4*e
^3/d/c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a-15/8*e^5/d^3/c^3*x/(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2)*a^2-5/4*e^4/d^2/c^2*x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.27712, size = 1216, normalized size = 5.05

$$\left[\frac{15\left(ac^2d^4e - 2a^2cd^2e^3 + a^3e^5 + \left(c^3d^5 - 2ac^2d^3e^2 + a^2cde^4\right)x\right)\sqrt{\frac{e}{cd}}\log\left(8c^2d^2e^2x^2 + c^2d^4 + 6acd^2e^2 + a^2e^4 + 8\left(c^2d^3e\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="f
ricas")
```

```
[Out] [1/16*(15*(a*c^2*d^4*e - 2*a^2*c*d^2*e^3 + a^3*e^5 + (c^3*d^5 - 2*a*c^2*d^3
*e^2 + a^2*c*d*e^4)*x)*sqrt(e/(c*d))*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*
c*d^2*e^2 + a^2*e^4 + 8*(c^2*d^3*e + a*c*d*e^3)*x + 4*(2*c^2*d^2*e*x + c^2*
d^3 + a*c*d*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e/(c*d)))
 + 4*(2*c^2*d^2*e^2*x^2 - 8*c^2*d^4 + 25*a*c*d^2*e^2 - 15*a^2*e^4 + (9*c^2*
d^3*e - 5*a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d
^4*x + a*c^3*d^3*e), -1/8*(15*(a*c^2*d^4*e - 2*a^2*c*d^2*e^3 + a^3*e^5 + (c
^3*d^5 - 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*x)*sqrt(-e/(c*d))*arctan(1/2*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-e/(
c*d))/(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x)) - 2*(2*c^2*d^2*e^2*x^2
 - 8*c^2*d^4 + 25*a*c*d^2*e^2 - 15*a^2*e^4 + (9*c^2*d^3*e - 5*a*c*d*e^3)*x)
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^4*x + a*c^3*d^3*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^4}{((d+ex)(ae+cdx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral((d + e*x)**4/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Giac [B] time = 1.31402, size = 648, normalized size = 2.69

$$\frac{\left(\left(\frac{2\left(c^4d^6e^5-2\,ac^3d^4e^7+a^2c^2d^2e^9\right)x}{c^5d^7e^2-2\,ac^4d^5e^4+a^2c^3d^3e^6}+\frac{11\,c^4d^7e^4-27\,ac^3d^5e^6+21\,a^2c^2d^3e^8-5\,a^3cde^{10}}{c^5d^7e^2-2\,ac^4d^5e^4+a^2c^3d^3e^6}\right)x+\frac{c^4d^8e^3+18\,ac^3d^6e^5-54\,a^2c^2d^4e^7+50\,a^3cd^2e^9-15\,a^4e^{11}}{c^5d^7e^2-2\,ac^4d^5e^4+a^2c^3d^3e^6}\right)x-\frac{8\,c^4d^9e^2}{}}{4\sqrt{cdx^2e+ade+\left(cd^2+ae^2\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="g
iac")
```

```
[Out] 1/4*(((2*(c^4*d^6*e^5 - 2*a*c^3*d^4*e^7 + a^2*c^2*d^2*e^9)*x/(c^5*d^7*e^2 -
 2*a*c^4*d^5*e^4 + a^2*c^3*d^3*e^6) + (11*c^4*d^7*e^4 - 27*a*c^3*d^5*e^6 +
21*a^2*c^2*d^3*e^8 - 5*a^3*c*d*e^10)/(c^5*d^7*e^2 - 2*a*c^4*d^5*e^4 + a^2*c
^3*d^3*e^6))*x + (c^4*d^8*e^3 + 18*a*c^3*d^6*e^5 - 54*a^2*c^2*d^4*e^7 + 50*
a^3*c*d^2*e^9 - 15*a^4*e^11)/(c^5*d^7*e^2 - 2*a*c^4*d^5*e^4 + a^2*c^3*d^3*e
^6))*x - (8*c^4*d^9*e^2 - 41*a*c^3*d^7*e^4 + 73*a^2*c^2*d^5*e^6 - 55*a^3*c*
d^3*e^8 + 15*a^4*d*e^10)/(c^5*d^7*e^2 - 2*a*c^4*d^5*e^4 + a^2*c^3*d^3*e^6))
/sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x) - 15/8*(c^2*d^4*e - 2*a*c*d^2*
e^3 + a^2*e^5)*sqrt(c*d)*e^(-1/2)*log(abs(-sqrt(c*d)*c*d^2*e^(1/2) - 2*(sqr
t(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x))*c*d*e - sqr
t(c*d)*a*e^(5/2)))/(c^4*d^4)
```

3.1956 $\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=180

$$\frac{3e\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^2d^2}+\frac{3\sqrt{e}\left(cd^2-ae^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2c^{5/2}d^{5/2}}-\frac{2(d+ex)^2}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+\ldots}}$$

```
[Out] (-2*(d + e*x)^2)/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (3*e*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c^2*d^2) + (3*Sqrt[e]*(c*d^2 -
 a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*c^(5/2)*d^(5/2))
```

Rubi [A] time = 0.0991254, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {668, 640, 621, 206}

$$\frac{3e\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^2d^2}+\frac{3\sqrt{e}\left(cd^2-ae^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2c^{5/2}d^{5/2}}-\frac{2(d+ex)^2}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```


```
[Out] (-2*(d + e*x)^2)/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (3*e*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c^2*d^2) + (3*Sqrt[e]*(c*d^2 -
 a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*c^(5/2)*d^(5/2))
```


Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2(d+ex)^2}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{(3e)\int \frac{d+ex}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{cd}$$

$$= -\frac{2(d+ex)^2}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{3e\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{c^2d^2} + \frac{\left(3e\left(cd^2\right.\right.}{}$$

$$= -\frac{2(d+ex)^2}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{3e\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{c^2d^2} + \frac{\left(3e\left(cd^2\right.\right.}{}$$

$$= -\frac{2(d+ex)^2}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{3e\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{c^2d^2} + \frac{3\sqrt{e}\left(cd\right.}{}$$

Mathematica [C] time = 0.0510216, size = 93, normalized size = 0.52

$$-\frac{2(d+ex)^2\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{cd\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{3/2}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (-2*(d + e*x)^2*Hypergeometric2F1[-3/2, -1/2, 1/2, (e*(a*e + c*d*x))/(-(c*d
^2) + a*e^2)])/(c*d*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(3/2)*Sqrt[(a*e + c*d
*x)*(d + e*x)])
```

Maple [B] time = 0.056, size = 1047, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] 2*d^3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)-9/4*d/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-
3/2*e/c*x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-9/4*d^5*c/(-a^2*e^4+2*a*c
*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+3/2*e/c*ln((1/2*a
*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2))/(d*e*c)^(1/2)+2*e^2/d/c^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a-5/2
*e^2*d^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
```

```
^(1/2)*a+e^2*x^2/d/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/4*e^4/d^3/c^
3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2-3/4*e^8/d^3/c^3/(-a^2*e^4+2*a
*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^4+1/2*e^6/d/c
^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
*a^3+e^4*d/c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)*a^2-3/2*e^3/d^2/c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/
2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a-3/2*e^7/d^2/c^2
/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x
*a^3+3/2*e^3/d^2/c^2*x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a+5/2*e^5/c/
(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*
a^2-1/2*e^3*d^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)*x*a-9/2*e*d^4*c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.82326, size = 932, normalized size = 5.18

$$\left[\frac{3\left(acd^2e - a^2e^3 + \left(c^2d^3 - acde^2\right)x\right)\sqrt{\frac{e}{cd}}\log\left(8\,c^2d^2e^2x^2 + c^2d^4 + 6\,acd^2e^2 + a^2e^4 + 8\left(c^2d^3e + acde^3\right)x + 4\left(2\,c^2d^2ex + \right.\right.}{4\left(c^3d^3x + ac^2d^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="f
ricas")
```

```
[Out] [1/4*(3*(a*c*d^2*e - a^2*e^3 + (c^2*d^3 - a*c*d*e^2)*x)*sqrt(e/(c*d))*log(8
*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 8*(c^2*d^3*e + a*c*d
*e^3)*x + 4*(2*c^2*d^2*e*x + c^2*d^3 + a*c*d*e^2)*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x)*sqrt(e/(c*d))) + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*(c*d*e*x - 2*c*d^2 + 3*a*e^2))/(c^3*d^3*x + a*c^2*d^2*e), -1/2*(3*(a
*c*d^2*e - a^2*e^3 + (c^2*d^3 - a*c*d*e^2)*x)*sqrt(-e/(c*d))*arctan(1/2*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-
e/(c*d))/(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x)) - 2*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*(c*d*e*x - 2*c*d^2 + 3*a*e^2))/(c^3*d^3*x + a*
c^2*d^2*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^3}{((d+ex)(ae+cdx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral((d + e*x)**3/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Giac [B] time = 1.28745, size = 478, normalized size = 2.66

$$\frac{\left(\frac{\left(c^3d^5e^3-2\,ac^2d^3e^5+a^2cde^7\right)x}{c^4d^6e-2\,ac^3d^4e^3+a^2c^2d^2e^5}-\frac{c^3d^6e^2-5\,ac^2d^4e^4+7\,a^2cd^2e^6-3\,a^3e^8}{c^4d^6e-2\,ac^3d^4e^3+a^2c^2d^2e^5}\right)x-\frac{2\,c^3d^7e-7\,ac^2d^5e^3+8\,a^2cd^3e^5-3\,a^3de^7}{c^4d^6e-2\,ac^3d^4e^3+a^2c^2d^2e^5}}{\sqrt{cdx^2e+ade+\left(cd^2+ae^2\right)x}}-\frac{3\left(cd^2e-ae^3\right)\sqrt{cd}e^{\left(-\frac{1}{2}\right)}\log\left(\left|-\sqrt{\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="g
iac")
```

```
[Out] (((c^3*d^5*e^3 - 2*a*c^2*d^3*e^5 + a^2*c*d*e^7)*x/(c^4*d^6*e - 2*a*c^3*d^4*
e^3 + a^2*c^2*d^2*e^5) - (c^3*d^6*e^2 - 5*a*c^2*d^4*e^4 + 7*a^2*c*d^2*e^6 -
 3*a^3*e^8)/(c^4*d^6*e - 2*a*c^3*d^4*e^3 + a^2*c^2*d^2*e^5))*x - (2*c^3*d^7
*e - 7*a*c^2*d^5*e^3 + 8*a^2*c*d^3*e^5 - 3*a^3*d*e^7)/(c^4*d^6*e - 2*a*c^3*
d^4*e^3 + a^2*c^2*d^2*e^5))/sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x) - 3
/2*(c*d^2*e - a*e^3)*sqrt(c*d)*e^(-1/2)*log(abs(-sqrt(c*d)*c*d^2*e^(1/2) -
2*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x))*c*d*e
 - sqrt(c*d)*a*e^(5/2)))/(c^3*d^3)
```

3.1957 $\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=125

$$\frac{\sqrt{e}\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{c^{3/2}d^{3/2}}-\frac{2(d+ex)}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

```
[Out] (-2*(d + e*x))/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (Sqrt[e]
*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(c^(3/2)*d^(3/2))
```

Rubi [A] time = 0.0582064, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {652, 621, 206}

$$\frac{\sqrt{e}\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{c^{3/2}d^{3/2}}-\frac{2(d+ex)}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (-2*(d + e*x))/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (Sqrt[e]
*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(c^(3/2)*d^(3/2))
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))^2*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[(e*(d + e*x)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[
(e^2*(p + 2))/(c*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2(d+ex)}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{e\int \frac{1}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{cd}$$

$$= -\frac{2(d+ex)}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{(2e)\,\text{Subst}\left(\int \frac{1}{4cde-x^2}\,dx, x, \frac{cd^2+ae^2+2cdex}{\sqrt{ade+\left(cd^2+ae^2\right)x+c}}\right.}{cd}$$

$$= -\frac{2(d+ex)}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{\sqrt{e}\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{c^{3/2}d^{3/2}}$$

Mathematica [A] time = 0.356946, size = 164, normalized size = 1.31

$$\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{cd^2-ae^2}\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)-2(cd)^{3/2}(d+ex)}{(cd)^{5/2}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (-2*(c*d)^(3/2)*(d + e*x) + 2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - a*e^2]*S
qrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqr
t[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/((c*d)^(5
/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [B] time = 0.051, size = 717, normalized size = 5.7

$$-\frac{ex}{cd}\frac{1}{\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2}}+\frac{ae^2}{2c^2d^2}\frac{1}{\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2}}-\frac{3}{2c}\frac{1}{\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2}}+\frac{}{cd\left(-a^2e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] -e*x/d/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*e^2/d^2/c^2/(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a-3/2/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)+e^5/d/c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2)*x*a^2-2*e^3*d/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)*x*a-3*e*d^3*c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+1/2*e^6/d^2/c^2/(-a^2*e^4+2*a*c*d^2*e^2-
c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^3-1/2*e^4/c/(-a^2*e^4+2*
a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2-5/2*e^2*d^
2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*
a-3/2*d^4*c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)+e/d/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+2*d^2*(2*c*d*e*x+a*e^2+c*d^2)/(4
*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.69143, size = 737, normalized size = 5.9

$$\left[\frac{(cdx+ae)\sqrt{\frac{e}{cd}}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4+8\left(c^2d^3e+acde^3\right)x+4\left(2\,c^2d^2ex+c^2d^3+acde^2\right)\sqrt{cdex^2+\cdots}\right.}{2\left(c^2d^2x+acde\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="f
ricas")
```

```
[Out] [1/2*((c*d*x + a*e)*sqrt(e/(c*d))*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d
^2*e^2 + a^2*e^4 + 8*(c^2*d^3*e + a*c*d*e^3)*x + 4*(2*c^2*d^2*e*x + c^2*d^3
 + a*c*d*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e/(c*d))) -
4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*x + a*c*d*e), -((c*
d*x + a*e)*sqrt(-e/(c*d))*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-e/(c*d))/(c*d*e^2*x^2 + a*d*e^2 + (
c*d^2*e + a*e^3)*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*
d^2*x + a*c*d*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^2}{((d+ex)(ae+cdx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral((d + e*x)**2/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Giac [B] time = 1.29553, size = 309, normalized size = 2.47

$$-\frac{2\left(\frac{\left(c^2d^4e-2\,acd^2e^3+a^2e^5\right)x}{c^3d^5-2\,ac^2d^3e^2+a^2cde^4}+\frac{c^2d^5-2\,acd^3e^2+a^2de^4}{c^3d^5-2\,ac^2d^3e^2+a^2cde^4}\right)}{\sqrt{cdx^2e+ade+\left(cd^2+ae^2\right)x}}-\frac{\sqrt{cde}^{\frac{1}{2}}\log\left(\left|-\sqrt{cd}cd^2e^{\frac{1}{2}}-2\left(\sqrt{cd}xe^{\frac{1}{2}}-\sqrt{cdx^2e+ade+\left(cd^2+ae^2\right)x}\right)\right.\right.}{c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="g
iac")
```

```
[Out] -2*((c^2*d^4*e - 2*a*c*d^2*e^3 + a^2*e^5)*x/(c^3*d^5 - 2*a*c^2*d^3*e^2 + a^
2*c*d*e^4) + (c^2*d^5 - 2*a*c*d^3*e^2 + a^2*d*e^4)/(c^3*d^5 - 2*a*c^2*d^3*e
^2 + a^2*c*d*e^4))/sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x) - sqrt(c*d)*
e^(1/2)*log(abs(-sqrt(c*d)*c*d^2*e^(1/2) - 2*(sqrt(c*d)*x*e^(1/2) - sqrt(c*
d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x))*c*d*e - sqrt(c*d)*a*e^(5/2)))/(c^2*d^
2)
```

3.1958 $\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=50

$$-\frac{2(d+ex)}{\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

```
[Out] (-2*(d + e*x))/((c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)
```

Rubi [A] time = 0.0249906, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {636}

$$-\frac{2(d+ex)}{\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (-2*(d + e*x))/((c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2(d+ex)}{\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

Mathematica [A] time = 0.0146645, size = 39, normalized size = 0.78

$$-\frac{2(d+ex)}{\left(cd^2-ae^2\right)\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (-2*(d + e*x))/((c*d^2 - a*e^2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] time = 0.044, size = 58, normalized size = 1.2

$$2\frac{(cdx+ae)(ex+d)^2}{(ae^2-cd^2)(cdex^2+ae^2x+cd^2x+ade)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] 2*(c*d*x+a*e)*(e*x+d)^2/(a*e^2-c*d^2)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/
2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.13993, size = 130, normalized size = 2.6

$$-\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{acd^2e-a^2e^3+(c^2d^3-acde^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="fri
cas")
```

```
[Out] -2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a*c*d^2*e - a^2*e^3 + (c^2*
d^3 - a*c*d*e^2)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{d+ex}{((d+ex)(ae+cdx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral((d + e*x)/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Giac [B] time = 1.22509, size = 147, normalized size = 2.94

$$-\frac{2\left(\frac{(cd^2e-ae^3)x}{c^2d^4-2\,acd^2e^2+a^2e^4}+\frac{cd^3-ade^2}{c^2d^4-2\,acd^2e^2+a^2e^4}\right)}{\sqrt{cdx^2e+ade+(cd^2+ae^2)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="gia
c")
```

```
[Out] -2*((c*d^2*e - a*e^3)*x/(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4) + (c*d^3 - a*d*
e^2)/(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4))/sqrt(c*d*x^2*e + a*d*e + (c*d^2 +
 a*e^2)*x)
```

3.1959 $\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{2\left(ae^2+cd^2+2cdex\right)}{\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

```
[Out] (-2*(c*d^2 + a*e^2 + 2*c*d*e*x))/((c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.0089736, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {613}

$$-\frac{2\left(ae^2+cd^2+2cdex\right)}{\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-3/2),x]
```

```
[Out] (-2*(c*d^2 + a*e^2 + 2*c*d*e*x))/((c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2\left(cd^2+ae^2+2cdex\right)}{\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

Mathematica [A] time = 0.0212348, size = 49, normalized size = 0.79

$$-\frac{2\left(ae^2+cd(d+2ex)\right)}{\left(cd^2-ae^2\right)^2\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-3/2),x]
```

```
[Out] (-2*(a*e^2 + c*d*(d + 2*e*x)))/((c*d^2 - a*e^2)^2*Sqrt[(a*e + c*d*x)*(d + e
*x)])
```

Maple [A] time = 0.043, size = 86, normalized size = 1.4

$$-2\frac{(cdx+ae)(ex+d)\left(2\,cdex+ae^{2}+cd^{2}\right)}{\left(a^{2}e^{4}-2\,acd^{2}e^{2}+c^{2}d^{4}\right)\left(cdex^{2}+ae^{2}x+cd^{2}x+ade\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] -2*(c*d*x+a*e)*(e*x+d)*(2*c*d*e*x+a*e^2+c*d^2)/(a^2*e^4-2*a*c*d^2*e^2+c^2*d
^4)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.55686, size = 305, normalized size = 4.92

$$-\frac{2\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}\left(2\,cdex+cd^{2}+ae^{2}\right)}{ac^{2}d^{5}e-2\,a^{2}cd^{3}e^{3}+a^{3}de^{5}+\left(c^{3}d^{5}e-2\,ac^{2}d^{3}e^{3}+a^{2}cde^{5}\right)x^{2}+\left(c^{3}d^{6}-ac^{2}d^{4}e^{2}-a^{2}cd^{2}e^{4}+a^{3}e^{6}\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)/
(a*c^2*d^5*e - 2*a^2*c*d^3*e^3 + a^3*d*e^5 + (c^3*d^5*e - 2*a*c^2*d^3*e^3 +
 a^2*c*d*e^5)*x^2 + (c^3*d^6 - a*c^2*d^4*e^2 - a^2*c*d^2*e^4 + a^3*e^6)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(ade+cdex^{2}+x\left(ae^{2}+cd^{2}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral((a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(-3/2), x)
```

Giac [A] time = 1.24905, size = 134, normalized size = 2.16

$$-\frac{2\left(\frac{2\,cdxe}{c^2d^4-2\,acd^2e^2+a^2e^4}+\frac{cd^2+ae^2}{c^2d^4-2\,acd^2e^2+a^2e^4}\right)}{\sqrt{cdx^2e+ade+\left(cd^2+ae^2\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(2*c*d*x*e/(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4) + (c*d^2 + a*e^2)/(c^2*d^
4 - 2*a*c*d^2*e^2 + a^2*e^4))/sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)
```

3.1960 $\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=121

$$\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8cd\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

```
[Out] 2/(3*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
 - (8*c*d*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^2 - a*e^2)^3*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.0386184, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {658, 613}

$$\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8cd\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```


```
[Out] 2/(3*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
 - (8*c*d*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^2 - a*e^2)^3*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])
```


Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{(4cd)\int \frac{1}{\left(ade+\left(c\right.\right.}}{3\left(c\right.}$$

$$=\frac{2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{8cd\,(}{3\left(cd^2-ae^2\right)^3}$$

Mathematica [A] time = 0.053194, size = 95, normalized size = 0.79

$$\frac{2a^2e^4-4acde^2(3d+2ex)-2c^2d^2\left(3d^2+12dex+8e^2x^2\right)}{3(d+ex)\left(cd^2-ae^2\right)^3\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (2*a^2*e^4 - 4*a*c*d*e^2*(3*d + 2*e*x) - 2*c^2*d^2*(3*d^2 + 12*d*e*x + 8*e^
2*x^2))/(3*(c*d^2 - a*e^2)^3*(d + e*x)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] time = 0.046, size = 138, normalized size = 1.1

$$-\frac{(2\,cdx+2\,ae)\left(-8\,c^2d^2e^2x^2-4\,acde^3x-12\,c^2d^3ex+a^2e^4-6\,acd^2e^2-3\,c^2d^4\right)}{3\,a^3e^6-9\,a^2cd^2e^4+9\,ac^2d^4e^2-3\,c^3d^6}\left(cdex^2+ae^2x+cd^2x+ade\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(-8*c^2*d^2*e^2*x^2-4*a*c*d*e^3*x-12*c^2*d^3*e*x+a^2*e^4-6
*a*c*d^2*e^2-3*c^2*d^4)/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2-c^3*d^6)/(
c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 14.6947, size = 599, normalized size = 4.95

$$-\frac{2\left(8\,c^2d^2e^2x^2+3\,c^2d^4+6\,acd^2e^2-a^2e^4+4\left(3\,c^2d^3e+acde^3\right)x\right)\sqrt{}}{3\left(ac^3d^8e-3\,a^2c^2d^6e^3+3\,a^3cd^4e^5-a^4d^2e^7+\left(c^4d^7e^2-3\,ac^3d^5e^4+3\,a^2c^2d^3e^6-a^3cde^8\right)x^3+\left(2\,c^4d^8e-5\,ac^3d^6e^3+3\,a^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="f
ricas")
```

```
[Out] -2/3*(8*c^2*d^2*e^2*x^2 + 3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4 + 4*(3*c^2*d^
3*e + a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a*c^3*d^8*
e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^4*d^7*e^2 - 3*a*
c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4*d^8*e - 5*a*c^3
```

```
*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2 + (c^4*d^9 - a*
c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d*e^8)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\left(d + ex\right)\left(ae + cdx\right)\right)^{\frac{3}{2}}\left(d + ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral(1/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\text{undef}, \text{undef}, \text{undef}, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="g
iac")
```

```
[Out] [undef, undef, undef, 1]
```

3.1961 $\int \frac{1}{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=181

$$-\frac{16c^2d^2\left(ae^2+cd^2+2cdex\right)}{5\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{4cd}{5(d+ex)\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{}{5(d+ex)^2\left(cd^2-\right.}$$

```
[Out] 2/(5*(c*d^2 - a*e^2)*(d + e*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]) + (4*c*d)/(5*(c*d^2 - a*e^2)^2*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2]) - (16*c^2*d^2*(c*d^2 + a*e^2 + 2*c*d*e*x))/(5*(c*d^2 - a*e^2)
^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.0713613, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {658, 613}

$$-\frac{16c^2d^2\left(ae^2+cd^2+2cdex\right)}{5\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{4cd}{5(d+ex)\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{}{5(d+ex)^2\left(cd^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] 2/(5*(c*d^2 - a*e^2)*(d + e*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]) + (4*c*d)/(5*(c*d^2 - a*e^2)^2*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2]) - (16*c^2*d^2*(c*d^2 + a*e^2 + 2*c*d*e*x))/(5*(c*d^2 - a*e^2)
^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2}{5\left(cd^2-ae^2\right)(d+ex)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{(6cd)\int \frac{}{(d+ex}}{}$$

$$= \frac{2}{5\left(cd^2-ae^2\right)(d+ex)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{}{5\left(cd^2-ae^2\right)}$$

$$= \frac{2}{5\left(cd^2-ae^2\right)(d+ex)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{}{5\left(cd^2-ae^2\right)}$$

Mathematica [A] time = 0.0633645, size = 136, normalized size = 0.75

$$-\frac{2\left(-a^2cde^4(5d+2ex)+a^3e^6+ac^2d^2e^2\left(15d^2+20dex+8e^2x^2\right)+c^3d^3\left(30d^2ex+5d^3+40de^2x^2+16e^3x^3\right)\right)}{5(d+ex)^2\left(cd^2-ae^2\right)^4\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*(a^3*e^6 - a^2*c*d*e^4*(5*d + 2*e*x) + a*c^2*d^2*e^2*(15*d^2 + 20*d*e*x
 + 8*e^2*x^2) + c^3*d^3*(5*d^3 + 30*d^2*e*x + 40*d*e^2*x^2 + 16*e^3*x^3)))/
(5*(c*d^2 - a*e^2)^4*(d + e*x)^2*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] time = 0.048, size = 216, normalized size = 1.2

$$-\frac{(2\,cdx+2\,ae)\left(16\,c^3d^3e^3x^3+8\,ac^2d^2e^4x^2+40\,c^3d^4e^2x^2-2\,a^2cde^5x+20\,ac^2d^3e^3x+30\,c^3d^5ex+a^3e^6-5\,a^2cd^2e^4+15\right.}{(5\,ex+5\,d)\left(a^4e^8-4\,a^3cd^2e^6+6\,a^2c^2d^4e^4-4\,ac^3d^6e^2+c^4d^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] -2/5*(c*d*x+a*e)*(16*c^3*d^3*e^3*x^3+8*a*c^2*d^2*e^4*x^2+40*c^3*d^4*e^2*x^2
-2*a^2*c*d*e^5*x+20*a*c^2*d^3*e^3*x+30*c^3*d^5*e*x+a^3*e^6-5*a^2*c*d^2*e^4+
15*a*c^2*d^4*e^2+5*c^3*d^6)/(e*x+d)/(a^4*e^8-4*a^3*c*d^2*e^6+6*a^2*c^2*d^4*
e^4-4*a*c^3*d^6*e^2+c^4*d^8)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 41.2815, size = 991, normalized size = 5.48

$$-\frac{2\left(16\,c^3d^3e^3x^3+5\,c^3d^6-\right.}{5\left(ac^4d^{11}e-4\,a^2c^3d^9e^3+6\,a^3c^2d^7e^5-4\,a^4cd^5e^7+a^5d^3e^9+\left(c^5d^9e^3-4\,ac^4d^7e^5+6\,a^2c^3d^5e^7-4\,a^3c^2d^3e^9+a^4cde^{11}\right)x^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"fricas")
```

```
[Out] -2/5*(16*c^3*d^3*e^3*x^3 + 5*c^3*d^6 + 15*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4 +
 a^3*e^6 + 8*(5*c^3*d^4*e^2 + a*c^2*d^2*e^4)*x^2 + 2*(15*c^3*d^5*e + 10*a*c
^2*d^3*e^3 - a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a
*c^4*d^11*e - 4*a^2*c^3*d^9*e^3 + 6*a^3*c^2*d^7*e^5 - 4*a^4*c*d^5*e^7 + a^5
*d^3*e^9 + (c^5*d^9*e^3 - 4*a*c^4*d^7*e^5 + 6*a^2*c^3*d^5*e^7 - 4*a^3*c^2*d
^3*e^9 + a^4*c*d*e^11)*x^4 + (3*c^5*d^10*e^2 - 11*a*c^4*d^8*e^4 + 14*a^2*c^
3*d^6*e^6 - 6*a^3*c^2*d^4*e^8 - a^4*c*d^2*e^10 + a^5*e^12)*x^3 + 3*(c^5*d^1
1*e - 3*a*c^4*d^9*e^3 + 2*a^2*c^3*d^7*e^5 + 2*a^3*c^2*d^5*e^7 - 3*a^4*c*d^3
*e^9 + a^5*d*e^11)*x^2 + (c^5*d^12 - a*c^4*d^10*e^2 - 6*a^2*c^3*d^8*e^4 + 1
4*a^3*c^2*d^6*e^6 - 11*a^4*c*d^4*e^8 + 3*a^5*d^2*e^10)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.1962 $\int \frac{1}{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=241

$$-\frac{128c^3d^3\left(ae^2+cd^2+2cdex\right)}{35\left(cd^2-ae^2\right)^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{32c^2d^2}{35(d+ex)\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{}{35(d+ex)^2}$$

```
[Out] 2/(7*(c*d^2 - a*e^2)*(d + e*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]) + (16*c*d)/(35*(c*d^2 - a*e^2)^2*(d + e*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2]) + (32*c^2*d^2)/(35*(c*d^2 - a*e^2)^3*(d + e*x)*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (128*c^3*d^3*(c*d^2 + a*e^2 + 2*c*d*e*
x))/(35*(c*d^2 - a*e^2)^5*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.111302, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {658, 613}

$$-\frac{128c^3d^3\left(ae^2+cd^2+2cdex\right)}{35\left(cd^2-ae^2\right)^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{32c^2d^2}{35(d+ex)\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{}{35(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```


```
[Out] 2/(7*(c*d^2 - a*e^2)*(d + e*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]) + (16*c*d)/(35*(c*d^2 - a*e^2)^2*(d + e*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2]) + (32*c^2*d^2)/(35*(c*d^2 - a*e^2)^3*(d + e*x)*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (128*c^3*d^3*(c*d^2 + a*e^2 + 2*c*d*e*
x))/(35*(c*d^2 - a*e^2)^5*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```


Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2}{7\left(cd^2-ae^2\right)(d+ex)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{(8cd)\int \frac{1}{(d+ex)^2(a}}{7(}$$

$$= \frac{2}{7\left(cd^2-ae^2\right)(d+ex)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{}{35\left(cd^2-ae^2\right)^2}$$

$$= \frac{2}{7\left(cd^2-ae^2\right)(d+ex)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{}{35\left(cd^2-ae^2\right)^2}$$

$$= \frac{2}{7\left(cd^2-ae^2\right)(d+ex)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{}{35\left(cd^2-ae^2\right)^2}$$

Mathematica [A] time = 0.0893301, size = 193, normalized size = 0.8

$$-\frac{2\left(-2a^2c^2d^2e^4\left(35d^2+28dex+8e^2x^2\right)+4a^3cde^6(7d+2ex)-5a^4e^8+4ac^3d^3e^2\left(70d^2ex+35d^3+56de^2x^2+16e^3x^3\right)+c\right.}{35(d+ex)^3\left(cd^2-ae^2\right)^5\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*(-5*a^4*e^8 + 4*a^3*c*d*e^6*(7*d + 2*e*x) - 2*a^2*c^2*d^2*e^4*(35*d^2 +
 28*d*e*x + 8*e^2*x^2) + 4*a*c^3*d^3*e^2*(35*d^3 + 70*d^2*e*x + 56*d*e^2*x^
2 + 16*e^3*x^3) + c^4*d^4*(35*d^4 + 280*d^3*e*x + 560*d^2*e^2*x^2 + 448*d*e
^3*x^3 + 128*e^4*x^4)))/(35*(c*d^2 - a*e^2)^5*(d + e*x)^3*Sqrt[(a*e + c*d*x
)*(d + e*x)])
```

Maple [A] time = 0.051, size = 307, normalized size = 1.3

$$-\frac{(2\,cdx+2\,ae)\left(-128\,c^4d^4e^4x^4-64\,ac^3d^3e^5x^3-448\,c^4d^5e^3x^3+16\,a^2c^2d^2e^6x^2-224\,ac^3d^4e^4x^2-560\,c^4d^6e^2x^2-8\,a^3cde^7x\right.}{35\left(a^5e^{10}-5\,a^4cd^2e^8+10\,a^3c^2d^4e^6-10\,a^2c^3d^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] -2/35*(c*d*x+a*e)*(-128*c^4*d^4*e^4*x^4-64*a*c^3*d^3*e^5*x^3-448*c^4*d^5*e^
3*x^3+16*a^2*c^2*d^2*e^6*x^2-224*a*c^3*d^4*e^4*x^2-560*c^4*d^6*e^2*x^2-8*a^
3*c*d*e^7*x+56*a^2*c^2*d^3*e^5*x-280*a*c^3*d^5*e^3*x-280*c^4*d^7*e*x+5*a^4*
e^8-28*a^3*c*d^2*e^6+70*a^2*c^2*d^4*e^4-140*a*c^3*d^6*e^2-35*c^4*d^8)/(e*x+
d)^2/(a^5*e^10-5*a^4*c*d^2*e^8+10*a^3*c^2*d^4*e^6-10*a^2*c^3*d^6*e^4+5*a*c^
4*d^8*e^2-c^5*d^10)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 115.071, size = 1509, normalized size = 6.26

$$-\frac{}{35\left(ac^5d^{14}e - 5\,a^2c^4d^{12}e^3 + 10\,a^3c^3d^{10}e^5 - 10\,a^4c^2d^8e^7 + 5\,a^5cd^6e^9 - a^6d^4e^{11} + \left(c^6d^{11}e^4 - 5\,ac^5d^9e^6 + 10\,a^2c^4d^7e^8 - 1\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"fricas")
```

```
[Out] -2/35*(128*c^4*d^4*e^4*x^4 + 35*c^4*d^8 + 140*a*c^3*d^6*e^2 - 70*a^2*c^2*d^
4*e^4 + 28*a^3*c*d^2*e^6 - 5*a^4*e^8 + 64*(7*c^4*d^5*e^3 + a*c^3*d^3*e^5)*x
^3 + 16*(35*c^4*d^6*e^2 + 14*a*c^3*d^4*e^4 - a^2*c^2*d^2*e^6)*x^2 + 8*(35*c
^4*d^7*e + 35*a*c^3*d^5*e^3 - 7*a^2*c^2*d^3*e^5 + a^3*c*d*e^7)*x)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a*c^5*d^14*e - 5*a^2*c^4*d^12*e^3 + 10*
a^3*c^3*d^10*e^5 - 10*a^4*c^2*d^8*e^7 + 5*a^5*c*d^6*e^9 - a^6*d^4*e^11 + (c
^6*d^11*e^4 - 5*a*c^5*d^9*e^6 + 10*a^2*c^4*d^7*e^8 - 10*a^3*c^3*d^5*e^10 +
5*a^4*c^2*d^3*e^12 - a^5*c*d*e^14)*x^5 + (4*c^6*d^12*e^3 - 19*a*c^5*d^10*e^
5 + 35*a^2*c^4*d^8*e^7 - 30*a^3*c^3*d^6*e^9 + 10*a^4*c^2*d^4*e^11 + a^5*c*d
^2*e^13 - a^6*e^15)*x^4 + 2*(3*c^6*d^13*e^2 - 13*a*c^5*d^11*e^4 + 20*a^2*c^
4*d^9*e^6 - 10*a^3*c^3*d^7*e^8 - 5*a^4*c^2*d^5*e^10 + 7*a^5*c*d^3*e^12 - 2*
a^6*d*e^14)*x^3 + 2*(2*c^6*d^14*e - 7*a*c^5*d^12*e^3 + 5*a^2*c^4*d^10*e^5 +
 10*a^3*c^3*d^8*e^7 - 20*a^4*c^2*d^6*e^9 + 13*a^5*c*d^4*e^11 - 3*a^6*d^2*e^
13)*x^2 + (c^6*d^15 - a*c^5*d^13*e^2 - 10*a^2*c^4*d^11*e^4 + 30*a^3*c^3*d^9
*e^6 - 35*a^4*c^2*d^7*e^8 + 19*a^5*c*d^5*e^10 - 4*a^6*d^3*e^12)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

[undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"giac")
```

```
[Out] [undef, undef, undef, 1]
```

3.1963 $\int \frac{1}{(d+ex)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=301

$$-\frac{256c^4d^4\left(ae^2+cd^2+2cdex\right)}{63\left(cd^2-ae^2\right)^6\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{64c^3d^3}{63(d+ex)\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{}{63(d+ex)^2}$$

```
[Out] 2/(9*(c*d^2 - a*e^2)*(d + e*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]) + (20*c*d)/(63*(c*d^2 - a*e^2)^2*(d + e*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2]) + (32*c^2*d^2)/(63*(c*d^2 - a*e^2)^3*(d + e*x)^2*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (64*c^3*d^3)/(63*(c*d^2 - a*e^2)^4*(
d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (256*c^4*d^4*(c*d^2
 + a*e^2 + 2*c*d*e*x))/(63*(c*d^2 - a*e^2)^6*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2])
```

Rubi [A] time = 0.158645, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {658, 613}

$$-\frac{256c^4d^4\left(ae^2+cd^2+2cdex\right)}{63\left(cd^2-ae^2\right)^6\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{64c^3d^3}{63(d+ex)\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{}{63(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```


```
[Out] 2/(9*(c*d^2 - a*e^2)*(d + e*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]) + (20*c*d)/(63*(c*d^2 - a*e^2)^2*(d + e*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2]) + (32*c^2*d^2)/(63*(c*d^2 - a*e^2)^3*(d + e*x)^2*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (64*c^3*d^3)/(63*(c*d^2 - a*e^2)^4*(
d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (256*c^4*d^4*(c*d^2
 + a*e^2 + 2*c*d*e*x))/(63*(c*d^2 - a*e^2)^6*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2])
```


Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}dx = \frac{2}{9\left(cd^2-ae^2\right)(d+ex)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{(10cd)\int \frac{}{(d+ex)^3(}}{9(}$$

$$= \frac{2}{9\left(cd^2-ae^2\right)(d+ex)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{}{63\left(cd^2-ae^2\right)^2}$$

$$= \frac{2}{9\left(cd^2-ae^2\right)(d+ex)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{}{63\left(cd^2-ae^2\right)^2}$$

$$= \frac{2}{9\left(cd^2-ae^2\right)(d+ex)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{}{63\left(cd^2-ae^2\right)^2}$$

$$= \frac{2}{9\left(cd^2-ae^2\right)(d+ex)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{}{63\left(cd^2-ae^2\right)^2}$$

Mathematica [A] time = 0.121169, size = 258, normalized size = 0.86

$$-\frac{2\left(2a^3c^2d^2e^6\left(63d^2+36dex+8e^2x^2\right)-2a^2c^3d^3e^4\left(126d^2ex+105d^3+72de^2x^2+16e^3x^3\right)-5a^4cde^8(9d+2ex)+7a^5e^{10}\right.}{63(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*(7*a^5*e^10 - 5*a^4*c*d*e^8*(9*d + 2*e*x) + 2*a^3*c^2*d^2*e^6*(63*d^2 +
 36*d*e*x + 8*e^2*x^2) - 2*a^2*c^3*d^3*e^4*(105*d^3 + 126*d^2*e*x + 72*d*e^
2*x^2 + 16*e^3*x^3) + a*c^4*d^4*e^2*(315*d^4 + 840*d^3*e*x + 1008*d^2*e^2*x
^2 + 576*d*e^3*x^3 + 128*e^4*x^4) + c^5*d^5*(63*d^5 + 630*d^4*e*x + 1680*d^
3*e^2*x^2 + 2016*d^2*e^3*x^3 + 1152*d*e^4*x^4 + 256*e^5*x^5)))/(63*(c*d^2 -
 a*e^2)^6*(d + e*x)^4*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] time = 0.051, size = 412, normalized size = 1.4

$$-\frac{(2\,cdx+2\,ae)\left(256\,c^5d^5e^5x^5+128\,ac^4d^4e^6x^4+1152\,c^5d^6e^4x^4-32\,a^2c^3d^3e^7x^3+576\,ac^4d^5e^5x^3+2016\,c^5d^7e^3x^3+16\,a^3c^2\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] -2/63*(c*d*x+a*e)*(256*c^5*d^5*e^5*x^5+128*a*c^4*d^4*e^6*x^4+1152*c^5*d^6*e
^4*x^4-32*a^2*c^3*d^3*e^7*x^3+576*a*c^4*d^5*e^5*x^3+2016*c^5*d^7*e^3*x^3+16
*a^3*c^2*d^2*e^8*x^2-144*a^2*c^3*d^4*e^6*x^2+1008*a*c^4*d^6*e^4*x^2+1680*c^
5*d^8*e^2*x^2-10*a^4*c*d*e^9*x+72*a^3*c^2*d^3*e^7*x-252*a^2*c^3*d^5*e^5*x+8
40*a*c^4*d^7*e^3*x+630*c^5*d^9*e*x+7*a^5*e^10-45*a^4*c*d^2*e^8+126*a^3*c^2*
d^4*e^6-210*a^2*c^3*d^6*e^4+315*a*c^4*d^8*e^2+63*c^5*d^10)/(e*x+d)^3/(a^6*e
^12-6*a^5*c*d^2*e^10+15*a^4*c^2*d^4*e^8-20*a^3*c^3*d^6*e^6+15*a^2*c^4*d^8*e
^4-6*a*c^5*d^10*e^2+c^6*d^12)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

[undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"giac")
```

```
[Out] [undef, undef, undef, 1]
```

3.1964 $\int \frac{(d+ex)^6}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=294

$$-\frac{14e(d+ex)^3}{3c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{35e^2(d+ex)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{6c^3d^3}+\frac{35e^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+a}}{4c^4d^4}$$

```
[Out] (-2*(d + e*x)^5)/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (1
4*e*(d + e*x)^3)/(3*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) +
(35*e^2*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*c^4
*d^4) + (35*e^2*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(6*c
^3*d^3) + (35*e^(3/2)*(c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)
/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/
(8*c^(9/2)*d^(9/2))
```

Rubi [A] time = 0.211751, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {668, 670, 640, 621, 206}

$$-\frac{14e(d+ex)^3}{3c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{35e^2(d+ex)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{6c^3d^3}+\frac{35e^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+a}}{4c^4d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^6/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```


```
[Out] (-2*(d + e*x)^5)/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (1
4*e*(d + e*x)^3)/(3*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) +
(35*e^2*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*c^4
*d^4) + (35*e^2*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(6*c
^3*d^3) + (35*e^(3/2)*(c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)
/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/
(8*c^(9/2)*d^(9/2))
```


Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^6}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^5}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{(7e)\int \frac{(d+ex)^4}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{3cd}$$

$$= -\frac{2(d+ex)^5}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{14e(d+ex)^3}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$= -\frac{2(d+ex)^5}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{14e(d+ex)^3}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$= -\frac{2(d+ex)^5}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{14e(d+ex)^3}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$= -\frac{2(d+ex)^5}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{14e(d+ex)^3}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$= -\frac{2(d+ex)^5}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{14e(d+ex)^3}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

Mathematica [C] time = 0.0689856, size = 112, normalized size = 0.38

$$-\frac{2\left(cd^2-ae^2\right)^3\sqrt{(d+ex)(ae+cdx)}\,{}_2F_1\left(-\frac{7}{2},-\frac{3}{2};-\frac{1}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{3c^4d^4(ae+cdx)^2\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^6/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (-2*(c*d^2 - a*e^2)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*Hypergeometric2F1[-7/2,
 -3/2, -1/2, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(3*c^4*d^4*(a*e + c*d*x
)^2*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])
```

Maple [B] time = 0.076, size = 4008, normalized size = 13.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] -35/24*e^3*d/c*x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-1165/192*e^6/d^2
/c^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^3-285/16*e^2*d^2/c*x^2/(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-35/384*e^10/d^6/c^6/(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(3/2)*a^5+637/384*e^8/d^4/c^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(3/2)*a^4+253/48*e*d^9*c^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)^2/(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1249/128*e^2*d^2/c^2/(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(3/2)*a-765/64*e*d^3/c*x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-
35/16*e^5/d^3/c^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2+35/16*e^7/d^5
/c^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^3+35/8*e^2*d^4/(-a^2*e^4+2*a
*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x-165/16*e^5*d^
5/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)*a^2+237/16*e^4/c^2*x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a-437/384*
e^2*d^6/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(3/2)*a-185/384*e^8/c^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(3/2)*a^4+1/2*e^5*x^5/d/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(3/2)-35/16*e^3/d/c^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a+253/384*d^
8*c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2
)+35/16*e*d/c^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+625/64*e^4/c^3/(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^2+17/4*e^4/c*x^4/(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(3/2)+35/8*e^2/c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(
1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+35/16*e*d^5/(-a
^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-35/8*
e^2/c^2*x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/128*d^4/c/(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(3/2)-115/4*e^4*d^6*c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)
^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a+35/8*e^10/d^4/c^4/(-a^2*e^4+
2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^4+65/64*
e^5*d^3/c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(3/2)*x*a^2+77/4*e^12/d^2/c^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)^2/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^5-415/64*e^9/d/c^3/(-a^2*e^4+2*a*c*d^2*
e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*a^4+265/48*e^7*d/c^2
/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x
*a^3-35/24*e^14/d^4/c^4/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)^2/(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2)*x*a^6+265/6*e^8*d^2/c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d
^4)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^3-415/8*e^10/c^2/(-a^2*e^
4+2*a*c*d^2*e^2-c^2*d^4)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^4-11
5/32*e^3*d^5/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(3/2)*x*a-35/4*e^6/c^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^2+65/8*e^6*d^4/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^
4)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^2+35/8*e^6/d^4/c^4*ln((1/2
*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2))/(d*e*c)^(1/2)*a^2-35/24*e^7/d^3/c^3*x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(3/2)*a^2+35/16*e^11/d^5/c^5/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^5+35/16*e^3*d^3/c/(-a^2*e^4+2*a*c*d^2*e^2
-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a+35/12*e^5/d/c^2*x^3/(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a-261/16*e^11/d/c^3/(-a^2*e^4+2*a*c*d^
```

```
2*e^2-c^2*d^4)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^5+45/32*e^5/d/c^
3*x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^2+35/4*e^4/d^2/c^3*x/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a-35/4*e^4/d^2/c^3*ln((1/2*a*e^2+1/2*c*d^2+
c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/
2)*a-7/4*e^6/d^2/c^2*x^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a-437/48*e
^3*d^7*c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2)*a-3/4*e^3*d/c^2*x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a-35/8*e
^5*d/c^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)*a^2+1255/384*e^6*d^2/c^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^3-165/128*e^4*d^4/c/(-a^2*e^4+2*a*c*d^2*e^2
-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^2-261/128*e^10/d^2/c^4/
(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^
5-147/16*e^6/d^2/c^3*x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^2+35/16*
e^9/d^3/c^4/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)*a^4-35/8*e^7/d/c^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)*a^3-35/192*e^13/d^5/c^5/(-a^2*e^4+2*a*c*d^2*e^2-
c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*a^6+77/32*e^11/d^3/c^4/(
-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*a
^5+35/64*e^9/d^5/c^5*x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^4+35/16*e^
8/d^4/c^4*x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^3-35/48*e^15/d^5/c^
5/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)*a^7+253/192*e*d^7*c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(3/2)*x-35/384*e^14/d^6/c^6/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^7+427/384*e^12/d^4/c^5/(-a^2*e^4+
2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^6-35/8*e^6
/d^4/c^4*x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2-185/48*e^9*d/c^2/(-a
^2*e^4+2*a*c*d^2*e^2-c^2*d^4)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^4
+1255/48*e^7*d^3/c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)^2/(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)*a^3-7/4*e^7/d^3/c^4*x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(3/2)*a^3+427/48*e^13/d^3/c^4/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)^2/(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^6+253/24*e^2*d^8*c^2/(-a^2*e^4+2*a*c*d^2*e^
2-c^2*d^4)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="m
axima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 27.4125, size = 1717, normalized size = 5.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="f
ricas")

[Out] [1/48*(105*(a^2*c^2*d^4*e^3 - 2*a^3*c*d^2*e^5 + a^4*e^7 + (c^4*d^6*e - 2*a*
c^3*d^4*e^3 + a^2*c^2*d^2*e^5)*x^2 + 2*(a*c^3*d^5*e^2 - 2*a^2*c^2*d^3*e^4 +
```

```
a^3*c*d*e^6)*x)*sqrt(e/(c*d))*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*
e^2 + a^2*e^4 + 8*(c^2*d^3*e + a*c*d*e^3)*x + 4*(2*c^2*d^2*e*x + c^2*d^3 +
a*c*d*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e/(c*d))) + 4*(
6*c^3*d^3*e^3*x^3 - 8*c^3*d^6 - 56*a*c^2*d^4*e^2 + 175*a^2*c*d^2*e^4 - 105*
a^3*e^6 + 3*(13*c^3*d^4*e^2 - 7*a*c^2*d^2*e^4)*x^2 - 2*(40*c^3*d^5*e - 119*
a*c^2*d^3*e^3 + 70*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x))/(c^6*d^6*x^2 + 2*a*c^5*d^5*e*x + a^2*c^4*d^4*e^2), -1/24*(105*(a^2*c^2
*d^4*e^3 - 2*a^3*c*d^2*e^5 + a^4*e^7 + (c^4*d^6*e - 2*a*c^3*d^4*e^3 + a^2*c
^2*d^2*e^5)*x^2 + 2*(a*c^3*d^5*e^2 - 2*a^2*c^2*d^3*e^4 + a^3*c*d*e^6)*x)*sq
rt(-e/(c*d))*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*
e*x + c*d^2 + a*e^2)*sqrt(-e/(c*d))/(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e
^3)*x)) - 2*(6*c^3*d^3*e^3*x^3 - 8*c^3*d^6 - 56*a*c^2*d^4*e^2 + 175*a^2*c*d
^2*e^4 - 105*a^3*e^6 + 3*(13*c^3*d^4*e^2 - 7*a*c^2*d^2*e^4)*x^2 - 2*(40*c^3
*d^5*e - 119*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x))/(c^6*d^6*x^2 + 2*a*c^5*d^5*e*x + a^2*c^4*d^4*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**6/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.5907, size = 1391, normalized size = 4.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="g
iac")
```

```
[Out] 1/12*((((3*(2*(c^7*d^11*e^8 - 4*a*c^6*d^9*e^10 + 6*a^2*c^5*d^7*e^12 - 4*a^3
*c^4*d^5*e^14 + a^4*c^3*d^3*e^16)*x/(c^8*d^12*e^3 - 4*a*c^7*d^10*e^5 + 6*a^
2*c^6*d^8*e^7 - 4*a^3*c^5*d^6*e^9 + a^4*c^4*d^4*e^11) + (17*c^7*d^12*e^7 -
75*a*c^6*d^10*e^9 + 130*a^2*c^5*d^8*e^11 - 110*a^3*c^4*d^6*e^13 + 45*a^4*c^
3*d^4*e^15 - 7*a^5*c^2*d^2*e^17)/(c^8*d^12*e^3 - 4*a*c^7*d^10*e^5 + 6*a^2*c
^6*d^8*e^7 - 4*a^3*c^5*d^6*e^9 + a^4*c^4*d^4*e^11))*x + 4*(c^7*d^13*e^6 + 4
5*a*c^6*d^11*e^8 - 225*a^2*c^5*d^9*e^10 + 430*a^3*c^4*d^7*e^12 - 405*a^4*c^
3*d^5*e^14 + 189*a^5*c^2*d^3*e^16 - 35*a^6*c*d*e^18)/(c^8*d^12*e^3 - 4*a*c^
7*d^10*e^5 + 6*a^2*c^6*d^8*e^7 - 4*a^3*c^5*d^6*e^9 + a^4*c^4*d^4*e^11))*x -
 3*(43*c^7*d^14*e^5 - 305*a*c^6*d^12*e^7 + 825*a^2*c^5*d^10*e^9 - 1075*a^3*
c^4*d^8*e^11 + 645*a^4*c^3*d^6*e^13 - 63*a^5*c^2*d^4*e^15 - 105*a^6*c*d^2*e
^17 + 35*a^7*e^19)/(c^8*d^12*e^3 - 4*a*c^7*d^10*e^5 + 6*a^2*c^6*d^8*e^7 - 4
*a^3*c^5*d^6*e^9 + a^4*c^4*d^4*e^11))*x - 6*(16*c^7*d^15*e^4 - 85*a*c^6*d^1
3*e^6 + 145*a^2*c^5*d^11*e^8 - 15*a^3*c^4*d^9*e^10 - 250*a^4*c^3*d^7*e^12 +
 329*a^5*c^2*d^5*e^14 - 175*a^6*c*d^3*e^16 + 35*a^7*d*e^18)/(c^8*d^12*e^3 -
 4*a*c^7*d^10*e^5 + 6*a^2*c^6*d^8*e^7 - 4*a^3*c^5*d^6*e^9 + a^4*c^4*d^4*e^1
1))*x - (8*c^7*d^16*e^3 + 24*a*c^6*d^14*e^5 - 351*a^2*c^5*d^12*e^7 + 1109*a
^3*c^4*d^10*e^9 - 1686*a^4*c^3*d^8*e^11 + 1386*a^5*c^2*d^6*e^13 - 595*a^6*c
*d^4*e^15 + 105*a^7*d^2*e^17)/(c^8*d^12*e^3 - 4*a*c^7*d^10*e^5 + 6*a^2*c^6*
```

```
d^8*e^7 - 4*a^3*c^5*d^6*e^9 + a^4*c^4*d^4*e^11))/(c*d*x^2*e + a*d*e + (c*d^
2 + a*e^2)*x)^(3/2) - 35/8*(c^2*d^4*e^2 - 2*a*c*d^2*e^4 + a^2*e^6)*sqrt(c*d
)*e^(-1/2)*log(abs(-sqrt(c*d)*c*d^2*e^(1/2) - 2*(sqrt(c*d)*x*e^(1/2) - sqrt
(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x))*c*d*e - sqrt(c*d)*a*e^(5/2)))/(c^5
*d^5)
```

3.1965 $\int \frac{(d+ex)^5}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=231

$$-\frac{10e(d+ex)^2}{3c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{5e^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^3d^3}+\frac{5e^{3/2}\left(cd^2-ae^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+a}}\right.}{2c^{7/2}d^{7/2}}$$

```
[Out] (-2*(d + e*x)^4)/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (1
0*e*(d + e*x)^2)/(3*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) +
(5*e^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c^3*d^3) + (5*e^(3/2)*
(c*d^2 - a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt
[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*c^(7/2)*d^(7/2))
```

Rubi [A] time = 0.139867, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {668, 640, 621, 206}

$$-\frac{10e(d+ex)^2}{3c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{5e^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^3d^3}+\frac{5e^{3/2}\left(cd^2-ae^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+a}}\right.}{2c^{7/2}d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^4)/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (1
0*e*(d + e*x)^2)/(3*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) +
(5*e^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c^3*d^3) + (5*e^(3/2)*
(c*d^2 - a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt
[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*c^(7/2)*d^(7/2))
```

Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^5}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^4}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{(5e)\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{3cd}$$

$$= -\frac{2(d+ex)^4}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{10e(d+ex)^2}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$= -\frac{2(d+ex)^4}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{10e(d+ex)^2}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$= -\frac{2(d+ex)^4}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{10e(d+ex)^2}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$= -\frac{2(d+ex)^4}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{10e(d+ex)^2}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

Mathematica [C] time = 0.0654721, size = 112, normalized size = 0.48

$$-\frac{2\left(cd^2-ae^2\right)^2\sqrt{(d+ex)(ae+cdx)}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{3c^3d^3(ae+cdx)^2\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (-2*(c*d^2 - a*e^2)^2*Sqrt[(a*e + c*d*x)*(d + e*x)]*Hypergeometric2F1[-5/2,
 -3/2, -1/2, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(3*c^3*d^3*(a*e + c*d*x
)^2*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])
```

Maple [B] time = 0.08, size = 3215, normalized size = 13.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] 19/96*d^7*c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(3/2)-5/6*e^3/c*x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+e^4*x^4/d/c
```

/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+5/96*e^8/d^5/c^5/(a*d*e+(a*e^2+c*d ^2)*x+c*d*e*x^2)^(3/2)*a^4+5/4*e/c^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-15/32*d^3/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+5/4*e*d^4/(-a^2*e^4+2 *a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-71/16*e^2*d/c ^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a-5/2*e^2/d/c^2*x/(a*d*e+(a*e^2+ c*d^2)*x+c*d*e*x^2)^(1/2)-5/4*e^5/d^4/c^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2) ^(1/2)*a^2+5/2*e^2/d/c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a* d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+19/12*e*d^8*c^2/(-a^2*e ^4+2*a*c*d^2*e^2-c^2*d^4)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+11/4*e^ 4/d/c^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^2-35/4*e^2*d/c*x^2/(a*d*e +(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-115/16*e*d^2/c*x/(a*d*e+(a*e^2+c*d^2)*x+c *d*e*x^2)^(3/2)-43/48*e^6/d^3/c^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a ^3+1/12*e^2*d^5/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d *e*x^2)^(3/2)*a-19/16*e^3/c^2*x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a+5 /2*e^2*d^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^ 2)^(1/2)*x+73/12*e^9/c^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)^2/(a*d*e+(a*e^2+c *d^2)*x+c*d*e*x^2)^(1/2)*a^4-97/12*e^5*d^4/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4) ^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2-5/2*e^8/d^3/c^3/(-a^2*e^4+2* a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^3-5/2*e^6/ d/c^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1 /2)*x*a^2-61/6*e^10/d/c^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)^2/(a*d*e+(a*e^2+ c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^4-11/6*e^4*d^5*c/(-a^2*e^4+2*a*c*d^2*e^2-c^2* d^4)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a+5/2*e^4*d/c/(-a^2*e^4+2* a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a+67/3*e^8*d /c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/ 2)*x*a^3+5/48*e^11/d^4/c^4/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c *d^2)*x+c*d*e*x^2)^(3/2)*x*a^5-61/48*e^9/d^2/c^3/(-a^2*e^4+2*a*c*d^2*e^2-c^ 2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*a^4+5/6*e^12/d^3/c^3/(-a^2 *e^4+2*a*c*d^2*e^2-c^2*d^4)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^5 -43/24*e^5*d^2/c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c* d*e*x^2)^(3/2)*x*a^2-5/2*e^7/d^2/c^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d* e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^3+11/16*e^5/d^2/c^3*x/(a*d*e+(a*e^2+c* d^2)*x+c*d*e*x^2)^(3/2)*a^2+1/2*e^6*d/c^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/ (a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^3-14/3*e^11/d^2/c^3/(-a^2*e^4+2*a *c*d^2*e^2-c^2*d^4)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^5-5/4*e^6/d ^3/c^3*x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^2+19/6*e^2*d^7*c^2/(-a ^2*e^4+2*a*c*d^2*e^2-c^2*d^4)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+1 9/48*e*d^6*c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e* x^2)^(3/2)*x+5/96*e^12/d^5/c^5/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e ^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^6-7/12*e^10/d^3/c^4/(-a^2*e^4+2*a*c*d^2*e^2- c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^5+73/96*e^8/d/c^3/(-a^2* e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^4+5/12 *e^13/d^4/c^4/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d *e*x^2)^(1/2)*a^6-5/16*e^7/d^4/c^4*x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^3+5/6*e^5/d^2/c^2*x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a-43/3*e^ 6*d^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^ (1/2)*x*a^2+67/24*e^7/c^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c* d^2)*x+c*d*e*x^2)^(3/2)*x*a^3-11/48*e^3*d^4/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*a+5/2*e^4/d^3/c^3*x/(a*d*e+(a*e ^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a-5/2*e^4/d^3/c^3*ln((1/2*a*e^2+1/2*c*d^2+c*d* e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a +2/3*e^3*d^6*c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)^2/(a*d*e+(a*e^2+c*d^2)*x+c* d*e*x^2)^(1/2)*a+4*e^4/d/c^2*x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a- 97/96*e^4*d^3/c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d *e*x^2)^(3/2)*a^2+4*e^7*d^2/c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)^2/(a*d*e+(a* e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^3+5/2*e^3*d^2/c/(-a^2*e^4+2*a*c*d^2*e^2-c^2 *d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a-5/4*e^9/d^4/c^4/(-a^2*e^4+2 *a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^4

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 12.2847, size = 1311, normalized size = 5.68

$$\left[\frac{15\left(a^2cd^2e^3 - a^3e^5 + \left(c^3d^4e - ac^2d^2e^3\right)x^2 + 2\left(ac^2d^3e^2 - a^2cde^4\right)x\right)\sqrt{\frac{e}{cd}}\log\left(8c^2d^2e^2x^2 + c^2d^4 + 6acd^2e^2 + a^2e^4 + 8(c\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="f
ricas")
```

```
[Out] [1/12*(15*(a^2*c*d^2*e^3 - a^3*e^5 + (c^3*d^4*e - a*c^2*d^2*e^3)*x^2 + 2*(a
*c^2*d^3*e^2 - a^2*c*d*e^4)*x)*sqrt(e/(c*d))*log(8*c^2*d^2*e^2*x^2 + c^2*d^
4 + 6*a*c*d^2*e^2 + a^2*e^4 + 8*(c^2*d^3*e + a*c*d*e^3)*x + 4*(2*c^2*d^2*e*
x + c^2*d^3 + a*c*d*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e
/(c*d))) + 4*(3*c^2*d^2*e^2*x^2 - 2*c^2*d^4 - 10*a*c*d^2*e^2 + 15*a^2*e^4 -
 2*(7*c^2*d^3*e - 10*a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x))/(c^5*d^5*x^2 + 2*a*c^4*d^4*e*x + a^2*c^3*d^3*e^2), -1/6*(15*(a^2*c*d^2
*e^3 - a^3*e^5 + (c^3*d^4*e - a*c^2*d^2*e^3)*x^2 + 2*(a*c^2*d^3*e^2 - a^2*c
*d*e^4)*x)*sqrt(-e/(c*d))*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-e/(c*d))/(c*d*e^2*x^2 + a*d*e^2 + (
c*d^2*e + a*e^3)*x)) - 2*(3*c^2*d^2*e^2*x^2 - 2*c^2*d^4 - 10*a*c*d^2*e^2 +
15*a^2*e^4 - 2*(7*c^2*d^3*e - 10*a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x))/(c^5*d^5*x^2 + 2*a*c^4*d^4*e*x + a^2*c^3*d^3*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.43827, size = 1116, normalized size = 4.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="g
iac")
```

```
[Out] 1/3*((((3*(c^6*d^10*e^6 - 4*a*c^5*d^8*e^8 + 6*a^2*c^4*d^6*e^10 - 4*a^3*c^3*
d^4*e^12 + a^4*c^2*d^2*e^14)*x/(c^7*d^11*e^2 - 4*a*c^6*d^9*e^4 + 6*a^2*c^5*
d^7*e^6 - 4*a^3*c^4*d^5*e^8 + a^4*c^3*d^3*e^10) - 4*(2*c^6*d^11*e^5 - 13*a*
c^5*d^9*e^7 + 32*a^2*c^4*d^7*e^9 - 38*a^3*c^3*d^5*e^11 + 22*a^4*c^2*d^3*e^1
3 - 5*a^5*c*d*e^15)/(c^7*d^11*e^2 - 4*a*c^6*d^9*e^4 + 6*a^2*c^5*d^7*e^6 - 4
*a^3*c^4*d^5*e^8 + a^4*c^3*d^3*e^10))*x - 3*(9*c^6*d^12*e^4 - 46*a*c^5*d^10
*e^6 + 89*a^2*c^4*d^8*e^8 - 76*a^3*c^3*d^6*e^10 + 19*a^4*c^2*d^4*e^12 + 10*
a^5*c*d^2*e^14 - 5*a^6*e^16)/(c^7*d^11*e^2 - 4*a*c^6*d^9*e^4 + 6*a^2*c^5*d^
7*e^6 - 4*a^3*c^4*d^5*e^8 + a^4*c^3*d^3*e^10))*x - 6*(3*c^6*d^13*e^3 - 12*a
*c^5*d^11*e^5 + 13*a^2*c^4*d^9*e^7 + 8*a^3*c^3*d^7*e^9 - 27*a^4*c^2*d^5*e^1
1 + 20*a^5*c*d^3*e^13 - 5*a^6*d*e^15)/(c^7*d^11*e^2 - 4*a*c^6*d^9*e^4 + 6*a
^2*c^5*d^7*e^6 - 4*a^3*c^4*d^5*e^8 + a^4*c^3*d^3*e^10))*x - (2*c^6*d^14*e^2
 + 2*a*c^5*d^12*e^4 - 43*a^2*c^4*d^10*e^6 + 112*a^3*c^3*d^8*e^8 - 128*a^4*c
^2*d^6*e^10 + 70*a^5*c*d^4*e^12 - 15*a^6*d^2*e^14)/(c^7*d^11*e^2 - 4*a*c^6*
d^9*e^4 + 6*a^2*c^5*d^7*e^6 - 4*a^3*c^4*d^5*e^8 + a^4*c^3*d^3*e^10))/(c*d*x
^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2) - 5/2*(c*d^2*e^2 - a*e^4)*sqrt(c*d)
*e^(-1/2)*log(abs(-sqrt(c*d)*c*d^2*e^(1/2) - 2*(sqrt(c*d)*x*e^(1/2) - sqrt(
c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x))*c*d*e - sqrt(c*d)*a*e^(5/2)))/(c^4*
d^4)
```

3.1966 $\int \frac{(d+ex)^4}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=172

$$-\frac{2e(d+ex)}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{e^{3/2}\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{c^{5/2}d^{5/2}}-\frac{2(d+ex)^3}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

```
[Out] (-2*(d + e*x)^3)/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (2
*e*(d + e*x))/(c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (e^(3
/2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(c^(5/2)*d^(5/2))
```

Rubi [A] time = 0.0887596, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {668, 652, 621, 206}

$$-\frac{2e(d+ex)}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{e^{3/2}\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{c^{5/2}d^{5/2}}-\frac{2(d+ex)^3}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```


```
[Out] (-2*(d + e*x)^3)/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (2
*e*(d + e*x))/(c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (e^(3
/2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(c^(5/2)*d^(5/2))
```


Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))^2*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[(e*(d + e*x)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[
(e^2*(p + 2))/(c*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^4}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^3}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{e\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{cd}$$

$$= -\frac{2(d+ex)^3}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2e(d+ex)}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+ \frac{e^2}{}$$

$$= -\frac{2(d+ex)^3}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2e(d+ex)}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+ \frac{(2}{}$$

$$= -\frac{2(d+ex)^3}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2e(d+ex)}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+ \frac{e^3}{}$$

Mathematica [C] time = 0.0720668, size = 98, normalized size = 0.57

$$-\frac{2((d+ex)(ae+cdx))^{3/2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{3cd(ae+cdx)^3\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (-2*((a*e + c*d*x)*(d + e*x))^(3/2)*Hypergeometric2F1[-3/2, -3/2, -1/2, (e*
(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(3*c*d*(a*e + c*d*x)^3*((c*d*(d + e*x))
/(c*d^2 - a*e^2))^(3/2))
```

Maple [B] time = 0.061, size = 2538, normalized size = 14.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] -1/48*e^6/d^4/c^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^3-31/8*e*d/c*x/
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+e^2*d^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2
*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x-7/24*e^6/c^2/(-a^2*e^4+2*a*
c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^3+5/16*e^2*d^4
/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a
-7/3*e^5*d^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)*a^2-1/3*e^3*x^3/d/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-e^
```

```
2/d^2/c^2*x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*e^3/d^3/c^3/(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a+e^2/d^2/c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*
e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)-1
/24*e^9/d^3/c^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(3/2)*x*a^4+2/3*e^7/d/c^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*a^3+16/3*e^4*d^4*c/(-a^2*e^4+2*a*c*d^2*e^
2-c^2*d^4)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a-5/4*e^5*d/c/(-a^2*
e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*a^2-1/
3*e^10/d^2/c^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)^2/(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)*x*a^4+e^6/d^2/c^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^2-11/16*d^2/c/(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(3/2)-1/48*d^6*c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(3/2)+1/2*e*d^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-25/16*e^2/c^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(3/2)*a-7/2*e^2/c*x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*e/d/c
^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/6*e*d^7*c^2/(-a^2*e^4+2*a*c*d^
2*e^2-c^2*d^4)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+7/16*e^4/d^2/c^3/(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^2+3/2*e^3*d/c/(-a^2*e^4+2*a*c*d^2*
e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a-1/6*e^11/d^3/c^3/(-a
^2*e^4+2*a*c*d^2*e^2-c^2*d^4)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^5
-1/3*e^2*d^6*c^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)^2/(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2)*x+5/2*e^3*d^5*c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)^2/(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a+1/8*e^5/d^3/c^3*x/(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(3/2)*a^2-7/24*e^4*d^2/c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^2-3/4*e^3/d/c^2*x/(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(3/2)*a+5/2*e^9/d/c^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)^2/(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^4-7/3*e^7*d/c/(-a^2*e^4+2*a*c*d^2*e^2
-c^2*d^4)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^3+1/2*e^4/d^2/c^2*x^2
/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a-1/24*e*d^5*c/(-a^2*e^4+2*a*c*d^2
*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x-1/48*e^10/d^4/c^4/(
-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^5
+16/3*e^8/c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)*x*a^3-10*e^6*d^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)^2/(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^2+2/3*e^3*d^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*
d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*a+2*e^4/c/(-a^2*e^4+2*a*c*d^
2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a+1/2*e^7/d^3/c^3/
(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^
3+3/2*e^5/d/c^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)*a^2+5/16*e^8/d^2/c^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="m
axima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 6.76715, size = 980, normalized size = 5.7

$$\left[\frac{3\left(c^2d^2ex^2+2\,acde^2x+a^2e^3\right)\sqrt{\frac{e}{cd}}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4+8\left(c^2d^3e+acde^3\right)x+4\left(2\,c^2d^2ex+c^2d^3+a\right.\right.}{6\left(c^4d^4x^2+2\,ac^3d^3ex+a^2c^2d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="f
ricas")
```

```
[Out] [1/6*(3*(c^2*d^2*e*x^2 + 2*a*c*d*e^2*x + a^2*e^3)*sqrt(e/(c*d))*log(8*c^2*d
^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 8*(c^2*d^3*e + a*c*d*e^3)*
x + 4*(2*c^2*d^2*e*x + c^2*d^3 + a*c*d*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*sqrt(e/(c*d))) - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*(4*c*d*e*x + c*d^2 + 3*a*e^2))/(c^4*d^4*x^2 + 2*a*c^3*d^3*e*x + a^2*c^2*d^
2*e^2), -1/3*(3*(c^2*d^2*e*x^2 + 2*a*c*d*e^2*x + a^2*e^3)*sqrt(-e/(c*d))*ar
ctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a
*e^2)*sqrt(-e/(c*d))/(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x)) + 2*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(4*c*d*e*x + c*d^2 + 3*a*e^2))/(c^
4*d^4*x^2 + 2*a*c^3*d^3*e*x + a^2*c^2*d^2*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.49888, size = 837, normalized size = 4.87

$$-\frac{2\left(\left(\left(\frac{4\left(c^5d^9e^3-4\,ac^4d^7e^5+6\,a^2c^3d^5e^7-4\,a^3c^2d^3e^9+a^4cde^{11}\right)x}{c^6d^{10}-4\,ac^5d^8e^2+6\,a^2c^4d^6e^4-4\,a^3c^3d^4e^6+a^4c^2d^2e^8}+\frac{3\left(3\,c^5d^{10}e^2-11\,ac^4d^8e^4+14\,a^2c^3d^6e^6-6\,a^3c^2d^4e^8-a^4cd^2e^{10}+a^5e^{12}\right)}{c^6d^{10}-4\,ac^5d^8e^2+6\,a^2c^4d^6e^4-4\,a^3c^3d^4e^6+a^4c^2d^2e^8}\right)x+\frac{6\left(c^5d^{11}e-3\,ac^4d^9e\right.}{c^6d^{10}-4\,ac^5\right.\right.}{3\left(cdx^2e+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="g
iac")
```

```
[Out] -2/3*(((4*(c^5*d^9*e^3 - 4*a*c^4*d^7*e^5 + 6*a^2*c^3*d^5*e^7 - 4*a^3*c^2*d^
3*e^9 + a^4*c*d*e^11)*x/(c^6*d^10 - 4*a*c^5*d^8*e^2 + 6*a^2*c^4*d^6*e^4 - 4
*a^3*c^3*d^4*e^6 + a^4*c^2*d^2*e^8) + 3*(3*c^5*d^10*e^2 - 11*a*c^4*d^8*e^4
+ 14*a^2*c^3*d^6*e^6 - 6*a^3*c^2*d^4*e^8 - a^4*c*d^2*e^10 + a^5*e^12)/(c^6*
d^10 - 4*a*c^5*d^8*e^2 + 6*a^2*c^4*d^6*e^4 - 4*a^3*c^3*d^4*e^6 + a^4*c^2*d^
2*e^8))*x + 6*(c^5*d^11*e - 3*a*c^4*d^9*e^3 + 2*a^2*c^3*d^7*e^5 + 2*a^3*c^2
*d^5*e^7 - 3*a^4*c*d^3*e^9 + a^5*d*e^11)/(c^6*d^10 - 4*a*c^5*d^8*e^2 + 6*a^
2*c^4*d^6*e^4 - 4*a^3*c^3*d^4*e^6 + a^4*c^2*d^2*e^8))*x + (c^5*d^12 - a*c^4
```

```
*d^10*e^2 - 6*a^2*c^3*d^8*e^4 + 14*a^3*c^2*d^6*e^6 - 11*a^4*c*d^4*e^8 + 3*a
^5*d^2*e^10)/(c^6*d^10 - 4*a*c^5*d^8*e^2 + 6*a^2*c^4*d^6*e^4 - 4*a^3*c^3*d^
4*e^6 + a^4*c^2*d^2*e^8))/(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2) - s
qrt(c*d)*e^(3/2)*log(abs(-sqrt(c*d)*c*d^2*e^(1/2) - 2*(sqrt(c*d)*x*e^(1/2)
- sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x))*c*d*e - sqrt(c*d)*a*e^(5/2))
)/(c^3*d^3)
```

3.1967 $\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=54

$$-\frac{2(d+ex)^3}{3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

```
[Out] (-2*(d + e*x)^3)/(3*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2))
```

Rubi [A] time = 0.0213717, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {650}

$$-\frac{2(d+ex)^3}{3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^3)/(3*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2))
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2(d+ex)^3}{3\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$

Mathematica [A] time = 0.0198118, size = 46, normalized size = 0.85

$$-\frac{2((d+ex)(ae+cdx))^{3/2}}{3\left(cd^2-ae^2\right)(ae+cdx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (-2*((a*e + c*d*x)*(d + e*x))^(3/2))/(3*(c*d^2 - a*e^2)*(a*e + c*d*x)^3)
```

Maple [A] time = 0.045, size = 58, normalized size = 1.1

$$\frac{(2\,cdx+2\,ae)\,(ex+d)^{4}}{3\,ae^{2}-3\,cd^{2}}\left(cdex^{2}+ae^{2}x+cd^{2}x+ade\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] 2/3*(c*d*x+a*e)*(e*x+d)^4/(a*e^2-c*d^2)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(
5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.69288, size = 205, normalized size = 3.8

$$-\frac{2\,\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}(ex+d)}{3\left(a^{2}cd^{2}e^{2}-a^{3}e^{4}+\left(c^{3}d^{4}-ac^{2}d^{2}e^{2}\right)x^{2}+2\left(ac^{2}d^{3}e-a^{2}cde^{3}\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="f
ricas")
```

```
[Out] -2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)/(a^2*c*d^2*e^2 -
 a^3*e^4 + (c^3*d^4 - a*c^2*d^2*e^2)*x^2 + 2*(a*c^2*d^3*e - a^2*c*d*e^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31512, size = 552, normalized size = 10.22

$$-\frac{2\left(\left(\left(\frac{(c^3d^6e^3-3\,ac^2d^4e^5+3\,a^2cd^2e^7-a^3e^9)x}{c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8}+\frac{3\,(c^3d^7e^2-3\,ac^2d^5e^4+3\,a^2cd^3e^6-a^3de^8)}{c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8}\right)x+\frac{3\,(c^3d^8e-3\,ac^2d^6e^3+3\,a^2cd^4e^5-a^3d^2e^7)}{c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8}\right)x+\cdots}{3\left(cdx^2e+ade+(cd^2+ae^2)x\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="g
iac")
```

```
[Out] -2/3*((((c^3*d^6*e^3 - 3*a*c^2*d^4*e^5 + 3*a^2*c*d^2*e^7 - a^3*e^9)*x/(c^4*
d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8) + 3*
(c^3*d^7*e^2 - 3*a*c^2*d^5*e^4 + 3*a^2*c*d^3*e^6 - a^3*d*e^8)/(c^4*d^8 - 4*
a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8))*x + 3*(c^3*
d^8*e - 3*a*c^2*d^6*e^3 + 3*a^2*c*d^4*e^5 - a^3*d^2*e^7)/(c^4*d^8 - 4*a*c^3
*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8))*x + (c^3*d^9 - 3
*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4 - a^3*d^3*e^6)/(c^4*d^8 - 4*a*c^3*d^6*e^2
+ 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8))/(c*d*x^2*e + a*d*e + (c*d
^2 + a*e^2)*x)^(3/2)
```

3.1968 $\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=116

$$\frac{2e\left(ae^2+cd^2+2cdex\right)}{3cd\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2(d+ex)}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

```
[Out] (-2*(d + e*x))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) + (2*e
*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*c*d*(c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.0432223, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {652, 613}

$$\frac{2e\left(ae^2+cd^2+2cdex\right)}{3cd\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2(d+ex)}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) + (2*e
*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*c*d*(c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))^2*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[(e*(d + e*x)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[
(e^2*(p + 2))/(c*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx &= -\frac{2(d+ex)}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{e\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{3cd}\\ &= -\frac{2(d+ex)}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{2e\left(cd^2+ae^2+2cdex\right)}{3cd\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)}}\end{aligned}$$

Mathematica [A] time = 0.032645, size = 59, normalized size = 0.51

$$-\frac{2(d+ex)^2\left(cd(d-2ex)-3ae^2\right)}{3\left(cd^2-ae^2\right)^2((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^2*(-3*a*e^2 + c*d*(d - 2*e*x)))/(3*(c*d^2 - a*e^2)^2*((a*e +
c*d*x)*(d + e*x))^(3/2))
```

Maple [A] time = 0.045, size = 90, normalized size = 0.8

$$\frac{(2\,cdx+2\,ae)\,(ex+d)^3\left(2\,cdex+3\,ae^2-cd^2\right)}{3\,a^2e^4-6\,acd^2e^2+3\,c^2d^4}\left(cdex^2+ae^2x+cd^2x+ade\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] 2/3*(c*d*x+a*e)*(e*x+d)^3*(2*c*d*e*x+3*a*e^2-c*d^2)/(a^2*e^4-2*a*c*d^2*e^2+
c^2*d^4)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 6.31967, size = 312, normalized size = 2.69

$$\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2\,cdex-cd^2+3\,ae^2\right)}{3\left(a^2c^2d^4e^2-2\,a^3cd^2e^4+a^4e^6+\left(c^4d^6-2\,ac^3d^4e^2+a^2c^2d^2e^4\right)x^2+2\left(ac^3d^5e-2\,a^2c^2d^3e^3+a^3cde^5\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="f
ricas")
```

```
[Out] 2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x - c*d^2 + 3*a*e^
2)/(a^2*c^2*d^4*e^2 - 2*a^3*c*d^2*e^4 + a^4*e^6 + (c^4*d^6 - 2*a*c^3*d^4*e^
2 + a^2*c^2*d^2*e^4)*x^2 + 2*(a*c^3*d^5*e - 2*a^2*c^2*d^3*e^3 + a^3*c*d*e^5
)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2967, size = 521, normalized size = 4.49

$$\frac{2\left(\left(\left(\frac{2\left(c^3d^5e^3-2\,ac^2d^3e^5+a^2cde^7\right)x}{c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8}+\frac{3\left(c^3d^6e^2-ac^2d^4e^4-a^2cd^2e^6+a^3e^8\right)}{c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8}\right)x+\frac{6\left(ac^2d^5e^3-2\,a^2cd^3e^5+a^3de^7\right)}{c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8}\right)x-\right.}{3\left(cdx^2e+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="g
iac")
```

```
[Out] 2/3*(((2*(c^3*d^5*e^3 - 2*a*c^2*d^3*e^5 + a^2*c*d*e^7)*x/(c^4*d^8 - 4*a*c^3
*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8) + 3*(c^3*d^6*e^2
- a*c^2*d^4*e^4 - a^2*c*d^2*e^6 + a^3*e^8)/(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a
^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8))*x + 6*(a*c^2*d^5*e^3 - 2*a^2*c
*d^3*e^5 + a^3*d*e^7)/(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^
3*c*d^2*e^6 + a^4*e^8))*x - (c^3*d^8 - 5*a*c^2*d^6*e^2 + 7*a^2*c*d^4*e^4 -
3*a^3*d^2*e^6)/(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2
*e^6 + a^4*e^8))/(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)
```

3.1969 $\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=118

$$\frac{8e\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2(d+ex)}{3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

```
[Out] (-2*(d + e*x))/(3*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2)) + (8*e*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^2 - a*e^2)^3*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.0498806, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {638, 613}

$$\frac{8e\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2(d+ex)}{3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x))/(3*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2)) + (8*e*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^2 - a*e^2)^3*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2(d+ex)}{3\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{(4e)\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,d}{3\left(cd^2-ae^2\right)}$$

$$=-\frac{2(d+ex)}{3\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{8e\left(cd^2+ae^2+2cd\right.}{3\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+\right.}}$$

Mathematica [A] time = 0.0566876, size = 90, normalized size = 0.76

$$\frac{2(d+ex)\left(3a^2e^4+6acde^2(d+2ex)+c^2d^2\left(-d^2+4dex+8e^2x^2\right)\right)}{3\left(cd^2-ae^2\right)^3\left((d+ex)(ae+cdx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (2*(d + e*x)*(3*a^2*e^4 + 6*a*c*d*e^2*(d + 2*e*x) + c^2*d^2*(-d^2 + 4*d*e*x
 + 8*e^2*x^2)))/(3*(c*d^2 - a*e^2)^3*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [A] time = 0.045, size = 146, normalized size = 1.2

$$-\frac{(2\,cdx+2\,ae)\,(ex+d)^2\left(8\,c^2d^2e^2x^2+12\,acde^3x+4\,c^2d^3ex+3\,a^2e^4+6\,acd^2e^2-c^2d^4\right)}{3\,a^3e^6-9\,a^2cd^2e^4+9\,ac^2d^4e^2-3\,c^3d^6}\left(cdex^2+ae^2x+cd^2x+ade\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(e*x+d)^2*(8*c^2*d^2*e^2*x^2+12*a*c*d*e^3*x+4*c^2*d^3*e*x+
3*a^2*e^4+6*a*c*d^2*e^2-c^2*d^4)/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2-c
^3*d^6)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 16.3236, size = 614, normalized size = 5.2

$$\frac{2\left(8\,c^2d^2e^2x^2-c^2d^4+6\,acd^2e^2+3\,a^2e^4+4\left(c^2d^3e+3\,acde^3\right.\right.}{3\left(a^2c^3d^7e^2-3\,a^3c^2d^5e^4+3\,a^4cd^3e^6-a^5de^8+\left(c^5d^8e-3\,ac^4d^6e^3+3\,a^2c^3d^4e^5-a^3c^2d^2e^7\right)x^3+\left(c^5d^9-ac^4d^7e^2-3\,a^2c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="fri
cas")
```

```
[Out] 2/3*(8*c^2*d^2*e^2*x^2 - c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4 + 4*(c^2*d^3*e
 + 3*a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a^2*c^3*d^7
*e^2 - 3*a^3*c^2*d^5*e^4 + 3*a^4*c*d^3*e^6 - a^5*d*e^8 + (c^5*d^8*e - 3*a*c
^4*d^6*e^3 + 3*a^2*c^3*d^4*e^5 - a^3*c^2*d^2*e^7)*x^3 + (c^5*d^9 - a*c^4*d^
```

```
7*e^2 - 3*a^2*c^3*d^5*e^4 + 5*a^3*c^2*d^3*e^6 - 2*a^4*c*d*e^8)*x^2 + (2*a*c
^4*d^8*e - 5*a^2*c^3*d^6*e^3 + 3*a^3*c^2*d^4*e^5 + a^4*c*d^2*e^7 - a^5*e^9)
*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25617, size = 497, normalized size = 4.21

$$\frac{2\left(\left(4\left(\frac{2\left(c^3d^4e^3-ac^2d^2e^5\right)x}{c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8}+\frac{3\left(c^3d^5e^2-a^2cde^6\right)}{c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8}\right)x+\frac{3\left(c^3d^6e+5\,ac^2d^4e^3-5\,a^2cd^2e^5-a^3e^7\right)}{c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8}\right)x-\cdots}{3\left(cdx^2e+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="gia
c")
```

```
[Out] 2/3*((4*(2*(c^3*d^4*e^3 - a*c^2*d^2*e^5)*x/(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a
^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8) + 3*(c^3*d^5*e^2 - a^2*c*d*e^6)
/(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8
))*x + 3*(c^3*d^6*e + 5*a*c^2*d^4*e^3 - 5*a^2*c*d^2*e^5 - a^3*e^7)/(c^4*d^8
 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8))*x - (c
^3*d^7 - 7*a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + 3*a^3*d*e^6)/(c^4*d^8 - 4*a*c^
3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8))/(c*d*x^2*e + a*
d*e + (c*d^2 + a*e^2)*x)^(3/2)
```

3.1970 $\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=132

$$\frac{16cde\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

```
[Out] (-2*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2)) + (16*c*d*e*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^
2 - a*e^2)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.024465, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {614, 613}

$$\frac{16cde\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-5/2),x]
```

```
[Out] (-2*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2)) + (16*c*d*e*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^
2 - a*e^2)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx &= -\frac{2\left(cd^2+ae^2+2cdex\right)}{3\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{(8cde)\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cde\right.}}{3\left(cd^2-ae^2\right)^2}\\ &= -\frac{2\left(cd^2+ae^2+2cdex\right)}{3\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{16cde\left(cd^2+ae^2\right.}{3\left(cd^2-ae^2\right)^4\sqrt{ade+\left(cd\right.}}\end{aligned}$$

Mathematica [A] time = 0.0657726, size = 132, normalized size = 1.

$$\frac{6a^2cde^4(3d+2ex)-2a^3e^6+6ac^2d^2e^2\left(3d^2+12dex+8e^2x^2\right)+2c^3d^3\left(6d^2ex-d^3+24de^2x^2+16e^3x^3\right)}{3\left(cd^2-ae^2\right)^4((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-5/2),x]
```

```
[Out] (-2*a^3*e^6 + 6*a^2*c*d*e^4*(3*d + 2*e*x) + 6*a*c^2*d^2*e^2*(3*d^2 + 12*d*e
*x + 8*e^2*x^2) + 2*c^3*d^3*(-d^3 + 6*d^2*e*x + 24*d*e^2*x^2 + 16*e^3*x^3))
/(3*(c*d^2 - a*e^2)^4*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [A] time = 0.045, size = 213, normalized size = 1.6

$$-\frac{(2\,cdx+2\,ae)\,(ex+d)\left(-16\,c^3d^3e^3x^3-24\,ac^2d^2e^4x^2-24\,c^3d^4e^2x^2-6\,a^2cde^5x-36\,ac^2d^3e^3x-6\,c^3d^5ex+a^3e^6-9\,a^2cd^2e\right.}{3\,a^4e^8-12\,a^3cd^2e^6+18\,a^2c^2d^4e^4-12\,ac^3d^6e^2+3\,c^4d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(e*x+d)*(-16*c^3*d^3*e^3*x^3-24*a*c^2*d^2*e^4*x^2-24*c^3*d
^4*e^2*x^2-6*a^2*c*d*e^5*x-36*a*c^2*d^3*e^3*x-6*c^3*d^5*e*x+a^3*e^6-9*a^2*c
*d^2*e^4-9*a*c^2*d^4*e^2+c^3*d^6)/(a^4*e^8-4*a^3*c*d^2*e^6+6*a^2*c^2*d^4*e^
4-4*a*c^3*d^6*e^2+c^4*d^8)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 58.0201, size = 972, normalized size = 7.36

$$\frac{2\left(16\,c^3d^3e^3x^3-c^3d^6+9\right.}{3\left(a^2c^4d^{10}e^2-4\,a^3c^3d^8e^4+6\,a^4c^2d^6e^6-4\,a^5cd^4e^8+a^6d^2e^{10}+\left(c^6d^{10}e^2-4\,ac^5d^8e^4+6\,a^2c^4d^6e^6-4\,a^3c^3d^4e^8+a^4c^2d^2e^{10}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(16*c^3*d^3*e^3*x^3 - c^3*d^6 + 9*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - a^3
*e^6 + 24*(c^3*d^4*e^2 + a*c^2*d^2*e^4)*x^2 + 6*(c^3*d^5*e + 6*a*c^2*d^3*e^
3 + a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a^2*c^4*d^
10*e^2 - 4*a^3*c^3*d^8*e^4 + 6*a^4*c^2*d^6*e^6 - 4*a^5*c*d^4*e^8 + a^6*d^2*
```

```
e^10 + (c^6*d^10*e^2 - 4*a*c^5*d^8*e^4 + 6*a^2*c^4*d^6*e^6 - 4*a^3*c^3*d^4*
e^8 + a^4*c^2*d^2*e^10)*x^4 + 2*(c^6*d^11*e - 3*a*c^5*d^9*e^3 + 2*a^2*c^4*d
^7*e^5 + 2*a^3*c^3*d^5*e^7 - 3*a^4*c^2*d^3*e^9 + a^5*c*d*e^11)*x^3 + (c^6*d
^12 - 9*a^2*c^4*d^8*e^4 + 16*a^3*c^3*d^6*e^6 - 9*a^4*c^2*d^4*e^8 + a^6*e^12
)*x^2 + 2*(a*c^5*d^11*e - 3*a^2*c^4*d^9*e^3 + 2*a^3*c^3*d^7*e^5 + 2*a^4*c^2
*d^5*e^7 - 3*a^5*c*d^3*e^9 + a^6*d*e^11)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.3058, size = 466, normalized size = 3.53

$$\frac{2\left(2\left(4\left(\frac{2\,c^3d^3xe^3}{c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8}+\frac{3\left(c^3d^4e^2+ac^2d^2e^4\right)}{c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8}\right)x+\frac{3\left(c^3d^5e+6\,ac^2d^3e^3+a^2cde^5\right)}{c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8}\right)\cdots}{3\left(cdx^2e+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(2*(4*(2*c^3*d^3*x*e^3/(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 -
 4*a^3*c*d^2*e^6 + a^4*e^8) + 3*(c^3*d^4*e^2 + a*c^2*d^2*e^4)/(c^4*d^8 - 4*
a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8))*x + 3*(c^3*
d^5*e + 6*a*c^2*d^3*e^3 + a^2*c*d*e^5)/(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c
^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8))*x - (c^3*d^6 - 9*a*c^2*d^4*e^2 - 9
*a^2*c*d^2*e^4 + a^3*e^6)/(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 -
4*a^3*c*d^2*e^6 + a^4*e^8))/(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)
```

3.1971 $\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=192

$$\frac{128c^2d^2e\left(ae^2+cd^2+2cdex\right)}{15\left(cd^2-ae^2\right)^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{16cd\left(ae^2+cd^2+2cdex\right)}{15\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}+\frac{}{5(d+ex)\left(cd^2-ae^2\right)(}$$

```
[Out] 2/(5*(c*d^2 - a*e^2)*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2
)) - (16*c*d*(c*d^2 + a*e^2 + 2*c*d*e*x))/(15*(c*d^2 - a*e^2)^3*(a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) + (128*c^2*d^2*e*(c*d^2 + a*e^2 + 2*c*d
*e*x))/(15*(c*d^2 - a*e^2)^5*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.0624783, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {658, 614, 613}

$$\frac{128c^2d^2e\left(ae^2+cd^2+2cdex\right)}{15\left(cd^2-ae^2\right)^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{16cd\left(ae^2+cd^2+2cdex\right)}{15\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}+\frac{}{5(d+ex)\left(cd^2-ae^2\right)(}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```


```
[Out] 2/(5*(c*d^2 - a*e^2)*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2
)) - (16*c*d*(c*d^2 + a*e^2 + 2*c*d*e*x))/(15*(c*d^2 - a*e^2)^3*(a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) + (128*c^2*d^2*e*(c*d^2 + a*e^2 + 2*c*d
*e*x))/(15*(c*d^2 - a*e^2)^5*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```


Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{2}{5\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{(8cd)\int \frac{}{(ade}}{5}$$

$$= \frac{2}{5\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{1}{15\left(cd^2-ae\right.}$$

$$= \frac{2}{5\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{1}{15\left(cd^2-ae\right.}$$

Mathematica [A] time = 0.0958727, size = 193, normalized size = 1.01

$$\frac{2\left(6a^2c^2d^2e^4\left(15d^2+20dex+8e^2x^2\right)-4a^3cde^6(5d+2ex)+3a^4e^8+12ac^3d^3e^2\left(30d^2ex+5d^3+40de^2x^2+16e^3x^3\right)+c\right.}{15(d+ex)\left(cd^2-ae^2\right)^5((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```

```
[Out] (2*(3*a^4*e^8 - 4*a^3*c*d*e^6*(5*d + 2*e*x) + 6*a^2*c^2*d^2*e^4*(15*d^2 + 2
0*d*e*x + 8*e^2*x^2) + 12*a*c^3*d^3*e^2*(5*d^3 + 30*d^2*e*x + 40*d*e^2*x^2
+ 16*e^3*x^3) + c^4*d^4*(-5*d^4 + 40*d^3*e*x + 240*d^2*e^2*x^2 + 320*d*e^3*
x^3 + 128*e^4*x^4)))/(15*(c*d^2 - a*e^2)^5*(d + e*x)*((a*e + c*d*x)*(d + e*
x))^(3/2))
```

Maple [A] time = 0.057, size = 300, normalized size = 1.6

$$-\frac{(2cdx+2ae)\left(128c^4d^4e^4x^4+192ac^3d^3e^5x^3+320c^4d^5e^3x^3+48a^2c^2d^2e^6x^2+480ac^3d^4e^4x^2+240c^4d^6e^2x^2-8a^3c\right.}{15a^5e^{10}-75a^4cd^2e^8+150a^3c^2d^4e^6-150}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] -2/15*(c*d*x+a*e)*(128*c^4*d^4*e^4*x^4+192*a*c^3*d^3*e^5*x^3+320*c^4*d^5*e^
3*x^3+48*a^2*c^2*d^2*e^6*x^2+480*a*c^3*d^4*e^4*x^2+240*c^4*d^6*e^2*x^2-8*a^
3*c*d*e^7*x+120*a^2*c^2*d^3*e^5*x+360*a*c^3*d^5*e^3*x+40*c^4*d^7*e*x+3*a^4*
e^8-20*a^3*c*d^2*e^6+90*a^2*c^2*d^4*e^4+60*a*c^3*d^6*e^2-5*c^4*d^8)/(a^5*e^
10-5*a^4*c*d^2*e^8+10*a^3*c^2*d^4*e^6-10*a^2*c^3*d^6*e^4+5*a*c^4*d^8*e^2-c^
5*d^10)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 172.752, size = 1566, normalized size = 8.16

$$\overline{15\left(a^2c^5d^{13}e^2 - 5\,a^3c^4d^{11}e^4 + 10\,a^4c^3d^9e^6 - 10\,a^5c^2d^7e^8 + 5\,a^6cd^5e^{10} - a^7d^3e^{12} + \left(c^7d^{12}e^3 - 5\,ac^6d^{10}e^5 + 10\,a^2c^5d^8e^7 - 10\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="f
ricas")
```

```
[Out] 2/15*(128*c^4*d^4*e^4*x^4 - 5*c^4*d^8 + 60*a*c^3*d^6*e^2 + 90*a^2*c^2*d^4*e
^4 - 20*a^3*c*d^2*e^6 + 3*a^4*e^8 + 64*(5*c^4*d^5*e^3 + 3*a*c^3*d^3*e^5)*x^
3 + 48*(5*c^4*d^6*e^2 + 10*a*c^3*d^4*e^4 + a^2*c^2*d^2*e^6)*x^2 + 8*(5*c^4*
d^7*e + 45*a*c^3*d^5*e^3 + 15*a^2*c^2*d^3*e^5 - a^3*c*d*e^7)*x)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a^2*c^5*d^13*e^2 - 5*a^3*c^4*d^11*e^4 + 1
0*a^4*c^3*d^9*e^6 - 10*a^5*c^2*d^7*e^8 + 5*a^6*c*d^5*e^10 - a^7*d^3*e^12 +
(c^7*d^12*e^3 - 5*a*c^6*d^10*e^5 + 10*a^2*c^5*d^8*e^7 - 10*a^3*c^4*d^6*e^9
+ 5*a^4*c^3*d^4*e^11 - a^5*c^2*d^2*e^13)*x^5 + (3*c^7*d^13*e^2 - 13*a*c^6*d
^11*e^4 + 20*a^2*c^5*d^9*e^6 - 10*a^3*c^4*d^7*e^8 - 5*a^4*c^3*d^5*e^10 + 7*
a^5*c^2*d^3*e^12 - 2*a^6*c*d*e^14)*x^4 + (3*c^7*d^14*e - 9*a*c^6*d^12*e^3 +
 a^2*c^5*d^10*e^5 + 25*a^3*c^4*d^8*e^7 - 35*a^4*c^3*d^6*e^9 + 17*a^5*c^2*d^
4*e^11 - a^6*c*d^2*e^13 - a^7*e^15)*x^3 + (c^7*d^15 + a*c^6*d^13*e^2 - 17*a
^2*c^5*d^11*e^4 + 35*a^3*c^4*d^9*e^6 - 25*a^4*c^3*d^7*e^8 - a^5*c^2*d^5*e^1
0 + 9*a^6*c*d^3*e^12 - 3*a^7*d*e^14)*x^2 + (2*a*c^6*d^14*e - 7*a^2*c^5*d^12
*e^3 + 5*a^3*c^4*d^10*e^5 + 10*a^4*c^3*d^8*e^7 - 20*a^5*c^2*d^6*e^9 + 13*a^
6*c*d^4*e^11 - 3*a^7*d^2*e^13)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="g
iac")
```

```
[Out] [undef, undef, undef, undef, undef, 1]
```

3.1972 $\int \frac{1}{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=252

$$\frac{256c^3d^3e\left(ae^2+cd^2+2cdex\right)}{21\left(cd^2-ae^2\right)^6\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{32c^2d^2\left(ae^2+cd^2+2cdex\right)}{21\left(cd^2-ae^2\right)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}+\frac{}{7(d+ex)\left(cd^2-ae^2\right.}$$

```
[Out] 2/(7*(c*d^2 - a*e^2)*(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3
/2)) + (4*c*d)/(7*(c*d^2 - a*e^2)^2*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(3/2)) - (32*c^2*d^2*(c*d^2 + a*e^2 + 2*c*d*e*x))/(21*(c*d^2 - a
*e^2)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) + (256*c^3*d^3*e*(c*
d^2 + a*e^2 + 2*c*d*e*x))/(21*(c*d^2 - a*e^2)^6*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])
```

Rubi [A] time = 0.101749, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {658, 614, 613}

$$\frac{256c^3d^3e\left(ae^2+cd^2+2cdex\right)}{21\left(cd^2-ae^2\right)^6\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{32c^2d^2\left(ae^2+cd^2+2cdex\right)}{21\left(cd^2-ae^2\right)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}+\frac{}{7(d+ex)\left(cd^2-ae^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```


```
[Out] 2/(7*(c*d^2 - a*e^2)*(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3
/2)) + (4*c*d)/(7*(c*d^2 - a*e^2)^2*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(3/2)) - (32*c^2*d^2*(c*d^2 + a*e^2 + 2*c*d*e*x))/(21*(c*d^2 - a
*e^2)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) + (256*c^3*d^3*e*(c*
d^2 + a*e^2 + 2*c*d*e*x))/(21*(c*d^2 - a*e^2)^6*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])
```


Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
```

```
NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}dx = \frac{2}{7\left(cd^2-ae^2\right)(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{(10cd)\int \frac{}{(d+e}}{}$$

$$= \frac{2}{7\left(cd^2-ae^2\right)(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{}{7\left(cd^2-ae^2\right)}$$

$$= \frac{2}{7\left(cd^2-ae^2\right)(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{}{7\left(cd^2-ae^2\right)}$$

$$= \frac{2}{7\left(cd^2-ae^2\right)(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{}{7\left(cd^2-ae^2\right)}$$

Mathematica [A] time = 0.125281, size = 259, normalized size = 1.03

$$\frac{2\left(-2a^3c^2d^2e^6\left(35d^2+28dex+8e^2x^2\right)+6a^2c^3d^3e^4\left(70d^2ex+35d^3+56de^2x^2+16e^3x^3\right)+3a^4cde^8(7d+2ex)-3a^5e^{10}+3}{21(d+ex)^2\left(cd^2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```

```
[Out] (2*(-3*a^5*e^10 + 3*a^4*c*d*e^8*(7*d + 2*e*x) - 2*a^3*c^2*d^2*e^6*(35*d^2 +
 28*d*e*x + 8*e^2*x^2) + 6*a^2*c^3*d^3*e^4*(35*d^3 + 70*d^2*e*x + 56*d*e^2*
x^2 + 16*e^3*x^3) + 3*a*c^4*d^4*e^2*(35*d^4 + 280*d^3*e*x + 560*d^2*e^2*x^2
 + 448*d*e^3*x^3 + 128*e^4*x^4) + c^5*d^5*(-7*d^5 + 70*d^4*e*x + 560*d^3*e^
2*x^2 + 1120*d^2*e^3*x^3 + 896*d*e^4*x^4 + 256*e^5*x^5)))/(21*(c*d^2 - a*e^
2)^6*(d + e*x)^2*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [A] time = 0.051, size = 412, normalized size = 1.6

$$-\frac{(2\,cdx+2\,ae)\left(-256\,c^5d^5e^5x^5-384\,ac^4d^4e^6x^4-896\,c^5d^6e^4x^4-96\,a^2c^3d^3e^7x^3-1344\,ac^4d^5e^5x^3-1120\,c^5d^7e^3x^3+16\,a^3c\right.}{(21\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] -2/21*(c*d*x+a*e)*(-256*c^5*d^5*e^5*x^5-384*a*c^4*d^4*e^6*x^4-896*c^5*d^6*e
^4*x^4-96*a^2*c^3*d^3*e^7*x^3-1344*a*c^4*d^5*e^5*x^3-1120*c^5*d^7*e^3*x^3+1
6*a^3*c^2*d^2*e^8*x^2-336*a^2*c^3*d^4*e^6*x^2-1680*a*c^4*d^6*e^4*x^2-560*c^
5*d^8*e^2*x^2-6*a^4*c*d*e^9*x+56*a^3*c^2*d^3*e^7*x-420*a^2*c^3*d^5*e^5*x-84
0*a*c^4*d^7*e^3*x-70*c^5*d^9*e*x+3*a^5*e^10-21*a^4*c*d^2*e^8+70*a^3*c^2*d^4
*e^6-210*a^2*c^3*d^6*e^4-105*a*c^4*d^8*e^2+7*c^5*d^10)/(e*x+d)/(a^6*e^12-6*
a^5*c*d^2*e^10+15*a^4*c^2*d^4*e^8-20*a^3*c^3*d^6*e^6+15*a^2*c^4*d^8*e^4-6*a
*c^5*d^10*e^2+c^6*d^12)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.1973 $\int \frac{1}{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=312

$$\frac{1024c^4d^4e\left(ae^2+cd^2+2cdex\right)}{63\left(cd^2-ae^2\right)^7\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{128c^3d^3\left(ae^2+cd^2+2cdex\right)}{63\left(cd^2-ae^2\right)^5\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}+\frac{}{21(d+ex)\left(cd^2-ae^2\right)^3}$$

```
[Out] 2/(9*(c*d^2 - a*e^2)*(d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3
/2)) + (8*c*d)/(21*(c*d^2 - a*e^2)^2*(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2)^(3/2)) + (16*c^2*d^2)/(21*(c*d^2 - a*e^2)^3*(d + e*x)*(a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (128*c^3*d^3*(c*d^2 + a*e^2 + 2*c
*d*e*x))/(63*(c*d^2 - a*e^2)^5*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2
)) + (1024*c^4*d^4*e*(c*d^2 + a*e^2 + 2*c*d*e*x))/(63*(c*d^2 - a*e^2)^7*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.151707, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {658, 614, 613}

$$\frac{1024c^4d^4e\left(ae^2+cd^2+2cdex\right)}{63\left(cd^2-ae^2\right)^7\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{128c^3d^3\left(ae^2+cd^2+2cdex\right)}{63\left(cd^2-ae^2\right)^5\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}+\frac{}{21(d+ex)\left(cd^2-ae^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```


```
[Out] 2/(9*(c*d^2 - a*e^2)*(d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3
/2)) + (8*c*d)/(21*(c*d^2 - a*e^2)^2*(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2)^(3/2)) + (16*c^2*d^2)/(21*(c*d^2 - a*e^2)^3*(d + e*x)*(a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (128*c^3*d^3*(c*d^2 + a*e^2 + 2*c
*d*e*x))/(63*(c*d^2 - a*e^2)^5*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2
)) + (1024*c^4*d^4*e*(c*d^2 + a*e^2 + 2*c*d*e*x))/(63*(c*d^2 - a*e^2)^7*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```


Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}dx = \frac{2}{9\left(cd^2-ae^2\right)(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{(4cd)\int -}{}$$

$$= \frac{2}{9\left(cd^2-ae^2\right)(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{}{21\left(cd^2-\right.}$$

$$= \frac{2}{9\left(cd^2-ae^2\right)(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{}{21\left(cd^2-\right.}$$

$$= \frac{2}{9\left(cd^2-ae^2\right)(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{}{21\left(cd^2-\right.}$$

$$= \frac{2}{9\left(cd^2-ae^2\right)(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{}{21\left(cd^2-\right.}$$

Mathematica [A] time = 0.153942, size = 336, normalized size = 1.08

$$\frac{2\left(3a^4c^2d^2e^8\left(63d^2+36dex+8e^2x^2\right)-4a^3c^3d^3e^6\left(126d^2ex+105d^3+72de^2x^2+16e^3x^3\right)+3a^2c^4d^4e^4\left(1008d^2e^2x^2+8\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```

```
[Out] (2*(7*a^6*e^12 - 6*a^5*c*d*e^10*(9*d + 2*e*x) + 3*a^4*c^2*d^2*e^8*(63*d^2 +
 36*d*e*x + 8*e^2*x^2) - 4*a^3*c^3*d^3*e^6*(105*d^3 + 126*d^2*e*x + 72*d*e^
2*x^2 + 16*e^3*x^3) + 3*a^2*c^4*d^4*e^4*(315*d^4 + 840*d^3*e*x + 1008*d^2*e
^2*x^2 + 576*d*e^3*x^3 + 128*e^4*x^4) + 6*a*c^5*d^5*e^2*(63*d^5 + 630*d^4*e
*x + 1680*d^3*e^2*x^2 + 2016*d^2*e^3*x^3 + 1152*d*e^4*x^4 + 256*e^5*x^5) +
c^6*d^6*(-21*d^6 + 252*d^5*e*x + 2520*d^4*e^2*x^2 + 6720*d^3*e^3*x^3 + 8064
*d^2*e^4*x^4 + 4608*d*e^5*x^5 + 1024*e^6*x^6)))/(63*(c*d^2 - a*e^2)^7*(d +
e*x)^3*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [A] time = 0.055, size = 536, normalized size = 1.7

$$-\frac{(2\,cdx+2\,ae)\left(1024\,c^6d^6e^6x^6+1536\,ac^5d^5e^7x^5+4608\,c^6d^7e^5x^5+384\,a^2c^4d^4e^8x^4+6912\,ac^5d^6e^6x^4+8064\,c^6d^8e^4x^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] -2/63*(c*d*x+a*e)*(1024*c^6*d^6*e^6*x^6+1536*a*c^5*d^5*e^7*x^5+4608*c^6*d^7
*e^5*x^5+384*a^2*c^4*d^4*e^8*x^4+6912*a*c^5*d^6*e^6*x^4+8064*c^6*d^8*e^4*x^
4-64*a^3*c^3*d^3*e^9*x^3+1728*a^2*c^4*d^5*e^7*x^3+12096*a*c^5*d^7*e^5*x^3+6
720*c^6*d^9*e^3*x^3+24*a^4*c^2*d^2*e^10*x^2-288*a^3*c^3*d^4*e^8*x^2+3024*a^
2*c^4*d^6*e^6*x^2+10080*a*c^5*d^8*e^4*x^2+2520*c^6*d^10*e^2*x^2-12*a^5*c*d*
e^11*x+108*a^4*c^2*d^3*e^9*x-504*a^3*c^3*d^5*e^7*x+2520*a^2*c^4*d^7*e^5*x+3
780*a*c^5*d^9*e^3*x+252*c^6*d^11*e*x+7*a^6*e^12-54*a^5*c*d^2*e^10+189*a^4*c
^2*d^4*e^8-420*a^3*c^3*d^6*e^6+945*a^2*c^4*d^8*e^4+378*a*c^5*d^10*e^2-21*c^
6*d^12)/(e*x+d)^2/(a^7*e^14-7*a^6*c*d^2*e^12+21*a^5*c^2*d^4*e^10-35*a^4*c^3
*d^6*e^8+35*a^3*c^4*d^8*e^6-21*a^2*c^5*d^10*e^4+7*a*c^6*d^12*e^2-c^7*d^14)/
(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm=
"giac")
```

```
[Out] [undef, undef, undef, undef, undef, 1]
```

3.1974 $\int \frac{d+ex}{\sqrt[3]{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=1485

result too large to display

```
[Out] (3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(2/3))/(4*c*d) + (3*(c*d^2 - a*e
^2)*Sqrt[(c*d^2 + a*e^2 + 2*c*d*e*x)^2]*Sqrt[(a*e^2 + c*d*(d + 2*e*x))^2])/
(2*2^(1/3)*c^(5/3)*d^(5/3)*e^(2/3)*(c*d^2 + a*e^2 + 2*c*d*e*x)*((1 + Sqrt[3
])*(c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(
d + e*x))^(1/3))) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*(c*d^2 - a*e^2)^(5/3)*Sqrt
[(c*d^2 + a*e^2 + 2*c*d*e*x)^2]*((c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^
(1/3)*e^(1/3)*((a*e + c*d*x)*(d + e*x))^(1/3))*Sqrt[((c*d^2 - a*e^2)^(4/3)
- 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*(c*d^2 - a*e^2)^(2/3)*((a*e + c*d*x)*(d +
 e*x))^(1/3) + 2*2^(1/3)*c^(2/3)*d^(2/3)*e^(2/3)*((a*e + c*d*x)*(d + e*x))^
(2/3))/((1 + Sqrt[3])*(c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/
3)*((a*e + c*d*x)*(d + e*x))^(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt[3])*(c*d
^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d + e*x
))^(1/3))/((1 + Sqrt[3])*(c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^
(1/3)*((a*e + c*d*x)*(d + e*x))^(1/3))], -7 - 4*Sqrt[3]])/(4*2^(1/3)*c^(5/3
)*d^(5/3)*e^(2/3)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[((c*d^2 - a*e^2)^(2/3)*(
(c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d +
 e*x))^(1/3)))/((1 + Sqrt[3])*(c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/
3)*e^(1/3)*((a*e + c*d*x)*(d + e*x))^(1/3))^2]*Sqrt[(a*e^2 + c*d*(d + 2*e*x
))^2]) + (3^(3/4)*(c*d^2 - a*e^2)^(5/3)*Sqrt[(c*d^2 + a*e^2 + 2*c*d*e*x)^2]
*((c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d
 + e*x))^(1/3))*Sqrt[((c*d^2 - a*e^2)^(4/3) - 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/
3)*(c*d^2 - a*e^2)^(2/3)*((a*e + c*d*x)*(d + e*x))^(1/3) + 2*2^(1/3)*c^(2/3
)*d^(2/3)*e^(2/3)*((a*e + c*d*x)*(d + e*x))^(2/3))/((1 + Sqrt[3])*(c*d^2 -
a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d + e*x))^(1
/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(
1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d + e*x))^(1/3))/((1 + Sqrt[3])*(c*d^2
 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d + e*x))
^(1/3))], -7 - 4*Sqrt[3]])/(2^(5/6)*c^(5/3)*d^(5/3)*e^(2/3)*(c*d^2 + a*e^2
+ 2*c*d*e*x)*Sqrt[((c*d^2 - a*e^2)^(2/3)*((c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c
^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d + e*x))^(1/3)))/((1 + Sqrt[3])*(c*
d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d + e*
x))^(1/3))^2]*Sqrt[(a*e^2 + c*d*(d + 2*e*x))^2])
```

Rubi [A] time = 2.23619, antiderivative size = 1485, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {640, 623, 303, 218, 1877}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3),x]
```

```
[Out] (3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(2/3))/(4*c*d) + (3*(c*d^2 - a*e
^2)*Sqrt[(c*d^2 + a*e^2 + 2*c*d*e*x)^2]*Sqrt[(a*e^2 + c*d*(d + 2*e*x))^2])/
(2*2^(1/3)*c^(5/3)*d^(5/3)*e^(2/3)*(c*d^2 + a*e^2 + 2*c*d*e*x)*((1 + Sqrt[3
])*(c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(
d + e*x))^(1/3))) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*(c*d^2 - a*e^2)^(5/3)*Sqrt
[(c*d^2 + a*e^2 + 2*c*d*e*x)^2]*((c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^
```

```
(1/3)*e^(1/3)*((a*e + c*d*x)*(d + e*x))^(1/3))*Sqrt[((c*d^2 - a*e^2)^(4/3)
- 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*(c*d^2 - a*e^2)^(2/3)*((a*e + c*d*x)*(d +
 e*x))^(1/3) + 2*2^(1/3)*c^(2/3)*d^(2/3)*e^(2/3)*((a*e + c*d*x)*(d + e*x))^
(2/3))/((1 + Sqrt[3])*(c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/
3)*((a*e + c*d*x)*(d + e*x))^(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt[3])*(c*d
^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d + e*x
))^(1/3))/((1 + Sqrt[3])*(c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^
(1/3)*((a*e + c*d*x)*(d + e*x))^(1/3))], -7 - 4*Sqrt[3]])/(4*2^(1/3)*c^(5/3
)*d^(5/3)*e^(2/3)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[((c*d^2 - a*e^2)^(2/3)*(
(c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d +
 e*x))^(1/3)))/((1 + Sqrt[3])*(c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/
3)*e^(1/3)*((a*e + c*d*x)*(d + e*x))^(1/3))^2]*Sqrt[(a*e^2 + c*d*(d + 2*e*x
))^2]) + (3^(3/4)*(c*d^2 - a*e^2)^(5/3)*Sqrt[(c*d^2 + a*e^2 + 2*c*d*e*x)^2]
*((c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d
 + e*x))^(1/3))*Sqrt[((c*d^2 - a*e^2)^(4/3) - 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/
3)*(c*d^2 - a*e^2)^(2/3)*((a*e + c*d*x)*(d + e*x))^(1/3) + 2*2^(1/3)*c^(2/3
)*d^(2/3)*e^(2/3)*((a*e + c*d*x)*(d + e*x))^(2/3))/((1 + Sqrt[3])*(c*d^2 -
a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d + e*x))^(1
/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(
1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d + e*x))^(1/3))/((1 + Sqrt[3])*(c*d^2
 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d + e*x))
^(1/3))], -7 - 4*Sqrt[3]])/(2^(5/6)*c^(5/3)*d^(5/3)*e^(2/3)*(c*d^2 + a*e^2
+ 2*c*d*e*x)*Sqrt[((c*d^2 - a*e^2)^(2/3)*((c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c
^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d + e*x))^(1/3)))/((1 + Sqrt[3])*(c*
d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d + e*
x))^(1/3))^2]*Sqrt[(a*e^2 + c*d*(d + 2*e*x))^2])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{d+ex}{\sqrt[3]{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{2/3}}{4cd} + \frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{\sqrt[3]{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{2d}$$

$$= \frac{3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{2/3}}{4cd} + \frac{\left(3\left(d^2-\frac{ae^2}{c}\right)\sqrt{\left(cd^2+ae^2+2cdex\right)^2}\right)\text{Subst}}{2d\left(cd\right.}$$

$$= \frac{3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{2/3}}{4cd} + \frac{\left(3\left(d^2-\frac{ae^2}{c}\right)\sqrt{\left(cd^2+ae^2+2cdex\right)^2}\right)\text{Subst}}{2\,2^{2/3}\sqrt[3]{c}d^{4/}}$$

$$= \frac{3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{2/3}}{4cd} + \frac{3\left(cd^2-ae^2\right)\sqrt{\left(cd^2+ae^2\right.}}{2\sqrt[3]{2}c^{5/3}d^{5/3}e^{2/3}\left(cd^2+ae^2+2cdex\right)\left(\left(1+\sqrt{3}\right.\right.}$$

Mathematica [C] time = 0.0587443, size = 88, normalized size = 0.06

$$\frac{3((d+ex)(ae+cdx))^{2/3}\,{}_2F_1\left(-\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{2cd\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3),x]
```

```
[Out] (3*((a*e + c*d*x)*(d + e*x))^(2/3)*Hypergeometric2F1[-2/3, 2/3, 5/3, (e*(a*
e + c*d*x))/(-(c*d^2) + a*e^2)])/(2*c*d*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(
2/3))
```

Maple [F] time = 0.992, size = 0, normalized size = 0.

$$\int (ex+d)\frac{1}{\sqrt[3]{ade+\left(ae^2+cd^2\right)x+cdex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x)
```

```
[Out] int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex+d}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x, algorithm="max
ima")
```

```
[Out] integrate((e*x + d)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{2}{3}}}{cdx+ae},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x, algorithm="fri
cas")
```

```
[Out] integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(2/3)/(c*d*x + a*e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d+ex}{\sqrt[3]{(d+ex)\,(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/3),x)
```

```
[Out] Integral((d + e*x)/((d + e*x)*(a*e + c*d*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex+d}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x, algorithm="gia
c")
```

```
[Out] integrate((e*x + d)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/3), x)
```

3.1975 $\int \frac{1}{\sqrt[3]{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=1432

result too large to display

```
[Out] (3*Sqrt[(c*d^2 + a*e^2 + 2*c*d*e*x)^2]*Sqrt[(a*e^2 + c*d*(d + 2*e*x))^2])/(
2^(1/3)*c^(2/3)*d^(2/3)*e^(2/3)*(c*d^2 + a*e^2 + 2*c*d*e*x)*((1 + Sqrt[3])*
(c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d +
 e*x))^(1/3))) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*(c*d^2 - a*e^2)^(2/3)*Sqrt[(c
*d^2 + a*e^2 + 2*c*d*e*x)^2]*((c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/
3)*e^(1/3)*((a*e + c*d*x)*(d + e*x))^(1/3))*Sqrt[((c*d^2 - a*e^2)^(4/3) - 2
^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*(c*d^2 - a*e^2)^(2/3)*((a*e + c*d*x)*(d + e*
x))^(1/3) + 2*2^(1/3)*c^(2/3)*d^(2/3)*e^(2/3)*((a*e + c*d*x)*(d + e*x))^(2/
3))/((1 + Sqrt[3])*(c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*
((a*e + c*d*x)*(d + e*x))^(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt[3])*(c*d^2
- a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d + e*x))^
(1/3))/((1 + Sqrt[3])*(c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/
3)*((a*e + c*d*x)*(d + e*x))^(1/3))], -7 - 4*Sqrt[3]])/(2*2^(1/3)*c^(2/3)*d
^(2/3)*e^(2/3)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[((c*d^2 - a*e^2)^(2/3)*((c*
d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d + e*
x))^(1/3)))/((1 + Sqrt[3])*(c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*
e^(1/3)*((a*e + c*d*x)*(d + e*x))^(1/3))^2]*Sqrt[(a*e^2 + c*d*(d + 2*e*x))^
2]) + (2^(1/6)*3^(3/4)*(c*d^2 - a*e^2)^(2/3)*Sqrt[(c*d^2 + a*e^2 + 2*c*d*e*
x)^2]*((c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*
x)*(d + e*x))^(1/3))*Sqrt[((c*d^2 - a*e^2)^(4/3) - 2^(2/3)*c^(1/3)*d^(1/3)*
e^(1/3)*(c*d^2 - a*e^2)^(2/3)*((a*e + c*d*x)*(d + e*x))^(1/3) + 2*2^(1/3)*c
^(2/3)*d^(2/3)*e^(2/3)*((a*e + c*d*x)*(d + e*x))^(2/3))/((1 + Sqrt[3])*(c*d
^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d + e*x
))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(c*d^2 - a*e^2)^(2/3) + 2^(2/3
)*c^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d + e*x))^(1/3))/((1 + Sqrt[3])*(
c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d +
e*x))^(1/3))], -7 - 4*Sqrt[3]])/(c^(2/3)*d^(2/3)*e^(2/3)*(c*d^2 + a*e^2 + 2
*c*d*e*x)*Sqrt[((c*d^2 - a*e^2)^(2/3)*((c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1
/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d + e*x))^(1/3)))/((1 + Sqrt[3])*(c*d^2
 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d + e*x))
^(1/3))^2]*Sqrt[(a*e^2 + c*d*(d + 2*e*x))^2])
```

Rubi [A] time = 1.3091, antiderivative size = 1432, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {623, 303, 218, 1877}

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(cd^2-ae^2\right)^{2/3}\sqrt{\left(cd^2+2cexd+ae^2\right)^2}\left(\left(cd^2-ae^2\right)^{2/3}+2^{2/3}\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{e}\sqrt[3]{(ae+cdx)(d+ex)}\right)\sqrt{\frac{\left(cd^2-ae^2\right)^{4/3}-\cdots}{\cdots}}}{2\sqrt[3]{2}c^{2/3}d^{2/3}e^{2/3}\left(cd^2+2cexd+ae^2\right)\sqrt{\frac{\left(cd^2-ae^2\right)^{2/3}\left(\cdots\right)}{\left(\left(1+\sqrt{3}\right)\left(cd\cdots\right.\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-1/3),x]
```

```
[Out] (3*Sqrt[(c*d^2 + a*e^2 + 2*c*d*e*x)^2]*Sqrt[(a*e^2 + c*d*(d + 2*e*x))^2])/(
2^(1/3)*c^(2/3)*d^(2/3)*e^(2/3)*(c*d^2 + a*e^2 + 2*c*d*e*x)*((1 + Sqrt[3])*
```

```
(c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d +
 e*x))^(1/3))) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*(c*d^2 - a*e^2)^(2/3)*Sqrt[(c
*d^2 + a*e^2 + 2*c*d*e*x)^2]*((c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/
3)*e^(1/3)*((a*e + c*d*x)*(d + e*x))^(1/3))*Sqrt[((c*d^2 - a*e^2)^(4/3) - 2
^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*(c*d^2 - a*e^2)^(2/3)*((a*e + c*d*x)*(d + e*
x))^(1/3) + 2*2^(1/3)*c^(2/3)*d^(2/3)*e^(2/3)*((a*e + c*d*x)*(d + e*x))^(2/
3))/((1 + Sqrt[3])*(c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*
((a*e + c*d*x)*(d + e*x))^(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt[3])*(c*d^2
- a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d + e*x))^
(1/3))/((1 + Sqrt[3])*(c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/
3)*((a*e + c*d*x)*(d + e*x))^(1/3))], -7 - 4*Sqrt[3]])/(2*2^(1/3)*c^(2/3)*d
^(2/3)*e^(2/3)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[((c*d^2 - a*e^2)^(2/3)*((c*
d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d + e*
x))^(1/3)))/((1 + Sqrt[3])*(c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*
e^(1/3)*((a*e + c*d*x)*(d + e*x))^(1/3))^2]*Sqrt[(a*e^2 + c*d*(d + 2*e*x))^
2]) + (2^(1/6)*3^(3/4)*(c*d^2 - a*e^2)^(2/3)*Sqrt[(c*d^2 + a*e^2 + 2*c*d*e*
x)^2]*((c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*
x)*(d + e*x))^(1/3))*Sqrt[((c*d^2 - a*e^2)^(4/3) - 2^(2/3)*c^(1/3)*d^(1/3)*
e^(1/3)*(c*d^2 - a*e^2)^(2/3)*((a*e + c*d*x)*(d + e*x))^(1/3) + 2*2^(1/3)*c
^(2/3)*d^(2/3)*e^(2/3)*((a*e + c*d*x)*(d + e*x))^(2/3))/((1 + Sqrt[3])*(c*d
^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d + e*x
))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(c*d^2 - a*e^2)^(2/3) + 2^(2/3
)*c^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d + e*x))^(1/3))/((1 + Sqrt[3])*(
c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d +
e*x))^(1/3))], -7 - 4*Sqrt[3]])/(c^(2/3)*d^(2/3)*e^(2/3)*(c*d^2 + a*e^2 + 2
*c*d*e*x)*Sqrt[((c*d^2 - a*e^2)^(2/3)*((c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1
/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d + e*x))^(1/3)))/((1 + Sqrt[3])*(c*d^2
 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*((a*e + c*d*x)*(d + e*x))
^(1/3))^2]*Sqrt[(a*e^2 + c*d*(d + 2*e*x))^2])
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
```

```
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{\left(3\sqrt{\left(cd^2+ae^2+2cdex\right)^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-4acd^2e^2+\left(cd^2+ae^2\right)^2+4cdex^3}}\,dx,x,\sqrt[3]{(ae+cd}\right.}{cd^2+ae^2+2cdex}$$

$$= \frac{\left(3\sqrt{\left(cd^2+ae^2+2cdex\right)^2}\right)\text{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)\left(cd^2-ae^2\right)^{2/3}+2^{2/3}\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{e}x}{\sqrt{-4acd^2e^2+\left(cd^2+ae^2\right)^2+4cdex^3}}\,dx,x,\sqrt[3]{(ae+}\right.}{2^{2/3}\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{e}\left(cd^2+ae^2+2cdex\right)}$$

$$= \frac{3\sqrt{\left(cd^2+ae^2+2cdex\right)^2}\sqrt{\left(ae^2+cd(d+2ex)\right)^2}}{\sqrt[3]{2}c^{2/3}d^{2/3}e^{2/3}\left(cd^2+ae^2+2cdex\right)\left(\left(1+\sqrt{3}\right)\left(cd^2-ae^2\right)^{2/3}+2^{2/3}\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{e}\sqrt[3]{(ae}\right.}$$

Mathematica [C] time = 0.0427169, size = 95, normalized size = 0.07

$$\frac{3\sqrt[3]{\frac{cd(d+ex)}{cd^2-ae^2}}((d+ex)(ae+cdx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{2cd(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-1/3),x]
```

```
[Out] (3*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(1/3)*((a*e + c*d*x)*(d + e*x))^(2/3)*
Hypergeometric2F1[1/3, 2/3, 5/3, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(2*
c*d*(d + e*x))
```

Maple [F] time = 2.056, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{ade+\left(ae^2+cd^2\right)x+cdex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x)
```

```
[Out] int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(-1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(-1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{ade + cdex^2 + x\left(ae^2 + cd^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/3),x)
```

```
[Out] Integral((a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(-1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(-1/3), x)
```

3.1976 $\int (d+ex)^{3/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right) dx$

Optimal. Leaf size=43

$$\frac{2}{7}(d+ex)^{7/2}\left(a-\frac{cd^2}{e^2}\right)+\frac{2cd(d+ex)^{9/2}}{9e^2}$$

```
[Out] (2*(a - (c*d^2)/e^2)*(d + e*x)^(7/2))/7 + (2*c*d*(d + e*x)^(9/2))/(9*e^2)
```

Rubi [A] time = 0.0199979, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$\frac{2}{7}(d+ex)^{7/2}\left(a-\frac{cd^2}{e^2}\right)+\frac{2cd(d+ex)^{9/2}}{9e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] (2*(a - (c*d^2)/e^2)*(d + e*x)^(7/2))/7 + (2*c*d*(d + e*x)^(9/2))/(9*e^2)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)dx &= \int (ae+cdx)(d+ex)^{5/2}\,dx\\
&= \int \left(\frac{\left(-cd^2+ae^2\right)(d+ex)^{5/2}}{e}+\frac{cd(d+ex)^{7/2}}{e}\right)dx\\
&= \frac{2}{7}\left(a-\frac{cd^2}{e^2}\right)(d+ex)^{7/2}+\frac{2cd(d+ex)^{9/2}}{9e^2}
\end{aligned}$$

Mathematica [A] time = 0.0337937, size = 34, normalized size = 0.79

$$\frac{2(d+ex)^{7/2}\left(9ae^2+cd(7ex-2d)\right)}{63e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] (2*(d + e*x)^(7/2)*(9*a*e^2 + c*d*(-2*d + 7*e*x)))/(63*e^2)
```

Maple [A] time = 0.042, size = 32, normalized size = 0.7

$$\frac{14\,cdex+18\,ae^2-4\,cd^2}{63\,e^2}(ex+d)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

```
[Out] 2/63*(e*x+d)^(7/2)*(7*c*d*e*x+9*a*e^2-2*c*d^2)/e^2
```

Maxima [A] time = 0.985135, size = 51, normalized size = 1.19

$$\frac{2\left(7\,(ex+d)^{\frac{9}{2}}cd-9\left(cd^2-ae^2\right)(ex+d)^{\frac{7}{2}}\right)}{63\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="max
ima")
```

```
[Out] 2/63*(7*(e*x + d)^(9/2)*c*d - 9*(c*d^2 - a*e^2)*(e*x + d)^(7/2))/e^2
```

Fricas [B] time = 2.33905, size = 213, normalized size = 4.95

$$\frac{2\left(7\,cde^4x^4-2\,cd^5+9\,ad^3e^2+\left(19\,cd^2e^3+9\,ae^5\right)x^3+3\left(5\,cd^3e^2+9\,ade^4\right)x^2+\left(cd^4e+27\,ad^2e^3\right)x\right)\sqrt{ex+d}}{63\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fri
cas")
```

```
[Out] 2/63*(7*c*d*e^4*x^4 - 2*c*d^5 + 9*a*d^3*e^2 + (19*c*d^2*e^3 + 9*a*e^5)*x^3
+ 3*(5*c*d^3*e^2 + 9*a*d*e^4)*x^2 + (c*d^4*e + 27*a*d^2*e^3)*x)*sqrt(e*x +
d)/e^2
```

Sympy [A] time = 12.7528, size = 235, normalized size = 5.47

$$ad^2e\left(\begin{cases}\sqrt{d}x & \text{for } e=0\\ \frac{2(d+ex)^{\frac{3}{2}}}{3e} & \text{otherwise}\end{cases}\right)+4ad\left(-\frac{d\,(d+ex)^{\frac{3}{2}}}{3}+\frac{(d+ex)^{\frac{5}{2}}}{5}\right)+2a\left(\frac{d^2\,(d+ex)^{\frac{3}{2}}}{3}-\frac{2d\,(d+ex)^{\frac{5}{2}}}{5}+\frac{(d+ex)^{\frac{7}{2}}}{7}\right)+\frac{2cd^3}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

```
[Out] a*d**2*e*Piecewise((sqrt(d)*x, Eq(e, 0)), (2*(d + e*x)**(3/2)/(3*e), True))
 + 4*a*d*(-d*(d + e*x)**(3/2)/3 + (d + e*x)**(5/2)/5) + 2*a*(d**2*(d + e*x)
**(3/2)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7) + 2*c*d**3*(-d*(d
+ e*x)**(3/2)/3 + (d + e*x)**(5/2)/5)/e**2 + 4*c*d**2*(d**2*(d + e*x)**(3/2
)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e**2 + 2*c*d*(-d**3*(d +
 e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d + e*x)**(7/2)/7 + (d +
e*x)**(9/2)/9)/e**2
```

Giac [B] time = 1.13888, size = 290, normalized size = 6.74

$$\frac{2}{315}\left(21\left(3\,(xe+d)^{\frac{5}{2}}-5\,(xe+d)^{\frac{3}{2}}d\right)cd^{3}e^{(-1)}+105\,(xe+d)^{\frac{3}{2}}ad^{2}e+6\left(15\,(xe+d)^{\frac{7}{2}}-42\,(xe+d)^{\frac{5}{2}}d+35\,(xe+d)^{\frac{3}{2}}d^{2}\right)cd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="gia
c")
```

```
[Out] 2/315*(21*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*c*d^3*e^(-1) + 105*(x*e
 + d)^(3/2)*a*d^2*e + 6*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*
e + d)^(3/2)*d^2)*c*d^2*e^(-1) + 42*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*
d)*a*d*e + (35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2
)*d^2 - 105*(x*e + d)^(3/2)*d^3)*c*d*e^(-1) + 3*(15*(x*e + d)^(7/2) - 42*(x
*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*a*e)*e^(-1)
```

3.1977 $\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx$

Optimal. Leaf size=43

$$\frac{2}{5}(d+ex)^{5/2}\left(a-\frac{cd^2}{e^2}\right)+\frac{2cd(d+ex)^{7/2}}{7e^2}$$

```
[Out] (2*(a - (c*d^2)/e^2)*(d + e*x)^(5/2))/5 + (2*c*d*(d + e*x)^(7/2))/(7*e^2)
```

Rubi [A] time = 0.0193963, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$\frac{2}{5}(d+ex)^{5/2}\left(a-\frac{cd^2}{e^2}\right)+\frac{2cd(d+ex)^{7/2}}{7e^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] (2*(a - (c*d^2)/e^2)*(d + e*x)^(5/2))/5 + (2*c*d*(d + e*x)^(7/2))/(7*e^2)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)dx&=\int (ae+cdx)(d+ex)^{3/2}\,dx\\&=\int \left(\frac{\left(-cd^2+ae^2\right)(d+ex)^{3/2}}{e}+\frac{cd(d+ex)^{5/2}}{e}\right)dx\\&=\frac{2}{5}\left(a-\frac{cd^2}{e^2}\right)(d+ex)^{5/2}+\frac{2cd(d+ex)^{7/2}}{7e^2}\end{aligned}$$

Mathematica [A] time = 0.0249812, size = 34, normalized size = 0.79

$$\frac{2(d+ex)^{5/2}\left(7ae^2+cd(5ex-2d)\right)}{35e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] (2*(d + e*x)^(5/2)*(7*a*e^2 + c*d*(-2*d + 5*e*x)))/(35*e^2)
```

Maple [A] time = 0.041, size = 32, normalized size = 0.7

$$\frac{10\,cdex + 14\,ae^2 - 4\,cd^2}{35\,e^2}(ex + d)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

```
[Out] 2/35*(e*x+d)^(5/2)*(5*c*d*e*x+7*a*e^2-2*c*d^2)/e^2
```

Maxima [A] time = 0.976588, size = 51, normalized size = 1.19

$$\frac{2\left(5\,(ex + d)^{\frac{7}{2}}cd - 7\left(cd^2 - ae^2\right)(ex + d)^{\frac{5}{2}}\right)}{35\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="max
ima")
```

```
[Out] 2/35*(5*(e*x + d)^(7/2)*c*d - 7*(c*d^2 - a*e^2)*(e*x + d)^(5/2))/e^2
```

Fricas [B] time = 2.27846, size = 163, normalized size = 3.79

$$\frac{2\left(5\,cde^3x^3 - 2\,cd^4 + 7\,ad^2e^2 + \left(8\,cd^2e^2 + 7\,ae^4\right)x^2 + \left(cd^3e + 14\,ade^3\right)x\right)\sqrt{ex + d}}{35\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fri
cas")
```

```
[Out] 2/35*(5*c*d*e^3*x^3 - 2*c*d^4 + 7*a*d^2*e^2 + (8*c*d^2*e^2 + 7*a*e^4)*x^2 +
 (c*d^3*e + 14*a*d*e^3)*x)*sqrt(e*x + d)/e^2
```

Sympy [A] time = 2.84052, size = 41, normalized size = 0.95

$$\frac{2\left(\frac{cd(d+ex)^{\frac{7}{2}}}{7e} + \frac{(d+ex)^{\frac{5}{2}}\left(ae^2-cd^2\right)}{5e}\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

```
[Out] 2*(c*d*(d + e*x)**(7/2)/(7*e) + (d + e*x)**(5/2)*(a*e**2 - c*d**2)/(5*e))/e
```

Giac [B] time = 1.13804, size = 157, normalized size = 3.65

$$\frac{2}{105}\left(7\left(3\,(xe+d)^{\frac{5}{2}}-5\,(xe+d)^{\frac{3}{2}}d\right)cd^2e^{(-1)}+35\,(xe+d)^{\frac{3}{2}}ade+\left(15\,(xe+d)^{\frac{7}{2}}-42\,(xe+d)^{\frac{5}{2}}d+35\,(xe+d)^{\frac{3}{2}}d^2\right)cde^{(-1)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="gia
c")
```

```
[Out] 2/105*(7*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*c*d^2*e^(-1) + 35*(x*e +
 d)^(3/2)*a*d*e + (15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)
^(3/2)*d^2)*c*d*e^(-1) + 7*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a*e)*e
^(-1)
```

3.1978 $\int \frac{ade+(cd^2+ae^2)x+cdex^2}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=43

$$\frac{2}{3}(d+ex)^{3/2}\left(a-\frac{cd^2}{e^2}\right)+\frac{2cd(d+ex)^{5/2}}{5e^2}$$

```
[Out] (2*(a - (c*d^2)/e^2)*(d + e*x)^(3/2))/3 + (2*c*d*(d + e*x)^(5/2))/(5*e^2)
```

Rubi [A] time = 0.0191914, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$\frac{2}{3}(d+ex)^{3/2}\left(a-\frac{cd^2}{e^2}\right)+\frac{2cd(d+ex)^{5/2}}{5e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/Sqrt[d + e*x],x]
```

```
[Out] (2*(a - (c*d^2)/e^2)*(d + e*x)^(3/2))/3 + (2*c*d*(d + e*x)^(5/2))/(5*e^2)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{ade+\left(cd^2+ae^2\right)x+cdex^2}{\sqrt{d+ex}}\,dx &= \int (ae+cdx)\sqrt{d+ex}\,dx\\ &= \int \left(\frac{\left(-cd^2+ae^2\right)\sqrt{d+ex}}{e}+\frac{cd(d+ex)^{3/2}}{e}\right)dx\\ &= \frac{2}{3}\left(a-\frac{cd^2}{e^2}\right)(d+ex)^{3/2}+\frac{2cd(d+ex)^{5/2}}{5e^2}\end{aligned}$$

Mathematica [A] time = 0.0224801, size = 34, normalized size = 0.79

$$\frac{2(d+ex)^{3/2}\left(5ae^2+cd(3ex-2d)\right)}{15e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/Sqrt[d + e*x],x]
```

```
[Out] (2*(d + e*x)^(3/2)*(5*a*e^2 + c*d*(-2*d + 3*e*x)))/(15*e^2)
```

Maple [A] time = 0.041, size = 32, normalized size = 0.7

$$\frac{6\,cdex+10\,ae^{2}-4\,cd^{2}}{15\,e^{2}}\,(ex+d)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(1/2),x)
```

```
[Out] 2/15*(e*x+d)^(3/2)*(3*c*d*e*x+5*a*e^2-2*c*d^2)/e^2
```

Maxima [B] time = 0.997965, size = 122, normalized size = 2.84

$$\frac{2\left(15\,\sqrt{ex+d}\,dae+\frac{\left(3\,(ex+d)^{\frac{5}{2}}-10\,(ex+d)^{\frac{3}{2}}d+15\,\sqrt{ex+d}d^{2}\right)cd}{e}+\frac{5\left(cd^{2}+ae^{2}\right)\left((ex+d)^{\frac{3}{2}}-3\,\sqrt{ex+d}d\right)}{e}\right)}{15\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(1/2),x, algorithm="max
ima")
```

```
[Out] 2/15*(15*sqrt(e*x + d)*a*d*e + (3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d +
15*sqrt(e*x + d)*d^2)*c*d/e + 5*(c*d^2 + a*e^2)*((e*x + d)^(3/2) - 3*sqrt(e
*x + d)*d)/e)/e
```

Fricas [A] time = 2.1406, size = 116, normalized size = 2.7

$$\frac{2\left(3\,cde^{2}x^{2}-2\,cd^{3}+5\,ade^{2}+\left(cd^{2}e+5\,ae^{3}\right)x\right)\sqrt{ex+d}}{15\,e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 2/15*(3*c*d*e^2*x^2 - 2*c*d^3 + 5*a*d*e^2 + (c*d^2*e + 5*a*e^3)*x)*sqrt(e*x
 + d)/e^2
```

Sympy [A] time = 22.5958, size = 221, normalized size = 5.14

$$\begin{cases} -\frac{\frac{2ad^{2}e}{\sqrt{d+ex}}+4ade\left(-\frac{d}{\sqrt{d+ex}}-\sqrt{d+ex}\right)+2ae\left(\frac{d^{2}}{\sqrt{d+ex}}+2d\sqrt{d+ex}-\frac{(d+ex)^{\frac{3}{2}}}{3}\right)+\frac{2cd^{3}\left(-\frac{d}{\sqrt{d+ex}}-\sqrt{d+ex}\right)}{e}+\frac{4cd^{2}\left(\frac{d^{2}}{\sqrt{d+ex}}+2d\sqrt{d+ex}-\frac{(d+ex)^{\frac{3}{2}}}{3}\right)}{e}+\frac{2cd\left(-\frac{d^{3}}{\sqrt{d+ex}}-3d^{2}\sqrt{d+ex}+d(d+ex)^{\frac{3}{2}}-\frac{(d+e}{5}\right.}{e}}{e} \\ \frac{cd^{\frac{3}{2}}x^{2}}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)/(e*x+d)**(1/2),x)
```

```
[Out] Piecewise((-(2*a*d**2*e/sqrt(d + e*x) + 4*a*d*e*(-d/sqrt(d + e*x) - sqrt(d
+ e*x)) + 2*a*e*(d**2/sqrt(d + e*x) + 2*d*sqrt(d + e*x) - (d + e*x)**(3/2)/
3) + 2*c*d**3*(-d/sqrt(d + e*x) - sqrt(d + e*x))/e + 4*c*d**2*(d**2/sqrt(d
+ e*x) + 2*d*sqrt(d + e*x) - (d + e*x)**(3/2)/3)/e + 2*c*d*(-d**3/sqrt(d +
e*x) - 3*d**2*sqrt(d + e*x) + d*(d + e*x)**(3/2) - (d + e*x)**(5/2)/5)/e)/e
, Ne(e, 0)), (c*d**(3/2)*x**2/2, True))
```

Giac [B] time = 1.1204, size = 151, normalized size = 3.51

$$\frac{2}{15}\left(5\left((xe+d)^{\frac{3}{2}}-3\sqrt{xe+d}d\right)cd^2e^{(-1)}+\left(3(xe+d)^{\frac{5}{2}}-10(xe+d)^{\frac{3}{2}}d+15\sqrt{xe+d}d^2\right)cde^{(-1)}+15\sqrt{xe+d}ade+5\left((x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(1/2),x, algorithm="gia
c")
```

```
[Out] 2/15*(5*((x*e + d)^(3/2) - 3*sqrt(x*e + d)*d)*c*d^2*e^(-1) + (3*(x*e + d)^(
5/2) - 10*(x*e + d)^(3/2)*d + 15*sqrt(x*e + d)*d^2)*c*d*e^(-1) + 15*sqrt(x*
e + d)*a*d*e + 5*((x*e + d)^(3/2) - 3*sqrt(x*e + d)*d)*a*e)*e^(-1)
```

3.1979 $\int \frac{ade+(cd^2+ae^2)x+cdex^2}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=41

$$2\sqrt{d+ex}\left(a-\frac{cd^2}{e^2}\right)+\frac{2cd(d+ex)^{3/2}}{3e^2}$$

```
[Out] 2*(a - (c*d^2)/e^2)*Sqrt[d + e*x] + (2*c*d*(d + e*x)^(3/2))/(3*e^2)
```

Rubi [A] time = 0.024613, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {24, 43}

$$2\sqrt{d+ex}\left(a-\frac{cd^2}{e^2}\right)+\frac{2cd(d+ex)^{3/2}}{3e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^(3/2),x]
```

```
[Out] 2*(a - (c*d^2)/e^2)*Sqrt[d + e*x] + (2*c*d*(d + e*x)^(3/2))/(3*e^2)
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{ade+\left(cd^2+ae^2\right)x+cdex^2}{(d+ex)^{3/2}}\,dx &= \frac{\int \frac{ae^3+cde^2x}{\sqrt{d+ex}}\,dx}{e^2}\\
&= \frac{\int \left(\frac{-cd^2e+ae^3}{\sqrt{d+ex}}+cde\sqrt{d+ex}\right)dx}{e^2}\\
&= 2\left(a-\frac{cd^2}{e^2}\right)\sqrt{d+ex}+\frac{2cd(d+ex)^{3/2}}{3e^2}
\end{aligned}$$

Mathematica [A] time = 0.0208033, size = 33, normalized size = 0.8

$$\frac{2\sqrt{d+ex}\left(3ae^2+cd(ex-2d)\right)}{3e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^(3/2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(3*a*e^2 + c*d*(-2*d + e*x)))/(3*e^2)
```

Maple [A] time = 0.042, size = 31, normalized size = 0.8

$$\frac{2\,cdex+6\,ae^2-4\,cd^2}{3\,e^2}\sqrt{ex+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(3/2),x)
```

```
[Out] 2/3*(e*x+d)^(1/2)*(c*d*e*x+3*a*e^2-2*c*d^2)/e^2
```

Maxima [A] time = 0.989093, size = 50, normalized size = 1.22

$$\frac{2\left((ex+d)^{\frac{3}{2}}cd-3\left(cd^2-ae^2\right)\sqrt{ex+d}\right)}{3\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(3/2),x, algorithm="max
ima")
```

```
[Out] 2/3*((e*x + d)^(3/2)*c*d - 3*(c*d^2 - a*e^2)*sqrt(e*x + d))/e^2
```

Fricas [A] time = 2.13835, size = 72, normalized size = 1.76

$$\frac{2\left(cdex-2\,cd^2+3\,ae^2\right)\sqrt{ex+d}}{3\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(3/2),x, algorithm="fri
cas")
```

```
[Out] 2/3*(c*d*e*x - 2*c*d^2 + 3*a*e^2)*sqrt(e*x + d)/e^2
```

Sympy [A] time = 9.61783, size = 124, normalized size = 3.02

$$\begin{cases} -\frac{\frac{2ade}{\sqrt{d+ex}}+2ae\left(-\frac{d}{\sqrt{d+ex}}-\sqrt{d+ex}\right)+\frac{2cd^2\left(-\frac{d}{\sqrt{d+ex}}-\sqrt{d+ex}\right)}{e}+\frac{2cd\left(\frac{d^2}{\sqrt{d+ex}}+2d\sqrt{d+ex}-\frac{(d+ex)^{\frac{3}{2}}}{3}\right)}{e}}{e} & \text{for } e\neq 0 \\ \frac{c\sqrt{d}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)/(e*x+d)**(3/2),x)
```

```
[Out] Piecewise((-(2*a*d*e/sqrt(d + e*x) + 2*a*e*(-d/sqrt(d + e*x) - sqrt(d + e*x
)) + 2*c*d**2*(-d/sqrt(d + e*x) - sqrt(d + e*x))/e + 2*c*d*(d**2/sqrt(d + e
*x) + 2*d*sqrt(d + e*x) - (d + e*x)**(3/2)/3)/e)/e, Ne(e, 0)), (c*sqrt(d)*x
**2/2, True))
```

Giac [A] time = 1.16205, size = 63, normalized size = 1.54

$$\frac{2}{3}\left((xe+d)^{\frac{3}{2}}cde^4 - 3\sqrt{xe+d}cd^2e^4 + 3\sqrt{xe+d}ae^6\right)e^{(-6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(3/2),x, algorithm="gia
c")
```

```
[Out] 2/3*((x*e + d)^(3/2)*c*d*e^4 - 3*sqrt(x*e + d)*c*d^2*e^4 + 3*sqrt(x*e + d)*
a*e^6)*e^(-6)
```

3.1980 $\int \frac{ade+(cd^2+ae^2)x+cdex^2}{(d+ex)^{5/2}} dx$

Optimal. Leaf size=39

$$\frac{2cd\sqrt{d+ex}}{e^2} - \frac{2\left(a-\frac{cd^2}{e^2}\right)}{\sqrt{d+ex}}$$

```
[Out] (-2*(a - (c*d^2)/e^2))/Sqrt[d + e*x] + (2*c*d*Sqrt[d + e*x])/e^2
```

Rubi [A] time = 0.0231704, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {24, 43}

$$\frac{2cd\sqrt{d+ex}}{e^2} - \frac{2\left(a-\frac{cd^2}{e^2}\right)}{\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^(5/2),x]
```

```
[Out] (-2*(a - (c*d^2)/e^2))/Sqrt[d + e*x] + (2*c*d*Sqrt[d + e*x])/e^2
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{ade+\left(cd^2+ae^2\right)x+cdex^2}{(d+ex)^{5/2}}\,dx &= \frac{\int \frac{ae^3+cde^2x}{(d+ex)^{3/2}}\,dx}{e^2}\\
&= \frac{\int \left(\frac{-cd^2e+ae^3}{(d+ex)^{3/2}}+\frac{cde}{\sqrt{d+ex}}\right)dx}{e^2}\\
&= -\frac{2\left(a-\frac{cd^2}{e^2}\right)}{\sqrt{d+ex}}+\frac{2cd\sqrt{d+ex}}{e^2}
\end{aligned}$$

Mathematica [A] time = 0.0185505, size = 31, normalized size = 0.79

$$\frac{2cd(2d+ex)-2ae^2}{e^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^(5/2),x]
```

```
[Out] (-2*a*e^2 + 2*c*d*(2*d + e*x))/(e^2*Sqrt[d + e*x])
```

Maple [A] time = 0.041, size = 31, normalized size = 0.8

$$-2\,{\frac{-cdex+a{e}^{2}-2\,c{d}^{2}}{\sqrt{ex+d}{e}^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(5/2),x)
```

```
[Out] -2/(e*x+d)^(1/2)*(-c*d*e*x+a*e^2-2*c*d^2)/e^2
```

Maxima [A] time = 1.00864, size = 57, normalized size = 1.46

$$\frac{2\,{\left(\frac{\sqrt{e x + d} c d}{e} + \frac{c d^{2} - a e^{2}}{\sqrt{e x + d} e}\right)}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(5/2),x, algorithm="max
ima")
```

```
[Out] 2*(sqrt(e*x + d)*c*d/e + (c*d^2 - a*e^2)/(sqrt(e*x + d)*e))/e
```

Fricas [A] time = 2.0821, size = 82, normalized size = 2.1

$$\frac{2 \,{\left(c d e x + 2 \, c d^{2} - a e^{2}\right)} \sqrt{e x + d}}{e^{3} x + d e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(5/2),x, algorithm="fri
cas")
```

```
[Out] 2*(c*d*e*x + 2*c*d^2 - a*e^2)*sqrt(e*x + d)/(e^3*x + d*e^2)
```

Sympy [A] time = 1.56779, size = 58, normalized size = 1.49

$$\begin{cases} - \frac{2 a}{\sqrt{d + e x}} + \frac{4 c d^{2}}{e^{2} \sqrt{d + e x}} + \frac{2 c d x}{e \sqrt{d + e x}} & \text{for}\: e \neq 0 \\\frac{c x^{2}}{2 \sqrt{d}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)/(e*x+d)**(5/2),x)
```

```
[Out] Piecewise((-2*a/sqrt(d + e*x) + 4*c*d**2/(e**2*sqrt(d + e*x)) + 2*c*d*x/(e*
sqrt(d + e*x)), Ne(e, 0)), (c*x**2/(2*sqrt(d)), True))
```

Giac [A] time = 1.16531, size = 68, normalized size = 1.74

$$2\sqrt{xe+d}cde^{(-2)}+\frac{2\left((xe+d)cd^2-(xe+d)ae^2\right)e^{(-2)}}{(xe+d)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(5/2),x, algorithm="gia
c")
```

```
[Out] 2*sqrt(x*e + d)*c*d*e^(-2) + 2*((x*e + d)*c*d^2 - (x*e + d)*a*e^2)*e^(-2)/(
x*e + d)^(3/2)
```

3.1981 $\int \frac{ade+\left(cd^2+ae^2\right)x+cdex^2}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{2\left(a-\frac{cd^2}{e^2}\right)}{3(d+ex)^{3/2}}-\frac{2cd}{e^2\sqrt{d+ex}}$$

[Out] (-2*(a - (c*d^2)/e^2))/(3*(d + e*x)^(3/2)) - (2*c*d)/(e^2*Sqrt[d + e*x])

Rubi [A] time = 0.0221359, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {24, 43}

$$-\frac{2\left(a-\frac{cd^2}{e^2}\right)}{3(d+ex)^{3/2}}-\frac{2cd}{e^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^(7/2),x]

[Out] (-2*(a - (c*d^2)/e^2))/(3*(d + e*x)^(3/2)) - (2*c*d)/(e^2*Sqrt[d + e*x])

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{ade+\left(cd^2+ae^2\right)x+cdex^2}{(d+ex)^{7/2}}\,dx &= \frac{\int \frac{ae^3+cde^2x}{(d+ex)^{5/2}}\,dx}{e^2}\\
&= \frac{\int \left(\frac{-cd^2e+ae^3}{(d+ex)^{5/2}}+\frac{cde}{(d+ex)^{3/2}}\right)dx}{e^2}\\
&= -\frac{2\left(a-\frac{cd^2}{e^2}\right)}{3(d+ex)^{3/2}}-\frac{2cd}{e^2\sqrt{d+ex}}
\end{aligned}$$

Mathematica [A] time = 0.0207019, size = 33, normalized size = 0.8

$$-\frac{2\left(ae^2+cd(2d+3ex)\right)}{3e^2(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^(7/2),x]
```

```
[Out] (-2*(a*e^2 + c*d*(2*d + 3*e*x)))/(3*e^2*(d + e*x)^(3/2))
```

Maple [A] time = 0.041, size = 31, normalized size = 0.8

$$-\frac{6\,cdex+2\,a{e}^{2}+4\,c{d}^{2}}{3\,{e}^{2}} \left(ex+d \right) ^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(7/2),x)
```

```
[Out] -2/3/(e*x+d)^(3/2)*(3*c*d*e*x+a*e^2+2*c*d^2)/e^2
```

Maxima [A] time = 1.03301, size = 45, normalized size = 1.1

$$-\frac{2\,{\left(3\,{\left(e x + d\right)} c d - c d^{2} + a e^{2}\right)}}{3\,{\left(e x + d\right)}^{\frac{3}{2}} e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(7/2),x, algorithm="max
ima")
```

```
[Out] -2/3*(3*(e*x + d)*c*d - c*d^2 + a*e^2)/((e*x + d)^(3/2)*e^2)
```

Fricas [A] time = 2.1431, size = 111, normalized size = 2.71

$$-\frac{2\,{\left(3\,c d e x + 2\,c d^{2} + a e^{2}\right)} \sqrt{e x + d}}{3\,{\left(e^{4} x^{2} + 2\,d e^{3} x + d^{2} e^{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(7/2),x, algorithm="fri
cas")
```

```
[Out] -2/3*(3*c*d*e*x + 2*c*d^2 + a*e^2)*sqrt(e*x + d)/(e^4*x^2 + 2*d*e^3*x + d^2
*e^2)
```

Sympy [A] time = 3.62371, size = 126, normalized size = 3.07

$$\begin{cases} - \frac{2 a e^{2}}{3 d e^{2} \sqrt{d + e x} + 3 e^{3} x \sqrt{d + e x}} - \frac{4 c d^{2}}{3 d e^{2} \sqrt{d + e x} + 3 e^{3} x \sqrt{d + e x}} - \frac{6 c d e x}{3 d e^{2} \sqrt{d + e x} + 3 e^{3} x \sqrt{d + e x}} & \text{for}\: e \neq 0 \\ \frac{c x^{2}}{2 d^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)/(e*x+d)**(7/2),x)
```

```
[Out] Piecewise((-2*a*e**2/(3*d*e**2*sqrt(d + e*x) + 3*e**3*x*sqrt(d + e*x)) - 4*
c*d**2/(3*d*e**2*sqrt(d + e*x) + 3*e**3*x*sqrt(d + e*x)) - 6*c*d*e*x/(3*d*e
**2*sqrt(d + e*x) + 3*e**3*x*sqrt(d + e*x)), Ne(e, 0)), (c*x**2/(2*d**(3/2)
), True))
```

Giac [A] time = 1.15023, size = 63, normalized size = 1.54

$$-\frac{2\left(3\,(xe+d)^{2}cd-(xe+d)cd^{2}+(xe+d)ae^{2}\right)e^{(-2)}}{3\,(xe+d)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(7/2),x, algorithm="gia
c")
```

```
[Out] -2/3*(3*(x*e + d)^2*c*d - (x*e + d)*c*d^2 + (x*e + d)*a*e^2)*e^(-2)/(x*e +
d)^(5/2)
```

3.1982 $\int \frac{ade+(cd^2+ae^2)x+cdex^2}{(d+ex)^{9/2}}\,dx$

Optimal. Leaf size=43

$$-\frac{2\left(a-\frac{cd^2}{e^2}\right)}{5(d+ex)^{5/2}}-\frac{2cd}{3e^2(d+ex)^{3/2}}$$

```
[Out] (-2*(a - (c*d^2)/e^2))/(5*(d + e*x)^(5/2)) - (2*c*d)/(3*e^2*(d + e*x)^(3/2)
)
```

Rubi [A] time = 0.0233258, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {24, 43}

$$-\frac{2\left(a-\frac{cd^2}{e^2}\right)}{5(d+ex)^{5/2}}-\frac{2cd}{3e^2(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^(9/2),x]
```

```
[Out] (-2*(a - (c*d^2)/e^2))/(5*(d + e*x)^(5/2)) - (2*c*d)/(3*e^2*(d + e*x)^(3/2)
)
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{ade+\left(cd^2+ae^2\right)x+cdex^2}{(d+ex)^{9/2}}\,dx&=\frac{\int \frac{ae^3+cde^2x}{(d+ex)^{7/2}}\,dx}{e^2}\\&=\frac{\int \left(\frac{-cd^2e+ae^3}{(d+ex)^{7/2}}+\frac{cde}{(d+ex)^{5/2}}\right)dx}{e^2}\\&=-\frac{2\left(a-\frac{cd^2}{e^2}\right)}{5(d+ex)^{5/2}}-\frac{2cd}{3e^2(d+ex)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0219119, size = 34, normalized size = 0.79

$$-\frac{2\left(3ae^2+cd(2d+5ex)\right)}{15e^2(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^(9/2),x]
```

```
[Out] (-2*(3*a*e^2 + c*d*(2*d + 5*e*x)))/(15*e^2*(d + e*x)^(5/2))
```

Maple [A] time = 0.042, size = 32, normalized size = 0.7

$$-\frac{10\,cdex+6\,a{e}^{2}+4\,c{d}^{2}}{15\,{e}^{2}} \left(ex+d \right) ^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(9/2),x)
```

```
[Out] -2/15/(e*x+d)^(5/2)*(5*c*d*e*x+3*a*e^2+2*c*d^2)/e^2
```

Maxima [A] time = 1.01439, size = 46, normalized size = 1.07

$$-\frac{2 \left(5 \,{\left(e x + d\right)} c d - 3 \, c d^{2} + 3 \, a e^{2}\right)}{15 \,{\left(e x + d\right)}^{\frac{5}{2}} e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(9/2),x, algorithm="max
ima")
```

```
[Out] -2/15*(5*(e*x + d)*c*d - 3*c*d^2 + 3*a*e^2)/((e*x + d)^(5/2)*e^2)
```

Fricas [A] time = 1.97147, size = 136, normalized size = 3.16

$$-\frac{2 \,{\left(5 \, c d e x + 2 \, c d^{2} + 3 \, a e^{2}\right)} \sqrt{e x + d}}{15 \,{\left(e^{5} x^{3} + 3 \, d e^{4} x^{2} + 3 \, d^{2} e^{3} x + d^{3} e^{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(9/2),x, algorithm="fri
cas")
```

```
[Out] -2/15*(5*c*d*e*x + 2*c*d^2 + 3*a*e^2)*sqrt(e*x + d)/(e^5*x^3 + 3*d*e^4*x^2
+ 3*d^2*e^3*x + d^3*e^2)
```

Sympy [A] time = 8.88199, size = 187, normalized size = 4.35

$$\begin{cases} - \frac{6 a e^{2}}{15 d^{2} e^{2} \sqrt{d + e x} + 30 d e^{3} x \sqrt{d + e x} + 15 e^{4} x^{2} \sqrt{d + e x}} - \frac{4 c d^{2}}{15 d^{2} e^{2} \sqrt{d + e x} + 30 d e^{3} x \sqrt{d + e x} + 15 e^{4} x^{2} \sqrt{d + e x}} - \frac{10 c d e x}{15 d^{2} e^{2} \sqrt{d + e x} + 30 d e^{3} x \sqrt{d + e x} + 15 e^{4} x^{2} \sqrt{d + e x}} & \text{for}\: e \neq 0 \\\frac{c x^{2}}{2 d^{\frac{5}{2}}} & \text{otherwis} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)/(e*x+d)**(9/2),x)
```

```
[Out] Piecewise((-6*a*e**2/(15*d**2*e**2*sqrt(d + e*x) + 30*d*e**3*x*sqrt(d + e*x
) + 15*e**4*x**2*sqrt(d + e*x)) - 4*c*d**2/(15*d**2*e**2*sqrt(d + e*x) + 30
*d*e**3*x*sqrt(d + e*x) + 15*e**4*x**2*sqrt(d + e*x)) - 10*c*d*e*x/(15*d**2
*e**2*sqrt(d + e*x) + 30*d*e**3*x*sqrt(d + e*x) + 15*e**4*x**2*sqrt(d + e*x
)), Ne(e, 0)), (c*x**2/(2*d**(5/2)), True))
```

Giac [A] time = 1.13672, size = 65, normalized size = 1.51

$$-\frac{2\left(5\,(xe+d)^2cd-3\,(xe+d)cd^2+3\,(xe+d)ae^2\right)e^{(-2)}}{15\,(xe+d)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(9/2),x, algorithm="gia
c")
```

```
[Out] -2/15*(5*(x*e + d)^2*c*d - 3*(x*e + d)*c*d^2 + 3*(x*e + d)*a*e^2)*e^(-2)/(x
*e + d)^(7/2)
```

3.1983 $\int \frac{ade+\left(cd^2+ae^2\right)x+cdex^2}{(d+ex)^{11/2}}\,dx$

Optimal. Leaf size=43

$$-\frac{2\left(a-\frac{cd^2}{e^2}\right)}{7(d+ex)^{7/2}}-\frac{2cd}{5e^2(d+ex)^{5/2}}$$

```
[Out] (-2*(a - (c*d^2)/e^2))/(7*(d + e*x)^(7/2)) - (2*c*d)/(5*e^2*(d + e*x)^(5/2)
)
```

Rubi [A] time = 0.0217398, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {24, 43}

$$-\frac{2\left(a-\frac{cd^2}{e^2}\right)}{7(d+ex)^{7/2}}-\frac{2cd}{5e^2(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^(11/2),x]
```

```
[Out] (-2*(a - (c*d^2)/e^2))/(7*(d + e*x)^(7/2)) - (2*c*d)/(5*e^2*(d + e*x)^(5/2)
)
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{ade+\left(cd^2+ae^2\right)x+cdex^2}{(d+ex)^{11/2}}\,dx &= \frac{\int \frac{ae^3+cde^2x}{(d+ex)^{9/2}}\,dx}{e^2}\\
&= \frac{\int \left(\frac{-cd^2e+ae^3}{(d+ex)^{9/2}}+\frac{cde}{(d+ex)^{7/2}}\right)dx}{e^2}\\
&= -\frac{2\left(a-\frac{cd^2}{e^2}\right)}{7(d+ex)^{7/2}}-\frac{2cd}{5e^2(d+ex)^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0226838, size = 34, normalized size = 0.79

$$-\frac{2\left(5ae^2+cd(2d+7ex)\right)}{35e^2(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^(11/2),x]
```

```
[Out] (-2*(5*a*e^2 + c*d*(2*d + 7*e*x)))/(35*e^2*(d + e*x)^(7/2))
```

Maple [A] time = 0.042, size = 32, normalized size = 0.7

$$-\frac{14\,cdex+10\,ae^2+4\,cd^2}{35\,e^2}\,(ex+d)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(11/2),x)
```

```
[Out] -2/35/(e*x+d)^(7/2)*(7*c*d*e*x+5*a*e^2+2*c*d^2)/e^2
```

Maxima [A] time = 0.996956, size = 46, normalized size = 1.07

$$-\frac{2\left(7\,(ex+d)cd-5\,cd^2+5\,ae^2\right)}{35\,(ex+d)^{\frac{7}{2}}e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(11/2),x, algorithm="ma
xima")
```

```
[Out] -2/35*(7*(e*x + d)*c*d - 5*c*d^2 + 5*a*e^2)/((e*x + d)^(7/2)*e^2)
```

Fricas [B] time = 1.84553, size = 158, normalized size = 3.67

$$-\frac{2\left(7\,cdex+2\,cd^2+5\,ae^2\right)\sqrt{ex+d}}{35\left(e^6x^4+4\,de^5x^3+6\,d^2e^4x^2+4\,d^3e^3x+d^4e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(11/2),x, algorithm="fr
icas")
```

```
[Out] -2/35*(7*c*d*e*x + 2*c*d^2 + 5*a*e^2)*sqrt(e*x + d)/(e^6*x^4 + 4*d*e^5*x^3
+ 6*d^2*e^4*x^2 + 4*d^3*e^3*x + d^4*e^2)
```

Sympy [A] time = 17.5803, size = 248, normalized size = 5.77

$$\begin{cases} -\frac{10ae^2}{35d^3e^2\sqrt{d+ex}+105d^2e^3x\sqrt{d+ex}+105de^4x^2\sqrt{d+ex}+35e^5x^3\sqrt{d+ex}} - \frac{4cd^2}{35d^3e^2\sqrt{d+ex}+105d^2e^3x\sqrt{d+ex}+105de^4x^2\sqrt{d+ex}+35e^5x^3\sqrt{d+ex}} - \frac{}{35d^3e^2\sqrt{d+ex}+1} \\ \frac{cx^2}{2d^{\frac{7}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)/(e*x+d)**(11/2),x)
```

```
[Out] Piecewise((-10*a*e**2/(35*d**3*e**2*sqrt(d + e*x) + 105*d**2*e**3*x*sqrt(d
+ e*x) + 105*d*e**4*x**2*sqrt(d + e*x) + 35*e**5*x**3*sqrt(d + e*x)) - 4*c*
d**2/(35*d**3*e**2*sqrt(d + e*x) + 105*d**2*e**3*x*sqrt(d + e*x) + 105*d*e*
*4*x**2*sqrt(d + e*x) + 35*e**5*x**3*sqrt(d + e*x)) - 14*c*d*e*x/(35*d**3*e
**2*sqrt(d + e*x) + 105*d**2*e**3*x*sqrt(d + e*x) + 105*d*e**4*x**2*sqrt(d
+ e*x) + 35*e**5*x**3*sqrt(d + e*x)), Ne(e, 0)), (c*x**2/(2*d**(7/2)), True
))
```

Giac [A] time = 1.15494, size = 65, normalized size = 1.51

$$-\frac{2\left(7\,(xe+d)^2cd-5\,(xe+d)cd^2+5\,(xe+d)ae^2\right)e^{(-2)}}{35\,(xe+d)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(11/2),x, algorithm="gi
ac")
```

```
[Out] -2/35*(7*(x*e + d)^2*c*d - 5*(x*e + d)*c*d^2 + 5*(x*e + d)*a*e^2)*e^(-2)/(x
*e + d)^(9/2)
```

3.1984 $\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2\,dx$

Optimal. Leaf size=83

$$-\frac{4cd(d+ex)^{9/2}\left(cd^2-ae^2\right)}{9e^3}+\frac{2(d+ex)^{7/2}\left(cd^2-ae^2\right)^2}{7e^3}+\frac{2c^2d^2(d+ex)^{11/2}}{11e^3}$$

```
[Out] (2*(c*d^2 - a*e^2)^2*(d + e*x)^(7/2))/(7*e^3) - (4*c*d*(c*d^2 - a*e^2)*(d +
 e*x)^(9/2))/(9*e^3) + (2*c^2*d^2*(d + e*x)^(11/2))/(11*e^3)
```

Rubi [A] time = 0.0577019, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {626, 43}

$$-\frac{4cd(d+ex)^{9/2}\left(cd^2-ae^2\right)}{9e^3}+\frac{2(d+ex)^{7/2}\left(cd^2-ae^2\right)^2}{7e^3}+\frac{2c^2d^2(d+ex)^{11/2}}{11e^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] (2*(c*d^2 - a*e^2)^2*(d + e*x)^(7/2))/(7*e^3) - (4*c*d*(c*d^2 - a*e^2)*(d +
 e*x)^(9/2))/(9*e^3) + (2*c^2*d^2*(d + e*x)^(11/2))/(11*e^3)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2\,dx &= \int (ae+cdx)^2(d+ex)^{5/2}\,dx\\ &= \int \left(\frac{\left(-cd^2+ae^2\right)^2(d+ex)^{5/2}}{e^2}-\frac{2cd\left(cd^2-ae^2\right)(d+ex)^{7/2}}{e^2}+\frac{c^2d^2(d+\right.\\ &= \frac{2\left(cd^2-ae^2\right)^2(d+ex)^{7/2}}{7e^3}-\frac{4cd\left(cd^2-ae^2\right)(d+ex)^{9/2}}{9e^3}+\frac{2c^2d^2(d+ex}{11e^3}\end{aligned}$$

Mathematica [A] time = 0.0557433, size = 67, normalized size = 0.81

$$\frac{2(d+ex)^{7/2}\left(99a^2e^4-22acde^2(2d-7ex)+c^2d^2\left(8d^2-28dex+63e^2x^2\right)\right)}{693e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] (2*(d + e*x)^(7/2)*(99*a^2*e^4 - 22*a*c*d*e^2*(2*d - 7*e*x) + c^2*d^2*(8*d^
2 - 28*d*e*x + 63*e^2*x^2)))/(693*e^3)
```

Maple [A] time = 0.046, size = 73, normalized size = 0.9

$$\frac{126\,c^2d^2x^2e^2+308\,acde^3x-56\,c^2d^3ex+198\,a^2e^4-88\,acd^2e^2+16\,c^2d^4}{693\,e^3}(ex+d)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2*(e*x+d)^(1/2),x)
```

```
[Out] 2/693*(e*x+d)^(7/2)*(63*c^2*d^2*e^2*x^2+154*a*c*d*e^3*x-28*c^2*d^3*e*x+99*a
^2*e^4-44*a*c*d^2*e^2+8*c^2*d^4)/e^3
```

Maxima [A] time = 0.98391, size = 108, normalized size = 1.3

$$\frac{2\left(63\,(ex+d)^{\frac{11}{2}}c^2d^2-154\left(c^2d^3-acde^2\right)(ex+d)^{\frac{9}{2}}+99\left(c^2d^4-2\,acd^2e^2+a^2e^4\right)(ex+d)^{\frac{7}{2}}\right)}{693\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2*(e*x+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] 2/693*(63*(e*x + d)^(11/2)*c^2*d^2 - 154*(c^2*d^3 - a*c*d*e^2)*(e*x + d)^(9
/2) + 99*(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*(e*x + d)^(7/2))/e^3
```

Fricas [B] time = 1.92318, size = 398, normalized size = 4.8

$$\frac{2\left(63\,c^2d^2e^5x^5+8\,c^2d^7-44\,acd^5e^2+99\,a^2d^3e^4+7\left(23\,c^2d^3e^4+22\,acde^6\right)x^4+\left(113\,c^2d^4e^3+418\,acd^2e^5+99\,a^2e^7\right)x^3+3\right.}{693\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2*(e*x+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 2/693*(63*c^2*d^2*e^5*x^5 + 8*c^2*d^7 - 44*a*c*d^5*e^2 + 99*a^2*d^3*e^4 + 7
*(23*c^2*d^3*e^4 + 22*a*c*d*e^6)*x^4 + (113*c^2*d^4*e^3 + 418*a*c*d^2*e^5 +
 99*a^2*e^7)*x^3 + 3*(c^2*d^5*e^2 + 110*a*c*d^3*e^4 + 99*a^2*d*e^6)*x^2 - (
4*c^2*d^6*e - 22*a*c*d^4*e^3 - 297*a^2*d^2*e^5)*x)*sqrt(e*x + d)/e^3
```

Sympy [A] time = 3.82981, size = 97, normalized size = 1.17

$$\frac{2\left(\frac{c^2d^2(d+ex)^{\frac{11}{2}}}{11e^2}+\frac{(d+ex)^{\frac{9}{2}}(2acde^2-2c^2d^3)}{9e^2}+\frac{(d+ex)^{\frac{7}{2}}(a^2e^4-2acd^2e^2+c^2d^4)}{7e^2}\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2*(e*x+d)**(1/2),x)
```

```
[Out] 2*(c**2*d**2*(d + e*x)**(11/2)/(11*e**2) + (d + e*x)**(9/2)*(2*a*c*d*e**2 -
 2*c**2*d**3)/(9*e**2) + (d + e*x)**(7/2)*(a**2*e**4 - 2*a*c*d**2*e**2 + c*
*2*d**4)/(7*e**2))/e
```

Giac [B] time = 1.16618, size = 529, normalized size = 6.37

$$\frac{2}{3465}\left(33\left(15(xe+d)^{\frac{7}{2}}-42(xe+d)^{\frac{5}{2}}d+35(xe+d)^{\frac{3}{2}}d^2\right)c^2d^4e^{(-2)}+22\left(35(xe+d)^{\frac{9}{2}}-135(xe+d)^{\frac{7}{2}}d+189(xe+d)^{\frac{5}{2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2*(e*x+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] 2/3465*(33*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*
d^2)*c^2*d^4*e^(-2) + 22*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*
(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*c^2*d^3*e^(-2) + 1155*(x*e +
 d)^(3/2)*a^2*d^2*e^2 + 462*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a*c*d
^3 + (315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*
d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*c^2*d^2*e^(-2) +
 132*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*a
*c*d^2 + 462*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^2*d*e^2 + 22*(35*(
x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e
 + d)^(3/2)*d^3)*a*c*d + 33*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35
*(x*e + d)^(3/2)*d^2)*a^2*e^2)*e^(-1)
```

3.1985 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=83

$$-\frac{4cd(d+ex)^{7/2}\left(cd^2-ae^2\right)}{7e^3}+\frac{2(d+ex)^{5/2}\left(cd^2-ae^2\right)^2}{5e^3}+\frac{2c^2d^2(d+ex)^{9/2}}{9e^3}$$

```
[Out] (2*(c*d^2 - a*e^2)^2*(d + e*x)^(5/2))/(5*e^3) - (4*c*d*(c*d^2 - a*e^2)*(d +
 e*x)^(7/2))/(7*e^3) + (2*c^2*d^2*(d + e*x)^(9/2))/(9*e^3)
```

Rubi [A] time = 0.0389183, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {626, 43}

$$-\frac{4cd(d+ex)^{7/2}\left(cd^2-ae^2\right)}{7e^3}+\frac{2(d+ex)^{5/2}\left(cd^2-ae^2\right)^2}{5e^3}+\frac{2c^2d^2(d+ex)^{9/2}}{9e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/Sqrt[d + e*x],x]
```

```
[Out] (2*(c*d^2 - a*e^2)^2*(d + e*x)^(5/2))/(5*e^3) - (4*c*d*(c*d^2 - a*e^2)*(d +
 e*x)^(7/2))/(7*e^3) + (2*c^2*d^2*(d + e*x)^(9/2))/(9*e^3)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{\sqrt{d+ex}}\,dx &= \int (ae+cdx)^2(d+ex)^{3/2}\,dx\\
&= \int \left(\frac{\left(-cd^2+ae^2\right)^2(d+ex)^{3/2}}{e^2}-\frac{2cd\left(cd^2-ae^2\right)(d+ex)^{5/2}}{e^2}+\frac{c^2d^2(d+ex)^{7/2}}{e^2}\right)dx\\
&= \frac{2\left(cd^2-ae^2\right)^2(d+ex)^{5/2}}{5e^3}-\frac{4cd\left(cd^2-ae^2\right)(d+ex)^{7/2}}{7e^3}+\frac{2c^2d^2(d+ex)^{9/2}}{9e^3}
\end{aligned}$$

Mathematica [A] time = 0.043057, size = 67, normalized size = 0.81

$$\frac{2(d+ex)^{5/2}\left(63a^2e^4+18acde^2(5ex-2d)+c^2d^2\left(8d^2-20dex+35e^2x^2\right)\right)}{315e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/Sqrt[d + e*x],x]
```

```
[Out] (2*(d + e*x)^(5/2)*(63*a^2*e^4 + 18*a*c*d*e^2*(-2*d + 5*e*x) + c^2*d^2*(8*d
^2 - 20*d*e*x + 35*e^2*x^2)))/(315*e^3)
```

Maple [A] time = 0.046, size = 73, normalized size = 0.9

$$\frac{70\,c^2d^2x^2e^2+180\,acde^3x-40\,c^2d^3ex+126\,a^2e^4-72\,acd^2e^2+16\,c^2d^4}{315\,e^3}(ex+d)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(1/2),x)
```

```
[Out] 2/315*(e*x+d)^(5/2)*(35*c^2*d^2*e^2*x^2+90*a*c*d*e^3*x-20*c^2*d^3*e*x+63*a^
2*e^4-36*a*c*d^2*e^2+8*c^2*d^4)/e^3
```

Maxima [B] time = 1.00856, size = 378, normalized size = 4.55

$$2\left(315\sqrt{ex+d}a^2d^2e^2+42\left(\frac{\left(3(ex+d)^{\frac{5}{2}}-10(ex+d)^{\frac{3}{2}}d+15\sqrt{ex+d}d^2\right)cd}{e}+\frac{5\left(cd^2+ae^2\right)\left((ex+d)^{\frac{3}{2}}-3\sqrt{ex+d}d\right)}{e}\right)ade+\frac{\left(35(ex+d)^{\frac{9}{2}}-180(ex+d)^{\frac{7}{2}}d+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] 2/315*(315*sqrt(e*x + d)*a^2*d^2*e^2 + 42*((3*(e*x + d)^(5/2) - 10*(e*x + d
)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*c*d/e + 5*(c*d^2 + a*e^2)*((e*x + d)^(3/2
) - 3*sqrt(e*x + d)*d)/e)*a*d*e + (35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)
*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*
d^4)*c^2*d^2/e^2 + 18*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x +
 d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*(c*d^2 + a*e^2)*c*d/e^2 + 21*(c*d^2 +
 a*e^2)^2*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)
/e^2)/e
```

Fricas [B] time = 1.86642, size = 316, normalized size = 3.81

$$\frac{2\left(35\,c^2d^2e^4x^4+8\,c^2d^6-36\,acd^4e^2+63\,a^2d^2e^4+10\left(5\,c^2d^3e^3+9\,acde^5\right)x^3+3\left(c^2d^4e^2+48\,acd^2e^4+21\,a^2e^6\right)x^2-2\left(\right.\right.}{315\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 2/315*(35*c^2*d^2*e^4*x^4 + 8*c^2*d^6 - 36*a*c*d^4*e^2 + 63*a^2*d^2*e^4 + 1
0*(5*c^2*d^3*e^3 + 9*a*c*d*e^5)*x^3 + 3*(c^2*d^4*e^2 + 48*a*c*d^2*e^4 + 21*
```

```
a^2*e^6)*x^2 - 2*(2*c^2*d^5*e - 9*a*c*d^3*e^3 - 63*a^2*d*e^5)*x)*sqrt(e*x +
 d)/e^3
```

Sympy [A] time = 85.4955, size = 631, normalized size = 7.6

$$\begin{cases} -\frac{\frac{2a^2d^3e^2}{\sqrt{d+ex}}+6a^2d^2e^2\left(-\frac{d}{\sqrt{d+ex}}-\sqrt{d+ex}\right)+6a^2de^2\left(\frac{d^2}{\sqrt{d+ex}}+2d\sqrt{d+ex}-\frac{(d+ex)^{\frac{3}{2}}}{3}\right)+2a^2e^2\left(-\frac{d^3}{\sqrt{d+ex}}-3d^2\sqrt{d+ex}+d(d+ex)^{\frac{3}{2}}-\frac{(d+ex)^{\frac{5}{2}}}{5}\right)+4acd^4\left(-\frac{d}{\sqrt{d+ex}}-\sqrt{d+ex}\right)+12acd^3\left(\right.}{} \\ \frac{c^2d^{\frac{7}{2}}x^3}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d)**(1/2),x)
```

```
[Out] Piecewise((-(2*a**2*d**3*e**2/sqrt(d + e*x) + 6*a**2*d**2*e**2*(-d/sqrt(d +
 e*x) - sqrt(d + e*x)) + 6*a**2*d*e**2*(d**2/sqrt(d + e*x) + 2*d*sqrt(d + e
*x) - (d + e*x)**(3/2)/3) + 2*a**2*e**2*(-d**3/sqrt(d + e*x) - 3*d**2*sqrt(
d + e*x) + d*(d + e*x)**(3/2) - (d + e*x)**(5/2)/5) + 4*a*c*d**4*(-d/sqrt(d
 + e*x) - sqrt(d + e*x)) + 12*a*c*d**3*(d**2/sqrt(d + e*x) + 2*d*sqrt(d + e
*x) - (d + e*x)**(3/2)/3) + 12*a*c*d**2*(-d**3/sqrt(d + e*x) - 3*d**2*sqrt(
d + e*x) + d*(d + e*x)**(3/2) - (d + e*x)**(5/2)/5) + 4*a*c*d*(d**4/sqrt(d
+ e*x) + 4*d**3*sqrt(d + e*x) - 2*d**2*(d + e*x)**(3/2) + 4*d*(d + e*x)**(5
/2)/5 - (d + e*x)**(7/2)/7) + 2*c**2*d**5*(d**2/sqrt(d + e*x) + 2*d*sqrt(d
+ e*x) - (d + e*x)**(3/2)/3)/e**2 + 6*c**2*d**4*(-d**3/sqrt(d + e*x) - 3*d*
*2*sqrt(d + e*x) + d*(d + e*x)**(3/2) - (d + e*x)**(5/2)/5)/e**2 + 6*c**2*d
**3*(d**4/sqrt(d + e*x) + 4*d**3*sqrt(d + e*x) - 2*d**2*(d + e*x)**(3/2) +
4*d*(d + e*x)**(5/2)/5 - (d + e*x)**(7/2)/7)/e**2 + 2*c**2*d**2*(-d**5/sqrt
(d + e*x) - 5*d**4*sqrt(d + e*x) + 10*d**3*(d + e*x)**(3/2)/3 - 2*d**2*(d +
 e*x)**(5/2) + 5*d*(d + e*x)**(7/2)/7 - (d + e*x)**(9/2)/9)/e**2)/e, Ne(e,
0)), (c**2*d**(7/2)*x**3/3, True))
```

Giac [B] time = 1.15654, size = 524, normalized size = 6.31

$$\frac{2}{315}\left(21\left(3(xe+d)^{\frac{5}{2}}-10(xe+d)^{\frac{3}{2}}d+15\sqrt{xe+d}d^2\right)c^2d^4e^{(-2)}+18\left(5(xe+d)^{\frac{7}{2}}-21(xe+d)^{\frac{5}{2}}d+35(xe+d)^{\frac{3}{2}}d^2-35\sqrt{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] 2/315*(21*(3*(x*e + d)^(5/2) - 10*(x*e + d)^(3/2)*d + 15*sqrt(x*e + d)*d^2)
*c^2*d^4*e^(-2) + 18*(5*(x*e + d)^(7/2) - 21*(x*e + d)^(5/2)*d + 35*(x*e +
d)^(3/2)*d^2 - 35*sqrt(x*e + d)*d^3)*c^2*d^3*e^(-2) + 210*((x*e + d)^(3/2)
- 3*sqrt(x*e + d)*d)*a*c*d^3 + (35*(x*e + d)^(9/2) - 180*(x*e + d)^(7/2)*d
+ 378*(x*e + d)^(5/2)*d^2 - 420*(x*e + d)^(3/2)*d^3 + 315*sqrt(x*e + d)*d^4
)*c^2*d^2*e^(-2) + 315*sqrt(x*e + d)*a^2*d^2*e^2 + 84*(3*(x*e + d)^(5/2) -
10*(x*e + d)^(3/2)*d + 15*sqrt(x*e + d)*d^2)*a*c*d^2 + 210*((x*e + d)^(3/2)
 - 3*sqrt(x*e + d)*d)*a^2*d*e^2 + 18*(5*(x*e + d)^(7/2) - 21*(x*e + d)^(5/2
)*d + 35*(x*e + d)^(3/2)*d^2 - 35*sqrt(x*e + d)*d^3)*a*c*d + 21*(3*(x*e + d
)^(5/2) - 10*(x*e + d)^(3/2)*d + 15*sqrt(x*e + d)*d^2)*a^2*e^2)*e^(-1)
```

3.1986 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^{3/2}} dx$

Optimal. Leaf size=83

$$-\frac{4cd(d+ex)^{5/2}\left(cd^2-ae^2\right)}{5e^3}+\frac{2(d+ex)^{3/2}\left(cd^2-ae^2\right)^2}{3e^3}+\frac{2c^2d^2(d+ex)^{7/2}}{7e^3}$$

```
[Out] (2*(c*d^2 - a*e^2)^2*(d + e*x)^(3/2))/(3*e^3) - (4*c*d*(c*d^2 - a*e^2)*(d +
 e*x)^(5/2))/(5*e^3) + (2*c^2*d^2*(d + e*x)^(7/2))/(7*e^3)
```

Rubi [A] time = 0.0391043, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {626, 43}

$$-\frac{4cd(d+ex)^{5/2}\left(cd^2-ae^2\right)}{5e^3}+\frac{2(d+ex)^{3/2}\left(cd^2-ae^2\right)^2}{3e^3}+\frac{2c^2d^2(d+ex)^{7/2}}{7e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^(3/2),x]
```

```
[Out] (2*(c*d^2 - a*e^2)^2*(d + e*x)^(3/2))/(3*e^3) - (4*c*d*(c*d^2 - a*e^2)*(d +
 e*x)^(5/2))/(5*e^3) + (2*c^2*d^2*(d + e*x)^(7/2))/(7*e^3)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^{3/2}} dx &= \int (ae+cdx)^2\sqrt{d+ex}\, dx\\
&= \int \left(\frac{\left(-cd^2+ae^2\right)^2\sqrt{d+ex}}{e^2}-\frac{2cd\left(cd^2-ae^2\right)(d+ex)^{3/2}}{e^2}+\frac{c^2d^2(d+ex)^{5/2}}{e^2}\right) dx\\
&= \frac{2\left(cd^2-ae^2\right)^2(d+ex)^{3/2}}{3e^3}-\frac{4cd\left(cd^2-ae^2\right)(d+ex)^{5/2}}{5e^3}+\frac{2c^2d^2(d+ex)^{7/2}}{7e^3}
\end{aligned}$$

Mathematica [A] time = 0.0396866, size = 67, normalized size = 0.81

$$\frac{2(d+ex)^{3/2}\left(35a^2e^4+14acde^2(3ex-2d)+c^2d^2\left(8d^2-12dex+15e^2x^2\right)\right)}{105e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^(3/2),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(35*a^2*e^4 + 14*a*c*d*e^2*(-2*d + 3*e*x) + c^2*d^2*(8*d
^2 - 12*d*e*x + 15*e^2*x^2)))/(105*e^3)
```

Maple [A] time = 0.043, size = 73, normalized size = 0.9

$$\frac{30\,c^2d^2x^2e^2+84\,acde^3x-24\,c^2d^3ex+70\,a^2e^4-56\,acd^2e^2+16\,c^2d^4}{105\,e^3}\,(ex+d)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(3/2),x)
```

```
[Out] 2/105*(e*x+d)^(3/2)*(15*c^2*d^2*e^2*x^2+42*a*c*d*e^3*x-12*c^2*d^3*e*x+35*a^
2*e^4-28*a*c*d^2*e^2+8*c^2*d^4)/e^3
```

Maxima [A] time = 1.03428, size = 108, normalized size = 1.3

$$\frac{2\left(15\,(ex+d)^{\frac{7}{2}}c^2d^2-42\left(c^2d^3-acde^2\right)(ex+d)^{\frac{5}{2}}+35\left(c^2d^4-2\,acd^2e^2+a^2e^4\right)(ex+d)^{\frac{3}{2}}\right)}{105\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(3/2),x, algorithm="m
axima")
```

```
[Out] 2/105*(15*(e*x + d)^(7/2)*c^2*d^2 - 42*(c^2*d^3 - a*c*d*e^2)*(e*x + d)^(5/2
) + 35*(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*(e*x + d)^(3/2))/e^3
```

Fricas [A] time = 1.92341, size = 236, normalized size = 2.84

$$\frac{2\left(15\,c^2d^2e^3x^3+8\,c^2d^5-28\,acd^3e^2+35\,a^2de^4+3\left(c^2d^3e^2+14\,acde^4\right)x^2-\left(4\,c^2d^4e-14\,acd^2e^3-35\,a^2e^5\right)x\right)\sqrt{ex+d}}{105\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 2/105*(15*c^2*d^2*e^3*x^3 + 8*c^2*d^5 - 28*a*c*d^3*e^2 + 35*a^2*d*e^4 + 3*(
c^2*d^3*e^2 + 14*a*c*d*e^4)*x^2 - (4*c^2*d^4*e - 14*a*c*d^2*e^3 - 35*a^2*e^
5)*x)*sqrt(e*x + d)/e^3
```

Sympy [A] time = 50.8447, size = 411, normalized size = 4.95

$$\begin{cases} -\frac{\frac{2a^2d^2e^2}{\sqrt{d+ex}}+4a^2de^2\left(-\frac{d}{\sqrt{d+ex}}-\sqrt{d+ex}\right)+2a^2e^2\left(\frac{d^2}{\sqrt{d+ex}}+2d\sqrt{d+ex}-\frac{(d+ex)^{\frac{3}{2}}}{3}\right)+4acd^3\left(-\frac{d}{\sqrt{d+ex}}-\sqrt{d+ex}\right)+8acd^2\left(\frac{d^2}{\sqrt{d+ex}}+2d\sqrt{d+ex}-\frac{(d+ex)^{\frac{3}{2}}}{3}\right)+4acd\left(-\frac{d^3}{\sqrt{d+ex}}-3d^2\sqrt{\ }\right.}{} \\ \frac{c^2d^{\frac{5}{2}}x^3}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d)**(3/2),x)
```

```
[Out] Piecewise((-(2*a**2*d**2*e**2/sqrt(d + e*x) + 4*a**2*d*e**2*(-d/sqrt(d + e*
x) - sqrt(d + e*x)) + 2*a**2*e**2*(d**2/sqrt(d + e*x) + 2*d*sqrt(d + e*x) -
 (d + e*x)**(3/2)/3) + 4*a*c*d**3*(-d/sqrt(d + e*x) - sqrt(d + e*x)) + 8*a*
c*d**2*(d**2/sqrt(d + e*x) + 2*d*sqrt(d + e*x) - (d + e*x)**(3/2)/3) + 4*a*
c*d*(-d**3/sqrt(d + e*x) - 3*d**2*sqrt(d + e*x) + d*(d + e*x)**(3/2) - (d +
 e*x)**(5/2)/5) + 2*c**2*d**4*(d**2/sqrt(d + e*x) + 2*d*sqrt(d + e*x) - (d
+ e*x)**(3/2)/3)/e**2 + 4*c**2*d**3*(-d**3/sqrt(d + e*x) - 3*d**2*sqrt(d +
e*x) + d*(d + e*x)**(3/2) - (d + e*x)**(5/2)/5)/e**2 + 2*c**2*d**2*(d**4/sq
rt(d + e*x) + 4*d**3*sqrt(d + e*x) - 2*d**2*(d + e*x)**(3/2) + 4*d*(d + e*x
)**(5/2)/5 - (d + e*x)**(7/2)/7)/e**2)/e, Ne(e, 0)), (c**2*d**(5/2)*x**3/3,
 True))
```

Giac [A] time = 1.20345, size = 143, normalized size = 1.72

$$\frac{2}{105}\left(15\,(xe+d)^{\frac{7}{2}}c^2d^2e^{18}-42\,(xe+d)^{\frac{5}{2}}c^2d^3e^{18}+35\,(xe+d)^{\frac{3}{2}}c^2d^4e^{18}+42\,(xe+d)^{\frac{5}{2}}acde^{20}-70\,(xe+d)^{\frac{3}{2}}acd^2e^{20}+35\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(3/2),x, algorithm="g
iac")
```

```
[Out] 2/105*(15*(x*e + d)^(7/2)*c^2*d^2*e^18 - 42*(x*e + d)^(5/2)*c^2*d^3*e^18 +
35*(x*e + d)^(3/2)*c^2*d^4*e^18 + 42*(x*e + d)^(5/2)*a*c*d*e^20 - 70*(x*e +
 d)^(3/2)*a*c*d^2*e^20 + 35*(x*e + d)^(3/2)*a^2*e^22)*e^(-21)
```

3.1987 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=81

$$-\frac{4cd(d+ex)^{3/2}\left(cd^2-ae^2\right)}{3e^3}+\frac{2\sqrt{d+ex}\left(cd^2-ae^2\right)^2}{e^3}+\frac{2c^2d^2(d+ex)^{5/2}}{5e^3}$$

```
[Out] (2*(c*d^2 - a*e^2)^2*Sqrt[d + e*x])/e^3 - (4*c*d*(c*d^2 - a*e^2)*(d + e*x)^
(3/2))/(3*e^3) + (2*c^2*d^2*(d + e*x)^(5/2))/(5*e^3)
```

Rubi [A] time = 0.0392762, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {626, 43}

$$-\frac{4cd(d+ex)^{3/2}\left(cd^2-ae^2\right)}{3e^3}+\frac{2\sqrt{d+ex}\left(cd^2-ae^2\right)^2}{e^3}+\frac{2c^2d^2(d+ex)^{5/2}}{5e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^(5/2),x]
```

```
[Out] (2*(c*d^2 - a*e^2)^2*Sqrt[d + e*x])/e^3 - (4*c*d*(c*d^2 - a*e^2)*(d + e*x)^
(3/2))/(3*e^3) + (2*c^2*d^2*(d + e*x)^(5/2))/(5*e^3)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^{5/2}}\,dx &= \int \frac{(ae+cdx)^2}{\sqrt{d+ex}}\,dx\\
&= \int \left(\frac{\left(-cd^2+ae^2\right)^2}{e^2\sqrt{d+ex}}-\frac{2cd\left(cd^2-ae^2\right)\sqrt{d+ex}}{e^2}+\frac{c^2d^2(d+ex)^{3/2}}{e^2}\right)dx\\
&= \frac{2\left(cd^2-ae^2\right)^2\sqrt{d+ex}}{e^3}-\frac{4cd\left(cd^2-ae^2\right)(d+ex)^{3/2}}{3e^3}+\frac{2c^2d^2(d+ex)^{5/2}}{5e^3}
\end{aligned}$$

Mathematica [A] time = 0.0392344, size = 66, normalized size = 0.81

$$\frac{2\sqrt{d+ex}\left(15a^2e^4+10acde^2(ex-2d)+c^2d^2\left(8d^2-4dex+3e^2x^2\right)\right)}{15e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^(5/2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(15*a^2*e^4 + 10*a*c*d*e^2*(-2*d + e*x) + c^2*d^2*(8*d^2 -
 4*d*e*x + 3*e^2*x^2)))/(15*e^3)
```

Maple [A] time = 0.045, size = 73, normalized size = 0.9

$$\frac{6\,c^{2}d^{2}x^{2}e^{2}+20\,acde^{3}x-8\,c^{2}d^{3}ex+30\,a^{2}e^{4}-40\,acd^{2}e^{2}+16\,c^{2}d^{4}}{15\,e^{3}}\sqrt{ex+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(5/2),x)
```

```
[Out] 2/15*(e*x+d)^(1/2)*(3*c^2*d^2*e^2*x^2+10*a*c*d*e^3*x-4*c^2*d^3*e*x+15*a^2*e
^4-20*a*c*d^2*e^2+8*c^2*d^4)/e^3
```

Maxima [A] time = 0.988448, size = 108, normalized size = 1.33

$$\frac{2\left(3\,(ex+d)^{\frac{5}{2}}c^{2}d^{2}-10\left(c^{2}d^{3}-acde^{2}\right)(ex+d)^{\frac{3}{2}}+15\left(c^{2}d^{4}-2\,acd^{2}e^{2}+a^{2}e^{4}\right)\sqrt{ex+d}\right)}{15\,e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(5/2),x, algorithm="m
axima")
```

```
[Out] 2/15*(3*(e*x + d)^(5/2)*c^2*d^2 - 10*(c^2*d^3 - a*c*d*e^2)*(e*x + d)^(3/2)
+ 15*(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(e*x + d))/e^3
```

Fricas [A] time = 1.85779, size = 162, normalized size = 2.

$$\frac{2\left(3\,c^{2}d^{2}e^{2}x^{2}+8\,c^{2}d^{4}-20\,acd^{2}e^{2}+15\,a^{2}e^{4}-2\left(2\,c^{2}d^{3}e-5\,acde^{3}\right)x\right)\sqrt{ex+d}}{15\,e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(5/2),x, algorithm="f
ricas")
```

```
[Out] 2/15*(3*c^2*d^2*e^2*x^2 + 8*c^2*d^4 - 20*a*c*d^2*e^2 + 15*a^2*e^4 - 2*(2*c^
2*d^3*e - 5*a*c*d*e^3)*x)*sqrt(e*x + d)/e^3
```

Sympy [A] time = 40.0993, size = 236, normalized size = 2.91

$$\begin{cases} -\frac{\frac{2a^2de^2}{\sqrt{d+ex}}+2a^2e^2\left(-\frac{d}{\sqrt{d+ex}}-\sqrt{d+ex}\right)+4acd^2\left(-\frac{d}{\sqrt{d+ex}}-\sqrt{d+ex}\right)+4acd\left(\frac{d^2}{\sqrt{d+ex}}+2d\sqrt{d+ex}-\frac{(d+ex)^{\frac{3}{2}}}{3}\right)+\frac{2c^2d^3\left(\frac{d^2}{\sqrt{d+ex}}+2d\sqrt{d+ex}-\frac{(d+ex)^{\frac{3}{2}}}{3}\right)}{e^2}+\frac{2c^2d^2\left(-\frac{d^3}{\sqrt{d+ex}}-3d^2\sqrt{d+ex}+d(d+e\right.}{e^2}}{e} \\ \frac{c^2d^{\frac{3}{2}}x^3}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d)**(5/2),x)
```

```
[Out] Piecewise((-(2*a**2*d*e**2/sqrt(d + e*x) + 2*a**2*e**2*(-d/sqrt(d + e*x) -
sqrt(d + e*x)) + 4*a*c*d**2*(-d/sqrt(d + e*x) - sqrt(d + e*x)) + 4*a*c*d*(d
**2/sqrt(d + e*x) + 2*d*sqrt(d + e*x) - (d + e*x)**(3/2)/3) + 2*c**2*d**3*(
d**2/sqrt(d + e*x) + 2*d*sqrt(d + e*x) - (d + e*x)**(3/2)/3)/e**2 + 2*c**2*
d**2*(-d**3/sqrt(d + e*x) - 3*d**2*sqrt(d + e*x) + d*(d + e*x)**(3/2) - (d
+ e*x)**(5/2)/5)/e**2)/e, Ne(e, 0)), (c**2*d**(3/2)*x**3/3, True))
```

Giac [A] time = 1.18047, size = 143, normalized size = 1.77

$$\frac{2}{15}\left(3\,(xe+d)^{\frac{5}{2}}c^2d^2e^{12}-10\,(xe+d)^{\frac{3}{2}}c^2d^3e^{12}+15\,\sqrt{xe+d}c^2d^4e^{12}+10\,(xe+d)^{\frac{3}{2}}acde^{14}-30\,\sqrt{xe+d}acd^2e^{14}+15\,\sqrt{xe+d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(5/2),x, algorithm="g
iac")
```

```
[Out] 2/15*(3*(x*e + d)^(5/2)*c^2*d^2*e^12 - 10*(x*e + d)^(3/2)*c^2*d^3*e^12 + 15
*sqrt(x*e + d)*c^2*d^4*e^12 + 10*(x*e + d)^(3/2)*a*c*d*e^14 - 30*sqrt(x*e +
 d)*a*c*d^2*e^14 + 15*sqrt(x*e + d)*a^2*e^16)*e^(-15)
```

3.1988 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^{7/2}} dx$

Optimal. Leaf size=79

$$-\frac{4cd\sqrt{d+ex}\left(cd^2-ae^2\right)}{e^3}-\frac{2\left(cd^2-ae^2\right)^2}{e^3\sqrt{d+ex}}+\frac{2c^2d^2(d+ex)^{3/2}}{3e^3}$$

```
[Out] (-2*(c*d^2 - a*e^2)^2)/(e^3*Sqrt[d + e*x]) - (4*c*d*(c*d^2 - a*e^2)*Sqrt[d
+ e*x])/e^3 + (2*c^2*d^2*(d + e*x)^(3/2))/(3*e^3)
```

Rubi [A] time = 0.0410338, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {626, 43}

$$-\frac{4cd\sqrt{d+ex}\left(cd^2-ae^2\right)}{e^3}-\frac{2\left(cd^2-ae^2\right)^2}{e^3\sqrt{d+ex}}+\frac{2c^2d^2(d+ex)^{3/2}}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^(7/2),x]
```

```
[Out] (-2*(c*d^2 - a*e^2)^2)/(e^3*Sqrt[d + e*x]) - (4*c*d*(c*d^2 - a*e^2)*Sqrt[d
+ e*x])/e^3 + (2*c^2*d^2*(d + e*x)^(3/2))/(3*e^3)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^{7/2}} dx &= \int \frac{(ae+cdx)^2}{(d+ex)^{3/2}} dx \\
&= \int \left(\frac{\left(-cd^2+ae^2\right)^2}{e^2(d+ex)^{3/2}}-\frac{2cd\left(cd^2-ae^2\right)}{e^2\sqrt{d+ex}}+\frac{c^2d^2\sqrt{d+ex}}{e^2}\right) dx \\
&= -\frac{2\left(cd^2-ae^2\right)^2}{e^3\sqrt{d+ex}}-\frac{4cd\left(cd^2-ae^2\right)\sqrt{d+ex}}{e^3}+\frac{2c^2d^2(d+ex)^{3/2}}{3e^3}
\end{aligned}$$

Mathematica [A] time = 0.0354436, size = 65, normalized size = 0.82

$$\frac{2\left(-3a^2e^4+6acde^2(2d+ex)+c^2d^2\left(-8d^2-4dex+e^2x^2\right)\right)}{3e^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^(7/2),x]
```

```
[Out] (2*(-3*a^2*e^4 + 6*a*c*d*e^2*(2*d + e*x) + c^2*d^2*(-8*d^2 - 4*d*e*x + e^2*
x^2)))/(3*e^3*Sqrt[d + e*x])
```

Maple [A] time = 0.044, size = 73, normalized size = 0.9

$$-\frac{-2\,c^2d^2x^2e^2-12\,acde^3x+8\,c^2d^3ex+6\,a^2e^4-24\,acd^2e^2+16\,c^2d^4}{3\,e^3}\frac{1}{\sqrt{ex+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(7/2),x)
```

```
[Out] -2/3/(e*x+d)^(1/2)*(-c^2*d^2*e^2*x^2-6*a*c*d*e^3*x+4*c^2*d^3*e*x+3*a^2*e^4-
12*a*c*d^2*e^2+8*c^2*d^4)/e^3
```

Maxima [A] time = 1.02826, size = 117, normalized size = 1.48

$$\frac{2\left(\frac{(ex+d)^{\frac{3}{2}}c^2d^2-6\left(c^2d^3-acde^2\right)\sqrt{ex+d}}{e^2}-\frac{3\left(c^2d^4-2\,acd^2e^2+a^2e^4\right)}{\sqrt{ex+d}e^2}\right)}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(7/2),x, algorithm="m
axima")
```

```
[Out] 2/3*(((e*x + d)^(3/2)*c^2*d^2 - 6*(c^2*d^3 - a*c*d*e^2)*sqrt(e*x + d))/e^2
 - 3*(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(sqrt(e*x + d)*e^2))/e
```

Fricas [A] time = 1.83272, size = 173, normalized size = 2.19

$$\frac{2\left(c^2d^2e^2x^2-8\,c^2d^4+12\,acd^2e^2-3\,a^2e^4-2\left(2\,c^2d^3e-3\,acde^3\right)x\right)\sqrt{ex+d}}{3\left(e^4x+de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(7/2),x, algorithm="f
ricas")
```

```
[Out] 2/3*(c^2*d^2*e^2*x^2 - 8*c^2*d^4 + 12*a*c*d^2*e^2 - 3*a^2*e^4 - 2*(2*c^2*d^
3*e - 3*a*c*d*e^3)*x)*sqrt(e*x + d)/(e^4*x + d*e^3)
```

Sympy [A] time = 5.87905, size = 133, normalized size = 1.68

$$\begin{cases} -\frac{2a^2e}{\sqrt{d+ex}} + \frac{8acd^2}{e\sqrt{d+ex}} + \frac{4acdx}{\sqrt{d+ex}} - \frac{16c^2d^4}{3e^3\sqrt{d+ex}} - \frac{8c^2d^3x}{3e^2\sqrt{d+ex}} + \frac{2c^2d^2x^2}{3e\sqrt{d+ex}} & \text{for } e \neq 0 \\ \frac{c^2\sqrt{d}x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d)**(7/2),x)
```

```
[Out] Piecewise((-2*a**2*e/sqrt(d + e*x) + 8*a*c*d**2/(e*sqrt(d + e*x)) + 4*a*c*d
*x/sqrt(d + e*x) - 16*c**2*d**4/(3*e**3*sqrt(d + e*x)) - 8*c**2*d**3*x/(3*e
**2*sqrt(d + e*x)) + 2*c**2*d**2*x**2/(3*e*sqrt(d + e*x)), Ne(e, 0)), (c**2
*sqrt(d)*x**3/3, True))
```

Giac [A] time = 1.13009, size = 155, normalized size = 1.96

$$\frac{2}{3}\left((xe+d)^{\frac{3}{2}}c^2d^2e^6 - 6\sqrt{xe+d}c^2d^3e^6 + 6\sqrt{xe+d}acde^8\right)e^{(-9)} - \frac{2\left((xe+d)^2c^2d^4 - 2(xe+d)^2acd^2e^2 + (xe+d)^2a^2e^4\right)e^{(}}{(xe+d)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(7/2),x, algorithm="g
iac")
```

```
[Out] 2/3*((x*e + d)^(3/2)*c^2*d^2*e^6 - 6*sqrt(x*e + d)*c^2*d^3*e^6 + 6*sqrt(x*e
 + d)*a*c*d*e^8)*e^(-9) - 2*((x*e + d)^2*c^2*d^4 - 2*(x*e + d)^2*a*c*d^2*e^
2 + (x*e + d)^2*a^2*e^4)*e^(-3)/(x*e + d)^(5/2)
```

3.1989 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^2}{(d+ex)^{9/2}}\,dx$

Optimal. Leaf size=79

$$\frac{4cd\left(cd^2-ae^2\right)}{e^3\sqrt{d+ex}}-\frac{2\left(cd^2-ae^2\right)^2}{3e^3(d+ex)^{3/2}}+\frac{2c^2d^2\sqrt{d+ex}}{e^3}$$

```
[Out] (-2*(c*d^2 - a*e^2)^2)/(3*e^3*(d + e*x)^(3/2)) + (4*c*d*(c*d^2 - a*e^2))/(e
^3*Sqrt[d + e*x]) + (2*c^2*d^2*Sqrt[d + e*x])/e^3
```

Rubi [A] time = 0.0394922, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {626, 43}

$$\frac{4cd\left(cd^2-ae^2\right)}{e^3\sqrt{d+ex}}-\frac{2\left(cd^2-ae^2\right)^2}{3e^3(d+ex)^{3/2}}+\frac{2c^2d^2\sqrt{d+ex}}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^(9/2),x]
```

```
[Out] (-2*(c*d^2 - a*e^2)^2)/(3*e^3*(d + e*x)^(3/2)) + (4*c*d*(c*d^2 - a*e^2))/(e
^3*Sqrt[d + e*x]) + (2*c^2*d^2*Sqrt[d + e*x])/e^3
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^{9/2}}\,dx &= \int \frac{(ae+cdx)^2}{(d+ex)^{5/2}}\,dx\\
&= \int \left(\frac{\left(-cd^2+ae^2\right)^2}{e^2(d+ex)^{5/2}}-\frac{2cd\left(cd^2-ae^2\right)}{e^2(d+ex)^{3/2}}+\frac{c^2d^2}{e^2\sqrt{d+ex}}\right)dx\\
&= -\frac{2\left(cd^2-ae^2\right)^2}{3e^3(d+ex)^{3/2}}+\frac{4cd\left(cd^2-ae^2\right)}{e^3\sqrt{d+ex}}+\frac{2c^2d^2\sqrt{d+ex}}{e^3}
\end{aligned}$$

Mathematica [A] time = 0.0391809, size = 68, normalized size = 0.86

$$\frac{-2a^2e^4-4acde^2(2d+3ex)+2c^2d^2\left(8d^2+12dex+3e^2x^2\right)}{3e^3(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^(9/2),x]
```

```
[Out] (-2*a^2*e^4 - 4*a*c*d*e^2*(2*d + 3*e*x) + 2*c^2*d^2*(8*d^2 + 12*d*e*x + 3*e
^2*x^2))/(3*e^3*(d + e*x)^(3/2))
```

Maple [A] time = 0.044, size = 72, normalized size = 0.9

$$-\frac{-6\,c^2d^2x^2e^2+12\,acde^3x-24\,c^2d^3ex+2\,a^2e^4+8\,acd^2e^2-16\,c^2d^4}{3\,e^3}(ex+d)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(9/2),x)
```

```
[Out] -2/3/(e*x+d)^(3/2)*(-3*c^2*d^2*e^2*x^2+6*a*c*d*e^3*x-12*c^2*d^3*e*x+a^2*e^4
+4*a*c*d^2*e^2-8*c^2*d^4)/e^3
```

Maxima [A] time = 1.0255, size = 113, normalized size = 1.43

$$\frac{2\left(\frac{3\sqrt{ex+d}c^2d^2}{e^2}-\frac{c^2d^4-2\,acd^2e^2+a^2e^4-6\left(c^2d^3-acde^2\right)(ex+d)}{(ex+d)^{\frac{3}{2}}e^2}\right)}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(9/2),x, algorithm="m
axima")
```

```
[Out] 2/3*(3*sqrt(e*x + d)*c^2*d^2/e^2 - (c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4 - 6*(
c^2*d^3 - a*c*d*e^2)*(e*x + d))/((e*x + d)^(3/2)*e^2))/e
```

Fricas [A] time = 1.82256, size = 190, normalized size = 2.41

$$\frac{2\left(3\,c^2d^2e^2x^2+8\,c^2d^4-4\,acd^2e^2-a^2e^4+6\left(2\,c^2d^3e-acde^3\right)x\right)\sqrt{ex+d}}{3\left(e^5x^2+2\,de^4x+d^2e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(9/2),x, algorithm="f
ricas")
```

```
[Out] 2/3*(3*c^2*d^2*e^2*x^2 + 8*c^2*d^4 - 4*a*c*d^2*e^2 - a^2*e^4 + 6*(2*c^2*d^3
*e - a*c*d*e^3)*x)*sqrt(e*x + d)/(e^5*x^2 + 2*d*e^4*x + d^2*e^3)
```

Sympy [A] time = 11.5154, size = 264, normalized size = 3.34

$$\begin{cases} -\frac{2a^2e^4}{3de^3\sqrt{d+ex}+3e^4x\sqrt{d+ex}}-\frac{8acd^2e^2}{3de^3\sqrt{d+ex}+3e^4x\sqrt{d+ex}}-\frac{12acde^3x}{3de^3\sqrt{d+ex}+3e^4x\sqrt{d+ex}}+\frac{16c^2d^4}{3de^3\sqrt{d+ex}+3e^4x\sqrt{d+ex}}+\frac{24c^2d^3ex}{3de^3\sqrt{d+ex}+3e^4x\sqrt{d+ex}}+\frac{6c^2}{3de^3\sqrt{d+e}} \\ \frac{c^2x^3}{3\sqrt{d}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d)**(9/2),x)
```

```
[Out] Piecewise((-2*a**2*e**4/(3*d*e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)) -
 8*a*c*d**2*e**2/(3*d*e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)) - 12*a*c
*d*e**3*x/(3*d*e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)) + 16*c**2*d**4/
(3*d*e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)) + 24*c**2*d**3*e*x/(3*d*e
**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)) + 6*c**2*d**2*e**2*x**2/(3*d*e*
*3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)), Ne(e, 0)), (c**2*x**3/(3*sqrt(d
)), True))
```

Giac [A] time = 1.16315, size = 150, normalized size = 1.9

$$2\sqrt{xe+d}c^2d^2e^{(-3)} + \frac{2\left(6(xe+d)^3c^2d^3 - (xe+d)^2c^2d^4 - 6(xe+d)^3acde^2 + 2(xe+d)^2acd^2e^2 - (xe+d)^2a^2e^4\right)e^{(-3)}}{3(xe+d)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(9/2),x, algorithm="g
iac")
```

```
[Out] 2*sqrt(x*e + d)*c^2*d^2*e^(-3) + 2/3*(6*(x*e + d)^3*c^2*d^3 - (x*e + d)^2*c
^2*d^4 - 6*(x*e + d)^3*a*c*d*e^2 + 2*(x*e + d)^2*a*c*d^2*e^2 - (x*e + d)^2*
a^2*e^4)*e^(-3)/(x*e + d)^(7/2)
```

3.1990 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^{11/2}}\,dx$

Optimal. Leaf size=81

$$\frac{4cd\left(cd^2-ae^2\right)}{3e^3(d+ex)^{3/2}}-\frac{2\left(cd^2-ae^2\right)^2}{5e^3(d+ex)^{5/2}}-\frac{2c^2d^2}{e^3\sqrt{d+ex}}$$

```
[Out] (-2*(c*d^2 - a*e^2)^2)/(5*e^3*(d + e*x)^(5/2)) + (4*c*d*(c*d^2 - a*e^2))/(3
*e^3*(d + e*x)^(3/2)) - (2*c^2*d^2)/(e^3*Sqrt[d + e*x])
```

Rubi [A] time = 0.0395864, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {626, 43}

$$\frac{4cd\left(cd^2-ae^2\right)}{3e^3(d+ex)^{3/2}}-\frac{2\left(cd^2-ae^2\right)^2}{5e^3(d+ex)^{5/2}}-\frac{2c^2d^2}{e^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^(11/2),x]
```

```
[Out] (-2*(c*d^2 - a*e^2)^2)/(5*e^3*(d + e*x)^(5/2)) + (4*c*d*(c*d^2 - a*e^2))/(3
*e^3*(d + e*x)^(3/2)) - (2*c^2*d^2)/(e^3*Sqrt[d + e*x])
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^{11/2}}\,dx &= \int \frac{(ae+cdx)^2}{(d+ex)^{7/2}}\,dx\\ &= \int \left(\frac{\left(-cd^2+ae^2\right)^2}{e^2(d+ex)^{7/2}}-\frac{2cd\left(cd^2-ae^2\right)}{e^2(d+ex)^{5/2}}+\frac{c^2d^2}{e^2(d+ex)^{3/2}}\right)dx\\ &= -\frac{2\left(cd^2-ae^2\right)^2}{5e^3(d+ex)^{5/2}}+\frac{4cd\left(cd^2-ae^2\right)}{3e^3(d+ex)^{3/2}}-\frac{2c^2d^2}{e^3\sqrt{d+ex}}\end{aligned}$$

Mathematica [A] time = 0.0370368, size = 67, normalized size = 0.83

$$-\frac{2\left(3a^2e^4+2acde^2(2d+5ex)+c^2d^2\left(8d^2+20dex+15e^2x^2\right)\right)}{15e^3(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^(11/2),x]
```

```
[Out] (-2*(3*a^2*e^4 + 2*a*c*d*e^2*(2*d + 5*e*x) + c^2*d^2*(8*d^2 + 20*d*e*x + 15
*e^2*x^2)))/(15*e^3*(d + e*x)^(5/2))
```

Maple [A] time = 0.044, size = 73, normalized size = 0.9

$$-\frac{30\,c^{2}d^{2}x^{2}e^{2}+20\,acde^{3}x+40\,c^{2}d^{3}ex+6\,a^{2}e^{4}+8\,acd^{2}e^{2}+16\,c^{2}d^{4}}{15\,e^{3}}\left(ex+d\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(11/2),x)
```

```
[Out] -2/15/(e*x+d)^(5/2)*(15*c^2*d^2*e^2*x^2+10*a*c*d*e^3*x+20*c^2*d^3*e*x+3*a^2
*e^4+4*a*c*d^2*e^2+8*c^2*d^4)/e^3
```

Maxima [A] time = 1.02547, size = 104, normalized size = 1.28

$$-\frac{2\left(15\left(ex+d\right)^{2}c^{2}d^{2}+3\,c^{2}d^{4}-6\,acd^{2}e^{2}+3\,a^{2}e^{4}-10\left(c^{2}d^{3}-acde^{2}\right)\left(ex+d\right)\right)}{15\left(ex+d\right)^{\frac{5}{2}}e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(11/2),x, algorithm="
maxima")
```

```
[Out] -2/15*(15*(e*x + d)^2*c^2*d^2 + 3*c^2*d^4 - 6*a*c*d^2*e^2 + 3*a^2*e^4 - 10*
(c^2*d^3 - a*c*d*e^2)*(e*x + d))/((e*x + d)^(5/2)*e^3)
```

Fricas [A] time = 1.72431, size = 220, normalized size = 2.72

$$-\frac{2\left(15\,c^{2}d^{2}e^{2}x^{2}+8\,c^{2}d^{4}+4\,acd^{2}e^{2}+3\,a^{2}e^{4}+10\left(2\,c^{2}d^{3}e+acde^{3}\right)x\right)\sqrt{ex+d}}{15\left(e^{6}x^{3}+3\,de^{5}x^{2}+3\,d^{2}e^{4}x+d^{3}e^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(11/2),x, algorithm="
fricas")
```

```
[Out] -2/15*(15*c^2*d^2*e^2*x^2 + 8*c^2*d^4 + 4*a*c*d^2*e^2 + 3*a^2*e^4 + 10*(2*c
^2*d^3*e + a*c*d*e^3)*x)*sqrt(e*x + d)/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x
 + d^3*e^3)
```

Sympy [A] time = 27.4767, size = 388, normalized size = 4.79

$$\begin{cases} -\frac{6a^2e^4}{15d^2e^3\sqrt{d+ex}+30de^4x\sqrt{d+ex}+15e^5x^2\sqrt{d+ex}} - \frac{8acd^2e^2}{15d^2e^3\sqrt{d+ex}+30de^4x\sqrt{d+ex}+15e^5x^2\sqrt{d+ex}} - \frac{20acde^3x}{15d^2e^3\sqrt{d+ex}+30de^4x\sqrt{d+ex}+15e^5x^2\sqrt{d+ex}} - \frac{}{15d^2e^3} \\ \frac{c^2x^3}{3d^{\frac{3}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d)**(11/2),x)
```

```
[Out] Piecewise((-6*a**2*e**4/(15*d**2*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d +
e*x) + 15*e**5*x**2*sqrt(d + e*x)) - 8*a*c*d**2*e**2/(15*d**2*e**3*sqrt(d +
 e*x) + 30*d*e**4*x*sqrt(d + e*x) + 15*e**5*x**2*sqrt(d + e*x)) - 20*a*c*d*
e**3*x/(15*d**2*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d + e*x) + 15*e**5*x*
*2*sqrt(d + e*x)) - 16*c**2*d**4/(15*d**2*e**3*sqrt(d + e*x) + 30*d*e**4*x*
sqrt(d + e*x) + 15*e**5*x**2*sqrt(d + e*x)) - 40*c**2*d**3*e*x/(15*d**2*e**
3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d + e*x) + 15*e**5*x**2*sqrt(d + e*x)) -
 30*c**2*d**2*e**2*x**2/(15*d**2*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d +
e*x) + 15*e**5*x**2*sqrt(d + e*x)), Ne(e, 0)), (c**2*x**3/(3*d**(3/2)), Tru
e))
```

Giac [A] time = 1.16608, size = 146, normalized size = 1.8

$$-\frac{2\left(15\,(xe+d)^4c^2d^2 - 10\,(xe+d)^3c^2d^3 + 3\,(xe+d)^2c^2d^4 + 10\,(xe+d)^3acde^2 - 6\,(xe+d)^2acd^2e^2 + 3\,(xe+d)^2a^2e^4\right)e^{(-3}}{15\,(xe+d)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(11/2),x, algorithm="
giac")
```

```
[Out] -2/15*(15*(x*e + d)^4*c^2*d^2 - 10*(x*e + d)^3*c^2*d^3 + 3*(x*e + d)^2*c^2*
d^4 + 10*(x*e + d)^3*a*c*d*e^2 - 6*(x*e + d)^2*a*c*d^2*e^2 + 3*(x*e + d)^2*
a^2*e^4)*e^(-3)/(x*e + d)^(9/2)
```

3.1991 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^{13/2}}\,dx$

Optimal. Leaf size=83

$$\frac{4cd\left(cd^2-ae^2\right)}{5e^3(d+ex)^{5/2}}-\frac{2\left(cd^2-ae^2\right)^2}{7e^3(d+ex)^{7/2}}-\frac{2c^2d^2}{3e^3(d+ex)^{3/2}}$$

```
[Out] (-2*(c*d^2 - a*e^2)^2)/(7*e^3*(d + e*x)^(7/2)) + (4*c*d*(c*d^2 - a*e^2))/(5
*e^3*(d + e*x)^(5/2)) - (2*c^2*d^2)/(3*e^3*(d + e*x)^(3/2))
```

Rubi [A] time = 0.0388828, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {626, 43}

$$\frac{4cd\left(cd^2-ae^2\right)}{5e^3(d+ex)^{5/2}}-\frac{2\left(cd^2-ae^2\right)^2}{7e^3(d+ex)^{7/2}}-\frac{2c^2d^2}{3e^3(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^(13/2),x]
```

```
[Out] (-2*(c*d^2 - a*e^2)^2)/(7*e^3*(d + e*x)^(7/2)) + (4*c*d*(c*d^2 - a*e^2))/(5
*e^3*(d + e*x)^(5/2)) - (2*c^2*d^2)/(3*e^3*(d + e*x)^(3/2))
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^{13/2}}\,dx&=\int \frac{(ae+cdx)^2}{(d+ex)^{9/2}}\,dx\\&=\int \left(\frac{\left(-cd^2+ae^2\right)^2}{e^2(d+ex)^{9/2}}-\frac{2cd\left(cd^2-ae^2\right)}{e^2(d+ex)^{7/2}}+\frac{c^2d^2}{e^2(d+ex)^{5/2}}\right)dx\\&=-\frac{2\left(cd^2-ae^2\right)^2}{7e^3(d+ex)^{7/2}}+\frac{4cd\left(cd^2-ae^2\right)}{5e^3(d+ex)^{5/2}}-\frac{2c^2d^2}{3e^3(d+ex)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0382827, size = 67, normalized size = 0.81

$$-\frac{2\left(15a^2e^4+6acde^2(2d+7ex)+c^2d^2\left(8d^2+28dex+35e^2x^2\right)\right)}{105e^3(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^(13/2),x]
```

```
[Out] (-2*(15*a^2*e^4 + 6*a*c*d*e^2*(2*d + 7*e*x) + c^2*d^2*(8*d^2 + 28*d*e*x + 3
5*e^2*x^2)))/(105*e^3*(d + e*x)^(7/2))
```

Maple [A] time = 0.045, size = 73, normalized size = 0.9

$$-\frac{70\,c^2d^2x^2e^2+84\,acde^3x+56\,c^2d^3ex+30\,a^2e^4+24\,acd^2e^2+16\,c^2d^4}{105\,e^3}\,(ex+d)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(13/2),x)
```

```
[Out] -2/105/(e*x+d)^(7/2)*(35*c^2*d^2*e^2*x^2+42*a*c*d*e^3*x+28*c^2*d^3*e*x+15*a
^2*e^4+12*a*c*d^2*e^2+8*c^2*d^4)/e^3
```

Maxima [A] time = 1.02218, size = 104, normalized size = 1.25

$$-\frac{2\left(35\,(ex+d)^2c^2d^2+15\,c^2d^4-30\,acd^2e^2+15\,a^2e^4-42\left(c^2d^3-acde^2\right)(ex+d)\right)}{105\,(ex+d)^{\frac{7}{2}}e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(13/2),x, algorithm="
maxima")
```

```
[Out] -2/105*(35*(e*x + d)^2*c^2*d^2 + 15*c^2*d^4 - 30*a*c*d^2*e^2 + 15*a^2*e^4 -
 42*(c^2*d^3 - a*c*d*e^2)*(e*x + d))/((e*x + d)^(7/2)*e^3)
```

Fricas [A] time = 1.89575, size = 248, normalized size = 2.99

$$-\frac{2\left(35\,c^2d^2e^2x^2+8\,c^2d^4+12\,acd^2e^2+15\,a^2e^4+14\left(2\,c^2d^3e+3\,acde^3\right)x\right)\sqrt{ex+d}}{105\left(e^7x^4+4\,de^6x^3+6\,d^2e^5x^2+4\,d^3e^4x+d^4e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(13/2),x, algorithm="
fricas")
```

```
[Out] -2/105*(35*c^2*d^2*e^2*x^2 + 8*c^2*d^4 + 12*a*c*d^2*e^2 + 15*a^2*e^4 + 14*(
2*c^2*d^3*e + 3*a*c*d*e^3)*x)*sqrt(e*x + d)/(e^7*x^4 + 4*d*e^6*x^3 + 6*d^2*
e^5*x^2 + 4*d^3*e^4*x + d^4*e^3)
```

Sympy [A] time = 65.3969, size = 510, normalized size = 6.14

$$\begin{cases} -\frac{30a^2e^4}{105d^3e^3\sqrt{d+ex}+315d^2e^4x\sqrt{d+ex}+315de^5x^2\sqrt{d+ex}+105e^6x^3\sqrt{d+ex}} - \frac{24acd^2e^2}{105d^3e^3\sqrt{d+ex}+315d^2e^4x\sqrt{d+ex}+315de^5x^2\sqrt{d+ex}+105e^6x^3\sqrt{d+ex}} - \frac{\quad}{105d^3e^3\sqrt{d+ex}} \\ \frac{c^2x^3}{3d^{\frac{5}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d)**(13/2),x)
```

```
[Out] Piecewise((-30*a**2*e**4/(105*d**3*e**3*sqrt(d + e*x) + 315*d**2*e**4*x*sqr
t(d + e*x) + 315*d*e**5*x**2*sqrt(d + e*x) + 105*e**6*x**3*sqrt(d + e*x)) -
 24*a*c*d**2*e**2/(105*d**3*e**3*sqrt(d + e*x) + 315*d**2*e**4*x*sqrt(d + e
*x) + 315*d*e**5*x**2*sqrt(d + e*x) + 105*e**6*x**3*sqrt(d + e*x)) - 84*a*c
*d*e**3*x/(105*d**3*e**3*sqrt(d + e*x) + 315*d**2*e**4*x*sqrt(d + e*x) + 31
5*d*e**5*x**2*sqrt(d + e*x) + 105*e**6*x**3*sqrt(d + e*x)) - 16*c**2*d**4/(
105*d**3*e**3*sqrt(d + e*x) + 315*d**2*e**4*x*sqrt(d + e*x) + 315*d*e**5*x*
*2*sqrt(d + e*x) + 105*e**6*x**3*sqrt(d + e*x)) - 56*c**2*d**3*e*x/(105*d**
3*e**3*sqrt(d + e*x) + 315*d**2*e**4*x*sqrt(d + e*x) + 315*d*e**5*x**2*sqrt
(d + e*x) + 105*e**6*x**3*sqrt(d + e*x)) - 70*c**2*d**2*e**2*x**2/(105*d**3
*e**3*sqrt(d + e*x) + 315*d**2*e**4*x*sqrt(d + e*x) + 315*d*e**5*x**2*sqrt(
d + e*x) + 105*e**6*x**3*sqrt(d + e*x)), Ne(e, 0)), (c**2*x**3/(3*d**(5/2))
, True))
```

Giac [A] time = 1.24375, size = 146, normalized size = 1.76

$$-\frac{2\left(35\,(xe+d)^4c^2d^2 - 42\,(xe+d)^3c^2d^3 + 15\,(xe+d)^2c^2d^4 + 42\,(xe+d)^3acde^2 - 30\,(xe+d)^2acd^2e^2 + 15\,(xe+d)^2a^2e^4\right)e^{(-}}{105\,(xe+d)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(13/2),x, algorithm="
giac")
```

```
[Out] -2/105*(35*(x*e + d)^4*c^2*d^2 - 42*(x*e + d)^3*c^2*d^3 + 15*(x*e + d)^2*c^
2*d^4 + 42*(x*e + d)^3*a*c*d*e^2 - 30*(x*e + d)^2*a*c*d^2*e^2 + 15*(x*e + d
)^2*a^2*e^4)*e^(-3)/(x*e + d)^(11/2)
```

3.1992 $\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3\,dx$

Optimal. Leaf size=119

$$-\frac{6c^2d^2(d+ex)^{13/2}\left(cd^2-ae^2\right)}{13e^4}+\frac{6cd(d+ex)^{11/2}\left(cd^2-ae^2\right)^2}{11e^4}-\frac{2(d+ex)^{9/2}\left(cd^2-ae^2\right)^3}{9e^4}+\frac{2c^3d^3(d+ex)^{15/2}}{15e^4}$$

```
[Out] (-2*(c*d^2 - a*e^2)^3*(d + e*x)^(9/2))/(9*e^4) + (6*c*d*(c*d^2 - a*e^2)^2*(
d + e*x)^(11/2))/(11*e^4) - (6*c^2*d^2*(c*d^2 - a*e^2)*(d + e*x)^(13/2))/(1
3*e^4) + (2*c^3*d^3*(d + e*x)^(15/2))/(15*e^4)
```

Rubi [A] time = 0.0816524, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {626, 43}

$$-\frac{6c^2d^2(d+ex)^{13/2}\left(cd^2-ae^2\right)}{13e^4}+\frac{6cd(d+ex)^{11/2}\left(cd^2-ae^2\right)^2}{11e^4}-\frac{2(d+ex)^{9/2}\left(cd^2-ae^2\right)^3}{9e^4}+\frac{2c^3d^3(d+ex)^{15/2}}{15e^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] (-2*(c*d^2 - a*e^2)^3*(d + e*x)^(9/2))/(9*e^4) + (6*c*d*(c*d^2 - a*e^2)^2*(
d + e*x)^(11/2))/(11*e^4) - (6*c^2*d^2*(c*d^2 - a*e^2)*(d + e*x)^(13/2))/(1
3*e^4) + (2*c^3*d^3*(d + e*x)^(15/2))/(15*e^4)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3\,dx=\int (ae+cdx)^3(d+ex)^{7/2}\,dx$$

$$=\int \left(\frac{\left(-cd^2+ae^2\right)^3(d+ex)^{7/2}}{e^3}+\frac{3cd\left(cd^2-ae^2\right)^2(d+ex)^{9/2}}{e^3}-\frac{3c^2d^2\left(c\right.}{}\right.$$

$$=-\frac{2\left(cd^2-ae^2\right)^3(d+ex)^{9/2}}{9e^4}+\frac{6cd\left(cd^2-ae^2\right)^2(d+ex)^{11/2}}{11e^4}-\frac{6c^2d^2\left(cd\right.}{}$$

Mathematica [A] time = 0.0915999, size = 111, normalized size = 0.93

$$\frac{2(d+ex)^{9/2}\left(-195a^2cde^4(2d-9ex)+715a^3e^6+15ac^2d^2e^2\left(8d^2-36dex+99e^2x^2\right)+c^3d^3\left(72d^2ex-16d^3-198de^2x^2\right.\right.}{6435e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] (2*(d + e*x)^(9/2)*(715*a^3*e^6 - 195*a^2*c*d*e^4*(2*d - 9*e*x) + 15*a*c^2*
d^2*e^2*(8*d^2 - 36*d*e*x + 99*e^2*x^2) + c^3*d^3*(-16*d^3 + 72*d^2*e*x - 1
98*d*e^2*x^2 + 429*e^3*x^3)))/(6435*e^4)
```

Maple [A] time = 0.046, size = 131, normalized size = 1.1

$$\frac{858\,x^3c^3d^3e^3+2970\,ac^2d^2e^4x^2-396\,c^3d^4e^2x^2+3510\,a^2cde^5x-1080\,ac^2d^3e^3x+144\,c^3d^5ex+1430\,a^3e^6-780\,a^2cd^2e^4+}{6435\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3*(e*x+d)^(1/2),x)
```

```
[Out] 2/6435*(e*x+d)^(9/2)*(429*c^3*d^3*e^3*x^3+1485*a*c^2*d^2*e^4*x^2-198*c^3*d^
4*e^2*x^2+1755*a^2*c*d*e^5*x-540*a*c^2*d^3*e^3*x+72*c^3*d^5*e*x+715*a^3*e^6
-390*a^2*c*d^2*e^4+120*a*c^2*d^4*e^2-16*c^3*d^6)/e^4
```

Maxima [A] time = 1.03659, size = 185, normalized size = 1.55

$$\frac{2\left(429\,(ex+d)^{\frac{15}{2}}c^3d^3-1485\left(c^3d^4-ac^2d^2e^2\right)(ex+d)^{\frac{13}{2}}+1755\left(c^3d^5-2\,ac^2d^3e^2+a^2cde^4\right)(ex+d)^{\frac{11}{2}}-715\left(c^3d^6-3\,ac^2\right.\right.}{6435\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3*(e*x+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] 2/6435*(429*(e*x + d)^(15/2)*c^3*d^3 - 1485*(c^3*d^4 - a*c^2*d^2*e^2)*(e*x
+ d)^(13/2) + 1755*(c^3*d^5 - 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*(e*x + d)^(11/
2) - 715*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*(e*x + d)^
(9/2))/e^4
```

Fricas [B] time = 1.91741, size = 743, normalized size = 6.24

$$\frac{2\left(429\,c^3d^3e^7x^7-16\,c^3d^{10}+120\,ac^2d^8e^2-390\,a^2cd^6e^4+715\,a^3d^4e^6+33\left(46\,c^3d^4e^6+45\,ac^2d^2e^8\right)x^6+9\left(206\,c^3d^5e^5+60\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3*(e*x+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 2/6435*(429*c^3*d^3*e^7*x^7 - 16*c^3*d^10 + 120*a*c^2*d^8*e^2 - 390*a^2*c*d
^6*e^4 + 715*a^3*d^4*e^6 + 33*(46*c^3*d^4*e^6 + 45*a*c^2*d^2*e^8)*x^6 + 9*(
206*c^3*d^5*e^5 + 600*a*c^2*d^3*e^7 + 195*a^2*c*d*e^9)*x^5 + 5*(160*c^3*d^6
*e^4 + 1374*a*c^2*d^4*e^6 + 1326*a^2*c*d^2*e^8 + 143*a^3*e^10)*x^4 + 5*(c^3
*d^7*e^3 + 636*a*c^2*d^5*e^5 + 1794*a^2*c*d^3*e^7 + 572*a^3*d*e^9)*x^3 - 3*
```

```
(2*c^3*d^8*e^2 - 15*a*c^2*d^6*e^4 - 1560*a^2*c*d^4*e^6 - 1430*a^3*d^2*e^8)*
x^2 + (8*c^3*d^9*e - 60*a*c^2*d^7*e^3 + 195*a^2*c*d^5*e^5 + 2860*a^3*d^3*e^
7)*x)*sqrt(e*x + d)/e^4
```

Sympy [A] time = 7.85638, size = 165, normalized size = 1.39

$$\frac{2\left(\frac{c^3d^3(d+ex)^{\frac{15}{2}}}{15e^3}+\frac{(d+ex)^{\frac{13}{2}}\left(3ac^2d^2e^2-3c^3d^4\right)}{13e^3}+\frac{(d+ex)^{\frac{11}{2}}\left(3a^2cde^4-6ac^2d^3e^2+3c^3d^5\right)}{11e^3}+\frac{(d+ex)^{\frac{9}{2}}\left(a^3e^6-3a^2cd^2e^4+3ac^2d^4e^2-c^3d^6\right)}{9e^3}\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3*(e*x+d)**(1/2),x)
```

```
[Out] 2*(c**3*d**3*(d + e*x)**(15/2)/(15*e**3) + (d + e*x)**(13/2)*(3*a*c**2*d**2
*e**2 - 3*c**3*d**4)/(13*e**3) + (d + e*x)**(11/2)*(3*a**2*c*d*e**4 - 6*a*c
**2*d**3*e**2 + 3*c**3*d**5)/(11*e**3) + (d + e*x)**(9/2)*(a**3*e**6 - 3*a*
*2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 - c**3*d**6)/(9*e**3))/e
```

Giac [B] time = 1.26956, size = 1258, normalized size = 10.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3*(e*x+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] 2/45045*(143*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5
/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*c^3*d^6*e^(-3) + 1287*(15*(x*e + d)^(7/2
) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*a*c^2*d^5*e^(-1) + 39*(3
15*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2
772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*c^3*d^5*e^(-3) + 9009*(
3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^2*c*d^4*e + 1287*(35*(x*e + d)^(
9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2
)*d^3)*a*c^2*d^4*e^(-1) + 15*(693*(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*
d + 10010*(x*e + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^
(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*c^3*d^4*e^(-3) + 15015*(x*e + d)^(3/2
)*a^3*d^3*e^3 + 3861*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e +
 d)^(3/2)*d^2)*a^2*c*d^3*e + 117*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/
2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)
^(3/2)*d^4)*a*c^2*d^3*e^(-1) + (3003*(x*e + d)^(15/2) - 20790*(x*e + d)^(13
/2)*d + 61425*(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525*(x*
e + d)^(7/2)*d^4 - 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*c
^3*d^3*e^(-3) + 9009*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^3*d^2*e^3
+ 1287*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^
2 - 105*(x*e + d)^(3/2)*d^3)*a^2*c*d^2*e + 15*(693*(x*e + d)^(13/2) - 4095*
(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3
+ 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*a*c^2*d^2*e^(-1) + 1
287*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*a^
3*d*e^3 + 39*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d
)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*a^2*c*d*
e + 143*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d
^2 - 105*(x*e + d)^(3/2)*d^3)*a^3*e^3)*e^(-1)
```

3.1993 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{\sqrt{d+ex}} dx$

Optimal. Leaf size=119

$$-\frac{6c^2d^2(d+ex)^{11/2}\left(cd^2-ae^2\right)}{11e^4}+\frac{2cd(d+ex)^{9/2}\left(cd^2-ae^2\right)^2}{3e^4}-\frac{2(d+ex)^{7/2}\left(cd^2-ae^2\right)^3}{7e^4}+\frac{2c^3d^3(d+ex)^{13/2}}{13e^4}$$

```
[Out] (-2*(c*d^2 - a*e^2)^3*(d + e*x)^(7/2))/(7*e^4) + (2*c*d*(c*d^2 - a*e^2)^2*(
d + e*x)^(9/2))/(3*e^4) - (6*c^2*d^2*(c*d^2 - a*e^2)*(d + e*x)^(11/2))/(11*
e^4) + (2*c^3*d^3*(d + e*x)^(13/2))/(13*e^4)
```

Rubi [A] time = 0.0569964, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {626, 43}

$$-\frac{6c^2d^2(d+ex)^{11/2}\left(cd^2-ae^2\right)}{11e^4}+\frac{2cd(d+ex)^{9/2}\left(cd^2-ae^2\right)^2}{3e^4}-\frac{2(d+ex)^{7/2}\left(cd^2-ae^2\right)^3}{7e^4}+\frac{2c^3d^3(d+ex)^{13/2}}{13e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/Sqrt[d + e*x],x]
```

```
[Out] (-2*(c*d^2 - a*e^2)^3*(d + e*x)^(7/2))/(7*e^4) + (2*c*d*(c*d^2 - a*e^2)^2*(
d + e*x)^(9/2))/(3*e^4) - (6*c^2*d^2*(c*d^2 - a*e^2)*(d + e*x)^(11/2))/(11*
e^4) + (2*c^3*d^3*(d + e*x)^(13/2))/(13*e^4)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{\sqrt{d+ex}} dx = \int (ae+cdx)^3(d+ex)^{5/2} dx$$

$$= \int \left(\frac{\left(-cd^2+ae^2\right)^3(d+ex)^{5/2}}{e^3}+\frac{3cd\left(cd^2-ae^2\right)^2(d+ex)^{7/2}}{e^3}-\frac{3c^2d^2\left(cd^2-ae^2\right)(d}{e^3}\right.$$

$$= -\frac{2\left(cd^2-ae^2\right)^3(d+ex)^{7/2}}{7e^4}+\frac{2cd\left(cd^2-ae^2\right)^2(d+ex)^{9/2}}{3e^4}-\frac{6c^2d^2\left(cd^2-ae^2\right)(d+}{11e^4}$$

Mathematica [A] time = 0.0779152, size = 98, normalized size = 0.82

$$\frac{2(d+ex)^{7/2}\left(-819c^2d^2(d+ex)^2\left(cd^2-ae^2\right)+1001cd(d+ex)\left(cd^2-ae^2\right)^2-429\left(cd^2-ae^2\right)^3+231c^3d^3(d+ex)^3\right)}{3003e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/Sqrt[d + e*x],x]
```

```
[Out] (2*(d + e*x)^(7/2)*(-429*(c*d^2 - a*e^2)^3 + 1001*c*d*(c*d^2 - a*e^2)^2*(d
+ e*x) - 819*c^2*d^2*(c*d^2 - a*e^2)*(d + e*x)^2 + 231*c^3*d^3*(d + e*x)^3)
)/(3003*e^4)
```

Maple [A] time = 0.045, size = 131, normalized size = 1.1

$$\frac{462\,x^3c^3d^3e^3+1638\,ac^2d^2e^4x^2-252\,c^3d^4e^2x^2+2002\,a^2cde^5x-728\,ac^2d^3e^3x+112\,c^3d^5ex+858\,a^3e^6-572\,a^2cd^2e^4+}{3003\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(1/2),x)
```

```
[Out] 2/3003*(e*x+d)^(7/2)*(231*c^3*d^3*e^3*x^3+819*a*c^2*d^2*e^4*x^2-126*c^3*d^4
*e^2*x^2+1001*a^2*c*d*e^5*x-364*a*c^2*d^3*e^3*x+56*c^3*d^5*e*x+429*a^3*e^6-
286*a^2*c*d^2*e^4+104*a*c^2*d^4*e^2-16*c^3*d^6)/e^4
```

Maxima [B] time = 1.01659, size = 825, normalized size = 6.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] 2/15015*(15015*sqrt(e*x + d)*a^3*d^3*e^3 + 3003*((3*(e*x + d)^(5/2) - 10*(e
*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*c*d/e + 5*(c*d^2 + a*e^2)*((e*x + d
)^(3/2) - 3*sqrt(e*x + d)*d)/e)*a^2*d^2*e^2 + 5*(231*(e*x + d)^(13/2) - 163
8*(e*x + d)^(11/2)*d + 5005*(e*x + d)^(9/2)*d^2 - 8580*(e*x + d)^(7/2)*d^3
+ 9009*(e*x + d)^(5/2)*d^4 - 6006*(e*x + d)^(3/2)*d^5 + 3003*sqrt(e*x + d)*
d^6)*c^3*d^3/e^3 + 143*((35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(
e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*c^2*d
^2/e^2 + 18*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*
d^2 - 35*sqrt(e*x + d)*d^3)*(c*d^2 + a*e^2)*c*d/e^2 + 21*(c*d^2 + a*e^2)^2*
(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)/e^2)*a*d*
e + 65*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d
^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d
)*d^5)*(c*d^2 + a*e^2)*c^2*d^2/e^3 + 143*(35*(e*x + d)^(9/2) - 180*(e*x + d
)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*
x + d)*d^4)*(c*d^2 + a*e^2)^2*c*d/e^3 + 429*(5*(e*x + d)^(7/2) - 21*(e*x +
d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*(c*d^2 + a*e^2)
^3/e^3)/e
```

Fricas [B] time = 1.91656, size = 620, normalized size = 5.21

$$\frac{2\left(231\,c^3d^3e^6x^6-16\,c^3d^9+104\,ac^2d^7e^2-286\,a^2cd^5e^4+429\,a^3d^3e^6+63\left(9\,c^3d^4e^5+13\,ac^2d^2e^7\right)x^5+7\left(53\,c^3d^5e^4+299\,a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 2/3003*(231*c^3*d^3*e^6*x^6 - 16*c^3*d^9 + 104*a*c^2*d^7*e^2 - 286*a^2*c*d^
5*e^4 + 429*a^3*d^3*e^6 + 63*(9*c^3*d^4*e^5 + 13*a*c^2*d^2*e^7)*x^5 + 7*(53
*c^3*d^5*e^4 + 299*a*c^2*d^3*e^6 + 143*a^2*c*d*e^8)*x^4 + (5*c^3*d^6*e^3 +
1469*a*c^2*d^4*e^5 + 2717*a^2*c*d^2*e^7 + 429*a^3*e^9)*x^3 - 3*(2*c^3*d^7*e
^2 - 13*a*c^2*d^5*e^4 - 715*a^2*c*d^3*e^6 - 429*a^3*d*e^8)*x^2 + (8*c^3*d^8
*e - 52*a*c^2*d^6*e^3 + 143*a^2*c*d^4*e^5 + 1287*a^3*d^2*e^7)*x)*sqrt(e*x +
 d)/e^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23625, size = 1254, normalized size = 10.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] 2/15015*(429*(5*(x*e + d)^(7/2) - 21*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)
*d^2 - 35*sqrt(x*e + d)*d^3)*c^3*d^6*e^(-3) + 3003*(3*(x*e + d)^(5/2) - 10*
(x*e + d)^(3/2)*d + 15*sqrt(x*e + d)*d^2)*a*c^2*d^5*e^(-1) + 143*(35*(x*e +
 d)^(9/2) - 180*(x*e + d)^(7/2)*d + 378*(x*e + d)^(5/2)*d^2 - 420*(x*e + d)
^(3/2)*d^3 + 315*sqrt(x*e + d)*d^4)*c^3*d^5*e^(-3) + 15015*((x*e + d)^(3/2)
 - 3*sqrt(x*e + d)*d)*a^2*c*d^4*e + 3861*(5*(x*e + d)^(7/2) - 21*(x*e + d)^
(5/2)*d + 35*(x*e + d)^(3/2)*d^2 - 35*sqrt(x*e + d)*d^3)*a*c^2*d^4*e^(-1) +
 65*(63*(x*e + d)^(11/2) - 385*(x*e + d)^(9/2)*d + 990*(x*e + d)^(7/2)*d^2
- 1386*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4 - 693*sqrt(x*e + d)*d
^5)*c^3*d^4*e^(-3) + 9009*(3*(x*e + d)^(5/2) - 10*(x*e + d)^(3/2)*d + 15*sq
rt(x*e + d)*d^2)*a^2*c*d^3*e + 429*(35*(x*e + d)^(9/2) - 180*(x*e + d)^(7/2
)*d + 378*(x*e + d)^(5/2)*d^2 - 420*(x*e + d)^(3/2)*d^3 + 315*sqrt(x*e + d)
*d^4)*a*c^2*d^3*e^(-1) + 5*(231*(x*e + d)^(13/2) - 1638*(x*e + d)^(11/2)*d
+ 5005*(x*e + d)^(9/2)*d^2 - 8580*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2
)*d^4 - 6006*(x*e + d)^(3/2)*d^5 + 3003*sqrt(x*e + d)*d^6)*c^3*d^3*e^(-3) +
```

```
 15015*sqrt(x*e + d)*a^3*d^3*e^3 + 15015*((x*e + d)^(3/2) - 3*sqrt(x*e + d)
*d)*a^3*d^2*e^3 + 3861*(5*(x*e + d)^(7/2) - 21*(x*e + d)^(5/2)*d + 35*(x*e
+ d)^(3/2)*d^2 - 35*sqrt(x*e + d)*d^3)*a^2*c*d^2*e + 65*(63*(x*e + d)^(11/2
) - 385*(x*e + d)^(9/2)*d + 990*(x*e + d)^(7/2)*d^2 - 1386*(x*e + d)^(5/2)*
d^3 + 1155*(x*e + d)^(3/2)*d^4 - 693*sqrt(x*e + d)*d^5)*a*c^2*d^2*e^(-1) +
3003*(3*(x*e + d)^(5/2) - 10*(x*e + d)^(3/2)*d + 15*sqrt(x*e + d)*d^2)*a^3*
d*e^3 + 143*(35*(x*e + d)^(9/2) - 180*(x*e + d)^(7/2)*d + 378*(x*e + d)^(5/
2)*d^2 - 420*(x*e + d)^(3/2)*d^3 + 315*sqrt(x*e + d)*d^4)*a^2*c*d*e + 429*(
5*(x*e + d)^(7/2) - 21*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2 - 35*sqrt
(x*e + d)*d^3)*a^3*e^3)*e^(-1)
```

3.1994 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=119

$$-\frac{2c^2d^2(d+ex)^{9/2}\left(cd^2-ae^2\right)}{3e^4}+\frac{6cd(d+ex)^{7/2}\left(cd^2-ae^2\right)^2}{7e^4}-\frac{2(d+ex)^{5/2}\left(cd^2-ae^2\right)^3}{5e^4}+\frac{2c^3d^3(d+ex)^{11/2}}{11e^4}$$

```
[Out] (-2*(c*d^2 - a*e^2)^3*(d + e*x)^(5/2))/(5*e^4) + (6*c*d*(c*d^2 - a*e^2)^2*(
d + e*x)^(7/2))/(7*e^4) - (2*c^2*d^2*(c*d^2 - a*e^2)*(d + e*x)^(9/2))/(3*e^
4) + (2*c^3*d^3*(d + e*x)^(11/2))/(11*e^4)
```

Rubi [A] time = 0.0576717, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {626, 43}

$$-\frac{2c^2d^2(d+ex)^{9/2}\left(cd^2-ae^2\right)}{3e^4}+\frac{6cd(d+ex)^{7/2}\left(cd^2-ae^2\right)^2}{7e^4}-\frac{2(d+ex)^{5/2}\left(cd^2-ae^2\right)^3}{5e^4}+\frac{2c^3d^3(d+ex)^{11/2}}{11e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^(3/2),x]
```

```
[Out] (-2*(c*d^2 - a*e^2)^3*(d + e*x)^(5/2))/(5*e^4) + (6*c*d*(c*d^2 - a*e^2)^2*(
d + e*x)^(7/2))/(7*e^4) - (2*c^2*d^2*(c*d^2 - a*e^2)*(d + e*x)^(9/2))/(3*e^
4) + (2*c^3*d^3*(d + e*x)^(11/2))/(11*e^4)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{3/2}}\,dx=\int (ae+cdx)^3(d+ex)^{3/2}\,dx$$

$$=\int \left(\frac{\left(-cd^2+ae^2\right)^3(d+ex)^{3/2}}{e^3}+\frac{3cd\left(cd^2-ae^2\right)^2(d+ex)^{5/2}}{e^3}-\frac{3c^2d^2\left(cd^2-ae^2\right)(d}{e^3}\right.$$

$$=-\frac{2\left(cd^2-ae^2\right)^3(d+ex)^{5/2}}{5e^4}+\frac{6cd\left(cd^2-ae^2\right)^2(d+ex)^{7/2}}{7e^4}-\frac{2c^2d^2\left(cd^2-ae^2\right)(d+}{3e^4}$$

Mathematica [A] time = 0.0712888, size = 98, normalized size = 0.82

$$\frac{2(d+ex)^{5/2}\left(-385c^2d^2(d+ex)^2\left(cd^2-ae^2\right)+495cd(d+ex)\left(cd^2-ae^2\right)^2-231\left(cd^2-ae^2\right)^3+105c^3d^3(d+ex)^3\right)}{1155e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^(3/2),x]
```

```
[Out] (2*(d + e*x)^(5/2)*(-231*(c*d^2 - a*e^2)^3 + 495*c*d*(c*d^2 - a*e^2)^2*(d +
 e*x) - 385*c^2*d^2*(c*d^2 - a*e^2)*(d + e*x)^2 + 105*c^3*d^3*(d + e*x)^3))
/(1155*e^4)
```

Maple [A] time = 0.043, size = 131, normalized size = 1.1

$$\frac{210\,x^3c^3d^3e^3+770\,ac^2d^2e^4x^2-140\,c^3d^4e^2x^2+990\,a^2cde^5x-440\,ac^2d^3e^3x+80\,c^3d^5ex+462\,a^3e^6-396\,a^2cd^2e^4+17\ldots}{1155\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(3/2),x)
```

```
[Out] 2/1155*(e*x+d)^(5/2)*(105*c^3*d^3*e^3*x^3+385*a*c^2*d^2*e^4*x^2-70*c^3*d^4*
e^2*x^2+495*a^2*c*d*e^5*x-220*a*c^2*d^3*e^3*x+40*c^3*d^5*e*x+231*a^3*e^6-19
8*a^2*c*d^2*e^4+88*a*c^2*d^4*e^2-16*c^3*d^6)/e^4
```

Maxima [A] time = 1.04634, size = 185, normalized size = 1.55

$$\frac{2\left(105\,(ex+d)^{\frac{11}{2}}c^3d^3-385\left(c^3d^4-ac^2d^2e^2\right)(ex+d)^{\frac{9}{2}}+495\left(c^3d^5-2\,ac^2d^3e^2+a^2cde^4\right)(ex+d)^{\frac{7}{2}}-231\left(c^3d^6-3\,ac^2d\ldots\right.\right.}{1155\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(3/2),x, algorithm="m
axima")
```

```
[Out] 2/1155*(105*(e*x + d)^(11/2)*c^3*d^3 - 385*(c^3*d^4 - a*c^2*d^2*e^2)*(e*x +
 d)^(9/2) + 495*(c^3*d^5 - 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*(e*x + d)^(7/2) -
 231*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*(e*x + d)^(5/2
))/e^4
```

Fricas [B] time = 1.83046, size = 498, normalized size = 4.18

$$\frac{2\left(105\,c^3d^3e^5x^5-16\,c^3d^8+88\,ac^2d^6e^2-198\,a^2cd^4e^4+231\,a^3d^2e^6+35\left(4\,c^3d^4e^4+11\,ac^2d^2e^6\right)x^4+5\left(c^3d^5e^3+110\,ac^2\ldots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 2/1155*(105*c^3*d^3*e^5*x^5 - 16*c^3*d^8 + 88*a*c^2*d^6*e^2 - 198*a^2*c*d^4
*e^4 + 231*a^3*d^2*e^6 + 35*(4*c^3*d^4*e^4 + 11*a*c^2*d^2*e^6)*x^4 + 5*(c^3
*d^5*e^3 + 110*a*c^2*d^3*e^5 + 99*a^2*c*d*e^7)*x^3 - 3*(2*c^3*d^6*e^2 - 11*
a*c^2*d^4*e^4 - 264*a^2*c*d^2*e^6 - 77*a^3*e^8)*x^2 + (8*c^3*d^7*e - 44*a*c
^2*d^5*e^3 + 99*a^2*c*d^3*e^5 + 462*a^3*d*e^7)*x)*sqrt(e*x + d)/e^4
```

Sympy [A] time = 90.1039, size = 971, normalized size = 8.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**(3/2),x)
```

```
[Out] Piecewise((-(2*a**3*d**3*e**3/sqrt(d + e*x) + 6*a**3*d**2*e**3*(-d/sqrt(d +
 e*x) - sqrt(d + e*x)) + 6*a**3*d*e**3*(d**2/sqrt(d + e*x) + 2*d*sqrt(d + e
*x) - (d + e*x)**(3/2)/3) + 2*a**3*e**3*(-d**3/sqrt(d + e*x) - 3*d**2*sqrt(
d + e*x) + d*(d + e*x)**(3/2) - (d + e*x)**(5/2)/5) + 6*a**2*c*d**4*e*(-d/s
qrt(d + e*x) - sqrt(d + e*x)) + 18*a**2*c*d**3*e*(d**2/sqrt(d + e*x) + 2*d*
sqrt(d + e*x) - (d + e*x)**(3/2)/3) + 18*a**2*c*d**2*e*(-d**3/sqrt(d + e*x)
 - 3*d**2*sqrt(d + e*x) + d*(d + e*x)**(3/2) - (d + e*x)**(5/2)/5) + 6*a**2
*c*d*e*(d**4/sqrt(d + e*x) + 4*d**3*sqrt(d + e*x) - 2*d**2*(d + e*x)**(3/2)
 + 4*d*(d + e*x)**(5/2)/5 - (d + e*x)**(7/2)/7) + 6*a*c**2*d**5*(d**2/sqrt(
d + e*x) + 2*d*sqrt(d + e*x) - (d + e*x)**(3/2)/3)/e + 18*a*c**2*d**4*(-d**
3/sqrt(d + e*x) - 3*d**2*sqrt(d + e*x) + d*(d + e*x)**(3/2) - (d + e*x)**(5
/2)/5)/e + 18*a*c**2*d**3*(d**4/sqrt(d + e*x) + 4*d**3*sqrt(d + e*x) - 2*d*
*2*(d + e*x)**(3/2) + 4*d*(d + e*x)**(5/2)/5 - (d + e*x)**(7/2)/7)/e + 6*a*
c**2*d**2*(-d**5/sqrt(d + e*x) - 5*d**4*sqrt(d + e*x) + 10*d**3*(d + e*x)**
(3/2)/3 - 2*d**2*(d + e*x)**(5/2) + 5*d*(d + e*x)**(7/2)/7 - (d + e*x)**(9/
2)/9)/e + 2*c**3*d**6*(-d**3/sqrt(d + e*x) - 3*d**2*sqrt(d + e*x) + d*(d +
e*x)**(3/2) - (d + e*x)**(5/2)/5)/e**3 + 6*c**3*d**5*(d**4/sqrt(d + e*x) +
4*d**3*sqrt(d + e*x) - 2*d**2*(d + e*x)**(3/2) + 4*d*(d + e*x)**(5/2)/5 - (
d + e*x)**(7/2)/7)/e**3 + 6*c**3*d**4*(-d**5/sqrt(d + e*x) - 5*d**4*sqrt(d
+ e*x) + 10*d**3*(d + e*x)**(3/2)/3 - 2*d**2*(d + e*x)**(5/2) + 5*d*(d + e*
x)**(7/2)/7 - (d + e*x)**(9/2)/9)/e**3 + 2*c**3*d**3*(d**6/sqrt(d + e*x) +
6*d**5*sqrt(d + e*x) - 5*d**4*(d + e*x)**(3/2) + 4*d**3*(d + e*x)**(5/2) -
15*d**2*(d + e*x)**(7/2)/7 + 2*d*(d + e*x)**(9/2)/3 - (d + e*x)**(11/2)/11)
/e**3)/e, Ne(e, 0)), (c**3*d**(9/2)*x**4/4, True))
```

Giac [A] time = 1.18415, size = 250, normalized size = 2.1

$$\frac{2}{1155}\Big(105\,(xe+d)^{\frac{11}{2}}c^3d^3e^{40} - 385\,(xe+d)^{\frac{9}{2}}c^3d^4e^{40} + 495\,(xe+d)^{\frac{7}{2}}c^3d^5e^{40} - 231\,(xe+d)^{\frac{5}{2}}c^3d^6e^{40} + 385\,(xe+d)^{\frac{9}{2}}ac^2d^2e^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(3/2),x, algorithm="g
iac")
```

```
[Out] 2/1155*(105*(x*e + d)^(11/2)*c^3*d^3*e^40 - 385*(x*e + d)^(9/2)*c^3*d^4*e^4
0 + 495*(x*e + d)^(7/2)*c^3*d^5*e^40 - 231*(x*e + d)^(5/2)*c^3*d^6*e^40 + 3
85*(x*e + d)^(9/2)*a*c^2*d^2*e^42 - 990*(x*e + d)^(7/2)*a*c^2*d^3*e^42 + 69
3*(x*e + d)^(5/2)*a*c^2*d^4*e^42 + 495*(x*e + d)^(7/2)*a^2*c*d*e^44 - 693*(
x*e + d)^(5/2)*a^2*c*d^2*e^44 + 231*(x*e + d)^(5/2)*a^3*e^46)*e^(-44)
```

3.1995 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=119

$$-\frac{6c^2d^2(d+ex)^{7/2}\left(cd^2-ae^2\right)}{7e^4}+\frac{6cd(d+ex)^{5/2}\left(cd^2-ae^2\right)^2}{5e^4}-\frac{2(d+ex)^{3/2}\left(cd^2-ae^2\right)^3}{3e^4}+\frac{2c^3d^3(d+ex)^{9/2}}{9e^4}$$

```
[Out] (-2*(c*d^2 - a*e^2)^3*(d + e*x)^(3/2))/(3*e^4) + (6*c*d*(c*d^2 - a*e^2)^2*(
d + e*x)^(5/2))/(5*e^4) - (6*c^2*d^2*(c*d^2 - a*e^2)*(d + e*x)^(7/2))/(7*e^
4) + (2*c^3*d^3*(d + e*x)^(9/2))/(9*e^4)
```

Rubi [A] time = 0.051909, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {626, 43}

$$-\frac{6c^2d^2(d+ex)^{7/2}\left(cd^2-ae^2\right)}{7e^4}+\frac{6cd(d+ex)^{5/2}\left(cd^2-ae^2\right)^2}{5e^4}-\frac{2(d+ex)^{3/2}\left(cd^2-ae^2\right)^3}{3e^4}+\frac{2c^3d^3(d+ex)^{9/2}}{9e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^(5/2),x]
```

```
[Out] (-2*(c*d^2 - a*e^2)^3*(d + e*x)^(3/2))/(3*e^4) + (6*c*d*(c*d^2 - a*e^2)^2*(
d + e*x)^(5/2))/(5*e^4) - (6*c^2*d^2*(c*d^2 - a*e^2)*(d + e*x)^(7/2))/(7*e^
4) + (2*c^3*d^3*(d + e*x)^(9/2))/(9*e^4)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{5/2}}\,dx=\int (ae+cdx)^3\sqrt{d+ex}\,dx$$

$$=\int \left(\frac{\left(-cd^2+ae^2\right)^3\sqrt{d+ex}}{e^3}+\frac{3cd\left(cd^2-ae^2\right)^2(d+ex)^{3/2}}{e^3}-\frac{3c^2d^2\left(cd^2-ae^2\right)(}{e^3}\right.$$

$$=-\frac{2\left(cd^2-ae^2\right)^3(d+ex)^{3/2}}{3e^4}+\frac{6cd\left(cd^2-ae^2\right)^2(d+ex)^{5/2}}{5e^4}-\frac{6c^2d^2\left(cd^2-ae^2\right)(}{7e^4}$$

Mathematica [A] time = 0.0666193, size = 98, normalized size = 0.82

$$\frac{2(d+ex)^{3/2}\left(-135c^2d^2(d+ex)^2\left(cd^2-ae^2\right)+189cd(d+ex)\left(cd^2-ae^2\right)^2-105\left(cd^2-ae^2\right)^3+35c^3d^3(d+ex)^3\right)}{315e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^(5/2),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(-105*(c*d^2 - a*e^2)^3 + 189*c*d*(c*d^2 - a*e^2)^2*(d +
 e*x) - 135*c^2*d^2*(c*d^2 - a*e^2)*(d + e*x)^2 + 35*c^3*d^3*(d + e*x)^3))/
(315*e^4)
```

Maple [A] time = 0.048, size = 131, normalized size = 1.1

$$\frac{70x^3c^3d^3e^3+270ac^2d^2e^4x^2-60c^3d^4e^2x^2+378a^2cde^5x-216ac^2d^3e^3x+48c^3d^5ex+210a^3e^6-252a^2cd^2e^4+144ac^2d}{315e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(5/2),x)
```

```
[Out] 2/315*(e*x+d)^(3/2)*(35*c^3*d^3*e^3*x^3+135*a*c^2*d^2*e^4*x^2-30*c^3*d^4*e^
2*x^2+189*a^2*c*d*e^5*x-108*a*c^2*d^3*e^3*x+24*c^3*d^5*e*x+105*a^3*e^6-126*
a^2*c*d^2*e^4+72*a*c^2*d^4*e^2-16*c^3*d^6)/e^4
```

Maxima [A] time = 1.0738, size = 185, normalized size = 1.55

$$\frac{2\left(35(ex+d)^{\frac{9}{2}}c^3d^3-135\left(c^3d^4-ac^2d^2e^2\right)(ex+d)^{\frac{7}{2}}+189\left(c^3d^5-2ac^2d^3e^2+a^2cde^4\right)(ex+d)^{\frac{5}{2}}-105\left(c^3d^6-3ac^2d^4e^2+\right.\right.}{315e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(5/2),x, algorithm="m
axima")
```

```
[Out] 2/315*(35*(e*x + d)^(9/2)*c^3*d^3 - 135*(c^3*d^4 - a*c^2*d^2*e^2)*(e*x + d)
^(7/2) + 189*(c^3*d^5 - 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*(e*x + d)^(5/2) - 10
5*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*(e*x + d)^(3/2))/
e^4
```

Fricas [A] time = 1.8748, size = 383, normalized size = 3.22

$$\frac{2\left(35c^3d^3e^4x^4-16c^3d^7+72ac^2d^5e^2-126a^2cd^3e^4+105a^3de^6+5\left(c^3d^4e^3+27ac^2d^2e^5\right)x^3-3\left(2c^3d^5e^2-9ac^2d^3e^4-63\right.\right.}{315e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(5/2),x, algorithm="f
ricas")
```

```
[Out] 2/315*(35*c^3*d^3*e^4*x^4 - 16*c^3*d^7 + 72*a*c^2*d^5*e^2 - 126*a^2*c*d^3*e
^4 + 105*a^3*d*e^6 + 5*(c^3*d^4*e^3 + 27*a*c^2*d^2*e^5)*x^3 - 3*(2*c^3*d^5*
e^2 - 9*a*c^2*d^3*e^4 - 63*a^2*c*d*e^6)*x^2 + (8*c^3*d^6*e - 36*a*c^2*d^4*e
^3 + 63*a^2*c*d^2*e^5 + 105*a^3*e^7)*x)*sqrt(e*x + d)/e^4
```

Sympy [A] time = 102.341, size = 644, normalized size = 5.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**(5/2),x)
```

```
[Out] Piecewise((-(2*a**3*d**2*e**3/sqrt(d + e*x) + 4*a**3*d*e**3*(-d/sqrt(d + e*
x) - sqrt(d + e*x)) + 2*a**3*e**3*(d**2/sqrt(d + e*x) + 2*d*sqrt(d + e*x) -
 (d + e*x)**(3/2)/3) + 6*a**2*c*d**3*e*(-d/sqrt(d + e*x) - sqrt(d + e*x)) +
 12*a**2*c*d**2*e*(d**2/sqrt(d + e*x) + 2*d*sqrt(d + e*x) - (d + e*x)**(3/2
)/3) + 6*a**2*c*d*e*(-d**3/sqrt(d + e*x) - 3*d**2*sqrt(d + e*x) + d*(d + e*
x)**(3/2) - (d + e*x)**(5/2)/5) + 6*a*c**2*d**4*(d**2/sqrt(d + e*x) + 2*d*s
qrt(d + e*x) - (d + e*x)**(3/2)/3)/e + 12*a*c**2*d**3*(-d**3/sqrt(d + e*x)
- 3*d**2*sqrt(d + e*x) + d*(d + e*x)**(3/2) - (d + e*x)**(5/2)/5)/e + 6*a*c
**2*d**2*(d**4/sqrt(d + e*x) + 4*d**3*sqrt(d + e*x) - 2*d**2*(d + e*x)**(3/
2) + 4*d*(d + e*x)**(5/2)/5 - (d + e*x)**(7/2)/7)/e + 2*c**3*d**5*(-d**3/sq
rt(d + e*x) - 3*d**2*sqrt(d + e*x) + d*(d + e*x)**(3/2) - (d + e*x)**(5/2)/
5)/e**3 + 4*c**3*d**4*(d**4/sqrt(d + e*x) + 4*d**3*sqrt(d + e*x) - 2*d**2*(
d + e*x)**(3/2) + 4*d*(d + e*x)**(5/2)/5 - (d + e*x)**(7/2)/7)/e**3 + 2*c**
3*d**3*(-d**5/sqrt(d + e*x) - 5*d**4*sqrt(d + e*x) + 10*d**3*(d + e*x)**(3/
2)/3 - 2*d**2*(d + e*x)**(5/2) + 5*d*(d + e*x)**(7/2)/7 - (d + e*x)**(9/2)/
9)/e**3)/e, Ne(e, 0)), (c**3*d**(7/2)*x**4/4, True))
```

Giac [A] time = 1.20444, size = 250, normalized size = 2.1

$$\frac{2}{315}\Big(35\,(xe+d)^{\frac{9}{2}}c^3d^3e^{32}-135\,(xe+d)^{\frac{7}{2}}c^3d^4e^{32}+189\,(xe+d)^{\frac{5}{2}}c^3d^5e^{32}-105\,(xe+d)^{\frac{3}{2}}c^3d^6e^{32}+135\,(xe+d)^{\frac{7}{2}}ac^2d^2e^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(5/2),x, algorithm="g
iac")
```

```
[Out] 2/315*(35*(x*e + d)^(9/2)*c^3*d^3*e^32 - 135*(x*e + d)^(7/2)*c^3*d^4*e^32 +
 189*(x*e + d)^(5/2)*c^3*d^5*e^32 - 105*(x*e + d)^(3/2)*c^3*d^6*e^32 + 135*
(x*e + d)^(7/2)*a*c^2*d^2*e^34 - 378*(x*e + d)^(5/2)*a*c^2*d^3*e^34 + 315*(
x*e + d)^(3/2)*a*c^2*d^4*e^34 + 189*(x*e + d)^(5/2)*a^2*c*d*e^36 - 315*(x*e
 + d)^(3/2)*a^2*c*d^2*e^36 + 105*(x*e + d)^(3/2)*a^3*e^38)*e^(-36)
```

3.1996 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{6c^2d^2(d+ex)^{5/2}\left(cd^2-ae^2\right)}{5e^4}+\frac{2cd(d+ex)^{3/2}\left(cd^2-ae^2\right)^2}{e^4}-\frac{2\sqrt{d+ex}\left(cd^2-ae^2\right)^3}{e^4}+\frac{2c^3d^3(d+ex)^{7/2}}{7e^4}$$

```
[Out] (-2*(c*d^2 - a*e^2)^3*Sqrt[d + e*x])/e^4 + (2*c*d*(c*d^2 - a*e^2)^2*(d + e*
x)^(3/2))/e^4 - (6*c^2*d^2*(c*d^2 - a*e^2)*(d + e*x)^(5/2))/(5*e^4) + (2*c^
3*d^3*(d + e*x)^(7/2))/(7*e^4)
```

Rubi [A] time = 0.0545124, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {626, 43}

$$-\frac{6c^2d^2(d+ex)^{5/2}\left(cd^2-ae^2\right)}{5e^4}+\frac{2cd(d+ex)^{3/2}\left(cd^2-ae^2\right)^2}{e^4}-\frac{2\sqrt{d+ex}\left(cd^2-ae^2\right)^3}{e^4}+\frac{2c^3d^3(d+ex)^{7/2}}{7e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^(7/2),x]
```

```
[Out] (-2*(c*d^2 - a*e^2)^3*Sqrt[d + e*x])/e^4 + (2*c*d*(c*d^2 - a*e^2)^2*(d + e*
x)^(3/2))/e^4 - (6*c^2*d^2*(c*d^2 - a*e^2)*(d + e*x)^(5/2))/(5*e^4) + (2*c^
3*d^3*(d + e*x)^(7/2))/(7*e^4)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{7/2}}\,dx=\int \frac{(ae+cdx)^3}{\sqrt{d+ex}}\,dx$$

$$=\int \left(\frac{\left(-cd^2+ae^2\right)^3}{e^3\sqrt{d+ex}}+\frac{3cd\left(cd^2-ae^2\right)^2\sqrt{d+ex}}{e^3}-\frac{3c^2d^2\left(cd^2-ae^2\right)(d+ex)^{3/2}}{e^3}+\frac{c}{}\right.$$

$$=-\frac{2\left(cd^2-ae^2\right)^3\sqrt{d+ex}}{e^4}+\frac{2cd\left(cd^2-ae^2\right)^2(d+ex)^{3/2}}{e^4}-\frac{6c^2d^2\left(cd^2-ae^2\right)(d+e}{5e^4}$$

Mathematica [A] time = 0.0695736, size = 110, normalized size = 0.96

$$\frac{2\sqrt{d+ex}\left(35a^2cde^4(ex-2d)+35a^3e^6+7ac^2d^2e^2\left(8d^2-4dex+3e^2x^2\right)+c^3d^3\left(8d^2ex-16d^3-6de^2x^2+5e^3x^3\right)\right)}{35e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^(7/2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(35*a^3*e^6 + 35*a^2*c*d*e^4*(-2*d + e*x) + 7*a*c^2*d^2*e^
2*(8*d^2 - 4*d*e*x + 3*e^2*x^2) + c^3*d^3*(-16*d^3 + 8*d^2*e*x - 6*d*e^2*x^
2 + 5*e^3*x^3)))/(35*e^4)
```

Maple [A] time = 0.043, size = 131, normalized size = 1.1

$$\frac{10\,x^3c^3d^3e^3+42\,ac^2d^2e^4x^2-12\,c^3d^4e^2x^2+70\,a^2cde^5x-56\,ac^2d^3e^3x+16\,c^3d^5ex+70\,a^3e^6-140\,a^2cd^2e^4+112\,ac^2d^4e}{35\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(7/2),x)
```

```
[Out] 2/35*(e*x+d)^(1/2)*(5*c^3*d^3*e^3*x^3+21*a*c^2*d^2*e^4*x^2-6*c^3*d^4*e^2*x^
2+35*a^2*c*d*e^5*x-28*a*c^2*d^3*e^3*x+8*c^3*d^5*e*x+35*a^3*e^6-70*a^2*c*d^2
*e^4+56*a*c^2*d^4*e^2-16*c^3*d^6)/e^4
```

Maxima [A] time = 1.01014, size = 185, normalized size = 1.61

$$\frac{2\left(5\,(ex+d)^{\frac{7}{2}}c^3d^3-21\left(c^3d^4-ac^2d^2e^2\right)(ex+d)^{\frac{5}{2}}+35\left(c^3d^5-2\,ac^2d^3e^2+a^2cde^4\right)(ex+d)^{\frac{3}{2}}-35\left(c^3d^6-3\,ac^2d^4e^2+3\right.}{35\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(7/2),x, algorithm="m
axima")
```

```
[Out] 2/35*(5*(e*x + d)^(7/2)*c^3*d^3 - 21*(c^3*d^4 - a*c^2*d^2*e^2)*(e*x + d)^(5
/2) + 35*(c^3*d^5 - 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*(e*x + d)^(3/2) - 35*(c^
3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(e*x + d))/e^4
```

Fricas [A] time = 1.94957, size = 275, normalized size = 2.39

$$\frac{2\left(5\,c^3d^3e^3x^3-16\,c^3d^6+56\,ac^2d^4e^2-70\,a^2cd^2e^4+35\,a^3e^6-3\left(2\,c^3d^4e^2-7\,ac^2d^2e^4\right)x^2+\left(8\,c^3d^5e-28\,ac^2d^3e^3+35\,a\right.\right.}{35\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(7/2),x, algorithm="f
ricas")
```

```
[Out] 2/35*(5*c^3*d^3*e^3*x^3 - 16*c^3*d^6 + 56*a*c^2*d^4*e^2 - 70*a^2*c*d^2*e^4
+ 35*a^3*e^6 - 3*(2*c^3*d^4*e^2 - 7*a*c^2*d^2*e^4)*x^2 + (8*c^3*d^5*e - 28*
```

```
a*c^2*d^3*e^3 + 35*a^2*c*d*e^5)*x)*sqrt(e*x + d)/e^4
```

Sympy [A] time = 138.656, size = 376, normalized size = 3.27

$$\begin{cases} -\frac{\frac{2a^3de^3}{\sqrt{d+ex}}+2a^3e^3\left(-\frac{d}{\sqrt{d+ex}}-\sqrt{d+ex}\right)+6a^2cd^2e\left(-\frac{d}{\sqrt{d+ex}}-\sqrt{d+ex}\right)+6a^2cde\left(\frac{d^2}{\sqrt{d+ex}}+2d\sqrt{d+ex}-\frac{(d+ex)^{\frac{3}{2}}}{3}\right)+\frac{6ac^2d^3\left(\frac{d^2}{\sqrt{d+ex}}+2d\sqrt{d+ex}-\frac{(d+ex)^{\frac{3}{2}}}{3}\right)}{e}+\frac{6ac^2d^2\left(-\frac{d^3}{\sqrt{d+ex}}-3d^2\sqrt{d+ex}\right.}{e}}{e} \\ \frac{c^3d^{\frac{5}{2}}x^4}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**(7/2),x)
```

```
[Out] Piecewise((-(2*a**3*d*e**3/sqrt(d + e*x) + 2*a**3*e**3*(-d/sqrt(d + e*x) -
sqrt(d + e*x)) + 6*a**2*c*d**2*e*(-d/sqrt(d + e*x) - sqrt(d + e*x)) + 6*a**
2*c*d*e*(d**2/sqrt(d + e*x) + 2*d*sqrt(d + e*x) - (d + e*x)**(3/2)/3) + 6*a
*c**2*d**3*(d**2/sqrt(d + e*x) + 2*d*sqrt(d + e*x) - (d + e*x)**(3/2)/3)/e
+ 6*a*c**2*d**2*(-d**3/sqrt(d + e*x) - 3*d**2*sqrt(d + e*x) + d*(d + e*x)**
(3/2) - (d + e*x)**(5/2)/5)/e + 2*c**3*d**4*(-d**3/sqrt(d + e*x) - 3*d**2*s
qrt(d + e*x) + d*(d + e*x)**(3/2) - (d + e*x)**(5/2)/5)/e**3 + 2*c**3*d**3*
(d**4/sqrt(d + e*x) + 4*d**3*sqrt(d + e*x) - 2*d**2*(d + e*x)**(3/2) + 4*d*
(d + e*x)**(5/2)/5 - (d + e*x)**(7/2)/7)/e**3)/e, Ne(e, 0)), (c**3*d**(5/2)
*x**4/4, True))
```

Giac [A] time = 1.2541, size = 250, normalized size = 2.17

$$\frac{2}{35}\left(5(xe+d)^{\frac{7}{2}}c^3d^3e^{24}-21(xe+d)^{\frac{5}{2}}c^3d^4e^{24}+35(xe+d)^{\frac{3}{2}}c^3d^5e^{24}-35\sqrt{xe+d}c^3d^6e^{24}+21(xe+d)^{\frac{5}{2}}ac^2d^2e^{26}-70(xe+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(7/2),x, algorithm="g
iac")
```

```
[Out] 2/35*(5*(x*e + d)^(7/2)*c^3*d^3*e^24 - 21*(x*e + d)^(5/2)*c^3*d^4*e^24 + 35
*(x*e + d)^(3/2)*c^3*d^5*e^24 - 35*sqrt(x*e + d)*c^3*d^6*e^24 + 21*(x*e + d
)^(5/2)*a*c^2*d^2*e^26 - 70*(x*e + d)^(3/2)*a*c^2*d^3*e^26 + 105*sqrt(x*e +
 d)*a*c^2*d^4*e^26 + 35*(x*e + d)^(3/2)*a^2*c*d*e^28 - 105*sqrt(x*e + d)*a^
2*c*d^2*e^28 + 35*sqrt(x*e + d)*a^3*e^30)*e^(-28)
```

3.1997 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{9/2}}\,dx$

Optimal. Leaf size=113

$$-\frac{2c^2d^2(d+ex)^{3/2}\left(cd^2-ae^2\right)}{e^4}+\frac{6cd\sqrt{d+ex}\left(cd^2-ae^2\right)^2}{e^4}+\frac{2\left(cd^2-ae^2\right)^3}{e^4\sqrt{d+ex}}+\frac{2c^3d^3(d+ex)^{5/2}}{5e^4}$$

```
[Out] (2*(c*d^2 - a*e^2)^3)/(e^4*Sqrt[d + e*x]) + (6*c*d*(c*d^2 - a*e^2)^2*Sqrt[d
 + e*x])/e^4 - (2*c^2*d^2*(c*d^2 - a*e^2)*(d + e*x)^(3/2))/e^4 + (2*c^3*d^3
*(d + e*x)^(5/2))/(5*e^4)
```

Rubi [A] time = 0.0515285, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {626, 43}

$$-\frac{2c^2d^2(d+ex)^{3/2}\left(cd^2-ae^2\right)}{e^4}+\frac{6cd\sqrt{d+ex}\left(cd^2-ae^2\right)^2}{e^4}+\frac{2\left(cd^2-ae^2\right)^3}{e^4\sqrt{d+ex}}+\frac{2c^3d^3(d+ex)^{5/2}}{5e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^(9/2),x]
```

```
[Out] (2*(c*d^2 - a*e^2)^3)/(e^4*Sqrt[d + e*x]) + (6*c*d*(c*d^2 - a*e^2)^2*Sqrt[d
 + e*x])/e^4 - (2*c^2*d^2*(c*d^2 - a*e^2)*(d + e*x)^(3/2))/e^4 + (2*c^3*d^3
*(d + e*x)^(5/2))/(5*e^4)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{9/2}}\,dx=\int \frac{(ae+cdx)^3}{(d+ex)^{3/2}}\,dx$$
$$=\int \left(\frac{\left(-cd^2+ae^2\right)^3}{e^3(d+ex)^{3/2}}+\frac{3cd\left(cd^2-ae^2\right)^2}{e^3\sqrt{d+ex}}-\frac{3c^2d^2\left(cd^2-ae^2\right)\sqrt{d+ex}}{e^3}+\frac{c^3d^3(d+}{e^3}\right.$$
$$=\frac{2\left(cd^2-ae^2\right)^3}{e^4\sqrt{d+ex}}+\frac{6cd\left(cd^2-ae^2\right)^2\sqrt{d+ex}}{e^4}-\frac{2c^2d^2\left(cd^2-ae^2\right)(d+ex)^{3/2}}{e^4}+2c$$

Mathematica [A] time = 0.0568238, size = 109, normalized size = 0.96

$$\frac{2\left(15a^2cde^4(2d+ex)-5a^3e^6-5ac^2d^2e^2\left(8d^2+4dex-e^2x^2\right)+c^3d^3\left(8d^2ex+16d^3-2de^2x^2+e^3x^3\right)\right)}{5e^4\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^(9/2),x]
```

```
[Out] (2*(-5*a^3*e^6 + 15*a^2*c*d*e^4*(2*d + e*x) - 5*a*c^2*d^2*e^2*(8*d^2 + 4*d*
e*x - e^2*x^2) + c^3*d^3*(16*d^3 + 8*d^2*e*x - 2*d*e^2*x^2 + e^3*x^3)))/(5*
e^4*Sqrt[d + e*x])
```

Maple [A] time = 0.046, size = 131, normalized size = 1.2

$$-\frac{-2x^3c^3d^3e^3-10ac^2d^2e^4x^2+4c^3d^4e^2x^2-30a^2cde^5x+40ac^2d^3e^3x-16c^3d^5ex+10a^3e^6-60a^2cd^2e^4+80ac^2d^4e^2-3}{5e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(9/2),x)
```

```
[Out] -2/5/(e*x+d)^(1/2)*(-c^3*d^3*e^3*x^3-5*a*c^2*d^2*e^4*x^2+2*c^3*d^4*e^2*x^2-
15*a^2*c*d*e^5*x+20*a*c^2*d^3*e^3*x-8*c^3*d^5*e*x+5*a^3*e^6-30*a^2*c*d^2*e^
4+40*a*c^2*d^4*e^2-16*c^3*d^6)/e^4
```

Maxima [A] time = 1.05078, size = 194, normalized size = 1.72

$$\frac{2\left(\frac{(ex+d)^{\frac{5}{2}}c^3d^3-5\left(c^3d^4-ac^2d^2e^2\right)(ex+d)^{\frac{3}{2}}+15\left(c^3d^5-2ac^2d^3e^2+a^2cde^4\right)\sqrt{ex+d}}{e^3}+\frac{5\left(c^3d^6-3ac^2d^4e^2+3a^2cd^2e^4-a^3e^6\right)}{\sqrt{ex+d}e^3}\right)}{5e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(9/2),x, algorithm="m
axima")
```

```
[Out] 2/5*(((e*x + d)^(5/2)*c^3*d^3 - 5*(c^3*d^4 - a*c^2*d^2*e^2)*(e*x + d)^(3/2)
 + 15*(c^3*d^5 - 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*sqrt(e*x + d))/e^3 + 5*(c^3
*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)/(sqrt(e*x + d)*e^3))/e
```

Fricas [A] time = 1.86707, size = 284, normalized size = 2.51

$$\frac{2\left(c^3d^3e^3x^3+16c^3d^6-40ac^2d^4e^2+30a^2cd^2e^4-5a^3e^6-\left(2c^3d^4e^2-5ac^2d^2e^4\right)x^2+\left(8c^3d^5e-20ac^2d^3e^3+15a^2cde^5\right)x\right)}{5\left(e^5x+de^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(9/2),x, algorithm="f
ricas")
```

```
[Out] 2/5*(c^3*d^3*e^3*x^3 + 16*c^3*d^6 - 40*a*c^2*d^4*e^2 + 30*a^2*c*d^2*e^4 - 5
*a^3*e^6 - (2*c^3*d^4*e^2 - 5*a*c^2*d^2*e^4)*x^2 + (8*c^3*d^5*e - 20*a*c^2*
d^3*e^3 + 15*a^2*c*d*e^5)*x)*sqrt(e*x + d)/(e^5*x + d*e^4)
```

Sympy [A] time = 16.4614, size = 230, normalized size = 2.04

$$\begin{cases} -\frac{2a^3e^2}{\sqrt{d+ex}} + \frac{12a^2cd^2}{\sqrt{d+ex}} + \frac{6a^2cdex}{\sqrt{d+ex}} - \frac{16ac^2d^4}{e^2\sqrt{d+ex}} - \frac{8ac^2d^3x}{e\sqrt{d+ex}} + \frac{2ac^2d^2x^2}{\sqrt{d+ex}} + \frac{32c^3d^6}{5e^4\sqrt{d+ex}} + \frac{16c^3d^5x}{5e^3\sqrt{d+ex}} - \frac{4c^3d^4x^2}{5e^2\sqrt{d+ex}} + \frac{2c^3d^3x^3}{5e\sqrt{d+ex}} & \text{for } e \neq 0 \\ \frac{c^3d^{\frac{3}{2}}x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**(9/2),x)
```

```
[Out] Piecewise((-2*a**3*e**2/sqrt(d + e*x) + 12*a**2*c*d**2/sqrt(d + e*x) + 6*a*
*2*c*d*e*x/sqrt(d + e*x) - 16*a*c**2*d**4/(e**2*sqrt(d + e*x)) - 8*a*c**2*d
**3*x/(e*sqrt(d + e*x)) + 2*a*c**2*d**2*x**2/sqrt(d + e*x) + 32*c**3*d**6/(
5*e**4*sqrt(d + e*x)) + 16*c**3*d**5*x/(5*e**3*sqrt(d + e*x)) - 4*c**3*d**4
*x**2/(5*e**2*sqrt(d + e*x)) + 2*c**3*d**3*x**3/(5*e*sqrt(d + e*x)), Ne(e,
0)), (c**3*d**(3/2)*x**4/4, True))
```

Giac [A] time = 1.21464, size = 263, normalized size = 2.33

$$\frac{2}{5}\left((xe+d)^{\frac{5}{2}}c^3d^3e^{16} - 5(xe+d)^{\frac{3}{2}}c^3d^4e^{16} + 15\sqrt{xe+d}c^3d^5e^{16} + 5(xe+d)^{\frac{3}{2}}ac^2d^2e^{18} - 30\sqrt{xe+d}ac^2d^3e^{18} + 15\sqrt{xe+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(9/2),x, algorithm="g
iac")
```

```
[Out] 2/5*((x*e + d)^(5/2)*c^3*d^3*e^16 - 5*(x*e + d)^(3/2)*c^3*d^4*e^16 + 15*sqr
t(x*e + d)*c^3*d^5*e^16 + 5*(x*e + d)^(3/2)*a*c^2*d^2*e^18 - 30*sqrt(x*e +
d)*a*c^2*d^3*e^18 + 15*sqrt(x*e + d)*a^2*c*d*e^20)*e^(-20) + 2*((x*e + d)^3
*c^3*d^6 - 3*(x*e + d)^3*a*c^2*d^4*e^2 + 3*(x*e + d)^3*a^2*c*d^2*e^4 - (x*e
 + d)^3*a^3*e^6)*e^(-4)/(x*e + d)^(7/2)
```

3.1998 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{11/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{6c^2d^2\sqrt{d+ex}\left(cd^2-ae^2\right)}{e^4}-\frac{6cd\left(cd^2-ae^2\right)^2}{e^4\sqrt{d+ex}}+\frac{2\left(cd^2-ae^2\right)^3}{3e^4(d+ex)^{3/2}}+\frac{2c^3d^3(d+ex)^{3/2}}{3e^4}$$

```
[Out] (2*(c*d^2 - a*e^2)^3)/(3*e^4*(d + e*x)^(3/2)) - (6*c*d*(c*d^2 - a*e^2)^2)/(
e^4*Sqrt[d + e*x]) - (6*c^2*d^2*(c*d^2 - a*e^2)*Sqrt[d + e*x])/e^4 + (2*c^3
*d^3*(d + e*x)^(3/2))/(3*e^4)
```

Rubi [A] time = 0.0520726, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {626, 43}

$$-\frac{6c^2d^2\sqrt{d+ex}\left(cd^2-ae^2\right)}{e^4}-\frac{6cd\left(cd^2-ae^2\right)^2}{e^4\sqrt{d+ex}}+\frac{2\left(cd^2-ae^2\right)^3}{3e^4(d+ex)^{3/2}}+\frac{2c^3d^3(d+ex)^{3/2}}{3e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^(11/2),x]
```

```
[Out] (2*(c*d^2 - a*e^2)^3)/(3*e^4*(d + e*x)^(3/2)) - (6*c*d*(c*d^2 - a*e^2)^2)/(
e^4*Sqrt[d + e*x]) - (6*c^2*d^2*(c*d^2 - a*e^2)*Sqrt[d + e*x])/e^4 + (2*c^3
*d^3*(d + e*x)^(3/2))/(3*e^4)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{11/2}}\,dx &= \int \frac{(ae+cdx)^3}{(d+ex)^{5/2}}\,dx\\
&= \int \left(\frac{\left(-cd^2+ae^2\right)^3}{e^3(d+ex)^{5/2}}+\frac{3cd\left(cd^2-ae^2\right)^2}{e^3(d+ex)^{3/2}}-\frac{3c^2d^2\left(cd^2-ae^2\right)}{e^3\sqrt{d+ex}}+\frac{c^3d^3\sqrt{d+ex}}{e^3}\right)dx\\
&= \frac{2\left(cd^2-ae^2\right)^3}{3e^4(d+ex)^{3/2}}-\frac{6cd\left(cd^2-ae^2\right)^2}{e^4\sqrt{d+ex}}-\frac{6c^2d^2\left(cd^2-ae^2\right)\sqrt{d+ex}}{e^4}+\frac{2c^3d^3(d+ex)^{3/2}}{3e^4}
\end{aligned}$$

Mathematica [A] time = 0.0598344, size = 110, normalized size = 0.96

$$-\frac{2\left(3a^2cde^4(2d+3ex)+a^3e^6-3ac^2d^2e^2\left(8d^2+12dex+3e^2x^2\right)+c^3d^3\left(24d^2ex+16d^3+6de^2x^2-e^3x^3\right)\right)}{3e^4(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^(11/2),x]
```

```
[Out] (-2*(a^3*e^6 + 3*a^2*c*d*e^4*(2*d + 3*e*x) - 3*a*c^2*d^2*e^2*(8*d^2 + 12*d*
e*x + 3*e^2*x^2) + c^3*d^3*(16*d^3 + 24*d^2*e*x + 6*d*e^2*x^2 - e^3*x^3)))/
(3*e^4*(d + e*x)^(3/2))
```

Maple [A] time = 0.044, size = 130, normalized size = 1.1

$$-\frac{-2x^3c^3d^3e^3-18ac^2d^2e^4x^2+12c^3d^4e^2x^2+18a^2cde^5x-72ac^2d^3e^3x+48c^3d^5ex+2a^3e^6+12a^2cd^2e^4-48ac^2d^4e^2}{3e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(11/2),x)
```

```
[Out] -2/3/(e*x+d)^(3/2)*(-c^3*d^3*e^3*x^3-9*a*c^2*d^2*e^4*x^2+6*c^3*d^4*e^2*x^2+
9*a^2*c*d*e^5*x-36*a*c^2*d^3*e^3*x+24*c^3*d^5*e*x+a^3*e^6+6*a^2*c*d^2*e^4-2
4*a*c^2*d^4*e^2+16*c^3*d^6)/e^4
```

Maxima [A] time = 1.01749, size = 190, normalized size = 1.65

$$\frac{2\left(\frac{(ex+d)^{\frac{3}{2}}c^3d^3-9\left(c^3d^4-ac^2d^2e^2\right)\sqrt{ex+d}}{e^3}+\frac{c^3d^6-3ac^2d^4e^2+3a^2cd^2e^4-a^3e^6-9\left(c^3d^5-2ac^2d^3e^2+a^2cde^4\right)(ex+d)}{(ex+d)^{\frac{3}{2}}e^3}\right)}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(11/2),x, algorithm="
maxima")
```

```
[Out] 2/3*(((e*x + d)^(3/2)*c^3*d^3 - 9*(c^3*d^4 - a*c^2*d^2*e^2)*sqrt(e*x + d))/
e^3 + (c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6 - 9*(c^3*d^5 -
 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*(e*x + d))/((e*x + d)^(3/2)*e^3))/e
```

Fricas [A] time = 1.87473, size = 305, normalized size = 2.65

$$\frac{2\left(c^3d^3e^3x^3-16c^3d^6+24ac^2d^4e^2-6a^2cd^2e^4-a^3e^6-3\left(2c^3d^4e^2-3ac^2d^2e^4\right)x^2-3\left(8c^3d^5e-12ac^2d^3e^3+3a^2cde^5\right)\right.}{3\left(e^6x^2+2de^5x+d^2e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(11/2),x, algorithm="
fricas")
```

```
[Out] 2/3*(c^3*d^3*e^3*x^3 - 16*c^3*d^6 + 24*a*c^2*d^4*e^2 - 6*a^2*c*d^2*e^4 - a^
3*e^6 - 3*(2*c^3*d^4*e^2 - 3*a*c^2*d^2*e^4)*x^2 - 3*(8*c^3*d^5*e - 12*a*c^2
*d^3*e^3 + 3*a^2*c*d*e^5)*x)*sqrt(e*x + d)/(e^6*x^2 + 2*d*e^5*x + d^2*e^4)
```

Sympy [A] time = 31.4873, size = 450, normalized size = 3.91

$$\begin{cases} -\frac{2a^3e^6}{3de^4\sqrt{d+ex}+3e^5x\sqrt{d+ex}} - \frac{12a^2cd^2e^4}{3de^4\sqrt{d+ex}+3e^5x\sqrt{d+ex}} - \frac{18a^2cde^5x}{3de^4\sqrt{d+ex}+3e^5x\sqrt{d+ex}} + \frac{48ac^2d^4e^2}{3de^4\sqrt{d+ex}+3e^5x\sqrt{d+ex}} + \frac{72ac^2d^3e^3x}{3de^4\sqrt{d+ex}+3e^5x\sqrt{d+ex}} + \frac{18ac^2d^2 \cdots}{3de^4\sqrt{d+ex}+3 \cdots} \\ \frac{c^3\sqrt{d}x^4}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**(11/2),x)
```

```
[Out] Piecewise((-2*a**3*e**6/(3*d*e**4*sqrt(d + e*x) + 3*e**5*x*sqrt(d + e*x)) -
 12*a**2*c*d**2*e**4/(3*d*e**4*sqrt(d + e*x) + 3*e**5*x*sqrt(d + e*x)) - 18
*a**2*c*d*e**5*x/(3*d*e**4*sqrt(d + e*x) + 3*e**5*x*sqrt(d + e*x)) + 48*a*c
**2*d**4*e**2/(3*d*e**4*sqrt(d + e*x) + 3*e**5*x*sqrt(d + e*x)) + 72*a*c**2
*d**3*e**3*x/(3*d*e**4*sqrt(d + e*x) + 3*e**5*x*sqrt(d + e*x)) + 18*a*c**2*
d**2*e**4*x**2/(3*d*e**4*sqrt(d + e*x) + 3*e**5*x*sqrt(d + e*x)) - 32*c**3*
d**6/(3*d*e**4*sqrt(d + e*x) + 3*e**5*x*sqrt(d + e*x)) - 48*c**3*d**5*e*x/(
3*d*e**4*sqrt(d + e*x) + 3*e**5*x*sqrt(d + e*x)) - 12*c**3*d**4*e**2*x**2/(
3*d*e**4*sqrt(d + e*x) + 3*e**5*x*sqrt(d + e*x)) + 2*c**3*d**3*e**3*x**3/(3
*d*e**4*sqrt(d + e*x) + 3*e**5*x*sqrt(d + e*x)), Ne(e, 0)), (c**3*sqrt(d)*x
**4/4, True))
```

Giac [A] time = 1.19244, size = 261, normalized size = 2.27

$$\frac{2}{3}\left((xe+d)^{\frac{3}{2}}c^3d^3e^8 - 9\sqrt{xe+d}c^3d^4e^8 + 9\sqrt{xe+d}ac^2d^2e^{10}\right)e^{(-12)} - \frac{2\left(9(xe+d)^4c^3d^5 - (xe+d)^3c^3d^6 - 18(xe+d)^4ac^2d^3 \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(11/2),x, algorithm="
giac")
```

```
[Out] 2/3*((x*e + d)^(3/2)*c^3*d^3*e^8 - 9*sqrt(x*e + d)*c^3*d^4*e^8 + 9*sqrt(x*e
 + d)*a*c^2*d^2*e^10)*e^(-12) - 2/3*(9*(x*e + d)^4*c^3*d^5 - (x*e + d)^3*c^
3*d^6 - 18*(x*e + d)^4*a*c^2*d^3*e^2 + 3*(x*e + d)^3*a*c^2*d^4*e^2 + 9*(x*e
 + d)^4*a^2*c*d*e^4 - 3*(x*e + d)^3*a^2*c*d^2*e^4 + (x*e + d)^3*a^3*e^6)*e^
(-4)/(x*e + d)^(9/2)
```

3.1999 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{13/2}}\,dx$

Optimal. Leaf size=113

$$\frac{6c^2d^2\left(cd^2-ae^2\right)}{e^4\sqrt{d+ex}}-\frac{2cd\left(cd^2-ae^2\right)^2}{e^4(d+ex)^{3/2}}+\frac{2\left(cd^2-ae^2\right)^3}{5e^4(d+ex)^{5/2}}+\frac{2c^3d^3\sqrt{d+ex}}{e^4}$$

```
[Out] (2*(c*d^2 - a*e^2)^3)/(5*e^4*(d + e*x)^(5/2)) - (2*c*d*(c*d^2 - a*e^2)^2)/(
e^4*(d + e*x)^(3/2)) + (6*c^2*d^2*(c*d^2 - a*e^2))/(e^4*Sqrt[d + e*x]) + (2
*c^3*d^3*Sqrt[d + e*x])/e^4
```

Rubi [A] time = 0.0530466, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {626, 43}

$$\frac{6c^2d^2\left(cd^2-ae^2\right)}{e^4\sqrt{d+ex}}-\frac{2cd\left(cd^2-ae^2\right)^2}{e^4(d+ex)^{3/2}}+\frac{2\left(cd^2-ae^2\right)^3}{5e^4(d+ex)^{5/2}}+\frac{2c^3d^3\sqrt{d+ex}}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^(13/2),x]
```

```
[Out] (2*(c*d^2 - a*e^2)^3)/(5*e^4*(d + e*x)^(5/2)) - (2*c*d*(c*d^2 - a*e^2)^2)/(
e^4*(d + e*x)^(3/2)) + (6*c^2*d^2*(c*d^2 - a*e^2))/(e^4*Sqrt[d + e*x]) + (2
*c^3*d^3*Sqrt[d + e*x])/e^4
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{13/2}}\,dx &= \int \frac{(ae+cdx)^3}{(d+ex)^{7/2}}\,dx\\
&= \int \left(\frac{\left(-cd^2+ae^2\right)^3}{e^3(d+ex)^{7/2}}+\frac{3cd\left(cd^2-ae^2\right)^2}{e^3(d+ex)^{5/2}}-\frac{3c^2d^2\left(cd^2-ae^2\right)}{e^3(d+ex)^{3/2}}+\frac{c^3d^3}{e^3\sqrt{d+ex}}\right)dx\\
&= \frac{2\left(cd^2-ae^2\right)^3}{5e^4(d+ex)^{5/2}}-\frac{2cd\left(cd^2-ae^2\right)^2}{e^4(d+ex)^{3/2}}+\frac{6c^2d^2\left(cd^2-ae^2\right)}{e^4\sqrt{d+ex}}+\frac{2c^3d^3\sqrt{d+ex}}{e^4}
\end{aligned}$$

Mathematica [A] time = 0.0561081, size = 109, normalized size = 0.96

$$-\frac{2\left(a^2cde^4(2d+5ex)+a^3e^6+ac^2d^2e^2\left(8d^2+20dex+15e^2x^2\right)-c^3d^3\left(40d^2ex+16d^3+30de^2x^2+5e^3x^3\right)\right)}{5e^4(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^(13/2),x]
```

```
[Out] (-2*(a^3*e^6 + a^2*c*d*e^4*(2*d + 5*e*x) + a*c^2*d^2*e^2*(8*d^2 + 20*d*e*x
+ 15*e^2*x^2) - c^3*d^3*(16*d^3 + 40*d^2*e*x + 30*d*e^2*x^2 + 5*e^3*x^3)))/
(5*e^4*(d + e*x)^(5/2))
```

Maple [A] time = 0.044, size = 130, normalized size = 1.2

$$-\frac{-10\,x^3c^3d^3e^3+30\,ac^2d^2e^4x^2-60\,c^3d^4e^2x^2+10\,a^2cde^5x+40\,ac^2d^3e^3x-80\,c^3d^5ex+2\,a^3e^6+4\,a^2cd^2e^4+16\,ac^2d^4e^2-3}{5\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(13/2),x)
```

```
[Out] -2/5/(e*x+d)^(5/2)*(-5*c^3*d^3*e^3*x^3+15*a*c^2*d^2*e^4*x^2-30*c^3*d^4*e^2*
x^2+5*a^2*c*d*e^5*x+20*a*c^2*d^3*e^3*x-40*c^3*d^5*e*x+a^3*e^6+2*a^2*c*d^2*e
^4+8*a*c^2*d^4*e^2-16*c^3*d^6)/e^4
```

Maxima [A] time = 1.04851, size = 189, normalized size = 1.67

$$\frac{2\left(\frac{5\sqrt{ex+d}c^3d^3}{e^3}+\frac{c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6+15\left(c^3d^4-ac^2d^2e^2\right)(ex+d)^2-5\left(c^3d^5-2\,ac^2d^3e^2+a^2cde^4\right)(ex+d)}{(ex+d)^{\frac{5}{2}}e^3}\right)}{5\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(13/2),x, algorithm="
maxima")
```

```
[Out] 2/5*(5*sqrt(e*x + d)*c^3*d^3/e^3 + (c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2
*e^4 - a^3*e^6 + 15*(c^3*d^4 - a*c^2*d^2*e^2)*(e*x + d)^2 - 5*(c^3*d^5 - 2*
a*c^2*d^3*e^2 + a^2*c*d*e^4)*(e*x + d))/((e*x + d)^(5/2)*e^3))/e
```

Fricas [A] time = 1.93331, size = 323, normalized size = 2.86

$$\frac{2\left(5\,c^3d^3e^3x^3+16\,c^3d^6-8\,ac^2d^4e^2-2\,a^2cd^2e^4-a^3e^6+15\left(2\,c^3d^4e^2-ac^2d^2e^4\right)x^2+5\left(8\,c^3d^5e-4\,ac^2d^3e^3-a^2cde^5\right)x\right)\sqrt{e}}{5\left(e^7x^3+3\,de^6x^2+3\,d^2e^5x+d^3e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(13/2),x, algorithm="
fricas")
```

```
[Out] 2/5*(5*c^3*d^3*e^3*x^3 + 16*c^3*d^6 - 8*a*c^2*d^4*e^2 - 2*a^2*c*d^2*e^4 - a
^3*e^6 + 15*(2*c^3*d^4*e^2 - a*c^2*d^2*e^4)*x^2 + 5*(8*c^3*d^5*e - 4*a*c^2*
d^3*e^3 - a^2*c*d*e^5)*x)*sqrt(e*x + d)/(e^7*x^3 + 3*d*e^6*x^2 + 3*d^2*e^5*
x + d^3*e^4)
```

Sympy [A] time = 47.2321, size = 654, normalized size = 5.79

$$\begin{cases} -\frac{2a^3e^6}{5d^2e^4\sqrt{d+ex}+10de^5x\sqrt{d+ex}+5e^6x^2\sqrt{d+ex}} - \frac{4a^2cd^2e^4}{5d^2e^4\sqrt{d+ex}+10de^5x\sqrt{d+ex}+5e^6x^2\sqrt{d+ex}} - \frac{10a^2cde^5x}{5d^2e^4\sqrt{d+ex}+10de^5x\sqrt{d+ex}+5e^6x^2\sqrt{d+ex}} - \frac{}{5d^2e^4\sqrt{d+ex}+1} \\ \frac{c^3x^4}{4\sqrt{d}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**(13/2),x)
```

```
[Out] Piecewise((-2*a**3*e**6/(5*d**2*e**4*sqrt(d + e*x) + 10*d*e**5*x*sqrt(d + e
*x) + 5*e**6*x**2*sqrt(d + e*x)) - 4*a**2*c*d**2*e**4/(5*d**2*e**4*sqrt(d +
 e*x) + 10*d*e**5*x*sqrt(d + e*x) + 5*e**6*x**2*sqrt(d + e*x)) - 10*a**2*c*
d*e**5*x/(5*d**2*e**4*sqrt(d + e*x) + 10*d*e**5*x*sqrt(d + e*x) + 5*e**6*x*
*2*sqrt(d + e*x)) - 16*a*c**2*d**4*e**2/(5*d**2*e**4*sqrt(d + e*x) + 10*d*e
**5*x*sqrt(d + e*x) + 5*e**6*x**2*sqrt(d + e*x)) - 40*a*c**2*d**3*e**3*x/(5
*d**2*e**4*sqrt(d + e*x) + 10*d*e**5*x*sqrt(d + e*x) + 5*e**6*x**2*sqrt(d +
 e*x)) - 30*a*c**2*d**2*e**4*x**2/(5*d**2*e**4*sqrt(d + e*x) + 10*d*e**5*x*
sqrt(d + e*x) + 5*e**6*x**2*sqrt(d + e*x)) + 32*c**3*d**6/(5*d**2*e**4*sqrt
(d + e*x) + 10*d*e**5*x*sqrt(d + e*x) + 5*e**6*x**2*sqrt(d + e*x)) + 80*c**
3*d**5*e*x/(5*d**2*e**4*sqrt(d + e*x) + 10*d*e**5*x*sqrt(d + e*x) + 5*e**6*
x**2*sqrt(d + e*x)) + 60*c**3*d**4*e**2*x**2/(5*d**2*e**4*sqrt(d + e*x) + 1
0*d*e**5*x*sqrt(d + e*x) + 5*e**6*x**2*sqrt(d + e*x)) + 10*c**3*d**3*e**3*x
**3/(5*d**2*e**4*sqrt(d + e*x) + 10*d*e**5*x*sqrt(d + e*x) + 5*e**6*x**2*sq
rt(d + e*x)), Ne(e, 0)), (c**3*x**4/(4*sqrt(d)), True))
```

Giac [A] time = 1.23598, size = 252, normalized size = 2.23

$$2\sqrt{xe+d}c^3d^3e^{(-4)} + \frac{2\left(15\,(xe+d)^5c^3d^4 - 5\,(xe+d)^4c^3d^5 + (xe+d)^3c^3d^6 - 15\,(xe+d)^5ac^2d^2e^2 + 10\,(xe+d)^4ac^2d^3e^2\right.}{5\,(xe+d)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(13/2),x, algorithm="
giac")
```

```
[Out] 2*sqrt(x*e + d)*c^3*d^3*e^(-4) + 2/5*(15*(x*e + d)^5*c^3*d^4 - 5*(x*e + d)^
4*c^3*d^5 + (x*e + d)^3*c^3*d^6 - 15*(x*e + d)^5*a*c^2*d^2*e^2 + 10*(x*e +
d)^4*a*c^2*d^3*e^2 - 3*(x*e + d)^3*a*c^2*d^4*e^2 - 5*(x*e + d)^4*a^2*c*d*e^
4 + 3*(x*e + d)^3*a^2*c*d^2*e^4 - (x*e + d)^3*a^3*e^6)*e^(-4)/(x*e + d)^(11
/2)
```

3.2000 $\int \frac{(d+ex)^{9/2}}{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx$

Optimal. Leaf size=180

$$\frac{2\sqrt{d+ex}\left(cd^2-ae^2\right)^3}{c^4d^4}+\frac{2(d+ex)^{3/2}\left(cd^2-ae^2\right)^2}{3c^3d^3}+\frac{2(d+ex)^{5/2}\left(cd^2-ae^2\right)}{5c^2d^2}-\frac{2\left(cd^2-ae^2\right)^{7/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{9/2}d^{9/2}}+\frac{2(d}{}$$

```
[Out] (2*(c*d^2 - a*e^2)^3*Sqrt[d + e*x])/(c^4*d^4) + (2*(c*d^2 - a*e^2)^2*(d + e
*x)^(3/2))/(3*c^3*d^3) + (2*(c*d^2 - a*e^2)*(d + e*x)^(5/2))/(5*c^2*d^2) +
(2*(d + e*x)^(7/2))/(7*c*d) - (2*(c*d^2 - a*e^2)^(7/2)*ArcTanh[(Sqrt[c]*Sqr
t[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(c^(9/2)*d^(9/2))
```

Rubi [A] time = 0.237441, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {626, 50, 63, 208}

$$\frac{2\sqrt{d+ex}\left(cd^2-ae^2\right)^3}{c^4d^4}+\frac{2(d+ex)^{3/2}\left(cd^2-ae^2\right)^2}{3c^3d^3}+\frac{2(d+ex)^{5/2}\left(cd^2-ae^2\right)}{5c^2d^2}-\frac{2\left(cd^2-ae^2\right)^{7/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{9/2}d^{9/2}}+\frac{2(d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(9/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] (2*(c*d^2 - a*e^2)^3*Sqrt[d + e*x])/(c^4*d^4) + (2*(c*d^2 - a*e^2)^2*(d + e
*x)^(3/2))/(3*c^3*d^3) + (2*(c*d^2 - a*e^2)*(d + e*x)^(5/2))/(5*c^2*d^2) +
(2*(d + e*x)^(7/2))/(7*c*d) - (2*(c*d^2 - a*e^2)^(7/2)*ArcTanh[(Sqrt[c]*Sqr
t[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(c^(9/2)*d^(9/2))
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{9/2}}{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx = \int \frac{(d+ex)^{7/2}}{ae+cdx}\,dx$$

$$= \frac{2(d+ex)^{7/2}}{7cd} + \frac{\left(cd^2-ae^2\right)\int \frac{(d+ex)^{5/2}}{ae+cdx}\,dx}{cd}$$

$$= \frac{2\left(cd^2-ae^2\right)(d+ex)^{5/2}}{5c^2d^2} + \frac{2(d+ex)^{7/2}}{7cd} + \frac{\left(cd^2-ae^2\right)^2\int \frac{(d+ex)^{3/2}}{ae+cdx}\,dx}{c^2d^2}$$

$$= \frac{2\left(cd^2-ae^2\right)^2(d+ex)^{3/2}}{3c^3d^3} + \frac{2\left(cd^2-ae^2\right)(d+ex)^{5/2}}{5c^2d^2} + \frac{2(d+ex)^{7/2}}{7cd} + \frac{\left(cd^2-ae^2\right)}{c^3}$$

$$= \frac{2\left(cd^2-ae^2\right)^3\sqrt{d+ex}}{c^4d^4} + \frac{2\left(cd^2-ae^2\right)^2(d+ex)^{3/2}}{3c^3d^3} + \frac{2\left(cd^2-ae^2\right)(d+ex)^{5/2}}{5c^2d^2} +$$

$$= \frac{2\left(cd^2-ae^2\right)^3\sqrt{d+ex}}{c^4d^4} + \frac{2\left(cd^2-ae^2\right)^2(d+ex)^{3/2}}{3c^3d^3} + \frac{2\left(cd^2-ae^2\right)(d+ex)^{5/2}}{5c^2d^2} +$$

$$= \frac{2\left(cd^2-ae^2\right)^3\sqrt{d+ex}}{c^4d^4} + \frac{2\left(cd^2-ae^2\right)^2(d+ex)^{3/2}}{3c^3d^3} + \frac{2\left(cd^2-ae^2\right)(d+ex)^{5/2}}{5c^2d^2} +$$

Mathematica [A] time = 0.289746, size = 175, normalized size = 0.97

$$\frac{2\left(cd^2-ae^2\right)\left(5\left(cd^2-ae^2\right)\left(\sqrt{c}\sqrt{d}\sqrt{d+ex}\left(cd(4d+ex)-3ae^2\right)-3\left(cd^2-ae^2\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)\right)+3c^{5/2}d^{5/2}(d+}{15c^{9/2}d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(9/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] (2*(d + e*x)^(7/2))/(7*c*d) + (2*(c*d^2 - a*e^2)*(3*c^(5/2)*d^(5/2)*(d + e*
x)^(5/2) + 5*(c*d^2 - a*e^2)*(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x]*(-3*a*e^2 + c*d
*(4*d + e*x)) - 3*(c*d^2 - a*e^2)^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e
*x])/Sqrt[c*d^2 - a*e^2]])))/(15*c^(9/2)*d^(9/2))
```

Maple [B] time = 0.213, size = 455, normalized size = 2.5

$$\frac{2}{7\,cd}(ex+d)^{\frac{7}{2}} - \frac{2\,ae^2}{5\,c^2d^2}(ex+d)^{\frac{5}{2}} + \frac{2}{5\,c}(ex+d)^{\frac{5}{2}} + \frac{2\,a^2e^4}{3\,c^3d^3}(ex+d)^{\frac{3}{2}} - \frac{4\,ae^2}{3\,c^2d}(ex+d)^{\frac{3}{2}} + \frac{2\,d}{3\,c}(ex+d)^{\frac{3}{2}} - 2\,\frac{a^3e^6\sqrt{ex+d}}{c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

```
[Out] 2/7*(e*x+d)^(7/2)/c/d-2/5/c^2/d^2*(e*x+d)^(5/2)*a*e^2+2/5/c*(e*x+d)^(5/2)+2
/3/c^3/d^3*(e*x+d)^(3/2)*a^2*e^4-4/3/c^2/d*(e*x+d)^(3/2)*a*e^2+2/3/c*d*(e*x
+d)^(3/2)-2/c^4/d^4*a^3*e^6*(e*x+d)^(1/2)+6/c^3/d^2*a^2*e^4*(e*x+d)^(1/2)-6
/c^2*a*e^2*(e*x+d)^(1/2)+2/c*d^2*(e*x+d)^(1/2)+2/c^4/d^4/((a*e^2-c*d^2)*c*d
```

```
)^(1/2)*arctan((e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1/2))*a^4*e^8-8/c^3/d
^2/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(
1/2))*a^3*e^6+12/c^2/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((e*x+d)^(1/2)*c*d/((a
*e^2-c*d^2)*c*d)^(1/2))*a^2*e^4-8/c*d^2/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((e
*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1/2))*a*e^2+2*d^4/((a*e^2-c*d^2)*c*d)^
(1/2)*arctan((e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.96166, size = 1081, normalized size = 6.01

$$\left[\frac{105\left(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6\right)\sqrt{\frac{cd^2-ae^2}{cd}}\log\left(\frac{cdex+2\,cd^2-ae^2-2\,\sqrt{ex+d}cd\sqrt{\frac{cd^2-ae^2}{cd}}}{cdx+ae}\right)+2\left(15\,c^3d^3e^3x^3+176\,c^3d^6-406\,a\right.}{105\,c^4d^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fri
cas")
```

```
[Out] [1/105*(105*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt((c
*d^2 - a*e^2)/(c*d))*log((c*d*e*x + 2*c*d^2 - a*e^2 - 2*sqrt(e*x + d)*c*d*s
qrt((c*d^2 - a*e^2)/(c*d)))/(c*d*x + a*e)) + 2*(15*c^3*d^3*e^3*x^3 + 176*c^
3*d^6 - 406*a*c^2*d^4*e^2 + 350*a^2*c*d^2*e^4 - 105*a^3*e^6 + 3*(22*c^3*d^4
*e^2 - 7*a*c^2*d^2*e^4)*x^2 + (122*c^3*d^5*e - 112*a*c^2*d^3*e^3 + 35*a^2*c
*d*e^5)*x)*sqrt(e*x + d))/(c^4*d^4), -2/105*(105*(c^3*d^6 - 3*a*c^2*d^4*e^2
 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-(c*d^2 - a*e^2)/(c*d))*arctan(-sqrt(e*x
 + d)*c*d*sqrt(-(c*d^2 - a*e^2)/(c*d))/(c*d^2 - a*e^2)) - (15*c^3*d^3*e^3*x
^3 + 176*c^3*d^6 - 406*a*c^2*d^4*e^2 + 350*a^2*c*d^2*e^4 - 105*a^3*e^6 + 3*
(22*c^3*d^4*e^2 - 7*a*c^2*d^2*e^4)*x^2 + (122*c^3*d^5*e - 112*a*c^2*d^3*e^3
 + 35*a^2*c*d*e^5)*x)*sqrt(e*x + d))/(c^4*d^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(9/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.2001 $\int \frac{(d+ex)^{7/2}}{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal. Leaf size=147

$$\frac{2\sqrt{d+ex}\left(cd^2-ae^2\right)^2}{c^3d^3}+\frac{2(d+ex)^{3/2}\left(cd^2-ae^2\right)}{3c^2d^2}-\frac{2\left(cd^2-ae^2\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{7/2}d^{7/2}}+\frac{2(d+ex)^{5/2}}{5cd}$$

```
[Out] (2*(c*d^2 - a*e^2)^2*Sqrt[d + e*x])/(c^3*d^3) + (2*(c*d^2 - a*e^2)*(d + e*x
)^(3/2))/(3*c^2*d^2) + (2*(d + e*x)^(5/2))/(5*c*d) - (2*(c*d^2 - a*e^2)^(5/
2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(c^(7/2)*d
^(7/2))
```

Rubi [A] time = 0.0917097, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {626, 50, 63, 208}

$$\frac{2\sqrt{d+ex}\left(cd^2-ae^2\right)^2}{c^3d^3}+\frac{2(d+ex)^{3/2}\left(cd^2-ae^2\right)}{3c^2d^2}-\frac{2\left(cd^2-ae^2\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{7/2}d^{7/2}}+\frac{2(d+ex)^{5/2}}{5cd}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```


```
[Out] (2*(c*d^2 - a*e^2)^2*Sqrt[d + e*x])/(c^3*d^3) + (2*(c*d^2 - a*e^2)*(d + e*x
)^(3/2))/(3*c^2*d^2) + (2*(d + e*x)^(5/2))/(5*c*d) - (2*(c*d^2 - a*e^2)^(5/
2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(c^(7/2)*d
^(7/2))
```


Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{7/2}}{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx=\int \frac{(d+ex)^{5/2}}{ae+cdx}\,dx$$

$$=\frac{2(d+ex)^{5/2}}{5cd}+\frac{\left(cd^2-ae^2\right)\int \frac{(d+ex)^{3/2}}{ae+cdx}\,dx}{cd}$$

$$=\frac{2\left(cd^2-ae^2\right)(d+ex)^{3/2}}{3c^2d^2}+\frac{2(d+ex)^{5/2}}{5cd}+\frac{\left(cd^2-ae^2\right)^2\int \frac{\sqrt{d+ex}}{ae+cdx}\,dx}{c^2d^2}$$

$$=\frac{2\left(cd^2-ae^2\right)^2\sqrt{d+ex}}{c^3d^3}+\frac{2\left(cd^2-ae^2\right)(d+ex)^{3/2}}{3c^2d^2}+\frac{2(d+ex)^{5/2}}{5cd}+\frac{\left(cd^2-ae^2\right)^3\int}{c}$$

$$=\frac{2\left(cd^2-ae^2\right)^2\sqrt{d+ex}}{c^3d^3}+\frac{2\left(cd^2-ae^2\right)(d+ex)^{3/2}}{3c^2d^2}+\frac{2(d+ex)^{5/2}}{5cd}+\frac{\left(2\left(cd^2-ae^2\right)\right.}{}$$

$$=\frac{2\left(cd^2-ae^2\right)^2\sqrt{d+ex}}{c^3d^3}+\frac{2\left(cd^2-ae^2\right)(d+ex)^{3/2}}{3c^2d^2}+\frac{2(d+ex)^{5/2}}{5cd}-\frac{2\left(cd^2-ae^2\right)^5}{}$$

Mathematica [A] time = 0.112306, size = 135, normalized size = 0.92

$$\frac{2\sqrt{d+ex}\left(15a^2e^4-5acde^2(7d+ex)+c^2d^2\left(23d^2+11dex+3e^2x^2\right)\right)}{15c^3d^3}-\frac{2\left(cd^2-ae^2\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{7/2}d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(15*a^2*e^4 - 5*a*c*d*e^2*(7*d + e*x) + c^2*d^2*(23*d^2 +
11*d*e*x + 3*e^2*x^2)))/(15*c^3*d^3) - (2*(c*d^2 - a*e^2)^(5/2)*ArcTanh[(Sq
rt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(c^(7/2)*d^(7/2))
```

Maple [B] time = 0.194, size = 324, normalized size = 2.2

$$\frac{2}{5cd}(ex+d)^{\frac{5}{2}}-\frac{2ae^2}{3c^2d^2}(ex+d)^{\frac{3}{2}}+\frac{2}{3c}(ex+d)^{\frac{3}{2}}+2\frac{a^2e^4\sqrt{ex+d}}{c^3d^3}-4\frac{ae^2\sqrt{ex+d}}{c^2d}+2\frac{d\sqrt{ex+d}}{c}-2\frac{a^3e^6}{c^3d^3\sqrt{\left(ae^2-cd^2\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

```
[Out] 2/5*(e*x+d)^(5/2)/c/d-2/3/c^2/d^2*(e*x+d)^(3/2)*a*e^2+2/3/c*(e*x+d)^(3/2)+2
/c^3/d^3*a^2*e^4*(e*x+d)^(1/2)-4/c^2/d*a*e^2*(e*x+d)^(1/2)+2/c*d*(e*x+d)^(1
/2)-2/c^3/d^3/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((e*x+d)^(1/2)*c*d/((a*e^2-c*
d^2)*c*d)^(1/2))*a^3*e^6+6/c^2/d/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((e*x+d)^(
1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1/2))*a^2*e^4-6/c*d/((a*e^2-c*d^2)*c*d)^(1/2)
*arctan((e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1/2))*a*e^2+2*d^3/((a*e^2-c*
d^2)*c*d)^(1/2)*arctan((e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.95522, size = 771, normalized size = 5.24

$$\left[\frac{15\left(c^2d^4 - 2\,acd^2e^2 + a^2e^4\right)\sqrt{\frac{cd^2-ae^2}{cd}}\log\left(\frac{cdex+2\,cd^2-ae^2-2\,\sqrt{ex+d}cd\sqrt{\frac{cd^2-ae^2}{cd}}}{cdx+ae}\right) + 2\left(3\,c^2d^2e^2x^2 + 23\,c^2d^4 - 35\,acd^2e^2 + 15\,a^2e^4 + \right.}{15\,c^3d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fri
cas")
```

```
[Out] [1/15*(15*(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*sqrt((c*d^2 - a*e^2)/(c*d))*l
og((c*d*e*x + 2*c*d^2 - a*e^2 - 2*sqrt(e*x + d)*c*d*sqrt((c*d^2 - a*e^2)/(c
*d)))/(c*d*x + a*e)) + 2*(3*c^2*d^2*e^2*x^2 + 23*c^2*d^4 - 35*a*c*d^2*e^2 +
 15*a^2*e^4 + (11*c^2*d^3*e - 5*a*c*d*e^3)*x)*sqrt(e*x + d))/(c^3*d^3), -2/
15*(15*(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c*d^2 - a*e^2)/(c*d))*arc
tan(-sqrt(e*x + d)*c*d*sqrt(-(c*d^2 - a*e^2)/(c*d))/(c*d^2 - a*e^2)) - (3*c
^2*d^2*e^2*x^2 + 23*c^2*d^4 - 35*a*c*d^2*e^2 + 15*a^2*e^4 + (11*c^2*d^3*e -
 5*a*c*d*e^3)*x)*sqrt(e*x + d))/(c^3*d^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.2002 $\int \frac{(d+ex)^{5/2}}{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal. Leaf size=114

$$\frac{2\sqrt{d+ex}\left(cd^2-ae^2\right)}{c^2d^2}-\frac{2\left(cd^2-ae^2\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{5/2}d^{5/2}}+\frac{2(d+ex)^{3/2}}{3cd}$$

```
[Out] (2*(c*d^2 - a*e^2)*Sqrt[d + e*x])/(c^2*d^2) + (2*(d + e*x)^(3/2))/(3*c*d) -
 (2*(c*d^2 - a*e^2)^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^
2 - a*e^2]])/(c^(5/2)*d^(5/2))
```

Rubi [A] time = 0.0712063, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {626, 50, 63, 208}

$$\frac{2\sqrt{d+ex}\left(cd^2-ae^2\right)}{c^2d^2}-\frac{2\left(cd^2-ae^2\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{5/2}d^{5/2}}+\frac{2(d+ex)^{3/2}}{3cd}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] (2*(c*d^2 - a*e^2)*Sqrt[d + e*x])/(c^2*d^2) + (2*(d + e*x)^(3/2))/(3*c*d) -
 (2*(c*d^2 - a*e^2)^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^
2 - a*e^2]])/(c^(5/2)*d^(5/2))
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx=\int \frac{(d+ex)^{3/2}}{ae+cdx}\,dx$$
$$=\frac{2(d+ex)^{3/2}}{3cd}+\frac{\left(cd^2-ae^2\right)\int \frac{\sqrt{d+ex}}{ae+cdx}\,dx}{cd}$$
$$=\frac{2\left(cd^2-ae^2\right)\sqrt{d+ex}}{c^2d^2}+\frac{2(d+ex)^{3/2}}{3cd}+\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{(ae+cdx)\sqrt{d+ex}}\,dx}{c^2d^2}$$
$$=\frac{2\left(cd^2-ae^2\right)\sqrt{d+ex}}{c^2d^2}+\frac{2(d+ex)^{3/2}}{3cd}+\frac{\left(2\left(cd^2-ae^2\right)^2\right)\mathrm{Subst}\left(\int \frac{1}{-\frac{cd^2}{e}+ae+\frac{cdx^2}{e}}\,dx\right.}{c^2d^2e}$$
$$=\frac{2\left(cd^2-ae^2\right)\sqrt{d+ex}}{c^2d^2}+\frac{2(d+ex)^{3/2}}{3cd}-\frac{2\left(cd^2-ae^2\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{5/2}d^{5/2}}$$

Mathematica [A] time = 0.0776083, size = 102, normalized size = 0.89

$$\frac{2\sqrt{d+ex}\left(cd(4d+ex)-3ae^2\right)}{3c^2d^2}-\frac{2\left(cd^2-ae^2\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{5/2}d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(-3*a*e^2 + c*d*(4*d + e*x)))/(3*c^2*d^2) - (2*(c*d^2 - a*
e^2)^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(c
^(5/2)*d^(5/2))
```

Maple [B] time = 0.192, size = 211, normalized size = 1.9

$$\frac{2}{3\,cd}(ex+d)^{\frac{3}{2}}-2\,\frac{ae^2\sqrt{ex+d}}{c^2d^2}+2\,\frac{\sqrt{ex+d}}{c}+2\,\frac{a^2e^4}{c^2d^2\sqrt{\left(ae^2-cd^2\right)cd}}\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{\left(ae^2-cd^2\right)cd}}\right)-4\,\frac{ae^2}{c\sqrt{\left(ae^2-cd^2\right)cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

```
[Out] 2/3*(e*x+d)^(3/2)/c/d-2/c^2/d^2*a*e^2*(e*x+d)^(1/2)+2/c*(e*x+d)^(1/2)+2/c^2
/d^2/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)
^(1/2))*a^2*e^4-4/c/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((e*x+d)^(1/2)*c*d/((a*
e^2-c*d^2)*c*d)^(1/2))*a*e^2+2*d^2/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((e*x+d)
^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="max
ima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.93921, size = 533, normalized size = 4.68

$$\left[\frac{3\left(cd^2-ae^2\right)\sqrt{\frac{cd^2-ae^2}{cd}}\log\left(\frac{cdex+2\,cd^2-ae^2-2\,\sqrt{ex+d}cd\sqrt{\frac{cd^2-ae^2}{cd}}}{cdx+ae}\right)+2\left(cdex+4\,cd^2-3\,ae^2\right)\sqrt{ex+d}}{3\,c^2d^2},-\frac{2\left(3\left(cd^2-ae^2\right)\sqrt{-\frac{cd^2-ae^2}{cd}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fri
cas")

[Out] [1/3*(3*(c*d^2 - a*e^2)*sqrt((c*d^2 - a*e^2)/(c*d))*log((c*d*e*x + 2*c*d^2
- a*e^2 - 2*sqrt(e*x + d)*c*d*sqrt((c*d^2 - a*e^2)/(c*d)))/(c*d*x + a*e)) +
2*(c*d*e*x + 4*c*d^2 - 3*a*e^2)*sqrt(e*x + d))/(c^2*d^2), -2/3*(3*(c*d^2 -
a*e^2)*sqrt(-(c*d^2 - a*e^2)/(c*d))*arctan(-sqrt(e*x + d)*c*d*sqrt(-(c*d^2
- a*e^2)/(c*d))/(c*d^2 - a*e^2)) - (c*d*e*x + 4*c*d^2 - 3*a*e^2)*sqrt(e*x
+ d))/(c^2*d^2)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="gia
c")

[Out] Timed out

3.2003 $\int \frac{(d+ex)^{3/2}}{ade+(cd^2+ae^2)x+cdex^2} dx$

Optimal. Leaf size=83

$$\frac{2\sqrt{d+ex}}{cd} - \frac{2\sqrt{cd^2-ae^2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{3/2}d^{3/2}}$$

```
[Out] (2*Sqrt[d + e*x])/(c*d) - (2*Sqrt[c*d^2 - a*e^2]*ArcTanh[(Sqrt[c]*Sqrt[d]*S
qrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(c^(3/2)*d^(3/2))
```

Rubi [A] time = 0.0546732, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {626, 50, 63, 208}

$$\frac{2\sqrt{d+ex}}{cd} - \frac{2\sqrt{cd^2-ae^2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{3/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x])/(c*d) - (2*Sqrt[c*d^2 - a*e^2]*ArcTanh[(Sqrt[c]*Sqrt[d]*S
qrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(c^(3/2)*d^(3/2))
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx=\int \frac{\sqrt{d+ex}}{ae+cdx}\,dx$$
$$=\frac{2\sqrt{d+ex}}{cd}+\frac{\left(cd^2-ae^2\right)\int \frac{1}{(ae+cdx)\sqrt{d+ex}}\,dx}{cd}$$
$$=\frac{2\sqrt{d+ex}}{cd}+\left(2\left(\frac{d}{e}-\frac{ae}{cd}\right)\right)\text{Subst}\left(\int \frac{1}{-\frac{cd^2}{e}+ae+\frac{cdx^2}{e}}\,dx,x,\sqrt{d+ex}\right)$$
$$=\frac{2\sqrt{d+ex}}{cd}-\frac{2\sqrt{cd^2-ae^2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{3/2}d^{3/2}}$$

Mathematica [A] time = 0.0340531, size = 83, normalized size = 1.

$$\frac{2\sqrt{d+ex}}{cd}-\frac{2\sqrt{cd^2-ae^2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{3/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x])/(c*d) - (2*Sqrt[c*d^2 - a*e^2]*ArcTanh[(Sqrt[c]*Sqrt[d]*S
qrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(c^(3/2)*d^(3/2))
```

Maple [A] time = 0.192, size = 122, normalized size = 1.5

$$2\,\frac{\sqrt{ex+d}}{cd}-2\,\frac{ae^2}{cd\sqrt{\left(ae^2-cd^2\right)cd}}\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{\left(ae^2-cd^2\right)cd}}\right)+2\,\frac{d}{\sqrt{\left(ae^2-cd^2\right)cd}}\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{\left(ae^2-cd^2\right)cd}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

```
[Out] 2*(e*x+d)^(1/2)/c/d-2/c/d/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((e*x+d)^(1/2)*c*
d/((a*e^2-c*d^2)*c*d)^(1/2))*a*e^2+2*d/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((e*
x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.97021, size = 385, normalized size = 4.64

$$\left[\frac{\sqrt{\frac{cd^2-ae^2}{cd}}\log\left(\frac{cdex+2\,cd^2-ae^2-2\,\sqrt{ex+d}cd\sqrt{\frac{cd^2-ae^2}{cd}}}{cdx+ae}\right)+2\,\sqrt{ex+d}}{cd}, -\frac{2\left(\sqrt{-\frac{cd^2-ae^2}{cd}}\arctan\left(-\frac{\sqrt{ex+d}cd\sqrt{-\frac{cd^2-ae^2}{cd}}}{cd^2-ae^2}\right)-\sqrt{ex+d}\right)}{cd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fri
cas")
```

```
[Out] [(sqrt((c*d^2 - a*e^2)/(c*d))*log((c*d*e*x + 2*c*d^2 - a*e^2 - 2*sqrt(e*x +
 d)*c*d*sqrt((c*d^2 - a*e^2)/(c*d)))/(c*d*x + a*e)) + 2*sqrt(e*x + d))/(c*d
), -2*(sqrt(-(c*d^2 - a*e^2)/(c*d))*arctan(-sqrt(e*x + d)*c*d*sqrt(-(c*d^2
- a*e^2)/(c*d))/(c*d^2 - a*e^2)) - sqrt(e*x + d))/(c*d)]
```

Sympy [A] time = 12.6772, size = 80, normalized size = 0.96

$$\frac{2\left(\frac{e\sqrt{d+ex}}{cd}-\frac{e\left(ae^2-cd^2\right)\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{\frac{ae^2-cd^2}{cd}}}\right)}{c^2d^2\sqrt{\frac{ae^2-cd^2}{cd}}}\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

```
[Out] 2*(e*sqrt(d + e*x)/(c*d) - e*(a*e**2 - c*d**2)*atan(sqrt(d + e*x)/sqrt((a*e
**2 - c*d**2)/(c*d)))/(c**2*d**2*sqrt((a*e**2 - c*d**2)/(c*d))))/e
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.2004 $\int \frac{\sqrt{d+ex}}{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal. Leaf size=65

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\sqrt{c}\sqrt{d}\sqrt{cd^2-ae^2}}$$

```
[Out] (-2*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(Sqrt[c]*
Sqrt[d]*Sqrt[c*d^2 - a*e^2])
```

Rubi [A] time = 0.0445752, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {626, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\sqrt{c}\sqrt{d}\sqrt{cd^2-ae^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(Sqrt[c]*
Sqrt[d]*Sqrt[c*d^2 - a*e^2])
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx=\int \frac{1}{(ae+cdx)\sqrt{d+ex}}\,dx$$
$$=\frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{cd^2}{e}+ae+\frac{cdx^2}{e}}\,dx,x,\sqrt{d+ex}\right)}{e}$$
$$=-\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\sqrt{c}\sqrt{d}\sqrt{cd^2-ae^2}}$$

Mathematica [A] time = 0.0161823, size = 65, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\sqrt{c}\sqrt{d}\sqrt{cd^2-ae^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(Sqrt[c]*
Sqrt[d]*Sqrt[c*d^2 - a*e^2])
```

Maple [A] time = 0.194, size = 48, normalized size = 0.7

$$2\,\frac{1}{\sqrt{\left(ae^2-cd^2\right)cd}}\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{\left(ae^2-cd^2\right)cd}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

```
[Out] 2/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1
/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83507, size = 325, normalized size = 5.

$$\left[\frac{\log\left(\frac{cdex+2\,cd^2-ae^2-2\,\sqrt{c^2d^3-acde^2}\sqrt{ex+d}}{cdx+ae}\right)}{\sqrt{c^2d^3-acde^2}},\ \frac{2\,\sqrt{-c^2d^3+acde^2}\arctan\left(\frac{\sqrt{-c^2d^3+acde^2}\sqrt{ex+d}}{cdex+cd^2}\right)}{c^2d^3-acde^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fri
cas")
```

```
[Out] [log((c*d*e*x + 2*c*d^2 - a*e^2 - 2*sqrt(c^2*d^3 - a*c*d*e^2)*sqrt(e*x + d)
)/(c*d*x + a*e))/sqrt(c^2*d^3 - a*c*d*e^2), 2*sqrt(-c^2*d^3 + a*c*d*e^2)*ar
ctan(sqrt(-c^2*d^3 + a*c*d*e^2)*sqrt(e*x + d)/(c*d*e*x + c*d^2))/(c^2*d^3 -
 a*c*d*e^2)]
```

Sympy [A] time = 2.71218, size = 48, normalized size = 0.74

$$\frac{2\,\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{\frac{ae^2-cd^2}{cd}}}\right)}{cd\sqrt{\frac{ae^2-cd^2}{cd}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

```
[Out] 2*atan(sqrt(d + e*x)/sqrt((a*e**2 - c*d**2)/(c*d)))/(c*d*sqrt((a*e**2 - c*d
**2)/(c*d)))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.2005 $\int \frac{1}{\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}\,dx$

Optimal. Leaf size=91

$$\frac{2}{\sqrt{d+ex}\left(cd^2-ae^2\right)}-\frac{2\sqrt{c}\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{3/2}}$$

```
[Out] 2/((c*d^2 - a*e^2)*Sqrt[d + e*x]) - (2*Sqrt[c]*Sqrt[d]*ArcTanh[(Sqrt[c]*Sqr
t[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(c*d^2 - a*e^2)^(3/2)
```

Rubi [A] time = 0.0830422, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {626, 51, 63, 208}

$$\frac{2}{\sqrt{d+ex}\left(cd^2-ae^2\right)}-\frac{2\sqrt{c}\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)),x]
```


```
[Out] 2/((c*d^2 - a*e^2)*Sqrt[d + e*x]) - (2*Sqrt[c]*Sqrt[d]*ArcTanh[(Sqrt[c]*Sqr
t[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(c*d^2 - a*e^2)^(3/2)
```


Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}\,dx = \int \frac{1}{(ae+cdx)(d+ex)^{3/2}}\,dx$$
$$= \frac{2}{\left(cd^2-ae^2\right)\sqrt{d+ex}} + \frac{(cd)\int \frac{1}{(ae+cdx)\sqrt{d+ex}}\,dx}{cd^2-ae^2}$$
$$= \frac{2}{\left(cd^2-ae^2\right)\sqrt{d+ex}} + \frac{(2cd)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{cd^2}{e}+ae+\frac{cdx^2}{e}}\,dx, x, \sqrt{d+ex}\right)}{e\left(cd^2-ae^2\right)}$$
$$= \frac{2}{\left(cd^2-ae^2\right)\sqrt{d+ex}} - \frac{2\sqrt{c}\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{3/2}}$$

Mathematica [C] time = 0.0151336, size = 55, normalized size = 0.6

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{cd(d+ex)}{cd^2-ae^2}\right)}{\sqrt{d+ex}\left(ae^2-cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, (c*d*(d + e*x))/(c*d^2 - a*e^2)])/((-(c
*d^2) + a*e^2)*Sqrt[d + e*x])
```

Maple [A] time = 0.195, size = 88, normalized size = 1.

$$-2\,\frac{cd}{\left(ae^2-cd^2\right)\sqrt{\left(ae^2-cd^2\right)cd}}\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{\left(ae^2-cd^2\right)cd}}\right)-2\,\frac{1}{\left(ae^2-cd^2\right)\sqrt{ex+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

```
[Out] -2*c*d/(a*e^2-c*d^2)/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((e*x+d)^(1/2)*c*d/((a
*e^2-c*d^2)*c*d)^(1/2))-2/(a*e^2-c*d^2)/(e*x+d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.85558, size = 529, normalized size = 5.81

$$\left[-\frac{(ex+d)\sqrt{\frac{cd}{cd^2-ae^2}}\log\left(\frac{cdex+2\,cd^2-ae^2+2\,(cd^2-ae^2)\sqrt{ex+d}\sqrt{\frac{cd}{cd^2-ae^2}}}{cdx+ae}\right)-2\,\sqrt{ex+d}}{cd^3-ade^2+(cd^2e-ae^3)x}, -\frac{2\left((ex+d)\sqrt{-\frac{cd}{cd^2-ae^2}}\arctan\left(-\frac{(cd^2-ae^2)\sqrt{ex}}{cdex}\right.\right.}{cd^3-ade^2+(cd^2e-a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="f
ricas")
```

```
[Out] [-((e*x + d)*sqrt(c*d/(c*d^2 - a*e^2))*log((c*d*e*x + 2*c*d^2 - a*e^2 + 2*(
c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(c*d/(c*d^2 - a*e^2)))/(c*d*x + a*e)) - 2*
sqrt(e*x + d))/(c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x), -2*((e*x + d)*sqrt(
-c*d/(c*d^2 - a*e^2))*arctan(-(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(-c*d/(c*d^
2 - a*e^2))/(c*d*e*x + c*d^2)) - sqrt(e*x + d))/(c*d^3 - a*d*e^2 + (c*d^2*e
 - a*e^3)*x)]
```

Sympy [A] time = 7.51262, size = 82, normalized size = 0.9

$$\frac{2cd\,\mathrm{atan}\left(\frac{1}{\sqrt{\frac{cd}{ae^2-cd^2}}\sqrt{d+ex}}\right)}{\sqrt{\frac{cd}{ae^2-cd^2}}\left(ae^2-cd^2\right)^2}-\frac{2}{\sqrt{d+ex}\left(ae^2-cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

```
[Out] 2*c*d*atan(1/(sqrt(c*d/(a*e**2 - c*d**2))*sqrt(d + e*x)))/(sqrt(c*d/(a*e**2
 - c*d**2))*(a*e**2 - c*d**2)**2) - 2/(sqrt(d + e*x)*(a*e**2 - c*d**2))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.2006 $\int \frac{1}{(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}\,dx$

Optimal. Leaf size=120

$$-\frac{2c^{3/2}d^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{5/2}}+\frac{2cd}{\sqrt{d+ex}\left(cd^2-ae^2\right)^2}+\frac{2}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

```
[Out] 2/(3*(c*d^2 - a*e^2)*(d + e*x)^(3/2)) + (2*c*d)/((c*d^2 - a*e^2)^2*Sqrt[d +
 e*x]) - (2*c^(3/2)*d^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*
d^2 - a*e^2]])/(c*d^2 - a*e^2)^(5/2)
```

Rubi [A] time = 0.0947309, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {626, 51, 63, 208}

$$-\frac{2c^{3/2}d^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{5/2}}+\frac{2cd}{\sqrt{d+ex}\left(cd^2-ae^2\right)^2}+\frac{2}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)),x]
```

```
[Out] 2/(3*(c*d^2 - a*e^2)*(d + e*x)^(3/2)) + (2*c*d)/((c*d^2 - a*e^2)^2*Sqrt[d +
 e*x]) - (2*c^(3/2)*d^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*
d^2 - a*e^2]])/(c*d^2 - a*e^2)^(5/2)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}dx=\int \frac{1}{(ae+cdx)(d+ex)^{5/2}}dx$$
$$=\frac{2}{3\left(cd^2-ae^2\right)(d+ex)^{3/2}}+\frac{(cd)\int \frac{1}{(ae+cdx)(d+ex)^{3/2}}dx}{cd^2-ae^2}$$
$$=\frac{2}{3\left(cd^2-ae^2\right)(d+ex)^{3/2}}+\frac{2cd}{\left(cd^2-ae^2\right)^2\sqrt{d+ex}}+\frac{\left(c^2d^2\right)\int \frac{1}{(ae+cdx)\sqrt{\ldots}}}{\left(cd^2-ae^2\right)^{\ldots}}$$
$$=\frac{2}{3\left(cd^2-ae^2\right)(d+ex)^{3/2}}+\frac{2cd}{\left(cd^2-ae^2\right)^2\sqrt{d+ex}}+\frac{\left(2c^2d^2\right)\text{Subst}\left(\int \ldots\right.}{e\ldots}$$
$$=\frac{2}{3\left(cd^2-ae^2\right)(d+ex)^{3/2}}+\frac{2cd}{\left(cd^2-ae^2\right)^2\sqrt{d+ex}}-\frac{2c^{3/2}d^{3/2}\tanh^{-1}\left(\ldots\right.}{\left(cd^2-ae^{2}\ldots\right.}$$

Mathematica [C] time = 0.0122831, size = 57, normalized size = 0.48

$$\frac{2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{cd(d+ex)}{cd^2-ae^2}\right)}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)),x]
```

```
[Out] (2*Hypergeometric2F1[-3/2, 1, -1/2, (c*d*(d + e*x))/(c*d^2 - a*e^2)])/(3*(c
*d^2 - a*e^2)*(d + e*x)^(3/2))
```

Maple [A] time = 0.197, size = 117, normalized size = 1.

$$-\frac{2}{3\,ae^2-3\,cd^2}(ex+d)^{-\frac{3}{2}}+2\,\frac{cd}{\left(ae^2-cd^2\right)^2\sqrt{ex+d}}+2\,\frac{c^2d^2}{\left(ae^2-cd^2\right)^2\sqrt{\left(ae^2-cd^2\right)cd}}\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{\left(ae^2-cd^2\right)cd}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

```
[Out] -2/3/(a*e^2-c*d^2)/(e*x+d)^(3/2)+2*c*d/(a*e^2-c*d^2)^2/(e*x+d)^(1/2)+2*c^2*
d^2/(a*e^2-c*d^2)^2/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((e*x+d)^(1/2)*c*d/((a*
e^2-c*d^2)*c*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.0197, size = 941, normalized size = 7.84

$$\left[\frac{3\left(cde^2x^2+2\,cd^2ex+cd^3\right)\sqrt{\frac{cd}{cd^2-ae^2}}\log\left(\frac{cdex+2\,cd^2-ae^2-2\left(cd^2-ae^2\right)\sqrt{ex+d}\sqrt{\frac{cd}{cd^2-ae^2}}}{cdx+ae}\right)+2\left(3\,cdex+4\,cd^2-ae^2\right)\sqrt{ex+d}}{3\left(c^2d^6-2\,acd^4e^2+a^2d^2e^4+\left(c^2d^4e^2-2\,acd^2e^4+a^2e^6\right)x^2+2\left(c^2d^5e-2\,acd^3e^3+a^2de^5\right)x\right)},-\frac{2\left(3\left(c\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="f
ricas")
```

```
[Out] [1/3*(3*(c*d*e^2*x^2 + 2*c*d^2*e*x + c*d^3)*sqrt(c*d/(c*d^2 - a*e^2))*log((
c*d*e*x + 2*c*d^2 - a*e^2 - 2*(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(c*d/(c*d^2
 - a*e^2)))/(c*d*x + a*e)) + 2*(3*c*d*e*x + 4*c*d^2 - a*e^2)*sqrt(e*x + d))
/(c^2*d^6 - 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^4*e^2 - 2*a*c*d^2*e^4 + a^
2*e^6)*x^2 + 2*(c^2*d^5*e - 2*a*c*d^3*e^3 + a^2*d*e^5)*x), -2/3*(3*(c*d*e^2
*x^2 + 2*c*d^2*e*x + c*d^3)*sqrt(-c*d/(c*d^2 - a*e^2))*arctan(-(c*d^2 - a*e
^2)*sqrt(e*x + d)*sqrt(-c*d/(c*d^2 - a*e^2))/(c*d*e*x + c*d^2)) - (3*c*d*e*
x + 4*c*d^2 - a*e^2)*sqrt(e*x + d))/(c^2*d^6 - 2*a*c*d^4*e^2 + a^2*d^2*e^4
+ (c^2*d^4*e^2 - 2*a*c*d^2*e^4 + a^2*e^6)*x^2 + 2*(c^2*d^5*e - 2*a*c*d^3*e^
3 + a^2*d*e^5)*x)]
```

Sympy [A] time = 9.90909, size = 107, normalized size = 0.89

$$\frac{2cd}{\sqrt{d+ex}\left(ae^2-cd^2\right)^2}+\frac{2cd\,\text{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{\frac{ae^2-cd^2}{cd}}}\right)}{\sqrt{\frac{ae^2-cd^2}{cd}}\left(ae^2-cd^2\right)^2}-\frac{2}{3\left(d+ex\right)^{\frac{3}{2}}\left(ae^2-cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

```
[Out] 2*c*d/(sqrt(d + e*x)*(a*e**2 - c*d**2)**2) + 2*c*d*atan(sqrt(d + e*x)/sqrt(
(a*e**2 - c*d**2)/(c*d)))/(sqrt((a*e**2 - c*d**2)/(c*d))*(a*e**2 - c*d**2)*
*2) - 2/(3*(d + e*x)**(3/2)*(a*e**2 - c*d**2))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.2007 $\int \frac{1}{(d+ex)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}\,dx$

Optimal. Leaf size=153

$$\frac{2c^2d^2}{\sqrt{d+ex}\left(cd^2-ae^2\right)^3}-\frac{2c^{5/2}d^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{7/2}}+\frac{2cd}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)^2}+\frac{2}{5(d+ex)^{5/2}\left(cd^2-ae^2\right)}$$

```
[Out] 2/(5*(c*d^2 - a*e^2)*(d + e*x)^(5/2)) + (2*c*d)/(3*(c*d^2 - a*e^2)^2*(d + e
*x)^(3/2)) + (2*c^2*d^2)/((c*d^2 - a*e^2)^3*Sqrt[d + e*x]) - (2*c^(5/2)*d^(
5/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(c*d^2 -
 a*e^2)^(7/2)
```

Rubi [A] time = 0.129555, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {626, 51, 63, 208}

$$\frac{2c^2d^2}{\sqrt{d+ex}\left(cd^2-ae^2\right)^3}-\frac{2c^{5/2}d^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{7/2}}+\frac{2cd}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)^2}+\frac{2}{5(d+ex)^{5/2}\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)),x]
```


```
[Out] 2/(5*(c*d^2 - a*e^2)*(d + e*x)^(5/2)) + (2*c*d)/(3*(c*d^2 - a*e^2)^2*(d + e
*x)^(3/2)) + (2*c^2*d^2)/((c*d^2 - a*e^2)^3*Sqrt[d + e*x]) - (2*c^(5/2)*d^(
5/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(c*d^2 -
 a*e^2)^(7/2)
```


Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}\,dx = \int \frac{1}{(ae+cdx)(d+ex)^{7/2}}\,dx$$

$$= \frac{2}{5\left(cd^2-ae^2\right)(d+ex)^{5/2}} + \frac{(cd)\int \frac{1}{(ae+cdx)(d+ex)^{5/2}}\,dx}{cd^2-ae^2}$$

$$= \frac{2}{5\left(cd^2-ae^2\right)(d+ex)^{5/2}} + \frac{2cd}{3\left(cd^2-ae^2\right)^2(d+ex)^{3/2}} + \frac{\left(c^2d^2\right)\int \frac{}{(ae+c}}{\left(cd^2-\right.}$$

$$= \frac{2}{5\left(cd^2-ae^2\right)(d+ex)^{5/2}} + \frac{2cd}{3\left(cd^2-ae^2\right)^2(d+ex)^{3/2}} + \frac{2c^2d^2}{\left(cd^2-ae^2\right)^3}$$

$$= \frac{2}{5\left(cd^2-ae^2\right)(d+ex)^{5/2}} + \frac{2cd}{3\left(cd^2-ae^2\right)^2(d+ex)^{3/2}} + \frac{2c^2d^2}{\left(cd^2-ae^2\right)^3}$$

$$= \frac{2}{5\left(cd^2-ae^2\right)(d+ex)^{5/2}} + \frac{2cd}{3\left(cd^2-ae^2\right)^2(d+ex)^{3/2}} + \frac{2c^2d^2}{\left(cd^2-ae^2\right)^3}$$

Mathematica [C] time = 0.0126572, size = 57, normalized size = 0.37

$$\frac{2\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\frac{cd(d+ex)}{cd^2-ae^2}\right)}{5(d+ex)^{5/2}\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)),x]
```

```
[Out] (2*Hypergeometric2F1[-5/2, 1, -3/2, (c*d*(d + e*x))/(c*d^2 - a*e^2)])/(5*(c
*d^2 - a*e^2)*(d + e*x)^(5/2))
```

Maple [A] time = 0.198, size = 146, normalized size = 1.

$$-\frac{2}{5\,ae^2-5\,cd^2}(ex+d)^{-\frac{5}{2}} - 2\,\frac{c^2d^2}{\left(ae^2-cd^2\right)^3\sqrt{ex+d}} + \frac{2\,cd}{3\left(ae^2-cd^2\right)^2}(ex+d)^{-\frac{3}{2}} - 2\,\frac{c^3d^3}{\left(ae^2-cd^2\right)^3\sqrt{\left(ae^2-cd^2\right)cd}}\,\text{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

```
[Out] -2/5/(a*e^2-c*d^2)/(e*x+d)^(5/2)-2*c^2*d^2/(a*e^2-c*d^2)^3/(e*x+d)^(1/2)+2/
3*c*d/(a*e^2-c*d^2)^2/(e*x+d)^(3/2)-2*c^3*d^3/(a*e^2-c*d^2)^3/((a*e^2-c*d^2
)*c*d)^(1/2)*arctan((e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.05397, size = 1555, normalized size = 10.16

$$\left[-\frac{15\left(c^2d^2e^3x^3+3\,c^2d^3e^2x^2+3\,c^2d^4ex+c^2d^5\right)\sqrt{\frac{cd}{cd^2-ae^2}}\log\left(\frac{cdex+2\,cd^2-ae^2+2\left(cd^2-ae^2\right)\sqrt{ex+d}\sqrt{\frac{cd}{cd^2-ae^2}}}{cdx+ae}\right)-2\left(15\,c^2d^2e^2x^2+23\right.}{15\left(c^3d^9-3\,ac^2d^7e^2+3\,a^2cd^5e^4-a^3d^3e^6+\left(c^3d^6e^3-3\,ac^2d^4e^5+3\,a^2cd^2e^7-a^3e^9\right)x^3+3\left(c^3d^7e^2-3\,ac^2d^5e^4+3\,a^2cd^3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="f
ricas")
```

```
[Out] [-1/15*(15*(c^2*d^2*e^3*x^3 + 3*c^2*d^3*e^2*x^2 + 3*c^2*d^4*e*x + c^2*d^5)*
sqrt(c*d/(c*d^2 - a*e^2))*log((c*d*e*x + 2*c*d^2 - a*e^2 + 2*(c*d^2 - a*e^2
)*sqrt(e*x + d)*sqrt(c*d/(c*d^2 - a*e^2)))/(c*d*x + a*e)) - 2*(15*c^2*d^2*e
^2*x^2 + 23*c^2*d^4 - 11*a*c*d^2*e^2 + 3*a^2*e^4 + 5*(7*c^2*d^3*e - a*c*d*e
^3)*x)*sqrt(e*x + d))/(c^3*d^9 - 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4 - a^3*d^
3*e^6 + (c^3*d^6*e^3 - 3*a*c^2*d^4*e^5 + 3*a^2*c*d^2*e^7 - a^3*e^9)*x^3 + 3
*(c^3*d^7*e^2 - 3*a*c^2*d^5*e^4 + 3*a^2*c*d^3*e^6 - a^3*d*e^8)*x^2 + 3*(c^3
*d^8*e - 3*a*c^2*d^6*e^3 + 3*a^2*c*d^4*e^5 - a^3*d^2*e^7)*x), -2/15*(15*(c^
2*d^2*e^3*x^3 + 3*c^2*d^3*e^2*x^2 + 3*c^2*d^4*e*x + c^2*d^5)*sqrt(-c*d/(c*d
^2 - a*e^2))*arctan(-(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(-c*d/(c*d^2 - a*e^2
))/(c*d*e*x + c*d^2)) - (15*c^2*d^2*e^2*x^2 + 23*c^2*d^4 - 11*a*c*d^2*e^2 +
 3*a^2*e^4 + 5*(7*c^2*d^3*e - a*c*d*e^3)*x)*sqrt(e*x + d))/(c^3*d^9 - 3*a*c
^2*d^7*e^2 + 3*a^2*c*d^5*e^4 - a^3*d^3*e^6 + (c^3*d^6*e^3 - 3*a*c^2*d^4*e^5
 + 3*a^2*c*d^2*e^7 - a^3*e^9)*x^3 + 3*(c^3*d^7*e^2 - 3*a*c^2*d^5*e^4 + 3*a^
2*c*d^3*e^6 - a^3*d*e^8)*x^2 + 3*(c^3*d^8*e - 3*a*c^2*d^6*e^3 + 3*a^2*c*d^4
*e^5 - a^3*d^2*e^7)*x)]
```

Sympy [A] time = 40.8523, size = 141, normalized size = 0.92

$$-\frac{2c^2d^2}{\sqrt{d+ex}\left(ae^2-cd^2\right)^3}-\frac{2c^2d^2\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{\frac{ae^2-cd^2}{cd}}}\right)}{\sqrt{\frac{ae^2-cd^2}{cd}}\left(ae^2-cd^2\right)^3}+\frac{2cd}{3\left(d+ex\right)^{\frac{3}{2}}\left(ae^2-cd^2\right)^2}-\frac{2}{5\left(d+ex\right)^{\frac{5}{2}}\left(ae^2-cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

```
[Out] -2*c**2*d**2/(sqrt(d + e*x)*(a*e**2 - c*d**2)**3) - 2*c**2*d**2*atan(sqrt(d
 + e*x)/sqrt((a*e**2 - c*d**2)/(c*d)))/(sqrt((a*e**2 - c*d**2)/(c*d))*(a*e*
*2 - c*d**2)**3) + 2*c*d/(3*(d + e*x)**(3/2)*(a*e**2 - c*d**2)**2) - 2/(5*(
d + e*x)**(5/2)*(a*e**2 - c*d**2))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.2008 $\int \frac{1}{(d+ex)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}\,dx$

Optimal. Leaf size=186

$$\frac{2c^3d^3}{\sqrt{d+ex}\left(cd^2-ae^2\right)^4}+\frac{2c^2d^2}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)^3}-\frac{2c^{7/2}d^{7/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{9/2}}+\frac{2cd}{5(d+ex)^{5/2}\left(cd^2-ae^2\right)^2}+\frac{}{7(d+ex)^7}$$

```
[Out] 2/(7*(c*d^2 - a*e^2)*(d + e*x)^(7/2)) + (2*c*d)/(5*(c*d^2 - a*e^2)^2*(d + e
*x)^(5/2)) + (2*c^2*d^2)/(3*(c*d^2 - a*e^2)^3*(d + e*x)^(3/2)) + (2*c^3*d^3
)/((c*d^2 - a*e^2)^4*Sqrt[d + e*x]) - (2*c^(7/2)*d^(7/2)*ArcTanh[(Sqrt[c]*S
qrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(c*d^2 - a*e^2)^(9/2)
```

Rubi [A] time = 0.163654, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {626, 51, 63, 208}

$$\frac{2c^3d^3}{\sqrt{d+ex}\left(cd^2-ae^2\right)^4}+\frac{2c^2d^2}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)^3}-\frac{2c^{7/2}d^{7/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{9/2}}+\frac{2cd}{5(d+ex)^{5/2}\left(cd^2-ae^2\right)^2}+\frac{}{7(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(7/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)),x]
```


```
[Out] 2/(7*(c*d^2 - a*e^2)*(d + e*x)^(7/2)) + (2*c*d)/(5*(c*d^2 - a*e^2)^2*(d + e
*x)^(5/2)) + (2*c^2*d^2)/(3*(c*d^2 - a*e^2)^3*(d + e*x)^(3/2)) + (2*c^3*d^3
)/((c*d^2 - a*e^2)^4*Sqrt[d + e*x]) - (2*c^(7/2)*d^(7/2)*ArcTanh[(Sqrt[c]*S
qrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(c*d^2 - a*e^2)^(9/2)
```


Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}dx = \int \frac{1}{(ae+cdx)(d+ex)^{9/2}}dx$$

$$= \frac{2}{7\left(cd^2-ae^2\right)(d+ex)^{7/2}} + \frac{(cd)\int \frac{1}{(ae+cdx)(d+ex)^{7/2}}dx}{cd^2-ae^2}$$

$$= \frac{2}{7\left(cd^2-ae^2\right)(d+ex)^{7/2}} + \frac{2cd}{5\left(cd^2-ae^2\right)^2(d+ex)^{5/2}} + \frac{\left(c^2d^2\right)\int \frac{}{(ae+c}}{\left(cd^2-\right.}$$

$$= \frac{2}{7\left(cd^2-ae^2\right)(d+ex)^{7/2}} + \frac{2cd}{5\left(cd^2-ae^2\right)^2(d+ex)^{5/2}} + \frac{2c^2}{3\left(cd^2-ae^2\right)^3}$$

$$= \frac{2}{7\left(cd^2-ae^2\right)(d+ex)^{7/2}} + \frac{2cd}{5\left(cd^2-ae^2\right)^2(d+ex)^{5/2}} + \frac{2c^2}{3\left(cd^2-ae^2\right)^3}$$

$$= \frac{2}{7\left(cd^2-ae^2\right)(d+ex)^{7/2}} + \frac{2cd}{5\left(cd^2-ae^2\right)^2(d+ex)^{5/2}} + \frac{2c^2}{3\left(cd^2-ae^2\right)^3}$$

$$= \frac{2}{7\left(cd^2-ae^2\right)(d+ex)^{7/2}} + \frac{2cd}{5\left(cd^2-ae^2\right)^2(d+ex)^{5/2}} + \frac{2c^2}{3\left(cd^2-ae^2\right)^3}$$

Mathematica [C] time = 0.0138336, size = 57, normalized size = 0.31

$$\frac{2\,{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};\frac{cd(d+ex)}{cd^2-ae^2}\right)}{7(d+ex)^{7/2}\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(7/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)),x]
```

```
[Out] (2*Hypergeometric2F1[-7/2, 1, -5/2, (c*d*(d + e*x))/(c*d^2 - a*e^2)])/(7*(c
*d^2 - a*e^2)*(d + e*x)^(7/2))
```

Maple [A] time = 0.199, size = 175, normalized size = 0.9

$$-\frac{2}{7\,ae^2-7\,cd^2}(ex+d)^{-\frac{7}{2}} - \frac{2\,c^2d^2}{3\left(ae^2-cd^2\right)^3}(ex+d)^{-\frac{3}{2}} + \frac{2\,cd}{5\left(ae^2-cd^2\right)^2}(ex+d)^{-\frac{5}{2}} + 2\frac{c^3d^3}{\left(ae^2-cd^2\right)^4\sqrt{ex+d}} + 2\frac{}{\left(ae^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

```
[Out] -2/7/(a*e^2-c*d^2)/(e*x+d)^(7/2)-2/3*c^2*d^2/(a*e^2-c*d^2)^3/(e*x+d)^(3/2)+
2/5*c*d/(a*e^2-c*d^2)^2/(e*x+d)^(5/2)+2*c^3*d^3/(a*e^2-c*d^2)^4/(e*x+d)^(1/
```

```
2)+2*c^4*d^4/(a*e^2-c*d^2)^4/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((e*x+d)^(1/2)
*c*d/((a*e^2-c*d^2)*c*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="m
axima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 2.03236, size = 2345, normalized size = 12.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="f
ricas")

[Out] [1/105*(105*(c^3*d^3*e^4*x^4 + 4*c^3*d^4*e^3*x^3 + 6*c^3*d^5*e^2*x^2 + 4*c^
3*d^6*e*x + c^3*d^7)*sqrt(c*d/(c*d^2 - a*e^2))*log((c*d*e*x + 2*c*d^2 - a*e
^2 - 2*(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(c*d/(c*d^2 - a*e^2)))/(c*d*x + a*
e)) + 2*(105*c^3*d^3*e^3*x^3 + 176*c^3*d^6 - 122*a*c^2*d^4*e^2 + 66*a^2*c*d
^2*e^4 - 15*a^3*e^6 + 35*(10*c^3*d^4*e^2 - a*c^2*d^2*e^4)*x^2 + 7*(58*c^3*d
^5*e - 16*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*x)*sqrt(e*x + d))/(c^4*d^12 - 4*a*
c^3*d^10*e^2 + 6*a^2*c^2*d^8*e^4 - 4*a^3*c*d^6*e^6 + a^4*d^4*e^8 + (c^4*d^8
*e^4 - 4*a*c^3*d^6*e^6 + 6*a^2*c^2*d^4*e^8 - 4*a^3*c*d^2*e^10 + a^4*e^12)*x
^4 + 4*(c^4*d^9*e^3 - 4*a*c^3*d^7*e^5 + 6*a^2*c^2*d^5*e^7 - 4*a^3*c*d^3*e^9
 + a^4*d*e^11)*x^3 + 6*(c^4*d^10*e^2 - 4*a*c^3*d^8*e^4 + 6*a^2*c^2*d^6*e^6
- 4*a^3*c*d^4*e^8 + a^4*d^2*e^10)*x^2 + 4*(c^4*d^11*e - 4*a*c^3*d^9*e^3 + 6
*a^2*c^2*d^7*e^5 - 4*a^3*c*d^5*e^7 + a^4*d^3*e^9)*x), -2/105*(105*(c^3*d^3*
e^4*x^4 + 4*c^3*d^4*e^3*x^3 + 6*c^3*d^5*e^2*x^2 + 4*c^3*d^6*e*x + c^3*d^7)*
sqrt(-c*d/(c*d^2 - a*e^2))*arctan(-(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(-c*d/
(c*d^2 - a*e^2))/(c*d*e*x + c*d^2)) - (105*c^3*d^3*e^3*x^3 + 176*c^3*d^6 -
122*a*c^2*d^4*e^2 + 66*a^2*c*d^2*e^4 - 15*a^3*e^6 + 35*(10*c^3*d^4*e^2 - a*
c^2*d^2*e^4)*x^2 + 7*(58*c^3*d^5*e - 16*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*x)*s
qrt(e*x + d))/(c^4*d^12 - 4*a*c^3*d^10*e^2 + 6*a^2*c^2*d^8*e^4 - 4*a^3*c*d^
6*e^6 + a^4*d^4*e^8 + (c^4*d^8*e^4 - 4*a*c^3*d^6*e^6 + 6*a^2*c^2*d^4*e^8 -
4*a^3*c*d^2*e^10 + a^4*e^12)*x^4 + 4*(c^4*d^9*e^3 - 4*a*c^3*d^7*e^5 + 6*a^2
*c^2*d^5*e^7 - 4*a^3*c*d^3*e^9 + a^4*d*e^11)*x^3 + 6*(c^4*d^10*e^2 - 4*a*c^
3*d^8*e^4 + 6*a^2*c^2*d^6*e^6 - 4*a^3*c*d^4*e^8 + a^4*d^2*e^10)*x^2 + 4*(c^
4*d^11*e - 4*a*c^3*d^9*e^3 + 6*a^2*c^2*d^7*e^5 - 4*a^3*c*d^5*e^7 + a^4*d^3*
e^9)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.2009 $\int \frac{(d+ex)^{13/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2} dx$

Optimal. Leaf size=210

$$\frac{9e(d+ex)^{5/2}\left(cd^2-ae^2\right)}{5c^3d^3}+\frac{3e(d+ex)^{3/2}\left(cd^2-ae^2\right)^2}{c^4d^4}+\frac{9e\sqrt{d+ex}\left(cd^2-ae^2\right)^3}{c^5d^5}-\frac{9e\left(cd^2-ae^2\right)^{7/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{11/2}d^{11/2}}-$$

```
[Out] (9*e*(c*d^2 - a*e^2)^3*Sqrt[d + e*x])/(c^5*d^5) + (3*e*(c*d^2 - a*e^2)^2*(d
 + e*x)^(3/2))/(c^4*d^4) + (9*e*(c*d^2 - a*e^2)*(d + e*x)^(5/2))/(5*c^3*d^3
) + (9*e*(d + e*x)^(7/2))/(7*c^2*d^2) - (d + e*x)^(9/2)/(c*d*(a*e + c*d*x))
 - (9*e*(c*d^2 - a*e^2)^(7/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[
c*d^2 - a*e^2]])/(c^(11/2)*d^(11/2))
```

Rubi [A] time = 0.176113, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {626, 47, 50, 63, 208}

$$\frac{9e(d+ex)^{5/2}\left(cd^2-ae^2\right)}{5c^3d^3}+\frac{3e(d+ex)^{3/2}\left(cd^2-ae^2\right)^2}{c^4d^4}+\frac{9e\sqrt{d+ex}\left(cd^2-ae^2\right)^3}{c^5d^5}-\frac{9e\left(cd^2-ae^2\right)^{7/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{11/2}d^{11/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(13/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] (9*e*(c*d^2 - a*e^2)^3*Sqrt[d + e*x])/(c^5*d^5) + (3*e*(c*d^2 - a*e^2)^2*(d
 + e*x)^(3/2))/(c^4*d^4) + (9*e*(c*d^2 - a*e^2)*(d + e*x)^(5/2))/(5*c^3*d^3
) + (9*e*(d + e*x)^(7/2))/(7*c^2*d^2) - (d + e*x)^(9/2)/(c*d*(a*e + c*d*x))
 - (9*e*(c*d^2 - a*e^2)^(7/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[
c*d^2 - a*e^2]])/(c^(11/2)*d^(11/2))
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{13/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx = \int \frac{(d+ex)^{9/2}}{(ae+cdx)^2}\,dx$$

$$= -\frac{(d+ex)^{9/2}}{cd(ae+cdx)} + \frac{(9e)\int \frac{(d+ex)^{7/2}}{ae+cdx}\,dx}{2cd}$$

$$= \frac{9e(d+ex)^{7/2}}{7c^2d^2} - \frac{(d+ex)^{9/2}}{cd(ae+cdx)} + \frac{\left(9e\left(cd^2-ae^2\right)\right)\int \frac{(d+ex)^{5/2}}{ae+cdx}\,dx}{2c^2d^2}$$

$$= \frac{9e\left(cd^2-ae^2\right)(d+ex)^{5/2}}{5c^3d^3} + \frac{9e(d+ex)^{7/2}}{7c^2d^2} - \frac{(d+ex)^{9/2}}{cd(ae+cdx)} + \frac{\left(9e\left(cd^2-ae^2\right)^2\right)\int}{2c^3d^3}$$

$$= \frac{3e\left(cd^2-ae^2\right)^2(d+ex)^{3/2}}{c^4d^4} + \frac{9e\left(cd^2-ae^2\right)(d+ex)^{5/2}}{5c^3d^3} + \frac{9e(d+ex)^{7/2}}{7c^2d^2} - \frac{(d+}{cd(ae}$$

$$= \frac{9e\left(cd^2-ae^2\right)^3\sqrt{d+ex}}{c^5d^5} + \frac{3e\left(cd^2-ae^2\right)^2(d+ex)^{3/2}}{c^4d^4} + \frac{9e\left(cd^2-ae^2\right)(d+ex)^{5}}{5c^3d^3}$$

$$= \frac{9e\left(cd^2-ae^2\right)^3\sqrt{d+ex}}{c^5d^5} + \frac{3e\left(cd^2-ae^2\right)^2(d+ex)^{3/2}}{c^4d^4} + \frac{9e\left(cd^2-ae^2\right)(d+ex)^{5}}{5c^3d^3}$$

$$= \frac{9e\left(cd^2-ae^2\right)^3\sqrt{d+ex}}{c^5d^5} + \frac{3e\left(cd^2-ae^2\right)^2(d+ex)^{3/2}}{c^4d^4} + \frac{9e\left(cd^2-ae^2\right)(d+ex)^{5}}{5c^3d^3}$$

Mathematica [C] time = 0.0251819, size = 59, normalized size = 0.28

$$\frac{2e(d+ex)^{11/2}\,{}_2F_1\left(2,\frac{11}{2};\frac{13}{2};-\frac{cd(d+ex)}{ae^2-cd^2}\right)}{11\left(ae^2-cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(13/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] (2*e*(d + e*x)^(11/2)*Hypergeometric2F1[2, 11/2, 13/2, -((c*d*(d + e*x))/(-
(c*d^2) + a*e^2))])/(11*(-(c*d^2) + a*e^2)^2)
```

Maple [B] time = 0.203, size = 628, normalized size = 3.

$$\frac{2e}{7c^2d^2}(ex+d)^{\frac{7}{2}} - \frac{4ae^3}{5c^3d^3}(ex+d)^{\frac{5}{2}} + \frac{4e}{5c^2d}(ex+d)^{\frac{5}{2}} + 2\frac{(ex+d)^{3/2}a^2e^5}{c^4d^4} - 4\frac{(ex+d)^{3/2}ae^3}{c^3d^2} + 2\frac{e(ex+d)^{3/2}}{c^2} - 8\frac{a^3e^7\sqrt{ex}}{c^5d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(13/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

```
[Out] 2/7*e*(e*x+d)^(7/2)/c^2/d^2-4/5/c^3/d^3*(e*x+d)^(5/2)*a*e^3+4/5*e/c^2/d*(e*
x+d)^(5/2)+2/c^4/d^4*(e*x+d)^(3/2)*a^2*e^5-4/c^3/d^2*(e*x+d)^(3/2)*a*e^3+2*
e/c^2*(e*x+d)^(3/2)-8/c^5/d^5*a^3*e^7*(e*x+d)^(1/2)+24/c^4/d^3*a^2*e^5*(e*x
+d)^(1/2)-24/c^3/d*a*e^3*(e*x+d)^(1/2)+8*e/c^2*d*(e*x+d)^(1/2)-1/c^5/d^5*(e
*x+d)^(1/2)/(c*d*e*x+a*e^2)*a^4*e^9+4/c^4/d^3*(e*x+d)^(1/2)/(c*d*e*x+a*e^2)
*a^3*e^7-6/c^3/d*(e*x+d)^(1/2)/(c*d*e*x+a*e^2)*a^2*e^5+4/c^2*d*(e*x+d)^(1/2
)/(c*d*e*x+a*e^2)*a*e^3-e/c*d^3*(e*x+d)^(1/2)/(c*d*e*x+a*e^2)+9/c^5/d^5/((a
*e^2-c*d^2)*c*d)^(1/2)*arctan((e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1/2))*
a^4*e^9-36/c^4/d^3/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((e*x+d)^(1/2)*c*d/((a*e
^2-c*d^2)*c*d)^(1/2))*a^3*e^7+54/c^3/d/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((e*
x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1/2))*a^2*e^5-36/c^2*d/((a*e^2-c*d^2)*c
*d)^(1/2)*arctan((e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1/2))*a*e^3+9*e/c*d
^3/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(
1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(13/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.03539, size = 1629, normalized size = 7.76

$$\left[\frac{315\left(ac^3d^6e^2 - 3a^2c^2d^4e^4 + 3a^3cd^2e^6 - a^4e^8 + \left(c^4d^7e - 3ac^3d^5e^3 + 3a^2c^2d^3e^5 - a^3cde^7\right)x\right)\sqrt{\frac{cd^2-ae^2}{cd}}\log\left(\frac{cdex+2cd^2-ae^2-2\sqrt{\cdot}}{cdx+\cdot}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(13/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="
fricas")
```

```
[Out] [1/70*(315*(a*c^3*d^6*e^2 - 3*a^2*c^2*d^4*e^4 + 3*a^3*c*d^2*e^6 - a^4*e^8 +
 (c^4*d^7*e - 3*a*c^3*d^5*e^3 + 3*a^2*c^2*d^3*e^5 - a^3*c*d*e^7)*x)*sqrt((c
*d^2 - a*e^2)/(c*d))*log((c*d*e*x + 2*c*d^2 - a*e^2 - 2*sqrt(e*x + d)*c*d*s
```

```
qrt((c*d^2 - a*e^2)/(c*d)))/(c*d*x + a*e)) + 2*(10*c^4*d^4*e^4*x^4 - 35*c^4
*d^8 + 528*a*c^3*d^6*e^2 - 1218*a^2*c^2*d^4*e^4 + 1050*a^3*c*d^2*e^6 - 315*
a^4*e^8 + 2*(29*c^4*d^5*e^3 - 9*a*c^3*d^3*e^5)*x^3 + 6*(26*c^4*d^6*e^2 - 23
*a*c^3*d^4*e^4 + 7*a^2*c^2*d^2*e^6)*x^2 + 2*(194*c^4*d^7*e - 426*a*c^3*d^5*
e^3 + 357*a^2*c^2*d^3*e^5 - 105*a^3*c*d*e^7)*x)*sqrt(e*x + d))/(c^6*d^6*x +
 a*c^5*d^5*e), -1/35*(315*(a*c^3*d^6*e^2 - 3*a^2*c^2*d^4*e^4 + 3*a^3*c*d^2*
e^6 - a^4*e^8 + (c^4*d^7*e - 3*a*c^3*d^5*e^3 + 3*a^2*c^2*d^3*e^5 - a^3*c*d*
e^7)*x)*sqrt(-(c*d^2 - a*e^2)/(c*d))*arctan(-sqrt(e*x + d)*c*d*sqrt(-(c*d^2
 - a*e^2)/(c*d))/(c*d^2 - a*e^2)) - (10*c^4*d^4*e^4*x^4 - 35*c^4*d^8 + 528*
a*c^3*d^6*e^2 - 1218*a^2*c^2*d^4*e^4 + 1050*a^3*c*d^2*e^6 - 315*a^4*e^8 + 2
*(29*c^4*d^5*e^3 - 9*a*c^3*d^3*e^5)*x^3 + 6*(26*c^4*d^6*e^2 - 23*a*c^3*d^4*
e^4 + 7*a^2*c^2*d^2*e^6)*x^2 + 2*(194*c^4*d^7*e - 426*a*c^3*d^5*e^3 + 357*a
^2*c^2*d^3*e^5 - 105*a^3*c*d*e^7)*x)*sqrt(e*x + d))/(c^6*d^6*x + a*c^5*d^5*
e)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(13/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(13/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="
giac")

[Out] Timed out
```

3.2010 $\int \frac{(d+ex)^{11/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$

Optimal. Leaf size=178

$$\frac{7e(d+ex)^{3/2}\left(cd^2-ae^2\right)}{3c^3d^3}+\frac{7e\sqrt{d+ex}\left(cd^2-ae^2\right)^2}{c^4d^4}-\frac{7e\left(cd^2-ae^2\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{9/2}d^{9/2}}-\frac{(d+ex)^{7/2}}{cd(ae+cdx)}+\frac{7e(d+ex)^{5/2}}{5c^2d^2}$$

```
[Out] (7*e*(c*d^2 - a*e^2)^2*Sqrt[d + e*x])/(c^4*d^4) + (7*e*(c*d^2 - a*e^2)*(d +
 e*x)^(3/2))/(3*c^3*d^3) + (7*e*(d + e*x)^(5/2))/(5*c^2*d^2) - (d + e*x)^(7
/2)/(c*d*(a*e + c*d*x)) - (7*e*(c*d^2 - a*e^2)^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[
d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(c^(9/2)*d^(9/2))
```

Rubi [A] time = 0.134765, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {626, 47, 50, 63, 208}

$$\frac{7e(d+ex)^{3/2}\left(cd^2-ae^2\right)}{3c^3d^3}+\frac{7e\sqrt{d+ex}\left(cd^2-ae^2\right)^2}{c^4d^4}-\frac{7e\left(cd^2-ae^2\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{9/2}d^{9/2}}-\frac{(d+ex)^{7/2}}{cd(ae+cdx)}+\frac{7e(d+ex)^{5/2}}{5c^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(11/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] (7*e*(c*d^2 - a*e^2)^2*Sqrt[d + e*x])/(c^4*d^4) + (7*e*(c*d^2 - a*e^2)*(d +
 e*x)^(3/2))/(3*c^3*d^3) + (7*e*(d + e*x)^(5/2))/(5*c^2*d^2) - (d + e*x)^(7
/2)/(c*d*(a*e + c*d*x)) - (7*e*(c*d^2 - a*e^2)^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[
d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(c^(9/2)*d^(9/2))
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{11/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx = \int \frac{(d+ex)^{7/2}}{(ae+cdx)^2}\,dx$$

$$= -\frac{(d+ex)^{7/2}}{cd(ae+cdx)} + \frac{(7e)\int \frac{(d+ex)^{5/2}}{ae+cdx}\,dx}{2cd}$$

$$= \frac{7e(d+ex)^{5/2}}{5c^2d^2} - \frac{(d+ex)^{7/2}}{cd(ae+cdx)} + \frac{\left(7e\left(cd^2-ae^2\right)\right)\int \frac{(d+ex)^{3/2}}{ae+cdx}\,dx}{2c^2d^2}$$

$$= \frac{7e\left(cd^2-ae^2\right)(d+ex)^{3/2}}{3c^3d^3} + \frac{7e(d+ex)^{5/2}}{5c^2d^2} - \frac{(d+ex)^{7/2}}{cd(ae+cdx)} + \frac{\left(7e\left(cd^2-ae^2\right)^2\right)\int}{2c^3d^3}$$

$$= \frac{7e\left(cd^2-ae^2\right)^2\sqrt{d+ex}}{c^4d^4} + \frac{7e\left(cd^2-ae^2\right)(d+ex)^{3/2}}{3c^3d^3} + \frac{7e(d+ex)^{5/2}}{5c^2d^2} - \frac{(d+ex}{cd(ae+}$$

$$= \frac{7e\left(cd^2-ae^2\right)^2\sqrt{d+ex}}{c^4d^4} + \frac{7e\left(cd^2-ae^2\right)(d+ex)^{3/2}}{3c^3d^3} + \frac{7e(d+ex)^{5/2}}{5c^2d^2} - \frac{(d+ex}{cd(ae+}$$

$$= \frac{7e\left(cd^2-ae^2\right)^2\sqrt{d+ex}}{c^4d^4} + \frac{7e\left(cd^2-ae^2\right)(d+ex)^{3/2}}{3c^3d^3} + \frac{7e(d+ex)^{5/2}}{5c^2d^2} - \frac{(d+ex}{cd(ae+}$$

Mathematica [C] time = 0.0186768, size = 59, normalized size = 0.33

$$\frac{2e(d+ex)^{9/2}\,{}_2F_1\left(2,\frac{9}{2};\frac{11}{2};-\frac{cd(d+ex)}{ae^2-cd^2}\right)}{9\left(ae^2-cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(11/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] (2*e*(d + e*x)^(9/2)*Hypergeometric2F1[2, 9/2, 11/2, -((c*d*(d + e*x))/(-(c
*d^2) + a*e^2))])/(9*(-(c*d^2) + a*e^2)^2)
```

Maple [B] time = 0.205, size = 457, normalized size = 2.6

$$\frac{2e}{5c^2d^2}(ex+d)^{\frac{5}{2}} - \frac{4ae^3}{3c^3d^3}(ex+d)^{\frac{3}{2}} + \frac{4e}{3c^2d}(ex+d)^{\frac{3}{2}} + 6\frac{a^2e^5\sqrt{ex+d}}{c^4d^4} - 12\frac{ae^3\sqrt{ex+d}}{c^3d^2} + 6\frac{e\sqrt{ex+d}}{c^2} + \frac{a^3e^7}{c^4d^4\left(cdex+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(11/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

```
[Out] 2/5*e*(e*x+d)^(5/2)/c^2/d^2-4/3/c^3/d^3*(e*x+d)^(3/2)*a*e^3+4/3*e/c^2/d*(e*
x+d)^(3/2)+6/c^4/d^4*a^2*e^5*(e*x+d)^(1/2)-12/c^3/d^2*a*e^3*(e*x+d)^(1/2)+6
*e/c^2*(e*x+d)^(1/2)+1/c^4/d^4*(e*x+d)^(1/2)/(c*d*e*x+a*e^2)*a^3*e^7-3/c^3/
d^2*(e*x+d)^(1/2)/(c*d*e*x+a*e^2)*a^2*e^5+3/c^2*(e*x+d)^(1/2)/(c*d*e*x+a*e^
2)*a*e^3-e/c*d^2*(e*x+d)^(1/2)/(c*d*e*x+a*e^2)-7/c^4/d^4/((a*e^2-c*d^2)*c*d
)^(1/2)*arctan((e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1/2))*a^3*e^7+21/c^3/
d^2/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^
(1/2))*a^2*e^5-21/c^2/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((e*x+d)^(1/2)*c*d/((
a*e^2-c*d^2)*c*d)^(1/2))*a*e^3+7*e/c*d^2/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((
e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(11/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.81165, size = 1214, normalized size = 6.82

$$\left[\frac{105\left(ac^2d^4e^2 - 2\,a^2cd^2e^4 + a^3e^6 + \left(c^3d^5e - 2\,ac^2d^3e^3 + a^2cde^5\right)x\right)\sqrt{\frac{cd^2-ae^2}{cd}}\log\left(\frac{cdex+2\,cd^2-ae^2-2\,\sqrt{ex+d}cd\sqrt{\frac{cd^2-ae^2}{cd}}}{cdx+ae}\right)+2\left(6\,c^3d^3\right.}{30\,(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(11/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="
fricas")
```

```
[Out] [1/30*(105*(a*c^2*d^4*e^2 - 2*a^2*c*d^2*e^4 + a^3*e^6 + (c^3*d^5*e - 2*a*c^
2*d^3*e^3 + a^2*c*d*e^5)*x)*sqrt((c*d^2 - a*e^2)/(c*d))*log((c*d*e*x + 2*c*
d^2 - a*e^2 - 2*sqrt(e*x + d)*c*d*sqrt((c*d^2 - a*e^2)/(c*d)))/(c*d*x + a*e
)) + 2*(6*c^3*d^3*e^3*x^3 - 15*c^3*d^6 + 161*a*c^2*d^4*e^2 - 245*a^2*c*d^2*
e^4 + 105*a^3*e^6 + 2*(16*c^3*d^4*e^2 - 7*a*c^2*d^2*e^4)*x^2 + 2*(58*c^3*d^
5*e - 84*a*c^2*d^3*e^3 + 35*a^2*c*d*e^5)*x)*sqrt(e*x + d))/(c^5*d^5*x + a*c
^4*d^4*e), -1/15*(105*(a*c^2*d^4*e^2 - 2*a^2*c*d^2*e^4 + a^3*e^6 + (c^3*d^5
*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x)*sqrt(-(c*d^2 - a*e^2)/(c*d))*arctan(
-sqrt(e*x + d)*c*d*sqrt(-(c*d^2 - a*e^2)/(c*d))/(c*d^2 - a*e^2)) - (6*c^3*d
^3*e^3*x^3 - 15*c^3*d^6 + 161*a*c^2*d^4*e^2 - 245*a^2*c*d^2*e^4 + 105*a^3*e
^6 + 2*(16*c^3*d^4*e^2 - 7*a*c^2*d^2*e^4)*x^2 + 2*(58*c^3*d^5*e - 84*a*c^2*
d^3*e^3 + 35*a^2*c*d*e^5)*x)*sqrt(e*x + d))/(c^5*d^5*x + a*c^4*d^4*e)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(11/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(11/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="
giac")
```

```
[Out] Timed out
```

3.2011 $\int \frac{(d+ex)^{9/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2} dx$

Optimal. Leaf size=144

$$\frac{5e\sqrt{d+ex}\left(cd^2-ae^2\right)}{c^3d^3}-\frac{5e\left(cd^2-ae^2\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{7/2}d^{7/2}}-\frac{(d+ex)^{5/2}}{cd(ae+cdx)}+\frac{5e(d+ex)^{3/2}}{3c^2d^2}$$

```
[Out] (5*e*(c*d^2 - a*e^2)*Sqrt[d + e*x])/(c^3*d^3) + (5*e*(d + e*x)^(3/2))/(3*c^
2*d^2) - (d + e*x)^(5/2)/(c*d*(a*e + c*d*x)) - (5*e*(c*d^2 - a*e^2)^(3/2)*A
rcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(c^(7/2)*d^(7/
2))
```

Rubi [A] time = 0.0953684, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {626, 47, 50, 63, 208}

$$\frac{5e\sqrt{d+ex}\left(cd^2-ae^2\right)}{c^3d^3}-\frac{5e\left(cd^2-ae^2\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{7/2}d^{7/2}}-\frac{(d+ex)^{5/2}}{cd(ae+cdx)}+\frac{5e(d+ex)^{3/2}}{3c^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(9/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] (5*e*(c*d^2 - a*e^2)*Sqrt[d + e*x])/(c^3*d^3) + (5*e*(d + e*x)^(3/2))/(3*c^
2*d^2) - (d + e*x)^(5/2)/(c*d*(a*e + c*d*x)) - (5*e*(c*d^2 - a*e^2)^(3/2)*A
rcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(c^(7/2)*d^(7/
2))
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{9/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx = \int \frac{(d+ex)^{5/2}}{(ae+cdx)^2}\,dx$$

$$= -\frac{(d+ex)^{5/2}}{cd(ae+cdx)} + \frac{(5e)\int \frac{(d+ex)^{3/2}}{ae+cdx}\,dx}{2cd}$$

$$= \frac{5e(d+ex)^{3/2}}{3c^2d^2} - \frac{(d+ex)^{5/2}}{cd(ae+cdx)} + \frac{\left(5e\left(cd^2-ae^2\right)\right)\int \frac{\sqrt{d+ex}}{ae+cdx}\,dx}{2c^2d^2}$$

$$= \frac{5e\left(cd^2-ae^2\right)\sqrt{d+ex}}{c^3d^3} + \frac{5e(d+ex)^{3/2}}{3c^2d^2} - \frac{(d+ex)^{5/2}}{cd(ae+cdx)} + \frac{\left(5e\left(cd^2-ae^2\right)^2\right)\int \frac{}{(}}{2c^3d^3}$$

$$= \frac{5e\left(cd^2-ae^2\right)\sqrt{d+ex}}{c^3d^3} + \frac{5e(d+ex)^{3/2}}{3c^2d^2} - \frac{(d+ex)^{5/2}}{cd(ae+cdx)} + \frac{\left(5\left(cd^2-ae^2\right)^2\right)\mathrm{Sub}}{}$$

$$= \frac{5e\left(cd^2-ae^2\right)\sqrt{d+ex}}{c^3d^3} + \frac{5e(d+ex)^{3/2}}{3c^2d^2} - \frac{(d+ex)^{5/2}}{cd(ae+cdx)} - \frac{5e\left(cd^2-ae^2\right)^{3/2}\tanh}{c^{7/2}d^{7/}}$$

Mathematica [C] time = 0.0175845, size = 59, normalized size = 0.41

$$\frac{2e(d+ex)^{7/2}\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};-\frac{cd(d+ex)}{ae^2-cd^2}\right)}{7\left(ae^2-cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(9/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] (2*e*(d + e*x)^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, -((c*d*(d + e*x))/(-(c*
d^2) + a*e^2))])/(7*(-(c*d^2) + a*e^2)^2)
```

Maple [B] time = 0.204, size = 314, normalized size = 2.2

$$\frac{2e}{3c^2d^2}(ex+d)^{\frac{3}{2}} - 4\frac{ae^3\sqrt{ex+d}}{c^3d^3} + 4\frac{e\sqrt{ex+d}}{c^2d} - \frac{a^2e^5}{c^3d^3\left(cdex+ae^2\right)}\sqrt{ex+d} + 2\frac{ae^3\sqrt{ex+d}}{c^2d\left(cdex+ae^2\right)} - \frac{de}{c\left(cdex+ae^2\right)}\sqrt{ex}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

```
[Out] 2/3*e*(e*x+d)^(3/2)/c^2/d^2-4/c^3/d^3*a*e^3*(e*x+d)^(1/2)+4*e/c^2/d*(e*x+d)
^(1/2)-1/c^3/d^3*(e*x+d)^(1/2)/(c*d*e*x+a*e^2)*a^2*e^5+2/c^2/d*(e*x+d)^(1/2
)/(c*d*e*x+a*e^2)*a*e^3-e/c*d*(e*x+d)^(1/2)/(c*d*e*x+a*e^2)+5/c^3/d^3/((a*e
^2-c*d^2)*c*d)^(1/2)*arctan((e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1/2))*a^
2*e^5-10/c^2/d/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((e*x+d)^(1/2)*c*d/((a*e^2-c
*d^2)*c*d)^(1/2))*a*e^3+5*e/c*d/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((e*x+d)^(1
/2)*c*d/((a*e^2-c*d^2)*c*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.97144, size = 855, normalized size = 5.94

$$\left[\frac{15\left(acd^2e^2 - a^2e^4 + \left(c^2d^3e - acde^3\right)x\right)\sqrt{\frac{cd^2-ae^2}{cd}}\log\left(\frac{cdex+2\,cd^2-ae^2-2\,\sqrt{ex+d}cd\sqrt{\frac{cd^2-ae^2}{cd}}}{cdx+ae}\right) + 2\left(2\,c^2d^2e^2x^2 - 3\,c^2d^4 + 20\,acd^2e^2 - \right.}{6\left(c^4d^4x + ac^3d^3e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="f
ricas")
```

```
[Out] [1/6*(15*(a*c*d^2*e^2 - a^2*e^4 + (c^2*d^3*e - a*c*d*e^3)*x)*sqrt((c*d^2 -
a*e^2)/(c*d))*log((c*d*e*x + 2*c*d^2 - a*e^2 - 2*sqrt(e*x + d)*c*d*sqrt((c*
d^2 - a*e^2)/(c*d)))/(c*d*x + a*e)) + 2*(2*c^2*d^2*e^2*x^2 - 3*c^2*d^4 + 20
*a*c*d^2*e^2 - 15*a^2*e^4 + 2*(7*c^2*d^3*e - 5*a*c*d*e^3)*x)*sqrt(e*x + d))
/(c^4*d^4*x + a*c^3*d^3*e), -1/3*(15*(a*c*d^2*e^2 - a^2*e^4 + (c^2*d^3*e -
a*c*d*e^3)*x)*sqrt(-(c*d^2 - a*e^2)/(c*d))*arctan(-sqrt(e*x + d)*c*d*sqrt(-
(c*d^2 - a*e^2)/(c*d))/(c*d^2 - a*e^2)) - (2*c^2*d^2*e^2*x^2 - 3*c^2*d^4 +
20*a*c*d^2*e^2 - 15*a^2*e^4 + 2*(7*c^2*d^3*e - 5*a*c*d*e^3)*x)*sqrt(e*x + d
))/(c^4*d^4*x + a*c^3*d^3*e)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(9/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.2012 $\int \frac{(d+ex)^{7/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$

Optimal. Leaf size=112

$$-\frac{3e\sqrt{cd^2-ae^2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{5/2}d^{5/2}}-\frac{(d+ex)^{3/2}}{cd(ae+cdx)}+\frac{3e\sqrt{d+ex}}{c^2d^2}$$

```
[Out] (3*e*Sqrt[d + e*x])/(c^2*d^2) - (d + e*x)^(3/2)/(c*d*(a*e + c*d*x)) - (3*e*
Sqrt[c*d^2 - a*e^2]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*
e^2]])/(c^(5/2)*d^(5/2))
```

Rubi [A] time = 0.0699287, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {626, 47, 50, 63, 208}

$$-\frac{3e\sqrt{cd^2-ae^2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{5/2}d^{5/2}}-\frac{(d+ex)^{3/2}}{cd(ae+cdx)}+\frac{3e\sqrt{d+ex}}{c^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] (3*e*Sqrt[d + e*x])/(c^2*d^2) - (d + e*x)^(3/2)/(c*d*(a*e + c*d*x)) - (3*e*
Sqrt[c*d^2 - a*e^2]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*
e^2]])/(c^(5/2)*d^(5/2))
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^{7/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx &= \int \frac{(d+ex)^{3/2}}{(ae+cdx)^2}\,dx\\
&= -\frac{(d+ex)^{3/2}}{cd(ae+cdx)}+\frac{(3e)\int \frac{\sqrt{d+ex}}{ae+cdx}\,dx}{2cd}\\
&= \frac{3e\sqrt{d+ex}}{c^2d^2}-\frac{(d+ex)^{3/2}}{cd(ae+cdx)}+\frac{\left(3e\left(cd^2-ae^2\right)\right)\int \frac{1}{(ae+cdx)\sqrt{d+ex}}\,dx}{2c^2d^2}\\
&= \frac{3e\sqrt{d+ex}}{c^2d^2}-\frac{(d+ex)^{3/2}}{cd(ae+cdx)}+\frac{\left(3\left(cd^2-ae^2\right)\right)\text{Subst}\left(\int \frac{1}{-\frac{cd^2}{e}+ae+\frac{cdx^2}{e}}\,dx,x,\sqrt{d+ex}\right)}{c^2d^2}\\
&= \frac{3e\sqrt{d+ex}}{c^2d^2}-\frac{(d+ex)^{3/2}}{cd(ae+cdx)}-\frac{3e\sqrt{cd^2-ae^2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{5/2}d^{5/2}}\end{aligned}$$

Mathematica [C] time = 0.0149621, size = 59, normalized size = 0.53

$$\frac{2e(d+ex)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};-\frac{cd(d+ex)}{ae^2-cd^2}\right)}{5\left(ae^2-cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] (2*e*(d + e*x)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, -((c*d*(d + e*x))/(-(c*
d^2) + a*e^2))])/(5*(-(c*d^2) + a*e^2)^2)
```

Maple [A] time = 0.203, size = 183, normalized size = 1.6

$$2\,\frac{e\sqrt{ex+d}}{c^2d^2}+\frac{ae^3}{c^2d^2\left(cdex+ae^2\right)}\sqrt{ex+d}-\frac{e}{c\left(cdex+ae^2\right)}\sqrt{ex+d}-3\,\frac{ae^3}{c^2d^2\sqrt{\left(ae^2-cd^2\right)cd}}\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{\left(ae^2-cd^2\right)cd}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

```
[Out] 2*e*(e*x+d)^(1/2)/c^2/d^2+1/c^2/d^2*(e*x+d)^(1/2)/(c*d*e*x+a*e^2)*a*e^3-e/c
*(e*x+d)^(1/2)/(c*d*e*x+a*e^2)-3/c^2/d^2/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((
```

```
e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1/2))*a*e^3+3*e/c/((a*e^2-c*d^2)*c*d)
^(1/2)*arctan((e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.96643, size = 576, normalized size = 5.14

$$\left[\frac{3\left(cdex+ae^2\right)\sqrt{\frac{cd^2-ae^2}{cd}}\log\left(\frac{cdex+2\,cd^2-ae^2-2\,\sqrt{ex+d}cd\sqrt{\frac{cd^2-ae^2}{cd}}}{cdx+ae}\right)+2\left(2\,cdex-cd^2+3\,ae^2\right)\sqrt{ex+d}}{2\left(c^3d^3x+ac^2d^2e\right)},-\frac{3\left(cdex+ae^2\right)\sqrt{-\frac{cd^2-ae^2}{cd}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="f
ricas")
```

```
[Out] [1/2*(3*(c*d*e*x + a*e^2)*sqrt((c*d^2 - a*e^2)/(c*d))*log((c*d*e*x + 2*c*d^
2 - a*e^2 - 2*sqrt(e*x + d)*c*d*sqrt((c*d^2 - a*e^2)/(c*d)))/(c*d*x + a*e))
 + 2*(2*c*d*e*x - c*d^2 + 3*a*e^2)*sqrt(e*x + d))/(c^3*d^3*x + a*c^2*d^2*e)
, -(3*(c*d*e*x + a*e^2)*sqrt(-(c*d^2 - a*e^2)/(c*d))*arctan(-sqrt(e*x + d)*
c*d*sqrt(-(c*d^2 - a*e^2)/(c*d))/(c*d^2 - a*e^2)) - (2*c*d*e*x - c*d^2 + 3*
a*e^2)*sqrt(e*x + d))/(c^3*d^3*x + a*c^2*d^2*e)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.2013 $\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2} dx$

Optimal. Leaf size=94

$$-\frac{e\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{3/2}d^{3/2}\sqrt{cd^2-ae^2}}-\frac{\sqrt{d+ex}}{cd(ae+cdx)}$$

```
[Out] -(Sqrt[d + e*x]/(c*d*(a*e + c*d*x))) - (e*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d +
 e*x])/Sqrt[c*d^2 - a*e^2]])/(c^(3/2)*d^(3/2)*Sqrt[c*d^2 - a*e^2])
```

Rubi [A] time = 0.0571539, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {626, 47, 63, 208}

$$-\frac{e\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{3/2}d^{3/2}\sqrt{cd^2-ae^2}}-\frac{\sqrt{d+ex}}{cd(ae+cdx)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] -(Sqrt[d + e*x]/(c*d*(a*e + c*d*x))) - (e*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d +
 e*x])/Sqrt[c*d^2 - a*e^2]])/(c^(3/2)*d^(3/2)*Sqrt[c*d^2 - a*e^2])
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx=\int \frac{\sqrt{d+ex}}{(ae+cdx)^2}\,dx$$
$$=-\frac{\sqrt{d+ex}}{cd(ae+cdx)}+\frac{e\int \frac{1}{(ae+cdx)\sqrt{d+ex}}\,dx}{2cd}$$
$$=-\frac{\sqrt{d+ex}}{cd(ae+cdx)}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{cd^2}{e}+ae+\frac{cdx^2}{e}}\,dx,x,\sqrt{d+ex}\right)}{cd}$$
$$=-\frac{\sqrt{d+ex}}{cd(ae+cdx)}-\frac{e\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{3/2}d^{3/2}\sqrt{cd^2-ae^2}}$$

Mathematica [A] time = 0.109552, size = 93, normalized size = 0.99

$$\frac{e\tan^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{ae^2-cd^2}}\right)}{c^{3/2}d^{3/2}\sqrt{ae^2-cd^2}}-\frac{\sqrt{d+ex}}{acde+c^2d^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] -(Sqrt[d + e*x]/(a*c*d*e + c^2*d^2*x)) + (e*ArcTan[(Sqrt[c]*Sqrt[d]*Sqrt[d
+ e*x])/Sqrt[-(c*d^2) + a*e^2]])/(c^(3/2)*d^(3/2)*Sqrt[-(c*d^2) + a*e^2])
```

Maple [A] time = 0.201, size = 84, normalized size = 0.9

$$-\frac{e}{cd\left(cdex+ae^2\right)}\sqrt{ex+d}+\frac{e}{cd}\arctan\left(cd\sqrt{ex+d}\frac{1}{\sqrt{\left(ae^2-cd^2\right)cd}}\right)\frac{1}{\sqrt{\left(ae^2-cd^2\right)cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

```
[Out] -e/d/c*(e*x+d)^(1/2)/(c*d*e*x+a*e^2)+e/d/c/((a*e^2-c*d^2)*c*d)^(1/2)*arctan
((e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.99687, size = 625, normalized size = 6.65

$$\left[\frac{\sqrt{c^2d^3-acde^2}\left(cdex+ae^2\right)\log\left(\frac{cdex+2\,cd^2-ae^2-2\,\sqrt{c^2d^3-acde^2}\sqrt{ex+d}}{cdx+ae}\right)-2\left(c^2d^3-acde^2\right)\sqrt{ex+d}}{2\left(ac^3d^4e-a^2c^2d^2e^3+\left(c^4d^5-ac^3d^3e^2\right)x\right)},\ \frac{\sqrt{-c^2d^3+acde^2}\left(cdex+ae^2\right)}{ac^3d^4e-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="f
ricas")
```

```
[Out] [1/2*(sqrt(c^2*d^3 - a*c*d*e^2)*(c*d*e*x + a*e^2)*log((c*d*e*x + 2*c*d^2 -
a*e^2 - 2*sqrt(c^2*d^3 - a*c*d*e^2)*sqrt(e*x + d))/(c*d*x + a*e)) - 2*(c^2*
d^3 - a*c*d*e^2)*sqrt(e*x + d))/(a*c^3*d^4*e - a^2*c^2*d^2*e^3 + (c^4*d^5 -
 a*c^3*d^3*e^2)*x), (sqrt(-c^2*d^3 + a*c*d*e^2)*(c*d*e*x + a*e^2)*arctan(sq
rt(-c^2*d^3 + a*c*d*e^2)*sqrt(e*x + d)/(c*d*e*x + c*d^2)) - (c^2*d^3 - a*c*
d*e^2)*sqrt(e*x + d))/(a*c^3*d^4*e - a^2*c^2*d^2*e^3 + (c^4*d^5 - a*c^3*d^3
*e^2)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.2014 $\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$

Optimal. Leaf size=101

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\sqrt{c}\sqrt{d}\left(cd^2-ae^2\right)^{3/2}}-\frac{\sqrt{d+ex}}{\left(cd^2-ae^2\right)(ae+cdx)}$$

```
[Out] -(Sqrt[d + e*x]/((c*d^2 - a*e^2)*(a*e + c*d*x))) + (e*ArcTanh[(Sqrt[c]*Sqrt
[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(Sqrt[c]*Sqrt[d]*(c*d^2 - a*e^2)^(
3/2))
```

Rubi [A] time = 0.0565612, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {626, 51, 63, 208}

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\sqrt{c}\sqrt{d}\left(cd^2-ae^2\right)^{3/2}}-\frac{\sqrt{d+ex}}{\left(cd^2-ae^2\right)(ae+cdx)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```


```
[Out] -(Sqrt[d + e*x]/((c*d^2 - a*e^2)*(a*e + c*d*x))) + (e*ArcTanh[(Sqrt[c]*Sqrt
[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(Sqrt[c]*Sqrt[d]*(c*d^2 - a*e^2)^(
3/2))
```


Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx=\int \frac{1}{(ae+cdx)^2\sqrt{d+ex}}\,dx$$
$$=-\frac{\sqrt{d+ex}}{\left(cd^2-ae^2\right)(ae+cdx)}-\frac{e\int \frac{1}{(ae+cdx)\sqrt{d+ex}}\,dx}{2\left(cd^2-ae^2\right)}$$
$$=-\frac{\sqrt{d+ex}}{\left(cd^2-ae^2\right)(ae+cdx)}-\frac{\text{Subst}\left(\int \frac{1}{-\frac{cd^2}{e}+ae+\frac{cdx^2}{e}}\,dx,x,\sqrt{d+ex}\right)}{cd^2-ae^2}$$
$$=-\frac{\sqrt{d+ex}}{\left(cd^2-ae^2\right)(ae+cdx)}+\frac{e\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\sqrt{c}\sqrt{d}\left(cd^2-ae^2\right)^{3/2}}$$

Mathematica [A] time = 0.0770845, size = 101, normalized size = 1.

$$\frac{e\tan^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{ae^2-cd^2}}\right)}{\sqrt{c}\sqrt{d}\left(ae^2-cd^2\right)^{3/2}}-\frac{\sqrt{d+ex}}{\left(cd^2-ae^2\right)(ae+cdx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] -(Sqrt[d + e*x]/((c*d^2 - a*e^2)*(a*e + c*d*x))) + (e*ArcTan[(Sqrt[c]*Sqrt[
d]*Sqrt[d + e*x])/Sqrt[-(c*d^2) + a*e^2]])/(Sqrt[c]*Sqrt[d]*(-(c*d^2) + a*e
^2)^(3/2))
```

Maple [A] time = 0.193, size = 99, normalized size = 1.

$$\frac{e}{\left(ae^2-cd^2\right)\left(cdex+ae^2\right)}\sqrt{ex+d}+\frac{e}{ae^2-cd^2}\arctan\left(cd\sqrt{ex+d}\frac{1}{\sqrt{\left(ae^2-cd^2\right)cd}}\right)\frac{1}{\sqrt{\left(ae^2-cd^2\right)cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

```
[Out] e*(e*x+d)^(1/2)/(a*e^2-c*d^2)/(c*d*e*x+a*e^2)+e/(a*e^2-c*d^2)/((a*e^2-c*d^2
)*c*d)^(1/2)*arctan((e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.84347, size = 725, normalized size = 7.18

$$\left[-\frac{\sqrt{c^2d^3 - acde^2}\left(cdex + ae^2\right)\log\left(\frac{cdex+2\,cd^2-ae^2-2\,\sqrt{c^2d^3-acde^2}\sqrt{ex+d}}{cdx+ae}\right) + 2\left(c^2d^3 - acde^2\right)\sqrt{ex+d}}{2\left(ac^3d^5e - 2\,a^2c^2d^3e^3 + a^3cde^5 + \left(c^4d^6 - 2\,ac^3d^4e^2 + a^2c^2d^2e^4\right)x\right)}, -\frac{\sqrt{-c^2d^3 + acde^2}\left(cdex \cdot\right.}{ac^3d^5e - 2\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="f
ricas")
```

```
[Out] [-1/2*(sqrt(c^2*d^3 - a*c*d*e^2)*(c*d*e*x + a*e^2)*log((c*d*e*x + 2*c*d^2 -
 a*e^2 - 2*sqrt(c^2*d^3 - a*c*d*e^2)*sqrt(e*x + d))/(c*d*x + a*e)) + 2*(c^2
*d^3 - a*c*d*e^2)*sqrt(e*x + d))/(a*c^3*d^5*e - 2*a^2*c^2*d^3*e^3 + a^3*c*d
*e^5 + (c^4*d^6 - 2*a*c^3*d^4*e^2 + a^2*c^2*d^2*e^4)*x), -(sqrt(-c^2*d^3 +
a*c*d*e^2)*(c*d*e*x + a*e^2)*arctan(sqrt(-c^2*d^3 + a*c*d*e^2)*sqrt(e*x + d
)/(c*d*e*x + c*d^2)) + (c^2*d^3 - a*c*d*e^2)*sqrt(e*x + d))/(a*c^3*d^5*e -
2*a^2*c^2*d^3*e^3 + a^3*c*d*e^5 + (c^4*d^6 - 2*a*c^3*d^4*e^2 + a^2*c^2*d^2*
e^4)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.2015 $\int \frac{\sqrt{d+ex}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$

Optimal. Leaf size=128

$$-\frac{3e}{\sqrt{d+ex}\left(cd^2-ae^2\right)^2}-\frac{1}{\sqrt{d+ex}\left(cd^2-ae^2\right)(ae+cdx)}+\frac{3\sqrt{c}\sqrt{d}e\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{5/2}}$$

```
[Out] (-3*e)/((c*d^2 - a*e^2)^2*Sqrt[d + e*x]) - 1/((c*d^2 - a*e^2)*(a*e + c*d*x)
*Sqrt[d + e*x]) + (3*Sqrt[c]*Sqrt[d]*e*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*
x])/Sqrt[c*d^2 - a*e^2]])/(c*d^2 - a*e^2)^(5/2)
```

Rubi [A] time = 0.0640229, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {626, 51, 63, 208}

$$-\frac{3e}{\sqrt{d+ex}\left(cd^2-ae^2\right)^2}-\frac{1}{\sqrt{d+ex}\left(cd^2-ae^2\right)(ae+cdx)}+\frac{3\sqrt{c}\sqrt{d}e\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] (-3*e)/((c*d^2 - a*e^2)^2*Sqrt[d + e*x]) - 1/((c*d^2 - a*e^2)*(a*e + c*d*x)
*Sqrt[d + e*x]) + (3*Sqrt[c]*Sqrt[d]*e*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*
x])/Sqrt[c*d^2 - a*e^2]])/(c*d^2 - a*e^2)^(5/2)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx = \int \frac{1}{(ae+cdx)^2(d+ex)^{3/2}}\,dx$$

$$= -\frac{1}{\left(cd^2-ae^2\right)(ae+cdx)\sqrt{d+ex}} - \frac{(3e)\int \frac{1}{(ae+cdx)(d+ex)^{3/2}}\,dx}{2\left(cd^2-ae^2\right)}$$

$$= -\frac{3e}{\left(cd^2-ae^2\right)^2\sqrt{d+ex}} - \frac{1}{\left(cd^2-ae^2\right)(ae+cdx)\sqrt{d+ex}} - \frac{(3cde)\int \frac{1}{(ae+cdx)\sqrt{d+e}}}{2\left(cd^2-ae^2\right)^2}$$

$$= -\frac{3e}{\left(cd^2-ae^2\right)^2\sqrt{d+ex}} - \frac{1}{\left(cd^2-ae^2\right)(ae+cdx)\sqrt{d+ex}} - \frac{(3cd)\,\mathrm{Subst}\left(\int \frac{}{-\frac{cd^2}{e}}\right.}{\left(cd^2\right.}$$

$$= -\frac{3e}{\left(cd^2-ae^2\right)^2\sqrt{d+ex}} - \frac{1}{\left(cd^2-ae^2\right)(ae+cdx)\sqrt{d+ex}} + \frac{3\sqrt{c}\sqrt{d}e\tanh^{-1}\left(\frac{\sqrt{}}{}\right.}{\left(cd^2-ae^2\right)^5}$$

Mathematica [C] time = 0.0145002, size = 57, normalized size = 0.45

$$-\frac{2e\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};-\frac{cd(d+ex)}{ae^2-cd^2}\right)}{\sqrt{d+ex}\left(ae^2-cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] (-2*e*Hypergeometric2F1[-1/2, 2, 1/2, -((c*d*(d + e*x))/(-(c*d^2) + a*e^2))
])/((-(c*d^2) + a*e^2)^2*Sqrt[d + e*x])
```

Maple [A] time = 0.236, size = 129, normalized size = 1.

$$-2\frac{e}{\left(ae^2-cd^2\right)^2\sqrt{ex+d}} - \frac{dec}{\left(ae^2-cd^2\right)^2\left(cdex+ae^2\right)}\sqrt{ex+d} - 3\frac{dec}{\left(ae^2-cd^2\right)^2\sqrt{\left(ae^2-cd^2\right)cd}}\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{\left(ae^2-cd^2\right)c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

```
[Out] -2*e/(a*e^2-c*d^2)^2/(e*x+d)^(1/2)-e*c*d/(a*e^2-c*d^2)^2*(e*x+d)^(1/2)/(c*d
*e*x+a*e^2)-3*e*c*d/(a*e^2-c*d^2)^2/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((e*x+d
)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.06362, size = 1010, normalized size = 7.89

$$\left[\frac{3\left(cde^2x^2 + ade^2 + \left(cd^2e + ae^3\right)x\right)\sqrt{\frac{cd}{cd^2-ae^2}}\log\left(\frac{cdex+2\,cd^2-ae^2+2\left(cd^2-ae^2\right)\sqrt{ex+d}\sqrt{\frac{cd}{cd^2-ae^2}}}{cdx+ae}\right) - 2\left(3\,cdex + cd^2 + 2\,ae^2\right)\sqrt{ex+d}}{2\left(ac^2d^5e - 2\,a^2cd^3e^3 + a^3de^5 + \left(c^3d^5e - 2\,ac^2d^3e^3 + a^2cde^5\right)x^2 + \left(c^3d^6 - ac^2d^4e^2 - a^2cd^2e^4 + a^3e^6\right)x\right)}, -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="f
ricas")
```

```
[Out] [1/2*(3*(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x)*sqrt(c*d/(c*d^2 - a*e
^2))*log((c*d*e*x + 2*c*d^2 - a*e^2 + 2*(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(
c*d/(c*d^2 - a*e^2)))/(c*d*x + a*e)) - 2*(3*c*d*e*x + c*d^2 + 2*a*e^2)*sqrt
(e*x + d))/(a*c^2*d^5*e - 2*a^2*c*d^3*e^3 + a^3*d*e^5 + (c^3*d^5*e - 2*a*c^
2*d^3*e^3 + a^2*c*d*e^5)*x^2 + (c^3*d^6 - a*c^2*d^4*e^2 - a^2*c*d^2*e^4 + a
^3*e^6)*x), (3*(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x)*sqrt(-c*d/(c*d
^2 - a*e^2))*arctan(-(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(-c*d/(c*d^2 - a*e^2
))/(c*d*e*x + c*d^2)) - (3*c*d*e*x + c*d^2 + 2*a*e^2)*sqrt(e*x + d))/(a*c^2
*d^5*e - 2*a^2*c*d^3*e^3 + a^3*d*e^5 + (c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c
*d*e^5)*x^2 + (c^3*d^6 - a*c^2*d^4*e^2 - a^2*c*d^2*e^4 + a^3*e^6)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.2016 $\int \frac{1}{\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$

Optimal. Leaf size=158

$$\frac{5c^{3/2}d^{3/2}e\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{7/2}}-\frac{5cde}{\sqrt{d+ex}\left(cd^2-ae^2\right)^3}-\frac{1}{(d+ex)^{3/2}\left(cd^2-ae^2\right)(ae+cdx)}-\frac{5e}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)^2}$$

```
[Out] (-5*e)/(3*(c*d^2 - a*e^2)^2*(d + e*x)^(3/2)) - 1/((c*d^2 - a*e^2)*(a*e + c*
d*x)*(d + e*x)^(3/2)) - (5*c*d*e)/((c*d^2 - a*e^2)^3*Sqrt[d + e*x]) + (5*c^
(3/2)*d^(3/2)*e*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]
])/(c*d^2 - a*e^2)^(7/2)
```

Rubi [A] time = 0.0927356, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {626, 51, 63, 208}

$$\frac{5c^{3/2}d^{3/2}e\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{7/2}}-\frac{5cde}{\sqrt{d+ex}\left(cd^2-ae^2\right)^3}-\frac{1}{(d+ex)^{3/2}\left(cd^2-ae^2\right)(ae+cdx)}-\frac{5e}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2),x]
```

```
[Out] (-5*e)/(3*(c*d^2 - a*e^2)^2*(d + e*x)^(3/2)) - 1/((c*d^2 - a*e^2)*(a*e + c*
d*x)*(d + e*x)^(3/2)) - (5*c*d*e)/((c*d^2 - a*e^2)^3*Sqrt[d + e*x]) + (5*c^
(3/2)*d^(3/2)*e*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]
])/(c*d^2 - a*e^2)^(7/2)
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx=\int \frac{1}{(ae+cdx)^2(d+ex)^{5/2}}\,dx$$

$$=-\frac{1}{\left(cd^2-ae^2\right)(ae+cdx)(d+ex)^{3/2}}-\frac{(5e)\int \frac{1}{(ae+cdx)(d+ex)^{5/2}}\,dx}{2\left(cd^2-ae^2\right)}$$

$$=-\frac{5e}{3\left(cd^2-ae^2\right)^2(d+ex)^{3/2}}-\frac{1}{\left(cd^2-ae^2\right)(ae+cdx)(d+ex)^{3/2}}-\frac{(5cde)\int}{2(}$$

$$=-\frac{5e}{3\left(cd^2-ae^2\right)^2(d+ex)^{3/2}}-\frac{1}{\left(cd^2-ae^2\right)(ae+cdx)(d+ex)^{3/2}}-\frac{}{(cd^2-a}$$

$$=-\frac{5e}{3\left(cd^2-ae^2\right)^2(d+ex)^{3/2}}-\frac{1}{\left(cd^2-ae^2\right)(ae+cdx)(d+ex)^{3/2}}-\frac{}{(cd^2-a}$$

$$=-\frac{5e}{3\left(cd^2-ae^2\right)^2(d+ex)^{3/2}}-\frac{1}{\left(cd^2-ae^2\right)(ae+cdx)(d+ex)^{3/2}}-\frac{}{(cd^2-a}$$

Mathematica [C] time = 0.0172592, size = 59, normalized size = 0.37

$$-\frac{2e\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};-\frac{cd(d+ex)}{ae^2-cd^2}\right)}{3(d+ex)^{3/2}\left(ae^2-cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2),x]
```

```
[Out] (-2*e*Hypergeometric2F1[-3/2, 2, -1/2, -((c*d*(d + e*x))/(-(c*d^2) + a*e^2)
)])/(3*(-(c*d^2) + a*e^2)^2*(d + e*x)^(3/2))
```

Maple [A] time = 0.204, size = 162, normalized size = 1.

$$-\frac{2e}{3\left(ae^2-cd^2\right)^2}(ex+d)^{-\frac{3}{2}}+4\frac{dec}{\left(ae^2-cd^2\right)^3\sqrt{ex+d}}+\frac{c^2ed^2}{\left(ae^2-cd^2\right)^3\left(cdex+ae^2\right)}\sqrt{ex+d}+5\frac{c^2ed^2}{\left(ae^2-cd^2\right)^3\sqrt{\left(ae^2-cd^2\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

```
[Out] -2/3*e/(a*e^2-c*d^2)^2/(e*x+d)^(3/2)+4*e/(a*e^2-c*d^2)^3*c*d/(e*x+d)^(1/2)+
e*c^2*d^2/(a*e^2-c*d^2)^3*(e*x+d)^(1/2)/(c*d*e*x+a*e^2)+5*e*c^2*d^2/(a*e^2-
c*d^2)^3/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*
c*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.11454, size = 1751, normalized size = 11.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm=
"fricas")
```

```
[Out] [-1/6*(15*(c^2*d^2*e^3*x^3 + a*c*d^3*e^2 + (2*c^2*d^3*e^2 + a*c*d*e^4)*x^2
+ (c^2*d^4*e + 2*a*c*d^2*e^3)*x)*sqrt(c*d/(c*d^2 - a*e^2))*log((c*d*e*x + 2
*c*d^2 - a*e^2 - 2*(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(c*d/(c*d^2 - a*e^2)))
/(c*d*x + a*e)) + 2*(15*c^2*d^2*e^2*x^2 + 3*c^2*d^4 + 14*a*c*d^2*e^2 - 2*a^
2*e^4 + 10*(2*c^2*d^3*e + a*c*d*e^3)*x)*sqrt(e*x + d))/(a*c^3*d^8*e - 3*a^2
*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^4*d^7*e^2 - 3*a*c^3*d^5*e
^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4*d^8*e - 5*a*c^3*d^6*e^3
+ 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2 + (c^4*d^9 - a*c^3*d^7*e
^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d*e^8)*x), 1/3*(15*(c^2*d^
2*e^3*x^3 + a*c*d^3*e^2 + (2*c^2*d^3*e^2 + a*c*d*e^4)*x^2 + (c^2*d^4*e + 2*
a*c*d^2*e^3)*x)*sqrt(-c*d/(c*d^2 - a*e^2))*arctan(-(c*d^2 - a*e^2)*sqrt(e*x
 + d)*sqrt(-c*d/(c*d^2 - a*e^2))/(c*d*e*x + c*d^2)) - (15*c^2*d^2*e^2*x^2 +
 3*c^2*d^4 + 14*a*c*d^2*e^2 - 2*a^2*e^4 + 10*(2*c^2*d^3*e + a*c*d*e^3)*x)*s
qrt(e*x + d))/(a*c^3*d^8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*
e^7 + (c^4*d^7*e^2 - 3*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3
 + (2*c^4*d^8*e - 5*a*c^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4
*e^9)*x^2 + (c^4*d^9 - a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6
- 2*a^4*d*e^8)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.2017 $\int \frac{1}{(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$

Optimal. Leaf size=192

$$-\frac{7c^2d^2e}{\sqrt{d+ex}\left(cd^2-ae^2\right)^4}+\frac{7c^{5/2}d^{5/2}e\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{9/2}}-\frac{7cde}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)^3}-\frac{1}{(d+ex)^{5/2}\left(cd^2-ae^2\right)(ae+cdx)}$$

```
[Out] (-7*e)/(5*(c*d^2 - a*e^2)^2*(d + e*x)^(5/2)) - 1/((c*d^2 - a*e^2)*(a*e + c*
d*x)*(d + e*x)^(5/2)) - (7*c*d*e)/(3*(c*d^2 - a*e^2)^3*(d + e*x)^(3/2)) - (
7*c^2*d^2*e)/((c*d^2 - a*e^2)^4*Sqrt[d + e*x]) + (7*c^(5/2)*d^(5/2)*e*ArcTa
nh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(c*d^2 - a*e^2)^(9
/2)
```

Rubi [A] time = 0.128608, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {626, 51, 63, 208}

$$-\frac{7c^2d^2e}{\sqrt{d+ex}\left(cd^2-ae^2\right)^4}+\frac{7c^{5/2}d^{5/2}e\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{9/2}}-\frac{7cde}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)^3}-\frac{1}{(d+ex)^{5/2}\left(cd^2-ae^2\right)(ae+cdx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2),x]
```


```
[Out] (-7*e)/(5*(c*d^2 - a*e^2)^2*(d + e*x)^(5/2)) - 1/((c*d^2 - a*e^2)*(a*e + c*
d*x)*(d + e*x)^(5/2)) - (7*c*d*e)/(3*(c*d^2 - a*e^2)^3*(d + e*x)^(3/2)) - (
7*c^2*d^2*e)/((c*d^2 - a*e^2)^4*Sqrt[d + e*x]) + (7*c^(5/2)*d^(5/2)*e*ArcTa
nh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(c*d^2 - a*e^2)^(9
/2)
```


Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx=\int \frac{1}{(ae+cdx)^2(d+ex)^{7/2}}\,dx$$

$$=-\frac{1}{\left(cd^2-ae^2\right)(ae+cdx)(d+ex)^{5/2}}-\frac{(7e)\int \frac{1}{(ae+cdx)(d+ex)^{7/2}}\,dx}{2\left(cd^2-ae^2\right)}$$

$$=-\frac{7e}{5\left(cd^2-ae^2\right)^2(d+ex)^{5/2}}-\frac{1}{\left(cd^2-ae^2\right)(ae+cdx)(d+ex)^{5/2}}-\frac{(7cde)}{}$$

$$=-\frac{7e}{5\left(cd^2-ae^2\right)^2(d+ex)^{5/2}}-\frac{1}{\left(cd^2-ae^2\right)(ae+cdx)(d+ex)^{5/2}}-\frac{}{3\left(cd^2\right.}$$

$$=-\frac{7e}{5\left(cd^2-ae^2\right)^2(d+ex)^{5/2}}-\frac{1}{\left(cd^2-ae^2\right)(ae+cdx)(d+ex)^{5/2}}-\frac{}{3\left(cd^2\right.}$$

$$=-\frac{7e}{5\left(cd^2-ae^2\right)^2(d+ex)^{5/2}}-\frac{1}{\left(cd^2-ae^2\right)(ae+cdx)(d+ex)^{5/2}}-\frac{}{3\left(cd^2\right.}$$

$$=-\frac{7e}{5\left(cd^2-ae^2\right)^2(d+ex)^{5/2}}-\frac{1}{\left(cd^2-ae^2\right)(ae+cdx)(d+ex)^{5/2}}-\frac{}{3\left(cd^2\right.}$$

Mathematica [C] time = 0.018472, size = 59, normalized size = 0.31

$$-\frac{2e\,{}_2F_1\left(-\frac{5}{2},2;-\frac{3}{2};-\frac{cd(d+ex)}{ae^2-cd^2}\right)}{5(d+ex)^{5/2}\left(ae^2-cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2),x]
```

```
[Out] (-2*e*Hypergeometric2F1[-5/2, 2, -3/2, -((c*d*(d + e*x))/(-(c*d^2) + a*e^2)
)])/(5*(-(c*d^2) + a*e^2)^2*(d + e*x)^(5/2))
```

Maple [A] time = 0.204, size = 193, normalized size = 1.

$$-\frac{2e}{5\left(ae^2-cd^2\right)^2}(ex+d)^{-\frac{5}{2}}-6\frac{c^2ed^2}{\left(ae^2-cd^2\right)^4\sqrt{ex+d}}+\frac{4dec}{3\left(ae^2-cd^2\right)^3}(ex+d)^{-\frac{3}{2}}-\frac{c^3ed^3}{\left(ae^2-cd^2\right)^4\left(cdex+ae^2\right)}\sqrt{ex+d}-7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

```
[Out] -2/5*e/(a*e^2-c*d^2)^2/(e*x+d)^(5/2)-6*e/(a*e^2-c*d^2)^4*c^2*d^2/(e*x+d)^(1
/2)+4/3*e/(a*e^2-c*d^2)^3*c*d/(e*x+d)^(3/2)-e*c^3*d^3/(a*e^2-c*d^2)^4*(e*x+
d)^(1/2)/(c*d*e*x+a*e^2)-7*e*c^3*d^3/(a*e^2-c*d^2)^4/((a*e^2-c*d^2)*c*d)^(1
/2)*arctan((e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.13286, size = 2678, normalized size = 13.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm=
"fricas")
```

```
[Out] [1/30*(105*(c^3*d^3*e^4*x^4 + a*c^2*d^5*e^2 + (3*c^3*d^4*e^3 + a*c^2*d^2*e^
5)*x^3 + 3*(c^3*d^5*e^2 + a*c^2*d^3*e^4)*x^2 + (c^3*d^6*e + 3*a*c^2*d^4*e^3
)*x)*sqrt(c*d/(c*d^2 - a*e^2))*log((c*d*e*x + 2*c*d^2 - a*e^2 + 2*(c*d^2 -
a*e^2)*sqrt(e*x + d)*sqrt(c*d/(c*d^2 - a*e^2)))/(c*d*x + a*e)) - 2*(105*c^3
*d^3*e^3*x^3 + 15*c^3*d^6 + 116*a*c^2*d^4*e^2 - 32*a^2*c*d^2*e^4 + 6*a^3*e^
6 + 35*(7*c^3*d^4*e^2 + 2*a*c^2*d^2*e^4)*x^2 + 7*(23*c^3*d^5*e + 24*a*c^2*d
^3*e^3 - 2*a^2*c*d*e^5)*x)*sqrt(e*x + d))/(a*c^4*d^11*e - 4*a^2*c^3*d^9*e^3
 + 6*a^3*c^2*d^7*e^5 - 4*a^4*c*d^5*e^7 + a^5*d^3*e^9 + (c^5*d^9*e^3 - 4*a*c
^4*d^7*e^5 + 6*a^2*c^3*d^5*e^7 - 4*a^3*c^2*d^3*e^9 + a^4*c*d*e^11)*x^4 + (3
*c^5*d^10*e^2 - 11*a*c^4*d^8*e^4 + 14*a^2*c^3*d^6*e^6 - 6*a^3*c^2*d^4*e^8 -
 a^4*c*d^2*e^10 + a^5*e^12)*x^3 + 3*(c^5*d^11*e - 3*a*c^4*d^9*e^3 + 2*a^2*c
^3*d^7*e^5 + 2*a^3*c^2*d^5*e^7 - 3*a^4*c*d^3*e^9 + a^5*d*e^11)*x^2 + (c^5*d
^12 - a*c^4*d^10*e^2 - 6*a^2*c^3*d^8*e^4 + 14*a^3*c^2*d^6*e^6 - 11*a^4*c*d^
4*e^8 + 3*a^5*d^2*e^10)*x), 1/15*(105*(c^3*d^3*e^4*x^4 + a*c^2*d^5*e^2 + (3
*c^3*d^4*e^3 + a*c^2*d^2*e^5)*x^3 + 3*(c^3*d^5*e^2 + a*c^2*d^3*e^4)*x^2 + (
c^3*d^6*e + 3*a*c^2*d^4*e^3)*x)*sqrt(-c*d/(c*d^2 - a*e^2))*arctan(-(c*d^2 -
 a*e^2)*sqrt(e*x + d)*sqrt(-c*d/(c*d^2 - a*e^2))/(c*d*e*x + c*d^2)) - (105*
c^3*d^3*e^3*x^3 + 15*c^3*d^6 + 116*a*c^2*d^4*e^2 - 32*a^2*c*d^2*e^4 + 6*a^3
*e^6 + 35*(7*c^3*d^4*e^2 + 2*a*c^2*d^2*e^4)*x^2 + 7*(23*c^3*d^5*e + 24*a*c^
2*d^3*e^3 - 2*a^2*c*d*e^5)*x)*sqrt(e*x + d))/(a*c^4*d^11*e - 4*a^2*c^3*d^9*
e^3 + 6*a^3*c^2*d^7*e^5 - 4*a^4*c*d^5*e^7 + a^5*d^3*e^9 + (c^5*d^9*e^3 - 4*
a*c^4*d^7*e^5 + 6*a^2*c^3*d^5*e^7 - 4*a^3*c^2*d^3*e^9 + a^4*c*d*e^11)*x^4 +
 (3*c^5*d^10*e^2 - 11*a*c^4*d^8*e^4 + 14*a^2*c^3*d^6*e^6 - 6*a^3*c^2*d^4*e^
8 - a^4*c*d^2*e^10 + a^5*e^12)*x^3 + 3*(c^5*d^11*e - 3*a*c^4*d^9*e^3 + 2*a^
2*c^3*d^7*e^5 + 2*a^3*c^2*d^5*e^7 - 3*a^4*c*d^3*e^9 + a^5*d*e^11)*x^2 + (c^
5*d^12 - a*c^4*d^10*e^2 - 6*a^2*c^3*d^8*e^4 + 14*a^3*c^2*d^6*e^6 - 11*a^4*c
*d^4*e^8 + 3*a^5*d^2*e^10)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.2018 $\int \frac{(d+ex)^{15/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal. Leaf size=222

$$\frac{21e^2(d+ex)^{3/2}\left(cd^2-ae^2\right)}{4c^4d^4}+\frac{63e^2\sqrt{d+ex}\left(cd^2-ae^2\right)^2}{4c^5d^5}-\frac{63e^2\left(cd^2-ae^2\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{4c^{11/2}d^{11/2}}-\frac{9e(d+ex)^{7/2}}{4c^2d^2(ae+cdx)}-$$

```
[Out] (63*e^2*(c*d^2 - a*e^2)^2*Sqrt[d + e*x])/(4*c^5*d^5) + (21*e^2*(c*d^2 - a*e
^2)*(d + e*x)^(3/2))/(4*c^4*d^4) + (63*e^2*(d + e*x)^(5/2))/(20*c^3*d^3) -
(9*e*(d + e*x)^(7/2))/(4*c^2*d^2*(a*e + c*d*x)) - (d + e*x)^(9/2)/(2*c*d*(a
*e + c*d*x)^2) - (63*e^2*(c*d^2 - a*e^2)^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqr
t[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(4*c^(11/2)*d^(11/2))
```

Rubi [A] time = 0.185903, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {626, 47, 50, 63, 208}

$$\frac{21e^2(d+ex)^{3/2}\left(cd^2-ae^2\right)}{4c^4d^4}+\frac{63e^2\sqrt{d+ex}\left(cd^2-ae^2\right)^2}{4c^5d^5}-\frac{63e^2\left(cd^2-ae^2\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{4c^{11/2}d^{11/2}}-\frac{9e(d+ex)^{7/2}}{4c^2d^2(ae+cdx)}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(15/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```


```
[Out] (63*e^2*(c*d^2 - a*e^2)^2*Sqrt[d + e*x])/(4*c^5*d^5) + (21*e^2*(c*d^2 - a*e
^2)*(d + e*x)^(3/2))/(4*c^4*d^4) + (63*e^2*(d + e*x)^(5/2))/(20*c^3*d^3) -
(9*e*(d + e*x)^(7/2))/(4*c^2*d^2*(a*e + c*d*x)) - (d + e*x)^(9/2)/(2*c*d*(a
*e + c*d*x)^2) - (63*e^2*(c*d^2 - a*e^2)^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqr
t[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(4*c^(11/2)*d^(11/2))
```


Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{15/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx = \int \frac{(d+ex)^{9/2}}{(ae+cdx)^3}\,dx$$

$$= -\frac{(d+ex)^{9/2}}{2cd(ae+cdx)^2}+\frac{(9e)\int \frac{(d+ex)^{7/2}}{(ae+cdx)^2}\,dx}{4cd}$$

$$= -\frac{9e(d+ex)^{7/2}}{4c^2d^2(ae+cdx)}-\frac{(d+ex)^{9/2}}{2cd(ae+cdx)^2}+\frac{\left(63e^2\right)\int \frac{(d+ex)^{5/2}}{ae+cdx}\,dx}{8c^2d^2}$$

$$= \frac{63e^2(d+ex)^{5/2}}{20c^3d^3}-\frac{9e(d+ex)^{7/2}}{4c^2d^2(ae+cdx)}-\frac{(d+ex)^{9/2}}{2cd(ae+cdx)^2}+\frac{\left(63e^2\left(cd^2-ae^2\right)\right)\int \frac{(d+ex)^{3}}{ae+cd}$$

$$= \frac{21e^2\left(cd^2-ae^2\right)(d+ex)^{3/2}}{4c^4d^4}+\frac{63e^2(d+ex)^{5/2}}{20c^3d^3}-\frac{9e(d+ex)^{7/2}}{4c^2d^2(ae+cdx)}-\frac{(d+ex)^{9/2}}{2cd(ae+cdx)^2}$$

$$= \frac{63e^2\left(cd^2-ae^2\right)^2\sqrt{d+ex}}{4c^5d^5}+\frac{21e^2\left(cd^2-ae^2\right)(d+ex)^{3/2}}{4c^4d^4}+\frac{63e^2(d+ex)^{5/2}}{20c^3d^3}-\frac{9e(}{4c^2d}$$

$$= \frac{63e^2\left(cd^2-ae^2\right)^2\sqrt{d+ex}}{4c^5d^5}+\frac{21e^2\left(cd^2-ae^2\right)(d+ex)^{3/2}}{4c^4d^4}+\frac{63e^2(d+ex)^{5/2}}{20c^3d^3}-\frac{9e(}{4c^2d}$$

$$= \frac{63e^2\left(cd^2-ae^2\right)^2\sqrt{d+ex}}{4c^5d^5}+\frac{21e^2\left(cd^2-ae^2\right)(d+ex)^{3/2}}{4c^4d^4}+\frac{63e^2(d+ex)^{5/2}}{20c^3d^3}-\frac{9e(}{4c^2d}$$

Mathematica [C] time = 0.0255233, size = 61, normalized size = 0.27

$$\frac{2e^2(d+ex)^{11/2}\,{}_2F_1\left(3,\frac{11}{2};\frac{13}{2};-\frac{cd(d+ex)}{ae^2-cd^2}\right)}{11\left(ae^2-cd^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(15/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] (2*e^2*(d + e*x)^(11/2)*Hypergeometric2F1[3, 11/2, 13/2, -((c*d*(d + e*x))/
(-(c*d^2) + a*e^2))])/(11*(-(c*d^2) + a*e^2)^3)
```

Maple [B] time = 0.239, size = 635, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(15/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

```
[Out] 2/5*e^2*(e*x+d)^(5/2)/c^3/d^3-2*e^4/c^4/d^4*(e*x+d)^(3/2)*a+2*e^2/c^3/d^2*(
e*x+d)^(3/2)+12*e^6/c^5/d^5*a^2*(e*x+d)^(1/2)-24*e^4/c^4/d^3*a*(e*x+d)^(1/2
)+12*e^2/c^3/d*(e*x+d)^(1/2)+17/4*e^8/c^4/d^4/(c*d*e*x+a*e^2)^2*(e*x+d)^(3/
2)*a^3-51/4*e^6/c^3/d^2/(c*d*e*x+a*e^2)^2*(e*x+d)^(3/2)*a^2+51/4*e^4/c^2/(c
*d*e*x+a*e^2)^2*(e*x+d)^(3/2)*a-17/4*e^2/c*d^2/(c*d*e*x+a*e^2)^2*(e*x+d)^(3
/2)+15/4*e^10/c^5/d^5/(c*d*e*x+a*e^2)^2*(e*x+d)^(1/2)*a^4-15*e^8/c^4/d^3/(c
*d*e*x+a*e^2)^2*(e*x+d)^(1/2)*a^3+45/2*e^6/c^3/d/(c*d*e*x+a*e^2)^2*(e*x+d)^
(1/2)*a^2-15*e^4/c^2*d/(c*d*e*x+a*e^2)^2*(e*x+d)^(1/2)*a+15/4*e^2/c*d^3/(c*
d*e*x+a*e^2)^2*(e*x+d)^(1/2)-63/4*e^8/c^5/d^5/((a*e^2-c*d^2)*c*d)^(1/2)*arc
tan((e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1/2))*a^3+189/4*e^6/c^4/d^3/((a*
e^2-c*d^2)*c*d)^(1/2)*arctan((e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1/2))*a
^2-189/4*e^4/c^3/d/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((e*x+d)^(1/2)*c*d/((a*e
^2-c*d^2)*c*d)^(1/2))*a+63/4*e^2/c^2*d/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((e*
x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(15/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.03045, size = 1748, normalized size = 7.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(15/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="
fricas")
```

```
[Out] [1/40*(315*(a^2*c^2*d^4*e^4 - 2*a^3*c*d^2*e^6 + a^4*e^8 + (c^4*d^6*e^2 - 2*
a*c^3*d^4*e^4 + a^2*c^2*d^2*e^6)*x^2 + 2*(a*c^3*d^5*e^3 - 2*a^2*c^2*d^3*e^5
 + a^3*c*d*e^7)*x)*sqrt((c*d^2 - a*e^2)/(c*d))*log((c*d*e*x + 2*c*d^2 - a*e
^2 - 2*sqrt(e*x + d)*c*d*sqrt((c*d^2 - a*e^2)/(c*d)))/(c*d*x + a*e)) + 2*(8
*c^4*d^4*e^4*x^4 - 10*c^4*d^8 - 45*a*c^3*d^6*e^2 + 483*a^2*c^2*d^4*e^4 - 73
5*a^3*c*d^2*e^6 + 315*a^4*e^8 + 8*(7*c^4*d^5*e^3 - 3*a*c^3*d^3*e^5)*x^3 + 2
4*(12*c^4*d^6*e^2 - 17*a*c^3*d^4*e^4 + 7*a^2*c^2*d^2*e^6)*x^2 - (85*c^4*d^7
*e - 831*a*c^3*d^5*e^3 + 1239*a^2*c^2*d^3*e^5 - 525*a^3*c*d*e^7)*x)*sqrt(e*
x + d))/(c^7*d^7*x^2 + 2*a*c^6*d^6*e*x + a^2*c^5*d^5*e^2), -1/20*(315*(a^2*
c^2*d^4*e^4 - 2*a^3*c*d^2*e^6 + a^4*e^8 + (c^4*d^6*e^2 - 2*a*c^3*d^4*e^4 +
a^2*c^2*d^2*e^6)*x^2 + 2*(a*c^3*d^5*e^3 - 2*a^2*c^2*d^3*e^5 + a^3*c*d*e^7)*
```

```
x)*sqrt(-(c*d^2 - a*e^2)/(c*d))*arctan(-sqrt(e*x + d)*c*d*sqrt(-(c*d^2 - a*
e^2)/(c*d))/(c*d^2 - a*e^2)) - (8*c^4*d^4*e^4*x^4 - 10*c^4*d^8 - 45*a*c^3*d
^6*e^2 + 483*a^2*c^2*d^4*e^4 - 735*a^3*c*d^2*e^6 + 315*a^4*e^8 + 8*(7*c^4*d
^5*e^3 - 3*a*c^3*d^3*e^5)*x^3 + 24*(12*c^4*d^6*e^2 - 17*a*c^3*d^4*e^4 + 7*a
^2*c^2*d^2*e^6)*x^2 - (85*c^4*d^7*e - 831*a*c^3*d^5*e^3 + 1239*a^2*c^2*d^3*
e^5 - 525*a^3*c*d*e^7)*x)*sqrt(e*x + d))/(c^7*d^7*x^2 + 2*a*c^6*d^6*e*x + a
^2*c^5*d^5*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(15/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(15/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="
giac")

[Out] Timed out
```

3.2019 $\int \frac{(d+ex)^{13/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal. Leaf size=186

$$\frac{35e^2\sqrt{d+ex}\left(cd^2-ae^2\right)}{4c^4d^4}-\frac{35e^2\left(cd^2-ae^2\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{4c^{9/2}d^{9/2}}-\frac{7e(d+ex)^{5/2}}{4c^2d^2(ae+cdx)}-\frac{(d+ex)^{7/2}}{2cd(ae+cdx)^2}+\frac{35e^2(d+ex}{12c^3d^3}$$

```
[Out] (35*e^2*(c*d^2 - a*e^2)*Sqrt[d + e*x])/(4*c^4*d^4) + (35*e^2*(d + e*x)^(3/2
))/(12*c^3*d^3) - (7*e*(d + e*x)^(5/2))/(4*c^2*d^2*(a*e + c*d*x)) - (d + e*
x)^(7/2)/(2*c*d*(a*e + c*d*x)^2) - (35*e^2*(c*d^2 - a*e^2)^(3/2)*ArcTanh[(S
qrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(4*c^(9/2)*d^(9/2))
```

Rubi [A] time = 0.137784, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {626, 47, 50, 63, 208}

$$\frac{35e^2\sqrt{d+ex}\left(cd^2-ae^2\right)}{4c^4d^4}-\frac{35e^2\left(cd^2-ae^2\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{4c^{9/2}d^{9/2}}-\frac{7e(d+ex)^{5/2}}{4c^2d^2(ae+cdx)}-\frac{(d+ex)^{7/2}}{2cd(ae+cdx)^2}+\frac{35e^2(d+ex}{12c^3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(13/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```


```
[Out] (35*e^2*(c*d^2 - a*e^2)*Sqrt[d + e*x])/(4*c^4*d^4) + (35*e^2*(d + e*x)^(3/2
))/(12*c^3*d^3) - (7*e*(d + e*x)^(5/2))/(4*c^2*d^2*(a*e + c*d*x)) - (d + e*
x)^(7/2)/(2*c*d*(a*e + c*d*x)^2) - (35*e^2*(c*d^2 - a*e^2)^(3/2)*ArcTanh[(S
qrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(4*c^(9/2)*d^(9/2))
```


Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{13/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx = \int \frac{(d+ex)^{7/2}}{(ae+cdx)^3}\,dx$$

$$= -\frac{(d+ex)^{7/2}}{2cd(ae+cdx)^2} + \frac{(7e)\int \frac{(d+ex)^{5/2}}{(ae+cdx)^2}\,dx}{4cd}$$

$$= -\frac{7e(d+ex)^{5/2}}{4c^2d^2(ae+cdx)} - \frac{(d+ex)^{7/2}}{2cd(ae+cdx)^2} + \frac{\left(35e^2\right)\int \frac{(d+ex)^{3/2}}{ae+cdx}\,dx}{8c^2d^2}$$

$$= \frac{35e^2(d+ex)^{3/2}}{12c^3d^3} - \frac{7e(d+ex)^{5/2}}{4c^2d^2(ae+cdx)} - \frac{(d+ex)^{7/2}}{2cd(ae+cdx)^2} + \frac{\left(35e^2\left(cd^2-ae^2\right)\right)\int \frac{\sqrt{d+ex}}{ae+cdx}}{8c^3d^3}$$

$$= \frac{35e^2\left(cd^2-ae^2\right)\sqrt{d+ex}}{4c^4d^4} + \frac{35e^2(d+ex)^{3/2}}{12c^3d^3} - \frac{7e(d+ex)^{5/2}}{4c^2d^2(ae+cdx)} - \frac{(d+ex)^{7/2}}{2cd(ae+cdx)^2} +$$

$$= \frac{35e^2\left(cd^2-ae^2\right)\sqrt{d+ex}}{4c^4d^4} + \frac{35e^2(d+ex)^{3/2}}{12c^3d^3} - \frac{7e(d+ex)^{5/2}}{4c^2d^2(ae+cdx)} - \frac{(d+ex)^{7/2}}{2cd(ae+cdx)^2} +$$

$$= \frac{35e^2\left(cd^2-ae^2\right)\sqrt{d+ex}}{4c^4d^4} + \frac{35e^2(d+ex)^{3/2}}{12c^3d^3} - \frac{7e(d+ex)^{5/2}}{4c^2d^2(ae+cdx)} - \frac{(d+ex)^{7/2}}{2cd(ae+cdx)^2} -$$

Mathematica [C] time = 0.0199576, size = 61, normalized size = 0.33

$$\frac{2e^2(d+ex)^{9/2}\,{}_2F_1\left(3,\frac{9}{2};\frac{11}{2};-\frac{cd(d+ex)}{ae^2-cd^2}\right)}{9\left(ae^2-cd^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(13/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] (2*e^2*(d + e*x)^(9/2)*Hypergeometric2F1[3, 9/2, 11/2, -((c*d*(d + e*x))/(-
(c*d^2) + a*e^2))])/(9*(-(c*d^2) + a*e^2)^3)
```

Maple [B] time = 0.204, size = 449, normalized size = 2.4

$$\frac{2e^2}{3c^3d^3}(ex+d)^{\frac{3}{2}} - 6\frac{e^4a\sqrt{ex+d}}{c^4d^4} + 6\frac{e^2\sqrt{ex+d}}{c^3d^2} - \frac{13e^6a^2}{4c^3d^3\left(cdex+ae^2\right)^2}(ex+d)^{\frac{3}{2}} + \frac{13e^4a}{2c^2d\left(cdex+ae^2\right)^2}(ex+d)^{\frac{3}{2}} - \frac{1}{4c\,(cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(13/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

```
[Out] 2/3*e^2*(e*x+d)^(3/2)/c^3/d^3-6*e^4/c^4/d^4*a*(e*x+d)^(1/2)+6*e^2/c^3/d^2*(
e*x+d)^(1/2)-13/4*e^6/c^3/d^3/(c*d*e*x+a*e^2)^2*(e*x+d)^(3/2)*a^2+13/2*e^4/
c^2/d/(c*d*e*x+a*e^2)^2*(e*x+d)^(3/2)*a-13/4*e^2/c*d/(c*d*e*x+a*e^2)^2*(e*x
+d)^(3/2)-11/4*e^8/c^4/d^4/(c*d*e*x+a*e^2)^2*(e*x+d)^(1/2)*a^3+33/4*e^6/c^3
/d^2/(c*d*e*x+a*e^2)^2*(e*x+d)^(1/2)*a^2-33/4*e^4/c^2/(c*d*e*x+a*e^2)^2*(e*
x+d)^(1/2)*a+11/4*e^2/c*d^2/(c*d*e*x+a*e^2)^2*(e*x+d)^(1/2)+35/4*e^6/c^4/d^
4/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1
/2))*a^2-35/2*e^4/c^3/d^2/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((e*x+d)^(1/2)*c*
d/((a*e^2-c*d^2)*c*d)^(1/2))*a+35/4*e^2/c^2/((a*e^2-c*d^2)*c*d)^(1/2)*arcta
n((e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(13/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.03103, size = 1289, normalized size = 6.93

$$\left[\frac{105\left(a^2cd^2e^4-a^3e^6+\left(c^3d^4e^2-ac^2d^2e^4\right)x^2+2\left(ac^2d^3e^3-a^2cde^5\right)x\right)\sqrt{\frac{cd^2-ae^2}{cd}}\log\left(\frac{cdex+2\,cd^2-ae^2-2\,\sqrt{ex+d}cd\sqrt{\frac{cd^2-ae^2}{cd}}}{cdx+ae}\right)+2\,(}{24\,(c^6d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(13/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="
fricas")
```

```
[Out] [1/24*(105*(a^2*c*d^2*e^4 - a^3*e^6 + (c^3*d^4*e^2 - a*c^2*d^2*e^4)*x^2 + 2
*(a*c^2*d^3*e^3 - a^2*c*d*e^5)*x)*sqrt((c*d^2 - a*e^2)/(c*d))*log((c*d*e*x
+ 2*c*d^2 - a*e^2 - 2*sqrt(e*x + d)*c*d*sqrt((c*d^2 - a*e^2)/(c*d)))/(c*d*x
 + a*e)) + 2*(8*c^3*d^3*e^3*x^3 - 6*c^3*d^6 - 21*a*c^2*d^4*e^2 + 140*a^2*c*
d^2*e^4 - 105*a^3*e^6 + 8*(10*c^3*d^4*e^2 - 7*a*c^2*d^2*e^4)*x^2 - (39*c^3*
d^5*e - 238*a*c^2*d^3*e^3 + 175*a^2*c*d*e^5)*x)*sqrt(e*x + d))/(c^6*d^6*x^2
 + 2*a*c^5*d^5*e*x + a^2*c^4*d^4*e^2), -1/12*(105*(a^2*c*d^2*e^4 - a^3*e^6
+ (c^3*d^4*e^2 - a*c^2*d^2*e^4)*x^2 + 2*(a*c^2*d^3*e^3 - a^2*c*d*e^5)*x)*sq
rt(-(c*d^2 - a*e^2)/(c*d))*arctan(-sqrt(e*x + d)*c*d*sqrt(-(c*d^2 - a*e^2)/
(c*d))/(c*d^2 - a*e^2)) - (8*c^3*d^3*e^3*x^3 - 6*c^3*d^6 - 21*a*c^2*d^4*e^2
 + 140*a^2*c*d^2*e^4 - 105*a^3*e^6 + 8*(10*c^3*d^4*e^2 - 7*a*c^2*d^2*e^4)*x
^2 - (39*c^3*d^5*e - 238*a*c^2*d^3*e^3 + 175*a^2*c*d*e^5)*x)*sqrt(e*x + d))
/(c^6*d^6*x^2 + 2*a*c^5*d^5*e*x + a^2*c^4*d^4*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(13/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(13/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="
giac")
```

```
[Out] Timed out
```

3.2020 $\int \frac{(d+ex)^{11/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal. Leaf size=152

$$-\frac{15e^2\sqrt{cd^2-ae^2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{4c^{7/2}d^{7/2}}-\frac{5e(d+ex)^{3/2}}{4c^2d^2(ae+cdx)}-\frac{(d+ex)^{5/2}}{2cd(ae+cdx)^2}+\frac{15e^2\sqrt{d+ex}}{4c^3d^3}$$

```
[Out] (15*e^2*Sqrt[d + e*x])/(4*c^3*d^3) - (5*e*(d + e*x)^(3/2))/(4*c^2*d^2*(a*e
+ c*d*x)) - (d + e*x)^(5/2)/(2*c*d*(a*e + c*d*x)^2) - (15*e^2*Sqrt[c*d^2 -
a*e^2]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(4*c^(
7/2)*d^(7/2))
```

Rubi [A] time = 0.100075, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {626, 47, 50, 63, 208}

$$-\frac{15e^2\sqrt{cd^2-ae^2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{4c^{7/2}d^{7/2}}-\frac{5e(d+ex)^{3/2}}{4c^2d^2(ae+cdx)}-\frac{(d+ex)^{5/2}}{2cd(ae+cdx)^2}+\frac{15e^2\sqrt{d+ex}}{4c^3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(11/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] (15*e^2*Sqrt[d + e*x])/(4*c^3*d^3) - (5*e*(d + e*x)^(3/2))/(4*c^2*d^2*(a*e
+ c*d*x)) - (d + e*x)^(5/2)/(2*c*d*(a*e + c*d*x)^2) - (15*e^2*Sqrt[c*d^2 -
a*e^2]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(4*c^(
7/2)*d^(7/2))
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{11/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx = \int \frac{(d+ex)^{5/2}}{(ae+cdx)^3}\,dx$$

$$= -\frac{(d+ex)^{5/2}}{2cd(ae+cdx)^2} + \frac{(5e)\int \frac{(d+ex)^{3/2}}{(ae+cdx)^2}\,dx}{4cd}$$

$$= -\frac{5e(d+ex)^{3/2}}{4c^2d^2(ae+cdx)} - \frac{(d+ex)^{5/2}}{2cd(ae+cdx)^2} + \frac{\left(15e^2\right)\int \frac{\sqrt{d+ex}}{ae+cdx}\,dx}{8c^2d^2}$$

$$= \frac{15e^2\sqrt{d+ex}}{4c^3d^3} - \frac{5e(d+ex)^{3/2}}{4c^2d^2(ae+cdx)} - \frac{(d+ex)^{5/2}}{2cd(ae+cdx)^2} + \frac{\left(15e^2\left(cd^2-ae^2\right)\right)\int \frac{1}{(ae+cdx)\sqrt{\ }}}{8c^3d^3}$$

$$= \frac{15e^2\sqrt{d+ex}}{4c^3d^3} - \frac{5e(d+ex)^{3/2}}{4c^2d^2(ae+cdx)} - \frac{(d+ex)^{5/2}}{2cd(ae+cdx)^2} + \frac{\left(15e\left(cd^2-ae^2\right)\right)\mathrm{Subst}\left(\int -\right.}{4c^3d}$$

$$= \frac{15e^2\sqrt{d+ex}}{4c^3d^3} - \frac{5e(d+ex)^{3/2}}{4c^2d^2(ae+cdx)} - \frac{(d+ex)^{5/2}}{2cd(ae+cdx)^2} - \frac{15e^2\sqrt{cd^2-ae^2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}}{\sqrt{cd}}\right.}{4c^{7/2}d^{7/2}}$$

Mathematica [C] time = 0.0181947, size = 61, normalized size = 0.4

$$\frac{2e^2(d+ex)^{7/2}\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};-\frac{cd(d+ex)}{ae^2-cd^2}\right)}{7\left(ae^2-cd^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(11/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] (2*e^2*(d + e*x)^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, -((c*d*(d + e*x))/(-(
c*d^2) + a*e^2))])/(7*(-(c*d^2) + a*e^2)^3)
```

Maple [B] time = 0.202, size = 288, normalized size = 1.9

$$2\frac{e^2\sqrt{ex+d}}{c^3d^3} + \frac{9e^4a}{4c^2d^2\left(cdex+ae^2\right)^2}(ex+d)^{\frac{3}{2}} - \frac{9e^2}{4c\left(cdex+ae^2\right)^2}(ex+d)^{\frac{3}{2}} + \frac{7e^6a^2}{4c^3d^3\left(cdex+ae^2\right)^2}\sqrt{ex+d} - \frac{7e^4a}{2c^2d\left(cdex+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(11/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

```
[Out] 2*e^2*(e*x+d)^(1/2)/c^3/d^3+9/4*e^4/c^2/d^2/(c*d*e*x+a*e^2)^2*(e*x+d)^(3/2)
*a-9/4*e^2/c/(c*d*e*x+a*e^2)^2*(e*x+d)^(3/2)+7/4*e^6/c^3/d^3/(c*d*e*x+a*e^2
)^2*(e*x+d)^(1/2)*a^2-7/2*e^4/c^2/d/(c*d*e*x+a*e^2)^2*(e*x+d)^(1/2)*a+7/4*e
^2/c*d/(c*d*e*x+a*e^2)^2*(e*x+d)^(1/2)-15/4*e^4/c^3/d^3/((a*e^2-c*d^2)*c*d)
^(1/2)*arctan((e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1/2))*a+15/4*e^2/c^2/d
/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1/
2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(11/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.00768, size = 892, normalized size = 5.87

$$\left[\frac{15\left(c^2d^2e^2x^2+2\,acde^3x+a^2e^4\right)\sqrt{\frac{cd^2-ae^2}{cd}}\log\left(\frac{cdex+2\,cd^2-ae^2-2\,\sqrt{ex+d}cd\sqrt{\frac{cd^2-ae^2}{cd}}}{cdx+ae}\right)+2\left(8\,c^2d^2e^2x^2-2\,c^2d^4-5\,acd^2e^2+15\,a\right.}{8\left(c^5d^5x^2+2\,ac^4d^4ex+a^2c^3d^3e^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(11/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="
fricas")
```

```
[Out] [1/8*(15*(c^2*d^2*e^2*x^2 + 2*a*c*d*e^3*x + a^2*e^4)*sqrt((c*d^2 - a*e^2)/(
c*d))*log((c*d*e*x + 2*c*d^2 - a*e^2 - 2*sqrt(e*x + d)*c*d*sqrt((c*d^2 - a*
e^2)/(c*d)))/(c*d*x + a*e)) + 2*(8*c^2*d^2*e^2*x^2 - 2*c^2*d^4 - 5*a*c*d^2*
e^2 + 15*a^2*e^4 - (9*c^2*d^3*e - 25*a*c*d*e^3)*x)*sqrt(e*x + d))/(c^5*d^5*
x^2 + 2*a*c^4*d^4*e*x + a^2*c^3*d^3*e^2), -1/4*(15*(c^2*d^2*e^2*x^2 + 2*a*c
*d*e^3*x + a^2*e^4)*sqrt(-(c*d^2 - a*e^2)/(c*d))*arctan(-sqrt(e*x + d)*c*d*
sqrt(-(c*d^2 - a*e^2)/(c*d))/(c*d^2 - a*e^2)) - (8*c^2*d^2*e^2*x^2 - 2*c^2*
d^4 - 5*a*c*d^2*e^2 + 15*a^2*e^4 - (9*c^2*d^3*e - 25*a*c*d*e^3)*x)*sqrt(e*x
 + d))/(c^5*d^5*x^2 + 2*a*c^4*d^4*e*x + a^2*c^3*d^3*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(11/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(11/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="
giac")
```

```
[Out] Timed out
```

3.2021 $\int \frac{(d+ex)^{9/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal. Leaf size=130

$$-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{4c^{5/2}d^{5/2}\sqrt{cd^2-ae^2}}-\frac{3e\sqrt{d+ex}}{4c^2d^2(ae+cdx)}-\frac{(d+ex)^{3/2}}{2cd(ae+cdx)^2}$$

```
[Out] (-3*e*Sqrt[d + e*x])/(4*c^2*d^2*(a*e + c*d*x)) - (d + e*x)^(3/2)/(2*c*d*(a*
e + c*d*x)^2) - (3*e^2*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 -
 a*e^2]])/(4*c^(5/2)*d^(5/2)*Sqrt[c*d^2 - a*e^2])
```

Rubi [A] time = 0.0809901, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {626, 47, 63, 208}

$$-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{4c^{5/2}d^{5/2}\sqrt{cd^2-ae^2}}-\frac{3e\sqrt{d+ex}}{4c^2d^2(ae+cdx)}-\frac{(d+ex)^{3/2}}{2cd(ae+cdx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(9/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] (-3*e*Sqrt[d + e*x])/(4*c^2*d^2*(a*e + c*d*x)) - (d + e*x)^(3/2)/(2*c*d*(a*
e + c*d*x)^2) - (3*e^2*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 -
 a*e^2]])/(4*c^(5/2)*d^(5/2)*Sqrt[c*d^2 - a*e^2])
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{9/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx=\int \frac{(d+ex)^{3/2}}{(ae+cdx)^3}\,dx$$

$$=-\frac{(d+ex)^{3/2}}{2cd(ae+cdx)^2}+\frac{(3e)\int \frac{\sqrt{d+ex}}{(ae+cdx)^2}\,dx}{4cd}$$

$$=-\frac{3e\sqrt{d+ex}}{4c^2d^2(ae+cdx)}-\frac{(d+ex)^{3/2}}{2cd(ae+cdx)^2}+\frac{\left(3e^2\right)\int \frac{1}{(ae+cdx)\sqrt{d+ex}}\,dx}{8c^2d^2}$$

$$=-\frac{3e\sqrt{d+ex}}{4c^2d^2(ae+cdx)}-\frac{(d+ex)^{3/2}}{2cd(ae+cdx)^2}+\frac{(3e)\,\text{Subst}\left(\int \frac{1}{-\frac{cd^2}{e}+ae+\frac{cdx^2}{e}}\,dx,x,\sqrt{d+ex}\right)}{4c^2d^2}$$

$$=-\frac{3e\sqrt{d+ex}}{4c^2d^2(ae+cdx)}-\frac{(d+ex)^{3/2}}{2cd(ae+cdx)^2}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{4c^{5/2}d^{5/2}\sqrt{cd^2-ae^2}}$$

Mathematica [A] time = 0.137595, size = 118, normalized size = 0.91

$$\frac{3e^2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{ae^2-cd^2}}\right)}{4c^{5/2}d^{5/2}\sqrt{ae^2-cd^2}}-\frac{\sqrt{d+ex}\left(3ae^2+cd(2d+5ex)\right)}{4c^2d^2(ae+cdx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(9/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] -(Sqrt[d + e*x]*(3*a*e^2 + c*d*(2*d + 5*e*x)))/(4*c^2*d^2*(a*e + c*d*x)^2)
+ (3*e^2*ArcTan[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[-(c*d^2) + a*e^2]])/(4
*c^(5/2)*d^(5/2)*Sqrt[-(c*d^2) + a*e^2])
```

Maple [A] time = 0.201, size = 149, normalized size = 1.2

$$-\frac{5e^2}{4\left(cdex+ae^2\right)^2dc}(ex+d)^{\frac{3}{2}}-\frac{3e^4a}{4\left(cdex+ae^2\right)^2c^2d^2}\sqrt{ex+d}+\frac{3e^2}{4\left(cdex+ae^2\right)^2c}\sqrt{ex+d}+\frac{3e^2}{4c^2d^2}\arctan\left(cd\sqrt{ex+d}\frac{}{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

```
[Out] -5/4*e^2/(c*d*e*x+a*e^2)^2/d/c*(e*x+d)^(3/2)-3/4*e^4/(c*d*e*x+a*e^2)^2/c^2/
d^2*(e*x+d)^(1/2)*a+3/4*e^2/(c*d*e*x+a*e^2)^2/c*(e*x+d)^(1/2)+3/4*e^2/c^2/d
^2/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(
1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.06872, size = 976, normalized size = 7.51

$$\left[\frac{3\left(c^2d^2e^2x^2+2\,acde^3x+a^2e^4\right)\sqrt{c^2d^3-acde^2}\,\log\left(\frac{cdex+2\,cd^2-ae^2-2\,\sqrt{c^2d^3-acde^2}\sqrt{ex+d}}{cdx+ae}\right)-2\left(2\,c^3d^5+ac^2d^3e^2-3\,a^2cde^4+5\cdots\right.}{8\left(a^2c^4d^5e^2-a^3c^3d^3e^4+\left(c^6d^7-ac^5d^5e^2\right)x^2+2\left(ac^5d^6e-a^2c^4d^4e^3\right)x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="f
ricas")
```

```
[Out] [1/8*(3*(c^2*d^2*e^2*x^2 + 2*a*c*d*e^3*x + a^2*e^4)*sqrt(c^2*d^3 - a*c*d*e^
2)*log((c*d*e*x + 2*c*d^2 - a*e^2 - 2*sqrt(c^2*d^3 - a*c*d*e^2)*sqrt(e*x +
d))/(c*d*x + a*e)) - 2*(2*c^3*d^5 + a*c^2*d^3*e^2 - 3*a^2*c*d*e^4 + 5*(c^3*
d^4*e - a*c^2*d^2*e^3)*x)*sqrt(e*x + d))/(a^2*c^4*d^5*e^2 - a^3*c^3*d^3*e^4
 + (c^6*d^7 - a*c^5*d^5*e^2)*x^2 + 2*(a*c^5*d^6*e - a^2*c^4*d^4*e^3)*x), 1/
4*(3*(c^2*d^2*e^2*x^2 + 2*a*c*d*e^3*x + a^2*e^4)*sqrt(-c^2*d^3 + a*c*d*e^2)
*arctan(sqrt(-c^2*d^3 + a*c*d*e^2)*sqrt(e*x + d)/(c*d*e*x + c*d^2)) - (2*c^
3*d^5 + a*c^2*d^3*e^2 - 3*a^2*c*d*e^4 + 5*(c^3*d^4*e - a*c^2*d^2*e^3)*x)*sq
rt(e*x + d))/(a^2*c^4*d^5*e^2 - a^3*c^3*d^3*e^4 + (c^6*d^7 - a*c^5*d^5*e^2)
*x^2 + 2*(a*c^5*d^6*e - a^2*c^4*d^4*e^3)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(9/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.2022 $\int \frac{(d+ex)^{7/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal. Leaf size=144

$$\frac{e^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{4c^{3/2}d^{3/2}\left(cd^2-ae^2\right)^{3/2}}-\frac{e\sqrt{d+ex}}{4cd\left(cd^2-ae^2\right)(ae+cdx)}-\frac{\sqrt{d+ex}}{2cd(ae+cdx)^2}$$

```
[Out] -Sqrt[d + e*x]/(2*c*d*(a*e + c*d*x)^2) - (e*Sqrt[d + e*x])/(4*c*d*(c*d^2 -
a*e^2)*(a*e + c*d*x)) + (e^2*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c
*d^2 - a*e^2]])/(4*c^(3/2)*d^(3/2)*(c*d^2 - a*e^2)^(3/2))
```

Rubi [A] time = 0.0882176, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {626, 47, 51, 63, 208}

$$\frac{e^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{4c^{3/2}d^{3/2}\left(cd^2-ae^2\right)^{3/2}}-\frac{e\sqrt{d+ex}}{4cd\left(cd^2-ae^2\right)(ae+cdx)}-\frac{\sqrt{d+ex}}{2cd(ae+cdx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] -Sqrt[d + e*x]/(2*c*d*(a*e + c*d*x)^2) - (e*Sqrt[d + e*x])/(4*c*d*(c*d^2 -
a*e^2)*(a*e + c*d*x)) + (e^2*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c
*d^2 - a*e^2]])/(4*c^(3/2)*d^(3/2)*(c*d^2 - a*e^2)^(3/2))
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{7/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx=\int \frac{\sqrt{d+ex}}{(ae+cdx)^3}\,dx$$
$$=-\frac{\sqrt{d+ex}}{2cd(ae+cdx)^2}+\frac{e\int \frac{1}{(ae+cdx)^2\sqrt{d+ex}}\,dx}{4cd}$$
$$=-\frac{\sqrt{d+ex}}{2cd(ae+cdx)^2}-\frac{e\sqrt{d+ex}}{4cd\left(cd^2-ae^2\right)(ae+cdx)}-\frac{e^2\int \frac{1}{(ae+cdx)\sqrt{d+ex}}\,dx}{8cd\left(cd^2-ae^2\right)}$$
$$=-\frac{\sqrt{d+ex}}{2cd(ae+cdx)^2}-\frac{e\sqrt{d+ex}}{4cd\left(cd^2-ae^2\right)(ae+cdx)}-\frac{e\,\mathrm{Subst}\left(\int \frac{1}{-\frac{cd^2}{e}+ae+\frac{cdx^2}{e}}\,dx,x,\right.}{4cd\left(cd^2-ae^2\right)}$$
$$=-\frac{\sqrt{d+ex}}{2cd(ae+cdx)^2}-\frac{e\sqrt{d+ex}}{4cd\left(cd^2-ae^2\right)(ae+cdx)}+\frac{e^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{4c^{3/2}d^{3/2}\left(cd^2-ae^2\right)^{3/2}}$$

Mathematica [C] time = 0.016056, size = 61, normalized size = 0.42

$$\frac{2e^2(d+ex)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};-\frac{cd(d+ex)}{ae^2-cd^2}\right)}{3\left(ae^2-cd^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] (2*e^2*(d + e*x)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, -((c*d*(d + e*x))/(-(
c*d^2) + a*e^2))])/(3*(-(c*d^2) + a*e^2)^3)
```

Maple [A] time = 0.201, size = 142, normalized size = 1.

$$\frac{e^2}{4\left(cdex+ae^2\right)^2\left(ae^2-cd^2\right)}(ex+d)^{\frac{3}{2}}-\frac{e^2}{4\left(cdex+ae^2\right)^2dc}\sqrt{ex+d}+\frac{e^2}{\left(4\,ae^2-4\,cd^2\right)cd}\arctan\left(cd\sqrt{ex+d}\frac{1}{\sqrt{\left(ae^2-c\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

```
[Out] 1/4*e^2/(c*d*e*x+a*e^2)^2/(a*e^2-c*d^2)*(e*x+d)^(3/2)-1/4*e^2/(c*d*e*x+a*e^
2)^2/d/c*(e*x+d)^(1/2)+1/4*e^2/(a*e^2-c*d^2)/c/d/((a*e^2-c*d^2)*c*d)^(1/2)*
arctan((e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.9764, size = 1129, normalized size = 7.84

$$\left[\frac{\left(c^2d^2e^2x^2 + 2\,acde^3x + a^2e^4\right)\sqrt{c^2d^3 - acde^2}\,\log\left(\frac{cdex+2\,cd^2-ae^2+2\,\sqrt{c^2d^3-acde^2}\,\sqrt{ex+d}}{cdx+ae}\right) - 2\left(2\,c^3d^5 - 3\,ac^2d^3e^2 + a^2cde^4 + \left(c^3d^4e\right.\right.}{8\left(a^2c^4d^6e^2 - 2\,a^3c^3d^4e^4 + a^4c^2d^2e^6 + \left(c^6d^8 - 2\,ac^5d^6e^2 + a^2c^4d^4e^4\right)x^2 + 2\left(ac^5d^7e - 2\,a^2c^4d^5e^3 + a^3c^3d^3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="f
ricas")
```

```
[Out] [1/8*((c^2*d^2*e^2*x^2 + 2*a*c*d*e^3*x + a^2*e^4)*sqrt(c^2*d^3 - a*c*d*e^2)
*log((c*d*e*x + 2*c*d^2 - a*e^2 + 2*sqrt(c^2*d^3 - a*c*d*e^2)*sqrt(e*x + d)
)/(c*d*x + a*e)) - 2*(2*c^3*d^5 - 3*a*c^2*d^3*e^2 + a^2*c*d*e^4 + (c^3*d^4*
e - a*c^2*d^2*e^3)*x)*sqrt(e*x + d))/(a^2*c^4*d^6*e^2 - 2*a^3*c^3*d^4*e^4 +
 a^4*c^2*d^2*e^6 + (c^6*d^8 - 2*a*c^5*d^6*e^2 + a^2*c^4*d^4*e^4)*x^2 + 2*(a
*c^5*d^7*e - 2*a^2*c^4*d^5*e^3 + a^3*c^3*d^3*e^5)*x), -1/4*((c^2*d^2*e^2*x^
2 + 2*a*c*d*e^3*x + a^2*e^4)*sqrt(-c^2*d^3 + a*c*d*e^2)*arctan(sqrt(-c^2*d^
3 + a*c*d*e^2)*sqrt(e*x + d)/(c*d*e*x + c*d^2)) + (2*c^3*d^5 - 3*a*c^2*d^3*
e^2 + a^2*c*d*e^4 + (c^3*d^4*e - a*c^2*d^2*e^3)*x)*sqrt(e*x + d))/(a^2*c^4*
d^6*e^2 - 2*a^3*c^3*d^4*e^4 + a^4*c^2*d^2*e^6 + (c^6*d^8 - 2*a*c^5*d^6*e^2
+ a^2*c^4*d^4*e^4)*x^2 + 2*(a*c^5*d^7*e - 2*a^2*c^4*d^5*e^3 + a^3*c^3*d^3*e
^5)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.2023 $\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal. Leaf size=146

$$\frac{3e\sqrt{d+ex}}{4\left(cd^2-ae^2\right)^2(ae+cdx)}-\frac{\sqrt{d+ex}}{2\left(cd^2-ae^2\right)(ae+cdx)^2}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{4\sqrt{c}\sqrt{d}\left(cd^2-ae^2\right)^{5/2}}$$

```
[Out] -Sqrt[d + e*x]/(2*(c*d^2 - a*e^2)*(a*e + c*d*x)^2) + (3*e*Sqrt[d + e*x])/(4
*(c*d^2 - a*e^2)^2*(a*e + c*d*x)) - (3*e^2*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d
+ e*x])/Sqrt[c*d^2 - a*e^2]])/(4*Sqrt[c]*Sqrt[d]*(c*d^2 - a*e^2)^(5/2))
```

Rubi [A] time = 0.0856611, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {626, 51, 63, 208}

$$\frac{3e\sqrt{d+ex}}{4\left(cd^2-ae^2\right)^2(ae+cdx)}-\frac{\sqrt{d+ex}}{2\left(cd^2-ae^2\right)(ae+cdx)^2}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{4\sqrt{c}\sqrt{d}\left(cd^2-ae^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] -Sqrt[d + e*x]/(2*(c*d^2 - a*e^2)*(a*e + c*d*x)^2) + (3*e*Sqrt[d + e*x])/(4
*(c*d^2 - a*e^2)^2*(a*e + c*d*x)) - (3*e^2*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d
+ e*x])/Sqrt[c*d^2 - a*e^2]])/(4*Sqrt[c]*Sqrt[d]*(c*d^2 - a*e^2)^(5/2))
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx=\int \frac{1}{(ae+cdx)^3\sqrt{d+ex}}\,dx$$

$$=-\frac{\sqrt{d+ex}}{2\left(cd^2-ae^2\right)(ae+cdx)^2}-\frac{(3e)\int \frac{1}{(ae+cdx)^2\sqrt{d+ex}}\,dx}{4\left(cd^2-ae^2\right)}$$

$$=-\frac{\sqrt{d+ex}}{2\left(cd^2-ae^2\right)(ae+cdx)^2}+\frac{3e\sqrt{d+ex}}{4\left(cd^2-ae^2\right)^2(ae+cdx)}+\frac{\left(3e^2\right)\int \frac{1}{(ae+cdx)\sqrt{d+ex}}\,d}{8\left(cd^2-ae^2\right)^2}$$

$$=-\frac{\sqrt{d+ex}}{2\left(cd^2-ae^2\right)(ae+cdx)^2}+\frac{3e\sqrt{d+ex}}{4\left(cd^2-ae^2\right)^2(ae+cdx)}+\frac{(3e)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{cd^2}{e}+ae}\right.}{4\left(cd^2\right.}$$

$$=-\frac{\sqrt{d+ex}}{2\left(cd^2-ae^2\right)(ae+cdx)^2}+\frac{3e\sqrt{d+ex}}{4\left(cd^2-ae^2\right)^2(ae+cdx)}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+e}}{\sqrt{cd^2-ae^2}}\right.}{4\sqrt{c}\sqrt{d}\left(cd^2-ae^2\right)^{5/}}$$

Mathematica [C] time = 0.0129869, size = 59, normalized size = 0.4

$$\frac{2e^2\sqrt{d+ex}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};-\frac{cd(d+ex)}{ae^2-cd^2}\right)}{\left(ae^2-cd^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] (2*e^2*Sqrt[d + e*x]*Hypergeometric2F1[1/2, 3, 3/2, -((c*d*(d + e*x))/(-(c*
d^2) + a*e^2))])/(-(c*d^2) + a*e^2)^3
```

Maple [A] time = 0.199, size = 144, normalized size = 1.

$$\frac{e^2}{\left(2ae^2-2cd^2\right)\left(cdex+ae^2\right)^2}\sqrt{ex+d}+\frac{3e^2}{4\left(ae^2-cd^2\right)^2\left(cdex+ae^2\right)}\sqrt{ex+d}+\frac{3e^2}{4\left(ae^2-cd^2\right)^2}\arctan\left(cd\sqrt{ex+d}\frac{}{\sqrt{(ae}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

```
[Out] 1/2*e^2*(e*x+d)^(1/2)/(a*e^2-c*d^2)/(c*d*e*x+a*e^2)^2+3/4*e^2/(a*e^2-c*d^2)
^2*(e*x+d)^(1/2)/(c*d*e*x+a*e^2)+3/4*e^2/(a*e^2-c*d^2)^2/((a*e^2-c*d^2)*c*d
)^(1/2)*arctan((e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.96957, size = 1295, normalized size = 8.87

$$\left[\frac{3\left(c^2d^2e^2x^2+2\,acde^3x+a^2e^4\right)\sqrt{c^2d^3-acde^2}\log\left(\frac{cdex+2\,cd^2-ae^2-2\,\sqrt{c^2d^3-acde^2}\sqrt{ex+d}}{cdx+ae}\right)-2\left(2\,c^3d^5-7\,ac^2d^3e^2+5\,a^2cde^4-3\left(\right.\right.}{8\left(a^2c^4d^7e^2-3\,a^3c^3d^5e^4+3\,a^4c^2d^3e^6-a^5cde^8+\left(c^6d^9-3\,ac^5d^7e^2+3\,a^2c^4d^5e^4-a^3c^3d^3e^6\right)x^2+2\left(ac^5d^8e-3\,a^2c^4d^6e^3+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="f
ricas")
```

```
[Out] [1/8*(3*(c^2*d^2*e^2*x^2 + 2*a*c*d*e^3*x + a^2*e^4)*sqrt(c^2*d^3 - a*c*d*e^
2)*log((c*d*e*x + 2*c*d^2 - a*e^2 - 2*sqrt(c^2*d^3 - a*c*d*e^2)*sqrt(e*x +
d))/(c*d*x + a*e)) - 2*(2*c^3*d^5 - 7*a*c^2*d^3*e^2 + 5*a^2*c*d*e^4 - 3*(c^
3*d^4*e - a*c^2*d^2*e^3)*x)*sqrt(e*x + d))/(a^2*c^4*d^7*e^2 - 3*a^3*c^3*d^5
*e^4 + 3*a^4*c^2*d^3*e^6 - a^5*c*d*e^8 + (c^6*d^9 - 3*a*c^5*d^7*e^2 + 3*a^2
*c^4*d^5*e^4 - a^3*c^3*d^3*e^6)*x^2 + 2*(a*c^5*d^8*e - 3*a^2*c^4*d^6*e^3 +
3*a^3*c^3*d^4*e^5 - a^4*c^2*d^2*e^7)*x), 1/4*(3*(c^2*d^2*e^2*x^2 + 2*a*c*d*
e^3*x + a^2*e^4)*sqrt(-c^2*d^3 + a*c*d*e^2)*arctan(sqrt(-c^2*d^3 + a*c*d*e^
2)*sqrt(e*x + d)/(c*d*e*x + c*d^2)) - (2*c^3*d^5 - 7*a*c^2*d^3*e^2 + 5*a^2*
c*d*e^4 - 3*(c^3*d^4*e - a*c^2*d^2*e^3)*x)*sqrt(e*x + d))/(a^2*c^4*d^7*e^2
- 3*a^3*c^3*d^5*e^4 + 3*a^4*c^2*d^3*e^6 - a^5*c*d*e^8 + (c^6*d^9 - 3*a*c^5*
d^7*e^2 + 3*a^2*c^4*d^5*e^4 - a^3*c^3*d^3*e^6)*x^2 + 2*(a*c^5*d^8*e - 3*a^2
*c^4*d^6*e^3 + 3*a^3*c^3*d^4*e^5 - a^4*c^2*d^2*e^7)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.2024 $\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3} \, dx$

Optimal. Leaf size=176

$$\frac{15e^2}{4\sqrt{d+ex}\left(cd^2-ae^2\right)^3}+\frac{5e}{4\sqrt{d+ex}\left(cd^2-ae^2\right)^2(ae+cdx)}-\frac{1}{2\sqrt{d+ex}\left(cd^2-ae^2\right)(ae+cdx)^2}-\frac{15\sqrt{c}\sqrt{d}e^2\tanh^{-1}\left(\cdots\right)}{4\left(cd^2-ae^2\right)^{7/2}}$$

```
[Out] (15*e^2)/(4*(c*d^2 - a*e^2)^3*Sqrt[d + e*x]) - 1/(2*(c*d^2 - a*e^2)*(a*e +
c*d*x)^2*Sqrt[d + e*x]) + (5*e)/(4*(c*d^2 - a*e^2)^2*(a*e + c*d*x)*Sqrt[d +
 e*x]) - (15*Sqrt[c]*Sqrt[d]*e^2*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sq
rt[c*d^2 - a*e^2]])/(4*(c*d^2 - a*e^2)^(7/2))
```

Rubi [A] time = 0.119063, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {626, 51, 63, 208}

$$\frac{15e^2}{4\sqrt{d+ex}\left(cd^2-ae^2\right)^3}+\frac{5e}{4\sqrt{d+ex}\left(cd^2-ae^2\right)^2(ae+cdx)}-\frac{1}{2\sqrt{d+ex}\left(cd^2-ae^2\right)(ae+cdx)^2}-\frac{15\sqrt{c}\sqrt{d}e^2\tanh^{-1}\left(\cdots\right)}{4\left(cd^2-ae^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] (15*e^2)/(4*(c*d^2 - a*e^2)^3*Sqrt[d + e*x]) - 1/(2*(c*d^2 - a*e^2)*(a*e +
c*d*x)^2*Sqrt[d + e*x]) + (5*e)/(4*(c*d^2 - a*e^2)^2*(a*e + c*d*x)*Sqrt[d +
 e*x]) - (15*Sqrt[c]*Sqrt[d]*e^2*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sq
rt[c*d^2 - a*e^2]])/(4*(c*d^2 - a*e^2)^(7/2))
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx = \int \frac{1}{(ae+cdx)^3(d+ex)^{3/2}}\,dx$$

$$= -\frac{1}{2\left(cd^2-ae^2\right)(ae+cdx)^2\sqrt{d+ex}} - \frac{(5e)\int \frac{1}{(ae+cdx)^2(d+ex)^{3/2}}\,dx}{4\left(cd^2-ae^2\right)}$$

$$= -\frac{1}{2\left(cd^2-ae^2\right)(ae+cdx)^2\sqrt{d+ex}} + \frac{5e}{4\left(cd^2-ae^2\right)^2(ae+cdx)\sqrt{d+ex}} + \frac{\left(15e^2\right)\int}{8}$$

$$= \frac{15e^2}{4\left(cd^2-ae^2\right)^3\sqrt{d+ex}} - \frac{1}{2\left(cd^2-ae^2\right)(ae+cdx)^2\sqrt{d+ex}} + \frac{5e}{4\left(cd^2-ae^2\right)^2(ae+}$$

$$= \frac{15e^2}{4\left(cd^2-ae^2\right)^3\sqrt{d+ex}} - \frac{1}{2\left(cd^2-ae^2\right)(ae+cdx)^2\sqrt{d+ex}} + \frac{5e}{4\left(cd^2-ae^2\right)^2(ae+}$$

$$= \frac{15e^2}{4\left(cd^2-ae^2\right)^3\sqrt{d+ex}} - \frac{1}{2\left(cd^2-ae^2\right)(ae+cdx)^2\sqrt{d+ex}} + \frac{5e}{4\left(cd^2-ae^2\right)^2(ae+}$$

Mathematica [C] time = 0.0154581, size = 59, normalized size = 0.34

$$-\frac{2e^2\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};-\frac{cd(d+ex)}{ae^2-cd^2}\right)}{\sqrt{d+ex}\left(ae^2-cd^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] (-2*e^2*Hypergeometric2F1[-1/2, 3, 1/2, -((c*d*(d + e*x))/(-(c*d^2) + a*e^2
))])/((-(c*d^2) + a*e^2)^3*Sqrt[d + e*x])
```

Maple [A] time = 0.207, size = 226, normalized size = 1.3

$$-2\,\frac{e^2}{\left(ae^2-cd^2\right)^3\sqrt{ex+d}} - \frac{7\,c^2d^2e^2}{4\,\left(ae^2-cd^2\right)^3\left(cdex+ae^2\right)^2}(ex+d)^{\frac{3}{2}} - \frac{9\,e^4cda}{4\,\left(ae^2-cd^2\right)^3\left(cdex+ae^2\right)^2}\sqrt{ex+d} + \frac{9\,c^2}{4\,\left(ae^2-cd^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

```
[Out] -2*e^2/(a*e^2-c*d^2)^3/(e*x+d)^(1/2)-7/4*e^2/(a*e^2-c*d^2)^3*c^2*d^2/(c*d*e
*x+a*e^2)^2*(e*x+d)^(3/2)-9/4*e^4/(a*e^2-c*d^2)^3*c*d/(c*d*e*x+a*e^2)^2*(e*
x+d)^(1/2)*a+9/4*e^2/(a*e^2-c*d^2)^3*c^2*d^3/(c*d*e*x+a*e^2)^2*(e*x+d)^(1/2
```

```
)-15/4*e^2/(a*e^2-c*d^2)^3*c*d/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((e*x+d)^(1/
2)*c*d/((a*e^2-c*d^2)*c*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.13636, size = 1767, normalized size = 10.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="f
ricas")
```

```
[Out] [1/8*(15*(c^2*d^2*e^3*x^3 + a^2*d*e^4 + (c^2*d^3*e^2 + 2*a*c*d*e^4)*x^2 + (
2*a*c*d^2*e^3 + a^2*e^5)*x)*sqrt(c*d/(c*d^2 - a*e^2))*log((c*d*e*x + 2*c*d^
2 - a*e^2 - 2*(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(c*d/(c*d^2 - a*e^2)))/(c*d
*x + a*e)) + 2*(15*c^2*d^2*e^2*x^2 - 2*c^2*d^4 + 9*a*c*d^2*e^2 + 8*a^2*e^4
+ 5*(c^2*d^3*e + 5*a*c*d*e^3)*x)*sqrt(e*x + d))/(a^2*c^3*d^7*e^2 - 3*a^3*c^
2*d^5*e^4 + 3*a^4*c*d^3*e^6 - a^5*d*e^8 + (c^5*d^8*e - 3*a*c^4*d^6*e^3 + 3*
a^2*c^3*d^4*e^5 - a^3*c^2*d^2*e^7)*x^3 + (c^5*d^9 - a*c^4*d^7*e^2 - 3*a^2*c
^3*d^5*e^4 + 5*a^3*c^2*d^3*e^6 - 2*a^4*c*d*e^8)*x^2 + (2*a*c^4*d^8*e - 5*a^
2*c^3*d^6*e^3 + 3*a^3*c^2*d^4*e^5 + a^4*c*d^2*e^7 - a^5*e^9)*x), -1/4*(15*(
c^2*d^2*e^3*x^3 + a^2*d*e^4 + (c^2*d^3*e^2 + 2*a*c*d*e^4)*x^2 + (2*a*c*d^2*
e^3 + a^2*e^5)*x)*sqrt(-c*d/(c*d^2 - a*e^2))*arctan(-(c*d^2 - a*e^2)*sqrt(e
*x + d)*sqrt(-c*d/(c*d^2 - a*e^2))/(c*d*e*x + c*d^2)) - (15*c^2*d^2*e^2*x^2
 - 2*c^2*d^4 + 9*a*c*d^2*e^2 + 8*a^2*e^4 + 5*(c^2*d^3*e + 5*a*c*d*e^3)*x)*s
qrt(e*x + d))/(a^2*c^3*d^7*e^2 - 3*a^3*c^2*d^5*e^4 + 3*a^4*c*d^3*e^6 - a^5*
d*e^8 + (c^5*d^8*e - 3*a*c^4*d^6*e^3 + 3*a^2*c^3*d^4*e^5 - a^3*c^2*d^2*e^7)
*x^3 + (c^5*d^9 - a*c^4*d^7*e^2 - 3*a^2*c^3*d^5*e^4 + 5*a^3*c^2*d^3*e^6 - 2
*a^4*c*d*e^8)*x^2 + (2*a*c^4*d^8*e - 5*a^2*c^3*d^6*e^3 + 3*a^3*c^2*d^4*e^5
+ a^4*c*d^2*e^7 - a^5*e^9)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.2025 $\int \frac{\sqrt{d+ex}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal. Leaf size=208

$$-\frac{35c^{3/2}d^{3/2}e^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{4\left(cd^2-ae^2\right)^{9/2}}+\frac{35cde^2}{4\sqrt{d+ex}\left(cd^2-ae^2\right)^4}+\frac{35e^2}{12(d+ex)^{3/2}\left(cd^2-ae^2\right)^3}+\frac{7e}{4(d+ex)^{3/2}\left(cd^2-ae^2\right)^2(a\ldots}$$

```
[Out] (35*e^2)/(12*(c*d^2 - a*e^2)^3*(d + e*x)^(3/2)) - 1/(2*(c*d^2 - a*e^2)*(a*e
 + c*d*x)^2*(d + e*x)^(3/2)) + (7*e)/(4*(c*d^2 - a*e^2)^2*(a*e + c*d*x)*(d
+ e*x)^(3/2)) + (35*c*d*e^2)/(4*(c*d^2 - a*e^2)^4*Sqrt[d + e*x]) - (35*c^(3
/2)*d^(3/2)*e^2*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]
])/(4*(c*d^2 - a*e^2)^(9/2))
```

Rubi [A] time = 0.137705, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {626, 51, 63, 208}

$$-\frac{35c^{3/2}d^{3/2}e^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{4\left(cd^2-ae^2\right)^{9/2}}+\frac{35cde^2}{4\sqrt{d+ex}\left(cd^2-ae^2\right)^4}+\frac{35e^2}{12(d+ex)^{3/2}\left(cd^2-ae^2\right)^3}+\frac{7e}{4(d+ex)^{3/2}\left(cd^2-ae^2\right)^2(a\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```



```
[Out] (35*e^2)/(12*(c*d^2 - a*e^2)^3*(d + e*x)^(3/2)) - 1/(2*(c*d^2 - a*e^2)*(a*e
 + c*d*x)^2*(d + e*x)^(3/2)) + (7*e)/(4*(c*d^2 - a*e^2)^2*(a*e + c*d*x)*(d
+ e*x)^(3/2)) + (35*c*d*e^2)/(4*(c*d^2 - a*e^2)^4*Sqrt[d + e*x]) - (35*c^(3
/2)*d^(3/2)*e^2*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]
])/(4*(c*d^2 - a*e^2)^(9/2))
```


Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx=\int \frac{1}{(ae+cdx)^3(d+ex)^{5/2}}\,dx$$

$$=-\frac{1}{2\left(cd^2-ae^2\right)(ae+cdx)^2(d+ex)^{3/2}}-\frac{(7e)\int \frac{1}{(ae+cdx)^2(d+ex)^{5/2}}\,dx}{4\left(cd^2-ae^2\right)}$$

$$=-\frac{1}{2\left(cd^2-ae^2\right)(ae+cdx)^2(d+ex)^{3/2}}+\frac{7e}{4\left(cd^2-ae^2\right)^2(ae+cdx)(d+ex)^{3/2}}+\frac{(35}{}$$

$$=\frac{35e^2}{12\left(cd^2-ae^2\right)^3(d+ex)^{3/2}}-\frac{1}{2\left(cd^2-ae^2\right)(ae+cdx)^2(d+ex)^{3/2}}+\frac{}{4\left(cd^2-ae^2\right)^2}$$

$$=\frac{35e^2}{12\left(cd^2-ae^2\right)^3(d+ex)^{3/2}}-\frac{1}{2\left(cd^2-ae^2\right)(ae+cdx)^2(d+ex)^{3/2}}+\frac{}{4\left(cd^2-ae^2\right)^2}$$

$$=\frac{35e^2}{12\left(cd^2-ae^2\right)^3(d+ex)^{3/2}}-\frac{1}{2\left(cd^2-ae^2\right)(ae+cdx)^2(d+ex)^{3/2}}+\frac{}{4\left(cd^2-ae^2\right)^2}$$

$$=\frac{35e^2}{12\left(cd^2-ae^2\right)^3(d+ex)^{3/2}}-\frac{1}{2\left(cd^2-ae^2\right)(ae+cdx)^2(d+ex)^{3/2}}+\frac{}{4\left(cd^2-ae^2\right)^2}$$

Mathematica [C] time = 0.0164422, size = 61, normalized size = 0.29

$$-\frac{2e^2\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};-\frac{cd(d+ex)}{ae^2-cd^2}\right)}{3(d+ex)^{3/2}\left(ae^2-cd^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] (-2*e^2*Hypergeometric2F1[-3/2, 3, -1/2, -((c*d*(d + e*x))/(-(c*d^2) + a*e^
2))])/(3*(-(c*d^2) + a*e^2)^3*(d + e*x)^(3/2))
```

Maple [A] time = 0.206, size = 262, normalized size = 1.3

$$-\frac{2\,e^2}{3\left(ae^2-cd^2\right)^3}(ex+d)^{-\frac{3}{2}}+6\,\frac{e^2cd}{\left(ae^2-cd^2\right)^4\sqrt{ex+d}}+\frac{11\,c^3d^3e^2}{4\left(ae^2-cd^2\right)^4\left(cdex+ae^2\right)^2}(ex+d)^{\frac{3}{2}}+\frac{13\,e^4c^2d^2a}{4\left(ae^2-cd^2\right)^4\left(cdex+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

```
[Out] -2/3*e^2/(a*e^2-c*d^2)^3/(e*x+d)^(3/2)+6*e^2/(a*e^2-c*d^2)^4*c*d/(e*x+d)^(1
/2)+11/4*e^2/(a*e^2-c*d^2)^4*c^3*d^3/(c*d*e*x+a*e^2)^2*(e*x+d)^(3/2)+13/4*e
^4/(a*e^2-c*d^2)^4*c^2*d^2/(c*d*e*x+a*e^2)^2*(e*x+d)^(1/2)*a-13/4*e^2/(a*e^
2-c*d^2)^4*c^3*d^4/(c*d*e*x+a*e^2)^2*(e*x+d)^(1/2)+35/4*e^2/(a*e^2-c*d^2)^4
*c^2*d^2/((a*e^2-c*d^2)*c*d)^(1/2)*arctan((e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*
c*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.15021, size = 2707, normalized size = 13.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="f
ricas")
```

```
[Out] [1/24*(105*(c^3*d^3*e^4*x^4 + a^2*c*d^3*e^4 + 2*(c^3*d^4*e^3 + a*c^2*d^2*e^
5)*x^3 + (c^3*d^5*e^2 + 4*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x^2 + 2*(a*c^2*d^4*e
^3 + a^2*c*d^2*e^5)*x)*sqrt(c*d/(c*d^2 - a*e^2))*log((c*d*e*x + 2*c*d^2 - a
*e^2 - 2*(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(c*d/(c*d^2 - a*e^2)))/(c*d*x +
a*e)) + 2*(105*c^3*d^3*e^3*x^3 - 6*c^3*d^6 + 39*a*c^2*d^4*e^2 + 80*a^2*c*d^
2*e^4 - 8*a^3*e^6 + 35*(4*c^3*d^4*e^2 + 5*a*c^2*d^2*e^4)*x^2 + 7*(3*c^3*d^5
*e + 34*a*c^2*d^3*e^3 + 8*a^2*c*d*e^5)*x)*sqrt(e*x + d))/(a^2*c^4*d^10*e^2
- 4*a^3*c^3*d^8*e^4 + 6*a^4*c^2*d^6*e^6 - 4*a^5*c*d^4*e^8 + a^6*d^2*e^10 +
(c^6*d^10*e^2 - 4*a*c^5*d^8*e^4 + 6*a^2*c^4*d^6*e^6 - 4*a^3*c^3*d^4*e^8 + a
^4*c^2*d^2*e^10)*x^4 + 2*(c^6*d^11*e - 3*a*c^5*d^9*e^3 + 2*a^2*c^4*d^7*e^5
+ 2*a^3*c^3*d^5*e^7 - 3*a^4*c^2*d^3*e^9 + a^5*c*d*e^11)*x^3 + (c^6*d^12 - 9
*a^2*c^4*d^8*e^4 + 16*a^3*c^3*d^6*e^6 - 9*a^4*c^2*d^4*e^8 + a^6*e^12)*x^2 +
 2*(a*c^5*d^11*e - 3*a^2*c^4*d^9*e^3 + 2*a^3*c^3*d^7*e^5 + 2*a^4*c^2*d^5*e^
7 - 3*a^5*c*d^3*e^9 + a^6*d*e^11)*x), -1/12*(105*(c^3*d^3*e^4*x^4 + a^2*c*d
^3*e^4 + 2*(c^3*d^4*e^3 + a*c^2*d^2*e^5)*x^3 + (c^3*d^5*e^2 + 4*a*c^2*d^3*e
^4 + a^2*c*d*e^6)*x^2 + 2*(a*c^2*d^4*e^3 + a^2*c*d^2*e^5)*x)*sqrt(-c*d/(c*d
^2 - a*e^2))*arctan(-(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(-c*d/(c*d^2 - a*e^2
))/(c*d*e*x + c*d^2)) - (105*c^3*d^3*e^3*x^3 - 6*c^3*d^6 + 39*a*c^2*d^4*e^2
 + 80*a^2*c*d^2*e^4 - 8*a^3*e^6 + 35*(4*c^3*d^4*e^2 + 5*a*c^2*d^2*e^4)*x^2
+ 7*(3*c^3*d^5*e + 34*a*c^2*d^3*e^3 + 8*a^2*c*d*e^5)*x)*sqrt(e*x + d))/(a^2
*c^4*d^10*e^2 - 4*a^3*c^3*d^8*e^4 + 6*a^4*c^2*d^6*e^6 - 4*a^5*c*d^4*e^8 + a
^6*d^2*e^10 + (c^6*d^10*e^2 - 4*a*c^5*d^8*e^4 + 6*a^2*c^4*d^6*e^6 - 4*a^3*c
^3*d^4*e^8 + a^4*c^2*d^2*e^10)*x^4 + 2*(c^6*d^11*e - 3*a*c^5*d^9*e^3 + 2*a^
2*c^4*d^7*e^5 + 2*a^3*c^3*d^5*e^7 - 3*a^4*c^2*d^3*e^9 + a^5*c*d*e^11)*x^3 +
 (c^6*d^12 - 9*a^2*c^4*d^8*e^4 + 16*a^3*c^3*d^6*e^6 - 9*a^4*c^2*d^4*e^8 + a
```

```
^6*e^12)*x^2 + 2*(a*c^5*d^11*e - 3*a^2*c^4*d^9*e^3 + 2*a^3*c^3*d^7*e^5 + 2*
a^4*c^2*d^5*e^7 - 3*a^5*c*d^3*e^9 + a^6*d*e^11)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.2026 $\int \frac{1}{\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal. Leaf size=244

$$\frac{63c^2d^2e^2}{4\sqrt{d+ex}\left(cd^2-ae^2\right)^5}-\frac{63c^{5/2}d^{5/2}e^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{4\left(cd^2-ae^2\right)^{11/2}}+\frac{21cde^2}{4(d+ex)^{3/2}\left(cd^2-ae^2\right)^4}+\frac{9e}{4(d+ex)^{5/2}\left(cd^2-ae^2\right)^2(ae+$$

```
[Out] (63*e^2)/(20*(c*d^2 - a*e^2)^3*(d + e*x)^(5/2)) - 1/(2*(c*d^2 - a*e^2)*(a*e
 + c*d*x)^2*(d + e*x)^(5/2)) + (9*e)/(4*(c*d^2 - a*e^2)^2*(a*e + c*d*x)*(d
+ e*x)^(5/2)) + (21*c*d*e^2)/(4*(c*d^2 - a*e^2)^4*(d + e*x)^(3/2)) + (63*c^
2*d^2*e^2)/(4*(c*d^2 - a*e^2)^5*Sqrt[d + e*x]) - (63*c^(5/2)*d^(5/2)*e^2*Ar
cTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(4*(c*d^2 - a*e
^2)^(11/2))
```

Rubi [A] time = 0.222973, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {626, 51, 63, 208}

$$\frac{63c^2d^2e^2}{4\sqrt{d+ex}\left(cd^2-ae^2\right)^5}-\frac{63c^{5/2}d^{5/2}e^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{4\left(cd^2-ae^2\right)^{11/2}}+\frac{21cde^2}{4(d+ex)^{3/2}\left(cd^2-ae^2\right)^4}+\frac{9e}{4(d+ex)^{5/2}\left(cd^2-ae^2\right)^2(ae+$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3),x]
```


```
[Out] (63*e^2)/(20*(c*d^2 - a*e^2)^3*(d + e*x)^(5/2)) - 1/(2*(c*d^2 - a*e^2)*(a*e
 + c*d*x)^2*(d + e*x)^(5/2)) + (9*e)/(4*(c*d^2 - a*e^2)^2*(a*e + c*d*x)*(d
+ e*x)^(5/2)) + (21*c*d*e^2)/(4*(c*d^2 - a*e^2)^4*(d + e*x)^(3/2)) + (63*c^
2*d^2*e^2)/(4*(c*d^2 - a*e^2)^5*Sqrt[d + e*x]) - (63*c^(5/2)*d^(5/2)*e^2*Ar
cTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(4*(c*d^2 - a*e
^2)^(11/2))
```


Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx = \int \frac{1}{(ae+cdx)^3(d+ex)^{7/2}}\,dx$$

$$= -\frac{1}{2\left(cd^2-ae^2\right)(ae+cdx)^2(d+ex)^{5/2}} - \frac{(9e)\int \frac{1}{(ae+cdx)^2(d+ex)^{7/2}}\,dx}{4\left(cd^2-ae^2\right)}$$

$$= -\frac{1}{2\left(cd^2-ae^2\right)(ae+cdx)^2(d+ex)^{5/2}} + \frac{9e}{4\left(cd^2-ae^2\right)^2(ae+cdx)(d+ex)^5}$$

$$= \frac{63e^2}{20\left(cd^2-ae^2\right)^3(d+ex)^{5/2}} - \frac{1}{2\left(cd^2-ae^2\right)(ae+cdx)^2(d+ex)^{5/2}} + \frac{}{4\left(cd^2\right.}$$

$$= \frac{63e^2}{20\left(cd^2-ae^2\right)^3(d+ex)^{5/2}} - \frac{1}{2\left(cd^2-ae^2\right)(ae+cdx)^2(d+ex)^{5/2}} + \frac{}{4\left(cd^2\right.}$$

$$= \frac{63e^2}{20\left(cd^2-ae^2\right)^3(d+ex)^{5/2}} - \frac{1}{2\left(cd^2-ae^2\right)(ae+cdx)^2(d+ex)^{5/2}} + \frac{}{4\left(cd^2\right.}$$

$$= \frac{63e^2}{20\left(cd^2-ae^2\right)^3(d+ex)^{5/2}} - \frac{1}{2\left(cd^2-ae^2\right)(ae+cdx)^2(d+ex)^{5/2}} + \frac{}{4\left(cd^2\right.}$$

$$= \frac{63e^2}{20\left(cd^2-ae^2\right)^3(d+ex)^{5/2}} - \frac{1}{2\left(cd^2-ae^2\right)(ae+cdx)^2(d+ex)^{5/2}} + \frac{}{4\left(cd^2\right.}$$

Mathematica [C] time = 0.0207404, size = 61, normalized size = 0.25

$$-\frac{2e^2\,{}_2F_1\left(-\frac{5}{2},3;-\frac{3}{2};-\frac{cd(d+ex)}{ae^2-cd^2}\right)}{5(d+ex)^{5/2}\left(ae^2-cd^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3),x]
```

```
[Out] (-2*e^2*Hypergeometric2F1[-5/2, 3, -3/2, -((c*d*(d + e*x))/(-(c*d^2) + a*e^
2))])/(5*(-(c*d^2) + a*e^2)^3*(d + e*x)^(5/2))
```

Maple [A] time = 0.234, size = 294, normalized size = 1.2

$$-\frac{2e^2}{5\left(ae^2-cd^2\right)^3}(ex+d)^{-\frac{5}{2}} - 12\frac{e^2c^2d^2}{\left(ae^2-cd^2\right)^5\sqrt{ex+d}} + 2\frac{e^2cd}{\left(ae^2-cd^2\right)^4(ex+d)^{3/2}} - \frac{15c^4d^4e^2}{4\left(ae^2-cd^2\right)^5\left(cdex+ae^2\right)^2}(ex+d)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

```
[Out] -2/5*e^2/(a*e^2-c*d^2)^3/(e*x+d)^(5/2)-12*e^2/(a*e^2-c*d^2)^5*c^2*d^2/(e*x+
d)^(1/2)+2*e^2/(a*e^2-c*d^2)^4*c*d/(e*x+d)^(3/2)-15/4*e^2/(a*e^2-c*d^2)^5*c
^4*d^4/(c*d*e*x+a*e^2)^2*(e*x+d)^(3/2)-17/4*e^4/(a*e^2-c*d^2)^5*c^3*d^3/(c*
d*e*x+a*e^2)^2*(e*x+d)^(1/2)*a+17/4*e^2/(a*e^2-c*d^2)^5*c^4*d^5/(c*d*e*x+a*
e^2)^2*(e*x+d)^(1/2)-63/4*e^2/(a*e^2-c*d^2)^5*c^3*d^3/((a*e^2-c*d^2)*c*d)^(
1/2)*arctan((e*x+d)^(1/2)*c*d/((a*e^2-c*d^2)*c*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.38589, size = 4073, normalized size = 16.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm=
"fricas")
```

```
[Out] [1/40*(315*(c^4*d^4*e^5*x^5 + a^2*c^2*d^5*e^4 + (3*c^4*d^5*e^4 + 2*a*c^3*d^
3*e^6)*x^4 + (3*c^4*d^6*e^3 + 6*a*c^3*d^4*e^5 + a^2*c^2*d^2*e^7)*x^3 + (c^4
*d^7*e^2 + 6*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6)*x^2 + (2*a*c^3*d^6*e^3 + 3*
a^2*c^2*d^4*e^5)*x)*sqrt(c*d/(c*d^2 - a*e^2))*log((c*d*e*x + 2*c*d^2 - a*e^
2 - 2*(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(c*d/(c*d^2 - a*e^2)))/(c*d*x + a*e
)) + 2*(315*c^4*d^4*e^4*x^4 - 10*c^4*d^8 + 85*a*c^3*d^6*e^2 + 288*a^2*c^2*d
^4*e^4 - 56*a^3*c*d^2*e^6 + 8*a^4*e^8 + 105*(7*c^4*d^5*e^3 + 5*a*c^3*d^3*e^
5)*x^3 + 21*(23*c^4*d^6*e^2 + 59*a*c^3*d^4*e^4 + 8*a^2*c^2*d^2*e^6)*x^2 + 3
*(15*c^4*d^7*e + 277*a*c^3*d^5*e^3 + 136*a^2*c^2*d^3*e^5 - 8*a^3*c*d*e^7)*x
)*sqrt(e*x + d))/(a^2*c^5*d^13*e^2 - 5*a^3*c^4*d^11*e^4 + 10*a^4*c^3*d^9*e^
6 - 10*a^5*c^2*d^7*e^8 + 5*a^6*c*d^5*e^10 - a^7*d^3*e^12 + (c^7*d^12*e^3 -
5*a*c^6*d^10*e^5 + 10*a^2*c^5*d^8*e^7 - 10*a^3*c^4*d^6*e^9 + 5*a^4*c^3*d^4*
e^11 - a^5*c^2*d^2*e^13)*x^5 + (3*c^7*d^13*e^2 - 13*a*c^6*d^11*e^4 + 20*a^2
*c^5*d^9*e^6 - 10*a^3*c^4*d^7*e^8 - 5*a^4*c^3*d^5*e^10 + 7*a^5*c^2*d^3*e^12
 - 2*a^6*c*d*e^14)*x^4 + (3*c^7*d^14*e - 9*a*c^6*d^12*e^3 + a^2*c^5*d^10*e^
5 + 25*a^3*c^4*d^8*e^7 - 35*a^4*c^3*d^6*e^9 + 17*a^5*c^2*d^4*e^11 - a^6*c*d
^2*e^13 - a^7*e^15)*x^3 + (c^7*d^15 + a*c^6*d^13*e^2 - 17*a^2*c^5*d^11*e^4
+ 35*a^3*c^4*d^9*e^6 - 25*a^4*c^3*d^7*e^8 - a^5*c^2*d^5*e^10 + 9*a^6*c*d^3*
e^12 - 3*a^7*d*e^14)*x^2 + (2*a*c^6*d^14*e - 7*a^2*c^5*d^12*e^3 + 5*a^3*c^4
*d^10*e^5 + 10*a^4*c^3*d^8*e^7 - 20*a^5*c^2*d^6*e^9 + 13*a^6*c*d^4*e^11 - 3
*a^7*d^2*e^13)*x), -1/20*(315*(c^4*d^4*e^5*x^5 + a^2*c^2*d^5*e^4 + (3*c^4*d
^5*e^4 + 2*a*c^3*d^3*e^6)*x^4 + (3*c^4*d^6*e^3 + 6*a*c^3*d^4*e^5 + a^2*c^2*
d^2*e^7)*x^3 + (c^4*d^7*e^2 + 6*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6)*x^2 + (2
```

```
*a*c^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5)*x)*sqrt(-c*d/(c*d^2 - a*e^2))*arctan(-(
c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(-c*d/(c*d^2 - a*e^2))/(c*d*e*x + c*d^2))
- (315*c^4*d^4*e^4*x^4 - 10*c^4*d^8 + 85*a*c^3*d^6*e^2 + 288*a^2*c^2*d^4*e^
4 - 56*a^3*c*d^2*e^6 + 8*a^4*e^8 + 105*(7*c^4*d^5*e^3 + 5*a*c^3*d^3*e^5)*x^
3 + 21*(23*c^4*d^6*e^2 + 59*a*c^3*d^4*e^4 + 8*a^2*c^2*d^2*e^6)*x^2 + 3*(15*
c^4*d^7*e + 277*a*c^3*d^5*e^3 + 136*a^2*c^2*d^3*e^5 - 8*a^3*c*d*e^7)*x)*sqr
t(e*x + d))/(a^2*c^5*d^13*e^2 - 5*a^3*c^4*d^11*e^4 + 10*a^4*c^3*d^9*e^6 - 1
0*a^5*c^2*d^7*e^8 + 5*a^6*c*d^5*e^10 - a^7*d^3*e^12 + (c^7*d^12*e^3 - 5*a*c
^6*d^10*e^5 + 10*a^2*c^5*d^8*e^7 - 10*a^3*c^4*d^6*e^9 + 5*a^4*c^3*d^4*e^11
- a^5*c^2*d^2*e^13)*x^5 + (3*c^7*d^13*e^2 - 13*a*c^6*d^11*e^4 + 20*a^2*c^5*
d^9*e^6 - 10*a^3*c^4*d^7*e^8 - 5*a^4*c^3*d^5*e^10 + 7*a^5*c^2*d^3*e^12 - 2*
a^6*c*d*e^14)*x^4 + (3*c^7*d^14*e - 9*a*c^6*d^12*e^3 + a^2*c^5*d^10*e^5 + 2
5*a^3*c^4*d^8*e^7 - 35*a^4*c^3*d^6*e^9 + 17*a^5*c^2*d^4*e^11 - a^6*c*d^2*e^
13 - a^7*e^15)*x^3 + (c^7*d^15 + a*c^6*d^13*e^2 - 17*a^2*c^5*d^11*e^4 + 35*
a^3*c^4*d^9*e^6 - 25*a^4*c^3*d^7*e^8 - a^5*c^2*d^5*e^10 + 9*a^6*c*d^3*e^12
- 3*a^7*d*e^14)*x^2 + (2*a*c^6*d^14*e - 7*a^2*c^5*d^12*e^3 + 5*a^3*c^4*d^10
*e^5 + 10*a^4*c^3*d^8*e^7 - 20*a^5*c^2*d^6*e^9 + 13*a^6*c*d^4*e^11 - 3*a^7*
d^2*e^13)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm=
"giac")

[Out] Timed out
```

3.2027 $\int (d+ex)^{7/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal. Leaf size=295

$$\frac{256\left(cd^2-ae^2\right)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3465c^5d^5(d+ex)^{3/2}}+\frac{128\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{1155c^4d^4\sqrt{d+ex}}+\frac{32\sqrt{d+ex}\,(cd}{}$$

```
[Out] (256*(c*d^2 - a*e^2)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3465
*c^5*d^5*(d + e*x)^(3/2)) + (128*(c*d^2 - a*e^2)^3*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(3/2))/(1155*c^4*d^4*Sqrt[d + e*x]) + (32*(c*d^2 - a*e^2)^2
*Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(231*c^3*d^3)
 + (16*(c*d^2 - a*e^2)*(d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(3/2))/(99*c^2*d^2) + (2*(d + e*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^(3/2))/(11*c*d)
```

Rubi [A] time = 0.262823, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {656, 648}

$$\frac{256\left(cd^2-ae^2\right)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3465c^5d^5(d+ex)^{3/2}}+\frac{128\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{1155c^4d^4\sqrt{d+ex}}+\frac{32\sqrt{d+ex}\,(cd}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```



```
[Out] (256*(c*d^2 - a*e^2)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3465
*c^5*d^5*(d + e*x)^(3/2)) + (128*(c*d^2 - a*e^2)^3*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(3/2))/(1155*c^4*d^4*Sqrt[d + e*x]) + (32*(c*d^2 - a*e^2)^2
*Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(231*c^3*d^3)
 + (16*(c*d^2 - a*e^2)*(d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(3/2))/(99*c^2*d^2) + (2*(d + e*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^(3/2))/(11*c*d)
```


Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int (d+ex)^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx = \frac{2(d+ex)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{11cd} + \frac{\left(8\left(d^2-\frac{ae^2}{c}\right)\right)\int (d+ex)^{5/}}{}$$

$$= \frac{16\left(cd^2-ae^2\right)(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{99c^2d^2} + \frac{2(d+ex)^{5/2}}{}$$

$$= \frac{32\left(cd^2-ae^2\right)^2\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{231c^3d^3} + \frac{16\left(cd^2-ae^2\right)}{}$$

$$= \frac{128\left(cd^2-ae^2\right)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{1155c^4d^4\sqrt{d+ex}} + \frac{32\left(cd^2-ae^2\right)^2\sqrt{d+}}{}$$

$$= \frac{256\left(cd^2-ae^2\right)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3465c^5d^5(d+ex)^{3/2}} + \frac{128\left(cd^2-ae^2\right)^3\left(ade\right.}{1155c}$$

Mathematica [A] time = 0.157107, size = 187, normalized size = 0.63

$$\frac{2((d+ex)(ae+cdx))^{3/2}\left(48a^2c^2d^2e^4\left(33d^2+22dex+5e^2x^2\right)-64a^3cde^6(11d+3ex)+128a^4e^8-8ac^3d^3e^2\left(297d^2ex+231\right.\right.}{3465c^5d^5(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(128*a^4*e^8 - 64*a^3*c*d*e^6*(11*d + 3*
e*x) + 48*a^2*c^2*d^2*e^4*(33*d^2 + 22*d*e*x + 5*e^2*x^2) - 8*a*c^3*d^3*e^2
*(231*d^3 + 297*d^2*e*x + 165*d*e^2*x^2 + 35*e^3*x^3) + c^4*d^4*(1155*d^4 +
 2772*d^3*e*x + 2970*d^2*e^2*x^2 + 1540*d*e^3*x^3 + 315*e^4*x^4)))/(3465*c^
5*d^5*(d + e*x)^(3/2))
```

Maple [A] time = 0.044, size = 243, normalized size = 0.8

$$\frac{(2\,cdx+2\,ae)\left(315\,e^4x^4c^4d^4-280\,ac^3d^3e^5x^3+1540\,c^4d^5e^3x^3+240\,a^2c^2d^2e^6x^2-1320\,ac^3d^4e^4x^2+2970\,c^4d^6e^2x^2-192\,a^3\right.}{34}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] 2/3465*(c*d*x+a*e)*(315*c^4*d^4*e^4*x^4-280*a*c^3*d^3*e^5*x^3+1540*c^4*d^5*
e^3*x^3+240*a^2*c^2*d^2*e^6*x^2-1320*a*c^3*d^4*e^4*x^2+2970*c^4*d^6*e^2*x^2
-192*a^3*c*d*e^7*x+1056*a^2*c^2*d^3*e^5*x-2376*a*c^3*d^5*e^3*x+2772*c^4*d^7
*e*x+128*a^4*e^8-704*a^3*c*d^2*e^6+1584*a^2*c^2*d^4*e^4-1848*a*c^3*d^6*e^2+
1155*c^4*d^8)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/c^5/d^5/(e*x+d)^(1/2)
```

Maxima [A] time = 1.09053, size = 400, normalized size = 1.36

$$\frac{2\left(315\,c^5d^5e^4x^5+1155\,ac^4d^8e-1848\,a^2c^3d^6e^3+1584\,a^3c^2d^4e^5-704\,a^4cd^2e^7+128\,a^5e^9+35\left(44\,c^5d^6e^3+ac^4d^4e^5\right)x^4+1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="maxima")
```

```
[Out] 2/3465*(315*c^5*d^5*e^4*x^5 + 1155*a*c^4*d^8*e - 1848*a^2*c^3*d^6*e^3 + 158
4*a^3*c^2*d^4*e^5 - 704*a^4*c*d^2*e^7 + 128*a^5*e^9 + 35*(44*c^5*d^6*e^3 +
a*c^4*d^4*e^5)*x^4 + 10*(297*c^5*d^7*e^2 + 22*a*c^4*d^5*e^4 - 4*a^2*c^3*d^3
*e^6)*x^3 + 6*(462*c^5*d^8*e + 99*a*c^4*d^6*e^3 - 44*a^2*c^3*d^4*e^5 + 8*a^
3*c^2*d^2*e^7)*x^2 + (1155*c^5*d^9 + 924*a*c^4*d^7*e^2 - 792*a^2*c^3*d^5*e^
4 + 352*a^3*c^2*d^3*e^6 - 64*a^4*c*d*e^8)*x)*sqrt(c*d*x + a*e)*(e*x + d)/(c
^5*d^5*e*x + c^5*d^6)
```

Fricas [A] time = 1.91222, size = 679, normalized size = 2.3

$$\frac{2\left(315\,c^5d^5e^4x^5+1155\,ac^4d^8e-1848\,a^2c^3d^6e^3+1584\,a^3c^2d^4e^5-704\,a^4cd^2e^7+128\,a^5e^9+35\left(44\,c^5d^6e^3+ac^4d^4e^5\right)x^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="fricas")
```

```
[Out] 2/3465*(315*c^5*d^5*e^4*x^5 + 1155*a*c^4*d^8*e - 1848*a^2*c^3*d^6*e^3 + 158
4*a^3*c^2*d^4*e^5 - 704*a^4*c*d^2*e^7 + 128*a^5*e^9 + 35*(44*c^5*d^6*e^3 +
a*c^4*d^4*e^5)*x^4 + 10*(297*c^5*d^7*e^2 + 22*a*c^4*d^5*e^4 - 4*a^2*c^3*d^3
*e^6)*x^3 + 6*(462*c^5*d^8*e + 99*a*c^4*d^6*e^3 - 44*a^2*c^3*d^4*e^5 + 8*a^
3*c^2*d^2*e^7)*x^2 + (1155*c^5*d^9 + 924*a*c^4*d^7*e^2 - 792*a^2*c^3*d^5*e^
4 + 352*a^3*c^2*d^3*e^6 - 64*a^4*c*d*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*d^5*e*x + c^5*d^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.2028 $\int (d+ex)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal. Leaf size=233

$$\frac{32\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{315c^4d^4(d+ex)^{3/2}}+\frac{16\left(cd^2-ae^2\right)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{105c^3d^3\sqrt{d+ex}}+\frac{4\sqrt{d+ex}\left(cd^2-ae^2\right.}{}$$

```
[Out] (32*(c*d^2 - a*e^2)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(315*c
^4*d^4*(d + e*x)^(3/2)) + (16*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(3/2))/(105*c^3*d^3*Sqrt[d + e*x]) + (4*(c*d^2 - a*e^2)*Sqrt[d
 + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(21*c^2*d^2) + (2*(d
 + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(9*c*d)
```

Rubi [A] time = 0.188651, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {656, 648}

$$\frac{32\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{315c^4d^4(d+ex)^{3/2}}+\frac{16\left(cd^2-ae^2\right)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{105c^3d^3\sqrt{d+ex}}+\frac{4\sqrt{d+ex}\left(cd^2-ae^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```


```
[Out] (32*(c*d^2 - a*e^2)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(315*c
^4*d^4*(d + e*x)^(3/2)) + (16*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(3/2))/(105*c^3*d^3*Sqrt[d + e*x]) + (4*(c*d^2 - a*e^2)*Sqrt[d
 + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(21*c^2*d^2) + (2*(d
 + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(9*c*d)
```


Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int (d+ex)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx = \frac{2(d+ex)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{9cd} + \frac{\left(2\left(d^2-\frac{ae^2}{c}\right)\right)\int (d+e\cdots}{\cdots}$$

$$= \frac{4\left(cd^2-ae^2\right)\sqrt{d+ex}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{21c^2d^2} + \frac{2(d+ex)^{3/2}\left(\cdots\right.}{\cdots}$$

$$= \frac{16\left(cd^2-ae^2\right)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{105c^3d^3\sqrt{d+ex}} + \frac{4\left(cd^2-ae^2\right)\sqrt{d+\cdots}}{\cdots}$$

$$= \frac{32\left(cd^2-ae^2\right)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{315c^4d^4(d+ex)^{3/2}} + \frac{16\left(cd^2-ae^2\right)^2\left(ad\cdots\right.}{105\cdots}$$

Mathematica [A] time = 0.105582, size = 131, normalized size = 0.56

$$\frac{2((d+ex)(ae+cdx))^{3/2}\left(24a^2cde^4(3d+ex)-16a^3e^6-6ac^2d^2e^2\left(21d^2+18dex+5e^2x^2\right)+c^3d^3\left(189d^2ex+105d^3+13\cdots\right.\right.}{315c^4d^4(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-16*a^3*e^6 + 24*a^2*c*d*e^4*(3*d + e*x
) - 6*a*c^2*d^2*e^2*(21*d^2 + 18*d*e*x + 5*e^2*x^2) + c^3*d^3*(105*d^3 + 18
9*d^2*e*x + 135*d*e^2*x^2 + 35*e^3*x^3)))/(315*c^4*d^4*(d + e*x)^(3/2))
```

Maple [A] time = 0.044, size = 168, normalized size = 0.7

$$-\frac{(2\,cdx+2\,ae)\left(-35\,e^3x^3c^3d^3+30\,ac^2d^2e^4x^2-135\,c^3d^4e^2x^2-24\,a^2cde^5x+108\,ac^2d^3e^3x-189\,c^3d^5ex+16\,a^3e^6-72\,a\cdots\right.}{315\,c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] -2/315*(c*d*x+a*e)*(-35*c^3*d^3*e^3*x^3+30*a*c^2*d^2*e^4*x^2-135*c^3*d^4*e^
2*x^2-24*a^2*c*d*e^5*x+108*a*c^2*d^3*e^3*x-189*c^3*d^5*e*x+16*a^3*e^6-72*a^
2*c*d^2*e^4+126*a*c^2*d^4*e^2-105*c^3*d^6)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e
)^(1/2)/c^4/d^4/(e*x+d)^(1/2)
```

Maxima [A] time = 1.12121, size = 285, normalized size = 1.22

$$\frac{2\left(35\,c^4d^4e^3x^4+105\,ac^3d^6e-126\,a^2c^2d^4e^3+72\,a^3cd^2e^5-16\,a^4e^7+5\left(27\,c^4d^5e^2+ac^3d^3e^4\right)x^3+3\left(63\,c^4d^6e+9\,ac^3d^4e\cdots\right.\right.}{315\left(c^4d^4ex+c^4d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="maxima")
```

```
[Out] 2/315*(35*c^4*d^4*e^3*x^4 + 105*a*c^3*d^6*e - 126*a^2*c^2*d^4*e^3 + 72*a^3*
c*d^2*e^5 - 16*a^4*e^7 + 5*(27*c^4*d^5*e^2 + a*c^3*d^3*e^4)*x^3 + 3*(63*c^4
*d^6*e + 9*a*c^3*d^4*e^3 - 2*a^2*c^2*d^2*e^5)*x^2 + (105*c^4*d^7 + 63*a*c^3
*d^5*e^2 - 36*a^2*c^2*d^3*e^4 + 8*a^3*c*d*e^6)*x)*sqrt(c*d*x + a*e)*(e*x +
d)/(c^4*d^4*e*x + c^4*d^5)
```

Fricas [A] time = 1.86278, size = 483, normalized size = 2.07

$$\frac{2\left(35\,c^4d^4e^3x^4+105\,ac^3d^6e-126\,a^2c^2d^4e^3+72\,a^3cd^2e^5-16\,a^4e^7+5\left(27\,c^4d^5e^2+ac^3d^3e^4\right)x^3+3\left(63\,c^4d^6e+9\,ac^3d^4e^3-\right.\right.}{315\left(c^4d^4ex+c^4d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="fricas")
```

```
[Out] 2/315*(35*c^4*d^4*e^3*x^4 + 105*a*c^3*d^6*e - 126*a^2*c^2*d^4*e^3 + 72*a^3*
c*d^2*e^5 - 16*a^4*e^7 + 5*(27*c^4*d^5*e^2 + a*c^3*d^3*e^4)*x^3 + 3*(63*c^4
*d^6*e + 9*a*c^3*d^4*e^3 - 2*a^2*c^2*d^2*e^5)*x^2 + (105*c^4*d^7 + 63*a*c^3
*d^5*e^2 - 36*a^2*c^2*d^3*e^4 + 8*a^3*c*d*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^4*d^4*e*x + c^4*d^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.2029 $\int (d+ex)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal. Leaf size=171

$$\frac{16\left(cd^2-ae^2\right)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{105c^3d^3(d+ex)^{3/2}}+\frac{8\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{35c^2d^2\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(x\left(ae^2+\right.\right.}{}$$

```
[Out] (16*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(105*c
^3*d^3*(d + e*x)^(3/2)) + (8*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^(3/2))/(35*c^2*d^2*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*(a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(7*c*d)
```

Rubi [A] time = 0.112851, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {656, 648}

$$\frac{16\left(cd^2-ae^2\right)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{105c^3d^3(d+ex)^{3/2}}+\frac{8\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{35c^2d^2\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(x\left(ae^2+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```


```
[Out] (16*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(105*c
^3*d^3*(d + e*x)^(3/2)) + (8*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^(3/2))/(35*c^2*d^2*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*(a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(7*c*d)
```


Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int (d+ex)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx=\frac{2\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{7cd}+\left(4\left(d^2-\frac{ae^2}{c}\right)\right)\int\sqrt{d+ex}$$

$$=\frac{8\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{35c^2d^2\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(ade+\left(cd^2\right.\right.}{7c}$$

$$=\frac{16\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{105c^3d^3(d+ex)^{3/2}}+\frac{8\left(cd^2-ae^2\right)\left(ade\right.}{35c^2}$$

Mathematica [A] time = 0.0736661, size = 88, normalized size = 0.51

$$\frac{2((d+ex)(ae+cdx))^{3/2}\left(8a^2e^4-4acde^2(7d+3ex)+c^2d^2\left(35d^2+42dex+15e^2x^2\right)\right)}{105c^3d^3(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(8*a^2*e^4 - 4*a*c*d*e^2*(7*d + 3*e*x) +
 c^2*d^2*(35*d^2 + 42*d*e*x + 15*e^2*x^2)))/(105*c^3*d^3*(d + e*x)^(3/2))
```

Maple [A] time = 0.044, size = 110, normalized size = 0.6

$$\frac{(2\,cdx+2\,ae)\left(15\,e^2x^2c^2d^2-12\,acde^3x+42\,c^2d^3ex+8\,a^2e^4-28\,acd^2e^2+35\,c^2d^4\right)}{105\,c^3d^3}\sqrt{cdex^2+ae^2x+cd^2x+ade}\frac{1}{\sqrt{ex+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] 2/105*(c*d*x+a*e)*(15*c^2*d^2*e^2*x^2-12*a*c*d*e^3*x+42*c^2*d^3*e*x+8*a^2*e
^4-28*a*c*d^2*e^2+35*c^2*d^4)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/c^3/d
^3/(e*x+d)^(1/2)
```

Maxima [A] time = 1.08813, size = 189, normalized size = 1.11

$$\frac{2\left(15\,c^3d^3e^2x^3+35\,ac^2d^4e-28\,a^2cd^2e^3+8\,a^3e^5+3\left(14\,c^3d^4e+ac^2d^2e^3\right)x^2+\left(35\,c^3d^5+14\,ac^2d^3e^2-4\,a^2cde^4\right)x\right)\sqrt{cdx+}}{105\left(c^3d^3ex+c^3d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="maxima")
```

```
[Out] 2/105*(15*c^3*d^3*e^2*x^3 + 35*a*c^2*d^4*e - 28*a^2*c*d^2*e^3 + 8*a^3*e^5 +
 3*(14*c^3*d^4*e + a*c^2*d^2*e^3)*x^2 + (35*c^3*d^5 + 14*a*c^2*d^3*e^2 - 4*
a^2*c*d*e^4)*x)*sqrt(c*d*x + a*e)*(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

Fricas [A] time = 1.84812, size = 336, normalized size = 1.96

$$\frac{2\left(15\,c^3d^3e^2x^3+35\,ac^2d^4e-28\,a^2cd^2e^3+8\,a^3e^5+3\left(14\,c^3d^4e+ac^2d^2e^3\right)x^2+\left(35\,c^3d^5+14\,ac^2d^3e^2-4\,a^2cde^4\right)x\right)\sqrt{cdex^2}}{105\left(c^3d^3ex+c^3d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="fricas")
```

```
[Out] 2/105*(15*c^3*d^3*e^2*x^3 + 35*a*c^2*d^4*e - 28*a^2*c*d^2*e^3 + 8*a^3*e^5 +
 3*(14*c^3*d^4*e + a*c^2*d^2*e^3)*x^2 + (35*c^3*d^5 + 14*a*c^2*d^3*e^2 - 4*
a^2*c*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(
c^3*d^3*e*x + c^3*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.2030 $\int \sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal. Leaf size=109

$$\frac{4(cd^2-ae^2)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{15c^2d^2(d+ex)^{3/2}}+\frac{2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{5cd\sqrt{d+ex}}$$

```
[Out] (4*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(15*c^2*d
^2*(d + e*x)^(3/2)) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*
c*d*Sqrt[d + e*x])
```

Rubi [A] time = 0.0602184, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {656, 648}

$$\frac{4(cd^2-ae^2)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{15c^2d^2(d+ex)^{3/2}}+\frac{2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{5cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (4*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(15*c^2*d
^2*(d + e*x)^(3/2)) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*
c*d*Sqrt[d + e*x])
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx=\frac{2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{5cd\sqrt{d+ex}}+\frac{\left(2\left(d^2-\frac{ae^2}{c}\right)\right)\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,d}{5d}$$

$$=\frac{4(cd^2-ae^2)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{15c^2d^2(d+ex)^{3/2}}+\frac{2\left(ade+(cd^2+ae^2)x+c\right.}{5cd\sqrt{d+ex}}$$

Mathematica [A] time = 0.0479747, size = 55, normalized size = 0.5

$$\frac{2((d+ex)(ae+cdx))^{3/2}\left(cd(5d+3ex)-2ae^2\right)}{15c^2d^2(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-2*a*e^2 + c*d*(5*d + 3*e*x)))/(15*c^2*
d^2*(d + e*x)^(3/2))
```

Maple [A] time = 0.042, size = 69, normalized size = 0.6

$$-\frac{(2\,cdx+2\,ae)\left(-3\,cdex+2\,ae^{2}-5\,cd^{2}\right)}{15\,c^{2}d^{2}}\sqrt{cdex^{2}+ae^{2}x+cd^{2}x+ade}\frac{1}{\sqrt{ex+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] -2/15*(c*d*x+a*e)*(-3*c*d*e*x+2*a*e^2-5*c*d^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a
*d*e)^(1/2)/c^2/d^2/(e*x+d)^(1/2)
```

Maxima [A] time = 1.08072, size = 112, normalized size = 1.03

$$\frac{2\left(3\,c^{2}d^{2}ex^{2}+5\,acd^{2}e-2\,a^{2}e^{3}+\left(5\,c^{2}d^{3}+acde^{2}\right)x\right)\sqrt{cdx+ae}(ex+d)}{15\left(c^{2}d^{2}ex+c^{2}d^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="maxima")
```

```
[Out] 2/15*(3*c^2*d^2*e*x^2 + 5*a*c*d^2*e - 2*a^2*e^3 + (5*c^2*d^3 + a*c*d*e^2)*x
)*sqrt(c*d*x + a*e)*(e*x + d)/(c^2*d^2*e*x + c^2*d^3)
```

Fricas [A] time = 1.80272, size = 216, normalized size = 1.98

$$\frac{2\left(3\,c^{2}d^{2}ex^{2}+5\,acd^{2}e-2\,a^{2}e^{3}+\left(5\,c^{2}d^{3}+acde^{2}\right)x\right)\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}\sqrt{ex+d}}{15\left(c^{2}d^{2}ex+c^{2}d^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="fricas")
```

```
[Out] 2/15*(3*c^2*d^2*e*x^2 + 5*a*c*d^2*e - 2*a^2*e^3 + (5*c^2*d^3 + a*c*d*e^2)*x
)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^2*d^2*e*x +
c^2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{(d+ex)(ae+cdx)}\sqrt{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))*sqrt(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d), x)
```

3.2031 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=48

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3cd(d+ex)^{3/2}}$$

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*c*d*(d + e*x)^(3/2))
```

Rubi [A] time = 0.0214836, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {648}

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3cd(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/Sqrt[d + e*x],x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*c*d*(d + e*x)^(3/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3cd(d+ex)^{3/2}}$$

Mathematica [A] time = 0.024581, size = 37, normalized size = 0.77

$$\frac{2((d+ex)(ae+cdx))^{3/2}}{3cd(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/Sqrt[d + e*x],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2))/(3*c*d*(d + e*x)^(3/2))
```

Maple [A] time = 0.04, size = 50, normalized size = 1.

$$\frac{2\,cdx+2\,ae}{3\,cd}\sqrt{cdex^2+ae^2x+cd^2x+ade}\frac{1}{\sqrt{ex+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] 2/3*(c*d*x+a*e)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/d/c/(e*x+d)^(1/2)
```

Maxima [A] time = 1.02892, size = 24, normalized size = 0.5

$$\frac{2\,(cdx+ae)^{\frac{3}{2}}}{3\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorith
m="maxima")
```

```
[Out] 2/3*(c*d*x + a*e)^(3/2)/(c*d)
```

Fricas [A] time = 1.76137, size = 128, normalized size = 2.67

$$\frac{2\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}(cdx+ae)\sqrt{ex+d}}{3\left(cdex+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorith
m="fricas")
```

```
[Out] 2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*x + a*e)*sqrt(e*x + d)
/(c*d*e*x + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{(d+ex)\,(ae+cdx)}}{\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/sqrt(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/sqrt(e*x + d), x)
```

3.2032 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=128

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e\sqrt{d+ex}}-\frac{2\sqrt{cd^2-ae^2}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{e^{3/2}}$$

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e*Sqrt[d + e*x]) - (2*Sqrt
[c*d^2 - a*e^2]*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/e^(3/2)
```

Rubi [A] time = 0.0978779, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {664, 660, 205}

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e\sqrt{d+ex}}-\frac{2\sqrt{cd^2-ae^2}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^(3/2),x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e*Sqrt[d + e*x]) - (2*Sqrt
[c*d^2 - a*e^2]*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/e^(3/2)
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(d+ex)^{3/2}}dx = \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e\sqrt{d+ex}} - \frac{\left(2cd^2e-e\left(cd^2+ae^2\right)\right)\int \frac{1}{\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x}}}{e^2}$$

$$= \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e\sqrt{d+ex}} - \left(2\left(cd^2-ae^2\right)\right)\text{Subst}\left(\int \frac{1}{2cd^2e-e\left(cd^2+ae\right.}\right.$$

$$= \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e\sqrt{d+ex}} - \frac{2\sqrt{cd^2-ae^2}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cd^2-ae^2}\sqrt{d+ex}}\right)}{e^{3/2}}$$

Mathematica [A] time = 0.119412, size = 105, normalized size = 0.82

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(\sqrt{e}-\frac{\sqrt{cd^2-ae^2}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd^2-ae^2}}\right)}{\sqrt{ae+cdx}}\right)}{e^{3/2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^(3/2),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[e] - (Sqrt[c*d^2 - a*e^2]*ArcTan[(Sq
rt[e]*Sqrt[a*e + c*d*x])/Sqrt[c*d^2 - a*e^2]])/Sqrt[a*e + c*d*x]))/(e^(3/2)
*Sqrt[d + e*x])
```

Maple [A] time = 0.26, size = 163, normalized size = 1.3

$$-2\frac{\sqrt{cdex^2+ae^2x+cd^2x+ade}}{\sqrt{ex+d}\sqrt{cdx+ae}e\sqrt{\left(ae^2-cd^2\right)e}}\left(\text{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)ae^2-\text{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)cd^2-\sqrt{cdx+ae}\sqrt{(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(3/2),x)
```

```
[Out] -2*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/(e*x+d)^(1/2)*(arctanh(e*(c*d*x+
a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*a*e^2-arctanh(e*(c*d*x+a*e)^(1/2)/((a*e
^2-c*d^2)*e)^(1/2))*c*d^2-(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2))/(c*d*x
+a*e)^(1/2)/e/((a*e^2-c*d^2)*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{(ex+d)^{\frac{3}{2}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(3/2),x, algorith
m="maxima")
```

[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(e*x + d)^(3/2), x)

Fricas [A] time = 1.94932, size = 659, normalized size = 5.15

$$\left[\frac{(ex+d)\sqrt{-\frac{cd^2-ae^2}{e}}\log\left(-\frac{cde^2x^2+2\,ae^3x-cd^3+2\,ade^2-2\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+de}\sqrt{-\frac{cd^2-ae^2}{e}}}{e^2x^2+2\,dex+d^2}\right)+2\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex}}{e^2x+de}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(3/2),x, algorith m="fricas")

[Out] [((e*x + d)*sqrt(-(c*d^2 - a*e^2)/e)*log(-(c*d*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2 - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*e *sqrt(-(c*d^2 - a*e^2)/e))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e^2*x + d*e), 2*((e*x + d)*sqrt((c*d^2 - a*e^2)/e)*arctan(sqrt(e*x + d)*sqrt((c*d^2 - a*e^2)/e)/sqrt(c*d*e* x^2 + a*d*e + (c*d^2 + a*e^2)*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e^2*x + d*e)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(d+ex)\,(ae+cdx)}}{(d+ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(3/2),x)

[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x)**(3/2), x)

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(3/2),x, algorith m="giac")

[Out] Timed out

3.2033 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=129

$$\frac{cd\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{e^{3/2}\sqrt{cd^2-ae^2}}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)^{3/2}}$$

```
[Out] -(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(e*(d + e*x)^(3/2))) + (c*d*A
rcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a
*e^2]*Sqrt[d + e*x])])/(e^(3/2)*Sqrt[c*d^2 - a*e^2])
```

Rubi [A] time = 0.0732543, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {662, 660, 205}

$$\frac{cd\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{e^{3/2}\sqrt{cd^2-ae^2}}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^(5/2),x]
```

```
[Out] -(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(e*(d + e*x)^(3/2))) + (c*d*A
rcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a
*e^2]*Sqrt[d + e*x])])/(e^(3/2)*Sqrt[c*d^2 - a*e^2])
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(d+ex)^{5/2}}\,dx = -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e(d+ex)^{3/2}} + \frac{(cd)\int \frac{1}{\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{2e}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e(d+ex)^{3/2}} + (cd)\,\text{Subst}\left(\int \frac{1}{2cd^2e-e\left(cd^2+ae^2\right)+e^2x^2}\,dx,$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e(d+ex)^{3/2}} + \frac{cd\tan^{-1}\left(\frac{\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cd^2-ae^2}\sqrt{d+ex}}\right)}{e^{3/2}\sqrt{cd^2-ae^2}}$$

Mathematica [A] time = 0.159372, size = 112, normalized size = 0.87

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{cd(d+ex)\tan^{-1}\left(\frac{\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd^2-ae^2}}\right)}{\sqrt{cd^2-ae^2}\sqrt{ae+cdx}}-\sqrt{e}\right)}{e^{3/2}(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^(5/2),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-Sqrt[e] + (c*d*(d + e*x)*ArcTan[(Sqrt[e]*S
qrt[a*e + c*d*x])/Sqrt[c*d^2 - a*e^2]])/(Sqrt[c*d^2 - a*e^2]*Sqrt[a*e + c*d
*x])))/(e^(3/2)*(d + e*x)^(3/2))
```

Maple [A] time = 0.244, size = 163, normalized size = 1.3

$$\frac{1}{e}\left(-\text{Artanh}\left(e\sqrt{cdx+ae}\frac{1}{\sqrt{\left(ae^2-cd^2\right)e}}\right)xcde-\text{Artanh}\left(e\sqrt{cdx+ae}\frac{1}{\sqrt{\left(ae^2-cd^2\right)e}}\right)cd^2-\sqrt{cdx+ae}\sqrt{\left(ae^2-cd^2\right)e}\right)\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(5/2),x)
```

```
[Out] (-arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*x*c*d*e-arctanh(e*(c
*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*c*d^2-(c*d*x+a*e)^(1/2)*((a*e^2-c*
d^2)*e)^(1/2))*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/(e*x+d)^(3/2)/(c*d*x
+a*e)^(1/2)/e/((a*e^2-c*d^2)*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(5/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(e*x + d)^(5/2), x)
```

Fricas [B] time = 1.87339, size = 1004, normalized size = 7.78

$$\left[-\frac{\left(cde^2x^2 + 2\,cd^2ex + cd^3\right)\sqrt{-cd^2e + ae^3}\log\left(-\frac{cde^2x^2+2\,ae^3x-cd^3+2\,ade^2-2\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{-cd^2e+ae^3}\sqrt{ex+d}}{e^2x^2+2\,dex+d^2}\right) + 2\,\sqrt{cdex^2}}{2\left(cd^4e^2 - ad^2e^4 + \left(cd^2e^4 - ae^6\right)x^2 + 2\left(cd^3e^3 - ade^5\right)x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(5/2),x, algorith
m="fricas")
```

```
[Out] [-1/2*((c*d*e^2*x^2 + 2*c*d^2*e*x + c*d^3)*sqrt(-c*d^2*e + a*e^3)*log(-(c*d
*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2 - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*sqrt(-c*d^2*e + a*e^3)*sqrt(e*x + d))/(e^2*x^2 + 2*d*e*x + d^
2)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d^2*e - a*e^3)*sqrt(
e*x + d))/(c*d^4*e^2 - a*d^2*e^4 + (c*d^2*e^4 - a*e^6)*x^2 + 2*(c*d^3*e^3 -
 a*d*e^5)*x), -((c*d*e^2*x^2 + 2*c*d^2*e*x + c*d^3)*sqrt(c*d^2*e - a*e^3)*a
rctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d^2*e - a*e^3)*sqr
t(e*x + d)/(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x)) + sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*(c*d^2*e - a*e^3)*sqrt(e*x + d))/(c*d^4*e^2 -
a*d^2*e^4 + (c*d^2*e^4 - a*e^6)*x^2 + 2*(c*d^3*e^3 - a*d*e^5)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(5/2),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.2034 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=199

$$\frac{c^2d^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{4e^{3/2}\left(cd^2-ae^2\right)^{3/2}}+\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4e(d+ex)^{3/2}\left(cd^2-ae^2\right)}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2e(d+ex)^{5/2}}$$

```
[Out] -Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*e*(d + e*x)^(5/2)) + (c*d*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*e*(c*d^2 - a*e^2)*(d + e*x)^
(3/2)) + (c^2*d^2*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(4*e^(3/2)*(c*d^2 - a*e^2)^(3/2))
```

Rubi [A] time = 0.131257, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {662, 672, 660, 205}

$$\frac{c^2d^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{4e^{3/2}\left(cd^2-ae^2\right)^{3/2}}+\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4e(d+ex)^{3/2}\left(cd^2-ae^2\right)}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2e(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^(7/2),x]
```

```
[Out] -Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*e*(d + e*x)^(5/2)) + (c*d*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*e*(c*d^2 - a*e^2)*(d + e*x)^
(3/2)) + (c^2*d^2*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(4*e^(3/2)*(c*d^2 - a*e^2)^(3/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*(m + 2*p + 2))/((m + p + 1)*(2*c*d - b*e)), Int[(d
 + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && Ne
Q[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
```

```
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(d+ex)^{7/2}}\,dx = -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2e(d+ex)^{5/2}} + \frac{(cd)\int \frac{1}{(d+ex)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{4e}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2e(d+ex)^{5/2}} + \frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e\left(cd^2-ae^2\right)(d+ex)^{3/2}} + \frac{\left(c^2d^2\right)\int}{}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2e(d+ex)^{5/2}} + \frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e\left(cd^2-ae^2\right)(d+ex)^{3/2}} + \frac{\left(c^2d^2\right)\mathrm{Su}}{}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2e(d+ex)^{5/2}} + \frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e\left(cd^2-ae^2\right)(d+ex)^{3/2}} + \frac{c^2d^2\tan}{4}$$

Mathematica [C] time = 0.0355337, size = 83, normalized size = 0.42

$$\frac{2c^2d^2((d+ex)(ae+cdx))^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^(7/2),x]
```

```
[Out] (2*c^2*d^2*((a*e + c*d*x)*(d + e*x))^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, (
e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(3*(c*d^2 - a*e^2)^3*(d + e*x)^(3/2))
```

Maple [A] time = 0.261, size = 292, normalized size = 1.5

$$\frac{1}{\left(4\,ae^2-4\,cd^2\right)e}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(\mathrm{Artanh}\left(e\sqrt{cdx+ae}\frac{1}{\sqrt{\left(ae^2-cd^2\right)e}}\right)x^2c^2d^2e^2+2\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+a}}{\sqrt{\left(ae^2-cd\right.}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(7/2),x)
```

```
[Out] 1/4*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(arctanh(e*(c*d*x+a*e)^(1/2)/((
a*e^2-c*d^2)*e)^(1/2))*x^2*c^2*d^2*e^2+2*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^
2-c*d^2)*e)^(1/2))*x*c^2*d^3*e+arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e
)^(1/2))*c^2*d^4-x*c*d*e*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)-2*((a*e^
2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*a*e^2+((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*
```

```
e)^(1/2)*c*d^2)/(e*x+d)^(5/2)/(c*d*x+a*e)^(1/2)/(a*e^2-c*d^2)/e/((a*e^2-c*d
^2)*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{(ex + d)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(7/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(e*x + d)^(7/2), x)
```

Fricas [B] time = 2.13503, size = 1528, normalized size = 7.68

$$\left[-\frac{\left(c^2d^2e^3x^3 + 3\,c^2d^3e^2x^2 + 3\,c^2d^4ex + c^2d^5\right)\sqrt{-cd^2e + ae^3}\log\left(-\frac{cde^2x^2+2\,ae^3x-cd^3+2\,ade^2-2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{-cd^2e+ae^3}\sqrt{ex+d}}{e^2x^2+2\,dex+d^2}\right.}{8\left(c^2d^7e^2 - 2\,acd^5e^4 + a^2d^3e^6 + \left(c^2d^4e^5 - 2\,acd^2e^7 + a^2e^9\right)x^3 + 3\left(c^2d^5e^4 - 2\,acd\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(7/2),x, algorith
m="fricas")
```

```
[Out] [-1/8*((c^2*d^2*e^3*x^3 + 3*c^2*d^3*e^2*x^2 + 3*c^2*d^4*e*x + c^2*d^5)*sqrt
(-c*d^2*e + a*e^3)*log(-(c*d*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2 - 2*sq
rt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d^2*e + a*e^3)*sqrt(e*x +
 d))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(c^2*d^4*e - 3*a*c*d^2*e^3 + 2*a^2*e^5
- (c^2*d^3*e^2 - a*c*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(e*x + d))/(c^2*d^7*e^2 - 2*a*c*d^5*e^4 + a^2*d^3*e^6 + (c^2*d^4*e^5 -
2*a*c*d^2*e^7 + a^2*e^9)*x^3 + 3*(c^2*d^5*e^4 - 2*a*c*d^3*e^6 + a^2*d*e^8)*
x^2 + 3*(c^2*d^6*e^3 - 2*a*c*d^4*e^5 + a^2*d^2*e^7)*x), -1/4*((c^2*d^2*e^3*
x^3 + 3*c^2*d^3*e^2*x^2 + 3*c^2*d^4*e*x + c^2*d^5)*sqrt(c*d^2*e - a*e^3)*ar
ctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d^2*e - a*e^3)*sqrt
(e*x + d)/(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x)) + (c^2*d^4*e - 3*a
*c*d^2*e^3 + 2*a^2*e^5 - (c^2*d^3*e^2 - a*c*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^7*e^2 - 2*a*c*d^5*e^4 + a^2*d^
3*e^6 + (c^2*d^4*e^5 - 2*a*c*d^2*e^7 + a^2*e^9)*x^3 + 3*(c^2*d^5*e^4 - 2*a*
c*d^3*e^6 + a^2*d*e^8)*x^2 + 3*(c^2*d^6*e^3 - 2*a*c*d^4*e^5 + a^2*d^2*e^7)*
x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(7/2),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.2035 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(d+ex)^{9/2}}\,dx$

Optimal. Leaf size=264

$$\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8e(d+ex)^{3/2}\left(cd^2-ae^2\right)^2}+\frac{c^3d^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{8e^{3/2}\left(cd^2-ae^2\right)^{5/2}}+\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{12e(d+ex)^{5/2}\left(cd^2-ae^2\right)}-\frac{\sqrt{x\left(ae^2+\right.}}{3e}$$

```
[Out] -Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(3*e*(d + e*x)^(7/2)) + (c*d*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*e*(c*d^2 - a*e^2)*(d + e*x)
^(5/2)) + (c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*e*(c*d^2
 - a*e^2)^2*(d + e*x)^(3/2)) + (c^3*d^3*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(8*e^(3/2)*
(c*d^2 - a*e^2)^(5/2))
```

Rubi [A] time = 0.176774, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {662, 672, 660, 205}

$$\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8e(d+ex)^{3/2}\left(cd^2-ae^2\right)^2}+\frac{c^3d^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{8e^{3/2}\left(cd^2-ae^2\right)^{5/2}}+\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{12e(d+ex)^{5/2}\left(cd^2-ae^2\right)}-\frac{\sqrt{x\left(ae^2+\right.}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^(9/2),x]
```



```
[Out] -Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(3*e*(d + e*x)^(7/2)) + (c*d*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*e*(c*d^2 - a*e^2)*(d + e*x)
^(5/2)) + (c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*e*(c*d^2
 - a*e^2)^2*(d + e*x)^(3/2)) + (c^3*d^3*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(8*e^(3/2)*
(c*d^2 - a*e^2)^(5/2))
```


Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*(m + 2*p + 2))/((m + p + 1)*(2*c*d - b*e)), Int[(d
 + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && Ne
Q[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(d+ex)^{9/2}}\,dx = -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3e(d+ex)^{7/2}}+\frac{(cd)\int \frac{1}{(d+ex)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{6e}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3e(d+ex)^{7/2}}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12e\left(cd^2-ae^2\right)(d+ex)^{5/2}}+\frac{\left(c^2d^2\right)\int \cdots}{\cdots}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3e(d+ex)^{7/2}}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12e\left(cd^2-ae^2\right)(d+ex)^{5/2}}+\frac{c^2d^2\sqrt{ad\cdots}}{8e\left(c\cdots\right.}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3e(d+ex)^{7/2}}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12e\left(cd^2-ae^2\right)(d+ex)^{5/2}}+\frac{c^2d^2\sqrt{ad\cdots}}{8e\left(c\cdots\right.}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3e(d+ex)^{7/2}}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12e\left(cd^2-ae^2\right)(d+ex)^{5/2}}+\frac{c^2d^2\sqrt{ad\cdots}}{8e\left(c\cdots\right.}$$

Mathematica [C] time = 0.0371872, size = 83, normalized size = 0.31

$$\frac{2c^3d^3((d+ex)(ae+cdx))^{3/2}\,{}_2F_1\left(\frac{3}{2},4;\frac{5}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^(9/2),x]
```

```
[Out] (2*c^3*d^3*((a*e + c*d*x)*(d + e*x))^(3/2)*Hypergeometric2F1[3/2, 4, 5/2, (
e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(3*(c*d^2 - a*e^2)^4*(d + e*x)^(3/2))
```

Maple [A] time = 0.251, size = 457, normalized size = 1.7

$$-\frac{1}{24\,e\left(ae^2-cd^2\right)^2}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(3\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)x^3c^3d^3e^3+9\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)x^2\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(9/2),x)
```

```
[Out] -1/24*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh(e*(c*d*x+a*e)^(1/2
)/((a*e^2-c*d^2)*e)^(1/2))*x^3*c^3*d^3*e^3+9*arctanh(e*(c*d*x+a*e)^(1/2)/((
a*e^2-c*d^2)*e)^(1/2))*x^2*c^3*d^4*e^2+9*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^
2-c*d^2)*e)^(1/2))*x*c^3*d^5*e+3*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)
*e)^(1/2))*c^3*d^6-3*x^2*c^2*d^2*e^2*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1
/2)+2*x*a*c*d*e^3*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)-8*x*c^2*d^3*e*(
c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+8*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a
*e)^(1/2)*a^2*e^4-14*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d^2*e^2+
3*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^4)/(e*x+d)^(7/2)/((a*e^2-
c*d^2)*e)^(1/2)/e/(a*e^2-c*d^2)^2/(c*d*x+a*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{(ex + d)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(9/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(e*x + d)^(9/2), x)
```

Fricas [B] time = 2.04197, size = 2246, normalized size = 8.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(9/2),x, algorith
m="fricas")
```

```
[Out] [-1/48*(3*(c^3*d^3*e^4*x^4 + 4*c^3*d^4*e^3*x^3 + 6*c^3*d^5*e^2*x^2 + 4*c^3*
d^6*e*x + c^3*d^7)*sqrt(-c*d^2*e + a*e^3)*log(-(c*d*e^2*x^2 + 2*a*e^3*x - c
*d^3 + 2*a*d*e^2 - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d^
2*e + a*e^3)*sqrt(e*x + d))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(3*c^3*d^6*e - 1
7*a*c^2*d^4*e^3 + 22*a^2*c*d^2*e^5 - 8*a^3*e^7 - 3*(c^3*d^4*e^3 - a*c^2*d^2
*e^5)*x^2 - 2*(4*c^3*d^5*e^2 - 5*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x)*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^10*e^2 - 3*a*c^2*d^
8*e^4 + 3*a^2*c*d^6*e^6 - a^3*d^4*e^8 + (c^3*d^6*e^6 - 3*a*c^2*d^4*e^8 + 3*
a^2*c*d^2*e^10 - a^3*e^12)*x^4 + 4*(c^3*d^7*e^5 - 3*a*c^2*d^5*e^7 + 3*a^2*c
*d^3*e^9 - a^3*d*e^11)*x^3 + 6*(c^3*d^8*e^4 - 3*a*c^2*d^6*e^6 + 3*a^2*c*d^4
*e^8 - a^3*d^2*e^10)*x^2 + 4*(c^3*d^9*e^3 - 3*a*c^2*d^7*e^5 + 3*a^2*c*d^5*e
^7 - a^3*d^3*e^9)*x), -1/24*(3*(c^3*d^3*e^4*x^4 + 4*c^3*d^4*e^3*x^3 + 6*c^3
*d^5*e^2*x^2 + 4*c^3*d^6*e*x + c^3*d^7)*sqrt(c*d^2*e - a*e^3)*arctan(sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d^2*e - a*e^3)*sqrt(e*x + d)/(
c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x)) + (3*c^3*d^6*e - 17*a*c^2*d^4
*e^3 + 22*a^2*c*d^2*e^5 - 8*a^3*e^7 - 3*(c^3*d^4*e^3 - a*c^2*d^2*e^5)*x^2 -
 2*(4*c^3*d^5*e^2 - 5*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x)*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^10*e^2 - 3*a*c^2*d^8*e^4 + 3*a
^2*c*d^6*e^6 - a^3*d^4*e^8 + (c^3*d^6*e^6 - 3*a*c^2*d^4*e^8 + 3*a^2*c*d^2*e
^10 - a^3*e^12)*x^4 + 4*(c^3*d^7*e^5 - 3*a*c^2*d^5*e^7 + 3*a^2*c*d^3*e^9 -
a^3*d*e^11)*x^3 + 6*(c^3*d^8*e^4 - 3*a*c^2*d^6*e^6 + 3*a^2*c*d^4*e^8 - a^3*
d^2*e^10)*x^2 + 4*(c^3*d^9*e^3 - 3*a*c^2*d^7*e^5 + 3*a^2*c*d^5*e^7 - a^3*d^
```

```
3*e^9)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(9/2),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.2036 $\int (d+ex)^{5/2} \left(ade + \left(cd^2 + ae^2\right) x + cdex^2\right)^{3/2} dx$

Optimal. Leaf size=295

$$\frac{256\left(cd^2-ae^2\right)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{15015c^5d^5(d+ex)^{5/2}}+\frac{128\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3003c^4d^4(d+ex)^{3/2}}+\frac{32\left(cd^2-ae^2\right)^2\left(x\right.}{42}$$

```
[Out] (256*(c*d^2 - a*e^2)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(1501
5*c^5*d^5*(d + e*x)^(5/2)) + (128*(c*d^2 - a*e^2)^3*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(5/2))/(3003*c^4*d^4*(d + e*x)^(3/2)) + (32*(c*d^2 - a*e^2
)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(429*c^3*d^3*Sqrt[d + e*
x]) + (16*(c*d^2 - a*e^2)*Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(5/2))/(143*c^2*d^2) + (2*(d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2)^(5/2))/(13*c*d)
```

Rubi [A] time = 0.296466, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {656, 648}

$$\frac{256\left(cd^2-ae^2\right)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{15015c^5d^5(d+ex)^{5/2}}+\frac{128\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3003c^4d^4(d+ex)^{3/2}}+\frac{32\left(cd^2-ae^2\right)^2\left(x\right.}{42}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```


```
[Out] (256*(c*d^2 - a*e^2)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(1501
5*c^5*d^5*(d + e*x)^(5/2)) + (128*(c*d^2 - a*e^2)^3*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(5/2))/(3003*c^4*d^4*(d + e*x)^(3/2)) + (32*(c*d^2 - a*e^2
)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(429*c^3*d^3*Sqrt[d + e*
x]) + (16*(c*d^2 - a*e^2)*Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(5/2))/(143*c^2*d^2) + (2*(d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2)^(5/2))/(13*c*d)
```


Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int (d+ex)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}dx = \frac{2(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{13cd} + \frac{\left(8\left(d^2-\frac{ae^2}{c}\right)\right)\int (d$$

$$= \frac{16\left(cd^2-ae^2\right)\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{143c^2d^2} + \frac{2(d+ex}{}$$

$$= \frac{32\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{429c^3d^3\sqrt{d+ex}} + \frac{16\left(cd^2-ae^2\right)\sqrt{}}{}$$

$$= \frac{128\left(cd^2-ae^2\right)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3003c^4d^4(d+ex)^{3/2}} + \frac{32\left(cd^2-ae^2\right)^2}{}$$

$$= \frac{256\left(cd^2-ae^2\right)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{15015c^5d^5(d+ex)^{5/2}} + \frac{128\left(cd^2-ae^2\right)}{}$$

Mathematica [A] time = 0.180637, size = 187, normalized size = 0.63

$$\frac{2((d+ex)(ae+cdx))^{5/2}\left(16a^2c^2d^2e^4\left(143d^2+130dex+35e^2x^2\right)-64a^3cde^6(13d+5ex)+128a^4e^8-8ac^3d^3e^2\left(715d^2ex\right.\right.}{15015c^5d^5(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(128*a^4*e^8 - 64*a^3*c*d*e^6*(13*d + 5*
e*x) + 16*a^2*c^2*d^2*e^4*(143*d^2 + 130*d*e*x + 35*e^2*x^2) - 8*a*c^3*d^3*
e^2*(429*d^3 + 715*d^2*e*x + 455*d*e^2*x^2 + 105*e^3*x^3) + c^4*d^4*(3003*d
^4 + 8580*d^3*e*x + 10010*d^2*e^2*x^2 + 5460*d*e^3*x^3 + 1155*e^4*x^4)))/(1
5015*c^5*d^5*(d + e*x)^(5/2))
```

Maple [A] time = 0.046, size = 243, normalized size = 0.8

$$\frac{(2\,cdx+2\,ae)\left(1155\,e^4x^4c^4d^4-840\,ac^3d^3e^5x^3+5460\,c^4d^5e^3x^3+560\,a^2c^2d^2e^6x^2-3640\,ac^3d^4e^4x^2+10010\,c^4d^6e^2x^2-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] 2/15015*(c*d*x+a*e)*(1155*c^4*d^4*e^4*x^4-840*a*c^3*d^3*e^5*x^3+5460*c^4*d^
5*e^3*x^3+560*a^2*c^2*d^2*e^6*x^2-3640*a*c^3*d^4*e^4*x^2+10010*c^4*d^6*e^2*
x^2-320*a^3*c*d*e^7*x+2080*a^2*c^2*d^3*e^5*x-5720*a*c^3*d^5*e^3*x+8580*c^4*
d^7*e*x+128*a^4*e^8-832*a^3*c*d^2*e^6+2288*a^2*c^2*d^4*e^4-3432*a*c^3*d^6*e
^2+3003*c^4*d^8)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)/c^5/d^5/(e*x+d)^(3
/2)
```

Maxima [A] time = 1.1146, size = 504, normalized size = 1.71

$$\frac{2\left(1155\,c^6d^6e^4x^6+3003\,a^2c^4d^8e^2-3432\,a^3c^3d^6e^4+2288\,a^4c^2d^4e^6-832\,a^5cd^2e^8+128\,a^6e^{10}+210\left(26\,c^6d^7e^3+7\,ac^5\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="maxima")
```

```
[Out] 2/15015*(1155*c^6*d^6*e^4*x^6 + 3003*a^2*c^4*d^8*e^2 - 3432*a^3*c^3*d^6*e^4
 + 2288*a^4*c^2*d^4*e^6 - 832*a^5*c*d^2*e^8 + 128*a^6*e^10 + 210*(26*c^6*d^
7*e^3 + 7*a*c^5*d^5*e^5)*x^5 + 35*(286*c^6*d^8*e^2 + 208*a*c^5*d^6*e^4 + a^
2*c^4*d^4*e^6)*x^4 + 20*(429*c^6*d^9*e + 715*a*c^5*d^7*e^3 + 13*a^2*c^4*d^5
*e^5 - 2*a^3*c^3*d^3*e^7)*x^3 + 3*(1001*c^6*d^10 + 4576*a*c^5*d^8*e^2 + 286
*a^2*c^4*d^6*e^4 - 104*a^3*c^3*d^4*e^6 + 16*a^4*c^2*d^2*e^8)*x^2 + 2*(3003*
a*c^5*d^9*e + 858*a^2*c^4*d^7*e^3 - 572*a^3*c^3*d^5*e^5 + 208*a^4*c^2*d^3*e
^7 - 32*a^5*c*d*e^9)*x)*sqrt(c*d*x + a*e)*(e*x + d)/(c^5*d^5*e*x + c^5*d^6)
```

Fricas [A] time = 1.89956, size = 853, normalized size = 2.89

$$2\left(1155\,c^{6}d^{6}e^{4}x^{6}+3003\,a^{2}c^{4}d^{8}e^{2}-3432\,a^{3}c^{3}d^{6}e^{4}+2288\,a^{4}c^{2}d^{4}e^{6}-832\,a^{5}cd^{2}e^{8}+128\,a^{6}e^{10}+210\left(26\,c^{6}d^{7}e^{3}+7\,ac^{5}d^{5}e^{5}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="fricas")
```

```
[Out] 2/15015*(1155*c^6*d^6*e^4*x^6 + 3003*a^2*c^4*d^8*e^2 - 3432*a^3*c^3*d^6*e^4
 + 2288*a^4*c^2*d^4*e^6 - 832*a^5*c*d^2*e^8 + 128*a^6*e^10 + 210*(26*c^6*d^
7*e^3 + 7*a*c^5*d^5*e^5)*x^5 + 35*(286*c^6*d^8*e^2 + 208*a*c^5*d^6*e^4 + a^
2*c^4*d^4*e^6)*x^4 + 20*(429*c^6*d^9*e + 715*a*c^5*d^7*e^3 + 13*a^2*c^4*d^5
*e^5 - 2*a^3*c^3*d^3*e^7)*x^3 + 3*(1001*c^6*d^10 + 4576*a*c^5*d^8*e^2 + 286
*a^2*c^4*d^6*e^4 - 104*a^3*c^3*d^4*e^6 + 16*a^4*c^2*d^2*e^8)*x^2 + 2*(3003*
a*c^5*d^9*e + 858*a^2*c^4*d^7*e^3 - 572*a^3*c^3*d^5*e^5 + 208*a^4*c^2*d^3*e
^7 - 32*a^5*c*d*e^9)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*
x + d)/(c^5*d^5*e*x + c^5*d^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.2037 $\int (d+ex)^{3/2} \left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2} dx$

Optimal. Leaf size=233

$$\frac{32\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{1155c^4d^4(d+ex)^{5/2}}+\frac{16\left(cd^2-ae^2\right)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{231c^3d^3(d+ex)^{3/2}}+\frac{4\left(cd^2-ae^2\right)\left(x\left(ae^2\right.\right.}{33c^2}$$

```
[Out] (32*(c*d^2 - a*e^2)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(1155*
c^4*d^4*(d + e*x)^(5/2)) + (16*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2)^(5/2))/(231*c^3*d^3*(d + e*x)^(3/2)) + (4*(c*d^2 - a*e^2)*(a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(33*c^2*d^2*Sqrt[d + e*x]) + (2
*Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(11*c*d)
```

Rubi [A] time = 0.207986, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {656, 648}

$$\frac{32\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{1155c^4d^4(d+ex)^{5/2}}+\frac{16\left(cd^2-ae^2\right)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{231c^3d^3(d+ex)^{3/2}}+\frac{4\left(cd^2-ae^2\right)\left(x\left(ae^2\right.\right.}{33c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (32*(c*d^2 - a*e^2)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(1155*
c^4*d^4*(d + e*x)^(5/2)) + (16*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2)^(5/2))/(231*c^3*d^3*(d + e*x)^(3/2)) + (4*(c*d^2 - a*e^2)*(a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(33*c^2*d^2*Sqrt[d + e*x]) + (2
*Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(11*c*d)
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int (d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx = \frac{2\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11cd} + \frac{\left(6\left(d^2-\frac{ae^2}{c}\right)\right)\int\sqrt{d+\cdots}}{\cdots}$$

$$= \frac{4\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{33c^2d^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(ade+\left(\cdots\right.\right.}{\cdots}$$

$$= \frac{16\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{231c^3d^3(d+ex)^{3/2}} + \frac{4\left(cd^2-ae^2\right)\left(a\cdots\right.}{3\cdots}$$

$$= \frac{32\left(cd^2-ae^2\right)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{1155c^4d^4(d+ex)^{5/2}} + \frac{16\left(cd^2-ae^2\right)^2\left(\cdots\right.}{2\cdots}$$

Mathematica [A] time = 0.125224, size = 132, normalized size = 0.57

$$\frac{2((d+ex)(ae+cdx))^{5/2}\left(8a^2cde^4(11d+5ex)-16a^3e^6-2ac^2d^2e^2\left(99d^2+110dex+35e^2x^2\right)+c^3d^3\left(495d^2ex+231d^3+\cdots\right.\right.}{1155c^4d^4(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-16*a^3*e^6 + 8*a^2*c*d*e^4*(11*d + 5*e
*x) - 2*a*c^2*d^2*e^2*(99*d^2 + 110*d*e*x + 35*e^2*x^2) + c^3*d^3*(231*d^3
+ 495*d^2*e*x + 385*d*e^2*x^2 + 105*e^3*x^3)))/(1155*c^4*d^4*(d + e*x)^(5/2
))
```

Maple [A] time = 0.046, size = 168, normalized size = 0.7

$$-\frac{(2\,cdx+2\,ae)\left(-105\,e^3x^3c^3d^3+70\,ac^2d^2e^4x^2-385\,c^3d^4e^2x^2-40\,a^2cde^5x+220\,ac^2d^3e^3x-495\,c^3d^5ex+16\,a^3e^6-88\cdots\right.}{1155\,c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] -2/1155*(c*d*x+a*e)*(-105*c^3*d^3*e^3*x^3+70*a*c^2*d^2*e^4*x^2-385*c^3*d^4*
e^2*x^2-40*a^2*c*d*e^5*x+220*a*c^2*d^3*e^3*x-495*c^3*d^5*e*x+16*a^3*e^6-88*
a^2*c*d^2*e^4+198*a*c^2*d^4*e^2-231*c^3*d^6)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d
*e)^(3/2)/c^4/d^4/(e*x+d)^(3/2)
```

Maxima [A] time = 1.08584, size = 369, normalized size = 1.58

$$\frac{2\left(105\,c^5d^5e^3x^5+231\,a^2c^3d^6e^2-198\,a^3c^2d^4e^4+88\,a^4cd^2e^6-16\,a^5e^8+35\left(11\,c^5d^6e^2+4\,ac^4d^4e^4\right)x^4+5\left(99\,c^5d^7e+11\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="maxima")
```

```
[Out] 2/1155*(105*c^5*d^5*e^3*x^5 + 231*a^2*c^3*d^6*e^2 - 198*a^3*c^2*d^4*e^4 + 8
8*a^4*c*d^2*e^6 - 16*a^5*e^8 + 35*(11*c^5*d^6*e^2 + 4*a*c^4*d^4*e^4)*x^4 +
5*(99*c^5*d^7*e + 110*a*c^4*d^5*e^3 + a^2*c^3*d^3*e^5)*x^3 + 3*(77*c^5*d^8
+ 264*a*c^4*d^6*e^2 + 11*a^2*c^3*d^4*e^4 - 2*a^3*c^2*d^2*e^6)*x^2 + (462*a*
c^4*d^7*e + 99*a^2*c^3*d^5*e^3 - 44*a^3*c^2*d^3*e^5 + 8*a^4*c*d*e^7)*x)*sqr
t(c*d*x + a*e)*(e*x + d)/(c^4*d^4*e*x + c^4*d^5)
```

Fricas [A] time = 1.9506, size = 614, normalized size = 2.64

$$\frac{2\left(105\,c^5d^5e^3x^5+231\,a^2c^3d^6e^2-198\,a^3c^2d^4e^4+88\,a^4cd^2e^6-16\,a^5e^8+35\left(11\,c^5d^6e^2+4\,ac^4d^4e^4\right)x^4+5\left(99\,c^5d^7e+110\,a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="fricas")
```

```
[Out] 2/1155*(105*c^5*d^5*e^3*x^5 + 231*a^2*c^3*d^6*e^2 - 198*a^3*c^2*d^4*e^4 + 8
8*a^4*c*d^2*e^6 - 16*a^5*e^8 + 35*(11*c^5*d^6*e^2 + 4*a*c^4*d^4*e^4)*x^4 +
5*(99*c^5*d^7*e + 110*a*c^4*d^5*e^3 + a^2*c^3*d^3*e^5)*x^3 + 3*(77*c^5*d^8
+ 264*a*c^4*d^6*e^2 + 11*a^2*c^3*d^4*e^4 - 2*a^3*c^2*d^2*e^6)*x^2 + (462*a*
c^4*d^7*e + 99*a^2*c^3*d^5*e^3 - 44*a^3*c^2*d^3*e^5 + 8*a^4*c*d*e^7)*x)*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^4*d^4*e*x + c^4*d
^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.2038 $\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx$

Optimal. Leaf size=171

$$\frac{8\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{63c^2d^2(d+ex)^{3/2}}+\frac{16\left(cd^2-ae^2\right)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{315c^3d^3(d+ex)^{5/2}}+\frac{2\left(x\left(ae^2+cd^2\right)+\cdots\right.}{9cd\sqrt{d+\cdots}}$$

```
[Out] (16*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(315*c
^3*d^3*(d + e*x)^(5/2)) + (8*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^(5/2))/(63*c^2*d^2*(d + e*x)^(3/2)) + (2*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(5/2))/(9*c*d*Sqrt[d + e*x])
```

Rubi [A] time = 0.121992, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {656, 648}

$$\frac{8\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{63c^2d^2(d+ex)^{3/2}}+\frac{16\left(cd^2-ae^2\right)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{315c^3d^3(d+ex)^{5/2}}+\frac{2\left(x\left(ae^2+cd^2\right)+\cdots\right.}{9cd\sqrt{d+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (16*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(315*c
^3*d^3*(d + e*x)^(5/2)) + (8*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^(5/2))/(63*c^2*d^2*(d + e*x)^(3/2)) + (2*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(5/2))/(9*c*d*Sqrt[d + e*x])
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{9cd\sqrt{d+ex}} + \frac{\left(4\left(d^2-\frac{ae^2}{c}\right)\right)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}{\sqrt{d+ex}}}{9d}$$

$$= \frac{8\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{63c^2d^2(d+ex)^{3/2}} + \frac{2\left(ade+\left(cd^2+ae^2\right)x\right.}{9cd\sqrt{d+ex}}$$

$$= \frac{16\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{315c^3d^3(d+ex)^{5/2}} + \frac{8\left(cd^2-ae^2\right)\left(ade+\right.}{63c^2d^2}$$

Mathematica [A] time = 0.0921158, size = 88, normalized size = 0.51

$$\frac{2((d+ex)(ae+cdx))^{5/2}\left(8a^2e^4-4acde^2(9d+5ex)+c^2d^2\left(63d^2+90dex+35e^2x^2\right)\right)}{315c^3d^3(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(8*a^2*e^4 - 4*a*c*d*e^2*(9*d + 5*e*x) +
 c^2*d^2*(63*d^2 + 90*d*e*x + 35*e^2*x^2)))/(315*c^3*d^3*(d + e*x)^(5/2))
```

Maple [A] time = 0.046, size = 110, normalized size = 0.6

$$\frac{(2\,cdx+2\,ae)\left(35\,e^2x^2c^2d^2-20\,acde^3x+90\,c^2d^3ex+8\,a^2e^4-36\,acd^2e^2+63\,c^2d^4\right)}{315\,c^3d^3}\left(cdex^2+ae^2x+cd^2x+ade\right)^{\frac{3}{2}}(ex+d)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] 2/315*(c*d*x+a*e)*(35*c^2*d^2*e^2*x^2-20*a*c*d*e^3*x+90*c^2*d^3*e*x+8*a^2*e
^4-36*a*c*d^2*e^2+63*c^2*d^4)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)/c^3/d
^3/(e*x+d)^(3/2)
```

Maxima [A] time = 1.09035, size = 255, normalized size = 1.49

$$\frac{2\left(35\,c^4d^4e^2x^4+63\,a^2c^2d^4e^2-36\,a^3cd^2e^4+8\,a^4e^6+10\left(9\,c^4d^5e+5\,ac^3d^3e^3\right)x^3+3\left(21\,c^4d^6+48\,ac^3d^4e^2+a^2c^2d^2e^4\right)x^2+\right.}{315\left(c^3d^3ex+c^3d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="maxima")
```

```
[Out] 2/315*(35*c^4*d^4*e^2*x^4 + 63*a^2*c^2*d^4*e^2 - 36*a^3*c*d^2*e^4 + 8*a^4*e
^6 + 10*(9*c^4*d^5*e + 5*a*c^3*d^3*e^3)*x^3 + 3*(21*c^4*d^6 + 48*a*c^3*d^4*
e^2 + a^2*c^2*d^2*e^4)*x^2 + 2*(63*a*c^3*d^5*e + 9*a^2*c^2*d^3*e^3 - 2*a^3*
c*d*e^5)*x)*sqrt(c*d*x + a*e)*(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

Fricas [A] time = 1.88297, size = 432, normalized size = 2.53

$$\frac{2\left(35\,c^4d^4e^2x^4+63\,a^2c^2d^4e^2-36\,a^3cd^2e^4+8\,a^4e^6+10\left(9\,c^4d^5e+5\,ac^3d^3e^3\right)x^3+3\left(21\,c^4d^6+48\,ac^3d^4e^2+a^2c^2d^2e^4\right)x}{315\left(c^3d^3ex+c^3d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="fricas")
```

```
[Out] 2/315*(35*c^4*d^4*e^2*x^4 + 63*a^2*c^2*d^4*e^2 - 36*a^3*c*d^2*e^4 + 8*a^4*e
^6 + 10*(9*c^4*d^5*e + 5*a*c^3*d^3*e^3)*x^3 + 3*(21*c^4*d^6 + 48*a*c^3*d^4*
e^2 + a^2*c^2*d^2*e^4)*x^2 + 2*(63*a*c^3*d^5*e + 9*a^2*c^2*d^3*e^3 - 2*a^3*
c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^3*
d^3*e*x + c^3*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.2039 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=109

$$\frac{4\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{35c^2d^2(d+ex)^{5/2}}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7cd(d+ex)^{3/2}}$$

```
[Out] (4*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(35*c^2*d
^2*(d + e*x)^(5/2)) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(7*
c*d*(d + e*x)^(3/2))
```

Rubi [A] time = 0.0610188, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {656, 648}

$$\frac{4\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{35c^2d^2(d+ex)^{5/2}}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7cd(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/Sqrt[d + e*x],x]
```

```
[Out] (4*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(35*c^2*d
^2*(d + e*x)^(5/2)) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(7*
c*d*(d + e*x)^(3/2))
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{\sqrt{d+ex}}\,dx &= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7cd(d+ex)^{3/2}}+\frac{\left(2\left(d^2-\frac{ae^2}{c}\right)\right)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx}{7d}\\ &= \frac{4\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{35c^2d^2(d+ex)^{5/2}}+\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7cd(d+ex)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0630417, size = 55, normalized size = 0.5

$$\frac{2((d+ex)(ae+cdx))^{5/2}\left(cd(7d+5ex)-2ae^2\right)}{35c^2d^2(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/Sqrt[d + e*x],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-2*a*e^2 + c*d*(7*d + 5*e*x)))/(35*c^2*
d^2*(d + e*x)^(5/2))
```

Maple [A] time = 0.043, size = 69, normalized size = 0.6

$$-\frac{(2\,cdx+2\,ae)\left(-5\,cdex+2\,ae^2-7\,cd^2\right)}{35\,c^2d^2}\left(cdex^2+ae^2x+cd^2x+ade\right)^{\frac{3}{2}}(ex+d)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(1/2),x)
```

```
[Out] -2/35*(c*d*x+a*e)*(-5*c*d*e*x+2*a*e^2-7*c*d^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a
*d*e)^(3/2)/c^2/d^2/(e*x+d)^(3/2)
```

Maxima [A] time = 1.06772, size = 132, normalized size = 1.21

$$\frac{2\left(5\,c^3d^3ex^3+7\,a^2cd^2e^2-2\,a^3e^4+\left(7\,c^3d^4+8\,ac^2d^2e^2\right)x^2+\left(14\,ac^2d^3e+a^2cde^3\right)x\right)\sqrt{cdx+ae}}{35\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(1/2),x, algorith
m="maxima")
```

```
[Out] 2/35*(5*c^3*d^3*e*x^3 + 7*a^2*c*d^2*e^2 - 2*a^3*e^4 + (7*c^3*d^4 + 8*a*c^2*
d^2*e^2)*x^2 + (14*a*c^2*d^3*e + a^2*c*d*e^3)*x)*sqrt(c*d*x + a*e)/(c^2*d^2
)
```

Fricas [A] time = 1.82388, size = 279, normalized size = 2.56

$$\frac{2\left(5\,c^3d^3ex^3+7\,a^2cd^2e^2-2\,a^3e^4+\left(7\,c^3d^4+8\,ac^2d^2e^2\right)x^2+\left(14\,ac^2d^3e+a^2cde^3\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+}}{35\left(c^2d^2ex+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(1/2),x, algorith
m="fricas")
```

```
[Out] 2/35*(5*c^3*d^3*e*x^3 + 7*a^2*c*d^2*e^2 - 2*a^3*e^4 + (7*c^3*d^4 + 8*a*c^2*
d^2*e^2)*x^2 + (14*a*c^2*d^3*e + a^2*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (
```

```
c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^2*d^2*e*x + c^2*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(1/2),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.2040 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=48

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5cd(d+ex)^{5/2}}$$

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*c*d*(d + e*x)^(5/2))
```

Rubi [A] time = 0.0245674, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {648}

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5cd(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(3/2),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*c*d*(d + e*x)^(5/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5cd(d+ex)^{5/2}}$$

Mathematica [A] time = 0.033448, size = 37, normalized size = 0.77

$$\frac{2((d+ex)(ae+cdx))^{5/2}}{5cd(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(3/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2))/(5*c*d*(d + e*x)^(5/2))
```

Maple [A] time = 0.042, size = 50, normalized size = 1.

$$\frac{2\,cdx+2\,ae}{5\,cd}\left(cdex^2+ae^2x+cd^2x+ade\right)^{\frac{3}{2}}(ex+d)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x)
```

```
[Out] 2/5*(c*d*x+a*e)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)/d/c/(e*x+d)^(3/2)
```

Maxima [A] time = 1.03958, size = 58, normalized size = 1.21

$$\frac{2\left(c^2d^2x^2 + 2\,acdex + a^2e^2\right)\sqrt{cdx + ae}}{5\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorith
m="maxima")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)/(c*d)
```

Fricas [A] time = 1.84927, size = 161, normalized size = 3.35

$$\frac{2\left(c^2d^2x^2 + 2\,acdex + a^2e^2\right)\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\sqrt{ex + d}}{5\left(cdex + cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorith
m="fricas")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(e*x + d)/(c*d*e*x + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((d + ex)\,(ae + cdx)\right)^{\frac{3}{2}}}{(d + ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(d + e*x)**(3/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.2041 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=181

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^{3/2}}+\frac{2\left(a-\frac{cd^2}{e^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}}+\frac{2\left(cd^2-ae^2\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right.}{e^{5/2}}$$

```
[Out] (2*(a - (c*d^2)/e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d +
e*x] + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*e*(d + e*x)^(3/
2)) + (2*(c*d^2 - a*e^2)^(3/2)*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/e^(5/2)
```

Rubi [A] time = 0.16295, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {664, 660, 205}

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^{3/2}}+\frac{2\left(a-\frac{cd^2}{e^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}}+\frac{2\left(cd^2-ae^2\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right.}{e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(5/2),x]
```

```
[Out] (2*(a - (c*d^2)/e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d +
e*x] + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*e*(d + e*x)^(3/
2)) + (2*(c*d^2 - a*e^2)^(3/2)*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/e^(5/2)
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{5/2}}dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e(d+ex)^{3/2}}-\frac{\left(2cd^2e-e\left(cd^2+ae^2\right)\right)\int\frac{\sqrt{ade+\left(cd^2+ae^2\right)x\cdot}}{(d+ex)^{3/2}}}{e^2}$$

$$=\frac{2\left(a-\frac{cd^2}{e^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}+\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/}}{3e(d+ex)^{3/2}}$$

$$=\frac{2\left(a-\frac{cd^2}{e^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}+\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/}}{3e(d+ex)^{3/2}}$$

$$=\frac{2\left(a-\frac{cd^2}{e^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}+\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/}}{3e(d+ex)^{3/2}}$$

Mathematica [A] time = 0.240027, size = 137, normalized size = 0.76

$$\frac{2\sqrt{d+ex}\sqrt{ae+cdx}\left(3\left(cd^2-ae^2\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd^2-ae^2}}\right)+\sqrt{e}\sqrt{ae+cdx}\left(4ae^2+cd(ex-3d)\right)\right)}{3e^{5/2}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(5/2),x]
```

```
[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[e]*Sqrt[a*e + c*d*x]*(4*a*e^2 + c*
d*(-3*d + e*x)) + 3*(c*d^2 - a*e^2)^(3/2)*ArcTan[(Sqrt[e]*Sqrt[a*e + c*d*x]
)/Sqrt[c*d^2 - a*e^2]]))/(3*e^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] time = 0.245, size = 275, normalized size = 1.5

$$-\frac{2}{3e^2}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(3\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)a^2e^4-6\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)acd^2e^2+3\,\mathrm{Artanh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(5/2),x)
```

```
[Out] -2/3*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh(e*(c*d*x+a*e)^(1/2)
/((a*e^2-c*d^2)*e)^(1/2))*a^2*e^4-6*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d
^2)*e)^(1/2))*a*c*d^2*e^2+3*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(
1/2))*c^2*d^4-x*c*d*e*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)-4*((a*e^2-c
*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*a*e^2+3*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e
)^(1/2)*c*d^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/e^2/((a*e^2-c*d^2)*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(5/2),x, algorith
m="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/(e*x + d)^(5/2), x)
```

Fricas [A] time = 1.82005, size = 841, normalized size = 4.65

$$\left[\frac{3\left(cd^3 - ade^2 + \left(cd^2e - ae^3\right)x\right)\sqrt{-\frac{cd^2-ae^2}{e}}\log\left(-\frac{cde^2x^2+2\,ae^3x-cd^3+2\,ade^2+2\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}e\sqrt{-\frac{cd^2-ae^2}{e}}}{e^2x^2+2\,dex+d^2}\right) + 2\,\sqrt{cdex^2 + \cdots}}{3\left(e^3x + de^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(5/2),x, algorith
m="fricas")
```

```
[Out] [1/3*(3*(c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)*sqrt(-(c*d^2 - a*e^2)/e)*lo
g(-(c*d*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2 + 2*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*e*sqrt(-(c*d^2 - a*e^2)/e))/(e^2*x^2 + 2
*d*e*x + d^2)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*e*x - 3
*c*d^2 + 4*a*e^2)*sqrt(e*x + d))/(e^3*x + d*e^2), -2/3*(3*(c*d^3 - a*d*e^2
+ (c*d^2*e - a*e^3)*x)*sqrt((c*d^2 - a*e^2)/e)*arctan(sqrt(e*x + d)*sqrt((c
*d^2 - a*e^2)/e)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)) - sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*e*x - 3*c*d^2 + 4*a*e^2)*sqrt(e*x + d
))/(e^3*x + d*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(5/2),x, algorith
m="giac")
```

```
[Out] Exception raised: AttributeError
```

3.2042 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=175

$$-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{e(d+ex)^{5/2}}+\frac{3cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^2\sqrt{d+ex}}-\frac{3cd\sqrt{cd^2-ae^2}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{e^{5/2}}$$

```
[Out] (3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e^2*Sqrt[d + e*x]) - (
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(e*(d + e*x)^(5/2)) - (3*c*d*S
qrt[c*d^2 - a*e^2]*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/e^(5/2)
```

Rubi [A] time = 0.11926, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {662, 664, 660, 205}

$$-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{e(d+ex)^{5/2}}+\frac{3cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^2\sqrt{d+ex}}-\frac{3cd\sqrt{cd^2-ae^2}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(7/2),x]
```

```
[Out] (3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e^2*Sqrt[d + e*x]) - (
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(e*(d + e*x)^(5/2)) - (3*c*d*S
qrt[c*d^2 - a*e^2]*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/e^(5/2)
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{7/2}}dx=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{e(d+ex)^{5/2}}+\frac{(3cd)\int\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(d+ex)^{3/2}}dx}{2e}$$

$$=\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^2\sqrt{d+ex}}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{e(d+ex)^{5/2}}-\frac{(3cd\left(cd\cdots\right.}{\cdots}$$

$$=\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^2\sqrt{d+ex}}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{e(d+ex)^{5/2}}-\frac{(3cd\left(cd\cdots\right.}{\cdots}$$

$$=\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^2\sqrt{d+ex}}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{e(d+ex)^{5/2}}-\frac{3cd\sqrt{cd^2\cdots}}{\cdots}$$

Mathematica [C] time = 0.0514579, size = 79, normalized size = 0.45

$$\frac{2cd((d+ex)(ae+cdx))^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{5(d+ex)^{5/2}\left(cd^2-ae^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(7/2),x]
```

```
[Out] (2*c*d*((a*e + c*d*x)*(d + e*x))^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, (e*(a
*e + c*d*x))/(-(c*d^2) + a*e^2)])/(5*(c*d^2 - a*e^2)^2*(d + e*x)^(5/2))
```

Maple [B] time = 0.247, size = 314, normalized size = 1.8

$$\frac{1}{e^2}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(-3\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)xacde^3+3\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)xc^2d^3e-3\,\mathrm{Artanh}\left(\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(7/2),x)
```

```
[Out] (c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/(e*x+d)^(3/2)*(-3*arctanh(e*(c*d*x+
a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*x*a*c*d*e^3+3*arctanh(e*(c*d*x+a*e)^(1/
2)/((a*e^2-c*d^2)*e)^(1/2))*x*c^2*d^3*e-3*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e
^2-c*d^2)*e)^(1/2))*a*c*d^2*e^2+3*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2
)*e)^(1/2))*c^2*d^4+2*x*c*d*e*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)-((a
*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*a*e^2+3*((a*e^2-c*d^2)*e)^(1/2)*(c*d
*x+a*e)^(1/2)*c*d^2)/(c*d*x+a*e)^(1/2)/e^2/((a*e^2-c*d^2)*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(7/2),x, algorith
m="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/(e*x + d)^(7/2), x)
```

Fricas [A] time = 2.01484, size = 867, normalized size = 4.95

$$\left[\frac{3\left(cde^2x^2 + 2\,cd^2ex + cd^3\right)\sqrt{-\frac{cd^2-ae^2}{e}}\log\left(-\frac{cde^2x^2+2\,ae^3x-cd^3+2\,ade^2-2\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}e\sqrt{-\frac{cd^2-ae^2}{e}}}{e^2x^2+2\,dex+d^2}\right) + 2\,\sqrt{cdex^2 + a}}{2\left(e^4x^2 + 2\,de^3x + d^2e^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(7/2),x, algorith
m="fricas")
```

```
[Out] [1/2*(3*(c*d*e^2*x^2 + 2*c*d^2*e*x + c*d^3)*sqrt(-(c*d^2 - a*e^2)/e)*log(-(
c*d*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2 - 2*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x)*sqrt(e*x + d)*e*sqrt(-(c*d^2 - a*e^2)/e))/(e^2*x^2 + 2*d*e
*x + d^2)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + 3*c
*d^2 - a*e^2)*sqrt(e*x + d))/(e^4*x^2 + 2*d*e^3*x + d^2*e^2), (3*(c*d*e^2*x
^2 + 2*c*d^2*e*x + c*d^3)*sqrt((c*d^2 - a*e^2)/e)*arctan(sqrt(e*x + d)*sqrt
((c*d^2 - a*e^2)/e)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)) + sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + 3*c*d^2 - a*e^2)*sqrt(e*x
+ d))/(e^4*x^2 + 2*d*e^3*x + d^2*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(7/2),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.2043 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{9/2}}\,dx$

Optimal. Leaf size=185

$$\frac{3c^2d^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{4e^{5/2}\sqrt{cd^2-ae^2}}-\frac{3cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4e^2(d+ex)^{3/2}}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2e(d+ex)^{7/2}}$$

```
[Out] (-3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*e^2*(d + e*x)^(3/2)
) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(2*e*(d + e*x)^(7/2)) + (
3*c^2*d^2*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqr
t[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(4*e^(5/2)*Sqrt[c*d^2 - a*e^2])
```

Rubi [A] time = 0.117634, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {662, 660, 205}

$$\frac{3c^2d^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{4e^{5/2}\sqrt{cd^2-ae^2}}-\frac{3cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4e^2(d+ex)^{3/2}}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2e(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(9/2),x]
```

```
[Out] (-3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*e^2*(d + e*x)^(3/2)
) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(2*e*(d + e*x)^(7/2)) + (
3*c^2*d^2*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqr
t[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(4*e^(5/2)*Sqrt[c*d^2 - a*e^2])
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{9/2}}dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2e(d+ex)^{7/2}} + \frac{(3cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(d+ex)^{5/2}}dx}{4e}$$

$$= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e^2(d+ex)^{3/2}} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2e(d+ex)^{7/2}} + \frac{(3c^2d^2}{}$$

$$= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e^2(d+ex)^{3/2}} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2e(d+ex)^{7/2}} + \frac{(3c^2d^2}{}$$

$$= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e^2(d+ex)^{3/2}} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2e(d+ex)^{7/2}} + \frac{3c^2d^2\,\mathrm{t}}{}$$

Mathematica [A] time = 0.178477, size = 176, normalized size = 0.95

$$\frac{3c^2d^2(d+ex)^2\sqrt{ae+cdx}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd^2-ae^2}}\right)-\sqrt{e}\sqrt{cd^2-ae^2}\left(2a^2e^3+acde(3d+7ex)+c^2d^2x(3d+5ex)\right)}{4e^{5/2}(d+ex)^{3/2}\sqrt{cd^2-ae^2}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(9/2),x]
```

```
[Out] (-(Sqrt[e]*Sqrt[c*d^2 - a*e^2]*(2*a^2*e^3 + c^2*d^2*x*(3*d + 5*e*x) + a*c*d
*e*(3*d + 7*e*x))) + 3*c^2*d^2*Sqrt[a*e + c*d*x]*(d + e*x)^2*ArcTan[(Sqrt[e
]*Sqrt[a*e + c*d*x])/Sqrt[c*d^2 - a*e^2]])/(4*e^(5/2)*Sqrt[c*d^2 - a*e^2]*(
d + e*x)^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] time = 0.247, size = 281, normalized size = 1.5

$$-\frac{1}{4e^2}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(3\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)x^2c^2d^2e^2+6\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)xc^2d^3e+3\,\mathrm{Artanh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(9/2),x)
```

```
[Out] -1/4*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh(e*(c*d*x+a*e)^(1/2)
/((a*e^2-c*d^2)*e)^(1/2))*x^2*c^2*d^2*e^2+6*arctanh(e*(c*d*x+a*e)^(1/2)/((a
*e^2-c*d^2)*e)^(1/2))*x*c^2*d^3*e+3*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d
^2)*e)^(1/2))*c^2*d^4+5*x*c*d*e*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+2
*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*a*e^2+3*((a*e^2-c*d^2)*e)^(1/2)*
(c*d*x+a*e)^(1/2)*c*d^2)/(e*x+d)^(5/2)/(c*d*x+a*e)^(1/2)/e^2/((a*e^2-c*d^2)
*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{9}{2}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(9/2),x, algorith
m="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/(e*x + d)^(9/2), x)
```

Fricas [A] time = 2.0764, size = 1323, normalized size = 7.15

$$\left[-\frac{3\left(c^2d^2e^3x^3+3c^2d^3e^2x^2+3c^2d^4ex+c^2d^5\right)\sqrt{-cd^2e+ae^3}\log\left(-\frac{cde^2x^2+2ae^3x-cd^3+2ade^2-2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{-cd^2e+ae^3}\cdots}{e^2x^2+2dex+d^2}\right.}{8\left(cd^5e^3-ad^3e^5+\left(cd^2e^6-ae^8\right)x^3+3\left(cd^3e^5-\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(9/2),x, algorith
m="fricas")
```

```
[Out] [-1/8*(3*(c^2*d^2*e^3*x^3 + 3*c^2*d^3*e^2*x^2 + 3*c^2*d^4*e*x + c^2*d^5)*sq
rt(-c*d^2*e + a*e^3)*log(-(c*d*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2 - 2*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d^2*e + a*e^3)*sqrt(e*x
 + d))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(3*c^2*d^4*e - a*c*d^2*e^3 - 2*a^2*e^
5 + 5*(c^2*d^3*e^2 - a*c*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*sqrt(e*x + d))/(c*d^5*e^3 - a*d^3*e^5 + (c*d^2*e^6 - a*e^8)*x^3 + 3*(c*
d^3*e^5 - a*d*e^7)*x^2 + 3*(c*d^4*e^4 - a*d^2*e^6)*x), -1/4*(3*(c^2*d^2*e^3
*x^3 + 3*c^2*d^3*e^2*x^2 + 3*c^2*d^4*e*x + c^2*d^5)*sqrt(c*d^2*e - a*e^3)*a
rctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d^2*e - a*e^3)*sqr
t(e*x + d)/(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x)) + (3*c^2*d^4*e -
a*c*d^2*e^3 - 2*a^2*e^5 + 5*(c^2*d^3*e^2 - a*c*d*e^4)*x)*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c*d^5*e^3 - a*d^3*e^5 + (c*d^2*e^
6 - a*e^8)*x^3 + 3*(c*d^3*e^5 - a*d*e^7)*x^2 + 3*(c*d^4*e^4 - a*d^2*e^6)*x)
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(9/2),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.2044 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{11/2}}\,dx$

Optimal. Leaf size=250

$$\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8e^2(d+ex)^{3/2}\left(cd^2-ae^2\right)}+\frac{c^3d^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{8e^{5/2}\left(cd^2-ae^2\right)^{3/2}}-\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4e^2(d+ex)^{5/2}}-\frac{\left(x\left(ae^2\right.\right.}{}$$

```
[Out] -(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*e^2*(d + e*x)^(5/2))
+ (c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*e^2*(c*d^2 - a*e
^2)*(d + e*x)^(3/2)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(3*e*(
d + e*x)^(9/2)) + (c^3*d^3*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(8*e^(5/2)*(c*d^2 - a*e^
2)^(3/2))
```

Rubi [A] time = 0.184425, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {662, 672, 660, 205}

$$\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8e^2(d+ex)^{3/2}\left(cd^2-ae^2\right)}+\frac{c^3d^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{8e^{5/2}\left(cd^2-ae^2\right)^{3/2}}-\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4e^2(d+ex)^{5/2}}-\frac{\left(x\left(ae^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(11/2),x]
```


```
[Out] -(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*e^2*(d + e*x)^(5/2))
+ (c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*e^2*(c*d^2 - a*e
^2)*(d + e*x)^(3/2)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(3*e*(
d + e*x)^(9/2)) + (c^3*d^3*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(8*e^(5/2)*(c*d^2 - a*e^
2)^(3/2))
```


Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*(m + 2*p + 2))/((m + p + 1)*(2*c*d - b*e)), Int[(d
 + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && Ne
Q[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{11/2}}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e(d+ex)^{9/2}} + \frac{(cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(d+ex)^{7/2}}\,dx}{2e}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e^2(d+ex)^{5/2}} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e(d+ex)^{9/2}} + \frac{\left(c^2d^2\right)\int}{}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e^2(d+ex)^{5/2}} + \frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8e^2\left(cd^2-ae^2\right)(d+ex)^{3/2}} - \frac{(ade+}{}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e^2(d+ex)^{5/2}} + \frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8e^2\left(cd^2-ae^2\right)(d+ex)^{3/2}} - \frac{(ade+}{}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e^2(d+ex)^{5/2}} + \frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8e^2\left(cd^2-ae^2\right)(d+ex)^{3/2}} - \frac{(ade+}{}$$

Mathematica [C] time = 0.0527971, size = 83, normalized size = 0.33

$$\frac{2c^3d^3((d+ex)(ae+cdx))^{5/2}\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{5(d+ex)^{5/2}\left(cd^2-ae^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(11/2),x]
```

```
[Out] (2*c^3*d^3*((a*e + c*d*x)*(d + e*x))^(5/2)*Hypergeometric2F1[5/2, 4, 7/2, (
e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(5*(c*d^2 - a*e^2)^4*(d + e*x)^(5/2))
```

Maple [B] time = 0.26, size = 457, normalized size = 1.8

$$\frac{1}{\left(24\,ae^2-24\,cd^2\right)e^2}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(3\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)x^3c^3d^3e^3+9\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)x^2c^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(11/2),x)
```

```
[Out] 1/24*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh(e*(c*d*x+a*e)^(1/2)
/((a*e^2-c*d^2)*e)^(1/2))*x^3*c^3*d^3*e^3+9*arctanh(e*(c*d*x+a*e)^(1/2)/((a
*e^2-c*d^2)*e)^(1/2))*x^2*c^3*d^4*e^2+9*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2
-c*d^2)*e)^(1/2))*x*c^3*d^5*e+3*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*
e)^(1/2))*c^3*d^6-3*x^2*c^2*d^2*e^2*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/
2)-14*x*a*c*d*e^3*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+8*x*c^2*d^3*e*(
c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)-8*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a
*e)^(1/2)*a^2*e^4+2*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d^2*e^2+3
*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^4)/(e*x+d)^(7/2)/(c*d*x+a*
e)^(1/2)/(a*e^2-c*d^2)/e^2/((a*e^2-c*d^2)*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(11/2),x, algorit
hm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/(e*x + d)^(11/2), x
)
```

Fricas [B] time = 2.09629, size = 1968, normalized size = 7.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(11/2),x, algorit
hm="fricas")
```

```
[Out] [-1/48*(3*(c^3*d^3*e^4*x^4 + 4*c^3*d^4*e^3*x^3 + 6*c^3*d^5*e^2*x^2 + 4*c^3*
d^6*e*x + c^3*d^7)*sqrt(-c*d^2*e + a*e^3)*log(-(c*d*e^2*x^2 + 2*a*e^3*x - c
*d^3 + 2*a*d*e^2 - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d^
2*e + a*e^3)*sqrt(e*x + d))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(3*c^3*d^6*e - a
*c^2*d^4*e^3 - 10*a^2*c*d^2*e^5 + 8*a^3*e^7 - 3*(c^3*d^4*e^3 - a*c^2*d^2*e^
5)*x^2 + 2*(4*c^3*d^5*e^2 - 11*a*c^2*d^3*e^4 + 7*a^2*c*d*e^6)*x)*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^8*e^3 - 2*a*c*d^6*e
^5 + a^2*d^4*e^7 + (c^2*d^4*e^7 - 2*a*c*d^2*e^9 + a^2*e^11)*x^4 + 4*(c^2*d^
5*e^6 - 2*a*c*d^3*e^8 + a^2*d*e^10)*x^3 + 6*(c^2*d^6*e^5 - 2*a*c*d^4*e^7 +
a^2*d^2*e^9)*x^2 + 4*(c^2*d^7*e^4 - 2*a*c*d^5*e^6 + a^2*d^3*e^8)*x), -1/24*
(3*(c^3*d^3*e^4*x^4 + 4*c^3*d^4*e^3*x^3 + 6*c^3*d^5*e^2*x^2 + 4*c^3*d^6*e*x
 + c^3*d^7)*sqrt(c*d^2*e - a*e^3)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*sqrt(c*d^2*e - a*e^3)*sqrt(e*x + d)/(c*d*e^2*x^2 + a*d*e^2 + (c*d
^2*e + a*e^3)*x)) + (3*c^3*d^6*e - a*c^2*d^4*e^3 - 10*a^2*c*d^2*e^5 + 8*a^3
*e^7 - 3*(c^3*d^4*e^3 - a*c^2*d^2*e^5)*x^2 + 2*(4*c^3*d^5*e^2 - 11*a*c^2*d^
3*e^4 + 7*a^2*c*d*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(
e*x + d))/(c^2*d^8*e^3 - 2*a*c*d^6*e^5 + a^2*d^4*e^7 + (c^2*d^4*e^7 - 2*a*c
*d^2*e^9 + a^2*e^11)*x^4 + 4*(c^2*d^5*e^6 - 2*a*c*d^3*e^8 + a^2*d*e^10)*x^3
 + 6*(c^2*d^6*e^5 - 2*a*c*d^4*e^7 + a^2*d^2*e^9)*x^2 + 4*(c^2*d^7*e^4 - 2*a
*c*d^5*e^6 + a^2*d^3*e^8)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(11/2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

3.2045 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{13/2}}\,dx$

Optimal. Leaf size=315

$$\frac{3c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64e^2(d+ex)^{3/2}\left(cd^2-ae^2\right)^2}+\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{32e^2(d+ex)^{5/2}\left(cd^2-ae^2\right)}+\frac{3c^4d^4\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{64e^{5/2}\left(cd^2-ae^2\right)^{5/2}}-\frac{cd\ldots}{}$$

```
[Out] -(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*e^2*(d + e*x)^(7/2))
+ (c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*e^2*(c*d^2 - a*
e^2)*(d + e*x)^(5/2)) + (3*c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(64*e^2*(c*d^2 - a*e^2)^2*(d + e*x)^(3/2)) - (a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^(3/2)/(4*e*(d + e*x)^(11/2)) + (3*c^4*d^4*ArcTan[(Sqrt[e]*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*
x])])/(64*e^(5/2)*(c*d^2 - a*e^2)^(5/2))
```

Rubi [A] time = 0.22298, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {662, 672, 660, 205}

$$\frac{3c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64e^2(d+ex)^{3/2}\left(cd^2-ae^2\right)^2}+\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{32e^2(d+ex)^{5/2}\left(cd^2-ae^2\right)}+\frac{3c^4d^4\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{64e^{5/2}\left(cd^2-ae^2\right)^{5/2}}-\frac{cd\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(13/2),x]
```


```
[Out] -(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*e^2*(d + e*x)^(7/2))
+ (c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*e^2*(c*d^2 - a*
e^2)*(d + e*x)^(5/2)) + (3*c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(64*e^2*(c*d^2 - a*e^2)^2*(d + e*x)^(3/2)) - (a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^(3/2)/(4*e*(d + e*x)^(11/2)) + (3*c^4*d^4*ArcTan[(Sqrt[e]*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*
x])])/(64*e^(5/2)*(c*d^2 - a*e^2)^(5/2))
```


Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*(m + 2*p + 2))/((m + p + 1)*(2*c*d - b*e)), Int[(d
 + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && Ne
Q[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{13/2}}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4e(d+ex)^{11/2}}+\frac{(3cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(d+ex)^{9/2}}\,dx}{8e}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8e^2(d+ex)^{7/2}}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4e(d+ex)^{11/2}}+\frac{\left(c^2d^2\right)\int}{}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8e^2(d+ex)^{7/2}}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32e^2\left(cd^2-ae^2\right)(d+ex)^{5/2}}-\frac{\left(ade+\right.}{}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8e^2(d+ex)^{7/2}}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32e^2\left(cd^2-ae^2\right)(d+ex)^{5/2}}+\frac{3c^3d^3}{64}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8e^2(d+ex)^{7/2}}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32e^2\left(cd^2-ae^2\right)(d+ex)^{5/2}}+\frac{3c^3d^3}{64}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8e^2(d+ex)^{7/2}}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32e^2\left(cd^2-ae^2\right)(d+ex)^{5/2}}+\frac{3c^3d^3}{64}$$

Mathematica [C] time = 0.0532264, size = 83, normalized size = 0.26

$$\frac{2c^4d^4((d+ex)(ae+cdx))^{5/2}\,{}_2F_1\left(\frac{5}{2},5;\frac{7}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{5(d+ex)^{5/2}\left(cd^2-ae^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(13/2),x]
```

```
[Out] (2*c^4*d^4*((a*e + c*d*x)*(d + e*x))^(5/2)*Hypergeometric2F1[5/2, 5, 7/2, (
e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(5*(c*d^2 - a*e^2)^5*(d + e*x)^(5/2))
```

Maple [B] time = 0.257, size = 662, normalized size = 2.1

$$-\frac{1}{64\,e^2\left(ae^2-cd^2\right)^2}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(3\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)x^4c^4d^4e^4+12\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)x^3c^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(13/2),x)
```

```
[Out] -1/64*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh(e*(c*d*x+a*e)^(1/2
)/((a*e^2-c*d^2)*e)^(1/2))*x^4*c^4*d^4*e^4+12*arctanh(e*(c*d*x+a*e)^(1/2)/(
(a*e^2-c*d^2)*e)^(1/2))*x^3*c^4*d^5*e^3+18*arctanh(e*(c*d*x+a*e)^(1/2)/((a*
e^2-c*d^2)*e)^(1/2))*x^2*c^4*d^6*e^2+12*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2
-c*d^2)*e)^(1/2))*x*c^4*d^7*e-3*x^3*c^3*d^3*e^3*(c*d*x+a*e)^(1/2)*((a*e^2-c
*d^2)*e)^(1/2)+3*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*c^4*d
^8+2*x^2*a*c^2*d^2*e^4*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)-11*x^2*c^3
*d^4*e^2*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+24*x*a^2*c*d*e^5*(c*d*x+
a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)-44*x*a*c^2*d^3*e^3*(c*d*x+a*e)^(1/2)*((a
*e^2-c*d^2)*e)^(1/2)+11*x*c^3*d^5*e*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/
2)+16*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*a^3*e^6-24*((a*e^2-c*d^2)*e
)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c*d^2*e^4+2*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*
e)^(1/2)*a*c^2*d^4*e^2+3*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*c^3*d^6)
/(e*x+d)^(9/2)/((a*e^2-c*d^2)*e)^(1/2)/e^2/(a*e^2-c*d^2)^2/(c*d*x+a*e)^(1/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex + d)^{\frac{13}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(13/2),x, algorit
hm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/(e*x + d)^(13/2), x
)
```

Fricas [B] time = 2.18837, size = 2807, normalized size = 8.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(13/2),x, algorit
hm="fricas")
```

```
[Out] [-1/128*(3*(c^4*d^4*e^5*x^5 + 5*c^4*d^5*e^4*x^4 + 10*c^4*d^6*e^3*x^3 + 10*c
^4*d^7*e^2*x^2 + 5*c^4*d^8*e*x + c^4*d^9)*sqrt(-c*d^2*e + a*e^3)*log(-(c*d*
e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2 - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*sqrt(-c*d^2*e + a*e^3)*sqrt(e*x + d))/(e^2*x^2 + 2*d*e*x + d^2
)) + 2*(3*c^4*d^8*e - a*c^3*d^6*e^3 - 26*a^2*c^2*d^4*e^5 + 40*a^3*c*d^2*e^7
 - 16*a^4*e^9 - 3*(c^4*d^5*e^4 - a*c^3*d^3*e^6)*x^3 - (11*c^4*d^6*e^3 - 13*
a*c^3*d^4*e^5 + 2*a^2*c^2*d^2*e^7)*x^2 + (11*c^4*d^7*e^2 - 55*a*c^3*d^5*e^4
 + 68*a^2*c^2*d^3*e^6 - 24*a^3*c*d*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d))/(c^3*d^11*e^3 - 3*a*c^2*d^9*e^5 + 3*a^2*c*d^7*e^
7 - a^3*d^5*e^9 + (c^3*d^6*e^8 - 3*a*c^2*d^4*e^10 + 3*a^2*c*d^2*e^12 - a^3*
e^14)*x^5 + 5*(c^3*d^7*e^7 - 3*a*c^2*d^5*e^9 + 3*a^2*c*d^3*e^11 - a^3*d*e^1
```

```
3)*x^4 + 10*(c^3*d^8*e^6 - 3*a*c^2*d^6*e^8 + 3*a^2*c*d^4*e^10 - a^3*d^2*e^1
2)*x^3 + 10*(c^3*d^9*e^5 - 3*a*c^2*d^7*e^7 + 3*a^2*c*d^5*e^9 - a^3*d^3*e^11
)*x^2 + 5*(c^3*d^10*e^4 - 3*a*c^2*d^8*e^6 + 3*a^2*c*d^6*e^8 - a^3*d^4*e^10)
*x), -1/64*(3*(c^4*d^4*e^5*x^5 + 5*c^4*d^5*e^4*x^4 + 10*c^4*d^6*e^3*x^3 + 1
0*c^4*d^7*e^2*x^2 + 5*c^4*d^8*e*x + c^4*d^9)*sqrt(c*d^2*e - a*e^3)*arctan(s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d^2*e - a*e^3)*sqrt(e*x +
 d)/(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x)) + (3*c^4*d^8*e - a*c^3*d
^6*e^3 - 26*a^2*c^2*d^4*e^5 + 40*a^3*c*d^2*e^7 - 16*a^4*e^9 - 3*(c^4*d^5*e^
4 - a*c^3*d^3*e^6)*x^3 - (11*c^4*d^6*e^3 - 13*a*c^3*d^4*e^5 + 2*a^2*c^2*d^2
*e^7)*x^2 + (11*c^4*d^7*e^2 - 55*a*c^3*d^5*e^4 + 68*a^2*c^2*d^3*e^6 - 24*a^
3*c*d*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c
^3*d^11*e^3 - 3*a*c^2*d^9*e^5 + 3*a^2*c*d^7*e^7 - a^3*d^5*e^9 + (c^3*d^6*e^
8 - 3*a*c^2*d^4*e^10 + 3*a^2*c*d^2*e^12 - a^3*e^14)*x^5 + 5*(c^3*d^7*e^7 -
3*a*c^2*d^5*e^9 + 3*a^2*c*d^3*e^11 - a^3*d*e^13)*x^4 + 10*(c^3*d^8*e^6 - 3*
a*c^2*d^6*e^8 + 3*a^2*c*d^4*e^10 - a^3*d^2*e^12)*x^3 + 10*(c^3*d^9*e^5 - 3*
a*c^2*d^7*e^7 + 3*a^2*c*d^5*e^9 - a^3*d^3*e^11)*x^2 + 5*(c^3*d^10*e^4 - 3*a
*c^2*d^8*e^6 + 3*a^2*c*d^6*e^8 - a^3*d^4*e^10)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(13/2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

3.2046 $\int (d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx$

Optimal. Leaf size=295

$$\frac{256\left(cd^2-ae^2\right)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{45045c^5d^5(d+ex)^{7/2}}+\frac{128\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{6435c^4d^4(d+ex)^{5/2}}+\frac{32\left(cd^2-ae^2\right)^{2}}{}$$

```
[Out] (256*(c*d^2 - a*e^2)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(4504
5*c^5*d^5*(d + e*x)^(7/2)) + (128*(c*d^2 - a*e^2)^3*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(7/2))/(6435*c^4*d^4*(d + e*x)^(5/2)) + (32*(c*d^2 - a*e^2
)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(715*c^3*d^3*(d + e*x)^(
3/2)) + (16*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/
(195*c^2*d^2*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2)^(7/2))/(15*c*d)
```

Rubi [A] time = 0.265003, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {656, 648}

$$\frac{256\left(cd^2-ae^2\right)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{45045c^5d^5(d+ex)^{7/2}}+\frac{128\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{6435c^4d^4(d+ex)^{5/2}}+\frac{32\left(cd^2-ae^2\right)^{2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (256*(c*d^2 - a*e^2)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(4504
5*c^5*d^5*(d + e*x)^(7/2)) + (128*(c*d^2 - a*e^2)^3*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(7/2))/(6435*c^4*d^4*(d + e*x)^(5/2)) + (32*(c*d^2 - a*e^2
)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(715*c^3*d^3*(d + e*x)^(
3/2)) + (16*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/
(195*c^2*d^2*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2)^(7/2))/(15*c*d)
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int (d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}dx = \frac{2\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{15cd} + \frac{\left(8\left(d^2-\frac{ae^2}{c}\right)\right)\int\sqrt{d+ex}$$

$$= \frac{16\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{195c^2d^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(ade+\left(c\right.\right.}{1}$$

$$= \frac{32\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{715c^3d^3(d+ex)^{3/2}} + \frac{16\left(cd^2-ae^2\right)\left(ade\right.}{195}$$

$$= \frac{128\left(cd^2-ae^2\right)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{6435c^4d^4(d+ex)^{5/2}} + \frac{32\left(cd^2-ae^2\right)^2\left(a\right.}{715}$$

$$= \frac{256\left(cd^2-ae^2\right)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{45045c^5d^5(d+ex)^{7/2}} + \frac{128\left(cd^2-ae^2\right)^3\left(\right.}{643}$$

Mathematica [A] time = 0.153624, size = 197, normalized size = 0.67

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\left(48a^2c^2d^2e^4\left(65d^2+70dex+21e^2x^2\right)-64a^3cde^6(15d+7ex)+128a^4e^8-8ac^3d^3e^2\left(1365\right.\right.}{45045c^5d^5\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(128*a^4*e^8 - 64*a^3*c*d*
e^6*(15*d + 7*e*x) + 48*a^2*c^2*d^2*e^4*(65*d^2 + 70*d*e*x + 21*e^2*x^2) -
8*a*c^3*d^3*e^2*(715*d^3 + 1365*d^2*e*x + 945*d*e^2*x^2 + 231*e^3*x^3) + c^
4*d^4*(6435*d^4 + 20020*d^3*e*x + 24570*d^2*e^2*x^2 + 13860*d*e^3*x^3 + 300
3*e^4*x^4)))/(45045*c^5*d^5*Sqrt[d + e*x])
```

Maple [A] time = 0.045, size = 243, normalized size = 0.8

$$\frac{(2\,cdx+2\,ae)\left(3003\,e^4x^4c^4d^4-1848\,ac^3d^3e^5x^3+13860\,c^4d^5e^3x^3+1008\,a^2c^2d^2e^6x^2-7560\,ac^3d^4e^4x^2+24570\,c^4d^6e^2x^2-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] 2/45045*(c*d*x+a*e)*(3003*c^4*d^4*e^4*x^4-1848*a*c^3*d^3*e^5*x^3+13860*c^4*
d^5*e^3*x^3+1008*a^2*c^2*d^2*e^6*x^2-7560*a*c^3*d^4*e^4*x^2+24570*c^4*d^6*e
^2*x^2-448*a^3*c*d*e^7*x+3360*a^2*c^2*d^3*e^5*x-10920*a*c^3*d^5*e^3*x+20020
*c^4*d^7*e*x+128*a^4*e^8-960*a^3*c*d^2*e^6+3120*a^2*c^2*d^4*e^4-5720*a*c^3*
d^6*e^2+6435*c^4*d^8)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)/c^5/d^5/(e*x+
d)^(5/2)
```

Maxima [A] time = 1.20802, size = 605, normalized size = 2.05

$$\frac{2\left(3003\,c^7d^7e^4x^7+6435\,a^3c^4d^8e^3-5720\,a^4c^3d^6e^5+3120\,a^5c^2d^4e^7-960\,a^6cd^2e^9+128\,a^7e^{11}+231\left(60\,c^7d^8e^3+31\,ac^6d^6\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="maxima")
```

```
[Out] 2/45045*(3003*c^7*d^7*e^4*x^7 + 6435*a^3*c^4*d^8*e^3 - 5720*a^4*c^3*d^6*e^5
 + 3120*a^5*c^2*d^4*e^7 - 960*a^6*c*d^2*e^9 + 128*a^7*e^11 + 231*(60*c^7*d^
8*e^3 + 31*a*c^6*d^6*e^5)*x^6 + 63*(390*c^7*d^9*e^2 + 540*a*c^6*d^7*e^4 + 7
1*a^2*c^5*d^5*e^6)*x^5 + 35*(572*c^7*d^10*e + 1794*a*c^6*d^8*e^3 + 636*a^2*
c^5*d^6*e^5 + a^3*c^4*d^4*e^7)*x^4 + 5*(1287*c^7*d^11 + 10868*a*c^6*d^9*e^2
 + 8814*a^2*c^5*d^7*e^4 + 60*a^3*c^4*d^5*e^6 - 8*a^4*c^3*d^3*e^8)*x^3 + 3*(
6435*a*c^6*d^10*e + 14300*a^2*c^5*d^8*e^3 + 390*a^3*c^4*d^6*e^5 - 120*a^4*c
^3*d^4*e^7 + 16*a^5*c^2*d^2*e^9)*x^2 + (19305*a^2*c^5*d^9*e^2 + 2860*a^3*c^
4*d^7*e^4 - 1560*a^4*c^3*d^5*e^6 + 480*a^5*c^2*d^3*e^8 - 64*a^6*c*d*e^10)*x
)*sqrt(c*d*x + a*e)*(e*x + d)/(c^5*d^5*e*x + c^5*d^6)
```

Fricas [A] time = 1.9239, size = 1026, normalized size = 3.48

$$\frac{2\left(3003\,c^7d^7e^4x^7 + 6435\,a^3c^4d^8e^3 - 5720\,a^4c^3d^6e^5 + 3120\,a^5c^2d^4e^7 - 960\,a^6cd^2e^9 + 128\,a^7e^{11} + 231\left(60\,c^7d^8e^3 + 31\,ac^6\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="fricas")
```

```
[Out] 2/45045*(3003*c^7*d^7*e^4*x^7 + 6435*a^3*c^4*d^8*e^3 - 5720*a^4*c^3*d^6*e^5
 + 3120*a^5*c^2*d^4*e^7 - 960*a^6*c*d^2*e^9 + 128*a^7*e^11 + 231*(60*c^7*d^
8*e^3 + 31*a*c^6*d^6*e^5)*x^6 + 63*(390*c^7*d^9*e^2 + 540*a*c^6*d^7*e^4 + 7
1*a^2*c^5*d^5*e^6)*x^5 + 35*(572*c^7*d^10*e + 1794*a*c^6*d^8*e^3 + 636*a^2*
c^5*d^6*e^5 + a^3*c^4*d^4*e^7)*x^4 + 5*(1287*c^7*d^11 + 10868*a*c^6*d^9*e^2
 + 8814*a^2*c^5*d^7*e^4 + 60*a^3*c^4*d^5*e^6 - 8*a^4*c^3*d^3*e^8)*x^3 + 3*(
6435*a*c^6*d^10*e + 14300*a^2*c^5*d^8*e^3 + 390*a^3*c^4*d^6*e^5 - 120*a^4*c
^3*d^4*e^7 + 16*a^5*c^2*d^2*e^9)*x^2 + (19305*a^2*c^5*d^9*e^2 + 2860*a^3*c^
4*d^7*e^4 - 1560*a^4*c^3*d^5*e^6 + 480*a^5*c^2*d^3*e^8 - 64*a^6*c*d*e^10)*x
)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*d^5*e*x +
c^5*d^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.2047 $\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx$

Optimal. Leaf size=233

$$\frac{12\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{143c^2d^2(d+ex)^{3/2}}+\frac{16\left(cd^2-ae^2\right)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{429c^3d^3(d+ex)^{5/2}}+\frac{32\left(cd^2-ae^2\right)^3\left(x\right.}{300}$$

```
[Out] (32*(c*d^2 - a*e^2)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(3003*
c^4*d^4*(d + e*x)^(7/2)) + (16*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2)^(7/2))/(429*c^3*d^3*(d + e*x)^(5/2)) + (12*(c*d^2 - a*e^2)*(a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(143*c^2*d^2*(d + e*x)^(3/2))
+ (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(13*c*d*Sqrt[d + e*x])
```

Rubi [A] time = 0.179461, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {656, 648}

$$\frac{12\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{143c^2d^2(d+ex)^{3/2}}+\frac{16\left(cd^2-ae^2\right)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{429c^3d^3(d+ex)^{5/2}}+\frac{32\left(cd^2-ae^2\right)^3\left(x\right.}{300}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (32*(c*d^2 - a*e^2)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(3003*
c^4*d^4*(d + e*x)^(7/2)) + (16*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2)^(7/2))/(429*c^3*d^3*(d + e*x)^(5/2)) + (12*(c*d^2 - a*e^2)*(a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(143*c^2*d^2*(d + e*x)^(3/2))
+ (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(13*c*d*Sqrt[d + e*x])
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{13cd\sqrt{d+ex}}+\frac{\left(6\left(d^2-\frac{ae^2}{c}\right)\right)\int\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}{\sqrt{d+ex}}}{13d}$$

$$=\frac{12\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{143c^2d^2(d+ex)^{3/2}}+\frac{2\left(ade+\left(cd^2+ae^2\right)x\right.}{13cd\sqrt{d+e}}$$

$$=\frac{16\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{429c^3d^3(d+ex)^{5/2}}+\frac{12\left(cd^2-ae^2\right)\left(ade+\right.}{143c^2d}$$

$$=\frac{32\left(cd^2-ae^2\right)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{3003c^4d^4(d+ex)^{7/2}}+\frac{16\left(cd^2-ae^2\right)^2\left(ade\right.}{429c^3d}$$

Mathematica [A] time = 0.121546, size = 142, normalized size = 0.61

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\left(8a^2cde^4(13d+7ex)-16a^3e^6-2ac^2d^2e^2\left(143d^2+182dex+63e^2x^2\right)+c^3d^3\left(1001d^2ex+\right.\right.}{3003c^4d^4\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-16*a^3*e^6 + 8*a^2*c*d*e
^4*(13*d + 7*e*x) - 2*a*c^2*d^2*e^2*(143*d^2 + 182*d*e*x + 63*e^2*x^2) + c^
3*d^3*(429*d^3 + 1001*d^2*e*x + 819*d*e^2*x^2 + 231*e^3*x^3)))/(3003*c^4*d^
4*Sqrt[d + e*x])
```

Maple [A] time = 0.047, size = 168, normalized size = 0.7

$$-\frac{(2\,cdx+2\,ae)\left(-231\,e^3x^3c^3d^3+126\,ac^2d^2e^4x^2-819\,c^3d^4e^2x^2-56\,a^2cde^5x+364\,ac^2d^3e^3x-1001\,c^3d^5ex+16\,a^3e^6-104\right.}{3003\,c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] -2/3003*(c*d*x+a*e)*(-231*c^3*d^3*e^3*x^3+126*a*c^2*d^2*e^4*x^2-819*c^3*d^4
*e^2*x^2-56*a^2*c*d*e^5*x+364*a*c^2*d^3*e^3*x-1001*c^3*d^5*e*x+16*a^3*e^6-1
04*a^2*c*d^2*e^4+286*a*c^2*d^4*e^2-429*c^3*d^6)*(c*d*e*x^2+a*e^2*x+c*d^2*x+
a*d*e)^(5/2)/c^4/d^4/(e*x+d)^(5/2)
```

Maxima [A] time = 1.13945, size = 452, normalized size = 1.94

$$2\left(231\,c^6d^6e^3x^6+429\,a^3c^3d^6e^3-286\,a^4c^2d^4e^5+104\,a^5cd^2e^7-16\,a^6e^9+63\left(13\,c^6d^7e^2+9\,ac^5d^5e^4\right)x^5+7\left(143\,c^6d^8e+29\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="maxima")
```

```
[Out] 2/3003*(231*c^6*d^6*e^3*x^6 + 429*a^3*c^3*d^6*e^3 - 286*a^4*c^2*d^4*e^5 + 1
04*a^5*c*d^2*e^7 - 16*a^6*e^9 + 63*(13*c^6*d^7*e^2 + 9*a*c^5*d^5*e^4)*x^5 +
 7*(143*c^6*d^8*e + 299*a*c^5*d^6*e^3 + 53*a^2*c^4*d^4*e^5)*x^4 + (429*c^6*
d^9 + 2717*a*c^5*d^7*e^2 + 1469*a^2*c^4*d^5*e^4 + 5*a^3*c^3*d^3*e^6)*x^3 +
3*(429*a*c^5*d^8*e + 715*a^2*c^4*d^6*e^3 + 13*a^3*c^3*d^4*e^5 - 2*a^4*c^2*d
^2*e^7)*x^2 + (1287*a^2*c^4*d^7*e^2 + 143*a^3*c^3*d^5*e^4 - 52*a^4*c^2*d^3*
e^6 + 8*a^5*c*d*e^8)*x)*sqrt(c*d*x + a*e)*(e*x + d)/(c^4*d^4*e*x + c^4*d^5)
```

Fricas [A] time = 1.88676, size = 752, normalized size = 3.23

$$\frac{2\left(231\,c^6d^6e^3x^6+429\,a^3c^3d^6e^3-286\,a^4c^2d^4e^5+104\,a^5cd^2e^7-16\,a^6e^9+63\left(13\,c^6d^7e^2+9\,ac^5d^5e^4\right)x^5+7\left(143\,c^6d^8e+\cdots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="fricas")
```

```
[Out] 2/3003*(231*c^6*d^6*e^3*x^6 + 429*a^3*c^3*d^6*e^3 - 286*a^4*c^2*d^4*e^5 + 1
04*a^5*c*d^2*e^7 - 16*a^6*e^9 + 63*(13*c^6*d^7*e^2 + 9*a*c^5*d^5*e^4)*x^5 +
 7*(143*c^6*d^8*e + 299*a*c^5*d^6*e^3 + 53*a^2*c^4*d^4*e^5)*x^4 + (429*c^6*
d^9 + 2717*a*c^5*d^7*e^2 + 1469*a^2*c^4*d^5*e^4 + 5*a^3*c^3*d^3*e^6)*x^3 +
3*(429*a*c^5*d^8*e + 715*a^2*c^4*d^6*e^3 + 13*a^3*c^3*d^4*e^5 - 2*a^4*c^2*d
^2*e^7)*x^2 + (1287*a^2*c^4*d^7*e^2 + 143*a^3*c^3*d^5*e^4 - 52*a^4*c^2*d^3*
e^6 + 8*a^5*c*d*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*
x + d)/(c^4*d^4*e*x + c^4*d^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.2048 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=171

$$\frac{8\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{99c^2d^2(d+ex)^{5/2}}+\frac{16\left(cd^2-ae^2\right)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{693c^3d^3(d+ex)^{7/2}}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11cd(d+ex)^{3/2}}$$

```
[Out] (16*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(693*c
^3*d^3*(d + e*x)^(7/2)) + (8*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^(7/2))/(99*c^2*d^2*(d + e*x)^(5/2)) + (2*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(7/2))/(11*c*d*(d + e*x)^(3/2))
```

Rubi [A] time = 0.114086, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {656, 648}

$$\frac{8\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{99c^2d^2(d+ex)^{5/2}}+\frac{16\left(cd^2-ae^2\right)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{693c^3d^3(d+ex)^{7/2}}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11cd(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/Sqrt[d + e*x],x]
```

```
[Out] (16*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(693*c
^3*d^3*(d + e*x)^(7/2)) + (8*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^(7/2))/(99*c^2*d^2*(d + e*x)^(5/2)) + (2*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(7/2))/(11*c*d*(d + e*x)^(3/2))
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{\sqrt{d+ex}}dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{11cd(d+ex)^{3/2}} + \frac{\left(4\left(d^2-\frac{ae^2}{c}\right)\right)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{3/2}}d}{11d}$$

$$= \frac{8\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{99c^2d^2(d+ex)^{5/2}} + \frac{2\left(ade+\left(cd^2+ae^2\right)x+cde\right.}{11cd(d+ex)^{3/2}}$$

$$= \frac{16\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{693c^3d^3(d+ex)^{7/2}} + \frac{8\left(cd^2-ae^2\right)\left(ade+\left(cd^2\right.\right.}{99c^2d^2(d+e}$$

Mathematica [A] time = 0.0844179, size = 98, normalized size = 0.57

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\left(8a^2e^4-4acde^2(11d+7ex)+c^2d^2\left(99d^2+154dex+63e^2x^2\right)\right)}{693c^3d^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/Sqrt[d + e*x],x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(8*a^2*e^4 - 4*a*c*d*e^2*(
11*d + 7*e*x) + c^2*d^2*(99*d^2 + 154*d*e*x + 63*e^2*x^2)))/(693*c^3*d^3*Sq
rt[d + e*x])
```

Maple [A] time = 0.046, size = 110, normalized size = 0.6

$$\frac{(2\,cdx+2\,ae)\left(63\,e^2x^2c^2d^2-28\,acde^3x+154\,c^2d^3ex+8\,a^2e^4-44\,acd^2e^2+99\,c^2d^4\right)}{693\,c^3d^3}\left(cdex^2+ae^2x+cd^2x+ade\right)^{\frac{5}{2}}(ex$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(1/2),x)
```

```
[Out] 2/693*(c*d*x+a*e)*(63*c^2*d^2*e^2*x^2-28*a*c*d*e^3*x+154*c^2*d^3*e*x+8*a^2*
e^4-44*a*c*d^2*e^2+99*c^2*d^4)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)/c^3/
d^3/(e*x+d)^(5/2)
```

Maxima [A] time = 1.04736, size = 293, normalized size = 1.71

$$\frac{2\left(63\,c^5d^5e^2x^5+99\,a^3c^2d^4e^3-44\,a^4cd^2e^5+8\,a^5e^7+7\left(22\,c^5d^6e+23\,ac^4d^4e^3\right)x^4+\left(99\,c^5d^7+418\,ac^4d^5e^2+113\,a^2c^3d\right.\right.}{693\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(1/2),x, algorith
m="maxima")
```

```
[Out] 2/693*(63*c^5*d^5*e^2*x^5 + 99*a^3*c^2*d^4*e^3 - 44*a^4*c*d^2*e^5 + 8*a^5*e
^7 + 7*(22*c^5*d^6*e + 23*a*c^4*d^4*e^3)*x^4 + (99*c^5*d^7 + 418*a*c^4*d^5*
e^2 + 113*a^2*c^3*d^3*e^4)*x^3 + 3*(99*a*c^4*d^6*e + 110*a^2*c^3*d^4*e^3 +
a^3*c^2*d^2*e^5)*x^2 + (297*a^2*c^3*d^5*e^2 + 22*a^3*c^2*d^3*e^4 - 4*a^4*c*
```

```
d*e^6)*x)*sqrt(c*d*x + a*e)/(c^3*d^3)
```

Fricas [A] time = 1.88095, size = 531, normalized size = 3.11

$$\frac{2\left(63\,c^5d^5e^2x^5+99\,a^3c^2d^4e^3-44\,a^4cd^2e^5+8\,a^5e^7+7\left(22\,c^5d^6e+23\,ac^4d^4e^3\right)x^4+\left(99\,c^5d^7+418\,ac^4d^5e^2+113\,a^2c^3d^3e^4\right)\dots}{693\left(\dots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(1/2),x, algorith
m="fricas")
```

```
[Out] 2/693*(63*c^5*d^5*e^2*x^5 + 99*a^3*c^2*d^4*e^3 - 44*a^4*c*d^2*e^5 + 8*a^5*e
^7 + 7*(22*c^5*d^6*e + 23*a*c^4*d^4*e^3)*x^4 + (99*c^5*d^7 + 418*a*c^4*d^5*
e^2 + 113*a^2*c^3*d^3*e^4)*x^3 + 3*(99*a*c^4*d^6*e + 110*a^2*c^3*d^4*e^3 +
a^3*c^2*d^2*e^5)*x^2 + (297*a^2*c^3*d^5*e^2 + 22*a^3*c^2*d^3*e^4 - 4*a^4*c*
d*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^3*d^
3*e*x + c^3*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(1/2),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.2049 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=109

$$\frac{4\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{63c^2d^2(d+ex)^{7/2}}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9cd(d+ex)^{5/2}}$$

```
[Out] (4*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(63*c^2*d
^2*(d + e*x)^(7/2)) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(9*
c*d*(d + e*x)^(5/2))
```

Rubi [A] time = 0.0572526, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {656, 648}

$$\frac{4\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{63c^2d^2(d+ex)^{7/2}}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9cd(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(3/2),x]
```

```
[Out] (4*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(63*c^2*d
^2*(d + e*x)^(7/2)) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(9*
c*d*(d + e*x)^(5/2))
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{3/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{9cd(d+ex)^{5/2}}+\frac{\left(2\left(d^2-\frac{ae^2}{c}\right)\right)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,d}{9d}$$

$$=\frac{4\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{63c^2d^2(d+ex)^{7/2}}+\frac{2\left(ade+\left(cd^2+ae^2\right)x+cde\right.}{9cd(d+ex)^{5/2}}$$

Mathematica [A] time = 0.0561066, size = 65, normalized size = 0.6

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\left(cd(9d+7ex)-2ae^2\right)}{63c^2d^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(3/2),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-2*a*e^2 + c*d*(9*d + 7*e
*x)))/(63*c^2*d^2*Sqrt[d + e*x])
```

Maple [A] time = 0.053, size = 69, normalized size = 0.6

$$-\frac{(2\,cdx+2\,ae)\left(-7\,cdex+2\,ae^2-9\,cd^2\right)}{63\,c^2d^2}\left(cdex^2+ae^2x+cd^2x+ade\right)^{\frac{5}{2}}(ex+d)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(3/2),x)
```

```
[Out] -2/63*(c*d*x+a*e)*(-7*c*d*e*x+2*a*e^2-9*c*d^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a
*d*e)^(5/2)/c^2/d^2/(e*x+d)^(5/2)
```

Maxima [A] time = 1.09768, size = 178, normalized size = 1.63

$$\frac{2\left(7\,c^4d^4ex^4+9\,a^3cd^2e^3-2\,a^4e^5+\left(9\,c^4d^5+19\,ac^3d^3e^2\right)x^3+3\left(9\,ac^3d^4e+5\,a^2c^2d^2e^3\right)x^2+\left(27\,a^2c^2d^3e^2+a^3cde^4\right)x\right)\sqrt{cdx}}{63\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(3/2),x, algorith
m="maxima")
```

```
[Out] 2/63*(7*c^4*d^4*e*x^4 + 9*a^3*c*d^2*e^3 - 2*a^4*e^5 + (9*c^4*d^5 + 19*a*c^3
*d^3*e^2)*x^3 + 3*(9*a*c^3*d^4*e + 5*a^2*c^2*d^2*e^3)*x^2 + (27*a^2*c^2*d^3
*e^2 + a^3*c*d*e^4)*x)*sqrt(c*d*x + a*e)/(c^2*d^2)
```

Fricas [A] time = 1.82335, size = 346, normalized size = 3.17

$$\frac{2\left(7\,c^4d^4ex^4+9\,a^3cd^2e^3-2\,a^4e^5+\left(9\,c^4d^5+19\,ac^3d^3e^2\right)x^3+3\left(9\,ac^3d^4e+5\,a^2c^2d^2e^3\right)x^2+\left(27\,a^2c^2d^3e^2+a^3cde^4\right)x\right)\sqrt{cde}}{63\left(c^2d^2ex+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(3/2),x, algorith
m="fricas")
```

```
[Out] 2/63*(7*c^4*d^4*e*x^4 + 9*a^3*c*d^2*e^3 - 2*a^4*e^5 + (9*c^4*d^5 + 19*a*c^3
*d^3*e^2)*x^3 + 3*(9*a*c^3*d^4*e + 5*a^2*c^2*d^2*e^3)*x^2 + (27*a^2*c^2*d^3
```

```
*e^2 + a^3*c*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x
 + d)/(c^2*d^2*e*x + c^2*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(3/2),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.2050 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=48

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7cd(d+ex)^{7/2}}$$

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(7*c*d*(d + e*x)^(7/2))
```

Rubi [A] time = 0.0213256, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {648}

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7cd(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(5/2),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(7*c*d*(d + e*x)^(7/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7cd(d+ex)^{7/2}}$$

Mathematica [A] time = 0.0419199, size = 37, normalized size = 0.77

$$\frac{2((d+ex)(ae+cdx))^{7/2}}{7cd(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(5/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(7/2))/(7*c*d*(d + e*x)^(7/2))
```

Maple [A] time = 0.043, size = 50, normalized size = 1.

$$\frac{2\,cdx+2\,ae}{7\,cd}\left(cdex^2+ae^2x+cd^2x+ade\right)^{\frac{5}{2}}(ex+d)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x)
```

```
[Out] 2/7*(c*d*x+a*e)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)/d/c/(e*x+d)^(5/2)
```

Maxima [A] time = 1.07677, size = 81, normalized size = 1.69

$$\frac{2\left(c^{3}d^{3}x^{3}+3\,ac^{2}d^{2}ex^{2}+3\,a^{2}cde^{2}x+a^{3}e^{3}\right)\sqrt{cdx+ae}}{7\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorith
m="maxima")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*
x + a*e)/(c*d)
```

Fricas [B] time = 1.87704, size = 193, normalized size = 4.02

$$\frac{2\left(c^{3}d^{3}x^{3}+3\,ac^{2}d^{2}ex^{2}+3\,a^{2}cde^{2}x+a^{3}e^{3}\right)\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}\sqrt{ex+d}}{7\left(cdex+cd^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorith
m="fricas")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c*d*e*x + c*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorith
m="giac")
```

```
[Out] Exception raised: AttributeError
```

3.2051 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=240

$$\frac{2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^3\sqrt{d+ex}}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e(d+ex)^{5/2}}+\frac{2\left(a-\frac{cd^2}{e^2}\right)\left(x\left(ae^2+cd^2\right)+ade+\right.}{3(d+ex)^{3/2}}$$

```
[Out] (2*(c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e^3*Sqrt
[d + e*x]) + (2*(a - (c*d^2)/e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2))/(3*(d + e*x)^(3/2)) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2
))/(5*e*(d + e*x)^(5/2)) - (2*(c*d^2 - a*e^2)^(5/2)*ArcTan[(Sqrt[e]*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])
/e^(7/2)
```

Rubi [A] time = 0.213578, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {664, 660, 205}

$$\frac{2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^3\sqrt{d+ex}}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e(d+ex)^{5/2}}+\frac{2\left(a-\frac{cd^2}{e^2}\right)\left(x\left(ae^2+cd^2\right)+ade+\right.}{3(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(7/2),x]
```

```
[Out] (2*(c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e^3*Sqrt
[d + e*x]) + (2*(a - (c*d^2)/e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2))/(3*(d + e*x)^(3/2)) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2
))/(5*e*(d + e*x)^(5/2)) - (2*(c*d^2 - a*e^2)^(5/2)*ArcTan[(Sqrt[e]*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])
/e^(7/2)
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{7/2}}\,dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5e(d+ex)^{5/2}} - \frac{\left(2cd^2e-e\left(cd^2+ae^2\right)\right)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex\right.}{(d+ex)^{5/2}}}{e^2}$$

$$= \frac{2\left(a-\frac{cd^2}{e^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3(d+ex)^{3/2}} + \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5e(d+ex)^{5/2}}$$

$$= \frac{2\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^3\sqrt{d+ex}} + \frac{2\left(a-\frac{cd^2}{e^2}\right)\left(ade+\left(cd^2+ae^2\right)x+\right.}{3(d+ex)^{3/2}}$$

$$= \frac{2\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^3\sqrt{d+ex}} + \frac{2\left(a-\frac{cd^2}{e^2}\right)\left(ade+\left(cd^2+ae^2\right)x+\right.}{3(d+ex)^{3/2}}$$

$$= \frac{2\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^3\sqrt{d+ex}} + \frac{2\left(a-\frac{cd^2}{e^2}\right)\left(ade+\left(cd^2+ae^2\right)x+\right.}{3(d+ex)^{3/2}}$$

Mathematica [A] time = 0.329395, size = 152, normalized size = 0.63

$$\frac{((d+ex)(ae+cdx))^{5/2}\left(\frac{10\left(ae^2-cd^2\right)\left(4ae^2+cd(ex-3d)\right)}{3e^2(ae+cdx)^2}-\frac{10\left(cd^2-ae^2\right)^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd^2-ae^2}}\right)}{e^{5/2}(ae+cdx)^{5/2}}+2\right)}{5e(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(7/2),x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^(5/2)*(2 + (10*(-(c*d^2) + a*e^2)*(4*a*e^2 + c*d
*(-3*d + e*x)))/(3*e^2*(a*e + c*d*x)^2) - (10*(c*d^2 - a*e^2)^(5/2)*ArcTan[
(Sqrt[e]*Sqrt[a*e + c*d*x])/Sqrt[c*d^2 - a*e^2]])/(e^(5/2)*(a*e + c*d*x)^(5
/2))))/(5*e*(d + e*x)^(5/2))
```

Maple [B] time = 0.245, size = 437, normalized size = 1.8

$$-\frac{2}{15\,e^3}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(15\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)a^3e^6-45\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)a^2cd^2e^4+45\,\mathrm{Artan}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(7/2),x)
```

```
[Out] -2/15*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh(e*(c*d*x+a*e)^(1/
2)/((a*e^2-c*d^2)*e)^(1/2))*a^3*e^6-45*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-
c*d^2)*e)^(1/2))*a^2*c*d^2*e^4+45*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2
)*e)^(1/2))*a*c^2*d^4*e^2-15*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^
(1/2))*c^3*d^6-3*x^2*c^2*d^2*e^2*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)-
11*x*a*c*d*e^3*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+5*x*c^2*d^3*e*(c*d
*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)-23*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e
```

```
)^(1/2)*a^2*e^4+35*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d^2*e^2-15
*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^4)/(e*x+d)^(1/2)/(c*d*x+a*
e)^(1/2)/e^3/((a*e^2-c*d^2)*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{\left(ex + d\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(7/2),x, algorith
m="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/(e*x + d)^(7/2), x)
```

Fricas [A] time = 2.01394, size = 1131, normalized size = 4.71

$$\left[\frac{15\left(c^2d^5 - 2\,acd^3e^2 + a^2de^4 + \left(c^2d^4e - 2\,acd^2e^3 + a^2e^5\right)x\right)\sqrt{-\frac{cd^2-ae^2}{e}}\log\left(-\frac{cde^2x^2+2\,ae^3x-cd^3+2\,ade^2-2\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{\ldots}}{e^2x^2+2\,dex+d^2}\right.}{15\,(\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(7/2),x, algorith
m="fricas")
```

```
[Out] [1/15*(15*(c^2*d^5 - 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e - 2*a*c*d^2*e^3
 + a^2*e^5)*x)*sqrt(-(c*d^2 - a*e^2)/e)*log(-(c*d*e^2*x^2 + 2*a*e^3*x - c*d
^3 + 2*a*d*e^2 - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d
)*e*sqrt(-(c*d^2 - a*e^2)/e))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(3*c^2*d^2*e^2
*x^2 + 15*c^2*d^4 - 35*a*c*d^2*e^2 + 23*a^2*e^4 - (5*c^2*d^3*e - 11*a*c*d*e
^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e^4*x +
d*e^3), 2/15*(15*(c^2*d^5 - 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e - 2*a*c*
d^2*e^3 + a^2*e^5)*x)*sqrt((c*d^2 - a*e^2)/e)*arctan(sqrt(e*x + d)*sqrt((c*
d^2 - a*e^2)/e)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)) + (3*c^2*d^2*e
^2*x^2 + 15*c^2*d^4 - 35*a*c*d^2*e^2 + 23*a^2*e^4 - (5*c^2*d^3*e - 11*a*c*d
*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e^4*x
+ d*e^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(7/2),x, algorith
m="giac")
```

```
[Out] Exception raised: AttributeError
```

3.2052 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{9/2}}\,dx$

Optimal. Leaf size=233

$$-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{e(d+ex)^{7/2}}+\frac{5cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e^2(d+ex)^{3/2}}+\frac{5cd\left(a-\frac{cd^2}{e^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex}}{e\sqrt{d+ex}}$$

```
[Out] (5*c*d*(a - (c*d^2)/e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e*Sq
rt[d + e*x]) + (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*e^2
*(d + e*x)^(3/2)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(e*(d + e
*x)^(7/2)) + (5*c*d*(c*d^2 - a*e^2)^(3/2)*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/e^(7/2)
```

Rubi [A] time = 0.181769, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {662, 664, 660, 205}

$$-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{e(d+ex)^{7/2}}+\frac{5cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e^2(d+ex)^{3/2}}+\frac{5cd\left(a-\frac{cd^2}{e^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex}}{e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(9/2),x]
```


```
[Out] (5*c*d*(a - (c*d^2)/e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e*Sq
rt[d + e*x]) + (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*e^2
*(d + e*x)^(3/2)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(e*(d + e
*x)^(7/2)) + (5*c*d*(c*d^2 - a*e^2)^(3/2)*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/e^(7/2)
```


Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
```

```
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{9/2}}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{e(d+ex)^{7/2}}+\frac{(5cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{5/2}}\,dx}{2e}$$

$$=\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e^2(d+ex)^{3/2}}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{e(d+ex)^{7/2}}-\frac{(5cd\left(\right.}{}$$

$$=-\frac{5cd\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^3\sqrt{d+ex}}+\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cde\right.}{3e^2(d+ex)^{3/2}}$$

$$=-\frac{5cd\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^3\sqrt{d+ex}}+\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cde\right.}{3e^2(d+ex)^{3/2}}$$

$$=-\frac{5cd\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^3\sqrt{d+ex}}+\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cde\right.}{3e^2(d+ex)^{3/2}}$$

Mathematica [C] time = 0.0704535, size = 79, normalized size = 0.34

$$\frac{2cd((d+ex)(ae+cdx))^{7/2}\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{7(d+ex)^{7/2}\left(cd^2-ae^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(9/2),x]
```

```
[Out] (2*c*d*((a*e + c*d*x)*(d + e*x))^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, (e*(a
*e + c*d*x))/(-(c*d^2) + a*e^2)])/(7*(c*d^2 - a*e^2)^2*(d + e*x)^(7/2))
```

Maple [B] time = 0.281, size = 521, normalized size = 2.2

$$-\frac{1}{3e^3}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(15\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)xa^2cde^5-30\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)xac^2d^3e^3+15\,\mathrm{Ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(9/2),x)
```

```
[Out] -1/3*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh(e*(c*d*x+a*e)^(1/2
)/((a*e^2-c*d^2)*e)^(1/2))*x*a^2*c*d*e^5-30*arctanh(e*(c*d*x+a*e)^(1/2)/((a
```

```
*e^2-c*d^2)*e)^(1/2))*x*a*c^2*d^3*e^3+15*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^
2-c*d^2)*e)^(1/2))*x*c^3*d^5*e+15*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2
)*e)^(1/2))*a^2*c*d^2*e^4-30*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^
(1/2))*a*c^2*d^4*e^2+15*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2)
)*c^3*d^6-2*x^2*c^2*d^2*e^2*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)-14*x*
a*c*d*e^3*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+10*x*c^2*d^3*e*(c*d*x+a
*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+3*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/
2)*a^2*e^4-20*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d^2*e^2+15*((a*
e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^4)/(e*x+d)^(3/2)/(c*d*x+a*e)^(1
/2)/e^3/((a*e^2-c*d^2)*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(9/2),x, algorith
m="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/(e*x + d)^(9/2), x)
```

Fricas [A] time = 2.0712, size = 1195, normalized size = 5.13

$$\left[\frac{15\left(c^2d^5 - acd^3e^2 + \left(c^2d^3e^2 - acde^4\right)x^2 + 2\left(c^2d^4e - acd^2e^3\right)x\right)\sqrt{-\frac{cd^2-ae^2}{e}}\log\left(-\frac{cde^2x^2+2\,ae^3x-cd^3+2\,ade^2+2\,\sqrt{cdex^2+ade+\left(cd^2+\cdots\right.}}{e^2x^2+2\,dex+d^2}\cdots\right.}{6\left(e^5x^2\cdots\right.}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(9/2),x, algorith
m="fricas")
```

```
[Out] [1/6*(15*(c^2*d^5 - a*c*d^3*e^2 + (c^2*d^3*e^2 - a*c*d*e^4)*x^2 + 2*(c^2*d^
4*e - a*c*d^2*e^3)*x)*sqrt(-(c*d^2 - a*e^2)/e)*log(-(c*d*e^2*x^2 + 2*a*e^3*
x - c*d^3 + 2*a*d*e^2 + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(
e*x + d)*e*sqrt(-(c*d^2 - a*e^2)/e))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(2*c^2*
d^2*e^2*x^2 - 15*c^2*d^4 + 20*a*c*d^2*e^2 - 3*a^2*e^4 - 2*(5*c^2*d^3*e - 7*
a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e
^5*x^2 + 2*d*e^4*x + d^2*e^3), -1/3*(15*(c^2*d^5 - a*c*d^3*e^2 + (c^2*d^3*e
^2 - a*c*d*e^4)*x^2 + 2*(c^2*d^4*e - a*c*d^2*e^3)*x)*sqrt((c*d^2 - a*e^2)/e
)*arctan(sqrt(e*x + d)*sqrt((c*d^2 - a*e^2)/e)/sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)) - (2*c^2*d^2*e^2*x^2 - 15*c^2*d^4 + 20*a*c*d^2*e^2 - 3*a^2
*e^4 - 2*(5*c^2*d^3*e - 7*a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*sqrt(e*x + d))/(e^5*x^2 + 2*d*e^4*x + d^2*e^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(9/2),x, algorith
m="giac")
```

```
[Out] Exception raised: AttributeError
```

3.2053 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{11/2}} dx$

Optimal. Leaf size=236

$$\frac{15c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4e^3\sqrt{d+ex}}-\frac{15c^2d^2\sqrt{cd^2-ae^2}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{4e^{7/2}}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)}{2e(d+ex)^{9/2}}$$

```
[Out] (15*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*e^3*Sqrt[d + e*
x]) - (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(4*e^2*(d + e*x
)^(5/2)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(2*e*(d + e*x)^(9/
2)) - (15*c^2*d^2*Sqrt[c*d^2 - a*e^2]*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(4*e^(7/2))
```

Rubi [A] time = 0.154452, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {662, 664, 660, 205}

$$\frac{15c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4e^3\sqrt{d+ex}}-\frac{15c^2d^2\sqrt{cd^2-ae^2}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{4e^{7/2}}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)}{2e(d+ex)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(11/2),x]
```


```
[Out] (15*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*e^3*Sqrt[d + e*
x]) - (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(4*e^2*(d + e*x
)^(5/2)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(2*e*(d + e*x)^(9/
2)) - (15*c^2*d^2*Sqrt[c*d^2 - a*e^2]*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(4*e^(7/2))
```


Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
```

```
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{11/2}}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{2e(d+ex)^{9/2}}+\frac{(5cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx}{4e}$$

$$= -\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4e^2(d+ex)^{5/2}}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{2e(d+ex)^{9/2}}+\frac{(15c$$

$$= \frac{15c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e^3\sqrt{d+ex}}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4e^2(d+ex)^{5/2}}-$$

$$= \frac{15c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e^3\sqrt{d+ex}}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4e^2(d+ex)^{5/2}}-$$

$$= \frac{15c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e^3\sqrt{d+ex}}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4e^2(d+ex)^{5/2}}-$$

Mathematica [C] time = 0.0730468, size = 83, normalized size = 0.35

$$\frac{2c^2d^2((d+ex)(ae+cdx))^{7/2}\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{7(d+ex)^{7/2}\left(cd^2-ae^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(11/2),x]
```

```
[Out] (2*c^2*d^2*((a*e + c*d*x)*(d + e*x))^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, (
e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(7*(c*d^2 - a*e^2)^3*(d + e*x)^(7/2))
```

Maple [B] time = 0.286, size = 527, normalized size = 2.2

$$-\frac{1}{4e^3}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(15\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)x^2ac^2d^2e^4-15\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)x^2c^3d^4e^2+30\,A\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(11/2),x)
```

```
[Out] -1/4*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh(e*(c*d*x+a*e)^(1/2
)/((a*e^2-c*d^2)*e)^(1/2))*x^2*a*c^2*d^2*e^4-15*arctanh(e*(c*d*x+a*e)^(1/2)
```

```
/((a*e^2-c*d^2)*e)^(1/2))*x^2*c^3*d^4*e^2+30*arctanh(e*(c*d*x+a*e)^(1/2)/((
a*e^2-c*d^2)*e)^(1/2))*x*a*c^2*d^3*e^3-30*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e
^2-c*d^2)*e)^(1/2))*x*c^3*d^5*e+15*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^
2)*e)^(1/2))*a*c^2*d^4*e^2-15*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)
^(1/2))*c^3*d^6-8*x^2*c^2*d^2*e^2*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)
+9*x*a*c*d*e^3*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)-25*x*c^2*d^3*e*(c*
d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+2*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e
)^(1/2)*a^2*e^4+5*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d^2*e^2-15*
((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^4)/(e*x+d)^(5/2)/(c*d*x+a*e
)^(1/2)/e^3/((a*e^2-c*d^2)*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(11/2),x, algorit
hm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/(e*x + d)^(11/2), x
)
```

Fricas [A] time = 2.49685, size = 1166, normalized size = 4.94

$$\left[\frac{15\left(c^2d^2e^3x^3+3c^2d^3e^2x^2+3c^2d^4ex+c^2d^5\right)\sqrt{-\frac{cd^2-ae^2}{e}}\log\left(-\frac{cde^2x^2+2ae^3x-cd^3+2ade^2-2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}e\sqrt{-\frac{cd^2-ae^2}{e}}}{e^2x^2+2dex+d^2}\right.}{8\left(e^6x^3+3de^5x^2+3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(11/2),x, algorit
hm="fricas")
```

```
[Out] [1/8*(15*(c^2*d^2*e^3*x^3 + 3*c^2*d^3*e^2*x^2 + 3*c^2*d^4*e*x + c^2*d^5)*sq
rt(-(c*d^2 - a*e^2)/e)*log(-(c*d*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2 -
2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*e*sqrt(-(c*d^2
- a*e^2)/e))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(8*c^2*d^2*e^2*x^2 + 15*c^2*d^4
 - 5*a*c*d^2*e^2 - 2*a^2*e^4 + (25*c^2*d^3*e - 9*a*c*d*e^3)*x)*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e^6*x^3 + 3*d*e^5*x^2 + 3*d
^2*e^4*x + d^3*e^3), 1/4*(15*(c^2*d^2*e^3*x^3 + 3*c^2*d^3*e^2*x^2 + 3*c^2*d
^4*e*x + c^2*d^5)*sqrt((c*d^2 - a*e^2)/e)*arctan(sqrt(e*x + d)*sqrt((c*d^2
- a*e^2)/e)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)) + (8*c^2*d^2*e^2*x
^2 + 15*c^2*d^4 - 5*a*c*d^2*e^2 - 2*a^2*e^4 + (25*c^2*d^3*e - 9*a*c*d*e^3)*
x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e^6*x^3 + 3*
d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(11/2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

3.2054 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{13/2}}\,dx$

Optimal. Leaf size=236

$$-\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8e^3(d+ex)^{3/2}}+\frac{5c^3d^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{8e^{7/2}\sqrt{cd^2-ae^2}}-\frac{5cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{12e^2(d+ex)^{7/2}}-$$

```
[Out] (-5*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*e^3*(d + e*x)^(
3/2)) - (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(12*e^2*(d +
e*x)^(7/2)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(3*e*(d + e*x)^
(11/2)) + (5*c^3*d^3*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(8*e^(7/2)*Sqrt[c*d^2 - a*e^2]
)
```

Rubi [A] time = 0.151789, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {662, 660, 205}

$$-\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8e^3(d+ex)^{3/2}}+\frac{5c^3d^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{8e^{7/2}\sqrt{cd^2-ae^2}}-\frac{5cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{12e^2(d+ex)^{7/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(13/2),x]
```

```
[Out] (-5*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*e^3*(d + e*x)^(
3/2)) - (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(12*e^2*(d +
e*x)^(7/2)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(3*e*(d + e*x)^
(11/2)) + (5*c^3*d^3*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(8*e^(7/2)*Sqrt[c*d^2 - a*e^2]
)
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{13/2}}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3e(d+ex)^{11/2}} + \frac{(5cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{9/2}}\,dx}{6e}$$

$$= -\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12e^2(d+ex)^{7/2}} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3e(d+ex)^{11/2}} + \frac{(5c^2}{}$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8e^3(d+ex)^{3/2}} - \frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12e^2(d+ex)^{7/2}} - ($$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8e^3(d+ex)^{3/2}} - \frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12e^2(d+ex)^{7/2}} - ($$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8e^3(d+ex)^{3/2}} - \frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12e^2(d+ex)^{7/2}} - ($$

Mathematica [A] time = 0.258554, size = 220, normalized size = 0.93

$$\frac{15c^3d^3(d+ex)^3\sqrt{ae+cdx}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd^2-ae^2}}\right) - \sqrt{e}\sqrt{cd^2-ae^2}\left(2a^2cde^3(5d+17ex)+8a^3e^5+ac^2d^2e\left(15d^2+50dex+59e^2\right.\right.}{24e^{7/2}(d+ex)^{5/2}\sqrt{cd^2-ae^2}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(13/2),x]
```

```
[Out] (-(Sqrt[e]*Sqrt[c*d^2 - a*e^2]*(8*a^3*e^5 + 2*a^2*c*d*e^3*(5*d + 17*e*x) +
c^3*d^3*x*(15*d^2 + 40*d*e*x + 33*e^2*x^2) + a*c^2*d^2*e*(15*d^2 + 50*d*e*x
 + 59*e^2*x^2))) + 15*c^3*d^3*Sqrt[a*e + c*d*x]*(d + e*x)^3*ArcTan[(Sqrt[e]
*Sqrt[a*e + c*d*x])/Sqrt[c*d^2 - a*e^2]])/(24*e^(7/2)*Sqrt[c*d^2 - a*e^2]*(
d + e*x)^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [B] time = 0.247, size = 443, normalized size = 1.9

$$-\frac{1}{24\,e^3}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(15\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)x^3c^3d^3e^3+45\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)x^2c^3d^4e^2+45\,\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(13/2),x)
```

```
[Out] -1/24*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh(e*(c*d*x+a*e)^(1/
2)/((a*e^2-c*d^2)*e)^(1/2))*x^3*c^3*d^3*e^3+45*arctanh(e*(c*d*x+a*e)^(1/2)/
((a*e^2-c*d^2)*e)^(1/2))*x^2*c^3*d^4*e^2+45*arctanh(e*(c*d*x+a*e)^(1/2)/((a
*e^2-c*d^2)*e)^(1/2))*x*c^3*d^5*e+15*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*
d^2)*e)^(1/2))*c^3*d^6+33*x^2*c^2*d^2*e^2*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*
e)^(1/2)+26*x*a*c*d*e^3*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+40*x*c^2*
d^3*e*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+8*((a*e^2-c*d^2)*e)^(1/2)*(
c*d*x+a*e)^(1/2)*a^2*e^4+10*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d
```

```
^2*e^2+15*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^4)/(e*x+d)^(7/2)/
(c*d*x+a*e)^(1/2)/e^3/((a*e^2-c*d^2)*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{\left(ex + d\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(13/2),x, algorit
hm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/(e*x + d)^(13/2), x
)
```

Fricas [B] time = 1.99794, size = 1709, normalized size = 7.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(13/2),x, algorit
hm="fricas")
```

```
[Out] [-1/48*(15*(c^3*d^3*e^4*x^4 + 4*c^3*d^4*e^3*x^3 + 6*c^3*d^5*e^2*x^2 + 4*c^3
*d^6*e*x + c^3*d^7)*sqrt(-c*d^2*e + a*e^3)*log(-(c*d*e^2*x^2 + 2*a*e^3*x -
c*d^3 + 2*a*d*e^2 - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d
^2*e + a*e^3)*sqrt(e*x + d))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(15*c^3*d^6*e -
 5*a*c^2*d^4*e^3 - 2*a^2*c*d^2*e^5 - 8*a^3*e^7 + 33*(c^3*d^4*e^3 - a*c^2*d^
2*e^5)*x^2 + 2*(20*c^3*d^5*e^2 - 7*a*c^2*d^3*e^4 - 13*a^2*c*d*e^6)*x)*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c*d^6*e^4 - a*d^4*e^
6 + (c*d^2*e^8 - a*e^10)*x^4 + 4*(c*d^3*e^7 - a*d*e^9)*x^3 + 6*(c*d^4*e^6 -
 a*d^2*e^8)*x^2 + 4*(c*d^5*e^5 - a*d^3*e^7)*x), -1/24*(15*(c^3*d^3*e^4*x^4
+ 4*c^3*d^4*e^3*x^3 + 6*c^3*d^5*e^2*x^2 + 4*c^3*d^6*e*x + c^3*d^7)*sqrt(c*d
^2*e - a*e^3)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d^2
*e - a*e^3)*sqrt(e*x + d)/(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x)) +
(15*c^3*d^6*e - 5*a*c^2*d^4*e^3 - 2*a^2*c*d^2*e^5 - 8*a^3*e^7 + 33*(c^3*d^4
*e^3 - a*c^2*d^2*e^5)*x^2 + 2*(20*c^3*d^5*e^2 - 7*a*c^2*d^3*e^4 - 13*a^2*c*
d*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c*d^6
*e^4 - a*d^4*e^6 + (c*d^2*e^8 - a*e^10)*x^4 + 4*(c*d^3*e^7 - a*d*e^9)*x^3 +
 6*(c*d^4*e^6 - a*d^2*e^8)*x^2 + 4*(c*d^5*e^5 - a*d^3*e^7)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(13/2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

3.2055 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{15/2}}\,dx$

Optimal. Leaf size=301

$$\frac{5c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64e^3(d+ex)^{3/2}\left(cd^2-ae^2\right)}-\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{32e^3(d+ex)^{5/2}}+\frac{5c^4d^4\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{64e^{7/2}\left(cd^2-ae^2\right)^{3/2}}-5c$$

```
[Out] (-5*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*e^3*(d + e*x)^
(5/2)) + (5*c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*e^3*(c
*d^2 - a*e^2)*(d + e*x)^(3/2)) - (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2))/(24*e^2*(d + e*x)^(9/2)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(5/2)/(4*e*(d + e*x)^(13/2)) + (5*c^4*d^4*ArcTan[(Sqrt[e]*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(64*e
^(7/2)*(c*d^2 - a*e^2)^(3/2))
```

Rubi [A] time = 0.234159, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {662, 672, 660, 205}

$$\frac{5c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64e^3(d+ex)^{3/2}\left(cd^2-ae^2\right)}-\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{32e^3(d+ex)^{5/2}}+\frac{5c^4d^4\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{64e^{7/2}\left(cd^2-ae^2\right)^{3/2}}-5c$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(15/2),x]
```

```
[Out] (-5*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*e^3*(d + e*x)^
(5/2)) + (5*c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*e^3*(c
*d^2 - a*e^2)*(d + e*x)^(3/2)) - (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2))/(24*e^2*(d + e*x)^(9/2)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(5/2)/(4*e*(d + e*x)^(13/2)) + (5*c^4*d^4*ArcTan[(Sqrt[e]*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(64*e
^(7/2)*(c*d^2 - a*e^2)^(3/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*(m + 2*p + 2))/((m + p + 1)*(2*c*d - b*e)), Int[(d
 + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && Ne
Q[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{15/2}}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4e(d+ex)^{13/2}} + \frac{(5cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{11/2}}\,dx}{8e}$$

$$= -\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24e^2(d+ex)^{9/2}} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4e(d+ex)^{13/2}} + \frac{(5c^2 \ldots}{\ldots}$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32e^3(d+ex)^{5/2}} - \frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24e^2(d+ex)^{9/2}} - \ldots$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32e^3(d+ex)^{5/2}} + \frac{5c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64e^3\left(cd^2-ae^2\right)(d+ex)^{3/2}} - \ldots$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32e^3(d+ex)^{5/2}} + \frac{5c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64e^3\left(cd^2-ae^2\right)(d+ex)^{3/2}} - \ldots$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32e^3(d+ex)^{5/2}} + \frac{5c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64e^3\left(cd^2-ae^2\right)(d+ex)^{3/2}} - \ldots$$

Mathematica [C] time = 0.0728909, size = 83, normalized size = 0.28

$$\frac{2c^4d^4((d+ex)(ae+cdx))^{7/2}\,{}_2F_1\left(\frac{7}{2},5;\frac{9}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{7(d+ex)^{7/2}\left(cd^2-ae^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(15/2),x]
```

```
[Out] (2*c^4*d^4*((a*e + c*d*x)*(d + e*x))^(7/2)*Hypergeometric2F1[7/2, 5, 9/2, (
e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(7*(c*d^2 - a*e^2)^5*(d + e*x)^(7/2))
```

Maple [B] time = 0.292, size = 662, normalized size = 2.2

$$\frac{1}{192\,e^3\left(ae^2-cd^2\right)}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(15\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)x^4c^4d^4e^4+60\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)x^3c^4 \ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(15/2),x)
```

```
[Out] 1/192*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh(e*(c*d*x+a*e)^(1/
2)/((a*e^2-c*d^2)*e)^(1/2))*x^4*c^4*d^4*e^4+60*arctanh(e*(c*d*x+a*e)^(1/2)/
((a*e^2-c*d^2)*e)^(1/2))*x^3*c^4*d^5*e^3+90*arctanh(e*(c*d*x+a*e)^(1/2)/((a
*e^2-c*d^2)*e)^(1/2))*x^2*c^4*d^6*e^2+60*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^
2-c*d^2)*e)^(1/2))*x*c^4*d^7*e-15*x^3*c^3*d^3*e^3*(c*d*x+a*e)^(1/2)*((a*e^2
-c*d^2)*e)^(1/2)+15*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*c^
4*d^8-118*x^2*a*c^2*d^2*e^4*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+73*x^
2*c^3*d^4*e^2*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)-136*x*a^2*c*d*e^5*(
c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+36*x*a*c^2*d^3*e^3*(c*d*x+a*e)^(1/
2)*((a*e^2-c*d^2)*e)^(1/2)+55*x*c^3*d^5*e*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*
e)^(1/2)-48*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*a^3*e^6+8*((a*e^2-c*d
^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c*d^2*e^4+10*((a*e^2-c*d^2)*e)^(1/2)*(c*
d*x+a*e)^(1/2)*a*c^2*d^4*e^2+15*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*c
^3*d^6)/(e*x+d)^(9/2)/(c*d*x+a*e)^(1/2)/e^3/(a*e^2-c*d^2)/((a*e^2-c*d^2)*e)
^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(15/2),x, algorit
hm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/(e*x + d)^(15/2), x
)
```

Fricas [B] time = 2.06199, size = 2495, normalized size = 8.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(15/2),x, algorit
hm="fricas")
```

```
[Out] [-1/384*(15*(c^4*d^4*e^5*x^5 + 5*c^4*d^5*e^4*x^4 + 10*c^4*d^6*e^3*x^3 + 10*
c^4*d^7*e^2*x^2 + 5*c^4*d^8*e*x + c^4*d^9)*sqrt(-c*d^2*e + a*e^3)*log(-(c*d
*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2 - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*sqrt(-c*d^2*e + a*e^3)*sqrt(e*x + d))/(e^2*x^2 + 2*d*e*x + d^
2)) + 2*(15*c^4*d^8*e - 5*a*c^3*d^6*e^3 - 2*a^2*c^2*d^4*e^5 - 56*a^3*c*d^2*
e^7 + 48*a^4*e^9 - 15*(c^4*d^5*e^4 - a*c^3*d^3*e^6)*x^3 + (73*c^4*d^6*e^3 -
 191*a*c^3*d^4*e^5 + 118*a^2*c^2*d^2*e^7)*x^2 + (55*c^4*d^7*e^2 - 19*a*c^3*
d^5*e^4 - 172*a^2*c^2*d^3*e^6 + 136*a^3*c*d*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^9*e^4 - 2*a*c*d^7*e^6 + a^2*d^5*
e^8 + (c^2*d^4*e^9 - 2*a*c*d^2*e^11 + a^2*e^13)*x^5 + 5*(c^2*d^5*e^8 - 2*a*
c*d^3*e^10 + a^2*d*e^12)*x^4 + 10*(c^2*d^6*e^7 - 2*a*c*d^4*e^9 + a^2*d^2*e^
```

```
11)*x^3 + 10*(c^2*d^7*e^6 - 2*a*c*d^5*e^8 + a^2*d^3*e^10)*x^2 + 5*(c^2*d^8*
e^5 - 2*a*c*d^6*e^7 + a^2*d^4*e^9)*x), -1/192*(15*(c^4*d^4*e^5*x^5 + 5*c^4*
d^5*e^4*x^4 + 10*c^4*d^6*e^3*x^3 + 10*c^4*d^7*e^2*x^2 + 5*c^4*d^8*e*x + c^4
*d^9)*sqrt(c*d^2*e - a*e^3)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*sqrt(c*d^2*e - a*e^3)*sqrt(e*x + d)/(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e +
 a*e^3)*x)) + (15*c^4*d^8*e - 5*a*c^3*d^6*e^3 - 2*a^2*c^2*d^4*e^5 - 56*a^3*
c*d^2*e^7 + 48*a^4*e^9 - 15*(c^4*d^5*e^4 - a*c^3*d^3*e^6)*x^3 + (73*c^4*d^6
*e^3 - 191*a*c^3*d^4*e^5 + 118*a^2*c^2*d^2*e^7)*x^2 + (55*c^4*d^7*e^2 - 19*
a*c^3*d^5*e^4 - 172*a^2*c^2*d^3*e^6 + 136*a^3*c*d*e^8)*x)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^9*e^4 - 2*a*c*d^7*e^6 + a^
2*d^5*e^8 + (c^2*d^4*e^9 - 2*a*c*d^2*e^11 + a^2*e^13)*x^5 + 5*(c^2*d^5*e^8
- 2*a*c*d^3*e^10 + a^2*d*e^12)*x^4 + 10*(c^2*d^6*e^7 - 2*a*c*d^4*e^9 + a^2*
d^2*e^11)*x^3 + 10*(c^2*d^7*e^6 - 2*a*c*d^5*e^8 + a^2*d^3*e^10)*x^2 + 5*(c^
2*d^8*e^5 - 2*a*c*d^6*e^7 + a^2*d^4*e^9)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(15/2),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(15/2),x, algorit
hm="giac")

[Out] Timed out
```

3.2056 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{17/2}}\,dx$

Optimal. Leaf size=366

$$\frac{3c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128e^3(d+ex)^{3/2}\left(cd^2-ae^2\right)^2}+\frac{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64e^3(d+ex)^{5/2}\left(cd^2-ae^2\right)}-\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{16e^3(d+ex)^{7/2}}+\frac{3c^5d^5}{}$$

```
[Out] -(c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(16*e^3*(d + e*x)^(7
/2)) + (c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*e^3*(c*d^2
 - a*e^2)*(d + e*x)^(5/2)) + (3*c^4*d^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])/(128*e^3*(c*d^2 - a*e^2)^2*(d + e*x)^(3/2)) - (c*d*(a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(8*e^2*(d + e*x)^(11/2)) - (a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2)^(5/2)/(5*e*(d + e*x)^(15/2)) + (3*c^5*d^5*ArcTan[(
Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*S
qrt[d + e*x])])/(128*e^(7/2)*(c*d^2 - a*e^2)^(5/2))
```

Rubi [A] time = 0.279515, antiderivative size = 366, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {662, 672, 660, 205}

$$\frac{3c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128e^3(d+ex)^{3/2}\left(cd^2-ae^2\right)^2}+\frac{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64e^3(d+ex)^{5/2}\left(cd^2-ae^2\right)}-\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{16e^3(d+ex)^{7/2}}+\frac{3c^5d^5}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(17/2),x]
```



```
[Out] -(c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(16*e^3*(d + e*x)^(7
/2)) + (c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*e^3*(c*d^2
 - a*e^2)*(d + e*x)^(5/2)) + (3*c^4*d^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])/(128*e^3*(c*d^2 - a*e^2)^2*(d + e*x)^(3/2)) - (c*d*(a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(8*e^2*(d + e*x)^(11/2)) - (a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2)^(5/2)/(5*e*(d + e*x)^(15/2)) + (3*c^5*d^5*ArcTan[(
Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*S
qrt[d + e*x])])/(128*e^(7/2)*(c*d^2 - a*e^2)^(5/2))
```


Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*(m + 2*p + 2))/((m + p + 1)*(2*c*d - b*e)), Int[(d
 + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && Ne
Q[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{17/2}}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5e(d+ex)^{15/2}}+\frac{(cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{13/2}}\,dx}{2e}$$

$$= -\frac{cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{8e^2(d+ex)^{11/2}}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5e(d+ex)^{15/2}}+\frac{\left(3c^2d\right.}{}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16e^3(d+ex)^{7/2}}-\frac{cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{8e^2(d+ex)^{11/2}}-\frac{\left(ad\right.}{}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16e^3(d+ex)^{7/2}}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64e^3\left(cd^2-ae^2\right)(d+ex)^{5/2}}-\frac{cd\left(\right.}{}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16e^3(d+ex)^{7/2}}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64e^3\left(cd^2-ae^2\right)(d+ex)^{5/2}}+\frac{3c^4}{1}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16e^3(d+ex)^{7/2}}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64e^3\left(cd^2-ae^2\right)(d+ex)^{5/2}}+\frac{3c^4}{1}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16e^3(d+ex)^{7/2}}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64e^3\left(cd^2-ae^2\right)(d+ex)^{5/2}}+\frac{3c^4}{1}$$

Mathematica [C] time = 0.0747819, size = 83, normalized size = 0.23

$$\frac{2c^5d^5((d+ex)(ae+cdx))^{7/2}\,{}_2F_1\left(\frac{7}{2},6;\frac{9}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{7(d+ex)^{7/2}\left(cd^2-ae^2\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(17/2),x]
```

```
[Out] (2*c^5*d^5*((a*e + c*d*x)*(d + e*x))^(7/2)*Hypergeometric2F1[7/2, 6, 9/2, (
e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(7*(c*d^2 - a*e^2)^6*(d + e*x)^(7/2))
```

Maple [B] time = 0.278, size = 910, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(17/2),x)
```

```
[Out] -1/640*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh(e*(c*d*x+a*e)^(1
/2)/((a*e^2-c*d^2)*e)^(1/2))*x^5*c^5*d^5*e^5+75*arctanh(e*(c*d*x+a*e)^(1/2)
/((a*e^2-c*d^2)*e)^(1/2))*x^4*c^5*d^6*e^4+150*arctanh(e*(c*d*x+a*e)^(1/2)/(
(a*e^2-c*d^2)*e)^(1/2))*x^3*c^5*d^7*e^3+150*arctanh(e*(c*d*x+a*e)^(1/2)/((a
*e^2-c*d^2)*e)^(1/2))*x^2*c^5*d^8*e^2-15*x^4*c^4*d^4*e^4*(c*d*x+a*e)^(1/2)*
((a*e^2-c*d^2)*e)^(1/2)+75*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1
/2))*x*c^5*d^9*e+10*x^3*a*c^3*d^3*e^5*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(
1/2)-70*x^3*c^4*d^5*e^3*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+15*arctan
h(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*c^5*d^10+248*x^2*a^2*c^2*d^2
*e^6*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)-466*x^2*a*c^3*d^4*e^4*(c*d*x
+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+128*x^2*c^4*d^6*e^2*(c*d*x+a*e)^(1/2)*(
(a*e^2-c*d^2)*e)^(1/2)+336*x*a^3*c*d*e^7*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e
)^(1/2)-512*x*a^2*c^2*d^3*e^5*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+46*
x*a*c^3*d^5*e^3*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+70*x*c^4*d^7*e*(c
*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+128*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+
a*e)^(1/2)*a^4*e^8-176*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*a^3*c*d^2*
e^6+8*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c^2*d^4*e^4+10*((a*e^2-
c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*a*c^3*d^6*e^2+15*((a*e^2-c*d^2)*e)^(1/2)*
(c*d*x+a*e)^(1/2)*c^4*d^8)/(e*x+d)^(11/2)/((a*e^2-c*d^2)*e)^(1/2)/e^3/(a*e^
2-c*d^2)^2/(c*d*x+a*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)^{\frac{17}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(17/2),x, algorit
hm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/(e*x + d)^(17/2), x
)
```

Fricas [B] time = 2.30276, size = 3484, normalized size = 9.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(17/2),x, algorit
hm="fricas")
```

```
[Out] [-1/1280*(15*(c^5*d^5*e^6*x^6 + 6*c^5*d^6*e^5*x^5 + 15*c^5*d^7*e^4*x^4 + 20
*c^5*d^8*e^3*x^3 + 15*c^5*d^9*e^2*x^2 + 6*c^5*d^10*e*x + c^5*d^11)*sqrt(-c*
d^2*e + a*e^3)*log(-(c*d*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2 - 2*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d^2*e + a*e^3)*sqrt(e*x + d))
/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(15*c^5*d^10*e - 5*a*c^4*d^8*e^3 - 2*a^2*c^
3*d^6*e^5 - 184*a^3*c^2*d^4*e^7 + 304*a^4*c*d^2*e^9 - 128*a^5*e^11 - 15*(c^
5*d^6*e^5 - a*c^4*d^4*e^7)*x^4 - 10*(7*c^5*d^7*e^4 - 8*a*c^4*d^5*e^6 + a^2*
c^3*d^3*e^8)*x^3 + 2*(64*c^5*d^8*e^3 - 297*a*c^4*d^6*e^5 + 357*a^2*c^3*d^4*
e^7 - 124*a^3*c^2*d^2*e^9)*x^2 + 2*(35*c^5*d^9*e^2 - 12*a*c^4*d^7*e^4 - 279
*a^2*c^3*d^5*e^6 + 424*a^3*c^2*d^3*e^8 - 168*a^4*c*d*e^10)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^12*e^4 - 3*a*c^2*d^10*
e^6 + 3*a^2*c*d^8*e^8 - a^3*d^6*e^10 + (c^3*d^6*e^10 - 3*a*c^2*d^4*e^12 + 3
*a^2*c*d^2*e^14 - a^3*e^16)*x^6 + 6*(c^3*d^7*e^9 - 3*a*c^2*d^5*e^11 + 3*a^2
*c*d^3*e^13 - a^3*d*e^15)*x^5 + 15*(c^3*d^8*e^8 - 3*a*c^2*d^6*e^10 + 3*a^2*
c*d^4*e^12 - a^3*d^2*e^14)*x^4 + 20*(c^3*d^9*e^7 - 3*a*c^2*d^7*e^9 + 3*a^2*
c*d^5*e^11 - a^3*d^3*e^13)*x^3 + 15*(c^3*d^10*e^6 - 3*a*c^2*d^8*e^8 + 3*a^2
*c*d^6*e^10 - a^3*d^4*e^12)*x^2 + 6*(c^3*d^11*e^5 - 3*a*c^2*d^9*e^7 + 3*a^2
*c*d^7*e^9 - a^3*d^5*e^11)*x), -1/640*(15*(c^5*d^5*e^6*x^6 + 6*c^5*d^6*e^5*
x^5 + 15*c^5*d^7*e^4*x^4 + 20*c^5*d^8*e^3*x^3 + 15*c^5*d^9*e^2*x^2 + 6*c^5*
d^10*e*x + c^5*d^11)*sqrt(c*d^2*e - a*e^3)*arctan(sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x)*sqrt(c*d^2*e - a*e^3)*sqrt(e*x + d)/(c*d*e^2*x^2 + a*d*e
^2 + (c*d^2*e + a*e^3)*x)) + (15*c^5*d^10*e - 5*a*c^4*d^8*e^3 - 2*a^2*c^3*d
^6*e^5 - 184*a^3*c^2*d^4*e^7 + 304*a^4*c*d^2*e^9 - 128*a^5*e^11 - 15*(c^5*d
^6*e^5 - a*c^4*d^4*e^7)*x^4 - 10*(7*c^5*d^7*e^4 - 8*a*c^4*d^5*e^6 + a^2*c^3
*d^3*e^8)*x^3 + 2*(64*c^5*d^8*e^3 - 297*a*c^4*d^6*e^5 + 357*a^2*c^3*d^4*e^7
 - 124*a^3*c^2*d^2*e^9)*x^2 + 2*(35*c^5*d^9*e^2 - 12*a*c^4*d^7*e^4 - 279*a^
2*c^3*d^5*e^6 + 424*a^3*c^2*d^3*e^8 - 168*a^4*c*d*e^10)*x)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^12*e^4 - 3*a*c^2*d^10*e^6
 + 3*a^2*c*d^8*e^8 - a^3*d^6*e^10 + (c^3*d^6*e^10 - 3*a*c^2*d^4*e^12 + 3*a^
2*c*d^2*e^14 - a^3*e^16)*x^6 + 6*(c^3*d^7*e^9 - 3*a*c^2*d^5*e^11 + 3*a^2*c*
d^3*e^13 - a^3*d*e^15)*x^5 + 15*(c^3*d^8*e^8 - 3*a*c^2*d^6*e^10 + 3*a^2*c*d
^4*e^12 - a^3*d^2*e^14)*x^4 + 20*(c^3*d^9*e^7 - 3*a*c^2*d^7*e^9 + 3*a^2*c*d
^5*e^11 - a^3*d^3*e^13)*x^3 + 15*(c^3*d^10*e^6 - 3*a*c^2*d^8*e^8 + 3*a^2*c*
d^6*e^10 - a^3*d^4*e^12)*x^2 + 6*(c^3*d^11*e^5 - 3*a*c^2*d^9*e^7 + 3*a^2*c*
d^7*e^9 - a^3*d^5*e^11)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(17/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(17/2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

3.2057 $\int \frac{(d+ex)^{7/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=233

$$\frac{32\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{35c^4d^4\sqrt{d+ex}}+\frac{16\sqrt{d+ex}\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{35c^3d^3}+\frac{12(d+ex)^{3/2}\left(cd\right.}{}$$

```
[Out] (32*(c*d^2 - a*e^2)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(35*c^4*
d^4*Sqrt[d + e*x]) + (16*(c*d^2 - a*e^2)^2*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2])/(35*c^3*d^3) + (12*(c*d^2 - a*e^2)*(d + e*x)^(3/
2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(35*c^2*d^2) + (2*(d + e*x)
^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(7*c*d)
```

Rubi [A] time = 0.178182, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {656, 648}

$$\frac{32\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{35c^4d^4\sqrt{d+ex}}+\frac{16\sqrt{d+ex}\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{35c^3d^3}+\frac{12(d+ex)^{3/2}\left(cd\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (32*(c*d^2 - a*e^2)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(35*c^4*
d^4*Sqrt[d + e*x]) + (16*(c*d^2 - a*e^2)^2*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2])/(35*c^3*d^3) + (12*(c*d^2 - a*e^2)*(d + e*x)^(3/
2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(35*c^2*d^2) + (2*(d + e*x)
^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(7*c*d)
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{7/2}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2(d+ex)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7cd} + \frac{\left(6\left(d^2-\frac{ae^2}{c}\right)\right)\int \frac{(d+ex)^{5/2}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}}{7d}$$

$$= \frac{12\left(cd^2-ae^2\right)(d+ex)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35c^2d^2} + \frac{2(d+ex)^{5/2}\sqrt{ade+(}}{7c}$$

$$= \frac{16\left(cd^2-ae^2\right)^2\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35c^3d^3} + \frac{12\left(cd^2-ae^2\right)(d+ex)}{}$$

$$= \frac{32\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35c^4d^4\sqrt{d+ex}} + \frac{16\left(cd^2-ae^2\right)^2\sqrt{d+ex}\sqrt{ade+}}{35c^3d^3}$$

Mathematica [A] time = 0.105574, size = 131, normalized size = 0.56

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(8a^2cde^4(7d+ex)-16a^3e^6-2ac^2d^2e^2\left(35d^2+14dex+3e^2x^2\right)+c^3d^3\left(35d^2ex+35d^3+21de^2x^2\right.\right.}{35c^4d^4\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-16*a^3*e^6 + 8*a^2*c*d*e^4*(7*d + e*x) -
 2*a*c^2*d^2*e^2*(35*d^2 + 14*d*e*x + 3*e^2*x^2) + c^3*d^3*(35*d^3 + 35*d^2
*e*x + 21*d*e^2*x^2 + 5*e^3*x^3)))/(35*c^4*d^4*Sqrt[d + e*x])
```

Maple [A] time = 0.045, size = 168, normalized size = 0.7

$$-\frac{(2\,cdx+2\,ae)\left(-5\,e^3x^3c^3d^3+6\,ac^2d^2e^4x^2-21\,c^3d^4e^2x^2-8\,a^2cde^5x+28\,ac^2d^3e^3x-35\,c^3d^5ex+16\,a^3e^6-56\,a^2cd^2e^4\right.}{35\,c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] -2/35*(c*d*x+a*e)*(-5*c^3*d^3*e^3*x^3+6*a*c^2*d^2*e^4*x^2-21*c^3*d^4*e^2*x^
2-8*a^2*c*d*e^5*x+28*a*c^2*d^3*e^3*x-35*c^3*d^5*e*x+16*a^3*e^6-56*a^2*c*d^2
*e^4+70*a*c^2*d^4*e^2-35*c^3*d^6)*(e*x+d)^(1/2)/c^4/d^4/(c*d*e*x^2+a*e^2*x+
c*d^2*x+a*d*e)^(1/2)
```

Maxima [A] time = 1.03328, size = 259, normalized size = 1.11

$$\frac{2\left(5\,c^4d^4e^3x^4+35\,ac^3d^6e-70\,a^2c^2d^4e^3+56\,a^3cd^2e^5-16\,a^4e^7+\left(21\,c^4d^5e^2-ac^3d^3e^4\right)x^3+\left(35\,c^4d^6e-7\,ac^3d^4e^3+2\,a\right.\right.}{35\,\sqrt{cdx+ae}c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="maxima")
```

```
[Out] 2/35*(5*c^4*d^4*e^3*x^4 + 35*a*c^3*d^6*e - 70*a^2*c^2*d^4*e^3 + 56*a^3*c*d^
2*e^5 - 16*a^4*e^7 + (21*c^4*d^5*e^2 - a*c^3*d^3*e^4)*x^3 + (35*c^4*d^6*e -
 7*a*c^3*d^4*e^3 + 2*a^2*c^2*d^2*e^5)*x^2 + (35*c^4*d^7 - 35*a*c^3*d^5*e^2
+ 28*a^2*c^2*d^3*e^4 - 8*a^3*c*d*e^6)*x)/(sqrt(c*d*x + a*e)*c^4*d^4)
```

Fricas [A] time = 1.81674, size = 362, normalized size = 1.55

$$\frac{2\left(5c^3d^3e^3x^3+35c^3d^6-70ac^2d^4e^2+56a^2cd^2e^4-16a^3e^6+3\left(7c^3d^4e^2-2ac^2d^2e^4\right)x^2+\left(35c^3d^5e-28ac^2d^3e^3+8a^2cd\right.\right.}{35\left(c^4d^4ex+c^4d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="fricas")
```

```
[Out] 2/35*(5*c^3*d^3*e^3*x^3 + 35*c^3*d^6 - 70*a*c^2*d^4*e^2 + 56*a^2*c*d^2*e^4
- 16*a^3*e^6 + 3*(7*c^3*d^4*e^2 - 2*a*c^2*d^2*e^4)*x^2 + (35*c^3*d^5*e - 28
*a*c^2*d^3*e^3 + 8*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*sqrt(e*x + d)/(c^4*d^4*e*x + c^4*d^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{7}{2}}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((e*x + d)^(7/2)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x), x)
```

3.2058 $\int \frac{(d+ex)^{5/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=171

$$\frac{16\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{15c^3d^3\sqrt{d+ex}}+\frac{8\sqrt{d+ex}\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{15c^2d^2}+\frac{2(d+ex)^{3/2}\sqrt{x}}{}$$

```
[Out] (16*(c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(15*c^3*
d^3*Sqrt[d + e*x]) + (8*(c*d^2 - a*e^2)*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])/(15*c^2*d^2) + (2*(d + e*x)^(3/2)*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2])/(5*c*d)
```

Rubi [A] time = 0.113826, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {656, 648}

$$\frac{16\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{15c^3d^3\sqrt{d+ex}}+\frac{8\sqrt{d+ex}\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{15c^2d^2}+\frac{2(d+ex)^{3/2}\sqrt{x}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```



```
[Out] (16*(c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(15*c^3*
d^3*Sqrt[d + e*x]) + (8*(c*d^2 - a*e^2)*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])/(15*c^2*d^2) + (2*(d + e*x)^(3/2)*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2])/(5*c*d)
```


Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}dx = \frac{2(d+ex)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{5cd} + \frac{\left(4\left(d^2-\frac{ae^2}{c}\right)\right)\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}d}{5d}$$

$$= \frac{8(cd^2-ae^2)\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{15c^2d^2} + \frac{2(d+ex)^{3/2}\sqrt{ade+(cd^2+a}}{5cd}$$

$$= \frac{16(cd^2-ae^2)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{15c^3d^3\sqrt{d+ex}} + \frac{8(cd^2-ae^2)\sqrt{d+ex}\sqrt{ade+(cd^2}}{15c^2d^2}$$

Mathematica [A] time = 0.0726049, size = 87, normalized size = 0.51

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(8a^2e^4-4acde^2(5d+ex)+c^2d^2\left(15d^2+10dex+3e^2x^2\right)\right)}{15c^3d^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(8*a^2*e^4 - 4*a*c*d*e^2*(5*d + e*x) + c^2
*d^2*(15*d^2 + 10*d*e*x + 3*e^2*x^2)))/(15*c^3*d^3*Sqrt[d + e*x])
```

Maple [A] time = 0.043, size = 110, normalized size = 0.6

$$\frac{(2\,cdx+2\,ae)\left(3\,e^2x^2c^2d^2-4\,acde^3x+10\,c^2d^3ex+8\,a^2e^4-20\,acd^2e^2+15\,c^2d^4\right)}{15\,c^3d^3}\sqrt{ex+d}\frac{1}{\sqrt{cdex^2+ae^2x+cd^2x+ade}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] 2/15*(c*d*x+a*e)*(3*c^2*d^2*e^2*x^2-4*a*c*d*e^3*x+10*c^2*d^3*e*x+8*a^2*e^4-
20*a*c*d^2*e^2+15*c^2*d^4)*(e*x+d)^(1/2)/c^3/d^3/(c*d*e*x^2+a*e^2*x+c*d^2*x
+a*d*e)^(1/2)
```

Maxima [A] time = 1.05119, size = 165, normalized size = 0.96

$$\frac{2\left(3\,c^3d^3e^2x^3+15\,ac^2d^4e-20\,a^2cd^2e^3+8\,a^3e^5+\left(10\,c^3d^4e-ac^2d^2e^3\right)x^2+\left(15\,c^3d^5-10\,ac^2d^3e^2+4\,a^2cde^4\right)x\right)}{15\,\sqrt{cdx+ae}c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="maxima")
```

```
[Out] 2/15*(3*c^3*d^3*e^2*x^3 + 15*a*c^2*d^4*e - 20*a^2*c*d^2*e^3 + 8*a^3*e^5 + (
10*c^3*d^4*e - a*c^2*d^2*e^3)*x^2 + (15*c^3*d^5 - 10*a*c^2*d^3*e^2 + 4*a^2*
c*d*e^4)*x)/(sqrt(c*d*x + a*e)*c^3*d^3)
```

Fricas [A] time = 1.7451, size = 248, normalized size = 1.45

$$\frac{2\left(3c^2d^2e^2x^2+15c^2d^4-20acd^2e^2+8a^2e^4+2\left(5c^2d^3e-2acde^3\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}}{15\left(c^3d^3ex+c^3d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="fricas")
```

```
[Out] 2/15*(3*c^2*d^2*e^2*x^2 + 15*c^2*d^4 - 20*a*c*d^2*e^2 + 8*a^2*e^4 + 2*(5*c^
2*d^3*e - 2*a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(
e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((e*x + d)^(5/2)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x), x)
```

3.2059 $\int \frac{(d+ex)^{3/2}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=109

$$\frac{4\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3c^2d^2\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd}$$

```
[Out] (4*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^2*d^2*
Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(3*c*d)
```

Rubi [A] time = 0.055953, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {656, 648}

$$\frac{4\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3c^2d^2\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (4*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^2*d^2*
Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(3*c*d)
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx=\frac{2\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3cd}+\frac{\left(2\left(d^2-\frac{ae^2}{c}\right)\right)\int \frac{\sqrt{d+ex}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{3d}$$

$$=\frac{4\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3c^2d^2\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}{3cd}$$

Mathematica [A] time = 0.0413709, size = 54, normalized size = 0.5

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(cd(3d+ex)-2ae^2\right)}{3c^2d^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-2*a*e^2 + c*d*(3*d + e*x)))/(3*c^2*d^2*S
qrt[d + e*x])
```

Maple [A] time = 0.041, size = 69, normalized size = 0.6

$$-\frac{(2\,cdx+2\,ae)\left(-cdex+2\,ae^2-3\,cd^2\right)}{3\,c^2d^2}\sqrt{ex+d}\frac{1}{\sqrt{cdex^2+ae^2x+cd^2x+ade}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(-c*d*e*x+2*a*e^2-3*c*d^2)*(e*x+d)^(1/2)/c^2/d^2/(c*d*e*x^
2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

Maxima [A] time = 1.09584, size = 88, normalized size = 0.81

$$\frac{2\left(c^2d^2ex^2+3\,acd^2e-2\,a^2e^3+\left(3\,c^2d^3-acde^2\right)x\right)}{3\,\sqrt{cdx+ae}c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="maxima")
```

```
[Out] 2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)/(
sqrt(c*d*x + a*e)*c^2*d^2)
```

Fricas [A] time = 1.81897, size = 158, normalized size = 1.45

$$\frac{2\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(cdex+3\,cd^2-2\,ae^2\right)\sqrt{ex+d}}{3\,\left(c^2d^2ex+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="fricas")
```

```
[Out] 2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*e*x + 3*c*d^2 - 2*a*e^
2)*sqrt(e*x + d)/(c^2*d^2*e*x + c^2*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((e*x + d)^(3/2)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x), x)
```

3.2060 $\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=46

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*Sqrt[d + e*x])
```

Rubi [A] time = 0.019753, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {648}

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*Sqrt[d + e*x])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx=\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cd\sqrt{d+ex}}$$

Mathematica [A] time = 0.0173811, size = 35, normalized size = 0.76

$$\frac{2\sqrt{(d+ex)(ae+cdx)}}{cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)])/(c*d*Sqrt[d + e*x])
```

Maple [A] time = 0.042, size = 50, normalized size = 1.1

$$2\frac{(cdx+ae)\sqrt{ex+d}}{cd\sqrt{cde\,x^2+ae^2x+cd^2x+ade}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] 2*(c*d*x+a*e)*(e*x+d)^(1/2)/d/c/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

Maxima [A] time = 1.07004, size = 24, normalized size = 0.52

$$\frac{2\sqrt{cdx+ae}}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="maxima")
```

```
[Out] 2*sqrt(c*d*x + a*e)/(c*d)
```

Fricas [A] time = 1.96065, size = 107, normalized size = 2.33

$$\frac{2\sqrt{cde\,x^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}}{cdex+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="fricas")
```

```
[Out] 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c*d*e*x + c*d^
2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.2061 $\int \frac{1}{\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=84

$$\frac{2\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{\sqrt{e}\sqrt{cd^2-ae^2}}$$

```
[Out] (2*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2
 - a*e^2]*Sqrt[d + e*x])])/(Sqrt[e]*Sqrt[c*d^2 - a*e^2])
```

Rubi [A] time = 0.0411245, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {660, 205}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{\sqrt{e}\sqrt{cd^2-ae^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (2*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2
 - a*e^2]*Sqrt[d + e*x])])/(Sqrt[e]*Sqrt[c*d^2 - a*e^2])
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = (2e)\,\text{Subst}\left(\int \frac{1}{2cd^2e-e\left(cd^2+ae^2\right)+e^2x^2}\,dx, x, \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+}}{\sqrt{d+ex}}\right.$$

$$=\frac{2\tan^{-1}\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cd^2-ae^2}\sqrt{d+ex}}\right)}{\sqrt{e}\sqrt{cd^2-ae^2}}$$

Mathematica [A] time = 0.0351118, size = 97, normalized size = 1.15

$$\frac{2\sqrt{d+ex}\sqrt{ae+cdx}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd^2-ae^2}}\right)}{\sqrt{e}\sqrt{cd^2-ae^2}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTan[(Sqrt[e]*Sqrt[a*e + c*d*x])/Sqrt[
c*d^2 - a*e^2]])/(Sqrt[e]*Sqrt[c*d^2 - a*e^2]*Sqrt[(a*e + c*d*x)*(d + e*x)]
)
```

Maple [A] time = 0.247, size = 91, normalized size = 1.1

$$-2\,{\frac {\sqrt {cdex^{2}+ae^{2}x+cd^{2}x+ade}}{\sqrt {ex+d}\sqrt {cdx+ae}\sqrt {\left(ae^{2}-cd^{2} \right) e}}{\rm Artanh} \left({\frac {e\sqrt {cdx+ae}}{\sqrt {\left(ae^{2}-cd^{2} \right) e}}} \right) }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] -2/(e*x+d)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/(c*d*x+a*e)^(1/2)/
((a*e^2-c*d^2)*e)^(1/2)*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cdex^{2} + ade + \left(cd^{2} + ae^{2}\right)x}\sqrt{ex + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate(1/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)), x)
```

Fricas [A] time = 2.2922, size = 500, normalized size = 5.95

$$\left[-\frac{\sqrt{-cd^{2}e + ae^{3}}\log\left(-\frac{cde^{2}x^{2} + 2\,ae^{3}x - cd^{3} + 2\,ade^{2} - 2\,\sqrt{cdex^{2} + ade + \left(cd^{2} + ae^{2}\right)x}\sqrt{-cd^{2}e + ae^{3}}\sqrt{ex + d}}{e^{2}x^{2} + 2\,dex + d^{2}}\right)}{cd^{2}e - ae^{3}}, -\frac{2\arctan\left(\frac{\sqrt{cdex^{2} + ade + \left(cd^{2} + ae^{2}\right)x}\sqrt{cd^{2}e \cdots}}{cde^{2}x^{2} + ade^{2} + \left(cd^{2}e + a \cdots\right.}\right)}{\sqrt{cd^{2}e - ae^{3}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algori
thm="fricas")
```

```
[Out] [-sqrt(-c*d^2*e + a*e^3)*log(-(c*d*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2
- 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d^2*e + a*e^3)*sqrt
(e*x + d))/(e^2*x^2 + 2*d*e*x + d^2))/(c*d^2*e - a*e^3), -2*arctan(sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d^2*e - a*e^3)*sqrt(e*x + d)/(c*
```

```
d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x))/sqrt(c*d^2*e - a*e^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(d+ex)\,(ae+cdx)}\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((d + e*x)*(a*e + c*d*x))*sqrt(d + e*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algori
thm="giac")
```

```
[Out] Timed out
```

3.2062 $\int \frac{1}{(d+ex)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=139

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)^{3/2}\left(cd^2-ae^2\right)}+\frac{cd\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{\sqrt{e}\left(cd^2-ae^2\right)^{3/2}}$$

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d^2 - a*e^2)*(d + e*x)^(3/2
)) + (c*d*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqr
t[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(Sqrt[e]*(c*d^2 - a*e^2)^(3/2))
```

Rubi [A] time = 0.0766746, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {672, 660, 205}

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)^{3/2}\left(cd^2-ae^2\right)}+\frac{cd\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{\sqrt{e}\left(cd^2-ae^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d^2 - a*e^2)*(d + e*x)^(3/2
)) + (c*d*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqr
t[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(Sqrt[e]*(c*d^2 - a*e^2)^(3/2))
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*(m + 2*p + 2))/((m + p + 1)*(2*c*d - b*e)), Int[(d
 + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && Ne
Q[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(cd^2-ae^2)\,(d+ex)^{3/2}} + \frac{(cd)\int \frac{1}{\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{2\,(cd^2-ae^2)}$$

$$= \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(cd^2-ae^2)\,(d+ex)^{3/2}} + \frac{(cde)\,\text{Subst}\left(\int \frac{1}{2cd^2e-e(cd^2+ae^2)+e^2x^2}\,dx, x\right.}{cd^2-ae^2}$$

$$= \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(cd^2-ae^2)\,(d+ex)^{3/2}} + \frac{cd\tan^{-1}\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cd^2-ae^2}\sqrt{d+ex}}\right)}{\sqrt{e}\,(cd^2-ae^2)^{3/2}}$$

Mathematica [A] time = 0.0832971, size = 135, normalized size = 0.97

$$\frac{\sqrt{e}\sqrt{cd^2-ae^2}(ae+cdx)+cd(d+ex)\sqrt{ae+cdx}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd^2-ae^2}}\right)}{\sqrt{e}\sqrt{d+ex}\,(cd^2-ae^2)^{3/2}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x
]
```

```
[Out] (Sqrt[e]*Sqrt[c*d^2 - a*e^2]*(a*e + c*d*x) + c*d*Sqrt[a*e + c*d*x]*(d + e*x
)*ArcTan[(Sqrt[e]*Sqrt[a*e + c*d*x])/Sqrt[c*d^2 - a*e^2]])/(Sqrt[e]*(c*d^2
- a*e^2)^(3/2)*Sqrt[d + e*x]*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] time = 0.242, size = 172, normalized size = 1.2

$$\frac{1}{ae^2-cd^2}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(\text{Artanh}\left(e\sqrt{cdx+ae}\frac{1}{\sqrt{(ae^2-cd^2)\,e}}\right)xcde+\text{Artanh}\left(e\sqrt{cdx+ae}\frac{1}{\sqrt{(ae^2-cd^2)\,e}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] (c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^
2-c*d^2)*e)^(1/2))*x*c*d*e+arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1
/2))*c*d^2-(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2))/(e*x+d)^(3/2)/(c*d*x+
a*e)^(1/2)/(a*e^2-c*d^2)/((a*e^2-c*d^2)*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate(1/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)^(3/2)),
x)
```

Fricas [B] time = 2.31989, size = 1160, normalized size = 8.35

$$\left[\frac{\left(cde^2x^2 + 2\,cd^2ex + cd^3\right)\sqrt{-cd^2e + ae^3}\log\left(-\frac{cde^2x^2+2\,ae^3x-cd^3+2\,ade^2+2\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{-cd^2e+ae^3}\sqrt{ex+d}}{e^2x^2+2\,dex+d^2}\right) + 2\,\sqrt{cdex^2 +}}{2\left(c^2d^6e - 2\,acd^4e^3 + a^2d^2e^5 + \left(c^2d^4e^3 - 2\,acd^2e^5 + a^2e^7\right)x^2 + 2\left(c^2d^5e^2 - 2\,acd^3e^4 + a\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algori
thm="fricas")
```

```
[Out] [1/2*((c*d*e^2*x^2 + 2*c*d^2*e*x + c*d^3)*sqrt(-c*d^2*e + a*e^3)*log(-(c*d*
e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2 + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*sqrt(-c*d^2*e + a*e^3)*sqrt(e*x + d))/(e^2*x^2 + 2*d*e*x + d^2
)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d^2*e - a*e^3)*sqrt(e
*x + d))/(c^2*d^6*e - 2*a*c*d^4*e^3 + a^2*d^2*e^5 + (c^2*d^4*e^3 - 2*a*c*d^
2*e^5 + a^2*e^7)*x^2 + 2*(c^2*d^5*e^2 - 2*a*c*d^3*e^4 + a^2*d*e^6)*x), -((c
*d*e^2*x^2 + 2*c*d^2*e*x + c*d^3)*sqrt(c*d^2*e - a*e^3)*arctan(sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d^2*e - a*e^3)*sqrt(e*x + d)/(c*d*e^
2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x)) - sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*(c*d^2*e - a*e^3)*sqrt(e*x + d))/(c^2*d^6*e - 2*a*c*d^4*e^3 + a^
2*d^2*e^5 + (c^2*d^4*e^3 - 2*a*c*d^2*e^5 + a^2*e^7)*x^2 + 2*(c^2*d^5*e^2 -
2*a*c*d^3*e^4 + a^2*d*e^6)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(d + ex)(ae + cdx)}\,(d + ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)**(3/2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algori
thm="giac")
```

```
[Out] Timed out
```

3.2063 $\int \frac{1}{(d+ex)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=207

$$\frac{3c^2d^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{4\sqrt{e}\left(cd^2-ae^2\right)^{5/2}}+\frac{3cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4(d+ex)^{3/2}\left(cd^2-ae^2\right)^2}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2(d+ex)^{5/2}\left(cd^2-ae^2\right)}$$

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*(c*d^2 - a*e^2)*(d + e*x)^(5
/2)) + (3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*(c*d^2 - a*e^
2)^2*(d + e*x)^(3/2)) + (3*c^2*d^2*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(4*Sqrt[e]*(c*d^
2 - a*e^2)^(5/2))
```

Rubi [A] time = 0.113309, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {672, 660, 205}

$$\frac{3c^2d^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{4\sqrt{e}\left(cd^2-ae^2\right)^{5/2}}+\frac{3cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4(d+ex)^{3/2}\left(cd^2-ae^2\right)^2}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2(d+ex)^{5/2}\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*(c*d^2 - a*e^2)*(d + e*x)^(5
/2)) + (3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*(c*d^2 - a*e^
2)^2*(d + e*x)^(3/2)) + (3*c^2*d^2*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(4*Sqrt[e]*(c*d^
2 - a*e^2)^(5/2))
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*(m + 2*p + 2))/((m + p + 1)*(2*c*d - b*e)), Int[(d
 + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && Ne
Q[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2\left(cd^2-ae^2\right)(d+ex)^{5/2}} + \frac{(3cd)\int \frac{1}{(d+ex)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,c}{4\left(cd^2-ae^2\right)}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2\left(cd^2-ae^2\right)(d+ex)^{5/2}} + \frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4\left(cd^2-ae^2\right)^2(d+ex)^{3/2}} +$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2\left(cd^2-ae^2\right)(d+ex)^{5/2}} + \frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4\left(cd^2-ae^2\right)^2(d+ex)^{3/2}} +$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2\left(cd^2-ae^2\right)(d+ex)^{5/2}} + \frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4\left(cd^2-ae^2\right)^2(d+ex)^{3/2}} +$$

Mathematica [C] time = 0.0316992, size = 81, normalized size = 0.39

$$\frac{2c^2d^2\sqrt{(d+ex)(ae+cdx)}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{\sqrt{d+ex}\left(cd^2-ae^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x
]
```

```
[Out] (2*c^2*d^2*Sqrt[(a*e + c*d*x)*(d + e*x)]*Hypergeometric2F1[1/2, 3, 3/2, (e*
(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/((c*d^2 - a*e^2)^3*Sqrt[d + e*x])
```

Maple [A] time = 0.243, size = 292, normalized size = 1.4

$$-\frac{1}{4\left(ae^2-cd^2\right)^2}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(3\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)x^2c^2d^2e^2+6\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)xc^2d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] -1/4*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh(e*(c*d*x+a*e)^(1/2)
/((a*e^2-c*d^2)*e)^(1/2))*x^2*c^2*d^2*e^2+6*arctanh(e*(c*d*x+a*e)^(1/2)/((a
*e^2-c*d^2)*e)^(1/2))*x*c^2*d^3*e+3*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d
^2)*e)^(1/2))*c^2*d^4-3*x*c*d*e*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+2
*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*a*e^2-5*((a*e^2-c*d^2)*e)^(1/2)*
(c*d*x+a*e)^(1/2)*c*d^2)/(e*x+d)^(5/2)/(c*d*x+a*e)^(1/2)/(a*e^2-c*d^2)^2/((
a*e^2-c*d^2)*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(ex + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate(1/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)^(5/2)),
x)
```

Fricas [B] time = 2.28048, size = 1758, normalized size = 8.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algori
thm="fricas")
```

```
[Out] [-1/8*(3*(c^2*d^2*e^3*x^3 + 3*c^2*d^3*e^2*x^2 + 3*c^2*d^4*e*x + c^2*d^5)*sq
rt(-c*d^2*e + a*e^3)*log(-(c*d*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2 - 2*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d^2*e + a*e^3)*sqrt(e*x
 + d))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(5*c^2*d^4*e - 7*a*c*d^2*e^3 + 2*a^2*
e^5 + 3*(c^2*d^3*e^2 - a*c*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)*sqrt(e*x + d))/(c^3*d^9*e - 3*a*c^2*d^7*e^3 + 3*a^2*c*d^5*e^5 - a^3*d
^3*e^7 + (c^3*d^6*e^4 - 3*a*c^2*d^4*e^6 + 3*a^2*c*d^2*e^8 - a^3*e^10)*x^3 +
 3*(c^3*d^7*e^3 - 3*a*c^2*d^5*e^5 + 3*a^2*c*d^3*e^7 - a^3*d*e^9)*x^2 + 3*(c
^3*d^8*e^2 - 3*a*c^2*d^6*e^4 + 3*a^2*c*d^4*e^6 - a^3*d^2*e^8)*x), -1/4*(3*(
c^2*d^2*e^3*x^3 + 3*c^2*d^3*e^2*x^2 + 3*c^2*d^4*e*x + c^2*d^5)*sqrt(c*d^2*e
 - a*e^3)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d^2*e -
 a*e^3)*sqrt(e*x + d)/(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x)) - (5*c
^2*d^4*e - 7*a*c*d^2*e^3 + 2*a^2*e^5 + 3*(c^2*d^3*e^2 - a*c*d*e^4)*x)*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^9*e - 3*a*c^2*
d^7*e^3 + 3*a^2*c*d^5*e^5 - a^3*d^3*e^7 + (c^3*d^6*e^4 - 3*a*c^2*d^4*e^6 +
3*a^2*c*d^2*e^8 - a^3*e^10)*x^3 + 3*(c^3*d^7*e^3 - 3*a*c^2*d^5*e^5 + 3*a^2*
c*d^3*e^7 - a^3*d*e^9)*x^2 + 3*(c^3*d^8*e^2 - 3*a*c^2*d^6*e^4 + 3*a^2*c*d^4
*e^6 - a^3*d^2*e^8)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algori
thm="giac")
```

```
[Out] sage0*x
```

3.2064 $\int \frac{1}{(d+ex)^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=269

$$\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8(d+ex)^{3/2}\left(cd^2-ae^2\right)^3}+\frac{5c^3d^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{8\sqrt{e}\left(cd^2-ae^2\right)^{7/2}}+\frac{5cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{12(d+ex)^{5/2}\left(cd^2-ae^2\right)^2}+\frac{\sqrt{x(ae}}{3(d}$$

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(3*(c*d^2 - a*e^2)*(d + e*x)^(7
/2)) + (5*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*(c*d^2 - a*e
^2)^2*(d + e*x)^(5/2)) + (5*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(8*(c*d^2 - a*e^2)^3*(d + e*x)^(3/2)) + (5*c^3*d^3*ArcTan[(Sqrt[e]*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*
x])])/(8*Sqrt[e]*(c*d^2 - a*e^2)^(7/2))
```

Rubi [A] time = 0.166055, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {672, 660, 205}

$$\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8(d+ex)^{3/2}\left(cd^2-ae^2\right)^3}+\frac{5c^3d^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{8\sqrt{e}\left(cd^2-ae^2\right)^{7/2}}+\frac{5cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{12(d+ex)^{5/2}\left(cd^2-ae^2\right)^2}+\frac{\sqrt{x(ae}}{3(d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(3*(c*d^2 - a*e^2)*(d + e*x)^(7
/2)) + (5*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*(c*d^2 - a*e
^2)^2*(d + e*x)^(5/2)) + (5*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(8*(c*d^2 - a*e^2)^3*(d + e*x)^(3/2)) + (5*c^3*d^3*ArcTan[(Sqrt[e]*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*
x])])/(8*Sqrt[e]*(c*d^2 - a*e^2)^(7/2))
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*(m + 2*p + 2))/((m + p + 1)*(2*c*d - b*e)), Int[(d
 + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && Ne
Q[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3\left(cd^2-ae^2\right)(d+ex)^{7/2}} + \frac{(5cd)\int \frac{1}{(d+ex)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,d}{6\left(cd^2-ae^2\right)}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3\left(cd^2-ae^2\right)(d+ex)^{7/2}} + \frac{5cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12\left(cd^2-ae^2\right)^2(d+ex)^{5/2}} +$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3\left(cd^2-ae^2\right)(d+ex)^{7/2}} + \frac{5cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12\left(cd^2-ae^2\right)^2(d+ex)^{5/2}} +$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3\left(cd^2-ae^2\right)(d+ex)^{7/2}} + \frac{5cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12\left(cd^2-ae^2\right)^2(d+ex)^{5/2}} +$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3\left(cd^2-ae^2\right)(d+ex)^{7/2}} + \frac{5cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12\left(cd^2-ae^2\right)^2(d+ex)^{5/2}} +$$

Mathematica [C] time = 0.0318227, size = 81, normalized size = 0.3

$$\frac{2c^3d^3\sqrt{(d+ex)(ae+cdx)}\,{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{\sqrt{d+ex}\left(cd^2-ae^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x
]
```

```
[Out] (2*c^3*d^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*Hypergeometric2F1[1/2, 4, 3/2, (e*
(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/((c*d^2 - a*e^2)^4*Sqrt[d + e*x])
```

Maple [A] time = 0.243, size = 454, normalized size = 1.7

$$\frac{1}{24\left(ae^2-cd^2\right)^3}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(15\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)x^3c^3d^3e^3+45\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] 1/24*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh(e*(c*d*x+a*e)^(1/2
)/((a*e^2-c*d^2)*e)^(1/2))*x^3*c^3*d^3*e^3+45*arctanh(e*(c*d*x+a*e)^(1/2)/(
(a*e^2-c*d^2)*e)^(1/2))*x^2*c^3*d^4*e^2+45*arctanh(e*(c*d*x+a*e)^(1/2)/((a*
e^2-c*d^2)*e)^(1/2))*x*c^3*d^5*e+15*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d
^2)*e)^(1/2))*c^3*d^6-15*x^2*c^2*d^2*e^2*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e
)^(1/2)+10*x*a*c*d*e^3*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)-40*x*c^2*d
^3*e*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)-8*((a*e^2-c*d^2)*e)^(1/2)*(c
```

```
*d*x+a*e)^(1/2)*a^2*e^4+26*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d^
2*e^2-33*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^4)/(e*x+d)^(7/2)/(
c*d*x+a*e)^(1/2)/(a*e^2-c*d^2)^3/((a*e^2-c*d^2)*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(ex + d)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate(1/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)^(7/2)),
x)
```

Fricas [B] time = 2.4491, size = 2535, normalized size = 9.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algori
thm="fricas")
```

```
[Out] [-1/48*(15*(c^3*d^3*e^4*x^4 + 4*c^3*d^4*e^3*x^3 + 6*c^3*d^5*e^2*x^2 + 4*c^3
*d^6*e*x + c^3*d^7)*sqrt(-c*d^2*e + a*e^3)*log(-(c*d*e^2*x^2 + 2*a*e^3*x -
c*d^3 + 2*a*d*e^2 - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d
^2*e + a*e^3)*sqrt(e*x + d))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(33*c^3*d^6*e -
 59*a*c^2*d^4*e^3 + 34*a^2*c*d^2*e^5 - 8*a^3*e^7 + 15*(c^3*d^4*e^3 - a*c^2*
d^2*e^5)*x^2 + 10*(4*c^3*d^5*e^2 - 5*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x)*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^4*d^12*e - 4*a*c^3*
d^10*e^3 + 6*a^2*c^2*d^8*e^5 - 4*a^3*c*d^6*e^7 + a^4*d^4*e^9 + (c^4*d^8*e^5
 - 4*a*c^3*d^6*e^7 + 6*a^2*c^2*d^4*e^9 - 4*a^3*c*d^2*e^11 + a^4*e^13)*x^4 +
 4*(c^4*d^9*e^4 - 4*a*c^3*d^7*e^6 + 6*a^2*c^2*d^5*e^8 - 4*a^3*c*d^3*e^10 +
a^4*d*e^12)*x^3 + 6*(c^4*d^10*e^3 - 4*a*c^3*d^8*e^5 + 6*a^2*c^2*d^6*e^7 - 4
*a^3*c*d^4*e^9 + a^4*d^2*e^11)*x^2 + 4*(c^4*d^11*e^2 - 4*a*c^3*d^9*e^4 + 6*
a^2*c^2*d^7*e^6 - 4*a^3*c*d^5*e^8 + a^4*d^3*e^10)*x), -1/24*(15*(c^3*d^3*e^
4*x^4 + 4*c^3*d^4*e^3*x^3 + 6*c^3*d^5*e^2*x^2 + 4*c^3*d^6*e*x + c^3*d^7)*sq
rt(c*d^2*e - a*e^3)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt
(c*d^2*e - a*e^3)*sqrt(e*x + d)/(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*
x)) - (33*c^3*d^6*e - 59*a*c^2*d^4*e^3 + 34*a^2*c*d^2*e^5 - 8*a^3*e^7 + 15*
(c^3*d^4*e^3 - a*c^2*d^2*e^5)*x^2 + 10*(4*c^3*d^5*e^2 - 5*a*c^2*d^3*e^4 + a
^2*c*d*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(
c^4*d^12*e - 4*a*c^3*d^10*e^3 + 6*a^2*c^2*d^8*e^5 - 4*a^3*c*d^6*e^7 + a^4*d
^4*e^9 + (c^4*d^8*e^5 - 4*a*c^3*d^6*e^7 + 6*a^2*c^2*d^4*e^9 - 4*a^3*c*d^2*e
^11 + a^4*e^13)*x^4 + 4*(c^4*d^9*e^4 - 4*a*c^3*d^7*e^6 + 6*a^2*c^2*d^5*e^8
- 4*a^3*c*d^3*e^10 + a^4*d*e^12)*x^3 + 6*(c^4*d^10*e^3 - 4*a*c^3*d^8*e^5 +
6*a^2*c^2*d^6*e^7 - 4*a^3*c*d^4*e^9 + a^4*d^2*e^11)*x^2 + 4*(c^4*d^11*e^2 -
 4*a*c^3*d^9*e^4 + 6*a^2*c^2*d^7*e^6 - 4*a^3*c*d^5*e^8 + a^4*d^3*e^10)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algori
thm="giac")
```

```
[Out] sage0*x
```

3.2065 $\int \frac{(d+ex)^{7/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=171

$$\frac{8(d+ex)^{3/2}\left(cd^2-ae^2\right)}{3c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{16\sqrt{d+ex}\left(cd^2-ae^2\right)^2}{3c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{2(d+ex)^{5/2}}{3cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

```
[Out] (-16*(c*d^2 - a*e^2)^2*Sqrt[d + e*x])/(3*c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2]) + (8*(c*d^2 - a*e^2)*(d + e*x)^(3/2))/(3*c^2*d^2*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (2*(d + e*x)^(5/2))/(3*c*d*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.116254, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {656, 648}

$$\frac{8(d+ex)^{3/2}\left(cd^2-ae^2\right)}{3c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{16\sqrt{d+ex}\left(cd^2-ae^2\right)^2}{3c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{2(d+ex)^{5/2}}{3cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (-16*(c*d^2 - a*e^2)^2*Sqrt[d + e*x])/(3*c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2]) + (8*(c*d^2 - a*e^2)*(d + e*x)^(3/2))/(3*c^2*d^2*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (2*(d + e*x)^(5/2))/(3*c*d*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{7/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2(d+ex)^{5/2}}{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{\left(4\left(d^2-\frac{ae^2}{c}\right)\right)\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{3d}$$

$$= \frac{8\left(cd^2-ae^2\right)(d+ex)^{3/2}}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{2(d+ex)^{5/2}}{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \ldots$$

$$= -\frac{16\left(cd^2-ae^2\right)^2\sqrt{d+ex}}{3c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{8\left(cd^2-ae^2\right)(d+ex)^{3/2}}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \ldots$$

Mathematica [A] time = 0.0599896, size = 87, normalized size = 0.51

$$-\frac{2\sqrt{d+ex}\left(8a^2e^4+4acde^2(ex-3d)+c^2d^2\left(3d^2-6dex-e^2x^2\right)\right)}{3c^3d^3\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(8*a^2*e^4 + 4*a*c*d*e^2*(-3*d + e*x) + c^2*d^2*(3*d^2 -
6*d*e*x - e^2*x^2)))/(3*c^3*d^3*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] time = 0.044, size = 110, normalized size = 0.6

$$-\frac{(2\,cdx+2\,ae)\left(-e^2x^2c^2d^2+4\,acde^3x-6\,c^2d^3ex+8\,a^2e^4-12\,acd^2e^2+3\,c^2d^4\right)}{3\,c^3d^3}(ex+d)^{\frac{3}{2}}\left(cdex^2+ae^2x+cd^2x+ade\right)^{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(-c^2*d^2*e^2*x^2+4*a*c*d*e^3*x-6*c^2*d^3*e*x+8*a^2*e^4-12
*a*c*d^2*e^2+3*c^2*d^4)*(e*x+d)^(3/2)/c^3/d^3/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*
d*e)^(3/2)
```

Maxima [A] time = 1.05948, size = 107, normalized size = 0.63

$$\frac{2\left(c^2d^2e^2x^2-3\,c^2d^4+12\,acd^2e^2-8\,a^2e^4+2\left(3\,c^2d^3e-2\,acde^3\right)x\right)}{3\sqrt{cdx+ae}c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="maxima")
```

```
[Out] 2/3*(c^2*d^2*e^2*x^2 - 3*c^2*d^4 + 12*a*c*d^2*e^2 - 8*a^2*e^4 + 2*(3*c^2*d^
3*e - 2*a*c*d*e^3)*x)/(sqrt(c*d*x + a*e)*c^3*d^3)
```

Fricas [A] time = 2.18425, size = 292, normalized size = 1.71

$$\frac{2\left(c^2d^2e^2x^2-3\,c^2d^4+12\,acd^2e^2-8\,a^2e^4+2\left(3\,c^2d^3e-2\,acde^3\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}}{3\left(c^4d^4ex^2+ac^3d^4e+\left(c^4d^5+ac^3d^3e^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="fricas")
```

```
[Out] 2/3*(c^2*d^2*e^2*x^2 - 3*c^2*d^4 + 12*a*c*d^2*e^2 - 8*a^2*e^4 + 2*(3*c^2*d^
3*e - 2*a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x
+ d)/(c^4*d^4*e*x^2 + a*c^3*d^4*e + (c^4*d^5 + a*c^3*d^3*e^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="giac")
```

```
[Out] sage0*x
```

3.2066 $\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=105

$$\frac{2(d+ex)^{3/2}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{4\sqrt{d+ex}\left(cd^2-ae^2\right)}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

```
[Out] (-4*(c*d^2 - a*e^2)*Sqrt[d + e*x])/(c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2]) + (2*(d + e*x)^(3/2))/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2])
```

Rubi [A] time = 0.0581765, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {656, 648}

$$\frac{2(d+ex)^{3/2}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{4\sqrt{d+ex}\left(cd^2-ae^2\right)}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (-4*(c*d^2 - a*e^2)*Sqrt[d + e*x])/(c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2]) + (2*(d + e*x)^(3/2))/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2])
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2(d+ex)^{3/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{\left(2\left(2cd^2e-e\left(cd^2+ae^2\right)\right)\right)\int \frac{(d+ex)}{\left(ade+\left(cd^2+ae^2\right)\right.}}{cde}$$

$$=-\frac{4\left(cd^2-ae^2\right)\sqrt{d+ex}}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{2(d+ex)^{3/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

Mathematica [A] time = 0.0339236, size = 51, normalized size = 0.49

$$-\frac{2\sqrt{d+ex}\left(cd(d-ex)-2ae^2\right)}{c^2d^2\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(-2*a*e^2 + c*d*(d - e*x)))/(c^2*d^2*Sqrt[(a*e + c*d*x)*(
d + e*x)])
```

Maple [A] time = 0.041, size = 68, normalized size = 0.7

$$2\,\frac{(cdx+ae)\left(cdex+2\,ae^2-cd^2\right)(ex+d)^{3/2}}{c^2d^2\left(cdex^2+ae^2x+cd^2x+ade\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] 2*(c*d*x+a*e)*(c*d*e*x+2*a*e^2-c*d^2)*(e*x+d)^(3/2)/c^2/d^2/(c*d*e*x^2+a*e^
2*x+c*d^2*x+a*d*e)^(3/2)
```

Maxima [A] time = 1.12429, size = 49, normalized size = 0.47

$$\frac{2\left(cdex-cd^2+2\,ae^2\right)}{\sqrt{cdx+ae}c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="maxima")
```

```
[Out] 2*(c*d*e*x - c*d^2 + 2*a*e^2)/(sqrt(c*d*x + a*e)*c^2*d^2)
```

Fricas [A] time = 2.04646, size = 201, normalized size = 1.91

$$\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(cdex-cd^2+2\,ae^2\right)\sqrt{ex+d}}{c^3d^3ex^2+ac^2d^3e+\left(c^3d^4+ac^2d^2e^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="fricas")
```

```
[Out] 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*e*x - c*d^2 + 2*a*e^2)*s
qrt(e*x + d)/(c^3*d^3*e*x^2 + a*c^2*d^3*e + (c^3*d^4 + a*c^2*d^2*e^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="giac")
```

```
[Out] sage0*x
```

3.2067 $\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=46

$$-\frac{2\sqrt{d+ex}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

```
[Out] (-2*Sqrt[d + e*x])/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.0205415, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {648}

$$-\frac{2\sqrt{d+ex}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x])/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2\sqrt{d+ex}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

Mathematica [A] time = 0.0131777, size = 35, normalized size = 0.76

$$-\frac{2\sqrt{d+ex}}{cd\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x])/(c*d*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] time = 0.042, size = 50, normalized size = 1.1

$$-2\frac{(cdx+ae)(ex+d)^{3/2}}{cd\left(cdex^2+ae^2x+cd^2x+ade\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] -2*(c*d*x+a*e)*(e*x+d)^(3/2)/d/c/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

Maxima [A] time = 1.09472, size = 24, normalized size = 0.52

$$-\frac{2}{\sqrt{cdx+ae}cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="maxima")
```

```
[Out] -2/(sqrt(c*d*x + a*e)*c*d)
```

Fricas [A] time = 2.1129, size = 157, normalized size = 3.41

$$-\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}}{c^2d^2ex^2+acd^2e+\left(c^2d^3+acde^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="fricas")
```

```
[Out] -2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^2*d^2*e*x^2
 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(d+ex)^{\frac{3}{2}}}{((d+ex)(ae+cdx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="giac")
```

```
[Out] sage0*x
```

3.2068 $\int \frac{\sqrt{d+ex}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=139

$$-\frac{2\sqrt{d+ex}}{\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{3/2}}$$

```
[Out] (-2*Sqrt[d + e*x])/((c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]) - (2*Sqrt[e]*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(c*d^2 - a*e^2)^(3/2)
```

Rubi [A] time = 0.0941894, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {666, 660, 205}

$$-\frac{2\sqrt{d+ex}}{\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x])/((c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]) - (2*Sqrt[e]*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(c*d^2 - a*e^2)^(3/2)
```

Rule 666

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((2*c*d - b*e)*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/(e*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[((2*c*d - b*e)*(m + 2*p + 2))/((p + 1)*(b^2
- 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0
] && LtQ[p, -1] && LtQ[0, m, 1] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{e\int \frac{1}{\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{cd^2-ae^2}$$

$$= -\frac{2\sqrt{d+ex}}{\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(2e^2\right)\text{Subst}\left(\int \frac{1}{2cd^2e-e\left(cd^2+ae^2\right)+e^2x^2}\right.}{cd^2-ae}$$

$$= -\frac{2\sqrt{d+ex}}{\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{2\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cd^2-ae^2}\sqrt{d+ex}}\right)}{\left(cd^2-ae^2\right)^{3/2}}$$

Mathematica [C] time = 0.0214777, size = 75, normalized size = 0.54

$$-\frac{2\sqrt{d+ex}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{\left(cd^2-ae^2\right)\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*Hypergeometric2F1[-1/2, 1, 1/2, (e*(a*e + c*d*x))/(-(c*d^
2) + a*e^2)])/((c*d^2 - a*e^2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] time = 0.207, size = 136, normalized size = 1.

$$-2\,\frac{\sqrt{cdex^2+ae^2x+cd^2x+ade}}{\sqrt{ex+d}\,(cdx+ae)\left(ae^2-cd^2\right)\sqrt{\left(ae^2-cd^2\right)e}}\left(e\,\text{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)\sqrt{cdx+ae}-\sqrt{\left(ae^2-cd^2\right)e}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] -2*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(e*arctanh(e*(c*d*x+a*e)^(1/2)/(
(a*e^2-c*d^2)*e)^(1/2))*(c*d*x+a*e)^(1/2)-((a*e^2-c*d^2)*e)^(1/2))/(e*x+d)^
(1/2)/(c*d*x+a*e)/(a*e^2-c*d^2)/((a*e^2-c*d^2)*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2), x)
```

Fricas [A] time = 2.15886, size = 1019, normalized size = 7.33

$$\left[-\frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)\sqrt{-\frac{e}{cd^2-ae^2}}\log\left(-\frac{cde^2x^2+2\,ae^3x-cd^3+2\,ade^2+2\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(cd^2-ae^2\right)\sqrt{ex+d}\sqrt{-\frac{e}{cd^2-ae^2}}}{e^2x^2+2\,dex+d^2}\right)+2\ldots}{acd^3e-a^2de^3+\left(c^2d^3e-acde^3\right)x^2+\left(c^2d^4-a^2e^4\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="fricas")
```

```
[Out] [-((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-e/(c*d^2 - a*e^2))*log(-(c
*d*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2 + 2*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(-e/(c*d^2 - a*e^2)))/(e^
2*x^2 + 2*d*e*x + d^2)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqr
t(e*x + d))/(a*c*d^3*e - a^2*d*e^3 + (c^2*d^3*e - a*c*d*e^3)*x^2 + (c^2*d^4
 - a^2*e^4)*x), -2*((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e/(c*d^2 -
 a*e^2))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d^2 - a*e^2
)*sqrt(e*x + d)*sqrt(e/(c*d^2 - a*e^2))/(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e +
 a*e^3)*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a
*c*d^3*e - a^2*d*e^3 + (c^2*d^3*e - a*c*d*e^3)*x^2 + (c^2*d^4 - a^2*e^4)*x)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}}{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="giac")
```

```
[Out] sage0*x
```

3.2069 $\int \frac{1}{\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=196

$$-\frac{3cd\sqrt{d+ex}}{\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{1}{\sqrt{d+ex}\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{3cd\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x(ae}}{\sqrt{d}}\right.}{\left(cd^2-ae^2\right.}$$

```
[Out] 1/((c*d^2 - a*e^2)*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]) - (3*c*d*Sqrt[d + e*x])/((c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2]) - (3*c*d*Sqrt[e]*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(c*d^2 - a*e^2)^(5
/2)
```

Rubi [A] time = 0.131511, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {672, 666, 660, 205}

$$-\frac{3cd\sqrt{d+ex}}{\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{1}{\sqrt{d+ex}\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{3cd\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x(ae}}{\sqrt{d}}\right.}{\left(cd^2-ae^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```


```
[Out] 1/((c*d^2 - a*e^2)*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]) - (3*c*d*Sqrt[d + e*x])/((c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2]) - (3*c*d*Sqrt[e]*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(c*d^2 - a*e^2)^(5
/2)
```


Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*(m + 2*p + 2))/((m + p + 1)*(2*c*d - b*e)), Int[(d
 + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && Ne
Q[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 666

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((2*c*d - b*e)*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/(e*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[((2*c*d - b*e)*(m + 2*p + 2))/((p + 1)*(b^2
- 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0
] && LtQ[p, -1] && LtQ[0, m, 1] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
```

```
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{1}{\left(cd^2-ae^2\right)\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{(3cd)\int \frac{}{\left(ade+\left(cd^2\right.\right.}}{2\left(cd^2\right.}$$

$$= \frac{1}{\left(cd^2-ae^2\right)\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{3}{\left(cd^2-ae^2\right)^2\sqrt{a}}$$

$$= \frac{1}{\left(cd^2-ae^2\right)\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{3}{\left(cd^2-ae^2\right)^2\sqrt{a}}$$

$$= \frac{1}{\left(cd^2-ae^2\right)\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{3}{\left(cd^2-ae^2\right)^2\sqrt{a}}$$

Mathematica [C] time = 0.023945, size = 77, normalized size = 0.39

$$-\frac{2cd\sqrt{d+ex}\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{\left(cd^2-ae^2\right)^2\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x
]
```

```
[Out] (-2*c*d*Sqrt[d + e*x]*Hypergeometric2F1[-1/2, 2, 1/2, (e*(a*e + c*d*x))/(-(
c*d^2) + a*e^2)])/((c*d^2 - a*e^2)^2*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] time = 0.214, size = 235, normalized size = 1.2

$$\frac{1}{(cdx+ae)\left(ae^2-cd^2\right)^2}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(3\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)\sqrt{cdx+a}excde^2+3\,\mathrm{Artanh}\left(\frac{e\sqrt{cd}}{\sqrt{\left(ae^2\right.}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] (c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh(e*(c*d*x+a*e)^(1/2)/((a*
e^2-c*d^2)*e)^(1/2))*(c*d*x+a*e)^(1/2)*x*c*d*e^2+3*arctanh(e*(c*d*x+a*e)^(1
```

```
/2)/((a*e^2-c*d^2)*e)^(1/2))*(c*d*x+a*e)^(1/2)*c*d^2*e-3*((a*e^2-c*d^2)*e)^
(1/2)*x*c*d*e-((a*e^2-c*d^2)*e)^(1/2)*a*e^2-2*((a*e^2-c*d^2)*e)^(1/2)*c*d^2
)/(e*x+d)^(3/2)/(c*d*x+a*e)/(a*e^2-c*d^2)^2/((a*e^2-c*d^2)*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}} \sqrt{ex + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algori
thm="maxima")
```

```
[Out] integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*sqrt(e*x + d)),
x)
```

Fricas [B] time = 2.25803, size = 1555, normalized size = 7.93

$$\left[\frac{3\left(c^2d^2e^2x^3 + acd^3e + \left(2\,c^2d^3e + acde^3\right)x^2 + \left(c^2d^4 + 2\,acd^2e^2\right)x\right)\sqrt{-\frac{e}{cd^2-ae^2}}\log\left(-\frac{cde^2x^2+2\,ae^3x-cd^3+2\,ade^2-2\,\sqrt{cdex^2+ade+\left(cd^2+ae\right.}}{e^2x^2+2\,dex+d^2}\right.}{2\left(ac^2d^6e - 2\,a^2cd^4e^3 + a^3d^2e^5 + \left(c^3d^5e^2 - 2\,ac^2d^3e^4 + a^2cde^6\right)x^3 + \left(2\,c^3d^6e - 3\,a\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algori
thm="fricas")
```

```
[Out] [1/2*(3*(c^2*d^2*e^2*x^3 + a*c*d^3*e + (2*c^2*d^3*e + a*c*d*e^3)*x^2 + (c^2
*d^4 + 2*a*c*d^2*e^2)*x)*sqrt(-e/(c*d^2 - a*e^2))*log(-(c*d*e^2*x^2 + 2*a*e
^3*x - c*d^3 + 2*a*d*e^2 - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c
*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(-e/(c*d^2 - a*e^2)))/(e^2*x^2 + 2*d*e*x +
d^2)) - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(3*c*d*e*x + 2*c*d^2
+ a*e^2)*sqrt(e*x + d))/(a*c^2*d^6*e - 2*a^2*c*d^4*e^3 + a^3*d^2*e^5 + (c^3
*d^5*e^2 - 2*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x^3 + (2*c^3*d^6*e - 3*a*c^2*d^4*
e^3 + a^3*e^7)*x^2 + (c^3*d^7 - 3*a^2*c*d^3*e^4 + 2*a^3*d*e^6)*x), -(3*(c^2
*d^2*e^2*x^3 + a*c*d^3*e + (2*c^2*d^3*e + a*c*d*e^3)*x^2 + (c^2*d^4 + 2*a*c
*d^2*e^2)*x)*sqrt(e/(c*d^2 - a*e^2))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(e/(c*d^2 - a*e^2))/(c*d*e^
2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*(3*c*d*e*x + 2*c*d^2 + a*e^2)*sqrt(e*x + d))/(a*c^2*d^6*e - 2*a^
2*c*d^4*e^3 + a^3*d^2*e^5 + (c^3*d^5*e^2 - 2*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x
^3 + (2*c^3*d^6*e - 3*a*c^2*d^4*e^3 + a^3*e^7)*x^2 + (c^3*d^7 - 3*a^2*c*d^3
*e^4 + 2*a^3*d*e^6)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\left(d + ex\right)\left(ae + cdx\right)\right)^{\frac{3}{2}} \sqrt{d + ex}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral(1/(((d + e*x)*(a*e + c*d*x))**(3/2)*sqrt(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algori
thm="giac")
```

```
[Out] sage0*x
```

3.2070 $\int \frac{1}{(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=269

$$-\frac{15c^2d^2\sqrt{d+ex}}{4\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{15c^2d^2\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{4\left(cd^2-ae^2\right)^{7/2}}+\frac{5cd}{4\sqrt{d+ex}\left(cd^2-ae^2\right)^2\sqrt{x\left(ae\right.}}$$

```
[Out] 1/(2*(c*d^2 - a*e^2)*(d + e*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]) + (5*c*d)/(4*(c*d^2 - a*e^2)^2*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2]) - (15*c^2*d^2*Sqrt[d + e*x])/(4*(c*d^2 - a*e^2)^3*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (15*c^2*d^2*Sqrt[e]*ArcTan[(Sqr
t[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt
[d + e*x])])/(4*(c*d^2 - a*e^2)^(7/2))
```

Rubi [A] time = 0.17749, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {672, 666, 660, 205}

$$-\frac{15c^2d^2\sqrt{d+ex}}{4\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{15c^2d^2\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{4\left(cd^2-ae^2\right)^{7/2}}+\frac{5cd}{4\sqrt{d+ex}\left(cd^2-ae^2\right)^2\sqrt{x\left(ae\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] 1/(2*(c*d^2 - a*e^2)*(d + e*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]) + (5*c*d)/(4*(c*d^2 - a*e^2)^2*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2]) - (15*c^2*d^2*Sqrt[d + e*x])/(4*(c*d^2 - a*e^2)^3*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (15*c^2*d^2*Sqrt[e]*ArcTan[(Sqr
t[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt
[d + e*x])])/(4*(c*d^2 - a*e^2)^(7/2))
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*(m + 2*p + 2))/((m + p + 1)*(2*c*d - b*e)), Int[(d
 + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && Ne
Q[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 666

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((2*c*d - b*e)*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/(e*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[((2*c*d - b*e)*(m + 2*p + 2))/((p + 1)*(b^2
- 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0
] && LtQ[p, -1] && LtQ[0, m, 1] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{1}{2\left(cd^2-ae^2\right)(d+ex)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{(5cd)\int -}{}$$

$$= \frac{1}{2\left(cd^2-ae^2\right)(d+ex)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{}{4\left(cd^2-\right.}$$

$$= \frac{1}{2\left(cd^2-ae^2\right)(d+ex)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{}{4\left(cd^2-\right.}$$

$$= \frac{1}{2\left(cd^2-ae^2\right)(d+ex)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{}{4\left(cd^2-\right.}$$

$$= \frac{1}{2\left(cd^2-ae^2\right)(d+ex)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{}{4\left(cd^2-\right.}$$

Mathematica [C] time = 0.0272629, size = 81, normalized size = 0.3

$$-\frac{2c^2d^2\sqrt{d+ex}\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{\left(cd^2-ae^2\right)^3\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))
,x]
```

```
[Out] (-2*c^2*d^2*Sqrt[d + e*x]*Hypergeometric2F1[-1/2, 3, 1/2, (e*(a*e + c*d*x))
/(-(c*d^2) + a*e^2)])/((c*d^2 - a*e^2)^3*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] time = 0.248, size = 384, normalized size = 1.4

$$-\frac{1}{(4\,cdx+4\,ae)\left(ae^2-cd^2\right)^3}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(15\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)\sqrt{cdx+ae}x^2c^2d^2e^3+30\,\mathrm{Artan}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] -1/4*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh(e*(c*d*x+a*e)^(1/2
)/((a*e^2-c*d^2)*e)^(1/2))*(c*d*x+a*e)^(1/2)*x^2*c^2*d^2*e^3+30*arctanh(e*(
c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*(c*d*x+a*e)^(1/2)*x*c^2*d^3*e^2+1
5*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*(c*d*x+a*e)^(1/2)*c^
2*d^4*e-15*((a*e^2-c*d^2)*e)^(1/2)*x^2*c^2*d^2*e^2-5*((a*e^2-c*d^2)*e)^(1/2
)*x*a*c*d*e^3-25*((a*e^2-c*d^2)*e)^(1/2)*x*c^2*d^3*e+2*((a*e^2-c*d^2)*e)^(1
/2)*a^2*e^4-9*((a*e^2-c*d^2)*e)^(1/2)*a*c*d^2*e^2-8*((a*e^2-c*d^2)*e)^(1/2)
*c^2*d^4)/(e*x+d)^(5/2)/(c*d*x+a*e)/(a*e^2-c*d^2)^3/((a*e^2-c*d^2)*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}(ex + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algori
thm="maxima")
```

```
[Out] integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)^(3/2))
, x)
```

Fricas [B] time = 2.47221, size = 2273, normalized size = 8.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algori
thm="fricas")
```

```
[Out] [1/8*(15*(c^3*d^3*e^3*x^4 + a*c^2*d^5*e + (3*c^3*d^4*e^2 + a*c^2*d^2*e^4)*x
^3 + 3*(c^3*d^5*e + a*c^2*d^3*e^3)*x^2 + (c^3*d^6 + 3*a*c^2*d^4*e^2)*x)*sqr
t(-e/(c*d^2 - a*e^2))*log(-(c*d*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2 - 2
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d^2 - a*e^2)*sqrt(e*x + d)*
sqrt(-e/(c*d^2 - a*e^2)))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(15*c^2*d^2*e^2*x^
2 + 8*c^2*d^4 + 9*a*c*d^2*e^2 - 2*a^2*e^4 + 5*(5*c^2*d^3*e + a*c*d*e^3)*x)*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a*c^3*d^9*e - 3
*a^2*c^2*d^7*e^3 + 3*a^3*c*d^5*e^5 - a^4*d^3*e^7 + (c^4*d^7*e^3 - 3*a*c^3*d
^5*e^5 + 3*a^2*c^2*d^3*e^7 - a^3*c*d*e^9)*x^4 + (3*c^4*d^8*e^2 - 8*a*c^3*d^
6*e^4 + 6*a^2*c^2*d^4*e^6 - a^4*e^10)*x^3 + 3*(c^4*d^9*e - 2*a*c^3*d^7*e^3
+ 2*a^3*c*d^3*e^7 - a^4*d*e^9)*x^2 + (c^4*d^10 - 6*a^2*c^2*d^6*e^4 + 8*a^3*
c*d^4*e^6 - 3*a^4*d^2*e^8)*x), -1/4*(15*(c^3*d^3*e^3*x^4 + a*c^2*d^5*e + (3
*c^3*d^4*e^2 + a*c^2*d^2*e^4)*x^3 + 3*(c^3*d^5*e + a*c^2*d^3*e^3)*x^2 + (c^
3*d^6 + 3*a*c^2*d^4*e^2)*x)*sqrt(e/(c*d^2 - a*e^2))*arctan(-sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(e/(c*d^2 -
a*e^2))/(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x)) + (15*c^2*d^2*e^2*x^
2 + 8*c^2*d^4 + 9*a*c*d^2*e^2 - 2*a^2*e^4 + 5*(5*c^2*d^3*e + a*c*d*e^3)*x)*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a*c^3*d^9*e - 3
*a^2*c^2*d^7*e^3 + 3*a^3*c*d^5*e^5 - a^4*d^3*e^7 + (c^4*d^7*e^3 - 3*a*c^3*d
^5*e^5 + 3*a^2*c^2*d^3*e^7 - a^3*c*d*e^9)*x^4 + (3*c^4*d^8*e^2 - 8*a*c^3*d^
6*e^4 + 6*a^2*c^2*d^4*e^6 - a^4*e^10)*x^3 + 3*(c^4*d^9*e - 2*a*c^3*d^7*e^3
```

```
+ 2*a^3*c*d^3*e^7 - a^4*d*e^9)*x^2 + (c^4*d^10 - 6*a^2*c^2*d^6*e^4 + 8*a^3*
c*d^4*e^6 - 3*a^4*d^2*e^8)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((d+ex)\left(ae+cdx\right)\right)^{\frac{3}{2}}\left(d+ex\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral(1/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, 2]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algori
thm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, undef, undef, 2]
```

3.2071 $\int \frac{1}{(d+ex)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=331

$$-\frac{35c^3d^3\sqrt{d+ex}}{8\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{35c^2d^2}{24\sqrt{d+ex}\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{35c^3d^3\sqrt{e}\tan^{-1}}{8(c}$$

```
[Out] 1/(3*(c*d^2 - a*e^2)*(d + e*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]) + (7*c*d)/(12*(c*d^2 - a*e^2)^2*(d + e*x)^(3/2)*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2]) + (35*c^2*d^2)/(24*(c*d^2 - a*e^2)^3*Sqrt[d + e*x]
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (35*c^3*d^3*Sqrt[d + e*x])/
(8*(c*d^2 - a*e^2)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (35*c^3
*d^3*Sqrt[e]*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(
Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(8*(c*d^2 - a*e^2)^(9/2))
```

Rubi [A] time = 0.246489, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {672, 666, 660, 205}

$$-\frac{35c^3d^3\sqrt{d+ex}}{8\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{35c^2d^2}{24\sqrt{d+ex}\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{35c^3d^3\sqrt{e}\tan^{-1}}{8(c}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```


```
[Out] 1/(3*(c*d^2 - a*e^2)*(d + e*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]) + (7*c*d)/(12*(c*d^2 - a*e^2)^2*(d + e*x)^(3/2)*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2]) + (35*c^2*d^2)/(24*(c*d^2 - a*e^2)^3*Sqrt[d + e*x]
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (35*c^3*d^3*Sqrt[d + e*x])/
(8*(c*d^2 - a*e^2)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (35*c^3
*d^3*Sqrt[e]*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(
Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(8*(c*d^2 - a*e^2)^(9/2))
```


Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*(m + 2*p + 2))/((m + p + 1)*(2*c*d - b*e)), Int[(d
 + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && Ne
Q[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 666

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((2*c*d - b*e)*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/(e*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[((2*c*d - b*e)*(m + 2*p + 2))/((p + 1)*(b^2
- 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0
] && LtQ[p, -1] && LtQ[0, m, 1] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{1}{3\left(cd^2-ae^2\right)(d+ex)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{(7cd)\int \cdots}{\cdots}$$

$$= \frac{1}{3\left(cd^2-ae^2\right)(d+ex)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{\cdots}{12\left(cd^2-\cdots\right.}$$

$$= \frac{1}{3\left(cd^2-ae^2\right)(d+ex)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{\cdots}{12\left(cd^2-\cdots\right.}$$

$$= \frac{1}{3\left(cd^2-ae^2\right)(d+ex)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{\cdots}{12\left(cd^2-\cdots\right.}$$

$$= \frac{1}{3\left(cd^2-ae^2\right)(d+ex)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{\cdots}{12\left(cd^2-\cdots\right.}$$

$$= \frac{1}{3\left(cd^2-ae^2\right)(d+ex)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{\cdots}{12\left(cd^2-\cdots\right.}$$

Mathematica [C] time = 0.0259689, size = 81, normalized size = 0.24

$$-\frac{2c^3d^3\sqrt{d+ex}\,{}_2F_1\left(-\frac{1}{2},4;\frac{1}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{\left(cd^2-ae^2\right)^4\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))
,x]
```

```
[Out] (-2*c^3*d^3*Sqrt[d + e*x]*Hypergeometric2F1[-1/2, 4, 1/2, (e*(a*e + c*d*x))
/(-(c*d^2) + a*e^2)])/((c*d^2 - a*e^2)^4*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] time = 0.251, size = 559, normalized size = 1.7

$$\frac{1}{\left(24\,cdx+24\,ae\right)\left(ae^{2}-cd^{2}\right)^{4}}\sqrt{cdex^{2}+ae^{2}x+cd^{2}x+ade}\left(105\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^{2}-cd^{2}\right)e}}\right)\sqrt{cdx+ae}\,x^{3}c^{3}d^{3}e^{4}+315\,\mathrm{Artan}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] 1/24*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(105*arctanh(e*(c*d*x+a*e)^(1/
2)/((a*e^2-c*d^2)*e)^(1/2))*(c*d*x+a*e)^(1/2)*x^3*c^3*d^3*e^4+315*arctanh(e
*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*(c*d*x+a*e)^(1/2)*x^2*c^3*d^4*e
^3+315*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*(c*d*x+a*e)^(1/
2)*x*c^3*d^5*e^2-105*((a*e^2-c*d^2)*e)^(1/2)*x^3*c^3*d^3*e^3+105*arctanh(e*
(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*(c*d*x+a*e)^(1/2)*c^3*d^6*e-35*(
(a*e^2-c*d^2)*e)^(1/2)*x^2*a*c^2*d^2*e^4-280*((a*e^2-c*d^2)*e)^(1/2)*x^2*c^
3*d^4*e^2+14*((a*e^2-c*d^2)*e)^(1/2)*x*a^2*c*d*e^5-98*((a*e^2-c*d^2)*e)^(1/
2)*x*a*c^2*d^3*e^3-231*((a*e^2-c*d^2)*e)^(1/2)*x*c^3*d^5*e-8*((a*e^2-c*d^2)
*e)^(1/2)*a^3*e^6+38*((a*e^2-c*d^2)*e)^(1/2)*a^2*c*d^2*e^4-87*((a*e^2-c*d^2
)*e)^(1/2)*a*c^2*d^4*e^2-48*((a*e^2-c*d^2)*e)^(1/2)*c^3*d^6)/(e*x+d)^(7/2)/
(c*d*x+a*e)/(a*e^2-c*d^2)^4/((a*e^2-c*d^2)*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{\frac{3}{2}}(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algori
thm="maxima")
```

```
[Out] integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)^(5/2))
, x)
```

Fricas [B] time = 2.46908, size = 3243, normalized size = 9.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algori
thm="fricas")
```

```
[Out] [1/48*(105*(c^4*d^4*e^4*x^5 + a*c^3*d^7*e + (4*c^4*d^5*e^3 + a*c^3*d^3*e^5)
*x^4 + 2*(3*c^4*d^6*e^2 + 2*a*c^3*d^4*e^4)*x^3 + 2*(2*c^4*d^7*e + 3*a*c^3*d
^5*e^3)*x^2 + (c^4*d^8 + 4*a*c^3*d^6*e^2)*x)*sqrt(-e/(c*d^2 - a*e^2))*log(-
(c*d*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2 - 2*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(-e/(c*d^2 - a*e^2)))/(
e^2*x^2 + 2*d*e*x + d^2)) - 2*(105*c^3*d^3*e^3*x^3 + 48*c^3*d^6 + 87*a*c^2*
d^4*e^2 - 38*a^2*c*d^2*e^4 + 8*a^3*e^6 + 35*(8*c^3*d^4*e^2 + a*c^2*d^2*e^4)
*x^2 + 7*(33*c^3*d^5*e + 14*a*c^2*d^3*e^3 - 2*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^
```

```
2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a*c^4*d^12*e - 4*a^2*c^3*d^1
0*e^3 + 6*a^3*c^2*d^8*e^5 - 4*a^4*c*d^6*e^7 + a^5*d^4*e^9 + (c^5*d^9*e^4 -
4*a*c^4*d^7*e^6 + 6*a^2*c^3*d^5*e^8 - 4*a^3*c^2*d^3*e^10 + a^4*c*d*e^12)*x^
5 + (4*c^5*d^10*e^3 - 15*a*c^4*d^8*e^5 + 20*a^2*c^3*d^6*e^7 - 10*a^3*c^2*d^
4*e^9 + a^5*e^13)*x^4 + 2*(3*c^5*d^11*e^2 - 10*a*c^4*d^9*e^4 + 10*a^2*c^3*d
^7*e^6 - 5*a^4*c*d^3*e^10 + 2*a^5*d*e^12)*x^3 + 2*(2*c^5*d^12*e - 5*a*c^4*d
^10*e^3 + 10*a^3*c^2*d^6*e^7 - 10*a^4*c*d^4*e^9 + 3*a^5*d^2*e^11)*x^2 + (c^
5*d^13 - 10*a^2*c^3*d^9*e^4 + 20*a^3*c^2*d^7*e^6 - 15*a^4*c*d^5*e^8 + 4*a^5
*d^3*e^10)*x), -1/24*(105*(c^4*d^4*e^4*x^5 + a*c^3*d^7*e + (4*c^4*d^5*e^3 +
 a*c^3*d^3*e^5)*x^4 + 2*(3*c^4*d^6*e^2 + 2*a*c^3*d^4*e^4)*x^3 + 2*(2*c^4*d^
7*e + 3*a*c^3*d^5*e^3)*x^2 + (c^4*d^8 + 4*a*c^3*d^6*e^2)*x)*sqrt(e/(c*d^2 -
 a*e^2))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d^2 - a*e^2
)*sqrt(e*x + d)*sqrt(e/(c*d^2 - a*e^2))/(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e +
 a*e^3)*x)) + (105*c^3*d^3*e^3*x^3 + 48*c^3*d^6 + 87*a*c^2*d^4*e^2 - 38*a^2
*c*d^2*e^4 + 8*a^3*e^6 + 35*(8*c^3*d^4*e^2 + a*c^2*d^2*e^4)*x^2 + 7*(33*c^3
*d^5*e + 14*a*c^2*d^3*e^3 - 2*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d
^2 + a*e^2)*x)*sqrt(e*x + d))/(a*c^4*d^12*e - 4*a^2*c^3*d^10*e^3 + 6*a^3*c^
2*d^8*e^5 - 4*a^4*c*d^6*e^7 + a^5*d^4*e^9 + (c^5*d^9*e^4 - 4*a*c^4*d^7*e^6
+ 6*a^2*c^3*d^5*e^8 - 4*a^3*c^2*d^3*e^10 + a^4*c*d*e^12)*x^5 + (4*c^5*d^10*
e^3 - 15*a*c^4*d^8*e^5 + 20*a^2*c^3*d^6*e^7 - 10*a^3*c^2*d^4*e^9 + a^5*e^13
)*x^4 + 2*(3*c^5*d^11*e^2 - 10*a*c^4*d^9*e^4 + 10*a^2*c^3*d^7*e^6 - 5*a^4*c
*d^3*e^10 + 2*a^5*d*e^12)*x^3 + 2*(2*c^5*d^12*e - 5*a*c^4*d^10*e^3 + 10*a^3
*c^2*d^6*e^7 - 10*a^4*c*d^4*e^9 + 3*a^5*d^2*e^11)*x^2 + (c^5*d^13 - 10*a^2*
c^3*d^9*e^4 + 20*a^3*c^2*d^7*e^6 - 15*a^4*c*d^5*e^8 + 4*a^5*d^3*e^10)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, undef, undef, 2]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algori
thm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, undef, undef, 2]
```

3.2072 $\int \frac{1}{(d+ex)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=393

$$-\frac{315c^4d^4\sqrt{d+ex}}{64\left(cd^2-ae^2\right)^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{105c^3d^3}{64\sqrt{d+ex}\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\quad}{32(d+ex)^{3/2}\left(\cdots\right.}$$

```
[Out] 1/(4*(c*d^2 - a*e^2)*(d + e*x)^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]) + (3*c*d)/(8*(c*d^2 - a*e^2)^2*(d + e*x)^(5/2)*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2]) + (21*c^2*d^2)/(32*(c*d^2 - a*e^2)^3*(d + e*x)^(3/2
)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (105*c^3*d^3)/(64*(c*d^2 -
 a*e^2)^4*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (315
*c^4*d^4*Sqrt[d + e*x])/(64*(c*d^2 - a*e^2)^5*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2]) - (315*c^4*d^4*Sqrt[e]*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(64*(c*d^2 -
 a*e^2)^(11/2))
```

Rubi [A] time = 0.31957, antiderivative size = 393, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {672, 666, 660, 205}

$$-\frac{315c^4d^4\sqrt{d+ex}}{64\left(cd^2-ae^2\right)^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{105c^3d^3}{64\sqrt{d+ex}\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\quad}{32(d+ex)^{3/2}\left(\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(7/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] 1/(4*(c*d^2 - a*e^2)*(d + e*x)^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]) + (3*c*d)/(8*(c*d^2 - a*e^2)^2*(d + e*x)^(5/2)*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2]) + (21*c^2*d^2)/(32*(c*d^2 - a*e^2)^3*(d + e*x)^(3/2
)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (105*c^3*d^3)/(64*(c*d^2 -
 a*e^2)^4*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (315
*c^4*d^4*Sqrt[d + e*x])/(64*(c*d^2 - a*e^2)^5*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2]) - (315*c^4*d^4*Sqrt[e]*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(64*(c*d^2 -
 a*e^2)^(11/2))
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*(m + 2*p + 2))/((m + p + 1)*(2*c*d - b*e)), Int[(d
 + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && Ne
Q[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 666

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((2*c*d - b*e)*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/(e*(p
```

```
+ 1)*(b^2 - 4*a*c)), x] - Dist[((2*c*d - b*e)*(m + 2*p + 2))/((p + 1)*(b^2
- 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0
] && LtQ[p, -1] && LtQ[0, m, 1] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{1}{4\left(cd^2-ae^2\right)(d+ex)^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{(9cd)\int -}{}$$

$$= \frac{1}{4\left(cd^2-ae^2\right)(d+ex)^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{}{8\left(cd^2-\right.}$$

$$= \frac{1}{4\left(cd^2-ae^2\right)(d+ex)^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{}{8\left(cd^2-\right.}$$

$$= \frac{1}{4\left(cd^2-ae^2\right)(d+ex)^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{}{8\left(cd^2-\right.}$$

$$= \frac{1}{4\left(cd^2-ae^2\right)(d+ex)^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{}{8\left(cd^2-\right.}$$

$$= \frac{1}{4\left(cd^2-ae^2\right)(d+ex)^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{}{8\left(cd^2-\right.}$$

$$= \frac{1}{4\left(cd^2-ae^2\right)(d+ex)^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{}{8\left(cd^2-\right.}$$

Mathematica [C] time = 0.0299, size = 81, normalized size = 0.21

$$-\frac{2c^4d^4\sqrt{d+ex}\,{}_2F_1\left(-\frac{1}{2},5;\frac{1}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{\left(cd^2-ae^2\right)^5\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(7/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))
,x]
```

```
[Out] (-2*c^4*d^4*Sqrt[d + e*x]*Hypergeometric2F1[-1/2, 5, 1/2, (e*(a*e + c*d*x))
/(-(c*d^2) + a*e^2)])/((c*d^2 - a*e^2)^5*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [B] time = 0.224, size = 767, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] -1/64*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(315*arctanh(e*(c*d*x+a*e)^(1
/2)/((a*e^2-c*d^2)*e)^(1/2))*(c*d*x+a*e)^(1/2)*x^4*c^4*d^4*e^5+1260*arctanh
(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*(c*d*x+a*e)^(1/2)*x^3*c^4*d^5
*e^4+1890*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*(c*d*x+a*e)^
(1/2)*x^2*c^4*d^6*e^3-315*((a*e^2-c*d^2)*e)^(1/2)*x^4*c^4*d^4*e^4+1260*arct
anh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*(c*d*x+a*e)^(1/2)*x*c^4*d^
7*e^2-105*((a*e^2-c*d^2)*e)^(1/2)*x^3*a*c^3*d^3*e^5-1155*((a*e^2-c*d^2)*e)^
(1/2)*x^3*c^4*d^5*e^3+315*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/
2))*(c*d*x+a*e)^(1/2)*c^4*d^8*e+42*((a*e^2-c*d^2)*e)^(1/2)*x^2*a^2*c^2*d^2*
e^6-399*((a*e^2-c*d^2)*e)^(1/2)*x^2*a*c^3*d^4*e^4-1533*((a*e^2-c*d^2)*e)^(1
/2)*x^2*c^4*d^6*e^2-24*((a*e^2-c*d^2)*e)^(1/2)*x*a^3*c*d*e^7+156*((a*e^2-c*
d^2)*e)^(1/2)*x*a^2*c^2*d^3*e^5-555*((a*e^2-c*d^2)*e)^(1/2)*x*a*c^3*d^5*e^3
-837*((a*e^2-c*d^2)*e)^(1/2)*x*c^4*d^7*e+16*((a*e^2-c*d^2)*e)^(1/2)*a^4*e^8
-88*((a*e^2-c*d^2)*e)^(1/2)*a^3*c*d^2*e^6+210*((a*e^2-c*d^2)*e)^(1/2)*a^2*c
^2*d^4*e^4-325*((a*e^2-c*d^2)*e)^(1/2)*a*c^3*d^6*e^2-128*((a*e^2-c*d^2)*e)^
(1/2)*c^4*d^8)/(e*x+d)^(9/2)/(c*d*x+a*e)/(a*e^2-c*d^2)^5/((a*e^2-c*d^2)*e)^
(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}} (ex + d)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algori
thm="maxima")
```

```
[Out] integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)^(7/2))
, x)
```

Fricas [B] time = 2.67082, size = 4335, normalized size = 11.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algori
thm="fricas")
```

```
[Out] [1/128*(315*(c^5*d^5*e^5*x^6 + a*c^4*d^9*e + (5*c^5*d^6*e^4 + a*c^4*d^4*e^6
)*x^5 + 5*(2*c^5*d^7*e^3 + a*c^4*d^5*e^5)*x^4 + 10*(c^5*d^8*e^2 + a*c^4*d^6
*e^4)*x^3 + 5*(c^5*d^9*e + 2*a*c^4*d^7*e^3)*x^2 + (c^5*d^10 + 5*a*c^4*d^8*e
^2)*x)*sqrt(-e/(c*d^2 - a*e^2))*log(-(c*d*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a
*d*e^2 - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d^2 - a*e^2)*sqrt
(e*x + d)*sqrt(-e/(c*d^2 - a*e^2)))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(315*c^4
*d^4*e^4*x^4 + 128*c^4*d^8 + 325*a*c^3*d^6*e^2 - 210*a^2*c^2*d^4*e^4 + 88*a
^3*c*d^2*e^6 - 16*a^4*e^8 + 105*(11*c^4*d^5*e^3 + a*c^3*d^3*e^5)*x^3 + 21*(
73*c^4*d^6*e^2 + 19*a*c^3*d^4*e^4 - 2*a^2*c^2*d^2*e^6)*x^2 + 3*(279*c^4*d^7
*e + 185*a*c^3*d^5*e^3 - 52*a^2*c^2*d^3*e^5 + 8*a^3*c*d*e^7)*x)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a*c^5*d^15*e - 5*a^2*c^4*d
^13*e^3 + 10*a^3*c^3*d^11*e^5 - 10*a^4*c^2*d^9*e^7 + 5*a^5*c*d^7*e^9 - a^6*
d^5*e^11 + (c^6*d^11*e^5 - 5*a*c^5*d^9*e^7 + 10*a^2*c^4*d^7*e^9 - 10*a^3*c^
3*d^5*e^11 + 5*a^4*c^2*d^3*e^13 - a^5*c*d*e^15)*x^6 + (5*c^6*d^12*e^4 - 24*
a*c^5*d^10*e^6 + 45*a^2*c^4*d^8*e^8 - 40*a^3*c^3*d^6*e^10 + 15*a^4*c^2*d^4*
e^12 - a^6*e^16)*x^5 + 5*(2*c^6*d^13*e^3 - 9*a*c^5*d^11*e^5 + 15*a^2*c^4*d^
9*e^7 - 10*a^3*c^3*d^7*e^9 + 3*a^5*c*d^3*e^13 - a^6*d*e^15)*x^4 + 10*(c^6*d
^14*e^2 - 4*a*c^5*d^12*e^4 + 5*a^2*c^4*d^10*e^6 - 5*a^4*c^2*d^6*e^10 + 4*a^
5*c*d^4*e^12 - a^6*d^2*e^14)*x^3 + 5*(c^6*d^15*e - 3*a*c^5*d^13*e^3 + 10*a^
3*c^3*d^9*e^7 - 15*a^4*c^2*d^7*e^9 + 9*a^5*c*d^5*e^11 - 2*a^6*d^3*e^13)*x^2
 + (c^6*d^16 - 15*a^2*c^4*d^12*e^4 + 40*a^3*c^3*d^10*e^6 - 45*a^4*c^2*d^8*e
^8 + 24*a^5*c*d^6*e^10 - 5*a^6*d^4*e^12)*x), -1/64*(315*(c^5*d^5*e^5*x^6 +
a*c^4*d^9*e + (5*c^5*d^6*e^4 + a*c^4*d^4*e^6)*x^5 + 5*(2*c^5*d^7*e^3 + a*c^
4*d^5*e^5)*x^4 + 10*(c^5*d^8*e^2 + a*c^4*d^6*e^4)*x^3 + 5*(c^5*d^9*e + 2*a*
c^4*d^7*e^3)*x^2 + (c^5*d^10 + 5*a*c^4*d^8*e^2)*x)*sqrt(e/(c*d^2 - a*e^2))*
arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d^2 - a*e^2)*sqrt(e*
x + d)*sqrt(e/(c*d^2 - a*e^2))/(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x
)) + (315*c^4*d^4*e^4*x^4 + 128*c^4*d^8 + 325*a*c^3*d^6*e^2 - 210*a^2*c^2*d
^4*e^4 + 88*a^3*c*d^2*e^6 - 16*a^4*e^8 + 105*(11*c^4*d^5*e^3 + a*c^3*d^3*e^
5)*x^3 + 21*(73*c^4*d^6*e^2 + 19*a*c^3*d^4*e^4 - 2*a^2*c^2*d^2*e^6)*x^2 + 3
*(279*c^4*d^7*e + 185*a*c^3*d^5*e^3 - 52*a^2*c^2*d^3*e^5 + 8*a^3*c*d*e^7)*x
)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a*c^5*d^15*e
- 5*a^2*c^4*d^13*e^3 + 10*a^3*c^3*d^11*e^5 - 10*a^4*c^2*d^9*e^7 + 5*a^5*c*d
^7*e^9 - a^6*d^5*e^11 + (c^6*d^11*e^5 - 5*a*c^5*d^9*e^7 + 10*a^2*c^4*d^7*e^
9 - 10*a^3*c^3*d^5*e^11 + 5*a^4*c^2*d^3*e^13 - a^5*c*d*e^15)*x^6 + (5*c^6*d
^12*e^4 - 24*a*c^5*d^10*e^6 + 45*a^2*c^4*d^8*e^8 - 40*a^3*c^3*d^6*e^10 + 15
*a^4*c^2*d^4*e^12 - a^6*e^16)*x^5 + 5*(2*c^6*d^13*e^3 - 9*a*c^5*d^11*e^5 +
15*a^2*c^4*d^9*e^7 - 10*a^3*c^3*d^7*e^9 + 3*a^5*c*d^3*e^13 - a^6*d*e^15)*x^
4 + 10*(c^6*d^14*e^2 - 4*a*c^5*d^12*e^4 + 5*a^2*c^4*d^10*e^6 - 5*a^4*c^2*d^
6*e^10 + 4*a^5*c*d^4*e^12 - a^6*d^2*e^14)*x^3 + 5*(c^6*d^15*e - 3*a*c^5*d^1
3*e^3 + 10*a^3*c^3*d^9*e^7 - 15*a^4*c^2*d^7*e^9 + 9*a^5*c*d^5*e^11 - 2*a^6*
d^3*e^13)*x^2 + (c^6*d^16 - 15*a^2*c^4*d^12*e^4 + 40*a^3*c^3*d^10*e^6 - 45*
a^4*c^2*d^8*e^8 + 24*a^5*c*d^6*e^10 - 5*a^6*d^4*e^12)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, 2]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algori
thm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, undef, undef, 2]
```

3.2073 $\int \frac{(d+ex)^{7/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=107

$$\frac{4(d+ex)^{3/2}\left(cd^2-ae^2\right)}{3c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}-\frac{2(d+ex)^{5/2}}{cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

```
[Out] (4*(c*d^2 - a*e^2)*(d + e*x)^(3/2))/(3*c^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2)^(3/2)) - (2*(d + e*x)^(5/2))/(c*d*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(3/2))
```

Rubi [A] time = 0.0650133, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {656, 648}

$$\frac{4(d+ex)^{3/2}\left(cd^2-ae^2\right)}{3c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}-\frac{2(d+ex)^{5/2}}{cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (4*(c*d^2 - a*e^2)*(d + e*x)^(3/2))/(3*c^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2)^(3/2)) - (2*(d + e*x)^(5/2))/(c*d*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(3/2))
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{7/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2(d+ex)^{5/2}}{cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{\left(2\left(2cd^2e-e\left(cd^2+ae^2\right)\right)\right)\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx}{cde}$$

$$=\frac{4\left(cd^2-ae^2\right)(d+ex)^{3/2}}{3c^2d^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2(d+ex)^{5/2}}{cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$

Mathematica [A] time = 0.0455718, size = 53, normalized size = 0.5

$$-\frac{2(d+ex)^{3/2}\left(2ae^2+cd(d+3ex)\right)}{3c^2d^2((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(2*a*e^2 + c*d*(d + 3*e*x)))/(3*c^2*d^2*((a*e + c*d*x)*
(d + e*x))^(3/2))
```

Maple [A] time = 0.043, size = 68, normalized size = 0.6

$$-\frac{(2\,cdx+2\,ae)\left(3\,cdex+2\,ae^2+cd^2\right)}{3\,c^2d^2}(ex+d)^{\frac{5}{2}}\left(cdex^2+ae^2x+cd^2x+ade\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(3*c*d*e*x+2*a*e^2+c*d^2)*(e*x+d)^(5/2)/c^2/d^2/(c*d*e*x^2
+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

Maxima [A] time = 1.11136, size = 68, normalized size = 0.64

$$-\frac{2\left(3\,cdex+cd^2+2\,ae^2\right)}{3\left(c^3d^3x+ac^2d^2e\right)\sqrt{cdx+ae}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="maxima")
```

```
[Out] -2/3*(3*c*d*e*x + c*d^2 + 2*a*e^2)/((c^3*d^3*x + a*c^2*d^2*e)*sqrt(c*d*x +
a*e))
```

Fricas [A] time = 2.07758, size = 270, normalized size = 2.52

$$-\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(3\,cdex+cd^2+2\,ae^2\right)\sqrt{ex+d}}{3\left(c^4d^4ex^3+a^2c^2d^3e^2+\left(c^4d^5+2\,ac^3d^3e^2\right)x^2+\left(2\,ac^3d^4e+a^2c^2d^2e^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="fricas")
```

```
[Out] -2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(3*c*d*e*x + c*d^2 + 2*a*e
^2)*sqrt(e*x + d)/(c^4*d^4*e*x^3 + a^2*c^2*d^3*e^2 + (c^4*d^5 + 2*a*c^3*d^3
```

```
*e^2)*x^2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="giac")
```

```
[Out] sage0*x
```

3.2074 $\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=48

$$-\frac{2(d+ex)^{3/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

```
[Out] (-2*(d + e*x)^(3/2))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))
```

Rubi [A] time = 0.0217219, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {648}

$$-\frac{2(d+ex)^{3/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2(d+ex)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$

Mathematica [A] time = 0.0288432, size = 37, normalized size = 0.77

$$-\frac{2(d+ex)^{3/2}}{3cd((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*c*d*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [A] time = 0.041, size = 50, normalized size = 1.

$$-\frac{2\,cdx+2\,ae}{3\,cd}(ex+d)^{\frac{5}{2}}\left(cdex^2+ae^2x+cd^2x+ade\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(e*x+d)^(5/2)/d/c/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

Maxima [A] time = 1.06331, size = 38, normalized size = 0.79

$$-\frac{2}{3\left(c^{2}d^{2}x+acde\right)\sqrt{cdx+ae}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="maxima")
```

```
[Out] -2/3/((c^2*d^2*x + a*c*d*e)*sqrt(c*d*x + a*e))
```

Fricas [B] time = 2.1005, size = 221, normalized size = 4.6

$$-\frac{2\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}\sqrt{ex+d}}{3\left(c^{3}d^{3}ex^{3}+a^{2}cd^{2}e^{2}+\left(c^{3}d^{4}+2\,ac^{2}d^{2}e^{2}\right)x^{2}+\left(2\,ac^{2}d^{3}e+a^{2}cde^{3}\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="fricas")
```

```
[Out] -2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x
^3 + a^2*c*d^2*e^2 + (c^3*d^4 + 2*a*c^2*d^2*e^2)*x^2 + (2*a*c^2*d^3*e + a^2
*c*d*e^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_{0}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="giac")
```

```
[Out] sage0*x
```

3.2075 $\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=251

$$\frac{2e\sqrt{d+ex}}{\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2e}{3cd\sqrt{d+ex}\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{}}{3cd\left(x\left(ae^2+cd^2\right.\right.}$$

```
[Out] (-2*Sqrt[d + e*x])/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) -
(2*e)/(3*c*d*(c*d^2 - a*e^2)*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2]) + (2*e*Sqrt[d + e*x])/((c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2]) + (2*e^(3/2)*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(c*d^2 - a*e^2
)^(5/2)
```

Rubi [A] time = 0.17421, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {668, 672, 666, 660, 205}

$$\frac{2e\sqrt{d+ex}}{\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2e}{3cd\sqrt{d+ex}\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{}}{3cd\left(x\left(ae^2+cd^2\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```


```
[Out] (-2*Sqrt[d + e*x])/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) -
(2*e)/(3*c*d*(c*d^2 - a*e^2)*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2]) + (2*e*Sqrt[d + e*x])/((c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2]) + (2*e^(3/2)*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(c*d^2 - a*e^2
)^(5/2)
```


Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*(m + 2*p + 2))/((m + p + 1)*(2*c*d - b*e)), Int[(d
 + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && Ne
Q[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 666

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((2*c*d - b*e)*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/(e*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[((2*c*d - b*e)*(m + 2*p + 2))/((p + 1)*(b^2
- 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0
] && LtQ[p, -1] && LtQ[0, m, 1] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2\sqrt{d+ex}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{(2e)\int \frac{1}{\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{3cd}$$

$$=-\frac{2\sqrt{d+ex}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2e}{3cd\left(cd^2-ae^2\right)\sqrt{d+ex}\sqrt{ade+\left(cd^2+\right.}}$$

$$=-\frac{2\sqrt{d+ex}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2e}{3cd\left(cd^2-ae^2\right)\sqrt{d+ex}\sqrt{ade+\left(cd^2+\right.}}$$

$$=-\frac{2\sqrt{d+ex}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2e}{3cd\left(cd^2-ae^2\right)\sqrt{d+ex}\sqrt{ade+\left(cd^2+\right.}}$$

$$=-\frac{2\sqrt{d+ex}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2e}{3cd\left(cd^2-ae^2\right)\sqrt{d+ex}\sqrt{ade+\left(cd^2+\right.}}$$

Mathematica [C] time = 0.0245399, size = 77, normalized size = 0.31

$$\frac{2(d+ex)^{3/2}\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{3\left(ae^2-cd^2\right)((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (2*(d + e*x)^(3/2)*Hypergeometric2F1[-3/2, 1, -1/2, (e*(a*e + c*d*x))/(-(c*
d^2) + a*e^2)])/(3*(-(c*d^2) + a*e^2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [A] time = 0.208, size = 234, normalized size = 0.9

$$-\frac{2}{3\,(cdx+ae)^2\,(ae^2-cd^2)^2}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(3\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{(ae^2-cd^2)\,e}}\right)\sqrt{cdx+ae}xcde^2+3\,\mathrm{Artanh}\left(\frac{e}{\sqrt{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] -2/3*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh(e*(c*d*x+a*e)^(1/2)
/((a*e^2-c*d^2)*e)^(1/2))*(c*d*x+a*e)^(1/2)*x*c*d*e^2+3*arctanh(e*(c*d*x+a*
e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*a*e^3*(c*d*x+a*e)^(1/2)-3*((a*e^2-c*d^2)*
e)^(1/2)*x*c*d*e-4*((a*e^2-c*d^2)*e)^(1/2)*a*e^2+((a*e^2-c*d^2)*e)^(1/2)*c*
d^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^2/(a*e^2-c*d^2)^2/((a*e^2-c*d^2)*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2), x)
```

Fricas [A] time = 2.29472, size = 1592, normalized size = 6.34

$$\left[\frac{3\left(c^2d^2e^2x^3+a^2de^3+\left(c^2d^3e+2\,acde^3\right)x^2+\left(2\,acd^2e^2+a^2e^4\right)x\right)\sqrt{-\frac{e}{cd^2-ae^2}}\log\left(-\frac{cde^2x^2+2\,ae^3x-cd^3+2\,ade^2+2\,\sqrt{cdex^2+ade+(cd}}{e^2x^2+2\,dex+}\right.}{3\left(a^2c^2d^5e^2-2\,a^3cd^3e^4+a^4de^6+\left(c^4d^6e-2\,ac^3d^4e^3+a^2c^2d^2e^5\right)x^3+\left(c^4d^7-3\,a^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="fricas")
```

```
[Out] [1/3*(3*(c^2*d^2*e^2*x^3 + a^2*d*e^3 + (c^2*d^3*e + 2*a*c*d*e^3)*x^2 + (2*a
*c*d^2*e^2 + a^2*e^4)*x)*sqrt(-e/(c*d^2 - a*e^2))*log(-(c*d*e^2*x^2 + 2*a*e
^3*x - c*d^3 + 2*a*d*e^2 + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c
*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(-e/(c*d^2 - a*e^2)))/(e^2*x^2 + 2*d*e*x +
d^2)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(3*c*d*e*x - c*d^2 +
4*a*e^2)*sqrt(e*x + d))/(a^2*c^2*d^5*e^2 - 2*a^3*c*d^3*e^4 + a^4*d*e^6 + (c
^4*d^6*e - 2*a*c^3*d^4*e^3 + a^2*c^2*d^2*e^5)*x^3 + (c^4*d^7 - 3*a^2*c^2*d^
3*e^4 + 2*a^3*c*d*e^6)*x^2 + (2*a*c^3*d^6*e - 3*a^2*c^2*d^4*e^3 + a^4*e^7)*
x), 2/3*(3*(c^2*d^2*e^2*x^3 + a^2*d*e^3 + (c^2*d^3*e + 2*a*c*d*e^3)*x^2 + (
2*a*c*d^2*e^2 + a^2*e^4)*x)*sqrt(e/(c*d^2 - a*e^2))*arctan(-sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(e/(c*d^2 -
```

```
a*e^2))/(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x)) + sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*(3*c*d*e*x - c*d^2 + 4*a*e^2)*sqrt(e*x + d))/(a^2
*c^2*d^5*e^2 - 2*a^3*c*d^3*e^4 + a^4*d*e^6 + (c^4*d^6*e - 2*a*c^3*d^4*e^3 +
 a^2*c^2*d^2*e^5)*x^3 + (c^4*d^7 - 3*a^2*c^2*d^3*e^4 + 2*a^3*c*d*e^6)*x^2 +
 (2*a*c^3*d^6*e - 3*a^2*c^2*d^4*e^3 + a^4*e^7)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="giac")

[Out] sage0*x
```

3.2076 $\int \frac{\sqrt{d+ex}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=257

$$\frac{5cde\sqrt{d+ex}}{\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{5e}{3\sqrt{d+ex}\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\cdots}{3\left(cd^2-ae^2\right)\left(x\left(\cdots\right.\right.}$$

```
[Out] (-2*Sqrt[d + e*x])/(3*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2)) - (5*e)/(3*(c*d^2 - a*e^2)^2*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2]) + (5*c*d*e*Sqrt[d + e*x])/((c*d^2 - a*e^2)^3*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (5*c*d*e^(3/2)*ArcTan[(Sqrt[e]*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x]
)])/(c*d^2 - a*e^2)^(7/2)
```

Rubi [A] time = 0.209254, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {666, 672, 660, 205}

$$\frac{5cde\sqrt{d+ex}}{\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{5e}{3\sqrt{d+ex}\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\cdots}{3\left(cd^2-ae^2\right)\left(x\left(\cdots\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (-2*Sqrt[d + e*x])/(3*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2)) - (5*e)/(3*(c*d^2 - a*e^2)^2*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2]) + (5*c*d*e*Sqrt[d + e*x])/((c*d^2 - a*e^2)^3*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (5*c*d*e^(3/2)*ArcTan[(Sqrt[e]*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x]
)])/(c*d^2 - a*e^2)^(7/2)
```

Rule 666

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((2*c*d - b*e)*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/(e*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[((2*c*d - b*e)*(m + 2*p + 2))/((p + 1)*(b^2
- 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0
] && LtQ[p, -1] && LtQ[0, m, 1] && IntegerQ[2*p]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*(m + 2*p + 2))/((m + p + 1)*(2*c*d - b*e)), Int[(d
 + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && Ne
Q[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}dx = -\frac{2\sqrt{d+ex}}{3\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{(5e)\int \frac{1}{\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}}{3\left(cd^2-ae^2\right)}$$

$$= -\frac{2\sqrt{d+ex}}{3\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{5e}{3\left(cd^2-ae^2\right)^2\sqrt{d+ex}\sqrt{ade+}}$$

$$= -\frac{2\sqrt{d+ex}}{3\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{5e}{3\left(cd^2-ae^2\right)^2\sqrt{d+ex}\sqrt{ade+}}$$

$$= -\frac{2\sqrt{d+ex}}{3\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{5e}{3\left(cd^2-ae^2\right)^2\sqrt{d+ex}\sqrt{ade+}}$$

$$= -\frac{2\sqrt{d+ex}}{3\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{5e}{3\left(cd^2-ae^2\right)^2\sqrt{d+ex}\sqrt{ade+}}$$

Mathematica [C] time = 0.0312925, size = 79, normalized size = 0.31

$$-\frac{2cd(d+ex)^{3/2}\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{3\left(cd^2-ae^2\right)^2((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (-2*c*d*(d + e*x)^(3/2)*Hypergeometric2F1[-3/2, 2, -1/2, (e*(a*e + c*d*x))/
(-(c*d^2) + a*e^2)])/(3*(c*d^2 - a*e^2)^2*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [A] time = 0.248, size = 433, normalized size = 1.7

$$\frac{1}{3\,(cdx+ae)^2\left(ae^2-cd^2\right)^3}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(15\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^2-cd^2\right)e}}\right)\sqrt{cdx+ae}x^2c^2d^2e^3+15\,\mathrm{Artanh}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] 1/3*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh(e*(c*d*x+a*e)^(1/2)
/((a*e^2-c*d^2)*e)^(1/2))*(c*d*x+a*e)^(1/2)*x^2*c^2*d^2*e^3+15*arctanh(e*(c
*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*x*a*c*d*e^4*(c*d*x+a*e)^(1/2)+15*a
rctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*(c*d*x+a*e)^(1/2)*x*c^2
*d^3*e^2+15*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*a*c*d^2*e^
3*(c*d*x+a*e)^(1/2)-15*((a*e^2-c*d^2)*e)^(1/2)*x^2*c^2*d^2*e^2-20*((a*e^2-c
*d^2)*e)^(1/2)*x*a*c*d*e^3-10*((a*e^2-c*d^2)*e)^(1/2)*x*c^2*d^3*e-3*((a*e^2
-c*d^2)*e)^(1/2)*a^2*e^4-14*((a*e^2-c*d^2)*e)^(1/2)*a*c*d^2*e^2+2*((a*e^2-c
*d^2)*e)^(1/2)*c^2*d^4)/(e*x+d)^(3/2)/(c*d*x+a*e)^2/(a*e^2-c*d^2)^3/((a*e^2
-c*d^2)*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2), x)
```

Fricas [B] time = 2.15481, size = 2452, normalized size = 9.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="fricas")
```

```
[Out] [-1/6*(15*(c^3*d^3*e^3*x^4 + a^2*c*d^3*e^3 + 2*(c^3*d^4*e^2 + a*c^2*d^2*e^4
)*x^3 + (c^3*d^5*e + 4*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x^2 + 2*(a*c^2*d^4*e^2
+ a^2*c*d^2*e^4)*x)*sqrt(-e/(c*d^2 - a*e^2))*log(-(c*d*e^2*x^2 + 2*a*e^3*x
- c*d^3 + 2*a*d*e^2 - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d^2
- a*e^2)*sqrt(e*x + d)*sqrt(-e/(c*d^2 - a*e^2)))/(e^2*x^2 + 2*d*e*x + d^2))
 - 2*(15*c^2*d^2*e^2*x^2 - 2*c^2*d^4 + 14*a*c*d^2*e^2 + 3*a^2*e^4 + 10*(c^2
*d^3*e + 2*a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e
*x + d))/(a^2*c^3*d^8*e^2 - 3*a^3*c^2*d^6*e^4 + 3*a^4*c*d^4*e^6 - a^5*d^2*e
^8 + (c^5*d^8*e^2 - 3*a*c^4*d^6*e^4 + 3*a^2*c^3*d^4*e^6 - a^3*c^2*d^2*e^8)*
x^4 + 2*(c^5*d^9*e - 2*a*c^4*d^7*e^3 + 2*a^3*c^2*d^3*e^7 - a^4*c*d*e^9)*x^3
 + (c^5*d^10 + a*c^4*d^8*e^2 - 8*a^2*c^3*d^6*e^4 + 8*a^3*c^2*d^4*e^6 - a^4*
c*d^2*e^8 - a^5*e^10)*x^2 + 2*(a*c^4*d^9*e - 2*a^2*c^3*d^7*e^3 + 2*a^4*c*d^
3*e^7 - a^5*d*e^9)*x), 1/3*(15*(c^3*d^3*e^3*x^4 + a^2*c*d^3*e^3 + 2*(c^3*d^
4*e^2 + a*c^2*d^2*e^4)*x^3 + (c^3*d^5*e + 4*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x^
2 + 2*(a*c^2*d^4*e^2 + a^2*c*d^2*e^4)*x)*sqrt(e/(c*d^2 - a*e^2))*arctan(-sq
rt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d^2 - a*e^2)*sqrt(e*x + d)*sqr
t(e/(c*d^2 - a*e^2))/(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x)) + (15*c
^2*d^2*e^2*x^2 - 2*c^2*d^4 + 14*a*c*d^2*e^2 + 3*a^2*e^4 + 10*(c^2*d^3*e + 2
*a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(
```

```
a^2*c^3*d^8*e^2 - 3*a^3*c^2*d^6*e^4 + 3*a^4*c*d^4*e^6 - a^5*d^2*e^8 + (c^5*
d^8*e^2 - 3*a*c^4*d^6*e^4 + 3*a^2*c^3*d^4*e^6 - a^3*c^2*d^2*e^8)*x^4 + 2*(c
^5*d^9*e - 2*a*c^4*d^7*e^3 + 2*a^3*c^2*d^3*e^7 - a^4*c*d*e^9)*x^3 + (c^5*d^
10 + a*c^4*d^8*e^2 - 8*a^2*c^3*d^6*e^4 + 8*a^3*c^2*d^4*e^6 - a^4*c*d^2*e^8
- a^5*e^10)*x^2 + 2*(a*c^4*d^9*e - 2*a^2*c^3*d^7*e^3 + 2*a^4*c*d^3*e^7 - a^
5*d*e^9)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="giac")
```

```
[Out] sage0*x
```

3.2077 $\int \frac{1}{\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}} dx$

Optimal. Leaf size=329

$$\frac{35c^2d^2e\sqrt{d+ex}}{4\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{35c^2d^2e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{4\left(cd^2-ae^2\right)^{9/2}}-\frac{35c}{12\sqrt{d+ex}\left(cd^2-ae^2\right)^3\sqrt{x}}$$

```
[Out] 1/(2*(c*d^2 - a*e^2)*Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(3/2)) - (7*c*d*Sqrt[d + e*x])/(6*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(3/2)) - (35*c*d*e)/(12*(c*d^2 - a*e^2)^3*Sqrt[d + e*x]*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (35*c^2*d^2*e*Sqrt[d + e*x])/(
4*(c*d^2 - a*e^2)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (35*c^2*
d^2*e^(3/2)*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(S
qrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(4*(c*d^2 - a*e^2)^(9/2))
```

Rubi [A] time = 0.280659, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {672, 666, 660, 205}

$$\frac{35c^2d^2e\sqrt{d+ex}}{4\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{35c^2d^2e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{4\left(cd^2-ae^2\right)^{9/2}}-\frac{35c}{12\sqrt{d+ex}\left(cd^2-ae^2\right)^3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```


```
[Out] 1/(2*(c*d^2 - a*e^2)*Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(3/2)) - (7*c*d*Sqrt[d + e*x])/(6*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(3/2)) - (35*c*d*e)/(12*(c*d^2 - a*e^2)^3*Sqrt[d + e*x]*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (35*c^2*d^2*e*Sqrt[d + e*x])/(
4*(c*d^2 - a*e^2)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (35*c^2*
d^2*e^(3/2)*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(S
qrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(4*(c*d^2 - a*e^2)^(9/2))
```


Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*(m + 2*p + 2))/((m + p + 1)*(2*c*d - b*e)), Int[(d
 + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && Ne
Q[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 666

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((2*c*d - b*e)*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/(e*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[((2*c*d - b*e)*(m + 2*p + 2))/((p + 1)*(b^2
- 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0
] && LtQ[p, -1] && LtQ[0, m, 1] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{1}{2\left(cd^2-ae^2\right)\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{(7cd)\int \frac{}{(ade+(c\cdots}}{4\,(c\cdots}$$

$$= \frac{1}{2\left(cd^2-ae^2\right)\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{}{6\left(cd^2-ae^2\right)^2\cdots}$$

$$= \frac{1}{2\left(cd^2-ae^2\right)\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{}{6\left(cd^2-ae^2\right)^2\cdots}$$

$$= \frac{1}{2\left(cd^2-ae^2\right)\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{}{6\left(cd^2-ae^2\right)^2\cdots}$$

$$= \frac{1}{2\left(cd^2-ae^2\right)\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{}{6\left(cd^2-ae^2\right)^2\cdots}$$

$$= \frac{1}{2\left(cd^2-ae^2\right)\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{}{6\left(cd^2-ae^2\right)^2\cdots}$$

Mathematica [C] time = 0.0323592, size = 83, normalized size = 0.25

$$-\frac{2c^2d^2(d+ex)^{3/2}\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{3\left(cd^2-ae^2\right)^3((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x
]
```

```
[Out] (-2*c^2*d^2*(d + e*x)^(3/2)*Hypergeometric2F1[-3/2, 3, -1/2, (e*(a*e + c*d*
x))/(-(c*d^2) + a*e^2)])/(3*(c*d^2 - a*e^2)^3*((a*e + c*d*x)*(d + e*x))^(3/
2))
```

Maple [B] time = 0.22, size = 668, normalized size = 2.

$$-\frac{1}{12\left(cdx+ae\right)^{2}\left(ae^{2}-cd^{2}\right)^{4}}\sqrt{cdex^{2}+ae^{2}x+cd^{2}x+ade}\left(105\,\mathrm{Artanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(ae^{2}-cd^{2}\right)e}}\right)\sqrt{cdx+ae}\,x^{3}c^{3}d^{3}e^{4}+105\,\mathrm{Ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] -1/12*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(105*arctanh(e*(c*d*x+a*e)^(1
/2)/((a*e^2-c*d^2)*e)^(1/2))*(c*d*x+a*e)^(1/2)*x^3*c^3*d^3*e^4+105*arctanh(
e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*x^2*a*c^2*d^2*e^5*(c*d*x+a*e)^
(1/2)+210*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*(c*d*x+a*e)^
(1/2)*x^2*c^3*d^4*e^3+210*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/
2))*x*a*c^2*d^3*e^4*(c*d*x+a*e)^(1/2)+105*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e
^2-c*d^2)*e)^(1/2))*(c*d*x+a*e)^(1/2)*x*c^3*d^5*e^2-105*((a*e^2-c*d^2)*e)^(
1/2)*x^3*c^3*d^3*e^3+105*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2
))*a*c^2*d^4*e^3*(c*d*x+a*e)^(1/2)-140*((a*e^2-c*d^2)*e)^(1/2)*x^2*a*c^2*d^
2*e^4-175*((a*e^2-c*d^2)*e)^(1/2)*x^2*c^3*d^4*e^2-21*((a*e^2-c*d^2)*e)^(1/2
)*x*a^2*c*d*e^5-238*((a*e^2-c*d^2)*e)^(1/2)*x*a*c^2*d^3*e^3-56*((a*e^2-c*d^
2)*e)^(1/2)*x*c^3*d^5*e+6*((a*e^2-c*d^2)*e)^(1/2)*a^3*e^6-39*((a*e^2-c*d^2)
*e)^(1/2)*a^2*c*d^2*e^4-80*((a*e^2-c*d^2)*e)^(1/2)*a*c^2*d^4*e^2+8*((a*e^2-
c*d^2)*e)^(1/2)*c^3*d^6)/(e*x+d)^(5/2)/(c*d*x+a*e)^2/(a*e^2-c*d^2)^4/((a*e^
2-c*d^2)*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{\frac{5}{2}}\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algori
thm="maxima")
```

```
[Out] integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*sqrt(e*x + d)),
x)
```

Fricas [B] time = 2.079, size = 3603, normalized size = 10.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algori
thm="fricas")
```

```
[Out] [1/24*(105*(c^4*d^4*e^4*x^5 + a^2*c^2*d^5*e^3 + (3*c^4*d^5*e^3 + 2*a*c^3*d^
3*e^5)*x^4 + (3*c^4*d^6*e^2 + 6*a*c^3*d^4*e^4 + a^2*c^2*d^2*e^6)*x^3 + (c^4
*d^7*e + 6*a*c^3*d^5*e^3 + 3*a^2*c^2*d^3*e^5)*x^2 + (2*a*c^3*d^6*e^2 + 3*a^
2*c^2*d^4*e^4)*x)*sqrt(-e/(c*d^2 - a*e^2))*log(-(c*d*e^2*x^2 + 2*a*e^3*x -
c*d^3 + 2*a*d*e^2 + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d^2 -
```

```
a*e^2)*sqrt(e*x + d)*sqrt(-e/(c*d^2 - a*e^2)))/(e^2*x^2 + 2*d*e*x + d^2)) +
 2*(105*c^3*d^3*e^3*x^3 - 8*c^3*d^6 + 80*a*c^2*d^4*e^2 + 39*a^2*c*d^2*e^4 -
 6*a^3*e^6 + 35*(5*c^3*d^4*e^2 + 4*a*c^2*d^2*e^4)*x^2 + 7*(8*c^3*d^5*e + 34
*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*sqrt(e*x + d))/(a^2*c^4*d^11*e^2 - 4*a^3*c^3*d^9*e^4 + 6*a^4*c^2*d^7*e^
6 - 4*a^5*c*d^5*e^8 + a^6*d^3*e^10 + (c^6*d^10*e^3 - 4*a*c^5*d^8*e^5 + 6*a^
2*c^4*d^6*e^7 - 4*a^3*c^3*d^4*e^9 + a^4*c^2*d^2*e^11)*x^5 + (3*c^6*d^11*e^2
 - 10*a*c^5*d^9*e^4 + 10*a^2*c^4*d^7*e^6 - 5*a^4*c^2*d^3*e^10 + 2*a^5*c*d*e
^12)*x^4 + (3*c^6*d^12*e - 6*a*c^5*d^10*e^3 - 5*a^2*c^4*d^8*e^5 + 20*a^3*c^
3*d^6*e^7 - 15*a^4*c^2*d^4*e^9 + 2*a^5*c*d^2*e^11 + a^6*e^13)*x^3 + (c^6*d^
13 + 2*a*c^5*d^11*e^2 - 15*a^2*c^4*d^9*e^4 + 20*a^3*c^3*d^7*e^6 - 5*a^4*c^2
*d^5*e^8 - 6*a^5*c*d^3*e^10 + 3*a^6*d*e^12)*x^2 + (2*a*c^5*d^12*e - 5*a^2*c
^4*d^10*e^3 + 10*a^4*c^2*d^6*e^7 - 10*a^5*c*d^4*e^9 + 3*a^6*d^2*e^11)*x), 1
/12*(105*(c^4*d^4*e^4*x^5 + a^2*c^2*d^5*e^3 + (3*c^4*d^5*e^3 + 2*a*c^3*d^3*
e^5)*x^4 + (3*c^4*d^6*e^2 + 6*a*c^3*d^4*e^4 + a^2*c^2*d^2*e^6)*x^3 + (c^4*d
^7*e + 6*a*c^3*d^5*e^3 + 3*a^2*c^2*d^3*e^5)*x^2 + (2*a*c^3*d^6*e^2 + 3*a^2*
c^2*d^4*e^4)*x)*sqrt(e/(c*d^2 - a*e^2))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x)*(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(e/(c*d^2 - a*e^2))/(c*d
*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x)) + (105*c^3*d^3*e^3*x^3 - 8*c^3*d
^6 + 80*a*c^2*d^4*e^2 + 39*a^2*c*d^2*e^4 - 6*a^3*e^6 + 35*(5*c^3*d^4*e^2 +
4*a*c^2*d^2*e^4)*x^2 + 7*(8*c^3*d^5*e + 34*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*x
)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a^2*c^4*d^11*
e^2 - 4*a^3*c^3*d^9*e^4 + 6*a^4*c^2*d^7*e^6 - 4*a^5*c*d^5*e^8 + a^6*d^3*e^1
0 + (c^6*d^10*e^3 - 4*a*c^5*d^8*e^5 + 6*a^2*c^4*d^6*e^7 - 4*a^3*c^3*d^4*e^9
 + a^4*c^2*d^2*e^11)*x^5 + (3*c^6*d^11*e^2 - 10*a*c^5*d^9*e^4 + 10*a^2*c^4*
d^7*e^6 - 5*a^4*c^2*d^3*e^10 + 2*a^5*c*d*e^12)*x^4 + (3*c^6*d^12*e - 6*a*c^
5*d^10*e^3 - 5*a^2*c^4*d^8*e^5 + 20*a^3*c^3*d^6*e^7 - 15*a^4*c^2*d^4*e^9 +
2*a^5*c*d^2*e^11 + a^6*e^13)*x^3 + (c^6*d^13 + 2*a*c^5*d^11*e^2 - 15*a^2*c^
4*d^9*e^4 + 20*a^3*c^3*d^7*e^6 - 5*a^4*c^2*d^5*e^8 - 6*a^5*c*d^3*e^10 + 3*a
^6*d*e^12)*x^2 + (2*a*c^5*d^12*e - 5*a^2*c^4*d^10*e^3 + 10*a^4*c^2*d^6*e^7
- 10*a^5*c*d^4*e^9 + 3*a^6*d^2*e^11)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algori
thm="giac")
```

```
[Out] sage0*x
```

3.2078 $\int \frac{1}{(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=395

$$\frac{105c^3d^3e\sqrt{d+ex}}{8\left(cd^2-ae^2\right)^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{35c^2d^2e}{8\sqrt{d+ex}\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{}{4\left(cd^2-ae^2\right)^3\left(x\right.}$$

```
[Out] 1/(3*(c*d^2 - a*e^2)*(d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)) + (3*c*d)/(4*(c*d^2 - a*e^2)^2*Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2)) - (7*c^2*d^2*Sqrt[d + e*x])/(4*(c*d^2 - a*e^2)^3*
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (35*c^2*d^2*e)/(8*(c*d^2 -
 a*e^2)^4*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (105
*c^3*d^3*e*Sqrt[d + e*x])/(8*(c*d^2 - a*e^2)^5*Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2]) + (105*c^3*d^3*e^(3/2)*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(8*(c*d^2 -
 a*e^2)^(11/2))
```

Rubi [A] time = 0.348835, antiderivative size = 395, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {672, 666, 660, 205}

$$\frac{105c^3d^3e\sqrt{d+ex}}{8\left(cd^2-ae^2\right)^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{35c^2d^2e}{8\sqrt{d+ex}\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{}{4\left(cd^2-ae^2\right)^3\left(x\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```


```
[Out] 1/(3*(c*d^2 - a*e^2)*(d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)) + (3*c*d)/(4*(c*d^2 - a*e^2)^2*Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2)) - (7*c^2*d^2*Sqrt[d + e*x])/(4*(c*d^2 - a*e^2)^3*
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (35*c^2*d^2*e)/(8*(c*d^2 -
 a*e^2)^4*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (105
*c^3*d^3*e*Sqrt[d + e*x])/(8*(c*d^2 - a*e^2)^5*Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2]) + (105*c^3*d^3*e^(3/2)*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(8*(c*d^2 -
 a*e^2)^(11/2))
```


Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*(m + 2*p + 2))/((m + p + 1)*(2*c*d - b*e)), Int[(d
 + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && Ne
Q[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 666

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((2*c*d - b*e)*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/(e*(p
```

```
+ 1)*(b^2 - 4*a*c)), x] - Dist[((2*c*d - b*e)*(m + 2*p + 2))/((p + 1)*(b^2
- 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0
] && LtQ[p, -1] && LtQ[0, m, 1] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{1}{3\left(cd^2-ae^2\right)(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{(3cd)\int \ldots}{\ldots}$$

$$= \frac{1}{3\left(cd^2-ae^2\right)(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{\ldots}{4\left(cd^2-a\ldots\right.}$$

$$= \frac{1}{3\left(cd^2-ae^2\right)(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{\ldots}{4\left(cd^2-a\ldots\right.}$$

$$= \frac{1}{3\left(cd^2-ae^2\right)(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{\ldots}{4\left(cd^2-a\ldots\right.}$$

$$= \frac{1}{3\left(cd^2-ae^2\right)(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{\ldots}{4\left(cd^2-a\ldots\right.}$$

$$= \frac{1}{3\left(cd^2-ae^2\right)(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{\ldots}{4\left(cd^2-a\ldots\right.}$$

$$= \frac{1}{3\left(cd^2-ae^2\right)(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{\ldots}{4\left(cd^2-a\ldots\right.}$$

Mathematica [C] time = 0.0352229, size = 83, normalized size = 0.21

$$-\frac{2c^3d^3(d+ex)^{3/2}\,{}_2F_1\left(-\frac{3}{2},4;-\frac{1}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{3\left(cd^2-ae^2\right)^4((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))
,x]
```

```
[Out] (-2*c^3*d^3*(d + e*x)^(3/2)*Hypergeometric2F1[-3/2, 4, -1/2, (e*(a*e + c*d*
x))/(-(c*d^2) + a*e^2)])/(3*(c*d^2 - a*e^2)^4*((a*e + c*d*x)*(d + e*x))^(3/
2))
```

Maple [B] time = 0.22, size = 930, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] 1/24*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(315*arctanh(e*(c*d*x+a*e)^(1/
2)/((a*e^2-c*d^2)*e)^(1/2))*(c*d*x+a*e)^(1/2)*x^4*c^4*d^4*e^5+315*arctanh(e
*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*x^3*a*c^3*d^3*e^6*(c*d*x+a*e)^(
1/2)+945*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*(c*d*x+a*e)^(
1/2)*x^3*c^4*d^5*e^4+945*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2
))*x^2*a*c^3*d^4*e^5*(c*d*x+a*e)^(1/2)+945*arctanh(e*(c*d*x+a*e)^(1/2)/((a*
e^2-c*d^2)*e)^(1/2))*(c*d*x+a*e)^(1/2)*x^2*c^4*d^6*e^3-315*((a*e^2-c*d^2)*e
)^(1/2)*x^4*c^4*d^4*e^4+945*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(
1/2))*x*a*c^3*d^5*e^4*(c*d*x+a*e)^(1/2)+315*arctanh(e*(c*d*x+a*e)^(1/2)/((a
*e^2-c*d^2)*e)^(1/2))*(c*d*x+a*e)^(1/2)*x*c^4*d^7*e^2-420*((a*e^2-c*d^2)*e)
^(1/2)*x^3*a*c^3*d^3*e^5-840*((a*e^2-c*d^2)*e)^(1/2)*x^3*c^4*d^5*e^3+315*ar
ctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*a*c^3*d^6*e^3*(c*d*x+a*e
)^(1/2)-63*((a*e^2-c*d^2)*e)^(1/2)*x^2*a^2*c^2*d^2*e^6-1134*((a*e^2-c*d^2)*
e)^(1/2)*x^2*a*c^3*d^4*e^4-693*((a*e^2-c*d^2)*e)^(1/2)*x^2*c^4*d^6*e^2+18*(
(a*e^2-c*d^2)*e)^(1/2)*x*a^3*c*d*e^7-180*((a*e^2-c*d^2)*e)^(1/2)*x*a^2*c^2*
d^3*e^5-954*((a*e^2-c*d^2)*e)^(1/2)*x*a*c^3*d^5*e^3-144*((a*e^2-c*d^2)*e)^(
1/2)*x*c^4*d^7*e-8*((a*e^2-c*d^2)*e)^(1/2)*a^4*e^8+50*((a*e^2-c*d^2)*e)^(1/
2)*a^3*c*d^2*e^6-165*((a*e^2-c*d^2)*e)^(1/2)*a^2*c^2*d^4*e^4-208*((a*e^2-c*
d^2)*e)^(1/2)*a*c^3*d^6*e^2+16*((a*e^2-c*d^2)*e)^(1/2)*c^4*d^8)/(e*x+d)^(7/
2)/(c*d*x+a*e)^2/(a*e^2-c*d^2)^5/((a*e^2-c*d^2)*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}} (ex + d)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algori
thm="maxima")
```

```
[Out] integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(e*x + d)^(3/2))
, x)
```

Fricas [B] time = 2.22492, size = 4867, normalized size = 12.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algori
thm="fricas")
```

```
[Out] [1/48*(315*(c^5*d^5*e^5*x^6 + a^2*c^3*d^7*e^3 + 2*(2*c^5*d^6*e^4 + a*c^4*d^
4*e^6)*x^5 + (6*c^5*d^7*e^3 + 8*a*c^4*d^5*e^5 + a^2*c^3*d^3*e^7)*x^4 + 4*(c
^5*d^8*e^2 + 3*a*c^4*d^6*e^4 + a^2*c^3*d^4*e^6)*x^3 + (c^5*d^9*e + 8*a*c^4*
d^7*e^3 + 6*a^2*c^3*d^5*e^5)*x^2 + 2*(a*c^4*d^8*e^2 + 2*a^2*c^3*d^6*e^4)*x)
*sqrt(-e/(c*d^2 - a*e^2))*log(-(c*d*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2
 + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d^2 - a*e^2)*sqrt(e*x +
 d)*sqrt(-e/(c*d^2 - a*e^2)))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(315*c^4*d^4*e
^4*x^4 - 16*c^4*d^8 + 208*a*c^3*d^6*e^2 + 165*a^2*c^2*d^4*e^4 - 50*a^3*c*d^
2*e^6 + 8*a^4*e^8 + 420*(2*c^4*d^5*e^3 + a*c^3*d^3*e^5)*x^3 + 63*(11*c^4*d^
6*e^2 + 18*a*c^3*d^4*e^4 + a^2*c^2*d^2*e^6)*x^2 + 18*(8*c^4*d^7*e + 53*a*c^
3*d^5*e^3 + 10*a^2*c^2*d^3*e^5 - a^3*c*d*e^7)*x)*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a^2*c^5*d^14*e^2 - 5*a^3*c^4*d^12*e^4 + 1
0*a^4*c^3*d^10*e^6 - 10*a^5*c^2*d^8*e^8 + 5*a^6*c*d^6*e^10 - a^7*d^4*e^12 +
 (c^7*d^12*e^4 - 5*a*c^6*d^10*e^6 + 10*a^2*c^5*d^8*e^8 - 10*a^3*c^4*d^6*e^1
0 + 5*a^4*c^3*d^4*e^12 - a^5*c^2*d^2*e^14)*x^6 + 2*(2*c^7*d^13*e^3 - 9*a*c^
6*d^11*e^5 + 15*a^2*c^5*d^9*e^7 - 10*a^3*c^4*d^7*e^9 + 3*a^5*c^2*d^3*e^13 -
 a^6*c*d*e^15)*x^5 + (6*c^7*d^14*e^2 - 22*a*c^6*d^12*e^4 + 21*a^2*c^5*d^10*
e^6 + 15*a^3*c^4*d^8*e^8 - 40*a^4*c^3*d^6*e^10 + 24*a^5*c^2*d^4*e^12 - 3*a^
6*c*d^2*e^14 - a^7*e^16)*x^4 + 4*(c^7*d^15*e - 2*a*c^6*d^13*e^3 - 4*a^2*c^5
*d^11*e^5 + 15*a^3*c^4*d^9*e^7 - 15*a^4*c^3*d^7*e^9 + 4*a^5*c^2*d^5*e^11 +
2*a^6*c*d^3*e^13 - a^7*d*e^15)*x^3 + (c^7*d^16 + 3*a*c^6*d^14*e^2 - 24*a^2*
c^5*d^12*e^4 + 40*a^3*c^4*d^10*e^6 - 15*a^4*c^3*d^8*e^8 - 21*a^5*c^2*d^6*e^
10 + 22*a^6*c*d^4*e^12 - 6*a^7*d^2*e^14)*x^2 + 2*(a*c^6*d^15*e - 3*a^2*c^5*
d^13*e^3 + 10*a^4*c^3*d^9*e^7 - 15*a^5*c^2*d^7*e^9 + 9*a^6*c*d^5*e^11 - 2*a
^7*d^3*e^13)*x), 1/24*(315*(c^5*d^5*e^5*x^6 + a^2*c^3*d^7*e^3 + 2*(2*c^5*d^
6*e^4 + a*c^4*d^4*e^6)*x^5 + (6*c^5*d^7*e^3 + 8*a*c^4*d^5*e^5 + a^2*c^3*d^3
*e^7)*x^4 + 4*(c^5*d^8*e^2 + 3*a*c^4*d^6*e^4 + a^2*c^3*d^4*e^6)*x^3 + (c^5*
d^9*e + 8*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5)*x^2 + 2*(a*c^4*d^8*e^2 + 2*a^2
*c^3*d^6*e^4)*x)*sqrt(e/(c*d^2 - a*e^2))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(e/(c*d^2 - a*e^2))/(c*
d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x)) + (315*c^4*d^4*e^4*x^4 - 16*c^4
*d^8 + 208*a*c^3*d^6*e^2 + 165*a^2*c^2*d^4*e^4 - 50*a^3*c*d^2*e^6 + 8*a^4*e
^8 + 420*(2*c^4*d^5*e^3 + a*c^3*d^3*e^5)*x^3 + 63*(11*c^4*d^6*e^2 + 18*a*c^
3*d^4*e^4 + a^2*c^2*d^2*e^6)*x^2 + 18*(8*c^4*d^7*e + 53*a*c^3*d^5*e^3 + 10*
a^2*c^2*d^3*e^5 - a^3*c*d*e^7)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*sqrt(e*x + d))/(a^2*c^5*d^14*e^2 - 5*a^3*c^4*d^12*e^4 + 10*a^4*c^3*d^10*
e^6 - 10*a^5*c^2*d^8*e^8 + 5*a^6*c*d^6*e^10 - a^7*d^4*e^12 + (c^7*d^12*e^4
- 5*a*c^6*d^10*e^6 + 10*a^2*c^5*d^8*e^8 - 10*a^3*c^4*d^6*e^10 + 5*a^4*c^3*d
^4*e^12 - a^5*c^2*d^2*e^14)*x^6 + 2*(2*c^7*d^13*e^3 - 9*a*c^6*d^11*e^5 + 15
*a^2*c^5*d^9*e^7 - 10*a^3*c^4*d^7*e^9 + 3*a^5*c^2*d^3*e^13 - a^6*c*d*e^15)*
x^5 + (6*c^7*d^14*e^2 - 22*a*c^6*d^12*e^4 + 21*a^2*c^5*d^10*e^6 + 15*a^3*c^
4*d^8*e^8 - 40*a^4*c^3*d^6*e^10 + 24*a^5*c^2*d^4*e^12 - 3*a^6*c*d^2*e^14 -
a^7*e^16)*x^4 + 4*(c^7*d^15*e - 2*a*c^6*d^13*e^3 - 4*a^2*c^5*d^11*e^5 + 15*
a^3*c^4*d^9*e^7 - 15*a^4*c^3*d^7*e^9 + 4*a^5*c^2*d^5*e^11 + 2*a^6*c*d^3*e^1
3 - a^7*d*e^15)*x^3 + (c^7*d^16 + 3*a*c^6*d^14*e^2 - 24*a^2*c^5*d^12*e^4 +
40*a^3*c^4*d^10*e^6 - 15*a^4*c^3*d^8*e^8 - 21*a^5*c^2*d^6*e^10 + 22*a^6*c*d
^4*e^12 - 6*a^7*d^2*e^14)*x^2 + 2*(a*c^6*d^15*e - 3*a^2*c^5*d^13*e^3 + 10*a
^4*c^3*d^9*e^7 - 15*a^5*c^2*d^7*e^9 + 9*a^6*c*d^5*e^11 - 2*a^7*d^3*e^13)*x)
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, 2]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algori
thm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, undef, undef, undef, undef, undef, unde
f, 2]
```

3.2079 $\int \frac{1}{(d+ex)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=457

$$\frac{1155c^4d^4e\sqrt{d+ex}}{64\left(cd^2-ae^2\right)^6\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{385c^3d^3e}{64\sqrt{d+ex}\left(cd^2-ae^2\right)^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{7}{32\left(cd^2-ae^2\right)^4\left(\right.}$$

```
[Out] 1/(4*(c*d^2 - a*e^2)*(d + e*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)) + (11*c*d)/(24*(c*d^2 - a*e^2)^2*(d + e*x)^(3/2)*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(3/2)) + (33*c^2*d^2)/(32*(c*d^2 - a*e^2)^3*Sqrt[d +
 e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (77*c^3*d^3*Sqrt[d +
 e*x])/(32*(c*d^2 - a*e^2)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))
 - (385*c^3*d^3*e)/(64*(c*d^2 - a*e^2)^5*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2]) + (1155*c^4*d^4*e*Sqrt[d + e*x])/(64*(c*d^2 - a*e^
2)^6*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (1155*c^4*d^4*e^(3/2)*A
rcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a
*e^2]*Sqrt[d + e*x])])/(64*(c*d^2 - a*e^2)^(13/2))
```

Rubi [A] time = 0.501198, antiderivative size = 457, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {672, 666, 660, 205}

$$\frac{1155c^4d^4e\sqrt{d+ex}}{64\left(cd^2-ae^2\right)^6\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{385c^3d^3e}{64\sqrt{d+ex}\left(cd^2-ae^2\right)^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{7}{32\left(cd^2-ae^2\right)^4\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```


```
[Out] 1/(4*(c*d^2 - a*e^2)*(d + e*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)) + (11*c*d)/(24*(c*d^2 - a*e^2)^2*(d + e*x)^(3/2)*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(3/2)) + (33*c^2*d^2)/(32*(c*d^2 - a*e^2)^3*Sqrt[d +
 e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (77*c^3*d^3*Sqrt[d +
 e*x])/(32*(c*d^2 - a*e^2)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))
 - (385*c^3*d^3*e)/(64*(c*d^2 - a*e^2)^5*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2]) + (1155*c^4*d^4*e*Sqrt[d + e*x])/(64*(c*d^2 - a*e^
2)^6*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (1155*c^4*d^4*e^(3/2)*A
rcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a
*e^2]*Sqrt[d + e*x])])/(64*(c*d^2 - a*e^2)^(13/2))
```


Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*(m + 2*p + 2))/((m + p + 1)*(2*c*d - b*e)), Int[(d
 + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && Ne
Q[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 666

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((2*c*d - b*e)*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/(e*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[((2*c*d - b*e)*(m + 2*p + 2))/((p + 1)*(b^2
- 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0
] && LtQ[p, -1] && LtQ[0, m, 1] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{1}{4\left(cd^2-ae^2\right)(d+ex)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{(11cd)\cdots}{\cdots}$$

$$= \frac{1}{4\left(cd^2-ae^2\right)(d+ex)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{\cdots}{24\left(cd^{\cdots}\right.\cdots}$$

$$= \frac{1}{4\left(cd^2-ae^2\right)(d+ex)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{\cdots}{24\left(cd^{\cdots}\right.\cdots}$$

$$= \frac{1}{4\left(cd^2-ae^2\right)(d+ex)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{\cdots}{24\left(cd^{\cdots}\right.\cdots}$$

$$= \frac{1}{4\left(cd^2-ae^2\right)(d+ex)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{\cdots}{24\left(cd^{\cdots}\right.\cdots}$$

$$= \frac{1}{4\left(cd^2-ae^2\right)(d+ex)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{\cdots}{24\left(cd^{\cdots}\right.\cdots}$$

$$= \frac{1}{4\left(cd^2-ae^2\right)(d+ex)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{\cdots}{24\left(cd^{\cdots}\right.\cdots}$$

$$= \frac{1}{4\left(cd^2-ae^2\right)(d+ex)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{\cdots}{24\left(cd^{\cdots}\right.\cdots}$$

Mathematica [C] time = 0.0411061, size = 83, normalized size = 0.18

$$-\frac{2c^4d^4(d+ex)^{3/2}\,{}_2F_1\left(-\frac{3}{2},5;-\frac{1}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{3\left(cd^2-ae^2\right)^5((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))
,x]
```

```
[Out] (-2*c^4*d^4*(d + e*x)^(3/2)*Hypergeometric2F1[-3/2, 5, -1/2, (e*(a*e + c*d*
x))/(-(c*d^2) + a*e^2)])/(3*(c*d^2 - a*e^2)^5*((a*e + c*d*x)*(d + e*x))^(3/
2))
```

Maple [B] time = 0.227, size = 1225, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] -1/192*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(-12705*((a*e^2-c*d^2)*e)^(1
/2)*x^4*c^5*d^6*e^4-16863*((a*e^2-c*d^2)*e)^(1/2)*x^3*c^5*d^7*e^3-9207*((a*
e^2-c*d^2)*e)^(1/2)*x^2*c^5*d^8*e^2-1408*((a*e^2-c*d^2)*e)^(1/2)*x*c^5*d^9*
e-328*((a*e^2-c*d^2)*e)^(1/2)*a^4*c*d^2*e^8+1030*((a*e^2-c*d^2)*e)^(1/2)*a^
3*c^2*d^4*e^6-2295*((a*e^2-c*d^2)*e)^(1/2)*a^2*c^3*d^6*e^4-3465*((a*e^2-c*d
^2)*e)^(1/2)*x^5*c^5*d^5*e^5+3465*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2
)*e)^(1/2))*x^4*a*c^4*d^4*e^7*(c*d*x+a*e)^(1/2)+13860*arctanh(e*(c*d*x+a*e)
^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*x^3*a*c^4*d^5*e^6*(c*d*x+a*e)^(1/2)+20790*a
rctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*x^2*a*c^4*d^6*e^5*(c*d*
x+a*e)^(1/2)+13860*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*x*a
*c^4*d^7*e^4*(c*d*x+a*e)^(1/2)-2048*((a*e^2-c*d^2)*e)^(1/2)*a*c^4*d^8*e^2+1
28*((a*e^2-c*d^2)*e)^(1/2)*c^5*d^10+48*((a*e^2-c*d^2)*e)^(1/2)*a^5*e^10-462
0*((a*e^2-c*d^2)*e)^(1/2)*x^4*a*c^4*d^4*e^6-693*((a*e^2-c*d^2)*e)^(1/2)*x^3
*a^2*c^3*d^3*e^7+198*((a*e^2-c*d^2)*e)^(1/2)*x^2*a^3*c^2*d^2*e^8-2673*((a*e
^2-c*d^2)*e)^(1/2)*x^2*a^2*c^3*d^4*e^6-88*((a*e^2-c*d^2)*e)^(1/2)*x*a^4*c*d
*e^9+748*((a*e^2-c*d^2)*e)^(1/2)*x*a^3*c^2*d^3*e^7-17094*((a*e^2-c*d^2)*e)^
(1/2)*x^3*a*c^4*d^5*e^5-22968*((a*e^2-c*d^2)*e)^(1/2)*x^2*a*c^4*d^6*e^4-127
82*((a*e^2-c*d^2)*e)^(1/2)*x*a*c^4*d^7*e^3-3795*((a*e^2-c*d^2)*e)^(1/2)*x*a
^2*c^3*d^5*e^5+3465*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*x^
5*c^5*d^5*e^6*(c*d*x+a*e)^(1/2)+13860*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c
*d^2)*e)^(1/2))*x^4*c^5*d^6*e^5*(c*d*x+a*e)^(1/2)+20790*arctanh(e*(c*d*x+a*
e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*x^3*c^5*d^7*e^4*(c*d*x+a*e)^(1/2)+13860*a
rctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*x^2*c^5*d^8*e^3*(c*d*x+
a*e)^(1/2)+3465*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*x*c^5*
d^9*e^2*(c*d*x+a*e)^(1/2)+3465*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e
)^(1/2))*a*c^4*d^8*e^3*(c*d*x+a*e)^(1/2))/(e*x+d)^(9/2)/(c*d*x+a*e)^2/(a*e^
2-c*d^2)^6/((a*e^2-c*d^2)*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algori
thm="maxima")
```

```
[Out] integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(e*x + d)^(5/2))
, x)
```

Fricas [B] time = 2.45268, size = 6433, normalized size = 14.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algori
thm="fricas")
```

```
[Out] [1/384*(3465*(c^6*d^6*e^6*x^7 + a^2*c^4*d^9*e^3 + (5*c^6*d^7*e^5 + 2*a*c^5*
d^5*e^7)*x^6 + (10*c^6*d^8*e^4 + 10*a*c^5*d^6*e^6 + a^2*c^4*d^4*e^8)*x^5 +
5*(2*c^6*d^9*e^3 + 4*a*c^5*d^7*e^5 + a^2*c^4*d^5*e^7)*x^4 + 5*(c^6*d^10*e^2
 + 4*a*c^5*d^8*e^4 + 2*a^2*c^4*d^6*e^6)*x^3 + (c^6*d^11*e + 10*a*c^5*d^9*e^
3 + 10*a^2*c^4*d^7*e^5)*x^2 + (2*a*c^5*d^10*e^2 + 5*a^2*c^4*d^8*e^4)*x)*sqr
t(-e/(c*d^2 - a*e^2))*log(-(c*d*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2 + 2
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d^2 - a*e^2)*sqrt(e*x + d)*
sqrt(-e/(c*d^2 - a*e^2)))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(3465*c^5*d^5*e^5*
x^5 - 128*c^5*d^10 + 2048*a*c^4*d^8*e^2 + 2295*a^2*c^3*d^6*e^4 - 1030*a^3*c
^2*d^4*e^6 + 328*a^4*c*d^2*e^8 - 48*a^5*e^10 + 1155*(11*c^5*d^6*e^4 + 4*a*c
^4*d^4*e^6)*x^4 + 231*(73*c^5*d^7*e^3 + 74*a*c^4*d^5*e^5 + 3*a^2*c^3*d^3*e^
7)*x^3 + 99*(93*c^5*d^8*e^2 + 232*a*c^4*d^6*e^4 + 27*a^2*c^3*d^4*e^6 - 2*a^
3*c^2*d^2*e^8)*x^2 + 11*(128*c^5*d^9*e + 1162*a*c^4*d^7*e^3 + 345*a^2*c^3*d
^5*e^5 - 68*a^3*c^2*d^3*e^7 + 8*a^4*c*d*e^9)*x)*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a^2*c^6*d^17*e^2 - 6*a^3*c^5*d^15*e^4 + 15
*a^4*c^4*d^13*e^6 - 20*a^5*c^3*d^11*e^8 + 15*a^6*c^2*d^9*e^10 - 6*a^7*c*d^7
*e^12 + a^8*d^5*e^14 + (c^8*d^14*e^5 - 6*a*c^7*d^12*e^7 + 15*a^2*c^6*d^10*e
^9 - 20*a^3*c^5*d^8*e^11 + 15*a^4*c^4*d^6*e^13 - 6*a^5*c^3*d^4*e^15 + a^6*c
^2*d^2*e^17)*x^7 + (5*c^8*d^15*e^4 - 28*a*c^7*d^13*e^6 + 63*a^2*c^6*d^11*e^
8 - 70*a^3*c^5*d^9*e^10 + 35*a^4*c^4*d^7*e^12 - 7*a^6*c^2*d^3*e^16 + 2*a^7*
c*d*e^18)*x^6 + (10*c^8*d^16*e^3 - 50*a*c^7*d^14*e^5 + 91*a^2*c^6*d^12*e^7
 - 56*a^3*c^5*d^10*e^9 - 35*a^4*c^4*d^8*e^11 + 70*a^5*c^3*d^6*e^13 - 35*a^6*
c^2*d^4*e^15 + 4*a^7*c*d^2*e^17 + a^8*e^19)*x^5 + 5*(2*c^8*d^17*e^2 - 8*a*c
^7*d^15*e^4 + 7*a^2*c^6*d^13*e^6 + 14*a^3*c^5*d^11*e^8 - 35*a^4*c^4*d^9*e^1
0 + 28*a^5*c^3*d^7*e^12 - 7*a^6*c^2*d^5*e^14 - 2*a^7*c*d^3*e^16 + a^8*d*e^1
8)*x^4 + 5*(c^8*d^18*e - 2*a*c^7*d^16*e^3 - 7*a^2*c^6*d^14*e^5 + 28*a^3*c^5
*d^12*e^7 - 35*a^4*c^4*d^10*e^9 + 14*a^5*c^3*d^8*e^11 + 7*a^6*c^2*d^6*e^13
 - 8*a^7*c*d^4*e^15 + 2*a^8*d^2*e^17)*x^3 + (c^8*d^19 + 4*a*c^7*d^17*e^2 - 3
5*a^2*c^6*d^15*e^4 + 70*a^3*c^5*d^13*e^6 - 35*a^4*c^4*d^11*e^8 - 56*a^5*c^3
*d^9*e^10 + 91*a^6*c^2*d^7*e^12 - 50*a^7*c*d^5*e^14 + 10*a^8*d^3*e^16)*x^2
 + (2*a*c^7*d^18*e - 7*a^2*c^6*d^16*e^3 + 35*a^4*c^4*d^12*e^7 - 70*a^5*c^3*d
^10*e^9 + 63*a^6*c^2*d^8*e^11 - 28*a^7*c*d^6*e^13 + 5*a^8*d^4*e^15)*x), 1/1
92*(3465*(c^6*d^6*e^6*x^7 + a^2*c^4*d^9*e^3 + (5*c^6*d^7*e^5 + 2*a*c^5*d^5*
e^7)*x^6 + (10*c^6*d^8*e^4 + 10*a*c^5*d^6*e^6 + a^2*c^4*d^4*e^8)*x^5 + 5*(2
*c^6*d^9*e^3 + 4*a*c^5*d^7*e^5 + a^2*c^4*d^5*e^7)*x^4 + 5*(c^6*d^10*e^2 + 4
*a*c^5*d^8*e^4 + 2*a^2*c^4*d^6*e^6)*x^3 + (c^6*d^11*e + 10*a*c^5*d^9*e^3 +
10*a^2*c^4*d^7*e^5)*x^2 + (2*a*c^5*d^10*e^2 + 5*a^2*c^4*d^8*e^4)*x)*sqrt(e/
(c*d^2 - a*e^2))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d^2
 - a*e^2)*sqrt(e*x + d)*sqrt(e/(c*d^2 - a*e^2))/(c*d*e^2*x^2 + a*d*e^2 + (c
*d^2*e + a*e^3)*x)) + (3465*c^5*d^5*e^5*x^5 - 128*c^5*d^10 + 2048*a*c^4*d^8
```

```
*e^2 + 2295*a^2*c^3*d^6*e^4 - 1030*a^3*c^2*d^4*e^6 + 328*a^4*c*d^2*e^8 - 48
*a^5*e^10 + 1155*(11*c^5*d^6*e^4 + 4*a*c^4*d^4*e^6)*x^4 + 231*(73*c^5*d^7*e
^3 + 74*a*c^4*d^5*e^5 + 3*a^2*c^3*d^3*e^7)*x^3 + 99*(93*c^5*d^8*e^2 + 232*a
*c^4*d^6*e^4 + 27*a^2*c^3*d^4*e^6 - 2*a^3*c^2*d^2*e^8)*x^2 + 11*(128*c^5*d^
9*e + 1162*a*c^4*d^7*e^3 + 345*a^2*c^3*d^5*e^5 - 68*a^3*c^2*d^3*e^7 + 8*a^4
*c*d*e^9)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a^
2*c^6*d^17*e^2 - 6*a^3*c^5*d^15*e^4 + 15*a^4*c^4*d^13*e^6 - 20*a^5*c^3*d^11
*e^8 + 15*a^6*c^2*d^9*e^10 - 6*a^7*c*d^7*e^12 + a^8*d^5*e^14 + (c^8*d^14*e^
5 - 6*a*c^7*d^12*e^7 + 15*a^2*c^6*d^10*e^9 - 20*a^3*c^5*d^8*e^11 + 15*a^4*c
^4*d^6*e^13 - 6*a^5*c^3*d^4*e^15 + a^6*c^2*d^2*e^17)*x^7 + (5*c^8*d^15*e^4
- 28*a*c^7*d^13*e^6 + 63*a^2*c^6*d^11*e^8 - 70*a^3*c^5*d^9*e^10 + 35*a^4*c^
4*d^7*e^12 - 7*a^6*c^2*d^3*e^16 + 2*a^7*c*d*e^18)*x^6 + (10*c^8*d^16*e^3 -
50*a*c^7*d^14*e^5 + 91*a^2*c^6*d^12*e^7 - 56*a^3*c^5*d^10*e^9 - 35*a^4*c^4*
d^8*e^11 + 70*a^5*c^3*d^6*e^13 - 35*a^6*c^2*d^4*e^15 + 4*a^7*c*d^2*e^17 + a
^8*e^19)*x^5 + 5*(2*c^8*d^17*e^2 - 8*a*c^7*d^15*e^4 + 7*a^2*c^6*d^13*e^6 +
14*a^3*c^5*d^11*e^8 - 35*a^4*c^4*d^9*e^10 + 28*a^5*c^3*d^7*e^12 - 7*a^6*c^2
*d^5*e^14 - 2*a^7*c*d^3*e^16 + a^8*d*e^18)*x^4 + 5*(c^8*d^18*e - 2*a*c^7*d^
16*e^3 - 7*a^2*c^6*d^14*e^5 + 28*a^3*c^5*d^12*e^7 - 35*a^4*c^4*d^10*e^9 + 1
4*a^5*c^3*d^8*e^11 + 7*a^6*c^2*d^6*e^13 - 8*a^7*c*d^4*e^15 + 2*a^8*d^2*e^17
)*x^3 + (c^8*d^19 + 4*a*c^7*d^17*e^2 - 35*a^2*c^6*d^15*e^4 + 70*a^3*c^5*d^1
3*e^6 - 35*a^4*c^4*d^11*e^8 - 56*a^5*c^3*d^9*e^10 + 91*a^6*c^2*d^7*e^12 - 5
0*a^7*c*d^5*e^14 + 10*a^8*d^3*e^16)*x^2 + (2*a*c^7*d^18*e - 7*a^2*c^6*d^16*
e^3 + 35*a^4*c^4*d^12*e^7 - 70*a^5*c^3*d^10*e^9 + 63*a^6*c^2*d^8*e^11 - 28*
a^7*c*d^6*e^13 + 5*a^8*d^4*e^15)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, undef, undef, undef, undef, undef, undef, 2]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algori
thm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, undef, undef, undef, undef, undef, unde
f, 2]
```

3.2080 $\int \frac{1}{(d+ex)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=519

$$\frac{3003c^5d^5e\sqrt{d+ex}}{128\left(cd^2-ae^2\right)^7\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{1001c^4d^4e}{128\sqrt{d+ex}\left(cd^2-ae^2\right)^6\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{}{320\left(cd^2-\right.}$$

```
[Out] 1/(5*(c*d^2 - a*e^2)*(d + e*x)^(7/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)) + (13*c*d)/(40*(c*d^2 - a*e^2)^2*(d + e*x)^(5/2)*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(3/2)) + (143*c^2*d^2)/(240*(c*d^2 - a*e^2)^3*(d + e
*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) + (429*c^3*d^3)/(3
20*(c*d^2 - a*e^2)^4*Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(3/2)) - (1001*c^4*d^4*Sqrt[d + e*x])/(320*(c*d^2 - a*e^2)^5*(a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (1001*c^4*d^4*e)/(128*(c*d^2 - a*e^2)^6*
Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (3003*c^5*d^5*
e*Sqrt[d + e*x])/(128*(c*d^2 - a*e^2)^7*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]) + (3003*c^5*d^5*e^(3/2)*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(128*(c*d^2 - a*e
^2)^(15/2))
```

Rubi [A] time = 0.598339, antiderivative size = 519, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {672, 666, 660, 205}

$$\frac{3003c^5d^5e\sqrt{d+ex}}{128\left(cd^2-ae^2\right)^7\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{1001c^4d^4e}{128\sqrt{d+ex}\left(cd^2-ae^2\right)^6\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{}{320\left(cd^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(7/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```


```
[Out] 1/(5*(c*d^2 - a*e^2)*(d + e*x)^(7/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)) + (13*c*d)/(40*(c*d^2 - a*e^2)^2*(d + e*x)^(5/2)*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(3/2)) + (143*c^2*d^2)/(240*(c*d^2 - a*e^2)^3*(d + e
*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) + (429*c^3*d^3)/(3
20*(c*d^2 - a*e^2)^4*Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(3/2)) - (1001*c^4*d^4*Sqrt[d + e*x])/(320*(c*d^2 - a*e^2)^5*(a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (1001*c^4*d^4*e)/(128*(c*d^2 - a*e^2)^6*
Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (3003*c^5*d^5*
e*Sqrt[d + e*x])/(128*(c*d^2 - a*e^2)^7*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]) + (3003*c^5*d^5*e^(3/2)*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(128*(c*d^2 - a*e
^2)^(15/2))
```


Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*(m + 2*p + 2))/((m + p + 1)*(2*c*d - b*e)), Int[(d
 + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && Ne
```

```
Q[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 666

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((2*c*d - b*e)*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/(e*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[((2*c*d - b*e)*(m + 2*p + 2))/((p + 1)*(b^2
- 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0
] && LtQ[p, -1] && LtQ[0, m, 1] && IntegerQ[2*p]
```

Rule 660

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Dist[2*e, Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}dx = \frac{1}{5\left(cd^2-ae^2\right)(d+ex)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{(13cd)}{}$$

$$= \frac{1}{5\left(cd^2-ae^2\right)(d+ex)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{40\left(cd\right.}$$

$$= \frac{1}{5\left(cd^2-ae^2\right)(d+ex)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{40\left(cd\right.}$$

$$= \frac{1}{5\left(cd^2-ae^2\right)(d+ex)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{40\left(cd\right.}$$

$$= \frac{1}{5\left(cd^2-ae^2\right)(d+ex)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{40\left(cd\right.}$$

$$= \frac{1}{5\left(cd^2-ae^2\right)(d+ex)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{40\left(cd\right.}$$

$$= \frac{1}{5\left(cd^2-ae^2\right)(d+ex)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{40\left(cd\right.}$$

$$= \frac{1}{5\left(cd^2-ae^2\right)(d+ex)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{40\left(cd\right.}$$

$$= \frac{1}{5\left(cd^2-ae^2\right)(d+ex)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{40\left(cd\right.}$$

Mathematica [C] time = 0.0379719, size = 83, normalized size = 0.16

$$-\frac{2c^5d^5(d+ex)^{3/2}\,{}_2F_1\left(-\frac{3}{2},6;-\frac{1}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{3\left(cd^2-ae^2\right)^6((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(7/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))
,x]
```

```
[Out] (-2*c^5*d^5*(d + e*x)^(3/2)*Hypergeometric2F1[-3/2, 6, -1/2, (e*(a*e + c*d*
x))/(-(c*d^2) + a*e^2)])/(3*(c*d^2 - a*e^2)^6*((a*e + c*d*x)*(d + e*x))^(3/
2))
```

Maple [B] time = 0.259, size = 1553, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] 1/1920*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(-45045*((a*e^2-c*d^2)*e)^(1
/2)*x^6*c^6*d^6*e^6-210210*((a*e^2-c*d^2)*e)^(1/2)*x^5*c^6*d^7*e^5-384384*(
(a*e^2-c*d^2)*e)^(1/2)*x^4*c^6*d^8*e^4-338910*((a*e^2-c*d^2)*e)^(1/2)*x^3*c
^6*d^9*e^3-137995*((a*e^2-c*d^2)*e)^(1/2)*x^2*c^6*d^10*e^2-24320*((a*e^2-c*
d^2)*e)^(1/2)*a*c^5*d^10*e^2+2928*((a*e^2-c*d^2)*e)^(1/2)*a^5*c*d^2*e^10-10
024*((a*e^2-c*d^2)*e)^(1/2)*a^4*c^2*d^4*e^8+21070*((a*e^2-c*d^2)*e)^(1/2)*a
^3*c^3*d^6*e^6-35595*((a*e^2-c*d^2)*e)^(1/2)*a^2*c^4*d^8*e^4-16640*((a*e^2-
c*d^2)*e)^(1/2)*x*c^6*d^11*e+1280*((a*e^2-c*d^2)*e)^(1/2)*c^6*d^12-384*((a*
e^2-c*d^2)*e)^(1/2)*a^6*e^12+225225*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d
^2)*e)^(1/2))*x*a*c^5*d^9*e^4*(c*d*x+a*e)^(1/2)+45045*arctanh(e*(c*d*x+a*e)
^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*x^5*a*c^5*d^5*e^8*(c*d*x+a*e)^(1/2)+225225*
arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*x^4*a*c^5*d^6*e^7*(c*d
*x+a*e)^(1/2)+450450*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*x
^3*a*c^5*d^7*e^6*(c*d*x+a*e)^(1/2)+450450*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e
^2-c*d^2)*e)^(1/2))*x^2*a*c^5*d^8*e^5*(c*d*x+a*e)^(1/2)+225225*arctanh(e*(c
*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*x^5*c^6*d^7*e^6*(c*d*x+a*e)^(1/2)+
450450*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*x^4*c^6*d^8*e^5
*(c*d*x+a*e)^(1/2)+450450*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/
2))*x^3*c^6*d^9*e^4*(c*d*x+a*e)^(1/2)+225225*arctanh(e*(c*d*x+a*e)^(1/2)/((
a*e^2-c*d^2)*e)^(1/2))*x^2*c^6*d^10*e^3*(c*d*x+a*e)^(1/2)+45045*arctanh(e*(
c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*x*c^6*d^11*e^2*(c*d*x+a*e)^(1/2)-
60060*((a*e^2-c*d^2)*e)^(1/2)*x^5*a*c^5*d^5*e^7-282282*((a*e^2-c*d^2)*e)^(1
/2)*x^4*a*c^5*d^6*e^6-520806*((a*e^2-c*d^2)*e)^(1/2)*x^3*a*c^5*d^7*e^5-4647
50*((a*e^2-c*d^2)*e)^(1/2)*x^2*a*c^5*d^8*e^4-192790*((a*e^2-c*d^2)*e)^(1/2)
*x*a*c^5*d^9*e^3-9009*((a*e^2-c*d^2)*e)^(1/2)*x^4*a^2*c^4*d^4*e^8+2574*((a*
e^2-c*d^2)*e)^(1/2)*x^3*a^3*c^3*d^3*e^9-43758*((a*e^2-c*d^2)*e)^(1/2)*x^3*a
^2*c^4*d^5*e^7-1144*((a*e^2-c*d^2)*e)^(1/2)*x^2*a^4*c^2*d^2*e^10+12298*((a*
e^2-c*d^2)*e)^(1/2)*x^2*a^3*c^3*d^4*e^8-84084*((a*e^2-c*d^2)*e)^(1/2)*x^2*a
^2*c^4*d^6*e^6+624*((a*e^2-c*d^2)*e)^(1/2)*x*a^5*c*d*e^11-5408*((a*e^2-c*d^
2)*e)^(1/2)*x*a^4*c^2*d^3*e^9+23114*((a*e^2-c*d^2)*e)^(1/2)*x*a^3*c^3*d^5*e
^7-79170*((a*e^2-c*d^2)*e)^(1/2)*x*a^2*c^4*d^7*e^5+45045*arctanh(e*(c*d*x+a
*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*x^6*c^6*d^6*e^7*(c*d*x+a*e)^(1/2)+45045*
arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*a*c^5*d^10*e^3*(c*d*x+
a*e)^(1/2))/(e*x+d)^(11/2)/(c*d*x+a*e)^2/(a*e^2-c*d^2)^7/((a*e^2-c*d^2)*e)^
(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}} (ex + d)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algori
thm="maxima")
```

```
[Out] integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(e*x + d)^(7/2))
, x)
```

Fricas [B] time = 2.80526, size = 8296, normalized size = 15.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algori
thm="fricas")
```

```
[Out] [1/3840*(45045*(c^7*d^7*e^7*x^8 + a^2*c^5*d^11*e^3 + 2*(3*c^7*d^8*e^6 + a*c
^6*d^6*e^8)*x^7 + (15*c^7*d^9*e^5 + 12*a*c^6*d^7*e^7 + a^2*c^5*d^5*e^9)*x^6
 + 2*(10*c^7*d^10*e^4 + 15*a*c^6*d^8*e^6 + 3*a^2*c^5*d^6*e^8)*x^5 + 5*(3*c^
7*d^11*e^3 + 8*a*c^6*d^9*e^5 + 3*a^2*c^5*d^7*e^7)*x^4 + 2*(3*c^7*d^12*e^2 +
 15*a*c^6*d^10*e^4 + 10*a^2*c^5*d^8*e^6)*x^3 + (c^7*d^13*e + 12*a*c^6*d^11*
e^3 + 15*a^2*c^5*d^9*e^5)*x^2 + 2*(a*c^6*d^12*e^2 + 3*a^2*c^5*d^10*e^4)*x)*
sqrt(-e/(c*d^2 - a*e^2))*log(-(c*d*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2
+ 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d^2 - a*e^2)*sqrt(e*x +
d)*sqrt(-e/(c*d^2 - a*e^2)))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(45045*c^6*d^6*
e^6*x^6 - 1280*c^6*d^12 + 24320*a*c^5*d^10*e^2 + 35595*a^2*c^4*d^8*e^4 - 21
070*a^3*c^3*d^6*e^6 + 10024*a^4*c^2*d^4*e^8 - 2928*a^5*c*d^2*e^10 + 384*a^6
*e^12 + 30030*(7*c^6*d^7*e^5 + 2*a*c^5*d^5*e^7)*x^5 + 3003*(128*c^6*d^8*e^4
 + 94*a*c^5*d^6*e^6 + 3*a^2*c^4*d^4*e^8)*x^4 + 858*(395*c^6*d^9*e^3 + 607*a
*c^5*d^7*e^5 + 51*a^2*c^4*d^5*e^7 - 3*a^3*c^3*d^3*e^9)*x^3 + 143*(965*c^6*d
^10*e^2 + 3250*a*c^5*d^8*e^4 + 588*a^2*c^4*d^6*e^6 - 86*a^3*c^3*d^4*e^8 + 8
*a^4*c^2*d^2*e^10)*x^2 + 26*(640*c^6*d^11*e + 7415*a*c^5*d^9*e^3 + 3045*a^2
*c^4*d^7*e^5 - 889*a^3*c^3*d^5*e^7 + 208*a^4*c^2*d^3*e^9 - 24*a^5*c*d*e^11)
*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a^2*c^7*d^2
0*e^2 - 7*a^3*c^6*d^18*e^4 + 21*a^4*c^5*d^16*e^6 - 35*a^5*c^4*d^14*e^8 + 35
*a^6*c^3*d^12*e^10 - 21*a^7*c^2*d^10*e^12 + 7*a^8*c*d^8*e^14 - a^9*d^6*e^16
 + (c^9*d^16*e^6 - 7*a*c^8*d^14*e^8 + 21*a^2*c^7*d^12*e^10 - 35*a^3*c^6*d^1
0*e^12 + 35*a^4*c^5*d^8*e^14 - 21*a^5*c^4*d^6*e^16 + 7*a^6*c^3*d^4*e^18 - a
^7*c^2*d^2*e^20)*x^8 + 2*(3*c^9*d^17*e^5 - 20*a*c^8*d^15*e^7 + 56*a^2*c^7*d
^13*e^9 - 84*a^3*c^6*d^11*e^11 + 70*a^4*c^5*d^9*e^13 - 28*a^5*c^4*d^7*e^15
+ 4*a^7*c^2*d^3*e^19 - a^8*c*d*e^21)*x^7 + (15*c^9*d^18*e^4 - 93*a*c^8*d^16
*e^6 + 232*a^2*c^7*d^14*e^8 - 280*a^3*c^6*d^12*e^10 + 126*a^4*c^5*d^10*e^12
 + 70*a^5*c^4*d^8*e^14 - 112*a^6*c^3*d^6*e^16 + 48*a^7*c^2*d^4*e^18 - 5*a^8
*c*d^2*e^20 - a^9*e^22)*x^6 + 2*(10*c^9*d^19*e^3 - 55*a*c^8*d^17*e^5 + 108*
a^2*c^7*d^15*e^7 - 56*a^3*c^6*d^13*e^9 - 112*a^4*c^5*d^11*e^11 + 210*a^5*c^
4*d^9*e^13 - 140*a^6*c^3*d^7*e^15 + 32*a^7*c^2*d^5*e^17 + 6*a^8*c*d^3*e^19
- 3*a^9*d*e^21)*x^5 + 5*(3*c^9*d^20*e^2 - 13*a*c^8*d^18*e^4 + 10*a^2*c^7*d^
16*e^6 + 42*a^3*c^6*d^14*e^8 - 112*a^4*c^5*d^12*e^10 + 112*a^5*c^4*d^10*e^1
2 - 42*a^6*c^3*d^8*e^14 - 10*a^7*c^2*d^6*e^16 + 13*a^8*c*d^4*e^18 - 3*a^9*d
^2*e^20)*x^4 + 2*(3*c^9*d^21*e - 6*a*c^8*d^19*e^3 - 32*a^2*c^7*d^17*e^5 + 1
40*a^3*c^6*d^15*e^7 - 210*a^4*c^5*d^13*e^9 + 112*a^5*c^4*d^11*e^11 + 56*a^6
*c^3*d^9*e^13 - 108*a^7*c^2*d^7*e^15 + 55*a^8*c*d^5*e^17 - 10*a^9*d^3*e^19)
*x^3 + (c^9*d^22 + 5*a*c^8*d^20*e^2 - 48*a^2*c^7*d^18*e^4 + 112*a^3*c^6*d^1
6*e^6 - 70*a^4*c^5*d^14*e^8 - 126*a^5*c^4*d^12*e^10 + 280*a^6*c^3*d^10*e^12
 - 232*a^7*c^2*d^8*e^14 + 93*a^8*c*d^6*e^16 - 15*a^9*d^4*e^18)*x^2 + 2*(a*c
^8*d^21*e - 4*a^2*c^7*d^19*e^3 + 28*a^4*c^5*d^15*e^7 - 70*a^5*c^4*d^13*e^9
+ 84*a^6*c^3*d^11*e^11 - 56*a^7*c^2*d^9*e^13 + 20*a^8*c*d^7*e^15 - 3*a^9*d^
5*e^17)*x), 1/1920*(45045*(c^7*d^7*e^7*x^8 + a^2*c^5*d^11*e^3 + 2*(3*c^7*d^
8*e^6 + a*c^6*d^6*e^8)*x^7 + (15*c^7*d^9*e^5 + 12*a*c^6*d^7*e^7 + a^2*c^5*d
^5*e^9)*x^6 + 2*(10*c^7*d^10*e^4 + 15*a*c^6*d^8*e^6 + 3*a^2*c^5*d^6*e^8)*x^
5 + 5*(3*c^7*d^11*e^3 + 8*a*c^6*d^9*e^5 + 3*a^2*c^5*d^7*e^7)*x^4 + 2*(3*c^7
*d^12*e^2 + 15*a*c^6*d^10*e^4 + 10*a^2*c^5*d^8*e^6)*x^3 + (c^7*d^13*e + 12*
a*c^6*d^11*e^3 + 15*a^2*c^5*d^9*e^5)*x^2 + 2*(a*c^6*d^12*e^2 + 3*a^2*c^5*d^
10*e^4)*x)*sqrt(e/(c*d^2 - a*e^2))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(e/(c*d^2 - a*e^2))/(c*d*e^2*
x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x)) + (45045*c^6*d^6*e^6*x^6 - 1280*c^6*d
```

```
^12 + 24320*a*c^5*d^10*e^2 + 35595*a^2*c^4*d^8*e^4 - 21070*a^3*c^3*d^6*e^6
+ 10024*a^4*c^2*d^4*e^8 - 2928*a^5*c*d^2*e^10 + 384*a^6*e^12 + 30030*(7*c^6
*d^7*e^5 + 2*a*c^5*d^5*e^7)*x^5 + 3003*(128*c^6*d^8*e^4 + 94*a*c^5*d^6*e^6
+ 3*a^2*c^4*d^4*e^8)*x^4 + 858*(395*c^6*d^9*e^3 + 607*a*c^5*d^7*e^5 + 51*a^
2*c^4*d^5*e^7 - 3*a^3*c^3*d^3*e^9)*x^3 + 143*(965*c^6*d^10*e^2 + 3250*a*c^5
*d^8*e^4 + 588*a^2*c^4*d^6*e^6 - 86*a^3*c^3*d^4*e^8 + 8*a^4*c^2*d^2*e^10)*x
^2 + 26*(640*c^6*d^11*e + 7415*a*c^5*d^9*e^3 + 3045*a^2*c^4*d^7*e^5 - 889*a
^3*c^3*d^5*e^7 + 208*a^4*c^2*d^3*e^9 - 24*a^5*c*d*e^11)*x)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a^2*c^7*d^20*e^2 - 7*a^3*c^6*d^
18*e^4 + 21*a^4*c^5*d^16*e^6 - 35*a^5*c^4*d^14*e^8 + 35*a^6*c^3*d^12*e^10 -
 21*a^7*c^2*d^10*e^12 + 7*a^8*c*d^8*e^14 - a^9*d^6*e^16 + (c^9*d^16*e^6 - 7
*a*c^8*d^14*e^8 + 21*a^2*c^7*d^12*e^10 - 35*a^3*c^6*d^10*e^12 + 35*a^4*c^5*
d^8*e^14 - 21*a^5*c^4*d^6*e^16 + 7*a^6*c^3*d^4*e^18 - a^7*c^2*d^2*e^20)*x^8
 + 2*(3*c^9*d^17*e^5 - 20*a*c^8*d^15*e^7 + 56*a^2*c^7*d^13*e^9 - 84*a^3*c^6
*d^11*e^11 + 70*a^4*c^5*d^9*e^13 - 28*a^5*c^4*d^7*e^15 + 4*a^7*c^2*d^3*e^19
 - a^8*c*d*e^21)*x^7 + (15*c^9*d^18*e^4 - 93*a*c^8*d^16*e^6 + 232*a^2*c^7*d
^14*e^8 - 280*a^3*c^6*d^12*e^10 + 126*a^4*c^5*d^10*e^12 + 70*a^5*c^4*d^8*e^
14 - 112*a^6*c^3*d^6*e^16 + 48*a^7*c^2*d^4*e^18 - 5*a^8*c*d^2*e^20 - a^9*e^
22)*x^6 + 2*(10*c^9*d^19*e^3 - 55*a*c^8*d^17*e^5 + 108*a^2*c^7*d^15*e^7 - 5
6*a^3*c^6*d^13*e^9 - 112*a^4*c^5*d^11*e^11 + 210*a^5*c^4*d^9*e^13 - 140*a^6
*c^3*d^7*e^15 + 32*a^7*c^2*d^5*e^17 + 6*a^8*c*d^3*e^19 - 3*a^9*d*e^21)*x^5
+ 5*(3*c^9*d^20*e^2 - 13*a*c^8*d^18*e^4 + 10*a^2*c^7*d^16*e^6 + 42*a^3*c^6*
d^14*e^8 - 112*a^4*c^5*d^12*e^10 + 112*a^5*c^4*d^10*e^12 - 42*a^6*c^3*d^8*e
^14 - 10*a^7*c^2*d^6*e^16 + 13*a^8*c*d^4*e^18 - 3*a^9*d^2*e^20)*x^4 + 2*(3*
c^9*d^21*e - 6*a*c^8*d^19*e^3 - 32*a^2*c^7*d^17*e^5 + 140*a^3*c^6*d^15*e^7
- 210*a^4*c^5*d^13*e^9 + 112*a^5*c^4*d^11*e^11 + 56*a^6*c^3*d^9*e^13 - 108*
a^7*c^2*d^7*e^15 + 55*a^8*c*d^5*e^17 - 10*a^9*d^3*e^19)*x^3 + (c^9*d^22 + 5
*a*c^8*d^20*e^2 - 48*a^2*c^7*d^18*e^4 + 112*a^3*c^6*d^16*e^6 - 70*a^4*c^5*d
^14*e^8 - 126*a^5*c^4*d^12*e^10 + 280*a^6*c^3*d^10*e^12 - 232*a^7*c^2*d^8*e
^14 + 93*a^8*c*d^6*e^16 - 15*a^9*d^4*e^18)*x^2 + 2*(a*c^8*d^21*e - 4*a^2*c^
7*d^19*e^3 + 28*a^4*c^5*d^15*e^7 - 70*a^5*c^4*d^13*e^9 + 84*a^6*c^3*d^11*e^
11 - 56*a^7*c^2*d^9*e^13 + 20*a^8*c*d^7*e^15 - 3*a^9*d^5*e^17)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, undef, undef, undef, undef, undef, undef, 2]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algori
thm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, undef, undef, undef, undef, undef, unde
f, 2]
```

3.2081 $\int \frac{1}{\sqrt{d+ex}\sqrt{d^2-e^2x^2}} dx$

Optimal. Leaf size=52

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{\sqrt{2}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{d}e}$$

```
[Out] -((Sqrt[2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/(Sqrt[2]*Sqrt[d]*Sqrt[d + e*x])])/(S
qrt[d]*e))
```

Rubi [A] time = 0.0241019, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {661, 208}

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{\sqrt{2}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{d}e}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/(Sqrt[2]*Sqrt[d]*Sqrt[d + e*x])])/(S
qrt[d]*e))
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{d+ex}\sqrt{d^2-e^2x^2}} dx &= (2e)\,\text{Subst}\left(\int \frac{1}{-2de^2+e^2x^2} dx, x, \frac{\sqrt{d^2-e^2x^2}}{\sqrt{d+ex}}\right)\\ &= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{\sqrt{2}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{d}e}\end{aligned}$$

Mathematica [A] time = 0.0515172, size = 52, normalized size = 1.

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{\sqrt{2}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{d}e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/(Sqrt[2]*Sqrt[d]*Sqrt[d + e*x])])/(S
qrt[d]*e))
```

Maple [A] time = 0.164, size = 58, normalized size = 1.1

$$-\frac{\sqrt{2}}{e}\sqrt{-e^2x^2+d^2}\operatorname{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{-ex+d}\frac{1}{\sqrt{d}}\right)\frac{1}{\sqrt{ex+d}}\frac{1}{\sqrt{-ex+d}}\frac{1}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/(e*x+d)^(1/2)*(-e^2*x^2+d^2)^(1/2)/(-e*x+d)^(1/2)/e*2^(1/2)/d^(1/2)*arct
anh(1/2*(-e*x+d)^(1/2)*2^(1/2)/d^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-e^2x^2+d^2}\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-e^2*x^2 + d^2)*sqrt(e*x + d)), x)
```

Fricas [A] time = 1.87935, size = 333, normalized size = 6.4

$$\left[\frac{\sqrt{2}\log\left(-\frac{e^2x^2-2\,dex+2\,\sqrt{2}\sqrt{-e^2x^2+d^2}\sqrt{ex+d}\sqrt{d}-3\,d^2}{e^2x^2+2\,dex+d^2}\right)}{2\,\sqrt{d}e}, -\frac{\sqrt{2}\sqrt{-\frac{1}{d}}\arctan\left(\frac{\sqrt{2}\sqrt{-e^2x^2+d^2}\sqrt{ex+d}d\sqrt{-\frac{1}{d}}}{e^2x^2-d^2}\right)}{e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*log(-(e^2*x^2 - 2*d*e*x + 2*sqrt(2)*sqrt(-e^2*x^2 + d^2)*sqrt(
e*x + d)*sqrt(d) - 3*d^2)/(e^2*x^2 + 2*d*e*x + d^2))/(sqrt(d)*e), -sqrt(2)*
sqrt(-1/d)*arctan(sqrt(2)*sqrt(-e^2*x^2 + d^2)*sqrt(e*x + d)*d*sqrt(-1/d)/(
e^2*x^2 - d^2))/e]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(-d+ex)(d+ex)}\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-d + e*x)*(d + e*x))*sqrt(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-e^2x^2+d^2}\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-e^2*x^2 + d^2)*sqrt(e*x + d)), x)
```

3.2082 $\int \frac{1}{\sqrt{-d+ex}\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{\sqrt{2}\sqrt{d}\sqrt{ex-d}}\right)}{\sqrt{d}e}$$

```
[Out] (Sqrt[2]*ArcTan[Sqrt[d^2 - e^2*x^2]/(Sqrt[2]*Sqrt[d]*Sqrt[-d + e*x])])/(Sqr
t[d]*e)
```

Rubi [A] time = 0.02212, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {661, 205}

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{\sqrt{2}\sqrt{d}\sqrt{ex-d}}\right)}{\sqrt{d}e}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-d + e*x]*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[2]*ArcTan[Sqrt[d^2 - e^2*x^2]/(Sqrt[2]*Sqrt[d]*Sqrt[-d + e*x])])/(Sqr
t[d]*e)
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-d+ex}\sqrt{d^2-e^2x^2}}\,dx &= (2e)\,\text{Subst}\left(\int \frac{1}{2de^2+e^2x^2}\,dx, x, \frac{\sqrt{d^2-e^2x^2}}{\sqrt{-d+ex}}\right)\\ &= \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{\sqrt{2}\sqrt{d}\sqrt{-d+ex}}\right)}{\sqrt{d}e}\end{aligned}$$

Mathematica [A] time = 0.0634732, size = 53, normalized size = 1.

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{\sqrt{2}\sqrt{d}\sqrt{ex-d}}\right)}{\sqrt{d}e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-d + e*x]*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[2]*ArcTan[Sqrt[d^2 - e^2*x^2]/(Sqrt[2]*Sqrt[d]*Sqrt[-d + e*x])])/(Sqr
t[d]*e)
```

Maple [A] time = 0.166, size = 63, normalized size = 1.2

$$\frac{\sqrt{2}}{e}\sqrt{-e^2x^2+d^2}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-ex-d}\frac{1}{\sqrt{d}}\right)\frac{1}{\sqrt{ex-d}}\frac{1}{\sqrt{-ex-d}}\frac{1}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x-d)^(1/2)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] 1/(e*x-d)^(1/2)*(-e^2*x^2+d^2)^(1/2)/(-e*x-d)^(1/2)/e*2^(1/2)/d^(1/2)*arcta
n(1/2*(-e*x-d)^(1/2)*2^(1/2)/d^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-e^2x^2+d^2}\sqrt{ex-d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x-d)^(1/2)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-e^2*x^2 + d^2)*sqrt(e*x - d)), x)
```

Fricas [A] time = 1.89183, size = 332, normalized size = 6.26

$$\left[\frac{\sqrt{2}\sqrt{-\frac{1}{d}}\log\left(-\frac{e^2x^2+2\,dex-2\,\sqrt{2}\sqrt{-e^2x^2+d^2}\sqrt{ex-d}d\sqrt{-\frac{1}{d}}-3\,d^2}{e^2x^2-2\,dex+d^2}\right)}{2\,e},\ \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-e^2x^2+d^2}\sqrt{ex-d}\sqrt{d}}{e^2x^2-d^2}\right)}{\sqrt{d}e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x-d)^(1/2)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*sqrt(-1/d)*log(-(e^2*x^2 + 2*d*e*x - 2*sqrt(2)*sqrt(-e^2*x^2 +
 d^2)*sqrt(e*x - d)*d*sqrt(-1/d) - 3*d^2)/(e^2*x^2 - 2*d*e*x + d^2))/e, sqr
t(2)*arctan(sqrt(2)*sqrt(-e^2*x^2 + d^2)*sqrt(e*x - d)*sqrt(d)/(e^2*x^2 - d
^2))/(sqrt(d)*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(-d+ex)(d+ex)}\sqrt{-d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x-d)**(1/2)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-d + e*x)*(d + e*x))*sqrt(-d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-e^2x^2 + d^2}\sqrt{ex - d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x-d)^(1/2)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-e^2*x^2 + d^2)*sqrt(e*x - d)), x)
```

3.2083 $\int \frac{(d+ex)^{2/3}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=566

$$\frac{3^{3/4}(d+ex)^{2/3}\left(cd^2-ae^2\right)^{2/3}\sqrt{ade+cd^2x}\left(\sqrt[3]{cd^2-ae^2}-\sqrt[3]{c}d^{2/3}\sqrt[3]{\frac{ex}{d}+1}\right)\sqrt{\frac{\sqrt[3]{c}d^{2/3}\sqrt[3]{\frac{ex}{d}+1}\sqrt[3]{cd^2-ae^2}+\left(cd^2-ae^2\right)^{2/3}+c^{2/3}d^{4/3}\left(\frac{ex}{d}+1\right)^{2}}{\left(\sqrt[3]{cd^2-ae^2}-\left(1+\sqrt{3}\right)\sqrt[3]{c}d^{2/3}\sqrt[3]{\frac{ex}{d}+1}\right)^2}}}{4cde\sqrt{d(ae+cdx)}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\sqrt{-\frac{\sqrt[3]{c}d^{2/3}\sqrt[3]{\frac{ex}{d}+1}\left(\sqrt[3]{cd^2-ae^2}\right.}{\left(\sqrt[3]{cd^2-ae^2}-\left(1+\sqrt{3}\right)\sqrt[3]{}\right.}}}$$

```
[Out] (3*(a*e + c*d*x)*(d + e*x)^(2/3))/(2*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]) + (3^(3/4)*(c*d^2 - a*e^2)^(2/3)*Sqrt[a*d*e + c*d^2*x]*(d + e*x)
^(2/3)*((c*d^2 - a*e^2)^(1/3) - c^(1/3)*d^(2/3)*(1 + (e*x)/d)^(1/3))*Sqrt[(
(c*d^2 - a*e^2)^(2/3) + c^(1/3)*d^(2/3)*(c*d^2 - a*e^2)^(1/3)*(1 + (e*x)/d)
^(1/3) + c^(2/3)*d^(4/3)*(1 + (e*x)/d)^(2/3))/((c*d^2 - a*e^2)^(1/3) - (1 +
 Sqrt[3])*c^(1/3)*d^(2/3)*(1 + (e*x)/d)^(1/3))^2]*EllipticF[ArcCos[((c*d^2
- a*e^2)^(1/3) - (1 - Sqrt[3])*c^(1/3)*d^(2/3)*(1 + (e*x)/d)^(1/3))/((c*d^2
 - a*e^2)^(1/3) - (1 + Sqrt[3])*c^(1/3)*d^(2/3)*(1 + (e*x)/d)^(1/3))], (2 +
 Sqrt[3])/4])/(4*c*d*e*Sqrt[d*(a*e + c*d*x)]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2]*Sqrt[-((c^(1/3)*d^(2/3)*(1 + (e*x)/d)^(1/3)*((c*d^2 - a*e^2)^
(1/3) - c^(1/3)*d^(2/3)*(1 + (e*x)/d)^(1/3)))/((c*d^2 - a*e^2)^(1/3) - (1 +
 Sqrt[3])*c^(1/3)*d^(2/3)*(1 + (e*x)/d)^(1/3))^2)])
```

Rubi [A] time = 0.765203, antiderivative size = 566, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {679, 677, 50, 63, 225}

$$\frac{3^{3/4}(d+ex)^{2/3}\left(cd^2-ae^2\right)^{2/3}\sqrt{ade+cd^2x}\left(\sqrt[3]{cd^2-ae^2}-\sqrt[3]{c}d^{2/3}\sqrt[3]{\frac{ex}{d}+1}\right)\sqrt{\frac{\sqrt[3]{c}d^{2/3}\sqrt[3]{\frac{ex}{d}+1}\sqrt[3]{cd^2-ae^2}+\left(cd^2-ae^2\right)^{2/3}+c^{2/3}d^{4/3}\left(\frac{ex}{d}+1\right)^{2}}{\left(\sqrt[3]{cd^2-ae^2}-\left(1+\sqrt{3}\right)\sqrt[3]{c}d^{2/3}\sqrt[3]{\frac{ex}{d}+1}\right)^2}}}{4cde\sqrt{d(ae+cdx)}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\sqrt{-\frac{\sqrt[3]{c}d^{2/3}\sqrt[3]{\frac{ex}{d}+1}\left(\sqrt[3]{cd^2-ae^2}-\sqrt[3]{c}\right.}{\left(\sqrt[3]{cd^2-ae^2}-\left(1+\sqrt{3}\right)\sqrt[3]{c}d^{2/3}\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(2/3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```


```
[Out] (3*(a*e + c*d*x)*(d + e*x)^(2/3))/(2*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]) + (3^(3/4)*(c*d^2 - a*e^2)^(2/3)*Sqrt[a*d*e + c*d^2*x]*(d + e*x)
^(2/3)*((c*d^2 - a*e^2)^(1/3) - c^(1/3)*d^(2/3)*(1 + (e*x)/d)^(1/3))*Sqrt[(
(c*d^2 - a*e^2)^(2/3) + c^(1/3)*d^(2/3)*(c*d^2 - a*e^2)^(1/3)*(1 + (e*x)/d)
^(1/3) + c^(2/3)*d^(4/3)*(1 + (e*x)/d)^(2/3))/((c*d^2 - a*e^2)^(1/3) - (1 +
 Sqrt[3])*c^(1/3)*d^(2/3)*(1 + (e*x)/d)^(1/3))^2]*EllipticF[ArcCos[((c*d^2
- a*e^2)^(1/3) - (1 - Sqrt[3])*c^(1/3)*d^(2/3)*(1 + (e*x)/d)^(1/3))/((c*d^2
 - a*e^2)^(1/3) - (1 + Sqrt[3])*c^(1/3)*d^(2/3)*(1 + (e*x)/d)^(1/3))], (2 +
 Sqrt[3])/4])/(4*c*d*e*Sqrt[d*(a*e + c*d*x)]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2]*Sqrt[-((c^(1/3)*d^(2/3)*(1 + (e*x)/d)^(1/3)*((c*d^2 - a*e^2)^
(1/3) - c^(1/3)*d^(2/3)*(1 + (e*x)/d)^(1/3)))/((c*d^2 - a*e^2)^(1/3) - (1 +
 Sqrt[3])*c^(1/3)*d^(2/3)*(1 + (e*x)/d)^(1/3))^2)])
```


Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(d^IntPart[m]*(d + e*x)^FracPart[m])/(1 + (e*x)/d)^FracPart[m
```

```
], Int[(1 + (e*x)/d)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !Intege
rQ[p] &&  !(IntegerQ[m] || GtQ[d, 0])
```

Rule 677

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(d^m*(a + b*x + c*x^2)^FracPart[p])/((1 + (e*x)/d)^FracPart[p
]*(a/d + (c*x)/e)^FracPart[p]), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || GtQ[d, 0]) &&  !(
IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{(d+ex)^{2/3}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}dx = \frac{(d+ex)^{2/3}\int \frac{\left(1+\frac{ex}{d}\right)^{2/3}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}dx}{\left(1+\frac{ex}{d}\right)^{2/3}}$$

$$= \frac{\left(\sqrt{ade+cd^2x}(d+ex)^{2/3}\right)\int \frac{\sqrt[6]{1+\frac{ex}{d}}}{\sqrt{ade+cd^2x}}dx}{\sqrt[6]{1+\frac{ex}{d}}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$= \frac{3(ae+cdx)(d+ex)^{2/3}}{2cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{\left(\left(1-\frac{ae^2}{cd^2}\right)\sqrt{ade+cd^2x}(d+ex)^{2/3}\right)\int \frac{}{\sqrt{ade+}}}{4\sqrt[6]{1+\frac{ex}{d}}\sqrt{ade+\left(cd^2+ae^2\right)x+}}$$

$$= \frac{3(ae+cdx)(d+ex)^{2/3}}{2cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{\left(3d\left(1-\frac{ae^2}{cd^2}\right)\sqrt{ade+cd^2x}(d+ex)^{2/3}\right)\text{Sub}}{2e\sqrt[6]{1+\frac{ex}{d}}\sqrt{ade+\left(cd^2\right.}}$$

$$= \frac{3(ae+cdx)(d+ex)^{2/3}}{2cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{3^{3/4}\left(cd^2-ae^2\right)^{2/3}\sqrt{ade+cd^2x}(d+ex)^{2/3}\left(\right.}{}$$

Mathematica [C] time = 0.0612044, size = 95, normalized size = 0.17

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{cd\sqrt[3]{d+ex}\sqrt[6]{\frac{cd(d+ex)}{cd^2-ae^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(2/3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*Hypergeometric2F1[-1/6, 1/2, 3/2, (e*(a*e
+ c*d*x))/(-(c*d^2) + a*e^2)])/(c*d*(d + e*x)^(1/3)*((c*d*(d + e*x))/(c*d^2
 - a*e^2))^(1/6))
```

Maple [F] time = 0.326, size = 0, normalized size = 0.

$$\int (ex+d)^{\frac{2}{3}}\frac{1}{\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(2/3)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] int((e*x+d)^(2/3)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{2}{3}}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(2/3)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate((e*x + d)^(2/3)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(ex+d)^{\frac{2}{3}}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(2/3)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="fricas")
```

```
[Out] integral((e*x + d)^(2/3)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(2/3)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{2}{3}}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(2/3)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((e*x + d)^(2/3)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x), x)
```

3.2084 $\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3 dx$

Optimal. Leaf size=130

$$-\frac{3c^2d^2\left(cd^2-ae^2\right)(d+ex)^{m+6}}{e^4(m+6)}-\frac{\left(cd^2-ae^2\right)^3(d+ex)^{m+4}}{e^4(m+4)}+\frac{3cd\left(cd^2-ae^2\right)^2(d+ex)^{m+5}}{e^4(m+5)}+\frac{c^3d^3(d+ex)^{m+7}}{e^4(m+7)}$$

```
[Out] -(((c*d^2 - a*e^2)^3*(d + e*x)^(4 + m))/(e^4*(4 + m))) + (3*c*d*(c*d^2 - a*
e^2)^2*(d + e*x)^(5 + m))/(e^4*(5 + m)) - (3*c^2*d^2*(c*d^2 - a*e^2)*(d + e
*x)^(6 + m))/(e^4*(6 + m)) + (c^3*d^3*(d + e*x)^(7 + m))/(e^4*(7 + m))
```

Rubi [A] time = 0.0765587, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$-\frac{3c^2d^2\left(cd^2-ae^2\right)(d+ex)^{m+6}}{e^4(m+6)}-\frac{\left(cd^2-ae^2\right)^3(d+ex)^{m+4}}{e^4(m+4)}+\frac{3cd\left(cd^2-ae^2\right)^2(d+ex)^{m+5}}{e^4(m+5)}+\frac{c^3d^3(d+ex)^{m+7}}{e^4(m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] -(((c*d^2 - a*e^2)^3*(d + e*x)^(4 + m))/(e^4*(4 + m))) + (3*c*d*(c*d^2 - a*
e^2)^2*(d + e*x)^(5 + m))/(e^4*(5 + m)) - (3*c^2*d^2*(c*d^2 - a*e^2)*(d + e
*x)^(6 + m))/(e^4*(6 + m)) + (c^3*d^3*(d + e*x)^(7 + m))/(e^4*(7 + m))
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3 dx = \int (ae+cdx)^3(d+ex)^{3+m}\,dx$$

$$= \int \left(\frac{\left(-cd^2+ae^2\right)^3(d+ex)^{3+m}}{e^3}+\frac{3cd\left(cd^2-ae^2\right)^2(d+ex)^{4+m}}{e^3}-\frac{3c^2d}{\ldots}\right.$$

$$= -\frac{\left(cd^2-ae^2\right)^3(d+ex)^{4+m}}{e^4(4+m)}+\frac{3cd\left(cd^2-ae^2\right)^2(d+ex)^{5+m}}{e^4(5+m)}-\frac{3c^2d^2\left(c\ldots\right.}{}$$

Mathematica [A] time = 0.0909152, size = 114, normalized size = 0.88

$$\frac{(d+ex)^{m+4}\left(-\frac{3c^2d^2(d+ex)^2\left(cd^2-ae^2\right)}{m+6}+\frac{3cd(d+ex)\left(cd^2-ae^2\right)^2}{m+5}-\frac{\left(cd^2-ae^2\right)^3}{m+4}+\frac{c^3d^3(d+ex)^3}{m+7}\right)}{e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] ((d + e*x)^(4 + m)*(-((c*d^2 - a*e^2)^3/(4 + m)) + (3*c*d*(c*d^2 - a*e^2)^2
*(d + e*x))/(5 + m) - (3*c^2*d^2*(c*d^2 - a*e^2)*(d + e*x)^2)/(6 + m) + (c^
3*d^3*(d + e*x)^3)/(7 + m)))/e^4
```

Maple [B] time = 0.047, size = 436, normalized size = 3.4

$$\frac{(ex+d)^{4+m}\left(c^3d^3e^3m^3x^3+3\,ac^2d^2e^4m^3x^2+15\,c^3d^3e^3m^2x^3+3\,a^2cde^5m^3x+48\,ac^2d^2e^4m^2x^2-3\,c^3d^4e^2m^2x^2+74\,c^3d^3e^3m\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

```
[Out] (e*x+d)^(4+m)*(c^3*d^3*e^3*m^3*x^3+3*a*c^2*d^2*e^4*m^3*x^2+15*c^3*d^3*e^3*m
^2*x^3+3*a^2*c*d*e^5*m^3*x+48*a*c^2*d^2*e^4*m^2*x^2-3*c^3*d^4*e^2*m^2*x^2+7
4*c^3*d^3*e^3*m*x^3+a^3*e^6*m^3+51*a^2*c*d*e^5*m^2*x-6*a*c^2*d^3*e^3*m^2*x+
249*a*c^2*d^2*e^4*m*x^2-27*c^3*d^4*e^2*m*x^2+120*c^3*d^3*e^3*x^3+18*a^3*e^6
*m^2-3*a^2*c*d^2*e^4*m^2+282*a^2*c*d*e^5*m*x-66*a*c^2*d^3*e^3*m*x+420*a*c^2
*d^2*e^4*x^2+6*c^3*d^5*e*m*x-60*c^3*d^4*e^2*x^2+107*a^3*e^6*m-39*a^2*c*d^2*
e^4*m+504*a^2*c*d*e^5*x+6*a*c^2*d^4*e^2*m-168*a*c^2*d^3*e^3*x+24*c^3*d^5*e*
x+210*a^3*e^6-126*a^2*c*d^2*e^4+42*a*c^2*d^4*e^2-6*c^3*d^6)/e^4/(m^4+22*m^3
+179*m^2+638*m+840)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.03512, size = 2418, normalized size = 18.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="frica
s")
```

```
[Out] (a^3*d^4*e^6*m^3 - 6*c^3*d^10 + 42*a*c^2*d^8*e^2 - 126*a^2*c*d^6*e^4 + 210*
a^3*d^4*e^6 + (c^3*d^3*e^7*m^3 + 15*c^3*d^3*e^7*m^2 + 74*c^3*d^3*e^7*m + 12
0*c^3*d^3*e^7)*x^7 + (420*c^3*d^4*e^6 + 420*a*c^2*d^2*e^8 + (4*c^3*d^4*e^6
+ 3*a*c^2*d^2*e^8)*m^3 + 3*(19*c^3*d^4*e^6 + 16*a*c^2*d^2*e^8)*m^2 + (269*c
```

```
^3*d^4*e^6 + 249*a*c^2*d^2*e^8)*m)*x^6 + 3*(168*c^3*d^5*e^5 + 504*a*c^2*d^3
*e^7 + 168*a^2*c*d*e^9 + (2*c^3*d^5*e^5 + 4*a*c^2*d^3*e^7 + a^2*c*d*e^9)*m^
3 + (26*c^3*d^5*e^5 + 62*a*c^2*d^3*e^7 + 17*a^2*c*d*e^9)*m^2 + 2*(57*c^3*d^
5*e^5 + 155*a*c^2*d^3*e^7 + 47*a^2*c*d*e^9)*m)*x^5 + (210*c^3*d^6*e^4 + 189
0*a*c^2*d^4*e^6 + 1890*a^2*c*d^2*e^8 + 210*a^3*e^10 + (4*c^3*d^6*e^4 + 18*a
*c^2*d^4*e^6 + 12*a^2*c*d^2*e^8 + a^3*e^10)*m^3 + 3*(14*c^3*d^6*e^4 + 88*a*
c^2*d^4*e^6 + 67*a^2*c*d^2*e^8 + 6*a^3*e^10)*m^2 + (158*c^3*d^6*e^4 + 1236*
a*c^2*d^4*e^6 + 1089*a^2*c*d^2*e^8 + 107*a^3*e^10)*m)*x^4 + (840*a*c^2*d^5*
e^5 + 2520*a^2*c*d^3*e^7 + 840*a^3*d*e^9 + (c^3*d^7*e^3 + 12*a*c^2*d^5*e^5
+ 18*a^2*c*d^3*e^7 + 4*a^3*d*e^9)*m^3 + 3*(c^3*d^7*e^3 + 52*a*c^2*d^5*e^5 +
 98*a^2*c*d^3*e^7 + 24*a^3*d*e^9)*m^2 + 2*(c^3*d^7*e^3 + 312*a*c^2*d^5*e^5
+ 768*a^2*c*d^3*e^7 + 214*a^3*d*e^9)*m)*x^3 - 3*(a^2*c*d^6*e^4 - 6*a^3*d^4*
e^6)*m^2 + 3*(420*a^2*c*d^4*e^6 + 420*a^3*d^2*e^8 + (a*c^2*d^6*e^4 + 4*a^2*
c*d^4*e^6 + 2*a^3*d^2*e^8)*m^3 - (c^3*d^8*e^2 - 8*a*c^2*d^6*e^4 - 62*a^2*c*
d^4*e^6 - 36*a^3*d^2*e^8)*m^2 - (c^3*d^8*e^2 - 7*a*c^2*d^6*e^4 - 298*a^2*c*
d^4*e^6 - 214*a^3*d^2*e^8)*m)*x^2 + (6*a*c^2*d^8*e^2 - 39*a^2*c*d^6*e^4 + 1
07*a^3*d^4*e^6)*m + (840*a^3*d^3*e^7 + (3*a^2*c*d^5*e^5 + 4*a^3*d^3*e^7)*m^
3 - 3*(2*a*c^2*d^7*e^3 - 13*a^2*c*d^5*e^5 - 24*a^3*d^3*e^7)*m^2 + 2*(3*c^3*
d^9*e - 21*a*c^2*d^7*e^3 + 63*a^2*c*d^5*e^5 + 214*a^3*d^3*e^7)*m)*x)*(e*x +
 d)^m/(e^4*m^4 + 22*e^4*m^3 + 179*e^4*m^2 + 638*e^4*m + 840*e^4)
```

Sympy [A] time = 13.7935, size = 7844, normalized size = 60.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

```
[Out] Piecewise((c**3*d**6*d**m*x**4/4, Eq(e, 0)), (-8*a**3*d**3*e**6/(30*d**6*e*
*4 + 90*d**5*e**5*x + 90*d**4*e**6*x**2 + 30*d**3*e**7*x**3) + 6*a**3*d**2*
e**7*x/(30*d**6*e**4 + 90*d**5*e**5*x + 90*d**4*e**6*x**2 + 30*d**3*e**7*x*
*3) + 6*a**3*d*e**8*x**2/(30*d**6*e**4 + 90*d**5*e**5*x + 90*d**4*e**6*x**2
 + 30*d**3*e**7*x**3) + 2*a**3*e**9*x**3/(30*d**6*e**4 + 90*d**5*e**5*x + 9
0*d**4*e**6*x**2 + 30*d**3*e**7*x**3) - 3*a**2*c*d**5*e**4/(30*d**6*e**4 +
90*d**5*e**5*x + 90*d**4*e**6*x**2 + 30*d**3*e**7*x**3) - 9*a**2*c*d**4*e**
5*x/(30*d**6*e**4 + 90*d**5*e**5*x + 90*d**4*e**6*x**2 + 30*d**3*e**7*x**3)
 + 36*a**2*c*d**3*e**6*x**2/(30*d**6*e**4 + 90*d**5*e**5*x + 90*d**4*e**6*x
**2 + 30*d**3*e**7*x**3) + 12*a**2*c*d**2*e**7*x**3/(30*d**6*e**4 + 90*d**5
*e**5*x + 90*d**4*e**6*x**2 + 30*d**3*e**7*x**3) + 30*a*c**2*d**4*e**5*x**3
/(30*d**6*e**4 + 90*d**5*e**5*x + 90*d**4*e**6*x**2 + 30*d**3*e**7*x**3) +
30*c**3*d**9*log(d/e + x)/(30*d**6*e**4 + 90*d**5*e**5*x + 90*d**4*e**6*x**
2 + 30*d**3*e**7*x**3) + 11*c**3*d**9/(30*d**6*e**4 + 90*d**5*e**5*x + 90*d
**4*e**6*x**2 + 30*d**3*e**7*x**3) + 90*c**3*d**8*e*x*log(d/e + x)/(30*d**6
*e**4 + 90*d**5*e**5*x + 90*d**4*e**6*x**2 + 30*d**3*e**7*x**3) + 3*c**3*d*
*8*e*x/(30*d**6*e**4 + 90*d**5*e**5*x + 90*d**4*e**6*x**2 + 30*d**3*e**7*x*
*3) + 90*c**3*d**7*e**2*x**2*log(d/e + x)/(30*d**6*e**4 + 90*d**5*e**5*x +
90*d**4*e**6*x**2 + 30*d**3*e**7*x**3) - 42*c**3*d**7*e**2*x**2/(30*d**6*e*
*4 + 90*d**5*e**5*x + 90*d**4*e**6*x**2 + 30*d**3*e**7*x**3) + 30*c**3*d**6
*e**3*x**3*log(d/e + x)/(30*d**6*e**4 + 90*d**5*e**5*x + 90*d**4*e**6*x**2
+ 30*d**3*e**7*x**3) - 44*c**3*d**6*e**3*x**3/(30*d**6*e**4 + 90*d**5*e**5*
x + 90*d**4*e**6*x**2 + 30*d**3*e**7*x**3), Eq(m, -7)), (-7*a**3*d**2*e**6/
(20*d**4*e**4 + 40*d**3*e**5*x + 20*d**2*e**6*x**2) + 6*a**3*d*e**7*x/(20*d
**4*e**4 + 40*d**3*e**5*x + 20*d**2*e**6*x**2) + 3*a**3*e**8*x**2/(20*d**4*
e**4 + 40*d**3*e**5*x + 20*d**2*e**6*x**2) - 3*a**2*c*d**4*e**4/(20*d**4*e*
*4 + 40*d**3*e**5*x + 20*d**2*e**6*x**2) - 6*a**2*c*d**3*e**5*x/(20*d**4*e*
*4 + 40*d**3*e**5*x + 20*d**2*e**6*x**2) + 27*a**2*c*d**2*e**6*x**2/(20*d**
```

4*e**4 + 40*d**3*e**5*x + 20*d**2*e**6*x**2) + 60*a*c**2*d**6*e**2*log(d/e + x)/(20*d**4*e**4 + 40*d**3*e**5*x + 20*d**2*e**6*x**2) + 27*a*c**2*d**6*e**2/(20*d**4*e**4 + 40*d**3*e**5*x + 20*d**2*e**6*x**2) + 120*a*c**2*d**5*e**3*x*log(d/e + x)/(20*d**4*e**4 + 40*d**3*e**5*x + 20*d**2*e**6*x**2) - 6*a*c**2*d**5*e**3*x/(20*d**4*e**4 + 40*d**3*e**5*x + 20*d**2*e**6*x**2) + 60*a*c**2*d**4*e**4*x**2*log(d/e + x)/(20*d**4*e**4 + 40*d**3*e**5*x + 20*d**2*e**6*x**2) - 63*a*c**2*d**4*e**4*x**2/(20*d**4*e**4 + 40*d**3*e**5*x + 20*d**2*e**6*x**2) - 60*c**3*d**8*log(d/e + x)/(20*d**4*e**4 + 40*d**3*e**5*x + 20*d**2*e**6*x**2) - 27*c**3*d**8/(20*d**4*e**4 + 40*d**3*e**5*x + 20*d**2*e**6*x**2) - 120*c**3*d**7*e*x*log(d/e + x)/(20*d**4*e**4 + 40*d**3*e**5*x + 20*d**2*e**6*x**2) + 6*c**3*d**7*e*x/(20*d**4*e**4 + 40*d**3*e**5*x + 20*d**2*e**6*x**2) - 60*c**3*d**6*e**2*x**2*log(d/e + x)/(20*d**4*e**4 + 40*d**3*e**5*x + 20*d**2*e**6*x**2) + 63*c**3*d**6*e**2*x**2/(20*d**4*e**4 + 40*d**3*e**5*x + 20*d**2*e**6*x**2) + 20*c**3*d**5*e**3*x**3/(20*d**4*e**4 + 40*d**3*e**5*x + 20*d**2*e**6*x**2), Eq(m, -6)), (-2*a**3*d*e**6/(4*d**2*e**4 + 4*d*e**5*x) + 2*a**3*e**7*x/(4*d**2*e**4 + 4*d*e**5*x) + 12*a**2*c*d**3*e**4*log(d/e + x)/(4*d**2*e**4 + 4*d*e**5*x) + 6*a**2*c*d**3*e**4/(4*d**2*e**4 + 4*d*e**5*x) + 12*a**2*c*d**2*e**5*x*log(d/e + x)/(4*d**2*e**4 + 4*d*e**5*x) - 6*a**2*c*d**2*e**5*x/(4*d**2*e**4 + 4*d*e**5*x) - 24*a*c**2*d**5*e**2*log(d/e + x)/(4*d**2*e**4 + 4*d*e**5*x) - 14*a*c**2*d**5*e**2/(4*d**2*e**4 + 4*d*e**5*x) - 24*a*c**2*d**4*e**3*x*log(d/e + x)/(4*d**2*e**4 + 4*d*e**5*x) + 10*a*c**2*d**4*e**3*x/(4*d**2*e**4 + 4*d*e**5*x) + 12*a*c**2*d**3*e**4*x**2/(4*d**2*e**4 + 4*d*e**5*x) + 12*c**3*d**7*log(d/e + x)/(4*d**2*e**4 + 4*d*e**5*x) + 7*c**3*d**7/(4*d**2*e**4 + 4*d*e**5*x) + 12*c**3*d**6*e*x*log(d/e + x)/(4*d**2*e**4 + 4*d*e**5*x) - 5*c**3*d**6*e*x/(4*d**2*e**4 + 4*d*e**5*x) - 6*c**3*d**5*e**2*x**2/(4*d**2*e**4 + 4*d*e**5*x) + 2*c**3*d**4*e**3*x**3/(4*d**2*e**4 + 4*d*e**5*x), Eq(m, -5)), (a**3*e**2*log(d/e + x) - 3*a**2*c*d**2*log(d/e + x) + 3*a**2*c*d*e*x + 3*a*c**2*d**4*log(d/e + x)/e**2 - 3*a*c**2*d**3*x/e + 3*a*c**2*d**2*x**2/2 - c**3*d**6*log(d/e + x)/e**4 + c**3*d**5*x/e**3 - c**3*d**4*x**2/(2*e**2) + c**3*d**3*x**3/(3*e), Eq(m, -4)), (a**3*d**4*e**6*m**3*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 18*a**3*d**4*e**6*m**2*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 107*a**3*d**4*e**6*m*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 210*a**3*d**4*e**6*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 4*a**3*d**3*e**7*m**3*x*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 72*a**3*d**3*e**7*m**2*x*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 428*a**3*d**3*e**7*m*x*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 840*a**3*d**3*e**7*x*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 6*a**3*d**2*e**8*m**3*x**2*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 108*a**3*d**2*e**8*m**2*x**2*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 642*a**3*d**2*e**8*m*x**2*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 1260*a**3*d**2*e**8*x**2*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 4*a**3*d*e**9*m**3*x**3*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 72*a**3*d*e**9*m**2*x**3*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 428*a**3*d*e**9*m*x**3*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 840*a**3*d*e**9*x**3*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + a**3*e**10*m**3*x**4*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 18*a**3*e**10*m**2*x**4*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 107*a**3*e**10*m*x**4*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 210*a**3*e**10*x**4*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) - 3*a**2*c*d**6*e**4*m**2*(d + e*x)**m/(e**4*m**4 + 22*e**4

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*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) - 39*a**2*c*d**6*e**4*m*(d +
 e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4)
 - 126*a**2*c*d**6*e**4*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m
**2 + 638*e**4*m + 840*e**4) + 3*a**2*c*d**5*e**5*m**3*x*(d + e*x)**m/(e**4
*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 39*a**2*c*d
**5*e**5*m**2*x*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 63
8*e**4*m + 840*e**4) + 126*a**2*c*d**5*e**5*m*x*(d + e*x)**m/(e**4*m**4 + 2
2*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 12*a**2*c*d**4*e**6*
m**3*x**2*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4
*m + 840*e**4) + 186*a**2*c*d**4*e**6*m**2*x**2*(d + e*x)**m/(e**4*m**4 + 2
2*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 894*a**2*c*d**4*e**6
*m*x**2*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m
 + 840*e**4) + 1260*a**2*c*d**4*e**6*x**2*(d + e*x)**m/(e**4*m**4 + 22*e**4
*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 18*a**2*c*d**3*e**7*m**3*x
**3*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 8
40*e**4) + 294*a**2*c*d**3*e**7*m**2*x**3*(d + e*x)**m/(e**4*m**4 + 22*e**4
*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 1536*a**2*c*d**3*e**7*m*x*
*3*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 84
0*e**4) + 2520*a**2*c*d**3*e**7*x**3*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3
 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 12*a**2*c*d**2*e**8*m**3*x**4*(
d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e*
*4) + 201*a**2*c*d**2*e**8*m**2*x**4*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3
 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 1089*a**2*c*d**2*e**8*m*x**4*(d
 + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**
4) + 1890*a**2*c*d**2*e**8*x**4*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 17
9*e**4*m**2 + 638*e**4*m + 840*e**4) + 3*a**2*c*d*e**9*m**3*x**5*(d + e*x)*
*m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 51*
a**2*c*d*e**9*m**2*x**5*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m
**2 + 638*e**4*m + 840*e**4) + 282*a**2*c*d*e**9*m*x**5*(d + e*x)**m/(e**4*
m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 504*a**2*c*d
*e**9*x**5*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**
4*m + 840*e**4) + 6*a*c**2*d**8*e**2*m*(d + e*x)**m/(e**4*m**4 + 22*e**4*m*
*3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 42*a*c**2*d**8*e**2*(d + e*x)
**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) - 6*
a*c**2*d**7*e**3*m**2*x*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m
**2 + 638*e**4*m + 840*e**4) - 42*a*c**2*d**7*e**3*m*x*(d + e*x)**m/(e**4*m
**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 3*a*c**2*d**6
*e**4*m**3*x**2*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 63
8*e**4*m + 840*e**4) + 24*a*c**2*d**6*e**4*m**2*x**2*(d + e*x)**m/(e**4*m**
4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 21*a*c**2*d**6*
e**4*m*x**2*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e*
*4*m + 840*e**4) + 12*a*c**2*d**5*e**5*m**3*x**3*(d + e*x)**m/(e**4*m**4 +
22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 156*a*c**2*d**5*e**
5*m**2*x**3*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e*
*4*m + 840*e**4) + 624*a*c**2*d**5*e**5*m*x**3*(d + e*x)**m/(e**4*m**4 + 22
*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 840*a*c**2*d**5*e**5*
x**3*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m +
840*e**4) + 18*a*c**2*d**4*e**6*m**3*x**4*(d + e*x)**m/(e**4*m**4 + 22*e**4
*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 264*a*c**2*d**4*e**6*m**2*
x**4*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m +
840*e**4) + 1236*a*c**2*d**4*e**6*m*x**4*(d + e*x)**m/(e**4*m**4 + 22*e**4*
m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 1890*a*c**2*d**4*e**6*x**4*
(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e
**4) + 12*a*c**2*d**3*e**7*m**3*x**5*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3
 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 186*a*c**2*d**3*e**7*m**2*x**5*
(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e
**4) + 930*a*c**2*d**3*e**7*m*x**5*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 +
 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 1512*a*c**2*d**3*e**7*x**5*(d + e
*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) +
```

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 3*a*c**2*d**2*e**8*m**3*x**6*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*
e**4*m**2 + 638*e**4*m + 840*e**4) + 48*a*c**2*d**2*e**8*m**2*x**6*(d + e*x
)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 2
49*a*c**2*d**2*e**8*m*x**6*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**
4*m**2 + 638*e**4*m + 840*e**4) + 420*a*c**2*d**2*e**8*x**6*(d + e*x)**m/(e
**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) - 6*c**3*d
**10*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m +
840*e**4) + 6*c**3*d**9*e*m*x*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*
e**4*m**2 + 638*e**4*m + 840*e**4) - 3*c**3*d**8*e**2*m**2*x**2*(d + e*x)**
m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) - 3*c*
*3*d**8*e**2*m*x**2*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2
+ 638*e**4*m + 840*e**4) + c**3*d**7*e**3*m**3*x**3*(d + e*x)**m/(e**4*m**4
 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 3*c**3*d**7*e**3
*m**2*x**3*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**
4*m + 840*e**4) + 2*c**3*d**7*e**3*m*x**3*(d + e*x)**m/(e**4*m**4 + 22*e**4
*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 4*c**3*d**6*e**4*m**3*x**4
*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*
e**4) + 42*c**3*d**6*e**4*m**2*x**4*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3
+ 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 158*c**3*d**6*e**4*m*x**4*(d + e
*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) +
 210*c**3*d**6*e**4*x**4*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*
m**2 + 638*e**4*m + 840*e**4) + 6*c**3*d**5*e**5*m**3*x**5*(d + e*x)**m/(e*
*4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 78*c**3*d
**5*e**5*m**2*x**5*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 +
 638*e**4*m + 840*e**4) + 342*c**3*d**5*e**5*m*x**5*(d + e*x)**m/(e**4*m**4
 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 504*c**3*d**5*e*
*5*x**5*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m
 + 840*e**4) + 4*c**3*d**4*e**6*m**3*x**6*(d + e*x)**m/(e**4*m**4 + 22*e**4
*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 57*c**3*d**4*e**6*m**2*x**
6*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840
*e**4) + 269*c**3*d**4*e**6*m*x**6*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 +
 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 420*c**3*d**4*e**6*x**6*(d + e*x)
**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + c*
*3*d**3*e**7*m**3*x**7*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m*
*2 + 638*e**4*m + 840*e**4) + 15*c**3*d**3*e**7*m**2*x**7*(d + e*x)**m/(e**
4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4) + 74*c**3*d*
*3*e**7*m*x**7*(d + e*x)**m/(e**4*m**4 + 22*e**4*m**3 + 179*e**4*m**2 + 638
*e**4*m + 840*e**4) + 120*c**3*d**3*e**7*x**7*(d + e*x)**m/(e**4*m**4 + 22*
e**4*m**3 + 179*e**4*m**2 + 638*e**4*m + 840*e**4), True))
```

Giac [B] time = 1.25022, size = 2696, normalized size = 20.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac"
)
```

```
[Out] ((x*e + d)^m*c^3*d^3*m^3*x^7*e^7 + 4*(x*e + d)^m*c^3*d^4*m^3*x^6*e^6 + 6*(x
*e + d)^m*c^3*d^5*m^3*x^5*e^5 + 4*(x*e + d)^m*c^3*d^6*m^3*x^4*e^4 + (x*e +
d)^m*c^3*d^7*m^3*x^3*e^3 + 15*(x*e + d)^m*c^3*d^3*m^2*x^7*e^7 + 57*(x*e + d
)^m*c^3*d^4*m^2*x^6*e^6 + 78*(x*e + d)^m*c^3*d^5*m^2*x^5*e^5 + 42*(x*e + d)
^m*c^3*d^6*m^2*x^4*e^4 + 3*(x*e + d)^m*c^3*d^7*m^2*x^3*e^3 - 3*(x*e + d)^m*
c^3*d^8*m^2*x^2*e^2 + 3*(x*e + d)^m*a*c^2*d^2*m^3*x^6*e^8 + 12*(x*e + d)^m*
a*c^2*d^3*m^3*x^5*e^7 + 74*(x*e + d)^m*c^3*d^3*m*x^7*e^7 + 18*(x*e + d)^m*a
*c^2*d^4*m^3*x^4*e^6 + 269*(x*e + d)^m*c^3*d^4*m*x^6*e^6 + 12*(x*e + d)^m*a
```

```
*c^2*d^5*m^3*x^3*e^5 + 342*(x*e + d)^m*c^3*d^5*m*x^5*e^5 + 3*(x*e + d)^m*a*
c^2*d^6*m^3*x^2*e^4 + 158*(x*e + d)^m*c^3*d^6*m*x^4*e^4 + 2*(x*e + d)^m*c^3
*d^7*m*x^3*e^3 - 3*(x*e + d)^m*c^3*d^8*m*x^2*e^2 + 6*(x*e + d)^m*c^3*d^9*m*
x*e + 48*(x*e + d)^m*a*c^2*d^2*m^2*x^6*e^8 + 186*(x*e + d)^m*a*c^2*d^3*m^2*
x^5*e^7 + 120*(x*e + d)^m*c^3*d^3*x^7*e^7 + 264*(x*e + d)^m*a*c^2*d^4*m^2*x
^4*e^6 + 420*(x*e + d)^m*c^3*d^4*x^6*e^6 + 156*(x*e + d)^m*a*c^2*d^5*m^2*x^
3*e^5 + 504*(x*e + d)^m*c^3*d^5*x^5*e^5 + 24*(x*e + d)^m*a*c^2*d^6*m^2*x^2*
e^4 + 210*(x*e + d)^m*c^3*d^6*x^4*e^4 - 6*(x*e + d)^m*a*c^2*d^7*m^2*x*e^3 -
 6*(x*e + d)^m*c^3*d^10 + 3*(x*e + d)^m*a^2*c*d*m^3*x^5*e^9 + 12*(x*e + d)^
m*a^2*c*d^2*m^3*x^4*e^8 + 249*(x*e + d)^m*a*c^2*d^2*m*x^6*e^8 + 18*(x*e + d
)^m*a^2*c*d^3*m^3*x^3*e^7 + 930*(x*e + d)^m*a*c^2*d^3*m*x^5*e^7 + 12*(x*e +
 d)^m*a^2*c*d^4*m^3*x^2*e^6 + 1236*(x*e + d)^m*a*c^2*d^4*m*x^4*e^6 + 3*(x*e
 + d)^m*a^2*c*d^5*m^3*x*e^5 + 624*(x*e + d)^m*a*c^2*d^5*m*x^3*e^5 + 21*(x*e
 + d)^m*a*c^2*d^6*m*x^2*e^4 - 42*(x*e + d)^m*a*c^2*d^7*m*x*e^3 + 6*(x*e + d
)^m*a*c^2*d^8*m*e^2 + 51*(x*e + d)^m*a^2*c*d*m^2*x^5*e^9 + 201*(x*e + d)^m*
a^2*c*d^2*m^2*x^4*e^8 + 420*(x*e + d)^m*a*c^2*d^2*x^6*e^8 + 294*(x*e + d)^m
*a^2*c*d^3*m^2*x^3*e^7 + 1512*(x*e + d)^m*a*c^2*d^3*x^5*e^7 + 186*(x*e + d)
^m*a^2*c*d^4*m^2*x^2*e^6 + 1890*(x*e + d)^m*a*c^2*d^4*x^4*e^6 + 39*(x*e + d
)^m*a^2*c*d^5*m^2*x*e^5 + 840*(x*e + d)^m*a*c^2*d^5*x^3*e^5 - 3*(x*e + d)^m
*a^2*c*d^6*m^2*e^4 + 42*(x*e + d)^m*a*c^2*d^8*e^2 + (x*e + d)^m*a^3*m^3*x^4
*e^10 + 4*(x*e + d)^m*a^3*d*m^3*x^3*e^9 + 282*(x*e + d)^m*a^2*c*d*m*x^5*e^9
 + 6*(x*e + d)^m*a^3*d^2*m^3*x^2*e^8 + 1089*(x*e + d)^m*a^2*c*d^2*m*x^4*e^8
 + 4*(x*e + d)^m*a^3*d^3*m^3*x*e^7 + 1536*(x*e + d)^m*a^2*c*d^3*m*x^3*e^7 +
 (x*e + d)^m*a^3*d^4*m^3*e^6 + 894*(x*e + d)^m*a^2*c*d^4*m*x^2*e^6 + 126*(x
*e + d)^m*a^2*c*d^5*m*x*e^5 - 39*(x*e + d)^m*a^2*c*d^6*m*e^4 + 18*(x*e + d)
^m*a^3*m^2*x^4*e^10 + 72*(x*e + d)^m*a^3*d*m^2*x^3*e^9 + 504*(x*e + d)^m*a^
2*c*d*x^5*e^9 + 108*(x*e + d)^m*a^3*d^2*m^2*x^2*e^8 + 1890*(x*e + d)^m*a^2*
c*d^2*x^4*e^8 + 72*(x*e + d)^m*a^3*d^3*m^2*x*e^7 + 2520*(x*e + d)^m*a^2*c*d
^3*x^3*e^7 + 18*(x*e + d)^m*a^3*d^4*m^2*e^6 + 1260*(x*e + d)^m*a^2*c*d^4*x^
2*e^6 - 126*(x*e + d)^m*a^2*c*d^6*e^4 + 107*(x*e + d)^m*a^3*m*x^4*e^10 + 42
8*(x*e + d)^m*a^3*d*m*x^3*e^9 + 642*(x*e + d)^m*a^3*d^2*m*x^2*e^8 + 428*(x*
e + d)^m*a^3*d^3*m*x*e^7 + 107*(x*e + d)^m*a^3*d^4*m*e^6 + 210*(x*e + d)^m*
a^3*x^4*e^10 + 840*(x*e + d)^m*a^3*d*x^3*e^9 + 1260*(x*e + d)^m*a^3*d^2*x^2
*e^8 + 840*(x*e + d)^m*a^3*d^3*x*e^7 + 210*(x*e + d)^m*a^3*d^4*e^6)/(m^4*e^
4 + 22*m^3*e^4 + 179*m^2*e^4 + 638*m*e^4 + 840*e^4)
```

3.2085 $\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2 dx$

Optimal. Leaf size=90

$$\frac{\left(cd^2-ae^2\right)^2 (d+ex)^{m+3}}{e^3(m+3)}-\frac{2cd\left(cd^2-ae^2\right)(d+ex)^{m+4}}{e^3(m+4)}+\frac{c^2d^2(d+ex)^{m+5}}{e^3(m+5)}$$

```
[Out] ((c*d^2 - a*e^2)^2*(d + e*x)^(3 + m))/(e^3*(3 + m)) - (2*c*d*(c*d^2 - a*e^2
)*(d + e*x)^(4 + m))/(e^3*(4 + m)) + (c^2*d^2*(d + e*x)^(5 + m))/(e^3*(5 +
m))
```

Rubi [A] time = 0.0503699, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 43}

$$\frac{\left(cd^2-ae^2\right)^2 (d+ex)^{m+3}}{e^3(m+3)}-\frac{2cd\left(cd^2-ae^2\right)(d+ex)^{m+4}}{e^3(m+4)}+\frac{c^2d^2(d+ex)^{m+5}}{e^3(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] ((c*d^2 - a*e^2)^2*(d + e*x)^(3 + m))/(e^3*(3 + m)) - (2*c*d*(c*d^2 - a*e^2
)*(d + e*x)^(4 + m))/(e^3*(4 + m)) + (c^2*d^2*(d + e*x)^(5 + m))/(e^3*(5 +
m))
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2 dx &= \int (ae+cdx)^2(d+ex)^{2+m}\,dx\\ &= \int \left(\frac{\left(-cd^2+ae^2\right)^2(d+ex)^{2+m}}{e^2}-\frac{2cd\left(cd^2-ae^2\right)(d+ex)^{3+m}}{e^2}+\frac{c^2d^2(d+\right.\\ &= \frac{\left(cd^2-ae^2\right)^2(d+ex)^{3+m}}{e^3(3+m)}-\frac{2cd\left(cd^2-ae^2\right)(d+ex)^{4+m}}{e^3(4+m)}+\frac{c^2d^2(d+ex)^{5+}}{e^3(5+m)}\end{aligned}$$

Mathematica [A] time = 0.0829189, size = 79, normalized size = 0.88

$$\frac{(d+ex)^{m+3}\left(-\frac{2cd(d+ex)\left(cd^2-ae^2\right)}{m+4}+\frac{\left(cd^2-ae^2\right)^2}{m+3}+\frac{c^2d^2(d+ex)^2}{m+5}\right)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] ((d + e*x)^(3 + m)*((c*d^2 - a*e^2)^2/(3 + m) - (2*c*d*(c*d^2 - a*e^2)*(d +
 e*x))/(4 + m) + (c^2*d^2*(d + e*x)^2)/(5 + m)))/e^3
```

Maple [B] time = 0.046, size = 183, normalized size = 2.

$$\frac{(ex+d)^{3+m}\left(c^2d^2e^2m^2x^2+2\,acde^3m^2x+7\,c^2d^2e^2mx^2+a^2e^4m^2+16\,acde^3mx-2\,c^2d^3emx+12\,c^2d^2x^2e^2+9\,a^2e^4m-\right.}{e^3\left(m^3+12\,m^2+47\,m+60\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

```
[Out] (e*x+d)^(3+m)*(c^2*d^2*e^2*m^2*x^2+2*a*c*d*e^3*m^2*x+7*c^2*d^2*e^2*m*x^2+a^
2*e^4*m^2+16*a*c*d*e^3*m*x-2*c^2*d^3*e*m*x+12*c^2*d^2*e^2*x^2+9*a^2*e^4*m-2
*a*c*d^2*e^2*m+30*a*c*d*e^3*x-6*c^2*d^3*e*x+20*a^2*e^4-10*a*c*d^2*e^2+2*c^2
*d^4)/e^3/(m^3+12*m^2+47*m+60)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.86772, size = 979, normalized size = 10.88

$$\frac{\left(a^2d^3e^4m^2+2\,c^2d^7-10\,acd^5e^2+20\,a^2d^3e^4+\left(c^2d^2e^5m^2+7\,c^2d^2e^5m+12\,c^2d^2e^5\right)x^5+\left(30\,c^2d^3e^4+30\,acde^6+\left(3\,c^2d^3\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="frica
s")
```

```
[Out] (a^2*d^3*e^4*m^2 + 2*c^2*d^7 - 10*a*c*d^5*e^2 + 20*a^2*d^3*e^4 + (c^2*d^2*e
^5*m^2 + 7*c^2*d^2*e^5*m + 12*c^2*d^2*e^5)*x^5 + (30*c^2*d^3*e^4 + 30*a*c*d
*e^6 + (3*c^2*d^3*e^4 + 2*a*c*d*e^6)*m^2 + (19*c^2*d^3*e^4 + 16*a*c*d*e^6)*
m)*x^4 + (20*c^2*d^4*e^3 + 80*a*c*d^2*e^5 + 20*a^2*e^7 + (3*c^2*d^4*e^3 + 6
*a*c*d^2*e^5 + a^2*e^7)*m^2 + (15*c^2*d^4*e^3 + 46*a*c*d^2*e^5 + 9*a^2*e^7)
*m)*x^3 + (60*a*c*d^3*e^4 + 60*a^2*d*e^6 + (c^2*d^5*e^2 + 6*a*c*d^3*e^4 + 3
*a^2*d*e^6)*m^2 + (c^2*d^5*e^2 + 42*a*c*d^3*e^4 + 27*a^2*d*e^6)*m)*x^2 - (2
*a*c*d^5*e^2 - 9*a^2*d^3*e^4)*m + (60*a^2*d^2*e^5 + (2*a*c*d^4*e^3 + 3*a^2*
d^2*e^5)*m^2 - (2*c^2*d^6*e - 10*a*c*d^4*e^3 - 27*a^2*d^2*e^5)*m)*x)*(e*x +
```

```
d)^m/(e^3*m^3 + 12*e^3*m^2 + 47*e^3*m + 60*e^3)
```

Sympy [A] time = 4.90235, size = 2839, normalized size = 31.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

```
[Out] Piecewise((c**2*d**4*d**m*x**3/3, Eq(e, 0)), (-5*a**2*d**2*e**4/(12*d**4*e*
*3 + 24*d**3*e**4*x + 12*d**2*e**5*x**2) + 2*a**2*d*e**5*x/(12*d**4*e**3 +
24*d**3*e**4*x + 12*d**2*e**5*x**2) + a**2*e**6*x**2/(12*d**4*e**3 + 24*d**
3*e**4*x + 12*d**2*e**5*x**2) - 2*a*c*d**4*e**2/(12*d**4*e**3 + 24*d**3*e**
4*x + 12*d**2*e**5*x**2) - 4*a*c*d**3*e**3*x/(12*d**4*e**3 + 24*d**3*e**4*x
 + 12*d**2*e**5*x**2) + 10*a*c*d**2*e**4*x**2/(12*d**4*e**3 + 24*d**3*e**4*
x + 12*d**2*e**5*x**2) + 12*c**2*d**6*log(d/e + x)/(12*d**4*e**3 + 24*d**3*
e**4*x + 12*d**2*e**5*x**2) + 7*c**2*d**6/(12*d**4*e**3 + 24*d**3*e**4*x +
12*d**2*e**5*x**2) + 24*c**2*d**5*e*x*log(d/e + x)/(12*d**4*e**3 + 24*d**3*
e**4*x + 12*d**2*e**5*x**2) + 2*c**2*d**5*e*x/(12*d**4*e**3 + 24*d**3*e**4*
x + 12*d**2*e**5*x**2) + 12*c**2*d**4*e**2*x**2*log(d/e + x)/(12*d**4*e**3
+ 24*d**3*e**4*x + 12*d**2*e**5*x**2) - 11*c**2*d**4*e**2*x**2/(12*d**4*e**
3 + 24*d**3*e**4*x + 12*d**2*e**5*x**2), Eq(m, -5)), (-2*a**2*d*e**4/(3*d**
2*e**3 + 3*d*e**4*x) + a**2*e**5*x/(3*d**2*e**3 + 3*d*e**4*x) + 6*a*c*d**3*
e**2*log(d/e + x)/(3*d**2*e**3 + 3*d*e**4*x) + 4*a*c*d**3*e**2/(3*d**2*e**3
 + 3*d*e**4*x) + 6*a*c*d**2*e**3*x*log(d/e + x)/(3*d**2*e**3 + 3*d*e**4*x)
- 2*a*c*d**2*e**3*x/(3*d**2*e**3 + 3*d*e**4*x) - 6*c**2*d**5*log(d/e + x)/(
3*d**2*e**3 + 3*d*e**4*x) - 5*c**2*d**5/(3*d**2*e**3 + 3*d*e**4*x) - 6*c**2
*d**4*e*x*log(d/e + x)/(3*d**2*e**3 + 3*d*e**4*x) + c**2*d**4*e*x/(3*d**2*e
**3 + 3*d*e**4*x) + 3*c**2*d**3*e**2*x**2/(3*d**2*e**3 + 3*d*e**4*x), Eq(m,
 -4)), (a**2*e*log(d/e + x) - 2*a*c*d**2*log(d/e + x)/e + 2*a*c*d*x + c**2*
d**4*log(d/e + x)/e**3 - c**2*d**3*x/e**2 + c**2*d**2*x**2/(2*e), Eq(m, -3)
), (a**2*d**3*e**4*m**2*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*m
+ 60*e**3) + 9*a**2*d**3*e**4*m*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47
*e**3*m + 60*e**3) + 20*a**2*d**3*e**4*(d + e*x)**m/(e**3*m**3 + 12*e**3*m*
*2 + 47*e**3*m + 60*e**3) + 3*a**2*d**2*e**5*m**2*x*(d + e*x)**m/(e**3*m**3
 + 12*e**3*m**2 + 47*e**3*m + 60*e**3) + 27*a**2*d**2*e**5*m*x*(d + e*x)**m
/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3) + 60*a**2*d**2*e**5*x*(d
+ e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3) + 3*a**2*d*e**6*
m**2*x**2*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3) + 2
7*a**2*d*e**6*m*x**2*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*m + 6
0*e**3) + 60*a**2*d*e**6*x**2*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e
**3*m + 60*e**3) + a**2*e**7*m**2*x**3*(d + e*x)**m/(e**3*m**3 + 12*e**3*m*
*2 + 47*e**3*m + 60*e**3) + 9*a**2*e**7*m*x**3*(d + e*x)**m/(e**3*m**3 + 12
*e**3*m**2 + 47*e**3*m + 60*e**3) + 20*a**2*e**7*x**3*(d + e*x)**m/(e**3*m*
*3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3) - 2*a*c*d**5*e**2*m*(d + e*x)**m/(
e**3*m**3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3) - 10*a*c*d**5*e**2*(d + e*x
)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3) + 2*a*c*d**4*e**3*m**
2*x*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3) + 10*a*c*
d**4*e**3*m*x*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3)
 + 6*a*c*d**3*e**4*m**2*x**2*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e*
*3*m + 60*e**3) + 42*a*c*d**3*e**4*m*x**2*(d + e*x)**m/(e**3*m**3 + 12*e**3
*m**2 + 47*e**3*m + 60*e**3) + 60*a*c*d**3*e**4*x**2*(d + e*x)**m/(e**3*m**
3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3) + 6*a*c*d**2*e**5*m**2*x**3*(d + e*
x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3) + 46*a*c*d**2*e**5*m
*x**3*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3) + 80*a*
c*d**2*e**5*x**3*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*m + 60*e*
```

```
*3) + 2*a*c*d*e**6*m**2*x**4*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e*
*3*m + 60*e**3) + 16*a*c*d*e**6*m*x**4*(d + e*x)**m/(e**3*m**3 + 12*e**3*m*
*2 + 47*e**3*m + 60*e**3) + 30*a*c*d*e**6*x**4*(d + e*x)**m/(e**3*m**3 + 12
*e**3*m**2 + 47*e**3*m + 60*e**3) + 2*c**2*d**7*(d + e*x)**m/(e**3*m**3 + 1
2*e**3*m**2 + 47*e**3*m + 60*e**3) - 2*c**2*d**6*e*m*x*(d + e*x)**m/(e**3*m
**3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3) + c**2*d**5*e**2*m**2*x**2*(d + e
*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3) + c**2*d**5*e**2*m*
x**2*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3) + 3*c**2
*d**4*e**3*m**2*x**3*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*m + 6
0*e**3) + 15*c**2*d**4*e**3*m*x**3*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**2 +
 47*e**3*m + 60*e**3) + 20*c**2*d**4*e**3*x**3*(d + e*x)**m/(e**3*m**3 + 12
*e**3*m**2 + 47*e**3*m + 60*e**3) + 3*c**2*d**3*e**4*m**2*x**4*(d + e*x)**m
/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3) + 19*c**2*d**3*e**4*m*x**
4*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3) + 30*c**2*d
**3*e**4*x**4*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3)
 + c**2*d**2*e**5*m**2*x**5*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e**
3*m + 60*e**3) + 7*c**2*d**2*e**5*m*x**5*(d + e*x)**m/(e**3*m**3 + 12*e**3*
m**2 + 47*e**3*m + 60*e**3) + 12*c**2*d**2*e**5*x**5*(d + e*x)**m/(e**3*m**
3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3), True))
```

Giac [B] time = 1.19131, size = 1085, normalized size = 12.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac"
)
```

```
[Out] ((x*e + d)^m*c^2*d^2*m^2*x^5*e^5 + 3*(x*e + d)^m*c^2*d^3*m^2*x^4*e^4 + 3*(x
*e + d)^m*c^2*d^4*m^2*x^3*e^3 + (x*e + d)^m*c^2*d^5*m^2*x^2*e^2 + 7*(x*e +
d)^m*c^2*d^2*m*x^5*e^5 + 19*(x*e + d)^m*c^2*d^3*m*x^4*e^4 + 15*(x*e + d)^m*
c^2*d^4*m*x^3*e^3 + (x*e + d)^m*c^2*d^5*m*x^2*e^2 - 2*(x*e + d)^m*c^2*d^6*m
*x*e + 2*(x*e + d)^m*a*c*d*m^2*x^4*e^6 + 6*(x*e + d)^m*a*c*d^2*m^2*x^3*e^5
+ 12*(x*e + d)^m*c^2*d^2*x^5*e^5 + 6*(x*e + d)^m*a*c*d^3*m^2*x^2*e^4 + 30*(
x*e + d)^m*c^2*d^3*x^4*e^4 + 2*(x*e + d)^m*a*c*d^4*m^2*x*e^3 + 20*(x*e + d)
^m*c^2*d^4*x^3*e^3 + 2*(x*e + d)^m*c^2*d^7 + 16*(x*e + d)^m*a*c*d*m*x^4*e^6
 + 46*(x*e + d)^m*a*c*d^2*m*x^3*e^5 + 42*(x*e + d)^m*a*c*d^3*m*x^2*e^4 + 10
*(x*e + d)^m*a*c*d^4*m*x*e^3 - 2*(x*e + d)^m*a*c*d^5*m*e^2 + (x*e + d)^m*a^
2*m^2*x^3*e^7 + 3*(x*e + d)^m*a^2*d*m^2*x^2*e^6 + 30*(x*e + d)^m*a*c*d*x^4*
e^6 + 3*(x*e + d)^m*a^2*d^2*m^2*x*e^5 + 80*(x*e + d)^m*a*c*d^2*x^3*e^5 + (x
*e + d)^m*a^2*d^3*m^2*e^4 + 60*(x*e + d)^m*a*c*d^3*x^2*e^4 - 10*(x*e + d)^m
*a*c*d^5*e^2 + 9*(x*e + d)^m*a^2*m*x^3*e^7 + 27*(x*e + d)^m*a^2*d*m*x^2*e^6
 + 27*(x*e + d)^m*a^2*d^2*m*x*e^5 + 9*(x*e + d)^m*a^2*d^3*m*e^4 + 20*(x*e +
 d)^m*a^2*x^3*e^7 + 60*(x*e + d)^m*a^2*d*x^2*e^6 + 60*(x*e + d)^m*a^2*d^2*x
*e^5 + 20*(x*e + d)^m*a^2*d^3*e^4)/(m^3*e^3 + 12*m^2*e^3 + 47*m*e^3 + 60*e^
3)
```

3.2086 $\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx$

Optimal. Leaf size=52

$$\frac{cd(d+ex)^{m+3}}{e^2(m+3)}-\frac{\left(cd^2-ae^2\right)(d+ex)^{m+2}}{e^2(m+2)}$$

```
[Out] -(((c*d^2 - a*e^2)*(d + e*x)^(2 + m))/(e^2*(2 + m))) + (c*d*(d + e*x)^(3 + m))/(e^2*(3 + m))
```

Rubi [A] time = 0.025133, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {626, 43}

$$\frac{cd(d+ex)^{m+3}}{e^2(m+3)}-\frac{\left(cd^2-ae^2\right)(d+ex)^{m+2}}{e^2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] -(((c*d^2 - a*e^2)*(d + e*x)^(2 + m))/(e^2*(2 + m))) + (c*d*(d + e*x)^(3 + m))/(e^2*(3 + m))
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx &= \int (ae+cdx)(d+ex)^{1+m}\,dx\\ &= \int \left(\frac{\left(-cd^2+ae^2\right)(d+ex)^{1+m}}{e}+\frac{cd(d+ex)^{2+m}}{e}\right)dx\\ &= -\frac{\left(cd^2-ae^2\right)(d+ex)^{2+m}}{e^2(2+m)}+\frac{cd(d+ex)^{3+m}}{e^2(3+m)}\end{aligned}$$

Mathematica [A] time = 0.0361307, size = 45, normalized size = 0.87

$$\frac{(d+ex)^{m+2}\left(ae^2(m+3)+cd(e(m+2)x-d)\right)}{e^2(m+2)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] ((d + e*x)^(2 + m)*(a*e^2*(3 + m) + c*d*(-d + e*(2 + m)*x)))/(e^2*(2 + m)*(
3 + m))
```

Maple [A] time = 0.045, size = 55, normalized size = 1.1

$$\frac{(ex+d)^{2+m}\left(cdemx+ae^2m+2\,cdex+3\,ae^2-cd^2\right)}{e^2\left(m^2+5\,m+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

```
[Out] (e*x+d)^(2+m)*(c*d*e*m*x+a*e^2*m+2*c*d*e*x+3*a*e^2-c*d^2)/e^2/(m^2+5*m+6)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.93392, size = 277, normalized size = 5.33

$$\frac{\left(ad^2e^2m-cd^4+3\,ad^2e^2+\left(cde^3m+2\,cde^3\right)x^3+\left(3\,cd^2e^2+3\,ae^4+\left(2\,cd^2e^2+ae^4\right)m\right)x^2+\left(6\,ade^3+\left(cd^3e+2\,ade^3\right)m\right)}{e^2m^2+5\,e^2m+6\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fricas"
)
```

```
[Out] (a*d^2*e^2*m - c*d^4 + 3*a*d^2*e^2 + (c*d*e^3*m + 2*c*d*e^3)*x^3 + (3*c*d^2
*e^2 + 3*a*e^4 + (2*c*d^2*e^2 + a*e^4)*m)*x^2 + (6*a*d*e^3 + (c*d^3*e + 2*a
*d*e^3)*m)*x)*(e*x + d)^m/(e^2*m^2 + 5*e^2*m + 6*e^2)
```

Sympy [A] time = 1.3552, size = 556, normalized size = 10.69

$$\begin{cases} \frac{cd^2d^mx^2}{2} \\ -\frac{ae^2}{de^2+e^3x}+\frac{cd^2\log\left(\frac{d}{e}+x\right)}{de^2+e^3x}+\frac{cd^2}{de^2+e^3x}+\frac{cdex\log\left(\frac{d}{e}+x\right)}{de^2+e^3x} \\ a\log\left(\frac{d}{e}+x\right)-\frac{cd^2\log\left(\frac{d}{e}+x\right)}{e^2}+\frac{cdx}{e} \\ \frac{ad^2e^2m(d+ex)^m}{e^2m^2+5e^2m+6e^2}+\frac{3ad^2e^2(d+ex)^m}{e^2m^2+5e^2m+6e^2}+\frac{2ade^3mx(d+ex)^m}{e^2m^2+5e^2m+6e^2}+\frac{6ade^3x(d+ex)^m}{e^2m^2+5e^2m+6e^2}+\frac{ae^4mx^2(d+ex)^m}{e^2m^2+5e^2m+6e^2}+\frac{3ae^4x^2(d+ex)^m}{e^2m^2+5e^2m+6e^2}-\frac{cd^4(d+ex)^m}{e^2m^2+5e^2m+6e^2}+\frac{cd^3em}{e^2m^2+5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

```
[Out] Piecewise((c*d**2*d**m*x**2/2, Eq(e, 0)), (-a*e**2/(d*e**2 + e**3*x) + c*d*
*2*log(d/e + x)/(d*e**2 + e**3*x) + c*d**2/(d*e**2 + e**3*x) + c*d*e*x*log(
d/e + x)/(d*e**2 + e**3*x), Eq(m, -3)), (a*log(d/e + x) - c*d**2*log(d/e +
x)/e**2 + c*d*x/e, Eq(m, -2)), (a*d**2*e**2*m*(d + e*x)**m/(e**2*m**2 + 5*e
**2*m + 6*e**2) + 3*a*d**2*e**2*(d + e*x)**m/(e**2*m**2 + 5*e**2*m + 6*e**2
) + 2*a*d*e**3*m*x*(d + e*x)**m/(e**2*m**2 + 5*e**2*m + 6*e**2) + 6*a*d*e**
3*x*(d + e*x)**m/(e**2*m**2 + 5*e**2*m + 6*e**2) + a*e**4*m*x**2*(d + e*x)*
*m/(e**2*m**2 + 5*e**2*m + 6*e**2) + 3*a*e**4*x**2*(d + e*x)**m/(e**2*m**2
+ 5*e**2*m + 6*e**2) - c*d**4*(d + e*x)**m/(e**2*m**2 + 5*e**2*m + 6*e**2)
+ c*d**3*e*m*x*(d + e*x)**m/(e**2*m**2 + 5*e**2*m + 6*e**2) + 2*c*d**2*e**2
*m*x**2*(d + e*x)**m/(e**2*m**2 + 5*e**2*m + 6*e**2) + 3*c*d**2*e**2*x**2*(
d + e*x)**m/(e**2*m**2 + 5*e**2*m + 6*e**2) + c*d*e**3*m*x**3*(d + e*x)**m/
(e**2*m**2 + 5*e**2*m + 6*e**2) + 2*c*d*e**3*x**3*(d + e*x)**m/(e**2*m**2 +
 5*e**2*m + 6*e**2), True))
```

Giac [B] time = 1.12733, size = 296, normalized size = 5.69

$$\frac{(xe+d)^m cdmx^3e^3 + 2\,(xe+d)^m cd^2mx^2e^2 + (xe+d)^m cd^3mxe + 2\,(xe+d)^m cdx^3e^3 + 3\,(xe+d)^m cd^2x^2e^2 - (xe+d)^m cd^4 +}{m^2e^2 + 5\,me}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*c*d*m*x^3*e^3 + 2*(x*e + d)^m*c*d^2*m*x^2*e^2 + (x*e + d)^m*c*
d^3*m*x*e + 2*(x*e + d)^m*c*d*x^3*e^3 + 3*(x*e + d)^m*c*d^2*x^2*e^2 - (x*e
+ d)^m*c*d^4 + (x*e + d)^m*a*m*x^2*e^4 + 2*(x*e + d)^m*a*d*m*x*e^3 + (x*e +
 d)^m*a*d^2*m*e^2 + 3*(x*e + d)^m*a*x^2*e^4 + 6*(x*e + d)^m*a*d*x*e^3 + 3*(
x*e + d)^m*a*d^2*e^2)/(m^2*e^2 + 5*m*e^2 + 6*e^2)
```

3.2087 $\int \frac{(d+ex)^m}{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal. Leaf size=54

$$-\frac{(d+ex)^m\,{}_2F_1\left(1,m;m+1;\frac{cd(d+ex)}{cd^2-ae^2}\right)}{m\left(cd^2-ae^2\right)}$$

```
[Out] -(((d + e*x)^m*Hypergeometric2F1[1, m, 1 + m, (c*d*(d + e*x))/(c*d^2 - a*e^
2)])/((c*d^2 - a*e^2)*m))
```

Rubi [A] time = 0.0243076, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 68}

$$-\frac{(d+ex)^m\,{}_2F_1\left(1,m;m+1;\frac{cd(d+ex)}{cd^2-ae^2}\right)}{m\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] -(((d + e*x)^m*Hypergeometric2F1[1, m, 1 + m, (c*d*(d + e*x))/(c*d^2 - a*e^
2)])/((c*d^2 - a*e^2)*m))
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^m}{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx &= \int \frac{(d+ex)^{-1+m}}{ae+cdx}\,dx\\ &= -\frac{(d+ex)^m\,{}_2F_1\left(1,m;1+m;\frac{cd(d+ex)}{cd^2-ae^2}\right)}{\left(cd^2-ae^2\right)m}\end{aligned}$$

Mathematica [A] time = 0.012819, size = 54, normalized size = 1.

$$-\frac{(d+ex)^m\,{}_2F_1\left(1,m;m+1;\frac{cd(d+ex)}{cd^2-ae^2}\right)}{m\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

```
[Out] -(((d + e*x)^m*Hypergeometric2F1[1, m, 1 + m, (c*d*(d + e*x))/(c*d^2 - a*e^
2)])/((c*d^2 - a*e^2)*m))
```

Maple [F] time = 1.179, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{ade+\left(ae^2+cd^2\right)x+cdex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

```
[Out] int((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{cdex^2+ade+\left(cd^2+ae^2\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^m}{cdex^2+ade+\left(cd^2+ae^2\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fricas"
)
```

```
[Out] integral((e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m}{(d+ex)\,(ae+cdx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

```
[Out] Integral((d + e*x)**m/((d + e*x)*(a*e + c*d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{cdex^2+ade+\left(cd^2+ae^2\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x), x)
```

3.2088 $\int \frac{(d+ex)^m}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2} dx$

Optimal. Leaf size=61

$$-\frac{e(d+ex)^{m-1}\,{}_2F_1\left(2,m-1;m;\frac{cd(d+ex)}{cd^2-ae^2}\right)}{(1-m)\left(cd^2-ae^2\right)^2}$$

```
[Out] -((e*(d + e*x)^(-1 + m)*Hypergeometric2F1[2, -1 + m, m, (c*d*(d + e*x))/(c*
d^2 - a*e^2)])/((c*d^2 - a*e^2)^2*(1 - m)))
```

Rubi [A] time = 0.0223567, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 68}

$$-\frac{e(d+ex)^{m-1}\,{}_2F_1\left(2,m-1;m;\frac{cd(d+ex)}{cd^2-ae^2}\right)}{(1-m)\left(cd^2-ae^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] -((e*(d + e*x)^(-1 + m)*Hypergeometric2F1[2, -1 + m, m, (c*d*(d + e*x))/(c*
d^2 - a*e^2)])/((c*d^2 - a*e^2)^2*(1 - m)))
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^m}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx &= \int \frac{(d+ex)^{-2+m}}{(ae+cdx)^2}\,dx\\ &= -\frac{e(d+ex)^{-1+m}\,{}_2F_1\left(2,-1+m;m;\frac{cd(d+ex)}{cd^2-ae^2}\right)}{\left(cd^2-ae^2\right)^2(1-m)}\end{aligned}$$

Mathematica [A] time = 0.0230798, size = 59, normalized size = 0.97

$$\frac{e(d+ex)^{m-1}\,{}_2F_1\left(2,m-1;m;-\frac{cd(d+ex)}{ae^2-cd^2}\right)}{(m-1)\left(ae^2-cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

```
[Out] (e*(d + e*x)^(-1 + m)*Hypergeometric2F1[2, -1 + m, m, -((c*d*(d + e*x))/(-(
c*d^2) + a*e^2))])/((-(c*d^2) + a*e^2)^2*(-1 + m))
```

Maple [F] time = 1.269, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(ade+\left(ae^2+cd^2\right)x+cdex^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

```
[Out] int((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="maxim
a")
```

```
[Out] integrate((e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(ex+d)^m}{c^2d^2e^2x^4+a^2d^2e^2+2\left(c^2d^3e+acde^3\right)x^3+\left(c^2d^4+4\,acd^2e^2+a^2e^4\right)x^2+2\left(acd^3e+a^2de^3\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="frica
s")
```

```
[Out] integral((e*x + d)^m/(c^2*d^2*e^2*x^4 + a^2*d^2*e^2 + 2*(c^2*d^3*e + a*c*d*
e^3)*x^3 + (c^2*d^4 + 4*a*c*d^2*e^2 + a^2*e^4)*x^2 + 2*(a*c*d^3*e + a^2*d*e
^3)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac"
)
```

```
[Out] integrate((e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^2, x)
```

3.2089 $\int \frac{(d+ex)^m}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3} dx$

Optimal. Leaf size=64

$$\frac{e^2(d+ex)^{m-2}\, {}_2F_1\left(3,m-2;m-1;\frac{cd(d+ex)}{cd^2-ae^2}\right)}{(2-m)\left(cd^2-ae^2\right)^3}$$

```
[Out] (e^2*(d + e*x)^(-2 + m)*Hypergeometric2F1[3, -2 + m, -1 + m, (c*d*(d + e*x)
)/(c*d^2 - a*e^2)])/((c*d^2 - a*e^2)^3*(2 - m))
```

Rubi [A] time = 0.0272594, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 68}

$$\frac{e^2(d+ex)^{m-2}\, {}_2F_1\left(3,m-2;m-1;\frac{cd(d+ex)}{cd^2-ae^2}\right)}{(2-m)\left(cd^2-ae^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] (e^2*(d + e*x)^(-2 + m)*Hypergeometric2F1[3, -2 + m, -1 + m, (c*d*(d + e*x)
)/(c*d^2 - a*e^2)])/((c*d^2 - a*e^2)^3*(2 - m))
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^m}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\, dx &= \int \frac{(d+ex)^{-3+m}}{(ae+cdx)^3}\, dx\\ &= \frac{e^2(d+ex)^{-2+m}\, {}_2F_1\left(3,-2+m;-1+m;\frac{cd(d+ex)}{cd^2-ae^2}\right)}{\left(cd^2-ae^2\right)^3(2-m)}\end{aligned}$$

Mathematica [A] time = 0.0228041, size = 63, normalized size = 0.98

$$\frac{e^2(d+ex)^{m-2}\, {}_2F_1\left(3,m-2;m-1;-\frac{cd(d+ex)}{ae^2-cd^2}\right)}{(m-2)\left(ae^2-cd^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

```
[Out] (e^2*(d + e*x)^(-2 + m)*Hypergeometric2F1[3, -2 + m, -1 + m, -((c*d*(d + e*
x))/(-(c*d^2) + a*e^2))])/((-(c*d^2) + a*e^2)^3*(-2 + m))
```

Maple [F] time = 1.609, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(ade+\left(ae^2+cd^2\right)x+cdex^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

```
[Out] int((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxim
a")
```

```
[Out] integrate((e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^m}{c^3d^3e^3x^6+a^3d^3e^3+3\left(c^3d^4e^2+ac^2d^2e^4\right)x^5+3\left(c^3d^5e+3\,ac^2d^3e^3+a^2cde^5\right)x^4+\left(c^3d^6+9\,ac^2d^4e^2+9\,a^2cd^2e^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="frica
s")
```

```
[Out] integral((e*x + d)^m/(c^3*d^3*e^3*x^6 + a^3*d^3*e^3 + 3*(c^3*d^4*e^2 + a*c^
2*d^2*e^4)*x^5 + 3*(c^3*d^5*e + 3*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x^4 + (c^3*d
^6 + 9*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 + a^3*e^6)*x^3 + 3*(a*c^2*d^5*e + 3*
a^2*c*d^3*e^3 + a^3*d*e^5)*x^2 + 3*(a^2*c*d^4*e^2 + a^3*d^2*e^4)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac"
)
```

```
[Out] integrate((e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^3, x)
```

3.2090 $\int \frac{(d+ex)^m}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx$

Optimal. Leaf size=65

$$-\frac{e^3(d+ex)^{m-3}\,{}_2F_1\left(4,m-3;m-2;\frac{cd(d+ex)}{cd^2-ae^2}\right)}{(3-m)\left(cd^2-ae^2\right)^4}$$

```
[Out] -((e^3*(d + e*x)^(-3 + m)*Hypergeometric2F1[4, -3 + m, -2 + m, (c*d*(d + e*
x))/(c*d^2 - a*e^2)])/((c*d^2 - a*e^2)^4*(3 - m)))
```

Rubi [A] time = 0.0262358, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {626, 68}

$$-\frac{e^3(d+ex)^{m-3}\,{}_2F_1\left(4,m-3;m-2;\frac{cd(d+ex)}{cd^2-ae^2}\right)}{(3-m)\left(cd^2-ae^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

```
[Out] -((e^3*(d + e*x)^(-3 + m)*Hypergeometric2F1[4, -3 + m, -2 + m, (c*d*(d + e*
x))/(c*d^2 - a*e^2)])/((c*d^2 - a*e^2)^4*(3 - m)))
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^m}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx &= \int \frac{(d+ex)^{-4+m}}{(ae+cdx)^4}\,dx\\ &= -\frac{e^3(d+ex)^{-3+m}\,{}_2F_1\left(4,-3+m;-2+m;\frac{cd(d+ex)}{cd^2-ae^2}\right)}{\left(cd^2-ae^2\right)^4(3-m)}\end{aligned}$$

Mathematica [A] time = 0.0278587, size = 63, normalized size = 0.97

$$\frac{e^3(d+ex)^{m-3}\,{}_2F_1\left(4,m-3;m-2;-\frac{cd(d+ex)}{ae^2-cd^2}\right)}{(m-3)\left(ae^2-cd^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

```
[Out] (e^3*(d + e*x)^(-3 + m)*Hypergeometric2F1[4, -3 + m, -2 + m, -((c*d*(d + e*
x))/(-(c*d^2) + a*e^2))])/((-(c*d^2) + a*e^2)^4*(-3 + m))
```

Maple [F] time = 1.294, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(ade+\left(ae^2+cd^2\right)x+cdex^2\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x)
```

```
[Out] int((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="maxim
a")
```

```
[Out] integrate((e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{}{c^4d^4e^4x^8+a^4d^4e^4+4\left(c^4d^5e^3+ac^3d^3e^5\right)x^7+2\left(3\,c^4d^6e^2+8\,ac^3d^4e^4+3\,a^2c^2d^2e^6\right)x^6+4\left(c^4d^7e+6\,ac^3d^5e^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="frica
s")
```

```
[Out] integral((e*x + d)^m/(c^4*d^4*e^4*x^8 + a^4*d^4*e^4 + 4*(c^4*d^5*e^3 + a*c^
3*d^3*e^5)*x^7 + 2*(3*c^4*d^6*e^2 + 8*a*c^3*d^4*e^4 + 3*a^2*c^2*d^2*e^6)*x^
6 + 4*(c^4*d^7*e + 6*a*c^3*d^5*e^3 + 6*a^2*c^2*d^3*e^5 + a^3*c*d*e^7)*x^5 +
 (c^4*d^8 + 16*a*c^3*d^6*e^2 + 36*a^2*c^2*d^4*e^4 + 16*a^3*c*d^2*e^6 + a^4*
e^8)*x^4 + 4*(a*c^3*d^7*e + 6*a^2*c^2*d^5*e^3 + 6*a^3*c*d^3*e^5 + a^4*d*e^7
)*x^3 + 2*(3*a^2*c^2*d^6*e^2 + 8*a^3*c*d^4*e^4 + 3*a^4*d^2*e^6)*x^2 + 4*(a^
3*c*d^5*e^3 + a^4*d^3*e^5)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="giac"
)
```

```
[Out] integrate((e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^4, x)
```

3.2091 $\int(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx$

Optimal. Leaf size=100

$$-\frac{(d+ex)^{m+1}(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\,{}_2F_1\left(1,m+2p+2;m+p+2;\frac{cd(d+ex)}{cd^2-ae^2}\right)}{(m+p+1)\left(cd^2-ae^2\right)}$$

```
[Out] -(((a*e + c*d*x)*(d + e*x)^(1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
p*Hypergeometric2F1[1, 2 + m + 2*p, 2 + m + p, (c*d*(d + e*x))/(c*d^2 - a*e
^2)])/((c*d^2 - a*e^2)*(1 + m + p)))
```

Rubi [A] time = 0.104822, antiderivative size = 123, normalized size of antiderivative = 1.23, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {679, 677, 70, 69}

$$\frac{(d+ex)^m(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-m-p}\,{}_2F_1\left(-m-p,p+1;p+2;-\frac{e(ae+cdx)}{cd^2-ae^2}\right)}{cd(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

```
[Out] ((a*e + c*d*x)*(d + e*x)^m*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(-m - p)*(a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p*Hypergeometric2F1[-m - p, 1 + p, 2 + p
, -((e*(a*e + c*d*x))/(c*d^2 - a*e^2))])/(c*d*(1 + p))
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(d^IntPart[m]*(d + e*x)^FracPart[m])/(1 + (e*x)/d)^FracPart[m
], Int[(1 + (e*x)/d)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !Intege
rQ[p] &&  !(IntegerQ[m] || GtQ[d, 0])
```

Rule 677

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(d^m*(a + b*x + c*x^2)^FracPart[p])/((1 + (e*x)/d)^FracPart[p
]*(a/d + (c*x)/e)^FracPart[p]), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || GtQ[d, 0]) &&  !(
IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
```

```
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = \left((d+ex)^m\left(1+\frac{ex}{d}\right)^{-m}\right)\int\left(1+\frac{ex}{d}\right)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx$$
$$=\left(\left(ade+cd^2x\right)^{-p}(d+ex)^m\left(1+\frac{ex}{d}\right)^{-m-p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\right)$$
$$=\left(\left(ade+cd^2x\right)^{-p}(d+ex)^m\left(\frac{cd^2\left(1+\frac{ex}{d}\right)}{cd^2-ae^2}\right)^{-m-p}\left(ade+\left(cd^2+ae^2\right)x+cde\right.\right.$$
$$=\frac{(ae+cdx)(d+ex)^m\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-m-p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,{}_2F_1\left(-\right.}{cd(1+p)}$$

Mathematica [A] time = 0.0539297, size = 107, normalized size = 1.07

$$\frac{(d+ex)^{m-1}((d+ex)(ae+cdx))^{p+1}\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-m-p}\,{}_2F_1\left(-m-p,p+1;p+2;\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{cd(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

```
[Out] ((d + e*x)^(-1 + m)*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(-m - p)*((a*e + c*d*
x)*(d + e*x))^(1 + p)*Hypergeometric2F1[-m - p, 1 + p, 2 + p, (e*(a*e + c*d
*x))/(-(c*d^2) + a*e^2)])/(c*d*(1 + p))
```

Maple [F] time = 1.319, size = 0, normalized size = 0.

$$\int (ex+d)^m\left(ade+\left(ae^2+cd^2\right)x+cdex^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x)
```

```
[Out] int((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm="maxim
a")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p*(e*x + d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^p (ex + d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm="frica
s")
```

```
[Out] integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p*(e*x + d)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^p (ex + d)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm="giac"
)
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p*(e*x + d)^m, x)
```

3.2092 $\int (d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx$

Optimal. Leaf size=95

$$-\frac{(d+ex)^4(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p {}_2F_1\left(1,2p+5;p+5;\frac{cd(d+ex)}{cd^2-ae^2}\right)}{(p+4)\left(cd^2-ae^2\right)}$$

```
[Out] -(((a*e + c*d*x)*(d + e*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p*Hype
rgeometric2F1[1, 5 + 2*p, 5 + p, (c*d*(d + e*x))/(c*d^2 - a*e^2)])/((c*d^2
- a*e^2)*(4 + p)))
```

Rubi [A] time = 0.0811374, antiderivative size = 124, normalized size of antiderivative = 1.31, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {677, 70, 69}

$$\frac{\left(cd^2-ae^2\right)^3(ae+cdx)\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-p}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p {}_2F_1\left(-p-3,p+1;p+2;-\frac{e(ae+cdx)}{cd^2-ae^2}\right)}{c^4d^4(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

```
[Out] ((c*d^2 - a*e^2)^3*(a*e + c*d*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p*
Hypergeometric2F1[-3 - p, 1 + p, 2 + p, -((e*(a*e + c*d*x))/(c*d^2 - a*e^2)
)])/(c^4*d^4*(1 + p)*((c*d*(d + e*x))/(c*d^2 - a*e^2))^p)
```

Rule 677

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(d^m*(a + b*x + c*x^2)^FracPart[p])/((1 + (e*x)/d)^FracPart[p
]*(a/d + (c*x)/e)^FracPart[p]), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || GtQ[d, 0]) &&  !(
IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p dx = \left(d^3(ae+cdx)^{-p}\left(1+\frac{ex}{d}\right)^{-p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\right)\int (ae+cd$$

$$= \frac{\left(\left(cd-\frac{ae^2}{d}\right)^3 (ae+cdx)^{-p}\left(\frac{cd\left(1+\frac{ex}{d}\right)}{cd-\frac{ae^2}{d}}\right)^{-p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\right)}{c^3}$$

$$= \frac{\left(cd^2-ae^2\right)^3 (ae+cdx)\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p {}_2F_1}{c^4d^4(1+p)}$$

Mathematica [A] time = 0.0527345, size = 112, normalized size = 1.18

$$\frac{\left(cd^2-ae^2\right)^3 (ae+cdx)\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-p}((d+ex)(ae+cdx))^p {}_2F_1\left(-p-3,p+1;p+2;\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{c^4d^4(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

```
[Out] ((c*d^2 - a*e^2)^3*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^p*Hypergeometric
2F1[-3 - p, 1 + p, 2 + p, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(c^4*d^4*(
1 + p)*((c*d*(d + e*x))/(c*d^2 - a*e^2))^p)
```

Maple [F] time = 1.162, size = 0, normalized size = 0.

$$\int (ex+d)^3 \left(ade+\left(ae^2+cd^2\right)x+cdex^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x)
```

```
[Out] int((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm="maxim
a")
```

```
[Out] integrate((e*x + d)^3*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm="frica
s")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3 \left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm="giac"
)
```

```
[Out] integrate((e*x + d)^3*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p, x)
```

3.2093 $\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx$

Optimal. Leaf size=95

$$-\frac{(d+ex)^3(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\,{}_2F_1\left(1,2(p+2);p+4;\frac{cd(d+ex)}{cd^2-ae^2}\right)}{(p+3)\left(cd^2-ae^2\right)}$$

```
[Out] -(((a*e + c*d*x)*(d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p*Hype
rgeometric2F1[1, 2*(2 + p), 4 + p, (c*d*(d + e*x))/(c*d^2 - a*e^2)])/((c*d^
2 - a*e^2)*(3 + p)))
```

Rubi [A] time = 0.0679863, antiderivative size = 124, normalized size of antiderivative = 1.31, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {677, 70, 69}

$$\frac{\left(cd^2-ae^2\right)^2(ae+cdx)\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-p}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\,{}_2F_1\left(-p-2,p+1;p+2;-\frac{e(ae+cdx)}{cd^2-ae^2}\right)}{c^3d^3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

```
[Out] ((c*d^2 - a*e^2)^2*(a*e + c*d*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p*
Hypergeometric2F1[-2 - p, 1 + p, 2 + p, -((e*(a*e + c*d*x))/(c*d^2 - a*e^2)
)])/(c^3*d^3*(1 + p)*((c*d*(d + e*x))/(c*d^2 - a*e^2))^p)
```

Rule 677

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(d^m*(a + b*x + c*x^2)^FracPart[p])/((1 + (e*x)/d)^FracPart[p
]*(a/d + (c*x)/e)^FracPart[p]), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || GtQ[d, 0]) &&  !(
IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p dx = \left(d^2(ae+cdx)^{-p}\left(1+\frac{ex}{d}\right)^{-p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\right)\int (ae+cdx)^p$$

$$= \frac{\left(\left(cd-\frac{ae^2}{d}\right)^2 (ae+cdx)^{-p}\left(\frac{cd\left(1+\frac{ex}{d}\right)}{cd-\frac{ae^2}{d}}\right)^{-p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\right)\int (\ldots}{c^2}$$

$$= \frac{\left(cd^2-ae^2\right)^2 (ae+cdx)\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p {}_2F_1\left(-\ldots\right.}{c^3d^3(1+p)}$$

Mathematica [A] time = 0.0430902, size = 112, normalized size = 1.18

$$\frac{\left(cd^2-ae^2\right)^2 (ae+cdx)\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-p}((d+ex)(ae+cdx))^p {}_2F_1\left(-p-2,p+1;p+2;\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{c^3d^3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

```
[Out] ((c*d^2 - a*e^2)^2*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^p*Hypergeometric
2F1[-2 - p, 1 + p, 2 + p, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(c^3*d^3*(
1 + p)*((c*d*(d + e*x))/(c*d^2 - a*e^2))^p)
```

Maple [F] time = 1.089, size = 0, normalized size = 0.

$$\int (ex+d)^2 \left(ade+\left(ae^2+cd^2\right)x+cdex^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x)
```

```
[Out] int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^2\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm="maxim
a")
```

```
[Out] integrate((e*x + d)^2*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^2+2\,dex+d^2\right)\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm="frica
s")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^
p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^2 \left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm="giac"
)
```

```
[Out] integrate((e*x + d)^2*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p, x)
```

3.2094 $\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx$

Optimal. Leaf size=95

$$-\frac{(d+ex)^2(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\,{}_2F_1\left(1,2p+3;p+3;\frac{cd(d+ex)}{cd^2-ae^2}\right)}{(p+2)\left(cd^2-ae^2\right)}$$

```
[Out] -(((a*e + c*d*x)*(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p*Hype
rgeometric2F1[1, 3 + 2*p, 3 + p, (c*d*(d + e*x))/(c*d^2 - a*e^2)])/((c*d^2
- a*e^2)*(2 + p)))
```

Rubi [A] time = 0.0576793, antiderivative size = 152, normalized size of antiderivative = 1.6, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {640, 624}

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{2cd(p+1)}-\frac{\left(-\frac{e(ae+cdx)}{cd^2-ae^2}\right)^{-p-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}\,{}_2F_1\left(-p,p+1;p+2;\frac{cd(d+ex)}{cd^2-ae^2}\right)}{2cd(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

```
[Out] (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 + p)/(2*c*d*(1 + p)) - ((-((e*(a
*e + c*d*x))/(c*d^2 - a*e^2)))^(-1 - p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (c*d*(d + e*x))/(c*d^2 - a
*e^2)])/(2*c*d*(1 + p))
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[4*p]
```

Rubi steps

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx=\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1+p}}{2cd(1+p)}+\frac{\left(d^2-\frac{ae^2}{c}\right)\int\left(ade+\left(cd^2+ae^2\right)x+cd\right.}{2d}$$

$$=\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1+p}}{2cd(1+p)}-\frac{\left(-\frac{e(ae+cdx)}{cd^2-ae^2}\right)^{-1-p}\left(ade+\left(cd^2+ae^2\right)x+\right.}{2cd}$$

Mathematica [A] time = 0.0521781, size = 94, normalized size = 0.99

$$\frac{\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-p-1}((d+ex)(ae+cdx))^{p+1}\,{}_2F_1\left(-p-1,p+1;p+2;\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{cd(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

```
[Out] (((c*d*(d + e*x))/(c*d^2 - a*e^2))^(-1 - p)*((a*e + c*d*x)*(d + e*x))^(1 +
p)*Hypergeometric2F1[-1 - p, 1 + p, 2 + p, (e*(a*e + c*d*x))/(-(c*d^2) + a*
e^2)])/(c*d*(1 + p))
```

Maple [F] time = 0.972, size = 0, normalized size = 0.

$$\int (ex+d)\left(ade+\left(ae^2+cd^2\right)x+cdex^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x)
```

```
[Out] int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm="maxima"
)
```

```
[Out] integrate((e*x + d)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((ex+d)\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm="fricas"
)
```

```
[Out] integral((e*x + d)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p, x)
```

3.2095 $\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx$

Optimal. Leaf size=113

$$-\frac{\left(-\frac{e(ae+cdx)}{cd^2-ae^2}\right)^{-p-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}{}_2F_1\left(-p,p+1;p+2;\frac{cd(d+ex)}{cd^2-ae^2}\right)}{(p+1)\left(cd^2-ae^2\right)}$$

```
[Out] -(((-((e*(a*e + c*d*x))/(c*d^2 - a*e^2)))^(-1 - p)*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (c*d*(d + e*x))
/(c*d^2 - a*e^2)])/((c*d^2 - a*e^2)*(1 + p)))
```

Rubi [A] time = 0.017553, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {624}

$$-\frac{\left(-\frac{e(ae+cdx)}{cd^2-ae^2}\right)^{-p-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}{}_2F_1\left(-p,p+1;p+2;\frac{cd(d+ex)}{cd^2-ae^2}\right)}{(p+1)\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

```
[Out] -(((-((e*(a*e + c*d*x))/(c*d^2 - a*e^2)))^(-1 - p)*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (c*d*(d + e*x))
/(c*d^2 - a*e^2)])/((c*d^2 - a*e^2)*(1 + p)))
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[4*p]
```

Rubi steps

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx=-\frac{\left(-\frac{e(ae+cdx)}{cd^2-ae^2}\right)^{-1-p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1+p}{}_2F_1\left(-p,1+p;2+p;\frac{cd(d+ex)}{cd^2-ae^2}\right)}{\left(cd^2-ae^2\right)(1+p)}$$

Mathematica [A] time = 0.0248334, size = 96, normalized size = 0.85

$$\frac{(ae+cdx)\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-p}((d+ex)(ae+cdx))^p\,{}_2F_1\left(-p,p+1;p+2;\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{cd(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

```
[Out] ((a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^p*Hypergeometric2F1[-p, 1 + p, 2 +
 p, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(c*d*(1 + p)*((c*d*(d + e*x))/(c
```

```
*d^2 - a*e^2))^p)
```

Maple [F] time = 0.953, size = 0, normalized size = 0.

$$\int \left(ade+\left(ae^{2}+cd^{2}\right)x+cde\,x^{2}\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x)
```

```
[Out] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cde\,x^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(cde\,x^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm="fricas")
```

```
[Out] integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ade+cde\,x^{2}+x\left(ae^{2}+cd^{2}\right)\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**p,x)
```

```
[Out] Integral((a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cde\,x^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p, x)
```

3.2096 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p}{d+ex}\,dx$

Optimal. Leaf size=93

$$\frac{\left(-\frac{e(ae+cdx)}{cd^2-ae^2}\right)^{-p}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p {}_2F_1\left(-p,p;p+1;\frac{cd(d+ex)}{cd^2-ae^2}\right)}{ep}$$

```
[Out] ((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p*Hypergeometric2F1[-p, p, 1 + p,
(c*d*(d + e*x))/(c*d^2 - a*e^2)])/(e*p*(-((e*(a*e + c*d*x))/(c*d^2 - a*e^2)
))^p)
```

Rubi [A] time = 0.0683955, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {677, 70, 69}

$$\frac{\left(-\frac{e(ae+cdx)}{cd^2-ae^2}\right)^{-p}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p {}_2F_1\left(-p,p;p+1;\frac{cd(d+ex)}{cd^2-ae^2}\right)}{ep}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p/(d + e*x),x]
```

```
[Out] ((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p*Hypergeometric2F1[-p, p, 1 + p,
(c*d*(d + e*x))/(c*d^2 - a*e^2)])/(e*p*(-((e*(a*e + c*d*x))/(c*d^2 - a*e^2)
))^p)
```

Rule 677

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(d^m*(a + b*x + c*x^2)^FracPart[p])/((1 + (e*x)/d)^FracPart[p
]*(a/d + (c*x)/e)^FracPart[p]), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || GtQ[d, 0]) &&  !(
IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p}{d+ex}\,dx = \frac{\left((ae+cdx)^{-p}\left(1+\frac{ex}{d}\right)^{-p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\right)\int(ae+cdx)^p\left(1+\frac{ex}{d}\right)}{d}$$

$$= \frac{\left(\left(\frac{e(ae+cdx)}{d\left(-cd+\frac{ae^2}{d}\right)}\right)^{-p}\left(1+\frac{ex}{d}\right)^{-p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\right)\int\left(1+\frac{ex}{d}\right)^{-1+p}\left(-\frac{c}{cd^2}\right.}{d}$$

$$= \frac{\left(-\frac{e(ae+cdx)}{cd^2-ae^2}\right)^{-p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p {}_2F_1\left(-p,p;1+p;\frac{cd(d+ex)}{cd^2-ae^2}\right)}{ep}$$

Mathematica [A] time = 0.0267257, size = 81, normalized size = 0.87

$$\frac{\left(\frac{e(ae+cdx)}{ae^2-cd^2}\right)^{-p}((d+ex)(ae+cdx))^p {}_2F_1\left(-p,p;p+1;\frac{cd(d+ex)}{cd^2-ae^2}\right)}{ep}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p/(d + e*x),x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^p*Hypergeometric2F1[-p, p, 1 + p, (c*d*(d + e*x)
)/(c*d^2 - a*e^2)])/(e*p*((e*(a*e + c*d*x))/(-(c*d^2) + a*e^2))^p)
```

Maple [F] time = 1.174, size = 0, normalized size = 0.

$$\int \frac{\left(ade+\left(ae^2+cd^2\right)x+cdex^2\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/(e*x+d),x)
```

```
[Out] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/(e*x+d),x, algorithm="maxima"
)
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^p}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/(e*x+d),x, algorithm="fricas"
)
```

```
[Out] integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{((d + ex)(ae + cdx))^p}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**p/(e*x+d),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**p/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^p}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p/(e*x + d), x)
```

3.2097 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=92

$$\frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p {}_2F_1\left(1,2p;p;\frac{cd(d+ex)}{cd^2-ae^2}\right)}{(1-p)(d+ex)\left(cd^2-ae^2\right)}$$

```
[Out] ((a*e + c*d*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p*Hypergeometric2F1[
1, 2*p, p, (c*d*(d + e*x))/(c*d^2 - a*e^2)])/((c*d^2 - a*e^2)*(1 - p)*(d +
e*x))
```

Rubi [A] time = 0.0700642, antiderivative size = 120, normalized size of antiderivative = 1.3, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {677, 70, 69}

$$\frac{cd(ae+cdx)\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-p}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p {}_2F_1\left(2-p,p+1;p+2;-\frac{e(ae+cdx)}{cd^2-ae^2}\right)}{(p+1)\left(cd^2-ae^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p/(d + e*x)^2,x]
```

```
[Out] (c*d*(a*e + c*d*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p*Hypergeometric
2F1[2 - p, 1 + p, 2 + p, -((e*(a*e + c*d*x))/(c*d^2 - a*e^2))])/((c*d^2 - a
*e^2)^2*(1 + p)*((c*d*(d + e*x))/(c*d^2 - a*e^2))^p)
```

Rule 677

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(d^m*(a + b*x + c*x^2)^FracPart[p])/((1 + (e*x)/d)^FracPart[p
]*(a/d + (c*x)/e)^FracPart[p]), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || GtQ[d, 0]) &&  !(
IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p}{(d+ex)^2}\,dx = \frac{\left((ae+cdx)^{-p}\left(1+\frac{ex}{d}\right)^{-p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\right)\int (ae+cdx)^p\left(1+\frac{ex}{d}\right)^{-2+p}}{d^2}$$

$$= \frac{\left(c^2(ae+cdx)^{-p}\left(\frac{cd\left(1+\frac{ex}{d}\right)}{cd-\frac{ae^2}{d}}\right)^{-p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\right)\int (ae+cdx)^p\left(\frac{cd^2}{cd^2-ae^2}\right.}{\left(cd-\frac{ae^2}{d}\right)^2}$$

$$= \frac{cd(ae+cdx)\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p {}_2F_1\left(2-p,1+p;2+p;-\right.}{\left(cd^2-ae^2\right)^2(1+p)}$$

Mathematica [A] time = 0.0376085, size = 108, normalized size = 1.17

$$\frac{cd(ae+cdx)\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-p}((d+ex)(ae+cdx))^p {}_2F_1\left(2-p,p+1;p+2;\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{(p+1)\left(cd^2-ae^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p/(d + e*x)^2,x]
```

```
[Out] (c*d*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^p*Hypergeometric2F1[2 - p, 1 +
 p, 2 + p, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/((c*d^2 - a*e^2)^2*(1 + p
)*((c*d*(d + e*x))/(c*d^2 - a*e^2))^p)
```

Maple [F] time = 1.165, size = 0, normalized size = 0.

$$\int \frac{\left(ade+\left(ae^2+cd^2\right)x+cdex^2\right)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/(e*x+d)^2,x)
```

```
[Out] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/(e*x+d)^2,x, algorithm="maxim
a")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p/(e*x + d)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^p}{e^2x^2 + 2\,dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/(e*x+d)^2,x, algorithm="frica
s")
```

```
[Out] integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p/(e^2*x^2 + 2*d*e*x + d^2
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**p/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^p}{(ex + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/(e*x+d)^2,x, algorithm="giac"
)
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p/(e*x + d)^2, x)
```

3.2098 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p}{(d+ex)^3}\,dx$

Optimal. Leaf size=96

$$\frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\,{}_2F_1\left(1,2p-1;p-1;\frac{cd(d+ex)}{cd^2-ae^2}\right)}{(2-p)(d+ex)^2\left(cd^2-ae^2\right)}$$

```
[Out] ((a*e + c*d*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p*Hypergeometric2F1[
1, -1 + 2*p, -1 + p, (c*d*(d + e*x))/(c*d^2 - a*e^2)])/((c*d^2 - a*e^2)*(2
- p)*(d + e*x)^2)
```

Rubi [A] time = 0.0684541, antiderivative size = 124, normalized size of antiderivative = 1.29, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {677, 70, 69}

$$\frac{c^2d^2(ae+cdx)\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-p}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\,{}_2F_1\left(3-p,p+1;p+2;-\frac{e(ae+cdx)}{cd^2-ae^2}\right)}{(p+1)\left(cd^2-ae^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p/(d + e*x)^3,x]
```

```
[Out] (c^2*d^2*(a*e + c*d*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p*Hypergeome
tric2F1[3 - p, 1 + p, 2 + p, -((e*(a*e + c*d*x))/(c*d^2 - a*e^2))])/((c*d^2
 - a*e^2)^3*(1 + p)*((c*d*(d + e*x))/(c*d^2 - a*e^2))^p)
```

Rule 677

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(d^m*(a + b*x + c*x^2)^FracPart[p])/((1 + (e*x)/d)^FracPart[p
]*(a/d + (c*x)/e)^FracPart[p]), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || GtQ[d, 0]) &&  !(
IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p}{(d+ex)^3}\,dx = \frac{\left((ae+cdx)^{-p}\left(1+\frac{ex}{d}\right)^{-p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\right)\int (ae+cdx)^p\left(1+\frac{ex}{d}\right)}{d^3}$$

$$= \frac{\left(c^3(ae+cdx)^{-p}\left(\frac{cd\left(1+\frac{ex}{d}\right)}{cd-\frac{ae^2}{d}}\right)^{-p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\right)\int (ae+cdx)^p\left(\frac{cd}{cd^2-}\right.}{\left(cd-\frac{ae^2}{d}\right)^3}$$

$$= \frac{c^2d^2(ae+cdx)\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,{}_2F_1\left(3-p,1+p;2+\right.}{\left(cd^2-ae^2\right)^3(1+p)}$$

Mathematica [A] time = 0.041563, size = 112, normalized size = 1.17

$$\frac{c^2d^2(ae+cdx)\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-p}((d+ex)(ae+cdx))^p\,{}_2F_1\left(3-p,p+1;p+2;\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{(p+1)\left(cd^2-ae^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p/(d + e*x)^3,x]
```

```
[Out] (c^2*d^2*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^p*Hypergeometric2F1[3 - p,
 1 + p, 2 + p, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/((c*d^2 - a*e^2)^3*(1
 + p)*((c*d*(d + e*x))/(c*d^2 - a*e^2))^p)
```

Maple [F] time = 1.201, size = 0, normalized size = 0.

$$\int \frac{\left(ade+\left(ae^2+cd^2\right)x+cdex^2\right)^p}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/(e*x+d)^3,x)
```

```
[Out] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/(e*x+d)^3,x, algorithm="maxim
a")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p/(e*x + d)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/(e*x+d)^3,x, algorithm="frica
s")
```

```
[Out] integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p/(e^3*x^3 + 3*d*e^2*x^2 +
 3*d^2*e*x + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**p/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/(e*x+d)^3,x, algorithm="giac"
)
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p/(e*x + d)^3, x)
```

3.2099 $\int (d+ex)^{-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx$

Optimal. Leaf size=113

$$\frac{(d+ex)^{-2p}(ae+cdx)\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^p\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\,{}_2F_1\left(p,p+1;p+2;-\frac{e(ae+cdx)}{cd^2-ae^2}\right)}{cd(p+1)}$$

```
[Out] ((a*e + c*d*x)*((c*d*(d + e*x))/(c*d^2 - a*e^2))^p*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^p*Hypergeometric2F1[p, 1 + p, 2 + p, -((e*(a*e + c*d*x))/(c
*d^2 - a*e^2))])/(c*d*(1 + p)*(d + e*x)^(2*p))
```

Rubi [A] time = 0.0940342, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {679, 677, 70, 69}

$$\frac{(d+ex)^{-2p}(ae+cdx)\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^p\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\,{}_2F_1\left(p,p+1;p+2;-\frac{e(ae+cdx)}{cd^2-ae^2}\right)}{cd(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p/(d + e*x)^(2*p),x]
```

```
[Out] ((a*e + c*d*x)*((c*d*(d + e*x))/(c*d^2 - a*e^2))^p*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^p*Hypergeometric2F1[p, 1 + p, 2 + p, -((e*(a*e + c*d*x))/(c
*d^2 - a*e^2))])/(c*d*(1 + p)*(d + e*x)^(2*p))
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(d^IntPart[m]*(d + e*x)^FracPart[m])/(1 + (e*x)/d)^FracPart[m
], Int[(1 + (e*x)/d)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !Intege
rQ[p] &&  !(IntegerQ[m] || GtQ[d, 0])
```

Rule 677

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(d^m*(a + b*x + c*x^2)^FracPart[p])/((1 + (e*x)/d)^FracPart[p
]*(a/d + (c*x)/e)^FracPart[p]), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || GtQ[d, 0]) &&  !(
IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
```

```
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (d+ex)^{-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = \left((d+ex)^{-2p}\left(1+\frac{ex}{d}\right)^{2p}\right)\int\left(1+\frac{ex}{d}\right)^{-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p$$

$$= \left(\left(ade+cd^2x\right)^{-p}(d+ex)^{-2p}\left(1+\frac{ex}{d}\right)^p\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\right)$$

$$= \left(\left(ade+cd^2x\right)^{-p}(d+ex)^{-2p}\left(\frac{cd^2\left(1+\frac{ex}{d}\right)}{cd^2-ae^2}\right)^p\left(ade+\left(cd^2+ae^2\right)x+cdex\right.\right.$$

$$= \frac{(ae+cdx)(d+ex)^{-2p}\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^p\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,{}_2F_1\left(p,\right.}{cd(1+p)}$$

Mathematica [A] time = 0.0439543, size = 97, normalized size = 0.86

$$\frac{(d+ex)^{-2p-1}\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^p((d+ex)(ae+cdx))^{p+1}\,{}_2F_1\left(p,p+1;p+2;\frac{e(ae+cdx)}{ae^2-cd^2}\right)}{cd(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p/(d + e*x)^(2*p),x]
```

```
[Out] ((d + e*x)^(-1 - 2*p)*((c*d*(d + e*x))/(c*d^2 - a*e^2))^p*((a*e + c*d*x)*(d
 + e*x))^(1 + p)*Hypergeometric2F1[p, 1 + p, 2 + p, (e*(a*e + c*d*x))/(-(c*
d^2) + a*e^2)])/(c*d*(1 + p))
```

Maple [F] time = 1.261, size = 0, normalized size = 0.

$$\int\frac{\left(ade+\left(ae^2+cd^2\right)x+cdex^2\right)^p}{(ex+d)^{2p}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/((e*x+d)^(2*p)),x)
```

```
[Out] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/((e*x+d)^(2*p)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p}{(ex+d)^{2p}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/((e*x+d)^(2*p)),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p/(e*x + d)^(2*p), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^p}{(ex + d)^{2p}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/((e*x+d)^(2*p)),x, algorithm=
"fricas")
```

```
[Out] integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p/(e*x + d)^(2*p), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**p/((e*x+d)**(2*p)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^p}{(ex + d)^{2p}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/((e*x+d)^(2*p)),x, algorithm=
"giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p/(e*x + d)^(2*p), x)
```

3.2100 $\int (d+ex)^{-1-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx$

Optimal. Leaf size=107

$$-\frac{(d+ex)^{-2p}\left(-\frac{e(ae+cdx)}{cd^2-ae^2}\right)^{-p}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\,{}_2F_1\left(-p,-p;1-p;\frac{cd(d+ex)}{cd^2-ae^2}\right)}{ep}$$

```
[Out] -(((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p*Hypergeometric2F1[-p, -p, 1 -
p, (c*d*(d + e*x))/(c*d^2 - a*e^2)])/(e*p*(-((e*(a*e + c*d*x))/(c*d^2 - a*e
^2)))^p*(d + e*x)^(2*p)))
```

Rubi [A] time = 0.095885, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {679, 677, 70, 69}

$$-\frac{(d+ex)^{-2p}\left(-\frac{e(ae+cdx)}{cd^2-ae^2}\right)^{-p}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\,{}_2F_1\left(-p,-p;1-p;\frac{cd(d+ex)}{cd^2-ae^2}\right)}{ep}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(-1 - 2*p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

```
[Out] -(((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p*Hypergeometric2F1[-p, -p, 1 -
p, (c*d*(d + e*x))/(c*d^2 - a*e^2)])/(e*p*(-((e*(a*e + c*d*x))/(c*d^2 - a*e
^2)))^p*(d + e*x)^(2*p)))
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(d^IntPart[m]*(d + e*x)^FracPart[m])/(1 + (e*x)/d)^FracPart[m
], Int[(1 + (e*x)/d)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !Intege
rQ[p] &&  !(IntegerQ[m] || GtQ[d, 0])
```

Rule 677

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(d^m*(a + b*x + c*x^2)^FracPart[p])/((1 + (e*x)/d)^FracPart[p
]*(a/d + (c*x)/e)^FracPart[p]), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || GtQ[d, 0]) &&  !(
IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
```

```
a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (d+ex)^{-1-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = \frac{\left((d+ex)^{-2p}\left(1+\frac{ex}{d}\right)^{2p}\right)\int\left(1+\frac{ex}{d}\right)^{-1-2p}\left(ade+\left(cd^2+ae^2\right)x+cd\right.}{d}$$

$$= \frac{\left(\left(ade+cd^2x\right)^{-p}(d+ex)^{-2p}\left(1+\frac{ex}{d}\right)^p\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.\right.}{d}$$

$$= \frac{\left(\left(\frac{e\left(ade+cd^2x\right)}{d\left(-cd^2+ae^2\right)}\right)^{-p}(d+ex)^{-2p}\left(1+\frac{ex}{d}\right)^p\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.\right.}{d}$$

$$= -\frac{\left(-\frac{e(ae+cdx)}{cd^2-ae^2}\right)^{-p}(d+ex)^{-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,{}_2F_1\left(-\right.}{ep}$$

Mathematica [A] time = 0.0222295, size = 95, normalized size = 0.89

$$-\frac{(d+ex)^{-2p}\left(\frac{e(ae+cdx)}{ae^2-cd^2}\right)^{-p}((d+ex)(ae+cdx))^p\,{}_2F_1\left(-p,-p;1-p;\frac{cd(d+ex)}{cd^2-ae^2}\right)}{ep}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(-1 - 2*p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

```
[Out] -((((a*e + c*d*x)*(d + e*x))^p*Hypergeometric2F1[-p, -p, 1 - p, (c*d*(d + e
*x))/(c*d^2 - a*e^2)])/(e*p*((e*(a*e + c*d*x))/(-(c*d^2) + a*e^2))^p*(d + e
*x)^(2*p)))
```

Maple [F] time = 1.219, size = 0, normalized size = 0.

$$\int (ex+d)^{-1-2p}\left(ade+\left(ae^2+cd^2\right)x+cdex^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(-1-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x)
```

```
[Out] int((e*x+d)^(-1-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p(ex+d)^{-2p-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-1-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm
="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p*(e*x + d)^(-2*p - 1), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^p (ex + d)^{-2\,p-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-1-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm
="fricas")
```

```
[Out] integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p*(e*x + d)^(-2*p - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(-1-2*p)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^p (ex + d)^{-2\,p-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-1-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm
="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p*(e*x + d)^(-2*p - 1), x
)
```

3.2101 $\int (d+ex)^{-2-2p} \left(ade + \left(cd^2 + ae^2\right) x + cdex^2\right)^p \, dx$

Optimal. Leaf size=60

$$\frac{(d+ex)^{-2(p+1)} \left(x \left(ae^2 + cd^2\right) + ade + cdex^2\right)^{p+1}}{(p+1) \left(cd^2 - ae^2\right)}$$

```
[Out] (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 + p)/((c*d^2 - a*e^2)*(1 + p)*(d
 + e*x)^(2*(1 + p)))
```

Rubi [A] time = 0.0162893, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {650}

$$\frac{(d+ex)^{-2(p+1)} \left(x \left(ae^2 + cd^2\right) + ade + cdex^2\right)^{p+1}}{(p+1) \left(cd^2 - ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(-2 - 2*p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

```
[Out] (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 + p)/((c*d^2 - a*e^2)*(1 + p)*(d
 + e*x)^(2*(1 + p)))
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int (d+ex)^{-2-2p} \left(ade + \left(cd^2 + ae^2\right) x + cdex^2\right)^p \, dx = \frac{(d+ex)^{-2(1+p)} \left(ade + \left(cd^2 + ae^2\right) x + cdex^2\right)^{1+p}}{\left(cd^2 - ae^2\right) (1+p)}$$

Mathematica [A] time = 0.0286327, size = 49, normalized size = 0.82

$$\frac{(d+ex)^{-2(p+1)} ((d+ex)(ae+cdx))^{p+1}}{(p+1) \left(cd^2 - ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(-2 - 2*p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

```
[Out] ((a*e + c*d*x)*(d + e*x))^(1 + p)/((c*d^2 - a*e^2)*(1 + p)*(d + e*x)^(2*(1
+ p)))
```

Maple [A] time = 0.043, size = 75, normalized size = 1.3

$$-\frac{(cdx+ae)(ex+d)^{-1-2p}\left(cde\,x^{2}+ae^{2}x+cd^{2}x+ade\right)^{p}}{ae^{2}p-cd^{2}p+ae^{2}-cd^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(-2-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x)
```

```
[Out] -(c*d*x+a*e)*(e*x+d)^(-1-2*p)/(a*e^2*p-c*d^2*p+a*e^2-c*d^2)*(c*d*e*x^2+a*e^
2*x+c*d^2*x+a*d*e)^p
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cde\,x^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{p}(ex+d)^{-2\,p-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-2-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm
="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p*(e*x + d)^(-2*p - 2), x
)
```

Fricas [A] time = 2.22, size = 189, normalized size = 3.15

$$\frac{\left(cde\,x^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)\left(cde\,x^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{p}(ex+d)^{-2\,p-2}}{cd^{2}-ae^{2}+\left(cd^{2}-ae^{2}\right)p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-2-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm
="fricas")
```

```
[Out] (c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)^p*(e*x + d)^(-2*p - 2)/(c*d^2 - a*e^2 + (c*d^2 - a*e^2)*p)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(-2-2*p)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^p (ex + d)^{-2p-2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-2-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm
="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p*(e*x + d)^(-2*p - 2), x
)
```

3.2102 $\int (d+ex)^{-3-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx$

Optimal. Leaf size=128

$$\frac{(d+ex)^{-2p-3}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{(p+2)\left(cd^2-ae^2\right)}+\frac{cd(d+ex)^{-2(p+1)}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{(p+1)(p+2)\left(cd^2-ae^2\right)^2}$$

```
[Out] ((d + e*x)^(-3 - 2*p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 + p))/((c*
d^2 - a*e^2)*(2 + p)) + (c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 + p
))/((c*d^2 - a*e^2)^2*(1 + p)*(2 + p)*(d + e*x)^(2*(1 + p)))
```

Rubi [A] time = 0.0524983, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {658, 650}

$$\frac{(d+ex)^{-2p-3}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{(p+2)\left(cd^2-ae^2\right)}+\frac{cd(d+ex)^{-2(p+1)}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{(p+1)(p+2)\left(cd^2-ae^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(-3 - 2*p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

```
[Out] ((d + e*x)^(-3 - 2*p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 + p))/((c*
d^2 - a*e^2)*(2 + p)) + (c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 + p
))/((c*d^2 - a*e^2)^2*(1 + p)*(2 + p)*(d + e*x)^(2*(1 + p)))
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int (d+ex)^{-3-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx=\frac{(d+ex)^{-3-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1+p}}{\left(cd^2-ae^2\right)(2+p)}+\frac{(cd)\int (d+ex)^{-2-2p}}{(c}$$

$$=\frac{(d+ex)^{-3-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1+p}}{\left(cd^2-ae^2\right)(2+p)}+\frac{cd(d+ex)^{-2(1+p)}(a}{(cd^2-}$$

Mathematica [A] time = 0.0554549, size = 76, normalized size = 0.59

$$\frac{(d+ex)^{-2p-3}((d+ex)(ae+cdx))^{p+1}\left(cd(d(p+2)+ex)-ae^2(p+1)\right)}{(p+1)(p+2)\left(cd^2-ae^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(-3 - 2*p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

```
[Out] ((d + e*x)^(-3 - 2*p)*((a*e + c*d*x)*(d + e*x))^(1 + p)*(-(a*e^2*(1 + p)) +
 c*d*(d*(2 + p) + e*x)))/((c*d^2 - a*e^2)^2*(1 + p)*(2 + p))
```

Maple [A] time = 0.043, size = 170, normalized size = 1.3

$$-\frac{(cdx+ae)(ex+d)^{-2-2p}\left(ae^2p-cd^2p-cdex+ae^2-2cd^2\right)\left(cdex^2+ae^2x+cd^2x+ade\right)^p}{a^2e^4p^2-2acd^2e^2p^2+c^2d^4p^2+3a^2e^4p-6acd^2e^2p+3c^2d^4p+2a^2e^4-4acd^2e^2+2c^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(-3-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x)
```

```
[Out] -(c*d*x+a*e)*(e*x+d)^(-2-2*p)*(a*e^2*p-c*d^2*p-c*d*e*x+a*e^2-2*c*d^2)*(c*d*
e*x^2+a*e^2*x+c*d^2*x+a*d*e)^p/(a^2*e^4*p^2-2*a*c*d^2*e^2*p^2+c^2*d^4*p^2+3
*a^2*e^4*p-6*a*c*d^2*e^2*p+3*c^2*d^4*p+2*a^2*e^4-4*a*c*d^2*e^2+2*c^2*d^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p(ex+d)^{-2p-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-3-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm
="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p*(e*x + d)^(-2*p - 3), x
)
```

Fricas [A] time = 2.30756, size = 505, normalized size = 3.95

$$\frac{\left(c^2d^2e^2x^3+2acd^3e-a^2de^3+\left(3c^2d^3e+\left(c^2d^3e-acde^3\right)p\right)x^2+\left(acd^3e-a^2de^3\right)p+\left(2c^2d^4+2acd^2e^2-a^2e^4+\left(c^2d^4-\right.\right.\right.}{2c^2d^4-4acd^2e^2+2a^2e^4+\left(c^2d^4-2acd^2e^2+a^2e^4\right)p^2+3\left(c^2d^4-2acd^2e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-3-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm
="fricas")
```

```
[Out] (c^2*d^2*e^2*x^3 + 2*a*c*d^3*e - a^2*d*e^3 + (3*c^2*d^3*e + (c^2*d^3*e - a*
c*d*e^3)*p)*x^2 + (a*c*d^3*e - a^2*d*e^3)*p + (2*c^2*d^4 + 2*a*c*d^2*e^2 -
```

```
a^2*e^4 + (c^2*d^4 - a^2*e^4)*p)*x)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
^p*(e*x + d)^(-2*p - 3)/(2*c^2*d^4 - 4*a*c*d^2*e^2 + 2*a^2*e^4 + (c^2*d^4 -
 2*a*c*d^2*e^2 + a^2*e^4)*p^2 + 3*(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*p)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(-3-2*p)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^p (ex + d)^{-2p-3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-3-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm
="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p*(e*x + d)^(-2*p - 3), x
)
```

3.2103 $\int (d+ex)^{-4-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx$

Optimal. Leaf size=206

$$\frac{2c^2d^2(d+ex)^{-2(p+1)}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{(p+1)(p+2)(p+3)\left(cd^2-ae^2\right)^3}+\frac{2cd(d+ex)^{-2p-3}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{(p+2)(p+3)\left(cd^2-ae^2\right)^2}+\frac{(d+ex)^{-}}{}$$

```
[Out] (2*c*d*(d + e*x)^(-3 - 2*p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 + p)
)/((c*d^2 - a*e^2)^2*(2 + p)*(3 + p)) + (2*c^2*d^2*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(1 + p))/((c*d^2 - a*e^2)^3*(1 + p)*(2 + p)*(3 + p)*(d + e*
x)^(2*(1 + p))) + (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 + p)/((c*d^2 -
 a*e^2)*(3 + p)*(d + e*x)^(2*(2 + p)))
```

Rubi [A] time = 0.0931066, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {658, 650}

$$\frac{2c^2d^2(d+ex)^{-2(p+1)}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{(p+1)(p+2)(p+3)\left(cd^2-ae^2\right)^3}+\frac{2cd(d+ex)^{-2p-3}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{(p+2)(p+3)\left(cd^2-ae^2\right)^2}+\frac{(d+ex)^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(-4 - 2*p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

```
[Out] (2*c*d*(d + e*x)^(-3 - 2*p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 + p)
)/((c*d^2 - a*e^2)^2*(2 + p)*(3 + p)) + (2*c^2*d^2*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(1 + p))/((c*d^2 - a*e^2)^3*(1 + p)*(2 + p)*(3 + p)*(d + e*
x)^(2*(1 + p))) + (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 + p)/((c*d^2 -
 a*e^2)*(3 + p)*(d + e*x)^(2*(2 + p)))
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int (d+ex)^{-4-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = \frac{(d+ex)^{-2(2+p)}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1+p}}{\left(cd^2-ae^2\right)(3+p)} + \frac{(2cd)\int (d+ex)^{-3\cdots}}{\cdots}$$

$$= \frac{2cd(d+ex)^{-3-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1+p}}{\left(cd^2-ae^2\right)^2(2+p)(3+p)} + \frac{(d+ex)^{-2(2+p)}\left(\cdots\right.}{\cdots}$$

$$= \frac{2cd(d+ex)^{-3-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1+p}}{\left(cd^2-ae^2\right)^2(2+p)(3+p)} + \frac{2c^2d^2(d+ex)^{-2\cdots}}{\left(cd^2-\cdots\right.}$$

Mathematica [A] time = 0.0906075, size = 131, normalized size = 0.64

$$\frac{(d+ex)^{-2(p+2)}((d+ex)(ae+cdx))^{p+1}\left(a^2e^4\left(p^2+3p+2\right)-2acde^2(p+1)(d(p+3)+ex)+c^2d^2\left(d^2\left(p^2+5p+6\right)+2de(p\cdots\right.\right.}{(p+1)(p+2)(p+3)\left(cd^2-ae^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(-4 - 2*p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^(1 + p)*(a^2*e^4*(2 + 3*p + p^2) - 2*a*c*d*e^2*(
1 + p)*(d*(3 + p) + e*x) + c^2*d^2*(d^2*(6 + 5*p + p^2) + 2*d*e*(3 + p)*x +
 2*e^2*x^2)))/((c*d^2 - a*e^2)^3*(1 + p)*(2 + p)*(3 + p)*(d + e*x)^(2*(2 +
p)))
```

Maple [A] time = 0.044, size = 381, normalized size = 1.9

$$-\frac{(cdx+ae)(ex+d)^{-3-2p}\left(a^2e^4p^2-2acd^2e^2p^2-2acde^3px+c^2d^4p^2+2c^2d^3epx+2c^2d^2e^2x^2+3a^2e^4p-8acd^2e^2p-2ac\cdots\right.}{a^3e^6p^3-3a^2cd^2e^4p^3+3ac^2d^4e^2p^3-c^3d^6p^3+6a^3e^6p^2-18a^2cd^2e^4p^2+18ac^2d^4e^2p^2-6c^3d^6p^2+11a^3e^6p-33\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(-4-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x)
```

```
[Out] -(c*d*x+a*e)*(e*x+d)^(-3-2*p)*(a^2*e^4*p^2-2*a*c*d^2*e^2*p^2-2*a*c*d*e^3*p*
x+c^2*d^4*p^2+2*c^2*d^3*e*p*x+2*c^2*d^2*e^2*x^2+3*a^2*e^4*p-8*a*c*d^2*e^2*p
-2*a*c*d*e^3*x+5*c^2*d^4*p+6*c^2*d^3*e*x+2*a^2*e^4-6*a*c*d^2*e^2+6*c^2*d^4)
*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^p/(a^3*e^6*p^3-3*a^2*c*d^2*e^4*p^3+3*a*c
^2*d^4*e^2*p^3-c^3*d^6*p^3+6*a^3*e^6*p^2-18*a^2*c*d^2*e^4*p^2+18*a*c^2*d^4*
e^2*p^2-6*c^3*d^6*p^2+11*a^3*e^6*p-33*a^2*c*d^2*e^4*p+33*a*c^2*d^4*e^2*p-11
*c^3*d^6*p+6*a^3*e^6-18*a^2*c*d^2*e^4+18*a*c^2*d^4*e^2-6*c^3*d^6)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p(ex+d)^{-2p-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-4-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm
="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p*(e*x + d)^(-2*p - 4), x
)
```

Fricas [B] time = 2.40401, size = 1145, normalized size = 5.56

$$\frac{\left(2 c^3 d^3 e^3 x^4 + 6 a c^2 d^5 e - 6 a^2 c d^3 e^3 + 2 a^3 d e^5 + 2 \left(4 c^3 d^4 e^2 + \left(c^3 d^4 e^2 - a c^2 d^2 e^4\right) p\right) x^3 + \left(a c^2 d^5 e - 2 a^2 c d^3 e^3 + a^3 d e^5\right) p^2 + \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-4-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm
="fricas")
```

```
[Out] (2*c^3*d^3*e^3*x^4 + 6*a*c^2*d^5*e - 6*a^2*c*d^3*e^3 + 2*a^3*d*e^5 + 2*(4*c
^3*d^4*e^2 + (c^3*d^4*e^2 - a*c^2*d^2*e^4)*p)*x^3 + (a*c^2*d^5*e - 2*a^2*c*
d^3*e^3 + a^3*d*e^5)*p^2 + (12*c^3*d^5*e + (c^3*d^5*e - 2*a*c^2*d^3*e^3 + a
^2*c*d*e^5)*p^2 + (7*c^3*d^5*e - 8*a*c^2*d^3*e^3 + a^2*c*d*e^5)*p)*x^2 + (5
*a*c^2*d^5*e - 8*a^2*c*d^3*e^3 + 3*a^3*d*e^5)*p + (6*c^3*d^6 + 6*a*c^2*d^4*
e^2 - 6*a^2*c*d^2*e^4 + 2*a^3*e^6 + (c^3*d^6 - a*c^2*d^4*e^2 - a^2*c*d^2*e^
4 + a^3*e^6)*p^2 + (5*c^3*d^6 - a*c^2*d^4*e^2 - 7*a^2*c*d^2*e^4 + 3*a^3*e^6
)*p)*x)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p*(e*x + d)^(-2*p - 4)/(6*c
^3*d^6 - 18*a*c^2*d^4*e^2 + 18*a^2*c*d^2*e^4 - 6*a^3*e^6 + (c^3*d^6 - 3*a*c
^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*p^3 + 6*(c^3*d^6 - 3*a*c^2*d^4*e^2
+ 3*a^2*c*d^2*e^4 - a^3*e^6)*p^2 + 11*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*
d^2*e^4 - a^3*e^6)*p)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(-4-2*p)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^p (ex+d)^{-2p-4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-4-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm
="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p*(e*x + d)^(-2*p - 4), x
)
```

3.2104 $\int (d+ex)^{-5-2p} \left(ade + \left(cd^2 + ae^2\right) x + cdex^2\right)^p \, dx$

Optimal. Leaf size=288

$$\frac{6c^2d^2(d+ex)^{-2p-3}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{(p+2)(p+3)(p+4)\left(cd^2-ae^2\right)^3}+\frac{6c^3d^3(d+ex)^{-2(p+1)}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{(p+1)(p+2)(p+3)(p+4)\left(cd^2-ae^2\right)^4}+\frac{(d+ex)^{-2}}{}$$

```
[Out] ((d + e*x)^(-5 - 2*p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 + p))/((c*
d^2 - a*e^2)*(4 + p)) + (6*c^2*d^2*(d + e*x)^(-3 - 2*p)*(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2)^(1 + p))/((c*d^2 - a*e^2)^3*(2 + p)*(3 + p)*(4 + p)) +
 (6*c^3*d^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 + p))/((c*d^2 - a*e^
2)^4*(1 + p)*(2 + p)*(3 + p)*(4 + p)*(d + e*x)^(2*(1 + p))) + (3*c*d*(a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 + p))/((c*d^2 - a*e^2)^2*(3 + p)*(4 +
p)*(d + e*x)^(2*(2 + p)))
```

Rubi [A] time = 0.164065, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {658, 650}

$$\frac{6c^2d^2(d+ex)^{-2p-3}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{(p+2)(p+3)(p+4)\left(cd^2-ae^2\right)^3}+\frac{6c^3d^3(d+ex)^{-2(p+1)}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{(p+1)(p+2)(p+3)(p+4)\left(cd^2-ae^2\right)^4}+\frac{(d+ex)^{-2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(-5 - 2*p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

```
[Out] ((d + e*x)^(-5 - 2*p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 + p))/((c*
d^2 - a*e^2)*(4 + p)) + (6*c^2*d^2*(d + e*x)^(-3 - 2*p)*(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2)^(1 + p))/((c*d^2 - a*e^2)^3*(2 + p)*(3 + p)*(4 + p)) +
 (6*c^3*d^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 + p))/((c*d^2 - a*e^
2)^4*(1 + p)*(2 + p)*(3 + p)*(4 + p)*(d + e*x)^(2*(1 + p))) + (3*c*d*(a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 + p))/((c*d^2 - a*e^2)^2*(3 + p)*(4 +
p)*(d + e*x)^(2*(2 + p)))
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int (d+ex)^{-5-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = \frac{(d+ex)^{-5-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1+p}}{\left(cd^2-ae^2\right)(4+p)} + \frac{(3cd)\int (d+ex)^{\cdots}}{\cdots}$$
$$= \frac{(d+ex)^{-5-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1+p}}{\left(cd^2-ae^2\right)(4+p)} + \frac{3cd(d+ex)^{-2(2+\cdots}}{\left(cd^{\cdots}\right.}$$
$$= \frac{(d+ex)^{-5-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1+p}}{\left(cd^2-ae^2\right)(4+p)} + \frac{6c^2d^2(d+ex)^{-3\cdots}}{\left(cd^2-\cdots\right.}$$
$$= \frac{(d+ex)^{-5-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1+p}}{\left(cd^2-ae^2\right)(4+p)} + \frac{6c^2d^2(d+ex)^{-3\cdots}}{\left(cd^2-\cdots\right.}$$

Mathematica [A] time = 0.149112, size = 217, normalized size = 0.75

$$\frac{(d+ex)^{-2p-5}((d+ex)(ae+cdx))^{p+1}\left(3a^2cde^4\left(p^2+3p+2\right)(d(p+4)+ex)-a^3e^6\left(p^3+6p^2+11p+6\right)-3ac^2d^2e^2(p+\cdots\right.}{(p+1)(p+2)\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(-5 - 2*p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

```
[Out] ((d + e*x)^(-5 - 2*p)*((a*e + c*d*x)*(d + e*x))^(1 + p)*(-(a^3*e^6*(6 + 11*
p + 6*p^2 + p^3)) + 3*a^2*c*d*e^4*(2 + 3*p + p^2)*(d*(4 + p) + e*x) - 3*a*c
^2*d^2*e^2*(1 + p)*(d^2*(12 + 7*p + p^2) + 2*d*e*(4 + p)*x + 2*e^2*x^2) + c
^3*d^3*(d^3*(24 + 26*p + 9*p^2 + p^3) + 3*d^2*e*(12 + 7*p + p^2)*x + 6*d*e^
2*(4 + p)*x^2 + 6*e^3*x^3)))/((c*d^2 - a*e^2)^4*(1 + p)*(2 + p)*(3 + p)*(4
+ p))
```

Maple [B] time = 0.048, size = 745, normalized size = 2.6

$$-\frac{(cdx+ae)(ex+d)^{-4-2p}\left(a^3e^6p^3-3\,a^2cd^2e^4p^3-3\,a^2cde^5p^2x+3\,ac^2d^4e^2p^3+6\,ac^2d^3e^3p^2x+6\,ac^2d^2e^4px^2-c^3d^6p^3-\cdots\right.}{a^4e^8p^4-4\,a^3cd^2e^6p^4+6\,a^2c^2d^4e^4p^4-4\,ac^3d^6e^2p^4+c^4d^8p^4+1\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(-5-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x)
```

```
[Out] -(c*d*x+a*e)*(e*x+d)^(-4-2*p)*(a^3*e^6*p^3-3*a^2*c*d^2*e^4*p^3-3*a^2*c*d*e^
5*p^2*x+3*a*c^2*d^4*e^2*p^3+6*a*c^2*d^3*e^3*p^2*x+6*a*c^2*d^2*e^4*p*x^2-c^3
*d^6*p^3-3*c^3*d^5*e*p^2*x-6*c^3*d^4*e^2*p*x^2-6*c^3*d^3*e^3*x^3+6*a^3*e^6*
p^2-21*a^2*c*d^2*e^4*p^2-9*a^2*c*d*e^5*p*x+24*a*c^2*d^4*e^2*p^2+30*a*c^2*d^
3*e^3*p*x+6*a*c^2*d^2*e^4*x^2-9*c^3*d^6*p^2-21*c^3*d^5*e*p*x-24*c^3*d^4*e^2
*x^2+11*a^3*e^6*p-42*a^2*c*d^2*e^4*p-6*a^2*c*d*e^5*x+57*a*c^2*d^4*e^2*p+24*
a*c^2*d^3*e^3*x-26*c^3*d^6*p-36*c^3*d^5*e*x+6*a^3*e^6-24*a^2*c*d^2*e^4+36*a
*c^2*d^4*e^2-24*c^3*d^6)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^p/(a^4*e^8*p^4-4
*a^3*c*d^2*e^6*p^4+6*a^2*c^2*d^4*e^4*p^4-4*a*c^3*d^6*e^2*p^4+c^4*d^8*p^4+10
*a^4*e^8*p^3-40*a^3*c*d^2*e^6*p^3+60*a^2*c^2*d^4*e^4*p^3-40*a*c^3*d^6*e^2*p
^3+10*c^4*d^8*p^3+35*a^4*e^8*p^2-140*a^3*c*d^2*e^6*p^2+210*a^2*c^2*d^4*e^4*
p^2-140*a*c^3*d^6*e^2*p^2+35*c^4*d^8*p^2+50*a^4*e^8*p-200*a^3*c*d^2*e^6*p+3
00*a^2*c^2*d^4*e^4*p-200*a*c^3*d^6*e^2*p+50*c^4*d^8*p+24*a^4*e^8-96*a^3*c*d
^2*e^6+144*a^2*c^2*d^4*e^4-96*a*c^3*d^6*e^2+24*c^4*d^8)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^p (ex + d)^{-2p-5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-5-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm
="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p*(e*x + d)^(-2*p - 5), x
)
```

Fricas [B] time = 2.48054, size = 2107, normalized size = 7.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-5-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm
="fricas")
```

```
[Out] (6*c^4*d^4*e^4*x^5 + 24*a*c^3*d^7*e - 36*a^2*c^2*d^5*e^3 + 24*a^3*c*d^3*e^5
 - 6*a^4*d*e^7 + 6*(5*c^4*d^5*e^3 + (c^4*d^5*e^3 - a*c^3*d^3*e^5)*p)*x^4 +
(a*c^3*d^7*e - 3*a^2*c^2*d^5*e^3 + 3*a^3*c*d^3*e^5 - a^4*d*e^7)*p^3 + 3*(20
*c^4*d^6*e^2 + (c^4*d^6*e^2 - 2*a*c^3*d^4*e^4 + a^2*c^2*d^2*e^6)*p^2 + (9*c
^4*d^6*e^2 - 10*a*c^3*d^4*e^4 + a^2*c^2*d^2*e^6)*p)*x^3 + 3*(3*a*c^3*d^7*e
- 8*a^2*c^2*d^5*e^3 + 7*a^3*c*d^3*e^5 - 2*a^4*d*e^7)*p^2 + (60*c^4*d^7*e +
(c^4*d^7*e - 3*a*c^3*d^5*e^3 + 3*a^2*c^2*d^3*e^5 - a^3*c*d*e^7)*p^3 + 3*(4*
c^4*d^7*e - 9*a*c^3*d^5*e^3 + 6*a^2*c^2*d^3*e^5 - a^3*c*d*e^7)*p^2 + (47*c^
4*d^7*e - 60*a*c^3*d^5*e^3 + 15*a^2*c^2*d^3*e^5 - 2*a^3*c*d*e^7)*p)*x^2 + (
26*a*c^3*d^7*e - 57*a^2*c^2*d^5*e^3 + 42*a^3*c*d^3*e^5 - 11*a^4*d*e^7)*p +
(24*c^4*d^8 + 24*a*c^3*d^6*e^2 - 36*a^2*c^2*d^4*e^4 + 24*a^3*c*d^2*e^6 - 6*
a^4*e^8 + (c^4*d^8 - 2*a*c^3*d^6*e^2 + 2*a^3*c*d^2*e^6 - a^4*e^8)*p^3 + 3*(
3*c^4*d^8 - 4*a*c^3*d^6*e^2 - 3*a^2*c^2*d^4*e^4 + 6*a^3*c*d^2*e^6 - 2*a^4*e
^8)*p^2 + (26*c^4*d^8 - 10*a*c^3*d^6*e^2 - 45*a^2*c^2*d^4*e^4 + 40*a^3*c*d^
2*e^6 - 11*a^4*e^8)*p)*x)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p*(e*x +
d)^(-2*p - 5)/(24*c^4*d^8 - 96*a*c^3*d^6*e^2 + 144*a^2*c^2*d^4*e^4 - 96*a^3
*c*d^2*e^6 + 24*a^4*e^8 + (c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 -
4*a^3*c*d^2*e^6 + a^4*e^8)*p^4 + 10*(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*
d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*p^3 + 35*(c^4*d^8 - 4*a*c^3*d^6*e^2 +
6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*p^2 + 50*(c^4*d^8 - 4*a*c^3*
d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*p)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(-5-2*p)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^p (ex + d)^{-2p-5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-5-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm
="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p*(e*x + d)^(-2*p - 5), x
)
```

3.2105 $\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$

Optimal. Leaf size=54

$$\frac{(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cd(1-m)}$$

```
[Out] ((d + e*x)^(-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*(
1 - m))
```

Rubi [A] time = 0.018182, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {648}

$$\frac{(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cd(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] ((d + e*x)^(-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*(
1 - m))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int (d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\frac{(d+ex)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cd(1-m)}$$

Mathematica [A] time = 0.0263788, size = 42, normalized size = 0.78

$$-\frac{(d+ex)^{m-1}((d+ex)(ae+cdx))^{1-m}}{cd(m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] -(((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m))/(c*d*(-1 + m)))
```

Maple [A] time = 0.04, size = 57, normalized size = 1.1

$$-\frac{(cdx+ae)\,(ex+d)^m}{cd\,(-1+m)\left(cdex^2+ae^2x+cd^2x+ade\right)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] -(c*d*x+a*e)/c/d/(-1+m)*(e*x+d)^m/((c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^m)
```

Maxima [A] time = 1.01008, size = 45, normalized size = 0.83

$$-\frac{cdx + ae}{(cdx + ae)^m cd(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="max
ima")
```

```
[Out] -(c*d*x + a*e)/((c*d*x + a*e)^m*c*d*(m - 1))
```

Fricas [A] time = 2.15617, size = 116, normalized size = 2.15

$$-\frac{(cdx + ae)(ex + d)^m}{(cdm - cd)\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="fri
cas")
```

```
[Out] -(c*d*x + a*e)*(e*x + d)^m/((c*d*m - c*d)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)^m)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14092, size = 117, normalized size = 2.17

$$-\frac{(xe + d)^m cdx e^{(-m\log(cdx+ae) - m\log(xe+d))} + (xe + d)^m a e^{(-m\log(cdx+ae) - m\log(xe+d) + 1)}}{cdm - cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="gia
c")
```

```
[Out] -((x*e + d)^m*c*d*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + (x*e + d)^m*
a*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1))/(c*d*m - c*d)
```

3.2106 $\int (d+ex)^{-p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx$

Optimal. Leaf size=52

$$\frac{(d+ex)^{-p-1} \left(x \left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{cd(p+1)}$$

```
[Out] ((d + e*x)^(-1 - p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 + p))/(c*d*(
1 + p))
```

Rubi [A] time = 0.0160988, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {648}

$$\frac{(d+ex)^{-p-1} \left(x \left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{cd(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p/(d + e*x)^p,x]
```

```
[Out] ((d + e*x)^(-1 - p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 + p))/(c*d*(
1 + p))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int (d+ex)^{-p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx = \frac{(d+ex)^{-1-p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1+p}}{cd(1+p)}$$

Mathematica [A] time = 0.0206808, size = 41, normalized size = 0.79

$$\frac{(d+ex)^{-p-1}((d+ex)(ae+cdx))^{p+1}}{cd(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p/(d + e*x)^p,x]
```

```
[Out] ((d + e*x)^(-1 - p)*((a*e + c*d*x)*(d + e*x))^(1 + p))/(c*d*(1 + p))
```

Maple [A] time = 0.043, size = 56, normalized size = 1.1

$$\frac{(cdx+ae)\left(cdex^2+ae^2x+cd^2x+ade\right)^p}{cd\left(1+p\right)(ex+d)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/((e*x+d)^p),x)
```

```
[Out] (c*d*x+a*e)/c/d/(1+p)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^p/((e*x+d)^p)
```

Maxima [A] time = 1.02852, size = 41, normalized size = 0.79

$$\frac{(cdx + ae)(cdx + ae)^p}{cd(p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/((e*x+d)^p),x, algorithm="max
ima")
```

```
[Out] (c*d*x + a*e)*(c*d*x + a*e)^p/(c*d*(p + 1))
```

Fricas [A] time = 2.13223, size = 115, normalized size = 2.21

$$\frac{(cdx + ae)\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^p}{\left(cdp + cd\right)(ex + d)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/((e*x+d)^p),x, algorithm="fri
cas")
```

```
[Out] (c*d*x + a*e)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p/((c*d*p + c*d)*(e*x
 + d)^p)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**p/((e*x+d)**p),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14888, size = 115, normalized size = 2.21

$$\frac{\frac{cdxe^{\left(p\log(cdx+ae)+p\log(xe+d)\right)}}{(xe+d)^p} + \frac{ae^{\left(p\log(cdx+ae)+p\log(xe+d)+1\right)}}{(xe+d)^p}}{cdp + cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/((e*x+d)^p),x, algorithm="gia
c")
```

```
[Out] (c*d*x*e^(p*log(c*d*x + a*e) + p*log(x*e + d))/(x*e + d)^p + a*e^(p*log(c*d
*x + a*e) + p*log(x*e + d) + 1)/(x*e + d)^p)/(c*d*p + c*d)
```

3.2107 $\int (d+ex)^4 \left(a+bx+cx^2\right) \, dx$

Optimal. Leaf size=69

$$\frac{(d+ex)^5\left(ae^2-bde+cd^2\right)}{5e^3}-\frac{(d+ex)^6(2cd-be)}{6e^3}+\frac{c(d+ex)^7}{7e^3}$$

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(d + e*x)^5)/(5*e^3) - ((2*c*d - b*e)*(d + e*x)^6)
/(6*e^3) + (c*(d + e*x)^7)/(7*e^3)
```

Rubi [A] time = 0.0982364, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {698}

$$\frac{(d+ex)^5\left(ae^2-bde+cd^2\right)}{5e^3}-\frac{(d+ex)^6(2cd-be)}{6e^3}+\frac{c(d+ex)^7}{7e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4*(a + b*x + c*x^2),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(d + e*x)^5)/(5*e^3) - ((2*c*d - b*e)*(d + e*x)^6)
/(6*e^3) + (c*(d + e*x)^7)/(7*e^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int (d+ex)^4\left(a+bx+cx^2\right)\,dx &= \int \left(\frac{\left(cd^2-bde+ae^2\right)(d+ex)^4}{e^2}+\frac{(-2cd+be)(d+ex)^5}{e^2}+\frac{c(d+ex)^6}{e^2}\right)dx\\ &= \frac{\left(cd^2-bde+ae^2\right)(d+ex)^5}{5e^3}-\frac{(2cd-be)(d+ex)^6}{6e^3}+\frac{c(d+ex)^7}{7e^3}\end{aligned}$$

Mathematica [A] time = 0.0388166, size = 135, normalized size = 1.96

$$\frac{1}{5}e^2x^5\left(ae^2+4bde+6cd^2\right)+\frac{1}{2}dex^4\left(2ae^2+3bde+2cd^2\right)+\frac{1}{3}d^2x^3\left(6ae^2+4bde+cd^2\right)+\frac{1}{2}d^3x^2(4ae+bd)+ad^4x+\frac{1}{6}e^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4*(a + b*x + c*x^2),x]
```

```
[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^2)/2 + (d^2*(c*d^2 + 4*b*d*e + 6*a*e^2)*x^3)
/3 + (d*e*(2*c*d^2 + 3*b*d*e + 2*a*e^2)*x^4)/2 + (e^2*(6*c*d^2 + 4*b*d*e +
a*e^2)*x^5)/5 + (e^3*(4*c*d + b*e)*x^6)/6 + (c*e^4*x^7)/7
```

Maple [B] time = 0.039, size = 136, normalized size = 2.

$$\frac{e^{4}c x^{7}}{7}+\frac{\left(e^{4}b+4\,d e^{3}c\right)x^{6}}{6}+\frac{\left(e^{4}a+4\,d e^{3}b+6\,d^{2}e^{2}c\right)x^{5}}{5}+\frac{\left(4\,a d e^{3}+6\,d^{2}e^{2}b+4\,c d^{3}e\right)x^{4}}{4}+\frac{\left(6\,a d^{2}e^{2}+4\,d^{3}e b+c d^{4}\right)x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(c*x^2+b*x+a),x)
```

```
[Out] 1/7*e^4*c*x^7+1/6*(b*e^4+4*c*d*e^3)*x^6+1/5*(a*e^4+4*b*d*e^3+6*c*d^2*e^2)*x
^5+1/4*(4*a*d*e^3+6*b*d^2*e^2+4*c*d^3*e)*x^4+1/3*(6*a*d^2*e^2+4*b*d^3*e+c*d
^4)*x^3+1/2*(4*a*d^3*e+b*d^4)*x^2+d^4*a*x
```

Maxima [B] time = 0.978365, size = 182, normalized size = 2.64

$$\frac{1}{7}\,c e^{4}x^{7}+\frac{1}{6}\left(4\,c d e^{3}+b e^{4}\right)x^{6}+a d^{4}x+\frac{1}{5}\left(6\,c d^{2}e^{2}+4\,b d e^{3}+a e^{4}\right)x^{5}+\frac{1}{2}\left(2\,c d^{3}e+3\,b d^{2}e^{2}+2\,a d e^{3}\right)x^{4}+\frac{1}{3}\left(c d^{4}+4\,b d^{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 1/7*c*e^4*x^7 + 1/6*(4*c*d*e^3 + b*e^4)*x^6 + a*d^4*x + 1/5*(6*c*d^2*e^2 +
4*b*d*e^3 + a*e^4)*x^5 + 1/2*(2*c*d^3*e + 3*b*d^2*e^2 + 2*a*d*e^3)*x^4 + 1/
3*(c*d^4 + 4*b*d^3*e + 6*a*d^2*e^2)*x^3 + 1/2*(b*d^4 + 4*a*d^3*e)*x^2
```

Fricas [B] time = 1.77728, size = 331, normalized size = 4.8

$$\frac{1}{7}x^{7}e^{4}c+\frac{2}{3}x^{6}e^{3}dc+\frac{1}{6}x^{6}e^{4}b+\frac{6}{5}x^{5}e^{2}d^{2}c+\frac{4}{5}x^{5}e^{3}db+\frac{1}{5}x^{5}e^{4}a+x^{4}ed^{3}c+\frac{3}{2}x^{4}e^{2}d^{2}b+x^{4}e^{3}da+\frac{1}{3}x^{3}d^{4}c+\frac{4}{3}x^{3}ed^{3}b+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*e^4*c + 2/3*x^6*e^3*d*c + 1/6*x^6*e^4*b + 6/5*x^5*e^2*d^2*c + 4/5*x
^5*e^3*d*b + 1/5*x^5*e^4*a + x^4*e*d^3*c + 3/2*x^4*e^2*d^2*b + x^4*e^3*d*a
+ 1/3*x^3*d^4*c + 4/3*x^3*e*d^3*b + 2*x^3*e^2*d^2*a + 1/2*x^2*d^4*b + 2*x^2
*e*d^3*a + x*d^4*a
```

Sympy [B] time = 0.090638, size = 146, normalized size = 2.12

$$ad^{4}x+\frac{ce^{4}x^{7}}{7}+x^{6}\left(\frac{be^{4}}{6}+\frac{2cde^{3}}{3}\right)+x^{5}\left(\frac{ae^{4}}{5}+\frac{4bde^{3}}{5}+\frac{6cd^{2}e^{2}}{5}\right)+x^{4}\left(ade^{3}+\frac{3bd^{2}e^{2}}{2}+cd^{3}e\right)+x^{3}\left(2ad^{2}e^{2}+\frac{4bd^{3}e}{3}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(c*x**2+b*x+a),x)
```

```
[Out] a*d**4*x + c*e**4*x**7/7 + x**6*(b*e**4/6 + 2*c*d*e**3/3) + x**5*(a*e**4/5
+ 4*b*d*e**3/5 + 6*c*d**2*e**2/5) + x**4*(a*d*e**3 + 3*b*d**2*e**2/2 + c*d*
```

```
*3*e) + x**3*(2*a*d**2*e**2 + 4*b*d**3*e/3 + c*d**4/3) + x**2*(2*a*d**3*e +
 b*d**4/2)
```

Giac [B] time = 1.10771, size = 189, normalized size = 2.74

$$\frac{1}{7}cx^7e^4 + \frac{2}{3}cdx^6e^3 + \frac{6}{5}cd^2x^5e^2 + cd^3x^4e + \frac{1}{3}cd^4x^3 + \frac{1}{6}bx^6e^4 + \frac{4}{5}bdx^5e^3 + \frac{3}{2}bd^2x^4e^2 + \frac{4}{3}bd^3x^3e + \frac{1}{2}bd^4x^2 + \frac{1}{5}ax^5e^4 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/7*c*x^7*e^4 + 2/3*c*d*x^6*e^3 + 6/5*c*d^2*x^5*e^2 + c*d^3*x^4*e + 1/3*c*d
^4*x^3 + 1/6*b*x^6*e^4 + 4/5*b*d*x^5*e^3 + 3/2*b*d^2*x^4*e^2 + 4/3*b*d^3*x^
3*e + 1/2*b*d^4*x^2 + 1/5*a*x^5*e^4 + a*d*x^4*e^3 + 2*a*d^2*x^3*e^2 + 2*a*d
^3*x^2*e + a*d^4*x
```

3.2108 $\int (d+ex)^3 \left(a+bx+cx^2\right) dx$

Optimal. Leaf size=69

$$\frac{(d+ex)^4 \left(ae^2-bde+cd^2\right)}{4e^3}-\frac{(d+ex)^5(2cd-be)}{5e^3}+\frac{c(d+ex)^6}{6e^3}$$

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(d + e*x)^4)/(4*e^3) - ((2*c*d - b*e)*(d + e*x)^5)
/(5*e^3) + (c*(d + e*x)^6)/(6*e^3)
```

Rubi [A] time = 0.0691618, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {698}

$$\frac{(d+ex)^4 \left(ae^2-bde+cd^2\right)}{4e^3}-\frac{(d+ex)^5(2cd-be)}{5e^3}+\frac{c(d+ex)^6}{6e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*x + c*x^2),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(d + e*x)^4)/(4*e^3) - ((2*c*d - b*e)*(d + e*x)^5)
/(5*e^3) + (c*(d + e*x)^6)/(6*e^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int (d+ex)^3 \left(a+bx+cx^2\right) dx &= \int \left(\frac{\left(cd^2-bde+ae^2\right)(d+ex)^3}{e^2}+\frac{(-2cd+be)(d+ex)^4}{e^2}+\frac{c(d+ex)^5}{e^2}\right) dx\\ &= \frac{\left(cd^2-bde+ae^2\right)(d+ex)^4}{4e^3}-\frac{(2cd-be)(d+ex)^5}{5e^3}+\frac{c(d+ex)^6}{6e^3}\end{aligned}$$

Mathematica [A] time = 0.0245141, size = 104, normalized size = 1.51

$$\frac{1}{4}ex^4\left(ae^2+3bde+3cd^2\right)+\frac{1}{3}dx^3\left(3ae^2+3bde+cd^2\right)+\frac{1}{2}d^2x^2(3ae+bd)+ad^3x+\frac{1}{5}e^2x^5(be+3cd)+\frac{1}{6}ce^3x^6$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*x + c*x^2),x]
```

```
[Out] a*d^3*x + (d^2*(b*d + 3*a*e)*x^2)/2 + (d*(c*d^2 + 3*b*d*e + 3*a*e^2)*x^3)/3
 + (e*(3*c*d^2 + 3*b*d*e + a*e^2)*x^4)/4 + (e^2*(3*c*d + b*e)*x^5)/5 + (c*e
^3*x^6)/6
```

Maple [A] time = 0.04, size = 103, normalized size = 1.5

$$\frac{e^{3}c\,x^{6}}{6}+\frac{\left(e^{3}b+3\,d\,e^{2}c\right)x^{5}}{5}+\frac{\left(a\,e^{3}+3\,d\,e^{2}b+3\,d^{2}e\,c\right)x^{4}}{4}+\frac{\left(3\,a\,d\,e^{2}+3\,d^{2}e\,b+c\,d^{3}\right)x^{3}}{3}+\frac{\left(3\,d^{2}e\,a+d^{3}b\right)x^{2}}{2}+d^{3}a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x+a),x)
```

```
[Out] 1/6*e^3*c*x^6+1/5*(b*e^3+3*c*d*e^2)*x^5+1/4*(a*e^3+3*b*d*e^2+3*c*d^2*e)*x^4
+1/3*(3*a*d*e^2+3*b*d^2*e+c*d^3)*x^3+1/2*(3*a*d^2*e+b*d^3)*x^2+d^3*a*x
```

Maxima [A] time = 0.975367, size = 138, normalized size = 2.

$$\frac{1}{6}\,c e^{3} x^{6}+\frac{1}{5}\left(3\,c d e^{2}+b e^{3}\right)x^{5}+a d^{3} x+\frac{1}{4}\left(3\,c d^{2} e+3\,b d e^{2}+a e^{3}\right)x^{4}+\frac{1}{3}\left(c d^{3}+3\,b d^{2} e+3\,a d e^{2}\right)x^{3}+\frac{1}{2}\left(b d^{3}+3\,a d^{2} e\right)x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 1/6*c*e^3*x^6 + 1/5*(3*c*d*e^2 + b*e^3)*x^5 + a*d^3*x + 1/4*(3*c*d^2*e + 3*
b*d*e^2 + a*e^3)*x^4 + 1/3*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*x^3 + 1/2*(b*d^3
 + 3*a*d^2*e)*x^2
```

Fricas [A] time = 1.75739, size = 255, normalized size = 3.7

$$\frac{1}{6}x^{6}e^{3}c+\frac{3}{5}x^{5}e^{2}dc+\frac{1}{5}x^{5}e^{3}b+\frac{3}{4}x^{4}ed^{2}c+\frac{3}{4}x^{4}e^{2}db+\frac{1}{4}x^{4}e^{3}a+\frac{1}{3}x^{3}d^{3}c+x^{3}ed^{2}b+x^{3}e^{2}da+\frac{1}{2}x^{2}d^{3}b+\frac{3}{2}x^{2}ed^{2}a+xd^{3}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*e^3*c + 3/5*x^5*e^2*d*c + 1/5*x^5*e^3*b + 3/4*x^4*e*d^2*c + 3/4*x^4
*e^2*d*b + 1/4*x^4*e^3*a + 1/3*x^3*d^3*c + x^3*e*d^2*b + x^3*e^2*d*a + 1/2*
x^2*d^3*b + 3/2*x^2*e*d^2*a + x*d^3*a
```

Sympy [A] time = 0.07866, size = 110, normalized size = 1.59

$$ad^{3}x+\frac{ce^{3}x^{6}}{6}+x^{5}\left(\frac{be^{3}}{5}+\frac{3cde^{2}}{5}\right)+x^{4}\left(\frac{ae^{3}}{4}+\frac{3bde^{2}}{4}+\frac{3cd^{2}e}{4}\right)+x^{3}\left(ade^{2}+bd^{2}e+\frac{cd^{3}}{3}\right)+x^{2}\left(\frac{3ad^{2}e}{2}+\frac{bd^{3}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x+a),x)
```

```
[Out] a*d**3*x + c*e**3*x**6/6 + x**5*(b*e**3/5 + 3*c*d*e**2/5) + x**4*(a*e**3/4
+ 3*b*d*e**2/4 + 3*c*d**2*e/4) + x**3*(a*d*e**2 + b*d**2*e + c*d**3/3) + x*
*2*(3*a*d**2*e/2 + b*d**3/2)
```

Giac [A] time = 1.12474, size = 144, normalized size = 2.09

$$\frac{1}{6}cx^6e^3+\frac{3}{5}cdx^5e^2+\frac{3}{4}cd^2x^4e+\frac{1}{3}cd^3x^3+\frac{1}{5}bx^5e^3+\frac{3}{4}bdx^4e^2+bd^2x^3e+\frac{1}{2}bd^3x^2+\frac{1}{4}ax^4e^3+adx^3e^2+\frac{3}{2}ad^2x^2e+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/6*c*x^6*e^3 + 3/5*c*d*x^5*e^2 + 3/4*c*d^2*x^4*e + 1/3*c*d^3*x^3 + 1/5*b*x
^5*e^3 + 3/4*b*d*x^4*e^2 + b*d^2*x^3*e + 1/2*b*d^3*x^2 + 1/4*a*x^4*e^3 + a*
d*x^3*e^2 + 3/2*a*d^2*x^2*e + a*d^3*x
```

3.2109 $\int (d+ex)^2 \left(a+bx+cx^2\right) dx$

Optimal. Leaf size=69

$$\frac{(d+ex)^3\left(ae^2-bde+cd^2\right)}{3e^3}-\frac{(d+ex)^4(2cd-be)}{4e^3}+\frac{c(d+ex)^5}{5e^3}$$

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(d + e*x)^3)/(3*e^3) - ((2*c*d - b*e)*(d + e*x)^4)
/(4*e^3) + (c*(d + e*x)^5)/(5*e^3)
```

Rubi [A] time = 0.0538831, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {698}

$$\frac{(d+ex)^3\left(ae^2-bde+cd^2\right)}{3e^3}-\frac{(d+ex)^4(2cd-be)}{4e^3}+\frac{c(d+ex)^5}{5e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*x + c*x^2),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(d + e*x)^3)/(3*e^3) - ((2*c*d - b*e)*(d + e*x)^4)
/(4*e^3) + (c*(d + e*x)^5)/(5*e^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int (d+ex)^2\left(a+bx+cx^2\right) dx &= \int \left(\frac{\left(cd^2-bde+ae^2\right)(d+ex)^2}{e^2}+\frac{(-2cd+be)(d+ex)^3}{e^2}+\frac{c(d+ex)^4}{e^2}\right) dx\\ &= \frac{\left(cd^2-bde+ae^2\right)(d+ex)^3}{3e^3}-\frac{(2cd-be)(d+ex)^4}{4e^3}+\frac{c(d+ex)^5}{5e^3}\end{aligned}$$

Mathematica [A] time = 0.020477, size = 73, normalized size = 1.06

$$\frac{1}{3}x^3\left(ae^2+2bde+cd^2\right)+\frac{1}{2}dx^2(2ae+bd)+ad^2x+\frac{1}{4}ex^4(be+2cd)+\frac{1}{5}ce^2x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*x + c*x^2),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^2)/2 + ((c*d^2 + 2*b*d*e + a*e^2)*x^3)/3 + (e*
(2*c*d + b*e)*x^4)/4 + (c*e^2*x^5)/5
```

Maple [A] time = 0.041, size = 70, normalized size = 1.

$$\frac{ce^2x^5}{5}+\frac{(e^2b+2\,dec)\,x^4}{4}+\frac{(ae^2+2\,bde+cd^2)\,x^3}{3}+\frac{(2\,ade+d^2b)\,x^2}{2}+ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+b*x+a),x)
```

```
[Out] 1/5*c*e^2*x^5+1/4*(b*e^2+2*c*d*e)*x^4+1/3*(a*e^2+2*b*d*e+c*d^2)*x^3+1/2*(2*
a*d*e+b*d^2)*x^2+a*d^2*x
```

Maxima [A] time = 0.99918, size = 93, normalized size = 1.35

$$\frac{1}{5}\,ce^2x^5+\frac{1}{4}\left(2\,cde+be^2\right)x^4+ad^2x+\frac{1}{3}\left(cd^2+2\,bde+ae^2\right)x^3+\frac{1}{2}\left(bd^2+2\,ade\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 1/5*c*e^2*x^5 + 1/4*(2*c*d*e + b*e^2)*x^4 + a*d^2*x + 1/3*(c*d^2 + 2*b*d*e
+ a*e^2)*x^3 + 1/2*(b*d^2 + 2*a*d*e)*x^2
```

Fricas [A] time = 1.77474, size = 180, normalized size = 2.61

$$\frac{1}{5}x^5e^2c+\frac{1}{2}x^4edc+\frac{1}{4}x^4e^2b+\frac{1}{3}x^3d^2c+\frac{2}{3}x^3edb+\frac{1}{3}x^3e^2a+\frac{1}{2}x^2d^2b+x^2eda+xd^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/5*x^5*e^2*c + 1/2*x^4*e*d*c + 1/4*x^4*e^2*b + 1/3*x^3*d^2*c + 2/3*x^3*e*d
*b + 1/3*x^3*e^2*a + 1/2*x^2*d^2*b + x^2*e*d*a + x*d^2*a
```

Sympy [A] time = 0.072641, size = 73, normalized size = 1.06

$$ad^2x+\frac{ce^2x^5}{5}+x^4\left(\frac{be^2}{4}+\frac{cde}{2}\right)+x^3\left(\frac{ae^2}{3}+\frac{2bde}{3}+\frac{cd^2}{3}\right)+x^2\left(ade+\frac{bd^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+b*x+a),x)
```

```
[Out] a*d**2*x + c*e**2*x**5/5 + x**4*(b*e**2/4 + c*d*e/2) + x**3*(a*e**2/3 + 2*b
*d*e/3 + c*d**2/3) + x**2*(a*d*e + b*d**2/2)
```

Giac [A] time = 1.10094, size = 101, normalized size = 1.46

$$\frac{1}{5}\,cx^5e^2+\frac{1}{2}\,cdx^4e+\frac{1}{3}\,cd^2x^3+\frac{1}{4}\,bx^4e^2+\frac{2}{3}\,bdx^3e+\frac{1}{2}\,bd^2x^2+\frac{1}{3}\,ax^3e^2+adx^2e+ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/5*c*x^5*e^2 + 1/2*c*d*x^4*e + 1/3*c*d^2*x^3 + 1/4*b*x^4*e^2 + 2/3*b*d*x^3
*e + 1/2*b*d^2*x^2 + 1/3*a*x^3*e^2 + a*d*x^2*e + a*d^2*x
```

3.2110 $\int (d+ex)\left(a+bx+cx^2\right)\,dx$

Optimal. Leaf size=42

$$\frac{1}{2}x^2(ae+bd)+adx+\frac{1}{3}x^3(be+cd)+\frac{1}{4}cex^4$$

```
[Out] a*d*x + ((b*d + a*e)*x^2)/2 + ((c*d + b*e)*x^3)/3 + (c*e*x^4)/4
```

Rubi [A] time = 0.028922, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {631}

$$\frac{1}{2}x^2(ae+bd)+adx+\frac{1}{3}x^3(be+cd)+\frac{1}{4}cex^4$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*x + c*x^2),x]
```

```
[Out] a*d*x + ((b*d + a*e)*x^2)/2 + ((c*d + b*e)*x^3)/3 + (c*e*x^4)/4
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(a+bx+cx^2\right)\,dx &= \int \left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right)\,dx\\ &= adx+\frac{1}{2}(bd+ae)x^2+\frac{1}{3}(cd+be)x^3+\frac{1}{4}cex^4\end{aligned}$$

Mathematica [A] time = 0.0129499, size = 42, normalized size = 1.

$$\frac{1}{2}x^2(ae+bd)+adx+\frac{1}{3}x^3(be+cd)+\frac{1}{4}cex^4$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*x + c*x^2),x]
```

```
[Out] a*d*x + ((b*d + a*e)*x^2)/2 + ((c*d + b*e)*x^3)/3 + (c*e*x^4)/4
```

Maple [A] time = 0.04, size = 37, normalized size = 0.9

$$adx+\frac{(ae+bd)\,x^2}{2}+\frac{(be+cd)\,x^3}{3}+\frac{cex^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+b*x+a),x)
```

```
[Out] a*d*x+1/2*(a*e+b*d)*x^2+1/3*(b*e+c*d)*x^3+1/4*c*e*x^4
```

Maxima [A] time = 0.967219, size = 49, normalized size = 1.17

$$\frac{1}{4}\,cex^4 + \frac{1}{3}\,(cd + be)x^3 + adx + \frac{1}{2}\,(bd + ae)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*c*e*x^4 + 1/3*(c*d + b*e)*x^3 + a*d*x + 1/2*(b*d + a*e)*x^2
```

Fricas [A] time = 1.81696, size = 104, normalized size = 2.48

$$\frac{1}{4}x^4ec + \frac{1}{3}x^3dc + \frac{1}{3}x^3eb + \frac{1}{2}x^2db + \frac{1}{2}x^2ea + xda$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*e*c + 1/3*x^3*d*c + 1/3*x^3*e*b + 1/2*x^2*d*b + 1/2*x^2*e*a + x*d*a
```

Sympy [A] time = 0.063283, size = 39, normalized size = 0.93

$$adx + \frac{cex^4}{4} + x^3\left(\frac{be}{3} + \frac{cd}{3}\right) + x^2\left(\frac{ae}{2} + \frac{bd}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+b*x+a),x)
```

```
[Out] a*d*x + c*e*x**4/4 + x**3*(b*e/3 + c*d/3) + x**2*(a*e/2 + b*d/2)
```

Giac [A] time = 1.08335, size = 58, normalized size = 1.38

$$\frac{1}{4}\,cx^4e + \frac{1}{3}\,cdx^3 + \frac{1}{3}\,bx^3e + \frac{1}{2}\,bdx^2 + \frac{1}{2}\,ax^2e + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*c*x^4*e + 1/3*c*d*x^3 + 1/3*b*x^3*e + 1/2*b*d*x^2 + 1/2*a*x^2*e + a*d*x
```

3.2111 $\int (a+bx+cx^2)\,dx$

Optimal. Leaf size=20

$$ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

[Out] a*x + (b*x^2)/2 + (c*x^3)/3

Rubi [A] time = 0.0039018, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Int[a + b*x + c*x^2,x]

[Out] a*x + (b*x^2)/2 + (c*x^3)/3

Rubi steps

$$\int (a+bx+cx^2)\,dx=ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Mathematica [A] time = 0.0000391, size = 20, normalized size = 1.

$$ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[a + b*x + c*x^2,x]

[Out] a*x + (b*x^2)/2 + (c*x^3)/3

Maple [A] time = 0.039, size = 17, normalized size = 0.9

$$ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(c*x^2+b*x+a,x)

[Out] a*x+1/2*b*x^2+1/3*c*x^3

Maxima [A] time = 1.01306, size = 22, normalized size = 1.1

$$\frac{1}{3}\,cx^3 + \frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2+b*x+a,x, algorithm="maxima")
```

```
[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Fricas [A] time = 1.71683, size = 39, normalized size = 1.95

$$\frac{1}{3}x^3c + \frac{1}{2}x^2b + xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2+b*x+a,x, algorithm="fricas")
```

```
[Out] 1/3*x^3*c + 1/2*x^2*b + x*a
```

Sympy [A] time = 0.054928, size = 15, normalized size = 0.75

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x**2+b*x+a,x)
```

```
[Out] a*x + b*x**2/2 + c*x**3/3
```

Giac [A] time = 1.13015, size = 22, normalized size = 1.1

$$\frac{1}{3}\,cx^3 + \frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2+b*x+a,x, algorithm="giac")
```

```
[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

3.2112 $\int \frac{a+bx+cx^2}{d+ex}\,dx$

Optimal. Leaf size=52

$$\frac{\log(d+ex)\left(ae^2-bde+cd^2\right)}{e^3}-\frac{x(cd-be)}{e^2}+\frac{cx^2}{2e}$$

```
[Out] -(((c*d - b*e)*x)/e^2) + (c*x^2)/(2*e) + ((c*d^2 - b*d*e + a*e^2)*Log[d + e
*x])/e^3
```

Rubi [A] time = 0.043259, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {698}

$$\frac{\log(d+ex)\left(ae^2-bde+cd^2\right)}{e^3}-\frac{x(cd-be)}{e^2}+\frac{cx^2}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(d + e*x),x]
```

```
[Out] -(((c*d - b*e)*x)/e^2) + (c*x^2)/(2*e) + ((c*d^2 - b*d*e + a*e^2)*Log[d + e
*x])/e^3
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{d+ex}\,dx &= \int \left(\frac{-cd+be}{e^2}+\frac{cx}{e}+\frac{cd^2-bde+ae^2}{e^2(d+ex)}\right)dx\\ &= -\frac{(cd-be)x}{e^2}+\frac{cx^2}{2e}+\frac{\left(cd^2-bde+ae^2\right)\log(d+ex)}{e^3}\end{aligned}$$

Mathematica [A] time = 0.0163879, size = 48, normalized size = 0.92

$$\frac{2\log(d+ex)\left(e(ae-bd)+cd^2\right)+ex(2be-2cd+cex)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(d + e*x),x]
```

```
[Out] (e*x*(-2*c*d + 2*b*e + c*e*x) + 2*(c*d^2 + e*(-(b*d) + a*e))*Log[d + e*x])/
(2*e^3)
```

Maple [A] time = 0.042, size = 63, normalized size = 1.2

$$\frac{cx^2}{2e}+\frac{bx}{e}-\frac{cdx}{e^2}+\frac{\ln(ex+d)\,a}{e}-\frac{\ln(ex+d)\,bd}{e^2}+\frac{\ln(ex+d)\,cd^2}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d),x)
```

```
[Out] 1/2*c*x^2/e+1/e*x*b-c*d*x/e^2+1/e*ln(e*x+d)*a-1/e^2*ln(e*x+d)*b*d+1/e^3*ln(
e*x+d)*c*d^2
```

Maxima [A] time = 1.01299, size = 68, normalized size = 1.31

$$\frac{cex^2-2(cd-be)x}{2e^2}+\frac{(cd^2-bde+ae^2)\log(ex+d)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*(c*e*x^2 - 2*(c*d - b*e)*x)/e^2 + (c*d^2 - b*d*e + a*e^2)*log(e*x + d)/
e^3
```

Fricas [A] time = 2.02152, size = 113, normalized size = 2.17

$$\frac{ce^2x^2-2(cde-be^2)x+2(cd^2-bde+ae^2)\log(ex+d)}{2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/2*(c*e^2*x^2 - 2*(c*d*e - b*e^2)*x + 2*(c*d^2 - b*d*e + a*e^2)*log(e*x +
d))/e^3
```

Sympy [A] time = 0.36413, size = 44, normalized size = 0.85

$$\frac{cx^2}{2e}+\frac{x(be-cd)}{e^2}+\frac{(ae^2-bde+cd^2)\log(d+ex)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d),x)
```

```
[Out] c*x**2/(2*e) + x*(b*e - c*d)/e**2 + (a*e**2 - b*d*e + c*d**2)*log(d + e*x)/
e**3
```

Giac [A] time = 1.08984, size = 69, normalized size = 1.33

$$\left(cd^2 - bde + ae^2\right)e^{(-3)}\log\left(|xe + d|\right) + \frac{1}{2}\left(cx^2e - 2\,cdx + 2\,bxe\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d),x, algorithm="giac")
```

```
[Out] (c*d^2 - b*d*e + a*e^2)*e^(-3)*log(abs(x*e + d)) + 1/2*(c*x^2*e - 2*c*d*x +
 2*b*x*e)*e^(-2)
```

3.2113 $\int \frac{a+bx+cx^2}{(d+ex)^2}\,dx$

Optimal. Leaf size=55

$$-\frac{ae^2-bde+cd^2}{e^3(d+ex)}-\frac{(2cd-be)\log(d+ex)}{e^3}+\frac{cx}{e^2}$$

```
[Out] (c*x)/e^2 - (c*d^2 - b*d*e + a*e^2)/(e^3*(d + e*x)) - ((2*c*d - b*e)*Log[d
+ e*x])/e^3
```

Rubi [A] time = 0.0454211, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {698}

$$-\frac{ae^2-bde+cd^2}{e^3(d+ex)}-\frac{(2cd-be)\log(d+ex)}{e^3}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(d + e*x)^2,x]
```

```
[Out] (c*x)/e^2 - (c*d^2 - b*d*e + a*e^2)/(e^3*(d + e*x)) - ((2*c*d - b*e)*Log[d
+ e*x])/e^3
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{(d+ex)^2}\,dx &= \int \left(\frac{c}{e^2}+\frac{cd^2-bde+ae^2}{e^2(d+ex)^2}+\frac{-2cd+be}{e^2(d+ex)}\right)dx\\ &= \frac{cx}{e^2}-\frac{cd^2-bde+ae^2}{e^3(d+ex)}-\frac{(2cd-be)\log(d+ex)}{e^3}\end{aligned}$$

Mathematica [A] time = 0.0273447, size = 49, normalized size = 0.89

$$\frac{-\frac{ae^2-bde+cd^2}{d+ex}+(be-2cd)\log(d+ex)+cex}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(d + e*x)^2,x]
```

```
[Out] (c*e*x - (c*d^2 - b*d*e + a*e^2)/(d + e*x) + (-2*c*d + b*e)*Log[d + e*x])/e
^3
```

Maple [A] time = 0.049, size = 74, normalized size = 1.4

$$\frac{cx}{e^2}+\frac{\ln(ex+d)\,b}{e^2}-2\frac{cd\ln(ex+d)}{e^3}-\frac{a}{e(ex+d)}+\frac{bd}{e^2(ex+d)}-\frac{cd^2}{e^3(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^2,x)
```

```
[Out] c*x/e^2+1/e^2*ln(e*x+d)*b-2*c*d*ln(e*x+d)/e^3-1/e/(e*x+d)*a+1/e^2/(e*x+d)*b
*d-1/e^3/(e*x+d)*c*d^2
```

Maxima [A] time = 0.994857, size = 78, normalized size = 1.42

$$-\frac{cd^2-bde+ae^2}{e^4x+de^3}+\frac{cx}{e^2}-\frac{(2\,cd-be)\log(ex+d)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(c*d^2 - b*d*e + a*e^2)/(e^4*x + d*e^3) + c*x/e^2 - (2*c*d - b*e)*log(e*x
+ d)/e^3
```

Fricas [A] time = 1.96706, size = 159, normalized size = 2.89

$$\frac{ce^2x^2+cdex-cd^2+bde-ae^2-\left(2\,cd^2-bde+\left(2\,cde-be^2\right)x\right)\log(ex+d)}{e^4x+de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] (c*e^2*x^2 + c*d*e*x - c*d^2 + b*d*e - a*e^2 - (2*c*d^2 - b*d*e + (2*c*d*e
- b*e^2)*x)*log(e*x + d))/(e^4*x + d*e^3)
```

Sympy [A] time = 0.509098, size = 49, normalized size = 0.89

$$\frac{cx}{e^2}-\frac{ae^2-bde+cd^2}{de^3+e^4x}+\frac{(be-2cd)\log(d+ex)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**2,x)
```

```
[Out] c*x/e**2 - (a*e**2 - b*d*e + c*d**2)/(d*e**3 + e**4*x) + (b*e - 2*c*d)*log(
d + e*x)/e**3
```

Giac [A] time = 1.10336, size = 143, normalized size = 2.6

$$-\left(e^{(-1)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^2}\right)-\frac{de^{(-1)}}{xe+d}\right)be^{(-1)}+\left(2\,de^{(-3)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^2}\right)+(xe+d)e^{(-3)}-\frac{d^2e^{(-3)}}{xe+d}\right)c-\frac{ae^{(-1)}}{xe+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -(e^(-1)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2) - d*e^(-1)/(x*e + d))*b*e^(-1
) + (2*d*e^(-3)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2) + (x*e + d)*e^(-3) - d
^2*e^(-3)/(x*e + d))*c - a*e^(-1)/(x*e + d)
```

3.2114 $\int \frac{a+bx+cx^2}{(d+ex)^3}\,dx$

Optimal. Leaf size=62

$$-\frac{ae^2-bde+cd^2}{2e^3(d+ex)^2}+\frac{2cd-be}{e^3(d+ex)}+\frac{c\log(d+ex)}{e^3}$$

```
[Out] -(c*d^2 - b*d*e + a*e^2)/(2*e^3*(d + e*x)^2) + (2*c*d - b*e)/(e^3*(d + e*x)
) + (c*Log[d + e*x])/e^3
```

Rubi [A] time = 0.045073, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {698}

$$-\frac{ae^2-bde+cd^2}{2e^3(d+ex)^2}+\frac{2cd-be}{e^3(d+ex)}+\frac{c\log(d+ex)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(d + e*x)^3,x]
```

```
[Out] -(c*d^2 - b*d*e + a*e^2)/(2*e^3*(d + e*x)^2) + (2*c*d - b*e)/(e^3*(d + e*x)
) + (c*Log[d + e*x])/e^3
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{(d+ex)^3}\,dx &= \int \left(\frac{cd^2-bde+ae^2}{e^2(d+ex)^3}+\frac{-2cd+be}{e^2(d+ex)^2}+\frac{c}{e^2(d+ex)}\right)dx\\ &= -\frac{cd^2-bde+ae^2}{2e^3(d+ex)^2}+\frac{2cd-be}{e^3(d+ex)}+\frac{c\log(d+ex)}{e^3}\end{aligned}$$

Mathematica [A] time = 0.020726, size = 57, normalized size = 0.92

$$\frac{-e(ae+bd+2bex)+cd(3d+4ex)+2c(d+ex)^2\log(d+ex)}{2e^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(d + e*x)^3,x]
```

```
[Out] (c*d*(3*d + 4*e*x) - e*(b*d + a*e + 2*b*e*x) + 2*c*(d + e*x)^2*Log[d + e*x]
)/(2*e^3*(d + e*x)^2)
```

Maple [A] time = 0.045, size = 83, normalized size = 1.3

$$-\frac{a}{2e(ex+d)^2}+\frac{bd}{2e^2(ex+d)^2}-\frac{cd^2}{2e^3(ex+d)^2}+\frac{c\ln(ex+d)}{e^3}-\frac{b}{e^2(ex+d)}+2\frac{cd}{e^3(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^3,x)
```

```
[Out] -1/2/e/(e*x+d)^2*a+1/2/e^2/(e*x+d)^2*b*d-1/2/e^3/(e*x+d)^2*c*d^2+c*ln(e*x+d
)/e^3-1/e^2/(e*x+d)*b+2*c*d/e^3/(e*x+d)
```

Maxima [A] time = 0.983436, size = 96, normalized size = 1.55

$$\frac{3cd^2-bde-ae^2+2(2cde-be^2)x}{2(e^5x^2+2de^4x+d^2e^3)}+\frac{c\log(ex+d)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(3*c*d^2 - b*d*e - a*e^2 + 2*(2*c*d*e - b*e^2)*x)/(e^5*x^2 + 2*d*e^4*x
+ d^2*e^3) + c*log(e*x + d)/e^3
```

Fricas [A] time = 1.98594, size = 184, normalized size = 2.97

$$\frac{3cd^2-bde-ae^2+2(2cde-be^2)x+2(ce^2x^2+2cdex+cd^2)\log(ex+d)}{2(e^5x^2+2de^4x+d^2e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*c*d^2 - b*d*e - a*e^2 + 2*(2*c*d*e - b*e^2)*x + 2*(c*e^2*x^2 + 2*c*d
*e*x + c*d^2)*log(e*x + d))/(e^5*x^2 + 2*d*e^4*x + d^2*e^3)
```

Sympy [A] time = 0.69464, size = 68, normalized size = 1.1

$$\frac{c\log(d+ex)}{e^3}-\frac{ae^2+bde-3cd^2+x(2be^2-4cde)}{2d^2e^3+4de^4x+2e^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**3,x)
```

```
[Out] c*log(d + e*x)/e**3 - (a*e**2 + b*d*e - 3*c*d**2 + x*(2*b*e**2 - 4*c*d*e))/
(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2)
```

Giac [A] time = 1.15949, size = 81, normalized size = 1.31

$$ce^{(-3)}\log\left(|xe+d|\right)+\frac{\left(2\,(2\,cd-be)x+\left(3\,cd^2-bde-ae^2\right)e^{(-1)}\right)e^{(-2)}}{2\,(xe+d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] c*e^(-3)*log(abs(x*e + d)) + 1/2*(2*(2*c*d - b*e)*x + (3*c*d^2 - b*d*e - a*
e^2)*e^(-1))*e^(-2)/(x*e + d)^2
```

3.2115 $\int \frac{a+bx+cx^2}{(d+ex)^4}\,dx$

Optimal. Leaf size=67

$$-\frac{ae^2-bde+cd^2}{3e^3(d+ex)^3}+\frac{2cd-be}{2e^3(d+ex)^2}-\frac{c}{e^3(d+ex)}$$

```
[Out] -(c*d^2 - b*d*e + a*e^2)/(3*e^3*(d + e*x)^3) + (2*c*d - b*e)/(2*e^3*(d + e*
x)^2) - c/(e^3*(d + e*x))
```

Rubi [A] time = 0.0443593, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {698}

$$-\frac{ae^2-bde+cd^2}{3e^3(d+ex)^3}+\frac{2cd-be}{2e^3(d+ex)^2}-\frac{c}{e^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(d + e*x)^4,x]
```

```
[Out] -(c*d^2 - b*d*e + a*e^2)/(3*e^3*(d + e*x)^3) + (2*c*d - b*e)/(2*e^3*(d + e*
x)^2) - c/(e^3*(d + e*x))
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{(d+ex)^4}\,dx &= \int \left(\frac{cd^2-bde+ae^2}{e^2(d+ex)^4}+\frac{-2cd+be}{e^2(d+ex)^3}+\frac{c}{e^2(d+ex)^2}\right)dx\\ &= -\frac{cd^2-bde+ae^2}{3e^3(d+ex)^3}+\frac{2cd-be}{2e^3(d+ex)^2}-\frac{c}{e^3(d+ex)}\end{aligned}$$

Mathematica [A] time = 0.0194428, size = 50, normalized size = 0.75

$$-\frac{e(2ae+b(d+3ex))+2c\left(d^2+3dex+3e^2x^2\right)}{6e^3(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(d + e*x)^4,x]
```

```
[Out] -(2*c*(d^2 + 3*d*e*x + 3*e^2*x^2) + e*(2*a*e + b*(d + 3*e*x)))/(6*e^3*(d +
e*x)^3)
```

Maple [A] time = 0.045, size = 63, normalized size = 0.9

$$-\frac{be-2\,cd}{2\,e^3\,(ex+d)^2}-\frac{c}{e^3\,(ex+d)}-\frac{ae^2-bde+cd^2}{3\,e^3\,(ex+d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^4,x)
```

```
[Out] -1/2*(b*e-2*c*d)/e^3/(e*x+d)^2-c/e^3/(e*x+d)-1/3*(a*e^2-b*d*e+c*d^2)/e^3/(e
*x+d)^3
```

Maxima [A] time = 0.992108, size = 104, normalized size = 1.55

$$-\frac{6\,ce^2x^2+2\,cd^2+bde+2\,ae^2+3\,\left(2\,cde+be^2\right)x}{6\,\left(e^6x^3+3\,de^5x^2+3\,d^2e^4x+d^3e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(6*c*e^2*x^2 + 2*c*d^2 + b*d*e + 2*a*e^2 + 3*(2*c*d*e + b*e^2)*x)/(e^6
*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3)
```

Fricas [A] time = 2.029, size = 162, normalized size = 2.42

$$-\frac{6\,ce^2x^2+2\,cd^2+bde+2\,ae^2+3\,\left(2\,cde+be^2\right)x}{6\,\left(e^6x^3+3\,de^5x^2+3\,d^2e^4x+d^3e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(6*c*e^2*x^2 + 2*c*d^2 + b*d*e + 2*a*e^2 + 3*(2*c*d*e + b*e^2)*x)/(e^6
*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3)
```

Sympy [A] time = 1.02867, size = 82, normalized size = 1.22

$$-\frac{2ae^2+bde+2cd^2+6ce^2x^2+x\left(3be^2+6cde\right)}{6d^3e^3+18d^2e^4x+18de^5x^2+6e^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**4,x)
```

```
[Out] -(2*a*e**2 + b*d*e + 2*c*d**2 + 6*c*e**2*x**2 + x*(3*b*e**2 + 6*c*d*e))/(6*
d**3*e**3 + 18*d**2*e**4*x + 18*d*e**5*x**2 + 6*e**6*x**3)
```

Giac [A] time = 1.13463, size = 68, normalized size = 1.01

$$-\frac{\left(6\,cx^2e^2+6\,cdxe+2\,cd^2+3\,bxe^2+bde+2\,ae^2\right)e^{(-3)}}{6\,(xe+d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -1/6*(6*c*x^2*e^2 + 6*c*d*x*e + 2*c*d^2 + 3*b*x*e^2 + b*d*e + 2*a*e^2)*e^(-
3)/(x*e + d)^3
```

3.2116 $\int \frac{a+bx+cx^2}{(d+ex)^5} dx$

Optimal. Leaf size=69

$$-\frac{ae^2-bde+cd^2}{4e^3(d+ex)^4}+\frac{2cd-be}{3e^3(d+ex)^3}-\frac{c}{2e^3(d+ex)^2}$$

```
[Out] -(c*d^2 - b*d*e + a*e^2)/(4*e^3*(d + e*x)^4) + (2*c*d - b*e)/(3*e^3*(d + e*
x)^3) - c/(2*e^3*(d + e*x)^2)
```

Rubi [A] time = 0.0464884, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {698}

$$-\frac{ae^2-bde+cd^2}{4e^3(d+ex)^4}+\frac{2cd-be}{3e^3(d+ex)^3}-\frac{c}{2e^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(d + e*x)^5,x]
```

```
[Out] -(c*d^2 - b*d*e + a*e^2)/(4*e^3*(d + e*x)^4) + (2*c*d - b*e)/(3*e^3*(d + e*
x)^3) - c/(2*e^3*(d + e*x)^2)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{(d+ex)^5}\,dx &= \int \left(\frac{cd^2-bde+ae^2}{e^2(d+ex)^5}+\frac{-2cd+be}{e^2(d+ex)^4}+\frac{c}{e^2(d+ex)^3}\right)dx\\ &= -\frac{cd^2-bde+ae^2}{4e^3(d+ex)^4}+\frac{2cd-be}{3e^3(d+ex)^3}-\frac{c}{2e^3(d+ex)^2}\end{aligned}$$

Mathematica [A] time = 0.0182494, size = 49, normalized size = 0.71

$$-\frac{e(3ae+b(d+4ex))+c\left(d^2+4dex+6e^2x^2\right)}{12e^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(d + e*x)^5,x]
```

```
[Out] -(c*(d^2 + 4*d*e*x + 6*e^2*x^2) + e*(3*a*e + b*(d + 4*e*x)))/(12*e^3*(d + e
*x)^4)
```

Maple [A] time = 0.045, size = 63, normalized size = 0.9

$$-\frac{ae^2-bde+cd^2}{4\,e^3\,(ex+d)^4}-\frac{c}{2\,e^3\,(ex+d)^2}-\frac{be-2\,cd}{3\,e^3\,(ex+d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^5,x)
```

```
[Out] -1/4*(a*e^2-b*d*e+c*d^2)/e^3/(e*x+d)^4-1/2*c/e^3/(e*x+d)^2-1/3*(b*e-2*c*d)/
e^3/(e*x+d)^3
```

Maxima [A] time = 1.00307, size = 116, normalized size = 1.68

$$-\frac{6\,ce^2x^2+cd^2+bde+3\,ae^2+4\left(cde+be^2\right)x}{12\left(e^7x^4+4\,de^6x^3+6\,d^2e^5x^2+4\,d^3e^4x+d^4e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(6*c*e^2*x^2 + c*d^2 + b*d*e + 3*a*e^2 + 4*(c*d*e + b*e^2)*x)/(e^7*x^
4 + 4*d*e^6*x^3 + 6*d^2*e^5*x^2 + 4*d^3*e^4*x + d^4*e^3)
```

Fricas [A] time = 1.899, size = 180, normalized size = 2.61

$$-\frac{6\,ce^2x^2+cd^2+bde+3\,ae^2+4\left(cde+be^2\right)x}{12\left(e^7x^4+4\,de^6x^3+6\,d^2e^5x^2+4\,d^3e^4x+d^4e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^5,x, algorithm="fricas")
```

```
[Out] -1/12*(6*c*e^2*x^2 + c*d^2 + b*d*e + 3*a*e^2 + 4*(c*d*e + b*e^2)*x)/(e^7*x^
4 + 4*d*e^6*x^3 + 6*d^2*e^5*x^2 + 4*d^3*e^4*x + d^4*e^3)
```

Sympy [A] time = 1.49977, size = 92, normalized size = 1.33

$$-\frac{3ae^2+bde+cd^2+6ce^2x^2+x\left(4be^2+4cde\right)}{12d^4e^3+48d^3e^4x+72d^2e^5x^2+48de^6x^3+12e^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**5,x)
```

```
[Out] -(3*a*e**2 + b*d*e + c*d**2 + 6*c*e**2*x**2 + x*(4*b*e**2 + 4*c*d*e))/(12*d
**4*e**3 + 48*d**3*e**4*x + 72*d**2*e**5*x**2 + 48*d*e**6*x**3 + 12*e**7*x**
*4)
```

Giac [A] time = 1.07926, size = 116, normalized size = 1.68

$$-\frac{1}{12}\left(\frac{6\,ce^{(-2)}}{(xe+d)^2}-\frac{8\,cde^{(-2)}}{(xe+d)^3}+\frac{3\,cd^2e^{(-2)}}{(xe+d)^4}+\frac{4\,be^{(-1)}}{(xe+d)^3}-\frac{3\,bde^{(-1)}}{(xe+d)^4}+\frac{3\,a}{(xe+d)^4}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] -1/12*(6*c*e^(-2)/(x*e + d)^2 - 8*c*d*e^(-2)/(x*e + d)^3 + 3*c*d^2*e^(-2)/(
x*e + d)^4 + 4*b*e^(-1)/(x*e + d)^3 - 3*b*d*e^(-1)/(x*e + d)^4 + 3*a/(x*e +
 d)^4)*e^(-1)
```

3.2117 $\int \frac{a+bx+cx^2}{(d+ex)^6}\,dx$

Optimal. Leaf size=69

$$-\frac{ae^2-bde+cd^2}{5e^3(d+ex)^5}+\frac{2cd-be}{4e^3(d+ex)^4}-\frac{c}{3e^3(d+ex)^3}$$

```
[Out] -(c*d^2 - b*d*e + a*e^2)/(5*e^3*(d + e*x)^5) + (2*c*d - b*e)/(4*e^3*(d + e*
x)^4) - c/(3*e^3*(d + e*x)^3)
```

Rubi [A] time = 0.0442678, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {698}

$$-\frac{ae^2-bde+cd^2}{5e^3(d+ex)^5}+\frac{2cd-be}{4e^3(d+ex)^4}-\frac{c}{3e^3(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(d + e*x)^6,x]
```

```
[Out] -(c*d^2 - b*d*e + a*e^2)/(5*e^3*(d + e*x)^5) + (2*c*d - b*e)/(4*e^3*(d + e*
x)^4) - c/(3*e^3*(d + e*x)^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{(d+ex)^6}\,dx&=\int \left(\frac{cd^2-bde+ae^2}{e^2(d+ex)^6}+\frac{-2cd+be}{e^2(d+ex)^5}+\frac{c}{e^2(d+ex)^4}\right)dx\\&=-\frac{cd^2-bde+ae^2}{5e^3(d+ex)^5}+\frac{2cd-be}{4e^3(d+ex)^4}-\frac{c}{3e^3(d+ex)^3}\end{aligned}$$

Mathematica [A] time = 0.0204608, size = 51, normalized size = 0.74

$$-\frac{3e(4ae+b(d+5ex))+2c\left(d^2+5dex+10e^2x^2\right)}{60e^3(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(d + e*x)^6,x]
```

```
[Out] -(2*c*(d^2 + 5*d*e*x + 10*e^2*x^2) + 3*e*(4*a*e + b*(d + 5*e*x)))/(60*e^3*(
d + e*x)^5)
```

Maple [A] time = 0.044, size = 63, normalized size = 0.9

$$-\frac{be-2\,cd}{4\,e^{3}\,(ex+d)^{4}}-\frac{ae^{2}-bde+cd^{2}}{5\,e^{3}\,(ex+d)^{5}}-\frac{c}{3\,e^{3}\,(ex+d)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^6,x)
```

```
[Out] -1/4*(b*e-2*c*d)/e^3/(e*x+d)^4-1/5*(a*e^2-b*d*e+c*d^2)/e^3/(e*x+d)^5-1/3*c/
e^3/(e*x+d)^3
```

Maxima [A] time = 1.00236, size = 136, normalized size = 1.97

$$-\frac{20\,ce^{2}x^{2}+2\,cd^{2}+3\,bde+12\,ae^{2}+5\,\left(2\,cde+3\,be^{2}\right)x}{60\,\left(e^{8}x^{5}+5\,de^{7}x^{4}+10\,d^{2}e^{6}x^{3}+10\,d^{3}e^{5}x^{2}+5\,d^{4}e^{4}x+d^{5}e^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^6,x, algorithm="maxima")
```

```
[Out] -1/60*(20*c*e^2*x^2 + 2*c*d^2 + 3*b*d*e + 12*a*e^2 + 5*(2*c*d*e + 3*b*e^2)*
x)/(e^8*x^5 + 5*d*e^7*x^4 + 10*d^2*e^6*x^3 + 10*d^3*e^5*x^2 + 5*d^4*e^4*x +
 d^5*e^3)
```

Fricas [A] time = 2.01564, size = 217, normalized size = 3.14

$$-\frac{20\,ce^{2}x^{2}+2\,cd^{2}+3\,bde+12\,ae^{2}+5\,\left(2\,cde+3\,be^{2}\right)x}{60\,\left(e^{8}x^{5}+5\,de^{7}x^{4}+10\,d^{2}e^{6}x^{3}+10\,d^{3}e^{5}x^{2}+5\,d^{4}e^{4}x+d^{5}e^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^6,x, algorithm="fricas")
```

```
[Out] -1/60*(20*c*e^2*x^2 + 2*c*d^2 + 3*b*d*e + 12*a*e^2 + 5*(2*c*d*e + 3*b*e^2)*
x)/(e^8*x^5 + 5*d*e^7*x^4 + 10*d^2*e^6*x^3 + 10*d^3*e^5*x^2 + 5*d^4*e^4*x +
 d^5*e^3)
```

Sympy [A] time = 2.10594, size = 107, normalized size = 1.55

$$-\frac{12ae^{2}+3bde+2cd^{2}+20ce^{2}x^{2}+x\left(15be^{2}+10cde\right)}{60d^{5}e^{3}+300d^{4}e^{4}x+600d^{3}e^{5}x^{2}+600d^{2}e^{6}x^{3}+300de^{7}x^{4}+60e^{8}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**6,x)
```

```
[Out] -(12*a*e**2 + 3*b*d*e + 2*c*d**2 + 20*c*e**2*x**2 + x*(15*b*e**2 + 10*c*d*e
))/(60*d**5*e**3 + 300*d**4*e**4*x + 600*d**3*e**5*x**2 + 600*d**2*e**6*x**
```

```
3 + 300*d*e**7*x**4 + 60*e**8*x**5)
```

Giac [A] time = 1.11487, size = 69, normalized size = 1.

$$-\frac{\left(20\,cx^2e^2+10\,cdxe+2\,cd^2+15\,bxe^2+3\,bde+12\,ae^2\right)e^{(-3)}}{60\,(xe+d)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^6,x, algorithm="giac")
```

```
[Out] -1/60*(20*c*x^2*e^2 + 10*c*d*x*e + 2*c*d^2 + 15*b*x*e^2 + 3*b*d*e + 12*a*e^
2)*e^(-3)/(x*e + d)^5
```

3.2118 $\int (d+ex)^4 \left(a+bx+cx^2\right)^2 dx$

Optimal. Leaf size=156

$$\frac{(d+ex)^7\left(-2ce(3bd-ae)+b^2e^2+6c^2d^2\right)}{7e^5}-\frac{(d+ex)^6(2cd-be)\left(ae^2-bde+cd^2\right)}{3e^5}+\frac{(d+ex)^5\left(ae^2-bde+cd^2\right)^2}{5e^5}-\frac{c(\ldots}{}$$

```
[Out] ((c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^5)/(5*e^5) - ((2*c*d - b*e)*(c*d^2 - b
*d*e + a*e^2)*(d + e*x)^6)/(3*e^5) + ((6*c^2*d^2 + b^2*e^2 - 2*c*e*(3*b*d -
 a*e))*(d + e*x)^7)/(7*e^5) - (c*(2*c*d - b*e)*(d + e*x)^8)/(4*e^5) + (c^2*
(d + e*x)^9)/(9*e^5)
```

Rubi [A] time = 0.249701, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$\frac{(d+ex)^7\left(-2ce(3bd-ae)+b^2e^2+6c^2d^2\right)}{7e^5}-\frac{(d+ex)^6(2cd-be)\left(ae^2-bde+cd^2\right)}{3e^5}+\frac{(d+ex)^5\left(ae^2-bde+cd^2\right)^2}{5e^5}-\frac{c(\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^5)/(5*e^5) - ((2*c*d - b*e)*(c*d^2 - b
*d*e + a*e^2)*(d + e*x)^6)/(3*e^5) + ((6*c^2*d^2 + b^2*e^2 - 2*c*e*(3*b*d -
 a*e))*(d + e*x)^7)/(7*e^5) - (c*(2*c*d - b*e)*(d + e*x)^8)/(4*e^5) + (c^2*
(d + e*x)^9)/(9*e^5)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^4\left(a+bx+cx^2\right)^2 dx=\int \left(\frac{\left(cd^2-bde+ae^2\right)^2(d+ex)^4}{e^4}+\frac{2(-2cd+be)\left(cd^2-bde+ae^2\right)(d+ex)^5}{e^4}+\frac{\left(6c^2d^{\ldots}\right.}{}\right.$$

$$=\frac{\left(cd^2-bde+ae^2\right)^2(d+ex)^5}{5e^5}-\frac{(2cd-be)\left(cd^2-bde+ae^2\right)(d+ex)^6}{3e^5}+\frac{\left(6c^2d^2+b^2e^{\ldots}\right.}{}$$

Mathematica [A] time = 0.0797288, size = 283, normalized size = 1.81

$$\frac{1}{5}x^5\left(e^2\left(a^2e^2+8abde+6b^2d^2\right)+4cd^2e(3ae+2bd)+c^2d^4\right)+\frac{1}{2}dx^4\left(2a^2e^3+6abde^2+4acd^2e+2b^2d^2e+bcd^3\right)+a^2d^4\ldots$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4*(a + b*x + c*x^2)^2,x]
```

```
[Out] a^2*d^4*x + a*d^3*(b*d + 2*a*e)*x^2 + (d^2*(b^2*d^2 + 8*a*b*d*e + 2*a*(c*d^
2 + 3*a*e^2))*x^3)/3 + (d*(b*c*d^3 + 2*b^2*d^2*e + 4*a*c*d^2*e + 6*a*b*d*e^
2 + 2*a^2*e^3)*x^4)/2 + ((c^2*d^4 + 4*c*d^2*e*(2*b*d + 3*a*e) + e^2*(6*b^2*
d^2 + 8*a*b*d*e + a^2*e^2))*x^5)/5 + (e*(2*c^2*d^3 + b*e^2*(2*b*d + a*e) +
2*c*d*e*(3*b*d + 2*a*e))*x^6)/3 + (e^2*(6*c^2*d^2 + b^2*e^2 + 2*c*e*(4*b*d
+ a*e))*x^7)/7 + (c*e^3*(2*c*d + b*e)*x^8)/4 + (c^2*e^4*x^9)/9
```

Maple [A] time = 0.038, size = 283, normalized size = 1.8

$$\frac{e^4c^2x^9}{9}+\frac{\left(2\,e^4bc+4\,de^3c^2\right)x^8}{8}+\frac{\left(6\,d^2e^2c^2+8\,de^3bc+e^4\left(2\,ac+b^2\right)\right)x^7}{7}+\frac{\left(4\,d^3ec^2+12\,d^2e^2bc+4\,de^3\left(2\,ac+b^2\right)+2\,e^4a\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(c*x^2+b*x+a)^2,x)
```

```
[Out] 1/9*e^4*c^2*x^9+1/8*(2*b*c*e^4+4*c^2*d*e^3)*x^8+1/7*(6*d^2*e^2*c^2+8*d*e^3*
b*c+e^4*(2*a*c+b^2))*x^7+1/6*(4*d^3*e*c^2+12*d^2*e^2*b*c+4*d*e^3*(2*a*c+b^2
)+2*e^4*a*b)*x^6+1/5*(c^2*d^4+8*d^3*e*b*c+6*d^2*e^2*(2*a*c+b^2)+8*d*e^3*a*b
+a^2*e^4)*x^5+1/4*(2*d^4*b*c+4*d^3*e*(2*a*c+b^2)+12*d^2*e^2*a*b+4*d*e^3*a^2
)*x^4+1/3*(d^4*(2*a*c+b^2)+8*d^3*e*a*b+6*d^2*e^2*a^2)*x^3+1/2*(4*a^2*d^3*e+
2*a*b*d^4)*x^2+d^4*a^2*x
```

Maxima [A] time = 0.975, size = 374, normalized size = 2.4

$$\frac{1}{9}\,c^2e^4x^9+\frac{1}{4}\left(2\,c^2de^3+bce^4\right)x^8+\frac{1}{7}\left(6\,c^2d^2e^2+8\,bcde^3+\left(b^2+2\,ac\right)e^4\right)x^7+a^2d^4x+\frac{1}{3}\left(2\,c^2d^3e+6\,bcd^2e^2+abe^4+2\left(b^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/9*c^2*e^4*x^9 + 1/4*(2*c^2*d*e^3 + b*c*e^4)*x^8 + 1/7*(6*c^2*d^2*e^2 + 8*
b*c*d*e^3 + (b^2 + 2*a*c)*e^4)*x^7 + a^2*d^4*x + 1/3*(2*c^2*d^3*e + 6*b*c*d
^2*e^2 + a*b*e^4 + 2*(b^2 + 2*a*c)*d*e^3)*x^6 + 1/5*(c^2*d^4 + 8*b*c*d^3*e
+ 8*a*b*d*e^3 + a^2*e^4 + 6*(b^2 + 2*a*c)*d^2*e^2)*x^5 + 1/2*(b*c*d^4 + 6*a
*b*d^2*e^2 + 2*a^2*d*e^3 + 2*(b^2 + 2*a*c)*d^3*e)*x^4 + 1/3*(8*a*b*d^3*e +
6*a^2*d^2*e^2 + (b^2 + 2*a*c)*d^4)*x^3 + (a*b*d^4 + 2*a^2*d^3*e)*x^2
```

Fricas [B] time = 1.7284, size = 756, normalized size = 4.85

$$\frac{1}{9}x^9e^4c^2+\frac{1}{2}x^8e^3dc^2+\frac{1}{4}x^8e^4cb+\frac{6}{7}x^7e^2d^2c^2+\frac{8}{7}x^7e^3dcb+\frac{1}{7}x^7e^4b^2+\frac{2}{7}x^7e^4ca+\frac{2}{3}x^6ed^3c^2+2x^6e^2d^2cb+\frac{2}{3}x^6e^3db^2+\frac{4}{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*e^4*c^2 + 1/2*x^8*e^3*d*c^2 + 1/4*x^8*e^4*c*b + 6/7*x^7*e^2*d^2*c^2
 + 8/7*x^7*e^3*d*c*b + 1/7*x^7*e^4*b^2 + 2/7*x^7*e^4*c*a + 2/3*x^6*e*d^3*c^
2 + 2*x^6*e^2*d^2*c*b + 2/3*x^6*e^3*d*b^2 + 4/3*x^6*e^3*d*c*a + 1/3*x^6*e^4
*b*a + 1/5*x^5*d^4*c^2 + 8/5*x^5*e*d^3*c*b + 6/5*x^5*e^2*d^2*b^2 + 12/5*x^5
*e^2*d^2*c*a + 8/5*x^5*e^3*d*b*a + 1/5*x^5*e^4*a^2 + 1/2*x^4*d^4*c*b + x^4*
```

```
e*d^3*b^2 + 2*x^4*e*d^3*c*a + 3*x^4*e^2*d^2*b*a + x^4*e^3*d*a^2 + 1/3*x^3*d
^4*b^2 + 2/3*x^3*d^4*c*a + 8/3*x^3*e*d^3*b*a + 2*x^3*e^2*d^2*a^2 + x^2*d^4*
b*a + 2*x^2*e*d^3*a^2 + x*d^4*a^2
```

Sympy [B] time = 0.109907, size = 337, normalized size = 2.16

$$a^2d^4x + \frac{c^2e^4x^9}{9} + x^8\left(\frac{bce^4}{4} + \frac{c^2de^3}{2}\right) + x^7\left(\frac{2ace^4}{7} + \frac{b^2e^4}{7} + \frac{8bcde^3}{7} + \frac{6c^2d^2e^2}{7}\right) + x^6\left(\frac{abe^4}{3} + \frac{4acde^3}{3} + \frac{2b^2de^3}{3} + 2bcd^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(c*x**2+b*x+a)**2,x)
```

```
[Out] a**2*d**4*x + c**2*e**4*x**9/9 + x**8*(b*c*e**4/4 + c**2*d*e**3/2) + x**7*(
2*a*c*e**4/7 + b**2*e**4/7 + 8*b*c*d*e**3/7 + 6*c**2*d**2*e**2/7) + x**6*(a
*b*e**4/3 + 4*a*c*d*e**3/3 + 2*b**2*d*e**3/3 + 2*b*c*d**2*e**2 + 2*c**2*d**
3*e/3) + x**5*(a**2*e**4/5 + 8*a*b*d*e**3/5 + 12*a*c*d**2*e**2/5 + 6*b**2*d
**2*e**2/5 + 8*b*c*d**3*e/5 + c**2*d**4/5) + x**4*(a**2*d*e**3 + 3*a*b*d**2
*e**2 + 2*a*c*d**3*e + b**2*d**3*e + b*c*d**4/2) + x**3*(2*a**2*d**2*e**2 +
 8*a*b*d**3*e/3 + 2*a*c*d**4/3 + b**2*d**4/3) + x**2*(2*a**2*d**3*e + a*b*d
**4)
```

Giac [B] time = 1.1046, size = 443, normalized size = 2.84

$$\frac{1}{9}c^2x^9e^4 + \frac{1}{2}c^2dx^8e^3 + \frac{6}{7}c^2d^2x^7e^2 + \frac{2}{3}c^2d^3x^6e + \frac{1}{5}c^2d^4x^5 + \frac{1}{4}bcx^8e^4 + \frac{8}{7}bcdx^7e^3 + 2\,bcd^2x^6e^2 + \frac{8}{5}bcd^3x^5e + \frac{1}{2}bcd^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*c^2*x^9*e^4 + 1/2*c^2*d*x^8*e^3 + 6/7*c^2*d^2*x^7*e^2 + 2/3*c^2*d^3*x^6
*e + 1/5*c^2*d^4*x^5 + 1/4*b*c*x^8*e^4 + 8/7*b*c*d*x^7*e^3 + 2*b*c*d^2*x^6*
e^2 + 8/5*b*c*d^3*x^5*e + 1/2*b*c*d^4*x^4 + 1/7*b^2*x^7*e^4 + 2/7*a*c*x^7*e
^4 + 2/3*b^2*d*x^6*e^3 + 4/3*a*c*d*x^6*e^3 + 6/5*b^2*d^2*x^5*e^2 + 12/5*a*c
*d^2*x^5*e^2 + b^2*d^3*x^4*e + 2*a*c*d^3*x^4*e + 1/3*b^2*d^4*x^3 + 2/3*a*c*
d^4*x^3 + 1/3*a*b*x^6*e^4 + 8/5*a*b*d*x^5*e^3 + 3*a*b*d^2*x^4*e^2 + 8/3*a*b
*d^3*x^3*e + a*b*d^4*x^2 + 1/5*a^2*x^5*e^4 + a^2*d*x^4*e^3 + 2*a^2*d^2*x^3*
e^2 + 2*a^2*d^3*x^2*e + a^2*d^4*x
```

3.2119 $\int (d+ex)^3 \left(a+bx+cx^2\right)^2 \, dx$

Optimal. Leaf size=156

$$\frac{(d+ex)^6\left(-2ce(3bd-ae)+b^2e^2+6c^2d^2\right)}{6e^5}-\frac{2(d+ex)^5(2cd-be)\left(ae^2-bde+cd^2\right)}{5e^5}+\frac{(d+ex)^4\left(ae^2-bde+cd^2\right)^2}{4e^5}-\frac{2c(d$$

```
[Out] ((c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^4)/(4*e^5) - (2*(2*c*d - b*e)*(c*d^2 -
 b*d*e + a*e^2)*(d + e*x)^5)/(5*e^5) + ((6*c^2*d^2 + b^2*e^2 - 2*c*e*(3*b*d
 - a*e))*(d + e*x)^6)/(6*e^5) - (2*c*(2*c*d - b*e)*(d + e*x)^7)/(7*e^5) + (
c^2*(d + e*x)^8)/(8*e^5)
```

Rubi [A] time = 0.174232, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$\frac{(d+ex)^6\left(-2ce(3bd-ae)+b^2e^2+6c^2d^2\right)}{6e^5}-\frac{2(d+ex)^5(2cd-be)\left(ae^2-bde+cd^2\right)}{5e^5}+\frac{(d+ex)^4\left(ae^2-bde+cd^2\right)^2}{4e^5}-\frac{2c(d$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^4)/(4*e^5) - (2*(2*c*d - b*e)*(c*d^2 -
 b*d*e + a*e^2)*(d + e*x)^5)/(5*e^5) + ((6*c^2*d^2 + b^2*e^2 - 2*c*e*(3*b*d
 - a*e))*(d + e*x)^6)/(6*e^5) - (2*c*(2*c*d - b*e)*(d + e*x)^7)/(7*e^5) + (
c^2*(d + e*x)^8)/(8*e^5)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^3\left(a+bx+cx^2\right)^2 dx = \int \left(\frac{\left(cd^2-bde+ae^2\right)^2(d+ex)^3}{e^4}+\frac{2(-2cd+be)\left(cd^2-bde+ae^2\right)(d+ex)^4}{e^4}+\frac{\left(6c^2d^2+\right.}{}$$

$$=\frac{\left(cd^2-bde+ae^2\right)^2(d+ex)^4}{4e^5}-\frac{2(2cd-be)\left(cd^2-bde+ae^2\right)(d+ex)^5}{5e^5}+\frac{\left(6c^2d^2+b^2e^2-\right.}{}$$

Mathematica [A] time = 0.0646776, size = 223, normalized size = 1.43

$$\frac{1}{4}x^4\left(a^2e^3+6abde^2+6acd^2e+3b^2d^2e+2bcd^3\right)+a^2d^3x+\frac{1}{6}ex^6\left(2ce(ae+3bd)+b^2e^2+3c^2d^2\right)+\frac{1}{3}dx^3\left(6abde+a\left(3ae^2\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*x + c*x^2)^2,x]
```

```
[Out] a^2*d^3*x + (a*d^2*(2*b*d + 3*a*e)*x^2)/2 + (d*(b^2*d^2 + 6*a*b*d*e + a*(2*
c*d^2 + 3*a*e^2))*x^3)/3 + ((2*b*c*d^3 + 3*b^2*d^2*e + 6*a*c*d^2*e + 6*a*b*
d*e^2 + a^2*e^3)*x^4)/4 + ((c^2*d^3 + 6*c*d*e*(b*d + a*e) + b*e^2*(3*b*d +
2*a*e))*x^5)/5 + (e*(3*c^2*d^2 + b^2*e^2 + 2*c*e*(3*b*d + a*e))*x^6)/6 + (c
*e^2*(3*c*d + 2*b*e)*x^7)/7 + (c^2*e^3*x^8)/8
```

Maple [A] time = 0.039, size = 219, normalized size = 1.4

$$\frac{c^2e^3x^8}{8}+\frac{\left(2\,e^3bc+3\,de^2c^2\right)x^7}{7}+\frac{\left(3\,d^2ec^2+6\,de^2bc+e^3\left(2\,ac+b^2\right)\right)x^6}{6}+\frac{\left(c^2d^3+6\,d^2ebc+3\,de^2\left(2\,ac+b^2\right)+2\,abe^3\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x+a)^2,x)
```

```
[Out] 1/8*c^2*e^3*x^8+1/7*(2*b*c*e^3+3*c^2*d*e^2)*x^7+1/6*(3*d^2*e*c^2+6*d*e^2*b*
c+e^3*(2*a*c+b^2))*x^6+1/5*(c^2*d^3+6*d^2*e*b*c+3*d*e^2*(2*a*c+b^2)+2*a*b*e
^3)*x^5+1/4*(2*d^3*b*c+3*d^2*e*(2*a*c+b^2)+6*d*e^2*a*b+a^2*e^3)*x^4+1/3*(d^
3*(2*a*c+b^2)+6*d^2*e*a*b+3*d*e^2*a^2)*x^3+1/2*(3*a^2*d^2*e+2*a*b*d^3)*x^2+
d^3*a^2*x
```

Maxima [A] time = 0.977418, size = 294, normalized size = 1.88

$$\frac{1}{8}\,c^2e^3x^8+\frac{1}{7}\left(3\,c^2de^2+2\,bce^3\right)x^7+\frac{1}{6}\left(3\,c^2d^2e+6\,bcde^2+\left(b^2+2\,ac\right)e^3\right)x^6+a^2d^3x+\frac{1}{5}\left(c^2d^3+6\,bcd^2e+2\,abe^3+3\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/8*c^2*e^3*x^8 + 1/7*(3*c^2*d*e^2 + 2*b*c*e^3)*x^7 + 1/6*(3*c^2*d^2*e + 6*
b*c*d*e^2 + (b^2 + 2*a*c)*e^3)*x^6 + a^2*d^3*x + 1/5*(c^2*d^3 + 6*b*c*d^2*e
 + 2*a*b*e^3 + 3*(b^2 + 2*a*c)*d*e^2)*x^5 + 1/4*(2*b*c*d^3 + 6*a*b*d*e^2 +
a^2*e^3 + 3*(b^2 + 2*a*c)*d^2*e)*x^4 + 1/3*(6*a*b*d^2*e + 3*a^2*d*e^2 + (b^
2 + 2*a*c)*d^3)*x^3 + 1/2*(2*a*b*d^3 + 3*a^2*d^2*e)*x^2
```

Fricas [A] time = 1.80195, size = 587, normalized size = 3.76

$$\frac{1}{8}x^8e^3c^2+\frac{3}{7}x^7e^2dc^2+\frac{2}{7}x^7e^3cb+\frac{1}{2}x^6ed^2c^2+x^6e^2dcb+\frac{1}{6}x^6e^3b^2+\frac{1}{3}x^6e^3ca+\frac{1}{5}x^5d^3c^2+\frac{6}{5}x^5ed^2cb+\frac{3}{5}x^5e^2db^2+\frac{6}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*e^3*c^2 + 3/7*x^7*e^2*d*c^2 + 2/7*x^7*e^3*c*b + 1/2*x^6*e*d^2*c^2 +
 x^6*e^2*d*c*b + 1/6*x^6*e^3*b^2 + 1/3*x^6*e^3*c*a + 1/5*x^5*d^3*c^2 + 6/5*
x^5*e*d^2*c*b + 3/5*x^5*e^2*d*b^2 + 6/5*x^5*e^2*d*c*a + 2/5*x^5*e^3*b*a + 1
/2*x^4*d^3*c*b + 3/4*x^4*e*d^2*b^2 + 3/2*x^4*e*d^2*c*a + 3/2*x^4*e^2*d*b*a
+ 1/4*x^4*e^3*a^2 + 1/3*x^3*d^3*b^2 + 2/3*x^3*d^3*c*a + 2*x^3*e*d^2*b*a + x
^3*e^2*d*a^2 + x^2*d^3*b*a + 3/2*x^2*e*d^2*a^2 + x*d^3*a^2
```

Sympy [A] time = 0.098688, size = 260, normalized size = 1.67

$$a^2d^3x + \frac{c^2e^3x^8}{8} + x^7\left(\frac{2bce^3}{7} + \frac{3c^2de^2}{7}\right) + x^6\left(\frac{ace^3}{3} + \frac{b^2e^3}{6} + bcde^2 + \frac{c^2d^2e}{2}\right) + x^5\left(\frac{2abe^3}{5} + \frac{6acde^2}{5} + \frac{3b^2de^2}{5} + \frac{6bcd^2e}{5} + \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x+a)**2,x)
```

```
[Out] a**2*d**3*x + c**2*e**3*x**8/8 + x**7*(2*b*c*e**3/7 + 3*c**2*d*e**2/7) + x*
*6*(a*c*e**3/3 + b**2*e**3/6 + b*c*d*e**2 + c**2*d**2*e/2) + x**5*(2*a*b*e*
*3/5 + 6*a*c*d*e**2/5 + 3*b**2*d*e**2/5 + 6*b*c*d**2*e/5 + c**2*d**3/5) + x
**4*(a**2*e**3/4 + 3*a*b*d*e**2/2 + 3*a*c*d**2*e/2 + 3*b**2*d**2*e/4 + b*c*
d**3/2) + x**3*(a**2*d*e**2 + 2*a*b*d**2*e + 2*a*c*d**3/3 + b**2*d**3/3) +
x**2*(3*a**2*d**2*e/2 + a*b*d**3)
```

Giac [A] time = 1.09897, size = 342, normalized size = 2.19

$$\frac{1}{8}c^2x^8e^3 + \frac{3}{7}c^2dx^7e^2 + \frac{1}{2}c^2d^2x^6e + \frac{1}{5}c^2d^3x^5 + \frac{2}{7}bcx^7e^3 + bcdx^6e^2 + \frac{6}{5}bcd^2x^5e + \frac{1}{2}bcd^3x^4 + \frac{1}{6}b^2x^6e^3 + \frac{1}{3}acx^6e^3 + \frac{3}{5}b^2 \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*c^2*x^8*e^3 + 3/7*c^2*d*x^7*e^2 + 1/2*c^2*d^2*x^6*e + 1/5*c^2*d^3*x^5 +
 2/7*b*c*x^7*e^3 + b*c*d*x^6*e^2 + 6/5*b*c*d^2*x^5*e + 1/2*b*c*d^3*x^4 + 1/
6*b^2*x^6*e^3 + 1/3*a*c*x^6*e^3 + 3/5*b^2*d*x^5*e^2 + 6/5*a*c*d*x^5*e^2 + 3
/4*b^2*d^2*x^4*e + 3/2*a*c*d^2*x^4*e + 1/3*b^2*d^3*x^3 + 2/3*a*c*d^3*x^3 +
2/5*a*b*x^5*e^3 + 3/2*a*b*d*x^4*e^2 + 2*a*b*d^2*x^3*e + a*b*d^3*x^2 + 1/4*a
^2*x^4*e^3 + a^2*d*x^3*e^2 + 3/2*a^2*d^2*x^2*e + a^2*d^3*x
```

3.2120 $\int (d+ex)^2 \left(a+bx+cx^2\right)^2 \, dx$

Optimal. Leaf size=156

$$\frac{(d+ex)^5 \left(-2ce(3bd-ae)+b^2e^2+6c^2d^2\right)}{5e^5}-\frac{(d+ex)^4(2cd-be)\left(ae^2-bde+cd^2\right)}{2e^5}+\frac{(d+ex)^3\left(ae^2-bde+cd^2\right)^2}{3e^5}-\frac{c(\cdots}{}$$

```
[Out] ((c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^3)/(3*e^5) - ((2*c*d - b*e)*(c*d^2 - b
*d*e + a*e^2)*(d + e*x)^4)/(2*e^5) + ((6*c^2*d^2 + b^2*e^2 - 2*c*e*(3*b*d -
 a*e))*(d + e*x)^5)/(5*e^5) - (c*(2*c*d - b*e)*(d + e*x)^6)/(3*e^5) + (c^2*
(d + e*x)^7)/(7*e^5)
```

Rubi [A] time = 0.140405, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$\frac{(d+ex)^5 \left(-2ce(3bd-ae)+b^2e^2+6c^2d^2\right)}{5e^5}-\frac{(d+ex)^4(2cd-be)\left(ae^2-bde+cd^2\right)}{2e^5}+\frac{(d+ex)^3\left(ae^2-bde+cd^2\right)^2}{3e^5}-\frac{c(\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^3)/(3*e^5) - ((2*c*d - b*e)*(c*d^2 - b
*d*e + a*e^2)*(d + e*x)^4)/(2*e^5) + ((6*c^2*d^2 + b^2*e^2 - 2*c*e*(3*b*d -
 a*e))*(d + e*x)^5)/(5*e^5) - (c*(2*c*d - b*e)*(d + e*x)^6)/(3*e^5) + (c^2*
(d + e*x)^7)/(7*e^5)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^2 \left(a+bx+cx^2\right)^2 \, dx = \int \left(\frac{\left(cd^2-bde+ae^2\right)^2 (d+ex)^2}{e^4}+\frac{2(-2cd+be)\left(cd^2-bde+ae^2\right)(d+ex)^3}{e^4}+\frac{\left(6c^2d^\cdots\right.}{}\right.$$

$$=\frac{\left(cd^2-bde+ae^2\right)^2 (d+ex)^3}{3e^5}-\frac{(2cd-be)\left(cd^2-bde+ae^2\right)(d+ex)^4}{2e^5}+\frac{\left(6c^2d^2+b^2e^\cdots\right.}{}$$

Mathematica [A] time = 0.0432366, size = 153, normalized size = 0.98

$$\frac{1}{3}x^3\left(a^2e^2+4abde+2acd^2+b^2d^2\right)+a^2d^2x+\frac{1}{5}x^5\left(2ace^2+b^2e^2+4bcde+c^2d^2\right)+\frac{1}{2}x^4\left(abe^2+2acde+b^2de+bcd^2\right)\cdots$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*x + c*x^2)^2,x]
```

```
[Out] a^2*d^2*x + a*d*(b*d + a*e)*x^2 + ((b^2*d^2 + 2*a*c*d^2 + 4*a*b*d*e + a^2*e
^2)*x^3)/3 + ((b*c*d^2 + b^2*d*e + 2*a*c*d*e + a*b*e^2)*x^4)/2 + ((c^2*d^2
+ 4*b*c*d*e + b^2*e^2 + 2*a*c*e^2)*x^5)/5 + (c*e*(c*d + b*e)*x^6)/3 + (c^2*
e^2*x^7)/7
```

Maple [A] time = 0.039, size = 155, normalized size = 1.

$$\frac{c^2e^2x^7}{7}+\frac{\left(2\,e^2bc+2\,c^2de\right)x^6}{6}+\frac{\left(c^2d^2+4\,bcde+e^2\left(2\,ac+b^2\right)\right)x^5}{5}+\frac{\left(2\,d^2bc+2\,de\left(2\,ac+b^2\right)+2\,e^2ab\right)x^4}{4}+\frac{\left(d^2\left(2\,ac+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+b*x+a)^2,x)
```

```
[Out] 1/7*c^2*e^2*x^7+1/6*(2*b*c*e^2+2*c^2*d*e)*x^6+1/5*(c^2*d^2+4*b*c*d*e+e^2*(2
*a*c+b^2))*x^5+1/4*(2*d^2*b*c+2*d*e*(2*a*c+b^2)+2*e^2*a*b)*x^4+1/3*(d^2*(2*
a*c+b^2)+4*a*b*d*e+a^2*e^2)*x^3+1/2*(2*a^2*d*e+2*a*b*d^2)*x^2+a^2*d^2*x
```

Maxima [A] time = 0.955823, size = 197, normalized size = 1.26

$$\frac{1}{7}\,c^2e^2x^7+\frac{1}{3}\left(c^2de+bce^2\right)x^6+\frac{1}{5}\left(c^2d^2+4\,bcde+\left(b^2+2\,ac\right)e^2\right)x^5+a^2d^2x+\frac{1}{2}\left(bcd^2+abe^2+\left(b^2+2\,ac\right)de\right)x^4+\frac{1}{3}\left(4\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/7*c^2*e^2*x^7 + 1/3*(c^2*d*e + b*c*e^2)*x^6 + 1/5*(c^2*d^2 + 4*b*c*d*e +
(b^2 + 2*a*c)*e^2)*x^5 + a^2*d^2*x + 1/2*(b*c*d^2 + a*b*e^2 + (b^2 + 2*a*c)
*d*e)*x^4 + 1/3*(4*a*b*d*e + a^2*e^2 + (b^2 + 2*a*c)*d^2)*x^3 + (a*b*d^2 +
a^2*d*e)*x^2
```

Fricas [A] time = 1.72716, size = 412, normalized size = 2.64

$$\frac{1}{7}x^7e^2c^2+\frac{1}{3}x^6edc^2+\frac{1}{3}x^6e^2cb+\frac{1}{5}x^5d^2c^2+\frac{4}{5}x^5edcb+\frac{1}{5}x^5e^2b^2+\frac{2}{5}x^5e^2ca+\frac{1}{2}x^4d^2cb+\frac{1}{2}x^4edb^2+x^4edca+\frac{1}{2}x^4e^2ba+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*e^2*c^2 + 1/3*x^6*e*d*c^2 + 1/3*x^6*e^2*c*b + 1/5*x^5*d^2*c^2 + 4/5
*x^5*e*d*c*b + 1/5*x^5*e^2*b^2 + 2/5*x^5*e^2*c*a + 1/2*x^4*d^2*c*b + 1/2*x^
4*e*d*b^2 + x^4*e*d*c*a + 1/2*x^4*e^2*b*a + 1/3*x^3*d^2*b^2 + 2/3*x^3*d^2*c
*a + 4/3*x^3*e*d*b*a + 1/3*x^3*e^2*a^2 + x^2*d^2*b*a + x^2*e*d*a^2 + x*d^2*
a^2
```

Sympy [A] time = 0.089446, size = 173, normalized size = 1.11

$$a^2d^2x+\frac{c^2e^2x^7}{7}+x^6\left(\frac{bce^2}{3}+\frac{c^2de}{3}\right)+x^5\left(\frac{2ace^2}{5}+\frac{b^2e^2}{5}+\frac{4bcde}{5}+\frac{c^2d^2}{5}\right)+x^4\left(\frac{abe^2}{2}+acde+\frac{b^2de}{2}+\frac{bcd^2}{2}\right)+x^3\left(\frac{a^2e^2}{3}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+b*x+a)**2,x)
```

```
[Out] a**2*d**2*x + c**2*e**2*x**7/7 + x**6*(b*c*e**2/3 + c**2*d*e/3) + x**5*(2*a
*c*e**2/5 + b**2*e**2/5 + 4*b*c*d*e/5 + c**2*d**2/5) + x**4*(a*b*e**2/2 + a
*c*d*e + b**2*d*e/2 + b*c*d**2/2) + x**3*(a**2*e**2/3 + 4*a*b*d*e/3 + 2*a*c
*d**2/3 + b**2*d**2/3) + x**2*(a**2*d*e + a*b*d**2)
```

Giac [A] time = 1.08653, size = 240, normalized size = 1.54

$$\frac{1}{7}c^2x^7e^2 + \frac{1}{3}c^2dx^6e + \frac{1}{5}c^2d^2x^5 + \frac{1}{3}bcx^6e^2 + \frac{4}{5}bcdx^5e + \frac{1}{2}bcd^2x^4 + \frac{1}{5}b^2x^5e^2 + \frac{2}{5}acx^5e^2 + \frac{1}{2}b^2dx^4e + acdx^4e + \frac{1}{3}b^2d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/7*c^2*x^7*e^2 + 1/3*c^2*d*x^6*e + 1/5*c^2*d^2*x^5 + 1/3*b*c*x^6*e^2 + 4/5
*b*c*d*x^5*e + 1/2*b*c*d^2*x^4 + 1/5*b^2*x^5*e^2 + 2/5*a*c*x^5*e^2 + 1/2*b^
2*d*x^4*e + a*c*d*x^4*e + 1/3*b^2*d^2*x^3 + 2/3*a*c*d^2*x^3 + 1/2*a*b*x^4*e
^2 + 4/3*a*b*d*x^3*e + a*b*d^2*x^2 + 1/3*a^2*x^3*e^2 + a^2*d*x^2*e + a^2*d^
2*x
```

3.2121 $\int (d+ex)\left(a+bx+cx^2\right)^2\,dx$

Optimal. Leaf size=96

$$a^2dx+\frac{1}{4}x^4\left(2ace+b^2e+2bcd\right)+\frac{1}{3}x^3\left(2abe+2acd+b^2d\right)+\frac{1}{2}ax^2(ae+2bd)+\frac{1}{5}cx^5(2be+cd)+\frac{1}{6}c^2ex^6$$

```
[Out] a^2*d*x + (a*(2*b*d + a*e)*x^2)/2 + ((b^2*d + 2*a*c*d + 2*a*b*e)*x^3)/3 + (
(2*b*c*d + b^2*e + 2*a*c*e)*x^4)/4 + (c*(c*d + 2*b*e)*x^5)/5 + (c^2*e*x^6)/
6
```

Rubi [A] time = 0.0748955, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {631}

$$a^2dx+\frac{1}{4}x^4\left(2ace+b^2e+2bcd\right)+\frac{1}{3}x^3\left(2abe+2acd+b^2d\right)+\frac{1}{2}ax^2(ae+2bd)+\frac{1}{5}cx^5(2be+cd)+\frac{1}{6}c^2ex^6$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*x + c*x^2)^2,x]
```

```
[Out] a^2*d*x + (a*(2*b*d + a*e)*x^2)/2 + ((b^2*d + 2*a*c*d + 2*a*b*e)*x^3)/3 + (
(2*b*c*d + b^2*e + 2*a*c*e)*x^4)/4 + (c*(c*d + 2*b*e)*x^5)/5 + (c^2*e*x^6)/
6
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\int (d+ex)\left(a+bx+cx^2\right)^2\,dx=\int \left(a^2d+a(2bd+ae)x+\left(b^2d+2acd+2abe\right)x^2+\left(2bcd+b^2e+2ace\right)x^3+c(cd+2be)x$$
$$=a^2dx+\frac{1}{2}a(2bd+ae)x^2+\frac{1}{3}\left(b^2d+2acd+2abe\right)x^3+\frac{1}{4}\left(2bcd+b^2e+2ace\right)x^4+\frac{1}{5}c(cd+$$

Mathematica [A] time = 0.0204831, size = 96, normalized size = 1.

$$a^2dx+\frac{1}{4}x^4\left(2ace+b^2e+2bcd\right)+\frac{1}{3}x^3\left(2abe+2acd+b^2d\right)+\frac{1}{2}ax^2(ae+2bd)+\frac{1}{5}cx^5(2be+cd)+\frac{1}{6}c^2ex^6$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*x + c*x^2)^2,x]
```

```
[Out] a^2*d*x + (a*(2*b*d + a*e)*x^2)/2 + ((b^2*d + 2*a*c*d + 2*a*b*e)*x^3)/3 + (
(2*b*c*d + b^2*e + 2*a*c*e)*x^4)/4 + (c*(c*d + 2*b*e)*x^5)/5 + (c^2*e*x^6)/
6
```

Maple [A] time = 0.04, size = 91, normalized size = 1.

$$\frac{c^2ex^6}{6}+\frac{\left(2\,bce+c^2d\right)x^5}{5}+\frac{\left(2\,bcd+e\left(2\,ac+b^2\right)\right)x^4}{4}+\frac{\left(\left(2\,ac+b^2\right)d+2\,aeb\right)x^3}{3}+\frac{\left(a^2e+2\,abd\right)x^2}{2}+a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+b*x+a)^2,x)
```

```
[Out] 1/6*c^2*e*x^6+1/5*(2*b*c*e+c^2*d)*x^5+1/4*(2*b*c*d+e*(2*a*c+b^2))*x^4+1/3*(
(2*a*c+b^2)*d+2*a*e*b)*x^3+1/2*(a^2*e+2*a*b*d)*x^2+a^2*d*x
```

Maxima [A] time = 0.967785, size = 122, normalized size = 1.27

$$\frac{1}{6}\,c^2ex^6+\frac{1}{5}\left(c^2d+2\,bce\right)x^5+\frac{1}{4}\left(2\,bcd+\left(b^2+2\,ac\right)e\right)x^4+a^2dx+\frac{1}{3}\left(2\,abe+\left(b^2+2\,ac\right)d\right)x^3+\frac{1}{2}\left(2\,abd+a^2e\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*c^2*e*x^6 + 1/5*(c^2*d + 2*b*c*e)*x^5 + 1/4*(2*b*c*d + (b^2 + 2*a*c)*e)
*x^4 + a^2*d*x + 1/3*(2*a*b*e + (b^2 + 2*a*c)*d)*x^3 + 1/2*(2*a*b*d + a^2*e
)*x^2
```

Fricas [A] time = 1.70081, size = 244, normalized size = 2.54

$$\frac{1}{6}x^6ec^2+\frac{1}{5}x^5dc^2+\frac{2}{5}x^5ecb+\frac{1}{2}x^4dcb+\frac{1}{4}x^4eb^2+\frac{1}{2}x^4eca+\frac{1}{3}x^3db^2+\frac{2}{3}x^3dca+\frac{2}{3}x^3eba+x^2dba+\frac{1}{2}x^2ea^2+xda^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*x^6*e*c^2 + 1/5*x^5*d*c^2 + 2/5*x^5*e*c*b + 1/2*x^4*d*c*b + 1/4*x^4*e*b
^2 + 1/2*x^4*e*c*a + 1/3*x^3*d*b^2 + 2/3*x^3*d*c*a + 2/3*x^3*e*b*a + x^2*d*
b*a + 1/2*x^2*e*a^2 + x*d*a^2
```

Sympy [A] time = 0.077819, size = 100, normalized size = 1.04

$$a^2dx+\frac{c^2ex^6}{6}+x^5\left(\frac{2bce}{5}+\frac{c^2d}{5}\right)+x^4\left(\frac{ace}{2}+\frac{b^2e}{4}+\frac{bcd}{2}\right)+x^3\left(\frac{2abe}{3}+\frac{2acd}{3}+\frac{b^2d}{3}\right)+x^2\left(\frac{a^2e}{2}+abd\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+b*x+a)**2,x)
```

```
[Out] a**2*d*x + c**2*e*x**6/6 + x**5*(2*b*c*e/5 + c**2*d/5) + x**4*(a*c*e/2 + b*
*2*e/4 + b*c*d/2) + x**3*(2*a*b*e/3 + 2*a*c*d/3 + b**2*d/3) + x**2*(a**2*e/
2 + a*b*d)
```

<u>Giac</u> [A] time = 1.11165, size = 142, normalized size = 1.48

$$\frac{1}{6}c^2x^6e + \frac{1}{5}c^2dx^5 + \frac{2}{5}bcx^5e + \frac{1}{2}bcdx^4 + \frac{1}{4}b^2x^4e + \frac{1}{2}acx^4e + \frac{1}{3}b^2dx^3 + \frac{2}{3}acdx^3 + \frac{2}{3}abx^3e + abdx^2 + \frac{1}{2}a^2x^2e + a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*c^2*x^6*e + 1/5*c^2*d*x^5 + 2/5*b*c*x^5*e + 1/2*b*c*d*x^4 + 1/4*b^2*x^4
*e + 1/2*a*c*x^4*e + 1/3*b^2*d*x^3 + 2/3*a*c*d*x^3 + 2/3*a*b*x^3*e + a*b*d*
x^2 + 1/2*a^2*x^2*e + a^2*d*x
```

3.2122 $\int \left(a+bx+cx^2\right)^2 dx$

Optimal. Leaf size=46

$$a^2x+\frac{1}{3}x^3\left(2ac+b^2\right)+abx^2+\frac{1}{2}bcx^4+\frac{c^2x^5}{5}$$

```
[Out] a^2*x + a*b*x^2 + ((b^2 + 2*a*c)*x^3)/3 + (b*c*x^4)/2 + (c^2*x^5)/5
```

Rubi [A] time = 0.0229114, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {611}

$$a^2x+\frac{1}{3}x^3\left(2ac+b^2\right)+abx^2+\frac{1}{2}bcx^4+\frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2,x]
```

```
[Out] a^2*x + a*b*x^2 + ((b^2 + 2*a*c)*x^3)/3 + (b*c*x^4)/2 + (c^2*x^5)/5
```

Rule 611

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && IGtQ[p, 0] && (EqQ[a, 0] ||  !PerfectSquareQ[b^2 - 4*a*c])
```

Rubi steps

$$\begin{aligned}\int \left(a+bx+cx^2\right)^2 dx &= \int \left(a^2+2abx+b^2\left(1+\frac{2ac}{b^2}\right)x^2+2bcx^3+c^2x^4\right) dx\\ &= a^2x+abx^2+\frac{1}{3}\left(b^2+2ac\right)x^3+\frac{1}{2}bcx^4+\frac{c^2x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.0062207, size = 46, normalized size = 1.

$$a^2x+\frac{1}{3}x^3\left(2ac+b^2\right)+abx^2+\frac{1}{2}bcx^4+\frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2,x]
```

```
[Out] a^2*x + a*b*x^2 + ((b^2 + 2*a*c)*x^3)/3 + (b*c*x^4)/2 + (c^2*x^5)/5
```

Maple [A] time = 0.039, size = 41, normalized size = 0.9

$$a^2x+abx^2+\frac{\left(2\,ac+b^2\right)x^3}{3}+\frac{bcx^4}{2}+\frac{c^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2,x)
```

```
[Out] a^2*x+a*b*x^2+1/3*(2*a*c+b^2)*x^3+1/2*b*c*x^4+1/5*c^2*x^5
```

Maxima [A] time = 0.979495, size = 61, normalized size = 1.33

$$\frac{1}{5}c^2x^5 + \frac{1}{2}bcx^4 + \frac{1}{3}b^2x^3 + a^2x + \frac{1}{3}\left(2cx^3 + 3bx^2\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/5*c^2*x^5 + 1/2*b*c*x^4 + 1/3*b^2*x^3 + a^2*x + 1/3*(2*c*x^3 + 3*b*x^2)*a
```

Fricas [A] time = 1.70207, size = 99, normalized size = 2.15

$$\frac{1}{5}x^5c^2 + \frac{1}{2}x^4cb + \frac{1}{3}x^3b^2 + \frac{2}{3}x^3ca + x^2ba + xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*c^2 + 1/2*x^4*c*b + 1/3*x^3*b^2 + 2/3*x^3*c*a + x^2*b*a + x*a^2
```

Sympy [A] time = 0.065928, size = 42, normalized size = 0.91

$$a^2x + abx^2 + \frac{bcx^4}{2} + \frac{c^2x^5}{5} + x^3\left(\frac{2ac}{3} + \frac{b^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2,x)
```

```
[Out] a**2*x + a*b*x**2 + b*c*x**4/2 + c**2*x**5/5 + x**3*(2*a*c/3 + b**2/3)
```

Giac [A] time = 1.09587, size = 57, normalized size = 1.24

$$\frac{1}{5}c^2x^5 + \frac{1}{2}bcx^4 + \frac{1}{3}b^2x^3 + \frac{2}{3}acx^3 + abx^2 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/5*c^2*x^5 + 1/2*b*c*x^4 + 1/3*b^2*x^3 + 2/3*a*c*x^3 + a*b*x^2 + a^2*x
```

3.2123 $\int \frac{(a+bx+cx^2)^2}{d+ex}\,dx$

Optimal. Leaf size=129

$$\frac{x^2\left(-2ce(bd-ae)+b^2e^2+c^2d^2\right)}{2e^3}-\frac{x(cd-be)\left(cd^2-e(bd-2ae)\right)}{e^4}+\frac{\log(d+ex)\left(ae^2-bde+cd^2\right)^2}{e^5}-\frac{cx^3(cd-2be)}{3e^2}+$$

```
[Out] -(((c*d - b*e)*(c*d^2 - e*(b*d - 2*a*e))*x)/e^4) + ((c^2*d^2 + b^2*e^2 - 2*
c*e*(b*d - a*e))*x^2)/(2*e^3) - (c*(c*d - 2*b*e)*x^3)/(3*e^2) + (c^2*x^4)/(
4*e) + ((c*d^2 - b*d*e + a*e^2)^2*Log[d + e*x])/e^5
```

Rubi [A] time = 0.155409, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$\frac{x^2\left(-2ce(bd-ae)+b^2e^2+c^2d^2\right)}{2e^3}-\frac{x(cd-be)\left(cd^2-e(bd-2ae)\right)}{e^4}+\frac{\log(d+ex)\left(ae^2-bde+cd^2\right)^2}{e^5}-\frac{cx^3(cd-2be)}{3e^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(d + e*x),x]
```

```
[Out] -(((c*d - b*e)*(c*d^2 - e*(b*d - 2*a*e))*x)/e^4) + ((c^2*d^2 + b^2*e^2 - 2*
c*e*(b*d - a*e))*x^2)/(2*e^3) - (c*(c*d - 2*b*e)*x^3)/(3*e^2) + (c^2*x^4)/(
4*e) + ((c*d^2 - b*d*e + a*e^2)^2*Log[d + e*x])/e^5
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^2}{d+ex}\,dx=\int \left(\frac{(cd-be)\left(-cd^2+e(bd-2ae)\right)}{e^4}+\frac{\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x}{e^3}-\frac{c(cd-2be)x^2}{e^2}+\frac{c^2x^3}{e}+\right.$$

$$=-\frac{(cd-be)\left(cd^2-e(bd-2ae)\right)x}{e^4}+\frac{\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^2}{2e^3}-\frac{c(cd-2be)x^3}{3e^2}+\frac{c^2x^4}{4e}+$$

Mathematica [A] time = 0.0629943, size = 128, normalized size = 0.99

$$\frac{ex\left(4ce\left(3ae(ex-2d)+b\left(6d^2-3dex+2e^2x^2\right)\right)+6be^2(4ae-2bd+bex)+c^2\left(6d^2ex-12d^3-4de^2x^2+3e^3x^3\right)\right)+12\,\mathrm{l}}{12e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(d + e*x),x]
```

```
[Out] (e*x*(6*b*e^2*(-2*b*d + 4*a*e + b*e*x) + c^2*(-12*d^3 + 6*d^2*e*x - 4*d*e^2
*x^2 + 3*e^3*x^3) + 4*c*e*(3*a*e*(-2*d + e*x) + b*(6*d^2 - 3*d*e*x + 2*e^2*
x^2))) + 12*(c*d^2 + e*(-(b*d) + a*e))^2*Log[d + e*x])/(12*e^5)
```

Maple [A] time = 0.042, size = 221, normalized size = 1.7

$$\frac{c^2x^4}{4e}+\frac{2bcx^3}{3e}-\frac{c^2dx^3}{3e^2}+\frac{acx^2}{e}+\frac{b^2x^2}{2e}-\frac{bcx^2d}{e^2}+\frac{c^2d^2x^2}{2e^3}+2\frac{abx}{e}-2\frac{acdx}{e^2}-\frac{b^2dx}{e^2}+2\frac{d^2bcx}{e^3}-\frac{c^2d^3x}{e^4}+\frac{\ln(ex+d)a^2}{e}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(e*x+d),x)
```

```
[Out] 1/4*c^2*x^4/e+2/3/e*x^3*b*c-1/3*c^2*d*x^3/e^2+1/e*x^2*a*c+1/2/e*x^2*b^2-1/e
^2*x^2*b*c*d+1/2/e^3*x^2*c^2*d^2+2/e*a*b*x-2/e^2*a*d*c*x-1/e^2*b^2*d*x+2/e^
3*b*c*d^2*x-1/e^4*c^2*d^3*x+1/e*ln(e*x+d)*a^2-2/e^2*ln(e*x+d)*d*a*b+2/e^3*l
n(e*x+d)*a*c*d^2+d^2/e^3*ln(e*x+d)*b^2-2*d^3/e^4*ln(e*x+d)*b*c+d^4/e^5*ln(e
*x+d)*c^2
```

Maxima [A] time = 1.00364, size = 227, normalized size = 1.76

$$\frac{3c^2e^3x^4-4\left(c^2de^2-2bce^3\right)x^3+6\left(c^2d^2e-2bcde^2+\left(b^2+2ac\right)e^3\right)x^2-12\left(c^2d^3-2bcd^2e-2abe^3+\left(b^2+2ac\right)de^2\right)x}{12e^4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/12*(3*c^2*e^3*x^4 - 4*(c^2*d*e^2 - 2*b*c*e^3)*x^3 + 6*(c^2*d^2*e - 2*b*c*
d*e^2 + (b^2 + 2*a*c)*e^3)*x^2 - 12*(c^2*d^3 - 2*b*c*d^2*e - 2*a*b*e^3 + (b
^2 + 2*a*c)*d*e^2)*x)/e^4 + (c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4
+ (b^2 + 2*a*c)*d^2*e^2)*log(e*x + d)/e^5
```

Fricas [A] time = 2.01063, size = 369, normalized size = 2.86

$$\frac{3c^2e^4x^4-4\left(c^2de^3-2bce^4\right)x^3+6\left(c^2d^2e^2-2bcde^3+\left(b^2+2ac\right)e^4\right)x^2-12\left(c^2d^3e-2bcd^2e^2-2abe^4+\left(b^2+2ac\right)de^3\right)x}{12e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/12*(3*c^2*e^4*x^4 - 4*(c^2*d*e^3 - 2*b*c*e^4)*x^3 + 6*(c^2*d^2*e^2 - 2*b*
c*d*e^3 + (b^2 + 2*a*c)*e^4)*x^2 - 12*(c^2*d^3*e - 2*b*c*d^2*e^2 - 2*a*b*e^
4 + (b^2 + 2*a*c)*d*e^3)*x + 12*(c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*
e^4 + (b^2 + 2*a*c)*d^2*e^2)*log(e*x + d))/e^5
```

Sympy [A] time = 0.610411, size = 144, normalized size = 1.12

$$\frac{c^2x^4}{4e}+\frac{x^3\left(2bce-c^2d\right)}{3e^2}+\frac{x^2\left(2ace^2+b^2e^2-2bcde+c^2d^2\right)}{2e^3}+\frac{x\left(2abe^3-2acde^2-b^2de^2+2bcd^2e-c^2d^3\right)}{e^4}+\frac{\left(ae^2-bde+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(e*x+d),x)
```

```
[Out] c**2*x**4/(4*e) + x**3*(2*b*c*e - c**2*d)/(3*e**2) + x**2*(2*a*c*e**2 + b**
2*e**2 - 2*b*c*d*e + c**2*d**2)/(2*e**3) + x*(2*a*b*e**3 - 2*a*c*d*e**2 - b
**2*d*e**2 + 2*b*c*d**2*e - c**2*d**3)/e**4 + (a*e**2 - b*d*e + c*d**2)**2*
log(d + e*x)/e**5
```

Giac [A] time = 1.1144, size = 243, normalized size = 1.88

$$\left(c^2d^4 - 2\,bcd^3e + b^2d^2e^2 + 2\,acd^2e^2 - 2\,abde^3 + a^2e^4\right)e^{(-5)}\log\left(|xe + d|\right) + \frac{1}{12}\left(3\,c^2x^4e^3 - 4\,c^2dx^3e^2 + 6\,c^2d^2x^2e - 12\,c^2d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d),x, algorithm="giac")
```

```
[Out] (c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^
4)*e^(-5)*log(abs(x*e + d)) + 1/12*(3*c^2*x^4*e^3 - 4*c^2*d*x^3*e^2 + 6*c^2
*d^2*x^2*e - 12*c^2*d^3*x + 8*b*c*x^3*e^3 - 12*b*c*d*x^2*e^2 + 24*b*c*d^2*x
*e + 6*b^2*x^2*e^3 + 12*a*c*x^2*e^3 - 12*b^2*d*x*e^2 - 24*a*c*d*x*e^2 + 24*
a*b*x*e^3)*e^(-4)
```

3.2124 $\int \frac{(a+bx+cx^2)^2}{(d+ex)^2}\,dx$

Optimal. Leaf size=131

$$\frac{x\left(-2ce(2bd-ae)+b^2e^2+3c^2d^2\right)}{e^4}-\frac{\left(ae^2-bde+cd^2\right)^2}{e^5(d+ex)}-\frac{2(2cd-be)\log(d+ex)\left(ae^2-bde+cd^2\right)}{e^5}-\frac{cx^2(cd-be)}{e^3}+\frac{c^2x^3}{3e^2}$$

```
[Out] ((3*c^2*d^2 + b^2*e^2 - 2*c*e*(2*b*d - a*e))*x)/e^4 - (c*(c*d - b*e)*x^2)/e
^3 + (c^2*x^3)/(3*e^2) - (c*d^2 - b*d*e + a*e^2)^2/(e^5*(d + e*x)) - (2*(2*
c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*Log[d + e*x])/e^5
```

Rubi [A] time = 0.141459, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$\frac{x\left(-2ce(2bd-ae)+b^2e^2+3c^2d^2\right)}{e^4}-\frac{\left(ae^2-bde+cd^2\right)^2}{e^5(d+ex)}-\frac{2(2cd-be)\log(d+ex)\left(ae^2-bde+cd^2\right)}{e^5}-\frac{cx^2(cd-be)}{e^3}+\frac{c^2x^3}{3e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(d + e*x)^2,x]
```

```
[Out] ((3*c^2*d^2 + b^2*e^2 - 2*c*e*(2*b*d - a*e))*x)/e^4 - (c*(c*d - b*e)*x^2)/e
^3 + (c^2*x^3)/(3*e^2) - (c*d^2 - b*d*e + a*e^2)^2/(e^5*(d + e*x)) - (2*(2*
c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*Log[d + e*x])/e^5
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int\frac{\left(a+bx+cx^2\right)^2}{(d+ex)^2}\,dx&=\int\left(\frac{3c^2d^2+b^2e^2-2ce(2bd-ae)}{e^4}-\frac{2c(cd-be)x}{e^3}+\frac{c^2x^2}{e^2}+\frac{\left(cd^2-bde+ae^2\right)^2}{e^4(d+ex)^2}+\frac{2(-2cd+be)\left(cd^2-bde+ae^2\right)}{e^4(d+ex)}\right)dx\\&=\frac{\left(3c^2d^2+b^2e^2-2ce(2bd-ae)\right)x}{e^4}-\frac{c(cd-be)x^2}{e^3}+\frac{c^2x^3}{3e^2}-\frac{\left(cd^2-bde+ae^2\right)^2}{e^5(d+ex)}-\frac{2(2cd-be)\left(cd^2-bde+ae^2\right)\log(d+ex)}{e^5}\end{aligned}$$

Mathematica [A] time = 0.111307, size = 127, normalized size = 0.97

$$\frac{3ex\left(2ce(ae-2bd)+b^2e^2+3c^2d^2\right)-\frac{3\left(e(ae-bd)+cd^2\right)^2}{d+ex}-6(2cd-be)\log(d+ex)\left(e(ae-bd)+cd^2\right)+3ce^2x^2(be-cd)+c^2e^3x^3}{3e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(d + e*x)^2,x]
```

```
[Out] (3*e*(3*c^2*d^2 + b^2*e^2 + 2*c*e*(-2*b*d + a*e))*x + 3*c*e^2*(-(c*d) + b*e
)*x^2 + c^2*e^3*x^3 - (3*(c*d^2 + e*(-(b*d) + a*e))^2)/(d + e*x) - 6*(2*c*d
 - b*e)*(c*d^2 + e*(-(b*d) + a*e))*Log[d + e*x])/(3*e^5)
```

Maple [A] time = 0.047, size = 246, normalized size = 1.9

$$\frac{c^2x^3}{3e^2}+\frac{bcx^2}{e^2}-\frac{c^2dx^2}{e^3}+2\frac{acx}{e^2}+\frac{b^2x}{e^2}-4\frac{bcdx}{e^3}+3\frac{c^2d^2x}{e^4}+2\frac{\ln(ex+d)\,ab}{e^2}-4\frac{\ln(ex+d)\,adc}{e^3}-2\frac{\ln(ex+d)\,b^2d}{e^3}+6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(e*x+d)^2,x)
```

```
[Out] 1/3*c^2*x^3/e^2+1/e^2*x^2*b*c-c^2*d*x^2/e^3+2/e^2*a*c*x+b^2*x/e^2-4/e^3*b*c
*d*x+3/e^4*c^2*d^2*x+2/e^2*ln(e*x+d)*a*b-4/e^3*ln(e*x+d)*a*d*c-2/e^3*ln(e*x
+d)*b^2*d+6/e^4*ln(e*x+d)*d^2*b*c-4/e^5*ln(e*x+d)*c^2*d^3-1/e/(e*x+d)*a^2+2
/e^2/(e*x+d)*d*a*b-2/e^3/(e*x+d)*a*c*d^2-1/e^3/(e*x+d)*b^2*d^2+2/e^4/(e*x+d
)*d^3*b*c-1/e^5/(e*x+d)*c^2*d^4
```

Maxima [A] time = 1.01908, size = 236, normalized size = 1.8

$$-\frac{c^2d^4-2\,bcd^3e-2\,abde^3+a^2e^4+\left(b^2+2\,ac\right)d^2e^2}{e^6x+de^5}+\frac{c^2e^2x^3-3\left(c^2de-bce^2\right)x^2+3\left(3\,c^2d^2-4\,bcde+\left(b^2+2\,ac\right)e^2\right)x}{3\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e^2)/(e
^6*x + d*e^5) + 1/3*(c^2*e^2*x^3 - 3*(c^2*d*e - b*c*e^2)*x^2 + 3*(3*c^2*d^2
 - 4*b*c*d*e + (b^2 + 2*a*c)*e^2)*x)/e^4 - 2*(2*c^2*d^3 - 3*b*c*d^2*e - a*b
*e^3 + (b^2 + 2*a*c)*d*e^2)*log(e*x + d)/e^5
```

Fricas [B] time = 2.10929, size = 548, normalized size = 4.18

$$\frac{c^2e^4x^4-3\,c^2d^4+6\,bcd^3e+6\,abde^3-3\,a^2e^4-3\left(b^2+2\,ac\right)d^2e^2-\left(2\,c^2de^3-3\,bce^4\right)x^3+3\left(2\,c^2d^2e^2-3\,bcde^3+\left(b^2+2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(c^2*e^4*x^4 - 3*c^2*d^4 + 6*b*c*d^3*e + 6*a*b*d*e^3 - 3*a^2*e^4 - 3*(b
^2 + 2*a*c)*d^2*e^2 - (2*c^2*d*e^3 - 3*b*c*e^4)*x^3 + 3*(2*c^2*d^2*e^2 - 3*
b*c*d*e^3 + (b^2 + 2*a*c)*e^4)*x^2 + 3*(3*c^2*d^3*e - 4*b*c*d^2*e^2 + (b^2
+ 2*a*c)*d*e^3)*x - 6*(2*c^2*d^4 - 3*b*c*d^3*e - a*b*d*e^3 + (b^2 + 2*a*c)*
d^2*e^2 + (2*c^2*d^3*e - 3*b*c*d^2*e^2 - a*b*e^4 + (b^2 + 2*a*c)*d*e^3)*x)*
log(e*x + d))/(e^6*x + d*e^5)
```

Sympy [A] time = 1.18507, size = 167, normalized size = 1.27

$$\frac{c^2x^3}{3e^2} - \frac{a^2e^4 - 2abde^3 + 2acd^2e^2 + b^2d^2e^2 - 2bcd^3e + c^2d^4}{de^5 + e^6x} + \frac{x^2\left(bce - c^2d\right)}{e^3} + \frac{x\left(2ace^2 + b^2e^2 - 4bcde + 3c^2d^2\right)}{e^4} + \frac{2\,(be -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(e*x+d)**2,x)
```

```
[Out] c**2*x**3/(3*e**2) - (a**2*e**4 - 2*a*b*d*e**3 + 2*a*c*d**2*e**2 + b**2*d**
2*e**2 - 2*b*c*d**3*e + c**2*d**4)/(d*e**5 + e**6*x) + x**2*(b*c*e - c**2*d
)/e**3 + x*(2*a*c*e**2 + b**2*e**2 - 4*b*c*d*e + 3*c**2*d**2)/e**4 + 2*(b*e
 - 2*c*d)*(a*e**2 - b*d*e + c*d**2)*log(d + e*x)/e**5
```

Giac [A] time = 1.11703, size = 336, normalized size = 2.56

$$\frac{1}{3}\left(c^2 - \frac{3\left(2\,c^2de - bce^2\right)e^{(-1)}}{xe + d} + \frac{3\left(6\,c^2d^2e^2 - 6\,bcde^3 + b^2e^4 + 2\,ace^4\right)e^{(-2)}}{(xe + d)^2}\right)(xe + d)^3e^{(-5)} + 2\left(2\,c^2d^3 - 3\,bcd^2e + b^2de^2 + 2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/3*(c^2 - 3*(2*c^2*d*e - b*c*e^2)*e^(-1)/(x*e + d) + 3*(6*c^2*d^2*e^2 - 6*
b*c*d*e^3 + b^2*e^4 + 2*a*c*e^4)*e^(-2)/(x*e + d)^2)*(x*e + d)^3*e^(-5) + 2
*(2*c^2*d^3 - 3*b*c*d^2*e + b^2*d*e^2 + 2*a*c*d*e^2 - a*b*e^3)*e^(-5)*log(a
bs(x*e + d)*e^(-1)/(x*e + d)^2) - (c^2*d^4*e^3/(x*e + d) - 2*b*c*d^3*e^4/(x
*e + d) + b^2*d^2*e^5/(x*e + d) + 2*a*c*d^2*e^5/(x*e + d) - 2*a*b*d*e^6/(x*
e + d) + a^2*e^7/(x*e + d))*e^(-8)
```

3.2125 $\int \frac{(a+bx+cx^2)^2}{(d+ex)^3}\,dx$

Optimal. Leaf size=138

$$\frac{\log(d+ex)\left(-2ce(3bd-ae)+b^2e^2+6c^2d^2\right)}{e^5}-\frac{\left(ae^2-bde+cd^2\right)^2}{2e^5(d+ex)^2}+\frac{2(2cd-be)\left(ae^2-bde+cd^2\right)}{e^5(d+ex)}-\frac{cx(3cd-2be)}{e^4}+$$

```
[Out] -((c*(3*c*d - 2*b*e)*x)/e^4) + (c^2*x^2)/(2*e^3) - (c*d^2 - b*d*e + a*e^2)^
2/(2*e^5*(d + e*x)^2) + (2*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2))/(e^5*(d +
 e*x)) + ((6*c^2*d^2 + b^2*e^2 - 2*c*e*(3*b*d - a*e))*Log[d + e*x])/e^5
```

Rubi [A] time = 0.135546, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$\frac{\log(d+ex)\left(-2ce(3bd-ae)+b^2e^2+6c^2d^2\right)}{e^5}-\frac{\left(ae^2-bde+cd^2\right)^2}{2e^5(d+ex)^2}+\frac{2(2cd-be)\left(ae^2-bde+cd^2\right)}{e^5(d+ex)}-\frac{cx(3cd-2be)}{e^4}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(d + e*x)^3,x]
```

```
[Out] -((c*(3*c*d - 2*b*e)*x)/e^4) + (c^2*x^2)/(2*e^3) - (c*d^2 - b*d*e + a*e^2)^
2/(2*e^5*(d + e*x)^2) + (2*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2))/(e^5*(d +
 e*x)) + ((6*c^2*d^2 + b^2*e^2 - 2*c*e*(3*b*d - a*e))*Log[d + e*x])/e^5
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^2}{(d+ex)^3}\,dx=\int \left(-\frac{c(3cd-2be)}{e^4}+\frac{c^2x}{e^3}+\frac{\left(cd^2-bde+ae^2\right)^2}{e^4(d+ex)^3}+\frac{2(-2cd+be)\left(cd^2-bde+ae^2\right)}{e^4(d+ex)^2}+\frac{6c^2d^2+b^2}{e}\right.$$

$$=-\frac{c(3cd-2be)x}{e^4}+\frac{c^2x^2}{2e^3}-\frac{\left(cd^2-bde+ae^2\right)^2}{2e^5(d+ex)^2}+\frac{2(2cd-be)\left(cd^2-bde+ae^2\right)}{e^5(d+ex)}+\frac{\left(6c^2d^2+b^2e^2\right.}{}$$

Mathematica [A] time = 0.0741781, size = 176, normalized size = 1.28

$$\frac{2(d+ex)^2\log(d+ex)\left(2ce(ae-3bd)+b^2e^2+6c^2d^2\right)+2ce\left(ade(3d+4ex)+b\left(-4d^2ex-5d^3+4de^2x^2+2e^3x^3\right)\right)+e^2}{2e^5(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(d + e*x)^3,x]
```

```
[Out] (e^2*(b*d - a*e)*(3*b*d + a*e + 4*b*e*x) + c^2*(7*d^4 + 2*d^3*e*x - 11*d^2*
e^2*x^2 - 4*d*e^3*x^3 + e^4*x^4) + 2*c*e*(a*d*e*(3*d + 4*e*x) + b*(-5*d^3 -
 4*d^2*e*x + 4*d*e^2*x^2 + 2*e^3*x^3)) + 2*(6*c^2*d^2 + b^2*e^2 + 2*c*e*(-3
*b*d + a*e))*(d + e*x)^2*Log[d + e*x])/(2*e^5*(d + e*x)^2)
```

Maple [A] time = 0.048, size = 266, normalized size = 1.9

$$\frac{c^2x^2}{2e^3}+2\frac{bcx}{e^3}-3\frac{c^2dx}{e^4}-\frac{a^2}{2e(ex+d)^2}+\frac{abd}{e^2(ex+d)^2}-\frac{acd^2}{e^3(ex+d)^2}-\frac{b^2d^2}{2e^3(ex+d)^2}+\frac{d^3bc}{e^4(ex+d)^2}-\frac{c^2d^4}{2e^5(ex+d)^2}+2\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(e*x+d)^3,x)
```

```
[Out] 1/2*c^2*x^2/e^3+2*c/e^3*x*b-3*c^2*d*x/e^4-1/2/e/(e*x+d)^2*a^2+1/e^2/(e*x+d)
^2*d*a*b-1/e^3/(e*x+d)^2*a*c*d^2-1/2/e^3/(e*x+d)^2*d^2*b^2+1/e^4/(e*x+d)^2*
d^3*b*c-1/2/e^5/(e*x+d)^2*c^2*d^4+2/e^3*ln(e*x+d)*a*c+b^2*ln(e*x+d)/e^3-6/e
^4*ln(e*x+d)*b*c*d+6/e^5*ln(e*x+d)*c^2*d^2-2/e^2/(e*x+d)*a*b+4/e^3/(e*x+d)*
a*d*c+2/e^3/(e*x+d)*b^2*d-6/e^4/(e*x+d)*d^2*b*c+4/e^5/(e*x+d)*c^2*d^3
```

Maxima [A] time = 1.02102, size = 250, normalized size = 1.81

$$\frac{7c^2d^4-10bcd^3e-2abde^3-a^2e^4+3\left(b^2+2ac\right)d^2e^2+4\left(2c^2d^3e-3bcd^2e^2-abe^4+\left(b^2+2ac\right)de^3\right)x}{2\left(e^7x^2+2de^6x+d^2e^5\right)}+\frac{c^2ex^2-2\left(3c^2d\right.}{2e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(7*c^2*d^4 - 10*b*c*d^3*e - 2*a*b*d*e^3 - a^2*e^4 + 3*(b^2 + 2*a*c)*d^2
*e^2 + 4*(2*c^2*d^3*e - 3*b*c*d^2*e^2 - a*b*e^4 + (b^2 + 2*a*c)*d*e^3)*x)/(
e^7*x^2 + 2*d*e^6*x + d^2*e^5) + 1/2*(c^2*e*x^2 - 2*(3*c^2*d - 2*b*c*e)*x)/
e^4 + (6*c^2*d^2 - 6*b*c*d*e + (b^2 + 2*a*c)*e^2)*log(e*x + d)/e^5
```

Fricas [B] time = 2.10841, size = 602, normalized size = 4.36

$$c^2e^4x^4+7c^2d^4-10bcd^3e-2abde^3-a^2e^4+3\left(b^2+2ac\right)d^2e^2-4\left(c^2de^3-bce^4\right)x^3-\left(11c^2d^2e^2-8bcde^3\right)x^2+2\left(c^2d^3e-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(c^2*e^4*x^4 + 7*c^2*d^4 - 10*b*c*d^3*e - 2*a*b*d*e^3 - a^2*e^4 + 3*(b^
2 + 2*a*c)*d^2*e^2 - 4*(c^2*d*e^3 - b*c*e^4)*x^3 - (11*c^2*d^2*e^2 - 8*b*c*
d*e^3)*x^2 + 2*(c^2*d^3*e - 4*b*c*d^2*e^2 - 2*a*b*e^4 + 2*(b^2 + 2*a*c)*d*e
^3)*x + 2*(6*c^2*d^4 - 6*b*c*d^3*e + (b^2 + 2*a*c)*d^2*e^2 + (6*c^2*d^2*e^2
 - 6*b*c*d*e^3 + (b^2 + 2*a*c)*e^4)*x^2 + 2*(6*c^2*d^3*e - 6*b*c*d^2*e^2 +
(b^2 + 2*a*c)*d*e^3)*x)*log(e*x + d))/(e^7*x^2 + 2*d*e^6*x + d^2*e^5)
```

Sympy [A] time = 2.88416, size = 209, normalized size = 1.51

$$\frac{c^2x^2}{2e^3} - \frac{a^2e^4 + 2abde^3 - 6acd^2e^2 - 3b^2d^2e^2 + 10bcd^3e - 7c^2d^4 + x\left(4abe^4 - 8acde^3 - 4b^2de^3 + 12bcd^2e^2 - 8c^2d^3e\right)}{2d^2e^5 + 4de^6x + 2e^7x^2} + \frac{x\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(e*x+d)**3,x)
```

```
[Out] c**2*x**2/(2*e**3) - (a**2*e**4 + 2*a*b*d*e**3 - 6*a*c*d**2*e**2 - 3*b**2*d
**2*e**2 + 10*b*c*d**3*e - 7*c**2*d**4 + x*(4*a*b*e**4 - 8*a*c*d*e**3 - 4*b
**2*d*e**3 + 12*b*c*d**2*e**2 - 8*c**2*d**3*e))/(2*d**2*e**5 + 4*d*e**6*x +
 2*e**7*x**2) + x*(2*b*c*e - 3*c**2*d)/e**4 + (2*a*c*e**2 + b**2*e**2 - 6*b
*c*d*e + 6*c**2*d**2)*log(d + e*x)/e**5
```

Giac [A] time = 1.13492, size = 238, normalized size = 1.72

$$\left(6\,c^2d^2 - 6\,bcde + b^2e^2 + 2\,ace^2\right)e^{(-5)}\log\left(|xe + d|\right) + \frac{1}{2}\left(c^2x^2e^3 - 6\,c^2dxe^2 + 4\,bcxe^3\right)e^{(-6)} + \frac{\left(7\,c^2d^4 - 10\,bcd^3e + 3\,b^2d^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] (6*c^2*d^2 - 6*b*c*d*e + b^2*e^2 + 2*a*c*e^2)*e^(-5)*log(abs(x*e + d)) + 1/
2*(c^2*x^2*e^3 - 6*c^2*d*x*e^2 + 4*b*c*x*e^3)*e^(-6) + 1/2*(7*c^2*d^4 - 10*
b*c*d^3*e + 3*b^2*d^2*e^2 + 6*a*c*d^2*e^2 - 2*a*b*d*e^3 - a^2*e^4 + 4*(2*c^
2*d^3*e - 3*b*c*d^2*e^2 + b^2*d*e^3 + 2*a*c*d*e^3 - a*b*e^4)*x)*e^(-5)/(x*e
 + d)^2
```

3.2126 $\int \frac{(a+bx+cx^2)^2}{(d+ex)^4} dx$

Optimal. Leaf size=139

$$-\frac{-2ce(3bd-ae)+b^2e^2+6c^2d^2}{e^5(d+ex)}+\frac{(2cd-be)\left(ae^2-bde+cd^2\right)}{e^5(d+ex)^2}-\frac{\left(ae^2-bde+cd^2\right)^2}{3e^5(d+ex)^3}-\frac{2c(2cd-be)\log(d+ex)}{e^5}+\frac{c^2x}{e^4}$$

```
[Out] (c^2*x)/e^4 - (c*d^2 - b*d*e + a*e^2)^2/(3*e^5*(d + e*x)^3) + ((2*c*d - b*e
)*(c*d^2 - b*d*e + a*e^2))/(e^5*(d + e*x)^2) - (6*c^2*d^2 + b^2*e^2 - 2*c*e
*(3*b*d - a*e))/(e^5*(d + e*x)) - (2*c*(2*c*d - b*e)*Log[d + e*x])/e^5
```

Rubi [A] time = 0.122614, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$-\frac{-2ce(3bd-ae)+b^2e^2+6c^2d^2}{e^5(d+ex)}+\frac{(2cd-be)\left(ae^2-bde+cd^2\right)}{e^5(d+ex)^2}-\frac{\left(ae^2-bde+cd^2\right)^2}{3e^5(d+ex)^3}-\frac{2c(2cd-be)\log(d+ex)}{e^5}+\frac{c^2x}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(d + e*x)^4,x]
```

```
[Out] (c^2*x)/e^4 - (c*d^2 - b*d*e + a*e^2)^2/(3*e^5*(d + e*x)^3) + ((2*c*d - b*e
)*(c*d^2 - b*d*e + a*e^2))/(e^5*(d + e*x)^2) - (6*c^2*d^2 + b^2*e^2 - 2*c*e
*(3*b*d - a*e))/(e^5*(d + e*x)) - (2*c*(2*c*d - b*e)*Log[d + e*x])/e^5
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^2}{(d+ex)^4}\,dx = \int \left(\frac{c^2}{e^4}+\frac{\left(cd^2-bde+ae^2\right)^2}{e^4(d+ex)^4}+\frac{2(-2cd+be)\left(cd^2-bde+ae^2\right)}{e^4(d+ex)^3}+\frac{6c^2d^2+b^2e^2-2ce(3bd-ae)}{e^4(d+ex)^2}-\right.$$

$$=\frac{c^2x}{e^4}-\frac{\left(cd^2-bde+ae^2\right)^2}{3e^5(d+ex)^3}+\frac{(2cd-be)\left(cd^2-bde+ae^2\right)}{e^5(d+ex)^2}-\frac{6c^2d^2+b^2e^2-2ce(3bd-ae)}{e^5(d+ex)}-\frac{2c(2c}{}$$

Mathematica [A] time = 0.0832742, size = 176, normalized size = 1.27

$$\frac{-e^2\left(a^2e^2+abe(d+3ex)+b^2\left(d^2+3dex+3e^2x^2\right)\right)+ce\left(bd\left(11d^2+27dex+18e^2x^2\right)-2ae\left(d^2+3dex+3e^2x^2\right)\right)-6c(d+}{3e^5(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(d + e*x)^4,x]
```

```
[Out] (c^2*(-13*d^4 - 27*d^3*e*x - 9*d^2*e^2*x^2 + 9*d*e^3*x^3 + 3*e^4*x^4) - e^2
*(a^2*e^2 + a*b*e*(d + 3*e*x) + b^2*(d^2 + 3*d*e*x + 3*e^2*x^2)) + c*e*(-2*
a*e*(d^2 + 3*d*e*x + 3*e^2*x^2) + b*d*(11*d^2 + 27*d*e*x + 18*e^2*x^2)) - 6
*c*(2*c*d - b*e)*(d + e*x)^3*Log[d + e*x])/(3*e^5*(d + e*x)^3)
```

Maple [B] time = 0.046, size = 279, normalized size = 2.

$$\frac{c^2x}{e^4} - \frac{a^2}{3\,e\,(ex+d)^3} + \frac{2\,abd}{3\,e^2\,(ex+d)^3} - \frac{2\,acd^2}{3\,e^3\,(ex+d)^3} - \frac{b^2d^2}{3\,e^3\,(ex+d)^3} + \frac{2\,d^3bc}{3\,e^4\,(ex+d)^3} - \frac{c^2d^4}{3\,e^5\,(ex+d)^3} - \frac{ab}{e^2\,(ex+d)^2} + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(e*x+d)^4,x)
```

```
[Out] c^2*x/e^4-1/3/e/(e*x+d)^3*a^2+2/3/e^2/(e*x+d)^3*d*a*b-2/3/e^3/(e*x+d)^3*a*c
*d^2-1/3/e^3/(e*x+d)^3*d^2*b^2+2/3/e^4/(e*x+d)^3*d^3*b*c-1/3/e^5/(e*x+d)^3*
c^2*d^4-1/e^2/(e*x+d)^2*a*b+2/e^3/(e*x+d)^2*a*d*c+1/e^3/(e*x+d)^2*b^2*d-3/e
^4/(e*x+d)^2*d^2*b*c+2/e^5/(e*x+d)^2*c^2*d^3+2*c/e^4*ln(e*x+d)*b-4*c^2*d*ln
(e*x+d)/e^5-2/e^3/(e*x+d)*a*c-1/e^3/(e*x+d)*b^2+6/e^4/(e*x+d)*b*c*d-6/e^5/(
e*x+d)*c^2*d^2
```

Maxima [A] time = 0.993387, size = 262, normalized size = 1.88

$$-\frac{13\,c^2d^4 - 11\,bcd^3e + abde^3 + a^2e^4 + \left(b^2 + 2\,ac\right)d^2e^2 + 3\left(6\,c^2d^2e^2 - 6\,bcde^3 + \left(b^2 + 2\,ac\right)e^4\right)x^2 + 3\left(10\,c^2d^3e - 9\,bcd^2e\right.}{3\left(e^8x^3 + 3\,de^7x^2 + 3\,d^2e^6x + d^3e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(13*c^2*d^4 - 11*b*c*d^3*e + a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e
^2 + 3*(6*c^2*d^2*e^2 - 6*b*c*d*e^3 + (b^2 + 2*a*c)*e^4)*x^2 + 3*(10*c^2*d^
3*e - 9*b*c*d^2*e^2 + a*b*e^4 + (b^2 + 2*a*c)*d*e^3)*x)/(e^8*x^3 + 3*d*e^7*
x^2 + 3*d^2*e^6*x + d^3*e^5) + c^2*x/e^4 - 2*(2*c^2*d - b*c*e)*log(e*x + d)
/e^5
```

Fricas [B] time = 1.94339, size = 578, normalized size = 4.16

$$\frac{3\,c^2e^4x^4 + 9\,c^2de^3x^3 - 13\,c^2d^4 + 11\,bcd^3e - abde^3 - a^2e^4 - \left(b^2 + 2\,ac\right)d^2e^2 - 3\left(3\,c^2d^2e^2 - 6\,bcde^3 + \left(b^2 + 2\,ac\right)e^4\right)x^2 -}{3(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*c^2*e^4*x^4 + 9*c^2*d*e^3*x^3 - 13*c^2*d^4 + 11*b*c*d^3*e - a*b*d*e^
3 - a^2*e^4 - (b^2 + 2*a*c)*d^2*e^2 - 3*(3*c^2*d^2*e^2 - 6*b*c*d*e^3 + (b^2
 + 2*a*c)*e^4)*x^2 - 3*(9*c^2*d^3*e - 9*b*c*d^2*e^2 + a*b*e^4 + (b^2 + 2*a*
c)*d*e^3)*x - 6*(2*c^2*d^4 - b*c*d^3*e + (2*c^2*d*e^3 - b*c*e^4)*x^3 + 3*(2
*c^2*d^2*e^2 - b*c*d*e^3)*x^2 + 3*(2*c^2*d^3*e - b*c*d^2*e^2)*x)*log(e*x +
```

```
d))/(e^8*x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3*e^5)
```

Sympy [A] time = 7.45517, size = 218, normalized size = 1.57

$$\frac{c^2x}{e^4}+\frac{2c(be-2cd)\log(d+ex)}{e^5}-\frac{a^2e^4+abde^3+2acd^2e^2+b^2d^2e^2-11bcd^3e+13c^2d^4+x^2\left(6ace^4+3b^2e^4-18bcde^3+1\right.}{3d^3e^5+9d^2e^6x+9de^7x^2+3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(e*x+d)**4,x)
```

```
[Out] c**2*x/e**4 + 2*c*(b*e - 2*c*d)*log(d + e*x)/e**5 - (a**2*e**4 + a*b*d*e**3
 + 2*a*c*d**2*e**2 + b**2*d**2*e**2 - 11*b*c*d**3*e + 13*c**2*d**4 + x**2*(
6*a*c*e**4 + 3*b**2*e**4 - 18*b*c*d*e**3 + 18*c**2*d**2*e**2) + x*(3*a*b*e*
*4 + 6*a*c*d*e**3 + 3*b**2*d*e**3 - 27*b*c*d**2*e**2 + 30*c**2*d**3*e))/(3*
d**3*e**5 + 9*d**2*e**6*x + 9*d*e**7*x**2 + 3*e**8*x**3)
```

Giac [A] time = 1.10542, size = 230, normalized size = 1.65

$$c^2xe^{(-4)}-2\left(2c^2d-bce\right)e^{(-5)}\log(|xe+d|)-\frac{\left(13c^2d^4-11bcd^3e+b^2d^2e^2+2acd^2e^2+abde^3+3\left(6c^2d^2e^2-6bcde^3+b^2e\right.\right.}{3(xe}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] c^2*x*e^(-4) - 2*(2*c^2*d - b*c*e)*e^(-5)*log(abs(x*e + d)) - 1/3*(13*c^2*d
^4 - 11*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 + a*b*d*e^3 + 3*(6*c^2*d^2*
e^2 - 6*b*c*d*e^3 + b^2*e^4 + 2*a*c*e^4)*x^2 + a^2*e^4 + 3*(10*c^2*d^3*e -
9*b*c*d^2*e^2 + b^2*d*e^3 + 2*a*c*d*e^3 + a*b*e^4)*x)*e^(-5)/(x*e + d)^3
```

3.2127 $\int \frac{(a+bx+cx^2)^2}{(d+ex)^5} dx$

Optimal. Leaf size=150

$$-\frac{-2ce(3bd-ae)+b^2e^2+6c^2d^2}{2e^5(d+ex)^2}+\frac{2(2cd-be)\left(ae^2-bde+cd^2\right)}{3e^5(d+ex)^3}-\frac{\left(ae^2-bde+cd^2\right)^2}{4e^5(d+ex)^4}+\frac{2c(2cd-be)}{e^5(d+ex)}+\frac{c^2\log(d+ex)}{e^5}$$

```
[Out] -(c*d^2 - b*d*e + a*e^2)^2/(4*e^5*(d + e*x)^4) + (2*(2*c*d - b*e)*(c*d^2 -
b*d*e + a*e^2))/(3*e^5*(d + e*x)^3) - (6*c^2*d^2 + b^2*e^2 - 2*c*e*(3*b*d -
 a*e))/(2*e^5*(d + e*x)^2) + (2*c*(2*c*d - b*e))/(e^5*(d + e*x)) + (c^2*Log
[d + e*x])/e^5
```

Rubi [A] time = 0.12168, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$-\frac{-2ce(3bd-ae)+b^2e^2+6c^2d^2}{2e^5(d+ex)^2}+\frac{2(2cd-be)\left(ae^2-bde+cd^2\right)}{3e^5(d+ex)^3}-\frac{\left(ae^2-bde+cd^2\right)^2}{4e^5(d+ex)^4}+\frac{2c(2cd-be)}{e^5(d+ex)}+\frac{c^2\log(d+ex)}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(d + e*x)^5,x]
```



```
[Out] -(c*d^2 - b*d*e + a*e^2)^2/(4*e^5*(d + e*x)^4) + (2*(2*c*d - b*e)*(c*d^2 -
b*d*e + a*e^2))/(3*e^5*(d + e*x)^3) - (6*c^2*d^2 + b^2*e^2 - 2*c*e*(3*b*d -
 a*e))/(2*e^5*(d + e*x)^2) + (2*c*(2*c*d - b*e))/(e^5*(d + e*x)) + (c^2*Log
[d + e*x])/e^5
```


Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^2}{(d+ex)^5}\,dx = \int \left(\frac{\left(cd^2-bde+ae^2\right)^2}{e^4(d+ex)^5}+\frac{2(-2cd+be)\left(cd^2-bde+ae^2\right)}{e^4(d+ex)^4}+\frac{6c^2d^2+b^2e^2-2ce(3bd-ae)}{e^4(d+ex)^3}-\frac{2c}{e}\right.$$

$$= -\frac{\left(cd^2-bde+ae^2\right)^2}{4e^5(d+ex)^4}+\frac{2(2cd-be)\left(cd^2-bde+ae^2\right)}{3e^5(d+ex)^3}-\frac{6c^2d^2+b^2e^2-2ce(3bd-ae)}{2e^5(d+ex)^2}+\frac{2c(2cd}{e^5(d}$$

Mathematica [A] time = 0.0728061, size = 170, normalized size = 1.13

$$\frac{-e^2\left(3a^2e^2+2abe(d+4ex)+b^2\left(d^2+4dex+6e^2x^2\right)\right)-2ce\left(ae\left(d^2+4dex+6e^2x^2\right)+3b\left(4d^2ex+d^3+6de^2x^2+4e^3x\right.\right.}{12e^5(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(d + e*x)^5,x]
```

```
[Out] (c^2*d*(25*d^3 + 88*d^2*e*x + 108*d*e^2*x^2 + 48*e^3*x^3) - e^2*(3*a^2*e^2
+ 2*a*b*e*(d + 4*e*x) + b^2*(d^2 + 4*d*e*x + 6*e^2*x^2)) - 2*c*e*(a*e*(d^2
+ 4*d*e*x + 6*e^2*x^2) + 3*b*(d^3 + 4*d^2*e*x + 6*d*e^2*x^2 + 4*e^3*x^3)) +
 12*c^2*(d + e*x)^4*Log[d + e*x])/(12*e^5*(d + e*x)^4)
```

Maple [A] time = 0.048, size = 287, normalized size = 1.9

$$-\frac{a^2}{4\,e\,(ex+d)^4}+\frac{abd}{2\,e^2\,(ex+d)^4}-\frac{acd^2}{2\,e^3\,(ex+d)^4}-\frac{b^2d^2}{4\,e^3\,(ex+d)^4}+\frac{d^3bc}{2\,e^4\,(ex+d)^4}-\frac{c^2d^4}{4\,e^5\,(ex+d)^4}-\frac{2\,ab}{3\,e^2\,(ex+d)^3}+\frac{4\,a}{3\,e^3\,(ex}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(e*x+d)^5,x)
```

```
[Out] -1/4/e/(e*x+d)^4*a^2+1/2/e^2/(e*x+d)^4*d*a*b-1/2/e^3/(e*x+d)^4*a*c*d^2-1/4/
e^3/(e*x+d)^4*b^2*d^2+1/2/e^4/(e*x+d)^4*d^3*b*c-1/4/e^5/(e*x+d)^4*c^2*d^4-2
/3/e^2/(e*x+d)^3*a*b+4/3/e^3/(e*x+d)^3*a*d*c+2/3/e^3/(e*x+d)^3*b^2*d-2/e^4/
(e*x+d)^3*d^2*b*c+4/3/e^5/(e*x+d)^3*c^2*d^3-1/e^3/(e*x+d)^2*a*c-1/2*b^2/e^3
/(e*x+d)^2+3/e^4/(e*x+d)^2*b*c*d-3/e^5/(e*x+d)^2*c^2*d^2+c^2*ln(e*x+d)/e^5-
2*c/e^4/(e*x+d)*b+4*c^2*d/e^5/(e*x+d)
```

Maxima [A] time = 1.00033, size = 290, normalized size = 1.93

$$\frac{25\,c^2d^4-6\,bcd^3e-2\,abde^3-3\,a^2e^4-\left(b^2+2\,ac\right)d^2e^2+24\left(2\,c^2de^3-bce^4\right)x^3+6\left(18\,c^2d^2e^2-6\,bcde^3-\left(b^2+2\,ac\right)e^4\right)x^2+}{12\left(e^9x^4+4\,de^8x^3+6\,d^2e^7x^2+4\,d^3e^6x+d^4e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^5,x, algorithm="maxima")
```

```
[Out] 1/12*(25*c^2*d^4 - 6*b*c*d^3*e - 2*a*b*d*e^3 - 3*a^2*e^4 - (b^2 + 2*a*c)*d^
2*e^2 + 24*(2*c^2*d*e^3 - b*c*e^4)*x^3 + 6*(18*c^2*d^2*e^2 - 6*b*c*d*e^3 -
(b^2 + 2*a*c)*e^4)*x^2 + 4*(22*c^2*d^3*e - 6*b*c*d^2*e^2 - 2*a*b*e^4 - (b^2
 + 2*a*c)*d*e^3)*x)/(e^9*x^4 + 4*d*e^8*x^3 + 6*d^2*e^7*x^2 + 4*d^3*e^6*x +
d^4*e^5) + c^2*log(e*x + d)/e^5
```

Fricas [A] time = 2.02727, size = 548, normalized size = 3.65

$$\frac{25\,c^2d^4-6\,bcd^3e-2\,abde^3-3\,a^2e^4-\left(b^2+2\,ac\right)d^2e^2+24\left(2\,c^2de^3-bce^4\right)x^3+6\left(18\,c^2d^2e^2-6\,bcde^3-\left(b^2+2\,ac\right)e^4\right)x^2+}{12\left(e^9x^4+4\,de^8x^3+6\,d^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(25*c^2*d^4 - 6*b*c*d^3*e - 2*a*b*d*e^3 - 3*a^2*e^4 - (b^2 + 2*a*c)*d^
2*e^2 + 24*(2*c^2*d*e^3 - b*c*e^4)*x^3 + 6*(18*c^2*d^2*e^2 - 6*b*c*d*e^3 -
(b^2 + 2*a*c)*e^4)*x^2 + 4*(22*c^2*d^3*e - 6*b*c*d^2*e^2 - 2*a*b*e^4 - (b^2
```

```
+ 2*a*c)*d*e^3)*x + 12*(c^2*e^4*x^4 + 4*c^2*d*e^3*x^3 + 6*c^2*d^2*e^2*x^2
+ 4*c^2*d^3*e*x + c^2*d^4)*log(e*x + d))/(e^9*x^4 + 4*d*e^8*x^3 + 6*d^2*e^7
*x^2 + 4*d^3*e^6*x + d^4*e^5)
```

Sympy [A] time = 17.1165, size = 238, normalized size = 1.59

$$\frac{c^2\log(d+ex)}{e^5} - \frac{3a^2e^4 + 2abde^3 + 2acd^2e^2 + b^2d^2e^2 + 6bcd^3e - 25c^2d^4 + x^3\left(24bce^4 - 48c^2de^3\right) + x^2\left(12ace^4 + 6b^2e^4 + \cdots\right.}{12d^4e^5 + 48d^3e^6x + 72d^2e^7x^2 + 48de\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(e*x+d)**5,x)
```

```
[Out] c**2*log(d + e*x)/e**5 - (3*a**2*e**4 + 2*a*b*d*e**3 + 2*a*c*d**2*e**2 + b*
*2*d**2*e**2 + 6*b*c*d**3*e - 25*c**2*d**4 + x**3*(24*b*c*e**4 - 48*c**2*d*
e**3) + x**2*(12*a*c*e**4 + 6*b**2*e**4 + 36*b*c*d*e**3 - 108*c**2*d**2*e**
2) + x*(8*a*b*e**4 + 8*a*c*d*e**3 + 4*b**2*d*e**3 + 24*b*c*d**2*e**2 - 88*c
**2*d**3*e))/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8
*x**3 + 12*e**9*x**4)
```

Giac [B] time = 1.12609, size = 410, normalized size = 2.73

$$-c^2e^{(-5)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^2}\right) + \frac{1}{12}\left(\frac{48\,c^2de^{15}}{xe+d} - \frac{36\,c^2d^2e^{15}}{(xe+d)^2} + \frac{16\,c^2d^3e^{15}}{(xe+d)^3} - \frac{3\,c^2d^4e^{15}}{(xe+d)^4} - \frac{24\,bce^{16}}{xe+d} + \frac{36\,bcde^{16}}{(xe+d)^2} - \frac{24\,bcd^2e^{16}}{(xe+d)^3} \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] -c^2*e^(-5)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2) + 1/12*(48*c^2*d*e^15/(x*e
 + d) - 36*c^2*d^2*e^15/(x*e + d)^2 + 16*c^2*d^3*e^15/(x*e + d)^3 - 3*c^2*d
^4*e^15/(x*e + d)^4 - 24*b*c*e^16/(x*e + d) + 36*b*c*d*e^16/(x*e + d)^2 - 2
4*b*c*d^2*e^16/(x*e + d)^3 + 6*b*c*d^3*e^16/(x*e + d)^4 - 6*b^2*e^17/(x*e +
 d)^2 - 12*a*c*e^17/(x*e + d)^2 + 8*b^2*d*e^17/(x*e + d)^3 + 16*a*c*d*e^17/
(x*e + d)^3 - 3*b^2*d^2*e^17/(x*e + d)^4 - 6*a*c*d^2*e^17/(x*e + d)^4 - 8*a
*b*e^18/(x*e + d)^3 + 6*a*b*d*e^18/(x*e + d)^4 - 3*a^2*e^19/(x*e + d)^4)*e^
(-20)
```

3.2128 $\int \frac{(a+bx+cx^2)^2}{(d+ex)^6} dx$

Optimal. Leaf size=151

$$-\frac{-2ce(3bd-ae)+b^2e^2+6c^2d^2}{3e^5(d+ex)^3}+\frac{(2cd-be)\left(ae^2-bde+cd^2\right)}{2e^5(d+ex)^4}-\frac{\left(ae^2-bde+cd^2\right)^2}{5e^5(d+ex)^5}+\frac{c(2cd-be)}{e^5(d+ex)^2}-\frac{c^2}{e^5(d+ex)}$$

```
[Out] -(c*d^2 - b*d*e + a*e^2)^2/(5*e^5*(d + e*x)^5) + ((2*c*d - b*e)*(c*d^2 - b*
d*e + a*e^2))/(2*e^5*(d + e*x)^4) - (6*c^2*d^2 + b^2*e^2 - 2*c*e*(3*b*d - a
*e))/(3*e^5*(d + e*x)^3) + (c*(2*c*d - b*e))/(e^5*(d + e*x)^2) - c^2/(e^5*(
d + e*x))
```

Rubi [A] time = 0.111422, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$-\frac{-2ce(3bd-ae)+b^2e^2+6c^2d^2}{3e^5(d+ex)^3}+\frac{(2cd-be)\left(ae^2-bde+cd^2\right)}{2e^5(d+ex)^4}-\frac{\left(ae^2-bde+cd^2\right)^2}{5e^5(d+ex)^5}+\frac{c(2cd-be)}{e^5(d+ex)^2}-\frac{c^2}{e^5(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(d + e*x)^6,x]
```

```
[Out] -(c*d^2 - b*d*e + a*e^2)^2/(5*e^5*(d + e*x)^5) + ((2*c*d - b*e)*(c*d^2 - b*
d*e + a*e^2))/(2*e^5*(d + e*x)^4) - (6*c^2*d^2 + b^2*e^2 - 2*c*e*(3*b*d - a
*e))/(3*e^5*(d + e*x)^3) + (c*(2*c*d - b*e))/(e^5*(d + e*x)^2) - c^2/(e^5*(
d + e*x))
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^2}{(d+ex)^6}\,dx=\int \left(\frac{\left(cd^2-bde+ae^2\right)^2}{e^4(d+ex)^6}+\frac{2(-2cd+be)\left(cd^2-bde+ae^2\right)}{e^4(d+ex)^5}+\frac{6c^2d^2+b^2e^2-2ce(3bd-ae)}{e^4(d+ex)^4}-\frac{2c(2c}{e^4(d}\right.$$
$$=-\frac{\left(cd^2-bde+ae^2\right)^2}{5e^5(d+ex)^5}+\frac{(2cd-be)\left(cd^2-bde+ae^2\right)}{2e^5(d+ex)^4}-\frac{6c^2d^2+b^2e^2-2ce(3bd-ae)}{3e^5(d+ex)^3}+\frac{c(2cd-be}{e^5(d+ex)^2}$$

Mathematica [A] time = 0.0722244, size = 160, normalized size = 1.06

$$-\frac{e^2\left(6a^2e^2+3abe(d+5ex)+b^2\left(d^2+5dex+10e^2x^2\right)\right)+ce\left(2ae\left(d^2+5dex+10e^2x^2\right)+3b\left(5d^2ex+d^3+10de^2x^2+10e^3\right.\right.}{30e^5(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(d + e*x)^6,x]
```

```
[Out] -(6*c^2*(d^4 + 5*d^3*e*x + 10*d^2*e^2*x^2 + 10*d*e^3*x^3 + 5*e^4*x^4) + e^2
*(6*a^2*e^2 + 3*a*b*e*(d + 5*e*x) + b^2*(d^2 + 5*d*e*x + 10*e^2*x^2)) + c*e
*(2*a*e*(d^2 + 5*d*e*x + 10*e^2*x^2) + 3*b*(d^3 + 5*d^2*e*x + 10*d*e^2*x^2
+ 10*e^3*x^3)))/(30*e^5*(d + e*x)^5)
```

Maple [A] time = 0.044, size = 195, normalized size = 1.3

$$-\frac{2abe^3-4ade^2c-2b^2de^2+6d^2ebc-4c^2d^3}{4e^5(ex+d)^4}-\frac{2ace^2+b^2e^2-6bcde+6c^2d^2}{3e^5(ex+d)^3}-\frac{c(be-2cd)}{e^5(ex+d)^2}-\frac{c^2}{e^5(ex+d)}-\frac{a^2e^4-2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(e*x+d)^6,x)
```

```
[Out] -1/4*(2*a*b*e^3-4*a*c*d*e^2-2*b^2*d*e^2+6*b*c*d^2*e-4*c^2*d^3)/e^5/(e*x+d)^
4-1/3*(2*a*c*e^2+b^2*e^2-6*b*c*d*e+6*c^2*d^2)/e^5/(e*x+d)^3-c*(b*e-2*c*d)/e
^5/(e*x+d)^2-c^2/e^5/(e*x+d)-1/5*(a^2*e^4-2*a*b*d*e^3+2*a*c*d^2*e^2+b^2*d^2
*e^2-2*b*c*d^3*e+c^2*d^4)/e^5/(e*x+d)^5
```

Maxima [A] time = 0.987873, size = 296, normalized size = 1.96

$$-\frac{30c^2e^4x^4+6c^2d^4+3bcd^3e+3abde^3+6a^2e^4+(b^2+2ac)d^2e^2+30(2c^2de^3+bce^4)x^3+10(6c^2d^2e^2+3bcde^3+(}{30(e^{10}x^5+5de^9x^4+10d^2e^8x^3+10d^3e^7x^2+5d^4e^6x+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^6,x, algorithm="maxima")
```

```
[Out] -1/30*(30*c^2*e^4*x^4 + 6*c^2*d^4 + 3*b*c*d^3*e + 3*a*b*d*e^3 + 6*a^2*e^4 +
 (b^2 + 2*a*c)*d^2*e^2 + 30*(2*c^2*d*e^3 + b*c*e^4)*x^3 + 10*(6*c^2*d^2*e^2
 + 3*b*c*d*e^3 + (b^2 + 2*a*c)*e^4)*x^2 + 5*(6*c^2*d^3*e + 3*b*c*d^2*e^2 +
3*a*b*e^4 + (b^2 + 2*a*c)*d*e^3)*x)/(e^10*x^5 + 5*d*e^9*x^4 + 10*d^2*e^8*x^
3 + 10*d^3*e^7*x^2 + 5*d^4*e^6*x + d^5*e^5)
```

Fricas [A] time = 2.03934, size = 466, normalized size = 3.09

$$-\frac{30c^2e^4x^4+6c^2d^4+3bcd^3e+3abde^3+6a^2e^4+(b^2+2ac)d^2e^2+30(2c^2de^3+bce^4)x^3+10(6c^2d^2e^2+3bcde^3+(}{30(e^{10}x^5+5de^9x^4+10d^2e^8x^3+10d^3e^7x^2+5d^4e^6x+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^6,x, algorithm="fricas")
```

```
[Out] -1/30*(30*c^2*e^4*x^4 + 6*c^2*d^4 + 3*b*c*d^3*e + 3*a*b*d*e^3 + 6*a^2*e^4 +
 (b^2 + 2*a*c)*d^2*e^2 + 30*(2*c^2*d*e^3 + b*c*e^4)*x^3 + 10*(6*c^2*d^2*e^2
 + 3*b*c*d*e^3 + (b^2 + 2*a*c)*e^4)*x^2 + 5*(6*c^2*d^3*e + 3*b*c*d^2*e^2 +
3*a*b*e^4 + (b^2 + 2*a*c)*d*e^3)*x)/(e^10*x^5 + 5*d*e^9*x^4 + 10*d^2*e^8*x^
3 + 10*d^3*e^7*x^2 + 5*d^4*e^6*x + d^5*e^5)
```

Sympy [A] time = 38.7147, size = 250, normalized size = 1.66

$$-\frac{6a^2e^4+3abde^3+2acd^2e^2+b^2d^2e^2+3bcd^3e+6c^2d^4+30c^2e^4x^4+x^3\left(30bce^4+60c^2de^3\right)+x^2\left(20ace^4+10b^2e^4+30bcd\right.}{30d^5e^5+150d^4e^6x+300d^3e^7x^2+300d^2e^8x^3+150de^9x^4+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(e*x+d)**6,x)
```

```
[Out] -(6*a**2*e**4 + 3*a*b*d*e**3 + 2*a*c*d**2*e**2 + b**2*d**2*e**2 + 3*b*c*d**
3*e + 6*c**2*d**4 + 30*c**2*e**4*x**4 + x**3*(30*b*c*e**4 + 60*c**2*d*e**3)
 + x**2*(20*a*c*e**4 + 10*b**2*e**4 + 30*b*c*d*e**3 + 60*c**2*d**2*e**2) +
x*(15*a*b*e**4 + 10*a*c*d*e**3 + 5*b**2*d*e**3 + 15*b*c*d**2*e**2 + 30*c**2
*d**3*e))/(30*d**5*e**5 + 150*d**4*e**6*x + 300*d**3*e**7*x**2 + 300*d**2*e
**8*x**3 + 150*d*e**9*x**4 + 30*e**10*x**5)
```

Giac [A] time = 1.11779, size = 242, normalized size = 1.6

$$-\frac{\left(30\,c^2x^4e^4+60\,c^2dx^3e^3+60\,c^2d^2x^2e^2+30\,c^2d^3xe+6\,c^2d^4+30\,bcx^3e^4+30\,bcdx^2e^3+15\,bcd^2xe^2+3\,bcd^3e+10\,b^2x^2e^4\right.}{30\,(xe+d)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^6,x, algorithm="giac")
```

```
[Out] -1/30*(30*c^2*x^4*e^4 + 60*c^2*d*x^3*e^3 + 60*c^2*d^2*x^2*e^2 + 30*c^2*d^3*
x*e + 6*c^2*d^4 + 30*b*c*x^3*e^4 + 30*b*c*d*x^2*e^3 + 15*b*c*d^2*x*e^2 + 3*
b*c*d^3*e + 10*b^2*x^2*e^4 + 20*a*c*x^2*e^4 + 5*b^2*d*x*e^3 + 10*a*c*d*x*e^
3 + b^2*d^2*e^2 + 2*a*c*d^2*e^2 + 15*a*b*x*e^4 + 3*a*b*d*e^3 + 6*a^2*e^4)*e
^(-5)/(x*e + d)^5
```

3.2129 $\int \frac{(a+bx+cx^2)^2}{(d+ex)^7}\,dx$

Optimal. Leaf size=156

$$-\frac{-2ce(3bd-ae)+b^2e^2+6c^2d^2}{4e^5(d+ex)^4}+\frac{2(2cd-be)\left(ae^2-bde+cd^2\right)}{5e^5(d+ex)^5}-\frac{\left(ae^2-bde+cd^2\right)^2}{6e^5(d+ex)^6}+\frac{2c(2cd-be)}{3e^5(d+ex)^3}-\frac{c^2}{2e^5(d+ex)^2}$$

```
[Out] -(c*d^2 - b*d*e + a*e^2)^2/(6*e^5*(d + e*x)^6) + (2*(2*c*d - b*e)*(c*d^2 -
b*d*e + a*e^2))/(5*e^5*(d + e*x)^5) - (6*c^2*d^2 + b^2*e^2 - 2*c*e*(3*b*d -
 a*e))/(4*e^5*(d + e*x)^4) + (2*c*(2*c*d - b*e))/(3*e^5*(d + e*x)^3) - c^2/
(2*e^5*(d + e*x)^2)
```

Rubi [A] time = 0.109025, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$-\frac{-2ce(3bd-ae)+b^2e^2+6c^2d^2}{4e^5(d+ex)^4}+\frac{2(2cd-be)\left(ae^2-bde+cd^2\right)}{5e^5(d+ex)^5}-\frac{\left(ae^2-bde+cd^2\right)^2}{6e^5(d+ex)^6}+\frac{2c(2cd-be)}{3e^5(d+ex)^3}-\frac{c^2}{2e^5(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(d + e*x)^7,x]
```

```
[Out] -(c*d^2 - b*d*e + a*e^2)^2/(6*e^5*(d + e*x)^6) + (2*(2*c*d - b*e)*(c*d^2 -
b*d*e + a*e^2))/(5*e^5*(d + e*x)^5) - (6*c^2*d^2 + b^2*e^2 - 2*c*e*(3*b*d -
 a*e))/(4*e^5*(d + e*x)^4) + (2*c*(2*c*d - b*e))/(3*e^5*(d + e*x)^3) - c^2/
(2*e^5*(d + e*x)^2)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^2}{(d+ex)^7}\,dx=\int \left(\frac{\left(cd^2-bde+ae^2\right)^2}{e^4(d+ex)^7}+\frac{2(-2cd+be)\left(cd^2-bde+ae^2\right)}{e^4(d+ex)^6}+\frac{6c^2d^2+b^2e^2-2ce(3bd-ae)}{e^4(d+ex)^5}-\frac{2c}{e}\right.$$

$$=-\frac{\left(cd^2-bde+ae^2\right)^2}{6e^5(d+ex)^6}+\frac{2(2cd-be)\left(cd^2-bde+ae^2\right)}{5e^5(d+ex)^5}-\frac{6c^2d^2+b^2e^2-2ce(3bd-ae)}{4e^5(d+ex)^4}+\frac{2c(2cd}{3e^5(d}$$

Mathematica [A] time = 0.0636362, size = 159, normalized size = 1.02

$$-\frac{e^2\left(10a^2e^2+4abe(d+6ex)+b^2\left(d^2+6dex+15e^2x^2\right)\right)+2ce\left(ae\left(d^2+6dex+15e^2x^2\right)+b\left(6d^2ex+d^3+15de^2x^2+2\right.\right.}{60e^5(d+ex)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(d + e*x)^7,x]
```

```
[Out] -(2*c^2*(d^4 + 6*d^3*e*x + 15*d^2*e^2*x^2 + 20*d*e^3*x^3 + 15*e^4*x^4) + e^
2*(10*a^2*e^2 + 4*a*b*e*(d + 6*e*x) + b^2*(d^2 + 6*d*e*x + 15*e^2*x^2)) + 2
*c*e*(a*e*(d^2 + 6*d*e*x + 15*e^2*x^2) + b*(d^3 + 6*d^2*e*x + 15*d*e^2*x^2
+ 20*e^3*x^3)))/(60*e^5*(d + e*x)^6)
```

Maple [A] time = 0.046, size = 195, normalized size = 1.3

$$-\frac{2ace^2+b^2e^2-6bcde+6c^2d^2}{4e^5(ex+d)^4}-\frac{2c(be-2cd)}{3e^5(ex+d)^3}-\frac{c^2}{2e^5(ex+d)^2}-\frac{a^2e^4-2de^3ab+2acd^2e^2+b^2d^2e^2-2d^3ebc+c^2d^4}{6e^5(ex+d)^6}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(e*x+d)^7,x)
```

```
[Out] -1/4*(2*a*c*e^2+b^2*e^2-6*b*c*d*e+6*c^2*d^2)/e^5/(e*x+d)^4-2/3*c*(b*e-2*c*d
)/e^5/(e*x+d)^3-1/2*c^2/e^5/(e*x+d)^2-1/6*(a^2*e^4-2*a*b*d*e^3+2*a*c*d^2*e^
2+b^2*d^2*e^2-2*b*c*d^3*e+c^2*d^4)/e^5/(e*x+d)^6-1/5*(2*a*b*e^3-4*a*c*d*e^2
-2*b^2*d*e^2+6*b*c*d^2*e-4*c^2*d^3)/e^5/(e*x+d)^5
```

Maxima [A] time = 1.00818, size = 309, normalized size = 1.98

$$-\frac{30c^2e^4x^4+2c^2d^4+2bcd^3e+4abde^3+10a^2e^4+(b^2+2ac)d^2e^2+40(c^2de^3+bce^4)x^3+15(2c^2d^2e^2+2bcde^3+(b^2+\cdots}{60(e^{11}x^6+6de^{10}x^5+15d^2e^9x^4+20d^3e^8x^3+15d^4e^7x^2+6d^5e^6\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(30*c^2*e^4*x^4 + 2*c^2*d^4 + 2*b*c*d^3*e + 4*a*b*d*e^3 + 10*a^2*e^4
+ (b^2 + 2*a*c)*d^2*e^2 + 40*(c^2*d*e^3 + b*c*e^4)*x^3 + 15*(2*c^2*d^2*e^2
+ 2*b*c*d*e^3 + (b^2 + 2*a*c)*e^4)*x^2 + 6*(2*c^2*d^3*e + 2*b*c*d^2*e^2 + 4
*a*b*e^4 + (b^2 + 2*a*c)*d*e^3)*x)/(e^11*x^6 + 6*d*e^10*x^5 + 15*d^2*e^9*x^
4 + 20*d^3*e^8*x^3 + 15*d^4*e^7*x^2 + 6*d^5*e^6*x + d^6*e^5)
```

Fricas [A] time = 2.02112, size = 489, normalized size = 3.13

$$-\frac{30c^2e^4x^4+2c^2d^4+2bcd^3e+4abde^3+10a^2e^4+(b^2+2ac)d^2e^2+40(c^2de^3+bce^4)x^3+15(2c^2d^2e^2+2bcde^3+(b^2+\cdots}{60(e^{11}x^6+6de^{10}x^5+15d^2e^9x^4+20d^3e^8x^3+15d^4e^7x^2+6d^5e^6\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^7,x, algorithm="fricas")
```

```
[Out] -1/60*(30*c^2*e^4*x^4 + 2*c^2*d^4 + 2*b*c*d^3*e + 4*a*b*d*e^3 + 10*a^2*e^4
+ (b^2 + 2*a*c)*d^2*e^2 + 40*(c^2*d*e^3 + b*c*e^4)*x^3 + 15*(2*c^2*d^2*e^2
+ 2*b*c*d*e^3 + (b^2 + 2*a*c)*e^4)*x^2 + 6*(2*c^2*d^3*e + 2*b*c*d^2*e^2 + 4
*a*b*e^4 + (b^2 + 2*a*c)*d*e^3)*x)/(e^11*x^6 + 6*d*e^10*x^5 + 15*d^2*e^9*x^
4 + 20*d^3*e^8*x^3 + 15*d^4*e^7*x^2 + 6*d^5*e^6*x + d^6*e^5)
```

Sympy [A] time = 85.4175, size = 262, normalized size = 1.68

$$-\frac{10a^2e^4 + 4abde^3 + 2acd^2e^2 + b^2d^2e^2 + 2bcd^3e + 2c^2d^4 + 30c^2e^4x^4 + x^3\left(40bce^4 + 40c^2de^3\right) + x^2\left(30ace^4 + 15b^2e^4 + 3\right.}{60d^6e^5 + 360d^5e^6x + 900d^4e^7x^2 + 1200d^3e^8x^3 + 900d^2e^9x^4 + }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(e*x+d)**7,x)
```

```
[Out] -(10*a**2*e**4 + 4*a*b*d*e**3 + 2*a*c*d**2*e**2 + b**2*d**2*e**2 + 2*b*c*d*
*3*e + 2*c**2*d**4 + 30*c**2*e**4*x**4 + x**3*(40*b*c*e**4 + 40*c**2*d*e**3
) + x**2*(30*a*c*e**4 + 15*b**2*e**4 + 30*b*c*d*e**3 + 30*c**2*d**2*e**2) +
 x*(24*a*b*e**4 + 12*a*c*d*e**3 + 6*b**2*d*e**3 + 12*b*c*d**2*e**2 + 12*c**
2*d**3*e))/(60*d**6*e**5 + 360*d**5*e**6*x + 900*d**4*e**7*x**2 + 1200*d**3
*e**8*x**3 + 900*d**2*e**9*x**4 + 360*d*e**10*x**5 + 60*e**11*x**6)
```

Giac [A] time = 1.09241, size = 242, normalized size = 1.55

$$-\frac{\left(30\,c^2x^4e^4 + 40\,c^2dx^3e^3 + 30\,c^2d^2x^2e^2 + 12\,c^2d^3xe + 2\,c^2d^4 + 40\,bcx^3e^4 + 30\,bcdx^2e^3 + 12\,bcd^2xe^2 + 2\,bcd^3e + 15\,b^2x\right.}{60\,(xe + d)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^7,x, algorithm="giac")
```

```
[Out] -1/60*(30*c^2*x^4*e^4 + 40*c^2*d*x^3*e^3 + 30*c^2*d^2*x^2*e^2 + 12*c^2*d^3*
x*e + 2*c^2*d^4 + 40*b*c*x^3*e^4 + 30*b*c*d*x^2*e^3 + 12*b*c*d^2*x*e^2 + 2*
b*c*d^3*e + 15*b^2*x^2*e^4 + 30*a*c*x^2*e^4 + 6*b^2*d*x*e^3 + 12*a*c*d*x*e^
3 + b^2*d^2*e^2 + 2*a*c*d^2*e^2 + 24*a*b*x*e^4 + 4*a*b*d*e^3 + 10*a^2*e^4)*
e^(-5)/(x*e + d)^6
```

3.2130 $\int \frac{(a+bx+cx^2)^2}{(d+ex)^8}\,dx$

Optimal. Leaf size=156

$$-\frac{-2ce(3bd-ae)+b^2e^2+6c^2d^2}{5e^5(d+ex)^5}+\frac{(2cd-be)\left(ae^2-bde+cd^2\right)}{3e^5(d+ex)^6}-\frac{\left(ae^2-bde+cd^2\right)^2}{7e^5(d+ex)^7}+\frac{c(2cd-be)}{2e^5(d+ex)^4}-\frac{c^2}{3e^5(d+ex)^3}$$

```
[Out] -(c*d^2 - b*d*e + a*e^2)^2/(7*e^5*(d + e*x)^7) + ((2*c*d - b*e)*(c*d^2 - b*
d*e + a*e^2))/(3*e^5*(d + e*x)^6) - (6*c^2*d^2 + b^2*e^2 - 2*c*e*(3*b*d - a
*e))/(5*e^5*(d + e*x)^5) + (c*(2*c*d - b*e))/(2*e^5*(d + e*x)^4) - c^2/(3*e
^5*(d + e*x)^3)
```

Rubi [A] time = 0.1081, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$-\frac{-2ce(3bd-ae)+b^2e^2+6c^2d^2}{5e^5(d+ex)^5}+\frac{(2cd-be)\left(ae^2-bde+cd^2\right)}{3e^5(d+ex)^6}-\frac{\left(ae^2-bde+cd^2\right)^2}{7e^5(d+ex)^7}+\frac{c(2cd-be)}{2e^5(d+ex)^4}-\frac{c^2}{3e^5(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(d + e*x)^8,x]
```

```
[Out] -(c*d^2 - b*d*e + a*e^2)^2/(7*e^5*(d + e*x)^7) + ((2*c*d - b*e)*(c*d^2 - b*
d*e + a*e^2))/(3*e^5*(d + e*x)^6) - (6*c^2*d^2 + b^2*e^2 - 2*c*e*(3*b*d - a
*e))/(5*e^5*(d + e*x)^5) + (c*(2*c*d - b*e))/(2*e^5*(d + e*x)^4) - c^2/(3*e
^5*(d + e*x)^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx+cx^2\right)^2}{(d+ex)^8}\,dx &= \int \left(\frac{\left(cd^2-bde+ae^2\right)^2}{e^4(d+ex)^8}+\frac{2(-2cd+be)\left(cd^2-bde+ae^2\right)}{e^4(d+ex)^7}+\frac{6c^2d^2+b^2e^2-2ce(3bd-ae)}{e^4(d+ex)^6}-\frac{2c(2c}{e^4(d}\right.\\ &= -\frac{\left(cd^2-bde+ae^2\right)^2}{7e^5(d+ex)^7}+\frac{(2cd-be)\left(cd^2-bde+ae^2\right)}{3e^5(d+ex)^6}-\frac{6c^2d^2+b^2e^2-2ce(3bd-ae)}{5e^5(d+ex)^5}+\frac{c(2cd-be}{2e^5(d+ex}\end{aligned}$$

Mathematica [A] time = 0.067985, size = 161, normalized size = 1.03

$$-\frac{2e^2\left(15a^2e^2+5abe(d+7ex)+b^2\left(d^2+7dex+21e^2x^2\right)\right)+ce\left(4ae\left(d^2+7dex+21e^2x^2\right)+3b\left(7d^2ex+d^3+21de^2x^2+35\right.\right.}{210e^5(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(d + e*x)^8,x]
```

```
[Out] -(2*c^2*(d^4 + 7*d^3*e*x + 21*d^2*e^2*x^2 + 35*d*e^3*x^3 + 35*e^4*x^4) + 2*
e^2*(15*a^2*e^2 + 5*a*b*e*(d + 7*e*x) + b^2*(d^2 + 7*d*e*x + 21*e^2*x^2)) +
 c*e*(4*a*e*(d^2 + 7*d*e*x + 21*e^2*x^2) + 3*b*(d^3 + 7*d^2*e*x + 21*d*e^2*
x^2 + 35*e^3*x^3)))/(210*e^5*(d + e*x)^7)
```

Maple [A] time = 0.046, size = 195, normalized size = 1.3

$$-\frac{c(be-2cd)}{2e^5(ex+d)^4}-\frac{c^2}{3e^5(ex+d)^3}-\frac{2abe^3-4ade^2c-2b^2de^2+6d^2ebc-4c^2d^3}{6e^5(ex+d)^6}-\frac{a^2e^4-2de^3ab+2acd^2e^2+b^2d^2e^2-}{7e^5(ex+d)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(e*x+d)^8,x)
```

```
[Out] -1/2*c*(b*e-2*c*d)/e^5/(e*x+d)^4-1/3*c^2/e^5/(e*x+d)^3-1/6*(2*a*b*e^3-4*a*c
*d*e^2-2*b^2*d*e^2+6*b*c*d^2*e-4*c^2*d^3)/e^5/(e*x+d)^6-1/7*(a^2*e^4-2*a*b*
d*e^3+2*a*c*d^2*e^2+b^2*d^2*e^2-2*b*c*d^3*e+c^2*d^4)/e^5/(e*x+d)^7-1/5*(2*a
*c*e^2+b^2*e^2-6*b*c*d*e+6*c^2*d^2)/e^5/(e*x+d)^5
```

Maxima [A] time = 1.06428, size = 331, normalized size = 2.12

$$-\frac{70c^2e^4x^4+2c^2d^4+3bcd^3e+10abde^3+30a^2e^4+2\left(b^2+2ac\right)d^2e^2+35\left(2c^2de^3+3bce^4\right)x^3+21\left(2c^2d^2e^2+3bcd\right.}{210\left(e^{12}x^7+7de^{11}x^6+21d^2e^{10}x^5+35d^3e^9x^4+35d^4e^8x^3+21\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^8,x, algorithm="maxima")
```

```
[Out] -1/210*(70*c^2*e^4*x^4 + 2*c^2*d^4 + 3*b*c*d^3*e + 10*a*b*d*e^3 + 30*a^2*e^
4 + 2*(b^2 + 2*a*c)*d^2*e^2 + 35*(2*c^2*d*e^3 + 3*b*c*e^4)*x^3 + 21*(2*c^2*
d^2*e^2 + 3*b*c*d*e^3 + 2*(b^2 + 2*a*c)*e^4)*x^2 + 7*(2*c^2*d^3*e + 3*b*c*d
^2*e^2 + 10*a*b*e^4 + 2*(b^2 + 2*a*c)*d*e^3)*x)/(e^12*x^7 + 7*d*e^11*x^6 +
21*d^2*e^10*x^5 + 35*d^3*e^9*x^4 + 35*d^4*e^8*x^3 + 21*d^5*e^7*x^2 + 7*d^6*
e^6*x + d^7*e^5)
```

Fricas [A] time = 2.013, size = 531, normalized size = 3.4

$$-\frac{70c^2e^4x^4+2c^2d^4+3bcd^3e+10abde^3+30a^2e^4+2\left(b^2+2ac\right)d^2e^2+35\left(2c^2de^3+3bce^4\right)x^3+21\left(2c^2d^2e^2+3bcd\right.}{210\left(e^{12}x^7+7de^{11}x^6+21d^2e^{10}x^5+35d^3e^9x^4+35d^4e^8x^3+21\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^8,x, algorithm="fricas")
```

```
[Out] -1/210*(70*c^2*e^4*x^4 + 2*c^2*d^4 + 3*b*c*d^3*e + 10*a*b*d*e^3 + 30*a^2*e^
4 + 2*(b^2 + 2*a*c)*d^2*e^2 + 35*(2*c^2*d*e^3 + 3*b*c*e^4)*x^3 + 21*(2*c^2*
d^2*e^2 + 3*b*c*d*e^3 + 2*(b^2 + 2*a*c)*e^4)*x^2 + 7*(2*c^2*d^3*e + 3*b*c*d
^2*e^2 + 10*a*b*e^4 + 2*(b^2 + 2*a*c)*d*e^3)*x)/(e^12*x^7 + 7*d*e^11*x^6 +
```

```
21*d^2*e^10*x^5 + 35*d^3*e^9*x^4 + 35*d^4*e^8*x^3 + 21*d^5*e^7*x^2 + 7*d^6*
e^6*x + d^7*e^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(e*x+d)**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09075, size = 243, normalized size = 1.56

$$-\frac{\left(70\,c^2x^4e^4+70\,c^2dx^3e^3+42\,c^2d^2x^2e^2+14\,c^2d^3xe+2\,c^2d^4+105\,bcx^3e^4+63\,bcdx^2e^3+21\,bcd^2xe^2+3\,bcd^3e+42\,b^2x^2e^4\right.}{210\,(xe+d)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^8,x, algorithm="giac")
```

```
[Out] -1/210*(70*c^2*x^4*e^4 + 70*c^2*d*x^3*e^3 + 42*c^2*d^2*x^2*e^2 + 14*c^2*d^3
*x*e + 2*c^2*d^4 + 105*b*c*x^3*e^4 + 63*b*c*d*x^2*e^3 + 21*b*c*d^2*x*e^2 +
3*b*c*d^3*e + 42*b^2*x^2*e^4 + 84*a*c*x^2*e^4 + 14*b^2*d*x*e^3 + 28*a*c*d*x
*e^3 + 2*b^2*d^2*e^2 + 4*a*c*d^2*e^2 + 70*a*b*x*e^4 + 10*a*b*d*e^3 + 30*a^2
*e^4)*e^(-5)/(x*e + d)^7
```

3.2131 $\int (d+ex)^4 \left(a+bx+cx^2\right)^3 dx$

Optimal. Leaf size=272

$$\frac{c(d+ex)^9 \left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{3e^7}-\frac{(d+ex)^8(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{8e^7}+\frac{3(d+ex)^7 \left(ae^2 -\right.}{}$$

```
[Out] ((c*d^2 - b*d*e + a*e^2)^3*(d + e*x)^5)/(5*e^7) - ((2*c*d - b*e)*(c*d^2 - b
*d*e + a*e^2)^2*(d + e*x)^6)/(2*e^7) + (3*(c*d^2 - b*d*e + a*e^2)*(5*c^2*d^
2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^7)/(7*e^7) - ((2*c*d - b*e)*(10*
c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*(d + e*x)^8)/(8*e^7) + (c*(5*c^2
*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^9)/(3*e^7) - (3*c^2*(2*c*d -
b*e)*(d + e*x)^10)/(10*e^7) + (c^3*(d + e*x)^11)/(11*e^7)
```

Rubi [A] time = 0.508922, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$\frac{c(d+ex)^9 \left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{3e^7}-\frac{(d+ex)^8(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{8e^7}+\frac{3(d+ex)^7 \left(ae^2 -\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4*(a + b*x + c*x^2)^3,x]
```



```
[Out] ((c*d^2 - b*d*e + a*e^2)^3*(d + e*x)^5)/(5*e^7) - ((2*c*d - b*e)*(c*d^2 - b
*d*e + a*e^2)^2*(d + e*x)^6)/(2*e^7) + (3*(c*d^2 - b*d*e + a*e^2)*(5*c^2*d^
2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^7)/(7*e^7) - ((2*c*d - b*e)*(10*
c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*(d + e*x)^8)/(8*e^7) + (c*(5*c^2
*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^9)/(3*e^7) - (3*c^2*(2*c*d -
b*e)*(d + e*x)^10)/(10*e^7) + (c^3*(d + e*x)^11)/(11*e^7)
```


Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^4 \left(a+bx+cx^2\right)^3 dx = \int \left(\frac{\left(cd^2-bde+ae^2\right)^3 (d+ex)^4}{e^6}+\frac{3(-2cd+be)\left(cd^2-bde+ae^2\right)^2 (d+ex)^5}{e^6}+\frac{3\left(cd\right.}{}\right.$$

$$=\frac{\left(cd^2-bde+ae^2\right)^3 (d+ex)^5}{5e^7}-\frac{(2cd-be)\left(cd^2-bde+ae^2\right)^2 (d+ex)^6}{2e^7}+\frac{3\left(cd^2-bde\right.}{}$$

Mathematica [A] time = 0.154939, size = 497, normalized size = 1.83

$$\frac{1}{7}x^7 \left(3ce^2 \left(a^2e^2+8abde+6b^2d^2\right)+b^2e^3(3ae+4bd)+6c^2d^2e(3ae+2bd)+c^3d^4\right)+\frac{1}{2}x^6 \left(b\left(a^2e^4+12acd^2e^2+c^2d^4\right)+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4*(a + b*x + c*x^2)^3,x]
```

```
[Out] a^3*d^4*x + (a^2*d^3*(3*b*d + 4*a*e)*x^2)/2 + a*d^2*(b^2*d^2 + 4*a*b*d*e +
a*(c*d^2 + 2*a*e^2))*x^3 + (d*(b^3*d^3 + 12*a*b^2*d^2*e + 4*a^2*e*(3*c*d^2
+ a*e^2) + 6*a*b*d*(c*d^2 + 3*a*e^2))*x^4)/4 + ((4*b^3*d^3*e + 12*a*b*d*e*(
2*c*d^2 + a*e^2) + 3*b^2*(c*d^4 + 6*a*d^2*e^2) + a*(3*c^2*d^4 + 18*a*c*d^2*
e^2 + a^2*e^4))*x^5)/5 + ((2*b^3*d^2*e^2 + 4*a*c*d*e*(c*d^2 + a*e^2) + 4*b^
2*(c*d^3*e + a*d*e^3) + b*(c^2*d^4 + 12*a*c*d^2*e^2 + a^2*e^4))*x^6)/2 + ((
c^3*d^4 + 6*c^2*d^2*e*(2*b*d + 3*a*e) + b^2*e^3*(4*b*d + 3*a*e) + 3*c*e^2*(
6*b^2*d^2 + 8*a*b*d*e + a^2*e^2))*x^7)/7 + (e*(4*c^3*d^3 + b^3*e^3 + 6*b*c*
e^2*(2*b*d + a*e) + 6*c^2*d*e*(3*b*d + 2*a*e))*x^8)/8 + (c*e^2*(2*c^2*d^2 +
 b^2*e^2 + c*e*(4*b*d + a*e))*x^9)/3 + (c^2*e^3*(4*c*d + 3*b*e)*x^10)/10 +
(c^3*e^4*x^11)/11
```

Maple [B] time = 0.04, size = 631, normalized size = 2.3

$$\frac{c^3e^4x^{11}}{11}+\frac{\left(3\,e^4bc^2+4\,de^3c^3\right)x^{10}}{10}+\frac{\left(6\,d^2e^2c^3+12\,de^3bc^2+e^4\left(ac^2+2\,b^2c+c\left(2\,ac+b^2\right)\right)\right)x^9}{9}+\frac{\left(4\,d^3ec^3+18\,d^2e^2bc^2+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(c*x^2+b*x+a)^3,x)
```

```
[Out] 1/11*c^3*e^4*x^11+1/10*(3*b*c^2*e^4+4*c^3*d*e^3)*x^10+1/9*(6*d^2*e^2*c^3+12
*d*e^3*b*c^2+e^4*(a*c^2+2*b^2*c+c*(2*a*c+b^2)))*x^9+1/8*(4*d^3*e*c^3+18*d^2
*e^2*b*c^2+4*d*e^3*(a*c^2+2*b^2*c+c*(2*a*c+b^2))+e^4*(4*a*b*c+b*(2*a*c+b^2)
))*x^8+1/7*(c^3*d^4+12*d^3*e*b*c^2+6*d^2*e^2*(a*c^2+2*b^2*c+c*(2*a*c+b^2))+
4*d*e^3*(4*a*b*c+b*(2*a*c+b^2))+e^4*(a*(2*a*c+b^2)+2*b^2*a+a^2*c))*x^7+1/6*
(3*d^4*b*c^2+4*d^3*e*(a*c^2+2*b^2*c+c*(2*a*c+b^2))+6*d^2*e^2*(4*a*b*c+b*(2*
a*c+b^2))+4*d*e^3*(a*(2*a*c+b^2)+2*b^2*a+a^2*c)+3*e^4*b*a^2)*x^6+1/5*(d^4*(
a*c^2+2*b^2*c+c*(2*a*c+b^2))+4*d^3*e*(4*a*b*c+b*(2*a*c+b^2))+6*d^2*e^2*(a*(
2*a*c+b^2)+2*b^2*a+a^2*c)+12*d*e^3*b*a^2+e^4*a^3)*x^5+1/4*(d^4*(4*a*b*c+b*(
2*a*c+b^2))+4*d^3*e*(a*(2*a*c+b^2)+2*b^2*a+a^2*c)+18*d^2*e^2*b*a^2+4*d*e^3*
a^3)*x^4+1/3*(d^4*(a*(2*a*c+b^2)+2*b^2*a+a^2*c)+12*d^3*e*b*a^2+6*d^2*e^2*a^
3)*x^3+1/2*(4*a^3*d^3*e+3*a^2*b*d^4)*x^2+d^4*a^3*x
```

Maxima [A] time = 1.00174, size = 653, normalized size = 2.4

$$\frac{1}{11}\,c^3e^4x^{11}+\frac{1}{10}\left(4\,c^3de^3+3\,bc^2e^4\right)x^{10}+\frac{1}{3}\left(2\,c^3d^2e^2+4\,bc^2de^3+\left(b^2c+ac^2\right)e^4\right)x^9+\frac{1}{8}\left(4\,c^3d^3e+18\,bc^2d^2e^2+12\left(b^2c+\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/11*c^3*e^4*x^11 + 1/10*(4*c^3*d*e^3 + 3*b*c^2*e^4)*x^10 + 1/3*(2*c^3*d^2*
e^2 + 4*b*c^2*d*e^3 + (b^2*c + a*c^2)*e^4)*x^9 + 1/8*(4*c^3*d^3*e + 18*b*c^
2*d^2*e^2 + 12*(b^2*c + a*c^2)*d*e^3 + (b^3 + 6*a*b*c)*e^4)*x^8 + a^3*d^4*x
 + 1/7*(c^3*d^4 + 12*b*c^2*d^3*e + 18*(b^2*c + a*c^2)*d^2*e^2 + 4*(b^3 + 6*
a*b*c)*d*e^3 + 3*(a*b^2 + a^2*c)*e^4)*x^7 + 1/2*(b*c^2*d^4 + a^2*b*e^4 + 4*
(b^2*c + a*c^2)*d^3*e + 2*(b^3 + 6*a*b*c)*d^2*e^2 + 4*(a*b^2 + a^2*c)*d*e^3
)*x^6 + 1/5*(12*a^2*b*d*e^3 + a^3*e^4 + 3*(b^2*c + a*c^2)*d^4 + 4*(b^3 + 6*
```

```
a*b*c)*d^3*e + 18*(a*b^2 + a^2*c)*d^2*e^2)*x^5 + 1/4*(18*a^2*b*d^2*e^2 + 4*
a^3*d*e^3 + (b^3 + 6*a*b*c)*d^4 + 12*(a*b^2 + a^2*c)*d^3*e)*x^4 + (4*a^2*b*
d^3*e + 2*a^3*d^2*e^2 + (a*b^2 + a^2*c)*d^4)*x^3 + 1/2*(3*a^2*b*d^4 + 4*a^3
*d^3*e)*x^2
```

Fricas [B] time = 1.72891, size = 1377, normalized size = 5.06

$$\frac{1}{11}x^{11}e^4c^3 + \frac{2}{5}x^{10}e^3dc^3 + \frac{3}{10}x^{10}e^4c^2b + \frac{2}{3}x^9e^2d^2c^3 + \frac{4}{3}x^9e^3dc^2b + \frac{1}{3}x^9e^4cb^2 + \frac{1}{3}x^9e^4c^2a + \frac{1}{2}x^8ed^3c^3 + \frac{9}{4}x^8e^2d^2c^2b + \frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*e^4*c^3 + 2/5*x^10*e^3*d*c^3 + 3/10*x^10*e^4*c^2*b + 2/3*x^9*e^2*
d^2*c^3 + 4/3*x^9*e^3*d*c^2*b + 1/3*x^9*e^4*c*b^2 + 1/3*x^9*e^4*c^2*a + 1/2
*x^8*e*d^3*c^3 + 9/4*x^8*e^2*d^2*c^2*b + 3/2*x^8*e^3*d*c*b^2 + 1/8*x^8*e^4*
b^3 + 3/2*x^8*e^3*d*c^2*a + 3/4*x^8*e^4*c*b*a + 1/7*x^7*d^4*c^3 + 12/7*x^7*
e*d^3*c^2*b + 18/7*x^7*e^2*d^2*c*b^2 + 4/7*x^7*e^3*d*b^3 + 18/7*x^7*e^2*d^2
*c^2*a + 24/7*x^7*e^3*d*c*b*a + 3/7*x^7*e^4*b^2*a + 3/7*x^7*e^4*c*a^2 + 1/2
*x^6*d^4*c^2*b + 2*x^6*e*d^3*c*b^2 + x^6*e^2*d^2*b^3 + 2*x^6*e*d^3*c^2*a +
6*x^6*e^2*d^2*c*b*a + 2*x^6*e^3*d*b^2*a + 2*x^6*e^3*d*c*a^2 + 1/2*x^6*e^4*b
*a^2 + 3/5*x^5*d^4*c*b^2 + 4/5*x^5*e*d^3*b^3 + 3/5*x^5*d^4*c^2*a + 24/5*x^5
*e*d^3*c*b*a + 18/5*x^5*e^2*d^2*b^2*a + 18/5*x^5*e^2*d^2*c*a^2 + 12/5*x^5*e
^3*d*b*a^2 + 1/5*x^5*e^4*a^3 + 1/4*x^4*d^4*b^3 + 3/2*x^4*d^4*c*b*a + 3*x^4*
e*d^3*b^2*a + 3*x^4*e*d^3*c*a^2 + 9/2*x^4*e^2*d^2*b*a^2 + x^4*e^3*d*a^3 + x
^3*d^4*b^2*a + x^3*d^4*c*a^2 + 4*x^3*e*d^3*b*a^2 + 2*x^3*e^2*d^2*a^3 + 3/2*
x^2*d^4*b*a^2 + 2*x^2*e*d^3*a^3 + x*d^4*a^3
```

Sympy [B] time = 0.149958, size = 620, normalized size = 2.28

$$a^3d^4x + \frac{c^3e^4x^{11}}{11} + x^{10}\left(\frac{3bc^2e^4}{10} + \frac{2c^3de^3}{5}\right) + x^9\left(\frac{ac^2e^4}{3} + \frac{b^2ce^4}{3} + \frac{4bc^2de^3}{3} + \frac{2c^3d^2e^2}{3}\right) + x^8\left(\frac{3abce^4}{4} + \frac{3ac^2de^3}{2} + \frac{b^3e^4}{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(c*x**2+b*x+a)**3,x)
```

```
[Out] a**3*d**4*x + c**3*e**4*x**11/11 + x**10*(3*b*c**2*e**4/10 + 2*c**3*d*e**3/
5) + x**9*(a*c**2*e**4/3 + b**2*c*e**4/3 + 4*b*c**2*d*e**3/3 + 2*c**3*d**2*
e**2/3) + x**8*(3*a*b*c*e**4/4 + 3*a*c**2*d*e**3/2 + b**3*e**4/8 + 3*b**2*c
*d*e**3/2 + 9*b*c**2*d**2*e**2/4 + c**3*d**3*e/2) + x**7*(3*a**2*c*e**4/7 +
 3*a*b**2*e**4/7 + 24*a*b*c*d*e**3/7 + 18*a*c**2*d**2*e**2/7 + 4*b**3*d*e**
3/7 + 18*b**2*c*d**2*e**2/7 + 12*b*c**2*d**3*e/7 + c**3*d**4/7) + x**6*(a**
2*b*e**4/2 + 2*a**2*c*d*e**3 + 2*a*b**2*d*e**3 + 6*a*b*c*d**2*e**2 + 2*a*c*
*2*d**3*e + b**3*d**2*e**2 + 2*b**2*c*d**3*e + b*c**2*d**4/2) + x**5*(a**3*
e**4/5 + 12*a**2*b*d*e**3/5 + 18*a**2*c*d**2*e**2/5 + 18*a*b**2*d**2*e**2/5
 + 24*a*b*c*d**3*e/5 + 3*a*c**2*d**4/5 + 4*b**3*d**3*e/5 + 3*b**2*c*d**4/5)
 + x**4*(a**3*d*e**3 + 9*a**2*b*d**2*e**2/2 + 3*a**2*c*d**3*e + 3*a*b**2*d*
*3*e + 3*a*b*c*d**4/2 + b**3*d**4/4) + x**3*(2*a**3*d**2*e**2 + 4*a**2*b*d*
*3*e + a**2*c*d**4 + a*b**2*d**4) + x**2*(2*a**3*d**3*e + 3*a**2*b*d**4/2)
```

Giac [B] time = 1.12191, size = 815, normalized size = 3.

$$\frac{1}{11}c^3x^{11}e^4+\frac{2}{5}c^3dx^{10}e^3+\frac{2}{3}c^3d^2x^9e^2+\frac{1}{2}c^3d^3x^8e+\frac{1}{7}c^3d^4x^7+\frac{3}{10}bc^2x^{10}e^4+\frac{4}{3}bc^2dx^9e^3+\frac{9}{4}bc^2d^2x^8e^2+\frac{12}{7}bc^2d^3x^7e+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/11*c^3*x^11*e^4 + 2/5*c^3*d*x^10*e^3 + 2/3*c^3*d^2*x^9*e^2 + 1/2*c^3*d^3*
x^8*e + 1/7*c^3*d^4*x^7 + 3/10*b*c^2*x^10*e^4 + 4/3*b*c^2*d*x^9*e^3 + 9/4*b
*c^2*d^2*x^8*e^2 + 12/7*b*c^2*d^3*x^7*e + 1/2*b*c^2*d^4*x^6 + 1/3*b^2*c*x^9
*e^4 + 1/3*a*c^2*x^9*e^4 + 3/2*b^2*c*d*x^8*e^3 + 3/2*a*c^2*d*x^8*e^3 + 18/7
*b^2*c*d^2*x^7*e^2 + 18/7*a*c^2*d^2*x^7*e^2 + 2*b^2*c*d^3*x^6*e + 2*a*c^2*d
^3*x^6*e + 3/5*b^2*c*d^4*x^5 + 3/5*a*c^2*d^4*x^5 + 1/8*b^3*x^8*e^4 + 3/4*a*
b*c*x^8*e^4 + 4/7*b^3*d*x^7*e^3 + 24/7*a*b*c*d*x^7*e^3 + b^3*d^2*x^6*e^2 +
6*a*b*c*d^2*x^6*e^2 + 4/5*b^3*d^3*x^5*e + 24/5*a*b*c*d^3*x^5*e + 1/4*b^3*d^
4*x^4 + 3/2*a*b*c*d^4*x^4 + 3/7*a*b^2*x^7*e^4 + 3/7*a^2*c*x^7*e^4 + 2*a*b^2
*d*x^6*e^3 + 2*a^2*c*d*x^6*e^3 + 18/5*a*b^2*d^2*x^5*e^2 + 18/5*a^2*c*d^2*x^
5*e^2 + 3*a*b^2*d^3*x^4*e + 3*a^2*c*d^3*x^4*e + a*b^2*d^4*x^3 + a^2*c*d^4*x
^3 + 1/2*a^2*b*x^6*e^4 + 12/5*a^2*b*d*x^5*e^3 + 9/2*a^2*b*d^2*x^4*e^2 + 4*a
^2*b*d^3*x^3*e + 3/2*a^2*b*d^4*x^2 + 1/5*a^3*x^5*e^4 + a^3*d*x^4*e^3 + 2*a^
3*d^2*x^3*e^2 + 2*a^3*d^3*x^2*e + a^3*d^4*x
```

3.2132 $\int (d+ex)^3 \left(a+bx+cx^2\right)^3 \, dx$

Optimal. Leaf size=272

$$\frac{3c(d+ex)^8\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{8e^7}-\frac{(d+ex)^7(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{7e^7}+\frac{(d+ex)^6\left(ae^2-\right.}{}$$

```
[Out] ((c*d^2 - b*d*e + a*e^2)^3*(d + e*x)^4)/(4*e^7) - (3*(2*c*d - b*e)*(c*d^2 -
 b*d*e + a*e^2)^2*(d + e*x)^5)/(5*e^7) + ((c*d^2 - b*d*e + a*e^2)*(5*c^2*d^
2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^6)/(2*e^7) - ((2*c*d - b*e)*(10*
c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*(d + e*x)^7)/(7*e^7) + (3*c*(5*c
^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^8)/(8*e^7) - (c^2*(2*c*d -
b*e)*(d + e*x)^9)/(3*e^7) + (c^3*(d + e*x)^10)/(10*e^7)
```

Rubi [A] time = 0.367624, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$\frac{3c(d+ex)^8\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{8e^7}-\frac{(d+ex)^7(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{7e^7}+\frac{(d+ex)^6\left(ae^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*x + c*x^2)^3,x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)^3*(d + e*x)^4)/(4*e^7) - (3*(2*c*d - b*e)*(c*d^2 -
 b*d*e + a*e^2)^2*(d + e*x)^5)/(5*e^7) + ((c*d^2 - b*d*e + a*e^2)*(5*c^2*d^
2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^6)/(2*e^7) - ((2*c*d - b*e)*(10*
c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*(d + e*x)^7)/(7*e^7) + (3*c*(5*c
^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^8)/(8*e^7) - (c^2*(2*c*d -
b*e)*(d + e*x)^9)/(3*e^7) + (c^3*(d + e*x)^10)/(10*e^7)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^3\left(a+bx+cx^2\right)^3\,dx=\int\left(\frac{\left(cd^2-bde+ae^2\right)^3(d+ex)^3}{e^6}+\frac{3(-2cd+be)\left(cd^2-bde+ae^2\right)^2(d+ex)^4}{e^6}+\frac{3\left(cd\right.}{}\right.$$

$$=\frac{\left(cd^2-bde+ae^2\right)^3(d+ex)^4}{4e^7}-\frac{3(2cd-be)\left(cd^2-bde+ae^2\right)^2(d+ex)^5}{5e^7}+\frac{\left(cd^2-bde\right.}{}$$

Mathematica [A] time = 0.136436, size = 372, normalized size = 1.37

$$\frac{1}{4}x^4\left(a^2e\left(ae^2+9cd^2\right)+9ab^2d^2e+3abd\left(3ae^2+2cd^2\right)+b^3d^3\right)+\frac{3}{2}a^2d^2x^2(ae+bd)+a^3d^3x+\frac{3}{8}cex^8\left(ce(ae+3bd)+b^2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*x + c*x^2)^3,x]
```

```
[Out] a^3*d^3*x + (3*a^2*d^2*(b*d + a*e)*x^2)/2 + a*d*(b^2*d^2 + 3*a*b*d*e + a*(c
*d^2 + a*e^2))*x^3 + ((b^3*d^3 + 9*a*b^2*d^2*e + a^2*e*(9*c*d^2 + a*e^2) +
3*a*b*d*(2*c*d^2 + 3*a*e^2))*x^4)/4 + (3*(b^3*d^2*e + a*b*e*(6*c*d^2 + a*e^
2) + a*c*d*(c*d^2 + 3*a*e^2) + b^2*(c*d^3 + 3*a*d*e^2))*x^5)/5 + ((b^3*d*e^
2 + a*c*e*(3*c*d^2 + a*e^2) + b*c*d*(c*d^2 + 6*a*e^2) + b^2*(3*c*d^2*e + a*
e^3))*x^6)/2 + ((c^3*d^3 + b^3*e^3 + 9*c^2*d*e*(b*d + a*e) + 3*b*c*e^2*(3*b
*d + 2*a*e))*x^7)/7 + (3*c*e*(c^2*d^2 + b^2*e^2 + c*e*(3*b*d + a*e))*x^8)/8
 + (c^2*e^2*(c*d + b*e)*x^9)/3 + (c^3*e^3*x^10)/10
```

Maple [A] time = 0.039, size = 495, normalized size = 1.8

$$\frac{c^3e^3x^{10}}{10}+\frac{\left(3\,e^3bc^2+3\,de^2c^3\right)x^9}{9}+\frac{\left(3\,d^2ec^3+9\,de^2bc^2+e^3\left(ac^2+2\,b^2c+c\left(2\,ac+b^2\right)\right)\right)x^8}{8}+\frac{\left(c^3d^3+9\,bc^2d^2e+3\,de^2\left(ac\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x+a)^3,x)
```

```
[Out] 1/10*c^3*e^3*x^10+1/9*(3*b*c^2*e^3+3*c^3*d*e^2)*x^9+1/8*(3*d^2*e*c^3+9*d*e^
2*b*c^2+e^3*(a*c^2+2*b^2*c+c*(2*a*c+b^2)))*x^8+1/7*(c^3*d^3+9*b*c^2*d^2*e+3
*d*e^2*(a*c^2+2*b^2*c+c*(2*a*c+b^2))+e^3*(4*a*b*c+b*(2*a*c+b^2)))*x^7+1/6*(
3*d^3*b*c^2+3*d^2*e*(a*c^2+2*b^2*c+c*(2*a*c+b^2))+3*d*e^2*(4*a*b*c+b*(2*a*c
+b^2))+e^3*(a*(2*a*c+b^2)+2*b^2*a+a^2*c))*x^6+1/5*(d^3*(a*c^2+2*b^2*c+c*(2*
a*c+b^2))+3*d^2*e*(4*a*b*c+b*(2*a*c+b^2))+3*d*e^2*(a*(2*a*c+b^2)+2*b^2*a+a^
2*c)+3*e^3*b*a^2)*x^5+1/4*(d^3*(4*a*b*c+b*(2*a*c+b^2))+3*d^2*e*(a*(2*a*c+b^
2)+2*b^2*a+a^2*c)+9*d*e^2*a^2*b+a^3*e^3)*x^4+1/3*(d^3*(a*(2*a*c+b^2)+2*b^2*
a+a^2*c)+9*d^2*e*b*a^2+3*d*e^2*a^3)*x^3+1/2*(3*a^3*d^2*e+3*a^2*b*d^3)*x^2+a
^3*d^3*x
```

Maxima [A] time = 1.0093, size = 495, normalized size = 1.82

$$\frac{1}{10}\,c^3e^3x^{10}+\frac{1}{3}\left(c^3de^2+bc^2e^3\right)x^9+\frac{3}{8}\left(c^3d^2e+3\,bc^2de^2+\left(b^2c+ac^2\right)e^3\right)x^8+\frac{1}{7}\left(c^3d^3+9\,bc^2d^2e+9\left(b^2c+ac^2\right)de^2+\left(b^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/10*c^3*e^3*x^10 + 1/3*(c^3*d*e^2 + b*c^2*e^3)*x^9 + 3/8*(c^3*d^2*e + 3*b*
c^2*d*e^2 + (b^2*c + a*c^2)*e^3)*x^8 + 1/7*(c^3*d^3 + 9*b*c^2*d^2*e + 9*(b^
2*c + a*c^2)*d*e^2 + (b^3 + 6*a*b*c)*e^3)*x^7 + a^3*d^3*x + 1/2*(b*c^2*d^3
+ 3*(b^2*c + a*c^2)*d^2*e + (b^3 + 6*a*b*c)*d*e^2 + (a*b^2 + a^2*c)*e^3)*x^
6 + 3/5*(a^2*b*e^3 + (b^2*c + a*c^2)*d^3 + (b^3 + 6*a*b*c)*d^2*e + 3*(a*b^2
 + a^2*c)*d*e^2)*x^5 + 1/4*(9*a^2*b*d*e^2 + a^3*e^3 + (b^3 + 6*a*b*c)*d^3 +
 9*(a*b^2 + a^2*c)*d^2*e)*x^4 + (3*a^2*b*d^2*e + a^3*d*e^2 + (a*b^2 + a^2*c
)*d^3)*x^3 + 3/2*(a^2*b*d^3 + a^3*d^2*e)*x^2
```

Fricas [A] time = 1.74845, size = 1075, normalized size = 3.95

$$\frac{1}{10}x^{10}e^3c^3+\frac{1}{3}x^9e^2dc^3+\frac{1}{3}x^9e^3c^2b+\frac{3}{8}x^8ed^2c^3+\frac{9}{8}x^8e^2dc^2b+\frac{3}{8}x^8e^3cb^2+\frac{3}{8}x^8e^3c^2a+\frac{1}{7}x^7d^3c^3+\frac{9}{7}x^7ed^2c^2b+\frac{9}{7}x^7e^2d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*e^3*c^3 + 1/3*x^9*e^2*d*c^3 + 1/3*x^9*e^3*c^2*b + 3/8*x^8*e*d^2*c
^3 + 9/8*x^8*e^2*d*c^2*b + 3/8*x^8*e^3*c*b^2 + 3/8*x^8*e^3*c^2*a + 1/7*x^7*
d^3*c^3 + 9/7*x^7*e*d^2*c^2*b + 9/7*x^7*e^2*d*c*b^2 + 1/7*x^7*e^3*b^3 + 9/7
*x^7*e^2*d*c^2*a + 6/7*x^7*e^3*c*b*a + 1/2*x^6*d^3*c^2*b + 3/2*x^6*e*d^2*c*
b^2 + 1/2*x^6*e^2*d*b^3 + 3/2*x^6*e*d^2*c^2*a + 3*x^6*e^2*d*c*b*a + 1/2*x^6
*e^3*b^2*a + 1/2*x^6*e^3*c*a^2 + 3/5*x^5*d^3*c*b^2 + 3/5*x^5*e*d^2*b^3 + 3/
5*x^5*d^3*c^2*a + 18/5*x^5*e*d^2*c*b*a + 9/5*x^5*e^2*d*b^2*a + 9/5*x^5*e^2*
d*c*a^2 + 3/5*x^5*e^3*b*a^2 + 1/4*x^4*d^3*b^3 + 3/2*x^4*d^3*c*b*a + 9/4*x^4
*e*d^2*b^2*a + 9/4*x^4*e*d^2*c*a^2 + 9/4*x^4*e^2*d*b*a^2 + 1/4*x^4*e^3*a^3
+ x^3*d^3*b^2*a + x^3*d^3*c*a^2 + 3*x^3*e*d^2*b*a^2 + x^3*e^2*d*a^3 + 3/2*x
^2*d^3*b*a^2 + 3/2*x^2*e*d^2*a^3 + x*d^3*a^3
```

Sympy [A] time = 0.141338, size = 484, normalized size = 1.78

$$a^3d^3x+\frac{c^3e^3x^{10}}{10}+x^9\left(\frac{bc^2e^3}{3}+\frac{c^3de^2}{3}\right)+x^8\left(\frac{3ac^2e^3}{8}+\frac{3b^2ce^3}{8}+\frac{9bc^2de^2}{8}+\frac{3c^3d^2e}{8}\right)+x^7\left(\frac{6abce^3}{7}+\frac{9ac^2de^2}{7}+\frac{b^3e^3}{7}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x+a)**3,x)
```

```
[Out] a**3*d**3*x + c**3*e**3*x**10/10 + x**9*(b*c**2*e**3/3 + c**3*d*e**2/3) + x
**8*(3*a*c**2*e**3/8 + 3*b**2*c*e**3/8 + 9*b*c**2*d*e**2/8 + 3*c**3*d**2*e/
8) + x**7*(6*a*b*c*e**3/7 + 9*a*c**2*d*e**2/7 + b**3*e**3/7 + 9*b**2*c*d*e*
*2/7 + 9*b*c**2*d**2*e/7 + c**3*d**3/7) + x**6*(a**2*c*e**3/2 + a*b**2*e**3
/2 + 3*a*b*c*d*e**2 + 3*a*c**2*d**2*e/2 + b**3*d*e**2/2 + 3*b**2*c*d**2*e/2
 + b*c**2*d**3/2) + x**5*(3*a**2*b*e**3/5 + 9*a**2*c*d*e**2/5 + 9*a*b**2*d*
e**2/5 + 18*a*b*c*d**2*e/5 + 3*a*c**2*d**3/5 + 3*b**3*d**2*e/5 + 3*b**2*c*d
**3/5) + x**4*(a**3*e**3/4 + 9*a**2*b*d*e**2/4 + 9*a**2*c*d**2*e/4 + 9*a*b*
*2*d**2*e/4 + 3*a*b*c*d**3/2 + b**3*d**3/4) + x**3*(a**3*d*e**2 + 3*a**2*b*
d**2*e + a**2*c*d**3 + a*b**2*d**3) + x**2*(3*a**3*d**2*e/2 + 3*a**2*b*d**3
/2)
```

Giac [A] time = 1.13292, size = 633, normalized size = 2.33

$$\frac{1}{10}c^3x^{10}e^3+\frac{1}{3}c^3dx^9e^2+\frac{3}{8}c^3d^2x^8e+\frac{1}{7}c^3d^3x^7+\frac{1}{3}bc^2x^9e^3+\frac{9}{8}bc^2dx^8e^2+\frac{9}{7}bc^2d^2x^7e+\frac{1}{2}bc^2d^3x^6+\frac{3}{8}b^2cx^8e^3+\frac{3}{8}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/10*c^3*x^10*e^3 + 1/3*c^3*d*x^9*e^2 + 3/8*c^3*d^2*x^8*e + 1/7*c^3*d^3*x^7
 + 1/3*b*c^2*x^9*e^3 + 9/8*b*c^2*d*x^8*e^2 + 9/7*b*c^2*d^2*x^7*e + 1/2*b*c^
2*d^3*x^6 + 3/8*b^2*c*x^8*e^3 + 3/8*a*c^2*x^8*e^3 + 9/7*b^2*c*d*x^7*e^2 + 9
/7*a*c^2*d*x^7*e^2 + 3/2*b^2*c*d^2*x^6*e + 3/2*a*c^2*d^2*x^6*e + 3/5*b^2*c*
```

```
d^3*x^5 + 3/5*a*c^2*d^3*x^5 + 1/7*b^3*x^7*e^3 + 6/7*a*b*c*x^7*e^3 + 1/2*b^3
*d*x^6*e^2 + 3*a*b*c*d*x^6*e^2 + 3/5*b^3*d^2*x^5*e + 18/5*a*b*c*d^2*x^5*e +
 1/4*b^3*d^3*x^4 + 3/2*a*b*c*d^3*x^4 + 1/2*a*b^2*x^6*e^3 + 1/2*a^2*c*x^6*e^
3 + 9/5*a*b^2*d*x^5*e^2 + 9/5*a^2*c*d*x^5*e^2 + 9/4*a*b^2*d^2*x^4*e + 9/4*a
^2*c*d^2*x^4*e + a*b^2*d^3*x^3 + a^2*c*d^3*x^3 + 3/5*a^2*b*x^5*e^3 + 9/4*a^
2*b*d*x^4*e^2 + 3*a^2*b*d^2*x^3*e + 3/2*a^2*b*d^3*x^2 + 1/4*a^3*x^4*e^3 + a
^3*d*x^3*e^2 + 3/2*a^3*d^2*x^2*e + a^3*d^3*x
```

3.2133 $\int (d+ex)^2 \left(a+bx+cx^2\right)^3 \, dx$

Optimal. Leaf size=272

$$\frac{3c(d+ex)^7\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{7e^7}-\frac{(d+ex)^6(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{6e^7}+\frac{3(d+ex)^5\left(ae^2\right.}{}$$

```
[Out] ((c*d^2 - b*d*e + a*e^2)^3*(d + e*x)^3)/(3*e^7) - (3*(2*c*d - b*e)*(c*d^2 -
 b*d*e + a*e^2)^2*(d + e*x)^4)/(4*e^7) + (3*(c*d^2 - b*d*e + a*e^2)*(5*c^2*
d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^5)/(5*e^7) - ((2*c*d - b*e)*(1
0*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*(d + e*x)^6)/(6*e^7) + (3*c*(5
*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^7)/(7*e^7) - (3*c^2*(2*c*
d - b*e)*(d + e*x)^8)/(8*e^7) + (c^3*(d + e*x)^9)/(9*e^7)
```

Rubi [A] time = 0.271228, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$\frac{3c(d+ex)^7\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{7e^7}-\frac{(d+ex)^6(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{6e^7}+\frac{3(d+ex)^5\left(ae^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*x + c*x^2)^3,x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)^3*(d + e*x)^3)/(3*e^7) - (3*(2*c*d - b*e)*(c*d^2 -
 b*d*e + a*e^2)^2*(d + e*x)^4)/(4*e^7) + (3*(c*d^2 - b*d*e + a*e^2)*(5*c^2*
d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^5)/(5*e^7) - ((2*c*d - b*e)*(1
0*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*(d + e*x)^6)/(6*e^7) + (3*c*(5
*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^7)/(7*e^7) - (3*c^2*(2*c*
d - b*e)*(d + e*x)^8)/(8*e^7) + (c^3*(d + e*x)^9)/(9*e^7)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^2\left(a+bx+cx^2\right)^3 dx = \int \left(\frac{\left(cd^2-bde+ae^2\right)^3(d+ex)^2}{e^6}+\frac{3(-2cd+be)\left(cd^2-bde+ae^2\right)^2(d+ex)^3}{e^6}+\frac{3\left(cd\right.}{}\right.$$

$$=\frac{\left(cd^2-bde+ae^2\right)^3(d+ex)^3}{3e^7}-\frac{3(2cd-be)\left(cd^2-bde+ae^2\right)^2(d+ex)^4}{4e^7}+\frac{3\left(cd^2-bd\right.}{}$$

Mathematica [A] time = 0.0948533, size = 282, normalized size = 1.04

$$\frac{1}{4}x^4\left(6a^2cde+6ab^2de+3ab\left(ae^2+2cd^2\right)+b^3d^2\right)+\frac{1}{2}a^2dx^2(2ae+3bd)+a^3d^2x+\frac{1}{7}cx^7\left(3ce(ae+2bd)+3b^2e^2+c^2d^2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*x + c*x^2)^3,x]
```

```
[Out] a^3*d^2*x + (a^2*d*(3*b*d + 2*a*e)*x^2)/2 + (a*(3*b^2*d^2 + 6*a*b*d*e + a*(
3*c*d^2 + a*e^2))*x^3)/3 + ((b^3*d^2 + 6*a*b^2*d*e + 6*a^2*c*d*e + 3*a*b*(2
*c*d^2 + a*e^2))*x^4)/4 + ((2*b^3*d*e + 12*a*b*c*d*e + 3*b^2*(c*d^2 + a*e^2
) + 3*a*c*(c*d^2 + a*e^2))*x^5)/5 + ((6*b^2*c*d*e + 6*a*c^2*d*e + b^3*e^2 +
 3*b*c*(c*d^2 + 2*a*e^2))*x^6)/6 + (c*(c^2*d^2 + 3*b^2*e^2 + 3*c*e*(2*b*d +
 a*e))*x^7)/7 + (c^2*e*(2*c*d + 3*b*e)*x^8)/8 + (c^3*e^2*x^9)/9
```

Maple [A] time = 0.041, size = 359, normalized size = 1.3

$$\frac{c^3e^2x^9}{9}+\frac{\left(3\,e^2bc^2+2\,dec^3\right)x^8}{8}+\frac{\left(d^2c^3+6\,debc^2+e^2\left(ac^2+2\,b^2c+c\left(2\,ac+b^2\right)\right)\right)x^7}{7}+\frac{\left(3\,d^2bc^2+2\,de\left(ac^2+2\,b^2c+c\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+b*x+a)^3,x)
```

```
[Out] 1/9*c^3*e^2*x^9+1/8*(3*b*c^2*e^2+2*c^3*d*e)*x^8+1/7*(d^2*c^3+6*d*e*b*c^2+e^
2*(a*c^2+2*b^2*c+c*(2*a*c+b^2)))*x^7+1/6*(3*d^2*b*c^2+2*d*e*(a*c^2+2*b^2*c+
c*(2*a*c+b^2))+e^2*(4*a*b*c+b*(2*a*c+b^2)))*x^6+1/5*(d^2*(a*c^2+2*b^2*c+c*(
2*a*c+b^2))+2*d*e*(4*a*b*c+b*(2*a*c+b^2))+e^2*(a*(2*a*c+b^2)+2*b^2*a+a^2*c)
)*x^5+1/4*(d^2*(4*a*b*c+b*(2*a*c+b^2))+2*d*e*(a*(2*a*c+b^2)+2*b^2*a+a^2*c)+
3*a^2*b*e^2)*x^4+1/3*(d^2*(a*(2*a*c+b^2)+2*b^2*a+a^2*c)+6*d*e*b*a^2+e^2*a^3
)*x^3+1/2*(2*a^3*d*e+3*a^2*b*d^2)*x^2+a^3*d^2*x
```

Maxima [A] time = 0.978291, size = 371, normalized size = 1.36

$$\frac{1}{9}\,c^3e^2x^9+\frac{1}{8}\left(2\,c^3de+3\,bc^2e^2\right)x^8+\frac{1}{7}\left(c^3d^2+6\,bc^2de+3\left(b^2c+ac^2\right)e^2\right)x^7+\frac{1}{6}\left(3\,bc^2d^2+6\left(b^2c+ac^2\right)de+\left(b^3+6\,abc\right)e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/9*c^3*e^2*x^9 + 1/8*(2*c^3*d*e + 3*b*c^2*e^2)*x^8 + 1/7*(c^3*d^2 + 6*b*c^
2*d*e + 3*(b^2*c + a*c^2)*e^2)*x^7 + 1/6*(3*b*c^2*d^2 + 6*(b^2*c + a*c^2)*d
*e + (b^3 + 6*a*b*c)*e^2)*x^6 + a^3*d^2*x + 1/5*(3*(b^2*c + a*c^2)*d^2 + 2*
(b^3 + 6*a*b*c)*d*e + 3*(a*b^2 + a^2*c)*e^2)*x^5 + 1/4*(3*a^2*b*e^2 + (b^3
+ 6*a*b*c)*d^2 + 6*(a*b^2 + a^2*c)*d*e)*x^4 + 1/3*(6*a^2*b*d*e + a^3*e^2 +
3*(a*b^2 + a^2*c)*d^2)*x^3 + 1/2*(3*a^2*b*d^2 + 2*a^3*d*e)*x^2
```

Fricas [A] time = 1.80794, size = 743, normalized size = 2.73

$$\frac{1}{9}x^9e^2c^3+\frac{1}{4}x^8edc^3+\frac{3}{8}x^8e^2c^2b+\frac{1}{7}x^7d^2c^3+\frac{6}{7}x^7edc^2b+\frac{3}{7}x^7e^2cb^2+\frac{3}{7}x^7e^2c^2a+\frac{1}{2}x^6d^2c^2b+x^6edcb^2+\frac{1}{6}x^6e^2b^3+x^6edc^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*e^2*c^3 + 1/4*x^8*e*d*c^3 + 3/8*x^8*e^2*c^2*b + 1/7*x^7*d^2*c^3 + 6
/7*x^7*e*d*c^2*b + 3/7*x^7*e^2*c*b^2 + 3/7*x^7*e^2*c^2*a + 1/2*x^6*d^2*c^2*
b + x^6*e*d*c*b^2 + 1/6*x^6*e^2*b^3 + x^6*e*d*c^2*a + x^6*e^2*c*b*a + 3/5*x
^5*d^2*c*b^2 + 2/5*x^5*e*d*b^3 + 3/5*x^5*d^2*c^2*a + 12/5*x^5*e*d*c*b*a + 3
/5*x^5*e^2*b^2*a + 3/5*x^5*e^2*c*a^2 + 1/4*x^4*d^2*b^3 + 3/2*x^4*d^2*c*b*a
+ 3/2*x^4*e*d*b^2*a + 3/2*x^4*e*d*c*a^2 + 3/4*x^4*e^2*b*a^2 + x^3*d^2*b^2*a
 + x^3*d^2*c*a^2 + 2*x^3*e*d*b*a^2 + 1/3*x^3*e^2*a^3 + 3/2*x^2*d^2*b*a^2 +
x^2*e*d*a^3 + x*d^2*a^3
```

Sympy [A] time = 0.116787, size = 332, normalized size = 1.22

$$a^3d^2x + \frac{c^3e^2x^9}{9} + x^8\left(\frac{3bc^2e^2}{8} + \frac{c^3de}{4}\right) + x^7\left(\frac{3ac^2e^2}{7} + \frac{3b^2ce^2}{7} + \frac{6bc^2de}{7} + \frac{c^3d^2}{7}\right) + x^6\left(abce^2 + ac^2de + \frac{b^3e^2}{6} + b^2cde + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+b*x+a)**3,x)
```

```
[Out] a**3*d**2*x + c**3*e**2*x**9/9 + x**8*(3*b*c**2*e**2/8 + c**3*d*e/4) + x**7
*(3*a*c**2*e**2/7 + 3*b**2*c*e**2/7 + 6*b*c**2*d*e/7 + c**3*d**2/7) + x**6*
(a*b*c*e**2 + a*c**2*d*e + b**3*e**2/6 + b**2*c*d*e + b*c**2*d**2/2) + x**5
*(3*a**2*c*e**2/5 + 3*a*b**2*e**2/5 + 12*a*b*c*d*e/5 + 3*a*c**2*d**2/5 + 2*
b**3*d*e/5 + 3*b**2*c*d**2/5) + x**4*(3*a**2*b*e**2/4 + 3*a**2*c*d*e/2 + 3*
a*b**2*d*e/2 + 3*a*b*c*d**2/2 + b**3*d**2/4) + x**3*(a**3*e**2/3 + 2*a**2*b
*d*e + a**2*c*d**2 + a*b**2*d**2) + x**2*(a**3*d*e + 3*a**2*b*d**2/2)
```

Giac [A] time = 1.1392, size = 446, normalized size = 1.64

$$\frac{1}{9}c^3x^9e^2 + \frac{1}{4}c^3dx^8e + \frac{1}{7}c^3d^2x^7 + \frac{3}{8}bc^2x^8e^2 + \frac{6}{7}bc^2dx^7e + \frac{1}{2}bc^2d^2x^6 + \frac{3}{7}b^2cx^7e^2 + \frac{3}{7}ac^2x^7e^2 + b^2cdx^6e + ac^2dx^6e +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/9*c^3*x^9*e^2 + 1/4*c^3*d*x^8*e + 1/7*c^3*d^2*x^7 + 3/8*b*c^2*x^8*e^2 + 6
/7*b*c^2*d*x^7*e + 1/2*b*c^2*d^2*x^6 + 3/7*b^2*c*x^7*e^2 + 3/7*a*c^2*x^7*e^
2 + b^2*c*d*x^6*e + a*c^2*d*x^6*e + 3/5*b^2*c*d^2*x^5 + 3/5*a*c^2*d^2*x^5 +
 1/6*b^3*x^6*e^2 + a*b*c*x^6*e^2 + 2/5*b^3*d*x^5*e + 12/5*a*b*c*d*x^5*e + 1
/4*b^3*d^2*x^4 + 3/2*a*b*c*d^2*x^4 + 3/5*a*b^2*x^5*e^2 + 3/5*a^2*c*x^5*e^2
+ 3/2*a*b^2*d*x^4*e + 3/2*a^2*c*d*x^4*e + a*b^2*d^2*x^3 + a^2*c*d^2*x^3 + 3
/4*a^2*b*x^4*e^2 + 2*a^2*b*d*x^3*e + 3/2*a^2*b*d^2*x^2 + 1/3*a^3*x^3*e^2 +
a^3*d*x^2*e + a^3*d^2*x
```

3.2134 $\int (d+ex)\left(a+bx+cx^2\right)^3\,dx$

Optimal. Leaf size=161

$$\frac{1}{4}x^4\left(3a^2ce+3ab^2e+6abcd+b^3d\right)+\frac{1}{2}a^2x^2(ae+3bd)+a^3dx+\frac{1}{5}x^5\left(6abce+3ac^2d+3b^2cd+b^3e\right)+\frac{1}{2}cx^6\left(ace+b^2e+b\right.$$

```
[Out] a^3*d*x + (a^2*(3*b*d + a*e)*x^2)/2 + a*(b^2*d + a*c*d + a*b*e)*x^3 + ((b^3
*d + 6*a*b*c*d + 3*a*b^2*e + 3*a^2*c*e)*x^4)/4 + ((3*b^2*c*d + 3*a*c^2*d +
b^3*e + 6*a*b*c*e)*x^5)/5 + (c*(b*c*d + b^2*e + a*c*e)*x^6)/2 + (c^2*(c*d +
 3*b*e)*x^7)/7 + (c^3*e*x^8)/8
```

Rubi [A] time = 0.157483, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {631}

$$\frac{1}{4}x^4\left(3a^2ce+3ab^2e+6abcd+b^3d\right)+\frac{1}{2}a^2x^2(ae+3bd)+a^3dx+\frac{1}{5}x^5\left(6abce+3ac^2d+3b^2cd+b^3e\right)+\frac{1}{2}cx^6\left(ace+b^2e+b\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*x + c*x^2)^3,x]
```

```
[Out] a^3*d*x + (a^2*(3*b*d + a*e)*x^2)/2 + a*(b^2*d + a*c*d + a*b*e)*x^3 + ((b^3
*d + 6*a*b*c*d + 3*a*b^2*e + 3*a^2*c*e)*x^4)/4 + ((3*b^2*c*d + 3*a*c^2*d +
b^3*e + 6*a*b*c*e)*x^5)/5 + (c*(b*c*d + b^2*e + a*c*e)*x^6)/2 + (c^2*(c*d +
 3*b*e)*x^7)/7 + (c^3*e*x^8)/8
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\int (d+ex)\left(a+bx+cx^2\right)^3\,dx=\int \left(a^3d+a^2(3bd+ae)x+3a\left(b^2d+acd+abe\right)x^2+\left(b^3d+6abcd+3ab^2e+3a^2ce\right)x^3+\right.$$

$$=a^3dx+\frac{1}{2}a^2(3bd+ae)x^2+a\left(b^2d+acd+abe\right)x^3+\frac{1}{4}\left(b^3d+6abcd+3ab^2e+3a^2ce\right)x^4-$$

Mathematica [A] time = 0.037271, size = 161, normalized size = 1.

$$\frac{1}{4}x^4\left(3a^2ce+3ab^2e+6abcd+b^3d\right)+\frac{1}{2}a^2x^2(ae+3bd)+a^3dx+\frac{1}{5}x^5\left(6abce+3ac^2d+3b^2cd+b^3e\right)+\frac{1}{2}cx^6\left(ace+b^2e+b\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*x + c*x^2)^3,x]
```

```
[Out] a^3*d*x + (a^2*(3*b*d + a*e)*x^2)/2 + a*(b^2*d + a*c*d + a*b*e)*x^3 + ((b^3
*d + 6*a*b*c*d + 3*a*b^2*e + 3*a^2*c*e)*x^4)/4 + ((3*b^2*c*d + 3*a*c^2*d +
b^3*e + 6*a*b*c*e)*x^5)/5 + (c*(b*c*d + b^2*e + a*c*e)*x^6)/2 + (c^2*(c*d +
```

```
3*b*e)*x^7)/7 + (c^3*e*x^8)/8
```

Maple [A] time = 0.04, size = 223, normalized size = 1.4

$$\frac{c^3ex^8}{8}+\frac{(3\,ebc^2+dc^3)\,x^7}{7}+\frac{(3\,dbc^2+e\,(ac^2+2\,b^2c+c\,(2\,ac+b^2)))\,x^6}{6}+\frac{(d\,(ac^2+2\,b^2c+c\,(2\,ac+b^2))+e\,(4\,abc+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+b*x+a)^3,x)
```

```
[Out] 1/8*c^3*e*x^8+1/7*(3*b*c^2*e+c^3*d)*x^7+1/6*(3*d*b*c^2+e*(a*c^2+2*b^2*c+c*(
2*a*c+b^2)))*x^6+1/5*(d*(a*c^2+2*b^2*c+c*(2*a*c+b^2))+e*(4*a*b*c+b*(2*a*c+b
^2)))*x^5+1/4*(d*(4*a*b*c+b*(2*a*c+b^2))+e*(a*(2*a*c+b^2)+2*b^2*a+a^2*c))*x
^4+1/3*(d*(a*(2*a*c+b^2)+2*b^2*a+a^2*c)+3*a^2*b*e)*x^3+1/2*(a^3*e+3*a^2*b*d
)*x^2+a^3*d*x
```

Maxima [A] time = 1.00462, size = 220, normalized size = 1.37

$$\frac{1}{8}\,c^3ex^8+\frac{1}{7}\left(c^3d+3\,bc^2e\right)x^7+\frac{1}{2}\left(bc^2d+\left(b^2c+ac^2\right)e\right)x^6+\frac{1}{5}\left(3\left(b^2c+ac^2\right)d+\left(b^3+6\,abc\right)e\right)x^5+a^3dx+\frac{1}{4}\left(\left(b^3+6\,a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*c^3*e*x^8 + 1/7*(c^3*d + 3*b*c^2*e)*x^7 + 1/2*(b*c^2*d + (b^2*c + a*c^2
)*e)*x^6 + 1/5*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*x^5 + a^3*d*x + 1/
4*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*x^4 + (a^2*b*e + (a*b^2 + a^2*c
)*d)*x^3 + 1/2*(3*a^2*b*d + a^3*e)*x^2
```

Fricas [A] time = 1.78398, size = 444, normalized size = 2.76

$$\frac{1}{8}x^8ec^3+\frac{1}{7}x^7dc^3+\frac{3}{7}x^7ec^2b+\frac{1}{2}x^6dc^2b+\frac{1}{2}x^6ecb^2+\frac{1}{2}x^6ec^2a+\frac{3}{5}x^5dcb^2+\frac{1}{5}x^5eb^3+\frac{3}{5}x^5dc^2a+\frac{6}{5}x^5ecba+\frac{1}{4}x^4db^3+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*e*c^3 + 1/7*x^7*d*c^3 + 3/7*x^7*e*c^2*b + 1/2*x^6*d*c^2*b + 1/2*x^6
*e*c*b^2 + 1/2*x^6*e*c^2*a + 3/5*x^5*d*c*b^2 + 1/5*x^5*e*b^3 + 3/5*x^5*d*c^
2*a + 6/5*x^5*e*c*b*a + 1/4*x^4*d*b^3 + 3/2*x^4*d*c*b*a + 3/4*x^4*e*b^2*a +
 3/4*x^4*e*c*a^2 + x^3*d*b^2*a + x^3*d*c*a^2 + x^3*e*b*a^2 + 3/2*x^2*d*b*a^
2 + 1/2*x^2*e*a^3 + x*d*a^3
```

Sympy [A] time = 0.097579, size = 190, normalized size = 1.18

$$a^3dx+\frac{c^3ex^8}{8}+x^7\left(\frac{3bc^2e}{7}+\frac{c^3d}{7}\right)+x^6\left(\frac{ac^2e}{2}+\frac{b^2ce}{2}+\frac{bc^2d}{2}\right)+x^5\left(\frac{6abce}{5}+\frac{3ac^2d}{5}+\frac{b^3e}{5}+\frac{3b^2cd}{5}\right)+x^4\left(\frac{3a^2ce}{4}+\frac{3ab}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+b*x+a)**3,x)
```

```
[Out] a**3*d*x + c**3*e*x**8/8 + x**7*(3*b*c**2*e/7 + c**3*d/7) + x**6*(a*c**2*e/
2 + b**2*c*e/2 + b*c**2*d/2) + x**5*(6*a*b*c*e/5 + 3*a*c**2*d/5 + b**3*e/5
+ 3*b**2*c*d/5) + x**4*(3*a**2*c*e/4 + 3*a*b**2*e/4 + 3*a*b*c*d/2 + b**3*d/
4) + x**3*(a**2*b*e + a**2*c*d + a*b**2*d) + x**2*(a**3*e/2 + 3*a**2*b*d/2)
```

Giac [A] time = 1.07771, size = 266, normalized size = 1.65

$$\frac{1}{8}c^3x^8e + \frac{1}{7}c^3dx^7 + \frac{3}{7}bc^2x^7e + \frac{1}{2}bc^2dx^6 + \frac{1}{2}b^2cx^6e + \frac{1}{2}ac^2x^6e + \frac{3}{5}b^2cdx^5 + \frac{3}{5}ac^2dx^5 + \frac{1}{5}b^3x^5e + \frac{6}{5}abcx^5e + \frac{1}{4}b^3dx^4 -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*c^3*x^8*e + 1/7*c^3*d*x^7 + 3/7*b*c^2*x^7*e + 1/2*b*c^2*d*x^6 + 1/2*b^2
*c*x^6*e + 1/2*a*c^2*x^6*e + 3/5*b^2*c*d*x^5 + 3/5*a*c^2*d*x^5 + 1/5*b^3*x^
5*e + 6/5*a*b*c*x^5*e + 1/4*b^3*d*x^4 + 3/2*a*b*c*d*x^4 + 3/4*a*b^2*x^4*e +
 3/4*a^2*c*x^4*e + a*b^2*d*x^3 + a^2*c*d*x^3 + a^2*b*x^3*e + 3/2*a^2*b*d*x^
2 + 1/2*a^3*x^2*e + a^3*d*x
```

3.2135 $\int \left(a+bx+cx^2\right)^3 \, dx$

Optimal. Leaf size=81

$$\frac{3}{2}a^2bx^2+a^3x+\frac{3}{5}cx^5\left(ac+b^2\right)+\frac{1}{4}bx^4\left(6ac+b^2\right)+ax^3\left(ac+b^2\right)+\frac{1}{2}bc^2x^6+\frac{c^3x^7}{7}$$

```
[Out] a^3*x + (3*a^2*b*x^2)/2 + a*(b^2 + a*c)*x^3 + (b*(b^2 + 6*a*c)*x^4)/4 + (3*
c*(b^2 + a*c)*x^5)/5 + (b*c^2*x^6)/2 + (c^3*x^7)/7
```

Rubi [A] time = 0.0557333, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {611}

$$\frac{3}{2}a^2bx^2+a^3x+\frac{3}{5}cx^5\left(ac+b^2\right)+\frac{1}{4}bx^4\left(6ac+b^2\right)+ax^3\left(ac+b^2\right)+\frac{1}{2}bc^2x^6+\frac{c^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3,x]
```

```
[Out] a^3*x + (3*a^2*b*x^2)/2 + a*(b^2 + a*c)*x^3 + (b*(b^2 + 6*a*c)*x^4)/4 + (3*
c*(b^2 + a*c)*x^5)/5 + (b*c^2*x^6)/2 + (c^3*x^7)/7
```

Rule 611

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && IGtQ[p, 0] && (EqQ[a, 0] ||  !PerfectSquareQ[b^2 - 4*a*c])
```

Rubi steps

$$\begin{aligned}\int \left(a+bx+cx^2\right)^3 \, dx &= \int \left(a^3+3a^2bx+3ab^2\left(1+\frac{ac}{b^2}\right)x^2+b^3\left(1+\frac{6ac}{b^2}\right)x^3+3b^2c\left(1+\frac{ac}{b^2}\right)x^4+3bc^2x^5+c^3x^6\right) dx\\ &= a^3x+\frac{3}{2}a^2bx^2+a\left(b^2+ac\right)x^3+\frac{1}{4}b\left(b^2+6ac\right)x^4+\frac{3}{5}c\left(b^2+ac\right)x^5+\frac{1}{2}bc^2x^6+\frac{c^3x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.0128007, size = 81, normalized size = 1.

$$\frac{3}{2}a^2bx^2+a^3x+\frac{3}{5}cx^5\left(ac+b^2\right)+\frac{1}{4}bx^4\left(6ac+b^2\right)+ax^3\left(ac+b^2\right)+\frac{1}{2}bc^2x^6+\frac{c^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3,x]
```

```
[Out] a^3*x + (3*a^2*b*x^2)/2 + a*(b^2 + a*c)*x^3 + (b*(b^2 + 6*a*c)*x^4)/4 + (3*
c*(b^2 + a*c)*x^5)/5 + (b*c^2*x^6)/2 + (c^3*x^7)/7
```

Maple [A] time = 0.039, size = 108, normalized size = 1.3

$$\frac{c^3x^7}{7}+\frac{bc^2x^6}{2}+\frac{\left(ac^2+2\,b^2c+c\left(2\,ac+b^2\right)\right)x^5}{5}+\frac{\left(4\,abc+b\left(2\,ac+b^2\right)\right)x^4}{4}+\frac{\left(a\left(2\,ac+b^2\right)+2\,b^2a+a^2c\right)x^3}{3}+\frac{3\,a^2b}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3,x)
```

```
[Out] 1/7*c^3*x^7+1/2*b*c^2*x^6+1/5*(a*c^2+2*b^2*c+c*(2*a*c+b^2))*x^5+1/4*(4*a*b*
c+b*(2*a*c+b^2))*x^4+1/3*(a*(2*a*c+b^2)+2*b^2*a+a^2*c)*x^3+3/2*a^2*b*x^2+x*
a^3
```

Maxima [A] time = 1.01875, size = 115, normalized size = 1.42

$$\frac{1}{7}c^3x^7 + \frac{1}{2}bc^2x^6 + \frac{3}{5}b^2cx^5 + \frac{1}{4}b^3x^4 + a^3x + \frac{1}{2}\left(2\,cx^3 + 3\,bx^2\right)a^2 + \frac{1}{10}\left(6\,c^2x^5 + 15\,bcx^4 + 10\,b^2x^3\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/7*c^3*x^7 + 1/2*b*c^2*x^6 + 3/5*b^2*c*x^5 + 1/4*b^3*x^4 + a^3*x + 1/2*(2*
c*x^3 + 3*b*x^2)*a^2 + 1/10*(6*c^2*x^5 + 15*b*c*x^4 + 10*b^2*x^3)*a
```

Fricas [A] time = 1.69686, size = 188, normalized size = 2.32

$$\frac{1}{7}x^7c^3 + \frac{1}{2}x^6c^2b + \frac{3}{5}x^5cb^2 + \frac{3}{5}x^5c^2a + \frac{1}{4}x^4b^3 + \frac{3}{2}x^4cba + x^3b^2a + x^3ca^2 + \frac{3}{2}x^2ba^2 + xa^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*c^3 + 1/2*x^6*c^2*b + 3/5*x^5*c*b^2 + 3/5*x^5*c^2*a + 1/4*x^4*b^3 +
 3/2*x^4*c*b*a + x^3*b^2*a + x^3*c*a^2 + 3/2*x^2*b*a^2 + x*a^3
```

Sympy [A] time = 0.078463, size = 85, normalized size = 1.05

$$a^3x + \frac{3a^2bx^2}{2} + \frac{bc^2x^6}{2} + \frac{c^3x^7}{7} + x^5\left(\frac{3ac^2}{5} + \frac{3b^2c}{5}\right) + x^4\left(\frac{3abc}{2} + \frac{b^3}{4}\right) + x^3\left(a^2c + ab^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3,x)
```

```
[Out] a**3*x + 3*a**2*b*x**2/2 + b*c**2*x**6/2 + c**3*x**7/7 + x**5*(3*a*c**2/5 +
 3*b**2*c/5) + x**4*(3*a*b*c/2 + b**3/4) + x**3*(a**2*c + a*b**2)
```

Giac [A] time = 1.07453, size = 111, normalized size = 1.37

$$\frac{1}{7}c^3x^7 + \frac{1}{2}bc^2x^6 + \frac{3}{5}b^2cx^5 + \frac{3}{5}ac^2x^5 + \frac{1}{4}b^3x^4 + \frac{3}{2}abcx^4 + ab^2x^3 + a^2cx^3 + \frac{3}{2}a^2bx^2 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/7*c^3*x^7 + 1/2*b*c^2*x^6 + 3/5*b^2*c*x^5 + 3/5*a*c^2*x^5 + 1/4*b^3*x^4 +
 3/2*a*b*c*x^4 + a*b^2*x^3 + a^2*c*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

3.2136 $\int \frac{(a+bx+cx^2)^3}{d+ex}\,dx$

Optimal. Leaf size=260

$$\frac{3c(d+ex)^4\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{4e^7}-\frac{(d+ex)^3(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{3e^7}+\frac{3(d+ex)^2\left(ae^2-b\right.}{}$$

```
[Out] (-3*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)^2*x)/e^6 + (3*(c*d^2 - b*d*e + a*
e^2)*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^2)/(2*e^7) - ((2*c
*d - b*e)*(10*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*(d + e*x)^3)/(3*e^
7) + (3*c*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^4)/(4*e^7) -
(3*c^2*(2*c*d - b*e)*(d + e*x)^5)/(5*e^7) + (c^3*(d + e*x)^6)/(6*e^7) + ((c
*d^2 - b*d*e + a*e^2)^3*Log[d + e*x])/e^7
```

Rubi [A] time = 0.32271, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$\frac{3c(d+ex)^4\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{4e^7}-\frac{(d+ex)^3(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{3e^7}+\frac{3(d+ex)^2\left(ae^2-b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(d + e*x),x]
```

```
[Out] (-3*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)^2*x)/e^6 + (3*(c*d^2 - b*d*e + a*
e^2)*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^2)/(2*e^7) - ((2*c
*d - b*e)*(10*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*(d + e*x)^3)/(3*e^
7) + (3*c*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^4)/(4*e^7) -
(3*c^2*(2*c*d - b*e)*(d + e*x)^5)/(5*e^7) + (c^3*(d + e*x)^6)/(6*e^7) + ((c
*d^2 - b*d*e + a*e^2)^3*Log[d + e*x])/e^7
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx+cx^2\right)^3}{d+ex}\,dx &= \int \left(\frac{3(-2cd+be)\left(cd^2-bde+ae^2\right)^2}{e^6}+\frac{\left(cd^2-bde+ae^2\right)^3}{e^6(d+ex)}+\frac{3\left(cd^2-bde+ae^2\right)\left(5c^2d^2-5bcde+\right.}{e^6}\right.\\ &= -\frac{3(2cd-be)\left(cd^2-bde+ae^2\right)^2x}{e^6}+\frac{3\left(cd^2-bde+ae^2\right)\left(5c^2d^2+b^2e^2-ce(5bd-ae)\right)(d+ex)^2}{2e^7}-\end{aligned}$$

Mathematica [A] time = 0.159707, size = 308, normalized size = 1.18

$$\frac{ex\left(15ce^2\left(6a^2e^2(ex-2d)+4abe\left(6d^2-3dex+2e^2x^2\right)+b^2\left(6d^2ex-12d^3-4de^2x^2+3e^3x^3\right)\right)+10be^3\left(18a^2e^2+9abe(ex-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(d + e*x),x]
```

```
[Out] (e*x*(c^3*(-60*d^5 + 30*d^4*e*x - 20*d^3*e^2*x^2 + 15*d^2*e^3*x^3 - 12*d*e^
4*x^4 + 10*e^5*x^5) + 10*b*e^3*(18*a^2*e^2 + 9*a*b*e*(-2*d + e*x) + b^2*(6*
d^2 - 3*d*e*x + 2*e^2*x^2)) + 15*c*e^2*(6*a^2*e^2*(-2*d + e*x) + 4*a*b*e*(6
*d^2 - 3*d*e*x + 2*e^2*x^2) + b^2*(-12*d^3 + 6*d^2*e*x - 4*d*e^2*x^2 + 3*e^
3*x^3)) + 3*c^2*e*(5*a*e*(-12*d^3 + 6*d^2*e*x - 4*d*e^2*x^2 + 3*e^3*x^3) +
b*(60*d^4 - 30*d^3*e*x + 20*d^2*e^2*x^2 - 15*d*e^3*x^3 + 12*e^4*x^4))) + 60
*(c*d^2 + e*(-(b*d) + a*e))^3*Log[d + e*x])/(60*e^7)
```

Maple [B] time = 0.044, size = 546, normalized size = 2.1

$$\frac{3\,bx^5c^2}{5\,e}+\frac{x^2c^3d^4}{2\,e^5}+6\,\frac{abcd^2x}{e^3}-3\,\frac{abx^2cd}{e^2}-6\,\frac{\ln\left(ex+d\right)abcd^3}{e^4}+3\,\frac{ba^2x}{e}-\frac{\ln\left(ex+d\right)b^3d^3}{e^4}+\frac{\ln\left(ex+d\right)c^3d^6}{e^7}+\frac{c^3x^6}{6\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(e*x+d),x)
```

```
[Out] 3/5/e*x^5*b*c^2+1/2/e^5*x^2*c^3*d^4+6/e^3*a*b*c*d^2*x-3/e^2*x^2*a*b*c*d-6/e
^4*ln(e*x+d)*a*b*c*d^3+3/e*b*a^2*x-1/e^4*ln(e*x+d)*b^3*d^3+1/e^7*ln(e*x+d)*
c^3*d^6+1/6*c^3*x^6/e+3/2/e*x^2*a^2*c+1/4/e^3*x^4*c^3*d^2-1/2/e^2*x^2*b^3*d
+3/4/e*x^4*b^2*c+1/e^3*b^3*d^2*x+3/2/e*x^2*a*b^2-3/e^2*ln(e*x+d)*a^2*b*d+2/
e*x^3*a*b*c+3/4/e*x^4*a*c^2+1/e*ln(e*x+d)*a^3+1/3/e*x^3*b^3-3/e^2*a*b^2*d*x
-3/e^4*a*c^2*d^3*x-3/e^4*b^2*c*d^3*x-3/2/e^4*x^2*b*c^2*d^3-3/e^2*a^2*c*d*x-
1/3/e^4*x^3*c^3*d^3-1/e^6*c^3*d^5*x-3/4/e^2*x^4*b*c^2*d+3/e^3*ln(e*x+d)*a^2
*c*d^2-1/e^2*x^3*b^2*c*d+3/2/e^3*x^2*a*c^2*d^2+3/2/e^3*x^2*b^2*c*d^2+3/e^5*
b*c^2*d^4*x-1/e^2*x^3*a*c^2*d+1/e^3*x^3*b*c^2*d^2+3/e^3*ln(e*x+d)*a*b^2*d^2
+3/e^5*ln(e*x+d)*a*c^2*d^4+3/e^5*ln(e*x+d)*b^2*c*d^4-3/e^6*ln(e*x+d)*b*c^2*
d^5-1/5*c^3*d*x^5/e^2
```

Maxima [A] time = 0.968411, size = 541, normalized size = 2.08

$$10\,c^3e^5x^6-12\left(c^3de^4-3\,bc^2e^5\right)x^5+15\left(c^3d^2e^3-3\,bc^2de^4+3\left(b^2c+ac^2\right)e^5\right)x^4-20\left(c^3d^3e^2-3\,bc^2d^2e^3+3\left(b^2c+ac^2\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/60*(10*c^3*e^5*x^6 - 12*(c^3*d*e^4 - 3*b*c^2*e^5)*x^5 + 15*(c^3*d^2*e^3 -
 3*b*c^2*d*e^4 + 3*(b^2*c + a*c^2)*e^5)*x^4 - 20*(c^3*d^3*e^2 - 3*b*c^2*d^2
*e^3 + 3*(b^2*c + a*c^2)*d*e^4 - (b^3 + 6*a*b*c)*e^5)*x^3 + 30*(c^3*d^4*e -
 3*b*c^2*d^3*e^2 + 3*(b^2*c + a*c^2)*d^2*e^3 - (b^3 + 6*a*b*c)*d*e^4 + 3*(a
*b^2 + a^2*c)*e^5)*x^2 - 60*(c^3*d^5 - 3*b*c^2*d^4*e - 3*a^2*b*e^5 + 3*(b^2
*c + a*c^2)*d^3*e^2 - (b^3 + 6*a*b*c)*d^2*e^3 + 3*(a*b^2 + a^2*c)*d*e^4)*x)
/e^6 + (c^3*d^6 - 3*b*c^2*d^5*e - 3*a^2*b*d*e^5 + a^3*e^6 + 3*(b^2*c + a*c^
2)*d^4*e^2 - (b^3 + 6*a*b*c)*d^3*e^3 + 3*(a*b^2 + a^2*c)*d^2*e^4)*log(e*x +
 d)/e^7
```

Fricas [A] time = 2.02334, size = 829, normalized size = 3.19

$$\frac{10\,c^3e^6x^6 - 12\left(c^3de^5 - 3\,bc^2e^6\right)x^5 + 15\left(c^3d^2e^4 - 3\,bc^2de^5 + 3\left(b^2c + ac^2\right)e^6\right)x^4 - 20\left(c^3d^3e^3 - 3\,bc^2d^2e^4 + 3\left(b^2c + ac^2\right)de^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/60*(10*c^3*e^6*x^6 - 12*(c^3*d*e^5 - 3*b*c^2*e^6)*x^5 + 15*(c^3*d^2*e^4 -
 3*b*c^2*d*e^5 + 3*(b^2*c + a*c^2)*e^6)*x^4 - 20*(c^3*d^3*e^3 - 3*b*c^2*d^2
*e^4 + 3*(b^2*c + a*c^2)*d*e^5 - (b^3 + 6*a*b*c)*e^6)*x^3 + 30*(c^3*d^4*e^2
 - 3*b*c^2*d^3*e^3 + 3*(b^2*c + a*c^2)*d^2*e^4 - (b^3 + 6*a*b*c)*d*e^5 + 3*
(a*b^2 + a^2*c)*e^6)*x^2 - 60*(c^3*d^5*e - 3*b*c^2*d^4*e^2 - 3*a^2*b*e^6 +
3*(b^2*c + a*c^2)*d^3*e^3 - (b^3 + 6*a*b*c)*d^2*e^4 + 3*(a*b^2 + a^2*c)*d*e
^5)*x + 60*(c^3*d^6 - 3*b*c^2*d^5*e - 3*a^2*b*d*e^5 + a^3*e^6 + 3*(b^2*c +
a*c^2)*d^4*e^2 - (b^3 + 6*a*b*c)*d^3*e^3 + 3*(a*b^2 + a^2*c)*d^2*e^4)*log(e
*x + d))/e^7
```

Sympy [A] time = 1.1198, size = 376, normalized size = 1.45

$$\frac{c^3x^6}{6e} + \frac{x^5\left(3bc^2e - c^3d\right)}{5e^2} + \frac{x^4\left(3ac^2e^2 + 3b^2ce^2 - 3bc^2de + c^3d^2\right)}{4e^3} + \frac{x^3\left(6abce^3 - 3ac^2de^2 + b^3e^3 - 3b^2cde^2 + 3bc^2d^2e - c^3d\right.}{3e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(e*x+d),x)
```

```
[Out] c**3*x**6/(6*e) + x**5*(3*b*c**2*e - c**3*d)/(5*e**2) + x**4*(3*a*c**2*e**2
 + 3*b**2*c*e**2 - 3*b*c**2*d*e + c**3*d**2)/(4*e**3) + x**3*(6*a*b*c*e**3
- 3*a*c**2*d*e**2 + b**3*e**3 - 3*b**2*c*d*e**2 + 3*b*c**2*d**2*e - c**3*d*
*3)/(3*e**4) + x**2*(3*a**2*c*e**4 + 3*a*b**2*e**4 - 6*a*b*c*d*e**3 + 3*a*c
**2*d**2*e**2 - b**3*d*e**3 + 3*b**2*c*d**2*e**2 - 3*b*c**2*d**3*e + c**3*d
**4)/(2*e**5) + x*(3*a**2*b*e**5 - 3*a**2*c*d*e**4 - 3*a*b**2*d*e**4 + 6*a*
b*c*d**2*e**3 - 3*a*c**2*d**3*e**2 + b**3*d**2*e**3 - 3*b**2*c*d**3*e**2 +
3*b*c**2*d**4*e - c**3*d**5)/e**6 + (a*e**2 - b*d*e + c*d**2)**3*log(d + e*
x)/e**7
```

Giac [A] time = 1.12113, size = 621, normalized size = 2.39

$$\left(c^3d^6 - 3\,bc^2d^5e + 3\,b^2cd^4e^2 + 3\,ac^2d^4e^2 - b^3d^3e^3 - 6\,abcd^3e^3 + 3\,ab^2d^2e^4 + 3\,a^2cd^2e^4 - 3\,a^2bde^5 + a^3e^6\right)e^{(-7)}\log\left(|xe + d|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d),x, algorithm="giac")
```

```
[Out] (c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3
- 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3
*e^6)*e^(-7)*log(abs(x*e + d)) + 1/60*(10*c^3*x^6*e^5 - 12*c^3*d*x^5*e^4 +
15*c^3*d^2*x^4*e^3 - 20*c^3*d^3*x^3*e^2 + 30*c^3*d^4*x^2*e - 60*c^3*d^5*x +
 36*b*c^2*x^5*e^5 - 45*b*c^2*d*x^4*e^4 + 60*b*c^2*d^2*x^3*e^3 - 90*b*c^2*d^
3*x^2*e^2 + 180*b*c^2*d^4*x*e + 45*b^2*c*x^4*e^5 + 45*a*c^2*x^4*e^5 - 60*b^
2*c*d*x^3*e^4 - 60*a*c^2*d*x^3*e^4 + 90*b^2*c*d^2*x^2*e^3 + 90*a*c^2*d^2*x^
```

```
2*e^3 - 180*b^2*c*d^3*x*e^2 - 180*a*c^2*d^3*x*e^2 + 20*b^3*x^3*e^5 + 120*a*
b*c*x^3*e^5 - 30*b^3*d*x^2*e^4 - 180*a*b*c*d*x^2*e^4 + 60*b^3*d^2*x*e^3 + 3
60*a*b*c*d^2*x*e^3 + 90*a*b^2*x^2*e^5 + 90*a^2*c*x^2*e^5 - 180*a*b^2*d*x*e^
4 - 180*a^2*c*d*x*e^4 + 180*a^2*b*x*e^5)*e^(-6)
```

3.2137 $\int \frac{(a+bx+cx^2)^3}{(d+ex)^2} dx$

Optimal. Leaf size=256

$$\frac{c(d+ex)^3\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{e^7}-\frac{(d+ex)^2(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{2e^7}+\frac{3x\left(ae^2-bde+cd^2\right)}{}$$

```
[Out] (3*(c*d^2 - b*d*e + a*e^2)*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*x)/e^6
 - (c*d^2 - b*d*e + a*e^2)^3/(e^7*(d + e*x)) - ((2*c*d - b*e)*(10*c^2*d^2 +
 b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*(d + e*x)^2)/(2*e^7) + (c*(5*c^2*d^2 + b^
2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^3)/e^7 - (3*c^2*(2*c*d - b*e)*(d + e*x
)^4)/(4*e^7) + (c^3*(d + e*x)^5)/(5*e^7) - (3*(2*c*d - b*e)*(c*d^2 - b*d*e
+ a*e^2)^2*Log[d + e*x])/e^7
```

Rubi [A] time = 0.320798, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$\frac{c(d+ex)^3\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{e^7}-\frac{(d+ex)^2(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{2e^7}+\frac{3x\left(ae^2-bde+cd^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(d + e*x)^2,x]
```

```
[Out] (3*(c*d^2 - b*d*e + a*e^2)*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*x)/e^6
 - (c*d^2 - b*d*e + a*e^2)^3/(e^7*(d + e*x)) - ((2*c*d - b*e)*(10*c^2*d^2 +
 b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*(d + e*x)^2)/(2*e^7) + (c*(5*c^2*d^2 + b^
2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^3)/e^7 - (3*c^2*(2*c*d - b*e)*(d + e*x
)^4)/(4*e^7) + (c^3*(d + e*x)^5)/(5*e^7) - (3*(2*c*d - b*e)*(c*d^2 - b*d*e
+ a*e^2)^2*Log[d + e*x])/e^7
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^3}{(d+ex)^2}\,dx=\int \left(\frac{3\left(cd^2-bde+ae^2\right)\left(5c^2d^2-5bcde+b^2e^2+ace^2\right)}{e^6}+\frac{\left(cd^2-bde+ae^2\right)^3}{e^6(d+ex)^2}+\frac{3(-2cd+be)\left(cd^2\right.}{e^6(d+}\right.$$

$$=\frac{3\left(cd^2-bde+ae^2\right)\left(5c^2d^2+b^2e^2-ce(5bd-ae)\right)x}{e^6}-\frac{\left(cd^2-bde+ae^2\right)^3}{e^7(d+ex)}-\frac{(2cd-be)\left(10c^2d^2+\right.}{}$$

Mathematica [A] time = 0.103753, size = 255, normalized size = 1.

$$\frac{20ex\left(3ce^2\left(a^2e^2-4abde+3b^2d^2\right)+b^2e^3(3ae-2bd)+3c^2d^2e(3ae-4bd)+5c^3d^4\right)+20ce^3x^3\left(ce(ae-2bd)+b^2e^2+c^2d^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(d + e*x)^2,x]
```

```
[Out] (20*e*(5*c^3*d^4 + 3*c^2*d^2*e*(-4*b*d + 3*a*e) + b^2*e^3*(-2*b*d + 3*a*e)
+ 3*c*e^2*(3*b^2*d^2 - 4*a*b*d*e + a^2*e^2))*x + 10*e^2*(-(c*d) + b*e)*(4*c
^2*d^2 + b^2*e^2 + c*e*(-5*b*d + 6*a*e))*x^2 + 20*c*e^3*(c^2*d^2 + b^2*e^2
+ c*e*(-2*b*d + a*e))*x^3 + 5*c^2*e^4*(-2*c*d + 3*b*e)*x^4 + 4*c^3*e^5*x^5
- (20*(c*d^2 + e*(-(b*d) + a*e))^3)/(d + e*x) - 60*(2*c*d - b*e)*(c*d^2 + e
*(-(b*d) + a*e))^2*Log[d + e*x])/(20*e^7)
```

Maple [B] time = 0.05, size = 585, normalized size = 2.3

$$18\,\frac{\ln(ex+d)\,abcd^2}{e^4}-3\,\frac{ax^2c^2d}{e^3}-3\,\frac{b^2x^2cd}{e^3}+\frac{c^3x^5}{5\,e^2}+9\,\frac{ac^2d^2x}{e^4}+9\,\frac{cb^2d^2x}{e^4}-12\,\frac{d^3bc^2x}{e^5}+3\,\frac{abx^2c}{e^2}-6\,\frac{dc\ln(ex+d)\,a^2}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(e*x+d)^2,x)
```

```
[Out] 18/e^4*ln(e*x+d)*a*b*c*d^2-3/e^3*x^2*a*c^2*d-3/e^3*x^2*b^2*c*d+1/5*c^3*x^5/
e^2+9/e^4*a*c^2*d^2*x+9/e^4*b^2*c*d^2*x-12/e^5*d^3*b*c^2*x+3/e^2*x^2*a*b*c-
6/e^3*ln(e*x+d)*a^2*c*d-6/e^3*ln(e*x+d)*a*b^2*d-12/e^5*ln(e*x+d)*a*c^2*d^3-
12/e^5*ln(e*x+d)*b^2*c*d^3+15/e^6*ln(e*x+d)*b*c^2*d^4+3/e^2/(e*x+d)*d*a^2*b
-3/e^3/(e*x+d)*a^2*c*d^2-3/e^3/(e*x+d)*a*b^2*d^2-3/e^5/(e*x+d)*a*c^2*d^4+3/
e^6/(e*x+d)*b*c^2*d^5-2/e^3*x^3*b*c^2*d-1/e^7/(e*x+d)*c^3*d^6+3/e^2*ln(e*x+
d)*a^2*b+3/e^4*ln(e*x+d)*b^3*d^2-6/e^7*ln(e*x+d)*c^3*d^5+1/e^4/(e*x+d)*b^3*
d^3+3/e^2*b^2*a*x-2/e^3*b^3*d*x+5/e^6*c^3*d^4*x+3/4/e^2*x^4*b*c^2+1/e^2*x^3
*a*c^2+1/e^2*x^3*b^2*c+1/e^4*x^3*c^3*d^2-2/e^5*x^2*c^3*d^3+3/e^2*a^2*c*x-1/
e/(e*x+d)*a^3-3/e^5/(e*x+d)*b^2*c*d^4+1/2/e^2*x^2*b^3+9/2/e^4*x^2*b*c^2*d^2
+6/e^4/(e*x+d)*a*b*c*d^3-12/e^3*a*b*c*d*x-1/2*c^3*d*x^4/e^3
```

Maxima [A] time = 0.989355, size = 554, normalized size = 2.16

$$-\frac{c^3d^6-3\,bc^2d^5e-3\,a^2bde^5+a^3e^6+3\left(b^2c+ac^2\right)d^4e^2-\left(b^3+6\,abc\right)d^3e^3+3\left(ab^2+a^2c\right)d^2e^4}{e^8x+de^7}+\frac{4\,c^3e^4x^5-5\left(2\,c^3de^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(c^3*d^6 - 3*b*c^2*d^5*e - 3*a^2*b*d*e^5 + a^3*e^6 + 3*(b^2*c + a*c^2)*d^4
*e^2 - (b^3 + 6*a*b*c)*d^3*e^3 + 3*(a*b^2 + a^2*c)*d^2*e^4)/(e^8*x + d*e^7)
 + 1/20*(4*c^3*e^4*x^5 - 5*(2*c^3*d*e^3 - 3*b*c^2*e^4)*x^4 + 20*(c^3*d^2*e^
2 - 2*b*c^2*d*e^3 + (b^2*c + a*c^2)*e^4)*x^3 - 10*(4*c^3*d^3*e - 9*b*c^2*d^
2*e^2 + 6*(b^2*c + a*c^2)*d*e^3 - (b^3 + 6*a*b*c)*e^4)*x^2 + 20*(5*c^3*d^4
- 12*b*c^2*d^3*e + 9*(b^2*c + a*c^2)*d^2*e^2 - 2*(b^3 + 6*a*b*c)*d*e^3 + 3*
(a*b^2 + a^2*c)*e^4)*x)/e^6 - 3*(2*c^3*d^5 - 5*b*c^2*d^4*e - a^2*b*e^5 + 4*
(b^2*c + a*c^2)*d^3*e^2 - (b^3 + 6*a*b*c)*d^2*e^3 + 2*(a*b^2 + a^2*c)*d*e^4
)*log(e*x + d)/e^7
```

Fricas [B] time = 2.11092, size = 1197, normalized size = 4.68

$$\frac{4\,c^3e^6x^6 - 20\,c^3d^6 + 60\,bc^2d^5e + 60\,a^2bde^5 - 20\,a^3e^6 - 60\left(b^2c + ac^2\right)d^4e^2 + 20\left(b^3 + 6\,abc\right)d^3e^3 - 60\left(ab^2 + a^2c\right)d^2e^4 - 3\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/20*(4*c^3*e^6*x^6 - 20*c^3*d^6 + 60*b*c^2*d^5*e + 60*a^2*b*d*e^5 - 20*a^3
*e^6 - 60*(b^2*c + a*c^2)*d^4*e^2 + 20*(b^3 + 6*a*b*c)*d^3*e^3 - 60*(a*b^2
+ a^2*c)*d^2*e^4 - 3*(2*c^3*d*e^5 - 5*b*c^2*e^6)*x^5 + 5*(2*c^3*d^2*e^4 - 5
*b*c^2*d*e^5 + 4*(b^2*c + a*c^2)*e^6)*x^4 - 10*(2*c^3*d^3*e^3 - 5*b*c^2*d^2
*e^4 + 4*(b^2*c + a*c^2)*d*e^5 - (b^3 + 6*a*b*c)*e^6)*x^3 + 30*(2*c^3*d^4*e
^2 - 5*b*c^2*d^3*e^3 + 4*(b^2*c + a*c^2)*d^2*e^4 - (b^3 + 6*a*b*c)*d*e^5 +
2*(a*b^2 + a^2*c)*e^6)*x^2 + 20*(5*c^3*d^5*e - 12*b*c^2*d^4*e^2 + 9*(b^2*c
+ a*c^2)*d^3*e^3 - 2*(b^3 + 6*a*b*c)*d^2*e^4 + 3*(a*b^2 + a^2*c)*d*e^5)*x -
 60*(2*c^3*d^6 - 5*b*c^2*d^5*e - a^2*b*d*e^5 + 4*(b^2*c + a*c^2)*d^4*e^2 -
(b^3 + 6*a*b*c)*d^3*e^3 + 2*(a*b^2 + a^2*c)*d^2*e^4 + (2*c^3*d^5*e - 5*b*c^
2*d^4*e^2 - a^2*b*e^6 + 4*(b^2*c + a*c^2)*d^3*e^3 - (b^3 + 6*a*b*c)*d^2*e^4
 + 2*(a*b^2 + a^2*c)*d*e^5)*x)*log(e*x + d))/(e^8*x + d*e^7)
```

Sympy [A] time = 2.64275, size = 403, normalized size = 1.57

$$\frac{c^3x^5}{5e^2} - \frac{a^3e^6 - 3a^2bde^5 + 3a^2cd^2e^4 + 3ab^2d^2e^4 - 6abcd^3e^3 + 3ac^2d^4e^2 - b^3d^3e^3 + 3b^2cd^4e^2 - 3bc^2d^5e + c^3d^6}{de^7 + e^8x} + \frac{x^4\left(3bc^2e - \right.}{4e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(e*x+d)**2,x)
```

```
[Out] c**3*x**5/(5*e**2) - (a**3*e**6 - 3*a**2*b*d*e**5 + 3*a**2*c*d**2*e**4 + 3*
a*b**2*d**2*e**4 - 6*a*b*c*d**3*e**3 + 3*a*c**2*d**4*e**2 - b**3*d**3*e**3
+ 3*b**2*c*d**4*e**2 - 3*b*c**2*d**5*e + c**3*d**6)/(d*e**7 + e**8*x) + x**
4*(3*b*c**2*e - 2*c**3*d)/(4*e**3) + x**3*(a*c**2*e**2 + b**2*c*e**2 - 2*b*
c**2*d*e + c**3*d**2)/e**4 + x**2*(6*a*b*c*e**3 - 6*a*c**2*d*e**2 + b**3*e*
*3 - 6*b**2*c*d*e**2 + 9*b*c**2*d**2*e - 4*c**3*d**3)/(2*e**5) + x*(3*a**2*
c*e**4 + 3*a*b**2*e**4 - 12*a*b*c*d*e**3 + 9*a*c**2*d**2*e**2 - 2*b**3*d*e*
*3 + 9*b**2*c*d**2*e**2 - 12*b*c**2*d**3*e + 5*c**3*d**4)/e**6 + 3*(b*e - 2
*c*d)*(a*e**2 - b*d*e + c*d**2)**2*log(d + e*x)/e**7
```

Giac [B] time = 1.1272, size = 730, normalized size = 2.85

$$\frac{1}{20}\left(4\,c^3 - \frac{15\left(2\,c^3de - bc^2e^2\right)e^{(-1)}}{xe + d} + \frac{20\left(5\,c^3d^2e^2 - 5\,bc^2de^3 + b^2ce^4 + ac^2e^4\right)e^{(-2)}}{(xe + d)^2} - \frac{10\left(20\,c^3d^3e^3 - 30\,bc^2d^2e^4 + 12\,b^2cde\right.}{(xe +}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/20*(4*c^3 - 15*(2*c^3*d*e - b*c^2*e^2)*e^(-1)/(x*e + d) + 20*(5*c^3*d^2*e
^2 - 5*b*c^2*d*e^3 + b^2*c*e^4 + a*c^2*e^4)*e^(-2)/(x*e + d)^2 - 10*(20*c^3
*d^3*e^3 - 30*b*c^2*d^2*e^4 + 12*b^2*c*d*e^5 + 12*a*c^2*d*e^5 - b^3*e^6 - 6
```

```
*a*b*c*e^6)*e^(-3)/(x*e + d)^3 + 60*(5*c^3*d^4*e^4 - 10*b*c^2*d^3*e^5 + 6*b
^2*c*d^2*e^6 + 6*a*c^2*d^2*e^6 - b^3*d*e^7 - 6*a*b*c*d*e^7 + a*b^2*e^8 + a^
2*c*e^8)*e^(-4)/(x*e + d)^4)*(x*e + d)^5*e^(-7) + 3*(2*c^3*d^5 - 5*b*c^2*d^
4*e + 4*b^2*c*d^3*e^2 + 4*a*c^2*d^3*e^2 - b^3*d^2*e^3 - 6*a*b*c*d^2*e^3 + 2
*a*b^2*d*e^4 + 2*a^2*c*d*e^4 - a^2*b*e^5)*e^(-7)*log(abs(x*e + d)*e^(-1)/(x
*e + d)^2) - (c^3*d^6*e^5/(x*e + d) - 3*b*c^2*d^5*e^6/(x*e + d) + 3*b^2*c*d
^4*e^7/(x*e + d) + 3*a*c^2*d^4*e^7/(x*e + d) - b^3*d^3*e^8/(x*e + d) - 6*a*
b*c*d^3*e^8/(x*e + d) + 3*a*b^2*d^2*e^9/(x*e + d) + 3*a^2*c*d^2*e^9/(x*e +
d) - 3*a^2*b*d*e^10/(x*e + d) + a^3*e^11/(x*e + d))*e^(-12)
```

3.2138 $\int \frac{(a+bx+cx^2)^3}{(d+ex)^3} dx$

Optimal. Leaf size=255

$$\frac{3cx^2\left(-ce(3bd-ae)+b^2e^2+2c^2d^2\right)}{2e^5}-\frac{x\left(-9c^2de(2bd-ae)+3bce^2(3bd-2ae)-b^3e^3+10c^3d^3\right)}{e^6}+\frac{3\log(d+ex)\left(ae^2-bd\right.}{}$$

```
[Out] -(((10*c^3*d^3 - b^3*e^3 + 3*b*c*e^2*(3*b*d - 2*a*e) - 9*c^2*d*e*(2*b*d - a
*e))*x)/e^6) + (3*c*(2*c^2*d^2 + b^2*e^2 - c*e*(3*b*d - a*e))*x^2)/(2*e^5)
- (c^2*(c*d - b*e)*x^3)/e^4 + (c^3*x^4)/(4*e^3) - (c*d^2 - b*d*e + a*e^2)^3
/(2*e^7*(d + e*x)^2) + (3*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)^2)/(e^7*(d
+ e*x)) + (3*(c*d^2 - b*d*e + a*e^2)*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*
e))*Log[d + e*x])/e^7
```

Rubi [A] time = 0.343087, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$\frac{3cx^2\left(-ce(3bd-ae)+b^2e^2+2c^2d^2\right)}{2e^5}-\frac{x\left(-9c^2de(2bd-ae)+3bce^2(3bd-2ae)-b^3e^3+10c^3d^3\right)}{e^6}+\frac{3\log(d+ex)\left(ae^2-bd\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(d + e*x)^3,x]
```

```
[Out] -(((10*c^3*d^3 - b^3*e^3 + 3*b*c*e^2*(3*b*d - 2*a*e) - 9*c^2*d*e*(2*b*d - a
*e))*x)/e^6) + (3*c*(2*c^2*d^2 + b^2*e^2 - c*e*(3*b*d - a*e))*x^2)/(2*e^5)
- (c^2*(c*d - b*e)*x^3)/e^4 + (c^3*x^4)/(4*e^3) - (c*d^2 - b*d*e + a*e^2)^3
/(2*e^7*(d + e*x)^2) + (3*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)^2)/(e^7*(d
+ e*x)) + (3*(c*d^2 - b*d*e + a*e^2)*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*
e))*Log[d + e*x])/e^7
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^3}{(d+ex)^3}dx=\int \left(\frac{-10c^3d^3+b^3e^3-3bce^2(3bd-2ae)+9c^2de(2bd-ae)}{e^6}+\frac{3c\left(2c^2d^2+b^2e^2-ce(3bd-ae)\right)x}{e^5}-\right.$$

$$=-\frac{\left(10c^3d^3-b^3e^3+3bce^2(3bd-2ae)-9c^2de(2bd-ae)\right)x}{e^6}+\frac{3c\left(2c^2d^2+b^2e^2-ce(3bd-ae)\right)x^2}{2e^5}-$$

Mathematica [A] time = 0.12979, size = 265, normalized size = 1.04

$$\frac{12\log(d+ex)\left(ce^2\left(a^2e^2-6abde+6b^2d^2\right)+b^2e^3(ae-bd)+2c^2d^2e(3ae-5bd)+5c^3d^4\right)+6ce^2x^2\left(ce(ae-3bd)+b^2e^2+2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(d + e*x)^3,x]
```

```
[Out] (4*e*(-10*c^3*d^3 + b^3*e^3 + 9*c^2*d*e*(2*b*d - a*e) + 3*b*c*e^2*(-3*b*d +
 2*a*e))*x + 6*c*e^2*(2*c^2*d^2 + b^2*e^2 + c*e*(-3*b*d + a*e))*x^2 + 4*c^2
*e^3*(-(c*d) + b*e)*x^3 + c^3*e^4*x^4 - (2*(c*d^2 + e*(-(b*d) + a*e))^3)/(d
 + e*x)^2 + (12*(2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e))^2)/(d + e*x) + 12*
(5*c^3*d^4 + b^2*e^3*(-(b*d) + a*e) + 2*c^2*d^2*e*(-5*b*d + 3*a*e) + c*e^2*
(6*b^2*d^2 - 6*a*b*d*e + a^2*e^2))*Log[d + e*x])/(4*e^7)
```

Maple [B] time = 0.053, size = 624, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(e*x+d)^3,x)
```

```
[Out] 1/4*c^3*x^4/e^3+1/e^3*x^3*b*c^2+3/2/e^3*x^2*a*c^2+3/2/e^3*x^2*b^2*c+3/e^5*x
^2*c^3*d^2-10/e^6*c^3*d^3*x+1/2/e^4/(e*x+d)^2*b^3*d^3-1/2/e^7/(e*x+d)^2*c^3
*d^6-18/e^4*ln(e*x+d)*a*b*c*d-18/e^4/(e*x+d)*a*b*c*d^2+1/e^3*b^3*x-1/2/e/(e
*x+d)^2*a^3+3/e^3*ln(e*x+d)*a^2*c+3/e^3*ln(e*x+d)*a*b^2-3/e^4*ln(e*x+d)*b^3
*d+15/e^7*ln(e*x+d)*c^3*d^4-3/e^2/(e*x+d)*a^2*b-3/e^4/(e*x+d)*b^3*d^2+6/e^7
/(e*x+d)*c^3*d^5-c^3*d*x^3/e^4-9/e^4*c^2*a*d*x-9/e^4*b^2*c*d*x+6/e^3/(e*x+d
)*a*b^2*d+18/e^5*ln(e*x+d)*a*c^2*d^2+12/e^5/(e*x+d)*a*c^2*d^3+12/e^5/(e*x+d
)*b^2*c*d^3-15/e^6/(e*x+d)*b*c^2*d^4+18/e^5*ln(e*x+d)*b^2*c*d^2-30/e^6*ln(e
*x+d)*d^3*b*c^2+6/e^3/(e*x+d)*a^2*c*d-3/2/e^3/(e*x+d)^2*d^2*a*b^2-3/2/e^5/(
e*x+d)^2*a*c^2*d^4-3/2/e^5/(e*x+d)^2*b^2*c*d^4+3/2/e^6/(e*x+d)^2*b*c^2*d^5+
6/e^3*a*b*c*x-9/2/e^4*x^2*b*c^2*d+18/e^5*b*c^2*d^2*x+3/2/e^2/(e*x+d)^2*d*a^
2*b-3/2/e^3/(e*x+d)^2*a^2*c*d^2+3/e^4/(e*x+d)^2*a*b*c*d^3
```

Maxima [A] time = 1.05871, size = 563, normalized size = 2.21

$$\frac{11\,c^3d^6 - 27\,bc^2d^5e - 3\,a^2bde^5 - a^3e^6 + 21\left(b^2c + ac^2\right)d^4e^2 - 5\left(b^3 + 6\,abc\right)d^3e^3 + 9\left(ab^2 + a^2c\right)d^2e^4 + 6\left(2\,c^3d^5e - 5\,bc^2\right.}{2\left(e^9x^2 + 2\,de^8x + d^2e^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(11*c^3*d^6 - 27*b*c^2*d^5*e - 3*a^2*b*d*e^5 - a^3*e^6 + 21*(b^2*c + a*
c^2)*d^4*e^2 - 5*(b^3 + 6*a*b*c)*d^3*e^3 + 9*(a*b^2 + a^2*c)*d^2*e^4 + 6*(2
*c^3*d^5*e - 5*b*c^2*d^4*e^2 - a^2*b*e^6 + 4*(b^2*c + a*c^2)*d^3*e^3 - (b^3
 + 6*a*b*c)*d^2*e^4 + 2*(a*b^2 + a^2*c)*d*e^5)*x)/(e^9*x^2 + 2*d*e^8*x + d^
2*e^7) + 1/4*(c^3*e^3*x^4 - 4*(c^3*d*e^2 - b*c^2*e^3)*x^3 + 6*(2*c^3*d^2*e
- 3*b*c^2*d*e^2 + (b^2*c + a*c^2)*e^3)*x^2 - 4*(10*c^3*d^3 - 18*b*c^2*d^2*e
 + 9*(b^2*c + a*c^2)*d*e^2 - (b^3 + 6*a*b*c)*e^3)*x)/e^6 + 3*(5*c^3*d^4 - 1
0*b*c^2*d^3*e + 6*(b^2*c + a*c^2)*d^2*e^2 - (b^3 + 6*a*b*c)*d*e^3 + (a*b^2
+ a^2*c)*e^4)*log(e*x + d)/e^7
```

Fricas [B] time = 2.06215, size = 1318, normalized size = 5.17

$$\frac{c^3e^6x^6 + 22\,c^3d^6 - 54\,bc^2d^5e - 6\,a^2bde^5 - 2\,a^3e^6 + 42\left(b^2c + ac^2\right)d^4e^2 - 10\left(b^3 + 6\,abc\right)d^3e^3 + 18\left(ab^2 + a^2c\right)d^2e^4 - 2\left(c^3de\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(c^3*e^6*x^6 + 22*c^3*d^6 - 54*b*c^2*d^5*e - 6*a^2*b*d*e^5 - 2*a^3*e^6
+ 42*(b^2*c + a*c^2)*d^4*e^2 - 10*(b^3 + 6*a*b*c)*d^3*e^3 + 18*(a*b^2 + a^2
*c)*d^2*e^4 - 2*(c^3*d*e^5 - 2*b*c^2*e^6)*x^5 + (5*c^3*d^2*e^4 - 10*b*c^2*d
*e^5 + 6*(b^2*c + a*c^2)*e^6)*x^4 - 4*(5*c^3*d^3*e^3 - 10*b*c^2*d^2*e^4 + 6
*(b^2*c + a*c^2)*d*e^5 - (b^3 + 6*a*b*c)*e^6)*x^3 - 2*(34*c^3*d^4*e^2 - 63*
b*c^2*d^3*e^3 + 33*(b^2*c + a*c^2)*d^2*e^4 - 4*(b^3 + 6*a*b*c)*d*e^5)*x^2 -
 4*(4*c^3*d^5*e - 3*b*c^2*d^4*e^2 + 3*a^2*b*e^6 - 3*(b^2*c + a*c^2)*d^3*e^3
 + 2*(b^3 + 6*a*b*c)*d^2*e^4 - 6*(a*b^2 + a^2*c)*d*e^5)*x + 12*(5*c^3*d^6 -
 10*b*c^2*d^5*e + 6*(b^2*c + a*c^2)*d^4*e^2 - (b^3 + 6*a*b*c)*d^3*e^3 + (a*
b^2 + a^2*c)*d^2*e^4 + (5*c^3*d^4*e^2 - 10*b*c^2*d^3*e^3 + 6*(b^2*c + a*c^2
)*d^2*e^4 - (b^3 + 6*a*b*c)*d*e^5 + (a*b^2 + a^2*c)*e^6)*x^2 + 2*(5*c^3*d^5
*e - 10*b*c^2*d^4*e^2 + 6*(b^2*c + a*c^2)*d^3*e^3 - (b^3 + 6*a*b*c)*d^2*e^4
 + (a*b^2 + a^2*c)*d*e^5)*x)*log(e*x + d))/(e^9*x^2 + 2*d*e^8*x + d^2*e^7)
```

Sympy [A] time = 9.01999, size = 457, normalized size = 1.79

$$\frac{c^3x^4}{4e^3} - \frac{a^3e^6 + 3a^2bde^5 - 9a^2cd^2e^4 - 9ab^2d^2e^4 + 30abcd^3e^3 - 21ac^2d^4e^2 + 5b^3d^3e^3 - 21b^2cd^4e^2 + 27bc^2d^5e - 11c^3d^6 + x\left(6\right.}{2d^2e^7 + 4de^8x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(e*x+d)**3,x)
```

```
[Out] c**3*x**4/(4*e**3) - (a**3*e**6 + 3*a**2*b*d*e**5 - 9*a**2*c*d**2*e**4 - 9*
a*b**2*d**2*e**4 + 30*a*b*c*d**3*e**3 - 21*a*c**2*d**4*e**2 + 5*b**3*d**3*e
**3 - 21*b**2*c*d**4*e**2 + 27*b*c**2*d**5*e - 11*c**3*d**6 + x*(6*a**2*b*e
**6 - 12*a**2*c*d*e**5 - 12*a*b**2*d*e**5 + 36*a*b*c*d**2*e**4 - 24*a*c**2*
d**3*e**3 + 6*b**3*d**2*e**4 - 24*b**2*c*d**3*e**3 + 30*b*c**2*d**4*e**2 -
12*c**3*d**5*e))/(2*d**2*e**7 + 4*d*e**8*x + 2*e**9*x**2) + x**3*(b*c**2*e
- c**3*d)/e**4 + x**2*(3*a*c**2*e**2 + 3*b**2*c*e**2 - 9*b*c**2*d*e + 6*c**
3*d**2)/(2*e**5) + x*(6*a*b*c*e**3 - 9*a*c**2*d*e**2 + b**3*e**3 - 9*b**2*c
*d*e**2 + 18*b*c**2*d**2*e - 10*c**3*d**3)/e**6 + 3*(a*e**2 - b*d*e + c*d**
2)*(a*c*e**2 + b**2*e**2 - 5*b*c*d*e + 5*c**2*d**2)*log(d + e*x)/e**7
```

Giac [A] time = 1.10103, size = 583, normalized size = 2.29

$$3\left(5\,c^3d^4 - 10\,bc^2d^3e + 6\,b^2cd^2e^2 + 6\,ac^2d^2e^2 - b^3de^3 - 6\,abcde^3 + ab^2e^4 + a^2ce^4\right)e^{(-7)}\log\left(|xe + d|\right) + \frac{1}{4}\left(c^3x^4e^9 - 4\,c^3dx^3e^8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 3*(5*c^3*d^4 - 10*b*c^2*d^3*e + 6*b^2*c*d^2*e^2 + 6*a*c^2*d^2*e^2 - b^3*d*e
^3 - 6*a*b*c*d*e^3 + a*b^2*e^4 + a^2*c*e^4)*e^(-7)*log(abs(x*e + d)) + 1/4*
```

```
(c^3*x^4*e^9 - 4*c^3*d*x^3*e^8 + 12*c^3*d^2*x^2*e^7 - 40*c^3*d^3*x*e^6 + 4*
b*c^2*x^3*e^9 - 18*b*c^2*d*x^2*e^8 + 72*b*c^2*d^2*x*e^7 + 6*b^2*c*x^2*e^9 +
 6*a*c^2*x^2*e^9 - 36*b^2*c*d*x*e^8 - 36*a*c^2*d*x*e^8 + 4*b^3*x*e^9 + 24*a
*b*c*x*e^9)*e^(-12) + 1/2*(11*c^3*d^6 - 27*b*c^2*d^5*e + 21*b^2*c*d^4*e^2 +
 21*a*c^2*d^4*e^2 - 5*b^3*d^3*e^3 - 30*a*b*c*d^3*e^3 + 9*a*b^2*d^2*e^4 + 9*
a^2*c*d^2*e^4 - 3*a^2*b*d*e^5 - a^3*e^6 + 6*(2*c^3*d^5*e - 5*b*c^2*d^4*e^2
+ 4*b^2*c*d^3*e^3 + 4*a*c^2*d^3*e^3 - b^3*d^2*e^4 - 6*a*b*c*d^2*e^4 + 2*a*b
^2*d*e^5 + 2*a^2*c*d*e^5 - a^2*b*e^6)*x)*e^(-7)/(x*e + d)^2
```

3.2139 $\int \frac{(a+bx+cx^2)^3}{(d+ex)^4} dx$

Optimal. Leaf size=251

$$-\frac{3\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)\left(ae^2-bde+cd^2\right)}{e^7(d+ex)}+\frac{cx\left(-3ce(4bd-ae)+3b^2e^2+10c^2d^2\right)}{e^6}-\frac{(2cd-be)\log(d+ex)\left(-2\right.}{}$$

```
[Out] (c*(10*c^2*d^2 + 3*b^2*e^2 - 3*c*e*(4*b*d - a*e))*x)/e^6 - (c^2*(4*c*d - 3*
b*e)*x^2)/(2*e^5) + (c^3*x^3)/(3*e^4) - (c*d^2 - b*d*e + a*e^2)^3/(3*e^7*(d
 + e*x)^3) + (3*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)^2)/(2*e^7*(d + e*x)^2
) - (3*(c*d^2 - b*d*e + a*e^2)*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e)))/(
e^7*(d + e*x)) - ((2*c*d - b*e)*(10*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*
e))*Log[d + e*x])/e^7
```

Rubi [A] time = 0.296033, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$-\frac{3\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)\left(ae^2-bde+cd^2\right)}{e^7(d+ex)}+\frac{cx\left(-3ce(4bd-ae)+3b^2e^2+10c^2d^2\right)}{e^6}-\frac{(2cd-be)\log(d+ex)\left(-2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(d + e*x)^4,x]
```

```
[Out] (c*(10*c^2*d^2 + 3*b^2*e^2 - 3*c*e*(4*b*d - a*e))*x)/e^6 - (c^2*(4*c*d - 3*
b*e)*x^2)/(2*e^5) + (c^3*x^3)/(3*e^4) - (c*d^2 - b*d*e + a*e^2)^3/(3*e^7*(d
 + e*x)^3) + (3*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)^2)/(2*e^7*(d + e*x)^2
) - (3*(c*d^2 - b*d*e + a*e^2)*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e)))/(
e^7*(d + e*x)) - ((2*c*d - b*e)*(10*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*
e))*Log[d + e*x])/e^7
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^3}{(d+ex)^4}\,dx = \int \left(\frac{c\left(10c^2d^2+3b^2e^2-3ce(4bd-ae)\right)}{e^6}-\frac{c^2(4cd-3be)x}{e^5}+\frac{c^3x^2}{e^4}+\frac{\left(cd^2-bde+ae^2\right)^3}{e^6(d+ex)^4}+\frac{3(-2cd}{}\right.$$

$$=\frac{c\left(10c^2d^2+3b^2e^2-3ce(4bd-ae)\right)x}{e^6}-\frac{c^2(4cd-3be)x^2}{2e^5}+\frac{c^3x^3}{3e^4}-\frac{\left(cd^2-bde+ae^2\right)^3}{3e^7(d+ex)^3}+\frac{3(2cd-b}{2}$$

Mathematica [A] time = 0.115325, size = 260, normalized size = 1.04

$$\frac{-\frac{18\left(ce^2\left(a^2e^2-6abde+6b^2d^2\right)+b^2e^3(ae-bd)+2c^2d^2e(3ae-5bd)+5c^3d^4\right)}{d+ex}+6cex\left(3ce(ae-4bd)+3b^2e^2+10c^2d^2\right)-6(2cd-be)\log(d+ex)\left(2\right.}{6e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(d + e*x)^4,x]
```

```
[Out] (6*c*e*(10*c^2*d^2 + 3*b^2*e^2 + 3*c*e*(-4*b*d + a*e))*x + 3*c^2*e^2*(-4*c*
d + 3*b*e)*x^2 + 2*c^3*e^3*x^3 - (2*(c*d^2 + e*(-(b*d) + a*e))^3)/(d + e*x)
^3 + (9*(2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e))^2)/(d + e*x)^2 - (18*(5*c^
3*d^4 + b^2*e^3*(-(b*d) + a*e) + 2*c^2*d^2*e*(-5*b*d + 3*a*e) + c*e^2*(6*b^
2*d^2 - 6*a*b*d*e + a^2*e^2)))/(d + e*x) - 6*(2*c*d - b*e)*(10*c^2*d^2 + b^
2*e^2 + 2*c*e*(-5*b*d + 3*a*e))*Log[d + e*x])/(6*e^7)
```

Maple [B] time = 0.054, size = 653, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(e*x+d)^4,x)
```

```
[Out] 3/e^3/(e*x+d)^2*a^2*c*d+3/e^3/(e*x+d)^2*a*b^2*d+6/e^5/(e*x+d)^2*a*c^2*d^3+6
/e^5/(e*x+d)^2*b^2*c*d^3-15/2/e^6/(e*x+d)^2*b*c^2*d^4+1/3*c^3*x^3/e^4+1/e^2
/(e*x+d)^3*d*a^2*b+6/e^4*ln(e*x+d)*a*b*c-12/e^5*ln(e*x+d)*c^2*a*d-18/e^5/(e
*x+d)*a*c^2*d^2-18/e^5/(e*x+d)*b^2*c*d^2-12/e^5*ln(e*x+d)*b^2*c*d+30/e^6*ln
(e*x+d)*b*c^2*d^2-1/e^5/(e*x+d)^3*b^2*c*d^4-1/e^5/(e*x+d)^3*a*c^2*d^4-1/e^3
/(e*x+d)^3*a^2*c*d^2-1/e^3/(e*x+d)^3*d^2*a*b^2-12*c^2/e^5*b*d*x+3/e^4/(e*x+
d)*b^3*d-15/e^7/(e*x+d)*c^3*d^4+3/2*c^2/e^4*x^2*b+3*c^2/e^4*a*x+3*c/e^4*b^2
*x+10*c^3/e^6*d^2*x+1/3/e^4/(e*x+d)^3*d^3*b^3-1/3/e^7/(e*x+d)^3*c^3*d^6-3/2
/e^2/(e*x+d)^2*a^2*b-3/2/e^4/(e*x+d)^2*b^3*d^2+3/e^7/(e*x+d)^2*c^3*d^5-20/e
^7*ln(e*x+d)*c^3*d^3-3/e^3/(e*x+d)*a^2*c-3/e^3/(e*x+d)*a*b^2+30/e^6/(e*x+d)
*d^3*b*c^2+1/e^6/(e*x+d)^3*b*c^2*d^5-1/3/e/(e*x+d)^3*a^3+1/e^4*ln(e*x+d)*b^
3-9/e^4/(e*x+d)^2*a*b*c*d^2+18/e^4/(e*x+d)*a*b*c*d+2/e^4/(e*x+d)^3*d^3*a*b*
c-2*c^3*d*x^2/e^5
```

Maxima [A] time = 1.08161, size = 581, normalized size = 2.31

$$-\frac{74\,c^3d^6 - 141\,bc^2d^5e + 3\,a^2bde^5 + 2\,a^3e^6 + 78\left(b^2c + ac^2\right)d^4e^2 - 11\left(b^3 + 6\,abc\right)d^3e^3 + 6\left(ab^2 + a^2c\right)d^2e^4 + 18\left(5\,c^3d^4e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(74*c^3*d^6 - 141*b*c^2*d^5*e + 3*a^2*b*d*e^5 + 2*a^3*e^6 + 78*(b^2*c
+ a*c^2)*d^4*e^2 - 11*(b^3 + 6*a*b*c)*d^3*e^3 + 6*(a*b^2 + a^2*c)*d^2*e^4 +
 18*(5*c^3*d^4*e^2 - 10*b*c^2*d^3*e^3 + 6*(b^2*c + a*c^2)*d^2*e^4 - (b^3 +
6*a*b*c)*d*e^5 + (a*b^2 + a^2*c)*e^6)*x^2 + 9*(18*c^3*d^5*e - 35*b*c^2*d^4*
e^2 + a^2*b*e^6 + 20*(b^2*c + a*c^2)*d^3*e^3 - 3*(b^3 + 6*a*b*c)*d^2*e^4 +
2*(a*b^2 + a^2*c)*d*e^5)*x)/(e^10*x^3 + 3*d*e^9*x^2 + 3*d^2*e^8*x + d^3*e^7
) + 1/6*(2*c^3*e^2*x^3 - 3*(4*c^3*d*e - 3*b*c^2*e^2)*x^2 + 6*(10*c^3*d^2 -
12*b*c^2*d*e + 3*(b^2*c + a*c^2)*e^2)*x)/e^6 - (20*c^3*d^3 - 30*b*c^2*d^2*e
 + 12*(b^2*c + a*c^2)*d*e^2 - (b^3 + 6*a*b*c)*e^3)*log(e*x + d)/e^7
```

Fricas [B] time = 2.04135, size = 1393, normalized size = 5.55

$$\frac{2\,c^3e^6x^6 - 74\,c^3d^6 + 141\,bc^2d^5e - 3\,a^2bde^5 - 2\,a^3e^6 - 78\,\left(b^2c + ac^2\right)d^4e^2 + 11\,\left(b^3 + 6\,abc\right)d^3e^3 - 6\,\left(ab^2 + a^2c\right)d^2e^4 - 3\,(2\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*c^3*e^6*x^6 - 74*c^3*d^6 + 141*b*c^2*d^5*e - 3*a^2*b*d*e^5 - 2*a^3*e
^6 - 78*(b^2*c + a*c^2)*d^4*e^2 + 11*(b^3 + 6*a*b*c)*d^3*e^3 - 6*(a*b^2 + a
^2*c)*d^2*e^4 - 3*(2*c^3*d*e^5 - 3*b*c^2*e^6)*x^5 + 3*(10*c^3*d^2*e^4 - 15*
b*c^2*d*e^5 + 6*(b^2*c + a*c^2)*e^6)*x^4 + (146*c^3*d^3*e^3 - 189*b*c^2*d^2
*e^4 + 54*(b^2*c + a*c^2)*d*e^5)*x^3 + 3*(26*c^3*d^4*e^2 - 9*b*c^2*d^3*e^3
- 18*(b^2*c + a*c^2)*d^2*e^4 + 6*(b^3 + 6*a*b*c)*d*e^5 - 6*(a*b^2 + a^2*c)*
e^6)*x^2 - 3*(34*c^3*d^5*e - 81*b*c^2*d^4*e^2 + 3*a^2*b*e^6 + 54*(b^2*c + a
*c^2)*d^3*e^3 - 9*(b^3 + 6*a*b*c)*d^2*e^4 + 6*(a*b^2 + a^2*c)*d*e^5)*x - 6*
(20*c^3*d^6 - 30*b*c^2*d^5*e + 12*(b^2*c + a*c^2)*d^4*e^2 - (b^3 + 6*a*b*c)
*d^3*e^3 + (20*c^3*d^3*e^3 - 30*b*c^2*d^2*e^4 + 12*(b^2*c + a*c^2)*d*e^5 -
(b^3 + 6*a*b*c)*e^6)*x^3 + 3*(20*c^3*d^4*e^2 - 30*b*c^2*d^3*e^3 + 12*(b^2*c
 + a*c^2)*d^2*e^4 - (b^3 + 6*a*b*c)*d*e^5)*x^2 + 3*(20*c^3*d^5*e - 30*b*c^2
*d^4*e^2 + 12*(b^2*c + a*c^2)*d^3*e^3 - (b^3 + 6*a*b*c)*d^2*e^4)*x)*log(e*x
 + d))/(e^10*x^3 + 3*d*e^9*x^2 + 3*d^2*e^8*x + d^3*e^7)
```

Sympy [B] time = 34.2345, size = 498, normalized size = 1.98

$$\frac{c^3x^3}{3e^4} - \frac{2a^3e^6 + 3a^2bde^5 + 6a^2cd^2e^4 + 6ab^2d^2e^4 - 66abcd^3e^3 + 78ac^2d^4e^2 - 11b^3d^3e^3 + 78b^2cd^4e^2 - 141bc^2d^5e + 74c^3d^6 +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(e*x+d)**4,x)
```

```
[Out] c**3*x**3/(3*e**4) - (2*a**3*e**6 + 3*a**2*b*d*e**5 + 6*a**2*c*d**2*e**4 +
6*a*b**2*d**2*e**4 - 66*a*b*c*d**3*e**3 + 78*a*c**2*d**4*e**2 - 11*b**3*d**
3*e**3 + 78*b**2*c*d**4*e**2 - 141*b*c**2*d**5*e + 74*c**3*d**6 + x**2*(18*
a**2*c*e**6 + 18*a*b**2*e**6 - 108*a*b*c*d*e**5 + 108*a*c**2*d**2*e**4 - 18
*b**3*d*e**5 + 108*b**2*c*d**2*e**4 - 180*b*c**2*d**3*e**3 + 90*c**3*d**4*e
**2) + x*(9*a**2*b*e**6 + 18*a**2*c*d*e**5 + 18*a*b**2*d*e**5 - 162*a*b*c*d
**2*e**4 + 180*a*c**2*d**3*e**3 - 27*b**3*d**2*e**4 + 180*b**2*c*d**3*e**3
- 315*b*c**2*d**4*e**2 + 162*c**3*d**5*e))/(6*d**3*e**7 + 18*d**2*e**8*x +
18*d*e**9*x**2 + 6*e**10*x**3) + x**2*(3*b*c**2*e - 4*c**3*d)/(2*e**5) + x*
(3*a*c**2*e**2 + 3*b**2*c*e**2 - 12*b*c**2*d*e + 10*c**3*d**2)/e**6 + (b*e
- 2*c*d)*(6*a*c*e**2 + b**2*e**2 - 10*b*c*d*e + 10*c**2*d**2)*log(d + e*x)/
e**7
```

Giac [A] time = 1.11308, size = 572, normalized size = 2.28

$$-\left(20\,c^3d^3 - 30\,bc^2d^2e + 12\,b^2cde^2 + 12\,ac^2de^2 - b^3e^3 - 6\,abce^3\right)e^{(-7)}\log\left(|xe + d|\right) + \frac{1}{6}\left(2\,c^3x^3e^8 - 12\,c^3dx^2e^7 + 60\,c^3d^2xe^6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -(20*c^3*d^3 - 30*b*c^2*d^2*e + 12*b^2*c*d*e^2 + 12*a*c^2*d*e^2 - b^3*e^3 -
 6*a*b*c*e^3)*e^(-7)*log(abs(x*e + d)) + 1/6*(2*c^3*x^3*e^8 - 12*c^3*d*x^2*
e^7 + 60*c^3*d^2*x*e^6 + 9*b*c^2*x^2*e^8 - 72*b*c^2*d*x*e^7 + 18*b^2*c*x*e^
8 + 18*a*c^2*x*e^8)*e^(-12) - 1/6*(74*c^3*d^6 - 141*b*c^2*d^5*e + 78*b^2*c*
d^4*e^2 + 78*a*c^2*d^4*e^2 - 11*b^3*d^3*e^3 - 66*a*b*c*d^3*e^3 + 6*a*b^2*d^
2*e^4 + 6*a^2*c*d^2*e^4 + 3*a^2*b*d*e^5 + 2*a^3*e^6 + 18*(5*c^3*d^4*e^2 - 1
0*b*c^2*d^3*e^3 + 6*b^2*c*d^2*e^4 + 6*a*c^2*d^2*e^4 - b^3*d*e^5 - 6*a*b*c*d
*e^5 + a*b^2*e^6 + a^2*c*e^6)*x^2 + 9*(18*c^3*d^5*e - 35*b*c^2*d^4*e^2 + 20
*b^2*c*d^3*e^3 + 20*a*c^2*d^3*e^3 - 3*b^3*d^2*e^4 - 18*a*b*c*d^2*e^4 + 2*a*
b^2*d*e^5 + 2*a^2*c*d*e^5 + a^2*b*e^6)*x)*e^(-7)/(x*e + d)^3
```

3.2140 $\int \frac{(a+bx+cx^2)^3}{(d+ex)^5} dx$

Optimal. Leaf size=251

$$\frac{(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{e^7(d+ex)}-\frac{3\left(ae^2-bde+cd^2\right)\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{2e^7(d+ex)^2}+\frac{3c\log(d+ex)\left(-ce(\right.}{}$$

```
[Out] -((c^2*(5*c*d - 3*b*e)*x)/e^6) + (c^3*x^2)/(2*e^5) - (c*d^2 - b*d*e + a*e^2
)^3/(4*e^7*(d + e*x)^4) + ((2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)^2)/(e^7*(d
 + e*x)^3) - (3*(c*d^2 - b*d*e + a*e^2)*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d -
 a*e)))/(2*e^7*(d + e*x)^2) + ((2*c*d - b*e)*(10*c^2*d^2 + b^2*e^2 - 2*c*e*
(5*b*d - 3*a*e)))/(e^7*(d + e*x)) + (3*c*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d
- a*e))*Log[d + e*x])/e^7
```

Rubi [A] time = 0.274186, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$\frac{(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{e^7(d+ex)}-\frac{3\left(ae^2-bde+cd^2\right)\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{2e^7(d+ex)^2}+\frac{3c\log(d+ex)\left(-ce(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(d + e*x)^5,x]
```

```
[Out] -((c^2*(5*c*d - 3*b*e)*x)/e^6) + (c^3*x^2)/(2*e^5) - (c*d^2 - b*d*e + a*e^2
)^3/(4*e^7*(d + e*x)^4) + ((2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)^2)/(e^7*(d
 + e*x)^3) - (3*(c*d^2 - b*d*e + a*e^2)*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d -
 a*e)))/(2*e^7*(d + e*x)^2) + ((2*c*d - b*e)*(10*c^2*d^2 + b^2*e^2 - 2*c*e*
(5*b*d - 3*a*e)))/(e^7*(d + e*x)) + (3*c*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d
- a*e))*Log[d + e*x])/e^7
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^3}{(d+ex)^5} dx = \int \left(-\frac{c^2(5cd-3be)}{e^6}+\frac{c^3x}{e^5}+\frac{(cd^2-bde+ae^2)^3}{e^6(d+ex)^5}+\frac{3(-2cd+be)(cd^2-bde+ae^2)^2}{e^6(d+ex)^4}+\frac{3(cd^2-bd}{}\right.$$

$$=-\frac{c^2(5cd-3be)x}{e^6}+\frac{c^3x^2}{2e^5}-\frac{(cd^2-bde+ae^2)^3}{4e^7(d+ex)^4}+\frac{(2cd-be)(cd^2-bde+ae^2)^2}{e^7(d+ex)^3}-\frac{3(cd^2-bde+a}{}$$

Mathematica [A] time = 0.175801, size = 402, normalized size = 1.6

$$\frac{-ce^2\left(a^2e^2\left(d^2+4dex+6e^2x^2\right)+6abe\left(4d^2ex+d^3+6de^2x^2+4e^3x^3\right)+b^2(-d)\left(88d^2ex+25d^3+108de^2x^2+48e^3x^3\right)\right)-e}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(d + e*x)^5,x]
```

```
[Out] (c^3*(57*d^6 + 168*d^5*e*x + 132*d^4*e^2*x^2 - 32*d^3*e^3*x^3 - 68*d^2*e^4*
x^4 - 12*d*e^5*x^5 + 2*e^6*x^6) - e^3*(a^3*e^3 + a^2*b*e^2*(d + 4*e*x) + a*
b^2*e*(d^2 + 4*d*e*x + 6*e^2*x^2) + b^3*(d^3 + 4*d^2*e*x + 6*d*e^2*x^2 + 4*
e^3*x^3)) - c*e^2*(a^2*e^2*(d^2 + 4*d*e*x + 6*e^2*x^2) + 6*a*b*e*(d^3 + 4*d
^2*e*x + 6*d*e^2*x^2 + 4*e^3*x^3) - b^2*d*(25*d^3 + 88*d^2*e*x + 108*d*e^2*
x^2 + 48*e^3*x^3)) + c^2*e*(a*d*e*(25*d^3 + 88*d^2*e*x + 108*d*e^2*x^2 + 48
*e^3*x^3) - b*(77*d^5 + 248*d^4*e*x + 252*d^3*e^2*x^2 + 48*d^2*e^3*x^3 - 48
*d*e^4*x^4 - 12*e^5*x^5)) + 12*c*(5*c^2*d^2 + b^2*e^2 + c*e*(-5*b*d + a*e))
*(d + e*x)^4*Log[d + e*x])/(4*e^7*(d + e*x)^4)
```

Maple [B] time = 0.051, size = 678, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(e*x+d)^5,x)
```

```
[Out] 1/2*c^3*x^2/e^5+1/4/e^4/(e*x+d)^4*d^3*b^3-1/4/e^7/(e*x+d)^4*c^3*d^6-1/e^2/(
e*x+d)^3*a^2*b-1/e^4/(e*x+d)^3*b^3*d^2+2/e^7/(e*x+d)^3*c^3*d^5+3*c^2/e^5*x*
b+3*c^2/e^5*ln(e*x+d)*a+3*c/e^5*ln(e*x+d)*b^2+15*c^3/e^7*ln(e*x+d)*d^2+20/e
^7/(e*x+d)*c^3*d^3-6/e^4/(e*x+d)^3*a*b*c*d^2+9/e^4/(e*x+d)^2*a*b*c*d-3/4/e^
5/(e*x+d)^4*d^4*b^2*c+3/4/e^6/(e*x+d)^4*b*c^2*d^5+2/e^3/(e*x+d)^3*a^2*c*d+2
/e^3/(e*x+d)^3*a*b^2*d+4/e^5/(e*x+d)^3*a*c^2*d^3+4/e^5/(e*x+d)^3*b^2*c*d^3-
5/e^6/(e*x+d)^3*b*c^2*d^4-9/e^5/(e*x+d)^2*a*c^2*d^2-1/e^4/(e*x+d)*b^3-1/4/e
/(e*x+d)^4*a^3-9/e^5/(e*x+d)^2*b^2*c*d^2-3/4/e^3/(e*x+d)^4*a^2*c*d^2+15/e^6
/(e*x+d)^2*d^3*b*c^2-15*c^2/e^6*ln(e*x+d)*b*d-6/e^4/(e*x+d)*a*b*c+12/e^5/(e
*x+d)*c^2*a*d+12/e^5/(e*x+d)*b^2*c*d-30/e^6/(e*x+d)*b*c^2*d^2+3/4/e^2/(e*x+
d)^4*d*a^2*b-3/2/e^3/(e*x+d)^2*a^2*c-3/2/e^3/(e*x+d)^2*a*b^2+3/2/e^4/(e*x+d
)^2*b^3*d-15/2/e^7/(e*x+d)^2*c^3*d^4+3/2/e^4/(e*x+d)^4*d^3*a*b*c-3/4/e^3/(e
*x+d)^4*d^2*a*b^2-3/4/e^5/(e*x+d)^4*a*c^2*d^4-5*c^3*d*x/e^6
```

Maxima [A] time = 1.04103, size = 595, normalized size = 2.37

$$\frac{57\,c^3d^6 - 77\,bc^2d^5e - a^2bde^5 - a^3e^6 + 25\left(b^2c + ac^2\right)d^4e^2 - \left(b^3 + 6\,abc\right)d^3e^3 - \left(ab^2 + a^2c\right)d^2e^4 + 4\left(20\,c^3d^3e^3 - 30\,bc^2d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^5,x, algorithm="maxima")
```

```
[Out] 1/4*(57*c^3*d^6 - 77*b*c^2*d^5*e - a^2*b*d*e^5 - a^3*e^6 + 25*(b^2*c + a*c^
2)*d^4*e^2 - (b^3 + 6*a*b*c)*d^3*e^3 - (a*b^2 + a^2*c)*d^2*e^4 + 4*(20*c^3*
d^3*e^3 - 30*b*c^2*d^2*e^4 + 12*(b^2*c + a*c^2)*d*e^5 - (b^3 + 6*a*b*c)*e^6
)*x^3 + 6*(35*c^3*d^4*e^2 - 50*b*c^2*d^3*e^3 + 18*(b^2*c + a*c^2)*d^2*e^4 -
 (b^3 + 6*a*b*c)*d*e^5 - (a*b^2 + a^2*c)*e^6)*x^2 + 4*(47*c^3*d^5*e - 65*b*
c^2*d^4*e^2 - a^2*b*e^6 + 22*(b^2*c + a*c^2)*d^3*e^3 - (b^3 + 6*a*b*c)*d^2*
e^4 - (a*b^2 + a^2*c)*d*e^5)*x)/(e^11*x^4 + 4*d*e^10*x^3 + 6*d^2*e^9*x^2 +
4*d^3*e^8*x + d^4*e^7) + 1/2*(c^3*e*x^2 - 2*(5*c^3*d - 3*b*c^2*e)*x)/e^6 +
```

```
3*(5*c^3*d^2 - 5*b*c^2*d*e + (b^2*c + a*c^2)*e^2)*log(e*x + d)/e^7
```

Fricas [B] time = 1.98217, size = 1308, normalized size = 5.21

$$\frac{2\,c^3e^6x^6 + 57\,c^3d^6 - 77\,bc^2d^5e - a^2bde^5 - a^3e^6 + 25\left(b^2c + ac^2\right)d^4e^2 - \left(b^3 + 6\,abc\right)d^3e^3 - \left(ab^2 + a^2c\right)d^2e^4 - 12\left(c^3de^5 - bc\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^5,x, algorithm="fricas")
```

```
[Out] 1/4*(2*c^3*e^6*x^6 + 57*c^3*d^6 - 77*b*c^2*d^5*e - a^2*b*d*e^5 - a^3*e^6 +
25*(b^2*c + a*c^2)*d^4*e^2 - (b^3 + 6*a*b*c)*d^3*e^3 - (a*b^2 + a^2*c)*d^2*
e^4 - 12*(c^3*d*e^5 - b*c^2*e^6)*x^5 - 4*(17*c^3*d^2*e^4 - 12*b*c^2*d*e^5)*
x^4 - 4*(8*c^3*d^3*e^3 + 12*b*c^2*d^2*e^4 - 12*(b^2*c + a*c^2)*d*e^5 + (b^3
 + 6*a*b*c)*e^6)*x^3 + 6*(22*c^3*d^4*e^2 - 42*b*c^2*d^3*e^3 + 18*(b^2*c + a
*c^2)*d^2*e^4 - (b^3 + 6*a*b*c)*d*e^5 - (a*b^2 + a^2*c)*e^6)*x^2 + 4*(42*c^
3*d^5*e - 62*b*c^2*d^4*e^2 - a^2*b*e^6 + 22*(b^2*c + a*c^2)*d^3*e^3 - (b^3
+ 6*a*b*c)*d^2*e^4 - (a*b^2 + a^2*c)*d*e^5)*x + 12*(5*c^3*d^6 - 5*b*c^2*d^5
*e + (b^2*c + a*c^2)*d^4*e^2 + (5*c^3*d^2*e^4 - 5*b*c^2*d*e^5 + (b^2*c + a*
c^2)*e^6)*x^4 + 4*(5*c^3*d^3*e^3 - 5*b*c^2*d^2*e^4 + (b^2*c + a*c^2)*d*e^5)
*x^3 + 6*(5*c^3*d^4*e^2 - 5*b*c^2*d^3*e^3 + (b^2*c + a*c^2)*d^2*e^4)*x^2 +
4*(5*c^3*d^5*e - 5*b*c^2*d^4*e^2 + (b^2*c + a*c^2)*d^3*e^3)*x)*log(e*x + d)
)/(e^11*x^4 + 4*d*e^10*x^3 + 6*d^2*e^9*x^2 + 4*d^3*e^8*x + d^4*e^7)
```

Sympy [B] time = 141.101, size = 518, normalized size = 2.06

$$\frac{c^3x^2}{2e^5} + \frac{3c\left(ace^2 + b^2e^2 - 5bcde + 5c^2d^2\right)\log\left(d + ex\right)}{e^7} - \frac{a^3e^6 + a^2bde^5 + a^2cd^2e^4 + ab^2d^2e^4 + 6abcd^3e^3 - 25ac^2d^4e^2 + b^3d^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(e*x+d)**5,x)
```

```
[Out] c**3*x**2/(2*e**5) + 3*c*(a*c*e**2 + b**2*e**2 - 5*b*c*d*e + 5*c**2*d**2)*l
og(d + e*x)/e**7 - (a**3*e**6 + a**2*b*d*e**5 + a**2*c*d**2*e**4 + a*b**2*d
**2*e**4 + 6*a*b*c*d**3*e**3 - 25*a*c**2*d**4*e**2 + b**3*d**3*e**3 - 25*b*
*2*c*d**4*e**2 + 77*b*c**2*d**5*e - 57*c**3*d**6 + x**3*(24*a*b*c*e**6 - 48
*a*c**2*d*e**5 + 4*b**3*e**6 - 48*b**2*c*d*e**5 + 120*b*c**2*d**2*e**4 - 80
*c**3*d**3*e**3) + x**2*(6*a**2*c*e**6 + 6*a*b**2*e**6 + 36*a*b*c*d*e**5 -
108*a*c**2*d**2*e**4 + 6*b**3*d*e**5 - 108*b**2*c*d**2*e**4 + 300*b*c**2*d*
*3*e**3 - 210*c**3*d**4*e**2) + x*(4*a**2*b*e**6 + 4*a**2*c*d*e**5 + 4*a*b*
*2*d*e**5 + 24*a*b*c*d**2*e**4 - 88*a*c**2*d**3*e**3 + 4*b**3*d**2*e**4 - 8
8*b**2*c*d**3*e**3 + 260*b*c**2*d**4*e**2 - 188*c**3*d**5*e))/(4*d**4*e**7
+ 16*d**3*e**8*x + 24*d**2*e**9*x**2 + 16*d*e**10*x**3 + 4*e**11*x**4) + x*
(3*b*c**2*e - 5*c**3*d)/e**6
```

Giac [B] time = 1.16593, size = 927, normalized size = 3.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] 1/2*(c^3 - 6*(2*c^3*d*e - b*c^2*e^2)*e^(-1)/(x*e + d))*(x*e + d)^2*e^(-7) -
 3*(5*c^3*d^2 - 5*b*c^2*d*e + b^2*c*e^2 + a*c^2*e^2)*e^(-7)*log(abs(x*e + d
)*e^(-1)/(x*e + d)^2) + 1/4*(80*c^3*d^3*e^29/(x*e + d) - 30*c^3*d^4*e^29/(x
*e + d)^2 + 8*c^3*d^5*e^29/(x*e + d)^3 - c^3*d^6*e^29/(x*e + d)^4 - 120*b*c
^2*d^2*e^30/(x*e + d) + 60*b*c^2*d^3*e^30/(x*e + d)^2 - 20*b*c^2*d^4*e^30/(
x*e + d)^3 + 3*b*c^2*d^5*e^30/(x*e + d)^4 + 48*b^2*c*d*e^31/(x*e + d) + 48*
a*c^2*d*e^31/(x*e + d) - 36*b^2*c*d^2*e^31/(x*e + d)^2 - 36*a*c^2*d^2*e^31/
(x*e + d)^2 + 16*b^2*c*d^3*e^31/(x*e + d)^3 + 16*a*c^2*d^3*e^31/(x*e + d)^3
 - 3*b^2*c*d^4*e^31/(x*e + d)^4 - 3*a*c^2*d^4*e^31/(x*e + d)^4 - 4*b^3*e^32
/(x*e + d) - 24*a*b*c*e^32/(x*e + d) + 6*b^3*d*e^32/(x*e + d)^2 + 36*a*b*c*
d*e^32/(x*e + d)^2 - 4*b^3*d^2*e^32/(x*e + d)^3 - 24*a*b*c*d^2*e^32/(x*e +
d)^3 + b^3*d^3*e^32/(x*e + d)^4 + 6*a*b*c*d^3*e^32/(x*e + d)^4 - 6*a*b^2*e^
33/(x*e + d)^2 - 6*a^2*c*e^33/(x*e + d)^2 + 8*a*b^2*d*e^33/(x*e + d)^3 + 8*
a^2*c*d*e^33/(x*e + d)^3 - 3*a*b^2*d^2*e^33/(x*e + d)^4 - 3*a^2*c*d^2*e^33/
(x*e + d)^4 - 4*a^2*b*e^34/(x*e + d)^3 + 3*a^2*b*d*e^34/(x*e + d)^4 - a^3*e
^35/(x*e + d)^4)*e^(-36)
```

3.2141 $\int \frac{(a+bx+cx^2)^3}{(d+ex)^6}\,dx$

Optimal. Leaf size=256

$$-\frac{3c\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{e^7(d+ex)}+\frac{(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{2e^7(d+ex)^2}-\frac{\left(ae^2-bde+cd^2\right)\left(-ce(5bd-ae)+\right.}{e^7(d+ex)^3}$$

```
[Out] (c^3*x)/e^6 - (c*d^2 - b*d*e + a*e^2)^3/(5*e^7*(d + e*x)^5) + (3*(2*c*d - b
*e)*(c*d^2 - b*d*e + a*e^2)^2)/(4*e^7*(d + e*x)^4) - ((c*d^2 - b*d*e + a*e^
2)*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e)))/(e^7*(d + e*x)^3) + ((2*c*d -
 b*e)*(10*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e)))/(2*e^7*(d + e*x)^2) -
 (3*c*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e)))/(e^7*(d + e*x)) - (3*c^2*(
2*c*d - b*e)*Log[d + e*x])/e^7
```

Rubi [A] time = 0.244684, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$-\frac{3c\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{e^7(d+ex)}+\frac{(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{2e^7(d+ex)^2}-\frac{\left(ae^2-bde+cd^2\right)\left(-ce(5bd-ae)+\right.}{e^7(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(d + e*x)^6,x]
```

```
[Out] (c^3*x)/e^6 - (c*d^2 - b*d*e + a*e^2)^3/(5*e^7*(d + e*x)^5) + (3*(2*c*d - b
*e)*(c*d^2 - b*d*e + a*e^2)^2)/(4*e^7*(d + e*x)^4) - ((c*d^2 - b*d*e + a*e^
2)*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e)))/(e^7*(d + e*x)^3) + ((2*c*d -
 b*e)*(10*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e)))/(2*e^7*(d + e*x)^2) -
 (3*c*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e)))/(e^7*(d + e*x)) - (3*c^2*(
2*c*d - b*e)*Log[d + e*x])/e^7
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^3}{(d+ex)^6}\,dx=\int \left(\frac{c^3}{e^6}+\frac{\left(cd^2-bde+ae^2\right)^3}{e^6(d+ex)^6}+\frac{3(-2cd+be)\left(cd^2-bde+ae^2\right)^2}{e^6(d+ex)^5}+\frac{3\left(cd^2-bde+ae^2\right)\left(5c^2d^2-5b\right.}{e^6(d+ex)^4}\right.$$

$$=\frac{c^3x}{e^6}-\frac{\left(cd^2-bde+ae^2\right)^3}{5e^7(d+ex)^5}+\frac{3(2cd-be)\left(cd^2-bde+ae^2\right)^2}{4e^7(d+ex)^4}-\frac{\left(cd^2-bde+ae^2\right)\left(5c^2d^2+b^2e^2-c\right.}{e^7(d+ex)^3}$$

Mathematica [A] time = 0.255258, size = 396, normalized size = 1.55

$$-\frac{2ce^2\left(a^2e^2\left(d^2+5dex+10e^2x^2\right)+3abe\left(5d^2ex+d^3+10de^2x^2+10e^3x^3\right)+6b^2\left(10d^2e^2x^2+5d^3ex+d^4+10de^3x^3+5e^4x\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(d + e*x)^6,x]
```

```
[Out] -(2*c^3*(87*d^6 + 375*d^5*e*x + 600*d^4*e^2*x^2 + 400*d^3*e^3*x^3 + 50*d^2*
e^4*x^4 - 50*d*e^5*x^5 - 10*e^6*x^6) + e^3*(4*a^3*e^3 + 3*a^2*b*e^2*(d + 5*
e*x) + 2*a*b^2*e*(d^2 + 5*d*e*x + 10*e^2*x^2) + b^3*(d^3 + 5*d^2*e*x + 10*d
*e^2*x^2 + 10*e^3*x^3)) + 2*c*e^2*(a^2*e^2*(d^2 + 5*d*e*x + 10*e^2*x^2) + 3
*a*b*e*(d^3 + 5*d^2*e*x + 10*d*e^2*x^2 + 10*e^3*x^3) + 6*b^2*(d^4 + 5*d^3*e
*x + 10*d^2*e^2*x^2 + 10*d*e^3*x^3 + 5*e^4*x^4)) + c^2*e*(12*a*e*(d^4 + 5*d
^3*e*x + 10*d^2*e^2*x^2 + 10*d*e^3*x^3 + 5*e^4*x^4) - b*d*(137*d^4 + 625*d^
3*e*x + 1100*d^2*e^2*x^2 + 900*d*e^3*x^3 + 300*e^4*x^4)) + 60*c^2*(2*c*d -
b*e)*(d + e*x)^5*Log[d + e*x])/(20*e^7*(d + e*x)^5)
```

Maple [B] time = 0.051, size = 688, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(e*x+d)^6,x)
```

```
[Out] -3*c/e^5/(e*x+d)*b^2-15*c^3/e^7/(e*x+d)*d^2-3/4/e^2/(e*x+d)^4*a^2*b-3/4/e^4
/(e*x+d)^4*b^3*d^2+3/2/e^7/(e*x+d)^4*c^3*d^5-1/e^3/(e*x+d)^3*a^2*c-1/e^3/(e
*x+d)^3*a*b^2+1/e^4/(e*x+d)^3*b^3*d-5/e^7/(e*x+d)^3*c^3*d^4+10/e^7/(e*x+d)^
2*c^3*d^3+3*c^2/e^6*ln(e*x+d)*b-3*c^2/e^5/(e*x+d)*a+6/e^4/(e*x+d)^3*a*b*c*d
+3/5/e^2/(e*x+d)^5*d*a^2*b-3/5/e^3/(e*x+d)^5*a^2*c*d^2-9/2/e^4/(e*x+d)^4*a*
b*c*d^2-3/5/e^3/(e*x+d)^5*d^2*a*b^2-3/5/e^5/(e*x+d)^5*a*c^2*d^4-3/5/e^5/(e*
x+d)^5*d^4*b^2*c+10/e^6/(e*x+d)^3*b*c^2*d^3-3/e^4/(e*x+d)^2*a*b*c+6/e^5/(e*
x+d)^2*c^2*a*d+6/e^5/(e*x+d)^2*b^2*c*d-15/e^6/(e*x+d)^2*b*c^2*d^2+3/2/e^3/(
e*x+d)^4*a^2*c*d+3/2/e^3/(e*x+d)^4*a*b^2*d-6/e^5/(e*x+d)^3*a*c^2*d^2-6/e^5/
(e*x+d)^3*b^2*c*d^2+3/e^5/(e*x+d)^4*a*c^2*d^3+3/e^5/(e*x+d)^4*b^2*c*d^3-15/
4/e^6/(e*x+d)^4*b*c^2*d^4+1/5/e^4/(e*x+d)^5*d^3*b^3-1/5/e^7/(e*x+d)^5*c^3*d
^6-1/5/e/(e*x+d)^5*a^3-1/2/e^4/(e*x+d)^2*b^3+3/5/e^6/(e*x+d)^5*d^5*b*c^2+15
*c^2/e^6/(e*x+d)*b*d+c^3*x/e^6+6/5/e^4/(e*x+d)^5*d^3*a*b*c-6*c^3*d*ln(e*x+d
)/e^7
```

Maxima [A] time = 1.06975, size = 606, normalized size = 2.37

$$-\frac{174\,c^3d^6 - 137\,bc^2d^5e + 3\,a^2bde^5 + 4\,a^3e^6 + 12\left(b^2c + ac^2\right)d^4e^2 + \left(b^3 + 6\,abc\right)d^3e^3 + 2\left(ab^2 + a^2c\right)d^2e^4 + 60\left(5\,c^3d^2e^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^6,x, algorithm="maxima")
```

```
[Out] -1/20*(174*c^3*d^6 - 137*b*c^2*d^5*e + 3*a^2*b*d*e^5 + 4*a^3*e^6 + 12*(b^2*
c + a*c^2)*d^4*e^2 + (b^3 + 6*a*b*c)*d^3*e^3 + 2*(a*b^2 + a^2*c)*d^2*e^4 +
60*(5*c^3*d^2*e^4 - 5*b*c^2*d*e^5 + (b^2*c + a*c^2)*e^6)*x^4 + 10*(100*c^3*
d^3*e^3 - 90*b*c^2*d^2*e^4 + 12*(b^2*c + a*c^2)*d*e^5 + (b^3 + 6*a*b*c)*e^6
)*x^3 + 10*(130*c^3*d^4*e^2 - 110*b*c^2*d^3*e^3 + 12*(b^2*c + a*c^2)*d^2*e^
4 + (b^3 + 6*a*b*c)*d*e^5 + 2*(a*b^2 + a^2*c)*e^6)*x^2 + 5*(154*c^3*d^5*e -
 125*b*c^2*d^4*e^2 + 3*a^2*b*e^6 + 12*(b^2*c + a*c^2)*d^3*e^3 + (b^3 + 6*a*
```

```
b*c)*d^2*e^4 + 2*(a*b^2 + a^2*c)*d*e^5)*x)/(e^12*x^5 + 5*d*e^11*x^4 + 10*d^
2*e^10*x^3 + 10*d^3*e^9*x^2 + 5*d^4*e^8*x + d^5*e^7) + c^3*x/e^6 - 3*(2*c^3
*d - b*c^2*e)*log(e*x + d)/e^7
```

Fricas [B] time = 2.09305, size = 1247, normalized size = 4.87

$$\frac{20\,c^3e^6x^6+100\,c^3de^5x^5-174\,c^3d^6+137\,bc^2d^5e-3\,a^2bde^5-4\,a^3e^6-12\left(b^2c+ac^2\right)d^4e^2-\left(b^3+6\,abc\right)d^3e^3-2\left(ab^2+a^2c\dots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^6,x, algorithm="fricas")
```

```
[Out] 1/20*(20*c^3*e^6*x^6 + 100*c^3*d*e^5*x^5 - 174*c^3*d^6 + 137*b*c^2*d^5*e -
3*a^2*b*d*e^5 - 4*a^3*e^6 - 12*(b^2*c + a*c^2)*d^4*e^2 - (b^3 + 6*a*b*c)*d^
3*e^3 - 2*(a*b^2 + a^2*c)*d^2*e^4 - 20*(5*c^3*d^2*e^4 - 15*b*c^2*d*e^5 + 3*
(b^2*c + a*c^2)*e^6)*x^4 - 10*(80*c^3*d^3*e^3 - 90*b*c^2*d^2*e^4 + 12*(b^2*
c + a*c^2)*d*e^5 + (b^3 + 6*a*b*c)*e^6)*x^3 - 10*(120*c^3*d^4*e^2 - 110*b*c
^2*d^3*e^3 + 12*(b^2*c + a*c^2)*d^2*e^4 + (b^3 + 6*a*b*c)*d*e^5 + 2*(a*b^2
+ a^2*c)*e^6)*x^2 - 5*(150*c^3*d^5*e - 125*b*c^2*d^4*e^2 + 3*a^2*b*e^6 + 12
*(b^2*c + a*c^2)*d^3*e^3 + (b^3 + 6*a*b*c)*d^2*e^4 + 2*(a*b^2 + a^2*c)*d*e^
5)*x - 60*(2*c^3*d^6 - b*c^2*d^5*e + (2*c^3*d*e^5 - b*c^2*e^6)*x^5 + 5*(2*c
^3*d^2*e^4 - b*c^2*d*e^5)*x^4 + 10*(2*c^3*d^3*e^3 - b*c^2*d^2*e^4)*x^3 + 10
*(2*c^3*d^4*e^2 - b*c^2*d^3*e^3)*x^2 + 5*(2*c^3*d^5*e - b*c^2*d^4*e^2)*x)*l
og(e*x + d))/(e^12*x^5 + 5*d*e^11*x^4 + 10*d^2*e^10*x^3 + 10*d^3*e^9*x^2 +
5*d^4*e^8*x + d^5*e^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(e*x+d)**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13207, size = 560, normalized size = 2.19

$$c^3xe^{(-6)}-3\left(2\,c^3d-bc^2e\right)e^{(-7)}\log\left(|xe+d|\right)-\frac{\left(174\,c^3d^6-137\,bc^2d^5e+12\,b^2cd^4e^2+12\,ac^2d^4e^2+b^3d^3e^3+6\,abcd^3e^3+2\,a\dots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^6,x, algorithm="giac")
```

```
[Out] c^3*x*e^(-6) - 3*(2*c^3*d - b*c^2*e)*e^(-7)*log(abs(x*e + d)) - 1/20*(174*c
^3*d^6 - 137*b*c^2*d^5*e + 12*b^2*c*d^4*e^2 + 12*a*c^2*d^4*e^2 + b^3*d^3*e^
3 + 6*a*b*c*d^3*e^3 + 2*a*b^2*d^2*e^4 + 2*a^2*c*d^2*e^4 + 60*(5*c^3*d^2*e^4
 - 5*b*c^2*d*e^5 + b^2*c*e^6 + a*c^2*e^6)*x^4 + 3*a^2*b*d*e^5 + 10*(100*c^3
*d^3*e^3 - 90*b*c^2*d^2*e^4 + 12*b^2*c*d*e^5 + 12*a*c^2*d*e^5 + b^3*e^6 + 6
```

```
*a*b*c*e^6)*x^3 + 4*a^3*e^6 + 10*(130*c^3*d^4*e^2 - 110*b*c^2*d^3*e^3 + 12*
b^2*c*d^2*e^4 + 12*a*c^2*d^2*e^4 + b^3*d*e^5 + 6*a*b*c*d*e^5 + 2*a*b^2*e^6
+ 2*a^2*c*e^6)*x^2 + 5*(154*c^3*d^5*e - 125*b*c^2*d^4*e^2 + 12*b^2*c*d^3*e^
3 + 12*a*c^2*d^3*e^3 + b^3*d^2*e^4 + 6*a*b*c*d^2*e^4 + 2*a*b^2*d*e^5 + 2*a^
2*c*d*e^5 + 3*a^2*b*e^6)*x)*e^(-7)/(x*e + d)^5
```

3.2142 $\int \frac{(a+bx+cx^2)^3}{(d+ex)^7} dx$

Optimal. Leaf size=266

$$-\frac{3c\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{2e^7(d+ex)^2}+\frac{(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{3e^7(d+ex)^3}-\frac{3\left(ae^2-bde+cd^2\right)\left(-ce(5bd-ae)\right.}{4e^7(d+ex)^4}$$

```
[Out] -(c*d^2 - b*d*e + a*e^2)^3/(6*e^7*(d + e*x)^6) + (3*(2*c*d - b*e)*(c*d^2 -
b*d*e + a*e^2)^2)/(5*e^7*(d + e*x)^5) - (3*(c*d^2 - b*d*e + a*e^2)*(5*c^2*d
^2 + b^2*e^2 - c*e*(5*b*d - a*e)))/(4*e^7*(d + e*x)^4) + ((2*c*d - b*e)*(10
*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e)))/(3*e^7*(d + e*x)^3) - (3*c*(5*
c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e)))/(2*e^7*(d + e*x)^2) + (3*c^2*(2*c*d
 - b*e))/(e^7*(d + e*x)) + (c^3*Log[d + e*x])/e^7
```

Rubi [A] time = 0.226392, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$-\frac{3c\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{2e^7(d+ex)^2}+\frac{(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{3e^7(d+ex)^3}-\frac{3\left(ae^2-bde+cd^2\right)\left(-ce(5bd-ae)\right.}{4e^7(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(d + e*x)^7,x]
```

```
[Out] -(c*d^2 - b*d*e + a*e^2)^3/(6*e^7*(d + e*x)^6) + (3*(2*c*d - b*e)*(c*d^2 -
b*d*e + a*e^2)^2)/(5*e^7*(d + e*x)^5) - (3*(c*d^2 - b*d*e + a*e^2)*(5*c^2*d
^2 + b^2*e^2 - c*e*(5*b*d - a*e)))/(4*e^7*(d + e*x)^4) + ((2*c*d - b*e)*(10
*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e)))/(3*e^7*(d + e*x)^3) - (3*c*(5*
c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e)))/(2*e^7*(d + e*x)^2) + (3*c^2*(2*c*d
 - b*e))/(e^7*(d + e*x)) + (c^3*Log[d + e*x])/e^7
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^3}{(d+ex)^7} dx = \int \left(\frac{(cd^2-bde+ae^2)^3}{e^6(d+ex)^7}+\frac{3(-2cd+be)(cd^2-bde+ae^2)^2}{e^6(d+ex)^6}+\frac{3(cd^2-bde+ae^2)(5c^2d^2-5bcde+}{e^6(d+ex)^5}\right.$$
$$=-\frac{(cd^2-bde+ae^2)^3}{6e^7(d+ex)^6}+\frac{3(2cd-be)(cd^2-bde+ae^2)^2}{5e^7(d+ex)^5}-\frac{3(cd^2-bde+ae^2)(5c^2d^2+b^2e^2-ce(5b}{4e^7(d+ex)^4}$$

Mathematica [A] time = 0.166499, size = 385, normalized size = 1.45

$$\frac{-3ce^2\left(a^2e^2\left(d^2+6dex+15e^2x^2\right)+2abe\left(6d^2ex+d^3+15de^2x^2+20e^3x^3\right)+2b^2\left(15d^2e^2x^2+6d^3ex+d^4+20de^3x^3+15e\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(d + e*x)^7,x]
```

```
[Out] (c^3*d*(147*d^5 + 822*d^4*e*x + 1875*d^3*e^2*x^2 + 2200*d^2*e^3*x^3 + 1350*
d*e^4*x^4 + 360*e^5*x^5) - e^3*(10*a^3*e^3 + 6*a^2*b*e^2*(d + 6*e*x) + 3*a*
b^2*e*(d^2 + 6*d*e*x + 15*e^2*x^2) + b^3*(d^3 + 6*d^2*e*x + 15*d*e^2*x^2 +
20*e^3*x^3)) - 3*c*e^2*(a^2*e^2*(d^2 + 6*d*e*x + 15*e^2*x^2) + 2*a*b*e*(d^3
 + 6*d^2*e*x + 15*d*e^2*x^2 + 20*e^3*x^3) + 2*b^2*(d^4 + 6*d^3*e*x + 15*d^2
*e^2*x^2 + 20*d*e^3*x^3 + 15*e^4*x^4)) - 6*c^2*e*(a*e*(d^4 + 6*d^3*e*x + 15
*d^2*e^2*x^2 + 20*d*e^3*x^3 + 15*e^4*x^4) + 5*b*(d^5 + 6*d^4*e*x + 15*d^3*e
^2*x^2 + 20*d^2*e^3*x^3 + 15*d*e^4*x^4 + 6*e^5*x^5)) + 60*c^3*(d + e*x)^6*L
og[d + e*x])/(60*e^7*(d + e*x)^6)
```

Maple [B] time = 0.051, size = 695, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(e*x+d)^7,x)
```

```
[Out] -1/6/e/(e*x+d)^6*a^3-1/3/e^4/(e*x+d)^3*b^3+9/2/e^4/(e*x+d)^4*c*a*b*d+4/e^5/
(e*x+d)^3*c^2*a*d+6/5/e^7/(e*x+d)^5*c^3*d^5-3*c^2/e^6/(e*x+d)*b-3/4/e^3/(e*
x+d)^4*b^2*a+3/4/e^4/(e*x+d)^4*b^3*d-15/4/e^7/(e*x+d)^4*c^3*d^4+20/3/e^7/(e
*x+d)^3*c^3*d^3+4/e^5/(e*x+d)^3*b^2*c*d-10/e^6/(e*x+d)^3*b*c^2*d^2+15/2*c^2
/e^6/(e*x+d)^2*b*d+1/2/e^2/(e*x+d)^6*b*a^2*d+1/e^4/(e*x+d)^6*d^3*a*b*c-18/5
/e^4/(e*x+d)^5*d^2*a*b*c-9/2/e^5/(e*x+d)^4*a*c^2*d^2-3/e^6/(e*x+d)^5*d^4*b*
c^2-1/2/e^5/(e*x+d)^6*a*c^2*d^4-1/2/e^5/(e*x+d)^6*d^4*b^2*c+1/2/e^6/(e*x+d)
^6*b*c^2*d^5+6/5/e^3/(e*x+d)^5*a^2*c*d+6/5/e^3/(e*x+d)^5*a*b^2*d+12/5/e^5/(
e*x+d)^5*a*c^2*d^3+12/5/e^5/(e*x+d)^5*d^3*b^2*c-1/2/e^3/(e*x+d)^6*a^2*c*d^2
-1/2/e^3/(e*x+d)^6*a*b^2*d^2-3/4/e^3/(e*x+d)^4*a^2*c-3/2*c^2/e^5/(e*x+d)^2*
a-3/2*c/e^5/(e*x+d)^2*b^2-15/2*c^3/e^7/(e*x+d)^2*d^2+1/6/e^4/(e*x+d)^6*b^3*
d^3-1/6/e^7/(e*x+d)^6*c^3*d^6-3/5/e^2/(e*x+d)^5*b*a^2-3/5/e^4/(e*x+d)^5*b^3
*d^2-9/2/e^5/(e*x+d)^4*c*b^2*d^2+15/2/e^6/(e*x+d)^4*d^3*b*c^2-2/e^4/(e*x+d)
^3*a*b*c+c^3*ln(e*x+d)/e^7+6*c^3*d/e^7/(e*x+d)
```

Maxima [A] time = 1.15611, size = 633, normalized size = 2.38

$$147\,c^3d^6 - 30\,bc^2d^5e - 6\,a^2bde^5 - 10\,a^3e^6 - 6\left(b^2c + ac^2\right)d^4e^2 - \left(b^3 + 6\,abc\right)d^3e^3 - 3\left(ab^2 + a^2c\right)d^2e^4 + 180\left(2\,c^3de^5 - b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^7,x, algorithm="maxima")
```

```
[Out] 1/60*(147*c^3*d^6 - 30*b*c^2*d^5*e - 6*a^2*b*d*e^5 - 10*a^3*e^6 - 6*(b^2*c
+ a*c^2)*d^4*e^2 - (b^3 + 6*a*b*c)*d^3*e^3 - 3*(a*b^2 + a^2*c)*d^2*e^4 + 18
0*(2*c^3*d*e^5 - b*c^2*e^6)*x^5 + 90*(15*c^3*d^2*e^4 - 5*b*c^2*d*e^5 - (b^2
*c + a*c^2)*e^6)*x^4 + 20*(110*c^3*d^3*e^3 - 30*b*c^2*d^2*e^4 - 6*(b^2*c +
a*c^2)*d*e^5 - (b^3 + 6*a*b*c)*e^6)*x^3 + 15*(125*c^3*d^4*e^2 - 30*b*c^2*d^
3*e^3 - 6*(b^2*c + a*c^2)*d^2*e^4 - (b^3 + 6*a*b*c)*d*e^5 - 3*(a*b^2 + a^2*
c)*e^6)*x^2 + 6*(137*c^3*d^5*e - 30*b*c^2*d^4*e^2 - 6*a^2*b*e^6 - 6*(b^2*c
```

```
+ a*c^2)*d^3*e^3 - (b^3 + 6*a*b*c)*d^2*e^4 - 3*(a*b^2 + a^2*c)*d*e^5)*x)/(e
^13*x^6 + 6*d*e^12*x^5 + 15*d^2*e^11*x^4 + 20*d^3*e^10*x^3 + 15*d^4*e^9*x^2
 + 6*d^5*e^8*x + d^6*e^7) + c^3*log(e*x + d)/e^7
```

Fricas [B] time = 1.95055, size = 1135, normalized size = 4.27

$$\frac{147\,c^3d^6 - 30\,bc^2d^5e - 6\,a^2bde^5 - 10\,a^3e^6 - 6\left(b^2c + ac^2\right)d^4e^2 - \left(b^3 + 6\,abc\right)d^3e^3 - 3\left(ab^2 + a^2c\right)d^2e^4 + 180\left(2\,c^3de^5 - bc^2e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^7,x, algorithm="fricas")
```

```
[Out] 1/60*(147*c^3*d^6 - 30*b*c^2*d^5*e - 6*a^2*b*d*e^5 - 10*a^3*e^6 - 6*(b^2*c
+ a*c^2)*d^4*e^2 - (b^3 + 6*a*b*c)*d^3*e^3 - 3*(a*b^2 + a^2*c)*d^2*e^4 + 18
0*(2*c^3*d*e^5 - b*c^2*e^6)*x^5 + 90*(15*c^3*d^2*e^4 - 5*b*c^2*d*e^5 - (b^2
*c + a*c^2)*e^6)*x^4 + 20*(110*c^3*d^3*e^3 - 30*b*c^2*d^2*e^4 - 6*(b^2*c +
a*c^2)*d*e^5 - (b^3 + 6*a*b*c)*e^6)*x^3 + 15*(125*c^3*d^4*e^2 - 30*b*c^2*d^
3*e^3 - 6*(b^2*c + a*c^2)*d^2*e^4 - (b^3 + 6*a*b*c)*d*e^5 - 3*(a*b^2 + a^2*
c)*e^6)*x^2 + 6*(137*c^3*d^5*e - 30*b*c^2*d^4*e^2 - 6*a^2*b*e^6 - 6*(b^2*c
+ a*c^2)*d^3*e^3 - (b^3 + 6*a*b*c)*d^2*e^4 - 3*(a*b^2 + a^2*c)*d*e^5)*x + 6
0*(c^3*e^6*x^6 + 6*c^3*d*e^5*x^5 + 15*c^3*d^2*e^4*x^4 + 20*c^3*d^3*e^3*x^3
+ 15*c^3*d^4*e^2*x^2 + 6*c^3*d^5*e*x + c^3*d^6)*log(e*x + d))/(e^13*x^6 + 6
*d*e^12*x^5 + 15*d^2*e^11*x^4 + 20*d^3*e^10*x^3 + 15*d^4*e^9*x^2 + 6*d^5*e^
8*x + d^6*e^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(e*x+d)**7,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13533, size = 574, normalized size = 2.16

$$c^3e^{(-7)}\log\left(|xe+d|\right) + \frac{\left(180\left(2\,c^3de^4 - bc^2e^5\right)x^5 + 90\left(15\,c^3d^2e^3 - 5\,bc^2de^4 - b^2ce^5 - ac^2e^5\right)x^4 + 20\left(110\,c^3d^3e^2 - 30\,bc^2d^2e^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^7,x, algorithm="giac")
```

```
[Out] c^3*e^(-7)*log(abs(x*e + d)) + 1/60*(180*(2*c^3*d*e^4 - b*c^2*e^5)*x^5 + 90
*(15*c^3*d^2*e^3 - 5*b*c^2*d*e^4 - b^2*c*e^5 - a*c^2*e^5)*x^4 + 20*(110*c^3
*d^3*e^2 - 30*b*c^2*d^2*e^3 - 6*b^2*c*d*e^4 - 6*a*c^2*d*e^4 - b^3*e^5 - 6*a
*b*c*e^5)*x^3 + 15*(125*c^3*d^4*e - 30*b*c^2*d^3*e^2 - 6*b^2*c*d^2*e^3 - 6*
a*c^2*d^2*e^3 - b^3*d*e^4 - 6*a*b*c*d*e^4 - 3*a*b^2*e^5 - 3*a^2*c*e^5)*x^2
+ 6*(137*c^3*d^5 - 30*b*c^2*d^4*e - 6*b^2*c*d^3*e^2 - 6*a*c^2*d^3*e^2 - b^3
```

```
*d^2*e^3 - 6*a*b*c*d^2*e^3 - 3*a*b^2*d*e^4 - 3*a^2*c*d*e^4 - 6*a^2*b*e^5)*x
 + (147*c^3*d^6 - 30*b*c^2*d^5*e - 6*b^2*c*d^4*e^2 - 6*a*c^2*d^4*e^2 - b^3*
d^3*e^3 - 6*a*b*c*d^3*e^3 - 3*a*b^2*d^2*e^4 - 3*a^2*c*d^2*e^4 - 6*a^2*b*d*e
^5 - 10*a^3*e^6)*e^(-1))*e^(-6)/(x*e + d)^6
```

3.2143 $\int \frac{(a+bx+cx^2)^3}{(d+ex)^8} dx$

Optimal. Leaf size=268

$$-\frac{c\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{e^7(d+ex)^3}+\frac{(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{4e^7(d+ex)^4}-\frac{3\left(ae^2-bde+cd^2\right)\left(-ce(5bd-ae)+\right.}{5e^7(d+ex)^5}$$

```
[Out] -(c*d^2 - b*d*e + a*e^2)^3/(7*e^7*(d + e*x)^7) + ((2*c*d - b*e)*(c*d^2 - b*
d*e + a*e^2)^2)/(2*e^7*(d + e*x)^6) - (3*(c*d^2 - b*d*e + a*e^2)*(5*c^2*d^2
 + b^2*e^2 - c*e*(5*b*d - a*e)))/(5*e^7*(d + e*x)^5) + ((2*c*d - b*e)*(10*c
^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e)))/(4*e^7*(d + e*x)^4) - (c*(5*c^2*
d^2 + b^2*e^2 - c*e*(5*b*d - a*e)))/(e^7*(d + e*x)^3) + (3*c^2*(2*c*d - b*e
))/(2*e^7*(d + e*x)^2) - c^3/(e^7*(d + e*x))
```

Rubi [A] time = 0.207406, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$-\frac{c\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{e^7(d+ex)^3}+\frac{(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{4e^7(d+ex)^4}-\frac{3\left(ae^2-bde+cd^2\right)\left(-ce(5bd-ae)+\right.}{5e^7(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(d + e*x)^8,x]
```

```
[Out] -(c*d^2 - b*d*e + a*e^2)^3/(7*e^7*(d + e*x)^7) + ((2*c*d - b*e)*(c*d^2 - b*
d*e + a*e^2)^2)/(2*e^7*(d + e*x)^6) - (3*(c*d^2 - b*d*e + a*e^2)*(5*c^2*d^2
 + b^2*e^2 - c*e*(5*b*d - a*e)))/(5*e^7*(d + e*x)^5) + ((2*c*d - b*e)*(10*c
^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e)))/(4*e^7*(d + e*x)^4) - (c*(5*c^2*
d^2 + b^2*e^2 - c*e*(5*b*d - a*e)))/(e^7*(d + e*x)^3) + (3*c^2*(2*c*d - b*e
))/(2*e^7*(d + e*x)^2) - c^3/(e^7*(d + e*x))
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^3}{(d+ex)^8}\,dx=\int\left(\frac{\left(cd^2-bde+ae^2\right)^3}{e^6(d+ex)^8}+\frac{3(-2cd+be)\left(cd^2-bde+ae^2\right)^2}{e^6(d+ex)^7}+\frac{3\left(cd^2-bde+ae^2\right)\left(5c^2d^2-5bcde+\right.}{e^6(d+ex)^6}\right.$$

$$=-\frac{\left(cd^2-bde+ae^2\right)^3}{7e^7(d+ex)^7}+\frac{(2cd-be)\left(cd^2-bde+ae^2\right)^2}{2e^7(d+ex)^6}-\frac{3\left(cd^2-bde+ae^2\right)\left(5c^2d^2+b^2e^2-ce(5bc\right.}{5e^7(d+ex)^5}$$

Mathematica [A] time = 0.142781, size = 377, normalized size = 1.41

$$-\frac{2ce^2\left(2a^2e^2\left(d^2+7dex+21e^2x^2\right)+3abe\left(7d^2ex+d^3+21de^2x^2+35e^3x^3\right)+2b^2\left(21d^2e^2x^2+7d^3ex+d^4+35de^3x^3+35\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(d + e*x)^8,x]
```

```
[Out] -(20*c^3*(d^6 + 7*d^5*e*x + 21*d^4*e^2*x^2 + 35*d^3*e^3*x^3 + 35*d^2*e^4*x^
4 + 21*d*e^5*x^5 + 7*e^6*x^6) + e^3*(20*a^3*e^3 + 10*a^2*b*e^2*(d + 7*e*x)
+ 4*a*b^2*e*(d^2 + 7*d*e*x + 21*e^2*x^2) + b^3*(d^3 + 7*d^2*e*x + 21*d*e^2*
x^2 + 35*e^3*x^3)) + 2*c*e^2*(2*a^2*e^2*(d^2 + 7*d*e*x + 21*e^2*x^2) + 3*a*
b*e*(d^3 + 7*d^2*e*x + 21*d*e^2*x^2 + 35*e^3*x^3) + 2*b^2*(d^4 + 7*d^3*e*x
+ 21*d^2*e^2*x^2 + 35*d*e^3*x^3 + 35*e^4*x^4)) + 2*c^2*e*(2*a*e*(d^4 + 7*d^
3*e*x + 21*d^2*e^2*x^2 + 35*d*e^3*x^3 + 35*e^4*x^4) + 5*b*(d^5 + 7*d^4*e*x
+ 21*d^3*e^2*x^2 + 35*d^2*e^3*x^3 + 35*d*e^4*x^4 + 21*e^5*x^5)))/(140*e^7*(
d + e*x)^7)
```

Maple [A] time = 0.049, size = 461, normalized size = 1.7

$$-\frac{6\,abce^3-12\,c^2ade^2+b^3e^3-12\,b^2cde^2+30\,bc^2d^2e-20\,c^3d^3}{4\,e^7\,(ex+d)^4}-\frac{c\left(ace^2+b^2e^2-5\,bcde+5\,c^2d^2\right)}{e^7\,(ex+d)^3}-\frac{3\,c^2\,(be-2\,cd)}{2\,e^7\,(ex+d)^2}-\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(e*x+d)^8,x)
```

```
[Out] -1/4*(6*a*b*c*e^3-12*a*c^2*d*e^2+b^3*e^3-12*b^2*c*d*e^2+30*b*c^2*d^2*e-20*c
^3*d^3)/e^7/(e*x+d)^4-c*(a*c*e^2+b^2*e^2-5*b*c*d*e+5*c^2*d^2)/e^7/(e*x+d)^3
-3/2*c^2*(b*e-2*c*d)/e^7/(e*x+d)^2-1/6*(3*a^2*b*e^5-6*a^2*c*d*e^4-6*a*b^2*d
*e^4+18*a*b*c*d^2*e^3-12*a*c^2*d^3*e^2+3*b^3*d^2*e^3-12*b^2*c*d^3*e^2+15*b*
c^2*d^4*e-6*c^3*d^5)/e^7/(e*x+d)^6-1/5*(3*a^2*c*e^4+3*a*b^2*e^4-18*a*b*c*d*
e^3+18*a*c^2*d^2*e^2-3*b^3*d*e^3+18*b^2*c*d^2*e^2-30*b*c^2*d^3*e+15*c^3*d^4
)/e^7/(e*x+d)^5-c^3/e^7/(e*x+d)-1/7*(a^3*e^6-3*a^2*b*d*e^5+3*a^2*c*d^2*e^4+
3*a*b^2*d^2*e^4-6*a*b*c*d^3*e^3+3*a*c^2*d^4*e^2-b^3*d^3*e^3+3*b^2*c*d^4*e^2
-3*b*c^2*d^5*e+c^3*d^6)/e^7/(e*x+d)^7
```

Maxima [A] time = 1.18849, size = 637, normalized size = 2.38

$$-\frac{140\,c^3e^6x^6+20\,c^3d^6+10\,bc^2d^5e+10\,a^2bde^5+20\,a^3e^6+4\left(b^2c+ac^2\right)d^4e^2+\left(b^3+6\,abc\right)d^3e^3+4\left(ab^2+a^2c\right)d^2e^4+\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^8,x, algorithm="maxima")
```

```
[Out] -1/140*(140*c^3*e^6*x^6 + 20*c^3*d^6 + 10*b*c^2*d^5*e + 10*a^2*b*d*e^5 + 20
*a^3*e^6 + 4*(b^2*c + a*c^2)*d^4*e^2 + (b^3 + 6*a*b*c)*d^3*e^3 + 4*(a*b^2 +
 a^2*c)*d^2*e^4 + 210*(2*c^3*d*e^5 + b*c^2*e^6)*x^5 + 70*(10*c^3*d^2*e^4 +
5*b*c^2*d*e^5 + 2*(b^2*c + a*c^2)*e^6)*x^4 + 35*(20*c^3*d^3*e^3 + 10*b*c^2*
d^2*e^4 + 4*(b^2*c + a*c^2)*d*e^5 + (b^3 + 6*a*b*c)*e^6)*x^3 + 21*(20*c^3*d
^4*e^2 + 10*b*c^2*d^3*e^3 + 4*(b^2*c + a*c^2)*d^2*e^4 + (b^3 + 6*a*b*c)*d*e
^5 + 4*(a*b^2 + a^2*c)*e^6)*x^2 + 7*(20*c^3*d^5*e + 10*b*c^2*d^4*e^2 + 10*a
^2*b*e^6 + 4*(b^2*c + a*c^2)*d^3*e^3 + (b^3 + 6*a*b*c)*d^2*e^4 + 4*(a*b^2 +
 a^2*c)*d*e^5)*x)/(e^14*x^7 + 7*d*e^13*x^6 + 21*d^2*e^12*x^5 + 35*d^3*e^11*
x^4 + 35*d^4*e^10*x^3 + 21*d^5*e^9*x^2 + 7*d^6*e^8*x + d^7*e^7)
```

Fricas [A] time = 2.14285, size = 999, normalized size = 3.73

$$-\frac{140\,c^3e^6x^6 + 20\,c^3d^6 + 10\,bc^2d^5e + 10\,a^2bde^5 + 20\,a^3e^6 + 4\left(b^2c + ac^2\right)d^4e^2 + \left(b^3 + 6\,abc\right)d^3e^3 + 4\left(ab^2 + a^2c\right)d^2e^4 + 210}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^8,x, algorithm="fricas")
```

```
[Out] -1/140*(140*c^3*e^6*x^6 + 20*c^3*d^6 + 10*b*c^2*d^5*e + 10*a^2*b*d*e^5 + 20
*a^3*e^6 + 4*(b^2*c + a*c^2)*d^4*e^2 + (b^3 + 6*a*b*c)*d^3*e^3 + 4*(a*b^2 +
 a^2*c)*d^2*e^4 + 210*(2*c^3*d*e^5 + b*c^2*e^6)*x^5 + 70*(10*c^3*d^2*e^4 +
5*b*c^2*d*e^5 + 2*(b^2*c + a*c^2)*e^6)*x^4 + 35*(20*c^3*d^3*e^3 + 10*b*c^2*
d^2*e^4 + 4*(b^2*c + a*c^2)*d*e^5 + (b^3 + 6*a*b*c)*e^6)*x^3 + 21*(20*c^3*d
^4*e^2 + 10*b*c^2*d^3*e^3 + 4*(b^2*c + a*c^2)*d^2*e^4 + (b^3 + 6*a*b*c)*d*e
^5 + 4*(a*b^2 + a^2*c)*e^6)*x^2 + 7*(20*c^3*d^5*e + 10*b*c^2*d^4*e^2 + 10*a
^2*b*e^6 + 4*(b^2*c + a*c^2)*d^3*e^3 + (b^3 + 6*a*b*c)*d^2*e^4 + 4*(a*b^2 +
 a^2*c)*d*e^5)*x)/(e^14*x^7 + 7*d*e^13*x^6 + 21*d^2*e^12*x^5 + 35*d^3*e^11*
x^4 + 35*d^4*e^10*x^3 + 21*d^5*e^9*x^2 + 7*d^6*e^8*x + d^7*e^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(e*x+d)**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09907, size = 618, normalized size = 2.31

$$-\frac{\left(140\,c^3x^6e^6 + 420\,c^3dx^5e^5 + 700\,c^3d^2x^4e^4 + 700\,c^3d^3x^3e^3 + 420\,c^3d^4x^2e^2 + 140\,c^3d^5xe + 20\,c^3d^6 + 210\,bc^2x^5e^6 + 350\,bc^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^8,x, algorithm="giac")
```

```
[Out] -1/140*(140*c^3*x^6*e^6 + 420*c^3*d*x^5*e^5 + 700*c^3*d^2*x^4*e^4 + 700*c^3
*d^3*x^3*e^3 + 420*c^3*d^4*x^2*e^2 + 140*c^3*d^5*x*e + 20*c^3*d^6 + 210*b*c
^2*x^5*e^6 + 350*b*c^2*d*x^4*e^5 + 350*b*c^2*d^2*x^3*e^4 + 210*b*c^2*d^3*x^
2*e^3 + 70*b*c^2*d^4*x*e^2 + 10*b*c^2*d^5*e + 140*b^2*c*x^4*e^6 + 140*a*c^2
*x^4*e^6 + 140*b^2*c*d*x^3*e^5 + 140*a*c^2*d*x^3*e^5 + 84*b^2*c*d^2*x^2*e^4
 + 84*a*c^2*d^2*x^2*e^4 + 28*b^2*c*d^3*x*e^3 + 28*a*c^2*d^3*x*e^3 + 4*b^2*c
*d^4*e^2 + 4*a*c^2*d^4*e^2 + 35*b^3*x^3*e^6 + 210*a*b*c*x^3*e^6 + 21*b^3*d*
x^2*e^5 + 126*a*b*c*d*x^2*e^5 + 7*b^3*d^2*x*e^4 + 42*a*b*c*d^2*x*e^4 + b^3*
d^3*e^3 + 6*a*b*c*d^3*e^3 + 84*a*b^2*x^2*e^6 + 84*a^2*c*x^2*e^6 + 28*a*b^2*
d*x*e^5 + 28*a^2*c*d*x*e^5 + 4*a*b^2*d^2*e^4 + 4*a^2*c*d^2*e^4 + 70*a^2*b*x
*e^6 + 10*a^2*b*d*e^5 + 20*a^3*e^6)*e^(-7)/(x*e + d)^7
```

3.2144 $\int \frac{(a+bx+cx^2)^3}{(d+ex)^9}\,dx$

Optimal. Leaf size=269

$$-\frac{3c\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{4e^7(d+ex)^4}+\frac{(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{5e^7(d+ex)^5}-\frac{\left(ae^2-bde+cd^2\right)\left(-ce(5bd-ae\right.}{2e^7(d+ex)^6}$$

```
[Out] -(c*d^2 - b*d*e + a*e^2)^3/(8*e^7*(d + e*x)^8) + (3*(2*c*d - b*e)*(c*d^2 -
b*d*e + a*e^2)^2)/(7*e^7*(d + e*x)^7) - ((c*d^2 - b*d*e + a*e^2)*(5*c^2*d^2
 + b^2*e^2 - c*e*(5*b*d - a*e)))/(2*e^7*(d + e*x)^6) + ((2*c*d - b*e)*(10*c
^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e)))/(5*e^7*(d + e*x)^5) - (3*c*(5*c^
2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e)))/(4*e^7*(d + e*x)^4) + (c^2*(2*c*d - b
*e))/(e^7*(d + e*x)^3) - c^3/(2*e^7*(d + e*x)^2)
```

Rubi [A] time = 0.22413, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$-\frac{3c\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{4e^7(d+ex)^4}+\frac{(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{5e^7(d+ex)^5}-\frac{\left(ae^2-bde+cd^2\right)\left(-ce(5bd-ae\right.}{2e^7(d+ex)^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(d + e*x)^9,x]
```

```
[Out] -(c*d^2 - b*d*e + a*e^2)^3/(8*e^7*(d + e*x)^8) + (3*(2*c*d - b*e)*(c*d^2 -
b*d*e + a*e^2)^2)/(7*e^7*(d + e*x)^7) - ((c*d^2 - b*d*e + a*e^2)*(5*c^2*d^2
 + b^2*e^2 - c*e*(5*b*d - a*e)))/(2*e^7*(d + e*x)^6) + ((2*c*d - b*e)*(10*c
^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e)))/(5*e^7*(d + e*x)^5) - (3*c*(5*c^
2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e)))/(4*e^7*(d + e*x)^4) + (c^2*(2*c*d - b
*e))/(e^7*(d + e*x)^3) - c^3/(2*e^7*(d + e*x)^2)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^3}{(d+ex)^9}\,dx=\int \left(\frac{\left(cd^2-bde+ae^2\right)^3}{e^6(d+ex)^9}+\frac{3(-2cd+be)\left(cd^2-bde+ae^2\right)^2}{e^6(d+ex)^8}+\frac{3\left(cd^2-bde+ae^2\right)\left(5c^2d^2-5bc\right.}{e^6(d+ex)^7}\right.$$

$$=-\frac{\left(cd^2-bde+ae^2\right)^3}{8e^7(d+ex)^8}+\frac{3(2cd-be)\left(cd^2-bde+ae^2\right)^2}{7e^7(d+ex)^7}-\frac{\left(cd^2-bde+ae^2\right)\left(5c^2d^2+b^2e^2-ce(\right.}{2e^7(d+ex)^6}$$

Mathematica [A] time = 0.153889, size = 375, normalized size = 1.39

$$-\frac{ce^2\left(5a^2e^2\left(d^2+8dex+28e^2x^2\right)+6abe\left(8d^2ex+d^3+28de^2x^2+56e^3x^3\right)+3b^2\left(28d^2e^2x^2+8d^3ex+d^4+56de^3x^3+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(d + e*x)^9,x]
```

```
[Out] -(5*c^3*(d^6 + 8*d^5*e*x + 28*d^4*e^2*x^2 + 56*d^3*e^3*x^3 + 70*d^2*e^4*x^4
 + 56*d*e^5*x^5 + 28*e^6*x^6) + e^3*(35*a^3*e^3 + 15*a^2*b*e^2*(d + 8*e*x)
+ 5*a*b^2*e*(d^2 + 8*d*e*x + 28*e^2*x^2) + b^3*(d^3 + 8*d^2*e*x + 28*d*e^2*
x^2 + 56*e^3*x^3)) + c*e^2*(5*a^2*e^2*(d^2 + 8*d*e*x + 28*e^2*x^2) + 6*a*b*
e*(d^3 + 8*d^2*e*x + 28*d*e^2*x^2 + 56*e^3*x^3) + 3*b^2*(d^4 + 8*d^3*e*x +
28*d^2*e^2*x^2 + 56*d*e^3*x^3 + 70*e^4*x^4)) + c^2*e*(3*a*e*(d^4 + 8*d^3*e*
x + 28*d^2*e^2*x^2 + 56*d*e^3*x^3 + 70*e^4*x^4) + 5*b*(d^5 + 8*d^4*e*x + 28
*d^3*e^2*x^2 + 56*d^2*e^3*x^3 + 70*d*e^4*x^4 + 56*e^5*x^5)))/(280*e^7*(d +
e*x)^8)
```

Maple [A] time = 0.05, size = 461, normalized size = 1.7

$$-\frac{3\,c\left(ace^2+b^2e^2-5\,bcde+5\,c^2d^2\right)}{4\,e^7\,(ex+d)^4}-\frac{c^2\,(be-2\,cd)}{e^7\,(ex+d)^3}-\frac{c^3}{2\,e^7\,(ex+d)^2}-\frac{a^3e^6-3\,ba^2de^5+3\,a^2cd^2e^4+3\,ab^2d^2e^4-6\,d^3abce^3}{8\,e^7\,(e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(e*x+d)^9,x)
```

```
[Out] -3/4*c*(a*c*e^2+b^2*e^2-5*b*c*d*e+5*c^2*d^2)/e^7/(e*x+d)^4-c^2*(b*e-2*c*d)/
e^7/(e*x+d)^3-1/2*c^3/e^7/(e*x+d)^2-1/8*(a^3*e^6-3*a^2*b*d*e^5+3*a^2*c*d^2*
e^4+3*a*b^2*d^2*e^4-6*a*b*c*d^3*e^3+3*a*c^2*d^4*e^2-b^3*d^3*e^3+3*b^2*c*d^4
*e^2-3*b*c^2*d^5*e+c^3*d^6)/e^7/(e*x+d)^8-1/6*(3*a^2*c*e^4+3*a*b^2*e^4-18*a
*b*c*d*e^3+18*a*c^2*d^2*e^2-3*b^3*d*e^3+18*b^2*c*d^2*e^2-30*b*c^2*d^3*e+15*
c^3*d^4)/e^7/(e*x+d)^6-1/5*(6*a*b*c*e^3-12*a*c^2*d*e^2+b^3*e^3-12*b^2*c*d*e
^2+30*b*c^2*d^2*e-20*c^3*d^3)/e^7/(e*x+d)^5-1/7*(3*a^2*b*e^5-6*a^2*c*d*e^4-
6*a*b^2*d*e^4+18*a*b*c*d^2*e^3-12*a*c^2*d^3*e^2+3*b^3*d^2*e^3-12*b^2*c*d^3*
e^2+15*b*c^2*d^4*e-6*c^3*d^5)/e^7/(e*x+d)^7
```

Maxima [A] time = 1.10903, size = 651, normalized size = 2.42

$$-\frac{140\,c^3e^6x^6+5\,c^3d^6+5\,bc^2d^5e+15\,a^2bde^5+35\,a^3e^6+3\left(b^2c+ac^2\right)d^4e^2+\left(b^3+6\,abc\right)d^3e^3+5\left(ab^2+a^2c\right)d^2e^4+280\,(c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^9,x, algorithm="maxima")
```

```
[Out] -1/280*(140*c^3*e^6*x^6 + 5*c^3*d^6 + 5*b*c^2*d^5*e + 15*a^2*b*d*e^5 + 35*a
^3*e^6 + 3*(b^2*c + a*c^2)*d^4*e^2 + (b^3 + 6*a*b*c)*d^3*e^3 + 5*(a*b^2 + a
^2*c)*d^2*e^4 + 280*(c^3*d*e^5 + b*c^2*e^6)*x^5 + 70*(5*c^3*d^2*e^4 + 5*b*c
^2*d*e^5 + 3*(b^2*c + a*c^2)*e^6)*x^4 + 56*(5*c^3*d^3*e^3 + 5*b*c^2*d^2*e^4
 + 3*(b^2*c + a*c^2)*d*e^5 + (b^3 + 6*a*b*c)*e^6)*x^3 + 28*(5*c^3*d^4*e^2 +
 5*b*c^2*d^3*e^3 + 3*(b^2*c + a*c^2)*d^2*e^4 + (b^3 + 6*a*b*c)*d*e^5 + 5*(a
*b^2 + a^2*c)*e^6)*x^2 + 8*(5*c^3*d^5*e + 5*b*c^2*d^4*e^2 + 15*a^2*b*e^6 +
3*(b^2*c + a*c^2)*d^3*e^3 + (b^3 + 6*a*b*c)*d^2*e^4 + 5*(a*b^2 + a^2*c)*d*e
^5)*x)/(e^15*x^8 + 8*d*e^14*x^7 + 28*d^2*e^13*x^6 + 56*d^3*e^12*x^5 + 70*d^
4*e^11*x^4 + 56*d^5*e^10*x^3 + 28*d^6*e^9*x^2 + 8*d^7*e^8*x + d^8*e^7)
```

Fricas [A] time = 2.10881, size = 1008, normalized size = 3.75

$$-\frac{140\,c^3e^6x^6+5\,c^3d^6+5\,bc^2d^5e+15\,a^2bde^5+35\,a^3e^6+3\left(b^2c+ac^2\right)d^4e^2+\left(b^3+6\,abc\right)d^3e^3+5\left(ab^2+a^2c\right)d^2e^4+28}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^9,x, algorithm="fricas")
```

```
[Out] -1/280*(140*c^3*e^6*x^6 + 5*c^3*d^6 + 5*b*c^2*d^5*e + 15*a^2*b*d*e^5 + 35*a
^3*e^6 + 3*(b^2*c + a*c^2)*d^4*e^2 + (b^3 + 6*a*b*c)*d^3*e^3 + 5*(a*b^2 + a
^2*c)*d^2*e^4 + 280*(c^3*d*e^5 + b*c^2*e^6)*x^5 + 70*(5*c^3*d^2*e^4 + 5*b*c
^2*d*e^5 + 3*(b^2*c + a*c^2)*e^6)*x^4 + 56*(5*c^3*d^3*e^3 + 5*b*c^2*d^2*e^4
 + 3*(b^2*c + a*c^2)*d*e^5 + (b^3 + 6*a*b*c)*e^6)*x^3 + 28*(5*c^3*d^4*e^2 +
 5*b*c^2*d^3*e^3 + 3*(b^2*c + a*c^2)*d^2*e^4 + (b^3 + 6*a*b*c)*d*e^5 + 5*(a
*b^2 + a^2*c)*e^6)*x^2 + 8*(5*c^3*d^5*e + 5*b*c^2*d^4*e^2 + 15*a^2*b*e^6 +
3*(b^2*c + a*c^2)*d^3*e^3 + (b^3 + 6*a*b*c)*d^2*e^4 + 5*(a*b^2 + a^2*c)*d*e
^5)*x)/(e^15*x^8 + 8*d*e^14*x^7 + 28*d^2*e^13*x^6 + 56*d^3*e^12*x^5 + 70*d^
4*e^11*x^4 + 56*d^5*e^10*x^3 + 28*d^6*e^9*x^2 + 8*d^7*e^8*x + d^8*e^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(e*x+d)**9,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10212, size = 618, normalized size = 2.3

$$-\frac{\left(140\,c^3x^6e^6+280\,c^3dx^5e^5+350\,c^3d^2x^4e^4+280\,c^3d^3x^3e^3+140\,c^3d^4x^2e^2+40\,c^3d^5xe+5\,c^3d^6+280\,bc^2x^5e^6+350\,bc}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^9,x, algorithm="giac")
```

```
[Out] -1/280*(140*c^3*x^6*e^6 + 280*c^3*d*x^5*e^5 + 350*c^3*d^2*x^4*e^4 + 280*c^3
*d^3*x^3*e^3 + 140*c^3*d^4*x^2*e^2 + 40*c^3*d^5*x*e + 5*c^3*d^6 + 280*b*c^2
*x^5*e^6 + 350*b*c^2*d*x^4*e^5 + 280*b*c^2*d^2*x^3*e^4 + 140*b*c^2*d^3*x^2*
e^3 + 40*b*c^2*d^4*x*e^2 + 5*b*c^2*d^5*e + 210*b^2*c*x^4*e^6 + 210*a*c^2*x^
4*e^6 + 168*b^2*c*d*x^3*e^5 + 168*a*c^2*d*x^3*e^5 + 84*b^2*c*d^2*x^2*e^4 +
84*a*c^2*d^2*x^2*e^4 + 24*b^2*c*d^3*x*e^3 + 24*a*c^2*d^3*x*e^3 + 3*b^2*c*d^
4*e^2 + 3*a*c^2*d^4*e^2 + 56*b^3*x^3*e^6 + 336*a*b*c*x^3*e^6 + 28*b^3*d*x^2
*e^5 + 168*a*b*c*d*x^2*e^5 + 8*b^3*d^2*x*e^4 + 48*a*b*c*d^2*x*e^4 + b^3*d^3
*e^3 + 6*a*b*c*d^3*e^3 + 140*a*b^2*x^2*e^6 + 140*a^2*c*x^2*e^6 + 40*a*b^2*d
*x*e^5 + 40*a^2*c*d*x*e^5 + 5*a*b^2*d^2*e^4 + 5*a^2*c*d^2*e^4 + 120*a^2*b*x
*e^6 + 15*a^2*b*d*e^5 + 35*a^3*e^6)*e^(-7)/(x*e + d)^8
```

3.2145 $\int \frac{(a+bx+cx^2)^3}{(d+ex)^{10}}\,dx$

Optimal. Leaf size=272

$$-\frac{3c\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{5e^7(d+ex)^5}+\frac{(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{6e^7(d+ex)^6}-\frac{3\left(ae^2-bde+cd^2\right)\left(-ce(5bd-ae)\right.}{7e^7(d+ex)^7}$$

```
[Out] -(c*d^2 - b*d*e + a*e^2)^3/(9*e^7*(d + e*x)^9) + (3*(2*c*d - b*e)*(c*d^2 -
b*d*e + a*e^2)^2)/(8*e^7*(d + e*x)^8) - (3*(c*d^2 - b*d*e + a*e^2)*(5*c^2*d
^2 + b^2*e^2 - c*e*(5*b*d - a*e)))/(7*e^7*(d + e*x)^7) + ((2*c*d - b*e)*(10
*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e)))/(6*e^7*(d + e*x)^6) - (3*c*(5*
c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e)))/(5*e^7*(d + e*x)^5) + (3*c^2*(2*c*d
 - b*e))/(4*e^7*(d + e*x)^4) - c^3/(3*e^7*(d + e*x)^3)
```

Rubi [A] time = 0.209361, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$-\frac{3c\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{5e^7(d+ex)^5}+\frac{(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{6e^7(d+ex)^6}-\frac{3\left(ae^2-bde+cd^2\right)\left(-ce(5bd-ae)\right.}{7e^7(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(d + e*x)^10,x]
```

```
[Out] -(c*d^2 - b*d*e + a*e^2)^3/(9*e^7*(d + e*x)^9) + (3*(2*c*d - b*e)*(c*d^2 -
b*d*e + a*e^2)^2)/(8*e^7*(d + e*x)^8) - (3*(c*d^2 - b*d*e + a*e^2)*(5*c^2*d
^2 + b^2*e^2 - c*e*(5*b*d - a*e)))/(7*e^7*(d + e*x)^7) + ((2*c*d - b*e)*(10
*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e)))/(6*e^7*(d + e*x)^6) - (3*c*(5*
c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e)))/(5*e^7*(d + e*x)^5) + (3*c^2*(2*c*d
 - b*e))/(4*e^7*(d + e*x)^4) - c^3/(3*e^7*(d + e*x)^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^3}{(d+ex)^{10}}\,dx = \int \left(\frac{\left(cd^2-bde+ae^2\right)^3}{e^6(d+ex)^{10}}+\frac{3(-2cd+be)\left(cd^2-bde+ae^2\right)^2}{e^6(d+ex)^9}+\frac{3\left(cd^2-bde+ae^2\right)\left(5c^2d^2-5bcde+\right.}{e^6(d+ex)^8}\right.$$

$$= -\frac{\left(cd^2-bde+ae^2\right)^3}{9e^7(d+ex)^9}+\frac{3(2cd-be)\left(cd^2-bde+ae^2\right)^2}{8e^7(d+ex)^8}-\frac{3\left(cd^2-bde+ae^2\right)\left(5c^2d^2+b^2e^2-ce(5\right.}{7e^7(d+ex)^7}$$

Mathematica [A] time = 0.151566, size = 378, normalized size = 1.39

$$-\frac{6ce^2\left(5a^2e^2\left(d^2+9dex+36e^2x^2\right)+5abe\left(9d^2ex+d^3+36de^2x^2+84e^3x^3\right)+2b^2\left(36d^2e^2x^2+9d^3ex+d^4+84de^3x^3+12\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(d + e*x)^10,x]
```

```
[Out] -(10*c^3*(d^6 + 9*d^5*e*x + 36*d^4*e^2*x^2 + 84*d^3*e^3*x^3 + 126*d^2*e^4*x
^4 + 126*d*e^5*x^5 + 84*e^6*x^6) + 5*e^3*(56*a^3*e^3 + 21*a^2*b*e^2*(d + 9*
e*x) + 6*a*b^2*e*(d^2 + 9*d*e*x + 36*e^2*x^2) + b^3*(d^3 + 9*d^2*e*x + 36*d
*e^2*x^2 + 84*e^3*x^3)) + 6*c*e^2*(5*a^2*e^2*(d^2 + 9*d*e*x + 36*e^2*x^2) +
 5*a*b*e*(d^3 + 9*d^2*e*x + 36*d*e^2*x^2 + 84*e^3*x^3) + 2*b^2*(d^4 + 9*d^3
*e*x + 36*d^2*e^2*x^2 + 84*d*e^3*x^3 + 126*e^4*x^4)) + 3*c^2*e*(4*a*e*(d^4
+ 9*d^3*e*x + 36*d^2*e^2*x^2 + 84*d*e^3*x^3 + 126*e^4*x^4) + 5*b*(d^5 + 9*d
^4*e*x + 36*d^3*e^2*x^2 + 84*d^2*e^3*x^3 + 126*d*e^4*x^4 + 126*e^5*x^5)))/(
2520*e^7*(d + e*x)^9)
```

Maple [A] time = 0.052, size = 461, normalized size = 1.7

$$-\frac{3c^2(be-2cd)}{4e^7(ex+d)^4}-\frac{c^3}{3e^7(ex+d)^3}-\frac{3ba^2e^5-6a^2cde^4-6ab^2de^4+18d^2abce^3-12ac^2d^3e^2+3b^3d^2e^3-12d^3b^2ce^2+1}{8e^7(ex+d)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(e*x+d)^10,x)
```

```
[Out] -3/4*c^2*(b*e-2*c*d)/e^7/(e*x+d)^4-1/3*c^3/e^7/(e*x+d)^3-1/8*(3*a^2*b*e^5-6
*a^2*c*d*e^4-6*a*b^2*d*e^4+18*a*b*c*d^2*e^3-12*a*c^2*d^3*e^2+3*b^3*d^2*e^3-
12*b^2*c*d^3*e^2+15*b*c^2*d^4*e-6*c^3*d^5)/e^7/(e*x+d)^8-1/9*(a^3*e^6-3*a^2
*b*d*e^5+3*a^2*c*d^2*e^4+3*a*b^2*d^2*e^4-6*a*b*c*d^3*e^3+3*a*c^2*d^4*e^2-b^
3*d^3*e^3+3*b^2*c*d^4*e^2-3*b*c^2*d^5*e+c^3*d^6)/e^7/(e*x+d)^9-1/6*(6*a*b*c
*e^3-12*a*c^2*d*e^2+b^3*e^3-12*b^2*c*d*e^2+30*b*c^2*d^2*e-20*c^3*d^3)/e^7/(
e*x+d)^6-3/5*c*(a*c*e^2+b^2*e^2-5*b*c*d*e+5*c^2*d^2)/e^7/(e*x+d)^5-1/7*(3*a
^2*c*e^4+3*a*b^2*e^4-18*a*b*c*d*e^3+18*a*c^2*d^2*e^2-3*b^3*d*e^3+18*b^2*c*d
^2*e^2-30*b*c^2*d^3*e+15*c^3*d^4)/e^7/(e*x+d)^7
```

Maxima [A] time = 1.09346, size = 674, normalized size = 2.48

$$-\frac{840c^3e^6x^6+10c^3d^6+15bc^2d^5e+105a^2bde^5+280a^3e^6+12\left(b^2c+ac^2\right)d^4e^2+5\left(b^3+6abc\right)d^3e^3+30\left(ab^2+a^2c\right)d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^10,x, algorithm="maxima")
```

```
[Out] -1/2520*(840*c^3*e^6*x^6 + 10*c^3*d^6 + 15*b*c^2*d^5*e + 105*a^2*b*d*e^5 +
280*a^3*e^6 + 12*(b^2*c + a*c^2)*d^4*e^2 + 5*(b^3 + 6*a*b*c)*d^3*e^3 + 30*(
a*b^2 + a^2*c)*d^2*e^4 + 630*(2*c^3*d*e^5 + 3*b*c^2*e^6)*x^5 + 126*(10*c^3*
d^2*e^4 + 15*b*c^2*d*e^5 + 12*(b^2*c + a*c^2)*e^6)*x^4 + 84*(10*c^3*d^3*e^3
 + 15*b*c^2*d^2*e^4 + 12*(b^2*c + a*c^2)*d*e^5 + 5*(b^3 + 6*a*b*c)*e^6)*x^3
 + 36*(10*c^3*d^4*e^2 + 15*b*c^2*d^3*e^3 + 12*(b^2*c + a*c^2)*d^2*e^4 + 5*(
b^3 + 6*a*b*c)*d*e^5 + 30*(a*b^2 + a^2*c)*e^6)*x^2 + 9*(10*c^3*d^5*e + 15*b
*c^2*d^4*e^2 + 105*a^2*b*e^6 + 12*(b^2*c + a*c^2)*d^3*e^3 + 5*(b^3 + 6*a*b*
c)*d^2*e^4 + 30*(a*b^2 + a^2*c)*d*e^5)*x)/(e^16*x^9 + 9*d*e^15*x^8 + 36*d^2
*e^14*x^7 + 84*d^3*e^13*x^6 + 126*d^4*e^12*x^5 + 126*d^5*e^11*x^4 + 84*d^6*
e^10*x^3 + 36*d^7*e^9*x^2 + 9*d^8*e^8*x + d^9*e^7)
```

Fricas [A] time = 1.97955, size = 1083, normalized size = 3.98

$$-\frac{840\,c^3e^6x^6+10\,c^3d^6+15\,bc^2d^5e+105\,a^2bde^5+280\,a^3e^6+12\left(b^2c+ac^2\right)d^4e^2+5\left(b^3+6\,abc\right)d^3e^3+30\left(ab^2+a^2c\right)d^2e^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^10,x, algorithm="fricas")
```

```
[Out] -1/2520*(840*c^3*e^6*x^6 + 10*c^3*d^6 + 15*b*c^2*d^5*e + 105*a^2*b*d*e^5 +
280*a^3*e^6 + 12*(b^2*c + a*c^2)*d^4*e^2 + 5*(b^3 + 6*a*b*c)*d^3*e^3 + 30*(
a*b^2 + a^2*c)*d^2*e^4 + 630*(2*c^3*d*e^5 + 3*b*c^2*e^6)*x^5 + 126*(10*c^3*
d^2*e^4 + 15*b*c^2*d*e^5 + 12*(b^2*c + a*c^2)*e^6)*x^4 + 84*(10*c^3*d^3*e^3
 + 15*b*c^2*d^2*e^4 + 12*(b^2*c + a*c^2)*d*e^5 + 5*(b^3 + 6*a*b*c)*e^6)*x^3
 + 36*(10*c^3*d^4*e^2 + 15*b*c^2*d^3*e^3 + 12*(b^2*c + a*c^2)*d^2*e^4 + 5*(
b^3 + 6*a*b*c)*d*e^5 + 30*(a*b^2 + a^2*c)*e^6)*x^2 + 9*(10*c^3*d^5*e + 15*b
*c^2*d^4*e^2 + 105*a^2*b*e^6 + 12*(b^2*c + a*c^2)*d^3*e^3 + 5*(b^3 + 6*a*b*
c)*d^2*e^4 + 30*(a*b^2 + a^2*c)*d*e^5)*x)/(e^16*x^9 + 9*d*e^15*x^8 + 36*d^2
*e^14*x^7 + 84*d^3*e^13*x^6 + 126*d^4*e^12*x^5 + 126*d^5*e^11*x^4 + 84*d^6*
e^10*x^3 + 36*d^7*e^9*x^2 + 9*d^8*e^8*x + d^9*e^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(e*x+d)**10,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1227, size = 620, normalized size = 2.28

$$-\frac{\left(840\,c^3x^6e^6+1260\,c^3dx^5e^5+1260\,c^3d^2x^4e^4+840\,c^3d^3x^3e^3+360\,c^3d^4x^2e^2+90\,c^3d^5xe+10\,c^3d^6+1890\,bc^2x^5e^6+1890}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^10,x, algorithm="giac")
```

```
[Out] -1/2520*(840*c^3*x^6*e^6 + 1260*c^3*d*x^5*e^5 + 1260*c^3*d^2*x^4*e^4 + 840*
c^3*d^3*x^3*e^3 + 360*c^3*d^4*x^2*e^2 + 90*c^3*d^5*x*e + 10*c^3*d^6 + 1890*
b*c^2*x^5*e^6 + 1890*b*c^2*d*x^4*e^5 + 1260*b*c^2*d^2*x^3*e^4 + 540*b*c^2*d
^3*x^2*e^3 + 135*b*c^2*d^4*x*e^2 + 15*b*c^2*d^5*e + 1512*b^2*c*x^4*e^6 + 15
12*a*c^2*x^4*e^6 + 1008*b^2*c*d*x^3*e^5 + 1008*a*c^2*d*x^3*e^5 + 432*b^2*c*
d^2*x^2*e^4 + 432*a*c^2*d^2*x^2*e^4 + 108*b^2*c*d^3*x*e^3 + 108*a*c^2*d^3*x
*e^3 + 12*b^2*c*d^4*e^2 + 12*a*c^2*d^4*e^2 + 420*b^3*x^3*e^6 + 2520*a*b*c*x
^3*e^6 + 180*b^3*d*x^2*e^5 + 1080*a*b*c*d*x^2*e^5 + 45*b^3*d^2*x*e^4 + 270*
a*b*c*d^2*x*e^4 + 5*b^3*d^3*e^3 + 30*a*b*c*d^3*e^3 + 1080*a*b^2*x^2*e^6 + 1
080*a^2*c*x^2*e^6 + 270*a*b^2*d*x*e^5 + 270*a^2*c*d*x*e^5 + 30*a*b^2*d^2*e^
4 + 30*a^2*c*d^2*e^4 + 945*a^2*b*x*e^6 + 105*a^2*b*d*e^5 + 280*a^3*e^6)*e^(
-7)/(x*e + d)^9
```

3.2146 $\int (d+ex)^4 \left(a+bx+cx^2\right)^4 dx$

Optimal. Leaf size=443

$$\frac{(d+ex)^9 \left(6c^2e^2 \left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4\right)}{9e^9}+\frac{2c^2(d+ex)^{11}}{}$$

```
[Out] ((c*d^2 - b*d*e + a*e^2)^4*(d + e*x)^5)/(5*e^9) - (2*(2*c*d - b*e)*(c*d^2 -
 b*d*e + a*e^2)^3*(d + e*x)^6)/(3*e^9) + (2*(c*d^2 - b*d*e + a*e^2)^2*(14*c
^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e))*(d + e*x)^7)/(7*e^9) - ((2*c*d -
b*e)*(c*d^2 - b*d*e + a*e^2)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e))*(d
 + e*x)^8)/(2*e^9) + ((70*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(5*b*d - 3*a*e) -
 20*c^3*d^2*e*(7*b*d - 3*a*e) + 6*c^2*e^2*(15*b^2*d^2 - 10*a*b*d*e + a^2*e^
2))*(d + e*x)^9)/(9*e^9) - (2*c*(2*c*d - b*e)*(7*c^2*d^2 + b^2*e^2 - c*e*(7
*b*d - 3*a*e))*(d + e*x)^10)/(5*e^9) + (2*c^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c
*e*(7*b*d - a*e))*(d + e*x)^11)/(11*e^9) - (c^3*(2*c*d - b*e)*(d + e*x)^12)
/(3*e^9) + (c^4*(d + e*x)^13)/(13*e^9)
```

Rubi [A] time = 0.870615, antiderivative size = 443, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$\frac{(d+ex)^9 \left(6c^2e^2 \left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4\right)}{9e^9}+\frac{2c^2(d+ex)^{11}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4*(a + b*x + c*x^2)^4,x]
```


```
[Out] ((c*d^2 - b*d*e + a*e^2)^4*(d + e*x)^5)/(5*e^9) - (2*(2*c*d - b*e)*(c*d^2 -
 b*d*e + a*e^2)^3*(d + e*x)^6)/(3*e^9) + (2*(c*d^2 - b*d*e + a*e^2)^2*(14*c
^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e))*(d + e*x)^7)/(7*e^9) - ((2*c*d -
b*e)*(c*d^2 - b*d*e + a*e^2)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e))*(d
 + e*x)^8)/(2*e^9) + ((70*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(5*b*d - 3*a*e) -
 20*c^3*d^2*e*(7*b*d - 3*a*e) + 6*c^2*e^2*(15*b^2*d^2 - 10*a*b*d*e + a^2*e^
2))*(d + e*x)^9)/(9*e^9) - (2*c*(2*c*d - b*e)*(7*c^2*d^2 + b^2*e^2 - c*e*(7
*b*d - 3*a*e))*(d + e*x)^10)/(5*e^9) + (2*c^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c
*e*(7*b*d - a*e))*(d + e*x)^11)/(11*e^9) - (c^3*(2*c*d - b*e)*(d + e*x)^12)
/(3*e^9) + (c^4*(d + e*x)^13)/(13*e^9)
```


Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^4 \left(a+bx+cx^2\right)^4 dx = \int \left(\frac{\left(cd^2-bde+ae^2\right)^4 (d+ex)^4}{e^8} + \frac{4(-2cd+be)\left(cd^2-bde+ae^2\right)^3 (d+ex)^5}{e^8} + \frac{2\left(cd^2-\right.}{} \right.$$
$$= \frac{\left(cd^2-bde+ae^2\right)^4 (d+ex)^5}{5e^9} - \frac{2(2cd-be)\left(cd^2-bde+ae^2\right)^3 (d+ex)^6}{3e^9} + \frac{2\left(cd^2-bde+\right.}{}$$

Mathematica [A] time = 0.26029, size = 766, normalized size = 1.73

$$\frac{1}{9}x^9\left(6c^2e^2\left(a^2e^2+8abde+6b^2d^2\right)+4b^2ce^3(3ae+4bd)+8c^3d^2e(3ae+2bd)+b^4e^4+c^4d^4\right)+\frac{1}{2}x^8\left(bc\left(3a^2e^4+18acd^2e^2+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4*(a + b*x + c*x^2)^4,x]
```

```
[Out] a^4*d^4*x + 2*a^3*d^3*(b*d + a*e)*x^2 + (2*a^2*d^2*(3*b^2*d^2 + 8*a*b*d*e +
 a*(2*c*d^2 + 3*a*e^2))*x^3)/3 + a*d*(b^3*d^3 + 6*a*b^2*d^2*e + a^2*e*(4*c*
d^2 + a*e^2) + 3*a*b*d*(c*d^2 + 2*a*e^2))*x^4 + ((b^4*d^4 + 16*a*b^3*d^3*e
+ 16*a^2*b*d*e*(3*c*d^2 + a*e^2) + 12*a*b^2*d^2*(c*d^2 + 3*a*e^2) + a^2*(6*
c^2*d^4 + 24*a*c*d^2*e^2 + a^2*e^4))*x^5)/5 + (2*(b^4*d^3*e + 6*a*b^2*d*e*(
2*c*d^2 + a*e^2) + 2*a^2*c*d*e*(3*c*d^2 + 2*a*e^2) + b^3*(c*d^4 + 6*a*d^2*e
^2) + a*b*(3*c^2*d^4 + 18*a*c*d^2*e^2 + a^2*e^4))*x^6)/3 + (2*(3*b^4*d^2*e^
2 + 24*a*b*c*d*e*(c*d^2 + a*e^2) + 8*b^3*(c*d^3*e + a*d*e^3) + 2*a*c*(c^2*d
^4 + 9*a*c*d^2*e^2 + a^2*e^4) + 3*b^2*(c^2*d^4 + 12*a*c*d^2*e^2 + a^2*e^4))
*x^7)/7 + ((b^4*d*e^3 + 6*b^2*c*d*e*(c*d^2 + 2*a*e^2) + 2*a*c^2*d*e*(2*c*d^
2 + 3*a*e^2) + b^3*(6*c*d^2*e^2 + a*e^4) + b*c*(c^2*d^4 + 18*a*c*d^2*e^2 +
3*a^2*e^4))*x^8)/2 + ((c^4*d^4 + b^4*e^4 + 8*c^3*d^2*e*(2*b*d + 3*a*e) + 4*
b^2*c*e^3*(4*b*d + 3*a*e) + 6*c^2*e^2*(6*b^2*d^2 + 8*a*b*d*e + a^2*e^2))*x^
9)/9 + (2*c*e*(c^3*d^3 + b^3*e^3 + 3*b*c*e^2*(2*b*d + a*e) + 2*c^2*d*e*(3*b
*d + 2*a*e))*x^10)/5 + (2*c^2*e^2*(3*c^2*d^2 + 3*b^2*e^2 + 2*c*e*(4*b*d + a
*e))*x^11)/11 + (c^3*e^3*(c*d + b*e)*x^12)/3 + (c^4*e^4*x^13)/13
```

Maple [B] time = 0.041, size = 949, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(c*x^2+b*x+a)^4,x)
```

```
[Out] 1/13*c^4*e^4*x^13+1/12*(4*b*c^3*e^4+4*c^4*d*e^3)*x^12+1/11*(6*d^2*e^2*c^4+1
6*d*e^3*b*c^3+e^4*(2*(2*a*c+b^2)*c^2+4*b^2*c^2))*x^11+1/10*(4*d^3*e*c^4+24*
d^2*e^2*b*c^3+4*d*e^3*(2*(2*a*c+b^2)*c^2+4*b^2*c^2)+e^4*(4*b*a*c^2+4*(2*a*c
+b^2)*b*c))*x^10+1/9*(c^4*d^4+16*b*c^3*d^3*e+6*d^2*e^2*(2*(2*a*c+b^2)*c^2+4
*b^2*c^2)+4*d*e^3*(4*b*a*c^2+4*(2*a*c+b^2)*b*c)+e^4*(2*a^2*c^2+8*a*c*b^2+(2
*a*c+b^2)^2))*x^9+1/8*(4*d^4*b*c^3+4*d^3*e*(2*(2*a*c+b^2)*c^2+4*b^2*c^2)+6*
d^2*e^2*(4*b*a*c^2+4*(2*a*c+b^2)*b*c)+4*d*e^3*(2*a^2*c^2+8*a*c*b^2+(2*a*c+b
^2)^2)+e^4*(4*a^2*b*c+4*a*b*(2*a*c+b^2)))*x^8+1/7*(d^4*(2*(2*a*c+b^2)*c^2+4
*b^2*c^2)+4*d^3*e*(4*b*a*c^2+4*(2*a*c+b^2)*b*c)+6*d^2*e^2*(2*a^2*c^2+8*a*c*
b^2+(2*a*c+b^2)^2)+4*d*e^3*(4*a^2*b*c+4*a*b*(2*a*c+b^2))+e^4*(2*a^2*(2*a*c+
b^2)+4*b^2*a^2))*x^7+1/6*(d^4*(4*b*a*c^2+4*(2*a*c+b^2)*b*c)+4*d^3*e*(2*a^2*
c^2+8*a*c*b^2+(2*a*c+b^2)^2)+6*d^2*e^2*(4*a^2*b*c+4*a*b*(2*a*c+b^2))+4*d*e^
3*(2*a^2*(2*a*c+b^2)+4*b^2*a^2)+4*e^4*a^3*b)*x^6+1/5*(d^4*(2*a^2*c^2+8*a*c*
```

```
b^2+(2*a*c+b^2)^2)+4*d^3*e*(4*a^2*b*c+4*a*b*(2*a*c+b^2))+6*d^2*e^2*(2*a^2*(
2*a*c+b^2)+4*b^2*a^2)+16*a^3*b*d*e^3+a^4*e^4)*x^5+1/4*(d^4*(4*a^2*b*c+4*a*b
*(2*a*c+b^2))+4*d^3*e*(2*a^2*(2*a*c+b^2)+4*b^2*a^2)+24*d^2*e^2*a^3*b+4*d*e^
3*a^4)*x^4+1/3*(d^4*(2*a^2*(2*a*c+b^2)+4*b^2*a^2)+16*d^3*e*a^3*b+6*d^2*e^2*
a^4)*x^3+1/2*(4*a^4*d^3*e+4*a^3*b*d^4)*x^2+a^4*d^4*x
```

Maxima [A] time = 1.11645, size = 1029, normalized size = 2.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/13*c^4*e^4*x^13 + 1/3*(c^4*d*e^3 + b*c^3*e^4)*x^12 + 2/11*(3*c^4*d^2*e^2
+ 8*b*c^3*d*e^3 + (3*b^2*c^2 + 2*a*c^3)*e^4)*x^11 + 2/5*(c^4*d^3*e + 6*b*c^
3*d^2*e^2 + 2*(3*b^2*c^2 + 2*a*c^3)*d*e^3 + (b^3*c + 3*a*b*c^2)*e^4)*x^10 +
 1/9*(c^4*d^4 + 16*b*c^3*d^3*e + 12*(3*b^2*c^2 + 2*a*c^3)*d^2*e^2 + 16*(b^3
*c + 3*a*b*c^2)*d*e^3 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*e^4)*x^9 + a^4*d^4*x
 + 1/2*(b*c^3*d^4 + 2*(3*b^2*c^2 + 2*a*c^3)*d^3*e + 6*(b^3*c + 3*a*b*c^2)*d
^2*e^2 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d*e^3 + (a*b^3 + 3*a^2*b*c)*e^4)*x^
8 + 2/7*((3*b^2*c^2 + 2*a*c^3)*d^4 + 8*(b^3*c + 3*a*b*c^2)*d^3*e + 3*(b^4 +
 12*a*b^2*c + 6*a^2*c^2)*d^2*e^2 + 8*(a*b^3 + 3*a^2*b*c)*d*e^3 + (3*a^2*b^2
 + 2*a^3*c)*e^4)*x^7 + 2/3*(a^3*b*e^4 + (b^3*c + 3*a*b*c^2)*d^4 + (b^4 + 12
*a*b^2*c + 6*a^2*c^2)*d^3*e + 6*(a*b^3 + 3*a^2*b*c)*d^2*e^2 + 2*(3*a^2*b^2
+ 2*a^3*c)*d*e^3)*x^6 + 1/5*(16*a^3*b*d*e^3 + a^4*e^4 + (b^4 + 12*a*b^2*c +
 6*a^2*c^2)*d^4 + 16*(a*b^3 + 3*a^2*b*c)*d^3*e + 12*(3*a^2*b^2 + 2*a^3*c)*d
^2*e^2)*x^5 + (6*a^3*b*d^2*e^2 + a^4*d*e^3 + (a*b^3 + 3*a^2*b*c)*d^4 + 2*(3
*a^2*b^2 + 2*a^3*c)*d^3*e)*x^4 + 2/3*(8*a^3*b*d^3*e + 3*a^4*d^2*e^2 + (3*a^
2*b^2 + 2*a^3*c)*d^4)*x^3 + 2*(a^3*b*d^4 + a^4*d^3*e)*x^2
```

Fricas [B] time = 1.72971, size = 2202, normalized size = 4.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*e^4*c^4 + 1/3*x^12*e^3*d*c^4 + 1/3*x^12*e^4*c^3*b + 6/11*x^11*e^2
*d^2*c^4 + 16/11*x^11*e^3*d*c^3*b + 6/11*x^11*e^4*c^2*b^2 + 4/11*x^11*e^4*c
^3*a + 2/5*x^10*e*d^3*c^4 + 12/5*x^10*e^2*d^2*c^3*b + 12/5*x^10*e^3*d*c^2*b
^2 + 2/5*x^10*e^4*c*b^3 + 8/5*x^10*e^3*d*c^3*a + 6/5*x^10*e^4*c^2*b*a + 1/9
*x^9*d^4*c^4 + 16/9*x^9*e*d^3*c^3*b + 4*x^9*e^2*d^2*c^2*b^2 + 16/9*x^9*e^3*
d*c*b^3 + 1/9*x^9*e^4*b^4 + 8/3*x^9*e^2*d^2*c^3*a + 16/3*x^9*e^3*d*c^2*b*a
+ 4/3*x^9*e^4*c*b^2*a + 2/3*x^9*e^4*c^2*a^2 + 1/2*x^8*d^4*c^3*b + 3*x^8*e*d
^3*c^2*b^2 + 3*x^8*e^2*d^2*c*b^3 + 1/2*x^8*e^3*d*b^4 + 2*x^8*e*d^3*c^3*a +
9*x^8*e^2*d^2*c^2*b*a + 6*x^8*e^3*d*c*b^2*a + 1/2*x^8*e^4*b^3*a + 3*x^8*e^3
*d*c^2*a^2 + 3/2*x^8*e^4*c*b*a^2 + 6/7*x^7*d^4*c^2*b^2 + 16/7*x^7*e*d^3*c*b
^3 + 6/7*x^7*e^2*d^2*b^4 + 4/7*x^7*d^4*c^3*a + 48/7*x^7*e*d^3*c^2*b*a + 72/
7*x^7*e^2*d^2*c*b^2*a + 16/7*x^7*e^3*d*b^3*a + 36/7*x^7*e^2*d^2*c^2*a^2 + 4
8/7*x^7*e^3*d*c*b*a^2 + 6/7*x^7*e^4*b^2*a^2 + 4/7*x^7*e^4*c*a^3 + 2/3*x^6*d
^4*c*b^3 + 2/3*x^6*e*d^3*b^4 + 2*x^6*d^4*c^2*b*a + 8*x^6*e*d^3*c*b^2*a + 4*
x^6*e^2*d^2*b^3*a + 4*x^6*e*d^3*c^2*a^2 + 12*x^6*e^2*d^2*c*b*a^2 + 4*x^6*e^
3*d*b^2*a^2 + 8/3*x^6*e^3*d*c*a^3 + 2/3*x^6*e^4*b*a^3 + 1/5*x^5*d^4*b^4 + 1
```

```
2/5*x^5*d^4*c*b^2*a + 16/5*x^5*e*d^3*b^3*a + 6/5*x^5*d^4*c^2*a^2 + 48/5*x^5
*e*d^3*c*b*a^2 + 36/5*x^5*e^2*d^2*b^2*a^2 + 24/5*x^5*e^2*d^2*c*a^3 + 16/5*x
^5*e^3*d*b*a^3 + 1/5*x^5*e^4*a^4 + x^4*d^4*b^3*a + 3*x^4*d^4*c*b*a^2 + 6*x^
4*e*d^3*b^2*a^2 + 4*x^4*e*d^3*c*a^3 + 6*x^4*e^2*d^2*b*a^3 + x^4*e^3*d*a^4 +
 2*x^3*d^4*b^2*a^2 + 4/3*x^3*d^4*c*a^3 + 16/3*x^3*e*d^3*b*a^3 + 2*x^3*e^2*d
^2*a^4 + 2*x^2*d^4*b*a^3 + 2*x^2*e*d^3*a^4 + x*d^4*a^4
```

Sympy [B] time = 0.18071, size = 998, normalized size = 2.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(c*x**2+b*x+a)**4,x)
```

```
[Out] a**4*d**4*x + c**4*e**4*x**13/13 + x**12*(b*c**3*e**4/3 + c**4*d*e**3/3) +
x**11*(4*a*c**3*e**4/11 + 6*b**2*c**2*e**4/11 + 16*b*c**3*d*e**3/11 + 6*c**
4*d**2*e**2/11) + x**10*(6*a*b*c**2*e**4/5 + 8*a*c**3*d*e**3/5 + 2*b**3*c*e
**4/5 + 12*b**2*c**2*d*e**3/5 + 12*b*c**3*d**2*e**2/5 + 2*c**4*d**3*e/5) +
x**9*(2*a**2*c**2*e**4/3 + 4*a*b**2*c*e**4/3 + 16*a*b*c**2*d*e**3/3 + 8*a*c
**3*d**2*e**2/3 + b**4*e**4/9 + 16*b**3*c*d*e**3/9 + 4*b**2*c**2*d**2*e**2
+ 16*b*c**3*d**3*e/9 + c**4*d**4/9) + x**8*(3*a**2*b*c*e**4/2 + 3*a**2*c**2
*d*e**3 + a*b**3*e**4/2 + 6*a*b**2*c*d*e**3 + 9*a*b*c**2*d**2*e**2 + 2*a*c*
*3*d**3*e + b**4*d*e**3/2 + 3*b**3*c*d**2*e**2 + 3*b**2*c**2*d**3*e + b*c**
3*d**4/2) + x**7*(4*a**3*c*e**4/7 + 6*a**2*b**2*e**4/7 + 48*a**2*b*c*d*e**3
/7 + 36*a**2*c**2*d**2*e**2/7 + 16*a*b**3*d*e**3/7 + 72*a*b**2*c*d**2*e**2/
7 + 48*a*b*c**2*d**3*e/7 + 4*a*c**3*d**4/7 + 6*b**4*d**2*e**2/7 + 16*b**3*c
*d**3*e/7 + 6*b**2*c**2*d**4/7) + x**6*(2*a**3*b*e**4/3 + 8*a**3*c*d*e**3/3
 + 4*a**2*b**2*d*e**3 + 12*a**2*b*c*d**2*e**2 + 4*a**2*c**2*d**3*e + 4*a*b*
*3*d**2*e**2 + 8*a*b**2*c*d**3*e + 2*a*b*c**2*d**4 + 2*b**4*d**3*e/3 + 2*b*
*3*c*d**4/3) + x**5*(a**4*e**4/5 + 16*a**3*b*d*e**3/5 + 24*a**3*c*d**2*e**2
/5 + 36*a**2*b**2*d**2*e**2/5 + 48*a**2*b*c*d**3*e/5 + 6*a**2*c**2*d**4/5 +
 16*a*b**3*d**3*e/5 + 12*a*b**2*c*d**4/5 + b**4*d**4/5) + x**4*(a**4*d*e**3
 + 6*a**3*b*d**2*e**2 + 4*a**3*c*d**3*e + 6*a**2*b**2*d**3*e + 3*a**2*b*c*d
**4 + a*b**3*d**4) + x**3*(2*a**4*d**2*e**2 + 16*a**3*b*d**3*e/3 + 4*a**3*c
*d**4/3 + 2*a**2*b**2*d**4) + x**2*(2*a**4*d**3*e + 2*a**3*b*d**4)
```

Giac [B] time = 1.15786, size = 1311, normalized size = 2.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(c*x^2+b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/13*c^4*x^13*e^4 + 1/3*c^4*d*x^12*e^3 + 6/11*c^4*d^2*x^11*e^2 + 2/5*c^4*d^
3*x^10*e + 1/9*c^4*d^4*x^9 + 1/3*b*c^3*x^12*e^4 + 16/11*b*c^3*d*x^11*e^3 +
12/5*b*c^3*d^2*x^10*e^2 + 16/9*b*c^3*d^3*x^9*e + 1/2*b*c^3*d^4*x^8 + 6/11*b
^2*c^2*x^11*e^4 + 4/11*a*c^3*x^11*e^4 + 12/5*b^2*c^2*d*x^10*e^3 + 8/5*a*c^3
*d*x^10*e^3 + 4*b^2*c^2*d^2*x^9*e^2 + 8/3*a*c^3*d^2*x^9*e^2 + 3*b^2*c^2*d^3
*x^8*e + 2*a*c^3*d^3*x^8*e + 6/7*b^2*c^2*d^4*x^7 + 4/7*a*c^3*d^4*x^7 + 2/5*
b^3*c*x^10*e^4 + 6/5*a*b*c^2*x^10*e^4 + 16/9*b^3*c*d*x^9*e^3 + 16/3*a*b*c^2
*d*x^9*e^3 + 3*b^3*c*d^2*x^8*e^2 + 9*a*b*c^2*d^2*x^8*e^2 + 16/7*b^3*c*d^3*x
^7*e + 48/7*a*b*c^2*d^3*x^7*e + 2/3*b^3*c*d^4*x^6 + 2*a*b*c^2*d^4*x^6 + 1/9
```

*b^4*x^9*e^4 + 4/3*a*b^2*c*x^9*e^4 + 2/3*a^2*c^2*x^9*e^4 + 1/2*b^4*d*x^8*e^3 + 6*a*b^2*c*d*x^8*e^3 + 3*a^2*c^2*d*x^8*e^3 + 6/7*b^4*d^2*x^7*e^2 + 72/7*a*b^2*c*d^2*x^7*e^2 + 36/7*a^2*c^2*d^2*x^7*e^2 + 2/3*b^4*d^3*x^6*e + 8*a*b^2*c*d^3*x^6*e + 4*a^2*c^2*d^3*x^6*e + 1/5*b^4*d^4*x^5 + 12/5*a*b^2*c*d^4*x^5 + 6/5*a^2*c^2*d^4*x^5 + 1/2*a*b^3*x^8*e^4 + 3/2*a^2*b*c*x^8*e^4 + 16/7*a*b^3*d*x^7*e^3 + 48/7*a^2*b*c*d*x^7*e^3 + 4*a*b^3*d^2*x^6*e^2 + 12*a^2*b*c*d^2*x^6*e^2 + 16/5*a*b^3*d^3*x^5*e + 48/5*a^2*b*c*d^3*x^5*e + a*b^3*d^4*x^4 + 3*a^2*b*c*d^4*x^4 + 6/7*a^2*b^2*x^7*e^4 + 4/7*a^3*c*x^7*e^4 + 4*a^2*b^2*d*x^6*e^3 + 8/3*a^3*c*d*x^6*e^3 + 36/5*a^2*b^2*d^2*x^5*e^2 + 24/5*a^3*c*d^2*x^5*e^2 + 6*a^2*b^2*d^3*x^4*e + 4*a^3*c*d^3*x^4*e + 2*a^2*b^2*d^4*x^3 + 4/3*a^3*c*d^4*x^3 + 2/3*a^3*b*x^6*e^4 + 16/5*a^3*b*d*x^5*e^3 + 6*a^3*b*d^2*x^4*e^2 + 16/3*a^3*b*d^3*x^3*e + 2*a^3*b*d^4*x^2 + 1/5*a^4*x^5*e^4 + a^4*d*x^4*e^3 + 2*a^4*d^2*x^3*e^2 + 2*a^4*d^3*x^2*e + a^4*d^4*x

3.2147 $\int (d+ex)^3 \left(a+bx+cx^2\right)^4 dx$

Optimal. Leaf size=443

$$\frac{(d+ex)^8 \left(6c^2e^2 \left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4\right)}{8e^9}+\frac{c^2(d+ex)^{10} \left(-2c\right.}{}$$

```
[Out] ((c*d^2 - b*d*e + a*e^2)^4*(d + e*x)^4)/(4*e^9) - (4*(2*c*d - b*e)*(c*d^2 -
 b*d*e + a*e^2)^3*(d + e*x)^5)/(5*e^9) + ((c*d^2 - b*d*e + a*e^2)^2*(14*c^2
*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e))*(d + e*x)^6)/(3*e^9) - (4*(2*c*d -
b*e)*(c*d^2 - b*d*e + a*e^2)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e))*(d
 + e*x)^7)/(7*e^9) + ((70*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(5*b*d - 3*a*e) -
 20*c^3*d^2*e*(7*b*d - 3*a*e) + 6*c^2*e^2*(15*b^2*d^2 - 10*a*b*d*e + a^2*e^
2))*(d + e*x)^8)/(8*e^9) - (4*c*(2*c*d - b*e)*(7*c^2*d^2 + b^2*e^2 - c*e*(7
*b*d - 3*a*e))*(d + e*x)^9)/(9*e^9) + (c^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*
(7*b*d - a*e))*(d + e*x)^10)/(5*e^9) - (4*c^3*(2*c*d - b*e)*(d + e*x)^11)/(
11*e^9) + (c^4*(d + e*x)^12)/(12*e^9)
```

Rubi [A] time = 0.673569, antiderivative size = 443, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$\frac{(d+ex)^8 \left(6c^2e^2 \left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4\right)}{8e^9}+\frac{c^2(d+ex)^{10} \left(-2c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*x + c*x^2)^4,x]
```


```
[Out] ((c*d^2 - b*d*e + a*e^2)^4*(d + e*x)^4)/(4*e^9) - (4*(2*c*d - b*e)*(c*d^2 -
 b*d*e + a*e^2)^3*(d + e*x)^5)/(5*e^9) + ((c*d^2 - b*d*e + a*e^2)^2*(14*c^2
*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e))*(d + e*x)^6)/(3*e^9) - (4*(2*c*d -
b*e)*(c*d^2 - b*d*e + a*e^2)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e))*(d
 + e*x)^7)/(7*e^9) + ((70*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(5*b*d - 3*a*e) -
 20*c^3*d^2*e*(7*b*d - 3*a*e) + 6*c^2*e^2*(15*b^2*d^2 - 10*a*b*d*e + a^2*e^
2))*(d + e*x)^8)/(8*e^9) - (4*c*(2*c*d - b*e)*(7*c^2*d^2 + b^2*e^2 - c*e*(7
*b*d - 3*a*e))*(d + e*x)^9)/(9*e^9) + (c^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*
(7*b*d - a*e))*(d + e*x)^10)/(5*e^9) - (4*c^3*(2*c*d - b*e)*(d + e*x)^11)/(
11*e^9) + (c^4*(d + e*x)^12)/(12*e^9)
```


Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^3 \left(a+bx+cx^2\right)^4 dx = \int \left(\frac{\left(cd^2-bde+ae^2\right)^4 (d+ex)^3}{e^8} + \frac{4(-2cd+be)\left(cd^2-bde+ae^2\right)^3 (d+ex)^4}{e^8} + \frac{2\left(cd\right.}{}\right.$$
$$= \frac{\left(cd^2-bde+ae^2\right)^4 (d+ex)^4}{4e^9} - \frac{4(2cd-be)\left(cd^2-bde+ae^2\right)^3 (d+ex)^5}{5e^9} + \frac{\left(cd^2-bde\right.}{}$$

Mathematica [A] time = 0.203673, size = 611, normalized size = 1.38

$$\frac{1}{6}x^6 \left(2a^2ce\left(2ae^2+9cd^2\right)+4b^3\left(3ade^2+cd^3\right)+6ab^2e\left(ae^2+6cd^2\right)+12abcd\left(3ae^2+cd^2\right)+3b^4d^2e\right)+\frac{1}{5}x^5\left(4a^2be\left(ae^2\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*x + c*x^2)^4,x]
```

```
[Out] a^4*d^3*x + (a^3*d^2*(4*b*d + 3*a*e)*x^2)/2 + (a^2*d*(6*b^2*d^2 + 12*a*b*d*
e + a*(4*c*d^2 + 3*a*e^2))*x^3)/3 + (a*(4*b^3*d^3 + 18*a*b^2*d^2*e + 12*a*b
*d*(c*d^2 + a*e^2) + a^2*e*(12*c*d^2 + a*e^2))*x^4)/4 + ((b^4*d^3 + 12*a*b^
3*d^2*e + 4*a^2*b*e*(9*c*d^2 + a*e^2) + 6*a^2*c*d*(c*d^2 + 2*a*e^2) + 6*a*b
^2*d*(2*c*d^2 + 3*a*e^2))*x^5)/5 + ((3*b^4*d^2*e + 6*a*b^2*e*(6*c*d^2 + a*e
^2) + 2*a^2*c*e*(9*c*d^2 + 2*a*e^2) + 12*a*b*c*d*(c*d^2 + 3*a*e^2) + 4*b^3*
(c*d^3 + 3*a*d*e^2))*x^6)/6 + ((3*b^4*d*e^2 + 12*a*b*c*e*(3*c*d^2 + a*e^2)
+ 6*b^2*c*d*(c*d^2 + 6*a*e^2) + 2*a*c^2*d*(2*c*d^2 + 9*a*e^2) + 4*b^3*(3*c*
d^2*e + a*e^3))*x^7)/7 + ((12*b^3*c*d*e^2 + b^4*e^3 + 6*a*c^2*e*(2*c*d^2 +
a*e^2) + 6*b^2*c*e*(3*c*d^2 + 2*a*e^2) + 4*b*c^2*d*(c*d^2 + 9*a*e^2))*x^8)/
8 + (c*(c^3*d^3 + 4*b^3*e^3 + 12*c^2*d*e*(b*d + a*e) + 6*b*c*e^2*(3*b*d + 2
*a*e))*x^9)/9 + (c^2*e*(3*c^2*d^2 + 6*b^2*e^2 + 4*c*e*(3*b*d + a*e))*x^10)/
10 + (c^3*e^2*(3*c*d + 4*b*e)*x^11)/11 + (c^4*e^3*x^12)/12
```

Maple [A] time = 0.041, size = 747, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x+a)^4,x)
```

```
[Out] 1/12*c^4*e^3*x^12+1/11*(4*b*c^3*e^3+3*c^4*d*e^2)*x^11+1/10*(3*d^2*e*c^4+12*
d*e^2*b*c^3+e^3*(2*(2*a*c+b^2)*c^2+4*b^2*c^2))*x^10+1/9*(d^3*c^4+12*d^2*e*b
*c^3+3*d*e^2*(2*(2*a*c+b^2)*c^2+4*b^2*c^2)+e^3*(4*b*a*c^2+4*(2*a*c+b^2)*b*c
))*x^9+1/8*(4*d^3*b*c^3+3*d^2*e*(2*(2*a*c+b^2)*c^2+4*b^2*c^2)+3*d*e^2*(4*b*
a*c^2+4*(2*a*c+b^2)*b*c)+e^3*(2*a^2*c^2+8*a*c*b^2+(2*a*c+b^2)^2))*x^8+1/7*(
d^3*(2*(2*a*c+b^2)*c^2+4*b^2*c^2)+3*d^2*e*(4*b*a*c^2+4*(2*a*c+b^2)*b*c)+3*d
*e^2*(2*a^2*c^2+8*a*c*b^2+(2*a*c+b^2)^2)+e^3*(4*a^2*b*c+4*a*b*(2*a*c+b^2)))
*x^7+1/6*(d^3*(4*b*a*c^2+4*(2*a*c+b^2)*b*c)+3*d^2*e*(2*a^2*c^2+8*a*c*b^2+(2
*a*c+b^2)^2)+3*d*e^2*(4*a^2*b*c+4*a*b*(2*a*c+b^2))+e^3*(2*a^2*(2*a*c+b^2)+4
*b^2*a^2))*x^6+1/5*(d^3*(2*a^2*c^2+8*a*c*b^2+(2*a*c+b^2)^2)+3*d^2*e*(4*a^2*
b*c+4*a*b*(2*a*c+b^2))+3*d*e^2*(2*a^2*(2*a*c+b^2)+4*b^2*a^2)+4*a^3*b*e^3)*x
^5+1/4*(d^3*(4*a^2*b*c+4*a*b*(2*a*c+b^2))+3*d^2*e*(2*a^2*(2*a*c+b^2)+4*b^2*
a^2)+12*d*e^2*a^3*b+e^3*a^4)*x^4+1/3*(d^3*(2*a^2*(2*a*c+b^2)+4*b^2*a^2)+12*
d^2*e*a^3*b+3*d*e^2*a^4)*x^3+1/2*(3*a^4*d^2*e+4*a^3*b*d^3)*x^2+a^4*d^3*x
```

Maxima [A] time = 0.999234, size = 829, normalized size = 1.87

$$\frac{1}{12}c^4e^3x^{12} + \frac{1}{11}\left(3c^4de^2 + 4bc^3e^3\right)x^{11} + \frac{1}{10}\left(3c^4d^2e + 12bc^3de^2 + 2\left(3b^2c^2 + 2ac^3\right)e^3\right)x^{10} + \frac{1}{9}\left(c^4d^3 + 12bc^3d^2e + 6\left(3b^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/12*c^4*e^3*x^12 + 1/11*(3*c^4*d*e^2 + 4*b*c^3*e^3)*x^11 + 1/10*(3*c^4*d^2
*e + 12*b*c^3*d*e^2 + 2*(3*b^2*c^2 + 2*a*c^3)*e^3)*x^10 + 1/9*(c^4*d^3 + 12
*b*c^3*d^2*e + 6*(3*b^2*c^2 + 2*a*c^3)*d*e^2 + 4*(b^3*c + 3*a*b*c^2)*e^3)*x
^9 + 1/8*(4*b*c^3*d^3 + 6*(3*b^2*c^2 + 2*a*c^3)*d^2*e + 12*(b^3*c + 3*a*b*c
^2)*d*e^2 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*e^3)*x^8 + a^4*d^3*x + 1/7*(2*(3
*b^2*c^2 + 2*a*c^3)*d^3 + 12*(b^3*c + 3*a*b*c^2)*d^2*e + 3*(b^4 + 12*a*b^2*
c + 6*a^2*c^2)*d*e^2 + 4*(a*b^3 + 3*a^2*b*c)*e^3)*x^7 + 1/6*(4*(b^3*c + 3*a
*b*c^2)*d^3 + 3*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^2*e + 12*(a*b^3 + 3*a^2*b*
c)*d*e^2 + 2*(3*a^2*b^2 + 2*a^3*c)*e^3)*x^6 + 1/5*(4*a^3*b*e^3 + (b^4 + 12*
a*b^2*c + 6*a^2*c^2)*d^3 + 12*(a*b^3 + 3*a^2*b*c)*d^2*e + 6*(3*a^2*b^2 + 2*
a^3*c)*d*e^2)*x^5 + 1/4*(12*a^3*b*d*e^2 + a^4*e^3 + 4*(a*b^3 + 3*a^2*b*c)*d
^3 + 6*(3*a^2*b^2 + 2*a^3*c)*d^2*e)*x^4 + 1/3*(12*a^3*b*d^2*e + 3*a^4*d*e^2
 + 2*(3*a^2*b^2 + 2*a^3*c)*d^3)*x^3 + 1/2*(4*a^3*b*d^3 + 3*a^4*d^2*e)*x^2
```

Fricas [A] time = 1.73839, size = 1704, normalized size = 3.85

$$\frac{1}{12}x^{12}e^3c^4 + \frac{3}{11}x^{11}e^2dc^4 + \frac{4}{11}x^{11}e^3c^3b + \frac{3}{10}x^{10}ed^2c^4 + \frac{6}{5}x^{10}e^2dc^3b + \frac{3}{5}x^{10}e^3c^2b^2 + \frac{2}{5}x^{10}e^3c^3a + \frac{1}{9}x^9d^3c^4 + \frac{4}{3}x^9ed^2c^3b + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*e^3*c^4 + 3/11*x^11*e^2*d*c^4 + 4/11*x^11*e^3*c^3*b + 3/10*x^10*e
*d^2*c^4 + 6/5*x^10*e^2*d*c^3*b + 3/5*x^10*e^3*c^2*b^2 + 2/5*x^10*e^3*c^3*a
 + 1/9*x^9*d^3*c^4 + 4/3*x^9*e*d^2*c^3*b + 2*x^9*e^2*d*c^2*b^2 + 4/9*x^9*e^
3*c*b^3 + 4/3*x^9*e^2*d*c^3*a + 4/3*x^9*e^3*c^2*b*a + 1/2*x^8*d^3*c^3*b + 9
/4*x^8*e*d^2*c^2*b^2 + 3/2*x^8*e^2*d*c*b^3 + 1/8*x^8*e^3*b^4 + 3/2*x^8*e*d^
2*c^3*a + 9/2*x^8*e^2*d*c^2*b*a + 3/2*x^8*e^3*c*b^2*a + 3/4*x^8*e^3*c^2*a^2
 + 6/7*x^7*d^3*c^2*b^2 + 12/7*x^7*e*d^2*c*b^3 + 3/7*x^7*e^2*d*b^4 + 4/7*x^7
*d^3*c^3*a + 36/7*x^7*e*d^2*c^2*b*a + 36/7*x^7*e^2*d*c*b^2*a + 4/7*x^7*e^3*
b^3*a + 18/7*x^7*e^2*d*c^2*a^2 + 12/7*x^7*e^3*c*b*a^2 + 2/3*x^6*d^3*c*b^3 +
 1/2*x^6*e*d^2*b^4 + 2*x^6*d^3*c^2*b*a + 6*x^6*e*d^2*c*b^2*a + 2*x^6*e^2*d*
b^3*a + 3*x^6*e*d^2*c^2*a^2 + 6*x^6*e^2*d*c*b*a^2 + x^6*e^3*b^2*a^2 + 2/3*x
^6*e^3*c*a^3 + 1/5*x^5*d^3*b^4 + 12/5*x^5*d^3*c*b^2*a + 12/5*x^5*e*d^2*b^3*
a + 6/5*x^5*d^3*c^2*a^2 + 36/5*x^5*e*d^2*c*b*a^2 + 18/5*x^5*e^2*d*b^2*a^2 +
 12/5*x^5*e^2*d*c*a^3 + 4/5*x^5*e^3*b*a^3 + x^4*d^3*b^3*a + 3*x^4*d^3*c*b*a
^2 + 9/2*x^4*e*d^2*b^2*a^2 + 3*x^4*e*d^2*c*a^3 + 3*x^4*e^2*d*b*a^3 + 1/4*x^
4*e^3*a^4 + 2*x^3*d^3*b^2*a^2 + 4/3*x^3*d^3*c*a^3 + 4*x^3*e*d^2*b*a^3 + x^3
*e^2*d*a^4 + 2*x^2*d^3*b*a^3 + 3/2*x^2*e*d^2*a^4 + x*d^3*a^4
```

Sympy [A] time = 0.151718, size = 777, normalized size = 1.75

$$a^4d^3x + \frac{c^4e^3x^{12}}{12} + x^{11}\left(\frac{4bc^3e^3}{11} + \frac{3c^4de^2}{11}\right) + x^{10}\left(\frac{2ac^3e^3}{5} + \frac{3b^2c^2e^3}{5} + \frac{6bc^3de^2}{5} + \frac{3c^4d^2e}{10}\right) + x^9\left(\frac{4abc^2e^3}{3} + \frac{4ac^3de^2}{3} + \frac{4b^3}{9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x+a)**4,x)
```

```
[Out] a**4*d**3*x + c**4*e**3*x**12/12 + x**11*(4*b*c**3*e**3/11 + 3*c**4*d*e**2/
11) + x**10*(2*a*c**3*e**3/5 + 3*b**2*c**2*e**3/5 + 6*b*c**3*d*e**2/5 + 3*c
**4*d**2*e/10) + x**9*(4*a*b*c**2*e**3/3 + 4*a*c**3*d*e**2/3 + 4*b**3*c*e**
3/9 + 2*b**2*c**2*d*e**2 + 4*b*c**3*d**2*e/3 + c**4*d**3/9) + x**8*(3*a**2*
c**2*e**3/4 + 3*a*b**2*c*e**3/2 + 9*a*b*c**2*d*e**2/2 + 3*a*c**3*d**2*e/2 +
 b**4*e**3/8 + 3*b**3*c*d*e**2/2 + 9*b**2*c**2*d**2*e/4 + b*c**3*d**3/2) +
x**7*(12*a**2*b*c*e**3/7 + 18*a**2*c**2*d*e**2/7 + 4*a*b**3*e**3/7 + 36*a*b
**2*c*d*e**2/7 + 36*a*b*c**2*d**2*e/7 + 4*a*c**3*d**3/7 + 3*b**4*d*e**2/7 +
 12*b**3*c*d**2*e/7 + 6*b**2*c**2*d**3/7) + x**6*(2*a**3*c*e**3/3 + a**2*b*
*2*e**3 + 6*a**2*b*c*d*e**2 + 3*a**2*c**2*d**2*e + 2*a*b**3*d*e**2 + 6*a*b*
*2*c*d**2*e + 2*a*b*c**2*d**3 + b**4*d**2*e/2 + 2*b**3*c*d**3/3) + x**5*(4*
a**3*b*e**3/5 + 12*a**3*c*d*e**2/5 + 18*a**2*b**2*d*e**2/5 + 36*a**2*b*c*d*
*2*e/5 + 6*a**2*c**2*d**3/5 + 12*a*b**3*d**2*e/5 + 12*a*b**2*c*d**3/5 + b**
4*d**3/5) + x**4*(a**4*e**3/4 + 3*a**3*b*d*e**2 + 3*a**3*c*d**2*e + 9*a**2*
b**2*d**2*e/2 + 3*a**2*b*c*d**3 + a*b**3*d**3) + x**3*(a**4*d*e**2 + 4*a**3
*b*d**2*e + 4*a**3*c*d**3/3 + 2*a**2*b**2*d**3) + x**2*(3*a**4*d**2*e/2 + 2
*a**3*b*d**3)
```

Giac [A] time = 1.103, size = 1018, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/12*c^4*x^12*e^3 + 3/11*c^4*d*x^11*e^2 + 3/10*c^4*d^2*x^10*e + 1/9*c^4*d^3
*x^9 + 4/11*b*c^3*x^11*e^3 + 6/5*b*c^3*d*x^10*e^2 + 4/3*b*c^3*d^2*x^9*e + 1
/2*b*c^3*d^3*x^8 + 3/5*b^2*c^2*x^10*e^3 + 2/5*a*c^3*x^10*e^3 + 2*b^2*c^2*d*
x^9*e^2 + 4/3*a*c^3*d*x^9*e^2 + 9/4*b^2*c^2*d^2*x^8*e + 3/2*a*c^3*d^2*x^8*e
 + 6/7*b^2*c^2*d^3*x^7 + 4/7*a*c^3*d^3*x^7 + 4/9*b^3*c*x^9*e^3 + 4/3*a*b*c^
2*x^9*e^3 + 3/2*b^3*c*d*x^8*e^2 + 9/2*a*b*c^2*d*x^8*e^2 + 12/7*b^3*c*d^2*x^
7*e + 36/7*a*b*c^2*d^2*x^7*e + 2/3*b^3*c*d^3*x^6 + 2*a*b*c^2*d^3*x^6 + 1/8*
b^4*x^8*e^3 + 3/2*a*b^2*c*x^8*e^3 + 3/4*a^2*c^2*x^8*e^3 + 3/7*b^4*d*x^7*e^2
 + 36/7*a*b^2*c*d*x^7*e^2 + 18/7*a^2*c^2*d*x^7*e^2 + 1/2*b^4*d^2*x^6*e + 6*
a*b^2*c*d^2*x^6*e + 3*a^2*c^2*d^2*x^6*e + 1/5*b^4*d^3*x^5 + 12/5*a*b^2*c*d^
3*x^5 + 6/5*a^2*c^2*d^3*x^5 + 4/7*a*b^3*x^7*e^3 + 12/7*a^2*b*c*x^7*e^3 + 2*
a*b^3*d*x^6*e^2 + 6*a^2*b*c*d*x^6*e^2 + 12/5*a*b^3*d^2*x^5*e + 36/5*a^2*b*c
*d^2*x^5*e + a*b^3*d^3*x^4 + 3*a^2*b*c*d^3*x^4 + a^2*b^2*x^6*e^3 + 2/3*a^3*
c*x^6*e^3 + 18/5*a^2*b^2*d*x^5*e^2 + 12/5*a^3*c*d*x^5*e^2 + 9/2*a^2*b^2*d^2
*x^4*e + 3*a^3*c*d^2*x^4*e + 2*a^2*b^2*d^3*x^3 + 4/3*a^3*c*d^3*x^3 + 4/5*a^
3*b*x^5*e^3 + 3*a^3*b*d*x^4*e^2 + 4*a^3*b*d^2*x^3*e + 2*a^3*b*d^3*x^2 + 1/4
*a^4*x^4*e^3 + a^4*d*x^3*e^2 + 3/2*a^4*d^2*x^2*e + a^4*d^3*x
```

3.2148 $\int (d+ex)^2 \left(a+bx+cx^2\right)^4 \, dx$

Optimal. Leaf size=441

$$\frac{(d+ex)^7 \left(6c^2e^2 \left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4\right)}{7e^9}+\frac{2c^2(d+ex)^9 \left(-2\right.}{}$$

```
[Out] ((c*d^2 - b*d*e + a*e^2)^4*(d + e*x)^3)/(3*e^9) - ((2*c*d - b*e)*(c*d^2 - b
*d*e + a*e^2)^3*(d + e*x)^4)/e^9 + (2*(c*d^2 - b*d*e + a*e^2)^2*(14*c^2*d^2
 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e))*(d + e*x)^5)/(5*e^9) - (2*(2*c*d - b*e)
*(c*d^2 - b*d*e + a*e^2)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e))*(d + e
*x)^6)/(3*e^9) + ((70*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(5*b*d - 3*a*e) - 20*
c^3*d^2*e*(7*b*d - 3*a*e) + 6*c^2*e^2*(15*b^2*d^2 - 10*a*b*d*e + a^2*e^2))*
(d + e*x)^7)/(7*e^9) - (c*(2*c*d - b*e)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d -
 3*a*e))*(d + e*x)^8)/(2*e^9) + (2*c^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b
*d - a*e))*(d + e*x)^9)/(9*e^9) - (2*c^3*(2*c*d - b*e)*(d + e*x)^10)/(5*e^9
) + (c^4*(d + e*x)^11)/(11*e^9)
```

Rubi [A] time = 0.523082, antiderivative size = 441, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$\frac{(d+ex)^7 \left(6c^2e^2 \left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4\right)}{7e^9}+\frac{2c^2(d+ex)^9 \left(-2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*x + c*x^2)^4,x]
```


```
[Out] ((c*d^2 - b*d*e + a*e^2)^4*(d + e*x)^3)/(3*e^9) - ((2*c*d - b*e)*(c*d^2 - b
*d*e + a*e^2)^3*(d + e*x)^4)/e^9 + (2*(c*d^2 - b*d*e + a*e^2)^2*(14*c^2*d^2
 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e))*(d + e*x)^5)/(5*e^9) - (2*(2*c*d - b*e)
*(c*d^2 - b*d*e + a*e^2)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e))*(d + e
*x)^6)/(3*e^9) + ((70*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(5*b*d - 3*a*e) - 20*
c^3*d^2*e*(7*b*d - 3*a*e) + 6*c^2*e^2*(15*b^2*d^2 - 10*a*b*d*e + a^2*e^2))*
(d + e*x)^7)/(7*e^9) - (c*(2*c*d - b*e)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d -
 3*a*e))*(d + e*x)^8)/(2*e^9) + (2*c^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b
*d - a*e))*(d + e*x)^9)/(9*e^9) - (2*c^3*(2*c*d - b*e)*(d + e*x)^10)/(5*e^9
) + (c^4*(d + e*x)^11)/(11*e^9)
```


Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^2\left(a+bx+cx^2\right)^4 dx = \int\left(\frac{\left(cd^2-bde+ae^2\right)^4(d+ex)^2}{e^8}+\frac{4(-2cd+be)\left(cd^2-bde+ae^2\right)^3(d+ex)^3}{e^8}+\frac{2\left(cd\right.}{}\right.$$
$$=\frac{\left(cd^2-bde+ae^2\right)^4(d+ex)^3}{3e^9}-\frac{(2cd-be)\left(cd^2-bde+ae^2\right)^3(d+ex)^4}{e^9}+\frac{2\left(cd^2-bde\right.}{}$$

Mathematica [A] time = 0.135433, size = 428, normalized size = 0.97

$$\frac{1}{3}x^6\left(6a^2c^2de+2b^3\left(ae^2+cd^2\right)+12ab^2cde+6abc\left(ae^2+cd^2\right)+b^4de\right)+\frac{1}{5}x^5\left(24a^2bcde+2a^2c\left(2ae^2+3cd^2\right)+6ab^2\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*x + c*x^2)^4,x]
```

```
[Out] a^4*d^2*x + a^3*d*(2*b*d + a*e)*x^2 + (a^2*(6*b^2*d^2 + 8*a*b*d*e + a*(4*c*
d^2 + a*e^2))*x^3)/3 + a*(b^3*d^2 + 3*a*b^2*d*e + 2*a^2*c*d*e + a*b*(3*c*d^
2 + a*e^2))*x^4 + ((b^4*d^2 + 8*a*b^3*d*e + 24*a^2*b*c*d*e + 6*a*b^2*(2*c*d
^2 + a*e^2) + 2*a^2*c*(3*c*d^2 + 2*a*e^2))*x^5)/5 + ((b^4*d*e + 12*a*b^2*c*
d*e + 6*a^2*c^2*d*e + 2*b^3*(c*d^2 + a*e^2) + 6*a*b*c*(c*d^2 + a*e^2))*x^6)
/3 + ((8*b^3*c*d*e + 24*a*b*c^2*d*e + b^4*e^2 + 6*b^2*c*(c*d^2 + 2*a*e^2) +
 2*a*c^2*(2*c*d^2 + 3*a*e^2))*x^7)/7 + (c*(3*b^2*c*d*e + 2*a*c^2*d*e + b^3*
e^2 + b*c*(c*d^2 + 3*a*e^2))*x^8)/2 + (c^2*(c^2*d^2 + 6*b^2*e^2 + 4*c*e*(2*
b*d + a*e))*x^9)/9 + (c^3*e*(c*d + 2*b*e)*x^10)/5 + (c^4*e^2*x^11)/11
```

Maple [A] time = 0.041, size = 545, normalized size = 1.2

$$\frac{c^4e^2x^{11}}{11}+\frac{\left(4e^2bc^3+2dec^4\right)x^{10}}{10}+\frac{\left(d^2c^4+8debc^3+e^2\left(2\left(2ac+b^2\right)c^2+4b^2c^2\right)\right)x^9}{9}+\frac{\left(4d^2bc^3+2de\left(2\left(2ac+b^2\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+b*x+a)^4,x)
```

```
[Out] 1/11*c^4*e^2*x^11+1/10*(4*b*c^3*e^2+2*c^4*d*e)*x^10+1/9*(d^2*c^4+8*d*e*b*c^
3+e^2*(2*(2*a*c+b^2)*c^2+4*b^2*c^2))*x^9+1/8*(4*d^2*b*c^3+2*d*e*(2*(2*a*c+b
^2)*c^2+4*b^2*c^2)+e^2*(4*b*a*c^2+4*(2*a*c+b^2)*b*c))*x^8+1/7*(d^2*(2*(2*a*
c+b^2)*c^2+4*b^2*c^2)+2*d*e*(4*b*a*c^2+4*(2*a*c+b^2)*b*c)+e^2*(2*a^2*c^2+8*
a*c*b^2+(2*a*c+b^2)^2))*x^7+1/6*(d^2*(4*b*a*c^2+4*(2*a*c+b^2)*b*c)+2*d*e*(2
*a^2*c^2+8*a*c*b^2+(2*a*c+b^2)^2)+e^2*(4*a^2*b*c+4*a*b*(2*a*c+b^2)))*x^6+1/
5*(d^2*(2*a^2*c^2+8*a*c*b^2+(2*a*c+b^2)^2)+2*d*e*(4*a^2*b*c+4*a*b*(2*a*c+b^
2))+e^2*(2*a^2*(2*a*c+b^2)+4*b^2*a^2))*x^5+1/4*(d^2*(4*a^2*b*c+4*a*b*(2*a*c
+b^2))+2*d*e*(2*a^2*(2*a*c+b^2)+4*b^2*a^2)+4*e^2*a^3*b)*x^4+1/3*(d^2*(2*a^2
*(2*a*c+b^2)+4*b^2*a^2)+8*d*e*a^3*b+e^2*a^4)*x^3+1/2*(2*a^4*d*e+4*a^3*b*d^2
)*x^2+a^4*d^2*x
```

Maxima [A] time = 0.967854, size = 589, normalized size = 1.34

$$\frac{1}{11}c^4e^2x^{11}+\frac{1}{5}\left(c^4de+2bc^3e^2\right)x^{10}+\frac{1}{9}\left(c^4d^2+8bc^3de+2\left(3b^2c^2+2ac^3\right)e^2\right)x^9+\frac{1}{2}\left(bc^3d^2+\left(3b^2c^2+2ac^3\right)de+\left(b^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/11*c^4*e^2*x^11 + 1/5*(c^4*d*e + 2*b*c^3*e^2)*x^10 + 1/9*(c^4*d^2 + 8*b*c
^3*d*e + 2*(3*b^2*c^2 + 2*a*c^3)*e^2)*x^9 + 1/2*(b*c^3*d^2 + (3*b^2*c^2 + 2
*a*c^3)*d*e + (b^3*c + 3*a*b*c^2)*e^2)*x^8 + 1/7*(2*(3*b^2*c^2 + 2*a*c^3)*d
^2 + 8*(b^3*c + 3*a*b*c^2)*d*e + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*e^2)*x^7 +
a^4*d^2*x + 1/3*(2*(b^3*c + 3*a*b*c^2)*d^2 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)
*d*e + 2*(a*b^3 + 3*a^2*b*c)*e^2)*x^6 + 1/5*((b^4 + 12*a*b^2*c + 6*a^2*c^2)
*d^2 + 8*(a*b^3 + 3*a^2*b*c)*d*e + 2*(3*a^2*b^2 + 2*a^3*c)*e^2)*x^5 + (a^3*
b*e^2 + (a*b^3 + 3*a^2*b*c)*d^2 + (3*a^2*b^2 + 2*a^3*c)*d*e)*x^4 + 1/3*(8*a
^3*b*d*e + a^4*e^2 + 2*(3*a^2*b^2 + 2*a^3*c)*d^2)*x^3 + (2*a^3*b*d^2 + a^4*
d*e)*x^2
```

Fricas [A] time = 1.74155, size = 1195, normalized size = 2.71

$$\frac{1}{11}x^{11}e^2c^4 + \frac{1}{5}x^{10}edc^4 + \frac{2}{5}x^{10}e^2c^3b + \frac{1}{9}x^9d^2c^4 + \frac{8}{9}x^9edc^3b + \frac{2}{3}x^9e^2c^2b^2 + \frac{4}{9}x^9e^2c^3a + \frac{1}{2}x^8d^2c^3b + \frac{3}{2}x^8edc^2b^2 + \frac{1}{2}x^8e^2cb^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*e^2*c^4 + 1/5*x^10*e*d*c^4 + 2/5*x^10*e^2*c^3*b + 1/9*x^9*d^2*c^4
 + 8/9*x^9*e*d*c^3*b + 2/3*x^9*e^2*c^2*b^2 + 4/9*x^9*e^2*c^3*a + 1/2*x^8*d^
2*c^3*b + 3/2*x^8*e*d*c^2*b^2 + 1/2*x^8*e^2*c*b^3 + x^8*e*d*c^3*a + 3/2*x^8
*e^2*c^2*b*a + 6/7*x^7*d^2*c^2*b^2 + 8/7*x^7*e*d*c*b^3 + 1/7*x^7*e^2*b^4 +
4/7*x^7*d^2*c^3*a + 24/7*x^7*e*d*c^2*b*a + 12/7*x^7*e^2*c*b^2*a + 6/7*x^7*e
^2*c^2*a^2 + 2/3*x^6*d^2*c*b^3 + 1/3*x^6*e*d*b^4 + 2*x^6*d^2*c^2*b*a + 4*x^
6*e*d*c*b^2*a + 2/3*x^6*e^2*b^3*a + 2*x^6*e*d*c^2*a^2 + 2*x^6*e^2*c*b*a^2 +
 1/5*x^5*d^2*b^4 + 12/5*x^5*d^2*c*b^2*a + 8/5*x^5*e*d*b^3*a + 6/5*x^5*d^2*c
^2*a^2 + 24/5*x^5*e*d*c*b*a^2 + 6/5*x^5*e^2*b^2*a^2 + 4/5*x^5*e^2*c*a^3 + x
^4*d^2*b^3*a + 3*x^4*d^2*c*b*a^2 + 3*x^4*e*d*b^2*a^2 + 2*x^4*e*d*c*a^3 + x^
4*e^2*b*a^3 + 2*x^3*d^2*b^2*a^2 + 4/3*x^3*d^2*c*a^3 + 8/3*x^3*e*d*b*a^3 + 1
/3*x^3*e^2*a^4 + 2*x^2*d^2*b*a^3 + x^2*e*d*a^4 + x*d^2*a^4
```

Sympy [A] time = 0.133133, size = 537, normalized size = 1.22

$$a^4d^2x + \frac{c^4e^2x^{11}}{11} + x^{10}\left(\frac{2bc^3e^2}{5} + \frac{c^4de}{5}\right) + x^9\left(\frac{4ac^3e^2}{9} + \frac{2b^2c^2e^2}{3} + \frac{8bc^3de}{9} + \frac{c^4d^2}{9}\right) + x^8\left(\frac{3abc^2e^2}{2} + ac^3de + \frac{b^3ce^2}{2} + \frac{3b^2c}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+b*x+a)**4,x)
```

```
[Out] a**4*d**2*x + c**4*e**2*x**11/11 + x**10*(2*b*c**3*e**2/5 + c**4*d*e/5) + x
**9*(4*a*c**3*e**2/9 + 2*b**2*c**2*e**2/3 + 8*b*c**3*d*e/9 + c**4*d**2/9) +
 x**8*(3*a*b*c**2*e**2/2 + a*c**3*d*e + b**3*c*e**2/2 + 3*b**2*c**2*d*e/2 +
 b*c**3*d**2/2) + x**7*(6*a**2*c**2*e**2/7 + 12*a*b**2*c*e**2/7 + 24*a*b*c*
*2*d*e/7 + 4*a*c**3*d**2/7 + b**4*e**2/7 + 8*b**3*c*d*e/7 + 6*b**2*c**2*d**
2/7) + x**6*(2*a**2*b*c*e**2 + 2*a**2*c**2*d*e + 2*a*b**3*e**2/3 + 4*a*b**2
*c*d*e + 2*a*b*c**2*d**2 + b**4*d*e/3 + 2*b**3*c*d**2/3) + x**5*(4*a**3*c*e
**2/5 + 6*a**2*b**2*e**2/5 + 24*a**2*b*c*d*e/5 + 6*a**2*c**2*d**2/5 + 8*a*b
**3*d*e/5 + 12*a*b**2*c*d**2/5 + b**4*d**2/5) + x**4*(a**3*b*e**2 + 2*a**3*
c*d*e + 3*a**2*b**2*d*e + 3*a**2*b*c*d**2 + a*b**3*d**2) + x**3*(a**4*e**2/
```

```
3 + 8*a**3*b*d*e/3 + 4*a**3*c*d**2/3 + 2*a**2*b**2*d**2) + x**2*(a**4*d*e +
 2*a**3*b*d**2)
```

Giac [A] time = 1.09229, size = 725, normalized size = 1.64

$$\frac{1}{11}c^4x^{11}e^2 + \frac{1}{5}c^4dx^{10}e + \frac{1}{9}c^4d^2x^9 + \frac{2}{5}bc^3x^{10}e^2 + \frac{8}{9}bc^3dx^9e + \frac{1}{2}bc^3d^2x^8 + \frac{2}{3}b^2c^2x^9e^2 + \frac{4}{9}ac^3x^9e^2 + \frac{3}{2}b^2c^2dx^8e + ac^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/11*c^4*x^11*e^2 + 1/5*c^4*d*x^10*e + 1/9*c^4*d^2*x^9 + 2/5*b*c^3*x^10*e^2
 + 8/9*b*c^3*d*x^9*e + 1/2*b*c^3*d^2*x^8 + 2/3*b^2*c^2*x^9*e^2 + 4/9*a*c^3*
x^9*e^2 + 3/2*b^2*c^2*d*x^8*e + a*c^3*d*x^8*e + 6/7*b^2*c^2*d^2*x^7 + 4/7*a
*c^3*d^2*x^7 + 1/2*b^3*c*x^8*e^2 + 3/2*a*b*c^2*x^8*e^2 + 8/7*b^3*c*d*x^7*e
+ 24/7*a*b*c^2*d*x^7*e + 2/3*b^3*c*d^2*x^6 + 2*a*b*c^2*d^2*x^6 + 1/7*b^4*x^
7*e^2 + 12/7*a*b^2*c*x^7*e^2 + 6/7*a^2*c^2*x^7*e^2 + 1/3*b^4*d*x^6*e + 4*a*
b^2*c*d*x^6*e + 2*a^2*c^2*d*x^6*e + 1/5*b^4*d^2*x^5 + 12/5*a*b^2*c*d^2*x^5
+ 6/5*a^2*c^2*d^2*x^5 + 2/3*a*b^3*x^6*e^2 + 2*a^2*b*c*x^6*e^2 + 8/5*a*b^3*d
*x^5*e + 24/5*a^2*b*c*d*x^5*e + a*b^3*d^2*x^4 + 3*a^2*b*c*d^2*x^4 + 6/5*a^2
*b^2*x^5*e^2 + 4/5*a^3*c*x^5*e^2 + 3*a^2*b^2*d*x^4*e + 2*a^3*c*d*x^4*e + 2*
a^2*b^2*d^2*x^3 + 4/3*a^3*c*d^2*x^3 + a^3*b*x^4*e^2 + 8/3*a^3*b*d*x^3*e + 2
*a^3*b*d^2*x^2 + 1/3*a^4*x^3*e^2 + a^4*d*x^2*e + a^4*d^2*x
```

3.2149 $\int (d+ex)\left(a+bx+cx^2\right)^4 dx$

Optimal. Leaf size=268

$$\frac{1}{6}x^6\left(6a^2c^2e+12ab^2ce+12abc^2d+4b^3cd+b^4e\right)+\frac{1}{5}x^5\left(12a^2bce+6a^2c^2d+12ab^2cd+4ab^3e+b^4d\right)+\frac{1}{2}ax^4\left(2a^2ce+3ab\right.$$

```
[Out] a^4*d*x + (a^3*(4*b*d + a*e)*x^2)/2 + (2*a^2*(3*b^2*d + 2*a*c*d + 2*a*b*e)*
x^3)/3 + (a*(2*b^3*d + 6*a*b*c*d + 3*a*b^2*e + 2*a^2*c*e)*x^4)/2 + ((b^4*d
+ 12*a*b^2*c*d + 6*a^2*c^2*d + 4*a*b^3*e + 12*a^2*b*c*e)*x^5)/5 + ((4*b^3*c
*d + 12*a*b*c^2*d + b^4*e + 12*a*b^2*c*e + 6*a^2*c^2*e)*x^6)/6 + (2*c*(3*b^
2*c*d + 2*a*c^2*d + 2*b^3*e + 6*a*b*c*e)*x^7)/7 + (c^2*(2*b*c*d + 3*b^2*e +
 2*a*c*e)*x^8)/4 + (c^3*(c*d + 4*b*e)*x^9)/9 + (c^4*e*x^10)/10
```

Rubi [A] time = 0.290969, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {631}

$$\frac{1}{6}x^6\left(6a^2c^2e+12ab^2ce+12abc^2d+4b^3cd+b^4e\right)+\frac{1}{5}x^5\left(12a^2bce+6a^2c^2d+12ab^2cd+4ab^3e+b^4d\right)+\frac{1}{2}ax^4\left(2a^2ce+3ab\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*x + c*x^2)^4,x]
```


```
[Out] a^4*d*x + (a^3*(4*b*d + a*e)*x^2)/2 + (2*a^2*(3*b^2*d + 2*a*c*d + 2*a*b*e)*
x^3)/3 + (a*(2*b^3*d + 6*a*b*c*d + 3*a*b^2*e + 2*a^2*c*e)*x^4)/2 + ((b^4*d
+ 12*a*b^2*c*d + 6*a^2*c^2*d + 4*a*b^3*e + 12*a^2*b*c*e)*x^5)/5 + ((4*b^3*c
*d + 12*a*b*c^2*d + b^4*e + 12*a*b^2*c*e + 6*a^2*c^2*e)*x^6)/6 + (2*c*(3*b^
2*c*d + 2*a*c^2*d + 2*b^3*e + 6*a*b*c*e)*x^7)/7 + (c^2*(2*b*c*d + 3*b^2*e +
 2*a*c*e)*x^8)/4 + (c^3*(c*d + 4*b*e)*x^9)/9 + (c^4*e*x^10)/10
```


Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\int (d+ex)\left(a+bx+cx^2\right)^4 dx = \int \left(a^4d+a^3(4bd+ae)x+2a^2\left(3b^2d+2acd+2abe\right)x^2+2a\left(2b^3d+6abcd+3ab^2e+2a\right.\right.$$

$$= a^4dx+\frac{1}{2}a^3(4bd+ae)x^2+\frac{2}{3}a^2\left(3b^2d+2acd+2abe\right)x^3+\frac{1}{2}a\left(2b^3d+6abcd+3ab^2e+2\right.$$

Mathematica [A] time = 0.0694483, size = 268, normalized size = 1.

$$\frac{1}{6}x^6\left(6a^2c^2e+12ab^2ce+12abc^2d+4b^3cd+b^4e\right)+\frac{1}{5}x^5\left(12a^2bce+6a^2c^2d+12ab^2cd+4ab^3e+b^4d\right)+\frac{1}{2}ax^4\left(2a^2ce+3ab\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*x + c*x^2)^4,x]
```

```
[Out] a^4*d*x + (a^3*(4*b*d + a*e)*x^2)/2 + (2*a^2*(3*b^2*d + 2*a*c*d + 2*a*b*e)*
x^3)/3 + (a*(2*b^3*d + 6*a*b*c*d + 3*a*b^2*e + 2*a^2*c*e)*x^4)/2 + ((b^4*d
+ 12*a*b^2*c*d + 6*a^2*c^2*d + 4*a*b^3*e + 12*a^2*b*c*e)*x^5)/5 + ((4*b^3*c
*d + 12*a*b*c^2*d + b^4*e + 12*a*b^2*c*e + 6*a^2*c^2*e)*x^6)/6 + (2*c*(3*b^
2*c*d + 2*a*c^2*d + 2*b^3*e + 6*a*b*c*e)*x^7)/7 + (c^2*(2*b*c*d + 3*b^2*e +
 2*a*c*e)*x^8)/4 + (c^3*(c*d + 4*b*e)*x^9)/9 + (c^4*e*x^10)/10
```

Maple [A] time = 0.038, size = 343, normalized size = 1.3

$$\frac{c^4ex^{10}}{10}+\frac{\left(4\,ebc^3+dc^4\right)x^9}{9}+\frac{\left(4\,dbc^3+e\left(2\,\left(2\,ac+b^2\right)c^2+4\,b^2c^2\right)\right)x^8}{8}+\frac{\left(d\left(2\,\left(2\,ac+b^2\right)c^2+4\,b^2c^2\right)+e\left(4\,bac^2+4\right.\right.}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+b*x+a)^4,x)
```

```
[Out] 1/10*c^4*e*x^10+1/9*(4*b*c^3*e+c^4*d)*x^9+1/8*(4*d*b*c^3+e*(2*(2*a*c+b^2)*c
^2+4*b^2*c^2))*x^8+1/7*(d*(2*(2*a*c+b^2)*c^2+4*b^2*c^2)+e*(4*b*a*c^2+4*(2*a
*c+b^2)*b*c))*x^7+1/6*(d*(4*b*a*c^2+4*(2*a*c+b^2)*b*c)+e*(2*a^2*c^2+8*a*c*b
^2+(2*a*c+b^2)^2))*x^6+1/5*(d*(2*a^2*c^2+8*a*c*b^2+(2*a*c+b^2)^2)+e*(4*a^2*
b*c+4*a*b*(2*a*c+b^2)))*x^5+1/4*(d*(4*a^2*b*c+4*a*b*(2*a*c+b^2))+e*(2*a^2*(
2*a*c+b^2)+4*b^2*a^2))*x^4+1/3*(d*(2*a^2*(2*a*c+b^2)+4*b^2*a^2)+4*e*a^3*b)*
x^3+1/2*(a^4*e+4*a^3*b*d)*x^2+a^4*d*x
```

Maxima [A] time = 1.01288, size = 373, normalized size = 1.39

$$\frac{1}{10}\,c^4ex^{10}+\frac{1}{9}\left(c^4d+4\,bc^3e\right)x^9+\frac{1}{4}\left(2\,bc^3d+\left(3\,b^2c^2+2\,ac^3\right)e\right)x^8+\frac{2}{7}\left(\left(3\,b^2c^2+2\,ac^3\right)d+2\left(b^3c+3\,abc^2\right)e\right)x^7+\frac{1}{6}\left(4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/10*c^4*e*x^10 + 1/9*(c^4*d + 4*b*c^3*e)*x^9 + 1/4*(2*b*c^3*d + (3*b^2*c^2
 + 2*a*c^3)*e)*x^8 + 2/7*((3*b^2*c^2 + 2*a*c^3)*d + 2*(b^3*c + 3*a*b*c^2)*e
)*x^7 + 1/6*(4*(b^3*c + 3*a*b*c^2)*d + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*e)*x^
6 + a^4*d*x + 1/5*((b^4 + 12*a*b^2*c + 6*a^2*c^2)*d + 4*(a*b^3 + 3*a^2*b*c)
*e)*x^5 + 1/2*(2*(a*b^3 + 3*a^2*b*c)*d + (3*a^2*b^2 + 2*a^3*c)*e)*x^4 + 2/3
*(2*a^3*b*e + (3*a^2*b^2 + 2*a^3*c)*d)*x^3 + 1/2*(4*a^3*b*d + a^4*e)*x^2
```

Fricas [A] time = 1.75107, size = 713, normalized size = 2.66

$$\frac{1}{10}x^{10}ec^4+\frac{1}{9}x^9dc^4+\frac{4}{9}x^9ec^3b+\frac{1}{2}x^8dc^3b+\frac{3}{4}x^8ec^2b^2+\frac{1}{2}x^8ec^3a+\frac{6}{7}x^7dc^2b^2+\frac{4}{7}x^7ecb^3+\frac{4}{7}x^7dc^3a+\frac{12}{7}x^7ec^2ba+\frac{2}{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*e*c^4 + 1/9*x^9*d*c^4 + 4/9*x^9*e*c^3*b + 1/2*x^8*d*c^3*b + 3/4*x
^8*e*c^2*b^2 + 1/2*x^8*e*c^3*a + 6/7*x^7*d*c^2*b^2 + 4/7*x^7*e*c*b^3 + 4/7*
x^7*d*c^3*a + 12/7*x^7*e*c^2*b*a + 2/3*x^6*d*c*b^3 + 1/6*x^6*e*b^4 + 2*x^6*
```

```
d*c^2*b*a + 2*x^6*e*c*b^2*a + x^6*e*c^2*a^2 + 1/5*x^5*d*b^4 + 12/5*x^5*d*c*
b^2*a + 4/5*x^5*e*b^3*a + 6/5*x^5*d*c^2*a^2 + 12/5*x^5*e*c*b*a^2 + x^4*d*b^
3*a + 3*x^4*d*c*b*a^2 + 3/2*x^4*e*b^2*a^2 + x^4*e*c*a^3 + 2*x^3*d*b^2*a^2 +
 4/3*x^3*d*c*a^3 + 4/3*x^3*e*b*a^3 + 2*x^2*d*b*a^3 + 1/2*x^2*e*a^4 + x*d*a^
4
```

Sympy [A] time = 0.113585, size = 313, normalized size = 1.17

$$a^4dx + \frac{c^4ex^{10}}{10} + x^9\left(\frac{4bc^3e}{9} + \frac{c^4d}{9}\right) + x^8\left(\frac{ac^3e}{2} + \frac{3b^2c^2e}{4} + \frac{bc^3d}{2}\right) + x^7\left(\frac{12abc^2e}{7} + \frac{4ac^3d}{7} + \frac{4b^3ce}{7} + \frac{6b^2c^2d}{7}\right) + x^6\left(a^2c^2e +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+b*x+a)**4,x)
```

```
[Out] a**4*d*x + c**4*e*x**10/10 + x**9*(4*b*c**3*e/9 + c**4*d/9) + x**8*(a*c**3*
e/2 + 3*b**2*c**2*e/4 + b*c**3*d/2) + x**7*(12*a*b*c**2*e/7 + 4*a*c**3*d/7
+ 4*b**3*c*e/7 + 6*b**2*c**2*d/7) + x**6*(a**2*c**2*e + 2*a*b**2*c*e + 2*a*
b*c**2*d + b**4*e/6 + 2*b**3*c*d/3) + x**5*(12*a**2*b*c*e/5 + 6*a**2*c**2*d
/5 + 4*a*b**3*e/5 + 12*a*b**2*c*d/5 + b**4*d/5) + x**4*(a**3*c*e + 3*a**2*b
**2*e/2 + 3*a**2*b*c*d + a*b**3*d) + x**3*(4*a**3*b*e/3 + 4*a**3*c*d/3 + 2*
a**2*b**2*d) + x**2*(a**4*e/2 + 2*a**3*b*d)
```

Giac [A] time = 1.09179, size = 435, normalized size = 1.62

$$\frac{1}{10}c^4x^{10}e + \frac{1}{9}c^4dx^9 + \frac{4}{9}bc^3x^9e + \frac{1}{2}bc^3dx^8 + \frac{3}{4}b^2c^2x^8e + \frac{1}{2}ac^3x^8e + \frac{6}{7}b^2c^2dx^7 + \frac{4}{7}ac^3dx^7 + \frac{4}{7}b^3cx^7e + \frac{12}{7}abc^2x^7e + \frac{2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/10*c^4*x^10*e + 1/9*c^4*d*x^9 + 4/9*b*c^3*x^9*e + 1/2*b*c^3*d*x^8 + 3/4*b
^2*c^2*x^8*e + 1/2*a*c^3*x^8*e + 6/7*b^2*c^2*d*x^7 + 4/7*a*c^3*d*x^7 + 4/7*
b^3*c*x^7*e + 12/7*a*b*c^2*x^7*e + 2/3*b^3*c*d*x^6 + 2*a*b*c^2*d*x^6 + 1/6*
b^4*x^6*e + 2*a*b^2*c*x^6*e + a^2*c^2*x^6*e + 1/5*b^4*d*x^5 + 12/5*a*b^2*c*
d*x^5 + 6/5*a^2*c^2*d*x^5 + 4/5*a*b^3*x^5*e + 12/5*a^2*b*c*x^5*e + a*b^3*d*
x^4 + 3*a^2*b*c*d*x^4 + 3/2*a^2*b^2*x^4*e + a^3*c*x^4*e + 2*a^2*b^2*d*x^3 +
 4/3*a^3*c*d*x^3 + 4/3*a^3*b*x^3*e + 2*a^3*b*d*x^2 + 1/2*a^4*x^2*e + a^4*d*
x
```

3.2150 $\int \left(a+bx+cx^2\right)^4 dx$

Optimal. Leaf size=133

$$\frac{1}{5}x^5\left(6a^2c^2+12ab^2c+b^4\right)+\frac{2}{3}a^2x^3\left(2ac+3b^2\right)+2a^3bx^2+a^4x+\frac{2}{7}c^2x^7\left(2ac+3b^2\right)+\frac{2}{3}bcx^6\left(3ac+b^2\right)+abx^4\left(3ac+\right.$$

```
[Out] a^4*x + 2*a^3*b*x^2 + (2*a^2*(3*b^2 + 2*a*c)*x^3)/3 + a*b*(b^2 + 3*a*c)*x^4
 + ((b^4 + 12*a*b^2*c + 6*a^2*c^2)*x^5)/5 + (2*b*c*(b^2 + 3*a*c)*x^6)/3 + (
2*c^2*(3*b^2 + 2*a*c)*x^7)/7 + (b*c^3*x^8)/2 + (c^4*x^9)/9
```

Rubi [A] time = 0.116593, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {611}

$$\frac{1}{5}x^5\left(6a^2c^2+12ab^2c+b^4\right)+\frac{2}{3}a^2x^3\left(2ac+3b^2\right)+2a^3bx^2+a^4x+\frac{2}{7}c^2x^7\left(2ac+3b^2\right)+\frac{2}{3}bcx^6\left(3ac+b^2\right)+abx^4\left(3ac+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^4,x]
```

```
[Out] a^4*x + 2*a^3*b*x^2 + (2*a^2*(3*b^2 + 2*a*c)*x^3)/3 + a*b*(b^2 + 3*a*c)*x^4
 + ((b^4 + 12*a*b^2*c + 6*a^2*c^2)*x^5)/5 + (2*b*c*(b^2 + 3*a*c)*x^6)/3 + (
2*c^2*(3*b^2 + 2*a*c)*x^7)/7 + (b*c^3*x^8)/2 + (c^4*x^9)/9
```

Rule 611

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && IGtQ[p, 0] && (EqQ[a, 0] ||  !PerfectSquareQ[b^2 - 4*a*c])
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^4 dx=\int \left(a^4+4a^3bx+6a^2b^2\left(1+\frac{2ac}{3b^2}\right)x^2+4ab^3\left(1+\frac{3ac}{b^2}\right)x^3+b^4\left(1+\frac{6ac\left(2b^2+ac\right)}{b^4}\right)x^4+4b^3c\right.$$

$$=a^4x+2a^3bx^2+\frac{2}{3}a^2\left(3b^2+2ac\right)x^3+ab\left(b^2+3ac\right)x^4+\frac{1}{5}\left(b^4+12ab^2c+6a^2c^2\right)x^5+\frac{2}{3}bc\left(b^2\right.$$

Mathematica [A] time = 0.0191627, size = 133, normalized size = 1.

$$\frac{1}{5}x^5\left(6a^2c^2+12ab^2c+b^4\right)+\frac{2}{3}a^2x^3\left(2ac+3b^2\right)+2a^3bx^2+a^4x+\frac{2}{7}c^2x^7\left(2ac+3b^2\right)+\frac{2}{3}bcx^6\left(3ac+b^2\right)+abx^4\left(3ac+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^4,x]
```

```
[Out] a^4*x + 2*a^3*b*x^2 + (2*a^2*(3*b^2 + 2*a*c)*x^3)/3 + a*b*(b^2 + 3*a*c)*x^4
 + ((b^4 + 12*a*b^2*c + 6*a^2*c^2)*x^5)/5 + (2*b*c*(b^2 + 3*a*c)*x^6)/3 + (
2*c^2*(3*b^2 + 2*a*c)*x^7)/7 + (b*c^3*x^8)/2 + (c^4*x^9)/9
```

Maple [A] time = 0.041, size = 168, normalized size = 1.3

$$\frac{c^4x^9}{9}+\frac{bc^3x^8}{2}+\frac{\left(2\left(2ac+b^2\right)c^2+4b^2c^2\right)x^7}{7}+\frac{\left(4bac^2+4\left(2ac+b^2\right)bc\right)x^6}{6}+\frac{\left(2a^2c^2+8acb^2+\left(2ac+b^2\right)^2\right)x^5}{5}+\frac{(4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^4,x)
```

```
[Out] 1/9*c^4*x^9+1/2*b*c^3*x^8+1/7*(2*(2*a*c+b^2)*c^2+4*b^2*c^2)*x^7+1/6*(4*b*a*
c^2+4*(2*a*c+b^2)*b*c)*x^6+1/5*(2*a^2*c^2+8*a*c*b^2+(2*a*c+b^2)^2)*x^5+1/4*
(4*a^2*b*c+4*a*b*(2*a*c+b^2))*x^4+1/3*(2*a^2*(2*a*c+b^2)+4*b^2*a^2)*x^3+2*a
^3*b*x^2+a^4*x
```

Maxima [A] time = 0.998654, size = 184, normalized size = 1.38

$$\frac{1}{9}c^4x^9+\frac{1}{2}bc^3x^8+\frac{6}{7}b^2c^2x^7+\frac{2}{3}b^3cx^6+\frac{1}{5}b^4x^5+a^4x+\frac{2}{3}\left(2cx^3+3bx^2\right)a^3+\frac{1}{5}\left(6c^2x^5+15bcx^4+10b^2x^3\right)a^2+\frac{1}{35}(20$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/9*c^4*x^9 + 1/2*b*c^3*x^8 + 6/7*b^2*c^2*x^7 + 2/3*b^3*c*x^6 + 1/5*b^4*x^5
 + a^4*x + 2/3*(2*c*x^3 + 3*b*x^2)*a^3 + 1/5*(6*c^2*x^5 + 15*b*c*x^4 + 10*b
^2*x^3)*a^2 + 1/35*(20*c^3*x^7 + 70*b*c^2*x^6 + 84*b^2*c*x^5 + 35*b^3*x^4)*
a
```

Fricas [A] time = 1.51721, size = 308, normalized size = 2.32

$$\frac{1}{9}x^9c^4+\frac{1}{2}x^8c^3b+\frac{6}{7}x^7c^2b^2+\frac{4}{7}x^7c^3a+\frac{2}{3}x^6cb^3+2x^6c^2ba+\frac{1}{5}x^5b^4+\frac{12}{5}x^5cb^2a+\frac{6}{5}x^5c^2a^2+x^4b^3a+3x^4cba^2+2x^3b^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*c^4 + 1/2*x^8*c^3*b + 6/7*x^7*c^2*b^2 + 4/7*x^7*c^3*a + 2/3*x^6*c*b
^3 + 2*x^6*c^2*b*a + 1/5*x^5*b^4 + 12/5*x^5*c*b^2*a + 6/5*x^5*c^2*a^2 + x^4
*b^3*a + 3*x^4*c*b*a^2 + 2*x^3*b^2*a^2 + 4/3*x^3*c*a^3 + 2*x^2*b*a^3 + x*a^
4
```

Sympy [A] time = 0.082604, size = 141, normalized size = 1.06

$$a^4x+2a^3bx^2+\frac{bc^3x^8}{2}+\frac{c^4x^9}{9}+x^7\left(\frac{4ac^3}{7}+\frac{6b^2c^2}{7}\right)+x^6\left(2abc^2+\frac{2b^3c}{3}\right)+x^5\left(\frac{6a^2c^2}{5}+\frac{12ab^2c}{5}+\frac{b^4}{5}\right)+x^4\left(3a^2bc+ab^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**4,x)
```

```
[Out] a**4*x + 2*a**3*b*x**2 + b*c**3*x**8/2 + c**4*x**9/9 + x**7*(4*a*c**3/7 + 6
*b**2*c**2/7) + x**6*(2*a*b*c**2 + 2*b**3*c/3) + x**5*(6*a**2*c**2/5 + 12*a
*b**2*c/5 + b**4/5) + x**4*(3*a**2*b*c + a*b**3) + x**3*(4*a**3*c/3 + 2*a**
2*b**2)
```

Giac [A] time = 1.08395, size = 186, normalized size = 1.4

$$\frac{1}{9}c^4x^9 + \frac{1}{2}bc^3x^8 + \frac{6}{7}b^2c^2x^7 + \frac{4}{7}ac^3x^7 + \frac{2}{3}b^3cx^6 + 2\,abc^2x^6 + \frac{1}{5}b^4x^5 + \frac{12}{5}ab^2cx^5 + \frac{6}{5}a^2c^2x^5 + ab^3x^4 + 3\,a^2bcx^4 + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/9*c^4*x^9 + 1/2*b*c^3*x^8 + 6/7*b^2*c^2*x^7 + 4/7*a*c^3*x^7 + 2/3*b^3*c*x
^6 + 2*a*b*c^2*x^6 + 1/5*b^4*x^5 + 12/5*a*b^2*c*x^5 + 6/5*a^2*c^2*x^5 + a*b
^3*x^4 + 3*a^2*b*c*x^4 + 2*a^2*b^2*x^3 + 4/3*a^3*c*x^3 + 2*a^3*b*x^2 + a^4*
x
```

3.2151 $\int \frac{\left(a+bx+cx^2\right)^4}{d+ex}\,dx$

Optimal. Leaf size=428

$$\frac{(d+ex)^4\left(6c^2e^2\left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4\right)}{4e^9}+\frac{c^2(d+ex)^6\left(-2cc\right.}{}$$

```
[Out] (-4*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)^3*x)/e^8 + ((c*d^2 - b*d*e + a*e^
2)^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e))*(d + e*x)^2)/e^9 - (4*(
2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*
a*e))*(d + e*x)^3)/(3*e^9) + ((70*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(5*b*d -
3*a*e) - 20*c^3*d^2*e*(7*b*d - 3*a*e) + 6*c^2*e^2*(15*b^2*d^2 - 10*a*b*d*e
+ a^2*e^2))*(d + e*x)^4)/(4*e^9) - (4*c*(2*c*d - b*e)*(7*c^2*d^2 + b^2*e^2
- c*e*(7*b*d - 3*a*e))*(d + e*x)^5)/(5*e^9) + (c^2*(14*c^2*d^2 + 3*b^2*e^2
- 2*c*e*(7*b*d - a*e))*(d + e*x)^6)/(3*e^9) - (4*c^3*(2*c*d - b*e)*(d + e*x
)^7)/(7*e^9) + (c^4*(d + e*x)^8)/(8*e^9) + ((c*d^2 - b*d*e + a*e^2)^4*Log[d
 + e*x])/e^9
```

Rubi [A] time = 0.708334, antiderivative size = 428, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$\frac{(d+ex)^4\left(6c^2e^2\left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4\right)}{4e^9}+\frac{c^2(d+ex)^6\left(-2cc\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^4/(d + e*x),x]
```

```
[Out] (-4*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)^3*x)/e^8 + ((c*d^2 - b*d*e + a*e^
2)^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e))*(d + e*x)^2)/e^9 - (4*(
2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*
a*e))*(d + e*x)^3)/(3*e^9) + ((70*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(5*b*d -
3*a*e) - 20*c^3*d^2*e*(7*b*d - 3*a*e) + 6*c^2*e^2*(15*b^2*d^2 - 10*a*b*d*e
+ a^2*e^2))*(d + e*x)^4)/(4*e^9) - (4*c*(2*c*d - b*e)*(7*c^2*d^2 + b^2*e^2
- c*e*(7*b*d - 3*a*e))*(d + e*x)^5)/(5*e^9) + (c^2*(14*c^2*d^2 + 3*b^2*e^2
- 2*c*e*(7*b*d - a*e))*(d + e*x)^6)/(3*e^9) - (4*c^3*(2*c*d - b*e)*(d + e*x
)^7)/(7*e^9) + (c^4*(d + e*x)^8)/(8*e^9) + ((c*d^2 - b*d*e + a*e^2)^4*Log[d
 + e*x])/e^9
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^4}{d+ex}\,dx = \int \left(\frac{4(-2cd+be)\left(cd^2-bde+ae^2\right)^3}{e^8} + \frac{\left(cd^2-bde+ae^2\right)^4}{e^8(d+ex)} + \frac{2\left(cd^2-bde+ae^2\right)^2\left(14c^2d^2+3b\right.}{e^8}\right.$$

$$= -\frac{4(2cd-be)\left(cd^2-bde+ae^2\right)^3 x}{e^8} + \frac{\left(cd^2-bde+ae^2\right)^2\left(14c^2d^2+3b^2e^2-2ce(7bd-ae)\right)(d+}{e^9}$$

Mathematica [A] time = 0.404188, size = 616, normalized size = 1.44

$$\frac{x\left(84c^2e^2\left(5a^2e^2\left(6d^2ex-12d^3-4de^2x^2+3e^3x^3\right)+2abe\left(20d^2e^2x^2-30d^3ex+60d^4-15de^3x^3+12e^4x^4\right)+b^2\left(-20d^3\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^4/(d + e*x),x]
```

```
[Out] (x*(c^4*(-840*d^7 + 420*d^6*e*x - 280*d^5*e^2*x^2 + 210*d^4*e^3*x^3 - 168*d
^3*e^4*x^4 + 140*d^2*e^5*x^5 - 120*d*e^6*x^6 + 105*e^7*x^7) + 70*b*e^4*(48*
a^3*e^3 + 36*a^2*b*e^2*(-2*d + e*x) + 8*a*b^2*e*(6*d^2 - 3*d*e*x + 2*e^2*x^
2) + b^3*(-12*d^3 + 6*d^2*e*x - 4*d*e^2*x^2 + 3*e^3*x^3)) + 56*c*e^3*(30*a^
3*e^3*(-2*d + e*x) + 30*a^2*b*e^2*(6*d^2 - 3*d*e*x + 2*e^2*x^2) + 15*a*b^2*
e*(-12*d^3 + 6*d^2*e*x - 4*d*e^2*x^2 + 3*e^3*x^3) + b^3*(60*d^4 - 30*d^3*e*
x + 20*d^2*e^2*x^2 - 15*d*e^3*x^3 + 12*e^4*x^4)) + 84*c^2*e^2*(5*a^2*e^2*(-
12*d^3 + 6*d^2*e*x - 4*d*e^2*x^2 + 3*e^3*x^3) + 2*a*b*e*(60*d^4 - 30*d^3*e*
x + 20*d^2*e^2*x^2 - 15*d*e^3*x^3 + 12*e^4*x^4) + b^2*(-60*d^5 + 30*d^4*e*x
 - 20*d^3*e^2*x^2 + 15*d^2*e^3*x^3 - 12*d*e^4*x^4 + 10*e^5*x^5)) + 8*c^3*e*
(7*a*e*(-60*d^5 + 30*d^4*e*x - 20*d^3*e^2*x^2 + 15*d^2*e^3*x^3 - 12*d*e^4*x
^4 + 10*e^5*x^5) + b*(420*d^6 - 210*d^5*e*x + 140*d^4*e^2*x^2 - 105*d^3*e^3
*x^3 + 84*d^2*e^4*x^4 - 70*d*e^5*x^5 + 60*e^6*x^6))))/(840*e^8) + ((c*d^2 +
 e*(-(b*d) + a*e))^4*Log[d + e*x])/e^9
```

Maple [B] time = 0.048, size = 1096, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^4/(e*x+d),x)
```

```
[Out] 1/e*ln(e*x+d)*a^4+1/4/e*x^4*b^4+1/8/e*c^4*x^8-12/e^4*ln(e*x+d)*a^2*b*c*d^3+
6/e^7*ln(e*x+d)*b^2*c^2*d^6-4/e^8*ln(e*x+d)*b*c^3*d^7-4/e^2*ln(e*x+d)*a^3*b
*d+4/e^3*ln(e*x+d)*a^3*c*d^2+6/e^3*ln(e*x+d)*a^2*b^2*d^2+6/e^5*ln(e*x+d)*a^
2*c^2*d^4-4/e^4*ln(e*x+d)*a*b^3*d^3-4/3/e^4*x^3*a*c^3*d^3-6/5/e^2*x^5*b^2*c
^2*d+4/5/e^3*x^5*b*c^3*d^2+4/e^7*ln(e*x+d)*a*c^3*d^6-4/e^6*ln(e*x+d)*b^3*c*
d^5-3/e^2*x^4*a*b*c^2*d-12/e^4*a*b^2*c*d^3*x+1/e*x^6*b^2*c^2-1/e^4*b^4*d^3*
x+1/e^5*ln(e*x+d)*b^4*d^4+1/e^9*ln(e*x+d)*c^4*d^8-1/7/e^2*x^7*c^4*d+4/7/e*x
^7*b*c^3-1/e^8*c^4*d^7*x+4/3/e*x^3*a*b^3+1/2/e^3*x^2*b^4*d^2+3/e*x^2*a^2*b^
2-1/3/e^6*x^3*c^4*d^5+2/e*x^2*a^3*c-1/3/e^2*x^3*b^4*d+1/2/e^7*x^2*c^4*d^6+4
/e*a^3*b*x-4/e^6*a*c^3*d^5*x+4/e^5*b^3*c*d^4*x+3/2/e*x^4*a^2*c^2-1/5/e^4*x^
5*c^4*d^3+1/4/e^5*x^4*c^4*d^4+1/6/e^3*x^6*c^4*d^2+4/5/e*x^5*b^3*c+2/3/e*x^6
*a*c^3+4/3/e^3*x^3*b^3*c*d^2-2/e^4*x^3*b^2*c^2*d^3+3/e^3*x^2*a^2*c^2*d^2+3/
e^5*x^2*b^2*c^2*d^4-6/e^6*b^2*c^2*d^5*x+4/e^7*b*c^3*d^6*x-2/e^6*x^2*b*c^3*d
^5-4/e^2*a^3*c*d*x+4/e^3*a*b^3*d^2*x-2/e^4*x^2*b^3*c*d^3-2/3/e^2*x^6*b*c^3*
d+12/5/e*x^5*a*b*c^2+4/3/e^5*x^3*b*c^3*d^4+3/e*x^4*a*b^2*c-2/e^2*x^2*a*b^3*
```

```
d+2/e^5*x^2*a*c^3*d^4+4/e*x^3*a^2*b*c-2/e^2*x^3*a^2*c^2*d-1/e^2*x^4*b^3*c*d
+1/e^3*x^4*a*c^3*d^2-6/e^2*a^2*b^2*d*x-6/e^4*a^2*c^2*d^3*x-4/5/e^2*x^5*a*c^
3*d+3/2/e^3*x^4*b^2*c^2*d^2-1/e^4*x^4*b*c^3*d^3+12/e^5*ln(e*x+d)*a*b^2*c*d^
4-12/e^6*ln(e*x+d)*a*b*c^2*d^5+12/e^5*a*b*c^2*d^4*x-6/e^2*x^2*a^2*b*c*d+6/e
^3*x^2*a*b^2*c*d^2-6/e^4*x^2*a*b*c^2*d^3-4/e^2*x^3*a*b^2*c*d+4/e^3*x^3*a*b*
c^2*d^2+12/e^3*a^2*b*c*d^2*x
```

Maxima [A] time = 1.05015, size = 1076, normalized size = 2.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/840*(105*c^4*e^7*x^8 - 120*(c^4*d*e^6 - 4*b*c^3*e^7)*x^7 + 140*(c^4*d^2*e
^5 - 4*b*c^3*d*e^6 + 2*(3*b^2*c^2 + 2*a*c^3)*e^7)*x^6 - 168*(c^4*d^3*e^4 -
4*b*c^3*d^2*e^5 + 2*(3*b^2*c^2 + 2*a*c^3)*d*e^6 - 4*(b^3*c + 3*a*b*c^2)*e^7
)*x^5 + 210*(c^4*d^4*e^3 - 4*b*c^3*d^3*e^4 + 2*(3*b^2*c^2 + 2*a*c^3)*d^2*e^
5 - 4*(b^3*c + 3*a*b*c^2)*d*e^6 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*e^7)*x^4 -
 280*(c^4*d^5*e^2 - 4*b*c^3*d^4*e^3 + 2*(3*b^2*c^2 + 2*a*c^3)*d^3*e^4 - 4*(
b^3*c + 3*a*b*c^2)*d^2*e^5 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d*e^6 - 4*(a*b^
3 + 3*a^2*b*c)*e^7)*x^3 + 420*(c^4*d^6*e - 4*b*c^3*d^5*e^2 + 2*(3*b^2*c^2 +
 2*a*c^3)*d^4*e^3 - 4*(b^3*c + 3*a*b*c^2)*d^3*e^4 + (b^4 + 12*a*b^2*c + 6*a
^2*c^2)*d^2*e^5 - 4*(a*b^3 + 3*a^2*b*c)*d*e^6 + 2*(3*a^2*b^2 + 2*a^3*c)*e^7
)*x^2 - 840*(c^4*d^7 - 4*b*c^3*d^6*e - 4*a^3*b*e^7 + 2*(3*b^2*c^2 + 2*a*c^3
)*d^5*e^2 - 4*(b^3*c + 3*a*b*c^2)*d^4*e^3 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*
d^3*e^4 - 4*(a*b^3 + 3*a^2*b*c)*d^2*e^5 + 2*(3*a^2*b^2 + 2*a^3*c)*d*e^6)*x)
/e^8 + (c^4*d^8 - 4*b*c^3*d^7*e - 4*a^3*b*d*e^7 + a^4*e^8 + 2*(3*b^2*c^2 +
2*a*c^3)*d^6*e^2 - 4*(b^3*c + 3*a*b*c^2)*d^5*e^3 + (b^4 + 12*a*b^2*c + 6*a^
2*c^2)*d^4*e^4 - 4*(a*b^3 + 3*a^2*b*c)*d^3*e^5 + 2*(3*a^2*b^2 + 2*a^3*c)*d^
2*e^6)*log(e*x + d)/e^9
```

Fricas [A] time = 1.77465, size = 1662, normalized size = 3.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/840*(105*c^4*e^8*x^8 - 120*(c^4*d*e^7 - 4*b*c^3*e^8)*x^7 + 140*(c^4*d^2*e
^6 - 4*b*c^3*d*e^7 + 2*(3*b^2*c^2 + 2*a*c^3)*e^8)*x^6 - 168*(c^4*d^3*e^5 -
4*b*c^3*d^2*e^6 + 2*(3*b^2*c^2 + 2*a*c^3)*d*e^7 - 4*(b^3*c + 3*a*b*c^2)*e^8
)*x^5 + 210*(c^4*d^4*e^4 - 4*b*c^3*d^3*e^5 + 2*(3*b^2*c^2 + 2*a*c^3)*d^2*e^
6 - 4*(b^3*c + 3*a*b*c^2)*d*e^7 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*e^8)*x^4 -
 280*(c^4*d^5*e^3 - 4*b*c^3*d^4*e^4 + 2*(3*b^2*c^2 + 2*a*c^3)*d^3*e^5 - 4*(
b^3*c + 3*a*b*c^2)*d^2*e^6 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d*e^7 - 4*(a*b^
3 + 3*a^2*b*c)*e^8)*x^3 + 420*(c^4*d^6*e^2 - 4*b*c^3*d^5*e^3 + 2*(3*b^2*c^2
 + 2*a*c^3)*d^4*e^4 - 4*(b^3*c + 3*a*b*c^2)*d^3*e^5 + (b^4 + 12*a*b^2*c + 6
*a^2*c^2)*d^2*e^6 - 4*(a*b^3 + 3*a^2*b*c)*d*e^7 + 2*(3*a^2*b^2 + 2*a^3*c)*e
^8)*x^2 - 840*(c^4*d^7*e - 4*b*c^3*d^6*e^2 - 4*a^3*b*e^8 + 2*(3*b^2*c^2 + 2
*a*c^3)*d^5*e^3 - 4*(b^3*c + 3*a*b*c^2)*d^4*e^4 + (b^4 + 12*a*b^2*c + 6*a^2
*c^2)*d^3*e^5 - 4*(a*b^3 + 3*a^2*b*c)*d^2*e^6 + 2*(3*a^2*b^2 + 2*a^3*c)*d*e
^7)*x + 840*(c^4*d^8 - 4*b*c^3*d^7*e - 4*a^3*b*d*e^7 + a^4*e^8 + 2*(3*b^2*c
```

```
^2 + 2*a*c^3)*d^6*e^2 - 4*(b^3*c + 3*a*b*c^2)*d^5*e^3 + (b^4 + 12*a*b^2*c +
 6*a^2*c^2)*d^4*e^4 - 4*(a*b^3 + 3*a^2*b*c)*d^3*e^5 + 2*(3*a^2*b^2 + 2*a^3*
c)*d^2*e^6)*log(e*x + d))/e^9
```

Sympy [A] time = 1.83222, size = 796, normalized size = 1.86

$$\frac{c^4x^8}{8e}+\frac{x^7\left(4bc^3e-c^4d\right)}{7e^2}+\frac{x^6\left(4ac^3e^2+6b^2c^2e^2-4bc^3de+c^4d^2\right)}{6e^3}+\frac{x^5\left(12abc^2e^3-4ac^3de^2+4b^3ce^3-6b^2c^2de^2+4bc\right.}{5e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**4/(e*x+d),x)
```

```
[Out] c**4*x**8/(8*e) + x**7*(4*b*c**3*e - c**4*d)/(7*e**2) + x**6*(4*a*c**3*e**2
 + 6*b**2*c**2*e**2 - 4*b*c**3*d*e + c**4*d**2)/(6*e**3) + x**5*(12*a*b*c**
2*e**3 - 4*a*c**3*d*e**2 + 4*b**3*c*e**3 - 6*b**2*c**2*d*e**2 + 4*b*c**3*d*
*2*e - c**4*d**3)/(5*e**4) + x**4*(6*a**2*c**2*e**4 + 12*a*b**2*c*e**4 - 12
*a*b*c**2*d*e**3 + 4*a*c**3*d**2*e**2 + b**4*e**4 - 4*b**3*c*d*e**3 + 6*b**
2*c**2*d**2*e**2 - 4*b*c**3*d**3*e + c**4*d**4)/(4*e**5) + x**3*(12*a**2*b*
c*e**5 - 6*a**2*c**2*d*e**4 + 4*a*b**3*e**5 - 12*a*b**2*c*d*e**4 + 12*a*b*c
**2*d**2*e**3 - 4*a*c**3*d**3*e**2 - b**4*d*e**4 + 4*b**3*c*d**2*e**3 - 6*b
**2*c**2*d**3*e**2 + 4*b*c**3*d**4*e - c**4*d**5)/(3*e**6) + x**2*(4*a**3*c
*e**6 + 6*a**2*b**2*e**6 - 12*a**2*b*c*d*e**5 + 6*a**2*c**2*d**2*e**4 - 4*a
*b**3*d*e**5 + 12*a*b**2*c*d**2*e**4 - 12*a*b*c**2*d**3*e**3 + 4*a*c**3*d**
4*e**2 + b**4*d**2*e**4 - 4*b**3*c*d**3*e**3 + 6*b**2*c**2*d**4*e**2 - 4*b*
c**3*d**5*e + c**4*d**6)/(2*e**7) + x*(4*a**3*b*e**7 - 4*a**3*c*d*e**6 - 6*
a**2*b**2*d*e**6 + 12*a**2*b*c*d**2*e**5 - 6*a**2*c**2*d**3*e**4 + 4*a*b**3
*d**2*e**5 - 12*a*b**2*c*d**3*e**4 + 12*a*b*c**2*d**4*e**3 - 4*a*c**3*d**5*
e**2 - b**4*d**3*e**4 + 4*b**3*c*d**4*e**3 - 6*b**2*c**2*d**5*e**2 + 4*b*c*
*3*d**6*e - c**4*d**7)/e**8 + (a*e**2 - b*d*e + c*d**2)**4*log(d + e*x)/e**
9
```

Giac [B] time = 1.16693, size = 1276, normalized size = 2.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d),x, algorithm="giac")
```

```
[Out] (c^4*d^8 - 4*b*c^3*d^7*e + 6*b^2*c^2*d^6*e^2 + 4*a*c^3*d^6*e^2 - 4*b^3*c*d^
5*e^3 - 12*a*b*c^2*d^5*e^3 + b^4*d^4*e^4 + 12*a*b^2*c*d^4*e^4 + 6*a^2*c^2*d
^4*e^4 - 4*a*b^3*d^3*e^5 - 12*a^2*b*c*d^3*e^5 + 6*a^2*b^2*d^2*e^6 + 4*a^3*c
*d^2*e^6 - 4*a^3*b*d*e^7 + a^4*e^8)*e^(-9)*log(abs(x*e + d)) + 1/840*(105*c
^4*x^8*e^7 - 120*c^4*d*x^7*e^6 + 140*c^4*d^2*x^6*e^5 - 168*c^4*d^3*x^5*e^4
+ 210*c^4*d^4*x^4*e^3 - 280*c^4*d^5*x^3*e^2 + 420*c^4*d^6*x^2*e - 840*c^4*d
^7*x + 480*b*c^3*x^7*e^7 - 560*b*c^3*d*x^6*e^6 + 672*b*c^3*d^2*x^5*e^5 - 84
0*b*c^3*d^3*x^4*e^4 + 1120*b*c^3*d^4*x^3*e^3 - 1680*b*c^3*d^5*x^2*e^2 + 336
0*b*c^3*d^6*x*e + 840*b^2*c^2*x^6*e^7 + 560*a*c^3*x^6*e^7 - 1008*b^2*c^2*d*
x^5*e^6 - 672*a*c^3*d*x^5*e^6 + 1260*b^2*c^2*d^2*x^4*e^5 + 840*a*c^3*d^2*x^
4*e^5 - 1680*b^2*c^2*d^3*x^3*e^4 - 1120*a*c^3*d^3*x^3*e^4 + 2520*b^2*c^2*d^
4*x^2*e^3 + 1680*a*c^3*d^4*x^2*e^3 - 5040*b^2*c^2*d^5*x*e^2 - 3360*a*c^3*d^
5*x*e^2 + 672*b^3*c*x^5*e^7 + 2016*a*b*c^2*x^5*e^7 - 840*b^3*c*d*x^4*e^6 -
2520*a*b*c^2*d*x^4*e^6 + 1120*b^3*c*d^2*x^3*e^5 + 3360*a*b*c^2*d^2*x^3*e^5
```

- 1680*b^3*c*d^3*x^2*e^4 - 5040*a*b*c^2*d^3*x^2*e^4 + 3360*b^3*c*d^4*x*e^3
+ 10080*a*b*c^2*d^4*x*e^3 + 210*b^4*x^4*e^7 + 2520*a*b^2*c*x^4*e^7 + 1260*a
^2*c^2*x^4*e^7 - 280*b^4*d*x^3*e^6 - 3360*a*b^2*c*d*x^3*e^6 - 1680*a^2*c^2*
d*x^3*e^6 + 420*b^4*d^2*x^2*e^5 + 5040*a*b^2*c*d^2*x^2*e^5 + 2520*a^2*c^2*d
^2*x^2*e^5 - 840*b^4*d^3*x*e^4 - 10080*a*b^2*c*d^3*x*e^4 - 5040*a^2*c^2*d^3
*x*e^4 + 1120*a*b^3*x^3*e^7 + 3360*a^2*b*c*x^3*e^7 - 1680*a*b^3*d*x^2*e^6 -
 5040*a^2*b*c*d*x^2*e^6 + 3360*a*b^3*d^2*x*e^5 + 10080*a^2*b*c*d^2*x*e^5 +
2520*a^2*b^2*x^2*e^7 + 1680*a^3*c*x^2*e^7 - 5040*a^2*b^2*d*x*e^6 - 3360*a^3
*c*d*x*e^6 + 3360*a^3*b*x*e^7)*e^(-8)

3.2152 $\int \frac{(a+bx+cx^2)^4}{(d+ex)^2} dx$

Optimal. Leaf size=426

$$\frac{(d+ex)^3\left(6c^2e^2\left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4\right)}{3e^9}+\frac{2c^2(d+ex)^5\left(\right.}{}$$

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e))
*x)/e^8 - (c*d^2 - b*d*e + a*e^2)^4/(e^9*(d + e*x)) - (2*(2*c*d - b*e)*(c*d
^2 - b*d*e + a*e^2)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e))*(d + e*x)^2
)/e^9 + ((70*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(5*b*d - 3*a*e) - 20*c^3*d^2*e
*(7*b*d - 3*a*e) + 6*c^2*e^2*(15*b^2*d^2 - 10*a*b*d*e + a^2*e^2))*(d + e*x)
^3)/(3*e^9) - (c*(2*c*d - b*e)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e))*
(d + e*x)^4)/e^9 + (2*c^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e))*(d
 + e*x)^5)/(5*e^9) - (2*c^3*(2*c*d - b*e)*(d + e*x)^6)/(3*e^9) + (c^4*(d +
e*x)^7)/(7*e^9) - (4*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)^3*Log[d + e*x])/
e^9
```

Rubi [A] time = 0.721856, antiderivative size = 426, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$\frac{(d+ex)^3\left(6c^2e^2\left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4\right)}{3e^9}+\frac{2c^2(d+ex)^5\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^4/(d + e*x)^2,x]
```


```
[Out] (2*(c*d^2 - b*d*e + a*e^2)^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e))
*x)/e^8 - (c*d^2 - b*d*e + a*e^2)^4/(e^9*(d + e*x)) - (2*(2*c*d - b*e)*(c*d
^2 - b*d*e + a*e^2)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e))*(d + e*x)^2
)/e^9 + ((70*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(5*b*d - 3*a*e) - 20*c^3*d^2*e
*(7*b*d - 3*a*e) + 6*c^2*e^2*(15*b^2*d^2 - 10*a*b*d*e + a^2*e^2))*(d + e*x)
^3)/(3*e^9) - (c*(2*c*d - b*e)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e))*
(d + e*x)^4)/e^9 + (2*c^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e))*(d
 + e*x)^5)/(5*e^9) - (2*c^3*(2*c*d - b*e)*(d + e*x)^6)/(3*e^9) + (c^4*(d +
e*x)^7)/(7*e^9) - (4*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)^3*Log[d + e*x])/
e^9
```


Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^4}{(d+ex)^2}\,dx = \int \left(\frac{2\left(cd^2-bde+ae^2\right)^2\left(14c^2d^2+3b^2e^2-2ce(7bd-ae)\right)}{e^8} + \frac{\left(cd^2-bde+ae^2\right)^4}{e^8(d+ex)^2} + \frac{4(-2cd+be)\left(\right.}{e^8(}\right.$$
$$= \frac{2\left(cd^2-bde+ae^2\right)^2\left(14c^2d^2+3b^2e^2-2ce(7bd-ae)\right)x}{e^8} - \frac{\left(cd^2-bde+ae^2\right)^4}{e^9(d+ex)} - \frac{2(2cd-be)\left(cd^2\right.}{}$$

Mathematica [A] time = 0.339387, size = 780, normalized size = 1.83

$$\frac{21c^2e^2\left(10a^2e^2\left(6d^2e^2x^2+9d^3ex-3d^4-2de^3x^3+e^4x^4\right)+5abe\left(-30d^3e^2x^2+10d^2e^3x^3-48d^4ex+12d^5-5de^4x^4+3e^5x^5\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^4/(d + e*x)^2,x]
```

```
[Out] (c^4*(-105*d^8 + 735*d^7*e*x + 420*d^6*e^2*x^2 - 140*d^5*e^3*x^3 + 70*d^4*e
^4*x^4 - 42*d^3*e^5*x^5 + 28*d^2*e^6*x^6 - 20*d*e^7*x^7 + 15*e^8*x^8) + 35*
e^4*(12*a^3*b*d*e^3 - 3*a^4*e^4 + 18*a^2*b^2*e^2*(-d^2 + d*e*x + e^2*x^2) +
 6*a*b^3*e*(2*d^3 - 4*d^2*e*x - 3*d*e^2*x^2 + e^3*x^3) + b^4*(-3*d^4 + 9*d^
3*e*x + 6*d^2*e^2*x^2 - 2*d*e^3*x^3 + e^4*x^4)) + 35*c*e^3*(12*a^3*e^3*(-d^
2 + d*e*x + e^2*x^2) + 18*a^2*b*e^2*(2*d^3 - 4*d^2*e*x - 3*d*e^2*x^2 + e^3*
x^3) + 12*a*b^2*e*(-3*d^4 + 9*d^3*e*x + 6*d^2*e^2*x^2 - 2*d*e^3*x^3 + e^4*x
^4) + b^3*(12*d^5 - 48*d^4*e*x - 30*d^3*e^2*x^2 + 10*d^2*e^3*x^3 - 5*d*e^4*
x^4 + 3*e^5*x^5)) + 21*c^2*e^2*(10*a^2*e^2*(-3*d^4 + 9*d^3*e*x + 6*d^2*e^2*
x^2 - 2*d*e^3*x^3 + e^4*x^4) + 5*a*b*e*(12*d^5 - 48*d^4*e*x - 30*d^3*e^2*x^
2 + 10*d^2*e^3*x^3 - 5*d*e^4*x^4 + 3*e^5*x^5) + b^2*(-30*d^6 + 150*d^5*e*x
+ 90*d^4*e^2*x^2 - 30*d^3*e^3*x^3 + 15*d^2*e^4*x^4 - 9*d*e^5*x^5 + 6*e^6*x^
6)) + 7*c^3*e*(6*a*e*(-10*d^6 + 50*d^5*e*x + 30*d^4*e^2*x^2 - 10*d^3*e^3*x^
3 + 5*d^2*e^4*x^4 - 3*d*e^5*x^5 + 2*e^6*x^6) + b*(60*d^7 - 360*d^6*e*x - 21
0*d^5*e^2*x^2 + 70*d^4*e^3*x^3 - 35*d^3*e^4*x^4 + 21*d^2*e^5*x^5 - 14*d*e^6
*x^6 + 10*e^7*x^7)) - 420*(2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e))^3*(d + e
*x)*Log[d + e*x])/(105*e^9*(d + e*x))
```

Maple [B] time = 0.055, size = 1159, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^4/(e*x+d)^2,x)
```

```
[Out] 4/e^2*x^3*a*b^2*c-2/e^3*x^4*a*c^3*d+1/7*c^4*x^7/e^2-6/e^7/(e*x+d)*b^2*c^2*d
^6+4/e^8/(e*x+d)*b*c^3*d^7-6/e^3/(e*x+d)*a^2*b^2*d^2-6/e^5/(e*x+d)*a^2*c^2*
d^4+4/e^4/(e*x+d)*a*b^3*d^3-4/e^7/(e*x+d)*a*c^3*d^6+4/e^6/(e*x+d)*b^3*c*d^5
+12/e^4*ln(e*x+d)*a*b^3*d^2-24/e^7*ln(e*x+d)*a*c^3*d^5+20/e^6*ln(e*x+d)*b^3
*c*d^4-36/e^7*ln(e*x+d)*b^2*c^2*d^5+28/e^8*ln(e*x+d)*b*c^3*d^6+4/e^2/(e*x+d
)*d*a^3*b-4/e^3/(e*x+d)*a^3*c*d^2-4/e^5*ln(e*x+d)*b^4*d^3-8/e^9*ln(e*x+d)*c
^4*d^7-1/e^5/(e*x+d)*b^4*d^4-1/e^9/(e*x+d)*c^4*d^8+1/e^2*x^4*b^3*c+2/3/e^2*
x^6*b*c^3+2/e^2*x^3*a^2*c^2+5/3/e^6*x^3*c^4*d^4+2/e^2*x^2*a*b^3-1/e^3*x^2*b
^4*d-3/e^7*x^2*c^4*d^5+4/e^2*a^3*c*x+6/e^2*b^2*a^2*x+3/e^4*b^4*d^2*x+4/5/e^
2*x^5*a*c^3+6/5/e^2*x^5*b^2*c^2+3/5/e^4*x^5*c^4*d^2-1/e^5*x^4*c^4*d^3+7/e^8
*c^4*d^6*x+4/e^2*ln(e*x+d)*a^3*b-8/e^3*a*b^3*d*x+20/e^6*a*c^3*d^4*x-16/e^5*
```

```
b^3*c*d^3*x+30/e^6*b^2*c^2*d^4*x-24/e^7*b*c^3*d^5*x+4/e^4*x^3*a*c^3*d^2-8/3
/e^3*x^3*b^3*c*d-3/e^3*x^4*b^2*c^2*d+3/e^4*x^4*b*c^3*d^2-8/e^5*x^2*a*c^3*d^
3+6/e^4*x^2*b^3*c*d^2-12/e^5*x^2*b^2*c^2*d^3+3/e^2*x^4*a*b*c^2-8/5/e^3*x^5*
b*c^3*d-8/e^3*ln(e*x+d)*a^3*c*d-12/e^3*ln(e*x+d)*a^2*b^2*d-24/e^5*ln(e*x+d)
*a^2*c^2*d^3-6/e^3*x^2*a^2*c^2*d+6/e^2*x^2*a^2*b*c+6/e^4*x^3*b^2*c^2*d^2-16
/3/e^5*x^3*b*c^3*d^3+10/e^6*x^2*b*c^3*d^4+18/e^4*a^2*c^2*d^2*x+1/3/e^2*x^3*
b^4-1/e/(e*x+d)*a^4-24/e^3*a^2*b*c*d*x+36/e^4*a*b^2*c*d^2*x-48/e^5*a*b*c^2*
d^3*x-12/e^3*x^2*a*b^2*c*d+18/e^4*x^2*a*b*c^2*d^2-8/e^3*x^3*a*b*c^2*d+36/e^
4*ln(e*x+d)*a^2*b*c*d^2-48/e^5*ln(e*x+d)*a*b^2*c*d^3+60/e^6*ln(e*x+d)*a*b*c
^2*d^4+12/e^4/(e*x+d)*a^2*b*c*d^3-12/e^5/(e*x+d)*a*b^2*c*d^4+12/e^6/(e*x+d)
*a*b*c^2*d^5-1/3*c^4*d*x^6/e^3
```

Maxima [A] time = 1.04409, size = 1089, normalized size = 2.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(c^4*d^8 - 4*b*c^3*d^7*e - 4*a^3*b*d*e^7 + a^4*e^8 + 2*(3*b^2*c^2 + 2*a*c^
3)*d^6*e^2 - 4*(b^3*c + 3*a*b*c^2)*d^5*e^3 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)
*d^4*e^4 - 4*(a*b^3 + 3*a^2*b*c)*d^3*e^5 + 2*(3*a^2*b^2 + 2*a^3*c)*d^2*e^6)
/(e^10*x + d*e^9) + 1/105*(15*c^4*e^6*x^7 - 35*(c^4*d*e^5 - 2*b*c^3*e^6)*x^
6 + 21*(3*c^4*d^2*e^4 - 8*b*c^3*d*e^5 + 2*(3*b^2*c^2 + 2*a*c^3)*e^6)*x^5 -
105*(c^4*d^3*e^3 - 3*b*c^3*d^2*e^4 + (3*b^2*c^2 + 2*a*c^3)*d*e^5 - (b^3*c +
 3*a*b*c^2)*e^6)*x^4 + 35*(5*c^4*d^4*e^2 - 16*b*c^3*d^3*e^3 + 6*(3*b^2*c^2
+ 2*a*c^3)*d^2*e^4 - 8*(b^3*c + 3*a*b*c^2)*d*e^5 + (b^4 + 12*a*b^2*c + 6*a^
2*c^2)*e^6)*x^3 - 105*(3*c^4*d^5*e - 10*b*c^3*d^4*e^2 + 4*(3*b^2*c^2 + 2*a*
c^3)*d^3*e^3 - 6*(b^3*c + 3*a*b*c^2)*d^2*e^4 + (b^4 + 12*a*b^2*c + 6*a^2*c^
2)*d*e^5 - 2*(a*b^3 + 3*a^2*b*c)*e^6)*x^2 + 105*(7*c^4*d^6 - 24*b*c^3*d^5*e
 + 10*(3*b^2*c^2 + 2*a*c^3)*d^4*e^2 - 16*(b^3*c + 3*a*b*c^2)*d^3*e^3 + 3*(b
^4 + 12*a*b^2*c + 6*a^2*c^2)*d^2*e^4 - 8*(a*b^3 + 3*a^2*b*c)*d*e^5 + 2*(3*a
^2*b^2 + 2*a^3*c)*e^6)*x)/e^8 - 4*(2*c^4*d^7 - 7*b*c^3*d^6*e - a^3*b*e^7 +
3*(3*b^2*c^2 + 2*a*c^3)*d^5*e^2 - 5*(b^3*c + 3*a*b*c^2)*d^4*e^3 + (b^4 + 12
*a*b^2*c + 6*a^2*c^2)*d^3*e^4 - 3*(a*b^3 + 3*a^2*b*c)*d^2*e^5 + (3*a^2*b^2
+ 2*a^3*c)*d*e^6)*log(e*x + d)/e^9
```

Fricas [B] time = 1.8496, size = 2288, normalized size = 5.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/105*(15*c^4*e^8*x^8 - 105*c^4*d^8 + 420*b*c^3*d^7*e + 420*a^3*b*d*e^7 - 1
05*a^4*e^8 - 210*(3*b^2*c^2 + 2*a*c^3)*d^6*e^2 + 420*(b^3*c + 3*a*b*c^2)*d^
5*e^3 - 105*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^4*e^4 + 420*(a*b^3 + 3*a^2*b*c
)*d^3*e^5 - 210*(3*a^2*b^2 + 2*a^3*c)*d^2*e^6 - 10*(2*c^4*d*e^7 - 7*b*c^3*e
^8)*x^7 + 14*(2*c^4*d^2*e^6 - 7*b*c^3*d*e^7 + 3*(3*b^2*c^2 + 2*a*c^3)*e^8)*
x^6 - 21*(2*c^4*d^3*e^5 - 7*b*c^3*d^2*e^6 + 3*(3*b^2*c^2 + 2*a*c^3)*d*e^7 -
 5*(b^3*c + 3*a*b*c^2)*e^8)*x^5 + 35*(2*c^4*d^4*e^4 - 7*b*c^3*d^3*e^5 + 3*(
3*b^2*c^2 + 2*a*c^3)*d^2*e^6 - 5*(b^3*c + 3*a*b*c^2)*d*e^7 + (b^4 + 12*a*b^
2*c + 6*a^2*c^2)*e^8)*x^4 - 70*(2*c^4*d^5*e^3 - 7*b*c^3*d^4*e^4 + 3*(3*b^2*
```

```
c^2 + 2*a*c^3)*d^3*e^5 - 5*(b^3*c + 3*a*b*c^2)*d^2*e^6 + (b^4 + 12*a*b^2*c
+ 6*a^2*c^2)*d*e^7 - 3*(a*b^3 + 3*a^2*b*c)*e^8)*x^3 + 210*(2*c^4*d^6*e^2 -
7*b*c^3*d^5*e^3 + 3*(3*b^2*c^2 + 2*a*c^3)*d^4*e^4 - 5*(b^3*c + 3*a*b*c^2)*d
^3*e^5 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^2*e^6 - 3*(a*b^3 + 3*a^2*b*c)*d*e
^7 + (3*a^2*b^2 + 2*a^3*c)*e^8)*x^2 + 105*(7*c^4*d^7*e - 24*b*c^3*d^6*e^2 +
 10*(3*b^2*c^2 + 2*a*c^3)*d^5*e^3 - 16*(b^3*c + 3*a*b*c^2)*d^4*e^4 + 3*(b^4
 + 12*a*b^2*c + 6*a^2*c^2)*d^3*e^5 - 8*(a*b^3 + 3*a^2*b*c)*d^2*e^6 + 2*(3*a
^2*b^2 + 2*a^3*c)*d*e^7)*x - 420*(2*c^4*d^8 - 7*b*c^3*d^7*e - a^3*b*d*e^7 +
 3*(3*b^2*c^2 + 2*a*c^3)*d^6*e^2 - 5*(b^3*c + 3*a*b*c^2)*d^5*e^3 + (b^4 + 1
2*a*b^2*c + 6*a^2*c^2)*d^4*e^4 - 3*(a*b^3 + 3*a^2*b*c)*d^3*e^5 + (3*a^2*b^2
 + 2*a^3*c)*d^2*e^6 + (2*c^4*d^7*e - 7*b*c^3*d^6*e^2 - a^3*b*e^8 + 3*(3*b^2
*c^2 + 2*a*c^3)*d^5*e^3 - 5*(b^3*c + 3*a*b*c^2)*d^4*e^4 + (b^4 + 12*a*b^2*c
 + 6*a^2*c^2)*d^3*e^5 - 3*(a*b^3 + 3*a^2*b*c)*d^2*e^6 + (3*a^2*b^2 + 2*a^3*
c)*d*e^7)*x)*log(e*x + d))/(e^10*x + d*e^9)
```

Sympy [A] time = 5.0012, size = 824, normalized size = 1.93

$$\frac{c^4x^7}{7e^2} - \frac{a^4e^8 - 4a^3bde^7 + 4a^3cd^2e^6 + 6a^2b^2d^2e^6 - 12a^2bcd^3e^5 + 6a^2c^2d^4e^4 - 4ab^3d^3e^5 + 12ab^2cd^4e^4 - 12abc^2d^5e^3 + 4ac^3d^6 \cdots}{de^9 + e^{10}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**4/(e*x+d)**2,x)
```

```
[Out] c**4*x**7/(7*e**2) - (a**4*e**8 - 4*a**3*b*d*e**7 + 4*a**3*c*d**2*e**6 + 6*
a**2*b**2*d**2*e**6 - 12*a**2*b*c*d**3*e**5 + 6*a**2*c**2*d**4*e**4 - 4*a*b
**3*d**3*e**5 + 12*a*b**2*c*d**4*e**4 - 12*a*b*c**2*d**5*e**3 + 4*a*c**3*d*
*6*e**2 + b**4*d**4*e**4 - 4*b**3*c*d**5*e**3 + 6*b**2*c**2*d**6*e**2 - 4*b
*c**3*d**7*e + c**4*d**8)/(d*e**9 + e**10*x) + x**6*(2*b*c**3*e - c**4*d)/(
3*e**3) + x**5*(4*a*c**3*e**2 + 6*b**2*c**2*e**2 - 8*b*c**3*d*e + 3*c**4*d*
*2)/(5*e**4) + x**4*(3*a*b*c**2*e**3 - 2*a*c**3*d*e**2 + b**3*c*e**3 - 3*b*
*2*c**2*d*e**2 + 3*b*c**3*d**2*e - c**4*d**3)/e**5 + x**3*(6*a**2*c**2*e**4
 + 12*a*b**2*c*e**4 - 24*a*b*c**2*d*e**3 + 12*a*c**3*d**2*e**2 + b**4*e**4
- 8*b**3*c*d*e**3 + 18*b**2*c**2*d**2*e**2 - 16*b*c**3*d**3*e + 5*c**4*d**4
)/(3*e**6) + x**2*(6*a**2*b*c*e**5 - 6*a**2*c**2*d*e**4 + 2*a*b**3*e**5 - 1
2*a*b**2*c*d*e**4 + 18*a*b*c**2*d**2*e**3 - 8*a*c**3*d**3*e**2 - b**4*d*e**
4 + 6*b**3*c*d**2*e**3 - 12*b**2*c**2*d**3*e**2 + 10*b*c**3*d**4*e - 3*c**4
*d**5)/e**7 + x*(4*a**3*c*e**6 + 6*a**2*b**2*e**6 - 24*a**2*b*c*d*e**5 + 18
*a**2*c**2*d**2*e**4 - 8*a*b**3*d*e**5 + 36*a*b**2*c*d**2*e**4 - 48*a*b*c**
2*d**3*e**3 + 20*a*c**3*d**4*e**2 + 3*b**4*d**2*e**4 - 16*b**3*c*d**3*e**3
+ 30*b**2*c**2*d**4*e**2 - 24*b*c**3*d**5*e + 7*c**4*d**6)/e**8 + 4*(b*e -
2*c*d)*(a*e**2 - b*d*e + c*d**2)**3*log(d + e*x)/e**9
```

Giac [B] time = 1.09843, size = 1368, normalized size = 3.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/105*(15*c^4 - 70*(2*c^4*d*e - b*c^3*e^2)*e^(-1)/(x*e + d) + 42*(14*c^4*d^
2*e^2 - 14*b*c^3*d*e^3 + 3*b^2*c^2*e^4 + 2*a*c^3*e^4)*e^(-2)/(x*e + d)^2 -
105*(14*c^4*d^3*e^3 - 21*b*c^3*d^2*e^4 + 9*b^2*c^2*d*e^5 + 6*a*c^3*d*e^5 -
```

```
b^3*c*e^6 - 3*a*b*c^2*e^6)*e^(-3)/(x*e + d)^3 + 35*(70*c^4*d^4*e^4 - 140*b*
c^3*d^3*e^5 + 90*b^2*c^2*d^2*e^6 + 60*a*c^3*d^2*e^6 - 20*b^3*c*d*e^7 - 60*a
*b*c^2*d*e^7 + b^4*e^8 + 12*a*b^2*c*e^8 + 6*a^2*c^2*e^8)*e^(-4)/(x*e + d)^4
 - 210*(14*c^4*d^5*e^5 - 35*b*c^3*d^4*e^6 + 30*b^2*c^2*d^3*e^7 + 20*a*c^3*d
^3*e^7 - 10*b^3*c*d^2*e^8 - 30*a*b*c^2*d^2*e^8 + b^4*d*e^9 + 12*a*b^2*c*d*e
^9 + 6*a^2*c^2*d*e^9 - a*b^3*e^10 - 3*a^2*b*c*e^10)*e^(-5)/(x*e + d)^5 + 21
0*(14*c^4*d^6*e^6 - 42*b*c^3*d^5*e^7 + 45*b^2*c^2*d^4*e^8 + 30*a*c^3*d^4*e^
8 - 20*b^3*c*d^3*e^9 - 60*a*b*c^2*d^3*e^9 + 3*b^4*d^2*e^10 + 36*a*b^2*c*d^2
*e^10 + 18*a^2*c^2*d^2*e^10 - 6*a*b^3*d*e^11 - 18*a^2*b*c*d*e^11 + 3*a^2*b^
2*e^12 + 2*a^3*c*e^12)*e^(-6)/(x*e + d)^6)*(x*e + d)^7*e^(-9) + 4*(2*c^4*d^
7 - 7*b*c^3*d^6*e + 9*b^2*c^2*d^5*e^2 + 6*a*c^3*d^5*e^2 - 5*b^3*c*d^4*e^3 -
 15*a*b*c^2*d^4*e^3 + b^4*d^3*e^4 + 12*a*b^2*c*d^3*e^4 + 6*a^2*c^2*d^3*e^4
- 3*a*b^3*d^2*e^5 - 9*a^2*b*c*d^2*e^5 + 3*a^2*b^2*d*e^6 + 2*a^3*c*d*e^6 - a
^3*b*e^7)*e^(-9)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2) - (c^4*d^8*e^7/(x*e +
 d) - 4*b*c^3*d^7*e^8/(x*e + d) + 6*b^2*c^2*d^6*e^9/(x*e + d) + 4*a*c^3*d^6
*e^9/(x*e + d) - 4*b^3*c*d^5*e^10/(x*e + d) - 12*a*b*c^2*d^5*e^10/(x*e + d)
 + b^4*d^4*e^11/(x*e + d) + 12*a*b^2*c*d^4*e^11/(x*e + d) + 6*a^2*c^2*d^4*e
^11/(x*e + d) - 4*a*b^3*d^3*e^12/(x*e + d) - 12*a^2*b*c*d^3*e^12/(x*e + d)
+ 6*a^2*b^2*d^2*e^13/(x*e + d) + 4*a^3*c*d^2*e^13/(x*e + d) - 4*a^3*b*d*e^1
4/(x*e + d) + a^4*e^15/(x*e + d))*e^(-16)
```

3.2153 $\int \frac{(a+bx+cx^2)^4}{(d+ex)^3} dx$

Optimal. Leaf size=430

$$\frac{(d+ex)^2\left(6c^2e^2\left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4\right)}{2e^9}+\frac{c^2(d+ex)^4\left(-2c\right.}{}$$

```
[Out] (-4*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d
 - 3*a*e))*x)/e^8 - (c*d^2 - b*d*e + a*e^2)^4/(2*e^9*(d + e*x)^2) + (4*(2*c
*d - b*e)*(c*d^2 - b*d*e + a*e^2)^3)/(e^9*(d + e*x)) + ((70*c^4*d^4 + b^4*e
^4 - 4*b^2*c*e^3*(5*b*d - 3*a*e) - 20*c^3*d^2*e*(7*b*d - 3*a*e) + 6*c^2*e^2
*(15*b^2*d^2 - 10*a*b*d*e + a^2*e^2))*(d + e*x)^2)/(2*e^9) - (4*c*(2*c*d -
b*e)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e))*(d + e*x)^3)/(3*e^9) + (c^
2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e))*(d + e*x)^4)/(2*e^9) - (4*
c^3*(2*c*d - b*e)*(d + e*x)^5)/(5*e^9) + (c^4*(d + e*x)^6)/(6*e^9) + (2*(c*
d^2 - b*d*e + a*e^2)^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e))*Log[d
 + e*x])/e^9
```

Rubi [A] time = 0.666783, antiderivative size = 430, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$\frac{(d+ex)^2\left(6c^2e^2\left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4\right)}{2e^9}+\frac{c^2(d+ex)^4\left(-2c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^4/(d + e*x)^3,x]
```


```
[Out] (-4*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d
 - 3*a*e))*x)/e^8 - (c*d^2 - b*d*e + a*e^2)^4/(2*e^9*(d + e*x)^2) + (4*(2*c
*d - b*e)*(c*d^2 - b*d*e + a*e^2)^3)/(e^9*(d + e*x)) + ((70*c^4*d^4 + b^4*e
^4 - 4*b^2*c*e^3*(5*b*d - 3*a*e) - 20*c^3*d^2*e*(7*b*d - 3*a*e) + 6*c^2*e^2
*(15*b^2*d^2 - 10*a*b*d*e + a^2*e^2))*(d + e*x)^2)/(2*e^9) - (4*c*(2*c*d -
b*e)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e))*(d + e*x)^3)/(3*e^9) + (c^
2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e))*(d + e*x)^4)/(2*e^9) - (4*
c^3*(2*c*d - b*e)*(d + e*x)^5)/(5*e^9) + (c^4*(d + e*x)^6)/(6*e^9) + (2*(c*
d^2 - b*d*e + a*e^2)^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e))*Log[d
 + e*x])/e^9
```


Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^4}{(d+ex)^3}\,dx = \int \left(\frac{4(2cd-be)(cd^2-bde+ae^2)(-7c^2d^2+7bcde-b^2e^2-3ace^2)}{e^8} + \frac{(cd^2-bde+ae^2)^4}{e^8(d+ex)^3} + \frac{4(\cdots}{\cdots} \right.$$

$$= -\frac{4(2cd-be)(cd^2-bde+ae^2)(7c^2d^2+b^2e^2-ce(7bd-3ae))x}{e^8} - \frac{(cd^2-bde+ae^2)^4}{2e^9(d+ex)^2} + \frac{4(2cd\cdots}{\cdots}$$

Mathematica [A] time = 0.204605, size = 440, normalized size = 1.02

$$\frac{15e^2x^2\left(6c^2e^2\left(a^2e^2-6abde+6b^2d^2\right)-12b^2ce^3(bd-ae)-8c^3d^2e(5bd-3ae)+b^4e^4+15c^4d^4\right)+30ex\left(-6c^2de^2\left(3a^2e^2\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^4/(d + e*x)^3,x]
```

```
[Out] (30*e*(-21*c^4*d^5 + 20*c^3*d^3*e*(3*b*d - 2*a*e) + b^3*e^4*(-3*b*d + 4*a*e
) + 12*b*c*e^3*(2*b^2*d^2 - 3*a*b*d*e + a^2*e^2) - 6*c^2*d*e^2*(10*b^2*d^2
- 12*a*b*d*e + 3*a^2*e^2))*x + 15*e^2*(15*c^4*d^4 + b^4*e^4 - 8*c^3*d^2*e*(
5*b*d - 3*a*e) - 12*b^2*c*e^3*(b*d - a*e) + 6*c^2*e^2*(6*b^2*d^2 - 6*a*b*d*
e + a^2*e^2))*x^2 + 20*c*e^3*(-(c*d) + b*e)*(5*c^2*d^2 + 2*b^2*e^2 + c*e*(-
7*b*d + 6*a*e))*x^3 + 15*c^2*e^4*(3*c^2*d^2 + 3*b^2*e^2 + 2*c*e*(-3*b*d + a
*e))*x^4 + 6*c^3*e^5*(-3*c*d + 4*b*e)*x^5 + 5*c^4*e^6*x^6 - (15*(c*d^2 + e*
(-(b*d) + a*e))^4)/(d + e*x)^2 + (120*(2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*
e))^3)/(d + e*x) + 60*(14*c^2*d^2 + 3*b^2*e^2 + 2*c*e*(-7*b*d + a*e))*(c*d^
2 + e*(-(b*d) + a*e))^2*Log[d + e*x])/(30*e^9)
```

Maple [B] time = 0.058, size = 1216, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^4/(e*x+d)^3,x)
```

```
[Out] 4/e^3*x^3*a*b*c^2-4/e^4*x^3*a*c^3*d-3/e^3/(e*x+d)^2*d^2*a^2*b^2+90/e^7*ln(e
*x+d)*b^2*c^2*d^4-84/e^8*ln(e*x+d)*b*c^3*d^5+8/e^3/(e*x+d)*a^3*c*d+12/e^3/(
e*x+d)*a^2*b^2*d+24/e^5/(e*x+d)*a^2*c^2*d^3-12/e^4/(e*x+d)*a*b^3*d^2+24/e^7
/(e*x+d)*a*c^3*d^5-20/e^6/(e*x+d)*b^3*c*d^4+36/e^7/(e*x+d)*b^2*c^2*d^5-28/e
^8/(e*x+d)*b*c^3*d^6-3/e^5/(e*x+d)^2*a^2*c^2*d^4+2/e^4/(e*x+d)^2*a*b^3*d^3-
2/e^7/(e*x+d)^2*a*c^3*d^6+2/e^6/(e*x+d)^2*b^3*c*d^5-3/e^7/(e*x+d)^2*b^2*c^2
*d^6+2/e^8/(e*x+d)^2*b*c^3*d^7+36/e^5*ln(e*x+d)*a^2*c^2*d^2-6/e^4*x^3*b^2*c
^2*d-3/e^4*x^4*b*c^3*d+12/e^3*a^2*b*c*x-18/e^4*a^2*c^2*d*x-40/e^6*a*c^3*d^3
*x+24/e^5*b^3*c*d^2*x-60/e^6*b^2*c^2*d^3*x+60/e^7*d^4*b*c^3*x+12/e^5*x^2*a*
c^3*d^2-6/e^4*x^2*b^3*c*d+18/e^5*x^2*b^2*c^2*d^2-20/e^6*x^2*b*c^3*d^3+8/e^5
*x^3*b*c^3*d^2+6/e^3*x^2*a*b^2*c+2/e^2/(e*x+d)^2*d*a^3*b-2/e^3/(e*x+d)^2*a^
3*c*d^2-12/e^4*ln(e*x+d)*a*b^3*d+60/e^7*ln(e*x+d)*a*c^3*d^4-40/e^6*ln(e*x+d
)*b^3*c*d^3+1/6/e^3*c^4*x^6-1/2/e/(e*x+d)^2*a^4+1/2*b^4*x^2/e^3-3/5/e^4*x^5
*c^4*d+3/2/e^3*x^4*b^2*c^2+3/2/e^5*x^4*c^4*d^2+4/3/e^3*x^3*b^3*c-10/3/e^6*x
^3*c^4*d^3+3/e^3*x^2*a^2*c^2-3/e^4*b^4*d*x+15/2/e^7*x^2*c^4*d^4+4/e^3*a*b^3
*x-21/e^8*c^4*d^5*x-1/2/e^5/(e*x+d)^2*b^4*d^4-1/2/e^9/(e*x+d)^2*c^4*d^8+4/e
^3*ln(e*x+d)*a^3*c+6/e^3*ln(e*x+d)*a^2*b^2+6/e^5*ln(e*x+d)*b^4*d^2+28/e^9*l
n(e*x+d)*c^4*d^6-4/e^2/(e*x+d)*a^3*b+4/e^5/(e*x+d)*b^4*d^3+8/e^9/(e*x+d)*c^
```

```
4*d^7+1/e^3*x^4*a*c^3+4/5/e^3*x^5*b*c^3-36/e^4/(e*x+d)*a^2*b*c*d^2+48/e^5/(
e*x+d)*a*b^2*c*d^3-60/e^6/(e*x+d)*a*b*c^2*d^4+72/e^5*a*b*c^2*d^2*x-36/e^4*a
*b^2*c*d*x-18/e^4*x^2*a*b*c^2*d+6/e^4/(e*x+d)^2*a^2*b*c*d^3-6/e^5/(e*x+d)^2
*a*b^2*c*d^4+6/e^6/(e*x+d)^2*a*b*c^2*d^5-36/e^4*ln(e*x+d)*a^2*b*c*d+72/e^5*
ln(e*x+d)*a*b^2*c*d^2-120/e^6*ln(e*x+d)*a*b*c^2*d^3
```

Maxima [A] time = 1.12175, size = 1106, normalized size = 2.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(15*c^4*d^8 - 52*b*c^3*d^7*e - 4*a^3*b*d*e^7 - a^4*e^8 + 22*(3*b^2*c^2
+ 2*a*c^3)*d^6*e^2 - 36*(b^3*c + 3*a*b*c^2)*d^5*e^3 + 7*(b^4 + 12*a*b^2*c +
 6*a^2*c^2)*d^4*e^4 - 20*(a*b^3 + 3*a^2*b*c)*d^3*e^5 + 6*(3*a^2*b^2 + 2*a^3
*c)*d^2*e^6 + 8*(2*c^4*d^7*e - 7*b*c^3*d^6*e^2 - a^3*b*e^8 + 3*(3*b^2*c^2 +
 2*a*c^3)*d^5*e^3 - 5*(b^3*c + 3*a*b*c^2)*d^4*e^4 + (b^4 + 12*a*b^2*c + 6*a
^2*c^2)*d^3*e^5 - 3*(a*b^3 + 3*a^2*b*c)*d^2*e^6 + (3*a^2*b^2 + 2*a^3*c)*d*e
^7)*x)/(e^11*x^2 + 2*d*e^10*x + d^2*e^9) + 1/30*(5*c^4*e^5*x^6 - 6*(3*c^4*d
*e^4 - 4*b*c^3*e^5)*x^5 + 15*(3*c^4*d^2*e^3 - 6*b*c^3*d*e^4 + (3*b^2*c^2 +
2*a*c^3)*e^5)*x^4 - 20*(5*c^4*d^3*e^2 - 12*b*c^3*d^2*e^3 + 3*(3*b^2*c^2 + 2
*a*c^3)*d*e^4 - 2*(b^3*c + 3*a*b*c^2)*e^5)*x^3 + 15*(15*c^4*d^4*e - 40*b*c^
3*d^3*e^2 + 12*(3*b^2*c^2 + 2*a*c^3)*d^2*e^3 - 12*(b^3*c + 3*a*b*c^2)*d*e^4
 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*e^5)*x^2 - 30*(21*c^4*d^5 - 60*b*c^3*d^4*
e + 20*(3*b^2*c^2 + 2*a*c^3)*d^3*e^2 - 24*(b^3*c + 3*a*b*c^2)*d^2*e^3 + 3*(
b^4 + 12*a*b^2*c + 6*a^2*c^2)*d*e^4 - 4*(a*b^3 + 3*a^2*b*c)*e^5)*x)/e^8 + 2
*(14*c^4*d^6 - 42*b*c^3*d^5*e + 15*(3*b^2*c^2 + 2*a*c^3)*d^4*e^2 - 20*(b^3*
c + 3*a*b*c^2)*d^3*e^3 + 3*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^2*e^4 - 6*(a*b^
3 + 3*a^2*b*c)*d*e^5 + (3*a^2*b^2 + 2*a^3*c)*e^6)*log(e*x + d)/e^9
```

Fricas [B] time = 1.96845, size = 2573, normalized size = 5.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/30*(5*c^4*e^8*x^8 + 225*c^4*d^8 - 780*b*c^3*d^7*e - 60*a^3*b*d*e^7 - 15*a
^4*e^8 + 330*(3*b^2*c^2 + 2*a*c^3)*d^6*e^2 - 540*(b^3*c + 3*a*b*c^2)*d^5*e^
3 + 105*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^4*e^4 - 300*(a*b^3 + 3*a^2*b*c)*d^
3*e^5 + 90*(3*a^2*b^2 + 2*a^3*c)*d^2*e^6 - 8*(c^4*d*e^7 - 3*b*c^3*e^8)*x^7
+ (14*c^4*d^2*e^6 - 42*b*c^3*d*e^7 + 15*(3*b^2*c^2 + 2*a*c^3)*e^8)*x^6 - 2*
(14*c^4*d^3*e^5 - 42*b*c^3*d^2*e^6 + 15*(3*b^2*c^2 + 2*a*c^3)*d*e^7 - 20*(b
^3*c + 3*a*b*c^2)*e^8)*x^5 + 5*(14*c^4*d^4*e^4 - 42*b*c^3*d^3*e^5 + 15*(3*b
^2*c^2 + 2*a*c^3)*d^2*e^6 - 20*(b^3*c + 3*a*b*c^2)*d*e^7 + 3*(b^4 + 12*a*b^
2*c + 6*a^2*c^2)*e^8)*x^4 - 20*(14*c^4*d^5*e^3 - 42*b*c^3*d^4*e^4 + 15*(3*b
^2*c^2 + 2*a*c^3)*d^3*e^5 - 20*(b^3*c + 3*a*b*c^2)*d^2*e^6 + 3*(b^4 + 12*a*
b^2*c + 6*a^2*c^2)*d*e^7 - 6*(a*b^3 + 3*a^2*b*c)*e^8)*x^3 - 15*(69*c^4*d^6*
e^2 - 200*b*c^3*d^5*e^3 + 68*(3*b^2*c^2 + 2*a*c^3)*d^4*e^4 - 84*(b^3*c + 3*
a*b*c^2)*d^3*e^5 + 11*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^2*e^6 - 16*(a*b^3 +
3*a^2*b*c)*d*e^7)*x^2 - 30*(13*c^4*d^7*e - 32*b*c^3*d^6*e^2 + 4*a^3*b*e^8 +
 8*(3*b^2*c^2 + 2*a*c^3)*d^5*e^3 - 4*(b^3*c + 3*a*b*c^2)*d^4*e^4 - (b^4 + 1
```

```
2*a*b^2*c + 6*a^2*c^2)*d^3*e^5 + 8*(a*b^3 + 3*a^2*b*c)*d^2*e^6 - 4*(3*a^2*b
^2 + 2*a^3*c)*d*e^7)*x + 60*(14*c^4*d^8 - 42*b*c^3*d^7*e + 15*(3*b^2*c^2 +
2*a*c^3)*d^6*e^2 - 20*(b^3*c + 3*a*b*c^2)*d^5*e^3 + 3*(b^4 + 12*a*b^2*c + 6
*a^2*c^2)*d^4*e^4 - 6*(a*b^3 + 3*a^2*b*c)*d^3*e^5 + (3*a^2*b^2 + 2*a^3*c)*d
^2*e^6 + (14*c^4*d^6*e^2 - 42*b*c^3*d^5*e^3 + 15*(3*b^2*c^2 + 2*a*c^3)*d^4*
e^4 - 20*(b^3*c + 3*a*b*c^2)*d^3*e^5 + 3*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^2
*e^6 - 6*(a*b^3 + 3*a^2*b*c)*d*e^7 + (3*a^2*b^2 + 2*a^3*c)*e^8)*x^2 + 2*(14
*c^4*d^7*e - 42*b*c^3*d^6*e^2 + 15*(3*b^2*c^2 + 2*a*c^3)*d^5*e^3 - 20*(b^3*
c + 3*a*b*c^2)*d^4*e^4 + 3*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^3*e^5 - 6*(a*b^
3 + 3*a^2*b*c)*d^2*e^6 + (3*a^2*b^2 + 2*a^3*c)*d*e^7)*x)*log(e*x + d))/(e^1
1*x^2 + 2*d*e^10*x + d^2*e^9)
```

Sympy [B] time = 22.8442, size = 892, normalized size = 2.07

$$\frac{c^4x^6}{6e^3} - \frac{a^4e^8 + 4a^3bde^7 - 12a^3cd^2e^6 - 18a^2b^2d^2e^6 + 60a^2bcd^3e^5 - 42a^2c^2d^4e^4 + 20ab^3d^3e^5 - 84ab^2cd^4e^4 + 108abc^2d^5e^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**4/(e*x+d)**3,x)
```

```
[Out] c**4*x**6/(6*e**3) - (a**4*e**8 + 4*a**3*b*d*e**7 - 12*a**3*c*d**2*e**6 - 1
8*a**2*b**2*d**2*e**6 + 60*a**2*b*c*d**3*e**5 - 42*a**2*c**2*d**4*e**4 + 20
*a*b**3*d**3*e**5 - 84*a*b**2*c*d**4*e**4 + 108*a*b*c**2*d**5*e**3 - 44*a*c
**3*d**6*e**2 - 7*b**4*d**4*e**4 + 36*b**3*c*d**5*e**3 - 66*b**2*c**2*d**6*
e**2 + 52*b*c**3*d**7*e - 15*c**4*d**8 + x*(8*a**3*b*e**8 - 16*a**3*c*d*e**
7 - 24*a**2*b**2*d*e**7 + 72*a**2*b*c*d**2*e**6 - 48*a**2*c**2*d**3*e**5 +
24*a*b**3*d**2*e**6 - 96*a*b**2*c*d**3*e**5 + 120*a*b*c**2*d**4*e**4 - 48*a
*c**3*d**5*e**3 - 8*b**4*d**3*e**5 + 40*b**3*c*d**4*e**4 - 72*b**2*c**2*d**
5*e**3 + 56*b*c**3*d**6*e**2 - 16*c**4*d**7*e))/(2*d**2*e**9 + 4*d*e**10*x
+ 2*e**11*x**2) + x**5*(4*b*c**3*e - 3*c**4*d)/(5*e**4) + x**4*(2*a*c**3*e*
*2 + 3*b**2*c**2*e**2 - 6*b*c**3*d*e + 3*c**4*d**2)/(2*e**5) + x**3*(12*a*b
*c**2*e**3 - 12*a*c**3*d*e**2 + 4*b**3*c*e**3 - 18*b**2*c**2*d*e**2 + 24*b*
c**3*d**2*e - 10*c**4*d**3)/(3*e**6) + x**2*(6*a**2*c**2*e**4 + 12*a*b**2*c
*e**4 - 36*a*b*c**2*d*e**3 + 24*a*c**3*d**2*e**2 + b**4*e**4 - 12*b**3*c*d*
e**3 + 36*b**2*c**2*d**2*e**2 - 40*b*c**3*d**3*e + 15*c**4*d**4)/(2*e**7) +
 x*(12*a**2*b*c*e**5 - 18*a**2*c**2*d*e**4 + 4*a*b**3*e**5 - 36*a*b**2*c*d*
e**4 + 72*a*b*c**2*d**2*e**3 - 40*a*c**3*d**3*e**2 - 3*b**4*d*e**4 + 24*b**
3*c*d**2*e**3 - 60*b**2*c**2*d**3*e**2 + 60*b*c**3*d**4*e - 21*c**4*d**5)/e
**8 + 2*(a*e**2 - b*d*e + c*d**2)**2*(2*a*c*e**2 + 3*b**2*e**2 - 14*b*c*d*e
 + 14*c**2*d**2)*log(d + e*x)/e**9
```

Giac [B] time = 1.12606, size = 1202, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 2*(14*c^4*d^6 - 42*b*c^3*d^5*e + 45*b^2*c^2*d^4*e^2 + 30*a*c^3*d^4*e^2 - 20
*b^3*c*d^3*e^3 - 60*a*b*c^2*d^3*e^3 + 3*b^4*d^2*e^4 + 36*a*b^2*c*d^2*e^4 +
18*a^2*c^2*d^2*e^4 - 6*a*b^3*d*e^5 - 18*a^2*b*c*d*e^5 + 3*a^2*b^2*e^6 + 2*a
^3*c*e^6)*e^(-9)*log(abs(x*e + d)) + 1/30*(5*c^4*x^6*e^15 - 18*c^4*d*x^5*e^
```

14 + 45*c^4*d^2*x^4*e^13 - 100*c^4*d^3*x^3*e^12 + 225*c^4*d^4*x^2*e^11 - 630*c^4*d^5*x*e^10 + 24*b*c^3*x^5*e^15 - 90*b*c^3*d*x^4*e^14 + 240*b*c^3*d^2*x^3*e^13 - 600*b*c^3*d^3*x^2*e^12 + 1800*b*c^3*d^4*x*e^11 + 45*b^2*c^2*x^4*e^15 + 30*a*c^3*x^4*e^15 - 180*b^2*c^2*d*x^3*e^14 - 120*a*c^3*d*x^3*e^14 + 540*b^2*c^2*d^2*x^2*e^13 + 360*a*c^3*d^2*x^2*e^13 - 1800*b^2*c^2*d^3*x*e^12 - 1200*a*c^3*d^3*x*e^12 + 40*b^3*c*x^3*e^15 + 120*a*b*c^2*x^3*e^15 - 180*b^3*c*d*x^2*e^14 - 540*a*b*c^2*d*x^2*e^14 + 720*b^3*c*d^2*x*e^13 + 2160*a*b*c^2*d^2*x*e^13 + 15*b^4*x^2*e^15 + 180*a*b^2*c*x^2*e^15 + 90*a^2*c^2*x^2*e^15 - 90*b^4*d*x*e^14 - 1080*a*b^2*c*d*x*e^14 - 540*a^2*c^2*d*x*e^14 + 120*a*b^3*x*e^15 + 360*a^2*b*c*x*e^15)*e^(-18) + 1/2*(15*c^4*d^8 - 52*b*c^3*d^7*e + 66*b^2*c^2*d^6*e^2 + 44*a*c^3*d^6*e^2 - 36*b^3*c*d^5*e^3 - 108*a*b*c^2*d^5*e^3 + 7*b^4*d^4*e^4 + 84*a*b^2*c*d^4*e^4 + 42*a^2*c^2*d^4*e^4 - 20*a*b^3*d^3*e^5 - 60*a^2*b*c*d^3*e^5 + 18*a^2*b^2*d^2*e^6 + 12*a^3*c*d^2*e^6 - 4*a^3*b*d*e^7 - a^4*e^8 + 8*(2*c^4*d^7*e - 7*b*c^3*d^6*e^2 + 9*b^2*c^2*d^5*e^3 + 6*a*c^3*d^5*e^3 - 5*b^3*c*d^4*e^4 - 15*a*b*c^2*d^4*e^4 + b^4*d^3*e^5 + 12*a*b^2*c*d^3*e^5 + 6*a^2*c^2*d^3*e^5 - 3*a*b^3*d^2*e^6 - 9*a^2*b*c*d^2*e^6 + 3*a^2*b^2*d*e^7 + 2*a^3*c*d*e^7 - a^3*b*e^8)*x)*e^(-9)/(x*e + d)^2

3.2154 $\int \frac{(a+bx+cx^2)^4}{(d+ex)^4}\,dx$

Optimal. Leaf size=417

$$\frac{x\left(6c^2e^2\left(a^2e^2-8abde+10b^2d^2\right)-4b^2ce^3(4bd-3ae)-40c^3d^2e(2bd-ae)+b^4e^4+35c^4d^4\right)}{e^8}+\frac{2c^2x^3\left(-2ce(4bd-ae)+\right.}{3e^6}$$

```
[Out] ((35*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(4*b*d - 3*a*e) - 40*c^3*d^2*e*(2*b*d
- a*e) + 6*c^2*e^2*(10*b^2*d^2 - 8*a*b*d*e + a^2*e^2))*x)/e^8 - (2*c*(5*c^3
*d^3 - b^3*e^3 - 2*c^2*d*e*(5*b*d - 2*a*e) + 3*b*c*e^2*(2*b*d - a*e))*x^2)/
e^7 + (2*c^2*(5*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(4*b*d - a*e))*x^3)/(3*e^6) - (
c^3*(c*d - b*e)*x^4)/e^5 + (c^4*x^5)/(5*e^4) - (c*d^2 - b*d*e + a*e^2)^4/(3
*e^9*(d + e*x)^3) + (2*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)^3)/(e^9*(d + e
*x)^2) - (2*(c*d^2 - b*d*e + a*e^2)^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*
d - a*e)))/(e^9*(d + e*x)) - (4*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*(7*c^
2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e))*Log[d + e*x])/e^9
```

Rubi [A] time = 0.651569, antiderivative size = 417, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$\frac{x\left(6c^2e^2\left(a^2e^2-8abde+10b^2d^2\right)-4b^2ce^3(4bd-3ae)-40c^3d^2e(2bd-ae)+b^4e^4+35c^4d^4\right)}{e^8}+\frac{2c^2x^3\left(-2ce(4bd-ae)+\right.}{3e^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^4/(d + e*x)^4,x]
```

```
[Out] ((35*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(4*b*d - 3*a*e) - 40*c^3*d^2*e*(2*b*d
- a*e) + 6*c^2*e^2*(10*b^2*d^2 - 8*a*b*d*e + a^2*e^2))*x)/e^8 - (2*c*(5*c^3
*d^3 - b^3*e^3 - 2*c^2*d*e*(5*b*d - 2*a*e) + 3*b*c*e^2*(2*b*d - a*e))*x^2)/
e^7 + (2*c^2*(5*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(4*b*d - a*e))*x^3)/(3*e^6) - (
c^3*(c*d - b*e)*x^4)/e^5 + (c^4*x^5)/(5*e^4) - (c*d^2 - b*d*e + a*e^2)^4/(3
*e^9*(d + e*x)^3) + (2*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)^3)/(e^9*(d + e
*x)^2) - (2*(c*d^2 - b*d*e + a*e^2)^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*
d - a*e)))/(e^9*(d + e*x)) - (4*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*(7*c^
2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e))*Log[d + e*x])/e^9
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^4}{(d+ex)^4}\,dx=\int \left(\frac{35c^4d^4+b^4e^4-4b^2ce^3(4bd-3ae)-40c^3d^2e(2bd-ae)+6c^2e^2\left(10b^2d^2-8abde+a^2e^2\right)}{e^8}-\right.$$

$$=\frac{\left(35c^4d^4+b^4e^4-4b^2ce^3(4bd-3ae)-40c^3d^2e(2bd-ae)+6c^2e^2\left(10b^2d^2-8abde+a^2e^2\right)\right)x}{e^8}-$$

Mathematica [A] time = 0.22621, size = 425, normalized size = 1.02

$$15ex\left(6c^2e^2\left(a^2e^2-8abde+10b^2d^2\right)-4b^2ce^3(4bd-3ae)+40c^3d^2e(ae-2bd)+b^4e^4+35c^4d^4\right)-60(2cd-be)\log(d+ex)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^4/(d + e*x)^4,x]
```

```
[Out] (15*e*(35*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(4*b*d - 3*a*e) + 40*c^3*d^2*e*(-
2*b*d + a*e) + 6*c^2*e^2*(10*b^2*d^2 - 8*a*b*d*e + a^2*e^2))*x + 30*c*e^2*(
-5*c^3*d^3 + b^3*e^3 + 2*c^2*d*e*(5*b*d - 2*a*e) + 3*b*c*e^2*(-2*b*d + a*e)
)*x^2 + 10*c^2*e^3*(5*c^2*d^2 + 3*b^2*e^2 + 2*c*e*(-4*b*d + a*e))*x^3 + 15*
c^3*e^4*(-(c*d) + b*e)*x^4 + 3*c^4*e^5*x^5 - (5*(c*d^2 + e*(-(b*d) + a*e))^
4)/(d + e*x)^3 + (30*(2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e))^3)/(d + e*x)^
2 - (30*(14*c^2*d^2 + 3*b^2*e^2 + 2*c*e*(-7*b*d + a*e))*(c*d^2 + e*(-(b*d)
+ a*e))^2)/(d + e*x) - 60*(2*c*d - b*e)*(7*c^3*d^4 - 2*c^2*d^2*e*(7*b*d - 5
*a*e) + b^2*e^3*(-(b*d) + a*e) + c*e^2*(8*b^2*d^2 - 10*a*b*d*e + 3*a^2*e^2)
)*Log[d + e*x])/(15*e^9)
```

Maple [B] time = 0.059, size = 1265, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^4/(e*x+d)^4,x)
```

```
[Out] 6/e^4*x^2*a*b*c^2+4/3/e^8/(e*x+d)^3*b*c^3*d^7+4/e^3/(e*x+d)^2*a^3*c*d+6/e^3
/(e*x+d)^2*a^2*b^2*d+12/e^5/(e*x+d)^2*a^2*c^2*d^3-6/e^4/(e*x+d)^2*a*b^3*d^2
+12/e^7/(e*x+d)^2*a*c^3*d^5-10/e^6/(e*x+d)^2*b^3*c*d^4+18/e^7/(e*x+d)^2*b^2
*c^2*d^5-14/e^8/(e*x+d)^2*b*c^3*d^6-8/e^5*x^2*a*c^3*d-12/e^5*x^2*b^2*c^2*d+
20/e^6*x^2*b*c^3*d^2+12/e^4*a*c*b^2*x+40/e^6*c^3*a*d^2*x-16/e^5*b^3*c*d*x+6
0/e^6*b^2*c^2*d^2*x-36/e^5/(e*x+d)*a^2*c^2*d^2+12/e^4/(e*x+d)*a*b^3*d-60/e^
7/(e*x+d)*a*c^3*d^4+40/e^6/(e*x+d)*b^3*c*d^3-90/e^7/(e*x+d)*b^2*c^2*d^4+84/
e^8/(e*x+d)*b*c^3*d^5-2/e^5/(e*x+d)^3*a^2*c^2*d^4-1/3/e/(e*x+d)^3*a^4+1/5*c
^4*x^5/e^4+1/e^4*x^4*b*c^3-1/e^5*x^4*c^4*d+4/3/e^4*x^3*a*c^3+2/e^4*x^3*b^2*
c^2+10/3/e^6*x^3*c^4*d^2+2/e^4*x^2*b^3*c-10/e^7*x^2*c^4*d^3+6/e^4*c^2*a^2*x
+35/e^8*c^4*d^4*x-1/3/e^5/(e*x+d)^3*b^4*d^4-1/3/e^9/(e*x+d)^3*c^4*d^8-2/e^2
/(e*x+d)^2*a^3*b+2/e^5/(e*x+d)^2*b^4*d^3+4/e^9/(e*x+d)^2*c^4*d^7+4/e^4*ln(e
*x+d)*a*b^3-4/e^5*ln(e*x+d)*b^4*d-56/e^9*ln(e*x+d)*c^4*d^5-4/e^3/(e*x+d)*a^
3*c-16/3/e^5*x^3*b*c^3*d+4/3/e^4/(e*x+d)^3*d^3*a*b^3-4/3/e^7/(e*x+d)^3*a*c^
3*d^6+4/3/e^6/(e*x+d)^3*b^3*c*d^5+12/e^4*ln(e*x+d)*a^2*b*c-24/e^5*ln(e*x+d)
*a^2*c^2*d-80/e^7*ln(e*x+d)*a*c^3*d^3+40/e^6*ln(e*x+d)*b^3*c*d^2-120/e^7*ln
(e*x+d)*b^2*c^2*d^3+140/e^8*ln(e*x+d)*b*c^3*d^4-80/e^7*b*c^3*d^3*x+4/3/e^2/
(e*x+d)^3*d*a^3*b-4/3/e^3/(e*x+d)^3*a^3*c*d^2-2/e^3/(e*x+d)^3*d^2*a^2*b^2-2
/e^7/(e*x+d)^3*b^2*c^2*d^6-6/e^3/(e*x+d)*a^2*b^2-6/e^5/(e*x+d)*b^4*d^2-28/e
^9/(e*x+d)*c^4*d^6+b^4*x/e^4+4/e^4/(e*x+d)^3*d^3*a^2*b*c-4/e^5/(e*x+d)^3*a*
b^2*c*d^4+4/e^6/(e*x+d)^3*a*b*c^2*d^5-18/e^4/(e*x+d)^2*a^2*b*c*d^2+24/e^5/(
e*x+d)^2*a*b^2*c*d^3-30/e^6/(e*x+d)^2*a*b*c^2*d^4-48/e^5*ln(e*x+d)*a*b^2*c*
d+120/e^6*ln(e*x+d)*a*b*c^2*d^2+36/e^4/(e*x+d)*a^2*b*c*d-72/e^5/(e*x+d)*a*b
^2*c*d^2+120/e^6/(e*x+d)*a*b*c^2*d^3-48/e^5*a*b*c^2*d*x
```

Maxima [B] time = 1.20639, size = 1116, normalized size = 2.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(73*c^4*d^8 - 214*b*c^3*d^7*e + 2*a^3*b*d*e^7 + a^4*e^8 + 74*(3*b^2*c^
2 + 2*a*c^3)*d^6*e^2 - 94*(b^3*c + 3*a*b*c^2)*d^5*e^3 + 13*(b^4 + 12*a*b^2*
c + 6*a^2*c^2)*d^4*e^4 - 22*(a*b^3 + 3*a^2*b*c)*d^3*e^5 + 2*(3*a^2*b^2 + 2*
a^3*c)*d^2*e^6 + 6*(14*c^4*d^6*e^2 - 42*b*c^3*d^5*e^3 + 15*(3*b^2*c^2 + 2*a
*c^3)*d^4*e^4 - 20*(b^3*c + 3*a*b*c^2)*d^3*e^5 + 3*(b^4 + 12*a*b^2*c + 6*a^
2*c^2)*d^2*e^6 - 6*(a*b^3 + 3*a^2*b*c)*d*e^7 + (3*a^2*b^2 + 2*a^3*c)*e^8)*x
^2 + 6*(26*c^4*d^7*e - 77*b*c^3*d^6*e^2 + a^3*b*e^8 + 27*(3*b^2*c^2 + 2*a*c
^3)*d^5*e^3 - 35*(b^3*c + 3*a*b*c^2)*d^4*e^4 + 5*(b^4 + 12*a*b^2*c + 6*a^2*
c^2)*d^3*e^5 - 9*(a*b^3 + 3*a^2*b*c)*d^2*e^6 + (3*a^2*b^2 + 2*a^3*c)*d*e^7)
*x)/(e^12*x^3 + 3*d*e^11*x^2 + 3*d^2*e^10*x + d^3*e^9) + 1/15*(3*c^4*e^4*x^
5 - 15*(c^4*d*e^3 - b*c^3*e^4)*x^4 + 10*(5*c^4*d^2*e^2 - 8*b*c^3*d*e^3 + (3
*b^2*c^2 + 2*a*c^3)*e^4)*x^3 - 30*(5*c^4*d^3*e - 10*b*c^3*d^2*e^2 + 2*(3*b^
2*c^2 + 2*a*c^3)*d*e^3 - (b^3*c + 3*a*b*c^2)*e^4)*x^2 + 15*(35*c^4*d^4 - 80
*b*c^3*d^3*e + 20*(3*b^2*c^2 + 2*a*c^3)*d^2*e^2 - 16*(b^3*c + 3*a*b*c^2)*d*
e^3 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*e^4)*x)/e^8 - 4*(14*c^4*d^5 - 35*b*c^3
*d^4*e + 10*(3*b^2*c^2 + 2*a*c^3)*d^3*e^2 - 10*(b^3*c + 3*a*b*c^2)*d^2*e^3
+ (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d*e^4 - (a*b^3 + 3*a^2*b*c)*e^5)*log(e*x +
 d)/e^9
```

Fricas [B] time = 1.91431, size = 2708, normalized size = 6.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(3*c^4*e^8*x^8 - 365*c^4*d^8 + 1070*b*c^3*d^7*e - 10*a^3*b*d*e^7 - 5*a
^4*e^8 - 370*(3*b^2*c^2 + 2*a*c^3)*d^6*e^2 + 470*(b^3*c + 3*a*b*c^2)*d^5*e^
3 - 65*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^4*e^4 + 110*(a*b^3 + 3*a^2*b*c)*d^3
*e^5 - 10*(3*a^2*b^2 + 2*a^3*c)*d^2*e^6 - 3*(2*c^4*d*e^7 - 5*b*c^3*e^8)*x^7
 + (14*c^4*d^2*e^6 - 35*b*c^3*d*e^7 + 10*(3*b^2*c^2 + 2*a*c^3)*e^8)*x^6 - 3
*(14*c^4*d^3*e^5 - 35*b*c^3*d^2*e^6 + 10*(3*b^2*c^2 + 2*a*c^3)*d*e^7 - 10*(
b^3*c + 3*a*b*c^2)*e^8)*x^5 + 15*(14*c^4*d^4*e^4 - 35*b*c^3*d^3*e^5 + 10*(3
*b^2*c^2 + 2*a*c^3)*d^2*e^6 - 10*(b^3*c + 3*a*b*c^2)*d*e^7 + (b^4 + 12*a*b^
2*c + 6*a^2*c^2)*e^8)*x^4 + 5*(235*c^4*d^5*e^3 - 556*b*c^3*d^4*e^4 + 146*(3
*b^2*c^2 + 2*a*c^3)*d^3*e^5 - 126*(b^3*c + 3*a*b*c^2)*d^2*e^6 + 9*(b^4 + 12
*a*b^2*c + 6*a^2*c^2)*d*e^7)*x^3 + 15*(67*c^4*d^6*e^2 - 136*b*c^3*d^5*e^3 +
 26*(3*b^2*c^2 + 2*a*c^3)*d^4*e^4 - 6*(b^3*c + 3*a*b*c^2)*d^3*e^5 - 3*(b^4
+ 12*a*b^2*c + 6*a^2*c^2)*d^2*e^6 + 12*(a*b^3 + 3*a^2*b*c)*d*e^7 - 2*(3*a^2
*b^2 + 2*a^3*c)*e^8)*x^2 - 15*(17*c^4*d^7*e - 74*b*c^3*d^6*e^2 + 2*a^3*b*e^
8 + 34*(3*b^2*c^2 + 2*a*c^3)*d^5*e^3 - 54*(b^3*c + 3*a*b*c^2)*d^4*e^4 + 9*(
b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^3*e^5 - 18*(a*b^3 + 3*a^2*b*c)*d^2*e^6 + 2*
(3*a^2*b^2 + 2*a^3*c)*d*e^7)*x - 60*(14*c^4*d^8 - 35*b*c^3*d^7*e + 10*(3*b^
2*c^2 + 2*a*c^3)*d^6*e^2 - 10*(b^3*c + 3*a*b*c^2)*d^5*e^3 + (b^4 + 12*a*b^2
*c + 6*a^2*c^2)*d^4*e^4 - (a*b^3 + 3*a^2*b*c)*d^3*e^5 + (14*c^4*d^5*e^3 - 3
5*b*c^3*d^4*e^4 + 10*(3*b^2*c^2 + 2*a*c^3)*d^3*e^5 - 10*(b^3*c + 3*a*b*c^2)
*d^2*e^6 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d*e^7 - (a*b^3 + 3*a^2*b*c)*e^8)*
x^3 + 3*(14*c^4*d^6*e^2 - 35*b*c^3*d^5*e^3 + 10*(3*b^2*c^2 + 2*a*c^3)*d^4*e
```

```
^4 - 10*(b^3*c + 3*a*b*c^2)*d^3*e^5 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^2*e^
6 - (a*b^3 + 3*a^2*b*c)*d*e^7)*x^2 + 3*(14*c^4*d^7*e - 35*b*c^3*d^6*e^2 + 1
0*(3*b^2*c^2 + 2*a*c^3)*d^5*e^3 - 10*(b^3*c + 3*a*b*c^2)*d^4*e^4 + (b^4 + 1
2*a*b^2*c + 6*a^2*c^2)*d^3*e^5 - (a*b^3 + 3*a^2*b*c)*d^2*e^6)*x)*log(e*x +
d))/(e^12*x^3 + 3*d*e^11*x^2 + 3*d^2*e^10*x + d^3*e^9)
```

Sympy [B] time = 111.556, size = 933, normalized size = 2.24

$$\frac{c^4x^5}{5e^4} - \frac{a^4e^8 + 2a^3bde^7 + 4a^3cd^2e^6 + 6a^2b^2d^2e^6 - 66a^2bcd^3e^5 + 78a^2c^2d^4e^4 - 22ab^3d^3e^5 + 156ab^2cd^4e^4 - 282abc^2d^5e^3 + 1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**4/(e*x+d)**4,x)
```

```
[Out] c**4*x**5/(5*e**4) - (a**4*e**8 + 2*a**3*b*d*e**7 + 4*a**3*c*d**2*e**6 + 6*
a**2*b**2*d**2*e**6 - 66*a**2*b*c*d**3*e**5 + 78*a**2*c**2*d**4*e**4 - 22*a
*b**3*d**3*e**5 + 156*a*b**2*c*d**4*e**4 - 282*a*b*c**2*d**5*e**3 + 148*a*c
**3*d**6*e**2 + 13*b**4*d**4*e**4 - 94*b**3*c*d**5*e**3 + 222*b**2*c**2*d**
6*e**2 - 214*b*c**3*d**7*e + 73*c**4*d**8 + x**2*(12*a**3*c*e**8 + 18*a**2*
b**2*e**8 - 108*a**2*b*c*d*e**7 + 108*a**2*c**2*d**2*e**6 - 36*a*b**3*d*e**
7 + 216*a*b**2*c*d**2*e**6 - 360*a*b*c**2*d**3*e**5 + 180*a*c**3*d**4*e**4
+ 18*b**4*d**2*e**6 - 120*b**3*c*d**3*e**5 + 270*b**2*c**2*d**4*e**4 - 252*
b*c**3*d**5*e**3 + 84*c**4*d**6*e**2) + x*(6*a**3*b*e**8 + 12*a**3*c*d*e**7
 + 18*a**2*b**2*d*e**7 - 162*a**2*b*c*d**2*e**6 + 180*a**2*c**2*d**3*e**5 -
 54*a*b**3*d**2*e**6 + 360*a*b**2*c*d**3*e**5 - 630*a*b*c**2*d**4*e**4 + 32
4*a*c**3*d**5*e**3 + 30*b**4*d**3*e**5 - 210*b**3*c*d**4*e**4 + 486*b**2*c*
*2*d**5*e**3 - 462*b*c**3*d**6*e**2 + 156*c**4*d**7*e))/(3*d**3*e**9 + 9*d*
*2*e**10*x + 9*d*e**11*x**2 + 3*e**12*x**3) + x**4*(b*c**3*e - c**4*d)/e**5
 + x**3*(4*a*c**3*e**2 + 6*b**2*c**2*e**2 - 16*b*c**3*d*e + 10*c**4*d**2)/(
3*e**6) + x**2*(6*a*b*c**2*e**3 - 8*a*c**3*d*e**2 + 2*b**3*c*e**3 - 12*b**2
*c**2*d*e**2 + 20*b*c**3*d**2*e - 10*c**4*d**3)/e**7 + x*(6*a**2*c**2*e**4
+ 12*a*b**2*c*e**4 - 48*a*b*c**2*d*e**3 + 40*a*c**3*d**2*e**2 + b**4*e**4 -
 16*b**3*c*d*e**3 + 60*b**2*c**2*d**2*e**2 - 80*b*c**3*d**3*e + 35*c**4*d**
4)/e**8 + 4*(b*e - 2*c*d)*(a*e**2 - b*d*e + c*d**2)*(3*a*c*e**2 + b**2*e**2
 - 7*b*c*d*e + 7*c**2*d**2)*log(d + e*x)/e**9
```

Giac [B] time = 1.10175, size = 1168, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -4*(14*c^4*d^5 - 35*b*c^3*d^4*e + 30*b^2*c^2*d^3*e^2 + 20*a*c^3*d^3*e^2 - 1
0*b^3*c*d^2*e^3 - 30*a*b*c^2*d^2*e^3 + b^4*d*e^4 + 12*a*b^2*c*d*e^4 + 6*a^2
*c^2*d*e^4 - a*b^3*e^5 - 3*a^2*b*c*e^5)*e^(-9)*log(abs(x*e + d)) + 1/15*(3*
c^4*x^5*e^16 - 15*c^4*d*x^4*e^15 + 50*c^4*d^2*x^3*e^14 - 150*c^4*d^3*x^2*e^
13 + 525*c^4*d^4*x*e^12 + 15*b*c^3*x^4*e^16 - 80*b*c^3*d*x^3*e^15 + 300*b*c
^3*d^2*x^2*e^14 - 1200*b*c^3*d^3*x*e^13 + 30*b^2*c^2*x^3*e^16 + 20*a*c^3*x^
3*e^16 - 180*b^2*c^2*d*x^2*e^15 - 120*a*c^3*d*x^2*e^15 + 900*b^2*c^2*d^2*x*
e^14 + 600*a*c^3*d^2*x*e^14 + 30*b^3*c*x^2*e^16 + 90*a*b*c^2*x^2*e^16 - 240
*b^3*c*d*x*e^15 - 720*a*b*c^2*d*x*e^15 + 15*b^4*x*e^16 + 180*a*b^2*c*x*e^16
```

```
 + 90*a^2*c^2*x*e^16)*e^(-20) - 1/3*(73*c^4*d^8 - 214*b*c^3*d^7*e + 222*b^2
*c^2*d^6*e^2 + 148*a*c^3*d^6*e^2 - 94*b^3*c*d^5*e^3 - 282*a*b*c^2*d^5*e^3 +
 13*b^4*d^4*e^4 + 156*a*b^2*c*d^4*e^4 + 78*a^2*c^2*d^4*e^4 - 22*a*b^3*d^3*e
^5 - 66*a^2*b*c*d^3*e^5 + 6*a^2*b^2*d^2*e^6 + 4*a^3*c*d^2*e^6 + 2*a^3*b*d*e
^7 + a^4*e^8 + 6*(14*c^4*d^6*e^2 - 42*b*c^3*d^5*e^3 + 45*b^2*c^2*d^4*e^4 +
30*a*c^3*d^4*e^4 - 20*b^3*c*d^3*e^5 - 60*a*b*c^2*d^3*e^5 + 3*b^4*d^2*e^6 +
36*a*b^2*c*d^2*e^6 + 18*a^2*c^2*d^2*e^6 - 6*a*b^3*d*e^7 - 18*a^2*b*c*d*e^7
+ 3*a^2*b^2*e^8 + 2*a^3*c*e^8)*x^2 + 6*(26*c^4*d^7*e - 77*b*c^3*d^6*e^2 + 8
1*b^2*c^2*d^5*e^3 + 54*a*c^3*d^5*e^3 - 35*b^3*c*d^4*e^4 - 105*a*b*c^2*d^4*e
^4 + 5*b^4*d^3*e^5 + 60*a*b^2*c*d^3*e^5 + 30*a^2*c^2*d^3*e^5 - 9*a*b^3*d^2*
e^6 - 27*a^2*b*c*d^2*e^6 + 3*a^2*b^2*d*e^7 + 2*a^3*c*d*e^7 + a^3*b*e^8)*x)*
e^(-9)/(x*e + d)^3
```

3.2155 $\int \frac{(a+bx+cx^2)^4}{(d+ex)^5}\,dx$

Optimal. Leaf size=426

$$\frac{\log(d+ex)\left(6c^2e^2\left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4\right)}{e^9}+\frac{c^2x^2\left(-4ce(5bd\right.}{}$$

```
[Out] -((c*(35*c^3*d^3 - 4*b^3*e^3 + 6*b*c*e^2*(5*b*d - 2*a*e) - 20*c^2*d*e*(3*b*
d - a*e))*x)/e^8) + (c^2*(15*c^2*d^2 + 6*b^2*e^2 - 4*c*e*(5*b*d - a*e))*x^2
)/(2*e^7) - (c^3*(5*c*d - 4*b*e)*x^3)/(3*e^6) + (c^4*x^4)/(4*e^5) - (c*d^2
- b*d*e + a*e^2)^4/(4*e^9*(d + e*x)^4) + (4*(2*c*d - b*e)*(c*d^2 - b*d*e +
a*e^2)^3)/(3*e^9*(d + e*x)^3) - ((c*d^2 - b*d*e + a*e^2)^2*(14*c^2*d^2 + 3*
b^2*e^2 - 2*c*e*(7*b*d - a*e)))/(e^9*(d + e*x)^2) + (4*(2*c*d - b*e)*(c*d^2
 - b*d*e + a*e^2)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e)))/(e^9*(d + e*
x)) + ((70*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(5*b*d - 3*a*e) - 20*c^3*d^2*e*(
7*b*d - 3*a*e) + 6*c^2*e^2*(15*b^2*d^2 - 10*a*b*d*e + a^2*e^2))*Log[d + e*x
])/e^9
```

Rubi [A] time = 0.634799, antiderivative size = 426, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$\frac{\log(d+ex)\left(6c^2e^2\left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4\right)}{e^9}+\frac{c^2x^2\left(-4ce(5bd\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^4/(d + e*x)^5,x]
```


```
[Out] -((c*(35*c^3*d^3 - 4*b^3*e^3 + 6*b*c*e^2*(5*b*d - 2*a*e) - 20*c^2*d*e*(3*b*
d - a*e))*x)/e^8) + (c^2*(15*c^2*d^2 + 6*b^2*e^2 - 4*c*e*(5*b*d - a*e))*x^2
)/(2*e^7) - (c^3*(5*c*d - 4*b*e)*x^3)/(3*e^6) + (c^4*x^4)/(4*e^5) - (c*d^2
- b*d*e + a*e^2)^4/(4*e^9*(d + e*x)^4) + (4*(2*c*d - b*e)*(c*d^2 - b*d*e +
a*e^2)^3)/(3*e^9*(d + e*x)^3) - ((c*d^2 - b*d*e + a*e^2)^2*(14*c^2*d^2 + 3*
b^2*e^2 - 2*c*e*(7*b*d - a*e)))/(e^9*(d + e*x)^2) + (4*(2*c*d - b*e)*(c*d^2
 - b*d*e + a*e^2)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e)))/(e^9*(d + e*
x)) + ((70*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(5*b*d - 3*a*e) - 20*c^3*d^2*e*(
7*b*d - 3*a*e) + 6*c^2*e^2*(15*b^2*d^2 - 10*a*b*d*e + a^2*e^2))*Log[d + e*x
])/e^9
```


Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^4}{(d+ex)^5}\,dx = \int \left(\frac{c\left(-35c^3d^3+4b^3e^3-6bce^2(5bd-2ae)+20c^2de(3bd-ae)\right)}{e^8} + \frac{c^2\left(15c^2d^2+6b^2e^2-4ce(5\right.}{e^7} \right.$$

$$= -\frac{c\left(35c^3d^3-4b^3e^3+6bce^2(5bd-2ae)-20c^2de(3bd-ae)\right)x}{e^8} + \frac{c^2\left(15c^2d^2+6b^2e^2-4ce(5bd\right.}{2e^7}$$

Mathematica [A] time = 0.205907, size = 430, normalized size = 1.01

$$\frac{\frac{48(2cd-be)\left(ce^2\left(3a^2e^2-10abde+8b^2d^2\right)+b^2e^3(ae-bd)-2c^2d^2e(7bd-5ae)+7c^3d^4\right)}{d+ex}+12\log(d+ex)\left(6c^2e^2\left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^4/(d + e*x)^5,x]
```

```
[Out] (12*c*e*(-35*c^3*d^3 + 4*b^3*e^3 - 6*b*c*e^2*(5*b*d - 2*a*e) + 20*c^2*d*e*(
3*b*d - a*e))*x + 6*c^2*e^2*(15*c^2*d^2 + 6*b^2*e^2 + 4*c*e*(-5*b*d + a*e))
*x^2 + 4*c^3*e^3*(-5*c*d + 4*b*e)*x^3 + 3*c^4*e^4*x^4 - (3*(c*d^2 + e*(-(b*
d) + a*e))^4)/(d + e*x)^4 + (16*(2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e))^3)
/(d + e*x)^3 - (12*(14*c^2*d^2 + 3*b^2*e^2 + 2*c*e*(-7*b*d + a*e))*(c*d^2 +
 e*(-(b*d) + a*e))^2)/(d + e*x)^2 + (48*(2*c*d - b*e)*(7*c^3*d^4 - 2*c^2*d^
2*e*(7*b*d - 5*a*e) + b^2*e^3*(-(b*d) + a*e) + c*e^2*(8*b^2*d^2 - 10*a*b*d*
e + 3*a^2*e^2)))/(d + e*x) + 12*(70*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(5*b*d
- 3*a*e) - 20*c^3*d^2*e*(7*b*d - 3*a*e) + 6*c^2*e^2*(15*b^2*d^2 - 10*a*b*d*
e + a^2*e^2))*Log[d + e*x])/(12*e^9)
```

Maple [B] time = 0.059, size = 1306, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^4/(e*x+d)^5,x)
```

```
[Out] b^4*ln(e*x+d)/e^5+24/e^5/(e*x+d)*a^2*c^2*d+80/e^7/(e*x+d)*a*c^3*d^3-40/e^6/
(e*x+d)*b^3*c*d^2+120/e^7/(e*x+d)*b^2*c^2*d^3-140/e^8/(e*x+d)*b*c^3*d^4-10*
c^3/e^6*x^2*b*d+12*c^2/e^5*a*b*x+8/e^5/(e*x+d)^3*a^2*c^2*d^3-4/e^4/(e*x+d)^
3*a*b^3*d^2+8/e^7/(e*x+d)^3*a*c^3*d^5-20/3/e^6/(e*x+d)^3*b^3*c*d^4+12/e^7/(
e*x+d)^3*b^2*c^2*d^5-28/3/e^8/(e*x+d)^3*b*c^3*d^6-18/e^5/(e*x+d)^2*a^2*c^2*
d^2+6/e^4/(e*x+d)^2*a*b^3*d-30/e^7/(e*x+d)^2*a*c^3*d^4+20/e^6/(e*x+d)^2*b^3
*c*d^3-45/e^7/(e*x+d)^2*b^2*c^2*d^4+42/e^8/(e*x+d)^2*b*c^3*d^5+12/e^5*ln(e*
x+d)*a*b^2*c+60/e^7*ln(e*x+d)*c^3*a*d^2-20/e^6*ln(e*x+d)*b^3*c*d+90/e^7*ln(
e*x+d)*b^2*c^2*d^2-20*c^3/e^6*a*d*x-30*c^2/e^6*b^2*d*x+60*c^3/e^7*b*d^2*x+1
/e^2/(e*x+d)^4*d*a^3*b-1/e^3/(e*x+d)^4*a^3*c*d^2-3/2/e^3/(e*x+d)^4*d^2*a^2*
b^2-3/2/e^5/(e*x+d)^4*a^2*c^2*d^4+1/e^4/(e*x+d)^4*d^3*a*b^3-1/e^7/(e*x+d)^4
*a*c^3*d^6-140/e^8*ln(e*x+d)*b*c^3*d^3-12/e^4/(e*x+d)*a^2*b*c-1/4/e/(e*x+d)
^4*a^4+1/e^6/(e*x+d)^4*b^3*c*d^5-3/2/e^7/(e*x+d)^4*b^2*c^2*d^6+1/e^8/(e*x+d
)^4*b*c^3*d^7+8/3/e^3/(e*x+d)^3*a^3*c*d+4/e^3/(e*x+d)^3*a^2*b^2*d+1/4*c^4*x
^4/e^5-2/e^3/(e*x+d)^2*a^3*c-3/e^3/(e*x+d)^2*a^2*b^2+4/3*c^3/e^5*x^3*b-3/e^
5/(e*x+d)^2*b^4*d^2-14/e^9/(e*x+d)^2*c^4*d^6+6/e^5*ln(e*x+d)*c^2*a^2+70/e^9
*ln(e*x+d)*c^4*d^4-4/e^4/(e*x+d)*a*b^3+4/e^5/(e*x+d)*b^4*d+56/e^9/(e*x+d)*c
^4*d^5-5/3*c^4/e^6*x^3*d+2*c^3/e^5*x^2*a+3*c^2/e^5*x^2*b^2+15/2*c^4/e^7*x^2
```

```
*d^2+4*c/e^5*b^3*x-35*c^4/e^8*d^3*x-1/4/e^5/(e*x+d)^4*d^4*b^4-1/4/e^9/(e*x+
d)^4*c^4*d^8-4/3/e^2/(e*x+d)^3*a^3*b+4/3/e^5/(e*x+d)^3*b^4*d^3+8/3/e^9/(e*x
+d)^3*c^4*d^7+3/e^6/(e*x+d)^4*a*b*c^2*d^5-12/e^4/(e*x+d)^3*a^2*b*c*d^2+16/e
^5/(e*x+d)^3*a*b^2*c*d^3-20/e^6/(e*x+d)^3*a*b*c^2*d^4+3/e^4/(e*x+d)^4*d^3*a
^2*b*c-3/e^5/(e*x+d)^4*d^4*a*b^2*c+18/e^4/(e*x+d)^2*a^2*b*c*d-36/e^5/(e*x+d
)^2*a*b^2*c*d^2+60/e^6/(e*x+d)^2*a*b*c^2*d^3-60/e^6*ln(e*x+d)*a*b*c^2*d+48/
e^5/(e*x+d)*a*b^2*c*d-120/e^6/(e*x+d)*a*b*c^2*d^2
```

Maxima [B] time = 1.15388, size = 1138, normalized size = 2.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^5,x, algorithm="maxima")
```

```
[Out] 1/12*(533*c^4*d^8 - 1276*b*c^3*d^7*e - 4*a^3*b*d*e^7 - 3*a^4*e^8 + 342*(3*b
^2*c^2 + 2*a*c^3)*d^6*e^2 - 308*(b^3*c + 3*a*b*c^2)*d^5*e^3 + 25*(b^4 + 12*
a*b^2*c + 6*a^2*c^2)*d^4*e^4 - 12*(a*b^3 + 3*a^2*b*c)*d^3*e^5 - 2*(3*a^2*b^
2 + 2*a^3*c)*d^2*e^6 + 48*(14*c^4*d^5*e^3 - 35*b*c^3*d^4*e^4 + 10*(3*b^2*c^
2 + 2*a*c^3)*d^3*e^5 - 10*(b^3*c + 3*a*b*c^2)*d^2*e^6 + (b^4 + 12*a*b^2*c +
 6*a^2*c^2)*d*e^7 - (a*b^3 + 3*a^2*b*c)*e^8)*x^3 + 12*(154*c^4*d^6*e^2 - 37
8*b*c^3*d^5*e^3 + 105*(3*b^2*c^2 + 2*a*c^3)*d^4*e^4 - 100*(b^3*c + 3*a*b*c^
2)*d^3*e^5 + 9*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^2*e^6 - 6*(a*b^3 + 3*a^2*b*
c)*d*e^7 - (3*a^2*b^2 + 2*a^3*c)*e^8)*x^2 + 8*(214*c^4*d^7*e - 518*b*c^3*d^
6*e^2 - 2*a^3*b*e^8 + 141*(3*b^2*c^2 + 2*a*c^3)*d^5*e^3 - 130*(b^3*c + 3*a*
b*c^2)*d^4*e^4 + 11*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^3*e^5 - 6*(a*b^3 + 3*a
^2*b*c)*d^2*e^6 - (3*a^2*b^2 + 2*a^3*c)*d*e^7)*x)/(e^13*x^4 + 4*d*e^12*x^3
+ 6*d^2*e^11*x^2 + 4*d^3*e^10*x + d^4*e^9) + 1/12*(3*c^4*e^3*x^4 - 4*(5*c^4
*d*e^2 - 4*b*c^3*e^3)*x^3 + 6*(15*c^4*d^2*e - 20*b*c^3*d*e^2 + 2*(3*b^2*c^2
 + 2*a*c^3)*e^3)*x^2 - 12*(35*c^4*d^3 - 60*b*c^3*d^2*e + 10*(3*b^2*c^2 + 2*
a*c^3)*d*e^2 - 4*(b^3*c + 3*a*b*c^2)*e^3)*x)/e^8 + (70*c^4*d^4 - 140*b*c^3*
d^3*e + 30*(3*b^2*c^2 + 2*a*c^3)*d^2*e^2 - 20*(b^3*c + 3*a*b*c^2)*d*e^3 + (
b^4 + 12*a*b^2*c + 6*a^2*c^2)*e^4)*log(e*x + d)/e^9
```

Fricas [B] time = 1.85231, size = 2774, normalized size = 6.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(3*c^4*e^8*x^8 + 533*c^4*d^8 - 1276*b*c^3*d^7*e - 4*a^3*b*d*e^7 - 3*a^
4*e^8 + 342*(3*b^2*c^2 + 2*a*c^3)*d^6*e^2 - 308*(b^3*c + 3*a*b*c^2)*d^5*e^3
 + 25*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^4*e^4 - 12*(a*b^3 + 3*a^2*b*c)*d^3*e
^5 - 2*(3*a^2*b^2 + 2*a^3*c)*d^2*e^6 - 8*(c^4*d*e^7 - 2*b*c^3*e^8)*x^7 + 4*
(7*c^4*d^2*e^6 - 14*b*c^3*d*e^7 + 3*(3*b^2*c^2 + 2*a*c^3)*e^8)*x^6 - 24*(7*
c^4*d^3*e^5 - 14*b*c^3*d^2*e^6 + 3*(3*b^2*c^2 + 2*a*c^3)*d*e^7 - 2*(b^3*c +
 3*a*b*c^2)*e^8)*x^5 - (1217*c^4*d^4*e^4 - 2224*b*c^3*d^3*e^5 + 408*(3*b^2*
c^2 + 2*a*c^3)*d^2*e^6 - 192*(b^3*c + 3*a*b*c^2)*d*e^7)*x^4 - 4*(377*c^4*d^
5*e^3 - 544*b*c^3*d^4*e^4 + 48*(3*b^2*c^2 + 2*a*c^3)*d^3*e^5 + 48*(b^3*c +
3*a*b*c^2)*d^2*e^6 - 12*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d*e^7 + 12*(a*b^3 +
3*a^2*b*c)*e^8)*x^3 + 6*(43*c^4*d^6*e^2 - 296*b*c^3*d^5*e^3 + 132*(3*b^2*c^
2 + 2*a*c^3)*d^4*e^4 - 168*(b^3*c + 3*a*b*c^2)*d^3*e^5 + 18*(b^4 + 12*a*b^2
```

```
*c + 6*a^2*c^2)*d^2*e^6 - 12*(a*b^3 + 3*a^2*b*c)*d*e^7 - 2*(3*a^2*b^2 + 2*a
^3*c)*e^8)*x^2 + 4*(323*c^4*d^7*e - 856*b*c^3*d^6*e^2 - 4*a^3*b*e^8 + 252*(
3*b^2*c^2 + 2*a*c^3)*d^5*e^3 - 248*(b^3*c + 3*a*b*c^2)*d^4*e^4 + 22*(b^4 +
12*a*b^2*c + 6*a^2*c^2)*d^3*e^5 - 12*(a*b^3 + 3*a^2*b*c)*d^2*e^6 - 2*(3*a^2
*b^2 + 2*a^3*c)*d*e^7)*x + 12*(70*c^4*d^8 - 140*b*c^3*d^7*e + 30*(3*b^2*c^2
 + 2*a*c^3)*d^6*e^2 - 20*(b^3*c + 3*a*b*c^2)*d^5*e^3 + (b^4 + 12*a*b^2*c +
6*a^2*c^2)*d^4*e^4 + (70*c^4*d^4*e^4 - 140*b*c^3*d^3*e^5 + 30*(3*b^2*c^2 +
2*a*c^3)*d^2*e^6 - 20*(b^3*c + 3*a*b*c^2)*d*e^7 + (b^4 + 12*a*b^2*c + 6*a^2
*c^2)*e^8)*x^4 + 4*(70*c^4*d^5*e^3 - 140*b*c^3*d^4*e^4 + 30*(3*b^2*c^2 + 2*
a*c^3)*d^3*e^5 - 20*(b^3*c + 3*a*b*c^2)*d^2*e^6 + (b^4 + 12*a*b^2*c + 6*a^2
*c^2)*d*e^7)*x^3 + 6*(70*c^4*d^6*e^2 - 140*b*c^3*d^5*e^3 + 30*(3*b^2*c^2 +
2*a*c^3)*d^4*e^4 - 20*(b^3*c + 3*a*b*c^2)*d^3*e^5 + (b^4 + 12*a*b^2*c + 6*a
^2*c^2)*d^2*e^6)*x^2 + 4*(70*c^4*d^7*e - 140*b*c^3*d^6*e^2 + 30*(3*b^2*c^2
+ 2*a*c^3)*d^5*e^3 - 20*(b^3*c + 3*a*b*c^2)*d^4*e^4 + (b^4 + 12*a*b^2*c + 6
*a^2*c^2)*d^3*e^5)*x)*log(e*x + d))/(e^13*x^4 + 4*d*e^12*x^3 + 6*d^2*e^11*x
^2 + 4*d^3*e^10*x + d^4*e^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**4/(e*x+d)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16583, size = 1732, normalized size = 4.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] 1/12*(3*c^4 - 16*(2*c^4*d*e - b*c^3*e^2)*e^(-1)/(x*e + d) + 12*(14*c^4*d^2*
e^2 - 14*b*c^3*d*e^3 + 3*b^2*c^2*e^4 + 2*a*c^3*e^4)*e^(-2)/(x*e + d)^2 - 48
*(14*c^4*d^3*e^3 - 21*b*c^3*d^2*e^4 + 9*b^2*c^2*d*e^5 + 6*a*c^3*d*e^5 - b^3
*c*e^6 - 3*a*b*c^2*e^6)*e^(-3)/(x*e + d)^3)*(x*e + d)^4*e^(-9) - (70*c^4*d^
4 - 140*b*c^3*d^3*e + 90*b^2*c^2*d^2*e^2 + 60*a*c^3*d^2*e^2 - 20*b^3*c*d*e^
3 - 60*a*b*c^2*d*e^3 + b^4*e^4 + 12*a*b^2*c*e^4 + 6*a^2*c^2*e^4)*e^(-9)*log
(abs(x*e + d)*e^(-1)/(x*e + d)^2) + 1/12*(672*c^4*d^5*e^43/(x*e + d) - 168*
c^4*d^6*e^43/(x*e + d)^2 + 32*c^4*d^7*e^43/(x*e + d)^3 - 3*c^4*d^8*e^43/(x*
e + d)^4 - 1680*b*c^3*d^4*e^44/(x*e + d) + 504*b*c^3*d^5*e^44/(x*e + d)^2 -
 112*b*c^3*d^6*e^44/(x*e + d)^3 + 12*b*c^3*d^7*e^44/(x*e + d)^4 + 1440*b^2*
c^2*d^3*e^45/(x*e + d) + 960*a*c^3*d^3*e^45/(x*e + d) - 540*b^2*c^2*d^4*e^4
5/(x*e + d)^2 - 360*a*c^3*d^4*e^45/(x*e + d)^2 + 144*b^2*c^2*d^5*e^45/(x*e
+ d)^3 + 96*a*c^3*d^5*e^45/(x*e + d)^3 - 18*b^2*c^2*d^6*e^45/(x*e + d)^4 -
12*a*c^3*d^6*e^45/(x*e + d)^4 - 480*b^3*c*d^2*e^46/(x*e + d) - 1440*a*b*c^2
*d^2*e^46/(x*e + d) + 240*b^3*c*d^3*e^46/(x*e + d)^2 + 720*a*b*c^2*d^3*e^46
/(x*e + d)^2 - 80*b^3*c*d^4*e^46/(x*e + d)^3 - 240*a*b*c^2*d^4*e^46/(x*e +
d)^3 + 12*b^3*c*d^5*e^46/(x*e + d)^4 + 36*a*b*c^2*d^5*e^46/(x*e + d)^4 + 48
*b^4*d*e^47/(x*e + d) + 576*a*b^2*c*d*e^47/(x*e + d) + 288*a^2*c^2*d*e^47/(
x*e + d) - 36*b^4*d^2*e^47/(x*e + d)^2 - 432*a*b^2*c*d^2*e^47/(x*e + d)^2 -
```

```
 216*a^2*c^2*d^2*e^47/(x*e + d)^2 + 16*b^4*d^3*e^47/(x*e + d)^3 + 192*a*b^2
*c*d^3*e^47/(x*e + d)^3 + 96*a^2*c^2*d^3*e^47/(x*e + d)^3 - 3*b^4*d^4*e^47/
(x*e + d)^4 - 36*a*b^2*c*d^4*e^47/(x*e + d)^4 - 18*a^2*c^2*d^4*e^47/(x*e +
d)^4 - 48*a*b^3*e^48/(x*e + d) - 144*a^2*b*c*e^48/(x*e + d) + 72*a*b^3*d*e^
48/(x*e + d)^2 + 216*a^2*b*c*d*e^48/(x*e + d)^2 - 48*a*b^3*d^2*e^48/(x*e +
d)^3 - 144*a^2*b*c*d^2*e^48/(x*e + d)^3 + 12*a*b^3*d^3*e^48/(x*e + d)^4 + 3
6*a^2*b*c*d^3*e^48/(x*e + d)^4 - 36*a^2*b^2*e^49/(x*e + d)^2 - 24*a^3*c*e^4
9/(x*e + d)^2 + 48*a^2*b^2*d*e^49/(x*e + d)^3 + 32*a^3*c*d*e^49/(x*e + d)^3
 - 18*a^2*b^2*d^2*e^49/(x*e + d)^4 - 12*a^3*c*d^2*e^49/(x*e + d)^4 - 16*a^3
*b*e^50/(x*e + d)^3 + 12*a^3*b*d*e^50/(x*e + d)^4 - 3*a^4*e^51/(x*e + d)^4)
*e^(-52)
```

3.2156 $\int \frac{(a+bx+cx^2)^4}{(d+ex)^6} dx$

Optimal. Leaf size=414

$$-\frac{6c^2e^2\left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4}{e^9(d+ex)}+\frac{c^2x\left(-4ce(6bd-ae)+6\right.}{e^8}$$

```
[Out] (c^2*(21*c^2*d^2 + 6*b^2*e^2 - 4*c*e*(6*b*d - a*e))*x)/e^8 - (c^3*(3*c*d -
2*b*e)*x^2)/e^7 + (c^4*x^3)/(3*e^6) - (c*d^2 - b*d*e + a*e^2)^4/(5*e^9*(d +
 e*x)^5) + ((2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)^3)/(e^9*(d + e*x)^4) - (2
*(c*d^2 - b*d*e + a*e^2)^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e)))/
(3*e^9*(d + e*x)^3) + (2*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*(7*c^2*d^2 +
 b^2*e^2 - c*e*(7*b*d - 3*a*e)))/(e^9*(d + e*x)^2) - (70*c^4*d^4 + b^4*e^4
- 4*b^2*c*e^3*(5*b*d - 3*a*e) - 20*c^3*d^2*e*(7*b*d - 3*a*e) + 6*c^2*e^2*(1
5*b^2*d^2 - 10*a*b*d*e + a^2*e^2))/(e^9*(d + e*x)) - (4*c*(2*c*d - b*e)*(7*
c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e))*Log[d + e*x])/e^9
```

Rubi [A] time = 0.557145, antiderivative size = 414, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$-\frac{6c^2e^2\left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4}{e^9(d+ex)}+\frac{c^2x\left(-4ce(6bd-ae)+6\right.}{e^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^4/(d + e*x)^6,x]
```


```
[Out] (c^2*(21*c^2*d^2 + 6*b^2*e^2 - 4*c*e*(6*b*d - a*e))*x)/e^8 - (c^3*(3*c*d -
2*b*e)*x^2)/e^7 + (c^4*x^3)/(3*e^6) - (c*d^2 - b*d*e + a*e^2)^4/(5*e^9*(d +
 e*x)^5) + ((2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)^3)/(e^9*(d + e*x)^4) - (2
*(c*d^2 - b*d*e + a*e^2)^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e)))/
(3*e^9*(d + e*x)^3) + (2*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*(7*c^2*d^2 +
 b^2*e^2 - c*e*(7*b*d - 3*a*e)))/(e^9*(d + e*x)^2) - (70*c^4*d^4 + b^4*e^4
- 4*b^2*c*e^3*(5*b*d - 3*a*e) - 20*c^3*d^2*e*(7*b*d - 3*a*e) + 6*c^2*e^2*(1
5*b^2*d^2 - 10*a*b*d*e + a^2*e^2))/(e^9*(d + e*x)) - (4*c*(2*c*d - b*e)*(7*
c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e))*Log[d + e*x])/e^9
```


Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^4}{(d+ex)^6} dx = \int \left(\frac{c^2\left(21c^2d^2+6b^2e^2-4ce(6bd-ae)\right)}{e^8} - \frac{2c^3(3cd-2be)x}{e^7} + \frac{c^4x^2}{e^6} + \frac{\left(cd^2-bde+ae^2\right)^4}{e^8(d+ex)^6} + \frac{4(}{}\right.$$

$$= \frac{c^2\left(21c^2d^2+6b^2e^2-4ce(6bd-ae)\right)x}{e^8} - \frac{c^3(3cd-2be)x^2}{e^7} + \frac{c^4x^3}{3e^6} - \frac{\left(cd^2-bde+ae^2\right)^4}{5e^9(d+ex)^5} + \frac{(2cd}{}$$

Mathematica [A] time = 0.233856, size = 419, normalized size = 1.01

$$\frac{-\frac{15\left(6c^2e^2\left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4\right)}{d+ex}+\frac{30(2cd-be)\left(ce^2\left(3a^2e^2-10abde+8b^2d^2\right)+b^2e^3(ae-bd)-2c^2d^2e(7bd-5ae)\right.}{(d+ex)^2}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^4/(d + e*x)^6,x]
```

```
[Out] (15*c^2*e*(21*c^2*d^2 + 6*b^2*e^2 + 4*c*e*(-6*b*d + a*e))*x + 15*c^3*e^2*(-
3*c*d + 2*b*e)*x^2 + 5*c^4*e^3*x^3 - (3*(c*d^2 + e*(-(b*d) + a*e))^4)/(d +
e*x)^5 + (15*(2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e))^3)/(d + e*x)^4 - (10*
(14*c^2*d^2 + 3*b^2*e^2 + 2*c*e*(-7*b*d + a*e))*(c*d^2 + e*(-(b*d) + a*e))^
2)/(d + e*x)^3 + (30*(2*c*d - b*e)*(7*c^3*d^4 - 2*c^2*d^2*e*(7*b*d - 5*a*e)
 + b^2*e^3*(-(b*d) + a*e) + c*e^2*(8*b^2*d^2 - 10*a*b*d*e + 3*a^2*e^2)))/(d
 + e*x)^2 - (15*(70*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(5*b*d - 3*a*e) - 20*c^
3*d^2*e*(7*b*d - 3*a*e) + 6*c^2*e^2*(15*b^2*d^2 - 10*a*b*d*e + a^2*e^2)))/(
d + e*x) - 60*c*(2*c*d - b*e)*(7*c^2*d^2 + b^2*e^2 + c*e*(-7*b*d + 3*a*e))*
Log[d + e*x])/(15*e^9)
```

Maple [B] time = 0.06, size = 1341, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^4/(e*x+d)^6,x)
```

```
[Out] 4*c/e^6*ln(e*x+d)*b^3-56*c^4/e^9*ln(e*x+d)*d^3-1/5/e^5/(e*x+d)^5*d^4*b^4-1/
5/e^9/(e*x+d)^5*c^4*d^8-6/e^5/(e*x+d)*c^2*a^2-70/e^9/(e*x+d)*c^4*d^4+6*c^2/
e^6*b^2*x+21*c^4/e^8*d^2*x-1/e^2/(e*x+d)^4*a^3*b+1/e^5/(e*x+d)^4*b^4*d^3+2/
e^9/(e*x+d)^4*c^4*d^7-4/3/e^3/(e*x+d)^3*a^3*c-2/e^3/(e*x+d)^3*a^2*b^2-2/e^5
/(e*x+d)^3*b^4*d^2-28/3/e^9/(e*x+d)^3*c^4*d^6-2/e^4/(e*x+d)^2*a*b^3+2/e^5/(
e*x+d)^2*b^4*d+28/e^9/(e*x+d)^2*c^4*d^5+2*c^3/e^6*x^2*b-3*c^4/e^7*x^2*d+4*c
^3/e^6*a*x-30/e^7/(e*x+d)^3*b^2*c^2*d^4+28/e^8/(e*x+d)^3*b*c^3*d^5-6/e^4/(e
*x+d)^2*a^2*b*c+12/e^5/(e*x+d)^2*a^2*c^2*d+40/e^7/(e*x+d)^2*a*c^3*d^3-20/e^
6/(e*x+d)^2*b^3*c*d^2+60/e^7/(e*x+d)^2*b^2*c^2*d^3-24*c^3/e^7*b*d*x+2/e^3/(
e*x+d)^4*a^3*c*d+3/e^3/(e*x+d)^4*a^2*b^2*d+6/e^5/(e*x+d)^4*a^2*c^2*d^3-3/e^
4/(e*x+d)^4*a*b^3*d^2+6/e^7/(e*x+d)^4*a*c^3*d^5-5/e^6/(e*x+d)^4*b^3*c*d^4+9
/e^7/(e*x+d)^4*b^2*c^2*d^5-7/e^8/(e*x+d)^4*b*c^3*d^6-12/e^5/(e*x+d)^3*a^2*c
^2*d^2+4/e^4/(e*x+d)^3*a*b^3*d-20/e^7/(e*x+d)^3*a*c^3*d^4+40/3/e^6/(e*x+d)^
3*b^3*c*d^3+12*c^2/e^6*ln(e*x+d)*a*b-24*c^3/e^7*ln(e*x+d)*a*d-1/5/e/(e*x+d)
^5*a^4-1/e^5/(e*x+d)*b^4-70/e^8/(e*x+d)^2*b*c^3*d^4-12/e^5/(e*x+d)*a*b^2*c-
60/e^7/(e*x+d)*c^3*a*d^2+20/e^6/(e*x+d)*b^3*c*d-90/e^7/(e*x+d)*b^2*c^2*d^2+
140/e^8/(e*x+d)*b*c^3*d^3+1/3*c^4*x^3/e^6-36*c^2/e^7*ln(e*x+d)*b^2*d+84*c^3
/e^8*ln(e*x+d)*b*d^2+4/5/e^2/(e*x+d)^5*d*a^3*b-4/5/e^3/(e*x+d)^5*a^3*c*d^2-
6/5/e^3/(e*x+d)^5*d^2*a^2*b^2-6/5/e^5/(e*x+d)^5*a^2*c^2*d^4+4/5/e^4/(e*x+d)
^5*d^3*a*b^3-4/5/e^7/(e*x+d)^5*a*c^3*d^6+4/5/e^6/(e*x+d)^5*d^5*b^3*c-6/5/e^
7/(e*x+d)^5*b^2*c^2*d^6+4/5/e^8/(e*x+d)^5*b*c^3*d^7+12/e^4/(e*x+d)^3*a^2*b*
c*d-24/e^5/(e*x+d)^3*a*b^2*c*d^2+40/e^6/(e*x+d)^3*a*b*c^2*d^3+24/e^5/(e*x+d
)^2*a*b^2*c*d-60/e^6/(e*x+d)^2*a*b*c^2*d^2+12/5/e^4/(e*x+d)^5*d^3*a^2*b*c+1
2/5/e^6/(e*x+d)^5*d^5*a*b*c^2+60/e^6/(e*x+d)*a*b*c^2*d-12/5/e^5/(e*x+d)^5*d
^4*a*b^2*c-9/e^4/(e*x+d)^4*a^2*b*c*d^2+12/e^5/(e*x+d)^4*a*b^2*c*d^3-15/e^6/
(e*x+d)^4*a*b*c^2*d^4
```

Maxima [B] time = 1.13759, size = 1148, normalized size = 2.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^6,x, algorithm="maxima")
```

```
[Out] -1/15*(743*c^4*d^8 - 1377*b*c^3*d^7*e + 3*a^3*b*d*e^7 + 3*a^4*e^8 + 261*(3*
b^2*c^2 + 2*a*c^3)*d^6*e^2 - 137*(b^3*c + 3*a*b*c^2)*d^5*e^3 + 3*(b^4 + 12*
a*b^2*c + 6*a^2*c^2)*d^4*e^4 + 3*(a*b^3 + 3*a^2*b*c)*d^3*e^5 + (3*a^2*b^2 +
 2*a^3*c)*d^2*e^6 + 15*(70*c^4*d^4*e^4 - 140*b*c^3*d^3*e^5 + 30*(3*b^2*c^2
+ 2*a*c^3)*d^2*e^6 - 20*(b^3*c + 3*a*b*c^2)*d*e^7 + (b^4 + 12*a*b^2*c + 6*a
^2*c^2)*e^8)*x^4 + 30*(126*c^4*d^5*e^3 - 245*b*c^3*d^4*e^4 + 50*(3*b^2*c^2
+ 2*a*c^3)*d^3*e^5 - 30*(b^3*c + 3*a*b*c^2)*d^2*e^6 + (b^4 + 12*a*b^2*c + 6
*a^2*c^2)*d*e^7 + (a*b^3 + 3*a^2*b*c)*e^8)*x^3 + 10*(518*c^4*d^6*e^2 - 987*
b*c^3*d^5*e^3 + 195*(3*b^2*c^2 + 2*a*c^3)*d^4*e^4 - 110*(b^3*c + 3*a*b*c^2)
*d^3*e^5 + 3*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^2*e^6 + 3*(a*b^3 + 3*a^2*b*c)
*d*e^7 + (3*a^2*b^2 + 2*a^3*c)*e^8)*x^2 + 5*(638*c^4*d^7*e - 1197*b*c^3*d^6
*e^2 + 3*a^3*b*e^8 + 231*(3*b^2*c^2 + 2*a*c^3)*d^5*e^3 - 125*(b^3*c + 3*a*b
*c^2)*d^4*e^4 + 3*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^3*e^5 + 3*(a*b^3 + 3*a^2
*b*c)*d^2*e^6 + (3*a^2*b^2 + 2*a^3*c)*d*e^7)*x)/(e^14*x^5 + 5*d*e^13*x^4 +
10*d^2*e^12*x^3 + 10*d^3*e^11*x^2 + 5*d^4*e^10*x + d^5*e^9) + 1/3*(c^4*e^2*
x^3 - 3*(3*c^4*d*e - 2*b*c^3*e^2)*x^2 + 3*(21*c^4*d^2 - 24*b*c^3*d*e + 2*(3
*b^2*c^2 + 2*a*c^3)*e^2)*x)/e^8 - 4*(14*c^4*d^3 - 21*b*c^3*d^2*e + 3*(3*b^2
*c^2 + 2*a*c^3)*d*e^2 - (b^3*c + 3*a*b*c^2)*e^3)*log(e*x + d)/e^9
```

Fricas [B] time = 1.77927, size = 2645, normalized size = 6.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^6,x, algorithm="fricas")
```

```
[Out] 1/15*(5*c^4*e^8*x^8 - 743*c^4*d^8 + 1377*b*c^3*d^7*e - 3*a^3*b*d*e^7 - 3*a^
4*e^8 - 261*(3*b^2*c^2 + 2*a*c^3)*d^6*e^2 + 137*(b^3*c + 3*a*b*c^2)*d^5*e^3
 - 3*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^4*e^4 - 3*(a*b^3 + 3*a^2*b*c)*d^3*e^5
 - (3*a^2*b^2 + 2*a^3*c)*d^2*e^6 - 10*(2*c^4*d*e^7 - 3*b*c^3*e^8)*x^7 + 10*
(14*c^4*d^2*e^6 - 21*b*c^3*d*e^7 + 3*(3*b^2*c^2 + 2*a*c^3)*e^8)*x^6 + 25*(4
7*c^4*d^3*e^5 - 60*b*c^3*d^2*e^6 + 6*(3*b^2*c^2 + 2*a*c^3)*d*e^7)*x^5 + 5*(
335*c^4*d^4*e^4 - 240*b*c^3*d^3*e^5 - 30*(3*b^2*c^2 + 2*a*c^3)*d^2*e^6 + 60
*(b^3*c + 3*a*b*c^2)*d*e^7 - 3*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*e^8)*x^4 - 10
*(85*c^4*d^5*e^3 - 390*b*c^3*d^4*e^4 + 120*(3*b^2*c^2 + 2*a*c^3)*d^3*e^5 -
90*(b^3*c + 3*a*b*c^2)*d^2*e^6 + 3*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d*e^7 + 3
*(a*b^3 + 3*a^2*b*c)*e^8)*x^3 - 10*(365*c^4*d^6*e^2 - 810*b*c^3*d^5*e^3 + 1
80*(3*b^2*c^2 + 2*a*c^3)*d^4*e^4 - 110*(b^3*c + 3*a*b*c^2)*d^3*e^5 + 3*(b^4
 + 12*a*b^2*c + 6*a^2*c^2)*d^2*e^6 + 3*(a*b^3 + 3*a^2*b*c)*d*e^7 + (3*a^2*b
^2 + 2*a^3*c)*e^8)*x^2 - 5*(575*c^4*d^7*e - 1125*b*c^3*d^6*e^2 + 3*a^3*b*e^
8 + 225*(3*b^2*c^2 + 2*a*c^3)*d^5*e^3 - 125*(b^3*c + 3*a*b*c^2)*d^4*e^4 + 3
*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^3*e^5 + 3*(a*b^3 + 3*a^2*b*c)*d^2*e^6 + (
3*a^2*b^2 + 2*a^3*c)*d*e^7)*x - 60*(14*c^4*d^8 - 21*b*c^3*d^7*e + 3*(3*b^2*
c^2 + 2*a*c^3)*d^6*e^2 - (b^3*c + 3*a*b*c^2)*d^5*e^3 + (14*c^4*d^3*e^5 - 21
*b*c^3*d^2*e^6 + 3*(3*b^2*c^2 + 2*a*c^3)*d*e^7 - (b^3*c + 3*a*b*c^2)*e^8)*x
^5 + 5*(14*c^4*d^4*e^4 - 21*b*c^3*d^3*e^5 + 3*(3*b^2*c^2 + 2*a*c^3)*d^2*e^6
```

```
 - (b^3*c + 3*a*b*c^2)*d*e^7)*x^4 + 10*(14*c^4*d^5*e^3 - 21*b*c^3*d^4*e^4 +
 3*(3*b^2*c^2 + 2*a*c^3)*d^3*e^5 - (b^3*c + 3*a*b*c^2)*d^2*e^6)*x^3 + 10*(1
4*c^4*d^6*e^2 - 21*b*c^3*d^5*e^3 + 3*(3*b^2*c^2 + 2*a*c^3)*d^4*e^4 - (b^3*c
 + 3*a*b*c^2)*d^3*e^5)*x^2 + 5*(14*c^4*d^7*e - 21*b*c^3*d^6*e^2 + 3*(3*b^2*
c^2 + 2*a*c^3)*d^5*e^3 - (b^3*c + 3*a*b*c^2)*d^4*e^4)*x)*log(e*x + d))/(e^1
4*x^5 + 5*d*e^13*x^4 + 10*d^2*e^12*x^3 + 10*d^3*e^11*x^2 + 5*d^4*e^10*x + d
^5*e^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**4/(e*x+d)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.12174, size = 1135, normalized size = 2.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^6,x, algorithm="giac")
```

```
[Out] -4*(14*c^4*d^3 - 21*b*c^3*d^2*e + 9*b^2*c^2*d*e^2 + 6*a*c^3*d*e^2 - b^3*c*e
^3 - 3*a*b*c^2*e^3)*e^(-9)*log(abs(x*e + d)) + 1/3*(c^4*x^3*e^12 - 9*c^4*d*
x^2*e^11 + 63*c^4*d^2*x*e^10 + 6*b*c^3*x^2*e^12 - 72*b*c^3*d*x*e^11 + 18*b^
2*c^2*x*e^12 + 12*a*c^3*x*e^12)*e^(-18) - 1/15*(743*c^4*d^8 - 1377*b*c^3*d^
7*e + 783*b^2*c^2*d^6*e^2 + 522*a*c^3*d^6*e^2 - 137*b^3*c*d^5*e^3 - 411*a*b
*c^2*d^5*e^3 + 3*b^4*d^4*e^4 + 36*a*b^2*c*d^4*e^4 + 18*a^2*c^2*d^4*e^4 + 3*
a*b^3*d^3*e^5 + 9*a^2*b*c*d^3*e^5 + 3*a^2*b^2*d^2*e^6 + 2*a^3*c*d^2*e^6 + 3
*a^3*b*d*e^7 + 15*(70*c^4*d^4*e^4 - 140*b*c^3*d^3*e^5 + 90*b^2*c^2*d^2*e^6
+ 60*a*c^3*d^2*e^6 - 20*b^3*c*d*e^7 - 60*a*b*c^2*d*e^7 + b^4*e^8 + 12*a*b^2
*c*e^8 + 6*a^2*c^2*e^8)*x^4 + 3*a^4*e^8 + 30*(126*c^4*d^5*e^3 - 245*b*c^3*d
^4*e^4 + 150*b^2*c^2*d^3*e^5 + 100*a*c^3*d^3*e^5 - 30*b^3*c*d^2*e^6 - 90*a*
b*c^2*d^2*e^6 + b^4*d*e^7 + 12*a*b^2*c*d*e^7 + 6*a^2*c^2*d*e^7 + a*b^3*e^8
+ 3*a^2*b*c*e^8)*x^3 + 10*(518*c^4*d^6*e^2 - 987*b*c^3*d^5*e^3 + 585*b^2*c^
2*d^4*e^4 + 390*a*c^3*d^4*e^4 - 110*b^3*c*d^3*e^5 - 330*a*b*c^2*d^3*e^5 + 3
*b^4*d^2*e^6 + 36*a*b^2*c*d^2*e^6 + 18*a^2*c^2*d^2*e^6 + 3*a*b^3*d*e^7 + 9*
a^2*b*c*d*e^7 + 3*a^2*b^2*e^8 + 2*a^3*c*e^8)*x^2 + 5*(638*c^4*d^7*e - 1197*
b*c^3*d^6*e^2 + 693*b^2*c^2*d^5*e^3 + 462*a*c^3*d^5*e^3 - 125*b^3*c*d^4*e^4
 - 375*a*b*c^2*d^4*e^4 + 3*b^4*d^3*e^5 + 36*a*b^2*c*d^3*e^5 + 18*a^2*c^2*d^
3*e^5 + 3*a*b^3*d^2*e^6 + 9*a^2*b*c*d^2*e^6 + 3*a^2*b^2*d*e^7 + 2*a^3*c*d*e
^7 + 3*a^3*b*e^8)*x)*e^(-9)/(x*e + d)^5
```

3.2157 $\int \frac{(a+bx+cx^2)^4}{(d+ex)^7}\,dx$

Optimal. Leaf size=426

$$-\frac{6c^2e^2\left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4}{2e^9(d+ex)^2}+\frac{4c(2cd-be)\left(-ce(7bd-\right.}{e^9(d+}$$

```
[Out] -((c^3*(7*c*d - 4*b*e)*x)/e^8) + (c^4*x^2)/(2*e^7) - (c*d^2 - b*d*e + a*e^2
)^4/(6*e^9*(d + e*x)^6) + (4*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)^3)/(5*e^
9*(d + e*x)^5) - ((c*d^2 - b*d*e + a*e^2)^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e
*(7*b*d - a*e)))/(2*e^9*(d + e*x)^4) + (4*(2*c*d - b*e)*(c*d^2 - b*d*e + a*
e^2)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e)))/(3*e^9*(d + e*x)^3) - (70
*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(5*b*d - 3*a*e) - 20*c^3*d^2*e*(7*b*d - 3*
a*e) + 6*c^2*e^2*(15*b^2*d^2 - 10*a*b*d*e + a^2*e^2))/(2*e^9*(d + e*x)^2) +
 (4*c*(2*c*d - b*e)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e)))/(e^9*(d +
e*x)) + (2*c^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e))*Log[d + e*x])
/e^9
```

Rubi [A] time = 0.507144, antiderivative size = 426, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$-\frac{6c^2e^2\left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4}{2e^9(d+ex)^2}+\frac{4c(2cd-be)\left(-ce(7bd-\right.}{e^9(d+}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^4/(d + e*x)^7,x]
```


```
[Out] -((c^3*(7*c*d - 4*b*e)*x)/e^8) + (c^4*x^2)/(2*e^7) - (c*d^2 - b*d*e + a*e^2
)^4/(6*e^9*(d + e*x)^6) + (4*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)^3)/(5*e^
9*(d + e*x)^5) - ((c*d^2 - b*d*e + a*e^2)^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e
*(7*b*d - a*e)))/(2*e^9*(d + e*x)^4) + (4*(2*c*d - b*e)*(c*d^2 - b*d*e + a*
e^2)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e)))/(3*e^9*(d + e*x)^3) - (70
*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(5*b*d - 3*a*e) - 20*c^3*d^2*e*(7*b*d - 3*
a*e) + 6*c^2*e^2*(15*b^2*d^2 - 10*a*b*d*e + a^2*e^2))/(2*e^9*(d + e*x)^2) +
 (4*c*(2*c*d - b*e)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e)))/(e^9*(d +
e*x)) + (2*c^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e))*Log[d + e*x])
/e^9
```


Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^4}{(d+ex)^7}\,dx = \int \left(-\frac{c^3(7cd-4be)}{e^8} + \frac{c^4x}{e^7} + \frac{(cd^2-bde+ae^2)^4}{e^8(d+ex)^7} + \frac{4(-2cd+be)(cd^2-bde+ae^2)^3}{e^8(d+ex)^6} + \frac{2(cd^2-bd}{}\right.$$

$$= -\frac{c^3(7cd-4be)x}{e^8} + \frac{c^4x^2}{2e^7} - \frac{(cd^2-bde+ae^2)^4}{6e^9(d+ex)^6} + \frac{4(2cd-be)(cd^2-bde+ae^2)^3}{5e^9(d+ex)^5} - \frac{(cd^2-bde+ae}{}$$

Mathematica [A] time = 0.3502, size = 764, normalized size = 1.79

$$\frac{-3c^2e^2\left(2a^2e^2\left(15d^2e^2x^2+6d^3ex+d^4+20de^3x^3+15e^4x^4\right)+20abe\left(15d^3e^2x^2+20d^2e^3x^3+6d^4ex+d^5+15de^4x^4+6e^5x\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^4/(d + e*x)^7,x]
```

```
[Out] (c^4*(1023*d^8 + 5298*d^7*e*x + 10725*d^6*e^2*x^2 + 10100*d^5*e^3*x^3 + 337
5*d^4*e^4*x^4 - 1170*d^3*e^5*x^5 - 1035*d^2*e^6*x^6 - 120*d*e^7*x^7 + 15*e^
8*x^8) - e^4*(5*a^4*e^4 + 4*a^3*b*e^3*(d + 6*e*x) + 3*a^2*b^2*e^2*(d^2 + 6*
d*e*x + 15*e^2*x^2) + 2*a*b^3*e*(d^3 + 6*d^2*e*x + 15*d*e^2*x^2 + 20*e^3*x^
3) + b^4*(d^4 + 6*d^3*e*x + 15*d^2*e^2*x^2 + 20*d*e^3*x^3 + 15*e^4*x^4)) -
2*c*e^3*(a^3*e^3*(d^2 + 6*d*e*x + 15*e^2*x^2) + 3*a^2*b*e^2*(d^3 + 6*d^2*e*
x + 15*d*e^2*x^2 + 20*e^3*x^3) + 6*a*b^2*e*(d^4 + 6*d^3*e*x + 15*d^2*e^2*x^
2 + 20*d*e^3*x^3 + 15*e^4*x^4) + 10*b^3*(d^5 + 6*d^4*e*x + 15*d^3*e^2*x^2 +
 20*d^2*e^3*x^3 + 15*d*e^4*x^4 + 6*e^5*x^5)) - 3*c^2*e^2*(2*a^2*e^2*(d^4 +
6*d^3*e*x + 15*d^2*e^2*x^2 + 20*d*e^3*x^3 + 15*e^4*x^4) + 20*a*b*e*(d^5 + 6
*d^4*e*x + 15*d^3*e^2*x^2 + 20*d^2*e^3*x^3 + 15*d*e^4*x^4 + 6*e^5*x^5) - b^
2*d*(147*d^5 + 822*d^4*e*x + 1875*d^3*e^2*x^2 + 2200*d^2*e^3*x^3 + 1350*d*e
^4*x^4 + 360*e^5*x^5)) + 2*c^3*e*(a*d*e*(147*d^5 + 822*d^4*e*x + 1875*d^3*e
^2*x^2 + 2200*d^2*e^3*x^3 + 1350*d*e^4*x^4 + 360*e^5*x^5) - b*(669*d^7 + 35
94*d^6*e*x + 7725*d^5*e^2*x^2 + 8200*d^4*e^3*x^3 + 4050*d^3*e^4*x^4 + 360*d
^2*e^5*x^5 - 360*d*e^6*x^6 - 60*e^7*x^7)) + 60*c^2*(14*c^2*d^2 + 3*b^2*e^2
+ 2*c*e*(-7*b*d + a*e))*(d + e*x)^6*Log[d + e*x])/(30*e^9*(d + e*x)^6)
```

Maple [B] time = 0.056, size = 1364, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^4/(e*x+d)^7,x)
```

```
[Out] -140/3/e^8/(e*x+d)^3*b*c^3*d^4-6/e^5/(e*x+d)^2*a*b^2*c+28*c^4/e^9*ln(e*x+d)
*d^2-1/6/e^5/(e*x+d)^6*d^4*b^4-1/6/e^9/(e*x+d)^6*c^4*d^8-4/5/e^2/(e*x+d)^5*
a^3*b+4/5/e^5/(e*x+d)^5*b^4*d^3+8/5/e^9/(e*x+d)^5*c^4*d^7-4*c/e^6/(e*x+d)*b
^3+56*c^4/e^9/(e*x+d)*d^3-1/e^3/(e*x+d)^4*a^3*c-3/2/e^3/(e*x+d)^4*a^2*b^2-3
/2/e^5/(e*x+d)^4*b^4*d^2-7/e^9/(e*x+d)^4*c^4*d^6-4/3/e^4/(e*x+d)^3*a*b^3-1/
e^5/(e*x+d)^6*a^2*c^2*d^4+2/3/e^4/(e*x+d)^6*d^3*a*b^3-2/3/e^7/(e*x+d)^6*a*c
^3*d^6+2/3/e^6/(e*x+d)^6*d^5*b^3*c-1/e^7/(e*x+d)^6*d^6*b^2*c^2+2/3/e^8/(e*x
+d)^6*b*c^3*d^7+8/5/e^3/(e*x+d)^5*a^3*c*d+12/5/e^3/(e*x+d)^5*a^2*b^2*d+24/5
/e^5/(e*x+d)^5*a^2*c^2*d^3-12/5/e^4/(e*x+d)^5*a*b^3*d^2+24/5/e^7/(e*x+d)^5*
a*c^3*d^5-4/e^6/(e*x+d)^5*b^3*c*d^4+36/5/e^7/(e*x+d)^5*b^2*c^2*d^5-28/5/e^8
/(e*x+d)^5*b*c^3*d^6+1/2*c^4*x^2/e^7-1/2/e^5/(e*x+d)^2*b^4-1/6/e/(e*x+d)^6*
a^4+4/3/e^5/(e*x+d)^3*b^4*d+56/3/e^9/(e*x+d)^3*c^4*d^5-3/e^5/(e*x+d)^2*c^2*
```

```
a^2-35/e^9/(e*x+d)^2*c^4*d^4+4*c^3/e^7*ln(e*x+d)*a+6*c^2/e^7*ln(e*x+d)*b^2+
4*c^3/e^7*x*b-7*c^4/e^8*x*d-30/e^7/(e*x+d)^2*c^3*a*d^2+10/e^6/(e*x+d)^2*b^3
*c*d-45/e^7/(e*x+d)^2*b^2*c^2*d^2+70/e^8/(e*x+d)^2*b*c^3*d^3-9/e^5/(e*x+d)^
4*a^2*c^2*d^2+3/e^4/(e*x+d)^4*a*b^3*d-15/e^7/(e*x+d)^4*a*c^3*d^4+10/e^6/(e*
x+d)^4*b^3*c*d^3-45/2/e^7/(e*x+d)^4*b^2*c^2*d^4+21/e^8/(e*x+d)^4*b*c^3*d^5-
4/e^4/(e*x+d)^3*a^2*b*c+8/e^5/(e*x+d)^3*a^2*c^2*d+80/3/e^7/(e*x+d)^3*a*c^3*
d^3-40/3/e^6/(e*x+d)^3*b^3*c*d^2+40/e^7/(e*x+d)^3*b^2*c^2*d^3-28*c^3/e^8*ln
(e*x+d)*b*d+2/3/e^2/(e*x+d)^6*d*a^3*b-2/3/e^3/(e*x+d)^6*a^3*c*d^2-1/e^3/(e*
x+d)^6*d^2*a^2*b^2-12*c^2/e^6/(e*x+d)*a*b+24*c^3/e^7/(e*x+d)*a*d+36*c^2/e^7
/(e*x+d)*b^2*d-84*c^3/e^8/(e*x+d)*b*d^2-2/e^5/(e*x+d)^6*d^4*a*b^2*c+2/e^6/(
e*x+d)^6*d^5*a*b*c^2-36/5/e^4/(e*x+d)^5*a^2*b*c*d^2+48/5/e^5/(e*x+d)^5*a*b^
2*c*d^3+9/e^4/(e*x+d)^4*a^2*b*c*d-18/e^5/(e*x+d)^4*a*b^2*c*d^2-12/e^6/(e*x+
d)^5*a*b*c^2*d^4+30/e^6/(e*x+d)^2*a*b*c^2*d+2/e^4/(e*x+d)^6*d^3*a^2*b*c-40/
e^6/(e*x+d)^3*a*b*c^2*d^2+30/e^6/(e*x+d)^4*a*b*c^2*d^3+16/e^5/(e*x+d)^3*a*b
^2*c*d
```

Maxima [B] time = 1.14516, size = 1169, normalized size = 2.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^7,x, algorithm="maxima")
```

```
[Out] 1/30*(1023*c^4*d^8 - 1338*b*c^3*d^7*e - 4*a^3*b*d*e^7 - 5*a^4*e^8 + 147*(3*
b^2*c^2 + 2*a*c^3)*d^6*e^2 - 20*(b^3*c + 3*a*b*c^2)*d^5*e^3 - (b^4 + 12*a*b
^2*c + 6*a^2*c^2)*d^4*e^4 - 2*(a*b^3 + 3*a^2*b*c)*d^3*e^5 - (3*a^2*b^2 + 2*
a^3*c)*d^2*e^6 + 120*(14*c^4*d^3*e^5 - 21*b*c^3*d^2*e^6 + 3*(3*b^2*c^2 + 2*
a*c^3)*d*e^7 - (b^3*c + 3*a*b*c^2)*e^8)*x^5 + 15*(490*c^4*d^4*e^4 - 700*b*c
^3*d^3*e^5 + 90*(3*b^2*c^2 + 2*a*c^3)*d^2*e^6 - 20*(b^3*c + 3*a*b*c^2)*d*e^
7 - (b^4 + 12*a*b^2*c + 6*a^2*c^2)*e^8)*x^4 + 20*(658*c^4*d^5*e^3 - 910*b*c
^3*d^4*e^4 + 110*(3*b^2*c^2 + 2*a*c^3)*d^3*e^5 - 20*(b^3*c + 3*a*b*c^2)*d^2
*e^6 - (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d*e^7 - 2*(a*b^3 + 3*a^2*b*c)*e^8)*x^
3 + 15*(798*c^4*d^6*e^2 - 1078*b*c^3*d^5*e^3 + 125*(3*b^2*c^2 + 2*a*c^3)*d^
4*e^4 - 20*(b^3*c + 3*a*b*c^2)*d^3*e^5 - (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^2
*e^6 - 2*(a*b^3 + 3*a^2*b*c)*d*e^7 - (3*a^2*b^2 + 2*a^3*c)*e^8)*x^2 + 6*(91
8*c^4*d^7*e - 1218*b*c^3*d^6*e^2 - 4*a^3*b*e^8 + 137*(3*b^2*c^2 + 2*a*c^3)*
d^5*e^3 - 20*(b^3*c + 3*a*b*c^2)*d^4*e^4 - (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d
^3*e^5 - 2*(a*b^3 + 3*a^2*b*c)*d^2*e^6 - (3*a^2*b^2 + 2*a^3*c)*d*e^7)*x)/(e
^15*x^6 + 6*d*e^14*x^5 + 15*d^2*e^13*x^4 + 20*d^3*e^12*x^3 + 15*d^4*e^11*x^
2 + 6*d^5*e^10*x + d^6*e^9) + 1/2*(c^4*e*x^2 - 2*(7*c^4*d - 4*b*c^3*e)*x)/e
^8 + 2*(14*c^4*d^2 - 14*b*c^3*d*e + (3*b^2*c^2 + 2*a*c^3)*e^2)*log(e*x + d)
/e^9
```

Fricas [B] time = 1.76318, size = 2498, normalized size = 5.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^7,x, algorithm="fricas")
```

```
[Out] 1/30*(15*c^4*e^8*x^8 + 1023*c^4*d^8 - 1338*b*c^3*d^7*e - 4*a^3*b*d*e^7 - 5*
a^4*e^8 + 147*(3*b^2*c^2 + 2*a*c^3)*d^6*e^2 - 20*(b^3*c + 3*a*b*c^2)*d^5*e^
3 - (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^4*e^4 - 2*(a*b^3 + 3*a^2*b*c)*d^3*e^5
```

```
- (3*a^2*b^2 + 2*a^3*c)*d^2*e^6 - 120*(c^4*d*e^7 - b*c^3*e^8)*x^7 - 45*(23*
c^4*d^2*e^6 - 16*b*c^3*d*e^7)*x^6 - 30*(39*c^4*d^3*e^5 + 24*b*c^3*d^2*e^6 -
 12*(3*b^2*c^2 + 2*a*c^3)*d*e^7 + 4*(b^3*c + 3*a*b*c^2)*e^8)*x^5 + 15*(225*
c^4*d^4*e^4 - 540*b*c^3*d^3*e^5 + 90*(3*b^2*c^2 + 2*a*c^3)*d^2*e^6 - 20*(b^
3*c + 3*a*b*c^2)*d*e^7 - (b^4 + 12*a*b^2*c + 6*a^2*c^2)*e^8)*x^4 + 20*(505*
c^4*d^5*e^3 - 820*b*c^3*d^4*e^4 + 110*(3*b^2*c^2 + 2*a*c^3)*d^3*e^5 - 20*(b
^3*c + 3*a*b*c^2)*d^2*e^6 - (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d*e^7 - 2*(a*b^3
 + 3*a^2*b*c)*e^8)*x^3 + 15*(715*c^4*d^6*e^2 - 1030*b*c^3*d^5*e^3 + 125*(3*
b^2*c^2 + 2*a*c^3)*d^4*e^4 - 20*(b^3*c + 3*a*b*c^2)*d^3*e^5 - (b^4 + 12*a*b
^2*c + 6*a^2*c^2)*d^2*e^6 - 2*(a*b^3 + 3*a^2*b*c)*d*e^7 - (3*a^2*b^2 + 2*a^
3*c)*e^8)*x^2 + 6*(883*c^4*d^7*e - 1198*b*c^3*d^6*e^2 - 4*a^3*b*e^8 + 137*(
3*b^2*c^2 + 2*a*c^3)*d^5*e^3 - 20*(b^3*c + 3*a*b*c^2)*d^4*e^4 - (b^4 + 12*a
*b^2*c + 6*a^2*c^2)*d^3*e^5 - 2*(a*b^3 + 3*a^2*b*c)*d^2*e^6 - (3*a^2*b^2 +
2*a^3*c)*d*e^7)*x + 60*(14*c^4*d^8 - 14*b*c^3*d^7*e + (3*b^2*c^2 + 2*a*c^3)
*d^6*e^2 + (14*c^4*d^2*e^6 - 14*b*c^3*d*e^7 + (3*b^2*c^2 + 2*a*c^3)*e^8)*x^
6 + 6*(14*c^4*d^3*e^5 - 14*b*c^3*d^2*e^6 + (3*b^2*c^2 + 2*a*c^3)*d*e^7)*x^5
 + 15*(14*c^4*d^4*e^4 - 14*b*c^3*d^3*e^5 + (3*b^2*c^2 + 2*a*c^3)*d^2*e^6)*x
^4 + 20*(14*c^4*d^5*e^3 - 14*b*c^3*d^4*e^4 + (3*b^2*c^2 + 2*a*c^3)*d^3*e^5)
*x^3 + 15*(14*c^4*d^6*e^2 - 14*b*c^3*d^5*e^3 + (3*b^2*c^2 + 2*a*c^3)*d^4*e^
4)*x^2 + 6*(14*c^4*d^7*e - 14*b*c^3*d^6*e^2 + (3*b^2*c^2 + 2*a*c^3)*d^5*e^3
)*x)*log(e*x + d))/(e^15*x^6 + 6*d*e^14*x^5 + 15*d^2*e^13*x^4 + 20*d^3*e^12
*x^3 + 15*d^4*e^11*x^2 + 6*d^5*e^10*x + d^6*e^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**4/(e*x+d)**7,x)

[Out] Timed out
```

Giac [B] time = 1.12314, size = 1137, normalized size = 2.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^7,x, algorithm="giac")

[Out] 2*(14*c^4*d^2 - 14*b*c^3*d*e + 3*b^2*c^2*e^2 + 2*a*c^3*e^2)*e^(-9)*log(abs(
x*e + d)) + 1/2*(c^4*x^2*e^7 - 14*c^4*d*x*e^6 + 8*b*c^3*x*e^7)*e^(-14) + 1/
30*(1023*c^4*d^8 - 1338*b*c^3*d^7*e + 441*b^2*c^2*d^6*e^2 + 294*a*c^3*d^6*e
^2 - 20*b^3*c*d^5*e^3 - 60*a*b*c^2*d^5*e^3 - b^4*d^4*e^4 - 12*a*b^2*c*d^4*e
^4 - 6*a^2*c^2*d^4*e^4 - 2*a*b^3*d^3*e^5 - 6*a^2*b*c*d^3*e^5 - 3*a^2*b^2*d^
2*e^6 - 2*a^3*c*d^2*e^6 + 120*(14*c^4*d^3*e^5 - 21*b*c^3*d^2*e^6 + 9*b^2*c^
2*d*e^7 + 6*a*c^3*d*e^7 - b^3*c*e^8 - 3*a*b*c^2*e^8)*x^5 - 4*a^3*b*d*e^7 +
15*(490*c^4*d^4*e^4 - 700*b*c^3*d^3*e^5 + 270*b^2*c^2*d^2*e^6 + 180*a*c^3*d
^2*e^6 - 20*b^3*c*d*e^7 - 60*a*b*c^2*d*e^7 - b^4*e^8 - 12*a*b^2*c*e^8 - 6*a
^2*c^2*e^8)*x^4 - 5*a^4*e^8 + 20*(658*c^4*d^5*e^3 - 910*b*c^3*d^4*e^4 + 330
*b^2*c^2*d^3*e^5 + 220*a*c^3*d^3*e^5 - 20*b^3*c*d^2*e^6 - 60*a*b*c^2*d^2*e^
6 - b^4*d*e^7 - 12*a*b^2*c*d*e^7 - 6*a^2*c^2*d*e^7 - 2*a*b^3*e^8 - 6*a^2*b*
c*e^8)*x^3 + 15*(798*c^4*d^6*e^2 - 1078*b*c^3*d^5*e^3 + 375*b^2*c^2*d^4*e^4
```

```
 + 250*a*c^3*d^4*e^4 - 20*b^3*c*d^3*e^5 - 60*a*b*c^2*d^3*e^5 - b^4*d^2*e^6
- 12*a*b^2*c*d^2*e^6 - 6*a^2*c^2*d^2*e^6 - 2*a*b^3*d*e^7 - 6*a^2*b*c*d*e^7
- 3*a^2*b^2*e^8 - 2*a^3*c*e^8)*x^2 + 6*(918*c^4*d^7*e - 1218*b*c^3*d^6*e^2
+ 411*b^2*c^2*d^5*e^3 + 274*a*c^3*d^5*e^3 - 20*b^3*c*d^4*e^4 - 60*a*b*c^2*d
^4*e^4 - b^4*d^3*e^5 - 12*a*b^2*c*d^3*e^5 - 6*a^2*c^2*d^3*e^5 - 2*a*b^3*d^2
*e^6 - 6*a^2*b*c*d^2*e^6 - 3*a^2*b^2*d*e^7 - 2*a^3*c*d*e^7 - 4*a^3*b*e^8)*x
)*e^(-9)/(x*e + d)^6
```

3.2158 $\int \frac{(a+bx+cx^2)^4}{(d+ex)^8}\,dx$

Optimal. Leaf size=424

$$-\frac{6c^2e^2\left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4}{3e^9(d+ex)^3}-\frac{2c^2\left(-2ce(7bd-ae)+3b^2e\right.}{e^9(d+ex)}$$

```
[Out] (c^4*x)/e^8 - (c*d^2 - b*d*e + a*e^2)^4/(7*e^9*(d + e*x)^7) + (2*(2*c*d - b
*e)*(c*d^2 - b*d*e + a*e^2)^3)/(3*e^9*(d + e*x)^6) - (2*(c*d^2 - b*d*e + a*
e^2)^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e)))/(5*e^9*(d + e*x)^5)
+ ((2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d
- 3*a*e)))/(e^9*(d + e*x)^4) - (70*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(5*b*d -
 3*a*e) - 20*c^3*d^2*e*(7*b*d - 3*a*e) + 6*c^2*e^2*(15*b^2*d^2 - 10*a*b*d*e
 + a^2*e^2))/(3*e^9*(d + e*x)^3) + (2*c*(2*c*d - b*e)*(7*c^2*d^2 + b^2*e^2
- c*e*(7*b*d - 3*a*e)))/(e^9*(d + e*x)^2) - (2*c^2*(14*c^2*d^2 + 3*b^2*e^2
- 2*c*e*(7*b*d - a*e)))/(e^9*(d + e*x)) - (4*c^3*(2*c*d - b*e)*Log[d + e*x]
)/e^9
```

Rubi [A] time = 0.481811, antiderivative size = 424, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$-\frac{6c^2e^2\left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4}{3e^9(d+ex)^3}-\frac{2c^2\left(-2ce(7bd-ae)+3b^2e\right.}{e^9(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^4/(d + e*x)^8,x]
```


```
[Out] (c^4*x)/e^8 - (c*d^2 - b*d*e + a*e^2)^4/(7*e^9*(d + e*x)^7) + (2*(2*c*d - b
*e)*(c*d^2 - b*d*e + a*e^2)^3)/(3*e^9*(d + e*x)^6) - (2*(c*d^2 - b*d*e + a*
e^2)^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e)))/(5*e^9*(d + e*x)^5)
+ ((2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d
- 3*a*e)))/(e^9*(d + e*x)^4) - (70*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(5*b*d -
 3*a*e) - 20*c^3*d^2*e*(7*b*d - 3*a*e) + 6*c^2*e^2*(15*b^2*d^2 - 10*a*b*d*e
 + a^2*e^2))/(3*e^9*(d + e*x)^3) + (2*c*(2*c*d - b*e)*(7*c^2*d^2 + b^2*e^2
- c*e*(7*b*d - 3*a*e)))/(e^9*(d + e*x)^2) - (2*c^2*(14*c^2*d^2 + 3*b^2*e^2
- 2*c*e*(7*b*d - a*e)))/(e^9*(d + e*x)) - (4*c^3*(2*c*d - b*e)*Log[d + e*x]
)/e^9
```


Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^4}{(d+ex)^8}\,dx = \int \left(\frac{c^4}{e^8} + \frac{\left(cd^2-bde+ae^2\right)^4}{e^8(d+ex)^8} + \frac{4(-2cd+be)\left(cd^2-bde+ae^2\right)^3}{e^8(d+ex)^7} + \frac{2\left(cd^2-bde+ae^2\right)^2\left(14c^2d\right.}{e^8(d+}\right.$$

$$= \frac{c^4x}{e^8} - \frac{\left(cd^2-bde+ae^2\right)^4}{7e^9(d+ex)^7} + \frac{2(2cd-be)\left(cd^2-bde+ae^2\right)^3}{3e^9(d+ex)^6} - \frac{2\left(cd^2-bde+ae^2\right)^2\left(14c^2d^2+3\right.}{5e^9(d+ex)}$$

Mathematica [A] time = 0.548716, size = 748, normalized size = 1.76

$$-\frac{6c^2e^2\left(a^2e^2\left(21d^2e^2x^2+7d^3ex+d^4+35de^3x^3+35e^4x^4\right)+5abe\left(21d^3e^2x^2+35d^2e^3x^3+7d^4ex+d^5+35de^4x^4+21e\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^4/(d + e*x)^8,x]
```

```
[Out] -(c^4*(1443*d^8 + 9261*d^7*e*x + 24843*d^6*e^2*x^2 + 35525*d^5*e^3*x^3 + 28
175*d^4*e^4*x^4 + 11025*d^3*e^5*x^5 + 735*d^2*e^6*x^6 - 735*d*e^7*x^7 - 105
*e^8*x^8) + e^4*(15*a^4*e^4 + 10*a^3*b*e^3*(d + 7*e*x) + 6*a^2*b^2*e^2*(d^2
 + 7*d*e*x + 21*e^2*x^2) + 3*a*b^3*e*(d^3 + 7*d^2*e*x + 21*d*e^2*x^2 + 35*e
^3*x^3) + b^4*(d^4 + 7*d^3*e*x + 21*d^2*e^2*x^2 + 35*d*e^3*x^3 + 35*e^4*x^4
)) + c*e^3*(4*a^3*e^3*(d^2 + 7*d*e*x + 21*e^2*x^2) + 9*a^2*b*e^2*(d^3 + 7*d
^2*e*x + 21*d*e^2*x^2 + 35*e^3*x^3) + 12*a*b^2*e*(d^4 + 7*d^3*e*x + 21*d^2*
e^2*x^2 + 35*d*e^3*x^3 + 35*e^4*x^4) + 10*b^3*(d^5 + 7*d^4*e*x + 21*d^3*e^2
*x^2 + 35*d^2*e^3*x^3 + 35*d*e^4*x^4 + 21*e^5*x^5)) + 6*c^2*e^2*(a^2*e^2*(d
^4 + 7*d^3*e*x + 21*d^2*e^2*x^2 + 35*d*e^3*x^3 + 35*e^4*x^4) + 5*a*b*e*(d^5
 + 7*d^4*e*x + 21*d^3*e^2*x^2 + 35*d^2*e^3*x^3 + 35*d*e^4*x^4 + 21*e^5*x^5)
 + 15*b^2*(d^6 + 7*d^5*e*x + 21*d^4*e^2*x^2 + 35*d^3*e^3*x^3 + 35*d^2*e^4*x
^4 + 21*d*e^5*x^5 + 7*e^6*x^6)) + c^3*e*(60*a*e*(d^6 + 7*d^5*e*x + 21*d^4*e
^2*x^2 + 35*d^3*e^3*x^3 + 35*d^2*e^4*x^4 + 21*d*e^5*x^5 + 7*e^6*x^6) - b*d*
(1089*d^6 + 7203*d^5*e*x + 20139*d^4*e^2*x^2 + 30625*d^3*e^3*x^3 + 26950*d^
2*e^4*x^4 + 13230*d*e^5*x^5 + 2940*e^6*x^6)) + 420*c^3*(2*c*d - b*e)*(d + e
*x)^7*Log[d + e*x])/(105*e^9*(d + e*x)^7)
```

Maple [B] time = 0.054, size = 1374, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^4/(e*x+d)^8,x)
```

```
[Out] -3/e^4/(e*x+d)^4*a^2*b*c+6/e^5/(e*x+d)^4*a^2*c^2*d+2/3/e^5/(e*x+d)^6*b^4*d^
3+4/3/e^9/(e*x+d)^6*c^4*d^7-4/5/e^3/(e*x+d)^5*a^3*c-6/5/e^3/(e*x+d)^5*a^2*b
^2-6/5/e^5/(e*x+d)^5*b^4*d^2-28/5/e^9/(e*x+d)^5*c^4*d^6-4*c^3/e^7/(e*x+d)*a
-6*c^2/e^7/(e*x+d)*b^2-28*c^4/e^9/(e*x+d)*d^2+14/e^9/(e*x+d)^4*c^4*d^5-2/e^
5/(e*x+d)^3*c^2*a^2-70/3/e^9/(e*x+d)^3*c^4*d^4-2*c/e^6/(e*x+d)^2*b^3+28*c^4
/e^9/(e*x+d)^2*d^3+4*c^3/e^8*ln(e*x+d)*b-8*c^4/e^9*ln(e*x+d)*d-1/e^4/(e*x+d
)^4*a*b^3+1/e^5/(e*x+d)^4*b^4*d-1/7/e^5/(e*x+d)^7*d^4*b^4-1/7/e^9/(e*x+d)^7
*c^4*d^8-2/3/e^2/(e*x+d)^6*a^3*b+36/5/e^4/(e*x+d)^5*a^2*b*c*d-72/5/e^5/(e*x
+d)^5*a*b^2*c*d^2-35/e^8/(e*x+d)^4*b*c^3*d^4-4/e^5/(e*x+d)^3*a*b^2*c-20/e^7
/(e*x+d)^3*c^3*a*d^2+28*c^3/e^8/(e*x+d)*b*d+4/3/e^3/(e*x+d)^6*a^3*c*d+2/e^3
/(e*x+d)^6*a^2*b^2*d+4/e^5/(e*x+d)^6*a^2*c^2*d^3-2/e^4/(e*x+d)^6*a*b^3*d^2+
4/e^7/(e*x+d)^6*a*c^3*d^5-10/3/e^6/(e*x+d)^6*b^3*c*d^4-14/3/e^8/(e*x+d)^6*b
```

```
*c^3*d^6-36/5/e^5/(e*x+d)^5*a^2*c^2*d^2+20/e^7/(e*x+d)^4*a*c^3*d^3-10/e^6/(
e*x+d)^4*b^3*c*d^2+4/7/e^2/(e*x+d)^7*d*a^3*b-4/7/e^3/(e*x+d)^7*a^3*c*d^2-6/
7/e^3/(e*x+d)^7*d^2*a^2*b^2-6/7/e^5/(e*x+d)^7*a^2*c^2*d^4+4/7/e^4/(e*x+d)^7
*d^3*a*b^3+12/5/e^4/(e*x+d)^5*a*b^3*d-12/e^7/(e*x+d)^5*a*c^3*d^4+8/e^6/(e*x
+d)^5*b^3*c*d^3-18/e^7/(e*x+d)^5*b^2*c^2*d^4+84/5/e^8/(e*x+d)^5*b*c^3*d^5+2
0/e^6/(e*x+d)^3*a*b*c^2*d-6/e^4/(e*x+d)^6*a^2*b*c*d^2+8/e^5/(e*x+d)^6*a*b^2
*c*d^3-10/e^6/(e*x+d)^6*a*b*c^2*d^4-30/e^6/(e*x+d)^4*a*b*c^2*d^2-1/7/e/(e*x
+d)^7*a^4-1/3/e^5/(e*x+d)^3*b^4-4/7/e^7/(e*x+d)^7*a*c^3*d^6+4/7/e^6/(e*x+d)
^7*d^5*b^3*c-6/7/e^7/(e*x+d)^7*d^6*b^2*c^2+4/7/e^8/(e*x+d)^7*d^7*b*c^3+20/3
/e^6/(e*x+d)^3*b^3*c*d-30/e^7/(e*x+d)^3*b^2*c^2*d^2+140/3/e^8/(e*x+d)^3*b*c
^3*d^3-6*c^2/e^6/(e*x+d)^2*a*b+12*c^3/e^7/(e*x+d)^2*a*d+18*c^2/e^7/(e*x+d)^
2*b^2*d-42*c^3/e^8/(e*x+d)^2*b*d^2+30/e^7/(e*x+d)^4*b^2*c^2*d^3+24/e^6/(e*x
+d)^5*a*b*c^2*d^3+6/e^7/(e*x+d)^6*b^2*c^2*d^5+12/e^5/(e*x+d)^4*a*b^2*c*d+12
/7/e^4/(e*x+d)^7*d^3*a^2*b*c-12/7/e^5/(e*x+d)^7*d^4*a*b^2*c+12/7/e^6/(e*x+d
)^7*d^5*a*b*c^2+c^4*x/e^8
```

Maxima [B] time = 1.12256, size = 1177, normalized size = 2.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^8,x, algorithm="maxima")
```

```
[Out] -1/105*(1443*c^4*d^8 - 1089*b*c^3*d^7*e + 10*a^3*b*d*e^7 + 15*a^4*e^8 + 30*
(3*b^2*c^2 + 2*a*c^3)*d^6*e^2 + 10*(b^3*c + 3*a*b*c^2)*d^5*e^3 + (b^4 + 12*
a*b^2*c + 6*a^2*c^2)*d^4*e^4 + 3*(a*b^3 + 3*a^2*b*c)*d^3*e^5 + 2*(3*a^2*b^2
 + 2*a^3*c)*d^2*e^6 + 210*(14*c^4*d^2*e^6 - 14*b*c^3*d*e^7 + (3*b^2*c^2 + 2
*a*c^3)*e^8)*x^6 + 210*(70*c^4*d^3*e^5 - 63*b*c^3*d^2*e^6 + 3*(3*b^2*c^2 +
2*a*c^3)*d*e^7 + (b^3*c + 3*a*b*c^2)*e^8)*x^5 + 35*(910*c^4*d^4*e^4 - 770*b
*c^3*d^3*e^5 + 30*(3*b^2*c^2 + 2*a*c^3)*d^2*e^6 + 10*(b^3*c + 3*a*b*c^2)*d*
e^7 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*e^8)*x^4 + 35*(1078*c^4*d^5*e^3 - 875*
b*c^3*d^4*e^4 + 30*(3*b^2*c^2 + 2*a*c^3)*d^3*e^5 + 10*(b^3*c + 3*a*b*c^2)*d
^2*e^6 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d*e^7 + 3*(a*b^3 + 3*a^2*b*c)*e^8)*
x^3 + 21*(1218*c^4*d^6*e^2 - 959*b*c^3*d^5*e^3 + 30*(3*b^2*c^2 + 2*a*c^3)*d
^4*e^4 + 10*(b^3*c + 3*a*b*c^2)*d^3*e^5 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^
2*e^6 + 3*(a*b^3 + 3*a^2*b*c)*d*e^7 + 2*(3*a^2*b^2 + 2*a^3*c)*e^8)*x^2 + 7*
(1338*c^4*d^7*e - 1029*b*c^3*d^6*e^2 + 10*a^3*b*e^8 + 30*(3*b^2*c^2 + 2*a*c
^3)*d^5*e^3 + 10*(b^3*c + 3*a*b*c^2)*d^4*e^4 + (b^4 + 12*a*b^2*c + 6*a^2*c^
2)*d^3*e^5 + 3*(a*b^3 + 3*a^2*b*c)*d^2*e^6 + 2*(3*a^2*b^2 + 2*a^3*c)*d*e^7)
*x)/(e^16*x^7 + 7*d*e^15*x^6 + 21*d^2*e^14*x^5 + 35*d^3*e^13*x^4 + 35*d^4*e
^12*x^3 + 21*d^5*e^11*x^2 + 7*d^6*e^10*x + d^7*e^9) + c^4*x/e^8 - 4*(2*c^4*
d - b*c^3*e)*log(e*x + d)/e^9
```

Fricas [B] time = 1.80653, size = 2275, normalized size = 5.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^8,x, algorithm="fricas")
```

```
[Out] 1/105*(105*c^4*e^8*x^8 + 735*c^4*d*e^7*x^7 - 1443*c^4*d^8 + 1089*b*c^3*d^7*
e - 10*a^3*b*d*e^7 - 15*a^4*e^8 - 30*(3*b^2*c^2 + 2*a*c^3)*d^6*e^2 - 10*(b^
3*c + 3*a*b*c^2)*d^5*e^3 - (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^4*e^4 - 3*(a*b^
```

```
3 + 3*a^2*b*c)*d^3*e^5 - 2*(3*a^2*b^2 + 2*a^3*c)*d^2*e^6 - 105*(7*c^4*d^2*e
^6 - 28*b*c^3*d*e^7 + 2*(3*b^2*c^2 + 2*a*c^3)*e^8)*x^6 - 105*(105*c^4*d^3*e
^5 - 126*b*c^3*d^2*e^6 + 6*(3*b^2*c^2 + 2*a*c^3)*d*e^7 + 2*(b^3*c + 3*a*b*c
^2)*e^8)*x^5 - 35*(805*c^4*d^4*e^4 - 770*b*c^3*d^3*e^5 + 30*(3*b^2*c^2 + 2*
a*c^3)*d^2*e^6 + 10*(b^3*c + 3*a*b*c^2)*d*e^7 + (b^4 + 12*a*b^2*c + 6*a^2*c
^2)*e^8)*x^4 - 35*(1015*c^4*d^5*e^3 - 875*b*c^3*d^4*e^4 + 30*(3*b^2*c^2 + 2
*a*c^3)*d^3*e^5 + 10*(b^3*c + 3*a*b*c^2)*d^2*e^6 + (b^4 + 12*a*b^2*c + 6*a^
2*c^2)*d*e^7 + 3*(a*b^3 + 3*a^2*b*c)*e^8)*x^3 - 21*(1183*c^4*d^6*e^2 - 959*
b*c^3*d^5*e^3 + 30*(3*b^2*c^2 + 2*a*c^3)*d^4*e^4 + 10*(b^3*c + 3*a*b*c^2)*d
^3*e^5 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^2*e^6 + 3*(a*b^3 + 3*a^2*b*c)*d*e
^7 + 2*(3*a^2*b^2 + 2*a^3*c)*e^8)*x^2 - 7*(1323*c^4*d^7*e - 1029*b*c^3*d^6*
e^2 + 10*a^3*b*e^8 + 30*(3*b^2*c^2 + 2*a*c^3)*d^5*e^3 + 10*(b^3*c + 3*a*b*c
^2)*d^4*e^4 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^3*e^5 + 3*(a*b^3 + 3*a^2*b*c
)*d^2*e^6 + 2*(3*a^2*b^2 + 2*a^3*c)*d*e^7)*x - 420*(2*c^4*d^8 - b*c^3*d^7*e
 + (2*c^4*d*e^7 - b*c^3*e^8)*x^7 + 7*(2*c^4*d^2*e^6 - b*c^3*d*e^7)*x^6 + 21
*(2*c^4*d^3*e^5 - b*c^3*d^2*e^6)*x^5 + 35*(2*c^4*d^4*e^4 - b*c^3*d^3*e^5)*x
^4 + 35*(2*c^4*d^5*e^3 - b*c^3*d^4*e^4)*x^3 + 21*(2*c^4*d^6*e^2 - b*c^3*d^5
*e^3)*x^2 + 7*(2*c^4*d^7*e - b*c^3*d^6*e^2)*x)*log(e*x + d))/(e^16*x^7 + 7*
d*e^15*x^6 + 21*d^2*e^14*x^5 + 35*d^3*e^13*x^4 + 35*d^4*e^12*x^3 + 21*d^5*e
^11*x^2 + 7*d^6*e^10*x + d^7*e^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**4/(e*x+d)**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.15361, size = 1125, normalized size = 2.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^8,x, algorithm="giac")
```

```
[Out] c^4*x*e^(-8) - 4*(2*c^4*d - b*c^3*e)*e^(-9)*log(abs(x*e + d)) - 1/105*(1443
*c^4*d^8 - 1089*b*c^3*d^7*e + 90*b^2*c^2*d^6*e^2 + 60*a*c^3*d^6*e^2 + 10*b^
3*c*d^5*e^3 + 30*a*b*c^2*d^5*e^3 + b^4*d^4*e^4 + 12*a*b^2*c*d^4*e^4 + 6*a^2
*c^2*d^4*e^4 + 3*a*b^3*d^3*e^5 + 9*a^2*b*c*d^3*e^5 + 210*(14*c^4*d^2*e^6 -
14*b*c^3*d*e^7 + 3*b^2*c^2*e^8 + 2*a*c^3*e^8)*x^6 + 6*a^2*b^2*d^2*e^6 + 4*a
^3*c*d^2*e^6 + 210*(70*c^4*d^3*e^5 - 63*b*c^3*d^2*e^6 + 9*b^2*c^2*d*e^7 + 6
*a*c^3*d*e^7 + b^3*c*e^8 + 3*a*b*c^2*e^8)*x^5 + 10*a^3*b*d*e^7 + 35*(910*c^
4*d^4*e^4 - 770*b*c^3*d^3*e^5 + 90*b^2*c^2*d^2*e^6 + 60*a*c^3*d^2*e^6 + 10*
b^3*c*d*e^7 + 30*a*b*c^2*d*e^7 + b^4*e^8 + 12*a*b^2*c*e^8 + 6*a^2*c^2*e^8)*
x^4 + 15*a^4*e^8 + 35*(1078*c^4*d^5*e^3 - 875*b*c^3*d^4*e^4 + 90*b^2*c^2*d^
3*e^5 + 60*a*c^3*d^3*e^5 + 10*b^3*c*d^2*e^6 + 30*a*b*c^2*d^2*e^6 + b^4*d*e^
7 + 12*a*b^2*c*d*e^7 + 6*a^2*c^2*d*e^7 + 3*a*b^3*e^8 + 9*a^2*b*c*e^8)*x^3 +
 21*(1218*c^4*d^6*e^2 - 959*b*c^3*d^5*e^3 + 90*b^2*c^2*d^4*e^4 + 60*a*c^3*d
^4*e^4 + 10*b^3*c*d^3*e^5 + 30*a*b*c^2*d^3*e^5 + b^4*d^2*e^6 + 12*a*b^2*c*d
^2*e^6 + 6*a^2*c^2*d^2*e^6 + 3*a*b^3*d*e^7 + 9*a^2*b*c*d*e^7 + 6*a^2*b^2*e^
```

8 + 4*a^3*c*e^8)*x^2 + 7*(1338*c^4*d^7*e - 1029*b*c^3*d^6*e^2 + 90*b^2*c^2*d^5*e^3 + 60*a*c^3*d^5*e^3 + 10*b^3*c*d^4*e^4 + 30*a*b*c^2*d^4*e^4 + b^4*d^3*e^5 + 12*a*b^2*c*d^3*e^5 + 6*a^2*c^2*d^3*e^5 + 3*a*b^3*d^2*e^6 + 9*a^2*b*c*d^2*e^6 + 6*a^2*b^2*d*e^7 + 4*a^3*c*d*e^7 + 10*a^3*b*e^8)*x)*e^(-9)/(x*e + d)^7

3.2159 $\int \frac{(a+bx+cx^2)^4}{(d+ex)^9}\,dx$

Optimal. Leaf size=435

$$-\frac{6c^2e^2\left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4}{4e^9(d+ex)^4}-\frac{c^2\left(-2ce(7bd-ae)+3b\right.}{e^9(d+ex)^2}$$

```
[Out] -(c*d^2 - b*d*e + a*e^2)^4/(8*e^9*(d + e*x)^8) + (4*(2*c*d - b*e)*(c*d^2 -
b*d*e + a*e^2)^3)/(7*e^9*(d + e*x)^7) - ((c*d^2 - b*d*e + a*e^2)^2*(14*c^2*
d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e)))/(3*e^9*(d + e*x)^6) + (4*(2*c*d - b
*e)*(c*d^2 - b*d*e + a*e^2)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e)))/(5
*e^9*(d + e*x)^5) - (70*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(5*b*d - 3*a*e) - 2
0*c^3*d^2*e*(7*b*d - 3*a*e) + 6*c^2*e^2*(15*b^2*d^2 - 10*a*b*d*e + a^2*e^2)
)/(4*e^9*(d + e*x)^4) + (4*c*(2*c*d - b*e)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*
d - 3*a*e)))/(3*e^9*(d + e*x)^3) - (c^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*
b*d - a*e)))/(e^9*(d + e*x)^2) + (4*c^3*(2*c*d - b*e))/(e^9*(d + e*x)) + (c
^4*Log[d + e*x])/e^9
```

Rubi [A] time = 0.462165, antiderivative size = 435, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$-\frac{6c^2e^2\left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4}{4e^9(d+ex)^4}-\frac{c^2\left(-2ce(7bd-ae)+3b\right.}{e^9(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^4/(d + e*x)^9,x]
```

```
[Out] -(c*d^2 - b*d*e + a*e^2)^4/(8*e^9*(d + e*x)^8) + (4*(2*c*d - b*e)*(c*d^2 -
b*d*e + a*e^2)^3)/(7*e^9*(d + e*x)^7) - ((c*d^2 - b*d*e + a*e^2)^2*(14*c^2*
d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e)))/(3*e^9*(d + e*x)^6) + (4*(2*c*d - b
*e)*(c*d^2 - b*d*e + a*e^2)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e)))/(5
*e^9*(d + e*x)^5) - (70*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(5*b*d - 3*a*e) - 2
0*c^3*d^2*e*(7*b*d - 3*a*e) + 6*c^2*e^2*(15*b^2*d^2 - 10*a*b*d*e + a^2*e^2)
)/(4*e^9*(d + e*x)^4) + (4*c*(2*c*d - b*e)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*
d - 3*a*e)))/(3*e^9*(d + e*x)^3) - (c^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*
b*d - a*e)))/(e^9*(d + e*x)^2) + (4*c^3*(2*c*d - b*e))/(e^9*(d + e*x)) + (c
^4*Log[d + e*x])/e^9
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^4}{(d+ex)^9}\,dx = \int \left(\frac{(cd^2-bde+ae^2)^4}{e^8(d+ex)^9} + \frac{4(-2cd+be)(cd^2-bde+ae^2)^3}{e^8(d+ex)^8} + \frac{2(cd^2-bde+ae^2)^2(14c^2d^2+3b^2e^2}{e^8(d+ex)^7}\right.$$

$$= -\frac{(cd^2-bde+ae^2)^4}{8e^9(d+ex)^8} + \frac{4(2cd-be)(cd^2-bde+ae^2)^3}{7e^9(d+ex)^7} - \frac{(cd^2-bde+ae^2)^2(14c^2d^2+3b^2e^2-2ce}{3e^9(d+ex)^6}$$

Mathematica [A] time = 0.352392, size = 740, normalized size = 1.7

$$-6c^2e^2\left(3a^2e^2\left(28d^2e^2x^2+8d^3ex+d^4+56de^3x^3+70e^4x^4\right)+10abe\left(28d^3e^2x^2+56d^2e^3x^3+8d^4ex+d^5+70de^4x^4+56e^5\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^4/(d + e*x)^9,x]
```

```
[Out] (c^4*d*(2283*d^7 + 17424*d^6*e*x + 57624*d^5*e^2*x^2 + 107408*d^4*e^3*x^3 +
 122500*d^3*e^4*x^4 + 86240*d^2*e^5*x^5 + 35280*d*e^6*x^6 + 6720*e^7*x^7) -
 3*e^4*(35*a^4*e^4 + 20*a^3*b*e^3*(d + 8*e*x) + 10*a^2*b^2*e^2*(d^2 + 8*d*e
*x + 28*e^2*x^2) + 4*a*b^3*e*(d^3 + 8*d^2*e*x + 28*d*e^2*x^2 + 56*e^3*x^3)
+ b^4*(d^4 + 8*d^3*e*x + 28*d^2*e^2*x^2 + 56*d*e^3*x^3 + 70*e^4*x^4)) - 4*c
*e^3*(5*a^3*e^3*(d^2 + 8*d*e*x + 28*e^2*x^2) + 9*a^2*b*e^2*(d^3 + 8*d^2*e*x
 + 28*d*e^2*x^2 + 56*e^3*x^3) + 9*a*b^2*e*(d^4 + 8*d^3*e*x + 28*d^2*e^2*x^2
 + 56*d*e^3*x^3 + 70*e^4*x^4) + 5*b^3*(d^5 + 8*d^4*e*x + 28*d^3*e^2*x^2 + 5
6*d^2*e^3*x^3 + 70*d*e^4*x^4 + 56*e^5*x^5)) - 6*c^2*e^2*(3*a^2*e^2*(d^4 + 8
*d^3*e*x + 28*d^2*e^2*x^2 + 56*d*e^3*x^3 + 70*e^4*x^4) + 10*a*b*e*(d^5 + 8*
d^4*e*x + 28*d^3*e^2*x^2 + 56*d^2*e^3*x^3 + 70*d*e^4*x^4 + 56*e^5*x^5) + 15
*b^2*(d^6 + 8*d^5*e*x + 28*d^4*e^2*x^2 + 56*d^3*e^3*x^3 + 70*d^2*e^4*x^4 +
56*d*e^5*x^5 + 28*e^6*x^6)) - 60*c^3*e*(a*e*(d^6 + 8*d^5*e*x + 28*d^4*e^2*x
^2 + 56*d^3*e^3*x^3 + 70*d^2*e^4*x^4 + 56*d*e^5*x^5 + 28*e^6*x^6) + 7*b*(d^
7 + 8*d^6*e*x + 28*d^5*e^2*x^2 + 56*d^4*e^3*x^3 + 70*d^3*e^4*x^4 + 56*d^2*e
^5*x^5 + 28*d*e^6*x^6 + 8*e^7*x^7)) + 840*c^4*(d + e*x)^8*Log[d + e*x])/(84
0*e^9*(d + e*x)^8)
```

Maple [B] time = 0.053, size = 1382, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^4/(e*x+d)^9,x)
```

```
[Out] -4*c^3/e^8/(e*x+d)*b+8*c^4/e^9/(e*x+d)*d-45/2/e^7/(e*x+d)^4*b^2*c^2*d^2+35/
e^8/(e*x+d)^4*b*c^3*d^3-4*c^2/e^6/(e*x+d)^3*a*b+8*c^3/e^7/(e*x+d)^3*a*d+12*
c^2/e^7/(e*x+d)^3*b^2*d-28*c^3/e^8/(e*x+d)^3*b*d^2+14*c^3/e^8/(e*x+d)^2*b*d
+1/2/e^2/(e*x+d)^8*a^3*b*d-15/e^7/(e*x+d)^4*c^3*a*d^2+5/e^6/(e*x+d)^4*b^3*c
*d+48/5/e^5/(e*x+d)^5*d*a*b^2*c-1/4*b^4/e^5/(e*x+d)^4+36/7/e^7/(e*x+d)^7*d^
5*b^2*c^2-4/e^8/(e*x+d)^7*b*c^3*d^6-3/e^5/(e*x+d)^4*a*c*b^2+1/2/e^8/(e*x+d)
^8*b*c^3*d^7-6/e^5/(e*x+d)^6*a^2*c^2*d^2+2/e^4/(e*x+d)^6*a*d*b^3-10/e^7/(e*
x+d)^6*a*c^3*d^4+20/3/e^6/(e*x+d)^6*d^3*b^3*c-15/e^7/(e*x+d)^6*d^4*b^2*c^2+
14/e^8/(e*x+d)^6*b*c^3*d^5-12/5/e^4/(e*x+d)^5*a^2*b*c+24/5/e^5/(e*x+d)^5*d*
a^2*c^2-1/2/e^3/(e*x+d)^8*a^3*c*d^2-3/4/e^3/(e*x+d)^8*a^2*b^2*d^2-3/4/e^5/(
e*x+d)^8*a^2*c^2*d^4+1/2/e^4/(e*x+d)^8*d^3*a*b^3-1/2/e^7/(e*x+d)^8*a*c^3*d^
6+1/2/e^6/(e*x+d)^8*d^5*b^3*c-3/4/e^7/(e*x+d)^8*d^6*b^2*c^2+8/7/e^3/(e*x+d)
```

```
^7*a^3*c*d+12/7/e^3/(e*x+d)^7*d*a^2*b^2+24/7/e^5/(e*x+d)^7*a^2*c^2*d^3-12/e
^5/(e*x+d)^6*a*b^2*c*d^2+20/e^6/(e*x+d)^6*d^3*a*b*c^2-1/8/e/(e*x+d)^8*a^4-4
/7/e^2/(e*x+d)^7*a^3*b+4/7/e^5/(e*x+d)^7*b^4*d^3+8/7/e^9/(e*x+d)^7*c^4*d^7-
3/2/e^5/(e*x+d)^4*c^2*a^2-35/2/e^9/(e*x+d)^4*c^4*d^4-4/3*c/e^6/(e*x+d)^3*b^
3+56/3*c^4/e^9/(e*x+d)^3*d^3-2*c^3/e^7/(e*x+d)^2*a-3*c^2/e^7/(e*x+d)^2*b^2-
14*c^4/e^9/(e*x+d)^2*d^2-1/8/e^5/(e*x+d)^8*b^4*d^4-1/8/e^9/(e*x+d)^8*c^4*d^
8-2/3/e^3/(e*x+d)^6*a^3*c-1/e^3/(e*x+d)^6*b^2*a^2-1/e^5/(e*x+d)^6*b^4*d^2-1
4/3/e^9/(e*x+d)^6*c^4*d^6-4/5/e^4/(e*x+d)^5*a*b^3+4/5/e^5/(e*x+d)^5*d*b^4+5
6/5/e^9/(e*x+d)^5*c^4*d^5+16/e^7/(e*x+d)^5*d^3*a*c^3-8/e^6/(e*x+d)^5*d^2*b^
3*c+24/e^7/(e*x+d)^5*d^3*b^2*c^2-28/e^8/(e*x+d)^5*d^4*b*c^3-12/7/e^4/(e*x+d
)^7*d^2*a*b^3+24/7/e^7/(e*x+d)^7*a*c^3*d^5-20/7/e^6/(e*x+d)^7*d^4*b^3*c-24/
e^6/(e*x+d)^5*d^2*a*b*c^2+6/e^4/(e*x+d)^6*d*a^2*b*c-3/2/e^5/(e*x+d)^8*d^4*a
*b^2*c+3/2/e^6/(e*x+d)^8*d^5*a*b*c^2+3/2/e^4/(e*x+d)^8*d^3*a^2*b*c+48/7/e^5
/(e*x+d)^7*d^3*a*b^2*c-60/7/e^6/(e*x+d)^7*d^4*a*b*c^2-36/7/e^4/(e*x+d)^7*c*
b*a^2*d^2+15/e^6/(e*x+d)^4*a*b*c^2*d+c^4*ln(e*x+d)/e^9
```

Maxima [B] time = 1.13881, size = 1207, normalized size = 2.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^9,x, algorithm="maxima")
```

```
[Out] 1/840*(2283*c^4*d^8 - 420*b*c^3*d^7*e - 60*a^3*b*d*e^7 - 105*a^4*e^8 - 30*(
3*b^2*c^2 + 2*a*c^3)*d^6*e^2 - 20*(b^3*c + 3*a*b*c^2)*d^5*e^3 - 3*(b^4 + 12
*a*b^2*c + 6*a^2*c^2)*d^4*e^4 - 12*(a*b^3 + 3*a^2*b*c)*d^3*e^5 - 10*(3*a^2*
b^2 + 2*a^3*c)*d^2*e^6 + 3360*(2*c^4*d*e^7 - b*c^3*e^8)*x^7 + 840*(42*c^4*d
^2*e^6 - 14*b*c^3*d*e^7 - (3*b^2*c^2 + 2*a*c^3)*e^8)*x^6 + 560*(154*c^4*d^3
*e^5 - 42*b*c^3*d^2*e^6 - 3*(3*b^2*c^2 + 2*a*c^3)*d*e^7 - 2*(b^3*c + 3*a*b*
c^2)*e^8)*x^5 + 70*(1750*c^4*d^4*e^4 - 420*b*c^3*d^3*e^5 - 30*(3*b^2*c^2 +
2*a*c^3)*d^2*e^6 - 20*(b^3*c + 3*a*b*c^2)*d*e^7 - 3*(b^4 + 12*a*b^2*c + 6*a
^2*c^2)*e^8)*x^4 + 56*(1918*c^4*d^5*e^3 - 420*b*c^3*d^4*e^4 - 30*(3*b^2*c^2
 + 2*a*c^3)*d^3*e^5 - 20*(b^3*c + 3*a*b*c^2)*d^2*e^6 - 3*(b^4 + 12*a*b^2*c
+ 6*a^2*c^2)*d*e^7 - 12*(a*b^3 + 3*a^2*b*c)*e^8)*x^3 + 28*(2058*c^4*d^6*e^2
 - 420*b*c^3*d^5*e^3 - 30*(3*b^2*c^2 + 2*a*c^3)*d^4*e^4 - 20*(b^3*c + 3*a*b
*c^2)*d^3*e^5 - 3*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^2*e^6 - 12*(a*b^3 + 3*a^
2*b*c)*d*e^7 - 10*(3*a^2*b^2 + 2*a^3*c)*e^8)*x^2 + 8*(2178*c^4*d^7*e - 420*
b*c^3*d^6*e^2 - 60*a^3*b*e^8 - 30*(3*b^2*c^2 + 2*a*c^3)*d^5*e^3 - 20*(b^3*c
 + 3*a*b*c^2)*d^4*e^4 - 3*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^3*e^5 - 12*(a*b^
3 + 3*a^2*b*c)*d^2*e^6 - 10*(3*a^2*b^2 + 2*a^3*c)*d*e^7)*x)/(e^17*x^8 + 8*d
*e^16*x^7 + 28*d^2*e^15*x^6 + 56*d^3*e^14*x^5 + 70*d^4*e^13*x^4 + 56*d^5*e^
12*x^3 + 28*d^6*e^11*x^2 + 8*d^7*e^10*x + d^8*e^9) + c^4*log(e*x + d)/e^9
```

Fricas [B] time = 1.78933, size = 2138, normalized size = 4.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^9,x, algorithm="fricas")
```

```
[Out] 1/840*(2283*c^4*d^8 - 420*b*c^3*d^7*e - 60*a^3*b*d*e^7 - 105*a^4*e^8 - 30*(
3*b^2*c^2 + 2*a*c^3)*d^6*e^2 - 20*(b^3*c + 3*a*b*c^2)*d^5*e^3 - 3*(b^4 + 12
*a*b^2*c + 6*a^2*c^2)*d^4*e^4 - 12*(a*b^3 + 3*a^2*b*c)*d^3*e^5 - 10*(3*a^2*
```

```
b^2 + 2*a^3*c)*d^2*e^6 + 3360*(2*c^4*d*e^7 - b*c^3*e^8)*x^7 + 840*(42*c^4*d
^2*e^6 - 14*b*c^3*d*e^7 - (3*b^2*c^2 + 2*a*c^3)*e^8)*x^6 + 560*(154*c^4*d^3
*e^5 - 42*b*c^3*d^2*e^6 - 3*(3*b^2*c^2 + 2*a*c^3)*d*e^7 - 2*(b^3*c + 3*a*b*
c^2)*e^8)*x^5 + 70*(1750*c^4*d^4*e^4 - 420*b*c^3*d^3*e^5 - 30*(3*b^2*c^2 +
2*a*c^3)*d^2*e^6 - 20*(b^3*c + 3*a*b*c^2)*d*e^7 - 3*(b^4 + 12*a*b^2*c + 6*a
^2*c^2)*e^8)*x^4 + 56*(1918*c^4*d^5*e^3 - 420*b*c^3*d^4*e^4 - 30*(3*b^2*c^2
 + 2*a*c^3)*d^3*e^5 - 20*(b^3*c + 3*a*b*c^2)*d^2*e^6 - 3*(b^4 + 12*a*b^2*c
+ 6*a^2*c^2)*d*e^7 - 12*(a*b^3 + 3*a^2*b*c)*e^8)*x^3 + 28*(2058*c^4*d^6*e^2
 - 420*b*c^3*d^5*e^3 - 30*(3*b^2*c^2 + 2*a*c^3)*d^4*e^4 - 20*(b^3*c + 3*a*b
*c^2)*d^3*e^5 - 3*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^2*e^6 - 12*(a*b^3 + 3*a^
2*b*c)*d*e^7 - 10*(3*a^2*b^2 + 2*a^3*c)*e^8)*x^2 + 8*(2178*c^4*d^7*e - 420*
b*c^3*d^6*e^2 - 60*a^3*b*e^8 - 30*(3*b^2*c^2 + 2*a*c^3)*d^5*e^3 - 20*(b^3*c
 + 3*a*b*c^2)*d^4*e^4 - 3*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^3*e^5 - 12*(a*b^
3 + 3*a^2*b*c)*d^2*e^6 - 10*(3*a^2*b^2 + 2*a^3*c)*d*e^7)*x + 840*(c^4*e^8*x
^8 + 8*c^4*d*e^7*x^7 + 28*c^4*d^2*e^6*x^6 + 56*c^4*d^3*e^5*x^5 + 70*c^4*d^4
*e^4*x^4 + 56*c^4*d^5*e^3*x^3 + 28*c^4*d^6*e^2*x^2 + 8*c^4*d^7*e*x + c^4*d^
8)*log(e*x + d))/(e^17*x^8 + 8*d*e^16*x^7 + 28*d^2*e^15*x^6 + 56*d^3*e^14*x
^5 + 70*d^4*e^13*x^4 + 56*d^5*e^12*x^3 + 28*d^6*e^11*x^2 + 8*d^7*e^10*x + d
^8*e^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**4/(e*x+d)**9,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10996, size = 1138, normalized size = 2.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^9,x, algorithm="giac")
```

```
[Out] c^4*e^(-9)*log(abs(x*e + d)) + 1/840*(3360*(2*c^4*d*e^6 - b*c^3*e^7)*x^7 +
840*(42*c^4*d^2*e^5 - 14*b*c^3*d*e^6 - 3*b^2*c^2*e^7 - 2*a*c^3*e^7)*x^6 + 5
60*(154*c^4*d^3*e^4 - 42*b*c^3*d^2*e^5 - 9*b^2*c^2*d*e^6 - 6*a*c^3*d*e^6 -
2*b^3*c*e^7 - 6*a*b*c^2*e^7)*x^5 + 70*(1750*c^4*d^4*e^3 - 420*b*c^3*d^3*e^4
 - 90*b^2*c^2*d^2*e^5 - 60*a*c^3*d^2*e^5 - 20*b^3*c*d*e^6 - 60*a*b*c^2*d*e^
6 - 3*b^4*e^7 - 36*a*b^2*c*e^7 - 18*a^2*c^2*e^7)*x^4 + 56*(1918*c^4*d^5*e^2
 - 420*b*c^3*d^4*e^3 - 90*b^2*c^2*d^3*e^4 - 60*a*c^3*d^3*e^4 - 20*b^3*c*d^2
*e^5 - 60*a*b*c^2*d^2*e^5 - 3*b^4*d*e^6 - 36*a*b^2*c*d*e^6 - 18*a^2*c^2*d*e
^6 - 12*a*b^3*e^7 - 36*a^2*b*c*e^7)*x^3 + 28*(2058*c^4*d^6*e - 420*b*c^3*d^
5*e^2 - 90*b^2*c^2*d^4*e^3 - 60*a*c^3*d^4*e^3 - 20*b^3*c*d^3*e^4 - 60*a*b*c
^2*d^3*e^4 - 3*b^4*d^2*e^5 - 36*a*b^2*c*d^2*e^5 - 18*a^2*c^2*d^2*e^5 - 12*a
*b^3*d*e^6 - 36*a^2*b*c*d*e^6 - 30*a^2*b^2*e^7 - 20*a^3*c*e^7)*x^2 + 8*(217
8*c^4*d^7 - 420*b*c^3*d^6*e - 90*b^2*c^2*d^5*e^2 - 60*a*c^3*d^5*e^2 - 20*b^
3*c*d^4*e^3 - 60*a*b*c^2*d^4*e^3 - 3*b^4*d^3*e^4 - 36*a*b^2*c*d^3*e^4 - 18*
a^2*c^2*d^3*e^4 - 12*a*b^3*d^2*e^5 - 36*a^2*b*c*d^2*e^5 - 30*a^2*b^2*d*e^6
- 20*a^3*c*d*e^6 - 60*a^3*b*e^7)*x + (2283*c^4*d^8 - 420*b*c^3*d^7*e - 90*b
```

```
^2*c^2*d^6*e^2 - 60*a*c^3*d^6*e^2 - 20*b^3*c*d^5*e^3 - 60*a*b*c^2*d^5*e^3 -
 3*b^4*d^4*e^4 - 36*a*b^2*c*d^4*e^4 - 18*a^2*c^2*d^4*e^4 - 12*a*b^3*d^3*e^5
 - 36*a^2*b*c*d^3*e^5 - 30*a^2*b^2*d^2*e^6 - 20*a^3*c*d^2*e^6 - 60*a^3*b*d*
e^7 - 105*a^4*e^8)*e^(-1))*e^(-8)/(x*e + d)^8
```

3.2160 $\int \frac{(a+bx+cx^2)^4}{(d+ex)^{10}} dx$

Optimal. Leaf size=436

$$-\frac{6c^2e^2\left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4}{5e^9(d+ex)^5}-\frac{2c^2\left(-2ce(7bd-ae)+3b^2e\right.}{3e^9(d+ex)^3}$$

```
[Out] -(c*d^2 - b*d*e + a*e^2)^4/(9*e^9*(d + e*x)^9) + ((2*c*d - b*e)*(c*d^2 - b*
d*e + a*e^2)^3)/(2*e^9*(d + e*x)^8) - (2*(c*d^2 - b*d*e + a*e^2)^2*(14*c^2*
d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e)))/(7*e^9*(d + e*x)^7) + (2*(2*c*d - b
*e)*(c*d^2 - b*d*e + a*e^2)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e)))/(3
*e^9*(d + e*x)^6) - (70*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(5*b*d - 3*a*e) - 2
0*c^3*d^2*e*(7*b*d - 3*a*e) + 6*c^2*e^2*(15*b^2*d^2 - 10*a*b*d*e + a^2*e^2)
)/(5*e^9*(d + e*x)^5) + (c*(2*c*d - b*e)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d
- 3*a*e)))/(e^9*(d + e*x)^4) - (2*c^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*
d - a*e)))/(3*e^9*(d + e*x)^3) + (2*c^3*(2*c*d - b*e))/(e^9*(d + e*x)^2) -
c^4/(e^9*(d + e*x))
```

Rubi [A] time = 0.410188, antiderivative size = 436, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$-\frac{6c^2e^2\left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4}{5e^9(d+ex)^5}-\frac{2c^2\left(-2ce(7bd-ae)+3b^2e\right.}{3e^9(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^4/(d + e*x)^10,x]
```


```
[Out] -(c*d^2 - b*d*e + a*e^2)^4/(9*e^9*(d + e*x)^9) + ((2*c*d - b*e)*(c*d^2 - b*
d*e + a*e^2)^3)/(2*e^9*(d + e*x)^8) - (2*(c*d^2 - b*d*e + a*e^2)^2*(14*c^2*
d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e)))/(7*e^9*(d + e*x)^7) + (2*(2*c*d - b
*e)*(c*d^2 - b*d*e + a*e^2)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e)))/(3
*e^9*(d + e*x)^6) - (70*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(5*b*d - 3*a*e) - 2
0*c^3*d^2*e*(7*b*d - 3*a*e) + 6*c^2*e^2*(15*b^2*d^2 - 10*a*b*d*e + a^2*e^2)
)/(5*e^9*(d + e*x)^5) + (c*(2*c*d - b*e)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d
- 3*a*e)))/(e^9*(d + e*x)^4) - (2*c^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*
d - a*e)))/(3*e^9*(d + e*x)^3) + (2*c^3*(2*c*d - b*e))/(e^9*(d + e*x)^2) -
c^4/(e^9*(d + e*x))
```


Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^4}{(d+ex)^{10}}\,dx = \int \left(\frac{(cd^2-bde+ae^2)^4}{e^8(d+ex)^{10}} + \frac{4(-2cd+be)(cd^2-bde+ae^2)^3}{e^8(d+ex)^9} + \frac{2(cd^2-bde+ae^2)^2(14c^2d^2+3b}{e^8(d+ex)^8}\right.$$

$$= -\frac{(cd^2-bde+ae^2)^4}{9e^9(d+ex)^9} + \frac{(2cd-be)(cd^2-bde+ae^2)^3}{2e^9(d+ex)^8} - \frac{2(cd^2-bde+ae^2)^2(14c^2d^2+3b^2e^2-}{7e^9(d+ex)^7}$$

Mathematica [A] time = 0.330883, size = 730, normalized size = 1.67

$$-\frac{3c^2e^2\left(2a^2e^2\left(36d^2e^2x^2+9d^3ex+d^4+84de^3x^3+126e^4x^4\right)+5abe\left(36d^3e^2x^2+84d^2e^3x^3+9d^4ex+d^5+126de^4x^4+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^4/(d + e*x)^10,x]
```

```
[Out] -(70*c^4*(d^8 + 9*d^7*e*x + 36*d^6*e^2*x^2 + 84*d^5*e^3*x^3 + 126*d^4*e^4*x
^4 + 126*d^3*e^5*x^5 + 84*d^2*e^6*x^6 + 36*d*e^7*x^7 + 9*e^8*x^8) + e^4*(70
*a^4*e^4 + 35*a^3*b*e^3*(d + 9*e*x) + 15*a^2*b^2*e^2*(d^2 + 9*d*e*x + 36*e^
2*x^2) + 5*a*b^3*e*(d^3 + 9*d^2*e*x + 36*d*e^2*x^2 + 84*e^3*x^3) + b^4*(d^4
 + 9*d^3*e*x + 36*d^2*e^2*x^2 + 84*d*e^3*x^3 + 126*e^4*x^4)) + c*e^3*(10*a^
3*e^3*(d^2 + 9*d*e*x + 36*e^2*x^2) + 15*a^2*b*e^2*(d^3 + 9*d^2*e*x + 36*d*e
^2*x^2 + 84*e^3*x^3) + 12*a*b^2*e*(d^4 + 9*d^3*e*x + 36*d^2*e^2*x^2 + 84*d*
e^3*x^3 + 126*e^4*x^4) + 5*b^3*(d^5 + 9*d^4*e*x + 36*d^3*e^2*x^2 + 84*d^2*e
^3*x^3 + 126*d*e^4*x^4 + 126*e^5*x^5)) + 3*c^2*e^2*(2*a^2*e^2*(d^4 + 9*d^3*
e*x + 36*d^2*e^2*x^2 + 84*d*e^3*x^3 + 126*e^4*x^4) + 5*a*b*e*(d^5 + 9*d^4*e
*x + 36*d^3*e^2*x^2 + 84*d^2*e^3*x^3 + 126*d*e^4*x^4 + 126*e^5*x^5) + 5*b^2
*(d^6 + 9*d^5*e*x + 36*d^4*e^2*x^2 + 84*d^3*e^3*x^3 + 126*d^2*e^4*x^4 + 126
*d*e^5*x^5 + 84*e^6*x^6)) + 5*c^3*e*(2*a*e*(d^6 + 9*d^5*e*x + 36*d^4*e^2*x^
2 + 84*d^3*e^3*x^3 + 126*d^2*e^4*x^4 + 126*d*e^5*x^5 + 84*e^6*x^6) + 7*b*(d
^7 + 9*d^6*e*x + 36*d^5*e^2*x^2 + 84*d^4*e^3*x^3 + 126*d^3*e^4*x^4 + 126*d^
2*e^5*x^5 + 84*d*e^6*x^6 + 36*e^7*x^7)))/(630*e^9*(d + e*x)^9)
```

Maple [B] time = 0.05, size = 914, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^4/(e*x+d)^10,x)
```

```
[Out] -1/7*(4*a^3*c*e^6+6*a^2*b^2*e^6-36*a^2*b*c*d*e^5+36*a^2*c^2*d^2*e^4-12*a*b^
3*d*e^5+72*a*b^2*c*d^2*e^4-120*a*b*c^2*d^3*e^3+60*a*c^3*d^4*e^2+6*b^4*d^2*e
^4-40*b^3*c*d^3*e^3+90*b^2*c^2*d^4*e^2-84*b*c^3*d^5*e+28*c^4*d^6)/e^9/(e*x+
d)^7-c*(3*a*b*c*e^3-6*a*c^2*d*e^2+b^3*e^3-9*b^2*c*d*e^2+21*b*c^2*d^2*e-14*c
^3*d^3)/e^9/(e*x+d)^4-2/3*c^2*(2*a*c*e^2+3*b^2*e^2-14*b*c*d*e+14*c^2*d^2)/e
^9/(e*x+d)^3-2*c^3*(b*e-2*c*d)/e^9/(e*x+d)^2-1/8*(4*a^3*b*e^7-8*a^3*c*d*e^6
-12*a^2*b^2*d*e^6+36*a^2*b*c*d^2*e^5-24*a^2*c^2*d^3*e^4+12*a*b^3*d^2*e^5-48
*a*b^2*c*d^3*e^4+60*a*b*c^2*d^4*e^3-24*a*c^3*d^5*e^2-4*b^4*d^3*e^4+20*b^3*c
*d^4*e^3-36*b^2*c^2*d^5*e^2+28*b*c^3*d^6*e-8*c^4*d^7)/e^9/(e*x+d)^8-1/9*(a^
4*e^8-4*a^3*b*d*e^7+4*a^3*c*d^2*e^6+6*a^2*b^2*d^2*e^6-12*a^2*b*c*d^3*e^5+6*
a^2*c^2*d^4*e^4-4*a*b^3*d^3*e^5+12*a*b^2*c*d^4*e^4-12*a*b*c^2*d^5*e^3+4*a*c
^3*d^6*e^2+b^4*d^4*e^4-4*b^3*c*d^5*e^3+6*b^2*c^2*d^6*e^2-4*b*c^3*d^7*e+c^4*
d^8)/e^9/(e*x+d)^9-1/6*(12*a^2*b*c*e^5-24*a^2*c^2*d*e^4+4*a*b^3*e^5-48*a*b^
```

```
2*c*d*e^4+120*a*b*c^2*d^2*e^3-80*a*c^3*d^3*e^2-4*b^4*d*e^4+40*b^3*c*d^2*e^3
-120*b^2*c^2*d^3*e^2+140*b*c^3*d^4*e-56*c^4*d^5)/e^9/(e*x+d)^6-1/5*(6*a^2*c
^2*e^4+12*a*b^2*c*e^4-60*a*b*c^2*d*e^3+60*a*c^3*d^2*e^2+b^4*e^4-20*b^3*c*d*
e^3+90*b^2*c^2*d^2*e^2-140*b*c^3*d^3*e+70*c^4*d^4)/e^9/(e*x+d)^5-c^4/e^9/(e
*x+d)
```

Maxima [B] time = 1.20995, size = 1206, normalized size = 2.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^10,x, algorithm="maxima")
```

```
[Out] -1/630*(630*c^4*e^8*x^8 + 70*c^4*d^8 + 35*b*c^3*d^7*e + 35*a^3*b*d*e^7 + 70
*a^4*e^8 + 5*(3*b^2*c^2 + 2*a*c^3)*d^6*e^2 + 5*(b^3*c + 3*a*b*c^2)*d^5*e^3
+ (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^4*e^4 + 5*(a*b^3 + 3*a^2*b*c)*d^3*e^5 +
5*(3*a^2*b^2 + 2*a^3*c)*d^2*e^6 + 1260*(2*c^4*d*e^7 + b*c^3*e^8)*x^7 + 420*
(14*c^4*d^2*e^6 + 7*b*c^3*d*e^7 + (3*b^2*c^2 + 2*a*c^3)*e^8)*x^6 + 630*(14*
c^4*d^3*e^5 + 7*b*c^3*d^2*e^6 + (3*b^2*c^2 + 2*a*c^3)*d*e^7 + (b^3*c + 3*a*
b*c^2)*e^8)*x^5 + 126*(70*c^4*d^4*e^4 + 35*b*c^3*d^3*e^5 + 5*(3*b^2*c^2 + 2
*a*c^3)*d^2*e^6 + 5*(b^3*c + 3*a*b*c^2)*d*e^7 + (b^4 + 12*a*b^2*c + 6*a^2*c
^2)*e^8)*x^4 + 84*(70*c^4*d^5*e^3 + 35*b*c^3*d^4*e^4 + 5*(3*b^2*c^2 + 2*a*c
^3)*d^3*e^5 + 5*(b^3*c + 3*a*b*c^2)*d^2*e^6 + (b^4 + 12*a*b^2*c + 6*a^2*c^2
)*d*e^7 + 5*(a*b^3 + 3*a^2*b*c)*e^8)*x^3 + 36*(70*c^4*d^6*e^2 + 35*b*c^3*d^
5*e^3 + 5*(3*b^2*c^2 + 2*a*c^3)*d^4*e^4 + 5*(b^3*c + 3*a*b*c^2)*d^3*e^5 + (
b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^2*e^6 + 5*(a*b^3 + 3*a^2*b*c)*d*e^7 + 5*(3*
a^2*b^2 + 2*a^3*c)*e^8)*x^2 + 9*(70*c^4*d^7*e + 35*b*c^3*d^6*e^2 + 35*a^3*b
*e^8 + 5*(3*b^2*c^2 + 2*a*c^3)*d^5*e^3 + 5*(b^3*c + 3*a*b*c^2)*d^4*e^4 + (b
^4 + 12*a*b^2*c + 6*a^2*c^2)*d^3*e^5 + 5*(a*b^3 + 3*a^2*b*c)*d^2*e^6 + 5*(3
*a^2*b^2 + 2*a^3*c)*d*e^7)*x)/(e^18*x^9 + 9*d*e^17*x^8 + 36*d^2*e^16*x^7 +
84*d^3*e^15*x^6 + 126*d^4*e^14*x^5 + 126*d^5*e^13*x^4 + 84*d^6*e^12*x^3 + 3
6*d^7*e^11*x^2 + 9*d^8*e^10*x + d^9*e^9)
```

Fricas [B] time = 1.73577, size = 1879, normalized size = 4.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^10,x, algorithm="fricas")
```

```
[Out] -1/630*(630*c^4*e^8*x^8 + 70*c^4*d^8 + 35*b*c^3*d^7*e + 35*a^3*b*d*e^7 + 70
*a^4*e^8 + 5*(3*b^2*c^2 + 2*a*c^3)*d^6*e^2 + 5*(b^3*c + 3*a*b*c^2)*d^5*e^3
+ (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^4*e^4 + 5*(a*b^3 + 3*a^2*b*c)*d^3*e^5 +
5*(3*a^2*b^2 + 2*a^3*c)*d^2*e^6 + 1260*(2*c^4*d*e^7 + b*c^3*e^8)*x^7 + 420*
(14*c^4*d^2*e^6 + 7*b*c^3*d*e^7 + (3*b^2*c^2 + 2*a*c^3)*e^8)*x^6 + 630*(14*
c^4*d^3*e^5 + 7*b*c^3*d^2*e^6 + (3*b^2*c^2 + 2*a*c^3)*d*e^7 + (b^3*c + 3*a*
b*c^2)*e^8)*x^5 + 126*(70*c^4*d^4*e^4 + 35*b*c^3*d^3*e^5 + 5*(3*b^2*c^2 + 2
*a*c^3)*d^2*e^6 + 5*(b^3*c + 3*a*b*c^2)*d*e^7 + (b^4 + 12*a*b^2*c + 6*a^2*c
^2)*e^8)*x^4 + 84*(70*c^4*d^5*e^3 + 35*b*c^3*d^4*e^4 + 5*(3*b^2*c^2 + 2*a*c
^3)*d^3*e^5 + 5*(b^3*c + 3*a*b*c^2)*d^2*e^6 + (b^4 + 12*a*b^2*c + 6*a^2*c^2
)*d*e^7 + 5*(a*b^3 + 3*a^2*b*c)*e^8)*x^3 + 36*(70*c^4*d^6*e^2 + 35*b*c^3*d^
5*e^3 + 5*(3*b^2*c^2 + 2*a*c^3)*d^4*e^4 + 5*(b^3*c + 3*a*b*c^2)*d^3*e^5 + (
b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^2*e^6 + 5*(a*b^3 + 3*a^2*b*c)*d*e^7 + 5*(3*
```

```
a^2*b^2 + 2*a^3*c)*e^8)*x^2 + 9*(70*c^4*d^7*e + 35*b*c^3*d^6*e^2 + 35*a^3*b
*e^8 + 5*(3*b^2*c^2 + 2*a*c^3)*d^5*e^3 + 5*(b^3*c + 3*a*b*c^2)*d^4*e^4 + (b
^4 + 12*a*b^2*c + 6*a^2*c^2)*d^3*e^5 + 5*(a*b^3 + 3*a^2*b*c)*d^2*e^6 + 5*(3
*a^2*b^2 + 2*a^3*c)*d*e^7)*x)/(e^18*x^9 + 9*d*e^17*x^8 + 36*d^2*e^16*x^7 +
84*d^3*e^15*x^6 + 126*d^4*e^14*x^5 + 126*d^5*e^13*x^4 + 84*d^6*e^12*x^3 + 3
6*d^7*e^11*x^2 + 9*d^8*e^10*x + d^9*e^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**4/(e*x+d)**10,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.12578, size = 1274, normalized size = 2.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^10,x, algorithm="giac")
```

```
[Out] -1/630*(630*c^4*x^8*e^8 + 2520*c^4*d*x^7*e^7 + 5880*c^4*d^2*x^6*e^6 + 8820*
c^4*d^3*x^5*e^5 + 8820*c^4*d^4*x^4*e^4 + 5880*c^4*d^5*x^3*e^3 + 2520*c^4*d^
6*x^2*e^2 + 630*c^4*d^7*x*e + 70*c^4*d^8 + 1260*b*c^3*x^7*e^8 + 2940*b*c^3*
d*x^6*e^7 + 4410*b*c^3*d^2*x^5*e^6 + 4410*b*c^3*d^3*x^4*e^5 + 2940*b*c^3*d^
4*x^3*e^4 + 1260*b*c^3*d^5*x^2*e^3 + 315*b*c^3*d^6*x*e^2 + 35*b*c^3*d^7*e +
 1260*b^2*c^2*x^6*e^8 + 840*a*c^3*x^6*e^8 + 1890*b^2*c^2*d*x^5*e^7 + 1260*a
*c^3*d*x^5*e^7 + 1890*b^2*c^2*d^2*x^4*e^6 + 1260*a*c^3*d^2*x^4*e^6 + 1260*b
^2*c^2*d^3*x^3*e^5 + 840*a*c^3*d^3*x^3*e^5 + 540*b^2*c^2*d^4*x^2*e^4 + 360*
a*c^3*d^4*x^2*e^4 + 135*b^2*c^2*d^5*x*e^3 + 90*a*c^3*d^5*x*e^3 + 15*b^2*c^2
*d^6*e^2 + 10*a*c^3*d^6*e^2 + 630*b^3*c*x^5*e^8 + 1890*a*b*c^2*x^5*e^8 + 63
0*b^3*c*d*x^4*e^7 + 1890*a*b*c^2*d*x^4*e^7 + 420*b^3*c*d^2*x^3*e^6 + 1260*a
*b*c^2*d^2*x^3*e^6 + 180*b^3*c*d^3*x^2*e^5 + 540*a*b*c^2*d^3*x^2*e^5 + 45*b
^3*c*d^4*x*e^4 + 135*a*b*c^2*d^4*x*e^4 + 5*b^3*c*d^5*e^3 + 15*a*b*c^2*d^5*e
^3 + 126*b^4*x^4*e^8 + 1512*a*b^2*c*x^4*e^8 + 756*a^2*c^2*x^4*e^8 + 84*b^4*
d*x^3*e^7 + 1008*a*b^2*c*d*x^3*e^7 + 504*a^2*c^2*d*x^3*e^7 + 36*b^4*d^2*x^2
*e^6 + 432*a*b^2*c*d^2*x^2*e^6 + 216*a^2*c^2*d^2*x^2*e^6 + 9*b^4*d^3*x*e^5
+ 108*a*b^2*c*d^3*x*e^5 + 54*a^2*c^2*d^3*x*e^5 + b^4*d^4*e^4 + 12*a*b^2*c*d
^4*e^4 + 6*a^2*c^2*d^4*e^4 + 420*a*b^3*x^3*e^8 + 1260*a^2*b*c*x^3*e^8 + 180
*a*b^3*d*x^2*e^7 + 540*a^2*b*c*d*x^2*e^7 + 45*a*b^3*d^2*x*e^6 + 135*a^2*b*c
*d^2*x*e^6 + 5*a*b^3*d^3*e^5 + 15*a^2*b*c*d^3*e^5 + 540*a^2*b^2*x^2*e^8 + 3
60*a^3*c*x^2*e^8 + 135*a^2*b^2*d*x*e^7 + 90*a^3*c*d*x*e^7 + 15*a^2*b^2*d^2*
e^6 + 10*a^3*c*d^2*e^6 + 315*a^3*b*x*e^8 + 35*a^3*b*d*e^7 + 70*a^4*e^8)*e^(
-9)/(x*e + d)^9
```

3.2161 $\int \frac{(a+bx+cx^2)^4}{(d+ex)^{11}} dx$

Optimal. Leaf size=443

$$-\frac{6c^2e^2\left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4}{6e^9(d+ex)^6}-\frac{c^2\left(-2ce(7bd-ae)+3b^2e^2\right.}{2e^9(d+ex)^4}$$

```
[Out] -(c*d^2 - b*d*e + a*e^2)^4/(10*e^9*(d + e*x)^10) + (4*(2*c*d - b*e)*(c*d^2
- b*d*e + a*e^2)^3)/(9*e^9*(d + e*x)^9) - ((c*d^2 - b*d*e + a*e^2)^2*(14*c^
2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e)))/(4*e^9*(d + e*x)^8) + (4*(2*c*d -
 b*e)*(c*d^2 - b*d*e + a*e^2)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e)))/
(7*e^9*(d + e*x)^7) - (70*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(5*b*d - 3*a*e) -
 20*c^3*d^2*e*(7*b*d - 3*a*e) + 6*c^2*e^2*(15*b^2*d^2 - 10*a*b*d*e + a^2*e^
2))/(6*e^9*(d + e*x)^6) + (4*c*(2*c*d - b*e)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*
b*d - 3*a*e)))/(5*e^9*(d + e*x)^5) - (c^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(
7*b*d - a*e)))/(2*e^9*(d + e*x)^4) + (4*c^3*(2*c*d - b*e))/(3*e^9*(d + e*x)
^3) - c^4/(2*e^9*(d + e*x)^2)
```

Rubi [A] time = 0.40089, antiderivative size = 443, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$-\frac{6c^2e^2\left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4}{6e^9(d+ex)^6}-\frac{c^2\left(-2ce(7bd-ae)+3b^2e^2\right.}{2e^9(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^4/(d + e*x)^11,x]
```


```
[Out] -(c*d^2 - b*d*e + a*e^2)^4/(10*e^9*(d + e*x)^10) + (4*(2*c*d - b*e)*(c*d^2
- b*d*e + a*e^2)^3)/(9*e^9*(d + e*x)^9) - ((c*d^2 - b*d*e + a*e^2)^2*(14*c^
2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e)))/(4*e^9*(d + e*x)^8) + (4*(2*c*d -
 b*e)*(c*d^2 - b*d*e + a*e^2)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e)))/
(7*e^9*(d + e*x)^7) - (70*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(5*b*d - 3*a*e) -
 20*c^3*d^2*e*(7*b*d - 3*a*e) + 6*c^2*e^2*(15*b^2*d^2 - 10*a*b*d*e + a^2*e^
2))/(6*e^9*(d + e*x)^6) + (4*c*(2*c*d - b*e)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*
b*d - 3*a*e)))/(5*e^9*(d + e*x)^5) - (c^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(
7*b*d - a*e)))/(2*e^9*(d + e*x)^4) + (4*c^3*(2*c*d - b*e))/(3*e^9*(d + e*x)
^3) - c^4/(2*e^9*(d + e*x)^2)
```


Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^4}{(d+ex)^{11}}\,dx = \int \left(\frac{(cd^2-bde+ae^2)^4}{e^8(d+ex)^{11}} + \frac{4(-2cd+be)(cd^2-bde+ae^2)^3}{e^8(d+ex)^{10}} + \frac{2(cd^2-bde+ae^2)^2(14c^2d^2+3b}{e^8(d+ex)^9}\right.$$

$$= -\frac{(cd^2-bde+ae^2)^4}{10e^9(d+ex)^{10}} + \frac{4(2cd-be)(cd^2-bde+ae^2)^3}{9e^9(d+ex)^9} - \frac{(cd^2-bde+ae^2)^2(14c^2d^2+3b^2e^2-}{4e^9(d+ex)^8}$$

Mathematica [A] time = 0.296293, size = 731, normalized size = 1.65

$$-\frac{3c^2e^2\left(2a^2e^2\left(45d^2e^2x^2+10d^3ex+d^4+120de^3x^3+210e^4x^4\right)+4abe\left(45d^3e^2x^2+120d^2e^3x^3+10d^4ex+d^5+210de^4x\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^4/(d + e*x)^11,x]
```

```
[Out] -(14*c^4*(d^8 + 10*d^7*e*x + 45*d^6*e^2*x^2 + 120*d^5*e^3*x^3 + 210*d^4*e^4
*x^4 + 252*d^3*e^5*x^5 + 210*d^2*e^6*x^6 + 120*d*e^7*x^7 + 45*e^8*x^8) + e^
4*(126*a^4*e^4 + 56*a^3*b*e^3*(d + 10*e*x) + 21*a^2*b^2*e^2*(d^2 + 10*d*e*x
 + 45*e^2*x^2) + 6*a*b^3*e*(d^3 + 10*d^2*e*x + 45*d*e^2*x^2 + 120*e^3*x^3)
+ b^4*(d^4 + 10*d^3*e*x + 45*d^2*e^2*x^2 + 120*d*e^3*x^3 + 210*e^4*x^4)) +
2*c*e^3*(7*a^3*e^3*(d^2 + 10*d*e*x + 45*e^2*x^2) + 9*a^2*b*e^2*(d^3 + 10*d^
2*e*x + 45*d*e^2*x^2 + 120*e^3*x^3) + 6*a*b^2*e*(d^4 + 10*d^3*e*x + 45*d^2*
e^2*x^2 + 120*d*e^3*x^3 + 210*e^4*x^4) + 2*b^3*(d^5 + 10*d^4*e*x + 45*d^3*e
^2*x^2 + 120*d^2*e^3*x^3 + 210*d*e^4*x^4 + 252*e^5*x^5)) + 3*c^2*e^2*(2*a^2
*e^2*(d^4 + 10*d^3*e*x + 45*d^2*e^2*x^2 + 120*d*e^3*x^3 + 210*e^4*x^4) + 4*
a*b*e*(d^5 + 10*d^4*e*x + 45*d^3*e^2*x^2 + 120*d^2*e^3*x^3 + 210*d*e^4*x^4
+ 252*e^5*x^5) + 3*b^2*(d^6 + 10*d^5*e*x + 45*d^4*e^2*x^2 + 120*d^3*e^3*x^3
 + 210*d^2*e^4*x^4 + 252*d*e^5*x^5 + 210*e^6*x^6)) + 2*c^3*e*(3*a*e*(d^6 +
10*d^5*e*x + 45*d^4*e^2*x^2 + 120*d^3*e^3*x^3 + 210*d^2*e^4*x^4 + 252*d*e^5
*x^5 + 210*e^6*x^6) + 7*b*(d^7 + 10*d^6*e*x + 45*d^5*e^2*x^2 + 120*d^4*e^3*
x^3 + 210*d^3*e^4*x^4 + 252*d^2*e^5*x^5 + 210*d*e^6*x^6 + 120*e^7*x^7)))/(1
260*e^9*(d + e*x)^10)
```

Maple [B] time = 0.048, size = 914, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^4/(e*x+d)^11,x)
```

```
[Out] -1/7*(12*a^2*b*c*e^5-24*a^2*c^2*d*e^4+4*a*b^3*e^5-48*a*b^2*c*d*e^4+120*a*b*
c^2*d^2*e^3-80*a*c^3*d^3*e^2-4*b^4*d*e^4+40*b^3*c*d^2*e^3-120*b^2*c^2*d^3*e
^2+140*b*c^3*d^4*e-56*c^4*d^5)/e^9/(e*x+d)^7-1/2*c^2*(2*a*c*e^2+3*b^2*e^2-1
4*b*c*d*e+14*c^2*d^2)/e^9/(e*x+d)^4-1/10*(a^4*e^8-4*a^3*b*d*e^7+4*a^3*c*d^2
*e^6+6*a^2*b^2*d^2*e^6-12*a^2*b*c*d^3*e^5+6*a^2*c^2*d^4*e^4-4*a*b^3*d^3*e^5
+12*a*b^2*c*d^4*e^4-12*a*b*c^2*d^5*e^3+4*a*c^3*d^6*e^2+b^4*d^4*e^4-4*b^3*c*
d^5*e^3+6*b^2*c^2*d^6*e^2-4*b*c^3*d^7*e+c^4*d^8)/e^9/(e*x+d)^10-4/3*c^3*(b*
e-2*c*d)/e^9/(e*x+d)^3-1/2*c^4/e^9/(e*x+d)^2-1/8*(4*a^3*c*e^6+6*a^2*b^2*e^6
-36*a^2*b*c*d*e^5+36*a^2*c^2*d^2*e^4-12*a*b^3*d*e^5+72*a*b^2*c*d^2*e^4-120*
a*b*c^2*d^3*e^3+60*a*c^3*d^4*e^2+6*b^4*d^2*e^4-40*b^3*c*d^3*e^3+90*b^2*c^2*
d^4*e^2-84*b*c^3*d^5*e+28*c^4*d^6)/e^9/(e*x+d)^8-1/9*(4*a^3*b*e^7-8*a^3*c*d
*e^6-12*a^2*b^2*d*e^6+36*a^2*b*c*d^2*e^5-24*a^2*c^2*d^3*e^4+12*a*b^3*d^2*e^
```

```
5-48*a*b^2*c*d^3*e^4+60*a*b*c^2*d^4*e^3-24*a*c^3*d^5*e^2-4*b^4*d^3*e^4+20*b
^3*c*d^4*e^3-36*b^2*c^2*d^5*e^2+28*b*c^3*d^6*e-8*c^4*d^7)/e^9/(e*x+d)^9-1/6
*(6*a^2*c^2*e^4+12*a*b^2*c*e^4-60*a*b*c^2*d*e^3+60*a*c^3*d^2*e^2+b^4*e^4-20
*b^3*c*d*e^3+90*b^2*c^2*d^2*e^2-140*b*c^3*d^3*e+70*c^4*d^4)/e^9/(e*x+d)^6-4
/5*c*(3*a*b*c*e^3-6*a*c^2*d*e^2+b^3*e^3-9*b^2*c*d*e^2+21*b*c^2*d^2*e-14*c^3
*d^3)/e^9/(e*x+d)^5
```

Maxima [B] time = 1.18858, size = 1223, normalized size = 2.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^11,x, algorithm="maxima")
```

```
[Out] -1/1260*(630*c^4*e^8*x^8 + 14*c^4*d^8 + 14*b*c^3*d^7*e + 56*a^3*b*d*e^7 + 1
26*a^4*e^8 + 3*(3*b^2*c^2 + 2*a*c^3)*d^6*e^2 + 4*(b^3*c + 3*a*b*c^2)*d^5*e^
3 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^4*e^4 + 6*(a*b^3 + 3*a^2*b*c)*d^3*e^5
+ 7*(3*a^2*b^2 + 2*a^3*c)*d^2*e^6 + 1680*(c^4*d*e^7 + b*c^3*e^8)*x^7 + 210*
(14*c^4*d^2*e^6 + 14*b*c^3*d*e^7 + 3*(3*b^2*c^2 + 2*a*c^3)*e^8)*x^6 + 252*(
14*c^4*d^3*e^5 + 14*b*c^3*d^2*e^6 + 3*(3*b^2*c^2 + 2*a*c^3)*d*e^7 + 4*(b^3*
c + 3*a*b*c^2)*e^8)*x^5 + 210*(14*c^4*d^4*e^4 + 14*b*c^3*d^3*e^5 + 3*(3*b^2
*c^2 + 2*a*c^3)*d^2*e^6 + 4*(b^3*c + 3*a*b*c^2)*d*e^7 + (b^4 + 12*a*b^2*c +
 6*a^2*c^2)*e^8)*x^4 + 120*(14*c^4*d^5*e^3 + 14*b*c^3*d^4*e^4 + 3*(3*b^2*c^
2 + 2*a*c^3)*d^3*e^5 + 4*(b^3*c + 3*a*b*c^2)*d^2*e^6 + (b^4 + 12*a*b^2*c +
6*a^2*c^2)*d*e^7 + 6*(a*b^3 + 3*a^2*b*c)*e^8)*x^3 + 45*(14*c^4*d^6*e^2 + 14
*b*c^3*d^5*e^3 + 3*(3*b^2*c^2 + 2*a*c^3)*d^4*e^4 + 4*(b^3*c + 3*a*b*c^2)*d^
3*e^5 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^2*e^6 + 6*(a*b^3 + 3*a^2*b*c)*d*e^
7 + 7*(3*a^2*b^2 + 2*a^3*c)*e^8)*x^2 + 10*(14*c^4*d^7*e + 14*b*c^3*d^6*e^2
+ 56*a^3*b*e^8 + 3*(3*b^2*c^2 + 2*a*c^3)*d^5*e^3 + 4*(b^3*c + 3*a*b*c^2)*d^
4*e^4 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^3*e^5 + 6*(a*b^3 + 3*a^2*b*c)*d^2*
e^6 + 7*(3*a^2*b^2 + 2*a^3*c)*d*e^7)*x)/(e^19*x^10 + 10*d*e^18*x^9 + 45*d^2
*e^17*x^8 + 120*d^3*e^16*x^7 + 210*d^4*e^15*x^6 + 252*d^5*e^14*x^5 + 210*d^
6*e^13*x^4 + 120*d^7*e^12*x^3 + 45*d^8*e^11*x^2 + 10*d^9*e^10*x + d^10*e^9)
```

Fricas [B] time = 1.83915, size = 1926, normalized size = 4.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^11,x, algorithm="fricas")
```

```
[Out] -1/1260*(630*c^4*e^8*x^8 + 14*c^4*d^8 + 14*b*c^3*d^7*e + 56*a^3*b*d*e^7 + 1
26*a^4*e^8 + 3*(3*b^2*c^2 + 2*a*c^3)*d^6*e^2 + 4*(b^3*c + 3*a*b*c^2)*d^5*e^
3 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^4*e^4 + 6*(a*b^3 + 3*a^2*b*c)*d^3*e^5
+ 7*(3*a^2*b^2 + 2*a^3*c)*d^2*e^6 + 1680*(c^4*d*e^7 + b*c^3*e^8)*x^7 + 210*
(14*c^4*d^2*e^6 + 14*b*c^3*d*e^7 + 3*(3*b^2*c^2 + 2*a*c^3)*e^8)*x^6 + 252*(
14*c^4*d^3*e^5 + 14*b*c^3*d^2*e^6 + 3*(3*b^2*c^2 + 2*a*c^3)*d*e^7 + 4*(b^3*
c + 3*a*b*c^2)*e^8)*x^5 + 210*(14*c^4*d^4*e^4 + 14*b*c^3*d^3*e^5 + 3*(3*b^2
*c^2 + 2*a*c^3)*d^2*e^6 + 4*(b^3*c + 3*a*b*c^2)*d*e^7 + (b^4 + 12*a*b^2*c +
 6*a^2*c^2)*e^8)*x^4 + 120*(14*c^4*d^5*e^3 + 14*b*c^3*d^4*e^4 + 3*(3*b^2*c^
2 + 2*a*c^3)*d^3*e^5 + 4*(b^3*c + 3*a*b*c^2)*d^2*e^6 + (b^4 + 12*a*b^2*c +
6*a^2*c^2)*d*e^7 + 6*(a*b^3 + 3*a^2*b*c)*e^8)*x^3 + 45*(14*c^4*d^6*e^2 + 14
*b*c^3*d^5*e^3 + 3*(3*b^2*c^2 + 2*a*c^3)*d^4*e^4 + 4*(b^3*c + 3*a*b*c^2)*d^
```

```
3*e^5 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^2*e^6 + 6*(a*b^3 + 3*a^2*b*c)*d*e^
7 + 7*(3*a^2*b^2 + 2*a^3*c)*e^8)*x^2 + 10*(14*c^4*d^7*e + 14*b*c^3*d^6*e^2
+ 56*a^3*b*e^8 + 3*(3*b^2*c^2 + 2*a*c^3)*d^5*e^3 + 4*(b^3*c + 3*a*b*c^2)*d^
4*e^4 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^3*e^5 + 6*(a*b^3 + 3*a^2*b*c)*d^2*
e^6 + 7*(3*a^2*b^2 + 2*a^3*c)*d*e^7)*x)/(e^19*x^10 + 10*d*e^18*x^9 + 45*d^2
*e^17*x^8 + 120*d^3*e^16*x^7 + 210*d^4*e^15*x^6 + 252*d^5*e^14*x^5 + 210*d^
6*e^13*x^4 + 120*d^7*e^12*x^3 + 45*d^8*e^11*x^2 + 10*d^9*e^10*x + d^10*e^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**4/(e*x+d)**11,x)

[Out] Timed out
```

Giac [B] time = 1.1216, size = 1274, normalized size = 2.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^11,x, algorithm="giac")

[Out] -1/1260*(630*c^4*x^8*e^8 + 1680*c^4*d*x^7*e^7 + 2940*c^4*d^2*x^6*e^6 + 3528
*c^4*d^3*x^5*e^5 + 2940*c^4*d^4*x^4*e^4 + 1680*c^4*d^5*x^3*e^3 + 630*c^4*d^
6*x^2*e^2 + 140*c^4*d^7*x*e + 14*c^4*d^8 + 1680*b*c^3*x^7*e^8 + 2940*b*c^3*
d*x^6*e^7 + 3528*b*c^3*d^2*x^5*e^6 + 2940*b*c^3*d^3*x^4*e^5 + 1680*b*c^3*d^
4*x^3*e^4 + 630*b*c^3*d^5*x^2*e^3 + 140*b*c^3*d^6*x*e^2 + 14*b*c^3*d^7*e +
1890*b^2*c^2*x^6*e^8 + 1260*a*c^3*x^6*e^8 + 2268*b^2*c^2*d*x^5*e^7 + 1512*a
*c^3*d*x^5*e^7 + 1890*b^2*c^2*d^2*x^4*e^6 + 1260*a*c^3*d^2*x^4*e^6 + 1080*b
^2*c^2*d^3*x^3*e^5 + 720*a*c^3*d^3*x^3*e^5 + 405*b^2*c^2*d^4*x^2*e^4 + 270*
a*c^3*d^4*x^2*e^4 + 90*b^2*c^2*d^5*x*e^3 + 60*a*c^3*d^5*x*e^3 + 9*b^2*c^2*d
^6*e^2 + 6*a*c^3*d^6*e^2 + 1008*b^3*c*x^5*e^8 + 3024*a*b*c^2*x^5*e^8 + 840*
b^3*c*d*x^4*e^7 + 2520*a*b*c^2*d*x^4*e^7 + 480*b^3*c*d^2*x^3*e^6 + 1440*a*b
*c^2*d^2*x^3*e^6 + 180*b^3*c*d^3*x^2*e^5 + 540*a*b*c^2*d^3*x^2*e^5 + 40*b^3
*c*d^4*x*e^4 + 120*a*b*c^2*d^4*x*e^4 + 4*b^3*c*d^5*e^3 + 12*a*b*c^2*d^5*e^3
 + 210*b^4*x^4*e^8 + 2520*a*b^2*c*x^4*e^8 + 1260*a^2*c^2*x^4*e^8 + 120*b^4*
d*x^3*e^7 + 1440*a*b^2*c*d*x^3*e^7 + 720*a^2*c^2*d*x^3*e^7 + 45*b^4*d^2*x^2
*e^6 + 540*a*b^2*c*d^2*x^2*e^6 + 270*a^2*c^2*d^2*x^2*e^6 + 10*b^4*d^3*x*e^5
 + 120*a*b^2*c*d^3*x*e^5 + 60*a^2*c^2*d^3*x*e^5 + b^4*d^4*e^4 + 12*a*b^2*c*
d^4*e^4 + 6*a^2*c^2*d^4*e^4 + 720*a*b^3*x^3*e^8 + 2160*a^2*b*c*x^3*e^8 + 27
0*a*b^3*d*x^2*e^7 + 810*a^2*b*c*d*x^2*e^7 + 60*a*b^3*d^2*x*e^6 + 180*a^2*b*
c*d^2*x*e^6 + 6*a*b^3*d^3*e^5 + 18*a^2*b*c*d^3*e^5 + 945*a^2*b^2*x^2*e^8 +
630*a^3*c*x^2*e^8 + 210*a^2*b^2*d*x*e^7 + 140*a^3*c*d*x*e^7 + 21*a^2*b^2*d^
2*e^6 + 14*a^3*c*d^2*e^6 + 560*a^3*b*x*e^8 + 56*a^3*b*d*e^7 + 126*a^4*e^8)*
e^(-9)/(x*e + d)^10
```

3.2162 $\int \frac{(a+bx+cx^2)^4}{(d+ex)^{12}}\,dx$

Optimal. Leaf size=440

$$-\frac{6c^2e^2\left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4}{7e^9(d+ex)^7}-\frac{2c^2\left(-2ce(7bd-ae)+3b^2e\right.}{5e^9(d+ex)^5}$$

```
[Out] -(c*d^2 - b*d*e + a*e^2)^4/(11*e^9*(d + e*x)^11) + (2*(2*c*d - b*e)*(c*d^2
- b*d*e + a*e^2)^3)/(5*e^9*(d + e*x)^10) - (2*(c*d^2 - b*d*e + a*e^2)^2*(14
*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e)))/(9*e^9*(d + e*x)^9) + ((2*c*d
- b*e)*(c*d^2 - b*d*e + a*e^2)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e)))
/(2*e^9*(d + e*x)^8) - (70*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(5*b*d - 3*a*e)
- 20*c^3*d^2*e*(7*b*d - 3*a*e) + 6*c^2*e^2*(15*b^2*d^2 - 10*a*b*d*e + a^2*e
^2))/(7*e^9*(d + e*x)^7) + (2*c*(2*c*d - b*e)*(7*c^2*d^2 + b^2*e^2 - c*e*(7
*b*d - 3*a*e)))/(3*e^9*(d + e*x)^6) - (2*c^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*
e*(7*b*d - a*e)))/(5*e^9*(d + e*x)^5) + (c^3*(2*c*d - b*e))/(e^9*(d + e*x)^
4) - c^4/(3*e^9*(d + e*x)^3)
```

Rubi [A] time = 0.396922, antiderivative size = 440, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$-\frac{6c^2e^2\left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4}{7e^9(d+ex)^7}-\frac{2c^2\left(-2ce(7bd-ae)+3b^2e\right.}{5e^9(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^4/(d + e*x)^12,x]
```

```
[Out] -(c*d^2 - b*d*e + a*e^2)^4/(11*e^9*(d + e*x)^11) + (2*(2*c*d - b*e)*(c*d^2
- b*d*e + a*e^2)^3)/(5*e^9*(d + e*x)^10) - (2*(c*d^2 - b*d*e + a*e^2)^2*(14
*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e)))/(9*e^9*(d + e*x)^9) + ((2*c*d
- b*e)*(c*d^2 - b*d*e + a*e^2)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e)))
/(2*e^9*(d + e*x)^8) - (70*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(5*b*d - 3*a*e)
- 20*c^3*d^2*e*(7*b*d - 3*a*e) + 6*c^2*e^2*(15*b^2*d^2 - 10*a*b*d*e + a^2*e
^2))/(7*e^9*(d + e*x)^7) + (2*c*(2*c*d - b*e)*(7*c^2*d^2 + b^2*e^2 - c*e*(7
*b*d - 3*a*e)))/(3*e^9*(d + e*x)^6) - (2*c^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*
e*(7*b*d - a*e)))/(5*e^9*(d + e*x)^5) + (c^3*(2*c*d - b*e))/(e^9*(d + e*x)^
4) - c^4/(3*e^9*(d + e*x)^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^4}{(d+ex)^{12}}\,dx = \int \left(\frac{(cd^2-bde+ae^2)^4}{e^8(d+ex)^{12}} + \frac{4(-2cd+be)(cd^2-bde+ae^2)^3}{e^8(d+ex)^{11}} + \frac{2(cd^2-bde+ae^2)^2(14c^2d^2+3b}{e^8(d+ex)^{10}}\right.$$

$$= -\frac{(cd^2-bde+ae^2)^4}{11e^9(d+ex)^{11}} + \frac{2(2cd-be)(cd^2-bde+ae^2)^3}{5e^9(d+ex)^{10}} - \frac{2(cd^2-bde+ae^2)^2(14c^2d^2+3b^2e^2}{9e^9(d+ex)^9}$$

Mathematica [A] time = 0.324083, size = 731, normalized size = 1.66

$$-\frac{6c^2e^2\left(3a^2e^2\left(55d^2e^2x^2+11d^3ex+d^4+165de^3x^3+330e^4x^4\right)+5abe\left(55d^3e^2x^2+165d^2e^3x^3+11d^4ex+d^5+330de^4x\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^4/(d + e*x)^12,x]
```

```
[Out] -(14*c^4*(d^8 + 11*d^7*e*x + 55*d^6*e^2*x^2 + 165*d^5*e^3*x^3 + 330*d^4*e^4
*x^4 + 462*d^3*e^5*x^5 + 462*d^2*e^6*x^6 + 330*d*e^7*x^7 + 165*e^8*x^8) + 3
*e^4*(210*a^4*e^4 + 84*a^3*b*e^3*(d + 11*e*x) + 28*a^2*b^2*e^2*(d^2 + 11*d*
e*x + 55*e^2*x^2) + 7*a*b^3*e*(d^3 + 11*d^2*e*x + 55*d*e^2*x^2 + 165*e^3*x^
3) + b^4*(d^4 + 11*d^3*e*x + 55*d^2*e^2*x^2 + 165*d*e^3*x^3 + 330*e^4*x^4))
 + c*e^3*(56*a^3*e^3*(d^2 + 11*d*e*x + 55*e^2*x^2) + 63*a^2*b*e^2*(d^3 + 11
*d^2*e*x + 55*d*e^2*x^2 + 165*e^3*x^3) + 36*a*b^2*e*(d^4 + 11*d^3*e*x + 55*
d^2*e^2*x^2 + 165*d*e^3*x^3 + 330*e^4*x^4) + 10*b^3*(d^5 + 11*d^4*e*x + 55*
d^3*e^2*x^2 + 165*d^2*e^3*x^3 + 330*d*e^4*x^4 + 462*e^5*x^5)) + 6*c^2*e^2*(
3*a^2*e^2*(d^4 + 11*d^3*e*x + 55*d^2*e^2*x^2 + 165*d*e^3*x^3 + 330*e^4*x^4)
 + 5*a*b*e*(d^5 + 11*d^4*e*x + 55*d^3*e^2*x^2 + 165*d^2*e^3*x^3 + 330*d*e^4
*x^4 + 462*e^5*x^5) + 3*b^2*(d^6 + 11*d^5*e*x + 55*d^4*e^2*x^2 + 165*d^3*e^
3*x^3 + 330*d^2*e^4*x^4 + 462*d*e^5*x^5 + 462*e^6*x^6)) + 3*c^3*e*(4*a*e*(d
^6 + 11*d^5*e*x + 55*d^4*e^2*x^2 + 165*d^3*e^3*x^3 + 330*d^2*e^4*x^4 + 462*
d*e^5*x^5 + 462*e^6*x^6) + 7*b*(d^7 + 11*d^6*e*x + 55*d^5*e^2*x^2 + 165*d^4
*e^3*x^3 + 330*d^3*e^4*x^4 + 462*d^2*e^5*x^5 + 462*d*e^6*x^6 + 330*e^7*x^7)
))/(6930*e^9*(d + e*x)^11)
```

Maple [B] time = 0.05, size = 914, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^4/(e*x+d)^12,x)
```

```
[Out] -1/7*(6*a^2*c^2*e^4+12*a*b^2*c*e^4-60*a*b*c^2*d*e^3+60*a*c^3*d^2*e^2+b^4*e^
4-20*b^3*c*d*e^3+90*b^2*c^2*d^2*e^2-140*b*c^3*d^3*e+70*c^4*d^4)/e^9/(e*x+d)
^7-c^3*(b*e-2*c*d)/e^9/(e*x+d)^4-1/10*(4*a^3*b*e^7-8*a^3*c*d*e^6-12*a^2*b^2
*d*e^6+36*a^2*b*c*d^2*e^5-24*a^2*c^2*d^3*e^4+12*a*b^3*d^2*e^5-48*a*b^2*c*d^
3*e^4+60*a*b*c^2*d^4*e^3-24*a*c^3*d^5*e^2-4*b^4*d^3*e^4+20*b^3*c*d^4*e^3-36
*b^2*c^2*d^5*e^2+28*b*c^3*d^6*e-8*c^4*d^7)/e^9/(e*x+d)^10-1/3*c^4/e^9/(e*x+
d)^3-1/8*(12*a^2*b*c*e^5-24*a^2*c^2*d*e^4+4*a*b^3*e^5-48*a*b^2*c*d*e^4+120*
a*b*c^2*d^2*e^3-80*a*c^3*d^3*e^2-4*b^4*d*e^4+40*b^3*c*d^2*e^3-120*b^2*c^2*d
^3*e^2+140*b*c^3*d^4*e-56*c^4*d^5)/e^9/(e*x+d)^8-1/9*(4*a^3*c*e^6+6*a^2*b^2
*e^6-36*a^2*b*c*d*e^5+36*a^2*c^2*d^2*e^4-12*a*b^3*d*e^5+72*a*b^2*c*d^2*e^4-
120*a*b*c^2*d^3*e^3+60*a*c^3*d^4*e^2+6*b^4*d^2*e^4-40*b^3*c*d^3*e^3+90*b^2*
c^2*d^4*e^2-84*b*c^3*d^5*e+28*c^4*d^6)/e^9/(e*x+d)^9-2/3*c*(3*a*b*c*e^3-6*a
```

```
*c^2*d*e^2+b^3*e^3-9*b^2*c*d*e^2+21*b*c^2*d^2*e-14*c^3*d^3)/e^9/(e*x+d)^6-2
/5*c^2*(2*a*c*e^2+3*b^2*e^2-14*b*c*d*e+14*c^2*d^2)/e^9/(e*x+d)^5-1/11*(a^4*
e^8-4*a^3*b*d*e^7+4*a^3*c*d^2*e^6+6*a^2*b^2*d^2*e^6-12*a^2*b*c*d^3*e^5+6*a^
2*c^2*d^4*e^4-4*a*b^3*d^3*e^5+12*a*b^2*c*d^4*e^4-12*a*b*c^2*d^5*e^3+4*a*c^3
*d^6*e^2+b^4*d^4*e^4-4*b^3*c*d^5*e^3+6*b^2*c^2*d^6*e^2-4*b*c^3*d^7*e+c^4*d^
8)/e^9/(e*x+d)^11
```

Maxima [B] time = 1.19663, size = 1247, normalized size = 2.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^12,x, algorithm="maxima")
```

```
[Out] -1/6930*(2310*c^4*e^8*x^8 + 14*c^4*d^8 + 21*b*c^3*d^7*e + 252*a^3*b*d*e^7 +
 630*a^4*e^8 + 6*(3*b^2*c^2 + 2*a*c^3)*d^6*e^2 + 10*(b^3*c + 3*a*b*c^2)*d^5
*e^3 + 3*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^4*e^4 + 21*(a*b^3 + 3*a^2*b*c)*d^
3*e^5 + 28*(3*a^2*b^2 + 2*a^3*c)*d^2*e^6 + 2310*(2*c^4*d*e^7 + 3*b*c^3*e^8)
*x^7 + 462*(14*c^4*d^2*e^6 + 21*b*c^3*d*e^7 + 6*(3*b^2*c^2 + 2*a*c^3)*e^8)*
x^6 + 462*(14*c^4*d^3*e^5 + 21*b*c^3*d^2*e^6 + 6*(3*b^2*c^2 + 2*a*c^3)*d*e^
7 + 10*(b^3*c + 3*a*b*c^2)*e^8)*x^5 + 330*(14*c^4*d^4*e^4 + 21*b*c^3*d^3*e^
5 + 6*(3*b^2*c^2 + 2*a*c^3)*d^2*e^6 + 10*(b^3*c + 3*a*b*c^2)*d*e^7 + 3*(b^4
 + 12*a*b^2*c + 6*a^2*c^2)*e^8)*x^4 + 165*(14*c^4*d^5*e^3 + 21*b*c^3*d^4*e^
4 + 6*(3*b^2*c^2 + 2*a*c^3)*d^3*e^5 + 10*(b^3*c + 3*a*b*c^2)*d^2*e^6 + 3*(b
^4 + 12*a*b^2*c + 6*a^2*c^2)*d*e^7 + 21*(a*b^3 + 3*a^2*b*c)*e^8)*x^3 + 55*(
14*c^4*d^6*e^2 + 21*b*c^3*d^5*e^3 + 6*(3*b^2*c^2 + 2*a*c^3)*d^4*e^4 + 10*(b
^3*c + 3*a*b*c^2)*d^3*e^5 + 3*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^2*e^6 + 21*(
a*b^3 + 3*a^2*b*c)*d*e^7 + 28*(3*a^2*b^2 + 2*a^3*c)*e^8)*x^2 + 11*(14*c^4*d
^7*e + 21*b*c^3*d^6*e^2 + 252*a^3*b*e^8 + 6*(3*b^2*c^2 + 2*a*c^3)*d^5*e^3 +
 10*(b^3*c + 3*a*b*c^2)*d^4*e^4 + 3*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^3*e^5
+ 21*(a*b^3 + 3*a^2*b*c)*d^2*e^6 + 28*(3*a^2*b^2 + 2*a^3*c)*d*e^7)*x)/(e^20
*x^11 + 11*d*e^19*x^10 + 55*d^2*e^18*x^9 + 165*d^3*e^17*x^8 + 330*d^4*e^16*
x^7 + 462*d^5*e^15*x^6 + 462*d^6*e^14*x^5 + 330*d^7*e^13*x^4 + 165*d^8*e^12
*x^3 + 55*d^9*e^11*x^2 + 11*d^10*e^10*x + d^11*e^9)
```

Fricas [B] time = 1.77729, size = 1995, normalized size = 4.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^12,x, algorithm="fricas")
```

```
[Out] -1/6930*(2310*c^4*e^8*x^8 + 14*c^4*d^8 + 21*b*c^3*d^7*e + 252*a^3*b*d*e^7 +
 630*a^4*e^8 + 6*(3*b^2*c^2 + 2*a*c^3)*d^6*e^2 + 10*(b^3*c + 3*a*b*c^2)*d^5
*e^3 + 3*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^4*e^4 + 21*(a*b^3 + 3*a^2*b*c)*d^
3*e^5 + 28*(3*a^2*b^2 + 2*a^3*c)*d^2*e^6 + 2310*(2*c^4*d*e^7 + 3*b*c^3*e^8)
*x^7 + 462*(14*c^4*d^2*e^6 + 21*b*c^3*d*e^7 + 6*(3*b^2*c^2 + 2*a*c^3)*e^8)*
x^6 + 462*(14*c^4*d^3*e^5 + 21*b*c^3*d^2*e^6 + 6*(3*b^2*c^2 + 2*a*c^3)*d*e^
7 + 10*(b^3*c + 3*a*b*c^2)*e^8)*x^5 + 330*(14*c^4*d^4*e^4 + 21*b*c^3*d^3*e^
5 + 6*(3*b^2*c^2 + 2*a*c^3)*d^2*e^6 + 10*(b^3*c + 3*a*b*c^2)*d*e^7 + 3*(b^4
 + 12*a*b^2*c + 6*a^2*c^2)*e^8)*x^4 + 165*(14*c^4*d^5*e^3 + 21*b*c^3*d^4*e^
4 + 6*(3*b^2*c^2 + 2*a*c^3)*d^3*e^5 + 10*(b^3*c + 3*a*b*c^2)*d^2*e^6 + 3*(b
^4 + 12*a*b^2*c + 6*a^2*c^2)*d*e^7 + 21*(a*b^3 + 3*a^2*b*c)*e^8)*x^3 + 55*(
```

```
14*c^4*d^6*e^2 + 21*b*c^3*d^5*e^3 + 6*(3*b^2*c^2 + 2*a*c^3)*d^4*e^4 + 10*(b
^3*c + 3*a*b*c^2)*d^3*e^5 + 3*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^2*e^6 + 21*(
a*b^3 + 3*a^2*b*c)*d*e^7 + 28*(3*a^2*b^2 + 2*a^3*c)*e^8)*x^2 + 11*(14*c^4*d
^7*e + 21*b*c^3*d^6*e^2 + 252*a^3*b*e^8 + 6*(3*b^2*c^2 + 2*a*c^3)*d^5*e^3 +
 10*(b^3*c + 3*a*b*c^2)*d^4*e^4 + 3*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^3*e^5
+ 21*(a*b^3 + 3*a^2*b*c)*d^2*e^6 + 28*(3*a^2*b^2 + 2*a^3*c)*d*e^7)*x)/(e^20
*x^11 + 11*d*e^19*x^10 + 55*d^2*e^18*x^9 + 165*d^3*e^17*x^8 + 330*d^4*e^16*
x^7 + 462*d^5*e^15*x^6 + 462*d^6*e^14*x^5 + 330*d^7*e^13*x^4 + 165*d^8*e^12
*x^3 + 55*d^9*e^11*x^2 + 11*d^10*e^10*x + d^11*e^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**4/(e*x+d)**12,x)

[Out] Timed out
```

Giac [B] time = 1.11203, size = 1276, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x+d)^12,x, algorithm="giac")

[Out] -1/6930*(2310*c^4*x^8*e^8 + 4620*c^4*d*x^7*e^7 + 6468*c^4*d^2*x^6*e^6 + 646
8*c^4*d^3*x^5*e^5 + 4620*c^4*d^4*x^4*e^4 + 2310*c^4*d^5*x^3*e^3 + 770*c^4*d
^6*x^2*e^2 + 154*c^4*d^7*x*e + 14*c^4*d^8 + 6930*b*c^3*x^7*e^8 + 9702*b*c^3
*d*x^6*e^7 + 9702*b*c^3*d^2*x^5*e^6 + 6930*b*c^3*d^3*x^4*e^5 + 3465*b*c^3*d
^4*x^3*e^4 + 1155*b*c^3*d^5*x^2*e^3 + 231*b*c^3*d^6*x*e^2 + 21*b*c^3*d^7*e
+ 8316*b^2*c^2*x^6*e^8 + 5544*a*c^3*x^6*e^8 + 8316*b^2*c^2*d*x^5*e^7 + 5544
*a*c^3*d*x^5*e^7 + 5940*b^2*c^2*d^2*x^4*e^6 + 3960*a*c^3*d^2*x^4*e^6 + 2970
*b^2*c^2*d^3*x^3*e^5 + 1980*a*c^3*d^3*x^3*e^5 + 990*b^2*c^2*d^4*x^2*e^4 + 6
60*a*c^3*d^4*x^2*e^4 + 198*b^2*c^2*d^5*x*e^3 + 132*a*c^3*d^5*x*e^3 + 18*b^2
*c^2*d^6*e^2 + 12*a*c^3*d^6*e^2 + 4620*b^3*c*x^5*e^8 + 13860*a*b*c^2*x^5*e^
8 + 3300*b^3*c*d*x^4*e^7 + 9900*a*b*c^2*d*x^4*e^7 + 1650*b^3*c*d^2*x^3*e^6
+ 4950*a*b*c^2*d^2*x^3*e^6 + 550*b^3*c*d^3*x^2*e^5 + 1650*a*b*c^2*d^3*x^2*e
^5 + 110*b^3*c*d^4*x*e^4 + 330*a*b*c^2*d^4*x*e^4 + 10*b^3*c*d^5*e^3 + 30*a*
b*c^2*d^5*e^3 + 990*b^4*x^4*e^8 + 11880*a*b^2*c*x^4*e^8 + 5940*a^2*c^2*x^4*
e^8 + 495*b^4*d*x^3*e^7 + 5940*a*b^2*c*d*x^3*e^7 + 2970*a^2*c^2*d*x^3*e^7 +
 165*b^4*d^2*x^2*e^6 + 1980*a*b^2*c*d^2*x^2*e^6 + 990*a^2*c^2*d^2*x^2*e^6 +
 33*b^4*d^3*x*e^5 + 396*a*b^2*c*d^3*x*e^5 + 198*a^2*c^2*d^3*x*e^5 + 3*b^4*d
^4*e^4 + 36*a*b^2*c*d^4*e^4 + 18*a^2*c^2*d^4*e^4 + 3465*a*b^3*x^3*e^8 + 103
95*a^2*b*c*x^3*e^8 + 1155*a*b^3*d*x^2*e^7 + 3465*a^2*b*c*d*x^2*e^7 + 231*a*
b^3*d^2*x*e^6 + 693*a^2*b*c*d^2*x*e^6 + 21*a*b^3*d^3*e^5 + 63*a^2*b*c*d^3*e
^5 + 4620*a^2*b^2*x^2*e^8 + 3080*a^3*c*x^2*e^8 + 924*a^2*b^2*d*x*e^7 + 616*
a^3*c*d*x*e^7 + 84*a^2*b^2*d^2*e^6 + 56*a^3*c*d^2*e^6 + 2772*a^3*b*x*e^8 +
252*a^3*b*d*e^7 + 630*a^4*e^8)*e^(-9)/(x*e + d)^11
```

3.2163 $\int x^4 \left(3-4x+x^2\right)^2 \, dx$

Optimal. Leaf size=32

$$\frac{x^9}{9}-x^8+\frac{22x^7}{7}-4x^6+\frac{9x^5}{5}$$

```
[Out] (9*x^5)/5 - 4*x^6 + (22*x^7)/7 - x^8 + x^9/9
```

Rubi [A] time = 0.0137426, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {698}

$$\frac{x^9}{9}-x^8+\frac{22x^7}{7}-4x^6+\frac{9x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(3 - 4*x + x^2)^2,x]
```

```
[Out] (9*x^5)/5 - 4*x^6 + (22*x^7)/7 - x^8 + x^9/9
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned} \int x^4 \left(3-4x+x^2\right)^2 \, dx &= \int \left(9x^4-24x^5+22x^6-8x^7+x^8\right) dx \\ &= \frac{9x^5}{5}-4x^6+\frac{22x^7}{7}-x^8+\frac{x^9}{9} \end{aligned}$$

Mathematica [A] time = 0.0009931, size = 32, normalized size = 1.

$$\frac{x^9}{9}-x^8+\frac{22x^7}{7}-4x^6+\frac{9x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(3 - 4*x + x^2)^2,x]
```

```
[Out] (9*x^5)/5 - 4*x^6 + (22*x^7)/7 - x^8 + x^9/9
```

Maple [A] time = 0.038, size = 27, normalized size = 0.8

$$\frac{9\,x^5}{5}-4\,x^6+\frac{22\,x^7}{7}-x^8+\frac{x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^2-4*x+3)^2,x)
```

```
[Out] 9/5*x^5-4*x^6+22/7*x^7-x^8+1/9*x^9
```

Maxima [A] time = 0.971315, size = 35, normalized size = 1.09

$$\frac{1}{9}\,x^9 - x^8 + \frac{22}{7}\,x^7 - 4\,x^6 + \frac{9}{5}\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^2-4*x+3)^2,x, algorithm="maxima")
```

```
[Out] 1/9*x^9 - x^8 + 22/7*x^7 - 4*x^6 + 9/5*x^5
```

Fricas [A] time = 1.4604, size = 59, normalized size = 1.84

$$\frac{1}{9}x^9 - x^8 + \frac{22}{7}x^7 - 4x^6 + \frac{9}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^2-4*x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9 - x^8 + 22/7*x^7 - 4*x^6 + 9/5*x^5
```

Sympy [A] time = 0.062411, size = 26, normalized size = 0.81

$$\frac{x^9}{9} - x^8 + \frac{22x^7}{7} - 4x^6 + \frac{9x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(x**2-4*x+3)**2,x)
```

```
[Out] x**9/9 - x**8 + 22*x**7/7 - 4*x**6 + 9*x**5/5
```

Giac [A] time = 1.10492, size = 35, normalized size = 1.09

$$\frac{1}{9}\,x^9 - x^8 + \frac{22}{7}\,x^7 - 4\,x^6 + \frac{9}{5}\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^2-4*x+3)^2,x, algorithm="giac")
```

```
[Out] 1/9*x^9 - x^8 + 22/7*x^7 - 4*x^6 + 9/5*x^5
```

3.2164 $\int x^3 (3-4x+x^2)^2 \, dx$

Optimal. Leaf size=36

$$\frac{x^8}{8}-\frac{8x^7}{7}+\frac{11x^6}{3}-\frac{24x^5}{5}+\frac{9x^4}{4}$$

```
[Out] (9*x^4)/4 - (24*x^5)/5 + (11*x^6)/3 - (8*x^7)/7 + x^8/8
```

Rubi [A] time = 0.0125975, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {698}

$$\frac{x^8}{8}-\frac{8x^7}{7}+\frac{11x^6}{3}-\frac{24x^5}{5}+\frac{9x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(3 - 4*x + x^2)^2,x]
```

```
[Out] (9*x^4)/4 - (24*x^5)/5 + (11*x^6)/3 - (8*x^7)/7 + x^8/8
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int x^3 \left(3-4x+x^2\right)^2 \, dx &= \int \left(9x^3-24x^4+22x^5-8x^6+x^7\right) dx\\ &= \frac{9x^4}{4}-\frac{24x^5}{5}+\frac{11x^6}{3}-\frac{8x^7}{7}+\frac{x^8}{8}\end{aligned}$$

Mathematica [A] time = 0.0006064, size = 36, normalized size = 1.

$$\frac{x^8}{8}-\frac{8x^7}{7}+\frac{11x^6}{3}-\frac{24x^5}{5}+\frac{9x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(3 - 4*x + x^2)^2,x]
```

```
[Out] (9*x^4)/4 - (24*x^5)/5 + (11*x^6)/3 - (8*x^7)/7 + x^8/8
```

Maple [A] time = 0.039, size = 27, normalized size = 0.8

$$\frac{9\,x^4}{4}-\frac{24\,x^5}{5}+\frac{11\,x^6}{3}-\frac{8\,x^7}{7}+\frac{x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2-4*x+3)^2,x)
```

```
[Out] 9/4*x^4-24/5*x^5+11/3*x^6-8/7*x^7+1/8*x^8
```

Maxima [A] time = 0.99327, size = 35, normalized size = 0.97

$$\frac{1}{8}x^8 - \frac{8}{7}x^7 + \frac{11}{3}x^6 - \frac{24}{5}x^5 + \frac{9}{4}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2-4*x+3)^2,x, algorithm="maxima")
```

```
[Out] 1/8*x^8 - 8/7*x^7 + 11/3*x^6 - 24/5*x^5 + 9/4*x^4
```

Fricas [A] time = 1.48796, size = 69, normalized size = 1.92

$$\frac{1}{8}x^8 - \frac{8}{7}x^7 + \frac{11}{3}x^6 - \frac{24}{5}x^5 + \frac{9}{4}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2-4*x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/8*x^8 - 8/7*x^7 + 11/3*x^6 - 24/5*x^5 + 9/4*x^4
```

Sympy [A] time = 0.059099, size = 31, normalized size = 0.86

$$\frac{x^8}{8} - \frac{8x^7}{7} + \frac{11x^6}{3} - \frac{24x^5}{5} + \frac{9x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**2-4*x+3)**2,x)
```

```
[Out] x**8/8 - 8*x**7/7 + 11*x**6/3 - 24*x**5/5 + 9*x**4/4
```

Giac [A] time = 1.12255, size = 35, normalized size = 0.97

$$\frac{1}{8}x^8 - \frac{8}{7}x^7 + \frac{11}{3}x^6 - \frac{24}{5}x^5 + \frac{9}{4}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2-4*x+3)^2,x, algorithm="giac")
```

```
[Out] 1/8*x^8 - 8/7*x^7 + 11/3*x^6 - 24/5*x^5 + 9/4*x^4
```

3.2165 $\int x^2 (3-4x+x^2)^2 \, dx$

Optimal. Leaf size=32

$$\frac{x^7}{7}-\frac{4x^6}{3}+\frac{22x^5}{5}-6x^4+3x^3$$

```
[Out] 3*x^3 - 6*x^4 + (22*x^5)/5 - (4*x^6)/3 + x^7/7
```

Rubi [A] time = 0.0128501, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {698}

$$\frac{x^7}{7}-\frac{4x^6}{3}+\frac{22x^5}{5}-6x^4+3x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(3 - 4*x + x^2)^2,x]
```

```
[Out] 3*x^3 - 6*x^4 + (22*x^5)/5 - (4*x^6)/3 + x^7/7
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int x^2 (3-4x+x^2)^2 \, dx &= \int (9x^2-24x^3+22x^4-8x^5+x^6) \, dx\\ &= 3x^3-6x^4+\frac{22x^5}{5}-\frac{4x^6}{3}+\frac{x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.000748, size = 32, normalized size = 1.

$$\frac{x^7}{7}-\frac{4x^6}{3}+\frac{22x^5}{5}-6x^4+3x^3$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(3 - 4*x + x^2)^2,x]
```

```
[Out] 3*x^3 - 6*x^4 + (22*x^5)/5 - (4*x^6)/3 + x^7/7
```

Maple [A] time = 0.04, size = 27, normalized size = 0.8

$$3x^3-6x^4+\frac{22x^5}{5}-\frac{4x^6}{3}+\frac{x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^2-4*x+3)^2,x)
```

```
[Out] 3*x^3-6*x^4+22/5*x^5-4/3*x^6+1/7*x^7
```

Maxima [A] time = 1.02264, size = 35, normalized size = 1.09

$$\frac{1}{7}\,x^7 - \frac{4}{3}\,x^6 + \frac{22}{5}\,x^5 - 6\,x^4 + 3\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2-4*x+3)^2,x, algorithm="maxima")
```

```
[Out] 1/7*x^7 - 4/3*x^6 + 22/5*x^5 - 6*x^4 + 3*x^3
```

Fricas [A] time = 1.44467, size = 62, normalized size = 1.94

$$\frac{1}{7}x^7 - \frac{4}{3}x^6 + \frac{22}{5}x^5 - 6x^4 + 3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2-4*x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7 - 4/3*x^6 + 22/5*x^5 - 6*x^4 + 3*x^3
```

Sympy [A] time = 0.060042, size = 27, normalized size = 0.84

$$\frac{x^7}{7} - \frac{4x^6}{3} + \frac{22x^5}{5} - 6x^4 + 3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**2-4*x+3)**2,x)
```

```
[Out] x**7/7 - 4*x**6/3 + 22*x**5/5 - 6*x**4 + 3*x**3
```

Giac [A] time = 1.10361, size = 35, normalized size = 1.09

$$\frac{1}{7}\,x^7 - \frac{4}{3}\,x^6 + \frac{22}{5}\,x^5 - 6\,x^4 + 3\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2-4*x+3)^2,x, algorithm="giac")
```

```
[Out] 1/7*x^7 - 4/3*x^6 + 22/5*x^5 - 6*x^4 + 3*x^3
```

3.2166 $\int x\left(3-4x+x^2\right)^2\,dx$

Optimal. Leaf size=34

$$\frac{x^6}{6}-\frac{8x^5}{5}+\frac{11x^4}{2}-8x^3+\frac{9x^2}{2}$$

```
[Out] (9*x^2)/2 - 8*x^3 + (11*x^4)/2 - (8*x^5)/5 + x^6/6
```

Rubi [A] time = 0.0107675, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {631}

$$\frac{x^6}{6}-\frac{8x^5}{5}+\frac{11x^4}{2}-8x^3+\frac{9x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(3 - 4*x + x^2)^2,x]
```

```
[Out] (9*x^2)/2 - 8*x^3 + (11*x^4)/2 - (8*x^5)/5 + x^6/6
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(3-4x+x^2\right)^2\,dx&=\int\left(9x-24x^2+22x^3-8x^4+x^5\right)\,dx\\&=\frac{9x^2}{2}-8x^3+\frac{11x^4}{2}-\frac{8x^5}{5}+\frac{x^6}{6}\end{aligned}$$

Mathematica [A] time = 0.0010482, size = 34, normalized size = 1.

$$\frac{x^6}{6}-\frac{8x^5}{5}+\frac{11x^4}{2}-8x^3+\frac{9x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(3 - 4*x + x^2)^2,x]
```

```
[Out] (9*x^2)/2 - 8*x^3 + (11*x^4)/2 - (8*x^5)/5 + x^6/6
```

Maple [A] time = 0.04, size = 27, normalized size = 0.8

$$\frac{9\,x^2}{2}-8\,x^3+\frac{11\,x^4}{2}-\frac{8\,x^5}{5}+\frac{x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2-4*x+3)^2,x)
```

```
[Out] 9/2*x^2-8*x^3+11/2*x^4-8/5*x^5+1/6*x^6
```

Maxima [A] time = 1.0305, size = 35, normalized size = 1.03

$$\frac{1}{6}\,x^6-\frac{8}{5}\,x^5+\frac{11}{2}\,x^4-8\,x^3+\frac{9}{2}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-4*x+3)^2,x, algorithm="maxima")
```

```
[Out] 1/6*x^6 - 8/5*x^5 + 11/2*x^4 - 8*x^3 + 9/2*x^2
```

Fricas [A] time = 1.60726, size = 65, normalized size = 1.91

$$\frac{1}{6}x^6-\frac{8}{5}x^5+\frac{11}{2}x^4-8x^3+\frac{9}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-4*x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/6*x^6 - 8/5*x^5 + 11/2*x^4 - 8*x^3 + 9/2*x^2
```

Sympy [A] time = 0.059149, size = 29, normalized size = 0.85

$$\frac{x^6}{6}-\frac{8x^5}{5}+\frac{11x^4}{2}-8x^3+\frac{9x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2-4*x+3)**2,x)
```

```
[Out] x**6/6 - 8*x**5/5 + 11*x**4/2 - 8*x**3 + 9*x**2/2
```

Giac [A] time = 1.10512, size = 35, normalized size = 1.03

$$\frac{1}{6}\,x^6-\frac{8}{5}\,x^5+\frac{11}{2}\,x^4-8\,x^3+\frac{9}{2}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-4*x+3)^2,x, algorithm="giac")
```

```
[Out] 1/6*x^6 - 8/5*x^5 + 11/2*x^4 - 8*x^3 + 9/2*x^2
```

3.2167 $\int \left(3-4x+x^2\right)^2 \, dx$

Optimal. Leaf size=28

$$-\frac{1}{5}(3-x)^5-\frac{4}{3}(3-x)^3+(x-3)^4$$

```
[Out] (-4*(3 - x)^3)/3 - (3 - x)^5/5 + (-3 + x)^4
```

Rubi [A] time = 0.0106319, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {610, 43}

$$-\frac{1}{5}(3-x)^5-\frac{4}{3}(3-x)^3+(x-3)^4$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 4*x + x^2)^2,x]
```

```
[Out] (-4*(3 - x)^3)/3 - (3 - x)^5/5 + (-3 + x)^4
```

Rule 610

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Dist[1/c^p, Int[Simp[b/2 - q/2 + c*x, x]^p*Simp[b/2 + q/2 + c
*x, x]^p, x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(3-4x+x^2\right)^2 \, dx &= \int (-3+x)^2(-1+x)^2 \, dx\\ &= \int \left(4(-3+x)^2+4(-3+x)^3+(-3+x)^4\right) dx\\ &= -\frac{4}{3}(3-x)^3-\frac{1}{5}(3-x)^5+(-3+x)^4\end{aligned}$$

Mathematica [A] time = 0.0006362, size = 28, normalized size = 1.

$$\frac{x^5}{5}-2x^4+\frac{22x^3}{3}-12x^2+9x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 4*x + x^2)^2,x]
```

```
[Out] 9*x - 12*x^2 + (22*x^3)/3 - 2*x^4 + x^5/5
```

Maple [A] time = 0.039, size = 25, normalized size = 0.9

$$\frac{x^5}{5} - 2x^4 + \frac{22x^3}{3} - 12x^2 + 9x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-4*x+3)^2,x)
```

```
[Out] 1/5*x^5-2*x^4+22/3*x^3-12*x^2+9*x
```

Maxima [A] time = 0.978717, size = 32, normalized size = 1.14

$$\frac{1}{5}x^5 - 2x^4 + \frac{22}{3}x^3 - 12x^2 + 9x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4*x+3)^2,x, algorithm="maxima")
```

```
[Out] 1/5*x^5 - 2*x^4 + 22/3*x^3 - 12*x^2 + 9*x
```

Fricas [A] time = 1.4252, size = 58, normalized size = 2.07

$$\frac{1}{5}x^5 - 2x^4 + \frac{22}{3}x^3 - 12x^2 + 9x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4*x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/5*x^5 - 2*x^4 + 22/3*x^3 - 12*x^2 + 9*x
```

Sympy [A] time = 0.058233, size = 24, normalized size = 0.86

$$\frac{x^5}{5} - 2x^4 + \frac{22x^3}{3} - 12x^2 + 9x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-4*x+3)**2,x)
```

```
[Out] x**5/5 - 2*x**4 + 22*x**3/3 - 12*x**2 + 9*x
```

Giac [A] time = 1.12871, size = 32, normalized size = 1.14

$$\frac{1}{5}x^5 - 2x^4 + \frac{22}{3}x^3 - 12x^2 + 9x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4*x+3)^2,x, algorithm="giac")
```

```
[Out] 1/5*x^5 - 2*x^4 + 22/3*x^3 - 12*x^2 + 9*x
```

3.2168 $\int \frac{(3-4x+x^2)^2}{x} dx$

Optimal. Leaf size=27

$$\frac{x^4}{4}-\frac{8x^3}{3}+11x^2-24x+9\log(x)$$

```
[Out] -24*x + 11*x^2 - (8*x^3)/3 + x^4/4 + 9*Log[x]
```

Rubi [A] time = 0.0092649, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {698}

$$\frac{x^4}{4}-\frac{8x^3}{3}+11x^2-24x+9\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 4*x + x^2)^2/x,x]

[Out] -24*x + 11*x^2 - (8*x^3)/3 + x^4/4 + 9*Log[x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{(3-4x+x^2)^2}{x}\,dx &= \int \left(-24+\frac{9}{x}+22x-8x^2+x^3\right) dx\\ &= -24x+11x^2-\frac{8x^3}{3}+\frac{x^4}{4}+9\log(x)\end{aligned}$$

Mathematica [A] time = 0.0006272, size = 27, normalized size = 1.

$$\frac{x^4}{4}-\frac{8x^3}{3}+11x^2-24x+9\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 4*x + x^2)^2/x,x]

[Out] -24*x + 11*x^2 - (8*x^3)/3 + x^4/4 + 9*Log[x]
```

Maple [A] time = 0.038, size = 24, normalized size = 0.9

$$-24\,x+11\,x^2-\frac{8\,x^3}{3}+\frac{x^4}{4}+9\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-4*x+3)^2/x,x)
```

```
[Out] -24*x+11*x^2-8/3*x^3+1/4*x^4+9*ln(x)
```

Maxima [A] time = 0.960832, size = 31, normalized size = 1.15

$$\frac{1}{4}\,x^4 - \frac{8}{3}\,x^3 + 11\,x^2 - 24\,x + 9\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4*x+3)^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - 8/3*x^3 + 11*x^2 - 24*x + 9*log(x)
```

Fricas [A] time = 1.63101, size = 62, normalized size = 2.3

$$\frac{1}{4}\,x^4 - \frac{8}{3}\,x^3 + 11\,x^2 - 24\,x + 9\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4*x+3)^2/x,x, algorithm="fricas")
```

```
[Out] 1/4*x^4 - 8/3*x^3 + 11*x^2 - 24*x + 9*log(x)
```

Sympy [A] time = 0.082447, size = 24, normalized size = 0.89

$$\frac{x^4}{4} - \frac{8x^3}{3} + 11x^2 - 24x + 9\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-4*x+3)**2/x,x)
```

```
[Out] x**4/4 - 8*x**3/3 + 11*x**2 - 24*x + 9*log(x)
```

Giac [A] time = 1.14016, size = 32, normalized size = 1.19

$$\frac{1}{4}\,x^4 - \frac{8}{3}\,x^3 + 11\,x^2 - 24\,x + 9\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4*x+3)^2/x,x, algorithm="giac")
```

```
[Out] 1/4*x^4 - 8/3*x^3 + 11*x^2 - 24*x + 9*log(abs(x))
```

3.2169 $\int \frac{(3-4x+x^2)^2}{x^2} dx$

Optimal. Leaf size=25

$$\frac{x^3}{3} - 4x^2 + 22x - \frac{9}{x} - 24\log(x)$$

```
[Out] -9/x + 22*x - 4*x^2 + x^3/3 - 24*Log[x]
```

Rubi [A] time = 0.01157, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {698}

$$\frac{x^3}{3} - 4x^2 + 22x - \frac{9}{x} - 24\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 4*x + x^2)^2/x^2,x]

[Out] -9/x + 22*x - 4*x^2 + x^3/3 - 24*Log[x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned} \int \frac{(3-4x+x^2)^2}{x^2} dx &= \int \left(22 + \frac{9}{x^2} - \frac{24}{x} - 8x + x^2\right) dx \\ &= -\frac{9}{x} + 22x - 4x^2 + \frac{x^3}{3} - 24\log(x) \end{aligned}$$

Mathematica [A] time = 0.00066, size = 25, normalized size = 1.

$$\frac{x^3}{3} - 4x^2 + 22x - \frac{9}{x} - 24\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 4*x + x^2)^2/x^2,x]

[Out] -9/x + 22*x - 4*x^2 + x^3/3 - 24*Log[x]
```

Maple [A] time = 0.044, size = 24, normalized size = 1.

$$-9\,x^{-1} + 22\,x - 4\,x^2 + \frac{x^3}{3} - 24\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-4*x+3)^2/x^2,x)
```

```
[Out] -9/x+22*x-4*x^2+1/3*x^3-24*ln(x)
```

Maxima [A] time = 0.954974, size = 31, normalized size = 1.24

$$\frac{1}{3}x^3 - 4x^2 + 22x - \frac{9}{x} - 24\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4*x+3)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 - 4*x^2 + 22*x - 9/x - 24*log(x)
```

Fricas [A] time = 1.68577, size = 68, normalized size = 2.72

$$\frac{x^4 - 12x^3 + 66x^2 - 72x\log(x) - 27}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4*x+3)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(x^4 - 12*x^3 + 66*x^2 - 72*x*log(x) - 27)/x
```

Sympy [A] time = 0.087531, size = 20, normalized size = 0.8

$$\frac{x^3}{3} - 4x^2 + 22x - 24\log(x) - \frac{9}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-4*x+3)**2/x**2,x)
```

```
[Out] x**3/3 - 4*x**2 + 22*x - 24*log(x) - 9/x
```

Giac [A] time = 1.10401, size = 32, normalized size = 1.28

$$\frac{1}{3}x^3 - 4x^2 + 22x - \frac{9}{x} - 24\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4*x+3)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3 - 4*x^2 + 22*x - 9/x - 24*log(abs(x))
```

3.2170 $\int \frac{(3-4x+x^2)^2}{x^3} dx$

Optimal. Leaf size=27

$$\frac{x^2}{2} - \frac{9}{2x^2} - 8x + \frac{24}{x} + 22\log(x)$$

```
[Out] -9/(2*x^2) + 24/x - 8*x + x^2/2 + 22*Log[x]
```

Rubi [A] time = 0.0113032, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {698}

$$\frac{x^2}{2} - \frac{9}{2x^2} - 8x + \frac{24}{x} + 22\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 4*x + x^2)^2/x^3,x]
```

```
[Out] -9/(2*x^2) + 24/x - 8*x + x^2/2 + 22*Log[x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned} \int \frac{(3-4x+x^2)^2}{x^3} dx &= \int \left(-8 + \frac{9}{x^3} - \frac{24}{x^2} + \frac{22}{x} + x\right) dx \\ &= -\frac{9}{2x^2} + \frac{24}{x} - 8x + \frac{x^2}{2} + 22\log(x) \end{aligned}$$

Mathematica [A] time = 0.0007447, size = 27, normalized size = 1.

$$\frac{x^2}{2} - \frac{9}{2x^2} - 8x + \frac{24}{x} + 22\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 4*x + x^2)^2/x^3,x]
```

```
[Out] -9/(2*x^2) + 24/x - 8*x + x^2/2 + 22*Log[x]
```

Maple [A] time = 0.043, size = 24, normalized size = 0.9

$$-\frac{9}{2x^2} + 24x^{-1} - 8x + \frac{x^2}{2} + 22\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-4*x+3)^2/x^3,x)
```

```
[Out] -9/2/x^2+24/x-8*x+1/2*x^2+22*ln(x)
```

Maxima [A] time = 1.06307, size = 31, normalized size = 1.15

$$\frac{1}{2}x^2 - 8x + \frac{3(16x-3)}{2x^2} + 22\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4*x+3)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 8*x + 3/2*(16*x - 3)/x^2 + 22*log(x)
```

Fricas [A] time = 1.72387, size = 69, normalized size = 2.56

$$\frac{x^4 - 16x^3 + 44x^2\log(x) + 48x - 9}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4*x+3)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(x^4 - 16*x^3 + 44*x^2*log(x) + 48*x - 9)/x^2
```

Sympy [A] time = 0.09643, size = 22, normalized size = 0.81

$$\frac{x^2}{2} - 8x + 22\log(x) + \frac{48x-9}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-4*x+3)**2/x**3,x)
```

```
[Out] x**2/2 - 8*x + 22*log(x) + (48*x - 9)/(2*x**2)
```

Giac [A] time = 1.14068, size = 32, normalized size = 1.19

$$\frac{1}{2}x^2 - 8x + \frac{3(16x-3)}{2x^2} + 22\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4*x+3)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 8*x + 3/2*(16*x - 3)/x^2 + 22*log(abs(x))
```

3.2171 $\int \frac{(3-4x+x^2)^2}{x^4}\,dx$

Optimal. Leaf size=21

$$\frac{12}{x^2}-\frac{3}{x^3}+x-\frac{22}{x}-8\log(x)$$

```
[Out] -3/x^3 + 12/x^2 - 22/x + x - 8*Log[x]
```

Rubi [A] time = 0.0107885, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {698}

$$\frac{12}{x^2}-\frac{3}{x^3}+x-\frac{22}{x}-8\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 4*x + x^2)^2/x^4,x]
```

```
[Out] -3/x^3 + 12/x^2 - 22/x + x - 8*Log[x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{(3-4x+x^2)^2}{x^4}\,dx &= \int \left(1+\frac{9}{x^4}-\frac{24}{x^3}+\frac{22}{x^2}-\frac{8}{x}\right)dx\\ &= -\frac{3}{x^3}+\frac{12}{x^2}-\frac{22}{x}+x-8\log(x)\end{aligned}$$

Mathematica [A] time = 0.0006489, size = 21, normalized size = 1.

$$\frac{12}{x^2}-\frac{3}{x^3}+x-\frac{22}{x}-8\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 4*x + x^2)^2/x^4,x]
```

```
[Out] -3/x^3 + 12/x^2 - 22/x + x - 8*Log[x]
```

Maple [A] time = 0.045, size = 22, normalized size = 1.1

$$-3\,x^{-3}+12\,x^{-2}-22\,x^{-1}+x-8\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-4*x+3)^2/x^4,x)
```

```
[Out] -3/x^3+12/x^2-22/x+x-8*ln(x)
```

Maxima [A] time = 0.985285, size = 28, normalized size = 1.33

$$x - \frac{22\,x^2 - 12\,x + 3}{x^3} - 8\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4*x+3)^2/x^4,x, algorithm="maxima")
```

```
[Out] x - (22*x^2 - 12*x + 3)/x^3 - 8*log(x)
```

Fricas [A] time = 1.6889, size = 62, normalized size = 2.95

$$\frac{x^4 - 8\,x^3\log\left(x\right) - 22\,x^2 + 12\,x - 3}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4*x+3)^2/x^4,x, algorithm="fricas")
```

```
[Out] (x^4 - 8*x^3*log(x) - 22*x^2 + 12*x - 3)/x^3
```

Sympy [A] time = 0.100731, size = 19, normalized size = 0.9

$$x - 8\log\left(x\right) - \frac{22x^2 - 12x + 3}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-4*x+3)**2/x**4,x)
```

```
[Out] x - 8*log(x) - (22*x**2 - 12*x + 3)/x**3
```

Giac [A] time = 1.13196, size = 30, normalized size = 1.43

$$x - \frac{22\,x^2 - 12\,x + 3}{x^3} - 8\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4*x+3)^2/x^4,x, algorithm="giac")
```

```
[Out] x - (22*x^2 - 12*x + 3)/x^3 - 8*log(abs(x))
```

3.2172 $\int \frac{(3-4x+x^2)^2}{x^5} dx$

Optimal. Leaf size=25

$$-\frac{11}{x^2}+\frac{8}{x^3}-\frac{9}{4x^4}+\frac{8}{x}+\log(x)$$

```
[Out] -9/(4*x^4) + 8/x^3 - 11/x^2 + 8/x + Log[x]
```

Rubi [A] time = 0.0112481, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {698}

$$-\frac{11}{x^2}+\frac{8}{x^3}-\frac{9}{4x^4}+\frac{8}{x}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 4*x + x^2)^2/x^5,x]

[Out] -9/(4*x^4) + 8/x^3 - 11/x^2 + 8/x + Log[x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{(3-4x+x^2)^2}{x^5} dx &= \int \left(\frac{9}{x^5}-\frac{24}{x^4}+\frac{22}{x^3}-\frac{8}{x^2}+\frac{1}{x}\right) dx\\ &= -\frac{9}{4x^4}+\frac{8}{x^3}-\frac{11}{x^2}+\frac{8}{x}+\log(x)\end{aligned}$$

Mathematica [A] time = 0.0006945, size = 25, normalized size = 1.

$$-\frac{11}{x^2}+\frac{8}{x^3}-\frac{9}{4x^4}+\frac{8}{x}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 4*x + x^2)^2/x^5,x]

[Out] -9/(4*x^4) + 8/x^3 - 11/x^2 + 8/x + Log[x]
```

Maple [A] time = 0.043, size = 24, normalized size = 1.

$$-\frac{9}{4\,x^4}+8\,x^{-3}-11\,x^{-2}+8\,x^{-1}+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-4*x+3)^2/x^5,x)
```

```
[Out] -9/4/x^4+8/x^3-11/x^2+8/x+ln(x)
```

Maxima [A] time = 0.995359, size = 31, normalized size = 1.24

$$\frac{32\,x^3 - 44\,x^2 + 32\,x - 9}{4\,x^4} + \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4*x+3)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*(32*x^3 - 44*x^2 + 32*x - 9)/x^4 + log(x)
```

Fricas [A] time = 1.85166, size = 72, normalized size = 2.88

$$\frac{4\,x^4\log\left(x\right) + 32\,x^3 - 44\,x^2 + 32\,x - 9}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4*x+3)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(4*x^4*log(x) + 32*x^3 - 44*x^2 + 32*x - 9)/x^4
```

Sympy [A] time = 0.105897, size = 22, normalized size = 0.88

$$\log\left(x\right) + \frac{32x^3 - 44x^2 + 32x - 9}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-4*x+3)**2/x**5,x)
```

```
[Out] log(x) + (32*x**3 - 44*x**2 + 32*x - 9)/(4*x**4)
```

Giac [A] time = 1.13567, size = 32, normalized size = 1.28

$$\frac{32\,x^3 - 44\,x^2 + 32\,x - 9}{4\,x^4} + \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4*x+3)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/4*(32*x^3 - 44*x^2 + 32*x - 9)/x^4 + log(abs(x))
```

3.2173 $\int \frac{(3-4x+x^2)^2}{x^6} dx$

Optimal. Leaf size=30

$$\frac{4}{x^2}-\frac{22}{3x^3}+\frac{6}{x^4}-\frac{9}{5x^5}-\frac{1}{x}$$

```
[Out] -9/(5*x^5) + 6/x^4 - 22/(3*x^3) + 4/x^2 - x^(-1)
```

Rubi [A] time = 0.0116723, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {698}

$$\frac{4}{x^2}-\frac{22}{3x^3}+\frac{6}{x^4}-\frac{9}{5x^5}-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 4*x + x^2)^2/x^6,x]

[Out] -9/(5*x^5) + 6/x^4 - 22/(3*x^3) + 4/x^2 - x^(-1)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{(3-4x+x^2)^2}{x^6} dx &= \int \left(\frac{9}{x^6}-\frac{24}{x^5}+\frac{22}{x^4}-\frac{8}{x^3}+\frac{1}{x^2}\right) dx\\ &= -\frac{9}{5x^5}+\frac{6}{x^4}-\frac{22}{3x^3}+\frac{4}{x^2}-\frac{1}{x}\end{aligned}$$

Mathematica [A] time = 0.0006086, size = 30, normalized size = 1.

$$\frac{4}{x^2}-\frac{22}{3x^3}+\frac{6}{x^4}-\frac{9}{5x^5}-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 4*x + x^2)^2/x^6,x]

[Out] -9/(5*x^5) + 6/x^4 - 22/(3*x^3) + 4/x^2 - x^(-1)
```

Maple [A] time = 0.043, size = 27, normalized size = 0.9

$$-\frac{9}{5\,x^5}+6\,x^{-4}-\frac{22}{3\,x^3}+4\,x^{-2}-x^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-4*x+3)^2/x^6,x)
```

```
[Out] -9/5/x^5+6/x^4-22/3/x^3+4/x^2-1/x
```

Maxima [A] time = 1.00619, size = 34, normalized size = 1.13

$$-\frac{15\,x^4-60\,x^3+110\,x^2-90\,x+27}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4*x+3)^2/x^6,x, algorithm="maxima")
```

```
[Out] -1/15*(15*x^4 - 60*x^3 + 110*x^2 - 90*x + 27)/x^5
```

Fricas [A] time = 1.88511, size = 69, normalized size = 2.3

$$-\frac{15\,x^4-60\,x^3+110\,x^2-90\,x+27}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4*x+3)^2/x^6,x, algorithm="fricas")
```

```
[Out] -1/15*(15*x^4 - 60*x^3 + 110*x^2 - 90*x + 27)/x^5
```

Sympy [A] time = 0.105595, size = 26, normalized size = 0.87

$$-\frac{15x^4-60x^3+110x^2-90x+27}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-4*x+3)**2/x**6,x)
```

```
[Out] -(15*x**4 - 60*x**3 + 110*x**2 - 90*x + 27)/(15*x**5)
```

Giac [A] time = 1.11944, size = 34, normalized size = 1.13

$$-\frac{15\,x^4-60\,x^3+110\,x^2-90\,x+27}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4*x+3)^2/x^6,x, algorithm="giac")
```

```
[Out] -1/15*(15*x^4 - 60*x^3 + 110*x^2 - 90*x + 27)/x^5
```

3.2174 $\int \frac{(3-4x+x^2)^2}{x^7} \, dx$

Optimal. Leaf size=36

$$-\frac{1}{2x^2}+\frac{8}{3x^3}-\frac{11}{2x^4}+\frac{24}{5x^5}-\frac{3}{2x^6}$$

```
[Out] -3/(2*x^6) + 24/(5*x^5) - 11/(2*x^4) + 8/(3*x^3) - 1/(2*x^2)
```

Rubi [A] time = 0.0116563, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {698}

$$-\frac{1}{2x^2}+\frac{8}{3x^3}-\frac{11}{2x^4}+\frac{24}{5x^5}-\frac{3}{2x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 4*x + x^2)^2/x^7,x]
```

```
[Out] -3/(2*x^6) + 24/(5*x^5) - 11/(2*x^4) + 8/(3*x^3) - 1/(2*x^2)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{(3-4x+x^2)^2}{x^7} \, dx &= \int \left(\frac{9}{x^7}-\frac{24}{x^6}+\frac{22}{x^5}-\frac{8}{x^4}+\frac{1}{x^3}\right) dx\\ &= -\frac{3}{2x^6}+\frac{24}{5x^5}-\frac{11}{2x^4}+\frac{8}{3x^3}-\frac{1}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.0006213, size = 36, normalized size = 1.

$$-\frac{1}{2x^2}+\frac{8}{3x^3}-\frac{11}{2x^4}+\frac{24}{5x^5}-\frac{3}{2x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 4*x + x^2)^2/x^7,x]
```

```
[Out] -3/(2*x^6) + 24/(5*x^5) - 11/(2*x^4) + 8/(3*x^3) - 1/(2*x^2)
```

Maple [A] time = 0.041, size = 27, normalized size = 0.8

$$-\frac{3}{2\,x^6}+\frac{24}{5\,x^5}-\frac{11}{2\,x^4}+\frac{8}{3\,x^3}-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-4*x+3)^2/x^7,x)
```

```
[Out] -3/2/x^6+24/5/x^5-11/2/x^4+8/3/x^3-1/2/x^2
```

Maxima [A] time = 1.00922, size = 34, normalized size = 0.94

$$-\frac{15\,x^4-80\,x^3+165\,x^2-144\,x+45}{30\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4*x+3)^2/x^7,x, algorithm="maxima")
```

```
[Out] -1/30*(15*x^4 - 80*x^3 + 165*x^2 - 144*x + 45)/x^6
```

Fricas [A] time = 1.90917, size = 70, normalized size = 1.94

$$-\frac{15\,x^4-80\,x^3+165\,x^2-144\,x+45}{30\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4*x+3)^2/x^7,x, algorithm="fricas")
```

```
[Out] -1/30*(15*x^4 - 80*x^3 + 165*x^2 - 144*x + 45)/x^6
```

Sympy [A] time = 0.116787, size = 26, normalized size = 0.72

$$-\frac{15x^4-80x^3+165x^2-144x+45}{30x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-4*x+3)**2/x**7,x)
```

```
[Out] -(15*x**4 - 80*x**3 + 165*x**2 - 144*x + 45)/(30*x**6)
```

Giac [A] time = 1.10473, size = 34, normalized size = 0.94

$$-\frac{15\,x^4-80\,x^3+165\,x^2-144\,x+45}{30\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4*x+3)^2/x^7,x, algorithm="giac")
```

```
[Out] -1/30*(15*x^4 - 80*x^3 + 165*x^2 - 144*x + 45)/x^6
```

3.2175 $\int \frac{2+2x+x^2}{2+x} \, dx$

Optimal. Leaf size=14

$$\frac{x^2}{2} + 2\log(x+2)$$

```
[Out] x^2/2 + 2*Log[2 + x]
```

Rubi [A] time = 0.008025, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {698}

$$\frac{x^2}{2} + 2\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x + x^2)/(2 + x),x]

[Out] x^2/2 + 2*Log[2 + x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned} \int \frac{2+2x+x^2}{2+x} \, dx &= \int \left(x + \frac{2}{2+x}\right) dx \\ &= \frac{x^2}{2} + 2\log(2+x) \end{aligned}$$

Mathematica [A] time = 0.0034979, size = 15, normalized size = 1.07

$$\frac{1}{2}\left(x^2 + 4\log(x+2) - 4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 2*x + x^2)/(2 + x),x]

[Out] (-4 + x^2 + 4*Log[2 + x])/2
```

Maple [A] time = 0.039, size = 13, normalized size = 0.9

$$\frac{x^2}{2} + 2\ln(2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x+2)/(2+x),x)
```

```
[Out] 1/2*x^2+2*ln(2+x)
```

Maxima [A] time = 0.983942, size = 16, normalized size = 1.14

$$\frac{1}{2}x^2 + 2\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+2)/(2+x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 2*log(x + 2)
```

Fricas [A] time = 2.01117, size = 32, normalized size = 2.29

$$\frac{1}{2}x^2 + 2\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+2)/(2+x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 2*log(x + 2)
```

Sympy [A] time = 0.077718, size = 10, normalized size = 0.71

$$\frac{x^2}{2} + 2\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x+2)/(2+x),x)
```

```
[Out] x**2/2 + 2*log(x + 2)
```

Giac [A] time = 1.11415, size = 18, normalized size = 1.29

$$\frac{1}{2}x^2 + 2\log(|x+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+2)/(2+x),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 2*log(abs(x + 2))
```

3.2176 $\int \frac{5+4x+x^2}{-2+x} \, dx$

Optimal. Leaf size=19

$$\frac{x^2}{2} + 6x + 17\log(2 - x)$$

```
[Out] 6*x + x^2/2 + 17*Log[2 - x]
```

Rubi [A] time = 0.009546, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {698}

$$\frac{x^2}{2} + 6x + 17\log(2 - x)$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 4*x + x^2)/(-2 + x),x]

[Out] 6*x + x^2/2 + 17*Log[2 - x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned} \int \frac{5+4x+x^2}{-2+x} \, dx &= \int \left(6 + \frac{17}{-2+x} + x\right) dx \\ &= 6x + \frac{x^2}{2} + 17\log(2-x) \end{aligned}$$

Mathematica [A] time = 0.0033058, size = 18, normalized size = 0.95

$$\frac{x^2}{2} + 6x + 17\log(x - 2) - 14$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 4*x + x^2)/(-2 + x),x]

[Out] -14 + 6*x + x^2/2 + 17*Log[-2 + x]
```

Maple [A] time = 0.04, size = 16, normalized size = 0.8

$$\frac{x^2}{2} + 6x + 17\ln(-2 + x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+4*x+5)/(-2+x),x)
```

```
[Out] 1/2*x^2+6*x+17*ln(-2+x)
```

Maxima [A] time = 0.959847, size = 20, normalized size = 1.05

$$\frac{1}{2}\,x^2 + 6\,x + 17\,\log\left(x - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x+5)/(-2+x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 6*x + 17*log(x - 2)
```

Fricas [A] time = 1.92843, size = 42, normalized size = 2.21

$$\frac{1}{2}\,x^2 + 6\,x + 17\,\log\left(x - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x+5)/(-2+x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 6*x + 17*log(x - 2)
```

Sympy [A] time = 0.076976, size = 14, normalized size = 0.74

$$\frac{x^2}{2} + 6x + 17\log\left(x - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+4*x+5)/(-2+x),x)
```

```
[Out] x**2/2 + 6*x + 17*log(x - 2)
```

Giac [A] time = 1.13047, size = 22, normalized size = 1.16

$$\frac{1}{2}\,x^2 + 6\,x + 17\,\log\left(|x - 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x+5)/(-2+x),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 6*x + 17*log(abs(x - 2))
```

3.2177 $\int \frac{2+2x+x^2}{(1+x)^3} dx$

Optimal. Leaf size=14

$$\log(x+1) - \frac{1}{2(x+1)^2}$$

```
[Out] -1/(2*(1 + x)^2) + Log[1 + x]
```

Rubi [A] time = 0.0071718, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {683}

$$\log(x+1) - \frac{1}{2(x+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x + x^2)/(1 + x)^3,x]
```

```
[Out] -1/(2*(1 + x)^2) + Log[1 + x]
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{2+2x+x^2}{(1+x)^3} dx &= \int \left(\frac{1}{(1+x)^3} + \frac{1}{1+x}\right) dx \\ &= -\frac{1}{2(1+x)^2} + \log(1+x)\end{aligned}$$

Mathematica [A] time = 0.0050636, size = 14, normalized size = 1.

$$\log(x+1) - \frac{1}{2(x+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 2*x + x^2)/(1 + x)^3,x]
```

```
[Out] -1/(2*(1 + x)^2) + Log[1 + x]
```

Maple [A] time = 0.045, size = 13, normalized size = 0.9

$$-\frac{1}{2\,(1+x)^2} + \ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x+2)/(1+x)^3,x)
```

```
[Out] -1/2/(1+x)^2+ln(1+x)
```

Maxima [A] time = 0.990921, size = 23, normalized size = 1.64

$$-\frac{1}{2\left(x^{2}+2\,x+1\right)}+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+2)/(1+x)^3,x, algorithm="maxima")
```

```
[Out] -1/2/(x^2 + 2*x + 1) + log(x + 1)
```

Fricas [B] time = 1.81557, size = 76, normalized size = 5.43

$$\frac{2\left(x^{2}+2\,x+1\right)\log\left(x+1\right)-1}{2\left(x^{2}+2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+2)/(1+x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(x^2 + 2*x + 1)*log(x + 1) - 1)/(x^2 + 2*x + 1)
```

Sympy [A] time = 0.101164, size = 15, normalized size = 1.07

$$\log\left(x+1\right)-\frac{1}{2x^{2}+4x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x+2)/(1+x)**3,x)
```

```
[Out] log(x + 1) - 1/(2*x**2 + 4*x + 2)
```

Giac [A] time = 1.09139, size = 18, normalized size = 1.29

$$-\frac{1}{2\left(x+1\right)^{2}}+\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+2)/(1+x)^3,x, algorithm="giac")
```

```
[Out] -1/2/(x + 1)^2 + log(abs(x + 1))
```

3.2178 $\int \frac{3+3x+2x^2}{(1+x)^3}\,dx$

Optimal. Leaf size=19

$$\frac{1}{x+1}-\frac{1}{(x+1)^2}+2\log(x+1)$$

```
[Out] -(1 + x)^(-2) + (1 + x)^(-1) + 2*Log[1 + x]
```

Rubi [A] time = 0.0110944, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {698}

$$\frac{1}{x+1}-\frac{1}{(x+1)^2}+2\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 3*x + 2*x^2)/(1 + x)^3,x]

[Out] -(1 + x)^(-2) + (1 + x)^(-1) + 2*Log[1 + x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{3+3x+2x^2}{(1+x)^3}\,dx &= \int \left(\frac{2}{(1+x)^3}-\frac{1}{(1+x)^2}+\frac{2}{1+x}\right)dx\\ &= -\frac{1}{(1+x)^2}+\frac{1}{1+x}+2\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.007844, size = 19, normalized size = 1.

$$\frac{1}{x+1}-\frac{1}{(x+1)^2}+2\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 3*x + 2*x^2)/(1 + x)^3,x]

[Out] -(1 + x)^(-2) + (1 + x)^(-1) + 2*Log[1 + x]
```

Maple [A] time = 0.044, size = 20, normalized size = 1.1

$$-(1+x)^{-2}+(1+x)^{-1}+2\ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+3*x+3)/(1+x)^3,x)
```

```
[Out] -1/(1+x)^2+1/(1+x)+2*ln(1+x)
```

Maxima [A] time = 1.01557, size = 26, normalized size = 1.37

$$\frac{x}{x^2+2x+1}+2\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3*x+3)/(1+x)^3,x, algorithm="maxima")
```

```
[Out] x/(x^2 + 2*x + 1) + 2*log(x + 1)
```

Fricas [A] time = 1.65808, size = 70, normalized size = 3.68

$$\frac{2\left(x^2+2x+1\right)\log(x+1)+x}{x^2+2x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3*x+3)/(1+x)^3,x, algorithm="fricas")
```

```
[Out] (2*(x^2 + 2*x + 1)*log(x + 1) + x)/(x^2 + 2*x + 1)
```

Sympy [A] time = 0.091706, size = 15, normalized size = 0.79

$$\frac{x}{x^2+2x+1}+2\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+3*x+3)/(1+x)**3,x)
```

```
[Out] x/(x**2 + 2*x + 1) + 2*log(x + 1)
```

Giac [A] time = 1.07939, size = 20, normalized size = 1.05

$$\frac{x}{(x+1)^2}+2\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3*x+3)/(1+x)^3,x, algorithm="giac")
```

```
[Out] x/(x + 1)^2 + 2*log(abs(x + 1))
```

3.2179 $\int \frac{1+x+x^2}{x} \, dx$

Optimal. Leaf size=11

$$\frac{x^2}{2} + x + \log(x)$$

[Out] x + x^2/2 + Log[x]

Rubi [A] time = 0.0027441, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {14}

$$\frac{x^2}{2} + x + \log(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^2)/x,x]

[Out] x + x^2/2 + Log[x]

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x+x^2}{x} \, dx &= \int \left(1+\frac{1}{x}+x\right) dx \\ &= x+\frac{x^2}{2}+\log(x) \end{aligned}$$

Mathematica [A] time = 0.00061, size = 11, normalized size = 1.

$$\frac{x^2}{2} + x + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + x^2)/x,x]

[Out] x + x^2/2 + Log[x]

Maple [A] time = 0.039, size = 10, normalized size = 0.9

$$x + \frac{x^2}{2} + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/x,x)
```

```
[Out] x+1/2*x^2+ln(x)
```

Maxima [A] time = 0.997564, size = 12, normalized size = 1.09

$$\frac{1}{2}x^2 + x + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + x + log(x)
```

Fricas [A] time = 1.6929, size = 30, normalized size = 2.73

$$\frac{1}{2}x^2 + x + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x,x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + x + log(x)
```

Sympy [A] time = 0.073268, size = 8, normalized size = 0.73

$$\frac{x^2}{2} + x + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/x,x)
```

```
[Out] x**2/2 + x + log(x)
```

Giac [A] time = 1.14119, size = 14, normalized size = 1.27

$$\frac{1}{2}x^2 + x + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x,x, algorithm="giac")
```

```
[Out] 1/2*x^2 + x + log(abs(x))
```

3.2180 $\int \frac{9+6x+x^2}{x^2} dx$

Optimal. Leaf size=11

$$x - \frac{9}{x} + 6\log(x)$$

```
[Out] -9/x + x + 6*Log[x]
```

Rubi [A] time = 0.0048209, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {14}

$$x - \frac{9}{x} + 6\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(9 + 6*x + x^2)/x^2,x]
```

```
[Out] -9/x + x + 6*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{9+6x+x^2}{x^2} dx &= \int \left(1+\frac{9}{x^2}+\frac{6}{x}\right) dx\\ &= -\frac{9}{x}+x+6\log(x)\end{aligned}$$

Mathematica [A] time = 0.0006919, size = 11, normalized size = 1.

$$x - \frac{9}{x} + 6\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + 6*x + x^2)/x^2,x]
```

```
[Out] -9/x + x + 6*Log[x]
```

Maple [A] time = 0.044, size = 12, normalized size = 1.1

$$-9x^{-1} + x + 6\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+6*x+9)/x^2,x)
```

```
[Out] -9/x+x+6*ln(x)
```

Maxima [A] time = 0.975769, size = 15, normalized size = 1.36

$$x - \frac{9}{x} + 6\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+6*x+9)/x^2,x, algorithm="maxima")
```

```
[Out] x - 9/x + 6*log(x)
```

Fricas [A] time = 1.74804, size = 35, normalized size = 3.18

$$\frac{x^2 + 6\,x \log\left(x\right) - 9}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+6*x+9)/x^2,x, algorithm="fricas")
```

```
[Out] (x^2 + 6*x*log(x) - 9)/x
```

Sympy [A] time = 0.091302, size = 8, normalized size = 0.73

$$x + 6\log\left(x\right) - \frac{9}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+6*x+9)/x**2,x)
```

```
[Out] x + 6*log(x) - 9/x
```

Giac [A] time = 1.13119, size = 16, normalized size = 1.45

$$x - \frac{9}{x} + 6\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+6*x+9)/x^2,x, algorithm="giac")
```

```
[Out] x - 9/x + 6*log(abs(x))
```

3.2181 $\int \frac{1+2x+x^2}{x^4}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{x^2}-\frac{1}{3x^3}-\frac{1}{x}$$

```
[Out] -1/(3*x^3) - x^(-2) - x^(-1)
```

Rubi [A] time = 0.0047827, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {14}

$$-\frac{1}{x^2}-\frac{1}{3x^3}-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x + x^2)/x^4,x]

[Out] -1/(3*x^3) - x^(-2) - x^(-1)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x+x^2}{x^4}\,dx &= \int \left(\frac{1}{x^4}+\frac{2}{x^3}+\frac{1}{x^2}\right)dx\\ &= -\frac{1}{3x^3}-\frac{1}{x^2}-\frac{1}{x}\end{aligned}$$

Mathematica [A] time = 0.000654, size = 18, normalized size = 1.

$$-\frac{1}{x^2}-\frac{1}{3x^3}-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x + x^2)/x^4,x]

[Out] -1/(3*x^3) - x^(-2) - x^(-1)
```

Maple [A] time = 0.042, size = 17, normalized size = 0.9

$$-\frac{1}{3\,x^3}-x^{-2}-x^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x+1)/x^4,x)
```

```
[Out] -1/3/x^3-1/x^2-1/x
```

Maxima [A] time = 1.00815, size = 20, normalized size = 1.11

$$-\frac{3\,x^2+3\,x+1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+1)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*x^2 + 3*x + 1)/x^3
```

Fricas [A] time = 1.67771, size = 38, normalized size = 2.11

$$-\frac{3\,x^2+3\,x+1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+1)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*x^2 + 3*x + 1)/x^3
```

Sympy [A] time = 0.087984, size = 15, normalized size = 0.83

$$-\frac{3x^2+3x+1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x+1)/x**4,x)
```

```
[Out] -(3*x**2 + 3*x + 1)/(3*x**3)
```

Giac [A] time = 1.13123, size = 20, normalized size = 1.11

$$-\frac{3\,x^2+3\,x+1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+1)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*x^2 + 3*x + 1)/x^3
```

3.2182 $\int \frac{(d+ex)^4}{a+bx+cx^2}\,dx$

Optimal. Leaf size=243

$$-\frac{\left(2c^2e^2\left(a^2e^2+6abde+3b^2d^2\right)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\sqrt{b^2-4ac}}+\frac{e(2cd-be)}{}$$

```
[Out] (e^2*(6*c^2*d^2 + b^2*e^2 - c*e*(4*b*d + a*e))*x)/c^3 + (e^3*(4*c*d - b*e)*
x^2)/(2*c^2) + (e^4*x^3)/(3*c) - ((2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d +
 a*e) - 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*
e^2))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*Sqrt[b^2 - 4*a*c]) + (e*
(2*c*d - b*e)*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*Log[a + b*x + c*x^2
])/(2*c^4)
```

Rubi [A] time = 0.439051, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {701, 634, 618, 206, 628}

$$-\frac{\left(2c^2e^2\left(a^2e^2+6abde+3b^2d^2\right)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\sqrt{b^2-4ac}}+\frac{e(2cd-be)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a + b*x + c*x^2),x]
```

```
[Out] (e^2*(6*c^2*d^2 + b^2*e^2 - c*e*(4*b*d + a*e))*x)/c^3 + (e^3*(4*c*d - b*e)*
x^2)/(2*c^2) + (e^4*x^3)/(3*c) - ((2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d +
 a*e) - 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*
e^2))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*Sqrt[b^2 - 4*a*c]) + (e*
(2*c*d - b*e)*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*Log[a + b*x + c*x^2
])/(2*c^4)
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)^4}{a+bx+cx^2}\,dx = \int \left(\frac{e^2\left(6c^2d^2+b^2e^2-ce(4bd+ae)\right)}{c^3}+\frac{e^3(4cd-be)x}{c^2}+\frac{e^4x^2}{c}+\frac{c^3d^4-6ac^2d^2e^2-ab^2e^4+ace^3(4bd}{c^3}\right.$$

$$=\frac{e^2\left(6c^2d^2+b^2e^2-ce(4bd+ae)\right)x}{c^3}+\frac{e^3(4cd-be)x^2}{2c^2}+\frac{e^4x^3}{3c}+\frac{\int\frac{c^3d^4-6ac^2d^2e^2-ab^2e^4+ace^3(4bd+ae)+e(2cd-be}{a+bx+cx^2}}{c^3}$$

$$=\frac{e^2\left(6c^2d^2+b^2e^2-ce(4bd+ae)\right)x}{c^3}+\frac{e^3(4cd-be)x^2}{2c^2}+\frac{e^4x^3}{3c}+\frac{\left(e(2cd-be)\left(2c^2d^2+b^2e^2-2ce(bd+\right.\right.}{2c^4}$$

$$=\frac{e^2\left(6c^2d^2+b^2e^2-ce(4bd+ae)\right)x}{c^3}+\frac{e^3(4cd-be)x^2}{2c^2}+\frac{e^4x^3}{3c}+\frac{e(2cd-be)\left(2c^2d^2+b^2e^2-2ce(bd+a\right.}{2c^4}$$

$$=\frac{e^2\left(6c^2d^2+b^2e^2-ce(4bd+ae)\right)x}{c^3}+\frac{e^3(4cd-be)x^2}{2c^2}+\frac{e^4x^3}{3c}-\frac{\left(2c^4d^4+b^4e^4-4b^2ce^3(bd+ae)-4c^3\right.}{}$$

Mathematica [A] time = 0.192607, size = 240, normalized size = 0.99

$$\frac{\frac{6\left(2c^2e^2\left(a^2e^2+6abde+3b^2d^2\right)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+6ce^2x\left(-ce(ae+4bd)+b^2e^2+6c^2d^2\right)+3e(2cd-}{6c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a + b*x + c*x^2),x]
```

```
[Out] (6*c*e^2*(6*c^2*d^2 + b^2*e^2 - c*e*(4*b*d + a*e))*x + 3*c^2*e^3*(4*c*d - b
*e)*x^2 + 2*c^3*e^4*x^3 + (6*(2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e)
 - 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))
*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 3*e*(2*c*d -
b*e)*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*Log[a + x*(b + c*x)])/(6*c^4
)
```

Maple [B] time = 0.154, size = 595, normalized size = 2.5

$$\frac{e^4x^3}{3c}-\frac{e^4x^2b}{2c^2}+2\frac{de^3x^2}{c}-\frac{e^4ax}{c^2}+\frac{b^2e^4x}{c^3}-4\frac{e^3bdx}{c^2}+6\frac{d^2e^2x}{c}+\frac{\ln\left(cx^2+bx+a\right)abe^4}{c^3}-2\frac{\ln\left(cx^2+bx+a\right)ade^3}{c^2}-\frac{\ln\left(cx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(c*x^2+b*x+a),x)
```

```
[Out] 1/3*e^4*x^3/c-1/2*e^4/c^2*x^2*b+2*d*e^3*x^2/c-e^4/c^2*a*x+e^4/c^3*b^2*x-4*e
^3/c^2*b*d*x+6*e^2/c*d^2*x+1/c^3*ln(c*x^2+b*x+a)*a*b*e^4-2/c^2*ln(c*x^2+b*x
+a)*a*d*e^3-1/2/c^4*ln(c*x^2+b*x+a)*b^3*e^4+2/c^3*ln(c*x^2+b*x+a)*b^2*d*e^3
-3/c^2*ln(c*x^2+b*x+a)*b*d^2*e^2+2/c*ln(c*x^2+b*x+a)*d^3*e+2/c^2/(4*a*c-b^2
)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*e^4-4/c^3/(4*a*c-b^2)^(1/2)
*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*e^4+12/c^2/(4*a*c-b^2)^(1/2)*arc
tan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*d*e^3-12/c/(4*a*c-b^2)^(1/2)*arctan((2
*c*x+b)/(4*a*c-b^2)^(1/2))*a*d^2*e^2+2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(
4*a*c-b^2)^(1/2))*d^4+1/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^
(1/2))*b^4*e^4-4/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*
b^3*d*e^3+6/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*d
^2*e^2-4/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*d^3*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.20638, size = 1693, normalized size = 6.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(b^2*c^3 - 4*a*c^4)*e^4*x^3 + 3*(4*(b^2*c^3 - 4*a*c^4)*d*e^3 - (b^3
*c^2 - 4*a*b*c^3)*e^4)*x^2 + 3*(2*c^4*d^4 - 4*b*c^3*d^3*e + 6*(b^2*c^2 - 2*
a*c^3)*d^2*e^2 - 4*(b^3*c - 3*a*b*c^2)*d*e^3 + (b^4 - 4*a*b^2*c + 2*a^2*c^2
)*e^4)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2
- 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 6*(6*(b^2*c^3 - 4*a*c^4)*d^2*e^2
 - 4*(b^3*c^2 - 4*a*b*c^3)*d*e^3 + (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e^4)*x
 + 3*(4*(b^2*c^3 - 4*a*c^4)*d^3*e - 6*(b^3*c^2 - 4*a*b*c^3)*d^2*e^2 + 4*(b^
4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d*e^3 - (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e^4)
*log(c*x^2 + b*x + a))/(b^2*c^4 - 4*a*c^5), 1/6*(2*(b^2*c^3 - 4*a*c^4)*e^4*
x^3 + 3*(4*(b^2*c^3 - 4*a*c^4)*d*e^3 - (b^3*c^2 - 4*a*b*c^3)*e^4)*x^2 - 6*(
2*c^4*d^4 - 4*b*c^3*d^3*e + 6*(b^2*c^2 - 2*a*c^3)*d^2*e^2 - 4*(b^3*c - 3*a*
b*c^2)*d*e^3 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^4)*sqrt(-b^2 + 4*a*c)*arctan
(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 6*(6*(b^2*c^3 - 4*a*c^4)*
d^2*e^2 - 4*(b^3*c^2 - 4*a*b*c^3)*d*e^3 + (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)
*e^4)*x + 3*(4*(b^2*c^3 - 4*a*c^4)*d^3*e - 6*(b^3*c^2 - 4*a*b*c^3)*d^2*e^2
+ 4*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d*e^3 - (b^5 - 6*a*b^3*c + 8*a^2*b*c^
2)*e^4)*log(c*x^2 + b*x + a))/(b^2*c^4 - 4*a*c^5)]
```

Sympy [B] time = 6.10659, size = 1554, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(c*x**2+b*x+a),x)
```

```
[Out] (e*(b*e - 2*c*d)*(2*a*c*e**2 - b**2*e**2 + 2*b*c*d*e - 2*c**2*d**2)/(2*c**4
) - sqrt(-4*a*c + b**2)*(2*a**2*c**2*e**4 - 4*a*b**2*c*e**4 + 12*a*b*c**2*d
*e**3 - 12*a*c**3*d**2*e**2 + b**4*e**4 - 4*b**3*c*d*e**3 + 6*b**2*c**2*d**
2*e**2 - 4*b*c**3*d**3*e + 2*c**4*d**4)/(2*c**4*(4*a*c - b**2)))*log(x + (-
3*a**2*b*c*e**4 + 8*a**2*c**2*d*e**3 + a*b**3*e**4 - 4*a*b**2*c*d*e**3 + 6*
a*b*c**2*d**2*e**2 + 4*a*c**4*(e*(b*e - 2*c*d)*(2*a*c*e**2 - b**2*e**2 + 2*
b*c*d*e - 2*c**2*d**2)/(2*c**4) - sqrt(-4*a*c + b**2)*(2*a**2*c**2*e**4 - 4
*a*b**2*c*e**4 + 12*a*b*c**2*d*e**3 - 12*a*c**3*d**2*e**2 + b**4*e**4 - 4*b
**3*c*d*e**3 + 6*b**2*c**2*d**2*e**2 - 4*b*c**3*d**3*e + 2*c**4*d**4)/(2*c*
*4*(4*a*c - b**2))) - 8*a*c**3*d**3*e - b**2*c**3*(e*(b*e - 2*c*d)*(2*a*c*e
**2 - b**2*e**2 + 2*b*c*d*e - 2*c**2*d**2)/(2*c**4) - sqrt(-4*a*c + b**2)*(
2*a**2*c**2*e**4 - 4*a*b**2*c*e**4 + 12*a*b*c**2*d*e**3 - 12*a*c**3*d**2*e*
*2 + b**4*e**4 - 4*b**3*c*d*e**3 + 6*b**2*c**2*d**2*e**2 - 4*b*c**3*d**3*e
+ 2*c**4*d**4)/(2*c**4*(4*a*c - b**2))) + b*c**3*d**4)/(2*a**2*c**2*e**4 -
4*a*b**2*c*e**4 + 12*a*b*c**2*d*e**3 - 12*a*c**3*d**2*e**2 + b**4*e**4 - 4*
b**3*c*d*e**3 + 6*b**2*c**2*d**2*e**2 - 4*b*c**3*d**3*e + 2*c**4*d**4)) + (
e*(b*e - 2*c*d)*(2*a*c*e**2 - b**2*e**2 + 2*b*c*d*e - 2*c**2*d**2)/(2*c**4)
 + sqrt(-4*a*c + b**2)*(2*a**2*c**2*e**4 - 4*a*b**2*c*e**4 + 12*a*b*c**2*d*
e**3 - 12*a*c**3*d**2*e**2 + b**4*e**4 - 4*b**3*c*d*e**3 + 6*b**2*c**2*d**2
*e**2 - 4*b*c**3*d**3*e + 2*c**4*d**4)/(2*c**4*(4*a*c - b**2)))*log(x + (-3
*a**2*b*c*e**4 + 8*a**2*c**2*d*e**3 + a*b**3*e**4 - 4*a*b**2*c*d*e**3 + 6*a
*b*c**2*d**2*e**2 + 4*a*c**4*(e*(b*e - 2*c*d)*(2*a*c*e**2 - b**2*e**2 + 2*b
*c*d*e - 2*c**2*d**2)/(2*c**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2*e**4 - 4*
a*b**2*c*e**4 + 12*a*b*c**2*d*e**3 - 12*a*c**3*d**2*e**2 + b**4*e**4 - 4*b*
*3*c*d*e**3 + 6*b**2*c**2*d**2*e**2 - 4*b*c**3*d**3*e + 2*c**4*d**4)/(2*c**
4*(4*a*c - b**2))) - 8*a*c**3*d**3*e - b**2*c**3*(e*(b*e - 2*c*d)*(2*a*c*e*
*2 - b**2*e**2 + 2*b*c*d*e - 2*c**2*d**2)/(2*c**4) + sqrt(-4*a*c + b**2)*(2
*a**2*c**2*e**4 - 4*a*b**2*c*e**4 + 12*a*b*c**2*d*e**3 - 12*a*c**3*d**2*e**
2 + b**4*e**4 - 4*b**3*c*d*e**3 + 6*b**2*c**2*d**2*e**2 - 4*b*c**3*d**3*e +
 2*c**4*d**4)/(2*c**4*(4*a*c - b**2))) + b*c**3*d**4)/(2*a**2*c**2*e**4 - 4
*a*b**2*c*e**4 + 12*a*b*c**2*d*e**3 - 12*a*c**3*d**2*e**2 + b**4*e**4 - 4*b
**3*c*d*e**3 + 6*b**2*c**2*d**2*e**2 - 4*b*c**3*d**3*e + 2*c**4*d**4)) + e*
*4*x**3/(3*c) - x**2*(b*e**4 - 4*c*d*e**3)/(2*c**2) - x*(a*c*e**4 - b**2*e*
*4 + 4*b*c*d*e**3 - 6*c**2*d**2*e**2)/c**3
```

Giac [A] time = 1.12097, size = 358, normalized size = 1.47

$$\frac{2\,c^2x^3e^4 + 12\,c^2dx^2e^3 + 36\,c^2d^2xe^2 - 3\,bcx^2e^4 - 24\,bcdxe^3 + 6\,b^2xe^4 - 6\,acxe^4}{6\,c^3} + \frac{\left(4\,c^3d^3e - 6\,bc^2d^2e^2 + 4\,b^2cde^3 - 4\,ac^2de^3\right.}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/6*(2*c^2*x^3*e^4 + 12*c^2*d*x^2*e^3 + 36*c^2*d^2*x*e^2 - 3*b*c*x^2*e^4 -
24*b*c*d*x*e^3 + 6*b^2*x*e^4 - 6*a*c*x*e^4)/c^3 + 1/2*(4*c^3*d^3*e - 6*b*c^
2*d^2*e^2 + 4*b^2*c*d*e^3 - 4*a*c^2*d*e^3 - b^3*e^4 + 2*a*b*c*e^4)*log(c*x^
2 + b*x + a)/c^4 + (2*c^4*d^4 - 4*b*c^3*d^3*e + 6*b^2*c^2*d^2*e^2 - 12*a*c^
3*d^2*e^2 - 4*b^3*c*d*e^3 + 12*a*b*c^2*d*e^3 + b^4*e^4 - 4*a*b^2*c*e^4 + 2*
a^2*c^2*e^4)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^4
)
```

3.2183 $\int \frac{(d+ex)^3}{a+bx+cx^2} dx$

Optimal. Leaf size=151

$$\frac{e\left(-ce(ae+3bd)+b^2e^2+3c^2d^2\right)\log\left(a+bx+cx^2\right)}{2c^3}-\frac{(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{e^2...}{}$$

```
[Out] (e^2*(3*c*d - b*e)*x)/c^2 + (e^3*x^2)/(2*c) - ((2*c*d - b*e)*(c^2*d^2 + b^2
*e^2 - c*e*(b*d + 3*a*e))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt
[b^2 - 4*a*c]) + (e*(3*c^2*d^2 + b^2*e^2 - c*e*(3*b*d + a*e))*Log[a + b*x +
 c*x^2])/(2*c^3)
```

Rubi [A] time = 0.171566, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {701, 634, 618, 206, 628}

$$\frac{e\left(-ce(ae+3bd)+b^2e^2+3c^2d^2\right)\log\left(a+bx+cx^2\right)}{2c^3}-\frac{(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{e^2...}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + b*x + c*x^2),x]
```

```
[Out] (e^2*(3*c*d - b*e)*x)/c^2 + (e^3*x^2)/(2*c) - ((2*c*d - b*e)*(c^2*d^2 + b^2
*e^2 - c*e*(b*d + 3*a*e))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt
[b^2 - 4*a*c]) + (e*(3*c^2*d^2 + b^2*e^2 - c*e*(3*b*d + a*e))*Log[a + b*x +
 c*x^2])/(2*c^3)
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)^3}{a+bx+cx^2}dx = \int \left(\frac{e^2(3cd-be)}{c^2} + \frac{e^3x}{c} + \frac{c^2d^3-3acde^2+abe^3+e\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)x}{c^2\left(a+bx+cx^2\right)}\right)dx$$

$$= \frac{e^2(3cd-be)x}{c^2} + \frac{e^3x^2}{2c} + \frac{\int \frac{c^2d^3-3acde^2+abe^3+e\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)x}{a+bx+cx^2}dx}{c^2}$$

$$= \frac{e^2(3cd-be)x}{c^2} + \frac{e^3x^2}{2c} + \frac{\left(e\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)\right)\int \frac{b+2cx}{a+bx+cx^2}dx}{2c^3} + \frac{\left((2cd-be)\left(c^2d^2+b^2e^2-\right.\right.}{}$$

$$= \frac{e^2(3cd-be)x}{c^2} + \frac{e^3x^2}{2c} + \frac{e\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)\log\left(a+bx+cx^2\right)}{2c^3} - \frac{\left((2cd-be)\left(c^2d^2+b^2e\right.\right.}{}$$

$$= \frac{e^2(3cd-be)x}{c^2} + \frac{e^3x^2}{2c} - \frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}} + \frac{e\left(3c^2d^2+b^2e^2\right.}{}$$

Mathematica [A] time = 0.130507, size = 148, normalized size = 0.98

$$\frac{e\left(-ce(ae+3bd)+b^2e^2+3c^2d^2\right)\log(a+x(b+cx)) + \frac{2(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + 2ce^2x(3cd-be)+c^2e^3x^2}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + b*x + c*x^2),x]
```

```
[Out] (2*c*e^2*(3*c*d - b*e)*x + c^2*e^3*x^2 + (2*(2*c*d - b*e)*(c^2*d^2 + b^2*e^
2 - c*e*(b*d + 3*a*e))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 +
4*a*c] + e*(3*c^2*d^2 + b^2*e^2 - c*e*(3*b*d + a*e))*Log[a + x*(b + c*x)])/
(2*c^3)
```

Maple [B] time = 0.153, size = 366, normalized size = 2.4

$$\frac{e^3x^2}{2c} - \frac{e^3xb}{c^2} + 3\frac{de^2x}{c} - \frac{\ln\left(cx^2+bx+a\right)ae^3}{2c^2} + \frac{\ln\left(cx^2+bx+a\right)b^2e^3}{2c^3} - \frac{3d\ln\left(cx^2+bx+a\right)e^2b}{2c^2} + \frac{3\ln\left(cx^2+bx+a\right)d}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^2+b*x+a),x)
```

```
[Out] 1/2*e^3*x^2/c-e^3/c^2*x*b+3*d*e^2*x/c-1/2/c^2*ln(c*x^2+b*x+a)*a*e^3+1/2/c^3
*ln(c*x^2+b*x+a)*b^2*e^3-3/2/c^2*ln(c*x^2+b*x+a)*d*e^2*b+3/2/c*ln(c*x^2+b*x
+a)*d^2*e+3/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*e
^3-6/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*d*e^2+2/(4*a
*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*d^3-1/c^3/(4*a*c-b^2)^(1/
2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*e^3+3/c^2/(4*a*c-b^2)^(1/2)*arct
an((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*d*e^2-3/c/(4*a*c-b^2)^(1/2)*arctan((2*c
```

```
*x+b)/(4*a*c-b^2)^(1/2))*b*d^2*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.91266, size = 1107, normalized size = 7.33

$$\left[\frac{\left(b^2c^2-4\,ac^3\right)e^3x^2+\left(2\,c^3d^3-3\,bc^2d^2e+3\,\left(b^2c-2\,ac^2\right)de^2-\left(b^3-3\,abc\right)e^3\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}}{cx^2+bx+a}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*c^2 - 4*a*c^3)*e^3*x^2 + (2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c -
 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2
*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) +
2*(3*(b^2*c^2 - 4*a*c^3)*d*e^2 - (b^3*c - 4*a*b*c^2)*e^3)*x + (3*(b^2*c^2 -
 4*a*c^3)*d^2*e - 3*(b^3*c - 4*a*b*c^2)*d*e^2 + (b^4 - 5*a*b^2*c + 4*a^2*c^
2)*e^3)*log(c*x^2 + b*x + a))/(b^2*c^3 - 4*a*c^4), 1/2*((b^2*c^2 - 4*a*c^3)
*e^3*x^2 - 2*(2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3
- 3*a*b*c)*e^3)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(
b^2 - 4*a*c)) + 2*(3*(b^2*c^2 - 4*a*c^3)*d*e^2 - (b^3*c - 4*a*b*c^2)*e^3)*x
 + (3*(b^2*c^2 - 4*a*c^3)*d^2*e - 3*(b^3*c - 4*a*b*c^2)*d*e^2 + (b^4 - 5*a*
b^2*c + 4*a^2*c^2)*e^3)*log(c*x^2 + b*x + a))/(b^2*c^3 - 4*a*c^4)]
```

Sympy [B] time = 3.44919, size = 892, normalized size = 5.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**2+b*x+a),x)
```

```
[Out] (-e*(a*c*e**2 - b**2*e**2 + 3*b*c*d*e - 3*c**2*d**2)/(2*c**3) - sqrt(-4*a*c
 + b**2)*(b*e - 2*c*d)*(3*a*c*e**2 - b**2*e**2 + b*c*d*e - c**2*d**2)/(2*c*
*3*(4*a*c - b**2)))*log(x + (2*a**2*c*e**3 - a*b**2*e**3 + 3*a*b*c*d*e**2 +
 4*a*c**3*(-e*(a*c*e**2 - b**2*e**2 + 3*b*c*d*e - 3*c**2*d**2)/(2*c**3) - s
qrt(-4*a*c + b**2)*(b*e - 2*c*d)*(3*a*c*e**2 - b**2*e**2 + b*c*d*e - c**2*d
**2)/(2*c**3*(4*a*c - b**2))) - 6*a*c**2*d**2*e - b**2*c**2*(-e*(a*c*e**2 -
 b**2*e**2 + 3*b*c*d*e - 3*c**2*d**2)/(2*c**3) - sqrt(-4*a*c + b**2)*(b*e -
 2*c*d)*(3*a*c*e**2 - b**2*e**2 + b*c*d*e - c**2*d**2)/(2*c**3*(4*a*c - b**
2))) + b*c**2*d**3)/(3*a*b*c*e**3 - 6*a*c**2*d*e**2 - b**3*e**3 + 3*b**2*c*
```

```
d*e**2 - 3*b*c**2*d**2*e + 2*c**3*d**3)) + (-e*(a*c*e**2 - b**2*e**2 + 3*b*
c*d*e - 3*c**2*d**2)/(2*c**3) + sqrt(-4*a*c + b**2)*(b*e - 2*c*d)*(3*a*c*e*
*2 - b**2*e**2 + b*c*d*e - c**2*d**2)/(2*c**3*(4*a*c - b**2)))*log(x + (2*a
**2*c*e**3 - a*b**2*e**3 + 3*a*b*c*d*e**2 + 4*a*c**3*(-e*(a*c*e**2 - b**2*e
**2 + 3*b*c*d*e - 3*c**2*d**2)/(2*c**3) + sqrt(-4*a*c + b**2)*(b*e - 2*c*d)
*(3*a*c*e**2 - b**2*e**2 + b*c*d*e - c**2*d**2)/(2*c**3*(4*a*c - b**2))) -
6*a*c**2*d**2*e - b**2*c**2*(-e*(a*c*e**2 - b**2*e**2 + 3*b*c*d*e - 3*c**2*
d**2)/(2*c**3) + sqrt(-4*a*c + b**2)*(b*e - 2*c*d)*(3*a*c*e**2 - b**2*e**2
+ b*c*d*e - c**2*d**2)/(2*c**3*(4*a*c - b**2))) + b*c**2*d**3)/(3*a*b*c*e**
3 - 6*a*c**2*d*e**2 - b**3*e**3 + 3*b**2*c*d*e**2 - 3*b*c**2*d**2*e + 2*c**
3*d**3)) + e**3*x**2/(2*c) - x*(b*e**3 - 3*c*d*e**2)/c**2
```

Giac [A] time = 1.13914, size = 217, normalized size = 1.44

$$\frac{cx^2e^3 + 6\,cdxe^2 - 2\,bxe^3}{2\,c^2} + \frac{\left(3\,c^2d^2e - 3\,bcde^2 + b^2e^3 - ace^3\right)\log\left(cx^2 + bx + a\right)}{2\,c^3} + \frac{\left(2\,c^3d^3 - 3\,bc^2d^2e + 3\,b^2cde^2 - 6\,ac^2de^2\right.}{\sqrt{-b^2 + }}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(c*x^2*e^3 + 6*c*d*x*e^2 - 2*b*x*e^3)/c^2 + 1/2*(3*c^2*d^2*e - 3*b*c*d*
e^2 + b^2*e^3 - a*c*e^3)*log(c*x^2 + b*x + a)/c^3 + (2*c^3*d^3 - 3*b*c^2*d^
2*e + 3*b^2*c*d*e^2 - 6*a*c^2*d*e^2 - b^3*e^3 + 3*a*b*c*e^3)*arctan((2*c*x
+ b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

3.2184 $\int \frac{(d+ex)^2}{a+bx+cx^2} dx$

Optimal. Leaf size=101

$$-\frac{\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}+\frac{e(2cd-be)\log\left(a+bx+cx^2\right)}{2c^2}+\frac{e^2x}{c}$$

```
[Out] (e^2*x)/c - ((2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*ArcTanh[(b + 2*c*x)/
Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) + (e*(2*c*d - b*e)*Log[a + b*x
+ c*x^2])/(2*c^2)
```

Rubi [A] time = 0.12332, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {701, 634, 618, 206, 628}

$$-\frac{\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}+\frac{e(2cd-be)\log\left(a+bx+cx^2\right)}{2c^2}+\frac{e^2x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*x + c*x^2),x]
```

```
[Out] (e^2*x)/c - ((2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*ArcTanh[(b + 2*c*x)/
Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) + (e*(2*c*d - b*e)*Log[a + b*x
+ c*x^2])/(2*c^2)
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)^2}{a+bx+cx^2}\,dx = \int \left(\frac{e^2}{c}+\frac{cd^2-ae^2+e(2cd-be)x}{c\left(a+bx+cx^2\right)}\right)dx$$
$$= \frac{e^2x}{c}+\frac{\int \frac{cd^2-ae^2+e(2cd-be)x}{a+bx+cx^2}\,dx}{c}$$
$$= \frac{e^2x}{c}+\frac{(e(2cd-be))\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2}+\frac{\left(-be(2cd-be)+2c\left(cd^2-ae^2\right)\right)\int \frac{1}{a+bx+cx^2}\,dx}{2c^2}$$
$$= \frac{e^2x}{c}+\frac{e(2cd-be)\log\left(a+bx+cx^2\right)}{2c^2}-\frac{\left(-be(2cd-be)+2c\left(cd^2-ae^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b\right.}{c^2}$$
$$= \frac{e^2x}{c}-\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}+\frac{e(2cd-be)\log\left(a+bx+cx^2\right)}{2c^2}$$

Mathematica [A] time = 0.0674587, size = 101, normalized size = 1.

$$\frac{\frac{2\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+e(2cd-be)\log(a+x(b+cx))+2ce^2x}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*x + c*x^2),x]
```

```
[Out] (2*c*e^2*x + (2*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*ArcTan[(b + 2*c*x
)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + e*(2*c*d - b*e)*Log[a + x*(b +
c*x)])/(2*c^2)
```

Maple [B] time = 0.153, size = 207, normalized size = 2.1

$$\frac{e^2x}{c}-\frac{\ln\left(cx^2+bx+a\right)be^2}{2c^2}+\frac{d\ln\left(cx^2+bx+a\right)e}{c}-2\frac{ae^2}{c\sqrt{4ac-b^2}}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)+2\frac{d^2}{\sqrt{4ac-b^2}}\arctan\left(\frac{2cx+}{\sqrt{4ac-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^2+b*x+a),x)
```

```
[Out] e^2*x/c-1/2/c^2*ln(c*x^2+b*x+a)*b*e^2+1/c*ln(c*x^2+b*x+a)*d*e-2/c/(4*a*c-b^
2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*e^2+2/(4*a*c-b^2)^(1/2)*arct
an((2*c*x+b)/(4*a*c-b^2)^(1/2))*d^2+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b
)/(4*a*c-b^2)^(1/2))*b^2*e^2-2/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-
b^2)^(1/2))*b*d*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.15431, size = 713, normalized size = 7.06

$$\left[\frac{2\left(b^2c-4\,ac^2\right)e^2x-\left(2\,c^2d^2-2\,bcde+\left(b^2-2\,ac\right)e^2\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+\left(2\left(b^2c-4\,ac\right.\right.}{2\left(b^2c^2-4\,ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b^2*c - 4*a*c^2)*e^2*x - (2*c^2*d^2 - 2*b*c*d*e + (b^2 - 2*a*c)*e^
2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*
a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (2*(b^2*c - 4*a*c^2)*d*e - (b^3 - 4*
a*b*c)*e^2)*log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3), 1/2*(2*(b^2*c - 4*a*
c^2)*e^2*x - 2*(2*c^2*d^2 - 2*b*c*d*e + (b^2 - 2*a*c)*e^2)*sqrt(-b^2 + 4*a*
c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (2*(b^2*c - 4*a*
c^2)*d*e - (b^3 - 4*a*b*c)*e^2)*log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] time = 1.7585, size = 588, normalized size = 5.82

$$\left(-\frac{e\,(be-2cd)}{2c^2}-\frac{\sqrt{-4ac+b^2}\left(2ace^2-b^2e^2+2bcde-2c^2d^2\right)}{2c^2\left(4ac-b^2\right)}\right)\log\left(x+\frac{-abe^2-4ac^2\left(-\frac{e(be-2cd)}{2c^2}-\frac{\sqrt{-4ac+b^2}\left(2ace^2-b^2e^2+2b\right.}{2c^2\left(4ac-b^2\right)}\right.}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**2+b*x+a),x)
```

```
[Out] (-e*(b*e - 2*c*d)/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c*e**2 - b**2*e**2 +
2*b*c*d*e - 2*c**2*d**2)/(2*c**2*(4*a*c - b**2)))*log(x + (-a*b*e**2 - 4*a*
c**2*(-e*(b*e - 2*c*d)/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c*e**2 - b**2*e*
*2 + 2*b*c*d*e - 2*c**2*d**2)/(2*c**2*(4*a*c - b**2))) + 4*a*c*d*e + b**2*c
*(-e*(b*e - 2*c*d)/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c*e**2 - b**2*e**2 +
 2*b*c*d*e - 2*c**2*d**2)/(2*c**2*(4*a*c - b**2))) - b*c*d**2)/(2*a*c*e**2
- b**2*e**2 + 2*b*c*d*e - 2*c**2*d**2)) + (-e*(b*e - 2*c*d)/(2*c**2) + sqrt
(-4*a*c + b**2)*(2*a*c*e**2 - b**2*e**2 + 2*b*c*d*e - 2*c**2*d**2)/(2*c**2*
(4*a*c - b**2)))*log(x + (-a*b*e**2 - 4*a*c**2*(-e*(b*e - 2*c*d)/(2*c**2) +
 sqrt(-4*a*c + b**2)*(2*a*c*e**2 - b**2*e**2 + 2*b*c*d*e - 2*c**2*d**2)/(2*
c**2*(4*a*c - b**2))) + 4*a*c*d*e + b**2*c*(-e*(b*e - 2*c*d)/(2*c**2) + sqr
t(-4*a*c + b**2)*(2*a*c*e**2 - b**2*e**2 + 2*b*c*d*e - 2*c**2*d**2)/(2*c**2
*(4*a*c - b**2))) - b*c*d**2)/(2*a*c*e**2 - b**2*e**2 + 2*b*c*d*e - 2*c**2*
d**2)) + e**2*x/c
```

Giac [A] time = 1.10346, size = 135, normalized size = 1.34

$$\frac{xe^2}{c} + \frac{(2\,cde - be^2)\log\left(cx^2 + bx + a\right)}{2\,c^2} + \frac{\left(2\,c^2d^2 - 2\,bcde + b^2e^2 - 2\,ace^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2 + 4\,ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] x*e^2/c + 1/2*(2*c*d*e - b*e^2)*log(c*x^2 + b*x + a)/c^2 + (2*c^2*d^2 - 2*b
*c*d*e + b^2*e^2 - 2*a*c*e^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(
-b^2 + 4*a*c)*c^2)
```

3.2185 $\int \frac{d+ex}{a+bx+cx^2}\,dx$

Optimal. Leaf size=66

$$\frac{e\log\left(a+bx+cx^2\right)}{2c}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}$$

```
[Out] -(((2*c*d - b*e)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*
c])) + (e*Log[a + b*x + c*x^2])/(2*c)
```

Rubi [A] time = 0.0353811, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {634, 618, 206, 628}

$$\frac{e\log\left(a+bx+cx^2\right)}{2c}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*x + c*x^2),x]
```

```
[Out] -(((2*c*d - b*e)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*
c])) + (e*Log[a + b*x + c*x^2])/(2*c)
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{d+ex}{a+bx+cx^2}\,dx = \frac{e\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c} + \frac{(2cd-be)\int \frac{1}{a+bx+cx^2}\,dx}{2c}$$
$$= \frac{e\log\left(a+bx+cx^2\right)}{2c} - \frac{(2cd-be)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{c}$$
$$= -\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} + \frac{e\log\left(a+bx+cx^2\right)}{2c}$$

Mathematica [A] time = 0.0583736, size = 66, normalized size = 1.

$$\frac{e\log(a+x(b+cx)) - \frac{2(be-2cd)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*x + c*x^2),x]
```

```
[Out] ((-2*(-2*c*d + b*e)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a
*c] + e*Log[a + x*(b + c*x)])/(2*c)
```

Maple [A] time = 0.15, size = 93, normalized size = 1.4

$$\frac{e\ln\left(cx^2+bx+a\right)}{2c} + 2\frac{d}{\sqrt{4ac-b^2}}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right) - \frac{be}{c}\arctan\left((2cx+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^2+b*x+a),x)
```

```
[Out] 1/2*e*ln(c*x^2+b*x+a)/c+2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1
/2))*d-1/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*e/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.02945, size = 464, normalized size = 7.03

$$\left[\frac{\left(b^2-4ac\right)e\log\left(cx^2+bx+a\right) - \sqrt{b^2-4ac}(2cd-be)\log\left(\frac{2c^2x^2+2bcx+b^2-2ac+\sqrt{b^2-4ac}(2cx+b)}{cx^2+bx+a}\right)}{2\left(b^2c-4ac^2\right)}, \frac{\left(b^2-4ac\right)e\log\left(cx^2+bx+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2 - 4*a*c)*e*log(c*x^2 + b*x + a) - sqrt(b^2 - 4*a*c)*(2*c*d - b*e
)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(
c*x^2 + b*x + a)))/(b^2*c - 4*a*c^2), 1/2*((b^2 - 4*a*c)*e*log(c*x^2 + b*x
+ a) - 2*sqrt(-b^2 + 4*a*c)*(2*c*d - b*e)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x
 + b)/(b^2 - 4*a*c)))/(b^2*c - 4*a*c^2)]
```

Sympy [B] time = 0.661877, size = 280, normalized size = 4.24

$$\left(\frac{e}{2c} - \frac{\sqrt{-4ac+b^2}\,(be-2cd)}{2c\left(4ac-b^2\right)}\right)\log\left(x + \frac{-4ac\left(\frac{e}{2c} - \frac{\sqrt{-4ac+b^2}(be-2cd)}{2c(4ac-b^2)}\right) + 2ae + b^2\left(\frac{e}{2c} - \frac{\sqrt{-4ac+b^2}(be-2cd)}{2c(4ac-b^2)}\right) - bd}{be-2cd}\right) + \left(\frac{e}{2c} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**2+b*x+a),x)
```

```
[Out] (e/(2*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(2*c*(4*a*c - b**2)))*log(x +
(-4*a*c*(e/(2*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(2*c*(4*a*c - b**2)))
+ 2*a*e + b**2*(e/(2*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(2*c*(4*a*c - b
**2))) - b*d)/(b*e - 2*c*d)) + (e/(2*c) + sqrt(-4*a*c + b**2)*(b*e - 2*c*d)
/(2*c*(4*a*c - b**2)))*log(x + (-4*a*c*(e/(2*c) + sqrt(-4*a*c + b**2)*(b*e
- 2*c*d)/(2*c*(4*a*c - b**2))) + 2*a*e + b**2*(e/(2*c) + sqrt(-4*a*c + b**2
)*(b*e - 2*c*d)/(2*c*(4*a*c - b**2))) - b*d)/(b*e - 2*c*d))
```

Giac [A] time = 1.10449, size = 88, normalized size = 1.33

$$\frac{e\log\left(cx^2+bx+a\right)}{2c} + \frac{(2cd-be)\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*e*log(c*x^2 + b*x + a)/c + (2*c*d - b*e)*arctan((2*c*x + b)/sqrt(-b^2 +
 4*a*c))/(sqrt(-b^2 + 4*a*c)*c)
```

3.2186 $\int \frac{1}{a+bx+cx^2} dx$

Optimal. Leaf size=34

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] (-2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rubi [A] time = 0.0177614, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+bx+cx^2} dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2} dx, x, b+2cx\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}\end{aligned}$$

Mathematica [A] time = 0.0065942, size = 38, normalized size = 1.12

$$\frac{2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(-1),x]
```

```
[Out] (2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
```

Maple [A] time = 0.148, size = 35, normalized size = 1.

$$2\frac{1}{\sqrt{4ac-b^2}}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a),x)
```

```
[Out] 2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89937, size = 277, normalized size = 8.15

$$\left[\frac{\log\left(\frac{2c^2x^2+2bcx+b^2-2ac-\sqrt{b^2-4ac}(2cx+b)}{cx^2+bx+a}\right)}{\sqrt{b^2-4ac}}, -\frac{2\sqrt{-b^2+4ac}\arctan\left(-\frac{\sqrt{-b^2+4ac}(2cx+b)}{b^2-4ac}\right)}{b^2-4ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c
*x^2 + b*x + a))/sqrt(b^2 - 4*a*c), -2*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2
 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

Sympy [B] time = 0.196203, size = 124, normalized size = 3.65

$$-\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)+\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*sqr
t(-1/(4*a*c - b**2)) + b)/(2*c)) + sqrt(-1/(4*a*c - b**2))*log(x + (4*a*c*s
```

```
qrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))
```

Giac [A] time = 1.11425, size = 46, normalized size = 1.35

$$\frac{2\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

3.2187 $\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=122

$$-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}-\frac{e\log\left(a+bx+cx^2\right)}{2\left(ae^2-bde+cd^2\right)}+\frac{e\log(d+ex)}{ae^2-bde+cd^2}$$

```
[Out] -(((2*c*d - b*e)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]
*(c*d^2 - b*d*e + a*e^2))) + (e*Log[d + e*x])/(c*d^2 - b*d*e + a*e^2) - (e*
Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 0.0942328, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {705, 31, 634, 618, 206, 628}

$$-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}-\frac{e\log\left(a+bx+cx^2\right)}{2\left(ae^2-bde+cd^2\right)}+\frac{e\log(d+ex)}{ae^2-bde+cd^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + b*x + c*x^2)),x]
```

```
[Out] -(((2*c*d - b*e)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]
*(c*d^2 - b*d*e + a*e^2))) + (e*Log[d + e*x])/(c*d^2 - b*d*e + a*e^2) - (e*
Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2))
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)}\,dx &= \frac{\int \frac{cd-be-cex}{a+bx+cx^2}\,dx}{cd^2-bde+ae^2}+\frac{e^2\int \frac{1}{d+ex}\,dx}{cd^2-bde+ae^2}\\
&= \frac{e\log(d+ex)}{cd^2-bde+ae^2}-\frac{e\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2\left(cd^2-bde+ae^2\right)}+\frac{(2cd-be)\int \frac{1}{a+bx+cx^2}\,dx}{2\left(cd^2-bde+ae^2\right)}\\
&= \frac{e\log(d+ex)}{cd^2-bde+ae^2}-\frac{e\log\left(a+bx+cx^2\right)}{2\left(cd^2-bde+ae^2\right)}-\frac{(2cd-be)\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{cd^2-bde+ae^2}\\
&= -\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)}+\frac{e\log(d+ex)}{cd^2-bde+ae^2}-\frac{e\log\left(a+bx+cx^2\right)}{2\left(cd^2-bde+ae^2\right)}
\end{aligned}$$

Mathematica [A] time = 0.0806766, size = 105, normalized size = 0.86

$$\frac{e\sqrt{4ac-b^2}(\log(a+x(b+cx))-2\log(d+ex))+(2be-4cd)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\left(e(bd-ae)-cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + b*x + c*x^2)),x]
```

```
[Out] ((-4*c*d + 2*b*e)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*
c]*e*(-2*Log[d + e*x] + Log[a + x*(b + c*x)]))/(2*Sqrt[-b^2 + 4*a*c]*(-(c*d
^2) + e*(b*d - a*e)))
```

Maple [A] time = 0.155, size = 168, normalized size = 1.4

$$-\frac{e\ln\left(cx^2+bx+a\right)}{2\,ae^2-2\,bde+2\,cd^2}-\frac{be}{ae^2-bde+cd^2}\arctan\left((2\,cx+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+2\,\frac{cd}{\left(ae^2-bde+cd^2\right)\sqrt{4\,ac-b^2}}\mathrm{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+b*x+a),x)
```

```
[Out] -1/2*e*ln(c*x^2+b*x+a)/(a*e^2-b*d*e+c*d^2)-1/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2
)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*e+2/(a*e^2-b*d*e+c*d^2)/(4*a*
c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c*d+e*ln(e*x+d)/(a*e^2-b*d
*e+c*d^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.85503, size = 697, normalized size = 5.71

$$\left[-\frac{\left(b^2-4\,ac\right)e\log\left(cx^2+bx+a\right)-2\left(b^2-4\,ac\right)e\log\left(ex+d\right)+\sqrt{b^2-4\,ac}(2\,cd-be)\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx}{cx^2+bx+a}\right.}{2\left(\left(b^2c-4\,ac^2\right)d^2-\left(b^3-4\,abc\right)de+\left(ab^2-4\,a^2c\right)e^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 4*a*c)*e*log(c*x^2 + b*x + a) - 2*(b^2 - 4*a*c)*e*log(e*x + d
) + sqrt(b^2 - 4*a*c)*(2*c*d - b*e)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c
+ sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)))/((b^2*c - 4*a*c^2)*d^2
 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2), -1/2*((b^2 - 4*a*c)*e*log(
c*x^2 + b*x + a) - 2*(b^2 - 4*a*c)*e*log(e*x + d) + 2*sqrt(-b^2 + 4*a*c)*(2
*c*d - b*e)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)))/((b^2*c
- 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11311, size = 170, normalized size = 1.39

$$-\frac{e\log\left(cx^2+bx+a\right)}{2\left(cd^2-bde+ae^2\right)}+\frac{e^2\log\left(|xe+d|\right)}{cd^2e-bde^2+ae^3}+\frac{(2\,cd-be)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(cd^2-bde+ae^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*e*log(c*x^2 + b*x + a)/(c*d^2 - b*d*e + a*e^2) + e^2*log(abs(x*e + d))
/(c*d^2*e - b*d*e^2 + a*e^3) + (2*c*d - b*e)*arctan((2*c*x + b)/sqrt(-b^2 +
 4*a*c))/((c*d^2 - b*d*e + a*e^2)*sqrt(-b^2 + 4*a*c))
```

3.2188 $\int \frac{1}{(d+ex)^2(a+bx+cx^2)}\,dx$

Optimal. Leaf size=186

$$-\frac{\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)^2}-\frac{e(2cd-be)\log\left(a+bx+cx^2\right)}{2\left(ae^2-bde+cd^2\right)^2}-\frac{e}{(d+ex)\left(ae^2-bde+cd^2\right)}+\frac{e(2cd}{(ae^2}$$

```
[Out] -(e/((c*d^2 - b*d*e + a*e^2)*(d + e*x))) - ((2*c^2*d^2 + b^2*e^2 - 2*c*e*(b
*d + a*e))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^
2 - b*d*e + a*e^2)^2) + (e*(2*c*d - b*e)*Log[d + e*x])/(c*d^2 - b*d*e + a*e
^2)^2 - (e*(2*c*d - b*e)*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)^2
)
```

Rubi [A] time = 0.285323, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {709, 800, 634, 618, 206, 628}

$$-\frac{\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)^2}-\frac{e(2cd-be)\log\left(a+bx+cx^2\right)}{2\left(ae^2-bde+cd^2\right)^2}-\frac{e}{(d+ex)\left(ae^2-bde+cd^2\right)}+\frac{e(2cd}{(ae^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + b*x + c*x^2)),x]
```

```
[Out] -(e/((c*d^2 - b*d*e + a*e^2)*(d + e*x))) - ((2*c^2*d^2 + b^2*e^2 - 2*c*e*(b
*d + a*e))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^
2 - b*d*e + a*e^2)^2) + (e*(2*c*d - b*e)*Log[d + e*x])/(c*d^2 - b*d*e + a*e
^2)^2 - (e*(2*c*d - b*e)*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)^2
)
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+bx+cx^2\right)}\,dx = -\frac{e}{\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{\int \frac{cd-be-cex}{(d+ex)\left(a+bx+cx^2\right)}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{e}{\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{\int \left(-\frac{e^2(-2cd+be)}{\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{c^2d^2+b^2e^2-ce(2bd+ae)-ce(2cd-be)x}{\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)}\right) d}{cd^2-bde+ae^2}$$

$$= -\frac{e}{\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{e(2cd-be)\log(d+ex)}{\left(cd^2-bde+ae^2\right)^2}+\frac{\int \frac{c^2d^2+b^2e^2-ce(2bd+ae)-ce(2cd-be)}{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)^2}$$

$$= -\frac{e}{\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{e(2cd-be)\log(d+ex)}{\left(cd^2-bde+ae^2\right)^2}-\frac{(e(2cd-be))\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2\left(cd^2-bde+ae^2\right)^2}+$$

$$= -\frac{e}{\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{e(2cd-be)\log(d+ex)}{\left(cd^2-bde+ae^2\right)^2}-\frac{e(2cd-be)\log\left(a+bx+cx^2\right.}{2\left(cd^2-bde+ae^2\right)^2}$$

$$= -\frac{e}{\left(cd^2-bde+ae^2\right)(d+ex)}-\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)^2}+\frac{e(2c}{(c}$$

Mathematica [A] time = 0.224112, size = 151, normalized size = 0.81

$$\frac{\frac{2\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-\frac{2e\left(e(ae-bd)+cd^2\right)}{d+ex}+e(be-2cd)\log(a+x(b+cx))-2e(be-2cd)\log(d+ex)}{2\left(e(ae-bd)+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + b*x + c*x^2)),x]
```

```
[Out] ((-2*e*(c*d^2 + e*(-(b*d) + a*e)))/(d + e*x) + (2*(2*c^2*d^2 + b^2*e^2 - 2*
c*e*(b*d + a*e))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
 - 2*e*(-2*c*d + b*e)*Log[d + e*x] + e*(-2*c*d + b*e)*Log[a + x*(b + c*x)])
/(2*(c*d^2 + e*(-(b*d) + a*e))^2)
```

Maple [B] time = 0.171, size = 386, normalized size = 2.1

$$\frac{\ln\left(cx^2+bx+a\right)be^2}{2\left(ae^2-bde+cd^2\right)^2}-\frac{c\ln\left(cx^2+bx+a\right)de}{\left(ae^2-bde+cd^2\right)^2}-2\frac{ace^2}{\left(ae^2-bde+cd^2\right)^2\sqrt{4ac-b^2}}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)+\frac{b^2e^2}{\left(ae^2-bde+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^2+b*x+a),x)
```

```
[Out] 1/2/(a*e^2-b*d*e+c*d^2)^2*ln(c*x^2+b*x+a)*b*e^2-1/(a*e^2-b*d*e+c*d^2)^2*c*l
n(c*x^2+b*x+a)*d*e-2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))*a*c*e^2+1/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(1/2)*arc
tan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*e^2-2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2
)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c*d*e+2/(a*e^2-b*d*e+c*d^2)^2
/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^2*d^2-e/(a*e^2-b*d
*e+c*d^2)/(e*x+d)-e^2/(a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)*b+2*e/(a*e^2-b*d*e+c*
d^2)^2*ln(e*x+d)*c*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 19.6378, size = 2295, normalized size = 12.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(b^2*c - 4*a*c^2)*d^2*e - 2*(b^3 - 4*a*b*c)*d*e^2 + 2*(a*b^2 - 4*a
^2*c)*e^3 + (2*c^2*d^3 - 2*b*c*d^2*e + (b^2 - 2*a*c)*d*e^2 + (2*c^2*d^2*e -
 2*b*c*d*e^2 + (b^2 - 2*a*c)*e^3)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b
*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (2
*(b^2*c - 4*a*c^2)*d^2*e - (b^3 - 4*a*b*c)*d*e^2 + (2*(b^2*c - 4*a*c^2)*d*e
^2 - (b^3 - 4*a*b*c)*e^3)*x)*log(c*x^2 + b*x + a) - 2*(2*(b^2*c - 4*a*c^2)*
d^2*e - (b^3 - 4*a*b*c)*d*e^2 + (2*(b^2*c - 4*a*c^2)*d*e^2 - (b^3 - 4*a*b*c
)*e^3)*x)*log(e*x + d))/((b^2*c^2 - 4*a*c^3)*d^5 - 2*(b^3*c - 4*a*b*c^2)*d^
4*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d^2*e^3
 + (a^2*b^2 - 4*a^3*c)*d*e^4 + ((b^2*c^2 - 4*a*c^3)*d^4*e - 2*(b^3*c - 4*a*
b*c^2)*d^3*e^2 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^3 - 2*(a*b^3 - 4*a^2*b
*c)*d*e^4 + (a^2*b^2 - 4*a^3*c)*e^5)*x), -1/2*(2*(b^2*c - 4*a*c^2)*d^2*e -
2*(b^3 - 4*a*b*c)*d*e^2 + 2*(a*b^2 - 4*a^2*c)*e^3 + 2*(2*c^2*d^3 - 2*b*c*d^
2*e + (b^2 - 2*a*c)*d*e^2 + (2*c^2*d^2*e - 2*b*c*d*e^2 + (b^2 - 2*a*c)*e^3)
*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)
) + (2*(b^2*c - 4*a*c^2)*d^2*e - (b^3 - 4*a*b*c)*d*e^2 + (2*(b^2*c - 4*a*c^
```

```
2)*d*e^2 - (b^3 - 4*a*b*c)*e^3)*x)*log(c*x^2 + b*x + a) - 2*(2*(b^2*c - 4*a
*c^2)*d^2*e - (b^3 - 4*a*b*c)*d*e^2 + (2*(b^2*c - 4*a*c^2)*d*e^2 - (b^3 - 4
*a*b*c)*e^3)*x)*log(e*x + d))/((b^2*c^2 - 4*a*c^3)*d^5 - 2*(b^3*c - 4*a*b*c
^2)*d^4*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d
^2*e^3 + (a^2*b^2 - 4*a^3*c)*d*e^4 + ((b^2*c^2 - 4*a*c^3)*d^4*e - 2*(b^3*c
- 4*a*b*c^2)*d^3*e^2 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^3 - 2*(a*b^3 - 4
*a^2*b*c)*d*e^4 + (a^2*b^2 - 4*a^3*c)*e^5)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11342, size = 447, normalized size = 2.4

$$-\frac{\left(2\,c^2d^2e^2-2\,bcde^3+b^2e^4-2\,ace^4\right)\arctan\left(-\frac{\left(2\,cd-\frac{2\,cd^2}{xe+d}-be+\frac{2\,bde}{xe+d}-\frac{2\,ae^2}{xe+d}\right)e^{(-1)}}{\sqrt{-b^2+4\,ac}}\right)e^{(-2)}}{\left(c^2d^4-2\,bcd^3e+b^2d^2e^2+2\,acd^2e^2-2\,abde^3+a^2e^4\right)\sqrt{-b^2+4\,ac}}-\frac{\left(2\,cde-be^2\right)\log\left(-c+\frac{2\,cd}{xe+d}-\frac{cd^2}{(xe+d)^2}\right.}{2\left(c^2d^4-2\,bcd^3e+b^2d^2e^2+2\,ac\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -(2*c^2*d^2*e^2 - 2*b*c*d*e^3 + b^2*e^4 - 2*a*c*e^4)*arctan(-(2*c*d - 2*c*d
^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*a*e^2/(x*e + d))*e^(-1)/sqrt(-b^
2 + 4*a*c))*e^(-2)/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 -
2*a*b*d*e^3 + a^2*e^4)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*c*d*e - b*e^2)*log(-c +
 2*c*d/(x*e + d) - c*d^2/(x*e + d)^2 - b*e/(x*e + d) + b*d*e/(x*e + d)^2 -
a*e^2/(x*e + d)^2)/(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2
*a*b*d*e^3 + a^2*e^4) - e^3/((c*d^2*e^2 - b*d*e^3 + a*e^4)*(x*e + d))
```

3.2189 $\int \frac{1}{(d+ex)^3(a+bx+cx^2)}\,dx$

Optimal. Leaf size=272

$$-\frac{e\left(-ce(ae+3bd)+b^2e^2+3c^2d^2\right)\log\left(a+bx+cx^2\right)}{2\left(ae^2-bde+cd^2\right)^3}+\frac{e\log(d+ex)\left(-ce(ae+3bd)+b^2e^2+3c^2d^2\right)}{\left(ae^2-bde+cd^2\right)^3}-\frac{(2cd-be)\left(-ce(3a\right.}{\sqrt{b}}$$

```
[Out] -e/(2*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^2) - (e*(2*c*d - b*e))/((c*d^2 - b*
d*e + a*e^2)^2*(d + e*x)) - ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d +
3*a*e))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 -
 b*d*e + a*e^2)^3) + (e*(3*c^2*d^2 + b^2*e^2 - c*e*(3*b*d + a*e))*Log[d + e
*x])/(c*d^2 - b*d*e + a*e^2)^3 - (e*(3*c^2*d^2 + b^2*e^2 - c*e*(3*b*d + a*e
))*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)^3)
```

Rubi [A] time = 0.425177, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {709, 800, 634, 618, 206, 628}

$$-\frac{e\left(-ce(ae+3bd)+b^2e^2+3c^2d^2\right)\log\left(a+bx+cx^2\right)}{2\left(ae^2-bde+cd^2\right)^3}+\frac{e\log(d+ex)\left(-ce(ae+3bd)+b^2e^2+3c^2d^2\right)}{\left(ae^2-bde+cd^2\right)^3}-\frac{(2cd-be)\left(-ce(3a\right.}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + b*x + c*x^2)),x]
```

```
[Out] -e/(2*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^2) - (e*(2*c*d - b*e))/((c*d^2 - b*
d*e + a*e^2)^2*(d + e*x)) - ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d +
3*a*e))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 -
 b*d*e + a*e^2)^3) + (e*(3*c^2*d^2 + b^2*e^2 - c*e*(3*b*d + a*e))*Log[d + e
*x])/(c*d^2 - b*d*e + a*e^2)^3 - (e*(3*c^2*d^2 + b^2*e^2 - c*e*(3*b*d + a*e
))*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)^3)
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(a+bx+cx^2\right)}\,dx = -\frac{e}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} + \frac{\int \frac{cd-be-cex}{(d+ex)^2\left(a+bx+cx^2\right)}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{e}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} + \frac{\int \left(-\frac{e^2(-2cd+be)}{\left(cd^2-bde+ae^2\right)(d+ex)^2} + \frac{e^2\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)}{\left(cd^2-bde+ae^2\right)^2(d+ex)} + \frac{c^3 \cdots}{\cdots}\right.}{cd^2 - \cdots}$$

$$= -\frac{e}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{e(2cd-be)}{\left(cd^2-bde+ae^2\right)^2(d+ex)} + \frac{e\left(3c^2d^2+b^2e^2-ce(3\cdots\right.}{\left(cd^2-bd\cdots\right.}$$

$$= -\frac{e}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{e(2cd-be)}{\left(cd^2-bde+ae^2\right)^2(d+ex)} + \frac{e\left(3c^2d^2+b^2e^2-ce(3\cdots\right.}{\left(cd^2-bd\cdots\right.}$$

$$= -\frac{e}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{e(2cd-be)}{\left(cd^2-bde+ae^2\right)^2(d+ex)} + \frac{e\left(3c^2d^2+b^2e^2-ce(3\cdots\right.}{\left(cd^2-bd\cdots\right.}$$

$$= -\frac{e}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{e(2cd-be)}{\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{(2cd-be)\left(c^2d^2+b^2e^{\cdots}\right.}{\sqrt{b^2-4\cdots}}$$

Mathematica [A] time = 0.346421, size = 272, normalized size = 1.

$$\frac{e\log(d+ex)\left(-ce(ae+3bd)+b^2e^2+3c^2d^2\right)}{\left(e(ae-bd)+cd^2\right)^3} + \frac{e\left(ce(ae+3bd)-b^2e^2-3c^2d^2\right)\log(a+x(b+cx))}{2\left(e(ae-bd)+cd^2\right)^3} + \frac{(be-2cd)\left(-ce(3a\cdots\right.}{\sqrt{4a\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a + b*x + c*x^2)),x]
```

```
[Out] -e/(2*(c*d^2 + e*(-(b*d) + a*e))*(d + e*x)^2) + (e*(-2*c*d + b*e))/((c*d^2
+ e*(-(b*d) + a*e))^2*(d + e*x)) + ((-2*c*d + b*e)*(c^2*d^2 + b^2*e^2 - c*e
*(b*d + 3*a*e))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]
```

```
*(-(c*d^2) + e*(b*d - a*e))^3) + (e*(3*c^2*d^2 + b^2*e^2 - c*e*(3*b*d + a*e
))*Log[d + e*x])/(c*d^2 + e*(-(b*d) + a*e))^3 + (e*(-3*c^2*d^2 - b^2*e^2 +
c*e*(3*b*d + a*e))*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e))^3)
```

Maple [B] time = 0.162, size = 719, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*x^2+b*x+a),x)
```

```
[Out] 1/2/(a*e^2-b*d*e+c*d^2)^3*c*ln(c*x^2+b*x+a)*a*e^3-1/2/(a*e^2-b*d*e+c*d^2)^3
*ln(c*x^2+b*x+a)*b^2*e^3+3/2/(a*e^2-b*d*e+c*d^2)^3*c*ln(c*x^2+b*x+a)*d*e^2*
b-3/2/(a*e^2-b*d*e+c*d^2)^3*c^2*ln(c*x^2+b*x+a)*d^2*e+3/(a*e^2-b*d*e+c*d^2)
^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*c*e^3-6/(a*e^2
-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^2*a
*d*e^2-1/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^
2)^(1/2))*b^3*e^3+3/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b
)/(4*a*c-b^2)^(1/2))*b^2*c*d*e^2-3/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*
arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c^2*d^2*e+2/(a*e^2-b*d*e+c*d^2)^3/(4*
a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^3*d^3-1/2*e/(a*e^2-b*d
*e+c*d^2)/(e*x+d)^2+e^2/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)*b-2*e/(a*e^2-b*d*e+c*
d^2)^2/(e*x+d)*c*d-e^3/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*a*c+e^3/(a*e^2-b*d*e
+c*d^2)^3*ln(e*x+d)*b^2-3*e^2/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*b*c*d+3*e/(a*
e^2-b*d*e+c*d^2)^3*ln(e*x+d)*c^2*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.11838, size = 801, normalized size = 2.94

$$-\frac{\left(3\,c^{2}d^{2}e-3\,bcde^{2}+b^{2}e^{3}-ace^{3}\right)\log\left(cx^{2}+bx+a\right)}{2\left(c^{3}d^{6}-3\,bc^{2}d^{5}e+3\,b^{2}cd^{4}e^{2}+3\,ac^{2}d^{4}e^{2}-b^{3}d^{3}e^{3}-6\,abcd^{3}e^{3}+3\,ab^{2}d^{2}e^{4}+3\,a^{2}cd^{2}e^{4}-3\,a^{2}bde^{5}+a^{3}e^{6}\right)}+\frac{}{c^{3}d^{6}e-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(3*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3 - a*c*e^3)*log(c*x^2 + b*x + a)/(
c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 -
 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*
e^6) + (3*c^2*d^2*e^2 - 3*b*c*d*e^3 + b^2*e^4 - a*c*e^4)*log(abs(x*e + d))/
(c^3*d^6*e - 3*b*c^2*d^5*e^2 + 3*b^2*c*d^4*e^3 + 3*a*c^2*d^4*e^3 - b^3*d^3*
e^4 - 6*a*b*c*d^3*e^4 + 3*a*b^2*d^2*e^5 + 3*a^2*c*d^2*e^5 - 3*a^2*b*d*e^6 +
 a^3*e^7) + (2*c^3*d^3 - 3*b*c^2*d^2*e + 3*b^2*c*d*e^2 - 6*a*c^2*d*e^2 - b^
3*e^3 + 3*a*b*c*e^3)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((c^3*d^6 - 3*b
*c^2*d^5*e + 3*b^2*c*d^4*e^2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a*b*c*d^3*
e^3 + 3*a*b^2*d^2*e^4 + 3*a^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6)*sqrt(-b^
2 + 4*a*c)) - 1/2*(5*c^2*d^4*e - 8*b*c*d^3*e^2 + 3*b^2*d^2*e^3 + 6*a*c*d^2*
e^3 - 4*a*b*d*e^4 + a^2*e^5 + 2*(2*c^2*d^3*e^2 - 3*b*c*d^2*e^3 + b^2*d*e^4
+ 2*a*c*d*e^4 - a*b*e^5)*x)/((c*d^2 - b*d*e + a*e^2)^3*(x*e + d)^2)
```

3.2190 $\int \frac{(d+ex)^5}{\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=374

$$\frac{(2cd-be)\left(-2c^2e^2\left(15a^2e^2+10abde+b^2d^2\right)+4b^2ce^3(5ae+bd)-4c^3d^2e(bd-5ae)-3b^4e^4+2c^4d^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\left(b^2-4ac\right)^{3/2}}+$$

```
[Out] (e^2*(12*c^3*d^3 - 3*b^3*e^3 - 10*c^2*d*e*(b*d + 3*a*e) + b*c*e^2*(10*b*d +
 11*a*e))*x)/(c^3*(b^2 - 4*a*c)) + (e^3*(16*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(5*
b*d + 4*a*e))*x^2)/(2*c^2*(b^2 - 4*a*c)) + (e^4*(2*c*d - b*e)*x^3)/(c*(b^2
- 4*a*c)) - ((d + e*x)^4*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*(a
 + b*x + c*x^2)) + ((2*c*d - b*e)*(2*c^4*d^4 - 3*b^4*e^4 - 4*c^3*d^2*e*(b*d
 - 5*a*e) + 4*b^2*c*e^3*(b*d + 5*a*e) - 2*c^2*e^2*(b^2*d^2 + 10*a*b*d*e + 1
5*a^2*e^2))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*(b^2 - 4*a*c)^(3/2
)) + (e^3*(10*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(5*b*d + a*e))*Log[a + b*x + c*x^
2])/(2*c^4)
```

Rubi [A] time = 0.72549, antiderivative size = 374, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {738, 800, 634, 618, 206, 628}

$$\frac{(2cd-be)\left(-2c^2e^2\left(15a^2e^2+10abde+b^2d^2\right)+4b^2ce^3(5ae+bd)-4c^3d^2e(bd-5ae)-3b^4e^4+2c^4d^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\left(b^2-4ac\right)^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5/(a + b*x + c*x^2)^2,x]
```


```
[Out] (e^2*(12*c^3*d^3 - 3*b^3*e^3 - 10*c^2*d*e*(b*d + 3*a*e) + b*c*e^2*(10*b*d +
 11*a*e))*x)/(c^3*(b^2 - 4*a*c)) + (e^3*(16*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(5*
b*d + 4*a*e))*x^2)/(2*c^2*(b^2 - 4*a*c)) + (e^4*(2*c*d - b*e)*x^3)/(c*(b^2
- 4*a*c)) - ((d + e*x)^4*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*(a
 + b*x + c*x^2)) + ((2*c*d - b*e)*(2*c^4*d^4 - 3*b^4*e^4 - 4*c^3*d^2*e*(b*d
 - 5*a*e) + 4*b^2*c*e^3*(b*d + 5*a*e) - 2*c^2*e^2*(b^2*d^2 + 10*a*b*d*e + 1
5*a^2*e^2))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*(b^2 - 4*a*c)^(3/2
)) + (e^3*(10*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(5*b*d + a*e))*Log[a + b*x + c*x^
2])/(2*c^4)
```


Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
```

```
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)^5}{\left(a+bx+cx^2\right)^2}\,dx = -\frac{(d+ex)^4(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\int \frac{(d+ex)^3\left(2cd^2-e(5bd-8ae)-3e(2cd-be)x\right)}{a+bx+cx^2}\,dx}{-b^2+4ac}$$

$$= -\frac{(d+ex)^4(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\int\left(-\frac{e^2\left(12c^3d^3-3b^3e^3-10c^2de(bd+3ae)+bce^2(10bd+11ae)\right)}{c^3} - \frac{e^3\left(16c^2d\right.}{}\right.}{}$$

$$= \frac{e^2\left(12c^3d^3-3b^3e^3-10c^2de(bd+3ae)+bce^2(10bd+11ae)\right)x}{c^3\left(b^2-4ac\right)} + \frac{e^3\left(16c^2d^2+3b^2e^2-2ce(5bd\right.}{2c^2\left(b^2-4ac\right)}$$

$$= \frac{e^2\left(12c^3d^3-3b^3e^3-10c^2de(bd+3ae)+bce^2(10bd+11ae)\right)x}{c^3\left(b^2-4ac\right)} + \frac{e^3\left(16c^2d^2+3b^2e^2-2ce(5bd\right.}{2c^2\left(b^2-4ac\right)}$$

$$= \frac{e^2\left(12c^3d^3-3b^3e^3-10c^2de(bd+3ae)+bce^2(10bd+11ae)\right)x}{c^3\left(b^2-4ac\right)} + \frac{e^3\left(16c^2d^2+3b^2e^2-2ce(5bd\right.}{2c^2\left(b^2-4ac\right)}$$

$$= \frac{e^2\left(12c^3d^3-3b^3e^3-10c^2de(bd+3ae)+bce^2(10bd+11ae)\right)x}{c^3\left(b^2-4ac\right)} + \frac{e^3\left(16c^2d^2+3b^2e^2-2ce(5bd\right.}{2c^2\left(b^2-4ac\right)}$$

Mathematica [A] time = 0.708468, size = 422, normalized size = 1.13

$$\frac{\frac{2\left(-2b^2ce^2\left(2a^2e^3-5acde(d+2ex)+5c^2d^3x\right)+bc^2\left(5a^2e^4(3d+ex)-10acd^2e^2(d+3ex)-c^2d^4(d-5ex)\right)+2c^2\left(-5a^2cde^3(2d+ex)+a^3e^5+5ac^2d^3e(d+2ex)-c^3d^5x\right)-5b^3ce^3\left(\right.\right.}{\left(b^2-4ac\right)(a+x(b+cx))}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5/(a + b*x + c*x^2)^2,x]
```

```
[Out] (2*c*e^4*(5*c*d - 2*b*e)*x + c^2*e^5*x^2 + (2*(b^5*e^5*x + b^4*e^4*(a*e - 5
*c*d*x) - 5*b^3*c*e^3*(-2*c*d^2*x + a*e*(d + e*x)) - 2*b^2*c*e^2*(2*a^2*e^3
 + 5*c^2*d^3*x - 5*a*c*d*e*(d + 2*e*x)) + 2*c^2*(a^3*e^5 - c^3*d^5*x - 5*a^
2*c*d*e^3*(2*d + e*x) + 5*a*c^2*d^3*e*(d + 2*e*x)) + b*c^2*(-(c^2*d^4*(d -
5*e*x)) + 5*a^2*e^4*(3*d + e*x) - 10*a*c*d^2*e^2*(d + 3*e*x))))/((b^2 - 4*a
*c)*(a + x*(b + c*x))) + (2*(-2*c*d + b*e)*(-2*c^4*d^4 + 3*b^4*e^4 + 4*c^3*
d^2*e*(b*d - 5*a*e) - 4*b^2*c*e^3*(b*d + 5*a*e) + 2*c^2*e^2*(b^2*d^2 + 10*a
*b*d*e + 15*a^2*e^2))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c
)^(3/2) + e^3*(10*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(5*b*d + a*e))*Log[a + x*(b +
 c*x)])/(2*c^4)
```

Maple [B] time = 0.165, size = 1542, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5/(c*x^2+b*x+a)^2,x)
```

```
[Out] 1/2*e^5*x^2/c^2+30/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*b*d^2*e^3-20/c^2/(c*x^2+
b*x+a)/(4*a*c-b^2)*x*a*b^2*d*e^4-3/2/c^4/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^4*e^
5+3/c^4/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^5*e^5-10/(4
*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*d^4*e+40/(4*a*c-b^2)^
(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*d^3*a*e^2-10/(c*x^2+b*x+a)/(4*a*c
-b^2)*a*d^4*e+1/(c*x^2+b*x+a)/(4*a*c-b^2)*b*d^5-2*e^5/c^3*x*b+5*e^4/c^2*x*d
+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*d^5-20/c^2/(4*a*
c-b^2)*ln(c*x^2+b*x+a)*a*b*d*e^4+5/c^3/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*b^3*e^
5+5/c^3/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^4*d*e^4-5/c^2/(c*x^2+b*x+a)/(4*a*c-b^
2)*x*a^2*b*e^5+10/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a^2*d*e^4+2*c/(c*x^2+b*x+a)
/(4*a*c-b^2)*x*d^5-2/c^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a^3*e^5-4/c^2/(4*a*c-b^2
)*ln(c*x^2+b*x+a)*a^2*e^5+10/c/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b*d^3*e^2+5/c^3/
(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^3*d*e^4-10/c^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^
3*d^2*e^3+10/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^2*d^3*e^2-15/c^2/(c*x^2+b*x+a)
/(4*a*c-b^2)*a^2*b*d*e^4-10/c^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^2*d^2*e^3-10/
c^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4*d*e^4+10/c^2/
(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*d^2*b^3*e^3-1/c^4/(c*
x^2+b*x+a)/(4*a*c-b^2)*a*b^4*e^5-1/c^4/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^5*e^5+
20/c/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*d^2*e^3+4/c^3/(c*x^2+b*x+a)/(4*a*c-b^2)*
a^2*b^2*e^5+5/c^3/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^3*d*e^4+7/c^3/(4*a*c-b^2)*l
n(c*x^2+b*x+a)*a*b^2*e^5-5/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b*d^4*e-60/c/(4*a*c-
b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*d*a^2*e^4-5/c^2/(4*a*c-b^2)*
ln(c*x^2+b*x+a)*b^2*d^2*e^3+20/c/(4*a*c-b^2)*ln(c*x^2+b*x+a)*a*d^2*e^3+30/c
^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b*e^5-20/c^3/(
4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^3*e^5-20/(c*x^2+b*
x+a)/(4*a*c-b^2)*x*a*d^3*e^2-60/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c
-b^2)^(1/2))*a*b*d^2*e^3+60/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b
^2)^(1/2))*a*b^2*d*e^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.33202, size = 5738, normalized size = 15.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*e^5*x^4 - 2*(b^3*c^4 - 4*a*b*c^5
)*d^5 + 20*(a*b^2*c^4 - 4*a^2*c^5)*d^4*e - 20*(a*b^3*c^3 - 4*a^2*b*c^4)*d^3
*e^2 + 20*(a*b^4*c^2 - 6*a^2*b^2*c^3 + 8*a^3*c^4)*d^2*e^3 - 10*(a*b^5*c - 7
*a^2*b^3*c^2 + 12*a^3*b*c^3)*d*e^4 + 2*(a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^
2 - 8*a^4*c^3)*e^5 + (10*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d*e^4 - 3*(b^
5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e^5)*x^3 + (10*(b^5*c^2 - 8*a*b^3*c^3 +
 16*a^2*b*c^4)*d*e^4 - (4*b^6*c - 33*a*b^4*c^2 + 72*a^2*b^2*c^3 - 16*a^3*c^
4)*e^5)*x^2 - (4*a*c^5*d^5 - 10*a*b*c^4*d^4*e + 40*a^2*c^4*d^3*e^2 + 10*(a*
b^3*c^2 - 6*a^2*b*c^3)*d^2*e^3 - 10*(a*b^4*c - 6*a^2*b^2*c^2 + 6*a^3*c^3)*d
*e^4 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*e^5 + (4*c^6*d^5 - 10*b*c^5*
d^4*e + 40*a*c^5*d^3*e^2 + 10*(b^3*c^3 - 6*a*b*c^4)*d^2*e^3 - 10*(b^4*c^2 -
 6*a*b^2*c^3 + 6*a^2*c^4)*d*e^4 + (3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*e
^5)*x^2 + (4*b*c^5*d^5 - 10*b^2*c^4*d^4*e + 40*a*b*c^4*d^3*e^2 + 10*(b^4*c^
2 - 6*a*b^2*c^3)*d^2*e^3 - 10*(b^5*c - 6*a*b^3*c^2 + 6*a^2*b*c^3)*d*e^4 + (
3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*e^5)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x
^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x +
a)) - 2*(2*(b^2*c^5 - 4*a*c^6)*d^5 - 5*(b^3*c^4 - 4*a*b*c^5)*d^4*e + 10*(b^
4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3*e^2 - 10*(b^5*c^2 - 7*a*b^3*c^3 + 12*a
^2*b*c^4)*d^2*e^3 + 5*(b^6*c - 9*a*b^4*c^2 + 26*a^2*b^2*c^3 - 24*a^3*c^4)*d
*e^4 - (b^7 - 11*a*b^5*c + 41*a^2*b^3*c^2 - 52*a^3*b*c^3)*e^5)*x + (10*(a*b
^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2*e^3 - 10*(a*b^5*c - 8*a^2*b^3*c^2
+ 16*a^3*b*c^3)*d*e^4 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c
^3)*e^5 + (10*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d^2*e^3 - 10*(b^5*c^2 -
8*a*b^3*c^3 + 16*a^2*b*c^4)*d*e^4 + (3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^
3 - 32*a^3*c^4)*e^5)*x^2 + (10*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^2*e
^3 - 10*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d*e^4 + (3*b^7 - 26*a*b^5*c
+ 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*e^5)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^4 -
 8*a^2*b^2*c^5 + 16*a^3*c^6 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^2 + (b
^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x), 1/2*((b^4*c^3 - 8*a*b^2*c^4 + 16*a
^2*c^5)*e^5*x^4 - 2*(b^3*c^4 - 4*a*b*c^5)*d^5 + 20*(a*b^2*c^4 - 4*a^2*c^5)*
d^4*e - 20*(a*b^3*c^3 - 4*a^2*b*c^4)*d^3*e^2 + 20*(a*b^4*c^2 - 6*a^2*b^2*c^
3 + 8*a^3*c^4)*d^2*e^3 - 10*(a*b^5*c - 7*a^2*b^3*c^2 + 12*a^3*b*c^3)*d*e^4
+ 2*(a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^2 - 8*a^4*c^3)*e^5 + (10*(b^4*c^3 -
 8*a*b^2*c^4 + 16*a^2*c^5)*d*e^4 - 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)
*e^5)*x^3 + (10*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d*e^4 - (4*b^6*c - 3
3*a*b^4*c^2 + 72*a^2*b^2*c^3 - 16*a^3*c^4)*e^5)*x^2 + 2*(4*a*c^5*d^5 - 10*a
*b*c^4*d^4*e + 40*a^2*c^4*d^3*e^2 + 10*(a*b^3*c^2 - 6*a^2*b*c^3)*d^2*e^3 -
10*(a*b^4*c - 6*a^2*b^2*c^2 + 6*a^3*c^3)*d*e^4 + (3*a*b^5 - 20*a^2*b^3*c +
30*a^3*b*c^2)*e^5 + (4*c^6*d^5 - 10*b*c^5*d^4*e + 40*a*c^5*d^3*e^2 + 10*(b^
3*c^3 - 6*a*b*c^4)*d^2*e^3 - 10*(b^4*c^2 - 6*a*b^2*c^3 + 6*a^2*c^4)*d*e^4 +
 (3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*e^5)*x^2 + (4*b*c^5*d^5 - 10*b^2*c
^4*d^4*e + 40*a*b*c^4*d^3*e^2 + 10*(b^4*c^2 - 6*a*b^2*c^3)*d^2*e^3 - 10*(b^
5*c - 6*a*b^3*c^2 + 6*a^2*b*c^3)*d*e^4 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c
```

```
^2)*e^5)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2
- 4*a*c)) - 2*(2*(b^2*c^5 - 4*a*c^6)*d^5 - 5*(b^3*c^4 - 4*a*b*c^5)*d^4*e +
10*(b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3*e^2 - 10*(b^5*c^2 - 7*a*b^3*c^3
+ 12*a^2*b*c^4)*d^2*e^3 + 5*(b^6*c - 9*a*b^4*c^2 + 26*a^2*b^2*c^3 - 24*a^3*
c^4)*d*e^4 - (b^7 - 11*a*b^5*c + 41*a^2*b^3*c^2 - 52*a^3*b*c^3)*e^5)*x + (1
0*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2*e^3 - 10*(a*b^5*c - 8*a^2*b^
3*c^2 + 16*a^3*b*c^3)*d*e^4 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32
*a^4*c^3)*e^5 + (10*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d^2*e^3 - 10*(b^5*
c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d*e^4 + (3*b^6*c - 26*a*b^4*c^2 + 64*a^2*
b^2*c^3 - 32*a^3*c^4)*e^5)*x^2 + (10*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)
*d^2*e^3 - 10*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d*e^4 + (3*b^7 - 26*a*
b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*e^5)*x)*log(c*x^2 + b*x + a))/(a*b^4
*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^
2 + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x)]
```

Sympy [B] time = 26.3127, size = 2669, normalized size = 7.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5/(c*x**2+b*x+a)**2,x)
```

```
[Out] (-e**3*(2*a*c*e**2 - 3*b**2*e**2 + 10*b*c*d*e - 10*c**2*d**2)/(2*c**4) - sq
rt(-(4*a*c - b**2)**3)*(b*e - 2*c*d)*(30*a**2*c**2*e**4 - 20*a*b**2*c*e**4
+ 20*a*b*c**2*d*e**3 - 20*a*c**3*d**2*e**2 + 3*b**4*e**4 - 4*b**3*c*d*e**3
+ 2*b**2*c**2*d**2*e**2 + 4*b*c**3*d**3*e - 2*c**4*d**4)/(2*c**4*(64*a**3*c
**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (16*a**3*c**2*e**5
- 17*a**2*b**2*c*e**5 + 50*a**2*b*c**2*d*e**4 + 16*a**2*c**5*(-e**3*(2*a*c*
e**2 - 3*b**2*e**2 + 10*b*c*d*e - 10*c**2*d**2)/(2*c**4) - sqrt(-(4*a*c - b
**2)**3)*(b*e - 2*c*d)*(30*a**2*c**2*e**4 - 20*a*b**2*c*e**4 + 20*a*b*c**2*
d*e**3 - 20*a*c**3*d**2*e**2 + 3*b**4*e**4 - 4*b**3*c*d*e**3 + 2*b**2*c**2*
d**2*e**2 + 4*b*c**3*d**3*e - 2*c**4*d**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*
b**2*c**2 + 12*a*b**4*c - b**6))) - 80*a**2*c**3*d**2*e**3 + 3*a*b**4*e**5
- 10*a*b**3*c*d*e**4 - 8*a*b**2*c**4*(-e**3*(2*a*c*e**2 - 3*b**2*e**2 + 10*
b*c*d*e - 10*c**2*d**2)/(2*c**4) - sqrt(-(4*a*c - b**2)**3)*(b*e - 2*c*d)*(
30*a**2*c**2*e**4 - 20*a*b**2*c*e**4 + 20*a*b*c**2*d*e**3 - 20*a*c**3*d**2*
e**2 + 3*b**4*e**4 - 4*b**3*c*d*e**3 + 2*b**2*c**2*d**2*e**2 + 4*b*c**3*d**
3*e - 2*c**4*d**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
- b**6))) + 10*a*b**2*c**2*d**2*e**3 + 20*a*b*c**3*d**3*e**2 + b**4*c**3*(-
e**3*(2*a*c*e**2 - 3*b**2*e**2 + 10*b*c*d*e - 10*c**2*d**2)/(2*c**4) - sqrt
(-(4*a*c - b**2)**3)*(b*e - 2*c*d)*(30*a**2*c**2*e**4 - 20*a*b**2*c*e**4 +
20*a*b*c**2*d*e**3 - 20*a*c**3*d**2*e**2 + 3*b**4*e**4 - 4*b**3*c*d*e**3 +
2*b**2*c**2*d**2*e**2 + 4*b*c**3*d**3*e - 2*c**4*d**4)/(2*c**4*(64*a**3*c**
3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - 5*b**2*c**3*d**4*e + 2*b*c*
*4*d**5)/(30*a**2*b*c**2*e**5 - 60*a**2*c**3*d*e**4 - 20*a*b**3*c*e**5 + 60
*a*b**2*c**2*d*e**4 - 60*a*b*c**3*d**2*e**3 + 40*a*c**4*d**3*e**2 + 3*b**5*
e**5 - 10*b**4*c*d*e**4 + 10*b**3*c**2*d**2*e**3 - 10*b*c**4*d**4*e + 4*c**
5*d**5)) + (-e**3*(2*a*c*e**2 - 3*b**2*e**2 + 10*b*c*d*e - 10*c**2*d**2)/(2
*c**4) + sqrt(-(4*a*c - b**2)**3)*(b*e - 2*c*d)*(30*a**2*c**2*e**4 - 20*a*b
**2*c*e**4 + 20*a*b*c**2*d*e**3 - 20*a*c**3*d**2*e**2 + 3*b**4*e**4 - 4*b**
3*c*d*e**3 + 2*b**2*c**2*d**2*e**2 + 4*b*c**3*d**3*e - 2*c**4*d**4)/(2*c**4
*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (16*a**3
*c**2*e**5 - 17*a**2*b**2*c*e**5 + 50*a**2*b*c**2*d*e**4 + 16*a**2*c**5*(-e
**3*(2*a*c*e**2 - 3*b**2*e**2 + 10*b*c*d*e - 10*c**2*d**2)/(2*c**4) + sqrt(
-(4*a*c - b**2)**3)*(b*e - 2*c*d)*(30*a**2*c**2*e**4 - 20*a*b**2*c*e**4 + 2
0*a*b*c**2*d*e**3 - 20*a*c**3*d**2*e**2 + 3*b**4*e**4 - 4*b**3*c*d*e**3 + 2
```

```
*b**2*c**2*d**2*e**2 + 4*b*c**3*d**3*e - 2*c**4*d**4)/(2*c**4*(64*a**3*c**3
 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - 80*a**2*c**3*d**2*e**3 + 3*a
*b**4*e**5 - 10*a*b**3*c*d*e**4 - 8*a*b**2*c**4*(-e**3*(2*a*c*e**2 - 3*b**2
*e**2 + 10*b*c*d*e - 10*c**2*d**2)/(2*c**4) + sqrt(-(4*a*c - b**2)**3)*(b*e
 - 2*c*d)*(30*a**2*c**2*e**4 - 20*a*b**2*c*e**4 + 20*a*b*c**2*d*e**3 - 20*a
*c**3*d**2*e**2 + 3*b**4*e**4 - 4*b**3*c*d*e**3 + 2*b**2*c**2*d**2*e**2 + 4
*b*c**3*d**3*e - 2*c**4*d**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 1
2*a*b**4*c - b**6))) + 10*a*b**2*c**2*d**2*e**3 + 20*a*b*c**3*d**3*e**2 + b
**4*c**3*(-e**3*(2*a*c*e**2 - 3*b**2*e**2 + 10*b*c*d*e - 10*c**2*d**2)/(2*c
**4) + sqrt(-(4*a*c - b**2)**3)*(b*e - 2*c*d)*(30*a**2*c**2*e**4 - 20*a*b**
2*c*e**4 + 20*a*b*c**2*d*e**3 - 20*a*c**3*d**2*e**2 + 3*b**4*e**4 - 4*b**3*
c*d*e**3 + 2*b**2*c**2*d**2*e**2 + 4*b*c**3*d**3*e - 2*c**4*d**4)/(2*c**4*(
64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - 5*b**2*c**3*d**4
*e + 2*b*c**4*d**5)/(30*a**2*b*c**2*e**5 - 60*a**2*c**3*d*e**4 - 20*a*b**3*
c*e**5 + 60*a*b**2*c**2*d*e**4 - 60*a*b*c**3*d**2*e**3 + 40*a*c**4*d**3*e**
2 + 3*b**5*e**5 - 10*b**4*c*d*e**4 + 10*b**3*c**2*d**2*e**3 - 10*b*c**4*d**
4*e + 4*c**5*d**5)) - (2*a**3*c**2*e**5 - 4*a**2*b**2*c*e**5 + 15*a**2*b*c*
*2*d*e**4 - 20*a**2*c**3*d**2*e**3 + a*b**4*e**5 - 5*a*b**3*c*d*e**4 + 10*a
*b**2*c**2*d**2*e**3 - 10*a*b*c**3*d**3*e**2 + 10*a*c**4*d**4*e - b*c**4*d*
*5 + x*(5*a**2*b*c**2*e**5 - 10*a**2*c**3*d*e**4 - 5*a*b**3*c*e**5 + 20*a*b
**2*c**2*d*e**4 - 30*a*b*c**3*d**2*e**3 + 20*a*c**4*d**3*e**2 + b**5*e**5 -
 5*b**4*c*d*e**4 + 10*b**3*c**2*d**2*e**3 - 10*b**2*c**3*d**3*e**2 + 5*b*c*
*4*d**4*e - 2*c**5*d**5))/(4*a**2*c**5 - a*b**2*c**4 + x**2*(4*a*c**6 - b**
2*c**5) + x*(4*a*b*c**5 - b**3*c**4)) + e**5*x**2/(2*c**2) - x*(2*b*e**5 -
5*c*d*e**4)/c**3
```

Giac [A] time = 1.128, size = 690, normalized size = 1.84

$$-\frac{\left(4\,c^5d^5-10\,bc^4d^4e+40\,ac^4d^3e^2+10\,b^3c^2d^2e^3-60\,abc^3d^2e^3-10\,b^4cde^4+60\,ab^2c^2de^4-60\,a^2c^3de^4+3\,b^5e^5-20\,ab^{3}\cdots}{\left(b^2c^4-4\,ac^5\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -(4*c^5*d^5 - 10*b*c^4*d^4*e + 40*a*c^4*d^3*e^2 + 10*b^3*c^2*d^2*e^3 - 60*a
*b*c^3*d^2*e^3 - 10*b^4*c*d*e^4 + 60*a*b^2*c^2*d*e^4 - 60*a^2*c^3*d*e^4 + 3
*b^5*e^5 - 20*a*b^3*c*e^5 + 30*a^2*b*c^2*e^5)*arctan((2*c*x + b)/sqrt(-b^2
+ 4*a*c))/((b^2*c^4 - 4*a*c^5)*sqrt(-b^2 + 4*a*c)) + 1/2*(10*c^2*d^2*e^3 -
10*b*c*d*e^4 + 3*b^2*e^5 - 2*a*c*e^5)*log(c*x^2 + b*x + a)/c^4 + 1/2*(c^2*x
^2*e^5 + 10*c^2*d*x*e^4 - 4*b*c*x*e^5)/c^4 - (b*c^4*d^5 - 10*a*c^4*d^4*e +
10*a*b*c^3*d^3*e^2 - 10*a*b^2*c^2*d^2*e^3 + 20*a^2*c^3*d^2*e^3 + 5*a*b^3*c*
d*e^4 - 15*a^2*b*c^2*d*e^4 - a*b^4*e^5 + 4*a^2*b^2*c*e^5 - 2*a^3*c^2*e^5 +
(2*c^5*d^5 - 5*b*c^4*d^4*e + 10*b^2*c^3*d^3*e^2 - 20*a*c^4*d^3*e^2 - 10*b^3
*c^2*d^2*e^3 + 30*a*b*c^3*d^2*e^3 + 5*b^4*c*d*e^4 - 20*a*b^2*c^2*d*e^4 + 10
*a^2*c^3*d*e^4 - b^5*e^5 + 5*a*b^3*c*e^5 - 5*a^2*b*c^2*e^5)*x)/((c*x^2 + b*
x + a)*(b^2 - 4*a*c)*c^4)
```

3.2191 $\int \frac{(d+ex)^4}{(a+bx+cx^2)^2}\,dx$

Optimal. Leaf size=260

$$\frac{2e^2x\left(-ce(3ae+2bd)+b^2e^2+3c^2d^2\right)}{c^2\left(b^2-4ac\right)}+\frac{2\left(2b^2ce^3(3ae+bd)-4c^3d^2e(bd-3ae)-6ac^2e^3(ae+2bd)-b^4e^4+2c^4d^4\right)\tanh^{-1}}{c^3\left(b^2-4ac\right)^{3/2}}$$

```
[Out] (2*e^2*(3*c^2*d^2 + b^2*e^2 - c*e*(2*b*d + 3*a*e))*x)/(c^2*(b^2 - 4*a*c)) +
 (e^3*(2*c*d - b*e)*x^2)/(c*(b^2 - 4*a*c)) - ((d + e*x)^3*(b*d - 2*a*e + (2
*c*d - b*e)*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) + (2*(2*c^4*d^4 - b^4*e^4
 - 4*c^3*d^2*e*(b*d - 3*a*e) - 6*a*c^2*e^3*(2*b*d + a*e) + 2*b^2*c*e^3*(b*d
 + 3*a*e))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(3/2)
) + (e^3*(2*c*d - b*e)*Log[a + b*x + c*x^2])/c^3
```

Rubi [A] time = 0.567281, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {738, 800, 634, 618, 206, 628}

$$\frac{2e^2x\left(-ce(3ae+2bd)+b^2e^2+3c^2d^2\right)}{c^2\left(b^2-4ac\right)}+\frac{2\left(2b^2ce^3(3ae+bd)-4c^3d^2e(bd-3ae)-6ac^2e^3(ae+2bd)-b^4e^4+2c^4d^4\right)\tanh^{-1}}{c^3\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a + b*x + c*x^2)^2,x]
```

```
[Out] (2*e^2*(3*c^2*d^2 + b^2*e^2 - c*e*(2*b*d + 3*a*e))*x)/(c^2*(b^2 - 4*a*c)) +
 (e^3*(2*c*d - b*e)*x^2)/(c*(b^2 - 4*a*c)) - ((d + e*x)^3*(b*d - 2*a*e + (2
*c*d - b*e)*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) + (2*(2*c^4*d^4 - b^4*e^4
 - 4*c^3*d^2*e*(b*d - 3*a*e) - 6*a*c^2*e^3*(2*b*d + a*e) + 2*b^2*c*e^3*(b*d
 + 3*a*e))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(3/2)
) + (e^3*(2*c*d - b*e)*Log[a + b*x + c*x^2])/c^3
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)^4}{\left(a+bx+cx^2\right)^2}\,dx = -\frac{(d+ex)^3(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\int \frac{(d+ex)^2\left(2\left(cd^2-2bde+3ae^2\right)-2e(2cd-be)x\right)}{a+bx+cx^2}\,dx}{-b^2+4ac}$$

$$= -\frac{(d+ex)^3(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\int \left(-\frac{2e^2\left(3c^2d^2+b^2e^2-ce(2bd+3ae)\right)}{c^2} - \frac{2e^3(2cd-be)x}{c} + \frac{2\left(c^3d^4+ab^2e^4-\right.}{\ldots}\right.}{-b^2+4ac}$$

$$= \frac{2e^2\left(3c^2d^2+b^2e^2-ce(2bd+3ae)\right)x}{c^2\left(b^2-4ac\right)} + \frac{e^3(2cd-be)x^2}{c\left(b^2-4ac\right)} - \frac{(d+ex)^3(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \ldots$$

$$= \frac{2e^2\left(3c^2d^2+b^2e^2-ce(2bd+3ae)\right)x}{c^2\left(b^2-4ac\right)} + \frac{e^3(2cd-be)x^2}{c\left(b^2-4ac\right)} - \frac{(d+ex)^3(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \ldots$$

$$= \frac{2e^2\left(3c^2d^2+b^2e^2-ce(2bd+3ae)\right)x}{c^2\left(b^2-4ac\right)} + \frac{e^3(2cd-be)x^2}{c\left(b^2-4ac\right)} - \frac{(d+ex)^3(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \ldots$$

$$= \frac{2e^2\left(3c^2d^2+b^2e^2-ce(2bd+3ae)\right)x}{c^2\left(b^2-4ac\right)} + \frac{e^3(2cd-be)x^2}{c\left(b^2-4ac\right)} - \frac{(d+ex)^3(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \ldots$$

Mathematica [A] time = 0.495343, size = 298, normalized size = 1.15

$$\frac{\frac{-bc\left(-3a^2e^4+6acde^2(d+2ex)+c^2d^3(d-4ex)\right)-2c^2\left(a^2e^3(4d+ex)-2acd^2e(2d+3ex)+c^2d^4x\right)+2b^2ce^2\left(2ae(d+ex)-3cd^2x\right)+b^3e^3(4cdx-ae)-b^4e^4x}{\left(b^2-4ac\right)(a+x(b+cx))} - \frac{2\left(-2b^2ce^3(3ae+b\ldots\right.}{\ldots}}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a + b*x + c*x^2)^2,x]
```

```
[Out] (c*e^4*x + (-(b^4*e^4*x) + b^3*e^3*(-(a*e) + 4*c*d*x) + 2*b^2*c*e^2*(-3*c*d
^2*x + 2*a*e*(d + e*x)) - b*c*(-3*a^2*e^4 + c^2*d^3*(d - 4*e*x) + 6*a*c*d*e
```

```
^2*(d + 2*e*x)) - 2*c^2*(c^2*d^4*x + a^2*e^3*(4*d + e*x) - 2*a*c*d^2*e*(2*d
 + 3*e*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x))) - (2*(-2*c^4*d^4 + b^4*e^4 +
4*c^3*d^2*e*(b*d - 3*a*e) + 6*a*c^2*e^3*(2*b*d + a*e) - 2*b^2*c*e^3*(b*d +
3*a*e))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + e^3*
(2*c*d - b*e)*Log[a + x*(b + c*x)])/c^3
```

Maple [B] time = 0.161, size = 1037, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(c*x^2+b*x+a)^2,x)
```

```
[Out] 6/c/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b*d^2*e^2-4/c^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x
*a*b^2*e^4-4/c^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^3*d*e^3-4/c^2/(c*x^2+b*x+a)/
(4*a*c-b^2)*a*b^2*d*e^3-24/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)
^(1/2))*a*b*d*e^3-2/c^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2
))*b^4*e^4+24/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*d^2*e
^2+12/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*b*d*e^3+e^4*x/c^2-8/(4*a*c-b^2)^(3/2)
*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*d^3*e+2*c/(c*x^2+b*x+a)/(4*a*c-b^2)*
x*d^4+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*d^4+1/(c*x^
2+b*x+a)/(4*a*c-b^2)*d^4*b+6/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^2*d^2*e^2-3/c^
2/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*b*e^4-4/c^2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*a*b
*e^4+4/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*d*e^3-
2/c^2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^2*d*e^3+8/c/(4*a*c-b^2)*ln(c*x^2+b*x+a)
*a*d*e^3-12/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*e^4
+1/c^3/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^3*e^4-8/(c*x^2+b*x+a)/(4*a*c-b^2)*a*d^
3*e+1/c^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^3*e^4+12/c^2/(4*a*c-b^2)^(3/2)*arct
an((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*e^4+1/c^3/(c*x^2+b*x+a)/(4*a*c-b^2)*x
*b^4*e^4+2/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a^2*e^4-12/(c*x^2+b*x+a)/(4*a*c-b^
2)*x*a*d^2*e^2-4/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b*d^3*e+8/c/(c*x^2+b*x+a)/(4*a
*c-b^2)*a^2*d*e^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.73383, size = 3791, normalized size = 14.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^4*x^3 + (b^5*c - 8*a*b^3*c^2 + 16*
a^2*b*c^3)*e^4*x^2 - (b^3*c^3 - 4*a*b*c^4)*d^4 + 8*(a*b^2*c^3 - 4*a^2*c^4)*
d^3*e - 6*(a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e^2 + 4*(a*b^4*c - 6*a^2*b^2*c^2 +
8*a^3*c^3)*d*e^3 - (a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2)*e^4 + (2*a*c^4*d^4
- 4*a*b*c^3*d^3*e + 12*a^2*c^3*d^2*e^2 + 2*(a*b^3*c - 6*a^2*b*c^2)*d*e^3 -
(a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*e^4 + (2*c^5*d^4 - 4*b*c^4*d^3*e + 12*a*c
^4*d^2*e^2 + 2*(b^3*c^2 - 6*a*b*c^3)*d*e^3 - (b^4*c - 6*a*b^2*c^2 + 6*a^2*c
^3)*e^4)*x^2 + (2*b*c^4*d^4 - 4*b^2*c^3*d^3*e + 12*a*b*c^3*d^2*e^2 + 2*(b^4
*c - 6*a*b^2*c^2)*d*e^3 - (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*e^4)*x)*sqrt(b^2
- 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x
+ b))/(c*x^2 + b*x + a)) - (2*(b^2*c^4 - 4*a*c^5)*d^4 - 4*(b^3*c^3 - 4*a*b*
c^4)*d^3*e + 6*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2*e^2 - 4*(b^5*c - 7*a
*b^3*c^2 + 12*a^2*b*c^3)*d*e^3 + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^3
*c^3)*e^4)*x + (2*(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*d*e^3 - (a*b^5 - 8
*a^2*b^3*c + 16*a^3*b*c^2)*e^4 + (2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*
e^3 - (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^4)*x^2 + (2*(b^5*c - 8*a*b^3*c
^2 + 16*a^2*b*c^3)*d*e^3 - (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*e^4)*x)*log(c
*x^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b
^2*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x), ((b^4
*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^4*x^3 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*
c^3)*e^4*x^2 - (b^3*c^3 - 4*a*b*c^4)*d^4 + 8*(a*b^2*c^3 - 4*a^2*c^4)*d^3*e
- 6*(a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e^2 + 4*(a*b^4*c - 6*a^2*b^2*c^2 + 8*a^3*
c^3)*d*e^3 - (a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2)*e^4 + 2*(2*a*c^4*d^4 - 4*
a*b*c^3*d^3*e + 12*a^2*c^3*d^2*e^2 + 2*(a*b^3*c - 6*a^2*b*c^2)*d*e^3 - (a*b
^4 - 6*a^2*b^2*c + 6*a^3*c^2)*e^4 + (2*c^5*d^4 - 4*b*c^4*d^3*e + 12*a*c^4*d
^2*e^2 + 2*(b^3*c^2 - 6*a*b*c^3)*d*e^3 - (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*
e^4)*x^2 + (2*b*c^4*d^4 - 4*b^2*c^3*d^3*e + 12*a*b*c^3*d^2*e^2 + 2*(b^4*c -
 6*a*b^2*c^2)*d*e^3 - (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*e^4)*x)*sqrt(-b^2 + 4
*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (2*(b^2*c^4 -
 4*a*c^5)*d^4 - 4*(b^3*c^3 - 4*a*b*c^4)*d^3*e + 6*(b^4*c^2 - 6*a*b^2*c^3 +
8*a^2*c^4)*d^2*e^2 - 4*(b^5*c - 7*a*b^3*c^2 + 12*a^2*b*c^3)*d*e^3 + (b^6 -
9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*e^4)*x + (2*(a*b^4*c - 8*a^2*b^2*c
^2 + 16*a^3*c^3)*d*e^3 - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*e^4 + (2*(b^4
*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^3 - (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^
3)*e^4)*x^2 + (2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e^3 - (b^6 - 8*a*b^
4*c + 16*a^2*b^2*c^2)*e^4)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*
c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*
a*b^3*c^4 + 16*a^2*b*c^5)*x)]
```

Sympy [B] time = 13.0234, size = 1924, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(c*x**2+b*x+a)**2,x)
```

```
[Out] (-e**3*(b*e - 2*c*d)/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2*e**4 - 6*
a*b**2*c*e**4 + 12*a*b*c**2*d*e**3 - 12*a*c**3*d**2*e**2 + b**4*e**4 - 2*b*
*3*c*d*e**3 + 4*b*c**3*d**3*e - 2*c**4*d**4)/(c**3*(64*a**3*c**3 - 48*a**2*
b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a**2*b*c*e**4 - 16*a**2*c**4
*(-e**3*(b*e - 2*c*d)/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2*e**4 - 6
*a*b**2*c*e**4 + 12*a*b*c**2*d*e**3 - 12*a*c**3*d**2*e**2 + b**4*e**4 - 2*b
**3*c*d*e**3 + 4*b*c**3*d**3*e - 2*c**4*d**4)/(c**3*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6))) + 32*a**2*c**2*d*e**3 + 2*a*b**3*e**4 +
8*a*b**2*c**3*(-e**3*(b*e - 2*c*d)/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*
c**2*e**4 - 6*a*b**2*c*e**4 + 12*a*b*c**2*d*e**3 - 12*a*c**3*d**2*e**2 + b*
```

```
*4*e**4 - 2*b**3*c*d*e**3 + 4*b*c**3*d**3*e - 2*c**4*d**4)/(c**3*(64*a**3*c
**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - 4*a*b**2*c*d*e**3 - 12*a*
b*c**2*d**2*e**2 - b**4*c**2*(-e**3*(b*e - 2*c*d)/c**3 - sqrt(-(4*a*c - b**
2)**3)*(6*a**2*c**2*e**4 - 6*a*b**2*c*e**4 + 12*a*b*c**2*d*e**3 - 12*a*c**3
*d**2*e**2 + b**4*e**4 - 2*b**3*c*d*e**3 + 4*b*c**3*d**3*e - 2*c**4*d**4)/(
c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 4*b**2*c**
2*d**3*e - 2*b*c**3*d**4)/(12*a**2*c**2*e**4 - 12*a*b**2*c*e**4 + 24*a*b*c*
*2*d*e**3 - 24*a*c**3*d**2*e**2 + 2*b**4*e**4 - 4*b**3*c*d*e**3 + 8*b*c**3*
d**3*e - 4*c**4*d**4)) + (-e**3*(b*e - 2*c*d)/c**3 + sqrt(-(4*a*c - b**2)**
3)*(6*a**2*c**2*e**4 - 6*a*b**2*c*e**4 + 12*a*b*c**2*d*e**3 - 12*a*c**3*d**
2*e**2 + b**4*e**4 - 2*b**3*c*d*e**3 + 4*b*c**3*d**3*e - 2*c**4*d**4)/(c**3
*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a**
2*b*c*e**4 - 16*a**2*c**4*(-e**3*(b*e - 2*c*d)/c**3 + sqrt(-(4*a*c - b**2)*
*3)*(6*a**2*c**2*e**4 - 6*a*b**2*c*e**4 + 12*a*b*c**2*d*e**3 - 12*a*c**3*d*
*2*e**2 + b**4*e**4 - 2*b**3*c*d*e**3 + 4*b*c**3*d**3*e - 2*c**4*d**4)/(c**
3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 32*a**2*c**2*
d*e**3 + 2*a*b**3*e**4 + 8*a*b**2*c**3*(-e**3*(b*e - 2*c*d)/c**3 + sqrt(-(4
*a*c - b**2)**3)*(6*a**2*c**2*e**4 - 6*a*b**2*c*e**4 + 12*a*b*c**2*d*e**3 -
 12*a*c**3*d**2*e**2 + b**4*e**4 - 2*b**3*c*d*e**3 + 4*b*c**3*d**3*e - 2*c*
*4*d**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) -
4*a*b**2*c*d*e**3 - 12*a*b*c**2*d**2*e**2 - b**4*c**2*(-e**3*(b*e - 2*c*d)/
c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2*e**4 - 6*a*b**2*c*e**4 + 12*a*
b*c**2*d*e**3 - 12*a*c**3*d**2*e**2 + b**4*e**4 - 2*b**3*c*d*e**3 + 4*b*c**
3*d**3*e - 2*c**4*d**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4
*c - b**6))) + 4*b**2*c**2*d**3*e - 2*b*c**3*d**4)/(12*a**2*c**2*e**4 - 12*
a*b**2*c*e**4 + 24*a*b*c**2*d*e**3 - 24*a*c**3*d**2*e**2 + 2*b**4*e**4 - 4*
b**3*c*d*e**3 + 8*b*c**3*d**3*e - 4*c**4*d**4)) + (-3*a**2*b*c*e**4 + 8*a**
2*c**2*d*e**3 + a*b**3*e**4 - 4*a*b**2*c*d*e**3 + 6*a*b*c**2*d**2*e**2 - 8*
a*c**3*d**3*e + b*c**3*d**4 + x*(2*a**2*c**2*e**4 - 4*a*b**2*c*e**4 + 12*a*
b*c**2*d*e**3 - 12*a*c**3*d**2*e**2 + b**4*e**4 - 4*b**3*c*d*e**3 + 6*b**2*
c**2*d**2*e**2 - 4*b*c**3*d**3*e + 2*c**4*d**4))/(4*a**2*c**4 - a*b**2*c**3
 + x**2*(4*a*c**5 - b**2*c**4) + x*(4*a*b*c**4 - b**3*c**3)) + e**4*x/c**2
```

Giac [A] time = 1.13373, size = 479, normalized size = 1.84

$$-\frac{2\left(2\,c^4d^4-4\,bc^3d^3e+12\,ac^3d^2e^2+2\,b^3cde^3-12\,abc^2de^3-b^4e^4+6\,ab^2ce^4-6\,a^2c^2e^4\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2c^3-4\,ac^4\right)\sqrt{-b^2+4\,ac}}+\frac{xe^4}{c^2}+\frac{(2\,cd\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2*(2*c^4*d^4 - 4*b*c^3*d^3*e + 12*a*c^3*d^2*e^2 + 2*b^3*c*d*e^3 - 12*a*b*c
^2*d*e^3 - b^4*e^4 + 6*a*b^2*c*e^4 - 6*a^2*c^2*e^4)*arctan((2*c*x + b)/sqrt
(-b^2 + 4*a*c))/((b^2*c^3 - 4*a*c^4)*sqrt(-b^2 + 4*a*c)) + x*e^4/c^2 + (2*c
*d*e^3 - b*e^4)*log(c*x^2 + b*x + a)/c^3 - ((2*c^4*d^4 - 4*b*c^3*d^3*e + 6*
b^2*c^2*d^2*e^2 - 12*a*c^3*d^2*e^2 - 4*b^3*c*d*e^3 + 12*a*b*c^2*d*e^3 + b^4
*e^4 - 4*a*b^2*c*e^4 + 2*a^2*c^2*e^4)*x/c + (b*c^3*d^4 - 8*a*c^3*d^3*e + 6*
a*b*c^2*d^2*e^2 - 4*a*b^2*c*d*e^3 + 8*a^2*c^2*d*e^3 + a*b^3*e^4 - 3*a^2*b*c
*e^4)/c)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2)
```

3.2192 $\int \frac{(d+ex)^3}{\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=173

$$\frac{(2cd-be)\left(-2ce(bd-3ae)-b^2e^2+2c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\left(b^2-4ac\right)^{3/2}}+\frac{e^2x(2cd-be)}{c\left(b^2-4ac\right)}-\frac{(d+ex)^2(-2ae+x(2cd-be)+bd)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+$$

```
[Out] (e^2*(2*c*d - b*e)*x)/(c*(b^2 - 4*a*c)) - ((d + e*x)^2*(b*d - 2*a*e + (2*c*
d - b*e)*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) + ((2*c*d - b*e)*(2*c^2*d^2
- b^2*e^2 - 2*c*e*(b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c
^2*(b^2 - 4*a*c)^(3/2)) + (e^3*Log[a + b*x + c*x^2])/(2*c^2)
```

Rubi [A] time = 0.284977, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {738, 773, 634, 618, 206, 628}

$$\frac{(2cd-be)\left(-2ce(bd-3ae)-b^2e^2+2c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\left(b^2-4ac\right)^{3/2}}+\frac{e^2x(2cd-be)}{c\left(b^2-4ac\right)}-\frac{(d+ex)^2(-2ae+x(2cd-be)+bd)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + b*x + c*x^2)^2,x]
```

```
[Out] (e^2*(2*c*d - b*e)*x)/(c*(b^2 - 4*a*c)) - ((d + e*x)^2*(b*d - 2*a*e + (2*c*
d - b*e)*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) + ((2*c*d - b*e)*(2*c^2*d^2
- b^2*e^2 - 2*c*e*(b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c
^2*(b^2 - 4*a*c)^(3/2)) + (e^3*Log[a + b*x + c*x^2])/(2*c^2)
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(a+bx+cx^2\right)^2}\,dx = -\frac{(d+ex)^2(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\int \frac{(d+ex)\left(2cd^2-e(3bd-4ae)-e(2cd-be)x\right)}{a+bx+cx^2}\,dx}{-b^2+4ac}$$

$$= \frac{e^2(2cd-be)x}{c\left(b^2-4ac\right)} - \frac{(d+ex)^2(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{\int \frac{ae^2(2cd-be)+cd\left(2cd^2-e(3bd-4ae)\right)+\left(-cde(2cd-be)+be^2\right.}{a+bx+cx^2}}{c\left(b^2-4ac\right)}$$

$$= \frac{e^2(2cd-be)x}{c\left(b^2-4ac\right)} - \frac{(d+ex)^2(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{e^3\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2} - \frac{\left((2cd-be)\left(2c^2d^2-b^2e^2\right.\right.}{2c^2}$$

$$= \frac{e^2(2cd-be)x}{c\left(b^2-4ac\right)} - \frac{(d+ex)^2(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{e^3\log\left(a+bx+cx^2\right)}{2c^2} + \frac{\left((2cd-be)\left(2c^2d^2-\right.\right.}{}$$

$$= \frac{e^2(2cd-be)x}{c\left(b^2-4ac\right)} - \frac{(d+ex)^2(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{(2cd-be)\left(2c^2d^2-b^2e^2-2ce(bd-3ae)\right)\text{ta}}{c^2\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.340097, size = 201, normalized size = 1.16

$$\frac{\frac{2\left(-2c\left(a^2e^3-3acde(d+ex)+c^2d^3x\right)+b^2e^2(ae-3cdx)-bc\left(3ae^2(d+ex)+cd^2(d-3ex)\right)+b^3e^3x\right)}{\left(b^2-4ac\right)(a+x(b+cx))} + \frac{2(be-2cd)\left(2ce(bd-3ae)+b^2e^2-2c^2d^2\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}} + e^3\log(a+}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + b*x + c*x^2)^2,x]
```

```
[Out] ((2*(b^3*e^3*x + b^2*e^2*(a*e - 3*c*d*x) - 2*c*(a^2*e^3 + c^2*d^3*x - 3*a*c
*d*e*(d + e*x)) - b*c*(c*d^2*(d - 3*e*x) + 3*a*e^2*(d + e*x))))/((b^2 - 4*a
*c)*(a + x*(b + c*x))) + (2*(-2*c*d + b*e)*(-2*c^2*d^2 + b^2*e^2 + 2*c*e*(b
*d - 3*a*e))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) +
 e^3*Log[a + x*(b + c*x)])/(2*c^2)
```

Maple [B] time = 0.157, size = 416, normalized size = 2.4

$$\frac{1}{cx^2+bx+a}\left(\frac{\left(3\,abce^3-6\,c^2ade^2-b^3e^3+3\,b^2cde^2-3\,bc^2d^2e+2\,c^3d^3\right)x}{c^2\left(4\,ac-b^2\right)}+\frac{2\,a^2ce^3-ab^2e^3+3\,abcde^2-6\,ac^2d^2e+bc}{c^2\left(4\,ac-b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^2+b*x+a)^2,x)
```

```
[Out] ((3*a*b*c*e^3-6*a*c^2*d*e^2-b^3*e^3+3*b^2*c*d*e^2-3*b*c^2*d^2*e+2*c^3*d^3)/
c^2/(4*a*c-b^2)*x+(2*a^2*c*e^3-a*b^2*e^3+3*a*b*c*d*e^2-6*a*c^2*d^2*e+b*c^2*
d^3)/(4*a*c-b^2)/c^2)/(c*x^2+b*x+a)+2/c/(4*a*c-b^2)*ln(c*x^2+b*x+a)*a*e^3-1
/2/c^2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^2*e^3-6/c/(4*a*c-b^2)^(3/2)*arctan((2*
c*x+b)/(4*a*c-b^2)^(1/2))*a*b*e^3+12/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*
a*c-b^2)^(1/2))*a*d*e^2-6/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1
/2))*b*d^2*e+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*d^3+
1/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*e^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.94204, size = 2435, normalized size = 14.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(b^3*c^2 - 4*a*b*c^3)*d^3 - 12*(a*b^2*c^2 - 4*a^2*c^3)*d^2*e + 6*(
a*b^3*c - 4*a^2*b*c^2)*d*e^2 - 2*(a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*e^3 - (4
*a*c^3*d^3 - 6*a*b*c^2*d^2*e + 12*a^2*c^2*d*e^2 + (a*b^3 - 6*a^2*b*c)*e^3 +
 (4*c^4*d^3 - 6*b*c^3*d^2*e + 12*a*c^3*d*e^2 + (b^3*c - 6*a*b*c^2)*e^3)*x^2
 + (4*b*c^3*d^3 - 6*b^2*c^2*d^2*e + 12*a*b*c^2*d*e^2 + (b^4 - 6*a*b^2*c)*e^
3)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 -
 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(2*(b^2*c^3 - 4*a*c^4)*d^3 - 3*
(b^3*c^2 - 4*a*b*c^3)*d^2*e + 3*(b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d*e^2 - (
b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*e^3)*x - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3
)*e^3*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*e^3*x + (a*b^4 - 8*a^2*b^2*c +
 16*a^3*c^2)*e^3)*log(c*x^2 + b*x + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3
*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 +
16*a^2*b*c^4)*x), -1/2*(2*(b^3*c^2 - 4*a*b*c^3)*d^3 - 12*(a*b^2*c^2 - 4*a^2
*c^3)*d^2*e + 6*(a*b^3*c - 4*a^2*b*c^2)*d*e^2 - 2*(a*b^4 - 6*a^2*b^2*c + 8*
a^3*c^2)*e^3 - 2*(4*a*c^3*d^3 - 6*a*b*c^2*d^2*e + 12*a^2*c^2*d*e^2 + (a*b^3
 - 6*a^2*b*c)*e^3 + (4*c^4*d^3 - 6*b*c^3*d^2*e + 12*a*c^3*d*e^2 + (b^3*c -
6*a*b*c^2)*e^3)*x^2 + (4*b*c^3*d^3 - 6*b^2*c^2*d^2*e + 12*a*b*c^2*d*e^2 + (
```

```
b^4 - 6*a*b^2*c)*e^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c
*x + b)/(b^2 - 4*a*c)) + 2*(2*(b^2*c^3 - 4*a*c^4)*d^3 - 3*(b^3*c^2 - 4*a*b*
c^3)*d^2*e + 3*(b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d*e^2 - (b^5 - 7*a*b^3*c +
 12*a^2*b*c^2)*e^3)*x - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^3*x^2 + (b^5
- 8*a*b^3*c + 16*a^2*b*c^2)*e^3*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*e^3)
*log(c*x^2 + b*x + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 -
 8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x)]
```

Sympy [B] time = 5.31504, size = 1238, normalized size = 7.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**2+b*x+a)**2,x)
```

```
[Out] (e**3/(2*c**2) - sqrt(-(4*a*c - b**2)**3)*(b*e - 2*c*d)*(6*a*c*e**2 - b**2*
e**2 - 2*b*c*d*e + 2*c**2*d**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
 12*a*b**4*c - b**6)))*log(x + (-16*a**2*c**3*(e**3/(2*c**2) - sqrt(-(4*a*c
 - b**2)**3)*(b*e - 2*c*d)*(6*a*c*e**2 - b**2*e**2 - 2*b*c*d*e + 2*c**2*d**
2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 8*a*
*2*c*e**3 + 8*a*b**2*c**2*(e**3/(2*c**2) - sqrt(-(4*a*c - b**2)**3)*(b*e -
2*c*d)*(6*a*c*e**2 - b**2*e**2 - 2*b*c*d*e + 2*c**2*d**2)/(2*c**2*(64*a**3*
c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - a*b**2*e**3 - 6*a*b*c*d*
e**2 - b**4*c*(e**3/(2*c**2) - sqrt(-(4*a*c - b**2)**3)*(b*e - 2*c*d)*(6*a*
c*e**2 - b**2*e**2 - 2*b*c*d*e + 2*c**2*d**2)/(2*c**2*(64*a**3*c**3 - 48*a*
*2*b**2*c**2 + 12*a*b**4*c - b**6))) + 3*b**2*c*d**2*e - 2*b*c**2*d**3)/(6*
a*b*c*e**3 - 12*a*c**2*d*e**2 - b**3*e**3 + 6*b*c**2*d**2*e - 4*c**3*d**3))
 + (e**3/(2*c**2) + sqrt(-(4*a*c - b**2)**3)*(b*e - 2*c*d)*(6*a*c*e**2 - b*
*2*e**2 - 2*b*c*d*e + 2*c**2*d**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**
2 + 12*a*b**4*c - b**6)))*log(x + (-16*a**2*c**3*(e**3/(2*c**2) + sqrt(-(4*
a*c - b**2)**3)*(b*e - 2*c*d)*(6*a*c*e**2 - b**2*e**2 - 2*b*c*d*e + 2*c**2*
d**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 8
*a**2*c*e**3 + 8*a*b**2*c**2*(e**3/(2*c**2) + sqrt(-(4*a*c - b**2)**3)*(b*e
 - 2*c*d)*(6*a*c*e**2 - b**2*e**2 - 2*b*c*d*e + 2*c**2*d**2)/(2*c**2*(64*a*
*3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - a*b**2*e**3 - 6*a*b*c
*d*e**2 - b**4*c*(e**3/(2*c**2) + sqrt(-(4*a*c - b**2)**3)*(b*e - 2*c*d)*(6
*a*c*e**2 - b**2*e**2 - 2*b*c*d*e + 2*c**2*d**2)/(2*c**2*(64*a**3*c**3 - 48
*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 3*b**2*c*d**2*e - 2*b*c**2*d**3)/
(6*a*b*c*e**3 - 12*a*c**2*d*e**2 - b**3*e**3 + 6*b*c**2*d**2*e - 4*c**3*d**
3)) + (2*a**2*c*e**3 - a*b**2*e**3 + 3*a*b*c*d*e**2 - 6*a*c**2*d**2*e + b*c
**2*d**3 + x*(3*a*b*c*e**3 - 6*a*c**2*d*e**2 - b**3*e**3 + 3*b**2*c*d*e**2
- 3*b*c**2*d**2*e + 2*c**3*d**3))/(4*a**2*c**3 - a*b**2*c**2 + x**2*(4*a*c*
*4 - b**2*c**3) + x*(4*a*b*c**3 - b**3*c**2))
```

Giac [A] time = 1.11513, size = 319, normalized size = 1.84

$$-\frac{\left(4\,c^3d^3-6\,bc^2d^2e+12\,ac^2de^2+b^3e^3-6\,abce^3\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2c^2-4\,ac^3\right)\sqrt{-b^2+4\,ac}}+\frac{e^3\log\left(cx^2+bx+a\right)}{2\,c^2}-\frac{bc^2d^3-6\,ac^2d^2e+3\,abcde}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -(4*c^3*d^3 - 6*b*c^2*d^2*e + 12*a*c^2*d*e^2 + b^3*e^3 - 6*a*b*c*e^3)*arcta
n((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^3)*sqrt(-b^2 + 4*a*c))
+ 1/2*e^3*log(c*x^2 + b*x + a)/c^2 - (b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d
*e^2 - a*b^2*e^3 + 2*a^2*c*e^3 + (2*c^3*d^3 - 3*b*c^2*d^2*e + 3*b^2*c*d*e^2
 - 6*a*c^2*d*e^2 - b^3*e^3 + 3*a*b*c*e^3)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*
c)*c^2)
```

3.2193 $\int \frac{(d+ex)^2}{\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=99

$$\frac{4\left(ae^2-bde+cd^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{(d+ex)(-2ae+x(2cd-be)+bd)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

```
[Out] -(((d + e*x)*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*(a + b*x + c*x
^2))) + (4*(c*d^2 - b*d*e + a*e^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/
(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.0512849, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {722, 618, 206}

$$\frac{4\left(ae^2-bde+cd^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{(d+ex)(-2ae+x(2cd-be)+bd)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*x + c*x^2)^2,x]
```

```
[Out] -(((d + e*x)*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*(a + b*x + c*x
^2))) + (4*(c*d^2 - b*d*e + a*e^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/
(b^2 - 4*a*c)^(3/2)
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(a+bx+cx^2\right)^2}\,dx = -\frac{(d+ex)(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{\left(2\left(cd^2-bde+ae^2\right)\right)\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac}$$
$$= -\frac{(d+ex)(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\left(4\left(cd^2-bde+ae^2\right)\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{b^2-4ac}$$
$$= -\frac{(d+ex)(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{4\left(cd^2-bde+ae^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.134299, size = 128, normalized size = 1.29

$$\frac{abe^2-2ace(2d+ex)+b^2e^2x+bcd(d-2ex)+2c^2d^2x}{c\left(4ac-b^2\right)(a+x(b+cx))} + \frac{4\left(e(ae-bd)+cd^2\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*x + c*x^2)^2,x]
```

```
[Out] (a*b*e^2 + 2*c^2*d^2*x + b^2*e^2*x + b*c*d*(d - 2*e*x) - 2*a*c*e*(2*d + e*x
))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))) + (4*(c*d^2 + e*(-(b*d) + a*e))*Arc
Tan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [B] time = 0.155, size = 212, normalized size = 2.1

$$\frac{1}{cx^2+bx+a}\left(-\frac{\left(2\,ace^2-b^2e^2+2\,bcde-2\,c^2d^2\right)x}{c\left(4\,ac-b^2\right)} + \frac{abe^2-4\,acde+bcd^2}{c\left(4\,ac-b^2\right)}\right) + 4\,\frac{ae^2}{\left(4\,ac-b^2\right)^{3/2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right) - 4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^2+b*x+a)^2,x)
```

```
[Out] (-(2*a*c*e^2-b^2*e^2+2*b*c*d*e-2*c^2*d^2)/c/(4*a*c-b^2)*x+1/c*(a*b*e^2-4*a*
c*d*e+b*c*d^2)/(4*a*c-b^2))/(c*x^2+b*x+a)+4/(4*a*c-b^2)^(3/2)*arctan((2*c*x
+b)/(4*a*c-b^2)^(1/2))*a*e^2-4/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^
2)^(1/2))*b*d*e+4/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c*d
^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.83418, size = 1393, normalized size = 14.07

$$\left[-\frac{\left(b^3c-4\,abc^2\right)d^2-4\left(ab^2c-4\,a^2c^2\right)de+\left(ab^3-4\,a^2bc\right)e^2+2\left(ac^2d^2-abcde+a^2ce^2+\left(c^3d^2-bc^2de+ac^2e^2\right)x^2+\left(bc^2\right.\right.}{ab^4c-8\,a^2b^2c^2+16\,a^3c^3+\left(b^4c^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-((b^3*c - 4*a*b*c^2)*d^2 - 4*(a*b^2*c - 4*a^2*c^2)*d*e + (a*b^3 - 4*a^2*b
*c)*e^2 + 2*(a*c^2*d^2 - a*b*c*d*e + a^2*c*e^2 + (c^3*d^2 - b*c^2*d*e + a*c
^2*e^2)*x^2 + (b*c^2*d^2 - b^2*c*d*e + a*b*c*e^2)*x)*sqrt(b^2 - 4*a*c)*log(
(2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2
+ b*x + a)) + (2*(b^2*c^2 - 4*a*c^3)*d^2 - 2*(b^3*c - 4*a*b*c^2)*d*e + (b^4
 - 6*a*b^2*c + 8*a^2*c^2)*e^2)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (
b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c
^3)*x), -((b^3*c - 4*a*b*c^2)*d^2 - 4*(a*b^2*c - 4*a^2*c^2)*d*e + (a*b^3 -
4*a^2*b*c)*e^2 - 4*(a*c^2*d^2 - a*b*c*d*e + a^2*c*e^2 + (c^3*d^2 - b*c^2*d*
e + a*c^2*e^2)*x^2 + (b*c^2*d^2 - b^2*c*d*e + a*b*c*e^2)*x)*sqrt(-b^2 + 4*a
*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (2*(b^2*c^2 - 4
*a*c^3)*d^2 - 2*(b^3*c - 4*a*b*c^2)*d*e + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*e^2
)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^
2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x)]
```

Sympy [B] time = 1.73342, size = 517, normalized size = 5.22

$$-2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(ae^2-bde+cd^2\right)\log\left(x+\frac{-32a^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(ae^2-bde+cd^2\right)+16ab^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(ae^2-bde+cd\right.}{4ace^2-4bcde+4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**2+b*x+a)**2,x)
```

```
[Out] -2*sqrt(-1/(4*a*c - b**2)**3)*(a*e**2 - b*d*e + c*d**2)*log(x + (-32*a**2*c
**2*sqrt(-1/(4*a*c - b**2)**3)*(a*e**2 - b*d*e + c*d**2) + 16*a*b**2*c*sqrt
(-1/(4*a*c - b**2)**3)*(a*e**2 - b*d*e + c*d**2) + 2*a*b*e**2 - 2*b**4*sqrt
(-1/(4*a*c - b**2)**3)*(a*e**2 - b*d*e + c*d**2) - 2*b**2*d*e + 2*b*c*d**2)
/(4*a*c*e**2 - 4*b*c*d*e + 4*c**2*d**2)) + 2*sqrt(-1/(4*a*c - b**2)**3)*(a*
e**2 - b*d*e + c*d**2)*log(x + (32*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(a*
e**2 - b*d*e + c*d**2) - 16*a*b**2*c*sqrt(-1/(4*a*c - b**2)**3)*(a*e**2 - b
*d*e + c*d**2) + 2*a*b*e**2 + 2*b**4*sqrt(-1/(4*a*c - b**2)**3)*(a*e**2 - b
*d*e + c*d**2) - 2*b**2*d*e + 2*b*c*d**2)/(4*a*c*e**2 - 4*b*c*d*e + 4*c**2*
d**2)) - (-a*b*e**2 + 4*a*c*d*e - b*c*d**2 + x*(2*a*c*e**2 - b**2*e**2 + 2*
b*c*d*e - 2*c**2*d**2))/(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b**2*c**
2) + x*(4*a*b*c**2 - b**3*c))
```

Giac [A] time = 1.11813, size = 188, normalized size = 1.9

$$-\frac{4\left(cd^2-bde+ae^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}-\frac{2\,c^2d^2x-2\,bcdxe+bcd^2+b^2xe^2-2\,acxe^2-4\,acde+abe^2}{\left(b^2c-4\,ac^2\right)\left(cx^2+bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -4*(c*d^2 - b*d*e + a*e^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4
*a*c)*sqrt(-b^2 + 4*a*c)) - (2*c^2*d^2*x - 2*b*c*d*x*e + b*c*d^2 + b^2*x*e^
2 - 2*a*c*x*e^2 - 4*a*c*d*e + a*b*e^2)/((b^2*c - 4*a*c^2)*(c*x^2 + b*x + a)
)
```

3.2194 $\int \frac{d+ex}{(a+bx+cx^2)^2}\,dx$

Optimal. Leaf size=87

$$\frac{2(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{-2ae+x(2cd-be)+bd}{(b^2-4ac)(a+bx+cx^2)}$$

```
[Out] -((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (2*(
2*c*d - b*e)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.0383568, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {638, 618, 206}

$$\frac{2(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{-2ae+x(2cd-be)+bd}{(b^2-4ac)(a+bx+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*x + c*x^2)^2,x]
```

```
[Out] -((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (2*(
2*c*d - b*e)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex}{\left(a+bx+cx^2\right)^2}\,dx = -\frac{bd-2ae+(2cd-be)x}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{(2cd-be)\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac}$$

$$= -\frac{bd-2ae+(2cd-be)x}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{(2(2cd-be))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{b^2-4ac}$$

$$= -\frac{bd-2ae+(2cd-be)x}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{2(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.0823219, size = 88, normalized size = 1.01

$$\frac{\frac{2(be-2cd)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{2ae-bd+bex-2cdx}{a+x(b+cx)}}{b^2-4ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*x + c*x^2)^2,x]
```

```
[Out] ((-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(a + x*(b + c*x)) + (2*(-2*c*d + b*e)*A
rcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c)
```

Maple [A] time = 0.154, size = 118, normalized size = 1.4

$$\frac{bd-2\,ae+(-be+2\,cd)\,x}{\left(4\,ac-b^2\right)\left(cx^2+bx+a\right)} - 2\,\frac{be}{\left(4\,ac-b^2\right)^{3/2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right) + 4\,\frac{cd}{\left(4\,ac-b^2\right)^{3/2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^2+b*x+a)^2,x)
```

```
[Out] (b*d-2*a*e+(-b*e+2*c*d)*x)/(4*a*c-b^2)/(c*x^2+b*x+a)-2/(4*a*c-b^2)^(3/2)*ar
ctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*e+4/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/
(4*a*c-b^2)^(1/2))*c*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.90407, size = 988, normalized size = 11.36

$$\left[\frac{\left(2\,acd - abe + \left(2\,c^2d - bce\right)x^2 + \left(2\,bcd - b^2e\right)x\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right) - \left(b^3-4\,abc\right)d + 2\,\left(ab^2\right.}{ab^4 - 8\,a^2b^2c + 16\,a^3c^2 + \left(b^4c - 8\,ab^2c^2 + 16\,a^2c^3\right)x^2 + \left(b^5 - 8\,ab^3c + 16\,a^2bc\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [((2*a*c*d - a*b*e + (2*c^2*d - b*c*e)*x^2 + (2*b*c*d - b^2*e)*x)*sqrt(b^2
- 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x
+ b))/(c*x^2 + b*x + a)) - (b^3 - 4*a*b*c)*d + 2*(a*b^2 - 4*a^2*c)*e - (2*(
b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^
2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^
2)*x), (2*(2*a*c*d - a*b*e + (2*c^2*d - b*c*e)*x^2 + (2*b*c*d - b^2*e)*x)*s
qrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (
b^3 - 4*a*b*c)*d + 2*(a*b^2 - 4*a^2*c)*e - (2*(b^2*c - 4*a*c^2)*d - (b^3 -
4*a*b*c)*e)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 1
6*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)]
```

Sympy [B] time = 1.10025, size = 359, normalized size = 4.13

$$\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(be-2cd\right)\log\left(x + \frac{-16a^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(be-2cd\right) + 8ab^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(be-2cd\right) - b^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(be-2\right.}{2bce-4c^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**2+b*x+a)**2,x)
```

```
[Out] sqrt(-1/(4*a*c - b**2)**3)*(b*e - 2*c*d)*log(x + (-16*a**2*c**2*sqrt(-1/(4*
a*c - b**2)**3)*(b*e - 2*c*d) + 8*a*b**2*c*sqrt(-1/(4*a*c - b**2)**3)*(b*e
- 2*c*d) - b**4*sqrt(-1/(4*a*c - b**2)**3)*(b*e - 2*c*d) + b**2*e - 2*b*c*d
)/(2*b*c*e - 4*c**2*d)) - sqrt(-1/(4*a*c - b**2)**3)*(b*e - 2*c*d)*log(x +
(16*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(b*e - 2*c*d) - 8*a*b**2*c*sqrt(-1
/(4*a*c - b**2)**3)*(b*e - 2*c*d) + b**4*sqrt(-1/(4*a*c - b**2)**3)*(b*e -
2*c*d) + b**2*e - 2*b*c*d)/(2*b*c*e - 4*c**2*d)) - (2*a*e - b*d + x*(b*e -
2*c*d))/(4*a**2*c - a*b**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))
```

Giac [A] time = 1.14903, size = 134, normalized size = 1.54

$$-\frac{2\,(2\,cd-be)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}} - \frac{2\,cdx - bxe + bd - 2\,ae}{\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2*(2*c*d - b*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt
(-b^2 + 4*a*c)) - (2*c*d*x - b*x*e + b*d - 2*a*e)/((c*x^2 + b*x + a)*(b^2 -
 4*a*c))
```

3.2195 $\int \frac{1}{(a+bx+cx^2)^2}\,dx$

Optimal. Leaf size=66

$$\frac{4c\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

```
[Out] -((b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x)
/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.0278588, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {614, 618, 206}

$$\frac{4c\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(-2),x]
```

```
[Out] -((b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x)
/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx = -\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{(2c)\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac}$$

$$= -\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{(4c)\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{b^2-4ac}$$

$$= -\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{4c\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.0783541, size = 70, normalized size = 1.06

$$-\frac{\frac{4c\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{b+2cx}{a+x(b+cx)}}{b^2-4ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(-2),x]
```

```
[Out] -(((b + 2*c*x)/(a + x*(b + c*x)) + (4*c*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*
c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c))
```

Maple [A] time = 0.151, size = 68, normalized size = 1.

$$\frac{2\,cx+b}{\left(4\,ac-b^2\right)\left(cx^2+bx+a\right)} + 4\,\frac{c}{\left(4\,ac-b^2\right)^{3/2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^2,x)
```

```
[Out] (2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/
(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.87347, size = 745, normalized size = 11.29

$$\left[-\frac{b^3-4\,abc+2\left(c^2x^2+bcx+ac\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+2\left(b^2c-4\,ac^2\right)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)x^2+\left(b^5-8\,ab^3c+16\,a^2bc^2\right)x}, -\frac{b^3-4\,abc-4\left(c^2\right.}{ab^4-8\,a^2b^2c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-(b^3 - 4*a*b*c + 2*(c^2*x^2 + b*c*x + a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x
^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x +
a)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8
*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), -(b^3 -
 4*a*b*c - 4*(c^2*x^2 + b*c*x + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 +
 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 - 8*a^2*
b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^
3*c + 16*a^2*b*c^2)*x)]
```

Sympy [B] time = 0.709606, size = 265, normalized size = 4.02

$$-2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x+\frac{-32a^2c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+16ab^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-2b^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+2bc}{4c^2}\right)+2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**2,x)
```

```
[Out] -2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**2*c**3*sqrt(-1/(4*a*c - b**
2)**3) + 16*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - 2*b**4*c*sqrt(-1/(4*a*
c - b**2)**3) + 2*b*c)/(4*c**2)) + 2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (
32*a**2*c**3*sqrt(-1/(4*a*c - b**2)**3) - 16*a*b**2*c**2*sqrt(-1/(4*a*c - b
**2)**3) + 2*b**4*c*sqrt(-1/(4*a*c - b**2)**3) + 2*b*c)/(4*c**2)) + (b + 2*
c*x)/(4*a**2*c - a*b**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))
```

Giac [A] time = 1.10226, size = 103, normalized size = 1.56

$$-\frac{4\,c\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}-\frac{2\,cx+b}{\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -4*c*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*
c)) - (2*c*x + b)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

3.2196 $\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=224

$$\frac{(2cd-be)\left(-2ce(bd-3ae)-b^2e^2+2c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}\left(ae^2-bde+cd^2\right)^2}-\frac{2ace+b^2(-e)+cx(2cd-be)+bcd}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)\left(ae^2-bde+cd^2\right)}-\frac{e^3\log\left(a+b\right.}{2\left(ae^2-bde\right.}$$

```
[Out] -((b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)/((b^2 - 4*a*c)*(c*d^2 - b*d
*e + a*e^2)*(a + b*x + c*x^2))) + ((2*c*d - b*e)*(2*c^2*d^2 - b^2*e^2 - 2*c
*e*(b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3
/2)*(c*d^2 - b*d*e + a*e^2)^2) + (e^3*Log[d + e*x])/(c*d^2 - b*d*e + a*e^2)
^2 - (e^3*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)^2)
```

Rubi [A] time = 0.38123, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {740, 800, 634, 618, 206, 628}

$$\frac{(2cd-be)\left(-2ce(bd-3ae)-b^2e^2+2c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}\left(ae^2-bde+cd^2\right)^2}-\frac{2ace+b^2(-e)+cx(2cd-be)+bcd}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)\left(ae^2-bde+cd^2\right)}-\frac{e^3\log\left(a+b\right.}{2\left(ae^2-bde\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + b*x + c*x^2)^2),x]
```


```
[Out] -((b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)/((b^2 - 4*a*c)*(c*d^2 - b*d
*e + a*e^2)*(a + b*x + c*x^2))) + ((2*c*d - b*e)*(2*c^2*d^2 - b^2*e^2 - 2*c
*e*(b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3
/2)*(c*d^2 - b*d*e + a*e^2)^2) + (e^3*Log[d + e*x])/(c*d^2 - b*d*e + a*e^2)
^2 - (e^3*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)^2)
```


Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^2}\,dx = -\frac{bcd-b^2e+2ace+c(2cd-be)x}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)} - \frac{\int \frac{2c^2d^2-b^2e^2-ce(bd-4ae)+ce(2cd-be)x}{(d+ex)\left(a+bx+cx^2\right)}\,dx}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)} - \frac{\int \left(-\frac{\left(b^2-4ac\right)e^4}{\left(cd^2-bde+ae^2\right)(d+ex)} + \frac{2c^3d^3+b^3e^3-5abc}{\left(cd^2-\right.}\right.}{\left(b^2-4ac\right)\left(cd^2-\right.}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)} + \frac{e^3\log(d+ex)}{\left(cd^2-bde+ae^2\right)^2} - \frac{\int \frac{2c^3d^3+b^3e^3-5abc}{}}{\left(b^2-4a\right.}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)} + \frac{e^3\log(d+ex)}{\left(cd^2-bde+ae^2\right)^2} - \frac{e^3\int \frac{b+2cx}{a+bx+cx^2}}{2\left(cd^2-bde+a\right.}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)} + \frac{e^3\log(d+ex)}{\left(cd^2-bde+ae^2\right)^2} - \frac{e^3\log\left(a+bx+\right.}{2\left(cd^2-bde+\right.}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)} + \frac{(2cd-be)\left(2c^2d^2-b^2e^2-2ce(bd-3\right.}{\left(b^2-4ac\right)^{3/2}\left(cd^2-bde\right.}$$

Mathematica [A] time = 0.3572, size = 223, normalized size = 1.

$$\frac{(be-2cd)\left(2ce(bd-3ae)+b^2e^2-2c^2d^2\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}\left(e(ae-bd)+cd^2\right)^2} + \frac{2c(ae+cdx)+b^2(-e)+bc(d-ex)}{\left(b^2-4ac\right)(a+x(b+cx))\left(e(bd-ae)-cd^2\right)} + \frac{e^3\log(d}{\left(e(ae-bd)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + b*x + c*x^2)^2),x]
```

```
[Out] (-(b^2*e) + 2*c*(a*e + c*d*x) + b*c*(d - e*x))/((b^2 - 4*a*c)*(-(c*d^2) + e
*(b*d - a*e))*(a + x*(b + c*x))) + ((-2*c*d + b*e)*(-2*c^2*d^2 + b^2*e^2 +
```

```
2*c*e*(b*d - 3*a*e))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/((-b^2 + 4*a*c
)^(3/2)*(c*d^2 + e*(-(b*d) + a*e))^2) + (e^3*Log[d + e*x])/(c*d^2 + e*(-(b*
d) + a*e))^2 - (e^3*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e))^2)
```

Maple [B] time = 0.193, size = 1037, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+b*x+a)^2,x)
```

```
[Out] -1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)*c/(4*a*c-b^2)*x*a*b*e^3+2/(a*e^2-b*d
*e+c*d^2)^2/(c*x^2+b*x+a)*c^2/(4*a*c-b^2)*x*a*d*e^2+1/(a*e^2-b*d*e+c*d^2)^2
/(c*x^2+b*x+a)*c/(4*a*c-b^2)*x*b^2*d*e^2-3/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x
+a)*c^2/(4*a*c-b^2)*x*b*d^2*e+2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)*c^3/(4*
a*c-b^2)*x*d^3+2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*c*e^3-
1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^2*e^3-1/(a*e^2-b*d*e+
c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b*c*d*e^2+2/(a*e^2-b*d*e+c*d^2)^2/(c*x
^2+b*x+a)/(4*a*c-b^2)*a*c^2*d^2*e+1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*
a*c-b^2)*b^3*d*e^2-2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*b^2*c*
d^2*e+1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*b*c^2*d^3-2/(a*e^2-
b*d*e+c*d^2)^2/(4*a*c-b^2)*c*ln(c*x^2+b*x+a)*a*e^3+1/2/(a*e^2-b*d*e+c*d^2)^
2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^2*e^3-6/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(
3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*c*e^3+12/(a*e^2-b*d*e+c*d^2)^2
/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^2*a*d*e^2+1/(a*e^2
-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*e
^3-6/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2))*b*c^2*d^2*e+4/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b
)/(4*a*c-b^2)^(1/2))*c^3*d^3+e^3*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 159.644, size = 4417, normalized size = 19.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(b^3*c^2 - 4*a*b*c^3)*d^3 - 4*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d^
2*e + 2*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d*e^2 - 2*(a*b^4 - 6*a^2*b^2*c + 8*
a^3*c^2)*e^3 - (4*a*c^3*d^3 - 6*a*b*c^2*d^2*e + 12*a^2*c^2*d*e^2 + (a*b^3 -
```

```
 6*a^2*b*c)*e^3 + (4*c^4*d^3 - 6*b*c^3*d^2*e + 12*a*c^3*d*e^2 + (b^3*c - 6*
a*b*c^2)*e^3)*x^2 + (4*b*c^3*d^3 - 6*b^2*c^2*d^2*e + 12*a*b*c^2*d*e^2 + (b^
4 - 6*a*b^2*c)*e^3)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2
*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(2*(b^2*c^3 -
4*a*c^4)*d^3 - 3*(b^3*c^2 - 4*a*b*c^3)*d^2*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2
*c^3)*d*e^2 - (a*b^3*c - 4*a^2*b*c^2)*e^3)*x + ((b^4*c - 8*a*b^2*c^2 + 16*a
^2*c^3)*e^3*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*e^3*x + (a*b^4 - 8*a^2*b
^2*c + 16*a^3*c^2)*e^3)*log(c*x^2 + b*x + a) - 2*((b^4*c - 8*a*b^2*c^2 + 16
*a^2*c^3)*e^3*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*e^3*x + (a*b^4 - 8*a^2
*b^2*c + 16*a^3*c^2)*e^3)*log(e*x + d))/((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^
3*c^4)*d^4 - 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3*e + (a*b^6 - 6*
a^2*b^4*c + 32*a^4*c^3)*d^2*e^2 - 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*
d*e^3 + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*e^4 + ((b^4*c^3 - 8*a*b^2*c^4
+ 16*a^2*c^5)*d^4 - 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^3*e + (b^6*c
 - 6*a*b^4*c^2 + 32*a^3*c^4)*d^2*e^2 - 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*
b*c^3)*d*e^3 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*e^4)*x^2 + ((b^5*c^
2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^4 - 2*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c
^3)*d^3*e + (b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*d^2*e^2 - 2*(a*b^6 - 8*a^2*b^4
*c + 16*a^3*b^2*c^2)*d*e^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*e^4)*x)
, -1/2*(2*(b^3*c^2 - 4*a*b*c^3)*d^3 - 4*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d
^2*e + 2*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d*e^2 - 2*(a*b^4 - 6*a^2*b^2*c + 8
*a^3*c^2)*e^3 - 2*(4*a*c^3*d^3 - 6*a*b*c^2*d^2*e + 12*a^2*c^2*d*e^2 + (a*b^
3 - 6*a^2*b*c)*e^3 + (4*c^4*d^3 - 6*b*c^3*d^2*e + 12*a*c^3*d*e^2 + (b^3*c -
 6*a*b*c^2)*e^3)*x^2 + (4*b*c^3*d^3 - 6*b^2*c^2*d^2*e + 12*a*b*c^2*d*e^2 +
(b^4 - 6*a*b^2*c)*e^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*
c*x + b)/(b^2 - 4*a*c)) + 2*(2*(b^2*c^3 - 4*a*c^4)*d^3 - 3*(b^3*c^2 - 4*a*b
*c^3)*d^2*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d*e^2 - (a*b^3*c - 4*a^2*b*
c^2)*e^3)*x + ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^3*x^2 + (b^5 - 8*a*b^3*
c + 16*a^2*b*c^2)*e^3*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*e^3)*log(c*x^2
 + b*x + a) - 2*((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^3*x^2 + (b^5 - 8*a*b^
3*c + 16*a^2*b*c^2)*e^3*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*e^3)*log(e*x
 + d))/((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 - 2*(a*b^5*c - 8*a^2*b
^3*c^2 + 16*a^3*b*c^3)*d^3*e + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2*e^2 -
 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d*e^3 + (a^3*b^4 - 8*a^4*b^2*c +
16*a^5*c^2)*e^4 + ((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d^4 - 2*(b^5*c^2 -
8*a*b^3*c^3 + 16*a^2*b*c^4)*d^3*e + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*d^2*
e^2 - 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d*e^3 + (a^2*b^4*c - 8*a^3
*b^2*c^2 + 16*a^4*c^3)*e^4)*x^2 + ((b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d
^4 - 2*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*d^3*e + (b^7 - 6*a*b^5*c + 32
*a^3*b*c^3)*d^2*e^2 - 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*d*e^3 + (a^2
*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*e^4)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.10915, size = 635, normalized size = 2.83

$$-\frac{e^3 \log\left(cx^2+bx+a\right)}{2\left(c^2d^4-2\,bcd^3e+b^2d^2e^2+2\,acd^2e^2-2\,abde^3+a^2e^4\right)}+\frac{e^4\log\left(|xe+d|\right)}{c^2d^4e-2\,bcd^3e^2+b^2d^2e^3+2\,acd^2e^3-2\,abde^4+a^2e^5}-\frac{}{\left(b^2c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*e^3*log(c*x^2 + b*x + a)/(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*
d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4) + e^4*log(abs(x*e + d))/(c^2*d^4*e - 2*b*c
*d^3*e^2 + b^2*d^2*e^3 + 2*a*c*d^2*e^3 - 2*a*b*d*e^4 + a^2*e^5) - (4*c^3*d^
3 - 6*b*c^2*d^2*e + 12*a*c^2*d*e^2 + b^3*e^3 - 6*a*b*c*e^3)*arctan((2*c*x +
 b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b
*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*
d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4)*sqrt(-b^2 + 4*a*c)) -
(b*c^2*d^3 - 2*b^2*c*d^2*e + 2*a*c^2*d^2*e + b^3*d*e^2 - a*b*c*d*e^2 - a*b^
2*e^3 + 2*a^2*c*e^3 + (2*c^3*d^3 - 3*b*c^2*d^2*e + b^2*c*d*e^2 + 2*a*c^2*d*
e^2 - a*b*c*e^3)*x)/((c*d^2 - b*d*e + a*e^2)^2*(c*x^2 + b*x + a)*(b^2 - 4*a
*c))
```

3.2197 $\int \frac{1}{(d+ex)^2(a+bx+cx^2)^2}\,dx$

Optimal. Leaf size=344

$$-\frac{2e\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)}{\left(b^2-4ac\right)(d+ex)\left(ae^2-bde+cd^2\right)^2}+\frac{2\left(2b^2ce^3(3ae+bd)-4c^3d^2e(bd-3ae)-6ac^2e^3(ae+2bd)-b^4e^4+2c^4d^4\right)\text{t}}{\left(b^2-4ac\right)^{3/2}\left(ae^2-bde+cd^2\right)^3}$$

```
[Out] (-2*e*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e)))/((b^2 - 4*a*c)*(c*d^2 - b*d*
e + a*e^2)^2*(d + e*x)) - (b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)/((b
^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*(d + e*x)*(a + b*x + c*x^2)) + (2*(2*c^
4*d^4 - b^4*e^4 - 4*c^3*d^2*e*(b*d - 3*a*e) - 6*a*c^2*e^3*(2*b*d + a*e) + 2
*b^2*c*e^3*(b*d + 3*a*e))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4
*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)^3) + (2*e^3*(2*c*d - b*e)*Log[d + e*x])
/(c*d^2 - b*d*e + a*e^2)^3 - (e^3*(2*c*d - b*e)*Log[a + b*x + c*x^2])/(c*d^
2 - b*d*e + a*e^2)^3
```

Rubi [A] time = 0.652194, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {740, 800, 634, 618, 206, 628}

$$-\frac{2e\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)}{\left(b^2-4ac\right)(d+ex)\left(ae^2-bde+cd^2\right)^2}+\frac{2\left(2b^2ce^3(3ae+bd)-4c^3d^2e(bd-3ae)-6ac^2e^3(ae+2bd)-b^4e^4+2c^4d^4\right)\text{t}}{\left(b^2-4ac\right)^{3/2}\left(ae^2-bde+cd^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + b*x + c*x^2)^2),x]
```

```
[Out] (-2*e*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e)))/((b^2 - 4*a*c)*(c*d^2 - b*d*
e + a*e^2)^2*(d + e*x)) - (b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)/((b
^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*(d + e*x)*(a + b*x + c*x^2)) + (2*(2*c^
4*d^4 - b^4*e^4 - 4*c^3*d^2*e*(b*d - 3*a*e) - 6*a*c^2*e^3*(2*b*d + a*e) + 2
*b^2*c*e^3*(b*d + 3*a*e))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4
*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)^3) + (2*e^3*(2*c*d - b*e)*Log[d + e*x])
/(c*d^2 - b*d*e + a*e^2)^3 - (e^3*(2*c*d - b*e)*Log[a + b*x + c*x^2])/(c*d^
2 - b*d*e + a*e^2)^3
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
```

```
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+bx+cx^2\right)^2}\,dx = -\frac{bcd-b^2e+2ace+c(2cd-be)x}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)\left(a+bx+cx^2\right)} - \frac{\int \frac{2\left(c^2d^2-b^2e^2+3ace^2\right)+2ce(2cd-be)x}{(d+ex)^2\left(a+bx+cx^2\right)}\,d}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)\left(a+bx+cx^2\right)} - \frac{\int\left(\frac{2e^2\left(-c^2d^2-b^2e^2+ce(bd+3ae)\right)}{\left(cd^2-bde+ae^2\right)(d+ex)^2}+\frac{2(}{(c}\right.}{}$$

$$= -\frac{2e\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{bcd-b^2e+2ace+c(2cd-be)x}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)\left(a+bx\right.}$$

$$= -\frac{2e\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{bcd-b^2e+2ace+c(2cd-be)x}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)\left(a+bx\right.}$$

$$= -\frac{2e\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{bcd-b^2e+2ace+c(2cd-be)x}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)\left(a+bx\right.}$$

$$= -\frac{2e\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{bcd-b^2e+2ace+c(2cd-be)x}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)\left(a+bx\right.}$$

Mathematica [A] time = 0.74381, size = 339, normalized size = 0.99

$$\frac{bc\left(3ae^2-cd(d-2ex)\right)-2c^2\left(ae(2d-ex)+cd^2x\right)+b^2ce(2d-ex)+b^3\left(-e^2\right)}{\left(b^2-4ac\right)(a+x(b+cx))\left(e(ae-bd)+cd^2\right)^2} - \frac{2\left(2b^2ce^3(3ae+bd)-4c^3d^2e(bd-3ae)-\right.}{\left(4ac-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + b*x + c*x^2)^2),x]
```

```
[Out] -(e^3/((c*d^2 + e*(-(b*d) + a*e))^2*(d + e*x))) + (-(b^3*e^2) + b^2*c*e*(2*
d - e*x) + b*c*(3*a*e^2 - c*d*(d - 2*e*x)) - 2*c^2*(c*d^2*x + a*e*(2*d - e*
x)))/((b^2 - 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))^2*(a + x*(b + c*x))) - (2*(2
*c^4*d^4 - b^4*e^4 - 4*c^3*d^2*e*(b*d - 3*a*e) - 6*a*c^2*e^3*(2*b*d + a*e)
+ 2*b^2*c*e^3*(b*d + 3*a*e))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/((-b^2
 + 4*a*c)^(3/2)*(-(c*d^2) + e*(b*d - a*e))^3) - (2*e^3*(-2*c*d + b*e)*Log[d
 + e*x])/(c*d^2 + e*(-(b*d) + a*e))^3 + (e^3*(-2*c*d + b*e)*Log[a + x*(b +
c*x)])/(c*d^2 + e*(-(b*d) + a*e))^3
```

Maple [B] time = 0.174, size = 1617, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^2+b*x+a)^2,x)
```

```
[Out] -2*e^4/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*b+1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x
+a)*c/(4*a*c-b^2)*x*a*b^2*e^4-6/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-
b^2)*a*b*c^2*d^2*e^2-4/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^3/(4*a*c-b^2)*
x*b*d^3*e-1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c/(4*a*c-b^2)*x*b^3*d*e^3+1
/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^2*c*d*e^3-24/(a*e^2-b*
d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*c^2*
d*e^3+3/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^2/(4*a*c-b^2)*x*b^2*d^2*e^2+2
/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^4/(4*a*c-b^2)*x*d^4-e^3/(a*e^2-b*d*e
+c*d^2)^2/(e*x+d)-1/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^3*e
^4+4/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2))*c^4*d^4+4*e^3/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*c*d-2/(a*e^2-b*d*e+c*d^
2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4*e^4+4/(a*e^2
-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*c^2*d*e^3+4/(a*e^2-b*d*e+c*d^
2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a*c^3*d^3*e+3/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b
*x+a)/(4*a*c-b^2)*b^3*c*d^2*e^2-3/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*
c-b^2)*b^2*c^2*d^3*e+4/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*c*ln(c*x^2+b*x+a)*
a*b*e^4-8/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*c^2*ln(c*x^2+b*x+a)*a*d*e^3+2/(
a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*c*ln(c*x^2+b*x+a)*b^2*d*e^3+12/(a*e^2-b*d*
e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*c*e^
4+24/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2))*c^3*a*d^2*e^2+4/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x
+b)/(4*a*c-b^2)^(1/2))*b^3*c*d*e^3-8/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2
)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c^3*d^3*e-2/(a*e^2-b*d*e+c*d^2)^3/(
c*x^2+b*x+a)*c^2/(4*a*c-b^2)*x*a^2*e^4-3/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a
)/(4*a*c-b^2)*a^2*b*c*e^4+1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)
*a*b^3*e^4-1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*b^4*d*e^3+1/(a
*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*d^4*b*c^3-12/(a*e^2-b*d*e+c*d
^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^2*a^2*e^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18058, size = 1220, normalized size = 3.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 2*(2*c^4*d^4*e^2 - 4*b*c^3*d^3*e^3 + 12*a*c^3*d^2*e^4 + 2*b^3*c*d*e^5 - 12*
a*b*c^2*d*e^5 - b^4*e^6 + 6*a*b^2*c*e^6 - 6*a^2*c^2*e^6)*arctan(-(2*c*d - 2
*c*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*a*e^2/(x*e + d))*e^(-1)/sqrt
(-b^2 + 4*a*c))*e^(-2)/((b^2*c^3*d^6 - 4*a*c^4*d^6 - 3*b^3*c^2*d^5*e + 12*a
*b*c^3*d^5*e + 3*b^4*c*d^4*e^2 - 9*a*b^2*c^2*d^4*e^2 - 12*a^2*c^3*d^4*e^2 -
 b^5*d^3*e^3 - 2*a*b^3*c*d^3*e^3 + 24*a^2*b*c^2*d^3*e^3 + 3*a*b^4*d^2*e^4 -
 9*a^2*b^2*c*d^2*e^4 - 12*a^3*c^2*d^2*e^4 - 3*a^2*b^3*d*e^5 + 12*a^3*b*c*d*
e^5 + a^3*b^2*e^6 - 4*a^4*c*e^6)*sqrt(-b^2 + 4*a*c)) - (2*c*d*e^3 - b*e^4)*
log(-c + 2*c*d/(x*e + d) - c*d^2/(x*e + d)^2 - b*e/(x*e + d) + b*d*e/(x*e +
 d)^2 - a*e^2/(x*e + d)^2)/(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 + 3*a
*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a^2*c*d^
2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6) - e^7/((c^2*d^4*e^4 - 2*b*c*d^3*e^5 + b^2*
d^2*e^6 + 2*a*c*d^2*e^6 - 2*a*b*d*e^7 + a^2*e^8)*(x*e + d)) - ((2*c^4*d^3*e
 - 3*b*c^3*d^2*e^2 + 3*b^2*c^2*d*e^3 - 6*a*c^3*d*e^3 - b^3*c*e^4 + 3*a*b*c^
2*e^4)/(c*d^2 - b*d*e + a*e^2) - (2*c^4*d^4*e^2 - 4*b*c^3*d^3*e^3 + 6*b^2*c
^2*d^2*e^4 - 12*a*c^3*d^2*e^4 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5 + b^4*e^6
- 4*a*b^2*c*e^6 + 2*a^2*c^2*e^6)*e^(-1)/((c*d^2 - b*d*e + a*e^2)*(x*e + d))
)/((c*d^2 - b*d*e + a*e^2)^2*(b^2 - 4*a*c)*(c - 2*c*d/(x*e + d) + c*d^2/(x*
e + d)^2 + b*e/(x*e + d) - b*d*e/(x*e + d)^2 + a*e^2/(x*e + d)^2))
```

3.2198 $\int \frac{1}{(d+ex)^3\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=485

$$\frac{(2cd-be)\left(-2c^2e^2\left(15a^2e^2+10abde+b^2d^2\right)+4b^2ce^3(5ae+bd)-4c^3d^2e(bd-5ae)-3b^4e^4+2c^4d^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}\left(ae^2-bde+cd^2\right)^4}$$

```
[Out] -(e*(4*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(b*d + 2*a*e)))/(2*(b^2 - 4*a*c)*(c*d^2
- b*d*e + a*e^2)^2*(d + e*x)^2) - (e*(2*c*d - b*e)*(c^2*d^2 + 3*b^2*e^2 - c
*e*(b*d + 11*a*e)))/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)^3*(d + e*x)) - (
b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)/((b^2 - 4*a*c)*(c*d^2 - b*d*e
+ a*e^2)*(d + e*x)^2*(a + b*x + c*x^2)) + ((2*c*d - b*e)*(2*c^4*d^4 - 3*b^4
*e^4 - 4*c^3*d^2*e*(b*d - 5*a*e) + 4*b^2*c*e^3*(b*d + 5*a*e) - 2*c^2*e^2*(b
^2*d^2 + 10*a*b*d*e + 15*a^2*e^2))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/
((b^2 - 4*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)^4) + (e^3*(10*c^2*d^2 + 3*b^2*
e^2 - 2*c*e*(5*b*d + a*e))*Log[d + e*x])/(c*d^2 - b*d*e + a*e^2)^4 - (e^3*(
10*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(5*b*d + a*e))*Log[a + b*x + c*x^2])/(2*(c*d
^2 - b*d*e + a*e^2)^4)
```

Rubi [A] time = 1.05679, antiderivative size = 485, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {740, 800, 634, 618, 206, 628}

$$\frac{(2cd-be)\left(-2c^2e^2\left(15a^2e^2+10abde+b^2d^2\right)+4b^2ce^3(5ae+bd)-4c^3d^2e(bd-5ae)-3b^4e^4+2c^4d^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}\left(ae^2-bde+cd^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + b*x + c*x^2)^2),x]
```


```
[Out] -(e*(4*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(b*d + 2*a*e)))/(2*(b^2 - 4*a*c)*(c*d^2
- b*d*e + a*e^2)^2*(d + e*x)^2) - (e*(2*c*d - b*e)*(c^2*d^2 + 3*b^2*e^2 - c
*e*(b*d + 11*a*e)))/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)^3*(d + e*x)) - (
b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)/((b^2 - 4*a*c)*(c*d^2 - b*d*e
+ a*e^2)*(d + e*x)^2*(a + b*x + c*x^2)) + ((2*c*d - b*e)*(2*c^4*d^4 - 3*b^4
*e^4 - 4*c^3*d^2*e*(b*d - 5*a*e) + 4*b^2*c*e^3*(b*d + 5*a*e) - 2*c^2*e^2*(b
^2*d^2 + 10*a*b*d*e + 15*a^2*e^2))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/
((b^2 - 4*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)^4) + (e^3*(10*c^2*d^2 + 3*b^2*
e^2 - 2*c*e*(5*b*d + a*e))*Log[d + e*x])/(c*d^2 - b*d*e + a*e^2)^4 - (e^3*(
10*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(5*b*d + a*e))*Log[a + b*x + c*x^2])/(2*(c*d
^2 - b*d*e + a*e^2)^4)
```


Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(a+bx+cx^2\right)^2}\,dx = -\frac{bcd-b^2e+2ace+c(2cd-be)x}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)^2\left(a+bx+cx^2\right)} - \frac{\int \frac{2c^2d^2-3b^2e^2+ce(bd+8ae)+3ce(2cd-b}{(d+ex)^3\left(a+bx+cx^2\right)}}{\left(b^2-4ac\right)\left(cd^2-bde+ae\right.}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)^2\left(a+bx+cx^2\right)} - \frac{\int\left(\frac{e^2\left(-4c^2d^2-3b^2e^2+4ce(bd+2ae)\right)}{\left(cd^2-bde+ae^2\right)(d+ex)^3}+\right.}{}$$

$$= -\frac{e\left(4c^2d^2+3b^2e^2-4ce(bd+2ae)\right)}{2\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)^2(d+ex)^2} - \frac{e(2cd-be)\left(c^2d^2+3b^2e^2-ce(bd+11ae)\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)^3(d+ex)}$$

$$= -\frac{e\left(4c^2d^2+3b^2e^2-4ce(bd+2ae)\right)}{2\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)^2(d+ex)^2} - \frac{e(2cd-be)\left(c^2d^2+3b^2e^2-ce(bd+11ae)\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)^3(d+ex)}$$

$$= -\frac{e\left(4c^2d^2+3b^2e^2-4ce(bd+2ae)\right)}{2\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)^2(d+ex)^2} - \frac{e(2cd-be)\left(c^2d^2+3b^2e^2-ce(bd+11ae)\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)^3(d+ex)}$$

$$= -\frac{e\left(4c^2d^2+3b^2e^2-4ce(bd+2ae)\right)}{2\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)^2(d+ex)^2} - \frac{e(2cd-be)\left(c^2d^2+3b^2e^2-ce(bd+11ae)\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)^3(d+ex)}$$

Mathematica [A] time = 1.28918, size = 489, normalized size = 1.01

$$\frac{2c^2\left(-a^2e^3+3acde(d-ex)+c^2d^3x\right)+b^2ce\left(4ae^2-3cd(d-ex)\right)+bc^2\left(3ae^2(ex-3d)+cd^2(d-3ex)\right)+b^3ce^2(3d-ex)}{\left(b^2-4ac\right)(a+x(b+cx))\left(e(bd-ae)-cd^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a + b*x + c*x^2)^2),x]
```

```
[Out] -e^3/(2*(c*d^2 + e*(-(b*d) + a*e))^2*(d + e*x)^2) + (2*e^3*(-2*c*d + b*e))/
((c*d^2 + e*(-(b*d) + a*e))^3*(d + e*x)) + (-(b^4*e^3) + b^3*c*e^2*(3*d - e
*x) + b^2*c*e*(4*a*e^2 - 3*c*d*(d - e*x)) + 2*c^2*(-(a^2*e^3) + c^2*d^3*x +
 3*a*c*d*e*(d - e*x)) + b*c^2*(c*d^2*(d - 3*e*x) + 3*a*e^2*(-3*d + e*x)))/(
(b^2 - 4*a*c)*(-(c*d^2) + e*(b*d - a*e))^3*(a + x*(b + c*x))) + ((-2*c*d +
b*e)*(-2*c^4*d^4 + 3*b^4*e^4 + 4*c^3*d^2*e*(b*d - 5*a*e) - 4*b^2*c*e^3*(b*d
 + 5*a*e) + 2*c^2*e^2*(b^2*d^2 + 10*a*b*d*e + 15*a^2*e^2))*ArcTan[(b + 2*c*
x)/Sqrt[-b^2 + 4*a*c]])/((-b^2 + 4*a*c)^(3/2)*(c*d^2 + e*(-(b*d) + a*e))^4)
 + (e^3*(10*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(5*b*d + a*e))*Log[d + e*x])/(c*d^2
 + e*(-(b*d) + a*e))^4 - (e^3*(10*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(5*b*d + a*e)
)*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e))^4)
```

Maple [B] time = 0.177, size = 2554, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*x^2+b*x+a)^2,x)
```

```
[Out] 6/(a*e^2-b*d*e+c*d^2)^4/(c*x^2+b*x+a)/(4*a*c-b^2)*a*c^4*d^4*e-20/(a*e^2-b*d
*e+c*d^2)^4/(4*a*c-b^2)*c^3*ln(c*x^2+b*x+a)*a*d^2*e^3-5/(a*e^2-b*d*e+c*d^2)
^4/(4*a*c-b^2)*c*ln(c*x^2+b*x+a)*b^3*d*e^4+5/(a*e^2-b*d*e+c*d^2)^4/(4*a*c-b
^2)*c^2*ln(c*x^2+b*x+a)*b^2*d^2*e^3+10/(a*e^2-b*d*e+c*d^2)^4/(4*a*c-b^2)^(3
/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*c^2*d^2*e^3-10/(a*e^2-b*d*e+c*d
^2)^4/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c^4*d^4*e-10/
(a*e^2-b*d*e+c*d^2)^4/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
*b^4*c*d*e^4-60/(a*e^2-b*d*e+c*d^2)^4/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4
*a*c-b^2)^(1/2))*a^2*c^3*d*e^4-20/(a*e^2-b*d*e+c*d^2)^4/(4*a*c-b^2)^(3/2)*a
rctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^3*c*e^5+40/(a*e^2-b*d*e+c*d^2)^4/(4*
a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c^4*d^3*e^2-4/(a*e^2-b
*d*e+c*d^2)^4/(c*x^2+b*x+a)/(4*a*c-b^2)*b^2*c^3*d^4*e-4/(a*e^2-b*d*e+c*d^2)
^4/(c*x^2+b*x+a)/(4*a*c-b^2)*b^4*c*d^2*e^3+6/(a*e^2-b*d*e+c*d^2)^4/(c*x^2+b
*x+a)/(4*a*c-b^2)*b^3*c^2*d^3*e^2+4/(a*e^2-b*d*e+c*d^2)^4/(c*x^2+b*x+a)/(4*
a*c-b^2)*a^2*c^3*d^2*e^3-2*e^5/(a*e^2-b*d*e+c*d^2)^4*ln(e*x+d)*a*c+10*e^3/(
a*e^2-b*d*e+c*d^2)^4*ln(e*x+d)*c^2*d^2+3/(a*e^2-b*d*e+c*d^2)^4/(4*a*c-b^2)^
(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^5*e^5+4/(a*e^2-b*d*e+c*d^2)^4/(
4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^5*d^5-4*e^3/(a*e^2-b
*d*e+c*d^2)^3/(e*x+d)*c*d+6/(a*e^2-b*d*e+c*d^2)^4/(c*x^2+b*x+a)*c^3/(4*a*c-
b^2)*x*a*b*d^2*e^3+3/2/(a*e^2-b*d*e+c*d^2)^4/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^
4*e^5-6/(a*e^2-b*d*e+c*d^2)^4/(c*x^2+b*x+a)*c^3/(4*a*c-b^2)*x*d*a^2*e^4+3/(
a*e^2-b*d*e+c*d^2)^4/(c*x^2+b*x+a)*c^2/(4*a*c-b^2)*x*a^2*b*e^5-4/(a*e^2-b*d
*e+c*d^2)^4/(c*x^2+b*x+a)*c^2/(4*a*c-b^2)*x*d^2*b^3*e^3+6/(a*e^2-b*d*e+c*d^
2)^4/(c*x^2+b*x+a)*c^3/(4*a*c-b^2)*x*b^2*d^3*e^2-1/(a*e^2-b*d*e+c*d^2)^4/(c
*x^2+b*x+a)*c/(4*a*c-b^2)*x*a*b^3*e^5-4/(a*e^2-b*d*e+c*d^2)^4/(c*x^2+b*x+a)
*c^4/(4*a*c-b^2)*x*d^3*a*e^2+1/(a*e^2-b*d*e+c*d^2)^4/(c*x^2+b*x+a)*c/(4*a*c
```

```
-b^2)*x*b^4*d*e^4-5/(a*e^2-b*d*e+c*d^2)^4/(c*x^2+b*x+a)*c^4/(4*a*c-b^2)*x*b
*d^4*e-7/(a*e^2-b*d*e+c*d^2)^4/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*b*c^2*d*e^4-1/
(a*e^2-b*d*e+c*d^2)^4/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^3*c*d*e^4-1/2*e^3/(a*e^
2-b*d*e+c*d^2)^2/(e*x+d)^2+4/(a*e^2-b*d*e+c*d^2)^4/(c*x^2+b*x+a)/(4*a*c-b^2
)*a^2*b^2*c*e^5-7/(a*e^2-b*d*e+c*d^2)^4/(4*a*c-b^2)*c*ln(c*x^2+b*x+a)*a*b^2
*e^5+30/(a*e^2-b*d*e+c*d^2)^4/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2
)^(1/2))*a^2*b*c^2*e^5+10/(a*e^2-b*d*e+c*d^2)^4/(c*x^2+b*x+a)/(4*a*c-b^2)*a
*b^2*c^2*d^2*e^3-14/(a*e^2-b*d*e+c*d^2)^4/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b*c^3
*d^3*e^2+60/(a*e^2-b*d*e+c*d^2)^4/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c
-b^2)^(1/2))*a*b^2*c^2*d*e^4-60/(a*e^2-b*d*e+c*d^2)^4/(4*a*c-b^2)^(3/2)*arc
tan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*c^3*d^2*e^3+20/(a*e^2-b*d*e+c*d^2)^4/(
4*a*c-b^2)*c^2*ln(c*x^2+b*x+a)*a*b*d*e^4+2*e^4/(a*e^2-b*d*e+c*d^2)^3/(e*x+d
)*b+3*e^5/(a*e^2-b*d*e+c*d^2)^4*ln(e*x+d)*b^2+2/(a*e^2-b*d*e+c*d^2)^4/(c*x^
2+b*x+a)*c^5/(4*a*c-b^2)*x*d^5-2/(a*e^2-b*d*e+c*d^2)^4/(c*x^2+b*x+a)/(4*a*c
-b^2)*a^3*c^2*e^5-1/(a*e^2-b*d*e+c*d^2)^4/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^4*e
^5+1/(a*e^2-b*d*e+c*d^2)^4/(c*x^2+b*x+a)/(4*a*c-b^2)*b^5*d*e^4+1/(a*e^2-b*d
*e+c*d^2)^4/(c*x^2+b*x+a)/(4*a*c-b^2)*b*c^4*d^5+4/(a*e^2-b*d*e+c*d^2)^4/(4*
a*c-b^2)*c^2*ln(c*x^2+b*x+a)*a^2*e^5-10*e^4/(a*e^2-b*d*e+c*d^2)^4*ln(e*x+d)
*b*c*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17698, size = 2176, normalized size = 4.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(10*c^2*d^2*e^3 - 10*b*c*d*e^4 + 3*b^2*e^5 - 2*a*c*e^5)*log(c*x^2 + b*
x + a)/(c^4*d^8 - 4*b*c^3*d^7*e + 6*b^2*c^2*d^6*e^2 + 4*a*c^3*d^6*e^2 - 4*b
^3*c*d^5*e^3 - 12*a*b*c^2*d^5*e^3 + b^4*d^4*e^4 + 12*a*b^2*c*d^4*e^4 + 6*a^
2*c^2*d^4*e^4 - 4*a*b^3*d^3*e^5 - 12*a^2*b*c*d^3*e^5 + 6*a^2*b^2*d^2*e^6 +
4*a^3*c*d^2*e^6 - 4*a^3*b*d*e^7 + a^4*e^8) + (10*c^2*d^2*e^4 - 10*b*c*d*e^5
 + 3*b^2*e^6 - 2*a*c*e^6)*log(abs(x*e + d))/(c^4*d^8*e - 4*b*c^3*d^7*e^2 +
6*b^2*c^2*d^6*e^3 + 4*a*c^3*d^6*e^3 - 4*b^3*c*d^5*e^4 - 12*a*b*c^2*d^5*e^4
+ b^4*d^4*e^5 + 12*a*b^2*c*d^4*e^5 + 6*a^2*c^2*d^4*e^5 - 4*a*b^3*d^3*e^6 -
12*a^2*b*c*d^3*e^6 + 6*a^2*b^2*d^2*e^7 + 4*a^3*c*d^2*e^7 - 4*a^3*b*d*e^8 +
a^4*e^9) - (4*c^5*d^5 - 10*b*c^4*d^4*e + 40*a*c^4*d^3*e^2 + 10*b^3*c^2*d^2*
e^3 - 60*a*b*c^3*d^2*e^3 - 10*b^4*c*d*e^4 + 60*a*b^2*c^2*d*e^4 - 60*a^2*c^3
*d*e^4 + 3*b^5*e^5 - 20*a*b^3*c*e^5 + 30*a^2*b*c^2*e^5)*arctan((2*c*x + b)/
sqrt(-b^2 + 4*a*c))/((b^2*c^4*d^8 - 4*a*c^5*d^8 - 4*b^3*c^3*d^7*e + 16*a*b*
c^4*d^7*e + 6*b^4*c^2*d^6*e^2 - 20*a*b^2*c^3*d^6*e^2 - 16*a^2*c^4*d^6*e^2 -
 4*b^5*c*d^5*e^3 + 4*a*b^3*c^2*d^5*e^3 + 48*a^2*b*c^3*d^5*e^3 + b^6*d^4*e^4
 + 8*a*b^4*c*d^4*e^4 - 42*a^2*b^2*c^2*d^4*e^4 - 24*a^3*c^3*d^4*e^4 - 4*a*b^
5*d^3*e^5 + 4*a^2*b^3*c*d^3*e^5 + 48*a^3*b*c^2*d^3*e^5 + 6*a^2*b^4*d^2*e^6
- 20*a^3*b^2*c*d^2*e^6 - 16*a^4*c^2*d^2*e^6 - 4*a^3*b^3*d*e^7 + 16*a^4*b*c*
d*e^7 + a^4*b^2*e^8 - 4*a^5*c*e^8)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*b*c^4*d^7 -
 8*b^2*c^3*d^6*e + 12*a*c^4*d^6*e + 12*b^3*c^2*d^5*e^2 - 28*a*b*c^3*d^5*e^2
 - 8*b^4*c*d^4*e^3 + 29*a*b^2*c^2*d^4*e^3 - 28*a^2*c^3*d^4*e^3 + 2*b^5*d^3*
e^4 - 16*a*b^3*c*d^3*e^4 + 42*a^2*b*c^2*d^3*e^4 + 3*a*b^4*d^2*e^5 - 2*a^2*b
^2*c*d^2*e^5 - 44*a^3*c^2*d^2*e^5 - 6*a^2*b^3*d*e^6 + 24*a^3*b*c*d*e^6 + a^
3*b^2*e^7 - 4*a^4*c*e^7 + 2*(2*c^5*d^5*e^2 - 5*b*c^4*d^4*e^3 + 10*b^2*c^3*d
^3*e^4 - 20*a*c^4*d^3*e^4 - 10*b^3*c^2*d^2*e^5 + 30*a*b*c^3*d^2*e^5 + 3*b^4
*c*d*e^6 - 4*a*b^2*c^2*d*e^6 - 22*a^2*c^3*d*e^6 - 3*a*b^3*c*e^7 + 11*a^2*b*
c^2*e^7)*x^3 + (8*c^5*d^6*e - 18*b*c^4*d^5*e^2 + 25*b^2*c^3*d^4*e^3 - 40*a*
c^4*d^4*e^3 - 10*b^3*c^2*d^3*e^4 + 20*a*b*c^3*d^3*e^4 - 11*b^4*c*d^2*e^5 +
58*a*b^2*c^2*d^2*e^5 - 56*a^2*c^3*d^2*e^5 + 6*b^5*d*e^6 - 20*a*b^3*c*d*e^6
- 10*a^2*b*c^2*d*e^6 - 6*a*b^4*e^7 + 25*a^2*b^2*c*e^7 - 8*a^3*c^2*e^7)*x^2
+ (4*c^5*d^7 - 6*b*c^4*d^6*e - 4*b^2*c^3*d^5*e^2 + 16*a*c^4*d^5*e^2 + 25*b^
3*c^2*d^4*e^3 - 80*a*b*c^3*d^4*e^3 - 28*b^4*c*d^3*e^4 + 104*a*b^2*c^2*d^3*e
^4 - 28*a^2*c^3*d^3*e^4 + 9*b^5*d^2*e^5 - 28*a*b^3*c*d^2*e^5 - 14*a^2*b*c^2
*d^2*e^5 - 6*a*b^4*d*e^6 + 32*a^2*b^2*c*d*e^6 - 40*a^3*c^2*d*e^6 - 3*a^2*b^
3*e^7 + 12*a^3*b*c*e^7)*x)/((c*d^2 - b*d*e + a*e^2)^4*(c*x^2 + b*x + a)*(b^
2 - 4*a*c)*(x*e + d)^2)
```

3.2199 $\int \frac{x^7}{(a+bx+cx^2)^3} dx$

Optimal. Leaf size=294

$$\frac{3x^2(16a^2c^2-13ab^2c+2b^4)}{2c^3(b^2-4ac)^2}+\frac{3b(70a^2b^2c^2-70a^3c^3-21ab^4c+2b^6)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^5(b^2-4ac)^{5/2}}-\frac{bx^3(2b^2-11ac)}{c^2(b^2-4ac)^2}+\frac{3(2b^2-ac}{}$$

```
[Out] (-3*b*(2*b^2 - 9*a*c)*(b^2 - 3*a*c)*x)/(c^4*(b^2 - 4*a*c)^2) + (3*(2*b^4 -
13*a*b^2*c + 16*a^2*c^2)*x^2)/(2*c^3*(b^2 - 4*a*c)^2) - (b*(2*b^2 - 11*a*c)
*x^3)/(c^2*(b^2 - 4*a*c)^2) + (x^6*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a + b*x +
 c*x^2)^2) + (3*x^4*(a*(b^2 - 8*a*c) + b*(b^2 - 6*a*c)*x))/(2*c*(b^2 - 4*a*
c)^2*(a + b*x + c*x^2)) + (3*b*(2*b^6 - 21*a*b^4*c + 70*a^2*b^2*c^2 - 70*a^
3*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^5*(b^2 - 4*a*c)^(5/2)) +
(3*(2*b^2 - a*c)*Log[a + b*x + c*x^2])/(2*c^5)
```

Rubi [A] time = 0.424593, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {738, 818, 800, 634, 618, 206, 628}

$$\frac{3x^2(16a^2c^2-13ab^2c+2b^4)}{2c^3(b^2-4ac)^2}+\frac{3b(70a^2b^2c^2-70a^3c^3-21ab^4c+2b^6)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^5(b^2-4ac)^{5/2}}-\frac{bx^3(2b^2-11ac)}{c^2(b^2-4ac)^2}+\frac{3(2b^2-ac}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x + c*x^2)^3,x]
```


```
[Out] (-3*b*(2*b^2 - 9*a*c)*(b^2 - 3*a*c)*x)/(c^4*(b^2 - 4*a*c)^2) + (3*(2*b^4 -
13*a*b^2*c + 16*a^2*c^2)*x^2)/(2*c^3*(b^2 - 4*a*c)^2) - (b*(2*b^2 - 11*a*c)
*x^3)/(c^2*(b^2 - 4*a*c)^2) + (x^6*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a + b*x +
 c*x^2)^2) + (3*x^4*(a*(b^2 - 8*a*c) + b*(b^2 - 6*a*c)*x))/(2*c*(b^2 - 4*a*
c)^2*(a + b*x + c*x^2)) + (3*b*(2*b^6 - 21*a*b^4*c + 70*a^2*b^2*c^2 - 70*a^
3*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^5*(b^2 - 4*a*c)^(5/2)) +
(3*(2*b^2 - a*c)*Log[a + b*x + c*x^2])/(2*c^5)
```


Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
```

```
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^7}{\left(a+bx+cx^2\right)^3}\,dx = \frac{x^6(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{\int \frac{x^5(12a+3bx)}{\left(a+bx+cx^2\right)^2}dx}{2\left(b^2-4ac\right)}$$

$$= \frac{x^6(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3x^4\left(a\left(b^2-8ac\right)+b\left(b^2-6ac\right)x\right)}{2c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{\int \frac{x^3\left(12a\left(b^2-8ac\right)+6b\left(2b^2-11ac\right)x\right)}{a+bx+cx^2}}{2c\left(b^2-4ac\right)^2}$$

$$= \frac{x^6(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3x^4\left(a\left(b^2-8ac\right)+b\left(b^2-6ac\right)x\right)}{2c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{\int \left(\frac{6b\left(2b^2-9ac\right)\left(b^2-3ac\right)}{c^3} - \frac{6\left(2b^4-\right.}{}\right.}{}$$

$$= -\frac{3b\left(2b^2-9ac\right)\left(b^2-3ac\right)x}{c^4\left(b^2-4ac\right)^2} + \frac{3\left(2b^4-13ab^2c+16a^2c^2\right)x^2}{2c^3\left(b^2-4ac\right)^2} - \frac{b\left(2b^2-11ac\right)x^3}{c^2\left(b^2-4ac\right)^2} + \frac{x^6(2}{2\left(b^2-4ac\right)(}$$

$$= -\frac{3b\left(2b^2-9ac\right)\left(b^2-3ac\right)x}{c^4\left(b^2-4ac\right)^2} + \frac{3\left(2b^4-13ab^2c+16a^2c^2\right)x^2}{2c^3\left(b^2-4ac\right)^2} - \frac{b\left(2b^2-11ac\right)x^3}{c^2\left(b^2-4ac\right)^2} + \frac{x^6(2}{2\left(b^2-4ac\right)(}$$

$$= -\frac{3b\left(2b^2-9ac\right)\left(b^2-3ac\right)x}{c^4\left(b^2-4ac\right)^2} + \frac{3\left(2b^4-13ab^2c+16a^2c^2\right)x^2}{2c^3\left(b^2-4ac\right)^2} - \frac{b\left(2b^2-11ac\right)x^3}{c^2\left(b^2-4ac\right)^2} + \frac{x^6(2}{2\left(b^2-4ac\right)(}$$

$$= -\frac{3b\left(2b^2-9ac\right)\left(b^2-3ac\right)x}{c^4\left(b^2-4ac\right)^2} + \frac{3\left(2b^4-13ab^2c+16a^2c^2\right)x^2}{2c^3\left(b^2-4ac\right)^2} - \frac{b\left(2b^2-11ac\right)x^3}{c^2\left(b^2-4ac\right)^2} + \frac{x^6(2}{2\left(b^2-4ac\right)(}$$

Mathematica [A] time = 0.4444, size = 299, normalized size = 1.02

$$\frac{\frac{2a^2b^3c(7cx-3b)+a^3bc^2(9b-7cx)-2a^4c^3+ab^5(b-7cx)+b^7x}{\left(b^2-4ac\right)(a+x(b+cx))^2} - \frac{-182a^2b^3c^3x+88a^2b^4c^2-153a^3b^2c^3+126a^3bc^4x+48a^4c^4+70ab^5c^2x-17ab^6c-8b^7cx+b^8}{\left(b^2-4ac\right)^2(a+x(b+cx))} + \frac{6bc\left(-70a^2\right.}{}}{2c^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x + c*x^2)^3,x]
```

```
[Out] (-6*b*c^2*x + c^3*x^2 - (b^8 - 17*a*b^6*c + 88*a^2*b^4*c^2 - 153*a^3*b^2*c^
3 + 48*a^4*c^4 - 8*b^7*c*x + 70*a*b^5*c^2*x - 182*a^2*b^3*c^3*x + 126*a^3*b
*c^4*x)/((b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (-2*a^4*c^3 + b^7*x + a*b^5*(
b - 7*c*x) + a^3*b*c^2*(9*b - 7*c*x) + 2*a^2*b^3*c*(-3*b + 7*c*x))/((b^2 -
4*a*c)*(a + x*(b + c*x))^2) + (6*b*c*(-2*b^6 + 21*a*b^4*c - 70*a^2*b^2*c^2
+ 70*a^3*c^3)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2)
- 3*c*(-2*b^2 + a*c)*Log[a + x*(b + c*x)])/(2*c^6)
```

Maple [B] time = 0.166, size = 1187, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^2+b*x+a)^3,x)
```

```
[Out] -51/2/c^4/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^2+b*x+a)*a*b^4+60/c^3/(16*a^2*c
^2-8*a*b^2*c+b^4)*ln(c*x^2+b*x+a)*a^2*b^2-6/c^5/(16*a^2*c^2-8*a*b^2*c+b^4)/
```

```
(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^7+7/2/c^5/(c*x^2+b*
x+a)^2*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*b^6-24/c/(c*x^2+b*x+a)^2/(16*a^2*c^2-
8*a*b^2*c+b^4)*x^2*a^4+7/2/c^5/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x
^2*b^8-55/2/c^4/(c*x^2+b*x+a)^2*a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*b^4+4/c^4/(c
*x^2+b*x+a)^2*b^7/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-53/2/c^4/(c*x^2+b*x+a)^2/(
16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a*b^6-58/c^4/(c*x^2+b*x+a)^2*a^2*b^5/(16*a^2*
c^2-8*a*b^2*c+b^4)*x+136/c^3/(c*x^2+b*x+a)^2*a^3*b^3/(16*a^2*c^2-8*a*b^2*c+
b^4)*x-73/c^2/(c*x^2+b*x+a)^2*a^4*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x-63/c/(c*x^
2+b*x+a)^2*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*a^3+210/c^2/(16*a^2*c^2-8*a*b^2
*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^3*b+63/c^4/
(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2))*a*b^5-210/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*
c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b^3+91/c^2/(c*x^2+b*x+a)^2*b^3/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^3*a^2-35/c^3/(c*x^2+b*x+a)^2*b^5/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^3*a+27/2/c^2/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a^3*b^2+7/c^5
/(c*x^2+b*x+a)^2*a*b^7/(16*a^2*c^2-8*a*b^2*c+b^4)*x+47/c^3/(c*x^2+b*x+a)^2/
(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a^2*b^4+1/2/c^3*x^2-20/c^2/(c*x^2+b*x+a)^2*a
^5/(16*a^2*c^2-8*a*b^2*c+b^4)-24/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^2+b*
x+a)*a^3+3/c^5/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^2+b*x+a)*b^6-3/c^4*b*x+115
/2/c^3/(c*x^2+b*x+a)^2*a^4/(16*a^2*c^2-8*a*b^2*c+b^4)*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.02249, size = 4757, normalized size = 16.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/2*(7*a^2*b^8 - 83*a^3*b^6*c + 335*a^4*b^4*c^2 - 500*a^5*b^2*c^3 + 160*a^
6*c^4 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*x^6 - 4*(b^7
*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*x^5 - (11*b^8*c^2 - 13
4*a*b^6*c^3 + 552*a^2*b^4*c^4 - 800*a^3*b^2*c^5 + 128*a^4*c^6)*x^4 + 2*(b^9
*c - 20*a*b^7*c^2 + 147*a^2*b^5*c^3 - 475*a^3*b^3*c^4 + 572*a^4*b*c^5)*x^3
+ (7*b^10 - 93*a*b^8*c + 451*a^2*b^6*c^2 - 937*a^3*b^4*c^3 + 660*a^4*b^2*c^
4 + 128*a^5*c^5)*x^2 - 3*(2*a^2*b^7 - 21*a^3*b^5*c + 70*a^4*b^3*c^2 - 70*a^
5*b*c^3 + (2*b^7*c^2 - 21*a*b^5*c^3 + 70*a^2*b^3*c^4 - 70*a^3*b*c^5)*x^4 +
2*(2*b^8*c - 21*a*b^6*c^2 + 70*a^2*b^4*c^3 - 70*a^3*b^2*c^4)*x^3 + (2*b^9 -
 17*a*b^7*c + 28*a^2*b^5*c^2 + 70*a^3*b^3*c^3 - 140*a^4*b*c^4)*x^2 + 2*(2*a
*b^8 - 21*a^2*b^6*c + 70*a^3*b^4*c^2 - 70*a^4*b^2*c^3)*x)*sqrt(b^2 - 4*a*c)
*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c
*x^2 + b*x + a)) + 2*(7*a*b^9 - 89*a^2*b^7*c + 404*a^3*b^5*c^2 - 761*a^4*b^
3*c^3 + 484*a^5*b*c^4)*x + 3*(2*a^2*b^8 - 25*a^3*b^6*c + 108*a^4*b^4*c^2 -
176*a^5*b^2*c^3 + 64*a^6*c^4 + (2*b^8*c^2 - 25*a*b^6*c^3 + 108*a^2*b^4*c^4
- 176*a^3*b^2*c^5 + 64*a^4*c^6)*x^4 + 2*(2*b^9*c - 25*a*b^7*c^2 + 108*a^2*b
```

```
^5*c^3 - 176*a^3*b^3*c^4 + 64*a^4*b*c^5)*x^3 + (2*b^10 - 21*a*b^8*c + 58*a^
2*b^6*c^2 + 40*a^3*b^4*c^3 - 288*a^4*b^2*c^4 + 128*a^5*c^5)*x^2 + 2*(2*a*b^
9 - 25*a^2*b^7*c + 108*a^3*b^5*c^2 - 176*a^4*b^3*c^3 + 64*a^5*b*c^4)*x)*log
(c*x^2 + b*x + a))/(a^2*b^6*c^5 - 12*a^3*b^4*c^6 + 48*a^4*b^2*c^7 - 64*a^5*
c^8 + (b^6*c^7 - 12*a*b^4*c^8 + 48*a^2*b^2*c^9 - 64*a^3*c^10)*x^4 + 2*(b^7*
c^6 - 12*a*b^5*c^7 + 48*a^2*b^3*c^8 - 64*a^3*b*c^9)*x^3 + (b^8*c^5 - 10*a*b
^6*c^6 + 24*a^2*b^4*c^7 + 32*a^3*b^2*c^8 - 128*a^4*c^9)*x^2 + 2*(a*b^7*c^5
- 12*a^2*b^5*c^6 + 48*a^3*b^3*c^7 - 64*a^4*b*c^8)*x), 1/2*(7*a^2*b^8 - 83*a
^3*b^6*c + 335*a^4*b^4*c^2 - 500*a^5*b^2*c^3 + 160*a^6*c^4 + (b^6*c^4 - 12*
a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*x^6 - 4*(b^7*c^3 - 12*a*b^5*c^4 +
48*a^2*b^3*c^5 - 64*a^3*b*c^6)*x^5 - (11*b^8*c^2 - 134*a*b^6*c^3 + 552*a^2*
b^4*c^4 - 800*a^3*b^2*c^5 + 128*a^4*c^6)*x^4 + 2*(b^9*c - 20*a*b^7*c^2 + 14
7*a^2*b^5*c^3 - 475*a^3*b^3*c^4 + 572*a^4*b*c^5)*x^3 + (7*b^10 - 93*a*b^8*c
 + 451*a^2*b^6*c^2 - 937*a^3*b^4*c^3 + 660*a^4*b^2*c^4 + 128*a^5*c^5)*x^2 +
 6*(2*a^2*b^7 - 21*a^3*b^5*c + 70*a^4*b^3*c^2 - 70*a^5*b*c^3 + (2*b^7*c^2 -
 21*a*b^5*c^3 + 70*a^2*b^3*c^4 - 70*a^3*b*c^5)*x^4 + 2*(2*b^8*c - 21*a*b^6*
c^2 + 70*a^2*b^4*c^3 - 70*a^3*b^2*c^4)*x^3 + (2*b^9 - 17*a*b^7*c + 28*a^2*b
^5*c^2 + 70*a^3*b^3*c^3 - 140*a^4*b*c^4)*x^2 + 2*(2*a*b^8 - 21*a^2*b^6*c +
70*a^3*b^4*c^2 - 70*a^4*b^2*c^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 +
4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(7*a*b^9 - 89*a^2*b^7*c + 404*a^3*b^5
*c^2 - 761*a^4*b^3*c^3 + 484*a^5*b*c^4)*x + 3*(2*a^2*b^8 - 25*a^3*b^6*c + 1
08*a^4*b^4*c^2 - 176*a^5*b^2*c^3 + 64*a^6*c^4 + (2*b^8*c^2 - 25*a*b^6*c^3 +
 108*a^2*b^4*c^4 - 176*a^3*b^2*c^5 + 64*a^4*c^6)*x^4 + 2*(2*b^9*c - 25*a*b^
7*c^2 + 108*a^2*b^5*c^3 - 176*a^3*b^3*c^4 + 64*a^4*b*c^5)*x^3 + (2*b^10 - 2
1*a*b^8*c + 58*a^2*b^6*c^2 + 40*a^3*b^4*c^3 - 288*a^4*b^2*c^4 + 128*a^5*c^5
)*x^2 + 2*(2*a*b^9 - 25*a^2*b^7*c + 108*a^3*b^5*c^2 - 176*a^4*b^3*c^3 + 64*
a^5*b*c^4)*x)*log(c*x^2 + b*x + a))/(a^2*b^6*c^5 - 12*a^3*b^4*c^6 + 48*a^4*
b^2*c^7 - 64*a^5*c^8 + (b^6*c^7 - 12*a*b^4*c^8 + 48*a^2*b^2*c^9 - 64*a^3*c^
10)*x^4 + 2*(b^7*c^6 - 12*a*b^5*c^7 + 48*a^2*b^3*c^8 - 64*a^3*b*c^9)*x^3 +
(b^8*c^5 - 10*a*b^6*c^6 + 24*a^2*b^4*c^7 + 32*a^3*b^2*c^8 - 128*a^4*c^9)*x^
2 + 2*(a*b^7*c^5 - 12*a^2*b^5*c^6 + 48*a^3*b^3*c^7 - 64*a^4*b*c^8)*x)]
```

Sympy [B] time = 5.69539, size = 1875, normalized size = 6.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**2+b*x+a)**3,x)
```

```
[Out] -3*b*x/c**4 + (-3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b**2*c
**2 + 21*a*b**4*c - 2*b**6)/(2*c**5*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 +
 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 3*(a*c -
 2*b**2)/(2*c**5))*log(x + (96*a**4*c**3 - 159*a**3*b**2*c**2 + 64*a**3*c**
7*(-3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b**2*c**2 + 21*a*b
**4*c - 2*b**6)/(2*c**5*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b*
*4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 3*(a*c - 2*b**2)/(2*
c**5)) + 57*a**2*b**4*c - 48*a**2*b**2*c**6*(-3*b*sqrt(-(4*a*c - b**2)**5)*
(70*a**3*c**3 - 70*a**2*b**2*c**2 + 21*a*b**4*c - 2*b**6)/(2*c**5*(1024*a**
5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20
*a*b**8*c - b**10)) - 3*(a*c - 2*b**2)/(2*c**5)) - 6*a*b**6 + 12*a*b**4*c**
5*(-3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b**2*c**2 + 21*a*b
**4*c - 2*b**6)/(2*c**5*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b*
*4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 3*(a*c - 2*b**2)/(2*
c**5)) - b**6*c**4*(-3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b
**2*c**2 + 21*a*b**4*c - 2*b**6)/(2*c**5*(1024*a**5*c**5 - 1280*a**4*b**2*c
**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 3*(
```

```
a*c - 2*b**2)/(2*c**5)))/(210*a**3*b*c**3 - 210*a**2*b**3*c**2 + 63*a*b**5*
c - 6*b**7)) + (3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b**2*c
**2 + 21*a*b**4*c - 2*b**6)/(2*c**5*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 +
 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 3*(a*c -
 2*b**2)/(2*c**5))*log(x + (96*a**4*c**3 - 159*a**3*b**2*c**2 + 64*a**3*c**
7*(3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b**2*c**2 + 21*a*b*
*4*c - 2*b**6)/(2*c**5*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**
4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 3*(a*c - 2*b**2)/(2*c
**5)) + 57*a**2*b**4*c - 48*a**2*b**2*c**6*(3*b*sqrt(-(4*a*c - b**2)**5)*(7
0*a**3*c**3 - 70*a**2*b**2*c**2 + 21*a*b**4*c - 2*b**6)/(2*c**5*(1024*a**5*
c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a
*b**8*c - b**10)) - 3*(a*c - 2*b**2)/(2*c**5)) - 6*a*b**6 + 12*a*b**4*c**5*
(3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b**2*c**2 + 21*a*b**4
*c - 2*b**6)/(2*c**5*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*
c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 3*(a*c - 2*b**2)/(2*c**
5)) - b**6*c**4*(3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b**2*
c**2 + 21*a*b**4*c - 2*b**6)/(2*c**5*(1024*a**5*c**5 - 1280*a**4*b**2*c**4
+ 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) - 3*(a*c
- 2*b**2)/(2*c**5)))/(210*a**3*b*c**3 - 210*a**2*b**3*c**2 + 63*a*b**5*c -
6*b**7)) - (40*a**5*c**3 - 115*a**4*b**2*c**2 + 55*a**3*b**4*c - 7*a**2*b**
6 + x**3*(126*a**3*b*c**4 - 182*a**2*b**3*c**3 + 70*a*b**5*c**2 - 8*b**7*c)
 + x**2*(48*a**4*c**4 - 27*a**3*b**2*c**3 - 94*a**2*b**4*c**2 + 53*a*b**6*c
 - 7*b**8) + x*(146*a**4*b*c**3 - 272*a**3*b**3*c**2 + 116*a**2*b**5*c - 14
*a*b**7))/(32*a**4*c**7 - 16*a**3*b**2*c**6 + 2*a**2*b**4*c**5 + x**4*(32*a
**2*c**9 - 16*a*b**2*c**8 + 2*b**4*c**7) + x**3*(64*a**2*b*c**8 - 32*a*b**3
*c**7 + 4*b**5*c**6) + x**2*(64*a**3*c**8 - 12*a*b**4*c**6 + 2*b**6*c**5) +
 x*(64*a**3*b*c**7 - 32*a**2*b**3*c**6 + 4*a*b**5*c**5)) + x**2/(2*c**3)
```

Giac [A] time = 1.14117, size = 448, normalized size = 1.52

$$-\frac{3\left(2\,b^{7}-21\,ab^{5}c+70\,a^{2}b^{3}c^{2}-70\,a^{3}bc^{3}\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(b^{4}c^{5}-8\,ab^{2}c^{6}+16\,a^{2}c^{7}\right)\sqrt{-b^{2}+4\,ac}}+\frac{3\left(2\,b^{2}-ac\right)\log\left(cx^{2}+bx+a\right)}{2\,c^{5}}+\frac{c^{3}x^{2}-6\,bc^{2}x}{2\,c^{6}}+\frac{7\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] -3*(2*b^7 - 21*a*b^5*c + 70*a^2*b^3*c^2 - 70*a^3*b*c^3)*arctan((2*c*x + b)/
sqrt(-b^2 + 4*a*c))/((b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt(-b^2 + 4*a*c
)) + 3/2*(2*b^2 - a*c)*log(c*x^2 + b*x + a)/c^5 + 1/2*(c^3*x^2 - 6*b*c^2*x)
/c^6 + 1/2*(7*a^2*b^6 - 55*a^3*b^4*c + 115*a^4*b^2*c^2 - 40*a^5*c^3 + 2*(4*
b^7*c - 35*a*b^5*c^2 + 91*a^2*b^3*c^3 - 63*a^3*b*c^4)*x^3 + (7*b^8 - 53*a*b
^6*c + 94*a^2*b^4*c^2 + 27*a^3*b^2*c^3 - 48*a^4*c^4)*x^2 + 2*(7*a*b^7 - 58*
a^2*b^5*c + 136*a^3*b^3*c^2 - 73*a^4*b*c^3)*x)/((c*x^2 + b*x + a)^2*(b^2 -
4*a*c)^2*c^5)
```

3.2200 $\int \frac{x^6}{(a+bx+cx^2)^3} \, dx$

Optimal. Leaf size=238

$$\frac{3x\left(10a^2c^2-7ab^2c+b^4\right)}{c^3\left(b^2-4ac\right)^2}-\frac{3\left(30a^2b^2c^2-20a^3c^3-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\left(b^2-4ac\right)^{5/2}}-\frac{3bx^2\left(b^2-6ac\right)}{2c^2\left(b^2-4ac\right)^2}+\frac{x^5(2a+b}{2\left(b^2-4ac\right)(a+}$$

```
[Out] (3*(b^4 - 7*a*b^2*c + 10*a^2*c^2)*x)/(c^3*(b^2 - 4*a*c)^2) - (3*b*(b^2 - 6*
a*c)*x^2)/(2*c^2*(b^2 - 4*a*c)^2) + (x^5*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a +
 b*x + c*x^2)^2) + (x^3*(a*(b^2 - 10*a*c) + b*(b^2 - 7*a*c)*x))/(c*(b^2 - 4
*a*c)^2*(a + b*x + c*x^2)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3
*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*(b^2 - 4*a*c)^(5/2)) - (
3*b*Log[a + b*x + c*x^2])/(2*c^4)
```

Rubi [A] time = 0.287418, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {738, 818, 800, 634, 618, 206, 628}

$$\frac{3x\left(10a^2c^2-7ab^2c+b^4\right)}{c^3\left(b^2-4ac\right)^2}-\frac{3\left(30a^2b^2c^2-20a^3c^3-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\left(b^2-4ac\right)^{5/2}}-\frac{3bx^2\left(b^2-6ac\right)}{2c^2\left(b^2-4ac\right)^2}+\frac{x^5(2a+b}{2\left(b^2-4ac\right)(a+}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x + c*x^2)^3,x]
```

```
[Out] (3*(b^4 - 7*a*b^2*c + 10*a^2*c^2)*x)/(c^3*(b^2 - 4*a*c)^2) - (3*b*(b^2 - 6*
a*c)*x^2)/(2*c^2*(b^2 - 4*a*c)^2) + (x^5*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a +
 b*x + c*x^2)^2) + (x^3*(a*(b^2 - 10*a*c) + b*(b^2 - 7*a*c)*x))/(c*(b^2 - 4
*a*c)^2*(a + b*x + c*x^2)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3
*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*(b^2 - 4*a*c)^(5/2)) - (
3*b*Log[a + b*x + c*x^2])/(2*c^4)
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
```

```
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx+cx^2\right)^3}\,dx = \frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{\int \frac{x^4(10a+2bx)}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= \frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^3\left(a\left(b^2-10ac\right)+b\left(b^2-7ac\right)x\right)}{c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{\int \frac{x^2\left(6a\left(b^2-10ac\right)+6b\left(b^2-6ac\right)x\right)}{a+bx+cx^2}\,d}{2c\left(b^2-4ac\right)^2}$$

$$= \frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^3\left(a\left(b^2-10ac\right)+b\left(b^2-7ac\right)x\right)}{c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{\int\left(-\frac{6\left(b^4-7ab^2c+10a^2c^2\right)}{c^2} + \frac{6b\left(b^2\right.}{}\right.}{}$$

$$= \frac{3\left(b^4-7ab^2c+10a^2c^2\right)x}{c^3\left(b^2-4ac\right)^2} - \frac{3b\left(b^2-6ac\right)x^2}{2c^2\left(b^2-4ac\right)^2} + \frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^3\left(a\left(b^2-10ac\right)\right.}{c\left(b^2-4ac\right)^2\left(\right.}$$

$$= \frac{3\left(b^4-7ab^2c+10a^2c^2\right)x}{c^3\left(b^2-4ac\right)^2} - \frac{3b\left(b^2-6ac\right)x^2}{2c^2\left(b^2-4ac\right)^2} + \frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^3\left(a\left(b^2-10ac\right)\right.}{c\left(b^2-4ac\right)^2\left(\right.}$$

$$= \frac{3\left(b^4-7ab^2c+10a^2c^2\right)x}{c^3\left(b^2-4ac\right)^2} - \frac{3b\left(b^2-6ac\right)x^2}{2c^2\left(b^2-4ac\right)^2} + \frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^3\left(a\left(b^2-10ac\right)\right.}{c\left(b^2-4ac\right)^2\left(\right.}$$

$$= \frac{3\left(b^4-7ab^2c+10a^2c^2\right)x}{c^3\left(b^2-4ac\right)^2} - \frac{3b\left(b^2-6ac\right)x^2}{2c^2\left(b^2-4ac\right)^2} + \frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^3\left(a\left(b^2-10ac\right)\right.}{c\left(b^2-4ac\right)^2\left(\right.}$$

Mathematica [A] time = 0.38183, size = 260, normalized size = 1.09

$$\frac{\frac{a^2b^2c(5b-9cx)+a^3c^2(2cx-5b)-ab^4(b-6cx)+b^6(-x)}{\left(b^2-4ac\right)(a+x(b+cx))^2} + \frac{-102a^2b^2c^3x+61a^2b^3c^2-78a^3bc^3+36a^3c^4x+48ab^4c^2x-14ab^5c-6b^6cx+b^7}{\left(b^2-4ac\right)^2(a+x(b+cx))} + \frac{6c\left(30a^2b^2c^2-20a^3c^3-10ab^4c+\right.}{\left(4ac-b^2\right)^{5/}}}{2c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x + c*x^2)^3,x]
```

```
[Out] (2*c^2*x + (b^7 - 14*a*b^5*c + 61*a^2*b^3*c^2 - 78*a^3*b*c^3 - 6*b^6*c*x +
48*a*b^4*c^2*x - 102*a^2*b^2*c^3*x + 36*a^3*c^4*x)/((b^2 - 4*a*c)^2*(a + x*
(b + c*x))) + (-(b^6*x) + a^2*b^2*c*(5*b - 9*c*x) - a*b^4*(b - 6*c*x) + a^3
*c^2*(-5*b + 2*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))^2) + (6*c*(b^6 - 10*a
*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]
])/(-b^2 + 4*a*c)^(5/2) - 3*b*c*Log[a + x*(b + c*x)])/(2*c^5)
```

Maple [B] time = 0.181, size = 1040, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^2+b*x+a)^3,x)
```

```
[Out] x/c^3+18/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*a^3-51/c/(c*x^2+b*x
+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*a^2*b^2+24/c^2/(c*x^2+b*x+a)^2/(16*a^2
```

```
*c^2-8*a*b^2*c+b^4)*x^3*a*b^4-3/c^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b
^4)*x^3*b^6-21/c/(c*x^2+b*x+a)^2*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a^3-41/2/
c^2/(c*x^2+b*x+a)^2*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a^2+17/c^3/(c*x^2+b*
x+a)^2*b^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a-5/2/c^4/(c*x^2+b*x+a)^2*b^7/(16
*a^2*c^2-8*a*b^2*c+b^4)*x^2+14/c/(c*x^2+b*x+a)^2*a^4/(16*a^2*c^2-8*a*b^2*c+
b^4)*x-71/c^2/(c*x^2+b*x+a)^2*a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^2+38/c^3/(
c*x^2+b*x+a)^2*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^4-5/c^4/(c*x^2+b*x+a)^2*a
/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^6-29/c^2/(c*x^2+b*x+a)^2*b*a^4/(16*a^2*c^2-
8*a*b^2*c+b^4)+18/c^3/(c*x^2+b*x+a)^2*b^3*a^3/(16*a^2*c^2-8*a*b^2*c+b^4)-5/
2/c^4/(c*x^2+b*x+a)^2*b^5*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)-24/c^2/(16*a^2*c^2
-8*a*b^2*c+b^4)*ln(c*x^2+b*x+a)*a^2*b+12/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(
c*x^2+b*x+a)*a*b^3-3/2/c^4/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^2+b*x+a)*b^5-6
0/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^
2)^(1/2))*a^3+90/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2
*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b^2-30/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c
-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^4+3/c^4/(16*a^2*c^2-8*a
*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^2+b*x+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 2.72527, size = 4120, normalized size = 17.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^2+b*x+a)^3,x, algorithm="fricas")

[Out] [-1/2*(5*a^2*b^7 - 56*a^3*b^5*c + 202*a^4*b^3*c^2 - 232*a^5*b*c^3 - 2*(b^6*
c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^5 - 4*(b^7*c^2 - 12*a*b
^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^4 + 2*(2*b^8*c - 26*a*b^6*c^2 + 1
23*a^2*b^4*c^3 - 254*a^3*b^2*c^4 + 200*a^4*c^5)*x^3 + (5*b^9 - 58*a*b^7*c +
 225*a^2*b^5*c^2 - 314*a^3*b^3*c^3 + 88*a^4*b*c^4)*x^2 + 3*(a^2*b^6 - 10*a^
3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3 + (b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^
2*c^4 - 20*a^3*c^5)*x^4 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3
*b*c^4)*x^3 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c
^4)*x^2 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*x)*sqrt(
b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*
c*x + b))/(c*x^2 + b*x + a)) + 2*(5*a*b^8 - 59*a^2*b^6*c + 235*a^3*b^4*c^2
- 346*a^4*b^2*c^3 + 120*a^5*c^4)*x + 3*(a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3
*c^2 - 64*a^5*b*c^3 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c
^5)*x^4 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^3 +
(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^2 +
2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x)*log(c*x^2 + b
*x + a))/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7 + (b^6
*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)*x^4 + 2*(b^7*c^5 - 12*a*
b^5*c^6 + 48*a^2*b^3*c^7 - 64*a^3*b*c^8)*x^3 + (b^8*c^4 - 10*a*b^6*c^5 + 24
```

```
*a^2*b^4*c^6 + 32*a^3*b^2*c^7 - 128*a^4*c^8)*x^2 + 2*(a*b^7*c^4 - 12*a^2*b^
5*c^5 + 48*a^3*b^3*c^6 - 64*a^4*b*c^7)*x), -1/2*(5*a^2*b^7 - 56*a^3*b^5*c +
 202*a^4*b^3*c^2 - 232*a^5*b*c^3 - 2*(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c
^5 - 64*a^3*c^6)*x^5 - 4*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*
b*c^5)*x^4 + 2*(2*b^8*c - 26*a*b^6*c^2 + 123*a^2*b^4*c^3 - 254*a^3*b^2*c^4
+ 200*a^4*c^5)*x^3 + (5*b^9 - 58*a*b^7*c + 225*a^2*b^5*c^2 - 314*a^3*b^3*c^
3 + 88*a^4*b*c^4)*x^2 + 6*(a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5
*c^3 + (b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^4 + 2*(b^7*
c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*x^3 + (b^8 - 8*a*b^6*c +
10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*x^2 + 2*(a*b^7 - 10*a^2*b^5*c
 + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 +
 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(5*a*b^8 - 59*a^2*b^6*c + 235*a^3*b^
4*c^2 - 346*a^4*b^2*c^3 + 120*a^5*c^4)*x + 3*(a^2*b^7 - 12*a^3*b^5*c + 48*a
^4*b^3*c^2 - 64*a^5*b*c^3 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a
^3*b*c^5)*x^4 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*
x^3 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*
x^2 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x)*log(c*x
^2 + b*x + a))/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7
+ (b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)*x^4 + 2*(b^7*c^5 -
 12*a*b^5*c^6 + 48*a^2*b^3*c^7 - 64*a^3*b*c^8)*x^3 + (b^8*c^4 - 10*a*b^6*c^
5 + 24*a^2*b^4*c^6 + 32*a^3*b^2*c^7 - 128*a^4*c^8)*x^2 + 2*(a*b^7*c^4 - 12*
a^2*b^5*c^5 + 48*a^3*b^3*c^6 - 64*a^4*b*c^7)*x)]
```

Sympy [B] time = 4.3829, size = 1714, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**2+b*x+a)**3,x)
```

```
[Out] (-3*b/(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c*
*2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 64
0*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))*log(x + (-66
*a**3*b*c**2 - 64*a**3*c**6*(-3*b/(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20
*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**
5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b*
*8*c - b**10))) + 27*a**2*b**3*c + 48*a**2*b**2*c**5*(-3*b/(2*c**4) - 3*sqr
t(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**
6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160
*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) - 3*a*b**5 - 12*a*b**4*c**4*(-3*b/
(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 1
0*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3
*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) + b**6*c**3*(-3*b/
(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 1
0*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3
*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))))/(60*a**3*c**3 - 9
0*a**2*b**2*c**2 + 30*a*b**4*c - 3*b**6)) + (-3*b/(2*c**4) + 3*sqrt(-(4*a*c
 - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**
4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**
6*c**2 + 20*a*b**8*c - b**10)))*log(x + (-66*a**3*b*c**2 - 64*a**3*c**6*(-3
*b/(2*c**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2
+ 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a
**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) + 27*a**2*b**3*
c + 48*a**2*b**2*c**5*(-3*b/(2*c**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*
c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 12
80*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c -
```

```
 b**10))) - 3*a*b**5 - 12*a*b**4*c**4*(-3*b/(2*c**4) + 3*sqrt(-(4*a*c - b**
2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(102
4*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2
 + 20*a*b**8*c - b**10))) + b**6*c**3*(-3*b/(2*c**4) + 3*sqrt(-(4*a*c - b**
2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(102
4*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2
 + 20*a*b**8*c - b**10))))/(60*a**3*c**3 - 90*a**2*b**2*c**2 + 30*a*b**4*c
- 3*b**6)) + (-58*a**4*b*c**2 + 36*a**3*b**3*c - 5*a**2*b**5 + x**3*(36*a**
3*c**4 - 102*a**2*b**2*c**3 + 48*a*b**4*c**2 - 6*b**6*c) + x**2*(-42*a**3*b
*c**3 - 41*a**2*b**3*c**2 + 34*a*b**5*c - 5*b**7) + x*(28*a**4*c**3 - 142*a
**3*b**2*c**2 + 76*a**2*b**4*c - 10*a*b**6))/(32*a**4*c**6 - 16*a**3*b**2*c
**5 + 2*a**2*b**4*c**4 + x**4*(32*a**2*c**8 - 16*a*b**2*c**7 + 2*b**4*c**6)
 + x**3*(64*a**2*b*c**7 - 32*a*b**3*c**6 + 4*b**5*c**5) + x**2*(64*a**3*c**
7 - 12*a*b**4*c**5 + 2*b**6*c**4) + x*(64*a**3*b*c**6 - 32*a**2*b**3*c**5 +
 4*a*b**5*c**4)) + x/c**3
```

Giac [A] time = 1.12738, size = 381, normalized size = 1.6

$$\frac{3\left(b^6-10\,ab^4c+30\,a^2b^2c^2-20\,a^3c^3\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4c^4-8\,ab^2c^5+16\,a^2c^6\right)\sqrt{-b^2+4\,ac}}+\frac{x}{c^3}-\frac{3\,b\log\left(cx^2+bx+a\right)}{2\,c^4}-\frac{5\,a^2b^5-36\,a^3b^3c+58\,a^4bc^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan((2*c*x + b)/sqrt(
-b^2 + 4*a*c))/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*sqrt(-b^2 + 4*a*c)) +
x/c^3 - 3/2*b*log(c*x^2 + b*x + a)/c^4 - 1/2*(5*a^2*b^5 - 36*a^3*b^3*c + 58
*a^4*b*c^2 + 6*(b^6*c - 8*a*b^4*c^2 + 17*a^2*b^2*c^3 - 6*a^3*c^4)*x^3 + (5*
b^7 - 34*a*b^5*c + 41*a^2*b^3*c^2 + 42*a^3*b*c^3)*x^2 + 2*(5*a*b^6 - 38*a^2
*b^4*c + 71*a^3*b^2*c^2 - 14*a^4*c^3)*x)/((c*x^2 + b*x + a)^2*(b^2 - 4*a*c)
^2*c^4)
```

3.2201 $\int \frac{(d+ex)^5}{(a+bx+cx^2)^3}\,dx$

Optimal. Leaf size=388

$$-\frac{\left(20c^3de^2\left(3a^2e^2-3abde+b^2d^2\right)-30a^2bc^2e^5+10ab^3ce^5-10c^4d^3e(3bd-4ae)-b^5e^5+12c^5d^5\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\left(b^2-4ac\right)^{5/2}}-\frac{(d+}{}$$

```
[Out] -((e^2*(2*c*d - b*e)*(3*c^2*d^2 - b^2*e^2 - c*e*(3*b*d - 7*a*e))*x)/(c^2*(b
^2 - 4*a*c)^2)) - ((d + e*x)^4*(b*d - 2*a*e + (2*c*d - b*e)*x))/(2*(b^2 - 4
*a*c)*(a + b*x + c*x^2)^2) - ((d + e*x)^2*(8*a*c*e*(c*d^2 + 2*a*e^2) - 6*b*
c*d*(c*d^2 + 3*a*e^2) + b^2*(7*c*d^2*e - a*e^3) - (2*c*d - b*e)*(6*c^2*d^2
- b^2*e^2 - 2*c*e*(3*b*d - 5*a*e))*x))/(2*c*(b^2 - 4*a*c)^2*(a + b*x + c*x^
2)) - ((12*c^5*d^5 - b^5*e^5 + 10*a*b^3*c*e^5 - 30*a^2*b*c^2*e^5 - 10*c^4*d
^3*e*(3*b*d - 4*a*e) + 20*c^3*d*e^2*(b^2*d^2 - 3*a*b*d*e + 3*a^2*e^2))*ArcT
anh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(5/2)) + (e^5*Log[a
+ b*x + c*x^2])/(2*c^3)
```

Rubi [A] time = 1.11005, antiderivative size = 388, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {738, 818, 773, 634, 618, 206, 628}

$$-\frac{\left(20c^3de^2\left(3a^2e^2-3abde+b^2d^2\right)-30a^2bc^2e^5+10ab^3ce^5-10c^4d^3e(3bd-4ae)-b^5e^5+12c^5d^5\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\left(b^2-4ac\right)^{5/2}}-\frac{(d+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5/(a + b*x + c*x^2)^3,x]
```


```
[Out] -((e^2*(2*c*d - b*e)*(3*c^2*d^2 - b^2*e^2 - c*e*(3*b*d - 7*a*e))*x)/(c^2*(b
^2 - 4*a*c)^2)) - ((d + e*x)^4*(b*d - 2*a*e + (2*c*d - b*e)*x))/(2*(b^2 - 4
*a*c)*(a + b*x + c*x^2)^2) - ((d + e*x)^2*(8*a*c*e*(c*d^2 + 2*a*e^2) - 6*b*
c*d*(c*d^2 + 3*a*e^2) + b^2*(7*c*d^2*e - a*e^3) - (2*c*d - b*e)*(6*c^2*d^2
- b^2*e^2 - 2*c*e*(3*b*d - 5*a*e))*x))/(2*c*(b^2 - 4*a*c)^2*(a + b*x + c*x^
2)) - ((12*c^5*d^5 - b^5*e^5 + 10*a*b^3*c*e^5 - 30*a^2*b*c^2*e^5 - 10*c^4*d
^3*e*(3*b*d - 4*a*e) + 20*c^3*d*e^2*(b^2*d^2 - 3*a*b*d*e + 3*a^2*e^2))*ArcT
anh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(5/2)) + (e^5*Log[a
+ b*x + c*x^2])/(2*c^3)
```


Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
```

```
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)^5}{\left(a+bx+cx^2\right)^3}\,dx = -\frac{(d+ex)^4(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{\int \frac{(d+ex)^3\left(6cd^2-e(7bd-8ae)-e(2cd-be)x\right)}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{(d+ex)^4(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{(d+ex)^2\left(8ace\left(cd^2+2ae^2\right)-6bcd\left(cd^2+3ae^2\right)+b^2\left(7cd^2\right.\right.}{2c\left(b^2-4ac\right)^2\left(\right.}$$

$$= -\frac{e^2(2cd-be)\left(3c^2d^2-b^2e^2-ce(3bd-7ae)\right)x}{c^2\left(b^2-4ac\right)^2} - \frac{(d+ex)^4(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{(d+ex)^2\left(\right.}{}$$

$$= -\frac{e^2(2cd-be)\left(3c^2d^2-b^2e^2-ce(3bd-7ae)\right)x}{c^2\left(b^2-4ac\right)^2} - \frac{(d+ex)^4(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{(d+ex)^2\left(\right.}{}$$

$$= -\frac{e^2(2cd-be)\left(3c^2d^2-b^2e^2-ce(3bd-7ae)\right)x}{c^2\left(b^2-4ac\right)^2} - \frac{(d+ex)^4(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{(d+ex)^2\left(\right.}{}$$

$$= -\frac{e^2(2cd-be)\left(3c^2d^2-b^2e^2-ce(3bd-7ae)\right)x}{c^2\left(b^2-4ac\right)^2} - \frac{(d+ex)^4(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{(d+ex)^2\left(\right.}{}$$

Mathematica [A] time = 1.20017, size = 628, normalized size = 1.62

$$\frac{-2b^2ce^2\left(2a^2e^3-5acde(d+2ex)+5c^2d^3x\right)+bc^2\left(5a^2e^4(3d+ex)-10acd^2e^2(d+3ex)-c^2d^4(d-5ex)\right)+2c^2\left(-5a^2cde^3(2d+ex)+a^3e^5+5ac^2d^3e(d+2ex)-c^3d^5x\right)-5b^3ce^3\left(ae(d+e\right.}{\left(b^2-4ac\right)(a+x(b+cx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5/(a + b*x + c*x^2)^3,x]
```

```
[Out] ((b^5*e^5*x + b^4*e^4*(a*e - 5*c*d*x) - 5*b^3*c*e^3*(-2*c*d^2*x + a*e*(d +
e*x)) - 2*b^2*c*e^2*(2*a^2*e^3 + 5*c^2*d^3*x - 5*a*c*d*e*(d + 2*e*x)) + 2*c
^2*(a^3*e^5 - c^3*d^5*x - 5*a^2*c*d*e^3*(2*d + e*x) + 5*a*c^2*d^3*e*(d + 2*
e*x)) + b*c^2*(-(c^2*d^4*(d - 5*e*x)) + 5*a^2*e^4*(3*d + e*x) - 10*a*c*d^2*
e^2*(d + 3*e*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x))^2) + (-(b^6*e^5) + b^5*c
*e^4*(5*d + 4*e*x) + b^4*c*e^3*(11*a*e^2 - 10*c*d*(d + e*x)) + 10*b^3*c^2*e
^2*(c*d^3 - a*e^2*(4*d + 3*e*x)) + 4*c^3*(8*a^3*e^5 + 3*c^3*d^5*x + 10*a*c^
2*d^3*e^2*x - 5*a^2*c*d*e^3*(8*d + 5*e*x)) + 2*b*c^3*(3*c^2*d^4*(d - 5*e*x)
 + 10*a*c*d^2*e^2*(d - 3*e*x) + 5*a^2*e^4*(11*d + 5*e*x)) + b^2*c^2*e*(-39*
a^2*e^4 - 5*c^2*d^3*(3*d - 4*e*x) + 10*a*c*d*e^2*(5*d + 8*e*x)))/((b^2 - 4*
a*c)^2*(a + x*(b + c*x))) + (2*c*(2*c*d - b*e)*(6*c^4*d^4 + b^4*e^4 + 2*b^2
*c*e^3*(b*d - 5*a*e) - 4*c^3*d^2*e*(3*b*d - 5*a*e) + 2*c^2*e^2*(2*b^2*d^2 -
 10*a*b*d*e + 15*a^2*e^2))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 +
4*a*c)^(5/2) + c*e^5*Log[a + x*(b + c*x)])/(2*c^4)
```

Maple [B] time = 0.168, size = 1444, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5/(c*x^2+b*x+a)^3,x)
```

```
[Out] ((25*a^2*b*c^2*e^5-50*a^2*c^3*d*e^4-15*a*b^3*c*e^5+40*a*b^2*c^2*d*e^4-30*a*
b*c^3*d^2*e^3+20*a*c^4*d^3*e^2+2*b^5*e^5-5*b^4*c*d*e^4+10*b^2*c^3*d^3*e^2-1
5*b*c^4*d^4*e+6*c^5*d^5)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/2*(32*a^3*c^3
*e^5+11*a^2*b^2*c^2*e^5+10*a^2*b*c^3*d*e^4-160*a^2*c^4*d^2*e^3-19*a*b^4*c*e
^5+40*a*b^3*c^2*d*e^4-10*a*b^2*c^3*d^2*e^3+60*a*b*c^4*d^3*e^2+3*b^6*e^5-5*b
^5*c*d*e^4-10*b^4*c^2*d^2*e^3+30*b^3*c^3*d^3*e^2-45*b^2*c^4*d^4*e+18*b*c^5*
d^5)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^3*x^2+(31*a^3*b*c^2*e^5-30*a^3*c^3*d*e^4-
22*a^2*b^3*c*e^5+50*a^2*b^2*c^2*d*e^4-50*a^2*b*c^3*d^2*e^3-20*a^2*c^4*d^3*e
^2+3*a*b^5*e^5-5*a*b^4*c*d*e^4-10*a*b^3*c^2*d^2*e^3+50*a*b^2*c^3*d^3*e^2-25
*a*b*c^4*d^4*e+10*a*c^5*d^5-5*b^3*c^3*d^4*e+2*b^2*c^4*d^5)/(16*a^2*c^2-8*a*
b^2*c+b^4)/c^3*x+1/2/c^3*(24*a^4*c^2*e^5-21*a^3*b^2*c*e^5+50*a^3*b*c^2*d*e^
4-80*a^3*c^3*d^2*e^3+3*a^2*b^4*e^5-5*a^2*b^3*c*d*e^4-10*a^2*b^2*c^2*d^2*e^3
+60*a^2*b*c^3*d^3*e^2-40*a^2*c^4*d^4*e-5*a*b^2*c^3*d^4*e+10*a*b*c^4*d^5-b^3
*c^3*d^5)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+8/c/(16*a^2*c^2-8*a*b
^2*c+b^4)*ln(c*x^2+b*x+a)*a^2*e^5-4/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^2
+b*x+a)*a*b^2*e^5+1/2/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^2+b*x+a)*b^4*e^
5-30/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c
-b^2)^(1/2))*a^2*b*e^5+60/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arct
an((2*c*x+b)/(4*a*c-b^2)^(1/2))*d*a^2*e^4+10/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)
/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^3*e^5-60/(16*a^2
*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a
*b*d^2*e^3+40*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))*d^3*a*e^2+20/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(
1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*d^3*e^2-30*c/(16*a^2*c^2-8*a*b
^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*d^4*e+12*
c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^
2)^(1/2))*d^5-1/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*
c*x+b)/(4*a*c-b^2)^(1/2))*b^5*e^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.36605, size = 8014, normalized size = 20.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/2*((b^5*c^3 - 14*a*b^3*c^4 + 40*a^2*b*c^5)*d^5 + 5*(a*b^4*c^3 + 4*a^2*b
^2*c^4 - 32*a^3*c^5)*d^4*e - 60*(a^2*b^3*c^3 - 4*a^3*b*c^4)*d^3*e^2 + 10*(a
^2*b^4*c^2 + 4*a^3*b^2*c^3 - 32*a^4*c^4)*d^2*e^3 + 5*(a^2*b^5*c - 14*a^3*b^
3*c^2 + 40*a^4*b*c^3)*d*e^4 - 3*(a^2*b^6 - 11*a^3*b^4*c + 36*a^4*b^2*c^2 -
32*a^5*c^3)*e^5 - 2*(6*(b^2*c^6 - 4*a*c^7)*d^5 - 15*(b^3*c^5 - 4*a*b*c^6)*d
^4*e + 10*(b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^3*e^2 - 30*(a*b^3*c^4 - 4*a
^2*b*c^5)*d^2*e^3 - 5*(b^6*c^2 - 12*a*b^4*c^3 + 42*a^2*b^2*c^4 - 40*a^3*c^5
)*d*e^4 + (2*b^7*c - 23*a*b^5*c^2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^4)*e^5)*x^
```

3 - (18*(b^3*c^5 - 4*a*b*c^6)*d^5 - 45*(b^4*c^4 - 4*a*b^2*c^5)*d^4*e + 30*(
b^5*c^3 - 2*a*b^3*c^4 - 8*a^2*b*c^5)*d^3*e^2 - 10*(b^6*c^2 - 3*a*b^4*c^3 +
12*a^2*b^2*c^4 - 64*a^3*c^5)*d^2*e^3 - 5*(b^7*c - 12*a*b^5*c^2 + 30*a^2*b^3
*c^3 + 8*a^3*b*c^4)*d*e^4 + (3*b^8 - 31*a*b^6*c + 87*a^2*b^4*c^2 - 12*a^3*b
^2*c^3 - 128*a^4*c^4)*e^5)*x^2 + (12*a^2*c^5*d^5 - 30*a^2*b*c^4*d^4*e - 60*
a^3*b*c^3*d^2*e^3 + 60*a^4*c^3*d*e^4 + 20*(a^2*b^2*c^3 + 2*a^3*c^4)*d^3*e^2
 - (a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2)*e^5 + (12*c^7*d^5 - 30*b*c^6*d^4
*e - 60*a*b*c^5*d^2*e^3 + 60*a^2*c^5*d*e^4 + 20*(b^2*c^5 + 2*a*c^6)*d^3*e^2
 - (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*e^5)*x^4 + 2*(12*b*c^6*d^5 - 30*
b^2*c^5*d^4*e - 60*a*b^2*c^4*d^2*e^3 + 60*a^2*b*c^4*d*e^4 + 20*(b^3*c^4 + 2
*a*b*c^5)*d^3*e^2 - (b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*e^5)*x^3 + (12*
(b^2*c^5 + 2*a*c^6)*d^5 - 30*(b^3*c^4 + 2*a*b*c^5)*d^4*e + 20*(b^4*c^3 + 4*
a*b^2*c^4 + 4*a^2*c^5)*d^3*e^2 - 60*(a*b^3*c^3 + 2*a^2*b*c^4)*d^2*e^3 + 60*
(a^2*b^2*c^3 + 2*a^3*c^4)*d*e^4 - (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^
3*b*c^3)*e^5)*x^2 + 2*(12*a*b*c^5*d^5 - 30*a*b^2*c^4*d^4*e - 60*a^2*b^2*c^3
*d^2*e^3 + 60*a^3*b*c^3*d*e^4 + 20*(a*b^3*c^3 + 2*a^2*b*c^4)*d^3*e^2 - (a*b
^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*e^5)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^
2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a
)) - 2*(2*(b^4*c^4 + a*b^2*c^5 - 20*a^2*c^6)*d^5 - 5*(b^5*c^3 + a*b^3*c^4 -
 20*a^2*b*c^5)*d^4*e + 10*(5*a*b^4*c^3 - 22*a^2*b^2*c^4 + 8*a^3*c^5)*d^3*e^
2 - 10*(a*b^5*c^2 + a^2*b^3*c^3 - 20*a^3*b*c^4)*d^2*e^3 - 5*(a*b^6*c - 14*a
^2*b^4*c^2 + 46*a^3*b^2*c^3 - 24*a^4*c^4)*d*e^4 + (3*a*b^7 - 34*a^2*b^5*c +
 119*a^3*b^3*c^2 - 124*a^4*b*c^3)*e^5)*x - ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^
2*b^2*c^4 - 64*a^3*c^5)*e^5*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3
 - 64*a^3*b*c^4)*e^5*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c
^3 - 128*a^4*c^4)*e^5*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a
^4*b*c^3)*e^5*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^
5)*log(c*x^2 + b*x + a))/(a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 - 6
4*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*x^4 + 2*
(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*x^3 + (b^8*c^3 - 1
0*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6 - 128*a^4*c^7)*x^2 + 2*(a*b^7
*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^4*b*c^6)*x), -1/2*((b^5*c^3 -
 14*a*b^3*c^4 + 40*a^2*b*c^5)*d^5 + 5*(a*b^4*c^3 + 4*a^2*b^2*c^4 - 32*a^3*c
^5)*d^4*e - 60*(a^2*b^3*c^3 - 4*a^3*b*c^4)*d^3*e^2 + 10*(a^2*b^4*c^2 + 4*a^
3*b^2*c^3 - 32*a^4*c^4)*d^2*e^3 + 5*(a^2*b^5*c - 14*a^3*b^3*c^2 + 40*a^4*b*
c^3)*d*e^4 - 3*(a^2*b^6 - 11*a^3*b^4*c + 36*a^4*b^2*c^2 - 32*a^5*c^3)*e^5 -
 2*(6*(b^2*c^6 - 4*a*c^7)*d^5 - 15*(b^3*c^5 - 4*a*b*c^6)*d^4*e + 10*(b^4*c^
4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^3*e^2 - 30*(a*b^3*c^4 - 4*a^2*b*c^5)*d^2*e^3
 - 5*(b^6*c^2 - 12*a*b^4*c^3 + 42*a^2*b^2*c^4 - 40*a^3*c^5)*d*e^4 + (2*b^7*
c - 23*a*b^5*c^2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^4)*e^5)*x^3 - (18*(b^3*c^5
 - 4*a*b*c^6)*d^5 - 45*(b^4*c^4 - 4*a*b^2*c^5)*d^4*e + 30*(b^5*c^3 - 2*a*b^3
*c^4 - 8*a^2*b*c^5)*d^3*e^2 - 10*(b^6*c^2 - 3*a*b^4*c^3 + 12*a^2*b^2*c^4 -
64*a^3*c^5)*d^2*e^3 - 5*(b^7*c - 12*a*b^5*c^2 + 30*a^2*b^3*c^3 + 8*a^3*b*c^
4)*d*e^4 + (3*b^8 - 31*a*b^6*c + 87*a^2*b^4*c^2 - 12*a^3*b^2*c^3 - 128*a^4*
c^4)*e^5)*x^2 + 2*(12*a^2*c^5*d^5 - 30*a^2*b*c^4*d^4*e - 60*a^3*b*c^3*d^2*e
^3 + 60*a^4*c^3*d*e^4 + 20*(a^2*b^2*c^3 + 2*a^3*c^4)*d^3*e^2 - (a^2*b^5 - 1
0*a^3*b^3*c + 30*a^4*b*c^2)*e^5 + (12*c^7*d^5 - 30*b*c^6*d^4*e - 60*a*b*c^5
*d^2*e^3 + 60*a^2*c^5*d*e^4 + 20*(b^2*c^5 + 2*a*c^6)*d^3*e^2 - (b^5*c^2 - 1
0*a*b^3*c^3 + 30*a^2*b*c^4)*e^5)*x^4 + 2*(12*b*c^6*d^5 - 30*b^2*c^5*d^4*e -
 60*a*b^2*c^4*d^2*e^3 + 60*a^2*b*c^4*d*e^4 + 20*(b^3*c^4 + 2*a*b*c^5)*d^3*e
^2 - (b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*e^5)*x^3 + (12*(b^2*c^5 + 2*a*
c^6)*d^5 - 30*(b^3*c^4 + 2*a*b*c^5)*d^4*e + 20*(b^4*c^3 + 4*a*b^2*c^4 + 4*a
^2*c^5)*d^3*e^2 - 60*(a*b^3*c^3 + 2*a^2*b*c^4)*d^2*e^3 + 60*(a^2*b^2*c^3 +
2*a^3*c^4)*d*e^4 - (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*e^5)*x
^2 + 2*(12*a*b*c^5*d^5 - 30*a*b^2*c^4*d^4*e - 60*a^2*b^2*c^3*d^2*e^3 + 60*a
^3*b*c^3*d*e^4 + 20*(a*b^3*c^3 + 2*a^2*b*c^4)*d^3*e^2 - (a*b^6 - 10*a^2*b^4
*c + 30*a^3*b^2*c^2)*e^5)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*
(2*c*x + b)/(b^2 - 4*a*c)) - 2*(2*(b^4*c^4 + a*b^2*c^5 - 20*a^2*c^6)*d^5 -
5*(b^5*c^3 + a*b^3*c^4 - 20*a^2*b*c^5)*d^4*e + 10*(5*a*b^4*c^3 - 22*a^2*b^2

```
*c^4 + 8*a^3*c^5)*d^3*e^2 - 10*(a*b^5*c^2 + a^2*b^3*c^3 - 20*a^3*b*c^4)*d^2
*e^3 - 5*(a*b^6*c - 14*a^2*b^4*c^2 + 46*a^3*b^2*c^3 - 24*a^4*c^4)*d*e^4 + (
3*a*b^7 - 34*a^2*b^5*c + 119*a^3*b^3*c^2 - 124*a^4*b*c^3)*e^5)*x - ((b^6*c^
2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^5*x^4 + 2*(b^7*c - 12*a*b
^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^5*x^3 + (b^8 - 10*a*b^6*c + 24*a^
2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*e^5*x^2 + 2*(a*b^7 - 12*a^2*b^5*c
 + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*e^5*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*
b^2*c^2 - 64*a^5*c^3)*e^5)*log(c*x^2 + b*x + a))/(a^2*b^6*c^3 - 12*a^3*b^4*
c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^
7 - 64*a^3*c^8)*x^4 + 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b
*c^7)*x^3 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6 - 128
*a^4*c^7)*x^2 + 2*(a*b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^4*b*c
^6)*x)]
```

Sympy [B] time = 138.706, size = 3403, normalized size = 8.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5/(c*x**2+b*x+a)**3,x)
```

```
[Out] (e**5/(2*c**3) - sqrt(-(4*a*c - b**2)**5)*(b*e - 2*c*d)*(30*a**2*c**2*e**4
- 10*a*b**2*c*e**4 - 20*a*b*c**2*d*e**3 + 20*a*c**3*d**2*e**2 + b**4*e**4 +
 2*b**3*c*d*e**3 + 4*b**2*c**2*d**2*e**2 - 12*b*c**3*d**3*e + 6*c**4*d**4)/
(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a*
*2*b**6*c**2 + 20*a*b**8*c - b**10)))*log(x + (-64*a**3*c**5*(e**5/(2*c**3)
 - sqrt(-(4*a*c - b**2)**5)*(b*e - 2*c*d)*(30*a**2*c**2*e**4 - 10*a*b**2*c*
e**4 - 20*a*b*c**2*d*e**3 + 20*a*c**3*d**2*e**2 + b**4*e**4 + 2*b**3*c*d*e*
*3 + 4*b**2*c**2*d**2*e**2 - 12*b*c**3*d**3*e + 6*c**4*d**4)/(2*c**3*(1024*
a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 +
 20*a*b**8*c - b**10))) + 32*a**3*c**2*e**5 + 48*a**2*b**2*c**4*(e**5/(2*c*
*3) - sqrt(-(4*a*c - b**2)**5)*(b*e - 2*c*d)*(30*a**2*c**2*e**4 - 10*a*b**2
*c*e**4 - 20*a*b*c**2*d*e**3 + 20*a*c**3*d**2*e**2 + b**4*e**4 + 2*b**3*c*d
*e**3 + 4*b**2*c**2*d**2*e**2 - 12*b*c**3*d**3*e + 6*c**4*d**4)/(2*c**3*(10
24*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**
2 + 20*a*b**8*c - b**10))) - 9*a**2*b**2*c*e**5 - 30*a**2*b*c**2*d*e**4 - 1
2*a*b**4*c**3*(e**5/(2*c**3) - sqrt(-(4*a*c - b**2)**5)*(b*e - 2*c*d)*(30*a
**2*c**2*e**4 - 10*a*b**2*c*e**4 - 20*a*b*c**2*d*e**3 + 20*a*c**3*d**2*e**2
 + b**4*e**4 + 2*b**3*c*d*e**3 + 4*b**2*c**2*d**2*e**2 - 12*b*c**3*d**3*e +
 6*c**4*d**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4
*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) + a*b**4*e**5 + 30*a*b*
*2*c**2*d**2*e**3 - 20*a*b*c**3*d**3*e**2 + b**6*c**2*(e**5/(2*c**3) - sqrt
(-(4*a*c - b**2)**5)*(b*e - 2*c*d)*(30*a**2*c**2*e**4 - 10*a*b**2*c*e**4 -
20*a*b*c**2*d*e**3 + 20*a*c**3*d**2*e**2 + b**4*e**4 + 2*b**3*c*d*e**3 + 4*
b**2*c**2*d**2*e**2 - 12*b*c**3*d**3*e + 6*c**4*d**4)/(2*c**3*(1024*a**5*c*
*5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b
**8*c - b**10))) - 10*b**3*c**2*d**3*e**2 + 15*b**2*c**3*d**4*e - 6*b*c**4*
d**5)/(30*a**2*b*c**2*e**5 - 60*a**2*c**3*d*e**4 - 10*a*b**3*c*e**5 + 60*a*
b*c**3*d**2*e**3 - 40*a*c**4*d**3*e**2 + b**5*e**5 - 20*b**2*c**3*d**3*e**2
 + 30*b*c**4*d**4*e - 12*c**5*d**5)) + (e**5/(2*c**3) + sqrt(-(4*a*c - b**2
)**5)*(b*e - 2*c*d)*(30*a**2*c**2*e**4 - 10*a*b**2*c*e**4 - 20*a*b*c**2*d*e
**3 + 20*a*c**3*d**2*e**2 + b**4*e**4 + 2*b**3*c*d*e**3 + 4*b**2*c**2*d**2*
e**2 - 12*b*c**3*d**3*e + 6*c**4*d**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*
b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))
)*log(x + (-64*a**3*c**5*(e**5/(2*c**3) + sqrt(-(4*a*c - b**2)**5)*(b*e - 2
*c*d)*(30*a**2*c**2*e**4 - 10*a*b**2*c*e**4 - 20*a*b*c**2*d*e**3 + 20*a*c**
```

```
3*d**2*e**2 + b**4*e**4 + 2*b**3*c*d*e**3 + 4*b**2*c**2*d**2*e**2 - 12*b*c*
*3*d**3*e + 6*c**4*d**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 64
0*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) + 32*a**3*c*
*2*e**5 + 48*a**2*b**2*c**4*(e**5/(2*c**3) + sqrt(-(4*a*c - b**2)**5)*(b*e
- 2*c*d)*(30*a**2*c**2*e**4 - 10*a*b**2*c*e**4 - 20*a*b*c**2*d*e**3 + 20*a*
c**3*d**2*e**2 + b**4*e**4 + 2*b**3*c*d*e**3 + 4*b**2*c**2*d**2*e**2 - 12*b
*c**3*d**3*e + 6*c**4*d**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 +
 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) - 9*a**2*
b**2*c*e**5 - 30*a**2*b*c**2*d*e**4 - 12*a*b**4*c**3*(e**5/(2*c**3) + sqrt(
-(4*a*c - b**2)**5)*(b*e - 2*c*d)*(30*a**2*c**2*e**4 - 10*a*b**2*c*e**4 - 2
0*a*b*c**2*d*e**3 + 20*a*c**3*d**2*e**2 + b**4*e**4 + 2*b**3*c*d*e**3 + 4*b
**2*c**2*d**2*e**2 - 12*b*c**3*d**3*e + 6*c**4*d**4)/(2*c**3*(1024*a**5*c**
5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b*
*8*c - b**10))) + a*b**4*e**5 + 30*a*b**2*c**2*d**2*e**3 - 20*a*b*c**3*d**3
*e**2 + b**6*c**2*(e**5/(2*c**3) + sqrt(-(4*a*c - b**2)**5)*(b*e - 2*c*d)*(
30*a**2*c**2*e**4 - 10*a*b**2*c*e**4 - 20*a*b*c**2*d*e**3 + 20*a*c**3*d**2*
e**2 + b**4*e**4 + 2*b**3*c*d*e**3 + 4*b**2*c**2*d**2*e**2 - 12*b*c**3*d**3
*e + 6*c**4*d**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*
b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) - 10*b**3*c**2*d**3
*e**2 + 15*b**2*c**3*d**4*e - 6*b*c**4*d**5)/(30*a**2*b*c**2*e**5 - 60*a**2
*c**3*d*e**4 - 10*a*b**3*c*e**5 + 60*a*b*c**3*d**2*e**3 - 40*a*c**4*d**3*e*
*2 + b**5*e**5 - 20*b**2*c**3*d**3*e**2 + 30*b*c**4*d**4*e - 12*c**5*d**5))
 + (24*a**4*c**2*e**5 - 21*a**3*b**2*c*e**5 + 50*a**3*b*c**2*d*e**4 - 80*a*
*3*c**3*d**2*e**3 + 3*a**2*b**4*e**5 - 5*a**2*b**3*c*d*e**4 - 10*a**2*b**2*
c**2*d**2*e**3 + 60*a**2*b*c**3*d**3*e**2 - 40*a**2*c**4*d**4*e - 5*a*b**2*
c**3*d**4*e + 10*a*b*c**4*d**5 - b**3*c**3*d**5 + x**3*(50*a**2*b*c**3*e**5
 - 100*a**2*c**4*d*e**4 - 30*a*b**3*c**2*e**5 + 80*a*b**2*c**3*d*e**4 - 60*
a*b*c**4*d**2*e**3 + 40*a*c**5*d**3*e**2 + 4*b**5*c*e**5 - 10*b**4*c**2*d*e
**4 + 20*b**2*c**4*d**3*e**2 - 30*b*c**5*d**4*e + 12*c**6*d**5) + x**2*(32*
a**3*c**3*e**5 + 11*a**2*b**2*c**2*e**5 + 10*a**2*b*c**3*d*e**4 - 160*a**2*
c**4*d**2*e**3 - 19*a*b**4*c*e**5 + 40*a*b**3*c**2*d*e**4 - 10*a*b**2*c**3*
d**2*e**3 + 60*a*b*c**4*d**3*e**2 + 3*b**6*e**5 - 5*b**5*c*d*e**4 - 10*b**4
*c**2*d**2*e**3 + 30*b**3*c**3*d**3*e**2 - 45*b**2*c**4*d**4*e + 18*b*c**5*
d**5) + x*(62*a**3*b*c**2*e**5 - 60*a**3*c**3*d*e**4 - 44*a**2*b**3*c*e**5
+ 100*a**2*b**2*c**2*d*e**4 - 100*a**2*b*c**3*d**2*e**3 - 40*a**2*c**4*d**3
*e**2 + 6*a*b**5*e**5 - 10*a*b**4*c*d*e**4 - 20*a*b**3*c**2*d**2*e**3 + 100
*a*b**2*c**3*d**3*e**2 - 50*a*b*c**4*d**4*e + 20*a*c**5*d**5 - 10*b**3*c**3
*d**4*e + 4*b**2*c**4*d**5))/(32*a**4*c**5 - 16*a**3*b**2*c**4 + 2*a**2*b**
4*c**3 + x**4*(32*a**2*c**7 - 16*a*b**2*c**6 + 2*b**4*c**5) + x**3*(64*a**2
*b*c**6 - 32*a*b**3*c**5 + 4*b**5*c**4) + x**2*(64*a**3*c**6 - 12*a*b**4*c*
*4 + 2*b**6*c**3) + x*(64*a**3*b*c**5 - 32*a**2*b**3*c**4 + 4*a*b**5*c**3))
```

Giac [B] time = 1.13553, size = 1087, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] (12*c^5*d^5 - 30*b*c^4*d^4*e + 20*b^2*c^3*d^3*e^2 + 40*a*c^4*d^3*e^2 - 60*a
*b*c^3*d^2*e^3 + 60*a^2*c^3*d*e^4 - b^5*e^5 + 10*a*b^3*c*e^5 - 30*a^2*b*c^2
*e^5)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4*c^3 - 8*a*b^2*c^4 + 16*a
^2*c^5)*sqrt(-b^2 + 4*a*c)) + 1/2*e^5*log(c*x^2 + b*x + a)/c^3 - 1/2*(b^3*c
^3*d^5 - 10*a*b*c^4*d^5 + 5*a*b^2*c^3*d^4*e + 40*a^2*c^4*d^4*e - 60*a^2*b*c
^3*d^3*e^2 + 10*a^2*b^2*c^2*d^2*e^3 + 80*a^3*c^3*d^2*e^3 + 5*a^2*b^3*c*d*e^
4 - 50*a^3*b*c^2*d*e^4 - 3*a^2*b^4*e^5 + 21*a^3*b^2*c*e^5 - 24*a^4*c^2*e^5
```

```
- 2*(6*c^6*d^5 - 15*b*c^5*d^4*e + 10*b^2*c^4*d^3*e^2 + 20*a*c^5*d^3*e^2 - 3
0*a*b*c^4*d^2*e^3 - 5*b^4*c^2*d*e^4 + 40*a*b^2*c^3*d*e^4 - 50*a^2*c^4*d*e^4
 + 2*b^5*c*e^5 - 15*a*b^3*c^2*e^5 + 25*a^2*b*c^3*e^5)*x^3 - (18*b*c^5*d^5 -
 45*b^2*c^4*d^4*e + 30*b^3*c^3*d^3*e^2 + 60*a*b*c^4*d^3*e^2 - 10*b^4*c^2*d^
2*e^3 - 10*a*b^2*c^3*d^2*e^3 - 160*a^2*c^4*d^2*e^3 - 5*b^5*c*d*e^4 + 40*a*b
^3*c^2*d*e^4 + 10*a^2*b*c^3*d*e^4 + 3*b^6*e^5 - 19*a*b^4*c*e^5 + 11*a^2*b^2
*c^2*e^5 + 32*a^3*c^3*e^5)*x^2 - 2*(2*b^2*c^4*d^5 + 10*a*c^5*d^5 - 5*b^3*c^
3*d^4*e - 25*a*b*c^4*d^4*e + 50*a*b^2*c^3*d^3*e^2 - 20*a^2*c^4*d^3*e^2 - 10
*a*b^3*c^2*d^2*e^3 - 50*a^2*b*c^3*d^2*e^3 - 5*a*b^4*c*d*e^4 + 50*a^2*b^2*c^
2*d*e^4 - 30*a^3*c^3*d*e^4 + 3*a*b^5*e^5 - 22*a^2*b^3*c*e^5 + 31*a^3*b*c^2*
e^5)*x)/((c*x^2 + b*x + a)^2*(b^2 - 4*a*c)^2*c^3)
```

3.2202 $\int \frac{(d+ex)^4}{(a+bx+cx^2)^3}\,dx$

Optimal. Leaf size=169

$$\frac{3(d+ex)\left(ae^2-bde+cd^2\right)(-2ae+x(2cd-be)+bd)}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{12\left(ae^2-bde+cd^2\right)^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}-\frac{(d+ex)^3(-2ae+x(2cd\ldots}{2\left(b^2-4ac\right)\left(a+bx\ldots}$$

```
[Out] -((d + e*x)^3*(b*d - 2*a*e + (2*c*d - b*e)*x))/(2*(b^2 - 4*a*c)*(a + b*x +
c*x^2)^2) + (3*(c*d^2 - b*d*e + a*e^2)*(d + e*x)*(b*d - 2*a*e + (2*c*d - b*
e)*x))/((b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - (12*(c*d^2 - b*d*e + a*e^2)^2*
ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 0.122729, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {722, 618, 206}

$$\frac{3(d+ex)\left(ae^2-bde+cd^2\right)(-2ae+x(2cd-be)+bd)}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{12\left(ae^2-bde+cd^2\right)^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}-\frac{(d+ex)^3(-2ae+x(2cd\ldots}{2\left(b^2-4ac\right)\left(a+bx\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a + b*x + c*x^2)^3,x]
```


```
[Out] -((d + e*x)^3*(b*d - 2*a*e + (2*c*d - b*e)*x))/(2*(b^2 - 4*a*c)*(a + b*x +
c*x^2)^2) + (3*(c*d^2 - b*d*e + a*e^2)*(d + e*x)*(b*d - 2*a*e + (2*c*d - b*
e)*x))/((b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - (12*(c*d^2 - b*d*e + a*e^2)^2*
ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```


Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^4}{\left(a+bx+cx^2\right)^3}\,dx = -\frac{(d+ex)^3(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{\left(3\left(cd^2-bde+ae^2\right)\right)\int \frac{(d+ex)^2}{\left(a+bx+cx^2\right)^2}\,dx}{b^2-4ac}$$

$$= -\frac{(d+ex)^3(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3\left(cd^2-bde+ae^2\right)(d+ex)(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \cdots$$

$$= -\frac{(d+ex)^3(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3\left(cd^2-bde+ae^2\right)(d+ex)(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \cdots$$

$$= -\frac{(d+ex)^3(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3\left(cd^2-bde+ae^2\right)(d+ex)(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \cdots$$

Mathematica [B] time = 0.598989, size = 413, normalized size = 2.44

$$\frac{1}{2}\left(\frac{bc\left(-3a^2e^4+6acde^2(d+2ex)+c^2d^3(d-4ex)\right)+2c^2\left(a^2e^3(4d+ex)-2acd^2e(2d+3ex)+c^2d^4x\right)+2b^2ce^2\left(3cd^2x-\cdots\right.}{c^3\left(4ac-b^2\right)(a+x(b+cx))^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a + b*x + c*x^2)^3,x]
```

```
[Out] ((b^4*e^4*x + b^3*e^3*(a*e - 4*c*d*x) + 2*b^2*c*e^2*(3*c*d^2*x - 2*a*e*(d +
 e*x)) + b*c*(-3*a^2*e^4 + c^2*d^3*(d - 4*e*x) + 6*a*c*d*e^2*(d + 2*e*x)) +
 2*c^2*(c^2*d^4*x + a^2*e^3*(4*d + e*x) - 2*a*c*d^2*e*(2*d + 3*e*x)))/(c^3*
(-b^2 + 4*a*c)*(a + x*(b + c*x))^2) + (b^5*e^4 + 2*b^3*c*e^2*(3*c*d^2 - 4*a
*e^2) - 2*b^4*c*e^3*(2*d + e*x) + 2*b*c^2*(11*a^2*e^4 + 3*c^2*d^3*(d - 4*e*
x) + 6*a*c*d*e^2*(d - 2*e*x)) + 4*b^2*c^2*e*(-3*c*d^2*(d - e*x) + a*e^2*(5*
d + 4*e*x)) + 4*c^3*(3*c^2*d^4*x + 6*a*c*d^2*e^2*x - a^2*e^3*(16*d + 5*e*x)
))/(c^3*(b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (24*(c*d^2 + e*(-(b*d) + a*e))
^2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/2
```

Maple [B] time = 0.162, size = 932, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(c*x^2+b*x+a)^3,x)
```

```
[Out] (-(10*a^2*c^2*e^4-8*a*b^2*c*e^4+12*a*b*c^2*d*e^3-12*a*c^3*d^2*e^2+b^4*e^4-6
*b^2*c^2*d^2*e^2+12*b*c^3*d^3*e-6*c^4*d^4)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3
+1/2*(2*a^2*b*c^2*e^4-64*a^2*c^3*d*e^3+8*a*b^3*c*e^4-4*a*b^2*c^2*d*e^3+36*a
*b*c^3*d^2*e^2-b^5*e^4-4*b^4*c*d*e^3+18*b^3*c^2*d^2*e^2-36*b^2*c^3*d^3*e+18
*b*c^4*d^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^2*x^2-(6*a^3*c^2*e^4-10*a^2*b^2*c*
e^4+20*a^2*b*c^2*d*e^3+12*a^2*c^3*d^2*e^2+a*b^4*e^4+4*a*b^3*c*d*e^3-30*a*b^
2*c^2*d^2*e^2+20*a*b*c^3*d^3*e-10*a*c^4*d^4+4*b^3*c^2*d^3*e-2*b^2*c^3*d^4)/
(16*a^2*c^2-8*a*b^2*c+b^4)/c^2*x+1/2/c^2*(10*a^3*b*c*e^4-32*a^3*c^2*d*e^3-a
^2*b^3*e^4-4*a^2*b^2*c*d*e^3+36*a^2*b*c^2*d^2*e^2-32*a^2*c^3*d^3*e-4*a*b^2*
c^2*d^3*e+10*a*b*c^3*d^4-b^3*c^2*d^4)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*
```

```
x+a)^2+12/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*
a*c-b^2)^(1/2))*a^2*e^4-24/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arc
tan((2*c*x+b)/(4*a*c-b^2)^(1/2))*d*e^3*a*b+24/(16*a^2*c^2-8*a*b^2*c+b^4)/(4
*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c*d^2*e^2+12/(16*a^2*
c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^
2*d^2*e^2-24/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/
(4*a*c-b^2)^(1/2))*b*c*d^3*e+12/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2
)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^2*d^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 2.34423, size = 5349, normalized size = 31.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x+a)^3,x, algorithm="fricas")

[Out] [-1/2*((b^5*c^2 - 14*a*b^3*c^3 + 40*a^2*b*c^4)*d^4 + 4*(a*b^4*c^2 + 4*a^2*b
^2*c^3 - 32*a^3*c^4)*d^3*e - 36*(a^2*b^3*c^2 - 4*a^3*b*c^3)*d^2*e^2 + 4*(a^
2*b^4*c + 4*a^3*b^2*c^2 - 32*a^4*c^3)*d*e^3 + (a^2*b^5 - 14*a^3*b^3*c + 40*
a^4*b*c^2)*e^4 - 2*(6*(b^2*c^5 - 4*a*c^6)*d^4 - 12*(b^3*c^4 - 4*a*b*c^5)*d^
3*e + 6*(b^4*c^3 - 2*a*b^2*c^4 - 8*a^2*c^5)*d^2*e^2 - 12*(a*b^3*c^3 - 4*a^2
*b*c^4)*d*e^3 - (b^6*c - 12*a*b^4*c^2 + 42*a^2*b^2*c^3 - 40*a^3*c^4)*e^4)*x
^3 - (18*(b^3*c^4 - 4*a*b*c^5)*d^4 - 36*(b^4*c^3 - 4*a*b^2*c^4)*d^3*e + 18*
(b^5*c^2 - 2*a*b^3*c^3 - 8*a^2*b*c^4)*d^2*e^2 - 4*(b^6*c - 3*a*b^4*c^2 + 12
*a^2*b^2*c^3 - 64*a^3*c^4)*d*e^3 - (b^7 - 12*a*b^5*c + 30*a^2*b^3*c^2 + 8*a
^3*b*c^3)*e^4)*x^2 - 12*(a^2*c^4*d^4 - 2*a^2*b*c^3*d^3*e - 2*a^3*b*c^2*d*e^
3 + a^4*c^2*e^4 + (a^2*b^2*c^2 + 2*a^3*c^3)*d^2*e^2 + (c^6*d^4 - 2*b*c^5*d^
3*e - 2*a*b*c^4*d*e^3 + a^2*c^4*e^4 + (b^2*c^4 + 2*a*c^5)*d^2*e^2)*x^4 + 2*
(b*c^5*d^4 - 2*b^2*c^4*d^3*e - 2*a*b^2*c^3*d*e^3 + a^2*b*c^3*e^4 + (b^3*c^3
 + 2*a*b*c^4)*d^2*e^2)*x^3 + ((b^2*c^4 + 2*a*c^5)*d^4 - 2*(b^3*c^3 + 2*a*b*
c^4)*d^3*e + (b^4*c^2 + 4*a*b^2*c^3 + 4*a^2*c^4)*d^2*e^2 - 2*(a*b^3*c^2 + 2
*a^2*b*c^3)*d*e^3 + (a^2*b^2*c^2 + 2*a^3*c^3)*e^4)*x^2 + 2*(a*b*c^4*d^4 - 2
*a*b^2*c^3*d^3*e - 2*a^2*b^2*c^2*d*e^3 + a^3*b*c^2*e^4 + (a*b^3*c^2 + 2*a^2
*b*c^3)*d^2*e^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*
c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(2*(b^4*c^3 + a*b
^2*c^4 - 20*a^2*c^5)*d^4 - 4*(b^5*c^2 + a*b^3*c^3 - 20*a^2*b*c^4)*d^3*e + 6
*(5*a*b^4*c^2 - 22*a^2*b^2*c^3 + 8*a^3*c^4)*d^2*e^2 - 4*(a*b^5*c + a^2*b^3*
c^2 - 20*a^3*b*c^3)*d*e^3 - (a*b^6 - 14*a^2*b^4*c + 46*a^3*b^2*c^2 - 24*a^4
*c^3)*e^4)*x)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5 +
 (b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*x^4 + 2*(b^7*c^3 -
12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*x^3 + (b^8*c^2 - 10*a*b^6*c^3
 + 24*a^2*b^4*c^4 + 32*a^3*b^2*c^5 - 128*a^4*c^6)*x^2 + 2*(a*b^7*c^2 - 12*a
^2*b^5*c^3 + 48*a^3*b^3*c^4 - 64*a^4*b*c^5)*x), -1/2*((b^5*c^2 - 14*a*b^3*c
^3 + 40*a^2*b*c^4)*d^4 + 4*(a*b^4*c^2 + 4*a^2*b^2*c^3 - 32*a^3*c^4)*d^3*e -
```

```
 36*(a^2*b^3*c^2 - 4*a^3*b*c^3)*d^2*e^2 + 4*(a^2*b^4*c + 4*a^3*b^2*c^2 - 32
*a^4*c^3)*d*e^3 + (a^2*b^5 - 14*a^3*b^3*c + 40*a^4*b*c^2)*e^4 - 2*(6*(b^2*c
^5 - 4*a*c^6)*d^4 - 12*(b^3*c^4 - 4*a*b*c^5)*d^3*e + 6*(b^4*c^3 - 2*a*b^2*c
^4 - 8*a^2*c^5)*d^2*e^2 - 12*(a*b^3*c^3 - 4*a^2*b*c^4)*d*e^3 - (b^6*c - 12*
a*b^4*c^2 + 42*a^2*b^2*c^3 - 40*a^3*c^4)*e^4)*x^3 - (18*(b^3*c^4 - 4*a*b*c^
5)*d^4 - 36*(b^4*c^3 - 4*a*b^2*c^4)*d^3*e + 18*(b^5*c^2 - 2*a*b^3*c^3 - 8*a
^2*b*c^4)*d^2*e^2 - 4*(b^6*c - 3*a*b^4*c^2 + 12*a^2*b^2*c^3 - 64*a^3*c^4)*d
*e^3 - (b^7 - 12*a*b^5*c + 30*a^2*b^3*c^2 + 8*a^3*b*c^3)*e^4)*x^2 + 24*(a^2
*c^4*d^4 - 2*a^2*b*c^3*d^3*e - 2*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 + (a^2*b^2*c
^2 + 2*a^3*c^3)*d^2*e^2 + (c^6*d^4 - 2*b*c^5*d^3*e - 2*a*b*c^4*d*e^3 + a^2*
c^4*e^4 + (b^2*c^4 + 2*a*c^5)*d^2*e^2)*x^4 + 2*(b*c^5*d^4 - 2*b^2*c^4*d^3*e
 - 2*a*b^2*c^3*d*e^3 + a^2*b*c^3*e^4 + (b^3*c^3 + 2*a*b*c^4)*d^2*e^2)*x^3 +
 ((b^2*c^4 + 2*a*c^5)*d^4 - 2*(b^3*c^3 + 2*a*b*c^4)*d^3*e + (b^4*c^2 + 4*a*
b^2*c^3 + 4*a^2*c^4)*d^2*e^2 - 2*(a*b^3*c^2 + 2*a^2*b*c^3)*d*e^3 + (a^2*b^2
*c^2 + 2*a^3*c^3)*e^4)*x^2 + 2*(a*b*c^4*d^4 - 2*a*b^2*c^3*d^3*e - 2*a^2*b^2
*c^2*d*e^3 + a^3*b*c^2*e^4 + (a*b^3*c^2 + 2*a^2*b*c^3)*d^2*e^2)*x)*sqrt(-b^
2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 2*(2*(b^
4*c^3 + a*b^2*c^4 - 20*a^2*c^5)*d^4 - 4*(b^5*c^2 + a*b^3*c^3 - 20*a^2*b*c^4
)*d^3*e + 6*(5*a*b^4*c^2 - 22*a^2*b^2*c^3 + 8*a^3*c^4)*d^2*e^2 - 4*(a*b^5*c
 + a^2*b^3*c^2 - 20*a^3*b*c^3)*d*e^3 - (a*b^6 - 14*a^2*b^4*c + 46*a^3*b^2*c
^2 - 24*a^4*c^3)*e^4)*x)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 6
4*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*x^4 + 2*
(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*x^3 + (b^8*c^2 - 1
0*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2*c^5 - 128*a^4*c^6)*x^2 + 2*(a*b^7
*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 - 64*a^4*b*c^5)*x)]
```

Sympy [B] time = 38.2636, size = 1355, normalized size = 8.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(c*x**2+b*x+a)**3,x)
```

```
[Out] -6*sqrt(-1/(4*a*c - b**2)**5)*(a*e**2 - b*d*e + c*d**2)**2*log(x + (-384*a*
*3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(a*e**2 - b*d*e + c*d**2)**2 + 288*a**2*
b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(a*e**2 - b*d*e + c*d**2)**2 + 6*a**2*
b*e**4 - 72*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(a*e**2 - b*d*e + c*d**2)**
2 - 12*a*b**2*d*e**3 + 12*a*b*c*d**2*e**2 + 6*b**6*sqrt(-1/(4*a*c - b**2)**
5)*(a*e**2 - b*d*e + c*d**2)**2 + 6*b**3*d**2*e**2 - 12*b**2*c*d**3*e + 6*b
*c**2*d**4)/(12*a**2*c*e**4 - 24*a*b*c*d*e**3 + 24*a*c**2*d**2*e**2 + 12*b*
*2*c*d**2*e**2 - 24*b*c**2*d**3*e + 12*c**3*d**4)) + 6*sqrt(-1/(4*a*c - b**
2)**5)*(a*e**2 - b*d*e + c*d**2)**2*log(x + (384*a**3*c**3*sqrt(-1/(4*a*c -
 b**2)**5)*(a*e**2 - b*d*e + c*d**2)**2 - 288*a**2*b**2*c**2*sqrt(-1/(4*a*c
 - b**2)**5)*(a*e**2 - b*d*e + c*d**2)**2 + 6*a**2*b*e**4 + 72*a*b**4*c*sqr
t(-1/(4*a*c - b**2)**5)*(a*e**2 - b*d*e + c*d**2)**2 - 12*a*b**2*d*e**3 + 1
2*a*b*c*d**2*e**2 - 6*b**6*sqrt(-1/(4*a*c - b**2)**5)*(a*e**2 - b*d*e + c*d
**2)**2 + 6*b**3*d**2*e**2 - 12*b**2*c*d**3*e + 6*b*c**2*d**4)/(12*a**2*c*e
**4 - 24*a*b*c*d*e**3 + 24*a*c**2*d**2*e**2 + 12*b**2*c*d**2*e**2 - 24*b*c*
*2*d**3*e + 12*c**3*d**4)) - (-10*a**3*b*c*e**4 + 32*a**3*c**2*d*e**3 + a**
2*b**3*e**4 + 4*a**2*b**2*c*d*e**3 - 36*a**2*b*c**2*d**2*e**2 + 32*a**2*c**
3*d**3*e + 4*a*b**2*c**2*d**3*e - 10*a*b*c**3*d**4 + b**3*c**2*d**4 + x**3*
(20*a**2*c**3*e**4 - 16*a*b**2*c**2*e**4 + 24*a*b*c**3*d*e**3 - 24*a*c**4*d
**2*e**2 + 2*b**4*c*e**4 - 12*b**2*c**3*d**2*e**2 + 24*b*c**4*d**3*e - 12*c
**5*d**4) + x**2*(-2*a**2*b*c**2*e**4 + 64*a**2*c**3*d*e**3 - 8*a*b**3*c*e*
*4 + 4*a*b**2*c**2*d*e**3 - 36*a*b*c**3*d**2*e**2 + b**5*e**4 + 4*b**4*c*d*
e**3 - 18*b**3*c**2*d**2*e**2 + 36*b**2*c**3*d**3*e - 18*b*c**4*d**4) + x*(
```

```
12*a**3*c**2*e**4 - 20*a**2*b**2*c*e**4 + 40*a**2*b*c**2*d*e**3 + 24*a**2*c
**3*d**2*e**2 + 2*a*b**4*e**4 + 8*a*b**3*c*d*e**3 - 60*a*b**2*c**2*d**2*e**
2 + 40*a*b*c**3*d**3*e - 20*a*c**4*d**4 + 8*b**3*c**2*d**3*e - 4*b**2*c**3*
d**4))/(32*a**4*c**4 - 16*a**3*b**2*c**3 + 2*a**2*b**4*c**2 + x**4*(32*a**2
*c**6 - 16*a*b**2*c**5 + 2*b**4*c**4) + x**3*(64*a**2*b*c**5 - 32*a*b**3*c*
*4 + 4*b**5*c**3) + x**2*(64*a**3*c**5 - 12*a*b**4*c**3 + 2*b**6*c**2) + x*
(64*a**3*b*c**4 - 32*a**2*b**3*c**3 + 4*a*b**5*c**2))
```

Giac [B] time = 1.13065, size = 845, normalized size = 5.

$$\frac{12\left(c^2d^4 - 2\,bcd^3e + b^2d^2e^2 + 2\,acd^2e^2 - 2\,abde^3 + a^2e^4\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4 - 8\,ab^2c + 16\,a^2c^2\right)\sqrt{-b^2+4\,ac}} + \frac{12\,c^5d^4x^3 - 24\,bc^4d^3x^3e + 18\,bc^4d^4x^2 + 12\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 12*(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2
*e^4)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c^2
)*sqrt(-b^2 + 4*a*c)) + 1/2*(12*c^5*d^4*x^3 - 24*b*c^4*d^3*x^3*e + 18*b*c^4
*d^4*x^2 + 12*b^2*c^3*d^2*x^3*e^2 + 24*a*c^4*d^2*x^3*e^2 - 36*b^2*c^3*d^3*x
^2*e + 4*b^2*c^3*d^4*x + 20*a*c^4*d^4*x - 24*a*b*c^3*d*x^3*e^3 + 18*b^3*c^2
*d^2*x^2*e^2 + 36*a*b*c^3*d^2*x^2*e^2 - 8*b^3*c^2*d^3*x*e - 40*a*b*c^3*d^3*
x*e - b^3*c^2*d^4 + 10*a*b*c^3*d^4 - 2*b^4*c*x^3*e^4 + 16*a*b^2*c^2*x^3*e^4
 - 20*a^2*c^3*x^3*e^4 - 4*b^4*c*d*x^2*e^3 - 4*a*b^2*c^2*d*x^2*e^3 - 64*a^2*
c^3*d*x^2*e^3 + 60*a*b^2*c^2*d^2*x*e^2 - 24*a^2*c^3*d^2*x*e^2 - 4*a*b^2*c^2
*d^3*e - 32*a^2*c^3*d^3*e - b^5*x^2*e^4 + 8*a*b^3*c*x^2*e^4 + 2*a^2*b*c^2*x
^2*e^4 - 8*a*b^3*c*d*x*e^3 - 40*a^2*b*c^2*d*x*e^3 + 36*a^2*b*c^2*d^2*e^2 -
2*a*b^4*x*e^4 + 20*a^2*b^2*c*x*e^4 - 12*a^3*c^2*x*e^4 - 4*a^2*b^2*c*d*e^3 -
 32*a^3*c^2*d*e^3 - a^2*b^3*e^4 + 10*a^3*b*c*e^4)/((b^4*c^2 - 8*a*b^2*c^3 +
 16*a^2*c^4)*(c*x^2 + b*x + a)^2)
```

3.2203 $\int \frac{(d+ex)^3}{(a+bx+cx^2)^3} dx$

Optimal. Leaf size=158

$$-\frac{6(2cd-be)\left(ae^2-bde+cd^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}-\frac{(b+2cx)(d+ex)^3}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{3(d+ex)(2cd-be)(-2ae+x(2cd-}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right.}$$

```
[Out] -((b + 2*c*x)*(d + e*x)^3)/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*(2*c*
d - b*e)*(d + e*x)*(b*d - 2*a*e + (2*c*d - b*e)*x))/(2*(b^2 - 4*a*c)^2*(a +
 b*x + c*x^2)) - (6*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*ArcTanh[(b + 2*c*
x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 0.0882377, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {728, 722, 618, 206}

$$-\frac{6(2cd-be)\left(ae^2-bde+cd^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}-\frac{(b+2cx)(d+ex)^3}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{3(d+ex)(2cd-be)(-2ae+x(2cd-}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + b*x + c*x^2)^3,x]
```

```
[Out] -((b + 2*c*x)*(d + e*x)^3)/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*(2*c*
d - b*e)*(d + e*x)*(b*d - 2*a*e + (2*c*d - b*e)*x))/(2*(b^2 - 4*a*c)^2*(a +
 b*x + c*x^2)) - (6*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*ArcTanh[(b + 2*c*
x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 728

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] + Dist[(m*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(
d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(a+bx+cx^2\right)^3}\,dx = -\frac{(b+2cx)(d+ex)^3}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{(3(2cd-be))\int \frac{(d+ex)^2}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{(b+2cx)(d+ex)^3}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3(2cd-be)(d+ex)(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{\left(3(2cd-be)\left(cd^2\right.\right.}{}$$

$$= -\frac{(b+2cx)(d+ex)^3}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3(2cd-be)(d+ex)(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{\left(6(2cd-be)\left(cd^2\right.\right.}{}$$

$$= -\frac{(b+2cx)(d+ex)^3}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3(2cd-be)(d+ex)(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{6(2cd-be)\left(cd^2\right.}{}$$

Mathematica [A] time = 0.440086, size = 308, normalized size = 1.95

$$\frac{1}{2}\left(\frac{4c^2\left(-4a^2e^3+3acde^2x+3c^2d^3x\right)+b^2ce\left(5ae^2-9cd^2+6cdex\right)+6bc^2\left(ae^2(d-ex)+cd^2(d-3ex)\right)+3b^3cde^2+b^4\left(-e\right.}{c^2\left(b^2-4ac\right)^2(a+x(b+cx))}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + b*x + c*x^2)^3,x]
```

```
[Out] ((3*b^3*c*d*e^2 - b^4*e^3 + b^2*c*e*(-9*c*d^2 + 5*a*e^2 + 6*c*d*e*x) + 4*c^
2*(-4*a^2*e^3 + 3*c^2*d^3*x + 3*a*c*d*e^2*x) + 6*b*c^2*(c*d^2*(d - 3*e*x) +
 a*e^2*(d - e*x)))/(c^2*(b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (-(b^3*e^3*x)
+ b^2*e^2*(-(a*e) + 3*c*d*x) + 2*c*(a^2*e^3 + c^2*d^3*x - 3*a*c*d*e*(d + e*
x)) + b*c*(c*d^2*(d - 3*e*x) + 3*a*e^2*(d + e*x)))/(c^2*(-b^2 + 4*a*c)*(a +
 x*(b + c*x))^2) + (12*(2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e))*ArcTan[(b +
 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/2
```

Maple [B] time = 0.16, size = 695, normalized size = 4.4

$$\frac{1}{\left(cx^2+bx+a\right)^2}\left(-3\,\frac{c\left(abe^3-2\,ade^2c-b^2de^2+3\,bcd^2e-2\,c^2d^3\right)x^3}{16\,a^2c^2-8\,acb^2+b^4} - \frac{\left(16\,a^2c^2e^3+ab^2ce^3-18\,abc^2de^2+b^4e^3-9\,b^3cde^2\right.}{2\,c\left(16\,a^2c^2-8\,acb^2+b^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^2+b*x+a)^3,x)
```

```
[Out] (-3*c*(a*b*e^3-2*a*c*d*e^2-b^2*d*e^2+3*b*c*d^2*e-2*c^2*d^3)/(16*a^2*c^2-8*a
*b^2*c+b^4)*x^3-1/2*(16*a^2*c^2*e^3+a*b^2*c*e^3-18*a*b*c^2*d*e^2+b^4*e^3-9*
b^3*c*d*e^2+27*b^2*c^2*d^2*e-18*b*c^3*d^3)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2
```

```
-1/c*(5*a^2*b*c*e^3+6*a^2*c^2*d*e^2+a*b^3*e^3-15*a*b^2*c*d*e^2+15*a*b*c^2*d
^2*e-10*a*c^3*d^3+3*b^3*c*d^2*e-2*b^2*c^2*d^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x
-1/2/c*(8*a^3*c*e^3+a^2*b^2*e^3-18*a^2*b*c*d*e^2+24*a^2*c^2*d^2*e+3*a*b^2*c
*d^2*e-10*a*b*c^2*d^3+b^3*c*d^3)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^
2-6/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^
2)^(1/2))*a*b*e^3+12/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2
*c*x+b)/(4*a*c-b^2)^(1/2))*a*d*e^2*c+6/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^
2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*d*e^2-18/(16*a^2*c^2-8*a*b
^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c*d^2*e+1
2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)
^(1/2))*c^2*d^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.22842, size = 4259, normalized size = 26.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/2*((b^5*c - 14*a*b^3*c^2 + 40*a^2*b*c^3)*d^3 + 3*(a*b^4*c + 4*a^2*b^2*c
^2 - 32*a^3*c^3)*d^2*e - 18*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^2 + (a^2*b^4 + 4*
a^3*b^2*c - 32*a^4*c^2)*e^3 - 6*(2*(b^2*c^4 - 4*a*c^5)*d^3 - 3*(b^3*c^3 - 4
*a*b*c^4)*d^2*e + (b^4*c^2 - 2*a*b^2*c^3 - 8*a^2*c^4)*d*e^2 - (a*b^3*c^2 -
4*a^2*b*c^3)*e^3)*x^3 - (18*(b^3*c^3 - 4*a*b*c^4)*d^3 - 27*(b^4*c^2 - 4*a*b
^2*c^3)*d^2*e + 9*(b^5*c - 2*a*b^3*c^2 - 8*a^2*b*c^3)*d*e^2 - (b^6 - 3*a*b^
4*c + 12*a^2*b^2*c^2 - 64*a^3*c^3)*e^3)*x^2 + 6*(2*a^2*c^3*d^3 - 3*a^2*b*c^
2*d^2*e - a^3*b*c*e^3 + (2*c^5*d^3 - 3*b*c^4*d^2*e - a*b*c^3*e^3 + (b^2*c^3
 + 2*a*c^4)*d*e^2)*x^4 + (a^2*b^2*c + 2*a^3*c^2)*d*e^2 + 2*(2*b*c^4*d^3 - 3
*b^2*c^3*d^2*e - a*b^2*c^2*e^3 + (b^3*c^2 + 2*a*b*c^3)*d*e^2)*x^3 + (2*(b^2
*c^3 + 2*a*c^4)*d^3 - 3*(b^3*c^2 + 2*a*b*c^3)*d^2*e + (b^4*c + 4*a*b^2*c^2
+ 4*a^2*c^3)*d*e^2 - (a*b^3*c + 2*a^2*b*c^2)*e^3)*x^2 + 2*(2*a*b*c^3*d^3 -
3*a*b^2*c^2*d^2*e - a^2*b^2*c*e^3 + (a*b^3*c + 2*a^2*b*c^2)*d*e^2)*x)*sqrt(
b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*
c*x + b))/(c*x^2 + b*x + a)) - 2*(2*(b^4*c^2 + a*b^2*c^3 - 20*a^2*c^4)*d^3
- 3*(b^5*c + a*b^3*c^2 - 20*a^2*b*c^3)*d^2*e + 3*(5*a*b^4*c - 22*a^2*b^2*c^
2 + 8*a^3*c^3)*d*e^2 - (a*b^5 + a^2*b^3*c - 20*a^3*b*c^2)*e^3)*x)/(a^2*b^6*
c - 12*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^3 - 12*a*b^4*c^4
+ 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^4 + 2*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3
*c^4 - 64*a^3*b*c^5)*x^3 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*b^4*c^3 + 32*a^3*
b^2*c^4 - 128*a^4*c^5)*x^2 + 2*(a*b^7*c - 12*a^2*b^5*c^2 + 48*a^3*b^3*c^3 -
 64*a^4*b*c^4)*x), -1/2*((b^5*c - 14*a*b^3*c^2 + 40*a^2*b*c^3)*d^3 + 3*(a*b
^4*c + 4*a^2*b^2*c^2 - 32*a^3*c^3)*d^2*e - 18*(a^2*b^3*c - 4*a^3*b*c^2)*d*e
^2 + (a^2*b^4 + 4*a^3*b^2*c - 32*a^4*c^2)*e^3 - 6*(2*(b^2*c^4 - 4*a*c^5)*d^
3 - 3*(b^3*c^3 - 4*a*b*c^4)*d^2*e + (b^4*c^2 - 2*a*b^2*c^3 - 8*a^2*c^4)*d*e
```

```
^2 - (a*b^3*c^2 - 4*a^2*b*c^3)*e^3)*x^3 - (18*(b^3*c^3 - 4*a*b*c^4)*d^3 - 2
7*(b^4*c^2 - 4*a*b^2*c^3)*d^2*e + 9*(b^5*c - 2*a*b^3*c^2 - 8*a^2*b*c^3)*d*e
^2 - (b^6 - 3*a*b^4*c + 12*a^2*b^2*c^2 - 64*a^3*c^3)*e^3)*x^2 + 12*(2*a^2*c
^3*d^3 - 3*a^2*b*c^2*d^2*e - a^3*b*c*e^3 + (2*c^5*d^3 - 3*b*c^4*d^2*e - a*b
*c^3*e^3 + (b^2*c^3 + 2*a*c^4)*d*e^2)*x^4 + (a^2*b^2*c + 2*a^3*c^2)*d*e^2 +
 2*(2*b*c^4*d^3 - 3*b^2*c^3*d^2*e - a*b^2*c^2*e^3 + (b^3*c^2 + 2*a*b*c^3)*d
*e^2)*x^3 + (2*(b^2*c^3 + 2*a*c^4)*d^3 - 3*(b^3*c^2 + 2*a*b*c^3)*d^2*e + (b
^4*c + 4*a*b^2*c^2 + 4*a^2*c^3)*d*e^2 - (a*b^3*c + 2*a^2*b*c^2)*e^3)*x^2 +
2*(2*a*b*c^3*d^3 - 3*a*b^2*c^2*d^2*e - a^2*b^2*c*e^3 + (a*b^3*c + 2*a^2*b*c
^2)*d*e^2)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^
2 - 4*a*c)) - 2*(2*(b^4*c^2 + a*b^2*c^3 - 20*a^2*c^4)*d^3 - 3*(b^5*c + a*b^
3*c^2 - 20*a^2*b*c^3)*d^2*e + 3*(5*a*b^4*c - 22*a^2*b^2*c^2 + 8*a^3*c^3)*d*
e^2 - (a*b^5 + a^2*b^3*c - 20*a^3*b*c^2)*e^3)*x)/(a^2*b^6*c - 12*a^3*b^4*c^
2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5
- 64*a^3*c^6)*x^4 + 2*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c
^5)*x^3 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*b^4*c^3 + 32*a^3*b^2*c^4 - 128*a^4
*c^5)*x^2 + 2*(a*b^7*c - 12*a^2*b^5*c^2 + 48*a^3*b^3*c^3 - 64*a^4*b*c^4)*x)
]
```

Sympy [B] time = 10.7488, size = 1180, normalized size = 7.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**2+b*x+a)**3,x)
```

```
[Out] 3*sqrt(-1/(4*a*c - b**2)**5)*(b*e - 2*c*d)*(a*e**2 - b*d*e + c*d**2)*log(x
+ (-192*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(b*e - 2*c*d)*(a*e**2 - b*d*e
+ c*d**2) + 144*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(b*e - 2*c*d)*(a*
e**2 - b*d*e + c*d**2) - 36*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(b*e - 2*c*
d)*(a*e**2 - b*d*e + c*d**2) + 3*a*b**2*e**3 - 6*a*b*c*d*e**2 + 3*b**6*sqrt
(-1/(4*a*c - b**2)**5)*(b*e - 2*c*d)*(a*e**2 - b*d*e + c*d**2) - 3*b**3*d*e
**2 + 9*b**2*c*d**2*e - 6*b*c**2*d**3)/(6*a*b*c*e**3 - 12*a*c**2*d*e**2 - 6
*b**2*c*d*e**2 + 18*b*c**2*d**2*e - 12*c**3*d**3)) - 3*sqrt(-1/(4*a*c - b**
2)**5)*(b*e - 2*c*d)*(a*e**2 - b*d*e + c*d**2)*log(x + (192*a**3*c**3*sqrt(
-1/(4*a*c - b**2)**5)*(b*e - 2*c*d)*(a*e**2 - b*d*e + c*d**2) - 144*a**2*b*
*2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(b*e - 2*c*d)*(a*e**2 - b*d*e + c*d**2)
+ 36*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(b*e - 2*c*d)*(a*e**2 - b*d*e + c*
d**2) + 3*a*b**2*e**3 - 6*a*b*c*d*e**2 - 3*b**6*sqrt(-1/(4*a*c - b**2)**5)*
(b*e - 2*c*d)*(a*e**2 - b*d*e + c*d**2) - 3*b**3*d*e**2 + 9*b**2*c*d**2*e -
 6*b*c**2*d**3)/(6*a*b*c*e**3 - 12*a*c**2*d*e**2 - 6*b**2*c*d*e**2 + 18*b*c
**2*d**2*e - 12*c**3*d**3)) - (8*a**3*c*e**3 + a**2*b**2*e**3 - 18*a**2*b*c
*d*e**2 + 24*a**2*c**2*d**2*e + 3*a*b**2*c*d**2*e - 10*a*b*c**2*d**3 + b**3
*c*d**3 + x**3*(6*a*b*c**2*e**3 - 12*a*c**3*d*e**2 - 6*b**2*c**2*d*e**2 + 1
8*b*c**3*d**2*e - 12*c**4*d**3) + x**2*(16*a**2*c**2*e**3 + a*b**2*c*e**3 -
 18*a*b*c**2*d*e**2 + b**4*e**3 - 9*b**3*c*d*e**2 + 27*b**2*c**2*d**2*e - 1
8*b*c**3*d**3) + x*(10*a**2*b*c*e**3 + 12*a**2*c**2*d*e**2 + 2*a*b**3*e**3
- 30*a*b**2*c*d*e**2 + 30*a*b*c**2*d**2*e - 20*a*c**3*d**3 + 6*b**3*c*d**2*
e - 4*b**2*c**2*d**3))/(32*a**4*c**3 - 16*a**3*b**2*c**2 + 2*a**2*b**4*c +
x**4*(32*a**2*c**5 - 16*a*b**2*c**4 + 2*b**4*c**3) + x**3*(64*a**2*b*c**4 -
 32*a*b**3*c**3 + 4*b**5*c**2) + x**2*(64*a**3*c**4 - 12*a*b**4*c**2 + 2*b*
*6*c) + x*(64*a**3*b*c**3 - 32*a**2*b**3*c**2 + 4*a*b**5*c))
```

Giac [B] time = 1.19454, size = 602, normalized size = 3.81

$$\frac{6\left(2\,c^2d^3-3\,bcd^2e+b^2de^2+2\,acde^2-abe^3\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}+\frac{12\,c^4d^3x^3-18\,bc^3d^2x^3e+18\,bc^3d^3x^2+6\,b^2c^2dx^3e^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 6*(2*c^2*d^3 - 3*b*c*d^2*e + b^2*d*e^2 + 2*a*c*d*e^2 - a*b*e^3)*arctan((2*c
*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a
*c)) + 1/2*(12*c^4*d^3*x^3 - 18*b*c^3*d^2*x^3*e + 18*b*c^3*d^3*x^2 + 6*b^2*
c^2*d*x^3*e^2 + 12*a*c^3*d*x^3*e^2 - 27*b^2*c^2*d^2*x^2*e + 4*b^2*c^2*d^3*x
 + 20*a*c^3*d^3*x - 6*a*b*c^2*x^3*e^3 + 9*b^3*c*d*x^2*e^2 + 18*a*b*c^2*d*x^
2*e^2 - 6*b^3*c*d^2*x*e - 30*a*b*c^2*d^2*x*e - b^3*c*d^3 + 10*a*b*c^2*d^3 -
 b^4*x^2*e^3 - a*b^2*c*x^2*e^3 - 16*a^2*c^2*x^2*e^3 + 30*a*b^2*c*d*x*e^2 -
12*a^2*c^2*d*x*e^2 - 3*a*b^2*c*d^2*e - 24*a^2*c^2*d^2*e - 2*a*b^3*x*e^3 - 1
0*a^2*b*c*x*e^3 + 18*a^2*b*c*d*e^2 - a^2*b^2*e^3 - 8*a^3*c*e^3)/((b^4*c - 8
*a*b^2*c^2 + 16*a^2*c^3)*(c*x^2 + b*x + a)^2)
```

3.2204 $\int \frac{(d+ex)^2}{(a+bx+cx^2)^3}\,dx$

Optimal. Leaf size=198

$$-\frac{-x\left(-2ce(3bd-ae)+b^2e^2+6c^2d^2\right)-3b\left(ae^2+cd^2\right)+4acde+2b^2de}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{2\left(-2ce(3bd-ae)+b^2e^2+6c^2d^2\right)\tanh^{-1}\left(\frac{b+2}{\sqrt{b^2-}}\right.}{\left(b^2-4ac\right)^{5/2}}$$

```
[Out] -((d + e*x)*(b*d - 2*a*e + (2*c*d - b*e)*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*
x^2)^2) - (2*b^2*d*e + 4*a*c*d*e - 3*b*(c*d^2 + a*e^2) - (6*c^2*d^2 + b^2*e
^2 - 2*c*e*(3*b*d - a*e))*x)/((b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - (2*(6*c^
2*d^2 + b^2*e^2 - 2*c*e*(3*b*d - a*e))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c
]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 0.21828, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {738, 638, 618, 206}

$$-\frac{-x\left(-2ce(3bd-ae)+b^2e^2+6c^2d^2\right)-3b\left(ae^2+cd^2\right)+4acde+2b^2de}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{2\left(-2ce(3bd-ae)+b^2e^2+6c^2d^2\right)\tanh^{-1}\left(\frac{b+2}{\sqrt{b^2-}}\right.}{\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*x + c*x^2)^3,x]
```

```
[Out] -((d + e*x)*(b*d - 2*a*e + (2*c*d - b*e)*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*
x^2)^2) - (2*b^2*d*e + 4*a*c*d*e - 3*b*(c*d^2 + a*e^2) - (6*c^2*d^2 + b^2*e
^2 - 2*c*e*(3*b*d - a*e))*x)/((b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - (2*(6*c^
2*d^2 + b^2*e^2 - 2*c*e*(3*b*d - a*e))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c
]])/(b^2 - 4*a*c)^(5/2)
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(a+bx+cx^2\right)^3}\,dx = -\frac{(d+ex)(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{\int \frac{2\left(3cd^2-e(2bd-ae)\right)+2e(2cd-be)x}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{(d+ex)(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{2b^2de+4acde-3b\left(cd^2+ae^2\right)-\left(6c^2d^2+b^2e^2-2ce(3b\right.}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}$$

$$= -\frac{(d+ex)(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{2b^2de+4acde-3b\left(cd^2+ae^2\right)-\left(6c^2d^2+b^2e^2-2ce(3b\right.}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}$$

$$= -\frac{(d+ex)(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{2b^2de+4acde-3b\left(cd^2+ae^2\right)-\left(6c^2d^2+b^2e^2-2ce(3b\right.}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}$$

Mathematica [A] time = 0.309917, size = 203, normalized size = 1.03

$$\frac{1}{2}\left(\frac{(b+2cx)\left(2ce(ae-3bd)+b^2e^2+6c^2d^2\right)}{c\left(b^2-4ac\right)^2(a+x(b+cx))} + \frac{abe^2-2ace(2d+ex)+b^2e^2x+bcd(d-2ex)+2c^2d^2x}{c\left(4ac-b^2\right)(a+x(b+cx))^2} + \frac{4\left(2ce(ae-3b\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*x + c*x^2)^3,x]
```

```
[Out] (((6*c^2*d^2 + b^2*e^2 + 2*c*e*(-3*b*d + a*e))*(b + 2*c*x))/(c*(b^2 - 4*a*c
)^2*(a + x*(b + c*x))) + (a*b*e^2 + 2*c^2*d^2*x + b^2*e^2*x + b*c*d*(d - 2*
e*x) - 2*a*c*e*(2*d + e*x))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))^2) + (4*(6*
c^2*d^2 + b^2*e^2 + 2*c*e*(-3*b*d + a*e))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*
a*c]])/(-b^2 + 4*a*c)^(5/2))/2
```

Maple [B] time = 0.159, size = 508, normalized size = 2.6

$$\frac{1}{\left(cx^2+bx+a\right)^2}\left(\frac{c\left(2ace^2+b^2e^2-6bcde+6c^2d^2\right)x^3}{16a^2c^2-8acb^2+b^4} + \frac{3b\left(2ace^2+b^2e^2-6bcde+6c^2d^2\right)x^2}{32a^2c^2-16acb^2+2b^4} - \frac{\left(2a^2ce^2-5ab^2e^2+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^2+b*x+a)^3,x)
```

```
[Out] (c*(2*a*c*e^2+b^2*e^2-6*b*c*d*e+6*c^2*d^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+3
/2*b*(2*a*c*e^2+b^2*e^2-6*b*c*d*e+6*c^2*d^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2
-(2*a^2*c*e^2-5*a*b^2*e^2+10*a*b*c*d*e-10*a*c^2*d^2+2*b^3*d*e-2*b^2*c*d^2)/
```

```
(16*a^2*c^2-8*a*b^2*c+b^4)*x+1/2*(6*a^2*b*e^2-16*a^2*c*d*e-2*a*b^2*d*e+10*a
*b*c*d^2-b^3*d^2)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+4/(16*a^2*c^2
-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c*e
^2+2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b
^2)^(1/2))*b^2*e^2-12/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((
2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c*d*e+12/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^
2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^2*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.15503, size = 3270, normalized size = 16.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(6*(b^2*c^3 - 4*a*c^4)*d^2 - 6*(b^3*c^2 - 4*a*b*c^3)*d*e + (b^4*c -
 2*a*b^2*c^2 - 8*a^2*c^3)*e^2)*x^3 - (b^5 - 14*a*b^3*c + 40*a^2*b*c^2)*d^2
- 2*(a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2)*d*e + 6*(a^2*b^3 - 4*a^3*b*c)*e^2 +
3*(6*(b^3*c^2 - 4*a*b*c^3)*d^2 - 6*(b^4*c - 4*a*b^2*c^2)*d*e + (b^5 - 2*a*b
^3*c - 8*a^2*b*c^2)*e^2)*x^2 + 2*(6*a^2*c^2*d^2 - 6*a^2*b*c*d*e + (6*c^4*d^
2 - 6*b*c^3*d*e + (b^2*c^2 + 2*a*c^3)*e^2)*x^4 + 2*(6*b*c^3*d^2 - 6*b^2*c^2
*d*e + (b^3*c + 2*a*b*c^2)*e^2)*x^3 + (a^2*b^2 + 2*a^3*c)*e^2 + (6*(b^2*c^2
 + 2*a*c^3)*d^2 - 6*(b^3*c + 2*a*b*c^2)*d*e + (b^4 + 4*a*b^2*c + 4*a^2*c^2)
*e^2)*x^2 + 2*(6*a*b*c^2*d^2 - 6*a*b^2*c*d*e + (a*b^3 + 2*a^2*b*c)*e^2)*x)*
sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c
)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(2*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d
^2 - 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*d*e + (5*a*b^4 - 22*a^2*b^2*c + 8*a^3
*c^2)*e^2)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*
c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5
*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*
c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*
b^3*c^2 - 64*a^4*b*c^3)*x), 1/2*(2*(6*(b^2*c^3 - 4*a*c^4)*d^2 - 6*(b^3*c^2
- 4*a*b*c^3)*d*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*e^2)*x^3 - (b^5 - 14*a
*b^3*c + 40*a^2*b*c^2)*d^2 - 2*(a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2)*d*e + 6*(
a^2*b^3 - 4*a^3*b*c)*e^2 + 3*(6*(b^3*c^2 - 4*a*b*c^3)*d^2 - 6*(b^4*c - 4*a*
b^2*c^2)*d*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*e^2)*x^2 - 4*(6*a^2*c^2*d^2
- 6*a^2*b*c*d*e + (6*c^4*d^2 - 6*b*c^3*d*e + (b^2*c^2 + 2*a*c^3)*e^2)*x^4 +
 2*(6*b*c^3*d^2 - 6*b^2*c^2*d*e + (b^3*c + 2*a*b*c^2)*e^2)*x^3 + (a^2*b^2 +
 2*a^3*c)*e^2 + (6*(b^2*c^2 + 2*a*c^3)*d^2 - 6*(b^3*c + 2*a*b*c^2)*d*e + (b
^4 + 4*a*b^2*c + 4*a^2*c^2)*e^2)*x^2 + 2*(6*a*b*c^2*d^2 - 6*a*b^2*c*d*e + (
a*b^3 + 2*a^2*b*c)*e^2)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2
*c*x + b)/(b^2 - 4*a*c)) + 2*(2*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d^2 - 2*(b
^5 + a*b^3*c - 20*a^2*b*c^2)*d*e + (5*a*b^4 - 22*a^2*b^2*c + 8*a^3*c^2)*e^2
)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*
```

```
a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48
*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*
a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 -
 64*a^4*b*c^3)*x)]
```

Sympy [B] time = 3.86347, size = 1052, normalized size = 5.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**2+b*x+a)**3,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**5)*(2*a*c*e**2 + b**2*e**2 - 6*b*c*d*e + 6*c**2*d*
*2)*log(x + (-64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c*e**2 + b**2*e*
*2 - 6*b*c*d*e + 6*c**2*d**2) + 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5
)*(2*a*c*e**2 + b**2*e**2 - 6*b*c*d*e + 6*c**2*d**2) - 12*a*b**4*c*sqrt(-1/
(4*a*c - b**2)**5)*(2*a*c*e**2 + b**2*e**2 - 6*b*c*d*e + 6*c**2*d**2) + 2*a
*b*c*e**2 + b**6*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c*e**2 + b**2*e**2 - 6*b*c
*d*e + 6*c**2*d**2) + b**3*e**2 - 6*b**2*c*d*e + 6*b*c**2*d**2)/(4*a*c**2*e
**2 + 2*b**2*c*e**2 - 12*b*c**2*d*e + 12*c**3*d**2)) + sqrt(-1/(4*a*c - b**
2)**5)*(2*a*c*e**2 + b**2*e**2 - 6*b*c*d*e + 6*c**2*d**2)*log(x + (64*a**3*
c**3*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c*e**2 + b**2*e**2 - 6*b*c*d*e + 6*c**
2*d**2) - 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c*e**2 + b**2*e
**2 - 6*b*c*d*e + 6*c**2*d**2) + 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(2*
a*c*e**2 + b**2*e**2 - 6*b*c*d*e + 6*c**2*d**2) + 2*a*b*c*e**2 - b**6*sqrt(
-1/(4*a*c - b**2)**5)*(2*a*c*e**2 + b**2*e**2 - 6*b*c*d*e + 6*c**2*d**2) +
b**3*e**2 - 6*b**2*c*d*e + 6*b*c**2*d**2)/(4*a*c**2*e**2 + 2*b**2*c*e**2 -
12*b*c**2*d*e + 12*c**3*d**2)) + (6*a**2*b*e**2 - 16*a**2*c*d*e - 2*a*b**2*
d*e + 10*a*b*c*d**2 - b**3*d**2 + x**3*(4*a*c**2*e**2 + 2*b**2*c*e**2 - 12*
b*c**2*d*e + 12*c**3*d**2) + x**2*(6*a*b*c*e**2 + 3*b**3*e**2 - 18*b**2*c*d
*e + 18*b*c**2*d**2) + x*(-4*a**2*c*e**2 + 10*a*b**2*e**2 - 20*a*b*c*d*e +
20*a*c**2*d**2 - 4*b**3*d*e + 4*b**2*c*d**2))/(32*a**4*c**2 - 16*a**3*b**2*
c + 2*a**2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c**2) + x**3
*(64*a**2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**3 - 12*a*b
**4*c + 2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))
```

Giac [A] time = 1.10699, size = 414, normalized size = 2.09

$$\frac{2\left(6\,c^2d^2-6\,bcde+b^2e^2+2\,ace^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}+\frac{12\,c^3d^2x^3-12\,bc^2dx^3e+18\,bc^2d^2x^2+2\,b^2cx^3e^2+4\,ac^2x^3e^2-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 2*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2 + 2*a*c*e^2)*arctan((2*c*x + b)/sqrt(-b^
2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(12*c
^3*d^2*x^3 - 12*b*c^2*d*x^3*e + 18*b*c^2*d^2*x^2 + 2*b^2*c*x^3*e^2 + 4*a*c^
2*x^3*e^2 - 18*b^2*c*d*x^2*e + 4*b^2*c*d^2*x + 20*a*c^2*d^2*x + 3*b^3*x^2*e
^2 + 6*a*b*c*x^2*e^2 - 4*b^3*d*x*e - 20*a*b*c*d*x*e - b^3*d^2 + 10*a*b*c*d^
2 + 10*a*b^2*x*e^2 - 4*a^2*c*x*e^2 - 2*a*b^2*d*e - 16*a^2*c*d*e + 6*a^2*b*e
^2)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c*x^2 + b*x + a)^2)
```

3.2205 $\int \frac{d+ex}{\left(a+bx+cx^2\right)^3}\,dx$

Optimal. Leaf size=131

$$\frac{3(b+2cx)(2cd-be)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{-2ae+x(2cd-be)+bd}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{6c(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

```
[Out] -(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3
*(2*c*d - b*e)*(b + 2*c*x))/(2*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - (6*c*(2
*c*d - b*e)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 0.0498186, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {638, 614, 618, 206}

$$\frac{3(b+2cx)(2cd-be)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{-2ae+x(2cd-be)+bd}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{6c(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*x + c*x^2)^3,x]
```

```
[Out] -(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3
*(2*c*d - b*e)*(b + 2*c*x))/(2*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - (6*c*(2
*c*d - b*e)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex}{\left(a+bx+cx^2\right)^3}\,dx = -\frac{bd-2ae+(2cd-be)x}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{(3(2cd-be))\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{bd-2ae+(2cd-be)x}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3(2cd-be)(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{(3c(2cd-be))\int \frac{1}{a+bx+cx^2}\,dx}{\left(b^2-4ac\right)^2}$$

$$= -\frac{bd-2ae+(2cd-be)x}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3(2cd-be)(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{(6c(2cd-be))\,\text{Subst}\left(\int \frac{1}{b^2-4a}\right.}{\left(b^2-4ac\right)^2}$$

$$= -\frac{bd-2ae+(2cd-be)x}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3(2cd-be)(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{6c(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.138294, size = 128, normalized size = 0.98

$$\frac{\frac{\left(b^2-4ac\right)(2ae-bd+bex-2cdx)}{(a+x(b+cx))^2} - \frac{12c(be-2cd)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{3(b+2cx)(2cd-be)}{a+x(b+cx)}}{2\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*x + c*x^2)^3,x]
```

```
[Out] (((b^2 - 4*a*c)*(-(b*d) + 2*a*e - 2*c*d*x + b*e*x))/(a + x*(b + c*x))^2 + (
3*(2*c*d - b*e)*(b + 2*c*x))/(a + x*(b + c*x)) - (12*c*(-2*c*d + b*e)*ArcTa
n[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(b^2 - 4*a*c)^2)
```

Maple [A] time = 0.152, size = 242, normalized size = 1.9

$$\frac{bd-2\,ae+(-be+2\,cd)\,x}{\left(8\,ac-2\,b^2\right)\left(cx^2+bx+a\right)^2} - 3\,\frac{bcxe}{\left(4\,ac-b^2\right)^2\left(cx^2+bx+a\right)} + 6\,\frac{c^2xd}{\left(4\,ac-b^2\right)^2\left(cx^2+bx+a\right)} - \frac{3\,b^2e}{2\,\left(4\,ac-b^2\right)^2\left(cx^2+bx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^2+b*x+a)^3,x)
```

```
[Out] 1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(4*a*c-b^2)/(c*x^2+b*x+a)^2-3/(4*a*c-b^2)^2/
(c*x^2+b*x+a)*x*c*b*e+6/(4*a*c-b^2)^2/(c*x^2+b*x+a)*x*c^2*d-3/2/(4*a*c-b^2)
^2/(c*x^2+b*x+a)*b^2*e+3/(4*a*c-b^2)^2/(c*x^2+b*x+a)*b*c*d-6/(4*a*c-b^2)^(5
/2)*c*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*e+12/(4*a*c-b^2)^(5/2)*c^2*arct
an((2*c*x+b)/(4*a*c-b^2)^(1/2))*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.15295, size = 2340, normalized size = 17.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/2*(6*(2*(b^2*c^3 - 4*a*c^4)*d - (b^3*c^2 - 4*a*b*c^3)*e)*x^3 + 9*(2*(b^3
*c^2 - 4*a*b*c^3)*d - (b^4*c - 4*a*b^2*c^2)*e)*x^2 - 6*(2*a^2*c^2*d - a^2*b
*c*e + (2*c^4*d - b*c^3*e)*x^4 + 2*(2*b*c^3*d - b^2*c^2*e)*x^3 + (2*(b^2*c^
2 + 2*a*c^3)*d - (b^3*c + 2*a*b*c^2)*e)*x^2 + 2*(2*a*b*c^2*d - a*b^2*c*e)*x
)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a
*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b^5 - 14*a*b^3*c + 40*a^2*b*c^2)*d -
 (a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2)*e + 2*(2*(b^4*c + a*b^2*c^2 - 20*a^2*c^
3)*d - (b^5 + a*b^3*c - 20*a^2*b*c^2)*e)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^
4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*
c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (
b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(
a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x), 1/2*(6*(2*(b^2*c^
3 - 4*a*c^4)*d - (b^3*c^2 - 4*a*b*c^3)*e)*x^3 + 9*(2*(b^3*c^2 - 4*a*b*c^3)*
d - (b^4*c - 4*a*b^2*c^2)*e)*x^2 - 12*(2*a^2*c^2*d - a^2*b*c*e + (2*c^4*d -
 b*c^3*e)*x^4 + 2*(2*b*c^3*d - b^2*c^2*e)*x^3 + (2*(b^2*c^2 + 2*a*c^3)*d -
(b^3*c + 2*a*b*c^2)*e)*x^2 + 2*(2*a*b*c^2*d - a*b^2*c*e)*x)*sqrt(-b^2 + 4*a
*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^5 - 14*a*b^3
*c + 40*a^2*b*c^2)*d - (a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2)*e + 2*(2*(b^4*c +
 a*b^2*c^2 - 20*a^2*c^3)*d - (b^5 + a*b^3*c - 20*a^2*b*c^2)*e)*x)/(a^2*b^6
- 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48
*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 -
 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 -
128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)
*x)]
```

Sympy [B] time = 2.26577, size = 651, normalized size = 4.97

$$3c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(be-2cd\right)\log\left(x+\frac{-192a^3c^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(be-2cd\right)+144a^2b^2c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(be-2cd\right)-36ab^4c^2\sqrt{-\frac{1}{(4}}}{6bc^2e-12c^3d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**2+b*x+a)**3,x)
```

```
[Out] 3*c*sqrt(-1/(4*a*c - b**2)**5)*(b*e - 2*c*d)*log(x + (-192*a**3*c**4*sqrt(-
1/(4*a*c - b**2)**5)*(b*e - 2*c*d) + 144*a**2*b**2*c**3*sqrt(-1/(4*a*c - b*
*2)**5)*(b*e - 2*c*d) - 36*a*b**4*c**2*sqrt(-1/(4*a*c - b**2)**5)*(b*e - 2*
c*d) + 3*b**6*c*sqrt(-1/(4*a*c - b**2)**5)*(b*e - 2*c*d) + 3*b**2*c*e - 6*b
*c**2*d)/(6*b*c**2*e - 12*c**3*d)) - 3*c*sqrt(-1/(4*a*c - b**2)**5)*(b*e -
2*c*d)*log(x + (192*a**3*c**4*sqrt(-1/(4*a*c - b**2)**5)*(b*e - 2*c*d) - 14
```

```
4*a**2*b**2*c**3*sqrt(-1/(4*a*c - b**2)**5)*(b*e - 2*c*d) + 36*a*b**4*c**2*
sqrt(-1/(4*a*c - b**2)**5)*(b*e - 2*c*d) - 3*b**6*c*sqrt(-1/(4*a*c - b**2)*
*5)*(b*e - 2*c*d) + 3*b**2*c*e - 6*b*c**2*d)/(6*b*c**2*e - 12*c**3*d)) - (8
*a**2*c*e + a*b**2*e - 10*a*b*c*d + b**3*d + x**3*(6*b*c**2*e - 12*c**3*d)
+ x**2*(9*b**2*c*e - 18*b*c**2*d) + x*(10*a*b*c*e - 20*a*c**2*d + 2*b**3*e
- 4*b**2*c*d))/(32*a**4*c**2 - 16*a**3*b**2*c + 2*a**2*b**4 + x**4*(32*a**2
*c**4 - 16*a*b**2*c**3 + 2*b**4*c**2) + x**3*(64*a**2*b*c**3 - 32*a*b**3*c*
*2 + 4*b**5*c) + x**2*(64*a**3*c**3 - 12*a*b**4*c + 2*b**6) + x*(64*a**3*b*
c**2 - 32*a**2*b**3*c + 4*a*b**5))
```

Giac [A] time = 1.12226, size = 278, normalized size = 2.12

$$\frac{6\left(2\,c^2d-bce\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}+\frac{12\,c^3dx^3-6\,bc^2x^3e+18\,bc^2dx^2-9\,b^2cx^2e+4\,b^2cdx+20\,ac^2dx-2\,b^3xe-10\,ab}{2\left(b^4-8\,ab^2c+16\,a^2c^2\right)\left(cx^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 6*(2*c^2*d - b*c*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*
c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(12*c^3*d*x^3 - 6*b*c^2*x^3*e + 1
8*b*c^2*d*x^2 - 9*b^2*c*x^2*e + 4*b^2*c*d*x + 20*a*c^2*d*x - 2*b^3*x*e - 10
*a*b*c*x*e - b^3*d + 10*a*b*c*d - a*b^2*e - 8*a^2*c*e)/((b^4 - 8*a*b^2*c +
16*a^2*c^2)*(c*x^2 + b*x + a)^2)
```

3.2206 $\int \frac{1}{(a+bx+cx^2)^3} \, dx$

Optimal. Leaf size=101

$$-\frac{12c^2 \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{3c(b+2cx)}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{b+2cx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

```
[Out] -(b + 2*c*x)/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*c*(b + 2*c*x))/((b^
2 - 4*a*c)^2*(a + b*x + c*x^2)) - (12*c^2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*
a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 0.0361321, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {614, 618, 206}

$$-\frac{12c^2 \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{3c(b+2cx)}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{b+2cx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(-3),x]
```

```
[Out] -(b + 2*c*x)/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*c*(b + 2*c*x))/((b^
2 - 4*a*c)^2*(a + b*x + c*x^2)) - (12*c^2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*
a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx+cx^2\right)^3}\,dx = -\frac{b+2cx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{(3c)\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx}{b^2-4ac}$$

$$= -\frac{b+2cx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3c(b+2cx)}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{\left(6c^2\right)\int \frac{1}{a+bx+cx^2}\,dx}{\left(b^2-4ac\right)^2}$$

$$= -\frac{b+2cx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3c(b+2cx)}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{\left(12c^2\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x\right.}{\left(b^2-4ac\right)^2}$$

$$= -\frac{b+2cx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3c(b+2cx)}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{12c^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.103199, size = 97, normalized size = 0.96

$$\frac{\frac{24c^2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - \frac{(b+2cx)\left(-2c\left(5a+3cx^2\right)+b^2-6bcx\right)}{(a+x(b+cx))^2}}{2\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(-3),x]
```

```
[Out] (-(((b + 2*c*x)*(b^2 - 6*b*c*x - 2*c*(5*a + 3*c*x^2)))/(a + x*(b + c*x))^2)
 + (24*c^2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(
b^2 - 4*a*c)^2)
```

Maple [A] time = 0.154, size = 129, normalized size = 1.3

$$\frac{2\,cx+b}{\left(8\,ac-2\,b^2\right)\left(cx^2+bx+a\right)^2} + 6\,\frac{c^2x}{\left(4\,ac-b^2\right)^2\left(cx^2+bx+a\right)} + 3\,\frac{bc}{\left(4\,ac-b^2\right)^2\left(cx^2+bx+a\right)} + 12\,\frac{c^2}{\left(4\,ac-b^2\right)^{5/2}}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^3,x)
```

```
[Out] 1/2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^2+6*c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)
*x+3*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)*b+12*c^2/(4*a*c-b^2)^(5/2)*arctan((2*c*x
+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.19359, size = 1685, normalized size = 16.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(b^5 - 14*a*b^3*c + 40*a^2*b*c^2 - 12*(b^2*c^3 - 4*a*c^4)*x^3 - 18*(b
^3*c^2 - 4*a*b*c^3)*x^2 - 12*(c^4*x^4 + 2*b*c^3*x^3 + 2*a*b*c^2*x + a^2*c^2
 + (b^2*c^2 + 2*a*c^3)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^
2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 4*(b^4*c +
a*b^2*c^2 - 20*a^2*c^3)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7
*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c
+ 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^
5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x), -1/2*(b^5 - 14*a*b^3*c + 40*a^2*b*
c^2 - 12*(b^2*c^3 - 4*a*c^4)*x^3 - 18*(b^3*c^2 - 4*a*b*c^3)*x^2 + 24*(c^4*x
^4 + 2*b*c^3*x^3 + 2*a*b*c^2*x + a^2*c^2 + (b^2*c^2 + 2*a*c^3)*x^2)*sqrt(-b
^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 4*(b^4*
c + a*b^2*c^2 - 20*a^2*c^3)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 6
4*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*
(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^
6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^
2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]
```

Sympy [B] time = 1.36351, size = 474, normalized size = 4.69

$$-6c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x+\frac{-384a^3c^5\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+288a^2b^2c^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-72ab^4c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+6b^6c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+\cdots}{12c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**3,x)
```

```
[Out] -6*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-384*a**3*c**5*sqrt(-1/(4*a*c -
 b**2)**5) + 288*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5) - 72*a*b**4*c**3
*sqrt(-1/(4*a*c - b**2)**5) + 6*b**6*c**2*sqrt(-1/(4*a*c - b**2)**5) + 6*b*
c**2)/(12*c**3)) + 6*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (384*a**3*c**5
*sqrt(-1/(4*a*c - b**2)**5) - 288*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5)
 + 72*a*b**4*c**3*sqrt(-1/(4*a*c - b**2)**5) - 6*b**6*c**2*sqrt(-1/(4*a*c -
 b**2)**5) + 6*b*c**2)/(12*c**3)) + (10*a*b*c - b**3 + 18*b*c**2*x**2 + 12*
c**3*x**3 + x*(20*a*c**2 + 4*b**2*c))/(32*a**4*c**2 - 16*a**3*b**2*c + 2*a*
*2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c**2) + x**3*(64*a**
2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**3 - 12*a*b**4*c +
2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))
```

Giac [A] time = 1.09151, size = 184, normalized size = 1.82

$$\frac{12\,c^2\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}+\frac{12\,c^3x^3+18\,bc^2x^2+4\,b^2cx+20\,ac^2x-b^3+10\,abc}{2\left(b^4-8\,ab^2c+16\,a^2c^2\right)\left(cx^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 12*c^2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c^
2)*sqrt(-b^2 + 4*a*c)) + 1/2*(12*c^3*x^3 + 18*b*c^2*x^2 + 4*b^2*c*x + 20*a*
c^2*x - b^3 + 10*a*b*c)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c*x^2 + b*x + a)^2
)
```

3.2207 $\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^3} dx$

Optimal. Leaf size=429

$$-\frac{\left(20c^3de^2\left(3a^2e^2-3abde+b^2d^2\right)-30a^2bc^2e^5+10ab^3ce^5-10c^4d^3e(3bd-4ae)-b^5e^5+12c^5d^5\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}\left(ae^2-bde+cd^2\right)^3}-\frac{-2c}{}$$

```
[Out] -(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)/(2*(b^2 - 4*a*c)*(c*d^2 - b*
d*e + a*e^2)*(a + b*x + c*x^2)^2) - (3*a*c*e*(2*c*d - b*e)^2 - (b*c*d - b^2
*e + 2*a*c*e)*(6*c^2*d^2 - 2*b^2*e^2 - c*e*(3*b*d - 8*a*e)) - 2*c*(2*c*d -
b*e)*(3*c^2*d^2 - b^2*e^2 - c*e*(3*b*d - 7*a*e))*x)/(2*(b^2 - 4*a*c)^2*(c*d
^2 - b*d*e + a*e^2)^2*(a + b*x + c*x^2)) - ((12*c^5*d^5 - b^5*e^5 + 10*a*b^
3*c*e^5 - 30*a^2*b*c^2*e^5 - 10*c^4*d^3*e*(3*b*d - 4*a*e) + 20*c^3*d*e^2*(b
^2*d^2 - 3*a*b*d*e + 3*a^2*e^2))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((
b^2 - 4*a*c)^(5/2)*(c*d^2 - b*d*e + a*e^2)^3) + (e^5*Log[d + e*x])/(c*d^2 -
 b*d*e + a*e^2)^3 - (e^5*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)^3
)
```

Rubi [A] time = 1.01571, antiderivative size = 429, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {740, 822, 800, 634, 618, 206, 628}

$$-\frac{\left(20c^3de^2\left(3a^2e^2-3abde+b^2d^2\right)-30a^2bc^2e^5+10ab^3ce^5-10c^4d^3e(3bd-4ae)-b^5e^5+12c^5d^5\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}\left(ae^2-bde+cd^2\right)^3}-\frac{-2c}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + b*x + c*x^2)^3),x]
```


```
[Out] -(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)/(2*(b^2 - 4*a*c)*(c*d^2 - b*
d*e + a*e^2)*(a + b*x + c*x^2)^2) - (3*a*c*e*(2*c*d - b*e)^2 - (b*c*d - b^2
*e + 2*a*c*e)*(6*c^2*d^2 - 2*b^2*e^2 - c*e*(3*b*d - 8*a*e)) - 2*c*(2*c*d -
b*e)*(3*c^2*d^2 - b^2*e^2 - c*e*(3*b*d - 7*a*e))*x)/(2*(b^2 - 4*a*c)^2*(c*d
^2 - b*d*e + a*e^2)^2*(a + b*x + c*x^2)) - ((12*c^5*d^5 - b^5*e^5 + 10*a*b^
3*c*e^5 - 30*a^2*b*c^2*e^5 - 10*c^4*d^3*e*(3*b*d - 4*a*e) + 20*c^3*d*e^2*(b
^2*d^2 - 3*a*b*d*e + 3*a^2*e^2))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((
b^2 - 4*a*c)^(5/2)*(c*d^2 - b*d*e + a*e^2)^3) + (e^5*Log[d + e*x])/(c*d^2 -
 b*d*e + a*e^2)^3 - (e^5*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)^3
)
```


Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^3}\,dx = -\frac{bcd-b^2e+2ace+c(2cd-be)x}{2\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^2} - \frac{\int \frac{6c^2d^2-2b^2e^2-ce(3bd-8ae)+3ce(2cd-be)x}{(d+ex)\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{2\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^2} - \frac{3ace(2cd-be)^2-\left(bcd-b^2e+2ace\right)\left(\right.}{2\left(\right.}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{2\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^2} - \frac{3ace(2cd-be)^2-\left(bcd-b^2e+2ace\right)\left(\right.}{2\left(\right.}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{2\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^2} - \frac{3ace(2cd-be)^2-\left(bcd-b^2e+2ace\right)\left(\right.}{2\left(\right.}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{2\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^2} - \frac{3ace(2cd-be)^2-\left(bcd-b^2e+2ace\right)\left(\right.}{2\left(\right.}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{2\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^2} - \frac{3ace(2cd-be)^2-\left(bcd-b^2e+2ace\right)\left(\right.}{2\left(\right.}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{2\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^2} - \frac{3ace(2cd-be)^2-\left(bcd-b^2e+2ace\right)\left(\right.}{2\left(\right.}$$

Mathematica [A] time = 1.30148, size = 429, normalized size = 1.

$$\frac{1}{2}\left(\frac{4c^2\left(4a^2e^3+7acde^2x+3c^2d^3x\right)+b^2ce\left(cd(2ex-9d)-15ae^2\right)+2bc^2\left(7ae^2(d-ex)+3cd^2(d-3ex)\right)+b^3ce^2(d+2ex)}{\left(b^2-4ac\right)^2(a+x(b+cx))\left(e(ae-bd)+cd^2\right)^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + b*x + c*x^2)^3),x]
```

```
[Out] ((-(b^2*e) + 2*c*(a*e + c*d*x) + b*c*(d - e*x))/((b^2 - 4*a*c)*(-(c*d^2) +
e*(b*d - a*e))*(a + x*(b + c*x))^2) + (2*b^4*e^3 + b^3*c*e^2*(d + 2*e*x) +
4*c^2*(4*a^2*e^3 + 3*c^2*d^3*x + 7*a*c*d*e^2*x) + 2*b*c^2*(3*c*d^2*(d - 3*e
*x) + 7*a*e^2*(d - e*x)) + b^2*c*e*(-15*a*e^2 + c*d*(-9*d + 2*e*x)))/((b^2
- 4*a*c)^2*(c*d^2 + e*(-(b*d) + a*e))^2*(a + x*(b + c*x))) + (2*(-12*c^5*d^
5 + b^5*e^5 - 10*a*b^3*c*e^5 + 30*a^2*b*c^2*e^5 + 10*c^4*d^3*e*(3*b*d - 4*a
*e) - 20*c^3*d*e^2*(b^2*d^2 - 3*a*b*d*e + 3*a^2*e^2))*ArcTan[(b + 2*c*x)/Sq
rt[-b^2 + 4*a*c]])/((-b^2 + 4*a*c)^(5/2)*(-(c*d^2) + e*(b*d - a*e))^3) + (2
*e^5*Log[d + e*x])/(c*d^2 + e*(-(b*d) + a*e))^3 - (e^5*Log[a + x*(b + c*x)]
)/(c*d^2 + e*(-(b*d) + a*e))^3)/2
```

Maple [B] time = 0.178, size = 4701, normalized size = 11.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d)/(c*x^2+b*x+a)^3,x)

[Out] 15/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^3*d^3*e^2-2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^5*d*e^4+1/2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^4*d^2*e^3+2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a*b^4*e^5+1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^5*c*d^2*e^3+3/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^4*c^2*d^3*e^2-5/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^3*c^3*d^4*e-7/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*a^3*b*c^2*d*e^4-15/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2*c^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b*d^4*e-29/2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*a*b^2*c^3*d^4*e+28/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*a^2*c^4*d^3*e^2+6/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*a^2*b*c^3*d^3*e^2-1/2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*a*b^4*c*d^2*e^3+12/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*a*b^3*c^2*d^3*e^2+14/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2*c^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*d*a^2*e^4+10/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2*c^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^2*d^3*e^2-6/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*a^2*b^3*c*e^5+20/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2*c^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*d^3*a*e^2-1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^4*d*e^4-29/2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a^2*b^2*e^5+8/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2*c^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a^2*d^2*e^3-25/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*a^2*b^2*c^2*d^2*e^3+27/2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*a^2*b^3*c*d*e^4-1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*a^3*b*c^2*e^5+18/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*a^3*c^3*d*e^4+60/(a*e^2-b*d*e+c*d^2)^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*c^3*d*e^4+10/(a*e^2-b*d*e+c*d^2)^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^3*c*e^5-30/(a*e^2-b*d*e+c*d^2)^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c^4*d^4*e-30/(a*e^2-b*d*e+c*d^2)^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b*c^2*e^5+20/(a*e^2-b*d*e+c*d^2)^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*c^3*d^3*e^2+40/(a*e^2-b*d*e+c*d^2)^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c^4*d^3*e^2+1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*a*b^3*e^5-7/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*a^2*b*e^5+13/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a^2*b*d*e^4-49/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a*b^2*d^2*e^3-30/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2*c^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*a*b*d^2*e^3+8/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*a*b^2*d*e^4+26/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*a*b^2*c^3*d^3*e^2-18/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*a*b^3*c^2*d^2*e^3-60/(a*e^2-b*d*e+c*d^2)^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*c^3*d^2*e^3+16/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a*b^3*d*e^4-34/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*a^2*b*c^3*d^2*e^3+6/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*a*b^4*c*d*e^4+30/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2*c^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a*b*d^3*e^2-25/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*a*b*c^4*d^4*e+10/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*a^2*b^2*c^2*d*e^4+4/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*a^2*c^4*d^4*e-4

```
5/2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2*c^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^
2*b^2*d^4*e-1/2/(a*e^2-b*d*e+c*d^2)^3/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^2+b
*x+a)*b^4*e^5-2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c
+b^4)*a*b^5*d*e^4+5/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b
^2*c+b^4)*a*b*c^4*d^5-3/2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^2
-8*a*b^2*c+b^4)*b^5*c*d^3*e^2+3/2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16
*a^2*c^2-8*a*b^2*c+b^4)*b^4*c^2*d^4*e+4/(a*e^2-b*d*e+c*d^2)^3/(16*a^2*c^2-8
*a*b^2*c+b^4)*c*ln(c*x^2+b*x+a)*a*b^2*e^5+8/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*
x+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a^3*e^5+9/(a*e^2-b*d*e+c*d^2)^3/(
c*x^2+b*x+a)^2*c^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b*d^5+1/(a*e^2-b*d*e+c*d^
2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*a*b^5*e^5+10/(a*e^2-b*d*e
+c*d^2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*a*c^5*d^5+16/(a*e^2-
b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*a^3*c^3*d^2*e^3-1
/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^6*d*e
^4+2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^2
*c^4*d^5-21/2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b
^4)*a^3*b^2*c*e^5+12/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*
b^2*c+b^4)*a^4*c^2*e^5+3/2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16*a^2*c^
2-8*a*b^2*c+b^4)*a^2*b^4*e^5+1/2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^2/(16*
a^2*c^2-8*a*b^2*c+b^4)*b^6*d^2*e^3-1/2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^
2/(16*a^2*c^2-8*a*b^2*c+b^4)*b^3*c^3*d^5-1/(a*e^2-b*d*e+c*d^2)^3/(16*a^2*c^
2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^5*
e^5+12/(a*e^2-b*d*e+c*d^2)^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*a
rctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^5*d^5-8/(a*e^2-b*d*e+c*d^2)^3/(16*a^2*
c^2-8*a*b^2*c+b^4)*c^2*ln(c*x^2+b*x+a)*a^2*e^5+6/(a*e^2-b*d*e+c*d^2)^3/(c*x
^2+b*x+a)^2*c^6/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*d^5+e^5*ln(e*x+d)/(a*e^2-b*d
*e+c*d^2)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^3,x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16121, size = 1871, normalized size = 4.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*e^5*log(c*x^2 + b*x + a)/(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 +
3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a^2*c
*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6) + e^6*log(abs(x*e + d))/(c^3*d^6*e - 3*
b*c^2*d^5*e^2 + 3*b^2*c*d^4*e^3 + 3*a*c^2*d^4*e^3 - b^3*d^3*e^4 - 6*a*b*c*d
^3*e^4 + 3*a*b^2*d^2*e^5 + 3*a^2*c*d^2*e^5 - 3*a^2*b*d*e^6 + a^3*e^7) + (12
*c^5*d^5 - 30*b*c^4*d^4*e + 20*b^2*c^3*d^3*e^2 + 40*a*c^4*d^3*e^2 - 60*a*b*
c^3*d^2*e^3 + 60*a^2*c^3*d*e^4 - b^5*e^5 + 10*a*b^3*c*e^5 - 30*a^2*b*c^2*e^
5)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4*c^3*d^6 - 8*a*b^2*c^4*d^6 +
 16*a^2*c^5*d^6 - 3*b^5*c^2*d^5*e + 24*a*b^3*c^3*d^5*e - 48*a^2*b*c^4*d^5*e
 + 3*b^6*c*d^4*e^2 - 21*a*b^4*c^2*d^4*e^2 + 24*a^2*b^2*c^3*d^4*e^2 + 48*a^3
*c^4*d^4*e^2 - b^7*d^3*e^3 + 2*a*b^5*c*d^3*e^3 + 32*a^2*b^3*c^2*d^3*e^3 - 9
6*a^3*b*c^3*d^3*e^3 + 3*a*b^6*d^2*e^4 - 21*a^2*b^4*c*d^2*e^4 + 24*a^3*b^2*c
^2*d^2*e^4 + 48*a^4*c^3*d^2*e^4 - 3*a^2*b^5*d*e^5 + 24*a^3*b^3*c*d*e^5 - 48
*a^4*b*c^2*d*e^5 + a^3*b^4*e^6 - 8*a^4*b^2*c*e^6 + 16*a^5*c^2*e^6)*sqrt(-b^
2 + 4*a*c)) - 1/2*(b^3*c^3*d^5 - 10*a*b*c^4*d^5 - 3*b^4*c^2*d^4*e + 29*a*b^
2*c^3*d^4*e - 8*a^2*c^4*d^4*e + 3*b^5*c*d^3*e^2 - 24*a*b^3*c^2*d^3*e^2 - 12
*a^2*b*c^3*d^3*e^2 - b^6*d^2*e^3 + a*b^4*c*d^2*e^3 + 50*a^2*b^2*c^2*d^2*e^3
 - 32*a^3*c^3*d^2*e^3 + 4*a*b^5*d*e^4 - 27*a^2*b^3*c*d*e^4 + 14*a^3*b*c^2*d
*e^4 - 3*a^2*b^4*e^5 + 21*a^3*b^2*c*e^5 - 24*a^4*c^2*e^5 - 2*(6*c^6*d^5 - 1
5*b*c^5*d^4*e + 10*b^2*c^4*d^3*e^2 + 20*a*c^5*d^3*e^2 - 30*a*b*c^4*d^2*e^3
- b^4*c^2*d*e^4 + 8*a*b^2*c^3*d*e^4 + 14*a^2*c^4*d*e^4 + a*b^3*c^2*e^5 - 7*
a^2*b*c^3*e^5)*x^3 - (18*b*c^5*d^5 - 45*b^2*c^4*d^4*e + 30*b^3*c^3*d^3*e^2
+ 60*a*b*c^4*d^3*e^2 + b^4*c^2*d^2*e^3 - 98*a*b^2*c^3*d^2*e^3 + 16*a^2*c^4*
d^2*e^3 - 4*b^5*c*d*e^4 + 32*a*b^3*c^2*d*e^4 + 26*a^2*b*c^3*d*e^4 + 4*a*b^4
*c*e^5 - 29*a^2*b^2*c^2*e^5 + 16*a^3*c^3*e^5)*x^2 - 2*(2*b^2*c^4*d^5 + 10*a
*c^5*d^5 - 5*b^3*c^3*d^4*e - 25*a*b*c^4*d^4*e + 3*b^4*c^2*d^3*e^2 + 26*a*b^
2*c^3*d^3*e^2 + 28*a^2*c^4*d^3*e^2 + b^5*c*d^2*e^3 - 18*a*b^3*c^2*d^2*e^3 -
 34*a^2*b*c^3*d^2*e^3 - b^6*d*e^4 + 6*a*b^4*c*d*e^4 + 10*a^2*b^2*c^2*d*e^4
+ 18*a^3*c^3*d*e^4 + a*b^5*e^5 - 6*a^2*b^3*c*e^5 - a^3*b*c^2*e^5)*x)/((c*d^
2 - b*d*e + a*e^2)^3*(c*x^2 + b*x + a)^2*(b^2 - 4*a*c)^2)
```

3.2208 $\int \frac{1}{x^2(a+bx+cx^2)^3} dx$

Optimal. Leaf size=239

$$\frac{20a^2c^2+3bcx(b^2-6ac)-20ab^2c+3b^4}{2a^2x(b^2-4ac)^2(a+bx+cx^2)}-\frac{3(30a^2b^2c^2-20a^3c^3-10ab^4c+b^6)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4(b^2-4ac)^{5/2}}-\frac{3(b^2-5ac)(b^2-2a}{a^3x(b^2-4ac)^2}$$

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(a^3*(b^2 - 4*a*c)^2*x) + (b^2 - 2*a*c + b
*c*x)/(2*a*(b^2 - 4*a*c)*x*(a + b*x + c*x^2)^2) + (3*b^4 - 20*a*b^2*c + 20*
a^2*c^2 + 3*b*c*(b^2 - 6*a*c)*x)/(2*a^2*(b^2 - 4*a*c)^2*x*(a + b*x + c*x^2)
) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTanh[(b + 2*c*x)
/Sqrt[b^2 - 4*a*c]])/(a^4*(b^2 - 4*a*c)^(5/2)) - (3*b*Log[x])/a^4 + (3*b*Lo
g[a + b*x + c*x^2])/(2*a^4)
```

Rubi [A] time = 0.313631, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {740, 822, 800, 634, 618, 206, 628}

$$\frac{20a^2c^2+3bcx(b^2-6ac)-20ab^2c+3b^4}{2a^2x(b^2-4ac)^2(a+bx+cx^2)}-\frac{3(30a^2b^2c^2-20a^3c^3-10ab^4c+b^6)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4(b^2-4ac)^{5/2}}-\frac{3(b^2-5ac)(b^2-2a}{a^3x(b^2-4ac)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x + c*x^2)^3),x]
```


```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(a^3*(b^2 - 4*a*c)^2*x) + (b^2 - 2*a*c + b
*c*x)/(2*a*(b^2 - 4*a*c)*x*(a + b*x + c*x^2)^2) + (3*b^4 - 20*a*b^2*c + 20*
a^2*c^2 + 3*b*c*(b^2 - 6*a*c)*x)/(2*a^2*(b^2 - 4*a*c)^2*x*(a + b*x + c*x^2)
) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTanh[(b + 2*c*x)
/Sqrt[b^2 - 4*a*c]])/(a^4*(b^2 - 4*a*c)^(5/2)) - (3*b*Log[x])/a^4 + (3*b*Lo
g[a + b*x + c*x^2])/(2*a^4)
```


Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
```

```
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx+cx^2\right)^3}\,dx = \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} - \frac{\int \frac{-3b^2+10ac-4bcx}{x^2\left(a+bx+cx^2\right)^2}\,dx}{2a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc\left(b^2-6ac\right)x}{2a^2\left(b^2-4ac\right)^2x\left(a+bx+cx^2\right)} + \frac{\int \frac{6\left(b^2-5ac\right)\left(b^2-2a\right.}{x^2\left(a+b\right.}}{2a^2\left(b^2\right.}$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc\left(b^2-6ac\right)x}{2a^2\left(b^2-4ac\right)^2x\left(a+bx+cx^2\right)} + \frac{\int\left(\frac{6\left(b^2-5ac\right)\left(b^2-\right.}{ax^2}\right.}{}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{a^3\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc\left(b^2\right.}{2a^2\left(b^2-4ac\right)^2x\left(a+bx+\right.}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{a^3\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc\left(b^2\right.}{2a^2\left(b^2-4ac\right)^2x\left(a+bx+\right.}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{a^3\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc\left(b^2\right.}{2a^2\left(b^2-4ac\right)^2x\left(a+bx+\right.}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{a^3\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc\left(b^2\right.}{2a^2\left(b^2-4ac\right)^2x\left(a+bx+\right.}$$

Mathematica [A] time = 0.462256, size = 221, normalized size = 0.92

$$\frac{\frac{a^2\left(-3abc-2ac^2x+b^2cx+b^3\right)}{\left(4ac-b^2\right)(a+x(b+cx))^2} - \frac{a\left(46a^2bc^2+28a^2c^3x-26ab^2c^2x-29ab^3c+4b^4cx+4b^5\right)}{\left(b^2-4ac\right)^2(a+x(b+cx))} + \frac{6\left(30a^2b^2c^2-20a^3c^3-10ab^4c+b^6\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}} + 3b\log(a+x(b}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x + c*x^2)^3),x]
```

```
[Out] ((-2*a)/x + (a^2*(b^3 - 3*a*b*c + b^2*c*x - 2*a*c^2*x))/((-b^2 + 4*a*c)*(a
+ x*(b + c*x))^2) - (a*(4*b^5 - 29*a*b^3*c + 46*a^2*b*c^2 + 4*b^4*c*x - 26*
a*b^2*c^2*x + 28*a^2*c^3*x))/((b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (6*(b^6
- 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
4*a*c]])/(-b^2 + 4*a*c)^(5/2) - 6*b*Log[x] + 3*b*Log[a + x*(b + c*x)])/(2*a
^4)
```

Maple [B] time = 0.172, size = 954, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^2+b*x+a)^3,x)
```

```
[Out] -1/a^3/x-3*b*ln(x)/a^4-14/a/(c*x^2+b*x+a)^2*c^4/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^3+13/a^2/(c*x^2+b*x+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^2-2/a^3/(c*
```

```
x^2+b*x+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^4-37/a/(c*x^2+b*x+a)^2*b*
c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+55/2/a^2/(c*x^2+b*x+a)^2*b^3*c^2/(16*a^2
*c^2-8*a*b^2*c+b^4)*x^2-4/a^3/(c*x^2+b*x+a)^2*b^5*c/(16*a^2*c^2-8*a*b^2*c+b
^4)*x^2-18/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c^3-7/a/(c*x^2+b*x+
a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^2*c^2+12/a^2/(c*x^2+b*x+a)^2/(16*a^2*c^
2-8*a*b^2*c+b^4)*x*b^4*c-2/a^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x
*b^6-29/(c*x^2+b*x+a)^2*b/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2+18/a/(c*x^2+b*x+a)
^2*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c-5/2/a^2/(c*x^2+b*x+a)^2*b^5/(16*a^2*c^2
-8*a*b^2*c+b^4)+24/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*ln(c*x^2+b*x+a)*b-12/
a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c*ln(c*x^2+b*x+a)*b^3+3/2/a^4/(16*a^2*c^2-8*
a*b^2*c+b^4)*ln(c*x^2+b*x+a)*b^5-60/a/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2
)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^3+90/a^2/(16*a^2*c^2-8*a*b^2*
c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*c^2-30/a^3
/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^
(1/2))*b^4*c+3/a^4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))*b^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 8.99664, size = 4849, normalized size = 20.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^3,x, algorithm="fricas")

[Out] [-1/2*(2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*c
^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*x^4 + 3*(4*a*b^7*c - 45*
a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*x^3 + 2*(3*a*b^8 - 30*a^2*b^
6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*x^2 + 3*((b^6*c^2 - 10
*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^5 + 2*(b^7*c - 10*a*b^5*c^2 + 3
0*a^2*b^3*c^3 - 20*a^3*b*c^4)*x^4 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*
a^3*b^2*c^3 - 40*a^4*c^4)*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 -
20*a^4*b*c^3)*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*
x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*
a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4
*b^3*c^2 - 488*a^5*b*c^3)*x - 3*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 -
 64*a^3*b*c^5)*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*
c^4)*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*
c^4)*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x^2 +
 (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x)*log(c*x^2 + b*
x + a) + 6*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^5 +
2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^4 + (b^9 - 10*
a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^3 + 2*(a*b^8 -
 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x^2 + (a^2*b^7 - 12*a^3*b^
5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x)*log(x))/((a^4*b^6*c^2 - 12*a^5*b^4*
```

```
c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*x^5 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48
*a^6*b^3*c^3 - 64*a^7*b*c^4)*x^4 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2
 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*x^3 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b
^3*c^2 - 64*a^8*b*c^3)*x^2 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*
a^9*c^3)*x), -1/2*(2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3
+ 6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*x^4 + 3*(4*a
*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*x^3 + 2*(3*a*b^8
 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*x^2 + 6*((
b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^5 + 2*(b^7*c - 10*a
*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*x^4 + (b^8 - 8*a*b^6*c + 10*a^2*b
^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^
3*b^3*c^2 - 20*a^4*b*c^3)*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 -
20*a^5*c^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b
^2 - 4*a*c)) + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3
)*x - 3*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^5 + 2*(
b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^4 + (b^9 - 10*a*b
^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^3 + 2*(a*b^8 - 12
*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x^2 + (a^2*b^7 - 12*a^3*b^5*c
 + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x)*log(c*x^2 + b*x + a) + 6*((b^7*c^2 - 1
2*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^5 + 2*(b^8*c - 12*a*b^6*c^2
+ 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2
 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b
^4*c^2 - 64*a^4*b^2*c^3)*x^2 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 6
4*a^5*b*c^3)*x)*log(x))/((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 6
4*a^7*c^5)*x^5 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*
c^4)*x^4 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*
a^8*c^4)*x^3 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*x
^2 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*x)]
```

Sympy [B] time = 50.92, size = 5722, normalized size = 23.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**2+b*x+a)**3,x)
```

```
[Out] (3*b/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**
2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640
*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))*log(x + (-108
544*a**16*b*c**8*(3*b/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 -
 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**
4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10
)))**2 + 224768*a**15*b**3*c**7*(3*b/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*
(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*
c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a
*b**8*c - b**10)))**2 - 202752*a**14*b**5*c**6*(3*b/(2*a**4) - 3*sqrt(-(4*a
*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a
**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b
**6*c**2 + 20*a*b**8*c - b**10)))**2 + 104128*a**13*b**7*c**5*(3*b/(2*a**4)
 - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4
*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c*
*3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))**2 - 19200*a**13*c**9*(3*b
/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 +
10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**
3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) - 33320*a**12*b**
9*c**4*(3*b/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b
```

2*c2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**
4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))**2 - 4
4736*a**12*b**2*c**8*(3*b/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c*
*3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280
*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b
10))) + 6806*a11*b**11*c**3*(3*b/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*
(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*
c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a
*b**8*c - b**10)))**2 + 101232*a**11*b**4*c**7*(3*b/(2*a**4) - 3*sqrt(-(4*a
*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a
4*(1024*a5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b
6*c2 + 20*a*b**8*c - b**10))) - 867*a**10*b**13*c**2*(3*b/(2*a**4) - 3*
sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c -
b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 -
160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))**2 - 77268*a**10*b**6*c**6*(3*b
/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 +
10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**
3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) + 63*a**9*b**15*c
*(3*b/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c*
*2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 64
0*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))**2 + 31368*a
9*b8*c**5*(3*b/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30
*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b
2*c4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))
 - 57600*a**9*b*c**9 - 2*a**8*b**17*(3*b/(2*a**4) - 3*sqrt(-(4*a*c - b**2)*
5)(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a
5*c5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 +
20*a*b**8*c - b**10)))**2 - 7545*a**8*b**10*c**4*(3*b/(2*a**4) - 3*sqrt(-(4
*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2
*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2
*b**6*c**2 + 20*a*b**8*c - b**10))) + 842688*a**8*b**3*c**8 + 1086*a**7*b**
12*c**3*(3*b/(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*
b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c*
*4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) - 171
9216*a**7*b**5*c**7 - 87*a**6*b**14*c**2*(3*b/(2*a**4) - 3*sqrt(-(4*a*c - b
2)5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1
024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c*
*2 + 20*a*b**8*c - b**10))) + 1592964*a**6*b**7*c**6 + 3*a**5*b**16*c*(3*b/
(2*a**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 1
0*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3
*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) - 843048*a**5*b**9
*c**5 + 277245*a**4*b**11*c**4 - 57996*a**3*b**13*c**3 + 7542*a**2*b**15*c*
*2 - 558*a*b**17*c + 18*b**19)/(18000*a**9*c**10 + 333720*a**8*b**2*c**9 -
991980*a**7*b**4*c**8 + 1099710*a**6*b**6*c**7 - 651186*a**5*b**8*c**6 + 23
1795*a**4*b**10*c**5 - 51480*a**3*b**12*c**4 + 7020*a**2*b**14*c**3 - 540*a
*b**16*c**2 + 18*b**18*c)) + (3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20
*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**
5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b*
*8*c - b**10)))*log(x + (-108544*a**16*b*c**8*(3*b/(2*a**4) + 3*sqrt(-(4*a*
c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a*
4(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b*
*6*c**2 + 20*a*b**8*c - b**10)))**2 + 224768*a**15*b**3*c**7*(3*b/(2*a**4)
+ 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*
c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**
3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))**2 - 202752*a**14*b**5*c**6
*(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c*
*2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 64
0*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))**2 + 104128*
a**13*b**7*c**5*(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 -

30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4
*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)
))**2 - 19200*a**13*c**9*(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**
3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 -
1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c
 - b**10))) - 33320*a**12*b**9*c**4*(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)*
5)(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a
5*c5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 +
20*a*b**8*c - b**10)))**2 - 44736*a**12*b**2*c**8*(3*b/(2*a**4) + 3*sqrt(-(
4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(
2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**
2*b**6*c**2 + 20*a*b**8*c - b**10))) + 6806*a**11*b**11*c**3*(3*b/(2*a**4)
 + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*
c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**
3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))**2 + 101232*a**11*b**4*c**7
*(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c*
*2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 64
0*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) - 867*a**10*
b**13*c**2*(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a*
*2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2
*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))**2
 - 77268*a**10*b**6*c**6*(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**
3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 -
1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c
 - b**10))) + 63*a**9*b**15*c*(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)**5)*(2
0*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c*
*5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b
8*c - b10)))**2 + 31368*a**9*b**8*c**5*(3*b/(2*a**4) + 3*sqrt(-(4*a*c -
 b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*
(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*
c**2 + 20*a*b**8*c - b**10))) - 57600*a**9*b*c**9 - 2*a**8*b**17*(3*b/(2*a*
*4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b
4*c - b6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4
*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))**2 - 7545*a**8*b**10*c*
4(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*
c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 +
640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) + 842688*a
8*b3*c**8 + 1086*a**7*b**12*c**3*(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)
5)*(20*a3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*
a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 +
 20*a*b**8*c - b**10))) - 1719216*a**7*b**5*c**7 - 87*a**6*b**14*c**2*(3*b/
(2*a**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 1
0*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3
*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) + 1592964*a**6*b**
7*c**6 + 3*a**5*b**16*c*(3*b/(2*a**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3
*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*a**4*(1024*a**5*c**5 - 1
280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c
- b**10))) - 843048*a**5*b**9*c**5 + 277245*a**4*b**11*c**4 - 57996*a**3*b*
*13*c**3 + 7542*a**2*b**15*c**2 - 558*a*b**17*c + 18*b**19)/(18000*a**9*c**
10 + 333720*a**8*b**2*c**9 - 991980*a**7*b**4*c**8 + 1099710*a**6*b**6*c**7
 - 651186*a**5*b**8*c**6 + 231795*a**4*b**10*c**5 - 51480*a**3*b**12*c**4 +
 7020*a**2*b**14*c**3 - 540*a*b**16*c**2 + 18*b**18*c)) - (32*a**4*c**2 - 1
6*a**3*b**2*c + 2*a**2*b**4 + x**4*(60*a**2*c**4 - 42*a*b**2*c**3 + 6*b**4*
c**2) + x**3*(138*a**2*b*c**3 - 87*a*b**3*c**2 + 12*b**5*c) + x**2*(100*a**
3*c**3 + 14*a**2*b**2*c**2 - 36*a*b**4*c + 6*b**6) + x*(122*a**3*b*c**2 - 6
8*a**2*b**3*c + 9*a*b**5))/(x**5*(32*a**5*c**4 - 16*a**4*b**2*c**3 + 2*a**3
*b**4*c**2) + x**4*(64*a**5*b*c**3 - 32*a**4*b**3*c**2 + 4*a**3*b**5*c) + x
3*(64*a6*c**3 - 12*a**4*b**4*c + 2*a**3*b**6) + x**2*(64*a**6*b*c**2 -
32*a**5*b**3*c + 4*a**4*b**5) + x*(32*a**7*c**2 - 16*a**6*b**2*c + 2*a**5*b

```
**4)) - 3*b*log(x)/a**4
```

Giac [A] time = 1.12849, size = 417, normalized size = 1.74

$$\frac{3\left(b^{6}-10\,ab^{4}c+30\,a^{2}b^{2}c^{2}-20\,a^{3}c^{3}\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(a^{4}b^{4}-8\,a^{5}b^{2}c+16\,a^{6}c^{2}\right)\sqrt{-b^{2}+4\,ac}}+\frac{3\,b\log\left(cx^{2}+bx+a\right)}{2\,a^{4}}-\frac{3\,b\log\left(|x|\right)}{a^{4}}-\frac{2\,a^{3}b^{4}-16\,a^{4}b^{2}c+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan((2*c*x + b)/sqrt(
-b^2 + 4*a*c))/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*sqrt(-b^2 + 4*a*c)) +
3/2*b*log(c*x^2 + b*x + a)/a^4 - 3*b*log(abs(x))/a^4 - 1/2*(2*a^3*b^4 - 16*
a^4*b^2*c + 32*a^5*c^2 + 6*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*x^4 + 3
*(4*a*b^5*c - 29*a^2*b^3*c^2 + 46*a^3*b*c^3)*x^3 + 2*(3*a*b^6 - 18*a^2*b^4*
c + 7*a^3*b^2*c^2 + 50*a^4*c^3)*x^2 + (9*a^2*b^5 - 68*a^3*b^3*c + 122*a^4*b
*c^2)*x)/((c*x^2 + b*x + a)^2*(b^2 - 4*a*c)^2*a^4*x)
```

3.2209 $\int \frac{1}{x^3\left(a+bx+cx^2\right)^3}\,dx$

Optimal. Leaf size=306

$$\frac{24a^2c^2+2bcx\left(2b^2-11ac\right)-25ab^2c+4b^4}{2a^2x^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{3\left(16a^2c^2-13ab^2c+2b^4\right)}{2a^3x^2\left(b^2-4ac\right)^2}+\frac{3b\left(70a^2b^2c^2-70a^3c^3-21ab^4c+2b^6\right)\tanh^{-}}{a^5\left(b^2-4ac\right)^{5/2}}$$

```
[Out] (-3*(2*b^4 - 13*a*b^2*c + 16*a^2*c^2))/(2*a^3*(b^2 - 4*a*c)^2*x^2) + (3*b*(
2*b^2 - 9*a*c)*(b^2 - 3*a*c))/(a^4*(b^2 - 4*a*c)^2*x) + (b^2 - 2*a*c + b*c*
x)/(2*a*(b^2 - 4*a*c)*x^2*(a + b*x + c*x^2)^2) + (4*b^4 - 25*a*b^2*c + 24*a
^2*c^2 + 2*b*c*(2*b^2 - 11*a*c)*x)/(2*a^2*(b^2 - 4*a*c)^2*x^2*(a + b*x + c*
x^2)) + (3*b*(2*b^6 - 21*a*b^4*c + 70*a^2*b^2*c^2 - 70*a^3*c^3)*ArcTanh[(b
+ 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^5*(b^2 - 4*a*c)^(5/2)) + (3*(2*b^2 - a*c)*L
og[x])/a^5 - (3*(2*b^2 - a*c)*Log[a + b*x + c*x^2])/(2*a^5)
```

Rubi [A] time = 0.459514, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {740, 822, 800, 634, 618, 206, 628}

$$\frac{24a^2c^2+2bcx\left(2b^2-11ac\right)-25ab^2c+4b^4}{2a^2x^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{3\left(16a^2c^2-13ab^2c+2b^4\right)}{2a^3x^2\left(b^2-4ac\right)^2}+\frac{3b\left(70a^2b^2c^2-70a^3c^3-21ab^4c+2b^6\right)\tanh^{-}}{a^5\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x + c*x^2)^3),x]
```


```
[Out] (-3*(2*b^4 - 13*a*b^2*c + 16*a^2*c^2))/(2*a^3*(b^2 - 4*a*c)^2*x^2) + (3*b*(
2*b^2 - 9*a*c)*(b^2 - 3*a*c))/(a^4*(b^2 - 4*a*c)^2*x) + (b^2 - 2*a*c + b*c*
x)/(2*a*(b^2 - 4*a*c)*x^2*(a + b*x + c*x^2)^2) + (4*b^4 - 25*a*b^2*c + 24*a
^2*c^2 + 2*b*c*(2*b^2 - 11*a*c)*x)/(2*a^2*(b^2 - 4*a*c)^2*x^2*(a + b*x + c*
x^2)) + (3*b*(2*b^6 - 21*a*b^4*c + 70*a^2*b^2*c^2 - 70*a^3*c^3)*ArcTanh[(b
+ 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^5*(b^2 - 4*a*c)^(5/2)) + (3*(2*b^2 - a*c)*L
og[x])/a^5 - (3*(2*b^2 - a*c)*Log[a + b*x + c*x^2])/(2*a^5)
```


Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
```

```
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx+cx^2\right)^3}\,dx = \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)^2} - \frac{\int \frac{-4\left(b^2-3ac\right)-5bcx}{x^3\left(a+bx+cx^2\right)^2}\,dx}{2a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)^2} + \frac{4b^4-25ab^2c+24a^2c^2+2bc\left(2b^2-11ac\right)x}{2a^2\left(b^2-4ac\right)^2x^2\left(a+bx+cx^2\right)} + \frac{\int \frac{6\left(2b^4-13ab^2 \right.}{}}{2}$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)^2} + \frac{4b^4-25ab^2c+24a^2c^2+2bc\left(2b^2-11ac\right)x}{2a^2\left(b^2-4ac\right)^2x^2\left(a+bx+cx^2\right)} + \frac{\int \left(\frac{6\left(2b^4-13a \right.}{a}\right.}{}$$

$$= -\frac{3\left(2b^4-13ab^2c+16a^2c^2\right)}{2a^3\left(b^2-4ac\right)^2x^2} + \frac{3b\left(2b^2-9ac\right)\left(b^2-3ac\right)}{a^4\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)^2} +$$

$$= -\frac{3\left(2b^4-13ab^2c+16a^2c^2\right)}{2a^3\left(b^2-4ac\right)^2x^2} + \frac{3b\left(2b^2-9ac\right)\left(b^2-3ac\right)}{a^4\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)^2} +$$

$$= -\frac{3\left(2b^4-13ab^2c+16a^2c^2\right)}{2a^3\left(b^2-4ac\right)^2x^2} + \frac{3b\left(2b^2-9ac\right)\left(b^2-3ac\right)}{a^4\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)^2} +$$

$$= -\frac{3\left(2b^4-13ab^2c+16a^2c^2\right)}{2a^3\left(b^2-4ac\right)^2x^2} + \frac{3b\left(2b^2-9ac\right)\left(b^2-3ac\right)}{a^4\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)^2} +$$

Mathematica [A] time = 0.526325, size = 269, normalized size = 0.88

$$\frac{\frac{a^2\left(2a^2c^2-4ab^2c-3abc^2x+b^3cx+b^4\right)}{\left(b^2-4ac\right)(a+x(b+cx))^2} + \frac{a\left(97a^2b^2c^2+66a^2bc^3x-32a^3c^3-42ab^3c^2x-47ab^4c+6b^5cx+6b^6\right)}{\left(b^2-4ac\right)^2(a+x(b+cx))} - \frac{6b\left(70a^2b^2c^2-70a^3c^3-21ab^4c+2b^6\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}} -}{2a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x + c*x^2)^3),x]
```

```
[Out] (-(a^2/x^2) + (6*a*b)/x + (a^2*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x - 3*a
*b*c^2*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))^2) + (a*(6*b^6 - 47*a*b^4*c + 9
7*a^2*b^2*c^2 - 32*a^3*c^3 + 6*b^5*c*x - 42*a*b^3*c^2*x + 66*a^2*b*c^3*x))/
((b^2 - 4*a*c)^2*(a + x*(b + c*x))) - (6*b*(2*b^6 - 21*a*b^4*c + 70*a^2*b^2
*c^2 - 70*a^3*c^3)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(
5/2) + 6*(2*b^2 - a*c)*Log[x] + 3*(-2*b^2 + a*c)*Log[a + x*(b + c*x)])/(2*a
^5)
```

Maple [B] time = 0.171, size = 1110, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^2+b*x+a)^3,x)
```

```
[Out] 115/2/a/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*b^2*c^2-55/2/a^2/(c*x^2+
b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*b^4*c-60/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)
*c^2*ln(c*x^2+b*x+a)*b^2+51/2/a^4/(16*a^2*c^2-8*a*b^2*c+b^4)*c*ln(c*x^2+b*x
+a)*b^4-6/a^5/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)
/(4*a*c-b^2)^(1/2))*b^7-1/2/a^3/x^2-16/a/(c*x^2+b*x+a)^2*c^4/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^2+3/a^4/(c*x^2+b*x+a)^2*b^7/(16*a^2*c^2-8*a*b^2*c+b^4)*x+7/2
/a^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*b^6+24/a^2/(16*a^2*c^2-8*a*
b^2*c+b^4)*c^3*ln(c*x^2+b*x+a)-3/a^5/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^2+b*
x+a)*b^6-20/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^3+3/a^4*b/x-3/a^4*
ln(x)*c+6/a^5*ln(x)*b^2+33/a^2/(c*x^2+b*x+a)^2*b*c^4/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^3-21/a^3/(c*x^2+b*x+a)^2*b^3*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+3/a^
4/(c*x^2+b*x+a)^2*b^5*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+163/2/a^2/(c*x^2+b
*x+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^2-89/2/a^3/(c*x^2+b*x+a)^2*c^2
/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^4+6/a^4/(c*x^2+b*x+a)^2*c/(16*a^2*c^2-8*a
*b^2*c+b^4)*x^2*b^6+23/a/(c*x^2+b*x+a)^2*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c^3
+24/a^2/(c*x^2+b*x+a)^2*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c^2-20/a^3/(c*x^2+
b*x+a)^2*b^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c+210/a^2/(16*a^2*c^2-8*a*b^2*c+b
^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c^3-210/a^3/(16
*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2
))*b^3*c^2+63/a^4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*
x+b)/(4*a*c-b^2)^(1/2))*b^5*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 16.8099, size = 5724, normalized size = 18.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3 - 6*(2*a*b^7*c^
2 - 23*a^2*b^5*c^3 + 87*a^3*b^3*c^4 - 108*a^4*b*c^5)*x^5 - 3*(8*a*b^8*c - 9
4*a^2*b^6*c^2 + 369*a^3*b^4*c^3 - 500*a^4*b^2*c^4 + 64*a^5*c^5)*x^4 - 2*(6*
a*b^9 - 63*a^2*b^7*c + 188*a^3*b^5*c^2 - 25*a^4*b^3*c^3 - 412*a^5*b*c^4)*x^
3 - (18*a^2*b^8 - 217*a^3*b^6*c + 887*a^4*b^4*c^2 - 1300*a^5*b^2*c^3 + 288*
a^6*c^4)*x^2 + 3*((2*b^7*c^2 - 21*a*b^5*c^3 + 70*a^2*b^3*c^4 - 70*a^3*b*c^5
)*x^6 + 2*(2*b^8*c - 21*a*b^6*c^2 + 70*a^2*b^4*c^3 - 70*a^3*b^2*c^4)*x^5 +
(2*b^9 - 17*a*b^7*c + 28*a^2*b^5*c^2 + 70*a^3*b^3*c^3 - 140*a^4*b*c^4)*x^4
+ 2*(2*a*b^8 - 21*a^2*b^6*c + 70*a^3*b^4*c^2 - 70*a^4*b^2*c^3)*x^3 + (2*a^2
*b^7 - 21*a^3*b^5*c + 70*a^4*b^3*c^2 - 70*a^5*b*c^3)*x^2)*sqrt(b^2 - 4*a*c)
*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c
*x^2 + b*x + a)) - 4*(a^3*b^7 - 12*a^4*b^5*c + 48*a^5*b^3*c^2 - 64*a^6*b*c^
3)*x + 3*((2*b^8*c^2 - 25*a*b^6*c^3 + 108*a^2*b^4*c^4 - 176*a^3*b^2*c^5 + 6
4*a^4*c^6)*x^6 + 2*(2*b^9*c - 25*a*b^7*c^2 + 108*a^2*b^5*c^3 - 176*a^3*b^3*
c^4 + 64*a^4*b*c^5)*x^5 + (2*b^10 - 21*a*b^8*c + 58*a^2*b^6*c^2 + 40*a^3*b^
```

```
4*c^3 - 288*a^4*b^2*c^4 + 128*a^5*c^5)*x^4 + 2*(2*a*b^9 - 25*a^2*b^7*c + 10
8*a^3*b^5*c^2 - 176*a^4*b^3*c^3 + 64*a^5*b*c^4)*x^3 + (2*a^2*b^8 - 25*a^3*b
^6*c + 108*a^4*b^4*c^2 - 176*a^5*b^2*c^3 + 64*a^6*c^4)*x^2)*log(c*x^2 + b*x
 + a) - 6*((2*b^8*c^2 - 25*a*b^6*c^3 + 108*a^2*b^4*c^4 - 176*a^3*b^2*c^5 +
64*a^4*c^6)*x^6 + 2*(2*b^9*c - 25*a*b^7*c^2 + 108*a^2*b^5*c^3 - 176*a^3*b^3
*c^4 + 64*a^4*b*c^5)*x^5 + (2*b^10 - 21*a*b^8*c + 58*a^2*b^6*c^2 + 40*a^3*b
^4*c^3 - 288*a^4*b^2*c^4 + 128*a^5*c^5)*x^4 + 2*(2*a*b^9 - 25*a^2*b^7*c + 1
08*a^3*b^5*c^2 - 176*a^4*b^3*c^3 + 64*a^5*b*c^4)*x^3 + (2*a^2*b^8 - 25*a^3*
b^6*c + 108*a^4*b^4*c^2 - 176*a^5*b^2*c^3 + 64*a^6*c^4)*x^2)*log(x))/((a^5*
b^6*c^2 - 12*a^6*b^4*c^3 + 48*a^7*b^2*c^4 - 64*a^8*c^5)*x^6 + 2*(a^5*b^7*c
- 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*x^5 + (a^5*b^8 - 10*a^6*b
^6*c + 24*a^7*b^4*c^2 + 32*a^8*b^2*c^3 - 128*a^9*c^4)*x^4 + 2*(a^6*b^7 - 12
*a^7*b^5*c + 48*a^8*b^3*c^2 - 64*a^9*b*c^3)*x^3 + (a^7*b^6 - 12*a^8*b^4*c +
 48*a^9*b^2*c^2 - 64*a^10*c^3)*x^2), -1/2*(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*
b^2*c^2 - 64*a^7*c^3 - 6*(2*a*b^7*c^2 - 23*a^2*b^5*c^3 + 87*a^3*b^3*c^4 - 1
08*a^4*b*c^5)*x^5 - 3*(8*a*b^8*c - 94*a^2*b^6*c^2 + 369*a^3*b^4*c^3 - 500*a
^4*b^2*c^4 + 64*a^5*c^5)*x^4 - 2*(6*a*b^9 - 63*a^2*b^7*c + 188*a^3*b^5*c^2
- 25*a^4*b^3*c^3 - 412*a^5*b*c^4)*x^3 - (18*a^2*b^8 - 217*a^3*b^6*c + 887*a
^4*b^4*c^2 - 1300*a^5*b^2*c^3 + 288*a^6*c^4)*x^2 - 6*((2*b^7*c^2 - 21*a*b^5
*c^3 + 70*a^2*b^3*c^4 - 70*a^3*b*c^5)*x^6 + 2*(2*b^8*c - 21*a*b^6*c^2 + 70*
a^2*b^4*c^3 - 70*a^3*b^2*c^4)*x^5 + (2*b^9 - 17*a*b^7*c + 28*a^2*b^5*c^2 +
70*a^3*b^3*c^3 - 140*a^4*b*c^4)*x^4 + 2*(2*a*b^8 - 21*a^2*b^6*c + 70*a^3*b^
4*c^2 - 70*a^4*b^2*c^3)*x^3 + (2*a^2*b^7 - 21*a^3*b^5*c + 70*a^4*b^3*c^2 -
70*a^5*b*c^3)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b
)/(b^2 - 4*a*c)) - 4*(a^3*b^7 - 12*a^4*b^5*c + 48*a^5*b^3*c^2 - 64*a^6*b*c^
3)*x + 3*((2*b^8*c^2 - 25*a*b^6*c^3 + 108*a^2*b^4*c^4 - 176*a^3*b^2*c^5 + 6
4*a^4*c^6)*x^6 + 2*(2*b^9*c - 25*a*b^7*c^2 + 108*a^2*b^5*c^3 - 176*a^3*b^3*
c^4 + 64*a^4*b*c^5)*x^5 + (2*b^10 - 21*a*b^8*c + 58*a^2*b^6*c^2 + 40*a^3*b^
4*c^3 - 288*a^4*b^2*c^4 + 128*a^5*c^5)*x^4 + 2*(2*a*b^9 - 25*a^2*b^7*c + 10
8*a^3*b^5*c^2 - 176*a^4*b^3*c^3 + 64*a^5*b*c^4)*x^3 + (2*a^2*b^8 - 25*a^3*b
^6*c + 108*a^4*b^4*c^2 - 176*a^5*b^2*c^3 + 64*a^6*c^4)*x^2)*log(c*x^2 + b*x
 + a) - 6*((2*b^8*c^2 - 25*a*b^6*c^3 + 108*a^2*b^4*c^4 - 176*a^3*b^2*c^5 +
64*a^4*c^6)*x^6 + 2*(2*b^9*c - 25*a*b^7*c^2 + 108*a^2*b^5*c^3 - 176*a^3*b^3
*c^4 + 64*a^4*b*c^5)*x^5 + (2*b^10 - 21*a*b^8*c + 58*a^2*b^6*c^2 + 40*a^3*b
^4*c^3 - 288*a^4*b^2*c^4 + 128*a^5*c^5)*x^4 + 2*(2*a*b^9 - 25*a^2*b^7*c + 1
08*a^3*b^5*c^2 - 176*a^4*b^3*c^3 + 64*a^5*b*c^4)*x^3 + (2*a^2*b^8 - 25*a^3*
b^6*c + 108*a^4*b^4*c^2 - 176*a^5*b^2*c^3 + 64*a^6*c^4)*x^2)*log(x))/((a^5*
b^6*c^2 - 12*a^6*b^4*c^3 + 48*a^7*b^2*c^4 - 64*a^8*c^5)*x^6 + 2*(a^5*b^7*c
- 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*x^5 + (a^5*b^8 - 10*a^6*b
^6*c + 24*a^7*b^4*c^2 + 32*a^8*b^2*c^3 - 128*a^9*c^4)*x^4 + 2*(a^6*b^7 - 12
*a^7*b^5*c + 48*a^8*b^3*c^2 - 64*a^9*b*c^3)*x^3 + (a^7*b^6 - 12*a^8*b^4*c +
 48*a^9*b^2*c^2 - 64*a^10*c^3)*x^2)]
```

Sympy [B] time = 52.2431, size = 7465, normalized size = 24.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**2+b*x+a)**3,x)
```

```
[Out] (-3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b**2*c**2 + 21*a*b**
4*c - 2*b**6)/(2*a**5*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4
*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 3*(a*c - 2*b**2)/(2*a*
*5))*log(x + (98304*a**19*c**9*(-3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3
 - 70*a**2*b**2*c**2 + 21*a*b**4*c - 2*b**6)/(2*a**5*(1024*a**5*c**5 - 1280
*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b
```

```
**10)) + 3*(a*c - 2*b**2)/(2*a**5))**2 - 429056*a**18*b**2*c**8*(-3*b*sqrt(
-(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b**2*c**2 + 21*a*b**4*c - 2*b**
6)/(2*a**5*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160
*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 3*(a*c - 2*b**2)/(2*a**5))**2 + 6
45888*a**17*b**4*c**7*(-3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a**
2*b**2*c**2 + 21*a*b**4*c - 2*b**6)/(2*a**5*(1024*a**5*c**5 - 1280*a**4*b**
2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) +
3*(a*c - 2*b**2)/(2*a**5))**2 - 508032*a**16*b**6*c**6*(-3*b*sqrt(-(4*a*c -
 b**2)**5)*(70*a**3*c**3 - 70*a**2*b**2*c**2 + 21*a*b**4*c - 2*b**6)/(2*a**
5*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**
6*c**2 + 20*a*b**8*c - b**10)) + 3*(a*c - 2*b**2)/(2*a**5))**2 + 241376*a**
15*b**8*c**5*(-3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b**2*c*
*2 + 21*a*b**4*c - 2*b**6)/(2*a**5*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 +
640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 3*(a*c -
2*b**2)/(2*a**5))**2 + 147456*a**15*c**10*(-3*b*sqrt(-(4*a*c - b**2)**5)*(7
0*a**3*c**3 - 70*a**2*b**2*c**2 + 21*a*b**4*c - 2*b**6)/(2*a**5*(1024*a**5*
c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a
*b**8*c - b**10)) + 3*(a*c - 2*b**2)/(2*a**5)) - 73436*a**14*b**10*c**4*(-3
*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b**2*c**2 + 21*a*b**4*c
 - 2*b**6)/(2*a**5*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c*
*3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 3*(a*c - 2*b**2)/(2*a**5)
)**2 - 542016*a**14*b**2*c**9*(-3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3
- 70*a**2*b**2*c**2 + 21*a*b**4*c - 2*b**6)/(2*a**5*(1024*a**5*c**5 - 1280*
a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b*
*10)) + 3*(a*c - 2*b**2)/(2*a**5)) + 14479*a**13*b**12*c**3*(-3*b*sqrt(-(4*
a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b**2*c**2 + 21*a*b**4*c - 2*b**6)/(
2*a**5*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**
2*b**6*c**2 + 20*a*b**8*c - b**10)) + 3*(a*c - 2*b**2)/(2*a**5))**2 + 92376
0*a**13*b**4*c**8*(-3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b*
*2*c**2 + 21*a*b**4*c - 2*b**6)/(2*a**5*(1024*a**5*c**5 - 1280*a**4*b**2*c*
*4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 3*(a
*c - 2*b**2)/(2*a**5)) - 1797*a**12*b**14*c**2*(-3*b*sqrt(-(4*a*c - b**2)**
5)*(70*a**3*c**3 - 70*a**2*b**2*c**2 + 21*a*b**4*c - 2*b**6)/(2*a**5*(1024*
a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 +
 20*a*b**8*c - b**10)) + 3*(a*c - 2*b**2)/(2*a**5))**2 - 849948*a**12*b**6*
c**7*(-3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b**2*c**2 + 21*
a*b**4*c - 2*b**6)/(2*a**5*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3
*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 3*(a*c - 2*b**2)/
(2*a**5)) + 128*a**11*b**16*c*(-3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3
- 70*a**2*b**2*c**2 + 21*a*b**4*c - 2*b**6)/(2*a**5*(1024*a**5*c**5 - 1280*
a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b*
*10)) + 3*(a*c - 2*b**2)/(2*a**5))**2 + 464829*a**11*b**8*c**6*(-3*b*sqrt(-
(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b**2*c**2 + 21*a*b**4*c - 2*b**6
)/(2*a**5*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*
a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 3*(a*c - 2*b**2)/(2*a**5)) - 44236
8*a**11*c**11 - 4*a**10*b**18*(-3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3
- 70*a**2*b**2*c**2 + 21*a*b**4*c - 2*b**6)/(2*a**5*(1024*a**5*c**5 - 1280*
a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b*
*10)) + 3*(a*c - 2*b**2)/(2*a**5))**2 - 159318*a**10*b**10*c**5*(-3*b*sqrt(
-(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b**2*c**2 + 21*a*b**4*c - 2*b**
6)/(2*a**5*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160
*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 3*(a*c - 2*b**2)/(2*a**5)) + 4889
664*a**10*b**2*c**10 + 34731*a**9*b**12*c**4*(-3*b*sqrt(-(4*a*c - b**2)**5)
*(70*a**3*c**3 - 70*a**2*b**2*c**2 + 21*a*b**4*c - 2*b**6)/(2*a**5*(1024*a*
*5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 2
0*a*b**8*c - b**10)) + 3*(a*c - 2*b**2)/(2*a**5)) - 18774576*a**9*b**4*c**9
 - 4695*a**8*b**14*c**3*(-3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a
**2*b**2*c**2 + 21*a*b**4*c - 2*b**6)/(2*a**5*(1024*a**5*c**5 - 1280*a**4*b
**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))
```

```
+ 3*(a*c - 2*b**2)/(2*a**5)) + 35177868*a**8*b**6*c**8 + 360*a**7*b**16*c**
2*(-3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b**2*c**2 + 21*a*b
**4*c - 2*b**6)/(2*a**5*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b*
*4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 3*(a*c - 2*b**2)/(2*
a**5)) - 37219329*a**7*b**8*c**7 - 12*a**6*b**18*c*(-3*b*sqrt(-(4*a*c - b**
2)**5)*(70*a**3*c**3 - 70*a**2*b**2*c**2 + 21*a*b**4*c - 2*b**6)/(2*a**5*(1
024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c*
*2 + 20*a*b**8*c - b**10)) + 3*(a*c - 2*b**2)/(2*a**5)) + 24372684*a**6*b**
10*c**6 - 10403442*a**5*b**12*c**5 + 2958642*a**4*b**14*c**4 - 557838*a**3*
b**16*c**3 + 67140*a**2*b**18*c**2 - 4680*a*b**20*c + 144*b**22)/(1451520*a
**10*b*c**11 - 8300250*a**9*b**3*c**10 + 19711566*a**8*b**5*c**9 - 24401871
*a**7*b**7*c**8 + 17859492*a**6*b**9*c**7 - 8284248*a**5*b**11*c**6 + 25137
00*a**4*b**13*c**5 - 499338*a**3*b**15*c**4 + 62748*a**2*b**17*c**3 - 4536*
a*b**19*c**2 + 144*b**21*c)) + (3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3
- 70*a**2*b**2*c**2 + 21*a*b**4*c - 2*b**6)/(2*a**5*(1024*a**5*c**5 - 1280*
a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b*
*10)) + 3*(a*c - 2*b**2)/(2*a**5))*log(x + (98304*a**19*c**9*(3*b*sqrt(-(4*
a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b**2*c**2 + 21*a*b**4*c - 2*b**6)/(
2*a**5*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**
2*b**6*c**2 + 20*a*b**8*c - b**10)) + 3*(a*c - 2*b**2)/(2*a**5))**2 - 42905
6*a**18*b**2*c**8*(3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b**
2*c**2 + 21*a*b**4*c - 2*b**6)/(2*a**5*(1024*a**5*c**5 - 1280*a**4*b**2*c**
4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 3*(a*
c - 2*b**2)/(2*a**5))**2 + 645888*a**17*b**4*c**7*(3*b*sqrt(-(4*a*c - b**2)
**5)*(70*a**3*c**3 - 70*a**2*b**2*c**2 + 21*a*b**4*c - 2*b**6)/(2*a**5*(102
4*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2
 + 20*a*b**8*c - b**10)) + 3*(a*c - 2*b**2)/(2*a**5))**2 - 508032*a**16*b**
6*c**6*(3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b**2*c**2 + 21
*a*b**4*c - 2*b**6)/(2*a**5*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**
3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 3*(a*c - 2*b**2)
/(2*a**5))**2 + 241376*a**15*b**8*c**5*(3*b*sqrt(-(4*a*c - b**2)**5)*(70*a*
*3*c**3 - 70*a**2*b**2*c**2 + 21*a*b**4*c - 2*b**6)/(2*a**5*(1024*a**5*c**5
 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**
8*c - b**10)) + 3*(a*c - 2*b**2)/(2*a**5))**2 + 147456*a**15*c**10*(3*b*sqr
t(-(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b**2*c**2 + 21*a*b**4*c - 2*b
**6)/(2*a**5*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 1
60*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 3*(a*c - 2*b**2)/(2*a**5)) - 73
436*a**14*b**10*c**4*(3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*
b**2*c**2 + 21*a*b**4*c - 2*b**6)/(2*a**5*(1024*a**5*c**5 - 1280*a**4*b**2*
c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 3*
(a*c - 2*b**2)/(2*a**5))**2 - 542016*a**14*b**2*c**9*(3*b*sqrt(-(4*a*c - b*
*2)**5)*(70*a**3*c**3 - 70*a**2*b**2*c**2 + 21*a*b**4*c - 2*b**6)/(2*a**5*(
1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c
**2 + 20*a*b**8*c - b**10)) + 3*(a*c - 2*b**2)/(2*a**5)) + 14479*a**13*b**1
2*c**3*(3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b**2*c**2 + 21
*a*b**4*c - 2*b**6)/(2*a**5*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**
3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 3*(a*c - 2*b**2)
/(2*a**5))**2 + 923760*a**13*b**4*c**8*(3*b*sqrt(-(4*a*c - b**2)**5)*(70*a*
*3*c**3 - 70*a**2*b**2*c**2 + 21*a*b**4*c - 2*b**6)/(2*a**5*(1024*a**5*c**5
 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**
8*c - b**10)) + 3*(a*c - 2*b**2)/(2*a**5)) - 1797*a**12*b**14*c**2*(3*b*sqr
t(-(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b**2*c**2 + 21*a*b**4*c - 2*b
**6)/(2*a**5*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 1
60*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 3*(a*c - 2*b**2)/(2*a**5))**2 -
 849948*a**12*b**6*c**7*(3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a*
*2*b**2*c**2 + 21*a*b**4*c - 2*b**6)/(2*a**5*(1024*a**5*c**5 - 1280*a**4*b*
*2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) +
 3*(a*c - 2*b**2)/(2*a**5)) + 128*a**11*b**16*c*(3*b*sqrt(-(4*a*c - b**2)**
5)*(70*a**3*c**3 - 70*a**2*b**2*c**2 + 21*a*b**4*c - 2*b**6)/(2*a**5*(1024*
```

```
a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 +
 20*a*b**8*c - b**10)) + 3*(a*c - 2*b**2)/(2*a**5))**2 + 464829*a**11*b**8*
c**6*(3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b**2*c**2 + 21*a
*b**4*c - 2*b**6)/(2*a**5*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*
b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 3*(a*c - 2*b**2)/(
2*a**5)) - 442368*a**11*c**11 - 4*a**10*b**18*(3*b*sqrt(-(4*a*c - b**2)**5)
*(70*a**3*c**3 - 70*a**2*b**2*c**2 + 21*a*b**4*c - 2*b**6)/(2*a**5*(1024*a*
*5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 2
0*a*b**8*c - b**10)) + 3*(a*c - 2*b**2)/(2*a**5))**2 - 159318*a**10*b**10*c
**5*(3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b**2*c**2 + 21*a*
b**4*c - 2*b**6)/(2*a**5*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b
**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 3*(a*c - 2*b**2)/(2
*a**5)) + 4889664*a**10*b**2*c**10 + 34731*a**9*b**12*c**4*(3*b*sqrt(-(4*a*
c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b**2*c**2 + 21*a*b**4*c - 2*b**6)/(2*
a**5*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*
b**6*c**2 + 20*a*b**8*c - b**10)) + 3*(a*c - 2*b**2)/(2*a**5)) - 18774576*a
**9*b**4*c**9 - 4695*a**8*b**14*c**3*(3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3
*c**3 - 70*a**2*b**2*c**2 + 21*a*b**4*c - 2*b**6)/(2*a**5*(1024*a**5*c**5 -
 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*
c - b**10)) + 3*(a*c - 2*b**2)/(2*a**5)) + 35177868*a**8*b**6*c**8 + 360*a*
*7*b**16*c**2*(3*b*sqrt(-(4*a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b**2*c*
*2 + 21*a*b**4*c - 2*b**6)/(2*a**5*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 +
640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 3*(a*c -
2*b**2)/(2*a**5)) - 37219329*a**7*b**8*c**7 - 12*a**6*b**18*c*(3*b*sqrt(-(4
*a*c - b**2)**5)*(70*a**3*c**3 - 70*a**2*b**2*c**2 + 21*a*b**4*c - 2*b**6)/
(2*a**5*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a*
*2*b**6*c**2 + 20*a*b**8*c - b**10)) + 3*(a*c - 2*b**2)/(2*a**5)) + 2437268
4*a**6*b**10*c**6 - 10403442*a**5*b**12*c**5 + 2958642*a**4*b**14*c**4 - 55
7838*a**3*b**16*c**3 + 67140*a**2*b**18*c**2 - 4680*a*b**20*c + 144*b**22)/
(1451520*a**10*b*c**11 - 8300250*a**9*b**3*c**10 + 19711566*a**8*b**5*c**9
- 24401871*a**7*b**7*c**8 + 17859492*a**6*b**9*c**7 - 8284248*a**5*b**11*c*
*6 + 2513700*a**4*b**13*c**5 - 499338*a**3*b**15*c**4 + 62748*a**2*b**17*c*
*3 - 4536*a*b**19*c**2 + 144*b**21*c)) + (-16*a**5*c**2 + 8*a**4*b**2*c - a
**3*b**4 + x**5*(162*a**2*b*c**4 - 90*a*b**3*c**3 + 12*b**5*c**2) + x**4*(-
48*a**3*c**4 + 363*a**2*b**2*c**3 - 186*a*b**4*c**2 + 24*b**6*c) + x**3*(20
6*a**3*b*c**3 + 64*a**2*b**3*c**2 - 78*a*b**5*c + 12*b**7) + x**2*(-72*a**4
*c**3 + 307*a**3*b**2*c**2 - 145*a**2*b**4*c + 18*a*b**6) + x*(64*a**4*b*c*
*2 - 32*a**3*b**3*c + 4*a**2*b**5))/(x**6*(32*a**6*c**4 - 16*a**5*b**2*c**3
 + 2*a**4*b**4*c**2) + x**5*(64*a**6*b*c**3 - 32*a**5*b**3*c**2 + 4*a**4*b*
*5*c) + x**4*(64*a**7*c**3 - 12*a**5*b**4*c + 2*a**4*b**6) + x**3*(64*a**7*
b*c**2 - 32*a**6*b**3*c + 4*a**5*b**5) + x**2*(32*a**8*c**2 - 16*a**7*b**2*
c + 2*a**6*b**4)) - 3*(a*c - 2*b**2)*log(x + (-442368*a**11*c**11 + 4889664
*a**10*b**2*c**10 - 442368*a**10*c**10*(a*c - 2*b**2) - 18774576*a**9*b**4*
c**9 + 1626048*a**9*b**2*c**9*(a*c - 2*b**2) + 884736*a**9*c**9*(a*c - 2*b*
*2)**2 + 35177868*a**8*b**6*c**8 - 2771280*a**8*b**4*c**8*(a*c - 2*b**2) -
3861504*a**8*b**2*c**8*(a*c - 2*b**2)**2 - 37219329*a**7*b**8*c**7 + 254984
4*a**7*b**6*c**7*(a*c - 2*b**2) + 5812992*a**7*b**4*c**7*(a*c - 2*b**2)**2
+ 24372684*a**6*b**10*c**6 - 1394487*a**6*b**8*c**6*(a*c - 2*b**2) - 457228
8*a**6*b**6*c**6*(a*c - 2*b**2)**2 - 10403442*a**5*b**12*c**5 + 477954*a**5
*b**10*c**5*(a*c - 2*b**2) + 2172384*a**5*b**8*c**5*(a*c - 2*b**2)**2 + 295
8642*a**4*b**14*c**4 - 104193*a**4*b**12*c**4*(a*c - 2*b**2) - 660924*a**4*
b**10*c**4*(a*c - 2*b**2)**2 - 557838*a**3*b**16*c**3 + 14085*a**3*b**14*c*
*3*(a*c - 2*b**2) + 130311*a**3*b**12*c**3*(a*c - 2*b**2)**2 + 67140*a**2*b
**18*c**2 - 1080*a**2*b**16*c**2*(a*c - 2*b**2) - 16173*a**2*b**14*c**2*(a*
c - 2*b**2)**2 - 4680*a*b**20*c + 36*a*b**18*c*(a*c - 2*b**2) + 1152*a*b**1
6*c*(a*c - 2*b**2)**2 + 144*b**22 - 36*b**18*(a*c - 2*b**2)**2)/(1451520*a*
*10*b*c**11 - 8300250*a**9*b**3*c**10 + 19711566*a**8*b**5*c**9 - 24401871*
a**7*b**7*c**8 + 17859492*a**6*b**9*c**7 - 8284248*a**5*b**11*c**6 + 251370
0*a**4*b**13*c**5 - 499338*a**3*b**15*c**4 + 62748*a**2*b**17*c**3 - 4536*a
```

```
*b**19*c**2 + 144*b**21*c))/a**5
```

Giac [A] time = 1.12907, size = 554, normalized size = 1.81

$$-\frac{3\left(2\,b^{7}-21\,ab^{5}c+70\,a^{2}b^{3}c^{2}-70\,a^{3}bc^{3}\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(a^{5}b^{4}-8\,a^{6}b^{2}c+16\,a^{7}c^{2}\right)\sqrt{-b^{2}+4\,ac}}+\frac{12\,b^{5}c^{2}x^{5}-90\,ab^{3}c^{3}x^{5}+162\,a^{2}bc^{4}x^{5}+24\,b^{6}cx^{4}-186\,ab}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] -3*(2*b^7 - 21*a*b^5*c + 70*a^2*b^3*c^2 - 70*a^3*b*c^3)*arctan((2*c*x + b)/
sqrt(-b^2 + 4*a*c))/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt(-b^2 + 4*a*c
)) + 1/2*(12*b^5*c^2*x^5 - 90*a*b^3*c^3*x^5 + 162*a^2*b*c^4*x^5 + 24*b^6*c*
x^4 - 186*a*b^4*c^2*x^4 + 363*a^2*b^2*c^3*x^4 - 48*a^3*c^4*x^4 + 12*b^7*x^3
 - 78*a*b^5*c*x^3 + 64*a^2*b^3*c^2*x^3 + 206*a^3*b*c^3*x^3 + 18*a*b^6*x^2 -
 145*a^2*b^4*c*x^2 + 307*a^3*b^2*c^2*x^2 - 72*a^4*c^3*x^2 + 4*a^2*b^5*x - 3
2*a^3*b^3*c*x + 64*a^4*b*c^2*x - a^3*b^4 + 8*a^4*b^2*c - 16*a^5*c^2)/((a^4*
b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*(c*x^3 + b*x^2 + a*x)^2) - 3/2*(2*b^2 - a*c
)*log(c*x^2 + b*x + a)/a^5 + 3*(2*b^2 - a*c)*log(abs(x))/a^5
```

3.2210 $\int \frac{x^8}{\left(a+bx+cx^2\right)^4} dx$

Optimal. Leaf size=349

$$\frac{x^3\left(bx\left(122a^2c^2-39ab^2c+4b^4\right)+4a\left(35a^2c^2-9ab^2c+b^4\right)\right)}{3c^2\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)}-\frac{2bx^2\left(29a^2c^2-10ab^2c+b^4\right)}{c^3\left(b^2-4ac\right)^3}+\frac{4x\left(38a^2b^2c^2-35a^3c^3\right.}{c^4\left(b^2-4a\right.}$$

```
[Out] (4*(b^6 - 11*a*b^4*c + 38*a^2*b^2*c^2 - 35*a^3*c^3)*x)/(c^4*(b^2 - 4*a*c)^3
) - (2*b*(b^4 - 10*a*b^2*c + 29*a^2*c^2)*x^2)/(c^3*(b^2 - 4*a*c)^3) + (x^7*
(2*a + b*x))/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^3) + (x^5*(a*(b^2 - 14*a*c)
 + b*(b^2 - 9*a*c)*x))/(3*c*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^2) + (x^3*(4*
a*(b^4 - 9*a*b^2*c + 35*a^2*c^2) + b*(4*b^4 - 39*a*b^2*c + 122*a^2*c^2)*x))
/(3*c^2*(b^2 - 4*a*c)^3*(a + b*x + c*x^2)) - (4*(b^8 - 14*a*b^6*c + 70*a^2*
b^4*c^2 - 140*a^3*b^2*c^3 + 70*a^4*c^4)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*
c]])/(c^5*(b^2 - 4*a*c)^(7/2)) - (2*b*Log[a + b*x + c*x^2])/c^5
```

Rubi [A] time = 0.639975, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {738, 818, 800, 634, 618, 206, 628}

$$\frac{x^3\left(bx\left(122a^2c^2-39ab^2c+4b^4\right)+4a\left(35a^2c^2-9ab^2c+b^4\right)\right)}{3c^2\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)}-\frac{2bx^2\left(29a^2c^2-10ab^2c+b^4\right)}{c^3\left(b^2-4ac\right)^3}+\frac{4x\left(38a^2b^2c^2-35a^3c^3\right.}{c^4\left(b^2-4a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x + c*x^2)^4,x]
```

```
[Out] (4*(b^6 - 11*a*b^4*c + 38*a^2*b^2*c^2 - 35*a^3*c^3)*x)/(c^4*(b^2 - 4*a*c)^3
) - (2*b*(b^4 - 10*a*b^2*c + 29*a^2*c^2)*x^2)/(c^3*(b^2 - 4*a*c)^3) + (x^7*
(2*a + b*x))/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^3) + (x^5*(a*(b^2 - 14*a*c)
 + b*(b^2 - 9*a*c)*x))/(3*c*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^2) + (x^3*(4*
a*(b^4 - 9*a*b^2*c + 35*a^2*c^2) + b*(4*b^4 - 39*a*b^2*c + 122*a^2*c^2)*x))
/(3*c^2*(b^2 - 4*a*c)^3*(a + b*x + c*x^2)) - (4*(b^8 - 14*a*b^6*c + 70*a^2*
b^4*c^2 - 140*a^3*b^2*c^3 + 70*a^4*c^4)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*
c]])/(c^5*(b^2 - 4*a*c)^(7/2)) - (2*b*Log[a + b*x + c*x^2])/c^5
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
```

```
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx+cx^2\right)^4}\,dx = \frac{x^7(2a+bx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} - \frac{\int \frac{x^6(14a+2bx)}{\left(a+bx+cx^2\right)^3}\,dx}{3\left(b^2-4ac\right)}$$

$$= \frac{x^7(2a+bx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{x^5\left(a\left(b^2-14ac\right)+b\left(b^2-9ac\right)x\right)}{3c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} - \frac{\int \frac{x^4\left(10a\left(b^2-14ac\right)+4b\left(2b^2-13\right.\right.}{\left(a+bx+cx^2\right)^2}}{6c\left(b^2-4ac\right)^2}$$

$$= \frac{x^7(2a+bx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{x^5\left(a\left(b^2-14ac\right)+b\left(b^2-9ac\right)x\right)}{3c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} + \frac{x^3\left(4a\left(b^4-9ab^2c+35a\right.\right.}{3c^2\left(b^2-\right.}$$

$$= \frac{x^7(2a+bx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{x^5\left(a\left(b^2-14ac\right)+b\left(b^2-9ac\right)x\right)}{3c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} + \frac{x^3\left(4a\left(b^4-9ab^2c+35a\right.\right.}{3c^2\left(b^2-\right.}$$

$$= \frac{4\left(b^6-11ab^4c+38a^2b^2c^2-35a^3c^3\right)x}{c^4\left(b^2-4ac\right)^3} - \frac{2b\left(b^4-10ab^2c+29a^2c^2\right)x^2}{c^3\left(b^2-4ac\right)^3} + \frac{x^7(2a+bx)}{3\left(b^2-4ac\right)\left(a+bx\right.}$$

$$= \frac{4\left(b^6-11ab^4c+38a^2b^2c^2-35a^3c^3\right)x}{c^4\left(b^2-4ac\right)^3} - \frac{2b\left(b^4-10ab^2c+29a^2c^2\right)x^2}{c^3\left(b^2-4ac\right)^3} + \frac{x^7(2a+bx)}{3\left(b^2-4ac\right)\left(a+bx\right.}$$

$$= \frac{4\left(b^6-11ab^4c+38a^2b^2c^2-35a^3c^3\right)x}{c^4\left(b^2-4ac\right)^3} - \frac{2b\left(b^4-10ab^2c+29a^2c^2\right)x^2}{c^3\left(b^2-4ac\right)^3} + \frac{x^7(2a+bx)}{3\left(b^2-4ac\right)\left(a+bx\right.}$$

$$= \frac{4\left(b^6-11ab^4c+38a^2b^2c^2-35a^3c^3\right)x}{c^4\left(b^2-4ac\right)^3} - \frac{2b\left(b^4-10ab^2c+29a^2c^2\right)x^2}{c^3\left(b^2-4ac\right)^3} + \frac{x^7(2a+bx)}{3\left(b^2-4ac\right)\left(a+bx\right.}$$

Mathematica [A] time = 0.719013, size = 435, normalized size = 1.25

$$-\frac{6c\left(146a^2b^4c^3x-212a^3b^2c^4x-83a^2b^5c^2+198a^3b^3c^3-163a^4bc^4+58a^4c^5x-36ab^6c^2x+15ab^7c+3b^8cx-b^9\right)}{\left(b^2-4ac\right)^3(a+x(b+cx))} + \frac{-212a^2b^4c^3x+220a^3b^2c^4x+95a^2b^5c^2-202a^3b^3c^3+}{\left(b^2-4ac\right)^2(a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x + c*x^2)^4,x]
```

```
[Out] (3*c^3*x + (-(b^8*x) + a^2*b^4*c*(7*b - 20*c*x) - a*b^6*(b - 8*c*x) - 2*a^3
*b^2*c^2*(7*b - 8*c*x) + a^4*c^3*(7*b - 2*c*x))/((b^2 - 4*a*c)*(a + x*(b +
c*x))^3) + (b^9 - 17*a*b^7*c + 95*a^2*b^5*c^2 - 202*a^3*b^3*c^3 + 125*a^4*b
*c^4 - 7*b^8*c*x + 68*a*b^6*c^2*x - 212*a^2*b^4*c^3*x + 220*a^3*b^2*c^4*x -
 38*a^4*c^5*x)/((b^2 - 4*a*c)^2*(a + x*(b + c*x))^2) - (6*c*(-b^9 + 15*a*b^
7*c - 83*a^2*b^5*c^2 + 198*a^3*b^3*c^3 - 163*a^4*b*c^4 + 3*b^8*c*x - 36*a*b
^6*c^2*x + 146*a^2*b^4*c^3*x - 212*a^3*b^2*c^4*x + 58*a^4*c^5*x))/((b^2 - 4
*a*c)^3*(a + x*(b + c*x))) - (12*c^2*(b^8 - 14*a*b^6*c + 70*a^2*b^4*c^2 - 1
40*a^3*b^2*c^3 + 70*a^4*c^4)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2
+ 4*a*c)^(7/2) - 6*b*c^2*Log[a + x*(b + c*x)])/(3*c^7)
```

Maple [B] time = 0.173, size = 2336, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^2+b*x+a)^4,x)
```

```
[Out] -424/(c*x^2+b*x+a)^3/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^5*a^3*b^2
-280/c/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1/2)*arctan(
(2*c*x+b)/(4*a*c-b^2)^(1/2))*a^4-4/c^5/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*
c-b^6)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^8-49/c^4/(c*
x^2+b*x+a)^3*a^4*b^5/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)+10/c^4/(c*x
^2+b*x+a)^3*b^9/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^4-590/3/c^2/(c
*x^2+b*x+a)^3*a^6*b/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)+535/3/c^3/(c
*x^2+b*x+a)^3*a^5*b^3/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)+13/3/c^5/(
c*x^2+b*x+a)^3*a^3*b^7/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)+116*c/(c*
x^2+b*x+a)^3/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^5*a^4-94/(c*x^2+b
*x+a)^3*b/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^4*a^4+76/c/(c*x^2+b*
x+a)^3*a^6/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x-24/c^4/(64*a^3*c^3-
48*a^2*b^2*c^2+12*a*b^4*c-b^6)*ln(c*x^2+b*x+a)*a*b^5-128/c^2/(64*a^3*c^3-48
*a^2*b^2*c^2+12*a*b^4*c-b^6)*ln(c*x^2+b*x+a)*a^3*b+96/c^3/(64*a^3*c^3-48*a^
2*b^2*c^2+12*a*b^4*c-b^6)*ln(c*x^2+b*x+a)*a^2*b^3+544/3/(c*x^2+b*x+a)^3/(64
*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^3*a^5+2/c^5/(64*a^3*c^3-48*a^2*b^
2*c^2+12*a*b^4*c-b^6)*ln(c*x^2+b*x+a)*b^7+1/c^4*x+6/c^3/(c*x^2+b*x+a)^3/(64
*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^5*b^8+13/3/c^5/(c*x^2+b*x+a)^3/(6
4*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^3*b^10-452/c/(c*x^2+b*x+a)^3*b^3
/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^4*a^3+56/c^4/(64*a^3*c^3-48*a
^2*b^2*c^2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2))*a*b^6+560/c^2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(4*a*c-b^2)^
(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^3*b^2-68/3/c^3/(c*x^2+b*x+a)^3/
(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^3*a^2*b^6-280/c^3/(64*a^3*c^3-
48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^
2)^(1/2))*a^2*b^4-304/c/(c*x^2+b*x+a)^3*a^5*b/(64*a^3*c^3-48*a^2*b^2*c^2+12
*a*b^4*c-b^6)*x^2-387/c^2/(c*x^2+b*x+a)^3*a^4*b^3/(64*a^3*c^3-48*a^2*b^2*c^
2+12*a*b^4*c-b^6)*x^2+486/c^3/(c*x^2+b*x+a)^3*a^3*b^5/(64*a^3*c^3-48*a^2*b^
2*c^2+12*a*b^4*c-b^6)*x^2+13/c^5/(c*x^2+b*x+a)^3*a*b^9/(64*a^3*c^3-48*a^2*b
^2*c^2+12*a*b^4*c-b^6)*x^2+418/c^2/(c*x^2+b*x+a)^3*b^5/(64*a^3*c^3-48*a^2*b
^2*c^2+12*a*b^4*c-b^6)*x^4*a^2-114/c^3/(c*x^2+b*x+a)^3*b^7/(64*a^3*c^3-48*a
^2*b^2*c^2+12*a*b^4*c-b^6)*x^4*a-1078/c/(c*x^2+b*x+a)^3/(64*a^3*c^3-48*a^2*
b^2*c^2+12*a*b^4*c-b^6)*x^3*a^4*b^2+596/c^2/(c*x^2+b*x+a)^3/(64*a^3*c^3-48*
a^2*b^2*c^2+12*a*b^4*c-b^6)*x^3*a^3*b^4-150/c^4/(c*x^2+b*x+a)^3*a^3/(64*a^3
*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x*b^6-32/c^4/(c*x^2+b*x+a)^3/(64*a^3*c^
3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^3*a*b^8-143/c^4/(c*x^2+b*x+a)^3*a^2*b^7/
(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^2-694/c^2/(c*x^2+b*x+a)^3*a^5/
(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x*b^2+567/c^3/(c*x^2+b*x+a)^3*a^
4/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x*b^4+13/c^5/(c*x^2+b*x+a)^3*a
^2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x*b^8-72/c^2/(c*x^2+b*x+a)^3/
(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^5*a*b^6+292/c/(c*x^2+b*x+a)^3/
(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^5*a^2*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.87633, size = 7313, normalized size = 20.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/3*(13*a^3*b^9 - 199*a^4*b^7*c + 1123*a^5*b^5*c^2 - 2730*a^6*b^3*c^3 + 2
360*a^7*b*c^4 - 3*(b^8*c^4 - 16*a*b^6*c^5 + 96*a^2*b^4*c^6 - 256*a^3*b^2*c^
7 + 256*a^4*c^8)*x^7 - 9*(b^9*c^3 - 16*a*b^7*c^4 + 96*a^2*b^5*c^5 - 256*a^3
*b^3*c^6 + 256*a^4*b*c^7)*x^6 + 3*(3*b^10*c^2 - 51*a*b^8*c^3 + 340*a^2*b^6*
c^4 - 1112*a^3*b^4*c^5 + 1812*a^4*b^2*c^6 - 1232*a^5*c^7)*x^5 + 3*(9*b^11*c
 - 144*a*b^9*c^2 + 874*a^2*b^7*c^3 - 2444*a^3*b^5*c^4 + 2994*a^4*b^3*c^5 -
1160*a^5*b*c^6)*x^4 + (13*b^12 - 157*a*b^10*c + 451*a^2*b^8*c^2 + 1340*a^3*
b^6*c^3 - 8946*a^4*b^4*c^4 + 13480*a^5*b^2*c^5 - 4480*a^6*c^6)*x^3 + 3*(13*
a*b^11 - 198*a^2*b^9*c + 1106*a^3*b^7*c^2 - 2619*a^4*b^5*c^3 + 2012*a^5*b^3
*c^4 + 448*a^6*b*c^5)*x^2 + 6*(a^3*b^8 - 14*a^4*b^6*c + 70*a^5*b^4*c^2 - 14
0*a^6*b^2*c^3 + 70*a^7*c^4 + (b^8*c^3 - 14*a*b^6*c^4 + 70*a^2*b^4*c^5 - 140
*a^3*b^2*c^6 + 70*a^4*c^7)*x^6 + 3*(b^9*c^2 - 14*a*b^7*c^3 + 70*a^2*b^5*c^4
 - 140*a^3*b^3*c^5 + 70*a^4*b*c^6)*x^5 + 3*(b^10*c - 13*a*b^8*c^2 + 56*a^2*
b^6*c^3 - 70*a^3*b^4*c^4 - 70*a^4*b^2*c^5 + 70*a^5*c^6)*x^4 + (b^11 - 8*a*b
^9*c - 14*a^2*b^7*c^2 + 280*a^3*b^5*c^3 - 770*a^4*b^3*c^4 + 420*a^5*b*c^5)*
x^3 + 3*(a*b^10 - 13*a^2*b^8*c + 56*a^3*b^6*c^2 - 70*a^4*b^4*c^3 - 70*a^5*b
^2*c^4 + 70*a^6*c^5)*x^2 + 3*(a^2*b^9 - 14*a^3*b^7*c + 70*a^4*b^5*c^2 - 140
*a^5*b^3*c^3 + 70*a^6*b*c^4)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x
+ b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 3*(13*a
^2*b^10 - 203*a^3*b^8*c + 1183*a^4*b^6*c^2 - 3058*a^5*b^4*c^3 + 3108*a^6*b^
2*c^4 - 560*a^7*c^5)*x + 6*(a^3*b^9 - 16*a^4*b^7*c + 96*a^5*b^5*c^2 - 256*a
^6*b^3*c^3 + 256*a^7*b*c^4 + (b^9*c^3 - 16*a*b^7*c^4 + 96*a^2*b^5*c^5 - 256
*a^3*b^3*c^6 + 256*a^4*b*c^7)*x^6 + 3*(b^10*c^2 - 16*a*b^8*c^3 + 96*a^2*b^6
*c^4 - 256*a^3*b^4*c^5 + 256*a^4*b^2*c^6)*x^5 + 3*(b^11*c - 15*a*b^9*c^2 +
80*a^2*b^7*c^3 - 160*a^3*b^5*c^4 + 256*a^5*b*c^6)*x^4 + (b^12 - 10*a*b^10*c
 + 320*a^3*b^6*c^3 - 1280*a^4*b^4*c^4 + 1536*a^5*b^2*c^5)*x^3 + 3*(a*b^11 -
 15*a^2*b^9*c + 80*a^3*b^7*c^2 - 160*a^4*b^5*c^3 + 256*a^6*b*c^5)*x^2 + 3*(
a^2*b^10 - 16*a^3*b^8*c + 96*a^4*b^6*c^2 - 256*a^5*b^4*c^3 + 256*a^6*b^2*c^
4)*x)*log(c*x^2 + b*x + a))/(a^3*b^8*c^5 - 16*a^4*b^6*c^6 + 96*a^5*b^4*c^7
- 256*a^6*b^2*c^8 + 256*a^7*c^9 + (b^8*c^8 - 16*a*b^6*c^9 + 96*a^2*b^4*c^10
 - 256*a^3*b^2*c^11 + 256*a^4*c^12)*x^6 + 3*(b^9*c^7 - 16*a*b^7*c^8 + 96*a^
2*b^5*c^9 - 256*a^3*b^3*c^10 + 256*a^4*b*c^11)*x^5 + 3*(b^10*c^6 - 15*a*b^8
*c^7 + 80*a^2*b^6*c^8 - 160*a^3*b^4*c^9 + 256*a^5*c^11)*x^4 + (b^11*c^5 - 1
0*a*b^9*c^6 + 320*a^3*b^5*c^8 - 1280*a^4*b^3*c^9 + 1536*a^5*b*c^10)*x^3 + 3
*(a*b^10*c^5 - 15*a^2*b^8*c^6 + 80*a^3*b^6*c^7 - 160*a^4*b^4*c^8 + 256*a^6*
c^10)*x^2 + 3*(a^2*b^9*c^5 - 16*a^3*b^7*c^6 + 96*a^4*b^5*c^7 - 256*a^5*b^3*
c^8 + 256*a^6*b*c^9)*x), -1/3*(13*a^3*b^9 - 199*a^4*b^7*c + 1123*a^5*b^5*c^
2 - 2730*a^6*b^3*c^3 + 2360*a^7*b*c^4 - 3*(b^8*c^4 - 16*a*b^6*c^5 + 96*a^2*
b^4*c^6 - 256*a^3*b^2*c^7 + 256*a^4*c^8)*x^7 - 9*(b^9*c^3 - 16*a*b^7*c^4 +
96*a^2*b^5*c^5 - 256*a^3*b^3*c^6 + 256*a^4*b*c^7)*x^6 + 3*(3*b^10*c^2 - 51*
a*b^8*c^3 + 340*a^2*b^6*c^4 - 1112*a^3*b^4*c^5 + 1812*a^4*b^2*c^6 - 1232*a^
5*c^7)*x^5 + 3*(9*b^11*c - 144*a*b^9*c^2 + 874*a^2*b^7*c^3 - 2444*a^3*b^5*c
^4 + 2994*a^4*b^3*c^5 - 1160*a^5*b*c^6)*x^4 + (13*b^12 - 157*a*b^10*c + 451
*a^2*b^8*c^2 + 1340*a^3*b^6*c^3 - 8946*a^4*b^4*c^4 + 13480*a^5*b^2*c^5 - 44
80*a^6*c^6)*x^3 + 3*(13*a*b^11 - 198*a^2*b^9*c + 1106*a^3*b^7*c^2 - 2619*a^
4*b^5*c^3 + 2012*a^5*b^3*c^4 + 448*a^6*b*c^5)*x^2 + 12*(a^3*b^8 - 14*a^4*b^
6*c + 70*a^5*b^4*c^2 - 140*a^6*b^2*c^3 + 70*a^7*c^4 + (b^8*c^3 - 14*a*b^6*c
^4 + 70*a^2*b^4*c^5 - 140*a^3*b^2*c^6 + 70*a^4*c^7)*x^6 + 3*(b^9*c^2 - 14*a
*b^7*c^3 + 70*a^2*b^5*c^4 - 140*a^3*b^3*c^5 + 70*a^4*b*c^6)*x^5 + 3*(b^10*c
 - 13*a*b^8*c^2 + 56*a^2*b^6*c^3 - 70*a^3*b^4*c^4 - 70*a^4*b^2*c^5 + 70*a^5
```

```
*c^6)*x^4 + (b^11 - 8*a*b^9*c - 14*a^2*b^7*c^2 + 280*a^3*b^5*c^3 - 770*a^4*
b^3*c^4 + 420*a^5*b*c^5)*x^3 + 3*(a*b^10 - 13*a^2*b^8*c + 56*a^3*b^6*c^2 -
70*a^4*b^4*c^3 - 70*a^5*b^2*c^4 + 70*a^6*c^5)*x^2 + 3*(a^2*b^9 - 14*a^3*b^7
*c + 70*a^4*b^5*c^2 - 140*a^5*b^3*c^3 + 70*a^6*b*c^4)*x)*sqrt(-b^2 + 4*a*c)
*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 3*(13*a^2*b^10 - 2
03*a^3*b^8*c + 1183*a^4*b^6*c^2 - 3058*a^5*b^4*c^3 + 3108*a^6*b^2*c^4 - 560
*a^7*c^5)*x + 6*(a^3*b^9 - 16*a^4*b^7*c + 96*a^5*b^5*c^2 - 256*a^6*b^3*c^3
+ 256*a^7*b*c^4 + (b^9*c^3 - 16*a*b^7*c^4 + 96*a^2*b^5*c^5 - 256*a^3*b^3*c^
6 + 256*a^4*b*c^7)*x^6 + 3*(b^10*c^2 - 16*a*b^8*c^3 + 96*a^2*b^6*c^4 - 256*
a^3*b^4*c^5 + 256*a^4*b^2*c^6)*x^5 + 3*(b^11*c - 15*a*b^9*c^2 + 80*a^2*b^7*
c^3 - 160*a^3*b^5*c^4 + 256*a^5*b*c^6)*x^4 + (b^12 - 10*a*b^10*c + 320*a^3*
b^6*c^3 - 1280*a^4*b^4*c^4 + 1536*a^5*b^2*c^5)*x^3 + 3*(a*b^11 - 15*a^2*b^9
*c + 80*a^3*b^7*c^2 - 160*a^4*b^5*c^3 + 256*a^6*b*c^5)*x^2 + 3*(a^2*b^10 -
16*a^3*b^8*c + 96*a^4*b^6*c^2 - 256*a^5*b^4*c^3 + 256*a^6*b^2*c^4)*x)*log(c
*x^2 + b*x + a))/(a^3*b^8*c^5 - 16*a^4*b^6*c^6 + 96*a^5*b^4*c^7 - 256*a^6*b
^2*c^8 + 256*a^7*c^9 + (b^8*c^8 - 16*a*b^6*c^9 + 96*a^2*b^4*c^10 - 256*a^3*
b^2*c^11 + 256*a^4*c^12)*x^6 + 3*(b^9*c^7 - 16*a*b^7*c^8 + 96*a^2*b^5*c^9 -
 256*a^3*b^3*c^10 + 256*a^4*b*c^11)*x^5 + 3*(b^10*c^6 - 15*a*b^8*c^7 + 80*a
^2*b^6*c^8 - 160*a^3*b^4*c^9 + 256*a^5*c^11)*x^4 + (b^11*c^5 - 10*a*b^9*c^6
 + 320*a^3*b^5*c^8 - 1280*a^4*b^3*c^9 + 1536*a^5*b*c^10)*x^3 + 3*(a*b^10*c^
5 - 15*a^2*b^8*c^6 + 80*a^3*b^6*c^7 - 160*a^4*b^4*c^8 + 256*a^6*c^10)*x^2 +
 3*(a^2*b^9*c^5 - 16*a^3*b^7*c^6 + 96*a^4*b^5*c^7 - 256*a^5*b^3*c^8 + 256*a
^6*b*c^9)*x)]
```

Sympy [B] time = 11.6313, size = 2769, normalized size = 7.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**2+b*x+a)**4,x)
```

```
[Out] (-2*b/c**5 - 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3
+ 70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(c**5*(16384*a**7*c**7 - 28672*a*
*6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*
c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14)))*log(x + (-372*a**4*b*c
**3 - 256*a**4*c**8*(-2*b/c**5 - 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 -
 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(c**5*(16384*
a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c*
*4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14))) +
232*a**3*b**3*c**2 + 256*a**3*b**2*c**7*(-2*b/c**5 - 2*sqrt(-(4*a*c - b**2)
**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c +
 b**8)/(c**5*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5
 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b
**12*c - b**14))) - 52*a**2*b**5*c - 96*a**2*b**4*c**6*(-2*b/c**5 - 2*sqrt(
-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2
- 14*a*b**6*c + b**8)/(c**5*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504
*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**1
0*c**2 + 28*a*b**12*c - b**14))) + 4*a*b**7 + 16*a*b**6*c**5*(-2*b/c**5 - 2
*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b**4
*c**2 - 14*a*b**6*c + b**8)/(c**5*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 +
 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**
2*b**10*c**2 + 28*a*b**12*c - b**14))) - b**8*c**4*(-2*b/c**5 - 2*sqrt(-(4*
a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14
*a*b**6*c + b**8)/(c**5*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**
5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c*
*2 + 28*a*b**12*c - b**14))))/(280*a**4*c**4 - 560*a**3*b**2*c**3 + 280*a**
```

```
2*b**4*c**2 - 56*a*b**6*c + 4*b**8)) + (-2*b/c**5 + 2*sqrt(-(4*a*c - b**2)*
*7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c +
b**8)/(c**5*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5
- 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b*
*12*c - b**14)))*log(x + (-372*a**4*b*c**3 - 256*a**4*c**8*(-2*b/c**5 + 2*s
qrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c
**2 - 14*a*b**6*c + b**8)/(c**5*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 2
1504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*
b**10*c**2 + 28*a*b**12*c - b**14))) + 232*a**3*b**3*c**2 + 256*a**3*b**2*c
**7*(-2*b/c**5 + 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c
**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(c**5*(16384*a**7*c**7 - 2867
2*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b
**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14))) - 52*a**2*b**5*c -
 96*a**2*b**4*c**6*(-2*b/c**5 + 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 -
140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(c**5*(16384*a
**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**
4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14))) + 4
*a*b**7 + 16*a*b**6*c**5*(-2*b/c**5 + 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c
**4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(c**5*(1
6384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b*
*6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14)
)) - b**8*c**4*(-2*b/c**5 + 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140*
a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(c**5*(16384*a**7*
c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 +
2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14))))/(280*a
**4*c**4 - 560*a**3*b**2*c**3 + 280*a**2*b**4*c**2 - 56*a*b**6*c + 4*b**8))
 + (-590*a**6*b*c**3 + 535*a**5*b**3*c**2 - 147*a**4*b**5*c + 13*a**3*b**7
+ x**5*(348*a**4*c**6 - 1272*a**3*b**2*c**5 + 876*a**2*b**4*c**4 - 216*a*b*
*6*c**3 + 18*b**8*c**2) + x**4*(-282*a**4*b*c**5 - 1356*a**3*b**3*c**4 + 12
54*a**2*b**5*c**3 - 342*a*b**7*c**2 + 30*b**9*c) + x**3*(544*a**5*c**5 - 32
34*a**4*b**2*c**4 + 1788*a**3*b**4*c**3 - 68*a**2*b**6*c**2 - 96*a*b**8*c +
 13*b**10) + x**2*(-912*a**5*b*c**4 - 1161*a**4*b**3*c**3 + 1458*a**3*b**5*
c**2 - 429*a**2*b**7*c + 39*a*b**9) + x*(228*a**6*c**4 - 2082*a**5*b**2*c**
3 + 1701*a**4*b**4*c**2 - 450*a**3*b**6*c + 39*a**2*b**8))/(192*a**6*c**8 -
 144*a**5*b**2*c**7 + 36*a**4*b**4*c**6 - 3*a**3*b**6*c**5 + x**6*(192*a**3
*c**11 - 144*a**2*b**2*c**10 + 36*a*b**4*c**9 - 3*b**6*c**8) + x**5*(576*a*
*3*b*c**10 - 432*a**2*b**3*c**9 + 108*a*b**5*c**8 - 9*b**7*c**7) + x**4*(57
6*a**4*c**10 + 144*a**3*b**2*c**9 - 324*a**2*b**4*c**8 + 99*a*b**6*c**7 - 9
*b**8*c**6) + x**3*(1152*a**4*b*c**9 - 672*a**3*b**3*c**8 + 72*a**2*b**5*c*
*7 + 18*a*b**7*c**6 - 3*b**9*c**5) + x**2*(576*a**5*c**9 + 144*a**4*b**2*c*
*8 - 324*a**3*b**4*c**7 + 99*a**2*b**6*c**6 - 9*a*b**8*c**5) + x*(576*a**5*
b*c**8 - 432*a**4*b**3*c**7 + 108*a**3*b**5*c**6 - 9*a**2*b**7*c**5)) + x/c
**4
```

Giac [A] time = 1.12785, size = 630, normalized size = 1.81

$$\frac{4\left(b^8-14\,ab^6c+70\,a^2b^4c^2-140\,a^3b^2c^3+70\,a^4c^4\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^6c^5-12\,ab^4c^6+48\,a^2b^2c^7-64\,a^3c^8\right)\sqrt{-b^2+4\,ac}}+\frac{x}{c^4}-\frac{2\,b\log\left(cx^2+bx+a\right)}{c^5}-\frac{13\,a^3b^7-147\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^2+b*x+a)^4,x, algorithm="giac")
```

```
[Out] 4*(b^8 - 14*a*b^6*c + 70*a^2*b^4*c^2 - 140*a^3*b^2*c^3 + 70*a^4*c^4)*arctan
((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7
- 64*a^3*c^8)*sqrt(-b^2 + 4*a*c)) + x/c^4 - 2*b*log(c*x^2 + b*x + a)/c^5 -
```

```
1/3*(13*a^3*b^7 - 147*a^4*b^5*c + 535*a^5*b^3*c^2 - 590*a^6*b*c^3 + 6*(3*b^
8*c^2 - 36*a*b^6*c^3 + 146*a^2*b^4*c^4 - 212*a^3*b^2*c^5 + 58*a^4*c^6)*x^5
+ 6*(5*b^9*c - 57*a*b^7*c^2 + 209*a^2*b^5*c^3 - 226*a^3*b^3*c^4 - 47*a^4*b*
c^5)*x^4 + (13*b^10 - 96*a*b^8*c - 68*a^2*b^6*c^2 + 1788*a^3*b^4*c^3 - 3234
*a^4*b^2*c^4 + 544*a^5*c^5)*x^3 + 3*(13*a*b^9 - 143*a^2*b^7*c + 486*a^3*b^5
*c^2 - 387*a^4*b^3*c^3 - 304*a^5*b*c^4)*x^2 + 3*(13*a^2*b^8 - 150*a^3*b^6*c
 + 567*a^4*b^4*c^2 - 694*a^5*b^2*c^3 + 76*a^6*c^4)*x)/((c*x^2 + b*x + a)^3*
(b^2 - 4*a*c)^3*c^5)
```

3.2211 $\int \frac{x^7}{\left(a+bx+cx^2\right)^4} dx$

Optimal. Leaf size=291

$$\frac{x^2\left(bx\left(140a^2c^2-32ab^2c+3b^4\right)+3a\left(64a^2c^2-10ab^2c+b^4\right)\right)}{6c^2\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)}-\frac{bx\left(38a^2c^2-11ab^2c+b^4\right)}{c^3\left(b^2-4ac\right)^3}+\frac{b\left(70a^2b^2c^2-140a^3c^3-\right.}{c^4\left(b\right.}$$

```
[Out] -((b*(b^4 - 11*a*b^2*c + 38*a^2*c^2)*x)/(c^3*(b^2 - 4*a*c)^3)) + (x^6*(2*a
+ b*x))/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^3) + (x^4*(a*(b^2 - 24*a*c) + b*
(b^2 - 14*a*c)*x))/(6*c*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^2) + (x^2*(3*a*(b
^4 - 10*a*b^2*c + 64*a^2*c^2) + b*(3*b^4 - 32*a*b^2*c + 140*a^2*c^2)*x))/(6
*c^2*(b^2 - 4*a*c)^3*(a + b*x + c*x^2)) + (b*(b^6 - 14*a*b^4*c + 70*a^2*b^2
*c^2 - 140*a^3*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*(b^2 - 4*a
*c)^(7/2)) + Log[a + b*x + c*x^2]/(2*c^4)
```

Rubi [A] time = 0.50454, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {738, 818, 773, 634, 618, 206, 628}

$$\frac{x^2\left(bx\left(140a^2c^2-32ab^2c+3b^4\right)+3a\left(64a^2c^2-10ab^2c+b^4\right)\right)}{6c^2\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)}-\frac{bx\left(38a^2c^2-11ab^2c+b^4\right)}{c^3\left(b^2-4ac\right)^3}+\frac{b\left(70a^2b^2c^2-140a^3c^3-\right.}{c^4\left(b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x + c*x^2)^4,x]
```

```
[Out] -((b*(b^4 - 11*a*b^2*c + 38*a^2*c^2)*x)/(c^3*(b^2 - 4*a*c)^3)) + (x^6*(2*a
+ b*x))/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^3) + (x^4*(a*(b^2 - 24*a*c) + b*
(b^2 - 14*a*c)*x))/(6*c*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^2) + (x^2*(3*a*(b
^4 - 10*a*b^2*c + 64*a^2*c^2) + b*(3*b^4 - 32*a*b^2*c + 140*a^2*c^2)*x))/(6
*c^2*(b^2 - 4*a*c)^3*(a + b*x + c*x^2)) + (b*(b^6 - 14*a*b^4*c + 70*a^2*b^2
*c^2 - 140*a^3*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*(b^2 - 4*a
*c)^(7/2)) + Log[a + b*x + c*x^2]/(2*c^4)
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
```

```
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^7}{\left(a+bx+cx^2\right)^4}\,dx = \frac{x^6(2a+bx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} - \frac{\int \frac{x^5(12a+bx)}{\left(a+bx+cx^2\right)^3}\,dx}{3\left(b^2-4ac\right)}$$

$$= \frac{x^6(2a+bx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{x^4\left(a\left(b^2-24ac\right)+b\left(b^2-14ac\right)x\right)}{6c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} - \frac{\int \frac{x^3\left(4a\left(b^2-24ac\right)+b\left(3b^2-22a\right.\right.}{\left(a+bx+cx^2\right)^2}}{6c\left(b^2-4ac\right)^2}$$

$$= \frac{x^6(2a+bx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{x^4\left(a\left(b^2-24ac\right)+b\left(b^2-14ac\right)x\right)}{6c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} + \frac{x^2\left(3a\left(b^4-10ab^2c+6\right.\right.}{6c^2\left(b^2\right.}$$

$$= -\frac{b\left(b^4-11ab^2c+38a^2c^2\right)x}{c^3\left(b^2-4ac\right)^3} + \frac{x^6(2a+bx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{x^4\left(a\left(b^2-24ac\right)+b\left(b^2-14a\right.\right.}{6c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right.}$$

$$= -\frac{b\left(b^4-11ab^2c+38a^2c^2\right)x}{c^3\left(b^2-4ac\right)^3} + \frac{x^6(2a+bx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{x^4\left(a\left(b^2-24ac\right)+b\left(b^2-14a\right.\right.}{6c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right.}$$

$$= -\frac{b\left(b^4-11ab^2c+38a^2c^2\right)x}{c^3\left(b^2-4ac\right)^3} + \frac{x^6(2a+bx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{x^4\left(a\left(b^2-24ac\right)+b\left(b^2-14a\right.\right.}{6c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right.}$$

$$= -\frac{b\left(b^4-11ab^2c+38a^2c^2\right)x}{c^3\left(b^2-4ac\right)^3} + \frac{x^6(2a+bx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{x^4\left(a\left(b^2-24ac\right)+b\left(b^2-14a\right.\right.}{6c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right.}$$

Mathematica [A] time = 0.584542, size = 386, normalized size = 1.33

$$-\frac{3c\left(-266a^2b^3c^3x+191a^2b^4c^2-374a^3b^2c^3+308a^3bc^4x+192a^4c^4+70ab^5c^2x-40ab^6c-6b^7cx+3b^8\right)}{\left(b^2-4ac\right)^3(a+x(b+cx))} + \frac{2\left(2a^2b^3c(7cx-3b)+a^3bc^2(9b-7cx)-2a^4c^3+ab^5(b-7cx)+b^7\right.}{\left(b^2-4ac\right)(a+x(b+cx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x + c*x^2)^4,x]
```

```
[Out] ((-2*b^8 + 29*a*b^6*c - 139*a^2*b^4*c^2 + 233*a^3*b^2*c^3 - 72*a^4*c^4 + 11
*b^7*c*x - 98*a*b^5*c^2*x + 259*a^2*b^3*c^3*x - 182*a^3*b*c^4*x)/((b^2 - 4*
a*c)^2*(a + x*(b + c*x))^2) - (3*c*(3*b^8 - 40*a*b^6*c + 191*a^2*b^4*c^2 -
374*a^3*b^2*c^3 + 192*a^4*c^4 - 6*b^7*c*x + 70*a*b^5*c^2*x - 266*a^2*b^3*c^
3*x + 308*a^3*b*c^4*x))/((b^2 - 4*a*c)^3*(a + x*(b + c*x))) + (2*(-2*a^4*c^
3 + b^7*x + a*b^5*(b - 7*c*x) + a^3*b*c^2*(9*b - 7*c*x) + 2*a^2*b^3*c*(-3*b
 + 7*c*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x))^3) + (6*b*c^2*(b^6 - 14*a*b^4*
c + 70*a^2*b^2*c^2 - 140*a^3*c^3)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(
-b^2 + 4*a*c)^(7/2) + 3*c^2*Log[a + x*(b + c*x)])/(6*c^6)
```

Maple [B] time = 0.168, size = 1013, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^2+b*x+a)^4,x)
```

```
[Out] ((154*a^3*c^3-133*a^2*b^2*c^2+35*a*b^4*c-3*b^6)*b/c^2/(64*a^3*c^3-48*a^2*b^
2*c^2+12*a*b^4*c-b^6)*x^5+1/2*(192*a^4*c^4+242*a^3*b^2*c^3-341*a^2*b^4*c^2+
100*a*b^6*c-9*b^8)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/c^3*x^4+1/6*b
/c^4*(2272*a^4*c^4-1698*a^3*b^2*c^3+117*a^2*b^4*c^2+76*a*b^6*c-11*b^8)/(64*
a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^3+1/2/c^4*a*(288*a^4*c^4+152*a^3*b
^2*c^3-381*a^2*b^4*c^2+119*a*b^6*c-11*b^8)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*
b^4*c-b^6)*x^2+1/2*a^2*b*(428*a^3*c^3-460*a^2*b^2*c^2+126*a*b^4*c-11*b^6)/(
64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/c^4*x+1/6*(352*a^3*c^3-438*a^2*b^
2*c^2+124*a*b^4*c-11*b^6)*a^3/c^4/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6
))/(c*x^2+b*x+a)^3+32/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/c*ln(c*x^2
+b*x+a)*a^3-24/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/c^2*ln(c*x^2+b*x+
a)*a^2*b^2+6/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/c^3*ln(c*x^2+b*x+a)
*a*b^4-1/2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/c^4*ln(c*x^2+b*x+a)*b
^6-140/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/c/(4*a*c-b^2)^(1/2)*arcta
n((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^3*b+70/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4
*c-b^6)/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b^3-1
4/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/c^3/(4*a*c-b^2)^(1/2)*arctan((
2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^5+1/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b
^6)/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^7
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.6627, size = 6337, normalized size = 21.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [1/6*(11*a^3*b^8 - 168*a^4*b^6*c + 934*a^5*b^4*c^2 - 2104*a^6*b^2*c^3 + 140
8*a^7*c^4 + 6*(3*b^9*c^2 - 47*a*b^7*c^3 + 273*a^2*b^5*c^4 - 686*a^3*b^3*c^5
 + 616*a^4*b*c^6)*x^5 + 3*(9*b^10*c - 136*a*b^8*c^2 + 741*a^2*b^6*c^3 - 160
6*a^3*b^4*c^4 + 776*a^4*b^2*c^5 + 768*a^5*c^6)*x^4 + (11*b^11 - 120*a*b^9*c
 + 187*a^2*b^7*c^2 + 2166*a^3*b^5*c^3 - 9064*a^4*b^3*c^4 + 9088*a^5*b*c^5)*
x^3 + 3*(11*a*b^10 - 163*a^2*b^8*c + 857*a^3*b^6*c^2 - 1676*a^4*b^4*c^3 + 3
20*a^5*b^2*c^4 + 1152*a^6*c^5)*x^2 + 3*(a^3*b^7 - 14*a^4*b^5*c + 70*a^5*b^3
*c^2 - 140*a^6*b*c^3 + (b^7*c^3 - 14*a*b^5*c^4 + 70*a^2*b^3*c^5 - 140*a^3*b
*c^6)*x^6 + 3*(b^8*c^2 - 14*a*b^6*c^3 + 70*a^2*b^4*c^4 - 140*a^3*b^2*c^5)*x
^5 + 3*(b^9*c - 13*a*b^7*c^2 + 56*a^2*b^5*c^3 - 70*a^3*b^3*c^4 - 140*a^4*b*
c^5)*x^4 + (b^10 - 8*a*b^8*c - 14*a^2*b^6*c^2 + 280*a^3*b^4*c^3 - 840*a^4*b
^2*c^4)*x^3 + 3*(a*b^9 - 13*a^2*b^7*c + 56*a^3*b^5*c^2 - 70*a^4*b^3*c^3 - 1
40*a^5*b*c^4)*x^2 + 3*(a^2*b^8 - 14*a^3*b^6*c + 70*a^4*b^4*c^2 - 140*a^5*b^
2*c^3)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b
^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 3*(11*a^2*b^9 - 170*a^3*b^7*c
 + 964*a^4*b^5*c^2 - 2268*a^5*b^3*c^3 + 1712*a^6*b*c^4)*x + 3*(a^3*b^8 - 16
*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3 + 256*a^7*c^4 + (b^8*c^3 - 16
```

```
*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6 + 256*a^4*c^7)*x^6 + 3*(b^9*c
^2 - 16*a*b^7*c^3 + 96*a^2*b^5*c^4 - 256*a^3*b^3*c^5 + 256*a^4*b*c^6)*x^5 +
 3*(b^10*c - 15*a*b^8*c^2 + 80*a^2*b^6*c^3 - 160*a^3*b^4*c^4 + 256*a^5*c^6)
*x^4 + (b^11 - 10*a*b^9*c + 320*a^3*b^5*c^3 - 1280*a^4*b^3*c^4 + 1536*a^5*b
*c^5)*x^3 + 3*(a*b^10 - 15*a^2*b^8*c + 80*a^3*b^6*c^2 - 160*a^4*b^4*c^3 + 2
56*a^6*c^5)*x^2 + 3*(a^2*b^9 - 16*a^3*b^7*c + 96*a^4*b^5*c^2 - 256*a^5*b^3*
c^3 + 256*a^6*b*c^4)*x)*log(c*x^2 + b*x + a))/(a^3*b^8*c^4 - 16*a^4*b^6*c^5
 + 96*a^5*b^4*c^6 - 256*a^6*b^2*c^7 + 256*a^7*c^8 + (b^8*c^7 - 16*a*b^6*c^8
 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10 + 256*a^4*c^11)*x^6 + 3*(b^9*c^6 - 16*
a*b^7*c^7 + 96*a^2*b^5*c^8 - 256*a^3*b^3*c^9 + 256*a^4*b*c^10)*x^5 + 3*(b^1
0*c^5 - 15*a*b^8*c^6 + 80*a^2*b^6*c^7 - 160*a^3*b^4*c^8 + 256*a^5*c^10)*x^4
 + (b^11*c^4 - 10*a*b^9*c^5 + 320*a^3*b^5*c^7 - 1280*a^4*b^3*c^8 + 1536*a^5
*b*c^9)*x^3 + 3*(a*b^10*c^4 - 15*a^2*b^8*c^5 + 80*a^3*b^6*c^6 - 160*a^4*b^4
*c^7 + 256*a^6*c^9)*x^2 + 3*(a^2*b^9*c^4 - 16*a^3*b^7*c^5 + 96*a^4*b^5*c^6
- 256*a^5*b^3*c^7 + 256*a^6*b*c^8)*x), 1/6*(11*a^3*b^8 - 168*a^4*b^6*c + 93
4*a^5*b^4*c^2 - 2104*a^6*b^2*c^3 + 1408*a^7*c^4 + 6*(3*b^9*c^2 - 47*a*b^7*c
^3 + 273*a^2*b^5*c^4 - 686*a^3*b^3*c^5 + 616*a^4*b*c^6)*x^5 + 3*(9*b^10*c -
 136*a*b^8*c^2 + 741*a^2*b^6*c^3 - 1606*a^3*b^4*c^4 + 776*a^4*b^2*c^5 + 768
*a^5*c^6)*x^4 + (11*b^11 - 120*a*b^9*c + 187*a^2*b^7*c^2 + 2166*a^3*b^5*c^3
 - 9064*a^4*b^3*c^4 + 9088*a^5*b*c^5)*x^3 + 3*(11*a*b^10 - 163*a^2*b^8*c +
857*a^3*b^6*c^2 - 1676*a^4*b^4*c^3 + 320*a^5*b^2*c^4 + 1152*a^6*c^5)*x^2 +
6*(a^3*b^7 - 14*a^4*b^5*c + 70*a^5*b^3*c^2 - 140*a^6*b*c^3 + (b^7*c^3 - 14*
a*b^5*c^4 + 70*a^2*b^3*c^5 - 140*a^3*b*c^6)*x^6 + 3*(b^8*c^2 - 14*a*b^6*c^3
 + 70*a^2*b^4*c^4 - 140*a^3*b^2*c^5)*x^5 + 3*(b^9*c - 13*a*b^7*c^2 + 56*a^2
*b^5*c^3 - 70*a^3*b^3*c^4 - 140*a^4*b*c^5)*x^4 + (b^10 - 8*a*b^8*c - 14*a^2
*b^6*c^2 + 280*a^3*b^4*c^3 - 840*a^4*b^2*c^4)*x^3 + 3*(a*b^9 - 13*a^2*b^7*c
 + 56*a^3*b^5*c^2 - 70*a^4*b^3*c^3 - 140*a^5*b*c^4)*x^2 + 3*(a^2*b^8 - 14*a
^3*b^6*c + 70*a^4*b^4*c^2 - 140*a^5*b^2*c^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-
sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 3*(11*a^2*b^9 - 170*a^3*b^7
*c + 964*a^4*b^5*c^2 - 2268*a^5*b^3*c^3 + 1712*a^6*b*c^4)*x + 3*(a^3*b^8 -
16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3 + 256*a^7*c^4 + (b^8*c^3 -
16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6 + 256*a^4*c^7)*x^6 + 3*(b^9
*c^2 - 16*a*b^7*c^3 + 96*a^2*b^5*c^4 - 256*a^3*b^3*c^5 + 256*a^4*b*c^6)*x^5
 + 3*(b^10*c - 15*a*b^8*c^2 + 80*a^2*b^6*c^3 - 160*a^3*b^4*c^4 + 256*a^5*c^
6)*x^4 + (b^11 - 10*a*b^9*c + 320*a^3*b^5*c^3 - 1280*a^4*b^3*c^4 + 1536*a^5
*b*c^5)*x^3 + 3*(a*b^10 - 15*a^2*b^8*c + 80*a^3*b^6*c^2 - 160*a^4*b^4*c^3 +
 256*a^6*c^5)*x^2 + 3*(a^2*b^9 - 16*a^3*b^7*c + 96*a^4*b^5*c^2 - 256*a^5*b^
3*c^3 + 256*a^6*b*c^4)*x)*log(c*x^2 + b*x + a))/(a^3*b^8*c^4 - 16*a^4*b^6*c
^5 + 96*a^5*b^4*c^6 - 256*a^6*b^2*c^7 + 256*a^7*c^8 + (b^8*c^7 - 16*a*b^6*c
^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10 + 256*a^4*c^11)*x^6 + 3*(b^9*c^6 - 1
6*a*b^7*c^7 + 96*a^2*b^5*c^8 - 256*a^3*b^3*c^9 + 256*a^4*b*c^10)*x^5 + 3*(b
^10*c^5 - 15*a*b^8*c^6 + 80*a^2*b^6*c^7 - 160*a^3*b^4*c^8 + 256*a^5*c^10)*x
^4 + (b^11*c^4 - 10*a*b^9*c^5 + 320*a^3*b^5*c^7 - 1280*a^4*b^3*c^8 + 1536*a
^5*b*c^9)*x^3 + 3*(a*b^10*c^4 - 15*a^2*b^8*c^5 + 80*a^3*b^6*c^6 - 160*a^4*b
^4*c^7 + 256*a^6*c^9)*x^2 + 3*(a^2*b^9*c^4 - 16*a^3*b^7*c^5 + 96*a^4*b^5*c^
6 - 256*a^5*b^3*c^7 + 256*a^6*b*c^8)*x)]
```

Sympy [B] time = 7.28591, size = 2565, normalized size = 8.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**2+b*x+a)**4,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**7)*(140*a**3*c**3 - 70*a**2*b**2*c**2 + 14*a*b**4
*c - b**6)/(2*c**4*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**
```

```
4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 +
28*a*b**12*c - b**14)) + 1/(2*c**4))*log(x + (-256*a**4*c**7*(-b*sqrt(-(4*a
*c - b**2)**7)*(140*a**3*c**3 - 70*a**2*b**2*c**2 + 14*a*b**4*c - b**6)/(2*
c**4*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*
a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c -
 b**14)) + 1/(2*c**4)) + 128*a**4*c**3 + 256*a**3*b**2*c**6*(-b*sqrt(-(4*a*
c - b**2)**7)*(140*a**3*c**3 - 70*a**2*b**2*c**2 + 14*a*b**4*c - b**6)/(2*c
**4*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a
**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c -
b**14)) + 1/(2*c**4)) - 58*a**3*b**2*c**2 - 96*a**2*b**4*c**5*(-b*sqrt(-(4*
a*c - b**2)**7)*(140*a**3*c**3 - 70*a**2*b**2*c**2 + 14*a*b**4*c - b**6)/(2
*c**4*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960
*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c
 - b**14)) + 1/(2*c**4)) + 13*a**2*b**4*c + 16*a*b**6*c**4*(-b*sqrt(-(4*a*c
 - b**2)**7)*(140*a**3*c**3 - 70*a**2*b**2*c**2 + 14*a*b**4*c - b**6)/(2*c**
4*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**
4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b*
*14)) + 1/(2*c**4)) - a*b**6 - b**8*c**3*(-b*sqrt(-(4*a*c - b**2)**7)*(140*
a**3*c**3 - 70*a**2*b**2*c**2 + 14*a*b**4*c - b**6)/(2*c**4*(16384*a**7*c**
7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 224
0*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14)) + 1/(2*c**4
)))/(140*a**3*b*c**3 - 70*a**2*b**3*c**2 + 14*a*b**5*c - b**7)) + (b*sqrt(-
(4*a*c - b**2)**7)*(140*a**3*c**3 - 70*a**2*b**2*c**2 + 14*a*b**4*c - b**6)
/(2*c**4*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8
960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12
*c - b**14)) + 1/(2*c**4))*log(x + (-256*a**4*c**7*(b*sqrt(-(4*a*c - b**2)*
*7)*(140*a**3*c**3 - 70*a**2*b**2*c**2 + 14*a*b**4*c - b**6)/(2*c**4*(16384
*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c
**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14)) +
1/(2*c**4)) + 128*a**4*c**3 + 256*a**3*b**2*c**6*(b*sqrt(-(4*a*c - b**2)**7
)*(140*a**3*c**3 - 70*a**2*b**2*c**2 + 14*a*b**4*c - b**6)/(2*c**4*(16384*a
**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**
4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14)) + 1/
(2*c**4)) - 58*a**3*b**2*c**2 - 96*a**2*b**4*c**5*(b*sqrt(-(4*a*c - b**2)**
7)*(140*a**3*c**3 - 70*a**2*b**2*c**2 + 14*a*b**4*c - b**6)/(2*c**4*(16384*
a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c*
*4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14)) + 1
/(2*c**4)) + 13*a**2*b**4*c + 16*a*b**6*c**4*(b*sqrt(-(4*a*c - b**2)**7)*(1
40*a**3*c**3 - 70*a**2*b**2*c**2 + 14*a*b**4*c - b**6)/(2*c**4*(16384*a**7*
c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 +
2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14)) + 1/(2*c
**4)) - a*b**6 - b**8*c**3*(b*sqrt(-(4*a*c - b**2)**7)*(140*a**3*c**3 - 70*
a**2*b**2*c**2 + 14*a*b**4*c - b**6)/(2*c**4*(16384*a**7*c**7 - 28672*a**6*
b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**
3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14)) + 1/(2*c**4)))/(140*a**3*b
*c**3 - 70*a**2*b**3*c**2 + 14*a*b**5*c - b**7)) + (352*a**6*c**3 - 438*a**
5*b**2*c**2 + 124*a**4*b**4*c - 11*a**3*b**6 + x**5*(924*a**3*b*c**5 - 798*
a**2*b**3*c**4 + 210*a*b**5*c**3 - 18*b**7*c**2) + x**4*(576*a**4*c**5 + 72
6*a**3*b**2*c**4 - 1023*a**2*b**4*c**3 + 300*a*b**6*c**2 - 27*b**8*c) + x**
3*(2272*a**4*b*c**4 - 1698*a**3*b**3*c**3 + 117*a**2*b**5*c**2 + 76*a*b**7*
c - 11*b**9) + x**2*(864*a**5*c**4 + 456*a**4*b**2*c**3 - 1143*a**3*b**4*c*
*2 + 357*a**2*b**6*c - 33*a*b**8) + x*(1284*a**5*b*c**3 - 1380*a**4*b**3*c*
*2 + 378*a**3*b**5*c - 33*a**2*b**7))/(384*a**6*c**7 - 288*a**5*b**2*c**6 +
 72*a**4*b**4*c**5 - 6*a**3*b**6*c**4 + x**6*(384*a**3*c**10 - 288*a**2*b**
2*c**9 + 72*a*b**4*c**8 - 6*b**6*c**7) + x**5*(1152*a**3*b*c**9 - 864*a**2*
b**3*c**8 + 216*a*b**5*c**7 - 18*b**7*c**6) + x**4*(1152*a**4*c**9 + 288*a*
*3*b**2*c**8 - 648*a**2*b**4*c**7 + 198*a*b**6*c**6 - 18*b**8*c**5) + x**3*
(2304*a**4*b*c**8 - 1344*a**3*b**3*c**7 + 144*a**2*b**5*c**6 + 36*a*b**7*c*
*5 - 6*b**9*c**4) + x**2*(1152*a**5*c**8 + 288*a**4*b**2*c**7 - 648*a**3*b*
```

```
*4*c**6 + 198*a**2*b**6*c**5 - 18*a*b**8*c**4) + x*(1152*a**5*b*c**7 - 864*
a**4*b**3*c**6 + 216*a**3*b**5*c**5 - 18*a**2*b**7*c**4))
```

Giac [A] time = 1.12191, size = 564, normalized size = 1.94

$$-\frac{\left(b^7-14\,ab^5c+70\,a^2b^3c^2-140\,a^3bc^3\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^6c^4-12\,ab^4c^5+48\,a^2b^2c^6-64\,a^3c^7\right)\sqrt{-b^2+4\,ac}}+\frac{\log\left(cx^2+bx+a\right)}{2\,c^4}+\frac{11\,a^3b^6-124\,a^4b^4c+438\,a^5b^2c^2-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^2+b*x+a)^4,x, algorithm="giac")
```

```
[Out] -(b^7 - 14*a*b^5*c + 70*a^2*b^3*c^2 - 140*a^3*b*c^3)*arctan((2*c*x + b)/sqr
t(-b^2 + 4*a*c))/((b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*sq
rt(-b^2 + 4*a*c)) + 1/2*log(c*x^2 + b*x + a)/c^4 + 1/6*(11*a^3*b^6 - 124*a^
4*b^4*c + 438*a^5*b^2*c^2 - 352*a^6*c^3 + 6*(3*b^7*c^2 - 35*a*b^5*c^3 + 133
*a^2*b^3*c^4 - 154*a^3*b*c^5)*x^5 + 3*(9*b^8*c - 100*a*b^6*c^2 + 341*a^2*b^
4*c^3 - 242*a^3*b^2*c^4 - 192*a^4*c^5)*x^4 + (11*b^9 - 76*a*b^7*c - 117*a^2
*b^5*c^2 + 1698*a^3*b^3*c^3 - 2272*a^4*b*c^4)*x^3 + 3*(11*a*b^8 - 119*a^2*b
^6*c + 381*a^3*b^4*c^2 - 152*a^4*b^2*c^3 - 288*a^5*c^4)*x^2 + 3*(11*a^2*b^7
 - 126*a^3*b^5*c + 460*a^4*b^3*c^2 - 428*a^5*b*c^3)*x)/((c*x^2 + b*x + a)^3
*(b^2 - 4*a*c)^3*c^4)
```

3.2212 $\int \frac{x^6}{(a+bx+cx^2)^4} dx$

Optimal. Leaf size=146

$$\frac{10a^2x(2a+bx)}{(b^2-4ac)^3(a+bx+cx^2)}+\frac{40a^3\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{7/2}}+\frac{x^5(2a+bx)}{3(b^2-4ac)(a+bx+cx^2)^3}-\frac{5ax^3(2a+bx)}{3(b^2-4ac)^2(a+bx+cx^2)^2}$$

```
[Out] (x^5*(2*a + b*x))/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^3) - (5*a*x^3*(2*a + b
*x))/(3*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^2) + (10*a^2*x*(2*a + b*x))/((b^2
 - 4*a*c)^3*(a + b*x + c*x^2)) + (40*a^3*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a
*c]])/(b^2 - 4*a*c)^(7/2)
```

Rubi [A] time = 0.0831933, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {722, 618, 206}

$$\frac{10a^2x(2a+bx)}{(b^2-4ac)^3(a+bx+cx^2)}+\frac{40a^3\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{7/2}}+\frac{x^5(2a+bx)}{3(b^2-4ac)(a+bx+cx^2)^3}-\frac{5ax^3(2a+bx)}{3(b^2-4ac)^2(a+bx+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x + c*x^2)^4,x]
```

```
[Out] (x^5*(2*a + b*x))/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^3) - (5*a*x^3*(2*a + b
*x))/(3*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^2) + (10*a^2*x*(2*a + b*x))/((b^2
 - 4*a*c)^3*(a + b*x + c*x^2)) + (40*a^3*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a
*c]])/(b^2 - 4*a*c)^(7/2)
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx+cx^2\right)^4}\,dx = \frac{x^5(2a+bx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} - \frac{(10a)\int \frac{x^4}{\left(a+bx+cx^2\right)^3}dx}{3\left(b^2-4ac\right)}$$

$$= \frac{x^5(2a+bx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} - \frac{5ax^3(2a+bx)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} + \frac{\left(10a^2\right)\int \frac{x^2}{\left(a+bx+cx^2\right)^2}dx}{\left(b^2-4ac\right)^2}$$

$$= \frac{x^5(2a+bx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} - \frac{5ax^3(2a+bx)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} + \frac{10a^2x(2a+bx)}{\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)} -$$

$$= \frac{x^5(2a+bx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} - \frac{5ax^3(2a+bx)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} + \frac{10a^2x(2a+bx)}{\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)} +$$

$$= \frac{x^5(2a+bx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} - \frac{5ax^3(2a+bx)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} + \frac{10a^2x(2a+bx)}{\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)} +$$

Mathematica [B] time = 0.209053, size = 314, normalized size = 2.15

$$\frac{a^2b^2c(9cx-5b)+a^3c^2(5b-2cx)+ab^4(b-6cx)+b^6x}{3c^5\left(4ac-b^2\right)(a+x(b+cx))^3} + \frac{48a^2b^2c^3x-48a^2b^3c^2+74a^3bc^3-44a^3c^4x-12ab^4c^2x+12a}{c^4\left(4ac-b^2\right)^3(a+x(b+cx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x + c*x^2)^4,x]
```

```
[Out] (b^7 - 12*a*b^5*c + 48*a^2*b^3*c^2 - 59*a^3*b*c^3 - 4*b^6*c*x + 33*a*b^4*c^
2*x - 72*a^2*b^2*c^3*x + 26*a^3*c^4*x)/(3*c^5*(b^2 - 4*a*c)^2*(a + x*(b + c
*x))^2) + (-b^7 + 12*a*b^5*c - 48*a^2*b^3*c^2 + 74*a^3*b*c^3 + b^6*c*x - 12
*a*b^4*c^2*x + 48*a^2*b^2*c^3*x - 44*a^3*c^4*x)/(c^4*(-b^2 + 4*a*c)^3*(a +
x*(b + c*x))) + (b^6*x + a*b^4*(b - 6*c*x) + a^3*c^2*(5*b - 2*c*x) + a^2*b^
2*c*(-5*b + 9*c*x))/(3*c^5*(-b^2 + 4*a*c)*(a + x*(b + c*x))^3) + (40*a^3*Ar
cTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(7/2)
```

Maple [B] time = 0.165, size = 531, normalized size = 3.6

$$\frac{1}{\left(cx^2+bx+a\right)^3}\left(-\frac{\left(44\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6\right)x^5}{c\left(64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6\right)} - \frac{b\left(14\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6\right)x^4}{c^2\left(64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6\right)} - \frac{\left(160\,a^4c^4-28\right.}{3\left(64\,a^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^2+b*x+a)^4,x)
```

```
[Out] (-(44*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/c/(64*a^3*c^3-48*a^2*b^2*c^2+1
2*a*b^4*c-b^6)*x^5-b*(14*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/c^2/(64*a^3
*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^4-1/3/c^3*(160*a^4*c^4-286*a^3*b^2*c^
3+12*a^2*b^4*c^2+7*a*b^6*c-b^8)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*
x^3+b/c^3*a*(16*a^3*c^3+53*a^2*b^2*c^2-12*a*b^4*c+b^6)/(64*a^3*c^3-48*a^2*b
^2*c^2+12*a*b^4*c-b^6)*x^2-a^2*(20*a^3*c^3-66*a^2*b^2*c^2+13*a*b^4*c-b^6)/(
64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/c^3*x+1/3*(66*a^2*c^2-13*a*b^2*c+
```

```
b^4)*a^3*b/c^3/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6))/(c*x^2+b*x+a)^3+
40*a^3/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1/2)*arctan(
(2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.28475, size = 3584, normalized size = 24.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/3*(a^3*b^7 - 17*a^4*b^5*c + 118*a^5*b^3*c^2 - 264*a^6*b*c^3 + 3*(b^8*c^
2 - 16*a*b^6*c^3 + 96*a^2*b^4*c^4 - 236*a^3*b^2*c^5 + 176*a^4*c^6)*x^5 + 3*
(b^9*c - 16*a*b^7*c^2 + 96*a^2*b^5*c^3 - 206*a^3*b^3*c^4 + 56*a^4*b*c^5)*x^
4 + (b^10 - 11*a*b^8*c + 16*a^2*b^6*c^2 + 334*a^3*b^4*c^3 - 1304*a^4*b^2*c^
4 + 640*a^5*c^5)*x^3 + 3*(a*b^9 - 16*a^2*b^7*c + 101*a^3*b^5*c^2 - 196*a^4*
b^3*c^3 - 64*a^5*b*c^4)*x^2 + 60*(a^3*c^6*x^6 + 3*a^3*b*c^5*x^5 + 3*a^5*b*c
^3*x + a^6*c^3 + 3*(a^3*b^2*c^4 + a^4*c^5)*x^4 + (a^3*b^3*c^3 + 6*a^4*b*c^4
)*x^3 + 3*(a^4*b^2*c^3 + a^5*c^4)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2
*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) +
3*(a^2*b^8 - 17*a^3*b^6*c + 118*a^4*b^4*c^2 - 284*a^5*b^2*c^3 + 80*a^6*c^4)
*x)/(a^3*b^8*c^3 - 16*a^4*b^6*c^4 + 96*a^5*b^4*c^5 - 256*a^6*b^2*c^6 + 256*
a^7*c^7 + (b^8*c^6 - 16*a*b^6*c^7 + 96*a^2*b^4*c^8 - 256*a^3*b^2*c^9 + 256*
a^4*c^10)*x^6 + 3*(b^9*c^5 - 16*a*b^7*c^6 + 96*a^2*b^5*c^7 - 256*a^3*b^3*c^
8 + 256*a^4*b*c^9)*x^5 + 3*(b^10*c^4 - 15*a*b^8*c^5 + 80*a^2*b^6*c^6 - 160*
a^3*b^4*c^7 + 256*a^5*c^9)*x^4 + (b^11*c^3 - 10*a*b^9*c^4 + 320*a^3*b^5*c^6
 - 1280*a^4*b^3*c^7 + 1536*a^5*b*c^8)*x^3 + 3*(a*b^10*c^3 - 15*a^2*b^8*c^4
+ 80*a^3*b^6*c^5 - 160*a^4*b^4*c^6 + 256*a^6*c^8)*x^2 + 3*(a^2*b^9*c^3 - 16
*a^3*b^7*c^4 + 96*a^4*b^5*c^5 - 256*a^5*b^3*c^6 + 256*a^6*b*c^7)*x), -1/3*(
a^3*b^7 - 17*a^4*b^5*c + 118*a^5*b^3*c^2 - 264*a^6*b*c^3 + 3*(b^8*c^2 - 16*
a*b^6*c^3 + 96*a^2*b^4*c^4 - 236*a^3*b^2*c^5 + 176*a^4*c^6)*x^5 + 3*(b^9*c
- 16*a*b^7*c^2 + 96*a^2*b^5*c^3 - 206*a^3*b^3*c^4 + 56*a^4*b*c^5)*x^4 + (b^
10 - 11*a*b^8*c + 16*a^2*b^6*c^2 + 334*a^3*b^4*c^3 - 1304*a^4*b^2*c^4 + 640
*a^5*c^5)*x^3 + 3*(a*b^9 - 16*a^2*b^7*c + 101*a^3*b^5*c^2 - 196*a^4*b^3*c^3
 - 64*a^5*b*c^4)*x^2 - 120*(a^3*c^6*x^6 + 3*a^3*b*c^5*x^5 + 3*a^5*b*c^3*x +
 a^6*c^3 + 3*(a^3*b^2*c^4 + a^4*c^5)*x^4 + (a^3*b^3*c^3 + 6*a^4*b*c^4)*x^3
+ 3*(a^4*b^2*c^3 + a^5*c^4)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a
*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 3*(a^2*b^8 - 17*a^3*b^6*c + 118*a^4*b^4*c^
2 - 284*a^5*b^2*c^3 + 80*a^6*c^4)*x)/(a^3*b^8*c^3 - 16*a^4*b^6*c^4 + 96*a^5
*b^4*c^5 - 256*a^6*b^2*c^6 + 256*a^7*c^7 + (b^8*c^6 - 16*a*b^6*c^7 + 96*a^2
*b^4*c^8 - 256*a^3*b^2*c^9 + 256*a^4*c^10)*x^6 + 3*(b^9*c^5 - 16*a*b^7*c^6
+ 96*a^2*b^5*c^7 - 256*a^3*b^3*c^8 + 256*a^4*b*c^9)*x^5 + 3*(b^10*c^4 - 15*
a*b^8*c^5 + 80*a^2*b^6*c^6 - 160*a^3*b^4*c^7 + 256*a^5*c^9)*x^4 + (b^11*c^3
 - 10*a*b^9*c^4 + 320*a^3*b^5*c^6 - 1280*a^4*b^3*c^7 + 1536*a^5*b*c^8)*x^3
```

```
+ 3*(a*b^10*c^3 - 15*a^2*b^8*c^4 + 80*a^3*b^6*c^5 - 160*a^4*b^4*c^6 + 256*a
^6*c^8)*x^2 + 3*(a^2*b^9*c^3 - 16*a^3*b^7*c^4 + 96*a^4*b^5*c^5 - 256*a^5*b^
3*c^6 + 256*a^6*b*c^7)*x)]
```

Sympy [B] time = 4.36335, size = 938, normalized size = 6.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**2+b*x+a)**4,x)
```

```
[Out] -20*a**3*sqrt(-1/(4*a*c - b**2)**7)*log(x + (-5120*a**7*c**4*sqrt(-1/(4*a*c
 - b**2)**7) + 5120*a**6*b**2*c**3*sqrt(-1/(4*a*c - b**2)**7) - 1920*a**5*b
**4*c**2*sqrt(-1/(4*a*c - b**2)**7) + 320*a**4*b**6*c*sqrt(-1/(4*a*c - b**2
)**7) - 20*a**3*b**8*sqrt(-1/(4*a*c - b**2)**7) + 20*a**3*b)/(40*a**3*c)) +
 20*a**3*sqrt(-1/(4*a*c - b**2)**7)*log(x + (5120*a**7*c**4*sqrt(-1/(4*a*c
- b**2)**7) - 5120*a**6*b**2*c**3*sqrt(-1/(4*a*c - b**2)**7) + 1920*a**5*b*
*4*c**2*sqrt(-1/(4*a*c - b**2)**7) - 320*a**4*b**6*c*sqrt(-1/(4*a*c - b**2)
**7) + 20*a**3*b**8*sqrt(-1/(4*a*c - b**2)**7) + 20*a**3*b)/(40*a**3*c)) -
(-66*a**5*b*c**2 + 13*a**4*b**3*c - a**3*b**5 + x**5*(132*a**3*c**5 - 144*a
**2*b**2*c**4 + 36*a*b**4*c**3 - 3*b**6*c**2) + x**4*(42*a**3*b*c**4 - 144*
a**2*b**3*c**3 + 36*a*b**5*c**2 - 3*b**7*c) + x**3*(160*a**4*c**4 - 286*a**
3*b**2*c**3 + 12*a**2*b**4*c**2 + 7*a*b**6*c - b**8) + x**2*(-48*a**4*b*c**
3 - 159*a**3*b**3*c**2 + 36*a**2*b**5*c - 3*a*b**7) + x*(60*a**5*c**3 - 198
*a**4*b**2*c**2 + 39*a**3*b**4*c - 3*a**2*b**6))/(192*a**6*c**6 - 144*a**5*
b**2*c**5 + 36*a**4*b**4*c**4 - 3*a**3*b**6*c**3 + x**6*(192*a**3*c**9 - 14
4*a**2*b**2*c**8 + 36*a*b**4*c**7 - 3*b**6*c**6) + x**5*(576*a**3*b*c**8 -
432*a**2*b**3*c**7 + 108*a*b**5*c**6 - 9*b**7*c**5) + x**4*(576*a**4*c**8 +
 144*a**3*b**2*c**7 - 324*a**2*b**4*c**6 + 99*a*b**6*c**5 - 9*b**8*c**4) +
x**3*(1152*a**4*b*c**7 - 672*a**3*b**3*c**6 + 72*a**2*b**5*c**5 + 18*a*b**7
*c**4 - 3*b**9*c**3) + x**2*(576*a**5*c**7 + 144*a**4*b**2*c**6 - 324*a**3*
b**4*c**5 + 99*a**2*b**6*c**4 - 9*a*b**8*c**3) + x*(576*a**5*b*c**6 - 432*a
**4*b**3*c**5 + 108*a**3*b**5*c**4 - 9*a**2*b**7*c**3))
```

Giac [B] time = 1.14392, size = 521, normalized size = 3.57

$$-\frac{40\,a^3\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3\right)\sqrt{-b^2+4\,ac}}-\frac{3\,b^6c^2x^5-36\,ab^4c^3x^5+144\,a^2b^2c^4x^5-132\,a^3c^5x^5+3\,b^7cx^4-36}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^2+b*x+a)^4,x, algorithm="giac")
```

```
[Out] -40*a^3*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^6 - 12*a*b^4*c + 48*a^2*
b^2*c^2 - 64*a^3*c^3)*sqrt(-b^2 + 4*a*c)) - 1/3*(3*b^6*c^2*x^5 - 36*a*b^4*c
^3*x^5 + 144*a^2*b^2*c^4*x^5 - 132*a^3*c^5*x^5 + 3*b^7*c*x^4 - 36*a*b^5*c^2
*x^4 + 144*a^2*b^3*c^3*x^4 - 42*a^3*b*c^4*x^4 + b^8*x^3 - 7*a*b^6*c*x^3 - 1
2*a^2*b^4*c^2*x^3 + 286*a^3*b^2*c^3*x^3 - 160*a^4*c^4*x^3 + 3*a*b^7*x^2 - 3
6*a^2*b^5*c*x^2 + 159*a^3*b^3*c^2*x^2 + 48*a^4*b*c^3*x^2 + 3*a^2*b^6*x - 39
*a^3*b^4*c*x + 198*a^4*b^2*c^2*x - 60*a^5*c^3*x + a^3*b^5 - 13*a^4*b^3*c +
66*a^5*b*c^2)/((b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*(c*x^
2 + b*x + a)^3)
```

3.2213 $\int \frac{x^5}{(a+bx+cx^2)^4}\,dx$

Optimal. Leaf size=145

$$-\frac{20a^2b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{7/2}}-\frac{x^5(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^3}+\frac{5bx^3(2a+bx)}{6(b^2-4ac)^2(a+bx+cx^2)^2}-\frac{5abx(2a+bx)}{(b^2-4ac)^3(a+bx+cx^2)}$$

```
[Out] -(x^5*(b + 2*c*x))/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^3) + (5*b*x^3*(2*a +
b*x))/(6*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^2) - (5*a*b*x*(2*a + b*x))/((b^2
 - 4*a*c)^3*(a + b*x + c*x^2)) - (20*a^2*b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4
*a*c]])/(b^2 - 4*a*c)^(7/2)
```

Rubi [A] time = 0.0638838, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {728, 722, 618, 206}

$$-\frac{20a^2b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{7/2}}-\frac{x^5(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^3}+\frac{5bx^3(2a+bx)}{6(b^2-4ac)^2(a+bx+cx^2)^2}-\frac{5abx(2a+bx)}{(b^2-4ac)^3(a+bx+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x + c*x^2)^4,x]
```

```
[Out] -(x^5*(b + 2*c*x))/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^3) + (5*b*x^3*(2*a +
b*x))/(6*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^2) - (5*a*b*x*(2*a + b*x))/((b^2
 - 4*a*c)^3*(a + b*x + c*x^2)) - (20*a^2*b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4
*a*c]])/(b^2 - 4*a*c)^(7/2)
```

Rule 728

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] + Dist[(m*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(
d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5}{(a+bx+cx^2)^4}\,dx = -\frac{x^5(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^3} + \frac{(5b)\int \frac{x^4}{(a+bx+cx^2)^3}\,dx}{3(b^2-4ac)}$$

$$= -\frac{x^5(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^3} + \frac{5bx^3(2a+bx)}{6(b^2-4ac)^2(a+bx+cx^2)^2} - \frac{(5ab)\int \frac{x^2}{(a+bx+cx^2)^2}\,dx}{(b^2-4ac)^2}$$

$$= -\frac{x^5(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^3} + \frac{5bx^3(2a+bx)}{6(b^2-4ac)^2(a+bx+cx^2)^2} - \frac{5abx(2a+bx)}{(b^2-4ac)^3(a+bx+cx^2)} +$$

$$= -\frac{x^5(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^3} + \frac{5bx^3(2a+bx)}{6(b^2-4ac)^2(a+bx+cx^2)^2} - \frac{5abx(2a+bx)}{(b^2-4ac)^3(a+bx+cx^2)} -$$

$$= -\frac{x^5(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^3} + \frac{5bx^3(2a+bx)}{6(b^2-4ac)^2(a+bx+cx^2)^2} - \frac{5abx(2a+bx)}{(b^2-4ac)^3(a+bx+cx^2)} -$$

Mathematica [A] time = 0.278009, size = 266, normalized size = 1.83

$$\frac{1}{6}\left(-\frac{2(a^2bc(5cx-4b)+2a^3c^2+ab^3(b-5cx)+b^5x)}{c^4(4ac-b^2)(a+x(b+cx))^3} + \frac{3(38a^2b^2c^2-20a^2bc^3x-64a^3c^3-12ab^4c+b^6)}{c^3(4ac-b^2)^3(a+x(b+cx))} + \frac{-61a^2b^2c^2}{\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x + c*x^2)^4,x]
```

```
[Out] ((-2*b^6 + 19*a*b^4*c - 61*a^2*b^2*c^2 + 48*a^3*c^3 + 5*b^5*c*x - 40*a*b^3*
c^2*x + 70*a^2*b*c^3*x)/(c^4*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^2) + (3*(b^6
 - 12*a*b^4*c + 38*a^2*b^2*c^2 - 64*a^3*c^3 - 20*a^2*b*c^3*x))/(c^3*(-b^2 +
 4*a*c)^3*(a + x*(b + c*x))) - (2*(2*a^3*c^2 + b^5*x + a*b^3*(b - 5*c*x) +
a^2*b*c*(-4*b + 5*c*x)))/(c^4*(-b^2 + 4*a*c)*(a + x*(b + c*x))^3) - (120*a^
2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(7/2))/6
```

Maple [B] time = 0.163, size = 486, normalized size = 3.4

$$\frac{1}{(cx^2+bx+a)^3}\left(-10\frac{a^2bc^2x^5}{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6} - \frac{(64a^3c^3+2a^2b^2c^2+12ab^4c-b^6)x^4}{(128a^3c^3-96a^2b^2c^2+24ab^4c-2b^6)c} - \frac{b(224a^3c^3+62\ldots}{6c^2(64a^3c^3-\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^2+b*x+a)^4,x)
```

```
[Out] (-10*b*a^2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*c^2*x^5-1/2*(64*a^3*c
^3+2*a^2*b^2*c^2+12*a*b^4*c-b^6)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)
/c*x^4-1/6*b*(224*a^3*c^3+62*a^2*b^2*c^2+12*a*b^4*c-b^6)/c^2/(64*a^3*c^3-48
*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^3-1/2*a*(64*a^3*c^3+32*a^2*b^2*c^2+17*a*b^4*
c-b^6)/c^2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^2-1/2*a^2*b*(44*a^2
*c^2+18*a*b^2*c-b^4)/c^2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x-1/6*a
^3*(64*a^2*c^2+18*a*b^2*c-b^4)/c^2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^
6))/(c*x^2+b*x+a)^3-20*b*a^2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(4*
a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.21565, size = 3379, normalized size = 23.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/6*(a^3*b^6 - 22*a^4*b^4*c + 8*a^5*b^2*c^2 + 256*a^6*c^3 - 60*(a^2*b^3*c
^4 - 4*a^3*b*c^5)*x^5 + 3*(b^8*c - 16*a*b^6*c^2 + 46*a^2*b^4*c^3 - 56*a^3*b
^2*c^4 + 256*a^4*c^5)*x^4 + (b^9 - 16*a*b^7*c - 14*a^2*b^5*c^2 + 24*a^3*b^3
*c^3 + 896*a^4*b*c^4)*x^3 + 3*(a*b^8 - 21*a^2*b^6*c + 36*a^3*b^4*c^2 + 64*a
^4*b^2*c^3 + 256*a^5*c^4)*x^2 + 60*(a^2*b*c^5*x^6 + 3*a^2*b^2*c^4*x^5 + 3*a
^4*b^2*c^2*x + a^5*b*c^2 + 3*(a^2*b^3*c^3 + a^3*b*c^4)*x^4 + (a^2*b^4*c^2 +
 6*a^3*b^2*c^3)*x^3 + 3*(a^3*b^3*c^2 + a^4*b*c^3)*x^2)*sqrt(b^2 - 4*a*c)*lo
g((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^
2 + b*x + a)) + 3*(a^2*b^7 - 22*a^3*b^5*c + 28*a^4*b^3*c^2 + 176*a^5*b*c^3)
*x)/(a^3*b^8*c^2 - 16*a^4*b^6*c^3 + 96*a^5*b^4*c^4 - 256*a^6*b^2*c^5 + 256*
a^7*c^6 + (b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8 + 256*
a^4*c^9)*x^6 + 3*(b^9*c^4 - 16*a*b^7*c^5 + 96*a^2*b^5*c^6 - 256*a^3*b^3*c^7
 + 256*a^4*b*c^8)*x^5 + 3*(b^10*c^3 - 15*a*b^8*c^4 + 80*a^2*b^6*c^5 - 160*a
^3*b^4*c^6 + 256*a^5*c^8)*x^4 + (b^11*c^2 - 10*a*b^9*c^3 + 320*a^3*b^5*c^5
- 1280*a^4*b^3*c^6 + 1536*a^5*b*c^7)*x^3 + 3*(a*b^10*c^2 - 15*a^2*b^8*c^3 +
 80*a^3*b^6*c^4 - 160*a^4*b^4*c^5 + 256*a^6*c^7)*x^2 + 3*(a^2*b^9*c^2 - 16*
a^3*b^7*c^3 + 96*a^4*b^5*c^4 - 256*a^5*b^3*c^5 + 256*a^6*b*c^6)*x), -1/6*(a
^3*b^6 - 22*a^4*b^4*c + 8*a^5*b^2*c^2 + 256*a^6*c^3 - 60*(a^2*b^3*c^4 - 4*a
^3*b*c^5)*x^5 + 3*(b^8*c - 16*a*b^6*c^2 + 46*a^2*b^4*c^3 - 56*a^3*b^2*c^4 +
 256*a^4*c^5)*x^4 + (b^9 - 16*a*b^7*c - 14*a^2*b^5*c^2 + 24*a^3*b^3*c^3 + 8
96*a^4*b*c^4)*x^3 + 3*(a*b^8 - 21*a^2*b^6*c + 36*a^3*b^4*c^2 + 64*a^4*b^2*c
^3 + 256*a^5*c^4)*x^2 + 120*(a^2*b*c^5*x^6 + 3*a^2*b^2*c^4*x^5 + 3*a^4*b^2*
c^2*x + a^5*b*c^2 + 3*(a^2*b^3*c^3 + a^3*b*c^4)*x^4 + (a^2*b^4*c^2 + 6*a^3*
b^2*c^3)*x^3 + 3*(a^3*b^3*c^2 + a^4*b*c^3)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-
sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 3*(a^2*b^7 - 22*a^3*b^5*c +
 28*a^4*b^3*c^2 + 176*a^5*b*c^3)*x)/(a^3*b^8*c^2 - 16*a^4*b^6*c^3 + 96*a^5*
b^4*c^4 - 256*a^6*b^2*c^5 + 256*a^7*c^6 + (b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*
```

```
b^4*c^7 - 256*a^3*b^2*c^8 + 256*a^4*c^9)*x^6 + 3*(b^9*c^4 - 16*a*b^7*c^5 +
96*a^2*b^5*c^6 - 256*a^3*b^3*c^7 + 256*a^4*b*c^8)*x^5 + 3*(b^10*c^3 - 15*a*
b^8*c^4 + 80*a^2*b^6*c^5 - 160*a^3*b^4*c^6 + 256*a^5*c^8)*x^4 + (b^11*c^2 -
 10*a*b^9*c^3 + 320*a^3*b^5*c^5 - 1280*a^4*b^3*c^6 + 1536*a^5*b*c^7)*x^3 +
3*(a*b^10*c^2 - 15*a^2*b^8*c^3 + 80*a^3*b^6*c^4 - 160*a^4*b^4*c^5 + 256*a^6
*c^7)*x^2 + 3*(a^2*b^9*c^2 - 16*a^3*b^7*c^3 + 96*a^4*b^5*c^4 - 256*a^5*b^3*
c^5 + 256*a^6*b*c^6)*x)]
```

Sympy [B] time = 3.48953, size = 898, normalized size = 6.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**2+b*x+a)**4,x)
```

```
[Out] 10*a**2*b*sqrt(-1/(4*a*c - b**2)**7)*log(x + (-2560*a**6*b*c**4*sqrt(-1/(4*
a*c - b**2)**7) + 2560*a**5*b**3*c**3*sqrt(-1/(4*a*c - b**2)**7) - 960*a**4
*b**5*c**2*sqrt(-1/(4*a*c - b**2)**7) + 160*a**3*b**7*c*sqrt(-1/(4*a*c - b*
*2)**7) - 10*a**2*b**9*sqrt(-1/(4*a*c - b**2)**7) + 10*a**2*b**2)/(20*a**2*
b*c)) - 10*a**2*b*sqrt(-1/(4*a*c - b**2)**7)*log(x + (2560*a**6*b*c**4*sqrt
(-1/(4*a*c - b**2)**7) - 2560*a**5*b**3*c**3*sqrt(-1/(4*a*c - b**2)**7) + 9
60*a**4*b**5*c**2*sqrt(-1/(4*a*c - b**2)**7) - 160*a**3*b**7*c*sqrt(-1/(4*a
*c - b**2)**7) + 10*a**2*b**9*sqrt(-1/(4*a*c - b**2)**7) + 10*a**2*b**2)/(2
0*a**2*b*c)) - (64*a**5*c**2 + 18*a**4*b**2*c - a**3*b**4 + 60*a**2*b*c**4*
x**5 + x**4*(192*a**3*c**4 + 6*a**2*b**2*c**3 + 36*a*b**4*c**2 - 3*b**6*c)
+ x**3*(224*a**3*b*c**3 + 62*a**2*b**3*c**2 + 12*a*b**5*c - b**7) + x**2*(1
92*a**4*c**3 + 96*a**3*b**2*c**2 + 51*a**2*b**4*c - 3*a*b**6) + x*(132*a**4
*b*c**2 + 54*a**3*b**3*c - 3*a**2*b**5))/(384*a**6*c**5 - 288*a**5*b**2*c**
4 + 72*a**4*b**4*c**3 - 6*a**3*b**6*c**2 + x**6*(384*a**3*c**8 - 288*a**2*b
**2*c**7 + 72*a*b**4*c**6 - 6*b**6*c**5) + x**5*(1152*a**3*b*c**7 - 864*a**
2*b**3*c**6 + 216*a*b**5*c**5 - 18*b**7*c**4) + x**4*(1152*a**4*c**7 + 288*
a**3*b**2*c**6 - 648*a**2*b**4*c**5 + 198*a*b**6*c**4 - 18*b**8*c**3) + x**
3*(2304*a**4*b*c**6 - 1344*a**3*b**3*c**5 + 144*a**2*b**5*c**4 + 36*a*b**7*
c**3 - 6*b**9*c**2) + x**2*(1152*a**5*c**6 + 288*a**4*b**2*c**5 - 648*a**3*
b**4*c**4 + 198*a**2*b**6*c**3 - 18*a*b**8*c**2) + x*(1152*a**5*b*c**5 - 86
4*a**4*b**3*c**4 + 216*a**3*b**5*c**3 - 18*a**2*b**7*c**2))
```

Giac [B] time = 1.12755, size = 440, normalized size = 3.03

$$\frac{20\,a^2b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3\right)\sqrt{-b^2+4\,ac}}+\frac{60\,a^2bc^4x^5-3\,b^6cx^4+36\,ab^4c^2x^4+6\,a^2b^2c^3x^4+192\,a^3c^4x^4-b^7x^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2+b*x+a)^4,x, algorithm="giac")
```

```
[Out] 20*a^2*b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^6 - 12*a*b^4*c + 48*a^2
*b^2*c^2 - 64*a^3*c^3)*sqrt(-b^2 + 4*a*c)) + 1/6*(60*a^2*b*c^4*x^5 - 3*b^6*
c*x^4 + 36*a*b^4*c^2*x^4 + 6*a^2*b^2*c^3*x^4 + 192*a^3*c^4*x^4 - b^7*x^3 +
12*a*b^5*c*x^3 + 62*a^2*b^3*c^2*x^3 + 224*a^3*b*c^3*x^3 - 3*a*b^6*x^2 + 51*
a^2*b^4*c*x^2 + 96*a^3*b^2*c^2*x^2 + 192*a^4*c^3*x^2 - 3*a^2*b^5*x + 54*a^3
*b^3*c*x + 132*a^4*b*c^2*x - a^3*b^4 + 18*a^4*b^2*c + 64*a^5*c^2)/((b^6*c^2
 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*(c*x^2 + b*x + a)^3)
```

3.2214 $\int \frac{(d+ex)^4}{\left(a+bx+cx^2\right)^4}\,dx$

Optimal. Leaf size=259

$$-\frac{2(d+ex)\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)(-2ae+x(2cd-be)+bd)}{\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)}+\frac{8\left(ae^2-bde+cd^2\right)\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right.}{\left(b^2-4ac\right)^{7/2}}$$

```
[Out] -((b + 2*c*x)*(d + e*x)^4)/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^3) + ((d + e*
x)^3*(5*b*c*d - 2*b^2*e - 2*a*c*e + 5*c*(2*c*d - b*e)*x))/(3*(b^2 - 4*a*c)^
2*(a + b*x + c*x^2)^2) - (2*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d +
e*x)*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)^3*(a + b*x + c*x^2)) +
 (8*(c*d^2 - b*d*e + a*e^2)*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*ArcTa
nh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(7/2)
```

Rubi [A] time = 0.452233, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {736, 804, 722, 618, 206}

$$-\frac{2(d+ex)\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)(-2ae+x(2cd-be)+bd)}{\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)}+\frac{8\left(ae^2-bde+cd^2\right)\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right.}{\left(b^2-4ac\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a + b*x + c*x^2)^4,x]
```

```
[Out] -((b + 2*c*x)*(d + e*x)^4)/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^3) + ((d + e*
x)^3*(5*b*c*d - 2*b^2*e - 2*a*c*e + 5*c*(2*c*d - b*e)*x))/(3*(b^2 - 4*a*c)^
2*(a + b*x + c*x^2)^2) - (2*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d +
e*x)*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)^3*(a + b*x + c*x^2)) +
 (8*(c*d^2 - b*d*e + a*e^2)*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*ArcTa
nh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(7/2)
```

Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] - Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)
*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p + 1
), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 0] && (L
tQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c,
d, e, m, p, x]
```

Rule 804

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*
(b*f - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(m*(b*(
e*f + d*g) - 2*(c*d*f + a*e*g)))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m
- 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0] && LtQ[p, -1]
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^4}{\left(a+bx+cx^2\right)^4}\,dx = -\frac{(b+2cx)(d+ex)^4}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}+\frac{\int \frac{(d+ex)^3(-10cd+4be-2cex)}{\left(a+bx+cx^2\right)^3}\,dx}{3\left(b^2-4ac\right)}$$

$$= -\frac{(b+2cx)(d+ex)^4}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}+\frac{(d+ex)^3\left(5bcd-2b^2e-2ace+5c(2cd-be)x\right)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2}+\frac{\left(2\left(5c^2d^2\right.\right.}{}$$

$$= -\frac{(b+2cx)(d+ex)^4}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}+\frac{(d+ex)^3\left(5bcd-2b^2e-2ace+5c(2cd-be)x\right)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2}-\frac{2\left(5c^2d^2+\right.}{}$$

$$= -\frac{(b+2cx)(d+ex)^4}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}+\frac{(d+ex)^3\left(5bcd-2b^2e-2ace+5c(2cd-be)x\right)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2}-\frac{2\left(5c^2d^2+\right.}{}$$

$$= -\frac{(b+2cx)(d+ex)^4}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}+\frac{(d+ex)^3\left(5bcd-2b^2e-2ace+5c(2cd-be)x\right)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2}-\frac{2\left(5c^2d^2+\right.}{}$$

Mathematica [B] time = 1.07465, size = 572, normalized size = 2.21

$$\frac{1}{3}\left(\frac{6(b+2cx)\left(ce^2\left(a^2e^2-6abde+6b^2d^2\right)+b^2e^3(ae-bd)+2c^2d^2e(3ae-5bd)+5c^3d^4\right)}{c\left(4ac-b^2\right)^3(a+x(b+cx))}+\frac{bc\left(-3a^2e^4+6acde^2(d+2e\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a + b*x + c*x^2)^4,x]
```

```
[Out] ((6*(5*c^3*d^4 + b^2*e^3*(-(b*d) + a*e) + 2*c^2*d^2*e*(-5*b*d + 3*a*e) + c*
e^2*(6*b^2*d^2 - 6*a*b*d*e + a^2*e^2))*(b + 2*c*x))/(c*(-b^2 + 4*a*c)^3*(a
+ x*(b + c*x))) + (b^4*e^4*x + b^3*e^3*(a*e - 4*c*d*x) + 2*b^2*c*e^2*(3*c*d
^2*x - 2*a*e*(d + e*x)) + b*c*(-3*a^2*e^4 + c^2*d^3*(d - 4*e*x) + 6*a*c*d*e
```

```
^2*(d + 2*e*x)) + 2*c^2*(c^2*d^4*x + a^2*e^3*(4*d + e*x) - 2*a*c*d^2*e*(2*d
 + 3*e*x)))/(c^3*(-b^2 + 4*a*c)*(a + x*(b + c*x))^3) + (b^5*e^4 - b^4*c*e^3
*(4*d + e*x) + b*c^2*(17*a^2*e^4 + 5*c^2*d^3*(d - 4*e*x) + 6*a*c*d*e^2*(d -
 2*e*x)) + b^3*c*e^2*(-7*a*e^2 + 2*c*d*(3*d - e*x)) + 2*b^2*c^2*e*(a*e^2*(9
*d + 5*e*x) + c*d^2*(-5*d + 6*e*x)) + 2*c^3*(5*c^2*d^4*x + 6*a*c*d^2*e^2*x
- a^2*e^3*(24*d + 7*e*x)))/(c^3*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^2) + (24*
(5*c^3*d^4 + b^2*e^3*(-(b*d) + a*e) + 2*c^2*d^2*e*(-5*b*d + 3*a*e) + c*e^2*
(6*b^2*d^2 - 6*a*b*d*e + a^2*e^2))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/
(-b^2 + 4*a*c)^(7/2))/3
```

Maple [B] time = 0.162, size = 1666, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(c*x^2+b*x+a)^4,x)
```

```
[Out] (4*(a^2*c*e^4+a*b^2*e^4-6*a*b*c*d*e^3+6*a*c^2*d^2*e^2-b^3*d*e^3+6*b^2*c*d^2
*e^2-10*b*c^2*d^3*e+5*c^3*d^4)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*c
^2*x^5+10*(a^2*c*e^4+a*b^2*e^4-6*a*b*c*d*e^3+6*a*c^2*d^2*e^2-b^3*d*e^3+6*b^
2*c*d^2*e^2-10*b*c^2*d^3*e+5*c^3*d^4)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c
-b^6)*b*c*x^4-1/3*(32*a^3*c^3*e^4-102*a^2*b^2*c^2*e^4+192*a^2*b*c^3*d*e^3-1
92*a^2*c^4*d^2*e^2-10*a*b^4*c*e^4+164*a*b^3*c^2*d*e^3-324*a*b^2*c^3*d^2*e^2
+320*a*b*c^4*d^3*e-160*a*c^5*d^4-b^6*e^4+22*b^5*c*d*e^3-132*b^4*c^2*d^2*e^2
+220*b^3*c^3*d^3*e-110*b^2*c^4*d^4)/c/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c
-b^6)*x^3+(16*a^3*b*c^2*e^4-64*a^3*c^3*d*e^3+17*a^2*b^3*c*e^4-48*a^2*b^2*c^
2*d*e^3+96*a^2*b*c^3*d^2*e^2+a*b^5*e^4-34*a*b^4*c*d*e^3+102*a*b^3*c^2*d^2*e
^2-160*a*b^2*c^3*d^3*e+80*a*b*c^4*d^4+6*b^5*c*d^2*e^2-10*b^4*c^2*d^3*e+5*b^
3*c^3*d^4)/c/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^2-(4*a^4*c^2*e^4-
22*a^3*b^2*c*e^4+40*a^3*b*c^2*d*e^3+24*a^3*c^3*d^2*e^2-a^2*b^4*e^4+40*a^2*b
^3*c*d*e^3-132*a^2*b^2*c^2*d^2*e^2+88*a^2*b*c^3*d^3*e-44*a^2*c^4*d^4-6*a*b^
4*c*d^2*e^2+36*a*b^3*c^2*d^3*e-18*a*b^2*c^3*d^4-2*b^5*c*d^3*e+b^4*c^2*d^4)/
c/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x+1/3*(26*a^4*b*c*e^4-64*a^4*c
^2*d*e^3+a^3*b^3*e^4-44*a^3*b^2*c*d*e^3+156*a^3*b*c^2*d^2*e^2-128*a^3*c^3*d
^3*e+6*a^2*b^3*c*d^2*e^2-36*a^2*b^2*c^2*d^3*e+66*a^2*b*c^3*d^4+2*a*b^4*c*d^
3*e-13*a*b^3*c^2*d^4+b^5*c*d^4)/c/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6
))/(c*x^2+b*x+a)^3+8/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(4*a*c-b^2)
^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*c*e^4+8/(64*a^3*c^3-48*a^2*b
^2*c^2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)
)*a*b^2*e^4-48/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1/2)
*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*c*d*e^3+48/(64*a^3*c^3-48*a^2*b^2*
c^2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a
*c^2*d^2*e^2-8/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1/2)
*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*d*e^3+48/(64*a^3*c^3-48*a^2*b^2*c^
2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2
*c*d^2*e^2-80/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1/2)*
arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c^2*d^3*e+40/(64*a^3*c^3-48*a^2*b^2*c
^2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^
3*d^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.72364, size = 9528, normalized size = 36.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/3*(12*(5*(b^2*c^6 - 4*a*c^7)*d^4 - 10*(b^3*c^5 - 4*a*b*c^6)*d^3*e + 6*(
b^4*c^4 - 3*a*b^2*c^5 - 4*a^2*c^6)*d^2*e^2 - (b^5*c^3 + 2*a*b^3*c^4 - 24*a^
2*b*c^5)*d*e^3 + (a*b^4*c^3 - 3*a^2*b^2*c^4 - 4*a^3*c^5)*e^4)*x^5 + (b^7*c
- 17*a*b^5*c^2 + 118*a^2*b^3*c^3 - 264*a^3*b*c^4)*d^4 + 2*(a*b^6*c - 22*a^2
*b^4*c^2 + 8*a^3*b^2*c^3 + 256*a^4*c^4)*d^3*e + 6*(a^2*b^5*c + 22*a^3*b^3*c
^2 - 104*a^4*b*c^3)*d^2*e^2 - 4*(11*a^3*b^4*c - 28*a^4*b^2*c^2 - 64*a^5*c^3
)*d*e^3 + (a^3*b^5 + 22*a^4*b^3*c - 104*a^5*b*c^2)*e^4 + 30*(5*(b^3*c^5 - 4
*a*b*c^6)*d^4 - 10*(b^4*c^4 - 4*a*b^2*c^5)*d^3*e + 6*(b^5*c^3 - 3*a*b^3*c^4
 - 4*a^2*b*c^5)*d^2*e^2 - (b^6*c^2 + 2*a*b^4*c^3 - 24*a^2*b^2*c^4)*d*e^3 +
(a*b^5*c^2 - 3*a^2*b^3*c^3 - 4*a^3*b*c^4)*e^4)*x^4 + (10*(11*b^4*c^4 - 28*a
*b^2*c^5 - 64*a^2*c^6)*d^4 - 20*(11*b^5*c^3 - 28*a*b^3*c^4 - 64*a^2*b*c^5)*
d^3*e + 12*(11*b^6*c^2 - 17*a*b^4*c^3 - 92*a^2*b^2*c^4 - 64*a^3*c^5)*d^2*e^
2 - 2*(11*b^7*c + 38*a*b^5*c^2 - 232*a^2*b^3*c^3 - 384*a^3*b*c^4)*d*e^3 + (
b^8 + 6*a*b^6*c + 62*a^2*b^4*c^2 - 440*a^3*b^2*c^3 + 128*a^4*c^4)*e^4)*x^3
+ 3*(5*(b^5*c^3 + 12*a*b^3*c^4 - 64*a^2*b*c^5)*d^4 - 10*(b^6*c^2 + 12*a*b^4
*c^3 - 64*a^2*b^2*c^4)*d^3*e + 6*(b^7*c + 13*a*b^5*c^2 - 52*a^2*b^3*c^3 - 6
4*a^3*b*c^4)*d^2*e^2 - 2*(17*a*b^6*c - 44*a^2*b^4*c^2 - 64*a^3*b^2*c^3 - 12
8*a^4*c^4)*d*e^3 + (a*b^7 + 13*a^2*b^5*c - 52*a^3*b^3*c^2 - 64*a^4*b*c^3)*e
^4)*x^2 + 12*(5*a^3*c^4*d^4 - 10*a^3*b*c^3*d^3*e + (5*c^7*d^4 - 10*b*c^6*d^
3*e + 6*(b^2*c^5 + a*c^6)*d^2*e^2 - (b^3*c^4 + 6*a*b*c^5)*d*e^3 + (a*b^2*c^
4 + a^2*c^5)*e^4)*x^6 + 3*(5*b*c^6*d^4 - 10*b^2*c^5*d^3*e + 6*(b^3*c^4 + a*
b*c^5)*d^2*e^2 - (b^4*c^3 + 6*a*b^2*c^4)*d*e^3 + (a*b^3*c^3 + a^2*b*c^4)*e^
4)*x^5 + 6*(a^3*b^2*c^2 + a^4*c^3)*d^2*e^2 - (a^3*b^3*c + 6*a^4*b*c^2)*d*e^
3 + (a^4*b^2*c + a^5*c^2)*e^4 + 3*(5*(b^2*c^5 + a*c^6)*d^4 - 10*(b^3*c^4 +
a*b*c^5)*d^3*e + 6*(b^4*c^3 + 2*a*b^2*c^4 + a^2*c^5)*d^2*e^2 - (b^5*c^2 + 7
*a*b^3*c^3 + 6*a^2*b*c^4)*d*e^3 + (a*b^4*c^2 + 2*a^2*b^2*c^3 + a^3*c^4)*e^4
)*x^4 + (5*(b^3*c^4 + 6*a*b*c^5)*d^4 - 10*(b^4*c^3 + 6*a*b^2*c^4)*d^3*e + 6
*(b^5*c^2 + 7*a*b^3*c^3 + 6*a^2*b*c^4)*d^2*e^2 - (b^6*c + 12*a*b^4*c^2 + 36
*a^2*b^2*c^3)*d*e^3 + (a*b^5*c + 7*a^2*b^3*c^2 + 6*a^3*b*c^3)*e^4)*x^3 + 3*
(5*(a*b^2*c^4 + a^2*c^5)*d^4 - 10*(a*b^3*c^3 + a^2*b*c^4)*d^3*e + 6*(a*b^4*
c^2 + 2*a^2*b^2*c^3 + a^3*c^4)*d^2*e^2 - (a*b^5*c + 7*a^2*b^3*c^2 + 6*a^3*b
*c^3)*d*e^3 + (a^2*b^4*c + 2*a^3*b^2*c^2 + a^4*c^3)*e^4)*x^2 + 3*(5*a^2*b*c
^4*d^4 - 10*a^2*b^2*c^3*d^3*e + 6*(a^2*b^3*c^2 + a^3*b*c^3)*d^2*e^2 - (a^2*
b^4*c + 6*a^3*b^2*c^2)*d*e^3 + (a^3*b^3*c + a^4*b*c^2)*e^4)*x)*sqrt(b^2 - 4
*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b
))/(c*x^2 + b*x + a)) - 3*((b^6*c^2 - 22*a*b^4*c^3 + 28*a^2*b^2*c^4 + 176*a
^3*c^5)*d^4 - 2*(b^7*c - 22*a*b^5*c^2 + 28*a^2*b^3*c^3 + 176*a^3*b*c^4)*d^3
*e - 6*(a*b^6*c + 18*a^2*b^4*c^2 - 92*a^3*b^2*c^3 + 16*a^4*c^4)*d^2*e^2 + 4
0*(a^2*b^5*c - 3*a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^3 - (a^2*b^6 + 18*a^3*b^4*c
 - 92*a^4*b^2*c^2 + 16*a^5*c^3)*e^4)*x)/(a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^
5*b^4*c^3 - 256*a^6*b^2*c^4 + 256*a^7*c^5 + (b^8*c^4 - 16*a*b^6*c^5 + 96*a^
2*b^4*c^6 - 256*a^3*b^2*c^7 + 256*a^4*c^8)*x^6 + 3*(b^9*c^3 - 16*a*b^7*c^4
+ 96*a^2*b^5*c^5 - 256*a^3*b^3*c^6 + 256*a^4*b*c^7)*x^5 + 3*(b^10*c^2 - 15*
a*b^8*c^3 + 80*a^2*b^6*c^4 - 160*a^3*b^4*c^5 + 256*a^5*c^7)*x^4 + (b^11*c -
 10*a*b^9*c^2 + 320*a^3*b^5*c^4 - 1280*a^4*b^3*c^5 + 1536*a^5*b*c^6)*x^3 +
```

```
3*(a*b^10*c - 15*a^2*b^8*c^2 + 80*a^3*b^6*c^3 - 160*a^4*b^4*c^4 + 256*a^6*c
^6)*x^2 + 3*(a^2*b^9*c - 16*a^3*b^7*c^2 + 96*a^4*b^5*c^3 - 256*a^5*b^3*c^4
+ 256*a^6*b*c^5)*x), -1/3*(12*(5*(b^2*c^6 - 4*a*c^7)*d^4 - 10*(b^3*c^5 - 4*
a*b*c^6)*d^3*e + 6*(b^4*c^4 - 3*a*b^2*c^5 - 4*a^2*c^6)*d^2*e^2 - (b^5*c^3 +
 2*a*b^3*c^4 - 24*a^2*b*c^5)*d*e^3 + (a*b^4*c^3 - 3*a^2*b^2*c^4 - 4*a^3*c^5
)*e^4)*x^5 + (b^7*c - 17*a*b^5*c^2 + 118*a^2*b^3*c^3 - 264*a^3*b*c^4)*d^4 +
 2*(a*b^6*c - 22*a^2*b^4*c^2 + 8*a^3*b^2*c^3 + 256*a^4*c^4)*d^3*e + 6*(a^2*
b^5*c + 22*a^3*b^3*c^2 - 104*a^4*b*c^3)*d^2*e^2 - 4*(11*a^3*b^4*c - 28*a^4*
b^2*c^2 - 64*a^5*c^3)*d*e^3 + (a^3*b^5 + 22*a^4*b^3*c - 104*a^5*b*c^2)*e^4
+ 30*(5*(b^3*c^5 - 4*a*b*c^6)*d^4 - 10*(b^4*c^4 - 4*a*b^2*c^5)*d^3*e + 6*(b
^5*c^3 - 3*a*b^3*c^4 - 4*a^2*b*c^5)*d^2*e^2 - (b^6*c^2 + 2*a*b^4*c^3 - 24*a
^2*b^2*c^4)*d*e^3 + (a*b^5*c^2 - 3*a^2*b^3*c^3 - 4*a^3*b*c^4)*e^4)*x^4 + (1
0*(11*b^4*c^4 - 28*a*b^2*c^5 - 64*a^2*c^6)*d^4 - 20*(11*b^5*c^3 - 28*a*b^3*
c^4 - 64*a^2*b*c^5)*d^3*e + 12*(11*b^6*c^2 - 17*a*b^4*c^3 - 92*a^2*b^2*c^4
- 64*a^3*c^5)*d^2*e^2 - 2*(11*b^7*c + 38*a*b^5*c^2 - 232*a^2*b^3*c^3 - 384*
a^3*b*c^4)*d*e^3 + (b^8 + 6*a*b^6*c + 62*a^2*b^4*c^2 - 440*a^3*b^2*c^3 + 12
8*a^4*c^4)*e^4)*x^3 + 3*(5*(b^5*c^3 + 12*a*b^3*c^4 - 64*a^2*b*c^5)*d^4 - 10
*(b^6*c^2 + 12*a*b^4*c^3 - 64*a^2*b^2*c^4)*d^3*e + 6*(b^7*c + 13*a*b^5*c^2
- 52*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2*e^2 - 2*(17*a*b^6*c - 44*a^2*b^4*c^2 -
 64*a^3*b^2*c^3 - 128*a^4*c^4)*d*e^3 + (a*b^7 + 13*a^2*b^5*c - 52*a^3*b^3*c
^2 - 64*a^4*b*c^3)*e^4)*x^2 - 24*(5*a^3*c^4*d^4 - 10*a^3*b*c^3*d^3*e + (5*c
^7*d^4 - 10*b*c^6*d^3*e + 6*(b^2*c^5 + a*c^6)*d^2*e^2 - (b^3*c^4 + 6*a*b*c^
5)*d*e^3 + (a*b^2*c^4 + a^2*c^5)*e^4)*x^6 + 3*(5*b*c^6*d^4 - 10*b^2*c^5*d^3
*e + 6*(b^3*c^4 + a*b*c^5)*d^2*e^2 - (b^4*c^3 + 6*a*b^2*c^4)*d*e^3 + (a*b^3
*c^3 + a^2*b*c^4)*e^4)*x^5 + 6*(a^3*b^2*c^2 + a^4*c^3)*d^2*e^2 - (a^3*b^3*c
 + 6*a^4*b*c^2)*d*e^3 + (a^4*b^2*c + a^5*c^2)*e^4 + 3*(5*(b^2*c^5 + a*c^6)*
d^4 - 10*(b^3*c^4 + a*b*c^5)*d^3*e + 6*(b^4*c^3 + 2*a*b^2*c^4 + a^2*c^5)*d^
2*e^2 - (b^5*c^2 + 7*a*b^3*c^3 + 6*a^2*b*c^4)*d*e^3 + (a*b^4*c^2 + 2*a^2*b^
2*c^3 + a^3*c^4)*e^4)*x^4 + (5*(b^3*c^4 + 6*a*b*c^5)*d^4 - 10*(b^4*c^3 + 6*
a*b^2*c^4)*d^3*e + 6*(b^5*c^2 + 7*a*b^3*c^3 + 6*a^2*b*c^4)*d^2*e^2 - (b^6*c
 + 12*a*b^4*c^2 + 36*a^2*b^2*c^3)*d*e^3 + (a*b^5*c + 7*a^2*b^3*c^2 + 6*a^3*
b*c^3)*e^4)*x^3 + 3*(5*(a*b^2*c^4 + a^2*c^5)*d^4 - 10*(a*b^3*c^3 + a^2*b*c^
4)*d^3*e + 6*(a*b^4*c^2 + 2*a^2*b^2*c^3 + a^3*c^4)*d^2*e^2 - (a*b^5*c + 7*a
^2*b^3*c^2 + 6*a^3*b*c^3)*d*e^3 + (a^2*b^4*c + 2*a^3*b^2*c^2 + a^4*c^3)*e^4
)*x^2 + 3*(5*a^2*b*c^4*d^4 - 10*a^2*b^2*c^3*d^3*e + 6*(a^2*b^3*c^2 + a^3*b*
c^3)*d^2*e^2 - (a^2*b^4*c + 6*a^3*b^2*c^2)*d*e^3 + (a^3*b^3*c + a^4*b*c^2)*
e^4)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*
a*c)) - 3*((b^6*c^2 - 22*a*b^4*c^3 + 28*a^2*b^2*c^4 + 176*a^3*c^5)*d^4 - 2*
(b^7*c - 22*a*b^5*c^2 + 28*a^2*b^3*c^3 + 176*a^3*b*c^4)*d^3*e - 6*(a*b^6*c
+ 18*a^2*b^4*c^2 - 92*a^3*b^2*c^3 + 16*a^4*c^4)*d^2*e^2 + 40*(a^2*b^5*c - 3
*a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^3 - (a^2*b^6 + 18*a^3*b^4*c - 92*a^4*b^2*c^
2 + 16*a^5*c^3)*e^4)*x)/(a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*
a^6*b^2*c^4 + 256*a^7*c^5 + (b^8*c^4 - 16*a*b^6*c^5 + 96*a^2*b^4*c^6 - 256*
a^3*b^2*c^7 + 256*a^4*c^8)*x^6 + 3*(b^9*c^3 - 16*a*b^7*c^4 + 96*a^2*b^5*c^5
 - 256*a^3*b^3*c^6 + 256*a^4*b*c^7)*x^5 + 3*(b^10*c^2 - 15*a*b^8*c^3 + 80*a
^2*b^6*c^4 - 160*a^3*b^4*c^5 + 256*a^5*c^7)*x^4 + (b^11*c - 10*a*b^9*c^2 +
320*a^3*b^5*c^4 - 1280*a^4*b^3*c^5 + 1536*a^5*b*c^6)*x^3 + 3*(a*b^10*c - 15
*a^2*b^8*c^2 + 80*a^3*b^6*c^3 - 160*a^4*b^4*c^4 + 256*a^6*c^6)*x^2 + 3*(a^2
*b^9*c - 16*a^3*b^7*c^2 + 96*a^4*b^5*c^3 - 256*a^5*b^3*c^4 + 256*a^6*b*c^5)
*x)]
```

Sympy [B] time = 121.526, size = 2547, normalized size = 9.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(c*x**2+b*x+a)**4,x)
```

```
[Out] -4*sqrt(-1/(4*a*c - b**2)**7)*(a*e**2 - b*d*e + c*d**2)*(a*c*e**2 + b**2*e*
*2 - 5*b*c*d*e + 5*c**2*d**2)*log(x + (-1024*a**4*c**4*sqrt(-1/(4*a*c - b**
2)**7)*(a*e**2 - b*d*e + c*d**2)*(a*c*e**2 + b**2*e**2 - 5*b*c*d*e + 5*c**2
*d**2) + 1024*a**3*b**2*c**3*sqrt(-1/(4*a*c - b**2)**7)*(a*e**2 - b*d*e + c
*d**2)*(a*c*e**2 + b**2*e**2 - 5*b*c*d*e + 5*c**2*d**2) - 384*a**2*b**4*c**
2*sqrt(-1/(4*a*c - b**2)**7)*(a*e**2 - b*d*e + c*d**2)*(a*c*e**2 + b**2*e**
2 - 5*b*c*d*e + 5*c**2*d**2) + 4*a**2*b*c*e**4 + 64*a*b**6*c*sqrt(-1/(4*a*c
 - b**2)**7)*(a*e**2 - b*d*e + c*d**2)*(a*c*e**2 + b**2*e**2 - 5*b*c*d*e +
5*c**2*d**2) + 4*a*b**3*e**4 - 24*a*b**2*c*d*e**3 + 24*a*b*c**2*d**2*e**2 -
 4*b**8*sqrt(-1/(4*a*c - b**2)**7)*(a*e**2 - b*d*e + c*d**2)*(a*c*e**2 + b*
*2*e**2 - 5*b*c*d*e + 5*c**2*d**2) - 4*b**4*d*e**3 + 24*b**3*c*d**2*e**2 -
40*b**2*c**2*d**3*e + 20*b*c**3*d**4)/(8*a**2*c**2*e**4 + 8*a*b**2*c*e**4 -
 48*a*b*c**2*d*e**3 + 48*a*c**3*d**2*e**2 - 8*b**3*c*d*e**3 + 48*b**2*c**2*
d**2*e**2 - 80*b*c**3*d**3*e + 40*c**4*d**4)) + 4*sqrt(-1/(4*a*c - b**2)**7
)*(a*e**2 - b*d*e + c*d**2)*(a*c*e**2 + b**2*e**2 - 5*b*c*d*e + 5*c**2*d**2
)*log(x + (1024*a**4*c**4*sqrt(-1/(4*a*c - b**2)**7)*(a*e**2 - b*d*e + c*d*
*2)*(a*c*e**2 + b**2*e**2 - 5*b*c*d*e + 5*c**2*d**2) - 1024*a**3*b**2*c**3*
sqrt(-1/(4*a*c - b**2)**7)*(a*e**2 - b*d*e + c*d**2)*(a*c*e**2 + b**2*e**2
- 5*b*c*d*e + 5*c**2*d**2) + 384*a**2*b**4*c**2*sqrt(-1/(4*a*c - b**2)**7)*
(a*e**2 - b*d*e + c*d**2)*(a*c*e**2 + b**2*e**2 - 5*b*c*d*e + 5*c**2*d**2)
+ 4*a**2*b*c*e**4 - 64*a*b**6*c*sqrt(-1/(4*a*c - b**2)**7)*(a*e**2 - b*d*e
+ c*d**2)*(a*c*e**2 + b**2*e**2 - 5*b*c*d*e + 5*c**2*d**2) + 4*a*b**3*e**4
- 24*a*b**2*c*d*e**3 + 24*a*b*c**2*d**2*e**2 + 4*b**8*sqrt(-1/(4*a*c - b**2
)**7)*(a*e**2 - b*d*e + c*d**2)*(a*c*e**2 + b**2*e**2 - 5*b*c*d*e + 5*c**2*
d**2) - 4*b**4*d*e**3 + 24*b**3*c*d**2*e**2 - 40*b**2*c**2*d**3*e + 20*b*c*
*3*d**4)/(8*a**2*c**2*e**4 + 8*a*b**2*c*e**4 - 48*a*b*c**2*d*e**3 + 48*a*c*
*3*d**2*e**2 - 8*b**3*c*d*e**3 + 48*b**2*c**2*d**2*e**2 - 80*b*c**3*d**3*e
+ 40*c**4*d**4)) + (26*a**4*b*c*e**4 - 64*a**4*c**2*d*e**3 + a**3*b**3*e**4
 - 44*a**3*b**2*c*d*e**3 + 156*a**3*b*c**2*d**2*e**2 - 128*a**3*c**3*d**3*e
 + 6*a**2*b**3*c*d**2*e**2 - 36*a**2*b**2*c**2*d**3*e + 66*a**2*b*c**3*d**4
 + 2*a*b**4*c*d**3*e - 13*a*b**3*c**2*d**4 + b**5*c*d**4 + x**5*(12*a**2*c*
*4*e**4 + 12*a*b**2*c**3*e**4 - 72*a*b*c**4*d*e**3 + 72*a*c**5*d**2*e**2 -
12*b**3*c**3*d*e**3 + 72*b**2*c**4*d**2*e**2 - 120*b*c**5*d**3*e + 60*c**6*
d**4) + x**4*(30*a**2*b*c**3*e**4 + 30*a*b**3*c**2*e**4 - 180*a*b**2*c**3*d
*e**3 + 180*a*b*c**4*d**2*e**2 - 30*b**4*c**2*d*e**3 + 180*b**3*c**3*d**2*e
**2 - 300*b**2*c**4*d**3*e + 150*b*c**5*d**4) + x**3*(-32*a**3*c**3*e**4 +
102*a**2*b**2*c**2*e**4 - 192*a**2*b*c**3*d*e**3 + 192*a**2*c**4*d**2*e**2
+ 10*a*b**4*c*e**4 - 164*a*b**3*c**2*d*e**3 + 324*a*b**2*c**3*d**2*e**2 - 3
20*a*b*c**4*d**3*e + 160*a*c**5*d**4 + b**6*e**4 - 22*b**5*c*d*e**3 + 132*b
**4*c**2*d**2*e**2 - 220*b**3*c**3*d**3*e + 110*b**2*c**4*d**4) + x**2*(48*
a**3*b*c**2*e**4 - 192*a**3*c**3*d*e**3 + 51*a**2*b**3*c*e**4 - 144*a**2*b*
*2*c**2*d*e**3 + 288*a**2*b*c**3*d**2*e**2 + 3*a*b**5*e**4 - 102*a*b**4*c*d
*e**3 + 306*a*b**3*c**2*d**2*e**2 - 480*a*b**2*c**3*d**3*e + 240*a*b*c**4*d
**4 + 18*b**5*c*d**2*e**2 - 30*b**4*c**2*d**3*e + 15*b**3*c**3*d**4) + x*(-
12*a**4*c**2*e**4 + 66*a**3*b**2*c*e**4 - 120*a**3*b*c**2*d*e**3 - 72*a**3*
c**3*d**2*e**2 + 3*a**2*b**4*e**4 - 120*a**2*b**3*c*d*e**3 + 396*a**2*b**2*
c**2*d**2*e**2 - 264*a**2*b*c**3*d**3*e + 132*a**2*c**4*d**4 + 18*a*b**4*c*
d**2*e**2 - 108*a*b**3*c**2*d**3*e + 54*a*b**2*c**3*d**4 + 6*b**5*c*d**3*e
- 3*b**4*c**2*d**4))/(192*a**6*c**4 - 144*a**5*b**2*c**3 + 36*a**4*b**4*c**
2 - 3*a**3*b**6*c + x**6*(192*a**3*c**7 - 144*a**2*b**2*c**6 + 36*a*b**4*c*
*5 - 3*b**6*c**4) + x**5*(576*a**3*b*c**6 - 432*a**2*b**3*c**5 + 108*a*b**5
*c**4 - 9*b**7*c**3) + x**4*(576*a**4*c**6 + 144*a**3*b**2*c**5 - 324*a**2*
b**4*c**4 + 99*a*b**6*c**3 - 9*b**8*c**2) + x**3*(1152*a**4*b*c**5 - 672*a*
*3*b**3*c**4 + 72*a**2*b**5*c**3 + 18*a*b**7*c**2 - 3*b**9*c) + x**2*(576*a
**5*c**5 + 144*a**4*b**2*c**4 - 324*a**3*b**4*c**3 + 99*a**2*b**6*c**2 - 9*
a*b**8*c) + x*(576*a**5*b*c**4 - 432*a**4*b**3*c**3 + 108*a**3*b**5*c**2 -
9*a**2*b**7*c))
```

Giac [B] time = 1.14049, size = 1524, normalized size = 5.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x+a)^4,x, algorithm="giac")
```

```
[Out] -8*(5*c^3*d^4 - 10*b*c^2*d^3*e + 6*b^2*c*d^2*e^2 + 6*a*c^2*d^2*e^2 - b^3*d*
e^3 - 6*a*b*c*d*e^3 + a*b^2*e^4 + a^2*c*e^4)*arctan((2*c*x + b)/sqrt(-b^2 +
 4*a*c))/((b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-b^2 + 4*a*
c)) - 1/3*(60*c^6*d^4*x^5 - 120*b*c^5*d^3*x^5*e + 150*b*c^5*d^4*x^4 + 72*b^
2*c^4*d^2*x^5*e^2 + 72*a*c^5*d^2*x^5*e^2 - 300*b^2*c^4*d^3*x^4*e + 110*b^2*
c^4*d^4*x^3 + 160*a*c^5*d^4*x^3 - 12*b^3*c^3*d*x^5*e^3 - 72*a*b*c^4*d*x^5*e
^3 + 180*b^3*c^3*d^2*x^4*e^2 + 180*a*b*c^4*d^2*x^4*e^2 - 220*b^3*c^3*d^3*x^
3*e - 320*a*b*c^4*d^3*x^3*e + 15*b^3*c^3*d^4*x^2 + 240*a*b*c^4*d^4*x^2 + 12
*a*b^2*c^3*x^5*e^4 + 12*a^2*c^4*x^5*e^4 - 30*b^4*c^2*d*x^4*e^3 - 180*a*b^2*
c^3*d*x^4*e^3 + 132*b^4*c^2*d^2*x^3*e^2 + 324*a*b^2*c^3*d^2*x^3*e^2 + 192*a
^2*c^4*d^2*x^3*e^2 - 30*b^4*c^2*d^3*x^2*e - 480*a*b^2*c^3*d^3*x^2*e - 3*b^4
*c^2*d^4*x + 54*a*b^2*c^3*d^4*x + 132*a^2*c^4*d^4*x + 30*a*b^3*c^2*x^4*e^4
+ 30*a^2*b*c^3*x^4*e^4 - 22*b^5*c*d*x^3*e^3 - 164*a*b^3*c^2*d*x^3*e^3 - 192
*a^2*b*c^3*d*x^3*e^3 + 18*b^5*c*d^2*x^2*e^2 + 306*a*b^3*c^2*d^2*x^2*e^2 + 2
88*a^2*b*c^3*d^2*x^2*e^2 + 6*b^5*c*d^3*x*e - 108*a*b^3*c^2*d^3*x*e - 264*a^
2*b*c^3*d^3*x*e + b^5*c*d^4 - 13*a*b^3*c^2*d^4 + 66*a^2*b*c^3*d^4 + b^6*x^3
*e^4 + 10*a*b^4*c*x^3*e^4 + 102*a^2*b^2*c^2*x^3*e^4 - 32*a^3*c^3*x^3*e^4 -
102*a*b^4*c*d*x^2*e^3 - 144*a^2*b^2*c^2*d*x^2*e^3 - 192*a^3*c^3*d*x^2*e^3 +
 18*a*b^4*c*d^2*x*e^2 + 396*a^2*b^2*c^2*d^2*x*e^2 - 72*a^3*c^3*d^2*x*e^2 +
2*a*b^4*c*d^3*e - 36*a^2*b^2*c^2*d^3*e - 128*a^3*c^3*d^3*e + 3*a*b^5*x^2*e^
4 + 51*a^2*b^3*c*x^2*e^4 + 48*a^3*b*c^2*x^2*e^4 - 120*a^2*b^3*c*d*x*e^3 - 1
20*a^3*b*c^2*d*x*e^3 + 6*a^2*b^3*c*d^2*e^2 + 156*a^3*b*c^2*d^2*e^2 + 3*a^2*
b^4*x*e^4 + 66*a^3*b^2*c*x*e^4 - 12*a^4*c^2*x*e^4 - 44*a^3*b^2*c*d*e^3 - 64
*a^4*c^2*d*e^3 + a^3*b^3*e^4 + 26*a^4*b*c*e^4)/((b^6*c - 12*a*b^4*c^2 + 48*
a^2*b^2*c^3 - 64*a^3*c^4)*(c*x^2 + b*x + a)^3)
```

3.2215 $\int \frac{(d+ex)^3}{(a+bx+cx^2)^4} dx$

Optimal. Leaf size=306

$$-\frac{2x(2cd-be)\left(-ce(ae+15bd)+4b^2e^2+15c^2d^2\right)-b^2\left(11ae^3+25cd^2e\right)+6bcd\left(13ae^2+5cd^2\right)-16ace\left(ae^2+5cd^2\right)+}{3\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)}$$

```
[Out] -((b + 2*c*x)*(d + e*x)^3)/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^3) + ((d + e*
x)^2*(10*b*c*d - 3*b^2*e - 8*a*c*e + 10*c*(2*c*d - b*e)*x))/(6*(b^2 - 4*a*c
)^2*(a + b*x + c*x^2)^2) - (3*b^3*d*e^2 - 16*a*c*e*(5*c*d^2 + a*e^2) + 6*b*
c*d*(5*c*d^2 + 13*a*e^2) - b^2*(25*c*d^2*e + 11*a*e^3) + 2*(2*c*d - b*e)*(1
5*c^2*d^2 + 4*b^2*e^2 - c*e*(15*b*d + a*e))*x)/(3*(b^2 - 4*a*c)^3*(a + b*x
+ c*x^2)) + (2*(2*c*d - b*e)*(10*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))
*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(7/2)
```

Rubi [A] time = 0.429487, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {736, 820, 777, 618, 206}

$$-\frac{2x(2cd-be)\left(-ce(ae+15bd)+4b^2e^2+15c^2d^2\right)-b^2\left(11ae^3+25cd^2e\right)+6bcd\left(13ae^2+5cd^2\right)-16ace\left(ae^2+5cd^2\right)+}{3\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + b*x + c*x^2)^4,x]
```

```
[Out] -((b + 2*c*x)*(d + e*x)^3)/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^3) + ((d + e*
x)^2*(10*b*c*d - 3*b^2*e - 8*a*c*e + 10*c*(2*c*d - b*e)*x))/(6*(b^2 - 4*a*c
)^2*(a + b*x + c*x^2)^2) - (3*b^3*d*e^2 - 16*a*c*e*(5*c*d^2 + a*e^2) + 6*b*
c*d*(5*c*d^2 + 13*a*e^2) - b^2*(25*c*d^2*e + 11*a*e^3) + 2*(2*c*d - b*e)*(1
5*c^2*d^2 + 4*b^2*e^2 - c*e*(15*b*d + a*e))*x)/(3*(b^2 - 4*a*c)^3*(a + b*x
+ c*x^2)) + (2*(2*c*d - b*e)*(10*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))
*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(7/2)
```

Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] - Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)
*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p + 1
), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 0] && (L
tQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c,
d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*
(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p +
 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*Simp[g*
(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(
m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (I
ntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 777

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p +
1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(
2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(a+bx+cx^2\right)^4}\,dx = -\frac{(b+2cx)(d+ex)^3}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}+\frac{\int \frac{(d+ex)^2(-10cd+3be-4cex)}{\left(a+bx+cx^2\right)^3}\,dx}{3\left(b^2-4ac\right)}$$

$$= -\frac{(b+2cx)(d+ex)^3}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}+\frac{(d+ex)^2\left(10bcd-3b^2e-8ace+10c(2cd-be)x\right)}{6\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2}-\frac{\int \frac{(d+ex)\left(-2(3\right.}{}}{}$$

$$= -\frac{(b+2cx)(d+ex)^3}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}+\frac{(d+ex)^2\left(10bcd-3b^2e-8ace+10c(2cd-be)x\right)}{6\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2}-\frac{3b^3de^2-16}{}$$

$$= -\frac{(b+2cx)(d+ex)^3}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}+\frac{(d+ex)^2\left(10bcd-3b^2e-8ace+10c(2cd-be)x\right)}{6\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2}-\frac{3b^3de^2-16}{}$$

$$= -\frac{(b+2cx)(d+ex)^3}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}+\frac{(d+ex)^2\left(10bcd-3b^2e-8ace+10c(2cd-be)x\right)}{6\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2}-\frac{3b^3de^2-16}{}$$

Mathematica [A] time = 0.822817, size = 401, normalized size = 1.31

$$\frac{1}{6}\left(\frac{2\left(2c\left(a^2e^3-3acde(d+ex)+c^2d^3x\right)+b^2e^2(3cdx-ae)+bc\left(3ae^2(d+ex)+cd^2(d-3ex)\right)-b^3e^3x\right)}{c^2\left(4ac-b^2\right)(a+x(b+cx))^3}+\frac{4c^2\left(-6a^2e^3+3a\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + b*x + c*x^2)^4,x]
```

```
[Out] ((3*(2*c*d - b*e)*(10*c^2*d^2 + b^2*e^2 + 2*c*e*(-5*b*d + 3*a*e))*(b + 2*c*
x))/(c*(-b^2 + 4*a*c)^3*(a + x*(b + c*x))) + (2*(-(b^3*e^3*x) + b^2*e^2*(-(
a*e) + 3*c*d*x) + 2*c*(a^2*e^3 + c^2*d^3*x - 3*a*c*d*e*(d + e*x)) + b*c*(c*
d^2*(d - 3*e*x) + 3*a*e^2*(d + e*x))))/(c^2*(-b^2 + 4*a*c)*(a + x*(b + c*x)
)^3) + (-2*b^4*e^3 + b^3*c*e^2*(6*d - e*x) + 4*c^2*(-6*a^2*e^3 + 5*c^2*d^3*
x + 3*a*c*d*e^2*x) + 2*b*c^2*(5*c*d^2*(d - 3*e*x) + 3*a*e^2*(d - e*x)) + 3*
b^2*c*e*(3*a*e^2 + c*d*(-5*d + 4*e*x)))/(c^2*(b^2 - 4*a*c)^2*(a + x*(b + c*
x))^2) - (12*(-2*c*d + b*e)*(10*c^2*d^2 + b^2*e^2 + 2*c*e*(-5*b*d + 3*a*e))
*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(7/2))/6
```

Maple [B] time = 0.165, size = 1213, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^2+b*x+a)^4,x)
```

```
[Out] (-(6*a*b*c*e^3-12*a*c^2*d*e^2+b^3*e^3-12*b^2*c*d*e^2+30*b*c^2*d^2*e-20*c^3*
d^3)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*c^2*x^5-5/2*(6*a*b*c*e^3-12
*a*c^2*d*e^2+b^3*e^3-12*b^2*c*d*e^2+30*b*c^2*d^2*e-20*c^3*d^3)/(64*a^3*c^3-
48*a^2*b^2*c^2+12*a*b^4*c-b^6)*b*c*x^4-1/6*(16*a*c+11*b^2)*(6*a*b*c*e^3-12*
a*c^2*d*e^2+b^3*e^3-12*b^2*c*d*e^2+30*b*c^2*d^2*e-20*c^3*d^3)/(64*a^3*c^3-4
8*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^3-1/2*(32*a^3*c^2*e^3+24*a^2*b^2*c*e^3-96*a
^2*b*c^2*d*e^2+17*a*b^4*e^3-102*a*b^3*c*d*e^2+240*a*b^2*c^2*d^2*e-160*a*b*c
^3*d^3-6*b^5*d*e^2+15*b^4*c*d^2*e-10*b^3*c^2*d^3)/(64*a^3*c^3-48*a^2*b^2*c^
2+12*a*b^4*c-b^6)*x^2-1/2*(20*a^3*b*c*e^3+24*a^3*c^2*d*e^2+20*a^2*b^3*e^3-1
32*a^2*b^2*c*d*e^2+132*a^2*b*c^2*d^2*e-88*a^2*c^3*d^3-6*a*b^4*d*e^2+54*a*b^
3*c*d^2*e-36*a*b^2*c^2*d^3-3*b^5*d^2*e+2*b^4*c*d^3)/(64*a^3*c^3-48*a^2*b^2*
c^2+12*a*b^4*c-b^6)*x-1/6*(32*a^4*c*e^3+22*a^3*b^2*e^3-156*a^3*b*c*d*e^2+19
2*a^3*c^2*d^2*e-6*a^2*b^3*d*e^2+54*a^2*b^2*c*d^2*e-132*a^2*b*c^2*d^3-3*a*b^
4*d^2*e+26*a*b^3*c*d^3-2*b^5*d^3)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6
))/(c*x^2+b*x+a)^3-12/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(4*a*c-b^2
)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*c*e^3+24/(64*a^3*c^3-48*a^2
*b^2*c^2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/
2))*c^2*a*d*e^2-2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1
/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*e^3+24/(64*a^3*c^3-48*a^2*b^2*c
^2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^
2*c*d*e^2-60/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1/2)*a
rctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c^2*d^2*e+40/(64*a^3*c^3-48*a^2*b^2*c^
2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^3
*d^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.6895, size = 7605, normalized size = 24.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/6*(6*(20*(b^2*c^5 - 4*a*c^6)*d^3 - 30*(b^3*c^4 - 4*a*b*c^5)*d^2*e + 12*
(b^4*c^3 - 3*a*b^2*c^4 - 4*a^2*c^5)*d*e^2 - (b^5*c^2 + 2*a*b^3*c^3 - 24*a^2
*b*c^4)*e^3)*x^5 + 15*(20*(b^3*c^4 - 4*a*b*c^5)*d^3 - 30*(b^4*c^3 - 4*a*b^2
*c^4)*d^2*e + 12*(b^5*c^2 - 3*a*b^3*c^3 - 4*a^2*b*c^4)*d*e^2 - (b^6*c + 2*a
*b^4*c^2 - 24*a^2*b^2*c^3)*e^3)*x^4 + 2*(b^7 - 17*a*b^5*c + 118*a^2*b^3*c^2
 - 264*a^3*b*c^3)*d^3 + 3*(a*b^6 - 22*a^2*b^4*c + 8*a^3*b^2*c^2 + 256*a^4*c
^3)*d^2*e + 6*(a^2*b^5 + 22*a^3*b^3*c - 104*a^4*b*c^2)*d*e^2 - 2*(11*a^3*b^
4 - 28*a^4*b^2*c - 64*a^5*c^2)*e^3 + (20*(11*b^4*c^3 - 28*a*b^2*c^4 - 64*a^
2*c^5)*d^3 - 30*(11*b^5*c^2 - 28*a*b^3*c^3 - 64*a^2*b*c^4)*d^2*e + 12*(11*b
^6*c - 17*a*b^4*c^2 - 92*a^2*b^2*c^3 - 64*a^3*c^4)*d*e^2 - (11*b^7 + 38*a*b
^5*c - 232*a^2*b^3*c^2 - 384*a^3*b*c^3)*e^3)*x^3 + 3*(10*(b^5*c^2 + 12*a*b^
3*c^3 - 64*a^2*b*c^4)*d^3 - 15*(b^6*c + 12*a*b^4*c^2 - 64*a^2*b^2*c^3)*d^2*
e + 6*(b^7 + 13*a*b^5*c - 52*a^2*b^3*c^2 - 64*a^3*b*c^3)*d*e^2 - (17*a*b^6
- 44*a^2*b^4*c - 64*a^3*b^2*c^2 - 128*a^4*c^3)*e^3)*x^2 - 6*(20*a^3*c^3*d^3
 - 30*a^3*b*c^2*d^2*e + (20*c^6*d^3 - 30*b*c^5*d^2*e + 12*(b^2*c^4 + a*c^5)
*d*e^2 - (b^3*c^3 + 6*a*b*c^4)*e^3)*x^6 + 3*(20*b*c^5*d^3 - 30*b^2*c^4*d^2*
e + 12*(b^3*c^3 + a*b*c^4)*d*e^2 - (b^4*c^2 + 6*a*b^2*c^3)*e^3)*x^5 + 3*(20
*(b^2*c^4 + a*c^5)*d^3 - 30*(b^3*c^3 + a*b*c^4)*d^2*e + 12*(b^4*c^2 + 2*a*b
^2*c^3 + a^2*c^4)*d*e^2 - (b^5*c + 7*a*b^3*c^2 + 6*a^2*b*c^3)*e^3)*x^4 + 12
*(a^3*b^2*c + a^4*c^2)*d*e^2 - (a^3*b^3 + 6*a^4*b*c)*e^3 + (20*(b^3*c^3 + 6
*a*b*c^4)*d^3 - 30*(b^4*c^2 + 6*a*b^2*c^3)*d^2*e + 12*(b^5*c + 7*a*b^3*c^2
+ 6*a^2*b*c^3)*d*e^2 - (b^6 + 12*a*b^4*c + 36*a^2*b^2*c^2)*e^3)*x^3 + 3*(20
*(a*b^2*c^3 + a^2*c^4)*d^3 - 30*(a*b^3*c^2 + a^2*b*c^3)*d^2*e + 12*(a*b^4*c
 + 2*a^2*b^2*c^2 + a^3*c^3)*d*e^2 - (a*b^5 + 7*a^2*b^3*c + 6*a^3*b*c^2)*e^3
)*x^2 + 3*(20*a^2*b*c^3*d^3 - 30*a^2*b^2*c^2*d^2*e + 12*(a^2*b^3*c + a^3*b*
c^2)*d*e^2 - (a^2*b^4 + 6*a^3*b^2*c)*e^3)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x
^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x +
a)) - 3*(2*(b^6*c - 22*a*b^4*c^2 + 28*a^2*b^2*c^3 + 176*a^3*c^4)*d^3 - 3*(b
^7 - 22*a*b^5*c + 28*a^2*b^3*c^2 + 176*a^3*b*c^3)*d^2*e - 6*(a*b^6 + 18*a^2
*b^4*c - 92*a^3*b^2*c^2 + 16*a^4*c^3)*d*e^2 + 20*(a^2*b^5 - 3*a^3*b^3*c - 4
*a^4*b*c^2)*e^3)*x)/(a^3*b^8 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*
c^3 + 256*a^7*c^4 + (b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*
c^6 + 256*a^4*c^7)*x^6 + 3*(b^9*c^2 - 16*a*b^7*c^3 + 96*a^2*b^5*c^4 - 256*a
^3*b^3*c^5 + 256*a^4*b*c^6)*x^5 + 3*(b^10*c - 15*a*b^8*c^2 + 80*a^2*b^6*c^3
 - 160*a^3*b^4*c^4 + 256*a^5*c^6)*x^4 + (b^11 - 10*a*b^9*c + 320*a^3*b^5*c^
3 - 1280*a^4*b^3*c^4 + 1536*a^5*b*c^5)*x^3 + 3*(a*b^10 - 15*a^2*b^8*c + 80*
a^3*b^6*c^2 - 160*a^4*b^4*c^3 + 256*a^6*c^5)*x^2 + 3*(a^2*b^9 - 16*a^3*b^7*
c + 96*a^4*b^5*c^2 - 256*a^5*b^3*c^3 + 256*a^6*b*c^4)*x), -1/6*(6*(20*(b^2*
c^5 - 4*a*c^6)*d^3 - 30*(b^3*c^4 - 4*a*b*c^5)*d^2*e + 12*(b^4*c^3 - 3*a*b^2
*c^4 - 4*a^2*c^5)*d*e^2 - (b^5*c^2 + 2*a*b^3*c^3 - 24*a^2*b*c^4)*e^3)*x^5 +
 15*(20*(b^3*c^4 - 4*a*b*c^5)*d^3 - 30*(b^4*c^3 - 4*a*b^2*c^4)*d^2*e + 12*(
b^5*c^2 - 3*a*b^3*c^3 - 4*a^2*b*c^4)*d*e^2 - (b^6*c + 2*a*b^4*c^2 - 24*a^2*
b^2*c^3)*e^3)*x^4 + 2*(b^7 - 17*a*b^5*c + 118*a^2*b^3*c^2 - 264*a^3*b*c^3)*
d^3 + 3*(a*b^6 - 22*a^2*b^4*c + 8*a^3*b^2*c^2 + 256*a^4*c^3)*d^2*e + 6*(a^2
*b^5 + 22*a^3*b^3*c - 104*a^4*b*c^2)*d*e^2 - 2*(11*a^3*b^4 - 28*a^4*b^2*c -
 64*a^5*c^2)*e^3 + (20*(11*b^4*c^3 - 28*a*b^2*c^4 - 64*a^2*c^5)*d^3 - 30*(1
1*b^5*c^2 - 28*a*b^3*c^3 - 64*a^2*b*c^4)*d^2*e + 12*(11*b^6*c - 17*a*b^4*c^
2 - 92*a^2*b^2*c^3 - 64*a^3*c^4)*d*e^2 - (11*b^7 + 38*a*b^5*c - 232*a^2*b^3
*c^2 - 384*a^3*b*c^3)*e^3)*x^3 + 3*(10*(b^5*c^2 + 12*a*b^3*c^3 - 64*a^2*b*c
^4)*d^3 - 15*(b^6*c + 12*a*b^4*c^2 - 64*a^2*b^2*c^3)*d^2*e + 6*(b^7 + 13*a*
b^5*c - 52*a^2*b^3*c^2 - 64*a^3*b*c^3)*d*e^2 - (17*a*b^6 - 44*a^2*b^4*c - 6
```

```
4*a^3*b^2*c^2 - 128*a^4*c^3)*e^3)*x^2 - 12*(20*a^3*c^3*d^3 - 30*a^3*b*c^2*d
^2*e + (20*c^6*d^3 - 30*b*c^5*d^2*e + 12*(b^2*c^4 + a*c^5)*d*e^2 - (b^3*c^3
 + 6*a*b*c^4)*e^3)*x^6 + 3*(20*b*c^5*d^3 - 30*b^2*c^4*d^2*e + 12*(b^3*c^3 +
 a*b*c^4)*d*e^2 - (b^4*c^2 + 6*a*b^2*c^3)*e^3)*x^5 + 3*(20*(b^2*c^4 + a*c^5
)*d^3 - 30*(b^3*c^3 + a*b*c^4)*d^2*e + 12*(b^4*c^2 + 2*a*b^2*c^3 + a^2*c^4)
*d*e^2 - (b^5*c + 7*a*b^3*c^2 + 6*a^2*b*c^3)*e^3)*x^4 + 12*(a^3*b^2*c + a^4
*c^2)*d*e^2 - (a^3*b^3 + 6*a^4*b*c)*e^3 + (20*(b^3*c^3 + 6*a*b*c^4)*d^3 - 3
0*(b^4*c^2 + 6*a*b^2*c^3)*d^2*e + 12*(b^5*c + 7*a*b^3*c^2 + 6*a^2*b*c^3)*d*
e^2 - (b^6 + 12*a*b^4*c + 36*a^2*b^2*c^2)*e^3)*x^3 + 3*(20*(a*b^2*c^3 + a^2
*c^4)*d^3 - 30*(a*b^3*c^2 + a^2*b*c^3)*d^2*e + 12*(a*b^4*c + 2*a^2*b^2*c^2
+ a^3*c^3)*d*e^2 - (a*b^5 + 7*a^2*b^3*c + 6*a^3*b*c^2)*e^3)*x^2 + 3*(20*a^2
*b*c^3*d^3 - 30*a^2*b^2*c^2*d^2*e + 12*(a^2*b^3*c + a^3*b*c^2)*d*e^2 - (a^2
*b^4 + 6*a^3*b^2*c)*e^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(
2*c*x + b)/(b^2 - 4*a*c)) - 3*(2*(b^6*c - 22*a*b^4*c^2 + 28*a^2*b^2*c^3 + 1
76*a^3*c^4)*d^3 - 3*(b^7 - 22*a*b^5*c + 28*a^2*b^3*c^2 + 176*a^3*b*c^3)*d^2
*e - 6*(a*b^6 + 18*a^2*b^4*c - 92*a^3*b^2*c^2 + 16*a^4*c^3)*d*e^2 + 20*(a^2
*b^5 - 3*a^3*b^3*c - 4*a^4*b*c^2)*e^3)*x)/(a^3*b^8 - 16*a^4*b^6*c + 96*a^5*
b^4*c^2 - 256*a^6*b^2*c^3 + 256*a^7*c^4 + (b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*
b^4*c^5 - 256*a^3*b^2*c^6 + 256*a^4*c^7)*x^6 + 3*(b^9*c^2 - 16*a*b^7*c^3 +
96*a^2*b^5*c^4 - 256*a^3*b^3*c^5 + 256*a^4*b*c^6)*x^5 + 3*(b^10*c - 15*a*b^
8*c^2 + 80*a^2*b^6*c^3 - 160*a^3*b^4*c^4 + 256*a^5*c^6)*x^4 + (b^11 - 10*a*
b^9*c + 320*a^3*b^5*c^3 - 1280*a^4*b^3*c^4 + 1536*a^5*b*c^5)*x^3 + 3*(a*b^1
0 - 15*a^2*b^8*c + 80*a^3*b^6*c^2 - 160*a^4*b^4*c^3 + 256*a^6*c^5)*x^2 + 3*
(a^2*b^9 - 16*a^3*b^7*c + 96*a^4*b^5*c^2 - 256*a^5*b^3*c^3 + 256*a^6*b*c^4)
*x)]
```

Sympy [B] time = 27.1883, size = 2057, normalized size = 6.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**2+b*x+a)**4,x)
```

```
[Out] sqrt(-1/(4*a*c - b**2)**7)*(b*e - 2*c*d)*(6*a*c*e**2 + b**2*e**2 - 10*b*c*d
*e + 10*c**2*d**2)*log(x + (-256*a**4*c**4*sqrt(-1/(4*a*c - b**2)**7)*(b*e
- 2*c*d)*(6*a*c*e**2 + b**2*e**2 - 10*b*c*d*e + 10*c**2*d**2) + 256*a**3*b*
*2*c**3*sqrt(-1/(4*a*c - b**2)**7)*(b*e - 2*c*d)*(6*a*c*e**2 + b**2*e**2 -
10*b*c*d*e + 10*c**2*d**2) - 96*a**2*b**4*c**2*sqrt(-1/(4*a*c - b**2)**7)*(
b*e - 2*c*d)*(6*a*c*e**2 + b**2*e**2 - 10*b*c*d*e + 10*c**2*d**2) + 16*a*b*
*6*c*sqrt(-1/(4*a*c - b**2)**7)*(b*e - 2*c*d)*(6*a*c*e**2 + b**2*e**2 - 10*
b*c*d*e + 10*c**2*d**2) + 6*a*b**2*c*e**3 - 12*a*b*c**2*d*e**2 - b**8*sqrt(
-1/(4*a*c - b**2)**7)*(b*e - 2*c*d)*(6*a*c*e**2 + b**2*e**2 - 10*b*c*d*e +
10*c**2*d**2) + b**4*e**3 - 12*b**3*c*d*e**2 + 30*b**2*c**2*d**2*e - 20*b*c
**3*d**3)/(12*a*b*c**2*e**3 - 24*a*c**3*d*e**2 + 2*b**3*c*e**3 - 24*b**2*c*
*2*d*e**2 + 60*b*c**3*d**2*e - 40*c**4*d**3)) - sqrt(-1/(4*a*c - b**2)**7)*
(b*e - 2*c*d)*(6*a*c*e**2 + b**2*e**2 - 10*b*c*d*e + 10*c**2*d**2)*log(x +
(256*a**4*c**4*sqrt(-1/(4*a*c - b**2)**7)*(b*e - 2*c*d)*(6*a*c*e**2 + b**2*
e**2 - 10*b*c*d*e + 10*c**2*d**2) - 256*a**3*b**2*c**3*sqrt(-1/(4*a*c - b**
2)**7)*(b*e - 2*c*d)*(6*a*c*e**2 + b**2*e**2 - 10*b*c*d*e + 10*c**2*d**2) +
 96*a**2*b**4*c**2*sqrt(-1/(4*a*c - b**2)**7)*(b*e - 2*c*d)*(6*a*c*e**2 + b
**2*e**2 - 10*b*c*d*e + 10*c**2*d**2) - 16*a*b**6*c*sqrt(-1/(4*a*c - b**2)*
*7)*(b*e - 2*c*d)*(6*a*c*e**2 + b**2*e**2 - 10*b*c*d*e + 10*c**2*d**2) + 6*
a*b**2*c*e**3 - 12*a*b*c**2*d*e**2 + b**8*sqrt(-1/(4*a*c - b**2)**7)*(b*e -
 2*c*d)*(6*a*c*e**2 + b**2*e**2 - 10*b*c*d*e + 10*c**2*d**2) + b**4*e**3 -
12*b**3*c*d*e**2 + 30*b**2*c**2*d**2*e - 20*b*c**3*d**3)/(12*a*b*c**2*e**3
- 24*a*c**3*d*e**2 + 2*b**3*c*e**3 - 24*b**2*c**2*d*e**2 + 60*b*c**3*d**2*e
```

```
 - 40*c**4*d**3)) - (32*a**4*c*e**3 + 22*a**3*b**2*e**3 - 156*a**3*b*c*d*e*
*2 + 192*a**3*c**2*d**2*e - 6*a**2*b**3*d*e**2 + 54*a**2*b**2*c*d**2*e - 13
2*a**2*b*c**2*d**3 - 3*a*b**4*d**2*e + 26*a*b**3*c*d**3 - 2*b**5*d**3 + x**
5*(36*a*b*c**3*e**3 - 72*a*c**4*d*e**2 + 6*b**3*c**2*e**3 - 72*b**2*c**3*d*
e**2 + 180*b*c**4*d**2*e - 120*c**5*d**3) + x**4*(90*a*b**2*c**2*e**3 - 180
*a*b*c**3*d*e**2 + 15*b**4*c*e**3 - 180*b**3*c**2*d*e**2 + 450*b**2*c**3*d*
*2*e - 300*b*c**4*d**3) + x**3*(96*a**2*b*c**2*e**3 - 192*a**2*c**3*d*e**2
+ 82*a*b**3*c*e**3 - 324*a*b**2*c**2*d*e**2 + 480*a*b*c**3*d**2*e - 320*a*c
**4*d**3 + 11*b**5*e**3 - 132*b**4*c*d*e**2 + 330*b**3*c**2*d**2*e - 220*b*
*2*c**3*d**3) + x**2*(96*a**3*c**2*e**3 + 72*a**2*b**2*c*e**3 - 288*a**2*b*
c**2*d*e**2 + 51*a*b**4*e**3 - 306*a*b**3*c*d*e**2 + 720*a*b**2*c**2*d**2*e
 - 480*a*b*c**3*d**3 - 18*b**5*d*e**2 + 45*b**4*c*d**2*e - 30*b**3*c**2*d**
3) + x*(60*a**3*b*c*e**3 + 72*a**3*c**2*d*e**2 + 60*a**2*b**3*e**3 - 396*a*
*2*b**2*c*d*e**2 + 396*a**2*b*c**2*d**2*e - 264*a**2*c**3*d**3 - 18*a*b**4*
d*e**2 + 162*a*b**3*c*d**2*e - 108*a*b**2*c**2*d**3 - 9*b**5*d**2*e + 6*b**
4*c*d**3))/(384*a**6*c**3 - 288*a**5*b**2*c**2 + 72*a**4*b**4*c - 6*a**3*b*
*6 + x**6*(384*a**3*c**6 - 288*a**2*b**2*c**5 + 72*a*b**4*c**4 - 6*b**6*c**
3) + x**5*(1152*a**3*b*c**5 - 864*a**2*b**3*c**4 + 216*a*b**5*c**3 - 18*b**
7*c**2) + x**4*(1152*a**4*c**5 + 288*a**3*b**2*c**4 - 648*a**2*b**4*c**3 +
198*a*b**6*c**2 - 18*b**8*c) + x**3*(2304*a**4*b*c**4 - 1344*a**3*b**3*c**3
 + 144*a**2*b**5*c**2 + 36*a*b**7*c - 6*b**9) + x**2*(1152*a**5*c**4 + 288*
a**4*b**2*c**3 - 648*a**3*b**4*c**2 + 198*a**2*b**6*c - 18*a*b**8) + x*(115
2*a**5*b*c**3 - 864*a**4*b**3*c**2 + 216*a**3*b**5*c - 18*a**2*b**7))
```

Giac [B] time = 1.13938, size = 1118, normalized size = 3.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^4,x, algorithm="giac")
```

```
[Out] -2*(20*c^3*d^3 - 30*b*c^2*d^2*e + 12*b^2*c*d*e^2 + 12*a*c^2*d*e^2 - b^3*e^3
 - 6*a*b*c*e^3)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^6 - 12*a*b^4*c +
 48*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-b^2 + 4*a*c)) - 1/6*(120*c^5*d^3*x^5 -
180*b*c^4*d^2*x^5*e + 300*b*c^4*d^3*x^4 + 72*b^2*c^3*d*x^5*e^2 + 72*a*c^4*d
*x^5*e^2 - 450*b^2*c^3*d^2*x^4*e + 220*b^2*c^3*d^3*x^3 + 320*a*c^4*d^3*x^3
- 6*b^3*c^2*x^5*e^3 - 36*a*b*c^3*x^5*e^3 + 180*b^3*c^2*d*x^4*e^2 + 180*a*b*
c^3*d*x^4*e^2 - 330*b^3*c^2*d^2*x^3*e - 480*a*b*c^3*d^2*x^3*e + 30*b^3*c^2*
d^3*x^2 + 480*a*b*c^3*d^3*x^2 - 15*b^4*c*x^4*e^3 - 90*a*b^2*c^2*x^4*e^3 + 1
32*b^4*c*d*x^3*e^2 + 324*a*b^2*c^2*d*x^3*e^2 + 192*a^2*c^3*d*x^3*e^2 - 45*b
^4*c*d^2*x^2*e - 720*a*b^2*c^2*d^2*x^2*e - 6*b^4*c*d^3*x + 108*a*b^2*c^2*d^
3*x + 264*a^2*c^3*d^3*x - 11*b^5*x^3*e^3 - 82*a*b^3*c*x^3*e^3 - 96*a^2*b*c^
2*x^3*e^3 + 18*b^5*d*x^2*e^2 + 306*a*b^3*c*d*x^2*e^2 + 288*a^2*b*c^2*d*x^2*
e^2 + 9*b^5*d^2*x*e - 162*a*b^3*c*d^2*x*e - 396*a^2*b*c^2*d^2*x*e + 2*b^5*d
^3 - 26*a*b^3*c*d^3 + 132*a^2*b*c^2*d^3 - 51*a*b^4*x^2*e^3 - 72*a^2*b^2*c*x
^2*e^3 - 96*a^3*c^2*x^2*e^3 + 18*a*b^4*d*x*e^2 + 396*a^2*b^2*c*d*x*e^2 - 72
*a^3*c^2*d*x*e^2 + 3*a*b^4*d^2*e - 54*a^2*b^2*c*d^2*e - 192*a^3*c^2*d^2*e -
 60*a^2*b^3*x*e^3 - 60*a^3*b*c*x*e^3 + 6*a^2*b^3*d*e^2 + 156*a^3*b*c*d*e^2
- 22*a^3*b^2*e^3 - 32*a^4*c*e^3)/((b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a
^3*c^3)*(c*x^2 + b*x + a)^3)
```

3.2216 $\int \frac{(d+ex)^2}{(a+bx+cx^2)^4} dx$

Optimal. Leaf size=260

$$-\frac{2(b+2cx)\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)}-\frac{-2x\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)-5b\left(ae^2+cd^2\right)+8acde+3b^2de}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2}+$$

```
[Out] -((d + e*x)*(b*d - 2*a*e + (2*c*d - b*e)*x))/(3*(b^2 - 4*a*c)*(a + b*x + c*
x^2)^3) - (3*b^2*d*e + 8*a*c*d*e - 5*b*(c*d^2 + a*e^2) - 2*(5*c^2*d^2 + b^2
*e^2 - c*e*(5*b*d - a*e))*x)/(3*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^2) - (2*(
5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(b + 2*c*x))/((b^2 - 4*a*c)^3*(a +
 b*x + c*x^2)) + (8*c*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*ArcTanh[(b
+ 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(7/2)
```

Rubi [A] time = 0.261681, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {738, 638, 614, 618, 206}

$$-\frac{2(b+2cx)\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)}-\frac{-2x\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)-5b\left(ae^2+cd^2\right)+8acde+3b^2de}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*x + c*x^2)^4,x]
```


```
[Out] -((d + e*x)*(b*d - 2*a*e + (2*c*d - b*e)*x))/(3*(b^2 - 4*a*c)*(a + b*x + c*
x^2)^3) - (3*b^2*d*e + 8*a*c*d*e - 5*b*(c*d^2 + a*e^2) - 2*(5*c^2*d^2 + b^2
*e^2 - c*e*(5*b*d - a*e))*x)/(3*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^2) - (2*(
5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(b + 2*c*x))/((b^2 - 4*a*c)^3*(a +
 b*x + c*x^2)) + (8*c*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*ArcTanh[(b
+ 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(7/2)
```


Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(a+bx+cx^2\right)^4}\,dx = -\frac{(d+ex)(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} - \frac{\int \frac{2\left(5cd^2-e(3bd-ae)\right)+4e(2cd-be)x}{\left(a+bx+cx^2\right)^3}\,dx}{3\left(b^2-4ac\right)}$$

$$= -\frac{(d+ex)(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} - \frac{3b^2de+8acde-5b\left(cd^2+ae^2\right)-2\left(5c^2d^2+b^2e^2-ce(5bd-\right.}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2}$$

$$= -\frac{(d+ex)(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} - \frac{3b^2de+8acde-5b\left(cd^2+ae^2\right)-2\left(5c^2d^2+b^2e^2-ce(5bd-\right.}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2}$$

$$= -\frac{(d+ex)(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} - \frac{3b^2de+8acde-5b\left(cd^2+ae^2\right)-2\left(5c^2d^2+b^2e^2-ce(5bd-\right.}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2}$$

$$= -\frac{(d+ex)(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} - \frac{3b^2de+8acde-5b\left(cd^2+ae^2\right)-2\left(5c^2d^2+b^2e^2-ce(5bd-\right.}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2}$$

Mathematica [A] time = 0.562601, size = 258, normalized size = 0.99

$$\frac{1}{3}\left(-\frac{6(b+2cx)\left(ce(ae-5bd)+b^2e^2+5c^2d^2\right)}{\left(b^2-4ac\right)^3(a+x(b+cx))} + \frac{(b+2cx)\left(ce(ae-5bd)+b^2e^2+5c^2d^2\right)}{c\left(b^2-4ac\right)^2(a+x(b+cx))^2} + \frac{abe^2-2ace(2d+ex)+b^2e^2x+}{c\left(4ac-b^2\right)(a+x}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*x + c*x^2)^4,x]
```

```
[Out] (((5*c^2*d^2 + b^2*e^2 + c*e*(-5*b*d + a*e))*(b + 2*c*x))/(c*(b^2 - 4*a*c)^
2*(a + x*(b + c*x))^2) - (6*(5*c^2*d^2 + b^2*e^2 + c*e*(-5*b*d + a*e))*(b +
 2*c*x))/((b^2 - 4*a*c)^3*(a + x*(b + c*x))) + (a*b*e^2 + 2*c^2*d^2*x + b^2
*e^2*x + b*c*d*(d - 2*e*x) - 2*a*c*e*(2*d + e*x))/(c*(-b^2 + 4*a*c)*(a + x*
(b + c*x))^3) + (24*c*(5*c^2*d^2 + b^2*e^2 + c*e*(-5*b*d + a*e))*ArcTan[(b
+ 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(7/2))/3
```

Maple [B] time = 0.161, size = 850, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^2+b*x+a)^4,x)
```

```
[Out] (4*c^3*(a*c*e^2+b^2*e^2-5*b*c*d*e+5*c^2*d^2)/(64*a^3*c^3-48*a^2*b^2*c^2+12*
a*b^4*c-b^6)*x^5+10*c^2*(a*c*e^2+b^2*e^2-5*b*c*d*e+5*c^2*d^2)/(64*a^3*c^3-4
8*a^2*b^2*c^2+12*a*b^4*c-b^6)*b*x^4+2/3*(16*a*c+11*b^2)*c*(a*c*e^2+b^2*e^2-
5*b*c*d*e+5*c^2*d^2)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^3+b*(16*a
*c+b^2)*(a*c*e^2+b^2*e^2-5*b*c*d*e+5*c^2*d^2)/(64*a^3*c^3-48*a^2*b^2*c^2+12
*a*b^4*c-b^6)*x^2-(4*a^3*c^2*e^2-22*a^2*b^2*c*e^2+44*a^2*b*c^2*d*e-44*a^2*c
^3*d^2-a*b^4*e^2+18*a*b^3*c*d*e-18*a*b^2*c^2*d^2-b^5*d*e+b^4*c*d^2)/(64*a^3
*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x+1/3*(26*a^3*b*c*e^2-64*a^3*c^2*d*e+a^
2*b^3*e^2-18*a^2*b^2*c*d*e+66*a^2*b*c^2*d^2+a*b^4*d*e-13*a*b^3*c*d^2+b^5*d^
2)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6))/(c*x^2+b*x+a)^3+8*c^2/(64*a^
3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*
a*c-b^2)^(1/2))*a*e^2+8*c/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(4*a*c
-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*e^2-40*c^2/(64*a^3*c^3-
48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^
2)^(1/2))*b*d*e+40*c^3/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(4*a*c-b^
2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.44419, size = 5802, normalized size = 22.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/3*(12*(5*(b^2*c^5 - 4*a*c^6)*d^2 - 5*(b^3*c^4 - 4*a*b*c^5)*d*e + (b^4*c
^3 - 3*a*b^2*c^4 - 4*a^2*c^5)*e^2)*x^5 + 30*(5*(b^3*c^4 - 4*a*b*c^5)*d^2 -
5*(b^4*c^3 - 4*a*b^2*c^4)*d*e + (b^5*c^2 - 3*a*b^3*c^3 - 4*a^2*b*c^4)*e^2)*
x^4 + 2*(5*(11*b^4*c^3 - 28*a*b^2*c^4 - 64*a^2*c^5)*d^2 - 5*(11*b^5*c^2 - 2
8*a*b^3*c^3 - 64*a^2*b*c^4)*d*e + (11*b^6*c - 17*a*b^4*c^2 - 92*a^2*b^2*c^3
 - 64*a^3*c^4)*e^2)*x^3 + (b^7 - 17*a*b^5*c + 118*a^2*b^3*c^2 - 264*a^3*b*c
^3)*d^2 + (a*b^6 - 22*a^2*b^4*c + 8*a^3*b^2*c^2 + 256*a^4*c^3)*d*e + (a^2*b
^5 + 22*a^3*b^3*c - 104*a^4*b*c^2)*e^2 + 3*(5*(b^5*c^2 + 12*a*b^3*c^3 - 64*
a^2*b*c^4)*d^2 - 5*(b^6*c + 12*a*b^4*c^2 - 64*a^2*b^2*c^3)*d*e + (b^7 + 13*
```

```
a*b^5*c - 52*a^2*b^3*c^2 - 64*a^3*b*c^3)*e^2)*x^2 + 12*(5*a^3*c^3*d^2 - 5*a
^3*b*c^2*d*e + (5*c^6*d^2 - 5*b*c^5*d*e + (b^2*c^4 + a*c^5)*e^2)*x^6 + 3*(5
*b*c^5*d^2 - 5*b^2*c^4*d*e + (b^3*c^3 + a*b*c^4)*e^2)*x^5 + 3*(5*(b^2*c^4 +
 a*c^5)*d^2 - 5*(b^3*c^3 + a*b*c^4)*d*e + (b^4*c^2 + 2*a*b^2*c^3 + a^2*c^4)
*e^2)*x^4 + (5*(b^3*c^3 + 6*a*b*c^4)*d^2 - 5*(b^4*c^2 + 6*a*b^2*c^3)*d*e +
(b^5*c + 7*a*b^3*c^2 + 6*a^2*b*c^3)*e^2)*x^3 + (a^3*b^2*c + a^4*c^2)*e^2 +
3*(5*(a*b^2*c^3 + a^2*c^4)*d^2 - 5*(a*b^3*c^2 + a^2*b*c^3)*d*e + (a*b^4*c +
 2*a^2*b^2*c^2 + a^3*c^3)*e^2)*x^2 + 3*(5*a^2*b*c^3*d^2 - 5*a^2*b^2*c^2*d*e
 + (a^2*b^3*c + a^3*b*c^2)*e^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c
*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 3*((
b^6*c - 22*a*b^4*c^2 + 28*a^2*b^2*c^3 + 176*a^3*c^4)*d^2 - (b^7 - 22*a*b^5*
c + 28*a^2*b^3*c^2 + 176*a^3*b*c^3)*d*e - (a*b^6 + 18*a^2*b^4*c - 92*a^3*b^
2*c^2 + 16*a^4*c^3)*e^2)*x)/(a^3*b^8 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*
a^6*b^2*c^3 + 256*a^7*c^4 + (b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*
a^3*b^2*c^6 + 256*a^4*c^7)*x^6 + 3*(b^9*c^2 - 16*a*b^7*c^3 + 96*a^2*b^5*c^4
 - 256*a^3*b^3*c^5 + 256*a^4*b*c^6)*x^5 + 3*(b^10*c - 15*a*b^8*c^2 + 80*a^2
*b^6*c^3 - 160*a^3*b^4*c^4 + 256*a^5*c^6)*x^4 + (b^11 - 10*a*b^9*c + 320*a^
3*b^5*c^3 - 1280*a^4*b^3*c^4 + 1536*a^5*b*c^5)*x^3 + 3*(a*b^10 - 15*a^2*b^8
*c + 80*a^3*b^6*c^2 - 160*a^4*b^4*c^3 + 256*a^6*c^5)*x^2 + 3*(a^2*b^9 - 16*
a^3*b^7*c + 96*a^4*b^5*c^2 - 256*a^5*b^3*c^3 + 256*a^6*b*c^4)*x), -1/3*(12*
(5*(b^2*c^5 - 4*a*c^6)*d^2 - 5*(b^3*c^4 - 4*a*b*c^5)*d*e + (b^4*c^3 - 3*a*b
^2*c^4 - 4*a^2*c^5)*e^2)*x^5 + 30*(5*(b^3*c^4 - 4*a*b*c^5)*d^2 - 5*(b^4*c^3
 - 4*a*b^2*c^4)*d*e + (b^5*c^2 - 3*a*b^3*c^3 - 4*a^2*b*c^4)*e^2)*x^4 + 2*(5
*(11*b^4*c^3 - 28*a*b^2*c^4 - 64*a^2*c^5)*d^2 - 5*(11*b^5*c^2 - 28*a*b^3*c^
3 - 64*a^2*b*c^4)*d*e + (11*b^6*c - 17*a*b^4*c^2 - 92*a^2*b^2*c^3 - 64*a^3*
c^4)*e^2)*x^3 + (b^7 - 17*a*b^5*c + 118*a^2*b^3*c^2 - 264*a^3*b*c^3)*d^2 +
(a*b^6 - 22*a^2*b^4*c + 8*a^3*b^2*c^2 + 256*a^4*c^3)*d*e + (a^2*b^5 + 22*a^
3*b^3*c - 104*a^4*b*c^2)*e^2 + 3*(5*(b^5*c^2 + 12*a*b^3*c^3 - 64*a^2*b*c^4)
*d^2 - 5*(b^6*c + 12*a*b^4*c^2 - 64*a^2*b^2*c^3)*d*e + (b^7 + 13*a*b^5*c -
52*a^2*b^3*c^2 - 64*a^3*b*c^3)*e^2)*x^2 - 24*(5*a^3*c^3*d^2 - 5*a^3*b*c^2*d
*e + (5*c^6*d^2 - 5*b*c^5*d*e + (b^2*c^4 + a*c^5)*e^2)*x^6 + 3*(5*b*c^5*d^2
 - 5*b^2*c^4*d*e + (b^3*c^3 + a*b*c^4)*e^2)*x^5 + 3*(5*(b^2*c^4 + a*c^5)*d^
2 - 5*(b^3*c^3 + a*b*c^4)*d*e + (b^4*c^2 + 2*a*b^2*c^3 + a^2*c^4)*e^2)*x^4
+ (5*(b^3*c^3 + 6*a*b*c^4)*d^2 - 5*(b^4*c^2 + 6*a*b^2*c^3)*d*e + (b^5*c + 7
*a*b^3*c^2 + 6*a^2*b*c^3)*e^2)*x^3 + (a^3*b^2*c + a^4*c^2)*e^2 + 3*(5*(a*b^
2*c^3 + a^2*c^4)*d^2 - 5*(a*b^3*c^2 + a^2*b*c^3)*d*e + (a*b^4*c + 2*a^2*b^2
*c^2 + a^3*c^3)*e^2)*x^2 + 3*(5*a^2*b*c^3*d^2 - 5*a^2*b^2*c^2*d*e + (a^2*b^
3*c + a^3*b*c^2)*e^2)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c
*x + b)/(b^2 - 4*a*c)) - 3*((b^6*c - 22*a*b^4*c^2 + 28*a^2*b^2*c^3 + 176*a^
3*c^4)*d^2 - (b^7 - 22*a*b^5*c + 28*a^2*b^3*c^2 + 176*a^3*b*c^3)*d*e - (a*b
^6 + 18*a^2*b^4*c - 92*a^3*b^2*c^2 + 16*a^4*c^3)*e^2)*x)/(a^3*b^8 - 16*a^4*
b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3 + 256*a^7*c^4 + (b^8*c^3 - 16*a*b^
6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6 + 256*a^4*c^7)*x^6 + 3*(b^9*c^2 -
16*a*b^7*c^3 + 96*a^2*b^5*c^4 - 256*a^3*b^3*c^5 + 256*a^4*b*c^6)*x^5 + 3*(b
^10*c - 15*a*b^8*c^2 + 80*a^2*b^6*c^3 - 160*a^3*b^4*c^4 + 256*a^5*c^6)*x^4
+ (b^11 - 10*a*b^9*c + 320*a^3*b^5*c^3 - 1280*a^4*b^3*c^4 + 1536*a^5*b*c^5)
*x^3 + 3*(a*b^10 - 15*a^2*b^8*c + 80*a^3*b^6*c^2 - 160*a^4*b^4*c^3 + 256*a^
6*c^5)*x^2 + 3*(a^2*b^9 - 16*a^3*b^7*c + 96*a^4*b^5*c^2 - 256*a^5*b^3*c^3 +
 256*a^6*b*c^4)*x)]
```

Sympy [B] time = 7.96913, size = 1635, normalized size = 6.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**2+b*x+a)**4,x)
```

```
[Out] -4*c*sqrt(-1/(4*a*c - b**2)**7)*(a*c*e**2 + b**2*e**2 - 5*b*c*d*e + 5*c**2*
d**2)*log(x + (-1024*a**4*c**5*sqrt(-1/(4*a*c - b**2)**7)*(a*c*e**2 + b**2*
e**2 - 5*b*c*d*e + 5*c**2*d**2) + 1024*a**3*b**2*c**4*sqrt(-1/(4*a*c - b**2
)**7)*(a*c*e**2 + b**2*e**2 - 5*b*c*d*e + 5*c**2*d**2) - 384*a**2*b**4*c**3
*sqrt(-1/(4*a*c - b**2)**7)*(a*c*e**2 + b**2*e**2 - 5*b*c*d*e + 5*c**2*d**2
) + 64*a*b**6*c**2*sqrt(-1/(4*a*c - b**2)**7)*(a*c*e**2 + b**2*e**2 - 5*b*c
*d*e + 5*c**2*d**2) + 4*a*b*c**2*e**2 - 4*b**8*c*sqrt(-1/(4*a*c - b**2)**7)
*(a*c*e**2 + b**2*e**2 - 5*b*c*d*e + 5*c**2*d**2) + 4*b**3*c*e**2 - 20*b**2
*c**2*d*e + 20*b*c**3*d**2)/(8*a*c**3*e**2 + 8*b**2*c**2*e**2 - 40*b*c**3*d
*e + 40*c**4*d**2)) + 4*c*sqrt(-1/(4*a*c - b**2)**7)*(a*c*e**2 + b**2*e**2
- 5*b*c*d*e + 5*c**2*d**2)*log(x + (1024*a**4*c**5*sqrt(-1/(4*a*c - b**2)**
7)*(a*c*e**2 + b**2*e**2 - 5*b*c*d*e + 5*c**2*d**2) - 1024*a**3*b**2*c**4*s
qrt(-1/(4*a*c - b**2)**7)*(a*c*e**2 + b**2*e**2 - 5*b*c*d*e + 5*c**2*d**2)
+ 384*a**2*b**4*c**3*sqrt(-1/(4*a*c - b**2)**7)*(a*c*e**2 + b**2*e**2 - 5*b
*c*d*e + 5*c**2*d**2) - 64*a*b**6*c**2*sqrt(-1/(4*a*c - b**2)**7)*(a*c*e**2
 + b**2*e**2 - 5*b*c*d*e + 5*c**2*d**2) + 4*a*b*c**2*e**2 + 4*b**8*c*sqrt(-
1/(4*a*c - b**2)**7)*(a*c*e**2 + b**2*e**2 - 5*b*c*d*e + 5*c**2*d**2) + 4*b
**3*c*e**2 - 20*b**2*c**2*d*e + 20*b*c**3*d**2)/(8*a*c**3*e**2 + 8*b**2*c**
2*e**2 - 40*b*c**3*d*e + 40*c**4*d**2)) + (26*a**3*b*c*e**2 - 64*a**3*c**2*
d*e + a**2*b**3*e**2 - 18*a**2*b**2*c*d*e + 66*a**2*b*c**2*d**2 + a*b**4*d*
e - 13*a*b**3*c*d**2 + b**5*d**2 + x**5*(12*a*c**4*e**2 + 12*b**2*c**3*e**2
 - 60*b*c**4*d*e + 60*c**5*d**2) + x**4*(30*a*b*c**3*e**2 + 30*b**3*c**2*e*
*2 - 150*b**2*c**3*d*e + 150*b*c**4*d**2) + x**3*(32*a**2*c**3*e**2 + 54*a*
b**2*c**2*e**2 - 160*a*b*c**3*d*e + 160*a*c**4*d**2 + 22*b**4*c*e**2 - 110*
b**3*c**2*d*e + 110*b**2*c**3*d**2) + x**2*(48*a**2*b*c**2*e**2 + 51*a*b**3
*c*e**2 - 240*a*b**2*c**2*d*e + 240*a*b*c**3*d**2 + 3*b**5*e**2 - 15*b**4*c
*d*e + 15*b**3*c**2*d**2) + x*(-12*a**3*c**2*e**2 + 66*a**2*b**2*c*e**2 - 1
32*a**2*b*c**2*d*e + 132*a**2*c**3*d**2 + 3*a*b**4*e**2 - 54*a*b**3*c*d*e +
 54*a*b**2*c**2*d**2 + 3*b**5*d*e - 3*b**4*c*d**2))/(192*a**6*c**3 - 144*a*
*5*b**2*c**2 + 36*a**4*b**4*c - 3*a**3*b**6 + x**6*(192*a**3*c**6 - 144*a**
2*b**2*c**5 + 36*a*b**4*c**4 - 3*b**6*c**3) + x**5*(576*a**3*b*c**5 - 432*a
**2*b**3*c**4 + 108*a*b**5*c**3 - 9*b**7*c**2) + x**4*(576*a**4*c**5 + 144*
a**3*b**2*c**4 - 324*a**2*b**4*c**3 + 99*a*b**6*c**2 - 9*b**8*c) + x**3*(11
52*a**4*b*c**4 - 672*a**3*b**3*c**3 + 72*a**2*b**5*c**2 + 18*a*b**7*c - 3*b
**9) + x**2*(576*a**5*c**4 + 144*a**4*b**2*c**3 - 324*a**3*b**4*c**2 + 99*a
**2*b**6*c - 9*a*b**8) + x*(576*a**5*b*c**3 - 432*a**4*b**3*c**2 + 108*a**3
*b**5*c - 9*a**2*b**7))
```

Giac [B] time = 1.12732, size = 814, normalized size = 3.13

$$-\frac{8\left(5c^3d^2-5bc^2de+b^2ce^2+ac^2e^2\right)\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\left(b^6-12ab^4c+48a^2b^2c^2-64a^3c^3\right)\sqrt{-b^2+4ac}}-\frac{60c^5d^2x^5-60bc^4dx^5e+150bc^4d^2x^4+12b^2c^3x^5e^2+12ac}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^4,x, algorithm="giac")
```

```
[Out] -8*(5*c^3*d^2 - 5*b*c^2*d*e + b^2*c*e^2 + a*c^2*e^2)*arctan((2*c*x + b)/sqr
t(-b^2 + 4*a*c))/((b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-b^
2 + 4*a*c)) - 1/3*(60*c^5*d^2*x^5 - 60*b*c^4*d*x^5*e + 150*b*c^4*d^2*x^4 +
12*b^2*c^3*x^5*e^2 + 12*a*c^4*x^5*e^2 - 150*b^2*c^3*d*x^4*e + 110*b^2*c^3*d
^2*x^3 + 160*a*c^4*d^2*x^3 + 30*b^3*c^2*x^4*e^2 + 30*a*b*c^3*x^4*e^2 - 110*
b^3*c^2*d*x^3*e - 160*a*b*c^3*d*x^3*e + 15*b^3*c^2*d^2*x^2 + 240*a*b*c^3*d^
2*x^2 + 22*b^4*c*x^3*e^2 + 54*a*b^2*c^2*x^3*e^2 + 32*a^2*c^3*x^3*e^2 - 15*b
^4*c*d*x^2*e - 240*a*b^2*c^2*d*x^2*e - 3*b^4*c*d^2*x + 54*a*b^2*c^2*d^2*x +
 132*a^2*c^3*d^2*x + 3*b^5*x^2*e^2 + 51*a*b^3*c*x^2*e^2 + 48*a^2*b*c^2*x^2*
```

```
e^2 + 3*b^5*d*x*e - 54*a*b^3*c*d*x*e - 132*a^2*b*c^2*d*x*e + b^5*d^2 - 13*a
*b^3*c*d^2 + 66*a^2*b*c^2*d^2 + 3*a*b^4*x*e^2 + 66*a^2*b^2*c*x*e^2 - 12*a^3
*c^2*x*e^2 + a*b^4*d*e - 18*a^2*b^2*c*d*e - 64*a^3*c^2*d*e + a^2*b^3*e^2 +
26*a^3*b*c*e^2)/((b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*(c*x^2 +
b*x + a)^3)
```

3.2217 $\int \frac{d+ex}{(a+bx+cx^2)^4} dx$

Optimal. Leaf size=173

$$\frac{20c^2(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{7/2}} - \frac{5c(b+2cx)(2cd-be)}{(b^2-4ac)^3(a+bx+cx^2)} + \frac{5(b+2cx)(2cd-be)}{6(b^2-4ac)^2(a+bx+cx^2)^2} - \frac{-2ae+x(2cd-be)}{3(b^2-4ac)(a+bx+cx^2)^3}$$

```
[Out] -(b*d - 2*a*e + (2*c*d - b*e)*x)/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^3) + (5
*(2*c*d - b*e)*(b + 2*c*x))/(6*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^2) - (5*c*
(2*c*d - b*e)*(b + 2*c*x))/((b^2 - 4*a*c)^3*(a + b*x + c*x^2)) + (20*c^2*(2
*c*d - b*e)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(7/2)
```

Rubi [A] time = 0.0738182, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {638, 614, 618, 206}

$$\frac{20c^2(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{7/2}} - \frac{5c(b+2cx)(2cd-be)}{(b^2-4ac)^3(a+bx+cx^2)} + \frac{5(b+2cx)(2cd-be)}{6(b^2-4ac)^2(a+bx+cx^2)^2} - \frac{-2ae+x(2cd-be)}{3(b^2-4ac)(a+bx+cx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*x + c*x^2)^4,x]
```


```
[Out] -(b*d - 2*a*e + (2*c*d - b*e)*x)/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^3) + (5
*(2*c*d - b*e)*(b + 2*c*x))/(6*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^2) - (5*c*
(2*c*d - b*e)*(b + 2*c*x))/((b^2 - 4*a*c)^3*(a + b*x + c*x^2)) + (20*c^2*(2
*c*d - b*e)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(7/2)
```


Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex}{\left(a+bx+cx^2\right)^4}\,dx = -\frac{bd-2ae+(2cd-be)x}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} - \frac{(5(2cd-be))\int \frac{1}{\left(a+bx+cx^2\right)^3}\,dx}{3\left(b^2-4ac\right)}$$

$$= -\frac{bd-2ae+(2cd-be)x}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{5(2cd-be)(b+2cx)}{6\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} + \frac{(5c(2cd-be))\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx}{\left(b^2-4ac\right)^2}$$

$$= -\frac{bd-2ae+(2cd-be)x}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{5(2cd-be)(b+2cx)}{6\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} - \frac{5c(2cd-be)(b+2cx)}{\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)} - (1$$

$$= -\frac{bd-2ae+(2cd-be)x}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{5(2cd-be)(b+2cx)}{6\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} - \frac{5c(2cd-be)(b+2cx)}{\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)} + (2$$

$$= -\frac{bd-2ae+(2cd-be)x}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{5(2cd-be)(b+2cx)}{6\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} - \frac{5c(2cd-be)(b+2cx)}{\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)} + 20$$

Mathematica [A] time = 0.21643, size = 168, normalized size = 0.97

$$\frac{\frac{120c^2(be-2cd)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - \frac{5\left(b^2-4ac\right)(b+2cx)(be-2cd)}{(a+x(b+cx))^2} + \frac{2\left(b^2-4ac\right)^2(2ae-bd+bex-2cdx)}{(a+x(b+cx))^3} + \frac{30c(b+2cx)(be-2cd)}{a+x(b+cx)}}{6\left(b^2-4ac\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*x + c*x^2)^4,x]
```

```
[Out] ((2*(b^2 - 4*a*c)^2*(-(b*d) + 2*a*e - 2*c*d*x + b*e*x))/(a + x*(b + c*x))^3
 - (5*(b^2 - 4*a*c)*(-2*c*d + b*e)*(b + 2*c*x))/(a + x*(b + c*x))^2 + (30*c
*(-2*c*d + b*e)*(b + 2*c*x))/(a + x*(b + c*x)) + (120*c^2*(-2*c*d + b*e)*Ar
cTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(6*(b^2 - 4*a*c)^
3)
```

Maple [B] time = 0.153, size = 369, normalized size = 2.1

$$\frac{bd-2\,ae+(-be+2\,cd)\,x}{\left(12\,ac-3\,b^2\right)\left(cx^2+bx+a\right)^3} - \frac{5\,bcxe}{3\left(4\,ac-b^2\right)^2\left(cx^2+bx+a\right)^2} + \frac{10\,c^2xd}{3\left(4\,ac-b^2\right)^2\left(cx^2+bx+a\right)^2} - \frac{5\,b^2e}{6\left(4\,ac-b^2\right)^2\left(cx^2+b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^2+b*x+a)^4,x)
```

```
[Out] 1/3*(b*d-2*a*e+(-b*e+2*c*d)*x)/(4*a*c-b^2)/(c*x^2+b*x+a)^3-5/3/(4*a*c-b^2)^
2/(c*x^2+b*x+a)^2*x*c*b*e+10/3/(4*a*c-b^2)^2/(c*x^2+b*x+a)^2*x*c^2*d-5/6/(4
*a*c-b^2)^2/(c*x^2+b*x+a)^2*b^2*e+5/3/(4*a*c-b^2)^2/(c*x^2+b*x+a)^2*b*c*d-1
0/(4*a*c-b^2)^3*c^2/(c*x^2+b*x+a)*x*b*e+20/(4*a*c-b^2)^3*c^3/(c*x^2+b*x+a)*
x*d-5/(4*a*c-b^2)^3*c/(c*x^2+b*x+a)*b^2*e+10/(4*a*c-b^2)^3*c^2/(c*x^2+b*x+a
```

```
)*b*d-20/(4*a*c-b^2)^(7/2)*c^2*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*e+40/(
4*a*c-b^2)^(7/2)*c^3*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^4,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 2.34774, size = 4157, normalized size = 24.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^4,x, algorithm="fricas")

[Out] [-1/6*(60*(2*(b^2*c^5 - 4*a*c^6)*d - (b^3*c^4 - 4*a*b*c^5)*e)*x^5 + 150*(2*
(b^3*c^4 - 4*a*b*c^5)*d - (b^4*c^3 - 4*a*b^2*c^4)*e)*x^4 + 10*(2*(11*b^4*c^
3 - 28*a*b^2*c^4 - 64*a^2*c^5)*d - (11*b^5*c^2 - 28*a*b^3*c^3 - 64*a^2*b*c^
4)*e)*x^3 + 15*(2*(b^5*c^2 + 12*a*b^3*c^3 - 64*a^2*b*c^4)*d - (b^6*c + 12*a
*b^4*c^2 - 64*a^2*b^2*c^3)*e)*x^2 - 60*(2*a^3*c^3*d - a^3*b*c^2*e + (2*c^6*
d - b*c^5*e)*x^6 + 3*(2*b*c^5*d - b^2*c^4*e)*x^5 + 3*(2*(b^2*c^4 + a*c^5)*d
 - (b^3*c^3 + a*b*c^4)*e)*x^4 + (2*(b^3*c^3 + 6*a*b*c^4)*d - (b^4*c^2 + 6*a
*b^2*c^3)*e)*x^3 + 3*(2*(a*b^2*c^3 + a^2*c^4)*d - (a*b^3*c^2 + a^2*b*c^3)*e
)*x^2 + 3*(2*a^2*b*c^3*d - a^2*b^2*c^2*e)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x
^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x +
a)) + 2*(b^7 - 17*a*b^5*c + 118*a^2*b^3*c^2 - 264*a^3*b*c^3)*d + (a*b^6 - 2
2*a^2*b^4*c + 8*a^3*b^2*c^2 + 256*a^4*c^3)*e - 3*(2*(b^6*c - 22*a*b^4*c^2 +
 28*a^2*b^2*c^3 + 176*a^3*c^4)*d - (b^7 - 22*a*b^5*c + 28*a^2*b^3*c^2 + 176
*a^3*b*c^3)*e)*x)/(a^3*b^8 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^
3 + 256*a^7*c^4 + (b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^
6 + 256*a^4*c^7)*x^6 + 3*(b^9*c^2 - 16*a*b^7*c^3 + 96*a^2*b^5*c^4 - 256*a^3
*b^3*c^5 + 256*a^4*b*c^6)*x^5 + 3*(b^10*c - 15*a*b^8*c^2 + 80*a^2*b^6*c^3 -
 160*a^3*b^4*c^4 + 256*a^5*c^6)*x^4 + (b^11 - 10*a*b^9*c + 320*a^3*b^5*c^3
- 1280*a^4*b^3*c^4 + 1536*a^5*b*c^5)*x^3 + 3*(a*b^10 - 15*a^2*b^8*c + 80*a^
3*b^6*c^2 - 160*a^4*b^4*c^3 + 256*a^6*c^5)*x^2 + 3*(a^2*b^9 - 16*a^3*b^7*c
+ 96*a^4*b^5*c^2 - 256*a^5*b^3*c^3 + 256*a^6*b*c^4)*x), -1/6*(60*(2*(b^2*c^
5 - 4*a*c^6)*d - (b^3*c^4 - 4*a*b*c^5)*e)*x^5 + 150*(2*(b^3*c^4 - 4*a*b*c^5
)*d - (b^4*c^3 - 4*a*b^2*c^4)*e)*x^4 + 10*(2*(11*b^4*c^3 - 28*a*b^2*c^4 - 6
4*a^2*c^5)*d - (11*b^5*c^2 - 28*a*b^3*c^3 - 64*a^2*b*c^4)*e)*x^3 + 15*(2*(b
^5*c^2 + 12*a*b^3*c^3 - 64*a^2*b*c^4)*d - (b^6*c + 12*a*b^4*c^2 - 64*a^2*b^
2*c^3)*e)*x^2 - 120*(2*a^3*c^3*d - a^3*b*c^2*e + (2*c^6*d - b*c^5*e)*x^6 +
3*(2*b*c^5*d - b^2*c^4*e)*x^5 + 3*(2*(b^2*c^4 + a*c^5)*d - (b^3*c^3 + a*b*c
^4)*e)*x^4 + (2*(b^3*c^3 + 6*a*b*c^4)*d - (b^4*c^2 + 6*a*b^2*c^3)*e)*x^3 +
3*(2*(a*b^2*c^3 + a^2*c^4)*d - (a*b^3*c^2 + a^2*b*c^3)*e)*x^2 + 3*(2*a^2*b*
c^3*d - a^2*b^2*c^2*e)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*
c*x + b)/(b^2 - 4*a*c)) + 2*(b^7 - 17*a*b^5*c + 118*a^2*b^3*c^2 - 264*a^3*b
*c^3)*d + (a*b^6 - 22*a^2*b^4*c + 8*a^3*b^2*c^2 + 256*a^4*c^3)*e - 3*(2*(b^
6*c - 22*a*b^4*c^2 + 28*a^2*b^2*c^3 + 176*a^3*c^4)*d - (b^7 - 22*a*b^5*c +
28*a^2*b^3*c^2 + 176*a^3*b*c^3)*e)*x)/(a^3*b^8 - 16*a^4*b^6*c + 96*a^5*b^4*
```

```
c^2 - 256*a^6*b^2*c^3 + 256*a^7*c^4 + (b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*
c^5 - 256*a^3*b^2*c^6 + 256*a^4*c^7)*x^6 + 3*(b^9*c^2 - 16*a*b^7*c^3 + 96*a
^2*b^5*c^4 - 256*a^3*b^3*c^5 + 256*a^4*b*c^6)*x^5 + 3*(b^10*c - 15*a*b^8*c^
2 + 80*a^2*b^6*c^3 - 160*a^3*b^4*c^4 + 256*a^5*c^6)*x^4 + (b^11 - 10*a*b^9*
c + 320*a^3*b^5*c^3 - 1280*a^4*b^3*c^4 + 1536*a^5*b*c^5)*x^3 + 3*(a*b^10 -
15*a^2*b^8*c + 80*a^3*b^6*c^2 - 160*a^4*b^4*c^3 + 256*a^6*c^5)*x^2 + 3*(a^2
*b^9 - 16*a^3*b^7*c + 96*a^4*b^5*c^2 - 256*a^5*b^3*c^3 + 256*a^6*b*c^4)*x)]
```

Sympy [B] time = 4.3648, size = 1062, normalized size = 6.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**2+b*x+a)**4,x)
```

```
[Out] 10*c**2*sqrt(-1/(4*a*c - b**2)**7)*(b*e - 2*c*d)*log(x + (-2560*a**4*c**6*s
qrt(-1/(4*a*c - b**2)**7)*(b*e - 2*c*d) + 2560*a**3*b**2*c**5*sqrt(-1/(4*a*
c - b**2)**7)*(b*e - 2*c*d) - 960*a**2*b**4*c**4*sqrt(-1/(4*a*c - b**2)**7)
*(b*e - 2*c*d) + 160*a*b**6*c**3*sqrt(-1/(4*a*c - b**2)**7)*(b*e - 2*c*d) -
 10*b**8*c**2*sqrt(-1/(4*a*c - b**2)**7)*(b*e - 2*c*d) + 10*b**2*c**2*e - 2
0*b*c**3*d)/(20*b*c**3*e - 40*c**4*d)) - 10*c**2*sqrt(-1/(4*a*c - b**2)**7)
*(b*e - 2*c*d)*log(x + (2560*a**4*c**6*sqrt(-1/(4*a*c - b**2)**7)*(b*e - 2*
c*d) - 2560*a**3*b**2*c**5*sqrt(-1/(4*a*c - b**2)**7)*(b*e - 2*c*d) + 960*a
**2*b**4*c**4*sqrt(-1/(4*a*c - b**2)**7)*(b*e - 2*c*d) - 160*a*b**6*c**3*sq
rt(-1/(4*a*c - b**2)**7)*(b*e - 2*c*d) + 10*b**8*c**2*sqrt(-1/(4*a*c - b**2
)**7)*(b*e - 2*c*d) + 10*b**2*c**2*e - 20*b*c**3*d)/(20*b*c**3*e - 40*c**4*
d)) - (64*a**3*c**2*e + 18*a**2*b**2*c*e - 132*a**2*b*c**2*d - a*b**4*e + 2
6*a*b**3*c*d - 2*b**5*d + x**5*(60*b*c**4*e - 120*c**5*d) + x**4*(150*b**2*
c**3*e - 300*b*c**4*d) + x**3*(160*a*b*c**3*e - 320*a*c**4*d + 110*b**3*c**
2*e - 220*b**2*c**3*d) + x**2*(240*a*b**2*c**2*e - 480*a*b*c**3*d + 15*b**4
*c*e - 30*b**3*c**2*d) + x*(132*a**2*b*c**2*e - 264*a**2*c**3*d + 54*a*b**3
*c*e - 108*a*b**2*c**2*d - 3*b**5*e + 6*b**4*c*d))/(384*a**6*c**3 - 288*a**
5*b**2*c**2 + 72*a**4*b**4*c - 6*a**3*b**6 + x**6*(384*a**3*c**6 - 288*a**2
*b**2*c**5 + 72*a*b**4*c**4 - 6*b**6*c**3) + x**5*(1152*a**3*b*c**5 - 864*a
**2*b**3*c**4 + 216*a*b**5*c**3 - 18*b**7*c**2) + x**4*(1152*a**4*c**5 + 28
8*a**3*b**2*c**4 - 648*a**2*b**4*c**3 + 198*a*b**6*c**2 - 18*b**8*c) + x**3
*(2304*a**4*b*c**4 - 1344*a**3*b**3*c**3 + 144*a**2*b**5*c**2 + 36*a*b**7*c
 - 6*b**9) + x**2*(1152*a**5*c**4 + 288*a**4*b**2*c**3 - 648*a**3*b**4*c**2
 + 198*a**2*b**6*c - 18*a*b**8) + x*(1152*a**5*b*c**3 - 864*a**4*b**3*c**2
+ 216*a**3*b**5*c - 18*a**2*b**7))
```

Giac [B] time = 1.18469, size = 510, normalized size = 2.95

$$-\frac{20\left(2c^3d-bc^2e\right)\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\left(b^6-12ab^4c+48a^2b^2c^2-64a^3c^3\right)\sqrt{-b^2+4ac}}-\frac{120c^5dx^5-60bc^4x^5e+300bc^4dx^4-150b^2c^3x^4e+220b^2c^3dx^3+3\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^4,x, algorithm="giac")
```

```
[Out] -20*(2*c^3*d - b*c^2*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^6 - 12*a
*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-b^2 + 4*a*c)) - 1/6*(120*c^5*d*
```

```
x^5 - 60*b*c^4*x^5*e + 300*b*c^4*d*x^4 - 150*b^2*c^3*x^4*e + 220*b^2*c^3*d*
x^3 + 320*a*c^4*d*x^3 - 110*b^3*c^2*x^3*e - 160*a*b*c^3*x^3*e + 30*b^3*c^2*
d*x^2 + 480*a*b*c^3*d*x^2 - 15*b^4*c*x^2*e - 240*a*b^2*c^2*x^2*e - 6*b^4*c*
d*x + 108*a*b^2*c^2*d*x + 264*a^2*c^3*d*x + 3*b^5*x*e - 54*a*b^3*c*x*e - 13
2*a^2*b*c^2*x*e + 2*b^5*d - 26*a*b^3*c*d + 132*a^2*b*c^2*d + a*b^4*e - 18*a
^2*b^2*c*e - 64*a^3*c^2*e)/((b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3
)*(c*x^2 + b*x + a)^3)
```

3.2218 $\int \frac{1}{(a+bx+cx^2)^4}\,dx$

Optimal. Leaf size=136

$$-\frac{10c^2(b+2cx)}{(b^2-4ac)^3(a+bx+cx^2)}+\frac{40c^3\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{7/2}}+\frac{5c(b+2cx)}{3(b^2-4ac)^2(a+bx+cx^2)^2}-\frac{b+2cx}{3(b^2-4ac)(a+bx+cx^2)^3}$$

```
[Out] -(b + 2*c*x)/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^3) + (5*c*(b + 2*c*x))/(3*(
b^2 - 4*a*c)^2*(a + b*x + c*x^2)^2) - (10*c^2*(b + 2*c*x))/((b^2 - 4*a*c)^3
*(a + b*x + c*x^2)) + (40*c^3*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2
- 4*a*c)^(7/2)
```

Rubi [A] time = 0.050947, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {614, 618, 206}

$$-\frac{10c^2(b+2cx)}{(b^2-4ac)^3(a+bx+cx^2)}+\frac{40c^3\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{7/2}}+\frac{5c(b+2cx)}{3(b^2-4ac)^2(a+bx+cx^2)^2}-\frac{b+2cx}{3(b^2-4ac)(a+bx+cx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(-4),x]
```


```
[Out] -(b + 2*c*x)/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^3) + (5*c*(b + 2*c*x))/(3*(
b^2 - 4*a*c)^2*(a + b*x + c*x^2)^2) - (10*c^2*(b + 2*c*x))/((b^2 - 4*a*c)^3
*(a + b*x + c*x^2)) + (40*c^3*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2
- 4*a*c)^(7/2)
```


Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx+cx^2\right)^4}\,dx = -\frac{b+2cx}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} - \frac{(10c)\int \frac{1}{\left(a+bx+cx^2\right)^3}\,dx}{3\left(b^2-4ac\right)}$$

$$= -\frac{b+2cx}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{5c(b+2cx)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} + \frac{\left(10c^2\right)\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx}{\left(b^2-4ac\right)^2}$$

$$= -\frac{b+2cx}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{5c(b+2cx)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} - \frac{10c^2(b+2cx)}{\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)} -$$

$$= -\frac{b+2cx}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{5c(b+2cx)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} - \frac{10c^2(b+2cx)}{\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)} +$$

$$= -\frac{b+2cx}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{5c(b+2cx)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} - \frac{10c^2(b+2cx)}{\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)} +$$

Mathematica [A] time = 0.15553, size = 134, normalized size = 0.99

$$-\frac{\frac{120c^3\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - \frac{5c\left(b^2-4ac\right)(b+2cx)}{(a+x(b+cx))^2} + \frac{\left(b^2-4ac\right)^2(b+2cx)}{(a+x(b+cx))^3} + \frac{30c^2(b+2cx)}{a+x(b+cx)}}{3\left(b^2-4ac\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(-4),x]
```

```
[Out] -(((b^2 - 4*a*c)^2*(b + 2*c*x))/(a + x*(b + c*x))^3 - (5*c*(b^2 - 4*a*c)*(b
 + 2*c*x))/(a + x*(b + c*x))^2 + (30*c^2*(b + 2*c*x))/(a + x*(b + c*x)) + (
120*c^3*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(3*(b^2
 - 4*a*c)^3)
```

Maple [A] time = 0.153, size = 189, normalized size = 1.4

$$\frac{2\,cx+b}{\left(12\,ac-3\,b^2\right)\left(cx^2+bx+a\right)^3} + \frac{10\,c^2x}{3\left(4\,ac-b^2\right)^2\left(cx^2+bx+a\right)^2} + \frac{5\,bc}{3\left(4\,ac-b^2\right)^2\left(cx^2+bx+a\right)^2} + 20\,\frac{c^3x}{\left(4\,ac-b^2\right)^3\left(cx^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^4,x)
```

```
[Out] 1/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^3+10/3*c^2/(4*a*c-b^2)^2/(c*x^2+b*x
+a)^2*x+5/3*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^2*b+20*c^3/(4*a*c-b^2)^3/(c*x^2+b
*x+a)*x+10*c^2/(4*a*c-b^2)^3/(c*x^2+b*x+a)*b+40*c^3/(4*a*c-b^2)^(7/2)*arcta
n((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.24641, size = 2898, normalized size = 21.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/3*(b^7 - 17*a*b^5*c + 118*a^2*b^3*c^2 - 264*a^3*b*c^3 + 60*(b^2*c^5 - 4
*a*c^6)*x^5 + 150*(b^3*c^4 - 4*a*b*c^5)*x^4 + 10*(11*b^4*c^3 - 28*a*b^2*c^4
 - 64*a^2*c^5)*x^3 + 15*(b^5*c^2 + 12*a*b^3*c^3 - 64*a^2*b*c^4)*x^2 + 60*(c
^6*x^6 + 3*b*c^5*x^5 + 3*a^2*b*c^3*x + a^3*c^3 + 3*(b^2*c^4 + a*c^5)*x^4 +
(b^3*c^3 + 6*a*b*c^4)*x^3 + 3*(a*b^2*c^3 + a^2*c^4)*x^2)*sqrt(b^2 - 4*a*c)*
log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*
x^2 + b*x + a)) - 3*(b^6*c - 22*a*b^4*c^2 + 28*a^2*b^2*c^3 + 176*a^3*c^4)*x
)/(a^3*b^8 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3 + 256*a^7*c^4
+ (b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6 + 256*a^4*c^7)
*x^6 + 3*(b^9*c^2 - 16*a*b^7*c^3 + 96*a^2*b^5*c^4 - 256*a^3*b^3*c^5 + 256*a
^4*b*c^6)*x^5 + 3*(b^10*c - 15*a*b^8*c^2 + 80*a^2*b^6*c^3 - 160*a^3*b^4*c^4
 + 256*a^5*c^6)*x^4 + (b^11 - 10*a*b^9*c + 320*a^3*b^5*c^3 - 1280*a^4*b^3*c
^4 + 1536*a^5*b*c^5)*x^3 + 3*(a*b^10 - 15*a^2*b^8*c + 80*a^3*b^6*c^2 - 160*
a^4*b^4*c^3 + 256*a^6*c^5)*x^2 + 3*(a^2*b^9 - 16*a^3*b^7*c + 96*a^4*b^5*c^2
 - 256*a^5*b^3*c^3 + 256*a^6*b*c^4)*x), -1/3*(b^7 - 17*a*b^5*c + 118*a^2*b^
3*c^2 - 264*a^3*b*c^3 + 60*(b^2*c^5 - 4*a*c^6)*x^5 + 150*(b^3*c^4 - 4*a*b*c
^5)*x^4 + 10*(11*b^4*c^3 - 28*a*b^2*c^4 - 64*a^2*c^5)*x^3 + 15*(b^5*c^2 + 1
2*a*b^3*c^3 - 64*a^2*b*c^4)*x^2 - 120*(c^6*x^6 + 3*b*c^5*x^5 + 3*a^2*b*c^3*
x + a^3*c^3 + 3*(b^2*c^4 + a*c^5)*x^4 + (b^3*c^3 + 6*a*b*c^4)*x^3 + 3*(a*b^
2*c^3 + a^2*c^4)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x
+ b)/(b^2 - 4*a*c)) - 3*(b^6*c - 22*a*b^4*c^2 + 28*a^2*b^2*c^3 + 176*a^3*c^
4)*x)/(a^3*b^8 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3 + 256*a^7*
c^4 + (b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6 + 256*a^4*
c^7)*x^6 + 3*(b^9*c^2 - 16*a*b^7*c^3 + 96*a^2*b^5*c^4 - 256*a^3*b^3*c^5 + 2
56*a^4*b*c^6)*x^5 + 3*(b^10*c - 15*a*b^8*c^2 + 80*a^2*b^6*c^3 - 160*a^3*b^4
*c^4 + 256*a^5*c^6)*x^4 + (b^11 - 10*a*b^9*c + 320*a^3*b^5*c^3 - 1280*a^4*b
^3*c^4 + 1536*a^5*b*c^5)*x^3 + 3*(a*b^10 - 15*a^2*b^8*c + 80*a^3*b^6*c^2 -
160*a^4*b^4*c^3 + 256*a^6*c^5)*x^2 + 3*(a^2*b^9 - 16*a^3*b^7*c + 96*a^4*b^5
*c^2 - 256*a^5*b^3*c^3 + 256*a^6*b*c^4)*x)]
```

Sympy [B] time = 2.5775, size = 777, normalized size = 5.71

$$-20c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^7}}\log\left(x+\frac{-5120a^4c^7\sqrt{-\frac{1}{\left(4ac-b^2\right)^7}}+5120a^3b^2c^6\sqrt{-\frac{1}{\left(4ac-b^2\right)^7}}-1920a^2b^4c^5\sqrt{-\frac{1}{\left(4ac-b^2\right)^7}}+320ab^6c^4\sqrt{-}}{40c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**4,x)
```

```
[Out] -20*c**3*sqrt(-1/(4*a*c - b**2)**7)*log(x + (-5120*a**4*c**7*sqrt(-1/(4*a*c
 - b**2)**7) + 5120*a**3*b**2*c**6*sqrt(-1/(4*a*c - b**2)**7) - 1920*a**2*b
**4*c**5*sqrt(-1/(4*a*c - b**2)**7) + 320*a*b**6*c**4*sqrt(-1/(4*a*c - b**2
)**7) - 20*b**8*c**3*sqrt(-1/(4*a*c - b**2)**7) + 20*b*c**3)/(40*c**4)) + 2
0*c**3*sqrt(-1/(4*a*c - b**2)**7)*log(x + (5120*a**4*c**7*sqrt(-1/(4*a*c -
b**2)**7) - 5120*a**3*b**2*c**6*sqrt(-1/(4*a*c - b**2)**7) + 1920*a**2*b**4
*c**5*sqrt(-1/(4*a*c - b**2)**7) - 320*a*b**6*c**4*sqrt(-1/(4*a*c - b**2)**
7) + 20*b**8*c**3*sqrt(-1/(4*a*c - b**2)**7) + 20*b*c**3)/(40*c**4)) + (66*
a**2*b*c**2 - 13*a*b**3*c + b**5 + 150*b*c**4*x**4 + 60*c**5*x**5 + x**3*(1
60*a*c**4 + 110*b**2*c**3) + x**2*(240*a*b*c**3 + 15*b**3*c**2) + x*(132*a*
*2*c**3 + 54*a*b**2*c**2 - 3*b**4*c))/(192*a**6*c**3 - 144*a**5*b**2*c**2 +
 36*a**4*b**4*c - 3*a**3*b**6 + x**6*(192*a**3*c**6 - 144*a**2*b**2*c**5 +
36*a*b**4*c**4 - 3*b**6*c**3) + x**5*(576*a**3*b*c**5 - 432*a**2*b**3*c**4
+ 108*a*b**5*c**3 - 9*b**7*c**2) + x**4*(576*a**4*c**5 + 144*a**3*b**2*c**4
 - 324*a**2*b**4*c**3 + 99*a*b**6*c**2 - 9*b**8*c) + x**3*(1152*a**4*b*c**4
 - 672*a**3*b**3*c**3 + 72*a**2*b**5*c**2 + 18*a*b**7*c - 3*b**9) + x**2*(5
76*a**5*c**4 + 144*a**4*b**2*c**3 - 324*a**3*b**4*c**2 + 99*a**2*b**6*c - 9
*a*b**8) + x*(576*a**5*b*c**3 - 432*a**4*b**3*c**2 + 108*a**3*b**5*c - 9*a*
*2*b**7))
```

Giac [A] time = 1.09886, size = 297, normalized size = 2.18

$$-\frac{40\,c^3\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3\right)\sqrt{-b^2+4\,ac}}-\frac{60\,c^5x^5+150\,bc^4x^4+110\,b^2c^3x^3+160\,ac^4x^3+15\,b^3c^2x^2+240\,a}{3\left(b^6-12\,ab^4c+48\,a^2b^2c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^4,x, algorithm="giac")
```

```
[Out] -40*c^3*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^6 - 12*a*b^4*c + 48*a^2*
b^2*c^2 - 64*a^3*c^3)*sqrt(-b^2 + 4*a*c)) - 1/3*(60*c^5*x^5 + 150*b*c^4*x^4
 + 110*b^2*c^3*x^3 + 160*a*c^4*x^3 + 15*b^3*c^2*x^2 + 240*a*b*c^3*x^2 - 3*b
^4*c*x + 54*a*b^2*c^2*x + 132*a^2*c^3*x + b^5 - 13*a*b^3*c + 66*a^2*b*c^2)/
((b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*(c*x^2 + b*x + a)^3)
```

3.2219 $\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^4}\,dx$

Optimal. Leaf size=771

$$\frac{-2cx(2cd-be)\left(c^2e^2\left(38a^2e^2-32abde+7b^2d^2\right)+b^2ce^3(3bd-11ae)-4c^3d^2e(5bd-8ae)+b^4e^4+10c^4d^4\right)+2b^2c^2e\left(43a^2\right.}{2\left(b^2-4ac\right)^3\left(\right.}$$

```
[Out] -(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)/(3*(b^2 - 4*a*c)*(c*d^2 - b*
d*e + a*e^2)*(a + b*x + c*x^2)^3) - (5*a*c*e*(2*c*d - b*e)^2 - (b*c*d - b^2
*e + 2*a*c*e)*(10*c^2*d^2 - 3*b^2*e^2 - c*e*(5*b*d - 12*a*e)) - c*(2*c*d -
b*e)*(10*c^2*d^2 - 3*b^2*e^2 - 2*c*e*(5*b*d - 11*a*e))*x)/(6*(b^2 - 4*a*c)^
2*(c*d^2 - b*d*e + a*e^2)^2*(a + b*x + c*x^2)^2) + (b^5*c*d*e^4 + 2*b^6*e^5
 - 64*a^3*c^3*e^5 + b^4*c*e^3*(c*d^2 - 23*a*e^2) - 2*b^3*c^2*d*e^2*(17*c*d^
2 + 5*a*e^2) - 4*b*c^3*d*(5*c^2*d^4 + 16*a*c*d^2*e^2 + 19*a^2*e^4) + 2*b^2*
c^2*e*(25*c^2*d^4 + 48*a*c*d^2*e^2 + 43*a^2*e^4) - 2*c*(2*c*d - b*e)*(10*c^
4*d^4 + b^4*e^4 + b^2*c*e^3*(3*b*d - 11*a*e) - 4*c^3*d^2*e*(5*b*d - 8*a*e)
+ c^2*e^2*(7*b^2*d^2 - 32*a*b*d*e + 38*a^2*e^2))*x)/(2*(b^2 - 4*a*c)^3*(c*d
^2 - b*d*e + a*e^2)^3*(a + b*x + c*x^2)) + ((40*c^7*d^7 + b^7*e^7 - 14*a*b^
5*c*e^7 + 70*a^2*b^3*c^2*e^7 - 140*a^3*b*c^3*e^7 - 28*c^6*d^5*e*(5*b*d - 6*
a*e) + 28*c^5*d^3*e^2*(6*b^2*d^2 - 15*a*b*d*e + 10*a^2*e^2) - 70*c^4*d*e^3*
(b^3*d^3 - 4*a*b^2*d^2*e + 6*a^2*b*d*e^2 - 4*a^3*e^3))*ArcTanh[(b + 2*c*x)/
Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(7/2)*(c*d^2 - b*d*e + a*e^2)^4) + (e^7*
Log[d + e*x])/(c*d^2 - b*d*e + a*e^2)^4 - (e^7*Log[a + b*x + c*x^2])/(2*(c*
d^2 - b*d*e + a*e^2)^4)
```

Rubi [A] time = 7.29987, antiderivative size = 771, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {740, 822, 800, 634, 618, 206, 628}

$$\frac{-2cx(2cd-be)\left(c^2e^2\left(38a^2e^2-32abde+7b^2d^2\right)+b^2ce^3(3bd-11ae)-4c^3d^2e(5bd-8ae)+b^4e^4+10c^4d^4\right)+2b^2c^2e\left(43a^2\right.}{2\left(b^2-4ac\right)^3\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + b*x + c*x^2)^4),x]
```

```
[Out] -(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)/(3*(b^2 - 4*a*c)*(c*d^2 - b*
d*e + a*e^2)*(a + b*x + c*x^2)^3) - (5*a*c*e*(2*c*d - b*e)^2 - (b*c*d - b^2
*e + 2*a*c*e)*(10*c^2*d^2 - 3*b^2*e^2 - c*e*(5*b*d - 12*a*e)) - c*(2*c*d -
b*e)*(10*c^2*d^2 - 3*b^2*e^2 - 2*c*e*(5*b*d - 11*a*e))*x)/(6*(b^2 - 4*a*c)^
2*(c*d^2 - b*d*e + a*e^2)^2*(a + b*x + c*x^2)^2) + (b^5*c*d*e^4 + 2*b^6*e^5
 - 64*a^3*c^3*e^5 + b^4*c*e^3*(c*d^2 - 23*a*e^2) - 2*b^3*c^2*d*e^2*(17*c*d^
2 + 5*a*e^2) - 4*b*c^3*d*(5*c^2*d^4 + 16*a*c*d^2*e^2 + 19*a^2*e^4) + 2*b^2*
c^2*e*(25*c^2*d^4 + 48*a*c*d^2*e^2 + 43*a^2*e^4) - 2*c*(2*c*d - b*e)*(10*c^
4*d^4 + b^4*e^4 + b^2*c*e^3*(3*b*d - 11*a*e) - 4*c^3*d^2*e*(5*b*d - 8*a*e)
+ c^2*e^2*(7*b^2*d^2 - 32*a*b*d*e + 38*a^2*e^2))*x)/(2*(b^2 - 4*a*c)^3*(c*d
^2 - b*d*e + a*e^2)^3*(a + b*x + c*x^2)) + ((40*c^7*d^7 + b^7*e^7 - 14*a*b^
5*c*e^7 + 70*a^2*b^3*c^2*e^7 - 140*a^3*b*c^3*e^7 - 28*c^6*d^5*e*(5*b*d - 6*
a*e) + 28*c^5*d^3*e^2*(6*b^2*d^2 - 15*a*b*d*e + 10*a^2*e^2) - 70*c^4*d*e^3*
(b^3*d^3 - 4*a*b^2*d^2*e + 6*a^2*b*d*e^2 - 4*a^3*e^3))*ArcTanh[(b + 2*c*x)/
Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(7/2)*(c*d^2 - b*d*e + a*e^2)^4) + (e^7*
Log[d + e*x])/(c*d^2 - b*d*e + a*e^2)^4 - (e^7*Log[a + b*x + c*x^2])/(2*(c*
d^2 - b*d*e + a*e^2)^4)
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^4}\,dx = -\frac{bcd-b^2e+2ace+c(2cd-be)x}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^3} - \frac{\int \frac{10c^2d^2-3b^2e^2-ce(5bd-12ae)+5ce(2cd-be)x}{(d+ex)\left(a+bx+cx^2\right)^3}\,dx}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^3} - \frac{5ace(2cd-be)^2-\left(bcd-b^2e+2ace\right)\left(\right.}{6}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^3} - \frac{5ace(2cd-be)^2-\left(bcd-b^2e+2ace\right)\left(\right.}{6}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^3} - \frac{5ace(2cd-be)^2-\left(bcd-b^2e+2ace\right)\left(\right.}{6}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^3} - \frac{5ace(2cd-be)^2-\left(bcd-b^2e+2ace\right)\left(\right.}{6}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^3} - \frac{5ace(2cd-be)^2-\left(bcd-b^2e+2ace\right)\left(\right.}{6}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^3} - \frac{5ace(2cd-be)^2-\left(bcd-b^2e+2ace\right)\left(\right.}{6}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^3} - \frac{5ace(2cd-be)^2-\left(bcd-b^2e+2ace\right)\left(\right.}{6}$$

Mathematica [A] time = 3.6818, size = 769, normalized size = 1.

$$\frac{1}{6}\left(\frac{4c^2\left(6a^2e^3+11acde^2x+5c^2d^3x\right)+b^2ce\left(cd(4ex-15d)-23ae^2\right)+2bc^2\left(11ae^2(d-ex)+5cd^2(d-3ex)\right)+b^3ce^2(2d+3\right.}{\left(b^2-4ac\right)^2(a+x(b+cx))^2\left(e(ae-bd)+cd^2\right)^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + b*x + c*x^2)^4),x]
```

```
[Out] ((-2*b^2*e + 4*c*(a*e + c*d*x) + 2*b*c*(d - e*x))/((b^2 - 4*a*c)*(-(c*d^2)
+ e*(b*d - a*e))*(a + x*(b + c*x))^3) + (3*b^4*e^3 + b^3*c*e^2*(2*d + 3*e*x
) + 4*c^2*(6*a^2*e^3 + 5*c^2*d^3*x + 11*a*c*d*e^2*x) + 2*b*c^2*(5*c*d^2*(d
- 3*e*x) + 11*a*e^2*(d - e*x)) + b^2*c*e*(-23*a*e^2 + c*d*(-15*d + 4*e*x)))
/((b^2 - 4*a*c)^2*(c*d^2 + e*(-(b*d) + a*e))^2*(a + x*(b + c*x))^2) + (3*(-
2*b^6*e^5 - b^5*c*e^4*(d + 2*e*x) + 8*c^3*(8*a^3*e^5 + 5*c^3*d^5*x + 16*a*c
^2*d^3*e^2*x + 19*a^2*c*d*e^4*x) + 4*b*c^3*(5*c^2*d^4*(d - 5*e*x) + 16*a*c*
d^2*e^2*(d - 3*e*x) + 19*a^2*e^4*(d - e*x)) - b^4*c*e^3*(-23*a*e^2 + c*d*(d
 + 2*e*x)) + 2*b^3*c^2*e^2*(c*d^2*(17*d - e*x) + a*e^2*(5*d + 11*e*x)) + 2*
b^2*c^2*e*(-43*a^2*e^4 + 2*a*c*d*e^2*(-24*d + 5*e*x) + c^2*d^3*(-25*d + 34*
e*x))))/((b^2 - 4*a*c)^3*(-(c*d^2) + e*(b*d - a*e))^3*(a + x*(b + c*x))) +
(6*(40*c^7*d^7 + b^7*e^7 - 14*a*b^5*c*e^7 + 70*a^2*b^3*c^2*e^7 - 140*a^3*b*
c^3*e^7 - 28*c^6*d^5*e*(5*b*d - 6*a*e) + 28*c^5*d^3*e^2*(6*b^2*d^2 - 15*a*b
*d*e + 10*a^2*e^2) - 70*c^4*d*e^3*(b^3*d^3 - 4*a*b^2*d^2*e + 6*a^2*b*d*e^2
- 4*a^3*e^3))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/((-b^2 + 4*a*c)^(7/2)
*(c*d^2 + e*(-(b*d) + a*e))^4) + (6*e^7*Log[d + e*x])/(c*d^2 - b*d*e + a*e^
```

```
2)^4 - (3*e^7*Log[a + x*(b + c*x)])/(c*d^2 + e*(-(b*d) + a*e))^4)/6
```

Maple [B] time = 0.222, size = 14396, normalized size = 18.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+b*x+a)^4,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27588, size = 4361, normalized size = 5.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/2*e^7*log(c*x^2 + b*x + a)/(c^4*d^8 - 4*b*c^3*d^7*e + 6*b^2*c^2*d^6*e^2
+ 4*a*c^3*d^6*e^2 - 4*b^3*c*d^5*e^3 - 12*a*b*c^2*d^5*e^3 + b^4*d^4*e^4 + 12
*a*b^2*c*d^4*e^4 + 6*a^2*c^2*d^4*e^4 - 4*a*b^3*d^3*e^5 - 12*a^2*b*c*d^3*e^5
 + 6*a^2*b^2*d^2*e^6 + 4*a^3*c*d^2*e^6 - 4*a^3*b*d*e^7 + a^4*e^8) + e^8*log
(abs(x*e + d))/(c^4*d^8*e - 4*b*c^3*d^7*e^2 + 6*b^2*c^2*d^6*e^3 + 4*a*c^3*d
^6*e^3 - 4*b^3*c*d^5*e^4 - 12*a*b*c^2*d^5*e^4 + b^4*d^4*e^5 + 12*a*b^2*c*d^
4*e^5 + 6*a^2*c^2*d^4*e^5 - 4*a*b^3*d^3*e^6 - 12*a^2*b*c*d^3*e^6 + 6*a^2*b^
2*d^2*e^7 + 4*a^3*c*d^2*e^7 - 4*a^3*b*d*e^8 + a^4*e^9) - (40*c^7*d^7 - 140*
b*c^6*d^6*e + 168*b^2*c^5*d^5*e^2 + 168*a*c^6*d^5*e^2 - 70*b^3*c^4*d^4*e^3
- 420*a*b*c^5*d^4*e^3 + 280*a*b^2*c^4*d^3*e^4 + 280*a^2*c^5*d^3*e^4 - 420*a
^2*b*c^4*d^2*e^5 + 280*a^3*c^4*d*e^6 + b^7*e^7 - 14*a*b^5*c*e^7 + 70*a^2*b^
3*c^2*e^7 - 140*a^3*b*c^3*e^7)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^6
*c^4*d^8 - 12*a*b^4*c^5*d^8 + 48*a^2*b^2*c^6*d^8 - 64*a^3*c^7*d^8 - 4*b^7*c
^3*d^7*e + 48*a*b^5*c^4*d^7*e - 192*a^2*b^3*c^5*d^7*e + 256*a^3*b*c^6*d^7*e
 + 6*b^8*c^2*d^6*e^2 - 68*a*b^6*c^3*d^6*e^2 + 240*a^2*b^4*c^4*d^6*e^2 - 192
*a^3*b^2*c^5*d^6*e^2 - 256*a^4*c^6*d^6*e^2 - 4*b^9*c*d^5*e^3 + 36*a*b^7*c^2
*d^5*e^3 - 48*a^2*b^5*c^3*d^5*e^3 - 320*a^3*b^3*c^4*d^5*e^3 + 768*a^4*b*c^5
*d^5*e^3 + b^10*d^4*e^4 - 90*a^2*b^6*c^2*d^4*e^4 + 440*a^3*b^4*c^3*d^4*e^4
- 480*a^4*b^2*c^4*d^4*e^4 - 384*a^5*c^5*d^4*e^4 - 4*a*b^9*d^3*e^5 + 36*a^2*
b^7*c*d^3*e^5 - 48*a^3*b^5*c^2*d^3*e^5 - 320*a^4*b^3*c^3*d^3*e^5 + 768*a^5*
b*c^4*d^3*e^5 + 6*a^2*b^8*d^2*e^6 - 68*a^3*b^6*c*d^2*e^6 + 240*a^4*b^4*c^2*
d^2*e^6 - 192*a^5*b^2*c^3*d^2*e^6 - 256*a^6*c^4*d^2*e^6 - 4*a^3*b^7*d*e^7 +
 48*a^4*b^5*c*d*e^7 - 192*a^5*b^3*c^2*d*e^7 + 256*a^6*b*c^3*d*e^7 + a^4*b^6
*e^8 - 12*a^5*b^4*c*e^8 + 48*a^6*b^2*c^2*e^8 - 64*a^7*c^3*e^8)*sqrt(-b^2 +
4*a*c)) - 1/6*(2*b^5*c^4*d^7 - 26*a*b^3*c^5*d^7 + 132*a^2*b*c^6*d^7 - 8*b^6
*c^3*d^6*e + 103*a*b^4*c^4*d^6*e - 510*a^2*b^2*c^5*d^6*e + 64*a^3*c^6*d^6*e
 + 12*b^7*c^2*d^5*e^2 - 144*a*b^5*c^3*d^5*e^2 + 618*a^2*b^3*c^4*d^5*e^2 + 3
24*a^3*b*c^5*d^5*e^2 - 8*b^8*c*d^4*e^3 + 74*a*b^6*c^2*d^4*e^3 - 120*a^2*b^4
*c^3*d^4*e^3 - 1314*a^3*b^2*c^4*d^4*e^3 + 288*a^4*c^5*d^4*e^3 + 2*b^9*d^3*e
^4 + 2*a*b^7*c*d^3*e^4 - 216*a^2*b^5*c^2*d^3*e^4 + 1190*a^3*b^3*c^3*d^3*e^4
 + 156*a^4*b*c^4*d^3*e^4 - 9*a*b^8*d^2*e^5 + 78*a^2*b^6*c*d^2*e^5 - 51*a^3*
b^4*c^2*d^2*e^5 - 1242*a^4*b^2*c^3*d^2*e^5 + 576*a^5*c^4*d^2*e^5 + 18*a^2*b
^7*d*e^6 - 202*a^3*b^5*c*d*e^6 + 682*a^4*b^3*c^2*d*e^6 - 228*a^5*b*c^3*d*e^
6 - 11*a^3*b^6*e^7 + 124*a^4*b^4*c*e^7 - 438*a^5*b^2*c^2*e^7 + 352*a^6*c^3*
e^7 + 6*(20*c^9*d^7 - 70*b*c^8*d^6*e + 84*b^2*c^7*d^5*e^2 + 84*a*c^8*d^5*e^
2 - 35*b^3*c^6*d^4*e^3 - 210*a*b*c^7*d^4*e^3 + 140*a*b^2*c^6*d^3*e^4 + 140*
a^2*c^7*d^3*e^4 - 210*a^2*b*c^6*d^2*e^5 + b^6*c^3*d*e^6 - 12*a*b^4*c^4*d*e^
6 + 48*a^2*b^2*c^5*d*e^6 + 76*a^3*c^6*d*e^6 - a*b^5*c^3*e^7 + 11*a^2*b^3*c^
4*e^7 - 38*a^3*b*c^5*e^7)*x^5 + 3*(100*b*c^8*d^7 - 350*b^2*c^7*d^6*e + 420*
b^3*c^6*d^5*e^2 + 420*a*b*c^7*d^5*e^2 - 175*b^4*c^5*d^4*e^3 - 1050*a*b^2*c^
6*d^4*e^3 + 700*a*b^3*c^5*d^3*e^4 + 700*a^2*b*c^6*d^3*e^4 - b^6*c^3*d^2*e^5
 + 12*a*b^4*c^4*d^2*e^5 - 1098*a^2*b^2*c^5*d^2*e^5 + 64*a^3*c^6*d^2*e^5 + 6
*b^7*c^2*d*e^6 - 72*a*b^5*c^3*d*e^6 + 288*a^2*b^3*c^4*d*e^6 + 316*a^3*b*c^5
*d*e^6 - 6*a*b^6*c^2*e^7 + 67*a^2*b^4*c^3*e^7 - 238*a^3*b^2*c^4*e^7 + 64*a^
4*c^5*e^7)*x^4 + (220*b^2*c^7*d^7 + 320*a*c^8*d^7 - 770*b^3*c^6*d^6*e - 112
0*a*b*c^7*d^6*e + 924*b^4*c^5*d^5*e^2 + 2268*a*b^2*c^6*d^5*e^2 + 1344*a^2*c
^7*d^5*e^2 - 385*b^5*c^4*d^4*e^3 - 2870*a*b^3*c^5*d^4*e^3 - 3360*a^2*b*c^6*
d^4*e^3 + 2*b^6*c^3*d^3*e^4 + 1516*a*b^4*c^4*d^3*e^4 + 3876*a^2*b^2*c^5*d^3
*e^4 + 2112*a^3*c^6*d^3*e^4 - 9*b^7*c^2*d^2*e^5 + 108*a*b^5*c^3*d^2*e^5 - 2
742*a^2*b^3*c^4*d^2*e^5 - 2784*a^3*b*c^5*d^2*e^5 + 18*b^8*c*d*e^6 - 198*a*b
^6*c^2*d*e^6 + 648*a^2*b^4*c^3*d*e^6 + 1252*a^3*b^2*c^4*d*e^6 + 1088*a^4*c^
5*d*e^6 - 18*a*b^7*c*e^7 + 189*a^2*b^5*c^2*e^7 - 578*a^3*b^3*c^3*e^7 - 160*
a^4*b*c^4*e^7)*x^3 + 3*(10*b^3*c^6*d^7 + 160*a*b*c^7*d^7 - 35*b^4*c^5*d^6*e
 - 560*a*b^2*c^6*d^6*e + 42*b^5*c^4*d^5*e^2 + 714*a*b^3*c^5*d^5*e^2 + 672*a
^2*b*c^6*d^5*e^2 - 18*b^6*c^3*d^4*e^3 - 379*a*b^4*c^4*d^4*e^3 - 1704*a^2*b^
2*c^5*d^4*e^3 + 32*a^3*c^6*d^4*e^3 + 2*b^7*c^2*d^3*e^4 + 46*a*b^5*c^3*d^3*e
^4 + 1286*a^2*b^3*c^4*d^3*e^4 + 992*a^3*b*c^5*d^3*e^4 - 3*b^8*c*d^2*e^5 + 3
```

3*a*b^6*c^2*d^2*e^5 - 213*a^2*b^4*c^3*d^2*e^5 - 1632*a^3*b^2*c^4*d^2*e^5 + 192*a^4*c^5*d^2*e^5 + 2*b^9*d*e^6 - 12*a*b^7*c*d*e^6 - 48*a^2*b^5*c^2*d*e^6 + 518*a^3*b^3*c^3*d*e^6 + 352*a^4*b*c^4*d*e^6 - 2*a*b^8*e^7 + 13*a^2*b^6*c*e^7 + 27*a^3*b^4*c^2*e^7 - 328*a^4*b^2*c^3*e^7 + 160*a^5*c^4*e^7)*x^2 - 3*(2*b^4*c^5*d^7 - 36*a*b^2*c^6*d^7 - 88*a^2*c^7*d^7 - 7*b^5*c^4*d^6*e + 126*a*b^3*c^5*d^6*e + 308*a^2*b*c^6*d^6*e + 8*b^6*c^3*d^5*e^2 - 138*a*b^4*c^4*d^5*e^2 - 540*a^2*b^2*c^5*d^5*e^2 - 344*a^3*c^6*d^5*e^2 - 2*b^7*c^2*d^4*e^3 + 24*a*b^5*c^3*d^4*e^3 + 604*a^2*b^3*c^4*d^4*e^3 + 828*a^3*b*c^5*d^4*e^3 - 2*b^8*c*d^3*e^4 + 36*a*b^6*c^2*d^3*e^4 - 310*a^2*b^4*c^3*d^3*e^4 - 836*a^3*b^2*c^4*d^3*e^4 - 488*a^4*c^5*d^3*e^4 + b^9*d^2*e^5 - 6*a*b^7*c*d^2*e^5 - 45*a^2*b^5*c^2*d^2*e^5 + 602*a^3*b^3*c^3*d^2*e^5 + 540*a^4*b*c^4*d^2*e^5 - 6*a*b^8*d*e^6 + 66*a^2*b^6*c*d*e^6 - 202*a^3*b^4*c^2*d*e^6 - 156*a^4*b^2*c^3*d*e^6 - 232*a^5*c^4*d*e^6 + 5*a^2*b^7*e^7 - 54*a^3*b^5*c*e^7 + 172*a^4*b^3*c^2*e^7 - 44*a^5*b*c^3*e^7)*x)/((c*d^2 - b*d*e + a*e^2)^4*(c*x^2 + b*x + a)^3*(b^2 - 4*a*c)^3)

3.2220 $\int \frac{1}{x^2(a+bx+cx^2)^4}\,dx$

Optimal. Leaf size=352

$$\frac{2\left(3bcx\left(29a^2c^2-10ab^2c+b^4\right)+105a^2b^2c^2-70a^3c^3-32ab^4c+3b^6\right)}{3a^3x\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)}+\frac{2\left(7a^2c^2-7ab^2c+b^4\right)+bcx\left(2b^2-13ac\right)}{3a^2x\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2}-\frac{4(3\cdots}{\cdots}$$

```
[Out] (-4*(b^6 - 11*a*b^4*c + 38*a^2*b^2*c^2 - 35*a^3*c^3))/(a^4*(b^2 - 4*a*c)^3*
x) + (b^2 - 2*a*c + b*c*x)/(3*a*(b^2 - 4*a*c)*x*(a + b*x + c*x^2)^3) + (2*(
b^4 - 7*a*b^2*c + 7*a^2*c^2) + b*c*(2*b^2 - 13*a*c)*x)/(3*a^2*(b^2 - 4*a*c)
^2*x*(a + b*x + c*x^2)^2) + (2*(3*b^6 - 32*a*b^4*c + 105*a^2*b^2*c^2 - 70*a
^3*c^3 + 3*b*c*(b^4 - 10*a*b^2*c + 29*a^2*c^2)*x))/(3*a^3*(b^2 - 4*a*c)^3*x
*(a + b*x + c*x^2)) - (4*(b^8 - 14*a*b^6*c + 70*a^2*b^4*c^2 - 140*a^3*b^2*c
^3 + 70*a^4*c^4)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^5*(b^2 - 4*a*c)
^(7/2)) - (4*b*Log[x])/a^5 + (2*b*Log[a + b*x + c*x^2])/a^5
```

Rubi [A] time = 0.51991, antiderivative size = 352, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {740, 822, 800, 634, 618, 206, 628}

$$\frac{2\left(3bcx\left(29a^2c^2-10ab^2c+b^4\right)+105a^2b^2c^2-70a^3c^3-32ab^4c+3b^6\right)}{3a^3x\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)}+\frac{2\left(7a^2c^2-7ab^2c+b^4\right)+bcx\left(2b^2-13ac\right)}{3a^2x\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2}-\frac{4(3\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x + c*x^2)^4),x]
```


```
[Out] (-4*(b^6 - 11*a*b^4*c + 38*a^2*b^2*c^2 - 35*a^3*c^3))/(a^4*(b^2 - 4*a*c)^3*
x) + (b^2 - 2*a*c + b*c*x)/(3*a*(b^2 - 4*a*c)*x*(a + b*x + c*x^2)^3) + (2*(
b^4 - 7*a*b^2*c + 7*a^2*c^2) + b*c*(2*b^2 - 13*a*c)*x)/(3*a^2*(b^2 - 4*a*c)
^2*x*(a + b*x + c*x^2)^2) + (2*(3*b^6 - 32*a*b^4*c + 105*a^2*b^2*c^2 - 70*a
^3*c^3 + 3*b*c*(b^4 - 10*a*b^2*c + 29*a^2*c^2)*x))/(3*a^3*(b^2 - 4*a*c)^3*x
*(a + b*x + c*x^2)) - (4*(b^8 - 14*a*b^6*c + 70*a^2*b^4*c^2 - 140*a^3*b^2*c
^3 + 70*a^4*c^4)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^5*(b^2 - 4*a*c)
^(7/2)) - (4*b*Log[x])/a^5 + (2*b*Log[a + b*x + c*x^2])/a^5
```


Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
```

```
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx+cx^2\right)^4}\,dx = \frac{b^2-2ac+bcx}{3a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^3} - \frac{\int \frac{-2\left(2b^2-7ac\right)-6bcx}{x^2\left(a+bx+cx^2\right)^3}\,dx}{3a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{3a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^3} + \frac{2\left(b^4-7ab^2c+7a^2c^2\right)+bc\left(2b^2-13ac\right)x}{3a^2\left(b^2-4ac\right)^2x\left(a+bx+cx^2\right)^2} + \frac{\int \frac{4\left(3b^2-7ac\right)\left(b^2- \right.}{x^2(a}}{6a^2(}$$

$$= \frac{b^2-2ac+bcx}{3a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^3} + \frac{2\left(b^4-7ab^2c+7a^2c^2\right)+bc\left(2b^2-13ac\right)x}{3a^2\left(b^2-4ac\right)^2x\left(a+bx+cx^2\right)^2} + \frac{2\left(3b^6-32ab^4\right.}{}$$

$$= \frac{b^2-2ac+bcx}{3a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^3} + \frac{2\left(b^4-7ab^2c+7a^2c^2\right)+bc\left(2b^2-13ac\right)x}{3a^2\left(b^2-4ac\right)^2x\left(a+bx+cx^2\right)^2} + \frac{2\left(3b^6-32ab^4\right.}{}$$

$$= -\frac{4\left(b^6-11ab^4c+38a^2b^2c^2-35a^3c^3\right)}{a^4\left(b^2-4ac\right)^3x} + \frac{b^2-2ac+bcx}{3a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^3} + \frac{2\left(b^4-7ab^2c+7a\right.}{3a^2\left(b^2-4ac\right)}$$

$$= -\frac{4\left(b^6-11ab^4c+38a^2b^2c^2-35a^3c^3\right)}{a^4\left(b^2-4ac\right)^3x} + \frac{b^2-2ac+bcx}{3a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^3} + \frac{2\left(b^4-7ab^2c+7a\right.}{3a^2\left(b^2-4ac\right)}$$

$$= -\frac{4\left(b^6-11ab^4c+38a^2b^2c^2-35a^3c^3\right)}{a^4\left(b^2-4ac\right)^3x} + \frac{b^2-2ac+bcx}{3a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^3} + \frac{2\left(b^4-7ab^2c+7a\right.}{3a^2\left(b^2-4ac\right)}$$

$$= -\frac{4\left(b^6-11ab^4c+38a^2b^2c^2-35a^3c^3\right)}{a^4\left(b^2-4ac\right)^3x} + \frac{b^2-2ac+bcx}{3a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^3} + \frac{2\left(b^4-7ab^2c+7a\right.}{3a^2\left(b^2-4ac\right)}$$

Mathematica [A] time = 0.906965, size = 329, normalized size = 0.93

$$\frac{\frac{a^3\left(-3abc-2ac^2x+b^2cx+b^3\right)}{\left(4ac-b^2\right)(a+x(b+cx))^3} - \frac{a^2\left(35a^2bc^2+22a^2c^3x-20ab^2c^2x-22ab^3c+3b^4cx+3b^5\right)}{\left(b^2-4ac\right)^2(a+x(b+cx))^2} + \frac{3a\left(-104a^2b^2c^3x-124a^2b^3c^2+134a^3bc^3+76a^3c^4x+32ab^4c^2x+34ab^5c-3b^6\right.}{\left(b^2-4ac\right)^3(a+x(b+cx))}}{3a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x + c*x^2)^4),x]
```

```
[Out] ((-3*a)/x + (a^3*(b^3 - 3*a*b*c + b^2*c*x - 2*a*c^2*x))/((-b^2 + 4*a*c)*(a
+ x*(b + c*x))^3) - (a^2*(3*b^5 - 22*a*b^3*c + 35*a^2*b*c^2 + 3*b^4*c*x - 2
0*a*b^2*c^2*x + 22*a^2*c^3*x))/((b^2 - 4*a*c)^2*(a + x*(b + c*x))^2) + (3*a
*(-3*b^7 + 34*a*b^5*c - 124*a^2*b^3*c^2 + 134*a^3*b*c^3 - 3*b^6*c*x + 32*a*
b^4*c^2*x - 104*a^2*b^2*c^3*x + 76*a^3*c^4*x))/((b^2 - 4*a*c)^3*(a + x*(b +
 c*x))) - (12*(b^8 - 14*a*b^6*c + 70*a^2*b^4*c^2 - 140*a^3*b^2*c^3 + 70*a^4
*c^4)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(7/2) - 12*b*L
og[x] + 6*b*Log[a + x*(b + c*x)])/(3*a^5)
```

Maple [B] time = 0.175, size = 2162, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^2+b*x+a)^4,x)
```

```
[Out] -1/a^4/x+3/a^4/(c*x^2+b*x+a)^3*b^9/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^
6)*x^2-116*a/(c*x^2+b*x+a)^3/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x*c
^4+7/a^3/(c*x^2+b*x+a)^3/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x*b^8-4
/a^5/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1/2)*arctan((2
*c*x+b)/(4*a*c-b^2)^(1/2))*b^8+128/a^2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*
c-b^6)*c^3*ln(c*x^2+b*x+a)*b-590/3*a/(c*x^2+b*x+a)^3*b/(64*a^3*c^3-48*a^2*b
^2*c^2+12*a*b^4*c-b^6)*c^3-49/a/(c*x^2+b*x+a)^3*b^5/(64*a^3*c^3-48*a^2*b^2*
c^2+12*a*b^4*c-b^6)*c-76/a/(c*x^2+b*x+a)^3*c^6/(64*a^3*c^3-48*a^2*b^2*c^2+1
2*a*b^4*c-b^6)*x^5-496/(c*x^2+b*x+a)^3*b/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^
4*c-b^6)*x^2*c^4-166/(c*x^2+b*x+a)^3/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-
b^6)*x*b^2*c^3-280/a/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(4*a*c-b^2)
^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^4-96/a^3/(64*a^3*c^3-48*a^2*b^
2*c^2+12*a*b^4*c-b^6)*c^2*ln(c*x^2+b*x+a)*b^3+24/a^4/(64*a^3*c^3-48*a^2*b^2
*c^2+12*a*b^4*c-b^6)*c*ln(c*x^2+b*x+a)*b^5+535/3/(c*x^2+b*x+a)^3*b^3/(64*a^
3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*c^2-544/3/(c*x^2+b*x+a)^3*c^5/(64*a^3*
c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^3-2/a^5/(64*a^3*c^3-48*a^2*b^2*c^2+12*
a*b^4*c-b^6)*ln(c*x^2+b*x+a)*b^7+13/3/a^2/(c*x^2+b*x+a)^3*b^7/(64*a^3*c^3-4
8*a^2*b^2*c^2+12*a*b^4*c-b^6)-93/a^3/(c*x^2+b*x+a)^3*c^2/(64*a^3*c^3-48*a^2
*b^2*c^2+12*a*b^4*c-b^6)*x^3*b^6-98/a^3/(c*x^2+b*x+a)^3*b^5*c^3/(64*a^3*c^3
-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^4+9/a^4/(c*x^2+b*x+a)^3*b^7*c^2/(64*a^3*c
^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^4-102/a/(c*x^2+b*x+a)^3*c^4/(64*a^3*c^3
-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^3*b^2-30/a^2/(c*x^2+b*x+a)^3*b^5/(64*a^3*
c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^2*c^2-20/a^3/(c*x^2+b*x+a)^3*b^7/(64*a
^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^2*c+243/a/(c*x^2+b*x+a)^3/(64*a^3*c
^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x*b^4*c^2-75/a^2/(c*x^2+b*x+a)^3/(64*a^3*
c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x*b^6*c-280/a^3/(64*a^3*c^3-48*a^2*b^2*c
^2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^
4*c^2+9/a^4/(c*x^2+b*x+a)^3*c/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^
3*b^8+832/3/a^2/(c*x^2+b*x+a)^3*c^3/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b
^6)*x^3*b^4-286/a/(c*x^2+b*x+a)^3*b*c^5/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4
*c-b^6)*x^4+332/a^2/(c*x^2+b*x+a)^3*b^3*c^4/(64*a^3*c^3-48*a^2*b^2*c^2+12*a
*b^4*c-b^6)*x^4+3/a^4/(c*x^2+b*x+a)^3*c^3/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b
^4*c-b^6)*x^5*b^6+560/a^2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(4*a*c
-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*c^3+56/a^4/(64*a^3*c^3-
48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^
2)^(1/2))*b^6*c+397/a/(c*x^2+b*x+a)^3*b^3/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b
^4*c-b^6)*x^2*c^3+104/a^2/(c*x^2+b*x+a)^3*c^5/(64*a^3*c^3-48*a^2*b^2*c^2+12
*a*b^4*c-b^6)*x^5*b^2-32/a^3/(c*x^2+b*x+a)^3*c^4/(64*a^3*c^3-48*a^2*b^2*c^2
+12*a*b^4*c-b^6)*x^5*b^4-4*b*ln(x)/a^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 17.9034, size = 8609, normalized size = 24.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/3*(3*a^4*b^8 - 48*a^5*b^6*c + 288*a^6*b^4*c^2 - 768*a^7*b^2*c^3 + 768*a
^8*c^4 + 12*(a*b^8*c^3 - 15*a^2*b^6*c^4 + 82*a^3*b^4*c^5 - 187*a^4*b^2*c^6
+ 140*a^5*c^7)*x^6 + 6*(6*a*b^9*c^2 - 91*a^2*b^7*c^3 + 506*a^3*b^5*c^4 - 11
91*a^4*b^3*c^5 + 956*a^5*b*c^6)*x^5 + 2*(18*a*b^10*c - 261*a^2*b^8*c^2 + 13
34*a^3*b^6*c^3 - 2537*a^4*b^4*c^4 + 340*a^5*b^2*c^5 + 2240*a^6*c^6)*x^4 + 3
*(4*a*b^11 - 42*a^2*b^9*c + 50*a^3*b^7*c^2 + 837*a^4*b^5*c^3 - 3364*a^5*b^3
*c^4 + 3520*a^6*b*c^5)*x^3 + 3*(10*a^2*b^10 - 148*a^3*b^8*c + 783*a^4*b^6*c
^2 - 1618*a^5*b^4*c^3 + 548*a^6*b^2*c^4 + 1232*a^7*c^5)*x^2 + 6*((b^8*c^3 -
 14*a*b^6*c^4 + 70*a^2*b^4*c^5 - 140*a^3*b^2*c^6 + 70*a^4*c^7)*x^7 + 3*(b^9
*c^2 - 14*a*b^7*c^3 + 70*a^2*b^5*c^4 - 140*a^3*b^3*c^5 + 70*a^4*b*c^6)*x^6
+ 3*(b^10*c - 13*a*b^8*c^2 + 56*a^2*b^6*c^3 - 70*a^3*b^4*c^4 - 70*a^4*b^2*c
^5 + 70*a^5*c^6)*x^5 + (b^11 - 8*a*b^9*c - 14*a^2*b^7*c^2 + 280*a^3*b^5*c^3
 - 770*a^4*b^3*c^4 + 420*a^5*b*c^5)*x^4 + 3*(a*b^10 - 13*a^2*b^8*c + 56*a^3
*b^6*c^2 - 70*a^4*b^4*c^3 - 70*a^5*b^2*c^4 + 70*a^6*c^5)*x^3 + 3*(a^2*b^9 -
 14*a^3*b^7*c + 70*a^4*b^5*c^2 - 140*a^5*b^3*c^3 + 70*a^6*b*c^4)*x^2 + (a^3
*b^8 - 14*a^4*b^6*c + 70*a^5*b^4*c^2 - 140*a^6*b^2*c^3 + 70*a^7*c^4)*x)*sqr
t(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(
2*c*x + b))/(c*x^2 + b*x + a)) + (22*a^3*b^9 - 343*a^4*b^7*c + 1987*a^5*b^5
*c^2 - 5034*a^6*b^3*c^3 + 4664*a^7*b*c^4)*x - 6*((b^9*c^3 - 16*a*b^7*c^4 +
96*a^2*b^5*c^5 - 256*a^3*b^3*c^6 + 256*a^4*b*c^7)*x^7 + 3*(b^10*c^2 - 16*a*
b^8*c^3 + 96*a^2*b^6*c^4 - 256*a^3*b^4*c^5 + 256*a^4*b^2*c^6)*x^6 + 3*(b^11
*c - 15*a*b^9*c^2 + 80*a^2*b^7*c^3 - 160*a^3*b^5*c^4 + 256*a^5*b*c^6)*x^5 +
 (b^12 - 10*a*b^10*c + 320*a^3*b^6*c^3 - 1280*a^4*b^4*c^4 + 1536*a^5*b^2*c^
5)*x^4 + 3*(a*b^11 - 15*a^2*b^9*c + 80*a^3*b^7*c^2 - 160*a^4*b^5*c^3 + 256*
a^6*b*c^5)*x^3 + 3*(a^2*b^10 - 16*a^3*b^8*c + 96*a^4*b^6*c^2 - 256*a^5*b^4*
c^3 + 256*a^6*b^2*c^4)*x^2 + (a^3*b^9 - 16*a^4*b^7*c + 96*a^5*b^5*c^2 - 256
*a^6*b^3*c^3 + 256*a^7*b*c^4)*x)*log(c*x^2 + b*x + a) + 12*((b^9*c^3 - 16*a
*b^7*c^4 + 96*a^2*b^5*c^5 - 256*a^3*b^3*c^6 + 256*a^4*b*c^7)*x^7 + 3*(b^10*
c^2 - 16*a*b^8*c^3 + 96*a^2*b^6*c^4 - 256*a^3*b^4*c^5 + 256*a^4*b^2*c^6)*x^
6 + 3*(b^11*c - 15*a*b^9*c^2 + 80*a^2*b^7*c^3 - 160*a^3*b^5*c^4 + 256*a^5*b
*c^6)*x^5 + (b^12 - 10*a*b^10*c + 320*a^3*b^6*c^3 - 1280*a^4*b^4*c^4 + 1536
*a^5*b^2*c^5)*x^4 + 3*(a*b^11 - 15*a^2*b^9*c + 80*a^3*b^7*c^2 - 160*a^4*b^5
*c^3 + 256*a^6*b*c^5)*x^3 + 3*(a^2*b^10 - 16*a^3*b^8*c + 96*a^4*b^6*c^2 - 2
56*a^5*b^4*c^3 + 256*a^6*b^2*c^4)*x^2 + (a^3*b^9 - 16*a^4*b^7*c + 96*a^5*b^
5*c^2 - 256*a^6*b^3*c^3 + 256*a^7*b*c^4)*x)*log(x))/((a^5*b^8*c^3 - 16*a^6*
b^6*c^4 + 96*a^7*b^4*c^5 - 256*a^8*b^2*c^6 + 256*a^9*c^7)*x^7 + 3*(a^5*b^9*
c^2 - 16*a^6*b^7*c^3 + 96*a^7*b^5*c^4 - 256*a^8*b^3*c^5 + 256*a^9*b*c^6)*x^
6 + 3*(a^5*b^10*c - 15*a^6*b^8*c^2 + 80*a^7*b^6*c^3 - 160*a^8*b^4*c^4 + 256
*a^10*c^6)*x^5 + (a^5*b^11 - 10*a^6*b^9*c + 320*a^8*b^5*c^3 - 1280*a^9*b^3*
c^4 + 1536*a^10*b*c^5)*x^4 + 3*(a^6*b^10 - 15*a^7*b^8*c + 80*a^8*b^6*c^2 -
160*a^9*b^4*c^3 + 256*a^11*c^5)*x^3 + 3*(a^7*b^9 - 16*a^8*b^7*c + 96*a^9*b^
5*c^2 - 256*a^10*b^3*c^3 + 256*a^11*b*c^4)*x^2 + (a^8*b^8 - 16*a^9*b^6*c +
96*a^10*b^4*c^2 - 256*a^11*b^2*c^3 + 256*a^12*c^4)*x), -1/3*(3*a^4*b^8 - 48
*a^5*b^6*c + 288*a^6*b^4*c^2 - 768*a^7*b^2*c^3 + 768*a^8*c^4 + 12*(a*b^8*c^
3 - 15*a^2*b^6*c^4 + 82*a^3*b^4*c^5 - 187*a^4*b^2*c^6 + 140*a^5*c^7)*x^6 +
6*(6*a*b^9*c^2 - 91*a^2*b^7*c^3 + 506*a^3*b^5*c^4 - 1191*a^4*b^3*c^5 + 956*
a^5*b*c^6)*x^5 + 2*(18*a*b^10*c - 261*a^2*b^8*c^2 + 1334*a^3*b^6*c^3 - 2537
*a^4*b^4*c^4 + 340*a^5*b^2*c^5 + 2240*a^6*c^6)*x^4 + 3*(4*a*b^11 - 42*a^2*b
^9*c + 50*a^3*b^7*c^2 + 837*a^4*b^5*c^3 - 3364*a^5*b^3*c^4 + 3520*a^6*b*c^5
)*x^3 + 3*(10*a^2*b^10 - 148*a^3*b^8*c + 783*a^4*b^6*c^2 - 1618*a^5*b^4*c^3
 + 548*a^6*b^2*c^4 + 1232*a^7*c^5)*x^2 + 12*((b^8*c^3 - 14*a*b^6*c^4 + 70*a
^2*b^4*c^5 - 140*a^3*b^2*c^6 + 70*a^4*c^7)*x^7 + 3*(b^9*c^2 - 14*a*b^7*c^3
+ 70*a^2*b^5*c^4 - 140*a^3*b^3*c^5 + 70*a^4*b*c^6)*x^6 + 3*(b^10*c - 13*a*b
^8*c^2 + 56*a^2*b^6*c^3 - 70*a^3*b^4*c^4 - 70*a^4*b^2*c^5 + 70*a^5*c^6)*x^5
 + (b^11 - 8*a*b^9*c - 14*a^2*b^7*c^2 + 280*a^3*b^5*c^3 - 770*a^4*b^3*c^4 +
```

```
420*a^5*b*c^5)*x^4 + 3*(a*b^10 - 13*a^2*b^8*c + 56*a^3*b^6*c^2 - 70*a^4*b^
4*c^3 - 70*a^5*b^2*c^4 + 70*a^6*c^5)*x^3 + 3*(a^2*b^9 - 14*a^3*b^7*c + 70*a
^4*b^5*c^2 - 140*a^5*b^3*c^3 + 70*a^6*b*c^4)*x^2 + (a^3*b^8 - 14*a^4*b^6*c
+ 70*a^5*b^4*c^2 - 140*a^6*b^2*c^3 + 70*a^7*c^4)*x)*sqrt(-b^2 + 4*a*c)*arct
an(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (22*a^3*b^9 - 343*a^4*b
^7*c + 1987*a^5*b^5*c^2 - 5034*a^6*b^3*c^3 + 4664*a^7*b*c^4)*x - 6*((b^9*c^
3 - 16*a*b^7*c^4 + 96*a^2*b^5*c^5 - 256*a^3*b^3*c^6 + 256*a^4*b*c^7)*x^7 +
3*(b^10*c^2 - 16*a*b^8*c^3 + 96*a^2*b^6*c^4 - 256*a^3*b^4*c^5 + 256*a^4*b^2
*c^6)*x^6 + 3*(b^11*c - 15*a*b^9*c^2 + 80*a^2*b^7*c^3 - 160*a^3*b^5*c^4 + 2
56*a^5*b*c^6)*x^5 + (b^12 - 10*a*b^10*c + 320*a^3*b^6*c^3 - 1280*a^4*b^4*c^
4 + 1536*a^5*b^2*c^5)*x^4 + 3*(a*b^11 - 15*a^2*b^9*c + 80*a^3*b^7*c^2 - 160
*a^4*b^5*c^3 + 256*a^6*b*c^5)*x^3 + 3*(a^2*b^10 - 16*a^3*b^8*c + 96*a^4*b^6
*c^2 - 256*a^5*b^4*c^3 + 256*a^6*b^2*c^4)*x^2 + (a^3*b^9 - 16*a^4*b^7*c + 9
6*a^5*b^5*c^2 - 256*a^6*b^3*c^3 + 256*a^7*b*c^4)*x)*log(c*x^2 + b*x + a) +
12*((b^9*c^3 - 16*a*b^7*c^4 + 96*a^2*b^5*c^5 - 256*a^3*b^3*c^6 + 256*a^4*b*
c^7)*x^7 + 3*(b^10*c^2 - 16*a*b^8*c^3 + 96*a^2*b^6*c^4 - 256*a^3*b^4*c^5 +
256*a^4*b^2*c^6)*x^6 + 3*(b^11*c - 15*a*b^9*c^2 + 80*a^2*b^7*c^3 - 160*a^3*
b^5*c^4 + 256*a^5*b*c^6)*x^5 + (b^12 - 10*a*b^10*c + 320*a^3*b^6*c^3 - 1280
*a^4*b^4*c^4 + 1536*a^5*b^2*c^5)*x^4 + 3*(a*b^11 - 15*a^2*b^9*c + 80*a^3*b^
7*c^2 - 160*a^4*b^5*c^3 + 256*a^6*b*c^5)*x^3 + 3*(a^2*b^10 - 16*a^3*b^8*c +
 96*a^4*b^6*c^2 - 256*a^5*b^4*c^3 + 256*a^6*b^2*c^4)*x^2 + (a^3*b^9 - 16*a^
4*b^7*c + 96*a^5*b^5*c^2 - 256*a^6*b^3*c^3 + 256*a^7*b*c^4)*x)*log(x))/((a^
5*b^8*c^3 - 16*a^6*b^6*c^4 + 96*a^7*b^4*c^5 - 256*a^8*b^2*c^6 + 256*a^9*c^7
)*x^7 + 3*(a^5*b^9*c^2 - 16*a^6*b^7*c^3 + 96*a^7*b^5*c^4 - 256*a^8*b^3*c^5
+ 256*a^9*b*c^6)*x^6 + 3*(a^5*b^10*c - 15*a^6*b^8*c^2 + 80*a^7*b^6*c^3 - 16
0*a^8*b^4*c^4 + 256*a^10*c^6)*x^5 + (a^5*b^11 - 10*a^6*b^9*c + 320*a^8*b^5*
c^3 - 1280*a^9*b^3*c^4 + 1536*a^10*b*c^5)*x^4 + 3*(a^6*b^10 - 15*a^7*b^8*c
+ 80*a^8*b^6*c^2 - 160*a^9*b^4*c^3 + 256*a^11*c^5)*x^3 + 3*(a^7*b^9 - 16*a^
8*b^7*c + 96*a^9*b^5*c^2 - 256*a^10*b^3*c^3 + 256*a^11*b*c^4)*x^2 + (a^8*b^
8 - 16*a^9*b^6*c + 96*a^10*b^4*c^2 - 256*a^11*b^2*c^3 + 256*a^12*c^4)*x)]
```

Sympy [B] time = 114.677, size = 9418, normalized size = 26.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**2+b*x+a)**4,x)
```

```
[Out] (2*b/a**5 - 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 +
 70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(a**5*(16384*a**7*c**7 - 28672*a**
6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c
**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14)))*log(x + (-6864896*a**21
*b*c**11*(2*b/a**5 - 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b*
*2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(a**5*(16384*a**7*c**7 -
28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a*
*3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14)))**2 + 19451904*
a**20*b**3*c**10*(2*b/a**5 - 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140
*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(a**5*(16384*a**7
*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 +
 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14)))**2 - 2
4960000*a**19*b**5*c**9*(2*b/a**5 - 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**
4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(a**5*(163
84*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6
*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14)))
**2 + 19157248*a**18*b**7*c**8*(2*b/a**5 - 2*sqrt(-(4*a*c - b**2)**7)*(70*a
**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(a*
```

5(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a*
*4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b
14)))2 - 9777216*a**17*b**9*c**7*(2*b/a**5 - 2*sqrt(-(4*a*c - b**2)**7)
*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b**
8)/(a**5*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8
960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12
*c - b**14)))**2 - 1254400*a**17*c**12*(2*b/a**5 - 2*sqrt(-(4*a*c - b**2)**
7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b
8)/(a5*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 -
 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**
12*c - b**14))) + 3485552*a**16*b**11*c**6*(2*b/a**5 - 2*sqrt(-(4*a*c - b**
2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c
 + b**8)/(a**5*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c*
*5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a
*b**12*c - b**14)))**2 - 4017152*a**16*b**2*c**11*(2*b/a**5 - 2*sqrt(-(4*a*
c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a
*b**6*c + b**8)/(a**5*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*
b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2
 + 28*a*b**12*c - b**14))) - 886004*a**15*b**13*c**5*(2*b/a**5 - 2*sqrt(-(4
*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 1
4*a*b**6*c + b**8)/(a**5*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a*
*5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c
2 + 28*a*b12*c - b**14)))**2 + 12987008*a**15*b**4*c**10*(2*b/a**5 - 2*
sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*
c**2 - 14*a*b**6*c + b**8)/(a**5*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 +
21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2
*b**10*c**2 + 28*a*b**12*c - b**14))) + 160635*a**14*b**15*c**4*(2*b/a**5 -
 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b*
*4*c**2 - 14*a*b**6*c + b**8)/(a**5*(16384*a**7*c**7 - 28672*a**6*b**2*c**6
 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a
2*b10*c**2 + 28*a*b**12*c - b**14)))**2 - 14915520*a**14*b**6*c**9*(2*b
/a**5 - 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70*
a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(a**5*(16384*a**7*c**7 - 28672*a**6*b*
*2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3
 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14))) - 20362*a**13*b**17*c**3*(2
*b/a**5 - 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 7
0*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(a**5*(16384*a**7*c**7 - 28672*a**6*
b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**
3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14)))**2 + 9737948*a**13*b**8*c
8*(2*b/a5 - 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c*
*3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(a**5*(16384*a**7*c**7 - 28672
*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b*
*8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14))) + 1719*a**12*b**19*
c**2*(2*b/a**5 - 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c
3 + 70*a2*b**4*c**2 - 14*a*b**6*c + b**8)/(a**5*(16384*a**7*c**7 - 2867
2*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b
8*c3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14)))**2 - 4124656*a**12
*b**10*c**7*(2*b/a**5 - 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3
*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(a**5*(16384*a**7*c**7
 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240
*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14))) - 5017600*a
12*b*c12 - 87*a**11*b**21*c*(2*b/a**5 - 2*sqrt(-(4*a*c - b**2)**7)*(70*
a**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(a
5*(16384*a7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a
4*b6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c -
b**14)))**2 + 1194984*a**11*b**12*c**6*(2*b/a**5 - 2*sqrt(-(4*a*c - b**2)**
7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b
8)/(a5*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 -
 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**

12*c - b**14))) + 93769728*a**11*b**3*c**11 + 2*a**10*b**23*(2*b/a**5 - 2*s
qrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c
2 - 14*a*b6*c + b**8)/(a**5*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 2
1504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*
b**10*c**2 + 28*a*b**12*c - b**14)))**2 - 240632*a**10*b**14*c**5*(2*b/a**5
 - 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*
b**4*c**2 - 14*a*b**6*c + b**8)/(a**5*(16384*a**7*c**7 - 28672*a**6*b**2*c*
*6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336
*a**2*b**10*c**2 + 28*a*b**12*c - b**14))) - 259282432*a**10*b**5*c**10 + 3
3300*a**9*b**16*c**4*(2*b/a**5 - 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 -
 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(a**5*(16384*
a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c*
*4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14))) +
339697920*a**9*b**7*c**9 - 3032*a**8*b**18*c**3*(2*b/a**5 - 2*sqrt(-(4*a*c
- b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b
6*c + b8)/(a**5*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b*
*4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 +
 28*a*b**12*c - b**14))) - 267564176*a**8*b**9*c**8 + 164*a**7*b**20*c**2*(
2*b/a**5 - 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 +
70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(a**5*(16384*a**7*c**7 - 28672*a**6
*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c*
*3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14))) + 139936832*a**7*b**11*c
7 - 4*a6*b**22*c*(2*b/a**5 - 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 -
 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(a**5*(16384*
a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c*
*4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14))) -
50988896*a**6*b**13*c**6 + 13213536*a**5*b**15*c**5 - 2436960*a**4*b**17*c*
*4 + 313664*a**3*b**19*c**3 - 26848*a**2*b**21*c**2 + 1376*a*b**23*c - 32*b
25)/(1372000*a12*c**13 + 33055680*a**11*b**2*c**12 - 134248800*a**10*b*
*4*c**11 + 211721440*a**9*b**6*c**10 - 187538736*a**8*b**8*c**9 + 106627392
*a**7*b**10*c**8 - 41403488*a**6*b**12*c**7 + 11287584*a**5*b**14*c**6 - 21
70560*a**4*b**16*c**5 + 289408*a**3*b**18*c**4 - 25536*a**2*b**20*c**3 + 13
44*a*b**22*c**2 - 32*b**24*c)) + (2*b/a**5 + 2*sqrt(-(4*a*c - b**2)**7)*(70
*a**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(
a**5*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*
a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c -
 b**14)))*log(x + (-6864896*a**21*b*c**11*(2*b/a**5 + 2*sqrt(-(4*a*c - b**2
)**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c
+ b**8)/(a**5*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**
5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*
b**12*c - b**14)))**2 + 19451904*a**20*b**3*c**10*(2*b/a**5 + 2*sqrt(-(4*a*
c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a
*b**6*c + b**8)/(a**5*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*
b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2
 + 28*a*b**12*c - b**14)))**2 - 24960000*a**19*b**5*c**9*(2*b/a**5 + 2*sqrt
(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2
 - 14*a*b**6*c + b**8)/(a**5*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 2150
4*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**
10*c**2 + 28*a*b**12*c - b**14)))**2 + 19157248*a**18*b**7*c**8*(2*b/a**5 +
 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b*
*4*c**2 - 14*a*b**6*c + b**8)/(a**5*(16384*a**7*c**7 - 28672*a**6*b**2*c**6
 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a
2*b10*c**2 + 28*a*b**12*c - b**14)))**2 - 9777216*a**17*b**9*c**7*(2*b/
a**5 + 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70*a
2*b4*c**2 - 14*a*b**6*c + b**8)/(a**5*(16384*a**7*c**7 - 28672*a**6*b**
2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 -
 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14)))**2 - 1254400*a**17*c**12*(2*
b/a**5 + 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70
*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(a**5*(16384*a**7*c**7 - 28672*a**6*b

2*c6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3
 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14))) + 3485552*a**16*b**11*c**6
*(2*b/a**5 + 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3
+ 70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(a**5*(16384*a**7*c**7 - 28672*a*
*6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*
c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14)))**2 - 4017152*a**16*b**
2*c**11*(2*b/a**5 + 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**
2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(a**5*(16384*a**7*c**7 - 2
8672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**
3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14))) - 886004*a**15*
b**13*c**5*(2*b/a**5 + 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*
b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(a**5*(16384*a**7*c**7
 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*
a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14)))**2 + 1298700
8*a**15*b**4*c**10*(2*b/a**5 + 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 1
40*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(a**5*(16384*a*
*7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4
 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14))) + 16
0635*a**14*b**15*c**4*(2*b/a**5 + 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4
 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(a**5*(16384
*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c
4 + 2240*a3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14)))**
2 - 14915520*a**14*b**6*c**9*(2*b/a**5 + 2*sqrt(-(4*a*c - b**2)**7)*(70*a**
4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(a**5
*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4
*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**
14))) - 20362*a**13*b**17*c**3*(2*b/a**5 + 2*sqrt(-(4*a*c - b**2)**7)*(70*a
4*c4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(a*
5(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a*
*4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b
14)))2 + 9737948*a**13*b**8*c**8*(2*b/a**5 + 2*sqrt(-(4*a*c - b**2)**7)
*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b**
8)/(a**5*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8
960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12
*c - b**14))) + 1719*a**12*b**19*c**2*(2*b/a**5 + 2*sqrt(-(4*a*c - b**2)**7
)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b*
*8)/(a**5*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 -
8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**1
2*c - b**14)))**2 - 4124656*a**12*b**10*c**7*(2*b/a**5 + 2*sqrt(-(4*a*c - b
2)7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6
*c + b**8)/(a**5*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*
c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28
*a*b**12*c - b**14))) - 5017600*a**12*b*c**12 - 87*a**11*b**21*c*(2*b/a**5
+ 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b
4*c2 - 14*a*b**6*c + b**8)/(a**5*(16384*a**7*c**7 - 28672*a**6*b**2*c**
6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*
a**2*b**10*c**2 + 28*a*b**12*c - b**14)))**2 + 1194984*a**11*b**12*c**6*(2*
b/a**5 + 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70
*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(a**5*(16384*a**7*c**7 - 28672*a**6*b
2*c6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3
 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14))) + 93769728*a**11*b**3*c**1
1 + 2*a**10*b**23*(2*b/a**5 + 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 14
0*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(a**5*(16384*a**
7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4
+ 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14)))**2 -
240632*a**10*b**14*c**5*(2*b/a**5 + 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**
4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(a**5*(163
84*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6
*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14)))

```
 - 259282432*a**10*b**5*c**10 + 33300*a**9*b**16*c**4*(2*b/a**5 + 2*sqrt(-(
4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 -
14*a*b**6*c + b**8)/(a**5*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a
**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*
c**2 + 28*a*b**12*c - b**14))) + 339697920*a**9*b**7*c**9 - 3032*a**8*b**18
*c**3*(2*b/a**5 + 2*sqrt(-(4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*
c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(a**5*(16384*a**7*c**7 - 286
72*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*
b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c - b**14))) - 267564176*a**8*
b**9*c**8 + 164*a**7*b**20*c**2*(2*b/a**5 + 2*sqrt(-(4*a*c - b**2)**7)*(70*
a**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 - 14*a*b**6*c + b**8)/(a
**5*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a**5*b**4*c**5 - 8960*a
**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*c**2 + 28*a*b**12*c -
b**14))) + 139936832*a**7*b**11*c**7 - 4*a**6*b**22*c*(2*b/a**5 + 2*sqrt(-(
4*a*c - b**2)**7)*(70*a**4*c**4 - 140*a**3*b**2*c**3 + 70*a**2*b**4*c**2 -
14*a*b**6*c + b**8)/(a**5*(16384*a**7*c**7 - 28672*a**6*b**2*c**6 + 21504*a
**5*b**4*c**5 - 8960*a**4*b**6*c**4 + 2240*a**3*b**8*c**3 - 336*a**2*b**10*
c**2 + 28*a*b**12*c - b**14))) - 50988896*a**6*b**13*c**6 + 13213536*a**5*b
**15*c**5 - 2436960*a**4*b**17*c**4 + 313664*a**3*b**19*c**3 - 26848*a**2*b
**21*c**2 + 1376*a*b**23*c - 32*b**25)/(1372000*a**12*c**13 + 33055680*a**1
1*b**2*c**12 - 134248800*a**10*b**4*c**11 + 211721440*a**9*b**6*c**10 - 187
538736*a**8*b**8*c**9 + 106627392*a**7*b**10*c**8 - 41403488*a**6*b**12*c**
7 + 11287584*a**5*b**14*c**6 - 2170560*a**4*b**16*c**5 + 289408*a**3*b**18*
c**4 - 25536*a**2*b**20*c**3 + 1344*a*b**22*c**2 - 32*b**24*c)) - (192*a**6
*c**3 - 144*a**5*b**2*c**2 + 36*a**4*b**4*c - 3*a**3*b**6 + x**6*(420*a**3*
c**6 - 456*a**2*b**2*c**5 + 132*a*b**4*c**4 - 12*b**6*c**3) + x**5*(1434*a*
*3*b*c**5 - 1428*a**2*b**3*c**4 + 402*a*b**5*c**3 - 36*b**7*c**2) + x**4*(1
120*a**4*c**5 + 450*a**3*b**2*c**4 - 1156*a**2*b**4*c**3 + 378*a*b**6*c**2
 - 36*b**8*c) + x**3*(2640*a**4*b*c**4 - 1863*a**3*b**3*c**3 + 162*a**2*b**5
*c**2 + 78*a*b**7*c - 12*b**9) + x**2*(924*a**5*c**4 + 642*a**4*b**2*c**3 -
 1053*a**3*b**4*c**2 + 324*a**2*b**6*c - 30*a*b**8) + x*(1166*a**5*b*c**3 -
 967*a**4*b**3*c**2 + 255*a**3*b**5*c - 22*a**2*b**7))/(x**7*(192*a**7*c**6
 - 144*a**6*b**2*c**5 + 36*a**5*b**4*c**4 - 3*a**4*b**6*c**3) + x**6*(576*a
**7*b*c**5 - 432*a**6*b**3*c**4 + 108*a**5*b**5*c**3 - 9*a**4*b**7*c**2) +
x**5*(576*a**8*c**5 + 144*a**7*b**2*c**4 - 324*a**6*b**4*c**3 + 99*a**5*b**
6*c**2 - 9*a**4*b**8*c) + x**4*(1152*a**8*b*c**4 - 672*a**7*b**3*c**3 + 72*
a**6*b**5*c**2 + 18*a**5*b**7*c - 3*a**4*b**9) + x**3*(576*a**9*c**4 + 144*
a**8*b**2*c**3 - 324*a**7*b**4*c**2 + 99*a**6*b**6*c - 9*a**5*b**8) + x**2*
(576*a**9*b*c**3 - 432*a**8*b**3*c**2 + 108*a**7*b**5*c - 9*a**6*b**7) + x*
(192*a**10*c**3 - 144*a**9*b**2*c**2 + 36*a**8*b**4*c - 3*a**7*b**6)) - 4*b
*log(x)/a**5
```

Giac [A] time = 1.13254, size = 670, normalized size = 1.9

$$\frac{4\left(b^8-14\,ab^6c+70\,a^2b^4c^2-140\,a^3b^2c^3+70\,a^4c^4\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^5b^6-12\,a^6b^4c+48\,a^7b^2c^2-64\,a^8c^3\right)\sqrt{-b^2+4\,ac}}+\frac{2\,b\log\left(cx^2+bx+a\right)}{a^5}-\frac{4\,b\log\left(|x|\right)}{a^5}-\frac{3\,a^4b^6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^4,x, algorithm="giac")
```

```
[Out] 4*(b^8 - 14*a*b^6*c + 70*a^2*b^4*c^2 - 140*a^3*b^2*c^3 + 70*a^4*c^4)*arctan
((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2
- 64*a^8*c^3)*sqrt(-b^2 + 4*a*c)) + 2*b*log(c*x^2 + b*x + a)/a^5 - 4*b*log(
abs(x))/a^5 - 1/3*(3*a^4*b^6 - 36*a^5*b^4*c + 144*a^6*b^2*c^2 - 192*a^7*c^3
 + 12*(a*b^6*c^3 - 11*a^2*b^4*c^4 + 38*a^3*b^2*c^5 - 35*a^4*c^6)*x^6 + 6*(6
```

```
*a*b^7*c^2 - 67*a^2*b^5*c^3 + 238*a^3*b^3*c^4 - 239*a^4*b*c^5)*x^5 + 2*(18*
a*b^8*c - 189*a^2*b^6*c^2 + 578*a^3*b^4*c^3 - 225*a^4*b^2*c^4 - 560*a^5*c^5
)*x^4 + 3*(4*a*b^9 - 26*a^2*b^7*c - 54*a^3*b^5*c^2 + 621*a^4*b^3*c^3 - 880*
a^5*b*c^4)*x^3 + 3*(10*a^2*b^8 - 108*a^3*b^6*c + 351*a^4*b^4*c^2 - 214*a^5*
b^2*c^3 - 308*a^6*c^4)*x^2 + (22*a^3*b^7 - 255*a^4*b^5*c + 967*a^5*b^3*c^2
- 1166*a^6*b*c^3)*x)/((c*x^2 + b*x + a)^3*(b^2 - 4*a*c)^3*a^5*x)
```

3.2221 $\int \frac{(d+ex)^5}{\left(a+bx+cx^2\right)^5} dx$

Optimal. Leaf size=388

$$\frac{(d+ex)^3\left(-cx\left(-4ce(35bd-8ae)+27b^2e^2+140c^2d^2\right)-10bc\left(3ae^2+7cd^2\right)+28ac^2de+63b^2cde-10b^3e^2\right)}{12\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)^2}+\frac{5(d+e\ldots}{}$$

```
[Out] -((b + 2*c*x)*(d + e*x)^5)/(4*(b^2 - 4*a*c)*(a + b*x + c*x^2)^4) + ((d + e*
x)^4*(14*b*c*d - 5*b^2*e - 8*a*c*e + 14*c*(2*c*d - b*e)*x))/(12*(b^2 - 4*a*
c)^2*(a + b*x + c*x^2)^3) + ((d + e*x)^3*(63*b^2*c*d*e + 28*a*c^2*d*e - 10*
b^3*e^2 - 10*b*c*(7*c*d^2 + 3*a*e^2) - c*(140*c^2*d^2 + 27*b^2*e^2 - 4*c*e*
(35*b*d - 8*a*e))*x))/(12*(b^2 - 4*a*c)^3*(a + b*x + c*x^2)^2) + (5*(2*c*d
- b*e)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e))*(d + e*x)*(b*d - 2*a*e +
 (2*c*d - b*e)*x))/(2*(b^2 - 4*a*c)^4*(a + b*x + c*x^2)) - (10*(2*c*d - b*e
)*(c*d^2 - b*d*e + a*e^2)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e))*ArcTa
nh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(9/2)
```

Rubi [A] time = 0.561369, antiderivative size = 388, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {736, 820, 804, 722, 618, 206}

$$\frac{(d+ex)^3\left(-cx\left(-4ce(35bd-8ae)+27b^2e^2+140c^2d^2\right)-10bc\left(3ae^2+7cd^2\right)+28ac^2de+63b^2cde-10b^3e^2\right)}{12\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)^2}+\frac{5(d+e\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5/(a + b*x + c*x^2)^5,x]
```


```
[Out] -((b + 2*c*x)*(d + e*x)^5)/(4*(b^2 - 4*a*c)*(a + b*x + c*x^2)^4) + ((d + e*
x)^4*(14*b*c*d - 5*b^2*e - 8*a*c*e + 14*c*(2*c*d - b*e)*x))/(12*(b^2 - 4*a*
c)^2*(a + b*x + c*x^2)^3) + ((d + e*x)^3*(63*b^2*c*d*e + 28*a*c^2*d*e - 10*
b^3*e^2 - 10*b*c*(7*c*d^2 + 3*a*e^2) - c*(140*c^2*d^2 + 27*b^2*e^2 - 4*c*e*
(35*b*d - 8*a*e))*x))/(12*(b^2 - 4*a*c)^3*(a + b*x + c*x^2)^2) + (5*(2*c*d
- b*e)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e))*(d + e*x)*(b*d - 2*a*e +
 (2*c*d - b*e)*x))/(2*(b^2 - 4*a*c)^4*(a + b*x + c*x^2)) - (10*(2*c*d - b*e
)*(c*d^2 - b*d*e + a*e^2)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e))*ArcTa
nh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(9/2)
```


Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] - Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)
*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p + 1
), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 0] && (L
tQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c,
d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*
```

```
(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p +
 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*Simp[g*
(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(
m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (I
ntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 804

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*
(b*f - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(m*(b*(
e*f + d*g) - 2*(c*d*f + a*e*g)))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m
- 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0] && LtQ[p, -1]
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^5}{\left(a+bx+cx^2\right)^5}\,dx = -\frac{(b+2cx)(d+ex)^5}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4} + \frac{\int \frac{(d+ex)^4(-14cd+5be-4cex)}{\left(a+bx+cx^2\right)^4}\,dx}{4\left(b^2-4ac\right)}$$

$$= -\frac{(b+2cx)(d+ex)^5}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4} + \frac{(d+ex)^4\left(14bcd-5b^2e-8ace+14c(2cd-be)x\right)}{12\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^3} - \int \frac{(d+ex)^3(\cdots}{\cdots}$$

$$= -\frac{(b+2cx)(d+ex)^5}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4} + \frac{(d+ex)^4\left(14bcd-5b^2e-8ace+14c(2cd-be)x\right)}{12\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^3} + \frac{(d+ex)^3\cdots}{\cdots}$$

$$= -\frac{(b+2cx)(d+ex)^5}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4} + \frac{(d+ex)^4\left(14bcd-5b^2e-8ace+14c(2cd-be)x\right)}{12\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^3} + \frac{(d+ex)^3\cdots}{\cdots}$$

$$= -\frac{(b+2cx)(d+ex)^5}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4} + \frac{(d+ex)^4\left(14bcd-5b^2e-8ace+14c(2cd-be)x\right)}{12\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^3} + \frac{(d+ex)^3\cdots}{\cdots}$$

$$= -\frac{(b+2cx)(d+ex)^5}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4} + \frac{(d+ex)^4\left(14bcd-5b^2e-8ace+14c(2cd-be)x\right)}{12\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^3} + \frac{(d+ex)^3\cdots}{\cdots}$$

Mathematica [B] time = 2.69068, size = 985, normalized size = 2.54

$$\frac{1}{12}\left(\frac{30(2cd-be)\left(7c^3d^4-2c^2e(7bd-5ae)d^2+b^2e^3(ae-bd)+ce^2\left(8b^2d^2-10abed+3a^2e^2\right)\right)(b+2cx)}{c\left(b^2-4ac\right)^4(a+x(b+cx))} + \frac{3e^5b^6+5cde\cdots}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5/(a + b*x + c*x^2)^5,x]
```

```
[Out] ((30*(2*c*d - b*e)*(7*c^3*d^4 - 2*c^2*d^2*e*(7*b*d - 5*a*e) + b^2*e^3*(-(b*
d) + a*e) + c*e^2*(8*b^2*d^2 - 10*a*b*d*e + 3*a^2*e^2))*(b + 2*c*x))/(c*(b^
2 - 4*a*c)^4*(a + x*(b + c*x))) + (5*b^5*c*d*e^4 + 3*b^6*e^5 + 4*c^3*(-48*a
^3*e^5 + 35*c^3*d^5*x + 50*a*c^2*d^3*e^2*x + 15*a^2*c*d*e^4*x) + b^2*c^2*e*
(129*a^2*e^4 - 25*c^2*d^3*(7*d - 12*e*x) - 30*a*c*d*e^2*(5*d - 4*e*x)) + 10
*b*c^3*(7*c^2*d^4*(d - 5*e*x) + 10*a*c*d^2*e^2*(d - 3*e*x) + 3*a^2*e^4*(d -
 e*x)) + 10*b^3*c^2*e^2*(5*c*d^2*(3*d - 2*e*x) + a*e^2*(6*d - e*x)) + b^4*c
*e^3*(-41*a*e^2 + 10*c*d*(-5*d + e*x)))/(c^3*(-b^2 + 4*a*c)^3*(a + x*(b + c
*x))^2) - (3*(b^5*e^5*x + b^4*e^4*(a*e - 5*c*d*x) - 5*b^3*c*e^3*(-2*c*d^2*x
 + a*e*(d + e*x)) - 2*b^2*c*e^2*(2*a^2*e^3 + 5*c^2*d^3*x - 5*a*c*d*e*(d + 2
*e*x)) + 2*c^2*(a^3*e^5 - c^3*d^5*x - 5*a^2*c*d*e^3*(2*d + e*x) + 5*a*c^2*d
^3*e*(d + 2*e*x)) + b*c^2*(-(c^2*d^4*(d - 5*e*x)) + 5*a^2*e^4*(3*d + e*x) -
 10*a*c*d^2*e^2*(d + 3*e*x))))/(c^4*(-b^2 + 4*a*c)*(a + x*(b + c*x))^4) + (
-3*b^6*e^5 + 3*b^5*c*e^4*(5*d + 2*e*x) + b^4*c*e^3*(27*a*e^2 - 10*c*d*(3*d
+ e*x)) - 10*b^3*c^2*e^2*(5*a*e^2*(2*d + e*x) + c*d^2*(-3*d + 2*e*x)) + 4*c
^3*(16*a^3*e^5 + 7*c^3*d^5*x + 10*a*c^2*d^3*e^2*x - 5*a^2*c*d*e^3*(16*d + 9
*e*x)) + 2*b*c^3*(7*c^2*d^4*(d - 5*e*x) + 10*a*c*d^2*e^2*(d - 3*e*x) + 5*a^
2*e^4*(23*d + 9*e*x)) + b^2*c^2*e*(-83*a^2*e^4 + 5*c^2*d^3*(-7*d + 12*e*x)
+ 10*a*c*d*e^2*(13*d + 12*e*x)))/(c^4*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^3)
+ (120*(2*c*d - b*e)*(7*c^3*d^4 - 2*c^2*d^2*e*(7*b*d - 5*a*e) + b^2*e^3*(-(
b*d) + a*e) + c*e^2*(8*b^2*d^2 - 10*a*b*d*e + 3*a^2*e^2))*ArcTan[(b + 2*c*x
)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(9/2))/12
```

Maple [B] time = 0.171, size = 3092, normalized size = 8.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5/(c*x^2+b*x+a)^5,x)
```

```
[Out] (-5*(3*a^2*b*c*e^5-6*a^2*c^2*d*e^4+a*b^3*e^5-12*a*b^2*c*d*e^4+30*a*b*c^2*d^
2*e^3-20*a*c^3*d^3*e^2-b^4*d*e^4+10*b^3*c*d^2*e^3-30*b^2*c^2*d^3*e^2+35*b*c
^3*d^4*e-14*c^4*d^5)/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c
+b^8)*c^3*x^7-35/2*(3*a^2*b*c*e^5-6*a^2*c^2*d*e^4+a*b^3*e^5-12*a*b^2*c*d*e^
4+30*a*b*c^2*d^2*e^3-20*a*c^3*d^3*e^2-b^4*d*e^4+10*b^3*c*d^2*e^3-30*b^2*c^2
*d^3*e^2+35*b*c^3*d^4*e-14*c^4*d^5)/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4
*c^2-16*a*b^6*c+b^8)*b*c^2*x^6-5/3*c*(11*a*c+13*b^2)*(3*a^2*b*c*e^5-6*a^2*c
^2*d*e^4+a*b^3*e^5-12*a*b^2*c*d*e^4+30*a*b*c^2*d^2*e^3-20*a*c^3*d^3*e^2-b^4
*d*e^4+10*b^3*c*d^2*e^3-30*b^2*c^2*d^3*e^2+35*b*c^3*d^4*e-14*c^4*d^5)/(256*
a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)*x^5-1/12*(768*a^4*c^
4*e^5+882*a^3*b^2*c^3*e^5-3300*a^3*b*c^4*d*e^4+1213*a^2*b^4*c^2*e^5-7350*a^
2*b^3*c^3*d*e^4+16500*a^2*b^2*c^4*d^2*e^3-11000*a^2*b*c^5*d^3*e^2+77*a*b^6*
c*e^5-2050*a*b^5*c^2*d*e^4+9250*a*b^4*c^3*d^2*e^3-19000*a*b^3*c^4*d^3*e^2+1
9250*a*b^2*c^5*d^4*e-7700*a*b*c^6*d^5+3*b^8*e^5-125*b^7*c*d*e^4+1250*b^6*c^
2*d^2*e^3-3750*b^5*c^3*d^3*e^2+4375*b^4*c^4*d^4*e-1750*b^3*c^5*d^5)/c/(256*
a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)*x^4-1/3*(219*a^4*b*c
^3*e^5+330*a^4*c^4*d*e^4+376*a^3*b^3*c^2*e^5-2250*a^3*b^2*c^3*d*e^4+2190*a^
3*b*c^4*d^2*e^3-1460*a^3*c^5*d^3*e^2+110*a^2*b^5*c*e^5-1015*a^2*b^4*c^2*d*e
^4+3760*a^2*b^3*c^3*d^2*e^3-4210*a^2*b^2*c^4*d^3*e^2+2555*a^2*b*c^5*d^4*e-1
022*a^2*c^6*d^5+3*a*b^7*e^5-185*a*b^6*c*d*e^4+1100*a*b^5*c^2*d^2*e^3-3090*a
*b^4*c^3*d^3*e^2+3535*a*b^3*c^4*d^4*e-1414*a*b^2*c^5*d^5+30*b^7*c*d^2*e^3-9
0*b^6*c^2*d^3*e^2+105*b^5*c^3*d^4*e-42*b^4*c^4*d^5)/c/(256*a^4*c^4-256*a^3*
b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)*x^3-1/6*(256*a^5*c^3*e^5+401*a^4*b^2
*c^2*e^5-1570*a^4*b*c^3*d*e^4+2560*a^4*c^4*d^2*e^3+399*a^3*b^4*c*e^5-2540*a
^3*b^3*c^2*d*e^4+4010*a^3*b^2*c^3*d^2*e^3-4380*a^3*b*c^4*d^3*e^2+9*a^2*b^6*
e^5-645*a^2*b^5*c*d*e^4+3990*a^2*b^4*c^2*d^2*e^3-7130*a^2*b^3*c^3*d^3*e^2+7
665*a^2*b^2*c^4*d^4*e-3066*a^2*b*c^5*d^5+90*a*b^6*c*d^2*e^3-820*a*b^5*c^2*d
^3*e^2+980*a*b^4*c^3*d^4*e-392*a*b^3*c^4*d^5+30*b^7*c*d^3*e^2-35*b^6*c^2*d^
4*e+14*b^5*c^3*d^5)/c/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*
c+b^8)*x^2-1/3*(83*a^5*b*c^2*e^5+90*a^5*c^3*d*e^4+151*a^4*b^3*c*e^5-920*a^4
*b^2*c^2*d*e^4+830*a^4*b*c^3*d^2*e^3+300*a^4*c^4*d^3*e^2+3*a^3*b^5*e^5-235*
a^3*b^4*c*d*e^4+1510*a^3*b^3*c^2*d^2*e^3-2790*a^3*b^2*c^3*d^3*e^2+1395*a^3*
b*c^4*d^4*e-558*a^3*c^5*d^5+30*a^2*b^5*c*d^2*e^3-280*a^2*b^4*c^2*d^3*e^2+87
0*a^2*b^3*c^3*d^4*e-348*a^2*b^2*c^4*d^5+10*a*b^6*c*d^3*e^2-95*a*b^5*c^2*d^4
*e+38*a*b^4*c^3*d^5+5*b^7*c*d^4*e-2*b^6*c^2*d^5)/c/(256*a^4*c^4-256*a^3*b^2
*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)*x-1/12*(128*a^6*c^2*e^5+166*a^5*b^2*c*e
^5-1100*a^5*b*c^2*d*e^4+1280*a^5*c^3*d^2*e^3+3*a^4*b^4*e^5-250*a^4*b^3*c*d*
e^4+1660*a^4*b^2*c^2*d^2*e^3-3240*a^4*b*c^3*d^3*e^2+1920*a^4*c^4*d^4*e+30*a
^3*b^4*c*d^2*e^3-280*a^3*b^3*c^2*d^3*e^2+870*a^3*b^2*c^3*d^4*e-1116*a^3*b*c
^4*d^5+10*a^2*b^5*c*d^3*e^2-95*a^2*b^4*c^2*d^4*e+326*a^2*b^3*c^3*d^5+5*a*b^
6*c*d^4*e-50*a*b^5*c^2*d^5+3*b^7*c*d^5)/c/(256*a^4*c^4-256*a^3*b^2*c^3+96*a
^2*b^4*c^2-16*a*b^6*c+b^8))/(c*x^2+b*x+a)^4-30/(256*a^4*c^4-256*a^3*b^2*c^3
+96*a^2*b^4*c^2-16*a*b^6*c+b^8)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b
^2)^(1/2))*a^2*b*c*e^5+60/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*
b^6*c+b^8)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*d*a^2*c^2*
e^4-10/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)/(4*a*c-b
^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^3*e^5+120/(256*a^4*c^4-25
6*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)/(4*a*c-b^2)^(1/2)*arctan((2*c*
x+b)/(4*a*c-b^2)^(1/2))*a*b^2*c*d*e^4-300/(256*a^4*c^4-256*a^3*b^2*c^3+96*a
^2*b^4*c^2-16*a*b^6*c+b^8)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
```

```
1/2))*a*b*c^2*d^2*e^3+200/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*
b^6*c+b^8)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*d^3*a*c^3*
e^2+10/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)/(4*a*c-b
^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4*d*e^4-100/(256*a^4*c^4-25
6*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)/(4*a*c-b^2)^(1/2)*arctan((2*c*
x+b)/(4*a*c-b^2)^(1/2))*d^2*b^3*c*e^3+300/(256*a^4*c^4-256*a^3*b^2*c^3+96*a
^2*b^4*c^2-16*a*b^6*c+b^8)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2))*b^2*c^2*d^3*e^2-350/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*
b^6*c+b^8)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c^3*d^4*
e+140/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)/(4*a*c-b^
2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^4*d^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*x^2+b*x+a)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.91482, size = 18823, normalized size = 48.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*x^2+b*x+a)^5,x, algorithm="fricas")
```

```
[Out] [1/12*(60*(14*(b^2*c^8 - 4*a*c^9)*d^5 - 35*(b^3*c^7 - 4*a*b*c^8)*d^4*e + 10
*(3*b^4*c^6 - 10*a*b^2*c^7 - 8*a^2*c^8)*d^3*e^2 - 10*(b^5*c^5 - a*b^3*c^6 -
 12*a^2*b*c^7)*d^2*e^3 + (b^6*c^4 + 8*a*b^4*c^5 - 42*a^2*b^2*c^6 - 24*a^3*c
^7)*d*e^4 - (a*b^5*c^4 - a^2*b^3*c^5 - 12*a^3*b*c^6)*e^5)*x^7 + 210*(14*(b^
3*c^7 - 4*a*b*c^8)*d^5 - 35*(b^4*c^6 - 4*a*b^2*c^7)*d^4*e + 10*(3*b^5*c^5 -
 10*a*b^3*c^6 - 8*a^2*b*c^7)*d^3*e^2 - 10*(b^6*c^4 - a*b^4*c^5 - 12*a^2*b^2
*c^6)*d^2*e^3 + (b^7*c^3 + 8*a*b^5*c^4 - 42*a^2*b^3*c^5 - 24*a^3*b*c^6)*d*e
^4 - (a*b^6*c^3 - a^2*b^4*c^4 - 12*a^3*b^2*c^5)*e^5)*x^6 - (3*b^9*c - 62*a*
b^7*c^2 + 526*a^2*b^5*c^3 - 2420*a^3*b^3*c^4 + 4464*a^4*b*c^5)*d^5 - 5*(a*b
^8*c - 23*a^2*b^6*c^2 + 250*a^3*b^4*c^3 - 312*a^4*b^2*c^4 - 1536*a^5*c^5)*d
^4*e - 10*(a^2*b^7*c - 32*a^3*b^5*c^2 - 212*a^4*b^3*c^3 + 1296*a^5*b*c^4)*d
^3*e^2 - 10*(3*a^3*b^6*c + 154*a^4*b^4*c^2 - 536*a^5*b^2*c^3 - 512*a^6*c^4)
*d^2*e^3 + 50*(5*a^4*b^5*c + 2*a^5*b^3*c^2 - 88*a^6*b*c^3)*d*e^4 - (3*a^4*b
^6 + 154*a^5*b^4*c - 536*a^6*b^2*c^2 - 512*a^7*c^3)*e^5 + 20*(14*(13*b^4*c^
6 - 41*a*b^2*c^7 - 44*a^2*c^8)*d^5 - 35*(13*b^5*c^5 - 41*a*b^3*c^6 - 44*a^2
*b*c^7)*d^4*e + 10*(39*b^6*c^4 - 97*a*b^4*c^5 - 214*a^2*b^2*c^6 - 88*a^3*c^
7)*d^3*e^2 - 10*(13*b^7*c^3 - 2*a*b^5*c^4 - 167*a^2*b^3*c^5 - 132*a^3*b*c^6
)*d^2*e^3 + (13*b^8*c^2 + 115*a*b^6*c^3 - 458*a^2*b^4*c^4 - 774*a^3*b^2*c^5
 - 264*a^4*c^6)*d*e^4 - (13*a*b^7*c^2 - 2*a^2*b^5*c^3 - 167*a^3*b^3*c^4 - 1
32*a^4*b*c^5)*e^5)*x^5 + (350*(5*b^5*c^5 + 2*a*b^3*c^6 - 88*a^2*b*c^7)*d^5
- 875*(5*b^6*c^4 + 2*a*b^4*c^5 - 88*a^2*b^2*c^6)*d^4*e + 250*(15*b^7*c^3 +
16*a*b^5*c^4 - 260*a^2*b^3*c^5 - 176*a^3*b*c^6)*d^3*e^2 - 250*(5*b^8*c^2 +
17*a*b^6*c^3 - 82*a^2*b^4*c^4 - 264*a^3*b^2*c^5)*d^2*e^3 + 25*(5*b^9*c + 62
*a*b^7*c^2 - 34*a^2*b^5*c^3 - 1044*a^3*b^3*c^4 - 528*a^4*b*c^5)*d*e^4 - (3*
b^10 + 65*a*b^8*c + 905*a^2*b^6*c^2 - 3970*a^3*b^4*c^3 - 2760*a^4*b^2*c^4 -
```

3072*a^5*c^5)*e^5)*x^4 + 4*(14*(3*b^6*c^4 + 89*a*b^4*c^5 - 331*a^2*b^2*c^6 - 292*a^3*c^7)*d^5 - 35*(3*b^7*c^3 + 89*a*b^5*c^4 - 331*a^2*b^3*c^5 - 292*a^3*b*c^6)*d^4*e + 10*(9*b^8*c^2 + 273*a*b^6*c^3 - 815*a^2*b^4*c^4 - 1538*a^3*b^2*c^5 - 584*a^4*c^6)*d^3*e^2 - 10*(3*b^9*c + 98*a*b^7*c^2 - 64*a^2*b^5*c^3 - 1285*a^3*b^3*c^4 - 876*a^4*b*c^5)*d^2*e^3 + 5*(37*a*b^8*c + 55*a^2*b^6*c^2 - 362*a^3*b^4*c^3 - 1866*a^4*b^2*c^4 + 264*a^5*c^5)*d*e^4 - (3*a*b^9 + 98*a^2*b^7*c - 64*a^3*b^5*c^2 - 1285*a^4*b^3*c^3 - 876*a^5*b*c^4)*e^5)*x^3 - 2*(14*(b^7*c^3 - 32*a*b^5*c^4 - 107*a^2*b^3*c^5 + 876*a^3*b*c^6)*d^5 - 35*(b^8*c^2 - 32*a*b^6*c^3 - 107*a^2*b^4*c^4 + 876*a^3*b^2*c^5)*d^4*e + 10*(3*b^9*c - 94*a*b^7*c^2 - 385*a^2*b^5*c^3 + 2414*a^3*b^3*c^4 + 1752*a^4*b*c^5)*d^3*e^2 + 10*(9*a*b^8*c + 363*a^2*b^6*c^2 - 1195*a^3*b^4*c^3 - 1348*a^4*b^2*c^4 - 1024*a^5*c^5)*d^2*e^3 - 5*(129*a^2*b^7*c - 8*a^3*b^5*c^2 - 1718*a^4*b^3*c^3 - 1256*a^5*b*c^4)*d*e^4 + (9*a^2*b^8 + 363*a^3*b^6*c - 1195*a^4*b^4*c^2 - 1348*a^5*b^2*c^3 - 1024*a^6*c^4)*e^5)*x^2 - 60*(14*a^4*c^5*d^5 - 35*a^4*b*c^4*d^4*e + (14*c^9*d^5 - 35*b*c^8*d^4*e + 10*(3*b^2*c^7 + 2*a*c^8)*d^3*e^2 - 10*(b^3*c^6 + 3*a*b*c^7)*d^2*e^3 + (b^4*c^5 + 12*a*b^2*c^6 + 6*a^2*c^7)*d*e^4 - (a*b^3*c^5 + 3*a^2*b*c^6)*e^5)*x^8 + 4*(14*b*c^8*d^5 - 35*b^2*c^7*d^4*e + 10*(3*b^3*c^6 + 2*a*b*c^7)*d^3*e^2 - 10*(b^4*c^5 + 3*a*b^2*c^6)*d^2*e^3 + (b^5*c^4 + 12*a*b^3*c^5 + 6*a^2*b*c^6)*d*e^4 - (a*b^4*c^4 + 3*a^2*b^2*c^5)*e^5)*x^7 + 2*(14*(3*b^2*c^7 + 2*a*c^8)*d^5 - 35*(3*b^3*c^6 + 2*a*b*c^7)*d^4*e + 10*(9*b^4*c^5 + 12*a*b^2*c^6 + 4*a^2*c^7)*d^3*e^2 - 10*(3*b^5*c^4 + 11*a*b^3*c^5 + 6*a^2*b*c^6)*d^2*e^3 + (3*b^6*c^3 + 38*a*b^4*c^4 + 42*a^2*b^2*c^5 + 12*a^3*c^6)*d*e^4 - (3*a*b^5*c^3 + 11*a^2*b^3*c^4 + 6*a^3*b*c^5)*e^5)*x^6 + 10*(3*a^4*b^2*c^3 + 2*a^5*c^4)*d^3*e^2 - 10*(a^4*b^3*c^2 + 3*a^5*b*c^3)*d^2*e^3 + (a^4*b^4*c + 12*a^5*b^2*c^2 + 6*a^6*c^3)*d*e^4 - (a^5*b^3*c + 3*a^6*b*c^2)*e^5 + 4*(14*(b^3*c^6 + 3*a*b*c^7)*d^5 - 35*(b^4*c^5 + 3*a*b^2*c^6)*d^4*e + 10*(3*b^5*c^4 + 11*a*b^3*c^5 + 6*a^2*b*c^6)*d^3*e^2 - 10*(b^6*c^3 + 6*a*b^4*c^4 + 9*a^2*b^2*c^5)*d^2*e^3 + (b^7*c^2 + 15*a*b^5*c^3 + 42*a^2*b^3*c^4 + 18*a^3*b*c^5)*d*e^4 - (a*b^6*c^2 + 6*a^2*b^4*c^3 + 9*a^3*b^2*c^4)*e^5)*x^5 + (14*(b^4*c^5 + 12*a*b^2*c^6 + 6*a^2*c^7)*d^5 - 35*(b^5*c^4 + 12*a*b^3*c^5 + 6*a^2*b*c^6)*d^4*e + 10*(3*b^6*c^3 + 38*a*b^4*c^4 + 42*a^2*b^2*c^5 + 12*a^3*c^6)*d^3*e^2 - 10*(b^7*c^2 + 15*a*b^5*c^3 + 42*a^2*b^3*c^4 + 18*a^3*b*c^5)*d^2*e^3 + (b^8*c + 24*a*b^6*c^2 + 156*a^2*b^4*c^3 + 144*a^3*b^2*c^4 + 36*a^4*c^5)*d*e^4 - (a*b^7*c + 15*a^2*b^5*c^2 + 42*a^3*b^3*c^3 + 18*a^4*b*c^4)*e^5)*x^4 + 4*(14*(a*b^3*c^5 + 3*a^2*b*c^6)*d^5 - 35*(a*b^4*c^4 + 3*a^2*b^2*c^5)*d^4*e + 10*(3*a*b^5*c^3 + 11*a^2*b^3*c^4 + 6*a^3*b*c^5)*d^3*e^2 - 10*(a*b^6*c^2 + 6*a^2*b^4*c^3 + 9*a^3*b^2*c^4)*d^2*e^3 + (a*b^7*c + 15*a^2*b^5*c^2 + 42*a^3*b^3*c^3 + 18*a^4*b*c^4)*d*e^4 - (a^2*b^6*c + 6*a^3*b^4*c^2 + 9*a^4*b^2*c^3)*e^5)*x^3 + 2*(14*(3*a^2*b^2*c^5 + 2*a^3*c^6)*d^5 - 35*(3*a^2*b^3*c^4 + 2*a^3*b*c^5)*d^4*e + 10*(9*a^2*b^4*c^3 + 12*a^3*b^2*c^4 + 4*a^4*c^5)*d^3*e^2 - 10*(3*a^2*b^5*c^2 + 11*a^3*b^3*c^3 + 6*a^4*b*c^4)*d^2*e^3 + (3*a^2*b^6*c + 38*a^3*b^4*c^2 + 42*a^4*b^2*c^3 + 12*a^5*c^4)*d*e^4 - (3*a^3*b^5*c + 11*a^4*b^3*c^2 + 6*a^5*b*c^3)*e^5)*x^2 + 4*(14*a^3*b*c^5*d^5 - 35*a^3*b^2*c^4*d^4*e + 10*(3*a^3*b^3*c^3 + 2*a^4*b*c^4)*d^3*e^2 - 10*(a^3*b^4*c^2 + 3*a^4*b^2*c^3)*d^2*e^3 + (a^3*b^5*c + 12*a^4*b^3*c^2 + 6*a^5*b*c^3)*d*e^4 - (a^4*b^4*c + 3*a^5*b^2*c^2)*e^5)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 4*(2*(b^8*c^2 - 23*a*b^6*c^3 + 250*a^2*b^4*c^4 - 417*a^3*b^2*c^5 - 1116*a^4*c^6)*d^5 - 5*(b^9*c - 23*a*b^7*c^2 + 250*a^2*b^5*c^3 - 417*a^3*b^3*c^4 - 1116*a^4*b*c^5)*d^4*e - 10*(a*b^8*c - 32*a^2*b^6*c^2 - 167*a^3*b^4*c^3 + 1146*a^4*b^2*c^4 - 120*a^5*c^5)*d^3*e^2 - 10*(3*a^2*b^7*c + 139*a^3*b^5*c^2 - 521*a^4*b^3*c^3 - 332*a^5*b*c^4)*d^2*e^3 + 5*(47*a^3*b^6*c - 4*a^4*b^4*c^2 - 754*a^5*b^2*c^3 + 72*a^6*c^4)*d*e^4 - (3*a^3*b^7 + 139*a^4*b^5*c - 521*a^5*b^3*c^2 - 332*a^6*b*c^3)*e^5)*x)/(a^4*b^10*c - 20*a^5*b^8*c^2 + 160*a^6*b^6*c^3 - 640*a^7*b^4*c^4 + 1280*a^8*b^2*c^5 - 1024*a^9*c^6 + (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*x^8 + 4*(b^11*c^4 - 20*a*b^9*c^5 + 160*a^2*b^7*c^6 - 640*a^3*b^5*c^7 + 1280*a^4*b^3*c^8 - 1024*a^5*b*c^9)*x^7 + 2*(3*b^12*c^3 - 58*a*b^10*c^4 + 440*a^2*b^8*c^5 - 1600*a^3*b^6*c^6 +

2560*a^4*b^4*c^7 - 512*a^5*b^2*c^8 - 2048*a^6*c^9)*x^6 + 4*(b^13*c^2 - 17*
a*b^11*c^3 + 100*a^2*b^9*c^4 - 160*a^3*b^7*c^5 - 640*a^4*b^5*c^6 + 2816*a^5
*b^3*c^7 - 3072*a^6*b*c^8)*x^5 + (b^14*c - 8*a*b^12*c^2 - 74*a^2*b^10*c^3 +
1160*a^3*b^8*c^4 - 5440*a^4*b^6*c^5 + 10496*a^5*b^4*c^6 - 4608*a^6*b^2*c^7
- 6144*a^7*c^8)*x^4 + 4*(a*b^13*c - 17*a^2*b^11*c^2 + 100*a^3*b^9*c^3 - 16
0*a^4*b^7*c^4 - 640*a^5*b^5*c^5 + 2816*a^6*b^3*c^6 - 3072*a^7*b*c^7)*x^3 +
2*(3*a^2*b^12*c - 58*a^3*b^10*c^2 + 440*a^4*b^8*c^3 - 1600*a^5*b^6*c^4 + 25
60*a^6*b^4*c^5 - 512*a^7*b^2*c^6 - 2048*a^8*c^7)*x^2 + 4*(a^3*b^11*c - 20*a
^4*b^9*c^2 + 160*a^5*b^7*c^3 - 640*a^6*b^5*c^4 + 1280*a^7*b^3*c^5 - 1024*a^
8*b*c^6)*x), 1/12*(60*(14*(b^2*c^8 - 4*a*c^9)*d^5 - 35*(b^3*c^7 - 4*a*b*c^8
)*d^4*e + 10*(3*b^4*c^6 - 10*a*b^2*c^7 - 8*a^2*c^8)*d^3*e^2 - 10*(b^5*c^5 -
a*b^3*c^6 - 12*a^2*b*c^7)*d^2*e^3 + (b^6*c^4 + 8*a*b^4*c^5 - 42*a^2*b^2*c^
6 - 24*a^3*c^7)*d*e^4 - (a*b^5*c^4 - a^2*b^3*c^5 - 12*a^3*b*c^6)*e^5)*x^7 +
210*(14*(b^3*c^7 - 4*a*b*c^8)*d^5 - 35*(b^4*c^6 - 4*a*b^2*c^7)*d^4*e + 10*
(3*b^5*c^5 - 10*a*b^3*c^6 - 8*a^2*b*c^7)*d^3*e^2 - 10*(b^6*c^4 - a*b^4*c^5
- 12*a^2*b^2*c^6)*d^2*e^3 + (b^7*c^3 + 8*a*b^5*c^4 - 42*a^2*b^3*c^5 - 24*a^
3*b*c^6)*d*e^4 - (a*b^6*c^3 - a^2*b^4*c^4 - 12*a^3*b^2*c^5)*e^5)*x^6 - (3*b
^9*c - 62*a*b^7*c^2 + 526*a^2*b^5*c^3 - 2420*a^3*b^3*c^4 + 4464*a^4*b*c^5)*
d^5 - 5*(a*b^8*c - 23*a^2*b^6*c^2 + 250*a^3*b^4*c^3 - 312*a^4*b^2*c^4 - 153
6*a^5*c^5)*d^4*e - 10*(a^2*b^7*c - 32*a^3*b^5*c^2 - 212*a^4*b^3*c^3 + 1296*
a^5*b*c^4)*d^3*e^2 - 10*(3*a^3*b^6*c + 154*a^4*b^4*c^2 - 536*a^5*b^2*c^3 -
512*a^6*c^4)*d^2*e^3 + 50*(5*a^4*b^5*c + 2*a^5*b^3*c^2 - 88*a^6*b*c^3)*d*e^
4 - (3*a^4*b^6 + 154*a^5*b^4*c - 536*a^6*b^2*c^2 - 512*a^7*c^3)*e^5 + 20*(1
4*(13*b^4*c^6 - 41*a*b^2*c^7 - 44*a^2*c^8)*d^5 - 35*(13*b^5*c^5 - 41*a*b^3*
c^6 - 44*a^2*b*c^7)*d^4*e + 10*(39*b^6*c^4 - 97*a*b^4*c^5 - 214*a^2*b^2*c^6
- 88*a^3*c^7)*d^3*e^2 - 10*(13*b^7*c^3 - 2*a*b^5*c^4 - 167*a^2*b^3*c^5 - 1
32*a^3*b*c^6)*d^2*e^3 + (13*b^8*c^2 + 115*a*b^6*c^3 - 458*a^2*b^4*c^4 - 774
*a^3*b^2*c^5 - 264*a^4*c^6)*d*e^4 - (13*a*b^7*c^2 - 2*a^2*b^5*c^3 - 167*a^3
*b^3*c^4 - 132*a^4*b*c^5)*e^5)*x^5 + (350*(5*b^5*c^5 + 2*a*b^3*c^6 - 88*a^2
*b*c^7)*d^5 - 875*(5*b^6*c^4 + 2*a*b^4*c^5 - 88*a^2*b^2*c^6)*d^4*e + 250*(1
5*b^7*c^3 + 16*a*b^5*c^4 - 260*a^2*b^3*c^5 - 176*a^3*b*c^6)*d^3*e^2 - 250*(
5*b^8*c^2 + 17*a*b^6*c^3 - 82*a^2*b^4*c^4 - 264*a^3*b^2*c^5)*d^2*e^3 + 25*(
5*b^9*c + 62*a*b^7*c^2 - 34*a^2*b^5*c^3 - 1044*a^3*b^3*c^4 - 528*a^4*b*c^5)
*d*e^4 - (3*b^10 + 65*a*b^8*c + 905*a^2*b^6*c^2 - 3970*a^3*b^4*c^3 - 2760*a
^4*b^2*c^4 - 3072*a^5*c^5)*e^5)*x^4 + 4*(14*(3*b^6*c^4 + 89*a*b^4*c^5 - 331
*a^2*b^2*c^6 - 292*a^3*c^7)*d^5 - 35*(3*b^7*c^3 + 89*a*b^5*c^4 - 331*a^2*b^
3*c^5 - 292*a^3*b*c^6)*d^4*e + 10*(9*b^8*c^2 + 273*a*b^6*c^3 - 815*a^2*b^4*
c^4 - 1538*a^3*b^2*c^5 - 584*a^4*c^6)*d^3*e^2 - 10*(3*b^9*c + 98*a*b^7*c^2
- 64*a^2*b^5*c^3 - 1285*a^3*b^3*c^4 - 876*a^4*b*c^5)*d^2*e^3 + 5*(37*a*b^8*
c + 55*a^2*b^6*c^2 - 362*a^3*b^4*c^3 - 1866*a^4*b^2*c^4 + 264*a^5*c^5)*d*e^
4 - (3*a*b^9 + 98*a^2*b^7*c - 64*a^3*b^5*c^2 - 1285*a^4*b^3*c^3 - 876*a^5*b
*c^4)*e^5)*x^3 - 2*(14*(b^7*c^3 - 32*a*b^5*c^4 - 107*a^2*b^3*c^5 + 876*a^3*
b*c^6)*d^5 - 35*(b^8*c^2 - 32*a*b^6*c^3 - 107*a^2*b^4*c^4 + 876*a^3*b^2*c^5
)*d^4*e + 10*(3*b^9*c - 94*a*b^7*c^2 - 385*a^2*b^5*c^3 + 2414*a^3*b^3*c^4 +
1752*a^4*b*c^5)*d^3*e^2 + 10*(9*a*b^8*c + 363*a^2*b^6*c^2 - 1195*a^3*b^4*c
^3 - 1348*a^4*b^2*c^4 - 1024*a^5*c^5)*d^2*e^3 - 5*(129*a^2*b^7*c - 8*a^3*b^
5*c^2 - 1718*a^4*b^3*c^3 - 1256*a^5*b*c^4)*d*e^4 + (9*a^2*b^8 + 363*a^3*b^6
*c - 1195*a^4*b^4*c^2 - 1348*a^5*b^2*c^3 - 1024*a^6*c^4)*e^5)*x^2 - 120*(14
*a^4*c^5*d^5 - 35*a^4*b*c^4*d^4*e + (14*c^9*d^5 - 35*b*c^8*d^4*e + 10*(3*b^
2*c^7 + 2*a*c^8)*d^3*e^2 - 10*(b^3*c^6 + 3*a*b*c^7)*d^2*e^3 + (b^4*c^5 + 12
*a*b^2*c^6 + 6*a^2*c^7)*d*e^4 - (a*b^3*c^5 + 3*a^2*b*c^6)*e^5)*x^8 + 4*(14*
b*c^8*d^5 - 35*b^2*c^7*d^4*e + 10*(3*b^3*c^6 + 2*a*b*c^7)*d^3*e^2 - 10*(b^4
*c^5 + 3*a*b^2*c^6)*d^2*e^3 + (b^5*c^4 + 12*a*b^3*c^5 + 6*a^2*b*c^6)*d*e^4
- (a*b^4*c^4 + 3*a^2*b^2*c^5)*e^5)*x^7 + 2*(14*(3*b^2*c^7 + 2*a*c^8)*d^5 -
35*(3*b^3*c^6 + 2*a*b*c^7)*d^4*e + 10*(9*b^4*c^5 + 12*a*b^2*c^6 + 4*a^2*c^7
)*d^3*e^2 - 10*(3*b^5*c^4 + 11*a*b^3*c^5 + 6*a^2*b*c^6)*d^2*e^3 + (3*b^6*c^
3 + 38*a*b^4*c^4 + 42*a^2*b^2*c^5 + 12*a^3*c^6)*d*e^4 - (3*a*b^5*c^3 + 11*a
^2*b^3*c^4 + 6*a^3*b*c^5)*e^5)*x^6 + 10*(3*a^4*b^2*c^3 + 2*a^5*c^4)*d^3*e^2
- 10*(a^4*b^3*c^2 + 3*a^5*b*c^3)*d^2*e^3 + (a^4*b^4*c + 12*a^5*b^2*c^2 + 6

```
*a^6*c^3)*d*e^4 - (a^5*b^3*c + 3*a^6*b*c^2)*e^5 + 4*(14*(b^3*c^6 + 3*a*b*c^
7)*d^5 - 35*(b^4*c^5 + 3*a*b^2*c^6)*d^4*e + 10*(3*b^5*c^4 + 11*a*b^3*c^5 +
6*a^2*b*c^6)*d^3*e^2 - 10*(b^6*c^3 + 6*a*b^4*c^4 + 9*a^2*b^2*c^5)*d^2*e^3 +
 (b^7*c^2 + 15*a*b^5*c^3 + 42*a^2*b^3*c^4 + 18*a^3*b*c^5)*d*e^4 - (a*b^6*c^
2 + 6*a^2*b^4*c^3 + 9*a^3*b^2*c^4)*e^5)*x^5 + (14*(b^4*c^5 + 12*a*b^2*c^6 +
 6*a^2*c^7)*d^5 - 35*(b^5*c^4 + 12*a*b^3*c^5 + 6*a^2*b*c^6)*d^4*e + 10*(3*b
^6*c^3 + 38*a*b^4*c^4 + 42*a^2*b^2*c^5 + 12*a^3*c^6)*d^3*e^2 - 10*(b^7*c^2
+ 15*a*b^5*c^3 + 42*a^2*b^3*c^4 + 18*a^3*b*c^5)*d^2*e^3 + (b^8*c + 24*a*b^6
*c^2 + 156*a^2*b^4*c^3 + 144*a^3*b^2*c^4 + 36*a^4*c^5)*d*e^4 - (a*b^7*c + 1
5*a^2*b^5*c^2 + 42*a^3*b^3*c^3 + 18*a^4*b*c^4)*e^5)*x^4 + 4*(14*(a*b^3*c^5
+ 3*a^2*b*c^6)*d^5 - 35*(a*b^4*c^4 + 3*a^2*b^2*c^5)*d^4*e + 10*(3*a*b^5*c^3
 + 11*a^2*b^3*c^4 + 6*a^3*b*c^5)*d^3*e^2 - 10*(a*b^6*c^2 + 6*a^2*b^4*c^3 +
9*a^3*b^2*c^4)*d^2*e^3 + (a*b^7*c + 15*a^2*b^5*c^2 + 42*a^3*b^3*c^3 + 18*a^
4*b*c^4)*d*e^4 - (a^2*b^6*c + 6*a^3*b^4*c^2 + 9*a^4*b^2*c^3)*e^5)*x^3 + 2*(
14*(3*a^2*b^2*c^5 + 2*a^3*c^6)*d^5 - 35*(3*a^2*b^3*c^4 + 2*a^3*b*c^5)*d^4*e
 + 10*(9*a^2*b^4*c^3 + 12*a^3*b^2*c^4 + 4*a^4*c^5)*d^3*e^2 - 10*(3*a^2*b^5*
c^2 + 11*a^3*b^3*c^3 + 6*a^4*b*c^4)*d^2*e^3 + (3*a^2*b^6*c + 38*a^3*b^4*c^2
 + 42*a^4*b^2*c^3 + 12*a^5*c^4)*d*e^4 - (3*a^3*b^5*c + 11*a^4*b^3*c^2 + 6*a
^5*b*c^3)*e^5)*x^2 + 4*(14*a^3*b*c^5*d^5 - 35*a^3*b^2*c^4*d^4*e + 10*(3*a^3
*b^3*c^3 + 2*a^4*b*c^4)*d^3*e^2 - 10*(a^3*b^4*c^2 + 3*a^4*b^2*c^3)*d^2*e^3
+ (a^3*b^5*c + 12*a^4*b^3*c^2 + 6*a^5*b*c^3)*d*e^4 - (a^4*b^4*c + 3*a^5*b^2
*c^2)*e^5)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^
2 - 4*a*c)) + 4*(2*(b^8*c^2 - 23*a*b^6*c^3 + 250*a^2*b^4*c^4 - 417*a^3*b^2*
c^5 - 1116*a^4*c^6)*d^5 - 5*(b^9*c - 23*a*b^7*c^2 + 250*a^2*b^5*c^3 - 417*a
^3*b^3*c^4 - 1116*a^4*b*c^5)*d^4*e - 10*(a*b^8*c - 32*a^2*b^6*c^2 - 167*a^3
*b^4*c^3 + 1146*a^4*b^2*c^4 - 120*a^5*c^5)*d^3*e^2 - 10*(3*a^2*b^7*c + 139*
a^3*b^5*c^2 - 521*a^4*b^3*c^3 - 332*a^5*b*c^4)*d^2*e^3 + 5*(47*a^3*b^6*c -
4*a^4*b^4*c^2 - 754*a^5*b^2*c^3 + 72*a^6*c^4)*d*e^4 - (3*a^3*b^7 + 139*a^4*
b^5*c - 521*a^5*b^3*c^2 - 332*a^6*b*c^3)*e^5)*x)/(a^4*b^10*c - 20*a^5*b^8*c
^2 + 160*a^6*b^6*c^3 - 640*a^7*b^4*c^4 + 1280*a^8*b^2*c^5 - 1024*a^9*c^6 +
(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2
*c^9 - 1024*a^5*c^10)*x^8 + 4*(b^11*c^4 - 20*a*b^9*c^5 + 160*a^2*b^7*c^6 -
640*a^3*b^5*c^7 + 1280*a^4*b^3*c^8 - 1024*a^5*b*c^9)*x^7 + 2*(3*b^12*c^3 -
58*a*b^10*c^4 + 440*a^2*b^8*c^5 - 1600*a^3*b^6*c^6 + 2560*a^4*b^4*c^7 - 512
*a^5*b^2*c^8 - 2048*a^6*c^9)*x^6 + 4*(b^13*c^2 - 17*a*b^11*c^3 + 100*a^2*b^
9*c^4 - 160*a^3*b^7*c^5 - 640*a^4*b^5*c^6 + 2816*a^5*b^3*c^7 - 3072*a^6*b*c
^8)*x^5 + (b^14*c - 8*a*b^12*c^2 - 74*a^2*b^10*c^3 + 1160*a^3*b^8*c^4 - 544
0*a^4*b^6*c^5 + 10496*a^5*b^4*c^6 - 4608*a^6*b^2*c^7 - 6144*a^7*c^8)*x^4 +
4*(a*b^13*c - 17*a^2*b^11*c^2 + 100*a^3*b^9*c^3 - 160*a^4*b^7*c^4 - 640*a^5
*b^5*c^5 + 2816*a^6*b^3*c^6 - 3072*a^7*b*c^7)*x^3 + 2*(3*a^2*b^12*c - 58*a^
3*b^10*c^2 + 440*a^4*b^8*c^3 - 1600*a^5*b^6*c^4 + 2560*a^6*b^4*c^5 - 512*a^
7*b^2*c^6 - 2048*a^8*c^7)*x^2 + 4*(a^3*b^11*c - 20*a^4*b^9*c^2 + 160*a^5*b^
7*c^3 - 640*a^6*b^5*c^4 + 1280*a^7*b^3*c^5 - 1024*a^8*b*c^6)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5/(c*x**2+b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.15848, size = 3162, normalized size = 8.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*x^2+b*x+a)^5,x, algorithm="giac")
```

```
[Out] 10*(14*c^4*d^5 - 35*b*c^3*d^4*e + 30*b^2*c^2*d^3*e^2 + 20*a*c^3*d^3*e^2 - 1
0*b^3*c*d^2*e^3 - 30*a*b*c^2*d^2*e^3 + b^4*d*e^4 + 12*a*b^2*c*d*e^4 + 6*a^2
*c^2*d*e^4 - a*b^3*e^5 - 3*a^2*b*c*e^5)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*
c))/((b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4)*sq
rt(-b^2 + 4*a*c)) + 1/12*(840*c^8*d^5*x^7 - 2100*b*c^7*d^4*x^7*e + 2940*b*c
^7*d^5*x^6 + 1800*b^2*c^6*d^3*x^7*e^2 + 1200*a*c^7*d^3*x^7*e^2 - 7350*b^2*c
^6*d^4*x^6*e + 3640*b^2*c^6*d^5*x^5 + 3080*a*c^7*d^5*x^5 - 600*b^3*c^5*d^2*
x^7*e^3 - 1800*a*b*c^6*d^2*x^7*e^3 + 6300*b^3*c^5*d^3*x^6*e^2 + 4200*a*b*c^
6*d^3*x^6*e^2 - 9100*b^3*c^5*d^4*x^5*e - 7700*a*b*c^6*d^4*x^5*e + 1750*b^3*
c^5*d^5*x^4 + 7700*a*b*c^6*d^5*x^4 + 60*b^4*c^4*d*x^7*e^4 + 720*a*b^2*c^5*d
*x^7*e^4 + 360*a^2*c^6*d*x^7*e^4 - 2100*b^4*c^4*d^2*x^6*e^3 - 6300*a*b^2*c^
5*d^2*x^6*e^3 + 7800*b^4*c^4*d^3*x^5*e^2 + 11800*a*b^2*c^5*d^3*x^5*e^2 + 44
00*a^2*c^6*d^3*x^5*e^2 - 4375*b^4*c^4*d^4*x^4*e - 19250*a*b^2*c^5*d^4*x^4*e
 + 168*b^4*c^4*d^5*x^3 + 5656*a*b^2*c^5*d^5*x^3 + 4088*a^2*c^6*d^5*x^3 - 60
*a*b^3*c^4*x^7*e^5 - 180*a^2*b*c^5*x^7*e^5 + 210*b^5*c^3*d*x^6*e^4 + 2520*a
*b^3*c^4*d*x^6*e^4 + 1260*a^2*b*c^5*d*x^6*e^4 - 2600*b^5*c^3*d^2*x^5*e^3 -
10000*a*b^3*c^4*d^2*x^5*e^3 - 6600*a^2*b*c^5*d^2*x^5*e^3 + 3750*b^5*c^3*d^3
*x^4*e^2 + 19000*a*b^3*c^4*d^3*x^4*e^2 + 11000*a^2*b*c^5*d^3*x^4*e^2 - 420*
b^5*c^3*d^4*x^3*e - 14140*a*b^3*c^4*d^4*x^3*e - 10220*a^2*b*c^5*d^4*x^3*e -
 28*b^5*c^3*d^5*x^2 + 784*a*b^3*c^4*d^5*x^2 + 6132*a^2*b*c^5*d^5*x^2 - 210*
a*b^4*c^3*x^6*e^5 - 630*a^2*b^2*c^4*x^6*e^5 + 260*b^6*c^2*d*x^5*e^4 + 3340*
a*b^4*c^3*d*x^5*e^4 + 4200*a^2*b^2*c^4*d*x^5*e^4 + 1320*a^3*c^5*d*x^5*e^4 -
 1250*b^6*c^2*d^2*x^4*e^3 - 9250*a*b^4*c^3*d^2*x^4*e^3 - 16500*a^2*b^2*c^4*
d^2*x^4*e^3 + 360*b^6*c^2*d^3*x^3*e^2 + 12360*a*b^4*c^3*d^3*x^3*e^2 + 16840
*a^2*b^2*c^4*d^3*x^3*e^2 + 5840*a^3*c^5*d^3*x^3*e^2 + 70*b^6*c^2*d^4*x^2*e
- 1960*a*b^4*c^3*d^4*x^2*e - 15330*a^2*b^2*c^4*d^4*x^2*e + 8*b^6*c^2*d^5*x
- 152*a*b^4*c^3*d^5*x + 1392*a^2*b^2*c^4*d^5*x + 2232*a^3*c^5*d^5*x - 260*a
*b^5*c^2*x^5*e^5 - 1000*a^2*b^3*c^3*x^5*e^5 - 660*a^3*b*c^4*x^5*e^5 + 125*b
^7*c*d*x^4*e^4 + 2050*a*b^5*c^2*d*x^4*e^4 + 7350*a^2*b^3*c^3*d*x^4*e^4 + 33
00*a^3*b*c^4*d*x^4*e^4 - 120*b^7*c*d^2*x^3*e^3 - 4400*a*b^5*c^2*d^2*x^3*e^3
 - 15040*a^2*b^3*c^3*d^2*x^3*e^3 - 8760*a^3*b*c^4*d^2*x^3*e^3 - 60*b^7*c*d^
3*x^2*e^2 + 1640*a*b^5*c^2*d^3*x^2*e^2 + 14260*a^2*b^3*c^3*d^3*x^2*e^2 + 87
60*a^3*b*c^4*d^3*x^2*e^2 - 20*b^7*c*d^4*x*e + 380*a*b^5*c^2*d^4*x*e - 3480*
a^2*b^3*c^3*d^4*x*e - 5580*a^3*b*c^4*d^4*x*e - 3*b^7*c*d^5 + 50*a*b^5*c^2*d
^5 - 326*a^2*b^3*c^3*d^5 + 1116*a^3*b*c^4*d^5 - 3*b^8*x^4*e^5 - 77*a*b^6*c*
x^4*e^5 - 1213*a^2*b^4*c^2*x^4*e^5 - 882*a^3*b^2*c^3*x^4*e^5 - 768*a^4*c^4*
x^4*e^5 + 740*a*b^6*c*d*x^3*e^4 + 4060*a^2*b^4*c^2*d*x^3*e^4 + 9000*a^3*b^2
*c^3*d*x^3*e^4 - 1320*a^4*c^4*d*x^3*e^4 - 180*a*b^6*c*d^2*x^2*e^3 - 7980*a^
2*b^4*c^2*d^2*x^2*e^3 - 8020*a^3*b^2*c^3*d^2*x^2*e^3 - 5120*a^4*c^4*d^2*x^2
*e^3 - 40*a*b^6*c*d^3*x*e^2 + 1120*a^2*b^4*c^2*d^3*x*e^2 + 11160*a^3*b^2*c^
3*d^3*x*e^2 - 1200*a^4*c^4*d^3*x*e^2 - 5*a*b^6*c*d^4*e + 95*a^2*b^4*c^2*d^4
*e - 870*a^3*b^2*c^3*d^4*e - 1920*a^4*c^4*d^4*e - 12*a*b^7*x^3*e^5 - 440*a^
2*b^5*c*x^3*e^5 - 1504*a^3*b^3*c^2*x^3*e^5 - 876*a^4*b*c^3*x^3*e^5 + 1290*a
^2*b^5*c*d*x^2*e^4 + 5080*a^3*b^3*c^2*d*x^2*e^4 + 3140*a^4*b*c^3*d*x^2*e^4
- 120*a^2*b^5*c*d^2*x*e^3 - 6040*a^3*b^3*c^2*d^2*x*e^3 - 3320*a^4*b*c^3*d^2
*x*e^3 - 10*a^2*b^5*c*d^3*e^2 + 280*a^3*b^3*c^2*d^3*e^2 + 3240*a^4*b*c^3*d^
3*e^2 - 18*a^2*b^6*x^2*e^5 - 798*a^3*b^4*c*x^2*e^5 - 802*a^4*b^2*c^2*x^2*e^
5 - 512*a^5*c^3*x^2*e^5 + 940*a^3*b^4*c*d*x*e^4 + 3680*a^4*b^2*c^2*d*x*e^4
- 360*a^5*c^3*d*x*e^4 - 30*a^3*b^4*c*d^2*e^3 - 1660*a^4*b^2*c^2*d^2*e^3 - 1
280*a^5*c^3*d^2*e^3 - 12*a^3*b^5*x*e^5 - 604*a^4*b^3*c*x*e^5 - 332*a^5*b*c^
2*x*e^5 + 250*a^4*b^3*c*d*e^4 + 1100*a^5*b*c^2*d*e^4 - 3*a^4*b^4*e^5 - 166*
```

```
a^5*b^2*c*e^5 - 128*a^6*c^2*e^5)/((b^8*c - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 -
256*a^3*b^2*c^4 + 256*a^4*c^5)*(c*x^2 + b*x + a)^4)
```

3.2222 $\int \frac{(d+ex)^4}{\left(a+bx+cx^2\right)^5}\,dx$

Optimal. Leaf size=545

$$-\frac{-x\left(2c^2e^2\left(-18a^2e^2-40abde+305b^2d^2\right)-38b^2ce^3(5bd-ae)-40c^3d^2e(21bd-2ae)+19b^4e^4+420c^4d^4\right)-10bc\,(11a}{6\left(b^2-4ac\right)^4(a}$$

```
[Out] -((b + 2*c*x)*(d + e*x)^4)/(4*(b^2 - 4*a*c)*(a + b*x + c*x^2)^4) + ((d + e*
x)^3*(7*b*c*d - 2*b^2*e - 6*a*c*e + 7*c*(2*c*d - b*e)*x))/(6*(b^2 - 4*a*c)^
2*(a + b*x + c*x^2)^3) + ((d + e*x)^2*(28*b^2*c*d*e + 28*a*c^2*d*e - 3*b^3*
e^2 - b*c*(35*c*d^2 + 23*a*e^2) - c*(70*c^2*d^2 + 13*b^2*e^2 - 2*c*e*(35*b*
d - 9*a*e))*x))/(6*(b^2 - 4*a*c)^3*(a + b*x + c*x^2)^2) - (6*b^4*d*e^3 + 16
*a*c^2*d*e*(35*c*d^2 + 16*a*e^2) + 4*b^2*c*d*e*(70*c*d^2 + 83*a*e^2) - 5*b^
3*(19*c*d^2*e^2 + 5*a*e^4) - 10*b*c*(21*c^2*d^4 + 88*a*c*d^2*e^2 + 11*a^2*e
^4) - (420*c^4*d^4 + 19*b^4*e^4 - 40*c^3*d^2*e*(21*b*d - 2*a*e) - 38*b^2*c*
e^3*(5*b*d - a*e) + 2*c^2*e^2*(305*b^2*d^2 - 40*a*b*d*e - 18*a^2*e^2))*x)/(
6*(b^2 - 4*a*c)^4*(a + b*x + c*x^2)) - (2*(70*c^4*d^4 + b^4*e^4 - 4*b^2*c*e
^3*(5*b*d - 3*a*e) - 20*c^3*d^2*e*(7*b*d - 3*a*e) + 6*c^2*e^2*(15*b^2*d^2 -
 10*a*b*d*e + a^2*e^2))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*
c)^(9/2)
```

Rubi [A] time = 0.879508, antiderivative size = 545, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {736, 820, 777, 618, 206}

$$-\frac{-x\left(2c^2e^2\left(-18a^2e^2-40abde+305b^2d^2\right)-38b^2ce^3(5bd-ae)-40c^3d^2e(21bd-2ae)+19b^4e^4+420c^4d^4\right)-10bc\,(11a}{6\left(b^2-4ac\right)^4(a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a + b*x + c*x^2)^5,x]
```

```
[Out] -((b + 2*c*x)*(d + e*x)^4)/(4*(b^2 - 4*a*c)*(a + b*x + c*x^2)^4) + ((d + e*
x)^3*(7*b*c*d - 2*b^2*e - 6*a*c*e + 7*c*(2*c*d - b*e)*x))/(6*(b^2 - 4*a*c)^
2*(a + b*x + c*x^2)^3) + ((d + e*x)^2*(28*b^2*c*d*e + 28*a*c^2*d*e - 3*b^3*
e^2 - b*c*(35*c*d^2 + 23*a*e^2) - c*(70*c^2*d^2 + 13*b^2*e^2 - 2*c*e*(35*b*
d - 9*a*e))*x))/(6*(b^2 - 4*a*c)^3*(a + b*x + c*x^2)^2) - (6*b^4*d*e^3 + 16
*a*c^2*d*e*(35*c*d^2 + 16*a*e^2) + 4*b^2*c*d*e*(70*c*d^2 + 83*a*e^2) - 5*b^
3*(19*c*d^2*e^2 + 5*a*e^4) - 10*b*c*(21*c^2*d^4 + 88*a*c*d^2*e^2 + 11*a^2*e
^4) - (420*c^4*d^4 + 19*b^4*e^4 - 40*c^3*d^2*e*(21*b*d - 2*a*e) - 38*b^2*c*
e^3*(5*b*d - a*e) + 2*c^2*e^2*(305*b^2*d^2 - 40*a*b*d*e - 18*a^2*e^2))*x)/(
6*(b^2 - 4*a*c)^4*(a + b*x + c*x^2)) - (2*(70*c^4*d^4 + b^4*e^4 - 4*b^2*c*e
^3*(5*b*d - 3*a*e) - 20*c^3*d^2*e*(7*b*d - 3*a*e) + 6*c^2*e^2*(15*b^2*d^2 -
 10*a*b*d*e + a^2*e^2))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*
c)^(9/2)
```

Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] - Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)
*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p + 1
), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
```

```
- b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 0] && (L
tQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c,
d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*
(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p +
 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*Simp[g*
(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(
m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (I
ntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 777

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p +
1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(
2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^4}{\left(a+bx+cx^2\right)^5}\,dx = -\frac{(b+2cx)(d+ex)^4}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4} + \frac{\int \frac{(d+ex)^3(-14cd+4be-6cex)}{\left(a+bx+cx^2\right)^4}\,dx}{4\left(b^2-4ac\right)}$$

$$= -\frac{(b+2cx)(d+ex)^4}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4} + \frac{(d+ex)^3\left(7bcd-2b^2e-6ace+7c(2cd-be)x\right)}{6\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^3} - \frac{\int \frac{(d+ex)^2(-4\cdots}{\cdots}}{\cdots}$$

$$= -\frac{(b+2cx)(d+ex)^4}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4} + \frac{(d+ex)^3\left(7bcd-2b^2e-6ace+7c(2cd-be)x\right)}{6\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^3} + \frac{(d+ex)^2\left(\cdots\right.}{\cdots}$$

$$= -\frac{(b+2cx)(d+ex)^4}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4} + \frac{(d+ex)^3\left(7bcd-2b^2e-6ace+7c(2cd-be)x\right)}{6\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^3} + \frac{(d+ex)^2\left(\cdots\right.}{\cdots}$$

$$= -\frac{(b+2cx)(d+ex)^4}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4} + \frac{(d+ex)^3\left(7bcd-2b^2e-6ace+7c(2cd-be)x\right)}{6\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^3} + \frac{(d+ex)^2\left(\cdots\right.}{\cdots}$$

$$= -\frac{(b+2cx)(d+ex)^4}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4} + \frac{(d+ex)^3\left(7bcd-2b^2e-6ace+7c(2cd-be)x\right)}{6\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^3} + \frac{(d+ex)^2\left(\cdots\right.}{\cdots}$$

Mathematica [A] time = 1.88643, size = 713, normalized size = 1.31

$$\frac{1}{12}\left(\frac{6(b+2cx)\left(6c^2e^2\left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4\right)}{c\left(b^2-4ac\right)^4(a+x(b+cx))} + \frac{(b+2c\cdots}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a + b*x + c*x^2)^5,x]
```

```
[Out] (((70*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(5*b*d - 3*a*e) - 20*c^3*d^2*e*(7*b*d
 - 3*a*e) + 6*c^2*e^2*(15*b^2*d^2 - 10*a*b*d*e + a^2*e^2))*(b + 2*c*x))/(c^
2*(-b^2 + 4*a*c)^3*(a + x*(b + c*x))^2) + (6*(70*c^4*d^4 + b^4*e^4 - 4*b^2*
c*e^3*(5*b*d - 3*a*e) - 20*c^3*d^2*e*(7*b*d - 3*a*e) + 6*c^2*e^2*(15*b^2*d^
2 - 10*a*b*d*e + a^2*e^2))*(b + 2*c*x))/(c*(b^2 - 4*a*c)^4*(a + x*(b + c*x)
)) + (3*(b^4*e^4*x + b^3*e^3*(a*e - 4*c*d*x) + 2*b^2*c*e^2*(3*c*d^2*x - 2*a
*e*(d + e*x)) + b*c*(-3*a^2*e^4 + c^2*d^3*(d - 4*e*x) + 6*a*c*d*e^2*(d + 2*
e*x)) + 2*c^2*(c^2*d^4*x + a^2*e^3*(4*d + e*x) - 2*a*c*d^2*e*(2*d + 3*e*x))
))/(c^3*(-b^2 + 4*a*c)*(a + x*(b + c*x))^4) + (3*b^5*e^4 - 2*b^4*c*e^3*(6*d
 + e*x) + 2*b^3*c*e^2*(-10*a*e^2 + c*d*(9*d - 4*e*x)) + 2*b*c^2*(23*a^2*e^4
 + 7*c^2*d^3*(d - 4*e*x) + 6*a*c*d*e^2*(d - 2*e*x)) + 4*b^2*c^2*e*(a*e^2*(1
3*d + 6*e*x) + c*d^2*(-7*d + 9*e*x)) - 4*c^3*(-7*c^2*d^4*x - 6*a*c*d^2*e^2*
x + a^2*e^3*(32*d + 9*e*x)))/(c^3*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^3) + (2
4*(70*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(5*b*d - 3*a*e) - 20*c^3*d^2*e*(7*b*d
 - 3*a*e) + 6*c^2*e^2*(15*b^2*d^2 - 10*a*b*d*e + a^2*e^2))*ArcTan[(b + 2*c*
x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(9/2))/12
```

Maple [B] time = 0.169, size = 2430, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(c*x^2+b*x+a)^5,x)
```

```
[Out] ((6*a^2*c^2*e^4+12*a*b^2*c*e^4-60*a*b*c^2*d*e^3+60*a*c^3*d^2*e^2+b^4*e^4-20
*b^3*c*d*e^3+90*b^2*c^2*d^2*e^2-140*b*c^3*d^3*e+70*c^4*d^4)/(256*a^4*c^4-25
6*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)*c^3*x^7+7/2*(6*a^2*c^2*e^4+12*
a*b^2*c*e^4-60*a*b*c^2*d*e^3+60*a*c^3*d^2*e^2+b^4*e^4-20*b^3*c*d*e^3+90*b^2
*c^2*d^2*e^2-140*b*c^3*d^3*e+70*c^4*d^4)/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^
2*b^4*c^2-16*a*b^6*c+b^8)*b*c^2*x^6+1/3*c*(11*a*c+13*b^2)*(6*a^2*c^2*e^4+12
*a*b^2*c*e^4-60*a*b*c^2*d*e^3+60*a*c^3*d^2*e^2+b^4*e^4-20*b^3*c*d*e^3+90*b^
2*c^2*d^2*e^2-140*b*c^3*d^3*e+70*c^4*d^4)/(256*a^4*c^4-256*a^3*b^2*c^3+96*a
^2*b^4*c^2-16*a*b^6*c+b^8)*x^5+5/12*b*(22*a*c+5*b^2)*(6*a^2*c^2*e^4+12*a*b^
2*c*e^4-60*a*b*c^2*d*e^3+60*a*c^3*d^2*e^2+b^4*e^4-20*b^3*c*d*e^3+90*b^2*c^2
*d^2*e^2-140*b*c^3*d^3*e+70*c^4*d^4)/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^
4*c^2-16*a*b^6*c+b^8)*x^4-1/3*(66*a^4*c^3*e^4-450*a^3*b^2*c^2*e^4+876*a^3*b
*c^3*d*e^3-876*a^3*c^4*d^2*e^2-203*a^2*b^4*c*e^4+1504*a^2*b^3*c^2*d*e^3-252
6*a^2*b^2*c^3*d^2*e^2+2044*a^2*b*c^4*d^3*e-1022*a^2*c^5*d^4-37*a*b^6*e^4+44
0*a*b^5*c*d*e^3-1854*a*b^4*c^2*d^2*e^2+2828*a*b^3*c^3*d^3*e-1414*a*b^2*c^4*
d^4+12*b^7*d*e^3-54*b^6*c*d^2*e^2+84*b^5*c^2*d^3*e-42*b^4*c^3*d^4)/(256*a^4
*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)*x^3+1/6*(314*a^4*b*c^2*
e^4-1024*a^4*c^3*d*e^3+508*a^3*b^3*c*e^4-1604*a^3*b^2*c^2*d*e^3+2628*a^3*b*
c^3*d^2*e^2+129*a^2*b^5*e^4-1596*a^2*b^4*c*d*e^3+4278*a^2*b^3*c^2*d^2*e^2-6
132*a^2*b^2*c^3*d^3*e+3066*a^2*b*c^4*d^4-36*a*b^6*d*e^3+492*a*b^5*c*d^2*e^2
-784*a*b^4*c^2*d^3*e+392*a*b^3*c^3*d^4-18*b^7*d^2*e^2+28*b^6*c*d^3*e-14*b^5
*c^2*d^4)/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)*x^2-1
/3*(18*a^5*c^2*e^4-184*a^4*b^2*c*e^4+332*a^4*b*c^2*d*e^3+180*a^4*c^3*d^2*e^
2-47*a^3*b^4*e^4+604*a^3*b^3*c*d*e^3-1674*a^3*b^2*c^2*d^2*e^2+1116*a^3*b*c^
3*d^3*e-558*a^3*c^4*d^4+12*a^2*b^5*d*e^3-168*a^2*b^4*c*d^2*e^2+696*a^2*b^3*
c^2*d^3*e-348*a^2*b^2*c^3*d^4+6*a*b^6*d^2*e^2-76*a*b^5*c*d^3*e+38*a*b^4*c^2
*d^4+4*b^7*d^3*e-2*b^6*c*d^4)/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-1
6*a*b^6*c+b^8)*x+1/12*(220*a^5*b*c*e^4-512*a^5*c^2*d*e^3+50*a^4*b^3*e^4-664
*a^4*b^2*c*d*e^3+1944*a^4*b*c^2*d^2*e^2-1536*a^4*c^3*d^3*e-12*a^3*b^4*d*e^3
+168*a^3*b^3*c*d^2*e^2-696*a^3*b^2*c^2*d^3*e+1116*a^3*b*c^3*d^4-6*a^2*b^5*d
^2*e^2+76*a^2*b^4*c*d^3*e-326*a^2*b^3*c^2*d^4-4*a*b^6*d^3*e+50*a*b^5*c*d^4-
3*b^7*d^4)/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8))/(c*
x^2+b*x+a)^4+12/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)
/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^2*a^2*e^4+24/(256*
a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)/(4*a*c-b^2)^(1/2)*ar
ctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*c*e^4-120/(256*a^4*c^4-256*a^3*b^2*
c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*
c-b^2)^(1/2))*a*b*c^2*d*e^3+120/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2
-16*a*b^6*c+b^8)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^3*
a*d^2*e^2+2/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)/(4*
a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4*e^4-40/(256*a^4*c^4-
256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)/(4*a*c-b^2)^(1/2)*arctan((2*
c*x+b)/(4*a*c-b^2)^(1/2))*b^3*c*d*e^3+180/(256*a^4*c^4-256*a^3*b^2*c^3+96*a
^2*b^4*c^2-16*a*b^6*c+b^8)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2))*b^2*c^2*d^2*e^2-280/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*
b^6*c+b^8)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c^3*d^3*
e+140/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)/(4*a*c-b^
2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^4*d^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x+a)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.11802, size = 15663, normalized size = 28.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x+a)^5,x, algorithm="fricas")
```

```
[Out] [1/12*(12*(70*(b^2*c^7 - 4*a*c^8)*d^4 - 140*(b^3*c^6 - 4*a*b*c^7)*d^3*e + 3
0*(3*b^4*c^5 - 10*a*b^2*c^6 - 8*a^2*c^7)*d^2*e^2 - 20*(b^5*c^4 - a*b^3*c^5
- 12*a^2*b*c^6)*d*e^3 + (b^6*c^3 + 8*a*b^4*c^4 - 42*a^2*b^2*c^5 - 24*a^3*c^
6)*e^4)*x^7 + 42*(70*(b^3*c^6 - 4*a*b*c^7)*d^4 - 140*(b^4*c^5 - 4*a*b^2*c^6
)*d^3*e + 30*(3*b^5*c^4 - 10*a*b^3*c^5 - 8*a^2*b*c^6)*d^2*e^2 - 20*(b^6*c^3
 - a*b^4*c^4 - 12*a^2*b^2*c^5)*d*e^3 + (b^7*c^2 + 8*a*b^5*c^3 - 42*a^2*b^3*
c^4 - 24*a^3*b*c^5)*e^4)*x^6 + 4*(70*(13*b^4*c^5 - 41*a*b^2*c^6 - 44*a^2*c^
7)*d^4 - 140*(13*b^5*c^4 - 41*a*b^3*c^5 - 44*a^2*b*c^6)*d^3*e + 30*(39*b^6*
c^3 - 97*a*b^4*c^4 - 214*a^2*b^2*c^5 - 88*a^3*c^6)*d^2*e^2 - 20*(13*b^7*c^2
 - 2*a*b^5*c^3 - 167*a^2*b^3*c^4 - 132*a^3*b*c^5)*d*e^3 + (13*b^8*c + 115*a
*b^6*c^2 - 458*a^2*b^4*c^3 - 774*a^3*b^2*c^4 - 264*a^4*c^5)*e^4)*x^5 - (3*b
^9 - 62*a*b^7*c + 526*a^2*b^5*c^2 - 2420*a^3*b^3*c^3 + 4464*a^4*b*c^4)*d^4
- 4*(a*b^8 - 23*a^2*b^6*c + 250*a^3*b^4*c^2 - 312*a^4*b^2*c^3 - 1536*a^5*c^
4)*d^3*e - 6*(a^2*b^7 - 32*a^3*b^5*c - 212*a^4*b^3*c^2 + 1296*a^5*b*c^3)*d^
2*e^2 - 4*(3*a^3*b^6 + 154*a^4*b^4*c - 536*a^5*b^2*c^2 - 512*a^6*c^3)*d*e^3
 + 10*(5*a^4*b^5 + 2*a^5*b^3*c - 88*a^6*b*c^2)*e^4 + 5*(70*(5*b^5*c^4 + 2*a
*b^3*c^5 - 88*a^2*b*c^6)*d^4 - 140*(5*b^6*c^3 + 2*a*b^4*c^4 - 88*a^2*b^2*c^
5)*d^3*e + 30*(15*b^7*c^2 + 16*a*b^5*c^3 - 260*a^2*b^3*c^4 - 176*a^3*b*c^5)
*d^2*e^2 - 20*(5*b^8*c + 17*a*b^6*c^2 - 82*a^2*b^4*c^3 - 264*a^3*b^2*c^4)*d
*e^3 + (5*b^9 + 62*a*b^7*c - 34*a^2*b^5*c^2 - 1044*a^3*b^3*c^3 - 528*a^4*b*
c^4)*e^4)*x^4 + 4*(14*(3*b^6*c^3 + 89*a*b^4*c^4 - 331*a^2*b^2*c^5 - 292*a^3
*c^6)*d^4 - 28*(3*b^7*c^2 + 89*a*b^5*c^3 - 331*a^2*b^3*c^4 - 292*a^3*b*c^5)
*d^3*e + 6*(9*b^8*c + 273*a*b^6*c^2 - 815*a^2*b^4*c^3 - 1538*a^3*b^2*c^4 -
584*a^4*c^5)*d^2*e^2 - 4*(3*b^9 + 98*a*b^7*c - 64*a^2*b^5*c^2 - 1285*a^3*b^
3*c^3 - 876*a^4*b*c^4)*d*e^3 + (37*a*b^8 + 55*a^2*b^6*c - 362*a^3*b^4*c^2 -
 1866*a^4*b^2*c^3 + 264*a^5*c^4)*e^4)*x^3 - 2*(14*(b^7*c^2 - 32*a*b^5*c^3 -
 107*a^2*b^3*c^4 + 876*a^3*b*c^5)*d^4 - 28*(b^8*c - 32*a*b^6*c^2 - 107*a^2*
b^4*c^3 + 876*a^3*b^2*c^4)*d^3*e + 6*(3*b^9 - 94*a*b^7*c - 385*a^2*b^5*c^2
+ 2414*a^3*b^3*c^3 + 1752*a^4*b*c^4)*d^2*e^2 + 4*(9*a*b^8 + 363*a^2*b^6*c -
 1195*a^3*b^4*c^2 - 1348*a^4*b^2*c^3 - 1024*a^5*c^4)*d*e^3 - (129*a^2*b^7 -
 8*a^3*b^5*c - 1718*a^4*b^3*c^2 - 1256*a^5*b*c^3)*e^4)*x^2 + 12*(70*a^4*c^4
*d^4 - 140*a^4*b*c^3*d^3*e + (70*c^8*d^4 - 140*b*c^7*d^3*e + 30*(3*b^2*c^6
+ 2*a*c^7)*d^2*e^2 - 20*(b^3*c^5 + 3*a*b*c^6)*d*e^3 + (b^4*c^4 + 12*a*b^2*c
^5 + 6*a^2*c^6)*e^4)*x^8 + 4*(70*b*c^7*d^4 - 140*b^2*c^6*d^3*e + 30*(3*b^3*
c^5 + 2*a*b*c^6)*d^2*e^2 - 20*(b^4*c^4 + 3*a*b^2*c^5)*d*e^3 + (b^5*c^3 + 12
*a*b^3*c^4 + 6*a^2*b*c^5)*e^4)*x^7 + 2*(70*(3*b^2*c^6 + 2*a*c^7)*d^4 - 140*
(3*b^3*c^5 + 2*a*b*c^6)*d^3*e + 30*(9*b^4*c^4 + 12*a*b^2*c^5 + 4*a^2*c^6)*d
^2*e^2 - 20*(3*b^5*c^3 + 11*a*b^3*c^4 + 6*a^2*b*c^5)*d*e^3 + (3*b^6*c^2 + 3
8*a*b^4*c^3 + 42*a^2*b^2*c^4 + 12*a^3*c^5)*e^4)*x^6 + 4*(70*(b^3*c^5 + 3*a*
b*c^6)*d^4 - 140*(b^4*c^4 + 3*a*b^2*c^5)*d^3*e + 30*(3*b^5*c^3 + 11*a*b^3*c
^4 + 6*a^2*b*c^5)*d^2*e^2 - 20*(b^6*c^2 + 6*a*b^4*c^3 + 9*a^2*b^2*c^4)*d*e^
3 + (b^7*c + 15*a*b^5*c^2 + 42*a^2*b^3*c^3 + 18*a^3*b*c^4)*e^4)*x^5 + 30*(3
*a^4*b^2*c^2 + 2*a^5*c^3)*d^2*e^2 - 20*(a^4*b^3*c + 3*a^5*b*c^2)*d*e^3 + (a
^4*b^4 + 12*a^5*b^2*c + 6*a^6*c^2)*e^4 + (70*(b^4*c^4 + 12*a*b^2*c^5 + 6*a^
2*c^6)*d^4 - 140*(b^5*c^3 + 12*a*b^3*c^4 + 6*a^2*b*c^5)*d^3*e + 30*(3*b^6*c
```

^2 + 38*a*b^4*c^3 + 42*a^2*b^2*c^4 + 12*a^3*c^5)*d^2*e^2 - 20*(b^7*c + 15*a
*b^5*c^2 + 42*a^2*b^3*c^3 + 18*a^3*b*c^4)*d*e^3 + (b^8 + 24*a*b^6*c + 156*a
^2*b^4*c^2 + 144*a^3*b^2*c^3 + 36*a^4*c^4)*e^4)*x^4 + 4*(70*(a*b^3*c^4 + 3*
a^2*b*c^5)*d^4 - 140*(a*b^4*c^3 + 3*a^2*b^2*c^4)*d^3*e + 30*(3*a*b^5*c^2 +
11*a^2*b^3*c^3 + 6*a^3*b*c^4)*d^2*e^2 - 20*(a*b^6*c + 6*a^2*b^4*c^2 + 9*a^3
*b^2*c^3)*d*e^3 + (a*b^7 + 15*a^2*b^5*c + 42*a^3*b^3*c^2 + 18*a^4*b*c^3)*e^
4)*x^3 + 2*(70*(3*a^2*b^2*c^4 + 2*a^3*c^5)*d^4 - 140*(3*a^2*b^3*c^3 + 2*a^3
*b*c^4)*d^3*e + 30*(9*a^2*b^4*c^2 + 12*a^3*b^2*c^3 + 4*a^4*c^4)*d^2*e^2 - 2
0*(3*a^2*b^5*c + 11*a^3*b^3*c^2 + 6*a^4*b*c^3)*d*e^3 + (3*a^2*b^6 + 38*a^3*
b^4*c + 42*a^4*b^2*c^2 + 12*a^5*c^3)*e^4)*x^2 + 4*(70*a^3*b*c^4*d^4 - 140*a
^3*b^2*c^3*d^3*e + 30*(3*a^3*b^3*c^2 + 2*a^4*b*c^3)*d^2*e^2 - 20*(a^3*b^4*c
 + 3*a^4*b^2*c^2)*d*e^3 + (a^3*b^5 + 12*a^4*b^3*c + 6*a^5*b*c^2)*e^4)*x)*sq
rt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*
(2*c*x + b))/(c*x^2 + b*x + a)) + 4*(2*(b^8*c - 23*a*b^6*c^2 + 250*a^2*b^4*
c^3 - 417*a^3*b^2*c^4 - 1116*a^4*c^5)*d^4 - 4*(b^9 - 23*a*b^7*c + 250*a^2*b
^5*c^2 - 417*a^3*b^3*c^3 - 1116*a^4*b*c^4)*d^3*e - 6*(a*b^8 - 32*a^2*b^6*c
- 167*a^3*b^4*c^2 + 1146*a^4*b^2*c^3 - 120*a^5*c^4)*d^2*e^2 - 4*(3*a^2*b^7
+ 139*a^3*b^5*c - 521*a^4*b^3*c^2 - 332*a^5*b*c^3)*d*e^3 + (47*a^3*b^6 - 4*
a^4*b^4*c - 754*a^5*b^2*c^2 + 72*a^6*c^3)*e^4)*x)/(a^4*b^10 - 20*a^5*b^8*c
+ 160*a^6*b^6*c^2 - 640*a^7*b^4*c^3 + 1280*a^8*b^2*c^4 - 1024*a^9*c^5 + (b^
10*c^4 - 20*a*b^8*c^5 + 160*a^2*b^6*c^6 - 640*a^3*b^4*c^7 + 1280*a^4*b^2*c^
8 - 1024*a^5*c^9)*x^8 + 4*(b^11*c^3 - 20*a*b^9*c^4 + 160*a^2*b^7*c^5 - 640*
a^3*b^5*c^6 + 1280*a^4*b^3*c^7 - 1024*a^5*b*c^8)*x^7 + 2*(3*b^12*c^2 - 58*a
*b^10*c^3 + 440*a^2*b^8*c^4 - 1600*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 512*a^5
*b^2*c^7 - 2048*a^6*c^8)*x^6 + 4*(b^13*c - 17*a*b^11*c^2 + 100*a^2*b^9*c^3
- 160*a^3*b^7*c^4 - 640*a^4*b^5*c^5 + 2816*a^5*b^3*c^6 - 3072*a^6*b*c^7)*x^
5 + (b^14 - 8*a*b^12*c - 74*a^2*b^10*c^2 + 1160*a^3*b^8*c^3 - 5440*a^4*b^6*
c^4 + 10496*a^5*b^4*c^5 - 4608*a^6*b^2*c^6 - 6144*a^7*c^7)*x^4 + 4*(a*b^13
- 17*a^2*b^11*c + 100*a^3*b^9*c^2 - 160*a^4*b^7*c^3 - 640*a^5*b^5*c^4 + 281
6*a^6*b^3*c^5 - 3072*a^7*b*c^6)*x^3 + 2*(3*a^2*b^12 - 58*a^3*b^10*c + 440*a
^4*b^8*c^2 - 1600*a^5*b^6*c^3 + 2560*a^6*b^4*c^4 - 512*a^7*b^2*c^5 - 2048*a
^8*c^6)*x^2 + 4*(a^3*b^11 - 20*a^4*b^9*c + 160*a^5*b^7*c^2 - 640*a^6*b^5*c^
3 + 1280*a^7*b^3*c^4 - 1024*a^8*b*c^5)*x), 1/12*(12*(70*(b^2*c^7 - 4*a*c^8)
d^4 - 140(b^3*c^6 - 4*a*b*c^7)*d^3*e + 30*(3*b^4*c^5 - 10*a*b^2*c^6 - 8*a
^2*c^7)*d^2*e^2 - 20*(b^5*c^4 - a*b^3*c^5 - 12*a^2*b*c^6)*d*e^3 + (b^6*c^3
+ 8*a*b^4*c^4 - 42*a^2*b^2*c^5 - 24*a^3*c^6)*e^4)*x^7 + 42*(70*(b^3*c^6 - 4
*a*b*c^7)*d^4 - 140*(b^4*c^5 - 4*a*b^2*c^6)*d^3*e + 30*(3*b^5*c^4 - 10*a*b^
3*c^5 - 8*a^2*b*c^6)*d^2*e^2 - 20*(b^6*c^3 - a*b^4*c^4 - 12*a^2*b^2*c^5)*d*
e^3 + (b^7*c^2 + 8*a*b^5*c^3 - 42*a^2*b^3*c^4 - 24*a^3*b*c^5)*e^4)*x^6 + 4*
(70*(13*b^4*c^5 - 41*a*b^2*c^6 - 44*a^2*c^7)*d^4 - 140*(13*b^5*c^4 - 41*a*b
^3*c^5 - 44*a^2*b*c^6)*d^3*e + 30*(39*b^6*c^3 - 97*a*b^4*c^4 - 214*a^2*b^2*
c^5 - 88*a^3*c^6)*d^2*e^2 - 20*(13*b^7*c^2 - 2*a*b^5*c^3 - 167*a^2*b^3*c^4
- 132*a^3*b*c^5)*d*e^3 + (13*b^8*c + 115*a*b^6*c^2 - 458*a^2*b^4*c^3 - 774*
a^3*b^2*c^4 - 264*a^4*c^5)*e^4)*x^5 - (3*b^9 - 62*a*b^7*c + 526*a^2*b^5*c^2
 - 2420*a^3*b^3*c^3 + 4464*a^4*b*c^4)*d^4 - 4*(a*b^8 - 23*a^2*b^6*c + 250*a
^3*b^4*c^2 - 312*a^4*b^2*c^3 - 1536*a^5*c^4)*d^3*e - 6*(a^2*b^7 - 32*a^3*b^
5*c - 212*a^4*b^3*c^2 + 1296*a^5*b*c^3)*d^2*e^2 - 4*(3*a^3*b^6 + 154*a^4*b^
4*c - 536*a^5*b^2*c^2 - 512*a^6*c^3)*d*e^3 + 10*(5*a^4*b^5 + 2*a^5*b^3*c -
88*a^6*b*c^2)*e^4 + 5*(70*(5*b^5*c^4 + 2*a*b^3*c^5 - 88*a^2*b*c^6)*d^4 - 14
0*(5*b^6*c^3 + 2*a*b^4*c^4 - 88*a^2*b^2*c^5)*d^3*e + 30*(15*b^7*c^2 + 16*a*
b^5*c^3 - 260*a^2*b^3*c^4 - 176*a^3*b*c^5)*d^2*e^2 - 20*(5*b^8*c + 17*a*b^6
*c^2 - 82*a^2*b^4*c^3 - 264*a^3*b^2*c^4)*d*e^3 + (5*b^9 + 62*a*b^7*c - 34*a
^2*b^5*c^2 - 1044*a^3*b^3*c^3 - 528*a^4*b*c^4)*e^4)*x^4 + 4*(14*(3*b^6*c^3
+ 89*a*b^4*c^4 - 331*a^2*b^2*c^5 - 292*a^3*c^6)*d^4 - 28*(3*b^7*c^2 + 89*a*
b^5*c^3 - 331*a^2*b^3*c^4 - 292*a^3*b*c^5)*d^3*e + 6*(9*b^8*c + 273*a*b^6*c
^2 - 815*a^2*b^4*c^3 - 1538*a^3*b^2*c^4 - 584*a^4*c^5)*d^2*e^2 - 4*(3*b^9 +
 98*a*b^7*c - 64*a^2*b^5*c^2 - 1285*a^3*b^3*c^3 - 876*a^4*b*c^4)*d*e^3 + (3
7*a*b^8 + 55*a^2*b^6*c - 362*a^3*b^4*c^2 - 1866*a^4*b^2*c^3 + 264*a^5*c^4)*
e^4)*x^3 - 2*(14*(b^7*c^2 - 32*a*b^5*c^3 - 107*a^2*b^3*c^4 + 876*a^3*b*c^5)

```
*d^4 - 28*(b^8*c - 32*a*b^6*c^2 - 107*a^2*b^4*c^3 + 876*a^3*b^2*c^4)*d^3*e
+ 6*(3*b^9 - 94*a*b^7*c - 385*a^2*b^5*c^2 + 2414*a^3*b^3*c^3 + 1752*a^4*b*c
^4)*d^2*e^2 + 4*(9*a*b^8 + 363*a^2*b^6*c - 1195*a^3*b^4*c^2 - 1348*a^4*b^2*
c^3 - 1024*a^5*c^4)*d*e^3 - (129*a^2*b^7 - 8*a^3*b^5*c - 1718*a^4*b^3*c^2 -
 1256*a^5*b*c^3)*e^4)*x^2 - 24*(70*a^4*c^4*d^4 - 140*a^4*b*c^3*d^3*e + (70*
c^8*d^4 - 140*b*c^7*d^3*e + 30*(3*b^2*c^6 + 2*a*c^7)*d^2*e^2 - 20*(b^3*c^5
+ 3*a*b*c^6)*d*e^3 + (b^4*c^4 + 12*a*b^2*c^5 + 6*a^2*c^6)*e^4)*x^8 + 4*(70*
b*c^7*d^4 - 140*b^2*c^6*d^3*e + 30*(3*b^3*c^5 + 2*a*b*c^6)*d^2*e^2 - 20*(b^
4*c^4 + 3*a*b^2*c^5)*d*e^3 + (b^5*c^3 + 12*a*b^3*c^4 + 6*a^2*b*c^5)*e^4)*x^
7 + 2*(70*(3*b^2*c^6 + 2*a*c^7)*d^4 - 140*(3*b^3*c^5 + 2*a*b*c^6)*d^3*e + 3
0*(9*b^4*c^4 + 12*a*b^2*c^5 + 4*a^2*c^6)*d^2*e^2 - 20*(3*b^5*c^3 + 11*a*b^3
*c^4 + 6*a^2*b*c^5)*d*e^3 + (3*b^6*c^2 + 38*a*b^4*c^3 + 42*a^2*b^2*c^4 + 12
*a^3*c^5)*e^4)*x^6 + 4*(70*(b^3*c^5 + 3*a*b*c^6)*d^4 - 140*(b^4*c^4 + 3*a*b
^2*c^5)*d^3*e + 30*(3*b^5*c^3 + 11*a*b^3*c^4 + 6*a^2*b*c^5)*d^2*e^2 - 20*(b
^6*c^2 + 6*a*b^4*c^3 + 9*a^2*b^2*c^4)*d*e^3 + (b^7*c + 15*a*b^5*c^2 + 42*a^
2*b^3*c^3 + 18*a^3*b*c^4)*e^4)*x^5 + 30*(3*a^4*b^2*c^2 + 2*a^5*c^3)*d^2*e^2
 - 20*(a^4*b^3*c + 3*a^5*b*c^2)*d*e^3 + (a^4*b^4 + 12*a^5*b^2*c + 6*a^6*c^2
)*e^4 + (70*(b^4*c^4 + 12*a*b^2*c^5 + 6*a^2*c^6)*d^4 - 140*(b^5*c^3 + 12*a*
b^3*c^4 + 6*a^2*b*c^5)*d^3*e + 30*(3*b^6*c^2 + 38*a*b^4*c^3 + 42*a^2*b^2*c^
4 + 12*a^3*c^5)*d^2*e^2 - 20*(b^7*c + 15*a*b^5*c^2 + 42*a^2*b^3*c^3 + 18*a^
3*b*c^4)*d*e^3 + (b^8 + 24*a*b^6*c + 156*a^2*b^4*c^2 + 144*a^3*b^2*c^3 + 36
*a^4*c^4)*e^4)*x^4 + 4*(70*(a*b^3*c^4 + 3*a^2*b*c^5)*d^4 - 140*(a*b^4*c^3 +
 3*a^2*b^2*c^4)*d^3*e + 30*(3*a*b^5*c^2 + 11*a^2*b^3*c^3 + 6*a^3*b*c^4)*d^2
*e^2 - 20*(a*b^6*c + 6*a^2*b^4*c^2 + 9*a^3*b^2*c^3)*d*e^3 + (a*b^7 + 15*a^2
*b^5*c + 42*a^3*b^3*c^2 + 18*a^4*b*c^3)*e^4)*x^3 + 2*(70*(3*a^2*b^2*c^4 + 2
*a^3*c^5)*d^4 - 140*(3*a^2*b^3*c^3 + 2*a^3*b*c^4)*d^3*e + 30*(9*a^2*b^4*c^2
 + 12*a^3*b^2*c^3 + 4*a^4*c^4)*d^2*e^2 - 20*(3*a^2*b^5*c + 11*a^3*b^3*c^2 +
 6*a^4*b*c^3)*d*e^3 + (3*a^2*b^6 + 38*a^3*b^4*c + 42*a^4*b^2*c^2 + 12*a^5*c
^3)*e^4)*x^2 + 4*(70*a^3*b*c^4*d^4 - 140*a^3*b^2*c^3*d^3*e + 30*(3*a^3*b^3*
c^2 + 2*a^4*b*c^3)*d^2*e^2 - 20*(a^3*b^4*c + 3*a^4*b^2*c^2)*d*e^3 + (a^3*b^
5 + 12*a^4*b^3*c + 6*a^5*b*c^2)*e^4)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^
2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 4*(2*(b^8*c - 23*a*b^6*c^2 + 250*a^
2*b^4*c^3 - 417*a^3*b^2*c^4 - 1116*a^4*c^5)*d^4 - 4*(b^9 - 23*a*b^7*c + 250
*a^2*b^5*c^2 - 417*a^3*b^3*c^3 - 1116*a^4*b*c^4)*d^3*e - 6*(a*b^8 - 32*a^2*
b^6*c - 167*a^3*b^4*c^2 + 1146*a^4*b^2*c^3 - 120*a^5*c^4)*d^2*e^2 - 4*(3*a^
2*b^7 + 139*a^3*b^5*c - 521*a^4*b^3*c^2 - 332*a^5*b*c^3)*d*e^3 + (47*a^3*b^
6 - 4*a^4*b^4*c - 754*a^5*b^2*c^2 + 72*a^6*c^3)*e^4)*x)/(a^4*b^10 - 20*a^5*
b^8*c + 160*a^6*b^6*c^2 - 640*a^7*b^4*c^3 + 1280*a^8*b^2*c^4 - 1024*a^9*c^5
 + (b^10*c^4 - 20*a*b^8*c^5 + 160*a^2*b^6*c^6 - 640*a^3*b^4*c^7 + 1280*a^4*
b^2*c^8 - 1024*a^5*c^9)*x^8 + 4*(b^11*c^3 - 20*a*b^9*c^4 + 160*a^2*b^7*c^5
- 640*a^3*b^5*c^6 + 1280*a^4*b^3*c^7 - 1024*a^5*b*c^8)*x^7 + 2*(3*b^12*c^2
- 58*a*b^10*c^3 + 440*a^2*b^8*c^4 - 1600*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 5
12*a^5*b^2*c^7 - 2048*a^6*c^8)*x^6 + 4*(b^13*c - 17*a*b^11*c^2 + 100*a^2*b^
9*c^3 - 160*a^3*b^7*c^4 - 640*a^4*b^5*c^5 + 2816*a^5*b^3*c^6 - 3072*a^6*b*c
^7)*x^5 + (b^14 - 8*a*b^12*c - 74*a^2*b^10*c^2 + 1160*a^3*b^8*c^3 - 5440*a^
4*b^6*c^4 + 10496*a^5*b^4*c^5 - 4608*a^6*b^2*c^6 - 6144*a^7*c^7)*x^4 + 4*(a
*b^13 - 17*a^2*b^11*c + 100*a^3*b^9*c^2 - 160*a^4*b^7*c^3 - 640*a^5*b^5*c^4
 + 2816*a^6*b^3*c^5 - 3072*a^7*b*c^6)*x^3 + 2*(3*a^2*b^12 - 58*a^3*b^10*c +
 440*a^4*b^8*c^2 - 1600*a^5*b^6*c^3 + 2560*a^6*b^4*c^4 - 512*a^7*b^2*c^5 -
2048*a^8*c^6)*x^2 + 4*(a^3*b^11 - 20*a^4*b^9*c + 160*a^5*b^7*c^2 - 640*a^6*
b^5*c^3 + 1280*a^7*b^3*c^4 - 1024*a^8*b*c^5)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(c*x**2+b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17875, size = 2483, normalized size = 4.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x+a)^5,x, algorithm="giac")
```

```
[Out] 2*(70*c^4*d^4 - 140*b*c^3*d^3*e + 90*b^2*c^2*d^2*e^2 + 60*a*c^3*d^2*e^2 - 2
0*b^3*c*d*e^3 - 60*a*b*c^2*d*e^3 + b^4*e^4 + 12*a*b^2*c*e^4 + 6*a^2*c^2*e^4
)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^8 - 16*a*b^6*c + 96*a^2*b^4*c^
2 - 256*a^3*b^2*c^3 + 256*a^4*c^4)*sqrt(-b^2 + 4*a*c)) + 1/12*(840*c^7*d^4*
x^7 - 1680*b*c^6*d^3*x^7*e + 2940*b*c^6*d^4*x^6 + 1080*b^2*c^5*d^2*x^7*e^2
+ 720*a*c^6*d^2*x^7*e^2 - 5880*b^2*c^5*d^3*x^6*e + 3640*b^2*c^5*d^4*x^5 + 3
080*a*c^6*d^4*x^5 - 240*b^3*c^4*d*x^7*e^3 - 720*a*b*c^5*d*x^7*e^3 + 3780*b^
3*c^4*d^2*x^6*e^2 + 2520*a*b*c^5*d^2*x^6*e^2 - 7280*b^3*c^4*d^3*x^5*e - 616
0*a*b*c^5*d^3*x^5*e + 1750*b^3*c^4*d^4*x^4 + 7700*a*b*c^5*d^4*x^4 + 12*b^4*
c^3*x^7*e^4 + 144*a*b^2*c^4*x^7*e^4 + 72*a^2*c^5*x^7*e^4 - 840*b^4*c^3*d*x^
6*e^3 - 2520*a*b^2*c^4*d*x^6*e^3 + 4680*b^4*c^3*d^2*x^5*e^2 + 7080*a*b^2*c^
4*d^2*x^5*e^2 + 2640*a^2*c^5*d^2*x^5*e^2 - 3500*b^4*c^3*d^3*x^4*e - 15400*a
*b^2*c^4*d^3*x^4*e + 168*b^4*c^3*d^4*x^3 + 5656*a*b^2*c^4*d^4*x^3 + 4088*a^
2*c^5*d^4*x^3 + 42*b^5*c^2*x^6*e^4 + 504*a*b^3*c^3*x^6*e^4 + 252*a^2*b*c^4*
x^6*e^4 - 1040*b^5*c^2*d*x^5*e^3 - 4000*a*b^3*c^3*d*x^5*e^3 - 2640*a^2*b*c^
4*d*x^5*e^3 + 2250*b^5*c^2*d^2*x^4*e^2 + 11400*a*b^3*c^3*d^2*x^4*e^2 + 6600
*a^2*b*c^4*d^2*x^4*e^2 - 336*b^5*c^2*d^3*x^3*e - 11312*a*b^3*c^3*d^3*x^3*e
- 8176*a^2*b*c^4*d^3*x^3*e - 28*b^5*c^2*d^4*x^2 + 784*a*b^3*c^3*d^4*x^2 + 6
132*a^2*b*c^4*d^4*x^2 + 52*b^6*c*x^5*e^4 + 668*a*b^4*c^2*x^5*e^4 + 840*a^2*
b^2*c^3*x^5*e^4 + 264*a^3*c^4*x^5*e^4 - 500*b^6*c*d*x^4*e^3 - 3700*a*b^4*c^
2*d*x^4*e^3 - 6600*a^2*b^2*c^3*d*x^4*e^3 + 216*b^6*c*d^2*x^3*e^2 + 7416*a*b
^4*c^2*d^2*x^3*e^2 + 10104*a^2*b^2*c^3*d^2*x^3*e^2 + 3504*a^3*c^4*d^2*x^3*e
^2 + 56*b^6*c*d^3*x^2*e - 1568*a*b^4*c^2*d^3*x^2*e - 12264*a^2*b^2*c^3*d^3*
x^2*e + 8*b^6*c*d^4*x - 152*a*b^4*c^2*d^4*x + 1392*a^2*b^2*c^3*d^4*x + 2232
*a^3*c^4*d^4*x + 25*b^7*x^4*e^4 + 410*a*b^5*c*x^4*e^4 + 1470*a^2*b^3*c^2*x^
4*e^4 + 660*a^3*b*c^3*x^4*e^4 - 48*b^7*d*x^3*e^3 - 1760*a*b^5*c*d*x^3*e^3 -
 6016*a^2*b^3*c^2*d*x^3*e^3 - 3504*a^3*b*c^3*d*x^3*e^3 - 36*b^7*d^2*x^2*e^2
 + 984*a*b^5*c*d^2*x^2*e^2 + 8556*a^2*b^3*c^2*d^2*x^2*e^2 + 5256*a^3*b*c^3*
d^2*x^2*e^2 - 16*b^7*d^3*x*e + 304*a*b^5*c*d^3*x*e - 2784*a^2*b^3*c^2*d^3*x
*e - 4464*a^3*b*c^3*d^3*x*e - 3*b^7*d^4 + 50*a*b^5*c*d^4 - 326*a^2*b^3*c^2*
d^4 + 1116*a^3*b*c^3*d^4 + 148*a*b^6*x^3*e^4 + 812*a^2*b^4*c*x^3*e^4 + 1800
*a^3*b^2*c^2*x^3*e^4 - 264*a^4*c^3*x^3*e^4 - 72*a*b^6*d*x^2*e^3 - 3192*a^2*
b^4*c*d*x^2*e^3 - 3208*a^3*b^2*c^2*d*x^2*e^3 - 2048*a^4*c^3*d*x^2*e^3 - 24*
a*b^6*d^2*x*e^2 + 672*a^2*b^4*c*d^2*x*e^2 + 6696*a^3*b^2*c^2*d^2*x*e^2 - 72
0*a^4*c^3*d^2*x*e^2 - 4*a*b^6*d^3*e + 76*a^2*b^4*c*d^3*e - 696*a^3*b^2*c^2*
d^3*e - 1536*a^4*c^3*d^3*e + 258*a^2*b^5*x^2*e^4 + 1016*a^3*b^3*c*x^2*e^4 +
 628*a^4*b*c^2*x^2*e^4 - 48*a^2*b^5*d*x*e^3 - 2416*a^3*b^3*c*d*x*e^3 - 1328
*a^4*b*c^2*d*x*e^3 - 6*a^2*b^5*d^2*e^2 + 168*a^3*b^3*c*d^2*e^2 + 1944*a^4*b
*c^2*d^2*e^2 + 188*a^3*b^4*x*e^4 + 736*a^4*b^2*c*x*e^4 - 72*a^5*c^2*x*e^4 -
 12*a^3*b^4*d*e^3 - 664*a^4*b^2*c*d*e^3 - 512*a^5*c^2*d*e^3 + 50*a^4*b^3*e^
4 + 220*a^5*b*c*e^4)/((b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3
+ 256*a^4*c^4)*(c*x^2 + b*x + a)^4)
```

3.2223 $\int \frac{(d+ex)^3}{\left(a+bx+cx^2\right)^5}\,dx$

Optimal. Leaf size=378

$$\frac{5(b+2cx)(2cd-be)\left(-ce(7bd-3ae)+b^2e^2+7c^2d^2\right)}{2\left(b^2-4ac\right)^4\left(a+bx+cx^2\right)}-\frac{2x(2cd-be)\left(-ce(13ae+35bd)+12b^2e^2+35c^2d^2\right)-b^2\left(27ae\right.}{12\left(b^2-4ac\right.}$$

```
[Out] -((b + 2*c*x)*(d + e*x)^3)/(4*(b^2 - 4*a*c)*(a + b*x + c*x^2)^4) + ((d + e*
x)^2*(14*b*c*d - 3*b^2*e - 16*a*c*e + 14*c*(2*c*d - b*e)*x))/(12*(b^2 - 4*a
*c)^2*(a + b*x + c*x^2)^3) - (3*b^3*d*e^2 - 32*a*c*e*(7*c*d^2 + a*e^2) + 2*
b*c*d*(35*c*d^2 + 99*a*e^2) - b^2*(49*c*d^2*e + 27*a*e^3) + 2*(2*c*d - b*e)
*(35*c^2*d^2 + 12*b^2*e^2 - c*e*(35*b*d + 13*a*e))*x)/(12*(b^2 - 4*a*c)^3*(
a + b*x + c*x^2)^2) + (5*(2*c*d - b*e)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d -
3*a*e))*(b + 2*c*x))/(2*(b^2 - 4*a*c)^4*(a + b*x + c*x^2)) - (10*c*(2*c*d -
 b*e)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/Sqrt[
b^2 - 4*a*c]])/(b^2 - 4*a*c)^(9/2)
```

Rubi [A] time = 0.501609, antiderivative size = 378, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {736, 820, 777, 614, 618, 206}

$$\frac{5(b+2cx)(2cd-be)\left(-ce(7bd-3ae)+b^2e^2+7c^2d^2\right)}{2\left(b^2-4ac\right)^4\left(a+bx+cx^2\right)}-\frac{2x(2cd-be)\left(-ce(13ae+35bd)+12b^2e^2+35c^2d^2\right)-b^2\left(27ae\right.}{12\left(b^2-4ac\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + b*x + c*x^2)^5,x]
```

```
[Out] -((b + 2*c*x)*(d + e*x)^3)/(4*(b^2 - 4*a*c)*(a + b*x + c*x^2)^4) + ((d + e*
x)^2*(14*b*c*d - 3*b^2*e - 16*a*c*e + 14*c*(2*c*d - b*e)*x))/(12*(b^2 - 4*a
*c)^2*(a + b*x + c*x^2)^3) - (3*b^3*d*e^2 - 32*a*c*e*(7*c*d^2 + a*e^2) + 2*
b*c*d*(35*c*d^2 + 99*a*e^2) - b^2*(49*c*d^2*e + 27*a*e^3) + 2*(2*c*d - b*e)
*(35*c^2*d^2 + 12*b^2*e^2 - c*e*(35*b*d + 13*a*e))*x)/(12*(b^2 - 4*a*c)^3*(
a + b*x + c*x^2)^2) + (5*(2*c*d - b*e)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d -
3*a*e))*(b + 2*c*x))/(2*(b^2 - 4*a*c)^4*(a + b*x + c*x^2)) - (10*c*(2*c*d -
 b*e)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/Sqrt[
b^2 - 4*a*c]])/(b^2 - 4*a*c)^(9/2)
```

Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] - Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)
*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p + 1
), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 0] && (L
tQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c,
d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*
```

```
(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p +
 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*Simp[g*
(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(
m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (I
ntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 777

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p +
1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(
2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(a+bx+cx^2\right)^5}\,dx = -\frac{(b+2cx)(d+ex)^3}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4} + \frac{\int \frac{(d+ex)^2(-14cd+3be-8cex)}{\left(a+bx+cx^2\right)^4}\,dx}{4\left(b^2-4ac\right)}$$

$$= -\frac{(b+2cx)(d+ex)^3}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4} + \frac{(d+ex)^2\left(14bcd-3b^2e-16ace+14c(2cd-be)x\right)}{12\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^3} - \frac{\int \frac{(d+ex)}{}}{}$$

$$= -\frac{(b+2cx)(d+ex)^3}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4} + \frac{(d+ex)^2\left(14bcd-3b^2e-16ace+14c(2cd-be)x\right)}{12\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^3} - \frac{3b^3de^2}{}$$

$$= -\frac{(b+2cx)(d+ex)^3}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4} + \frac{(d+ex)^2\left(14bcd-3b^2e-16ace+14c(2cd-be)x\right)}{12\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^3} - \frac{3b^3de^2}{}$$

$$= -\frac{(b+2cx)(d+ex)^3}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4} + \frac{(d+ex)^2\left(14bcd-3b^2e-16ace+14c(2cd-be)x\right)}{12\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^3} - \frac{3b^3de^2}{}$$

$$= -\frac{(b+2cx)(d+ex)^3}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4} + \frac{(d+ex)^2\left(14bcd-3b^2e-16ace+14c(2cd-be)x\right)}{12\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^3} - \frac{3b^3de^2}{}$$

Mathematica [A] time = 1.30144, size = 467, normalized size = 1.24

$$\frac{1}{12}\left(\frac{4c^2\left(-8a^2e^3+3acde^2x+7c^2d^3x\right)+b^2ce\left(13ae^2-3cd(7d-6ex)\right)+2bc^2\left(3ae^2(d-ex)+7cd^2(d-3ex)\right)+b^3ce^2(9d}{c^2\left(b^2-4ac\right)^2(a+x(b+cx))^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + b*x + c*x^2)^5,x]
```

```
[Out] ((5*(2*c*d - b*e)*(7*c^2*d^2 + b^2*e^2 + c*e*(-7*b*d + 3*a*e))*(b + 2*c*x))
/(c*(-b^2 + 4*a*c)^3*(a + x*(b + c*x))^2) + (30*(2*c*d - b*e)*(7*c^2*d^2 +
b^2*e^2 + c*e*(-7*b*d + 3*a*e))*(b + 2*c*x))/((b^2 - 4*a*c)^4*(a + x*(b + c
*x))) + (-3*b^4*e^3 + b^3*c*e^2*(9*d - 2*e*x) + 4*c^2*(-8*a^2*e^3 + 7*c^2*d
^3*x + 3*a*c*d*e^2*x) + b^2*c*e*(13*a*e^2 - 3*c*d*(7*d - 6*e*x)) + 2*b*c^2*
(7*c*d^2*(d - 3*e*x) + 3*a*e^2*(d - e*x)))/(c^2*(b^2 - 4*a*c)^2*(a + x*(b +
 c*x))^3) + (3*(-(b^3*e^3*x) + b^2*e^2*(-(a*e) + 3*c*d*x) + 2*c*(a^2*e^3 +
c^2*d^3*x - 3*a*c*d*e*(d + e*x)) + b*c*(c*d^2*(d - 3*e*x) + 3*a*e^2*(d + e*
x))))/(c^2*(-b^2 + 4*a*c)*(a + x*(b + c*x))^4) + (120*c*(2*c*d - b*e)*(7*c^
2*d^2 + b^2*e^2 + c*e*(-7*b*d + 3*a*e))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*
c]])/(-b^2 + 4*a*c)^(9/2))/12
```

Maple [B] time = 0.173, size = 1742, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^2+b*x+a)^5,x)
```

```
[Out] (-5*c^4*(3*a*b*c*e^3-6*a*c^2*d*e^2+b^3*e^3-9*b^2*c*d*e^2+21*b*c^2*d^2*e-14*
c^3*d^3)/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)*x^7-35
```

```
/2*c^3*(3*a*b*c*e^3-6*a*c^2*d*e^2+b^3*e^3-9*b^2*c*d*e^2+21*b*c^2*d^2*e-14*c
^3*d^3)/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)*b*x^6-5
/3*c^2*(11*a*c+13*b^2)*(3*a*b*c*e^3-6*a*c^2*d*e^2+b^3*e^3-9*b^2*c*d*e^2+21*
b*c^2*d^2*e-14*c^3*d^3)/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^
6*c+b^8)*x^5-25/12*b*(22*a*c+5*b^2)*c*(3*a*b*c*e^3-6*a*c^2*d*e^2+b^3*e^3-9*
b^2*c*d*e^2+21*b*c^2*d^2*e-14*c^3*d^3)/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*
b^4*c^2-16*a*b^6*c+b^8)*x^4-1/3*(73*a^2*c^2+101*a*b^2*c+3*b^4)*(3*a*b*c*e^3
-6*a*c^2*d*e^2+b^3*e^3-9*b^2*c*d*e^2+21*b*c^2*d^2*e-14*c^3*d^3)/(256*a^4*c^
4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)*x^3-1/6*(256*a^4*c^3*e^3+4
01*a^3*b^2*c^2*e^3-1314*a^3*b*c^3*d*e^2+399*a^2*b^4*c*e^3-2139*a^2*b^3*c^2*
d*e^2+4599*a^2*b^2*c^3*d^2*e-3066*a^2*b*c^4*d^3+9*a*b^6*e^3-246*a*b^5*c*d*e
^2+588*a*b^4*c^2*d^2*e-392*a*b^3*c^3*d^3+9*b^7*d*e^2-21*b^6*c*d^2*e+14*b^5*
c^2*d^3)/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)*x^2-1/
3*(83*a^4*b*c^2*e^3+90*a^4*c^3*d*e^2+151*a^3*b^3*c*e^3-837*a^3*b^2*c^2*d*e^
2+837*a^3*b*c^3*d^2*e-558*a^3*c^4*d^3+3*a^2*b^5*e^3-84*a^2*b^4*c*d*e^2+522*
a^2*b^3*c^2*d^2*e-348*a^2*b^2*c^3*d^3+3*a*b^6*d*e^2-57*a*b^5*c*d^2*e+38*a*b
^4*c^2*d^3+3*b^7*d^2*e-2*b^6*c*d^3)/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4
*c^2-16*a*b^6*c+b^8)*x-1/12*(128*a^5*c^2*e^3+166*a^4*b^2*c*e^3-972*a^4*b*c^
2*d*e^2+1152*a^4*c^3*d^2*e+3*a^3*b^4*e^3-84*a^3*b^3*c*d*e^2+522*a^3*b^2*c^2
*d^2*e-1116*a^3*b*c^3*d^3+3*a^2*b^5*d*e^2-57*a^2*b^4*c*d^2*e+326*a^2*b^3*c^
2*d^3+3*a*b^6*d^2*e-50*a*b^5*c*d^3+3*b^7*d^3)/(256*a^4*c^4-256*a^3*b^2*c^3+
96*a^2*b^4*c^2-16*a*b^6*c+b^8))/(c*x^2+b*x+a)^4-30*c^2/(256*a^4*c^4-256*a^3
*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/
(4*a*c-b^2)^(1/2))*a*b*e^3+60*c^3/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c
^2-16*a*b^6*c+b^8)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*
d*e^2-10*c/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)/(4*a
*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*e^3+90*c^2/(256*a^4*c
^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)/(4*a*c-b^2)^(1/2)*arctan(
(2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*d*e^2-210*c^3/(256*a^4*c^4-256*a^3*b^2*c^3
+96*a^2*b^4*c^2-16*a*b^6*c+b^8)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b
^2)^(1/2))*b*d^2*e+140*c^4/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a
*b^6*c+b^8)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*d^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.44002, size = 12459, normalized size = 32.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^5,x, algorithm="fricas")
```

```
[Out] [1/12*(60*(14*(b^2*c^7 - 4*a*c^8)*d^3 - 21*(b^3*c^6 - 4*a*b*c^7)*d^2*e + 3*
(3*b^4*c^5 - 10*a*b^2*c^6 - 8*a^2*c^7)*d*e^2 - (b^5*c^4 - a*b^3*c^5 - 12*a^
2*b*c^6)*e^3)*x^7 + 210*(14*(b^3*c^6 - 4*a*b*c^7)*d^3 - 21*(b^4*c^5 - 4*a*b
^2*c^6)*d^2*e + 3*(3*b^5*c^4 - 10*a*b^3*c^5 - 8*a^2*b*c^6)*d*e^2 - (b^6*c^3
```

- a*b^4*c^4 - 12*a^2*b^2*c^5)*e^3)*x^6 + 20*(14*(13*b^4*c^5 - 41*a*b^2*c^6 - 44*a^2*c^7)*d^3 - 21*(13*b^5*c^4 - 41*a*b^3*c^5 - 44*a^2*b*c^6)*d^2*e + 3*(39*b^6*c^3 - 97*a*b^4*c^4 - 214*a^2*b^2*c^5 - 88*a^3*c^6)*d*e^2 - (13*b^7*c^2 - 2*a*b^5*c^3 - 167*a^2*b^3*c^4 - 132*a^3*b*c^5)*e^3)*x^5 + 25*(14*(5*b^5*c^4 + 2*a*b^3*c^5 - 88*a^2*b*c^6)*d^3 - 21*(5*b^6*c^3 + 2*a*b^4*c^4 - 88*a^2*b^2*c^5)*d^2*e + 3*(15*b^7*c^2 + 16*a*b^5*c^3 - 260*a^2*b^3*c^4 - 176*a^3*b*c^5)*d*e^2 - (5*b^8*c + 17*a*b^6*c^2 - 82*a^2*b^4*c^3 - 264*a^3*b^2*c^4)*e^3)*x^4 - (3*b^9 - 62*a*b^7*c + 526*a^2*b^5*c^2 - 2420*a^3*b^3*c^3 + 4464*a^4*b*c^4)*d^3 - 3*(a*b^8 - 23*a^2*b^6*c + 250*a^3*b^4*c^2 - 312*a^4*b^2*c^3 - 1536*a^5*c^4)*d^2*e - 3*(a^2*b^7 - 32*a^3*b^5*c - 212*a^4*b^3*c^2 + 1296*a^5*b*c^3)*d*e^2 - (3*a^3*b^6 + 154*a^4*b^4*c - 536*a^5*b^2*c^2 - 512*a^6*c^3)*e^3 + 4*(14*(3*b^6*c^3 + 89*a*b^4*c^4 - 331*a^2*b^2*c^5 - 292*a^3*c^6)*d^3 - 21*(3*b^7*c^2 + 89*a*b^5*c^3 - 331*a^2*b^3*c^4 - 292*a^3*b*c^5)*d^2*e + 3*(9*b^8*c + 273*a*b^6*c^2 - 815*a^2*b^4*c^3 - 1538*a^3*b^2*c^4 - 584*a^4*c^5)*d*e^2 - (3*b^9 + 98*a*b^7*c - 64*a^2*b^5*c^2 - 1285*a^3*b^3*c^3 - 876*a^4*b*c^4)*e^3)*x^3 - 2*(14*(b^7*c^2 - 32*a*b^5*c^3 - 107*a^2*b^3*c^4 + 876*a^3*b*c^5)*d^3 - 21*(b^8*c - 32*a*b^6*c^2 - 107*a^2*b^4*c^3 + 876*a^3*b^2*c^4)*d^2*e + 3*(3*b^9 - 94*a*b^7*c - 385*a^2*b^5*c^2 + 2414*a^3*b^3*c^3 + 1752*a^4*b*c^4)*d*e^2 + (9*a*b^8 + 363*a^2*b^6*c - 1195*a^3*b^4*c^2 - 1348*a^4*b^2*c^3 - 1024*a^5*c^4)*e^3)*x^2 - 60*(14*a^4*c^4*d^3 - 21*a^4*b*c^3*d^2*e + (14*c^8*d^3 - 21*b*c^7*d^2*e + 3*(3*b^2*c^6 + 2*a*c^7)*d*e^2 - (b^3*c^5 + 3*a*b*c^6)*e^3)*x^8 + 4*(14*b*c^7*d^3 - 21*b^2*c^6*d^2*e + 3*(3*b^3*c^5 + 2*a*b*c^6)*d*e^2 - (b^4*c^4 + 3*a*b^2*c^5)*e^3)*x^7 + 2*(14*(3*b^2*c^6 + 2*a*c^7)*d^3 - 21*(3*b^3*c^5 + 2*a*b*c^6)*d^2*e + 3*(9*b^4*c^4 + 12*a*b^2*c^5 + 4*a^2*c^6)*d*e^2 - (3*b^5*c^3 + 11*a*b^3*c^4 + 6*a^2*b*c^5)*e^3)*x^6 + 4*(14*(b^3*c^5 + 3*a*b*c^6)*d^3 - 21*(b^4*c^4 + 3*a*b^2*c^5)*d^2*e + 3*(3*b^5*c^3 + 11*a*b^3*c^4 + 6*a^2*b*c^5)*d*e^2 - (b^6*c^2 + 6*a*b^4*c^3 + 9*a^2*b^2*c^4)*e^3)*x^5 + (14*(b^4*c^4 + 12*a*b^2*c^5 + 6*a^2*c^6)*d^3 - 21*(b^5*c^3 + 12*a*b^3*c^4 + 6*a^2*b*c^5)*d^2*e + 3*(3*b^6*c^2 + 38*a*b^4*c^3 + 42*a^2*b^2*c^4 + 12*a^3*c^5)*d*e^2 - (b^7*c + 15*a*b^5*c^2 + 42*a^2*b^3*c^3 + 18*a^3*b*c^4)*e^3)*x^4 + 3*(3*a^4*b^2*c^2 + 2*a^5*c^3)*d*e^2 - (a^4*b^3*c + 3*a^5*b*c^2)*e^3 + 4*(14*(a*b^3*c^4 + 3*a^2*b*c^5)*d^3 - 21*(a*b^4*c^3 + 3*a^2*b^2*c^4)*d^2*e + 3*(3*a*b^5*c^2 + 11*a^2*b^3*c^3 + 6*a^3*b*c^4)*d*e^2 - (a*b^6*c + 6*a^2*b^4*c^2 + 9*a^3*b^2*c^3)*e^3)*x^3 + 2*(14*(3*a^2*b^2*c^4 + 2*a^3*c^5)*d^3 - 21*(3*a^2*b^3*c^3 + 2*a^3*b*c^4)*d^2*e + 3*(9*a^2*b^4*c^2 + 12*a^3*b^2*c^3 + 4*a^4*c^4)*d*e^2 - (3*a^2*b^5*c + 11*a^3*b^3*c^2 + 6*a^4*b*c^3)*e^3)*x^2 + 4*(14*a^3*b*c^4*d^3 - 21*a^3*b^2*c^3*d^2*e + 3*(3*a^3*b^3*c^2 + 2*a^4*b*c^3)*d*e^2 - (a^3*b^4*c + 3*a^4*b^2*c^2)*e^3)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 4*(2*(b^8*c - 23*a*b^6*c^2 + 250*a^2*b^4*c^3 - 417*a^3*b^2*c^4 - 1116*a^4*c^5)*d^3 - 3*(b^9 - 23*a*b^7*c + 250*a^2*b^5*c^2 - 417*a^3*b^3*c^3 - 1116*a^4*b*c^4)*d^2*e - 3*(a*b^8 - 32*a^2*b^6*c - 167*a^3*b^4*c^2 + 1146*a^4*b^2*c^3 - 120*a^5*c^4)*d*e^2 - (3*a^2*b^7 + 139*a^3*b^5*c - 521*a^4*b^3*c^2 - 332*a^5*b*c^3)*e^3)*x)/(a^4*b^10 - 20*a^5*b^8*c + 160*a^6*b^6*c^2 - 640*a^7*b^4*c^3 + 1280*a^8*b^2*c^4 - 1024*a^9*c^5 + (b^10*c^4 - 20*a*b^8*c^5 + 160*a^2*b^6*c^6 - 640*a^3*b^4*c^7 + 1280*a^4*b^2*c^8 - 1024*a^5*c^9)*x^8 + 4*(b^11*c^3 - 20*a*b^9*c^4 + 160*a^2*b^7*c^5 - 640*a^3*b^5*c^6 + 1280*a^4*b^3*c^7 - 1024*a^5*b*c^8)*x^7 + 2*(3*b^12*c^2 - 58*a*b^10*c^3 + 440*a^2*b^8*c^4 - 1600*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 512*a^5*b^2*c^7 - 2048*a^6*c^8)*x^6 + 4*(b^13*c - 17*a*b^11*c^2 + 100*a^2*b^9*c^3 - 160*a^3*b^7*c^4 - 640*a^4*b^5*c^5 + 2816*a^5*b^3*c^6 - 3072*a^6*b*c^7)*x^5 + (b^14 - 8*a*b^12*c - 74*a^2*b^10*c^2 + 1160*a^3*b^8*c^3 - 5440*a^4*b^6*c^4 + 10496*a^5*b^4*c^5 - 4608*a^6*b^2*c^6 - 6144*a^7*c^7)*x^4 + 4*(a*b^13 - 17*a^2*b^11*c + 100*a^3*b^9*c^2 - 160*a^4*b^7*c^3 - 640*a^5*b^5*c^4 + 2816*a^6*b^3*c^5 - 3072*a^7*b*c^6)*x^3 + 2*(3*a^2*b^12 - 58*a^3*b^10*c + 440*a^4*b^8*c^2 - 1600*a^5*b^6*c^3 + 2560*a^6*b^4*c^4 - 512*a^7*b^2*c^5 - 2048*a^8*c^6)*x^2 + 4*(a^3*b^11 - 20*a^4*b^9*c + 160*a^5*b^7*c^2 - 640*a^6*b^5*c^3 + 1280*a^7*b^3*c^4 - 1024*a^8*b*c^5)*x), 1/12*(60*(14*(b^2*c^7 - 4*a*c^8)*d^3 - 21*(b^3*c^6 - 4*a*b*c^7)*d^2*e + 3*(3*b^4*c^5 - 10*a*b^

2*c^6 - 8*a^2*c^7)*d*e^2 - (b^5*c^4 - a*b^3*c^5 - 12*a^2*b*c^6)*e^3)*x^7 + 210*(14*(b^3*c^6 - 4*a*b*c^7)*d^3 - 21*(b^4*c^5 - 4*a*b^2*c^6)*d^2*e + 3*(3*b^5*c^4 - 10*a*b^3*c^5 - 8*a^2*b*c^6)*d*e^2 - (b^6*c^3 - a*b^4*c^4 - 12*a^2*b^2*c^5)*e^3)*x^6 + 20*(14*(13*b^4*c^5 - 41*a*b^2*c^6 - 44*a^2*c^7)*d^3 - 21*(13*b^5*c^4 - 41*a*b^3*c^5 - 44*a^2*b*c^6)*d^2*e + 3*(39*b^6*c^3 - 97*a*b^4*c^4 - 214*a^2*b^2*c^5 - 88*a^3*c^6)*d*e^2 - (13*b^7*c^2 - 2*a*b^5*c^3 - 167*a^2*b^3*c^4 - 132*a^3*b*c^5)*e^3)*x^5 + 25*(14*(5*b^5*c^4 + 2*a*b^3*c^5 - 88*a^2*b*c^6)*d^3 - 21*(5*b^6*c^3 + 2*a*b^4*c^4 - 88*a^2*b^2*c^5)*d^2*e + 3*(15*b^7*c^2 + 16*a*b^5*c^3 - 260*a^2*b^3*c^4 - 176*a^3*b*c^5)*d*e^2 - (5*b^8*c + 17*a*b^6*c^2 - 82*a^2*b^4*c^3 - 264*a^3*b^2*c^4)*e^3)*x^4 - (3*b^9 - 62*a*b^7*c + 526*a^2*b^5*c^2 - 2420*a^3*b^3*c^3 + 4464*a^4*b*c^4)*d^3 - 3*(a*b^8 - 23*a^2*b^6*c + 250*a^3*b^4*c^2 - 312*a^4*b^2*c^3 - 1536*a^5*c^4)*d^2*e - 3*(a^2*b^7 - 32*a^3*b^5*c - 212*a^4*b^3*c^2 + 1296*a^5*b*c^3)*d*e^2 - (3*a^3*b^6 + 154*a^4*b^4*c - 536*a^5*b^2*c^2 - 512*a^6*c^3)*e^3 + 4*(14*(3*b^6*c^3 + 89*a*b^4*c^4 - 331*a^2*b^2*c^5 - 292*a^3*c^6)*d^3 - 21*(3*b^7*c^2 + 89*a*b^5*c^3 - 331*a^2*b^3*c^4 - 292*a^3*b*c^5)*d^2*e + 3*(9*b^8*c + 273*a*b^6*c^2 - 815*a^2*b^4*c^3 - 1538*a^3*b^2*c^4 - 584*a^4*c^5)*d*e^2 - (3*b^9 + 98*a*b^7*c - 64*a^2*b^5*c^2 - 1285*a^3*b^3*c^3 - 876*a^4*b*c^4)*e^3)*x^3 - 2*(14*(b^7*c^2 - 32*a*b^5*c^3 - 107*a^2*b^3*c^4 + 876*a^3*b*c^5)*d^3 - 21*(b^8*c - 32*a*b^6*c^2 - 107*a^2*b^4*c^3 + 876*a^3*b^2*c^4)*d^2*e + 3*(3*b^9 - 94*a*b^7*c - 385*a^2*b^5*c^2 + 2414*a^3*b^3*c^3 + 1752*a^4*b*c^4)*d*e^2 + (9*a*b^8 + 363*a^2*b^6*c - 1195*a^3*b^4*c^2 - 1348*a^4*b^2*c^3 - 1024*a^5*c^4)*e^3)*x^2 - 120*(14*a^4*c^4*d^3 - 21*a^4*b*c^3*d^2*e + (14*c^8*d^3 - 21*b*c^7*d^2*e + 3*(3*b^2*c^6 + 2*a*c^7)*d*e^2 - (b^3*c^5 + 3*a*b*c^6)*e^3)*x^8 + 4*(14*b*c^7*d^3 - 21*b^2*c^6*d^2*e + 3*(3*b^3*c^5 + 2*a*b*c^6)*d*e^2 - (b^4*c^4 + 3*a*b^2*c^5)*e^3)*x^7 + 2*(14*(3*b^2*c^6 + 2*a*c^7)*d^3 - 21*(3*b^3*c^5 + 2*a*b*c^6)*d^2*e + 3*(9*b^4*c^4 + 12*a*b^2*c^5 + 4*a^2*c^6)*d*e^2 - (3*b^5*c^3 + 11*a*b^3*c^4 + 6*a^2*b*c^5)*e^3)*x^6 + 4*(14*(b^3*c^5 + 3*a*b*c^6)*d^3 - 21*(b^4*c^4 + 3*a*b^2*c^5)*d^2*e + 3*(3*b^5*c^3 + 11*a*b^3*c^4 + 6*a^2*b*c^5)*d*e^2 - (b^6*c^2 + 6*a*b^4*c^3 + 9*a^2*b^2*c^4)*e^3)*x^5 + (14*(b^4*c^4 + 12*a*b^2*c^5 + 6*a^2*c^6)*d^3 - 21*(b^5*c^3 + 12*a*b^3*c^4 + 6*a^2*b*c^5)*d^2*e + 3*(3*b^6*c^2 + 38*a*b^4*c^3 + 42*a^2*b^2*c^4 + 12*a^3*c^5)*d*e^2 - (b^7*c + 15*a*b^5*c^2 + 42*a^2*b^3*c^3 + 18*a^3*b*c^4)*e^3)*x^4 + 3*(3*a^4*b^2*c^2 + 2*a^5*c^3)*d*e^2 - (a^4*b^3*c + 3*a^5*b*c^2)*e^3 + 4*(14*(a*b^3*c^4 + 3*a^2*b*c^5)*d^3 - 21*(a*b^4*c^3 + 3*a^2*b^2*c^4)*d^2*e + 3*(3*a*b^5*c^2 + 11*a^2*b^3*c^3 + 6*a^3*b*c^4)*d*e^2 - (a*b^6*c + 6*a^2*b^4*c^2 + 9*a^3*b^2*c^3)*e^3)*x^3 + 2*(14*(3*a^2*b^2*c^4 + 2*a^3*c^5)*d^3 - 21*(3*a^2*b^3*c^3 + 2*a^3*b*c^4)*d^2*e + 3*(9*a^2*b^4*c^2 + 12*a^3*b^2*c^3 + 4*a^4*c^4)*d*e^2 - (3*a^2*b^5*c + 11*a^3*b^3*c^2 + 6*a^4*b*c^3)*e^3)*x^2 + 4*(14*a^3*b*c^4*d^3 - 21*a^3*b^2*c^3*d^2*e + 3*(3*a^3*b^3*c^2 + 2*a^4*b*c^3)*d*e^2 - (a^3*b^4*c + 3*a^4*b^2*c^2)*e^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 4*(2*(b^8*c - 23*a*b^6*c^2 + 250*a^2*b^4*c^3 - 417*a^3*b^2*c^4 - 1116*a^4*c^5)*d^3 - 3*(b^9 - 23*a*b^7*c + 250*a^2*b^5*c^2 - 417*a^3*b^3*c^3 - 1116*a^4*b*c^4)*d^2*e - 3*(a*b^8 - 32*a^2*b^6*c - 167*a^3*b^4*c^2 + 1146*a^4*b^2*c^3 - 120*a^5*c^4)*d*e^2 - (3*a^2*b^7 + 139*a^3*b^5*c - 521*a^4*b^3*c^2 - 332*a^5*b*c^3)*e^3)*x)/(a^4*b^10 - 20*a^5*b^8*c + 160*a^6*b^6*c^2 - 640*a^7*b^4*c^3 + 1280*a^8*b^2*c^4 - 1024*a^9*c^5 + (b^10*c^4 - 20*a*b^8*c^5 + 160*a^2*b^6*c^6 - 640*a^3*b^4*c^7 + 1280*a^4*b^2*c^8 - 1024*a^5*c^9)*x^8 + 4*(b^11*c^3 - 20*a*b^9*c^4 + 160*a^2*b^7*c^5 - 640*a^3*b^5*c^6 + 1280*a^4*b^3*c^7 - 1024*a^5*b*c^8)*x^7 + 2*(3*b^12*c^2 - 58*a*b^10*c^3 + 440*a^2*b^8*c^4 - 1600*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 512*a^5*b^2*c^7 - 2048*a^6*c^8)*x^6 + 4*(b^13*c - 17*a*b^11*c^2 + 100*a^2*b^9*c^3 - 160*a^3*b^7*c^4 - 640*a^4*b^5*c^5 + 2816*a^5*b^3*c^6 - 3072*a^6*b*c^7)*x^5 + (b^14 - 8*a*b^12*c - 74*a^2*b^10*c^2 + 1160*a^3*b^8*c^3 - 5440*a^4*b^6*c^4 + 10496*a^5*b^4*c^5 - 4608*a^6*b^2*c^6 - 6144*a^7*c^7)*x^4 + 4*(a*b^13 - 17*a^2*b^11*c + 100*a^3*b^9*c^2 - 160*a^4*b^7*c^3 - 640*a^5*b^5*c^4 + 2816*a^6*b^3*c^5 - 3072*a^7*b*c^6)*x^3 + 2*(3*a^2*b^12 - 58*a^3*b^10*c + 440*a^4*b^8*c^2 - 1600*a^5*b^6*c^3 + 2560*a^6*b^4*c^4 - 512*a^7*b^2*c^5 - 2048*a^8*c^6)*x^2 + 4*(a^3*b^11 - 20*a^4*b^9*c +

```
160*a^5*b^7*c^2 - 640*a^6*b^5*c^3 + 1280*a^7*b^3*c^4 - 1024*a^8*b*c^5)*x)]
```

Sympy [B] time = 58.2152, size = 2994, normalized size = 7.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**2+b*x+a)**5,x)
```

```
[Out] 5*c*sqrt(-1/(4*a*c - b**2)**9)*(b*e - 2*c*d)*(3*a*c*e**2 + b**2*e**2 - 7*b*
c*d*e + 7*c**2*d**2)*log(x + (-5120*a**5*c**6*sqrt(-1/(4*a*c - b**2)**9)*(b
*e - 2*c*d)*(3*a*c*e**2 + b**2*e**2 - 7*b*c*d*e + 7*c**2*d**2) + 6400*a**4*
b**2*c**5*sqrt(-1/(4*a*c - b**2)**9)*(b*e - 2*c*d)*(3*a*c*e**2 + b**2*e**2
- 7*b*c*d*e + 7*c**2*d**2) - 3200*a**3*b**4*c**4*sqrt(-1/(4*a*c - b**2)**9)
*(b*e - 2*c*d)*(3*a*c*e**2 + b**2*e**2 - 7*b*c*d*e + 7*c**2*d**2) + 800*a**
2*b**6*c**3*sqrt(-1/(4*a*c - b**2)**9)*(b*e - 2*c*d)*(3*a*c*e**2 + b**2*e**
2 - 7*b*c*d*e + 7*c**2*d**2) - 100*a*b**8*c**2*sqrt(-1/(4*a*c - b**2)**9)*(
b*e - 2*c*d)*(3*a*c*e**2 + b**2*e**2 - 7*b*c*d*e + 7*c**2*d**2) + 15*a*b**2
*c**2*e**3 - 30*a*b*c**3*d*e**2 + 5*b**10*c*sqrt(-1/(4*a*c - b**2)**9)*(b*e
 - 2*c*d)*(3*a*c*e**2 + b**2*e**2 - 7*b*c*d*e + 7*c**2*d**2) + 5*b**4*c*e**
3 - 45*b**3*c**2*d*e**2 + 105*b**2*c**3*d**2*e - 70*b*c**4*d**3)/(30*a*b*c*
*3*e**3 - 60*a*c**4*d*e**2 + 10*b**3*c**2*e**3 - 90*b**2*c**3*d*e**2 + 210*
b*c**4*d**2*e - 140*c**5*d**3)) - 5*c*sqrt(-1/(4*a*c - b**2)**9)*(b*e - 2*c
*d)*(3*a*c*e**2 + b**2*e**2 - 7*b*c*d*e + 7*c**2*d**2)*log(x + (5120*a**5*c
**6*sqrt(-1/(4*a*c - b**2)**9)*(b*e - 2*c*d)*(3*a*c*e**2 + b**2*e**2 - 7*b*
c*d*e + 7*c**2*d**2) - 6400*a**4*b**2*c**5*sqrt(-1/(4*a*c - b**2)**9)*(b*e
- 2*c*d)*(3*a*c*e**2 + b**2*e**2 - 7*b*c*d*e + 7*c**2*d**2) + 3200*a**3*b**
4*c**4*sqrt(-1/(4*a*c - b**2)**9)*(b*e - 2*c*d)*(3*a*c*e**2 + b**2*e**2 - 7
*b*c*d*e + 7*c**2*d**2) - 800*a**2*b**6*c**3*sqrt(-1/(4*a*c - b**2)**9)*(b*
e - 2*c*d)*(3*a*c*e**2 + b**2*e**2 - 7*b*c*d*e + 7*c**2*d**2) + 100*a*b**8*
c**2*sqrt(-1/(4*a*c - b**2)**9)*(b*e - 2*c*d)*(3*a*c*e**2 + b**2*e**2 - 7*b
*c*d*e + 7*c**2*d**2) + 15*a*b**2*c**2*e**3 - 30*a*b*c**3*d*e**2 - 5*b**10*
c*sqrt(-1/(4*a*c - b**2)**9)*(b*e - 2*c*d)*(3*a*c*e**2 + b**2*e**2 - 7*b*c*
d*e + 7*c**2*d**2) + 5*b**4*c*e**3 - 45*b**3*c**2*d*e**2 + 105*b**2*c**3*d*
*2*e - 70*b*c**4*d**3)/(30*a*b*c**3*e**3 - 60*a*c**4*d*e**2 + 10*b**3*c**2*
e**3 - 90*b**2*c**3*d*e**2 + 210*b*c**4*d**2*e - 140*c**5*d**3)) - (128*a**
5*c**2*e**3 + 166*a**4*b**2*c*e**3 - 972*a**4*b*c**2*d*e**2 + 1152*a**4*c**
3*d**2*e + 3*a**3*b**4*e**3 - 84*a**3*b**3*c*d*e**2 + 522*a**3*b**2*c**2*d*
*2*e - 1116*a**3*b*c**3*d**3 + 3*a**2*b**5*d*e**2 - 57*a**2*b**4*c*d**2*e +
 326*a**2*b**3*c**2*d**3 + 3*a*b**6*d**2*e - 50*a*b**5*c*d**3 + 3*b**7*d**3
 + x**7*(180*a*b*c**5*e**3 - 360*a*c**6*d*e**2 + 60*b**3*c**4*e**3 - 540*b*
*2*c**5*d*e**2 + 1260*b*c**6*d**2*e - 840*c**7*d**3) + x**6*(630*a*b**2*c**
4*e**3 - 1260*a*b*c**5*d*e**2 + 210*b**4*c**3*e**3 - 1890*b**3*c**4*d*e**2
+ 4410*b**2*c**5*d**2*e - 2940*b*c**6*d**3) + x**5*(660*a**2*b*c**4*e**3 -
1320*a**2*c**5*d*e**2 + 1000*a*b**3*c**3*e**3 - 3540*a*b**2*c**4*d*e**2 + 4
620*a*b*c**5*d**2*e - 3080*a*c**6*d**3 + 260*b**5*c**2*e**3 - 2340*b**4*c**
3*d*e**2 + 5460*b**3*c**4*d**2*e - 3640*b**2*c**5*d**3) + x**4*(1650*a**2*b
**2*c**3*e**3 - 3300*a**2*b*c**4*d*e**2 + 925*a*b**4*c**2*e**3 - 5700*a*b**
3*c**3*d*e**2 + 11550*a*b**2*c**4*d**2*e - 7700*a*b*c**5*d**3 + 125*b**6*c*
e**3 - 1125*b**5*c**2*d*e**2 + 2625*b**4*c**3*d**2*e - 1750*b**3*c**4*d**3)
 + x**3*(876*a**3*b*c**3*e**3 - 1752*a**3*c**4*d*e**2 + 1504*a**2*b**3*c**2
*e**3 - 5052*a**2*b**2*c**3*d*e**2 + 6132*a**2*b*c**4*d**2*e - 4088*a**2*c*
*5*d**3 + 440*a*b**5*c*e**3 - 3708*a*b**4*c**2*d*e**2 + 8484*a*b**3*c**3*d*
*2*e - 5656*a*b**2*c**4*d**3 + 12*b**7*e**3 - 108*b**6*c*d*e**2 + 252*b**5*
c**2*d**2*e - 168*b**4*c**3*d**3) + x**2*(512*a**4*c**3*e**3 + 802*a**3*b**
2*c**2*e**3 - 2628*a**3*b*c**3*d*e**2 + 798*a**2*b**4*c*e**3 - 4278*a**2*b*
```

```
*3*c**2*d*e**2 + 9198*a**2*b**2*c**3*d**2*e - 6132*a**2*b*c**4*d**3 + 18*a*
b**6*e**3 - 492*a*b**5*c*d*e**2 + 1176*a*b**4*c**2*d**2*e - 784*a*b**3*c**3
*d**3 + 18*b**7*d*e**2 - 42*b**6*c*d**2*e + 28*b**5*c**2*d**3) + x*(332*a**
4*b*c**2*e**3 + 360*a**4*c**3*d*e**2 + 604*a**3*b**3*c*e**3 - 3348*a**3*b**
2*c**2*d*e**2 + 3348*a**3*b*c**3*d**2*e - 2232*a**3*c**4*d**3 + 12*a**2*b**
5*e**3 - 336*a**2*b**4*c*d*e**2 + 2088*a**2*b**3*c**2*d**2*e - 1392*a**2*b*
*2*c**3*d**3 + 12*a*b**6*d*e**2 - 228*a*b**5*c*d**2*e + 152*a*b**4*c**2*d**
3 + 12*b**7*d**2*e - 8*b**6*c*d**3))/(3072*a**8*c**4 - 3072*a**7*b**2*c**3
+ 1152*a**6*b**4*c**2 - 192*a**5*b**6*c + 12*a**4*b**8 + x**8*(3072*a**4*c*
*8 - 3072*a**3*b**2*c**7 + 1152*a**2*b**4*c**6 - 192*a*b**6*c**5 + 12*b**8*
c**4) + x**7*(12288*a**4*b*c**7 - 12288*a**3*b**3*c**6 + 4608*a**2*b**5*c**
5 - 768*a*b**7*c**4 + 48*b**9*c**3) + x**6*(12288*a**5*c**7 + 6144*a**4*b**
2*c**6 - 13824*a**3*b**4*c**5 + 6144*a**2*b**6*c**4 - 1104*a*b**8*c**3 + 72
*b**10*c**2) + x**5*(36864*a**5*b*c**6 - 24576*a**4*b**3*c**5 + 1536*a**3*b
**5*c**4 + 2304*a**2*b**7*c**3 - 624*a*b**9*c**2 + 48*b**11*c) + x**4*(1843
2*a**6*c**6 + 18432*a**5*b**2*c**5 - 26880*a**4*b**4*c**4 + 9600*a**3*b**6*
c**3 - 1080*a**2*b**8*c**2 - 48*a*b**10*c + 12*b**12) + x**3*(36864*a**6*b*
c**5 - 24576*a**5*b**3*c**4 + 1536*a**4*b**5*c**3 + 2304*a**3*b**7*c**2 - 6
24*a**2*b**9*c + 48*a*b**11) + x**2*(12288*a**7*c**5 + 6144*a**6*b**2*c**4
- 13824*a**5*b**4*c**3 + 6144*a**4*b**6*c**2 - 1104*a**3*b**8*c + 72*a**2*b
**10) + x*(12288*a**7*b*c**4 - 12288*a**6*b**3*c**3 + 4608*a**5*b**5*c**2 -
 768*a**4*b**7*c + 48*a**3*b**9))
```

Giac [B] time = 1.12628, size = 1889, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^5,x, algorithm="giac")
```

```
[Out] 10*(14*c^4*d^3 - 21*b*c^3*d^2*e + 9*b^2*c^2*d*e^2 + 6*a*c^3*d*e^2 - b^3*c*e
^3 - 3*a*b*c^2*e^3)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^8 - 16*a*b^6
*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4)*sqrt(-b^2 + 4*a*c)) +
1/12*(840*c^7*d^3*x^7 - 1260*b*c^6*d^2*x^7*e + 2940*b*c^6*d^3*x^6 + 540*b^2
*c^5*d*x^7*e^2 + 360*a*c^6*d*x^7*e^2 - 4410*b^2*c^5*d^2*x^6*e + 3640*b^2*c^
5*d^3*x^5 + 3080*a*c^6*d^3*x^5 - 60*b^3*c^4*x^7*e^3 - 180*a*b*c^5*x^7*e^3 +
 1890*b^3*c^4*d*x^6*e^2 + 1260*a*b*c^5*d*x^6*e^2 - 5460*b^3*c^4*d^2*x^5*e -
 4620*a*b*c^5*d^2*x^5*e + 1750*b^3*c^4*d^3*x^4 + 7700*a*b*c^5*d^3*x^4 - 210
*b^4*c^3*x^6*e^3 - 630*a*b^2*c^4*x^6*e^3 + 2340*b^4*c^3*d*x^5*e^2 + 3540*a*
b^2*c^4*d*x^5*e^2 + 1320*a^2*c^5*d*x^5*e^2 - 2625*b^4*c^3*d^2*x^4*e - 11550
*a*b^2*c^4*d^2*x^4*e + 168*b^4*c^3*d^3*x^3 + 5656*a*b^2*c^4*d^3*x^3 + 4088*
a^2*c^5*d^3*x^3 - 260*b^5*c^2*x^5*e^3 - 1000*a*b^3*c^3*x^5*e^3 - 660*a^2*b*
c^4*x^5*e^3 + 1125*b^5*c^2*d*x^4*e^2 + 5700*a*b^3*c^3*d*x^4*e^2 + 3300*a^2*
b*c^4*d*x^4*e^2 - 252*b^5*c^2*d^2*x^3*e - 8484*a*b^3*c^3*d^2*x^3*e - 6132*a
^2*b*c^4*d^2*x^3*e - 28*b^5*c^2*d^3*x^2 + 784*a*b^3*c^3*d^3*x^2 + 6132*a^2*
b*c^4*d^3*x^2 - 125*b^6*c*x^4*e^3 - 925*a*b^4*c^2*x^4*e^3 - 1650*a^2*b^2*c^
3*x^4*e^3 + 108*b^6*c*d*x^3*e^2 + 3708*a*b^4*c^2*d*x^3*e^2 + 5052*a^2*b^2*c
^3*d*x^3*e^2 + 1752*a^3*c^4*d*x^3*e^2 + 42*b^6*c*d^2*x^2*e - 1176*a*b^4*c^2
*d^2*x^2*e - 9198*a^2*b^2*c^3*d^2*x^2*e + 8*b^6*c*d^3*x - 152*a*b^4*c^2*d^3
*x + 1392*a^2*b^2*c^3*d^3*x + 2232*a^3*c^4*d^3*x - 12*b^7*x^3*e^3 - 440*a*b
^5*c*x^3*e^3 - 1504*a^2*b^3*c^2*x^3*e^3 - 876*a^3*b*c^3*x^3*e^3 - 18*b^7*d*
x^2*e^2 + 492*a*b^5*c*d*x^2*e^2 + 4278*a^2*b^3*c^2*d*x^2*e^2 + 2628*a^3*b*c
^3*d*x^2*e^2 - 12*b^7*d^2*x*e + 228*a*b^5*c*d^2*x*e - 2088*a^2*b^3*c^2*d^2*
x*e - 3348*a^3*b*c^3*d^2*x*e - 3*b^7*d^3 + 50*a*b^5*c*d^3 - 326*a^2*b^3*c^2
*d^3 + 1116*a^3*b*c^3*d^3 - 18*a*b^6*x^2*e^3 - 798*a^2*b^4*c*x^2*e^3 - 802*
a^3*b^2*c^2*x^2*e^3 - 512*a^4*c^3*x^2*e^3 - 12*a*b^6*d*x*e^2 + 336*a^2*b^4*
```

```
c*d*x*e^2 + 3348*a^3*b^2*c^2*d*x*e^2 - 360*a^4*c^3*d*x*e^2 - 3*a*b^6*d^2*e
+ 57*a^2*b^4*c*d^2*e - 522*a^3*b^2*c^2*d^2*e - 1152*a^4*c^3*d^2*e - 12*a^2*
b^5*x*e^3 - 604*a^3*b^3*c*x*e^3 - 332*a^4*b*c^2*x*e^3 - 3*a^2*b^5*d*e^2 + 8
4*a^3*b^3*c*d*e^2 + 972*a^4*b*c^2*d*e^2 - 3*a^3*b^4*e^3 - 166*a^4*b^2*c*e^3
 - 128*a^5*c^2*e^3)/((b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 +
 256*a^4*c^4)*(c*x^2 + b*x + a)^4)
```

3.2224 $\int \frac{(d+ex)^2}{\left(a+bx+cx^2\right)^5} dx$

Optimal. Leaf size=330

$$\frac{5c(b+2cx)\left(-2ce(7bd-ae)+3b^2e^2+14c^2d^2\right)}{2\left(b^2-4ac\right)^4\left(a+bx+cx^2\right)}-\frac{5(b+2cx)\left(-2ce(7bd-ae)+3b^2e^2+14c^2d^2\right)}{12\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)^2}-\frac{-x\left(-2ce(7bd-ae)+3\right.}{6}$$

```
[Out] -((d + e*x)*(b*d - 2*a*e + (2*c*d - b*e)*x))/(4*(b^2 - 4*a*c)*(a + b*x + c*
x^2)^4) - (4*b^2*d*e + 12*a*c*d*e - 7*b*(c*d^2 + a*e^2) - (14*c^2*d^2 + 3*b
^2*e^2 - 2*c*e*(7*b*d - a*e))*x)/(6*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^3) -
(5*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e))*(b + 2*c*x))/(12*(b^2 - 4
*a*c)^3*(a + b*x + c*x^2)^2) + (5*c*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d
- a*e))*(b + 2*c*x))/(2*(b^2 - 4*a*c)^4*(a + b*x + c*x^2)) - (10*c^2*(14*c^
2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a
*c]])/(b^2 - 4*a*c)^(9/2)
```

Rubi [A] time = 0.386843, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {738, 638, 614, 618, 206}

$$\frac{5c(b+2cx)\left(-2ce(7bd-ae)+3b^2e^2+14c^2d^2\right)}{2\left(b^2-4ac\right)^4\left(a+bx+cx^2\right)}-\frac{5(b+2cx)\left(-2ce(7bd-ae)+3b^2e^2+14c^2d^2\right)}{12\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)^2}-\frac{-x\left(-2ce(7bd-ae)+3\right.}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*x + c*x^2)^5,x]
```


```
[Out] -((d + e*x)*(b*d - 2*a*e + (2*c*d - b*e)*x))/(4*(b^2 - 4*a*c)*(a + b*x + c*
x^2)^4) - (4*b^2*d*e + 12*a*c*d*e - 7*b*(c*d^2 + a*e^2) - (14*c^2*d^2 + 3*b
^2*e^2 - 2*c*e*(7*b*d - a*e))*x)/(6*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^3) -
(5*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e))*(b + 2*c*x))/(12*(b^2 - 4
*a*c)^3*(a + b*x + c*x^2)^2) + (5*c*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d
- a*e))*(b + 2*c*x))/(2*(b^2 - 4*a*c)^4*(a + b*x + c*x^2)) - (10*c^2*(14*c^
2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a
*c]])/(b^2 - 4*a*c)^(9/2)
```


Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
```

```
NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(a+bx+cx^2\right)^5}\,dx = -\frac{(d+ex)(bd-2ae+(2cd-be)x)}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4} - \frac{\int \frac{2\left(7cd^2-e(4bd-ae)\right)+6e(2cd-be)x}{\left(a+bx+cx^2\right)^4}\,dx}{4\left(b^2-4ac\right)}$$

$$= -\frac{(d+ex)(bd-2ae+(2cd-be)x)}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4} - \frac{4b^2de+12acde-7b\left(cd^2+ae^2\right)-\left(14c^2d^2+3b^2e^2-2ce\right.}{6\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^3}$$

$$= -\frac{(d+ex)(bd-2ae+(2cd-be)x)}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4} - \frac{4b^2de+12acde-7b\left(cd^2+ae^2\right)-\left(14c^2d^2+3b^2e^2-2ce\right.}{6\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^3}$$

$$= -\frac{(d+ex)(bd-2ae+(2cd-be)x)}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4} - \frac{4b^2de+12acde-7b\left(cd^2+ae^2\right)-\left(14c^2d^2+3b^2e^2-2ce\right.}{6\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^3}$$

$$= -\frac{(d+ex)(bd-2ae+(2cd-be)x)}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4} - \frac{4b^2de+12acde-7b\left(cd^2+ae^2\right)-\left(14c^2d^2+3b^2e^2-2ce\right.}{6\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^3}$$

$$= -\frac{(d+ex)(bd-2ae+(2cd-be)x)}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4} - \frac{4b^2de+12acde-7b\left(cd^2+ae^2\right)-\left(14c^2d^2+3b^2e^2-2ce\right.}{6\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^3}$$

Mathematica [A] time = 0.908472, size = 326, normalized size = 0.99

$$\frac{1}{12}\left(\frac{30c(b+2cx)\left(2ce(ae-7bd)+3b^2e^2+14c^2d^2\right)}{\left(b^2-4ac\right)^4(a+x(b+cx))} - \frac{5(b+2cx)\left(2ce(ae-7bd)+3b^2e^2+14c^2d^2\right)}{\left(b^2-4ac\right)^3(a+x(b+cx))^2} + \frac{3\left(abe^2-2ace(2d\right.}{c\left(\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*x + c*x^2)^5,x]
```

```
[Out] (((14*c^2*d^2 + 3*b^2*e^2 + 2*c*e*(-7*b*d + a*e))*(b + 2*c*x))/(c*(b^2 - 4*
a*c)^2*(a + x*(b + c*x))^3) - (5*(14*c^2*d^2 + 3*b^2*e^2 + 2*c*e*(-7*b*d +
a*e))*(b + 2*c*x))/((b^2 - 4*a*c)^3*(a + x*(b + c*x))^2) + (30*c*(14*c^2*d^
2 + 3*b^2*e^2 + 2*c*e*(-7*b*d + a*e))*(b + 2*c*x))/((b^2 - 4*a*c)^4*(a + x*
(b + c*x))) + (3*(a*b*e^2 + 2*c^2*d^2*x + b^2*e^2*x + b*c*d*(d - 2*e*x) - 2
*a*c*e*(2*d + e*x)))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))^4) + (120*c^2*(14*
c^2*d^2 + 3*b^2*e^2 + 2*c*e*(-7*b*d + a*e))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
4*a*c]])/(-b^2 + 4*a*c)^(9/2))/12
```

Maple [B] time = 0.168, size = 1243, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^2+b*x+a)^5,x)
```

```
[Out] (5*c^5*(2*a*c*e^2+3*b^2*e^2-14*b*c*d*e+14*c^2*d^2)/(256*a^4*c^4-256*a^3*b^2
*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)*x^7+35/2*c^4*(2*a*c*e^2+3*b^2*e^2-14*b*
c*d*e+14*c^2*d^2)/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^
8)*b*x^6+5/3*c^3*(11*a*c+13*b^2)*(2*a*c*e^2+3*b^2*e^2-14*b*c*d*e+14*c^2*d^2
)/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)*x^5+25/12*b*(
22*a*c+5*b^2)*c^2*(2*a*c*e^2+3*b^2*e^2-14*b*c*d*e+14*c^2*d^2)/(256*a^4*c^4-
256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)*x^4+1/3*(73*a^2*c^2+101*a*b^
2*c+3*b^4)*c*(2*a*c*e^2+3*b^2*e^2-14*b*c*d*e+14*c^2*d^2)/(256*a^4*c^4-256*a
^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)*x^3+1/6*b*(219*a^2*c^2+28*a*b^2*c
-b^4)*(2*a*c*e^2+3*b^2*e^2-14*b*c*d*e+14*c^2*d^2)/(256*a^4*c^4-256*a^3*b^2*
c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)*x^2-1/3*(30*a^4*c^3*e^2-279*a^3*b^2*c^2*
e^2+558*a^3*b*c^3*d*e-558*a^3*c^4*d^2-28*a^2*b^4*c*e^2+348*a^2*b^3*c^2*d*e-
348*a^2*b^2*c^3*d^2+a*b^6*e^2-38*a*b^5*c*d*e+38*a*b^4*c^2*d^2+2*b^7*d*e-2*b
^6*c*d^2)/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)*x+1/1
2*(324*a^4*b*c^2*e^2-768*a^4*c^3*d*e+28*a^3*b^3*c*e^2-348*a^3*b^2*c^2*d*e+1
116*a^3*b*c^3*d^2-a^2*b^5*e^2+38*a^2*b^4*c*d*e-326*a^2*b^3*c^2*d^2-2*a*b^6*
d*e+50*a*b^5*c*d^2-3*b^7*d^2)/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-1
6*a*b^6*c+b^8))/(c*x^2+b*x+a)^4+20*c^3/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*
b^4*c^2-16*a*b^6*c+b^8)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2
))*a*e^2+30*c^2/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)
/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*e^2-140*c^3/(256
*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)/(4*a*c-b^2)^(1/2)*a
rctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*d*e+140*c^4/(256*a^4*c^4-256*a^3*b^2*c
^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c
-b^2)^(1/2))*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.38926, size = 9765, normalized size = 29.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^5,x, algorithm="fricas")
```

```
[Out] [1/12*(60*(14*(b^2*c^7 - 4*a*c^8)*d^2 - 14*(b^3*c^6 - 4*a*b*c^7)*d*e + (3*b
^4*c^5 - 10*a*b^2*c^6 - 8*a^2*c^7)*e^2)*x^7 + 210*(14*(b^3*c^6 - 4*a*b*c^7)
*d^2 - 14*(b^4*c^5 - 4*a*b^2*c^6)*d*e + (3*b^5*c^4 - 10*a*b^3*c^5 - 8*a^2*b
*c^6)*e^2)*x^6 + 20*(14*(13*b^4*c^5 - 41*a*b^2*c^6 - 44*a^2*c^7)*d^2 - 14*(
13*b^5*c^4 - 41*a*b^3*c^5 - 44*a^2*b*c^6)*d*e + (39*b^6*c^3 - 97*a*b^4*c^4
- 214*a^2*b^2*c^5 - 88*a^3*c^6)*e^2)*x^5 + 25*(14*(5*b^5*c^4 + 2*a*b^3*c^5
- 88*a^2*b*c^6)*d^2 - 14*(5*b^6*c^3 + 2*a*b^4*c^4 - 88*a^2*b^2*c^5)*d*e + (
15*b^7*c^2 + 16*a*b^5*c^3 - 260*a^2*b^3*c^4 - 176*a^3*b*c^5)*e^2)*x^4 + 4*(
14*(3*b^6*c^3 + 89*a*b^4*c^4 - 331*a^2*b^2*c^5 - 292*a^3*c^6)*d^2 - 14*(3*b
^7*c^2 + 89*a*b^5*c^3 - 331*a^2*b^3*c^4 - 292*a^3*b*c^5)*d*e + (9*b^8*c + 2
73*a*b^6*c^2 - 815*a^2*b^4*c^3 - 1538*a^3*b^2*c^4 - 584*a^4*c^5)*e^2)*x^3 -
 (3*b^9 - 62*a*b^7*c + 526*a^2*b^5*c^2 - 2420*a^3*b^3*c^3 + 4464*a^4*b*c^4)
*d^2 - 2*(a*b^8 - 23*a^2*b^6*c + 250*a^3*b^4*c^2 - 312*a^4*b^2*c^3 - 1536*a
^5*c^4)*d*e - (a^2*b^7 - 32*a^3*b^5*c - 212*a^4*b^3*c^2 + 1296*a^5*b*c^3)*e
^2 - 2*(14*(b^7*c^2 - 32*a*b^5*c^3 - 107*a^2*b^3*c^4 + 876*a^3*b*c^5)*d^2 -
 14*(b^8*c - 32*a*b^6*c^2 - 107*a^2*b^4*c^3 + 876*a^3*b^2*c^4)*d*e + (3*b^9
 - 94*a*b^7*c - 385*a^2*b^5*c^2 + 2414*a^3*b^3*c^3 + 1752*a^4*b*c^4)*e^2)*x
^2 + 60*(14*a^4*c^4*d^2 - 14*a^4*b*c^3*d*e + (14*c^8*d^2 - 14*b*c^7*d*e + (
3*b^2*c^6 + 2*a*c^7)*e^2)*x^8 + 4*(14*b*c^7*d^2 - 14*b^2*c^6*d*e + (3*b^3*c
^5 + 2*a*b*c^6)*e^2)*x^7 + 2*(14*(3*b^2*c^6 + 2*a*c^7)*d^2 - 14*(3*b^3*c^5
+ 2*a*b*c^6)*d*e + (9*b^4*c^4 + 12*a*b^2*c^5 + 4*a^2*c^6)*e^2)*x^6 + 4*(14*
(b^3*c^5 + 3*a*b*c^6)*d^2 - 14*(b^4*c^4 + 3*a*b^2*c^5)*d*e + (3*b^5*c^3 + 1
1*a*b^3*c^4 + 6*a^2*b*c^5)*e^2)*x^5 + (14*(b^4*c^4 + 12*a*b^2*c^5 + 6*a^2*c
^6)*d^2 - 14*(b^5*c^3 + 12*a*b^3*c^4 + 6*a^2*b*c^5)*d*e + (3*b^6*c^2 + 38*a
*b^4*c^3 + 42*a^2*b^2*c^4 + 12*a^3*c^5)*e^2)*x^4 + 4*(14*(a*b^3*c^4 + 3*a^2
*b*c^5)*d^2 - 14*(a*b^4*c^3 + 3*a^2*b^2*c^4)*d*e + (3*a*b^5*c^2 + 11*a^2*b^
3*c^3 + 6*a^3*b*c^4)*e^2)*x^3 + (3*a^4*b^2*c^2 + 2*a^5*c^3)*e^2 + 2*(14*(3*
a^2*b^2*c^4 + 2*a^3*c^5)*d^2 - 14*(3*a^2*b^3*c^3 + 2*a^3*b*c^4)*d*e + (9*a^
2*b^4*c^2 + 12*a^3*b^2*c^3 + 4*a^4*c^4)*e^2)*x^2 + 4*(14*a^3*b*c^4*d^2 - 14
*a^3*b^2*c^3*d*e + (3*a^3*b^3*c^2 + 2*a^4*b*c^3)*e^2)*x)*sqrt(b^2 - 4*a*c)*
log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*
x^2 + b*x + a)) + 4*(2*(b^8*c - 23*a*b^6*c^2 + 250*a^2*b^4*c^3 - 417*a^3*b^
2*c^4 - 1116*a^4*c^5)*d^2 - 2*(b^9 - 23*a*b^7*c + 250*a^2*b^5*c^2 - 417*a^3
*b^3*c^3 - 1116*a^4*b*c^4)*d*e - (a*b^8 - 32*a^2*b^6*c - 167*a^3*b^4*c^2 +
1146*a^4*b^2*c^3 - 120*a^5*c^4)*e^2)*x)/(a^4*b^10 - 20*a^5*b^8*c + 160*a^6*
b^6*c^2 - 640*a^7*b^4*c^3 + 1280*a^8*b^2*c^4 - 1024*a^9*c^5 + (b^10*c^4 - 2
0*a*b^8*c^5 + 160*a^2*b^6*c^6 - 640*a^3*b^4*c^7 + 1280*a^4*b^2*c^8 - 1024*a
^5*c^9)*x^8 + 4*(b^11*c^3 - 20*a*b^9*c^4 + 160*a^2*b^7*c^5 - 640*a^3*b^5*c^
6 + 1280*a^4*b^3*c^7 - 1024*a^5*b*c^8)*x^7 + 2*(3*b^12*c^2 - 58*a*b^10*c^3
+ 440*a^2*b^8*c^4 - 1600*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 512*a^5*b^2*c^7 -
 2048*a^6*c^8)*x^6 + 4*(b^13*c - 17*a*b^11*c^2 + 100*a^2*b^9*c^3 - 160*a^3*
b^7*c^4 - 640*a^4*b^5*c^5 + 2816*a^5*b^3*c^6 - 3072*a^6*b*c^7)*x^5 + (b^14
- 8*a*b^12*c - 74*a^2*b^10*c^2 + 1160*a^3*b^8*c^3 - 5440*a^4*b^6*c^4 + 1049
6*a^5*b^4*c^5 - 4608*a^6*b^2*c^6 - 6144*a^7*c^7)*x^4 + 4*(a*b^13 - 17*a^2*b
^11*c + 100*a^3*b^9*c^2 - 160*a^4*b^7*c^3 - 640*a^5*b^5*c^4 + 2816*a^6*b^3*
c^5 - 3072*a^7*b*c^6)*x^3 + 2*(3*a^2*b^12 - 58*a^3*b^10*c + 440*a^4*b^8*c^2
 - 1600*a^5*b^6*c^3 + 2560*a^6*b^4*c^4 - 512*a^7*b^2*c^5 - 2048*a^8*c^6)*x^
2 + 4*(a^3*b^11 - 20*a^4*b^9*c + 160*a^5*b^7*c^2 - 640*a^6*b^5*c^3 + 1280*a
^7*b^3*c^4 - 1024*a^8*b*c^5)*x), 1/12*(60*(14*(b^2*c^7 - 4*a*c^8)*d^2 - 14*
(b^3*c^6 - 4*a*b*c^7)*d*e + (3*b^4*c^5 - 10*a*b^2*c^6 - 8*a^2*c^7)*e^2)*x^7
 + 210*(14*(b^3*c^6 - 4*a*b*c^7)*d^2 - 14*(b^4*c^5 - 4*a*b^2*c^6)*d*e + (3*
```

```
b^5*c^4 - 10*a*b^3*c^5 - 8*a^2*b*c^6)*e^2)*x^6 + 20*(14*(13*b^4*c^5 - 41*a*
b^2*c^6 - 44*a^2*c^7)*d^2 - 14*(13*b^5*c^4 - 41*a*b^3*c^5 - 44*a^2*b*c^6)*d
*e + (39*b^6*c^3 - 97*a*b^4*c^4 - 214*a^2*b^2*c^5 - 88*a^3*c^6)*e^2)*x^5 +
25*(14*(5*b^5*c^4 + 2*a*b^3*c^5 - 88*a^2*b*c^6)*d^2 - 14*(5*b^6*c^3 + 2*a*b
^4*c^4 - 88*a^2*b^2*c^5)*d*e + (15*b^7*c^2 + 16*a*b^5*c^3 - 260*a^2*b^3*c^4
 - 176*a^3*b*c^5)*e^2)*x^4 + 4*(14*(3*b^6*c^3 + 89*a*b^4*c^4 - 331*a^2*b^2*
c^5 - 292*a^3*c^6)*d^2 - 14*(3*b^7*c^2 + 89*a*b^5*c^3 - 331*a^2*b^3*c^4 - 2
92*a^3*b*c^5)*d*e + (9*b^8*c + 273*a*b^6*c^2 - 815*a^2*b^4*c^3 - 1538*a^3*b
^2*c^4 - 584*a^4*c^5)*e^2)*x^3 - (3*b^9 - 62*a*b^7*c + 526*a^2*b^5*c^2 - 24
20*a^3*b^3*c^3 + 4464*a^4*b*c^4)*d^2 - 2*(a*b^8 - 23*a^2*b^6*c + 250*a^3*b^
4*c^2 - 312*a^4*b^2*c^3 - 1536*a^5*c^4)*d*e - (a^2*b^7 - 32*a^3*b^5*c - 212
*a^4*b^3*c^2 + 1296*a^5*b*c^3)*e^2 - 2*(14*(b^7*c^2 - 32*a*b^5*c^3 - 107*a^
2*b^3*c^4 + 876*a^3*b*c^5)*d^2 - 14*(b^8*c - 32*a*b^6*c^2 - 107*a^2*b^4*c^3
 + 876*a^3*b^2*c^4)*d*e + (3*b^9 - 94*a*b^7*c - 385*a^2*b^5*c^2 + 2414*a^3*
b^3*c^3 + 1752*a^4*b*c^4)*e^2)*x^2 - 120*(14*a^4*c^4*d^2 - 14*a^4*b*c^3*d*e
 + (14*c^8*d^2 - 14*b*c^7*d*e + (3*b^2*c^6 + 2*a*c^7)*e^2)*x^8 + 4*(14*b*c^
7*d^2 - 14*b^2*c^6*d*e + (3*b^3*c^5 + 2*a*b*c^6)*e^2)*x^7 + 2*(14*(3*b^2*c^
6 + 2*a*c^7)*d^2 - 14*(3*b^3*c^5 + 2*a*b*c^6)*d*e + (9*b^4*c^4 + 12*a*b^2*c
^5 + 4*a^2*c^6)*e^2)*x^6 + 4*(14*(b^3*c^5 + 3*a*b*c^6)*d^2 - 14*(b^4*c^4 +
3*a*b^2*c^5)*d*e + (3*b^5*c^3 + 11*a*b^3*c^4 + 6*a^2*b*c^5)*e^2)*x^5 + (14*
(b^4*c^4 + 12*a*b^2*c^5 + 6*a^2*c^6)*d^2 - 14*(b^5*c^3 + 12*a*b^3*c^4 + 6*a
^2*b*c^5)*d*e + (3*b^6*c^2 + 38*a*b^4*c^3 + 42*a^2*b^2*c^4 + 12*a^3*c^5)*e^
2)*x^4 + 4*(14*(a*b^3*c^4 + 3*a^2*b*c^5)*d^2 - 14*(a*b^4*c^3 + 3*a^2*b^2*c^
4)*d*e + (3*a*b^5*c^2 + 11*a^2*b^3*c^3 + 6*a^3*b*c^4)*e^2)*x^3 + (3*a^4*b^2
*c^2 + 2*a^5*c^3)*e^2 + 2*(14*(3*a^2*b^2*c^4 + 2*a^3*c^5)*d^2 - 14*(3*a^2*b
^3*c^3 + 2*a^3*b*c^4)*d*e + (9*a^2*b^4*c^2 + 12*a^3*b^2*c^3 + 4*a^4*c^4)*e^
2)*x^2 + 4*(14*a^3*b*c^4*d^2 - 14*a^3*b^2*c^3*d*e + (3*a^3*b^3*c^2 + 2*a^4*
b*c^3)*e^2)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b
^2 - 4*a*c)) + 4*(2*(b^8*c - 23*a*b^6*c^2 + 250*a^2*b^4*c^3 - 417*a^3*b^2*c
^4 - 1116*a^4*c^5)*d^2 - 2*(b^9 - 23*a*b^7*c + 250*a^2*b^5*c^2 - 417*a^3*b^
3*c^3 - 1116*a^4*b*c^4)*d*e - (a*b^8 - 32*a^2*b^6*c - 167*a^3*b^4*c^2 + 114
6*a^4*b^2*c^3 - 120*a^5*c^4)*e^2)*x)/(a^4*b^10 - 20*a^5*b^8*c + 160*a^6*b^6
*c^2 - 640*a^7*b^4*c^3 + 1280*a^8*b^2*c^4 - 1024*a^9*c^5 + (b^10*c^4 - 20*a
*b^8*c^5 + 160*a^2*b^6*c^6 - 640*a^3*b^4*c^7 + 1280*a^4*b^2*c^8 - 1024*a^5*
c^9)*x^8 + 4*(b^11*c^3 - 20*a*b^9*c^4 + 160*a^2*b^7*c^5 - 640*a^3*b^5*c^6 +
 1280*a^4*b^3*c^7 - 1024*a^5*b*c^8)*x^7 + 2*(3*b^12*c^2 - 58*a*b^10*c^3 + 4
40*a^2*b^8*c^4 - 1600*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 512*a^5*b^2*c^7 - 20
48*a^6*c^8)*x^6 + 4*(b^13*c - 17*a*b^11*c^2 + 100*a^2*b^9*c^3 - 160*a^3*b^7
*c^4 - 640*a^4*b^5*c^5 + 2816*a^5*b^3*c^6 - 3072*a^6*b*c^7)*x^5 + (b^14 - 8
*a*b^12*c - 74*a^2*b^10*c^2 + 1160*a^3*b^8*c^3 - 5440*a^4*b^6*c^4 + 10496*a
^5*b^4*c^5 - 4608*a^6*b^2*c^6 - 6144*a^7*c^7)*x^4 + 4*(a*b^13 - 17*a^2*b^11
*c + 100*a^3*b^9*c^2 - 160*a^4*b^7*c^3 - 640*a^5*b^5*c^4 + 2816*a^6*b^3*c^5
 - 3072*a^7*b*c^6)*x^3 + 2*(3*a^2*b^12 - 58*a^3*b^10*c + 440*a^4*b^8*c^2 -
1600*a^5*b^6*c^3 + 2560*a^6*b^4*c^4 - 512*a^7*b^2*c^5 - 2048*a^8*c^6)*x^2 +
 4*(a^3*b^11 - 20*a^4*b^9*c + 160*a^5*b^7*c^2 - 640*a^6*b^5*c^3 + 1280*a^7*
b^3*c^4 - 1024*a^8*b*c^5)*x)]
```

Sympy [B] time = 16.992, size = 2407, normalized size = 7.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**2+b*x+a)**5,x)
```

```
[Out] -5*c**2*sqrt(-1/(4*a*c - b**2)**9)*(2*a*c*e**2 + 3*b**2*e**2 - 14*b*c*d*e +
 14*c**2*d**2)*log(x + (-5120*a**5*c**7*sqrt(-1/(4*a*c - b**2)**9)*(2*a*c*e
```

```
**2 + 3*b**2*e**2 - 14*b*c*d*e + 14*c**2*d**2) + 6400*a**4*b**2*c**6*sqrt(-
1/(4*a*c - b**2)**9)*(2*a*c*e**2 + 3*b**2*e**2 - 14*b*c*d*e + 14*c**2*d**2)
 - 3200*a**3*b**4*c**5*sqrt(-1/(4*a*c - b**2)**9)*(2*a*c*e**2 + 3*b**2*e**2
 - 14*b*c*d*e + 14*c**2*d**2) + 800*a**2*b**6*c**4*sqrt(-1/(4*a*c - b**2)**
9)*(2*a*c*e**2 + 3*b**2*e**2 - 14*b*c*d*e + 14*c**2*d**2) - 100*a*b**8*c**3
*sqrt(-1/(4*a*c - b**2)**9)*(2*a*c*e**2 + 3*b**2*e**2 - 14*b*c*d*e + 14*c**
2*d**2) + 10*a*b*c**3*e**2 + 5*b**10*c**2*sqrt(-1/(4*a*c - b**2)**9)*(2*a*c
*e**2 + 3*b**2*e**2 - 14*b*c*d*e + 14*c**2*d**2) + 15*b**3*c**2*e**2 - 70*b
**2*c**3*d*e + 70*b*c**4*d**2)/(20*a*c**4*e**2 + 30*b**2*c**3*e**2 - 140*b*
c**4*d*e + 140*c**5*d**2)) + 5*c**2*sqrt(-1/(4*a*c - b**2)**9)*(2*a*c*e**2
+ 3*b**2*e**2 - 14*b*c*d*e + 14*c**2*d**2)*log(x + (5120*a**5*c**7*sqrt(-1/
(4*a*c - b**2)**9)*(2*a*c*e**2 + 3*b**2*e**2 - 14*b*c*d*e + 14*c**2*d**2) -
 6400*a**4*b**2*c**6*sqrt(-1/(4*a*c - b**2)**9)*(2*a*c*e**2 + 3*b**2*e**2 -
 14*b*c*d*e + 14*c**2*d**2) + 3200*a**3*b**4*c**5*sqrt(-1/(4*a*c - b**2)**9
)*(2*a*c*e**2 + 3*b**2*e**2 - 14*b*c*d*e + 14*c**2*d**2) - 800*a**2*b**6*c*
*4*sqrt(-1/(4*a*c - b**2)**9)*(2*a*c*e**2 + 3*b**2*e**2 - 14*b*c*d*e + 14*c
**2*d**2) + 100*a*b**8*c**3*sqrt(-1/(4*a*c - b**2)**9)*(2*a*c*e**2 + 3*b**2
*e**2 - 14*b*c*d*e + 14*c**2*d**2) + 10*a*b*c**3*e**2 - 5*b**10*c**2*sqrt(-
1/(4*a*c - b**2)**9)*(2*a*c*e**2 + 3*b**2*e**2 - 14*b*c*d*e + 14*c**2*d**2)
 + 15*b**3*c**2*e**2 - 70*b**2*c**3*d*e + 70*b*c**4*d**2)/(20*a*c**4*e**2 +
 30*b**2*c**3*e**2 - 140*b*c**4*d*e + 140*c**5*d**2)) + (324*a**4*b*c**2*e*
*2 - 768*a**4*c**3*d*e + 28*a**3*b**3*c*e**2 - 348*a**3*b**2*c**2*d*e + 111
6*a**3*b*c**3*d**2 - a**2*b**5*e**2 + 38*a**2*b**4*c*d*e - 326*a**2*b**3*c*
*2*d**2 - 2*a*b**6*d*e + 50*a*b**5*c*d**2 - 3*b**7*d**2 + x**7*(120*a*c**6*
e**2 + 180*b**2*c**5*e**2 - 840*b*c**6*d*e + 840*c**7*d**2) + x**6*(420*a*b
*c**5*e**2 + 630*b**3*c**4*e**2 - 2940*b**2*c**5*d*e + 2940*b*c**6*d**2) +
x**5*(440*a**2*c**5*e**2 + 1180*a*b**2*c**4*e**2 - 3080*a*b*c**5*d*e + 3080
*a*c**6*d**2 + 780*b**4*c**3*e**2 - 3640*b**3*c**4*d*e + 3640*b**2*c**5*d**
2) + x**4*(1100*a**2*b*c**4*e**2 + 1900*a*b**3*c**3*e**2 - 7700*a*b**2*c**4
*d*e + 7700*a*b*c**5*d**2 + 375*b**5*c**2*e**2 - 1750*b**4*c**3*d*e + 1750*
b**3*c**4*d**2) + x**3*(584*a**3*c**4*e**2 + 1684*a**2*b**2*c**3*e**2 - 408
8*a**2*b*c**4*d*e + 4088*a**2*c**5*d**2 + 1236*a*b**4*c**2*e**2 - 5656*a*b*
*3*c**3*d*e + 5656*a*b**2*c**4*d**2 + 36*b**6*c*e**2 - 168*b**5*c**2*d*e +
168*b**4*c**3*d**2) + x**2*(876*a**3*b*c**3*e**2 + 1426*a**2*b**3*c**2*e**2
 - 6132*a**2*b**2*c**3*d*e + 6132*a**2*b*c**4*d**2 + 164*a*b**5*c*e**2 - 78
4*a*b**4*c**2*d*e + 784*a*b**3*c**3*d**2 - 6*b**7*e**2 + 28*b**6*c*d*e - 28
*b**5*c**2*d**2) + x*(-120*a**4*c**3*e**2 + 1116*a**3*b**2*c**2*e**2 - 2232
*a**3*b*c**3*d*e + 2232*a**3*c**4*d**2 + 112*a**2*b**4*c*e**2 - 1392*a**2*b
**3*c**2*d*e + 1392*a**2*b**2*c**3*d**2 - 4*a*b**6*e**2 + 152*a*b**5*c*d*e
- 152*a*b**4*c**2*d**2 - 8*b**7*d*e + 8*b**6*c*d**2))/(3072*a**8*c**4 - 307
2*a**7*b**2*c**3 + 1152*a**6*b**4*c**2 - 192*a**5*b**6*c + 12*a**4*b**8 + x
**8*(3072*a**4*c**8 - 3072*a**3*b**2*c**7 + 1152*a**2*b**4*c**6 - 192*a*b**
6*c**5 + 12*b**8*c**4) + x**7*(12288*a**4*b*c**7 - 12288*a**3*b**3*c**6 + 4
608*a**2*b**5*c**5 - 768*a*b**7*c**4 + 48*b**9*c**3) + x**6*(12288*a**5*c**
7 + 6144*a**4*b**2*c**6 - 13824*a**3*b**4*c**5 + 6144*a**2*b**6*c**4 - 1104
*a*b**8*c**3 + 72*b**10*c**2) + x**5*(36864*a**5*b*c**6 - 24576*a**4*b**3*c
**5 + 1536*a**3*b**5*c**4 + 2304*a**2*b**7*c**3 - 624*a*b**9*c**2 + 48*b**1
1*c) + x**4*(18432*a**6*c**6 + 18432*a**5*b**2*c**5 - 26880*a**4*b**4*c**4
+ 9600*a**3*b**6*c**3 - 1080*a**2*b**8*c**2 - 48*a*b**10*c + 12*b**12) + x*
*3*(36864*a**6*b*c**5 - 24576*a**5*b**3*c**4 + 1536*a**4*b**5*c**3 + 2304*a
**3*b**7*c**2 - 624*a**2*b**9*c + 48*a*b**11) + x**2*(12288*a**7*c**5 + 614
4*a**6*b**2*c**4 - 13824*a**5*b**4*c**3 + 6144*a**4*b**6*c**2 - 1104*a**3*b
**8*c + 72*a**2*b**10) + x*(12288*a**7*b*c**4 - 12288*a**6*b**3*c**3 + 4608
*a**5*b**5*c**2 - 768*a**4*b**7*c + 48*a**3*b**9))
```

Giac [B] time = 1.16681, size = 1359, normalized size = 4.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^5,x, algorithm="giac")
```

```
[Out] 10*(14*c^4*d^2 - 14*b*c^3*d*e + 3*b^2*c^2*e^2 + 2*a*c^3*e^2)*arctan((2*c*x
+ b)/sqrt(-b^2 + 4*a*c))/((b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*
c^3 + 256*a^4*c^4)*sqrt(-b^2 + 4*a*c)) + 1/12*(840*c^7*d^2*x^7 - 840*b*c^6*
d*x^7*e + 2940*b*c^6*d^2*x^6 + 180*b^2*c^5*x^7*e^2 + 120*a*c^6*x^7*e^2 - 29
40*b^2*c^5*d*x^6*e + 3640*b^2*c^5*d^2*x^5 + 3080*a*c^6*d^2*x^5 + 630*b^3*c^
4*x^6*e^2 + 420*a*b*c^5*x^6*e^2 - 3640*b^3*c^4*d*x^5*e - 3080*a*b*c^5*d*x^5
*e + 1750*b^3*c^4*d^2*x^4 + 7700*a*b*c^5*d^2*x^4 + 780*b^4*c^3*x^5*e^2 + 11
80*a*b^2*c^4*x^5*e^2 + 440*a^2*c^5*x^5*e^2 - 1750*b^4*c^3*d*x^4*e - 7700*a*
b^2*c^4*d*x^4*e + 168*b^4*c^3*d^2*x^3 + 5656*a*b^2*c^4*d^2*x^3 + 4088*a^2*c
^5*d^2*x^3 + 375*b^5*c^2*x^4*e^2 + 1900*a*b^3*c^3*x^4*e^2 + 1100*a^2*b*c^4*
x^4*e^2 - 168*b^5*c^2*d*x^3*e - 5656*a*b^3*c^3*d*x^3*e - 4088*a^2*b*c^4*d*x
^3*e - 28*b^5*c^2*d^2*x^2 + 784*a*b^3*c^3*d^2*x^2 + 6132*a^2*b*c^4*d^2*x^2
+ 36*b^6*c*x^3*e^2 + 1236*a*b^4*c^2*x^3*e^2 + 1684*a^2*b^2*c^3*x^3*e^2 + 58
4*a^3*c^4*x^3*e^2 + 28*b^6*c*d*x^2*e - 784*a*b^4*c^2*d*x^2*e - 6132*a^2*b^2
*c^3*d*x^2*e + 8*b^6*c*d^2*x - 152*a*b^4*c^2*d^2*x + 1392*a^2*b^2*c^3*d^2*x
 + 2232*a^3*c^4*d^2*x - 6*b^7*x^2*e^2 + 164*a*b^5*c*x^2*e^2 + 1426*a^2*b^3*
c^2*x^2*e^2 + 876*a^3*b*c^3*x^2*e^2 - 8*b^7*d*x*e + 152*a*b^5*c*d*x*e - 139
2*a^2*b^3*c^2*d*x*e - 2232*a^3*b*c^3*d*x*e - 3*b^7*d^2 + 50*a*b^5*c*d^2 - 3
26*a^2*b^3*c^2*d^2 + 1116*a^3*b*c^3*d^2 - 4*a*b^6*x*e^2 + 112*a^2*b^4*c*x*e
^2 + 1116*a^3*b^2*c^2*x*e^2 - 120*a^4*c^3*x*e^2 - 2*a*b^6*d*e + 38*a^2*b^4*
c*d*e - 348*a^3*b^2*c^2*d*e - 768*a^4*c^3*d*e - a^2*b^5*e^2 + 28*a^3*b^3*c*
e^2 + 324*a^4*b*c^2*e^2)/((b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*
c^3 + 256*a^4*c^4)*(c*x^2 + b*x + a)^4)
```

3.2225 $\int \frac{d+ex}{(a+bx+cx^2)^5} dx$

Optimal. Leaf size=219

$$\frac{35c^2(b+2cx)(2cd-be)}{2(b^2-4ac)^4(a+bx+cx^2)} - \frac{70c^3(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{9/2}} - \frac{35c(b+2cx)(2cd-be)}{12(b^2-4ac)^3(a+bx+cx^2)^2} + \frac{7(b+2cx)(2c}{12(b^2-4ac)^2(a+}$$

```
[Out] -(b*d - 2*a*e + (2*c*d - b*e)*x)/(4*(b^2 - 4*a*c)*(a + b*x + c*x^2)^4) + (7
*(2*c*d - b*e)*(b + 2*c*x))/(12*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^3) - (35*
c*(2*c*d - b*e)*(b + 2*c*x))/(12*(b^2 - 4*a*c)^3*(a + b*x + c*x^2)^2) + (35
*c^2*(2*c*d - b*e)*(b + 2*c*x))/(2*(b^2 - 4*a*c)^4*(a + b*x + c*x^2)) - (70
*c^3*(2*c*d - b*e)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(9
/2)
```

Rubi [A] time = 0.0963824, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {638, 614, 618, 206}

$$\frac{35c^2(b+2cx)(2cd-be)}{2(b^2-4ac)^4(a+bx+cx^2)} - \frac{70c^3(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{9/2}} - \frac{35c(b+2cx)(2cd-be)}{12(b^2-4ac)^3(a+bx+cx^2)^2} + \frac{7(b+2cx)(2c}{12(b^2-4ac)^2(a+}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*x + c*x^2)^5,x]
```


```
[Out] -(b*d - 2*a*e + (2*c*d - b*e)*x)/(4*(b^2 - 4*a*c)*(a + b*x + c*x^2)^4) + (7
*(2*c*d - b*e)*(b + 2*c*x))/(12*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^3) - (35*
c*(2*c*d - b*e)*(b + 2*c*x))/(12*(b^2 - 4*a*c)^3*(a + b*x + c*x^2)^2) + (35
*c^2*(2*c*d - b*e)*(b + 2*c*x))/(2*(b^2 - 4*a*c)^4*(a + b*x + c*x^2)) - (70
*c^3*(2*c*d - b*e)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(9
/2)
```


Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex}{\left(a+bx+cx^2\right)^5}\,dx = -\frac{bd-2ae+(2cd-be)x}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4} - \frac{(7(2cd-be))\int \frac{1}{\left(a+bx+cx^2\right)^4}\,dx}{4\left(b^2-4ac\right)}$$

$$= -\frac{bd-2ae+(2cd-be)x}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4} + \frac{7(2cd-be)(b+2cx)}{12\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^3} + \frac{(35c(2cd-be))\int \frac{1}{\left(a+bx+cx^2\right)^3}\,dx}{6\left(b^2-4ac\right)^2}$$

$$= -\frac{bd-2ae+(2cd-be)x}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4} + \frac{7(2cd-be)(b+2cx)}{12\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^3} - \frac{35c(2cd-be)(b+2cx)}{12\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)^2}$$

$$= -\frac{bd-2ae+(2cd-be)x}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4} + \frac{7(2cd-be)(b+2cx)}{12\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^3} - \frac{35c(2cd-be)(b+2cx)}{12\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)^2}$$

$$= -\frac{bd-2ae+(2cd-be)x}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4} + \frac{7(2cd-be)(b+2cx)}{12\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^3} - \frac{35c(2cd-be)(b+2cx)}{12\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)^2}$$

$$= -\frac{bd-2ae+(2cd-be)x}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^4} + \frac{7(2cd-be)(b+2cx)}{12\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^3} - \frac{35c(2cd-be)(b+2cx)}{12\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)^2}$$

Mathematica [A] time = 0.30673, size = 209, normalized size = 0.95

$$\frac{-\frac{840c^3(be-2cd)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{35c\left(b^2-4ac\right)(b+2cx)(be-2cd)}{(a+x(b+cx))^2} - \frac{7\left(b^2-4ac\right)^2(b+2cx)(be-2cd)}{(a+x(b+cx))^3} + \frac{3\left(b^2-4ac\right)^3(2ae-bd+bex-2cdx)}{(a+x(b+cx))^4} + \frac{210c^2(b+2cx)(2cd-be)}{a+x(b+cx)}}{12\left(b^2-4ac\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*x + c*x^2)^5,x]
```

```
[Out] ((3*(b^2 - 4*a*c)^3*(-(b*d) + 2*a*e - 2*c*d*x + b*e*x))/(a + x*(b + c*x))^4
 - (7*(b^2 - 4*a*c)^2*(-2*c*d + b*e)*(b + 2*c*x))/(a + x*(b + c*x))^3 + (35
*c*(b^2 - 4*a*c)*(-2*c*d + b*e)*(b + 2*c*x))/(a + x*(b + c*x))^2 + (210*c^2
*(2*c*d - b*e)*(b + 2*c*x))/(a + x*(b + c*x)) - (840*c^3*(-2*c*d + b*e)*Arc
Tan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(12*(b^2 - 4*a*c)^
4)
```

Maple [B] time = 0.157, size = 496, normalized size = 2.3

$$\frac{bd-2\,ae+(-be+2\,cd)\,x}{\left(16\,ac-4\,b^2\right)\left(cx^2+bx+a\right)^4} - \frac{7\,bcxe}{6\,\left(4\,ac-b^2\right)^2\left(cx^2+bx+a\right)^3} + \frac{7\,c^2xd}{3\,\left(4\,ac-b^2\right)^2\left(cx^2+bx+a\right)^3} - \frac{7\,b^2e}{12\,\left(4\,ac-b^2\right)^2\left(cx^2+\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^2+b*x+a)^5,x)
```

```
[Out] 1/4*(b*d-2*a*e+(-b*e+2*c*d)*x)/(4*a*c-b^2)/(c*x^2+b*x+a)^4-7/6/(4*a*c-b^2)^
2/(c*x^2+b*x+a)^3*x*c*b*e+7/3/(4*a*c-b^2)^2/(c*x^2+b*x+a)^3*x*c^2*d-7/12/(4
*a*c-b^2)^2/(c*x^2+b*x+a)^3*b^2*e+7/6/(4*a*c-b^2)^2/(c*x^2+b*x+a)^3*b*c*d-3
5/6/(4*a*c-b^2)^3*c^2/(c*x^2+b*x+a)^2*x*b*e+35/3/(4*a*c-b^2)^3*c^3/(c*x^2+b
*x+a)^2*x*d-35/12/(4*a*c-b^2)^3*c/(c*x^2+b*x+a)^2*b^2*e+35/6/(4*a*c-b^2)^3*
c^2/(c*x^2+b*x+a)^2*b*d-35/(4*a*c-b^2)^4*c^3/(c*x^2+b*x+a)*x*b*e+70/(4*a*c-
b^2)^4*c^4/(c*x^2+b*x+a)*x*d-35/2/(4*a*c-b^2)^4*c^2/(c*x^2+b*x+a)*b^2*e+35/
(4*a*c-b^2)^4*c^3/(c*x^2+b*x+a)*b*d-70/(4*a*c-b^2)^(9/2)*c^3*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))*b*e+140/(4*a*c-b^2)^(9/2)*c^4*arctan((2*c*x+b)/(4*a*c
-b^2)^(1/2))*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.2368, size = 6942, normalized size = 31.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^5,x, algorithm="fricas")
```

```
[Out] [1/12*(420*(2*(b^2*c^7 - 4*a*c^8)*d - (b^3*c^6 - 4*a*b*c^7)*e)*x^7 + 1470*(
2*(b^3*c^6 - 4*a*b*c^7)*d - (b^4*c^5 - 4*a*b^2*c^6)*e)*x^6 + 140*(2*(13*b^4
*c^5 - 41*a*b^2*c^6 - 44*a^2*c^7)*d - (13*b^5*c^4 - 41*a*b^3*c^5 - 44*a^2*b
*c^6)*e)*x^5 + 175*(2*(5*b^5*c^4 + 2*a*b^3*c^5 - 88*a^2*b*c^6)*d - (5*b^6*c
^3 + 2*a*b^4*c^4 - 88*a^2*b^2*c^5)*e)*x^4 + 28*(2*(3*b^6*c^3 + 89*a*b^4*c^4
 - 331*a^2*b^2*c^5 - 292*a^3*c^6)*d - (3*b^7*c^2 + 89*a*b^5*c^3 - 331*a^2*b
^3*c^4 - 292*a^3*b*c^5)*e)*x^3 - 14*(2*(b^7*c^2 - 32*a*b^5*c^3 - 107*a^2*b^
3*c^4 + 876*a^3*b*c^5)*d - (b^8*c - 32*a*b^6*c^2 - 107*a^2*b^4*c^3 + 876*a^
3*b^2*c^4)*e)*x^2 - 420*(2*a^4*c^4*d - a^4*b*c^3*e + (2*c^8*d - b*c^7*e)*x^
8 + 4*(2*b*c^7*d - b^2*c^6*e)*x^7 + 2*(2*(3*b^2*c^6 + 2*a*c^7)*d - (3*b^3*c
^5 + 2*a*b*c^6)*e)*x^6 + 4*(2*(b^3*c^5 + 3*a*b*c^6)*d - (b^4*c^4 + 3*a*b^2*
c^5)*e)*x^5 + (2*(b^4*c^4 + 12*a*b^2*c^5 + 6*a^2*c^6)*d - (b^5*c^3 + 12*a*b
^3*c^4 + 6*a^2*b*c^5)*e)*x^4 + 4*(2*(a*b^3*c^4 + 3*a^2*b*c^5)*d - (a*b^4*c^
3 + 3*a^2*b^2*c^4)*e)*x^3 + 2*(2*(3*a^2*b^2*c^4 + 2*a^3*c^5)*d - (3*a^2*b^3
*c^3 + 2*a^3*b*c^4)*e)*x^2 + 4*(2*a^3*b*c^4*d - a^3*b^2*c^3*e)*x)*sqrt(b^2
- 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x
+ b))/(c*x^2 + b*x + a)) - (3*b^9 - 62*a*b^7*c + 526*a^2*b^5*c^2 - 2420*a^3
*b^3*c^3 + 4464*a^4*b*c^4)*d - (a*b^8 - 23*a^2*b^6*c + 250*a^3*b^4*c^2 - 31
2*a^4*b^2*c^3 - 1536*a^5*c^4)*e + 4*(2*(b^8*c - 23*a*b^6*c^2 + 250*a^2*b^4*
c^3 - 417*a^3*b^2*c^4 - 1116*a^4*c^5)*d - (b^9 - 23*a*b^7*c + 250*a^2*b^5*c
^2 - 417*a^3*b^3*c^3 - 1116*a^4*b*c^4)*e)*x)/(a^4*b^10 - 20*a^5*b^8*c + 160
*a^6*b^6*c^2 - 640*a^7*b^4*c^3 + 1280*a^8*b^2*c^4 - 1024*a^9*c^5 + (b^10*c^
4 - 20*a*b^8*c^5 + 160*a^2*b^6*c^6 - 640*a^3*b^4*c^7 + 1280*a^4*b^2*c^8 - 1
```

```
024*a^5*c^9)*x^8 + 4*(b^11*c^3 - 20*a*b^9*c^4 + 160*a^2*b^7*c^5 - 640*a^3*b
^5*c^6 + 1280*a^4*b^3*c^7 - 1024*a^5*b*c^8)*x^7 + 2*(3*b^12*c^2 - 58*a*b^10
*c^3 + 440*a^2*b^8*c^4 - 1600*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 512*a^5*b^2*
c^7 - 2048*a^6*c^8)*x^6 + 4*(b^13*c - 17*a*b^11*c^2 + 100*a^2*b^9*c^3 - 160
*a^3*b^7*c^4 - 640*a^4*b^5*c^5 + 2816*a^5*b^3*c^6 - 3072*a^6*b*c^7)*x^5 + (
b^14 - 8*a*b^12*c - 74*a^2*b^10*c^2 + 1160*a^3*b^8*c^3 - 5440*a^4*b^6*c^4 +
 10496*a^5*b^4*c^5 - 4608*a^6*b^2*c^6 - 6144*a^7*c^7)*x^4 + 4*(a*b^13 - 17*
a^2*b^11*c + 100*a^3*b^9*c^2 - 160*a^4*b^7*c^3 - 640*a^5*b^5*c^4 + 2816*a^6
*b^3*c^5 - 3072*a^7*b*c^6)*x^3 + 2*(3*a^2*b^12 - 58*a^3*b^10*c + 440*a^4*b^
8*c^2 - 1600*a^5*b^6*c^3 + 2560*a^6*b^4*c^4 - 512*a^7*b^2*c^5 - 2048*a^8*c^
6)*x^2 + 4*(a^3*b^11 - 20*a^4*b^9*c + 160*a^5*b^7*c^2 - 640*a^6*b^5*c^3 + 1
280*a^7*b^3*c^4 - 1024*a^8*b*c^5)*x), 1/12*(420*(2*(b^2*c^7 - 4*a*c^8)*d -
(b^3*c^6 - 4*a*b*c^7)*e)*x^7 + 1470*(2*(b^3*c^6 - 4*a*b*c^7)*d - (b^4*c^5 -
 4*a*b^2*c^6)*e)*x^6 + 140*(2*(13*b^4*c^5 - 41*a*b^2*c^6 - 44*a^2*c^7)*d -
(13*b^5*c^4 - 41*a*b^3*c^5 - 44*a^2*b*c^6)*e)*x^5 + 175*(2*(5*b^5*c^4 + 2*a
*b^3*c^5 - 88*a^2*b*c^6)*d - (5*b^6*c^3 + 2*a*b^4*c^4 - 88*a^2*b^2*c^5)*e)*
x^4 + 28*(2*(3*b^6*c^3 + 89*a*b^4*c^4 - 331*a^2*b^2*c^5 - 292*a^3*c^6)*d -
(3*b^7*c^2 + 89*a*b^5*c^3 - 331*a^2*b^3*c^4 - 292*a^3*b*c^5)*e)*x^3 - 14*(2
*(b^7*c^2 - 32*a*b^5*c^3 - 107*a^2*b^3*c^4 + 876*a^3*b*c^5)*d - (b^8*c - 32
*a*b^6*c^2 - 107*a^2*b^4*c^3 + 876*a^3*b^2*c^4)*e)*x^2 - 840*(2*a^4*c^4*d -
 a^4*b*c^3*e + (2*c^8*d - b*c^7*e)*x^8 + 4*(2*b*c^7*d - b^2*c^6*e)*x^7 + 2*
(2*(3*b^2*c^6 + 2*a*c^7)*d - (3*b^3*c^5 + 2*a*b*c^6)*e)*x^6 + 4*(2*(b^3*c^5
 + 3*a*b*c^6)*d - (b^4*c^4 + 3*a*b^2*c^5)*e)*x^5 + (2*(b^4*c^4 + 12*a*b^2*c
^5 + 6*a^2*c^6)*d - (b^5*c^3 + 12*a*b^3*c^4 + 6*a^2*b*c^5)*e)*x^4 + 4*(2*(a
*b^3*c^4 + 3*a^2*b*c^5)*d - (a*b^4*c^3 + 3*a^2*b^2*c^4)*e)*x^3 + 2*(2*(3*a^
2*b^2*c^4 + 2*a^3*c^5)*d - (3*a^2*b^3*c^3 + 2*a^3*b*c^4)*e)*x^2 + 4*(2*a^3*
b*c^4*d - a^3*b^2*c^3*e)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(
2*c*x + b)/(b^2 - 4*a*c)) - (3*b^9 - 62*a*b^7*c + 526*a^2*b^5*c^2 - 2420*a^
3*b^3*c^3 + 4464*a^4*b*c^4)*d - (a*b^8 - 23*a^2*b^6*c + 250*a^3*b^4*c^2 - 3
12*a^4*b^2*c^3 - 1536*a^5*c^4)*e + 4*(2*(b^8*c - 23*a*b^6*c^2 + 250*a^2*b^4
*c^3 - 417*a^3*b^2*c^4 - 1116*a^4*c^5)*d - (b^9 - 23*a*b^7*c + 250*a^2*b^5*
c^2 - 417*a^3*b^3*c^3 - 1116*a^4*b*c^4)*e)*x)/(a^4*b^10 - 20*a^5*b^8*c + 16
0*a^6*b^6*c^2 - 640*a^7*b^4*c^3 + 1280*a^8*b^2*c^4 - 1024*a^9*c^5 + (b^10*c
^4 - 20*a*b^8*c^5 + 160*a^2*b^6*c^6 - 640*a^3*b^4*c^7 + 1280*a^4*b^2*c^8 -
1024*a^5*c^9)*x^8 + 4*(b^11*c^3 - 20*a*b^9*c^4 + 160*a^2*b^7*c^5 - 640*a^3*
b^5*c^6 + 1280*a^4*b^3*c^7 - 1024*a^5*b*c^8)*x^7 + 2*(3*b^12*c^2 - 58*a*b^1
0*c^3 + 440*a^2*b^8*c^4 - 1600*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 512*a^5*b^2
*c^7 - 2048*a^6*c^8)*x^6 + 4*(b^13*c - 17*a*b^11*c^2 + 100*a^2*b^9*c^3 - 16
0*a^3*b^7*c^4 - 640*a^4*b^5*c^5 + 2816*a^5*b^3*c^6 - 3072*a^6*b*c^7)*x^5 +
(b^14 - 8*a*b^12*c - 74*a^2*b^10*c^2 + 1160*a^3*b^8*c^3 - 5440*a^4*b^6*c^4
+ 10496*a^5*b^4*c^5 - 4608*a^6*b^2*c^6 - 6144*a^7*c^7)*x^4 + 4*(a*b^13 - 17
*a^2*b^11*c + 100*a^3*b^9*c^2 - 160*a^4*b^7*c^3 - 640*a^5*b^5*c^4 + 2816*a^
6*b^3*c^5 - 3072*a^7*b*c^6)*x^3 + 2*(3*a^2*b^12 - 58*a^3*b^10*c + 440*a^4*b
^8*c^2 - 1600*a^5*b^6*c^3 + 2560*a^6*b^4*c^4 - 512*a^7*b^2*c^5 - 2048*a^8*c
^6)*x^2 + 4*(a^3*b^11 - 20*a^4*b^9*c + 160*a^5*b^7*c^2 - 640*a^6*b^5*c^3 +
1280*a^7*b^3*c^4 - 1024*a^8*b*c^5)*x)]
```

Sympy [B] time = 8.58564, size = 1564, normalized size = 7.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**2+b*x+a)**5,x)
```

```
[Out] 35*c**3*sqrt(-1/(4*a*c - b**2)**9)*(b*e - 2*c*d)*log(x + (-35840*a**5*c**8*
sqrt(-1/(4*a*c - b**2)**9)*(b*e - 2*c*d) + 44800*a**4*b**2*c**7*sqrt(-1/(4*
```

```
a*c - b**2)**9)*(b*e - 2*c*d) - 22400*a**3*b**4*c**6*sqrt(-1/(4*a*c - b**2)
**9)*(b*e - 2*c*d) + 5600*a**2*b**6*c**5*sqrt(-1/(4*a*c - b**2)**9)*(b*e -
2*c*d) - 700*a*b**8*c**4*sqrt(-1/(4*a*c - b**2)**9)*(b*e - 2*c*d) + 35*b**1
0*c**3*sqrt(-1/(4*a*c - b**2)**9)*(b*e - 2*c*d) + 35*b**2*c**3*e - 70*b*c**
4*d)/(70*b*c**4*e - 140*c**5*d)) - 35*c**3*sqrt(-1/(4*a*c - b**2)**9)*(b*e
- 2*c*d)*log(x + (35840*a**5*c**8*sqrt(-1/(4*a*c - b**2)**9)*(b*e - 2*c*d)
- 44800*a**4*b**2*c**7*sqrt(-1/(4*a*c - b**2)**9)*(b*e - 2*c*d) + 22400*a**
3*b**4*c**6*sqrt(-1/(4*a*c - b**2)**9)*(b*e - 2*c*d) - 5600*a**2*b**6*c**5*
sqrt(-1/(4*a*c - b**2)**9)*(b*e - 2*c*d) + 700*a*b**8*c**4*sqrt(-1/(4*a*c -
 b**2)**9)*(b*e - 2*c*d) - 35*b**10*c**3*sqrt(-1/(4*a*c - b**2)**9)*(b*e -
2*c*d) + 35*b**2*c**3*e - 70*b*c**4*d)/(70*b*c**4*e - 140*c**5*d)) - (384*a
**4*c**3*e + 174*a**3*b**2*c**2*e - 1116*a**3*b*c**3*d - 19*a**2*b**4*c*e +
 326*a**2*b**3*c**2*d + a*b**6*e - 50*a*b**5*c*d + 3*b**7*d + x**7*(420*b*c
**6*e - 840*c**7*d) + x**6*(1470*b**2*c**5*e - 2940*b*c**6*d) + x**5*(1540*
a*b*c**5*e - 3080*a*c**6*d + 1820*b**3*c**4*e - 3640*b**2*c**5*d) + x**4*(3
850*a*b**2*c**4*e - 7700*a*b*c**5*d + 875*b**4*c**3*e - 1750*b**3*c**4*d) +
 x**3*(2044*a**2*b*c**4*e - 4088*a**2*c**5*d + 2828*a*b**3*c**3*e - 5656*a*
b**2*c**4*d + 84*b**5*c**2*e - 168*b**4*c**3*d) + x**2*(3066*a**2*b**2*c**3
*e - 6132*a**2*b*c**4*d + 392*a*b**4*c**2*e - 784*a*b**3*c**3*d - 14*b**6*c
*e + 28*b**5*c**2*d) + x*(1116*a**3*b*c**3*e - 2232*a**3*c**4*d + 696*a**2*
b**3*c**2*e - 1392*a**2*b**2*c**3*d - 76*a*b**5*c*e + 152*a*b**4*c**2*d + 4
*b**7*e - 8*b**6*c*d))/(3072*a**8*c**4 - 3072*a**7*b**2*c**3 + 1152*a**6*b*
*4*c**2 - 192*a**5*b**6*c + 12*a**4*b**8 + x**8*(3072*a**4*c**8 - 3072*a**3
*b**2*c**7 + 1152*a**2*b**4*c**6 - 192*a*b**6*c**5 + 12*b**8*c**4) + x**7*(
12288*a**4*b*c**7 - 12288*a**3*b**3*c**6 + 4608*a**2*b**5*c**5 - 768*a*b**7
*c**4 + 48*b**9*c**3) + x**6*(12288*a**5*c**7 + 6144*a**4*b**2*c**6 - 13824
*a**3*b**4*c**5 + 6144*a**2*b**6*c**4 - 1104*a*b**8*c**3 + 72*b**10*c**2) +
 x**5*(36864*a**5*b*c**6 - 24576*a**4*b**3*c**5 + 1536*a**3*b**5*c**4 + 230
4*a**2*b**7*c**3 - 624*a*b**9*c**2 + 48*b**11*c) + x**4*(18432*a**6*c**6 +
18432*a**5*b**2*c**5 - 26880*a**4*b**4*c**4 + 9600*a**3*b**6*c**3 - 1080*a*
*2*b**8*c**2 - 48*a*b**10*c + 12*b**12) + x**3*(36864*a**6*b*c**5 - 24576*a
**5*b**3*c**4 + 1536*a**4*b**5*c**3 + 2304*a**3*b**7*c**2 - 624*a**2*b**9*c
 + 48*a*b**11) + x**2*(12288*a**7*c**5 + 6144*a**6*b**2*c**4 - 13824*a**5*b
**4*c**3 + 6144*a**4*b**6*c**2 - 1104*a**3*b**8*c + 72*a**2*b**10) + x*(122
88*a**7*b*c**4 - 12288*a**6*b**3*c**3 + 4608*a**5*b**5*c**2 - 768*a**4*b**7
*c + 48*a**3*b**9))
```

Giac [B] time = 1.14646, size = 826, normalized size = 3.77

$$\frac{70\left(2\,c^4d-bc^3e\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^8-16\,ab^6c+96\,a^2b^4c^2-256\,a^3b^2c^3+256\,a^4c^4\right)\sqrt{-b^2+4\,ac}}+\frac{840\,c^7dx^7-420\,bc^6x^7e+2940\,bc^6dx^6-1470\,b^2c^5x^6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^5,x, algorithm="giac")
```

```
[Out] 70*(2*c^4*d - b*c^3*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^8 - 16*a*
b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4)*sqrt(-b^2 + 4*a*c))
 + 1/12*(840*c^7*d*x^7 - 420*b*c^6*x^7*e + 2940*b*c^6*d*x^6 - 1470*b^2*c^5*
x^6*e + 3640*b^2*c^5*d*x^5 + 3080*a*c^6*d*x^5 - 1820*b^3*c^4*x^5*e - 1540*a
*b*c^5*x^5*e + 1750*b^3*c^4*d*x^4 + 7700*a*b*c^5*d*x^4 - 875*b^4*c^3*x^4*e
- 3850*a*b^2*c^4*x^4*e + 168*b^4*c^3*d*x^3 + 5656*a*b^2*c^4*d*x^3 + 4088*a^
2*c^5*d*x^3 - 84*b^5*c^2*x^3*e - 2828*a*b^3*c^3*x^3*e - 2044*a^2*b*c^4*x^3*
e - 28*b^5*c^2*d*x^2 + 784*a*b^3*c^3*d*x^2 + 6132*a^2*b*c^4*d*x^2 + 14*b^6*
c*x^2*e - 392*a*b^4*c^2*x^2*e - 3066*a^2*b^2*c^3*x^2*e + 8*b^6*c*d*x - 152*
a*b^4*c^2*d*x + 1392*a^2*b^2*c^3*d*x + 2232*a^3*c^4*d*x - 4*b^7*x*e + 76*a*
```

```
b^5*c*x*e - 696*a^2*b^3*c^2*x*e - 1116*a^3*b*c^3*x*e - 3*b^7*d + 50*a*b^5*c
*d - 326*a^2*b^3*c^2*d + 1116*a^3*b*c^3*d - a*b^6*e + 19*a^2*b^4*c*e - 174*
a^3*b^2*c^2*e - 384*a^4*c^3*e)/((b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^
3*b^2*c^3 + 256*a^4*c^4)*(c*x^2 + b*x + a)^4)
```

3.2226 $\int \frac{1}{(a+bx+cx^2)^5}\,dx$

Optimal. Leaf size=171

$$\frac{35c^3(b+2cx)}{(b^2-4ac)^4(a+bx+cx^2)}-\frac{35c^2(b+2cx)}{6(b^2-4ac)^3(a+bx+cx^2)^2}-\frac{140c^4\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{9/2}}+\frac{7c(b+2cx)}{6(b^2-4ac)^2(a+bx+cx^2)^3}$$

```
[Out] -(b + 2*c*x)/(4*(b^2 - 4*a*c)*(a + b*x + c*x^2)^4) + (7*c*(b + 2*c*x))/(6*(
b^2 - 4*a*c)^2*(a + b*x + c*x^2)^3) - (35*c^2*(b + 2*c*x))/(6*(b^2 - 4*a*c)
^3*(a + b*x + c*x^2)^2) + (35*c^3*(b + 2*c*x))/((b^2 - 4*a*c)^4*(a + b*x +
c*x^2)) - (140*c^4*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(9
/2)
```

Rubi [A] time = 0.0698263, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {614, 618, 206}

$$\frac{35c^3(b+2cx)}{(b^2-4ac)^4(a+bx+cx^2)}-\frac{35c^2(b+2cx)}{6(b^2-4ac)^3(a+bx+cx^2)^2}-\frac{140c^4\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{9/2}}+\frac{7c(b+2cx)}{6(b^2-4ac)^2(a+bx+cx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(-5),x]
```

```
[Out] -(b + 2*c*x)/(4*(b^2 - 4*a*c)*(a + b*x + c*x^2)^4) + (7*c*(b + 2*c*x))/(6*(
b^2 - 4*a*c)^2*(a + b*x + c*x^2)^3) - (35*c^2*(b + 2*c*x))/(6*(b^2 - 4*a*c)
^3*(a + b*x + c*x^2)^2) + (35*c^3*(b + 2*c*x))/((b^2 - 4*a*c)^4*(a + b*x +
c*x^2)) - (140*c^4*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(9
/2)
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx+cx^2)^5}\,dx = -\frac{b+2cx}{4(b^2-4ac)(a+bx+cx^2)^4} - \frac{(7c)\int \frac{1}{(a+bx+cx^2)^4}\,dx}{2(b^2-4ac)}$$

$$= -\frac{b+2cx}{4(b^2-4ac)(a+bx+cx^2)^4} + \frac{7c(b+2cx)}{6(b^2-4ac)^2(a+bx+cx^2)^3} + \frac{(35c^2)\int \frac{1}{(a+bx+cx^2)^3}\,dx}{3(b^2-4ac)^2}$$

$$= -\frac{b+2cx}{4(b^2-4ac)(a+bx+cx^2)^4} + \frac{7c(b+2cx)}{6(b^2-4ac)^2(a+bx+cx^2)^3} - \frac{35c^2(b+2cx)}{6(b^2-4ac)^3(a+bx+cx^2)^2} -$$

$$= -\frac{b+2cx}{4(b^2-4ac)(a+bx+cx^2)^4} + \frac{7c(b+2cx)}{6(b^2-4ac)^2(a+bx+cx^2)^3} - \frac{35c^2(b+2cx)}{6(b^2-4ac)^3(a+bx+cx^2)^2} +$$

$$= -\frac{b+2cx}{4(b^2-4ac)(a+bx+cx^2)^4} + \frac{7c(b+2cx)}{6(b^2-4ac)^2(a+bx+cx^2)^3} - \frac{35c^2(b+2cx)}{6(b^2-4ac)^3(a+bx+cx^2)^2} +$$

$$= -\frac{b+2cx}{4(b^2-4ac)(a+bx+cx^2)^4} + \frac{7c(b+2cx)}{6(b^2-4ac)^2(a+bx+cx^2)^3} - \frac{35c^2(b+2cx)}{6(b^2-4ac)^3(a+bx+cx^2)^2} +$$

Mathematica [A] time = 0.173295, size = 167, normalized size = 0.98

$$\frac{-\frac{70c^2(b^2-4ac)(b+2cx)}{(a+x(b+cx))^2} + \frac{1680c^4\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{14c(b^2-4ac)^2(b+2cx)}{(a+x(b+cx))^3} - \frac{3(b^2-4ac)^3(b+2cx)}{(a+x(b+cx))^4} + \frac{420c^3(b+2cx)}{a+x(b+cx)}}{12(b^2-4ac)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(-5),x]
```

```
[Out] ((-3*(b^2 - 4*a*c)^3*(b + 2*c*x))/(a + x*(b + c*x))^4 + (14*c*(b^2 - 4*a*c)
^2*(b + 2*c*x))/(a + x*(b + c*x))^3 - (70*c^2*(b^2 - 4*a*c)*(b + 2*c*x))/(a
 + x*(b + c*x))^2 + (420*c^3*(b + 2*c*x))/(a + x*(b + c*x)) + (1680*c^4*Arc
Tan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(12*(b^2 - 4*a*c)^
4)
```

Maple [A] time = 0.162, size = 249, normalized size = 1.5

$$\frac{2cx+b}{(16ac-4b^2)(cx^2+bx+a)^4} + \frac{7c^2x}{3(4ac-b^2)^2(cx^2+bx+a)^3} + \frac{7bc}{6(4ac-b^2)^2(cx^2+bx+a)^3} + \frac{35c^3x}{3(4ac-b^2)^3(cx^2+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^5,x)
```

```
[Out] 1/4*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^4+7/3*c^2/(4*a*c-b^2)^2/(c*x^2+b*x+
a)^3*x+7/6*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^3*b+35/3*c^3/(4*a*c-b^2)^3/(c*x^2+
b*x+a)^2*x+35/6*c^2/(4*a*c-b^2)^3/(c*x^2+b*x+a)^2*b+70*c^4/(4*a*c-b^2)^4/(c
*x^2+b*x+a)*x+35*c^3/(4*a*c-b^2)^4/(c*x^2+b*x+a)*b+140*c^4/(4*a*c-b^2)^(9/2
)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.14844, size = 4934, normalized size = 28.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^5,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*b^9 - 62*a*b^7*c + 526*a^2*b^5*c^2 - 2420*a^3*b^3*c^3 + 4464*a^4*
b*c^4 - 840*(b^2*c^7 - 4*a*c^8)*x^7 - 2940*(b^3*c^6 - 4*a*b*c^7)*x^6 - 280*
(13*b^4*c^5 - 41*a*b^2*c^6 - 44*a^2*c^7)*x^5 - 350*(5*b^5*c^4 + 2*a*b^3*c^5
 - 88*a^2*b*c^6)*x^4 - 56*(3*b^6*c^3 + 89*a*b^4*c^4 - 331*a^2*b^2*c^5 - 292
*a^3*c^6)*x^3 + 28*(b^7*c^2 - 32*a*b^5*c^3 - 107*a^2*b^3*c^4 + 876*a^3*b*c^
5)*x^2 - 840*(c^8*x^8 + 4*b*c^7*x^7 + 4*a^3*b*c^4*x + a^4*c^4 + 2*(3*b^2*c^
6 + 2*a*c^7)*x^6 + 4*(b^3*c^5 + 3*a*b*c^6)*x^5 + (b^4*c^4 + 12*a*b^2*c^5 +
6*a^2*c^6)*x^4 + 4*(a*b^3*c^4 + 3*a^2*b*c^5)*x^3 + 2*(3*a^2*b^2*c^4 + 2*a^3
*c^5)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(
b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 8*(b^8*c - 23*a*b^6*c^2 + 25
0*a^2*b^4*c^3 - 417*a^3*b^2*c^4 - 1116*a^4*c^5)*x)/(a^4*b^10 - 20*a^5*b^8*c
 + 160*a^6*b^6*c^2 - 640*a^7*b^4*c^3 + 1280*a^8*b^2*c^4 - 1024*a^9*c^5 + (b
^10*c^4 - 20*a*b^8*c^5 + 160*a^2*b^6*c^6 - 640*a^3*b^4*c^7 + 1280*a^4*b^2*c
^8 - 1024*a^5*c^9)*x^8 + 4*(b^11*c^3 - 20*a*b^9*c^4 + 160*a^2*b^7*c^5 - 640
*a^3*b^5*c^6 + 1280*a^4*b^3*c^7 - 1024*a^5*b*c^8)*x^7 + 2*(3*b^12*c^2 - 58*
a*b^10*c^3 + 440*a^2*b^8*c^4 - 1600*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 512*a^
5*b^2*c^7 - 2048*a^6*c^8)*x^6 + 4*(b^13*c - 17*a*b^11*c^2 + 100*a^2*b^9*c^3
 - 160*a^3*b^7*c^4 - 640*a^4*b^5*c^5 + 2816*a^5*b^3*c^6 - 3072*a^6*b*c^7)*x
^5 + (b^14 - 8*a*b^12*c - 74*a^2*b^10*c^2 + 1160*a^3*b^8*c^3 - 5440*a^4*b^6
*c^4 + 10496*a^5*b^4*c^5 - 4608*a^6*b^2*c^6 - 6144*a^7*c^7)*x^4 + 4*(a*b^13
 - 17*a^2*b^11*c + 100*a^3*b^9*c^2 - 160*a^4*b^7*c^3 - 640*a^5*b^5*c^4 + 28
16*a^6*b^3*c^5 - 3072*a^7*b*c^6)*x^3 + 2*(3*a^2*b^12 - 58*a^3*b^10*c + 440*
a^4*b^8*c^2 - 1600*a^5*b^6*c^3 + 2560*a^6*b^4*c^4 - 512*a^7*b^2*c^5 - 2048*
a^8*c^6)*x^2 + 4*(a^3*b^11 - 20*a^4*b^9*c + 160*a^5*b^7*c^2 - 640*a^6*b^5*c
^3 + 1280*a^7*b^3*c^4 - 1024*a^8*b*c^5)*x), -1/12*(3*b^9 - 62*a*b^7*c + 526
*a^2*b^5*c^2 - 2420*a^3*b^3*c^3 + 4464*a^4*b*c^4 - 840*(b^2*c^7 - 4*a*c^8)*
x^7 - 2940*(b^3*c^6 - 4*a*b*c^7)*x^6 - 280*(13*b^4*c^5 - 41*a*b^2*c^6 - 44*
a^2*c^7)*x^5 - 350*(5*b^5*c^4 + 2*a*b^3*c^5 - 88*a^2*b*c^6)*x^4 - 56*(3*b^6
*c^3 + 89*a*b^4*c^4 - 331*a^2*b^2*c^5 - 292*a^3*c^6)*x^3 + 28*(b^7*c^2 - 32
*a*b^5*c^3 - 107*a^2*b^3*c^4 + 876*a^3*b*c^5)*x^2 + 1680*(c^8*x^8 + 4*b*c^7
*x^7 + 4*a^3*b*c^4*x + a^4*c^4 + 2*(3*b^2*c^6 + 2*a*c^7)*x^6 + 4*(b^3*c^5 +
 3*a*b*c^6)*x^5 + (b^4*c^4 + 12*a*b^2*c^5 + 6*a^2*c^6)*x^4 + 4*(a*b^3*c^4 +
 3*a^2*b*c^5)*x^3 + 2*(3*a^2*b^2*c^4 + 2*a^3*c^5)*x^2)*sqrt(-b^2 + 4*a*c)*a
rctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 8*(b^8*c - 23*a*b^6*
c^2 + 250*a^2*b^4*c^3 - 417*a^3*b^2*c^4 - 1116*a^4*c^5)*x)/(a^4*b^10 - 20*a
^5*b^8*c + 160*a^6*b^6*c^2 - 640*a^7*b^4*c^3 + 1280*a^8*b^2*c^4 - 1024*a^9*
c^5 + (b^10*c^4 - 20*a*b^8*c^5 + 160*a^2*b^6*c^6 - 640*a^3*b^4*c^7 + 1280*a
```

```
^4*b^2*c^8 - 1024*a^5*c^9)*x^8 + 4*(b^11*c^3 - 20*a*b^9*c^4 + 160*a^2*b^7*c
^5 - 640*a^3*b^5*c^6 + 1280*a^4*b^3*c^7 - 1024*a^5*b*c^8)*x^7 + 2*(3*b^12*c
^2 - 58*a*b^10*c^3 + 440*a^2*b^8*c^4 - 1600*a^3*b^6*c^5 + 2560*a^4*b^4*c^6
- 512*a^5*b^2*c^7 - 2048*a^6*c^8)*x^6 + 4*(b^13*c - 17*a*b^11*c^2 + 100*a^2
*b^9*c^3 - 160*a^3*b^7*c^4 - 640*a^4*b^5*c^5 + 2816*a^5*b^3*c^6 - 3072*a^6*
b*c^7)*x^5 + (b^14 - 8*a*b^12*c - 74*a^2*b^10*c^2 + 1160*a^3*b^8*c^3 - 5440
*a^4*b^6*c^4 + 10496*a^5*b^4*c^5 - 4608*a^6*b^2*c^6 - 6144*a^7*c^7)*x^4 + 4
*(a*b^13 - 17*a^2*b^11*c + 100*a^3*b^9*c^2 - 160*a^4*b^7*c^3 - 640*a^5*b^5*
c^4 + 2816*a^6*b^3*c^5 - 3072*a^7*b*c^6)*x^3 + 2*(3*a^2*b^12 - 58*a^3*b^10*
c + 440*a^4*b^8*c^2 - 1600*a^5*b^6*c^3 + 2560*a^6*b^4*c^4 - 512*a^7*b^2*c^5
 - 2048*a^8*c^6)*x^2 + 4*(a^3*b^11 - 20*a^4*b^9*c + 160*a^5*b^7*c^2 - 640*a
^6*b^5*c^3 + 1280*a^7*b^3*c^4 - 1024*a^8*b*c^5)*x)]
```

Sympy [B] time = 4.93017, size = 1153, normalized size = 6.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**5,x)
```

```
[Out] -70*c**4*sqrt(-1/(4*a*c - b**2)**9)*log(x + (-71680*a**5*c**9*sqrt(-1/(4*a*
c - b**2)**9) + 89600*a**4*b**2*c**8*sqrt(-1/(4*a*c - b**2)**9) - 44800*a**
3*b**4*c**7*sqrt(-1/(4*a*c - b**2)**9) + 11200*a**2*b**6*c**6*sqrt(-1/(4*a*
c - b**2)**9) - 1400*a*b**8*c**5*sqrt(-1/(4*a*c - b**2)**9) + 70*b**10*c**4
*sqrt(-1/(4*a*c - b**2)**9) + 70*b*c**4)/(140*c**5)) + 70*c**4*sqrt(-1/(4*a
*c - b**2)**9)*log(x + (71680*a**5*c**9*sqrt(-1/(4*a*c - b**2)**9) - 89600*
a**4*b**2*c**8*sqrt(-1/(4*a*c - b**2)**9) + 44800*a**3*b**4*c**7*sqrt(-1/(4
*a*c - b**2)**9) - 11200*a**2*b**6*c**6*sqrt(-1/(4*a*c - b**2)**9) + 1400*a
*b**8*c**5*sqrt(-1/(4*a*c - b**2)**9) - 70*b**10*c**4*sqrt(-1/(4*a*c - b**2
)**9) + 70*b*c**4)/(140*c**5)) + (1116*a**3*b*c**3 - 326*a**2*b**3*c**2 + 5
0*a*b**5*c - 3*b**7 + 2940*b*c**6*x**6 + 840*c**7*x**7 + x**5*(3080*a*c**6
+ 3640*b**2*c**5) + x**4*(7700*a*b*c**5 + 1750*b**3*c**4) + x**3*(4088*a**2
*c**5 + 5656*a*b**2*c**4 + 168*b**4*c**3) + x**2*(6132*a**2*b*c**4 + 784*a*
b**3*c**3 - 28*b**5*c**2) + x*(2232*a**3*c**4 + 1392*a**2*b**2*c**3 - 152*a
*b**4*c**2 + 8*b**6*c))/(3072*a**8*c**4 - 3072*a**7*b**2*c**3 + 1152*a**6*b
**4*c**2 - 192*a**5*b**6*c + 12*a**4*b**8 + x**8*(3072*a**4*c**8 - 3072*a**
3*b**2*c**7 + 1152*a**2*b**4*c**6 - 192*a*b**6*c**5 + 12*b**8*c**4) + x**7*
(12288*a**4*b*c**7 - 12288*a**3*b**3*c**6 + 4608*a**2*b**5*c**5 - 768*a*b**
7*c**4 + 48*b**9*c**3) + x**6*(12288*a**5*c**7 + 6144*a**4*b**2*c**6 - 1382
4*a**3*b**4*c**5 + 6144*a**2*b**6*c**4 - 1104*a*b**8*c**3 + 72*b**10*c**2)
+ x**5*(36864*a**5*b*c**6 - 24576*a**4*b**3*c**5 + 1536*a**3*b**5*c**4 + 23
04*a**2*b**7*c**3 - 624*a*b**9*c**2 + 48*b**11*c) + x**4*(18432*a**6*c**6 +
 18432*a**5*b**2*c**5 - 26880*a**4*b**4*c**4 + 9600*a**3*b**6*c**3 - 1080*a
**2*b**8*c**2 - 48*a*b**10*c + 12*b**12) + x**3*(36864*a**6*b*c**5 - 24576*
a**5*b**3*c**4 + 1536*a**4*b**5*c**3 + 2304*a**3*b**7*c**2 - 624*a**2*b**9*
c + 48*a*b**11) + x**2*(12288*a**7*c**5 + 6144*a**6*b**2*c**4 - 13824*a**5*
b**4*c**3 + 6144*a**4*b**6*c**2 - 1104*a**3*b**8*c + 72*a**2*b**10) + x*(12
288*a**7*b*c**4 - 12288*a**6*b**3*c**3 + 4608*a**5*b**5*c**2 - 768*a**4*b**
7*c + 48*a**3*b**9))
```

Giac [B] time = 1.09955, size = 454, normalized size = 2.65

$$\frac{140\,c^4 \arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^8-16\,ab^6c+96\,a^2b^4c^2-256\,a^3b^2c^3+256\,a^4c^4\right)\sqrt{-b^2+4\,ac}}+\frac{840\,c^7x^7+2940\,bc^6x^6+3640\,b^2c^5x^5+3080\,ac^6x^5+1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^5,x, algorithm="giac")
```

```
[Out] 140*c^4*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^8 - 16*a*b^6*c + 96*a^2*
b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4)*sqrt(-b^2 + 4*a*c)) + 1/12*(840*c^
7*x^7 + 2940*b*c^6*x^6 + 3640*b^2*c^5*x^5 + 3080*a*c^6*x^5 + 1750*b^3*c^4*x
^4 + 7700*a*b*c^5*x^4 + 168*b^4*c^3*x^3 + 5656*a*b^2*c^4*x^3 + 4088*a^2*c^5
*x^3 - 28*b^5*c^2*x^2 + 784*a*b^3*c^3*x^2 + 6132*a^2*b*c^4*x^2 + 8*b^6*c*x
- 152*a*b^4*c^2*x + 1392*a^2*b^2*c^3*x + 2232*a^3*c^4*x - 3*b^7 + 50*a*b^5*
c - 326*a^2*b^3*c^2 + 1116*a^3*b*c^3)/((b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 -
 256*a^3*b^2*c^3 + 256*a^4*c^4)*(c*x^2 + b*x + a)^4)
```

3.2227 $\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^5}\,dx$

Optimal. Leaf size=1324

result too large to display

```
[Out] -(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)/(4*(b^2 - 4*a*c)*(c*d^2 - b*
d*e + a*e^2)*(a + b*x + c*x^2)^4) - (7*a*c*e*(2*c*d - b*e)^2 - (b*c*d - b^2
*e + 2*a*c*e)*(14*c^2*d^2 - 4*b^2*e^2 - c*e*(7*b*d - 16*a*e)) - 2*c*(2*c*d
- b*e)*(7*c^2*d^2 - 2*b^2*e^2 - c*e*(7*b*d - 15*a*e))*x)/(12*(b^2 - 4*a*c)^
2*(c*d^2 - b*d*e + a*e^2)^2*(a + b*x + c*x^2)^3) + (5*a*c*e*(2*c*d - b*e)^2
*(7*c^2*d^2 - 2*b^2*e^2 - c*e*(7*b*d - 15*a*e)) - (b*c*d - b^2*e + 2*a*c*e)
*(70*c^4*d^4 + 6*b^4*e^4 + 2*b^2*c*e^3*(5*b*d - 24*a*e) - 15*c^3*d^2*e*(7*b
*d - 10*a*e) + 3*c^2*e^2*(5*b^2*d^2 - 25*a*b*d*e + 32*a^2*e^2)) - 2*c*(2*c*
d - b*e)*(35*c^4*d^4 + 3*b^4*e^4 + 2*b^2*c*e^3*(5*b*d - 17*a*e) - 10*c^3*d^
2*e*(7*b*d - 11*a*e) + c^2*e^2*(25*b^2*d^2 - 110*a*b*d*e + 123*a^2*e^2))*x)
/(12*(b^2 - 4*a*c)^3*(c*d^2 - b*d*e + a*e^2)^3*(a + b*x + c*x^2)^2) + (b^7*
c*d*e^6 + 2*b^8*e^7 + 256*a^4*c^4*e^7 + b^6*c*e^5*(c*d^2 - 31*a*e^2) + b^5*
c^2*d*e^4*(c*d^2 - 14*a*e^2) - b^4*c^2*e^3*(125*c^2*d^4 + 13*a*c*d^2*e^2 -
178*a^2*e^4) + b^3*c^3*d*e^2*(295*c^2*d^4 + 492*a*c*d^2*e^2 + 69*a^2*e^4) +
 2*b*c^4*d*(35*c^3*d^6 + 145*a*c^2*d^4*e^2 + 233*a^2*c*d^2*e^4 + 187*a^3*e^
6) - b^2*c^3*e*(245*c^3*d^6 + 725*a*c^2*d^4*e^2 + 699*a^2*c*d^2*e^4 + 443*a
^3*e^6) + 2*c*(2*c*d - b*e)*(35*c^6*d^6 - b^6*e^6 - 5*c^5*d^4*e*(21*b*d - 2
9*a*e) - 3*b^4*c*e^5*(b*d - 5*a*e) - b^2*c^2*e^4*(7*b^2*d^2 - 44*a*b*d*e +
82*a^2*e^2) + c^4*d^2*e^2*(95*b^2*d^2 - 290*a*b*d*e + 233*a^2*e^2) - c^3*e^
3*(15*b^3*d^3 - 101*a*b^2*d^2*e + 233*a^2*b*d*e^2 - 187*a^3*e^3))*x)/(2*(b^
2 - 4*a*c)^4*(c*d^2 - b*d*e + a*e^2)^4*(a + b*x + c*x^2)) - ((140*c^9*d^9 -
 b^9*e^9 + 18*a*b^7*c*e^9 - 126*a^2*b^5*c^2*e^9 + 420*a^3*b^3*c^3*e^9 - 630
*a^4*b*c^4*e^9 - 90*c^8*d^7*e*(7*b*d - 8*a*e) + 72*c^7*d^5*e^2*(15*b^2*d^2
- 35*a*b*d*e + 21*a^2*e^2) - 84*c^6*d^3*e^3*(10*b^3*d^3 - 36*a*b^2*d^2*e +
45*a^2*b*d*e^2 - 20*a^3*e^3) + 252*c^5*d*e^4*(b^4*d^4 - 5*a*b^3*d^3*e + 10*
a^2*b^2*d^2*e^2 - 10*a^3*b*d*e^3 + 5*a^4*e^4))*ArcTanh[(b + 2*c*x)/Sqrt[b^2
 - 4*a*c]])/((b^2 - 4*a*c)^(9/2)*(c*d^2 - e*(b*d - a*e))^5) + (e^9*Log[d +
e*x])/(c*d^2 - b*d*e + a*e^2)^5 - (e^9*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*
d*e + a*e^2)^5)
```

Rubi [A] time = 13.9008, antiderivative size = 1324, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {740, 822, 800, 634, 618, 206, 628}

$$\frac{\log(d+ex)e^9}{\left(cd^2-bed+ae^2\right)^5}-\frac{\log\left(cx^2+bx+a\right)e^9}{2\left(cd^2-bed+ae^2\right)^5}-\frac{\left(140c^9d^9-90c^8e(7bd-8ae)d^7+72c^7e^2\left(15b^2d^2-35abed+21a^2e^2\right)d^5-84\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + b*x + c*x^2)^5),x]
```


```
[Out] -(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)/(4*(b^2 - 4*a*c)*(c*d^2 - b*
d*e + a*e^2)*(a + b*x + c*x^2)^4) - (7*a*c*e*(2*c*d - b*e)^2 - (b*c*d - b^2
*e + 2*a*c*e)*(14*c^2*d^2 - 4*b^2*e^2 - c*e*(7*b*d - 16*a*e)) - 2*c*(2*c*d
- b*e)*(7*c^2*d^2 - 2*b^2*e^2 - c*e*(7*b*d - 15*a*e))*x)/(12*(b^2 - 4*a*c)^
2*(c*d^2 - b*d*e + a*e^2)^2*(a + b*x + c*x^2)^3) + (5*a*c*e*(2*c*d - b*e)^2
*(7*c^2*d^2 - 2*b^2*e^2 - c*e*(7*b*d - 15*a*e)) - (b*c*d - b^2*e + 2*a*c*e)
*(70*c^4*d^4 + 6*b^4*e^4 + 2*b^2*c*e^3*(5*b*d - 24*a*e) - 15*c^3*d^2*e*(7*b
```

```
*d - 10*a*e) + 3*c^2*e^2*(5*b^2*d^2 - 25*a*b*d*e + 32*a^2*e^2)) - 2*c*(2*c*
d - b*e)*(35*c^4*d^4 + 3*b^4*e^4 + 2*b^2*c*e^3*(5*b*d - 17*a*e) - 10*c^3*d^
2*e*(7*b*d - 11*a*e) + c^2*e^2*(25*b^2*d^2 - 110*a*b*d*e + 123*a^2*e^2))*x)
/(12*(b^2 - 4*a*c)^3*(c*d^2 - b*d*e + a*e^2)^3*(a + b*x + c*x^2)^2) + (b^7*
c*d*e^6 + 2*b^8*e^7 + 256*a^4*c^4*e^7 + b^6*c*e^5*(c*d^2 - 31*a*e^2) + b^5*
c^2*d*e^4*(c*d^2 - 14*a*e^2) - b^4*c^2*e^3*(125*c^2*d^4 + 13*a*c*d^2*e^2 -
178*a^2*e^4) + b^3*c^3*d*e^2*(295*c^2*d^4 + 492*a*c*d^2*e^2 + 69*a^2*e^4) +
 2*b*c^4*d*(35*c^3*d^6 + 145*a*c^2*d^4*e^2 + 233*a^2*c*d^2*e^4 + 187*a^3*e^
6) - b^2*c^3*e*(245*c^3*d^6 + 725*a*c^2*d^4*e^2 + 699*a^2*c*d^2*e^4 + 443*a
^3*e^6) + 2*c*(2*c*d - b*e)*(35*c^6*d^6 - b^6*e^6 - 5*c^5*d^4*e*(21*b*d - 2
9*a*e) - 3*b^4*c*e^5*(b*d - 5*a*e) - b^2*c^2*e^4*(7*b^2*d^2 - 44*a*b*d*e +
82*a^2*e^2) + c^4*d^2*e^2*(95*b^2*d^2 - 290*a*b*d*e + 233*a^2*e^2) - c^3*e^
3*(15*b^3*d^3 - 101*a*b^2*d^2*e + 233*a^2*b*d*e^2 - 187*a^3*e^3))*x)/(2*(b^
2 - 4*a*c)^4*(c*d^2 - b*d*e + a*e^2)^4*(a + b*x + c*x^2)) - ((140*c^9*d^9 -
 b^9*e^9 + 18*a*b^7*c*e^9 - 126*a^2*b^5*c^2*e^9 + 420*a^3*b^3*c^3*e^9 - 630
*a^4*b*c^4*e^9 - 90*c^8*d^7*e*(7*b*d - 8*a*e) + 72*c^7*d^5*e^2*(15*b^2*d^2
- 35*a*b*d*e + 21*a^2*e^2) - 84*c^6*d^3*e^3*(10*b^3*d^3 - 36*a*b^2*d^2*e +
45*a^2*b*d*e^2 - 20*a^3*e^3) + 252*c^5*d*e^4*(b^4*d^4 - 5*a*b^3*d^3*e + 10*
a^2*b^2*d^2*e^2 - 10*a^3*b*d*e^3 + 5*a^4*e^4))*ArcTanh[(b + 2*c*x)/Sqrt[b^2
 - 4*a*c]])/((b^2 - 4*a*c)^(9/2)*(c*d^2 - e*(b*d - a*e))^5) + (e^9*Log[d +
e*x])/(c*d^2 - b*d*e + a*e^2)^5 - (e^9*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*
d*e + a*e^2)^5)
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
```

```
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^5}\,dx = -\frac{bcd-b^2e+2ace+c(2cd-be)x}{4\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^4} - \frac{\int \frac{14c^2d^2-4b^2e^2-ce(7bd-16ae)+7ce(2cd-be)x}{(d+ex)\left(a+bx+cx^2\right)^4}\,dx}{4\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{4\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^4} - \frac{7ace(2cd-be)^2-\left(bcd-b^2e+2ace\right)(}{12}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{4\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^4} - \frac{7ace(2cd-be)^2-\left(bcd-b^2e+2ace\right)(}{12}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{4\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^4} - \frac{7ace(2cd-be)^2-\left(bcd-b^2e+2ace\right)(}{12}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{4\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^4} - \frac{7ace(2cd-be)^2-\left(bcd-b^2e+2ace\right)(}{12}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{4\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^4} - \frac{7ace(2cd-be)^2-\left(bcd-b^2e+2ace\right)(}{12}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{4\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^4} - \frac{7ace(2cd-be)^2-\left(bcd-b^2e+2ace\right)(}{12}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{4\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^4} - \frac{7ace(2cd-be)^2-\left(bcd-b^2e+2ace\right)(}{12}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{4\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^4} - \frac{7ace(2cd-be)^2-\left(bcd-b^2e+2ace\right)(}{12}$$

Mathematica [A] time = 6.58957, size = 1601, normalized size = 1.21

$$\frac{\log(d+ex)e^9}{\left(cd^2-bed+ae^2\right)^5}-\frac{\log\left(cx^2+bx+a\right)e^9}{2\left(cd^2-bed+ae^2\right)^5}+\frac{\left(-140c^9d^9+630bc^8ed^8-720ac^8e^2d^7-1080b^2c^7e^2d^7+2520abc^7e^3d^6+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + b*x + c*x^2)^5),x]
```

```
[Out] (b*c*d - b^2*e + 2*a*c*e + 2*c^2*d*x - b*c*e*x)/(4*(-b^2 + 4*a*c)*(c*d^2 -
b*d*e + a*e^2)*(a + b*x + c*x^2)^4) + (14*b*c^3*d^3 - 21*b^2*c^2*d^2*e + 3*
b^3*c*d*e^2 + 30*a*b*c^2*d*e^2 + 4*b^4*e^3 - 31*a*b^2*c*e^3 + 32*a^2*c^2*e^
3 + 28*c^4*d^3*x - 42*b*c^3*d^2*e*x + 6*b^2*c^2*d*e^2*x + 60*a*c^3*d*e^2*x
+ 4*b^3*c*e^3*x - 30*a*b*c^2*e^3*x)/(12*(-b^2 + 4*a*c)^2*(c*d^2 - b*d*e + a
*e^2)^2*(a + b*x + c*x^2)^3) + (70*b*c^5*d^5 - 175*b^2*c^4*d^4*e + 120*b^3*
c^3*d^3*e^2 + 220*a*b*c^4*d^3*e^2 - 5*b^4*c^2*d^2*e^3 - 330*a*b^2*c^3*d^2*e
^3 - 4*b^5*c*d*e^4 + 42*a*b^3*c^2*d*e^4 + 246*a^2*b*c^3*d*e^4 - 6*b^6*e^5 +
 70*a*b^4*c*e^5 - 267*a^2*b^2*c^2*e^5 + 192*a^3*c^3*e^5 + 140*c^6*d^5*x - 3
50*b*c^5*d^4*e*x + 240*b^2*c^4*d^3*e^2*x + 440*a*c^5*d^3*e^2*x - 10*b^3*c^3
*d^2*e^3*x - 660*a*b*c^4*d^2*e^3*x - 8*b^4*c^2*d*e^4*x + 84*a*b^2*c^3*d*e^4
*x + 492*a^2*c^4*d*e^4*x - 6*b^5*c*e^5*x + 68*a*b^3*c^2*e^5*x - 246*a^2*b*c
^3*e^5*x)/(12*(-b^2 + 4*a*c)^3*(c*d^2 - b*d*e + a*e^2)^3*(a + b*x + c*x^2)^
2) + (70*b*c^7*d^7 - 245*b^2*c^6*d^6*e + 295*b^3*c^5*d^5*e^2 + 290*a*b*c^6*
d^5*e^2 - 125*b^4*c^4*d^4*e^3 - 725*a*b^2*c^5*d^4*e^3 + b^5*c^3*d^3*e^4 + 4
92*a*b^3*c^4*d^3*e^4 + 466*a^2*b*c^5*d^3*e^4 + b^6*c^2*d^2*e^5 - 13*a*b^4*c
^3*d^2*e^5 - 699*a^2*b^2*c^4*d^2*e^5 + b^7*c*d*e^6 - 14*a*b^5*c^2*d*e^6 + 6
9*a^2*b^3*c^3*d*e^6 + 374*a^3*b*c^4*d*e^6 + 2*b^8*e^7 - 31*a*b^6*c*e^7 + 17
8*a^2*b^4*c^2*e^7 - 443*a^3*b^2*c^3*e^7 + 256*a^4*c^4*e^7 + 140*c^8*d^7*x -
 490*b*c^7*d^6*e*x + 590*b^2*c^6*d^5*e^2*x + 580*a*c^7*d^5*e^2*x - 250*b^3*
c^5*d^4*e^3*x - 1450*a*b*c^6*d^4*e^3*x + 2*b^4*c^4*d^3*e^4*x + 984*a*b^2*c^
5*d^3*e^4*x + 932*a^2*c^6*d^3*e^4*x + 2*b^5*c^3*d^2*e^5*x - 26*a*b^3*c^4*d^
2*e^5*x - 1398*a^2*b*c^5*d^2*e^5*x + 2*b^6*c^2*d*e^6*x - 28*a*b^4*c^3*d*e^6
*x + 138*a^2*b^2*c^4*d*e^6*x + 748*a^3*c^5*d*e^6*x + 2*b^7*c*e^7*x - 30*a*b
^5*c^2*e^7*x + 164*a^2*b^3*c^3*e^7*x - 374*a^3*b*c^4*e^7*x)/(2*(-b^2 + 4*a*
c)^4*(c*d^2 - b*d*e + a*e^2)^4*(a + b*x + c*x^2)) + ((-140*c^9*d^9 + 630*b*
c^8*d^8*e - 1080*b^2*c^7*d^7*e^2 - 720*a*c^8*d^7*e^2 + 840*b^3*c^6*d^6*e^3
+ 2520*a*b*c^7*d^6*e^3 - 252*b^4*c^5*d^5*e^4 - 3024*a*b^2*c^6*d^5*e^4 - 151
2*a^2*c^7*d^5*e^4 + 1260*a*b^3*c^5*d^4*e^5 + 3780*a^2*b*c^6*d^4*e^5 - 2520*
a^2*b^2*c^5*d^3*e^6 - 1680*a^3*c^6*d^3*e^6 + 2520*a^3*b*c^5*d^2*e^7 - 1260*
a^4*c^5*d*e^8 + b^9*e^9 - 18*a*b^7*c*e^9 + 126*a^2*b^5*c^2*e^9 - 420*a^3*b^
3*c^3*e^9 + 630*a^4*b*c^4*e^9)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/((b^
2 - 4*a*c)^4*Sqrt[-b^2 + 4*a*c]*(-(c*d^2) + b*d*e - a*e^2)^5) + (e^9*Log[d
+ e*x])/(c*d^2 - b*d*e + a*e^2)^5 - (e^9*Log[a + b*x + c*x^2])/(2*(c*d^2 -
b*d*e + a*e^2)^5)
```

Maple [B] time = 0.213, size = 32834, normalized size = 24.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+b*x+a)^5,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.39818, size = 8597, normalized size = 6.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/2*e^9*log(c*x^2 + b*x + a)/(c^5*d^10 - 5*b*c^4*d^9*e + 10*b^2*c^3*d^8*e^
2 + 5*a*c^4*d^8*e^2 - 10*b^3*c^2*d^7*e^3 - 20*a*b*c^3*d^7*e^3 + 5*b^4*c*d^6
*e^4 + 30*a*b^2*c^2*d^6*e^4 + 10*a^2*c^3*d^6*e^4 - b^5*d^5*e^5 - 20*a*b^3*c
*d^5*e^5 - 30*a^2*b*c^2*d^5*e^5 + 5*a*b^4*d^4*e^6 + 30*a^2*b^2*c*d^4*e^6 +
10*a^3*c^2*d^4*e^6 - 10*a^2*b^3*d^3*e^7 - 20*a^3*b*c*d^3*e^7 + 10*a^3*b^2*d
^2*e^8 + 5*a^4*c*d^2*e^8 - 5*a^4*b*d*e^9 + a^5*e^10) + e^10*log(abs(x*e + d
))/(c^5*d^10*e - 5*b*c^4*d^9*e^2 + 10*b^2*c^3*d^8*e^3 + 5*a*c^4*d^8*e^3 - 1
0*b^3*c^2*d^7*e^4 - 20*a*b*c^3*d^7*e^4 + 5*b^4*c*d^6*e^5 + 30*a*b^2*c^2*d^6
*e^5 + 10*a^2*c^3*d^6*e^5 - b^5*d^5*e^6 - 20*a*b^3*c*d^5*e^6 - 30*a^2*b*c^2
*d^5*e^6 + 5*a*b^4*d^4*e^7 + 30*a^2*b^2*c*d^4*e^7 + 10*a^3*c^2*d^4*e^7 - 10
*a^2*b^3*d^3*e^8 - 20*a^3*b*c*d^3*e^8 + 10*a^3*b^2*d^2*e^9 + 5*a^4*c*d^2*e^
9 - 5*a^4*b*d*e^10 + a^5*e^11) + (140*c^9*d^9 - 630*b*c^8*d^8*e + 1080*b^2*
```

c^7*d^7*e^2 + 720*a*c^8*d^7*e^2 - 840*b^3*c^6*d^6*e^3 - 2520*a*b*c^7*d^6*e^
3 + 252*b^4*c^5*d^5*e^4 + 3024*a*b^2*c^6*d^5*e^4 + 1512*a^2*c^7*d^5*e^4 - 1
260*a*b^3*c^5*d^4*e^5 - 3780*a^2*b*c^6*d^4*e^5 + 2520*a^2*b^2*c^5*d^3*e^6 +
 1680*a^3*c^6*d^3*e^6 - 2520*a^3*b*c^5*d^2*e^7 + 1260*a^4*c^5*d*e^8 - b^9*e
^9 + 18*a*b^7*c*e^9 - 126*a^2*b^5*c^2*e^9 + 420*a^3*b^3*c^3*e^9 - 630*a^4*b
*c^4*e^9)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^8*c^5*d^10 - 16*a*b^6*
c^6*d^10 + 96*a^2*b^4*c^7*d^10 - 256*a^3*b^2*c^8*d^10 + 256*a^4*c^9*d^10 -
5*b^9*c^4*d^9*e + 80*a*b^7*c^5*d^9*e - 480*a^2*b^5*c^6*d^9*e + 1280*a^3*b^3
*c^7*d^9*e - 1280*a^4*b*c^8*d^9*e + 10*b^10*c^3*d^8*e^2 - 155*a*b^8*c^4*d^8
*e^2 + 880*a^2*b^6*c^5*d^8*e^2 - 2080*a^3*b^4*c^6*d^8*e^2 + 1280*a^4*b^2*c^
7*d^8*e^2 + 1280*a^5*c^8*d^8*e^2 - 10*b^11*c^2*d^7*e^3 + 140*a*b^9*c^3*d^7*
e^3 - 640*a^2*b^7*c^4*d^7*e^3 + 640*a^3*b^5*c^5*d^7*e^3 + 2560*a^4*b^3*c^6*
d^7*e^3 - 5120*a^5*b*c^7*d^7*e^3 + 5*b^12*c*d^6*e^4 - 50*a*b^10*c^2*d^6*e^4
 + 10*a^2*b^8*c^3*d^6*e^4 + 1440*a^3*b^6*c^4*d^6*e^4 - 5440*a^4*b^4*c^5*d^6
*e^4 + 5120*a^5*b^2*c^6*d^6*e^4 + 2560*a^6*c^7*d^6*e^4 - b^13*d^5*e^5 - 4*a
*b^11*c*d^5*e^5 + 194*a^2*b^9*c^2*d^5*e^5 - 1184*a^3*b^7*c^3*d^5*e^5 + 1984
*a^4*b^5*c^4*d^5*e^5 + 2560*a^5*b^3*c^5*d^5*e^5 - 7680*a^6*b*c^6*d^5*e^5 +
5*a*b^12*d^4*e^6 - 50*a^2*b^10*c*d^4*e^6 + 10*a^3*b^8*c^2*d^4*e^6 + 1440*a^
4*b^6*c^3*d^4*e^6 - 5440*a^5*b^4*c^4*d^4*e^6 + 5120*a^6*b^2*c^5*d^4*e^6 + 2
560*a^7*c^6*d^4*e^6 - 10*a^2*b^11*d^3*e^7 + 140*a^3*b^9*c*d^3*e^7 - 640*a^4
*b^7*c^2*d^3*e^7 + 640*a^5*b^5*c^3*d^3*e^7 + 2560*a^6*b^3*c^4*d^3*e^7 - 512
0*a^7*b*c^5*d^3*e^7 + 10*a^3*b^10*d^2*e^8 - 155*a^4*b^8*c*d^2*e^8 + 880*a^5
*b^6*c^2*d^2*e^8 - 2080*a^6*b^4*c^3*d^2*e^8 + 1280*a^7*b^2*c^4*d^2*e^8 + 12
80*a^8*c^5*d^2*e^8 - 5*a^4*b^9*d*e^9 + 80*a^5*b^7*c*d*e^9 - 480*a^6*b^5*c^2
*d*e^9 + 1280*a^7*b^3*c^3*d*e^9 - 1280*a^8*b*c^4*d*e^9 + a^5*b^8*e^10 - 16*
a^6*b^6*c*e^10 + 96*a^7*b^4*c^2*e^10 - 256*a^8*b^2*c^3*e^10 + 256*a^9*c^4*e
^10)*sqrt(-b^2 + 4*a*c)) - 1/12*(3*b^7*c^5*d^9 - 50*a*b^5*c^6*d^9 + 326*a^2
*b^3*c^7*d^9 - 1116*a^3*b*c^8*d^9 - 15*b^8*c^4*d^8*e + 249*a*b^6*c^5*d^8*e
- 1611*a^2*b^4*c^6*d^8*e + 5406*a^3*b^2*c^7*d^8*e - 384*a^4*c^8*d^8*e + 30*
b^9*c^3*d^7*e^2 - 480*a*b^7*c^4*d^7*e^2 + 2916*a^2*b^5*c^5*d^7*e^2 - 8688*a
^3*b^3*c^6*d^7*e^2 - 3984*a^4*b*c^7*d^7*e^2 - 30*b^10*c^2*d^6*e^3 + 430*a*b
^8*c^3*d^6*e^3 - 2080*a^2*b^6*c^4*d^6*e^3 + 3132*a^3*b^4*c^5*d^6*e^3 + 1906
4*a^4*b^2*c^6*d^6*e^3 - 2048*a^5*c^7*d^6*e^3 + 15*b^11*c*d^5*e^4 - 150*a*b^
9*c^2*d^5*e^4 + 4800*a^3*b^5*c^4*d^5*e^4 - 25500*a^4*b^3*c^5*d^5*e^4 - 4680
*a^5*b*c^6*d^5*e^4 - 3*b^12*d^4*e^5 - 15*a*b^10*c*d^4*e^5 + 639*a^2*b^8*c^2
*d^4*e^5 - 3984*a^3*b^6*c^3*d^4*e^5 + 7962*a^4*b^4*c^4*d^4*e^5 + 25524*a^5*
b^2*c^5*d^4*e^5 - 4608*a^6*c^6*d^4*e^5 + 16*a*b^11*d^3*e^6 - 154*a^2*b^9*c*
d^3*e^6 - 96*a^3*b^7*c^2*d^3*e^6 + 5780*a^4*b^5*c^3*d^3*e^6 - 24368*a^5*b^3
*c^4*d^3*e^6 - 1104*a^6*b*c^5*d^3*e^6 - 36*a^2*b^10*d^2*e^7 + 498*a^3*b^8*c
*d^2*e^7 - 2172*a^4*b^6*c^2*d^2*e^7 + 1044*a^5*b^4*c^3*d^2*e^7 + 17016*a^6*
b^2*c^4*d^2*e^7 - 6144*a^7*c^5*d^2*e^7 + 48*a^3*b^9*d*e^8 - 741*a^4*b^7*c*d
*e^8 + 4158*a^5*b^5*c^2*d*e^8 - 9354*a^6*b^3*c^3*d*e^8 + 2244*a^7*b*c^4*d*e
^8 - 25*a^4*b^8*e^9 + 385*a^5*b^6*c*e^9 - 2175*a^6*b^4*c^2*e^9 + 5150*a^7*b
^2*c^3*e^9 - 3200*a^8*c^4*e^9 - 12*(70*c^12*d^9 - 315*b*c^11*d^8*e + 540*b^
2*c^10*d^7*e^2 + 360*a*c^11*d^7*e^2 - 420*b^3*c^9*d^6*e^3 - 1260*a*b*c^10*d
^6*e^3 + 126*b^4*c^8*d^5*e^4 + 1512*a*b^2*c^9*d^5*e^4 + 756*a^2*c^10*d^5*e^
4 - 630*a*b^3*c^8*d^4*e^5 - 1890*a^2*b*c^9*d^4*e^5 + 1260*a^2*b^2*c^8*d^3*e
^6 + 840*a^3*c^9*d^3*e^6 - 1260*a^3*b*c^8*d^2*e^7 - b^8*c^4*d*e^8 + 16*a*b^
6*c^5*d*e^8 - 96*a^2*b^4*c^6*d*e^8 + 256*a^3*b^2*c^7*d*e^8 + 374*a^4*c^8*d*
e^8 + a*b^7*c^4*e^9 - 15*a^2*b^5*c^5*e^9 + 82*a^3*b^3*c^6*e^9 - 187*a^4*b*c
^7*e^9)*x^7 - 6*(490*b*c^11*d^9 - 2205*b^2*c^10*d^8*e + 3780*b^3*c^9*d^7*e^
2 + 2520*a*b*c^10*d^7*e^2 - 2940*b^4*c^8*d^6*e^3 - 8820*a*b^2*c^9*d^6*e^3 +
 882*b^5*c^7*d^5*e^4 + 10584*a*b^3*c^8*d^5*e^4 + 5292*a^2*b*c^9*d^5*e^4 - 4
410*a*b^4*c^7*d^4*e^5 - 13230*a^2*b^2*c^8*d^4*e^5 + 8820*a^2*b^3*c^7*d^3*e^
6 + 5880*a^3*b*c^8*d^3*e^6 + b^8*c^4*d^2*e^7 - 16*a*b^6*c^5*d^2*e^7 + 96*a^
2*b^4*c^6*d^2*e^7 - 9076*a^3*b^2*c^7*d^2*e^7 + 256*a^4*c^8*d^2*e^7 - 8*b^9*
c^3*d*e^8 + 128*a*b^7*c^4*d*e^8 - 768*a^2*b^5*c^5*d*e^8 + 2048*a^3*b^3*c^6*
d*e^8 + 2362*a^4*b*c^7*d*e^8 + 8*a*b^8*c^3*e^9 - 121*a^2*b^6*c^4*e^9 + 670*
a^3*b^4*c^5*e^9 - 1565*a^4*b^2*c^6*e^9 + 256*a^5*c^7*e^9)*x^6 - 4*(910*b^2*

c^10*d^9 + 770*a*c^11*d^9 - 4095*b^3*c^9*d^8*e - 3465*a*b*c^10*d^8*e + 7020
*b^4*c^8*d^7*e^2 + 10620*a*b^2*c^9*d^7*e^2 + 3960*a^2*c^10*d^7*e^2 - 5460*b
^5*c^7*d^6*e^3 - 21000*a*b^3*c^8*d^6*e^3 - 13860*a^2*b*c^9*d^6*e^3 + 1638*b
^6*c^6*d^5*e^4 + 21042*a*b^4*c^7*d^5*e^4 + 26460*a^2*b^2*c^8*d^5*e^4 + 8316
*a^3*c^9*d^5*e^4 - 8190*a*b^5*c^6*d^4*e^5 - 31500*a^2*b^3*c^7*d^4*e^5 - 207
90*a^3*b*c^8*d^4*e^5 - b^8*c^4*d^3*e^6 + 16*a*b^6*c^5*d^3*e^6 + 16284*a^2*b
^4*c^6*d^3*e^6 + 25036*a^3*b^2*c^7*d^3*e^6 + 8984*a^4*c^8*d^3*e^6 + 6*b^9*c
^3*d^2*e^7 - 96*a*b^7*c^4*d^2*e^7 + 576*a^2*b^5*c^5*d^2*e^7 - 17916*a^3*b^3
*c^6*d^2*e^7 - 12324*a^4*b*c^7*d^2*e^7 - 18*b^10*c^2*d*e^8 + 276*a*b^8*c^3*
d*e^8 - 1536*a^2*b^6*c^4*d*e^8 + 3456*a^3*b^4*c^5*d*e^8 + 6654*a^4*b^2*c^6*
d*e^8 + 3858*a^5*c^7*d*e^8 + 18*a*b^9*c^2*e^9 - 264*a^2*b^7*c^3*e^9 + 1381*
a^3*b^5*c^4*e^9 - 2809*a^4*b^3*c^5*e^9 - 777*a^5*b*c^6*e^9)*x^5 - (1750*b^3
*c^9*d^9 + 7700*a*b*c^10*d^9 - 7875*b^4*c^8*d^8*e - 34650*a*b^2*c^9*d^8*e +
13500*b^5*c^7*d^7*e^2 + 68400*a*b^3*c^8*d^7*e^2 + 39600*a^2*b*c^9*d^7*e^2
- 10500*b^6*c^6*d^6*e^3 - 77700*a*b^4*c^7*d^6*e^3 - 138600*a^2*b^2*c^8*d^6*
e^3 + 3150*b^7*c^5*d^5*e^4 + 51660*a*b^5*c^6*d^5*e^4 + 185220*a^2*b^3*c^7*d
^5*e^4 + 83160*a^3*b*c^8*d^5*e^4 + 3*b^8*c^4*d^4*e^5 - 15798*a*b^6*c^5*d^4*
e^5 - 116262*a^2*b^4*c^6*d^4*e^5 - 208668*a^3*b^2*c^7*d^4*e^5 + 768*a^4*c^8
*d^4*e^5 - 16*b^9*c^3*d^3*e^6 + 256*a*b^7*c^4*d^3*e^6 + 29964*a^2*b^5*c^5*d
^3*e^6 + 163696*a^3*b^3*c^6*d^3*e^6 + 88304*a^4*b*c^7*d^3*e^6 + 36*b^10*c^2
*d^2*e^7 - 552*a*b^8*c^3*d^2*e^7 + 3072*a^2*b^6*c^4*d^2*e^7 - 38412*a^3*b^4
*c^5*d^2*e^7 - 135528*a^4*b^2*c^6*d^2*e^7 + 6144*a^5*c^7*d^2*e^7 - 48*b^11*
c*d*e^8 + 624*a*b^9*c^2*d*e^8 - 2304*a^2*b^7*c^3*d*e^8 - 1536*a^3*b^5*c^4*d
*e^8 + 40326*a^4*b^3*c^5*d*e^8 + 32436*a^5*b*c^6*d*e^8 + 48*a*b^10*c*e^9 -
612*a^2*b^8*c^2*e^9 + 2272*a^3*b^6*c^3*e^9 + 473*a^4*b^4*c^4*e^9 - 20058*a^
5*b^2*c^5*e^9 + 5376*a^6*c^6*e^9)*x^4 - 4*(42*b^4*c^8*d^9 + 1414*a*b^2*c^9*
d^9 + 1022*a^2*c^10*d^9 - 189*b^5*c^7*d^8*e - 6363*a*b^3*c^8*d^8*e - 4599*a
^2*b*c^9*d^8*e + 324*b^6*c^6*d^7*e^2 + 11124*a*b^4*c^7*d^7*e^2 + 15156*a^2*
b^2*c^8*d^7*e^2 + 5256*a^3*c^9*d^7*e^2 - 252*b^7*c^5*d^6*e^3 - 9240*a*b^5*c
^6*d^6*e^3 - 31584*a^2*b^3*c^7*d^6*e^3 - 18396*a^3*b*c^8*d^6*e^3 + 75*b^8*c
^4*d^5*e^4 + 3462*a*b^6*c^5*d^5*e^4 + 32778*a^2*b^4*c^6*d^5*e^4 + 37500*a^3
*b^2*c^7*d^5*e^4 + 10884*a^4*c^8*d^5*e^4 + 3*b^9*c^3*d^4*e^5 - 426*a*b^7*c^
4*d^4*e^5 - 13572*a^2*b^5*c^5*d^4*e^5 - 48144*a^3*b^3*c^6*d^4*e^5 - 26826*a
^4*b*c^7*d^4*e^5 - 6*b^10*c^2*d^3*e^6 + 92*a*b^8*c^3*d^3*e^6 + 244*a^2*b^6*
c^4*d^3*e^6 + 27108*a^3*b^4*c^5*d^3*e^6 + 34852*a^4*b^2*c^6*d^3*e^6 + 11240
*a^5*c^7*d^3*e^6 + 6*b^11*c*d^2*e^7 - 78*a*b^9*c^2*d^2*e^7 + 288*a^2*b^7*c^
3*d^2*e^7 - 564*a^3*b^5*c^4*d^2*e^7 - 28524*a^4*b^3*c^5*d^2*e^7 - 13788*a^5
*b*c^6*d^2*e^7 - 3*b^12*d*e^8 + 12*a*b^10*c*d*e^8 + 270*a^2*b^8*c^2*d*e^8 -
2400*a^3*b^6*c^3*d*e^8 + 7098*a^4*b^4*c^4*d*e^8 + 8118*a^5*b^2*c^5*d*e^8 +
4590*a^6*c^6*d*e^8 + 3*a*b^11*e^9 - 15*a^2*b^9*c*e^9 - 204*a^3*b^7*c^2*e^9
+ 1882*a^4*b^5*c^3*e^9 - 5089*a^5*b^3*c^4*e^9 + 393*a^6*b*c^5*e^9)*x^3 + 2
*(14*b^5*c^7*d^9 - 392*a*b^3*c^8*d^9 - 3066*a^2*b*c^9*d^9 - 63*b^6*c^6*d^8*
e + 1764*a*b^4*c^7*d^8*e + 13797*a^2*b^2*c^8*d^8*e + 108*b^7*c^5*d^7*e^2 -
2952*a*b^5*c^6*d^7*e^2 - 25668*a^2*b^3*c^7*d^7*e^2 - 15768*a^3*b*c^8*d^7*e^
2 - 85*b^8*c^4*d^6*e^3 + 2116*a*b^6*c^5*d^6*e^3 + 25356*a^2*b^4*c^6*d^6*e^3
+ 55444*a^3*b^2*c^7*d^6*e^3 - 256*a^4*c^8*d^6*e^3 + 30*b^9*c^3*d^5*e^4 - 4
80*a*b^7*c^4*d^5*e^4 - 13374*a^2*b^5*c^5*d^5*e^4 - 71688*a^3*b^3*c^6*d^5*e^
4 - 31884*a^4*b*c^7*d^5*e^4 - 9*b^10*c^2*d^4*e^5 + 12*a*b^8*c^3*d^4*e^5 + 2
382*a^2*b^6*c^4*d^4*e^5 + 39906*a^3*b^4*c^5*d^4*e^5 + 82014*a^4*b^2*c^6*d^4
*e^5 - 1536*a^5*c^7*d^4*e^5 + 8*b^11*c*d^3*e^6 - 104*a*b^9*c^2*d^3*e^6 + 63
6*a^2*b^7*c^3*d^3*e^6 - 6632*a^3*b^5*c^4*d^3*e^6 - 63988*a^4*b^3*c^5*d^3*e^
6 - 30648*a^5*b*c^6*d^3*e^6 - 3*b^12*d^2*e^7 + 12*a*b^10*c*d^2*e^7 + 270*a^
2*b^8*c^2*d^2*e^7 - 2652*a^3*b^6*c^3*d^2*e^7 + 13776*a^4*b^4*c^4*d^2*e^7 +
50580*a^5*b^2*c^5*d^2*e^7 - 4608*a^6*c^6*d^2*e^7 + 24*a*b^11*d*e^8 - 312*a^
2*b^9*c*d*e^8 + 1152*a^3*b^7*c^2*d*e^8 + 894*a^4*b^5*c^3*d*e^8 - 15816*a^5*
b^3*c^4*d*e^8 - 9162*a^6*b*c^5*d*e^8 - 21*a^2*b^10*e^9 + 274*a^3*b^8*c*e^9
- 1078*a^4*b^6*c^2*e^9 + 150*a^5*b^4*c^3*e^9 + 7525*a^6*b^2*c^4*e^9 - 3328*
a^7*c^5*e^9)*x^2 - 4*(2*b^6*c^6*d^9 - 38*a*b^4*c^7*d^9 + 348*a^2*b^2*c^8*d^
9 + 558*a^3*c^9*d^9 - 9*b^7*c^5*d^8*e + 171*a*b^5*c^6*d^8*e - 1566*a^2*b^3*

c^7*d^8*e - 2511*a^3*b*c^8*d^8*e + 15*b^8*c^4*d^7*e^2 - 276*a*b^6*c^5*d^7*e
^2 + 2448*a^2*b^4*c^6*d^7*e^2 + 6204*a^3*b^2*c^7*d^7*e^2 + 2760*a^4*c^8*d^7
*e^2 - 10*b^9*c^3*d^6*e^3 + 160*a*b^7*c^4*d^6*e^3 - 1212*a^2*b^5*c^5*d^6*e^
3 - 10124*a^3*b^3*c^6*d^6*e^3 - 9532*a^4*b*c^7*d^6*e^3 + 30*a*b^8*c^3*d^5*e
^4 - 480*a^2*b^6*c^4*d^5*e^4 + 8802*a^3*b^4*c^5*d^5*e^4 + 15504*a^4*b^2*c^6
*d^5*e^4 + 5412*a^5*c^7*d^5*e^4 + 3*b^11*c*d^4*e^5 - 57*a*b^9*c^2*d^4*e^5 +
432*a^2*b^7*c^3*d^4*e^5 - 2010*a^3*b^5*c^4*d^4*e^5 - 15954*a^4*b^3*c^5*d^4
*e^5 - 12762*a^5*b*c^6*d^4*e^5 - b^12*d^3*e^6 + 4*a*b^10*c*d^3*e^6 + 126*a^
2*b^8*c^2*d^3*e^6 - 1460*a^3*b^6*c^3*d^3*e^6 + 8048*a^4*b^4*c^4*d^3*e^6 + 1
2684*a^5*b^2*c^5*d^3*e^6 + 5160*a^6*c^6*d^3*e^6 + 6*a*b^11*d^2*e^7 - 78*a^2
*b^9*c*d^2*e^7 + 252*a^3*b^7*c^2*d^2*e^7 + 876*a^4*b^5*c^3*d^2*e^7 - 9336*a
^5*b^3*c^4*d^2*e^7 - 5436*a^6*b*c^5*d^2*e^7 - 18*a^2*b^10*d*e^8 + 276*a^3*b
^8*c*d*e^8 - 1518*a^4*b^6*c^2*d*e^8 + 3114*a^5*b^4*c^3*d*e^8 + 1596*a^6*b^2
*c^4*d*e^8 + 1950*a^7*c^5*d*e^8 + 13*a^3*b^9*e^9 - 196*a^4*b^7*c*e^9 + 1068
*a^5*b^5*c^2*e^9 - 2324*a^6*b^3*c^3*e^9 + 689*a^7*b*c^4*e^9)*x)/((c*d^2 - b
*d*e + a*e^2)^5*(c*x^2 + b*x + a)^4*(b^2 - 4*a*c)^4)

3.2228 $\int \frac{1}{(d+ex)^2(a+bx+cx^2)^5} dx$

Optimal. Leaf size=1761

result too large to display

```
[Out] (5*e*(14*c^8*d^8 - b^8*e^8 - 4*c^7*d^6*e*(14*b*d - 19*a*e) + b^6*c*e^7*(b*d
 + 15*a*e) + b^4*c^2*e^6*(b^2*d^2 - 16*a*b*d*e - 82*a^2*e^2) + c^6*d^4*e^2*
(79*b^2*d^2 - 228*a*b*d*e + 176*a^2*e^2) - c^5*d^2*e^3*(41*b^3*d^3 - 197*a*
b^2*d^2*e + 352*a^2*b*d*e^2 - 244*a^3*e^3) + b^2*c^3*e^5*(b^3*d^3 - 15*a*b^
2*d^2*e + 95*a^2*b*d*e^2 + 187*a^3*e^3) + c^4*e^4*(b^4*d^4 - 14*a*b^3*d^3*e
 + 81*a^2*b^2*d^2*e^2 - 244*a^3*b*d*e^3 - 126*a^4*e^4)))/((b^2 - 4*a*c)^4*(
c*d^2 - b*d*e + a*e^2)^5*(d + e*x)) - (b*c*d - b^2*e + 2*a*c*e + c*(2*c*d -
 b*e)*x)/(4*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*(d + e*x)*(a + b*x + c*x^
2)^4) - (8*a*c*e*(2*c*d - b*e)^2 - (b*c*d - b^2*e + 2*a*c*e)*(14*c^2*d^2 -
5*b^2*e^2 - 6*c*e*(b*d - 3*a*e)) - c*(2*c*d - b*e)*(14*c^2*d^2 - 5*b^2*e^2
- 2*c*e*(7*b*d - 17*a*e))*x)/(12*(b^2 - 4*a*c)^2*(c*d^2 - b*d*e + a*e^2)^2*
(d + e*x)*(a + b*x + c*x^2)^3) + (3*a*c*e*(2*c*d - b*e)^2*(14*c^2*d^2 - 5*b
^2*e^2 - 2*c*e*(7*b*d - 17*a*e)) - (b*c*d - b^2*e + 2*a*c*e)*(70*c^4*d^4 +
10*b^4*e^4 - 2*c^3*d^2*e*(49*b*d - 78*a*e) + 5*b^2*c*e^3*(2*b*d - 15*a*e) +
 3*c^2*e^2*(b^2*d^2 - 18*a*b*d*e + 42*a^2*e^2)) - 5*c*(2*c*d - b*e)*(14*c^4
*d^4 + 2*b^4*e^4 + b^2*c*e^3*(5*b*d - 21*a*e) - 4*c^3*d^2*e*(7*b*d - 12*a*e
) + 3*c^2*e^2*(3*b^2*d^2 - 16*a*b*d*e + 22*a^2*e^2))*x)/(12*(b^2 - 4*a*c)^3
*(c*d^2 - b*d*e + a*e^2)^3*(d + e*x)*(a + b*x + c*x^2)^2) - (5*(2*a*c*e*(2*
c*d - b*e)^2*(14*c^4*d^4 + 2*b^4*e^4 + b^2*c*e^3*(5*b*d - 21*a*e) - 4*c^3*d
^2*e*(7*b*d - 12*a*e) + 3*c^2*e^2*(3*b^2*d^2 - 16*a*b*d*e + 22*a^2*e^2)) -
(b*c*d - b^2*e + 2*a*c*e)*(42*c^6*d^6 - 3*b^6*e^6 - 2*c^5*d^4*e*(49*b*d - 6
5*a*e) - 2*b^4*c*e^5*(b*d - 17*a*e) + b^2*c^2*e^4*(b^2*d^2 + 16*a*b*d*e - 1
23*a^2*e^2) + c^4*d^2*e^2*(55*b^2*d^2 - 164*a*b*d*e + 150*a^2*e^2) + 6*c^3*
e^3*(b^3*d^3 - 4*a*b^2*d^2*e - 3*a^2*b*d*e^2 + 21*a^3*e^3)) - 3*c*(2*c*d -
b*e)*(14*c^6*d^6 - b^6*e^6 - 2*c^5*d^4*e*(21*b*d - 31*a*e) - 2*b^4*c*e^5*(b
*d - 7*a*e) - b^2*c^2*e^4*(3*b^2*d^2 - 26*a*b*d*e + 69*a^2*e^2) + c^4*d^2*e
^2*(37*b^2*d^2 - 124*a*b*d*e + 114*a^2*e^2) - 2*c^3*e^3*(2*b^3*d^3 - 18*a*b
^2*d^2*e + 57*a^2*b*d*e^2 - 65*a^3*e^3))*x))/(6*(b^2 - 4*a*c)^4*(c*d^2 - b*
d*e + a*e^2)^4*(d + e*x)*(a + b*x + c*x^2)) - (5*(28*c^10*d^10 + b^10*e^10
- 20*c^9*d^8*e*(7*b*d - 9*a*e) - 252*a^4*c^5*e^9*(5*b*d + a*e) + 210*a^3*b^
2*c^4*e^9*(4*b*d + 3*a*e) - 84*a^2*b^4*c^3*e^9*(3*b*d + 5*a*e) + 18*a*b^6*c
^2*e^9*(2*b*d + 7*a*e) - 2*b^8*c*e^9*(b*d + 9*a*e) + 18*c^8*d^6*e^2*(15*b^2
*d^2 - 40*a*b*d*e + 28*a^2*e^2) - 24*c^7*d^4*e^3*(10*b^3*d^3 - 42*a*b^2*d^2
*e + 63*a^2*b*d*e^2 - 35*a^3*e^3) + 84*c^6*d^2*e^4*(b^4*d^4 - 6*a*b^3*d^3*e
 + 15*a^2*b^2*d^2*e^2 - 20*a^3*b*d*e^3 + 15*a^4*e^4))*ArcTanh[(b + 2*c*x)/S
qrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(9/2)*(c*d^2 - e*(b*d - a*e))^6) + (5*e^9
*(2*c*d - b*e)*Log[d + e*x])/(c*d^2 - b*d*e + a*e^2)^6 - (5*e^9*(2*c*d - b*
e)*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)^6)
```

Rubi [A] time = 15.1959, antiderivative size = 1761, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {740, 822, 800, 634, 618, 206, 628}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + b*x + c*x^2)^5),x]
```

```
[Out] (5*e*(14*c^8*d^8 - b^8*e^8 - 4*c^7*d^6*e*(14*b*d - 19*a*e) + b^6*c*e^7*(b*d
 + 15*a*e) + b^4*c^2*e^6*(b^2*d^2 - 16*a*b*d*e - 82*a^2*e^2) + c^6*d^4*e^2*
(79*b^2*d^2 - 228*a*b*d*e + 176*a^2*e^2) - c^5*d^2*e^3*(41*b^3*d^3 - 197*a*
b^2*d^2*e + 352*a^2*b*d*e^2 - 244*a^3*e^3) + b^2*c^3*e^5*(b^3*d^3 - 15*a*b^
2*d^2*e + 95*a^2*b*d*e^2 + 187*a^3*e^3) + c^4*e^4*(b^4*d^4 - 14*a*b^3*d^3*e
 + 81*a^2*b^2*d^2*e^2 - 244*a^3*b*d*e^3 - 126*a^4*e^4)))/((b^2 - 4*a*c)^4*(
c*d^2 - b*d*e + a*e^2)^5*(d + e*x)) - (b*c*d - b^2*e + 2*a*c*e + c*(2*c*d -
 b*e)*x)/(4*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*(d + e*x)*(a + b*x + c*x^
2)^4) - (8*a*c*e*(2*c*d - b*e)^2 - (b*c*d - b^2*e + 2*a*c*e)*(14*c^2*d^2 -
5*b^2*e^2 - 6*c*e*(b*d - 3*a*e)) - c*(2*c*d - b*e)*(14*c^2*d^2 - 5*b^2*e^2
- 2*c*e*(7*b*d - 17*a*e))*x)/(12*(b^2 - 4*a*c)^2*(c*d^2 - b*d*e + a*e^2)^2*
(d + e*x)*(a + b*x + c*x^2)^3) + (3*a*c*e*(2*c*d - b*e)^2*(14*c^2*d^2 - 5*b
^2*e^2 - 2*c*e*(7*b*d - 17*a*e)) - (b*c*d - b^2*e + 2*a*c*e)*(70*c^4*d^4 +
10*b^4*e^4 - 2*c^3*d^2*e*(49*b*d - 78*a*e) + 5*b^2*c*e^3*(2*b*d - 15*a*e) +
 3*c^2*e^2*(b^2*d^2 - 18*a*b*d*e + 42*a^2*e^2)) - 5*c*(2*c*d - b*e)*(14*c^4
*d^4 + 2*b^4*e^4 + b^2*c*e^3*(5*b*d - 21*a*e) - 4*c^3*d^2*e*(7*b*d - 12*a*e
) + 3*c^2*e^2*(3*b^2*d^2 - 16*a*b*d*e + 22*a^2*e^2))*x)/(12*(b^2 - 4*a*c)^3
*(c*d^2 - b*d*e + a*e^2)^3*(d + e*x)*(a + b*x + c*x^2)^2) - (5*(2*a*c*e*(2*
c*d - b*e)^2*(14*c^4*d^4 + 2*b^4*e^4 + b^2*c*e^3*(5*b*d - 21*a*e) - 4*c^3*d
^2*e*(7*b*d - 12*a*e) + 3*c^2*e^2*(3*b^2*d^2 - 16*a*b*d*e + 22*a^2*e^2)) -
(b*c*d - b^2*e + 2*a*c*e)*(42*c^6*d^6 - 3*b^6*e^6 - 2*c^5*d^4*e*(49*b*d - 6
5*a*e) - 2*b^4*c*e^5*(b*d - 17*a*e) + b^2*c^2*e^4*(b^2*d^2 + 16*a*b*d*e - 1
23*a^2*e^2) + c^4*d^2*e^2*(55*b^2*d^2 - 164*a*b*d*e + 150*a^2*e^2) + 6*c^3*
e^3*(b^3*d^3 - 4*a*b^2*d^2*e - 3*a^2*b*d*e^2 + 21*a^3*e^3)) - 3*c*(2*c*d -
b*e)*(14*c^6*d^6 - b^6*e^6 - 2*c^5*d^4*e*(21*b*d - 31*a*e) - 2*b^4*c*e^5*(b
*d - 7*a*e) - b^2*c^2*e^4*(3*b^2*d^2 - 26*a*b*d*e + 69*a^2*e^2) + c^4*d^2*e
^2*(37*b^2*d^2 - 124*a*b*d*e + 114*a^2*e^2) - 2*c^3*e^3*(2*b^3*d^3 - 18*a*b
^2*d^2*e + 57*a^2*b*d*e^2 - 65*a^3*e^3))*x))/(6*(b^2 - 4*a*c)^4*(c*d^2 - b*
d*e + a*e^2)^4*(d + e*x)*(a + b*x + c*x^2)) - (5*(28*c^10*d^10 + b^10*e^10
- 20*c^9*d^8*e*(7*b*d - 9*a*e) - 252*a^4*c^5*e^9*(5*b*d + a*e) + 210*a^3*b^
2*c^4*e^9*(4*b*d + 3*a*e) - 84*a^2*b^4*c^3*e^9*(3*b*d + 5*a*e) + 18*a*b^6*c
^2*e^9*(2*b*d + 7*a*e) - 2*b^8*c*e^9*(b*d + 9*a*e) + 18*c^8*d^6*e^2*(15*b^2
*d^2 - 40*a*b*d*e + 28*a^2*e^2) - 24*c^7*d^4*e^3*(10*b^3*d^3 - 42*a*b^2*d^2
*e + 63*a^2*b*d*e^2 - 35*a^3*e^3) + 84*c^6*d^2*e^4*(b^4*d^4 - 6*a*b^3*d^3*e
 + 15*a^2*b^2*d^2*e^2 - 20*a^3*b*d*e^3 + 15*a^4*e^4))*ArcTanh[(b + 2*c*x)/S
qrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(9/2)*(c*d^2 - e*(b*d - a*e))^6) + (5*e^9
*(2*c*d - b*e)*Log[d + e*x])/(c*d^2 - b*d*e + a*e^2)^6 - (5*e^9*(2*c*d - b*
e)*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)^6)
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
```

```
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+bx+cx^2\right)^5}\,dx = -\frac{bcd-b^2e+2ace+c(2cd-be)x}{4\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)\left(a+bx+cx^2\right)^4} - \frac{\int \frac{14c^2d^2-5b^2e^2-6ce(bd-3ae)+8}{(d+ex)^2\left(a+bx+cx^2\right)} }{4\left(b^2-4ac\right)\left(cd^2-bd\right.}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{4\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)\left(a+bx+cx^2\right)^4} - \frac{8ace(2cd-be)^2-\left(bcd-\right.}{}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{4\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)\left(a+bx+cx^2\right)^4} - \frac{8ace(2cd-be)^2-\left(bcd-\right.}{}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{4\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)\left(a+bx+cx^2\right)^4} - \frac{8ace(2cd-be)^2-\left(bcd-\right.}{}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x}{4\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)\left(a+bx+cx^2\right)^4} - \frac{8ace(2cd-be)^2-\left(bcd-\right.}{}$$

$$= \frac{5e\left(14c^8d^8-b^8e^8-4c^7d^6e(14bd-19ae)+b^6ce^7(bd+15ae)+b^4c^2e^6\left(b^2d^2-16abde\right.\right.}{}$$

$$= \frac{5e\left(14c^8d^8-b^8e^8-4c^7d^6e(14bd-19ae)+b^6ce^7(bd+15ae)+b^4c^2e^6\left(b^2d^2-16abde\right.\right.}{}$$

$$= \frac{5e\left(14c^8d^8-b^8e^8-4c^7d^6e(14bd-19ae)+b^6ce^7(bd+15ae)+b^4c^2e^6\left(b^2d^2-16abde\right.\right.}{}$$

$$= \frac{5e\left(14c^8d^8-b^8e^8-4c^7d^6e(14bd-19ae)+b^6ce^7(bd+15ae)+b^4c^2e^6\left(b^2d^2-16abde\right.\right.}{}$$

Mathematica [A] time = 7.12981, size = 2147, normalized size = 1.22

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + b*x + c*x^2)^5),x]
```

```
[Out] -(e^9/((c*d^2 - b*d*e + a*e^2)^5*(d + e*x))) + (b*c^2*d^2 - 2*b^2*c*d*e + 4
*a*c^2*d*e + b^3*e^2 - 3*a*b*c*e^2 + 2*c^3*d^2*x - 2*b*c^2*d*e*x + b^2*c*e^
2*x - 2*a*c^2*e^2*x)/(4*(-b^2 + 4*a*c)*(c*d^2 - b*d*e + a*e^2)^2*(a + b*x +
 c*x^2)^4) + (14*b*c^4*d^4 - 28*b^2*c^3*d^3*e + 9*b^3*c^2*d^2*e^2 + 48*a*b*
c^3*d^2*e^2 + 13*b^4*c*d*e^3 - 112*a*b^2*c^2*d*e^3 + 128*a^2*c^3*d*e^3 - 8*
b^5*e^4 + 59*a*b^3*c*e^4 - 94*a^2*b*c^2*e^4 + 28*c^5*d^4*x - 56*b*c^4*d^3*e
*x + 18*b^2*c^3*d^2*e^2*x + 96*a*c^4*d^2*e^2*x + 10*b^3*c^2*d*e^3*x - 96*a*
b*c^3*d*e^3*x - 8*b^4*c*e^4*x + 54*a*b^2*c^2*e^4*x - 60*a^2*c^3*e^4*x)/(12*
(-b^2 + 4*a*c)^2*(c*d^2 - b*d*e + a*e^2)^3*(a + b*x + c*x^2)^3) + (70*b*c^6
*d^6 - 210*b^2*c^5*d^5*e + 185*b^3*c^4*d^4*e^2 + 310*a*b*c^5*d^4*e^2 - 20*b
^4*c^3*d^3*e^3 - 620*a*b^2*c^4*d^3*e^3 - 15*b^5*c^2*d^2*e^4 + 180*a*b^3*c^3
*d^2*e^4 + 570*a^2*b*c^4*d^2*e^4 - 28*b^6*c*d*e^5 + 346*a*b^4*c^2*d*e^5 - 1
434*a^2*b^2*c^3*d*e^5 + 1152*a^3*c^4*d*e^5 + 18*b^7*e^6 - 206*a*b^5*c*e^6 +
 759*a^2*b^3*c^2*e^6 - 822*a^3*b*c^3*e^6 + 140*c^7*d^6*x - 420*b*c^6*d^5*e*
```

```
x + 370*b^2*c^5*d^4*e^2*x + 620*a*c^6*d^4*e^2*x - 40*b^3*c^4*d^3*e^3*x - 12
40*a*b*c^5*d^3*e^3*x - 30*b^4*c^3*d^2*e^4*x + 360*a*b^2*c^4*d^2*e^4*x + 114
0*a^2*c^5*d^2*e^4*x - 20*b^5*c^2*d*e^5*x + 260*a*b^3*c^3*d*e^5*x - 1140*a^2
*b*c^4*d*e^5*x + 18*b^6*c*e^6*x - 196*a*b^4*c^2*e^6*x + 654*a^2*b^2*c^3*e^6
*x - 492*a^3*c^4*e^6*x)/(12*(-b^2 + 4*a*c)^3*(c*d^2 - b*d*e + a*e^2)^4*(a +
 b*x + c*x^2)^2) + (70*b*c^8*d^8 - 280*b^2*c^7*d^7*e + 395*b^3*c^6*d^6*e^2
+ 380*a*b*c^7*d^6*e^2 - 205*b^4*c^5*d^5*e^3 - 1140*a*b^2*c^6*d^5*e^3 + 5*b^
5*c^4*d^4*e^4 + 985*a*b^3*c^5*d^4*e^4 + 880*a^2*b*c^6*d^4*e^4 + 5*b^6*c^3*d
^3*e^5 - 70*a*b^4*c^4*d^3*e^5 - 1760*a^2*b^2*c^5*d^3*e^5 + 5*b^7*c^2*d^2*e^
6 - 75*a*b^5*c^3*d^2*e^6 + 405*a^2*b^3*c^4*d^2*e^6 + 1220*a^3*b*c^5*d^2*e^6
 + 13*b^8*c*d*e^7 - 208*a*b^6*c^2*d*e^7 + 1243*a^2*b^4*c^3*d*e^7 - 3268*a^3
*b^2*c^4*d*e^7 + 2048*a^4*c^5*d*e^7 - 8*b^9*e^8 + 123*a*b^7*c*e^8 - 698*a^2
*b^5*c^2*e^8 + 1703*a^3*b^3*c^3*e^8 - 1398*a^4*b*c^4*e^8 + 140*c^9*d^8*x -
560*b*c^8*d^7*e*x + 790*b^2*c^7*d^6*e^2*x + 760*a*c^8*d^6*e^2*x - 410*b^3*c
^6*d^5*e^3*x - 2280*a*b*c^7*d^5*e^3*x + 10*b^4*c^5*d^4*e^4*x + 1970*a*b^2*c
^6*d^4*e^4*x + 1760*a^2*c^7*d^4*e^4*x + 10*b^5*c^4*d^3*e^5*x - 140*a*b^3*c^
5*d^3*e^5*x - 3520*a^2*b*c^6*d^3*e^5*x + 10*b^6*c^3*d^2*e^6*x - 150*a*b^4*c
^4*d^2*e^6*x + 810*a^2*b^2*c^5*d^2*e^6*x + 2440*a^3*c^6*d^2*e^6*x + 10*b^7*
c^2*d*e^7*x - 160*a*b^5*c^3*d*e^7*x + 950*a^2*b^3*c^4*d*e^7*x - 2440*a^3*b*
c^5*d*e^7*x - 8*b^8*c*e^8*x + 118*a*b^6*c^2*e^8*x - 628*a^2*b^4*c^3*e^8*x +
 1358*a^3*b^2*c^4*e^8*x - 748*a^4*c^5*e^8*x)/(2*(-b^2 + 4*a*c)^4*(c*d^2 - b
*d*e + a*e^2)^5*(a + b*x + c*x^2)) + (5*(28*c^10*d^10 - 140*b*c^9*d^9*e + 2
70*b^2*c^8*d^8*e^2 + 180*a*c^9*d^8*e^2 - 240*b^3*c^7*d^7*e^3 - 720*a*b*c^8*
d^7*e^3 + 84*b^4*c^6*d^6*e^4 + 1008*a*b^2*c^7*d^6*e^4 + 504*a^2*c^8*d^6*e^4
 - 504*a*b^3*c^6*d^5*e^5 - 1512*a^2*b*c^7*d^5*e^5 + 1260*a^2*b^2*c^6*d^4*e^
6 + 840*a^3*c^7*d^4*e^6 - 1680*a^3*b*c^6*d^3*e^7 + 1260*a^4*c^6*d^2*e^8 - 2
*b^9*c*d*e^9 + 36*a*b^7*c^2*d*e^9 - 252*a^2*b^5*c^3*d*e^9 + 840*a^3*b^3*c^4
*d*e^9 - 1260*a^4*b*c^5*d*e^9 + b^10*e^10 - 18*a*b^8*c*e^10 + 126*a^2*b^6*c
^2*e^10 - 420*a^3*b^4*c^3*e^10 + 630*a^4*b^2*c^4*e^10 - 252*a^5*c^5*e^10)*A
rcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/((b^2 - 4*a*c)^4*Sqrt[-b^2 + 4*a*c]*
(-(c*d^2) + b*d*e - a*e^2)^6) + (5*(2*c*d*e^9 - b*e^10)*Log[d + e*x])/(c*d^
2 - b*d*e + a*e^2)^6 - (5*(2*c*d*e^9 - b*e^10)*Log[a + b*x + c*x^2])/(2*(c*
d^2 - b*d*e + a*e^2)^6)
```

Maple [B] time = 0.227, size = 39599, normalized size = 22.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^2+b*x+a)^5,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x+a)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x+a)^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**2+b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 4.22153, size = 10855, normalized size = 6.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x+a)^5,x, algorithm="giac")
```

```
[Out] -5*(28*c^10*d^10*e^2 - 140*b*c^9*d^9*e^3 + 270*b^2*c^8*d^8*e^4 + 180*a*c^9*
d^8*e^4 - 240*b^3*c^7*d^7*e^5 - 720*a*b*c^8*d^7*e^5 + 84*b^4*c^6*d^6*e^6 +
1008*a*b^2*c^7*d^6*e^6 + 504*a^2*c^8*d^6*e^6 - 504*a*b^3*c^6*d^5*e^7 - 1512
*a^2*b*c^7*d^5*e^7 + 1260*a^2*b^2*c^6*d^4*e^8 + 840*a^3*c^7*d^4*e^8 - 1680*
a^3*b*c^6*d^3*e^9 + 1260*a^4*c^6*d^2*e^10 - 2*b^9*c*d*e^11 + 36*a*b^7*c^2*d
*e^11 - 252*a^2*b^5*c^3*d*e^11 + 840*a^3*b^3*c^4*d*e^11 - 1260*a^4*b*c^5*d*
e^11 + b^10*e^12 - 18*a*b^8*c*e^12 + 126*a^2*b^6*c^2*e^12 - 420*a^3*b^4*c^3
*e^12 + 630*a^4*b^2*c^4*e^12 - 252*a^5*c^5*e^12)*arctan(-(2*c*d - 2*c*d^2/(
x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*a*e^2/(x*e + d))*e^(-1)/sqrt(-b^2 +
4*a*c))*e^(-2)/((b^8*c^6*d^12 - 16*a*b^6*c^7*d^12 + 96*a^2*b^4*c^8*d^12 - 2
56*a^3*b^2*c^9*d^12 + 256*a^4*c^10*d^12 - 6*b^9*c^5*d^11*e + 96*a*b^7*c^6*d
^11*e - 576*a^2*b^5*c^7*d^11*e + 1536*a^3*b^3*c^8*d^11*e - 1536*a^4*b*c^9*d
^11*e + 15*b^10*c^4*d^10*e^2 - 234*a*b^8*c^5*d^10*e^2 + 1344*a^2*b^6*c^6*d^
10*e^2 - 3264*a^3*b^4*c^7*d^10*e^2 + 2304*a^4*b^2*c^8*d^10*e^2 + 1536*a^5*c
^9*d^10*e^2 - 20*b^11*c^3*d^9*e^3 + 290*a*b^9*c^4*d^9*e^3 - 1440*a^2*b^7*c^
5*d^9*e^3 + 2240*a^3*b^5*c^6*d^9*e^3 + 2560*a^4*b^3*c^7*d^9*e^3 - 7680*a^5*
b*c^8*d^9*e^3 + 15*b^12*c^2*d^8*e^4 - 180*a*b^10*c^3*d^8*e^4 + 495*a^2*b^8*
c^4*d^8*e^4 + 1680*a^3*b^6*c^5*d^8*e^4 - 10080*a^4*b^4*c^6*d^8*e^4 + 11520*
a^5*b^2*c^7*d^8*e^4 + 3840*a^6*c^8*d^8*e^4 - 6*b^13*c*d^7*e^5 + 36*a*b^11*c
^2*d^7*e^5 + 324*a^2*b^9*c^3*d^7*e^5 - 3264*a^3*b^7*c^4*d^7*e^5 + 8064*a^4*
b^5*c^5*d^7*e^5 - 15360*a^6*b*c^7*d^7*e^5 + b^14*d^6*e^6 + 14*a*b^12*c*d^6*
e^6 - 294*a^2*b^10*c^2*d^6*e^6 + 1204*a^3*b^8*c^3*d^6*e^6 + 896*a^4*b^6*c^4
*d^6*e^6 - 13440*a^5*b^4*c^5*d^6*e^6 + 17920*a^6*b^2*c^6*d^6*e^6 + 5120*a^7
*c^7*d^6*e^6 - 6*a*b^13*d^5*e^7 + 36*a^2*b^11*c*d^5*e^7 + 324*a^3*b^9*c^2*d
```

^5*e^7 - 3264*a^4*b^7*c^3*d^5*e^7 + 8064*a^5*b^5*c^4*d^5*e^7 - 15360*a^7*b*c^6*d^5*e^7 + 15*a^2*b^12*d^4*e^8 - 180*a^3*b^10*c*d^4*e^8 + 495*a^4*b^8*c^2*d^4*e^8 + 1680*a^5*b^6*c^3*d^4*e^8 - 10080*a^6*b^4*c^4*d^4*e^8 + 11520*a^7*b^2*c^5*d^4*e^8 + 3840*a^8*c^6*d^4*e^8 - 20*a^3*b^11*d^3*e^9 + 290*a^4*b^9*c*d^3*e^9 - 1440*a^5*b^7*c^2*d^3*e^9 + 2240*a^6*b^5*c^3*d^3*e^9 + 2560*a^7*b^3*c^4*d^3*e^9 - 7680*a^8*b*c^5*d^3*e^9 + 15*a^4*b^10*d^2*e^10 - 234*a^5*b^8*c*d^2*e^10 + 1344*a^6*b^6*c^2*d^2*e^10 - 3264*a^7*b^4*c^3*d^2*e^10 + 2304*a^8*b^2*c^4*d^2*e^10 + 1536*a^9*c^5*d^2*e^10 - 6*a^5*b^9*d*e^11 + 96*a^6*b^7*c*d*e^11 - 576*a^7*b^5*c^2*d*e^11 + 1536*a^8*b^3*c^3*d*e^11 - 1536*a^9*b*c^4*d*e^11 + a^6*b^8*e^12 - 16*a^7*b^6*c*e^12 + 96*a^8*b^4*c^2*e^12 - 256*a^9*b^2*c^3*e^12 + 256*a^10*c^4*e^12)*sqrt(-b^2 + 4*a*c)) - 5/2*(2*c*d*e^9 - b*e^10)*log(-c + 2*c*d/(x*e + d) - c*d^2/(x*e + d)^2 - b*e/(x*e + d) + b*d*e/(x*e + d)^2 - a*e^2/(x*e + d)^2)/(c^6*d^12 - 6*b*c^5*d^11*e + 15*b^2*c^4*d^10*e^2 + 6*a*c^5*d^10*e^2 - 20*b^3*c^3*d^9*e^3 - 30*a*b*c^4*d^9*e^3 + 15*b^4*c^2*d^8*e^4 + 60*a*b^2*c^3*d^8*e^4 + 15*a^2*c^4*d^8*e^4 - 6*b^5*c*d^7*e^5 - 60*a*b^3*c^2*d^7*e^5 - 60*a^2*b*c^3*d^7*e^5 + b^6*d^6*e^6 + 30*a*b^4*c*d^6*e^6 + 90*a^2*b^2*c^2*d^6*e^6 + 20*a^3*c^3*d^6*e^6 - 6*a*b^5*d^5*e^7 - 60*a^2*b^3*c*d^5*e^7 - 60*a^3*b*c^2*d^5*e^7 + 15*a^2*b^4*d^4*e^8 + 60*a^3*b^2*c*d^4*e^8 + 15*a^4*c^2*d^4*e^8 - 20*a^3*b^3*d^3*e^9 - 30*a^4*b*c*d^3*e^9 + 15*a^4*b^2*d^2*e^10 + 6*a^5*c*d^2*e^10 - 6*a^5*b*d*e^11 + a^6*e^12) - e^19/((c^5*d^10*e^10 - 5*b*c^4*d^9*e^11 + 10*b^2*c^3*d^8*e^12 + 5*a*c^4*d^8*e^12 - 10*b^3*c^2*d^7*e^13 - 20*a*b*c^3*d^7*e^13 + 5*b^4*c*d^6*e^14 + 30*a*b^2*c^2*d^6*e^14 + 10*a^2*c^3*d^6*e^14 - b^5*d^5*e^15 - 20*a*b^3*c*d^5*e^15 - 30*a^2*b*c^2*d^5*e^15 + 5*a*b^4*d^4*e^16 + 30*a^2*b^2*c*d^4*e^16 + 10*a^3*c^2*d^4*e^16 - 10*a^2*b^3*d^3*e^17 - 20*a^3*b*c*d^3*e^17 + 10*a^3*b^2*d^2*e^18 + 5*a^4*c*d^2*e^18 - 5*a^4*b*d*e^19 + a^5*e^20)*(x*e + d)) + 1/12*(840*c^13*d^9*e - 3780*b*c^12*d^8*e^2 + 6280*b^2*c^11*d^7*e^3 + 5120*a*c^12*d^7*e^3 - 4340*b^3*c^10*d^6*e^4 - 17920*a*b*c^11*d^6*e^4 + 738*b^4*c^9*d^5*e^5 + 20136*a*b^2*c^10*d^5*e^5 + 13488*a^2*c^11*d^5*e^5 + 185*b^5*c^8*d^4*e^6 - 5540*a*b^3*c^9*d^4*e^6 - 33720*a^2*b*c^10*d^4*e^6 + 80*b^6*c^7*d^3*e^7 - 1700*a*b^4*c^8*d^3*e^7 + 17880*a^2*b^2*c^9*d^3*e^7 + 21120*a^3*c^10*d^3*e^7 + 45*b^7*c^6*d^2*e^8 - 870*a*b^5*c^7*d^2*e^8 + 6900*a^2*b^3*c^8*d^2*e^8 - 31680*a^3*b*c^9*d^2*e^8 - 202*b^8*c^5*d*e^9 + 3142*a*b^6*c^6*d*e^9 - 17982*a^2*b^4*c^7*d*e^9 + 43352*a^3*b^2*c^8*d*e^9 - 27512*a^4*c^9*d*e^9 + 77*b^9*c^4*e^10 - 1184*a*b^7*c^5*e^10 + 6717*a^2*b^5*c^6*e^10 - 16396*a^3*b^3*c^7*e^10 + 13756*a^4*b*c^8*e^10 - 4*(1470*c^13*d^10*e^2 - 7350*b*c^12*d^9*e^3 + 14315*b^2*c^11*d^8*e^4 + 8890*a*c^12*d^8*e^4 - 13160*b^3*c^10*d^7*e^5 - 35560*a*b*c^11*d^7*e^5 + 5186*b^4*c^9*d^6*e^6 + 50632*a*b^2*c^10*d^6*e^6 + 23196*a^2*c^11*d^6*e^6 - 368*b^5*c^8*d^5*e^7 - 27436*a*b^3*c^9*d^5*e^7 - 69588*a^2*b*c^10*d^5*e^7 - 25*b^6*c^7*d^4*e^8 + 2140*a*b^4*c^8*d^4*e^8 + 60030*a^2*b^2*c^9*d^4*e^8 + 35940*a^3*c^10*d^4*e^8 + 10*b^7*c^6*d^3*e^9 - 40*a*b^5*c^7*d^3*e^9 - 4080*a^2*b^3*c^8*d^3*e^9 - 71880*a^3*b*c^9*d^3*e^9 - 422*b^8*c^5*d^2*e^10 + 6722*a*b^6*c^6*d^2*e^10 - 40272*a^2*b^4*c^7*d^2*e^10 + 111472*a^3*b^2*c^8*d^2*e^10 - 57562*a^4*c^9*d^2*e^10 + 344*b^9*c^4*d*e^11 - 5348*a*b^7*c^5*d*e^11 + 30714*a^2*b^5*c^6*d*e^11 - 75532*a^3*b^3*c^7*d*e^11 + 57562*a^4*b*c^8*d*e^11 - 77*b^10*c^3*e^12 + 1196*a*b^8*c^4*e^12 - 6894*a^2*b^6*c^5*e^12 + 17338*a^3*b^4*c^6*e^12 - 15793*a^4*b^2*c^7*e^12 + 1122*a^5*c^8*e^12)*e^(-1)/(x*e + d) + 2*(8820*c^13*d^11*e^3 - 48510*b*c^12*d^10*e^4 + 107660*b^2*c^11*d^9*e^5 + 54460*a*c^12*d^9*e^5 - 120645*b^3*c^10*d^8*e^6 - 245070*a*b*c^11*d^8*e^6 + 68512*b^4*c^9*d^7*e^7 + 417064*a*b^2*c^10*d^7*e^7 + 146152*a^2*c^11*d^7*e^7 - 16352*b^5*c^8*d^6*e^8 - 316064*a*b^3*c^9*d^6*e^8 - 511532*a^2*b*c^10*d^6*e^8 + 744*b^6*c^7*d^5*e^9 + 89184*a*b^4*c^8*d^5*e^9 + 591456*a^2*b^2*c^9*d^5*e^9 + 234456*a^3*c^10*d^5*e^9 + 65*b^7*c^6*d^4*e^10 - 4630*a*b^5*c^7*d^4*e^10 - 199810*a^2*b^3*c^8*d^4*e^10 - 586140*a^3*b*c^9*d^4*e^10 - 2816*b^8*c^5*d^3*e^11 + 44796*a*b^6*c^6*d^3*e^11 - 259516*a^2*b^4*c^7*d^3*e^11 + 958456*a^3*b^2*c^8*d^3*e^11 - 372316*a^4*c^9*d^3*e^11 + 3648*b^9*c^4*d^2*e^12 - 57216*a*b^7*c^5*d^2*e^12 + 333318*a^2*b^5*c^6*d^2*e^12 - 851544*a^3*b^3*c^7*d^2*e^12 + 558474*a^4*b*c^8*d^2*e^12 - 1588*b^10*c^3*d*e^13 + 24464*a*b^8*c^4*d*e^13 - 138496*a^2*b^6*c^5*d*e^13 +

331772*a^3*b^4*c^6*d*e^13 - 237772*a^4*b^2*c^7*d*e^13 - 33172*a^5*c^8*d*e^
13 + 231*b^11*c^2*e^14 - 3494*a*b^9*c^3*e^14 + 19214*a^2*b^7*c^4*e^14 - 435
00*a^3*b^5*c^5*e^14 + 25807*a^4*b^3*c^6*e^14 + 16586*a^5*b*c^7*e^14)*e^(-2)
/(x*e + d)^2 - 4*(7350*c^13*d^12*e^4 - 44100*b*c^12*d^11*e^5 + 109375*b^2*c
^11*d^10*e^6 + 47600*a*c^12*d^10*e^6 - 142625*b^3*c^10*d^9*e^7 - 238000*a*b
*c^11*d^9*e^7 + 101967*b^4*c^9*d^8*e^8 + 467889*a*b^2*c^10*d^8*e^8 + 135222
*a^2*c^11*d^8*e^8 - 37218*b^5*c^8*d^7*e^9 - 443556*a*b^3*c^9*d^7*e^9 - 5408
88*a^2*b*c^10*d^7*e^9 + 5487*b^6*c^7*d^6*e^10 + 194682*a*b^4*c^8*d^6*e^10 +
773718*a^2*b^2*c^9*d^6*e^10 + 230448*a^3*c^10*d^6*e^10 - 123*b^7*c^6*d^5*e
^11 - 31200*a*b^5*c^7*d^5*e^11 - 428046*a^2*b^3*c^8*d^5*e^11 - 691344*a^3*b
*c^9*d^5*e^11 - 2592*b^8*c^5*d^4*e^12 + 42087*a*b^6*c^6*d^4*e^12 - 174522*a
^2*b^4*c^7*d^4*e^12 + 1178802*a^3*b^2*c^8*d^4*e^12 - 314622*a^4*c^9*d^4*e^1
2 + 4668*b^9*c^4*d^3*e^13 - 73656*a*b^7*c^5*d^3*e^13 + 431418*a^2*b^5*c^6*d
^3*e^13 - 1205364*a^3*b^3*c^7*d^3*e^13 + 629244*a^4*b*c^8*d^3*e^13 - 3012*b
^10*c^3*d^2*e^14 + 46236*a*b^8*c^4*d^2*e^14 - 259404*a^2*b^6*c^5*d^2*e^14 +
606198*a^3*b^4*c^6*d^2*e^14 - 308373*a^4*b^2*c^7*d^2*e^14 - 130848*a^5*c^8
*d^2*e^14 + 823*b^11*c^2*d*e^15 - 12082*a*b^9*c^3*d*e^15 + 62502*a^2*b^7*c^
4*d*e^15 - 118740*a^3*b^5*c^5*d*e^15 - 6249*a^4*b^3*c^6*d*e^15 + 130848*a^5
*b*c^7*d*e^15 - 77*b^12*c*e^16 + 1025*a*b^10*c^2*e^16 - 4209*a^2*b^8*c^3*e^
16 + 1614*a^3*b^6*c^4*e^16 + 24843*a^4*b^4*c^5*e^16 - 38499*a^5*b^2*c^6*e^1
6 + 3858*a^6*c^7*e^16)*e^(-3)/(x*e + d)^3 + (29400*c^13*d^13*e^5 - 191100*b
*c^12*d^12*e^6 + 522200*b^2*c^11*d^11*e^7 + 204400*a*c^12*d^11*e^7 - 770000
*b^3*c^10*d^10*e^8 - 1124200*a*b*c^11*d^10*e^8 + 650450*b^4*c^9*d^9*e^9 + 2
496400*a*b^2*c^10*d^9*e^9 + 628200*a^2*c^11*d^9*e^9 - 305175*b^5*c^8*d^8*e^
10 - 2802300*a*b^3*c^9*d^8*e^10 - 2826900*a^2*b*c^10*d^8*e^10 + 69600*b^6*c
^7*d^7*e^11 + 1606200*a*b^4*c^8*d^7*e^11 + 4784400*a^2*b^2*c^9*d^7*e^11 + 1
159200*a^3*c^10*d^7*e^11 - 5250*b^7*c^6*d^6*e^12 - 413700*a*b^5*c^7*d^6*e^1
2 - 3553200*a^2*b^3*c^8*d^6*e^12 - 4057200*a^3*b*c^9*d^6*e^12 - 11544*b^8*c
^5*d^5*e^13 + 216204*a*b^6*c^6*d^5*e^13 - 56124*a^2*b^4*c^7*d^5*e^13 + 7256
064*a^3*b^2*c^8*d^5*e^13 - 1170264*a^4*c^9*d^5*e^13 + 26760*b^9*c^4*d^4*e^1
4 - 423960*a*b^7*c^5*d^4*e^14 + 2427210*a^2*b^5*c^6*d^4*e^14 - 7997160*a^3*
b^3*c^7*d^4*e^14 + 2925660*a^4*b*c^8*d^4*e^14 - 22980*b^10*c^3*d^3*e^15 + 3
52560*a*b^8*c^4*d^3*e^15 - 1972560*a^2*b^6*c^5*d^3*e^15 + 4653960*a^3*b^4*c
^6*d^3*e^15 - 1310760*a^4*b^2*c^7*d^3*e^15 - 1291920*a^5*c^8*d^3*e^15 + 903
5*b^11*c^2*d^2*e^16 - 129830*a*b^9*c^3*d^2*e^16 + 639630*a^2*b^7*c^4*d^2*e^
16 - 1012380*a^3*b^5*c^5*d^2*e^16 - 959520*a^4*b^3*c^6*d^2*e^16 + 1937880*a
^5*b*c^7*d^2*e^16 - 1550*b^12*c*d*e^17 + 19130*a*b^10*c^2*d*e^17 - 61470*a^
2*b^8*c^3*d*e^17 - 98580*a^3*b^6*c^4*d*e^17 + 801930*a^4*b^4*c^5*d*e^17 - 8
99280*a^5*b^2*c^6*d*e^17 - 46440*a^6*c^7*d*e^17 + 77*b^13*e^18 - 452*a*b^11
*c*e^18 - 4593*a^2*b^9*c^2*e^18 + 48048*a^3*b^7*c^3*e^18 - 143523*a^4*b^5*c
^4*e^18 + 126660*a^5*b^3*c^5*e^18 + 23220*a^6*b*c^6*e^18)*e^(-4)/(x*e + d)^
4 - 20*(882*c^13*d^14*e^6 - 6174*b*c^12*d^13*e^7 + 18389*b^2*c^11*d^12*e^8
+ 6706*a*c^12*d^12*e^8 - 30072*b^3*c^10*d^11*e^9 - 40236*a*b*c^11*d^11*e^9
+ 28936*b^4*c^9*d^10*e^10 + 99304*a*b^2*c^10*d^10*e^10 + 22690*a^2*c^11*d^1
0*e^10 - 16167*b^5*c^8*d^9*e^11 - 127690*a*b^3*c^9*d^9*e^11 - 113450*a^2*b*
c^10*d^9*e^11 + 4752*b^6*c^7*d^8*e^12 + 88479*a*b^4*c^8*d^8*e^12 + 220689*a
^2*b^2*c^9*d^8*e^12 + 46098*a^3*c^10*d^8*e^12 - 546*b^7*c^6*d^7*e^13 - 3037
2*a*b^5*c^7*d^7*e^13 - 202056*a^2*b^3*c^8*d^7*e^13 - 184392*a^3*b*c^9*d^7*e
^13 - 393*b^8*c^5*d^6*e^14 + 10110*a*b^6*c^6*d^6*e^14 + 45642*a^2*b^4*c^7*d
^6*e^14 + 349752*a^3*b^2*c^8*d^6*e^14 - 27066*a^4*c^9*d^6*e^14 + 1116*b^9*c
^4*d^5*e^15 - 17730*a*b^7*c^5*d^5*e^15 + 93780*a^2*b^5*c^6*d^5*e^15 - 40388
4*a^3*b^3*c^7*d^5*e^15 + 81198*a^4*b*c^8*d^5*e^15 - 1200*b^10*c^3*d^4*e^16
+ 18420*a*b^8*c^4*d^4*e^16 - 103035*a^2*b^6*c^5*d^4*e^16 + 255840*a^3*b^4*c
^6*d^4*e^16 - 6825*a^4*b^2*c^7*d^4*e^16 - 75738*a^5*c^8*d^4*e^16 + 611*b^11
*c^2*d^3*e^17 - 8642*a*b^9*c^3*d^3*e^17 + 40938*a^2*b^7*c^4*d^3*e^17 - 5366
4*a^3*b^5*c^5*d^3*e^17 - 121680*a^4*b^3*c^6*d^3*e^17 + 151476*a^5*b*c^7*d^3
*e^17 - 147*b^12*c*d^2*e^18 + 1695*a*b^10*c^2*d^2*e^18 - 3987*a^2*b^8*c^3*d
^2*e^18 - 19674*a^3*b^6*c^4*d^2*e^18 + 99270*a^4*b^4*c^5*d^2*e^18 - 85824*a
^5*b^2*c^6*d^2*e^18 - 18522*a^6*c^7*d^2*e^18 + 13*b^13*d*e^19 - 44*a*b^11*c

*d*e^19 - 1211*a^2*b^9*c^2*d*e^19 + 9924*a^3*b^7*c^3*d*e^19 - 24897*a^4*b^5
*c^4*d*e^19 + 10086*a^5*b^3*c^5*d*e^19 + 18522*a^6*b*c^6*d*e^19 - 13*a*b^12
*e^20 + 178*a^2*b^10*c*e^20 - 783*a^3*b^8*c^2*e^20 + 651*a^4*b^6*c^3*e^20 +
3417*a^5*b^4*c^4*e^20 - 6237*a^6*b^2*c^5*e^20 + 918*a^7*c^6*e^20)*e^(-5)/(
x*e + d)^5 + 30*(196*c^13*d^15*e^7 - 1470*b*c^12*d^14*e^8 + 4732*b^2*c^11*d
^13*e^9 + 1652*a*c^12*d^13*e^9 - 8463*b^3*c^10*d^12*e^10 - 10738*a*b*c^11*d
^12*e^10 + 9058*b^4*c^9*d^11*e^11 + 29092*a*b^2*c^10*d^11*e^11 + 6244*a^2*c
^11*d^11*e^11 - 5775*b^5*c^8*d^10*e^12 - 41888*a*b^3*c^9*d^10*e^12 - 34342*
a^2*b*c^10*d^10*e^12 + 2016*b^6*c^7*d^9*e^13 + 33558*a*b^4*c^8*d^9*e^13 + 7
5208*a^2*b^2*c^9*d^9*e^13 + 14196*a^3*c^10*d^9*e^13 - 294*b^7*c^6*d^8*e^14
- 14028*a*b^5*c^7*d^8*e^14 - 80871*a^2*b^3*c^8*d^8*e^14 - 63882*a^3*b*c^9*d
^8*e^14 - 102*b^8*c^5*d^7*e^15 + 3984*a*b^6*c^6*d^7*e^15 + 32208*a^2*b^4*c^
7*d^7*e^15 + 129768*a^3*b^2*c^8*d^7*e^15 - 2004*a^4*c^9*d^7*e^15 + 343*b^9*
c^4*d^6*e^16 - 5460*a*b^7*c^5*d^6*e^16 + 24276*a^2*b^5*c^6*d^6*e^16 - 15607
2*a^3*b^3*c^7*d^6*e^16 + 7014*a^4*b*c^8*d^6*e^16 - 444*b^10*c^3*d^5*e^17 +
6822*a*b^8*c^4*d^5*e^17 - 38196*a^2*b^6*c^5*d^5*e^17 + 104232*a^3*b^4*c^6*d
^5*e^17 + 25644*a^4*b^2*c^7*d^5*e^17 - 28932*a^5*c^8*d^5*e^17 + 277*b^11*c^
2*d^4*e^18 - 3874*a*b^9*c^3*d^4*e^18 + 17811*a^2*b^7*c^4*d^4*e^18 - 19458*a
^3*b^5*c^5*d^4*e^18 - 81645*a^4*b^3*c^6*d^4*e^18 + 72330*a^5*b*c^7*d^4*e^18
- 84*b^12*c*d^3*e^19 + 908*a*b^10*c^2*d^3*e^19 - 1332*a^2*b^8*c^3*d^3*e^19
- 16644*a^3*b^6*c^4*d^3*e^19 + 69390*a^4*b^4*c^5*d^3*e^19 - 45708*a^5*b^2*
c^6*d^3*e^19 - 17748*a^6*c^7*d^3*e^19 + 10*b^13*d^2*e^20 - 8*a*b^11*c*d^2*e
^20 - 1274*a^2*b^9*c^2*d^2*e^20 + 8976*a^3*b^7*c^3*d^2*e^20 - 18933*a^4*b^5
*c^4*d^2*e^20 - 3768*a^5*b^3*c^5*d^2*e^20 + 26622*a^6*b*c^6*d^2*e^20 - 20*a
*b^12*d*e^21 + 248*a^2*b^10*c*d*e^21 - 804*a^3*b^8*c^2*d*e^21 - 1272*a^4*b^
6*c^3*d*e^21 + 10626*a^5*b^4*c^4*d*e^21 - 12912*a^6*b^2*c^5*d*e^21 - 228*a^
7*c^6*d*e^21 + 10*a^2*b^11*e^22 - 156*a^3*b^9*c*e^22 + 903*a^4*b^7*c^2*e^22
- 2274*a^5*b^5*c^3*e^22 + 2019*a^6*b^3*c^4*e^22 + 114*a^7*b*c^5*e^22)*e^(-
6)/(x*e + d)^6 - 60*(14*c^13*d^16*e^8 - 112*b*c^12*d^15*e^9 + 387*b^2*c^11*
d^14*e^10 + 132*a*c^12*d^14*e^10 - 749*b^3*c^10*d^13*e^11 - 924*a*b*c^11*d^
13*e^11 + 877*b^4*c^9*d^12*e^12 + 2721*a*b^2*c^10*d^12*e^12 + 564*a^2*c^11*
d^12*e^12 - 621*b^5*c^8*d^11*e^13 - 4314*a*b^3*c^9*d^11*e^13 - 3384*a^2*b*c
^10*d^11*e^13 + 246*b^6*c^7*d^10*e^14 + 3879*a*b^4*c^8*d^10*e^14 + 8211*a^2
*b^2*c^9*d^10*e^14 + 1460*a^3*c^10*d^10*e^14 - 42*b^7*c^6*d^9*e^15 - 1872*a
*b^5*c^7*d^9*e^15 - 10035*a^2*b^3*c^8*d^9*e^15 - 7300*a^3*b*c^9*d^9*e^15 -
9*b^8*c^5*d^8*e^16 + 522*a*b^6*c^6*d^8*e^16 + 5292*a^2*b^4*c^7*d^8*e^16 + 1
5993*a^3*b^2*c^8*d^8*e^16 + 432*a^4*c^9*d^8*e^16 + 35*b^9*c^4*d^7*e^17 - 55
8*a*b^7*c^5*d^7*e^17 + 1818*a^2*b^5*c^6*d^7*e^17 - 20172*a^3*b^3*c^7*d^7*e^
17 - 1728*a^4*b*c^8*d^7*e^17 - 53*b^10*c^3*d^6*e^18 + 815*a*b^8*c^4*d^6*e^1
8 - 4567*a^2*b^6*c^5*d^6*e^18 + 14026*a^3*b^4*c^6*d^6*e^18 + 7249*a^4*b^2*c
^7*d^6*e^18 - 3380*a^5*c^8*d^6*e^18 + 39*b^11*c^2*d^5*e^19 - 540*a*b^9*c^3*
d^5*e^19 + 2415*a^2*b^7*c^4*d^5*e^19 - 2136*a^3*b^5*c^5*d^5*e^19 - 15699*a^
4*b^3*c^6*d^5*e^19 + 10140*a^5*b*c^7*d^5*e^19 - 14*b^12*c*d^4*e^20 + 141*a*
b^10*c^2*d^4*e^20 - 60*a^2*b^8*c^3*d^4*e^20 - 3705*a^3*b^6*c^4*d^4*e^20 + 1
3785*a^4*b^4*c^5*d^4*e^20 - 6357*a^5*b^2*c^6*d^4*e^20 - 4212*a^6*c^7*d^4*e^
20 + 2*b^13*d^3*e^21 + 4*a*b^11*c*d^3*e^21 - 326*a^2*b^9*c^2*d^3*e^21 + 203
6*a^3*b^7*c^3*d^3*e^21 - 3421*a^4*b^5*c^4*d^3*e^21 - 4186*a^5*b^3*c^5*d^3*e
^21 + 8424*a^6*b*c^6*d^3*e^21 - 6*a*b^12*d^2*e^22 + 66*a^2*b^10*c*d^2*e^22
- 114*a^3*b^8*c^2*d^2*e^22 - 1071*a^4*b^6*c^3*d^2*e^22 + 4623*a^5*b^4*c^4*d
^2*e^22 - 4071*a^6*b^2*c^5*d^2*e^22 - 1284*a^7*c^6*d^2*e^22 + 6*a^2*b^11*d*
e^23 - 88*a^3*b^9*c*d*e^23 + 453*a^4*b^7*c^2*d*e^23 - 840*a^5*b^5*c^3*d*e^2
3 - 141*a^6*b^3*c^4*d*e^23 + 1284*a^7*b*c^5*d*e^23 - 2*a^3*b^10*e^24 + 32*a
^4*b^8*c*e^24 - 193*a^5*b^6*c^2*e^24 + 526*a^6*b^4*c^3*e^24 - 581*a^7*b^2*c
^4*e^24 + 130*a^8*c^5*e^24)*e^(-7)/(x*e + d)^7)/((c*d^2 - b*d*e + a*e^2)^6*
(b^2 - 4*a*c)^4*(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + b*e/(x*e + d) -
b*d*e/(x*e + d)^2 + a*e^2/(x*e + d)^2)^4)

3.2229 $\int \frac{1}{(1+2x)(2+3x+5x^2)^3} dx$

Optimal. Leaf size=89

$$\frac{20x+37}{434\left(5x^2+3x+2\right)^2}+\frac{2(2290x+2609)}{47089\left(5x^2+3x+2\right)}-\frac{16}{343}\log\left(5x^2+3x+2\right)+\frac{32}{343}\log(2x+1)+\frac{125624\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{329623\sqrt{31}}$$

```
[Out] (37 + 20*x)/(434*(2 + 3*x + 5*x^2)^2) + (2*(2609 + 2290*x))/(47089*(2 + 3*x
 + 5*x^2)) + (125624*ArcTan[(3 + 10*x)/Sqrt[31]])/(329623*Sqrt[31]) + (32*L
og[1 + 2*x])/343 - (16*Log[2 + 3*x + 5*x^2])/343
```

Rubi [A] time = 0.0825689, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {740, 822, 800, 634, 618, 204, 628}

$$\frac{20x+37}{434\left(5x^2+3x+2\right)^2}+\frac{2(2290x+2609)}{47089\left(5x^2+3x+2\right)}-\frac{16}{343}\log\left(5x^2+3x+2\right)+\frac{32}{343}\log(2x+1)+\frac{125624\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{329623\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + 2*x)*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] (37 + 20*x)/(434*(2 + 3*x + 5*x^2)^2) + (2*(2609 + 2290*x))/(47089*(2 + 3*x
 + 5*x^2)) + (125624*ArcTan[(3 + 10*x)/Sqrt[31]])/(329623*Sqrt[31]) + (32*L
og[1 + 2*x])/343 - (16*Log[2 + 3*x + 5*x^2])/343
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1+2x)\left(2+3x+5x^2\right)^3}\,dx &= \frac{37+20x}{434\left(2+3x+5x^2\right)^2}+\frac{1}{434}\int \frac{308+120x}{(1+2x)\left(2+3x+5x^2\right)^2}\,dx\\
&= \frac{37+20x}{434\left(2+3x+5x^2\right)^2}+\frac{2(2609+2290x)}{47089\left(2+3x+5x^2\right)}+\frac{\int \frac{39912+18320x}{(1+2x)\left(2+3x+5x^2\right)}\,dx}{94178}\\
&= \frac{37+20x}{434\left(2+3x+5x^2\right)^2}+\frac{2(2609+2290x)}{47089\left(2+3x+5x^2\right)}+\frac{\int \left(\frac{123008}{7(1+2x)}-\frac{8(-4171+38440x)}{7\left(2+3x+5x^2\right)}\right)dx}{94178}\\
&= \frac{37+20x}{434\left(2+3x+5x^2\right)^2}+\frac{2(2609+2290x)}{47089\left(2+3x+5x^2\right)}+\frac{32}{343}\log(1+2x)-\frac{4\int \frac{-4171+38440x}{2+3x+5x^2}\,dx}{329623}\\
&= \frac{37+20x}{434\left(2+3x+5x^2\right)^2}+\frac{2(2609+2290x)}{47089\left(2+3x+5x^2\right)}+\frac{32}{343}\log(1+2x)-\frac{16}{343}\int \frac{3+10x}{2+3x+5x^2\cdots}\\
&= \frac{37+20x}{434\left(2+3x+5x^2\right)^2}+\frac{2(2609+2290x)}{47089\left(2+3x+5x^2\right)}+\frac{32}{343}\log(1+2x)-\frac{16}{343}\log\left(2+3x+5\cdots\right.\\
&= \frac{37+20x}{434\left(2+3x+5x^2\right)^2}+\frac{2(2609+2290x)}{47089\left(2+3x+5x^2\right)}+\frac{125624\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{329623\sqrt{31}}+\frac{32}{343}\log(1+2\cdots
\end{aligned}$$

Mathematica [A] time = 0.0961209, size = 78, normalized size = 0.88

$$\frac{8\left(\frac{217(45800x^3+79660x^2+53968x+28901)}{16(5x^2+3x+2)^2}-59582\log\left(4\left(5x^2+3x+2\right)\right)+119164\log(2x+1)+15703\sqrt{31}\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)\right)}{10218313}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + 2*x)*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] (8*((217*(28901 + 53968*x + 79660*x^2 + 45800*x^3))/(16*(2 + 3*x + 5*x^2)^2
) + 15703*Sqrt[31]*ArcTan[(3 + 10*x)/Sqrt[31]] + 119164*Log[1 + 2*x] - 5958
2*Log[4*(2 + 3*x + 5*x^2)]))/10218313
```

Maple [A] time = 0.049, size = 68, normalized size = 0.8

$$\frac{32\ln(1+2x)}{343}-\frac{25}{343\left(5x^2+3x+2\right)^2}\left(-\frac{6412x^3}{961}-\frac{55762x^2}{4805}-\frac{188888x}{24025}-\frac{202307}{48050}\right)-\frac{16\ln\left(5x^2+3x+2\right)}{343}+\frac{125\ldots}{10\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2*x)/(5*x^2+3*x+2)^3,x)
```

```
[Out] 32/343*ln(1+2*x)-25/343*(-6412/961*x^3-55762/4805*x^2-188888/24025*x-202307
/48050)/(5*x^2+3*x+2)^2-16/343*ln(5*x^2+3*x+2)+125624/10218313*arctan(1/31*
(3+10*x)*31^(1/2))*31^(1/2)
```

Maxima [A] time = 1.50388, size = 104, normalized size = 1.17

$$\frac{125624}{10218313}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+\frac{45800x^3+79660x^2+53968x+28901}{94178\left(25x^4+30x^3+29x^2+12x+4\right)}-\frac{16}{343}\log\left(5x^2+3x+2\right)+\frac{3\ldots}{3\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 125624/10218313*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1/94178*(45800*
x^3 + 79660*x^2 + 53968*x + 28901)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) -
16/343*log(5*x^2 + 3*x + 2) + 32/343*log(2*x + 1)
```

Fricas [A] time = 2.80941, size = 432, normalized size = 4.85

$$\frac{9938600x^3+251248\sqrt{31}\left(25x^4+30x^3+29x^2+12x+4\right)\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+17286220x^2-953312\left(25x\ldots\right.}{20436626\left(25\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/20436626*(9938600*x^3 + 251248*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x
+ 4)*arctan(1/31*sqrt(31)*(10*x + 3)) + 17286220*x^2 - 953312*(25*x^4 + 30*
x^3 + 29*x^2 + 12*x + 4)*log(5*x^2 + 3*x + 2) + 1906624*(25*x^4 + 30*x^3 +
29*x^2 + 12*x + 4)*log(2*x + 1) + 11711056*x + 6271517)/(25*x^4 + 30*x^3 +
29*x^2 + 12*x + 4)
```

Sympy [A] time = 0.240652, size = 90, normalized size = 1.01

$$\frac{45800x^3 + 79660x^2 + 53968x + 28901}{2354450x^4 + 2825340x^3 + 2731162x^2 + 1130136x + 376712} + \frac{32\log\left(x + \frac{1}{2}\right)}{343} - \frac{16\log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{343} + \frac{125624\sqrt{31}\,\mathrm{ata}}{102}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(5*x**2+3*x+2)**3,x)
```

```
[Out] (45800*x**3 + 79660*x**2 + 53968*x + 28901)/(2354450*x**4 + 2825340*x**3 +
2731162*x**2 + 1130136*x + 376712) + 32*log(x + 1/2)/343 - 16*log(x**2 + 3*
x/5 + 2/5)/343 + 125624*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/102
18313
```

Giac [A] time = 1.1083, size = 92, normalized size = 1.03

$$\frac{125624}{10218313}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) + \frac{45800x^3 + 79660x^2 + 53968x + 28901}{94178\left(5x^2 + 3x + 2\right)^2} - \frac{16}{343}\log\left(5x^2 + 3x + 2\right) + \frac{32}{343}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 125624/10218313*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1/94178*(45800*
x^3 + 79660*x^2 + 53968*x + 28901)/(5*x^2 + 3*x + 2)^2 - 16/343*log(5*x^2 +
 3*x + 2) + 32/343*log(abs(2*x + 1))
```

3.2230 $\int \frac{1}{(1+2x)^2(2+3x+5x^2)^3}\,dx$

Optimal. Leaf size=114

$$\frac{20x+37}{434(2x+1)\left(5x^2+3x+2\right)^2}+\frac{5820x+6427}{47089(2x+1)\left(5x^2+3x+2\right)}-\frac{192\log\left(5x^2+3x+2\right)}{2401}-\frac{51516}{329623(2x+1)}+\frac{384\log(2}{240}$$

```
[Out] -51516/(329623*(1 + 2*x)) + (37 + 20*x)/(434*(1 + 2*x)*(2 + 3*x + 5*x^2)^2)
 + (6427 + 5820*x)/(47089*(1 + 2*x)*(2 + 3*x + 5*x^2)) - (1065012*ArcTan[(3
 + 10*x)/Sqrt[31]])/(2307361*Sqrt[31]) + (384*Log[1 + 2*x])/2401 - (192*Log
[2 + 3*x + 5*x^2])/2401
```

Rubi [A] time = 0.0882789, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {740, 822, 800, 634, 618, 204, 628}

$$\frac{20x+37}{434(2x+1)\left(5x^2+3x+2\right)^2}+\frac{5820x+6427}{47089(2x+1)\left(5x^2+3x+2\right)}-\frac{192\log\left(5x^2+3x+2\right)}{2401}-\frac{51516}{329623(2x+1)}+\frac{384\log(2}{240}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + 2*x)^2*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] -51516/(329623*(1 + 2*x)) + (37 + 20*x)/(434*(1 + 2*x)*(2 + 3*x + 5*x^2)^2)
 + (6427 + 5820*x)/(47089*(1 + 2*x)*(2 + 3*x + 5*x^2)) - (1065012*ArcTan[(3
 + 10*x)/Sqrt[31]])/(2307361*Sqrt[31]) + (384*Log[1 + 2*x])/2401 - (192*Log
[2 + 3*x + 5*x^2])/2401
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(1+2x)^2\left(2+3x+5x^2\right)^3}\,dx = \frac{37+20x}{434(1+2x)\left(2+3x+5x^2\right)^2}+\frac{1}{434}\int \frac{382+160x}{(1+2x)^2\left(2+3x+5x^2\right)^2}\,dx$$

$$= \frac{37+20x}{434(1+2x)\left(2+3x+5x^2\right)^2}+\frac{6427+5820x}{47089(1+2x)\left(2+3x+5x^2\right)}+\frac{\int \frac{74796+46560x}{(1+2x)^2\left(2+3x+5x^2\right)}\,dx}{94178}$$

$$= \frac{37+20x}{434(1+2x)\left(2+3x+5x^2\right)^2}+\frac{6427+5820x}{47089(1+2x)\left(2+3x+5x^2\right)}+\frac{\int \left(\frac{206064}{7(1+2x)^2}+\frac{1476096}{49(1+2x)}\ldots\right.}{94\ldots}$$

$$= -\frac{51516}{329623(1+2x)}+\frac{37+20x}{434(1+2x)\left(2+3x+5x^2\right)^2}+\frac{6427+5820x}{47089(1+2x)\left(2+3x+5x^2\right)}+\ldots$$

$$= -\frac{51516}{329623(1+2x)}+\frac{37+20x}{434(1+2x)\left(2+3x+5x^2\right)^2}+\frac{6427+5820x}{47089(1+2x)\left(2+3x+5x^2\right)}+\ldots$$

$$= -\frac{51516}{329623(1+2x)}+\frac{37+20x}{434(1+2x)\left(2+3x+5x^2\right)^2}+\frac{6427+5820x}{47089(1+2x)\left(2+3x+5x^2\right)}+\ldots$$

$$= -\frac{51516}{329623(1+2x)}+\frac{37+20x}{434(1+2x)\left(2+3x+5x^2\right)^2}+\frac{6427+5820x}{47089(1+2x)\left(2+3x+5x^2\right)}-\ldots$$

Mathematica [A] time = 0.0665952, size = 98, normalized size = 0.86

$$\frac{4\left(-\frac{47089(270x-43)}{8\left(5x^2+3x+2\right)^2}-\frac{217(51910x-15179)}{4\left(5x^2+3x+2\right)}-1429968\log\left(4\left(5x^2+3x+2\right)\right)-\frac{1668296}{2x+1}+2859936\log(2x+1)-266253\sqrt{31}\,\mathrm{t}\right.}{71528191}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + 2*x)^2*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] (4*(-1668296/(1 + 2*x) - (47089*(-43 + 270*x))/(8*(2 + 3*x + 5*x^2)^2) - (2
17*(-15179 + 51910*x))/(4*(2 + 3*x + 5*x^2)) - 266253*Sqrt[31]*ArcTan[(3 +
10*x)/Sqrt[31]] + 2859936*Log[1 + 2*x] - 1429968*Log[4*(2 + 3*x + 5*x^2)]))
/71528191
```

Maple [A] time = 0.05, size = 77, normalized size = 0.7

$$-\frac{32}{343+686\,x}+\frac{384\,\ln(1+2\,x)}{2401}-\frac{25}{2401\,\left(5\,x^2+3\,x+2\right)^2}\left(\frac{72674\,x^3}{961}+\frac{111769\,x^2}{4805}+\frac{613046\,x}{24025}-\frac{490329}{48050}\right)-\frac{192\,\ln(5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2*x)^2/(5*x^2+3*x+2)^3,x)
```

```
[Out] -32/343/(1+2*x)+384/2401*ln(1+2*x)-25/2401*(72674/961*x^3+111769/4805*x^2+6
13046/24025*x-490329/48050)/(5*x^2+3*x+2)^2-192/2401*ln(5*x^2+3*x+2)-106501
2/71528191*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Maxima [A] time = 1.51454, size = 117, normalized size = 1.03

$$-\frac{1065012}{71528191}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right)-\frac{2575800\,x^4+2683560\,x^3+2293598\,x^2+773110\,x+175969}{659246\left(50\,x^5+85\,x^4+88\,x^3+53\,x^2+20\,x+4\right)}-\frac{192}{2401}\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^2/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] -1065012/71528191*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 1/659246*(257
5800*x^4 + 2683560*x^3 + 2293598*x^2 + 773110*x + 175969)/(50*x^5 + 85*x^4
+ 88*x^3 + 53*x^2 + 20*x + 4) - 192/2401*log(5*x^2 + 3*x + 2) + 384/2401*lo
g(2*x + 1)
```

Fricas [A] time = 2.97443, size = 517, normalized size = 4.54

$$-\frac{558948600\,x^4+582332520\,x^3+2130024\,\sqrt{31}\left(50\,x^5+85\,x^4+88\,x^3+53\,x^2+20\,x+4\right)\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^2/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] -1/143056382*(558948600*x^4 + 582332520*x^3 + 2130024*sqrt(31)*(50*x^5 + 85
*x^4 + 88*x^3 + 53*x^2 + 20*x + 4)*arctan(1/31*sqrt(31)*(10*x + 3)) + 49771
0766*x^2 + 11439744*(50*x^5 + 85*x^4 + 88*x^3 + 53*x^2 + 20*x + 4)*log(5*x^
2 + 3*x + 2) - 22879488*(50*x^5 + 85*x^4 + 88*x^3 + 53*x^2 + 20*x + 4)*log(
2*x + 1) + 167764870*x + 38185273)/(50*x^5 + 85*x^4 + 88*x^3 + 53*x^2 + 20*
x + 4)
```

Sympy [A] time = 0.250377, size = 100, normalized size = 0.88

$$-\frac{2575800x^4+2683560x^3+2293598x^2+773110x+175969}{32962300x^5+56035910x^4+58013648x^3+34940038x^2+13184920x+2636984}+\frac{384\log\left(x+\frac{1}{2}\right)}{2401}-\frac{192\log\left(x^2+\frac{3}{5}\right.}{2401}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)**2/(5*x**2+3*x+2)**3,x)
```

```
[Out] -(2575800*x**4 + 2683560*x**3 + 2293598*x**2 + 773110*x + 175969)/(32962300
*x**5 + 56035910*x**4 + 58013648*x**3 + 34940038*x**2 + 13184920*x + 263698
4) + 384*log(x + 1/2)/2401 - 192*log(x**2 + 3*x/5 + 2/5)/2401 - 1065012*sqr
t(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/71528191
```

Giac [A] time = 1.10521, size = 146, normalized size = 1.28

$$-\frac{1065012}{71528191}\sqrt{31}\arctan\left(-\frac{1}{31}\sqrt{31}\left(\frac{7}{2x+1}-2\right)\right)-\frac{32}{343(2x+1)}+\frac{4\left(\frac{1178375}{2x+1}-\frac{2320190}{(2x+1)^2}+\frac{87843}{(2x+1)^3}-1304250\right)}{2307361\left(\frac{4}{2x+1}-\frac{7}{(2x+1)^2}-5\right)^2}-\frac{192}{2401}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^2/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] -1065012/71528191*sqrt(31)*arctan(-1/31*sqrt(31)*(7/(2*x + 1) - 2)) - 32/34
3/(2*x + 1) + 4/2307361*(1178375/(2*x + 1) - 2320190/(2*x + 1)^2 + 87843/(2
*x + 1)^3 - 1304250)/(4/(2*x + 1) - 7/(2*x + 1)^2 - 5)^2 - 192/2401*log(-4/
(2*x + 1) + 7/(2*x + 1)^2 + 5)
```

3.2231 $\int \frac{1}{(1+2x)(2+3x+5x^2)^4}\,dx$

Optimal. Leaf size=110

$$\frac{20x+37}{651\left(5x^2+3x+2\right)^3}+\frac{4(203230x+180133)}{10218313\left(5x^2+3x+2\right)}+\frac{4(1805x+1983)}{141267\left(5x^2+3x+2\right)^2}-\frac{64\log\left(5x^2+3x+2\right)}{2401}+\frac{128\log(2x+1)}{2401}$$

```
[Out] (37 + 20*x)/(651*(2 + 3*x + 5*x^2)^3) + (4*(1983 + 1805*x))/(141267*(2 + 3*
x + 5*x^2)^2) + (4*(180133 + 203230*x))/(10218313*(2 + 3*x + 5*x^2)) + (190
07376*ArcTan[(3 + 10*x)/Sqrt[31]])/(71528191*Sqrt[31]) + (128*Log[1 + 2*x])
/2401 - (64*Log[2 + 3*x + 5*x^2])/2401
```

Rubi [A] time = 0.0992217, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {740, 822, 800, 634, 618, 204, 628}

$$\frac{20x+37}{651\left(5x^2+3x+2\right)^3}+\frac{4(203230x+180133)}{10218313\left(5x^2+3x+2\right)}+\frac{4(1805x+1983)}{141267\left(5x^2+3x+2\right)^2}-\frac{64\log\left(5x^2+3x+2\right)}{2401}+\frac{128\log(2x+1)}{2401}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + 2*x)*(2 + 3*x + 5*x^2)^4),x]
```

```
[Out] (37 + 20*x)/(651*(2 + 3*x + 5*x^2)^3) + (4*(1983 + 1805*x))/(141267*(2 + 3*
x + 5*x^2)^2) + (4*(180133 + 203230*x))/(10218313*(2 + 3*x + 5*x^2)) + (190
07376*ArcTan[(3 + 10*x)/Sqrt[31]])/(71528191*Sqrt[31]) + (128*Log[1 + 2*x])
/2401 - (64*Log[2 + 3*x + 5*x^2])/2401
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(1+2x)\left(2+3x+5x^2\right)^4}\,dx = \frac{37+20x}{651\left(2+3x+5x^2\right)^3}+\frac{1}{651}\int \frac{472+200x}{(1+2x)\left(2+3x+5x^2\right)^3}\,dx$$
$$= \frac{37+20x}{651\left(2+3x+5x^2\right)^3}+\frac{4(1983+1805x)}{141267\left(2+3x+5x^2\right)^2}+\frac{\int \frac{135576+86640x}{(1+2x)\left(2+3x+5x^2\right)^2}\,dx}{282534}$$
$$= \frac{37+20x}{651\left(2+3x+5x^2\right)^3}+\frac{4(1983+1805x)}{141267\left(2+3x+5x^2\right)^2}+\frac{4(180133+203230x)}{10218313\left(2+3x+5x^2\right)}+\frac{\int \frac{1631}{(1+}$$
$$= \frac{37+20x}{651\left(2+3x+5x^2\right)^3}+\frac{4(1983+1805x)}{141267\left(2+3x+5x^2\right)^2}+\frac{4(180133+203230x)}{10218313\left(2+3x+5x^2\right)}+\frac{\int\Big(\frac{457}{7(}}{}$$
$$= \frac{37+20x}{651\left(2+3x+5x^2\right)^3}+\frac{4(1983+1805x)}{141267\left(2+3x+5x^2\right)^2}+\frac{4(180133+203230x)}{10218313\left(2+3x+5x^2\right)}+\frac{128\,\mathrm{lo}}{2}$$
$$= \frac{37+20x}{651\left(2+3x+5x^2\right)^3}+\frac{4(1983+1805x)}{141267\left(2+3x+5x^2\right)^2}+\frac{4(180133+203230x)}{10218313\left(2+3x+5x^2\right)}+\frac{128\,\mathrm{lo}}{2}$$
$$= \frac{37+20x}{651\left(2+3x+5x^2\right)^3}+\frac{4(1983+1805x)}{141267\left(2+3x+5x^2\right)^2}+\frac{4(180133+203230x)}{10218313\left(2+3x+5x^2\right)}+\frac{128\,\mathrm{lo}}{2}$$
$$= \frac{37+20x}{651\left(2+3x+5x^2\right)^3}+\frac{4(1983+1805x)}{141267\left(2+3x+5x^2\right)^2}+\frac{4(180133+203230x)}{10218313\left(2+3x+5x^2\right)}+\frac{19007}{7}$$

Mathematica [A] time = 0.112337, size = 88, normalized size = 0.8

$$\frac{16\left(\frac{217\left(60969000x^5+127202700x^4+143405620x^3+105257844x^2+44933184x+13831165\right)}{16\left(5x^2+3x+2\right)^3}-11082252\log\left(4\left(5x^2+3x+2\right)\right)+22164504\,\mathrm{l}\right.}{6652121763}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + 2*x)*(2 + 3*x + 5*x^2)^4),x]
```

```
[Out] (16*((217*(13831165 + 44933184*x + 105257844*x^2 + 143405620*x^3 + 12720270
0*x^4 + 60969000*x^5))/(16*(2 + 3*x + 5*x^2)^3) + 3563883*Sqrt[31]*ArcTan[(
3 + 10*x)/Sqrt[31]] + 22164504*Log[1 + 2*x] - 11082252*Log[4*(2 + 3*x + 5*x
^2)]))/6652121763
```

Maple [A] time = 0.051, size = 78, normalized size = 0.7

$$\frac{128\ \ln(1+2\,x)}{2401}-\frac{125}{2401\,\left(5\,x^2+3\,x+2\right)^3}\left(-\frac{1138088\,x^5}{29791}-\frac{11872252\,x^4}{148955}-\frac{200767868\,x^3}{2234325}-\frac{245601636\,x^2}{3723875}-\frac{1048440}{372387}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2*x)/(5*x^2+3*x+2)^4,x)
```

```
[Out] 128/2401*ln(1+2*x)-125/2401*(-1138088/29791*x^5-11872252/148955*x^4-2007678
68/2234325*x^3-245601636/3723875*x^2-104844096/3723875*x-19363631/2234325)/
```

```
(5*x^2+3*x+2)^3-64/2401*ln(5*x^2+3*x+2)+19007376/2217373921*arctan(1/31*(3+
10*x)*31^(1/2))*31^(1/2)
```

Maxima [A] time = 1.51665, size = 131, normalized size = 1.19

$$\frac{19007376}{2217373921}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+\frac{60969000x^5+127202700x^4+143405620x^3+105257844x^2+4493318}{30654939\left(125x^6+225x^5+285x^4+207x^3+114x^2+36x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

```
[Out] 19007376/2217373921*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1/30654939*
(60969000*x^5 + 127202700*x^4 + 143405620*x^3 + 105257844*x^2 + 44933184*x
+ 13831165)/(125*x^6 + 225*x^5 + 285*x^4 + 207*x^3 + 114*x^2 + 36*x + 8) -
64/2401*log(5*x^2 + 3*x + 2) + 128/2401*log(2*x + 1)
```

Fricas [A] time = 2.38046, size = 633, normalized size = 5.75

$$\frac{13230273000x^5+27602985900x^4+31119019540x^3+57022128\sqrt{31}\left(125x^6+225x^5+285x^4+207x^3+114x^2+36\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

```
[Out] 1/6652121763*(13230273000*x^5 + 27602985900*x^4 + 31119019540*x^3 + 5702212
8*sqrt(31)*(125*x^6 + 225*x^5 + 285*x^4 + 207*x^3 + 114*x^2 + 36*x + 8)*arc
tan(1/31*sqrt(31)*(10*x + 3)) + 22840952148*x^2 - 177316032*(125*x^6 + 225*
x^5 + 285*x^4 + 207*x^3 + 114*x^2 + 36*x + 8)*log(5*x^2 + 3*x + 2) + 354632
064*(125*x^6 + 225*x^5 + 285*x^4 + 207*x^3 + 114*x^2 + 36*x + 8)*log(2*x +
1) + 9750500928*x + 3001362805)/(125*x^6 + 225*x^5 + 285*x^4 + 207*x^3 + 11
4*x^2 + 36*x + 8)
```

Sympy [A] time = 0.274918, size = 110, normalized size = 1.

$$\frac{60969000x^5+127202700x^4+143405620x^3+105257844x^2+44933184x+13831165}{3831867375x^6+6897361275x^5+8736657615x^4+6345572373x^3+3494663046x^2+1103577804x+245239512}+\frac{128}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(5*x**2+3*x+2)**4,x)
```

```
[Out] (60969000*x**5 + 127202700*x**4 + 143405620*x**3 + 105257844*x**2 + 4493318
4*x + 13831165)/(3831867375*x**6 + 6897361275*x**5 + 8736657615*x**4 + 6345
572373*x**3 + 3494663046*x**2 + 1103577804*x + 245239512) + 128*log(x + 1/2
)/2401 - 64*log(x**2 + 3*x/5 + 2/5)/2401 + 19007376*sqrt(31)*atan(10*sqrt(3
1)*x/31 + 3*sqrt(31)/31)/2217373921
```

Giac [A] time = 1.07708, size = 105, normalized size = 0.95

$$\frac{19007376}{2217373921}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+\frac{60969000x^5+127202700x^4+143405620x^3+105257844x^2+4493}{30654939\left(5x^2+3x+2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(5*x^2+3*x+2)^4,x, algorithm="giac")
```

```
[Out] 19007376/2217373921*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1/30654939*
(60969000*x^5 + 127202700*x^4 + 143405620*x^3 + 105257844*x^2 + 44933184*x
+ 13831165)/(5*x^2 + 3*x + 2)^3 - 64/2401*log(5*x^2 + 3*x + 2) + 128/2401*l
og(abs(2*x + 1))
```

3.2232 $\int \frac{1}{(1+2x)^2(2+3x+5x^2)^4}\,dx$

Optimal. Leaf size=142

$$\frac{20x+37}{651(2x+1)\left(5x^2+3x+2\right)^3}+\frac{2(603620x+504757)}{10218313(2x+1)\left(5x^2+3x+2\right)}+\frac{2820x+3047}{47089(2x+1)\left(5x^2+3x+2\right)^2}-\frac{1024\log\left(5x^2+3x\cdot\right.}{16807}$$

```
[Out] -6802312/(71528191*(1 + 2*x)) + (37 + 20*x)/(651*(1 + 2*x)*(2 + 3*x + 5*x^2
)^3) + (3047 + 2820*x)/(47089*(1 + 2*x)*(2 + 3*x + 5*x^2)^2) + (2*(504757 +
 603620*x))/(10218313*(1 + 2*x)*(2 + 3*x + 5*x^2)) - (116056984*ArcTan[(3 +
 10*x)/Sqrt[31]])/(500697337*Sqrt[31]) + (2048*Log[1 + 2*x])/16807 - (1024*
Log[2 + 3*x + 5*x^2])/16807
```

Rubi [A] time = 0.108502, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {740, 822, 800, 634, 618, 204, 628}

$$\frac{20x+37}{651(2x+1)\left(5x^2+3x+2\right)^3}+\frac{2(603620x+504757)}{10218313(2x+1)\left(5x^2+3x+2\right)}+\frac{2820x+3047}{47089(2x+1)\left(5x^2+3x+2\right)^2}-\frac{1024\log\left(5x^2+3x\cdot\right.}{16807}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + 2*x)^2*(2 + 3*x + 5*x^2)^4),x]
```


```
[Out] -6802312/(71528191*(1 + 2*x)) + (37 + 20*x)/(651*(1 + 2*x)*(2 + 3*x + 5*x^2
)^3) + (3047 + 2820*x)/(47089*(1 + 2*x)*(2 + 3*x + 5*x^2)^2) + (2*(504757 +
 603620*x))/(10218313*(1 + 2*x)*(2 + 3*x + 5*x^2)) - (116056984*ArcTan[(3 +
 10*x)/Sqrt[31]])/(500697337*Sqrt[31]) + (2048*Log[1 + 2*x])/16807 - (1024*
Log[2 + 3*x + 5*x^2])/16807
```


Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(1+2x)^2\left(2+3x+5x^2\right)^4}dx = \frac{37+20x}{651(1+2x)\left(2+3x+5x^2\right)^3}+\frac{1}{651}\int \frac{546+240x}{(1+2x)^2\left(2+3x+5x^2\right)^3}dx$$

$$= \frac{37+20x}{651(1+2x)\left(2+3x+5x^2\right)^3}+\frac{3047+2820x}{47089(1+2x)\left(2+3x+5x^2\right)^2}+\frac{\int \frac{192972+135360x}{(1+2x)^2\left(2+3x+5x^2\right)^2}dx}{282534}$$

$$= \frac{37+20x}{651(1+2x)\left(2+3x+5x^2\right)^3}+\frac{3047+2820x}{47089(1+2x)\left(2+3x+5x^2\right)^2}+\frac{2(504757+603}{10218313(1+2x)(2}$$

$$= \frac{37+20x}{651(1+2x)\left(2+3x+5x^2\right)^3}+\frac{3047+2820x}{47089(1+2x)\left(2+3x+5x^2\right)^2}+\frac{2(504757+603}{10218313(1+2x)(2}$$

$$= -\frac{6802312}{71528191(1+2x)}+\frac{37+20x}{651(1+2x)\left(2+3x+5x^2\right)^3}+\frac{3047+2820x}{47089(1+2x)\left(2+3x+5x^2\right)^2}+$$

$$= -\frac{6802312}{71528191(1+2x)}+\frac{37+20x}{651(1+2x)\left(2+3x+5x^2\right)^3}+\frac{3047+2820x}{47089(1+2x)\left(2+3x+5x^2\right)^2}+$$

$$= -\frac{6802312}{71528191(1+2x)}+\frac{37+20x}{651(1+2x)\left(2+3x+5x^2\right)^3}+\frac{3047+2820x}{47089(1+2x)\left(2+3x+5x^2\right)^2}+$$

$$= -\frac{6802312}{71528191(1+2x)}+\frac{37+20x}{651(1+2x)\left(2+3x+5x^2\right)^3}+\frac{3047+2820x}{47089(1+2x)\left(2+3x+5x^2\right)^2}+$$

Mathematica [A] time = 0.0901292, size = 119, normalized size = 0.84

$$\frac{8\left(-\frac{10218313(270x-43)}{8\left(5x^2+3x+2\right)^3}-\frac{651(3736330x-1739037)}{4\left(5x^2+3x+2\right)}-\frac{141267(27530x-7117)}{8\left(5x^2+3x+2\right)^2}-354632064\log\left(4\left(5x^2+3x+2\right)\right)-\frac{310303056}{2x+1}+7092641\right.}{46564852341}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + 2*x)^2*(2 + 3*x + 5*x^2)^4),x]
```

```
[Out] (8*(-310303056/(1 + 2*x) - (10218313*(-43 + 270*x))/(8*(2 + 3*x + 5*x^2)^3)
 - (141267*(-7117 + 27530*x))/(8*(2 + 3*x + 5*x^2)^2) - (651*(-1739037 + 37
36330*x))/(4*(2 + 3*x + 5*x^2)) - 43521369*Sqrt[31]*ArcTan[(3 + 10*x)/Sqrt[
31]] + 709264128*Log[1 + 2*x] - 354632064*Log[4*(2 + 3*x + 5*x^2)]))/465648
52341
```

Maple [A] time = 0.05, size = 87, normalized size = 0.6

$$-\frac{128}{2401+4802\,x}+\frac{2048\ln(1+2\,x)}{16807}-\frac{125}{16807\left(5\,x^2+3\,x+2\right)^3}\left(\frac{10461724\,x^5}{29791}+\frac{38423826\,x^4}{148955}+\frac{199128958\,x^3}{744775}-\frac{694498}{37238}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2*x)^2/(5*x^2+3*x+2)^4,x)
```

```
[Out] -128/2401/(1+2*x)+2048/16807*ln(1+2*x)-125/16807*(10461724/29791*x^5+384238
26/148955*x^4+199128958/744775*x^3-6944987/3723875*x^2-410739/744775*x-3711
96343/11171625)/(5*x^2+3*x+2)^3-1024/16807*ln(5*x^2+3*x+2)-116056984/155216
17447*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Maxima [A] time = 1.56445, size = 144, normalized size = 1.01

$$-\frac{116056984}{15521617447}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right)-\frac{2550867000\,x^6+3957759600\,x^5+4525420710\,x^4+2788779072\,x}{214584573\left(250\,x^7+575\,x^6+795\,x^5+699\,x^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^2/(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

```
[Out] -116056984/15521617447*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 1/214584
573*(2550867000*x^6 + 3957759600*x^5 + 4525420710*x^4 + 2788779072*x^3 + 12
99394083*x^2 + 304894531*x + 38489903)/(250*x^7 + 575*x^6 + 795*x^5 + 699*x
^4 + 435*x^3 + 186*x^2 + 52*x + 8) - 1024/16807*log(5*x^2 + 3*x + 2) + 2048
/16807*log(2*x + 1)
```

Fricas [A] time = 2.43371, size = 726, normalized size = 5.11

$$-\frac{553538139000\,x^6+858833833200\,x^5+982016294070\,x^4+605165058624\,x^3+348170952\sqrt{31}\left(250\,x^7+575\,x^6+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^2/(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

```
[Out] -1/46564852341*(553538139000*x^6 + 858833833200*x^5 + 982016294070*x^4 + 60
5165058624*x^3 + 348170952*sqrt(31)*(250*x^7 + 575*x^6 + 795*x^5 + 699*x^4
+ 435*x^3 + 186*x^2 + 52*x + 8)*arctan(1/31*sqrt(31)*(10*x + 3)) + 28196851
6011*x^2 + 2837056512*(250*x^7 + 575*x^6 + 795*x^5 + 699*x^4 + 435*x^3 + 18
6*x^2 + 52*x + 8)*log(5*x^2 + 3*x + 2) - 5674113024*(250*x^7 + 575*x^6 + 79
5*x^5 + 699*x^4 + 435*x^3 + 186*x^2 + 52*x + 8)*log(2*x + 1) + 66162113227*
x + 8352308951)/(250*x^7 + 575*x^6 + 795*x^5 + 699*x^4 + 435*x^3 + 186*x^2
+ 52*x + 8)
```

Sympy [A] time = 0.299741, size = 121, normalized size = 0.85

$$-\frac{2550867000x^6+3957759600x^5+4525420710x^4+2788779072x^3+1299394083x^2+304894531x}{53646143250x^7+123386129475x^6+170594735535x^5+149994616527x^4+93344289255x^3+39912730578x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)**2/(5*x**2+3*x+2)**4,x)
```

```
[Out] -(2550867000*x**6 + 3957759600*x**5 + 4525420710*x**4 + 2788779072*x**3 + 1
299394083*x**2 + 304894531*x + 38489903)/(53646143250*x**7 + 123386129475*x
**6 + 170594735535*x**5 + 149994616527*x**4 + 93344289255*x**3 + 3991273057
```

```
8*x**2 + 11158397796*x + 1716676584) + 2048*log(x + 1/2)/16807 - 1024*log(x
**2 + 3*x/5 + 2/5)/16807 - 116056984*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqr
t(31)/31)/15521617447
```

Giac [A] time = 1.09174, size = 170, normalized size = 1.2

$$-\frac{116056984}{15521617447}\sqrt{31}\arctan\left(-\frac{1}{31}\sqrt{31}\left(\frac{7}{2x+1}-2\right)\right)-\frac{128}{2401\,(2x+1)}-\frac{8\left(\frac{3841449975}{2x+1}-\frac{8833663680}{(2x+1)^2}+\frac{7499779568}{(2x+1)^3}-\frac{7050406230}{(2x+1)^4}\right.}{1502092011\left(\frac{4}{2x+1}-\frac{7}{(2x+1)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^2/(5*x^2+3*x+2)^4,x, algorithm="giac")
```

```
[Out] -116056984/15521617447*sqrt(31)*arctan(-1/31*sqrt(31)*(7/(2*x + 1) - 2)) -
128/2401/(2*x + 1) - 8/1502092011*(3841449975/(2*x + 1) - 8833663680/(2*x +
 1)^2 + 7499779568/(2*x + 1)^3 - 7050406230/(2*x + 1)^4 + 1291725897/(2*x +
 1)^5 - 2009265250)/(4/(2*x + 1) - 7/(2*x + 1)^2 - 5)^3 - 1024/16807*log(-4
/(2*x + 1) + 7/(2*x + 1)^2 + 5)
```

3.2233 $\int \frac{7-3x}{-5+2x+x^2}\,dx$

Optimal. Leaf size=47

$$-\frac{1}{6}\left(9-5\sqrt{6}\right)\log\left(x-\sqrt{6}+1\right)-\frac{1}{6}\left(9+5\sqrt{6}\right)\log\left(x+\sqrt{6}+1\right)$$

```
[Out] -((9 - 5*Sqrt[6])*Log[1 - Sqrt[6] + x])/6 - ((9 + 5*Sqrt[6])*Log[1 + Sqrt[6
] + x])/6
```

Rubi [A] time = 0.026857, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {632, 31}

$$-\frac{1}{6}\left(9-5\sqrt{6}\right)\log\left(x-\sqrt{6}+1\right)-\frac{1}{6}\left(9+5\sqrt{6}\right)\log\left(x+\sqrt{6}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(7 - 3*x)/(-5 + 2*x + x^2),x]
```

```
[Out] -((9 - 5*Sqrt[6])*Log[1 - Sqrt[6] + x])/6 - ((9 + 5*Sqrt[6])*Log[1 + Sqrt[6
] + x])/6
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{7-3x}{-5+2x+x^2}\,dx &= \frac{1}{6}\left(-9+5\sqrt{6}\right)\int \frac{1}{1-\sqrt{6}+x}\,dx-\frac{1}{6}\left(9+5\sqrt{6}\right)\int \frac{1}{1+\sqrt{6}+x}\,dx\\ &= -\frac{1}{6}\left(9-5\sqrt{6}\right)\log\left(1-\sqrt{6}+x\right)-\frac{1}{6}\left(9+5\sqrt{6}\right)\log\left(1+\sqrt{6}+x\right)\end{aligned}$$

Mathematica [A] time = 0.0375687, size = 47, normalized size = 1.

$$\frac{1}{6}\left(5\sqrt{6}-9\right)\log\left(-x+\sqrt{6}-1\right)+\frac{1}{6}\left(-9-5\sqrt{6}\right)\log\left(x+\sqrt{6}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 - 3*x)/(-5 + 2*x + x^2),x]
```

```
[Out] ((-9 + 5*Sqrt[6])*Log[-1 + Sqrt[6] - x])/6 + ((-9 - 5*Sqrt[6])*Log[1 + Sqrt
[6] + x])/6
```

Maple [A] time = 0.041, size = 29, normalized size = 0.6

$$-\frac{3\ln\left(x^2+2x-5\right)}{2}-\frac{5\sqrt{6}}{3}\operatorname{Artanh}\left(\frac{(2x+2)\sqrt{6}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7-3*x)/(x^2+2*x-5),x)
```

```
[Out] -3/2*ln(x^2+2*x-5)-5/3*6^(1/2)*arctanh(1/12*(2*x+2)*6^(1/2))
```

Maxima [A] time = 1.50902, size = 47, normalized size = 1.

$$\frac{5}{6}\sqrt{6}\log\left(\frac{x-\sqrt{6}+1}{x+\sqrt{6}+1}\right)-\frac{3}{2}\log\left(x^2+2x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7-3*x)/(x^2+2*x-5),x, algorithm="maxima")
```

```
[Out] 5/6*sqrt(6)*log((x - sqrt(6) + 1)/(x + sqrt(6) + 1)) - 3/2*log(x^2 + 2*x -
5)
```

Fricas [A] time = 2.27729, size = 151, normalized size = 3.21

$$\frac{5}{6}\sqrt{3}\sqrt{2}\log\left(-\frac{2\sqrt{3}\sqrt{2}(x+1)-x^2-2x-7}{x^2+2x-5}\right)-\frac{3}{2}\log\left(x^2+2x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7-3*x)/(x^2+2*x-5),x, algorithm="fricas")
```

```
[Out] 5/6*sqrt(3)*sqrt(2)*log(-(2*sqrt(3)*sqrt(2)*(x + 1) - x^2 - 2*x - 7)/(x^2 +
 2*x - 5)) - 3/2*log(x^2 + 2*x - 5)
```

Sympy [A] time = 0.111478, size = 44, normalized size = 0.94

$$-\left(\frac{3}{2}+\frac{5\sqrt{6}}{6}\right)\log\left(x+1+\sqrt{6}\right)-\left(\frac{3}{2}-\frac{5\sqrt{6}}{6}\right)\log\left(x-\sqrt{6}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7-3*x)/(x**2+2*x-5),x)
```

```
[Out] -(3/2 + 5*sqrt(6)/6)*log(x + 1 + sqrt(6)) - (3/2 - 5*sqrt(6)/6)*log(x - sqr
t(6) + 1)
```

Giac [A] time = 1.12449, size = 59, normalized size = 1.26

$$\frac{5}{6}\,\sqrt{6}\log\left(\frac{|2\,x-2\,\sqrt{6}+2|}{|2\,x+2\,\sqrt{6}+2|}\right)-\frac{3}{2}\,\log\left(|x^{2}+2\,x-5|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7-3*x)/(x^2+2*x-5),x, algorithm="giac")
```

```
[Out] 5/6*sqrt(6)*log(abs(2*x - 2*sqrt(6) + 2)/abs(2*x + 2*sqrt(6) + 2)) - 3/2*lo
g(abs(x^2 + 2*x - 5))
```

3.2234 $\int \frac{1}{(-1+x)(1+x+x^2)} dx$

Optimal. Leaf size=41

$$-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x]/3 - Log[1 + x + x^2]/6
```

Rubi [A] time = 0.0274052, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {705, 31, 634, 618, 204, 628}

$$-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x)*(1 + x + x^2)),x]
```

```
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x]/3 - Log[1 + x + x^2]/6
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-1+x)\left(1+x+x^2\right)}\,dx &= \frac{1}{3}\int \frac{1}{-1+x}\,dx+\frac{1}{3}\int \frac{-2-x}{1+x+x^2}\,dx\\ &= \frac{1}{3}\log(1-x)-\frac{1}{6}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1+x+x^2}\,dx\\ &= \frac{1}{3}\log(1-x)-\frac{1}{6}\log\left(1+x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\ &= -\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log(1-x)-\frac{1}{6}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0083459, size = 41, normalized size = 1.

$$-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x)*(1 + x + x^2)),x]

[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x]/3 - Log[1 + x + x^2]/6
```

Maple [A] time = 0.043, size = 33, normalized size = 0.8

$$\frac{\ln(-1+x)}{3}-\frac{\ln\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+x)/(x^2+x+1),x)

[Out] 1/3*ln(-1+x)-1/6*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.45487, size = 43, normalized size = 1.05

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(x^2 + x + 1) + 1/3*log
(x - 1)
```

Fricas [A] time = 2.26713, size = 113, normalized size = 2.76

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(x^2 + x + 1) + 1/3*log
(x - 1)
```

Sympy [A] time = 0.129621, size = 41, normalized size = 1.

$$\frac{\log\left(x-1\right)}{3}-\frac{\log\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x**2+x+1),x)
```

```
[Out] log(x - 1)/3 - log(x**2 + x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3
)/3
```

Giac [A] time = 1.10052, size = 45, normalized size = 1.1

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x^2+x+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(x^2 + x + 1) + 1/3*log
(abs(x - 1))
```

3.2235 $\int \frac{2\left(\left(\frac{a}{b}\right)^{\frac{1}{n}}-x\cos\left(\frac{(-1+2k)\pi}{n}\right)\right)}{\left(\frac{a}{b}\right)^{2/n}+x^2-2\left(\frac{a}{b}\right)^{\frac{1}{n}}x\cos\left(\frac{(-1+2k)\pi}{n}\right)}\,dx$

Optimal. Leaf size=114

$$2\sin\left(\frac{\pi-2\pi k}{n}\right)\tan^{-1}\left(\left(\frac{a}{b}\right)^{-1/n}\csc\left(\frac{\pi-2\pi k}{n}\right)\left(x-\left(\frac{a}{b}\right)^{\frac{1}{n}}\cos\left(\frac{\pi-2\pi k}{n}\right)\right)\right)-\cos\left(\frac{\pi-2\pi k}{n}\right)\log\left(-2x\left(\frac{a}{b}\right)^{\frac{1}{n}}\cos\left(\frac{\pi-
2\pi k}{n}\right.\right.$$

```
[Out] -(Cos[(Pi - 2*k*Pi)/n]*Log[(a/b)^(2/n) + x^2 - 2*(a/b)^n^(-1)*x*Cos[(Pi - 2
*k*Pi)/n]]) + 2*ArcTan[((x - (a/b)^n^(-1)*Cos[(Pi - 2*k*Pi)/n])*Csc[(Pi - 2
*k*Pi)/n])/(a/b)^n^(-1)]*Sin[(Pi - 2*k*Pi)/n]
```

Rubi [A] time = 0.261596, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 66, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.076, Rules used = {12, 634, 618, 204, 628}

$$2\sin\left(\frac{\pi-2\pi k}{n}\right)\tan^{-1}\left(\left(\frac{a}{b}\right)^{-1/n}\csc\left(\frac{\pi-2\pi k}{n}\right)\left(x-\left(\frac{a}{b}\right)^{\frac{1}{n}}\cos\left(\frac{\pi-2\pi k}{n}\right)\right)\right)-\cos\left(\frac{\pi-2\pi k}{n}\right)\log\left(-2x\left(\frac{a}{b}\right)^{\frac{1}{n}}\cos\left(\frac{\pi-
2\pi k}{n}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(2*((a/b)^n^(-1) - x*Cos[((-1 + 2*k)*Pi)/n]))/((a/b)^(2/n) + x^2 - 2*(a
/b)^n^(-1)*x*Cos[((-1 + 2*k)*Pi)/n]),x]
```

```
[Out] -(Cos[(Pi - 2*k*Pi)/n]*Log[(a/b)^(2/n) + x^2 - 2*(a/b)^n^(-1)*x*Cos[(Pi - 2
*k*Pi)/n]]) + 2*ArcTan[((x - (a/b)^n^(-1)*Cos[(Pi - 2*k*Pi)/n])*Csc[(Pi - 2
*k*Pi)/n])/(a/b)^n^(-1)]*Sin[(Pi - 2*k*Pi)/n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{2\left(\left(\frac{a}{b}\right)^{\frac{1}{n}}-x\cos\left(\frac{(-1+2k)\pi}{n}\right)\right)}{\left(\frac{a}{b}\right)^{2/n}+x^2-2\left(\frac{a}{b}\right)^{\frac{1}{n}}x\cos\left(\frac{(-1+2k)\pi}{n}\right)}\,dx = 2\int \frac{\left(\frac{a}{b}\right)^{\frac{1}{n}}-x\cos\left(\frac{(-1+2k)\pi}{n}\right)}{\left(\frac{a}{b}\right)^{2/n}+x^2-2\left(\frac{a}{b}\right)^{\frac{1}{n}}x\cos\left(\frac{(-1+2k)\pi}{n}\right)}\,dx$$

$$= -\left(\cos\left(\frac{(-1+2k)\pi}{n}\right)\int \frac{2x-2\left(\frac{a}{b}\right)^{\frac{1}{n}}\cos\left(\frac{(-1+2k)\pi}{n}\right)}{\left(\frac{a}{b}\right)^{2/n}+x^2-2\left(\frac{a}{b}\right)^{\frac{1}{n}}x\cos\left(\frac{(-1+2k)\pi}{n}\right)}\,dx\right)+\left(2\left(\frac{a}{b}\right)^{\frac{1}{n}}\right.$$

$$= -\cos\left(\frac{(1-2k)\pi}{n}\right)\log\left(\left(\frac{a}{b}\right)^{2/n}+x^2-2\left(\frac{a}{b}\right)^{\frac{1}{n}}x\cos\left(\frac{\pi-2k\pi}{n}\right)\right)+\left(2\left(-2\left(\frac{a}{b}\right.\right.\right.$$

$$= -\cos\left(\frac{(1-2k)\pi}{n}\right)\log\left(\left(\frac{a}{b}\right)^{2/n}+x^2-2\left(\frac{a}{b}\right)^{\frac{1}{n}}x\cos\left(\frac{\pi-2k\pi}{n}\right)\right)+2\tan^{-1}\left(\left(\right.\right.$$

Mathematica [A] time = 0.0716779, size = 111, normalized size = 0.97

$$2\left(\sin\left(\frac{\pi(2k-1)}{n}\right)\tan^{-1}\left(\frac{\tan\left(\frac{\pi(2k-1)}{2n}\right)\left(\left(\frac{a}{b}\right)^{\frac{1}{n}}+x\right)}{\left(\frac{a}{b}\right)^{\frac{1}{n}}-x}\right)-\frac{1}{2}\cos\left(\frac{\pi(2k-1)}{n}\right)\log\left(-2x\left(\frac{a}{b}\right)^{\frac{1}{n}}\cos\left(\frac{\pi(2k-1)}{n}\right)+\left(\frac{a}{b}\right)^{2/n}+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*((a/b)^n^(-1) - x*Cos[((-1 + 2*k)*Pi)/n]))/((a/b)^(2/n) + x^2
- 2*(a/b)^n^(-1)*x*Cos[((-1 + 2*k)*Pi)/n]),x]
```

```
[Out] 2*(-(Cos[((-1 + 2*k)*Pi)/n]*Log[(a/b)^(2/n) + x^2 - 2*(a/b)^n^(-1)*x*Cos[((
-1 + 2*k)*Pi)/n]])/2 + ArcTan[(((a/b)^n^(-1) + x)*Tan[((-1 + 2*k)*Pi)/(2*n)
])/((a/b)^n^(-1) - x)]*Sin[((-1 + 2*k)*Pi)/n])
```

Maple [B] time = 0.274, size = 311, normalized size = 2.7

$$-\cos\left(\frac{(2k-1)\pi}{n}\right)\ln\left(2\sqrt[n]{\frac{a}{b}}x\cos\left(\frac{(2k-1)\pi}{n}\right)-x^2-\left(\frac{a}{b}\right)^{2n^{-1}}\right)+2\arctan\left(1/2\left(2\sqrt[n]{\frac{a}{b}}\cos\left(\frac{(2k-1)\pi}{n}\right)-2x\right)\frac{}{\sqrt{\left(\frac{a}{b}\right)^{2n^{-}}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*((1/b*a)^(1/n)-x*cos(Pi*(2*k-1)/n))/((1/b*a)^(2/n)+x^2-2*(1/b*a)^(1/n
)*x*cos(Pi*(2*k-1)/n)),x)
```

```
[Out] -cos(Pi*(2*k-1)/n)*ln(2*(1/b*a)^(1/n)*x*cos(Pi*(2*k-1)/n)-x^2-(1/b*a)^(2/n)
)+2/((1/b*a)^(2/n)-((1/b*a)^(1/n))^2*cos(Pi*(2*k-1)/n)^2)^(1/2)*arctan(1/2*
(2*(1/b*a)^(1/n)*cos(Pi*(2*k-1)/n)-2*x)/((1/b*a)^(2/n)-((1/b*a)^(1/n))^2*co
```

```
s(Pi*(2*k-1)/n)^2)^(1/2))*cos(Pi*(2*k-1)/n)^2*(1/b*a)^(1/n)-2/((1/b*a)^(2/n
)-((1/b*a)^(1/n))^2*cos(Pi*(2*k-1)/n)^2)^(1/2)*arctan(1/2*(2*(1/b*a)^(1/n)*
cos(Pi*(2*k-1)/n)-2*x)/((1/b*a)^(2/n)-((1/b*a)^(1/n))^2*cos(Pi*(2*k-1)/n)^2
)^(1/2))*(1/b*a)^(1/n)
```

Maxima [A] time = 1.50017, size = 282, normalized size = 2.47

$$-\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)\log\left(-2x\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)+x^2+\left(\frac{a}{b}\right)^{\frac{2}{n}}\right)-\sqrt{\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)^2-1}\log\left(\frac{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)}{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*((a/b)^(1/n)-x*cos((-1+2*k)*pi/n))/((a/b)^(2/n)+x^2-2*(a/b)^(1/
n)*x*cos((-1+2*k)*pi/n)),x, algorithm="maxima")
```

```
[Out] -cos(2*pi*k/n - pi/n)*log(-2*x*(a/b)^(1/n)*cos(2*pi*k/n - pi/n) + x^2 + (a/
b)^(2/n)) - sqrt(cos(2*pi*k/n - pi/n)^2 - 1)*log(((a/b)^(1/n)*cos(2*pi*k/n
- pi/n) + sqrt(cos(2*pi*k/n - pi/n)^2 - 1)*(a/b)^(1/n) - x)/((a/b)^(1/n)*co
s(2*pi*k/n - pi/n) - sqrt(cos(2*pi*k/n - pi/n)^2 - 1)*(a/b)^(1/n) - x))
```

Fricas [A] time = 2.57633, size = 302, normalized size = 2.65

$$-\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)\log\left(-\frac{2\left(2x\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)-x^2-\left(\frac{a}{b}\right)^{\frac{2}{n}}\right)}{\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)+1}\right)-2\arctan\left(\frac{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)-x}{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\sin\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)}\right)\sin\left(\frac{2\pi k}{n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*((a/b)^(1/n)-x*cos((-1+2*k)*pi/n))/((a/b)^(2/n)+x^2-2*(a/b)^(1/
n)*x*cos((-1+2*k)*pi/n)),x, algorithm="fricas")
```

```
[Out] -cos(2*pi*k/n - pi/n)*log(-2*(2*x*(a/b)^(1/n)*cos(2*pi*k/n - pi/n) - x^2 -
(a/b)^(2/n))/(cos(2*pi*k/n - pi/n) + 1)) - 2*arctan(((a/b)^(1/n)*cos(2*pi*k
/n - pi/n) - x)/((a/b)^(1/n)*sin(2*pi*k/n - pi/n)))*sin(2*pi*k/n - pi/n)
```

Sympy [A] time = 1.07647, size = 177, normalized size = 1.55

$$-\left(-\sqrt{\left(\cos\left(\frac{\pi(2k-1)}{n}\right)-1\right)\left(\cos\left(\frac{\pi(2k-1)}{n}\right)+1\right)}+\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)\right)\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{n}}\left(-\sqrt{\left(\cos\left(\frac{\pi(2k-1)}{n}\right)-1\right)\left(\cos\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*((a/b)**(1/n)-x*cos((-1+2*k)*pi/n))/((a/b)**(2/n)+x**2-2*(a/b)*
*(1/n)*x*cos((-1+2*k)*pi/n)),x)
```

```
[Out] -(-sqrt((cos(pi*(2*k - 1)/n) - 1)*(cos(pi*(2*k - 1)/n) + 1)) + cos(2*pi*k/n
 - pi/n))*log(x - (a/b)**(1/n)*(-sqrt((cos(pi*(2*k - 1)/n) - 1)*(cos(pi*(2*
k - 1)/n) + 1)) + cos(2*pi*k/n - pi/n))) - (sqrt((cos(pi*(2*k - 1)/n) - 1)*
```

```
(cos(pi*(2*k - 1)/n) + 1)) + cos(2*pi*k/n - pi/n))*log(x - (a/b)**(1/n)*(sq
rt((cos(pi*(2*k - 1)/n) - 1)*(cos(pi*(2*k - 1)/n) + 1)) + cos(2*pi*k/n - pi
/n)))
```

Giac [A] time = 1.19875, size = 273, normalized size = 2.39

$$-\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)\log\left(-2x\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)+x^2+\left(\frac{a}{b}\right)^{\frac{2}{n}}\right)-\frac{2\left(\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)^2-\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\right)\arctan\left(-\frac{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}}{\sqrt{-\cos(}}\right.}{\sqrt{-\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)^2+1}\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*((a/b)^(1/n)-x*cos((-1+2*k)*pi/n))/((a/b)^(2/n)+x^2-2*(a/b)^(1/
n)*x*cos((-1+2*k)*pi/n)),x, algorithm="giac")
```

```
[Out] -cos(2*pi*k/n - pi/n)*log(-2*x*(a/b)^(1/n)*cos(2*pi*k/n - pi/n) + x^2 + (a/
b)^(2/n)) - 2*((a/b)^(1/n)*cos(2*pi*k/n - pi/n)^2 - (a/b)^(1/n))*arctan(-((
a/b)^(1/n)*cos(2*pi*k/n - pi/n) - x)/(sqrt(-cos(2*pi*k/n - pi/n)^2 + 1)*(a/
b)^(1/n)))/(sqrt(-cos(2*pi*k/n - pi/n)^2 + 1)*(a/b)^(1/n))
```

3.2236 $\int \frac{x^4}{2+13x+15x^2} \, dx$

Optimal. Leaf size=40

$$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}-\frac{16}{567}\log(3x+2)+\frac{\log(5x+1)}{4375}$$

```
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]
/4375
```

Rubi [A] time = 0.0172645, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {701, 632, 31}

$$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}-\frac{16}{567}\log(3x+2)+\frac{\log(5x+1)}{4375}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(2 + 13*x + 15*x^2),x]
```

```
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]
/4375
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{2+13x+15x^2} \, dx &= \int \left(\frac{139}{3375}-\frac{13x}{225}+\frac{x^2}{15}-\frac{278+1417x}{3375\left(2+13x+15x^2\right)}\right) dx \\
&= \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{\int \frac{278+1417x}{2+13x+15x^2} \, dx}{3375} \\
&= \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}+\frac{3}{875}\int \frac{1}{3+15x} \, dx-\frac{80}{189}\int \frac{1}{10+15x} \, dx \\
&= \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16}{567}\log(2+3x)+\frac{\log(1+5x)}{4375}
\end{aligned}$$

Mathematica [A] time = 0.0052034, size = 40, normalized size = 1.

$$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}-\frac{16}{567}\log(3x+2)+\frac{\log(5x+1)}{4375}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(2 + 13*x + 15*x^2),x]
```

```
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]
/4375
```

Maple [A] time = 0.044, size = 31, normalized size = 0.8

$$\frac{139\,x}{3375}-\frac{13\,x^2}{450}+\frac{x^3}{45}-\frac{16\,\ln(2+3\,x)}{567}+\frac{\ln(1+5\,x)}{4375}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(15*x^2+13*x+2),x)
```

```
[Out] 139/3375*x-13/450*x^2+1/45*x^3-16/567*ln(2+3*x)+1/4375*ln(1+5*x)
```

Maxima [A] time = 0.97224, size = 41, normalized size = 1.02

$$\frac{1}{45}\,x^3-\frac{13}{450}\,x^2+\frac{139}{3375}\,x+\frac{1}{4375}\,\log(5\,x+1)-\frac{16}{567}\,\log(3\,x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(15*x^2+13*x+2),x, algorithm="maxima")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x +
 2)
```

Fricas [A] time = 2.27832, size = 108, normalized size = 2.7

$$\frac{1}{45}\,x^3-\frac{13}{450}\,x^2+\frac{139}{3375}\,x+\frac{1}{4375}\,\log(5\,x+1)-\frac{16}{567}\,\log(3\,x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(15*x^2+13*x+2),x, algorithm="fricas")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x +
 2)
```

Sympy [A] time = 0.105138, size = 34, normalized size = 0.85

$$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}+\frac{\log\left(x+\frac{1}{5}\right)}{4375}-\frac{16\log\left(x+\frac{2}{3}\right)}{567}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(15*x**2+13*x+2),x)
```

```
[Out] x**3/45 - 13*x**2/450 + 139*x/3375 + log(x + 1/5)/4375 - 16*log(x + 2/3)/56
7
```

Giac [A] time = 1.09413, size = 43, normalized size = 1.08

$$\frac{1}{45}x^3 - \frac{13}{450}x^2 + \frac{139}{3375}x + \frac{1}{4375}\log(|5x+1|) - \frac{16}{567}\log(|3x+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(15*x^2+13*x+2),x, algorithm="giac")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(abs(5*x + 1)) - 16/567*log(
abs(3*x + 2))
```

3.2237 $\int \frac{x^3}{2+13x+15x^2}\,dx$

Optimal. Leaf size=33

$$\frac{x^2}{30}-\frac{13x}{225}+\frac{8}{189}\log(3x+2)-\frac{1}{875}\log(5x+1)$$

```
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Rubi [A] time = 0.0161926, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {701, 632, 31}

$$\frac{x^2}{30}-\frac{13x}{225}+\frac{8}{189}\log(3x+2)-\frac{1}{875}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(2 + 13*x + 15*x^2),x]

[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{2+13x+15x^2}\,dx &= \int \left(-\frac{13}{225}+\frac{x}{15}+\frac{26+139x}{225\left(2+13x+15x^2\right)}\right)dx\\
&= -\frac{13x}{225}+\frac{x^2}{30}+\frac{1}{225}\int \frac{26+139x}{2+13x+15x^2}\,dx\\
&= -\frac{13x}{225}+\frac{x^2}{30}-\frac{3}{175}\int \frac{1}{3+15x}\,dx+\frac{40}{63}\int \frac{1}{10+15x}\,dx\\
&= -\frac{13x}{225}+\frac{x^2}{30}+\frac{8}{189}\log(2+3x)-\frac{1}{875}\log(1+5x)
\end{aligned}$$

Mathematica [A] time = 0.0042784, size = 33, normalized size = 1.

$$\frac{x^2}{30}-\frac{13x}{225}+\frac{8}{189}\log(3x+2)-\frac{1}{875}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(2 + 13*x + 15*x^2),x]
```

```
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Maple [A] time = 0.044, size = 26, normalized size = 0.8

$$-\frac{13\,x}{225}+\frac{x^2}{30}+\frac{8\,\ln(2+3\,x)}{189}-\frac{\ln(1+5\,x)}{875}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(15*x^2+13*x+2),x)
```

```
[Out] -13/225*x+1/30*x^2+8/189*ln(2+3*x)-1/875*ln(1+5*x)
```

Maxima [A] time = 0.976913, size = 34, normalized size = 1.03

$$\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\log(5\,x+1)+\frac{8}{189}\,\log(3\,x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(15*x^2+13*x+2),x, algorithm="maxima")
```

```
[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

Fricas [A] time = 2.16127, size = 85, normalized size = 2.58

$$\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\log(5\,x+1)+\frac{8}{189}\,\log(3\,x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(15*x^2+13*x+2),x, algorithm="fricas")
```

```
[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

Sympy [A] time = 0.108456, size = 27, normalized size = 0.82

$$\frac{x^2}{30}-\frac{13x}{225}-\frac{\log\left(x+\frac{1}{5}\right)}{875}+\frac{8\log\left(x+\frac{2}{3}\right)}{189}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(15*x**2+13*x+2),x)
```

```
[Out] x**2/30 - 13*x/225 - log(x + 1/5)/875 + 8*log(x + 2/3)/189
```

Giac [A] time = 1.11101, size = 36, normalized size = 1.09

$$\frac{1}{30}x^2 - \frac{13}{225}x - \frac{1}{875}\log(|5x+1|) + \frac{8}{189}\log(|3x+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(15*x^2+13*x+2),x, algorithm="giac")
```

```
[Out] 1/30*x^2 - 13/225*x - 1/875*log(abs(5*x + 1)) + 8/189*log(abs(3*x + 2))
```

3.2238 $\int \frac{x^2}{2+13x+15x^2} dx$

Optimal. Leaf size=26

$$\frac{x}{15} - \frac{4}{63}\log(3x+2) + \frac{1}{175}\log(5x+1)$$

```
[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Rubi [A] time = 0.0124871, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {703, 632, 31}

$$\frac{x}{15} - \frac{4}{63}\log(3x+2) + \frac{1}{175}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 + 13*x + 15*x^2),x]
```

```
[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{2+13x+15x^2}\,dx &= \frac{x}{15}+\frac{1}{15}\int \frac{-2-13x}{2+13x+15x^2}\,dx\\ &= \frac{x}{15}+\frac{3}{35}\int \frac{1}{3+15x}\,dx-\frac{20}{21}\int \frac{1}{10+15x}\,dx\\ &= \frac{x}{15}-\frac{4}{63}\log(2+3x)+\frac{1}{175}\log(1+5x)\end{aligned}$$

Mathematica [A] time = 0.0034883, size = 26, normalized size = 1.

$$\frac{x}{15} - \frac{4}{63}\log(3x+2) + \frac{1}{175}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 + 13*x + 15*x^2),x]
```

```
[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Maple [A] time = 0.044, size = 21, normalized size = 0.8

$$\frac{x}{15}-\frac{4\ln\left(2+3x\right)}{63}+\frac{\ln\left(1+5x\right)}{175}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(15*x^2+13*x+2),x)
```

```
[Out] 1/15*x-4/63*ln(2+3*x)+1/175*ln(1+5*x)
```

Maxima [A] time = 0.985239, size = 27, normalized size = 1.04

$$\frac{1}{15}x+\frac{1}{175}\log\left(5x+1\right)-\frac{4}{63}\log\left(3x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(15*x^2+13*x+2),x, algorithm="maxima")
```

```
[Out] 1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

Fricas [A] time = 2.2354, size = 66, normalized size = 2.54

$$\frac{1}{15}x+\frac{1}{175}\log\left(5x+1\right)-\frac{4}{63}\log\left(3x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(15*x^2+13*x+2),x, algorithm="fricas")
```

```
[Out] 1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

Sympy [A] time = 0.102507, size = 20, normalized size = 0.77

$$\frac{x}{15}+\frac{\log\left(x+\frac{1}{5}\right)}{175}-\frac{4\log\left(x+\frac{2}{3}\right)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(15*x**2+13*x+2),x)
```

```
[Out] x/15 + log(x + 1/5)/175 - 4*log(x + 2/3)/63
```

Giac [A] time = 1.10693, size = 30, normalized size = 1.15

$$\frac{1}{15}\,x + \frac{1}{175}\,\log\left(|5\,x + 1|\right) - \frac{4}{63}\,\log\left(|3\,x + 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(15*x^2+13*x+2),x, algorithm="giac")
```

```
[Out] 1/15*x + 1/175*log(abs(5*x + 1)) - 4/63*log(abs(3*x + 2))
```

3.2239 $\int \frac{x}{2+13x+15x^2} dx$

Optimal. Leaf size=21

$$\frac{2}{21}\log(3x+2) - \frac{1}{35}\log(5x+1)$$

```
[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

Rubi [A] time = 0.0058873, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {632, 31}

$$\frac{2}{21}\log(3x+2) - \frac{1}{35}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x/(2 + 13*x + 15*x^2),x]
```

```
[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{2+13x+15x^2}\,dx &= -\left(\frac{3}{7}\int \frac{1}{3+15x}\,dx\right)+\frac{10}{7}\int \frac{1}{10+15x}\,dx\\ &= \frac{2}{21}\log(2+3x)-\frac{1}{35}\log(1+5x)\end{aligned}$$

Mathematica [A] time = 0.0032549, size = 21, normalized size = 1.

$$\frac{2}{21}\log(3x+2) - \frac{1}{35}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(2 + 13*x + 15*x^2),x]
```

```
[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

Maple [A] time = 0.043, size = 18, normalized size = 0.9

$$\frac{2\ln\left(2+3x\right)}{21}-\frac{\ln\left(1+5x\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(15*x^2+13*x+2),x)
```

```
[Out] 2/21*ln(2+3*x)-1/35*ln(1+5*x)
```

Maxima [A] time = 0.960248, size = 23, normalized size = 1.1

$$-\frac{1}{35}\log\left(5x+1\right)+\frac{2}{21}\log\left(3x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(15*x^2+13*x+2),x, algorithm="maxima")
```

```
[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

Fricas [A] time = 2.06817, size = 54, normalized size = 2.57

$$-\frac{1}{35}\log\left(5x+1\right)+\frac{2}{21}\log\left(3x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(15*x^2+13*x+2),x, algorithm="fricas")
```

```
[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

Sympy [A] time = 0.098034, size = 17, normalized size = 0.81

$$-\frac{\log\left(x+\frac{1}{5}\right)}{35}+\frac{2\log\left(x+\frac{2}{3}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(15*x**2+13*x+2),x)
```

```
[Out] -log(x + 1/5)/35 + 2*log(x + 2/3)/21
```

Giac [A] time = 1.12875, size = 26, normalized size = 1.24

$$-\frac{1}{35}\log\left(|5x+1|\right)+\frac{2}{21}\log\left(|3x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(15*x^2+13*x+2),x, algorithm="giac")
```

```
[Out] -1/35*log(abs(5*x + 1)) + 2/21*log(abs(3*x + 2))
```

3.2240 $\int \frac{1}{2+13x+15x^2} dx$

Optimal. Leaf size=21

$$\frac{1}{7}\log(5x+1)-\frac{1}{7}\log(3x+2)$$

```
[Out] -Log[2 + 3*x]/7 + Log[1 + 5*x]/7
```

Rubi [A] time = 0.0058359, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {616, 31}

$$\frac{1}{7}\log(5x+1)-\frac{1}{7}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 13*x + 15*x^2)^(-1),x]
```

```
[Out] -Log[2 + 3*x]/7 + Log[1 + 5*x]/7
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2+13x+15x^2}\,dx &= \frac{15}{7}\int \frac{1}{3+15x}\,dx-\frac{15}{7}\int \frac{1}{10+15x}\,dx\\ &= -\frac{1}{7}\log(2+3x)+\frac{1}{7}\log(1+5x)\end{aligned}$$

Mathematica [A] time = 0.0023779, size = 21, normalized size = 1.

$$\frac{1}{7}\log(5x+1)-\frac{1}{7}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 13*x + 15*x^2)^(-1),x]
```

```
[Out] -Log[2 + 3*x]/7 + Log[1 + 5*x]/7
```

Maple [A] time = 0.043, size = 18, normalized size = 0.9

$$-\frac{\ln\left(2+3x\right)}{7}+\frac{\ln\left(1+5x\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15*x^2+13*x+2),x)
```

```
[Out] -1/7*ln(2+3*x)+1/7*ln(1+5*x)
```

Maxima [A] time = 0.988588, size = 23, normalized size = 1.1

$$\frac{1}{7}\log\left(5x+1\right)-\frac{1}{7}\log\left(3x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15*x^2+13*x+2),x, algorithm="maxima")
```

```
[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

Fricas [A] time = 2.35735, size = 50, normalized size = 2.38

$$\frac{1}{7}\log\left(5x+1\right)-\frac{1}{7}\log\left(3x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15*x^2+13*x+2),x, algorithm="fricas")
```

```
[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

Sympy [A] time = 0.093426, size = 15, normalized size = 0.71

$$\frac{\log\left(x+\frac{1}{5}\right)}{7}-\frac{\log\left(x+\frac{2}{3}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15*x**2+13*x+2),x)
```

```
[Out] log(x + 1/5)/7 - log(x + 2/3)/7
```

Giac [A] time = 1.14799, size = 26, normalized size = 1.24

$$\frac{1}{7}\log\left(|5x+1|\right)-\frac{1}{7}\log\left(|3x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15*x^2+13*x+2),x, algorithm="giac")

[Out] 1/7*log(abs(5*x + 1)) - 1/7*log(abs(3*x + 2))
```

3.2241 $\int \frac{1}{x(2+13x+15x^2)} dx$

Optimal. Leaf size=27

$$\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

```
[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Rubi [A] time = 0.0119905, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {705, 29, 632, 31}

$$\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(2 + 13*x + 15*x^2)),x]
```

```
[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(2+13x+15x^2\right)}\,dx &= \frac{1}{2}\int \frac{1}{x}\,dx+\frac{1}{2}\int \frac{-13-15x}{2+13x+15x^2}\,dx\\ &= \frac{\log(x)}{2}+\frac{45}{14}\int \frac{1}{10+15x}\,dx-\frac{75}{7}\int \frac{1}{3+15x}\,dx\\ &= \frac{\log(x)}{2}+\frac{3}{14}\log(2+3x)-\frac{5}{7}\log(1+5x)\end{aligned}$$

Mathematica [A] time = 0.0043299, size = 27, normalized size = 1.

$$\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(2 + 13*x + 15*x^2)),x]
```

```
[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Maple [A] time = 0.045, size = 22, normalized size = 0.8

$$\frac{\ln(x)}{2}+\frac{3\ln(2+3x)}{14}-\frac{5\ln(1+5x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(15*x^2+13*x+2),x)
```

```
[Out] 1/2*ln(x)+3/14*ln(2+3*x)-5/7*ln(1+5*x)
```

Maxima [A] time = 0.969524, size = 28, normalized size = 1.04

$$-\frac{5}{7}\log(5x+1)+\frac{3}{14}\log(3x+2)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(15*x^2+13*x+2),x, algorithm="maxima")
```

```
[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

Fricas [A] time = 2.32869, size = 70, normalized size = 2.59

$$-\frac{5}{7}\log(5x+1)+\frac{3}{14}\log(3x+2)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(15*x^2+13*x+2),x, algorithm="fricas")
```

```
[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

Sympy [A] time = 0.126563, size = 24, normalized size = 0.89

$$\frac{\log(x)}{2}-\frac{5\log\left(x+\frac{1}{5}\right)}{7}+\frac{3\log\left(x+\frac{2}{3}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(15*x**2+13*x+2),x)
```

```
[Out] log(x)/2 - 5*log(x + 1/5)/7 + 3*log(x + 2/3)/14
```

Giac [A] time = 1.10404, size = 32, normalized size = 1.19

$$-\frac{5}{7}\log\left(|5x+1|\right)+\frac{3}{14}\log\left(|3x+2|\right)+\frac{1}{2}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(15*x^2+13*x+2),x, algorithm="giac")
```

```
[Out] -5/7*log(abs(5*x + 1)) + 3/14*log(abs(3*x + 2)) + 1/2*log(abs(x))
```

3.2242 $\int \frac{1}{x^2(2+13x+15x^2)}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)$$

```
[Out] -1/(2*x) - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

Rubi [A] time = 0.0259163, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {709, 800}

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(2 + 13*x + 15*x^2)),x]
```

```
[Out] -1/(2*x) - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(2+13x+15x^2\right)}\,dx &= -\frac{1}{2x}+\frac{1}{2}\int \frac{-13-15x}{x\left(2+13x+15x^2\right)}\,dx\\
&= -\frac{1}{2x}+\frac{1}{2}\int \left(-\frac{13}{2x}-\frac{27}{14(2+3x)}+\frac{250}{7(1+5x)}\right)dx\\
&= -\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(2+3x)+\frac{25}{7}\log(1+5x)
\end{aligned}$$

Mathematica [A] time = 0.0038619, size = 34, normalized size = 1.

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(2 + 13*x + 15*x^2)),x]
```

```
[Out] -1/(2*x) - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

Maple [A] time = 0.045, size = 27, normalized size = 0.8

$$-\frac{1}{2\,x}-\frac{13\,\ln\left(x\right)}{4}-\frac{9\,\ln\left(2+3\,x\right)}{28}+\frac{25\,\ln\left(1+5\,x\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(15*x^2+13*x+2),x)
```

```
[Out] -1/2/x-13/4*ln(x)-9/28*ln(2+3*x)+25/7*ln(1+5*x)
```

Maxima [A] time = 0.95531, size = 35, normalized size = 1.03

$$-\frac{1}{2\,x}+\frac{25}{7}\,\log\left(5\,x+1\right)-\frac{9}{28}\,\log\left(3\,x+2\right)-\frac{13}{4}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(15*x^2+13*x+2),x, algorithm="maxima")
```

```
[Out] -1/2/x + 25/7*log(5*x + 1) - 9/28*log(3*x + 2) - 13/4*log(x)
```

Fricas [A] time = 2.19871, size = 90, normalized size = 2.65

$$\frac{100\,x\log\left(5\,x+1\right)-9\,x\log\left(3\,x+2\right)-91\,x\log\left(x\right)-14}{28\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(15*x^2+13*x+2),x, algorithm="fricas")
```

```
[Out] 1/28*(100*x*log(5*x + 1) - 9*x*log(3*x + 2) - 91*x*log(x) - 14)/x
```

Sympy [A] time = 0.147424, size = 31, normalized size = 0.91

$$-\frac{13\log\left(x\right)}{4}+\frac{25\log\left(x+\frac{1}{5}\right)}{7}-\frac{9\log\left(x+\frac{2}{3}\right)}{28}-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(15*x**2+13*x+2),x)
```

```
[Out] -13*log(x)/4 + 25*log(x + 1/5)/7 - 9*log(x + 2/3)/28 - 1/(2*x)
```

Giac [A] time = 1.08573, size = 39, normalized size = 1.15

$$-\frac{1}{2x}+\frac{25}{7}\log\left(|5x+1|\right)-\frac{9}{28}\log\left(|3x+2|\right)-\frac{13}{4}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(15*x^2+13*x+2),x, algorithm="giac")
```

```
[Out] -1/2/x + 25/7*log(abs(5*x + 1)) - 9/28*log(abs(3*x + 2)) - 13/4*log(abs(x))
```

3.2243 $\int \frac{1}{x^3(2+13x+15x^2)}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

```
[Out] -1/(4*x^2) + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
+ 5*x])/7
```

Rubi [A] time = 0.0283698, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {709, 800}

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(2 + 13*x + 15*x^2)),x]
```

```
[Out] -1/(4*x^2) + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
+ 5*x])/7
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(2+13x+15x^2\right)}\,dx &= -\frac{1}{4x^2}+\frac{1}{2}\int \frac{-13-15x}{x^2\left(2+13x+15x^2\right)}\,dx\\ &= -\frac{1}{4x^2}+\frac{1}{2}\int \left(-\frac{13}{2x^2}+\frac{139}{4x}+\frac{81}{28(2+3x)}-\frac{1250}{7(1+5x)}\right)dx\\ &= -\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(2+3x)-\frac{125}{7}\log(1+5x)\end{aligned}$$

Mathematica [A] time = 0.0047596, size = 41, normalized size = 1.

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(2 + 13*x + 15*x^2)),x]
```

```
[Out] -1/(4*x^2) + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
+ 5*x])/7
```

Maple [A] time = 0.046, size = 32, normalized size = 0.8

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\ln(x)}{8}+\frac{27\ln(2+3x)}{56}-\frac{125\ln(1+5x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(15*x^2+13*x+2),x)
```

```
[Out] -1/4/x^2+13/4/x+139/8*ln(x)+27/56*ln(2+3*x)-125/7*ln(1+5*x)
```

Maxima [A] time = 0.962267, size = 42, normalized size = 1.02

$$\frac{13x-1}{4x^2}-\frac{125}{7}\log(5x+1)+\frac{27}{56}\log(3x+2)+\frac{139}{8}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(15*x^2+13*x+2),x, algorithm="maxima")
```

```
[Out] 1/4*(13*x - 1)/x^2 - 125/7*log(5*x + 1) + 27/56*log(3*x + 2) + 139/8*log(x)
```

Fricas [A] time = 2.22888, size = 117, normalized size = 2.85

$$-\frac{1000x^2\log(5x+1)-27x^2\log(3x+2)-973x^2\log(x)-182x+14}{56x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(15*x^2+13*x+2),x, algorithm="fricas")
```

```
[Out] -1/56*(1000*x^2*log(5*x + 1) - 27*x^2*log(3*x + 2) - 973*x^2*log(x) - 182*x
 + 14)/x^2
```

Sympy [A] time = 0.153982, size = 36, normalized size = 0.88

$$\frac{139\log(x)}{8}-\frac{125\log\left(x+\frac{1}{5}\right)}{7}+\frac{27\log\left(x+\frac{2}{3}\right)}{56}+\frac{13x-1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(15*x**2+13*x+2),x)
```

```
[Out] 139*log(x)/8 - 125*log(x + 1/5)/7 + 27*log(x + 2/3)/56 + (13*x - 1)/(4*x**2
)
```

Giac [A] time = 1.0889, size = 46, normalized size = 1.12

$$\frac{13\,x-1}{4\,x^2}-\frac{125}{7}\,\log\left(|5\,x+1|\right)+\frac{27}{56}\,\log\left(|3\,x+2|\right)+\frac{139}{8}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(15*x^2+13*x+2),x, algorithm="giac")
```

```
[Out] 1/4*(13*x - 1)/x^2 - 125/7*log(abs(5*x + 1)) + 27/56*log(abs(3*x + 2)) + 13
9/8*log(abs(x))
```

3.2244 $\int \frac{1}{x^4(2+13x+15x^2)}\,dx$

Optimal. Leaf size=48

$$\frac{13}{8x^2}-\frac{1}{6x^3}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

```
[Out] -1/(6*x^3) + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/
112 + (625*Log[1 + 5*x])/7
```

Rubi [A] time = 0.0374681, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {709, 800}

$$\frac{13}{8x^2}-\frac{1}{6x^3}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(2 + 13*x + 15*x^2)),x]
```

```
[Out] -1/(6*x^3) + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/
112 + (625*Log[1 + 5*x])/7
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(2+13x+15x^2\right)}\,dx &= -\frac{1}{6x^3}+\frac{1}{2}\int \frac{-13-15x}{x^3\left(2+13x+15x^2\right)}\,dx\\ &= -\frac{1}{6x^3}+\frac{1}{2}\int \left(-\frac{13}{2x^3}+\frac{139}{4x^2}-\frac{1417}{8x}-\frac{243}{56(2+3x)}+\frac{6250}{7(1+5x)}\right)dx\\ &= -\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(2+3x)+\frac{625}{7}\log(1+5x)\end{aligned}$$

Mathematica [A] time = 0.0044779, size = 48, normalized size = 1.

$$\frac{13}{8x^2}-\frac{1}{6x^3}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(2 + 13*x + 15*x^2)),x]
```

```
[Out] -1/(6*x^3) + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/
112 + (625*Log[1 + 5*x])/7
```

Maple [A] time = 0.047, size = 37, normalized size = 0.8

$$-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\ln(x)}{16}-\frac{81\ln(2+3x)}{112}+\frac{625\ln(1+5x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(15*x^2+13*x+2),x)
```

```
[Out] -1/6/x^3+13/8/x^2-139/8/x-1417/16*ln(x)-81/112*ln(2+3*x)+625/7*ln(1+5*x)
```

Maxima [A] time = 0.982733, size = 49, normalized size = 1.02

$$-\frac{417x^2-39x+4}{24x^3}+\frac{625}{7}\log(5x+1)-\frac{81}{112}\log(3x+2)-\frac{1417}{16}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(15*x^2+13*x+2),x, algorithm="maxima")
```

```
[Out] -1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(5*x + 1) - 81/112*log(3*x + 2) -
 1417/16*log(x)
```

Fricas [A] time = 2.27677, size = 138, normalized size = 2.88

$$\frac{30000x^3\log(5x+1)-243x^3\log(3x+2)-29757x^3\log(x)-5838x^2+546x-56}{336x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(15*x^2+13*x+2),x, algorithm="fricas")
```

```
[Out] 1/336*(30000*x^3*log(5*x + 1) - 243*x^3*log(3*x + 2) - 29757*x^3*log(x) - 5
838*x^2 + 546*x - 56)/x^3
```

Sympy [A] time = 0.16347, size = 41, normalized size = 0.85

$$-\frac{1417\log(x)}{16}+\frac{625\log\left(x+\frac{1}{5}\right)}{7}-\frac{81\log\left(x+\frac{2}{3}\right)}{112}-\frac{417x^2-39x+4}{24x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(15*x**2+13*x+2),x)
```

```
[Out] -1417*log(x)/16 + 625*log(x + 1/5)/7 - 81*log(x + 2/3)/112 - (417*x**2 - 39
*x + 4)/(24*x**3)
```

Giac [A] time = 1.10061, size = 53, normalized size = 1.1

$$-\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\,\log\left(|5\,x+1|\right)-\frac{81}{112}\,\log\left(|3\,x+2|\right)-\frac{1417}{16}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(15*x^2+13*x+2),x, algorithm="giac")
```

```
[Out] -1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(abs(5*x + 1)) - 81/112*log(abs(3
*x + 2)) - 1417/16*log(abs(x))
```

3.2245 $\int \frac{x^5}{2x+13x^2+15x^3}\,dx$

Optimal. Leaf size=40

$$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}-\frac{16}{567}\log(3x+2)+\frac{\log(5x+1)}{4375}$$

```
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]
/4375
```

Rubi [A] time = 0.0253351, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1585, 701, 632, 31}

$$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}-\frac{16}{567}\log(3x+2)+\frac{\log(5x+1)}{4375}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(2*x + 13*x^2 + 15*x^3),x]
```

```
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]
/4375
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^5}{2x+13x^2+15x^3}\,dx = \int \frac{x^4}{2+13x+15x^2}\,dx$$
$$= \int \left(\frac{139}{3375} - \frac{13x}{225} + \frac{x^2}{15} - \frac{278+1417x}{3375\left(2+13x+15x^2\right)}\right) dx$$
$$= \frac{139x}{3375} - \frac{13x^2}{450} + \frac{x^3}{45} - \frac{\int \frac{278+1417x}{2+13x+15x^2}\,dx}{3375}$$
$$= \frac{139x}{3375} - \frac{13x^2}{450} + \frac{x^3}{45} + \frac{3}{875}\int \frac{1}{3+15x}\,dx - \frac{80}{189}\int \frac{1}{10+15x}\,dx$$
$$= \frac{139x}{3375} - \frac{13x^2}{450} + \frac{x^3}{45} - \frac{16}{567}\log(2+3x) + \frac{\log(1+5x)}{4375}$$

Mathematica [A] time = 0.0043921, size = 40, normalized size = 1.

$$\frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} - \frac{16}{567}\log(3x+2) + \frac{\log(5x+1)}{4375}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(2*x + 13*x^2 + 15*x^3),x]
```

```
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]
/4375
```

Maple [A] time = 0.043, size = 31, normalized size = 0.8

$$\frac{139\,x}{3375} - \frac{13\,x^2}{450} + \frac{x^3}{45} - \frac{16\,\ln(2+3\,x)}{567} + \frac{\ln(1+5\,x)}{4375}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(15*x^3+13*x^2+2*x),x)
```

```
[Out] 139/3375*x-13/450*x^2+1/45*x^3-16/567*ln(2+3*x)+1/4375*ln(1+5*x)
```

Maxima [A] time = 0.988406, size = 41, normalized size = 1.02

$$\frac{1}{45}\,x^3 - \frac{13}{450}\,x^2 + \frac{139}{3375}\,x + \frac{1}{4375}\,\log(5\,x+1) - \frac{16}{567}\,\log(3\,x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(15*x^3+13*x^2+2*x),x, algorithm="maxima")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x +
 2)
```

Fricas [A] time = 2.29945, size = 108, normalized size = 2.7

$$\frac{1}{45}\,x^3 - \frac{13}{450}\,x^2 + \frac{139}{3375}\,x + \frac{1}{4375}\,\log(5\,x+1) - \frac{16}{567}\,\log(3\,x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(15*x^3+13*x^2+2*x),x, algorithm="fricas")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x +
 2)
```

Sympy [A] time = 0.111151, size = 34, normalized size = 0.85

$$\frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} + \frac{\log\left(x + \frac{1}{5}\right)}{4375} - \frac{16\log\left(x + \frac{2}{3}\right)}{567}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(15*x**3+13*x**2+2*x),x)
```

```
[Out] x**3/45 - 13*x**2/450 + 139*x/3375 + log(x + 1/5)/4375 - 16*log(x + 2/3)/56
7
```

Giac [A] time = 1.09997, size = 43, normalized size = 1.08

$$\frac{1}{45}x^3 - \frac{13}{450}x^2 + \frac{139}{3375}x + \frac{1}{4375}\log\left(|5x+1|\right) - \frac{16}{567}\log\left(|3x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(15*x^3+13*x^2+2*x),x, algorithm="giac")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(abs(5*x + 1)) - 16/567*log(
abs(3*x + 2))
```

3.2246 $\int \frac{x^4}{2x+13x^2+15x^3}\,dx$

Optimal. Leaf size=33

$$\frac{x^2}{30}-\frac{13x}{225}+\frac{8}{189}\log(3x+2)-\frac{1}{875}\log(5x+1)$$

```
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Rubi [A] time = 0.0233262, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1585, 701, 632, 31}

$$\frac{x^2}{30}-\frac{13x}{225}+\frac{8}{189}\log(3x+2)-\frac{1}{875}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(2*x + 13*x^2 + 15*x^3),x]
```

```
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{2x+13x^2+15x^3}\,dx &= \int \frac{x^3}{2+13x+15x^2}\,dx\\ &= \int \left(-\frac{13}{225}+\frac{x}{15}+\frac{26+139x}{225\left(2+13x+15x^2\right)}\right)dx\\ &= -\frac{13x}{225}+\frac{x^2}{30}+\frac{1}{225}\int \frac{26+139x}{2+13x+15x^2}\,dx\\ &= -\frac{13x}{225}+\frac{x^2}{30}-\frac{3}{175}\int \frac{1}{3+15x}\,dx+\frac{40}{63}\int \frac{1}{10+15x}\,dx\\ &= -\frac{13x}{225}+\frac{x^2}{30}+\frac{8}{189}\log(2+3x)-\frac{1}{875}\log(1+5x)\end{aligned}$$

Mathematica [A] time = 0.0040937, size = 33, normalized size = 1.

$$\frac{x^2}{30}-\frac{13x}{225}+\frac{8}{189}\log(3x+2)-\frac{1}{875}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(2*x + 13*x^2 + 15*x^3),x]
```

```
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Maple [A] time = 0.043, size = 26, normalized size = 0.8

$$-\frac{13\,x}{225}+\frac{x^2}{30}+\frac{8\ln(2+3\,x)}{189}-\frac{\ln(1+5\,x)}{875}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(15*x^3+13*x^2+2*x),x)
```

```
[Out] -13/225*x+1/30*x^2+8/189*ln(2+3*x)-1/875*ln(1+5*x)
```

Maxima [A] time = 1.05667, size = 34, normalized size = 1.03

$$\frac{1}{30}x^2-\frac{13}{225}x-\frac{1}{875}\log(5\,x+1)+\frac{8}{189}\log(3\,x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(15*x^3+13*x^2+2*x),x, algorithm="maxima")
```

```
[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

Fricas [A] time = 2.18498, size = 85, normalized size = 2.58

$$\frac{1}{30}x^2-\frac{13}{225}x-\frac{1}{875}\log(5\,x+1)+\frac{8}{189}\log(3\,x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(15*x^3+13*x^2+2*x),x, algorithm="fricas")
```

```
[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

Sympy [A] time = 0.110473, size = 27, normalized size = 0.82

$$\frac{x^2}{30} - \frac{13x}{225} - \frac{\log\left(x + \frac{1}{5}\right)}{875} + \frac{8\log\left(x + \frac{2}{3}\right)}{189}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(15*x**3+13*x**2+2*x),x)
```

```
[Out] x**2/30 - 13*x/225 - log(x + 1/5)/875 + 8*log(x + 2/3)/189
```

Giac [A] time = 1.09139, size = 36, normalized size = 1.09

$$\frac{1}{30}\,x^2 - \frac{13}{225}\,x - \frac{1}{875}\,\log\left(|5\,x + 1|\right) + \frac{8}{189}\,\log\left(|3\,x + 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(15*x^3+13*x^2+2*x),x, algorithm="giac")
```

```
[Out] 1/30*x^2 - 13/225*x - 1/875*log(abs(5*x + 1)) + 8/189*log(abs(3*x + 2))
```

3.2247 $\int \frac{x^3}{2x+13x^2+15x^3}\,dx$

Optimal. Leaf size=26

$$\frac{x}{15}-\frac{4}{63}\log(3x+2)+\frac{1}{175}\log(5x+1)$$

```
[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Rubi [A] time = 0.0191958, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1585, 703, 632, 31}

$$\frac{x}{15}-\frac{4}{63}\log(3x+2)+\frac{1}{175}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(2*x + 13*x^2 + 15*x^3),x]
```

```
[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^3}{2x+13x^2+15x^3}\,dx = \int \frac{x^2}{2+13x+15x^2}\,dx$$
$$= \frac{x}{15} + \frac{1}{15}\int \frac{-2-13x}{2+13x+15x^2}\,dx$$
$$= \frac{x}{15} + \frac{3}{35}\int \frac{1}{3+15x}\,dx - \frac{20}{21}\int \frac{1}{10+15x}\,dx$$
$$= \frac{x}{15} - \frac{4}{63}\log(2+3x) + \frac{1}{175}\log(1+5x)$$

Mathematica [A] time = 0.0035334, size = 26, normalized size = 1.

$$\frac{x}{15} - \frac{4}{63}\log(3x+2) + \frac{1}{175}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(2*x + 13*x^2 + 15*x^3),x]
```

```
[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Maple [A] time = 0.043, size = 21, normalized size = 0.8

$$\frac{x}{15} - \frac{4\ln(2+3x)}{63} + \frac{\ln(1+5x)}{175}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(15*x^3+13*x^2+2*x),x)
```

```
[Out] 1/15*x-4/63*ln(2+3*x)+1/175*ln(1+5*x)
```

Maxima [A] time = 1.0192, size = 27, normalized size = 1.04

$$\frac{1}{15}x + \frac{1}{175}\log(5x+1) - \frac{4}{63}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(15*x^3+13*x^2+2*x),x, algorithm="maxima")
```

```
[Out] 1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

Fricas [A] time = 2.00499, size = 66, normalized size = 2.54

$$\frac{1}{15}x + \frac{1}{175}\log(5x+1) - \frac{4}{63}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(15*x^3+13*x^2+2*x),x, algorithm="fricas")
```

```
[Out] 1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

Sympy [A] time = 0.10991, size = 20, normalized size = 0.77

$$\frac{x}{15} + \frac{\log\left(x + \frac{1}{5}\right)}{175} - \frac{4\log\left(x + \frac{2}{3}\right)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(15*x**3+13*x**2+2*x),x)
```

```
[Out] x/15 + log(x + 1/5)/175 - 4*log(x + 2/3)/63
```

Giac [A] time = 1.09935, size = 30, normalized size = 1.15

$$\frac{1}{15}\,x + \frac{1}{175}\,\log\left(|5\,x + 1|\right) - \frac{4}{63}\,\log\left(|3\,x + 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(15*x^3+13*x^2+2*x),x, algorithm="giac")
```

```
[Out] 1/15*x + 1/175*log(abs(5*x + 1)) - 4/63*log(abs(3*x + 2))
```

3.2248 $\int \frac{x^2}{2x+13x^2+15x^3}\,dx$

Optimal. Leaf size=21

$$\frac{2}{21}\log(3x+2)-\frac{1}{35}\log(5x+1)$$

```
[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

Rubi [A] time = 0.0132027, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1585, 632, 31}

$$\frac{2}{21}\log(3x+2)-\frac{1}{35}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2*x + 13*x^2 + 15*x^3),x]
```

```
[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{2x+13x^2+15x^3}\,dx &= \int \frac{x}{2+13x+15x^2}\,dx\\ &= -\left(\frac{3}{7}\int \frac{1}{3+15x}\,dx\right)+\frac{10}{7}\int \frac{1}{10+15x}\,dx\\ &= \frac{2}{21}\log(2+3x)-\frac{1}{35}\log(1+5x)\end{aligned}$$

Mathematica [A] time = 0.0029787, size = 21, normalized size = 1.

$$\frac{2}{21}\log(3x+2)-\frac{1}{35}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2*x + 13*x^2 + 15*x^3),x]
```

```
[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

Maple [A] time = 0.043, size = 18, normalized size = 0.9

$$\frac{2\ln(2+3x)}{21}-\frac{\ln(1+5x)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(15*x^3+13*x^2+2*x),x)
```

```
[Out] 2/21*ln(2+3*x)-1/35*ln(1+5*x)
```

Maxima [A] time = 1.01069, size = 23, normalized size = 1.1

$$-\frac{1}{35}\log(5x+1)+\frac{2}{21}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(15*x^3+13*x^2+2*x),x, algorithm="maxima")
```

```
[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

Fricas [A] time = 2.30499, size = 54, normalized size = 2.57

$$-\frac{1}{35}\log(5x+1)+\frac{2}{21}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(15*x^3+13*x^2+2*x),x, algorithm="fricas")
```

```
[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

Sympy [A] time = 0.101509, size = 17, normalized size = 0.81

$$-\frac{\log\left(x+\frac{1}{5}\right)}{35}+\frac{2\log\left(x+\frac{2}{3}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(15*x**3+13*x**2+2*x),x)
```

```
[Out] -log(x + 1/5)/35 + 2*log(x + 2/3)/21
```

Giac [A] time = 1.0854, size = 26, normalized size = 1.24

$$-\frac{1}{35}\log\left(|5x+1|\right)+\frac{2}{21}\log\left(|3x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(15*x^3+13*x^2+2*x),x, algorithm="giac")
```

```
[Out] -1/35*log(abs(5*x + 1)) + 2/21*log(abs(3*x + 2))
```

3.2249 $\int \frac{x}{2x+13x^2+15x^3} dx$

Optimal. Leaf size=21

$$\frac{1}{7}\log(5x+1)-\frac{1}{7}\log(3x+2)$$

```
[Out] -Log[2 + 3*x]/7 + Log[1 + 5*x]/7
```

Rubi [A] time = 0.0106777, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1585, 616, 31}

$$\frac{1}{7}\log(5x+1)-\frac{1}{7}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[x/(2*x + 13*x^2 + 15*x^3),x]

[Out] -Log[2 + 3*x]/7 + Log[1 + 5*x]/7
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{2x+13x^2+15x^3}\,dx &= \int \frac{1}{2+13x+15x^2}\,dx\\ &= \frac{15}{7}\int \frac{1}{3+15x}\,dx-\frac{15}{7}\int \frac{1}{10+15x}\,dx\\ &= -\frac{1}{7}\log(2+3x)+\frac{1}{7}\log(1+5x)\end{aligned}$$

Mathematica [A] time = 0.0025279, size = 21, normalized size = 1.

$$\frac{1}{7}\log(5x+1)-\frac{1}{7}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(2*x + 13*x^2 + 15*x^3),x]
```

```
[Out] -Log[2 + 3*x]/7 + Log[1 + 5*x]/7
```

Maple [A] time = 0.044, size = 18, normalized size = 0.9

$$-\frac{\ln(2+3x)}{7}+\frac{\ln(1+5x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(15*x^3+13*x^2+2*x),x)
```

```
[Out] -1/7*ln(2+3*x)+1/7*ln(1+5*x)
```

Maxima [A] time = 0.981315, size = 23, normalized size = 1.1

$$\frac{1}{7}\log(5x+1)-\frac{1}{7}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(15*x^3+13*x^2+2*x),x, algorithm="maxima")
```

```
[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

Fricas [A] time = 2.37545, size = 50, normalized size = 2.38

$$\frac{1}{7}\log(5x+1)-\frac{1}{7}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(15*x^3+13*x^2+2*x),x, algorithm="fricas")
```

```
[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

Sympy [A] time = 0.101533, size = 15, normalized size = 0.71

$$\frac{\log\left(x+\frac{1}{5}\right)}{7}-\frac{\log\left(x+\frac{2}{3}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(15*x**3+13*x**2+2*x),x)
```

```
[Out] log(x + 1/5)/7 - log(x + 2/3)/7
```

Giac [A] time = 1.11805, size = 26, normalized size = 1.24

$$\frac{1}{7}\log\left(|5x+1|\right)-\frac{1}{7}\log\left(|3x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(15*x^3+13*x^2+2*x),x, algorithm="giac")
```

```
[Out] 1/7*log(abs(5*x + 1)) - 1/7*log(abs(3*x + 2))
```

3.2250 $\int \frac{1}{2x+13x^2+15x^3}\,dx$

Optimal. Leaf size=27

$$\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

```
[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Rubi [A] time = 0.0139869, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1594, 705, 29, 632, 31}

$$\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + 13*x^2 + 15*x^3)^(-1),x]
```

```
[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2x+13x^2+15x^3}\,dx &= \int \frac{1}{x\left(2+13x+15x^2\right)}\,dx\\ &= \frac{1}{2}\int \frac{1}{x}\,dx+\frac{1}{2}\int \frac{-13-15x}{2+13x+15x^2}\,dx\\ &= \frac{\log(x)}{2}+\frac{45}{14}\int \frac{1}{10+15x}\,dx-\frac{75}{7}\int \frac{1}{3+15x}\,dx\\ &= \frac{\log(x)}{2}+\frac{3}{14}\log(2+3x)-\frac{5}{7}\log(1+5x)\end{aligned}$$

Mathematica [A] time = 0.0035077, size = 27, normalized size = 1.

$$\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + 13*x^2 + 15*x^3)^(-1),x]

[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Maple [A] time = 0.046, size = 22, normalized size = 0.8

$$\frac{\ln(x)}{2}+\frac{3\ln(2+3x)}{14}-\frac{5\ln(1+5x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15*x^3+13*x^2+2*x),x)

[Out] 1/2*ln(x)+3/14*ln(2+3*x)-5/7*ln(1+5*x)
```

Maxima [A] time = 0.985128, size = 28, normalized size = 1.04

$$-\frac{5}{7}\log(5x+1)+\frac{3}{14}\log(3x+2)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15*x^3+13*x^2+2*x),x, algorithm="maxima")

[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

Fricas [A] time = 2.49202, size = 70, normalized size = 2.59

$$-\frac{5}{7}\log(5x+1)+\frac{3}{14}\log(3x+2)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15*x^3+13*x^2+2*x),x, algorithm="fricas")
```

```
[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

Sympy [A] time = 0.128965, size = 24, normalized size = 0.89

$$\frac{\log(x)}{2} - \frac{5\log\left(x + \frac{1}{5}\right)}{7} + \frac{3\log\left(x + \frac{2}{3}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15*x**3+13*x**2+2*x),x)
```

```
[Out] log(x)/2 - 5*log(x + 1/5)/7 + 3*log(x + 2/3)/14
```

Giac [A] time = 1.12763, size = 32, normalized size = 1.19

$$-\frac{5}{7}\log(|5x + 1|) + \frac{3}{14}\log(|3x + 2|) + \frac{1}{2}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15*x^3+13*x^2+2*x),x, algorithm="giac")
```

```
[Out] -5/7*log(abs(5*x + 1)) + 3/14*log(abs(3*x + 2)) + 1/2*log(abs(x))
```

3.2251 $\int \frac{1}{x(2x+13x^2+15x^3)} dx$

Optimal. Leaf size=34

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)$$

```
[Out] -1/(2*x) - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

Rubi [A] time = 0.0281488, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1585, 709, 800}

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(2*x + 13*x^2 + 15*x^3)),x]
```

```
[Out] -1/(2*x) - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{x(2x+13x^2+15x^3)}\,dx = \int \frac{1}{x^2(2+13x+15x^2)}\,dx$$
$$= -\frac{1}{2x} + \frac{1}{2}\int \frac{-13-15x}{x(2+13x+15x^2)}\,dx$$
$$= -\frac{1}{2x} + \frac{1}{2}\int \left(-\frac{13}{2x} - \frac{27}{14(2+3x)} + \frac{250}{7(1+5x)}\right)dx$$
$$= -\frac{1}{2x} - \frac{13\log(x)}{4} - \frac{9}{28}\log(2+3x) + \frac{25}{7}\log(1+5x)$$

Mathematica [A] time = 0.0036394, size = 34, normalized size = 1.

$$-\frac{1}{2x} - \frac{13\log(x)}{4} - \frac{9}{28}\log(3x+2) + \frac{25}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(2*x + 13*x^2 + 15*x^3)),x]
```

```
[Out] -1/(2*x) - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

Maple [A] time = 0.048, size = 27, normalized size = 0.8

$$-\frac{1}{2x} - \frac{13\ln(x)}{4} - \frac{9\ln(2+3x)}{28} + \frac{25\ln(1+5x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(15*x^3+13*x^2+2*x),x)
```

```
[Out] -1/2/x-13/4*ln(x)-9/28*ln(2+3*x)+25/7*ln(1+5*x)
```

Maxima [A] time = 1.00272, size = 35, normalized size = 1.03

$$-\frac{1}{2x} + \frac{25}{7}\log(5x+1) - \frac{9}{28}\log(3x+2) - \frac{13}{4}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(15*x^3+13*x^2+2*x),x, algorithm="maxima")
```

```
[Out] -1/2/x + 25/7*log(5*x + 1) - 9/28*log(3*x + 2) - 13/4*log(x)
```

Fricas [A] time = 2.46814, size = 90, normalized size = 2.65

$$\frac{100\,x\log(5x+1) - 9\,x\log(3x+2) - 91\,x\log(x) - 14}{28\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(15*x^3+13*x^2+2*x),x, algorithm="fricas")
```

```
[Out] 1/28*(100*x*log(5*x + 1) - 9*x*log(3*x + 2) - 91*x*log(x) - 14)/x
```

Sympy [A] time = 0.143913, size = 31, normalized size = 0.91

$$-\frac{13\log(x)}{4}+\frac{25\log\left(x+\frac{1}{5}\right)}{7}-\frac{9\log\left(x+\frac{2}{3}\right)}{28}-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(15*x**3+13*x**2+2*x),x)
```

```
[Out] -13*log(x)/4 + 25*log(x + 1/5)/7 - 9*log(x + 2/3)/28 - 1/(2*x)
```

Giac [A] time = 1.13378, size = 39, normalized size = 1.15

$$-\frac{1}{2\,x}+\frac{25}{7}\,\log\left(|5\,x+1|\right)-\frac{9}{28}\,\log\left(|3\,x+2|\right)-\frac{13}{4}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(15*x^3+13*x^2+2*x),x, algorithm="giac")
```

```
[Out] -1/2/x + 25/7*log(abs(5*x + 1)) - 9/28*log(abs(3*x + 2)) - 13/4*log(abs(x))
```

3.2252 $\int \frac{1}{x^2(2x+13x^2+15x^3)}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

```
[Out] -1/(4*x^2) + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
+ 5*x])/7
```

Rubi [A] time = 0.0314923, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1585, 709, 800}

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(2*x + 13*x^2 + 15*x^3)),x]
```

```
[Out] -1/(4*x^2) + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
+ 5*x])/7
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(2x+13x^2+15x^3)}\,dx &= \int \frac{1}{x^3(2+13x+15x^2)}\,dx\\ &= -\frac{1}{4x^2}+\frac{1}{2}\int \frac{-13-15x}{x^2(2+13x+15x^2)}\,dx\\ &= -\frac{1}{4x^2}+\frac{1}{2}\int \left(-\frac{13}{2x^2}+\frac{139}{4x}+\frac{81}{28(2+3x)}-\frac{1250}{7(1+5x)}\right)dx\\ &= -\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(2+3x)-\frac{125}{7}\log(1+5x)\end{aligned}$$

Mathematica [A] time = 0.0042869, size = 41, normalized size = 1.

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(2*x + 13*x^2 + 15*x^3)),x]
```

```
[Out] -1/(4*x^2) + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
+ 5*x])/7
```

Maple [A] time = 0.045, size = 32, normalized size = 0.8

$$-\frac{1}{4\,x^2}+\frac{13}{4\,x}+\frac{139\,\ln(x)}{8}+\frac{27\,\ln(2+3\,x)}{56}-\frac{125\,\ln(1+5\,x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(15*x^3+13*x^2+2*x),x)
```

```
[Out] -1/4/x^2+13/4/x+139/8*ln(x)+27/56*ln(2+3*x)-125/7*ln(1+5*x)
```

Maxima [A] time = 1.00238, size = 42, normalized size = 1.02

$$\frac{13\,x-1}{4\,x^2}-\frac{125}{7}\log(5\,x+1)+\frac{27}{56}\log(3\,x+2)+\frac{139}{8}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(15*x^3+13*x^2+2*x),x, algorithm="maxima")
```

```
[Out] 1/4*(13*x - 1)/x^2 - 125/7*log(5*x + 1) + 27/56*log(3*x + 2) + 139/8*log(x)
```

Fricas [A] time = 2.23969, size = 117, normalized size = 2.85

$$-\frac{1000\,x^2\log(5\,x+1)-27\,x^2\log(3\,x+2)-973\,x^2\log(x)-182\,x+14}{56\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(15*x^3+13*x^2+2*x),x, algorithm="fricas")
```

```
[Out] -1/56*(1000*x^2*log(5*x + 1) - 27*x^2*log(3*x + 2) - 973*x^2*log(x) - 182*x
 + 14)/x^2
```

Sympy [A] time = 0.158388, size = 36, normalized size = 0.88

$$\frac{139\log(x)}{8} - \frac{125\log\left(x+\frac{1}{5}\right)}{7} + \frac{27\log\left(x+\frac{2}{3}\right)}{56} + \frac{13x-1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(15*x**3+13*x**2+2*x),x)
```

```
[Out] 139*log(x)/8 - 125*log(x + 1/5)/7 + 27*log(x + 2/3)/56 + (13*x - 1)/(4*x**2
)
```

Giac [A] time = 1.12023, size = 46, normalized size = 1.12

$$\frac{13\,x-1}{4\,x^2} - \frac{125}{7}\log\left(|5\,x+1|\right) + \frac{27}{56}\log\left(|3\,x+2|\right) + \frac{139}{8}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(15*x^3+13*x^2+2*x),x, algorithm="giac")
```

```
[Out] 1/4*(13*x - 1)/x^2 - 125/7*log(abs(5*x + 1)) + 27/56*log(abs(3*x + 2)) + 13
9/8*log(abs(x))
```

3.2253 $\int \frac{1}{x^3(2x+13x^2+15x^3)} dx$

Optimal. Leaf size=48

$$\frac{13}{8x^2} - \frac{1}{6x^3} - \frac{139}{8x} - \frac{1417\log(x)}{16} - \frac{81}{112}\log(3x+2) + \frac{625}{7}\log(5x+1)$$

```
[Out] -1/(6*x^3) + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/
112 + (625*Log[1 + 5*x])/7
```

Rubi [A] time = 0.034138, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1585, 709, 800}

$$\frac{13}{8x^2} - \frac{1}{6x^3} - \frac{139}{8x} - \frac{1417\log(x)}{16} - \frac{81}{112}\log(3x+2) + \frac{625}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(2*x + 13*x^2 + 15*x^3)),x]
```

```
[Out] -1/(6*x^3) + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/
112 + (625*Log[1 + 5*x])/7
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(2x+13x^2+15x^3\right)}\,dx &= \int \frac{1}{x^4\left(2+13x+15x^2\right)}\,dx\\ &= -\frac{1}{6x^3}+\frac{1}{2}\int \frac{-13-15x}{x^3\left(2+13x+15x^2\right)}\,dx\\ &= -\frac{1}{6x^3}+\frac{1}{2}\int \left(-\frac{13}{2x^3}+\frac{139}{4x^2}-\frac{1417}{8x}-\frac{243}{56(2+3x)}+\frac{6250}{7(1+5x)}\right)dx\\ &= -\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(2+3x)+\frac{625}{7}\log(1+5x)\end{aligned}$$

Mathematica [A] time = 0.0044358, size = 48, normalized size = 1.

$$\frac{13}{8x^2}-\frac{1}{6x^3}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(2*x + 13*x^2 + 15*x^3)),x]
```

```
[Out] -1/(6*x^3) + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/
112 + (625*Log[1 + 5*x])/7
```

Maple [A] time = 0.047, size = 37, normalized size = 0.8

$$-\frac{1}{6\,x^3}+\frac{13}{8\,x^2}-\frac{139}{8\,x}-\frac{1417\,\ln(x)}{16}-\frac{81\,\ln(2+3\,x)}{112}+\frac{625\,\ln(1+5\,x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(15*x^3+13*x^2+2*x),x)
```

```
[Out] -1/6/x^3+13/8/x^2-139/8/x-1417/16*ln(x)-81/112*ln(2+3*x)+625/7*ln(1+5*x)
```

Maxima [A] time = 0.974375, size = 49, normalized size = 1.02

$$-\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\log(5\,x+1)-\frac{81}{112}\log(3\,x+2)-\frac{1417}{16}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(15*x^3+13*x^2+2*x),x, algorithm="maxima")
```

```
[Out] -1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(5*x + 1) - 81/112*log(3*x + 2) -
 1417/16*log(x)
```

Fricas [A] time = 2.43815, size = 138, normalized size = 2.88

$$\frac{30000\,x^3\log(5\,x+1)-243\,x^3\log(3\,x+2)-29757\,x^3\log(x)-5838\,x^2+546\,x-56}{336\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(15*x^3+13*x^2+2*x),x, algorithm="fricas")
```

```
[Out] 1/336*(30000*x^3*log(5*x + 1) - 243*x^3*log(3*x + 2) - 29757*x^3*log(x) - 5
838*x^2 + 546*x - 56)/x^3
```

Sympy [A] time = 0.165996, size = 41, normalized size = 0.85

$$-\frac{1417\log(x)}{16}+\frac{625\log\left(x+\frac{1}{5}\right)}{7}-\frac{81\log\left(x+\frac{2}{3}\right)}{112}-\frac{417x^2-39x+4}{24x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(15*x**3+13*x**2+2*x),x)
```

```
[Out] -1417*log(x)/16 + 625*log(x + 1/5)/7 - 81*log(x + 2/3)/112 - (417*x**2 - 39
*x + 4)/(24*x**3)
```

Giac [A] time = 1.09253, size = 53, normalized size = 1.1

$$-\frac{417x^2-39x+4}{24x^3}+\frac{625}{7}\log\left(|5x+1|\right)-\frac{81}{112}\log\left(|3x+2|\right)-\frac{1417}{16}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(15*x^3+13*x^2+2*x),x, algorithm="giac")
```

```
[Out] -1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(abs(5*x + 1)) - 81/112*log(abs(3
*x + 2)) - 1417/16*log(abs(x))
```

3.2254 $\int \frac{x}{4+4x+x^2}\,dx$

Optimal. Leaf size=12

$$\frac{2}{x+2}+\log(x+2)$$

```
[Out] 2/(2 + x) + Log[2 + x]
```

Rubi [A] time = 0.0064843, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {27, 43}

$$\frac{2}{x+2}+\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[x/(4 + 4*x + x^2),x]
```

```
[Out] 2/(2 + x) + Log[2 + x]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{4+4x+x^2}\,dx &= \int \frac{x}{(2+x)^2}\,dx\\ &= \int \left(-\frac{2}{(2+x)^2}+\frac{1}{2+x}\right)dx\\ &= \frac{2}{2+x}+\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.0033978, size = 12, normalized size = 1.

$$\frac{2}{x+2}+\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(4 + 4*x + x^2),x]
```

```
[Out] 2/(2 + x) + Log[2 + x]
```

Maple [A] time = 0.042, size = 13, normalized size = 1.1

$$2\,(2+x)^{-1}+\ln\left(2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+4*x+4),x)
```

```
[Out] 2/(2+x)+ln(2+x)
```

Maxima [A] time = 0.956167, size = 16, normalized size = 1.33

$$\frac{2}{x+2}+\log\left(x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+4*x+4),x, algorithm="maxima")
```

```
[Out] 2/(x + 2) + log(x + 2)
```

Fricas [A] time = 2.34795, size = 46, normalized size = 3.83

$$\frac{(x+2)\log\left(x+2\right)+2}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+4*x+4),x, algorithm="fricas")
```

```
[Out] ((x + 2)*log(x + 2) + 2)/(x + 2)
```

Sympy [A] time = 0.080823, size = 8, normalized size = 0.67

$$\log\left(x+2\right)+\frac{2}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+4*x+4),x)
```

```
[Out] log(x + 2) + 2/(x + 2)
```

Giac [A] time = 1.1104, size = 18, normalized size = 1.5

$$\frac{2}{x+2}+\log\left(|x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+4*x+4),x, algorithm="giac")
```

```
[Out] 2/(x + 2) + log(abs(x + 2))
```

3.2255 $\int \frac{x}{5+2x+x^2} dx$

Optimal. Leaf size=26

$$\frac{1}{2}\log\left(x^2+2x+5\right)-\frac{1}{2}\tan^{-1}\left(\frac{x+1}{2}\right)$$

```
[Out] -ArcTan[(1 + x)/2]/2 + Log[5 + 2*x + x^2]/2
```

Rubi [A] time = 0.0134394, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {634, 618, 204, 628}

$$\frac{1}{2}\log\left(x^2+2x+5\right)-\frac{1}{2}\tan^{-1}\left(\frac{x+1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(5 + 2*x + x^2),x]

[Out] -ArcTan[(1 + x)/2]/2 + Log[5 + 2*x + x^2]/2
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{5+2x+x^2}\,dx &= \frac{1}{2}\int \frac{2+2x}{5+2x+x^2}\,dx-\int \frac{1}{5+2x+x^2}\,dx\\
&= \frac{1}{2}\log\left(5+2x+x^2\right)+2\,\text{Subst}\left(\int \frac{1}{-16-x^2}\,dx,x,2+2x\right)\\
&= -\frac{1}{2}\tan^{-1}\left(\frac{1+x}{2}\right)+\frac{1}{2}\log\left(5+2x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0035023, size = 26, normalized size = 1.

$$\frac{1}{2}\log\left(x^2+2x+5\right)-\frac{1}{2}\tan^{-1}\left(\frac{x+1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(5 + 2*x + x^2),x]
```

```
[Out] -ArcTan[(1 + x)/2]/2 + Log[5 + 2*x + x^2]/2
```

Maple [A] time = 0.04, size = 21, normalized size = 0.8

$$-\frac{1}{2}\arctan\left(\frac{1}{2}+\frac{x}{2}\right)+\frac{\ln\left(x^2+2x+5\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+2*x+5),x)
```

```
[Out] -1/2*arctan(1/2+1/2*x)+1/2*ln(x^2+2*x+5)
```

Maxima [A] time = 1.50598, size = 27, normalized size = 1.04

$$-\frac{1}{2}\arctan\left(\frac{1}{2}x+\frac{1}{2}\right)+\frac{1}{2}\log\left(x^2+2x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+2*x+5),x, algorithm="maxima")
```

```
[Out] -1/2*arctan(1/2*x + 1/2) + 1/2*log(x^2 + 2*x + 5)
```

Fricas [A] time = 2.44496, size = 69, normalized size = 2.65

$$-\frac{1}{2}\arctan\left(\frac{1}{2}x+\frac{1}{2}\right)+\frac{1}{2}\log\left(x^2+2x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+2*x+5),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(1/2*x + 1/2) + 1/2*log(x^2 + 2*x + 5)
```

Sympy [A] time = 0.103024, size = 20, normalized size = 0.77

$$\frac{\log\left(x^2+2x+5\right)}{2}-\frac{\operatorname{atan}\left(\frac{x}{2}+\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+2*x+5),x)
```

```
[Out] log(x**2 + 2*x + 5)/2 - atan(x/2 + 1/2)/2
```

Giac [A] time = 1.09128, size = 27, normalized size = 1.04

$$-\frac{1}{2}\arctan\left(\frac{1}{2}x+\frac{1}{2}\right)+\frac{1}{2}\log\left(x^2+2x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+2*x+5),x, algorithm="giac")
```

```
[Out] -1/2*arctan(1/2*x + 1/2) + 1/2*log(x^2 + 2*x + 5)
```

3.2256 $\int \frac{x}{6-5x+x^2} \, dx$

Optimal. Leaf size=17

$$3\log(3-x)-2\log(2-x)$$

```
[Out] -2*Log[2 - x] + 3*Log[3 - x]
```

Rubi [A] time = 0.00531, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {632, 31}

$$3\log(3-x)-2\log(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[x/(6 - 5*x + x^2),x]
```

```
[Out] -2*Log[2 - x] + 3*Log[3 - x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{x}{6-5x+x^2} \, dx &= -\left(2\int \frac{1}{-2+x} \, dx\right)+3\int \frac{1}{-3+x} \, dx \\ &= -2\log(2-x)+3\log(3-x) \end{aligned}$$

Mathematica [A] time = 0.003102, size = 17, normalized size = 1.

$$3\log(3-x)-2\log(2-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(6 - 5*x + x^2),x]
```

```
[Out] -2*Log[2 - x] + 3*Log[3 - x]
```

Maple [A] time = 0.046, size = 14, normalized size = 0.8

$$3\ln(-3+x)-2\ln(-2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2-5*x+6),x)
```

```
[Out] 3*ln(-3+x)-2*ln(-2+x)
```

Maxima [A] time = 0.949461, size = 18, normalized size = 1.06

$$-2\log(x-2)+3\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-5*x+6),x, algorithm="maxima")
```

```
[Out] -2*log(x - 2) + 3*log(x - 3)
```

Fricas [A] time = 2.69446, size = 41, normalized size = 2.41

$$-2\log(x-2)+3\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-5*x+6),x, algorithm="fricas")
```

```
[Out] -2*log(x - 2) + 3*log(x - 3)
```

Sympy [A] time = 0.096094, size = 12, normalized size = 0.71

$$3\log(x-3)-2\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2-5*x+6),x)
```

```
[Out] 3*log(x - 3) - 2*log(x - 2)
```

Giac [A] time = 1.09749, size = 20, normalized size = 1.18

$$-2\log(|x-2|)+3\log(|x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-5*x+6),x, algorithm="giac")
```

```
[Out] -2*log(abs(x - 2)) + 3*log(abs(x - 3))
```

3.2257 $\int \frac{x}{\left(2+2x+x^2\right)^2} \, dx$

Optimal. Leaf size=26

$$-\frac{x+2}{2\left(x^2+2x+2\right)}-\frac{1}{2}\tan^{-1}(x+1)$$

```
[Out] -(2 + x)/(2*(2 + 2*x + x^2)) - ArcTan[1 + x]/2
```

Rubi [A] time = 0.0065948, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {638, 617, 204}

$$-\frac{x+2}{2\left(x^2+2x+2\right)}-\frac{1}{2}\tan^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x/(2 + 2*x + x^2)^2,x]
```

```
[Out] -(2 + x)/(2*(2 + 2*x + x^2)) - ArcTan[1 + x]/2
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(2+2x+x^2\right)^2} \, dx &= -\frac{2+x}{2\left(2+2x+x^2\right)}-\frac{1}{2}\int \frac{1}{2+2x+x^2} \, dx\\
&= -\frac{2+x}{2\left(2+2x+x^2\right)}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1-x^2} \, dx,x,1+x\right)\\
&= -\frac{2+x}{2\left(2+2x+x^2\right)}-\frac{1}{2}\tan^{-1}(1+x)
\end{aligned}$$

Mathematica [A] time = 0.0115417, size = 28, normalized size = 1.08

$$\frac{-x-2}{2\left(x^2+2x+2\right)}-\frac{1}{2}\tan^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(2 + 2*x + x^2)^2,x]
```

```
[Out] (-2 - x)/(2*(2 + 2*x + x^2)) - ArcTan[1 + x]/2
```

Maple [A] time = 0.04, size = 25, normalized size = 1.

$$\frac{-2x-4}{4x^2+8x+8}-\frac{\arctan(1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+2*x+2)^2,x)
```

```
[Out] 1/4*(-2*x-4)/(x^2+2*x+2)-1/2*arctan(1+x)
```

Maxima [A] time = 1.45028, size = 30, normalized size = 1.15

$$-\frac{x+2}{2\left(x^2+2x+2\right)}-\frac{1}{2}\arctan(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+2*x+2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(x + 2)/(x^2 + 2*x + 2) - 1/2*arctan(x + 1)
```

Fricas [A] time = 2.58163, size = 84, normalized size = 3.23

$$-\frac{\left(x^2+2x+2\right)\arctan(x+1)+x+2}{2\left(x^2+2x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+2*x+2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((x^2 + 2*x + 2)*arctan(x + 1) + x + 2)/(x^2 + 2*x + 2)
```

Sympy [A] time = 0.11191, size = 20, normalized size = 0.77

$$-\frac{x+2}{2x^2+4x+4}-\frac{\operatorname{atan}(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+2*x+2)**2,x)
```

```
[Out] -(x + 2)/(2*x**2 + 4*x + 4) - atan(x + 1)/2
```

Giac [A] time = 1.13224, size = 30, normalized size = 1.15

$$-\frac{x+2}{2\left(x^2+2\,x+2\right)}-\frac{1}{2}\,\arctan\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+2*x+2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(x + 2)/(x^2 + 2*x + 2) - 1/2*arctan(x + 1)
```

3.2258 $\int \frac{x}{(1+x+x^2)^3} dx$

Optimal. Leaf size=54

$$-\frac{x+2}{6(x^2+x+1)^2}-\frac{2x+1}{6(x^2+x+1)}-\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -(2 + x)/(6*(1 + x + x^2)^2) - (1 + 2*x)/(6*(1 + x + x^2)) - (2*ArcTan[(1 +
 2*x)/Sqrt[3]])/(3*Sqrt[3])
```

Rubi [A] time = 0.0205593, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {638, 614, 618, 204}

$$-\frac{x+2}{6(x^2+x+1)^2}-\frac{2x+1}{6(x^2+x+1)}-\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x + x^2)^3,x]
```

```
[Out] -(2 + x)/(6*(1 + x + x^2)^2) - (1 + 2*x)/(6*(1 + x + x^2)) - (2*ArcTan[(1 +
 2*x)/Sqrt[3]])/(3*Sqrt[3])
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\left(1+x+x^2\right)^3}\,dx = -\frac{2+x}{6\left(1+x+x^2\right)^2}-\frac{1}{2}\int \frac{1}{\left(1+x+x^2\right)^2}\,dx$$
$$= -\frac{2+x}{6\left(1+x+x^2\right)^2}-\frac{1+2x}{6\left(1+x+x^2\right)}-\frac{1}{3}\int \frac{1}{1+x+x^2}\,dx$$
$$= -\frac{2+x}{6\left(1+x+x^2\right)^2}-\frac{1+2x}{6\left(1+x+x^2\right)}+\frac{2}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)$$
$$= -\frac{2+x}{6\left(1+x+x^2\right)^2}-\frac{1+2x}{6\left(1+x+x^2\right)}-\frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Mathematica [A] time = 0.0253443, size = 49, normalized size = 0.91

$$\frac{1}{18}\left(-\frac{3\left(2x^3+3x^2+4x+3\right)}{\left(x^2+x+1\right)^2}-4\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x + x^2)^3,x]
```

```
[Out] ((-3*(3 + 4*x + 3*x^2 + 2*x^3))/(1 + x + x^2)^2 - 4*Sqrt[3]*ArcTan[(1 + 2*x
)/Sqrt[3]])/18
```

Maple [A] time = 0.041, size = 48, normalized size = 0.9

$$\frac{-2-x}{6\left(x^2+x+1\right)^2}-\frac{1+2x}{6x^2+6x+6}-\frac{2\sqrt{3}}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+x+1)^3,x)
```

```
[Out] 1/6*(-2-x)/(x^2+x+1)^2-1/6*(1+2*x)/(x^2+x+1)-2/9*arctan(1/3*(1+2*x)*3^(1/2)
)*3^(1/2)
```

Maxima [A] time = 1.44099, size = 73, normalized size = 1.35

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{2x^3+3x^2+4x+3}{6\left(x^4+2x^3+3x^2+2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+1)^3,x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*(2*x^3 + 3*x^2 + 4*x + 3)/
(x^4 + 2*x^3 + 3*x^2 + 2*x + 1)
```

Fricas [A] time = 2.87343, size = 189, normalized size = 3.5

$$-\frac{6\,x^3+4\,\sqrt{3}\left(x^4+2\,x^3+3\,x^2+2\,x+1\right)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+9\,x^2+12\,x+9}{18\left(x^4+2\,x^3+3\,x^2+2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+1)^3,x, algorithm="fricas")
```

```
[Out] -1/18*(6*x^3 + 4*sqrt(3)*(x^4 + 2*x^3 + 3*x^2 + 2*x + 1)*arctan(1/3*sqrt(3)
*(2*x + 1)) + 9*x^2 + 12*x + 9)/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1)
```

Sympy [A] time = 0.135304, size = 63, normalized size = 1.17

$$-\frac{2x^3+3x^2+4x+3}{6x^4+12x^3+18x^2+12x+6}-\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+x+1)**3,x)
```

```
[Out] -(2*x**3 + 3*x**2 + 4*x + 3)/(6*x**4 + 12*x**3 + 18*x**2 + 12*x + 6) - 2*sq
rt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

Giac [A] time = 1.1185, size = 57, normalized size = 1.06

$$-\frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-\frac{2\,x^3+3\,x^2+4\,x+3}{6\left(x^2+x+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+1)^3,x, algorithm="giac")
```

```
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*(2*x^3 + 3*x^2 + 4*x + 3)/
(x^2 + x + 1)^2
```

3.2259 $\int \frac{x^2}{1+x+x^2} dx$

Optimal. Leaf size=32

$$-\frac{1}{2}\log\left(x^2+x+1\right)+x-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] x - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x + x^2]/2
```

Rubi [A] time = 0.0192937, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {703, 634, 618, 204, 628}

$$-\frac{1}{2}\log\left(x^2+x+1\right)+x-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + x + x^2),x]
```

```
[Out] x - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x + x^2]/2
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{1+x+x^2}\,dx &= x+\int \frac{-1-x}{1+x+x^2}\,dx\\
&= x-\frac{1}{2}\int \frac{1}{1+x+x^2}\,dx-\frac{1}{2}\int \frac{1+2x}{1+x+x^2}\,dx\\
&= x-\frac{1}{2}\log\left(1+x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&= x-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(1+x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0078181, size = 32, normalized size = 1.

$$
-\frac{1}{2}\log\left(x^2+x+1\right)+x-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + x + x^2),x]
```

```
[Out] x - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x + x^2]/2
```

Maple [A] time = 0.04, size = 28, normalized size = 0.9

$$
x-\frac{\ln\left(x^2+x+1\right)}{2}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+x+1),x)
```

```
[Out] x-1/2*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.50221, size = 36, normalized size = 1.12

$$
-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x-\frac{1}{2}\log\left(x^2+x+1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x - 1/2*log(x^2 + x + 1)
```

Fricas [A] time = 2.23919, size = 96, normalized size = 3.

$$
-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x-\frac{1}{2}\log\left(x^2+x+1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x - 1/2*log(x^2 + x + 1)
```

Sympy [A] time = 0.105458, size = 36, normalized size = 1.12

$$x - \frac{\log\left(x^2 + x + 1\right)}{2} - \frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+x+1),x)
```

```
[Out] x - log(x**2 + x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3
```

Giac [A] time = 1.0799, size = 36, normalized size = 1.12

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + x - \frac{1}{2}\log\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+x+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x - 1/2*log(x^2 + x + 1)
```

3.2260 $\int \frac{x^2}{2-3x+x^2} \, dx$

Optimal. Leaf size=18

$$x - \log(1-x) + 4\log(2-x)$$

```
[Out] x - Log[1 - x] + 4*Log[2 - x]
```

Rubi [A] time = 0.0079984, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {703, 632, 31}

$$x - \log(1-x) + 4\log(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 - 3*x + x^2),x]

[Out] x - Log[1 - x] + 4*Log[2 - x]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{2-3x+x^2} \, dx &= x + \int \frac{-2+3x}{2-3x+x^2} \, dx \\
&= x + 4\int \frac{1}{-2+x} \, dx - \int \frac{1}{-1+x} \, dx \\
&= x - \log(1-x) + 4\log(2-x)
\end{aligned}$$

Mathematica [A] time = 0.0034947, size = 18, normalized size = 1.

$$x - \log(1-x) + 4\log(2-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 - 3*x + x^2),x]
```

```
[Out] x - Log[1 - x] + 4*Log[2 - x]
```

Maple [A] time = 0.043, size = 15, normalized size = 0.8

$$x-\ln\left(-1+x\right)+4\ \ln\left(-2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2-3*x+2),x)
```

```
[Out] x-ln(-1+x)+4*ln(-2+x)
```

Maxima [A] time = 0.955964, size = 19, normalized size = 1.06

$$x-\log\left(x-1\right)+4\ \log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2-3*x+2),x, algorithm="maxima")
```

```
[Out] x - log(x - 1) + 4*log(x - 2)
```

Fricas [A] time = 2.25181, size = 42, normalized size = 2.33

$$x-\log\left(x-1\right)+4\ \log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2-3*x+2),x, algorithm="fricas")
```

```
[Out] x - log(x - 1) + 4*log(x - 2)
```

Sympy [A] time = 0.094126, size = 12, normalized size = 0.67

$$x+4\log\left(x-2\right)-\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2-3*x+2),x)
```

```
[Out] x + 4*log(x - 2) - log(x - 1)
```

Giac [A] time = 1.0922, size = 22, normalized size = 1.22

$$x - \log(|x - 1|) + 4 \log(|x - 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2-3*x+2),x, algorithm="giac")
```

```
[Out] x - log(abs(x - 1)) + 4*log(abs(x - 2))
```

3.2261 $\int \frac{x^2}{-6+x+x^2}\,dx$

Optimal. Leaf size=20

$$x+\frac{4}{5}\log(2-x)-\frac{9}{5}\log(x+3)$$

```
[Out] x + (4*Log[2 - x])/5 - (9*Log[3 + x])/5
```

Rubi [A] time = 0.0082817, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {703, 632, 31}

$$x+\frac{4}{5}\log(2-x)-\frac{9}{5}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(-6 + x + x^2),x]

[Out] x + (4*Log[2 - x])/5 - (9*Log[3 + x])/5
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{-6+x+x^2}\,dx &= x+\int \frac{6-x}{-6+x+x^2}\,dx\\ &= x+\frac{4}{5}\int \frac{1}{-2+x}\,dx-\frac{9}{5}\int \frac{1}{3+x}\,dx\\ &= x+\frac{4}{5}\log(2-x)-\frac{9}{5}\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.0032725, size = 20, normalized size = 1.

$$x+\frac{4}{5}\log(2-x)-\frac{9}{5}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(-6 + x + x^2),x]
```

```
[Out] x + (4*Log[2 - x])/5 - (9*Log[3 + x])/5
```

Maple [A] time = 0.044, size = 15, normalized size = 0.8

$$x-\frac{9\ln(3+x)}{5}+\frac{4\ln(-2+x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+x-6),x)
```

```
[Out] x-9/5*ln(3+x)+4/5*ln(-2+x)
```

Maxima [A] time = 0.980849, size = 19, normalized size = 0.95

$$x-\frac{9}{5}\log(x+3)+\frac{4}{5}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+x-6),x, algorithm="maxima")
```

```
[Out] x - 9/5*log(x + 3) + 4/5*log(x - 2)
```

Fricas [A] time = 2.26013, size = 50, normalized size = 2.5

$$x-\frac{9}{5}\log(x+3)+\frac{4}{5}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+x-6),x, algorithm="fricas")
```

```
[Out] x - 9/5*log(x + 3) + 4/5*log(x - 2)
```

Sympy [A] time = 0.097478, size = 17, normalized size = 0.85

$$x+\frac{4\log(x-2)}{5}-\frac{9\log(x+3)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+x-6),x)
```

```
[Out] x + 4*log(x - 2)/5 - 9*log(x + 3)/5
```

Giac [A] time = 1.09608, size = 22, normalized size = 1.1

$$x - \frac{9}{5}\log(|x + 3|) + \frac{4}{5}\log(|x - 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+x-6),x, algorithm="giac")
```

```
[Out] x - 9/5*log(abs(x + 3)) + 4/5*log(abs(x - 2))
```

3.2262 $\int \frac{x^2}{\left(2+2x+x^2\right)^2} \, dx$

Optimal. Leaf size=15

$$\frac{1}{x^2+2x+2}+\tan^{-1}(x+1)$$

```
[Out] (2 + 2*x + x^2)^(-1) + ArcTan[1 + x]
```

Rubi [A] time = 0.0076848, antiderivative size = 23, normalized size of antiderivative = 1.53, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {722, 617, 204}

$$\tan^{-1}(x+1)-\frac{x(x+2)}{2\left(x^2+2x+2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 + 2*x + x^2)^2,x]
```

```
[Out] -(x*(2 + x))/(2*(2 + 2*x + x^2)) + ArcTan[1 + x]
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 - b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(2+2x+x^2\right)^2} \, dx &= -\frac{x(2+x)}{2\left(2+2x+x^2\right)}+\int \frac{1}{2+2x+x^2} \, dx\\ &= -\frac{x(2+x)}{2\left(2+2x+x^2\right)}-\text{Subst}\left(\int \frac{1}{-1-x^2} \, dx,x,1+x\right)\\ &= -\frac{x(2+x)}{2\left(2+2x+x^2\right)}+\tan^{-1}(1+x)\end{aligned}$$

Mathematica [A] time = 0.0075959, size = 15, normalized size = 1.

$$\frac{1}{x^2+2x+2}+\tan^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 + 2*x + x^2)^2,x]
```

```
[Out] (2 + 2*x + x^2)^(-1) + ArcTan[1 + x]
```

Maple [A] time = 0.042, size = 16, normalized size = 1.1

$$\left(x^2+2x+2\right)^{-1}+\arctan(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+2*x+2)^2,x)
```

```
[Out] 1/(x^2+2*x+2)+arctan(1+x)
```

Maxima [A] time = 1.53619, size = 20, normalized size = 1.33

$$\frac{1}{x^2+2x+2}+\arctan(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+2*x+2)^2,x, algorithm="maxima")
```

```
[Out] 1/(x^2 + 2*x + 2) + arctan(x + 1)
```

Fricas [A] time = 2.12864, size = 72, normalized size = 4.8

$$\frac{\left(x^2+2x+2\right)\arctan(x+1)+1}{x^2+2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+2*x+2)^2,x, algorithm="fricas")
```

```
[Out] ((x^2 + 2*x + 2)*arctan(x + 1) + 1)/(x^2 + 2*x + 2)
```

Sympy [A] time = 0.108789, size = 14, normalized size = 0.93

$$\operatorname{atan}(x+1)+\frac{1}{x^2+2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+2*x+2)**2,x)
```

```
[Out] atan(x + 1) + 1/(x**2 + 2*x + 2)
```

Giac [A] time = 1.08865, size = 20, normalized size = 1.33

$$\frac{1}{x^2 + 2x + 2} + \arctan(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+2*x+2)^2,x, algorithm="giac")
```

```
[Out] 1/(x^2 + 2*x + 2) + arctan(x + 1)
```

3.2263 $\int \frac{x^3}{2-3x+x^2}\,dx$

Optimal. Leaf size=27

$$\frac{x^2}{2}+3x-\log(1-x)+8\log(2-x)$$

```
[Out] 3*x + x^2/2 - Log[1 - x] + 8*Log[2 - x]
```

Rubi [A] time = 0.0121766, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {701, 632, 31}

$$\frac{x^2}{2}+3x-\log(1-x)+8\log(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(2 - 3*x + x^2),x]
```

```
[Out] 3*x + x^2/2 - Log[1 - x] + 8*Log[2 - x]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{2-3x+x^2}\,dx &= \int \left(3+x-\frac{6-7x}{2-3x+x^2}\right)dx\\
&= 3x+\frac{x^2}{2}-\int \frac{6-7x}{2-3x+x^2}\,dx\\
&= 3x+\frac{x^2}{2}+8\int \frac{1}{-2+x}\,dx-\int \frac{1}{-1+x}\,dx\\
&= 3x+\frac{x^2}{2}-\log(1-x)+8\log(2-x)
\end{aligned}$$

Mathematica [A] time = 0.0035987, size = 27, normalized size = 1.

$$\frac{x^2}{2}+3x-\log(1-x)+8\log(2-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(2 - 3*x + x^2),x]
```

```
[Out] 3*x + x^2/2 - Log[1 - x] + 8*Log[2 - x]
```

Maple [A] time = 0.043, size = 22, normalized size = 0.8

$$\frac{x^2}{2}+3x-\ln(-1+x)+8\ln(-2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^2-3*x+2),x)
```

```
[Out] 1/2*x^2+3*x-ln(-1+x)+8*ln(-2+x)
```

Maxima [A] time = 0.974202, size = 28, normalized size = 1.04

$$\frac{1}{2}x^2+3x-\log(x-1)+8\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2-3*x+2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 3*x - log(x - 1) + 8*log(x - 2)
```

Fricas [A] time = 2.2626, size = 58, normalized size = 2.15

$$\frac{1}{2}x^2+3x-\log(x-1)+8\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2-3*x+2),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 3*x - log(x - 1) + 8*log(x - 2)
```

Sympy [A] time = 0.095846, size = 19, normalized size = 0.7

$$\frac{x^2}{2}+3x+8\log(x-2)-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**2-3*x+2),x)

[Out] x**2/2 + 3*x + 8*log(x - 2) - log(x - 1)
```

Giac [A] time = 1.09396, size = 31, normalized size = 1.15

$$\frac{1}{2}x^2 + 3x - \log(|x-1|) + 8\log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2-3*x+2),x, algorithm="giac")

[Out] 1/2*x^2 + 3*x - log(abs(x - 1)) + 8*log(abs(x - 2))
```

3.2264 $\int \frac{x^3}{1+2x+x^2} \, dx$

Optimal. Leaf size=22

$$\frac{x^2}{2}-2x+\frac{1}{x+1}+3\log(x+1)$$

```
[Out] -2*x + x^2/2 + (1 + x)^(-1) + 3*Log[1 + x]
```

Rubi [A] time = 0.0100248, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {27, 43}

$$\frac{x^2}{2}-2x+\frac{1}{x+1}+3\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + 2*x + x^2),x]
```

```
[Out] -2*x + x^2/2 + (1 + x)^(-1) + 3*Log[1 + x]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Cancel[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{1+2x+x^2} \, dx &= \int \frac{x^3}{(1+x)^2} \, dx\\ &= \int \left(-2+x-\frac{1}{(1+x)^2}+\frac{3}{1+x}\right) dx\\ &= -2x+\frac{x^2}{2}+\frac{1}{1+x}+3\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.008752, size = 26, normalized size = 1.18

$$\frac{1}{2}(x+1)^2-3(x+1)+\frac{1}{x+1}+3\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + 2*x + x^2),x]
```

```
[Out] (1 + x)^(-1) - 3*(1 + x) + (1 + x)^2/2 + 3*Log[1 + x]
```

Maple [A] time = 0.043, size = 21, normalized size = 1.

$$-2x+\frac{x^2}{2}+(1+x)^{-1}+3\ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^2+2*x+1),x)
```

```
[Out] -2*x+1/2*x^2+1/(1+x)+3*ln(1+x)
```

Maxima [A] time = 0.957131, size = 27, normalized size = 1.23

$$\frac{1}{2}x^2-2x+\frac{1}{x+1}+3\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+2*x+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 2*x + 1/(x + 1) + 3*log(x + 1)
```

Fricas [A] time = 2.24834, size = 81, normalized size = 3.68

$$\frac{x^3-3x^2+6(x+1)\log(x+1)-4x+2}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+2*x+1),x, algorithm="fricas")
```

```
[Out] 1/2*(x^3 - 3*x^2 + 6*(x + 1)*log(x + 1) - 4*x + 2)/(x + 1)
```

Sympy [A] time = 0.079324, size = 19, normalized size = 0.86

$$\frac{x^2}{2}-2x+3\log(x+1)+\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**2+2*x+1),x)
```

```
[Out] x**2/2 - 2*x + 3*log(x + 1) + 1/(x + 1)
```

Giac [A] time = 1.12364, size = 28, normalized size = 1.27

$$\frac{1}{2}x^2-2x+\frac{1}{x+1}+3\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+2*x+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 2*x + 1/(x + 1) + 3*log(abs(x + 1))
```

3.2265 $\int \frac{x^3}{1-2x+x^2}\,dx$

Optimal. Leaf size=26

$$\frac{x^2}{2}+2x+\frac{1}{1-x}+3\log(1-x)$$

```
[Out] (1 - x)^(-1) + 2*x + x^2/2 + 3*Log[1 - x]
```

Rubi [A] time = 0.0101849, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {27, 43}

$$\frac{x^2}{2}+2x+\frac{1}{1-x}+3\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 - 2*x + x^2),x]
```

```
[Out] (1 - x)^(-1) + 2*x + x^2/2 + 3*Log[1 - x]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{1-2x+x^2}\,dx &= \int \frac{x^3}{(-1+x)^2}\,dx\\ &= \int \left(2+\frac{1}{(-1+x)^2}+\frac{3}{-1+x}+x\right)dx\\ &= \frac{1}{1-x}+2x+\frac{x^2}{2}+3\log(1-x)\end{aligned}$$

Mathematica [A] time = 0.0074819, size = 25, normalized size = 0.96

$$\frac{1}{2}\left(x^2+4x-\frac{2}{x-1}+6\log(x-1)-5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 - 2*x + x^2),x]
```

```
[Out] (-5 - 2/(-1 + x) + 4*x + x^2 + 6*Log[-1 + x])/2
```

Maple [A] time = 0.046, size = 23, normalized size = 0.9

$$\frac{x^2}{2} + 2x + 3\ln(-1+x) - (-1+x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^2-2*x+1),x)
```

```
[Out] 1/2*x^2+2*x+3*ln(-1+x)-1/(-1+x)
```

Maxima [A] time = 1.02233, size = 30, normalized size = 1.15

$$\frac{1}{2}x^2 + 2x - \frac{1}{x-1} + 3\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2-2*x+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 2*x - 1/(x - 1) + 3*log(x - 1)
```

Fricas [A] time = 2.1213, size = 81, normalized size = 3.12

$$\frac{x^3 + 3x^2 + 6(x-1)\log(x-1) - 4x - 2}{2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2-2*x+1),x, algorithm="fricas")
```

```
[Out] 1/2*(x^3 + 3*x^2 + 6*(x - 1)*log(x - 1) - 4*x - 2)/(x - 1)
```

Sympy [A] time = 0.079259, size = 19, normalized size = 0.73

$$\frac{x^2}{2} + 2x + 3\log(x-1) - \frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**2-2*x+1),x)
```

```
[Out] x**2/2 + 2*x + 3*log(x - 1) - 1/(x - 1)
```

Giac [A] time = 1.09125, size = 31, normalized size = 1.19

$$\frac{1}{2}x^2 + 2x - \frac{1}{x-1} + 3\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2-2*x+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 2*x - 1/(x - 1) + 3*log(abs(x - 1))
```

3.2266 $\int \frac{x^4}{4+4x+x^2}\,dx$

Optimal. Leaf size=29

$$\frac{x^3}{3}-2x^2+12x-\frac{16}{x+2}-32\log(x+2)$$

```
[Out] 12*x - 2*x^2 + x^3/3 - 16/(2 + x) - 32*Log[2 + x]
```

Rubi [A] time = 0.0147971, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {27, 43}

$$\frac{x^3}{3}-2x^2+12x-\frac{16}{x+2}-32\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(4 + 4*x + x^2),x]
```

```
[Out] 12*x - 2*x^2 + x^3/3 - 16/(2 + x) - 32*Log[2 + x]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Cancel[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{4+4x+x^2}\,dx &= \int \frac{x^4}{(2+x)^2}\,dx\\ &= \int \left(12-4x+x^2+\frac{16}{(2+x)^2}-\frac{32}{2+x}\right)dx\\ &= 12x-2x^2+\frac{x^3}{3}-\frac{16}{2+x}-32\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.009439, size = 30, normalized size = 1.03

$$\frac{1}{3}\left(x^3-6x^2+36x-\frac{48}{x+2}-96\log(x+2)+104\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(4 + 4*x + x^2),x]
```

```
[Out] (104 + 36*x - 6*x^2 + x^3 - 48/(2 + x) - 96*Log[2 + x])/3
```

Maple [A] time = 0.044, size = 28, normalized size = 1.

$$12x - 2x^2 + \frac{x^3}{3} - 16(2+x)^{-1} - 32\ln(2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^2+4*x+4),x)
```

```
[Out] 12*x-2*x^2+1/3*x^3-16/(2+x)-32*ln(2+x)
```

Maxima [A] time = 0.985296, size = 36, normalized size = 1.24

$$\frac{1}{3}x^3 - 2x^2 + 12x - \frac{16}{x+2} - 32\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^2+4*x+4),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 - 2*x^2 + 12*x - 16/(x + 2) - 32*log(x + 2)
```

Fricas [A] time = 2.2301, size = 97, normalized size = 3.34

$$\frac{x^4 - 4x^3 + 24x^2 - 96(x+2)\log(x+2) + 72x - 48}{3(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^2+4*x+4),x, algorithm="fricas")
```

```
[Out] 1/3*(x^4 - 4*x^3 + 24*x^2 - 96*(x + 2)*log(x + 2) + 72*x - 48)/(x + 2)
```

Sympy [A] time = 0.08555, size = 24, normalized size = 0.83

$$\frac{x^3}{3} - 2x^2 + 12x - 32\log(x+2) - \frac{16}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**2+4*x+4),x)
```

```
[Out] x**3/3 - 2*x**2 + 12*x - 32*log(x + 2) - 16/(x + 2)
```

Giac [A] time = 1.11598, size = 38, normalized size = 1.31

$$\frac{1}{3}x^3 - 2x^2 + 12x - \frac{16}{x+2} - 32\log(|x+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^2+4*x+4),x, algorithm="giac")
```

```
[Out] 1/3*x^3 - 2*x^2 + 12*x - 16/(x + 2) - 32*log(abs(x + 2))
```

3.2267 $\int \frac{1}{x(1+x+x^2)} dx$

Optimal. Leaf size=33

$$-\frac{1}{2}\log\left(x^2+x+1\right)+\log(x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[x] - Log[1 + x + x^2]/2
```

Rubi [A] time = 0.0186396, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {705, 29, 634, 618, 204, 628}

$$-\frac{1}{2}\log\left(x^2+x+1\right)+\log(x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x + x^2)),x]
```

```
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[x] - Log[1 + x + x^2]/2
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(1+x+x^2)}\,dx &= \int \frac{1}{x}\,dx + \int \frac{-1-x}{1+x+x^2}\,dx\\ &= \log(x) - \frac{1}{2}\int \frac{1}{1+x+x^2}\,dx - \frac{1}{2}\int \frac{1+2x}{1+x+x^2}\,dx\\ &= \log(x) - \frac{1}{2}\log\left(1+x+x^2\right) + \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x\right)\\ &= -\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \log(x) - \frac{1}{2}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0072856, size = 33, normalized size = 1.

$$-\frac{1}{2}\log\left(x^2+x+1\right) + \log(x) - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x + x^2)),x]

[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[x] - Log[1 + x + x^2]/2
```

Maple [A] time = 0.043, size = 29, normalized size = 0.9

$$\ln(x) - \frac{\ln\left(x^2+x+1\right)}{2} - \frac{\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2+x+1),x)

[Out] ln(x)-1/2*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.46229, size = 38, normalized size = 1.15

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{2}\log\left(x^2+x+1\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+x+1),x, algorithm="maxima")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1) + log(x)
```

Fricas [A] time = 2.26684, size = 103, normalized size = 3.12

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{2}\log\left(x^2+x+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1) + log(x)
```

Sympy [A] time = 0.125574, size = 37, normalized size = 1.12

$$\log(x)-\frac{\log\left(x^2+x+1\right)}{2}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2+x+1),x)
```

```
[Out] log(x) - log(x**2 + x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3
```

Giac [A] time = 1.12207, size = 39, normalized size = 1.18

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{2}\log\left(x^2+x+1\right)+\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+x+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1) + log(abs
(x))
```

3.2268 $\int (d+ex)^{5/2} \left(a+bx+cx^2\right) dx$

Optimal. Leaf size=75

$$\frac{2(d+ex)^{7/2} \left(ae^2-bde+cd^2\right)}{7e^3}-\frac{2(d+ex)^{9/2}(2cd-be)}{9e^3}+\frac{2c(d+ex)^{11/2}}{11e^3}$$

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(7/2))/(7*e^3) - (2*(2*c*d - b*e)*(d +
 e*x)^(9/2))/(9*e^3) + (2*c*(d + e*x)^(11/2))/(11*e^3)
```

Rubi [A] time = 0.0381153, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$\frac{2(d+ex)^{7/2} \left(ae^2-bde+cd^2\right)}{7e^3}-\frac{2(d+ex)^{9/2}(2cd-be)}{9e^3}+\frac{2c(d+ex)^{11/2}}{11e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)*(a + b*x + c*x^2),x]
```

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(7/2))/(7*e^3) - (2*(2*c*d - b*e)*(d +
 e*x)^(9/2))/(9*e^3) + (2*c*(d + e*x)^(11/2))/(11*e^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int (d+ex)^{5/2} \left(a+bx+cx^2\right) dx &= \int \left(\frac{\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}}{e^2}+\frac{(-2cd+be)(d+ex)^{7/2}}{e^2}+\frac{c(d+ex)^{9/2}}{e^2}\right) dx\\ &= \frac{2\left(cd^2-bde+ae^2\right)(d+ex)^{7/2}}{7e^3}-\frac{2(2cd-be)(d+ex)^{9/2}}{9e^3}+\frac{2c(d+ex)^{11/2}}{11e^3}\end{aligned}$$

Mathematica [A] time = 0.0620969, size = 55, normalized size = 0.73

$$\frac{2(d+ex)^{7/2} \left(11e(9ae-2bd+7bex)+c\left(8d^2-28dex+63e^2x^2\right)\right)}{693e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)*(a + b*x + c*x^2),x]
```

```
[Out] (2*(d + e*x)^(7/2)*(11*e*(-2*b*d + 9*a*e + 7*b*e*x) + c*(8*d^2 - 28*d*e*x +
 63*e^2*x^2)))/(693*e^3)
```

Maple [A] time = 0.042, size = 53, normalized size = 0.7

$$\frac{126\,c e^{2} x^{2}+154\,b e^{2} x-56\,c d e x+198\,a e^{2}-44\,b d e+16\,c d^{2}}{693\,e^{3}}\,(e x+d)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(c*x^2+b*x+a),x)
```

```
[Out] 2/693*(e*x+d)^(7/2)*(63*c*e^2*x^2+77*b*e^2*x-28*c*d*e*x+99*a*e^2-22*b*d*e+8
*c*d^2)/e^3
```

Maxima [A] time = 1.04428, size = 80, normalized size = 1.07

$$\frac{2\left(63\,(e x+d)^{\frac{11}{2}} c-77\,(2\,c d-b e)(e x+d)^{\frac{9}{2}}+99\left(c d^{2}-b d e+a e^{2}\right)(e x+d)^{\frac{7}{2}}\right)}{693\,e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 2/693*(63*(e*x + d)^(11/2)*c - 77*(2*c*d - b*e)*(e*x + d)^(9/2) + 99*(c*d^2
 - b*d*e + a*e^2)*(e*x + d)^(7/2))/e^3
```

Fricas [B] time = 2.1901, size = 340, normalized size = 4.53

$$\frac{2\left(63\,c e^{5} x^{5}+8\,c d^{5}-22\,b d^{4} e+99\,a d^{3} e^{2}+7\left(23\,c d e^{4}+11\,b e^{5}\right)x^{4}+\left(113\,c d^{2} e^{3}+209\,b d e^{4}+99\,a e^{5}\right)x^{3}+3\left(c d^{3} e^{2}+55\,b\right.}{693\,e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 2/693*(63*c*e^5*x^5 + 8*c*d^5 - 22*b*d^4*e + 99*a*d^3*e^2 + 7*(23*c*d*e^4 +
 11*b*e^5)*x^4 + (113*c*d^2*e^3 + 209*b*d*e^4 + 99*a*e^5)*x^3 + 3*(c*d^3*e^
2 + 55*b*d^2*e^3 + 99*a*d*e^4)*x^2 - (4*c*d^4*e - 11*b*d^3*e^2 - 297*a*d^2*
e^3)*x)*sqrt(e*x + d)/e^3
```

Sympy [A] time = 3.35191, size = 326, normalized size = 4.35

$$\begin{cases} \frac{2ad^{3}\sqrt{d+ex}}{7e}+\frac{6ad^{2}x\sqrt{d+ex}}{7}+\frac{6adex^{2}\sqrt{d+ex}}{7}+\frac{2ae^{2}x^{3}\sqrt{d+ex}}{7}-\frac{4bd^{4}\sqrt{d+ex}}{63e^{2}}+\frac{2bd^{3}x\sqrt{d+ex}}{63e}+\frac{10bd^{2}x^{2}\sqrt{d+ex}}{21}+\frac{38bdex^{3}\sqrt{d+ex}}{63}+\frac{2be^{2}x^{4}\sqrt{d+ex}}{9} \\ d^{\frac{5}{2}}\left(ax+\frac{bx^{2}}{2}+\frac{cx^{3}}{3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(c*x**2+b*x+a),x)
```

```
[Out] Piecewise((2*a*d**3*sqrt(d + e*x)/(7*e) + 6*a*d**2*x*sqrt(d + e*x)/7 + 6*a*
d*e*x**2*sqrt(d + e*x)/7 + 2*a*e**2*x**3*sqrt(d + e*x)/7 - 4*b*d**4*sqrt(d
+ e*x)/(63*e**2) + 2*b*d**3*x*sqrt(d + e*x)/(63*e) + 10*b*d**2*x**2*sqrt(d
```

```
+ e*x)/21 + 38*b*d*e*x**3*sqrt(d + e*x)/63 + 2*b*e**2*x**4*sqrt(d + e*x)/9
+ 16*c*d**5*sqrt(d + e*x)/(693*e**3) - 8*c*d**4*x*sqrt(d + e*x)/(693*e**2)
+ 2*c*d**3*x**2*sqrt(d + e*x)/(231*e) + 226*c*d**2*x**3*sqrt(d + e*x)/693 +
 46*c*d*e*x**4*sqrt(d + e*x)/99 + 2*c*e**2*x**5*sqrt(d + e*x)/11, Ne(e, 0))
, (d**(5/2)*(a*x + b*x**2/2 + c*x**3/3), True))
```

Giac [B] time = 1.15002, size = 498, normalized size = 6.64

$$\frac{2}{3465}\left(231\left(3\,(xe+d)^{\frac{5}{2}}-5\,(xe+d)^{\frac{3}{2}}d\right)bd^2e^{(-1)}+33\left(15\,(xe+d)^{\frac{7}{2}}-42\,(xe+d)^{\frac{5}{2}}d+35\,(xe+d)^{\frac{3}{2}}d^2\right)cd^2e^{(-2)}+1155\,(xe+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 2/3465*(231*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*b*d^2*e^(-1) + 33*(15
*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*c*d^2*e^(
-2) + 1155*(x*e + d)^(3/2)*a*d^2 + 66*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5
/2)*d + 35*(x*e + d)^(3/2)*d^2)*b*d*e^(-1) + 22*(35*(x*e + d)^(9/2) - 135*(
x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*c*d*e
^(-2) + 462*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a*d + 11*(35*(x*e + d
)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(
3/2)*d^3)*b*e^(-1) + (315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*
(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*
c*e^(-2) + 33*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/
2)*d^2)*a)*e^(-1)
```

3.2269 $\int (d+ex)^{3/2}\left(a+bx+cx^2\right)dx$

Optimal. Leaf size=75

$$\frac{2(d+ex)^{5/2}\left(ae^2-bde+cd^2\right)}{5e^3}-\frac{2(d+ex)^{7/2}(2cd-be)}{7e^3}+\frac{2c(d+ex)^{9/2}}{9e^3}$$

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(5/2))/(5*e^3) - (2*(2*c*d - b*e)*(d +
 e*x)^(7/2))/(7*e^3) + (2*c*(d + e*x)^(9/2))/(9*e^3)
```

Rubi [A] time = 0.0291398, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$\frac{2(d+ex)^{5/2}\left(ae^2-bde+cd^2\right)}{5e^3}-\frac{2(d+ex)^{7/2}(2cd-be)}{7e^3}+\frac{2c(d+ex)^{9/2}}{9e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*(a + b*x + c*x^2),x]
```

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(5/2))/(5*e^3) - (2*(2*c*d - b*e)*(d +
 e*x)^(7/2))/(7*e^3) + (2*c*(d + e*x)^(9/2))/(9*e^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int (d+ex)^{3/2}\left(a+bx+cx^2\right)dx &= \int \left(\frac{\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}}{e^2}+\frac{(-2cd+be)(d+ex)^{5/2}}{e^2}+\frac{c(d+ex)^{7/2}}{e^2}\right)dx\\ &= \frac{2\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}}{5e^3}-\frac{2(2cd-be)(d+ex)^{7/2}}{7e^3}+\frac{2c(d+ex)^{9/2}}{9e^3}\end{aligned}$$

Mathematica [A] time = 0.0470921, size = 55, normalized size = 0.73

$$\frac{2(d+ex)^{5/2}\left(9e(7ae-2bd+5bex)+c\left(8d^2-20dex+35e^2x^2\right)\right)}{315e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)*(a + b*x + c*x^2),x]
```

```
[Out] (2*(d + e*x)^(5/2)*(9*e*(-2*b*d + 7*a*e + 5*b*e*x) + c*(8*d^2 - 20*d*e*x +
35*e^2*x^2)))/(315*e^3)
```

Maple [A] time = 0.041, size = 53, normalized size = 0.7

$$\frac{70\,ce^2x^2+90\,be^2x-40\,cdex+126\,ae^2-36\,bde+16\,cd^2}{315\,e^3}(ex+d)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(c*x^2+b*x+a),x)
```

```
[Out] 2/315*(e*x+d)^(5/2)*(35*c*e^2*x^2+45*b*e^2*x-20*c*d*e*x+63*a*e^2-18*b*d*e+8
*c*d^2)/e^3
```

Maxima [A] time = 0.970494, size = 80, normalized size = 1.07

$$\frac{2\left(35\,(ex+d)^{\frac{9}{2}}c-45\,(2\,cd-be)(ex+d)^{\frac{7}{2}}+63\,\left(cd^2-bde+ae^2\right)(ex+d)^{\frac{5}{2}}\right)}{315\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 2/315*(35*(e*x + d)^(9/2)*c - 45*(2*c*d - b*e)*(e*x + d)^(7/2) + 63*(c*d^2
- b*d*e + a*e^2)*(e*x + d)^(5/2))/e^3
```

Fricas [A] time = 2.24341, size = 266, normalized size = 3.55

$$\frac{2\left(35\,ce^4x^4+8\,cd^4-18\,bd^3e+63\,ad^2e^2+5\left(10\,cde^3+9\,be^4\right)x^3+3\left(cd^2e^2+24\,bde^3+21\,ae^4\right)x^2-\left(4\,cd^3e-9\,bd^2e^2-126\right.\right.}{315\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 2/315*(35*c*e^4*x^4 + 8*c*d^4 - 18*b*d^3*e + 63*a*d^2*e^2 + 5*(10*c*d*e^3 +
 9*b*e^4)*x^3 + 3*(c*d^2*e^2 + 24*b*d*e^3 + 21*a*e^4)*x^2 - (4*c*d^3*e - 9*
b*d^2*e^2 - 126*a*d*e^3)*x)*sqrt(e*x + d)/e^3
```

Sympy [A] time = 8.00577, size = 230, normalized size = 3.07

$$ad\left(\begin{cases}\sqrt{d}x & \text{for}\ e=0\\ \frac{2(d+ex)^{\frac{3}{2}}}{3e} & \text{otherwise}\end{cases}\right)+\frac{2a\left(-\frac{d(d+ex)^{\frac{3}{2}}}{3}+\frac{(d+ex)^{\frac{5}{2}}}{5}\right)}{e}+\frac{2bd\left(-\frac{d(d+ex)^{\frac{3}{2}}}{3}+\frac{(d+ex)^{\frac{5}{2}}}{5}\right)}{e^2}+\frac{2b\left(\frac{d^2(d+ex)^{\frac{3}{2}}}{3}-\frac{2d(d+ex)^{\frac{5}{2}}}{5}+\frac{(d+ex)^{\frac{7}{2}}}{7}\right.}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(c*x**2+b*x+a),x)
```

```
[Out] a*d*Piecewise((sqrt(d)*x, Eq(e, 0)), (2*(d + e*x)**(3/2)/(3*e), True)) + 2*
a*(-d*(d + e*x)**(3/2)/3 + (d + e*x)**(5/2)/5)/e + 2*b*d*(-d*(d + e*x)**(3/
2)/3 + (d + e*x)**(5/2)/5)/e**2 + 2*b*(d**2*(d + e*x)**(3/2)/3 - 2*d*(d + e
```

```
*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e**2 + 2*c*d*(d**2*(d + e*x)**(3/2)/3 -
2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e**3 + 2*c*(-d**3*(d + e*x)**(
3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d + e*x)**(7/2)/7 + (d + e*x)**(9
/2)/9)/e**3
```

Giac [B] time = 1.11779, size = 274, normalized size = 3.65

$$\frac{2}{315}\left(21\left(3\,(xe+d)^{\frac{5}{2}}-5\,(xe+d)^{\frac{3}{2}}d\right)bde^{(-1)}+3\left(15\,(xe+d)^{\frac{7}{2}}-42\,(xe+d)^{\frac{5}{2}}d+35\,(xe+d)^{\frac{3}{2}}d^{2}\right)cde^{(-2)}+105\,(xe+d)^{\frac{3}{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 2/315*(21*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*b*d*e^(-1) + 3*(15*(x*e
 + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*c*d*e^(-2) + 1
05*(x*e + d)^(3/2)*a*d + 3*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*
(x*e + d)^(3/2)*d^2)*b*e^(-1) + (35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d
 + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*c*e^(-2) + 21*(3*(x*e
 + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a)*e^(-1)
```

3.2270 $\int \sqrt{d+ex}\left(a+bx+cx^2\right)\,dx$

Optimal. Leaf size=75

$$\frac{2(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}{3e^3}-\frac{2(d+ex)^{5/2}(2cd-be)}{5e^3}+\frac{2c(d+ex)^{7/2}}{7e^3}$$

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(3/2))/(3*e^3) - (2*(2*c*d - b*e)*(d +
 e*x)^(5/2))/(5*e^3) + (2*c*(d + e*x)^(7/2))/(7*e^3)
```

Rubi [A] time = 0.0296745, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$\frac{2(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}{3e^3}-\frac{2(d+ex)^{5/2}(2cd-be)}{5e^3}+\frac{2c(d+ex)^{7/2}}{7e^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(a + b*x + c*x^2),x]
```

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(3/2))/(3*e^3) - (2*(2*c*d - b*e)*(d +
 e*x)^(5/2))/(5*e^3) + (2*c*(d + e*x)^(7/2))/(7*e^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \sqrt{d+ex}\left(a+bx+cx^2\right)\,dx &= \int \left(\frac{\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}{e^2}+\frac{(-2cd+be)(d+ex)^{3/2}}{e^2}+\frac{c(d+ex)^{5/2}}{e^2}\right)dx\\ &= \frac{2\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}}{3e^3}-\frac{2(2cd-be)(d+ex)^{5/2}}{5e^3}+\frac{2c(d+ex)^{7/2}}{7e^3}\end{aligned}$$

Mathematica [A] time = 0.0477906, size = 55, normalized size = 0.73

$$\frac{2(d+ex)^{3/2}\left(7e(5ae-2bd+3bex)+c\left(8d^2-12dex+15e^2x^2\right)\right)}{105e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*(a + b*x + c*x^2),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(7*e*(-2*b*d + 5*a*e + 3*b*e*x) + c*(8*d^2 - 12*d*e*x +
15*e^2*x^2)))/(105*e^3)
```

Maple [A] time = 0.041, size = 53, normalized size = 0.7

$$\frac{30 c e^2 x^2 + 42 b e^2 x - 24 c d e x + 70 a e^2 - 28 b d e + 16 c d^2}{105 e^3} (ex+d)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*(e*x+d)^(1/2),x)
```

```
[Out] 2/105*(e*x+d)^(3/2)*(15*c*e^2*x^2+21*b*e^2*x-12*c*d*e*x+35*a*e^2-14*b*d*e+8
*c*d^2)/e^3
```

Maxima [A] time = 0.97133, size = 80, normalized size = 1.07

$$\frac{2 \left(15 (ex+d)^{\frac{7}{2}} c - 21 (2 cd - be)(ex+d)^{\frac{5}{2}} + 35 \left(cd^2 - bde + ae^2\right)(ex+d)^{\frac{3}{2}}\right)}{105 e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*(e*x + d)^(7/2)*c - 21*(2*c*d - b*e)*(e*x + d)^(5/2) + 35*(c*d^2
- b*d*e + a*e^2)*(e*x + d)^(3/2))/e^3
```

Fricas [A] time = 2.24467, size = 193, normalized size = 2.57

$$\frac{2 \left(15 c e^3 x^3 + 8 c d^3 - 14 b d^2 e + 35 a d e^2 + 3 \left(c d e^2 + 7 b e^3\right) x^2 - \left(4 c d^2 e - 7 b d e^2 - 35 a e^3\right) x\right) \sqrt{ex+d}}{105 e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*c*e^3*x^3 + 8*c*d^3 - 14*b*d^2*e + 35*a*d*e^2 + 3*(c*d*e^2 + 7*b*
e^3)*x^2 - (4*c*d^2*e - 7*b*d*e^2 - 35*a*e^3)*x)*sqrt(e*x + d)/e^3
```

Sympy [A] time = 2.27761, size = 71, normalized size = 0.95

$$\frac{2 \left(\frac{c(d+ex)^{\frac{7}{2}}}{7e^2} + \frac{(d+ex)^{\frac{5}{2}} (be-2cd)}{5e^2} + \frac{(d+ex)^{\frac{3}{2}} \left(ae^2 - bde + cd^2\right)}{3e^2}\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*(e*x+d)**(1/2),x)
```

```
[Out] 2*(c*(d + e*x)**(7/2)/(7*e**2) + (d + e*x)**(5/2)*(b*e - 2*c*d)/(5*e**2) +
(d + e*x)**(3/2)*(a*e**2 - b*d*e + c*d**2)/(3*e**2))/e
```

Giac [A] time = 1.09711, size = 111, normalized size = 1.48

$$\frac{2}{105}\left(7\left(3(xe+d)^{\frac{5}{2}}-5(xe+d)^{\frac{3}{2}}d\right)be^{(-1)}+\left(15(xe+d)^{\frac{7}{2}}-42(xe+d)^{\frac{5}{2}}d+35(xe+d)^{\frac{3}{2}}d^2\right)ce^{(-2)}+35(xe+d)^{\frac{3}{2}}a\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(7*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*b*e^(-1) + (15*(x*e + d)
^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*c*e^(-2) + 35*(x*e
+ d)^(3/2)*a)*e^(-1)
```

3.2271 $\int \frac{a+bx+cx^2}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=73

$$\frac{2\sqrt{d+ex}\,(ae^2-bde+cd^2)}{e^3}-\frac{2(d+ex)^{3/2}(2cd-be)}{3e^3}+\frac{2c(d+ex)^{5/2}}{5e^3}$$

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x])/e^3 - (2*(2*c*d - b*e)*(d + e*x)^
(3/2))/(3*e^3) + (2*c*(d + e*x)^(5/2))/(5*e^3)
```

Rubi [A] time = 0.0299421, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$\frac{2\sqrt{d+ex}\,(ae^2-bde+cd^2)}{e^3}-\frac{2(d+ex)^{3/2}(2cd-be)}{3e^3}+\frac{2c(d+ex)^{5/2}}{5e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/Sqrt[d + e*x],x]
```

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x])/e^3 - (2*(2*c*d - b*e)*(d + e*x)^
(3/2))/(3*e^3) + (2*c*(d + e*x)^(5/2))/(5*e^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{\sqrt{d+ex}}\,dx &= \int \left(\frac{cd^2-bde+ae^2}{e^2\sqrt{d+ex}}+\frac{(-2cd+be)\sqrt{d+ex}}{e^2}+\frac{c(d+ex)^{3/2}}{e^2}\right)dx\\ &= \frac{2\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}{e^3}-\frac{2(2cd-be)(d+ex)^{3/2}}{3e^3}+\frac{2c(d+ex)^{5/2}}{5e^3}\end{aligned}$$

Mathematica [A] time = 0.0505959, size = 54, normalized size = 0.74

$$\frac{2\sqrt{d+ex}\left(5e(3ae-2bd+bex)+c\left(8d^2-4dex+3e^2x^2\right)\right)}{15e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[d + e*x]*(5*e*(-2*b*d + 3*a*e + b*e*x) + c*(8*d^2 - 4*d*e*x + 3*e^2
*x^2)))/(15*e^3)
```

Maple [A] time = 0.042, size = 53, normalized size = 0.7

$$\frac{6\,c e^{2} x^{2}+10\,b e^{2} x-8\,c d e x+30\,a e^{2}-20\,b d e+16\,c d^{2}}{15\,e^{3}}\sqrt{e x+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(1/2),x)
```

```
[Out] 2/15*(e*x+d)^(1/2)*(3*c*e^2*x^2+5*b*e^2*x-4*c*d*e*x+15*a*e^2-10*b*d*e+8*c*d
^2)/e^3
```

Maxima [A] time = 0.978035, size = 104, normalized size = 1.42

$$\frac{2\left(15\,\sqrt{e x+d}a+\frac{5\left((e x+d)^{\frac{3}{2}}-3\,\sqrt{e x+d}d\right)b}{e}+\frac{\left(3\,(e x+d)^{\frac{5}{2}}-10\,(e x+d)^{\frac{3}{2}}d+15\,\sqrt{e x+d}d^{2}\right)c}{e^{2}}\right)}{15\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(e*x + d)*a + 5*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*b/e + (3
*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*c/e^2)/e
```

Fricas [A] time = 2.27172, size = 127, normalized size = 1.74

$$\frac{2\left(3\,c e^{2} x^{2}+8\,c d^{2}-10\,b d e+15\,a e^{2}-\left(4\,c d e-5\,b e^{2}\right)x\right)\sqrt{e x+d}}{15\,e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*c*e^2*x^2 + 8*c*d^2 - 10*b*d*e + 15*a*e^2 - (4*c*d*e - 5*b*e^2)*x)*
sqrt(e*x + d)/e^3
```

Sympy [A] time = 8.2473, size = 223, normalized size = 3.05

$$\begin{cases} -\frac{\frac{2ad}{\sqrt{d+ex}}+2a\left(-\frac{d}{\sqrt{d+ex}}-\sqrt{d+ex}\right)+\frac{2bd\left(-\frac{d}{\sqrt{d+ex}}-\sqrt{d+ex}\right)}{e}+\frac{2b\left(\frac{d^{2}}{\sqrt{d+ex}}+2d\sqrt{d+ex}-\frac{(d+ex)^{\frac{3}{2}}}{3}\right)}{e}+\frac{2cd\left(\frac{d^{2}}{\sqrt{d+ex}}+2d\sqrt{d+ex}-\frac{(d+ex)^{\frac{3}{2}}}{3}\right)}{e^{2}}+\frac{2c\left(-\frac{d^{3}}{\sqrt{d+ex}}-3d^{2}\sqrt{d+ex}+d(d+ex)^{\frac{3}{2}}-\frac{(d+ex)^{\frac{5}{2}}}{5}\right)}{e^{2}}}{e} \\ \frac{ax+\frac{bx^{2}}{2}+\frac{cx^{3}}{3}}{\sqrt{d}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(1/2),x)
```

```
[Out] Piecewise((-(2*a*d/sqrt(d + e*x) + 2*a*(-d/sqrt(d + e*x) - sqrt(d + e*x)) +
 2*b*d*(-d/sqrt(d + e*x) - sqrt(d + e*x))/e + 2*b*(d**2/sqrt(d + e*x) + 2*d
*sqrt(d + e*x) - (d + e*x)**(3/2)/3)/e + 2*c*d*(d**2/sqrt(d + e*x) + 2*d*sq
rt(d + e*x) - (d + e*x)**(3/2)/3)/e**2 + 2*c*(-d**3/sqrt(d + e*x) - 3*d**2*
sqrt(d + e*x) + d*(d + e*x)**(3/2) - (d + e*x)**(5/2)/5)/e**2)/e, Ne(e, 0))
, ((a*x + b*x**2/2 + c*x**3/3)/sqrt(d), True))
```

Giac [A] time = 1.10022, size = 108, normalized size = 1.48

$$\frac{2}{15}\left(5\left((xe+d)^{\frac{3}{2}}-3\sqrt{xe+d}d\right)be^{(-1)}+\left(3(xe+d)^{\frac{5}{2}}-10(xe+d)^{\frac{3}{2}}d+15\sqrt{xe+d}d^2\right)ce^{(-2)}+15\sqrt{xe+d}a\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(5*((x*e + d)^(3/2) - 3*sqrt(x*e + d)*d)*b*e^(-1) + (3*(x*e + d)^(5/2)
 - 10*(x*e + d)^(3/2)*d + 15*sqrt(x*e + d)*d^2)*c*e^(-2) + 15*sqrt(x*e + d)
*a)*e^(-1)
```

3.2272 $\int \frac{a+bx+cx^2}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=71

$$-\frac{2\left(ae^2-bde+cd^2\right)}{e^3\sqrt{d+ex}}-\frac{2\sqrt{d+ex}(2cd-be)}{e^3}+\frac{2c(d+ex)^{3/2}}{3e^3}$$

```
[Out] (-2*(c*d^2 - b*d*e + a*e^2))/(e^3*Sqrt[d + e*x]) - (2*(2*c*d - b*e)*Sqrt[d
+ e*x])/e^3 + (2*c*(d + e*x)^(3/2))/(3*e^3)
```

Rubi [A] time = 0.0306339, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$-\frac{2\left(ae^2-bde+cd^2\right)}{e^3\sqrt{d+ex}}-\frac{2\sqrt{d+ex}(2cd-be)}{e^3}+\frac{2c(d+ex)^{3/2}}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(d + e*x)^(3/2),x]
```

```
[Out] (-2*(c*d^2 - b*d*e + a*e^2))/(e^3*Sqrt[d + e*x]) - (2*(2*c*d - b*e)*Sqrt[d
+ e*x])/e^3 + (2*c*(d + e*x)^(3/2))/(3*e^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{(d+ex)^{3/2}}\,dx &= \int \left(\frac{cd^2-bde+ae^2}{e^2(d+ex)^{3/2}}+\frac{-2cd+be}{e^2\sqrt{d+ex}}+\frac{c\sqrt{d+ex}}{e^2}\right)dx\\ &= -\frac{2\left(cd^2-bde+ae^2\right)}{e^3\sqrt{d+ex}}-\frac{2(2cd-be)\sqrt{d+ex}}{e^3}+\frac{2c(d+ex)^{3/2}}{3e^3}\end{aligned}$$

Mathematica [A] time = 0.050236, size = 54, normalized size = 0.76

$$\frac{6e(-ae+2bd+bex)+2c\left(-8d^2-4dex+e^2x^2\right)}{3e^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(d + e*x)^(3/2),x]
```

```
[Out] (6*e*(2*b*d - a*e + b*e*x) + 2*c*(-8*d^2 - 4*d*e*x + e^2*x^2))/(3*e^3*Sqrt[
d + e*x])
```

Maple [A] time = 0.042, size = 53, normalized size = 0.8

$$-\frac{-2 c e^2 x^2 - 6 b e^2 x + 8 c d e x + 6 a e^2 - 12 b d e + 16 c d^2}{3 e^3} \frac{1}{\sqrt{e x + d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(3/2),x)
```

```
[Out] -2/3/(e*x+d)^(1/2)*(-c*e^2*x^2-3*b*e^2*x+4*c*d*e*x+3*a*e^2-6*b*d*e+8*c*d^2)
/e^3
```

Maxima [A] time = 0.975398, size = 89, normalized size = 1.25

$$\frac{2 \left(\frac{(ex+d)^{\frac{3}{2}} c - 3 (2 c d - b e) \sqrt{ex+d}}{e^2} - \frac{3 \left(c d^2 - b d e + a e^2 \right)}{\sqrt{ex+d} e^2} \right)}{3 e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(((e*x + d)^(3/2)*c - 3*(2*c*d - b*e)*sqrt(e*x + d))/e^2 - 3*(c*d^2 - b
*d*e + a*e^2)/(sqrt(e*x + d)*e^2))/e
```

Fricas [A] time = 2.18648, size = 136, normalized size = 1.92

$$\frac{2 \left(c e^2 x^2 - 8 c d^2 + 6 b d e - 3 a e^2 - \left(4 c d e - 3 b e^2 \right) x \right) \sqrt{e x + d}}{3 \left(e^4 x + d e^3 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(c*e^2*x^2 - 8*c*d^2 + 6*b*d*e - 3*a*e^2 - (4*c*d*e - 3*b*e^2)*x)*sqrt(
e*x + d)/(e^4*x + d*e^3)
```

Sympy [A] time = 9.0996, size = 70, normalized size = 0.99

$$\frac{2c (d + ex)^{\frac{3}{2}}}{3e^3} + \frac{\sqrt{d + ex} (2be - 4cd)}{e^3} - \frac{2 \left(ae^2 - bde + cd^2 \right)}{e^3 \sqrt{d + ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(3/2),x)
```

```
[Out] 2*c*(d + e*x)**(3/2)/(3*e**3) + sqrt(d + e*x)*(2*b*e - 4*c*d)/e**3 - 2*(a*e
**2 - b*d*e + c*d**2)/(e**3*sqrt(d + e*x))
```

Giac [A] time = 1.1096, size = 99, normalized size = 1.39

$$\frac{2}{3}\left((xe+d)^{\frac{3}{2}}ce^6 - 6\sqrt{xe+d}cde^6 + 3\sqrt{xe+d}be^7\right)e^{(-9)} - \frac{2\left(cd^2 - bde + ae^2\right)e^{(-3)}}{\sqrt{xe+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*((x*e + d)^(3/2)*c*e^6 - 6*sqrt(x*e + d)*c*d*e^6 + 3*sqrt(x*e + d)*b*e^
7)*e^(-9) - 2*(c*d^2 - b*d*e + a*e^2)*e^(-3)/sqrt(x*e + d)
```

3.2273 $\int \frac{a+bx+cx^2}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=71

$$-\frac{2\left(ae^2-bde+cd^2\right)}{3e^3(d+ex)^{3/2}}+\frac{2(2cd-be)}{e^3\sqrt{d+ex}}+\frac{2c\sqrt{d+ex}}{e^3}$$

```
[Out] (-2*(c*d^2 - b*d*e + a*e^2))/(3*e^3*(d + e*x)^(3/2)) + (2*(2*c*d - b*e))/(e
^3*Sqrt[d + e*x]) + (2*c*Sqrt[d + e*x])/e^3
```

Rubi [A] time = 0.0321167, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$-\frac{2\left(ae^2-bde+cd^2\right)}{3e^3(d+ex)^{3/2}}+\frac{2(2cd-be)}{e^3\sqrt{d+ex}}+\frac{2c\sqrt{d+ex}}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(d + e*x)^(5/2),x]
```

```
[Out] (-2*(c*d^2 - b*d*e + a*e^2))/(3*e^3*(d + e*x)^(3/2)) + (2*(2*c*d - b*e))/(e
^3*Sqrt[d + e*x]) + (2*c*Sqrt[d + e*x])/e^3
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{(d+ex)^{5/2}}\,dx &= \int \left(\frac{cd^2-bde+ae^2}{e^2(d+ex)^{5/2}}+\frac{-2cd+be}{e^2(d+ex)^{3/2}}+\frac{c}{e^2\sqrt{d+ex}}\right)dx\\ &= -\frac{2\left(cd^2-bde+ae^2\right)}{3e^3(d+ex)^{3/2}}+\frac{2(2cd-be)}{e^3\sqrt{d+ex}}+\frac{2c\sqrt{d+ex}}{e^3}\end{aligned}$$

Mathematica [A] time = 0.0468586, size = 55, normalized size = 0.77

$$\frac{2c\left(8d^2+12dex+3e^2x^2\right)-2e(ae+2bd+3bex)}{3e^3(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(d + e*x)^(5/2),x]
```

```
[Out] (-2*e*(2*b*d + a*e + 3*b*e*x) + 2*c*(8*d^2 + 12*d*e*x + 3*e^2*x^2))/(3*e^3*
(d + e*x)^(3/2))
```

Maple [A] time = 0.042, size = 52, normalized size = 0.7

$$-\frac{-6\,c e^{2} x^{2}+6\,b e^{2} x-24\,c d e x+2\,a e^{2}+4\,b d e-16\,c d^{2}}{3\,e^{3}}\left(e x+d\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(5/2),x)
```

```
[Out] -2/3/(e*x+d)^(3/2)*(-3*c*e^2*x^2+3*b*e^2*x-12*c*d*e*x+a*e^2+2*b*d*e-8*c*d^2
)/e^3
```

Maxima [A] time = 1.03847, size = 85, normalized size = 1.2

$$\frac{2\left(\frac{3\,\sqrt{ex+d}c}{e^{2}}-\frac{cd^{2}-bde+ae^{2}-3\,(2\,cd-be)(ex+d)}{(ex+d)^{\frac{3}{2}}e^{2}}\right)}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(3*sqrt(e*x + d)*c/e^2 - (c*d^2 - b*d*e + a*e^2 - 3*(2*c*d - b*e)*(e*x
+ d))/((e*x + d)^(3/2)*e^2))/e
```

Fricas [A] time = 2.31682, size = 158, normalized size = 2.23

$$\frac{2\left(3\,c e^{2} x^{2}+8\,c d^{2}-2\,b d e-a e^{2}+3\left(4\,c d e-b e^{2}\right)x\right)\sqrt{e x+d}}{3\left(e^{5} x^{2}+2\,d e^{4} x+d^{2} e^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*c*e^2*x^2 + 8*c*d^2 - 2*b*d*e - a*e^2 + 3*(4*c*d*e - b*e^2)*x)*sqrt(
e*x + d)/(e^5*x^2 + 2*d*e^4*x + d^2*e^3)
```

Sympy [A] time = 1.298, size = 252, normalized size = 3.55

$$\begin{cases} -\frac{2ae^2}{3de^3\sqrt{d+ex}+3e^4x\sqrt{d+ex}} - \frac{4bde}{3de^3\sqrt{d+ex}+3e^4x\sqrt{d+ex}} - \frac{6be^2x}{3de^3\sqrt{d+ex}+3e^4x\sqrt{d+ex}} + \frac{16cd^2}{3de^3\sqrt{d+ex}+3e^4x\sqrt{d+ex}} + \frac{24cdex}{3de^3\sqrt{d+ex}+3e^4x\sqrt{d+ex}} + \frac{6ce^2x}{3de^3\sqrt{d+ex}+3} \\ \frac{ax+\frac{bx^2}{2}+\frac{cx^3}{3}}{d^{\frac{5}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(5/2),x)
```

```
[Out] Piecewise((-2*a*e**2/(3*d*e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)) - 4*
b*d*e/(3*d*e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)) - 6*b*e**2*x/(3*d*e
```

```
**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)) + 16*c*d**2/(3*d*e**3*sqrt(d +
e*x) + 3*e**4*x*sqrt(d + e*x)) + 24*c*d*e*x/(3*d*e**3*sqrt(d + e*x) + 3*e**
4*x*sqrt(d + e*x)) + 6*c*e**2*x**2/(3*d*e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(
d + e*x)), Ne(e, 0)), ((a*x + b*x**2/2 + c*x**3/3)/d**(5/2), True))
```

Giac [A] time = 1.10228, size = 86, normalized size = 1.21

$$2\sqrt{xe+d}ce^{(-3)} + \frac{2\left(6(xe+d)cd - cd^2 - 3(xe+d)be + bde - ae^2\right)e^{(-3)}}{3(xe+d)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x*e + d)*c*e^(-3) + 2/3*(6*(x*e + d)*c*d - c*d^2 - 3*(x*e + d)*b*e +
 b*d*e - a*e^2)*e^(-3)/(x*e + d)^(3/2)
```

3.2274 $\int \frac{a+bx+cx^2}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=73

$$-\frac{2\left(ae^2-bde+cd^2\right)}{5e^3(d+ex)^{5/2}}+\frac{2(2cd-be)}{3e^3(d+ex)^{3/2}}-\frac{2c}{e^3\sqrt{d+ex}}$$

```
[Out] (-2*(c*d^2 - b*d*e + a*e^2))/(5*e^3*(d + e*x)^(5/2)) + (2*(2*c*d - b*e))/(3
*e^3*(d + e*x)^(3/2)) - (2*c)/(e^3*Sqrt[d + e*x])
```

Rubi [A] time = 0.029924, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$-\frac{2\left(ae^2-bde+cd^2\right)}{5e^3(d+ex)^{5/2}}+\frac{2(2cd-be)}{3e^3(d+ex)^{3/2}}-\frac{2c}{e^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(d + e*x)^(7/2),x]
```

```
[Out] (-2*(c*d^2 - b*d*e + a*e^2))/(5*e^3*(d + e*x)^(5/2)) + (2*(2*c*d - b*e))/(3
*e^3*(d + e*x)^(3/2)) - (2*c)/(e^3*Sqrt[d + e*x])
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{(d+ex)^{7/2}}\,dx &= \int \left(\frac{cd^2-bde+ae^2}{e^2(d+ex)^{7/2}}+\frac{-2cd+be}{e^2(d+ex)^{5/2}}+\frac{c}{e^2(d+ex)^{3/2}}\right)dx\\ &= -\frac{2\left(cd^2-bde+ae^2\right)}{5e^3(d+ex)^{5/2}}+\frac{2(2cd-be)}{3e^3(d+ex)^{3/2}}-\frac{2c}{e^3\sqrt{d+ex}}\end{aligned}$$

Mathematica [A] time = 0.0490813, size = 54, normalized size = 0.74

$$-\frac{2\left(e(3ae+2bd+5bex)+c\left(8d^2+20dex+15e^2x^2\right)\right)}{15e^3(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(d + e*x)^(7/2),x]
```

```
[Out] (-2*(e*(2*b*d + 3*a*e + 5*b*e*x) + c*(8*d^2 + 20*d*e*x + 15*e^2*x^2)))/(15*
e^3*(d + e*x)^(5/2))
```

Maple [A] time = 0.042, size = 53, normalized size = 0.7

$$-\frac{30\,c e^2 x^2+10\,b e^2 x+40\,c d e x+6\,a e^2+4\,b d e+16\,c d^2}{15\,e^3}\,(e x+d)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(7/2),x)
```

```
[Out] -2/15/(e*x+d)^(5/2)*(15*c*e^2*x^2+5*b*e^2*x+20*c*d*e*x+3*a*e^2+2*b*d*e+8*c*
d^2)/e^3
```

Maxima [A] time = 0.994225, size = 76, normalized size = 1.04

$$-\frac{2\left(15\,(e x+d)^2 c+3\,c d^2-3\,b d e+3\,a e^2-5\,(2\,c d-b e)(e x+d)\right)}{15\,(e x+d)^{\frac{5}{2}} e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/15*(15*(e*x + d)^2*c + 3*c*d^2 - 3*b*d*e + 3*a*e^2 - 5*(2*c*d - b*e)*(e*
x + d))/((e*x + d)^(5/2)*e^3)
```

Fricas [A] time = 2.30313, size = 186, normalized size = 2.55

$$-\frac{2\left(15\,c e^2 x^2+8\,c d^2+2\,b d e+3\,a e^2+5\left(4\,c d e+b e^2\right)x\right)\sqrt{e x+d}}{15\left(e^6 x^3+3\,d e^5 x^2+3\,d^2 e^4 x+d^3 e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/15*(15*c*e^2*x^2 + 8*c*d^2 + 2*b*d*e + 3*a*e^2 + 5*(4*c*d*e + b*e^2)*x)*
sqrt(e*x + d)/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3)
```

Sympy [A] time = 3.04277, size = 376, normalized size = 5.15

$$\begin{cases} -\frac{6ae^2}{15d^2e^3\sqrt{d+ex}+30de^4x\sqrt{d+ex}+15e^5x^2\sqrt{d+ex}}-\frac{4bde}{15d^2e^3\sqrt{d+ex}+30de^4x\sqrt{d+ex}+15e^5x^2\sqrt{d+ex}}-\frac{10be^2x}{15d^2e^3\sqrt{d+ex}+30de^4x\sqrt{d+ex}+15e^5x^2\sqrt{d+ex}}-\frac{}{15d^2e^3\cdots} \\ \frac{ax+\frac{bx^2}{2}+\frac{cx^3}{3}}{d^{\frac{7}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(7/2),x)
```

```
[Out] Piecewise((-6*a*e**2/(15*d**2*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d + e*x
) + 15*e**5*x**2*sqrt(d + e*x)) - 4*b*d*e/(15*d**2*e**3*sqrt(d + e*x) + 30*
```

```
d*e**4*x*sqrt(d + e*x) + 15*e**5*x**2*sqrt(d + e*x)) - 10*b*e**2*x/(15*d**2
*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d + e*x) + 15*e**5*x**2*sqrt(d + e*x
)) - 16*c*d**2/(15*d**2*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d + e*x) + 15
*e**5*x**2*sqrt(d + e*x)) - 40*c*d*e*x/(15*d**2*e**3*sqrt(d + e*x) + 30*d*e
**4*x*sqrt(d + e*x) + 15*e**5*x**2*sqrt(d + e*x)) - 30*c*e**2*x**2/(15*d**2
*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d + e*x) + 15*e**5*x**2*sqrt(d + e*x
)), Ne(e, 0)), ((a*x + b*x**2/2 + c*x**3/3)/d**(7/2), True))
```

Giac [A] time = 1.13541, size = 84, normalized size = 1.15

$$-\frac{2\left(15\,(xe+d)^2c-10\,(xe+d)cd+3\,cd^2+5\,(xe+d)be-3\,bde+3\,ae^2\right)e^{(-3)}}{15\,(xe+d)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*(15*(x*e + d)^2*c - 10*(x*e + d)*c*d + 3*c*d^2 + 5*(x*e + d)*b*e - 3*
b*d*e + 3*a*e^2)*e^(-3)/(x*e + d)^(5/2)
```

3.2275 $\int (d+ex)^{5/2}\left(a+bx+cx^2\right)^2\,dx$

Optimal. Leaf size=166

$$\frac{2(d+ex)^{11/2}\left(-2ce(3bd-ae)+b^2e^2+6c^2d^2\right)}{11e^5}-\frac{4(d+ex)^{9/2}(2cd-be)\left(ae^2-bde+cd^2\right)}{9e^5}+\frac{2(d+ex)^{7/2}\left(ae^2-bde+cd^2\right)}{7e^5}$$

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^(7/2))/(7*e^5) - (4*(2*c*d - b*e)*(c
*d^2 - b*d*e + a*e^2)*(d + e*x)^(9/2))/(9*e^5) + (2*(6*c^2*d^2 + b^2*e^2 -
2*c*e*(3*b*d - a*e))*(d + e*x)^(11/2))/(11*e^5) - (4*c*(2*c*d - b*e)*(d + e
*x)^(13/2))/(13*e^5) + (2*c^2*(d + e*x)^(15/2))/(15*e^5)
```

Rubi [A] time = 0.092938, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {698}

$$\frac{2(d+ex)^{11/2}\left(-2ce(3bd-ae)+b^2e^2+6c^2d^2\right)}{11e^5}-\frac{4(d+ex)^{9/2}(2cd-be)\left(ae^2-bde+cd^2\right)}{9e^5}+\frac{2(d+ex)^{7/2}\left(ae^2-bde+cd^2\right)}{7e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)*(a + b*x + c*x^2)^2,x]
```

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^(7/2))/(7*e^5) - (4*(2*c*d - b*e)*(c
*d^2 - b*d*e + a*e^2)*(d + e*x)^(9/2))/(9*e^5) + (2*(6*c^2*d^2 + b^2*e^2 -
2*c*e*(3*b*d - a*e))*(d + e*x)^(11/2))/(11*e^5) - (4*c*(2*c*d - b*e)*(d + e
*x)^(13/2))/(13*e^5) + (2*c^2*(d + e*x)^(15/2))/(15*e^5)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^{5/2}\left(a+bx+cx^2\right)^2\,dx=\int\left(\frac{\left(cd^2-bde+ae^2\right)^2(d+ex)^{5/2}}{e^4}+\frac{2(-2cd+be)\left(cd^2-bde+ae^2\right)(d+ex)^{7/2}}{e^4}+\right.$$

$$=\frac{2\left(cd^2-bde+ae^2\right)^2(d+ex)^{7/2}}{7e^5}-\frac{4(2cd-be)\left(cd^2-bde+ae^2\right)(d+ex)^{9/2}}{9e^5}+\frac{2\left(6c^2\right.}{}$$

Mathematica [A] time = 0.163742, size = 173, normalized size = 1.04

$$\frac{2(d+ex)^{7/2}\left(65e^2\left(99a^2e^2+22abe(7ex-2d)+b^2\left(8d^2-28dex+63e^2x^2\right)\right)-10ce\left(3b\left(-56d^2ex+16d^3+126de^2x^2-\right.\right.\right.}{45045e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)*(a + b*x + c*x^2)^2,x]
```

```
[Out] (2*(d + e*x)^(7/2)*(c^2*(128*d^4 - 448*d^3*e*x + 1008*d^2*e^2*x^2 - 1848*d*
e^3*x^3 + 3003*e^4*x^4) + 65*e^2*(99*a^2*e^2 + 22*a*b*e*(-2*d + 7*e*x) + b^
2*(8*d^2 - 28*d*e*x + 63*e^2*x^2)) - 10*c*e*(-13*a*e*(8*d^2 - 28*d*e*x + 63
*e^2*x^2) + 3*b*(16*d^3 - 56*d^2*e*x + 126*d*e^2*x^2 - 231*e^3*x^3))))/(450
45*e^5)
```

Maple [A] time = 0.048, size = 194, normalized size = 1.2

$$6006\,c^2x^4e^4 + 13860\,bce^4x^3 - 3696\,c^2de^3x^3 + 16380\,ace^4x^2 + 8190\,b^2e^4x^2 - 7560\,bcde^3x^2 + 2016\,c^2d^2e^2x^2 + 20020\,abe^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(c*x^2+b*x+a)^2,x)
```

```
[Out] 2/45045*(e*x+d)^(7/2)*(3003*c^2*e^4*x^4+6930*b*c*e^4*x^3-1848*c^2*d*e^3*x^3
+8190*a*c*e^4*x^2+4095*b^2*e^4*x^2-3780*b*c*d*e^3*x^2+1008*c^2*d^2*e^2*x^2+
10010*a*b*e^4*x-3640*a*c*d*e^3*x-1820*b^2*d*e^3*x+1680*b*c*d^2*e^2*x-448*c^
2*d^3*e*x+6435*a^2*e^4-2860*a*b*d*e^3+1040*a*c*d^2*e^2+520*b^2*d^2*e^2-480*
b*c*d^3*e+128*c^2*d^4)/e^5
```

Maxima [A] time = 0.983419, size = 238, normalized size = 1.43

$$\frac{2\left(3003\,(ex+d)^{\frac{15}{2}}c^2 - 6930\left(2\,c^2d - bce\right)(ex+d)^{\frac{13}{2}} + 4095\left(6\,c^2d^2 - 6\,bcde + \left(b^2 + 2\,ac\right)e^2\right)(ex+d)^{\frac{11}{2}} - 10010\left(2\,c^2d^3 - 3\right.}{45045\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2/45045*(3003*(e*x + d)^(15/2)*c^2 - 6930*(2*c^2*d - b*c*e)*(e*x + d)^(13/2
) + 4095*(6*c^2*d^2 - 6*b*c*d*e + (b^2 + 2*a*c)*e^2)*(e*x + d)^(11/2) - 100
10*(2*c^2*d^3 - 3*b*c*d^2*e - a*b*e^3 + (b^2 + 2*a*c)*d*e^2)*(e*x + d)^(9/2
) + 6435*(c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2
*e^2)*(e*x + d)^(7/2))/e^5
```

Fricas [B] time = 2.35495, size = 860, normalized size = 5.18

$$2\left(3003\,c^2e^7x^7 + 128\,c^2d^7 - 480\,bcd^6e - 2860\,abd^4e^3 + 6435\,a^2d^3e^4 + 520\left(b^2 + 2\,ac\right)d^5e^2 + 231\left(31\,c^2de^6 + 30\,bce^7\right)x^6 + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 2/45045*(3003*c^2*e^7*x^7 + 128*c^2*d^7 - 480*b*c*d^6*e - 2860*a*b*d^4*e^3
+ 6435*a^2*d^3*e^4 + 520*(b^2 + 2*a*c)*d^5*e^2 + 231*(31*c^2*d*e^6 + 30*b*c
*e^7)*x^6 + 63*(71*c^2*d^2*e^5 + 270*b*c*d*e^6 + 65*(b^2 + 2*a*c)*e^7)*x^5
+ 35*(c^2*d^3*e^4 + 318*b*c*d^2*e^5 + 286*a*b*e^7 + 299*(b^2 + 2*a*c)*d*e^6
)*x^4 - 5*(8*c^2*d^4*e^3 - 30*b*c*d^3*e^4 - 5434*a*b*d*e^6 - 1287*a^2*e^7 -
 1469*(b^2 + 2*a*c)*d^2*e^5)*x^3 + 3*(16*c^2*d^5*e^2 - 60*b*c*d^4*e^3 + 715
0*a*b*d^2*e^5 + 6435*a^2*d*e^6 + 65*(b^2 + 2*a*c)*d^3*e^4)*x^2 - (64*c^2*d^
```

```
6*e - 240*b*c*d^5*e^2 - 1430*a*b*d^3*e^4 - 19305*a^2*d^2*e^5 + 260*(b^2 + 2
*a*c)*d^4*e^3)*x)*sqrt(e*x + d)/e^5
```

Sympy [A] time = 34.4345, size = 1129, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(c*x**2+b*x+a)**2,x)
```

```
[Out] a**2*d**2*Piecewise((sqrt(d)*x, Eq(e, 0)), (2*(d + e*x)**(3/2)/(3*e), True)
) + 4*a**2*d*(-d*(d + e*x)**(3/2)/3 + (d + e*x)**(5/2)/5)/e + 2*a**2*(d**2*
(d + e*x)**(3/2)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e + 4*a*b
*d**2*(-d*(d + e*x)**(3/2)/3 + (d + e*x)**(5/2)/5)/e**2 + 8*a*b*d*(d**2*(d
+ e*x)**(3/2)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e**2 + 4*a*b
*(-d**3*(d + e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d + e*x)**(7/
2)/7 + (d + e*x)**(9/2)/9)/e**2 + 4*a*c*d**2*(d**2*(d + e*x)**(3/2)/3 - 2*d
*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e**3 + 8*a*c*d*(-d**3*(d + e*x)**
(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d + e*x)**(7/2)/7 + (d + e*x)**(
9/2)/9)/e**3 + 4*a*c*(d**4*(d + e*x)**(3/2)/3 - 4*d**3*(d + e*x)**(5/2)/5 +
 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e*x)**(9/2)/9 + (d + e*x)**(11/2)/11)
/e**3 + 2*b**2*d**2*(d**2*(d + e*x)**(3/2)/3 - 2*d*(d + e*x)**(5/2)/5 + (d
+ e*x)**(7/2)/7)/e**3 + 4*b**2*d*(-d**3*(d + e*x)**(3/2)/3 + 3*d**2*(d + e*
x)**(5/2)/5 - 3*d*(d + e*x)**(7/2)/7 + (d + e*x)**(9/2)/9)/e**3 + 2*b**2*(d
**4*(d + e*x)**(3/2)/3 - 4*d**3*(d + e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/2
)/7 - 4*d*(d + e*x)**(9/2)/9 + (d + e*x)**(11/2)/11)/e**3 + 4*b*c*d**2*(-d*
*3*(d + e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d + e*x)**(7/2)/7
+ (d + e*x)**(9/2)/9)/e**4 + 8*b*c*d*(d**4*(d + e*x)**(3/2)/3 - 4*d**3*(d +
 e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e*x)**(9/2)/9 + (d +
e*x)**(11/2)/11)/e**4 + 4*b*c*(-d**5*(d + e*x)**(3/2)/3 + d**4*(d + e*x)**(
5/2) - 10*d**3*(d + e*x)**(7/2)/7 + 10*d**2*(d + e*x)**(9/2)/9 - 5*d*(d + e
*x)**(11/2)/11 + (d + e*x)**(13/2)/13)/e**4 + 2*c**2*d**2*(d**4*(d + e*x)**
(3/2)/3 - 4*d**3*(d + e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d +
e*x)**(9/2)/9 + (d + e*x)**(11/2)/11)/e**5 + 4*c**2*d*(-d**5*(d + e*x)**(3/
2)/3 + d**4*(d + e*x)**(5/2) - 10*d**3*(d + e*x)**(7/2)/7 + 10*d**2*(d + e*
x)**(9/2)/9 - 5*d*(d + e*x)**(11/2)/11 + (d + e*x)**(13/2)/13)/e**5 + 2*c**
2*(d**6*(d + e*x)**(3/2)/3 - 6*d**5*(d + e*x)**(5/2)/5 + 15*d**4*(d + e*x)*
*(7/2)/7 - 20*d**3*(d + e*x)**(9/2)/9 + 15*d**2*(d + e*x)**(11/2)/11 - 6*d*
(d + e*x)**(13/2)/13 + (d + e*x)**(15/2)/15)/e**5
```

Giac [B] time = 1.15802, size = 1353, normalized size = 8.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 2/45045*(6006*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a*b*d^2*e^(-1) + 42
9*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*b^2*
d^2*e^(-2) + 858*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^
(3/2)*d^2)*a*c*d^2*e^(-2) + 286*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d
 + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*b*c*d^2*e^(-3) + 13*(
```

315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*c^2*d^2*e^(-4) + 15015*(x*e + d)^(3/2)*a^2*d^2 + 1716*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*a*b*d*e^(-1) + 286*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*b^2*d*e^(-2) + 572*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a*c*d*e^(-2) + 52*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*b*c*d*e^(-3) + 10*(693*(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*c^2*d*e^(-4) + 6006*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^2*d + 286*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a*b*e^(-1) + 13*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*b^2*e^(-2) + 26*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*a*c*e^(-2) + 10*(693*(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*b*c*e^(-3) + (3003*(x*e + d)^(15/2) - 20790*(x*e + d)^(13/2)*d + 61425*(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525*(x*e + d)^(7/2)*d^4 - 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*c^2*e^(-4) + 429*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*a^2)*e^(-1)

3.2276 $\int (d+ex)^{3/2} \left(a+bx+cx^2\right)^2 \, dx$

Optimal. Leaf size=166

$$\frac{2(d+ex)^{9/2}\left(-2ce(3bd-ae)+b^2e^2+6c^2d^2\right)}{9e^5}-\frac{4(d+ex)^{7/2}(2cd-be)\left(ae^2-bde+cd^2\right)}{7e^5}+\frac{2(d+ex)^{5/2}\left(ae^2-bde+cd^2\right)}{5e^5}$$

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^(5/2))/(5*e^5) - (4*(2*c*d - b*e)*(c
*d^2 - b*d*e + a*e^2)*(d + e*x)^(7/2))/(7*e^5) + (2*(6*c^2*d^2 + b^2*e^2 -
2*c*e*(3*b*d - a*e))*(d + e*x)^(9/2))/(9*e^5) - (4*c*(2*c*d - b*e)*(d + e*x
)^(11/2))/(11*e^5) + (2*c^2*(d + e*x)^(13/2))/(13*e^5)
```

Rubi [A] time = 0.0733657, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {698}

$$\frac{2(d+ex)^{9/2}\left(-2ce(3bd-ae)+b^2e^2+6c^2d^2\right)}{9e^5}-\frac{4(d+ex)^{7/2}(2cd-be)\left(ae^2-bde+cd^2\right)}{7e^5}+\frac{2(d+ex)^{5/2}\left(ae^2-bde+cd^2\right)}{5e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*(a + b*x + c*x^2)^2,x]
```

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^(5/2))/(5*e^5) - (4*(2*c*d - b*e)*(c
*d^2 - b*d*e + a*e^2)*(d + e*x)^(7/2))/(7*e^5) + (2*(6*c^2*d^2 + b^2*e^2 -
2*c*e*(3*b*d - a*e))*(d + e*x)^(9/2))/(9*e^5) - (4*c*(2*c*d - b*e)*(d + e*x
)^(11/2))/(11*e^5) + (2*c^2*(d + e*x)^(13/2))/(13*e^5)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^{3/2}\left(a+bx+cx^2\right)^2\,dx=\int\left(\frac{\left(cd^2-bde+ae^2\right)^2(d+ex)^{3/2}}{e^4}+\frac{2(-2cd+be)\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}}{e^4}+\right.$$

$$=\frac{2\left(cd^2-bde+ae^2\right)^2(d+ex)^{5/2}}{5e^5}-\frac{4(2cd-be)\left(cd^2-bde+ae^2\right)(d+ex)^{7/2}}{7e^5}+\frac{2\left(6c^2\right.}{}$$

Mathematica [A] time = 0.149767, size = 174, normalized size = 1.05

$$\frac{2(d+ex)^{5/2}\left(143e^2\left(63a^2e^2+18abe(5ex-2d)+b^2\left(8d^2-20dex+35e^2x^2\right)\right)-26ce\left(3b\left(-40d^2ex+16d^3+70de^2x^2-\right.\right.\right.}{45045e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)*(a + b*x + c*x^2)^2,x]
```

```
[Out] (2*(d + e*x)^(5/2)*(3*c^2*(128*d^4 - 320*d^3*e*x + 560*d^2*e^2*x^2 - 840*d*
e^3*x^3 + 1155*e^4*x^4) + 143*e^2*(63*a^2*e^2 + 18*a*b*e*(-2*d + 5*e*x) + b
^2*(8*d^2 - 20*d*e*x + 35*e^2*x^2)) - 26*c*e*(-11*a*e*(8*d^2 - 20*d*e*x + 3
5*e^2*x^2) + 3*b*(16*d^3 - 40*d^2*e*x + 70*d*e^2*x^2 - 105*e^3*x^3))))/(450
45*e^5)
```

Maple [A] time = 0.046, size = 194, normalized size = 1.2

$$\frac{6930\,c^2x^4e^4+16380\,bce^4x^3-5040\,c^2de^3x^3+20020\,ace^4x^2+10010\,b^2e^4x^2-10920\,bcde^3x^2+3360\,c^2d^2e^2x^2+25740\,abe}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(c*x^2+b*x+a)^2,x)
```

```
[Out] 2/45045*(e*x+d)^(5/2)*(3465*c^2*e^4*x^4+8190*b*c*e^4*x^3-2520*c^2*d*e^3*x^3
+10010*a*c*e^4*x^2+5005*b^2*e^4*x^2-5460*b*c*d*e^3*x^2+1680*c^2*d^2*e^2*x^2
+12870*a*b*e^4*x-5720*a*c*d*e^3*x-2860*b^2*d*e^3*x+3120*b*c*d^2*e^2*x-960*c
^2*d^3*e*x+9009*a^2*e^4-5148*a*b*d*e^3+2288*a*c*d^2*e^2+1144*b^2*d^2*e^2-12
48*b*c*d^3*e+384*c^2*d^4)/e^5
```

Maxima [A] time = 0.978018, size = 238, normalized size = 1.43

$$\frac{2\left(3465\,(ex+d)^{\frac{13}{2}}c^2-8190\left(2\,c^2d-bce\right)(ex+d)^{\frac{11}{2}}+5005\left(6\,c^2d^2-6\,bcde+\left(b^2+2\,ac\right)e^2\right)(ex+d)^{\frac{9}{2}}-12870\left(2\,c^2d^3-3\right.}{45045\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2/45045*(3465*(e*x + d)^(13/2)*c^2 - 8190*(2*c^2*d - b*c*e)*(e*x + d)^(11/2
) + 5005*(6*c^2*d^2 - 6*b*c*d*e + (b^2 + 2*a*c)*e^2)*(e*x + d)^(9/2) - 1287
0*(2*c^2*d^3 - 3*b*c*d^2*e - a*b*e^3 + (b^2 + 2*a*c)*d*e^2)*(e*x + d)^(7/2)
 + 9009*(c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*
e^2)*(e*x + d)^(5/2))/e^5
```

Fricas [B] time = 2.28634, size = 724, normalized size = 4.36

$$\frac{2\left(3465\,c^2e^6x^6+384\,c^2d^6-1248\,bcd^5e-5148\,abd^3e^3+9009\,a^2d^2e^4+1144\left(b^2+2\,ac\right)d^4e^2+630\left(7\,c^2de^5+13\,bce^6\right)x^5+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 2/45045*(3465*c^2*e^6*x^6 + 384*c^2*d^6 - 1248*b*c*d^5*e - 5148*a*b*d^3*e^3
 + 9009*a^2*d^2*e^4 + 1144*(b^2 + 2*a*c)*d^4*e^2 + 630*(7*c^2*d*e^5 + 13*b*
c*e^6)*x^5 + 35*(3*c^2*d^2*e^4 + 312*b*c*d*e^5 + 143*(b^2 + 2*a*c)*e^6)*x^4
 - 10*(12*c^2*d^3*e^3 - 39*b*c*d^2*e^4 - 1287*a*b*e^6 - 715*(b^2 + 2*a*c)*d
*e^5)*x^3 + 3*(48*c^2*d^4*e^2 - 156*b*c*d^3*e^3 + 6864*a*b*d*e^5 + 3003*a^2
*e^6 + 143*(b^2 + 2*a*c)*d^2*e^4)*x^2 - 2*(96*c^2*d^5*e - 312*b*c*d^4*e^2 -
 1287*a*b*d^2*e^4 - 9009*a^2*d*e^5 + 286*(b^2 + 2*a*c)*d^3*e^3)*x)*sqrt(e*x
```

```
+ d)/e^5
```

Sympy [A] time = 21.058, size = 654, normalized size = 3.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(c*x**2+b*x+a)**2,x)
```

```
[Out] a**2*d*Piecewise((sqrt(d)*x, Eq(e, 0)), (2*(d + e*x)**(3/2)/(3*e), True)) +
 2*a**2*(-d*(d + e*x)**(3/2)/3 + (d + e*x)**(5/2)/5)/e + 4*a*b*d*(-d*(d + e
*x)**(3/2)/3 + (d + e*x)**(5/2)/5)/e**2 + 4*a*b*(d**2*(d + e*x)**(3/2)/3 -
2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e**2 + 4*a*c*d*(d**2*(d + e*x)
**(3/2)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e**3 + 4*a*c*(-d**
3*(d + e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d + e*x)**(7/2)/7 +
 (d + e*x)**(9/2)/9)/e**3 + 2*b**2*d*(d**2*(d + e*x)**(3/2)/3 - 2*d*(d + e*
x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e**3 + 2*b**2*(-d**3*(d + e*x)**(3/2)/3 +
 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d + e*x)**(7/2)/7 + (d + e*x)**(9/2)/9)/e
**3 + 4*b*c*d*(-d**3*(d + e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(
d + e*x)**(7/2)/7 + (d + e*x)**(9/2)/9)/e**4 + 4*b*c*(d**4*(d + e*x)**(3/2)
/3 - 4*d**3*(d + e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e*x)*
*(9/2)/9 + (d + e*x)**(11/2)/11)/e**4 + 2*c**2*d*(d**4*(d + e*x)**(3/2)/3 -
 4*d**3*(d + e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e*x)**(9/
2)/9 + (d + e*x)**(11/2)/11)/e**5 + 2*c**2*(-d**5*(d + e*x)**(3/2)/3 + d**4
*(d + e*x)**(5/2) - 10*d**3*(d + e*x)**(7/2)/7 + 10*d**2*(d + e*x)**(9/2)/9
 - 5*d*(d + e*x)**(11/2)/11 + (d + e*x)**(13/2)/13)/e**5
```

Giac [B] time = 1.16917, size = 786, normalized size = 4.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 2/45045*(6006*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a*b*d*e^(-1) + 429*
(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*b^2*d*
e^(-2) + 858*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2
)*d^2)*a*c*d*e^(-2) + 286*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189
*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*b*c*d*e^(-3) + 13*(315*(x*e
 + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*
e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*c^2*d*e^(-4) + 15015*(x*e + d)
^(3/2)*a^2*d + 858*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d
)^(3/2)*d^2)*a*b*e^(-1) + 143*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d +
 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*b^2*e^(-2) + 286*(35*(x
*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e
+ d)^(3/2)*d^3)*a*c*e^(-2) + 26*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2
)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^
(3/2)*d^4)*b*c*e^(-3) + 5*(693*(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d +
 10010*(x*e + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/
2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*c^2*e^(-4) + 3003*(3*(x*e + d)^(5/2) - 5
*(x*e + d)^(3/2)*d)*a^2)*e^(-1)
```

3.2277 $\int \sqrt{d+ex}\left(a+bx+cx^2\right)^2\,dx$

Optimal. Leaf size=166

$$\frac{2(d+ex)^{7/2}\left(-2ce(3bd-ae)+b^2e^2+6c^2d^2\right)}{7e^5}-\frac{4(d+ex)^{5/2}(2cd-be)\left(ae^2-bde+cd^2\right)}{5e^5}+\frac{2(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)^2}{3e^5}$$

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^(3/2))/(3*e^5) - (4*(2*c*d - b*e)*(c
*d^2 - b*d*e + a*e^2)*(d + e*x)^(5/2))/(5*e^5) + (2*(6*c^2*d^2 + b^2*e^2 -
2*c*e*(3*b*d - a*e))*(d + e*x)^(7/2))/(7*e^5) - (4*c*(2*c*d - b*e)*(d + e*x
)^(9/2))/(9*e^5) + (2*c^2*(d + e*x)^(11/2))/(11*e^5)
```

Rubi [A] time = 0.0724684, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {698}

$$\frac{2(d+ex)^{7/2}\left(-2ce(3bd-ae)+b^2e^2+6c^2d^2\right)}{7e^5}-\frac{4(d+ex)^{5/2}(2cd-be)\left(ae^2-bde+cd^2\right)}{5e^5}+\frac{2(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)^2}{3e^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(a + b*x + c*x^2)^2,x]
```

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^(3/2))/(3*e^5) - (4*(2*c*d - b*e)*(c
*d^2 - b*d*e + a*e^2)*(d + e*x)^(5/2))/(5*e^5) + (2*(6*c^2*d^2 + b^2*e^2 -
2*c*e*(3*b*d - a*e))*(d + e*x)^(7/2))/(7*e^5) - (4*c*(2*c*d - b*e)*(d + e*x
)^(9/2))/(9*e^5) + (2*c^2*(d + e*x)^(11/2))/(11*e^5)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \sqrt{d+ex}\left(a+bx+cx^2\right)^2\,dx=\int \left(\frac{\left(cd^2-bde+ae^2\right)^2\sqrt{d+ex}}{e^4}+\frac{2(-2cd+be)\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}}{e^4}+\frac{\left(6c^2d^2+\right.}{}\right.$$

$$=\frac{2\left(cd^2-bde+ae^2\right)^2(d+ex)^{3/2}}{3e^5}-\frac{4(2cd-be)\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}}{5e^5}+\frac{2\left(6c^2d^2+\right.}{}$$

Mathematica [A] time = 0.151066, size = 172, normalized size = 1.04

$$\frac{2(d+ex)^{3/2}\left(33e^2\left(35a^2e^2+14abe(3ex-2d)+b^2\left(8d^2-12dex+15e^2x^2\right)\right)-22ce\left(b\left(-24d^2ex+16d^3+30de^2x^2-35e^3x^3\right.\right.\right.}{3465e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*(a + b*x + c*x^2)^2,x]
```

```
[Out] (2*(d + e*x)^(3/2)*(c^2*(128*d^4 - 192*d^3*e*x + 240*d^2*e^2*x^2 - 280*d*e^
3*x^3 + 315*e^4*x^4) + 33*e^2*(35*a^2*e^2 + 14*a*b*e*(-2*d + 3*e*x) + b^2*(
8*d^2 - 12*d*e*x + 15*e^2*x^2)) - 22*c*e*(-3*a*e*(8*d^2 - 12*d*e*x + 15*e^2
*x^2) + b*(16*d^3 - 24*d^2*e*x + 30*d*e^2*x^2 - 35*e^3*x^3))))/(3465*e^5)
```

Maple [A] time = 0.043, size = 194, normalized size = 1.2

$$\frac{630\,c^2x^4e^4+1540\,bce^4x^3-560\,c^2de^3x^3+1980\,ace^4x^2+990\,b^2e^4x^2-1320\,bcde^3x^2+480\,c^2d^2e^2x^2+2772\,abe^4x-158\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2*(e*x+d)^(1/2),x)
```

```
[Out] 2/3465*(e*x+d)^(3/2)*(315*c^2*e^4*x^4+770*b*c*e^4*x^3-280*c^2*d*e^3*x^3+990
*a*c*e^4*x^2+495*b^2*e^4*x^2-660*b*c*d*e^3*x^2+240*c^2*d^2*e^2*x^2+1386*a*b
*e^4*x-792*a*c*d*e^3*x-396*b^2*d*e^3*x+528*b*c*d^2*e^2*x-192*c^2*d^3*e*x+11
55*a^2*e^4-924*a*b*d*e^3+528*a*c*d^2*e^2+264*b^2*d^2*e^2-352*b*c*d^3*e+128*
c^2*d^4)/e^5
```

Maxima [A] time = 0.971869, size = 238, normalized size = 1.43

$$\frac{2\left(315\,(ex+d)^{\frac{11}{2}}c^2-770\left(2\,c^2d-bce\right)(ex+d)^{\frac{9}{2}}+495\left(6\,c^2d^2-6\,bcde+\left(b^2+2\,ac\right)e^2\right)(ex+d)^{\frac{7}{2}}-1386\left(2\,c^2d^3-3\,bc\ldots\right.\right.}{3465\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3465*(315*(e*x + d)^(11/2)*c^2 - 770*(2*c^2*d - b*c*e)*(e*x + d)^(9/2) +
495*(6*c^2*d^2 - 6*b*c*d*e + (b^2 + 2*a*c)*e^2)*(e*x + d)^(7/2) - 1386*(2*c
^2*d^3 - 3*b*c*d^2*e - a*b*e^3 + (b^2 + 2*a*c)*d*e^2)*(e*x + d)^(5/2) + 115
5*(c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e^2)*(
e*x + d)^(3/2))/e^5
```

Fricas [A] time = 2.31287, size = 554, normalized size = 3.34

$$\frac{2\left(315\,c^2e^5x^5+128\,c^2d^5-352\,bcd^4e-924\,abd^2e^3+1155\,a^2de^4+264\left(b^2+2\,ac\right)d^3e^2+35\left(c^2de^4+22\,bce^5\right)x^4-5\left(8\,c\ldots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*c^2*e^5*x^5 + 128*c^2*d^5 - 352*b*c*d^4*e - 924*a*b*d^2*e^3 + 1
155*a^2*d*e^4 + 264*(b^2 + 2*a*c)*d^3*e^2 + 35*(c^2*d*e^4 + 22*b*c*e^5)*x^4
 - 5*(8*c^2*d^2*e^3 - 22*b*c*d*e^4 - 99*(b^2 + 2*a*c)*e^5)*x^3 + 3*(16*c^2*
d^3*e^2 - 44*b*c*d^2*e^3 + 462*a*b*e^5 + 33*(b^2 + 2*a*c)*d*e^4)*x^2 - (64*
c^2*d^4*e - 176*b*c*d^3*e^2 - 462*a*b*d*e^4 - 1155*a^2*e^5 + 132*(b^2 + 2*a
*c)*d^2*e^3)*x)*sqrt(e*x + d)/e^5
```

Sympy [A] time = 4.05104, size = 230, normalized size = 1.39

$$2\left(\frac{c^2(d+ex)^{\frac{11}{2}}}{11e^4} + \frac{(d+ex)^{\frac{9}{2}}\left(2bce-4c^2d\right)}{9e^4} + \frac{(d+ex)^{\frac{7}{2}}\left(2ace^2+b^2e^2-6bcde+6c^2d^2\right)}{7e^4} + \frac{(d+ex)^{\frac{5}{2}}\left(2abe^3-4acde^2-2b^2de^2+6bcd^2e-4c^2d^3\right)}{5e^4} + \frac{(d+ex)^{\frac{3}{2}}\left(a^2e^4-2abde^3+\cdots\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2*(e*x+d)**(1/2),x)
```

```
[Out] 2*(c**2*(d + e*x)**(11/2)/(11*e**4) + (d + e*x)**(9/2)*(2*b*c*e - 4*c**2*d)
/(9*e**4) + (d + e*x)**(7/2)*(2*a*c*e**2 + b**2*e**2 - 6*b*c*d*e + 6*c**2*d
**2)/(7*e**4) + (d + e*x)**(5/2)*(2*a*b*e**3 - 4*a*c*d*e**2 - 2*b**2*d*e**2
 + 6*b*c*d**2*e - 4*c**2*d**3)/(5*e**4) + (d + e*x)**(3/2)*(a**2*e**4 - 2*a
*b*d*e**3 + 2*a*c*d**2*e**2 + b**2*d**2*e**2 - 2*b*c*d**3*e + c**2*d**4)/(3
*e**4))/e
```

Giac [A] time = 1.16171, size = 338, normalized size = 2.04

$$\frac{2}{3465}\left(462\left(3\,(xe+d)^{\frac{5}{2}} - 5\,(xe+d)^{\frac{3}{2}}d\right)abe^{(-1)} + 33\left(15\,(xe+d)^{\frac{7}{2}} - 42\,(xe+d)^{\frac{5}{2}}d + 35\,(xe+d)^{\frac{3}{2}}d^2\right)b^2e^{(-2)} + 66\left(15\,(xe+\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3465*(462*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a*b*e^(-1) + 33*(15*(
x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*b^2*e^(-2)
+ 66*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*a
*c*e^(-2) + 22*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^
(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*b*c*e^(-3) + (315*(x*e + d)^(11/2) - 1
540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3
 + 1155*(x*e + d)^(3/2)*d^4)*c^2*e^(-4) + 1155*(x*e + d)^(3/2)*a^2)*e^(-1)
```

3.2278 $\int \frac{(a+bx+cx^2)^2}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=164

$$\frac{2(d+ex)^{5/2}\left(-2ce(3bd-ae)+b^2e^2+6c^2d^2\right)}{5e^5}-\frac{4(d+ex)^{3/2}(2cd-be)\left(ae^2-bde+cd^2\right)}{3e^5}+\frac{2\sqrt{d+ex}\left(ae^2-bde+cd^2\right)^2}{e^5}$$

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[d + e*x])/e^5 - (4*(2*c*d - b*e)*(c*d^2 -
 b*d*e + a*e^2)*(d + e*x)^(3/2))/(3*e^5) + (2*(6*c^2*d^2 + b^2*e^2 - 2*c*e*
(3*b*d - a*e))*(d + e*x)^(5/2))/(5*e^5) - (4*c*(2*c*d - b*e)*(d + e*x)^(7/2
))/(7*e^5) + (2*c^2*(d + e*x)^(9/2))/(9*e^5)
```

Rubi [A] time = 0.0727377, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {698}

$$\frac{2(d+ex)^{5/2}\left(-2ce(3bd-ae)+b^2e^2+6c^2d^2\right)}{5e^5}-\frac{4(d+ex)^{3/2}(2cd-be)\left(ae^2-bde+cd^2\right)}{3e^5}+\frac{2\sqrt{d+ex}\left(ae^2-bde+cd^2\right)^2}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/Sqrt[d + e*x],x]
```


```
[Out] (2*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[d + e*x])/e^5 - (4*(2*c*d - b*e)*(c*d^2 -
 b*d*e + a*e^2)*(d + e*x)^(3/2))/(3*e^5) + (2*(6*c^2*d^2 + b^2*e^2 - 2*c*e*
(3*b*d - a*e))*(d + e*x)^(5/2))/(5*e^5) - (4*c*(2*c*d - b*e)*(d + e*x)^(7/2
))/(7*e^5) + (2*c^2*(d + e*x)^(9/2))/(9*e^5)
```


Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^2}{\sqrt{d+ex}}\,dx=\int \left(\frac{\left(cd^2-bde+ae^2\right)^2}{e^4\sqrt{d+ex}}+\frac{2(-2cd+be)\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}{e^4}+\frac{\left(6c^2d^2+b^2e^2-2ce(3bd\right.}{e^4}\right.$$

$$=\frac{2\left(cd^2-bde+ae^2\right)^2\sqrt{d+ex}}{e^5}-\frac{4(2cd-be)\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}}{3e^5}+\frac{2\left(6c^2d^2+b^2e^2-\right.}{}$$

Mathematica [A] time = 0.160254, size = 172, normalized size = 1.05

$$\frac{2\sqrt{d+ex}\left(21e^2\left(15a^2e^2+10abe(ex-2d)+b^2\left(8d^2-4dex+3e^2x^2\right)\right)-6ce\left(3b\left(-8d^2ex+16d^3+6de^2x^2-5e^3x^3\right)-7a\right.\right.}{315e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[d + e*x]*(c^2*(128*d^4 - 64*d^3*e*x + 48*d^2*e^2*x^2 - 40*d*e^3*x^3
 + 35*e^4*x^4) + 21*e^2*(15*a^2*e^2 + 10*a*b*e*(-2*d + e*x) + b^2*(8*d^2 -
4*d*e*x + 3*e^2*x^2)) - 6*c*e*(-7*a*e*(8*d^2 - 4*d*e*x + 3*e^2*x^2) + 3*b*(
16*d^3 - 8*d^2*e*x + 6*d*e^2*x^2 - 5*e^3*x^3))))/(315*e^5)
```

Maple [A] time = 0.044, size = 194, normalized size = 1.2

$$\frac{70\,c^2x^4e^4+180\,bce^4x^3-80\,c^2de^3x^3+252\,ace^4x^2+126\,b^2e^4x^2-216\,bcde^3x^2+96\,c^2d^2e^2x^2+420\,abe^4x-336\,acde^3x-1\cdots}{315\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(e*x+d)^(1/2),x)
```

```
[Out] 2/315*(e*x+d)^(1/2)*(35*c^2*e^4*x^4+90*b*c*e^4*x^3-40*c^2*d*e^3*x^3+126*a*c
*e^4*x^2+63*b^2*e^4*x^2-108*b*c*d*e^3*x^2+48*c^2*d^2*e^2*x^2+210*a*b*e^4*x-
168*a*c*d*e^3*x-84*b^2*d*e^3*x+144*b*c*d^2*e^2*x-64*c^2*d^3*e*x+315*a^2*e^4
-420*a*b*d*e^3+336*a*c*d^2*e^2+168*b^2*d^2*e^2-288*b*c*d^3*e+128*c^2*d^4)/e
^5
```

Maxima [A] time = 1.01342, size = 320, normalized size = 1.95

$$\frac{2\left(315\,\sqrt{ex+d}a^2+42\,a\left(\frac{5\left((ex+d)^{\frac{3}{2}}-3\,\sqrt{ex+d}d\right)b}{e}+\frac{\left(3\,(ex+d)^{\frac{5}{2}}-10\,(ex+d)^{\frac{3}{2}}d+15\,\sqrt{ex+d}d^2\right)c}{e^2}\right)+\frac{21\left(3\,(ex+d)^{\frac{5}{2}}-10\,(ex+d)^{\frac{3}{2}}d+15\,\sqrt{ex+d}d^2\right)b^2}{e^2}+\frac{18\cdots}{\cdots}\right.}{315\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(315*sqrt(e*x + d)*a^2 + 42*a*(5*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d
)*b/e + (3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*c
/e^2) + 21*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2
)*b^2/e^2 + 18*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/
2)*d^2 - 35*sqrt(e*x + d)*d^3)*b*c/e^3 + (35*(e*x + d)^(9/2) - 180*(e*x + d
)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*
x + d)*d^4)*c^2/e^4)/e
```

Fricas [A] time = 2.25558, size = 412, normalized size = 2.51

$$\frac{2\left(35\,c^2e^4x^4+128\,c^2d^4-288\,bcd^3e-420\,abde^3+315\,a^2e^4+168\left(b^2+2\,ac\right)d^2e^2-10\left(4\,c^2de^3-9\,bce^4\right)x^3+3\left(16\,c^2d^2e^2\cdots\right.\right.}{315\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*c^2*e^4*x^4 + 128*c^2*d^4 - 288*b*c*d^3*e - 420*a*b*d*e^3 + 315*a
^2*e^4 + 168*(b^2 + 2*a*c)*d^2*e^2 - 10*(4*c^2*d*e^3 - 9*b*c*e^4)*x^3 + 3*(
```

```
16*c^2*d^2*e^2 - 36*b*c*d*e^3 + 21*(b^2 + 2*a*c)*e^4)*x^2 - 2*(32*c^2*d^3*e
 - 72*b*c*d^2*e^2 - 105*a*b*e^4 + 42*(b^2 + 2*a*c)*d*e^3)*x)*sqrt(e*x + d)/
e^5
```

Sympy [A] time = 53.3516, size = 644, normalized size = 3.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(e*x+d)**(1/2),x)
```

```
[Out] Piecewise((-(2*a**2*d/sqrt(d + e*x) + 2*a**2*(-d/sqrt(d + e*x) - sqrt(d + e
*x)) + 4*a*b*d*(-d/sqrt(d + e*x) - sqrt(d + e*x))/e + 4*a*b*(d**2/sqrt(d +
e*x) + 2*d*sqrt(d + e*x) - (d + e*x)**(3/2)/3)/e + 4*a*c*d*(d**2/sqrt(d + e
*x) + 2*d*sqrt(d + e*x) - (d + e*x)**(3/2)/3)/e**2 + 4*a*c*(-d**3/sqrt(d +
e*x) - 3*d**2*sqrt(d + e*x) + d*(d + e*x)**(3/2) - (d + e*x)**(5/2)/5)/e**2
 + 2*b**2*d*(d**2/sqrt(d + e*x) + 2*d*sqrt(d + e*x) - (d + e*x)**(3/2)/3)/e
**2 + 2*b**2*(-d**3/sqrt(d + e*x) - 3*d**2*sqrt(d + e*x) + d*(d + e*x)**(3/
2) - (d + e*x)**(5/2)/5)/e**2 + 4*b*c*d*(-d**3/sqrt(d + e*x) - 3*d**2*sqrt(
d + e*x) + d*(d + e*x)**(3/2) - (d + e*x)**(5/2)/5)/e**3 + 4*b*c*(d**4/sqrt
(d + e*x) + 4*d**3*sqrt(d + e*x) - 2*d**2*(d + e*x)**(3/2) + 4*d*(d + e*x)*
*(5/2)/5 - (d + e*x)**(7/2)/7)/e**3 + 2*c**2*d*(d**4/sqrt(d + e*x) + 4*d**3
*sqrt(d + e*x) - 2*d**2*(d + e*x)**(3/2) + 4*d*(d + e*x)**(5/2)/5 - (d + e*
x)**(7/2)/7)/e**4 + 2*c**2*(-d**5/sqrt(d + e*x) - 5*d**4*sqrt(d + e*x) + 10
*d**3*(d + e*x)**(3/2)/3 - 2*d**2*(d + e*x)**(5/2) + 5*d*(d + e*x)**(7/2)/7
 - (d + e*x)**(9/2)/9)/e**4)/e, Ne(e, 0)), ((a**2*x + a*b*x**2 + b*c*x**4/2
 + c**2*x**5/5 + x**3*(2*a*c + b**2)/3)/sqrt(d), True))
```

Giac [A] time = 1.1095, size = 335, normalized size = 2.04

$$\frac{2}{315}\left(210\left((xe+d)^{\frac{3}{2}}-3\sqrt{xe+d}d\right)abe^{(-1)}+21\left(3(xe+d)^{\frac{5}{2}}-10(xe+d)^{\frac{3}{2}}d+15\sqrt{xe+d}d^2\right)b^2e^{(-2)}+42\left(3(xe+d)^{\frac{5}{2}}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(210*((x*e + d)^(3/2) - 3*sqrt(x*e + d)*d)*a*b*e^(-1) + 21*(3*(x*e +
d)^(5/2) - 10*(x*e + d)^(3/2)*d + 15*sqrt(x*e + d)*d^2)*b^2*e^(-2) + 42*(3*
(x*e + d)^(5/2) - 10*(x*e + d)^(3/2)*d + 15*sqrt(x*e + d)*d^2)*a*c*e^(-2) +
 18*(5*(x*e + d)^(7/2) - 21*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2 - 35
*sqrt(x*e + d)*d^3)*b*c*e^(-3) + (35*(x*e + d)^(9/2) - 180*(x*e + d)^(7/2)*
d + 378*(x*e + d)^(5/2)*d^2 - 420*(x*e + d)^(3/2)*d^3 + 315*sqrt(x*e + d)*d
^4)*c^2*e^(-4) + 315*sqrt(x*e + d)*a^2)*e^(-1)
```

3.2279 $\int \frac{(a+bx+cx^2)^2}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=162

$$\frac{2(d+ex)^{3/2}\left(-2ce(3bd-ae)+b^2e^2+6c^2d^2\right)}{3e^5}-\frac{4\sqrt{d+ex}(2cd-be)\left(ae^2-bde+cd^2\right)}{e^5}-\frac{2\left(ae^2-bde+cd^2\right)^2}{e^5\sqrt{d+ex}}-\frac{4c(d+ex)^{\cdots}}{5\cdots}$$

```
[Out] (-2*(c*d^2 - b*d*e + a*e^2)^2)/(e^5*Sqrt[d + e*x]) - (4*(2*c*d - b*e)*(c*d^
2 - b*d*e + a*e^2)*Sqrt[d + e*x])/e^5 + (2*(6*c^2*d^2 + b^2*e^2 - 2*c*e*(3*
b*d - a*e))*(d + e*x)^(3/2))/(3*e^5) - (4*c*(2*c*d - b*e)*(d + e*x)^(5/2))/
(5*e^5) + (2*c^2*(d + e*x)^(7/2))/(7*e^5)
```

Rubi [A] time = 0.0700174, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {698}

$$\frac{2(d+ex)^{3/2}\left(-2ce(3bd-ae)+b^2e^2+6c^2d^2\right)}{3e^5}-\frac{4\sqrt{d+ex}(2cd-be)\left(ae^2-bde+cd^2\right)}{e^5}-\frac{2\left(ae^2-bde+cd^2\right)^2}{e^5\sqrt{d+ex}}-\frac{4c(d+ex)^{\cdots}}{5\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(d + e*x)^(3/2),x]
```

```
[Out] (-2*(c*d^2 - b*d*e + a*e^2)^2)/(e^5*Sqrt[d + e*x]) - (4*(2*c*d - b*e)*(c*d^
2 - b*d*e + a*e^2)*Sqrt[d + e*x])/e^5 + (2*(6*c^2*d^2 + b^2*e^2 - 2*c*e*(3*
b*d - a*e))*(d + e*x)^(3/2))/(3*e^5) - (4*c*(2*c*d - b*e)*(d + e*x)^(5/2))/
(5*e^5) + (2*c^2*(d + e*x)^(7/2))/(7*e^5)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^2}{(d+ex)^{3/2}}\,dx=\int \left(\frac{\left(cd^2-bde+ae^2\right)^2}{e^4(d+ex)^{3/2}}+\frac{2(-2cd+be)\left(cd^2-bde+ae^2\right)}{e^4\sqrt{d+ex}}+\frac{\left(6c^2d^2+b^2e^2-2ce(3bd-ae)\right)\sqrt{d+\cdots}}{e^4}\cdots\right.$$
$$=-\frac{2\left(cd^2-bde+ae^2\right)^2}{e^5\sqrt{d+ex}}-\frac{4(2cd-be)\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}{e^5}+\frac{2\left(6c^2d^2+b^2e^2-2ce(3bd-ae\cdots\right.}{3e^5}$$

Mathematica [A] time = 0.134387, size = 171, normalized size = 1.06

$$\frac{-70e^2\left(3a^2e^2-6abe(2d+ex)+b^2\left(8d^2+4dex-e^2x^2\right)\right)+28ce\left(5ae\left(-8d^2-4dex+e^2x^2\right)+3b\left(8d^2ex+16d^3-2de^2x^2+\cdots\right.\right.}{105e^5\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(d + e*x)^(3/2),x]
```

```
[Out] (-6*c^2*(128*d^4 + 64*d^3*e*x - 16*d^2*e^2*x^2 + 8*d*e^3*x^3 - 5*e^4*x^4) -
 70*e^2*(3*a^2*e^2 - 6*a*b*e*(2*d + e*x) + b^2*(8*d^2 + 4*d*e*x - e^2*x^2))
 + 28*c*e*(5*a*e*(-8*d^2 - 4*d*e*x + e^2*x^2) + 3*b*(16*d^3 + 8*d^2*e*x - 2
*d*e^2*x^2 + e^3*x^3)))/(105*e^5*Sqrt[d + e*x])
```

Maple [A] time = 0.045, size = 194, normalized size = 1.2

$$-\frac{-30\,c^2x^4e^4 - 84\,bce^4x^3 + 48\,c^2de^3x^3 - 140\,ace^4x^2 - 70\,b^2e^4x^2 + 168\,bcde^3x^2 - 96\,c^2d^2e^2x^2 - 420\,abe^4x + 560\,acde^3x}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(e*x+d)^(3/2),x)
```

```
[Out] -2/105/(e*x+d)^(1/2)*(-15*c^2*e^4*x^4-42*b*c*e^4*x^3+24*c^2*d*e^3*x^3-70*a*
c*e^4*x^2-35*b^2*e^4*x^2+84*b*c*d*e^3*x^2-48*c^2*d^2*e^2*x^2-210*a*b*e^4*x+
280*a*c*d*e^3*x+140*b^2*d*e^3*x-336*b*c*d^2*e^2*x+192*c^2*d^3*e*x+105*a^2*e
^4-420*a*b*d*e^3+560*a*c*d^2*e^2+280*b^2*d^2*e^2-672*b*c*d^3*e+384*c^2*d^4)
/e^5
```

Maxima [A] time = 0.971692, size = 248, normalized size = 1.53

$$\frac{2\left(\frac{15\,(ex+d)^{\frac{7}{2}}c^2-42\,\left(2\,c^2d-bce\right)(ex+d)^{\frac{5}{2}}+35\,\left(6\,c^2d^2-6\,bcde+\left(b^2+2\,ac\right)e^2\right)(ex+d)^{\frac{3}{2}}-210\,\left(2\,c^2d^3-3\,bcd^2e-abe^3+\left(b^2+2\,ac\right)de^2\right)\sqrt{ex+d}}{e^4}-\frac{105\,\left(c^2d^4-2\,bcd^3e-2\right.}{}\right.}{105\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/105*((15*(e*x + d)^(7/2)*c^2 - 42*(2*c^2*d - b*c*e)*(e*x + d)^(5/2) + 35*
(6*c^2*d^2 - 6*b*c*d*e + (b^2 + 2*a*c)*e^2)*(e*x + d)^(3/2) - 210*(2*c^2*d^
3 - 3*b*c*d^2*e - a*b*e^3 + (b^2 + 2*a*c)*d*e^2)*sqrt(e*x + d))/e^4 - 105*(
c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e^2)/(sqr
t(e*x + d)*e^4))/e
```

Fricas [A] time = 2.32187, size = 425, normalized size = 2.62

$$\frac{2\left(15\,c^2e^4x^4-384\,c^2d^4+672\,bcd^3e+420\,abde^3-105\,a^2e^4-280\,\left(b^2+2\,ac\right)d^2e^2-6\,\left(4\,c^2de^3-7\,bce^4\right)x^3+\left(48\,c^2d^2e^2\right.\right.}{105\,\left(e^6x+de^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*c^2*e^4*x^4 - 384*c^2*d^4 + 672*b*c*d^3*e + 420*a*b*d*e^3 - 105*a
^2*e^4 - 280*(b^2 + 2*a*c)*d^2*e^2 - 6*(4*c^2*d*e^3 - 7*b*c*e^4)*x^3 + (48*
c^2*d^2*e^2 - 84*b*c*d*e^3 + 35*(b^2 + 2*a*c)*e^4)*x^2 - 2*(96*c^2*d^3*e -
```

```
168*b*c*d^2*e^2 - 105*a*b*e^4 + 70*(b^2 + 2*a*c)*d*e^3)*x)*sqrt(e*x + d)/(e
^6*x + d*e^5)
```

Sympy [A] time = 29.9527, size = 182, normalized size = 1.12

$$\frac{2c^2(d+ex)^{\frac{7}{2}}}{7e^5}+\frac{(d+ex)^{\frac{5}{2}}\left(4bce-8c^2d\right)}{5e^5}+\frac{(d+ex)^{\frac{3}{2}}\left(4ace^2+2b^2e^2-12bcde+12c^2d^2\right)}{3e^5}+\frac{\sqrt{d+ex}\left(4abe^3-8acde^2-4b^2\right.}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(e*x+d)**(3/2),x)
```

```
[Out] 2*c**2*(d + e*x)**(7/2)/(7*e**5) + (d + e*x)**(5/2)*(4*b*c*e - 8*c**2*d)/(5
*e**5) + (d + e*x)**(3/2)*(4*a*c*e**2 + 2*b**2*e**2 - 12*b*c*d*e + 12*c**2*
d**2)/(3*e**5) + sqrt(d + e*x)*(4*a*b*e**3 - 8*a*c*d*e**2 - 4*b**2*d*e**2 +
 12*b*c*d**2*e - 8*c**2*d**3)/e**5 - 2*(a*e**2 - b*d*e + c*d**2)**2/(e**5*s
qrt(d + e*x))
```

Giac [A] time = 1.11763, size = 342, normalized size = 2.11

$$\frac{2}{105}\left(15(xe+d)^{\frac{7}{2}}c^2e^{30}-84(xe+d)^{\frac{5}{2}}c^2de^{30}+210(xe+d)^{\frac{3}{2}}c^2d^2e^{30}-420\sqrt{xe+d}c^2d^3e^{30}+42(xe+d)^{\frac{5}{2}}bce^{31}-210(xe+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] 2/105*(15*(x*e + d)^(7/2)*c^2*e^30 - 84*(x*e + d)^(5/2)*c^2*d*e^30 + 210*(x
*e + d)^(3/2)*c^2*d^2*e^30 - 420*sqrt(x*e + d)*c^2*d^3*e^30 + 42*(x*e + d)^
(5/2)*b*c*e^31 - 210*(x*e + d)^(3/2)*b*c*d*e^31 + 630*sqrt(x*e + d)*b*c*d^2
*e^31 + 35*(x*e + d)^(3/2)*b^2*e^32 + 70*(x*e + d)^(3/2)*a*c*e^32 - 210*sqr
t(x*e + d)*b^2*d*e^32 - 420*sqrt(x*e + d)*a*c*d*e^32 + 210*sqrt(x*e + d)*a*
b*e^33)*e^(-35) - 2*(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 -
2*a*b*d*e^3 + a^2*e^4)*e^(-5)/sqrt(x*e + d)
```

3.2280 $\int \frac{(a+bx+cx^2)^2}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=162

$$\frac{2\sqrt{d+ex}\left(-2ce(3bd-ae)+b^2e^2+6c^2d^2\right)}{e^5}+\frac{4(2cd-be)\left(ae^2-bde+cd^2\right)}{e^5\sqrt{d+ex}}-\frac{2\left(ae^2-bde+cd^2\right)^2}{3e^5(d+ex)^{3/2}}-\frac{4c(d+ex)^{3/2}(2cd}{3e^5}$$

```
[Out] (-2*(c*d^2 - b*d*e + a*e^2)^2)/(3*e^5*(d + e*x)^(3/2)) + (4*(2*c*d - b*e)*(
c*d^2 - b*d*e + a*e^2))/(e^5*Sqrt[d + e*x]) + (2*(6*c^2*d^2 + b^2*e^2 - 2*c
*e*(3*b*d - a*e))*Sqrt[d + e*x])/e^5 - (4*c*(2*c*d - b*e)*(d + e*x)^(3/2))/
(3*e^5) + (2*c^2*(d + e*x)^(5/2))/(5*e^5)
```

Rubi [A] time = 0.0745995, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {698}

$$\frac{2\sqrt{d+ex}\left(-2ce(3bd-ae)+b^2e^2+6c^2d^2\right)}{e^5}+\frac{4(2cd-be)\left(ae^2-bde+cd^2\right)}{e^5\sqrt{d+ex}}-\frac{2\left(ae^2-bde+cd^2\right)^2}{3e^5(d+ex)^{3/2}}-\frac{4c(d+ex)^{3/2}(2cd}{3e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(d + e*x)^(5/2),x]
```

```
[Out] (-2*(c*d^2 - b*d*e + a*e^2)^2)/(3*e^5*(d + e*x)^(3/2)) + (4*(2*c*d - b*e)*(
c*d^2 - b*d*e + a*e^2))/(e^5*Sqrt[d + e*x]) + (2*(6*c^2*d^2 + b^2*e^2 - 2*c
*e*(3*b*d - a*e))*Sqrt[d + e*x])/e^5 - (4*c*(2*c*d - b*e)*(d + e*x)^(3/2))/
(3*e^5) + (2*c^2*(d + e*x)^(5/2))/(5*e^5)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^2}{(d+ex)^{5/2}}\,dx=\int \left(\frac{\left(cd^2-bde+ae^2\right)^2}{e^4(d+ex)^{5/2}}+\frac{2(-2cd+be)\left(cd^2-bde+ae^2\right)}{e^4(d+ex)^{3/2}}+\frac{6c^2d^2+b^2e^2-2ce(3bd-ae)}{e^4\sqrt{d+ex}}-\frac{2c}{}\right.$$

$$=-\frac{2\left(cd^2-bde+ae^2\right)^2}{3e^5(d+ex)^{3/2}}+\frac{4(2cd-be)\left(cd^2-bde+ae^2\right)}{e^5\sqrt{d+ex}}+\frac{2\left(6c^2d^2+b^2e^2-2ce(3bd-ae)\right)\sqrt{d}}{e^5}$$

Mathematica [A] time = 0.128175, size = 170, normalized size = 1.05

$$\frac{2\left(-5e^2\left(a^2e^2+2abe(2d+3ex)+b^2\left(-\left(8d^2+12dex+3e^2x^2\right)\right)\right)+10ce\left(ae\left(8d^2+12dex+3e^2x^2\right)+b\left(-24d^2ex-16d^3\right.\right.\right.}{15e^5(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(d + e*x)^(5/2),x]
```

```
[Out] (2*(c^2*(128*d^4 + 192*d^3*e*x + 48*d^2*e^2*x^2 - 8*d*e^3*x^3 + 3*e^4*x^4)
- 5*e^2*(a^2*e^2 + 2*a*b*e*(2*d + 3*e*x) - b^2*(8*d^2 + 12*d*e*x + 3*e^2*x^
2)) + 10*c*e*(a*e*(8*d^2 + 12*d*e*x + 3*e^2*x^2) + b*(-16*d^3 - 24*d^2*e*x
- 6*d*e^2*x^2 + e^3*x^3))))/(15*e^5*(d + e*x)^(3/2))
```

Maple [A] time = 0.046, size = 194, normalized size = 1.2

$$-\frac{-6c^2x^4e^4 - 20bce^4x^3 + 16c^2de^3x^3 - 60ace^4x^2 - 30b^2e^4x^2 + 120bcde^3x^2 - 96c^2d^2e^2x^2 + 60abe^4x - 240acde^3x - 120b}{15e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(e*x+d)^(5/2),x)
```

```
[Out] -2/15/(e*x+d)^(3/2)*(-3*c^2*e^4*x^4-10*b*c*e^4*x^3+8*c^2*d*e^3*x^3-30*a*c*e
^4*x^2-15*b^2*e^4*x^2+60*b*c*d*e^3*x^2-48*c^2*d^2*e^2*x^2+30*a*b*e^4*x-120*
a*c*d*e^3*x-60*b^2*d*e^3*x+240*b*c*d^2*e^2*x-192*c^2*d^3*e*x+5*a^2*e^4+20*a
*b*d*e^3-80*a*c*d^2*e^2-40*b^2*d^2*e^2+160*b*c*d^3*e-128*c^2*d^4)/e^5
```

Maxima [A] time = 0.974549, size = 246, normalized size = 1.52

$$\frac{2\left(\frac{3(ex+d)^{\frac{5}{2}}c^2-10(2c^2d-bce)(ex+d)^{\frac{3}{2}}+15(6c^2d^2-6bcde+(b^2+2ac)e^2)\sqrt{ex+d}}{e^4}-\frac{5(c^2d^4-2bcd^3e-2abde^3+a^2e^4+(b^2+2ac)d^2e^2-6(2c^2d^3-3bcd^2e-abe^3+(b^2+}{(ex+d)^{\frac{3}{2}}e^4}\right.}{15e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*((3*(e*x + d)^(5/2)*c^2 - 10*(2*c^2*d - b*c*e)*(e*x + d)^(3/2) + 15*(6
*c^2*d^2 - 6*b*c*d*e + (b^2 + 2*a*c)*e^2)*sqrt(e*x + d))/e^4 - 5*(c^2*d^4 -
 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e^2 - 6*(2*c^2*d^3
 - 3*b*c*d^2*e - a*b*e^3 + (b^2 + 2*a*c)*d*e^2)*(e*x + d))/((e*x + d)^(3/2)
*e^4))/e
```

Fricas [A] time = 2.00105, size = 436, normalized size = 2.69

$$\frac{2\left(3c^2e^4x^4 + 128c^2d^4 - 160bcd^3e - 20abde^3 - 5a^2e^4 + 40(b^2+2ac)d^2e^2 - 2(4c^2de^3 - 5bce^4)x^3 + 3(16c^2d^2e^2 - 20bc\right.}{15(e^7x^2 + 2de^6x + d^2e^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*c^2*e^4*x^4 + 128*c^2*d^4 - 160*b*c*d^3*e - 20*a*b*d*e^3 - 5*a^2*e^
4 + 40*(b^2 + 2*a*c)*d^2*e^2 - 2*(4*c^2*d*e^3 - 5*b*c*e^4)*x^3 + 3*(16*c^2*
d^2*e^2 - 20*b*c*d*e^3 + 5*(b^2 + 2*a*c)*e^4)*x^2 + 6*(32*c^2*d^3*e - 40*b*
c*d^2*e^2 - 5*a*b*e^4 + 10*(b^2 + 2*a*c)*d*e^3)*x)*sqrt(e*x + d)/(e^7*x^2 +
```

```
2*d*e^6*x + d^2*e^5)
```

Sympy [A] time = 43.371, size = 160, normalized size = 0.99

$$\frac{2c^2(d+ex)^{\frac{5}{2}}}{5e^5}+\frac{(d+ex)^{\frac{3}{2}}\left(4bce-8c^2d\right)}{3e^5}+\frac{\sqrt{d+ex}\left(4ace^2+2b^2e^2-12bcde+12c^2d^2\right)}{e^5}-\frac{4(be-2cd)\left(ae^2-bde+cd^2\right)}{e^5\sqrt{d+ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(e*x+d)**(5/2),x)
```

```
[Out] 2*c**2*(d + e*x)**(5/2)/(5*e**5) + (d + e*x)**(3/2)*(4*b*c*e - 8*c**2*d)/(3
*e**5) + sqrt(d + e*x)*(4*a*c*e**2 + 2*b**2*e**2 - 12*b*c*d*e + 12*c**2*d**
2)/e**5 - 4*(b*e - 2*c*d)*(a*e**2 - b*d*e + c*d**2)/(e**5*sqrt(d + e*x)) -
2*(a*e**2 - b*d*e + c*d**2)**2/(3*e**5*(d + e*x)**(3/2))
```

Giac [A] time = 1.1425, size = 329, normalized size = 2.03

$$\frac{2}{15}\left(3(xe+d)^{\frac{5}{2}}c^2e^{20}-20(xe+d)^{\frac{3}{2}}c^2de^{20}+90\sqrt{xe+d}c^2d^2e^{20}+10(xe+d)^{\frac{3}{2}}bce^{21}-90\sqrt{xe+d}bcde^{21}+15\sqrt{xe+d}b^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*(x*e + d)^(5/2)*c^2*e^20 - 20*(x*e + d)^(3/2)*c^2*d*e^20 + 90*sqrt(
x*e + d)*c^2*d^2*e^20 + 10*(x*e + d)^(3/2)*b*c*e^21 - 90*sqrt(x*e + d)*b*c*
d*e^21 + 15*sqrt(x*e + d)*b^2*e^22 + 30*sqrt(x*e + d)*a*c*e^22)*e^(-25) + 2
/3*(12*(x*e + d)*c^2*d^3 - c^2*d^4 - 18*(x*e + d)*b*c*d^2*e + 2*b*c*d^3*e +
 6*(x*e + d)*b^2*d*e^2 + 12*(x*e + d)*a*c*d*e^2 - b^2*d^2*e^2 - 2*a*c*d^2*e
^2 - 6*(x*e + d)*a*b*e^3 + 2*a*b*d*e^3 - a^2*e^4)*e^(-5)/(x*e + d)^(3/2)
```

3.2281 $\int \frac{(a+bx+cx^2)^2}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=162

$$-\frac{2\left(-2ce(3bd-ae)+b^2e^2+6c^2d^2\right)}{e^5\sqrt{d+ex}}+\frac{4(2cd-be)\left(ae^2-bde+cd^2\right)}{3e^5(d+ex)^{3/2}}-\frac{2\left(ae^2-bde+cd^2\right)^2}{5e^5(d+ex)^{5/2}}-\frac{4c\sqrt{d+ex}(2cd-be)}{e^5}+\frac{2c^2(}{}$$

```
[Out] (-2*(c*d^2 - b*d*e + a*e^2)^2)/(5*e^5*(d + e*x)^(5/2)) + (4*(2*c*d - b*e)*(
c*d^2 - b*d*e + a*e^2))/(3*e^5*(d + e*x)^(3/2)) - (2*(6*c^2*d^2 + b^2*e^2 -
 2*c*e*(3*b*d - a*e)))/(e^5*Sqrt[d + e*x]) - (4*c*(2*c*d - b*e)*Sqrt[d + e*
x])/e^5 + (2*c^2*(d + e*x)^(3/2))/(3*e^5)
```

Rubi [A] time = 0.0708952, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {698}

$$-\frac{2\left(-2ce(3bd-ae)+b^2e^2+6c^2d^2\right)}{e^5\sqrt{d+ex}}+\frac{4(2cd-be)\left(ae^2-bde+cd^2\right)}{3e^5(d+ex)^{3/2}}-\frac{2\left(ae^2-bde+cd^2\right)^2}{5e^5(d+ex)^{5/2}}-\frac{4c\sqrt{d+ex}(2cd-be)}{e^5}+\frac{2c^2(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(d + e*x)^(7/2),x]
```

```
[Out] (-2*(c*d^2 - b*d*e + a*e^2)^2)/(5*e^5*(d + e*x)^(5/2)) + (4*(2*c*d - b*e)*(
c*d^2 - b*d*e + a*e^2))/(3*e^5*(d + e*x)^(3/2)) - (2*(6*c^2*d^2 + b^2*e^2 -
 2*c*e*(3*b*d - a*e)))/(e^5*Sqrt[d + e*x]) - (4*c*(2*c*d - b*e)*Sqrt[d + e*
x])/e^5 + (2*c^2*(d + e*x)^(3/2))/(3*e^5)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx+cx^2\right)^2}{(d+ex)^{7/2}}\,dx &= \int \left(\frac{\left(cd^2-bde+ae^2\right)^2}{e^4(d+ex)^{7/2}}+\frac{2(-2cd+be)\left(cd^2-bde+ae^2\right)}{e^4(d+ex)^{5/2}}+\frac{6c^2d^2+b^2e^2-2ce(3bd-ae)}{e^4(d+ex)^{3/2}}-\frac{2c(2c}{e^4\sqrt{}}\right.\\ &= -\frac{2\left(cd^2-bde+ae^2\right)^2}{5e^5(d+ex)^{5/2}}+\frac{4(2cd-be)\left(cd^2-bde+ae^2\right)}{3e^5(d+ex)^{3/2}}-\frac{2\left(6c^2d^2+b^2e^2-2ce(3bd-ae)\right)}{e^5\sqrt{d+ex}}-\frac{4c(2}{}\end{aligned}$$

Mathematica [A] time = 0.136638, size = 172, normalized size = 1.06

$$-\frac{2\left(3a^2e^4+2abe^3(2d+5ex)+2ace^2\left(8d^2+20dex+15e^2x^2\right)+b^2e^2\left(8d^2+20dex+15e^2x^2\right)-6bce\left(40d^2ex+16d^3+30d\right.\right.}{15e^5(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(d + e*x)^(7/2),x]
```

```
[Out] (-2*(3*a^2*e^4 + 2*a*b*e^3*(2*d + 5*e*x) + b^2*e^2*(8*d^2 + 20*d*e*x + 15*e
^2*x^2) + 2*a*c*e^2*(8*d^2 + 20*d*e*x + 15*e^2*x^2) - 6*b*c*e*(16*d^3 + 40*
d^2*e*x + 30*d*e^2*x^2 + 5*e^3*x^3) + c^2*(128*d^4 + 320*d^3*e*x + 240*d^2*
e^2*x^2 + 40*d*e^3*x^3 - 5*e^4*x^4)))/(15*e^5*(d + e*x)^(5/2))
```

Maple [A] time = 0.044, size = 194, normalized size = 1.2

$$-\frac{-10\,c^2x^4e^4-60\,bce^4x^3+80\,c^2de^3x^3+60\,ace^4x^2+30\,b^2e^4x^2-360\,bcde^3x^2+480\,c^2d^2e^2x^2+20\,abe^4x+80\,acde^3x+}{15\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(e*x+d)^(7/2),x)
```

```
[Out] -2/15/(e*x+d)^(5/2)*(-5*c^2*e^4*x^4-30*b*c*e^4*x^3+40*c^2*d*e^3*x^3+30*a*c*
e^4*x^2+15*b^2*e^4*x^2-180*b*c*d*e^3*x^2+240*c^2*d^2*e^2*x^2+10*a*b*e^4*x+4
0*a*c*d*e^3*x+20*b^2*d*e^3*x-240*b*c*d^2*e^2*x+320*c^2*d^3*e*x+3*a^2*e^4+4*
a*b*d*e^3+16*a*c*d^2*e^2+8*b^2*d^2*e^2-96*b*c*d^3*e+128*c^2*d^4)/e^5
```

Maxima [A] time = 0.964951, size = 250, normalized size = 1.54

$$\frac{2\left(\frac{5\left((ex+d)^{\frac{3}{2}}c^2-6\left(2\,c^2d-bce\right)\sqrt{ex+d}\right)}{e^4}-\frac{3\,c^2d^4-6\,bcd^3e-6\,abde^3+3\,a^2e^4+3\left(b^2+2\,ac\right)d^2e^2+15\left(6\,c^2d^2-6\,bcde+\left(b^2+2\,ac\right)e^2\right)(ex+d)^2-10\left(2\,c^2d^3-3\,bcd^2e-abe\right.}{(ex+d)^{\frac{5}{2}}e^4}\right.}{15\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*(5*((e*x + d)^(3/2)*c^2 - 6*(2*c^2*d - b*c*e)*sqrt(e*x + d))/e^4 - (3*
c^2*d^4 - 6*b*c*d^3*e - 6*a*b*d*e^3 + 3*a^2*e^4 + 3*(b^2 + 2*a*c)*d^2*e^2 +
 15*(6*c^2*d^2 - 6*b*c*d*e + (b^2 + 2*a*c)*e^2)*(e*x + d)^2 - 10*(2*c^2*d^3
 - 3*b*c*d^2*e - a*b*e^3 + (b^2 + 2*a*c)*d*e^2)*(e*x + d))/((e*x + d)^(5/2)
*e^4))/e
```

Fricas [A] time = 1.99409, size = 451, normalized size = 2.78

$$\frac{2\left(5\,c^2e^4x^4-128\,c^2d^4+96\,bcd^3e-4\,abde^3-3\,a^2e^4-8\left(b^2+2\,ac\right)d^2e^2-10\left(4\,c^2de^3-3\,bce^4\right)x^3-15\left(16\,c^2d^2e^2-12\,b\right.\right.}{15\left(e^8x^3+3\,de^7x^2+3\,d^2e^6x+d^3e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(5*c^2*e^4*x^4 - 128*c^2*d^4 + 96*b*c*d^3*e - 4*a*b*d*e^3 - 3*a^2*e^4
- 8*(b^2 + 2*a*c)*d^2*e^2 - 10*(4*c^2*d*e^3 - 3*b*c*e^4)*x^3 - 15*(16*c^2*d
^2*e^2 - 12*b*c*d*e^3 + (b^2 + 2*a*c)*e^4)*x^2 - 10*(32*c^2*d^3*e - 24*b*c*
d^2*e^2 + a*b*e^4 + 2*(b^2 + 2*a*c)*d*e^3)*x)*sqrt(e*x + d)/(e^8*x^3 + 3*d*
```

```
e^7*x^2 + 3*d^2*e^6*x + d^3*e^5)
```

Sympy [A] time = 3.81245, size = 1180, normalized size = 7.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(e*x+d)**(7/2),x)
```

```
[Out] Piecewise((-6*a**2*e**4/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d +
e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 8*a*b*d*e**3/(15*d**2*e**5*sqrt(d + e*
x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 20*a*b*e**4*
x/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sq
rt(d + e*x)) - 32*a*c*d**2*e**2/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*s
qrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 80*a*c*d*e**3*x/(15*d**2*e**5*
sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 6
0*a*c*e**4*x**2/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 1
5*e**7*x**2*sqrt(d + e*x)) - 16*b**2*d**2*e**2/(15*d**2*e**5*sqrt(d + e*x)
+ 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 40*b**2*d*e**3*
x/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sq
rt(d + e*x)) - 30*b**2*e**4*x**2/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*
sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) + 192*b*c*d**3*e/(15*d**2*e**5*
sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) + 4
80*b*c*d**2*e**2*x/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x)
+ 15*e**7*x**2*sqrt(d + e*x)) + 360*b*c*d*e**3*x**2/(15*d**2*e**5*sqrt(d +
e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) + 60*b*c*e**
4*x**3/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x*
*2*sqrt(d + e*x)) - 256*c**2*d**4/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x
*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 640*c**2*d**3*e*x/(15*d**2*e
**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x))
 - 480*c**2*d**2*e**2*x**2/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d
 + e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 80*c**2*d*e**3*x**3/(15*d**2*e**5*s
qrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) + 10
*c**2*e**4*x**4/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 1
5*e**7*x**2*sqrt(d + e*x)), Ne(e, 0)), ((a**2*x + a*b*x**2 + 2*a*c*x**3/3 +
 b**2*x**3/3 + b*c*x**4/2 + c**2*x**5/5)/d**(7/2), True))
```

Giac [A] time = 1.13767, size = 325, normalized size = 2.01

$$\frac{2}{3}\left((xe+d)^{\frac{3}{2}}c^2e^{10} - 12\sqrt{xe+d}c^2de^{10} + 6\sqrt{xe+d}bce^{11}\right)e^{(-15)} - \frac{2\left(90(xe+d)^2c^2d^2 - 20(xe+d)c^2d^3 + 3c^2d^4 - 90(xe+d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] 2/3*((x*e + d)^(3/2)*c^2*e^10 - 12*sqrt(x*e + d)*c^2*d*e^10 + 6*sqrt(x*e +
d)*b*c*e^11)*e^(-15) - 2/15*(90*(x*e + d)^2*c^2*d^2 - 20*(x*e + d)*c^2*d^3
+ 3*c^2*d^4 - 90*(x*e + d)^2*b*c*d*e + 30*(x*e + d)*b*c*d^2*e - 6*b*c*d^3*e
 + 15*(x*e + d)^2*b^2*e^2 + 30*(x*e + d)^2*a*c*e^2 - 10*(x*e + d)*b^2*d*e^2
 - 20*(x*e + d)*a*c*d*e^2 + 3*b^2*d^2*e^2 + 6*a*c*d^2*e^2 + 10*(x*e + d)*a*
b*e^3 - 6*a*b*d*e^3 + 3*a^2*e^4)*e^(-5)/(x*e + d)^(5/2)
```

3.2282 $\int (d+ex)^{5/2} \left(a+bx+cx^2\right)^3 \, dx$

Optimal. Leaf size=286

$$\frac{2c(d+ex)^{15/2}\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{5e^7}-\frac{2(d+ex)^{13/2}(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{13e^7}+\frac{6(d+ex)}{}$$

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)^3*(d + e*x)^(7/2))/(7*e^7) - (2*(2*c*d - b*e)*(c
*d^2 - b*d*e + a*e^2)^2*(d + e*x)^(9/2))/(3*e^7) + (6*(c*d^2 - b*d*e + a*e^
2)*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^(11/2))/(11*e^7) - (
2*(2*c*d - b*e)*(10*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*(d + e*x)^(1
3/2))/(13*e^7) + (2*c*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^(
15/2))/(5*e^7) - (6*c^2*(2*c*d - b*e)*(d + e*x)^(17/2))/(17*e^7) + (2*c^3*(
d + e*x)^(19/2))/(19*e^7)
```

Rubi [A] time = 0.180941, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {698}

$$\frac{2c(d+ex)^{15/2}\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{5e^7}-\frac{2(d+ex)^{13/2}(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{13e^7}+\frac{6(d+ex)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)*(a + b*x + c*x^2)^3,x]
```


```
[Out] (2*(c*d^2 - b*d*e + a*e^2)^3*(d + e*x)^(7/2))/(7*e^7) - (2*(2*c*d - b*e)*(c
*d^2 - b*d*e + a*e^2)^2*(d + e*x)^(9/2))/(3*e^7) + (6*(c*d^2 - b*d*e + a*e^
2)*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^(11/2))/(11*e^7) - (
2*(2*c*d - b*e)*(10*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*(d + e*x)^(1
3/2))/(13*e^7) + (2*c*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^(
15/2))/(5*e^7) - (6*c^2*(2*c*d - b*e)*(d + e*x)^(17/2))/(17*e^7) + (2*c^3*(
d + e*x)^(19/2))/(19*e^7)
```


Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^{5/2}\left(a+bx+cx^2\right)^3\,dx=\int\left(\frac{\left(cd^2-bde+ae^2\right)^3(d+ex)^{5/2}}{e^6}+\frac{3(-2cd+be)\left(cd^2-bde+ae^2\right)^2(d+ex)^{7/2}}{e^6}+\cdots\right.$$

$$=\frac{2\left(cd^2-bde+ae^2\right)^3(d+ex)^{7/2}}{7e^7}-\frac{2(2cd-be)\left(cd^2-bde+ae^2\right)^2(d+ex)^{9/2}}{3e^7}+\frac{6\left(cd\right.}{}$$

Mathematica [A] time = 0.909399, size = 321, normalized size = 1.12

$$2\left((d+ex)^{7/2}(a+x(b+cx))^3-\frac{2(d+ex)^{9/2}\left(-646ce^2\left(65a^2e^2(2d-9ex)-15abe\left(8d^2-36dex+99e^2x^2\right)+2b^2\left(-72d^2ex+16d^3+198de^2x^2-429e^3x^3\right)\right)+1615b\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)*(a + b*x + c*x^2)^3,x]
```

```
[Out] (2*((d + e*x)^(7/2)*(a + x*(b + c*x))^3 - (2*(d + e*x)^(9/2)*(-10*c^3*(256*
d^5 - 1152*d^4*e*x + 3168*d^3*e^2*x^2 - 6864*d^2*e^3*x^3 + 12870*d*e^4*x^4
- 21879*e^5*x^5) + 1615*b*e^3*(143*a^2*e^2 + 26*a*b*e*(-2*d + 9*e*x) + b^2*
(8*d^2 - 36*d*e*x + 99*e^2*x^2)) - 646*c*e^2*(65*a^2*e^2*(2*d - 9*e*x) - 15
*a*b*e*(8*d^2 - 36*d*e*x + 99*e^2*x^2) + 2*b^2*(16*d^3 - 72*d^2*e*x + 198*d
*e^2*x^2 - 429*e^3*x^3)) + 19*c^2*e*(68*a*e*(-16*d^3 + 72*d^2*e*x - 198*d*e
^2*x^2 + 429*e^3*x^3) + 5*b*(128*d^4 - 576*d^3*e*x + 1584*d^2*e^2*x^2 - 343
2*d*e^3*x^3 + 6435*e^4*x^4))))/(692835*e^6)))/(7*e)
```

Maple [A] time = 0.046, size = 495, normalized size = 1.7

$$\frac{510510\,c^3x^6e^6+1711710\,bc^2e^6x^5-360360\,c^3de^5x^5+1939938\,ac^2e^6x^4+1939938\,b^2ce^6x^4-1141140\,bc^2de^5x^4+240240\,c\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(c*x^2+b*x+a)^3,x)
```

```
[Out] 2/4849845*(e*x+d)^(7/2)*(255255*c^3*e^6*x^6+855855*b*c^2*e^6*x^5-180180*c^3
*d*e^5*x^5+969969*a*c^2*e^6*x^4+969969*b^2*c*e^6*x^4-570570*b*c^2*d*e^5*x^4
+120120*c^3*d^2*e^4*x^4+2238390*a*b*c*e^6*x^3-596904*a*c^2*d*e^5*x^3+373065
*b^3*e^6*x^3-596904*b^2*c*d*e^5*x^3+351120*b*c^2*d^2*e^4*x^3-73920*c^3*d^3*
e^3*x^3+1322685*a^2*c*e^6*x^2+1322685*a*b^2*e^6*x^2-1220940*a*b*c*d*e^5*x^2
+325584*a*c^2*d^2*e^4*x^2-203490*b^3*d*e^5*x^2+325584*b^2*c*d^2*e^4*x^2-191
520*b*c^2*d^3*e^3*x^2+40320*c^3*d^4*e^2*x^2+1616615*a^2*b*e^6*x-587860*a^2*
c*d*e^5*x-587860*a*b^2*d*e^5*x+542640*a*b*c*d^2*e^4*x-144704*a*c^2*d^3*e^3*
x+90440*b^3*d^2*e^4*x-144704*b^2*c*d^3*e^3*x+85120*b*c^2*d^4*e^2*x-17920*c^
3*d^5*e*x+692835*a^3*e^6-461890*a^2*b*d*e^5+167960*a^2*c*d^2*e^4+167960*a*b
^2*d^2*e^4-155040*a*b*c*d^3*e^3+41344*a*c^2*d^4*e^2-25840*b^3*d^3*e^3+41344
*b^2*c*d^4*e^2-24320*b*c^2*d^5*e+5120*c^3*d^6)/e^7
```

Maxima [A] time = 1.01909, size = 549, normalized size = 1.92

$$\frac{2\Big(255255\,(ex+d)^{\frac{19}{2}}c^3-855855\,\big(2\,c^3d-bc^2e\big)(ex+d)^{\frac{17}{2}}+969969\,\big(5\,c^3d^2-5\,bc^2de+\big(b^2c+ac^2\big)e^2\big)(ex+d)^{\frac{15}{2}}-373065\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 2/4849845*(255255*(e*x + d)^(19/2)*c^3 - 855855*(2*c^3*d - b*c^2*e)*(e*x +
d)^(17/2) + 969969*(5*c^3*d^2 - 5*b*c^2*d*e + (b^2*c + a*c^2)*e^2)*(e*x + d
)^(15/2) - 373065*(20*c^3*d^3 - 30*b*c^2*d^2*e + 12*(b^2*c + a*c^2)*d*e^2 -
 (b^3 + 6*a*b*c)*e^3)*(e*x + d)^(13/2) + 1322685*(5*c^3*d^4 - 10*b*c^2*d^3*
e + 6*(b^2*c + a*c^2)*d^2*e^2 - (b^3 + 6*a*b*c)*d*e^3 + (a*b^2 + a^2*c)*e^4
)*(e*x + d)^(11/2) - 1616615*(2*c^3*d^5 - 5*b*c^2*d^4*e - a^2*b*e^5 + 4*(b^
2*c + a*c^2)*d^3*e^2 - (b^3 + 6*a*b*c)*d^2*e^3 + 2*(a*b^2 + a^2*c)*d*e^4)*(
e*x + d)^(9/2) + 692835*(c^3*d^6 - 3*b*c^2*d^5*e - 3*a^2*b*d*e^5 + a^3*e^6
+ 3*(b^2*c + a*c^2)*d^4*e^2 - (b^3 + 6*a*b*c)*d^3*e^3 + 3*(a*b^2 + a^2*c)*d
```

```
^2*e^4)*(e*x + d)^(7/2))/e^7
```

Fricas [B] time = 2.03145, size = 1733, normalized size = 6.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 2/4849845*(255255*c^3*e^9*x^9 + 5120*c^3*d^9 - 24320*b*c^2*d^8*e - 461890*a
^2*b*d^4*e^5 + 692835*a^3*d^3*e^6 + 41344*(b^2*c + a*c^2)*d^7*e^2 - 25840*(
b^3 + 6*a*b*c)*d^6*e^3 + 167960*(a*b^2 + a^2*c)*d^5*e^4 + 45045*(13*c^3*d*e
^8 + 19*b*c^2*e^9)*x^8 + 3003*(115*c^3*d^2*e^7 + 665*b*c^2*d*e^8 + 323*(b^2
*c + a*c^2)*e^9)*x^7 + 231*(5*c^3*d^3*e^6 + 5225*b*c^2*d^2*e^7 + 10013*(b^2
*c + a*c^2)*d*e^8 + 1615*(b^3 + 6*a*b*c)*e^9)*x^6 - 63*(20*c^3*d^4*e^5 - 95
*b*c^2*d^3*e^6 - 22933*(b^2*c + a*c^2)*d^2*e^7 - 14535*(b^3 + 6*a*b*c)*d*e^
8 - 20995*(a*b^2 + a^2*c)*e^9)*x^5 + 35*(40*c^3*d^5*e^4 - 190*b*c^2*d^4*e^5
 + 46189*a^2*b*e^9 + 323*(b^2*c + a*c^2)*d^3*e^6 + 17119*(b^3 + 6*a*b*c)*d^
2*e^7 + 96577*(a*b^2 + a^2*c)*d*e^8)*x^4 - 5*(320*c^3*d^6*e^3 - 1520*b*c^2*
d^5*e^4 - 877591*a^2*b*d*e^8 - 138567*a^3*e^9 + 2584*(b^2*c + a*c^2)*d^4*e^
5 - 1615*(b^3 + 6*a*b*c)*d^3*e^6 - 474487*(a*b^2 + a^2*c)*d^2*e^7)*x^3 + 3*
(640*c^3*d^7*e^2 - 3040*b*c^2*d^6*e^3 + 1154725*a^2*b*d^2*e^7 + 692835*a^3*
d*e^8 + 5168*(b^2*c + a*c^2)*d^5*e^4 - 3230*(b^3 + 6*a*b*c)*d^4*e^5 + 20995
*(a*b^2 + a^2*c)*d^3*e^6)*x^2 - (2560*c^3*d^8*e - 12160*b*c^2*d^7*e^2 - 230
945*a^2*b*d^3*e^6 - 2078505*a^3*d^2*e^7 + 20672*(b^2*c + a*c^2)*d^6*e^3 - 1
2920*(b^3 + 6*a*b*c)*d^5*e^4 + 83980*(a*b^2 + a^2*c)*d^4*e^5)*x)*sqrt(e*x +
 d)/e^7
```

Sympy [A] time = 65.2801, size = 2363, normalized size = 8.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(c*x**2+b*x+a)**3,x)
```

```
[Out] a**3*d**2*Piecewise((sqrt(d)*x, Eq(e, 0)), (2*(d + e*x)**(3/2)/(3*e), True)
) + 4*a**3*d*(-d*(d + e*x)**(3/2)/3 + (d + e*x)**(5/2)/5)/e + 2*a**3*(d**2*
(d + e*x)**(3/2)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e + 6*a**
2*b*d**2*(-d*(d + e*x)**(3/2)/3 + (d + e*x)**(5/2)/5)/e**2 + 12*a**2*b*d*(d
**2*(d + e*x)**(3/2)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e**2
+ 6*a**2*b*(-d**3*(d + e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d +
 e*x)**(7/2)/7 + (d + e*x)**(9/2)/9)/e**2 + 6*a**2*c*d**2*(d**2*(d + e*x)**
(3/2)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e**3 + 12*a**2*c*d*(
-d**3*(d + e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d + e*x)**(7/2)
/7 + (d + e*x)**(9/2)/9)/e**3 + 6*a**2*c*(d**4*(d + e*x)**(3/2)/3 - 4*d**3*
(d + e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e*x)**(9/2)/9 + (
d + e*x)**(11/2)/11)/e**3 + 6*a*b**2*d**2*(d**2*(d + e*x)**(3/2)/3 - 2*d*(d
 + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e**3 + 12*a*b**2*d*(-d**3*(d + e*x)*
*(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d + e*x)**(7/2)/7 + (d + e*x)**
(9/2)/9)/e**3 + 6*a*b**2*(d**4*(d + e*x)**(3/2)/3 - 4*d**3*(d + e*x)**(5/2)
/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e*x)**(9/2)/9 + (d + e*x)**(11/2)
/11)/e**3 + 12*a*b*c*d**2*(-d**3*(d + e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/
```

```
2)/5 - 3*d*(d + e*x)**(7/2)/7 + (d + e*x)**(9/2)/9)/e**4 + 24*a*b*c*d*(d**4
*(d + e*x)**(3/2)/3 - 4*d**3*(d + e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/2)/7
 - 4*d*(d + e*x)**(9/2)/9 + (d + e*x)**(11/2)/11)/e**4 + 12*a*b*c*(-d**5*(d
 + e*x)**(3/2)/3 + d**4*(d + e*x)**(5/2) - 10*d**3*(d + e*x)**(7/2)/7 + 10*
d**2*(d + e*x)**(9/2)/9 - 5*d*(d + e*x)**(11/2)/11 + (d + e*x)**(13/2)/13)/
e**4 + 6*a*c**2*d**2*(d**4*(d + e*x)**(3/2)/3 - 4*d**3*(d + e*x)**(5/2)/5 +
 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e*x)**(9/2)/9 + (d + e*x)**(11/2)/11)
/e**5 + 12*a*c**2*d*(-d**5*(d + e*x)**(3/2)/3 + d**4*(d + e*x)**(5/2) - 10*
d**3*(d + e*x)**(7/2)/7 + 10*d**2*(d + e*x)**(9/2)/9 - 5*d*(d + e*x)**(11/2
)/11 + (d + e*x)**(13/2)/13)/e**5 + 6*a*c**2*(d**6*(d + e*x)**(3/2)/3 - 6*d
**5*(d + e*x)**(5/2)/5 + 15*d**4*(d + e*x)**(7/2)/7 - 20*d**3*(d + e*x)**(9
/2)/9 + 15*d**2*(d + e*x)**(11/2)/11 - 6*d*(d + e*x)**(13/2)/13 + (d + e*x)
**(15/2)/15)/e**5 + 2*b**3*d**2*(-d**3*(d + e*x)**(3/2)/3 + 3*d**2*(d + e*x
)**(5/2)/5 - 3*d*(d + e*x)**(7/2)/7 + (d + e*x)**(9/2)/9)/e**4 + 4*b**3*d*(
d**4*(d + e*x)**(3/2)/3 - 4*d**3*(d + e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/
2)/7 - 4*d*(d + e*x)**(9/2)/9 + (d + e*x)**(11/2)/11)/e**4 + 2*b**3*(-d**5*
(d + e*x)**(3/2)/3 + d**4*(d + e*x)**(5/2) - 10*d**3*(d + e*x)**(7/2)/7 + 1
0*d**2*(d + e*x)**(9/2)/9 - 5*d*(d + e*x)**(11/2)/11 + (d + e*x)**(13/2)/13
)/e**4 + 6*b**2*c*d**2*(d**4*(d + e*x)**(3/2)/3 - 4*d**3*(d + e*x)**(5/2)/5
 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e*x)**(9/2)/9 + (d + e*x)**(11/2)/1
1)/e**5 + 12*b**2*c*d*(-d**5*(d + e*x)**(3/2)/3 + d**4*(d + e*x)**(5/2) - 1
0*d**3*(d + e*x)**(7/2)/7 + 10*d**2*(d + e*x)**(9/2)/9 - 5*d*(d + e*x)**(11
/2)/11 + (d + e*x)**(13/2)/13)/e**5 + 6*b**2*c*(d**6*(d + e*x)**(3/2)/3 - 6
*d**5*(d + e*x)**(5/2)/5 + 15*d**4*(d + e*x)**(7/2)/7 - 20*d**3*(d + e*x)**
(9/2)/9 + 15*d**2*(d + e*x)**(11/2)/11 - 6*d*(d + e*x)**(13/2)/13 + (d + e*
x)**(15/2)/15)/e**5 + 6*b*c**2*d**2*(-d**5*(d + e*x)**(3/2)/3 + d**4*(d + e
*x)**(5/2) - 10*d**3*(d + e*x)**(7/2)/7 + 10*d**2*(d + e*x)**(9/2)/9 - 5*d*
(d + e*x)**(11/2)/11 + (d + e*x)**(13/2)/13)/e**6 + 12*b*c**2*d*(d**6*(d +
e*x)**(3/2)/3 - 6*d**5*(d + e*x)**(5/2)/5 + 15*d**4*(d + e*x)**(7/2)/7 - 20
*d**3*(d + e*x)**(9/2)/9 + 15*d**2*(d + e*x)**(11/2)/11 - 6*d*(d + e*x)**(1
3/2)/13 + (d + e*x)**(15/2)/15)/e**6 + 6*b*c**2*(-d**7*(d + e*x)**(3/2)/3 +
 7*d**6*(d + e*x)**(5/2)/5 - 3*d**5*(d + e*x)**(7/2) + 35*d**4*(d + e*x)**(
9/2)/9 - 35*d**3*(d + e*x)**(11/2)/11 + 21*d**2*(d + e*x)**(13/2)/13 - 7*d*
(d + e*x)**(15/2)/15 + (d + e*x)**(17/2)/17)/e**6 + 2*c**3*d**2*(d**6*(d +
e*x)**(3/2)/3 - 6*d**5*(d + e*x)**(5/2)/5 + 15*d**4*(d + e*x)**(7/2)/7 - 20
*d**3*(d + e*x)**(9/2)/9 + 15*d**2*(d + e*x)**(11/2)/11 - 6*d*(d + e*x)**(1
3/2)/13 + (d + e*x)**(15/2)/15)/e**7 + 4*c**3*d*(-d**7*(d + e*x)**(3/2)/3 +
 7*d**6*(d + e*x)**(5/2)/5 - 3*d**5*(d + e*x)**(7/2) + 35*d**4*(d + e*x)**(
9/2)/9 - 35*d**3*(d + e*x)**(11/2)/11 + 21*d**2*(d + e*x)**(13/2)/13 - 7*d*
(d + e*x)**(15/2)/15 + (d + e*x)**(17/2)/17)/e**7 + 2*c**3*(d**8*(d + e*x)*
*(3/2)/3 - 8*d**7*(d + e*x)**(5/2)/5 + 4*d**6*(d + e*x)**(7/2) - 56*d**5*(d
 + e*x)**(9/2)/9 + 70*d**4*(d + e*x)**(11/2)/11 - 56*d**3*(d + e*x)**(13/2)
/13 + 28*d**2*(d + e*x)**(15/2)/15 - 8*d*(d + e*x)**(17/2)/17 + (d + e*x)**
(19/2)/19)/e**7
```

Giac [B] time = 1.23712, size = 2830, normalized size = 9.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 2/14549535*(2909907*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^2*b*d^2*e^(
-1) + 415701*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2
)*d^2)*a*b^2*d^2*e^(-2) + 415701*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d
 + 35*(x*e + d)^(3/2)*d^2)*a^2*c*d^2*e^(-2) + 46189*(35*(x*e + d)^(9/2) - 1
```

35*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*b ^3*d^2*e^(-3) + 277134*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x *e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a*b*c*d^2*e^(-3) + 12597*(315* (x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772 *(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*b^2*c*d^2*e^(-4) + 12597*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*a*c^2*d^2*e^(-4) + 484 5*(693*(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d ^2 - 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^ (3/2)*d^5)*b*c^2*d^2*e^(-5) + 323*(3003*(x*e + d)^(15/2) - 20790*(x*e + d)^ (13/2)*d + 61425*(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525* (x*e + d)^(7/2)*d^4 - 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*c^3*d^2*e^(-6) + 4849845*(x*e + d)^(3/2)*a^3*d^2 + 831402*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*a^2*b*d*e^(-1) + 2771 34*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a*b^2*d*e^(-2) + 277134*(35*(x*e + d)^(9/2) - 135* (x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a^2* c*d*e^(-2) + 8398*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x* e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*b^3 *d*e^(-3) + 50388*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d + 2970*(x* e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4)*a*b *c*d*e^(-3) + 9690*(693*(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010* (x*e + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*b^2*c*d*e^(-4) + 9690*(693*(x*e + d)^(13/2) - 4 095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)* d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*a*c^2*d*e^(-4) + 1938*(3003*(x*e + d)^(15/2) - 20790*(x*e + d)^(13/2)*d + 61425*(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525*(x*e + d)^(7/2)*d^4 - 54054* (x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*b*c^2*d*e^(-5) + 266*(6435 *(x*e + d)^(17/2) - 51051*(x*e + d)^(15/2)*d + 176715*(x*e + d)^(13/2)*d^2 - 348075*(x*e + d)^(11/2)*d^3 + 425425*(x*e + d)^(9/2)*d^4 - 328185*(x*e + d)^(7/2)*d^5 + 153153*(x*e + d)^(5/2)*d^6 - 36465*(x*e + d)^(3/2)*d^7)*c^3* d*e^(-6) + 1939938*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^3*d + 138567 *(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 10 5*(x*e + d)^(3/2)*d^3)*a^2*b*e^(-1) + 12597*(315*(x*e + d)^(11/2) - 1540*(x *e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 115 5*(x*e + d)^(3/2)*d^4)*a*b^2*e^(-2) + 12597*(315*(x*e + d)^(11/2) - 1540*(x *e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 115 5*(x*e + d)^(3/2)*d^4)*a^2*c*e^(-2) + 1615*(693*(x*e + d)^(13/2) - 4095*(x* e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 + 9 009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*b^3*e^(-3) + 9690*(693* (x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12 870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d ^5)*a*b*c*e^(-3) + 969*(3003*(x*e + d)^(15/2) - 20790*(x*e + d)^(13/2)*d + 61425*(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525*(x*e + d)^(7/2)*d^4 - 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*b^2*c*e^(-4) + 969*(3003*(x*e + d)^(15/2) - 20790*(x*e + d)^(13/2)*d + 61425*(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525*(x*e + d)^(7/2)*d^4 - 54 054*(x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*a*c^2*e^(-4) + 399*(64 35*(x*e + d)^(17/2) - 51051*(x*e + d)^(15/2)*d + 176715*(x*e + d)^(13/2)*d^ 2 - 348075*(x*e + d)^(11/2)*d^3 + 425425*(x*e + d)^(9/2)*d^4 - 328185*(x*e + d)^(7/2)*d^5 + 153153*(x*e + d)^(5/2)*d^6 - 36465*(x*e + d)^(3/2)*d^7)*b* c^2*e^(-5) + 7*(109395*(x*e + d)^(19/2) - 978120*(x*e + d)^(17/2)*d + 38798 76*(x*e + d)^(15/2)*d^2 - 8953560*(x*e + d)^(13/2)*d^3 + 13226850*(x*e + d) ^(11/2)*d^4 - 12932920*(x*e + d)^(9/2)*d^5 + 8314020*(x*e + d)^(7/2)*d^6 - 3325608*(x*e + d)^(5/2)*d^7 + 692835*(x*e + d)^(3/2)*d^8)*c^3*e^(-6) + 1385 67*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*a^3)*e^(-1)

3.2283 $\int (d+ex)^{3/2} \left(a+bx+cx^2\right)^3 \, dx$

Optimal. Leaf size=286

$$\frac{6c(d+ex)^{13/2}\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{13e^7}-\frac{2(d+ex)^{11/2}(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{11e^7}+\frac{2(d+ex)^{9/2}}{}$$

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)^3*(d + e*x)^(5/2))/(5*e^7) - (6*(2*c*d - b*e)*(c
*d^2 - b*d*e + a*e^2)^2*(d + e*x)^(7/2))/(7*e^7) + (2*(c*d^2 - b*d*e + a*e^
2)*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^(9/2))/(3*e^7) - (2*
(2*c*d - b*e)*(10*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*(d + e*x)^(11/
2))/(11*e^7) + (6*c*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^(13
/2))/(13*e^7) - (2*c^2*(2*c*d - b*e)*(d + e*x)^(15/2))/(5*e^7) + (2*c^3*(d
+ e*x)^(17/2))/(17*e^7)
```

Rubi [A] time = 0.129118, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {698}

$$\frac{6c(d+ex)^{13/2}\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{13e^7}-\frac{2(d+ex)^{11/2}(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{11e^7}+\frac{2(d+ex)^{9/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*(a + b*x + c*x^2)^3,x]
```

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)^3*(d + e*x)^(5/2))/(5*e^7) - (6*(2*c*d - b*e)*(c
*d^2 - b*d*e + a*e^2)^2*(d + e*x)^(7/2))/(7*e^7) + (2*(c*d^2 - b*d*e + a*e^
2)*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^(9/2))/(3*e^7) - (2*
(2*c*d - b*e)*(10*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*(d + e*x)^(11/
2))/(11*e^7) + (6*c*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^(13
/2))/(13*e^7) - (2*c^2*(2*c*d - b*e)*(d + e*x)^(15/2))/(5*e^7) + (2*c^3*(d
+ e*x)^(17/2))/(17*e^7)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^{3/2}\left(a+bx+cx^2\right)^3 \, dx = \int \left(\frac{\left(cd^2-bde+ae^2\right)^3(d+ex)^{3/2}}{e^6}+\frac{3(-2cd+be)\left(cd^2-bde+ae^2\right)^2(d+ex)^{5/2}}{e^6}+\frac{3(c}{}\right.$$

$$=\frac{2\left(cd^2-bde+ae^2\right)^3(d+ex)^{5/2}}{5e^7}-\frac{6(2cd-be)\left(cd^2-bde+ae^2\right)^2(d+ex)^{7/2}}{7e^7}+\frac{2\left(cd^2-\right.}{}$$

Mathematica [A] time = 0.882017, size = 320, normalized size = 1.12

$$2\left((d+ex)^{5/2}(a+x(b+cx))^3-\frac{2(d+ex)^{7/2}\left(-34ce^2\left(143a^2e^2(2d-7ex)-39abe\left(8d^2-28dex+63e^2x^2\right)+6b^2\left(-56d^2ex+16d^3+126de^2x^2-231e^3x^3\right)\right)+221be^3(99}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)*(a + b*x + c*x^2)^3,x]
```

```
[Out] (2*((d + e*x)^(5/2)*(a + x*(b + c*x))^3 - (2*(d + e*x)^(7/2)*(-2*c^3*(256*d
^5 - 896*d^4*e*x + 2016*d^3*e^2*x^2 - 3696*d^2*e^3*x^3 + 6006*d*e^4*x^4 - 9
009*e^5*x^5) + 221*b*e^3*(99*a^2*e^2 + 22*a*b*e*(-2*d + 7*e*x) + b^2*(8*d^2
 - 28*d*e*x + 63*e^2*x^2)) - 34*c*e^2*(143*a^2*e^2*(2*d - 7*e*x) - 39*a*b*e
*(8*d^2 - 28*d*e*x + 63*e^2*x^2) + 6*b^2*(16*d^3 - 56*d^2*e*x + 126*d*e^2*x
^2 - 231*e^3*x^3)) + 17*c^2*e*(12*a*e*(-16*d^3 + 56*d^2*e*x - 126*d*e^2*x^2
 + 231*e^3*x^3) + b*(128*d^4 - 448*d^3*e*x + 1008*d^2*e^2*x^2 - 1848*d*e^3*
x^3 + 3003*e^4*x^4))))/(51051*e^6)))/(5*e)
```

Maple [A] time = 0.045, size = 495, normalized size = 1.7

$$\frac{30030\,c^3x^6e^6+102102\,bc^2e^6x^5-24024\,c^3de^5x^5+117810\,ac^2e^6x^4+117810\,b^2ce^6x^4-78540\,bc^2de^5x^4+18480\,c^3d^2e^4x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(c*x^2+b*x+a)^3,x)
```

```
[Out] 2/255255*(e*x+d)^(5/2)*(15015*c^3*e^6*x^6+51051*b*c^2*e^6*x^5-12012*c^3*d*e
^5*x^5+58905*a*c^2*e^6*x^4+58905*b^2*c*e^6*x^4-39270*b*c^2*d*e^5*x^4+9240*c
^3*d^2*e^4*x^4+139230*a*b*c*e^6*x^3-42840*a*c^2*d*e^5*x^3+23205*b^3*e^6*x^3
-42840*b^2*c*d*e^5*x^3+28560*b*c^2*d^2*e^4*x^3-6720*c^3*d^3*e^3*x^3+85085*a
^2*c*e^6*x^2+85085*a*b^2*e^6*x^2-92820*a*b*c*d*e^5*x^2+28560*a*c^2*d^2*e^4*
x^2-15470*b^3*d*e^5*x^2+28560*b^2*c*d^2*e^4*x^2-19040*b*c^2*d^3*e^3*x^2+448
0*c^3*d^4*e^2*x^2+109395*a^2*b*e^6*x-48620*a^2*c*d*e^5*x-48620*a*b^2*d*e^5*
x+53040*a*b*c*d^2*e^4*x-16320*a*c^2*d^3*e^3*x+8840*b^3*d^2*e^4*x-16320*b^2*
c*d^3*e^3*x+10880*b*c^2*d^4*e^2*x-2560*c^3*d^5*e*x+51051*a^3*e^6-43758*a^2*
b*d*e^5+19448*a^2*c*d^2*e^4+19448*a*b^2*d^2*e^4-21216*a*b*c*d^3*e^3+6528*a*
c^2*d^4*e^2-3536*b^3*d^3*e^3+6528*b^2*c*d^4*e^2-4352*b*c^2*d^5*e+1024*c^3*d
^6)/e^7
```

Maxima [A] time = 1.03337, size = 549, normalized size = 1.92

$$\frac{2\left(15015\,(ex+d)^{\frac{17}{2}}c^3-51051\left(2\,c^3d-bc^2e\right)(ex+d)^{\frac{15}{2}}+58905\left(5\,c^3d^2-5\,bc^2de+\left(b^2c+ac^2\right)e^2\right)(ex+d)^{\frac{13}{2}}-23205\,(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 2/255255*(15015*(e*x + d)^(17/2)*c^3 - 51051*(2*c^3*d - b*c^2*e)*(e*x + d)^
(15/2) + 58905*(5*c^3*d^2 - 5*b*c^2*d*e + (b^2*c + a*c^2)*e^2)*(e*x + d)^(1
3/2) - 23205*(20*c^3*d^3 - 30*b*c^2*d^2*e + 12*(b^2*c + a*c^2)*d*e^2 - (b^3
 + 6*a*b*c)*e^3)*(e*x + d)^(11/2) + 85085*(5*c^3*d^4 - 10*b*c^2*d^3*e + 6*(
b^2*c + a*c^2)*d^2*e^2 - (b^3 + 6*a*b*c)*d*e^3 + (a*b^2 + a^2*c)*e^4)*(e*x
+ d)^(9/2) - 109395*(2*c^3*d^5 - 5*b*c^2*d^4*e - a^2*b*e^5 + 4*(b^2*c + a*c
^2)*d^3*e^2 - (b^3 + 6*a*b*c)*d^2*e^3 + 2*(a*b^2 + a^2*c)*d*e^4)*(e*x + d)^
(7/2) + 51051*(c^3*d^6 - 3*b*c^2*d^5*e - 3*a^2*b*d*e^5 + a^3*e^6 + 3*(b^2*c
 + a*c^2)*d^4*e^2 - (b^3 + 6*a*b*c)*d^3*e^3 + 3*(a*b^2 + a^2*c)*d^2*e^4)*(e
```

```
*x + d)^(5/2))/e^7
```

Fricas [B] time = 2.04237, size = 1426, normalized size = 4.99

$$2\left(15015\,c^3e^8x^8+1024\,c^3d^8-4352\,bc^2d^7e-43758\,a^2bd^3e^5+51051\,a^3d^2e^6+6528\,\left(b^2c+ac^2\right)d^6e^2-3536\,\left(b^3+6\,abc\right)d^5e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 2/255255*(15015*c^3*e^8*x^8 + 1024*c^3*d^8 - 4352*b*c^2*d^7*e - 43758*a^2*b
*d^3*e^5 + 51051*a^3*d^2*e^6 + 6528*(b^2*c + a*c^2)*d^6*e^2 - 3536*(b^3 + 6
*a*b*c)*d^5*e^3 + 19448*(a*b^2 + a^2*c)*d^4*e^4 + 3003*(6*c^3*d*e^7 + 17*b*
c^2*e^8)*x^7 + 231*(c^3*d^2*e^6 + 272*b*c^2*d*e^7 + 255*(b^2*c + a*c^2)*e^8
)*x^6 - 21*(12*c^3*d^3*e^5 - 51*b*c^2*d^2*e^6 - 3570*(b^2*c + a*c^2)*d*e^7
- 1105*(b^3 + 6*a*b*c)*e^8)*x^5 + 35*(8*c^3*d^4*e^4 - 34*b*c^2*d^3*e^5 + 51
*(b^2*c + a*c^2)*d^2*e^6 + 884*(b^3 + 6*a*b*c)*d*e^7 + 2431*(a*b^2 + a^2*c)
*e^8)*x^4 - 5*(64*c^3*d^5*e^3 - 272*b*c^2*d^4*e^4 - 21879*a^2*b*e^8 + 408*(
b^2*c + a*c^2)*d^3*e^5 - 221*(b^3 + 6*a*b*c)*d^2*e^6 - 24310*(a*b^2 + a^2*c
)*d*e^7)*x^3 + 3*(128*c^3*d^6*e^2 - 544*b*c^2*d^5*e^3 + 58344*a^2*b*d*e^7 +
 17017*a^3*e^8 + 816*(b^2*c + a*c^2)*d^4*e^4 - 442*(b^3 + 6*a*b*c)*d^3*e^5
+ 2431*(a*b^2 + a^2*c)*d^2*e^6)*x^2 - (512*c^3*d^7*e - 2176*b*c^2*d^6*e^2 -
 21879*a^2*b*d^2*e^6 - 102102*a^3*d*e^7 + 3264*(b^2*c + a*c^2)*d^5*e^3 - 17
68*(b^3 + 6*a*b*c)*d^4*e^4 + 9724*(a*b^2 + a^2*c)*d^3*e^5)*x)*sqrt(e*x + d)
/e^7
```

Sympy [A] time = 39.5039, size = 1411, normalized size = 4.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(c*x**2+b*x+a)**3,x)
```

```
[Out] a**3*d*Piecewise((sqrt(d)*x, Eq(e, 0)), (2*(d + e*x)**(3/2)/(3*e), True)) +
 2*a**3*(-d*(d + e*x)**(3/2)/3 + (d + e*x)**(5/2)/5)/e + 6*a**2*b*d*(-d*(d
+ e*x)**(3/2)/3 + (d + e*x)**(5/2)/5)/e**2 + 6*a**2*b*(d**2*(d + e*x)**(3/2
)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e**2 + 6*a**2*c*d*(d**2*
(d + e*x)**(3/2)/3 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e**3 + 6*
a**2*c*(-d**3*(d + e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d + e*x
)**(7/2)/7 + (d + e*x)**(9/2)/9)/e**3 + 6*a*b**2*d*(d**2*(d + e*x)**(3/2)/3
 - 2*d*(d + e*x)**(5/2)/5 + (d + e*x)**(7/2)/7)/e**3 + 6*a*b**2*(-d**3*(d +
 e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 - 3*d*(d + e*x)**(7/2)/7 + (d +
e*x)**(9/2)/9)/e**3 + 12*a*b*c*d*(-d**3*(d + e*x)**(3/2)/3 + 3*d**2*(d + e*
x)**(5/2)/5 - 3*d*(d + e*x)**(7/2)/7 + (d + e*x)**(9/2)/9)/e**4 + 12*a*b*c*
(d**4*(d + e*x)**(3/2)/3 - 4*d**3*(d + e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7
/2)/7 - 4*d*(d + e*x)**(9/2)/9 + (d + e*x)**(11/2)/11)/e**4 + 6*a*c**2*d*(d
**4*(d + e*x)**(3/2)/3 - 4*d**3*(d + e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/2
)/7 - 4*d*(d + e*x)**(9/2)/9 + (d + e*x)**(11/2)/11)/e**5 + 6*a*c**2*(-d**5
*(d + e*x)**(3/2)/3 + d**4*(d + e*x)**(5/2) - 10*d**3*(d + e*x)**(7/2)/7 +
10*d**2*(d + e*x)**(9/2)/9 - 5*d*(d + e*x)**(11/2)/11 + (d + e*x)**(13/2)/1
3)/e**5 + 2*b**3*d*(-d**3*(d + e*x)**(3/2)/3 + 3*d**2*(d + e*x)**(5/2)/5 -
3*d*(d + e*x)**(7/2)/7 + (d + e*x)**(9/2)/9)/e**4 + 2*b**3*(d**4*(d + e*x)*
```

```
*(3/2)/3 - 4*d**3*(d + e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d +
 e*x)**(9/2)/9 + (d + e*x)**(11/2)/11)/e**4 + 6*b**2*c*d*(d**4*(d + e*x)**(
3/2)/3 - 4*d**3*(d + e*x)**(5/2)/5 + 6*d**2*(d + e*x)**(7/2)/7 - 4*d*(d + e
*x)**(9/2)/9 + (d + e*x)**(11/2)/11)/e**5 + 6*b**2*c*(-d**5*(d + e*x)**(3/2
)/3 + d**4*(d + e*x)**(5/2) - 10*d**3*(d + e*x)**(7/2)/7 + 10*d**2*(d + e*x
)**(9/2)/9 - 5*d*(d + e*x)**(11/2)/11 + (d + e*x)**(13/2)/13)/e**5 + 6*b*c*
*2*d*(-d**5*(d + e*x)**(3/2)/3 + d**4*(d + e*x)**(5/2) - 10*d**3*(d + e*x)*
*(7/2)/7 + 10*d**2*(d + e*x)**(9/2)/9 - 5*d*(d + e*x)**(11/2)/11 + (d + e*x
)**(13/2)/13)/e**6 + 6*b*c**2*(d**6*(d + e*x)**(3/2)/3 - 6*d**5*(d + e*x)**
(5/2)/5 + 15*d**4*(d + e*x)**(7/2)/7 - 20*d**3*(d + e*x)**(9/2)/9 + 15*d**2
*(d + e*x)**(11/2)/11 - 6*d*(d + e*x)**(13/2)/13 + (d + e*x)**(15/2)/15)/e*
*6 + 2*c**3*d*(d**6*(d + e*x)**(3/2)/3 - 6*d**5*(d + e*x)**(5/2)/5 + 15*d**
4*(d + e*x)**(7/2)/7 - 20*d**3*(d + e*x)**(9/2)/9 + 15*d**2*(d + e*x)**(11/
2)/11 - 6*d*(d + e*x)**(13/2)/13 + (d + e*x)**(15/2)/15)/e**7 + 2*c**3*(-d*
*7*(d + e*x)**(3/2)/3 + 7*d**6*(d + e*x)**(5/2)/5 - 3*d**5*(d + e*x)**(7/2)
 + 35*d**4*(d + e*x)**(9/2)/9 - 35*d**3*(d + e*x)**(11/2)/11 + 21*d**2*(d +
 e*x)**(13/2)/13 - 7*d*(d + e*x)**(15/2)/15 + (d + e*x)**(17/2)/17)/e**7
```

Giac [B] time = 1.1873, size = 1690, normalized size = 5.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 2/765765*(153153*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^2*b*d*e^(-1) +
 21879*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)
*a*b^2*d*e^(-2) + 21879*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*
e + d)^(3/2)*d^2)*a^2*c*d*e^(-2) + 2431*(35*(x*e + d)^(9/2) - 135*(x*e + d)
^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*b^3*d*e^(-3)
+ 14586*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d
^2 - 105*(x*e + d)^(3/2)*d^3)*a*b*c*d*e^(-3) + 663*(315*(x*e + d)^(11/2) -
1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^
3 + 1155*(x*e + d)^(3/2)*d^4)*b^2*c*d*e^(-4) + 663*(315*(x*e + d)^(11/2) -
1540*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^
3 + 1155*(x*e + d)^(3/2)*d^4)*a*c^2*d*e^(-4) + 255*(693*(x*e + d)^(13/2) -
4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)
*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*b*c^2*d*e^(-5)
+ 17*(3003*(x*e + d)^(15/2) - 20790*(x*e + d)^(13/2)*d + 61425*(x*e + d)^(1
1/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525*(x*e + d)^(7/2)*d^4 - 54054*(
x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*c^3*d*e^(-6) + 255255*(x*e
+ d)^(3/2)*a^3*d + 21879*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x
*e + d)^(3/2)*d^2)*a^2*b*e^(-1) + 7293*(35*(x*e + d)^(9/2) - 135*(x*e + d)^
(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*a*b^2*e^(-2) +
 7293*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2
 - 105*(x*e + d)^(3/2)*d^3)*a^2*c*e^(-2) + 221*(315*(x*e + d)^(11/2) - 1540
*(x*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 +
1155*(x*e + d)^(3/2)*d^4)*b^3*e^(-3) + 1326*(315*(x*e + d)^(11/2) - 1540*(x
*e + d)^(9/2)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 115
5*(x*e + d)^(3/2)*d^4)*a*b*c*e^(-3) + 255*(693*(x*e + d)^(13/2) - 4095*(x*e
 + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 + 90
09*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*b^2*c*e^(-4) + 255*(693*
(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d + 10010*(x*e + d)^(9/2)*d^2 - 12
870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 3003*(x*e + d)^(3/2)*d
^5)*a*c^2*e^(-4) + 51*(3003*(x*e + d)^(15/2) - 20790*(x*e + d)^(13/2)*d + 6
1425*(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9/2)*d^3 + 96525*(x*e + d)^(7
```

```
/2)*d^4 - 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*e + d)^(3/2)*d^6)*b*c^2*e^(-
5) + 7*(6435*(x*e + d)^(17/2) - 51051*(x*e + d)^(15/2)*d + 176715*(x*e + d)
^(13/2)*d^2 - 348075*(x*e + d)^(11/2)*d^3 + 425425*(x*e + d)^(9/2)*d^4 - 32
8185*(x*e + d)^(7/2)*d^5 + 153153*(x*e + d)^(5/2)*d^6 - 36465*(x*e + d)^(3/
2)*d^7)*c^3*e^(-6) + 51051*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^3)*e
^(-1)
```

3.2284 $\int \sqrt{d+ex}\left(a+bx+cx^2\right)^3\,dx$

Optimal. Leaf size=286

$$\frac{6c(d+ex)^{11/2}\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{11e^7}-\frac{2(d+ex)^{9/2}(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{9e^7}+\frac{6(d+ex)^7}{}$$

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)^3*(d + e*x)^(3/2))/(3*e^7) - (6*(2*c*d - b*e)*(c
*d^2 - b*d*e + a*e^2)^2*(d + e*x)^(5/2))/(5*e^7) + (6*(c*d^2 - b*d*e + a*e^
2)*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^(7/2))/(7*e^7) - (2*
(2*c*d - b*e)*(10*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*(d + e*x)^(9/2
))/(9*e^7) + (6*c*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^(11/2
))/(11*e^7) - (6*c^2*(2*c*d - b*e)*(d + e*x)^(13/2))/(13*e^7) + (2*c^3*(d +
 e*x)^(15/2))/(15*e^7)
```

Rubi [A] time = 0.129942, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {698}

$$\frac{6c(d+ex)^{11/2}\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{11e^7}-\frac{2(d+ex)^{9/2}(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{9e^7}+\frac{6(d+ex)^7}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(a + b*x + c*x^2)^3,x]
```

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)^3*(d + e*x)^(3/2))/(3*e^7) - (6*(2*c*d - b*e)*(c
*d^2 - b*d*e + a*e^2)^2*(d + e*x)^(5/2))/(5*e^7) + (6*(c*d^2 - b*d*e + a*e^
2)*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^(7/2))/(7*e^7) - (2*
(2*c*d - b*e)*(10*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*(d + e*x)^(9/2
))/(9*e^7) + (6*c*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^(11/2
))/(11*e^7) - (6*c^2*(2*c*d - b*e)*(d + e*x)^(13/2))/(13*e^7) + (2*c^3*(d +
 e*x)^(15/2))/(15*e^7)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \sqrt{d+ex}\left(a+bx+cx^2\right)^3\,dx=\int\left(\frac{\left(cd^2-bde+ae^2\right)^3\sqrt{d+ex}}{e^6}+\frac{3(-2cd+be)\left(cd^2-bde+ae^2\right)^2(d+ex)^{3/2}}{e^6}+\frac{3\left(c\right.}{}\right.$$

$$=\frac{2\left(cd^2-bde+ae^2\right)^3(d+ex)^{3/2}}{3e^7}-\frac{6(2cd-be)\left(cd^2-bde+ae^2\right)^2(d+ex)^{5/2}}{5e^7}+\frac{6\left(cd^2\right.}{}$$

Mathematica [A] time = 0.387385, size = 396, normalized size = 1.38

$$\frac{2(d+ex)^{3/2}\left(39ce^2\left(33a^2e^2\left(8d^2-12dex+15e^2x^2\right)+22abe\left(24d^2ex-16d^3-30de^2x^2+35e^3x^3\right)+b^2\left(240d^2e^2x^2-19\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*(a + b*x + c*x^2)^3,x]
```

```
[Out] (2*(d + e*x)^(3/2)*(c^3*(1024*d^6 - 1536*d^5*e*x + 1920*d^4*e^2*x^2 - 2240*
d^3*e^3*x^3 + 2520*d^2*e^4*x^4 - 2772*d*e^5*x^5 + 3003*e^6*x^6) + 143*e^3*(
105*a^3*e^3 + 63*a^2*b*e^2*(-2*d + 3*e*x) + 9*a*b^2*e*(8*d^2 - 12*d*e*x + 1
5*e^2*x^2) + b^3*(-16*d^3 + 24*d^2*e*x - 30*d*e^2*x^2 + 35*e^3*x^3)) + 39*c
*e^2*(33*a^2*e^2*(8*d^2 - 12*d*e*x + 15*e^2*x^2) + 22*a*b*e*(-16*d^3 + 24*d
^2*e*x - 30*d*e^2*x^2 + 35*e^3*x^3) + b^2*(128*d^4 - 192*d^3*e*x + 240*d^2*
e^2*x^2 - 280*d*e^3*x^3 + 315*e^4*x^4)) - 3*c^2*e*(-13*a*e*(128*d^4 - 192*d
^3*e*x + 240*d^2*e^2*x^2 - 280*d*e^3*x^3 + 315*e^4*x^4) + 5*b*(256*d^5 - 38
4*d^4*e*x + 480*d^3*e^2*x^2 - 560*d^2*e^3*x^3 + 630*d*e^4*x^4 - 693*e^5*x^5
))))/(45045*e^7)
```

Maple [A] time = 0.047, size = 495, normalized size = 1.7

$$\frac{6006\,c^3x^6e^6 + 20790\,bc^2e^6x^5 - 5544\,c^3de^5x^5 + 24570\,ac^2e^6x^4 + 24570\,b^2ce^6x^4 - 18900\,bc^2de^5x^4 + 5040\,c^3d^2e^4x^4 + 6006\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3*(e*x+d)^(1/2),x)
```

```
[Out] 2/45045*(e*x+d)^(3/2)*(3003*c^3*e^6*x^6+10395*b*c^2*e^6*x^5-2772*c^3*d*e^5*
x^5+12285*a*c^2*e^6*x^4+12285*b^2*c*e^6*x^4-9450*b*c^2*d*e^5*x^4+2520*c^3*d
^2*e^4*x^4+30030*a*b*c*e^6*x^3-10920*a*c^2*d*e^5*x^3+5005*b^3*e^6*x^3-10920
*b^2*c*d*e^5*x^3+8400*b*c^2*d^2*e^4*x^3-2240*c^3*d^3*e^3*x^3+19305*a^2*c*e^
6*x^2+19305*a*b^2*e^6*x^2-25740*a*b*c*d*e^5*x^2+9360*a*c^2*d^2*e^4*x^2-4290
*b^3*d*e^5*x^2+9360*b^2*c*d^2*e^4*x^2-7200*b*c^2*d^3*e^3*x^2+1920*c^3*d^4*e
^2*x^2+27027*a^2*b*e^6*x-15444*a^2*c*d*e^5*x-15444*a*b^2*d*e^5*x+20592*a*b*
c*d^2*e^4*x-7488*a*c^2*d^3*e^3*x+3432*b^3*d^2*e^4*x-7488*b^2*c*d^3*e^3*x+57
60*b*c^2*d^4*e^2*x-1536*c^3*d^5*e*x+15015*a^3*e^6-18018*a^2*b*d*e^5+10296*a
^2*c*d^2*e^4+10296*a*b^2*d^2*e^4-13728*a*b*c*d^3*e^3+4992*a*c^2*d^4*e^2-228
8*b^3*d^3*e^3+4992*b^2*c*d^4*e^2-3840*b*c^2*d^5*e+1024*c^3*d^6)/e^7
```

Maxima [A] time = 0.999574, size = 549, normalized size = 1.92

$$\frac{2\left(3003\,(ex+d)^{\frac{15}{2}}c^3 - 10395\left(2\,c^3d - bc^2e\right)(ex+d)^{\frac{13}{2}} + 12285\left(5\,c^3d^2 - 5\,bc^2de + \left(b^2c + ac^2\right)e^2\right)(ex+d)^{\frac{11}{2}} - 5005\left(20\,c^3\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(3003*(e*x + d)^(15/2)*c^3 - 10395*(2*c^3*d - b*c^2*e)*(e*x + d)^(1
3/2) + 12285*(5*c^3*d^2 - 5*b*c^2*d*e + (b^2*c + a*c^2)*e^2)*(e*x + d)^(11/
2) - 5005*(20*c^3*d^3 - 30*b*c^2*d^2*e + 12*(b^2*c + a*c^2)*d*e^2 - (b^3 +
6*a*b*c)*e^3)*(e*x + d)^(9/2) + 19305*(5*c^3*d^4 - 10*b*c^2*d^3*e + 6*(b^2*
c + a*c^2)*d^2*e^2 - (b^3 + 6*a*b*c)*d*e^3 + (a*b^2 + a^2*c)*e^4)*(e*x + d)
^(7/2) - 27027*(2*c^3*d^5 - 5*b*c^2*d^4*e - a^2*b*e^5 + 4*(b^2*c + a*c^2)*d
^3*e^2 - (b^3 + 6*a*b*c)*d^2*e^3 + 2*(a*b^2 + a^2*c)*d*e^4)*(e*x + d)^(5/2)
 + 15015*(c^3*d^6 - 3*b*c^2*d^5*e - 3*a^2*b*d*e^5 + a^3*e^6 + 3*(b^2*c + a*
c^2)*d^4*e^2 - (b^3 + 6*a*b*c)*d^3*e^3 + 3*(a*b^2 + a^2*c)*d^2*e^4)*(e*x +
```

```
d)^(3/2))/e^7
```

Fricas [A] time = 2.00285, size = 1170, normalized size = 4.09

$$\frac{2\left(3003\,c^3e^7x^7+1024\,c^3d^7-3840\,bc^2d^6e-18018\,a^2bd^2e^5+15015\,a^3de^6+4992\left(b^2c+ac^2\right)d^5e^2-2288\left(b^3+6\,abc\right)d^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3003*c^3*e^7*x^7 + 1024*c^3*d^7 - 3840*b*c^2*d^6*e - 18018*a^2*b*d
^2*e^5 + 15015*a^3*d*e^6 + 4992*(b^2*c + a*c^2)*d^5*e^2 - 2288*(b^3 + 6*a*b
*c)*d^4*e^3 + 10296*(a*b^2 + a^2*c)*d^3*e^4 + 231*(c^3*d*e^6 + 45*b*c^2*e^7
)*x^6 - 63*(4*c^3*d^2*e^5 - 15*b*c^2*d*e^6 - 195*(b^2*c + a*c^2)*e^7)*x^5 +
 35*(8*c^3*d^3*e^4 - 30*b*c^2*d^2*e^5 + 39*(b^2*c + a*c^2)*d*e^6 + 143*(b^3
 + 6*a*b*c)*e^7)*x^4 - 5*(64*c^3*d^4*e^3 - 240*b*c^2*d^3*e^4 + 312*(b^2*c +
 a*c^2)*d^2*e^5 - 143*(b^3 + 6*a*b*c)*d*e^6 - 3861*(a*b^2 + a^2*c)*e^7)*x^3
 + 3*(128*c^3*d^5*e^2 - 480*b*c^2*d^4*e^3 + 9009*a^2*b*e^7 + 624*(b^2*c + a
*c^2)*d^3*e^4 - 286*(b^3 + 6*a*b*c)*d^2*e^5 + 1287*(a*b^2 + a^2*c)*d*e^6)*x
^2 - (512*c^3*d^6*e - 1920*b*c^2*d^5*e^2 - 9009*a^2*b*d*e^6 - 15015*a^3*e^7
 + 2496*(b^2*c + a*c^2)*d^4*e^3 - 1144*(b^3 + 6*a*b*c)*d^3*e^4 + 5148*(a*b^
2 + a^2*c)*d^2*e^5)*x)*sqrt(e*x + d)/e^7
```

Sympy [A] time = 7.26919, size = 539, normalized size = 1.88

$$\frac{2\left(\frac{c^3(d+ex)^{\frac{15}{2}}}{15e^6}+\frac{(d+ex)^{\frac{13}{2}}\left(3bc^2e-6c^3d\right)}{13e^6}+\frac{(d+ex)^{\frac{11}{2}}\left(3ac^2e^2+3b^2ce^2-15bc^2de+15c^3d^2\right)}{11e^6}+\frac{(d+ex)^{\frac{9}{2}}\left(6abce^3-12ac^2de^2+b^3e^3-12b^2cde^2+30bc^2d^2e-20c^3d^3\right.}{9e^6}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3*(e*x+d)**(1/2),x)
```

```
[Out] 2*(c**3*(d + e*x)**(15/2)/(15*e**6) + (d + e*x)**(13/2)*(3*b*c**2*e - 6*c**
3*d)/(13*e**6) + (d + e*x)**(11/2)*(3*a*c**2*e**2 + 3*b**2*c*e**2 - 15*b*c*
*2*d*e + 15*c**3*d**2)/(11*e**6) + (d + e*x)**(9/2)*(6*a*b*c*e**3 - 12*a*c*
*2*d*e**2 + b**3*e**3 - 12*b**2*c*d*e**2 + 30*b*c**2*d**2*e - 20*c**3*d**3)
/(9*e**6) + (d + e*x)**(7/2)*(3*a**2*c*e**4 + 3*a*b**2*e**4 - 18*a*b*c*d*e*
*3 + 18*a*c**2*d**2*e**2 - 3*b**3*d*e**3 + 18*b**2*c*d**2*e**2 - 30*b*c**2*
d**3*e + 15*c**3*d**4)/(7*e**6) + (d + e*x)**(5/2)*(3*a**2*b*e**5 - 6*a**2*
c*d*e**4 - 6*a*b**2*d*e**4 + 18*a*b*c*d**2*e**3 - 12*a*c**2*d**3*e**2 + 3*b
**3*d**2*e**3 - 12*b**2*c*d**3*e**2 + 15*b*c**2*d**4*e - 6*c**3*d**5)/(5*e*
*6) + (d + e*x)**(3/2)*(a**3*e**6 - 3*a**2*b*d*e**5 + 3*a**2*c*d**2*e**4 +
3*a*b**2*d**2*e**4 - 6*a*b*c*d**3*e**3 + 3*a*c**2*d**4*e**2 - b**3*d**3*e**
3 + 3*b**2*c*d**4*e**2 - 3*b*c**2*d**5*e + c**3*d**6)/(3*e**6))/e
```

Giac [B] time = 1.12241, size = 752, normalized size = 2.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 2/45045*(9009*(3*(x*e + d)^(5/2) - 5*(x*e + d)^(3/2)*d)*a^2*b*e^(-1) + 1287
*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2)*a*b^2
*e^(-2) + 1287*(15*(x*e + d)^(7/2) - 42*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3
/2)*d^2)*a^2*c*e^(-2) + 143*(35*(x*e + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 1
89*(x*e + d)^(5/2)*d^2 - 105*(x*e + d)^(3/2)*d^3)*b^3*e^(-3) + 858*(35*(x*e
 + d)^(9/2) - 135*(x*e + d)^(7/2)*d + 189*(x*e + d)^(5/2)*d^2 - 105*(x*e +
d)^(3/2)*d^3)*a*b*c*e^(-3) + 39*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2
)*d + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^
(3/2)*d^4)*b^2*c*e^(-4) + 39*(315*(x*e + d)^(11/2) - 1540*(x*e + d)^(9/2)*d
 + 2970*(x*e + d)^(7/2)*d^2 - 2772*(x*e + d)^(5/2)*d^3 + 1155*(x*e + d)^(3/
2)*d^4)*a*c^2*e^(-4) + 15*(693*(x*e + d)^(13/2) - 4095*(x*e + d)^(11/2)*d +
 10010*(x*e + d)^(9/2)*d^2 - 12870*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/
2)*d^4 - 3003*(x*e + d)^(3/2)*d^5)*b*c^2*e^(-5) + (3003*(x*e + d)^(15/2) -
20790*(x*e + d)^(13/2)*d + 61425*(x*e + d)^(11/2)*d^2 - 100100*(x*e + d)^(9
/2)*d^3 + 96525*(x*e + d)^(7/2)*d^4 - 54054*(x*e + d)^(5/2)*d^5 + 15015*(x*
e + d)^(3/2)*d^6)*c^3*e^(-6) + 15015*(x*e + d)^(3/2)*a^3)*e^(-1)
```

3.2285 $\int \frac{(a+bx+cx^2)^3}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=282

$$\frac{2c(d+ex)^{9/2}\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{3e^7}-\frac{2(d+ex)^{7/2}(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{7e^7}+\frac{6(d+ex)^{5/2}}{}$$

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)^3*Sqrt[d + e*x])/e^7 - (2*(2*c*d - b*e)*(c*d^2 -
 b*d*e + a*e^2)^2*(d + e*x)^(3/2))/e^7 + (6*(c*d^2 - b*d*e + a*e^2)*(5*c^2*
d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^(5/2))/(5*e^7) - (2*(2*c*d - b
*e)*(10*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*(d + e*x)^(7/2))/(7*e^7)
 + (2*c*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^(9/2))/(3*e^7)
- (6*c^2*(2*c*d - b*e)*(d + e*x)^(11/2))/(11*e^7) + (2*c^3*(d + e*x)^(13/2)
)/(13*e^7)
```

Rubi [A] time = 0.127872, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {698}

$$\frac{2c(d+ex)^{9/2}\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{3e^7}-\frac{2(d+ex)^{7/2}(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{7e^7}+\frac{6(d+ex)^{5/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/Sqrt[d + e*x],x]
```


```
[Out] (2*(c*d^2 - b*d*e + a*e^2)^3*Sqrt[d + e*x])/e^7 - (2*(2*c*d - b*e)*(c*d^2 -
 b*d*e + a*e^2)^2*(d + e*x)^(3/2))/e^7 + (6*(c*d^2 - b*d*e + a*e^2)*(5*c^2*
d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^(5/2))/(5*e^7) - (2*(2*c*d - b
*e)*(10*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*(d + e*x)^(7/2))/(7*e^7)
 + (2*c*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^(9/2))/(3*e^7)
- (6*c^2*(2*c*d - b*e)*(d + e*x)^(11/2))/(11*e^7) + (2*c^3*(d + e*x)^(13/2)
)/(13*e^7)
```


Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^3}{\sqrt{d+ex}}\,dx=\int \left(\frac{\left(cd^2-bde+ae^2\right)^3}{e^6\sqrt{d+ex}}+\frac{3(-2cd+be)\left(cd^2-bde+ae^2\right)^2\sqrt{d+ex}}{e^6}+\frac{3\left(cd^2-bde+ae^2\right)\left(5c^2\right.}{}\right.$$

$$=\frac{2\left(cd^2-bde+ae^2\right)^3\sqrt{d+ex}}{e^7}-\frac{2(2cd-be)\left(cd^2-bde+ae^2\right)^2(d+ex)^{3/2}}{e^7}+\frac{6\left(cd^2-bde+ae\right.}{}$$

Mathematica [A] time = 0.559044, size = 317, normalized size = 1.12

$$\frac{2\sqrt{d+ex}(a+x(b+cx))^3}{e}-\frac{4(d+ex)^{3/2}\left(-286ce^2\left(21a^2e^2(2d-3ex)-9abe\left(8d^2-12dex+15e^2x^2\right)+b^2\left(-48d^2ex+3\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[d + e*x]*(a + x*(b + c*x))^3)/e - (4*(d + e*x)^(3/2)*(-10*c^3*(256*
d^5 - 384*d^4*e*x + 480*d^3*e^2*x^2 - 560*d^2*e^3*x^3 + 630*d*e^4*x^4 - 693
*e^5*x^5) + 429*b*e^3*(35*a^2*e^2 + 14*a*b*e*(-2*d + 3*e*x) + b^2*(8*d^2 -
12*d*e*x + 15*e^2*x^2)) - 286*c*e^2*(21*a^2*e^2*(2*d - 3*e*x) - 9*a*b*e*(8*
d^2 - 12*d*e*x + 15*e^2*x^2) + b^2*(32*d^3 - 48*d^2*e*x + 60*d*e^2*x^2 - 70
*e^3*x^3)) + 13*c^2*e*(44*a*e*(-16*d^3 + 24*d^2*e*x - 30*d*e^2*x^2 + 35*e^3
*x^3) + 5*b*(128*d^4 - 192*d^3*e*x + 240*d^2*e^2*x^2 - 280*d*e^3*x^3 + 315*
e^4*x^4))))/(15015*e^7)
```

Maple [A] time = 0.044, size = 495, normalized size = 1.8

$$\frac{2310\,c^3x^6e^6+8190\,bc^2e^6x^5-2520\,c^3de^5x^5+10010\,ac^2e^6x^4+10010\,b^2ce^6x^4-9100\,bc^2de^5x^4+2800\,c^3d^2e^4x^4+25740\,ab}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(e*x+d)^(1/2),x)
```

```
[Out] 2/15015*(e*x+d)^(1/2)*(1155*c^3*e^6*x^6+4095*b*c^2*e^6*x^5-1260*c^3*d*e^5*x
^5+5005*a*c^2*e^6*x^4+5005*b^2*c*e^6*x^4-4550*b*c^2*d*e^5*x^4+1400*c^3*d^2*
e^4*x^4+12870*a*b*c*e^6*x^3-5720*a*c^2*d*e^5*x^3+2145*b^3*e^6*x^3-5720*b^2*
c*d*e^5*x^3+5200*b*c^2*d^2*e^4*x^3-1600*c^3*d^3*e^3*x^3+9009*a^2*c*e^6*x^2+
9009*a*b^2*e^6*x^2-15444*a*b*c*d*e^5*x^2+6864*a*c^2*d^2*e^4*x^2-2574*b^3*d*
e^5*x^2+6864*b^2*c*d^2*e^4*x^2-6240*b*c^2*d^3*e^3*x^2+1920*c^3*d^4*e^2*x^2+
15015*a^2*b*e^6*x-12012*a^2*c*d*e^5*x-12012*a*b^2*d*e^5*x+20592*a*b*c*d^2*e
^4*x-9152*a*c^2*d^3*e^3*x+3432*b^3*d^2*e^4*x-9152*b^2*c*d^3*e^3*x+8320*b*c^
2*d^4*e^2*x-2560*c^3*d^5*e*x+15015*a^3*e^6-30030*a^2*b*d*e^5+24024*a^2*c*d^
2*e^4+24024*a*b^2*d^2*e^4-41184*a*b*c*d^3*e^3+18304*a*c^2*d^4*e^2-6864*b^3*
d^3*e^3+18304*b^2*c*d^4*e^2-16640*b*c^2*d^5*e+5120*c^3*d^6)/e^7
```

Maxima [B] time = 1.02919, size = 709, normalized size = 2.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15015*(15015*sqrt(e*x + d)*a^3 + 3003*a^2*(5*((e*x + d)^(3/2) - 3*sqrt(e*
x + d)*d)*b/e + (3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d
)*d^2)*c/e^2) + 143*a*(21*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sq
rt(e*x + d)*d^2)*b^2/e^2 + 18*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 3
5*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*b*c/e^3 + (35*(e*x + d)^(9/2)
 - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^
3 + 315*sqrt(e*x + d)*d^4)*c^2/e^4) + 429*(5*(e*x + d)^(7/2) - 21*(e*x + d)
^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*b^3/e^3 + 143*(35
*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e
*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*b^2*c/e^4 + 65*(63*(e*x + d)^(11
/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2
)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*b*c^2/e^5 + 5*(23
```

```
1*(e*x + d)^(13/2) - 1638*(e*x + d)^(11/2)*d + 5005*(e*x + d)^(9/2)*d^2 - 8
580*(e*x + d)^(7/2)*d^3 + 9009*(e*x + d)^(5/2)*d^4 - 6006*(e*x + d)^(3/2)*d
^5 + 3003*sqrt(e*x + d)*d^6)*c^3/e^6)/e
```

Fricas [A] time = 2.01929, size = 956, normalized size = 3.39

$$\frac{2\left(1155\,c^3e^6x^6+5120\,c^3d^6-16640\,bc^2d^5e-30030\,a^2bde^5+15015\,a^3e^6+18304\left(b^2c+ac^2\right)d^4e^2-6864\left(b^3+6\,abc\right)d^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15015*(1155*c^3*e^6*x^6 + 5120*c^3*d^6 - 16640*b*c^2*d^5*e - 30030*a^2*b*
d*e^5 + 15015*a^3*e^6 + 18304*(b^2*c + a*c^2)*d^4*e^2 - 6864*(b^3 + 6*a*b*c
)*d^3*e^3 + 24024*(a*b^2 + a^2*c)*d^2*e^4 - 315*(4*c^3*d*e^5 - 13*b*c^2*e^6
)*x^5 + 35*(40*c^3*d^2*e^4 - 130*b*c^2*d*e^5 + 143*(b^2*c + a*c^2)*e^6)*x^4
 - 5*(320*c^3*d^3*e^3 - 1040*b*c^2*d^2*e^4 + 1144*(b^2*c + a*c^2)*d*e^5 - 4
29*(b^3 + 6*a*b*c)*e^6)*x^3 + 3*(640*c^3*d^4*e^2 - 2080*b*c^2*d^3*e^3 + 228
8*(b^2*c + a*c^2)*d^2*e^4 - 858*(b^3 + 6*a*b*c)*d*e^5 + 3003*(a*b^2 + a^2*c
)*e^6)*x^2 - (2560*c^3*d^5*e - 8320*b*c^2*d^4*e^2 - 15015*a^2*b*e^6 + 9152*
(b^2*c + a*c^2)*d^3*e^3 - 3432*(b^3 + 6*a*b*c)*d^2*e^4 + 12012*(a*b^2 + a^2
*c)*d*e^5)*x)*sqrt(e*x + d)/e^7
```

Sympy [A] time = 120.098, size = 1406, normalized size = 4.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(e*x+d)**(1/2),x)
```

```
[Out] Piecewise((-(2*a**3*d/sqrt(d + e*x) + 2*a**3*(-d/sqrt(d + e*x) - sqrt(d + e
*x)) + 6*a**2*b*d*(-d/sqrt(d + e*x) - sqrt(d + e*x))/e + 6*a**2*b*(d**2/sqr
t(d + e*x) + 2*d*sqrt(d + e*x) - (d + e*x)**(3/2)/3)/e + 6*a**2*c*d*(d**2/s
qrt(d + e*x) + 2*d*sqrt(d + e*x) - (d + e*x)**(3/2)/3)/e**2 + 6*a**2*c*(-d*
*3/sqrt(d + e*x) - 3*d**2*sqrt(d + e*x) + d*(d + e*x)**(3/2) - (d + e*x)**(
5/2)/5)/e**2 + 6*a*b**2*d*(d**2/sqrt(d + e*x) + 2*d*sqrt(d + e*x) - (d + e*
x)**(3/2)/3)/e**2 + 6*a*b**2*(-d**3/sqrt(d + e*x) - 3*d**2*sqrt(d + e*x) +
d*(d + e*x)**(3/2) - (d + e*x)**(5/2)/5)/e**2 + 12*a*b*c*d*(-d**3/sqrt(d +
e*x) - 3*d**2*sqrt(d + e*x) + d*(d + e*x)**(3/2) - (d + e*x)**(5/2)/5)/e**3
 + 12*a*b*c*(d**4/sqrt(d + e*x) + 4*d**3*sqrt(d + e*x) - 2*d**2*(d + e*x)**
(3/2) + 4*d*(d + e*x)**(5/2)/5 - (d + e*x)**(7/2)/7)/e**3 + 6*a*c**2*d*(d**
4/sqrt(d + e*x) + 4*d**3*sqrt(d + e*x) - 2*d**2*(d + e*x)**(3/2) + 4*d*(d +
 e*x)**(5/2)/5 - (d + e*x)**(7/2)/7)/e**4 + 6*a*c**2*(-d**5/sqrt(d + e*x) -
 5*d**4*sqrt(d + e*x) + 10*d**3*(d + e*x)**(3/2)/3 - 2*d**2*(d + e*x)**(5/2
) + 5*d*(d + e*x)**(7/2)/7 - (d + e*x)**(9/2)/9)/e**4 + 2*b**3*d*(-d**3/sqr
t(d + e*x) - 3*d**2*sqrt(d + e*x) + d*(d + e*x)**(3/2) - (d + e*x)**(5/2)/5
)/e**3 + 2*b**3*(d**4/sqrt(d + e*x) + 4*d**3*sqrt(d + e*x) - 2*d**2*(d + e*
x)**(3/2) + 4*d*(d + e*x)**(5/2)/5 - (d + e*x)**(7/2)/7)/e**3 + 6*b**2*c*d*
(d**4/sqrt(d + e*x) + 4*d**3*sqrt(d + e*x) - 2*d**2*(d + e*x)**(3/2) + 4*d*
(d + e*x)**(5/2)/5 - (d + e*x)**(7/2)/7)/e**4 + 6*b**2*c*(-d**5/sqrt(d + e*
x) - 5*d**4*sqrt(d + e*x) + 10*d**3*(d + e*x)**(3/2)/3 - 2*d**2*(d + e*x)**
(5/2) + 5*d*(d + e*x)**(7/2)/7 - (d + e*x)**(9/2)/9)/e**4 + 6*b*c**2*d*(-d*
```

```
*5/sqrt(d + e*x) - 5*d**4*sqrt(d + e*x) + 10*d**3*(d + e*x)**(3/2)/3 - 2*d*
*2*(d + e*x)**(5/2) + 5*d*(d + e*x)**(7/2)/7 - (d + e*x)**(9/2)/9)/e**5 + 6
*b*c**2*(d**6/sqrt(d + e*x) + 6*d**5*sqrt(d + e*x) - 5*d**4*(d + e*x)**(3/2
) + 4*d**3*(d + e*x)**(5/2) - 15*d**2*(d + e*x)**(7/2)/7 + 2*d*(d + e*x)**(
9/2)/3 - (d + e*x)**(11/2)/11)/e**5 + 2*c**3*d*(d**6/sqrt(d + e*x) + 6*d**5
*sqrt(d + e*x) - 5*d**4*(d + e*x)**(3/2) + 4*d**3*(d + e*x)**(5/2) - 15*d**
2*(d + e*x)**(7/2)/7 + 2*d*(d + e*x)**(9/2)/3 - (d + e*x)**(11/2)/11)/e**6
+ 2*c**3*(-d**7/sqrt(d + e*x) - 7*d**6*sqrt(d + e*x) + 7*d**5*(d + e*x)**(3
/2) - 7*d**4*(d + e*x)**(5/2) + 5*d**3*(d + e*x)**(7/2) - 7*d**2*(d + e*x)*
*(9/2)/3 + 7*d*(d + e*x)**(11/2)/11 - (d + e*x)**(13/2)/13)/e**6)/e, Ne(e,
0)), ((a**3*x + 3*a**2*b*x**2/2 + b*c**2*x**6/2 + c**3*x**7/7 + x**5*(3*a*c
**2 + 3*b**2*c)/5 + x**4*(6*a*b*c + b**3)/4 + x**3*(3*a**2*c + 3*a*b**2)/3)
/sqrt(d), True))
```

Giac [B] time = 1.12624, size = 751, normalized size = 2.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15015*(15015*((x*e + d)^(3/2) - 3*sqrt(x*e + d)*d)*a^2*b*e^(-1) + 3003*(3
*(x*e + d)^(5/2) - 10*(x*e + d)^(3/2)*d + 15*sqrt(x*e + d)*d^2)*a*b^2*e^(-2
) + 3003*(3*(x*e + d)^(5/2) - 10*(x*e + d)^(3/2)*d + 15*sqrt(x*e + d)*d^2)*
a^2*c*e^(-2) + 429*(5*(x*e + d)^(7/2) - 21*(x*e + d)^(5/2)*d + 35*(x*e + d)
^(3/2)*d^2 - 35*sqrt(x*e + d)*d^3)*b^3*e^(-3) + 2574*(5*(x*e + d)^(7/2) - 2
1*(x*e + d)^(5/2)*d + 35*(x*e + d)^(3/2)*d^2 - 35*sqrt(x*e + d)*d^3)*a*b*c*
e^(-3) + 143*(35*(x*e + d)^(9/2) - 180*(x*e + d)^(7/2)*d + 378*(x*e + d)^(5
/2)*d^2 - 420*(x*e + d)^(3/2)*d^3 + 315*sqrt(x*e + d)*d^4)*b^2*c*e^(-4) + 1
43*(35*(x*e + d)^(9/2) - 180*(x*e + d)^(7/2)*d + 378*(x*e + d)^(5/2)*d^2 -
420*(x*e + d)^(3/2)*d^3 + 315*sqrt(x*e + d)*d^4)*a*c^2*e^(-4) + 65*(63*(x*e
 + d)^(11/2) - 385*(x*e + d)^(9/2)*d + 990*(x*e + d)^(7/2)*d^2 - 1386*(x*e
+ d)^(5/2)*d^3 + 1155*(x*e + d)^(3/2)*d^4 - 693*sqrt(x*e + d)*d^5)*b*c^2*e^
(-5) + 5*(231*(x*e + d)^(13/2) - 1638*(x*e + d)^(11/2)*d + 5005*(x*e + d)^(
9/2)*d^2 - 8580*(x*e + d)^(7/2)*d^3 + 9009*(x*e + d)^(5/2)*d^4 - 6006*(x*e
+ d)^(3/2)*d^5 + 3003*sqrt(x*e + d)*d^6)*c^3*e^(-6) + 15015*sqrt(x*e + d)*a
^3)*e^(-1)
```

3.2286 $\int \frac{(a+bx+cx^2)^3}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=280

$$\frac{6c(d+ex)^{7/2}\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{7e^7}-\frac{2(d+ex)^{5/2}(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{5e^7}+\frac{2(d+ex)^{3/2}}{}$$

```
[Out] (-2*(c*d^2 - b*d*e + a*e^2)^3)/(e^7*Sqrt[d + e*x]) - (6*(2*c*d - b*e)*(c*d^
2 - b*d*e + a*e^2)^2*Sqrt[d + e*x])/e^7 + (2*(c*d^2 - b*d*e + a*e^2)*(5*c^2
*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^(3/2))/e^7 - (2*(2*c*d - b*e)
*(10*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*(d + e*x)^(5/2))/(5*e^7) +
(6*c*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^(7/2))/(7*e^7) - (
2*c^2*(2*c*d - b*e)*(d + e*x)^(9/2))/(3*e^7) + (2*c^3*(d + e*x)^(11/2))/(11
*e^7)
```

Rubi [A] time = 0.127591, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {698}

$$\frac{6c(d+ex)^{7/2}\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{7e^7}-\frac{2(d+ex)^{5/2}(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{5e^7}+\frac{2(d+ex)^{3/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(d + e*x)^(3/2),x]
```


```
[Out] (-2*(c*d^2 - b*d*e + a*e^2)^3)/(e^7*Sqrt[d + e*x]) - (6*(2*c*d - b*e)*(c*d^
2 - b*d*e + a*e^2)^2*Sqrt[d + e*x])/e^7 + (2*(c*d^2 - b*d*e + a*e^2)*(5*c^2
*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^(3/2))/e^7 - (2*(2*c*d - b*e)
*(10*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*(d + e*x)^(5/2))/(5*e^7) +
(6*c*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^(7/2))/(7*e^7) - (
2*c^2*(2*c*d - b*e)*(d + e*x)^(9/2))/(3*e^7) + (2*c^3*(d + e*x)^(11/2))/(11
*e^7)
```


Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^3}{(d+ex)^{3/2}}\,dx=\int \left(\frac{(cd^2-bde+ae^2)^3}{e^6(d+ex)^{3/2}}+\frac{3(-2cd+be)(cd^2-bde+ae^2)^2}{e^6\sqrt{d+ex}}+\frac{3(cd^2-bde+ae^2)(5c^2d^2-5bc}{e^6}\right.$$

$$=-\frac{2(cd^2-bde+ae^2)^3}{e^7\sqrt{d+ex}}-\frac{6(2cd-be)(cd^2-bde+ae^2)^2\sqrt{d+ex}}{e^7}+\frac{2(cd^2-bde+ae^2)(5c^2d^2}{}$$

Mathematica [A] time = 0.411908, size = 394, normalized size = 1.41

$$-66ce^2\left(35a^2e^2\left(8d^2+4dex-e^2x^2\right)-42abe\left(8d^2ex+16d^3-2de^2x^2+e^3x^3\right)+3b^2\left(-16d^2e^2x^2+64d^3ex+128d^4+8de^3x\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(d + e*x)^(3/2),x]
```

```
[Out] (-10*c^3*(1024*d^6 + 512*d^5*e*x - 128*d^4*e^2*x^2 + 64*d^3*e^3*x^3 - 40*d^
2*e^4*x^4 + 28*d*e^5*x^5 - 21*e^6*x^6) + 462*e^3*(-5*a^3*e^3 + 15*a^2*b*e^2
*(2*d + e*x) + 5*a*b^2*e*(-8*d^2 - 4*d*e*x + e^2*x^2) + b^3*(16*d^3 + 8*d^2
*e*x - 2*d*e^2*x^2 + e^3*x^3)) - 66*c*e^2*(35*a^2*e^2*(8*d^2 + 4*d*e*x - e^
2*x^2) - 42*a*b*e*(16*d^3 + 8*d^2*e*x - 2*d*e^2*x^2 + e^3*x^3) + 3*b^2*(128
*d^4 + 64*d^3*e*x - 16*d^2*e^2*x^2 + 8*d*e^3*x^3 - 5*e^4*x^4)) + 22*c^2*e*(
9*a*e*(-128*d^4 - 64*d^3*e*x + 16*d^2*e^2*x^2 - 8*d*e^3*x^3 + 5*e^4*x^4) +
5*b*(256*d^5 + 128*d^4*e*x - 32*d^3*e^2*x^2 + 16*d^2*e^3*x^3 - 10*d*e^4*x^4
 + 7*e^5*x^5)))/(1155*e^7*Sqrt[d + e*x])
```

Maple [A] time = 0.045, size = 495, normalized size = 1.8

$$-\frac{-210\,c^3x^6e^6-770\,bc^2e^6x^5+280\,c^3de^5x^5-990\,ac^2e^6x^4-990\,b^2ce^6x^4+1100\,bc^2de^5x^4-400\,c^3d^2e^4x^4-2772\,abce^6x^3+\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(e*x+d)^(3/2),x)
```

```
[Out] -2/1155/(e*x+d)^(1/2)*(-105*c^3*e^6*x^6-385*b*c^2*e^6*x^5+140*c^3*d*e^5*x^5
-495*a*c^2*e^6*x^4-495*b^2*c*e^6*x^4+550*b*c^2*d*e^5*x^4-200*c^3*d^2*e^4*x^
4-1386*a*b*c*e^6*x^3+792*a*c^2*d*e^5*x^3-231*b^3*e^6*x^3+792*b^2*c*d*e^5*x^
3-880*b*c^2*d^2*e^4*x^3+320*c^3*d^3*e^3*x^3-1155*a^2*c*e^6*x^2-1155*a*b^2*e
^6*x^2+2772*a*b*c*d*e^5*x^2-1584*a*c^2*d^2*e^4*x^2+462*b^3*d*e^5*x^2-1584*b
^2*c*d^2*e^4*x^2+1760*b*c^2*d^3*e^3*x^2-640*c^3*d^4*e^2*x^2-3465*a^2*b*e^6*
x+4620*a^2*c*d*e^5*x+4620*a*b^2*d*e^5*x-11088*a*b*c*d^2*e^4*x+6336*a*c^2*d^
3*e^3*x-1848*b^3*d^2*e^4*x+6336*b^2*c*d^3*e^3*x-7040*b*c^2*d^4*e^2*x+2560*c
^3*d^5*e*x+1155*a^3*e^6-6930*a^2*b*d*e^5+9240*a^2*c*d^2*e^4+9240*a*b^2*d^2*
e^4-22176*a*b*c*d^3*e^3+12672*a*c^2*d^4*e^2-3696*b^3*d^3*e^3+12672*b^2*c*d^
4*e^2-14080*b*c^2*d^5*e+5120*c^3*d^6)/e^7
```

Maxima [A] time = 1.04633, size = 560, normalized size = 2.

$$2\left(\frac{105\,(ex+d)^{\frac{11}{2}}c^3-385\left(2\,c^3d-bc^2e\right)(ex+d)^{\frac{9}{2}}+495\left(5\,c^3d^2-5\,bc^2de+\left(b^2c+ac^2\right)e^2\right)(ex+d)^{\frac{7}{2}}-231\left(20\,c^3d^3-30\,bc^2d^2e+12\left(b^2c+ac^2\right)de^2-\left(b^3+6\,abc\right)e^3\right)(ex+d)^{\frac{5}{2}}+115\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/1155*((105*(e*x + d)^(11/2)*c^3 - 385*(2*c^3*d - b*c^2*e)*(e*x + d)^(9/2)
 + 495*(5*c^3*d^2 - 5*b*c^2*d*e + (b^2*c + a*c^2)*e^2)*(e*x + d)^(7/2) - 23
1*(20*c^3*d^3 - 30*b*c^2*d^2*e + 12*(b^2*c + a*c^2)*d*e^2 - (b^3 + 6*a*b*c)
```

```
*e^3)*(e*x + d)^(5/2) + 1155*(5*c^3*d^4 - 10*b*c^2*d^3*e + 6*(b^2*c + a*c^2
)*d^2*e^2 - (b^3 + 6*a*b*c)*d*e^3 + (a*b^2 + a^2*c)*e^4)*(e*x + d)^(3/2) -
3465*(2*c^3*d^5 - 5*b*c^2*d^4*e - a^2*b*e^5 + 4*(b^2*c + a*c^2)*d^3*e^2 - (
b^3 + 6*a*b*c)*d^2*e^3 + 2*(a*b^2 + a^2*c)*d*e^4)*sqrt(e*x + d))/e^6 - 1155
*(c^3*d^6 - 3*b*c^2*d^5*e - 3*a^2*b*d*e^5 + a^3*e^6 + 3*(b^2*c + a*c^2)*d^4
*e^2 - (b^3 + 6*a*b*c)*d^3*e^3 + 3*(a*b^2 + a^2*c)*d^2*e^4)/(sqrt(e*x + d)*
e^6))/e
```

Fricas [A] time = 2.01709, size = 950, normalized size = 3.39

$$\frac{2\left(105\,c^3e^6x^6 - 5120\,c^3d^6 + 14080\,bc^2d^5e + 6930\,a^2bde^5 - 1155\,a^3e^6 - 12672\left(b^2c + ac^2\right)d^4e^2 + 3696\left(b^3 + 6\,abc\right)d^3e^3 \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*c^3*e^6*x^6 - 5120*c^3*d^6 + 14080*b*c^2*d^5*e + 6930*a^2*b*d*e
^5 - 1155*a^3*e^6 - 12672*(b^2*c + a*c^2)*d^4*e^2 + 3696*(b^3 + 6*a*b*c)*d^
3*e^3 - 9240*(a*b^2 + a^2*c)*d^2*e^4 - 35*(4*c^3*d*e^5 - 11*b*c^2*e^6)*x^5
+ 5*(40*c^3*d^2*e^4 - 110*b*c^2*d*e^5 + 99*(b^2*c + a*c^2)*e^6)*x^4 - (320*
c^3*d^3*e^3 - 880*b*c^2*d^2*e^4 + 792*(b^2*c + a*c^2)*d*e^5 - 231*(b^3 + 6*
a*b*c)*e^6)*x^3 + (640*c^3*d^4*e^2 - 1760*b*c^2*d^3*e^3 + 1584*(b^2*c + a*c
^2)*d^2*e^4 - 462*(b^3 + 6*a*b*c)*d*e^5 + 1155*(a*b^2 + a^2*c)*e^6)*x^2 - (
2560*c^3*d^5*e - 7040*b*c^2*d^4*e^2 - 3465*a^2*b*e^6 + 6336*(b^2*c + a*c^2)
*d^3*e^3 - 1848*(b^3 + 6*a*b*c)*d^2*e^4 + 4620*(a*b^2 + a^2*c)*d*e^5)*x)*sq
rt(e*x + d)/(e^8*x + d*e^7)
```

Sympy [A] time = 99.2054, size = 428, normalized size = 1.53

$$\frac{2c^3(d+ex)^{\frac{11}{2}}}{11e^7} + \frac{(d+ex)^{\frac{9}{2}}\left(6bc^2e - 12c^3d\right)}{9e^7} + \frac{(d+ex)^{\frac{7}{2}}\left(6ac^2e^2 + 6b^2ce^2 - 30bc^2de + 30c^3d^2\right)}{7e^7} + \frac{(d+ex)^{\frac{5}{2}}\left(12abce^3 - \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(e*x+d)**(3/2),x)
```

```
[Out] 2*c**3*(d + e*x)**(11/2)/(11*e**7) + (d + e*x)**(9/2)*(6*b*c**2*e - 12*c**3
*d)/(9*e**7) + (d + e*x)**(7/2)*(6*a*c**2*e**2 + 6*b**2*c*e**2 - 30*b*c**2*
d*e + 30*c**3*d**2)/(7*e**7) + (d + e*x)**(5/2)*(12*a*b*c*e**3 - 24*a*c**2*
d*e**2 + 2*b**3*e**3 - 24*b**2*c*d*e**2 + 60*b*c**2*d**2*e - 40*c**3*d**3)/
(5*e**7) + (d + e*x)**(3/2)*(6*a**2*c*e**4 + 6*a*b**2*e**4 - 36*a*b*c*d*e**
3 + 36*a*c**2*d**2*e**2 - 6*b**3*d*e**3 + 36*b**2*c*d**2*e**2 - 60*b*c**2*d
**3*e + 30*c**3*d**4)/(3*e**7) + sqrt(d + e*x)*(6*a**2*b*e**5 - 12*a**2*c*d
*e**4 - 12*a*b**2*d*e**4 + 36*a*b*c*d**2*e**3 - 24*a*c**2*d**3*e**2 + 6*b**
3*d**2*e**3 - 24*b**2*c*d**3*e**2 + 30*b*c**2*d**4*e - 12*c**3*d**5)/e**7 -
 2*(a*e**2 - b*d*e + c*d**2)**3/(e**7*sqrt(d + e*x))
```

Giac [B] time = 1.1384, size = 852, normalized size = 3.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] 2/1155*(105*(x*e + d)^(11/2)*c^3*e^70 - 770*(x*e + d)^(9/2)*c^3*d*e^70 + 24
75*(x*e + d)^(7/2)*c^3*d^2*e^70 - 4620*(x*e + d)^(5/2)*c^3*d^3*e^70 + 5775*
(x*e + d)^(3/2)*c^3*d^4*e^70 - 6930*sqrt(x*e + d)*c^3*d^5*e^70 + 385*(x*e +
 d)^(9/2)*b*c^2*e^71 - 2475*(x*e + d)^(7/2)*b*c^2*d*e^71 + 6930*(x*e + d)^(
5/2)*b*c^2*d^2*e^71 - 11550*(x*e + d)^(3/2)*b*c^2*d^3*e^71 + 17325*sqrt(x*e
 + d)*b*c^2*d^4*e^71 + 495*(x*e + d)^(7/2)*b^2*c*e^72 + 495*(x*e + d)^(7/2)
*a*c^2*e^72 - 2772*(x*e + d)^(5/2)*b^2*c*d*e^72 - 2772*(x*e + d)^(5/2)*a*c^
2*d*e^72 + 6930*(x*e + d)^(3/2)*b^2*c*d^2*e^72 + 6930*(x*e + d)^(3/2)*a*c^2
*d^2*e^72 - 13860*sqrt(x*e + d)*b^2*c*d^3*e^72 - 13860*sqrt(x*e + d)*a*c^2*
d^3*e^72 + 231*(x*e + d)^(5/2)*b^3*e^73 + 1386*(x*e + d)^(5/2)*a*b*c*e^73 -
 1155*(x*e + d)^(3/2)*b^3*d*e^73 - 6930*(x*e + d)^(3/2)*a*b*c*d*e^73 + 3465
*sqrt(x*e + d)*b^3*d^2*e^73 + 20790*sqrt(x*e + d)*a*b*c*d^2*e^73 + 1155*(x*
e + d)^(3/2)*a*b^2*e^74 + 1155*(x*e + d)^(3/2)*a^2*c*e^74 - 6930*sqrt(x*e +
 d)*a*b^2*d*e^74 - 6930*sqrt(x*e + d)*a^2*c*d*e^74 + 3465*sqrt(x*e + d)*a^2
*b*e^75)*e^(-77) - 2*(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 + 3*a*c^2*d
^4*e^2 - b^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a^2*c*d^2*e^4
- 3*a^2*b*d*e^5 + a^3*e^6)*e^(-7)/sqrt(x*e + d)
```

3.2287 $\int \frac{(a+bx+cx^2)^3}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=282

$$\frac{6c(d+ex)^{5/2}\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{5e^7}-\frac{2(d+ex)^{3/2}(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{3e^7}+\frac{6\sqrt{d+ex}\,(}{}$$

```
[Out] (-2*(c*d^2 - b*d*e + a*e^2)^3)/(3*e^7*(d + e*x)^(3/2)) + (6*(2*c*d - b*e)*(
c*d^2 - b*d*e + a*e^2)^2)/(e^7*Sqrt[d + e*x]) + (6*(c*d^2 - b*d*e + a*e^2)*
(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*Sqrt[d + e*x])/e^7 - (2*(2*c*d -
b*e)*(10*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*(d + e*x)^(3/2))/(3*e^7
) + (6*c*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^(5/2))/(5*e^7)
 - (6*c^2*(2*c*d - b*e)*(d + e*x)^(7/2))/(7*e^7) + (2*c^3*(d + e*x)^(9/2))/
(9*e^7)
```

Rubi [A] time = 0.131565, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {698}

$$\frac{6c(d+ex)^{5/2}\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{5e^7}-\frac{2(d+ex)^{3/2}(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{3e^7}+\frac{6\sqrt{d+ex}\,(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(d + e*x)^(5/2),x]
```

```
[Out] (-2*(c*d^2 - b*d*e + a*e^2)^3)/(3*e^7*(d + e*x)^(3/2)) + (6*(2*c*d - b*e)*(
c*d^2 - b*d*e + a*e^2)^2)/(e^7*Sqrt[d + e*x]) + (6*(c*d^2 - b*d*e + a*e^2)*
(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*Sqrt[d + e*x])/e^7 - (2*(2*c*d -
b*e)*(10*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*(d + e*x)^(3/2))/(3*e^7
) + (6*c*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^(5/2))/(5*e^7)
 - (6*c^2*(2*c*d - b*e)*(d + e*x)^(7/2))/(7*e^7) + (2*c^3*(d + e*x)^(9/2))/
(9*e^7)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^3}{(d+ex)^{5/2}}\,dx=\int \left(\frac{(cd^2-bde+ae^2)^3}{e^6(d+ex)^{5/2}}+\frac{3(-2cd+be)(cd^2-bde+ae^2)^2}{e^6(d+ex)^{3/2}}+\frac{3(cd^2-bde+ae^2)(5c^2d^2-5bc}{e^6\sqrt{d+ex}}\right.$$

$$=-\frac{2(cd^2-bde+ae^2)^3}{3e^7(d+ex)^{3/2}}+\frac{6(2cd-be)(cd^2-bde+ae^2)^2}{e^7\sqrt{d+ex}}+\frac{6(cd^2-bde+ae^2)(5c^2d^2+b^2e^2-}{e^7}$$

Mathematica [A] time = 0.381602, size = 395, normalized size = 1.4

$$\frac{2\left(63ce^2\left(5a^2e^2\left(8d^2+12dex+3e^2x^2\right)+10abe\left(-24d^2ex-16d^3-6de^2x^2+e^3x^3\right)+b^2\left(48d^2e^2x^2+192d^3ex+128d^4-8d\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(d + e*x)^(5/2),x]
```

```
[Out] (2*(5*c^3*(1024*d^6 + 1536*d^5*e*x + 384*d^4*e^2*x^2 - 64*d^3*e^3*x^3 + 24*
d^2*e^4*x^4 - 12*d*e^5*x^5 + 7*e^6*x^6) - 105*e^3*(a^3*e^3 + 3*a^2*b*e^2*(2
*d + 3*e*x) - 3*a*b^2*e*(8*d^2 + 12*d*e*x + 3*e^2*x^2) + b^3*(16*d^3 + 24*d
^2*e*x + 6*d*e^2*x^2 - e^3*x^3)) + 63*c*e^2*(5*a^2*e^2*(8*d^2 + 12*d*e*x +
3*e^2*x^2) + 10*a*b*e*(-16*d^3 - 24*d^2*e*x - 6*d*e^2*x^2 + e^3*x^3) + b^2*
(128*d^4 + 192*d^3*e*x + 48*d^2*e^2*x^2 - 8*d*e^3*x^3 + 3*e^4*x^4)) - 9*c^2
*e*(-7*a*e*(128*d^4 + 192*d^3*e*x + 48*d^2*e^2*x^2 - 8*d*e^3*x^3 + 3*e^4*x^
4) + 5*b*(256*d^5 + 384*d^4*e*x + 96*d^3*e^2*x^2 - 16*d^2*e^3*x^3 + 6*d*e^4
*x^4 - 3*e^5*x^5))))/(315*e^7*(d + e*x)^(3/2))
```

Maple [A] time = 0.044, size = 495, normalized size = 1.8

$$-\frac{-70\,c^3x^6e^6-270\,bc^2e^6x^5+120\,c^3de^5x^5-378\,ac^2e^6x^4-378\,b^2ce^6x^4+540\,bc^2de^5x^4-240\,c^3d^2e^4x^4-1260\,abce^6x^3+10}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(e*x+d)^(5/2),x)
```

```
[Out] -2/315/(e*x+d)^(3/2)*(-35*c^3*e^6*x^6-135*b*c^2*e^6*x^5+60*c^3*d*e^5*x^5-18
9*a*c^2*e^6*x^4-189*b^2*c*e^6*x^4+270*b*c^2*d*e^5*x^4-120*c^3*d^2*e^4*x^4-6
30*a*b*c*e^6*x^3+504*a*c^2*d*e^5*x^3-105*b^3*e^6*x^3+504*b^2*c*d*e^5*x^3-72
0*b*c^2*d^2*e^4*x^3+320*c^3*d^3*e^3*x^3-945*a^2*c*e^6*x^2-945*a*b^2*e^6*x^2
+3780*a*b*c*d*e^5*x^2-3024*a*c^2*d^2*e^4*x^2+630*b^3*d*e^5*x^2-3024*b^2*c*d
^2*e^4*x^2+4320*b*c^2*d^3*e^3*x^2-1920*c^3*d^4*e^2*x^2+945*a^2*b*e^6*x-3780
*a^2*c*d*e^5*x-3780*a*b^2*d*e^5*x+15120*a*b*c*d^2*e^4*x-12096*a*c^2*d^3*e^3
*x+2520*b^3*d^2*e^4*x-12096*b^2*c*d^3*e^3*x+17280*b*c^2*d^4*e^2*x-7680*c^3*
d^5*e*x+105*a^3*e^6+630*a^2*b*d*e^5-2520*a^2*c*d^2*e^4-2520*a*b^2*d^2*e^4+1
0080*a*b*c*d^3*e^3-8064*a*c^2*d^4*e^2+1680*b^3*d^3*e^3-8064*b^2*c*d^4*e^2+1
1520*b*c^2*d^5*e-5120*c^3*d^6)/e^7
```

Maxima [A] time = 1.08639, size = 558, normalized size = 1.98

$$\frac{2\left(\frac{35\,(ex+d)^{\frac{9}{2}}c^3-135\left(2\,c^3d-bc^2e\right)(ex+d)^{\frac{7}{2}}+189\left(5\,c^3d^2-5\,bc^2de+\left(b^2c+ac^2\right)e^2\right)(ex+d)^{\frac{5}{2}}-105\left(20\,c^3d^3-30\,bc^2d^2e+12\left(b^2c+ac^2\right)de^2-\left(b^3+6\,abc\right)e^3\right)(ex+d)^{\frac{3}{2}}+945\,(5}{e^6}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/315*((35*(e*x + d)^(9/2)*c^3 - 135*(2*c^3*d - b*c^2*e)*(e*x + d)^(7/2) +
189*(5*c^3*d^2 - 5*b*c^2*d*e + (b^2*c + a*c^2)*e^2)*(e*x + d)^(5/2) - 105*(
20*c^3*d^3 - 30*b*c^2*d^2*e + 12*(b^2*c + a*c^2)*d*e^2 - (b^3 + 6*a*b*c)*e^
```

```
3)*(e*x + d)^(3/2) + 945*(5*c^3*d^4 - 10*b*c^2*d^3*e + 6*(b^2*c + a*c^2)*d^
2*e^2 - (b^3 + 6*a*b*c)*d*e^3 + (a*b^2 + a^2*c)*e^4)*sqrt(e*x + d))/e^6 - 1
05*(c^3*d^6 - 3*b*c^2*d^5*e - 3*a^2*b*d*e^5 + a^3*e^6 + 3*(b^2*c + a*c^2)*d
^4*e^2 - (b^3 + 6*a*b*c)*d^3*e^3 + 3*(a*b^2 + a^2*c)*d^2*e^4 - 9*(2*c^3*d^5
 - 5*b*c^2*d^4*e - a^2*b*e^5 + 4*(b^2*c + a*c^2)*d^3*e^2 - (b^3 + 6*a*b*c)*
d^2*e^3 + 2*(a*b^2 + a^2*c)*d*e^4)*(e*x + d))/((e*x + d)^(3/2)*e^6))/e
```

Fricas [A] time = 2.09166, size = 964, normalized size = 3.42

$$\frac{2\left(35\,c^3e^6x^6+5120\,c^3d^6-11520\,bc^2d^5e-630\,a^2bde^5-105\,a^3e^6+8064\left(b^2c+ac^2\right)d^4e^2-1680\left(b^3+6\,abc\right)d^3e^3+252\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*c^3*e^6*x^6 + 5120*c^3*d^6 - 11520*b*c^2*d^5*e - 630*a^2*b*d*e^5
- 105*a^3*e^6 + 8064*(b^2*c + a*c^2)*d^4*e^2 - 1680*(b^3 + 6*a*b*c)*d^3*e^3
 + 2520*(a*b^2 + a^2*c)*d^2*e^4 - 15*(4*c^3*d*e^5 - 9*b*c^2*e^6)*x^5 + 3*(4
0*c^3*d^2*e^4 - 90*b*c^2*d*e^5 + 63*(b^2*c + a*c^2)*e^6)*x^4 - (320*c^3*d^3
*e^3 - 720*b*c^2*d^2*e^4 + 504*(b^2*c + a*c^2)*d*e^5 - 105*(b^3 + 6*a*b*c)*
e^6)*x^3 + 3*(640*c^3*d^4*e^2 - 1440*b*c^2*d^3*e^3 + 1008*(b^2*c + a*c^2)*d
^2*e^4 - 210*(b^3 + 6*a*b*c)*d*e^5 + 315*(a*b^2 + a^2*c)*e^6)*x^2 + 3*(2560
*c^3*d^5*e - 5760*b*c^2*d^4*e^2 - 315*a^2*b*e^6 + 4032*(b^2*c + a*c^2)*d^3*
e^3 - 840*(b^3 + 6*a*b*c)*d^2*e^4 + 1260*(a*b^2 + a^2*c)*d*e^5)*x)*sqrt(e*x
 + d)/(e^9*x^2 + 2*d*e^8*x + d^2*e^7)
```

Sympy [A] time = 117.06, size = 348, normalized size = 1.23

$$\frac{2c^3\left(d+ex\right)^{\frac{9}{2}}}{9e^7}+\frac{\left(d+ex\right)^{\frac{7}{2}}\left(6bc^2e-12c^3d\right)}{7e^7}+\frac{\left(d+ex\right)^{\frac{5}{2}}\left(6ac^2e^2+6b^2ce^2-30bc^2de+30c^3d^2\right)}{5e^7}+\frac{\left(d+ex\right)^{\frac{3}{2}}\left(12abce^3-2\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(e*x+d)**(5/2),x)
```

```
[Out] 2*c**3*(d + e*x)**(9/2)/(9*e**7) + (d + e*x)**(7/2)*(6*b*c**2*e - 12*c**3*d
)/(7*e**7) + (d + e*x)**(5/2)*(6*a*c**2*e**2 + 6*b**2*c*e**2 - 30*b*c**2*d*
e + 30*c**3*d**2)/(5*e**7) + (d + e*x)**(3/2)*(12*a*b*c*e**3 - 24*a*c**2*d*
e**2 + 2*b**3*e**3 - 24*b**2*c*d*e**2 + 60*b*c**2*d**2*e - 40*c**3*d**3)/(3
*e**7) + sqrt(d + e*x)*(6*a**2*c*e**4 + 6*a*b**2*e**4 - 36*a*b*c*d*e**3 + 3
6*a*c**2*d**2*e**2 - 6*b**3*d*e**3 + 36*b**2*c*d**2*e**2 - 60*b*c**2*d**3*e
 + 30*c**3*d**4)/e**7 - 6*(b*e - 2*c*d)*(a*e**2 - b*d*e + c*d**2)**2/(e**7*
sqrt(d + e*x)) - 2*(a*e**2 - b*d*e + c*d**2)**3/(3*e**7*(d + e*x)**(3/2))
```

Giac [B] time = 1.14024, size = 826, normalized size = 2.93

$$\frac{2}{315}\left(35\left(xe+d\right)^{\frac{9}{2}}c^3e^{56}-270\left(xe+d\right)^{\frac{7}{2}}c^3de^{56}+945\left(xe+d\right)^{\frac{5}{2}}c^3d^2e^{56}-2100\left(xe+d\right)^{\frac{3}{2}}c^3d^3e^{56}+4725\sqrt{xe+d}c^3d^4e^{56}+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] 2/315*(35*(x*e + d)^(9/2)*c^3*e^56 - 270*(x*e + d)^(7/2)*c^3*d*e^56 + 945*(
x*e + d)^(5/2)*c^3*d^2*e^56 - 2100*(x*e + d)^(3/2)*c^3*d^3*e^56 + 4725*sqrt
(x*e + d)*c^3*d^4*e^56 + 135*(x*e + d)^(7/2)*b*c^2*e^57 - 945*(x*e + d)^(5/
2)*b*c^2*d*e^57 + 3150*(x*e + d)^(3/2)*b*c^2*d^2*e^57 - 9450*sqrt(x*e + d)*
b*c^2*d^3*e^57 + 189*(x*e + d)^(5/2)*b^2*c*e^58 + 189*(x*e + d)^(5/2)*a*c^2
*e^58 - 1260*(x*e + d)^(3/2)*b^2*c*d*e^58 - 1260*(x*e + d)^(3/2)*a*c^2*d*e^
58 + 5670*sqrt(x*e + d)*b^2*c*d^2*e^58 + 5670*sqrt(x*e + d)*a*c^2*d^2*e^58
+ 105*(x*e + d)^(3/2)*b^3*e^59 + 630*(x*e + d)^(3/2)*a*b*c*e^59 - 945*sqrt(
x*e + d)*b^3*d*e^59 - 5670*sqrt(x*e + d)*a*b*c*d*e^59 + 945*sqrt(x*e + d)*a
*b^2*e^60 + 945*sqrt(x*e + d)*a^2*c*e^60)*e^(-63) + 2/3*(18*(x*e + d)*c^3*d
^5 - c^3*d^6 - 45*(x*e + d)*b*c^2*d^4*e + 3*b*c^2*d^5*e + 36*(x*e + d)*b^2*
c*d^3*e^2 + 36*(x*e + d)*a*c^2*d^3*e^2 - 3*b^2*c*d^4*e^2 - 3*a*c^2*d^4*e^2
- 9*(x*e + d)*b^3*d^2*e^3 - 54*(x*e + d)*a*b*c*d^2*e^3 + b^3*d^3*e^3 + 6*a*
b*c*d^3*e^3 + 18*(x*e + d)*a*b^2*d*e^4 + 18*(x*e + d)*a^2*c*d*e^4 - 3*a*b^2
*d^2*e^4 - 3*a^2*c*d^2*e^4 - 9*(x*e + d)*a^2*b*e^5 + 3*a^2*b*d*e^5 - a^3*e^
6)*e^(-7)/(x*e + d)^(3/2)
```

3.2288 $\int \frac{(a+bx+cx^2)^3}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=278

$$\frac{2c(d+ex)^{3/2}\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{e^7}-\frac{2\sqrt{d+ex}(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{e^7}-\frac{6\left(ae^2-bde+\right.}{}$$

```
[Out] (-2*(c*d^2 - b*d*e + a*e^2)^3)/(5*e^7*(d + e*x)^(5/2)) + (2*(2*c*d - b*e)*(
c*d^2 - b*d*e + a*e^2)^2)/(e^7*(d + e*x)^(3/2)) - (6*(c*d^2 - b*d*e + a*e^2
)*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e)))/(e^7*Sqrt[d + e*x]) - (2*(2*c*
d - b*e)*(10*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*Sqrt[d + e*x])/e^7
+ (2*c*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^(3/2))/e^7 - (6*
c^2*(2*c*d - b*e)*(d + e*x)^(5/2))/(5*e^7) + (2*c^3*(d + e*x)^(7/2))/(7*e^7
)
```

Rubi [A] time = 0.134166, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {698}

$$\frac{2c(d+ex)^{3/2}\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{e^7}-\frac{2\sqrt{d+ex}(2cd-be)\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)}{e^7}-\frac{6\left(ae^2-bde+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(d + e*x)^(7/2),x]
```


```
[Out] (-2*(c*d^2 - b*d*e + a*e^2)^3)/(5*e^7*(d + e*x)^(5/2)) + (2*(2*c*d - b*e)*(
c*d^2 - b*d*e + a*e^2)^2)/(e^7*(d + e*x)^(3/2)) - (6*(c*d^2 - b*d*e + a*e^2
)*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e)))/(e^7*Sqrt[d + e*x]) - (2*(2*c*
d - b*e)*(10*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*Sqrt[d + e*x])/e^7
+ (2*c*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^(3/2))/e^7 - (6*
c^2*(2*c*d - b*e)*(d + e*x)^(5/2))/(5*e^7) + (2*c^3*(d + e*x)^(7/2))/(7*e^7
)
```


Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^3}{(d+ex)^{7/2}}\,dx=\int \left(\frac{\left(cd^2-bde+ae^2\right)^3}{e^6(d+ex)^{7/2}}+\frac{3(-2cd+be)\left(cd^2-bde+ae^2\right)^2}{e^6(d+ex)^{5/2}}+\frac{3\left(cd^2-bde+ae^2\right)\left(5c^2d^2-5bc\right.}{e^6(d+ex)^{3/2}}\right.$$

$$=-\frac{2\left(cd^2-bde+ae^2\right)^3}{5e^7(d+ex)^{5/2}}+\frac{2(2cd-be)\left(cd^2-bde+ae^2\right)^2}{e^7(d+ex)^{3/2}}-\frac{6\left(cd^2-bde+ae^2\right)\left(5c^2d^2+b^2e^2-\right.}{e^7\sqrt{d+ex}}$$

Mathematica [A] time = 0.364776, size = 391, normalized size = 1.41

$$-\frac{2\left(7ce^2\left(a^2e^2\left(8d^2+20dex+15e^2x^2\right)-6abe\left(40d^2ex+16d^3+30de^2x^2+5e^3x^3\right)+b^2\left(240d^2e^2x^2+320d^3ex+128d^4+\cdots\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(d + e*x)^(7/2),x]
```

```
[Out] (-2*(c^3*(1024*d^6 + 2560*d^5*e*x + 1920*d^4*e^2*x^2 + 320*d^3*e^3*x^3 - 40
*d^2*e^4*x^4 + 12*d*e^5*x^5 - 5*e^6*x^6) + 7*e^3*(a^3*e^3 + a^2*b*e^2*(2*d
+ 5*e*x) + a*b^2*e*(8*d^2 + 20*d*e*x + 15*e^2*x^2) - b^3*(16*d^3 + 40*d^2*e
*x + 30*d*e^2*x^2 + 5*e^3*x^3)) + 7*c*e^2*(a^2*e^2*(8*d^2 + 20*d*e*x + 15*e
^2*x^2) - 6*a*b*e*(16*d^3 + 40*d^2*e*x + 30*d*e^2*x^2 + 5*e^3*x^3) + b^2*(1
28*d^4 + 320*d^3*e*x + 240*d^2*e^2*x^2 + 40*d*e^3*x^3 - 5*e^4*x^4)) - 7*c^2
*e*(a*e*(-128*d^4 - 320*d^3*e*x - 240*d^2*e^2*x^2 - 40*d*e^3*x^3 + 5*e^4*x^
4) + b*(256*d^5 + 640*d^4*e*x + 480*d^3*e^2*x^2 + 80*d^2*e^3*x^3 - 10*d*e^4
*x^4 + 3*e^5*x^5))))/(35*e^7*(d + e*x)^(5/2))
```

Maple [A] time = 0.047, size = 495, normalized size = 1.8

$$-\frac{-10c^3x^6e^6-42bc^2e^6x^5+24c^3de^5x^5-70ac^2e^6x^4-70b^2ce^6x^4+140bc^2de^5x^4-80c^3d^2e^4x^4-420abce^6x^3+560ac^2de\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(e*x+d)^(7/2),x)
```

```
[Out] -2/35/(e*x+d)^(5/2)*(-5*c^3*e^6*x^6-21*b*c^2*e^6*x^5+12*c^3*d*e^5*x^5-35*a*
c^2*e^6*x^4-35*b^2*c*e^6*x^4+70*b*c^2*d*e^5*x^4-40*c^3*d^2*e^4*x^4-210*a*b*
c*e^6*x^3+280*a*c^2*d*e^5*x^3-35*b^3*e^6*x^3+280*b^2*c*d*e^5*x^3-560*b*c^2*
d^2*e^4*x^3+320*c^3*d^3*e^3*x^3+105*a^2*c*e^6*x^2+105*a*b^2*e^6*x^2-1260*a*
b*c*d*e^5*x^2+1680*a*c^2*d^2*e^4*x^2-210*b^3*d*e^5*x^2+1680*b^2*c*d^2*e^4*x
^2-3360*b*c^2*d^3*e^3*x^2+1920*c^3*d^4*e^2*x^2+35*a^2*b*e^6*x+140*a^2*c*d*e
^5*x+140*a*b^2*d*e^5*x-1680*a*b*c*d^2*e^4*x+2240*a*c^2*d^3*e^3*x-280*b^3*d^
2*e^4*x+2240*b^2*c*d^3*e^3*x-4480*b*c^2*d^4*e^2*x+2560*c^3*d^5*e*x+7*a^3*e^
6+14*a^2*b*d*e^5+56*a^2*c*d^2*e^4+56*a*b^2*d^2*e^4-672*a*b*c*d^3*e^3+896*a*
c^2*d^4*e^2-112*b^3*d^3*e^3+896*b^2*c*d^4*e^2-1792*b*c^2*d^5*e+1024*c^3*d^6
)/e^7
```

Maxima [A] time = 1.03108, size = 558, normalized size = 2.01

$$\frac{2\left(\frac{5(ex+d)^{\frac{7}{2}}c^3-21\left(2c^3d-bc^2e\right)(ex+d)^{\frac{5}{2}}+35\left(5c^3d^2-5bc^2de+\left(b^2c+ac^2\right)e^2\right)(ex+d)^{\frac{3}{2}}-35\left(20c^3d^3-30bc^2d^2e+12\left(b^2c+ac^2\right)de^2-\left(b^3+6abc\right)e^3\right)\sqrt{ex+d}}{e^6}-\frac{7\left(c^3d^6-3\cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/35*((5*(e*x + d)^(7/2)*c^3 - 21*(2*c^3*d - b*c^2*e)*(e*x + d)^(5/2) + 35*
(5*c^3*d^2 - 5*b*c^2*d*e + (b^2*c + a*c^2)*e^2)*(e*x + d)^(3/2) - 35*(20*c^
3*d^3 - 30*b*c^2*d^2*e + 12*(b^2*c + a*c^2)*d*e^2 - (b^3 + 6*a*b*c)*e^3)*sq
```

```
rt(e*x + d))/e^6 - 7*(c^3*d^6 - 3*b*c^2*d^5*e - 3*a^2*b*d*e^5 + a^3*e^6 + 3
*(b^2*c + a*c^2)*d^4*e^2 - (b^3 + 6*a*b*c)*d^3*e^3 + 3*(a*b^2 + a^2*c)*d^2*
e^4 + 15*(5*c^3*d^4 - 10*b*c^2*d^3*e + 6*(b^2*c + a*c^2)*d^2*e^2 - (b^3 + 6
*a*b*c)*d*e^3 + (a*b^2 + a^2*c)*e^4)*(e*x + d)^2 - 5*(2*c^3*d^5 - 5*b*c^2*d
^4*e - a^2*b*e^5 + 4*(b^2*c + a*c^2)*d^3*e^2 - (b^3 + 6*a*b*c)*d^2*e^3 + 2*
(a*b^2 + a^2*c)*d*e^4)*(e*x + d))/((e*x + d)^(5/2)*e^6))/e
```

Fricas [A] time = 1.99361, size = 950, normalized size = 3.42

$$2\left(5\,c^3e^6x^6 - 1024\,c^3d^6 + 1792\,bc^2d^5e - 14\,a^2bde^5 - 7\,a^3e^6 - 896\left(b^2c + ac^2\right)d^4e^2 + 112\left(b^3 + 6\,abc\right)d^3e^3 - 56\left(ab^2 + a^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*c^3*e^6*x^6 - 1024*c^3*d^6 + 1792*b*c^2*d^5*e - 14*a^2*b*d*e^5 - 7*
a^3*e^6 - 896*(b^2*c + a*c^2)*d^4*e^2 + 112*(b^3 + 6*a*b*c)*d^3*e^3 - 56*(a
*b^2 + a^2*c)*d^2*e^4 - 3*(4*c^3*d*e^5 - 7*b*c^2*e^6)*x^5 + 5*(8*c^3*d^2*e^
4 - 14*b*c^2*d*e^5 + 7*(b^2*c + a*c^2)*e^6)*x^4 - 5*(64*c^3*d^3*e^3 - 112*b
*c^2*d^2*e^4 + 56*(b^2*c + a*c^2)*d*e^5 - 7*(b^3 + 6*a*b*c)*e^6)*x^3 - 15*(
128*c^3*d^4*e^2 - 224*b*c^2*d^3*e^3 + 112*(b^2*c + a*c^2)*d^2*e^4 - 14*(b^3
 + 6*a*b*c)*d*e^5 + 7*(a*b^2 + a^2*c)*e^6)*x^2 - 5*(512*c^3*d^5*e - 896*b*c
^2*d^4*e^2 + 7*a^2*b*e^6 + 448*(b^2*c + a*c^2)*d^3*e^3 - 56*(b^3 + 6*a*b*c)
*d^2*e^4 + 28*(a*b^2 + a^2*c)*d*e^5)*x)*sqrt(e*x + d)/(e^10*x^3 + 3*d*e^9*x
^2 + 3*d^2*e^8*x + d^3*e^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(e*x+d)**(7/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.13184, size = 822, normalized size = 2.96

$$\frac{2}{35}\left(5\,(xe+d)^{\frac{7}{2}}c^3e^{42} - 42\,(xe+d)^{\frac{5}{2}}c^3de^{42} + 175\,(xe+d)^{\frac{3}{2}}c^3d^2e^{42} - 700\,\sqrt{xe+d}c^3d^3e^{42} + 21\,(xe+d)^{\frac{5}{2}}bc^2e^{43} - 175\,(xe\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] 2/35*(5*(x*e + d)^(7/2)*c^3*e^42 - 42*(x*e + d)^(5/2)*c^3*d*e^42 + 175*(x*e
 + d)^(3/2)*c^3*d^2*e^42 - 700*sqrt(x*e + d)*c^3*d^3*e^42 + 21*(x*e + d)^(5
/2)*b*c^2*e^43 - 175*(x*e + d)^(3/2)*b*c^2*d*e^43 + 1050*sqrt(x*e + d)*b*c^
2*d^2*e^43 + 35*(x*e + d)^(3/2)*b^2*c*e^44 + 35*(x*e + d)^(3/2)*a*c^2*e^44
- 420*sqrt(x*e + d)*b^2*c*d*e^44 - 420*sqrt(x*e + d)*a*c^2*d*e^44 + 35*sqrt
```

```
(x*e + d)*b^3*e^45 + 210*sqrt(x*e + d)*a*b*c*e^45)*e^(-49) - 2/5*(75*(x*e +
 d)^2*c^3*d^4 - 10*(x*e + d)*c^3*d^5 + c^3*d^6 - 150*(x*e + d)^2*b*c^2*d^3*
e + 25*(x*e + d)*b*c^2*d^4*e - 3*b*c^2*d^5*e + 90*(x*e + d)^2*b^2*c*d^2*e^2
 + 90*(x*e + d)^2*a*c^2*d^2*e^2 - 20*(x*e + d)*b^2*c*d^3*e^2 - 20*(x*e + d)
*a*c^2*d^3*e^2 + 3*b^2*c*d^4*e^2 + 3*a*c^2*d^4*e^2 - 15*(x*e + d)^2*b^3*d*e
^3 - 90*(x*e + d)^2*a*b*c*d*e^3 + 5*(x*e + d)*b^3*d^2*e^3 + 30*(x*e + d)*a*
b*c*d^2*e^3 - b^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 15*(x*e + d)^2*a*b^2*e^4 + 15
*(x*e + d)^2*a^2*c*e^4 - 10*(x*e + d)*a*b^2*d*e^4 - 10*(x*e + d)*a^2*c*d*e^
4 + 3*a*b^2*d^2*e^4 + 3*a^2*c*d^2*e^4 + 5*(x*e + d)*a^2*b*e^5 - 3*a^2*b*d*e
^5 + a^3*e^6)*e^(-7)/(x*e + d)^(5/2)
```

3.2289 $\int \frac{(d+ex)^{5/2}}{a+bx+cx^2} dx$

Optimal. Leaf size=459

$$-\frac{\sqrt{2}\left(-3c^2de\left(-d\sqrt{b^2-4ac}+2ae+bd\right)+ce^2\left(-3bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}+3abe+3b^2d\right)-b^2e^3\left(b-\sqrt{b^2-4ac}\right)\right.}{c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

```
[Out] (2*e*(2*c*d - b*e)*Sqrt[d + e*x])/c^2 + (2*e*(d + e*x)^(3/2))/(3*c) - (Sqrt
[2]*(2*c^3*d^3 - b^2*(b - Sqrt[b^2 - 4*a*c])*e^3 - 3*c^2*d*e*(b*d - Sqrt[b^
2 - 4*a*c]*d + 2*a*e) + c*e^2*(3*b^2*d - 3*b*Sqrt[b^2 - 4*a*c]*d + 3*a*b*e
- a*Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e]])/(c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b
 - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(2*c^3*d^3 - b^2*(b + Sqrt[b^2 - 4*a*c
])*e^3 - 3*c^2*d*e*(b*d + Sqrt[b^2 - 4*a*c]*d + 2*a*e) + c*e^2*(3*b^2*d + a
*Sqrt[b^2 - 4*a*c]*e + 3*b*(Sqrt[b^2 - 4*a*c]*d + a*e)))*ArcTanh[(Sqrt[2]*S
qrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(5/2)*Sq
rt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 4.43191, antiderivative size = 459, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {703, 824, 826, 1166, 208}

$$-\frac{\sqrt{2}\left(-3c^2de\left(-d\sqrt{b^2-4ac}+2ae+bd\right)+ce^2\left(-3bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}+3abe+3b^2d\right)-b^2e^3\left(b-\sqrt{b^2-4ac}\right)\right.}{c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(a + b*x + c*x^2),x]
```

```
[Out] (2*e*(2*c*d - b*e)*Sqrt[d + e*x])/c^2 + (2*e*(d + e*x)^(3/2))/(3*c) - (Sqrt
[2]*(2*c^3*d^3 - b^2*(b - Sqrt[b^2 - 4*a*c])*e^3 - 3*c^2*d*e*(b*d - Sqrt[b^
2 - 4*a*c]*d + 2*a*e) + c*e^2*(3*b^2*d - 3*b*Sqrt[b^2 - 4*a*c]*d + 3*a*b*e
- a*Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e]])/(c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b
 - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(2*c^3*d^3 - b^2*(b + Sqrt[b^2 - 4*a*c
])*e^3 - 3*c^2*d*e*(b*d + Sqrt[b^2 - 4*a*c]*d + 2*a*e) + c*e^2*(3*b^2*d + a
*Sqrt[b^2 - 4*a*c]*e + 3*b*(Sqrt[b^2 - 4*a*c]*d + a*e)))*ArcTanh[(Sqrt[2]*S
qrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(5/2)*Sq
rt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{a+bx+cx^2}\,dx = \frac{2e(d+ex)^{3/2}}{3c} + \frac{\int \frac{\sqrt{d+ex}\left(cd^2-ae^2+e(2cd-be)x\right)}{a+bx+cx^2}\,dx}{c}$$

$$= \frac{2e(2cd-be)\sqrt{d+ex}}{c^2} + \frac{2e(d+ex)^{3/2}}{3c} + \frac{\int \frac{c^2d^3-3acde^2+abe^3+e\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)x}{\sqrt{d+ex}\left(a+bx+cx^2\right)}\,dx}{c^2}$$

$$= \frac{2e(2cd-be)\sqrt{d+ex}}{c^2} + \frac{2e(d+ex)^{3/2}}{3c} + \frac{2\,\mathrm{Subst}\left(\int \frac{e\left(c^2d^3-3acde^2+abe^3\right)-de\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)+e\left(3c^2d^2+b^2\right.}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\right.}{c^2}$$

$$= \frac{2e(2cd-be)\sqrt{d+ex}}{c^2} + \frac{2e(d+ex)^{3/2}}{3c} + \frac{\left(2c^3d^3-b^2\left(b-\sqrt{b^2-4ac}\right)e^3-3c^2de\left(bd-\sqrt{b^2-4ac}d+\right.\right.}{}$$

$$= \frac{2e(2cd-be)\sqrt{d+ex}}{c^2} + \frac{2e(d+ex)^{3/2}}{3c} - \frac{\sqrt{2}\left(2c^3d^3-b^2\left(b-\sqrt{b^2-4ac}\right)e^3-3c^2de\left(bd-\sqrt{b^2-4ac}d\right.\right.}{c^{5/2}\sqrt{}}$$

Mathematica [A] time = 1.08186, size = 455, normalized size = 0.99

$$-\frac{\sqrt{2}\left(3c^2de\left(d\sqrt{b^2-4ac}-2ae-bd\right)+ce^2\left(-3bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}+3abe+3b^2d\right)+b^2e^3\left(\sqrt{b^2-4ac}-b\right)+2c^3d\right.}{c^{5/2}\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(a + b*x + c*x^2),x]
```

```
[Out] (-2*e*(-2*c*d + b*e)*Sqrt[d + e*x])/c^2 + (2*e*(d + e*x)^(3/2))/(3*c) - (Sq
rt[2]*(2*c^3*d^3 + b^2*(-b + Sqrt[b^2 - 4*a*c])*e^3 + 3*c^2*d*e*(-(b*d) + S
qrt[b^2 - 4*a*c]*d - 2*a*e) + c*e^2*(3*b^2*d - 3*b*Sqrt[b^2 - 4*a*c]*d + 3*
a*b*e - a*Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqr
t[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*
d + (-b + Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(2*c^3*d^3 - b^2*(b + Sqrt[b^2
- 4*a*c])*e^3 - 3*c^2*d*e*(b*d + Sqrt[b^2 - 4*a*c]*d + 2*a*e) + c*e^2*(3*b^
2*d + a*Sqrt[b^2 - 4*a*c]*e + 3*b*(Sqrt[b^2 - 4*a*c]*d + a*e)))*ArcTanh[(Sq
rt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(
5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Maple [B] time = 0.335, size = 1929, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(c*x^2+b*x+a),x)
```

```
[Out] 2/3*e*(e*x+d)^(3/2)/c-2/c^2*(e*x+d)^(1/2)*b*e^2+4*d*e*(e*x+d)^(1/2)/c-3/c/(
-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1
/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)
^(1/2))*a*b*e^4+6/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c
-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*
a*c-b^2))^(1/2))*c)^(1/2))*a*d*e^3+1/c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/(
(b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2
)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^3*e^4-3/c/(-e^2*(4*a*c-
b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((
e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2*
d*e^3+3/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/
2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^
(1/2))*c)^(1/2))*b*d^2*e^2-2*e*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c
*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-
2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d^3-1/c*2^(1/2)/((b*e-2*c*d+(-e^2
*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-
e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*e^3+1/c^2*2^(1/2)/((b*e-2*c*d+(-e^2*(4*
a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*
(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2*e^3-3/c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-
b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a
*c-b^2))^(1/2))*c)^(1/2))*b*d*e^2+3*e*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2)
)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b
^2))^(1/2))*c)^(1/2))*d^2-3/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d
+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+
2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b*e^4+6/(-e^2*(4*a*c-b^2))^(1/2
)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(
1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*d*e^3+1/c
^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*
c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1
/2))*c)^(1/2))*b^3*e^4-3/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-
e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c
*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2*d*e^3+3/(-e^2*(4*a*c-b^2))^(1/2)
*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1
/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d^2*e^2-2*
e*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))
*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(
```

```
1/2))*c)^(1/2))*d^3+1/c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(
1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))
*c)^(1/2))*a*e^3-1/c^2*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1
/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*
c)^(1/2))*b^2*e^3+3/c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/
2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c
)^(1/2))*b*d*e^2-3*e*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2
)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)
^(1/2))*d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/(c*x^2 + b*x + a), x)
```

Fricas [B] time = 17.1642, size = 13779, normalized size = 30.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(2)*c^2*sqrt((2*c^5*d^5 - 5*b*c^4*d^4*e + 10*(b^2*c^3 - 2*a*c^4)
*d^3*e^2 - 10*(b^3*c^2 - 3*a*b*c^3)*d^2*e^3 + 5*(b^4*c - 4*a*b^2*c^2 + 2*a^
2*c^3)*d*e^4 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^5 + (b^2*c^5 - 4*a*c^6)*sq
rt((25*c^8*d^8*e^2 - 100*b*c^7*d^7*e^3 + 100*(2*b^2*c^6 - a*c^7)*d^6*e^4 -
50*(5*b^3*c^5 - 6*a*b*c^6)*d^5*e^5 + 10*(21*b^4*c^4 - 43*a*b^2*c^5 + 11*a^2
*c^6)*d^4*e^6 - 20*(6*b^5*c^3 - 18*a*b^3*c^4 + 11*a^2*b*c^5)*d^3*e^7 + 5*(9
*b^6*c^2 - 36*a*b^4*c^3 + 36*a^2*b^2*c^4 - 4*a^3*c^5)*d^2*e^8 - 10*(b^7*c -
 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^9 + (b^8 - 6*a*b^6*c + 11*a
^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^10)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^
5 - 4*a*c^6))*log(sqrt(2)*(10*(b^2*c^5 - 4*a*c^6)*d^5*e^2 - 25*(b^3*c^4 - 4
*a*b*c^5)*d^4*e^3 + 10*(3*b^4*c^3 - 14*a*b^2*c^4 + 8*a^2*c^5)*d^3*e^4 - 10*
(2*b^5*c^2 - 11*a*b^3*c^3 + 12*a^2*b*c^4)*d^2*e^5 + (7*b^6*c - 44*a*b^4*c^2
 + 66*a^2*b^2*c^3 - 8*a^3*c^4)*d*e^6 - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 -
4*a^3*b*c^3)*e^7 - (2*(b^2*c^7 - 4*a*c^8)*d^2 - 2*(b^3*c^6 - 4*a*b*c^7)*d*e
 + (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*e^2)*sqrt((25*c^8*d^8*e^2 - 100*b*c^
7*d^7*e^3 + 100*(2*b^2*c^6 - a*c^7)*d^6*e^4 - 50*(5*b^3*c^5 - 6*a*b*c^6)*d^
5*e^5 + 10*(21*b^4*c^4 - 43*a*b^2*c^5 + 11*a^2*c^6)*d^4*e^6 - 20*(6*b^5*c^3
 - 18*a*b^3*c^4 + 11*a^2*b*c^5)*d^3*e^7 + 5*(9*b^6*c^2 - 36*a*b^4*c^3 + 36*
a^2*b^2*c^4 - 4*a^3*c^5)*d^2*e^8 - 10*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3
- 2*a^3*b*c^4)*d*e^9 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
a^4*c^4)*e^10)/(b^2*c^10 - 4*a*c^11)))*sqrt((2*c^5*d^5 - 5*b*c^4*d^4*e + 10
*(b^2*c^3 - 2*a*c^4)*d^3*e^2 - 10*(b^3*c^2 - 3*a*b*c^3)*d^2*e^3 + 5*(b^4*c
- 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^4 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^5 + (b
^2*c^5 - 4*a*c^6)*sqrt((25*c^8*d^8*e^2 - 100*b*c^7*d^7*e^3 + 100*(2*b^2*c^6
 - a*c^7)*d^6*e^4 - 50*(5*b^3*c^5 - 6*a*b*c^6)*d^5*e^5 + 10*(21*b^4*c^4 - 4
3*a*b^2*c^5 + 11*a^2*c^6)*d^4*e^6 - 20*(6*b^5*c^3 - 18*a*b^3*c^4 + 11*a^2*b
```

*c^5)*d^3*e^7 + 5*(9*b^6*c^2 - 36*a*b^4*c^3 + 36*a^2*b^2*c^4 - 4*a^3*c^5)*d
^2*e^8 - 10*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^9 + (b^
8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^10)/(b^2*c^10 -
 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) + 4*(5*c^6*d^8*e - 20*b*c^5*d^7*e^2 + 35*
b^2*c^4*d^6*e^3 - 35*b^3*c^3*d^5*e^4 + 7*(3*b^4*c^2 + a*b^2*c^3 - 2*a^2*c^4
)*d^4*e^5 - 7*(b^5*c + 2*a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e^6 + (b^6 + 9*a*b^4*
c - 15*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^7 - (2*a*b^5 - a^2*b^3*c - 8*a^3*b*c^
2)*d*e^8 + (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*e^9)*sqrt(e*x + d)) - 3*sqrt(2
)*c^2*sqrt((2*c^5*d^5 - 5*b*c^4*d^4*e + 10*(b^2*c^3 - 2*a*c^4)*d^3*e^2 - 10
*(b^3*c^2 - 3*a*b*c^3)*d^2*e^3 + 5*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^4
- (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^5 + (b^2*c^5 - 4*a*c^6)*sqrt((25*c^8*d^
8*e^2 - 100*b*c^7*d^7*e^3 + 100*(2*b^2*c^6 - a*c^7)*d^6*e^4 - 50*(5*b^3*c^5
 - 6*a*b*c^6)*d^5*e^5 + 10*(21*b^4*c^4 - 43*a*b^2*c^5 + 11*a^2*c^6)*d^4*e^6
 - 20*(6*b^5*c^3 - 18*a*b^3*c^4 + 11*a^2*b*c^5)*d^3*e^7 + 5*(9*b^6*c^2 - 36
*a*b^4*c^3 + 36*a^2*b^2*c^4 - 4*a^3*c^5)*d^2*e^8 - 10*(b^7*c - 5*a*b^5*c^2
+ 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^9 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
6*a^3*b^2*c^3 + a^4*c^4)*e^10)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))
log(-sqrt(2)(10*(b^2*c^5 - 4*a*c^6)*d^5*e^2 - 25*(b^3*c^4 - 4*a*b*c^5)*d^
4*e^3 + 10*(3*b^4*c^3 - 14*a*b^2*c^4 + 8*a^2*c^5)*d^3*e^4 - 10*(2*b^5*c^2 -
 11*a*b^3*c^3 + 12*a^2*b*c^4)*d^2*e^5 + (7*b^6*c - 44*a*b^4*c^2 + 66*a^2*b^
2*c^3 - 8*a^3*c^4)*d*e^6 - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)
e^7 - (2(b^2*c^7 - 4*a*c^8)*d^2 - 2*(b^3*c^6 - 4*a*b*c^7)*d*e + (b^4*c^5
- 6*a*b^2*c^6 + 8*a^2*c^7)*e^2)*sqrt((25*c^8*d^8*e^2 - 100*b*c^7*d^7*e^3 +
100*(2*b^2*c^6 - a*c^7)*d^6*e^4 - 50*(5*b^3*c^5 - 6*a*b*c^6)*d^5*e^5 + 10*(
21*b^4*c^4 - 43*a*b^2*c^5 + 11*a^2*c^6)*d^4*e^6 - 20*(6*b^5*c^3 - 18*a*b^3*
c^4 + 11*a^2*b*c^5)*d^3*e^7 + 5*(9*b^6*c^2 - 36*a*b^4*c^3 + 36*a^2*b^2*c^4
- 4*a^3*c^5)*d^2*e^8 - 10*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^
4)*d*e^9 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^1
0)/(b^2*c^10 - 4*a*c^11)))*sqrt((2*c^5*d^5 - 5*b*c^4*d^4*e + 10*(b^2*c^3 -
2*a*c^4)*d^3*e^2 - 10*(b^3*c^2 - 3*a*b*c^3)*d^2*e^3 + 5*(b^4*c - 4*a*b^2*c^
2 + 2*a^2*c^3)*d*e^4 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^5 + (b^2*c^5 - 4*a
*c^6)*sqrt((25*c^8*d^8*e^2 - 100*b*c^7*d^7*e^3 + 100*(2*b^2*c^6 - a*c^7)*d^
6*e^4 - 50*(5*b^3*c^5 - 6*a*b*c^6)*d^5*e^5 + 10*(21*b^4*c^4 - 43*a*b^2*c^5
+ 11*a^2*c^6)*d^4*e^6 - 20*(6*b^5*c^3 - 18*a*b^3*c^4 + 11*a^2*b*c^5)*d^3*e^
7 + 5*(9*b^6*c^2 - 36*a*b^4*c^3 + 36*a^2*b^2*c^4 - 4*a^3*c^5)*d^2*e^8 - 10*
(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^9 + (b^8 - 6*a*b^6*
c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^10)/(b^2*c^10 - 4*a*c^11)))
/(b^2*c^5 - 4*a*c^6)) + 4*(5*c^6*d^8*e - 20*b*c^5*d^7*e^2 + 35*b^2*c^4*d^6*
e^3 - 35*b^3*c^3*d^5*e^4 + 7*(3*b^4*c^2 + a*b^2*c^3 - 2*a^2*c^4)*d^4*e^5 -
7*(b^5*c + 2*a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e^6 + (b^6 + 9*a*b^4*c - 15*a^2*b
^2*c^2 - 8*a^3*c^3)*d^2*e^7 - (2*a*b^5 - a^2*b^3*c - 8*a^3*b*c^2)*d*e^8 + (
a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*e^9)*sqrt(e*x + d)) + 3*sqrt(2)*c^2*sqrt((
2*c^5*d^5 - 5*b*c^4*d^4*e + 10*(b^2*c^3 - 2*a*c^4)*d^3*e^2 - 10*(b^3*c^2 -
3*a*b*c^3)*d^2*e^3 + 5*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^4 - (b^5 - 5*a
*b^3*c + 5*a^2*b*c^2)*e^5 - (b^2*c^5 - 4*a*c^6)*sqrt((25*c^8*d^8*e^2 - 100*
b*c^7*d^7*e^3 + 100*(2*b^2*c^6 - a*c^7)*d^6*e^4 - 50*(5*b^3*c^5 - 6*a*b*c^6
)*d^5*e^5 + 10*(21*b^4*c^4 - 43*a*b^2*c^5 + 11*a^2*c^6)*d^4*e^6 - 20*(6*b^5
*c^3 - 18*a*b^3*c^4 + 11*a^2*b*c^5)*d^3*e^7 + 5*(9*b^6*c^2 - 36*a*b^4*c^3 +
 36*a^2*b^2*c^4 - 4*a^3*c^5)*d^2*e^8 - 10*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*
c^3 - 2*a^3*b*c^4)*d*e^9 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^
3 + a^4*c^4)*e^10)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(sqrt(2)
(10(b^2*c^5 - 4*a*c^6)*d^5*e^2 - 25*(b^3*c^4 - 4*a*b*c^5)*d^4*e^3 + 10*(3
*b^4*c^3 - 14*a*b^2*c^4 + 8*a^2*c^5)*d^3*e^4 - 10*(2*b^5*c^2 - 11*a*b^3*c^3
 + 12*a^2*b*c^4)*d^2*e^5 + (7*b^6*c - 44*a*b^4*c^2 + 66*a^2*b^2*c^3 - 8*a^3
*c^4)*d*e^6 - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e^7 + (2*(b^
2*c^7 - 4*a*c^8)*d^2 - 2*(b^3*c^6 - 4*a*b*c^7)*d*e + (b^4*c^5 - 6*a*b^2*c^6
 + 8*a^2*c^7)*e^2)*sqrt((25*c^8*d^8*e^2 - 100*b*c^7*d^7*e^3 + 100*(2*b^2*c^
6 - a*c^7)*d^6*e^4 - 50*(5*b^3*c^5 - 6*a*b*c^6)*d^5*e^5 + 10*(21*b^4*c^4 -
43*a*b^2*c^5 + 11*a^2*c^6)*d^4*e^6 - 20*(6*b^5*c^3 - 18*a*b^3*c^4 + 11*a^2*

```
b*c^5)*d^3*e^7 + 5*(9*b^6*c^2 - 36*a*b^4*c^3 + 36*a^2*b^2*c^4 - 4*a^3*c^5)*
d^2*e^8 - 10*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^9 + (b
^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^10)/(b^2*c^10
- 4*a*c^11)))*sqrt((2*c^5*d^5 - 5*b*c^4*d^4*e + 10*(b^2*c^3 - 2*a*c^4)*d^3*
e^2 - 10*(b^3*c^2 - 3*a*b*c^3)*d^2*e^3 + 5*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3
)*d*e^4 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^5 - (b^2*c^5 - 4*a*c^6)*sqrt((2
5*c^8*d^8*e^2 - 100*b*c^7*d^7*e^3 + 100*(2*b^2*c^6 - a*c^7)*d^6*e^4 - 50*(5
*b^3*c^5 - 6*a*b*c^6)*d^5*e^5 + 10*(21*b^4*c^4 - 43*a*b^2*c^5 + 11*a^2*c^6)
*d^4*e^6 - 20*(6*b^5*c^3 - 18*a*b^3*c^4 + 11*a^2*b*c^5)*d^3*e^7 + 5*(9*b^6*
c^2 - 36*a*b^4*c^3 + 36*a^2*b^2*c^4 - 4*a^3*c^5)*d^2*e^8 - 10*(b^7*c - 5*a*
b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^9 + (b^8 - 6*a*b^6*c + 11*a^2*b^
4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^10)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4
*a*c^6)) + 4*(5*c^6*d^8*e - 20*b*c^5*d^7*e^2 + 35*b^2*c^4*d^6*e^3 - 35*b^3*
c^3*d^5*e^4 + 7*(3*b^4*c^2 + a*b^2*c^3 - 2*a^2*c^4)*d^4*e^5 - 7*(b^5*c + 2*
a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e^6 + (b^6 + 9*a*b^4*c - 15*a^2*b^2*c^2 - 8*a^
3*c^3)*d^2*e^7 - (2*a*b^5 - a^2*b^3*c - 8*a^3*b*c^2)*d*e^8 + (a^2*b^4 - 3*a
^3*b^2*c + a^4*c^2)*e^9)*sqrt(e*x + d)) - 3*sqrt(2)*c^2*sqrt((2*c^5*d^5 - 5
*b*c^4*d^4*e + 10*(b^2*c^3 - 2*a*c^4)*d^3*e^2 - 10*(b^3*c^2 - 3*a*b*c^3)*d^
2*e^3 + 5*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^4 - (b^5 - 5*a*b^3*c + 5*a^
2*b*c^2)*e^5 - (b^2*c^5 - 4*a*c^6)*sqrt((25*c^8*d^8*e^2 - 100*b*c^7*d^7*e^3
 + 100*(2*b^2*c^6 - a*c^7)*d^6*e^4 - 50*(5*b^3*c^5 - 6*a*b*c^6)*d^5*e^5 + 1
0*(21*b^4*c^4 - 43*a*b^2*c^5 + 11*a^2*c^6)*d^4*e^6 - 20*(6*b^5*c^3 - 18*a*b
^3*c^4 + 11*a^2*b*c^5)*d^3*e^7 + 5*(9*b^6*c^2 - 36*a*b^4*c^3 + 36*a^2*b^2*c
^4 - 4*a^3*c^5)*d^2*e^8 - 10*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b
*c^4)*d*e^9 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*
e^10)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-sqrt(2)*(10*(b^2*c^
5 - 4*a*c^6)*d^5*e^2 - 25*(b^3*c^4 - 4*a*b*c^5)*d^4*e^3 + 10*(3*b^4*c^3 - 1
4*a*b^2*c^4 + 8*a^2*c^5)*d^3*e^4 - 10*(2*b^5*c^2 - 11*a*b^3*c^3 + 12*a^2*b*
c^4)*d^2*e^5 + (7*b^6*c - 44*a*b^4*c^2 + 66*a^2*b^2*c^3 - 8*a^3*c^4)*d*e^6
- (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e^7 + (2*(b^2*c^7 - 4*a*
c^8)*d^2 - 2*(b^3*c^6 - 4*a*b*c^7)*d*e + (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7
)*e^2)*sqrt((25*c^8*d^8*e^2 - 100*b*c^7*d^7*e^3 + 100*(2*b^2*c^6 - a*c^7)*d
^6*e^4 - 50*(5*b^3*c^5 - 6*a*b*c^6)*d^5*e^5 + 10*(21*b^4*c^4 - 43*a*b^2*c^5
 + 11*a^2*c^6)*d^4*e^6 - 20*(6*b^5*c^3 - 18*a*b^3*c^4 + 11*a^2*b*c^5)*d^3*e
^7 + 5*(9*b^6*c^2 - 36*a*b^4*c^3 + 36*a^2*b^2*c^4 - 4*a^3*c^5)*d^2*e^8 - 10
*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^9 + (b^8 - 6*a*b^6
*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^10)/(b^2*c^10 - 4*a*c^11))
)*sqrt((2*c^5*d^5 - 5*b*c^4*d^4*e + 10*(b^2*c^3 - 2*a*c^4)*d^3*e^2 - 10*(b^
3*c^2 - 3*a*b*c^3)*d^2*e^3 + 5*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^4 - (b
^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^5 - (b^2*c^5 - 4*a*c^6)*sqrt((25*c^8*d^8*e^
2 - 100*b*c^7*d^7*e^3 + 100*(2*b^2*c^6 - a*c^7)*d^6*e^4 - 50*(5*b^3*c^5 - 6
*a*b*c^6)*d^5*e^5 + 10*(21*b^4*c^4 - 43*a*b^2*c^5 + 11*a^2*c^6)*d^4*e^6 - 2
0*(6*b^5*c^3 - 18*a*b^3*c^4 + 11*a^2*b*c^5)*d^3*e^7 + 5*(9*b^6*c^2 - 36*a*b
^4*c^3 + 36*a^2*b^2*c^4 - 4*a^3*c^5)*d^2*e^8 - 10*(b^7*c - 5*a*b^5*c^2 + 7*
a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^9 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^
3*b^2*c^3 + a^4*c^4)*e^10)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) + 4
*(5*c^6*d^8*e - 20*b*c^5*d^7*e^2 + 35*b^2*c^4*d^6*e^3 - 35*b^3*c^3*d^5*e^4
+ 7*(3*b^4*c^2 + a*b^2*c^3 - 2*a^2*c^4)*d^4*e^5 - 7*(b^5*c + 2*a*b^3*c^2 -
4*a^2*b*c^3)*d^3*e^6 + (b^6 + 9*a*b^4*c - 15*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e
^7 - (2*a*b^5 - a^2*b^3*c - 8*a^3*b*c^2)*d*e^8 + (a^2*b^4 - 3*a^3*b^2*c + a
^4*c^2)*e^9)*sqrt(e*x + d)) + 4*(c*e^2*x + 7*c*d*e - 3*b*e^2)*sqrt(e*x + d)
)/c^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2290 $\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=322

$$-\frac{\sqrt{2}\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\left(-2ce\left(d\sqrt{b^2-4ac}\right.\right.}{}$$

```
[Out] (2*e*Sqrt[d + e*x])/c - (Sqrt[2]*(2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2
 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d
 + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(c^(3/2)*Sqrt[b^2 - 4*a*
c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(2*c^2*d^2 + b*(b +
Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(
Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c
^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 1.21587, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {703, 826, 1166, 208}

$$-\frac{\sqrt{2}\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\left(-2ce\left(d\sqrt{b^2-4ac}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(a + b*x + c*x^2),x]
```

```
[Out] (2*e*Sqrt[d + e*x])/c - (Sqrt[2]*(2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2
 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d
 + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(c^(3/2)*Sqrt[b^2 - 4*a*
c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(2*c^2*d^2 + b*(b +
Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(
Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c
^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2e\sqrt{d+ex}}{c}+\frac{\int \frac{cd^2-ae^2+e(2cd-be)x}{\sqrt{d+ex}(a+bx+cx^2)}\,dx}{c}$$

$$= \frac{2e\sqrt{d+ex}}{c}+\frac{2\,\mathrm{Subst}\left(\int \frac{-de(2cd-be)+e(cd^2-ae^2)+e(2cd-be)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx,x,\sqrt{d+ex}\right)}{c}$$

$$= \frac{2e\sqrt{d+ex}}{c}+\frac{\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\mathrm{Subst}\left(\int \frac{}{-\frac{1}{2}\sqrt{b^2-4ac}\,e+}\right.}{c\sqrt{b^2-4ac}}$$

$$= \frac{2e\sqrt{d+ex}}{c}-\frac{\sqrt{2}\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{}}{\sqrt{2cd-\left(b-\right.}}\right.}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

Mathematica [A] time = 0.773262, size = 317, normalized size = 0.98

$$\frac{\frac{\sqrt{2}\left(2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}-b\right)-2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}+\frac{\sqrt{2}\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{}{\sqrt{2c}}\right.}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[c]*e*Sqrt[d + e*x] + (Sqrt[2]*(-2*c^2*d^2 + b*(-b + Sqrt[b^2 - 4*a*
c])*e^2 + 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]
*Sqrt[d + e*x])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c
]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(2*c^2*d^2 + b*(b +
Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(
Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(S
qrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/c^(3/2)
```

Maple [B] time = 0.264, size = 1138, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*x^2+b*x+a),x)
```

```
[Out] 2*e*(e*x+d)^(1/2)/c+2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4
*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2
*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*e^3-1/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/(
(b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2
)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2*e^3+2/(-e^2*(4*a*c-b^
2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*
x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d*e^
2-2*e*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/
2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^
(1/2))*c)^(1/2))*d^2-1/c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(
1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c
)^(1/2))*b*e^2+2*e*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*a
rctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2
))*d+2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/
2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2)
)^(1/2))*c)^(1/2))*a*e^3-1/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+
(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2
*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2*e^3+2/(-e^2*(4*a*c-b^2))^(1/2)
*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1
/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d*e^2-2*e*
c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c
)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/
2))*c)^(1/2))*d^2+1/c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/
2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c
)^(1/2))*b*e^2-2*e*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*
arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(
1/2))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(c*x^2 + b*x + a), x)
```

Fricas [B] time = 3.77658, size = 5581, normalized size = 17.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2
 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3
*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b
```

```
^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*l
og(sqrt(2)*(3*(b^2*c^2 - 4*a*c^3)*d^2*e^2 - 3*(b^3*c - 4*a*b*c^2)*d*e^3 + (
b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^4 - (2*(b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*
a*b*c^4)*e)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3
)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b
^2*c^6 - 4*a*c^7)))*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d
*e^2 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b
*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5
+ (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4
)) - 4*(3*c^3*d^4*e - 6*b*c^2*d^3*e^2 + 2*(2*b^2*c + a*c^2)*d^2*e^3 - (b^3
+ 2*a*b*c)*d*e^4 + (a*b^2 - a^2*c)*e^5)*sqrt(e*x + d)) - sqrt(2)*c*sqrt((2*
c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 +
 (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2
- 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2
)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-sqrt(2)*(3*(b^2*c^2
- 4*a*c^3)*d^2*e^2 - 3*(b^3*c - 4*a*b*c^2)*d*e^3 + (b^4 - 5*a*b^2*c + 4*a^2
*c^2)*e^4 - (2*(b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e)*sqrt((9*c^4
*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c -
a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))*sqr
t((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*
e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2
*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^
2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 4*(3*c^3*d^4*e - 6
*b*c^2*d^3*e^2 + 2*(2*b^2*c + a*c^2)*d^2*e^3 - (b^3 + 2*a*b*c)*d*e^4 + (a*b
^2 - a^2*c)*e^5)*sqrt(e*x + d)) + sqrt(2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e
 + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4)*sq
rt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*
(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c
^7)))/(b^2*c^3 - 4*a*c^4))*log(sqrt(2)*(3*(b^2*c^2 - 4*a*c^3)*d^2*e^2 - 3*(
b^3*c - 4*a*b*c^2)*d*e^3 + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^4 + (2*(b^2*c^4
- 4*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*
e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 -
2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))*sqrt((2*c^3*d^3 - 3*b*c^2*d
^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4
)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4
- 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4
*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 4*(3*c^3*d^4*e - 6*b*c^2*d^3*e^2 + 2*(2*b^
2*c + a*c^2)*d^2*e^3 - (b^3 + 2*a*b*c)*d*e^4 + (a*b^2 - a^2*c)*e^5)*sqrt(e*
x + d)) - sqrt(2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d
*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b
*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5
+ (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4
))*log(-sqrt(2)*(3*(b^2*c^2 - 4*a*c^3)*d^2*e^2 - 3*(b^3*c - 4*a*b*c^2)*d*e^
3 + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^4 + (2*(b^2*c^4 - 4*a*c^5)*d - (b^3*c^3
 - 4*a*b*c^4)*e)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*
a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^
6)/(b^2*c^6 - 4*a*c^7)))*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c
^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 -
 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d
*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*
a*c^4)) - 4*(3*c^3*d^4*e - 6*b*c^2*d^3*e^2 + 2*(2*b^2*c + a*c^2)*d^2*e^3 -
(b^3 + 2*a*b*c)*d*e^4 + (a*b^2 - a^2*c)*e^5)*sqrt(e*x + d)) - 4*sqrt(e*x +
d)*e)/c
```

Sympy [B] time = 162.641, size = 1443, normalized size = 4.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+b*x+a),x)
```

```
[Out] -2*a*e**3*RootSum(_t**4*(256*a**3*c**2*e**6 - 128*a**2*b**2*c*e**6 - 256*a*
*2*b*c**2*d*e**5 + 256*a**2*c**3*d**2*e**4 + 16*a*b**4*e**6 + 128*a*b**3*c*
d*e**5 - 128*a*b**2*c**2*d**2*e**4 - 16*b**5*d*e**5 + 16*b**4*c*d**2*e**4)
+ _t**2*(-16*a*b*c*e**3 + 32*a*c**2*d*e**2 + 4*b**3*e**3 - 8*b**2*c*d*e**2)
 + c, Lambda(_t, _t*log(32*_t**3*a**2*b*e**5 - 64*_t**3*a**2*c*d*e**4 - 8*_
t**3*a*b**3*e**5/c - 16*_t**3*a*b**2*d*e**4 + 96*_t**3*a*b*c*d**2*e**3 - 64
*_t**3*a*c**2*d**3*e**2 + 8*_t**3*b**4*d*e**4/c - 24*_t**3*b**3*d**2*e**3 +
 16*_t**3*b**2*c*d**3*e**2 + 4*_t*a*e**2 - 2*_t*b**2*e**2/c + 4*_t*b*d*e -
4*_t*c*d**2 + sqrt(d + e*x))))/c + 2*b*d*e**2*RootSum(_t**4*(256*a**3*c**2*
e**6 - 128*a**2*b**2*c*e**6 - 256*a**2*b*c**2*d*e**5 + 256*a**2*c**3*d**2*e
**4 + 16*a*b**4*e**6 + 128*a*b**3*c*d*e**5 - 128*a*b**2*c**2*d**2*e**4 - 16
*b**5*d*e**5 + 16*b**4*c*d**2*e**4) + _t**2*(-16*a*b*c*e**3 + 32*a*c**2*d*e
**2 + 4*b**3*e**3 - 8*b**2*c*d*e**2) + c, Lambda(_t, _t*log(32*_t**3*a**2*b
*e**5 - 64*_t**3*a**2*c*d*e**4 - 8*_t**3*a*b**3*e**5/c - 16*_t**3*a*b**2*d*
e**4 + 96*_t**3*a*b*c*d**2*e**3 - 64*_t**3*a*c**2*d**3*e**2 + 8*_t**3*b**4*
d*e**4/c - 24*_t**3*b**3*d**2*e**3 + 16*_t**3*b**2*c*d**3*e**2 + 4*_t*a*e**
2 - 2*_t*b**2*e**2/c + 4*_t*b*d*e - 4*_t*c*d**2 + sqrt(d + e*x))))/c - 2*b*
e**2*RootSum(_t**4*(256*a**2*c**3*e**4 - 128*a*b**2*c**2*e**4 + 16*b**4*c*e
**4) + _t**2*(-16*a*b*c*e**3 + 32*a*c**2*d*e**2 + 4*b**3*e**3 - 8*b**2*c*d*
e**2) + a*e**2 - b*d*e + c*d**2, Lambda(_t, _t*log(64*_t**3*a*c**2*e**2 - 1
6*_t**3*b**2*c*e**2 - 2*_t*b*e + 4*_t*c*d + sqrt(d + e*x))))/c - 2*d**2*e*R
ootSum(_t**4*(256*a**3*c**2*e**6 - 128*a**2*b**2*c*e**6 - 256*a**2*b*c**2*d
*e**5 + 256*a**2*c**3*d**2*e**4 + 16*a*b**4*e**6 + 128*a*b**3*c*d*e**5 - 12
8*a*b**2*c**2*d**2*e**4 - 16*b**5*d*e**5 + 16*b**4*c*d**2*e**4) + _t**2*(-1
6*a*b*c*e**3 + 32*a*c**2*d*e**2 + 4*b**3*e**3 - 8*b**2*c*d*e**2) + c, Lambd
a(_t, _t*log(32*_t**3*a**2*b*e**5 - 64*_t**3*a**2*c*d*e**4 - 8*_t**3*a*b**3
*e**5/c - 16*_t**3*a*b**2*d*e**4 + 96*_t**3*a*b*c*d**2*e**3 - 64*_t**3*a*c*
*2*d**3*e**2 + 8*_t**3*b**4*d*e**4/c - 24*_t**3*b**3*d**2*e**3 + 16*_t**3*b
**2*c*d**3*e**2 + 4*_t*a*e**2 - 2*_t*b**2*e**2/c + 4*_t*b*d*e - 4*_t*c*d**2
 + sqrt(d + e*x)))) + 4*d*e*RootSum(_t**4*(256*a**2*c**3*e**4 - 128*a*b**2*
c**2*e**4 + 16*b**4*c*e**4) + _t**2*(-16*a*b*c*e**3 + 32*a*c**2*d*e**2 + 4*
b**3*e**3 - 8*b**2*c*d*e**2) + a*e**2 - b*d*e + c*d**2, Lambda(_t, _t*log(6
4*_t**3*a*c**2*e**2 - 16*_t**3*b**2*c*e**2 - 2*_t*b*e + 4*_t*c*d + sqrt(d +
 e*x)))) + 2*e*sqrt(d + e*x)/c
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2291 $\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=198

$$\frac{\sqrt{2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] -((Sqrt[2]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]
*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[c]*Sqrt[b^2
 - 4*a*c])) + (Sqrt[2]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqr
t[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt
[c]*Sqrt[b^2 - 4*a*c])
```

Rubi [A] time = 0.288592, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {699, 1130, 208}

$$\frac{\sqrt{2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] -((Sqrt[2]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]
*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[c]*Sqrt[b^2
 - 4*a*c])) + (Sqrt[2]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqr
t[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt
[c]*Sqrt[b^2 - 4*a*c])
```

Rule 699

```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Dist[2*e, Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*
x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx = (2e)\,\text{Subst}\left(\int \frac{x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)$$

$$= -\left(\left(-e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ace}+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x, \sqrt{d+ex}\right)\right)+\left(e-\frac{2cd-}{\sqrt{b^2-}}\right.$$

$$= -\frac{\sqrt{2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{c}\sqrt{b^2-4ac}} + \frac{\sqrt{2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\tanh^{-1}}{\sqrt{c}\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.467059, size = 175, normalized size = 0.88

$$\frac{\sqrt{2}\left(\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)-\sqrt{e\sqrt{b^2-4ac}-be+2cd}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)\right)}{\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] (Sqrt[2]*(-(Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*ArcTanh[(Sqrt[2]*Sqrt[c
]*Sqrt[d + e*x])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]]) + Sqrt[2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e]]))/(Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Maple [B] time = 0.253, size = 545, normalized size = 2.8

$$e^2\sqrt{2}b\arctan\left(c\sqrt{2}\sqrt{ex+d}\frac{1}{\sqrt{\left(be-2\,cd+\sqrt{-e^2\left(4\,ac-b^2\right)}\right)c}}\right)\frac{1}{\sqrt{-e^2\left(4\,ac-b^2\right)}}\frac{1}{\sqrt{\left(be-2\,cd+\sqrt{-e^2\left(4\,ac-b^2\right)}\right)c}}-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*x^2+b*x+a),x)
```

```
[Out] e^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*
c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2
))*c)^(1/2))*b-2*c*e/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*
a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*
(4*a*c-b^2))^(1/2))*c)^(1/2))*d+e*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1
/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))
^(1/2))*c)^(1/2))+e^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(
4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-
e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b-2*c*e/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/
((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(
1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d-e*2^(1/2)/((-b*e+2*
c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b
*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(c*x^2 + b*x + a), x)
```

Fricas [B] time = 2.41623, size = 1466, normalized size = 7.4

$$-\frac{1}{2}\sqrt{2}\sqrt{\frac{2\,cd-be+\left(b^2c-4\,ac^2\right)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\sqrt{2}\left(b^2c-4\,ac^2\right)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}\sqrt{\frac{2\,cd-be+\left(b^2c-4\,ac^2\right)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*
c^3)))/(b^2*c - 4*a*c^2))*log(sqrt(2)*(b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 -
 4*a*c^3))*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^
3)))/(b^2*c - 4*a*c^2)) + 2*sqrt(e*x + d)*e) + 1/2*sqrt(2)*sqrt((2*c*d - b*
e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*log
(-sqrt(2)*(b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3))*sqrt((2*c*d - b*e
 + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2)) + 2*
sqrt(e*x + d)*e) + 1/2*sqrt(2)*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e
^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*log(sqrt(2)*(b^2*c - 4*a*c^2)*s
qrt(e^2/(b^2*c^2 - 4*a*c^3))*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2
/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2)) + 2*sqrt(e*x + d)*e) - 1/2*sqrt(2
)*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2
*c - 4*a*c^2))*log(-sqrt(2)*(b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3))
*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*
c - 4*a*c^2)) + 2*sqrt(e*x + d)*e)
```

Sympy [A] time = 10.3997, size = 155, normalized size = 0.78

$$2e\,\mathrm{RootSum}\left(t^4\left(256a^2c^3e^4-128ab^2c^2e^4+16b^4ce^4\right)+t^2\left(-16abce^3+32ac^2de^2+4b^3e^3-8b^2cde^2\right)+ae^2-bde+cd^2,\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

```
[Out] 2*e*RootSum(_t**4*(256*a**2*c**3*e**4 - 128*a*b**2*c**2*e**4 + 16*b**4*c*e*
*4) + _t**2*(-16*a*b*c*e**3 + 32*a*c**2*d*e**2 + 4*b**3*e**3 - 8*b**2*c*d*e
**2) + a*e**2 - b*d*e + c*d**2, Lambda(_t, _t*log(64*_t**3*a*c**2*e**2 - 16
*_t**3*b**2*c*e**2 - 2*_t*b*e + 4*_t*c*d + sqrt(d + e*x))))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2292 $\int \frac{1}{\sqrt{d+ex}(a+bx+cx^2)} dx$

Optimal. Leaf size=199

$$\frac{2\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{2\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

```
[Out] (-2*Sqrt[2]*Sqrt[c]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b
 - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 -
4*a*c])*e]) + (2*Sqrt[2]*Sqrt[c]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sq
rt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 0.29574, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {707, 1093, 208}

$$\frac{2\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{2\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*Sqrt[2]*Sqrt[c]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b
 - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 -
4*a*c])*e]) + (2*Sqrt[2]*Sqrt[c]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sq
rt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e])
```

Rule 707

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Sym
bol] :> Dist[2*e, Subst[Int[1/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\left(a+bx+cx^2\right)}dx = (2e)\,\text{Subst}\left(\int \frac{1}{cd^2-bde+ae^2-(2cd-be)x^2+cx^4}dx, x, \sqrt{d+ex}\right)$$

$$= \frac{(2c)\,\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}e+\frac{1}{2}(-2cd+be)+cx^2}dx, x, \sqrt{d+ex}\right)}{\sqrt{b^2-4ac}} - \frac{(2c)\,\text{Subst}\left(\int \frac{1}{\frac{1}{2}\sqrt{b^2-4ac}e+\frac{1}{2}(-2cd+}\right.}{\sqrt{b^2-4a}}$$

$$= -\frac{2\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{2\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Mathematica [A] time = 0.534019, size = 176, normalized size = 0.88

$$\frac{2\sqrt{2}\sqrt{c}\left(\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} - \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[c]*(-(ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d -
b*e + Sqrt[b^2 - 4*a*c]*e]]/Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + Arc
Tanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
]]/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/Sqrt[b^2 - 4*a*c]
```

Maple [A] time = 0.242, size = 194, normalized size = 1.

$$-2\frac{ce\sqrt{2}}{\sqrt{-e^2\left(4ac-b^2\right)}\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\arctan\left(\frac{\sqrt{ex+d}c\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)-2\frac{}{\sqrt{-e^2\left(4ac-b^2\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(c*x^2+b*x+a),x)
```

```
[Out] -2*c*e/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2
))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(
1/2))*c)^(1/2))-2*c*e/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(
4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-
e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)\sqrt{ex+d}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)*sqrt(e*x + d)), x)
```

Fricas [B] time = 2.83425, size = 5416, normalized size = 27.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt((2*c*d - b*e + ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d
*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(e^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c -
4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2
*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*
a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2))*log(4*sqrt(e*x + d)*c*e + sqrt(2)*((b^
2 - 4*a*c)*e^2 - (2*(b^2*c^2 - 4*a*c^3)*d^3 - 3*(b^3*c - 4*a*b*c^2)*d^2*e +
 (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d*e^2 - (a*b^3 - 4*a^2*b*c)*e^3)*sqrt(e^2/((
b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8
*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))
)*sqrt((2*c*d - b*e + ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2
 - 4*a^2*c)*e^2)*sqrt(e^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*
d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3
 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e
+ (a*b^2 - 4*a^2*c)*e^2))) + 1/2*sqrt(2)*sqrt((2*c*d - b*e + ((b^2*c - 4*a*
c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(e^2/((b^2*c^2
- 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2
)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*
c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2))*log(4*sqrt
(e*x + d)*c*e - sqrt(2)*((b^2 - 4*a*c)*e^2 - (2*(b^2*c^2 - 4*a*c^3)*d^3 - 3
*(b^3*c - 4*a*b*c^2)*d^2*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d*e^2 - (a*b^3 -
 4*a^2*b*c)*e^3)*sqrt(e^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*
d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3
 + (a^2*b^2 - 4*a^3*c)*e^4)))*sqrt((2*c*d - b*e + ((b^2*c - 4*a*c^2)*d^2 -
(b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(e^2/((b^2*c^2 - 4*a*c^3)*
d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 -
 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2
)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2))) - 1/2*sqrt(2)*sqrt((
2*c*d - b*e - ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2
*c)*e^2)*sqrt(e^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e +
(b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*
b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2
 - 4*a^2*c)*e^2))*log(4*sqrt(e*x + d)*c*e + sqrt(2)*((b^2 - 4*a*c)*e^2 + (2
*(b^2*c^2 - 4*a*c^3)*d^3 - 3*(b^3*c - 4*a*b*c^2)*d^2*e + (b^4 - 2*a*b^2*c -
 8*a^2*c^2)*d*e^2 - (a*b^3 - 4*a^2*b*c)*e^3)*sqrt(e^2/((b^2*c^2 - 4*a*c^3)*
d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 -
 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))*sqrt((2*c*d - b*e
 - ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sq
rt(e^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*
b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3
*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)
*e^2))) + 1/2*sqrt(2)*sqrt((2*c*d - b*e - ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4
*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(e^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*
(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^
```

```
3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 -
(b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2))*log(4*sqrt(e*x + d)*c*e - sqr
t(2)*((b^2 - 4*a*c)*e^2 + (2*(b^2*c^2 - 4*a*c^3)*d^3 - 3*(b^3*c - 4*a*b*c^2
)*d^2*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d*e^2 - (a*b^3 - 4*a^2*b*c)*e^3)*sq
rt(e^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*
b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3
*c)*e^4)))*sqrt((2*c*d - b*e - ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e
 + (a*b^2 - 4*a^2*c)*e^2)*sqrt(e^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*
a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b
*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*
b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d+ex}\left(a+bx+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

```
[Out] Integral(1/(sqrt(d + e*x)*(a + b*x + c*x**2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2293 $\int \frac{1}{(d+ex)^{3/2}\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=310

$$-\frac{\sqrt{2}\sqrt{c}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{2}\sqrt{c}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(ae^2-bde+cd^2\right)}$$

```
[Out] (-2*e)/((c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x]) - (Sqrt[2]*Sqrt[c]*(2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c
*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt
[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) + (Sqrt[2]*Sqrt[c]*(2*c*d - (b -
 Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d -
 (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 0.728792, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {709, 826, 1166, 208}

$$-\frac{\sqrt{2}\sqrt{c}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{2}\sqrt{c}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*e)/((c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x]) - (Sqrt[2]*Sqrt[c]*(2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c
*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt
[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) + (Sqrt[2]*Sqrt[c]*(2*c*d - (b -
 Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d -
 (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2))
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
```

```
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\left(a+bx+cx^2\right)}\,dx = -\frac{2e}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}+\frac{\int \frac{cd-be-cex}{\sqrt{d+ex}\left(a+bx+cx^2\right)}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{2e}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}+\frac{2\,\mathrm{Subst}\left(\int \frac{cde+e(cd-be)-cex^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{cd^2-bde+ae^2}$$

$$= -\frac{2e}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}-\frac{\left(c\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\right)\mathrm{Subst}\left(\int \frac{1}{\frac{1}{2}\sqrt{b^2-4ac}e+\frac{1}{2}(-2cd\cdots}\right.}{\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{2e}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}-\frac{\sqrt{2}\sqrt{c}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4a\cdots}\right.}}\right.}{\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(cd^2-bde+ae^2\cdots\right.}$$

Mathematica [A] time = 0.412327, size = 273, normalized size = 0.88

$$\frac{2\left(-\frac{\sqrt{c}\left(e\left(\sqrt{b^2-4ac}+b\right)-2cd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}-\frac{\sqrt{c}\left(e\left(\sqrt{b^2-4ac}-b\right)+2cd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{e}{\sqrt{d+ex}}\right)}{e(bd-ae)-cd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*(a + b*x + c*x^2)),x]
```

```
[Out] (2*(e/Sqrt[d + e*x] - (Sqrt[c]*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*ArcTanh
[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/
(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) - (Sqr
t[c]*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d +
 e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]
*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])))/(-(c*d^2) + e*(b*d - a*e))
```

Maple [B] time = 0.258, size = 689, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(c*x^2+b*x+a),x)
```

```
[Out] -2*e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^(1/2)+c/(a*e^2-b*d*e+c*d^2)/(-e^2*(4*a*c-b
^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e
*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*e^2
-2*e*c^2/(a*e^2-b*d*e+c*d^2)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(
-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*
d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d-e*c/(a*e^2-b*d*e+c*d^2)*2^(1/2)/((b
*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/
((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))+c/(a*e^2-b*d*e+c*d^2)/(-e^2
*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)
*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^
(1/2))*b*e^2-2*e*c^2/(a*e^2-b*d*e+c*d^2)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/(
(-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1
/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d+e*c/(a*e^2-b*d*e+c*d
^2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)
^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + bx + a)(ex + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)*(e*x + d)^(3/2)), x)
```

Fricas [B] time = 6.49919, size = 23024, normalized size = 74.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*(c*d^3 - b*d^2*e + a*d*e^2 + (c*d^2*e - b*d*e^2 + a*e^3)*x)*sq
rt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)
*e^3 + ((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c
- 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e
^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*
d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3
*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2
*c + a^2*c^2)*e^6)/((b^2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^6)*d^11
*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*
a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^
```

5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^
5 + (b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)*d^6*
e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7 + 15*
(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5 - 5*a^4*b^3*
c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4 - 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^
10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (a^6*b^2 - 4*a^7*c)*e^12)))/((b^2*c^3
 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 -
4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 -
3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2
 - 4*a^4*c)*e^6))*log(sqrt(2)*(6*(b^2*c^3 - 4*a*c^4)*d^3*e^2 - 9*(b^3*c^2 -
 4*a*b*c^3)*d^2*e^3 + (5*b^4*c - 22*a*b^2*c^2 + 8*a^2*c^3)*d*e^4 - (b^5 - 5
*a*b^3*c + 4*a^2*b*c^2)*e^5 - (2*(b^2*c^5 - 4*a*c^6)*d^8 - 8*(b^3*c^4 - 4*a
*b*c^5)*d^7*e + (13*b^4*c^3 - 48*a*b^2*c^4 - 16*a^2*c^5)*d^6*e^2 - (11*b^5*
c^2 - 32*a*b^3*c^3 - 48*a^2*b*c^4)*d^5*e^3 + 5*(b^6*c - a*b^4*c^2 - 12*a^2*
b^2*c^3)*d^4*e^4 - (b^7 + 6*a*b^5*c - 40*a^2*b^3*c^2)*d^3*e^5 + (3*a*b^6 -
9*a^2*b^4*c - 16*a^3*b^2*c^2 + 16*a^4*c^3)*d^2*e^6 - (3*a^2*b^5 - 16*a^3*b^
3*c + 16*a^4*b*c^2)*d*e^7 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^8)*sqrt((
9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3
*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/((b^2*c^6 - 4*a*c^7)
d^12 - 6(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^
2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 + 15*
(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2 - 3
0*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 + (b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2
 - 340*a^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b
^3*c^2 - 40*a^4*b*c^3)*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*
d^4*e^8 - 10*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^
4 - 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (
a^6*b^2 - 4*a^7*c)*e^12)))*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a
*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 + ((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 -
 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*
a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e
^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6)*sqrt((9*c^4*d
^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*
b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/((b^2*c^6 - 4*a*c^7)*d^12 -
 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)*
d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 + 15*(b^6*c^
2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2 - 30*a^2*b
^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 + (b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 340*a
^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2
 - 40*a^4*b*c^3)*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8
 - 10*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4 - 18*
a^5*b^2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (a^6*b^2
 - 4*a^7*c)*e^12)))/((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*
e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2
*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3
 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6)) - 4*(3*c^4*d^2*e - 3*b*c^3*d
*e^2 + (b^2*c^2 - a*c^3)*e^3)*sqrt(e*x + d)) - sqrt(2)*(c*d^3 - b*d^2*e + a
*d*e^2 + (c*d^2*e - b*d*e^2 + a*e^3)*x)*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3
*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 + ((b^2*c^3 - 4*a*c^4)*d^6 -
 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^
2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a
^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6)*
sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 -
6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/((b^2*c^6 - 4*
a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5
 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3
 + 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c
^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 + (b^8 + 26*a*b^6*c - 30*a^2*b^
4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30

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*a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5
*c^3)*d^4*e^8 - 10*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*
a^4*b^4 - 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*e^
11 + (a^6*b^2 - 4*a^7*c)*e^12)))/((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*
a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b
^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4
- 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6))*log(-sqrt(2)*(6
*(b^2*c^3 - 4*a*c^4)*d^3*e^2 - 9*(b^3*c^2 - 4*a*b*c^3)*d^2*e^3 + (5*b^4*c -
 22*a*b^2*c^2 + 8*a^2*c^3)*d*e^4 - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e^5 - (2
*(b^2*c^5 - 4*a*c^6)*d^8 - 8*(b^3*c^4 - 4*a*b*c^5)*d^7*e + (13*b^4*c^3 - 48
*a*b^2*c^4 - 16*a^2*c^5)*d^6*e^2 - (11*b^5*c^2 - 32*a*b^3*c^3 - 48*a^2*b*c^
4)*d^5*e^3 + 5*(b^6*c - a*b^4*c^2 - 12*a^2*b^2*c^3)*d^4*e^4 - (b^7 + 6*a*b^
5*c - 40*a^2*b^3*c^2)*d^3*e^5 + (3*a*b^6 - 9*a^2*b^4*c - 16*a^3*b^2*c^2 + 1
6*a^4*c^3)*d^2*e^6 - (3*a^2*b^5 - 16*a^3*b^3*c + 16*a^4*b*c^2)*d*e^7 + (a^3
*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^8)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3
 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a
*b^2*c + a^2*c^2)*e^6)/((b^2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^6)*
d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3
- 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^
3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^
7*e^5 + (b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)*
d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7 +
 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5 - 5*a^4*
b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4 - 18*a^5*b^2*c - 8*a^6*c^2)*d^
2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (a^6*b^2 - 4*a^7*c)*e^12)))*sqrt(
(2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^
3 + ((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3
*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3
+ 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e
^5 + (a^3*b^2 - 4*a^4*c)*e^6)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5
*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c
+ a^2*c^2)*e^6)/((b^2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e
+ 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b
^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*
d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 +
 (b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6
 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7 + 15*(a^
2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5 - 5*a^4*b^3*c -
 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4 - 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10
- 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (a^6*b^2 - 4*a^7*c)*e^12)))/((b^2*c^3 -
4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a
^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a
^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 -
4*a^4*c)*e^6)) - 4*(3*c^4*d^2*e - 3*b*c^3*d*e^2 + (b^2*c^2 - a*c^3)*e^3)*sq
rt(e*x + d)) + sqrt(2)*(c*d^3 - b*d^2*e + a*d*e^2 + (c*d^2*e - b*d*e^2 + a*
e^3)*x)*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3
- 3*a*b*c)*e^3 - ((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e +
 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*
c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4
*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d
^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4
 - 2*a*b^2*c + a^2*c^2)*e^6)/((b^2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b
*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^
5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^4
- 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c
^4)*d^7*e^5 + (b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4
*c^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5
*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5 -
5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4 - 18*a^5*b^2*c - 8*a^6*c
```

^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (a^6*b^2 - 4*a^7*c)*e^12)))
/((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*
b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3
*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5
+ (a^3*b^2 - 4*a^4*c)*e^6))*log(sqrt(2)*(6*(b^2*c^3 - 4*a*c^4)*d^3*e^2 - 9*
(b^3*c^2 - 4*a*b*c^3)*d^2*e^3 + (5*b^4*c - 22*a*b^2*c^2 + 8*a^2*c^3)*d*e^4
- (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e^5 + (2*(b^2*c^5 - 4*a*c^6)*d^8 - 8*(b^3
*c^4 - 4*a*b*c^5)*d^7*e + (13*b^4*c^3 - 48*a*b^2*c^4 - 16*a^2*c^5)*d^6*e^2
- (11*b^5*c^2 - 32*a*b^3*c^3 - 48*a^2*b*c^4)*d^5*e^3 + 5*(b^6*c - a*b^4*c^2
 - 12*a^2*b^2*c^3)*d^4*e^4 - (b^7 + 6*a*b^5*c - 40*a^2*b^3*c^2)*d^3*e^5 + (
3*a*b^6 - 9*a^2*b^4*c - 16*a^3*b^2*c^2 + 16*a^4*c^3)*d^2*e^6 - (3*a^2*b^5 -
 16*a^3*b^3*c + 16*a^4*b*c^2)*d*e^7 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e
^8)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^
4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/((b^2*c^6
- 4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*
c^5 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9
e^3 + 15(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*b
^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 + (b^8 + 26*a*b^6*c - 30*a^
2*b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c
- 30*a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4
*a^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3
*(5*a^4*b^4 - 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*
d*e^11 + (a^6*b^2 - 4*a^7*c)*e^12)))*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b
^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 - ((b^2*c^3 - 4*a*c^4)*d^6 - 3*
(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 -
 (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*
c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6)*sqr
t((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(
b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/((b^2*c^6 - 4*a*c
^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8
*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 +
15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2
- 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 + (b^8 + 26*a*b^6*c - 30*a^2*b^4*c
^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^
3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^
3)*d^4*e^8 - 10*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4
*b^4 - 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11
+ (a^6*b^2 - 4*a^7*c)*e^12)))/((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b
*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*
c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3
*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6)) - 4*(3*c^4*d^2*e -
 3*b*c^3*d*e^2 + (b^2*c^2 - a*c^3)*e^3)*sqrt(e*x + d)) - sqrt(2)*(c*d^3 - b
*d^2*e + a*d*e^2 + (c*d^2*e - b*d*e^2 + a*e^3)*x)*sqrt((2*c^3*d^3 - 3*b*c^2
*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 - ((b^2*c^3 - 4*a*
c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c
^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b
^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^
4*c)*e^6)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*
d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/((b^
2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*
a*b^2*c^5 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^
5)*d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c +
 6*a*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 + (b^8 + 26*a*b^6*c -
 30*a^2*b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*
b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c
^2 - 4*a^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e
^9 + 3*(5*a^4*b^4 - 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6
*b*c)*d*e^11 + (a^6*b^2 - 4*a^7*c)*e^12)))/((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^
3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b

```
^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2
)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6))*log(-
sqrt(2)*(6*(b^2*c^3 - 4*a*c^4)*d^3*e^2 - 9*(b^3*c^2 - 4*a*b*c^3)*d^2*e^3 +
(5*b^4*c - 22*a*b^2*c^2 + 8*a^2*c^3)*d*e^4 - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2
)*e^5 + (2*(b^2*c^5 - 4*a*c^6)*d^8 - 8*(b^3*c^4 - 4*a*b*c^5)*d^7*e + (13*b^
4*c^3 - 48*a*b^2*c^4 - 16*a^2*c^5)*d^6*e^2 - (11*b^5*c^2 - 32*a*b^3*c^3 - 4
8*a^2*b*c^4)*d^5*e^3 + 5*(b^6*c - a*b^4*c^2 - 12*a^2*b^2*c^3)*d^4*e^4 - (b^
7 + 6*a*b^5*c - 40*a^2*b^3*c^2)*d^3*e^5 + (3*a*b^6 - 9*a^2*b^4*c - 16*a^3*b
^2*c^2 + 16*a^4*c^3)*d^2*e^6 - (3*a^2*b^5 - 16*a^3*b^3*c + 16*a^4*b*c^2)*d*
e^7 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^8)*sqrt((9*c^4*d^4*e^2 - 18*b*c
^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 +
(b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/((b^2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5 - 4
*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d^10*e^2 - 10*(
2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*
c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3
*b*c^4)*d^7*e^5 + (b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 340*a^3*b^2*c^3 - 80
*a^4*c^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*c^3)
*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8 - 10*(2*a^3*b^
5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4 - 18*a^5*b^2*c - 8*a
^6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (a^6*b^2 - 4*a^7*c)*e^1
2)))*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3
*a*b*c)*e^3 - ((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*
(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2
)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^
3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*
e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 -
2*a*b^2*c + a^2*c^2)*e^6)/((b^2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^
6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c
^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4
*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)
*d^7*e^5 + (b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c^
4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^
7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5 - 5*a
^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4 - 18*a^5*b^2*c - 8*a^6*c^2)
*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (a^6*b^2 - 4*a^7*c)*e^12)))/((
b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2
*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a
*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (
a^3*b^2 - 4*a^4*c)*e^6)) - 4*(3*c^4*d^2*e - 3*b*c^3*d*e^2 + (b^2*c^2 - a*c^
3)*e^3)*sqrt(e*x + d)) - 4*sqrt(e*x + d)*e)/(c*d^3 - b*d^2*e + a*d*e^2 + (c
*d^2*e - b*d*e^2 + a*e^3)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)^{\frac{3}{2}}\left(a+bx+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(c*x**2+b*x+a),x)
```

```
[Out] Integral(1/((d + e*x)**(3/2)*(a + b*x + c*x**2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2294 $\int \frac{1}{(d+ex)^{5/2}\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=414

$$-\frac{\sqrt{2}\sqrt{c}\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(ae^2-bde+cd^2\right)^2}+\frac{\sqrt{2}\sqrt{c}\left(-2ce\left(-d\cdots\right.\right.}{\cdots}$$

```
[Out] (-2*e)/(3*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(3/2)) - (2*e*(2*c*d - b*e))/((
c*d^2 - b*d*e + a*e^2)^2*Sqrt[d + e*x]) - (Sqrt[2]*Sqrt[c]*(2*c^2*d^2 + b*(
b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTa
nh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]]
)/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e
 + a*e^2)^2) + (Sqrt[2]*Sqrt[c]*(2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2
- 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d
+ e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2
*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)^2)
```

Rubi [A] time = 1.54601, antiderivative size = 414, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {709, 828, 826, 1166, 208}

$$-\frac{\sqrt{2}\sqrt{c}\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(ae^2-bde+cd^2\right)^2}+\frac{\sqrt{2}\sqrt{c}\left(-2ce\left(-d\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(5/2)*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*e)/(3*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(3/2)) - (2*e*(2*c*d - b*e))/((
c*d^2 - b*d*e + a*e^2)^2*Sqrt[d + e*x]) - (Sqrt[2]*Sqrt[c]*(2*c^2*d^2 + b*(
b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTa
nh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]]
)/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e
 + a*e^2)^2) + (Sqrt[2]*Sqrt[c]*(2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2
- 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d
+ e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2
*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)^2)
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{5/2}\left(a+bx+cx^2\right)}\,dx = -\frac{2e}{3\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} + \frac{\int \frac{cd-be-cex}{(d+ex)^{3/2}\left(a+bx+cx^2\right)}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{2e}{3\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} - \frac{2e(2cd-be)}{\left(cd^2-bde+ae^2\right)^2\sqrt{d+ex}} + \frac{\int \frac{c^2d^2+b^2e^2-ce(2bd+ae)-ce\ldots}{\sqrt{d+ex}\left(a+bx+cx^2\ldots\right.}}{\left(cd^2-bde+a\ldots\right.}$$

$$= -\frac{2e}{3\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} - \frac{2e(2cd-be)}{\left(cd^2-bde+ae^2\right)^2\sqrt{d+ex}} + \frac{2\,\mathrm{Subst}\left(\int \frac{cde(2cd-be)+\ldots}{cd\ldots}\right.}{\ldots}$$

$$= -\frac{2e}{3\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} - \frac{2e(2cd-be)}{\left(cd^2-bde+ae^2\right)^2\sqrt{d+ex}} - \frac{\left(c\left(2c^2d^2+b\left(b-\sqrt{b^2\ldots}\right.\right.\right.}{\ldots}$$

$$= -\frac{2e}{3\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} - \frac{2e(2cd-be)}{\left(cd^2-bde+ae^2\right)^2\sqrt{d+ex}} - \frac{\sqrt{2}\sqrt{c}\left(2c^2d^2+b\left(b+\ldots\right.\right.}{\sqrt{\ldots}}$$

Mathematica [A] time = 1.33603, size = 377, normalized size = 0.91

$$\frac{2\left(-\frac{3\sqrt{c}\left(\frac{\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\left(e(bd-ae)-cd^2\right)}\right.}{3\left(e(ae-bd)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(5/2)*(a + b*x + c*x^2)),x]
```

```
[Out] (2*(-(e/(d + e*x)^(3/2)) + (3*e*(-2*c*d + b*e))/((c*d^2 + e*(-(b*d) + a*e))
*Sqrt[d + e*x]) - (3*Sqrt[c]*(-(((2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2
 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d
 + e*x])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/Sqrt[2*c*d + (-b + Sqrt[
b^2 - 4*a*c])*e]) + ((2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*
d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqr
t[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e]))/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*(-(c*d^2) + e*(b*d - a*e)))))/(3*(c*d^2 +
e*(-(b*d) + a*e)))
```

Maple [B] time = 0.256, size = 1444, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(5/2)/(c*x^2+b*x+a),x)
```

```
[Out] -2/3*e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^(3/2)+2/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^(1
/2)*b*e^2-4*e/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^(1/2)*c*d+2/(a*e^2-b*d*e+c*d^2)
^2*c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2
))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(
1/2))*c)^(1/2))*a*e^3-1/(a*e^2-b*d*e+c*d^2)^2*c/(-e^2*(4*a*c-b^2))^(1/2)*2^
(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c
*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2*e^3+2/(a*e^2-b
*d*e+c*d^2)^2*c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c
-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*
a*c-b^2))^(1/2))*c)^(1/2))*b*d*e^2-2*e/(a*e^2-b*d*e+c*d^2)^2*c^3/(-e^2*(4*a
*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arcta
n((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d
^2+1/(a*e^2-b*d*e+c*d^2)^2*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*
c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2
))*c)^(1/2))*b*e^2-2*e/(a*e^2-b*d*e+c*d^2)^2*c^2*2^(1/2)/((b*e-2*c*d+(-e^2*
(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e
^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d+2/(a*e^2-b*d*e+c*d^2)^2*c^2/(-e^2*(4*a*c
-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan
h((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*
a*e^3-1/(a*e^2-b*d*e+c*d^2)^2*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c
```

```
*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*
e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2*e^3+2/(a*e^2-b*d*e+c*d^2)^2
*c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2)
)*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^
(1/2))*c)^(1/2))*b*d*e^2-2*e/(a*e^2-b*d*e+c*d^2)^2*c^3/(-e^2*(4*a*c-b^2))^(
1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d
)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d^2-1/(a
*e^2-b*d*e+c*d^2)^2*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/
2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c
)^(1/2))*b*e^2+2*e/(a*e^2-b*d*e+c*d^2)^2*c^2*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*
a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^
2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)\left(ex+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)*(e*x + d)^(5/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(5/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2295 $\int \frac{(d+ex)^{7/2}}{\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=691

$$\frac{e\sqrt{d+ex}\left(-2ce(5ae+bd)+3b^2e^2+2c^2d^2\right)}{c^2\left(b^2-4ac\right)}+\frac{\left(c^2e^2\left(-3bd\left(d\sqrt{b^2-4ac}+12ae\right)+2ae\left(13d\sqrt{b^2-4ac}-10ae\right)+3b^2d^2\right)-\right.}{}$$

```
[Out] (e*(2*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(b*d + 5*a*e))*Sqrt[d + e*x])/(c^2*(b^2 -
 4*a*c)) + (e*(2*c*d - b*e)*(d + e*x)^(3/2))/(c*(b^2 - 4*a*c)) - ((d + e*x)
^(5/2)*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) +
 ((8*c^4*d^4 - 3*b^3*(b - Sqrt[b^2 - 4*a*c])*e^4 - 2*c^3*d^2*e*(8*b*d - Sqr
t[b^2 - 4*a*c]*d - 18*a*e) + b*c*e^3*(5*b^2*d - 5*b*Sqrt[b^2 - 4*a*c]*d + 1
9*a*b*e - 13*a*Sqrt[b^2 - 4*a*c]*e) + c^2*e^2*(3*b^2*d^2 + 2*a*e*(13*Sqrt[b
^2 - 4*a*c]*d - 10*a*e) - 3*b*d*(Sqrt[b^2 - 4*a*c]*d + 12*a*e)))*ArcTanh[(S
qrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sq
rt[2]*c^(5/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])
- ((8*c^4*d^4 - 3*b^3*(b + Sqrt[b^2 - 4*a*c])*e^4 - 2*c^3*d^2*e*(8*b*d + Sq
rt[b^2 - 4*a*c]*d - 18*a*e) + b*c*e^3*(5*b^2*d + 5*b*Sqrt[b^2 - 4*a*c]*d +
19*a*b*e + 13*a*Sqrt[b^2 - 4*a*c]*e) + c^2*e^2*(3*b^2*d^2 + 3*b*d*(Sqrt[b^2
 - 4*a*c]*d - 12*a*e) - 2*a*e*(13*Sqrt[b^2 - 4*a*c]*d + 10*a*e)))*ArcTanh[(
Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(S
qrt[2]*c^(5/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 15.9623, antiderivative size = 691, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {738, 824, 826, 1166, 208}

$$\frac{e\sqrt{d+ex}\left(-2ce(5ae+bd)+3b^2e^2+2c^2d^2\right)}{c^2\left(b^2-4ac\right)}+\frac{\left(c^2e^2\left(-3bd\left(d\sqrt{b^2-4ac}+12ae\right)+2ae\left(13d\sqrt{b^2-4ac}-10ae\right)+3b^2d^2\right)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] (e*(2*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(b*d + 5*a*e))*Sqrt[d + e*x])/(c^2*(b^2 -
 4*a*c)) + (e*(2*c*d - b*e)*(d + e*x)^(3/2))/(c*(b^2 - 4*a*c)) - ((d + e*x)
^(5/2)*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) +
 ((8*c^4*d^4 - 3*b^3*(b - Sqrt[b^2 - 4*a*c])*e^4 - 2*c^3*d^2*e*(8*b*d - Sqr
t[b^2 - 4*a*c]*d - 18*a*e) + b*c*e^3*(5*b^2*d - 5*b*Sqrt[b^2 - 4*a*c]*d + 1
9*a*b*e - 13*a*Sqrt[b^2 - 4*a*c]*e) + c^2*e^2*(3*b^2*d^2 + 2*a*e*(13*Sqrt[b
^2 - 4*a*c]*d - 10*a*e) - 3*b*d*(Sqrt[b^2 - 4*a*c]*d + 12*a*e)))*ArcTanh[(S
qrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sq
rt[2]*c^(5/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])
- ((8*c^4*d^4 - 3*b^3*(b + Sqrt[b^2 - 4*a*c])*e^4 - 2*c^3*d^2*e*(8*b*d + Sq
rt[b^2 - 4*a*c]*d - 18*a*e) + b*c*e^3*(5*b^2*d + 5*b*Sqrt[b^2 - 4*a*c]*d +
19*a*b*e + 13*a*Sqrt[b^2 - 4*a*c]*e) + c^2*e^2*(3*b^2*d^2 + 3*b*d*(Sqrt[b^2
 - 4*a*c]*d - 12*a*e) - 2*a*e*(13*Sqrt[b^2 - 4*a*c]*d + 10*a*e)))*ArcTanh[(
Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(S
```

```
qrt[2]*c^(5/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{7/2}}{\left(a+bx+cx^2\right)^2}\,dx = -\frac{(d+ex)^{5/2}(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\int \frac{(d+ex)^{3/2}\left(\frac{1}{2}\left(4cd^2-7bde+10ae^2\right)-\frac{3}{2}e(2cd-be)x\right)}{a+bx+cx^2}\,dx}{-b^2+4ac}$$

$$= \frac{e(2cd-be)(d+ex)^{3/2}}{c\left(b^2-4ac\right)} - \frac{(d+ex)^{5/2}(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{\int \frac{\sqrt{d+ex}\left(\frac{1}{2}\left(4c^2d^3-3abe^3-cde(7bd-16ae)\right)-\cdots}{a+bx+cx^2}}{c\left(b^2-4a\cdots\right.}$$

$$= \frac{e\left(2c^2d^2+3b^2e^2-2ce(bd+5ae)\right)\sqrt{d+ex}}{c^2\left(b^2-4ac\right)} + \frac{e(2cd-be)(d+ex)^{3/2}}{c\left(b^2-4ac\right)} - \frac{(d+ex)^{5/2}(bd-2ae+(2c\cdots}{\left(b^2-4ac\right)\left(a+bx+c\cdots\right.}$$

$$= \frac{e\left(2c^2d^2+3b^2e^2-2ce(bd+5ae)\right)\sqrt{d+ex}}{c^2\left(b^2-4ac\right)} + \frac{e(2cd-be)(d+ex)^{3/2}}{c\left(b^2-4ac\right)} - \frac{(d+ex)^{5/2}(bd-2ae+(2c\cdots}{\left(b^2-4ac\right)\left(a+bx+c\cdots\right.}$$

$$= \frac{e\left(2c^2d^2+3b^2e^2-2ce(bd+5ae)\right)\sqrt{d+ex}}{c^2\left(b^2-4ac\right)} + \frac{e(2cd-be)(d+ex)^{3/2}}{c\left(b^2-4ac\right)} - \frac{(d+ex)^{5/2}(bd-2ae+(2c\cdots}{\left(b^2-4ac\right)\left(a+bx+c\cdots\right.}$$

$$= \frac{e\left(2c^2d^2+3b^2e^2-2ce(bd+5ae)\right)\sqrt{d+ex}}{c^2\left(b^2-4ac\right)} + \frac{e(2cd-be)(d+ex)^{3/2}}{c\left(b^2-4ac\right)} - \frac{(d+ex)^{5/2}(bd-2ae+(2c\cdots}{\left(b^2-4ac\right)\left(a+bx+c\cdots\right.}$$

Mathematica [A] time = 6.4917, size = 1116, normalized size = 1.62

$$-\frac{\left(-eb^2+cdb+2ace+c(2cd-be)x\right)(d+ex)^{9/2}}{\left(b^2-4ac\right)\left(cd^2-bed+ae^2\right)\left(cx^2+bx+a\right)}-\frac{2\left(\frac{2\left(\frac{2\left(\frac{4\left(\sqrt{2cd-be-\sqrt{b^2-4ac}e}\left(\frac{105}{32}c^2e(2cd-be)\left(cd^2-bed+ae^2\right)\left(c^2d^2-bced-3b^2e^2+13ace^2\right)-\frac{2c\left(\frac{105}{32}c^2e(c\cdots\right.}{\cdots}\right.\right.}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] -(((d + e*x)^(9/2)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4
*a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2))) - (-(e*(2*c*d - b*e)*(d +
 e*x)^(7/2)) + (2*(-7*c*e*(c*d^2 - e*(b*d - a*e))*(d + e*x)^(5/2) + (2*((-3
5*c*e*(2*c*d - b*e)*(c*d^2 - e*(b*d - a*e))*(d + e*x)^(3/2))/4 + (2*((-105*
c*e*(c*d^2 - b*d*e + a*e^2)*(2*c^2*d^2 - 2*b*c*d*e + 3*b^2*e^2 - 10*a*c*e^2
)*Sqrt[d + e*x])/8 + (4*((Sqrt[2*c*d - b*e - Sqrt[b^2 - 4*a*c]*e]*((105*c^2
*e*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*(c^2*d^2 - b*c*d*e - 3*b^2*e^2 + 1
3*a*c*e^2))/32 - ((-105*c^2*e*(2*c*d - b*e)*(-2*c*d + b*e)*(c*d^2 - b*d*e +
 a*e^2)*(c^2*d^2 - b*c*d*e - 3*b^2*e^2 + 13*a*c*e^2))/32 + 2*c*((-105*c^2*d
*e*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*(c^2*d^2 - b*c*d*e - 3*b^2*e^2 + 1
3*a*c*e^2))/32 + (105*c^2*e*(c*d^2 - b*d*e + a*e^2)*(4*c^3*d^4 - 7*b*c^2*d^
3*e + 18*a*c^2*d^2*e^2 - 5*a*b*c*d*e^3 + 3*a*b^2*e^4 - 10*a^2*c*e^4))/32))/
(Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d -
 b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[2]*Sqrt[c]*(-2*c*d + b*e + Sqrt[b^2 - 4
*a*c]*e)) + (Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*((105*c^2*e*(2*c*d - b
*e)*(c*d^2 - b*d*e + a*e^2)*(c^2*d^2 - b*c*d*e - 3*b^2*e^2 + 13*a*c*e^2))/3
2 + ((-105*c^2*e*(2*c*d - b*e)*(-2*c*d + b*e)*(c*d^2 - b*d*e + a*e^2)*(c^2*
d^2 - b*c*d*e - 3*b^2*e^2 + 13*a*c*e^2))/32 + 2*c*((-105*c^2*d*e*(2*c*d - b
```

```
*e)*(c*d^2 - b*d*e + a*e^2)*(c^2*d^2 - b*c*d*e - 3*b^2*e^2 + 13*a*c*e^2))/3
2 + (105*c^2*e*(c*d^2 - b*d*e + a*e^2)*(4*c^3*d^4 - 7*b*c^2*d^3*e + 18*a*c^
2*d^2*e^2 - 5*a*b*c*d*e^3 + 3*a*b^2*e^4 - 10*a^2*c*e^4))/32))/(Sqrt[b^2 - 4
*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - b*e + Sqrt[b
^2 - 4*a*c]*e]])/(Sqrt[2]*Sqrt[c]*(-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e))))/c
))/(3*c)))/(5*c)))/(7*c))/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))
```

Maple [B] time = 0.294, size = 3838, normalized size = 5.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/(c*x^2+b*x+a)^2,x)
```

```
[Out] -4*e*c^2/(4*a*c-b^2)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*
a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*
(4*a*c-b^2))^(1/2))*c)^(1/2))*d^4-13/2*e^4/c/(4*a*c-b^2)*2^(1/2)/((b*e-2*c*
d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2
*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b-3/2*e^3/(4*a*c-b^2)/(-e^2*(4*a
*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arct
anh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)
)*b^2*d^2-3/2*e^3/(4*a*c-b^2)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+
(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c
*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2*d^2+3/2*e^5/c^2/(4*a*c-b^2)/(-e^
2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)
*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1
/2))*b^4-19/2*e^5/c/(4*a*c-b^2)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c
*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*
e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b^2-5/2*e^4/c/(4*a*c-b^2)/(-e
^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2
)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(
1/2))*b^3*d+8*e^2*c/(4*a*c-b^2)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*
d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2
*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d^3-18*e^3*c/(4*a*c-b^2)/(-e^2*(
4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*a
rctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1
/2))*a*d^2-18*e^3*c/(4*a*c-b^2)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*
d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2
*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*d^2+18*e^4/(4*a*c-b^2)/(-e^2*(4*
a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arct
an((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*
a*b*d+18*e^4/(4*a*c-b^2)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^
2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d
+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b*d-19/2*e^5/c/(4*a*c-b^2)/(-e^2*(4*
a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arct
an((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*
a*b^2-5/2*e^4/c/(4*a*c-b^2)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(
-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*
c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^3*d+8*e^2*c/(4*a*c-b^2)/(-e^2*(4*
a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arc
tanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2
))*b*d^3+5/2*e^3/c/(4*a*c-b^2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2
))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))
^(1/2))*c)^(1/2))*b^2*d-5/2*e^3/c/(4*a*c-b^2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*
a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*
(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2*d+3/2*e^5/c^2/(4*a*c-b^2)/(-e^2*(4*a*c-b^
```

```
2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((
e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^4
-4*e*c^2/(4*a*c-b^2)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4
*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e
^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d^4+13/2*e^4/c/(4*a*c-b^2)*2^(1/2)/((-b*e+
2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((
-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b-13*e^3/(4*a*c-b^2)*2^(1/
2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*
2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*d+2*e^3/c^2*(e*x
+d)^(1/2)+13*e^3/(4*a*c-b^2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*
c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2
))*c)^(1/2))*a*d+3/2*e^4/c^2/(4*a*c-b^2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b
^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*
c-b^2))^(1/2))*c)^(1/2))*b^3-3/2*e^4/c^2/(4*a*c-b^2)*2^(1/2)/((-b*e+2*c*d+(
-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*
c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^3+3*e^3/c/(c*e^2*x^2+b*e^2*x+a*e^
2)/(4*a*c-b^2)*(e*x+d)^(3/2)*b^2*d-3/2*e^2/(4*a*c-b^2)*2^(1/2)/((b*e-2*c*d+
(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c
*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d^2+10*e^5/(4*a*c-b^2)/(-e^2*(4*a*
c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan
((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a^
2-3*e^3/c/(c*e^2*x^2+b*e^2*x+a*e^2)/(4*a*c-b^2)*(e*x+d)^(1/2)*b^2*d^2-e*c/(
4*a*c-b^2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(
(e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d^
3+e*c/(4*a*c-b^2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*ar
ctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)
)*d^3+3*e^4/c/(c*e^2*x^2+b*e^2*x+a*e^2)/(4*a*c-b^2)*(e*x+d)^(3/2)*a*b-e^5/c
^2/(c*e^2*x^2+b*e^2*x+a*e^2)/(4*a*c-b^2)*(e*x+d)^(1/2)*a*b^2+e^4/c^2/(c*e^2
*x^2+b*e^2*x+a*e^2)/(4*a*c-b^2)*(e*x+d)^(1/2)*b^3*d+10*e^5/(4*a*c-b^2)/(-e^
2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2
)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)
^(1/2))*a^2+3/2*e^2/(4*a*c-b^2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/
2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2)
)^(1/2))*c)^(1/2))*b*d^2-3*e^2/(c*e^2*x^2+b*e^2*x+a*e^2)/(4*a*c-b^2)*(e*x+d
)^(3/2)*b*d^2+4*e^2/(c*e^2*x^2+b*e^2*x+a*e^2)/(4*a*c-b^2)*(e*x+d)^(1/2)*b*d
^3-e^4/c^2/(c*e^2*x^2+b*e^2*x+a*e^2)/(4*a*c-b^2)*(e*x+d)^(3/2)*b^3+2*e*c/(c
*e^2*x^2+b*e^2*x+a*e^2)/(4*a*c-b^2)*(e*x+d)^(3/2)*d^3+2*e^5/c/(c*e^2*x^2+b*
e^2*x+a*e^2)/(4*a*c-b^2)*(e*x+d)^(1/2)*a^2-2*e*c/(c*e^2*x^2+b*e^2*x+a*e^2)/
(4*a*c-b^2)*(e*x+d)^(1/2)*d^4-6*e^3/(c*e^2*x^2+b*e^2*x+a*e^2)/(4*a*c-b^2)*(
e*x+d)^(3/2)*a*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{7}{2}}}{\left(cx^2+bx+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(7/2)/(c*x^2 + b*x + a)^2, x)
```

Fricas [B] time = 64.651, size = 22507, normalized size = 32.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2
 - 4*a*b*c^3)*x)*sqrt((32*c^7*d^7 - 112*b*c^6*d^6*e + 14*(7*b^2*c^5 + 20*a*
c^6)*d^5*e^2 + 35*(b^3*c^4 - 20*a*b*c^5)*d^4*e^3 - 70*(b^4*c^3 - 6*a*b^2*c^
4 - 8*a^2*c^5)*d^3*e^4 + 14*(b^5*c^2 + 5*a*b^3*c^3 - 60*a^2*b*c^4)*d^2*e^5
+ 7*(3*b^6*c - 40*a*b^4*c^2 + 150*a^2*b^2*c^3 - 120*a^3*c^4)*d*e^6 - (9*b^7
 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*e^7 + (b^6*c^5 - 12*a*b^4
*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((1225*c^8*d^8*e^6 - 4900*b*c^7*d^7
*e^7 + 980*(6*b^2*c^6 + 11*a*c^7)*d^6*e^8 - 490*(b^3*c^5 + 66*a*b*c^6)*d^5*
e^9 - 14*(241*b^4*c^4 - 2103*a*b^2*c^5 - 1569*a^2*c^6)*d^4*e^10 + 28*(66*b^
5*c^3 - 178*a*b^3*c^4 - 1569*a^2*b*c^5)*d^3*e^11 + 7*(27*b^6*c^2 - 1116*a*b
^4*c^3 + 5532*a^2*b^2*c^4 - 1100*a^3*c^5)*d^2*e^12 - 14*(27*b^7*c - 351*a*b
^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*d*e^13 + (81*b^8 - 918*a*b^6*c +
 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*e^14)/(b^6*c^10 - 12*a*
b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^
2*b^2*c^7 - 64*a^3*c^8))*log(sqrt(1/2)*(70*(b^4*c^6 - 8*a*b^2*c^7 + 16*a^2*
c^8)*d^6*e^4 - 210*(b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*d^5*e^5 + 21*(13*
b^6*c^4 - 106*a*b^4*c^5 + 224*a^2*b^2*c^6 - 32*a^3*c^7)*d^4*e^6 - 28*(7*b^7
*c^3 - 59*a*b^5*c^4 + 136*a^2*b^3*c^5 - 48*a^3*b*c^6)*d^3*e^7 - 6*(3*b^8*c^
2 - 146*a*b^6*c^3 + 1289*a^2*b^4*c^4 - 4072*a^3*b^2*c^5 + 4240*a^4*c^6)*d^2
*e^8 + 3*(27*b^9*c - 474*a*b^7*c^2 + 3026*a^2*b^5*c^3 - 8368*a^3*b^3*c^4 +
8480*a^4*b*c^5)*d*e^9 - (27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^
3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5)*e^10 - (8*(b^6*c^8 - 12*a*b^4
*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)*d^3 - 12*(b^7*c^7 - 12*a*b^5*c^8 + 48
*a^2*b^3*c^9 - 64*a^3*b*c^10)*d^2*e - 2*(b^8*c^6 - 28*a*b^6*c^7 + 240*a^2*b
^4*c^8 - 832*a^3*b^2*c^9 + 1024*a^4*c^10)*d*e^2 + (3*b^9*c^5 - 52*a*b^7*c^6
 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*e^3)*sqrt((1225*c^8*
d^8*e^6 - 4900*b*c^7*d^7*e^7 + 980*(6*b^2*c^6 + 11*a*c^7)*d^6*e^8 - 490*(b^
3*c^5 + 66*a*b*c^6)*d^5*e^9 - 14*(241*b^4*c^4 - 2103*a*b^2*c^5 - 1569*a^2*c
^6)*d^4*e^10 + 28*(66*b^5*c^3 - 178*a*b^3*c^4 - 1569*a^2*b*c^5)*d^3*e^11 +
7*(27*b^6*c^2 - 1116*a*b^4*c^3 + 5532*a^2*b^2*c^4 - 1100*a^3*c^5)*d^2*e^12
- 14*(27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*d*e^13 +
 (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)
*e^14)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt((3
2*c^7*d^7 - 112*b*c^6*d^6*e + 14*(7*b^2*c^5 + 20*a*c^6)*d^5*e^2 + 35*(b^3*c
^4 - 20*a*b*c^5)*d^4*e^3 - 70*(b^4*c^3 - 6*a*b^2*c^4 - 8*a^2*c^5)*d^3*e^4 +
 14*(b^5*c^2 + 5*a*b^3*c^3 - 60*a^2*b*c^4)*d^2*e^5 + 7*(3*b^6*c - 40*a*b^4*
c^2 + 150*a^2*b^2*c^3 - 120*a^3*c^4)*d*e^6 - (9*b^7 - 105*a*b^5*c + 385*a^2
*b^3*c^2 - 420*a^3*b*c^3)*e^7 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 -
64*a^3*c^8)*sqrt((1225*c^8*d^8*e^6 - 4900*b*c^7*d^7*e^7 + 980*(6*b^2*c^6 +
11*a*c^7)*d^6*e^8 - 490*(b^3*c^5 + 66*a*b*c^6)*d^5*e^9 - 14*(241*b^4*c^4 -
2103*a*b^2*c^5 - 1569*a^2*c^6)*d^4*e^10 + 28*(66*b^5*c^3 - 178*a*b^3*c^4 -
1569*a^2*b*c^5)*d^3*e^11 + 7*(27*b^6*c^2 - 1116*a*b^4*c^3 + 5532*a^2*b^2*c^
4 - 1100*a^3*c^5)*d^2*e^12 - 14*(27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^
3 - 550*a^3*b*c^4)*d*e^13 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550
*a^3*b^2*c^3 + 625*a^4*c^4)*e^14)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^
12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))
 - 2*(560*c^8*d^10*e^3 - 2800*b*c^7*d^9*e^4 + 7*(647*b^2*c^6 + 1012*a*c^7)*
d^8*e^5 - 28*(47*b^3*c^5 + 1012*a*b*c^6)*d^7*e^6 - (3329*b^4*c^4 - 35844*a*
b^2*c^5 - 27488*a^2*c^6)*d^6*e^7 + (2833*b^5*c^3 - 8356*a*b^3*c^4 - 82464*a
^2*b*c^5)*d^5*e^8 + (9*b^6*c^2 - 14273*a*b^4*c^3 + 77982*a^2*b^2*c^4 + 3346
4*a^3*c^5)*d^4*e^9 - (675*b^7*c - 9414*a*b^5*c^2 + 18524*a^2*b^3*c^3 + 6692
8*a^3*b*c^4)*d^3*e^10 + (189*b^8 - 999*a*b^6*c - 8127*a^2*b^4*c^2 + 40196*a
^3*b^2*c^3 + 10000*a^4*c^4)*d^2*e^11 - (378*a*b^7 - 3645*a^2*b^5*c + 6732*a
^3*b^3*c^2 + 10000*a^4*b*c^3)*d*e^12 + (189*a^2*b^6 - 1971*a^3*b^4*c + 5625
*a^4*b^2*c^2 - 2500*a^5*c^3)*e^13)*sqrt(e*x + d)) - sqrt(1/2)*(a*b^2*c^2 -
```

4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x)*sqrt((32*c^7
*d^7 - 112*b*c^6*d^6*e + 14*(7*b^2*c^5 + 20*a*c^6)*d^5*e^2 + 35*(b^3*c^4 -
20*a*b*c^5)*d^4*e^3 - 70*(b^4*c^3 - 6*a*b^2*c^4 - 8*a^2*c^5)*d^3*e^4 + 14*(
b^5*c^2 + 5*a*b^3*c^3 - 60*a^2*b*c^4)*d^2*e^5 + 7*(3*b^6*c - 40*a*b^4*c^2 +
150*a^2*b^2*c^3 - 120*a^3*c^4)*d*e^6 - (9*b^7 - 105*a*b^5*c + 385*a^2*b^3*
c^2 - 420*a^3*b*c^3)*e^7 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^
3*c^8)*sqrt((1225*c^8*d^8*e^6 - 4900*b*c^7*d^7*e^7 + 980*(6*b^2*c^6 + 11*a*
c^7)*d^6*e^8 - 490*(b^3*c^5 + 66*a*b*c^6)*d^5*e^9 - 14*(241*b^4*c^4 - 2103*
a*b^2*c^5 - 1569*a^2*c^6)*d^4*e^10 + 28*(66*b^5*c^3 - 178*a*b^3*c^4 - 1569*
a^2*b*c^5)*d^3*e^11 + 7*(27*b^6*c^2 - 1116*a*b^4*c^3 + 5532*a^2*b^2*c^4 - 1
100*a^3*c^5)*d^2*e^12 - 14*(27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 5
50*a^3*b*c^4)*d*e^13 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*
b^2*c^3 + 625*a^4*c^4)*e^14)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 -
64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*log(
-sqrt(1/2)*(70*(b^4*c^6 - 8*a*b^2*c^7 + 16*a^2*c^8)*d^6*e^4 - 210*(b^5*c^5
- 8*a*b^3*c^6 + 16*a^2*b*c^7)*d^5*e^5 + 21*(13*b^6*c^4 - 106*a*b^4*c^5 + 22
4*a^2*b^2*c^6 - 32*a^3*c^7)*d^4*e^6 - 28*(7*b^7*c^3 - 59*a*b^5*c^4 + 136*a^
2*b^3*c^5 - 48*a^3*b*c^6)*d^3*e^7 - 6*(3*b^8*c^2 - 146*a*b^6*c^3 + 1289*a^2
*b^4*c^4 - 4072*a^3*b^2*c^5 + 4240*a^4*c^6)*d^2*e^8 + 3*(27*b^9*c - 474*a*b
^7*c^2 + 3026*a^2*b^5*c^3 - 8368*a^3*b^3*c^4 + 8480*a^4*b*c^5)*d*e^9 - (27*
b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^3*b^4*c^3 + 11360*a^4*b^2*c^
4 - 4000*a^5*c^5)*e^10 - (8*(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*
a^3*c^11)*d^3 - 12*(b^7*c^7 - 12*a*b^5*c^8 + 48*a^2*b^3*c^9 - 64*a^3*b*c^10
)*d^2*e - 2*(b^8*c^6 - 28*a*b^6*c^7 + 240*a^2*b^4*c^8 - 832*a^3*b^2*c^9 + 1
024*a^4*c^10)*d*e^2 + (3*b^9*c^5 - 52*a*b^7*c^6 + 336*a^2*b^5*c^7 - 960*a^3
*b^3*c^8 + 1024*a^4*b*c^9)*e^3)*sqrt((1225*c^8*d^8*e^6 - 4900*b*c^7*d^7*e^7
+ 980*(6*b^2*c^6 + 11*a*c^7)*d^6*e^8 - 490*(b^3*c^5 + 66*a*b*c^6)*d^5*e^9
- 14*(241*b^4*c^4 - 2103*a*b^2*c^5 - 1569*a^2*c^6)*d^4*e^10 + 28*(66*b^5*c^
3 - 178*a*b^3*c^4 - 1569*a^2*b*c^5)*d^3*e^11 + 7*(27*b^6*c^2 - 1116*a*b^4*c
^3 + 5532*a^2*b^2*c^4 - 1100*a^3*c^5)*d^2*e^12 - 14*(27*b^7*c - 351*a*b^5*c
^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*d*e^13 + (81*b^8 - 918*a*b^6*c + 305
1*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*e^14)/(b^6*c^10 - 12*a*b^4*
c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt((32*c^7*d^7 - 112*b*c^6*d^6*e
+ 14*(7*b^2*c^5 + 20*a*c^6)*d^5*e^2 + 35*(b^3*c^4 - 20*a*b*c^5)*d^4*e^3 - 7
0*(b^4*c^3 - 6*a*b^2*c^4 - 8*a^2*c^5)*d^3*e^4 + 14*(b^5*c^2 + 5*a*b^3*c^3 -
60*a^2*b*c^4)*d^2*e^5 + 7*(3*b^6*c - 40*a*b^4*c^2 + 150*a^2*b^2*c^3 - 120*
a^3*c^4)*d*e^6 - (9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*e^
7 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((1225*c^8*d
^8*e^6 - 4900*b*c^7*d^7*e^7 + 980*(6*b^2*c^6 + 11*a*c^7)*d^6*e^8 - 490*(b^3
*c^5 + 66*a*b*c^6)*d^5*e^9 - 14*(241*b^4*c^4 - 2103*a*b^2*c^5 - 1569*a^2*c^
6)*d^4*e^10 + 28*(66*b^5*c^3 - 178*a*b^3*c^4 - 1569*a^2*b*c^5)*d^3*e^11 + 7
*(27*b^6*c^2 - 1116*a*b^4*c^3 + 5532*a^2*b^2*c^4 - 1100*a^3*c^5)*d^2*e^12 -
14*(27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*d*e^13 +
(81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*
e^14)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5
- 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)) - 2*(560*c^8*d^10*e^3 - 280
0*b*c^7*d^9*e^4 + 7*(647*b^2*c^6 + 1012*a*c^7)*d^8*e^5 - 28*(47*b^3*c^5 + 1
012*a*b*c^6)*d^7*e^6 - (3329*b^4*c^4 - 35844*a*b^2*c^5 - 27488*a^2*c^6)*d^6
*e^7 + (2833*b^5*c^3 - 8356*a*b^3*c^4 - 82464*a^2*b*c^5)*d^5*e^8 + (9*b^6*c
^2 - 14273*a*b^4*c^3 + 77982*a^2*b^2*c^4 + 33464*a^3*c^5)*d^4*e^9 - (675*b^
7*c - 9414*a*b^5*c^2 + 18524*a^2*b^3*c^3 + 66928*a^3*b*c^4)*d^3*e^10 + (189
*b^8 - 999*a*b^6*c - 8127*a^2*b^4*c^2 + 40196*a^3*b^2*c^3 + 10000*a^4*c^4)*
d^2*e^11 - (378*a*b^7 - 3645*a^2*b^5*c + 6732*a^3*b^3*c^2 + 10000*a^4*b*c^3
)*d*e^12 + (189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*a^5*c^3)
*e^13)*sqrt(e*x + d)) + sqrt(1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c
^4)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x)*sqrt((32*c^7*d^7 - 112*b*c^6*d^6*e + 14*
(7*b^2*c^5 + 20*a*c^6)*d^5*e^2 + 35*(b^3*c^4 - 20*a*b*c^5)*d^4*e^3 - 70*(b^
4*c^3 - 6*a*b^2*c^4 - 8*a^2*c^5)*d^3*e^4 + 14*(b^5*c^2 + 5*a*b^3*c^3 - 60*a
^2*b*c^4)*d^2*e^5 + 7*(3*b^6*c - 40*a*b^4*c^2 + 150*a^2*b^2*c^3 - 120*a^3*c

^4)*d*e^6 - (9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*e^7 - (
b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((1225*c^8*d^8*e^
6 - 4900*b*c^7*d^7*e^7 + 980*(6*b^2*c^6 + 11*a*c^7)*d^6*e^8 - 490*(b^3*c^5
+ 66*a*b*c^6)*d^5*e^9 - 14*(241*b^4*c^4 - 2103*a*b^2*c^5 - 1569*a^2*c^6)*d^
4*e^10 + 28*(66*b^5*c^3 - 178*a*b^3*c^4 - 1569*a^2*b*c^5)*d^3*e^11 + 7*(27*
b^6*c^2 - 1116*a*b^4*c^3 + 5532*a^2*b^2*c^4 - 1100*a^3*c^5)*d^2*e^12 - 14*(
27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*d*e^13 + (81*b
^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*e^14)
/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12
*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*log(sqrt(1/2)*(70*(b^4*c^6 - 8*a
*b^2*c^7 + 16*a^2*c^8)*d^6*e^4 - 210*(b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)
*d^5*e^5 + 21*(13*b^6*c^4 - 106*a*b^4*c^5 + 224*a^2*b^2*c^6 - 32*a^3*c^7)*d
^4*e^6 - 28*(7*b^7*c^3 - 59*a*b^5*c^4 + 136*a^2*b^3*c^5 - 48*a^3*b*c^6)*d^3
e^7 - 6(3*b^8*c^2 - 146*a*b^6*c^3 + 1289*a^2*b^4*c^4 - 4072*a^3*b^2*c^5 +
4240*a^4*c^6)*d^2*e^8 + 3*(27*b^9*c - 474*a*b^7*c^2 + 3026*a^2*b^5*c^3 - 8
368*a^3*b^3*c^4 + 8480*a^4*b*c^5)*d*e^9 - (27*b^10 - 459*a*b^8*c + 2961*a^2
*b^6*c^2 - 8818*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5)*e^10 + (8*(
b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)*d^3 - 12*(b^7*c^7 -
12*a*b^5*c^8 + 48*a^2*b^3*c^9 - 64*a^3*b*c^10)*d^2*e - 2*(b^8*c^6 - 28*a*b
^6*c^7 + 240*a^2*b^4*c^8 - 832*a^3*b^2*c^9 + 1024*a^4*c^10)*d*e^2 + (3*b^9*
c^5 - 52*a*b^7*c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*e^
3)*sqrt((1225*c^8*d^8*e^6 - 4900*b*c^7*d^7*e^7 + 980*(6*b^2*c^6 + 11*a*c^7)
*d^6*e^8 - 490*(b^3*c^5 + 66*a*b*c^6)*d^5*e^9 - 14*(241*b^4*c^4 - 2103*a*b^
2*c^5 - 1569*a^2*c^6)*d^4*e^10 + 28*(66*b^5*c^3 - 178*a*b^3*c^4 - 1569*a^2*
b*c^5)*d^3*e^11 + 7*(27*b^6*c^2 - 1116*a*b^4*c^3 + 5532*a^2*b^2*c^4 - 1100*
a^3*c^5)*d^2*e^12 - 14*(27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a
^3*b*c^4)*d*e^13 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*
c^3 + 625*a^4*c^4)*e^14)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a
^3*c^13)))*sqrt((32*c^7*d^7 - 112*b*c^6*d^6*e + 14*(7*b^2*c^5 + 20*a*c^6)*d
^5*e^2 + 35*(b^3*c^4 - 20*a*b*c^5)*d^4*e^3 - 70*(b^4*c^3 - 6*a*b^2*c^4 - 8*
a^2*c^5)*d^3*e^4 + 14*(b^5*c^2 + 5*a*b^3*c^3 - 60*a^2*b*c^4)*d^2*e^5 + 7*(3
*b^6*c - 40*a*b^4*c^2 + 150*a^2*b^2*c^3 - 120*a^3*c^4)*d*e^6 - (9*b^7 - 105
*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*e^7 - (b^6*c^5 - 12*a*b^4*c^6 +
48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((1225*c^8*d^8*e^6 - 4900*b*c^7*d^7*e^7 +
980*(6*b^2*c^6 + 11*a*c^7)*d^6*e^8 - 490*(b^3*c^5 + 66*a*b*c^6)*d^5*e^9 -
14*(241*b^4*c^4 - 2103*a*b^2*c^5 - 1569*a^2*c^6)*d^4*e^10 + 28*(66*b^5*c^3
- 178*a*b^3*c^4 - 1569*a^2*b*c^5)*d^3*e^11 + 7*(27*b^6*c^2 - 1116*a*b^4*c^3
+ 5532*a^2*b^2*c^4 - 1100*a^3*c^5)*d^2*e^12 - 14*(27*b^7*c - 351*a*b^5*c^2
+ 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*d*e^13 + (81*b^8 - 918*a*b^6*c + 3051*
a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*e^14)/(b^6*c^10 - 12*a*b^4*c^
11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*
c^7 - 64*a^3*c^8)) - 2*(560*c^8*d^10*e^3 - 2800*b*c^7*d^9*e^4 + 7*(647*b^2*
c^6 + 1012*a*c^7)*d^8*e^5 - 28*(47*b^3*c^5 + 1012*a*b*c^6)*d^7*e^6 - (3329*
b^4*c^4 - 35844*a*b^2*c^5 - 27488*a^2*c^6)*d^6*e^7 + (2833*b^5*c^3 - 8356*a
*b^3*c^4 - 82464*a^2*b*c^5)*d^5*e^8 + (9*b^6*c^2 - 14273*a*b^4*c^3 + 77982*
a^2*b^2*c^4 + 33464*a^3*c^5)*d^4*e^9 - (675*b^7*c - 9414*a*b^5*c^2 + 18524*
a^2*b^3*c^3 + 66928*a^3*b*c^4)*d^3*e^10 + (189*b^8 - 999*a*b^6*c - 8127*a^2
*b^4*c^2 + 40196*a^3*b^2*c^3 + 10000*a^4*c^4)*d^2*e^11 - (378*a*b^7 - 3645*
a^2*b^5*c + 6732*a^3*b^3*c^2 + 10000*a^4*b*c^3)*d*e^12 + (189*a^2*b^6 - 197
1*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*a^5*c^3)*e^13)*sqrt(e*x + d)) - sqrt(
1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2 - 4*a*b*c^
3)*x)*sqrt((32*c^7*d^7 - 112*b*c^6*d^6*e + 14*(7*b^2*c^5 + 20*a*c^6)*d^5*e^
2 + 35*(b^3*c^4 - 20*a*b*c^5)*d^4*e^3 - 70*(b^4*c^3 - 6*a*b^2*c^4 - 8*a^2*c
^5)*d^3*e^4 + 14*(b^5*c^2 + 5*a*b^3*c^3 - 60*a^2*b*c^4)*d^2*e^5 + 7*(3*b^6*
c - 40*a*b^4*c^2 + 150*a^2*b^2*c^3 - 120*a^3*c^4)*d*e^6 - (9*b^7 - 105*a*b^
5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*e^7 - (b^6*c^5 - 12*a*b^4*c^6 + 48*a
^2*b^2*c^7 - 64*a^3*c^8)*sqrt((1225*c^8*d^8*e^6 - 4900*b*c^7*d^7*e^7 + 980*
(6*b^2*c^6 + 11*a*c^7)*d^6*e^8 - 490*(b^3*c^5 + 66*a*b*c^6)*d^5*e^9 - 14*(2
41*b^4*c^4 - 2103*a*b^2*c^5 - 1569*a^2*c^6)*d^4*e^10 + 28*(66*b^5*c^3 - 178

```
*a*b^3*c^4 - 1569*a^2*b*c^5)*d^3*e^11 + 7*(27*b^6*c^2 - 1116*a*b^4*c^3 + 55
32*a^2*b^2*c^4 - 1100*a^3*c^5)*d^2*e^12 - 14*(27*b^7*c - 351*a*b^5*c^2 + 11
97*a^2*b^3*c^3 - 550*a^3*b*c^4)*d*e^13 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b
^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*e^14)/(b^6*c^10 - 12*a*b^4*c^11 +
48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 -
 64*a^3*c^8))*log(-sqrt(1/2)*(70*(b^4*c^6 - 8*a*b^2*c^7 + 16*a^2*c^8)*d^6*e
^4 - 210*(b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*d^5*e^5 + 21*(13*b^6*c^4 -
106*a*b^4*c^5 + 224*a^2*b^2*c^6 - 32*a^3*c^7)*d^4*e^6 - 28*(7*b^7*c^3 - 59*
a*b^5*c^4 + 136*a^2*b^3*c^5 - 48*a^3*b*c^6)*d^3*e^7 - 6*(3*b^8*c^2 - 146*a*
b^6*c^3 + 1289*a^2*b^4*c^4 - 4072*a^3*b^2*c^5 + 4240*a^4*c^6)*d^2*e^8 + 3*(
27*b^9*c - 474*a*b^7*c^2 + 3026*a^2*b^5*c^3 - 8368*a^3*b^3*c^4 + 8480*a^4*b
*c^5)*d*e^9 - (27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^3*b^4*c^3
+ 11360*a^4*b^2*c^4 - 4000*a^5*c^5)*e^10 + (8*(b^6*c^8 - 12*a*b^4*c^9 + 48*
a^2*b^2*c^10 - 64*a^3*c^11)*d^3 - 12*(b^7*c^7 - 12*a*b^5*c^8 + 48*a^2*b^3*c
^9 - 64*a^3*b*c^10)*d^2*e - 2*(b^8*c^6 - 28*a*b^6*c^7 + 240*a^2*b^4*c^8 - 8
32*a^3*b^2*c^9 + 1024*a^4*c^10)*d*e^2 + (3*b^9*c^5 - 52*a*b^7*c^6 + 336*a^2
*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*e^3)*sqrt((1225*c^8*d^8*e^6 -
4900*b*c^7*d^7*e^7 + 980*(6*b^2*c^6 + 11*a*c^7)*d^6*e^8 - 490*(b^3*c^5 + 66
*a*b*c^6)*d^5*e^9 - 14*(241*b^4*c^4 - 2103*a*b^2*c^5 - 1569*a^2*c^6)*d^4*e^
10 + 28*(66*b^5*c^3 - 178*a*b^3*c^4 - 1569*a^2*b*c^5)*d^3*e^11 + 7*(27*b^6*
c^2 - 1116*a*b^4*c^3 + 5532*a^2*b^2*c^4 - 1100*a^3*c^5)*d^2*e^12 - 14*(27*b
^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*d*e^13 + (81*b^8 -
 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*e^14)/(b^
6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt((32*c^7*d^7
- 112*b*c^6*d^6*e + 14*(7*b^2*c^5 + 20*a*c^6)*d^5*e^2 + 35*(b^3*c^4 - 20*a*
b*c^5)*d^4*e^3 - 70*(b^4*c^3 - 6*a*b^2*c^4 - 8*a^2*c^5)*d^3*e^4 + 14*(b^5*c
^2 + 5*a*b^3*c^3 - 60*a^2*b*c^4)*d^2*e^5 + 7*(3*b^6*c - 40*a*b^4*c^2 + 150*
a^2*b^2*c^3 - 120*a^3*c^4)*d*e^6 - (9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 -
 420*a^3*b*c^3)*e^7 - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8
)*sqrt((1225*c^8*d^8*e^6 - 4900*b*c^7*d^7*e^7 + 980*(6*b^2*c^6 + 11*a*c^7)*
d^6*e^8 - 490*(b^3*c^5 + 66*a*b*c^6)*d^5*e^9 - 14*(241*b^4*c^4 - 2103*a*b^2
*c^5 - 1569*a^2*c^6)*d^4*e^10 + 28*(66*b^5*c^3 - 178*a*b^3*c^4 - 1569*a^2*b
*c^5)*d^3*e^11 + 7*(27*b^6*c^2 - 1116*a*b^4*c^3 + 5532*a^2*b^2*c^4 - 1100*a
^3*c^5)*d^2*e^12 - 14*(27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^
3*b*c^4)*d*e^13 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c
^3 + 625*a^4*c^4)*e^14)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^
3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)) - 2*(560*
c^8*d^10*e^3 - 2800*b*c^7*d^9*e^4 + 7*(647*b^2*c^6 + 1012*a*c^7)*d^8*e^5 -
28*(47*b^3*c^5 + 1012*a*b*c^6)*d^7*e^6 - (3329*b^4*c^4 - 35844*a*b^2*c^5 -
27488*a^2*c^6)*d^6*e^7 + (2833*b^5*c^3 - 8356*a*b^3*c^4 - 82464*a^2*b*c^5)*
d^5*e^8 + (9*b^6*c^2 - 14273*a*b^4*c^3 + 77982*a^2*b^2*c^4 + 33464*a^3*c^5)
*d^4*e^9 - (675*b^7*c - 9414*a*b^5*c^2 + 18524*a^2*b^3*c^3 + 66928*a^3*b*c^
4)*d^3*e^10 + (189*b^8 - 999*a*b^6*c - 8127*a^2*b^4*c^2 + 40196*a^3*b^2*c^3
 + 10000*a^4*c^4)*d^2*e^11 - (378*a*b^7 - 3645*a^2*b^5*c + 6732*a^3*b^3*c^2
 + 10000*a^4*b*c^3)*d*e^12 + (189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c
^2 - 2500*a^5*c^3)*e^13)*sqrt(e*x + d)) + 2*(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*
a*b*c*d*e^2 - 2*(b^2*c - 4*a*c^2)*e^3*x^2 - (3*a*b^2 - 10*a^2*c)*e^3 + (2*c
^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (3*b^3 - 11*a*b*c)*e^3
)*x)*sqrt(e*x + d))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^3
*c^2 - 4*a*b*c^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.2296 $\int \frac{(d+ex)^{5/2}}{(a+bx+cx^2)^2}\,dx$

Optimal. Leaf size=504

$$\frac{\left(-2c^2de\left(-d\sqrt{b^2-4ac}-8ae+6bd\right)+2ce^2\left(-b\left(d\sqrt{b^2-4ac}+4ae\right)+3ae\sqrt{b^2-4ac}+b^2d\right)+b^2e^3\left(b-\sqrt{b^2-4ac}\right)+8\cdots}{\sqrt{2}c^{3/2}\left(b^2-4ac\right)^{3/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

```
[Out] (e*(2*c*d - b*e)*Sqrt[d + e*x])/(c*(b^2 - 4*a*c)) - ((d + e*x)^(3/2)*(b*d -
 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) + ((8*c^3*d^3
+ b^2*(b - Sqrt[b^2 - 4*a*c])*e^3 - 2*c^2*d*e*(6*b*d - Sqrt[b^2 - 4*a*c]*d
- 8*a*e) + 2*c*e^2*(b^2*d + 3*a*Sqrt[b^2 - 4*a*c]*e - b*(Sqrt[b^2 - 4*a*c]*
d + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt
[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[2*c*d - (b -
Sqrt[b^2 - 4*a*c])*e]) - ((8*c^3*d^3 + b^2*(b + Sqrt[b^2 - 4*a*c])*e^3 - 2*
c^2*d*e*(6*b*d + Sqrt[b^2 - 4*a*c]*d - 8*a*e) + 2*c*e^2*(b^2*d + b*Sqrt[b^2
 - 4*a*c]*d - 4*a*b*e - 3*a*Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*
Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(3/2)*(
b^2 - 4*a*c)^(3/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 4.61806, antiderivative size = 504, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {738, 824, 826, 1166, 208}

$$\frac{\left(-2c^2de\left(-d\sqrt{b^2-4ac}-8ae+6bd\right)+2ce^2\left(-b\left(d\sqrt{b^2-4ac}+4ae\right)+3ae\sqrt{b^2-4ac}+b^2d\right)+b^2e^3\left(b-\sqrt{b^2-4ac}\right)+8\cdots}{\sqrt{2}c^{3/2}\left(b^2-4ac\right)^{3/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] (e*(2*c*d - b*e)*Sqrt[d + e*x])/(c*(b^2 - 4*a*c)) - ((d + e*x)^(3/2)*(b*d -
 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) + ((8*c^3*d^3
+ b^2*(b - Sqrt[b^2 - 4*a*c])*e^3 - 2*c^2*d*e*(6*b*d - Sqrt[b^2 - 4*a*c]*d
- 8*a*e) + 2*c*e^2*(b^2*d + 3*a*Sqrt[b^2 - 4*a*c]*e - b*(Sqrt[b^2 - 4*a*c]*
d + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt
[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[2*c*d - (b -
Sqrt[b^2 - 4*a*c])*e]) - ((8*c^3*d^3 + b^2*(b + Sqrt[b^2 - 4*a*c])*e^3 - 2*
c^2*d*e*(6*b*d + Sqrt[b^2 - 4*a*c]*d - 8*a*e) + 2*c*e^2*(b^2*d + b*Sqrt[b^2
 - 4*a*c]*d - 4*a*b*e - 3*a*Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*
Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(3/2)*(
b^2 - 4*a*c)^(3/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
```

```
1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\left(a+bx+cx^2\right)^2}\,dx = -\frac{(d+ex)^{3/2}(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\int \frac{\sqrt{d+ex}\left(\frac{1}{2}\left(4cd^2-5bde+6ae^2\right)-\frac{1}{2}e(2cd-be)x\right)}{a+bx+cx^2}\,dx}{-b^2+4ac}$$

$$=\frac{e(2cd-be)\sqrt{d+ex}}{c\left(b^2-4ac\right)}-\frac{(d+ex)^{3/2}(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int \frac{\frac{1}{2}\left(4c^2d^3-abe^3-cde(5bd-8ae)\right)+\frac{1}{2}e\left(2c^2d^2-b^2e\right.}{\sqrt{d+ex}\left(a+bx+cx^2\right)}}{c\left(b^2-4ac\right)}$$

$$=\frac{e(2cd-be)\sqrt{d+ex}}{c\left(b^2-4ac\right)}-\frac{(d+ex)^{3/2}(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{2\,\mathrm{Subst}\left(\int \frac{\frac{1}{2}e\left(4c^2d^3-abe^3-cde(5bd-8ae)\right)-}{}\right.}{}$$

$$=\frac{e(2cd-be)\sqrt{d+ex}}{c\left(b^2-4ac\right)}-\frac{(d+ex)^{3/2}(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\left(8c^3d^3+b^2\left(b+\sqrt{b^2-4ac}\right)e^3-2c^2\right.}{}$$

$$=\frac{e(2cd-be)\sqrt{d+ex}}{c\left(b^2-4ac\right)}-\frac{(d+ex)^{3/2}(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\left(8c^3d^3+b^2\left(b-\sqrt{b^2-4ac}\right)e^3-2c^2\right.}{}$$

Mathematica [A] time = 5.79614, size = 549, normalized size = 1.09

$$\frac{1}{2}\left(e(ae-bd)+cd^2\right)\left(\frac{\sqrt{2}\left(\frac{\left(2c^2de\left(d\sqrt{b^2-4ac}+8ae-6bd\right)+2ce^2\left(-b\left(d\sqrt{b^2-4ac}+4ae\right)+3ae\sqrt{b^2-4ac}+b^2d\right)+b^2e^3\left(b-\sqrt{b^2-4ac}\right)+8c^3d^3\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}-\frac{\left(-2c^2de\left(\cdots\right.\right.}{\cdots}\right.}{c^{3/2}\sqrt{b^2-4ac}}\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] (e*(2*c*d - b*e)*(d + e*x)^(5/2) + ((d + e*x)^(7/2)*(b^2*e - 2*c*(a*e + c*d
*x) + b*c*(-d + e*x)))/(a + x*(b + c*x)) + ((c*d^2 + e*(-(b*d) + a*e))*((-2
*e*(-2*c*d + b*e)*Sqrt[d + e*x])/c + 4*e*(d + e*x)^(3/2) + (Sqrt[2]*(((8*c^
3*d^3 + b^2*(b - Sqrt[b^2 - 4*a*c])*e^3 + 2*c^2*d*e*(-6*b*d + Sqrt[b^2 - 4*
a*c]*d + 8*a*e) + 2*c*e^2*(b^2*d + 3*a*Sqrt[b^2 - 4*a*c]*e - b*(Sqrt[b^2 -
4*a*c]*d + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - b*
e + Sqrt[b^2 - 4*a*c]*e]])/Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e] - ((8*c
^3*d^3 + b^2*(b + Sqrt[b^2 - 4*a*c])*e^3 - 2*c^2*d*e*(6*b*d + Sqrt[b^2 - 4*
a*c]*d - 8*a*e) + 2*c*e^2*(b^2*d + b*Sqrt[b^2 - 4*a*c]*d - 4*a*b*e - 3*a*Sq
rt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e]])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/(c^(3/
2)*Sqrt[b^2 - 4*a*c])))/2)/((b^2 - 4*a*c)*(c*d^2 + e*(-(b*d) + a*e)))
```

Maple [B] time = 0.278, size = 2557, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(c*x^2+b*x+a)^2,x)
```

```
[Out] 4*e^4/(4*a*c-b^2)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c
-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*
a*c-b^2))^(1/2))*c)^(1/2))*a*b+6*e^2/(4*a*c-b^2)*c/(-e^2*(4*a*c-b^2))^(1/2)
*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2
)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d^2-8*e^3/(4*
a*c-b^2)*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))
^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-
b^2))^(1/2))*c)^(1/2))*a*d+6*e^2/(4*a*c-b^2)*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(
1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*
c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d^2-8*e^3/(4*a
*c-b^2)*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(
1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2)
)^(1/2))*c)^(1/2))*a*d-e^3/(4*a*c-b^2)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-
b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2
)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2*d-1/2*e^4/(4*a*c-b^2
)/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))
*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(
```

```
1/2))*c)^(1/2))*b^3-4*e/(4*a*c-b^2)*c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((
-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/
2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d^3-1/2*e^4/(4*a*c-b^2)
/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c
)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2)
)*c)^(1/2))*b^3-e/(4*a*c-b^2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/
2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2)
)^(1/2))*c)^(1/2))*d^2-1/2*e^3/(4*a*c-b^2)/c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a
*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(
4*a*c-b^2))^(1/2))*c)^(1/2))*b^2+e^4/(c*e^2*x^2+b*e^2*x+a*e^2)/c/(4*a*c-b^2
)*(e*x+d)^(1/2)*a*b-e^3/(c*e^2*x^2+b*e^2*x+a*e^2)/c/(4*a*c-b^2)*(e*x+d)^(1/
2)*b^2*d+1/2*e^3/(4*a*c-b^2)/c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2
))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))
^(1/2))*c)^(1/2))*b^2+4*e^4/(4*a*c-b^2)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((
-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/
2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b-e^3/(4*a*c-b^2)/(-e
^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2
)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(
1/2))*b^2*d-4*e/(4*a*c-b^2)*c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*
d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2
*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d^3-2*e^3/(c*e^2*x^2+b*e^2*x+a*e^2
)/(4*a*c-b^2)*(e*x+d)^(3/2)*a+e/(4*a*c-b^2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*
a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*
(4*a*c-b^2))^(1/2))*c)^(1/2))*d^2-e^2/(4*a*c-b^2)*2^(1/2)/((b*e-2*c*d+(-e^2
*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-
e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d+e^2/(4*a*c-b^2)*2^(1/2)/((-b*e+2*c*d+
(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2
*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d+e^3/(c*e^2*x^2+b*e^2*x+a*e^2)/
c/(4*a*c-b^2)*(e*x+d)^(3/2)*b^2-2*e^3/(c*e^2*x^2+b*e^2*x+a*e^2)/(4*a*c-b^2)
*(e*x+d)^(1/2)*a*d+3*e^3/(4*a*c-b^2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))
^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^
2))^(1/2))*c)^(1/2))*a-3*e^3/(4*a*c-b^2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-
b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4
*a*c-b^2))^(1/2))*c)^(1/2))*a-2*e^2/(c*e^2*x^2+b*e^2*x+a*e^2)/(4*a*c-b^2)*(
e*x+d)^(3/2)*b*d+2*e/(c*e^2*x^2+b*e^2*x+a*e^2)*c/(4*a*c-b^2)*(e*x+d)^(3/2)*
d^2+3*e^2/(c*e^2*x^2+b*e^2*x+a*e^2)/(4*a*c-b^2)*(e*x+d)^(1/2)*b*d^2-2*e/(c*
e^2*x^2+b*e^2*x+a*e^2)*c/(4*a*c-b^2)*(e*x+d)^(1/2)*d^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\left(cx^2+bx+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/(c*x^2 + b*x + a)^2, x)
```

Fricas [B] time = 5.06061, size = 11088, normalized size = 22.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*(a*b^2*c - 4*a^2*c^2 + (b^2*c^2 - 4*a*c^3)*x^2 + (b^3*c - 4*
a*b*c^2)*x)*sqrt((32*c^5*d^5 - 80*b*c^4*d^4*e + 10*(5*b^2*c^3 + 12*a*c^4)*d
^3*e^2 + 5*(b^3*c^2 - 36*a*b*c^3)*d^2*e^3 - 5*(b^4*c - 6*a*b^2*c^2 - 24*a^2
*c^3)*d*e^4 - (b^5 - 15*a*b^3*c + 60*a^2*b*c^2)*e^5 + (b^6*c^3 - 12*a*b^4*c
^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((25*c^4*d^4*e^6 - 50*b*c^3*d^3*e^7 +
 15*(b^2*c^2 + 6*a*c^3)*d^2*e^8 + 10*(b^3*c - 9*a*b*c^2)*d*e^9 + (b^4 - 18*
a*b^2*c + 81*a^2*c^2)*e^10)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a
^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log(sqrt(
1/2)*(10*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d^3*e^4 - 15*(b^5*c^2 - 8*a*b
^3*c^3 + 16*a^2*b*c^4)*d^2*e^5 + 3*(b^6*c - 2*a*b^4*c^2 - 32*a^2*b^2*c^3 +
96*a^3*c^4)*d*e^6 + (b^7 - 17*a*b^5*c + 88*a^2*b^3*c^2 - 144*a^3*b*c^3)*e^7
 - (8*(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*d^2 - 8*(b^7*c
^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*d*e - (b^8*c^3 - 24*a*b^
6*c^4 + 192*a^2*b^4*c^5 - 640*a^3*b^2*c^6 + 768*a^4*c^7)*e^2)*sqrt((25*c^4*
d^4*e^6 - 50*b*c^3*d^3*e^7 + 15*(b^2*c^2 + 6*a*c^3)*d^2*e^8 + 10*(b^3*c - 9
*a*b*c^2)*d*e^9 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*e^10)/(b^6*c^6 - 12*a*b^4
*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt((32*c^5*d^5 - 80*b*c^4*d^4*e + 1
0*(5*b^2*c^3 + 12*a*c^4)*d^3*e^2 + 5*(b^3*c^2 - 36*a*b*c^3)*d^2*e^3 - 5*(b^
4*c - 6*a*b^2*c^2 - 24*a^2*c^3)*d*e^4 - (b^5 - 15*a*b^3*c + 60*a^2*b*c^2)*e
^5 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((25*c^4*d^
4*e^6 - 50*b*c^3*d^3*e^7 + 15*(b^2*c^2 + 6*a*c^3)*d^2*e^8 + 10*(b^3*c - 9*a
*b*c^2)*d*e^9 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*e^10)/(b^6*c^6 - 12*a*b^4*c
^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^
5 - 64*a^3*c^6)) + 2*(80*c^5*d^6*e^3 - 240*b*c^4*d^5*e^4 + (199*b^2*c^3 + 4
04*a*c^4)*d^4*e^5 + 2*(b^3*c^2 - 404*a*b*c^3)*d^3*e^6 - 6*(6*b^4*c - 47*a*b
^2*c^2 - 108*a^2*c^3)*d^2*e^7 - (5*b^5 - 122*a*b^3*c + 648*a^2*b*c^2)*d*e^8
 + (5*a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*e^9)*sqrt(e*x + d)) - sqrt(1/2)*(
a*b^2*c - 4*a^2*c^2 + (b^2*c^2 - 4*a*c^3)*x^2 + (b^3*c - 4*a*b*c^2)*x)*sqrt
((32*c^5*d^5 - 80*b*c^4*d^4*e + 10*(5*b^2*c^3 + 12*a*c^4)*d^3*e^2 + 5*(b^3*
c^2 - 36*a*b*c^3)*d^2*e^3 - 5*(b^4*c - 6*a*b^2*c^2 - 24*a^2*c^3)*d*e^4 - (b
^5 - 15*a*b^3*c + 60*a^2*b*c^2)*e^5 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*
c^5 - 64*a^3*c^6)*sqrt((25*c^4*d^4*e^6 - 50*b*c^3*d^3*e^7 + 15*(b^2*c^2 + 6
*a*c^3)*d^2*e^8 + 10*(b^3*c - 9*a*b*c^2)*d*e^9 + (b^4 - 18*a*b^2*c + 81*a^2
*c^2)*e^10)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c
^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log(-sqrt(1/2)*(10*(b^4*c
^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d^3*e^4 - 15*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2
*b*c^4)*d^2*e^5 + 3*(b^6*c - 2*a*b^4*c^2 - 32*a^2*b^2*c^3 + 96*a^3*c^4)*d*e
^6 + (b^7 - 17*a*b^5*c + 88*a^2*b^3*c^2 - 144*a^3*b*c^3)*e^7 - (8*(b^6*c^5
- 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*d^2 - 8*(b^7*c^4 - 12*a*b^5*c
^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*d*e - (b^8*c^3 - 24*a*b^6*c^4 + 192*a^2
*b^4*c^5 - 640*a^3*b^2*c^6 + 768*a^4*c^7)*e^2)*sqrt((25*c^4*d^4*e^6 - 50*b*
c^3*d^3*e^7 + 15*(b^2*c^2 + 6*a*c^3)*d^2*e^8 + 10*(b^3*c - 9*a*b*c^2)*d*e^9
 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*e^10)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b
^2*c^8 - 64*a^3*c^9)))*sqrt((32*c^5*d^5 - 80*b*c^4*d^4*e + 10*(5*b^2*c^3 +
12*a*c^4)*d^3*e^2 + 5*(b^3*c^2 - 36*a*b*c^3)*d^2*e^3 - 5*(b^4*c - 6*a*b^2*c
^2 - 24*a^2*c^3)*d*e^4 - (b^5 - 15*a*b^3*c + 60*a^2*b*c^2)*e^5 + (b^6*c^3 -
 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((25*c^4*d^4*e^6 - 50*b*c^
3*d^3*e^7 + 15*(b^2*c^2 + 6*a*c^3)*d^2*e^8 + 10*(b^3*c - 9*a*b*c^2)*d*e^9 +
 (b^4 - 18*a*b^2*c + 81*a^2*c^2)*e^10)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2
*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)
) + 2*(80*c^5*d^6*e^3 - 240*b*c^4*d^5*e^4 + (199*b^2*c^3 + 404*a*c^4)*d^4*e
^5 + 2*(b^3*c^2 - 404*a*b*c^3)*d^3*e^6 - 6*(6*b^4*c - 47*a*b^2*c^2 - 108*a^
2*c^3)*d^2*e^7 - (5*b^5 - 122*a*b^3*c + 648*a^2*b*c^2)*d*e^8 + (5*a*b^4 - 8
1*a^2*b^2*c + 324*a^3*c^2)*e^9)*sqrt(e*x + d)) + sqrt(1/2)*(a*b^2*c - 4*a^2
*c^2 + (b^2*c^2 - 4*a*c^3)*x^2 + (b^3*c - 4*a*b*c^2)*x)*sqrt((32*c^5*d^5 -
80*b*c^4*d^4*e + 10*(5*b^2*c^3 + 12*a*c^4)*d^3*e^2 + 5*(b^3*c^2 - 36*a*b*c^
3)*d^2*e^3 - 5*(b^4*c - 6*a*b^2*c^2 - 24*a^2*c^3)*d*e^4 - (b^5 - 15*a*b^3*c
```

```
 + 60*a^2*b*c^2)*e^5 - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^
6)*sqrt((25*c^4*d^4*e^6 - 50*b*c^3*d^3*e^7 + 15*(b^2*c^2 + 6*a*c^3)*d^2*e^8
 + 10*(b^3*c - 9*a*b*c^2)*d*e^9 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*e^10)/(b^
6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c
^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log(sqrt(1/2)*(10*(b^4*c^3 - 8*a*b^2*c^4
 + 16*a^2*c^5)*d^3*e^4 - 15*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^2*e^5
+ 3*(b^6*c - 2*a*b^4*c^2 - 32*a^2*b^2*c^3 + 96*a^3*c^4)*d*e^6 + (b^7 - 17*a
*b^5*c + 88*a^2*b^3*c^2 - 144*a^3*b*c^3)*e^7 + (8*(b^6*c^5 - 12*a*b^4*c^6 +
 48*a^2*b^2*c^7 - 64*a^3*c^8)*d^2 - 8*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*
c^6 - 64*a^3*b*c^7)*d*e - (b^8*c^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 - 640*a
^3*b^2*c^6 + 768*a^4*c^7)*e^2)*sqrt((25*c^4*d^4*e^6 - 50*b*c^3*d^3*e^7 + 15
*(b^2*c^2 + 6*a*c^3)*d^2*e^8 + 10*(b^3*c - 9*a*b*c^2)*d*e^9 + (b^4 - 18*a*b
^2*c + 81*a^2*c^2)*e^10)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*
c^9)))*sqrt((32*c^5*d^5 - 80*b*c^4*d^4*e + 10*(5*b^2*c^3 + 12*a*c^4)*d^3*e^
2 + 5*(b^3*c^2 - 36*a*b*c^3)*d^2*e^3 - 5*(b^4*c - 6*a*b^2*c^2 - 24*a^2*c^3)
*d*e^4 - (b^5 - 15*a*b^3*c + 60*a^2*b*c^2)*e^5 - (b^6*c^3 - 12*a*b^4*c^4 +
48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((25*c^4*d^4*e^6 - 50*b*c^3*d^3*e^7 + 15*(
b^2*c^2 + 6*a*c^3)*d^2*e^8 + 10*(b^3*c - 9*a*b*c^2)*d*e^9 + (b^4 - 18*a*b^2
*c + 81*a^2*c^2)*e^10)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^
9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)) + 2*(80*c^5*d^
6*e^3 - 240*b*c^4*d^5*e^4 + (199*b^2*c^3 + 404*a*c^4)*d^4*e^5 + 2*(b^3*c^2
- 404*a*b*c^3)*d^3*e^6 - 6*(6*b^4*c - 47*a*b^2*c^2 - 108*a^2*c^3)*d^2*e^7 -
 (5*b^5 - 122*a*b^3*c + 648*a^2*b*c^2)*d*e^8 + (5*a*b^4 - 81*a^2*b^2*c + 32
4*a^3*c^2)*e^9)*sqrt(e*x + d)) - sqrt(1/2)*(a*b^2*c - 4*a^2*c^2 + (b^2*c^2
- 4*a*c^3)*x^2 + (b^3*c - 4*a*b*c^2)*x)*sqrt((32*c^5*d^5 - 80*b*c^4*d^4*e +
 10*(5*b^2*c^3 + 12*a*c^4)*d^3*e^2 + 5*(b^3*c^2 - 36*a*b*c^3)*d^2*e^3 - 5*(
b^4*c - 6*a*b^2*c^2 - 24*a^2*c^3)*d*e^4 - (b^5 - 15*a*b^3*c + 60*a^2*b*c^2)
*e^5 - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((25*c^4*
d^4*e^6 - 50*b*c^3*d^3*e^7 + 15*(b^2*c^2 + 6*a*c^3)*d^2*e^8 + 10*(b^3*c - 9
*a*b*c^2)*d*e^9 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*e^10)/(b^6*c^6 - 12*a*b^4
*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*
c^5 - 64*a^3*c^6))*log(-sqrt(1/2)*(10*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*
d^3*e^4 - 15*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^2*e^5 + 3*(b^6*c - 2*
a*b^4*c^2 - 32*a^2*b^2*c^3 + 96*a^3*c^4)*d*e^6 + (b^7 - 17*a*b^5*c + 88*a^2
*b^3*c^2 - 144*a^3*b*c^3)*e^7 + (8*(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7
 - 64*a^3*c^8)*d^2 - 8*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*
c^7)*d*e - (b^8*c^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 - 640*a^3*b^2*c^6 + 76
8*a^4*c^7)*e^2)*sqrt((25*c^4*d^4*e^6 - 50*b*c^3*d^3*e^7 + 15*(b^2*c^2 + 6*a
*c^3)*d^2*e^8 + 10*(b^3*c - 9*a*b*c^2)*d*e^9 + (b^4 - 18*a*b^2*c + 81*a^2*c
^2)*e^10)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt((32
*c^5*d^5 - 80*b*c^4*d^4*e + 10*(5*b^2*c^3 + 12*a*c^4)*d^3*e^2 + 5*(b^3*c^2
- 36*a*b*c^3)*d^2*e^3 - 5*(b^4*c - 6*a*b^2*c^2 - 24*a^2*c^3)*d*e^4 - (b^5 -
 15*a*b^3*c + 60*a^2*b*c^2)*e^5 - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5
- 64*a^3*c^6)*sqrt((25*c^4*d^4*e^6 - 50*b*c^3*d^3*e^7 + 15*(b^2*c^2 + 6*a*c
^3)*d^2*e^8 + 10*(b^3*c - 9*a*b*c^2)*d*e^9 + (b^4 - 18*a*b^2*c + 81*a^2*c^2
)*e^10)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 -
 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)) + 2*(80*c^5*d^6*e^3 - 240*b*c
^4*d^5*e^4 + (199*b^2*c^3 + 404*a*c^4)*d^4*e^5 + 2*(b^3*c^2 - 404*a*b*c^3)*
d^3*e^6 - 6*(6*b^4*c - 47*a*b^2*c^2 - 108*a^2*c^3)*d^2*e^7 - (5*b^5 - 122*a
*b^3*c + 648*a^2*b*c^2)*d*e^8 + (5*a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*e^9)
*sqrt(e*x + d)) - 2*(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (2*c^2*d^2 - 2*b*c*d*e
 + (b^2 - 2*a*c)*e^2)*x)*sqrt(e*x + d))/(a*b^2*c - 4*a^2*c^2 + (b^2*c^2 - 4
*a*c^3)*x^2 + (b^3*c - 4*a*b*c^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.2297 $\int \frac{(d+ex)^{3/2}}{(a+bx+cx^2)^2} dx$

Optimal. Leaf size=363

$$\frac{\left(-2ce\left(-d\sqrt{b^2-4ac}-2ae+4bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+8c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(-2ce\left(d\sqrt{b^2-4ac}-2ae\right.\right.}{\sqrt{\cdot}}$$

```
[Out] -((Sqrt[d + e*x]*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*(a + b*x +
 c*x^2))) + ((8*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(4*b*d - Sq
rt[b^2 - 4*a*c]*d - 2*a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*
c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqr
t[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((8*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a
*c])*e^2 - 2*c*e*(4*b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e))*ArcTanh[(Sqrt[2]*Sq
rt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqr
t[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 1.29296, antiderivative size = 363, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {738, 826, 1166, 208}

$$\frac{\left(-2ce\left(-d\sqrt{b^2-4ac}-2ae+4bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+8c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(-2ce\left(d\sqrt{b^2-4ac}-2ae\right.\right.}{\sqrt{\cdot}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(a + b*x + c*x^2)^2,x]
```


```
[Out] -((Sqrt[d + e*x]*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*(a + b*x +
 c*x^2))) + ((8*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(4*b*d - Sq
rt[b^2 - 4*a*c]*d - 2*a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*
c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqr
t[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((8*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a
*c])*e^2 - 2*c*e*(4*b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e))*ArcTanh[(Sqrt[2]*Sq
rt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqr
t[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```


Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\left(a+bx+cx^2\right)^2}\,dx = -\frac{\sqrt{d+ex}(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\int \frac{\frac{1}{2}\left(4cd^2-3bde+2ae^2\right)+\frac{1}{2}e(2cd-be)x}{\sqrt{d+ex}\left(a+bx+cx^2\right)}\,dx}{-b^2+4ac}$$

$$= -\frac{\sqrt{d+ex}(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{2\,\mathrm{Subst}\left(\int \frac{-\frac{1}{2}de(2cd-be)+\frac{1}{2}e\left(4cd^2-3bde+2ae^2\right)+\frac{1}{2}e(2cd-be)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x\right.}{b^2-4ac}$$

$$= -\frac{\sqrt{d+ex}(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\left(8c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(4bd-\sqrt{b^2-4ac}d\right.\right.}{2\left(b^2\right.}$$

$$= -\frac{\sqrt{d+ex}(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\left(8c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(4bd-\sqrt{b^2-4ac}d\right.\right.}{\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{2cd-\left(b-\right.}}$$

Mathematica [A] time = 3.18578, size = 418, normalized size = 1.15

$$\frac{\frac{1}{2}\left(e(ae-bd)+cd^2\right)\left(\frac{\sqrt{2}\left(\frac{\left(2ce\left(d\sqrt{b^2-4ac}+2ae-4bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+8c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}-\frac{\left(-2ce\left(d\sqrt{b^2-4ac}-2ae+4bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+8c^2d^2\right)\tanh}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right.}{\sqrt{c}\sqrt{b^2-4ac}}\right.}{\left(b^2-4ac\right)\left(e(ae-bd)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] (e*(2*c*d - b*e)*(d + e*x)^(3/2) + ((d + e*x)^(5/2)*(b^2*e - 2*c*(a*e + c*d
*x) + b*c*(-d + e*x)))/(a + x*(b + c*x)) + ((c*d^2 + e*(-(b*d) + a*e))*(4*e
*Sqrt[d + e*x] + (Sqrt[2]*(((8*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 + 2*
c*e*(-4*b*d + Sqrt[b^2 - 4*a*c]*d + 2*a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d
 + e*x])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/Sqrt[2*c*d + (-b + Sqrt[
b^2 - 4*a*c])*e] - ((8*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(4*b
*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/
Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e]))/(Sqrt[c]*Sqrt[b^2 - 4*a*c])))/2)/((b^2 - 4*a*c)*(c*d^2 + e*(-(b*d)
 + a*e)))
```

Maple [B] time = 0.293, size = 1503, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*x^2+b*x+a)^2,x)
```

```
[Out] -e^2/(c*e^2*x^2+b*e^2*x+a*e^2)/(4*a*c-b^2)*(e*x+d)^(3/2)*b+2*e/(c*e^2*x^2+b
*e^2*x+a*e^2)/(4*a*c-b^2)*(e*x+d)^(3/2)*c*d-2*e^3/(c*e^2*x^2+b*e^2*x+a*e^2)
/(4*a*c-b^2)*(e*x+d)^(1/2)*a+2*e^2/(c*e^2*x^2+b*e^2*x+a*e^2)/(4*a*c-b^2)*(e
*x+d)^(1/2)*b*d-2*e/(c*e^2*x^2+b*e^2*x+a*e^2)/(4*a*c-b^2)*(e*x+d)^(1/2)*c*d
^2-2*e^3/(4*a*c-b^2)*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(
4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^
2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a-1/2*e^3/(4*a*c-b^2)/(-e^2*(4*a*c-b^2))^(1
/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(
1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2+4*e^2/(4
*a*c-b^2)*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))
^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^
2))^(1/2))*c)^(1/2))*b*d-4*e/(4*a*c-b^2)*c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/
2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^
(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d^2-1/2*e^2/(4*a*c-b^
2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1
/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b+e/(4*a*c-b^
2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^
(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d-2*e^3/(4*
a*c-b^2)*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))
^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-
b^2))^(1/2))*c)^(1/2))*a-1/2*e^3/(4*a*c-b^2)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/
2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*
2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2+4*e^2/(4*a*c-b
^2)*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2
))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))
^(1/2))*c)^(1/2))*b*d-4*e/(4*a*c-b^2)*c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/
((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(
1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d^2+1/2*e^2/(4*a*c-b^
2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^
(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b-e/(4*a*c
-b^2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*
x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cx^2+bx+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(c*x^2 + b*x + a)^2, x)
```

Fricas [B] time = 2.45584, size = 5069, normalized size = 13.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*(a*b^2 - 4*a^2*c + (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x
)*sqrt((32*c^3*d^3 - 48*b*c^2*d^2*e + 6*(3*b^2*c + 4*a*c^2)*d*e^2 - (b^3 +
12*a*b*c)*e^3 + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*sqrt(e
^6/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)))/(b^6*c - 12*a*b
^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log(sqrt(1/2)*((b^4 - 8*a*b^2*c + 16
*a^2*c^2)*e^4 + 2*sqrt(e^6/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^
3*c^5))*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d - (b^7*
c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e))*sqrt((32*c^3*d^3 - 48
*b*c^2*d^2*e + 6*(3*b^2*c + 4*a*c^2)*d*e^2 - (b^3 + 12*a*b*c)*e^3 + (b^6*c
- 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*sqrt(e^6/(b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3
 - 64*a^3*c^4)) + (16*c^2*d^2*e^3 - 16*b*c*d*e^4 + (3*b^2 + 4*a*c)*e^5)*sqr
t(e*x + d)) - sqrt(1/2)*(a*b^2 - 4*a^2*c + (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4
*a*b*c)*x)*sqrt((32*c^3*d^3 - 48*b*c^2*d^2*e + 6*(3*b^2*c + 4*a*c^2)*d*e^2
- (b^3 + 12*a*b*c)*e^3 + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^
4)*sqrt(e^6/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)))/(b^6*c
 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log(-sqrt(1/2)*((b^4 - 8*a*
b^2*c + 16*a^2*c^2)*e^4 + 2*sqrt(e^6/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c
^4 - 64*a^3*c^5))*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)
*d - (b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e))*sqrt((32*c^
3*d^3 - 48*b*c^2*d^2*e + 6*(3*b^2*c + 4*a*c^2)*d*e^2 - (b^3 + 12*a*b*c)*e^3
 + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*sqrt(e^6/(b^6*c^2 -
 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)))/(b^6*c - 12*a*b^4*c^2 + 48*a
^2*b^2*c^3 - 64*a^3*c^4)) + (16*c^2*d^2*e^3 - 16*b*c*d*e^4 + (3*b^2 + 4*a*c
)*e^5)*sqrt(e*x + d)) + sqrt(1/2)*(a*b^2 - 4*a^2*c + (b^2*c - 4*a*c^2)*x^2
+ (b^3 - 4*a*b*c)*x)*sqrt((32*c^3*d^3 - 48*b*c^2*d^2*e + 6*(3*b^2*c + 4*a*c
^2)*d*e^2 - (b^3 + 12*a*b*c)*e^3 - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 -
 64*a^3*c^4)*sqrt(e^6/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5
)))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log(sqrt(1/2)*((b
^4 - 8*a*b^2*c + 16*a^2*c^2)*e^4 - 2*sqrt(e^6/(b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5))*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64
*a^3*c^5)*d - (b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e))*sq
rt((32*c^3*d^3 - 48*b*c^2*d^2*e + 6*(3*b^2*c + 4*a*c^2)*d*e^2 - (b^3 + 12*a
*b*c)*e^3 - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*sqrt(e^6/(
b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)))/(b^6*c - 12*a*b^4*c
^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)) + (16*c^2*d^2*e^3 - 16*b*c*d*e^4 + (3*b^
2 + 4*a*c)*e^5)*sqrt(e*x + d)) - sqrt(1/2)*(a*b^2 - 4*a^2*c + (b^2*c - 4*a*
```

```
c^2)*x^2 + (b^3 - 4*a*b*c)*x)*sqrt((32*c^3*d^3 - 48*b*c^2*d^2*e + 6*(3*b^2*
c + 4*a*c^2)*d*e^2 - (b^3 + 12*a*b*c)*e^3 - (b^6*c - 12*a*b^4*c^2 + 48*a^2*
b^2*c^3 - 64*a^3*c^4)*sqrt(e^6/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 6
4*a^3*c^5)))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log(-sqr
t(1/2)*((b^4 - 8*a*b^2*c + 16*a^2*c^2)*e^4 - 2*sqrt(e^6/(b^6*c^2 - 12*a*b^4
*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^
2*c^4 - 64*a^3*c^5)*d - (b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c
^4)*e))*sqrt((32*c^3*d^3 - 48*b*c^2*d^2*e + 6*(3*b^2*c + 4*a*c^2)*d*e^2 - (
b^3 + 12*a*b*c)*e^3 - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*
sqrt(e^6/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)))/(b^6*c -
12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)) + (16*c^2*d^2*e^3 - 16*b*c*d*e
^4 + (3*b^2 + 4*a*c)*e^5)*sqrt(e*x + d)) - 2*(b*d - 2*a*e + (2*c*d - b*e)*x
)*sqrt(e*x + d))/(a*b^2 - 4*a^2*c + (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)
*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+b*x+a)**2,x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="giac")

[Out] Timed out
```

3.2298 $\int \frac{\sqrt{d+ex}}{(a+bx+cx^2)^2} dx$

Optimal. Leaf size=287

$$-\frac{(b+2cx)\sqrt{d+ex}}{(b^2-4ac)(a+bx+cx^2)}+\frac{\sqrt{2}\sqrt{c}\left(4cd-e\left(2b-\sqrt{b^2-4ac}\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{(b^2-4ac)^{3/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{2}\sqrt{c}\left(4cd-e\left(\sqrt{b^2-4\right.\right.}{(b^2-4ac)^{3/2}}$$

```
[Out] -(((b + 2*c*x)*Sqrt[d + e*x])/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (Sqrt[2]
*Sqrt[c]*(4*c*d - (2*b - Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqr
t[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/((b^2 - 4*a*c)^(3/2)*
Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2]*Sqrt[c]*(4*c*d - (2*b +
 Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d -
 (b + Sqrt[b^2 - 4*a*c])*e]])/((b^2 - 4*a*c)^(3/2)*Sqrt[2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e])
```

Rubi [A] time = 0.671111, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {736, 826, 1166, 208}

$$-\frac{(b+2cx)\sqrt{d+ex}}{(b^2-4ac)(a+bx+cx^2)}+\frac{\sqrt{2}\sqrt{c}\left(4cd-e\left(2b-\sqrt{b^2-4ac}\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{(b^2-4ac)^{3/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{2}\sqrt{c}\left(4cd-e\left(\sqrt{b^2-4\right.\right.}{(b^2-4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(a + b*x + c*x^2)^2,x]
```

```
[Out] -(((b + 2*c*x)*Sqrt[d + e*x])/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (Sqrt[2]
*Sqrt[c]*(4*c*d - (2*b - Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqr
t[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/((b^2 - 4*a*c)^(3/2)*
Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2]*Sqrt[c]*(4*c*d - (2*b +
 Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d -
 (b + Sqrt[b^2 - 4*a*c])*e]])/((b^2 - 4*a*c)^(3/2)*Sqrt[2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e])
```

Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] - Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)
*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p + 1
), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 0] && (L
tQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c,
d, e, m, p, x]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
```

```
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\left(a+bx+cx^2\right)^2}\,dx = -\frac{(b+2cx)\sqrt{d+ex}}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{\int \frac{-2cd+\frac{be}{2}-cex}{\sqrt{d+ex}\left(a+bx+cx^2\right)}\,dx}{-b^2+4ac}$$

$$= -\frac{(b+2cx)\sqrt{d+ex}}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{2\,\text{Subst}\left(\int \frac{cde+e\left(-2cd+\frac{be}{2}\right)-cex^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{b^2-4ac}$$

$$= -\frac{(b+2cx)\sqrt{d+ex}}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{\left(c\left(4cd-\left(2b-\sqrt{b^2-4ac}\right)e\right)\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}e+\frac{1}{2}(-2cd+be)+cx^2}\right.}{\left(b^2-4ac\right)^{3/2}}$$

$$= -\frac{(b+2cx)\sqrt{d+ex}}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\sqrt{2}\sqrt{c}\left(4cd-\left(2b-\sqrt{b^2-4ac}\right)e\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \ldots$$

Mathematica [A] time = 0.651244, size = 283, normalized size = 0.99

$$\frac{-\frac{\sqrt{2}\sqrt{c}\left(e\left(\sqrt{b^2-4ac}-2b\right)+4cd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}+\frac{\sqrt{2}\sqrt{c}\left(4cd-e\left(\sqrt{b^2-4ac}+2b\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{(b+2cx)\sqrt{d+ex}}{a+x(b+cx)}}{4ac-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(a + b*x + c*x^2)^2,x]
```

```
[Out] (((b + 2*c*x)*Sqrt[d + e*x])/(a + x*(b + c*x)) - (Sqrt[2]*Sqrt[c]*(4*c*d +
(-2*b + Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[
2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-b +
Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*Sqrt[c]*(4*c*d - (2*b + Sqrt[b^2 - 4*a*c]
)*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/(-
```

```
b^2 + 4*a*c)
```

Maple [B] time = 0.329, size = 869, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*x^2+b*x+a)^2,x)
```

```
[Out] e/(4*a*c-b^2)/(-e^2*(4*a*c-b^2))^(1/2)*(-4*a*c*e^2+b^2*e^2)^(1/2)*(e*x+d)^(
1/2)/(e*x+1/2*b*e/c-1/2/c*(e^2*(-4*a*c+b^2))^(1/2))+2*e^2*c/(4*a*c-b^2)/(-e
^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/
2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c
)^(1/2))*b-4*e*c^2/(4*a*c-b^2)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*
d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e
+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d-e*c/(4*a*c-b^2)/(-e^2*(4*a*c-b
^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(
(e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*(-
4*a*c*e^2+b^2*e^2)^(1/2)+e/(4*a*c-b^2)/(-e^2*(4*a*c-b^2))^(1/2)*(-4*a*c*e^2
+b^2*e^2)^(1/2)*(e*x+d)^(1/2)/(e*x+1/2*b*e/c+1/2/c*(e^2*(-4*a*c+b^2))^(1/2)
)+2*e^2*c/(4*a*c-b^2)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4
*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2
*(4*a*c-b^2))^(1/2))*c)^(1/2))*b-4*e*c^2/(4*a*c-b^2)/(-e^2*(4*a*c-b^2))^(1/
2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1
/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d+e*c/(4*a*c-
b^2)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))
*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/
2))*c)^(1/2))*(-4*a*c*e^2+b^2*e^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\left(cx^2+bx+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(c*x^2 + b*x + a)^2, x)
```

Fricas [B] time = 3.28035, size = 13504, normalized size = 47.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*(a*b^2 - 4*a^2*c + (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x
)*sqrt((32*c^3*d^3 - 48*b*c^2*d^2*e + 6*(3*b^2*c + 4*a*c^2)*d*e^2 - (b^3 +
```

12*a*b*c)*e^3 + sqrt(e^6/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3
*c^5)*d^4 - 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3*e
+ (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2*e^
2 - 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d*e^3 + (a^2*b
^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*((b^6*c - 12*a*b^4*c
^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*d^2 - (b^7 - 12*a*b^5*c + 48*a^2*b^3*c^2
- 64*a^3*b*c^3)*d*e + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*
e^2))/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*d^2 - (b^7 - 12
*a*b^5*c + 48*a^2*b^3*c^2 - 64*a^3*b*c^3)*d*e + (a*b^6 - 12*a^2*b^4*c + 48*
a^3*b^2*c^2 - 64*a^4*c^3)*e^2))*log(sqrt(1/2)*(2*(b^4*c - 8*a*b^2*c^2 + 16*
a^2*c^3)*d*e^4 - (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*e^5 - (8*(b^6*c^3 - 12*a*
b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*d^4 - 16*(b^7*c^2 - 12*a*b^5*c^3 + 4
8*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^3*e + 3*(3*b^8*c - 32*a*b^6*c^2 + 96*a^2*b^
4*c^3 - 256*a^4*c^5)*d^2*e^2 - (b^9 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3 - 76
8*a^4*b*c^4)*d*e^3 + (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*
e^4)*sqrt(e^6/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 -
2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3*e + (b^8 - 10
*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2*e^2 - 2*(a*b^
7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d*e^3 + (a^2*b^6 - 12*a^3
*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)))*sqrt((32*c^3*d^3 - 48*b*c^2*d^
2*e + 6*(3*b^2*c + 4*a*c^2)*d*e^2 - (b^3 + 12*a*b*c)*e^3 + sqrt(e^6/((b^6*c
^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 - 2*(b^7*c - 12*a*b^5*
c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3*e + (b^8 - 10*a*b^6*c + 24*a^2*b^4
*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2*e^2 - 2*(a*b^7 - 12*a^2*b^5*c + 48
*a^3*b^3*c^2 - 64*a^4*b*c^3)*d*e^3 + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c
^2 - 64*a^5*c^3)*e^4))*((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4
)*d^2 - (b^7 - 12*a*b^5*c + 48*a^2*b^3*c^2 - 64*a^3*b*c^3)*d*e + (a*b^6 - 1
2*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))/((b^6*c - 12*a*b^4*c^2 + 4
8*a^2*b^2*c^3 - 64*a^3*c^4)*d^2 - (b^7 - 12*a*b^5*c + 48*a^2*b^3*c^2 - 64*a
^3*b*c^3)*d*e + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))
+ 2*(16*c^3*d^2*e^3 - 16*b*c^2*d*e^4 + (3*b^2*c + 4*a*c^2)*e^5)*sqrt(e*x +
d)) - sqrt(1/2)*(a*b^2 - 4*a^2*c + (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*
x)*sqrt((32*c^3*d^3 - 48*b*c^2*d^2*e + 6*(3*b^2*c + 4*a*c^2)*d*e^2 - (b^3 +
12*a*b*c)*e^3 + sqrt(e^6/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^
3*c^5)*d^4 - 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3*e
+ (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2*e
^2 - 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d*e^3 + (a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*((b^6*c - 12*a*b^4*
c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*d^2 - (b^7 - 12*a*b^5*c + 48*a^2*b^3*c^2
- 64*a^3*b*c^3)*d*e + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)
*e^2))/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*d^2 - (b^7 - 1
2*a*b^5*c + 48*a^2*b^3*c^2 - 64*a^3*b*c^3)*d*e + (a*b^6 - 12*a^2*b^4*c + 48
*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))*log(-sqrt(1/2)*(2*(b^4*c - 8*a*b^2*c^2 + 1
6*a^2*c^3)*d*e^4 - (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*e^5 - (8*(b^6*c^3 - 12*
a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*d^4 - 16*(b^7*c^2 - 12*a*b^5*c^3 +
48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^3*e + 3*(3*b^8*c - 32*a*b^6*c^2 + 96*a^2*
b^4*c^3 - 256*a^4*c^5)*d^2*e^2 - (b^9 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3 -
768*a^4*b*c^4)*d*e^3 + (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4
)*e^4)*sqrt(e^6/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4
- 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3*e + (b^8 -
10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2*e^2 - 2*(a*
b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d*e^3 + (a^2*b^6 - 12*a
^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)))*sqrt((32*c^3*d^3 - 48*b*c^2*
d^2*e + 6*(3*b^2*c + 4*a*c^2)*d*e^2 - (b^3 + 12*a*b*c)*e^3 + sqrt(e^6/((b^6
*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 - 2*(b^7*c - 12*a*b^
5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3*e + (b^8 - 10*a*b^6*c + 24*a^2*b
^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2*e^2 - 2*(a*b^7 - 12*a^2*b^5*c +
48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d*e^3 + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2
*c^2 - 64*a^5*c^3)*e^4))*((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c

```
^4)*d^2 - (b^7 - 12*a*b^5*c + 48*a^2*b^3*c^2 - 64*a^3*b*c^3)*d*e + (a*b^6 -
 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))/((b^6*c - 12*a*b^4*c^2 +
 48*a^2*b^2*c^3 - 64*a^3*c^4)*d^2 - (b^7 - 12*a*b^5*c + 48*a^2*b^3*c^2 - 64
*a^3*b*c^3)*d*e + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2)
) + 2*(16*c^3*d^2*e^3 - 16*b*c^2*d*e^4 + (3*b^2*c + 4*a*c^2)*e^5)*sqrt(e*x
+ d)) + sqrt(1/2)*(a*b^2 - 4*a^2*c + (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c
)*x)*sqrt((32*c^3*d^3 - 48*b*c^2*d^2*e + 6*(3*b^2*c + 4*a*c^2)*d*e^2 - (b^3
 + 12*a*b*c)*e^3 - sqrt(e^6/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*
a^3*c^5)*d^4 - 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3
*e + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2
*e^2 - 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d*e^3 + (a^
2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*((b^6*c - 12*a*b^
4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*d^2 - (b^7 - 12*a*b^5*c + 48*a^2*b^3*c
^2 - 64*a^3*b*c^3)*d*e + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^
3)*e^2))/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*d^2 - (b^7 -
 12*a*b^5*c + 48*a^2*b^3*c^2 - 64*a^3*b*c^3)*d*e + (a*b^6 - 12*a^2*b^4*c +
48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))*log(sqrt(1/2)*(2*(b^4*c - 8*a*b^2*c^2 +
16*a^2*c^3)*d*e^4 - (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*e^5 + (8*(b^6*c^3 - 12
*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*d^4 - 16*(b^7*c^2 - 12*a*b^5*c^3
+ 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^3*e + 3*(3*b^8*c - 32*a*b^6*c^2 + 96*a^2
*b^4*c^3 - 256*a^4*c^5)*d^2*e^2 - (b^9 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3 -
 768*a^4*b*c^4)*d*e^3 + (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^
4)*e^4)*sqrt(e^6/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^
4 - 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3*e + (b^8 -
 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2*e^2 - 2*(a
*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d*e^3 + (a^2*b^6 - 12*
a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)))*sqrt((32*c^3*d^3 - 48*b*c^2
*d^2*e + 6*(3*b^2*c + 4*a*c^2)*d*e^2 - (b^3 + 12*a*b*c)*e^3 - sqrt(e^6/((b^
6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 - 2*(b^7*c - 12*a*b
^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3*e + (b^8 - 10*a*b^6*c + 24*a^2*
b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2*e^2 - 2*(a*b^7 - 12*a^2*b^5*c +
 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d*e^3 + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^
2*c^2 - 64*a^5*c^3)*e^4))*((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*
c^4)*d^2 - (b^7 - 12*a*b^5*c + 48*a^2*b^3*c^2 - 64*a^3*b*c^3)*d*e + (a*b^6
- 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))/((b^6*c - 12*a*b^4*c^2
+ 48*a^2*b^2*c^3 - 64*a^3*c^4)*d^2 - (b^7 - 12*a*b^5*c + 48*a^2*b^3*c^2 - 6
4*a^3*b*c^3)*d*e + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2
)) + 2*(16*c^3*d^2*e^3 - 16*b*c^2*d*e^4 + (3*b^2*c + 4*a*c^2)*e^5)*sqrt(e*x
 + d)) - sqrt(1/2)*(a*b^2 - 4*a^2*c + (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*
c)*x)*sqrt((32*c^3*d^3 - 48*b*c^2*d^2*e + 6*(3*b^2*c + 4*a*c^2)*d*e^2 - (b^
3 + 12*a*b*c)*e^3 - sqrt(e^6/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64
*a^3*c^5)*d^4 - 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^
3*e + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^
2*e^2 - 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d*e^3 + (a
^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*((b^6*c - 12*a*b
^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*d^2 - (b^7 - 12*a*b^5*c + 48*a^2*b^3*
c^2 - 64*a^3*b*c^3)*d*e + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c
^3)*e^2))/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*d^2 - (b^7
- 12*a*b^5*c + 48*a^2*b^3*c^2 - 64*a^3*b*c^3)*d*e + (a*b^6 - 12*a^2*b^4*c +
 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))*log(-sqrt(1/2)*(2*(b^4*c - 8*a*b^2*c^2
+ 16*a^2*c^3)*d*e^4 - (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*e^5 + (8*(b^6*c^3 -
12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*d^4 - 16*(b^7*c^2 - 12*a*b^5*c^
3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^3*e + 3*(3*b^8*c - 32*a*b^6*c^2 + 96*a
^2*b^4*c^3 - 256*a^4*c^5)*d^2*e^2 - (b^9 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3
 - 768*a^4*b*c^4)*d*e^3 + (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*
c^4)*e^4)*sqrt(e^6/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*
d^4 - 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3*e + (b^8
 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2*e^2 - 2*
(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d*e^3 + (a^2*b^6 - 1
```

```
2*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)))*sqrt((32*c^3*d^3 - 48*b*c
^2*d^2*e + 6*(3*b^2*c + 4*a*c^2)*d*e^2 - (b^3 + 12*a*b*c)*e^3 - sqrt(e^6/((
b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 - 2*(b^7*c - 12*a
*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3*e + (b^8 - 10*a*b^6*c + 24*a^
2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2*e^2 - 2*(a*b^7 - 12*a^2*b^5*c
 + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d*e^3 + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*
b^2*c^2 - 64*a^5*c^3)*e^4))*((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^
3*c^4)*d^2 - (b^7 - 12*a*b^5*c + 48*a^2*b^3*c^2 - 64*a^3*b*c^3)*d*e + (a*b^
6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))/((b^6*c - 12*a*b^4*c^
2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*d^2 - (b^7 - 12*a*b^5*c + 48*a^2*b^3*c^2 -
 64*a^3*b*c^3)*d*e + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e
^2)) + 2*(16*c^3*d^2*e^3 - 16*b*c^2*d*e^4 + (3*b^2*c + 4*a*c^2)*e^5)*sqrt(e
*x + d)) - 2*(2*c*x + b)*sqrt(e*x + d))/(a*b^2 - 4*a^2*c + (b^2*c - 4*a*c^2
)*x^2 + (b^3 - 4*a*b*c)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.2299 $\int \frac{1}{\sqrt{d+ex}\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=428

$$\frac{\sqrt{c}\left(-2ce\left(-d\sqrt{b^2-4ac}-6ae+4bd\right)-be^2\left(\sqrt{b^2-4ac}+b\right)+8c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{c}\left(-2ce\left(d\sqrt{b^2-\right.\right.}{\sqrt{}}$$

```
[Out] -((Sqrt[d + e*x]*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a
*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2))) + (Sqrt[c]*(8*c^2*d^2 - b*(
b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(4*b*d - Sqrt[b^2 - 4*a*c]*d - 6*a*e))*A
rcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*
(c*d^2 - b*d*e + a*e^2)) - (Sqrt[c]*(8*c^2*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*
e^2 - 2*c*e*(4*b*d + Sqrt[b^2 - 4*a*c]*d - 6*a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]
*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*(b^2 - 4
*a*c)^(3/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)
)
```

Rubi [A] time = 1.47212, antiderivative size = 428, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {740, 826, 1166, 208}

$$\frac{\sqrt{c}\left(-2ce\left(-d\sqrt{b^2-4ac}-6ae+4bd\right)-be^2\left(\sqrt{b^2-4ac}+b\right)+8c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{c}\left(-2ce\left(d\sqrt{b^2-\right.\right.}{\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(a + b*x + c*x^2)^2),x]
```


```
[Out] -((Sqrt[d + e*x]*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a
*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2))) + (Sqrt[c]*(8*c^2*d^2 - b*(
b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(4*b*d - Sqrt[b^2 - 4*a*c]*d - 6*a*e))*A
rcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*
(c*d^2 - b*d*e + a*e^2)) - (Sqrt[c]*(8*c^2*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*
e^2 - 2*c*e*(4*b*d + Sqrt[b^2 - 4*a*c]*d - 6*a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]
*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*(b^2 - 4
*a*c)^(3/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)
)
```


Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
```

```
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\left(a+bx+cx^2\right)^2}\,dx = -\frac{\sqrt{d+ex}\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)} - \frac{\int \frac{\frac{1}{2}\left(4c^2d^2-b^2e^2-3ce(bd-2ae)\right)+\frac{1}{2}ce(2cd-be)x}{\sqrt{d+ex}\left(a+bx+cx^2\right)}\,dx}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{\sqrt{d+ex}\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)} - \frac{2\,\text{Subst}\left(\int \frac{-\frac{1}{2}cde(2cd-be)+\frac{1}{2}e\left(4c^2d^2-b^2e^2-3ce(\cdots\right.}{cd^2-bde+ae^2+(-2cd+\cdots}\right.}{\left(b^2-4ac\right)\left(cd^2-\cdots\right.}$$

$$= -\frac{\sqrt{d+ex}\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)} - \frac{\left(c\left(8c^2d^2-b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(\cdots\right.\right.\right.}{\cdots}$$

$$= -\frac{\sqrt{d+ex}\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)} + \frac{\sqrt{c}\left(8c^2d^2-b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(\cdots\right.\right.}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{2cd\cdots}}$$

Mathematica [A] time = 1.44925, size = 366, normalized size = 0.86

$$\frac{\frac{\sqrt{c}\left(\frac{\left(2ce\left(d\sqrt{b^2-4ac}+6ae-4bd\right)-be^2\left(\sqrt{b^2-4ac}+b\right)+8c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}-\frac{\left(-2ce\left(d\sqrt{b^2-4ac}-6ae+4bd\right)+be^2\left(\sqrt{b^2-4ac}-b\right)+8c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\sqrt{b^2-4ac}}+\frac{\sqrt{d\cdots}}{\cdots}}{\left(b^2-4ac\right)\left(e(ae-bd)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(a + b*x + c*x^2)^2),x]
```

```
[Out] ((Sqrt[d + e*x]*(b^2*e - 2*c*(a*e + c*d*x) + b*c*(-d + e*x)))/(a + x*(b + c
*x)) + (Sqrt[c]*(((8*c^2*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*e^2 + 2*c*e*(-4*b*
d + Sqrt[b^2 - 4*a*c]*d + 6*a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/S
qrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*
c])*e] - ((8*c^2*d^2 + b*(-b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(4*b*d + Sqrt
[b^2 - 4*a*c]*d - 6*a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e]])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/
(Sqrt[2]*Sqrt[b^2 - 4*a*c]))/((b^2 - 4*a*c)*(c*d^2 + e*(-(b*d) + a*e)))
```

Maple [B] time = 0.283, size = 1120, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)^2,x)
```

```
[Out] 2*e*c/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)/(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)
^(1/2))*(-4*a*c*e^2+b^2*e^2)^(1/2)*(e*x+d)^(1/2)/(e*x+1/2*b*e/c-1/2/c*(e^2*
(-4*a*c+b^2))^(1/2))+8*e^2*c^2/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)/(-2*b*e
+4*c*d+2*(-4*a*c*e^2+b^2*e^2)^(1/2))*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2)
)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c
-b^2))^(1/2))*c)^(1/2))*b-16*e*c^3/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)/(-2
*b*e+4*c*d+2*(-4*a*c*e^2+b^2*e^2)^(1/2))*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-
b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4
*a*c-b^2))^(1/2))*c)^(1/2))*d-12*e*c^2/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)
/(-2*b*e+4*c*d+2*(-4*a*c*e^2+b^2*e^2)^(1/2))*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*
a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^
2*(4*a*c-b^2))^(1/2))*c)^(1/2))*(-4*a*c*e^2+b^2*e^2)^(1/2)+2*e*c/(-e^2*(4*a
*c-b^2))^(1/2)/(4*a*c-b^2)/(-b*e+2*c*d-(-4*a*c*e^2+b^2*e^2)^(1/2))*(-4*a*c*
e^2+b^2*e^2)^(1/2)*(e*x+d)^(1/2)/(e*x+1/2*b*e/c+1/2/c*(e^2*(-4*a*c+b^2))^(1
/2))-8*e^2*c^2/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)/(2*b*e-4*c*d+2*(-4*a*c*
e^2+b^2*e^2)^(1/2))*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*
arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/
2))*b+16*e*c^3/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)/(2*b*e-4*c*d+2*(-4*a*c*
e^2+b^2*e^2)^(1/2))*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*
arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/
2))*d-12*e*c^2/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)/(2*b*e-4*c*d+2*(-4*a*c*
e^2+b^2*e^2)^(1/2))*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*
arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/
2))*(-4*a*c*e^2+b^2*e^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2 + bx + a\right)^2 \sqrt{ex + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^2*sqrt(e*x + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.2300 $\int \frac{1}{(d+ex)^{3/2}\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=604

$$-\frac{e\left(-2ce(5ae+bd)+3b^2e^2+2c^2d^2\right)}{\left(b^2-4ac\right)\sqrt{d+ex}\left(ae^2-bde+cd^2\right)^2}+\frac{\sqrt{c}\left(-2c^2de\left(-d\sqrt{b^2-4ac}-16ae+6bd\right)-2ce^2\left(bd\sqrt{b^2-4ac}+5ae\sqrt{b^2-4\right.\right.}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{2cd-e\left(b\right.}}$$

```
[Out] -((e*(2*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(b*d + 5*a*e)))/((b^2 - 4*a*c)*(c*d^2 -
 b*d*e + a*e^2)^2*Sqrt[d + e*x])) - (b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b
*e)*x)/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x]*(a + b*x + c*x^
2)) + (Sqrt[c]*(8*c^3*d^3 + 3*b^2*(b + Sqrt[b^2 - 4*a*c])*e^3 - 2*c^2*d*e*(
6*b*d - Sqrt[b^2 - 4*a*c]*d - 16*a*e) - 2*c*e^2*(b^2*d + b*Sqrt[b^2 - 4*a*c
]*d + 8*a*b*e + 5*a*Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d +
 e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/
2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)^2) - (Sq
rt[c]*(8*c^3*d^3 + 3*b^2*(b - Sqrt[b^2 - 4*a*c])*e^3 - 2*c^2*d*e*(6*b*d + S
qrt[b^2 - 4*a*c]*d - 16*a*e) - 2*c*e^2*(b^2*d - b*Sqrt[b^2 - 4*a*c]*d + 8*a
*b*e - 5*a*Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sq
rt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[2
*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)^2)
```

Rubi [A] time = 4.94267, antiderivative size = 604, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {740, 828, 826, 1166, 208}

$$-\frac{e\left(-2ce(5ae+bd)+3b^2e^2+2c^2d^2\right)}{\left(b^2-4ac\right)\sqrt{d+ex}\left(ae^2-bde+cd^2\right)^2}+\frac{\sqrt{c}\left(-2c^2de\left(-d\sqrt{b^2-4ac}-16ae+6bd\right)-2ce^2\left(bd\sqrt{b^2-4ac}+5ae\sqrt{b^2-4\right.\right.}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{2cd-e\left(b\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*(a + b*x + c*x^2)^2),x]
```

```
[Out] -((e*(2*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(b*d + 5*a*e)))/((b^2 - 4*a*c)*(c*d^2 -
 b*d*e + a*e^2)^2*Sqrt[d + e*x])) - (b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b
*e)*x)/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x]*(a + b*x + c*x^
2)) + (Sqrt[c]*(8*c^3*d^3 + 3*b^2*(b + Sqrt[b^2 - 4*a*c])*e^3 - 2*c^2*d*e*(
6*b*d - Sqrt[b^2 - 4*a*c]*d - 16*a*e) - 2*c*e^2*(b^2*d + b*Sqrt[b^2 - 4*a*c
]*d + 8*a*b*e + 5*a*Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d +
 e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/
2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)^2) - (Sq
rt[c]*(8*c^3*d^3 + 3*b^2*(b - Sqrt[b^2 - 4*a*c])*e^3 - 2*c^2*d*e*(6*b*d + S
qrt[b^2 - 4*a*c]*d - 16*a*e) - 2*c*e^2*(b^2*d - b*Sqrt[b^2 - 4*a*c]*d + 8*a
*b*e - 5*a*Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sq
rt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[2
*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)^2)
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
```

```
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\left(a+bx+cx^2\right)^2}\,dx = -\frac{bcd-b^2e+2ace+c(2cd-be)x}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{d+ex}\left(a+bx+cx^2\right)} - \frac{\int \frac{\frac{1}{2}\left(4c^2d^2-bcde-3b^2e^2+10ace^2\right)+ \cdots}{(d+ex)^{3/2}\left(a+bx+cx^2\right)} \cdots}{\left(b^2-4ac\right)\left(cd^2-bd \cdots\right.}$$

$$= -\frac{e\left(2c^2d^2+3b^2e^2-2ce(bd+5ae)\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)^2\sqrt{d+ex}} - \frac{bcd-b^2e+2ace+c(2cd-be) \cdots}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{d+ex}\,(a \cdots}$$

$$= -\frac{e\left(2c^2d^2+3b^2e^2-2ce(bd+5ae)\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)^2\sqrt{d+ex}} - \frac{bcd-b^2e+2ace+c(2cd-be) \cdots}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{d+ex}\,(a \cdots}$$

$$= -\frac{e\left(2c^2d^2+3b^2e^2-2ce(bd+5ae)\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)^2\sqrt{d+ex}} - \frac{bcd-b^2e+2ace+c(2cd-be) \cdots}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{d+ex}\,(a \cdots}$$

$$= -\frac{e\left(2c^2d^2+3b^2e^2-2ce(bd+5ae)\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)^2\sqrt{d+ex}} - \frac{bcd-b^2e+2ace+c(2cd-be) \cdots}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{d+ex}\,(a \cdots}$$

Mathematica [A] time = 2.93336, size = 552, normalized size = 0.91

$$\frac{\frac{e\left(2ce(5ae+bd)-3b^2e^2-2c^2d^2\right)}{\sqrt{d+ex}\left(e(ae-bd)+cd^2\right)} - \frac{\sqrt{c}\left(\frac{\left(2c^2de\left(d\sqrt{b^2-4ac}+16ae-6bd\right)-2ce^2\left(bd\sqrt{b^2-4ac}+5ae\sqrt{b^2-4ac}+8abe+b^2d\right)+3b^2e^3\left(\sqrt{b^2-4ac}+b\right)+8c^3d^3\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}} - \frac{\left(-2 \cdots\right.}{\cdots}\right.}{\sqrt{2}\sqrt{b^2-4ac}\left(e(bd \cdots\right.}}{\left(b^2-4ac\right)\left(e(ae - \cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*(a + b*x + c*x^2)^2),x]
```

```
[Out] ((e*(-2*c^2*d^2 - 3*b^2*e^2 + 2*c*e*(b*d + 5*a*e)))/((c*d^2 + e*(-(b*d) + a
*e))*Sqrt[d + e*x]) + (b^2*e - 2*c*(a*e + c*d*x) + b*c*(-d + e*x))/(Sqrt[d
+ e*x]*(a + x*(b + c*x))) - (Sqrt[c]*(((8*c^3*d^3 + 3*b^2*(b + Sqrt[b^2 - 4
*a*c])*e^3 + 2*c^2*d*e*(-6*b*d + Sqrt[b^2 - 4*a*c]*d + 16*a*e) - 2*c*e^2*(b
^2*d + b*Sqrt[b^2 - 4*a*c]*d + 8*a*b*e + 5*a*Sqrt[b^2 - 4*a*c]*e))*ArcTanh[
(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/S
qrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e] - ((8*c^3*d^3 + 3*b^2*(b - Sqrt[b^2
 - 4*a*c])*e^3 - 2*c^2*d*e*(6*b*d + Sqrt[b^2 - 4*a*c]*d - 16*a*e) + 2*c*e^2
*(-(b^2*d) + b*Sqrt[b^2 - 4*a*c]*d - 8*a*b*e + 5*a*Sqrt[b^2 - 4*a*c]*e))*Ar
cTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e]])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*(
-(c*d^2) + e*(b*d - a*e))))/((b^2 - 4*a*c)*(c*d^2 + e*(-(b*d) + a*e)))
```

Maple [B] time = 0.3, size = 3212, normalized size = 5.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)^2,x)

[Out] -4*e/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c^4/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)
/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^
(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d^3-4*e/(a*e^2-b*d*e
+c*d^2)^2/(4*a*c-b^2)*c^4/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^
2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(
-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d^3-e^2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2
)*c^2*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)
^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d-3/2*e^
4/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*
e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/(
(b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^3-3/2*e^4/(a*e^2-b*d*e+c*d
^2)^2/(4*a*c-b^2)*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*
a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^
2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^3+e^2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c
^2*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^
(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d-2*e^3/
(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^(1/2)+8*e^4/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)
c^2/(-e^2(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))
*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/
2))*c)^(1/2))*a*b+e^3/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c^2/(-e^2*(4*a*c-b^
2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*
x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2*d-
16*e^3/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c^3/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/
2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*
2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*d+2*e/(a*e^2-b*d
*e+c*d^2)^2/(c*e^2*x^2+b*e^2*x+a*e^2)*c^3/(4*a*c-b^2)*(e*x+d)^(3/2)*d^2-2*e
/(a*e^2-b*d*e+c*d^2)^2/(c*e^2*x^2+b*e^2*x+a*e^2)/(4*a*c-b^2)*(e*x+d)^(1/2)*
c^3*d^3-2*e^3/(a*e^2-b*d*e+c*d^2)^2/(c*e^2*x^2+b*e^2*x+a*e^2)*c^2/(4*a*c-b^
2)*(e*x+d)^(3/2)*a+e^3/(a*e^2-b*d*e+c*d^2)^2/(c*e^2*x^2+b*e^2*x+a*e^2)*c/(4
*a*c-b^2)*(e*x+d)^(3/2)*b^2-e/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c^3*2^(1/2)
/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^
(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d^2+e/(a*e^2-b*d*e+c
*d^2)^2/(4*a*c-b^2)*c^3*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1
/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)
^(1/2))*d^2-3*e^4/(a*e^2-b*d*e+c*d^2)^2/(c*e^2*x^2+b*e^2*x+a*e^2)/(4*a*c-b^
2)*(e*x+d)^(1/2)*a*b*c+3*e^2/(a*e^2-b*d*e+c*d^2)^2/(c*e^2*x^2+b*e^2*x+a*e^2
)/(4*a*c-b^2)*(e*x+d)^(1/2)*b*c^2*d^2-2*e^2/(a*e^2-b*d*e+c*d^2)^2/(c*e^2*x^
2+b*e^2*x+a*e^2)*c^2/(4*a*c-b^2)*(e*x+d)^(3/2)*b*d+e^3/(a*e^2-b*d*e+c*d^2)^
2/(4*a*c-b^2)*c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*
c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*
(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2*d-16*e^3/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2
)*c^3/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2)
)*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1
/2))*c)^(1/2))*a*d+6*e^2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c^3/(-e^2*(4*a*c
-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan
h((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*
b*d^2+6*e^3/(a*e^2-b*d*e+c*d^2)^2/(c*e^2*x^2+b*e^2*x+a*e^2)/(4*a*c-b^2)*(e*
x+d)^(1/2)*c^2*a*d-3*e^3/(a*e^2-b*d*e+c*d^2)^2/(c*e^2*x^2+b*e^2*x+a*e^2)/(4
*a*c-b^2)*(e*x+d)^(1/2)*b^2*c*d-3/2*e^3/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c
*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1
/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2-5*e^3/(a
*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c^2*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^
(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2
))^(1/2))*c)^(1/2))*a+3/2*e^3/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c*2^(1/2)/(
(b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2
)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2+5*e^3/(a*e^2-b*d*e+c*
d^2)^2/(4*a*c-b^2)*c^2*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1

```
/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*
c)^(1/2))*a+6*e^2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c^3/(-e^2*(4*a*c-b^2))^
(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)
^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d^2+8*e^
4/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((
-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/
2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b+e^4/(a*e^2-b*d*e+c*
d^2)^2/(c*e^2*x^2+b*e^2*x+a*e^2)/(4*a*c-b^2)*(e*x+d)^(1/2)*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^2(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^2*(e*x + d)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.2301 $\int \frac{1}{x^{5/2}(a+bx+cx^2)^2}\,dx$

Optimal. Leaf size=360

$$\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(28a^2c^2-29ab^2c-b\left(5b^2-19ac\right)\sqrt{b^2\right.}}{\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{}}}$$

```
[Out] -(5*b^2 - 14*a*c)/(3*a^2*(b^2 - 4*a*c)*x^(3/2)) + (b*(5*b^2 - 19*a*c))/(a^3
*(b^2 - 4*a*c)*Sqrt[x]) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^(3/2)*(a
 + b*x + c*x^2)) + (Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + b*(5*b^2 - 1
9*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^
2 - 4*a*c]]])/(Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 - (Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - b*(5*b^2 - 19*a*c)*Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(S
qrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 3.69344, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {740, 828, 826, 1166, 205}

$$\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(28a^2c^2-29ab^2c-b\left(5b^2-19ac\right)\sqrt{b^2\right.}}{\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x + c*x^2)^2),x]
```


```
[Out] -(5*b^2 - 14*a*c)/(3*a^2*(b^2 - 4*a*c)*x^(3/2)) + (b*(5*b^2 - 19*a*c))/(a^3
*(b^2 - 4*a*c)*Sqrt[x]) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^(3/2)*(a
 + b*x + c*x^2)) + (Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + b*(5*b^2 - 1
9*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^
2 - 4*a*c]]])/(Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 - (Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - b*(5*b^2 - 19*a*c)*Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(S
qrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```


Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
```

```
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(a+bx+cx^2\right)^2}\,dx = \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^{3/2}\left(a+bx+cx^2\right)} - \frac{\int \frac{\frac{1}{2}\left(-5b^2+14ac\right)-\frac{5bcx}{2}}{x^{5/2}\left(a+bx+cx^2\right)}\,dx}{a\left(b^2-4ac\right)}$$

$$= -\frac{5b^2-14ac}{3a^2\left(b^2-4ac\right)x^{3/2}} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^{3/2}\left(a+bx+cx^2\right)} - \frac{\int \frac{\frac{1}{2}b\left(5b^2-19ac\right)+\frac{1}{2}c\left(5b^2-14ac\right)x}{x^{3/2}\left(a+bx+cx^2\right)}\,dx}{a^2\left(b^2-4ac\right)}$$

$$= -\frac{5b^2-14ac}{3a^2\left(b^2-4ac\right)x^{3/2}} + \frac{b\left(5b^2-19ac\right)}{a^3\left(b^2-4ac\right)\sqrt{x}} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^{3/2}\left(a+bx+cx^2\right)} - \frac{\int \frac{1}{2}\left(-5b^4+24a \ldots\right.}{\ldots}$$

$$= -\frac{5b^2-14ac}{3a^2\left(b^2-4ac\right)x^{3/2}} + \frac{b\left(5b^2-19ac\right)}{a^3\left(b^2-4ac\right)\sqrt{x}} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^{3/2}\left(a+bx+cx^2\right)} - \frac{2\,\mathrm{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{5b^2-14ac}{3a^2\left(b^2-4ac\right)x^{3/2}} + \frac{b\left(5b^2-19ac\right)}{a^3\left(b^2-4ac\right)\sqrt{x}} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^{3/2}\left(a+bx+cx^2\right)} - \frac{\left(c\left(5b^4-29 \ldots\right.\right.}{\ldots}$$

$$= -\frac{5b^2-14ac}{3a^2\left(b^2-4ac\right)x^{3/2}} + \frac{b\left(5b^2-19ac\right)}{a^3\left(b^2-4ac\right)\sqrt{x}} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^{3/2}\left(a+bx+cx^2\right)} + \frac{\sqrt{c}\left(5b^4-2 \ldots\right.}{\ldots}$$

Mathematica [A] time = 0.91322, size = 337, normalized size = 0.94

$$\frac{-\frac{\sqrt{c}\left(\frac{\left(28a^2c^2-5b^3\sqrt{b^2-4ac}-29ab^2c+19abc\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}-\frac{\left(28a^2c^2+5b^3\sqrt{b^2-4ac}-29ab^2c-19abc\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}}+\frac{b\left(5b^2-19ac\right)}{a^2\sqrt{x}}+\cdots}{a\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x + c*x^2)^2),x]
```

```
[Out] ((-5*b^2 + 14*a*c)/(3*a*x^(3/2)) + (b*(5*b^2 - 19*a*c))/(a^2*Sqrt[x]) + (b^
2 - 2*a*c + b*c*x)/(x^(3/2)*(a + x*(b + c*x))) - (Sqrt[c]*(-(((5*b^4 - 29*a
*b^2*c + 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sq
rt[b^2 - 4*a*c]]) + ((5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - 5*b^3*Sqrt[b^2 - 4*
a*c] + 19*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(Sqrt[2]*a^2*Sqrt[b^2
- 4*a*c]))/(a*(b^2 - 4*a*c))
```

Maple [B] time = 0.226, size = 929, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(c*x^2+b*x+a)^2,x)
```

```
[Out] -2/3/a^2/x^(3/2)+4*b/a^3/x^(1/2)+3/a^2/(c*x^2+b*x+a)*b*c^2/(4*a*c-b^2)*x^(3
/2)-1/a^3/(c*x^2+b*x+a)*b^3*c/(4*a*c-b^2)*x^(3/2)-2/a/(c*x^2+b*x+a)/(4*a*c-
b^2)*x^(1/2)*c^2+4/a^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x^(1/2)*c*b^2-1/a^3/(c*x^2
+b*x+a)/(4*a*c-b^2)*x^(1/2)*b^4-19/2/a^2*c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(x^(1/2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))*b+5/2/a^3*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctanh(x^(1/2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3+14/a*c^3/
(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctanh(x^(1/2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-29/2/a^2*c^2/(4*
a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
nh(x^(1/2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2+5/2/a^3*c/(4*a*
c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(x^(1/2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4+19/2/a^2*c^2/(4*a
*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x^(1/2)*c*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b-5/2/a^3*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctan(x^(1/2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*b^3+14/a*c^3/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(x^(1/2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
-29/2/a^2*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctan(x^(1/2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2+5
/2/a^3*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctan(x^(1/2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\left(5\,b^4c-24\,ab^2c^2+14\,a^2c^3\right)x^{\frac{5}{2}}+3\left(5\,b^5-19\,ab^3c-5\,a^2bc^2\right)x^{\frac{3}{2}}+2\left(15\,ab^4-67\,a^2b^2c+28\,a^3c^2\right)\sqrt{x}+\frac{10\left(a^2b^3-4\,a^3bc\right)}{\sqrt{x}}\cdot}{3\left(a^5b^2-4\,a^6c+\left(a^4b^2c-4\,a^5c^2\right)x^2+\left(a^4b^3-4\,a^5bc\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(5*b^4*c - 24*a*b^2*c^2 + 14*a^2*c^3)*x^(5/2) + 3*(5*b^5 - 19*a*b^3*
c - 5*a^2*b*c^2)*x^(3/2) + 2*(15*a*b^4 - 67*a^2*b^2*c + 28*a^3*c^2)*sqrt(x)
 + 10*(a^2*b^3 - 4*a^3*b*c)/sqrt(x) - 2*(a^3*b^2 - 4*a^4*c)/x^(3/2))/(a^5*b
^2 - 4*a^6*c + (a^4*b^2*c - 4*a^5*c^2)*x^2 + (a^4*b^3 - 4*a^5*b*c)*x) + int
egrate(-1/2*((5*b^4*c - 24*a*b^2*c^2 + 14*a^2*c^3)*x^(3/2) + (5*b^5 - 29*a*
b^3*c + 33*a^2*b*c^2)*sqrt(x))/(a^5*b^2 - 4*a^6*c + (a^4*b^2*c - 4*a^5*c^2)
*x^2 + (a^4*b^3 - 4*a^5*b*c)*x), x)
```

Fricas [B] time = 8.58928, size = 7992, normalized size = 22.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*x^4 + (a^3*b^3 - 4*a^4*b*c)*x^3 +
 (a^4*b^2 - 4*a^5*c)*x^2)*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 -
2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^
2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630
*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*
b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^
4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))*log(sqrt(1/2)*(125*b^14 - 2425*a*b^12*
c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^
5*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7 - (5*a^7*b^11 - 94*a^8*b^9*c
+ 700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5
)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 +
76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b
^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*
a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c +
 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b
^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a
^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b
^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)) + 2*(1125*b^8*c^4 - 1232
5*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*sqrt(x)
) - 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*x^4 + (a^3*b^3 - 4*a^4*b*c)*x^3 +
(a^4*b^2 - 4*a^5*c)*x^2)*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2
415*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2
 - 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*
a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b
^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4
*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))*log(-sqrt(1/2)*(125*b^14 - 2425*a*b^12*
c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^
5*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7 - (5*a^7*b^11 - 94*a^8*b^9*c
+ 700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5
)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 +
```

```
76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b
^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*
a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c +
 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b
^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a
^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b
^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)) + 2*(1125*b^8*c^4 - 1232
5*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*sqrt(x)
) + 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*x^4 + (a^3*b^3 - 4*a^4*b*c)*x^3 +
(a^4*b^2 - 4*a^5*c)*x^2)*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2
415*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2
 - 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*
a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b
^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4
*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))*log(sqrt(1/2)*(125*b^14 - 2425*a*b^12*c
 + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5
*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7 + (5*a^7*b^11 - 94*a^8*b^9*c +
 700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)
*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 7
6686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^
4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a
^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a^7*b^6 - 12*a^8*b^4*c +
48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^
8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^
6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^
6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)) + 2*(1125*b^8*c^4 - 12325
*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*sqrt(x))
 - 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*x^4 + (a^3*b^3 - 4*a^4*b*c)*x^3 + (
a^4*b^2 - 4*a^5*c)*x^2)*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 24
15*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2
- 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a
^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^
6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*
c + 48*a^9*b^2*c^2 - 64*a^10*c^3))*log(-sqrt(1/2)*(125*b^14 - 2425*a*b^12*c
 + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5
*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7 + (5*a^7*b^11 - 94*a^8*b^9*c +
 700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)
*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 7
6686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^
4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a
^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a^7*b^6 - 12*a^8*b^4*c +
48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^
8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^
6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^
6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)) + 2*(1125*b^8*c^4 - 12325
*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*sqrt(x))
 - 2*(2*a^2*b^2 - 8*a^3*c - 3*(5*b^3*c - 19*a*b*c^2)*x^3 - (15*b^4 - 62*a*b
^2*c + 14*a^2*c^2)*x^2 - 10*(a*b^3 - 4*a^2*b*c)*x)*sqrt(x))/((a^3*b^2*c - 4
*a^4*c^2)*x^4 + (a^3*b^3 - 4*a^4*b*c)*x^3 + (a^4*b^2 - 4*a^5*c)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.2302 $\int \frac{(d+ex)^{7/2}}{\left(a+bx+cx^2\right)^3} \, dx$

Optimal. Leaf size=751

$$\frac{\sqrt{d+ex}\left(x(2cd-be)\left(-4ce(3bd-2ae)+b^2e^2+12c^2d^2\right)+b^2\left(-\left(ae^3+11cd^2e\right)\right)+12bcd\left(3ae^2+cd^2\right)-4ace\left(5ae^2+7cd^2\right.\right.}{4c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}$$

```
[Out] -((d + e*x)^(5/2)*(b*d - 2*a*e + (2*c*d - b*e)*x))/(2*(b^2 - 4*a*c)*(a + b*
x + c*x^2)^2) + (Sqrt[d + e*x]*(12*b*c*d*(c*d^2 + 3*a*e^2) - 4*a*c*e*(7*c*d
^2 + 5*a*e^2) - b^2*(11*c*d^2*e + a*e^3) + (2*c*d - b*e)*(12*c^2*d^2 + b^2*
e^2 - 4*c*e*(3*b*d - 2*a*e))*x))/(4*c*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) -
((96*c^4*d^4 - b^3*(b - Sqrt[b^2 - 4*a*c])*e^4 - 8*c^3*d^2*e*(24*b*d - 3*Sq
rt[b^2 - 4*a*c]*d - 19*a*e) - 2*b*c*e^3*(5*b^2*d - 5*b*Sqrt[b^2 - 4*a*c]*d
- 9*a*b*e + 8*a*Sqrt[b^2 - 4*a*c]*e) + 2*c^2*e^2*(53*b^2*d^2 + 4*a*e*(4*Sqr
t[b^2 - 4*a*c]*d + 5*a*e) - 2*b*d*(9*Sqrt[b^2 - 4*a*c]*d + 38*a*e)))*ArcTan
h[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])
/(4*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)^(5/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c]
)*e]) + ((96*c^4*d^4 - b^3*(b + Sqrt[b^2 - 4*a*c])*e^4 - 8*c^3*d^2*e*(24*b*
d + 3*Sqrt[b^2 - 4*a*c]*d - 19*a*e) - 2*b*c*e^3*(5*b^2*d + 5*b*Sqrt[b^2 - 4
*a*c]*d - 9*a*b*e - 8*a*Sqrt[b^2 - 4*a*c]*e) + 2*c^2*e^2*(53*b^2*d^2 + 2*b*
d*(9*Sqrt[b^2 - 4*a*c]*d - 38*a*e) - 4*a*e*(4*Sqrt[b^2 - 4*a*c]*d - 5*a*e))
)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e]])/(4*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)^(5/2)*Sqrt[2*c*d - (b + Sqrt[b^2
- 4*a*c])*e])
```

Rubi [A] time = 13.5123, antiderivative size = 751, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {738, 818, 826, 1166, 208}

$$\frac{\sqrt{d+ex}\left(x(2cd-be)\left(-4ce(3bd-2ae)+b^2e^2+12c^2d^2\right)+b^2\left(-\left(ae^3+11cd^2e\right)\right)+12bcd\left(3ae^2+cd^2\right)-4ace\left(5ae^2+7cd^2\right.\right.}{4c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/(a + b*x + c*x^2)^3,x]
```

```
[Out] -((d + e*x)^(5/2)*(b*d - 2*a*e + (2*c*d - b*e)*x))/(2*(b^2 - 4*a*c)*(a + b*
x + c*x^2)^2) + (Sqrt[d + e*x]*(12*b*c*d*(c*d^2 + 3*a*e^2) - 4*a*c*e*(7*c*d
^2 + 5*a*e^2) - b^2*(11*c*d^2*e + a*e^3) + (2*c*d - b*e)*(12*c^2*d^2 + b^2*
e^2 - 4*c*e*(3*b*d - 2*a*e))*x))/(4*c*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) -
((96*c^4*d^4 - b^3*(b - Sqrt[b^2 - 4*a*c])*e^4 - 8*c^3*d^2*e*(24*b*d - 3*Sq
rt[b^2 - 4*a*c]*d - 19*a*e) - 2*b*c*e^3*(5*b^2*d - 5*b*Sqrt[b^2 - 4*a*c]*d
- 9*a*b*e + 8*a*Sqrt[b^2 - 4*a*c]*e) + 2*c^2*e^2*(53*b^2*d^2 + 4*a*e*(4*Sqr
t[b^2 - 4*a*c]*d + 5*a*e) - 2*b*d*(9*Sqrt[b^2 - 4*a*c]*d + 38*a*e)))*ArcTan
h[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])
/(4*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)^(5/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c]
)*e]) + ((96*c^4*d^4 - b^3*(b + Sqrt[b^2 - 4*a*c])*e^4 - 8*c^3*d^2*e*(24*b*
d + 3*Sqrt[b^2 - 4*a*c]*d - 19*a*e) - 2*b*c*e^3*(5*b^2*d + 5*b*Sqrt[b^2 - 4
*a*c]*d - 9*a*b*e - 8*a*Sqrt[b^2 - 4*a*c]*e) + 2*c^2*e^2*(53*b^2*d^2 + 2*b*
```

```
d*(9*Sqrt[b^2 - 4*a*c]*d - 38*a*e) - 4*a*e*(4*Sqrt[b^2 - 4*a*c]*d - 5*a*e))
)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e]])/(4*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)^(5/2)*Sqrt[2*c*d - (b + Sqrt[b^2
- 4*a*c])*e])
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{7/2}}{\left(a+bx+cx^2\right)^3}\,dx = -\frac{(d+ex)^{5/2}(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{\int \frac{(d+ex)^{3/2}\left(\frac{1}{2}\left(12cd^2-11bde+10ae^2\right)+\frac{1}{2}e(2cd-be)x\right)}{\left(a+bx+cx^2\right)^2}dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{(d+ex)^{5/2}(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{\sqrt{d+ex}\left(12bcd\left(cd^2+3ae^2\right)-4ace\left(7cd^2+5ae^2\right)-b^2\left(1\right.\right.}{4c\left(b^2-4ac\right.}$$

$$= -\frac{(d+ex)^{5/2}(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{\sqrt{d+ex}\left(12bcd\left(cd^2+3ae^2\right)-4ace\left(7cd^2+5ae^2\right)-b^2\left(1\right.\right.}{4c\left(b^2-4ac\right.}$$

$$= -\frac{(d+ex)^{5/2}(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{\sqrt{d+ex}\left(12bcd\left(cd^2+3ae^2\right)-4ace\left(7cd^2+5ae^2\right)-b^2\left(1\right.\right.}{4c\left(b^2-4ac\right.}$$

$$= -\frac{(d+ex)^{5/2}(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{\sqrt{d+ex}\left(12bcd\left(cd^2+3ae^2\right)-4ace\left(7cd^2+5ae^2\right)-b^2\left(1\right.\right.}{4c\left(b^2-4ac\right.}$$

Mathematica [B] time = 6.69539, size = 17950, normalized size = 23.9

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)/(a + b*x + c*x^2)^3,x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.31, size = 5849, normalized size = 7.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/(c*x^2+b*x+a)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{7}{2}}}{\left(cx^2+bx+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(7/2)/(c*x^2 + b*x + a)^3, x)
```

Fricas [B] time = 15.9702, size = 19941, normalized size = 26.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(1/2)*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + (b^4*c^3 - 8*a*b^
2*c^4 + 16*a^2*c^5)*x^4 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^3 + (b
^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*
b*c^3)*x)*sqrt((4608*c^7*d^7 - 16128*b*c^6*d^6*e + 672*(31*b^2*c^5 + 20*a*c
^6)*d^5*e^2 - 1680*(7*b^3*c^4 + 20*a*b*c^5)*d^4*e^3 + 70*(35*b^4*c^3 + 392*
a*b^2*c^4 + 176*a^2*c^5)*d^3*e^4 + 21*(b^5*c^2 - 360*a*b^3*c^3 - 880*a^2*b*
c^4)*d^2*e^5 - 21*(b^6*c - 10*a*b^4*c^2 - 320*a^2*b^2*c^3 - 160*a^3*c^4)*d*
e^6 - (b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*e^7 + (b^10*c^3
 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 10
24*a^5*c^8)*sqrt((441*c^4*d^4*e^10 - 882*b*c^3*d^3*e^11 + 21*(19*b^2*c^2 +
50*a*c^3)*d^2*e^12 + 42*(b^3*c - 25*a*b*c^2)*d*e^13 + (b^4 - 50*a*b^2*c + 6
25*a^2*c^2)*e^14)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*
c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a
^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))*log(1/2*sq
rt(1/2)*(504*(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*d^4*e^6
 - 1008*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*d^3*e^7 +
3*(167*b^8*c^2 - 1664*a*b^6*c^3 + 3936*a^2*b^4*c^4 + 5632*a^3*b^2*c^5 - 217
60*a^4*c^6)*d^2*e^8 + 3*(b^9*c - 352*a*b^7*c^2 + 4128*a^2*b^5*c^3 - 16384*a
^3*b^3*c^4 + 21760*a^4*b*c^5)*d*e^9 - (b^10 - 17*a*b^8*c - 392*a^2*b^6*c^2
+ 5696*a^3*b^4*c^3 - 23680*a^4*b^2*c^4 + 32000*a^5*c^5)*e^10 - (96*(b^10*c^
6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 -
1024*a^5*c^11)*d^3 - 144*(b^11*c^5 - 20*a*b^9*c^6 + 160*a^2*b^7*c^7 - 640*a
^3*b^5*c^8 + 1280*a^4*b^3*c^9 - 1024*a^5*b*c^10)*d^2*e + 2*(23*b^12*c^4 - 4
08*a*b^10*c^5 + 2640*a^2*b^8*c^6 - 6400*a^3*b^6*c^7 - 3840*a^4*b^4*c^8 + 43
008*a^5*b^2*c^9 - 53248*a^6*c^10)*d*e^2 + (b^13*c^3 - 72*a*b^11*c^4 + 1200*
a^2*b^9*c^5 - 8960*a^3*b^7*c^6 + 34560*a^4*b^5*c^7 - 67584*a^5*b^3*c^8 + 53
248*a^6*b*c^9)*e^3)*sqrt((441*c^4*d^4*e^10 - 882*b*c^3*d^3*e^11 + 21*(19*b^
2*c^2 + 50*a*c^3)*d^2*e^12 + 42*(b^3*c - 25*a*b*c^2)*d*e^13 + (b^4 - 50*a*b
^2*c + 625*a^2*c^2)*e^14)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*
a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))*sqrt((4608*c^7*d^7 - 161
28*b*c^6*d^6*e + 672*(31*b^2*c^5 + 20*a*c^6)*d^5*e^2 - 1680*(7*b^3*c^4 + 20
*a*b*c^5)*d^4*e^3 + 70*(35*b^4*c^3 + 392*a*b^2*c^4 + 176*a^2*c^5)*d^3*e^4 +
 21*(b^5*c^2 - 360*a*b^3*c^3 - 880*a^2*b*c^4)*d^2*e^5 - 21*(b^6*c - 10*a*b^
4*c^2 - 320*a^2*b^2*c^3 - 160*a^3*c^4)*d*e^6 - (b^7 - 35*a*b^5*c + 280*a^2*
b^3*c^2 + 1680*a^3*b*c^3)*e^7 + (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5
- 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((441*c^4*d^4*e^10
 - 882*b*c^3*d^3*e^11 + 21*(19*b^2*c^2 + 50*a*c^3)*d^2*e^12 + 42*(b^3*c - 2
5*a*b*c^2)*d*e^13 + (b^4 - 50*a*b^2*c + 625*a^2*c^2)*e^14)/(b^10*c^6 - 20*a
*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5
*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 128
0*a^4*b^2*c^7 - 1024*a^5*c^8)) + (48384*c^7*d^8*e^5 - 193536*b*c^6*d^7*e^6
+ 432*(683*b^2*c^5 + 404*a*c^6)*d^6*e^7 - 432*(481*b^3*c^4 + 1212*a*b*c^5)*
d^5*e^8 + 9*(6841*b^4*c^3 + 60712*a*b^2*c^4 + 24016*a^2*c^5)*d^4*e^9 - 18*(
145*b^5*c^2 + 12232*a*b^3*c^3 + 24016*a^2*b*c^4)*d^3*e^10 - 2*(518*b^6*c -
10131*a*b^4*c^2 - 124608*a^2*b^2*c^3 - 50000*a^3*c^4)*d^2*e^11 - (35*b^7 -
2562*a*b^5*c + 33072*a^2*b^3*c^2 + 100000*a^3*b*c^3)*d*e^12 + (35*a*b^6 - 1
491*a^2*b^4*c + 15000*a^3*b^2*c^2 + 10000*a^4*c^3)*e^13)*sqrt(e*x + d)) - s
```

qrt(1/2)*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^3 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x)*sqrt((4608*c^7*d^7 - 16128*b*c^6*d^6*e + 672*(31*b^2*c^5 + 20*a*c^6)*d^5*e^2 - 1680*(7*b^3*c^4 + 20*a*b*c^5)*d^4*e^3 + 70*(35*b^4*c^3 + 392*a*b^2*c^4 + 176*a^2*c^5)*d^3*e^4 + 21*(b^5*c^2 - 360*a*b^3*c^3 - 880*a^2*b*c^4)*d^2*e^5 - 21*(b^6*c - 10*a*b^4*c^2 - 320*a^2*b^2*c^3 - 160*a^3*c^4)*d*e^6 - (b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*e^7 + (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((441*c^4*d^4*e^10 - 882*b*c^3*d^3*e^11 + 21*(19*b^2*c^2 + 50*a*c^3)*d^2*e^12 + 42*(b^3*c - 25*a*b*c^2)*d*e^13 + (b^4 - 50*a*b^2*c + 625*a^2*c^2)*e^14)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))*log(-1/2*sqrt(1/2)*(504*(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*d^4*e^6 - 1008*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*d^3*e^7 + 3*(167*b^8*c^2 - 1664*a*b^6*c^3 + 3936*a^2*b^4*c^4 + 5632*a^3*b^2*c^5 - 21760*a^4*c^6)*d^2*e^8 + 3*(b^9*c - 352*a*b^7*c^2 + 4128*a^2*b^5*c^3 - 16384*a^3*b^3*c^4 + 21760*a^4*b*c^5)*d*e^9 - (b^10 - 17*a*b^8*c - 392*a^2*b^6*c^2 + 5696*a^3*b^4*c^3 - 23680*a^4*b^2*c^4 + 32000*a^5*c^5)*e^10 - (96*(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)*d^3 - 144*(b^11*c^5 - 20*a*b^9*c^6 + 160*a^2*b^7*c^7 - 640*a^3*b^5*c^8 + 1280*a^4*b^3*c^9 - 1024*a^5*b*c^10)*d^2*e + 2*(23*b^12*c^4 - 408*a*b^10*c^5 + 2640*a^2*b^8*c^6 - 6400*a^3*b^6*c^7 - 3840*a^4*b^4*c^8 + 43008*a^5*b^2*c^9 - 53248*a^6*c^10)*d*e^2 + (b^13*c^3 - 72*a*b^11*c^4 + 1200*a^2*b^9*c^5 - 8960*a^3*b^7*c^6 + 34560*a^4*b^5*c^7 - 67584*a^5*b^3*c^8 + 53248*a^6*b*c^9)*e^3)*sqrt((441*c^4*d^4*e^10 - 882*b*c^3*d^3*e^11 + 21*(19*b^2*c^2 + 50*a*c^3)*d^2*e^12 + 42*(b^3*c - 25*a*b*c^2)*d*e^13 + (b^4 - 50*a*b^2*c + 625*a^2*c^2)*e^14)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))*sqrt((4608*c^7*d^7 - 16128*b*c^6*d^6*e + 672*(31*b^2*c^5 + 20*a*c^6)*d^5*e^2 - 1680*(7*b^3*c^4 + 20*a*b*c^5)*d^4*e^3 + 70*(35*b^4*c^3 + 392*a*b^2*c^4 + 176*a^2*c^5)*d^3*e^4 + 21*(b^5*c^2 - 360*a*b^3*c^3 - 880*a^2*b*c^4)*d^2*e^5 - 21*(b^6*c - 10*a*b^4*c^2 - 320*a^2*b^2*c^3 - 160*a^3*c^4)*d*e^6 - (b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*e^7 + (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((441*c^4*d^4*e^10 - 882*b*c^3*d^3*e^11 + 21*(19*b^2*c^2 + 50*a*c^3)*d^2*e^12 + 42*(b^3*c - 25*a*b*c^2)*d*e^13 + (b^4 - 50*a*b^2*c + 625*a^2*c^2)*e^14)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)) + (48384*c^7*d^8*e^5 - 193536*b*c^6*d^7*e^6 + 432*(683*b^2*c^5 + 404*a*c^6)*d^6*e^7 - 432*(481*b^3*c^4 + 1212*a*b*c^5)*d^5*e^8 + 9*(6841*b^4*c^3 + 60712*a*b^2*c^4 + 24016*a^2*c^5)*d^4*e^9 - 18*(145*b^5*c^2 + 12232*a*b^3*c^3 + 24016*a^2*b*c^4)*d^3*e^10 - 2*(518*b^6*c - 10131*a*b^4*c^2 - 124608*a^2*b^2*c^3 - 50000*a^3*c^4)*d^2*e^11 - (35*b^7 - 2562*a*b^5*c + 33072*a^2*b^3*c^2 + 100000*a^3*b*c^3)*d*e^12 + (35*a*b^6 - 1491*a^2*b^4*c + 15000*a^3*b^2*c^2 + 10000*a^4*c^3)*e^13)*sqrt(e*x + d)) + sqrt(1/2)*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^3 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x)*sqrt((4608*c^7*d^7 - 16128*b*c^6*d^6*e + 672*(31*b^2*c^5 + 20*a*c^6)*d^5*e^2 - 1680*(7*b^3*c^4 + 20*a*b*c^5)*d^4*e^3 + 70*(35*b^4*c^3 + 392*a*b^2*c^4 + 176*a^2*c^5)*d^3*e^4 + 21*(b^5*c^2 - 360*a*b^3*c^3 - 880*a^2*b*c^4)*d^2*e^5 - 21*(b^6*c - 10*a*b^4*c^2 - 320*a^2*b^2*c^3 - 160*a^3*c^4)*d*e^6 - (b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*e^7 - (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((441*c^4*d^4*e^10 - 882*b*c^3*d^3*e^11 + 21*(19*b^2*c^2 + 50*a*c^3)*d^2*e^12 + 42*(b^3*c - 25*a*b*c^2)*d*e^13 + (b^4 - 50*a*b^2*c + 625*a^2*c^2)*e^14)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^

4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))*log(1/2*sqrt(1/2)*(504*(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*d^4*e^6 - 1008*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*d^3*e^7 + 3*(167*b^8*c^2 - 1664*a*b^6*c^3 + 3936*a^2*b^4*c^4 + 5632*a^3*b^2*c^5 - 21760*a^4*c^6)*d^2*e^8 + 3*(b^9*c - 352*a*b^7*c^2 + 4128*a^2*b^5*c^3 - 16384*a^3*b^3*c^4 + 21760*a^4*b*c^5)*d*e^9 - (b^10 - 17*a*b^8*c - 392*a^2*b^6*c^2 + 5696*a^3*b^4*c^3 - 23680*a^4*b^2*c^4 + 32000*a^5*c^5)*e^10 + (96*(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)*d^3 - 144*(b^11*c^5 - 20*a*b^9*c^6 + 160*a^2*b^7*c^7 - 640*a^3*b^5*c^8 + 1280*a^4*b^3*c^9 - 1024*a^5*b*c^10)*d^2*e + 2*(23*b^12*c^4 - 408*a*b^10*c^5 + 2640*a^2*b^8*c^6 - 6400*a^3*b^6*c^7 - 3840*a^4*b^4*c^8 + 43008*a^5*b^2*c^9 - 53248*a^6*c^10)*d*e^2 + (b^13*c^3 - 72*a*b^11*c^4 + 1200*a^2*b^9*c^5 - 8960*a^3*b^7*c^6 + 34560*a^4*b^5*c^7 - 67584*a^5*b^3*c^8 + 53248*a^6*b*c^9)*e^3)*sqrt((441*c^4*d^4*e^10 - 882*b*c^3*d^3*e^11 + 21*(19*b^2*c^2 + 50*a*c^3)*d^2*e^12 + 42*(b^3*c - 25*a*b*c^2)*d*e^13 + (b^4 - 50*a*b^2*c + 625*a^2*c^2)*e^14)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))*sqrt((4608*c^7*d^7 - 16128*b*c^6*d^6*e + 672*(31*b^2*c^5 + 20*a*c^6)*d^5*e^2 - 1680*(7*b^3*c^4 + 20*a*b*c^5)*d^4*e^3 + 70*(35*b^4*c^3 + 392*a*b^2*c^4 + 176*a^2*c^5)*d^3*e^4 + 21*(b^5*c^2 - 360*a*b^3*c^3 - 880*a^2*b*c^4)*d^2*e^5 - 21*(b^6*c - 10*a*b^4*c^2 - 320*a^2*b^2*c^3 - 160*a^3*c^4)*d*e^6 - (b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*e^7 - (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((441*c^4*d^4*e^10 - 882*b*c^3*d^3*e^11 + 21*(19*b^2*c^2 + 50*a*c^3)*d^2*e^12 + 42*(b^3*c - 25*a*b*c^2)*d*e^13 + (b^4 - 50*a*b^2*c + 625*a^2*c^2)*e^14)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)) + (48384*c^7*d^8*e^5 - 193536*b*c^6*d^7*e^6 + 432*(683*b^2*c^5 + 404*a*c^6)*d^6*e^7 - 432*(481*b^3*c^4 + 1212*a*b*c^5)*d^5*e^8 + 9*(6841*b^4*c^3 + 60712*a*b^2*c^4 + 24016*a^2*c^5)*d^4*e^9 - 18*(145*b^5*c^2 + 12232*a*b^3*c^3 + 24016*a^2*b*c^4)*d^3*e^10 - 2*(518*b^6*c - 10131*a*b^4*c^2 - 124608*a^2*b^2*c^3 - 50000*a^3*c^4)*d^2*e^11 - (35*b^7 - 2562*a*b^5*c + 33072*a^2*b^3*c^2 + 100000*a^3*b*c^3)*d*e^12 + (35*a*b^6 - 1491*a^2*b^4*c + 15000*a^3*b^2*c^2 + 10000*a^4*c^3)*e^13)*sqrt(e*x + d)) - sqrt(1/2)*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^3 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x)*sqrt((4608*c^7*d^7 - 16128*b*c^6*d^6*e + 672*(31*b^2*c^5 + 20*a*c^6)*d^5*e^2 - 1680*(7*b^3*c^4 + 20*a*b*c^5)*d^4*e^3 + 70*(35*b^4*c^3 + 392*a*b^2*c^4 + 176*a^2*c^5)*d^3*e^4 + 21*(b^5*c^2 - 360*a*b^3*c^3 - 880*a^2*b*c^4)*d^2*e^5 - 21*(b^6*c - 10*a*b^4*c^2 - 320*a^2*b^2*c^3 - 160*a^3*c^4)*d*e^6 - (b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*e^7 - (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((441*c^4*d^4*e^10 - 882*b*c^3*d^3*e^11 + 21*(19*b^2*c^2 + 50*a*c^3)*d^2*e^12 + 42*(b^3*c - 25*a*b*c^2)*d*e^13 + (b^4 - 50*a*b^2*c + 625*a^2*c^2)*e^14)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))*log(-1/2*sqrt(1/2)*(504*(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*d^4*e^6 - 1008*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*d^3*e^7 + 3*(167*b^8*c^2 - 1664*a*b^6*c^3 + 3936*a^2*b^4*c^4 + 5632*a^3*b^2*c^5 - 21760*a^4*c^6)*d^2*e^8 + 3*(b^9*c - 352*a*b^7*c^2 + 4128*a^2*b^5*c^3 - 16384*a^3*b^3*c^4 + 21760*a^4*b*c^5)*d*e^9 - (b^10 - 17*a*b^8*c - 392*a^2*b^6*c^2 + 5696*a^3*b^4*c^3 - 23680*a^4*b^2*c^4 + 32000*a^5*c^5)*e^10 + (96*(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)*d^3 - 144*(b^11*c^5 - 20*a*b^9*c^6 + 160*a^2*b^7*c^7 - 640*a^3*b^5*c^8 + 1280*a^4*b^3*c^9 - 1024*a^5*b*c^10)*d^2*e + 2*(23*b^12*c^4 - 408*a*b^10*c^5 + 2640*a^2*b^8*c^6 - 6400*a^3*b^6*c^7 - 3840*a^4*b^4*c^8 + 43008*a^5*b^2*c^9 - 5

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3248*a^6*c^10)*d*e^2 + (b^13*c^3 - 72*a*b^11*c^4 + 1200*a^2*b^9*c^5 - 8960*
a^3*b^7*c^6 + 34560*a^4*b^5*c^7 - 67584*a^5*b^3*c^8 + 53248*a^6*b*c^9)*e^3)
*sqrt((441*c^4*d^4*e^10 - 882*b*c^3*d^3*e^11 + 21*(19*b^2*c^2 + 50*a*c^3)*d
^2*e^12 + 42*(b^3*c - 25*a*b*c^2)*d*e^13 + (b^4 - 50*a*b^2*c + 625*a^2*c^2)
*e^14)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*
a^4*b^2*c^10 - 1024*a^5*c^11)))*sqrt((4608*c^7*d^7 - 16128*b*c^6*d^6*e + 67
2*(31*b^2*c^5 + 20*a*c^6)*d^5*e^2 - 1680*(7*b^3*c^4 + 20*a*b*c^5)*d^4*e^3 +
 70*(35*b^4*c^3 + 392*a*b^2*c^4 + 176*a^2*c^5)*d^3*e^4 + 21*(b^5*c^2 - 360*
a*b^3*c^3 - 880*a^2*b*c^4)*d^2*e^5 - 21*(b^6*c - 10*a*b^4*c^2 - 320*a^2*b^2
*c^3 - 160*a^3*c^4)*d*e^6 - (b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*
b*c^3)*e^7 - (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 +
 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((441*c^4*d^4*e^10 - 882*b*c^3*d^3*e^
11 + 21*(19*b^2*c^2 + 50*a*c^3)*d^2*e^12 + 42*(b^3*c - 25*a*b*c^2)*d*e^13 +
 (b^4 - 50*a*b^2*c + 625*a^2*c^2)*e^14)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*
b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3
- 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 102
4*a^5*c^8)) + (48384*c^7*d^8*e^5 - 193536*b*c^6*d^7*e^6 + 432*(683*b^2*c^5
+ 404*a*c^6)*d^6*e^7 - 432*(481*b^3*c^4 + 1212*a*b*c^5)*d^5*e^8 + 9*(6841*b
^4*c^3 + 60712*a*b^2*c^4 + 24016*a^2*c^5)*d^4*e^9 - 18*(145*b^5*c^2 + 12232
*a*b^3*c^3 + 24016*a^2*b*c^4)*d^3*e^10 - 2*(518*b^6*c - 10131*a*b^4*c^2 - 1
24608*a^2*b^2*c^3 - 50000*a^3*c^4)*d^2*e^11 - (35*b^7 - 2562*a*b^5*c + 3307
2*a^2*b^3*c^2 + 100000*a^3*b*c^3)*d*e^12 + (35*a*b^6 - 1491*a^2*b^4*c + 150
00*a^3*b^2*c^2 + 10000*a^4*c^3)*e^13)*sqrt(e*x + d)) - 2*(36*a^2*b*c*d*e^2
- 2*(b^3*c - 10*a*b*c^2)*d^3 - (7*a*b^2*c + 44*a^2*c^2)*d^2*e - (a^2*b^2 +
20*a^3*c)*e^3 + (24*c^4*d^3 - 36*b*c^3*d^2*e + 2*(5*b^2*c^2 + 16*a*c^3)*d*e
^2 + (b^3*c - 16*a*b*c^2)*e^3)*x^3 + (36*b*c^3*d^3 - (55*b^2*c^2 - 4*a*c^3)
*d^2*e + 4*(4*b^3*c + 11*a*b*c^2)*d*e^2 - (b^4 + 5*a*b^2*c + 36*a^2*c^2)*e^
3)*x^2 + (8*(b^2*c^2 + 5*a*c^3)*d^3 - (13*b^3*c + 56*a*b*c^2)*d^2*e + 2*(29
*a*b^2*c - 8*a^2*c^2)*d*e^2 - 2*(a*b^3 + 14*a^2*b*c)*e^3)*x)*sqrt(e*x + d))
/(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*
c^5)*x^4 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^3 + (b^6*c - 6*a*b^4*
c^2 + 32*a^3*c^4)*x^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/(c*x**2+b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.2303 $\int \frac{(d+ex)^{5/2}}{\left(a+bx+cx^2\right)^3} \, dx$

Optimal. Leaf size=577

$$-\frac{3\sqrt{d+ex}\left(-x\left(-4ce(2bd-ae)+b^2e^2+8c^2d^2\right)-4b\left(ae^2+cd^2\right)+4acde+3b^2de\right)}{4\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{3\left(-8c^2de\left(-d\sqrt{b^2-4ac}-3ae+\right.\right.}{}$$

```
[Out] -((d + e*x)^(3/2)*(b*d - 2*a*e + (2*c*d - b*e)*x))/(2*(b^2 - 4*a*c)*(a + b*
x + c*x^2)^2) - (3*Sqrt[d + e*x]*(3*b^2*d*e + 4*a*c*d*e - 4*b*(c*d^2 + a*e^
2) - (8*c^2*d^2 + b^2*e^2 - 4*c*e*(2*b*d - a*e))*x))/(4*(b^2 - 4*a*c)^2*(a
+ b*x + c*x^2)) - (3*(32*c^3*d^3 - b^2*(b - Sqrt[b^2 - 4*a*c])*e^3 - 8*c^2*
d*e*(6*b*d - Sqrt[b^2 - 4*a*c]*d - 3*a*e) + 2*c*e^2*(9*b^2*d - 4*b*Sqrt[b^2
 - 4*a*c]*d - 6*a*b*e + 2*a*Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*
Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(4*Sqrt[2]*Sqrt[c]
*(b^2 - 4*a*c)^(5/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (3*(32*c^3*
d^3 - b^2*(b + Sqrt[b^2 - 4*a*c])*e^3 - 8*c^2*d*e*(6*b*d + Sqrt[b^2 - 4*a*c
]*d - 3*a*e) + 2*c*e^2*(9*b^2*d + 4*b*Sqrt[b^2 - 4*a*c]*d - 6*a*b*e - 2*a*S
qrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e]])/(4*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(5/2)*Sqrt[2*c
*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 5.05684, antiderivative size = 577, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {738, 820, 826, 1166, 208}

$$-\frac{3\sqrt{d+ex}\left(-x\left(-4ce(2bd-ae)+b^2e^2+8c^2d^2\right)-4b\left(ae^2+cd^2\right)+4acde+3b^2de\right)}{4\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{3\left(-8c^2de\left(-d\sqrt{b^2-4ac}-3ae+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(a + b*x + c*x^2)^3,x]
```

```
[Out] -((d + e*x)^(3/2)*(b*d - 2*a*e + (2*c*d - b*e)*x))/(2*(b^2 - 4*a*c)*(a + b*
x + c*x^2)^2) - (3*Sqrt[d + e*x]*(3*b^2*d*e + 4*a*c*d*e - 4*b*(c*d^2 + a*e^
2) - (8*c^2*d^2 + b^2*e^2 - 4*c*e*(2*b*d - a*e))*x))/(4*(b^2 - 4*a*c)^2*(a
+ b*x + c*x^2)) - (3*(32*c^3*d^3 - b^2*(b - Sqrt[b^2 - 4*a*c])*e^3 - 8*c^2*
d*e*(6*b*d - Sqrt[b^2 - 4*a*c]*d - 3*a*e) + 2*c*e^2*(9*b^2*d - 4*b*Sqrt[b^2
 - 4*a*c]*d - 6*a*b*e + 2*a*Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*
Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(4*Sqrt[2]*Sqrt[c]
*(b^2 - 4*a*c)^(5/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (3*(32*c^3*
d^3 - b^2*(b + Sqrt[b^2 - 4*a*c])*e^3 - 8*c^2*d*e*(6*b*d + Sqrt[b^2 - 4*a*c
]*d - 3*a*e) + 2*c*e^2*(9*b^2*d + 4*b*Sqrt[b^2 - 4*a*c]*d - 6*a*b*e - 2*a*S
qrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e]])/(4*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(5/2)*Sqrt[2*c
*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*
(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p +
 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*Simp[g*
(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(
m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (I
ntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\left(a+bx+cx^2\right)^3}dx = -\frac{(d+ex)^{3/2}(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{\int \frac{\sqrt{d+ex}\left(\frac{3}{2}\left(4cd^2-3bde+2ae^2\right)+\frac{3}{2}e(2cd-be)x\right)}{\left(a+bx+cx^2\right)^2}dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{(d+ex)^{3/2}(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{3\sqrt{d+ex}\left(3b^2de+4acde-4b\left(cd^2+ae^2\right)-\left(8c^2d^2+\right.\right.}{4\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}$$

$$= -\frac{(d+ex)^{3/2}(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{3\sqrt{d+ex}\left(3b^2de+4acde-4b\left(cd^2+ae^2\right)-\left(8c^2d^2+\right.\right.}{4\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}$$

$$= -\frac{(d+ex)^{3/2}(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{3\sqrt{d+ex}\left(3b^2de+4acde-4b\left(cd^2+ae^2\right)-\left(8c^2d^2+\right.\right.}{4\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}$$

$$= -\frac{(d+ex)^{3/2}(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{3\sqrt{d+ex}\left(3b^2de+4acde-4b\left(cd^2+ae^2\right)-\left(8c^2d^2+\right.\right.}{4\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}$$

Mathematica [A] time = 5.05538, size = 506, normalized size = 0.88

$$\frac{\sqrt{d+ex}\left(4b\left(3a^2e^2+ac\left(5d^2-9dex+4e^2x^2\right)+3c^2dx^2(3d-2ex)\right)+4c\left(-a^2e(7d+ex)+acx\left(10d^2+dex+3e^2x^2\right)+\right.\right.}{4\left(b^2-4ac\right)^2(a+x(b+cx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(a + b*x + c*x^2)^3,x]
```

```
[Out] (Sqrt[d + e*x]*(b^3*(-2*d^2 - 9*d*e*x + 5*e^2*x^2) + b^2*(a*e*(-5*d + 19*e*
x) + c*x*(8*d^2 - 37*d*e*x + 3*e^2*x^2)) + 4*c*(6*c^2*d^2*x^3 - a^2*e*(7*d
+ e*x) + a*c*x*(10*d^2 + d*e*x + 3*e^2*x^2)) + 4*b*(3*a^2*e^2 + 3*c^2*d*x^2
*(3*d - 2*e*x) + a*c*(5*d^2 - 9*d*e*x + 4*e^2*x^2))))/(4*(b^2 - 4*a*c)^2*(a
 + x*(b + c*x))^2) - (3*Sqrt[4*c*d + 2*(-b + Sqrt[b^2 - 4*a*c])*e]*(16*c^2*
d^2 + b*(3*b + 2*Sqrt[b^2 - 4*a*c])*e^2 + 4*c*e*(-4*b*d - Sqrt[b^2 - 4*a*c]
*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - b*e + Sqrt[
b^2 - 4*a*c]*e]])/(8*Sqrt[c]*(b^2 - 4*a*c)^(5/2)) + (3*Sqrt[4*c*d - 2*(b +
Sqrt[b^2 - 4*a*c])*e]*(16*c^2*d^2 + b*(3*b - 2*Sqrt[b^2 - 4*a*c])*e^2 + 4*c
*e*(-4*b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d +
e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(8*Sqrt[c]*(b^2 - 4*a*c)^(5
/2))
```

Maple [B] time = 0.287, size = 3925, normalized size = 6.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(c*x^2+b*x+a)^3,x)
```

[Out] -17*e^4/(c*e^2*x^2+b*e^2*x+a*e^2)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(3/2)*a*b*c*d-9*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*d-27/4*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2*d+18*e^4/(c*e^2*x^2+b*e^2*x+a*e^2)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(1/2)*a*b*c*d^2-12*e/(16*a^2*c^2-8*a*b^2*c+b^4)*c^3/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d^3-3*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d-12*e/(16*a^2*c^2-8*a*b^2*c+b^4)*c^3/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d^3+3*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d-27/4*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2*d+18*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d^2+9/2*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b+18*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d^2+9/2*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b-9*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*d-12*e^3/(c*e^2*x^2+b*e^2*x+a*e^2)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(1/2)*b^2*c*d^3+15*e^2/(c*e^2*x^2+b*e^2*x+a*e^2)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(1/2)*b*c^2*d^4+3/8*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^3+3/8*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^3+3/2*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a+3*e/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d^2-3/2*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a-3*e/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d^2-6*e^2/(c*e^2*x^2+b*e^2*x+a*e^2)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(7/2)*b*d+4*e^4/(c*e^2*x^2+b*e^2*x+a*e^2)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(5/2)*a*b*c-8*e^3/(c*e^2*x^2+b*e^2*x+a*e^2)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(5/2)*c^2*a*d-23/2*e^3/(c*e^2*x^2+b*e^2*x+a*e^2)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(5/2)*b^2*c*d+27*e^2/(c*e^2*x^2+b*e^2*x+a*e^2)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(5/2)*b*c^2*d^2+17*e^3/(c*e^2*x^2+b*e^2*x+a*e^2)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(3/2)*a*c^2*d^2+91/4*e^3/(c*e^2*x^2+b*e^2*x+a*e^2)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(3/2)*b^2*c*d^2-6*e^5/(c*e^2*x^2+b

```
*e^2*x+a*e^2)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(1/2)*a^2*c*d-6*e^5/(c*e
^2*x^2+b*e^2*x+a*e^2)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(1/2)*a*b^2*d-36
*e^2/(c*e^2*x^2+b*e^2*x+a*e^2)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(3/2)*b
*c^2*d^3-12*e^3/(c*e^2*x^2+b*e^2*x+a*e^2)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x
+d)^(1/2)*a*c^2*d^3+5/4*e^4/(c*e^2*x^2+b*e^2*x+a*e^2)^2/(16*a^2*c^2-8*a*b^2
*c+b^4)*(e*x+d)^(5/2)*b^3-19/4*e^4/(c*e^2*x^2+b*e^2*x+a*e^2)^2/(16*a^2*c^2-
8*a*b^2*c+b^4)*(e*x+d)^(3/2)*b^3*d+18*e/(c*e^2*x^2+b*e^2*x+a*e^2)^2/(16*a^2
*c^2-8*a*b^2*c+b^4)*(e*x+d)^(3/2)*c^3*d^4+3*e^6/(c*e^2*x^2+b*e^2*x+a*e^2)^2
/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(1/2)*a^2*b+3*e^4/(c*e^2*x^2+b*e^2*x+a*
e^2)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(1/2)*b^3*d^2-6*e/(c*e^2*x^2+b*e^
2*x+a*e^2)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(1/2)*c^3*d^5+3/8*e^3/(16*a
^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2
)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(
1/2))*b^2-3/8*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*
a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^
2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2+3*e^3/(c*e^2*x^2+b*e^2*x+a*e^2)^2*c^2/(
16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(7/2)*a-e^5/(c*e^2*x^2+b*e^2*x+a*e^2)^2/(
16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(3/2)*a^2*c+19/4*e^5/(c*e^2*x^2+b*e^2*x+a
*e^2)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(3/2)*a*b^2+3/4*e^3/(c*e^2*x^2+b
*e^2*x+a*e^2)^2*c/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(7/2)*b^2+6*e/(c*e^2*x
^2+b*e^2*x+a*e^2)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(7/2)*d^2-18*e/(
c*e^2*x^2+b*e^2*x+a*e^2)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(5/2)*c^3*d^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\left(cx^2+bx+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/(c*x^2 + b*x + a)^3, x)
```

Fricas [B] time = 4.53833, size = 11310, normalized size = 19.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(3*sqrt(1/2)*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^4*c^2 - 8*a*b^2*
c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 -
 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*sq
rt((512*c^5*d^5 - 1280*b*c^4*d^4*e + 160*(7*b^2*c^3 + 4*a*c^4)*d^3*e^2 - 80
*(5*b^3*c^2 + 12*a*b*c^3)*d^2*e^3 + 10*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3
)*d*e^4 - (b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*e^5 + (b^10*c - 20*a*b^8*c^2 +
160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)*sqrt(e
^10/(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4
*b^2*c^6 - 1024*a^5*c^7)))/(b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a
^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6))*log(27*sqrt(1/2)*(4*(b^6*c -
 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*d*e^6 - 2*(b^7 - 12*a*b^5*c +
48*a^2*b^3*c^2 - 64*a^3*b*c^3)*e^7 + sqrt(e^10/(b^10*c^2 - 20*a*b^8*c^3 + 1
```

60*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))*(16*(b
^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c
^7 - 1024*a^5*c^8)*d^2 - 16*(b^11*c^2 - 20*a*b^9*c^3 + 160*a^2*b^7*c^4 - 64
0*a^3*b^5*c^5 + 1280*a^4*b^3*c^6 - 1024*a^5*b*c^7)*d*e + (3*b^12*c - 56*a*b
^10*c^2 + 400*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 1280*a^4*b^4*c^5 + 2048*a^5*
b^2*c^6 - 4096*a^6*c^7)*e^2))*sqrt((512*c^5*d^5 - 1280*b*c^4*d^4*e + 160*(7
*b^2*c^3 + 4*a*c^4)*d^3*e^2 - 80*(5*b^3*c^2 + 12*a*b*c^3)*d^2*e^3 + 10*(5*b
^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d*e^4 - (b^5 + 40*a*b^3*c + 80*a^2*b*c^2)
*e^5 + (b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^
4*b^2*c^5 - 1024*a^5*c^6)*sqrt(e^10/(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*
c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7)))/(b^10*c - 20*a*b
^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^
6)) + 27*(256*c^4*d^4*e^5 - 512*b*c^3*d^3*e^6 + 48*(7*b^2*c^2 + 4*a*c^3)*d^
2*e^7 - 16*(5*b^3*c + 12*a*b*c^2)*d*e^8 + (5*b^4 + 40*a*b^2*c + 16*a^2*c^2)
*e^9)*sqrt(e*x + d)) - 3*sqrt(1/2)*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b
^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*
c^3)*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16
*a^3*b*c^2)*x)*sqrt((512*c^5*d^5 - 1280*b*c^4*d^4*e + 160*(7*b^2*c^3 + 4*a*
c^4)*d^3*e^2 - 80*(5*b^3*c^2 + 12*a*b*c^3)*d^2*e^3 + 10*(5*b^4*c + 40*a*b^2
*c^2 + 16*a^2*c^3)*d*e^4 - (b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*e^5 + (b^10*c
- 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 102
4*a^5*c^6)*sqrt(e^10/(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b
^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7)))/(b^10*c - 20*a*b^8*c^2 + 160*a^
2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6))*log(-27*sqr
t(1/2)*(4*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*d*e^6 - 2*(b
^7 - 12*a*b^5*c + 48*a^2*b^3*c^2 - 64*a^3*b*c^3)*e^7 + sqrt(e^10/(b^10*c^2
- 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 102
4*a^5*c^7))*(16*(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^
6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*d^2 - 16*(b^11*c^2 - 20*a*b^9*c^3 + 16
0*a^2*b^7*c^4 - 640*a^3*b^5*c^5 + 1280*a^4*b^3*c^6 - 1024*a^5*b*c^7)*d*e +
(3*b^12*c - 56*a*b^10*c^2 + 400*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 1280*a^4*b
^4*c^5 + 2048*a^5*b^2*c^6 - 4096*a^6*c^7)*e^2))*sqrt((512*c^5*d^5 - 1280*b*
c^4*d^4*e + 160*(7*b^2*c^3 + 4*a*c^4)*d^3*e^2 - 80*(5*b^3*c^2 + 12*a*b*c^3)
*d^2*e^3 + 10*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d*e^4 - (b^5 + 40*a*b^3
*c + 80*a^2*b*c^2)*e^5 + (b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3
*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)*sqrt(e^10/(b^10*c^2 - 20*a*b^8*
c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))
)/(b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2
*c^5 - 1024*a^5*c^6)) + 27*(256*c^4*d^4*e^5 - 512*b*c^3*d^3*e^6 + 48*(7*b^2
*c^2 + 4*a*c^3)*d^2*e^7 - 16*(5*b^3*c + 12*a*b*c^2)*d*e^8 + (5*b^4 + 40*a*b
^2*c + 16*a^2*c^2)*e^9)*sqrt(e*x + d)) + 3*sqrt(1/2)*(a^2*b^4 - 8*a^3*b^2*c
 + 16*a^4*c^2 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b
^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b^5
- 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*sqrt((512*c^5*d^5 - 1280*b*c^4*d^4*e + 160
*(7*b^2*c^3 + 4*a*c^4)*d^3*e^2 - 80*(5*b^3*c^2 + 12*a*b*c^3)*d^2*e^3 + 10*(
5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d*e^4 - (b^5 + 40*a*b^3*c + 80*a^2*b*c
^2)*e^5 - (b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280
*a^4*b^2*c^5 - 1024*a^5*c^6)*sqrt(e^10/(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b
^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7)))/(b^10*c - 20*
a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5
*c^6))*log(27*sqrt(1/2)*(4*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*
c^4)*d*e^6 - 2*(b^7 - 12*a*b^5*c + 48*a^2*b^3*c^2 - 64*a^3*b*c^3)*e^7 - sqr
t(e^10/(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*
a^4*b^2*c^6 - 1024*a^5*c^7))*(16*(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5
 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*d^2 - 16*(b^11*c^2 -
20*a*b^9*c^3 + 160*a^2*b^7*c^4 - 640*a^3*b^5*c^5 + 1280*a^4*b^3*c^6 - 1024*
a^5*b*c^7)*d*e + (3*b^12*c - 56*a*b^10*c^2 + 400*a^2*b^8*c^3 - 1280*a^3*b^6
*c^4 + 1280*a^4*b^4*c^5 + 2048*a^5*b^2*c^6 - 4096*a^6*c^7)*e^2))*sqrt((512*
c^5*d^5 - 1280*b*c^4*d^4*e + 160*(7*b^2*c^3 + 4*a*c^4)*d^3*e^2 - 80*(5*b^3*

```
c^2 + 12*a*b*c^3)*d^2*e^3 + 10*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d*e^4
- (b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*e^5 - (b^10*c - 20*a*b^8*c^2 + 160*a^2*
b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)*sqrt(e^10/(b^1
0*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6
 - 1024*a^5*c^7)))/(b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c
^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)) + 27*(256*c^4*d^4*e^5 - 512*b*c^3*d^
3*e^6 + 48*(7*b^2*c^2 + 4*a*c^3)*d^2*e^7 - 16*(5*b^3*c + 12*a*b*c^2)*d*e^8
+ (5*b^4 + 40*a*b^2*c + 16*a^2*c^2)*e^9)*sqrt(e*x + d)) - 3*sqrt(1/2)*(a^2*
b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 +
 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3
)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*sqrt((512*c^5*d^5 - 1280*
b*c^4*d^4*e + 160*(7*b^2*c^3 + 4*a*c^4)*d^3*e^2 - 80*(5*b^3*c^2 + 12*a*b*c^
3)*d^2*e^3 + 10*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d*e^4 - (b^5 + 40*a*b
^3*c + 80*a^2*b*c^2)*e^5 - (b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a
^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)*sqrt(e^10/(b^10*c^2 - 20*a*b^
8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7
)))/(b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b
^2*c^5 - 1024*a^5*c^6))*log(-27*sqrt(1/2)*(4*(b^6*c - 12*a*b^4*c^2 + 48*a^2
*b^2*c^3 - 64*a^3*c^4)*d*e^6 - 2*(b^7 - 12*a*b^5*c + 48*a^2*b^3*c^2 - 64*a^
3*b*c^3)*e^7 - sqrt(e^10/(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a
^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))*(16*(b^10*c^3 - 20*a*b^8*c^4
 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*d^2
 - 16*(b^11*c^2 - 20*a*b^9*c^3 + 160*a^2*b^7*c^4 - 640*a^3*b^5*c^5 + 1280*a
^4*b^3*c^6 - 1024*a^5*b*c^7)*d*e + (3*b^12*c - 56*a*b^10*c^2 + 400*a^2*b^8*
c^3 - 1280*a^3*b^6*c^4 + 1280*a^4*b^4*c^5 + 2048*a^5*b^2*c^6 - 4096*a^6*c^7
)*e^2))*sqrt((512*c^5*d^5 - 1280*b*c^4*d^4*e + 160*(7*b^2*c^3 + 4*a*c^4)*d^
3*e^2 - 80*(5*b^3*c^2 + 12*a*b*c^3)*d^2*e^3 + 10*(5*b^4*c + 40*a*b^2*c^2 +
16*a^2*c^3)*d*e^4 - (b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*e^5 - (b^10*c - 20*a*
b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c
^6)*sqrt(e^10/(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5
+ 1280*a^4*b^2*c^6 - 1024*a^5*c^7)))/(b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c
^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)) + 27*(256*c^4*d^4*
e^5 - 512*b*c^3*d^3*e^6 + 48*(7*b^2*c^2 + 4*a*c^3)*d^2*e^7 - 16*(5*b^3*c +
12*a*b*c^2)*d*e^8 + (5*b^4 + 40*a*b^2*c + 16*a^2*c^2)*e^9)*sqrt(e*x + d)) -
 2*(12*a^2*b*e^2 + 3*(8*c^3*d^2 - 8*b*c^2*d*e + (b^2*c + 4*a*c^2)*e^2)*x^3
- 2*(b^3 - 10*a*b*c)*d^2 - (5*a*b^2 + 28*a^2*c)*d*e + (36*b*c^2*d^2 - (37*b
^2*c - 4*a*c^2)*d*e + (5*b^3 + 16*a*b*c)*e^2)*x^2 + (8*(b^2*c + 5*a*c^2)*d^
2 - 9*(b^3 + 4*a*b*c)*d*e + (19*a*b^2 - 4*a^2*c)*e^2)*x)*sqrt(e*x + d))/(a^
2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4
 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c
^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(c*x**2+b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.2304 $\int \frac{(d+ex)^{3/2}}{\left(a+bx+cx^2\right)^3}\,dx$

Optimal. Leaf size=441

$$-\frac{3\sqrt{c}\left(-4ce\left(-d\sqrt{b^2-4ac}-ae+4bd\right)+be^2\left(3b-2\sqrt{b^2-4ac}\right)+16c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{2\sqrt{2}\left(b^2-4ac\right)^{5/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{3\sqrt{c}\left(-4ce\left(d\right.\right.}{}$$

```
[Out] -(Sqrt[d + e*x]*(b*d - 2*a*e + (2*c*d - b*e)*x))/(2*(b^2 - 4*a*c)*(a + b*x
+ c*x^2)^2) + (Sqrt[d + e*x]*(12*b*c*d - 7*b^2*e + 4*a*c*e + 12*c*(2*c*d -
b*e)*x))/(4*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - (3*Sqrt[c]*(16*c^2*d^2 + b
*(3*b - 2*Sqrt[b^2 - 4*a*c])*e^2 - 4*c*e*(4*b*d - Sqrt[b^2 - 4*a*c]*d - a*e
))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a
*c])*e]])/(2*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c
])*e]) + (3*Sqrt[c]*(16*c^2*d^2 + b*(3*b + 2*Sqrt[b^2 - 4*a*c])*e^2 - 4*c*e
*(4*b*d + Sqrt[b^2 - 4*a*c]*d - a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x
])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(2*Sqrt[2]*(b^2 - 4*a*c)^(5/2)
*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 1.94249, antiderivative size = 441, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {738, 822, 826, 1166, 208}

$$-\frac{3\sqrt{c}\left(-4ce\left(-d\sqrt{b^2-4ac}-ae+4bd\right)+be^2\left(3b-2\sqrt{b^2-4ac}\right)+16c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{2\sqrt{2}\left(b^2-4ac\right)^{5/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{3\sqrt{c}\left(-4ce\left(d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(a + b*x + c*x^2)^3,x]
```

```
[Out] -(Sqrt[d + e*x]*(b*d - 2*a*e + (2*c*d - b*e)*x))/(2*(b^2 - 4*a*c)*(a + b*x
+ c*x^2)^2) + (Sqrt[d + e*x]*(12*b*c*d - 7*b^2*e + 4*a*c*e + 12*c*(2*c*d -
b*e)*x))/(4*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - (3*Sqrt[c]*(16*c^2*d^2 + b
*(3*b - 2*Sqrt[b^2 - 4*a*c])*e^2 - 4*c*e*(4*b*d - Sqrt[b^2 - 4*a*c]*d - a*e
))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a
*c])*e]])/(2*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c
])*e]) + (3*Sqrt[c]*(16*c^2*d^2 + b*(3*b + 2*Sqrt[b^2 - 4*a*c])*e^2 - 4*c*e
*(4*b*d + Sqrt[b^2 - 4*a*c]*d - a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x
])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(2*Sqrt[2]*(b^2 - 4*a*c)^(5/2)
*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
```

```
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\left(a+bx+cx^2\right)^3}\,dx = -\frac{\sqrt{d+ex}(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{\int \frac{\frac{1}{2}\left(12cd^2-7bde+2ae^2\right)+\frac{5}{2}e(2cd-be)x}{\sqrt{d+ex}\left(a+bx+cx^2\right)^2}dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{\sqrt{d+ex}(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{\sqrt{d+ex}\left(12bcd-7b^2e+4ace+12c(2cd-be)x\right)}{4\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{\int \frac{3}{4}(}{}$$

$$= -\frac{\sqrt{d+ex}(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{\sqrt{d+ex}\left(12bcd-7b^2e+4ace+12c(2cd-be)x\right)}{4\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{\text{Sub}}{}$$

$$= -\frac{\sqrt{d+ex}(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{\sqrt{d+ex}\left(12bcd-7b^2e+4ace+12c(2cd-be)x\right)}{4\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{(3c(}{}$$

$$= -\frac{\sqrt{d+ex}(bd-2ae+(2cd-be)x)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{\sqrt{d+ex}\left(12bcd-7b^2e+4ace+12c(2cd-be)x\right)}{4\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{3\sqrt{c}}{}$$

Mathematica [B] time = 6.26712, size = 4707, normalized size = 10.67

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(a + b*x + c*x^2)^3,x]
```

```
[Out] -((d + e*x)^(5/2)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/(2*(b^2 -
4*a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2)^2) - (-(((d + e*x)^(5/2)*(
-(a*c*e*(2*c*d - b*e)^2)/2 + ((b*c*d - b^2*e + 2*a*c*e)*(12*c^2*d^2 + b^2*e
^2 - c*e*(11*b*d - 6*a*e)))/2 + c*(-(c*e*(b*d - 2*a*e)*(2*c*d - b*e))/2 + (
(2*c*d - b*e)*(12*c^2*d^2 + b^2*e^2 - c*e*(11*b*d - 6*a*e)))/2)*x))/((b^2 -
 4*a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2))) - (-(e*(2*c*d - b*e)*(1
2*c^2*d^2 + b^2*e^2 - 4*c*e*(3*b*d - 2*a*e))*(d + e*x)^(3/2))/2 + (2*((3*((
-3*c^2*d*e*(2*c*d - b*e)*(12*c^2*d^2 + b^2*e^2 - 4*c*e*(3*b*d - 2*a*e)))/4
+ (3*b*c*e^2*(2*c*d - b*e)*(12*c^2*d^2 + b^2*e^2 - 4*c*e*(3*b*d - 2*a*e)))/
4 + (3*c*e*(16*c^4*d^4 + b^4*e^4 - 7*b^2*c*e^3*(2*b*d - a*e) - 4*c^3*d^2*e*
(11*b*d - a*e) + c^2*e^2*(41*b^2*d^2 - 12*a*b*d*e - 4*a^2*e^2)))/4)*Sqrt[d
+ e*x])/c + (4*((Sqrt[2*c*d - b*e - Sqrt[b^2 - 4*a*c]*e]*(((3*c*e*((3*a*c*e
^2*(2*c*d - b*e)*(12*c^2*d^2 + b^2*e^2 - 4*c*e*(3*b*d - 2*a*e)))/4 + (3*c*d
*(16*c^4*d^4 + b^4*e^4 - 7*b^2*c*e^3*(2*b*d - a*e) - 4*c^3*d^2*e*(11*b*d -
a*e) + c^2*e^2*(41*b^2*d^2 - 12*a*b*d*e - 4*a^2*e^2)))/4))/2 + (3*c*d*((-3*
c^2*d*e*(2*c*d - b*e)*(12*c^2*d^2 + b^2*e^2 - 4*c*e*(3*b*d - 2*a*e)))/4 + (
3*b*c*e^2*(2*c*d - b*e)*(12*c^2*d^2 + b^2*e^2 - 4*c*e*(3*b*d - 2*a*e)))/4 +
 (3*c*e*(16*c^4*d^4 + b^4*e^4 - 7*b^2*c*e^3*(2*b*d - a*e) - 4*c^3*d^2*e*(11
*b*d - a*e) + c^2*e^2*(41*b^2*d^2 - 12*a*b*d*e - 4*a^2*e^2)))/4))/2 - (3*b*
e*((-3*c^2*d*e*(2*c*d - b*e)*(12*c^2*d^2 + b^2*e^2 - 4*c*e*(3*b*d - 2*a*e))
)/4 + (3*b*c*e^2*(2*c*d - b*e)*(12*c^2*d^2 + b^2*e^2 - 4*c*e*(3*b*d - 2*a*e
)))/4 + (3*c*e*(16*c^4*d^4 + b^4*e^4 - 7*b^2*c*e^3*(2*b*d - a*e) - 4*c^3*d^
2*e*(11*b*d - a*e) + c^2*e^2*(41*b^2*d^2 - 12*a*b*d*e - 4*a^2*e^2)))/4))/2)
/2 - (-((-2*c*d + b*e)*((3*c*e*((3*a*c*e^2*(2*c*d - b*e)*(12*c^2*d^2 + b^2*
e^2 - 4*c*e*(3*b*d - 2*a*e)))/4 + (3*c*d*(16*c^4*d^4 + b^4*e^4 - 7*b^2*c*e^
3*(2*b*d - a*e) - 4*c^3*d^2*e*(11*b*d - a*e) + c^2*e^2*(41*b^2*d^2 - 12*a*b
```

```
*d*e - 4*a^2*e^2)))/4))/2 + (3*c*d*((-3*c^2*d*e*(2*c*d - b*e)*(12*c^2*d^2 +
 b^2*e^2 - 4*c*e*(3*b*d - 2*a*e)))/4 + (3*b*c*e^2*(2*c*d - b*e)*(12*c^2*d^2
 + b^2*e^2 - 4*c*e*(3*b*d - 2*a*e)))/4 + (3*c*e*(16*c^4*d^4 + b^4*e^4 - 7*b
^2*c*e^3*(2*b*d - a*e) - 4*c^3*d^2*e*(11*b*d - a*e) + c^2*e^2*(41*b^2*d^2 -
 12*a*b*d*e - 4*a^2*e^2)))/4))/2 - (3*b*e*((-3*c^2*d*e*(2*c*d - b*e)*(12*c^
2*d^2 + b^2*e^2 - 4*c*e*(3*b*d - 2*a*e)))/4 + (3*b*c*e^2*(2*c*d - b*e)*(12*
c^2*d^2 + b^2*e^2 - 4*c*e*(3*b*d - 2*a*e)))/4 + (3*c*e*(16*c^4*d^4 + b^4*e^
4 - 7*b^2*c*e^3*(2*b*d - a*e) - 4*c^3*d^2*e*(11*b*d - a*e) + c^2*e^2*(41*b^
2*d^2 - 12*a*b*d*e - 4*a^2*e^2)))/4))/2))/2 + 2*c*((e*((3*c*d*((3*a*c*e^2*(
2*c*d - b*e)*(12*c^2*d^2 + b^2*e^2 - 4*c*e*(3*b*d - 2*a*e)))/4 + (3*c*d*(16
*c^4*d^4 + b^4*e^4 - 7*b^2*c*e^3*(2*b*d - a*e) - 4*c^3*d^2*e*(11*b*d - a*e)
 + c^2*e^2*(41*b^2*d^2 - 12*a*b*d*e - 4*a^2*e^2)))/4))/2 - (3*a*e*((-3*c^2*
d*e*(2*c*d - b*e)*(12*c^2*d^2 + b^2*e^2 - 4*c*e*(3*b*d - 2*a*e)))/4 + (3*b*
c*e^2*(2*c*d - b*e)*(12*c^2*d^2 + b^2*e^2 - 4*c*e*(3*b*d - 2*a*e)))/4 + (3*
c*e*(16*c^4*d^4 + b^4*e^4 - 7*b^2*c*e^3*(2*b*d - a*e) - 4*c^3*d^2*e*(11*b*d
 - a*e) + c^2*e^2*(41*b^2*d^2 - 12*a*b*d*e - 4*a^2*e^2)))/4))/2))/2 - (d*((
3*c*e*((3*a*c*e^2*(2*c*d - b*e)*(12*c^2*d^2 + b^2*e^2 - 4*c*e*(3*b*d - 2*a*
e)))/4 + (3*c*d*(16*c^4*d^4 + b^4*e^4 - 7*b^2*c*e^3*(2*b*d - a*e) - 4*c^3*d
^2*e*(11*b*d - a*e) + c^2*e^2*(41*b^2*d^2 - 12*a*b*d*e - 4*a^2*e^2)))/4))/2
 + (3*c*d*((-3*c^2*d*e*(2*c*d - b*e)*(12*c^2*d^2 + b^2*e^2 - 4*c*e*(3*b*d -
 2*a*e)))/4 + (3*b*c*e^2*(2*c*d - b*e)*(12*c^2*d^2 + b^2*e^2 - 4*c*e*(3*b*d
 - 2*a*e)))/4 + (3*c*e*(16*c^4*d^4 + b^4*e^4 - 7*b^2*c*e^3*(2*b*d - a*e) -
4*c^3*d^2*e*(11*b*d - a*e) + c^2*e^2*(41*b^2*d^2 - 12*a*b*d*e - 4*a^2*e^2))
)/4))/2 - (3*b*e*((-3*c^2*d*e*(2*c*d - b*e)*(12*c^2*d^2 + b^2*e^2 - 4*c*e*(
3*b*d - 2*a*e)))/4 + (3*b*c*e^2*(2*c*d - b*e)*(12*c^2*d^2 + b^2*e^2 - 4*c*e
*(3*b*d - 2*a*e)))/4 + (3*c*e*(16*c^4*d^4 + b^4*e^4 - 7*b^2*c*e^3*(2*b*d -
a*e) - 4*c^3*d^2*e*(11*b*d - a*e) + c^2*e^2*(41*b^2*d^2 - 12*a*b*d*e - 4*a^
2*e^2)))/4))/2))/2))/(Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d
 + e*x])/Sqrt[2*c*d - b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[2]*Sqrt[c]*(-2*c*d
 + b*e + Sqrt[b^2 - 4*a*c]*e)) + (Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*(
((3*c*e*((3*a*c*e^2*(2*c*d - b*e)*(12*c^2*d^2 + b^2*e^2 - 4*c*e*(3*b*d - 2*
a*e)))/4 + (3*c*d*(16*c^4*d^4 + b^4*e^4 - 7*b^2*c*e^3*(2*b*d - a*e) - 4*c^3
*d^2*e*(11*b*d - a*e) + c^2*e^2*(41*b^2*d^2 - 12*a*b*d*e - 4*a^2*e^2)))/4))
/2 + (3*c*d*((-3*c^2*d*e*(2*c*d - b*e)*(12*c^2*d^2 + b^2*e^2 - 4*c*e*(3*b*d
 - 2*a*e)))/4 + (3*b*c*e^2*(2*c*d - b*e)*(12*c^2*d^2 + b^2*e^2 - 4*c*e*(3*b
*d - 2*a*e)))/4 + (3*c*e*(16*c^4*d^4 + b^4*e^4 - 7*b^2*c*e^3*(2*b*d - a*e)
- 4*c^3*d^2*e*(11*b*d - a*e) + c^2*e^2*(41*b^2*d^2 - 12*a*b*d*e - 4*a^2*e^2
)))/4))/2 - (3*b*e*((-3*c^2*d*e*(2*c*d - b*e)*(12*c^2*d^2 + b^2*e^2 - 4*c*e
*(3*b*d - 2*a*e)))/4 + (3*b*c*e^2*(2*c*d - b*e)*(12*c^2*d^2 + b^2*e^2 - 4*c
*e*(3*b*d - 2*a*e)))/4 + (3*c*e*(16*c^4*d^4 + b^4*e^4 - 7*b^2*c*e^3*(2*b*d
- a*e) - 4*c^3*d^2*e*(11*b*d - a*e) + c^2*e^2*(41*b^2*d^2 - 12*a*b*d*e - 4*
a^2*e^2)))/4))/2)/2 + (-((-2*c*d + b*e)*((3*c*e*((3*a*c*e^2*(2*c*d - b*e)*(
12*c^2*d^2 + b^2*e^2 - 4*c*e*(3*b*d - 2*a*e)))/4 + (3*c*d*(16*c^4*d^4 + b^4
*e^4 - 7*b^2*c*e^3*(2*b*d - a*e) - 4*c^3*d^2*e*(11*b*d - a*e) + c^2*e^2*(41
*b^2*d^2 - 12*a*b*d*e - 4*a^2*e^2)))/4))/2 + (3*c*d*((-3*c^2*d*e*(2*c*d - b
*e)*(12*c^2*d^2 + b^2*e^2 - 4*c*e*(3*b*d - 2*a*e)))/4 + (3*b*c*e^2*(2*c*d -
 b*e)*(12*c^2*d^2 + b^2*e^2 - 4*c*e*(3*b*d - 2*a*e)))/4 + (3*c*e*(16*c^4*d^
4 + b^4*e^4 - 7*b^2*c*e^3*(2*b*d - a*e) - 4*c^3*d^2*e*(11*b*d - a*e) + c^2*
e^2*(41*b^2*d^2 - 12*a*b*d*e - 4*a^2*e^2)))/4))/2 - (3*b*e*((-3*c^2*d*e*(2*
c*d - b*e)*(12*c^2*d^2 + b^2*e^2 - 4*c*e*(3*b*d - 2*a*e)))/4 + (3*b*c*e^2*(
2*c*d - b*e)*(12*c^2*d^2 + b^2*e^2 - 4*c*e*(3*b*d - 2*a*e)))/4 + (3*c*e*(16
*c^4*d^4 + b^4*e^4 - 7*b^2*c*e^3*(2*b*d - a*e) - 4*c^3*d^2*e*(11*b*d - a*e)
 + c^2*e^2*(41*b^2*d^2 - 12*a*b*d*e - 4*a^2*e^2)))/4))/2))/2 + 2*c*((e*((3*
c*d*((3*a*c*e^2*(2*c*d - b*e)*(12*c^2*d^2 + b^2*e^2 - 4*c*e*(3*b*d - 2*a*e)
))/4 + (3*c*d*(16*c^4*d^4 + b^4*e^4 - 7*b^2*c*e^3*(2*b*d - a*e) - 4*c^3*d^2
*e*(11*b*d - a*e) + c^2*e^2*(41*b^2*d^2 - 12*a*b*d*e - 4*a^2*e^2)))/4))/2 -
 (3*a*e*((-3*c^2*d*e*(2*c*d - b*e)*(12*c^2*d^2 + b^2*e^2 - 4*c*e*(3*b*d - 2
*a*e)))/4 + (3*b*c*e^2*(2*c*d - b*e)*(12*c^2*d^2 + b^2*e^2 - 4*c*e*(3*b*d -
 2*a*e)))/4 + (3*c*e*(16*c^4*d^4 + b^4*e^4 - 7*b^2*c*e^3*(2*b*d - a*e) - 4*
```

```
c^3*d^2*e*(11*b*d - a*e) + c^2*e^2*(41*b^2*d^2 - 12*a*b*d*e - 4*a^2*e^2)))/
4))/2))/2 - (d*((3*c*e*((3*a*c*e^2*(2*c*d - b*e)*(12*c^2*d^2 + b^2*e^2 - 4*
c*e*(3*b*d - 2*a*e)))/4 + (3*c*d*(16*c^4*d^4 + b^4*e^4 - 7*b^2*c*e^3*(2*b*d
 - a*e) - 4*c^3*d^2*e*(11*b*d - a*e) + c^2*e^2*(41*b^2*d^2 - 12*a*b*d*e - 4
*a^2*e^2)))/4))/2 + (3*c*d*((-3*c^2*d*e*(2*c*d - b*e)*(12*c^2*d^2 + b^2*e^2
 - 4*c*e*(3*b*d - 2*a*e)))/4 + (3*b*c*e^2*(2*c*d - b*e)*(12*c^2*d^2 + b^2*e
^2 - 4*c*e*(3*b*d - 2*a*e)))/4 + (3*c*e*(16*c^4*d^4 + b^4*e^4 - 7*b^2*c*e^3
*(2*b*d - a*e) - 4*c^3*d^2*e*(11*b*d - a*e) + c^2*e^2*(41*b^2*d^2 - 12*a*b*
d*e - 4*a^2*e^2)))/4))/2 - (3*b*e*((-3*c^2*d*e*(2*c*d - b*e)*(12*c^2*d^2 +
b^2*e^2 - 4*c*e*(3*b*d - 2*a*e)))/4 + (3*b*c*e^2*(2*c*d - b*e)*(12*c^2*d^2
+ b^2*e^2 - 4*c*e*(3*b*d - 2*a*e)))/4 + (3*c*e*(16*c^4*d^4 + b^4*e^4 - 7*b^
2*c*e^3*(2*b*d - a*e) - 4*c^3*d^2*e*(11*b*d - a*e) + c^2*e^2*(41*b^2*d^2 -
12*a*b*d*e - 4*a^2*e^2)))/4))/2))/2))/(Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[
2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[2
]*Sqrt[c]*(-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e))))/c))/(3*c))/((b^2 - 4*a*c)
*(c*d^2 - b*d*e + a*e^2)))/(2*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))
```

Maple [B] time = 0.277, size = 2582, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*x^2+b*x+a)^3,x)
```

```
[Out] -27*e^2/(c*e^2*x^2+b*e^2*x+a*e^2)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(3/2
)*b*c^2*d^2-9/4*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-e^2*(4*a*c-b^2))^(1/2)*2
^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2
)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2-12*e/(16*a
^2*c^2-8*a*b^2*c+b^4)*c^3/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e
^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*
d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d^2+9*e^4/(c*e^2*x^2+b*e^2*x+a*e^2)^2
/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(1/2)*a*b*c*d-3*e^3/(16*a^2*c^2-8*a*b^2
*c+b^4)*c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))
^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^
2))^(1/2))*c)^(1/2))*a+12*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(-e^2*(4*a*c-b
^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(
(e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*
d+12*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((
b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)
/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d-9*e^3/(c*e^2*x^2+b*e^2
*x+a*e^2)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(1/2)*a*c^2*d^2-4*e^4/(c*e^2
*x^2+b*e^2*x+a*e^2)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(3/2)*a*b*c+8*e^3/
(c*e^2*x^2+b*e^2*x+a*e^2)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(3/2)*c^2*a*
d-5/4*e^4/(c*e^2*x^2+b*e^2*x+a*e^2)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(3
/2)*b^3+18*e^2/(c*e^2*x^2+b*e^2*x+a*e^2)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(
e*x+d)^(5/2)*b*d+3/2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((-b*e+2*c*d+
(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2
*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b+23/2*e^3/(c*e^2*x^2+b*e^2*x+a*e^
2)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(3/2)*b^2*c*d-9/4*e^3/(16*a^2*c^2-8
*a*b^2*c+b^4)*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b
^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*
c-b^2))^(1/2))*c)^(1/2))*b^2-12*e/(16*a^2*c^2-8*a*b^2*c+b^4)*c^3/(-e^2*(4*a
*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arcta
n((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d
^2-3*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((
-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/
```

```
2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a+12*e^2/(c*e^2*x^2+b*e
^2*x+a*e^2)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(1/2)*b*c^2*d^3-3/2*e^2/(1
6*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)
^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))
*c)^(1/2))*b+3*e/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*2^(1/2)/((b*e-2*c*d+(-e^2*(
4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^
2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d-3*e/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*2^(1/2
)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2
^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d-27/4*e^3/(c*e^2*x
^2+b*e^2*x+a*e^2)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(1/2)*b^2*c*d^2+e^3/
(c*e^2*x^2+b*e^2*x+a*e^2)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(5/2)*a-
19/4*e^3/(c*e^2*x^2+b*e^2*x+a*e^2)^2*c/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(
5/2)*b^2-18*e/(c*e^2*x^2+b*e^2*x+a*e^2)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*(e
*x+d)^(5/2)*d^2+18*e/(c*e^2*x^2+b*e^2*x+a*e^2)^2/(16*a^2*c^2-8*a*b^2*c+b^4)
*(e*x+d)^(3/2)*c^3*d^3+3/4*e^4/(c*e^2*x^2+b*e^2*x+a*e^2)^2/(16*a^2*c^2-8*a*
b^2*c+b^4)*(e*x+d)^(1/2)*b^3*d-6*e/(c*e^2*x^2+b*e^2*x+a*e^2)^2/(16*a^2*c^2-
8*a*b^2*c+b^4)*(e*x+d)^(1/2)*c^3*d^4-3*e^5/(c*e^2*x^2+b*e^2*x+a*e^2)^2/(16*
a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(1/2)*a^2*c-3/4*e^5/(c*e^2*x^2+b*e^2*x+a*e^2
)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(1/2)*a*b^2-3*e^2/(c*e^2*x^2+b*e^2*x
+a*e^2)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(7/2)*b+6*e/(c*e^2*x^2+b*e
^2*x+a*e^2)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*(e*x+d)^(7/2)*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+bx+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(c*x^2 + b*x + a)^3, x)
```

Fricas [B] time = 6.19812, size = 24935, normalized size = 56.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(3*sqrt(1/2)*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^4*c^2 - 8*a*b^2*
c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 -
 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*sq
rt((512*c^5*d^5 - 1280*b*c^4*d^4*e + 160*(7*b^2*c^3 + 4*a*c^4)*d^3*e^2 - 80
*(5*b^3*c^2 + 12*a*b*c^3)*d^2*e^3 + 10*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3
)*d*e^4 - (b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*e^5 + sqrt(e^10/((b^10*c^2 - 20
*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^
5*c^7)*d^4 - 2*(b^11*c - 20*a*b^9*c^2 + 160*a^2*b^7*c^3 - 640*a^3*b^5*c^4 +
 1280*a^4*b^3*c^5 - 1024*a^5*b*c^6)*d^3*e + (b^12 - 18*a*b^10*c + 120*a^2*b
^8*c^2 - 320*a^3*b^6*c^3 + 1536*a^5*b^2*c^5 - 2048*a^6*c^6)*d^2*e^2 - 2*(a*
b^11 - 20*a^2*b^9*c + 160*a^3*b^7*c^2 - 640*a^4*b^5*c^3 + 1280*a^5*b^3*c^4
- 1024*a^6*b*c^5)*d*e^3 + (a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*
a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*((b^10*c - 20*a*b^8*c^
```

2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)*d^
2 - (b^11 - 20*a*b^9*c + 160*a^2*b^7*c^2 - 640*a^3*b^5*c^3 + 1280*a^4*b^3*c
^4 - 1024*a^5*b*c^5)*d*e + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a
^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))/((b^10*c - 20*a*b^8*c^2
 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)*d^2
 - (b^11 - 20*a*b^9*c + 160*a^2*b^7*c^2 - 640*a^3*b^5*c^3 + 1280*a^4*b^3*c^
4 - 1024*a^5*b*c^5)*d*e + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^
4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))*log(27/2*sqrt(1/2)*(8*(b
^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2*e^6 - 8*(b^7*c - 1
2*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d*e^7 + (b^8 - 8*a*b^6*c + 128
*a^3*b^2*c^3 - 256*a^4*c^4)*e^8 - sqrt(e^10/((b^10*c^2 - 20*a*b^8*c^3 + 160
*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7)*d^4 - 2*(
b^11*c - 20*a*b^9*c^2 + 160*a^2*b^7*c^3 - 640*a^3*b^5*c^4 + 1280*a^4*b^3*c^
5 - 1024*a^5*b*c^6)*d^3*e + (b^12 - 18*a*b^10*c + 120*a^2*b^8*c^2 - 320*a^3
*b^6*c^3 + 1536*a^5*b^2*c^5 - 2048*a^6*c^6)*d^2*e^2 - 2*(a*b^11 - 20*a^2*b^
9*c + 160*a^3*b^7*c^2 - 640*a^4*b^5*c^3 + 1280*a^5*b^3*c^4 - 1024*a^6*b*c^5
)*d*e^3 + (a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 12
80*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*(32*(b^10*c^4 - 20*a*b^8*c^5 + 160*a^2
*b^6*c^6 - 640*a^3*b^4*c^7 + 1280*a^4*b^2*c^8 - 1024*a^5*c^9)*d^5 - 80*(b^1
1*c^3 - 20*a*b^9*c^4 + 160*a^2*b^7*c^5 - 640*a^3*b^5*c^6 + 1280*a^4*b^3*c^7
 - 1024*a^5*b*c^8)*d^4*e + 2*(33*b^12*c^2 - 632*a*b^10*c^3 + 4720*a^2*b^8*c
^4 - 16640*a^3*b^6*c^5 + 24320*a^4*b^4*c^6 + 2048*a^5*b^2*c^7 - 28672*a^6*c
^8)*d^3*e^2 - (19*b^13*c - 296*a*b^11*c^2 + 1360*a^2*b^9*c^3 + 1280*a^3*b^7
*c^4 - 29440*a^4*b^5*c^5 + 88064*a^5*b^3*c^6 - 86016*a^6*b*c^7)*d^2*e^3 + (
b^14 + 10*a*b^12*c - 416*a^2*b^10*c^2 + 3680*a^3*b^8*c^3 - 14080*a^4*b^6*c^
4 + 22016*a^5*b^4*c^5 - 24576*a^7*c^7)*d*e^4 - (a*b^13 - 8*a^2*b^11*c - 80*
a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 122
88*a^7*b*c^6)*e^5))*sqrt((512*c^5*d^5 - 1280*b*c^4*d^4*e + 160*(7*b^2*c^3 +
 4*a*c^4)*d^3*e^2 - 80*(5*b^3*c^2 + 12*a*b*c^3)*d^2*e^3 + 10*(5*b^4*c + 40*
a*b^2*c^2 + 16*a^2*c^3)*d*e^4 - (b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*e^5 + sqr
t(e^10/((b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280
*a^4*b^2*c^6 - 1024*a^5*c^7)*d^4 - 2*(b^11*c - 20*a*b^9*c^2 + 160*a^2*b^7*c
^3 - 640*a^3*b^5*c^4 + 1280*a^4*b^3*c^5 - 1024*a^5*b*c^6)*d^3*e + (b^12 - 1
8*a*b^10*c + 120*a^2*b^8*c^2 - 320*a^3*b^6*c^3 + 1536*a^5*b^2*c^5 - 2048*a^
6*c^6)*d^2*e^2 - 2*(a*b^11 - 20*a^2*b^9*c + 160*a^3*b^7*c^2 - 640*a^4*b^5*c
^3 + 1280*a^5*b^3*c^4 - 1024*a^6*b*c^5)*d*e^3 + (a^2*b^10 - 20*a^3*b^8*c +
160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*
((b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*
c^5 - 1024*a^5*c^6)*d^2 - (b^11 - 20*a*b^9*c + 160*a^2*b^7*c^2 - 640*a^3*b^
5*c^3 + 1280*a^4*b^3*c^4 - 1024*a^5*b*c^5)*d*e + (a*b^10 - 20*a^2*b^8*c + 1
60*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))/(
(b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c
^5 - 1024*a^5*c^6)*d^2 - (b^11 - 20*a*b^9*c + 160*a^2*b^7*c^2 - 640*a^3*b^5
*c^3 + 1280*a^4*b^3*c^4 - 1024*a^5*b*c^5)*d*e + (a*b^10 - 20*a^2*b^8*c + 16
0*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2)) +
27*(256*c^5*d^4*e^5 - 512*b*c^4*d^3*e^6 + 48*(7*b^2*c^3 + 4*a*c^4)*d^2*e^7
 - 16*(5*b^3*c^2 + 12*a*b*c^3)*d*e^8 + (5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)
*e^9)*sqrt(e*x + d)) - 3*sqrt(1/2)*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b
^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*
c^3)*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16
*a^3*b*c^2)*x)*sqrt((512*c^5*d^5 - 1280*b*c^4*d^4*e + 160*(7*b^2*c^3 + 4*a*
c^4)*d^3*e^2 - 80*(5*b^3*c^2 + 12*a*b*c^3)*d^2*e^3 + 10*(5*b^4*c + 40*a*b^2
*c^2 + 16*a^2*c^3)*d*e^4 - (b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*e^5 + sqrt(e^1
0/((b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*
b^2*c^6 - 1024*a^5*c^7)*d^4 - 2*(b^11*c - 20*a*b^9*c^2 + 160*a^2*b^7*c^3 -
640*a^3*b^5*c^4 + 1280*a^4*b^3*c^5 - 1024*a^5*b*c^6)*d^3*e + (b^12 - 18*a*b
^10*c + 120*a^2*b^8*c^2 - 320*a^3*b^6*c^3 + 1536*a^5*b^2*c^5 - 2048*a^6*c^6
)*d^2*e^2 - 2*(a*b^11 - 20*a^2*b^9*c + 160*a^3*b^7*c^2 - 640*a^4*b^5*c^3 +
1280*a^5*b^3*c^4 - 1024*a^6*b*c^5)*d*e^3 + (a^2*b^10 - 20*a^3*b^8*c + 160*a

^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*((b^1
0*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 -
 1024*a^5*c^6)*d^2 - (b^11 - 20*a*b^9*c + 160*a^2*b^7*c^2 - 640*a^3*b^5*c^3
 + 1280*a^4*b^3*c^4 - 1024*a^5*b*c^5)*d*e + (a*b^10 - 20*a^2*b^8*c + 160*a^
3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))/((b^10
*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 -
1024*a^5*c^6)*d^2 - (b^11 - 20*a*b^9*c + 160*a^2*b^7*c^2 - 640*a^3*b^5*c^3
+ 1280*a^4*b^3*c^4 - 1024*a^5*b*c^5)*d*e + (a*b^10 - 20*a^2*b^8*c + 160*a^3
*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))*log(-27
/2*sqrt(1/2)*(8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2*
e^6 - 8*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d*e^7 + (b^8
 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)*e^8 - sqrt(e^10/((b^10*c^2 -
20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*
a^5*c^7)*d^4 - 2*(b^11*c - 20*a*b^9*c^2 + 160*a^2*b^7*c^3 - 640*a^3*b^5*c^4
 + 1280*a^4*b^3*c^5 - 1024*a^5*b*c^6)*d^3*e + (b^12 - 18*a*b^10*c + 120*a^2
*b^8*c^2 - 320*a^3*b^6*c^3 + 1536*a^5*b^2*c^5 - 2048*a^6*c^6)*d^2*e^2 - 2*(
a*b^11 - 20*a^2*b^9*c + 160*a^3*b^7*c^2 - 640*a^4*b^5*c^3 + 1280*a^5*b^3*c^
4 - 1024*a^6*b*c^5)*d*e^3 + (a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 64
0*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*(32*(b^10*c^4 - 20*a
*b^8*c^5 + 160*a^2*b^6*c^6 - 640*a^3*b^4*c^7 + 1280*a^4*b^2*c^8 - 1024*a^5*
c^9)*d^5 - 80*(b^11*c^3 - 20*a*b^9*c^4 + 160*a^2*b^7*c^5 - 640*a^3*b^5*c^6
+ 1280*a^4*b^3*c^7 - 1024*a^5*b*c^8)*d^4*e + 2*(33*b^12*c^2 - 632*a*b^10*c^
3 + 4720*a^2*b^8*c^4 - 16640*a^3*b^6*c^5 + 24320*a^4*b^4*c^6 + 2048*a^5*b^2
*c^7 - 28672*a^6*c^8)*d^3*e^2 - (19*b^13*c - 296*a*b^11*c^2 + 1360*a^2*b^9*
c^3 + 1280*a^3*b^7*c^4 - 29440*a^4*b^5*c^5 + 88064*a^5*b^3*c^6 - 86016*a^6*
b*c^7)*d^2*e^3 + (b^14 + 10*a*b^12*c - 416*a^2*b^10*c^2 + 3680*a^3*b^8*c^3
- 14080*a^4*b^6*c^4 + 22016*a^5*b^4*c^5 - 24576*a^7*c^7)*d*e^4 - (a*b^13 -
8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336
*a^6*b^3*c^5 - 12288*a^7*b*c^6)*e^5))*sqrt((512*c^5*d^5 - 1280*b*c^4*d^4*e
+ 160*(7*b^2*c^3 + 4*a*c^4)*d^3*e^2 - 80*(5*b^3*c^2 + 12*a*b*c^3)*d^2*e^3 +
 10*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d*e^4 - (b^5 + 40*a*b^3*c + 80*a^
2*b*c^2)*e^5 + sqrt(e^10/((b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*
a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7)*d^4 - 2*(b^11*c - 20*a*b^9*c
^2 + 160*a^2*b^7*c^3 - 640*a^3*b^5*c^4 + 1280*a^4*b^3*c^5 - 1024*a^5*b*c^6)
*d^3*e + (b^12 - 18*a*b^10*c + 120*a^2*b^8*c^2 - 320*a^3*b^6*c^3 + 1536*a^5
*b^2*c^5 - 2048*a^6*c^6)*d^2*e^2 - 2*(a*b^11 - 20*a^2*b^9*c + 160*a^3*b^7*c
^2 - 640*a^4*b^5*c^3 + 1280*a^5*b^3*c^4 - 1024*a^6*b*c^5)*d*e^3 + (a^2*b^10
 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 10
24*a^7*c^5)*e^4))*((b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c
^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)*d^2 - (b^11 - 20*a*b^9*c + 160*a^2*b^
7*c^2 - 640*a^3*b^5*c^3 + 1280*a^4*b^3*c^4 - 1024*a^5*b*c^5)*d*e + (a*b^10
- 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 102
4*a^6*c^5)*e^2))/((b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^
4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)*d^2 - (b^11 - 20*a*b^9*c + 160*a^2*b^7
*c^2 - 640*a^3*b^5*c^3 + 1280*a^4*b^3*c^4 - 1024*a^5*b*c^5)*d*e + (a*b^10 -
 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024
*a^6*c^5)*e^2)) + 27*(256*c^5*d^4*e^5 - 512*b*c^4*d^3*e^6 + 48*(7*b^2*c^3 +
 4*a*c^4)*d^2*e^7 - 16*(5*b^3*c^2 + 12*a*b*c^3)*d*e^8 + (5*b^4*c + 40*a*b^2
*c^2 + 16*a^2*c^3)*e^9)*sqrt(e*x + d)) + 3*sqrt(1/2)*(a^2*b^4 - 8*a^3*b^2*c
 + 16*a^4*c^2 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b
^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b^5
- 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*sqrt((512*c^5*d^5 - 1280*b*c^4*d^4*e + 160
*(7*b^2*c^3 + 4*a*c^4)*d^3*e^2 - 80*(5*b^3*c^2 + 12*a*b*c^3)*d^2*e^3 + 10*(
5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d*e^4 - (b^5 + 40*a*b^3*c + 80*a^2*b*c
^2)*e^5 - sqrt(e^10/((b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b
^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7)*d^4 - 2*(b^11*c - 20*a*b^9*c^2 +
160*a^2*b^7*c^3 - 640*a^3*b^5*c^4 + 1280*a^4*b^3*c^5 - 1024*a^5*b*c^6)*d^3*
e + (b^12 - 18*a*b^10*c + 120*a^2*b^8*c^2 - 320*a^3*b^6*c^3 + 1536*a^5*b^2*
c^5 - 2048*a^6*c^6)*d^2*e^2 - 2*(a*b^11 - 20*a^2*b^9*c + 160*a^3*b^7*c^2 -

640*a^4*b^5*c^3 + 1280*a^5*b^3*c^4 - 1024*a^6*b*c^5)*d*e^3 + (a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*((b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)*d^2 - (b^11 - 20*a*b^9*c + 160*a^2*b^7*c^2 - 640*a^3*b^5*c^3 + 1280*a^4*b^3*c^4 - 1024*a^5*b*c^5)*d*e + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))/((b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)*d^2 - (b^11 - 20*a*b^9*c + 160*a^2*b^7*c^2 - 640*a^3*b^5*c^3 + 1280*a^4*b^3*c^4 - 1024*a^5*b*c^5)*d*e + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))*log(27/2*sqrt(1/2)*(8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2*e^6 - 8*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d*e^7 + (b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)*e^8 + sqrt(e^10/((b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7)*d^4 - 2*(b^11*c - 20*a*b^9*c^2 + 160*a^2*b^7*c^3 - 640*a^3*b^5*c^4 + 1280*a^4*b^3*c^5 - 1024*a^5*b*c^6)*d^3*e + (b^12 - 18*a*b^10*c + 120*a^2*b^8*c^2 - 320*a^3*b^6*c^3 + 1536*a^5*b^2*c^5 - 2048*a^6*c^6)*d^2*e^2 - 2*(a*b^11 - 20*a^2*b^9*c + 160*a^3*b^7*c^2 - 640*a^4*b^5*c^3 + 1280*a^5*b^3*c^4 - 1024*a^6*b*c^5)*d*e^3 + (a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*(32*(b^10*c^4 - 20*a*b^8*c^5 + 160*a^2*b^6*c^6 - 640*a^3*b^4*c^7 + 1280*a^4*b^2*c^8 - 1024*a^5*c^9)*d^5 - 80*(b^11*c^3 - 20*a*b^9*c^4 + 160*a^2*b^7*c^5 - 640*a^3*b^5*c^6 + 1280*a^4*b^3*c^7 - 1024*a^5*b*c^8)*d^4*e + 2*(33*b^12*c^2 - 632*a*b^10*c^3 + 4720*a^2*b^8*c^4 - 16640*a^3*b^6*c^5 + 24320*a^4*b^4*c^6 + 2048*a^5*b^2*c^7 - 28672*a^6*c^8)*d^3*e^2 - (19*b^13*c - 296*a*b^11*c^2 + 1360*a^2*b^9*c^3 + 1280*a^3*b^7*c^4 - 29440*a^4*b^5*c^5 + 88064*a^5*b^3*c^6 - 86016*a^6*b*c^7)*d^2*e^3 + (b^14 + 10*a*b^12*c - 416*a^2*b^10*c^2 + 3680*a^3*b^8*c^3 - 14080*a^4*b^6*c^4 + 22016*a^5*b^4*c^5 - 24576*a^7*c^7)*d*e^4 - (a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)*e^5))*sqrt((512*c^5*d^5 - 1280*b*c^4*d^4*e + 160*(7*b^2*c^3 + 4*a*c^4)*d^3*e^2 - 80*(5*b^3*c^2 + 12*a*b*c^3)*d^2*e^3 + 10*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d*e^4 - (b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*e^5 - sqrt(e^10/((b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7)*d^4 - 2*(b^11*c - 20*a*b^9*c^2 + 160*a^2*b^7*c^3 - 640*a^3*b^5*c^4 + 1280*a^4*b^3*c^5 - 1024*a^5*b*c^6)*d^3*e + (b^12 - 18*a*b^10*c + 120*a^2*b^8*c^2 - 320*a^3*b^6*c^3 + 1536*a^5*b^2*c^5 - 2048*a^6*c^6)*d^2*e^2 - 2*(a*b^11 - 20*a^2*b^9*c + 160*a^3*b^7*c^2 - 640*a^4*b^5*c^3 + 1280*a^5*b^3*c^4 - 1024*a^6*b*c^5)*d*e^3 + (a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*((b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)*d^2 - (b^11 - 20*a*b^9*c + 160*a^2*b^7*c^2 - 640*a^3*b^5*c^3 + 1280*a^4*b^3*c^4 - 1024*a^5*b*c^5)*d*e + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))/((b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)*d^2 - (b^11 - 20*a*b^9*c + 160*a^2*b^7*c^2 - 640*a^3*b^5*c^3 + 1280*a^4*b^3*c^4 - 1024*a^5*b*c^5)*d*e + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2)) + 27*(256*c^5*d^4*e^5 - 512*b*c^4*d^3*e^6 + 48*(7*b^2*c^3 + 4*a*c^4)*d^2*e^7 - 16*(5*b^3*c^2 + 12*a*b*c^3)*d*e^8 + (5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*e^9)*sqrt(e*x + d)) - 3*sqrt(1/2)*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*sqrt((512*c^5*d^5 - 1280*b*c^4*d^4*e + 160*(7*b^2*c^3 + 4*a*c^4)*d^3*e^2 - 80*(5*b^3*c^2 + 12*a*b*c^3)*d^2*e^3 + 10*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d*e^4 - (b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*e^5 - sqrt(e^10/((b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7)*d^4 - 2*(b^11*c - 20*a*b^9*c^2 + 160*a^2*b^7*c^3 - 640*a^3*b^5*c^4 + 1280*a^4*b^3*c^5 - 1024*a^5*b*c^6)*d^3*e + (b^12 - 18*a*b^10*c + 120*a^2*b^8*c^2 - 320*a^3*b^6*c^

```
3 + 1536*a^5*b^2*c^5 - 2048*a^6*c^6)*d^2*e^2 - 2*(a*b^11 - 20*a^2*b^9*c + 1
60*a^3*b^7*c^2 - 640*a^4*b^5*c^3 + 1280*a^5*b^3*c^4 - 1024*a^6*b*c^5)*d*e^3
 + (a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*
b^2*c^4 - 1024*a^7*c^5)*e^4))*((b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 6
40*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)*d^2 - (b^11 - 20*a*b^9*c
+ 160*a^2*b^7*c^2 - 640*a^3*b^5*c^3 + 1280*a^4*b^3*c^4 - 1024*a^5*b*c^5)*d*
e + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b
^2*c^4 - 1024*a^6*c^5)*e^2))/((b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 64
0*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)*d^2 - (b^11 - 20*a*b^9*c +
 160*a^2*b^7*c^2 - 640*a^3*b^5*c^3 + 1280*a^4*b^3*c^4 - 1024*a^5*b*c^5)*d*e
 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^
2*c^4 - 1024*a^6*c^5)*e^2))*log(-27/2*sqrt(1/2)*(8*(b^6*c^2 - 12*a*b^4*c^3
+ 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2*e^6 - 8*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b
^3*c^3 - 64*a^3*b*c^4)*d*e^7 + (b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4
*c^4)*e^8 + sqrt(e^10/((b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3
*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7)*d^4 - 2*(b^11*c - 20*a*b^9*c^2
+ 160*a^2*b^7*c^3 - 640*a^3*b^5*c^4 + 1280*a^4*b^3*c^5 - 1024*a^5*b*c^6)*d^
3*e + (b^12 - 18*a*b^10*c + 120*a^2*b^8*c^2 - 320*a^3*b^6*c^3 + 1536*a^5*b^
2*c^5 - 2048*a^6*c^6)*d^2*e^2 - 2*(a*b^11 - 20*a^2*b^9*c + 160*a^3*b^7*c^2
- 640*a^4*b^5*c^3 + 1280*a^5*b^3*c^4 - 1024*a^6*b*c^5)*d*e^3 + (a^2*b^10 -
20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*
a^7*c^5)*e^4))*(32*(b^10*c^4 - 20*a*b^8*c^5 + 160*a^2*b^6*c^6 - 640*a^3*b^4
*c^7 + 1280*a^4*b^2*c^8 - 1024*a^5*c^9)*d^5 - 80*(b^11*c^3 - 20*a*b^9*c^4 +
 160*a^2*b^7*c^5 - 640*a^3*b^5*c^6 + 1280*a^4*b^3*c^7 - 1024*a^5*b*c^8)*d^4
*e + 2*(33*b^12*c^2 - 632*a*b^10*c^3 + 4720*a^2*b^8*c^4 - 16640*a^3*b^6*c^5
 + 24320*a^4*b^4*c^6 + 2048*a^5*b^2*c^7 - 28672*a^6*c^8)*d^3*e^2 - (19*b^13
*c - 296*a*b^11*c^2 + 1360*a^2*b^9*c^3 + 1280*a^3*b^7*c^4 - 29440*a^4*b^5*c
^5 + 88064*a^5*b^3*c^6 - 86016*a^6*b*c^7)*d^2*e^3 + (b^14 + 10*a*b^12*c - 4
16*a^2*b^10*c^2 + 3680*a^3*b^8*c^3 - 14080*a^4*b^6*c^4 + 22016*a^5*b^4*c^5
- 24576*a^7*c^7)*d*e^4 - (a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4
*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)*e^5))*sq
rt((512*c^5*d^5 - 1280*b*c^4*d^4*e + 160*(7*b^2*c^3 + 4*a*c^4)*d^3*e^2 - 80
*(5*b^3*c^2 + 12*a*b*c^3)*d^2*e^3 + 10*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3
)*d*e^4 - (b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*e^5 - sqrt(e^10/((b^10*c^2 - 20
*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^
5*c^7)*d^4 - 2*(b^11*c - 20*a*b^9*c^2 + 160*a^2*b^7*c^3 - 640*a^3*b^5*c^4 +
 1280*a^4*b^3*c^5 - 1024*a^5*b*c^6)*d^3*e + (b^12 - 18*a*b^10*c + 120*a^2*b
^8*c^2 - 320*a^3*b^6*c^3 + 1536*a^5*b^2*c^5 - 2048*a^6*c^6)*d^2*e^2 - 2*(a*
b^11 - 20*a^2*b^9*c + 160*a^3*b^7*c^2 - 640*a^4*b^5*c^3 + 1280*a^5*b^3*c^4
- 1024*a^6*b*c^5)*d*e^3 + (a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*
a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*((b^10*c - 20*a*b^8*c^
2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)*d^
2 - (b^11 - 20*a*b^9*c + 160*a^2*b^7*c^2 - 640*a^3*b^5*c^3 + 1280*a^4*b^3*c
^4 - 1024*a^5*b*c^5)*d*e + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a
^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))/((b^10*c - 20*a*b^8*c^2
 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)*d^2
 - (b^11 - 20*a*b^9*c + 160*a^2*b^7*c^2 - 640*a^3*b^5*c^3 + 1280*a^4*b^3*c^
4 - 1024*a^5*b*c^5)*d*e + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^
4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2)) + 27*(256*c^5*d^4*e^5 -
512*b*c^4*d^3*e^6 + 48*(7*b^2*c^3 + 4*a*c^4)*d^2*e^7 - 16*(5*b^3*c^2 + 12*a
*b*c^3)*d*e^8 + (5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*e^9)*sqrt(e*x + d)) -
 2*(12*(2*c^3*d - b*c^2*e)*x^3 + (36*b*c^2*d - (19*b^2*c - 4*a*c^2)*e)*x^2
- 2*(b^3 - 10*a*b*c)*d - 3*(a*b^2 + 4*a^2*c)*e + (8*(b^2*c + 5*a*c^2)*d - (
5*b^3 + 16*a*b*c)*e)*x)*sqrt(e*x + d))/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2
+ (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^
2*b*c^3)*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c
+ 16*a^3*b*c^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.2305 $\int \frac{\sqrt{d+ex}}{\left(a+bx+cx^2\right)^3}\,dx$

Optimal. Leaf size=634

$$-\frac{\sqrt{d+ex}\left(-cx\left(-4ce(6bd-5ae)+b^2e^2+24c^2d^2\right)-4bc\left(2ae^2+3cd^2\right)-4ac^2de+13b^2cde+b^3\left(-e^2\right)\right)}{4\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{c}\left(-8c^2de\left(-3\right.\right.}{}$$

```
[Out] -((b + 2*c*x)*Sqrt[d + e*x])/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (Sqrt[
d + e*x]*(13*b^2*c*d*e - 4*a*c^2*d*e - b^3*e^2 - 4*b*c*(3*c*d^2 + 2*a*e^2)
- c*(24*c^2*d^2 + b^2*e^2 - 4*c*e*(6*b*d - 5*a*e))*x))/(4*(b^2 - 4*a*c)^2*(
c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2)) - (Sqrt[c]*(96*c^3*d^3 + b^2*(b +
 Sqrt[b^2 - 4*a*c])*e^3 - 8*c^2*d*e*(18*b*d - 3*Sqrt[b^2 - 4*a*c]*d - 13*a*
e) + 2*c*e^2*(23*b^2*d + 10*a*Sqrt[b^2 - 4*a*c]*e - 2*b*(6*Sqrt[b^2 - 4*a*c
]*d + 13*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - S
qrt[b^2 - 4*a*c])*e]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[2*c*d - (b - Sqr
t[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) + (Sqrt[c]*(96*c^3*d^3 + b^2*(b
 - Sqrt[b^2 - 4*a*c])*e^3 - 8*c^2*d*e*(18*b*d + 3*Sqrt[b^2 - 4*a*c]*d - 13*
a*e) + 2*c*e^2*(23*b^2*d + 12*b*Sqrt[b^2 - 4*a*c]*d - 26*a*b*e - 10*a*Sqrt[
b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[2*c*d - (b + Sq
rt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 4.97375, antiderivative size = 634, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {736, 822, 826, 1166, 208}

$$-\frac{\sqrt{d+ex}\left(-cx\left(-4ce(6bd-5ae)+b^2e^2+24c^2d^2\right)-4bc\left(2ae^2+3cd^2\right)-4ac^2de+13b^2cde+b^3\left(-e^2\right)\right)}{4\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{c}\left(-8c^2de\left(-3\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(a + b*x + c*x^2)^3,x]
```


```
[Out] -((b + 2*c*x)*Sqrt[d + e*x])/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (Sqrt[
d + e*x]*(13*b^2*c*d*e - 4*a*c^2*d*e - b^3*e^2 - 4*b*c*(3*c*d^2 + 2*a*e^2)
- c*(24*c^2*d^2 + b^2*e^2 - 4*c*e*(6*b*d - 5*a*e))*x))/(4*(b^2 - 4*a*c)^2*(
c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2)) - (Sqrt[c]*(96*c^3*d^3 + b^2*(b +
 Sqrt[b^2 - 4*a*c])*e^3 - 8*c^2*d*e*(18*b*d - 3*Sqrt[b^2 - 4*a*c]*d - 13*a*
e) + 2*c*e^2*(23*b^2*d + 10*a*Sqrt[b^2 - 4*a*c]*e - 2*b*(6*Sqrt[b^2 - 4*a*c
]*d + 13*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - S
qrt[b^2 - 4*a*c])*e]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[2*c*d - (b - Sqr
t[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) + (Sqrt[c]*(96*c^3*d^3 + b^2*(b
 - Sqrt[b^2 - 4*a*c])*e^3 - 8*c^2*d*e*(18*b*d + 3*Sqrt[b^2 - 4*a*c]*d - 13*
a*e) + 2*c*e^2*(23*b^2*d + 12*b*Sqrt[b^2 - 4*a*c]*d - 26*a*b*e - 10*a*Sqrt[
b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[2*c*d - (b + Sq
rt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2))
```

Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] - Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)
*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p + 1
), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 0] && (L
tQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c,
d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\left(a+bx+cx^2\right)^3}\,dx = -\frac{(b+2cx)\sqrt{d+ex}}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{\int \frac{-6cd+\frac{be}{2}-5cex}{\sqrt{d+ex}\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{(b+2cx)\sqrt{d+ex}}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{\sqrt{d+ex}\left(13b^2cde-4ac^2de-b^3e^2-4bc\left(3cd^2+2ae^2\right)-c\left(24c^2\right.\right.}{4\left(b^2-4ac\right)^2\left(cd^2-bde+ae^2\right)\left(a+bx\right.}$$

$$= -\frac{(b+2cx)\sqrt{d+ex}}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{\sqrt{d+ex}\left(13b^2cde-4ac^2de-b^3e^2-4bc\left(3cd^2+2ae^2\right)-c\left(24c^2\right.\right.}{4\left(b^2-4ac\right)^2\left(cd^2-bde+ae^2\right)\left(a+bx\right.}$$

$$= -\frac{(b+2cx)\sqrt{d+ex}}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{\sqrt{d+ex}\left(13b^2cde-4ac^2de-b^3e^2-4bc\left(3cd^2+2ae^2\right)-c\left(24c^2\right.\right.}{4\left(b^2-4ac\right)^2\left(cd^2-bde+ae^2\right)\left(a+bx\right.}$$

$$= -\frac{(b+2cx)\sqrt{d+ex}}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{\sqrt{d+ex}\left(13b^2cde-4ac^2de-b^3e^2-4bc\left(3cd^2+2ae^2\right)-c\left(24c^2\right.\right.}{4\left(b^2-4ac\right)^2\left(cd^2-bde+ae^2\right)\left(a+bx\right.}$$

Mathematica [A] time = 4.29806, size = 580, normalized size = 0.91

$$-\frac{\sqrt{d+ex}\left(4bc\left(2ae^2+3cd(d-2ex)\right)+4c^2\left(ae(d+5ex)+6cd^2x\right)+b^2ce(ex-13d)+b^3e^2\right)}{4\left(b^2-4ac\right)^2(a+x(b+cx))\left(e(bd-ae)-cd^2\right)} - \frac{\sqrt{c}\left(\frac{\left(8c^2de\left(3d\sqrt{b^2-4ac}+13ae-18bd\right.\right.}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(a + b*x + c*x^2)^3,x]
```

```
[Out] -((b + 2*c*x)*Sqrt[d + e*x])/(2*(b^2 - 4*a*c)*(a + x*(b + c*x))^2) - (Sqrt[
d + e*x]*(b^3*e^2 + b^2*c*e*(-13*d + e*x) + 4*b*c*(2*a*e^2 + 3*c*d*(d - 2*e
*x)) + 4*c^2*(6*c*d^2*x + a*e*(d + 5*e*x))))/(4*(b^2 - 4*a*c)^2*(-(c*d^2) +
 e*(b*d - a*e))*(a + x*(b + c*x))) - (Sqrt[c]*(((96*c^3*d^3 + b^2*(b + Sqrt
[b^2 - 4*a*c])*e^3 + 8*c^2*d*e*(-18*b*d + 3*Sqrt[b^2 - 4*a*c]*d + 13*a*e) +
 2*c*e^2*(23*b^2*d + 10*a*Sqrt[b^2 - 4*a*c]*e - 2*b*(6*Sqrt[b^2 - 4*a*c]*d
+ 13*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - b*e + Sqrt
[b^2 - 4*a*c]*e]])/Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e] - ((96*c^3*d^3
+ b^2*(b - Sqrt[b^2 - 4*a*c])*e^3 - 8*c^2*d*e*(18*b*d + 3*Sqrt[b^2 - 4*a*c]
*d - 13*a*e) + 2*c*e^2*(23*b^2*d + 12*b*Sqrt[b^2 - 4*a*c]*d - 26*a*b*e - 10
*a*Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e]])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/(
4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*(c*d^2 + e*(-(b*d) + a*e)))
```

Maple [B] time = 0.352, size = 3360, normalized size = 5.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*x^2+b*x+a)^3,x)
```

```
[Out] -40*e^3*c^3/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(4*b*e-8*c*d+4*(-4*a*c*e
^2+b^2*e^2)^(1/2))*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*a
rctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2
))*a-3*e^2*c/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(e*x+1/2*b*e/c-1/2/c*(e
^2*(-4*a*c+b^2))^(1/2))^2*(-4*a*c*e^2+b^2*e^2)^(1/2)/(-b*e+2*c*d+(-4*a*c*e^
2+b^2*e^2)^(1/2))*(e*x+d)^(3/2)*b+6*e*c^2/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b
^2)^2/(e*x+1/2*b*e/c-1/2/c*(e^2*(-4*a*c+b^2))^(1/2))^2*(-4*a*c*e^2+b^2*e^2)
^(1/2)/(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))*(e*x+d)^(3/2)*d+40*e^3*c^3/(
-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(-4*b*e+8*c*d+4*(-4*a*c*e^2+b^2*e^2)^
(1/2))*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x
+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a-34*e
^3*c^2/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(-4*b*e+8*c*d+4*(-4*a*c*e^2+b
^2*e^2)^(1/2))*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arct
anh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)
)*b^2-96*e*c^4/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(-4*b*e+8*c*d+4*(-4*a
*c*e^2+b^2*e^2)^(1/2))*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1
/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*
c)^(1/2))*d^2+96*e*c^4/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(4*b*e-8*c*d+
4*(-4*a*c*e^2+b^2*e^2)^(1/2))*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))
*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/
2))*c)^(1/2))*d^2+34*e^3*c^2/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(4*b*e-
8*c*d+4*(-4*a*c*e^2+b^2*e^2)^(1/2))*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^
(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2
))^(1/2))*c)^(1/2))*b^2+96*e^2*c^3/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(
-4*b*e+8*c*d+4*(-4*a*c*e^2+b^2*e^2)^(1/2))*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*
c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*
(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d+36*e^2*c^2/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*
c-b^2)^2/(-4*b*e+8*c*d+4*(-4*a*c*e^2+b^2*e^2)^(1/2))*2^(1/2)/((-b*e+2*c*d+(
-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*
c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*(-4*a*c*e^2+b^2*e^2)^(1/2)*b-72*e*c
^3/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(-4*b*e+8*c*d+4*(-4*a*c*e^2+b^2*e
^2)^(1/2))*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(
(e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*(-
4*a*c*e^2+b^2*e^2)^(1/2)*d-96*e^2*c^3/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^
2/(4*b*e-8*c*d+4*(-4*a*c*e^2+b^2*e^2)^(1/2))*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a
*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(
4*a*c-b^2))^(1/2))*c)^(1/2))*b*d+36*e^2*c^2/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c
-b^2)^2/(4*b*e-8*c*d+4*(-4*a*c*e^2+b^2*e^2)^(1/2))*2^(1/2)/((b*e-2*c*d+(-e^
2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(
-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*(-4*a*c*e^2+b^2*e^2)^(1/2)*b-3*e^2*c/(-e
^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(e*x+1/2*b*e/c+1/2/c*(e^2*(-4*a*c+b^2))
^(1/2))^2*(-4*a*c*e^2+b^2*e^2)^(1/2)/(-b*e+2*c*d-(-4*a*c*e^2+b^2*e^2)^(1/2)
)*(e*x+d)^(3/2)*b+3/2*e^2/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(e*x+1/2*b
*e/c+1/2/c*(e^2*(-4*a*c+b^2))^(1/2))^2*(-4*a*c*e^2+b^2*e^2)^(1/2)*(e*x+d)^(
1/2)*b+7*e^3*c/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(e*x+1/2*b*e/c-1/2/c*
(e^2*(-4*a*c+b^2))^(1/2))^2*(e*x+d)^(1/2)*a-7*e^3*c/(-e^2*(4*a*c-b^2))^(1/2
)/(4*a*c-b^2)^2/(e*x+1/2*b*e/c+1/2/c*(e^2*(-4*a*c+b^2))^(1/2))^2*(e*x+d)^(1
/2)*a+3/2*e^2/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(e*x+1/2*b*e/c-1/2/c*(
e^2*(-4*a*c+b^2))^(1/2))^2*(-4*a*c*e^2+b^2*e^2)^(1/2)*(e*x+d)^(1/2)*b-3*e*c
/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(e*x+1/2*b*e/c+1/2/c*(e^2*(-4*a*c+b
^2))^(1/2))^2*(-4*a*c*e^2+b^2*e^2)^(1/2)*(e*x+d)^(1/2)*d+10*e^3*c^2/(-e^2*(
4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(e*x+1/2*b*e/c+1/2/c*(e^2*(-4*a*c+b^2))^(1/
2))^2/(-b*e+2*c*d-(-4*a*c*e^2+b^2*e^2)^(1/2))*(e*x+d)^(3/2)*a-5/2*e^3*c/(-e
^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(e*x+1/2*b*e/c+1/2/c*(e^2*(-4*a*c+b^2))
^(1/2))^2/(-b*e+2*c*d-(-4*a*c*e^2+b^2*e^2)^(1/2))*(e*x+d)^(3/2)*b^2-3*e*c/(
-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(e*x+1/2*b*e/c-1/2/c*(e^2*(-4*a*c+b^2
))^(1/2))^2*(-4*a*c*e^2+b^2*e^2)^(1/2)*(e*x+d)^(1/2)*d-10*e^3*c^2/(-e^2*(4*
```

```
a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(e*x+1/2*b*e/c-1/2/c*(e^2*(-4*a*c+b^2))^(1/2)
)^2/(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))*(e*x+d)^(3/2)*a+5/2*e^3*c/(-e^2
*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(e*x+1/2*b*e/c-1/2/c*(e^2*(-4*a*c+b^2))^(
1/2))^2/(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))*(e*x+d)^(3/2)*b^2-72*e*c^3/
(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(4*b*e-8*c*d+4*(-4*a*c*e^2+b^2*e^2)^
(1/2))*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d
)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*(-4*a*c*e
^2+b^2*e^2)^(1/2)*d+6*e*c^2/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(e*x+1/2
*b*e/c+1/2/c*(e^2*(-4*a*c+b^2))^(1/2))^2*(-4*a*c*e^2+b^2*e^2)^(1/2)/(-b*e+2
*c*d-(-4*a*c*e^2+b^2*e^2)^(1/2))*(e*x+d)^(3/2)*d-7/4*e^3/(-e^2*(4*a*c-b^2))
^(1/2)/(4*a*c-b^2)^2/(e*x+1/2*b*e/c-1/2/c*(e^2*(-4*a*c+b^2))^(1/2))^2*(e*x+
d)^(1/2)*b^2+7/4*e^3/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(e*x+1/2*b*e/c+
1/2/c*(e^2*(-4*a*c+b^2))^(1/2))^2*(e*x+d)^(1/2)*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\left(cx^2+bx+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(c*x^2 + b*x + a)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.2306 $\int \frac{1}{\sqrt{d+ex}\left(a+bx+cx^2\right)^3}\,dx$

Optimal. Leaf size=835

$$-\frac{\sqrt{d+ex}\left(-eb^2+cdb+2ace+c(2cd-be)x\right)}{2\left(b^2-4ac\right)\left(cd^2-bed+ae^2\right)\left(cx^2+bx+a\right)^2}-\frac{3\sqrt{c}\left(32c^4d^4-8c^3e\left(8bd-\sqrt{b^2-4ac}\,d-9ae\right)d^2+b^3\left(b+\sqrt{b^2-4ac}\right)e\right.}{}$$

```
[Out] -(Sqrt[d + e*x]*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/(2*(b^2 - 4*
a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2)^2) - (Sqrt[d + e*x]*(5*a*c*e
*(2*c*d - b*e)^2 - (b*c*d - b^2*e + 2*a*c*e)*(12*c^2*d^2 - 3*b^2*e^2 - 7*c*
e*(b*d - 2*a*e)) - 3*c*(2*c*d - b*e)*(4*c^2*d^2 - b^2*e^2 - 4*c*e*(b*d - 2*
a*e))*x))/(4*(b^2 - 4*a*c)^2*(c*d^2 - b*d*e + a*e^2)^2*(a + b*x + c*x^2)) -
 (3*Sqrt[c]*(32*c^4*d^4 + b^3*(b + Sqrt[b^2 - 4*a*c])*e^4 - 8*c^3*d^2*e*(8*
b*d - Sqrt[b^2 - 4*a*c]*d - 9*a*e) + 2*b*c*e^3*(b^2*d + b*Sqrt[b^2 - 4*a*c]
*d - 5*a*b*e - 4*a*Sqrt[b^2 - 4*a*c]*e) + 2*c^2*e^2*(15*b^2*d^2 - 6*b*d*(Sq
rt[b^2 - 4*a*c]*d + 6*a*e) + 4*a*e*(2*Sqrt[b^2 - 4*a*c]*d + 7*a*e)))*ArcTan
h[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])
/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*
d^2 - b*d*e + a*e^2)^2) + (3*Sqrt[c]*(32*c^4*d^4 + b^3*(b - Sqrt[b^2 - 4*a*
c])*e^4 - 8*c^3*d^2*e*(8*b*d + Sqrt[b^2 - 4*a*c]*d - 9*a*e) + 2*c^2*e^2*(15
*b^2*d^2 - 4*a*e*(2*Sqrt[b^2 - 4*a*c]*d - 7*a*e) + 6*b*d*(Sqrt[b^2 - 4*a*c]
*d - 6*a*e)) + 2*b*c*e^3*(b^2*d + 4*a*Sqrt[b^2 - 4*a*c]*e - b*(Sqrt[b^2 - 4
*a*c]*d + 5*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - e*(b*d - a*e))^2)
```

Rubi [A] time = 13.9194, antiderivative size = 834, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {740, 822, 826, 1166, 208}

$$-\frac{\sqrt{d+ex}\left(-eb^2+cdb+2ace+c(2cd-be)x\right)}{2\left(b^2-4ac\right)\left(cd^2-bed+ae^2\right)\left(cx^2+bx+a\right)^2}-\frac{3\sqrt{c}\left(32c^4d^4-8c^3e\left(8bd-\sqrt{b^2-4ac}\,d-9ae\right)d^2+b^3\left(b+\sqrt{b^2-4ac}\right)e\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(a + b*x + c*x^2)^3),x]
```


```
[Out] -(Sqrt[d + e*x]*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/(2*(b^2 - 4*
a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2)^2) - (Sqrt[d + e*x]*(5*a*c*e
*(2*c*d - b*e)^2 - (b*c*d - b^2*e + 2*a*c*e)*(12*c^2*d^2 - 3*b^2*e^2 - 7*c*
e*(b*d - 2*a*e)) - 3*c*(2*c*d - b*e)*(4*c^2*d^2 - b^2*e^2 - 4*c*e*(b*d - 2*
a*e))*x))/(4*(b^2 - 4*a*c)^2*(c*d^2 - b*d*e + a*e^2)^2*(a + b*x + c*x^2)) -
 (3*Sqrt[c]*(32*c^4*d^4 + b^3*(b + Sqrt[b^2 - 4*a*c])*e^4 - 8*c^3*d^2*e*(8*
b*d - Sqrt[b^2 - 4*a*c]*d - 9*a*e) + 2*b*c*e^3*(b^2*d + b*Sqrt[b^2 - 4*a*c]
*d - 5*a*b*e - 4*a*Sqrt[b^2 - 4*a*c]*e) + 2*c^2*e^2*(15*b^2*d^2 - 6*b*d*(Sq
rt[b^2 - 4*a*c]*d + 6*a*e) + 4*a*e*(2*Sqrt[b^2 - 4*a*c]*d + 7*a*e)))*ArcTan
h[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])
/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*
d^2 - b*d*e + a*e^2)^2) + (3*Sqrt[c]*(32*c^4*d^4 + b^3*(b - Sqrt[b^2 - 4*a*
```

```
c])*e^4 - 8*c^3*d^2*e*(8*b*d + Sqrt[b^2 - 4*a*c]*d - 9*a*e) + 2*c^2*e^2*(15
*b^2*d^2 - 4*a*e*(2*Sqrt[b^2 - 4*a*c]*d - 7*a*e) + 6*b*d*(Sqrt[b^2 - 4*a*c]
*d - 6*a*e)) + 2*b*c*e^3*(b^2*d + 4*a*Sqrt[b^2 - 4*a*c]*e - b*(Sqrt[b^2 - 4
*a*c]*d + 5*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)^2)
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\left(a+bx+cx^2\right)^3}\,dx = -\frac{\sqrt{d+ex}\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{2\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^2} - \frac{\int \frac{\frac{1}{2}\left(12c^2d^2-3b^2e^2-7ce(bd-2ae)\right)+\frac{5}{2}ce(2cd-be)x}{\sqrt{d+ex}\left(a+bx+cx^2\right)^2}}{2\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{\sqrt{d+ex}\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{2\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^2} - \frac{\sqrt{d+ex}\left(5ace(2cd-be)^2-\left(bcd-b^2e\right.\right.}{}$$

$$= -\frac{\sqrt{d+ex}\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{2\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^2} - \frac{\sqrt{d+ex}\left(5ace(2cd-be)^2-\left(bcd-b^2e\right.\right.}{}$$

$$= -\frac{\sqrt{d+ex}\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{2\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^2} - \frac{\sqrt{d+ex}\left(5ace(2cd-be)^2-\left(bcd-b^2e\right.\right.}{}$$

$$= -\frac{\sqrt{d+ex}\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{2\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^2} - \frac{\sqrt{d+ex}\left(5ace(2cd-be)^2-\left(bcd-b^2e\right.\right.}{}$$

Mathematica [A] time = 6.20855, size = 1056, normalized size = 1.26

$$-\frac{\sqrt{d+ex}\left(-eb^2+cdb+2ace+c(2cd-be)x\right)}{2\left(b^2-4ac\right)\left(cd^2-bed+ae^2\right)\left(cx^2+bx+a\right)^2} - \frac{-\frac{\sqrt{d+ex}\left(-\frac{5}{2}ace(2cd-be)^2+\frac{1}{2}\left(-eb^2+cdb+2ace\right)\left(12c^2d^2-3b^2e^2-7ce(bd-2ae)\right)+c\left(\frac{1}{2}(2cd-be)\left(12c\right.\right.\right.}{\left(b^2-4ac\right)\left(cd^2-bed+ae^2\right)\left(cx^2+bx+a\right)}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(a + b*x + c*x^2)^3),x]
```

```
[Out] -(Sqrt[d + e*x]*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/(2*(b^2 - 4*
a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2)^2) - (-((Sqrt[d + e*x]*((-5*
a*c*e*(2*c*d - b*e)^2)/2 + ((b*c*d - b^2*e + 2*a*c*e)*(12*c^2*d^2 - 3*b^2*e
^2 - 7*c*e*(b*d - 2*a*e)))/2 + c*((-5*c*e*(b*d - 2*a*e)*(2*c*d - b*e))/2 +
((2*c*d - b*e)*(12*c^2*d^2 - 3*b^2*e^2 - 7*c*e*(b*d - 2*a*e)))/2)*x))/((b^2
 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2))) - (2*((Sqrt[2*c*d - b
*e - Sqrt[b^2 - 4*a*c]*e]*((3*c*e*(2*c*d - b*e)*(4*c^2*d^2 - b^2*e^2 - 4*c*
e*(b*d - 2*a*e)))/4 - ((-3*c*e*(2*c*d - b*e)*(-2*c*d + b*e)*(4*c^2*d^2 - b^
2*e^2 - 4*c*e*(b*d - 2*a*e)))/4 + 2*c*((-3*c*d*e*(2*c*d - b*e)*(4*c^2*d^2 -
 b^2*e^2 - 4*c*e*(b*d - 2*a*e)))/4 + (3*e*(16*c^4*d^4 + b^4*e^4 + b^2*c*e^3
*(2*b*d - 9*a*e) - 4*c^3*d^2*e*(7*b*d - 9*a*e) + c^2*e^2*(9*b^2*d^2 - 28*a*
b*d*e + 28*a^2*e^2)))/4))/(Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*S
qrt[d + e*x])/Sqrt[2*c*d - b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[2]*Sqrt[c]*(-
2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e)) + (Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]
*e]*((3*c*e*(2*c*d - b*e)*(4*c^2*d^2 - b^2*e^2 - 4*c*e*(b*d - 2*a*e)))/4 +
((-3*c*e*(2*c*d - b*e)*(-2*c*d + b*e)*(4*c^2*d^2 - b^2*e^2 - 4*c*e*(b*d - 2
*a*e)))/4 + 2*c*((-3*c*d*e*(2*c*d - b*e)*(4*c^2*d^2 - b^2*e^2 - 4*c*e*(b*d
- 2*a*e)))/4 + (3*e*(16*c^4*d^4 + b^4*e^4 + b^2*c*e^3*(2*b*d - 9*a*e) - 4*c
^3*d^2*e*(7*b*d - 9*a*e) + c^2*e^2*(9*b^2*d^2 - 28*a*b*d*e + 28*a^2*e^2)))/
4))/(Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c
```

```
*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[2]*Sqrt[c]*(-2*c*d + b*e - Sqrt[b^2
 - 4*a*c]*e))))/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)))/(2*(b^2 - 4*a*c)*(
c*d^2 - b*d*e + a*e^2))
```

Maple [B] time = 0.341, size = 4488, normalized size = 5.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)^3,x)
```

```
[Out] -96*e*c^5/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(-8*a*c*e^2+4*b^2*e^2-8*b*
c*d*e+8*c^2*d^2+4*(-4*a*c*e^2+b^2*e^2)^(1/2)*b*e-8*(-4*a*c*e^2+b^2*e^2)^(1/
2)*c*d)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+
d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d^2-3*e^
2*c^2/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(e*x+1/2*b*e/c+1/2/c*(e^2*(-4*
a*c+b^2))^(1/2))^2*(-4*a*c*e^2+b^2*e^2)^(1/2)/(-2*a*c*e^2+b^2*e^2-2*b*c*d*e
+2*c^2*d^2+(-4*a*c*e^2+b^2*e^2)^(1/2)*b*e-2*(-4*a*c*e^2+b^2*e^2)^(1/2)*c*d)
*(e*x+d)^(3/2)*b+3*e^2*c/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(e*x+1/2*b*
e/c-1/2/c*(e^2*(-4*a*c+b^2))^(1/2))^2*(-4*a*c*e^2+b^2*e^2)^(1/2)/(-b*e+2*c*
d+(-4*a*c*e^2+b^2*e^2)^(1/2))*(e*x+d)^(1/2)*b+6*e*c^3/(-e^2*(4*a*c-b^2))^(1
/2)/(4*a*c-b^2)^2/(e*x+1/2*b*e/c-1/2/c*(e^2*(-4*a*c+b^2))^(1/2))^2*(-4*a*c*
e^2+b^2*e^2)^(1/2)/(-2*a*c*e^2+b^2*e^2-2*b*c*d*e+2*c^2*d^2-(-4*a*c*e^2+b^2*
e^2)^(1/2)*b*e+2*(-4*a*c*e^2+b^2*e^2)^(1/2)*c*d)*(e*x+d)^(3/2)*d+3*e^2*c/(-
e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(e*x+1/2*b*e/c+1/2/c*(e^2*(-4*a*c+b^2)
)^(1/2))^2*(-4*a*c*e^2+b^2*e^2)^(1/2)/(-b*e+2*c*d-(-4*a*c*e^2+b^2*e^2)^(1/2
))*(e*x+d)^(1/2)*b-6*e*c^2/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(e*x+1/2*
b*e/c+1/2/c*(e^2*(-4*a*c+b^2))^(1/2))^2*(-4*a*c*e^2+b^2*e^2)^(1/2)/(-b*e+2*
c*d-(-4*a*c*e^2+b^2*e^2)^(1/2))*(e*x+d)^(1/2)*d+168*e^3*c^4/(-e^2*(4*a*c-b^
2))^(1/2)/(4*a*c-b^2)^2/(-8*a*c*e^2+4*b^2*e^2-8*b*c*d*e+8*c^2*d^2+4*(-4*a*c
*e^2+b^2*e^2)^(1/2)*b*e-8*(-4*a*c*e^2+b^2*e^2)^(1/2)*c*d)*2^(1/2)/((b*e-2*c
*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-
2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a-6*e*c^2/(-e^2*(4*a*c-b^2))^(1/2
)/(4*a*c-b^2)^2/(e*x+1/2*b*e/c-1/2/c*(e^2*(-4*a*c+b^2))^(1/2))^2*(-4*a*c*e^
2+b^2*e^2)^(1/2)/(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))*(e*x+d)^(1/2)*d-16
8*e^3*c^4/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(8*a*c*e^2-4*b^2*e^2+8*b*c
*d*e-8*c^2*d^2+4*(-4*a*c*e^2+b^2*e^2)^(1/2)*b*e-8*(-4*a*c*e^2+b^2*e^2)^(1/2
)*c*d)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x
+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a+96*e
*c^5/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(8*a*c*e^2-4*b^2*e^2+8*b*c*d*e-
8*c^2*d^2+4*(-4*a*c*e^2+b^2*e^2)^(1/2)*b*e-8*(-4*a*c*e^2+b^2*e^2)^(1/2)*c*d
)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(
1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d^2-3*e^2*c
^2/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(e*x+1/2*b*e/c-1/2/c*(e^2*(-4*a*c
+b^2))^(1/2))^2*(-4*a*c*e^2+b^2*e^2)^(1/2)/(-2*a*c*e^2+b^2*e^2-2*b*c*d*e+2*
c^2*d^2-(-4*a*c*e^2+b^2*e^2)^(1/2)*b*e+2*(-4*a*c*e^2+b^2*e^2)^(1/2)*c*d)*(e
*x+d)^(3/2)*b+66*e^3*c^3/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(8*a*c*e^2-
4*b^2*e^2+8*b*c*d*e-8*c^2*d^2+4*(-4*a*c*e^2+b^2*e^2)^(1/2)*b*e-8*(-4*a*c*e^
2+b^2*e^2)^(1/2)*c*d)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/
2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c
)^(1/2))*b^2+6*e*c^3/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(e*x+1/2*b*e/c+
1/2/c*(e^2*(-4*a*c+b^2))^(1/2))^2*(-4*a*c*e^2+b^2*e^2)^(1/2)/(-2*a*c*e^2+b^
2*e^2-2*b*c*d*e+2*c^2*d^2+(-4*a*c*e^2+b^2*e^2)^(1/2)*b*e-2*(-4*a*c*e^2+b^2*
e^2)^(1/2)*c*d)*(e*x+d)^(3/2)*d-66*e^3*c^3/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-
b^2)^2/(-8*a*c*e^2+4*b^2*e^2-8*b*c*d*e+8*c^2*d^2+4*(-4*a*c*e^2+b^2*e^2)^(1/
2)*b*e-8*(-4*a*c*e^2+b^2*e^2)^(1/2)*c*d)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b
```

```
^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*
c-b^2))^(1/2))*c)^(1/2))*b^2+9/2*e^3*c^2/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^
2)^2/(e*x+1/2*b*e/c-1/2/c*(e^2*(-4*a*c+b^2))^(1/2))^2/(-2*a*c*e^2+b^2*e^2-2
*b*c*d*e+2*c^2*d^2-(-4*a*c*e^2+b^2*e^2)^(1/2)*b*e+2*(-4*a*c*e^2+b^2*e^2)^(1
/2)*c*d)*(e*x+d)^(3/2)*b^2+22*e^3*c^2/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^
2/(e*x+1/2*b*e/c-1/2/c*(e^2*(-4*a*c+b^2))^(1/2))^2/(-b*e+2*c*d+(-4*a*c*e^2+
b^2*e^2)^(1/2))*(e*x+d)^(1/2)*a-11/2*e^3*c/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-
b^2)^2/(e*x+1/2*b*e/c-1/2/c*(e^2*(-4*a*c+b^2))^(1/2))^2/(-b*e+2*c*d+(-4*a*c
*e^2+b^2*e^2)^(1/2))*(e*x+d)^(1/2)*b^2-9/2*e^3*c^2/(-e^2*(4*a*c-b^2))^(1/2)
/(4*a*c-b^2)^2/(e*x+1/2*b*e/c+1/2/c*(e^2*(-4*a*c+b^2))^(1/2))^2/(-2*a*c*e^2
+b^2*e^2-2*b*c*d*e+2*c^2*d^2+(-4*a*c*e^2+b^2*e^2)^(1/2)*b*e-2*(-4*a*c*e^2+b
^2*e^2)^(1/2)*c*d)*(e*x+d)^(3/2)*b^2-22*e^3*c^2/(-e^2*(4*a*c-b^2))^(1/2)/(4
*a*c-b^2)^2/(e*x+1/2*b*e/c+1/2/c*(e^2*(-4*a*c+b^2))^(1/2))^2/(-b*e+2*c*d-(-
4*a*c*e^2+b^2*e^2)^(1/2))*(e*x+d)^(1/2)*a+11/2*e^3*c/(-e^2*(4*a*c-b^2))^(1/
2)/(4*a*c-b^2)^2/(e*x+1/2*b*e/c+1/2/c*(e^2*(-4*a*c+b^2))^(1/2))^2/(-b*e+2*c
*d-(-4*a*c*e^2+b^2*e^2)^(1/2))*(e*x+d)^(1/2)*b^2-18*e^3*c^3/(-e^2*(4*a*c-b^
2))^(1/2)/(4*a*c-b^2)^2/(e*x+1/2*b*e/c-1/2/c*(e^2*(-4*a*c+b^2))^(1/2))^2/(-
2*a*c*e^2+b^2*e^2-2*b*c*d*e+2*c^2*d^2-(-4*a*c*e^2+b^2*e^2)^(1/2)*b*e+2*(-4*
a*c*e^2+b^2*e^2)^(1/2)*c*d)*(e*x+d)^(3/2)*a+18*e^3*c^3/(-e^2*(4*a*c-b^2))^(
1/2)/(4*a*c-b^2)^2/(e*x+1/2*b*e/c+1/2/c*(e^2*(-4*a*c+b^2))^(1/2))^2/(-2*a*c
*e^2+b^2*e^2-2*b*c*d*e+2*c^2*d^2+(-4*a*c*e^2+b^2*e^2)^(1/2)*b*e-2*(-4*a*c*e
^2+b^2*e^2)^(1/2)*c*d)*(e*x+d)^(3/2)*a-60*e^2*c^3/(-e^2*(4*a*c-b^2))^(1/2)/
(4*a*c-b^2)^2/(-8*a*c*e^2+4*b^2*e^2-8*b*c*d*e+8*c^2*d^2+4*(-4*a*c*e^2+b^2*e
^2)^(1/2)*b*e-8*(-4*a*c*e^2+b^2*e^2)^(1/2)*c*d)*2^(1/2)/((b*e-2*c*d+(-e^2*(
4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^
2*(4*a*c-b^2))^(1/2))*c)^(1/2))*(-4*a*c*e^2+b^2*e^2)^(1/2)*b-60*e^2*c^3/(-e
^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(8*a*c*e^2-4*b^2*e^2+8*b*c*d*e-8*c^2*d^
2+4*(-4*a*c*e^2+b^2*e^2)^(1/2)*b*e-8*(-4*a*c*e^2+b^2*e^2)^(1/2)*c*d)*2^(1/2
)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2
^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*(-4*a*c*e^2+b^2*e^2
)^(1/2)*b-96*e^2*c^4/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(8*a*c*e^2-4*b^
2*e^2+8*b*c*d*e-8*c^2*d^2+4*(-4*a*c*e^2+b^2*e^2)^(1/2)*b*e-8*(-4*a*c*e^2+b^
2*e^2)^(1/2)*c*d)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*a
rctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1
/2))*b*d+120*e*c^4/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)^2/(8*a*c*e^2-4*b^2*
e^2+8*b*c*d*e-8*c^2*d^2+4*(-4*a*c*e^2+b^2*e^2)^(1/2)*b*e-8*(-4*a*c*e^2+b^2*
e^2)^(1/2)*c*d)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arc
tanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2
))*(-4*a*c*e^2+b^2*e^2)^(1/2)*d+96*e^2*c^4/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-
b^2)^2/(-8*a*c*e^2+4*b^2*e^2-8*b*c*d*e+8*c^2*d^2+4*(-4*a*c*e^2+b^2*e^2)^(1/
2)*b*e-8*(-4*a*c*e^2+b^2*e^2)^(1/2)*c*d)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b
^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*
c-b^2))^(1/2))*c)^(1/2))*b*d+120*e*c^4/(-e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)
^2/(-8*a*c*e^2+4*b^2*e^2-8*b*c*d*e+8*c^2*d^2+4*(-4*a*c*e^2+b^2*e^2)^(1/2)*b
*e-8*(-4*a*c*e^2+b^2*e^2)^(1/2)*c*d)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))
^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^
2))^(1/2))*c)^(1/2))*(-4*a*c*e^2+b^2*e^2)^(1/2)*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^3\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^3*sqrt(e*x + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(c*x**2+b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.2307 $\int \frac{\sqrt{d+ex}}{a+ibx+cx^2}\,dx$

Optimal. Leaf size=629

$$\frac{e\log\left(-\sqrt{d+ex}\sqrt{2\sqrt{c}\sqrt{cd^2-e(-ae+ibd)}-ibe+2cd}+\sqrt{cd^2-e(-ae+ibd)}+\sqrt{c}(d+ex)\right)}{2\sqrt{c}\sqrt{2\sqrt{c}\sqrt{cd^2-e(-ae+ibd)}-ibe+2cd}}-\frac{e\log\left(\sqrt{d+ex}\sqrt{2\sqrt{c}\sqrt{cd^2}}\right.}{2}$$

```
[Out] (e*ArcTanh[(Sqrt[2*c*d - I*b*e + 2*Sqrt[c]*Sqrt[c*d^2 - e*(I*b*d - a*e)]] -
 2*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - I*b*e - 2*Sqrt[c]*Sqrt[c*d^2 - e*(I*
b*d - a*e)]]])/(Sqrt[c]*Sqrt[2*c*d - I*b*e - 2*Sqrt[c]*Sqrt[c*d^2 - e*(I*b*
d - a*e)]]) - (e*ArcTanh[(Sqrt[2*c*d - I*b*e + 2*Sqrt[c]*Sqrt[c*d^2 - e*(I*
b*d - a*e)]] + 2*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - I*b*e - 2*Sqrt[c]*Sqrt
[c*d^2 - e*(I*b*d - a*e)]]])/(Sqrt[c]*Sqrt[2*c*d - I*b*e - 2*Sqrt[c]*Sqrt[c
*d^2 - e*(I*b*d - a*e)]]) + (e*Log[Sqrt[c*d^2 - e*(I*b*d - a*e)] - Sqrt[2*c
*d - I*b*e + 2*Sqrt[c]*Sqrt[c*d^2 - e*(I*b*d - a*e)]]*Sqrt[d + e*x] + Sqrt[
c]*(d + e*x)])/(2*Sqrt[c]*Sqrt[2*c*d - I*b*e + 2*Sqrt[c]*Sqrt[c*d^2 - e*(I*
b*d - a*e)]]) - (e*Log[Sqrt[c*d^2 - e*(I*b*d - a*e)] + Sqrt[2*c*d - I*b*e +
 2*Sqrt[c]*Sqrt[c*d^2 - e*(I*b*d - a*e)]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)
])/(2*Sqrt[c]*Sqrt[2*c*d - I*b*e + 2*Sqrt[c]*Sqrt[c*d^2 - e*(I*b*d - a*e)]]
)
```

Rubi [A] time = 1.22707, antiderivative size = 629, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {699, 1129, 634, 618, 206, 628}

$$\frac{e\log\left(-\sqrt{d+ex}\sqrt{2\sqrt{c}\sqrt{cd^2-e(-ae+ibd)}-ibe+2cd}+\sqrt{cd^2-e(-ae+ibd)}+\sqrt{c}(d+ex)\right)}{2\sqrt{c}\sqrt{2\sqrt{c}\sqrt{cd^2-e(-ae+ibd)}-ibe+2cd}}-\frac{e\log\left(\sqrt{d+ex}\sqrt{2\sqrt{c}\sqrt{cd^2}}\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(a + I*b*x + c*x^2),x]
```

```
[Out] (e*ArcTanh[(Sqrt[2*c*d - I*b*e + 2*Sqrt[c]*Sqrt[c*d^2 - e*(I*b*d - a*e)]] -
 2*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - I*b*e - 2*Sqrt[c]*Sqrt[c*d^2 - e*(I*
b*d - a*e)]]])/(Sqrt[c]*Sqrt[2*c*d - I*b*e - 2*Sqrt[c]*Sqrt[c*d^2 - e*(I*b*
d - a*e)]]) - (e*ArcTanh[(Sqrt[2*c*d - I*b*e + 2*Sqrt[c]*Sqrt[c*d^2 - e*(I*
b*d - a*e)]] + 2*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - I*b*e - 2*Sqrt[c]*Sqrt
[c*d^2 - e*(I*b*d - a*e)]]])/(Sqrt[c]*Sqrt[2*c*d - I*b*e - 2*Sqrt[c]*Sqrt[c
*d^2 - e*(I*b*d - a*e)]]) + (e*Log[Sqrt[c*d^2 - e*(I*b*d - a*e)] - Sqrt[2*c
*d - I*b*e + 2*Sqrt[c]*Sqrt[c*d^2 - e*(I*b*d - a*e)]]*Sqrt[d + e*x] + Sqrt[
c]*(d + e*x)])/(2*Sqrt[c]*Sqrt[2*c*d - I*b*e + 2*Sqrt[c]*Sqrt[c*d^2 - e*(I*
b*d - a*e)]]) - (e*Log[Sqrt[c*d^2 - e*(I*b*d - a*e)] + Sqrt[2*c*d - I*b*e +
 2*Sqrt[c]*Sqrt[c*d^2 - e*(I*b*d - a*e)]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)
])/(2*Sqrt[c]*Sqrt[2*c*d - I*b*e + 2*Sqrt[c]*Sqrt[c*d^2 - e*(I*b*d - a*e)]]
)
```

Rule 699

```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Dist[2*e, Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*
x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
```

```
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1129

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q =
 Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m - 1)/(q
 - r*x + x^2), x], x] - Dist[1/(2*c*r), Int[x^(m - 1)/(q + r*x + x^2), x],
x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 1] && LtQ[m, 3
] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{a+ibx+cx^2}\,dx = (2e)\,\text{Subst}\left(\int \frac{x^2}{cd^2-ibde+ae^2+(-2cd+ibe)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)$$

$$= \frac{e\,\text{Subst}\left(\int \frac{x}{\frac{\sqrt{cd^2-ibde+ae^2}}{\sqrt{c}}-\frac{\sqrt{2cd-ibe+2\sqrt{c}\sqrt{cd^2-ibde+ae^2}}\,x}{\sqrt{c}}+x^2}\,dx, x, \sqrt{d+ex}\right)}{\sqrt{c}\sqrt{2cd-ibe+2\sqrt{c}\sqrt{cd^2-e(ibd-ae)}}} - \frac{e\,\text{Subst}\left(\int \frac{x}{\frac{\sqrt{cd^2-ibde+ae^2}}{\sqrt{c}}+\frac{\sqrt{2cd-ibe+2\sqrt{c}}\cdots}{}\right.}{\sqrt{c}\sqrt{2cd-ibe+2\sqrt{c}}\cdots}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{cd^2-ibde+ae^2}}{\sqrt{c}}-\frac{\sqrt{2cd-ibe+2\sqrt{c}\sqrt{cd^2-ibde+ae^2}}\,x}{\sqrt{c}}+x^2}\,dx, x, \sqrt{d+ex}\right)}{2c} + \frac{e\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{cd^2-ibde+ae^2}}{\sqrt{c}}+\frac{\sqrt{2cd-ibe+2\sqrt{}}\cdots}{}\right.}{2c}$$

$$= \frac{e\log\left(\sqrt{cd^2-e(ibd-ae)}-\sqrt{2cd-ibe+2\sqrt{c}\sqrt{cd^2-e(ibd-ae)}}\sqrt{d+ex}+\sqrt{c}(d+ex)\right)}{2\sqrt{c}\sqrt{2cd-ibe+2\sqrt{c}\sqrt{cd^2-e(ibd-ae)}}} - \frac{e\log\left(\sqrt{}\cdots\right.}{}$$

$$= \frac{e\tanh^{-1}\left(\frac{\sqrt{c}\left(\frac{\sqrt{2cd-ibe+2\sqrt{c}\sqrt{cd^2-e(ibd-ae)}}}{\sqrt{c}}-2\sqrt{d+ex}\right)}{\sqrt{2cd-ibe-2\sqrt{c}\sqrt{cd^2-e(ibd-ae)}}}\right)}{\sqrt{c}\sqrt{2cd-ibe-2\sqrt{c}\sqrt{cd^2-e(ibd-ae)}}} - \frac{e\tanh^{-1}\left(\frac{\sqrt{c}\left(\frac{\sqrt{2cd-ibe+2\sqrt{c}\sqrt{cd^2-e(ibd-ae)}}}{\sqrt{c}}+2\sqrt{d+ex}\right)}{\sqrt{2cd-ibe-2\sqrt{c}\sqrt{cd^2-e(ibd-ae)}}}\right)}{\sqrt{c}\sqrt{2cd-ibe-2\sqrt{c}\sqrt{cd^2-e(ibd-ae)}}} + \frac{e\log\left(\cdots\right.}{}$$

Mathematica [A] time = 0.498221, size = 197, normalized size = 0.31

$$\frac{\sqrt{2}\left(\sqrt{2cd-e\left(\sqrt{-4ac-b^2}+ib\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{-4ac-b^2}+ib\right)}}\right)-\sqrt{e\sqrt{-4ac-b^2}-ibe+2cd}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{-4ac-b^2}-ibe+2cd}}\right)\right)}{\sqrt{c}\sqrt{-4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(a + I*b*x + c*x^2),x]
```

```
[Out] (Sqrt[2]*(-(Sqrt[2*c*d - I*b*e + Sqrt[-b^2 - 4*a*c]*e]*ArcTanh[(Sqrt[2]*Sqr
t[c]*Sqrt[d + e*x])/Sqrt[2*c*d - I*b*e + Sqrt[-b^2 - 4*a*c]*e]]) + Sqrt[2*c
*d - (I*b + Sqrt[-b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/
Sqrt[2*c*d - (I*b + Sqrt[-b^2 - 4*a*c])*e]]))/(Sqrt[c]*Sqrt[-b^2 - 4*a*c])
```

Maple [A] time = 0.379, size = 609, normalized size = 1.

$$\frac{e}{2}\ln\left((ex+d)\sqrt{c}-\sqrt{ex+d}\sqrt{2\sqrt{-c\left(ibde-ae^2-cd^2\right)}-ibe+2\,cd}+\sqrt{-ibde+ae^2+cd^2}\right)\frac{1}{\sqrt{2\sqrt{-c\left(ibde-ae^2-cd^2\right)}-\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(a+I*b*x+c*x^2),x)
```

```
[Out] 1/2*e/(2*(-c*(I*b*d*e-a*e^2-c*d^2))^(1/2)-I*b*e+2*c*d)^(1/2)/c^(1/2)*ln((e*
x+d)*c^(1/2)-(e*x+d)^(1/2)*(2*(-c*(I*b*d*e-a*e^2-c*d^2))^(1/2)-I*b*e+2*c*d)
^(1/2)+(-I*b*d*e+a*e^2+c*d^2)^(1/2))+e/c^(1/2)/(4*c^(1/2)*(-I*b*d*e+a*e^2+c
*d^2)^(1/2)-2*(-c*(I*b*d*e-a*e^2-c*d^2))^(1/2)+I*b*e-2*c*d)^(1/2)*arctan((2
*c^(1/2)*(e*x+d)^(1/2)-(2*(-c*(I*b*d*e-a*e^2-c*d^2))^(1/2)-I*b*e+2*c*d)^(1/
2))/(4*c^(1/2)*(-I*b*d*e+a*e^2+c*d^2)^(1/2)-2*(-c*(I*b*d*e-a*e^2-c*d^2))^(1
/2)+I*b*e-2*c*d)^(1/2))-1/2*e/(2*(-c*(I*b*d*e-a*e^2-c*d^2))^(1/2)-I*b*e+2*c
*d)^(1/2)/c^(1/2)*ln((e*x+d)*c^(1/2)+(e*x+d)^(1/2)*(2*(-c*(I*b*d*e-a*e^2-c*
d^2))^(1/2)-I*b*e+2*c*d)^(1/2)+(-I*b*d*e+a*e^2+c*d^2)^(1/2))+e/c^(1/2)/(4*c
^(1/2)*(-I*b*d*e+a*e^2+c*d^2)^(1/2)-2*(-c*(I*b*d*e-a*e^2-c*d^2))^(1/2)+I*b*
e-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*(-c*(I*b*d*e-a*e^2-c*d^2)
)^(1/2)-I*b*e+2*c*d)^(1/2))/(4*c^(1/2)*(-I*b*d*e+a*e^2+c*d^2)^(1/2)-2*(-c*(
I*b*d*e-a*e^2-c*d^2))^(1/2)+I*b*e-2*c*d)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{cx^2+i\,bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a+I*b*x+c*x^2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(c*x^2 + I*b*x + a), x)
```

Fricas [A] time = 2.79583, size = 1515, normalized size = 2.41

$$-\frac{1}{2}\sqrt{-\frac{4\,cd-2i\,be+2\left(b^2c+4\,ac^2\right)\sqrt{-\frac{e^2}{b^2c^2+4\,ac^3}}}{b^2c+4\,ac^2}}\log\left(\frac{\left(b^2c+4\,ac^2\right)\sqrt{-\frac{e^2}{b^2c^2+4\,ac^3}}\sqrt{-\frac{4\,cd-2i\,be+2\left(b^2c+4\,ac^2\right)\sqrt{-\frac{e^2}{b^2c^2+4\,ac^3}}}{b^2c+4\,ac^2}}+2}{2\,e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a+I*b*x+c*x^2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-(4*c*d - 2*I*b*e + 2*(b^2*c + 4*a*c^2)*sqrt(-e^2/(b^2*c^2 + 4*a*
c^3)))/(b^2*c + 4*a*c^2))*log(1/2*((b^2*c + 4*a*c^2)*sqrt(-e^2/(b^2*c^2 + 4
*a*c^3))*sqrt(-(4*c*d - 2*I*b*e + 2*(b^2*c + 4*a*c^2)*sqrt(-e^2/(b^2*c^2 +
4*a*c^3)))/(b^2*c + 4*a*c^2)) + 2*sqrt(e*x + d)*e)/e) + 1/2*sqrt(-(4*c*d -
2*I*b*e + 2*(b^2*c + 4*a*c^2)*sqrt(-e^2/(b^2*c^2 + 4*a*c^3)))/(b^2*c + 4*a*
c^2))*log(-1/2*((b^2*c + 4*a*c^2)*sqrt(-e^2/(b^2*c^2 + 4*a*c^3))*sqrt(-(4*c
*d - 2*I*b*e + 2*(b^2*c + 4*a*c^2)*sqrt(-e^2/(b^2*c^2 + 4*a*c^3)))/(b^2*c +
 4*a*c^2)) - 2*sqrt(e*x + d)*e)/e) + 1/2*sqrt(-(4*c*d - 2*I*b*e - 2*(b^2*c
+ 4*a*c^2)*sqrt(-e^2/(b^2*c^2 + 4*a*c^3)))/(b^2*c + 4*a*c^2))*log(1/2*((b^2
*c + 4*a*c^2)*sqrt(-e^2/(b^2*c^2 + 4*a*c^3))*sqrt(-(4*c*d - 2*I*b*e - 2*(b^
2*c + 4*a*c^2)*sqrt(-e^2/(b^2*c^2 + 4*a*c^3)))/(b^2*c + 4*a*c^2)) + 2*sqrt(
e*x + d)*e)/e) - 1/2*sqrt(-(4*c*d - 2*I*b*e - 2*(b^2*c + 4*a*c^2)*sqrt(-e^2
/(b^2*c^2 + 4*a*c^3)))/(b^2*c + 4*a*c^2))*log(-1/2*((b^2*c + 4*a*c^2)*sqrt(
-e^2/(b^2*c^2 + 4*a*c^3))*sqrt(-(4*c*d - 2*I*b*e - 2*(b^2*c + 4*a*c^2)*sqrt
(-e^2/(b^2*c^2 + 4*a*c^3)))/(b^2*c + 4*a*c^2)) - 2*sqrt(e*x + d)*e)/e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}}{a+ibx+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(a+I*b*x+c*x**2),x)
```

```
[Out] Integral(sqrt(d + e*x)/(a + I*b*x + c*x**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a+I*b*x+c*x^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.2308 $\int \frac{1}{\sqrt{d+ex}\left(a+ibx+cx^2\right)}\,dx$

Optimal. Leaf size=705

$$-\frac{e\log\left(-\sqrt{d+ex}\sqrt{2\sqrt{c}\sqrt{cd^2-e(-ae+ibd)}-ibe+2cd}+\sqrt{cd^2-e(-ae+ibd)}+\sqrt{c}(d+ex)\right)}{2\sqrt{cd^2-e(-ae+ibd)}\sqrt{2\sqrt{c}\sqrt{cd^2-e(-ae+ibd)}-ibe+2cd}}+\frac{e\log\left(\sqrt{d+ex}\sqrt{2\sqrt{c}\cdots\right.}{2\sqrt{cd^2\cdots}}$$

```
[Out] (e*ArcTanh[(Sqrt[2*c*d - I*b*e + 2*Sqrt[c]*Sqrt[c*d^2 - e*(I*b*d - a*e)]] -
 2*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - I*b*e - 2*Sqrt[c]*Sqrt[c*d^2 - e*(I*
b*d - a*e)]]])/(Sqrt[c*d^2 - e*(I*b*d - a*e)]*Sqrt[2*c*d - I*b*e - 2*Sqrt[c
]*Sqrt[c*d^2 - e*(I*b*d - a*e)]]) - (e*ArcTanh[(Sqrt[2*c*d - I*b*e + 2*Sqrt
[c]*Sqrt[c*d^2 - e*(I*b*d - a*e)]] + 2*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d -
I*b*e - 2*Sqrt[c]*Sqrt[c*d^2 - e*(I*b*d - a*e)]]])/(Sqrt[c*d^2 - e*(I*b*d -
 a*e)]*Sqrt[2*c*d - I*b*e - 2*Sqrt[c]*Sqrt[c*d^2 - e*(I*b*d - a*e)]]) - (e*
Log[Sqrt[c*d^2 - e*(I*b*d - a*e)] - Sqrt[2*c*d - I*b*e + 2*Sqrt[c]*Sqrt[c*d
^2 - e*(I*b*d - a*e)]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(2*Sqrt[c*d^2 -
e*(I*b*d - a*e)]*Sqrt[2*c*d - I*b*e + 2*Sqrt[c]*Sqrt[c*d^2 - e*(I*b*d - a*e
)]]) + (e*Log[Sqrt[c*d^2 - e*(I*b*d - a*e)] + Sqrt[2*c*d - I*b*e + 2*Sqrt[c
]*Sqrt[c*d^2 - e*(I*b*d - a*e)]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(2*Sqr
t[c*d^2 - e*(I*b*d - a*e)]*Sqrt[2*c*d - I*b*e + 2*Sqrt[c]*Sqrt[c*d^2 - e*(I
*b*d - a*e)]])
```

Rubi [A] time = 0.791481, antiderivative size = 705, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {707, 1094, 634, 618, 206, 628}

$$-\frac{e\log\left(-\sqrt{d+ex}\sqrt{2\sqrt{c}\sqrt{cd^2-e(-ae+ibd)}-ibe+2cd}+\sqrt{cd^2-e(-ae+ibd)}+\sqrt{c}(d+ex)\right)}{2\sqrt{cd^2-e(-ae+ibd)}\sqrt{2\sqrt{c}\sqrt{cd^2-e(-ae+ibd)}-ibe+2cd}}+\frac{e\log\left(\sqrt{d+ex}\sqrt{2\sqrt{c}\cdots\right.}{2\sqrt{cd^2\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(a + I*b*x + c*x^2)),x]
```

```
[Out] (e*ArcTanh[(Sqrt[2*c*d - I*b*e + 2*Sqrt[c]*Sqrt[c*d^2 - e*(I*b*d - a*e)]] -
 2*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - I*b*e - 2*Sqrt[c]*Sqrt[c*d^2 - e*(I*
b*d - a*e)]]])/(Sqrt[c*d^2 - e*(I*b*d - a*e)]*Sqrt[2*c*d - I*b*e - 2*Sqrt[c
]*Sqrt[c*d^2 - e*(I*b*d - a*e)]]) - (e*ArcTanh[(Sqrt[2*c*d - I*b*e + 2*Sqrt
[c]*Sqrt[c*d^2 - e*(I*b*d - a*e)]] + 2*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d -
I*b*e - 2*Sqrt[c]*Sqrt[c*d^2 - e*(I*b*d - a*e)]]])/(Sqrt[c*d^2 - e*(I*b*d -
 a*e)]*Sqrt[2*c*d - I*b*e - 2*Sqrt[c]*Sqrt[c*d^2 - e*(I*b*d - a*e)]]) - (e*
Log[Sqrt[c*d^2 - e*(I*b*d - a*e)] - Sqrt[2*c*d - I*b*e + 2*Sqrt[c]*Sqrt[c*d
^2 - e*(I*b*d - a*e)]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(2*Sqrt[c*d^2 -
e*(I*b*d - a*e)]*Sqrt[2*c*d - I*b*e + 2*Sqrt[c]*Sqrt[c*d^2 - e*(I*b*d - a*e
)]]) + (e*Log[Sqrt[c*d^2 - e*(I*b*d - a*e)] + Sqrt[2*c*d - I*b*e + 2*Sqrt[c
]*Sqrt[c*d^2 - e*(I*b*d - a*e)]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/(2*Sqr
t[c*d^2 - e*(I*b*d - a*e)]*Sqrt[2*c*d - I*b*e + 2*Sqrt[c]*Sqrt[c*d^2 - e*(I
*b*d - a*e)]])
```

Rule 707

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Sym
bol] :> Dist[2*e, Subst[Int[1/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
```

```
c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\left(a+ibx+cx^2\right)}dx = (2e)\,\text{Subst}\left(\int \frac{1}{cd^2-ibde+ae^2-(2cd-ibe)x^2+cx^4}dx, x, \sqrt{d+ex}\right)$$

$$= \frac{e\,\text{Subst}\left(\int \frac{\frac{\sqrt{2cd-ibe+2\sqrt{c}\sqrt{cd^2-ibde+ae^2}}}{\sqrt{c}}-x}{\frac{\sqrt{cd^2-ibde+ae^2}}{\sqrt{c}}-\frac{\sqrt{2cd-ibe+2\sqrt{c}\sqrt{cd^2-ibde+ae^2}}x}{\sqrt{c}}+x^2}dx, x, \sqrt{d+ex}\right)}{\sqrt{cd^2-e(ibd-ae)}\sqrt{2cd-ibe+2\sqrt{c}\sqrt{cd^2-e(ibd-ae)}}} + \frac{e\,\text{Subst}\left(\int \frac{}{\frac{\sqrt{cd^2-ibde}}{\sqrt{c}}}\right)}{\sqrt{cd^2-e(ibd-}}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{cd^2-ibde+ae^2}}{\sqrt{c}}-\frac{\sqrt{2cd-ibe+2\sqrt{c}\sqrt{cd^2-ibde+ae^2}}x}{\sqrt{c}}+x^2}dx, x, \sqrt{d+ex}\right)}{2\sqrt{c}\sqrt{cd^2-e(ibd-ae)}} + \frac{e\,\text{Subst}\left(\int \frac{}{\frac{\sqrt{cd^2-ibde}}{\sqrt{c}}}\right)}{}$$

$$= -\frac{e\log\left(\sqrt{cd^2-e(ibd-ae)}-\sqrt{2cd-ibe+2\sqrt{c}\sqrt{cd^2-e(ibd-ae)}}\sqrt{d+ex}+\sqrt{c}(d+e\right.}{2\sqrt{cd^2-e(ibd-ae)}\sqrt{2cd-ibe+2\sqrt{c}\sqrt{cd^2-e(ibd-ae)}}}$$

$$= \frac{e\tanh^{-1}\left(\frac{\sqrt{c}\left(\frac{\sqrt{2cd-ibe+2\sqrt{c}\sqrt{cd^2-e(ibd-ae)}}}{\sqrt{c}}-2\sqrt{d+ex}\right)}{\sqrt{2cd-ibe-2\sqrt{c}\sqrt{cd^2-e(ibd-ae)}}}\right)}{\sqrt{cd^2-e(ibd-ae)}\sqrt{2cd-ibe-2\sqrt{c}\sqrt{cd^2-e(ibd-ae)}}} - \frac{e\tanh^{-1}\left(\frac{\sqrt{c}\left(\sqrt{2cd-ibe+}\right.}{\sqrt{2cd-ib}}\right.}{\sqrt{cd^2-e(ibd-ae)}\sqrt{2cd-}}$$

Mathematica [A] time = 0.627007, size = 198, normalized size = 0.28

$$\frac{2\sqrt{2}\sqrt{c}\left(\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{-4ac-b^2}+ib\right)}}\right)}{\sqrt{2cd-e\left(\sqrt{-4ac-b^2}+ib\right)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{-4ac-b^2}-ibe+2cd}}\right)}{\sqrt{2cd+e\left(\sqrt{-4ac-b^2}-ib\right)}}\right)}{\sqrt{-4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(a + I*b*x + c*x^2)),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[c]*(-(ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d -
I*b*e + Sqrt[-b^2 - 4*a*c]*e]]/Sqrt[2*c*d + ((-I)*b + Sqrt[-b^2 - 4*a*c])*e
]) + ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (I*b + Sqrt[-b^2
- 4*a*c])*e]]/Sqrt[2*c*d - (I*b + Sqrt[-b^2 - 4*a*c])*e]))/Sqrt[-b^2 - 4*a*
c]
```

Maple [A] time = 0.38, size = 673, normalized size = 1.

$$-\frac{e}{2}\ln\left((ex+d)\sqrt{c}-\sqrt{ex+d}\sqrt{2\sqrt{-c\left(ibde-ae^2-cd^2\right)}-ibe+2cd}+\sqrt{-ibde+ae^2+cd^2}\right)\frac{1}{\sqrt{2\sqrt{-c\left(ibde-ae^2-cd^2\right.}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(a+I*b*x+c*x^2),x)
```

```
[Out] -1/2*e/(2*(-c*(I*b*d*e-a*e^2-c*d^2))^(1/2)-I*b*e+2*c*d)^(1/2)/(-I*b*d*e+a*e
^2+c*d^2)^(1/2)*ln((e*x+d)*c^(1/2)-(e*x+d)^(1/2)*(2*(-c*(I*b*d*e-a*e^2-c*d^
2))^(1/2)-I*b*e+2*c*d)^(1/2)+(-I*b*d*e+a*e^2+c*d^2)^(1/2))+e/(-I*b*d*e+a*e^
2+c*d^2)^(1/2)/(4*c^(1/2)*(-I*b*d*e+a*e^2+c*d^2)^(1/2)-2*(-c*(I*b*d*e-a*e^2
-c*d^2))^(1/2)+I*b*e-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)-(2*(-c*(I
*b*d*e-a*e^2-c*d^2))^(1/2)-I*b*e+2*c*d)^(1/2))/(4*c^(1/2)*(-I*b*d*e+a*e^2+c
*d^2)^(1/2)-2*(-c*(I*b*d*e-a*e^2-c*d^2))^(1/2)+I*b*e-2*c*d)^(1/2))+1/2*e/(2
*(-c*(I*b*d*e-a*e^2-c*d^2))^(1/2)-I*b*e+2*c*d)^(1/2)/(-I*b*d*e+a*e^2+c*d^2)
^(1/2)*ln((e*x+d)*c^(1/2)+(e*x+d)^(1/2)*(2*(-c*(I*b*d*e-a*e^2-c*d^2))^(1/2)
-I*b*e+2*c*d)^(1/2)+(-I*b*d*e+a*e^2+c*d^2)^(1/2))+e/(-I*b*d*e+a*e^2+c*d^2)^
(1/2)/(4*c^(1/2)*(-I*b*d*e+a*e^2+c*d^2)^(1/2)-2*(-c*(I*b*d*e-a*e^2-c*d^2))^
(1/2)+I*b*e-2*c*d)^(1/2)*arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*(-c*(I*b*d*e-a*
e^2-c*d^2))^(1/2)-I*b*e+2*c*d)^(1/2))/(4*c^(1/2)*(-I*b*d*e+a*e^2+c*d^2)^(1/
2)-2*(-c*(I*b*d*e-a*e^2-c*d^2))^(1/2)+I*b*e-2*c*d)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2 + i\,bx + a\right)\sqrt{ex + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(a+I*b*x+c*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + I*b*x + a)*sqrt(e*x + d)), x)
```

Fricas [B] time = 3.26354, size = 5848, normalized size = 8.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(a+I*b*x+c*x^2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-(4*c*d - 2*I*b*e + (2*(b^2*c + 4*a*c^2)*d^2 - (2*I*b^3 + 8*I*a*b
*c)*d*e + 2*(a*b^2 + 4*a^2*c)*e^2)*sqrt(-e^2/((b^2*c^2 + 4*a*c^3)*d^4 + (-2
*I*b^3*c - 8*I*a*b*c^2)*d^3*e - (b^4 + 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 + (-2
*I*a*b^3 - 8*I*a^2*b*c)*d*e^3 + (a^2*b^2 + 4*a^3*c)*e^4)))/((b^2*c + 4*a*c^
2)*d^2 + (-I*b^3 - 4*I*a*b*c)*d*e + (a*b^2 + 4*a^2*c)*e^2))*log(1/4*(4*sqrt
(e*x + d)*c*e - ((b^2 + 4*a*c)*e^2 + (2*(b^2*c^2 + 4*a*c^3)*d^3 - (3*I*b^3*
c + 12*I*a*b*c^2)*d^2*e - (b^4 + 2*a*b^2*c - 8*a^2*c^2)*d*e^2 - (I*a*b^3 +
4*I*a^2*b*c)*e^3)*sqrt(-e^2/((b^2*c^2 + 4*a*c^3)*d^4 + (-2*I*b^3*c - 8*I*a*
b*c^2)*d^3*e - (b^4 + 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 + (-2*I*a*b^3 - 8*I*a^
2*b*c)*d*e^3 + (a^2*b^2 + 4*a^3*c)*e^4)))*sqrt(-(4*c*d - 2*I*b*e + (2*(b^2*
c + 4*a*c^2)*d^2 - (2*I*b^3 + 8*I*a*b*c)*d*e + 2*(a*b^2 + 4*a^2*c)*e^2)*sqr
t(-e^2/((b^2*c^2 + 4*a*c^3)*d^4 + (-2*I*b^3*c - 8*I*a*b*c^2)*d^3*e - (b^4 +
 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 + (-2*I*a*b^3 - 8*I*a^2*b*c)*d*e^3 + (a^2*b
^2 + 4*a^3*c)*e^4)))/((b^2*c + 4*a*c^2)*d^2 + (-I*b^3 - 4*I*a*b*c)*d*e + (a
*b^2 + 4*a^2*c)*e^2)))/(c*e)) + 1/2*sqrt(-(4*c*d - 2*I*b*e + (2*(b^2*c + 4*
a*c^2)*d^2 - (2*I*b^3 + 8*I*a*b*c)*d*e + 2*(a*b^2 + 4*a^2*c)*e^2)*sqrt(-e^2
/((b^2*c^2 + 4*a*c^3)*d^4 + (-2*I*b^3*c - 8*I*a*b*c^2)*d^3*e - (b^4 + 2*a*b
^2*c - 8*a^2*c^2)*d^2*e^2 + (-2*I*a*b^3 - 8*I*a^2*b*c)*d*e^3 + (a^2*b^2 + 4
```

```
*a^3*c)*e^4)))/((b^2*c + 4*a*c^2)*d^2 + (-I*b^3 - 4*I*a*b*c)*d*e + (a*b^2 +
 4*a^2*c)*e^2))*log(1/4*(4*sqrt(e*x + d)*c*e + ((b^2 + 4*a*c)*e^2 + (2*(b^2
*c^2 + 4*a*c^3)*d^3 + (-3*I*b^3*c - 12*I*a*b*c^2)*d^2*e - (b^4 + 2*a*b^2*c
- 8*a^2*c^2)*d*e^2 + (-I*a*b^3 - 4*I*a^2*b*c)*e^3)*sqrt(-e^2/((b^2*c^2 + 4*
a*c^3)*d^4 + (-2*I*b^3*c - 8*I*a*b*c^2)*d^3*e - (b^4 + 2*a*b^2*c - 8*a^2*c^
2)*d^2*e^2 + (-2*I*a*b^3 - 8*I*a^2*b*c)*d*e^3 + (a^2*b^2 + 4*a^3*c)*e^4)))*
sqrt(-(4*c*d - 2*I*b*e + (2*(b^2*c + 4*a*c^2)*d^2 - (2*I*b^3 + 8*I*a*b*c)*d
*e + 2*(a*b^2 + 4*a^2*c)*e^2)*sqrt(-e^2/((b^2*c^2 + 4*a*c^3)*d^4 + (-2*I*b^
3*c - 8*I*a*b*c^2)*d^3*e - (b^4 + 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 + (-2*I*a*
b^3 - 8*I*a^2*b*c)*d*e^3 + (a^2*b^2 + 4*a^3*c)*e^4)))/((b^2*c + 4*a*c^2)*d^
2 + (-I*b^3 - 4*I*a*b*c)*d*e + (a*b^2 + 4*a^2*c)*e^2)))/(c*e)) + 1/2*sqrt(-
(4*c*d - 2*I*b*e - (2*(b^2*c + 4*a*c^2)*d^2 + (-2*I*b^3 - 8*I*a*b*c)*d*e +
2*(a*b^2 + 4*a^2*c)*e^2)*sqrt(-e^2/((b^2*c^2 + 4*a*c^3)*d^4 + (-2*I*b^3*c -
 8*I*a*b*c^2)*d^3*e - (b^4 + 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 + (-2*I*a*b^3 -
 8*I*a^2*b*c)*d*e^3 + (a^2*b^2 + 4*a^3*c)*e^4)))/((b^2*c + 4*a*c^2)*d^2 + (
-I*b^3 - 4*I*a*b*c)*d*e + (a*b^2 + 4*a^2*c)*e^2))*log(1/4*(4*sqrt(e*x + d)*
c*e + ((b^2 + 4*a*c)*e^2 - (2*(b^2*c^2 + 4*a*c^3)*d^3 - (3*I*b^3*c + 12*I*a
*b*c^2)*d^2*e - (b^4 + 2*a*b^2*c - 8*a^2*c^2)*d*e^2 - (I*a*b^3 + 4*I*a^2*b*
c)*e^3)*sqrt(-e^2/((b^2*c^2 + 4*a*c^3)*d^4 + (-2*I*b^3*c - 8*I*a*b*c^2)*d^3
*e - (b^4 + 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 + (-2*I*a*b^3 - 8*I*a^2*b*c)*d*e
^3 + (a^2*b^2 + 4*a^3*c)*e^4)))*sqrt(-(4*c*d - 2*I*b*e - (2*(b^2*c + 4*a*c^
2)*d^2 + (-2*I*b^3 - 8*I*a*b*c)*d*e + 2*(a*b^2 + 4*a^2*c)*e^2)*sqrt(-e^2/((
b^2*c^2 + 4*a*c^3)*d^4 + (-2*I*b^3*c - 8*I*a*b*c^2)*d^3*e - (b^4 + 2*a*b^2*
c - 8*a^2*c^2)*d^2*e^2 + (-2*I*a*b^3 - 8*I*a^2*b*c)*d*e^3 + (a^2*b^2 + 4*a^
3*c)*e^4)))/((b^2*c + 4*a*c^2)*d^2 + (-I*b^3 - 4*I*a*b*c)*d*e + (a*b^2 + 4*
a^2*c)*e^2)))/(c*e)) - 1/2*sqrt(-(4*c*d - 2*I*b*e - (2*(b^2*c + 4*a*c^2)*d^
2 + (-2*I*b^3 - 8*I*a*b*c)*d*e + 2*(a*b^2 + 4*a^2*c)*e^2)*sqrt(-e^2/((b^2*c
^2 + 4*a*c^3)*d^4 + (-2*I*b^3*c - 8*I*a*b*c^2)*d^3*e - (b^4 + 2*a*b^2*c - 8
*a^2*c^2)*d^2*e^2 + (-2*I*a*b^3 - 8*I*a^2*b*c)*d*e^3 + (a^2*b^2 + 4*a^3*c)*
e^4)))/((b^2*c + 4*a*c^2)*d^2 + (-I*b^3 - 4*I*a*b*c)*d*e + (a*b^2 + 4*a^2*c
)*e^2))*log(1/4*(4*sqrt(e*x + d)*c*e - ((b^2 + 4*a*c)*e^2 - (2*(b^2*c^2 + 4
*a*c^3)*d^3 + (-3*I*b^3*c - 12*I*a*b*c^2)*d^2*e - (b^4 + 2*a*b^2*c - 8*a^2*
c^2)*d*e^2 + (-I*a*b^3 - 4*I*a^2*b*c)*e^3)*sqrt(-e^2/((b^2*c^2 + 4*a*c^3)*d
^4 + (-2*I*b^3*c - 8*I*a*b*c^2)*d^3*e - (b^4 + 2*a*b^2*c - 8*a^2*c^2)*d^2*e
^2 + (-2*I*a*b^3 - 8*I*a^2*b*c)*d*e^3 + (a^2*b^2 + 4*a^3*c)*e^4)))*sqrt(-(4
*c*d - 2*I*b*e - (2*(b^2*c + 4*a*c^2)*d^2 + (-2*I*b^3 - 8*I*a*b*c)*d*e + 2*
(a*b^2 + 4*a^2*c)*e^2)*sqrt(-e^2/((b^2*c^2 + 4*a*c^3)*d^4 + (-2*I*b^3*c - 8
*I*a*b*c^2)*d^3*e - (b^4 + 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 + (-2*I*a*b^3 - 8
*I*a^2*b*c)*d*e^3 + (a^2*b^2 + 4*a^3*c)*e^4)))/((b^2*c + 4*a*c^2)*d^2 + (-I
*b^3 - 4*I*a*b*c)*d*e + (a*b^2 + 4*a^2*c)*e^2)))/(c*e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d + ex}\left(a + ibx + cx^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(a+I*b*x+c*x**2),x)
```

```
[Out] Integral(1/(sqrt(d + e*x)*(a + I*b*x + c*x**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(a+I*b*x+c*x^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.2309 $\int \frac{(1+2x)^{7/2}}{2+3x+5x^2}\,dx$

Optimal. Leaf size=279

$$\frac{4}{25}(2x+1)^{5/2}+\frac{16}{75}(2x+1)^{3/2}-\frac{76}{125}\sqrt{2x+1}-\frac{1}{125}\sqrt{\frac{1}{310}\left(168698+42875\sqrt{35}\right)}\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x\right.$$

```
[Out] (-76*Sqrt[1 + 2*x])/125 + (16*(1 + 2*x)^(3/2))/75 + (4*(1 + 2*x)^(5/2))/25
+ (Sqrt[(2*(-168698 + 42875*Sqrt[35]))/155]*ArcTan[(Sqrt[10*(2 + Sqrt[35])]
 - 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/125 - (Sqrt[(2*(-168698 + 4
2875*Sqrt[35]))/155]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] + 10*Sqrt[1 + 2*x])/Sq
rt[10*(-2 + Sqrt[35])]])/125 - (Sqrt[(168698 + 42875*Sqrt[35])/310]*Log[Sqr
t[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/125 + (Sqrt[(
168698 + 42875*Sqrt[35])/310]*Log[Sqrt[35] + Sqrt[10*(2 + Sqrt[35])]*Sqrt[1
 + 2*x] + 5*(1 + 2*x)])/125
```

Rubi [A] time = 0.60758, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {703, 824, 826, 1169, 634, 618, 204, 628}

$$\frac{4}{25}(2x+1)^{5/2}+\frac{16}{75}(2x+1)^{3/2}-\frac{76}{125}\sqrt{2x+1}-\frac{1}{125}\sqrt{\frac{1}{310}\left(168698+42875\sqrt{35}\right)}\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^(7/2)/(2 + 3*x + 5*x^2),x]
```


```
[Out] (-76*Sqrt[1 + 2*x])/125 + (16*(1 + 2*x)^(3/2))/75 + (4*(1 + 2*x)^(5/2))/25
+ (Sqrt[(2*(-168698 + 42875*Sqrt[35]))/155]*ArcTan[(Sqrt[10*(2 + Sqrt[35])]
 - 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/125 - (Sqrt[(2*(-168698 + 4
2875*Sqrt[35]))/155]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] + 10*Sqrt[1 + 2*x])/Sq
rt[10*(-2 + Sqrt[35])]])/125 - (Sqrt[(168698 + 42875*Sqrt[35])/310]*Log[Sqr
t[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/125 + (Sqrt[(
168698 + 42875*Sqrt[35])/310]*Log[Sqrt[35] + Sqrt[10*(2 + Sqrt[35])]*Sqrt[1
 + 2*x] + 5*(1 + 2*x)])/125
```


Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(1+2x)^{7/2}}{2+3x+5x^2}\,dx = \frac{4}{25}(1+2x)^{5/2} + \frac{1}{5}\int \frac{(1+2x)^{3/2}(-3+8x)}{2+3x+5x^2}\,dx$$
$$= \frac{16}{75}(1+2x)^{3/2} + \frac{4}{25}(1+2x)^{5/2} + \frac{1}{25}\int \frac{(-47-38x)\sqrt{1+2x}}{2+3x+5x^2}\,dx$$
$$= -\frac{76}{125}\sqrt{1+2x} + \frac{16}{75}(1+2x)^{3/2} + \frac{4}{25}(1+2x)^{5/2} + \frac{1}{125}\int \frac{-83-432x}{\sqrt{1+2x}\left(2+3x+5x^2\right)}\,dx$$
$$= -\frac{76}{125}\sqrt{1+2x} + \frac{16}{75}(1+2x)^{3/2} + \frac{4}{25}(1+2x)^{5/2} + \frac{2}{125}\,\text{Subst}\left(\int \frac{266-432x^2}{7-4x^2+5x^4}\,dx, x, \sqrt{1+2x}\right)$$
$$= -\frac{76}{125}\sqrt{1+2x} + \frac{16}{75}(1+2x)^{3/2} + \frac{4}{25}(1+2x)^{5/2} + \frac{\text{Subst}\left(\int \frac{266\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}-\left(266+432\sqrt{\frac{7}{5}}\right)x}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}x+x^2}\,dx, x, \sqrt{1\right.}{125\sqrt{14\left(2+\sqrt{35}\right)}}$$
$$= -\frac{76}{125}\sqrt{1+2x} + \frac{16}{75}(1+2x)^{3/2} + \frac{4}{25}(1+2x)^{5/2} + \frac{1}{625}\left(-216+19\sqrt{35}\right)\text{Subst}\left(\int \frac{1}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}}\left(2\right.}\right.$$
$$= -\frac{76}{125}\sqrt{1+2x} + \frac{16}{75}(1+2x)^{3/2} + \frac{4}{25}(1+2x)^{5/2} - \frac{1}{125}\sqrt{\frac{1}{310}\left(168698+42875\sqrt{35}\right)}\log\left(\sqrt{35}-\right.$$
$$= -\frac{76}{125}\sqrt{1+2x} + \frac{16}{75}(1+2x)^{3/2} + \frac{4}{25}(1+2x)^{5/2} + \frac{1}{125}\sqrt{\frac{2}{155}\left(-168698+42875\sqrt{35}\right)}\tan^{-1}\left(\sqrt{\right.}\right.$$

Mathematica [C] time = 0.315421, size = 134, normalized size = 0.48

$$\frac{2\left(620\sqrt{2x+1}\left(30x^2+50x-11\right)+3\sqrt{10-5i\sqrt{31}}\left(589+178i\sqrt{31}\right)\tanh^{-1}\left(\frac{\sqrt{10x+5}}{\sqrt{2-i\sqrt{31}}}\right)+3\sqrt{10+5i\sqrt{31}}\left(589-178i\sqrt{\right.}\right.}{58125}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^(7/2)/(2 + 3*x + 5*x^2),x]
```

```
[Out] (2*(620*Sqrt[1 + 2*x]*(-11 + 50*x + 30*x^2) + 3*Sqrt[10 - (5*I)*Sqrt[31]]*(
589 + (178*I)*Sqrt[31])*ArcTanh[Sqrt[5 + 10*x]/Sqrt[2 - I*Sqrt[31]]] + 3*Sq
rt[10 + (5*I)*Sqrt[31]]*(589 - (178*I)*Sqrt[31])*ArcTanh[Sqrt[5 + 10*x]/Sqr
t[2 + I*Sqrt[31]]]))/58125
```

Maple [B] time = 0.303, size = 634, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^(7/2)/(5*x^2+3*x+2),x)
```

```
[Out] 4/25*(1+2*x)^(5/2)+16/75*(1+2*x)^(3/2)-76/125*(1+2*x)^(1/2)+233/38750*ln(5^
(1/2)*7^(1/2)+10*x+5+(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2))*5^(
1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+89/3875*ln(5^(1/2)*7^(1/2)+10*x+5+(2*5^(1/
2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2))*7^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1
/2)-233/3875/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((10*(1+2*x)^(1/2)+5^(1/2)
*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*(2*5^(1/2)*7^(
```

```
1/2)+4)-178/3875/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((10*(1+2*x)^(1/2)+5^(
1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*(2
*5^(1/2)*7^(1/2)+4)*7^(1/2)+76/125/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((10
*(1+2*x)^(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2)-20)
^(1/2))*5^(1/2)*7^(1/2)-233/38750*ln(-(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(
1+2*x)^(1/2)+5^(1/2)*7^(1/2)+10*x+5)*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)-89
/3875*ln(-(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2)+5^(1/2)*7^(1/2)
+10*x+5)*7^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)-233/3875/(10*5^(1/2)*7^(1/2)-2
0)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/2))/(10
*5^(1/2)*7^(1/2)-20)^(1/2))*(2*5^(1/2)*7^(1/2)+4)-178/3875/(10*5^(1/2)*7^(1
/2)-20)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/2)
)/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)*7^(1/2)+76/1
25/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/
2)+10*(1+2*x)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*7^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x+1)^{\frac{7}{2}}}{5x^2+3x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(7/2)/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] integrate((2*x + 1)^(7/2)/(5*x^2 + 3*x + 2), x)
```

Fricas [B] time = 2.67357, size = 2519, normalized size = 9.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(7/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 1/1752203762968750*42875^(1/4)*sqrt(155)*(168698*sqrt(35)*sqrt(31) + 150062
5*sqrt(31))*sqrt(-14465853500*sqrt(35) + 128678593750)*log(26582500/34021*4
2875^(1/4)*sqrt(155)*(216*sqrt(35)*sqrt(31) + 665*sqrt(31))*sqrt(2*x + 1)*s
qrt(-14465853500*sqrt(35) + 128678593750) + 353314653125000*x + 35331465312
500*sqrt(35) + 176657326562500) - 1/1752203762968750*42875^(1/4)*sqrt(155)*
(168698*sqrt(35)*sqrt(31) + 1500625*sqrt(31))*sqrt(-14465853500*sqrt(35) +
128678593750)*log(-26582500/34021*42875^(1/4)*sqrt(155)*(216*sqrt(35)*sqrt(
31) + 665*sqrt(31))*sqrt(2*x + 1)*sqrt(-14465853500*sqrt(35) + 128678593750
) + 353314653125000*x + 35331465312500*sqrt(35) + 176657326562500) + 2/8307
03125*42875^(1/4)*sqrt(155)*sqrt(35)*sqrt(-14465853500*sqrt(35) + 128678593
750)*arctan(1/20446821407446226406 25*42875^(3/4)*sqrt(34021)*sqrt(217)*sqrt
(155)*sqrt(42875^(1/4)*sqrt(155)*(216*sqrt(35)*sqrt(31) + 665*sqrt(31))*sqr
t(2*x + 1)*sqrt(-14465853500*sqrt(35) + 128678593750) + 452181616250*x + 45
218161625*sqrt(35) + 226090808125)*(19*sqrt(35) + 216)*sqrt(-14465853500*sq
rt(35) + 128678593750) - 1/1582635656875*42875^(3/4)*sqrt(155)*sqrt(2*x + 1
)*(19*sqrt(35) + 216)*sqrt(-14465853500*sqrt(35) + 128678593750) - 1/31*sqr
t(35)*sqrt(31) - 2/31*sqrt(31)) + 2/830703125*42875^(1/4)*sqrt(155)*sqrt(35
)*sqrt(-14465853500*sqrt(35) + 128678593750)*arctan(1/7156387492606179242 18
750*42875^(3/4)*sqrt(34021)*sqrt(155)*sqrt(-26582500*42875^(1/4)*sqrt(155)*
(216*sqrt(35)*sqrt(31) + 665*sqrt(31))*sqrt(2*x + 1)*sqrt(-14465853500*sqrt
(35) + 128678593750) + 120201178139656250 00*x + 120201178139656250 0*sqrt(35
```

```
) + 6010058906982812500)*(19*sqrt(35) + 216)*sqrt(-14465853500*sqrt(35) + 1
28678593750) - 1/1582635656875*42875^(3/4)*sqrt(155)*sqrt(2*x + 1)*(19*sqrt
(35) + 216)*sqrt(-14465853500*sqrt(35) + 128678593750) + 1/31*sqrt(35)*sqrt
(31) + 2/31*sqrt(31)) + 8/375*(30*x^2 + 50*x - 11)*sqrt(2*x + 1)
```

Sympy [A] time = 61.4254, size = 109, normalized size = 0.39

$$\frac{4(2x+1)^{\frac{5}{2}}}{25}+\frac{16(2x+1)^{\frac{3}{2}}}{75}-\frac{76\sqrt{2x+1}}{125}-\frac{864\operatorname{RootSum}\left(1230080t^4+1984t^2+7,\left(t\mapsto t\log\left(9920t^3+8t+\sqrt{2x+\right.\right.\right.}{125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**(7/2)/(5*x**2+3*x+2),x)
```

```
[Out] 4*(2*x + 1)**(5/2)/25 + 16*(2*x + 1)**(3/2)/75 - 76*sqrt(2*x + 1)/125 - 864
*RootSum(1230080*_t**4 + 1984*_t**2 + 7, Lambda(_t, _t*log(9920*_t**3 + 8*_
t + sqrt(2*x + 1))))/125 + 532*RootSum(1722112*_t**4 + 1984*_t**2 + 5, Lamb
da(_t, _t*log(-27776*_t**3/5 + 108*_t/5 + sqrt(2*x + 1))))/125
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x+1)^{\frac{7}{2}}}{5x^2+3x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(7/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x + 1)^(7/2)/(5*x^2 + 3*x + 2), x)
```

3.2310 $\int \frac{(1+2x)^{5/2}}{2+3x+5x^2}\,dx$

Optimal. Leaf size=266

$$\frac{4}{15}(2x+1)^{3/2}+\frac{16}{25}\sqrt{2x+1}+\frac{1}{25}\sqrt{\frac{1}{310}\left(1225\sqrt{35}-7162\right)}\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}+\sqrt{35}\right)-\frac{1}{25}\sqrt{\frac{1}{310}}$$

```
[Out] (16*Sqrt[1 + 2*x])/25 + (4*(1 + 2*x)^(3/2))/15 + (Sqrt[(2*(7162 + 1225*Sqrt
[35]))/155]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] - 10*Sqrt[1 + 2*x])/Sqrt[10*(-2
 + Sqrt[35])]])/25 - (Sqrt[(2*(7162 + 1225*Sqrt[35]))/155]*ArcTan[(Sqrt[10*
(2 + Sqrt[35])] + 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/25 + (Sqrt[(
-7162 + 1225*Sqrt[35])/310]*Log[Sqrt[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 +
 2*x] + 5*(1 + 2*x)])/25 - (Sqrt[(-7162 + 1225*Sqrt[35])/310]*Log[Sqrt[35]
+ Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/25
```

Rubi [A] time = 0.436201, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {703, 824, 826, 1169, 634, 618, 204, 628}

$$\frac{4}{15}(2x+1)^{3/2}+\frac{16}{25}\sqrt{2x+1}+\frac{1}{25}\sqrt{\frac{1}{310}\left(1225\sqrt{35}-7162\right)}\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}+\sqrt{35}\right)-\frac{1}{25}\sqrt{\frac{1}{310}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^(5/2)/(2 + 3*x + 5*x^2),x]
```

```
[Out] (16*Sqrt[1 + 2*x])/25 + (4*(1 + 2*x)^(3/2))/15 + (Sqrt[(2*(7162 + 1225*Sqrt
[35]))/155]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] - 10*Sqrt[1 + 2*x])/Sqrt[10*(-2
 + Sqrt[35])]])/25 - (Sqrt[(2*(7162 + 1225*Sqrt[35]))/155]*ArcTan[(Sqrt[10*
(2 + Sqrt[35])] + 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/25 + (Sqrt[(
-7162 + 1225*Sqrt[35])/310]*Log[Sqrt[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 +
 2*x] + 5*(1 + 2*x)])/25 - (Sqrt[(-7162 + 1225*Sqrt[35])/310]*Log[Sqrt[35]
+ Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/25
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
```

```
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(1+2x)^{5/2}}{2+3x+5x^2}\,dx = \frac{4}{15}(1+2x)^{3/2} + \frac{1}{5}\int \frac{\sqrt{1+2x}(-3+8x)}{2+3x+5x^2}\,dx$$
$$= \frac{16}{25}\sqrt{1+2x} + \frac{4}{15}(1+2x)^{3/2} + \frac{1}{25}\int \frac{-47-38x}{\sqrt{1+2x}\left(2+3x+5x^2\right)}\,dx$$
$$= \frac{16}{25}\sqrt{1+2x} + \frac{4}{15}(1+2x)^{3/2} + \frac{2}{25}\,\text{Subst}\left(\int \frac{-56-38x^2}{7-4x^2+5x^4}\,dx, x, \sqrt{1+2x}\right)$$
$$= \frac{16}{25}\sqrt{1+2x} + \frac{4}{15}(1+2x)^{3/2} + \frac{\text{Subst}\left(\int \frac{-56\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}-\left(-56+38\sqrt{\frac{7}{5}}\right)x}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}x+x^2}\,dx, x, \sqrt{1+2x}\right)}{25\sqrt{14\left(2+\sqrt{35}\right)}} + \frac{\text{Subst}\left(\int \frac{-56\ldots}{\ldots}\right.}{\ldots}$$
$$= \frac{16}{25}\sqrt{1+2x} + \frac{4}{15}(1+2x)^{3/2} - \frac{1}{125}\sqrt{921+152\sqrt{35}}\,\text{Subst}\left(\int \frac{1}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}}\left(2+\sqrt{35}\right)x+x^2}\,dx, x, \sqrt{1\ldots}\right.$$
$$= \frac{16}{25}\sqrt{1+2x} + \frac{4}{15}(1+2x)^{3/2} + \frac{1}{25}\sqrt{\frac{1}{310}\left(-7162+1225\sqrt{35}\right)}\log\left(\sqrt{35}-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1+2x}+\ldots\right.$$
$$= \frac{16}{25}\sqrt{1+2x} + \frac{4}{15}(1+2x)^{3/2} + \frac{1}{25}\sqrt{\frac{2}{155}\left(7162+1225\sqrt{35}\right)}\tan^{-1}\left(\sqrt{\frac{5}{2\left(-2+\sqrt{35}\right)}}\left(\sqrt{\frac{2}{5}}\left(2+\sqrt{35}\ldots\right.\right.\right.$$

Mathematica [C] time = 0.276248, size = 133, normalized size = 0.5

$$\frac{2\left(310\sqrt{2x+1}(10x+17)+3i\sqrt{10-5i\sqrt{31}}\left(27\sqrt{31}+124i\right)\tanh^{-1}\left(\frac{\sqrt{10x+5}}{\sqrt{2-i\sqrt{31}}}\right)-3i\sqrt{10+5i\sqrt{31}}\left(27\sqrt{31}-124i\right)\tanh^{-1}\left(\ldots\right.\right.}{11625}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^(5/2)/(2 + 3*x + 5*x^2),x]
```

```
[Out] (2*(310*Sqrt[1 + 2*x]*(17 + 10*x) + (3*I)*Sqrt[10 - (5*I)*Sqrt[31]]*(124*I
+ 27*Sqrt[31])*ArcTanh[Sqrt[5 + 10*x]/Sqrt[2 - I*Sqrt[31]]] - (3*I)*Sqrt[10
 + (5*I)*Sqrt[31]]*(-124*I + 27*Sqrt[31])*ArcTanh[Sqrt[5 + 10*x]/Sqrt[2 + I
*Sqrt[31]]]))/11625
```

Maple [B] time = 0.078, size = 625, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^(5/2)/(5*x^2+3*x+2),x)
```

```
[Out] 4/15*(1+2*x)^(3/2)+16/25*(1+2*x)^(1/2)-89/3875*ln(5^(1/2)*7^(1/2)+10*x+5+(2
*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)
+4)^(1/2)+27/1550*ln(5^(1/2)*7^(1/2)+10*x+5+(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(
1/2)*(1+2*x)^(1/2))*7^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+178/775/(10*5^(1/2)
*7^(1/2)-20)^(1/2)*arctan((10*(1+2*x)^(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(
1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*(2*5^(1/2)*7^(1/2)+4)-27/775/(10*5^(1/
2)*7^(1/2)-20)^(1/2)*arctan((10*(1+2*x)^(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)
^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)*7^(1/2
```

```
)-16/25/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((10*(1+2*x)^(1/2)+5^(1/2)*(2*5
^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*7^(1/2)+89/
3875*ln(-(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2)+5^(1/2)*7^(1/2)+
10*x+5)*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)-27/1550*ln(-(2*5^(1/2)*7^(1/2)+
4)^(1/2)*5^(1/2)*(1+2*x)^(1/2)+5^(1/2)*7^(1/2)+10*x+5)*7^(1/2)*(2*5^(1/2)*7
^(1/2)+4)^(1/2)+178/775/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((-5^(1/2)*(2*5
^(1/2)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*(2
*5^(1/2)*7^(1/2)+4)-27/775/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((-5^(1/2)*(
2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))
*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)*7^(1/2)-16/25/(10*5^(1/2)*7^(1/2)-20)^(1/2)*
arctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/2))/(10*5^(1/2)*
7^(1/2)-20)^(1/2))*5^(1/2)*7^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x+1)^{\frac{5}{2}}}{5x^2+3x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(5/2)/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] integrate((2*x + 1)^(5/2)/(5*x^2 + 3*x + 2), x)
```

Fricas [B] time = 2.71444, size = 2144, normalized size = 8.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(5/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] -1/1673341250*sqrt(155)*35^(1/4)*sqrt(2)*(7162*sqrt(35)*sqrt(31) - 42875*sq
rt(31))*sqrt(7162*sqrt(35) + 42875)*log(26582500/199*sqrt(155)*35^(1/4)*sqr
t(2)*(19*sqrt(35)*sqrt(31) - 140*sqrt(31))*sqrt(2*x + 1)*sqrt(7162*sqrt(35)
 + 42875) + 288420125000*x + 28842012500*sqrt(35) + 144210062500) + 1/16733
41250*sqrt(155)*35^(1/4)*sqrt(2)*(7162*sqrt(35)*sqrt(31) - 42875*sqrt(31))*
sqrt(7162*sqrt(35) + 42875)*log(-26582500/199*sqrt(155)*35^(1/4)*sqrt(2)*(1
9*sqrt(35)*sqrt(31) - 140*sqrt(31))*sqrt(2*x + 1)*sqrt(7162*sqrt(35) + 4287
5) + 288420125000*x + 28842012500*sqrt(35) + 144210062500) + 2/135625*sqrt(
155)*35^(3/4)*sqrt(2)*sqrt(7162*sqrt(35) + 42875)*arctan(1/46619287225*sqrt
(217)*sqrt(199)*sqrt(155)*35^(3/4)*sqrt(2)*sqrt(sqrt(155)*35^(1/4)*sqrt(2)*
(19*sqrt(35)*sqrt(31) - 140*sqrt(31))*sqrt(2*x + 1)*sqrt(7162*sqrt(35) + 42
875) + 2159150*x + 215915*sqrt(35) + 1079575)*sqrt(7162*sqrt(35) + 42875)*(
4*sqrt(35) - 19) - 1/215915*sqrt(155)*35^(3/4)*sqrt(2)*sqrt(2*x + 1)*sqrt(7
162*sqrt(35) + 42875)*(4*sqrt(35) - 19) + 1/31*sqrt(35)*sqrt(31) + 2/31*sqr
t(31)) + 2/135625*sqrt(155)*35^(3/4)*sqrt(2)*sqrt(7162*sqrt(35) + 42875)*ar
ctan(1/16316750528750*sqrt(199)*sqrt(155)*35^(3/4)*sqrt(2)*sqrt(-26582500*s
qrt(155)*35^(1/4)*sqrt(2)*(19*sqrt(35)*sqrt(31) - 140*sqrt(31))*sqrt(2*x +
1)*sqrt(7162*sqrt(35) + 42875) + 57395604875000*x + 5739560487500*sqrt(35)
+ 28697802437500)*sqrt(7162*sqrt(35) + 42875)*(4*sqrt(35) - 19) - 1/215915*
sqrt(155)*35^(3/4)*sqrt(2)*sqrt(2*x + 1)*sqrt(7162*sqrt(35) + 42875)*(4*sqr
t(35) - 19) - 1/31*sqrt(35)*sqrt(31) - 2/31*sqrt(31)) + 4/75*(10*x + 17)*sq
rt(2*x + 1)
```

Sympy [A] time = 33.9032, size = 97, normalized size = 0.36

$$\frac{4(2x+1)^{\frac{3}{2}}}{15}+\frac{16\sqrt{2x+1}}{25}-\frac{76\,\mathrm{RootSum}\left(1230080t^4+1984t^2+7,\left(t\mapsto t\log\left(9920t^3+8t+\sqrt{2x+1}\right)\right)\right)}{25}-\frac{112\,\mathrm{RootSu}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**(5/2)/(5*x**2+3*x+2),x)
```

```
[Out] 4*(2*x + 1)**(3/2)/15 + 16*sqrt(2*x + 1)/25 - 76*RootSum(1230080*_t**4 + 19
84*_t**2 + 7, Lambda(_t, _t*log(9920*_t**3 + 8*_t + sqrt(2*x + 1))))/25 - 1
12*RootSum(1722112*_t**4 + 1984*_t**2 + 5, Lambda(_t, _t*log(-27776*_t**3/5
 + 108*_t/5 + sqrt(2*x + 1))))/25
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x+1)^{\frac{5}{2}}}{5x^2+3x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(5/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x + 1)^(5/2)/(5*x^2 + 3*x + 2), x)
```

3.2311 $\int \frac{(1+2x)^{3/2}}{2+3x+5x^2} dx$

Optimal. Leaf size=253

$$\frac{4}{5}\sqrt{2x+1}+\frac{1}{5}\sqrt{\frac{1}{310}\left(178+35\sqrt{35}\right)}\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}+\sqrt{35}\right)-\frac{1}{5}\sqrt{\frac{1}{310}\left(178+35\sqrt{35}\right)}\log\left(\right.$$

```
[Out] (4*Sqrt[1 + 2*x])/5 + (Sqrt[(2*(-178 + 35*Sqrt[35]))/155]*ArcTan[(Sqrt[10*(
2 + Sqrt[35])] - 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/5 - (Sqrt[(2*
(-178 + 35*Sqrt[35]))/155]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] + 10*Sqrt[1 + 2*
x])/Sqrt[10*(-2 + Sqrt[35])]])/5 + (Sqrt[(178 + 35*Sqrt[35])/310]*Log[Sqrt[
35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/5 - (Sqrt[(178
+ 35*Sqrt[35])/310]*Log[Sqrt[35] + Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] +
5*(1 + 2*x)])/5
```

Rubi [A] time = 0.357923, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {703, 826, 1169, 634, 618, 204, 628}

$$\frac{4}{5}\sqrt{2x+1}+\frac{1}{5}\sqrt{\frac{1}{310}\left(178+35\sqrt{35}\right)}\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}+\sqrt{35}\right)-\frac{1}{5}\sqrt{\frac{1}{310}\left(178+35\sqrt{35}\right)}\log\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^(3/2)/(2 + 3*x + 5*x^2),x]
```

```
[Out] (4*Sqrt[1 + 2*x])/5 + (Sqrt[(2*(-178 + 35*Sqrt[35]))/155]*ArcTan[(Sqrt[10*(
2 + Sqrt[35])] - 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/5 - (Sqrt[(2*
(-178 + 35*Sqrt[35]))/155]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] + 10*Sqrt[1 + 2*
x])/Sqrt[10*(-2 + Sqrt[35])]])/5 + (Sqrt[(178 + 35*Sqrt[35])/310]*Log[Sqrt[
35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/5 - (Sqrt[(178
+ 35*Sqrt[35])/310]*Log[Sqrt[35] + Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] +
5*(1 + 2*x)])/5
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
```

```
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+2x)^{3/2}}{2+3x+5x^2}\,dx &= \frac{4}{5}\sqrt{1+2x}+\frac{1}{5}\int \frac{-3+8x}{\sqrt{1+2x}\left(2+3x+5x^2\right)}\,dx\\
&= \frac{4}{5}\sqrt{1+2x}+\frac{2}{5}\operatorname{Subst}\left(\int \frac{-14+8x^2}{7-4x^2+5x^4}\,dx,x,\sqrt{1+2x}\right)\\
&= \frac{4}{5}\sqrt{1+2x}+\frac{\operatorname{Subst}\left(\int \frac{-14\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}-\left(-14-8\sqrt{\frac{7}{5}}\right)x}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}x+x^2}\,dx,x,\sqrt{1+2x}\right)}{5\sqrt{14\left(2+\sqrt{35}\right)}}+\frac{\operatorname{Subst}\left(\int \frac{-14\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}+\left(-14-8\sqrt{\frac{7}{5}}\right.}{\sqrt{\frac{7}{5}}+\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}x+x^2}\right.}{5\sqrt{14\left(2+\sqrt{35}\right.}}\\
&= \frac{4}{5}\sqrt{1+2x}+\frac{1}{25}\left(4-\sqrt{35}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}x+x^2}\,dx,x,\sqrt{1+2x}\right)+\frac{1}{25}\left(4-\sqrt{35}\right)\mathrm{S}\\
&= \frac{4}{5}\sqrt{1+2x}+\frac{1}{5}\sqrt{\frac{1}{310}\left(178+35\sqrt{35}\right)}\log\left(\sqrt{35}-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1+2x}+5(1+2x)\right)-\frac{1}{5}\sqrt{\frac{1}{310}\left(\right.}\\
&= \frac{4}{5}\sqrt{1+2x}+\frac{1}{5}\sqrt{\frac{2}{155}\left(-178+35\sqrt{35}\right)}\tan^{-1}\left(\sqrt{\frac{5}{2\left(-2+\sqrt{35}\right)}}\left(\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}-2\sqrt{1+2x}\right)\right)-\frac{1}{5}
\end{aligned}
$$

Mathematica [C] time = 0.265968, size = 128, normalized size = 0.51

$$
\frac{2}{775}\left(310\sqrt{2x+1}-i\sqrt{10-5i\sqrt{31}}\left(2\sqrt{31}-31i\right)\tanh^{-1}\left(\frac{\sqrt{10x+5}}{\sqrt{2-i\sqrt{31}}}\right)+i\sqrt{10+5i\sqrt{31}}\left(2\sqrt{31}+31i\right)\tanh^{-1}\left(\frac{\sqrt{10x+5}}{\sqrt{2+i\sqrt{31}}}\right.\right.
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^(3/2)/(2 + 3*x + 5*x^2),x]
```

```
[Out] (2*(310*Sqrt[1 + 2*x] - I*Sqrt[10 - (5*I)*Sqrt[31]]*(-31*I + 2*Sqrt[31])*Ar
cTanh[Sqrt[5 + 10*x]/Sqrt[2 - I*Sqrt[31]]] + I*Sqrt[10 + (5*I)*Sqrt[31]]*(3
1*I + 2*Sqrt[31])*ArcTanh[Sqrt[5 + 10*x]/Sqrt[2 + I*Sqrt[31]]]))/775
```

Maple [B] time = 0.073, size = 616, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^(3/2)/(5*x^2+3*x+2),x)
```

```
[Out] 4/5*(1+2*x)^(1/2)-27/1550*ln(5^(1/2)*7^(1/2)+10*x+5+(2*5^(1/2)*7^(1/2)+4)^(
1/2)*5^(1/2)*(1+2*x)^(1/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)-1/155*ln(5^
(1/2)*7^(1/2)+10*x+5+(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2))*7^(
1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+27/155/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arcta
n((10*(1+2*x)^(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2
)-20)^(1/2))*(2*5^(1/2)*7^(1/2)+4)+2/155/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arct
an((10*(1+2*x)^(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/
2)-20)^(1/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)*7^(1/2)-4/5/(10*5^(1/2)*7^(1/2)
-20)^(1/2)*arctan((10*(1+2*x)^(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(1
0*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*7^(1/2)+27/1550*ln(-(2*5^(1/2)*7^(1/2)
+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2)+5^(1/2)*7^(1/2)+10*x+5)*5^(1/2)*(2*5^(1/2)*
7^(1/2)+4)^(1/2)+1/155*ln(-(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2
)+5^(1/2)*7^(1/2)+10*x+5)*7^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+27/155/(10*5^
(1/2)*7^(1/2)-20)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+
2*x)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*(2*5^(1/2)*7^(1/2)+4)+2/155/(10*
5^(1/2)*7^(1/2)-20)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(
1+2*x)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)*
7^(1/2)-4/5/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/
2)+4)^(1/2)+10*(1+2*x)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*7^(1/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x+1)^{\frac{3}{2}}}{5x^2+3x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(3/2)/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] integrate((2*x + 1)^(3/2)/(5*x^2 + 3*x + 2), x)
```

Fricas [B] time = 2.55124, size = 2006, normalized size = 7.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(3/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] -1/1118363750*42875^(1/4)*sqrt(155)*(178*sqrt(35)*sqrt(31) + 1225*sqrt(31))
*sqrt(-12460*sqrt(35) + 85750)*log(620/19*42875^(1/4)*sqrt(155)*(4*sqrt(35)
*sqrt(31) + 35*sqrt(31))*sqrt(2*x + 1)*sqrt(-12460*sqrt(35) + 85750) + 2354
45000*x + 23544500*sqrt(35) + 117722500) + 1/1118363750*42875^(1/4)*sqrt(15
5)*(178*sqrt(35)*sqrt(31) + 1225*sqrt(31))*sqrt(-12460*sqrt(35) + 85750)*lo
g(-620/19*42875^(1/4)*sqrt(155)*(4*sqrt(35)*sqrt(31) + 35*sqrt(31))*sqrt(2*
x + 1)*sqrt(-12460*sqrt(35) + 85750) + 235445000*x + 23544500*sqrt(35) + 11
7722500) + 2/949375*42875^(1/4)*sqrt(155)*sqrt(35)*sqrt(-12460*sqrt(35) + 8
5750)*arctan(1/5205983256250*42875^(3/4)*sqrt(155)*sqrt(-620*42875^(1/4)*sq
rt(155)*(4*sqrt(35)*sqrt(31) + 35*sqrt(31))*sqrt(2*x + 1)*sqrt(-12460*sqrt(
35) + 85750) + 4473455000*x + 447345500*sqrt(35) + 2236727500)*(sqrt(35)*sq
rt(19) + 4*sqrt(19))*sqrt(-12460*sqrt(35) + 85750) - 1/25253375*42875^(3/4)
*sqrt(155)*sqrt(2*x + 1)*(sqrt(35) + 4)*sqrt(-12460*sqrt(35) + 85750) + 1/3
1*sqrt(35)*sqrt(31) + 2/31*sqrt(31)) + 2/949375*42875^(1/4)*sqrt(155)*sqrt(
35)*sqrt(-12460*sqrt(35) + 85750)*arctan(-1/25253375*42875^(3/4)*sqrt(155)*
sqrt(2*x + 1)*(sqrt(35) + 4)*sqrt(-12460*sqrt(35) + 85750) + 1/16793494375*
42875^(3/4)*sqrt(42875^(1/4)*sqrt(155)*(4*sqrt(35)*sqrt(31) + 35*sqrt(31))*
sqrt(2*x + 1)*sqrt(-12460*sqrt(35) + 85750) + 7215250*x + 721525*sqrt(35) +
 3607625)*(sqrt(35)*sqrt(19) + 4*sqrt(19))*sqrt(-12460*sqrt(35) + 85750) -
1/31*sqrt(35)*sqrt(31) - 2/31*sqrt(31)) + 4/5*sqrt(2*x + 1)
```

Sympy [A] time = 17.6129, size = 85, normalized size = 0.34

$$\frac{4\sqrt{2x+1}}{5} + \frac{16\operatorname{RootSum}\left(1230080t^4 + 1984t^2 + 7, \left(t \mapsto t\log\left(9920t^3 + 8t + \sqrt{2x+1}\right)\right)\right)}{5} - \frac{28\operatorname{RootSum}\left(1722112t^4 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**(3/2)/(5*x**2+3*x+2),x)
```

```
[Out] 4*sqrt(2*x + 1)/5 + 16*RootSum(1230080*_t**4 + 1984*_t**2 + 7, Lambda(_t, _
t*log(9920*_t**3 + 8*_t + sqrt(2*x + 1))))/5 - 28*RootSum(1722112*_t**4 + 1
984*_t**2 + 5, Lambda(_t, _t*log(-27776*_t**3/5 + 108*_t/5 + sqrt(2*x + 1))
))/5
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x+1)^{\frac{3}{2}}}{5x^2+3x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(3/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x + 1)^(3/2)/(5*x^2 + 3*x + 2), x)
```

3.2312 $\int \frac{\sqrt{1+2x}}{2+3x+5x^2} \, dx$

Optimal. Leaf size=222

$$\frac{\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}+\sqrt{35}\right)}{\sqrt{10\left(2+\sqrt{35}\right)}}-\frac{\log\left(5(2x+1)+\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}+\sqrt{35}\right)}{\sqrt{10\left(2+\sqrt{35}\right)}}-\sqrt{\frac{2}{5\left(\sqrt{35}-2\right)}}$$

```
[Out] -(Sqrt[2/(5*(-2 + Sqrt[35]))]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] - 10*Sqrt[1 +
 2*x])/Sqrt[10*(-2 + Sqrt[35])]]) + Sqrt[2/(5*(-2 + Sqrt[35]))]*ArcTan[(Sqr
t[10*(2 + Sqrt[35])] + 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]] + Log[Sq
rt[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)]/Sqrt[10*(2 +
Sqrt[35])] - Log[Sqrt[35] + Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 +
2*x)]/Sqrt[10*(2 + Sqrt[35])]
```

Rubi [A] time = 0.247047, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {699, 1127, 1161, 618, 204, 1164, 628}

$$\frac{\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}+\sqrt{35}\right)}{\sqrt{10\left(2+\sqrt{35}\right)}}-\frac{\log\left(5(2x+1)+\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}+\sqrt{35}\right)}{\sqrt{10\left(2+\sqrt{35}\right)}}-\sqrt{\frac{2}{5\left(\sqrt{35}-2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 2*x]/(2 + 3*x + 5*x^2),x]
```

```
[Out] -(Sqrt[2/(5*(-2 + Sqrt[35]))]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] - 10*Sqrt[1 +
 2*x])/Sqrt[10*(-2 + Sqrt[35])]]) + Sqrt[2/(5*(-2 + Sqrt[35]))]*ArcTan[(Sqr
t[10*(2 + Sqrt[35])] + 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]] + Log[Sq
rt[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)]/Sqrt[10*(2 +
Sqrt[35])] - Log[Sqrt[35] + Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 +
2*x)]/Sqrt[10*(2 + Sqrt[35])]
```

Rule 699

```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Dist[2*e, Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*
x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
```

```
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+2x}}{2+3x+5x^2}\,dx = 4\,\text{Subst}\left(\int \frac{x^2}{7-4x^2+5x^4}\,dx, x, \sqrt{1+2x}\right)$$
$$= -\left(2\,\text{Subst}\left(\int \frac{\sqrt{\frac{7}{5}}-x^2}{7-4x^2+5x^4}\,dx, x, \sqrt{1+2x}\right)\right) + 2\,\text{Subst}\left(\int \frac{\sqrt{\frac{7}{5}}+x^2}{7-4x^2+5x^4}\,dx, x, \sqrt{1+2x}\right)$$
$$= \frac{1}{5}\,\text{Subst}\left(\int \frac{1}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}x+x^2}\,dx, x, \sqrt{1+2x}\right) + \frac{1}{5}\,\text{Subst}\left(\int \frac{1}{\sqrt{\frac{7}{5}}+\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}x+x^2}\right.$$
$$= \frac{\log\left(\sqrt{35}-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1+2x}+5(1+2x)\right)}{\sqrt{10\left(2+\sqrt{35}\right)}} - \frac{\log\left(\sqrt{35}+\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1+2x}+5(1+2x\right.}{\sqrt{10\left(2+\sqrt{35}\right)}}$$
$$= -\sqrt{\frac{2}{5\left(-2+\sqrt{35}\right)}}\tan^{-1}\left(\sqrt{\frac{5}{2\left(-2+\sqrt{35}\right)}}\left(\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}-2\sqrt{1+2x}\right)\right) + \sqrt{\frac{2}{5\left(-2+\sqrt{35}\right)}}\tan^{-1}$$

Mathematica [C] time = 0.202769, size = 112, normalized size = 0.5

$$\frac{2\left(\sqrt{-2+i\sqrt{31}}\left(\sqrt{31}-2i\right)\tan^{-1}\left(\frac{\sqrt{10x+5}}{\sqrt{-2-i\sqrt{31}}}\right)+\sqrt{-2-i\sqrt{31}}\left(\sqrt{31}+2i\right)\tan^{-1}\left(\frac{\sqrt{10x+5}}{\sqrt{-2+i\sqrt{31}}}\right)\right)}{5\sqrt{217}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 2*x]/(2 + 3*x + 5*x^2),x]
```

```
[Out] (2*(Sqrt[-2 + I*Sqrt[31]]*(-2*I + Sqrt[31])*ArcTan[Sqrt[5 + 10*x]/Sqrt[-2 -
 I*Sqrt[31]]] + Sqrt[-2 - I*Sqrt[31]]*(2*I + Sqrt[31])*ArcTan[Sqrt[5 + 10*x
]/Sqrt[-2 + I*Sqrt[31]]]))/(5*Sqrt[217])
```

Maple [B] time = 0.087, size = 486, normalized size = 2.2

$$\frac{\sqrt{5}\sqrt{2\sqrt{5}\sqrt{7}+4}}{155}\ln\left(\sqrt{5}\sqrt{7}+10x+5+\sqrt{2\sqrt{5}\sqrt{7}+4}\sqrt{5}\sqrt{1+2x}\right)-\frac{4\sqrt{5}\sqrt{7}+8}{31\sqrt{10\sqrt{5}\sqrt{7}-20}}\arctan\left(\frac{1}{\sqrt{10\sqrt{5}\sqrt{7}-20}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^(1/2)/(5*x^2+3*x+2),x)
```

```
[Out] 1/155*ln(5^(1/2)*7^(1/2)+10*x+5+(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)
^(1/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)-2/31/(10*5^(1/2)*7^(1/2)-20)^(1
/2)*arctan((10*(1+2*x)^(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/
2)*7^(1/2)-20)^(1/2))*(2*5^(1/2)*7^(1/2)+4)-1/62*ln(5^(1/2)*7^(1/2)+10*x+5+
(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2))*7^(1/2)*(2*5^(1/2)*7^(1/
2)+4)^(1/2)+1/31/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((10*(1+2*x)^(1/2)+5^(
1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*(2
*5^(1/2)*7^(1/2)+4)*7^(1/2)-1/155*ln(-(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(
1+2*x)^(1/2)+5^(1/2)*7^(1/2)+10*x+5)*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)-2/
31/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/
2)+10*(1+2*x)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*(2*5^(1/2)*7^(1/2)+4)+1
/62*ln(-(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2)+5^(1/2)*7^(1/2)+1
0*x+5)*7^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+1/31/(10*5^(1/2)*7^(1/2)-20)^(1/
2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/2))/(10*5^(1/
2)*7^(1/2)-20)^(1/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)*7^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x+1}}{5x^2+3x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(1/2)/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x + 1)/(5*x^2 + 3*x + 2), x)
```

Fricas [B] time = 2.65576, size = 1442, normalized size = 6.5

$$\frac{1}{336350}\sqrt{155}35^{\frac{1}{4}}\left(2\sqrt{35}\sqrt{31}-35\sqrt{31}\right)\sqrt{4\sqrt{35}+70}\log\left(\frac{124}{7}\sqrt{155}35^{\frac{3}{4}}\sqrt{31}\sqrt{2x+1}\sqrt{4\sqrt{35}+70}+192200x+1922\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(1/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 1/336350*sqrt(155)*35^(1/4)*(2*sqrt(35)*sqrt(31) - 35*sqrt(31))*sqrt(4*sqrt
(35) + 70)*log(124/7*sqrt(155)*35^(3/4)*sqrt(31)*sqrt(2*x + 1)*sqrt(4*sqrt(
35) + 70) + 192200*x + 19220*sqrt(35) + 96100) - 1/336350*sqrt(155)*35^(1/4
)*(2*sqrt(35)*sqrt(31) - 35*sqrt(31))*sqrt(4*sqrt(35) + 70)*log(-124/7*sqrt
(155)*35^(3/4)*sqrt(31)*sqrt(2*x + 1)*sqrt(4*sqrt(35) + 70) + 192200*x + 19
220*sqrt(35) + 96100) - 2/5425*sqrt(155)*35^(3/4)*sqrt(4*sqrt(35) + 70)*arc
tan(1/1177225*sqrt(155)*35^(3/4)*sqrt(31)*sqrt(7)*sqrt(sqrt(155)*35^(3/4)*s
qrt(31)*sqrt(2*x + 1)*sqrt(4*sqrt(35) + 70) + 10850*x + 1085*sqrt(35) + 542
5)*sqrt(4*sqrt(35) + 70) - 1/1085*sqrt(155)*35^(3/4)*sqrt(2*x + 1)*sqrt(4*s
qrt(35) + 70) - 1/31*sqrt(35)*sqrt(31) - 2/31*sqrt(31)) - 2/5425*sqrt(155)*
35^(3/4)*sqrt(4*sqrt(35) + 70)*arctan(1/2354450*sqrt(155)*35^(3/4)*sqrt(7)*
sqrt(-124*sqrt(155)*35^(3/4)*sqrt(31)*sqrt(2*x + 1)*sqrt(4*sqrt(35) + 70) +
 1345400*x + 134540*sqrt(35) + 672700)*sqrt(4*sqrt(35) + 70) - 1/1085*sqrt(
155)*35^(3/4)*sqrt(2*x + 1)*sqrt(4*sqrt(35) + 70) + 1/31*sqrt(35)*sqrt(31)
+ 2/31*sqrt(31))
```

Sympy [A] time = 2.5134, size = 32, normalized size = 0.14

$$4\,\mathrm{RootSum}\left(1230080t^4+1984t^2+7,\left(t\mapsto t\log\left(9920t^3+8t+\sqrt{2x+1}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**(1/2)/(5*x**2+3*x+2),x)
```

```
[Out] 4*RootSum(1230080*_t**4 + 1984*_t**2 + 7, Lambda(_t, _t*log(9920*_t**3 + 8*
_t + sqrt(2*x + 1))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{2\,x+1}}{5\,x^2+3\,x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(1/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(2*x + 1)/(5*x^2 + 3*x + 2), x)
```

3.2313 $\int \frac{1}{\sqrt{1+2x}(2+3x+5x^2)}\,dx$

Optimal. Leaf size=218

$$-\frac{\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}+\sqrt{35}\right)}{\sqrt{14\left(2+\sqrt{35}\right)}}+\frac{\log\left(5(2x+1)+\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}+\sqrt{35}\right)}{\sqrt{14\left(2+\sqrt{35}\right)}}-\sqrt{\frac{2}{217}}\left(2+\right.$$

```
[Out] -(Sqrt[(2*(2 + Sqrt[35]))/217]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] - 10*Sqrt[1
+ 2*x])/Sqrt[10*(-2 + Sqrt[35])]]) + Sqrt[(2*(2 + Sqrt[35]))/217]*ArcTan[(S
qrt[10*(2 + Sqrt[35])] + 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]] - Log[
Sqrt[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)]/Sqrt[14*(2
+ Sqrt[35])] + Log[Sqrt[35] + Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1
+ 2*x)]/Sqrt[14*(2 + Sqrt[35])]
```

Rubi [A] time = 0.271973, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {707, 1094, 634, 618, 204, 628}

$$-\frac{\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}+\sqrt{35}\right)}{\sqrt{14\left(2+\sqrt{35}\right)}}+\frac{\log\left(5(2x+1)+\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}+\sqrt{35}\right)}{\sqrt{14\left(2+\sqrt{35}\right)}}-\sqrt{\frac{2}{217}}\left(2+\right.$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + 2*x]*(2 + 3*x + 5*x^2)),x]
```

```
[Out] -(Sqrt[(2*(2 + Sqrt[35]))/217]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] - 10*Sqrt[1
+ 2*x])/Sqrt[10*(-2 + Sqrt[35])]]) + Sqrt[(2*(2 + Sqrt[35]))/217]*ArcTan[(S
qrt[10*(2 + Sqrt[35])] + 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]] - Log[
Sqrt[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)]/Sqrt[14*(2
+ Sqrt[35])] + Log[Sqrt[35] + Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1
+ 2*x)]/Sqrt[14*(2 + Sqrt[35])]
```

Rule 707

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Sym
bol] :> Dist[2*e, Subst[Int[1/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+2x}\left(2+3x+5x^2\right)}\,dx = 4\,\text{Subst}\left(\int \frac{1}{7-4x^2+5x^4}\,dx, x, \sqrt{1+2x}\right)$$

$$= \sqrt{\frac{2}{7\left(2+\sqrt{35}\right)}}\,\text{Subst}\left(\int \frac{\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}-x}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}x+x^2}\,dx, x, \sqrt{1+2x}\right) + \sqrt{\frac{2}{7\left(2+\sqrt{35}\right)}}\,\text{S}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}x+x^2}\,dx, x, \sqrt{1+2x}\right)}{\sqrt{35}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\frac{7}{5}}+\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}x+x^2}\,dx, x, \sqrt{1+2}\right.}{\sqrt{35}}$$

$$= -\frac{\log\left(\sqrt{35}-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1+2x}+5(1+2x)\right)}{\sqrt{14\left(2+\sqrt{35}\right)}} + \frac{\log\left(\sqrt{35}+\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1+2\right.}{\sqrt{14\left(2+\sqrt{35}\right)}}$$

$$= -\sqrt{\frac{2}{7\left(-2+\sqrt{35}\right)}}\tan^{-1}\left(\sqrt{\frac{5}{2\left(-2+\sqrt{35}\right)}}\left(\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}-2\sqrt{1+2x}\right)\right) + \sqrt{\frac{2}{7\left(-2+\sqrt{\right.}}}$$

Mathematica [C] time = 0.246324, size = 112, normalized size = 0.51

$$\frac{2\left(\sqrt{2-i\sqrt{31}}\left(\sqrt{31}-2i\right)\tanh^{-1}\left(\frac{\sqrt{10x+5}}{\sqrt{2-i\sqrt{31}}}\right)+\sqrt{2+i\sqrt{31}}\left(\sqrt{31}+2i\right)\tanh^{-1}\left(\frac{\sqrt{10x+5}}{\sqrt{2+i\sqrt{31}}}\right)\right)}{7\sqrt{155}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + 2*x]*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (2*(Sqrt[2 - I*Sqrt[31]]*(-2*I + Sqrt[31])*ArcTanh[Sqrt[5 + 10*x]/Sqrt[2 -
I*Sqrt[31]]] + Sqrt[2 + I*Sqrt[31]]*(2*I + Sqrt[31])*ArcTanh[Sqrt[5 + 10*x]
/Sqrt[2 + I*Sqrt[31]]]))/(7*Sqrt[155])
```

Maple [B] time = 0.071, size = 607, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2*x)^(1/2)/(5*x^2+3*x+2),x)
```

```
[Out] 1/62*ln(5^(1/2)*7^(1/2)+10*x+5+(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^
(1/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)-1/217*ln(5^(1/2)*7^(1/2)+10*x+5+
(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2))*7^(1/2)*(2*5^(1/2)*7^(1/
2)+4)^(1/2)-5/31/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((10*(1+2*x)^(1/2)+5^(
1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*(2*5^(1/2)
*7^(1/2)+4)+2/217/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((10*(1+2*x)^(1/2)+5^
(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*(
2*5^(1/2)*7^(1/2)+4)*7^(1/2)+4/7/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((10*(
1+2*x)^(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(
1/2))*5^(1/2)*7^(1/2)-1/62*ln(-(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^
(1/2)+5^(1/2)*7^(1/2)+10*x+5)*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+1/217*ln(
-(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2)+5^(1/2)*7^(1/2)+10*x+5)*
7^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)-5/31/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arct
an((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/2))/(10*5^(1/2)*7^(1
/2)-20)^(1/2))*(2*5^(1/2)*7^(1/2)+4)+2/217/(10*5^(1/2)*7^(1/2)-20)^(1/2)*ar
ctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/2))/(10*5^(1/2)*7^
(1/2)-20)^(1/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)*7^(1/2)+4/7/(10*5^(1/2)*7^(1
/2)-20)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/2)
)/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*7^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)\sqrt{2\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(1/2)/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)*sqrt(2*x + 1)), x)
```

Fricas [B] time = 2.61098, size = 1427, normalized size = 6.55

$$-\frac{1}{470890}\sqrt{217}35^{\frac{1}{4}}\left(2\sqrt{35}\sqrt{31}-35\sqrt{31}\right)\sqrt{4\sqrt{35}+70}\log\left(4340\sqrt{217}35^{\frac{1}{4}}\sqrt{31}\sqrt{2\,x+1}\sqrt{4\sqrt{35}+70}+9417800\,x+9\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(1/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] -1/470890*sqrt(217)*35^(1/4)*(2*sqrt(35)*sqrt(31) - 35*sqrt(31))*sqrt(4*sqr
t(35) + 70)*log(4340*sqrt(217)*35^(1/4)*sqrt(31)*sqrt(2*x + 1)*sqrt(4*sqrt(
35) + 70) + 9417800*x + 941780*sqrt(35) + 4708900) + 1/470890*sqrt(217)*35^
(1/4)*(2*sqrt(35)*sqrt(31) - 35*sqrt(31))*sqrt(4*sqrt(35) + 70)*log(-4340*s
qrt(217)*35^(1/4)*sqrt(31)*sqrt(2*x + 1)*sqrt(4*sqrt(35) + 70) + 9417800*x
```

```
+ 941780*sqrt(35) + 4708900) - 2/7595*sqrt(217)*35^(3/4)*sqrt(4*sqrt(35) +
70)*arctan(1/235445*sqrt(1085)*sqrt(217)*35^(1/4)*sqrt(sqrt(217)*35^(1/4)*s
qrt(31)*sqrt(2*x + 1)*sqrt(4*sqrt(35) + 70) + 2170*x + 217*sqrt(35) + 1085)
*sqrt(4*sqrt(35) + 70) - 1/217*sqrt(217)*35^(1/4)*sqrt(2*x + 1)*sqrt(4*sqrt
(35) + 70) - 1/31*sqrt(35)*sqrt(31) - 2/31*sqrt(31)) - 2/7595*sqrt(217)*35^
(3/4)*sqrt(4*sqrt(35) + 70)*arctan(1/470890*sqrt(217)*35^(1/4)*sqrt(-4340*s
qrt(217)*35^(1/4)*sqrt(31)*sqrt(2*x + 1)*sqrt(4*sqrt(35) + 70) + 9417800*x
+ 941780*sqrt(35) + 4708900)*sqrt(4*sqrt(35) + 70) - 1/217*sqrt(217)*35^(1/
4)*sqrt(2*x + 1)*sqrt(4*sqrt(35) + 70) + 1/31*sqrt(35)*sqrt(31) + 2/31*sqrt
(31))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x+1}\left(5x^2+3x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)**(1/2)/(5*x**2+3*x+2),x)
```

```
[Out] Integral(1/(sqrt(2*x + 1)*(5*x**2 + 3*x + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)\sqrt{2\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(1/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)*sqrt(2*x + 1)), x)
```

3.2314 $\int \frac{1}{(1+2x)^{3/2}(2+3x+5x^2)}\,dx$

Optimal. Leaf size=253

$$-\frac{4}{7\sqrt{2x+1}}-\frac{1}{7}\sqrt{\frac{1}{434}\left(178+35\sqrt{35}\right)}\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}+\sqrt{35}\right)+\frac{1}{7}\sqrt{\frac{1}{434}\left(178+35\sqrt{35}\right)}\log$$

```
[Out] -4/(7*Sqrt[1 + 2*x]) + (Sqrt[(2*(-178 + 35*Sqrt[35]))/217]*ArcTan[(Sqrt[10*
(2 + Sqrt[35])] - 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/7 - (Sqrt[(2
*(-178 + 35*Sqrt[35]))/217]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] + 10*Sqrt[1 + 2
*x])/Sqrt[10*(-2 + Sqrt[35])]])/7 - (Sqrt[(178 + 35*Sqrt[35])/434]*Log[Sqrt
[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/7 + (Sqrt[(178
 + 35*Sqrt[35])/434]*Log[Sqrt[35] + Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] +
 5*(1 + 2*x)])/7
```

Rubi [A] time = 0.337402, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {709, 826, 1169, 634, 618, 204, 628}

$$-\frac{4}{7\sqrt{2x+1}}-\frac{1}{7}\sqrt{\frac{1}{434}\left(178+35\sqrt{35}\right)}\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}+\sqrt{35}\right)+\frac{1}{7}\sqrt{\frac{1}{434}\left(178+35\sqrt{35}\right)}\log$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + 2*x)^(3/2)*(2 + 3*x + 5*x^2)),x]
```


```
[Out] -4/(7*Sqrt[1 + 2*x]) + (Sqrt[(2*(-178 + 35*Sqrt[35]))/217]*ArcTan[(Sqrt[10*
(2 + Sqrt[35])] - 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/7 - (Sqrt[(2
*(-178 + 35*Sqrt[35]))/217]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] + 10*Sqrt[1 + 2
*x])/Sqrt[10*(-2 + Sqrt[35])]])/7 - (Sqrt[(178 + 35*Sqrt[35])/434]*Log[Sqrt
[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/7 + (Sqrt[(178
 + 35*Sqrt[35])/434]*Log[Sqrt[35] + Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] +
 5*(1 + 2*x)])/7
```


Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(1+2x)^{3/2}\left(2+3x+5x^2\right)}\,dx = -\frac{4}{7\sqrt{1+2x}}+\frac{1}{7}\int \frac{-1-10x}{\sqrt{1+2x}\left(2+3x+5x^2\right)}\,dx$$

$$= -\frac{4}{7\sqrt{1+2x}}+\frac{2}{7}\,\text{Subst}\left(\int \frac{8-10x^2}{7-4x^2+5x^4}\,dx,x,\sqrt{1+2x}\right)$$

$$= -\frac{4}{7\sqrt{1+2x}}+\frac{\text{Subst}\left(\int \frac{8\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}-\left(8+2\sqrt{35}\right)x}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}x+x^2}\,dx,x,\sqrt{1+2x}\right)}{7\sqrt{14\left(2+\sqrt{35}\right)}}+\frac{\text{Subst}\left(\int \frac{8\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}+\cdots}{\sqrt{\frac{7}{5}}+\sqrt{\frac{2}{5}\left(2+\cdots\right.}}\right.}{7\sqrt{14\left(\cdots\right.}}$$

$$= -\frac{4}{7\sqrt{1+2x}}-\frac{\left(4+\sqrt{35}\right)\text{Subst}\left(\int \frac{-\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}+2x}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}x+x^2}\,dx,x,\sqrt{1+2x}\right)}{7\sqrt{14\left(2+\sqrt{35}\right)}}+\frac{\left(4+\sqrt{35}\right)\text{Su}\cdots}{\cdots}$$

$$= -\frac{4}{7\sqrt{1+2x}}-\frac{1}{7}\sqrt{\frac{89}{217}+\frac{5\sqrt{35}}{62}}\log\left(\sqrt{35}-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1+2x}+5(1+2x)\right)+\frac{1}{7}\cdots$$

$$= -\frac{4}{7\sqrt{1+2x}}+\frac{1}{7}\sqrt{\frac{2}{217}\left(-178+35\sqrt{35}\right)}\tan^{-1}\left(\sqrt{\frac{5}{2\left(-2+\sqrt{35}\right)}}\left(\sqrt{\frac{2}{5}}\left(2+\sqrt{35}\right)-2\cdots\right.\right.$$

Mathematica [C] time = 0.45037, size = 122, normalized size = 0.48

$$\frac{2\left(-\frac{2170}{\sqrt{2x+1}}+\sqrt{10-5i\sqrt{31}}\left(124+27i\sqrt{31}\right)\tanh^{-1}\left(\frac{\sqrt{10x+5}}{\sqrt{2-i\sqrt{31}}}\right)+\sqrt{10+5i\sqrt{31}}\left(124-27i\sqrt{31}\right)\tanh^{-1}\left(\frac{\sqrt{10x+5}}{\sqrt{2+i\sqrt{31}}}\right)\right)}{7595}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + 2*x)^(3/2)*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (2*(-2170/Sqrt[1 + 2*x] + Sqrt[10 - (5*I)*Sqrt[31]]*(124 + (27*I)*Sqrt[31])
*ArcTanh[Sqrt[5 + 10*x]/Sqrt[2 - I*Sqrt[31]]] + Sqrt[10 + (5*I)*Sqrt[31]]*(
124 - (27*I)*Sqrt[31])*ArcTanh[Sqrt[5 + 10*x]/Sqrt[2 + I*Sqrt[31]]]))/7595
```

Maple [B] time = 0.077, size = 616, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2*x)^(3/2)/(5*x^2+3*x+2),x)
```

```
[Out] 1/217*ln(5^(1/2)*7^(1/2)+10*x+5+(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)
^(1/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+27/3038*ln(5^(1/2)*7^(1/2)+10*x
+5+(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2))*7^(1/2)*(2*5^(1/2)*7^
(1/2)+4)^(1/2)-10/217/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((10*(1+2*x)^(1/2
)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*(2*5^
(1/2)*7^(1/2)+4)-27/1519/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((10*(1+2*x)^(
1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^
(1/2)*(2*5^(1/2)*7^(1/2)+4)*7^(1/2)+16/49/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arc
tan((10*(1+2*x)^(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1
/2)-20)^(1/2))*5^(1/2)*7^(1/2)-1/217*ln(-(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2
)*(1+2*x)^(1/2)+5^(1/2)*7^(1/2)+10*x+5)*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)
-27/3038*ln(-(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2)+5^(1/2)*7^(1
/2)+10*x+5)*7^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)-10/217/(10*5^(1/2)*7^(1/2)-
20)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/2))/(1
0*5^(1/2)*7^(1/2)-20)^(1/2))*(2*5^(1/2)*7^(1/2)+4)-27/1519/(10*5^(1/2)*7^(1
/2)-20)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/2)
)/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)*7^(1/2)+16/4
9/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2
)+10*(1+2*x)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*7^(1/2)-4/7/(1+2
*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)\left(2\,x+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(3/2)/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)*(2*x + 1)^(3/2)), x)
```

Fricas [B] time = 2.77233, size = 2148, normalized size = 8.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(3/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 1/2191992950*(2356*42875^(1/4)*sqrt(217)*sqrt(35)*(2*x + 1)*sqrt(-12460*sqr
t(35) + 85750)*arctan(1/3644188279375*42875^(3/4)*sqrt(217)*sqrt(31)*sqrt(4
2875^(1/4)*sqrt(217)*(sqrt(35)*sqrt(31) + 4*sqrt(31))*sqrt(2*x + 1)*sqrt(-1
2460*sqrt(35) + 85750) + 1443050*x + 144305*sqrt(35) + 721525)*(4*sqrt(35)*
sqrt(19) + 35*sqrt(19))*sqrt(-12460*sqrt(35) + 85750) - 1/176773625*42875^(
3/4)*sqrt(217)*sqrt(2*x + 1)*(4*sqrt(35) + 35)*sqrt(-12460*sqrt(35) + 85750
) - 1/31*sqrt(35)*sqrt(31) - 2/31*sqrt(31)) + 2356*42875^(1/4)*sqrt(217)*sq
rt(35)*(2*x + 1)*sqrt(-12460*sqrt(35) + 85750)*arctan(1/255093179556250*428
75^(3/4)*sqrt(217)*sqrt(-151900*42875^(1/4)*sqrt(217)*(sqrt(35)*sqrt(31) +
4*sqrt(31))*sqrt(2*x + 1)*sqrt(-12460*sqrt(35) + 85750) + 219199295000*x +
21919929500*sqrt(35) + 109599647500)*(4*sqrt(35)*sqrt(19) + 35*sqrt(19))*sq
rt(-12460*sqrt(35) + 85750) - 1/176773625*42875^(3/4)*sqrt(217)*sqrt(2*x +
1)*(4*sqrt(35) + 35)*sqrt(-12460*sqrt(35) + 85750) + 1/31*sqrt(35)*sqrt(31)
 + 2/31*sqrt(31)) + 42875^(1/4)*sqrt(217)*(178*sqrt(35)*sqrt(31)*(2*x + 1)
+ 1225*sqrt(31)*(2*x + 1))*sqrt(-12460*sqrt(35) + 85750)*log(151900/19*4287
5^(1/4)*sqrt(217)*(sqrt(35)*sqrt(31) + 4*sqrt(31))*sqrt(2*x + 1)*sqrt(-1246
0*sqrt(35) + 85750) + 11536805000*x + 1153680500*sqrt(35) + 5768402500) - 4
2875^(1/4)*sqrt(217)*(178*sqrt(35)*sqrt(31)*(2*x + 1) + 1225*sqrt(31)*(2*x
+ 1))*sqrt(-12460*sqrt(35) + 85750)*log(-151900/19*42875^(1/4)*sqrt(217)*(s
qrt(35)*sqrt(31) + 4*sqrt(31))*sqrt(2*x + 1)*sqrt(-12460*sqrt(35) + 85750)
+ 11536805000*x + 1153680500*sqrt(35) + 5768402500) - 1252567400*sqrt(2*x +
 1))/(2*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x+1)^{\frac{3}{2}}\left(5x^2+3x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)**(3/2)/(5*x**2+3*x+2),x)
```

```
[Out] Integral(1/((2*x + 1)**(3/2)*(5*x**2 + 3*x + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)(2\,x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(3/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)*(2*x + 1)^(3/2)), x)
```

3.2315 $\int \frac{1}{(1+2x)^{5/2}(2+3x+5x^2)}\,dx$

Optimal. Leaf size=266

$$-\frac{16}{49\sqrt{2x+1}}-\frac{4}{21(2x+1)^{3/2}}-\frac{1}{49}\sqrt{\frac{1}{434}\left(1225\sqrt{35}-7162\right)}\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}+\sqrt{35}\right)+\frac{1}{49}\sqrt{}$$

```
[Out] -4/(21*(1 + 2*x)^(3/2)) - 16/(49*Sqrt[1 + 2*x]) + (Sqrt[(2*(7162 + 1225*Sqr
t[35]))/217]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] - 10*Sqrt[1 + 2*x])/Sqrt[10*(-
2 + Sqrt[35])]])/49 - (Sqrt[(2*(7162 + 1225*Sqrt[35]))/217]*ArcTan[(Sqrt[10
*(2 + Sqrt[35])] + 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/49 - (Sqrt[
(-7162 + 1225*Sqrt[35])/434]*Log[Sqrt[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1
+ 2*x] + 5*(1 + 2*x)])/49 + (Sqrt[(-7162 + 1225*Sqrt[35])/434]*Log[Sqrt[35]
 + Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/49
```

Rubi [A] time = 0.382157, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {709, 828, 826, 1169, 634, 618, 204, 628}

$$-\frac{16}{49\sqrt{2x+1}}-\frac{4}{21(2x+1)^{3/2}}-\frac{1}{49}\sqrt{\frac{1}{434}\left(1225\sqrt{35}-7162\right)}\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}+\sqrt{35}\right)+\frac{1}{49}\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + 2*x)^(5/2)*(2 + 3*x + 5*x^2)),x]
```

```
[Out] -4/(21*(1 + 2*x)^(3/2)) - 16/(49*Sqrt[1 + 2*x]) + (Sqrt[(2*(7162 + 1225*Sqr
t[35]))/217]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] - 10*Sqrt[1 + 2*x])/Sqrt[10*(-
2 + Sqrt[35])]])/49 - (Sqrt[(2*(7162 + 1225*Sqrt[35]))/217]*ArcTan[(Sqrt[10
*(2 + Sqrt[35])] + 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/49 - (Sqrt[
(-7162 + 1225*Sqrt[35])/434]*Log[Sqrt[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1
+ 2*x] + 5*(1 + 2*x)])/49 + (Sqrt[(-7162 + 1225*Sqrt[35])/434]*Log[Sqrt[35]
 + Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/49
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(1+2x)^{5/2}\left(2+3x+5x^2\right)}\,dx = -\frac{4}{21(1+2x)^{3/2}} + \frac{1}{7}\int \frac{-1-10x}{(1+2x)^{3/2}\left(2+3x+5x^2\right)}\,dx$$
$$= -\frac{4}{21(1+2x)^{3/2}} - \frac{16}{49\sqrt{1+2x}} + \frac{1}{49}\int \frac{-39-40x}{\sqrt{1+2x}\left(2+3x+5x^2\right)}\,dx$$
$$= -\frac{4}{21(1+2x)^{3/2}} - \frac{16}{49\sqrt{1+2x}} + \frac{2}{49}\,\text{Subst}\left(\int \frac{-38-40x^2}{7-4x^2+5x^4}\,dx, x, \sqrt{1+2x}\right)$$
$$= -\frac{4}{21(1+2x)^{3/2}} - \frac{16}{49\sqrt{1+2x}} + \frac{\text{Subst}\left(\int \frac{-38\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}-\left(-38+8\sqrt{35}\right)x}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}x+x^2}\,dx, x, \sqrt{1+2x}\right)}{49\sqrt{14\left(2+\sqrt{35}\right)}}$$
$$= -\frac{4}{21(1+2x)^{3/2}} - \frac{16}{49\sqrt{1+2x}} - \frac{\left(140+19\sqrt{35}\right)\text{Subst}\left(\int \frac{1}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}x+x^2}\,dx, x, \sqrt{\ldots}\right.}{1715}$$
$$= -\frac{4}{21(1+2x)^{3/2}} - \frac{16}{49\sqrt{1+2x}} - \frac{1}{49}\sqrt{\frac{1}{434}\left(-7162+1225\sqrt{35}\right)}\log\left(\sqrt{35}-\sqrt{10\left(2\ldots\right.}\right.$$
$$= -\frac{4}{21(1+2x)^{3/2}} - \frac{16}{49\sqrt{1+2x}} + \frac{1}{49}\sqrt{\frac{2}{217}\left(7162+1225\sqrt{35}\right)}\tan^{-1}\left(\sqrt{\frac{5}{2\left(-2+\sqrt{3\ldots}\right.}}\right.$$

Mathematica [C] time = 0.527477, size = 133, normalized size = 0.5

$$\frac{2\left(-\frac{2170(24x+19)}{(2x+1)^{3/2}}+3i\sqrt{10-5i\sqrt{31}}\left(178\sqrt{31}+589i\right)\tanh^{-1}\left(\frac{\sqrt{10x+5}}{\sqrt{2-i\sqrt{31}}}\right)-3i\sqrt{10+5i\sqrt{31}}\left(178\sqrt{31}-589i\right)\tanh^{-1}\left(\frac{\sqrt{10\ldots}}{\sqrt{2+\ldots}}\right.\right.}{159495}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + 2*x)^(5/2)*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (2*((-2170*(19 + 24*x))/(1 + 2*x)^(3/2) + (3*I)*Sqrt[10 - (5*I)*Sqrt[31]]*(
589*I + 178*Sqrt[31])*ArcTanh[Sqrt[5 + 10*x]/Sqrt[2 - I*Sqrt[31]]] - (3*I)*
Sqrt[10 + (5*I)*Sqrt[31]]*(-589*I + 178*Sqrt[31])*ArcTanh[Sqrt[5 + 10*x]/Sq
rt[2 + I*Sqrt[31]]]))/159495
```

Maple [B] time = 0.075, size = 625, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2*x)^(5/2)/(5*x^2+3*x+2),x)
```

```
[Out] -27/3038*ln(5^(1/2)*7^(1/2)+10*x+5+(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2
*x)^(1/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+89/10633*ln(5^(1/2)*7^(1/2)+
10*x+5+(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2))*7^(1/2)*(2*5^(1/2
)*7^(1/2)+4)^(1/2)+135/1519/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((10*(1+2*x
)^(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))
*(2*5^(1/2)*7^(1/2)+4)-178/10633/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((10*(
1+2*x)^(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(
```

```
1/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)*7^(1/2)-76/343/(10*5^(1/2)*7^(1/2)-20)^
(1/2)*arctan((10*(1+2*x)^(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(
1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*7^(1/2)+27/3038*ln(-(2*5^(1/2)*7^(1/2)+4)^(
1/2)*5^(1/2)*(1+2*x)^(1/2)+5^(1/2)*7^(1/2)+10*x+5)*5^(1/2)*(2*5^(1/2)*7^(1/
2)+4)^(1/2)-89/10633*ln(-(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2)+
5^(1/2)*7^(1/2)+10*x+5)*7^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+135/1519/(10*5^
(1/2)*7^(1/2)-20)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+
2*x)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*(2*5^(1/2)*7^(1/2)+4)-178/10633/
(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+
10*(1+2*x)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)
+4)*7^(1/2)-76/343/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2
)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)
*7^(1/2)-4/21/(1+2*x)^(3/2)-16/49/(1+2*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)\left(2\,x+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(5/2)/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)*(2*x + 1)^(5/2)), x)
```

Fricas [B] time = 2.7599, size = 2337, normalized size = 8.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(5/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] -1/13774945170*(74028*sqrt(217)*35^(3/4)*sqrt(2)*(4*x^2 + 4*x + 1)*sqrt(716
2*sqrt(35) + 42875)*arctan(1/326335010575*sqrt(1085)*sqrt(217)*sqrt(199)*35
^(3/4)*sqrt(2)*sqrt(sqrt(217)*35^(1/4)*sqrt(2)*(4*sqrt(35)*sqrt(31) - 19*sq
rt(31))*sqrt(2*x + 1)*sqrt(7162*sqrt(35) + 42875) + 431830*x + 43183*sqrt(3
5) + 215915)*sqrt(7162*sqrt(35) + 42875)*(19*sqrt(35) - 140) - 1/1511405*sq
rt(217)*35^(3/4)*sqrt(2)*sqrt(2*x + 1)*sqrt(7162*sqrt(35) + 42875)*(19*sqrt
(35) - 140) + 1/31*sqrt(35)*sqrt(31) + 2/31*sqrt(31)) + 74028*sqrt(217)*35^
(3/4)*sqrt(2)*(4*x^2 + 4*x + 1)*sqrt(7162*sqrt(35) + 42875)*arctan(1/799520
775908750*sqrt(217)*sqrt(199)*35^(3/4)*sqrt(2)*sqrt(-6512712500*sqrt(217)*3
5^(1/4)*sqrt(2)*(4*sqrt(35)*sqrt(31) - 19*sqrt(31))*sqrt(2*x + 1)*sqrt(7162
*sqrt(35) + 42875) + 2812384638875000*x + 281238463887500*sqrt(35) + 140619
2319437500)*sqrt(7162*sqrt(35) + 42875)*(19*sqrt(35) - 140) - 1/1511405*sqr
t(217)*35^(3/4)*sqrt(2)*sqrt(2*x + 1)*sqrt(7162*sqrt(35) + 42875)*(19*sqrt(
35) - 140) - 1/31*sqrt(35)*sqrt(31) - 2/31*sqrt(31)) + 3*sqrt(217)*35^(1/4)
*sqrt(2)*(7162*sqrt(35)*sqrt(31)*(4*x^2 + 4*x + 1) - 42875*sqrt(31)*(4*x^2
+ 4*x + 1))*sqrt(7162*sqrt(35) + 42875)*log(6512712500/199*sqrt(217)*35^(1/
4)*sqrt(2)*(4*sqrt(35)*sqrt(31) - 19*sqrt(31))*sqrt(2*x + 1)*sqrt(7162*sqrt
(35) + 42875) + 14132586125000*x + 1413258612500*sqrt(35) + 7066293062500)
- 3*sqrt(217)*35^(1/4)*sqrt(2)*(7162*sqrt(35)*sqrt(31)*(4*x^2 + 4*x + 1) -
42875*sqrt(31)*(4*x^2 + 4*x + 1))*sqrt(7162*sqrt(35) + 42875)*log(-65127125
00/199*sqrt(217)*35^(1/4)*sqrt(2)*(4*sqrt(35)*sqrt(31) - 19*sqrt(31))*sqrt(
```

```
2*x + 1)*sqrt(7162*sqrt(35) + 42875) + 14132586125000*x + 1413258612500*sqr
t(35) + 7066293062500) + 374828440*(24*x + 19)*sqrt(2*x + 1))/(4*x^2 + 4*x
+ 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x+1)^{\frac{5}{2}}\left(5x^2+3x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)**(5/2)/(5*x**2+3*x+2),x)
```

```
[Out] Integral(1/((2*x + 1)**(5/2)*(5*x**2 + 3*x + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)(2\,x+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(5/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)*(2*x + 1)^(5/2)), x)
```

3.2316 $\int \frac{(1+2x)^{7/2}}{(2+3x+5x^2)^2} dx$

Optimal. Leaf size=296

$$-\frac{(5-4x)(2x+1)^{5/2}}{31(5x^2+3x+2)}-\frac{8}{155}(2x+1)^{3/2}+\frac{604}{775}\sqrt{2x+1}+\frac{1}{775}\sqrt{\frac{1}{310}(5682718+968975\sqrt{35})}\log\left(5(2x+1)-\sqrt{10(2+\sqrt{}\right.$$

```
[Out] (604*Sqrt[1 + 2*x])/775 - (8*(1 + 2*x)^(3/2))/155 - ((5 - 4*x)*(1 + 2*x)^(5
/2))/(31*(2 + 3*x + 5*x^2)) + (Sqrt[(2*(-5682718 + 968975*Sqrt[35]))/155]*A
rcTan[(Sqrt[10*(2 + Sqrt[35])] - 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]
])/775 - (Sqrt[(2*(-5682718 + 968975*Sqrt[35]))/155]*ArcTan[(Sqrt[10*(2 + S
qrt[35])] + 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/775 + (Sqrt[(56827
18 + 968975*Sqrt[35])/310]*Log[Sqrt[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 +
2*x] + 5*(1 + 2*x)])/775 - (Sqrt[(5682718 + 968975*Sqrt[35])/310]*Log[Sqrt[
35] + Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/775
```

Rubi [A] time = 0.445667, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {738, 824, 826, 1169, 634, 618, 204, 628}

$$-\frac{(5-4x)(2x+1)^{5/2}}{31(5x^2+3x+2)}-\frac{8}{155}(2x+1)^{3/2}+\frac{604}{775}\sqrt{2x+1}+\frac{1}{775}\sqrt{\frac{1}{310}(5682718+968975\sqrt{35})}\log\left(5(2x+1)-\sqrt{10(2+\sqrt{}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^(7/2)/(2 + 3*x + 5*x^2)^2,x]
```


```
[Out] (604*Sqrt[1 + 2*x])/775 - (8*(1 + 2*x)^(3/2))/155 - ((5 - 4*x)*(1 + 2*x)^(5
/2))/(31*(2 + 3*x + 5*x^2)) + (Sqrt[(2*(-5682718 + 968975*Sqrt[35]))/155]*A
rcTan[(Sqrt[10*(2 + Sqrt[35])] - 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]
])/775 - (Sqrt[(2*(-5682718 + 968975*Sqrt[35]))/155]*ArcTan[(Sqrt[10*(2 + S
qrt[35])] + 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/775 + (Sqrt[(56827
18 + 968975*Sqrt[35])/310]*Log[Sqrt[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 +
2*x] + 5*(1 + 2*x)])/775 - (Sqrt[(5682718 + 968975*Sqrt[35])/310]*Log[Sqrt[
35] + Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/775
```


Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
```

```
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(1+2x)^{7/2}}{\left(2+3x+5x^2\right)^2}\,dx = -\frac{(5-4x)(1+2x)^{5/2}}{31\left(2+3x+5x^2\right)} + \frac{1}{31}\int \frac{(29-12x)(1+2x)^{3/2}}{2+3x+5x^2}\,dx$$

$$= -\frac{8}{155}(1+2x)^{3/2} - \frac{(5-4x)(1+2x)^{5/2}}{31\left(2+3x+5x^2\right)} + \frac{1}{155}\int \frac{\sqrt{1+2x}(193+302x)}{2+3x+5x^2}\,dx$$

$$= \frac{604}{775}\sqrt{1+2x} - \frac{8}{155}(1+2x)^{3/2} - \frac{(5-4x)(1+2x)^{5/2}}{31\left(2+3x+5x^2\right)} + \frac{1}{775}\int \frac{-243+1628x}{\sqrt{1+2x}\left(2+3x+5x^2\right)}\,dx$$

$$= \frac{604}{775}\sqrt{1+2x} - \frac{8}{155}(1+2x)^{3/2} - \frac{(5-4x)(1+2x)^{5/2}}{31\left(2+3x+5x^2\right)} + \frac{2}{775}\text{Subst}\left(\int \frac{-2114+1628x^2}{7-4x^2+5x^4}\,dx, x, \sqrt{1\right.$$

$$= \frac{604}{775}\sqrt{1+2x} - \frac{8}{155}(1+2x)^{3/2} - \frac{(5-4x)(1+2x)^{5/2}}{31\left(2+3x+5x^2\right)} + \frac{\text{Subst}\left(\int \frac{-2114\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}-\left(-2114-1628\sqrt{\frac{7}{5}}\right)x}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}x+x^2}\right.}{775\sqrt{14\left(2+\sqrt{35}\right)}}$$

$$= \frac{604}{775}\sqrt{1+2x} - \frac{8}{155}(1+2x)^{3/2} - \frac{(5-4x)(1+2x)^{5/2}}{31\left(2+3x+5x^2\right)} - \frac{\sqrt{1460631-245828\sqrt{35}}\,\text{Subst}\left(\int \frac{}{\sqrt{\frac{7}{5}}-\sqrt{}}\right.}{3875}$$

$$= \frac{604}{775}\sqrt{1+2x} - \frac{8}{155}(1+2x)^{3/2} - \frac{(5-4x)(1+2x)^{5/2}}{31\left(2+3x+5x^2\right)} + \frac{1}{775}\sqrt{\frac{1}{310}\left(5682718+968975\sqrt{35}\right)}\log\left(\right.$$

$$= \frac{604}{775}\sqrt{1+2x} - \frac{8}{155}(1+2x)^{3/2} - \frac{(5-4x)(1+2x)^{5/2}}{31\left(2+3x+5x^2\right)} + \frac{1}{775}\sqrt{\frac{2}{155}\left(-5682718+968975\sqrt{35}\right)}\tan$$

Mathematica [C] time = 0.433634, size = 199, normalized size = 0.67

$$\frac{1}{217}\left(\frac{(20x+37)(2x+1)^{9/2}}{5x^2+3x+2} - 8(2x+1)^{7/2} - 28(2x+1)^{5/2} - \frac{56}{5}(2x+1)^{3/2} + \frac{4228}{25}\sqrt{2x+1} - \frac{14i\left(\sqrt{2-i\sqrt{31}}\left(512\sqrt{31}-4\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^(7/2)/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] ((4228*Sqrt[1 + 2*x])/25 - (56*(1 + 2*x)^(3/2))/5 - 28*(1 + 2*x)^(5/2) - 8*
(1 + 2*x)^(7/2) + ((1 + 2*x)^(9/2)*(37 + 20*x))/(2 + 3*x + 5*x^2) - (((14*I
)/775)*(Sqrt[2 - I*Sqrt[31]]*(-4681*I + 512*Sqrt[31])*ArcTanh[Sqrt[5 + 10*x
]/Sqrt[2 - I*Sqrt[31]]] - Sqrt[2 + I*Sqrt[31]]*(4681*I + 512*Sqrt[31])*ArcT
anh[Sqrt[5 + 10*x]/Sqrt[2 + I*Sqrt[31]]]))/Sqrt[5])/217
```

Maple [B] time = 0.078, size = 651, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^(7/2)/(5*x^2+3*x+2)^2,x)
```

```
[Out] 16/25*(1+2*x)^(1/2)+16/25*(-89/310*(1+2*x)^(3/2)+189/620*(1+2*x)^(1/2))/((1
+2*x)^2-8/5*x+3/5)-3657/240250*ln(5^(1/2)*7^(1/2)+10*x+5+(2*5^(1/2)*7^(1/2)
+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)-256/24
025*ln(5^(1/2)*7^(1/2)+10*x+5+(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(
1/2))*7^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+3657/24025/(10*5^(1/2)*7^(1/2)-20
)^(1/2)*arctan((10*(1+2*x)^(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5
^(1/2)*7^(1/2)-20)^(1/2))*(2*5^(1/2)*7^(1/2)+4)+512/24025/(10*5^(1/2)*7^(1/
2)-20)^(1/2)*arctan((10*(1+2*x)^(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/
(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)*7^(1/2)-604/77
5/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((10*(1+2*x)^(1/2)+5^(1/2)*(2*5^(1/2)
*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*7^(1/2)+3657/2402
50*ln(-(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2)+5^(1/2)*7^(1/2)+10
*x+5)*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+256/24025*ln(-(2*5^(1/2)*7^(1/2)+
4)^(1/2)*5^(1/2)*(1+2*x)^(1/2)+5^(1/2)*7^(1/2)+10*x+5)*7^(1/2)*(2*5^(1/2)*7
^(1/2)+4)^(1/2)+3657/24025/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((-5^(1/2)*(
2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))
*(2*5^(1/2)*7^(1/2)+4)+512/24025/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((-5^(
1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^
(1/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)*7^(1/2)-604/775/(10*5^(1/2)*7^(1/2)-20
)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/2))/(10*
5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*7^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x+1)^{\frac{7}{2}}}{(5x^2+3x+2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(7/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] integrate((2*x + 1)^(7/2)/(5*x^2 + 3*x + 2)^2, x)
```

Fricas [B] time = 3.00175, size = 2880, normalized size = 9.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(7/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/977420404174943750*(16794436*21898835^(1/4)*sqrt(155)*sqrt(35)*(5*x^2 + 3
*x + 2)*sqrt(-11012823348100*sqrt(35) + 65723878543750)*arctan(1/6033269966
2225359002939375*21898835^(3/4)*sqrt(4369)*sqrt(3955)*sqrt(155)*sqrt(218988
35^(1/4)*sqrt(155)*(814*sqrt(35)*sqrt(31) + 5285*sqrt(31))*sqrt(2*x + 1)*sq
rt(-11012823348100*sqrt(35) + 65723878543750) + 40683471557750*x + 40683471
55775*sqrt(35) + 20341735778875)*(151*sqrt(35) + 814)*sqrt(-11012823348100*
sqrt(35) + 65723878543750) - 1/3218062600218025*21898835^(3/4)*sqrt(155)*sq
rt(2*x + 1)*(151*sqrt(35) + 814)*sqrt(-11012823348100*sqrt(35) + 6572387854
3750) - 1/31*sqrt(35)*sqrt(31) - 2/31*sqrt(31)) + 16794436*21898835^(1/4)*s
qrt(155)*sqrt(35)*(5*x^2 + 3*x + 2)*sqrt(-11012823348100*sqrt(35) + 6572387
8543750)*arctan(1/4223288976355775130205756250*21898835^(3/4)*sqrt(4369)*sq
rt(155)*sqrt(-19379500*21898835^(1/4)*sqrt(155)*(814*sqrt(35)*sqrt(31) + 52
85*sqrt(31))*sqrt(2*x + 1)*sqrt(-11012823348100*sqrt(35) + 65723878543750)
```

```
+ 7884253370534161250000*x + 788425337053416125000*sqrt(35) + 394212668526708
062500)*(151*sqrt(35) + 814)*sqrt(-11012823348100*sqrt(35) + 65723878543750
) - 1/3218062600218025*21898835^(3/4)*sqrt(155)*sqrt(2*x + 1)*(151*sqrt(35)
 + 814)*sqrt(-11012823348100*sqrt(35) + 65723878543750) + 1/31*sqrt(35)*sqr
t(31) + 2/31*sqrt(31)) - 21898835^(1/4)*sqrt(155)*(5682718*sqrt(35)*sqrt(31
)*(5*x^2 + 3*x + 2) + 33914125*sqrt(31)*(5*x^2 + 3*x + 2))*sqrt(-1101282334
8100*sqrt(35) + 65723878543750)*log(19379500/4369*21898835^(1/4)*sqrt(155)*
(814*sqrt(35)*sqrt(31) + 5285*sqrt(31))*sqrt(2*x + 1)*sqrt(-11012823348100*
sqrt(35) + 65723878543750) + 180458992230125000*x + 18045899223012500*sqrt(
35) + 90229496115062500) + 21898835^(1/4)*sqrt(155)*(5682718*sqrt(35)*sqrt(
31)*(5*x^2 + 3*x + 2) + 33914125*sqrt(31)*(5*x^2 + 3*x + 2))*sqrt(-11012823
348100*sqrt(35) + 65723878543750)*log(-19379500/4369*21898835^(1/4)*sqrt(15
5)*(814*sqrt(35)*sqrt(31) + 5285*sqrt(31))*sqrt(2*x + 1)*sqrt(-110128233481
00*sqrt(35) + 65723878543750) + 180458992230125000*x + 18045899223012500*sq
rt(35) + 90229496115062500) + 1261187618290250*(2480*x^2 + 1132*x + 1003)*s
qrt(2*x + 1))/(5*x^2 + 3*x + 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**(7/2)/(5*x**2+3*x+2)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x+1)^{\frac{7}{2}}}{\left(5x^2+3x+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(7/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] integrate((2*x + 1)^(7/2)/(5*x^2 + 3*x + 2)^2, x)
```

3.2317 $\int \frac{(1+2x)^{5/2}}{(2+3x+5x^2)^2} dx$

Optimal. Leaf size=283

$$-\frac{(5-4x)(2x+1)^{3/2}}{31\left(5x^2+3x+2\right)}-\frac{8}{155}\sqrt{2x+1}+\frac{1}{155}\sqrt{\frac{1}{310}\left(10325\sqrt{35}-32678\right)}\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}+\sqrt{3}\right.$$

```
[Out] (-8*Sqrt[1 + 2*x])/155 - ((5 - 4*x)*(1 + 2*x)^(3/2))/(31*(2 + 3*x + 5*x^2))
 - (Sqrt[(2*(32678 + 10325*Sqrt[35]))/155]*ArcTan[(Sqrt[10*(2 + Sqrt[35])]
- 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/155 + (Sqrt[(2*(32678 + 1032
5*Sqrt[35]))/155]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] + 10*Sqrt[1 + 2*x])/Sqrt[
10*(-2 + Sqrt[35])]])/155 + (Sqrt[(-32678 + 10325*Sqrt[35])/310]*Log[Sqrt[3
5] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/155 - (Sqrt[(-32
678 + 10325*Sqrt[35])/310]*Log[Sqrt[35] + Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 +
2*x] + 5*(1 + 2*x)])/155
```

Rubi [A] time = 0.410117, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {738, 824, 826, 1169, 634, 618, 204, 628}

$$-\frac{(5-4x)(2x+1)^{3/2}}{31\left(5x^2+3x+2\right)}-\frac{8}{155}\sqrt{2x+1}+\frac{1}{155}\sqrt{\frac{1}{310}\left(10325\sqrt{35}-32678\right)}\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}+\sqrt{3}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^(5/2)/(2 + 3*x + 5*x^2)^2,x]
```


```
[Out] (-8*Sqrt[1 + 2*x])/155 - ((5 - 4*x)*(1 + 2*x)^(3/2))/(31*(2 + 3*x + 5*x^2))
 - (Sqrt[(2*(32678 + 10325*Sqrt[35]))/155]*ArcTan[(Sqrt[10*(2 + Sqrt[35])]
- 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/155 + (Sqrt[(2*(32678 + 1032
5*Sqrt[35]))/155]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] + 10*Sqrt[1 + 2*x])/Sqrt[
10*(-2 + Sqrt[35])]])/155 + (Sqrt[(-32678 + 10325*Sqrt[35])/310]*Log[Sqrt[3
5] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/155 - (Sqrt[(-32
678 + 10325*Sqrt[35])/310]*Log[Sqrt[35] + Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 +
2*x] + 5*(1 + 2*x)])/155
```


Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
```

```
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(1+2x)^{5/2}}{\left(2+3x+5x^2\right)^2}\,dx = -\frac{(5-4x)(1+2x)^{3/2}}{31\left(2+3x+5x^2\right)} + \frac{1}{31}\int \frac{(19-4x)\sqrt{1+2x}}{2+3x+5x^2}\,dx$$

$$= -\frac{8}{155}\sqrt{1+2x} - \frac{(5-4x)(1+2x)^{3/2}}{31\left(2+3x+5x^2\right)} + \frac{1}{155}\int \frac{111+194x}{\sqrt{1+2x}\left(2+3x+5x^2\right)}\,dx$$

$$= -\frac{8}{155}\sqrt{1+2x} - \frac{(5-4x)(1+2x)^{3/2}}{31\left(2+3x+5x^2\right)} + \frac{2}{155}\text{Subst}\left(\int \frac{28+194x^2}{7-4x^2+5x^4}\,dx, x, \sqrt{1+2x}\right)$$

$$= -\frac{8}{155}\sqrt{1+2x} - \frac{(5-4x)(1+2x)^{3/2}}{31\left(2+3x+5x^2\right)} + \frac{\text{Subst}\left(\int \frac{28\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}-\left(28-194\sqrt{\frac{7}{5}}\right)x}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}x+x^2}\,dx, x, \sqrt{1+2x}\right)}{155\sqrt{14\left(2+\sqrt{35}\right)}} + \cdots$$

$$= -\frac{8}{155}\sqrt{1+2x} - \frac{(5-4x)(1+2x)^{3/2}}{31\left(2+3x+5x^2\right)} + \frac{1}{775}\left(97+2\sqrt{35}\right)\text{Subst}\left(\int \frac{1}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}x+\cdots}\right.$$

$$= -\frac{8}{155}\sqrt{1+2x} - \frac{(5-4x)(1+2x)^{3/2}}{31\left(2+3x+5x^2\right)} + \frac{1}{155}\sqrt{\frac{1}{310}\left(-32678+10325\sqrt{35}\right)}\log\left(\sqrt{35}-\sqrt{10\left(2\cdots\right.}\right.$$

$$= -\frac{8}{155}\sqrt{1+2x} - \frac{(5-4x)(1+2x)^{3/2}}{31\left(2+3x+5x^2\right)} - \frac{1}{155}\sqrt{\frac{2}{155}\left(32678+10325\sqrt{35}\right)}\tan^{-1}\left(\sqrt{\frac{5}{2\left(-2+\sqrt{3\cdots}\right.}}\right.$$

Mathematica [C] time = 0.672765, size = 141, normalized size = 0.5

$$\frac{-\frac{155\sqrt{2x+1}(54x+41)}{5x^2+3x+2} + 2\sqrt{10-5i\sqrt{31}}\left(62-101i\sqrt{31}\right)\tanh^{-1}\left(\frac{\sqrt{10x+5}}{\sqrt{2-i\sqrt{31}}}\right) + 2\sqrt{10+5i\sqrt{31}}\left(62+101i\sqrt{31}\right)\tanh^{-1}\left(\frac{\sqrt{10x+\cdots}}{\sqrt{2+i\sqrt{3\cdots}}}\right.}{24025}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^(5/2)/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] ((-155*Sqrt[1 + 2*x]*(41 + 54*x))/(2 + 3*x + 5*x^2) + 2*Sqrt[10 - (5*I)*Sqr
t[31]]*(62 - (101*I)*Sqrt[31])*ArcTanh[Sqrt[5 + 10*x]/Sqrt[2 - I*Sqrt[31]]]
 + 2*Sqrt[10 + (5*I)*Sqrt[31]]*(62 + (101*I)*Sqrt[31])*ArcTanh[Sqrt[5 + 10*
x]/Sqrt[2 + I*Sqrt[31]]])/24025
```

Maple [B] time = 0.076, size = 642, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^(5/2)/(5*x^2+3*x+2)^2,x)
```

```
[Out] 16*(-27/3100*(1+2*x)^(3/2)-7/1550*(1+2*x)^(1/2))/((1+2*x)^2-8/5*x+3/5)-101/
9610*ln(5^(1/2)*7^(1/2)+10*x+5+(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^
(1/2))*7^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+132/24025*ln(5^(1/2)*7^(1/2)+10*
x+5+(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2))*5^(1/2)*(2*5^(1/2)*7
^(1/2)+4)^(1/2)+101/4805/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((10*(1+2*x)^(
1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^
(1/2)*(2*5^(1/2)*7^(1/2)+4)*7^(1/2)-264/4805/(10*5^(1/2)*7^(1/2)-20)^(1/2)*
```

```
arctan((10*(1+2*x)^(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7
^(1/2)-20)^(1/2))*(2*5^(1/2)*7^(1/2)+4)+8/155/(10*5^(1/2)*7^(1/2)-20)^(1/2)
*arctan((10*(1+2*x)^(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*
7^(1/2)-20)^(1/2))*5^(1/2)*7^(1/2)+101/9610*ln(-(2*5^(1/2)*7^(1/2)+4)^(1/2)
*5^(1/2)*(1+2*x)^(1/2)+5^(1/2)*7^(1/2)+10*x+5)*7^(1/2)*(2*5^(1/2)*7^(1/2)+4
)^(1/2)-132/24025*ln(-(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2)+5^(
1/2)*7^(1/2)+10*x+5)*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+101/4805/(10*5^(1/
2)*7^(1/2)-20)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+2*x
)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)*7^(1/
2)-264/4805/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/
2)+4)^(1/2)+10*(1+2*x)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*(2*5^(1/2)*7^(
1/2)+4)+8/155/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(
1/2)+4)^(1/2)+10*(1+2*x)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*7^(1
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x+1)^{\frac{5}{2}}}{(5x^2+3x+2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] integrate((2*x + 1)^(5/2)/(5*x^2 + 3*x + 2)^2, x)
```

Fricas [B] time = 2.89147, size = 2557, normalized size = 9.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/199574966022250*(1149356*5969915^(1/4)*sqrt(826)*sqrt(155)*sqrt(35)*(5*x^
2 + 3*x + 2)*sqrt(32678*sqrt(35) + 361375)*arctan(1/328333851982428997 25*59
69915^(3/4)*sqrt(826)*sqrt(299)*sqrt(155)*sqrt(59)*sqrt(5969915^(1/4)*sqrt(
826)*sqrt(155)*(97*sqrt(35)*sqrt(31) - 70*sqrt(31))*sqrt(2*x + 1)*sqrt(3267
8*sqrt(35) + 361375) + 41534852450*x + 4153485245*sqrt(35) + 20767426225)*s
qrt(32678*sqrt(35) + 361375)*(2*sqrt(35) - 97) - 1/1715389406185*5969915^(3
/4)*sqrt(826)*sqrt(155)*sqrt(2*x + 1)*sqrt(32678*sqrt(35) + 361375)*(2*sqrt
(35) - 97) + 1/31*sqrt(35)*sqrt(31) + 2/31*sqrt(31)) + 1149356*5969915^(1/4
)*sqrt(826)*sqrt(155)*sqrt(35)*(5*x^2 + 3*x + 2)*sqrt(32678*sqrt(35) + 3613
75)*arctan(1/16416692599121449862 50*5969915^(3/4)*sqrt(826)*sqrt(299)*sqrt(
155)*sqrt(-147500*5969915^(1/4)*sqrt(826)*sqrt(155)*(97*sqrt(35)*sqrt(31) -
 70*sqrt(31))*sqrt(2*x + 1)*sqrt(32678*sqrt(35) + 361375) + 612639073637500
0*x + 612639073637500*sqrt(35) + 3063195368187500)*sqrt(32678*sqrt(35) + 36
1375)*(2*sqrt(35) - 97) - 1/1715389406185*5969915^(3/4)*sqrt(826)*sqrt(155)
*sqrt(2*x + 1)*sqrt(32678*sqrt(35) + 361375)*(2*sqrt(35) - 97) - 1/31*sqrt(
35)*sqrt(31) - 2/31*sqrt(31)) + 5969915^(1/4)*sqrt(826)*sqrt(155)*(32678*sq
rt(35)*sqrt(31)*(5*x^2 + 3*x + 2) - 361375*sqrt(31)*(5*x^2 + 3*x + 2))*sqrt
(32678*sqrt(35) + 361375)*log(147500/299*5969915^(1/4)*sqrt(826)*sqrt(155)*
(97*sqrt(35)*sqrt(31) - 70*sqrt(31))*sqrt(2*x + 1)*sqrt(32678*sqrt(35) + 36
1375) + 20489601125000*x + 2048960112500*sqrt(35) + 10244800562500) - 59699
```

```
15^(1/4)*sqrt(826)*sqrt(155)*(32678*sqrt(35)*sqrt(31)*(5*x^2 + 3*x + 2) - 3
61375*sqrt(31)*(5*x^2 + 3*x + 2))*sqrt(32678*sqrt(35) + 361375)*log(-147500
/299*5969915^(1/4)*sqrt(826)*sqrt(155)*(97*sqrt(35)*sqrt(31) - 70*sqrt(31))
*sqrt(2*x + 1)*sqrt(32678*sqrt(35) + 361375) + 20489601125000*x + 204896011
2500*sqrt(35) + 10244800562500) - 1287580425950*(54*x + 41)*sqrt(2*x + 1))/
(5*x^2 + 3*x + 2)
```

Sympy [A] time = 127.154, size = 246, normalized size = 0.87

$$-\frac{304(2x+1)^{\frac{3}{2}}}{5\left(-992x+620(2x+1)^2+372\right)}-\frac{896(2x+1)^{\frac{3}{2}}}{5\left(-6944x+4340(2x+1)^2+2604\right)}+\frac{608\sqrt{2x+1}}{25\left(-992x+620(2x+1)^2+372\right)}-\frac{}{25(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**(5/2)/(5*x**2+3*x+2)**2,x)
```

```
[Out] -304*(2*x + 1)**(3/2)/(5*(-992*x + 620*(2*x + 1)**2 + 372)) - 896*(2*x + 1)
**(3/2)/(5*(-6944*x + 4340*(2*x + 1)**2 + 2604)) + 608*sqrt(2*x + 1)/(25*(-
992*x + 620*(2*x + 1)**2 + 372)) - 12096*sqrt(2*x + 1)/(25*(-6944*x + 4340*
(2*x + 1)**2 + 2604)) - 304*RootSum(407144088666112*_t**4 + 3325152256*_t**
2 + 11045, Lambda(_t, _t*log(33312534528*_t**3/235 + 166784*_t/235 + sqrt(2
*x + 1))))/25 - 448*RootSum(19950060344639488*_t**4 + 498437272576*_t**2 +
10878125, Lambda(_t, _t*log(-11049511452672*_t**3/2205125 + 307918256*_t/22
05125 + sqrt(2*x + 1))))/25 + 64*RootSum(1722112*_t**4 + 1984*_t**2 + 5, La
mbda(_t, _t*log(-27776*_t**3/5 + 108*_t/5 + sqrt(2*x + 1))))/25 + 16*RootSu
m(1230080*_t**4 + 1984*_t**2 + 7, Lambda(_t, _t*log(9920*_t**3 + 8*_t + sqr
t(2*x + 1))))/5
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,x+1)^{\frac{5}{2}}}{\left(5\,x^2+3\,x+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] integrate((2*x + 1)^(5/2)/(5*x^2 + 3*x + 2)^2, x)
```

3.2318 $\int \frac{(1+2x)^{3/2}}{\left(2+3x+5x^2\right)^2} dx$

Optimal. Leaf size=270

$$-\frac{\sqrt{2x+1}(5-4x)}{31\left(5x^2+3x+2\right)}-\frac{1}{31}\sqrt{\frac{1}{310}\left(47\sqrt{35}-218\right)}\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}+\sqrt{35}\right)+\frac{1}{31}\sqrt{\frac{1}{310}\left(47\sqrt{35}-2\right.}$$

```
[Out] -((5 - 4*x)*Sqrt[1 + 2*x])/(31*(2 + 3*x + 5*x^2)) - (Sqrt[(2*(218 + 47*Sqrt
[35]))/155]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] - 10*Sqrt[1 + 2*x])/Sqrt[10*(-2
 + Sqrt[35])]])/31 + (Sqrt[(2*(218 + 47*Sqrt[35]))/155]*ArcTan[(Sqrt[10*(2
+ Sqrt[35])] + 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/31 - (Sqrt[(-21
8 + 47*Sqrt[35])/310]*Log[Sqrt[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x]
+ 5*(1 + 2*x)])/31 + (Sqrt[(-218 + 47*Sqrt[35])/310]*Log[Sqrt[35] + Sqrt[10
*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/31
```

Rubi [A] time = 0.363569, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {738, 826, 1169, 634, 618, 204, 628}

$$-\frac{\sqrt{2x+1}(5-4x)}{31\left(5x^2+3x+2\right)}-\frac{1}{31}\sqrt{\frac{1}{310}\left(47\sqrt{35}-218\right)}\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}+\sqrt{35}\right)+\frac{1}{31}\sqrt{\frac{1}{310}\left(47\sqrt{35}-2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^(3/2)/(2 + 3*x + 5*x^2)^2,x]
```


```
[Out] -((5 - 4*x)*Sqrt[1 + 2*x])/(31*(2 + 3*x + 5*x^2)) - (Sqrt[(2*(218 + 47*Sqrt
[35]))/155]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] - 10*Sqrt[1 + 2*x])/Sqrt[10*(-2
 + Sqrt[35])]])/31 + (Sqrt[(2*(218 + 47*Sqrt[35]))/155]*ArcTan[(Sqrt[10*(2
+ Sqrt[35])] + 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/31 - (Sqrt[(-21
8 + 47*Sqrt[35])/310]*Log[Sqrt[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x]
+ 5*(1 + 2*x)])/31 + (Sqrt[(-218 + 47*Sqrt[35])/310]*Log[Sqrt[35] + Sqrt[10
*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/31
```


Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(1+2x)^{3/2}}{\left(2+3x+5x^2\right)^2}\,dx = -\frac{(5-4x)\sqrt{1+2x}}{31\left(2+3x+5x^2\right)} + \frac{1}{31}\int \frac{9+4x}{\sqrt{1+2x}\left(2+3x+5x^2\right)}\,dx$$

$$= -\frac{(5-4x)\sqrt{1+2x}}{31\left(2+3x+5x^2\right)} + \frac{2}{31}\,\text{Subst}\left(\int \frac{14+4x^2}{7-4x^2+5x^4}\,dx, x, \sqrt{1+2x}\right)$$

$$= -\frac{(5-4x)\sqrt{1+2x}}{31\left(2+3x+5x^2\right)} + \frac{\text{Subst}\left(\int \frac{14\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}-\left(14-4\sqrt{\frac{7}{5}}\right)x}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}x+x^2}\,dx, x, \sqrt{1+2x}\right)}{31\sqrt{14\left(2+\sqrt{35}\right)}} + \frac{\text{Subst}\left(\int \frac{14\sqrt{\frac{2}{5}\left(2+\sqrt{\right.}\right.}}{\sqrt{\frac{7}{5}}+\sqrt{\frac{}{5}}}\right.}{31\sqrt{}}$$

$$= -\frac{(5-4x)\sqrt{1+2x}}{31\left(2+3x+5x^2\right)} + \frac{1}{155}\left(2+\sqrt{35}\right)\text{Subst}\left(\int \frac{1}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}x+x^2}\,dx, x, \sqrt{1+2x}\right)$$

$$= -\frac{(5-4x)\sqrt{1+2x}}{31\left(2+3x+5x^2\right)} - \frac{1}{31}\sqrt{\frac{1}{310}\left(-218+47\sqrt{35}\right)}\log\left(\sqrt{35}-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1+2x}+5(1+\right.$$

$$= -\frac{(5-4x)\sqrt{1+2x}}{31\left(2+3x+5x^2\right)} - \frac{1}{31}\sqrt{\frac{2}{155}\left(218+47\sqrt{35}\right)}\tan^{-1}\left(\sqrt{\frac{5}{2\left(-2+\sqrt{35}\right)}}\left(\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}-2\right.\right.$$

Mathematica [C] time = 0.718012, size = 141, normalized size = 0.52

$$\frac{\frac{155\sqrt{2x+1}(4x-5)}{5x^2+3x+2}+2\left(31-4i\sqrt{31}\right)\sqrt{10-5i\sqrt{31}}\tanh^{-1}\left(\frac{\sqrt{10x+5}}{\sqrt{2-i\sqrt{31}}}\right)+2\left(31+4i\sqrt{31}\right)\sqrt{10+5i\sqrt{31}}\tanh^{-1}\left(\frac{\sqrt{10x+5}}{\sqrt{2+i\sqrt{31}}}\right)}{4805}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^(3/2)/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] ((155*Sqrt[1 + 2*x]*(-5 + 4*x))/(2 + 3*x + 5*x^2) + 2*(31 - (4*I)*Sqrt[31])
*Sqrt[10 - (5*I)*Sqrt[31]]*ArcTanh[Sqrt[5 + 10*x]/Sqrt[2 - I*Sqrt[31]]] + 2
*(31 + (4*I)*Sqrt[31])*Sqrt[10 + (5*I)*Sqrt[31]]*ArcTanh[Sqrt[5 + 10*x]/Sqr
t[2 + I*Sqrt[31]]])/4805
```

Maple [B] time = 0.082, size = 642, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^(3/2)/(5*x^2+3*x+2)^2,x)
```

```
[Out] 16*(1/310*(1+2*x)^(3/2)-7/620*(1+2*x)^(1/2))/((1+2*x)^2-8/5*x+3/5)+39/9610*
ln(5^(1/2)*7^(1/2)+10*x+5+(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2)
)*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)-2/961*ln(5^(1/2)*7^(1/2)+10*x+5+(2*5^
(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2))*7^(1/2)*(2*5^(1/2)*7^(1/2)+4)
^(1/2)-39/961/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((10*(1+2*x)^(1/2)+5^(1/2
)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*(2*5^(1/2)*7^
(1/2)+4)+4/961/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((10*(1+2*x)^(1/2)+5^(1/
2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*(2*5
^(1/2)*7^(1/2)+4)*7^(1/2)+4/31/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((10*(1+
2*x)^(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/
2))*5^(1/2)*7^(1/2)-39/9610*ln(-(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)
^(1/2)+5^(1/2)*7^(1/2)+10*x+5)*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+2/961*ln
(-(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2)+5^(1/2)*7^(1/2)+10*x+5)
*7^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)-39/961/(10*5^(1/2)*7^(1/2)-20)^(1/2)*a
rctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/2))/(10*5^(1/2)*7
^(1/2)-20)^(1/2))*(2*5^(1/2)*7^(1/2)+4)+4/961/(10*5^(1/2)*7^(1/2)-20)^(1/2)
*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/2))/(10*5^(1/2)
*7^(1/2)-20)^(1/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)*7^(1/2)+4/31/(10*5^(1/2)*
7^(1/2)-20)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(
1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*7^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x+1)^{\frac{3}{2}}}{(5x^2+3x+2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] integrate((2*x + 1)^(3/2)/(5*x^2 + 3*x + 2)^2, x)
```

Fricas [B] time = 3.19425, size = 2195, normalized size = 8.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] -1/15191920450*(3844*77315^(1/4)*sqrt(155)*sqrt(35)*(5*x^2 + 3*x + 2)*sqrt(
20492*sqrt(35) + 154630)*arctan(1/3788913966425*77315^(3/4)*sqrt(155)*sqrt(
47)*sqrt(77315^(1/4)*sqrt(155)*(2*sqrt(35)*sqrt(31) - 35*sqrt(31))*sqrt(2*x
 + 1)*sqrt(20492*sqrt(35) + 154630) + 15808450*x + 1580845*sqrt(35) + 79042
25)*(sqrt(35)*sqrt(7) - 2*sqrt(7))*sqrt(20492*sqrt(35) + 154630) - 1/742997
15*77315^(3/4)*sqrt(155)*sqrt(2*x + 1)*sqrt(20492*sqrt(35) + 154630)*(sqrt(
35) - 2) + 1/31*sqrt(35)*sqrt(31) + 2/31*sqrt(31)) + 3844*77315^(1/4)*sqrt(
155)*sqrt(35)*(5*x^2 + 3*x + 2)*sqrt(20492*sqrt(35) + 154630)*arctan(1/7577
827932850*77315^(3/4)*sqrt(155)*sqrt(-188*77315^(1/4)*sqrt(155)*(2*sqrt(35)
*sqrt(31) - 35*sqrt(31))*sqrt(2*x + 1)*sqrt(20492*sqrt(35) + 154630) + 2971
988600*x + 297198860*sqrt(35) + 1485994300)*(sqrt(35)*sqrt(7) - 2*sqrt(7))*
sqrt(20492*sqrt(35) + 154630) - 1/74299715*77315^(3/4)*sqrt(155)*sqrt(2*x +
 1)*sqrt(20492*sqrt(35) + 154630)*(sqrt(35) - 2) - 1/31*sqrt(35)*sqrt(31) -
 2/31*sqrt(31)) - 77315^(1/4)*sqrt(155)*(218*sqrt(35)*sqrt(31)*(5*x^2 + 3*x
 + 2) - 1645*sqrt(31)*(5*x^2 + 3*x + 2))*sqrt(20492*sqrt(35) + 154630)*log(
188/7*77315^(1/4)*sqrt(155)*(2*sqrt(35)*sqrt(31) - 35*sqrt(31))*sqrt(2*x +
1)*sqrt(20492*sqrt(35) + 154630) + 424569800*x + 42456980*sqrt(35) + 212284
900) + 77315^(1/4)*sqrt(155)*(218*sqrt(35)*sqrt(31)*(5*x^2 + 3*x + 2) - 164
5*sqrt(31)*(5*x^2 + 3*x + 2))*sqrt(20492*sqrt(35) + 154630)*log(-188/7*7731
5^(1/4)*sqrt(155)*(2*sqrt(35)*sqrt(31) - 35*sqrt(31))*sqrt(2*x + 1)*sqrt(20
492*sqrt(35) + 154630) + 424569800*x + 42456980*sqrt(35) + 212284900) - 490
061950*(4*x - 5)*sqrt(2*x + 1))/(5*x^2 + 3*x + 2)
```

Sympy [A] time = 58.9013, size = 211, normalized size = 0.78

$$\frac{64(2x+1)^{\frac{3}{2}}}{-992x+620(2x+1)^2+372}-\frac{224(2x+1)^{\frac{3}{2}}}{-6944x+4340(2x+1)^2+2604}-\frac{128\sqrt{2x+1}}{5\left(-992x+620(2x+1)^2+372\right)}-\frac{30}{5\left(-6944x+4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**(3/2)/(5*x**2+3*x+2)**2,x)
```

```
[Out] 64*(2*x + 1)**(3/2)/(-992*x + 620*(2*x + 1)**2 + 372) - 224*(2*x + 1)**(3/2
)/(-6944*x + 4340*(2*x + 1)**2 + 2604) - 128*sqrt(2*x + 1)/(5*(-992*x + 620
*(2*x + 1)**2 + 372)) - 3024*sqrt(2*x + 1)/(5*(-6944*x + 4340*(2*x + 1)**2
+ 2604)) + 64*RootSum(407144088666112*_t**4 + 3325152256*_t**2 + 11045, Lam
bda(_t, _t*log(33312534528*_t**3/235 + 166784*_t/235 + sqrt(2*x + 1))))/5 -
 112*RootSum(19950060344639488*_t**4 + 498437272576*_t**2 + 10878125, Lambd
a(_t, _t*log(-11049511452672*_t**3/2205125 + 307918256*_t/2205125 + sqrt(2*
x + 1))))/5 + 16*RootSum(1722112*_t**4 + 1984*_t**2 + 5, Lambda(_t, _t*log(
-27776*_t**3/5 + 108*_t/5 + sqrt(2*x + 1))))/5
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x+1)^{\frac{3}{2}}}{(5x^2+3x+2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] integrate((2*x + 1)^(3/2)/(5*x^2 + 3*x + 2)^2, x)
```

3.2319 $\int \frac{\sqrt{1+2x}}{\left(2+3x+5x^2\right)^2} dx$

Optimal. Leaf size=270

$$\frac{\sqrt{2x+1}(10x+3)}{31\left(5x^2+3x+2\right)}+\frac{1}{31}\sqrt{\frac{1}{434}\left(47\sqrt{35}-218\right)}\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}+\sqrt{35}\right)-\frac{1}{31}\sqrt{\frac{1}{434}}\left(47\sqrt{35}-\right.$$

```
[Out] (Sqrt[1 + 2*x]*(3 + 10*x))/(31*(2 + 3*x + 5*x^2)) - (Sqrt[(2*(218 + 47*Sqrt
[35]))/217]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] - 10*Sqrt[1 + 2*x])/Sqrt[10*(-2
 + Sqrt[35])]])/31 + (Sqrt[(2*(218 + 47*Sqrt[35]))/217]*ArcTan[(Sqrt[10*(2
+ Sqrt[35])] + 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/31 + (Sqrt[(-21
8 + 47*Sqrt[35])/434]*Log[Sqrt[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x]
+ 5*(1 + 2*x)])/31 - (Sqrt[(-218 + 47*Sqrt[35])/434]*Log[Sqrt[35] + Sqrt[10
*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/31
```

Rubi [A] time = 0.34043, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {736, 826, 1169, 634, 618, 204, 628}

$$\frac{\sqrt{2x+1}(10x+3)}{31\left(5x^2+3x+2\right)}+\frac{1}{31}\sqrt{\frac{1}{434}\left(47\sqrt{35}-218\right)}\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}+\sqrt{35}\right)-\frac{1}{31}\sqrt{\frac{1}{434}}\left(47\sqrt{35}-\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 2*x]/(2 + 3*x + 5*x^2)^2,x]
```


```
[Out] (Sqrt[1 + 2*x]*(3 + 10*x))/(31*(2 + 3*x + 5*x^2)) - (Sqrt[(2*(218 + 47*Sqrt
[35]))/217]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] - 10*Sqrt[1 + 2*x])/Sqrt[10*(-2
 + Sqrt[35])]])/31 + (Sqrt[(2*(218 + 47*Sqrt[35]))/217]*ArcTan[(Sqrt[10*(2
+ Sqrt[35])] + 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/31 + (Sqrt[(-21
8 + 47*Sqrt[35])/434]*Log[Sqrt[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x]
+ 5*(1 + 2*x)])/31 - (Sqrt[(-218 + 47*Sqrt[35])/434]*Log[Sqrt[35] + Sqrt[10
*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/31
```


Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] - Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)
*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p + 1
), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 0] && (L
tQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c,
d, e, m, p, x]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+2x}}{\left(2+3x+5x^2\right)^2}\,dx = \frac{\sqrt{1+2x}(3+10x)}{31\left(2+3x+5x^2\right)} - \frac{1}{31}\int \frac{-7-10x}{\sqrt{1+2x}\left(2+3x+5x^2\right)}\,dx$$

$$= \frac{\sqrt{1+2x}(3+10x)}{31\left(2+3x+5x^2\right)} - \frac{2}{31}\,\mathrm{Subst}\left(\int \frac{-4-10x^2}{7-4x^2+5x^4}\,dx, x, \sqrt{1+2x}\right)$$

$$= \frac{\sqrt{1+2x}(3+10x)}{31\left(2+3x+5x^2\right)} - \frac{\mathrm{Subst}\left(\int \frac{-4\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}-\left(-4+2\sqrt{35}\right)x}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}x+x^2}\,dx, x, \sqrt{1+2x}\right)}{31\sqrt{14\left(2+\sqrt{35}\right)}} - \frac{\mathrm{Subst}\left(\int \frac{-4\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}+\ldots}{\sqrt{\frac{7}{5}}+\sqrt{\frac{2}{5}\left(2+\ldots\right.}}\right.}{31\sqrt{14\ldots}}$$

$$= \frac{\sqrt{1+2x}(3+10x)}{31\left(2+3x+5x^2\right)} + \frac{\left(35+2\sqrt{35}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}x+x^2}\,dx, x, \sqrt{1+2x}\right)}{1085} + \frac{\left(35+2\sqrt{35}\right)\mathrm{S}\ldots}{\ldots}$$

$$= \frac{\sqrt{1+2x}(3+10x)}{31\left(2+3x+5x^2\right)} + \frac{1}{31}\sqrt{\frac{1}{434}\left(-218+47\sqrt{35}\right)}\log\left(\sqrt{35}-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1+2x}+5(1+2x)\ldots\right.$$

$$= \frac{\sqrt{1+2x}(3+10x)}{31\left(2+3x+5x^2\right)} - \frac{1}{31}\sqrt{\frac{2}{217}\left(218+47\sqrt{35}\right)}\tan^{-1}\left(\sqrt{\frac{5}{2\left(-2+\sqrt{35}\right)}}\left(\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}-2\sqrt{1+\ldots}\right.\right.$$

Mathematica [C] time = 0.369282, size = 145, normalized size = 0.54

$$\frac{1}{31}\left(\frac{\sqrt{2x+1}(10x+3)}{5x^2+3x+2}+\frac{2\sqrt{10-5i\sqrt{31}}\left(62-39i\sqrt{31}\right)\tanh^{-1}\left(\frac{\sqrt{10x+5}}{\sqrt{2-i\sqrt{31}}}\right)+2\sqrt{10+5i\sqrt{31}}\left(62+39i\sqrt{31}\right)\tanh^{-1}\left(\frac{\sqrt{1}}{\sqrt{2}}\right.}{1085}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 2*x]/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] ((Sqrt[1 + 2*x]*(3 + 10*x))/(2 + 3*x + 5*x^2) + (2*Sqrt[10 - (5*I)*Sqrt[31]
]*(62 - (39*I)*Sqrt[31])*ArcTanh[Sqrt[5 + 10*x]/Sqrt[2 - I*Sqrt[31]]] + 2*S
qrt[10 + (5*I)*Sqrt[31]]*(62 + (39*I)*Sqrt[31])*ArcTanh[Sqrt[5 + 10*x]/Sqrt
[2 + I*Sqrt[31]]])/1085)/31
```

Maple [B] time = 0.243, size = 972, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^(1/2)/(5*x^2+3*x+2)^2,x)
```

```
[Out] 5/6727*(2/25*(-5425*7^(1/2)+2170*5^(1/2))/(2*5^(1/2)-5*7^(1/2))*(1+2*x)^(1/
2)+1/25*7^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)*(-1085*5^(1/2)+310*7^(1/2))/(2*
5^(1/2)-5*7^(1/2)))/(1/5*5^(1/2)*7^(1/2)+2*x+1+1/5*(2*5^(1/2)*7^(1/2)+4)^(1
/2)*5^(1/2)*(1+2*x)^(1/2))-109/6727/(2*5^(1/2)-5*7^(1/2))*ln(5+10*x+35^(1/2
)+(1+2*x)^(1/2)*(20+10*35^(1/2))^(1/2))*(2*35^(1/2)+4)^(1/2)*35^(1/2)+235/1
922/(2*5^(1/2)-5*7^(1/2))*ln(5+10*x+35^(1/2)+(1+2*x)^(1/2)*(20+10*35^(1/2))
^(1/2))*(2*35^(1/2)+4)^(1/2)+218/6727/(2*5^(1/2)-5*7^(1/2))/(-20+10*35^(1/2
))^(1/2)*arctan((10*(1+2*x)^(1/2)+(20+10*35^(1/2))^(1/2))/(-20+10*35^(1/2))
^(1/2))*(20+10*35^(1/2))^(1/2)*(2*35^(1/2)+4)^(1/2)*35^(1/2)-235/961/(2*5^(
1/2)-5*7^(1/2))/(-20+10*35^(1/2))^(1/2)*arctan((10*(1+2*x)^(1/2)+(20+10*35^
(1/2))^(1/2))/(-20+10*35^(1/2))^(1/2))*(20+10*35^(1/2))^(1/2)*(2*35^(1/2)+4
)^(1/2)-40/31/(2*5^(1/2)-5*7^(1/2))/(-20+10*35^(1/2))^(1/2)*arctan((10*(1+2
*x)^(1/2)+(20+10*35^(1/2))^(1/2))/(-20+10*35^(1/2))^(1/2))*5^(1/2)+80/217/(
2*5^(1/2)-5*7^(1/2))/(-20+10*35^(1/2))^(1/2)*arctan((10*(1+2*x)^(1/2)+(20+1
0*35^(1/2))^(1/2))/(-20+10*35^(1/2))^(1/2))*7^(1/2)-5/6727*(-2/25*(-5425*7^
(1/2)+2170*5^(1/2))/(2*5^(1/2)-5*7^(1/2))*(1+2*x)^(1/2)+1/25*7^(1/2)*(2*5^(
1/2)*7^(1/2)+4)^(1/2)*(-1085*5^(1/2)+310*7^(1/2))/(2*5^(1/2)-5*7^(1/2)))/(1
/5*5^(1/2)*7^(1/2)+2*x+1-1/5*(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1
/2))+109/6727/(2*5^(1/2)-5*7^(1/2))*ln(5+10*x+35^(1/2)-(1+2*x)^(1/2)*(20+10
*35^(1/2))^(1/2))*(2*35^(1/2)+4)^(1/2)*35^(1/2)-235/1922/(2*5^(1/2)-5*7^(1/
2))*ln(5+10*x+35^(1/2)-(1+2*x)^(1/2)*(20+10*35^(1/2))^(1/2))*(2*35^(1/2)+4)
^(1/2)+218/6727/(2*5^(1/2)-5*7^(1/2))/(-20+10*35^(1/2))^(1/2)*arctan((-(20+
10*35^(1/2))^(1/2)+10*(1+2*x)^(1/2))/(-20+10*35^(1/2))^(1/2))*(20+10*35^(1/
2))^(1/2)*(2*35^(1/2)+4)^(1/2)*35^(1/2)-235/961/(2*5^(1/2)-5*7^(1/2))/(-20+
10*35^(1/2))^(1/2)*arctan((-(20+10*35^(1/2))^(1/2)+10*(1+2*x)^(1/2))/(-20+1
0*35^(1/2))^(1/2))*(20+10*35^(1/2))^(1/2)*(2*35^(1/2)+4)^(1/2)-40/31/(2*5^(
1/2)-5*7^(1/2))/(-20+10*35^(1/2))^(1/2)*arctan((-(20+10*35^(1/2))^(1/2)+10*
(1+2*x)^(1/2))/(-20+10*35^(1/2))^(1/2))*5^(1/2)+80/217/(2*5^(1/2)-5*7^(1/2)
)/(-20+10*35^(1/2))^(1/2)*arctan((-(20+10*35^(1/2))^(1/2)+10*(1+2*x)^(1/2))
/(-20+10*35^(1/2))^(1/2))*7^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x+1}}{(5x^2+3x+2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x + 1)/(5*x^2 + 3*x + 2)^2, x)
```

Fricas [B] time = 3.09446, size = 2176, normalized size = 8.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/21268688630*(3844*77315^(1/4)*sqrt(217)*sqrt(35)*(5*x^2 + 3*x + 2)*sqrt(2
0492*sqrt(35) + 154630)*arctan(1/26522397764975*77315^(3/4)*sqrt(1645)*sqrt
(217)*sqrt(77315^(1/4)*sqrt(217)*(sqrt(35)*sqrt(31) - 2*sqrt(31))*sqrt(2*x
+ 1)*sqrt(20492*sqrt(35) + 154630) + 3161690*x + 316169*sqrt(35) + 1580845)
*sqrt(20492*sqrt(35) + 154630)*(2*sqrt(35) - 35) - 1/520098005*77315^(3/4)*
sqrt(217)*sqrt(2*x + 1)*sqrt(20492*sqrt(35) + 154630)*(2*sqrt(35) - 35) + 1
/31*sqrt(35)*sqrt(31) + 2/31*sqrt(31)) + 3844*77315^(1/4)*sqrt(217)*sqrt(35
)*(5*x^2 + 3*x + 2)*sqrt(20492*sqrt(35) + 154630)*arctan(1/53044795529950*7
7315^(3/4)*sqrt(217)*sqrt(-6580*77315^(1/4)*sqrt(217)*(sqrt(35)*sqrt(31) -
2*sqrt(31))*sqrt(2*x + 1)*sqrt(20492*sqrt(35) + 154630) + 20803920200*x + 2
080392020*sqrt(35) + 10401960100)*sqrt(20492*sqrt(35) + 154630)*(2*sqrt(35)
 - 35) - 1/520098005*77315^(3/4)*sqrt(217)*sqrt(2*x + 1)*sqrt(20492*sqrt(35
) + 154630)*(2*sqrt(35) - 35) - 1/31*sqrt(35)*sqrt(31) - 2/31*sqrt(31)) + 7
7315^(1/4)*sqrt(217)*(218*sqrt(35)*sqrt(31)*(5*x^2 + 3*x + 2) - 1645*sqrt(3
1)*(5*x^2 + 3*x + 2))*sqrt(20492*sqrt(35) + 154630)*log(6580*77315^(1/4)*sq
rt(217)*(sqrt(35)*sqrt(31) - 2*sqrt(31))*sqrt(2*x + 1)*sqrt(20492*sqrt(35)
+ 154630) + 20803920200*x + 2080392020*sqrt(35) + 10401960100) - 77315^(1/4
)*sqrt(217)*(218*sqrt(35)*sqrt(31)*(5*x^2 + 3*x + 2) - 1645*sqrt(31)*(5*x^2
 + 3*x + 2))*sqrt(20492*sqrt(35) + 154630)*log(-6580*77315^(1/4)*sqrt(217)*
(sqrt(35)*sqrt(31) - 2*sqrt(31))*sqrt(2*x + 1)*sqrt(20492*sqrt(35) + 154630
) + 20803920200*x + 2080392020*sqrt(35) + 10401960100) + 686086730*(10*x +
3)*sqrt(2*x + 1))/(5*x^2 + 3*x + 2)
```

Sympy [A] time = 5.22867, size = 83, normalized size = 0.31

$$\frac{80(2x+1)^{\frac{3}{2}}}{-992x+620(2x+1)^2+372} - \frac{32\sqrt{2x+1}}{-992x+620(2x+1)^2+372} + 16\,\mathrm{RootSum}\left(407144088666112t^4 + 3325152256t^2 + 11\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**(1/2)/(5*x**2+3*x+2)**2,x)
```

```
[Out] 80*(2*x + 1)**(3/2)/(-992*x + 620*(2*x + 1)**2 + 372) - 32*sqrt(2*x + 1)/(-
992*x + 620*(2*x + 1)**2 + 372) + 16*RootSum(407144088666112*_t**4 + 332515
2256*_t**2 + 11045, Lambda(_t, _t*log(33312534528*_t**3/235 + 166784*_t/235
 + sqrt(2*x + 1))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x+1}}{\left(5x^2+3x+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(2*x + 1)/(5*x^2 + 3*x + 2)^2, x)
```

3.2320 $\int \frac{1}{\sqrt{1+2x}\left(2+3x+5x^2\right)^2}\,dx$

Optimal. Leaf size=270

$$\frac{\sqrt{2x+1}(20x+37)}{217\left(5x^2+3x+2\right)}-\frac{1}{217}\sqrt{\frac{1}{434}\left(10325\sqrt{35}-32678\right)}\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}+\sqrt{35}\right)+\frac{1}{217}\sqrt{\frac{1}{434}\left(10\right.}$$

```
[Out] (Sqrt[1 + 2*x]*(37 + 20*x))/(217*(2 + 3*x + 5*x^2)) - (Sqrt[(2*(32678 + 103
25*Sqrt[35]))/217]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] - 10*Sqrt[1 + 2*x])/Sqrt
[10*(-2 + Sqrt[35])]])/217 + (Sqrt[(2*(32678 + 10325*Sqrt[35]))/217]*ArcTan
[(Sqrt[10*(2 + Sqrt[35])] + 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/21
7 - (Sqrt[(-32678 + 10325*Sqrt[35])/434]*Log[Sqrt[35] - Sqrt[10*(2 + Sqrt[3
5])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/217 + (Sqrt[(-32678 + 10325*Sqrt[35])/43
4]*Log[Sqrt[35] + Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/217
```

Rubi [A] time = 0.323751, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {740, 826, 1169, 634, 618, 204, 628}

$$\frac{\sqrt{2x+1}(20x+37)}{217\left(5x^2+3x+2\right)}-\frac{1}{217}\sqrt{\frac{1}{434}\left(10325\sqrt{35}-32678\right)}\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}+\sqrt{35}\right)+\frac{1}{217}\sqrt{\frac{1}{434}\left(10\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + 2*x]*(2 + 3*x + 5*x^2)^2),x]
```


```
[Out] (Sqrt[1 + 2*x]*(37 + 20*x))/(217*(2 + 3*x + 5*x^2)) - (Sqrt[(2*(32678 + 103
25*Sqrt[35]))/217]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] - 10*Sqrt[1 + 2*x])/Sqrt
[10*(-2 + Sqrt[35])]])/217 + (Sqrt[(2*(32678 + 10325*Sqrt[35]))/217]*ArcTan
[(Sqrt[10*(2 + Sqrt[35])] + 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/21
7 - (Sqrt[(-32678 + 10325*Sqrt[35])/434]*Log[Sqrt[35] - Sqrt[10*(2 + Sqrt[3
5])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/217 + (Sqrt[(-32678 + 10325*Sqrt[35])/43
4]*Log[Sqrt[35] + Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/217
```


Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
```

```
a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+2x}\left(2+3x+5x^2\right)^2}dx = \frac{\sqrt{1+2x}(37+20x)}{217\left(2+3x+5x^2\right)} + \frac{1}{217}\int \frac{107+20x}{\sqrt{1+2x}\left(2+3x+5x^2\right)}dx$$

$$= \frac{\sqrt{1+2x}(37+20x)}{217\left(2+3x+5x^2\right)} + \frac{2}{217}\text{Subst}\left(\int \frac{194+20x^2}{7-4x^2+5x^4}dx, x, \sqrt{1+2x}\right)$$

$$= \frac{\sqrt{1+2x}(37+20x)}{217\left(2+3x+5x^2\right)} + \frac{\text{Subst}\left(\int \frac{194\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}-\left(194-4\sqrt{35}\right)x}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}x+x^2}dx, x, \sqrt{1+2x}\right)}{217\sqrt{14\left(2+\sqrt{35}\right)}} + \frac{\text{Subst}\left(\int \frac{1}{}\right.}{}$$

$$= \frac{\sqrt{1+2x}(37+20x)}{217\left(2+3x+5x^2\right)} + \frac{\left(70+97\sqrt{35}\right)\text{Subst}\left(\int \frac{1}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}x+x^2}dx, x, \sqrt{1+2x}\right)}{7595} + \frac{(70}{}$$

$$= \frac{\sqrt{1+2x}(37+20x)}{217\left(2+3x+5x^2\right)} - \frac{1}{217}\sqrt{\frac{1}{434}\left(-32678+10325\sqrt{35}\right)}\log\left(\sqrt{35}-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1\right.}$$

$$= \frac{\sqrt{1+2x}(37+20x)}{217\left(2+3x+5x^2\right)} - \frac{1}{217}\sqrt{\frac{2}{217}\left(32678+10325\sqrt{35}\right)}\tan^{-1}\left(\sqrt{\frac{5}{2\left(-2+\sqrt{35}\right)}}\left(\sqrt{\frac{2}{5}}\left(2\right.\right.\right.$$

Mathematica [C] time = 0.308927, size = 166, normalized size = 0.61

$$\frac{1}{217}\left(\frac{\sqrt{2x+1}(20x+37)}{5x^2+3x+2} + \frac{2\sqrt{10-5i\sqrt{31}}\left(101\sqrt{31}-62i\right)\tanh^{-1}\left(\frac{\sqrt{10x+5}}{\sqrt{2-i\sqrt{31}}}\right)}{31\left(\sqrt{31}+2i\right)} + \frac{2\sqrt{10+5i\sqrt{31}}\left(101\sqrt{31}+62i\right)\tanh^{-1}\left(\right.}{31\left(\sqrt{31}-2i\right)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + 2*x]*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] ((Sqrt[1 + 2*x]*(37 + 20*x))/(2 + 3*x + 5*x^2) + (2*Sqrt[10 - (5*I)*Sqrt[31
]]*(-62*I + 101*Sqrt[31])*ArcTanh[Sqrt[5 + 10*x]/Sqrt[2 - I*Sqrt[31]]])/(31
*(2*I + Sqrt[31])) + (2*Sqrt[10 + (5*I)*Sqrt[31]]*(62*I + 101*Sqrt[31])*Arc
Tanh[Sqrt[5 + 10*x]/Sqrt[2 + I*Sqrt[31]]])/(31*(-2*I + Sqrt[31])))/217
```

Maple [B] time = 0.223, size = 968, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2*x)^(1/2)/(5*x^2+3*x+2)^2,x)
```

```
[Out] 5/47089*(2/37375*(-3244150*5^(1/2)*7^(1/2)+6488300)/(2*5^(1/2)-5*7^(1/2))*5
^(1/2)*(1+2*x)^(1/2)+1/7475/(2*5^(1/2)-5*7^(1/2))*(2*5^(1/2)*7^(1/2)+4)^(1/
2)*(-1946490*5^(1/2)*7^(1/2)+13949845))/(1/5*5^(1/2)*7^(1/2)+2*x+1+1/5*(2*5
^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2))-4063/94178/(2*5^(1/2)-5*7^(1
/2))*ln(5+10*x+35^(1/2)+(1+2*x)^(1/2)*(20+10*35^(1/2))^(1/2))*(2*35^(1/2)+4
)^(1/2)*35^(1/2)+1165/6727/(2*5^(1/2)-5*7^(1/2))*ln(5+10*x+35^(1/2)+(1+2*x)
^(1/2)*(20+10*35^(1/2))^(1/2))*(2*35^(1/2)+4)^(1/2)+4063/47089/(2*5^(1/2)-5
```

```
*7^(1/2))/(-20+10*35^(1/2))^(1/2)*arctan((10*(1+2*x)^(1/2)+(20+10*35^(1/2))
^(1/2))/(-20+10*35^(1/2))^(1/2))*(20+10*35^(1/2))^(1/2)*(2*35^(1/2)+4)^(1/2
)*35^(1/2)-2330/6727/(2*5^(1/2)-5*7^(1/2))/(-20+10*35^(1/2))^(1/2)*arctan((
10*(1+2*x)^(1/2)+(20+10*35^(1/2))^(1/2))/(-20+10*35^(1/2))^(1/2))*(20+10*35
^(1/2))^(1/2)*(2*35^(1/2)+4)^(1/2)-1940/217/(2*5^(1/2)-5*7^(1/2))/(-20+10*3
5^(1/2))^(1/2)*arctan((10*(1+2*x)^(1/2)+(20+10*35^(1/2))^(1/2))/(-20+10*35^
(1/2))^(1/2))*5^(1/2)+3880/1519/(2*5^(1/2)-5*7^(1/2))/(-20+10*35^(1/2))^(1/
2)*arctan((10*(1+2*x)^(1/2)+(20+10*35^(1/2))^(1/2))/(-20+10*35^(1/2))^(1/2)
)*7^(1/2)+5/47089*(2/37375*(-3244150*5^(1/2)*7^(1/2)+6488300)/(2*5^(1/2)-5*
7^(1/2))*5^(1/2)*(1+2*x)^(1/2)-1/7475/(2*5^(1/2)-5*7^(1/2))*(2*5^(1/2)*7^(1
/2)+4)^(1/2)*(-1946490*5^(1/2)*7^(1/2)+13949845))/(1/5*5^(1/2)*7^(1/2)+2*x+
1-1/5*(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2))+4063/94178/(2*5^(1
/2)-5*7^(1/2))*ln(5+10*x+35^(1/2)-(1+2*x)^(1/2)*(20+10*35^(1/2))^(1/2))*(2*
35^(1/2)+4)^(1/2)*35^(1/2)-1165/6727/(2*5^(1/2)-5*7^(1/2))*ln(5+10*x+35^(1/
2)-(1+2*x)^(1/2)*(20+10*35^(1/2))^(1/2))*(2*35^(1/2)+4)^(1/2)+4063/47089/(2
*5^(1/2)-5*7^(1/2))/(-20+10*35^(1/2))^(1/2)*arctan((-(20+10*35^(1/2))^(1/2)
+10*(1+2*x)^(1/2))/(-20+10*35^(1/2))^(1/2))*(20+10*35^(1/2))^(1/2)*(2*35^(1
/2)+4)^(1/2)*35^(1/2)-2330/6727/(2*5^(1/2)-5*7^(1/2))/(-20+10*35^(1/2))^(1/
2)*arctan((-(20+10*35^(1/2))^(1/2)+10*(1+2*x)^(1/2))/(-20+10*35^(1/2))^(1/2
))*(20+10*35^(1/2))^(1/2)*(2*35^(1/2)+4)^(1/2)-1940/217/(2*5^(1/2)-5*7^(1/2
))/(-20+10*35^(1/2))^(1/2)*arctan((-(20+10*35^(1/2))^(1/2)+10*(1+2*x)^(1/2)
)/(-20+10*35^(1/2))^(1/2))*5^(1/2)+3880/1519/(2*5^(1/2)-5*7^(1/2))/(-20+10*
35^(1/2))^(1/2)*arctan((-(20+10*35^(1/2))^(1/2)+10*(1+2*x)^(1/2))/(-20+10*3
5^(1/2))^(1/2))*7^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)^2\sqrt{2\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^2*sqrt(2*x + 1)), x)
```

Fricas [B] time = 3.06592, size = 2591, normalized size = 9.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] -1/391166933403610*(1149356*5969915^(1/4)*sqrt(826)*sqrt(217)*sqrt(35)*(5*x
^2 + 3*x + 2)*sqrt(32678*sqrt(35) + 361375)*arctan(1/22983369638770029807 5*
5969915^(3/4)*sqrt(826)*sqrt(299)*sqrt(295)*sqrt(217)*sqrt(5969915^(1/4)*sq
rt(826)*sqrt(217)*(2*sqrt(35)*sqrt(31) - 97*sqrt(31))*sqrt(2*x + 1)*sqrt(32
678*sqrt(35) + 361375) + 8306970490*x + 830697049*sqrt(35) + 4153485245)*sq
rt(32678*sqrt(35) + 361375)*(97*sqrt(35) - 70) - 1/12007725843295*5969915^(
3/4)*sqrt(826)*sqrt(217)*sqrt(2*x + 1)*sqrt(32678*sqrt(35) + 361375)*(97*sq
rt(35) - 70) + 1/31*sqrt(35)*sqrt(31) + 2/31*sqrt(31)) + 1149356*5969915^(1
/4)*sqrt(826)*sqrt(217)*sqrt(35)*(5*x^2 + 3*x + 2)*sqrt(32678*sqrt(35) + 36
1375)*arctan(1/804417937356951043262 50*5969915^(3/4)*sqrt(826)*sqrt(299)*sq
```

```
rt(217)*sqrt(-36137500*5969915^(1/4)*sqrt(826)*sqrt(217)*(2*sqrt(35)*sqrt(3
1) - 97*sqrt(31))*sqrt(2*x + 1)*sqrt(32678*sqrt(35) + 361375) + 30019314608
2375000*x + 300193146082375000*sqrt(35) + 150096573041187500)*sqrt(32678*sqr
t(35) + 361375)*(97*sqrt(35) - 70) - 1/12007725843295*5969915^(3/4)*sqrt(82
6)*sqrt(217)*sqrt(2*x + 1)*sqrt(32678*sqrt(35) + 361375)*(97*sqrt(35) - 70)
 - 1/31*sqrt(35)*sqrt(31) - 2/31*sqrt(31)) - 5969915^(1/4)*sqrt(826)*sqrt(2
17)*(32678*sqrt(35)*sqrt(31)*(5*x^2 + 3*x + 2) - 361375*sqrt(31)*(5*x^2 + 3
*x + 2))*sqrt(32678*sqrt(35) + 361375)*log(36137500/299*5969915^(1/4)*sqrt(
826)*sqrt(217)*(2*sqrt(35)*sqrt(31) - 97*sqrt(31))*sqrt(2*x + 1)*sqrt(32678
*sqrt(35) + 361375) + 1003990455125000*x + 100399045512500*sqrt(35) + 50199
5227562500) + 5969915^(1/4)*sqrt(826)*sqrt(217)*(32678*sqrt(35)*sqrt(31)*(5
*x^2 + 3*x + 2) - 361375*sqrt(31)*(5*x^2 + 3*x + 2))*sqrt(32678*sqrt(35) +
361375)*log(-36137500/299*5969915^(1/4)*sqrt(826)*sqrt(217)*(2*sqrt(35)*sqr
t(31) - 97*sqrt(31))*sqrt(2*x + 1)*sqrt(32678*sqrt(35) + 361375) + 10039904
55125000*x + 100399045512500*sqrt(35) + 501995227562500) - 1802612596330*(2
0*x + 37)*sqrt(2*x + 1))/(5*x^2 + 3*x + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x+1}\left(5x^2+3x+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)**(1/2)/(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral(1/(sqrt(2*x + 1)*(5*x**2 + 3*x + 2)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(5x^2+3x+2\right)^2\sqrt{2x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^2*sqrt(2*x + 1)), x)
```

3.2321 $\int \frac{1}{(1+2x)^{3/2}(2+3x+5x^2)^2}\,dx$

Optimal. Leaf size=283

$$\frac{20x+37}{217\sqrt{2x+1}\left(5x^2+3x+2\right)}-\frac{604}{1519\sqrt{2x+1}}-\frac{\sqrt{\frac{1}{434}\left(5682718+968975\sqrt{35}\right)}\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}\right.}{1519}$$

```
[Out] -604/(1519*Sqrt[1 + 2*x]) + (37 + 20*x)/(217*Sqrt[1 + 2*x]*(2 + 3*x + 5*x^2
)) + (Sqrt[(2*(-5682718 + 968975*Sqrt[35]))/217]*ArcTan[(Sqrt[10*(2 + Sqrt[
35])] - 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/1519 - (Sqrt[(2*(-5682
718 + 968975*Sqrt[35]))/217]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] + 10*Sqrt[1 +
2*x])/Sqrt[10*(-2 + Sqrt[35])]])/1519 - (Sqrt[(5682718 + 968975*Sqrt[35])/4
34]*Log[Sqrt[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/15
19 + (Sqrt[(5682718 + 968975*Sqrt[35])/434]*Log[Sqrt[35] + Sqrt[10*(2 + Sqr
t[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/1519
```

Rubi [A] time = 0.366829, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {740, 828, 826, 1169, 634, 618, 204, 628}

$$\frac{20x+37}{217\sqrt{2x+1}\left(5x^2+3x+2\right)}-\frac{604}{1519\sqrt{2x+1}}-\frac{\sqrt{\frac{1}{434}\left(5682718+968975\sqrt{35}\right)}\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}\right.}{1519}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + 2*x)^(3/2)*(2 + 3*x + 5*x^2)^2),x]
```


```
[Out] -604/(1519*Sqrt[1 + 2*x]) + (37 + 20*x)/(217*Sqrt[1 + 2*x]*(2 + 3*x + 5*x^2
)) + (Sqrt[(2*(-5682718 + 968975*Sqrt[35]))/217]*ArcTan[(Sqrt[10*(2 + Sqrt[
35])] - 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/1519 - (Sqrt[(2*(-5682
718 + 968975*Sqrt[35]))/217]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] + 10*Sqrt[1 +
2*x])/Sqrt[10*(-2 + Sqrt[35])]])/1519 - (Sqrt[(5682718 + 968975*Sqrt[35])/4
34]*Log[Sqrt[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/15
19 + (Sqrt[(5682718 + 968975*Sqrt[35])/434]*Log[Sqrt[35] + Sqrt[10*(2 + Sqr
t[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/1519
```


Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
```

```
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(1+2x)^{3/2}\left(2+3x+5x^2\right)^2}\,dx = \frac{37+20x}{217\sqrt{1+2x}\left(2+3x+5x^2\right)}+\frac{1}{217}\int \frac{181+60x}{(1+2x)^{3/2}\left(2+3x+5x^2\right)}\,dx$$

$$= -\frac{604}{1519\sqrt{1+2x}}+\frac{37+20x}{217\sqrt{1+2x}\left(2+3x+5x^2\right)}+\frac{\int \frac{59-1510x}{\sqrt{1+2x}(2+3x+5x^2)}\,dx}{1519}$$

$$= -\frac{604}{1519\sqrt{1+2x}}+\frac{37+20x}{217\sqrt{1+2x}\left(2+3x+5x^2\right)}+\frac{2\,\text{Subst}\left(\int \frac{1628-1510x^2}{7-4x^2+5x^4}\,dx, x, \sqrt{1+\right.}{1519}$$

$$= -\frac{604}{1519\sqrt{1+2x}}+\frac{37+20x}{217\sqrt{1+2x}\left(2+3x+5x^2\right)}+\frac{\text{Subst}\left(\int \frac{1628\sqrt{\frac{2}{5}(2+\sqrt{35})}-(1628+302}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}(2+\sqrt{35})}x+x^2}\right.}{1519\sqrt{14\,(2+\sqrt{}}}$$

$$= -\frac{604}{1519\sqrt{1+2x}}+\frac{37+20x}{217\sqrt{1+2x}\left(2+3x+5x^2\right)}+\frac{\left(-5285+814\sqrt{35}\right)\text{Subst}\left(\int \frac{}{\sqrt{\frac{7}{5}}}\right.}{5316}$$

$$= -\frac{604}{1519\sqrt{1+2x}}+\frac{37+20x}{217\sqrt{1+2x}\left(2+3x+5x^2\right)}-\frac{\sqrt{\frac{1}{434}\left(5682718+968975\sqrt{35}\right)}\,\text{lo}}{}$$

$$= -\frac{604}{1519\sqrt{1+2x}}+\frac{37+20x}{217\sqrt{1+2x}\left(2+3x+5x^2\right)}+\frac{\sqrt{\frac{2}{217}\left(-5682718+968975\sqrt{35}\right)}}{}$$

Mathematica [C] time = 0.392974, size = 158, normalized size = 0.56

$$\frac{1}{217}\left(\frac{20x+37}{\sqrt{2x+1}\left(5x^2+3x+2\right)}-\frac{604}{7\sqrt{2x+1}}+\frac{2\sqrt{10-5i\sqrt{31}}\left(25234+3657i\sqrt{31}\right)\tanh^{-1}\left(\frac{\sqrt{10x+5}}{\sqrt{2-i\sqrt{31}}}\right)+2\sqrt{10+5i\sqrt{31}}\,(}{7595}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + 2*x)^(3/2)*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] (-604/(7*Sqrt[1 + 2*x]) + (37 + 20*x)/(Sqrt[1 + 2*x]*(2 + 3*x + 5*x^2)) + (
2*Sqrt[10 - (5*I)*Sqrt[31]]*(25234 + (3657*I)*Sqrt[31])*ArcTanh[Sqrt[5 + 10
*x]/Sqrt[2 - I*Sqrt[31]]] + 2*Sqrt[10 + (5*I)*Sqrt[31]]*(25234 - (3657*I)*S
qrt[31])*ArcTanh[Sqrt[5 + 10*x]/Sqrt[2 + I*Sqrt[31]]])/7595)/217
```

Maple [B] time = 0.081, size = 651, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2*x)^(3/2)/(5*x^2+3*x+2)^2,x)
```

```
[Out] -16/49*(27/124*(1+2*x)^(3/2)-89/310*(1+2*x)^(1/2))/((1+2*x)^2-8/5*x+3/5)+25
6/47089*ln(5^(1/2)*7^(1/2)+10*x+5+(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*
x)^(1/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+3657/659246*ln(5^(1/2)*7^(1/2
```

```
)+10*x+5+(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2))*7^(1/2)*(2*5^(1
/2)*7^(1/2)+4)^(1/2)-2560/47089/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((10*(1
+2*x)^(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1
/2))*(2*5^(1/2)*7^(1/2)+4)-3657/329623/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan
((10*(1+2*x)^(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2)
-20)^(1/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)*7^(1/2)+3256/10633/(10*5^(1/2)*7^
(1/2)-20)^(1/2)*arctan((10*(1+2*x)^(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2
))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*7^(1/2)-256/47089*ln(-(2*5^(1/2)*
7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2)+5^(1/2)*7^(1/2)+10*x+5)*5^(1/2)*(2*5
^(1/2)*7^(1/2)+4)^(1/2)-3657/659246*ln(-(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)
*(1+2*x)^(1/2)+5^(1/2)*7^(1/2)+10*x+5)*7^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)-
2560/47089/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/2
)+4)^(1/2)+10*(1+2*x)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*(2*5^(1/2)*7^(1
/2)+4)-3657/329623/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2
)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)
*(2*5^(1/2)*7^(1/2)+4)*7^(1/2)+3256/10633/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arc
tan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/2))/(10*5^(1/2)*7^(
1/2)-20)^(1/2))*5^(1/2)*7^(1/2)-16/49/(1+2*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(5x^2+3x+2\right)^2(2x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^2*(2*x + 1)^(3/2)), x)
```

Fricas [B] time = 2.92854, size = 3001, normalized size = 10.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/2682041589056045650*(16794436*21898835^(1/4)*sqrt(217)*sqrt(35)*(10*x^3 +
 11*x^2 + 7*x + 2)*sqrt(-11012823348100*sqrt(35) + 65723878543750)*arctan(1
/8446577952711550260411 5125*21898835^(3/4)*sqrt(4369)*sqrt(791)*sqrt(217)*s
qrt(21898835^(1/4)*sqrt(217)*(151*sqrt(35)*sqrt(31) + 814*sqrt(31))*sqrt(2*
x + 1)*sqrt(-11012823348100*sqrt(35) + 65723878543750) + 8136694311550*x +
813669431155*sqrt(35) + 4068347155775)*(814*sqrt(35) + 5285)*sqrt(-11012823
348100*sqrt(35) + 65723878543750) - 1/22526438201526175*21898835^(3/4)*sqrt
(217)*sqrt(2*x + 1)*(814*sqrt(35) + 5285)*sqrt(-11012823348100*sqrt(35) + 6
5723878543750) - 1/31*sqrt(35)*sqrt(31) - 2/31*sqrt(31)) + 16794436*2189883
5^(1/4)*sqrt(217)*sqrt(35)*(10*x^3 + 11*x^2 + 7*x + 2)*sqrt(-11012823348100
*sqrt(35) + 65723878543750)*arctan(1/2069411598414329813800820562 50*2189883
5^(3/4)*sqrt(4369)*sqrt(217)*sqrt(-4747977500*21898835^(1/4)*sqrt(217)*(151
*sqrt(35)*sqrt(31) + 814*sqrt(31))*sqrt(2*x + 1)*sqrt(-11012823348100*sqrt(
35) + 65723878543750) + 386328415156173901250 00*x + 386328415156173901250 0*
sqrt(35) + 193164207578086950625 00)*(814*sqrt(35) + 5285)*sqrt(-11012823348
100*sqrt(35) + 65723878543750) - 1/22526438201526175*21898835^(3/4)*sqrt(21
```

```
7)*sqrt(2*x + 1)*(814*sqrt(35) + 5285)*sqrt(-11012823348100*sqrt(35) + 6572
3878543750) + 1/31*sqrt(35)*sqrt(31) + 2/31*sqrt(31)) + 21898835^(1/4)*sqrt
(217)*(5682718*sqrt(35)*sqrt(31)*(10*x^3 + 11*x^2 + 7*x + 2) + 33914125*sqr
t(31)*(10*x^3 + 11*x^2 + 7*x + 2))*sqrt(-11012823348100*sqrt(35) + 65723878
543750)*log(4747977500/4369*21898835^(1/4)*sqrt(217)*(151*sqrt(35)*sqrt(31)
 + 814*sqrt(31))*sqrt(2*x + 1)*sqrt(-11012823348100*sqrt(35) + 657238785437
50) + 8842490619276125000*x + 884249061927612500*sqrt(35) + 442124530963806
2500) - 21898835^(1/4)*sqrt(217)*(5682718*sqrt(35)*sqrt(31)*(10*x^3 + 11*x^
2 + 7*x + 2) + 33914125*sqrt(31)*(10*x^3 + 11*x^2 + 7*x + 2))*sqrt(-1101282
3348100*sqrt(35) + 65723878543750)*log(-4747977500/4369*21898835^(1/4)*sqrt
(217)*(151*sqrt(35)*sqrt(31) + 814*sqrt(31))*sqrt(2*x + 1)*sqrt(-1101282334
8100*sqrt(35) + 65723878543750) + 8842490619276125000*x + 88424906192761250
0*sqrt(35) + 4421245309638062500) - 1765662665606350*(3020*x^2 + 1672*x + 9
49)*sqrt(2*x + 1))/(10*x^3 + 11*x^2 + 7*x + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x+1)^{\frac{3}{2}}\left(5x^2+3x+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)**(3/2)/(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral(1/((2*x + 1)**(3/2)*(5*x**2 + 3*x + 2)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)^2(2\,x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^2*(2*x + 1)^(3/2)), x)
```

3.2322 $\int \frac{1}{(1+2x)^{5/2}\left(2+3x+5x^2\right)^2}\,dx$

Optimal. Leaf size=296

$$\frac{20x+37}{217(2x+1)^{3/2}\left(5x^2+3x+2\right)}-\frac{4680}{10633\sqrt{2x+1}}-\frac{820}{4557(2x+1)^{3/2}}-\frac{5\sqrt{\frac{1}{434}\left(2632525\sqrt{35}-12504542\right)}\log\left(5(2x+1)-\right.}{10633}$$

```
[Out] -820/(4557*(1 + 2*x)^(3/2)) - 4680/(10633*Sqrt[1 + 2*x]) + (37 + 20*x)/(217
*(1 + 2*x)^(3/2)*(2 + 3*x + 5*x^2)) + (5*Sqrt[(2*(12504542 + 2632525*Sqrt[3
5]))/217]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] - 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 +
 Sqrt[35])]])/10633 - (5*Sqrt[(2*(12504542 + 2632525*Sqrt[35]))/217]*ArcTan
[(Sqrt[10*(2 + Sqrt[35])] + 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/10
633 - (5*Sqrt[(-12504542 + 2632525*Sqrt[35])/434]*Log[Sqrt[35] - Sqrt[10*(2
 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/10633 + (5*Sqrt[(-12504542 + 26
32525*Sqrt[35])/434]*Log[Sqrt[35] + Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] +
 5*(1 + 2*x)])/10633
```

Rubi [A] time = 0.44318, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {740, 828, 826, 1169, 634, 618, 204, 628}

$$\frac{20x+37}{217(2x+1)^{3/2}\left(5x^2+3x+2\right)}-\frac{4680}{10633\sqrt{2x+1}}-\frac{820}{4557(2x+1)^{3/2}}-\frac{5\sqrt{\frac{1}{434}\left(2632525\sqrt{35}-12504542\right)}\log\left(5(2x+1)-\right.}{10633}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + 2*x)^(5/2)*(2 + 3*x + 5*x^2)^2),x]
```


```
[Out] -820/(4557*(1 + 2*x)^(3/2)) - 4680/(10633*Sqrt[1 + 2*x]) + (37 + 20*x)/(217
*(1 + 2*x)^(3/2)*(2 + 3*x + 5*x^2)) + (5*Sqrt[(2*(12504542 + 2632525*Sqrt[3
5]))/217]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] - 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 +
 Sqrt[35])]])/10633 - (5*Sqrt[(2*(12504542 + 2632525*Sqrt[35]))/217]*ArcTan
[(Sqrt[10*(2 + Sqrt[35])] + 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/10
633 - (5*Sqrt[(-12504542 + 2632525*Sqrt[35])/434]*Log[Sqrt[35] - Sqrt[10*(2
 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/10633 + (5*Sqrt[(-12504542 + 26
32525*Sqrt[35])/434]*Log[Sqrt[35] + Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] +
 5*(1 + 2*x)])/10633
```


Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(1+2x)^{5/2}\left(2+3x+5x^2\right)^2}\,dx = \frac{37+20x}{217(1+2x)^{3/2}\left(2+3x+5x^2\right)} + \frac{1}{217}\int \frac{255+100x}{(1+2x)^{5/2}\left(2+3x+5x^2\right)}\,dx$$

$$= -\frac{820}{4557(1+2x)^{3/2}} + \frac{37+20x}{217(1+2x)^{3/2}\left(2+3x+5x^2\right)} + \frac{\int \frac{145-2050x}{(1+2x)^{3/2}(2+3x+5x^2)}\,dx}{1519}$$

$$= -\frac{820}{4557(1+2x)^{3/2}} - \frac{4680}{10633\sqrt{1+2x}} + \frac{37+20x}{217(1+2x)^{3/2}\left(2+3x+5x^2\right)} + \frac{\int \frac{-8345-11700\ldots}{\sqrt{1+2x}(2+3x+5\ldots}}{10633}$$

$$= -\frac{820}{4557(1+2x)^{3/2}} - \frac{4680}{10633\sqrt{1+2x}} + \frac{37+20x}{217(1+2x)^{3/2}\left(2+3x+5x^2\right)} + \frac{2\,\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{820}{4557(1+2x)^{3/2}} - \frac{4680}{10633\sqrt{1+2x}} + \frac{37+20x}{217(1+2x)^{3/2}\left(2+3x+5x^2\right)} + \frac{\text{Subst}\left(\left(\int -998\ldots\right.\right.}{\ldots}$$

$$= -\frac{820}{4557(1+2x)^{3/2}} - \frac{4680}{10633\sqrt{1+2x}} + \frac{37+20x}{217(1+2x)^{3/2}\left(2+3x+5x^2\right)} + \frac{\left(5\left(499-234\ldots\right.\right.}{\ldots}$$

$$= -\frac{820}{4557(1+2x)^{3/2}} - \frac{4680}{10633\sqrt{1+2x}} + \frac{37+20x}{217(1+2x)^{3/2}\left(2+3x+5x^2\right)} - \frac{5\sqrt{-\frac{6252271}{217}+\ldots}}{\ldots}$$

$$= -\frac{820}{4557(1+2x)^{3/2}} - \frac{4680}{10633\sqrt{1+2x}} + \frac{37+20x}{217(1+2x)^{3/2}\left(2+3x+5x^2\right)} + \frac{5\sqrt{\frac{2}{7\left(-2+\sqrt{35}\right)}}\,(\ldots}{\ldots}$$

Mathematica [C] time = 0.51755, size = 176, normalized size = 0.59

$$\frac{1}{217}\left(\frac{20x+37}{(2x+1)^{3/2}\left(5x^2+3x+2\right)} - \frac{4680}{49\sqrt{2x+1}} - \frac{820}{21(2x+1)^{3/2}} + \frac{2i\sqrt{5}\left(\sqrt{2-i\sqrt{31}}\left(9188\sqrt{31}+15469i\right)\tanh^{-1}\left(\frac{\sqrt{10x+5}}{\sqrt{2-i\sqrt{31}}}\right)\ldots\right.}{106\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + 2*x)^(5/2)*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] (-820/(21*(1 + 2*x)^(3/2)) - 4680/(49*Sqrt[1 + 2*x]) + (37 + 20*x)/((1 + 2*
x)^(3/2)*(2 + 3*x + 5*x^2)) + ((2*I)/10633)*Sqrt[5]*(Sqrt[2 - I*Sqrt[31]]*(
15469*I + 9188*Sqrt[31])*ArcTanh[Sqrt[5 + 10*x]/Sqrt[2 - I*Sqrt[31]]] + (15
469*I - 9188*Sqrt[31])*Sqrt[2 + I*Sqrt[31]]*ArcTanh[Sqrt[5 + 10*x]/Sqrt[2 +
 I*Sqrt[31]]]))/217
```

Maple [B] time = 0.081, size = 660, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2*x)^(5/2)/(5*x^2+3*x+2)^2,x)
```

```
[Out] -16/343*(89/62*(1+2*x)^(3/2)+233/620*(1+2*x)^(1/2))/((1+2*x)^2-8/5*x+3/5)-4
835/659246*ln(5^(1/2)*7^(1/2)+10*x+5+(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1
+2*x)^(1/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+22970/2307361*ln(5^(1/2)*7
^(1/2)+10*x+5+(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2))*7^(1/2)*(2
*5^(1/2)*7^(1/2)+4)^(1/2)+24175/329623/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan
((10*(1+2*x)^(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2)
-20)^(1/2))*(2*5^(1/2)*7^(1/2)+4)-45940/2307361/(10*5^(1/2)*7^(1/2)-20)^(1/
2)*arctan((10*(1+2*x)^(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2
)*7^(1/2)-20)^(1/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)*7^(1/2)-9980/74431/(10*5
^(1/2)*7^(1/2)-20)^(1/2)*arctan((10*(1+2*x)^(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2
)+4)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*7^(1/2)+4835/659246*ln(-
(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2)+5^(1/2)*7^(1/2)+10*x+5)*5
^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)-22970/2307361*ln(-(2*5^(1/2)*7^(1/2)+4)^
(1/2)*5^(1/2)*(1+2*x)^(1/2)+5^(1/2)*7^(1/2)+10*x+5)*7^(1/2)*(2*5^(1/2)*7^(1
/2)+4)^(1/2)+24175/329623/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((-5^(1/2)*(2
*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*
(2*5^(1/2)*7^(1/2)+4)-45940/2307361/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((-
5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/2))/(10*5^(1/2)*7^(1/2)-2
0)^(1/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)*7^(1/2)-9980/74431/(10*5^(1/2)*7^(1
/2)-20)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/2)
)/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*7^(1/2)-16/147/(1+2*x)^(3/2)-128/3
43/(1+2*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)^2 (2\,x+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^2*(2*x + 1)^(5/2)), x)
```

Fricas [B] time = 2.62435, size = 3170, normalized size = 10.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] -1/107651010693660706O2*(620294748*161637035^(1/4)*sqrt(4298)*sqrt(217)*sqr
t(35)*(20*x^4 + 32*x^3 + 25*x^2 + 11*x + 2)*sqrt(12504542*sqrt(35) + 921383
75)*arctan(1/551523162491167237577442255*161637035^(3/4)*sqrt(4298)*sqrt(153
5)*sqrt(217)*sqrt(149)*sqrt(161637035^(1/4)*sqrt(4298)*sqrt(217)*(234*sqrt(
35)*sqrt(31) - 499*sqrt(31))*sqrt(2*x + 1)*sqrt(12504542*sqrt(35) + 9213837
5) + 7775911578470*x + 777591157847*sqrt(35) + 3887955789235)*sqrt(12504542
*sqrt(35) + 92138375)*(499*sqrt(35) - 8190) - 1/58486518937462105*161637035
^(3/4)*sqrt(4298)*sqrt(217)*sqrt(2*x + 1)*sqrt(12504542*sqrt(35) + 92138375
)*(499*sqrt(35) - 8190) + 1/31*sqrt(35)*sqrt(31) + 2/31*sqrt(31)) + 6202947
48*161637035^(1/4)*sqrt(4298)*sqrt(217)*sqrt(35)*(20*x^4 + 32*x^3 + 25*x^2
+ 11*x + 2)*sqrt(12504542*sqrt(35) + 92138375)*arctan(1/4729311118361759062
226567293750*161637035^(3/4)*sqrt(4298)*sqrt(217)*sqrt(149)*sqrt(-112869509
```

```
37500*161637035^(1/4)*sqrt(4298)*sqrt(217)*(234*sqrt(35)*sqrt(31) - 499*sqr
t(31))*sqrt(2*x + 1)*sqrt(12504542*sqrt(35) + 92138375) + 87766332480529071
315625000*x + 87766332480529071315625000*sqrt(35) + 438831662402645356578125
00)*sqrt(12504542*sqrt(35) + 92138375)*(499*sqrt(35) - 8190) - 1/5848651893
7462105*161637035^(3/4)*sqrt(4298)*sqrt(217)*sqrt(2*x + 1)*sqrt(12504542*sq
rt(35) + 92138375)*(499*sqrt(35) - 8190) - 1/31*sqrt(35)*sqrt(31) - 2/31*sq
rt(31)) + 3*161637035^(1/4)*sqrt(4298)*sqrt(217)*(12504542*sqrt(35)*sqrt(31
)*(20*x^4 + 32*x^3 + 25*x^2 + 11*x + 2) - 92138375*sqrt(31)*(20*x^4 + 32*x^
3 + 25*x^2 + 11*x + 2))*sqrt(12504542*sqrt(35) + 92138375)*log(112869509375
00/53789*161637035^(1/4)*sqrt(4298)*sqrt(217)*(234*sqrt(35)*sqrt(31) - 499*
sqrt(31))*sqrt(2*x + 1)*sqrt(12504542*sqrt(35) + 92138375) + 16316780843765
28125000*x + 163167808437652812500*sqrt(35) + 815839042188264062500) - 3*16
1637035^(1/4)*sqrt(4298)*sqrt(217)*(12504542*sqrt(35)*sqrt(31)*(20*x^4 + 32
*x^3 + 25*x^2 + 11*x + 2) - 92138375*sqrt(31)*(20*x^4 + 32*x^3 + 25*x^2 + 1
1*x + 2))*sqrt(12504542*sqrt(35) + 92138375)*log(-11286950937500/53789*1616
37035^(1/4)*sqrt(4298)*sqrt(217)*(234*sqrt(35)*sqrt(31) - 499*sqrt(31))*sqr
t(2*x + 1)*sqrt(12504542*sqrt(35) + 92138375) + 1631678084376528125000*x +
163167808437652812500*sqrt(35) + 815839042188264062500) + 337474562505598*(
140400*x^3 + 183140*x^2 + 112560*x + 34121)*sqrt(2*x + 1))/(20*x^4 + 32*x^3
 + 25*x^2 + 11*x + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x+1)^{\frac{5}{2}}\left(5x^2+3x+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)**(5/2)/(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral(1/((2*x + 1)**(5/2)*(5*x**2 + 3*x + 2)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(5x^2+3x+2\right)^2(2x+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^2*(2*x + 1)^(5/2)), x)
```

3.2323 $\int \frac{(1+2x)^{9/2}}{(2+3x+5x^2)^3} dx$

Optimal. Leaf size=313

$$-\frac{(5-4x)(2x+1)^{7/2}}{62\left(5x^2+3x+2\right)^2}-\frac{(1143-1088x)(2x+1)^{3/2}}{9610\left(5x^2+3x+2\right)}-\frac{1584\sqrt{2x+1}}{24025}+\frac{3\sqrt{\frac{1}{310}\left(64681225\sqrt{35}-250141922\right)}\log\left(5(2x+\right.}{48050}$$

```
[Out] (-1584*Sqrt[1 + 2*x])/24025 - ((5 - 4*x)*(1 + 2*x)^(7/2))/(62*(2 + 3*x + 5*
x^2)^2) - ((1143 - 1088*x)*(1 + 2*x)^(3/2))/(9610*(2 + 3*x + 5*x^2)) - (3*S
qrt[(250141922 + 64681225*Sqrt[35])/310]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] -
10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/24025 + (3*Sqrt[(250141922 + 6
4681225*Sqrt[35])/310]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] + 10*Sqrt[1 + 2*x])/
Sqrt[10*(-2 + Sqrt[35])]])/24025 + (3*Sqrt[(-250141922 + 64681225*Sqrt[35])
/310]*Log[Sqrt[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/
48050 - (3*Sqrt[(-250141922 + 64681225*Sqrt[35])/310]*Log[Sqrt[35] + Sqrt[1
0*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/48050
```

Rubi [A] time = 0.521533, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {738, 818, 824, 826, 1169, 634, 618, 204, 628}

$$-\frac{(5-4x)(2x+1)^{7/2}}{62\left(5x^2+3x+2\right)^2}-\frac{(1143-1088x)(2x+1)^{3/2}}{9610\left(5x^2+3x+2\right)}-\frac{1584\sqrt{2x+1}}{24025}+\frac{3\sqrt{\frac{1}{310}\left(64681225\sqrt{35}-250141922\right)}\log\left(5(2x+\right.}{48050}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^(9/2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (-1584*Sqrt[1 + 2*x])/24025 - ((5 - 4*x)*(1 + 2*x)^(7/2))/(62*(2 + 3*x + 5*
x^2)^2) - ((1143 - 1088*x)*(1 + 2*x)^(3/2))/(9610*(2 + 3*x + 5*x^2)) - (3*S
qrt[(250141922 + 64681225*Sqrt[35])/310]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] -
10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/24025 + (3*Sqrt[(250141922 + 6
4681225*Sqrt[35])/310]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] + 10*Sqrt[1 + 2*x])/
Sqrt[10*(-2 + Sqrt[35])]])/24025 + (3*Sqrt[(-250141922 + 64681225*Sqrt[35])
/310]*Log[Sqrt[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/
48050 - (3*Sqrt[(-250141922 + 64681225*Sqrt[35])/310]*Log[Sqrt[35] + Sqrt[1
0*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/48050
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
```

```
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(1+2x)^{9/2}}{\left(2+3x+5x^2\right)^3}\,dx = -\frac{(5-4x)(1+2x)^{7/2}}{62\left(2+3x+5x^2\right)^2}+\frac{1}{62}\int \frac{(47-4x)(1+2x)^{5/2}}{\left(2+3x+5x^2\right)^2}\,dx$$

$$= -\frac{(5-4x)(1+2x)^{7/2}}{62\left(2+3x+5x^2\right)^2}-\frac{(1143-1088x)(1+2x)^{3/2}}{9610\left(2+3x+5x^2\right)}-\frac{\int \frac{\sqrt{1+2x}(-4269+1584x)}{2+3x+5x^2}\,dx}{9610}$$

$$= -\frac{1584\sqrt{1+2x}}{24025}-\frac{(5-4x)(1+2x)^{7/2}}{62\left(2+3x+5x^2\right)^2}-\frac{(1143-1088x)(1+2x)^{3/2}}{9610\left(2+3x+5x^2\right)}-\frac{\int \frac{-27681-44274x}{\sqrt{1+2x}\left(2+3x+5x^2\right)}\,dx}{48050}$$

$$= -\frac{1584\sqrt{1+2x}}{24025}-\frac{(5-4x)(1+2x)^{7/2}}{62\left(2+3x+5x^2\right)^2}-\frac{(1143-1088x)(1+2x)^{3/2}}{9610\left(2+3x+5x^2\right)}-\frac{\text{Subst}\left(\int \frac{-11088-44274x^2}{7-4x^2+5x^4}\right.}{24025}$$

$$= -\frac{1584\sqrt{1+2x}}{24025}-\frac{(5-4x)(1+2x)^{7/2}}{62\left(2+3x+5x^2\right)^2}-\frac{(1143-1088x)(1+2x)^{3/2}}{9610\left(2+3x+5x^2\right)}-\frac{\text{Subst}\left(\int \frac{-11088\sqrt{\frac{2}{5}}(2+\sqrt{3}}{\sqrt{\frac{7}{5}}-\sqrt{}}\right.}{4805}$$

$$= -\frac{1584\sqrt{1+2x}}{24025}-\frac{(5-4x)(1+2x)^{7/2}}{62\left(2+3x+5x^2\right)^2}-\frac{(1143-1088x)(1+2x)^{3/2}}{9610\left(2+3x+5x^2\right)}-\frac{\left(3\left(9240-7379\sqrt{35}\right)\right)\,S}{24}$$

$$= -\frac{1584\sqrt{1+2x}}{24025}-\frac{(5-4x)(1+2x)^{7/2}}{62\left(2+3x+5x^2\right)^2}-\frac{(1143-1088x)(1+2x)^{3/2}}{9610\left(2+3x+5x^2\right)}+\frac{3\left(7379-264\sqrt{35}\right)\log}{4}$$

$$= -\frac{1584\sqrt{1+2x}}{24025}-\frac{(5-4x)(1+2x)^{7/2}}{62\left(2+3x+5x^2\right)^2}-\frac{(1143-1088x)(1+2x)^{3/2}}{9610\left(2+3x+5x^2\right)}-\frac{3\sqrt{\frac{1}{310}\left(250141922+64\right.}}{}$$

Mathematica [C] time = 0.697732, size = 236, normalized size = 0.75

$$\frac{-\frac{7(640x+409)(2x+1)^{11/2}}{5x^2+3x+2}+\frac{217(20x+37)(2x+1)^{11/2}}{\left(5x^2+3x+2\right)^2}+1792(2x+1)^{9/2}+1932(2x+1)^{7/2}-2352(2x+1)^{5/2}-\frac{47236}{5}(2x+1)^{3/2}-15}{94178}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^(9/2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] ((-155232*Sqrt[1 + 2*x])/25 - (47236*(1 + 2*x)^(3/2))/5 - 2352*(1 + 2*x)^(5
/2) + 1932*(1 + 2*x)^(7/2) + 1792*(1 + 2*x)^(9/2) + (217*(1 + 2*x)^(11/2)*(
37 + 20*x))/(2 + 3*x + 5*x^2)^2 - (7*(1 + 2*x)^(11/2)*(409 + 640*x))/(2 + 3
*x + 5*x^2) + (294*(Sqrt[2 - I*Sqrt[31]]*(8184 - (7907*I)*Sqrt[31])*ArcTanh
[Sqrt[5 + 10*x]/Sqrt[2 - I*Sqrt[31]]] + Sqrt[2 + I*Sqrt[31]]*(8184 + (7907*
I)*Sqrt[31])*ArcTanh[Sqrt[5 + 10*x]/Sqrt[2 + I*Sqrt[31]]]))/(775*Sqrt[5]))/
94178
```

Maple [B] time = 0.077, size = 662, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^(9/2)/(5*x^2+3*x+2)^3,x)
```

```
[Out] 1600*(-1723/768800*(1+2*x)^(7/2)-3833/4805000*(1+2*x)^(5/2)-14693/19220000*
(1+2*x)^(3/2)-4851/2402500*(1+2*x)^(1/2))/(5*(1+2*x)^2-8*x+3)^2+35997/74477
50*ln(5^(1/2)*7^(1/2)+10*x+5+(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1
/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)-23721/2979100*ln(5^(1/2)*7^(1/2)+1
0*x+5+(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2))*7^(1/2)*(2*5^(1/2)
*7^(1/2)+4)^(1/2)-35997/744775/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((10*(1+
2*x)^(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/
2))*(2*5^(1/2)*7^(1/2)+4)+23721/1489550/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arcta
n((10*(1+2*x)^(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2
)-20)^(1/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)*7^(1/2)+1584/24025/(10*5^(1/2)*7
^(1/2)-20)^(1/2)*arctan((10*(1+2*x)^(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/
2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*7^(1/2)-35997/7447750*ln(-(2*5^(
1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2)+5^(1/2)*7^(1/2)+10*x+5)*5^(1/2)
*(2*5^(1/2)*7^(1/2)+4)^(1/2)+23721/2979100*ln(-(2*5^(1/2)*7^(1/2)+4)^(1/2)*
5^(1/2)*(1+2*x)^(1/2)+5^(1/2)*7^(1/2)+10*x+5)*7^(1/2)*(2*5^(1/2)*7^(1/2)+4)
^(1/2)-35997/744775/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((-5^(1/2)*(2*5^(1/
2)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*(2*5^(
1/2)*7^(1/2)+4)+23721/1489550/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((-5^(1/2
)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/
2))*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)*7^(1/2)+1584/24025/(10*5^(1/2)*7^(1/2)-20
)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/2))/(10*
5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*7^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x+1)^{\frac{9}{2}}}{(5x^2+3x+2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(9/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] integrate((2*x + 1)^(9/2)/(5*x^2 + 3*x + 2)^3, x)
```

Fricas [B] time = 2.8415, size = 3374, normalized size = 10.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(9/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/44382950698994113517500*(19347824532*97578096035^(1/4)*sqrt(105602)*sqrt(
155)*sqrt(35)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(250141922*sqrt(35)
```

```
 + 2263842875)*arctan(1/21602528465119702171313226393834 25*97578096035^(3/4
)*sqrt(1677751)*sqrt(105602)*sqrt(7543)*sqrt(155)*sqrt(97578096035^(1/4)*sq
rt(105602)*sqrt(155)*(7379*sqrt(35)*sqrt(31) - 9240*sqrt(31))*sqrt(2*x + 1)
*sqrt(250141922*sqrt(35) + 2263842875) + 29796214090828850*x + 297962140908
2885*sqrt(35) + 14898107045414425)*sqrt(250141922*sqrt(35) + 2263842875)*(2
64*sqrt(35) - 7379) - 1/15732699002098541088 5*97578096035^(3/4)*sqrt(105602
)*sqrt(155)*sqrt(2*x + 1)*sqrt(250141922*sqrt(35) + 2263842875)*(264*sqrt(3
5) - 7379) + 1/31*sqrt(35)*sqrt(31) + 2/31*sqrt(31)) + 19347824532*97578096
035^(1/4)*sqrt(105602)*sqrt(155)*sqrt(35)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x
+ 4)*sqrt(250141922*sqrt(35) + 2263842875)*arctan(1/793892921093149054795 76
10699734086875*97578096035^(3/4)*sqrt(1677751)*sqrt(105602)*sqrt(155)*sqrt(
-101872929375*97578096035^(1/4)*sqrt(105602)*sqrt(155)*(7379*sqrt(35)*sqrt(
31) - 9240*sqrt(31))*sqrt(2*x + 1)*sqrt(250141922*sqrt(35) + 2263842875) +
303542761371738727126246875 0*x + 303542761371738727126246875*sqrt(35) + 151
771380685869363563123437 5)*sqrt(250141922*sqrt(35) + 2263842875)*(264*sqrt(
35) - 7379) - 1/157326990020985410885*97578096035^(3/4)*sqrt(105602)*sqrt(1
55)*sqrt(2*x + 1)*sqrt(250141922*sqrt(35) + 2263842875)*(264*sqrt(35) - 737
9) - 1/31*sqrt(35)*sqrt(31) - 2/31*sqrt(31)) + 3*97578096035^(1/4)*sqrt(105
602)*sqrt(155)*(250141922*sqrt(35)*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2 + 12*
x + 4) - 2263842875*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4))*sqrt(25
0141922*sqrt(35) + 2263842875)*log(101872929375/1677751*97578096035^(1/4)*s
qrt(105602)*sqrt(155)*(7379*sqrt(35)*sqrt(31) - 9240*sqrt(31))*sqrt(2*x + 1
)*sqrt(250141922*sqrt(35) + 2263842875) + 1809224142150645281250*x + 180922
414215064528125*sqrt(35) + 904612071075322640625) - 3*97578096035^(1/4)*sqr
t(105602)*sqrt(155)*(250141922*sqrt(35)*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2
+ 12*x + 4) - 2263842875*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4))*sq
rt(250141922*sqrt(35) + 2263842875)*log(-101872929375/1677751*97578096035^(
1/4)*sqrt(105602)*sqrt(155)*(7379*sqrt(35)*sqrt(31) - 9240*sqrt(31))*sqrt(2
*x + 1)*sqrt(250141922*sqrt(35) + 2263842875) + 1809224142150645281250*x +
180922414215064528125*sqrt(35) + 904612071075322640625) - 92368263681569435
0*(86150*x^3 + 144557*x^2 + 87291*x + 27977)*sqrt(2*x + 1))/(25*x^4 + 30*x^
3 + 29*x^2 + 12*x + 4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**(9/2)/(5*x**2+3*x+2)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x+1)^{\frac{9}{2}}}{(5x^2+3x+2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(9/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] integrate((2*x + 1)^(9/2)/(5*x^2 + 3*x + 2)^3, x)
```

3.2324 $\int \frac{(1+2x)^{7/2}}{(2+3x+5x^2)^3} dx$

Optimal. Leaf size=300

$$-\frac{(5-4x)(2x+1)^{5/2}}{62\left(5x^2+3x+2\right)^2}-\frac{(957-592x)\sqrt{2x+1}}{9610\left(5x^2+3x+2\right)}-\frac{\sqrt{\frac{1}{310}\left(1806875\sqrt{35}-9651062\right)}\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}\right.}{9610}$$

```
[Out] -((5 - 4*x)*(1 + 2*x)^(5/2))/(62*(2 + 3*x + 5*x^2)^2) - ((957 - 592*x)*Sqrt
[1 + 2*x])/(9610*(2 + 3*x + 5*x^2)) - (Sqrt[(9651062 + 1806875*Sqrt[35])/31
0]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] - 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[3
5])]])/4805 + (Sqrt[(9651062 + 1806875*Sqrt[35])/310]*ArcTan[(Sqrt[10*(2 +
Sqrt[35])] + 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/4805 - (Sqrt[(-96
51062 + 1806875*Sqrt[35])/310]*Log[Sqrt[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[
1 + 2*x] + 5*(1 + 2*x)])/9610 + (Sqrt[(-9651062 + 1806875*Sqrt[35])/310]*Lo
g[Sqrt[35] + Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/9610
```

Rubi [A] time = 0.438287, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {738, 818, 826, 1169, 634, 618, 204, 628}

$$-\frac{(5-4x)(2x+1)^{5/2}}{62\left(5x^2+3x+2\right)^2}-\frac{(957-592x)\sqrt{2x+1}}{9610\left(5x^2+3x+2\right)}-\frac{\sqrt{\frac{1}{310}\left(1806875\sqrt{35}-9651062\right)}\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}\right.}{9610}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^(7/2)/(2 + 3*x + 5*x^2)^3,x]
```


```
[Out] -((5 - 4*x)*(1 + 2*x)^(5/2))/(62*(2 + 3*x + 5*x^2)^2) - ((957 - 592*x)*Sqrt
[1 + 2*x])/(9610*(2 + 3*x + 5*x^2)) - (Sqrt[(9651062 + 1806875*Sqrt[35])/31
0]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] - 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[3
5])]])/4805 + (Sqrt[(9651062 + 1806875*Sqrt[35])/310]*ArcTan[(Sqrt[10*(2 +
Sqrt[35])] + 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/4805 - (Sqrt[(-96
51062 + 1806875*Sqrt[35])/310]*Log[Sqrt[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[
1 + 2*x] + 5*(1 + 2*x)])/9610 + (Sqrt[(-9651062 + 1806875*Sqrt[35])/310]*Lo
g[Sqrt[35] + Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/9610
```


Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
```

```
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(1+2x)^{7/2}}{\left(2+3x+5x^2\right)^3}\,dx = -\frac{(5-4x)(1+2x)^{5/2}}{62\left(2+3x+5x^2\right)^2}+\frac{1}{62}\int \frac{(1+2x)^{3/2}(37+4x)}{\left(2+3x+5x^2\right)^2}\,dx$$

$$= -\frac{(5-4x)(1+2x)^{5/2}}{62\left(2+3x+5x^2\right)^2}-\frac{(957-592x)\sqrt{1+2x}}{9610\left(2+3x+5x^2\right)}-\frac{\int \frac{-1797-1088x}{\sqrt{1+2x}(2+3x+5x^2)}\,dx}{9610}$$

$$= -\frac{(5-4x)(1+2x)^{5/2}}{62\left(2+3x+5x^2\right)^2}-\frac{(957-592x)\sqrt{1+2x}}{9610\left(2+3x+5x^2\right)}-\frac{\text{Subst}\left(\int \frac{-2506-1088x^2}{7-4x^2+5x^4}\,dx,x,\sqrt{1+2x}\right)}{4805}$$

$$= -\frac{(5-4x)(1+2x)^{5/2}}{62\left(2+3x+5x^2\right)^2}-\frac{(957-592x)\sqrt{1+2x}}{9610\left(2+3x+5x^2\right)}-\frac{\text{Subst}\left(\int \frac{-2506\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}-\left(-2506+1088\sqrt{\frac{7}{5}}\right)x}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}x+x^2}\,dx,x,\right.}{9610\sqrt{14\left(2+\sqrt{35}\right)}}$$

$$= -\frac{(5-4x)(1+2x)^{5/2}}{62\left(2+3x+5x^2\right)^2}-\frac{(957-592x)\sqrt{1+2x}}{9610\left(2+3x+5x^2\right)}+\frac{\sqrt{1417371+194752\sqrt{35}}\,\text{Subst}\left(\int \frac{1}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}\left(2+\sqrt{3}\right.}}\right.}{48050}$$

$$= -\frac{(5-4x)(1+2x)^{5/2}}{62\left(2+3x+5x^2\right)^2}-\frac{(957-592x)\sqrt{1+2x}}{9610\left(2+3x+5x^2\right)}-\frac{\sqrt{\frac{1}{310}\left(-9651062+1806875\sqrt{35}\right)}\log\left(\sqrt{35}-\sqrt{\right.}}{9610}$$

$$= -\frac{(5-4x)(1+2x)^{5/2}}{62\left(2+3x+5x^2\right)^2}-\frac{(957-592x)\sqrt{1+2x}}{9610\left(2+3x+5x^2\right)}-\frac{\sqrt{\frac{1}{310}\left(9651062+1806875\sqrt{35}\right)}\tan^{-1}\left(\sqrt{\frac{5}{2(-2+}}\right.}{4805}$$

Mathematica [C] time = 0.626067, size = 223, normalized size = 0.74

$$\frac{-\frac{5(400x+89)(2x+1)^{9/2}}{5x^2+3x+2}+\frac{217(20x+37)(2x+1)^{9/2}}{\left(5x^2+3x+2\right)^2}+800(2x+1)^{7/2}+196(2x+1)^{5/2}-2352(2x+1)^{3/2}-\frac{35084}{5}\sqrt{2x+1}+\frac{98\left(\sqrt{2-i\sqrt{31}}(5\right.}{}}{94178}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^(7/2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] ((-35084*Sqrt[1 + 2*x])/5 - 2352*(1 + 2*x)^(3/2) + 196*(1 + 2*x)^(5/2) + 80
0*(1 + 2*x)^(7/2) + (217*(1 + 2*x)^(9/2)*(37 + 20*x))/(2 + 3*x + 5*x^2)^2 -
 (5*(1 + 2*x)^(9/2)*(89 + 400*x))/(2 + 3*x + 5*x^2) + (98*(Sqrt[2 - I*Sqrt[
31]]*(5549 - (902*I)*Sqrt[31])*ArcTanh[Sqrt[5 + 10*x]/Sqrt[2 - I*Sqrt[31]]]
 + Sqrt[2 + I*Sqrt[31]]*(5549 + (902*I)*Sqrt[31])*ArcTanh[Sqrt[5 + 10*x]/Sq
rt[2 + I*Sqrt[31]]]))/(155*Sqrt[5]))/94178
```

Maple [B] time = 0.076, size = 662, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^(7/2)/(5*x^2+3*x+2)^3,x)
```

```
[Out] 1600*(17/24025*(1+2*x)^(7/2)-11789/3844000*(1+2*x)^(5/2)+1771/961000*(1+2*x
)^(3/2)-8771/3844000*(1+2*x)^(1/2))/(5*(1+2*x)^2-8*x+3)^2+7353/2979100*ln(5
^(1/2)*7^(1/2)+10*x+5+(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2))*5^
(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)-451/297910*ln(5^(1/2)*7^(1/2)+10*x+5+(2*5
^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2))*7^(1/2)*(2*5^(1/2)*7^(1/2)+4
)^(1/2)-7353/297910/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((10*(1+2*x)^(1/2)+
5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*(2*5^(1
/2)*7^(1/2)+4)+451/148955/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((10*(1+2*x)^
(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5
^(1/2)*(2*5^(1/2)*7^(1/2)+4)*7^(1/2)+358/4805/(10*5^(1/2)*7^(1/2)-20)^(1/2)
*arctan((10*(1+2*x)^(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*
7^(1/2)-20)^(1/2))*5^(1/2)*7^(1/2)-7353/2979100*ln(-(2*5^(1/2)*7^(1/2)+4)^(
1/2)*5^(1/2)*(1+2*x)^(1/2)+5^(1/2)*7^(1/2)+10*x+5)*5^(1/2)*(2*5^(1/2)*7^(1/
2)+4)^(1/2)+451/297910*ln(-(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2
)+5^(1/2)*7^(1/2)+10*x+5)*7^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)-7353/297910/(
10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+1
0*(1+2*x)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*(2*5^(1/2)*7^(1/2)+4)+451/1
48955/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^
(1/2)+10*(1+2*x)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*(2*5^(1/2)*7
^(1/2)+4)*7^(1/2)+358/4805/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((-5^(1/2)*(
2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))
*5^(1/2)*7^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x+1)^{\frac{7}{2}}}{\left(5x^2+3x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(7/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] integrate((2*x + 1)^(7/2)/(5*x^2 + 3*x + 2)^3, x)
```

Fricas [B] time = 2.71219, size = 2979, normalized size = 9.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(7/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] -1/157428509198663500*(102361876*121835^(1/4)*sqrt(155)*sqrt(118)*sqrt(35)*
(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(9651062*sqrt(35) + 63240625)*arc
tan(1/531479992814524674252 5*121835^(3/4)*sqrt(26629)*sqrt(413)*sqrt(155)*s
qrt(118)*sqrt(121835^(1/4)*sqrt(155)*sqrt(118)*(544*sqrt(35)*sqrt(31) - 626
5*sqrt(31))*sqrt(2*x + 1)*sqrt(9651062*sqrt(35) + 63240625) + 528443184850*
x + 52844318485*sqrt(35) + 264221592425)*sqrt(9651062*sqrt(35) + 63240625)*
(179*sqrt(35) - 544) - 1/3117814790615*121835^(3/4)*sqrt(155)*sqrt(118)*sqr
t(2*x + 1)*sqrt(9651062*sqrt(35) + 63240625)*(179*sqrt(35) - 544) + 1/31*sq
rt(35)*sqrt(31) + 2/31*sqrt(31)) + 102361876*121835^(1/4)*sqrt(155)*sqrt(11
8)*sqrt(35)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(9651062*sqrt(35) + 6
3240625)*arctan(1/232522496856354544498546875*121835^(3/4)*sqrt(26629)*sqrt(
155)*sqrt(118)*sqrt(-7905078125*121835^(1/4)*sqrt(155)*sqrt(118)*(544*sqrt(
```

```
35)*sqrt(31) - 6265*sqrt(31))*sqrt(2*x + 1)*sqrt(9651062*sqrt(35) + 6324062
5) + 417738466086306640625 0*x + 417738466086306640625*sqrt(35) + 2088692330
431533203125)*sqrt(9651062*sqrt(35) + 63240625)*(179*sqrt(35) - 544) - 1/31
17814790615*121835^(3/4)*sqrt(155)*sqrt(118)*sqrt(2*x + 1)*sqrt(9651062*sqr
t(35) + 63240625)*(179*sqrt(35) - 544) - 1/31*sqrt(35)*sqrt(31) - 2/31*sqrt
(31)) - 121835^(1/4)*sqrt(155)*sqrt(118)*(9651062*sqrt(35)*sqrt(31)*(25*x^4
 + 30*x^3 + 29*x^2 + 12*x + 4) - 63240625*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^
2 + 12*x + 4))*sqrt(9651062*sqrt(35) + 63240625)*log(7905078125/26629*12183
5^(1/4)*sqrt(155)*sqrt(118)*(544*sqrt(35)*sqrt(31) - 6265*sqrt(31))*sqrt(2*
x + 1)*sqrt(9651062*sqrt(35) + 63240625) + 156873508613281250*x + 156873508
61328125*sqrt(35) + 78436754306640625) + 121835^(1/4)*sqrt(155)*sqrt(118)*(
9651062*sqrt(35)*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) - 63240625*
sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4))*sqrt(9651062*sqrt(35) + 632
40625)*log(-7905078125/26629*121835^(1/4)*sqrt(155)*sqrt(118)*(544*sqrt(35)
*sqrt(31) - 6265*sqrt(31))*sqrt(2*x + 1)*sqrt(9651062*sqrt(35) + 63240625)
+ 156873508613281250*x + 15687350861328125*sqrt(35) + 78436754306640625) -
16381738730350*(5440*x^3 - 3629*x^2 - 4167*x - 2689)*sqrt(2*x + 1))/(25*x^4
 + 30*x^3 + 29*x^2 + 12*x + 4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**(7/2)/(5*x**2+3*x+2)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,x+1)^{\frac{7}{2}}}{\left(5\,x^2+3\,x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(7/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] integrate((2*x + 1)^(7/2)/(5*x^2 + 3*x + 2)^3, x)
```

3.2325 $\int \frac{(1+2x)^{5/2}}{\left(2+3x+5x^2\right)^3} dx$

Optimal. Leaf size=300

$$-\frac{(5-4x)(2x+1)^{3/2}}{62\left(5x^2+3x+2\right)^2}+\frac{3(78x+11)\sqrt{2x+1}}{1922\left(5x^2+3x+2\right)}+\frac{3\sqrt{\frac{1}{310}\left(2705\sqrt{35}-15082\right)}\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}+\right.}{1922}$$

```
[Out] -((5 - 4*x)*(1 + 2*x)^(3/2))/(62*(2 + 3*x + 5*x^2)^2) + (3*Sqrt[1 + 2*x]*(1
1 + 78*x))/(1922*(2 + 3*x + 5*x^2)) - (3*Sqrt[(15082 + 2705*Sqrt[35])/310]*
ArcTan[(Sqrt[10*(2 + Sqrt[35])] - 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])
]])/961 + (3*Sqrt[(15082 + 2705*Sqrt[35])/310]*ArcTan[(Sqrt[10*(2 + Sqrt[35
])] + 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/961 + (3*Sqrt[(-15082 +
2705*Sqrt[35])/310]*Log[Sqrt[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] +
5*(1 + 2*x)])/1922 - (3*Sqrt[(-15082 + 2705*Sqrt[35])/310]*Log[Sqrt[35] + S
qrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/1922
```

Rubi [A] time = 0.435021, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {738, 820, 826, 1169, 634, 618, 204, 628}

$$-\frac{(5-4x)(2x+1)^{3/2}}{62\left(5x^2+3x+2\right)^2}+\frac{3(78x+11)\sqrt{2x+1}}{1922\left(5x^2+3x+2\right)}+\frac{3\sqrt{\frac{1}{310}\left(2705\sqrt{35}-15082\right)}\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}+\right.}{1922}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^(5/2)/(2 + 3*x + 5*x^2)^3,x]
```


```
[Out] -((5 - 4*x)*(1 + 2*x)^(3/2))/(62*(2 + 3*x + 5*x^2)^2) + (3*Sqrt[1 + 2*x]*(1
1 + 78*x))/(1922*(2 + 3*x + 5*x^2)) - (3*Sqrt[(15082 + 2705*Sqrt[35])/310]*
ArcTan[(Sqrt[10*(2 + Sqrt[35])] - 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])
]])/961 + (3*Sqrt[(15082 + 2705*Sqrt[35])/310]*ArcTan[(Sqrt[10*(2 + Sqrt[35
])] + 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/961 + (3*Sqrt[(-15082 +
2705*Sqrt[35])/310]*Log[Sqrt[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] +
5*(1 + 2*x)])/1922 - (3*Sqrt[(-15082 + 2705*Sqrt[35])/310]*Log[Sqrt[35] + S
qrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/1922
```


Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*
(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p +
 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*Simp[g*
```

```
(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(
m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (I
ntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(1+2x)^{5/2}}{\left(2+3x+5x^2\right)^3}\,dx = -\frac{(5-4x)(1+2x)^{3/2}}{62\left(2+3x+5x^2\right)^2} + \frac{1}{62}\int \frac{\sqrt{1+2x}(27+12x)}{\left(2+3x+5x^2\right)^2}\,dx$$

$$= -\frac{(5-4x)(1+2x)^{3/2}}{62\left(2+3x+5x^2\right)^2} + \frac{3\sqrt{1+2x}(11+78x)}{1922\left(2+3x+5x^2\right)} + \frac{\int \frac{201+234x}{\sqrt{1+2x}(2+3x+5x^2)}\,dx}{1922}$$

$$= -\frac{(5-4x)(1+2x)^{3/2}}{62\left(2+3x+5x^2\right)^2} + \frac{3\sqrt{1+2x}(11+78x)}{1922\left(2+3x+5x^2\right)} + \frac{1}{961}\text{Subst}\left(\int \frac{168+234x^2}{7-4x^2+5x^4}\,dx, x, \sqrt{1+2x}\right)$$

$$= -\frac{(5-4x)(1+2x)^{3/2}}{62\left(2+3x+5x^2\right)^2} + \frac{3\sqrt{1+2x}(11+78x)}{1922\left(2+3x+5x^2\right)} + \frac{\text{Subst}\left(\int \frac{168\sqrt{\frac{2}{5}(2+\sqrt{35})}-\left(168-234\sqrt{\frac{7}{5}}\right)x}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}(2+\sqrt{35})}x+x^2}\,dx, x, \sqrt{1\right.}{1922\sqrt{14\left(2+\sqrt{35}\right)}}$$

$$= -\frac{(5-4x)(1+2x)^{3/2}}{62\left(2+3x+5x^2\right)^2} + \frac{3\sqrt{1+2x}(11+78x)}{1922\left(2+3x+5x^2\right)} + \frac{\left(3\left(39+4\sqrt{35}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}(2+\sqrt{35})}x+x^2}\right.}{9610}$$

$$= -\frac{(5-4x)(1+2x)^{3/2}}{62\left(2+3x+5x^2\right)^2} + \frac{3\sqrt{1+2x}(11+78x)}{1922\left(2+3x+5x^2\right)} + \frac{3\sqrt{\frac{1}{310}}\left(-15082+2705\sqrt{35}\right)\log\left(\sqrt{35}-\sqrt{10\right.}}{1922}$$

$$= -\frac{(5-4x)(1+2x)^{3/2}}{62\left(2+3x+5x^2\right)^2} + \frac{3\sqrt{1+2x}(11+78x)}{1922\left(2+3x+5x^2\right)} - \frac{3}{961}\sqrt{\frac{7541}{155}+\frac{541\sqrt{35}}{62}}\tan^{-1}\left(\sqrt{\frac{5}{2\left(-2+\sqrt{35\right.}}}\right.$$

Mathematica [C] time = 0.565355, size = 209, normalized size = 0.7

$$\frac{\frac{(480x+1973)(2x+1)^{7/2}}{5x^2+3x+2} + \frac{217(20x+37)(2x+1)^{7/2}}{\left(5x^2+3x+2\right)^2} - 192(2x+1)^{5/2} - 1540(2x+1)^{3/2} - 2352\sqrt{2x+1} + \frac{294\left(\sqrt{2-i\sqrt{31}}\left(124-47i\sqrt{31}\right)\tanh\right.}{}}{94178}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^(5/2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (-2352*Sqrt[1 + 2*x] - 1540*(1 + 2*x)^(3/2) - 192*(1 + 2*x)^(5/2) + (217*(1
 + 2*x)^(7/2)*(37 + 20*x))/(2 + 3*x + 5*x^2)^2 + ((1 + 2*x)^(7/2)*(1973 + 4
80*x))/(2 + 3*x + 5*x^2) + (294*(Sqrt[2 - I*Sqrt[31]]*(124 - (47*I)*Sqrt[31
])*ArcTanh[Sqrt[5 + 10*x]/Sqrt[2 - I*Sqrt[31]]] + Sqrt[2 + I*Sqrt[31]]*(124
 + (47*I)*Sqrt[31])*ArcTanh[Sqrt[5 + 10*x]/Sqrt[2 + I*Sqrt[31]]]))/(31*Sqrt
[5]))/94178
```

Maple [B] time = 0.083, size = 662, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^(5/2)/(5*x^2+3*x+2)^3,x)
```

```
[Out] 1600*(117/153760*(1+2*x)^(7/2)-4/4805*(1+2*x)^(5/2)+287/768800*(1+2*x)^(3/2
)-147/192200*(1+2*x)^(1/2))/(5*(1+2*x)^2-8*x+3)^2+327/297910*ln(5^(1/2)*7^(
1/2)+10*x+5+(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2))*5^(1/2)*(2*5
^(1/2)*7^(1/2)+4)^(1/2)-141/119164*ln(5^(1/2)*7^(1/2)+10*x+5+(2*5^(1/2)*7^(
1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2))*7^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)-32
7/29791/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((10*(1+2*x)^(1/2)+5^(1/2)*(2*5
^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*(2*5^(1/2)*7^(1/2)+
4)+141/59582/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((10*(1+2*x)^(1/2)+5^(1/2)
*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*(2*5^(
1/2)*7^(1/2)+4)*7^(1/2)+24/961/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((10*(1+
2*x)^(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/
2))*5^(1/2)*7^(1/2)-327/297910*ln(-(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2
*x)^(1/2)+5^(1/2)*7^(1/2)+10*x+5)*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+141/1
19164*ln(-(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2)+5^(1/2)*7^(1/2)
+10*x+5)*7^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)-327/29791/(10*5^(1/2)*7^(1/2)-
20)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/2))/(1
0*5^(1/2)*7^(1/2)-20)^(1/2))*(2*5^(1/2)*7^(1/2)+4)+141/59582/(10*5^(1/2)*7^
(1/2)-20)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/
2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)*7^(1/2)+24
/961/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(
1/2)+10*(1+2*x)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*7^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x+1)^{\frac{5}{2}}}{\left(5x^2+3x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] integrate((2*x + 1)^(5/2)/(5*x^2 + 3*x + 2)^3, x)
```

Fricas [B] time = 2.73719, size = 2738, normalized size = 9.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/1680488219973500*(357492*256095875^(1/4)*sqrt(155)*sqrt(35)*(25*x^4 + 30*
x^3 + 29*x^2 + 12*x + 4)*sqrt(81593620*sqrt(35) + 512191750)*arctan(1/69413
8872776299934375*256095875^(3/4)*sqrt(2705)*sqrt(217)*sqrt(155)*sqrt(256095
875^(1/4)*sqrt(155)*(39*sqrt(35)*sqrt(31) - 140*sqrt(31))*sqrt(2*x + 1)*sqr
t(81593620*sqrt(35) + 512191750) + 28204629250*x + 2820462925*sqrt(35) + 14
102314625)*sqrt(81593620*sqrt(35) + 512191750)*(4*sqrt(35) - 39) - 1/762935
2212125*256095875^(3/4)*sqrt(155)*sqrt(2*x + 1)*sqrt(81593620*sqrt(35) + 51
2191750)*(4*sqrt(35) - 39) + 1/31*sqrt(35)*sqrt(31) + 2/31*sqrt(31)) + 3574
92*256095875^(1/4)*sqrt(155)*sqrt(35)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
*sqrt(81593620*sqrt(35) + 512191750)*arctan(1/208241661832889980 3125*256095
875^(3/4)*sqrt(217)*sqrt(155)*sqrt(-24345*256095875^(1/4)*sqrt(155)*(39*sqr
t(35)*sqrt(31) - 140*sqrt(31))*sqrt(2*x + 1)*sqrt(81593620*sqrt(35) + 51219
1750) + 686641699091250*x + 68664169909125*sqrt(35) + 343320849545625)*sqrt
```

```
(81593620*sqrt(35) + 512191750)*(4*sqrt(35) - 39) - 1/7629352212125*2560958
75^(3/4)*sqrt(155)*sqrt(2*x + 1)*sqrt(81593620*sqrt(35) + 512191750)*(4*sqr
t(35) - 39) - 1/31*sqrt(35)*sqrt(31) - 2/31*sqrt(31)) + 3*256095875^(1/4)*s
qrt(155)*(15082*sqrt(35)*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) - 9
4675*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4))*sqrt(81593620*sqrt(35)
 + 512191750)*log(24345/217*256095875^(1/4)*sqrt(155)*(39*sqrt(35)*sqrt(31)
 - 140*sqrt(31))*sqrt(2*x + 1)*sqrt(81593620*sqrt(35) + 512191750) + 316424
7461250*x + 316424746125*sqrt(35) + 1582123730625) - 3*256095875^(1/4)*sqrt
(155)*(15082*sqrt(35)*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) - 9467
5*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4))*sqrt(81593620*sqrt(35) +
512191750)*log(-24345/217*256095875^(1/4)*sqrt(155)*(39*sqrt(35)*sqrt(31) -
 140*sqrt(31))*sqrt(2*x + 1)*sqrt(81593620*sqrt(35) + 512191750) + 31642474
61250*x + 316424746125*sqrt(35) + 1582123730625) + 874343506750*(1170*x^3 +
 1115*x^2 + 381*x - 89)*sqrt(2*x + 1))/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**(5/2)/(5*x**2+3*x+2)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x+1)^{\frac{5}{2}}}{(5x^2+3x+2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] integrate((2*x + 1)^(5/2)/(5*x^2 + 3*x + 2)^3, x)
```

3.2326 $\int \frac{(1+2x)^{3/2}}{\left(2+3x+5x^2\right)^3}\,dx$

Optimal. Leaf size=300

$$-\frac{\sqrt{2x+1}(5-4x)}{62\left(5x^2+3x+2\right)^2}+\frac{\sqrt{2x+1}(120x+67)}{1922\left(5x^2+3x+2\right)}-\frac{3\sqrt{\frac{1}{434}\left(2705\sqrt{35}-15082\right)}\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}+\sqrt{3}\right.}{1922}$$

```
[Out] -((5 - 4*x)*Sqrt[1 + 2*x])/(62*(2 + 3*x + 5*x^2)^2) + (Sqrt[1 + 2*x]*(67 +
120*x))/(1922*(2 + 3*x + 5*x^2)) - (3*Sqrt[(15082 + 2705*Sqrt[35])/434]*Arc
Tan[(Sqrt[10*(2 + Sqrt[35])] - 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])
/961 + (3*Sqrt[(15082 + 2705*Sqrt[35])/434]*ArcTan[(Sqrt[10*(2 + Sqrt[35])]
 + 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/961 - (3*Sqrt[(-15082 + 270
5*Sqrt[35])/434]*Log[Sqrt[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(
1 + 2*x)])/1922 + (3*Sqrt[(-15082 + 2705*Sqrt[35])/434]*Log[Sqrt[35] + Sqrt
[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/1922
```

Rubi [A] time = 0.421929, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {738, 822, 826, 1169, 634, 618, 204, 628}

$$-\frac{\sqrt{2x+1}(5-4x)}{62\left(5x^2+3x+2\right)^2}+\frac{\sqrt{2x+1}(120x+67)}{1922\left(5x^2+3x+2\right)}-\frac{3\sqrt{\frac{1}{434}\left(2705\sqrt{35}-15082\right)}\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}+\sqrt{3}\right.}{1922}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^(3/2)/(2 + 3*x + 5*x^2)^3,x]
```


```
[Out] -((5 - 4*x)*Sqrt[1 + 2*x])/(62*(2 + 3*x + 5*x^2)^2) + (Sqrt[1 + 2*x]*(67 +
120*x))/(1922*(2 + 3*x + 5*x^2)) - (3*Sqrt[(15082 + 2705*Sqrt[35])/434]*Arc
Tan[(Sqrt[10*(2 + Sqrt[35])] - 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])
/961 + (3*Sqrt[(15082 + 2705*Sqrt[35])/434]*ArcTan[(Sqrt[10*(2 + Sqrt[35])]
 + 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/961 - (3*Sqrt[(-15082 + 270
5*Sqrt[35])/434]*Log[Sqrt[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(
1 + 2*x)])/1922 + (3*Sqrt[(-15082 + 2705*Sqrt[35])/434]*Log[Sqrt[35] + Sqrt
[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/1922
```


Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
```

```
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(1+2x)^{3/2}}{(2+3x+5x^2)^3}\,dx = -\frac{(5-4x)\sqrt{1+2x}}{62(2+3x+5x^2)^2} + \frac{1}{62}\int \frac{17+20x}{\sqrt{1+2x}(2+3x+5x^2)^2}\,dx$$

$$= -\frac{(5-4x)\sqrt{1+2x}}{62(2+3x+5x^2)^2} + \frac{\sqrt{1+2x}(67+120x)}{1922(2+3x+5x^2)} + \frac{\int \frac{1239+840x}{\sqrt{1+2x}(2+3x+5x^2)}\,dx}{13454}$$

$$= -\frac{(5-4x)\sqrt{1+2x}}{62(2+3x+5x^2)^2} + \frac{\sqrt{1+2x}(67+120x)}{1922(2+3x+5x^2)} + \frac{\text{Subst}\left(\int \frac{1638+840x^2}{7-4x^2+5x^4}\,dx, x, \sqrt{1+2x}\right)}{6727}$$

$$= -\frac{(5-4x)\sqrt{1+2x}}{62(2+3x+5x^2)^2} + \frac{\sqrt{1+2x}(67+120x)}{1922(2+3x+5x^2)} + \frac{\text{Subst}\left(\int \frac{1638\sqrt{\frac{2}{5}(2+\sqrt{35})}-(1638-168\sqrt{35})x}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}(2+\sqrt{35})}x+x^2}\,dx, x, \sqrt{1-\right.}{13454\sqrt{14(2+\sqrt{35})}}$$

$$= -\frac{(5-4x)\sqrt{1+2x}}{62(2+3x+5x^2)^2} + \frac{\sqrt{1+2x}(67+120x)}{1922(2+3x+5x^2)} + \frac{(3(140+39\sqrt{35}))\,\text{Subst}\left(\int \frac{1}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}(2+\sqrt{35})}x+x^2}\, d\right.}{67270}$$

$$= -\frac{(5-4x)\sqrt{1+2x}}{62(2+3x+5x^2)^2} + \frac{\sqrt{1+2x}(67+120x)}{1922(2+3x+5x^2)} - \frac{3\sqrt{\frac{1}{434}(-15082+2705\sqrt{35})}\log\left(\sqrt{35}-\sqrt{10(2\right.}}{1922}$$

$$= -\frac{(5-4x)\sqrt{1+2x}}{62(2+3x+5x^2)^2} + \frac{\sqrt{1+2x}(67+120x)}{1922(2+3x+5x^2)} - \frac{3}{961}\sqrt{\frac{1}{434}(15082+2705\sqrt{35})}\tan^{-1}\left(\sqrt{\frac{5}{2(-2+}}\right.$$

Mathematica [C] time = 0.48492, size = 198, normalized size = 0.66

$$\frac{\frac{(2960x+4391)(2x+1)^{5/2}}{5x^2+3x+2} + \frac{217(20x+37)(2x+1)^{5/2}}{(5x^2+3x+2)^2} - 1184(2x+1)^{3/2} - 3276\sqrt{2x+1} + \frac{42\left(\sqrt{2-i\sqrt{31}}(1209-218i\sqrt{31})\tanh^{-1}\left(\frac{\sqrt{10x+5}}{\sqrt{2-i\sqrt{31}}}\right)+\sqrt{2+i\sqrt{31}}\right.}{31\sqrt{5}}}{94178}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^(3/2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (-3276*Sqrt[1 + 2*x] - 1184*(1 + 2*x)^(3/2) + (217*(1 + 2*x)^(5/2)*(37 + 20
*x))/(2 + 3*x + 5*x^2)^2 + ((1 + 2*x)^(5/2)*(4391 + 2960*x))/(2 + 3*x + 5*x
^2) + (42*(Sqrt[2 - I*Sqrt[31]]*(1209 - (218*I)*Sqrt[31])*ArcTanh[Sqrt[5 +
10*x]/Sqrt[2 - I*Sqrt[31]]] + Sqrt[2 + I*Sqrt[31]]*(1209 + (218*I)*Sqrt[31]
)*ArcTanh[Sqrt[5 + 10*x]/Sqrt[2 + I*Sqrt[31]]]))/(31*Sqrt[5]))/94178
```

Maple [B] time = 0.078, size = 662, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^(3/2)/(5*x^2+3*x+2)^3,x)
```

```
[Out] 1600*(3/7688*(1+2*x)^(7/2)-41/153760*(1+2*x)^(5/2)+4/4805*(1+2*x)^(3/2)-819
/768800*(1+2*x)^(1/2))/(5*(1+2*x)^2-8*x+3)^2+141/119164*ln(5^(1/2)*7^(1/2)+
```

```
10*x+5+(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2))*5^(1/2)*(2*5^(1/2
)*7^(1/2)+4)^(1/2)-327/417074*ln(5^(1/2)*7^(1/2)+10*x+5+(2*5^(1/2)*7^(1/2)+
4)^(1/2)*5^(1/2)*(1+2*x)^(1/2))*7^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)-705/595
82/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((10*(1+2*x)^(1/2)+5^(1/2)*(2*5^(1/2
)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*(2*5^(1/2)*7^(1/2)+4)+32
7/208537/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((10*(1+2*x)^(1/2)+5^(1/2)*(2*
5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*(2*5^(1/2)
*7^(1/2)+4)*7^(1/2)+234/6727/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((10*(1+2*
x)^(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2)
)*5^(1/2)*7^(1/2)-141/119164*ln(-(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x
)^(1/2)+5^(1/2)*7^(1/2)+10*x+5)*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+327/417
074*ln(-(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2)+5^(1/2)*7^(1/2)+1
0*x+5)*7^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)-705/59582/(10*5^(1/2)*7^(1/2)-20
)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/2))/(10*
5^(1/2)*7^(1/2)-20)^(1/2))*(2*5^(1/2)*7^(1/2)+4)+327/208537/(10*5^(1/2)*7^(
1/2)-20)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/2
))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)*7^(1/2)+234
/6727/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^
(1/2)+10*(1+2*x)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*7^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x+1)^{\frac{3}{2}}}{(5x^2+3x+2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] integrate((2*x + 1)^(3/2)/(5*x^2 + 3*x + 2)^3, x)
```

Fricas [B] time = 2.84599, size = 2778, normalized size = 9.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] -1/2352683507962900*(357492*256095875^(1/4)*sqrt(217)*sqrt(35)*(25*x^4 + 30
*x^3 + 29*x^2 + 12*x + 4)*sqrt(81593620*sqrt(35) + 512191750)*arctan(1/9717
94421886819908125*256095875^(3/4)*sqrt(3787)*sqrt(217)*sqrt(256095875^(1/4)
*sqrt(217)*(4*sqrt(35)*sqrt(31) - 39*sqrt(31))*sqrt(2*x + 1)*sqrt(81593620*
sqrt(35) + 512191750) + 5640925850*x + 564092585*sqrt(35) + 2820462925)*(39
*sqrt(35)*sqrt(31) - 140*sqrt(31))*sqrt(81593620*sqrt(35) + 512191750) - 1/
53405465484875*256095875^(3/4)*sqrt(217)*sqrt(2*x + 1)*sqrt(81593620*sqrt(3
5) + 512191750)*(39*sqrt(35) - 140) + 1/31*sqrt(35)*sqrt(31) + 2/31*sqrt(31
)) + 357492*256095875^(1/4)*sqrt(217)*sqrt(35)*(25*x^4 + 30*x^3 + 29*x^2 +
12*x + 4)*sqrt(81593620*sqrt(35) + 512191750)*arctan(1/1457691632830229862l
875*256095875^(3/4)*sqrt(217)*sqrt(-852075*256095875^(1/4)*sqrt(217)*(4*sqr
t(35)*sqrt(31) - 39*sqrt(31))*sqrt(2*x + 1)*sqrt(81593620*sqrt(35) + 512191
750) + 4806491893638750*x + 480649189363875*sqrt(35) + 2403245946819375)*(3
9*sqrt(35)*sqrt(31) - 140*sqrt(31))*sqrt(81593620*sqrt(35) + 512191750) - 1
/53405465484875*256095875^(3/4)*sqrt(217)*sqrt(2*x + 1)*sqrt(81593620*sqrt(
```

```
35) + 512191750)*(39*sqrt(35) - 140) - 1/31*sqrt(35)*sqrt(31) - 2/31*sqrt(3
1)) - 3*256095875^(1/4)*sqrt(217)*(15082*sqrt(35)*sqrt(31)*(25*x^4 + 30*x^3
 + 29*x^2 + 12*x + 4) - 94675*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4
))*sqrt(81593620*sqrt(35) + 512191750)*log(852075/31*256095875^(1/4)*sqrt(2
17)*(4*sqrt(35)*sqrt(31) - 39*sqrt(31))*sqrt(2*x + 1)*sqrt(81593620*sqrt(35
) + 512191750) + 155048125601250*x + 15504812560125*sqrt(35) + 775240628006
25) + 3*256095875^(1/4)*sqrt(217)*(15082*sqrt(35)*sqrt(31)*(25*x^4 + 30*x^3
 + 29*x^2 + 12*x + 4) - 94675*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4
))*sqrt(81593620*sqrt(35) + 512191750)*log(-852075/31*256095875^(1/4)*sqrt(
217)*(4*sqrt(35)*sqrt(31) - 39*sqrt(31))*sqrt(2*x + 1)*sqrt(81593620*sqrt(3
5) + 512191750) + 155048125601250*x + 15504812560125*sqrt(35) + 77524062800
625) - 1224080909450*(600*x^3 + 695*x^2 + 565*x - 21)*sqrt(2*x + 1))/(25*x^
4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

Sympy [A] time = 92.2554, size = 490, normalized size = 1.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**(3/2)/(5*x**2+3*x+2)**3,x)
```

```
[Out] 229120*(2*x + 1)**(7/2)/(-24109568*x + 5381600*(2*x + 1)**4 - 8610560*(2*x
+ 1)**3 + 18512704*(2*x + 1)**2 - 1506848) - 1774080*(2*x + 1)**(7/2)/(-168
766976*x + 37671200*(2*x + 1)**4 - 60273920*(2*x + 1)**3 + 129588928*(2*x +
 1)**2 - 10547936) - 259072*(2*x + 1)**(5/2)/(-24109568*x + 5381600*(2*x +
1)**4 - 8610560*(2*x + 1)**3 + 18512704*(2*x + 1)**2 - 1506848) - 940352*(2
*x + 1)**(5/2)/(-168766976*x + 37671200*(2*x + 1)**4 - 60273920*(2*x + 1)**
3 + 129588928*(2*x + 1)**2 - 10547936) + 3017984*(2*x + 1)**(3/2)/(5*(-2410
9568*x + 5381600*(2*x + 1)**4 - 8610560*(2*x + 1)**3 + 18512704*(2*x + 1)**
2 - 1506848)) - 6868736*(2*x + 1)**(3/2)/(5*(-168766976*x + 37671200*(2*x +
 1)**4 - 60273920*(2*x + 1)**3 + 129588928*(2*x + 1)**2 - 10547936)) + 128*
(2*x + 1)**(3/2)/(-6944*x + 4340*(2*x + 1)**2 + 2604) - 974848*sqrt(2*x + 1
)/(5*(-24109568*x + 5381600*(2*x + 1)**4 - 8610560*(2*x + 1)**3 + 18512704*
(2*x + 1)**2 - 1506848)) - 5403328*sqrt(2*x + 1)/(-168766976*x + 37671200*(
2*x + 1)**4 - 60273920*(2*x + 1)**3 + 129588928*(2*x + 1)**2 - 10547936) +
1728*sqrt(2*x + 1)/(5*(-6944*x + 4340*(2*x + 1)**2 + 2604)) + 256*RootSum(7
546593148740323163032780 8*_t**4 + 9053854476152406016*_t**2 + 333142578125,
 Lambda(_t, _t*log(216321170454022717 44*_t**3/158378125 + 10865340674816*_t
/1108646875 + sqrt(2*x + 1))))/5 - 448*RootSum(369783064288275834988606259 2
*_t**4 + 2111968303753265086464*_t**2 + 705698730253125, Lambda(_t, _t*log(
-3459438283411209322496*_t**3/1377792122625 + 251494140770688*_t/3572053651
25 + sqrt(2*x + 1))))/5 + 64*RootSum(19950060344639488*_t**4 + 498437272576
*_t**2 + 10878125, Lambda(_t, _t*log(-11049511452672*_t**3/2205125 + 307918
256*_t/2205125 + sqrt(2*x + 1))))/5
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x+1)^{\frac{3}{2}}}{\left(5x^2+3x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] integrate((2*x + 1)^(3/2)/(5*x^2 + 3*x + 2)^3, x)
```

3.2327 $\int \frac{\sqrt{1+2x}}{(2+3x+5x^2)^3} dx$

Optimal. Leaf size=300

$$\frac{\sqrt{2x+1}(10x+3)}{62(5x^2+3x+2)^2}+\frac{\sqrt{2x+1}(1790x+599)}{13454(5x^2+3x+2)}+\frac{\sqrt{\frac{1}{434}\left(1806875\sqrt{35}-9651062\right)}\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}\right.}{13454}$$

```
[Out] (Sqrt[1 + 2*x]*(3 + 10*x))/(62*(2 + 3*x + 5*x^2)^2) + (Sqrt[1 + 2*x]*(599 +
 1790*x))/(13454*(2 + 3*x + 5*x^2)) - (Sqrt[(9651062 + 1806875*Sqrt[35])/43
4]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] - 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[3
5])]])/6727 + (Sqrt[(9651062 + 1806875*Sqrt[35])/434]*ArcTan[(Sqrt[10*(2 +
Sqrt[35])] + 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/6727 + (Sqrt[(-96
51062 + 1806875*Sqrt[35])/434]*Log[Sqrt[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[
1 + 2*x] + 5*(1 + 2*x)])/13454 - (Sqrt[(-9651062 + 1806875*Sqrt[35])/434]*L
og[Sqrt[35] + Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/13454
```

Rubi [A] time = 0.384568, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {736, 822, 826, 1169, 634, 618, 204, 628}

$$\frac{\sqrt{2x+1}(10x+3)}{62(5x^2+3x+2)^2}+\frac{\sqrt{2x+1}(1790x+599)}{13454(5x^2+3x+2)}+\frac{\sqrt{\frac{1}{434}\left(1806875\sqrt{35}-9651062\right)}\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{2x+1}\right.}{13454}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 2*x]/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (Sqrt[1 + 2*x]*(3 + 10*x))/(62*(2 + 3*x + 5*x^2)^2) + (Sqrt[1 + 2*x]*(599 +
 1790*x))/(13454*(2 + 3*x + 5*x^2)) - (Sqrt[(9651062 + 1806875*Sqrt[35])/43
4]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] - 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[3
5])]])/6727 + (Sqrt[(9651062 + 1806875*Sqrt[35])/434]*ArcTan[(Sqrt[10*(2 +
Sqrt[35])] + 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/6727 + (Sqrt[(-96
51062 + 1806875*Sqrt[35])/434]*Log[Sqrt[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[
1 + 2*x] + 5*(1 + 2*x)])/13454 - (Sqrt[(-9651062 + 1806875*Sqrt[35])/434]*L
og[Sqrt[35] + Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/13454
```

Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] - Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)
*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p + 1
), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 0] && (L
tQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c,
d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
```

```
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+2x}}{\left(2+3x+5x^2\right)^3}\,dx = \frac{\sqrt{1+2x}(3+10x)}{62\left(2+3x+5x^2\right)^2} - \frac{1}{62}\int \frac{-27-50x}{\sqrt{1+2x}\left(2+3x+5x^2\right)^2}\,dx$$

$$= \frac{\sqrt{1+2x}(3+10x)}{62\left(2+3x+5x^2\right)^2} + \frac{\sqrt{1+2x}(599+1790x)}{13454\left(2+3x+5x^2\right)} - \frac{\int \frac{-1439-1790x}{\sqrt{1+2x}(2+3x+5x^2)}\,dx}{13454}$$

$$= \frac{\sqrt{1+2x}(3+10x)}{62\left(2+3x+5x^2\right)^2} + \frac{\sqrt{1+2x}(599+1790x)}{13454\left(2+3x+5x^2\right)} - \frac{\text{Subst}\left(\int \frac{-1088-1790x^2}{7-4x^2+5x^4}\,dx, x, \sqrt{1+2x}\right)}{6727}$$

$$= \frac{\sqrt{1+2x}(3+10x)}{62\left(2+3x+5x^2\right)^2} + \frac{\sqrt{1+2x}(599+1790x)}{13454\left(2+3x+5x^2\right)} - \frac{\text{Subst}\left(\int \frac{-1088\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}-\left(-1088+358\sqrt{35}\right)x}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}x+x^2}\,dx, x, \sqrt{\right.}{13454\sqrt{14\left(2+\sqrt{35}\right)}}$$

$$= \frac{\sqrt{1+2x}(3+10x)}{62\left(2+3x+5x^2\right)^2} + \frac{\sqrt{1+2x}(599+1790x)}{13454\left(2+3x+5x^2\right)} + \frac{\left(6265+544\sqrt{35}\right)\text{Subst}\left(\int \frac{1}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}x+x^2}\,d\right.}{470890}$$

$$= \frac{\sqrt{1+2x}(3+10x)}{62\left(2+3x+5x^2\right)^2} + \frac{\sqrt{1+2x}(599+1790x)}{13454\left(2+3x+5x^2\right)} + \frac{\sqrt{\frac{1}{434}\left(-9651062+1806875\sqrt{35}\right)}\log\left(\sqrt{35}-\sqrt{\right.}{13454}$$

$$= \frac{\sqrt{1+2x}(3+10x)}{62\left(2+3x+5x^2\right)^2} + \frac{\sqrt{1+2x}(599+1790x)}{13454\left(2+3x+5x^2\right)} - \frac{\sqrt{\frac{1}{434}\left(9651062+1806875\sqrt{35}\right)}\tan^{-1}\left(\sqrt{\frac{5}{2(-2+}}\right.}{6727}$$

Mathematica [C] time = 0.660843, size = 151, normalized size = 0.5

$$\frac{\frac{1085\sqrt{2x+1}\left(8950x^3+8365x^2+7547x+1849\right)}{\left(5x^2+3x+2\right)^2} + 2\sqrt{10-5i\sqrt{31}}\left(16864-7353i\sqrt{31}\right)\tanh^{-1}\left(\frac{\sqrt{10x+5}}{\sqrt{2-i\sqrt{31}}}\right) + 2\sqrt{10+5i\sqrt{31}}\left(16864+73\right.}{14597590}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 2*x]/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] ((1085*Sqrt[1 + 2*x]*(1849 + 7547*x + 8365*x^2 + 8950*x^3))/(2 + 3*x + 5*x^
2)^2 + 2*Sqrt[10 - (5*I)*Sqrt[31]]*(16864 - (7353*I)*Sqrt[31])*ArcTanh[Sqrt
[5 + 10*x]/Sqrt[2 - I*Sqrt[31]]] + 2*Sqrt[10 + (5*I)*Sqrt[31]]*(16864 + (73
53*I)*Sqrt[31])*ArcTanh[Sqrt[5 + 10*x]/Sqrt[2 + I*Sqrt[31]]])/14597590
```

Maple [B] time = 0.428, size = 1108, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^(1/2)/(5*x^2+3*x+2)^3,x)
```

```
[Out] 5/2919518*(2/21475*5^(1/2)*(-13012793430*5^(1/2)+6673227400*7^(1/2))/(-390+
40*5^(1/2)*7^(1/2))*(1+2*x)^(3/2)+1/107375/(-390+40*5^(1/2)*7^(1/2))*(-2145
87133600*5^(1/2)+114637845000*7^(1/2))*(2*5^(1/2)*7^(1/2)+4)^(1/2)*(1+2*x)+
```

```
2/107375*(-141628999400*5^(1/2)*7^(1/2)+440433008400)/(-390+40*5^(1/2)*7^(1
/2))*(1+2*x)^(1/2)+1/107375*(-76332028500*7^(1/2)+54802482000*5^(1/2))*(2*5
^(1/2)*7^(1/2)+4)^(1/2)/(-390+40*5^(1/2)*7^(1/2)))/(1/5*5^(1/2)*7^(1/2)+2*x
+1+1/5*(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2))^2-161475/417074/(
20*5^(1/2)*7^(1/2)-195)*ln(5+10*x+35^(1/2)+(1+2*x)^(1/2)*(20+10*35^(1/2))^(
1/2))*5^(1/2)*(2*35^(1/2)+4)^(1/2)+2065235/5839036/(20*5^(1/2)*7^(1/2)-195)
*ln(5+10*x+35^(1/2)+(1+2*x)^(1/2)*(20+10*35^(1/2))^(1/2))*(2*35^(1/2)+4)^(1
/2)*7^(1/2)+161475/208537/(20*5^(1/2)*7^(1/2)-195)/(-20+10*35^(1/2))^(1/2)*
arctan((10*(1+2*x)^(1/2)+(20+10*35^(1/2))^(1/2))/(-20+10*35^(1/2))^(1/2))*(
20+10*35^(1/2))^(1/2)*5^(1/2)*(2*35^(1/2)+4)^(1/2)-2065235/2919518/(20*5^(1
/2)*7^(1/2)-195)/(-20+10*35^(1/2))^(1/2)*arctan((10*(1+2*x)^(1/2)+(20+10*35
^(1/2))^(1/2))/(-20+10*35^(1/2))^(1/2))*(20+10*35^(1/2))^(1/2)*(2*35^(1/2)+
4)^(1/2)*7^(1/2)-212160/47089/(20*5^(1/2)*7^(1/2)-195)/(-20+10*35^(1/2))^(1
/2)*arctan((10*(1+2*x)^(1/2)+(20+10*35^(1/2))^(1/2))/(-20+10*35^(1/2))^(1/2
))*35^(1/2)+108800/6727/(20*5^(1/2)*7^(1/2)-195)/(-20+10*35^(1/2))^(1/2)*ar
ctan((10*(1+2*x)^(1/2)+(20+10*35^(1/2))^(1/2))/(-20+10*35^(1/2))^(1/2))-5/2
919518*(-2/21475*5^(1/2)*(-13012793430*5^(1/2)+6673227400*7^(1/2))/(-390+40
*5^(1/2)*7^(1/2))*(1+2*x)^(3/2)+1/107375/(-390+40*5^(1/2)*7^(1/2))*(-214587
133600*5^(1/2)+114637845000*7^(1/2))*(2*5^(1/2)*7^(1/2)+4)^(1/2)*(1+2*x)-2/
107375*(-141628999400*5^(1/2)*7^(1/2)+440433008400)/(-390+40*5^(1/2)*7^(1/2
))*(1+2*x)^(1/2)+1/107375*(-76332028500*7^(1/2)+54802482000*5^(1/2))*(2*5^(
1/2)*7^(1/2)+4)^(1/2)/(-390+40*5^(1/2)*7^(1/2)))/(1/5*5^(1/2)*7^(1/2)+2*x+1
-1/5*(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2))^2+161475/417074/(20
*5^(1/2)*7^(1/2)-195)*ln(5+10*x+35^(1/2)-(1+2*x)^(1/2)*(20+10*35^(1/2))^(1/
2))*5^(1/2)*(2*35^(1/2)+4)^(1/2)-2065235/5839036/(20*5^(1/2)*7^(1/2)-195)*l
n(5+10*x+35^(1/2)-(1+2*x)^(1/2)*(20+10*35^(1/2))^(1/2))*(2*35^(1/2)+4)^(1/2
)*7^(1/2)+161475/208537/(20*5^(1/2)*7^(1/2)-195)/(-20+10*35^(1/2))^(1/2)*ar
ctan((-(20+10*35^(1/2))^(1/2)+10*(1+2*x)^(1/2))/(-20+10*35^(1/2))^(1/2))*(2
0+10*35^(1/2))^(1/2)*5^(1/2)*(2*35^(1/2)+4)^(1/2)-2065235/2919518/(20*5^(1/
2)*7^(1/2)-195)/(-20+10*35^(1/2))^(1/2)*arctan((-(20+10*35^(1/2))^(1/2)+10*
(1+2*x)^(1/2))/(-20+10*35^(1/2))^(1/2))*(20+10*35^(1/2))^(1/2)*(2*35^(1/2)+
4)^(1/2)*7^(1/2)-212160/47089/(20*5^(1/2)*7^(1/2)-195)/(-20+10*35^(1/2))^(1
/2)*arctan((-(20+10*35^(1/2))^(1/2)+10*(1+2*x)^(1/2))/(-20+10*35^(1/2))^(1/
2))*35^(1/2)+108800/6727/(20*5^(1/2)*7^(1/2)-195)/(-20+10*35^(1/2))^(1/2)*a
rctan((-(20+10*35^(1/2))^(1/2)+10*(1+2*x)^(1/2))/(-20+10*35^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x+1}}{\left(5x^2+3x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(1/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x + 1)/(5*x^2 + 3*x + 2)^3, x)
```

Fricas [B] time = 2.59914, size = 3016, normalized size = 10.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(1/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/308559878029380460*(102361876*121835^(1/4)*sqrt(217)*sqrt(118)*sqrt(35)*(
25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(9651062*sqrt(35) + 63240625)*arct
an(1/3720359949701672719767 5*121835^(3/4)*sqrt(26629)*sqrt(2065)*sqrt(217)*
sqrt(118)*sqrt(121835^(1/4)*sqrt(217)*sqrt(118)*(179*sqrt(35)*sqrt(31) - 54
4*sqrt(31))*sqrt(2*x + 1)*sqrt(9651062*sqrt(35) + 63240625) + 105688636970*
x + 10568863697*sqrt(35) + 52844318485)*sqrt(9651062*sqrt(35) + 63240625)*(
544*sqrt(35) - 6265) - 1/21824703534305*121835^(3/4)*sqrt(217)*sqrt(118)*sq
rt(2*x + 1)*sqrt(9651062*sqrt(35) + 63240625)*(544*sqrt(35) - 6265) + 1/31*
sqrt(35)*sqrt(31) + 2/31*sqrt(31)) + 102361876*121835^(1/4)*sqrt(217)*sqrt(
118)*sqrt(35)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(9651062*sqrt(35) +
 63240625)*arctan(1/1139360234596137270428796875*121835^(3/4)*sqrt(26629)*s
qrt(217)*sqrt(118)*sqrt(-1936744140625*121835^(1/4)*sqrt(217)*sqrt(118)*(17
9*sqrt(35)*sqrt(31) - 544*sqrt(31))*sqrt(2*x + 1)*sqrt(9651062*sqrt(35) + 6
3240625) + 2046918483822902539062 50*x + 204691848382290253906 25*sqrt(35) +
102345924191145126953125)*sqrt(9651062*sqrt(35) + 63240625)*(544*sqrt(35) -
 6265) - 1/21824703534305*121835^(3/4)*sqrt(217)*sqrt(118)*sqrt(2*x + 1)*sq
rt(9651062*sqrt(35) + 63240625)*(544*sqrt(35) - 6265) - 1/31*sqrt(35)*sqrt(
31) - 2/31*sqrt(31)) + 121835^(1/4)*sqrt(217)*sqrt(118)*(9651062*sqrt(35)*s
qrt(31)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) - 63240625*sqrt(31)*(25*x^4 +
 30*x^3 + 29*x^2 + 12*x + 4))*sqrt(9651062*sqrt(35) + 63240625)*log(1936744
140625/26629*121835^(1/4)*sqrt(217)*sqrt(118)*(179*sqrt(35)*sqrt(31) - 544*
sqrt(31))*sqrt(2*x + 1)*sqrt(9651062*sqrt(35) + 63240625) + 768680192205078
1250*x + 768680192205078125*sqrt(35) + 3843400961025390625) - 121835^(1/4)*
sqrt(217)*sqrt(118)*(9651062*sqrt(35)*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2 +
12*x + 4) - 63240625*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4))*sqrt(9
651062*sqrt(35) + 63240625)*log(-1936744140625/26629*121835^(1/4)*sqrt(217)
*sqrt(118)*(179*sqrt(35)*sqrt(31) - 544*sqrt(31))*sqrt(2*x + 1)*sqrt(965106
2*sqrt(35) + 63240625) + 7686801922050781250*x + 768680192205078125*sqrt(35
) + 3843400961025390625) + 22934434222490*(8950*x^3 + 8365*x^2 + 7547*x + 1
849)*sqrt(2*x + 1))/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

Sympy [A] time = 8.83838, size = 199, normalized size = 0.66

$$\frac{286400\,(2x+1)^{\frac{7}{2}}}{-24109568x+5381600\,(2x+1)^4-8610560\,(2x+1)^3+18512704\,(2x+1)^2-1506848}-\frac{}{-24109568x+5381600\,(2x+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**(1/2)/(5*x**2+3*x+2)**3,x)
```

```
[Out] 286400*(2*x + 1)**(7/2)/(-24109568*x + 5381600*(2*x + 1)**4 - 8610560*(2*x
+ 1)**3 + 18512704*(2*x + 1)**2 - 1506848) - 323840*(2*x + 1)**(5/2)/(-2410
9568*x + 5381600*(2*x + 1)**4 - 8610560*(2*x + 1)**3 + 18512704*(2*x + 1)**
2 - 1506848) + 754496*(2*x + 1)**(3/2)/(-24109568*x + 5381600*(2*x + 1)**4
- 8610560*(2*x + 1)**3 + 18512704*(2*x + 1)**2 - 1506848) - 243712*sqrt(2*x
 + 1)/(-24109568*x + 5381600*(2*x + 1)**4 - 8610560*(2*x + 1)**3 + 18512704
*(2*x + 1)**2 - 1506848) + 64*RootSum(754659314874032316303278 08*_t**4 + 90
53854476152406016*_t**2 + 333142578125, Lambda(_t, _t*log(21632117045402271
744*_t**3/158378125 + 10865340674816*_t/1108646875 + sqrt(2*x + 1))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,x+1}}{\left(5\,x^2+3\,x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(1/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(2*x + 1)/(5*x^2 + 3*x + 2)^3, x)
```

3.2328 $\int \frac{1}{\sqrt{1+2x}\left(2+3x+5x^2\right)^3}\,dx$

Optimal. Leaf size=314

$$\frac{\sqrt{2x+1}(20x+37)}{434\left(5x^2+3x+2\right)^2}+\frac{\sqrt{2x+1}(7920x+9227)}{94178\left(5x^2+3x+2\right)}-\frac{3\sqrt{\frac{1}{434}\left(64681225\sqrt{35}-250141922\right)}\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\right.}{94178}$$

```
[Out] (Sqrt[1 + 2*x]*(37 + 20*x))/(434*(2 + 3*x + 5*x^2)^2) + (Sqrt[1 + 2*x]*(922
7 + 7920*x))/(94178*(2 + 3*x + 5*x^2)) - (3*Sqrt[(2 + Sqrt[35])/434]*(7379
+ 264*Sqrt[35])*ArcTan[(Sqrt[10*(2 + Sqrt[35])] - 10*Sqrt[1 + 2*x])/Sqrt[10
*(-2 + Sqrt[35])]])/47089 + (3*Sqrt[(2 + Sqrt[35])/434]*(7379 + 264*Sqrt[35
])*ArcTan[(Sqrt[10*(2 + Sqrt[35])] + 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[3
5])]])/47089 - (3*Sqrt[(-250141922 + 64681225*Sqrt[35])/434]*Log[Sqrt[35] -
 Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/94178 + (3*Sqrt[(-25
0141922 + 64681225*Sqrt[35])/434]*Log[Sqrt[35] + Sqrt[10*(2 + Sqrt[35])]*Sq
rt[1 + 2*x] + 5*(1 + 2*x)])/94178
```

Rubi [A] time = 0.386157, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {740, 822, 826, 1169, 634, 618, 204, 628}

$$\frac{\sqrt{2x+1}(20x+37)}{434\left(5x^2+3x+2\right)^2}+\frac{\sqrt{2x+1}(7920x+9227)}{94178\left(5x^2+3x+2\right)}-\frac{3\sqrt{\frac{1}{434}\left(64681225\sqrt{35}-250141922\right)}\log\left(5(2x+1)-\sqrt{10\left(2+\sqrt{35}\right)}\right.}{94178}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + 2*x]*(2 + 3*x + 5*x^2)^3),x]
```


```
[Out] (Sqrt[1 + 2*x]*(37 + 20*x))/(434*(2 + 3*x + 5*x^2)^2) + (Sqrt[1 + 2*x]*(922
7 + 7920*x))/(94178*(2 + 3*x + 5*x^2)) - (3*Sqrt[(2 + Sqrt[35])/434]*(7379
+ 264*Sqrt[35])*ArcTan[(Sqrt[10*(2 + Sqrt[35])] - 10*Sqrt[1 + 2*x])/Sqrt[10
*(-2 + Sqrt[35])]])/47089 + (3*Sqrt[(2 + Sqrt[35])/434]*(7379 + 264*Sqrt[35
])*ArcTan[(Sqrt[10*(2 + Sqrt[35])] + 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[3
5])]])/47089 - (3*Sqrt[(-250141922 + 64681225*Sqrt[35])/434]*Log[Sqrt[35] -
 Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/94178 + (3*Sqrt[(-25
0141922 + 64681225*Sqrt[35])/434]*Log[Sqrt[35] + Sqrt[10*(2 + Sqrt[35])]*Sq
rt[1 + 2*x] + 5*(1 + 2*x)])/94178
```


Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+2x}\left(2+3x+5x^2\right)^3}\,dx = \frac{\sqrt{1+2x}(37+20x)}{434\left(2+3x+5x^2\right)^2} + \frac{1}{434}\int \frac{271+100x}{\sqrt{1+2x}\left(2+3x+5x^2\right)^2}\,dx$$

$$= \frac{\sqrt{1+2x}(37+20x)}{434\left(2+3x+5x^2\right)^2} + \frac{\sqrt{1+2x}(9227+7920x)}{94178\left(2+3x+5x^2\right)} + \frac{\int \frac{26097+7920x}{\sqrt{1+2x}\left(2+3x+5x^2\right)}\,dx}{94178}$$

$$= \frac{\sqrt{1+2x}(37+20x)}{434\left(2+3x+5x^2\right)^2} + \frac{\sqrt{1+2x}(9227+7920x)}{94178\left(2+3x+5x^2\right)} + \frac{\text{Subst}\left(\int \frac{44274+7920x^2}{7-4x^2+5x^4}\,dx, x, \sqrt{1+2x}\right.}{47089}$$

$$= \frac{\sqrt{1+2x}(37+20x)}{434\left(2+3x+5x^2\right)^2} + \frac{\sqrt{1+2x}(9227+7920x)}{94178\left(2+3x+5x^2\right)} + \frac{\text{Subst}\left(\int \frac{44274\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}-\left(44274-1584\right.}{\sqrt{\frac{7}{5}}-\sqrt{\frac{2}{5}\left(2+\sqrt{35}\right)}x+x^2}\right.}{94178\sqrt{14\left(2+\sqrt{3\right.}}}$$

$$= \frac{\sqrt{1+2x}(37+20x)}{434\left(2+3x+5x^2\right)^2} + \frac{\sqrt{1+2x}(9227+7920x)}{94178\left(2+3x+5x^2\right)} + \frac{\left(3\left(9240+7379\sqrt{35}\right)\right)\text{Subst}\left(\int \frac{}{\sqrt{\frac{7}{5}}}\right.}{329623}$$

$$= \frac{\sqrt{1+2x}(37+20x)}{434\left(2+3x+5x^2\right)^2} + \frac{\sqrt{1+2x}(9227+7920x)}{94178\left(2+3x+5x^2\right)} - \frac{3\sqrt{\frac{1}{434}\left(-250141922+64681225\sqrt{3\right.}}}{}$$

$$= \frac{\sqrt{1+2x}(37+20x)}{434\left(2+3x+5x^2\right)^2} + \frac{\sqrt{1+2x}(9227+7920x)}{94178\left(2+3x+5x^2\right)} - \frac{3\sqrt{\frac{1}{434}\left(250141922+64681225\sqrt{35}\right)}}{}$$

Mathematica [C] time = 0.651586, size = 151, normalized size = 0.48

$$\frac{\frac{1085\sqrt{2x+1}\left(39600x^3+69895x^2+47861x+26483\right)}{\left(5x^2+3x+2\right)^2} + 6\sqrt{10-5i\sqrt{31}}\left(228749-23998i\sqrt{31}\right)\tanh^{-1}\left(\frac{\sqrt{10x+5}}{\sqrt{2-i\sqrt{31}}}\right) + 6\sqrt{10+5i\sqrt{31}}\left(2287\right.}{102183130}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + 2*x]*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] ((1085*Sqrt[1 + 2*x]*(26483 + 47861*x + 69895*x^2 + 39600*x^3))/(2 + 3*x +
5*x^2)^2 + 6*Sqrt[10 - (5*I)*Sqrt[31]]*(228749 - (23998*I)*Sqrt[31])*ArcTan
h[Sqrt[5 + 10*x]/Sqrt[2 - I*Sqrt[31]]] + 6*Sqrt[10 + (5*I)*Sqrt[31]]*(22874
9 + (23998*I)*Sqrt[31])*ArcTanh[Sqrt[5 + 10*x]/Sqrt[2 + I*Sqrt[31]]])/10218
3130
```

Maple [B] time = 0.444, size = 1100, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2*x)^(1/2)/(5*x^2+3*x+2)^3,x)
```

```
[Out] 5/20436626*(6/1353025*(-6045943503600+620096769600*5^(1/2)*7^(1/2))/(-390+4
0*5^(1/2)*7^(1/2))*(1+2*x)^(3/2)+1/6765125/(-390+40*5^(1/2)*7^(1/2))*(-9104
```

```
8526818200*5^(1/2)+65791327714000*7^(1/2))*(2*5^(1/2)*7^(1/2)+4)^(1/2)*(1+2
*x)+2/6765125*(-59423591568600*5^(1/2)*7^(1/2)+320925328420550)/(-390+40*5^
(1/2)*7^(1/2))*(1+2*x)^(1/2)+1/13530250*(-123371070933600*7^(1/2)+152992435
939000*5^(1/2))*(2*5^(1/2)*7^(1/2)+4)^(1/2)/(-390+40*5^(1/2)*7^(1/2)))/(1/5
*5^(1/2)*7^(1/2)+2*x+1+1/5*(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2
))^2-6065475/5839036/(20*5^(1/2)*7^(1/2)-195)*ln(5+10*x+35^(1/2)+(1+2*x)^(1
/2)*(20+10*35^(1/2))^(1/2))*5^(1/2)*(2*35^(1/2)+4)^(1/2)+15321765/20436626/
(20*5^(1/2)*7^(1/2)-195)*ln(5+10*x+35^(1/2)+(1+2*x)^(1/2)*(20+10*35^(1/2))^
(1/2))*(2*35^(1/2)+4)^(1/2)*7^(1/2)+6065475/2919518/(20*5^(1/2)*7^(1/2)-195
)/(-20+10*35^(1/2))^(1/2)*arctan((10*(1+2*x)^(1/2)+(20+10*35^(1/2))^(1/2))/
(-20+10*35^(1/2))^(1/2))*(20+10*35^(1/2))^(1/2)*5^(1/2)*(2*35^(1/2)+4)^(1/2
)-15321765/10218313/(20*5^(1/2)*7^(1/2)-195)/(-20+10*35^(1/2))^(1/2)*arctan
((10*(1+2*x)^(1/2)+(20+10*35^(1/2))^(1/2))/(-20+10*35^(1/2))^(1/2))*(20+10*
35^(1/2))^(1/2)*(2*35^(1/2)+4)^(1/2)*7^(1/2)-8633430/329623/(20*5^(1/2)*7^(
1/2)-195)/(-20+10*35^(1/2))^(1/2)*arctan((10*(1+2*x)^(1/2)+(20+10*35^(1/2))
^(1/2))/(-20+10*35^(1/2))^(1/2))*35^(1/2)+4427400/47089/(20*5^(1/2)*7^(1/2)
-195)/(-20+10*35^(1/2))^(1/2)*arctan((10*(1+2*x)^(1/2)+(20+10*35^(1/2))^(1/
2))/(-20+10*35^(1/2))^(1/2))-5/20436626*(-6/1353025*(-6045943503600+6200967
69600*5^(1/2)*7^(1/2))/(-390+40*5^(1/2)*7^(1/2))*(1+2*x)^(3/2)+1/6765125/(-
390+40*5^(1/2)*7^(1/2))*(-91048526818200*5^(1/2)+65791327714000*7^(1/2))*(2
*5^(1/2)*7^(1/2)+4)^(1/2)*(1+2*x)-2/6765125*(-59423591568600*5^(1/2)*7^(1/2
)+320925328420550)/(-390+40*5^(1/2)*7^(1/2))*(1+2*x)^(1/2)+1/13530250*(-123
371070933600*7^(1/2)+152992435939000*5^(1/2))*(2*5^(1/2)*7^(1/2)+4)^(1/2)/(
-390+40*5^(1/2)*7^(1/2)))/(1/5*5^(1/2)*7^(1/2)+2*x+1-1/5*(2*5^(1/2)*7^(1/2)
+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2))^2+6065475/5839036/(20*5^(1/2)*7^(1/2)-195)
*ln(5+10*x+35^(1/2)-(1+2*x)^(1/2)*(20+10*35^(1/2))^(1/2))*5^(1/2)*(2*35^(1/
2)+4)^(1/2)-15321765/20436626/(20*5^(1/2)*7^(1/2)-195)*ln(5+10*x+35^(1/2)-(
1+2*x)^(1/2)*(20+10*35^(1/2))^(1/2))*(2*35^(1/2)+4)^(1/2)*7^(1/2)+6065475/2
919518/(20*5^(1/2)*7^(1/2)-195)/(-20+10*35^(1/2))^(1/2)*arctan((-(20+10*35^
(1/2))^(1/2)+10*(1+2*x)^(1/2))/(-20+10*35^(1/2))^(1/2))*(20+10*35^(1/2))^(1
/2)*5^(1/2)*(2*35^(1/2)+4)^(1/2)-15321765/10218313/(20*5^(1/2)*7^(1/2)-195)
/(-20+10*35^(1/2))^(1/2)*arctan((-(20+10*35^(1/2))^(1/2)+10*(1+2*x)^(1/2))/
(-20+10*35^(1/2))^(1/2))*(20+10*35^(1/2))^(1/2)*(2*35^(1/2)+4)^(1/2)*7^(1/2
)-8633430/329623/(20*5^(1/2)*7^(1/2)-195)/(-20+10*35^(1/2))^(1/2)*arctan((-
(20+10*35^(1/2))^(1/2)+10*(1+2*x)^(1/2))/(-20+10*35^(1/2))^(1/2))*35^(1/2)+
4427400/47089/(20*5^(1/2)*7^(1/2)-195)/(-20+10*35^(1/2))^(1/2)*arctan((-(20
+10*35^(1/2))^(1/2)+10*(1+2*x)^(1/2))/(-20+10*35^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(5x^2+3x+2\right)^3\sqrt{2x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(1/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^3*sqrt(2*x + 1)), x)
```

Fricas [B] time = 2.83957, size = 3406, normalized size = 10.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(1/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] -1/1217868167180398474920 20*(19347824532*97578096035^(1/4)*sqrt(105602)*sqr
t(217)*sqrt(35)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(250141922*sqrt(3
5) + 2263842875)*arctan(1/151217699255837915199192584756839 75*97578096035^(
3/4)*sqrt(1677751)*sqrt(105602)*sqrt(37715)*sqrt(217)*sqrt(97578096035^(1/4
)*sqrt(105602)*sqrt(217)*(264*sqrt(35)*sqrt(31) - 7379*sqrt(31))*sqrt(2*x +
 1)*sqrt(250141922*sqrt(35) + 2263842875) + 5959242818165770*x + 5959242818
16577*sqrt(35) + 2979621409082885)*sqrt(250141922*sqrt(35) + 2263842875)*(7
379*sqrt(35) - 9240) - 1/110128893014689787619 5*97578096035^(3/4)*sqrt(1056
02)*sqrt(217)*sqrt(2*x + 1)*sqrt(250141922*sqrt(35) + 2263842875)*(7379*sqr
t(35) - 9240) + 1/31*sqrt(35)*sqrt(31) + 2/31*sqrt(31)) + 19347824532*97578
096035^(1/4)*sqrt(105602)*sqrt(217)*sqrt(35)*(25*x^4 + 30*x^3 + 29*x^2 + 12
*x + 4)*sqrt(250141922*sqrt(35) + 2263842875)*arctan(1/38900753133564303684
992292428697025687 5*97578096035^(3/4)*sqrt(1677751)*sqrt(105602)*sqrt(217)*
sqrt(-24958867696875*97578096035^(1/4)*sqrt(105602)*sqrt(217)*(264*sqrt(35)
*sqrt(31) - 7379*sqrt(31))*sqrt(2*x + 1)*sqrt(250141922*sqrt(35) + 22638428
75) + 14873595307215197629186096875 0*x + 1487359530721519762918609687 5*sqrt
(35) + 743679765360759881459304843 75)*sqrt(250141922*sqrt(35) + 2263842875)
*(7379*sqrt(35) - 9240) - 1/110128893014689787619 5*97578096035^(3/4)*sqrt(1
05602)*sqrt(217)*sqrt(2*x + 1)*sqrt(250141922*sqrt(35) + 2263842875)*(7379*
sqrt(35) - 9240) - 1/31*sqrt(35)*sqrt(31) - 2/31*sqrt(31)) - 3*97578096035^
(1/4)*sqrt(105602)*sqrt(217)*(250141922*sqrt(35)*sqrt(31)*(25*x^4 + 30*x^3
+ 29*x^2 + 12*x + 4) - 2263842875*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x
 + 4))*sqrt(250141922*sqrt(35) + 2263842875)*log(24958867696875/1677751*975
78096035^(1/4)*sqrt(105602)*sqrt(217)*(264*sqrt(35)*sqrt(31) - 7379*sqrt(31
))*sqrt(2*x + 1)*sqrt(250141922*sqrt(35) + 2263842875) + 886519829653816187
81250*x + 8865198296538161878125*sqrt(35) + 4432599148269080939062 5) + 3*97
578096035^(1/4)*sqrt(105602)*sqrt(217)*(250141922*sqrt(35)*sqrt(31)*(25*x^4
 + 30*x^3 + 29*x^2 + 12*x + 4) - 2263842875*sqrt(31)*(25*x^4 + 30*x^3 + 29*
x^2 + 12*x + 4))*sqrt(250141922*sqrt(35) + 2263842875)*log(-24958867696875/
1677751*97578096035^(1/4)*sqrt(105602)*sqrt(217)*(264*sqrt(35)*sqrt(31) - 7
379*sqrt(31))*sqrt(2*x + 1)*sqrt(250141922*sqrt(35) + 2263842875) + 8865198
2965381618781250*x + 8865198296538161878125*sqrt(35) + 44325991482690809390
625) - 1293155691541972090*(39600*x^3 + 69895*x^2 + 47861*x + 26483)*sqrt(2
*x + 1))/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x+1}\left(5x^2+3x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)**(1/2)/(5*x**2+3*x+2)**3,x)
```

```
[Out] Integral(1/(sqrt(2*x + 1)*(5*x**2 + 3*x + 2)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(5x^2+3x+2\right)^3\sqrt{2x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(1/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^3*sqrt(2*x + 1)), x)
```

3.2329 $\int \frac{1}{(1+2x)^{3/2}(2+3x+5x^2)^3}\,dx$

Optimal. Leaf size=313

$$\frac{20x+37}{434\sqrt{2x+1}\,(5x^2+3x+2)^2}+\frac{5(2080x+2329)}{94178\sqrt{2x+1}\,(5x^2+3x+2)}-\frac{81090}{329623\sqrt{2x+1}}-\frac{15\sqrt{\frac{1}{434}\left(2257111762+387427075\sqrt{\right.}}}{}$$

```
[Out] -81090/(329623*Sqrt[1 + 2*x]) + (37 + 20*x)/(434*Sqrt[1 + 2*x]*(2 + 3*x + 5
*x^2)^2) + (5*(2329 + 2080*x))/(94178*Sqrt[1 + 2*x]*(2 + 3*x + 5*x^2)) - (1
5*Sqrt[(-2257111762 + 387427075*Sqrt[35])/434]*ArcTan[(Sqrt[10*(2 + Sqrt[35
])] - 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/329623 + (15*Sqrt[(-2257
111762 + 387427075*Sqrt[35])/434]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] + 10*Sqrt
[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/329623 - (15*Sqrt[(2257111762 + 38742
7075*Sqrt[35])/434]*Log[Sqrt[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] +
5*(1 + 2*x)])/659246 + (15*Sqrt[(2257111762 + 387427075*Sqrt[35])/434]*Log[
Sqrt[35] + Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/659246
```

Rubi [A] time = 0.423517, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {740, 822, 828, 826, 1169, 634, 618, 204, 628}

$$\frac{20x+37}{434\sqrt{2x+1}\,(5x^2+3x+2)^2}+\frac{5(2080x+2329)}{94178\sqrt{2x+1}\,(5x^2+3x+2)}-\frac{81090}{329623\sqrt{2x+1}}-\frac{15\sqrt{\frac{1}{434}\left(2257111762+387427075\sqrt{\right.}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + 2*x)^(3/2)*(2 + 3*x + 5*x^2)^3),x]
```



```
[Out] -81090/(329623*Sqrt[1 + 2*x]) + (37 + 20*x)/(434*Sqrt[1 + 2*x]*(2 + 3*x + 5
*x^2)^2) + (5*(2329 + 2080*x))/(94178*Sqrt[1 + 2*x]*(2 + 3*x + 5*x^2)) - (1
5*Sqrt[(-2257111762 + 387427075*Sqrt[35])/434]*ArcTan[(Sqrt[10*(2 + Sqrt[35
])] - 10*Sqrt[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/329623 + (15*Sqrt[(-2257
111762 + 387427075*Sqrt[35])/434]*ArcTan[(Sqrt[10*(2 + Sqrt[35])] + 10*Sqrt
[1 + 2*x])/Sqrt[10*(-2 + Sqrt[35])]])/329623 - (15*Sqrt[(2257111762 + 38742
7075*Sqrt[35])/434]*Log[Sqrt[35] - Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] +
5*(1 + 2*x)])/659246 + (15*Sqrt[(2257111762 + 387427075*Sqrt[35])/434]*Log[
Sqrt[35] + Sqrt[10*(2 + Sqrt[35])]*Sqrt[1 + 2*x] + 5*(1 + 2*x)])/659246
```


Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
```

```
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(1+2x)^{3/2}\left(2+3x+5x^2\right)^3}\,dx = \frac{37+20x}{434\sqrt{1+2x}\left(2+3x+5x^2\right)^2}+\frac{1}{434}\int \frac{345+140x}{(1+2x)^{3/2}\left(2+3x+5x^2\right)^2}\,dx$$

$$= \frac{37+20x}{434\sqrt{1+2x}\left(2+3x+5x^2\right)^2}+\frac{5(2329+2080x)}{94178\sqrt{1+2x}\left(2+3x+5x^2\right)}+\frac{\int \frac{56145+31200x}{(1+2x)^{3/2}\left(2+3x+5x^2\right)}\,d}{94178}$$

$$= -\frac{81090}{329623\sqrt{1+2x}}+\frac{37+20x}{434\sqrt{1+2x}\left(2+3x+5x^2\right)^2}+\frac{5(2329+2080x)}{94178\sqrt{1+2x}\left(2+3x+5x^2\right)}+$$

$$= -\frac{81090}{329623\sqrt{1+2x}}+\frac{37+20x}{434\sqrt{1+2x}\left(2+3x+5x^2\right)^2}+\frac{5(2329+2080x)}{94178\sqrt{1+2x}\left(2+3x+5x^2\right)}+$$

$$= -\frac{81090}{329623\sqrt{1+2x}}+\frac{37+20x}{434\sqrt{1+2x}\left(2+3x+5x^2\right)^2}+\frac{5(2329+2080x)}{94178\sqrt{1+2x}\left(2+3x+5x^2\right)}+$$

$$= -\frac{81090}{329623\sqrt{1+2x}}+\frac{37+20x}{434\sqrt{1+2x}\left(2+3x+5x^2\right)^2}+\frac{5(2329+2080x)}{94178\sqrt{1+2x}\left(2+3x+5x^2\right)}-$$

$$= -\frac{81090}{329623\sqrt{1+2x}}+\frac{37+20x}{434\sqrt{1+2x}\left(2+3x+5x^2\right)^2}+\frac{5(2329+2080x)}{94178\sqrt{1+2x}\left(2+3x+5x^2\right)}-$$

$$= -\frac{81090}{329623\sqrt{1+2x}}+\frac{37+20x}{434\sqrt{1+2x}\left(2+3x+5x^2\right)^2}+\frac{5(2329+2080x)}{94178\sqrt{1+2x}\left(2+3x+5x^2\right)}-$$

Mathematica [C] time = 0.870828, size = 156, normalized size = 0.5

$$\frac{-\frac{217\left(4054500x^4+4501400x^3+4077245x^2+1525635x+429487\right)}{\sqrt{2x+1}\left(5x^2+3x+2\right)^2}+6\sqrt{10-5i\sqrt{31}}\left(560852+58421i\sqrt{31}\right)\tanh^{-1}\left(\frac{\sqrt{10x+5}}{\sqrt{2-i\sqrt{31}}}\right)+6\sqrt{10+5i}}{143056382}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + 2*x)^(3/2)*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] ((-217*(429487 + 1525635*x + 4077245*x^2 + 4501400*x^3 + 4054500*x^4))/(Sqr
t[1 + 2*x]*(2 + 3*x + 5*x^2)^2) + 6*Sqrt[10 - (5*I)*Sqrt[31]]*(560852 + (58
421*I)*Sqrt[31])*ArcTanh[Sqrt[5 + 10*x]/Sqrt[2 - I*Sqrt[31]]] + 6*Sqrt[10 +
 (5*I)*Sqrt[31]]*(560852 - (58421*I)*Sqrt[31])*ArcTanh[Sqrt[5 + 10*x]/Sqrt[
2 + I*Sqrt[31]]])/143056382
```

Maple [B] time = 0.082, size = 671, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2*x)^(3/2)/(5*x^2+3*x+2)^3,x)
```

```
[Out] -1600/343*(9793/30752*(1+2*x)^(7/2)-14343/19220*(1+2*x)^(5/2)+762223/768800
*(1+2*x)^(3/2)-170877/192200*(1+2*x)^(1/2))/(5*(1+2*x)^2-8*x+3)^2+95145/204
36626*ln(5^(1/2)*7^(1/2)+10*x+5+(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)
^(1/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+876315/286112764*ln(5^(1/2)*7^(
1/2)+10*x+5+(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2))*7^(1/2)*(2*5
^(1/2)*7^(1/2)+4)^(1/2)-475725/10218313/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arcta
n((10*(1+2*x)^(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2
)-20)^(1/2))*(2*5^(1/2)*7^(1/2)+4)-876315/143056382/(10*5^(1/2)*7^(1/2)-20)
^(1/2)*arctan((10*(1+2*x)^(1/2)+5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2))/(10*5^
(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)*7^(1/2)+542760/23073
61/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((10*(1+2*x)^(1/2)+5^(1/2)*(2*5^(1/2
)*7^(1/2)+4)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*7^(1/2)-95145/20
436626*ln(-(2*5^(1/2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2)+5^(1/2)*7^(1/2
)+10*x+5)*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)-876315/286112764*ln(-(2*5^(1/
2)*7^(1/2)+4)^(1/2)*5^(1/2)*(1+2*x)^(1/2)+5^(1/2)*7^(1/2)+10*x+5)*7^(1/2)*(
2*5^(1/2)*7^(1/2)+4)^(1/2)-475725/10218313/(10*5^(1/2)*7^(1/2)-20)^(1/2)*ar
ctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/2))/(10*5^(1/2)*7^
(1/2)-20)^(1/2))*(2*5^(1/2)*7^(1/2)+4)-876315/143056382/(10*5^(1/2)*7^(1/2)
-20)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4)^(1/2)+10*(1+2*x)^(1/2))/(
10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*(2*5^(1/2)*7^(1/2)+4)*7^(1/2)+542760/
2307361/(10*5^(1/2)*7^(1/2)-20)^(1/2)*arctan((-5^(1/2)*(2*5^(1/2)*7^(1/2)+4
)^(1/2)+10*(1+2*x)^(1/2))/(10*5^(1/2)*7^(1/2)-20)^(1/2))*5^(1/2)*7^(1/2)-64
/343/(1+2*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(5x^2+3x+2\right)^3(2x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^3*(2*x + 1)^(3/2)), x)
```

Fricas [B] time = 2.95809, size = 3787, normalized size = 12.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] -1/4920973343123441812446014 0*(26636371428*3500868535115^(1/4)*sqrt(217)*sq
rt(35)*(50*x^5 + 85*x^4 + 88*x^3 + 53*x^2 + 20*x + 4)*sqrt(-174893241579951
2300*sqrt(35) + 10506981691013893750)*arctan(1/150900176830169173856778767 1
747064606625*3500868535115^(3/4)*sqrt(2309779)*sqrt(316267)*sqrt(217)*sqrt(
3500868535115^(1/4)*sqrt(217)*(2703*sqrt(35)*sqrt(31) + 18092*sqrt(31))*sqr
t(2*x + 1)*sqrt(-1748932415799512300*sqrt(35) + 10506981691013893750) + 171
9941911827268850*x + 171994191182726885*sqrt(35) + 859970955913634425)*(180
```

```
92*sqrt(35) + 94605)*sqrt(-1748932415799512300*sqrt(35) + 10506981691013893
750) - 1/190386304019756193084 0325*3500868535115^(3/4)*sqrt(217)*sqrt(2*x +
 1)*(18092*sqrt(35) + 94605)*sqrt(-1748932415799512300*sqrt(35) + 105069816
91013893750) - 1/31*sqrt(35)*sqrt(31) - 2/31*sqrt(31)) + 26636371428*350086
8535115^(1/4)*sqrt(217)*sqrt(35)*(50*x^5 + 85*x^4 + 88*x^3 + 53*x^2 + 20*x
+ 4)*sqrt(-1748932415799512300*sqrt(35) + 10506981691013893750)*arctan(1/27
7279074925435856961830984683523121467343 75*3500868535115^(3/4)*sqrt(2309779
)*sqrt(217)*sqrt(-106784587546875*3500868535115^(1/4)*sqrt(217)*(2703*sqrt(
35)*sqrt(31) + 18092*sqrt(31))*sqrt(2*x + 1)*sqrt(-1748932415799512300*sqrt
(35) + 10506981691013893750) + 18366328765905855251575260234375 0*x + 183663
287659058552515752602 34375*sqrt(35) + 91831643829529276257876301171875)*(18
092*sqrt(35) + 94605)*sqrt(-1748932415799512300*sqrt(35) + 1050698169101389
3750) - 1/190386304019756193084 0325*3500868535115^(3/4)*sqrt(217)*sqrt(2*x
+ 1)*(18092*sqrt(35) + 94605)*sqrt(-1748932415799512300*sqrt(35) + 10506981
691013893750) + 1/31*sqrt(35)*sqrt(31) + 2/31*sqrt(31)) - 3*3500868535115^(
1/4)*sqrt(217)*(2257111762*sqrt(35)*sqrt(31)*(50*x^5 + 85*x^4 + 88*x^3 + 53
*x^2 + 20*x + 4) + 13559947625*sqrt(31)*(50*x^5 + 85*x^4 + 88*x^3 + 53*x^2
+ 20*x + 4))*sqrt(-1748932415799512300*sqrt(35) + 10506981691013893750)*log
(106784587546875/2309779*3500868535115^(1/4)*sqrt(217)*(2703*sqrt(35)*sqrt(
31) + 18092*sqrt(31))*sqrt(2*x + 1)*sqrt(-1748932415799512300*sqrt(35) + 10
506981691013893750) + 7951552406488177116328125 0*x + 79515524064881771163 28
125*sqrt(35) + 39757762032440885581640625) + 3*3500868535115^(1/4)*sqrt(217
)*(2257111762*sqrt(35)*sqrt(31)*(50*x^5 + 85*x^4 + 88*x^3 + 53*x^2 + 20*x +
 4) + 13559947625*sqrt(31)*(50*x^5 + 85*x^4 + 88*x^3 + 53*x^2 + 20*x + 4))*
sqrt(-1748932415799512300*sqrt(35) + 10506981691013893750)*log(-10678458754
6875/2309779*3500868535115^(1/4)*sqrt(217)*(2703*sqrt(35)*sqrt(31) + 18092*
sqrt(31))*sqrt(2*x + 1)*sqrt(-1748932415799512300*sqrt(35) + 10506981691013
893750) + 79515524064881771163281250*x + 79515524064881771163 28125*sqrt(35)
 + 39757762032440885581640625) + 74645478973303468090*(4054500*x^4 + 450140
0*x^3 + 4077245*x^2 + 1525635*x + 429487)*sqrt(2*x + 1))/(50*x^5 + 85*x^4 +
 88*x^3 + 53*x^2 + 20*x + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x+1)^{\frac{3}{2}}\left(5x^2+3x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)**(3/2)/(5*x**2+3*x+2)**3,x)
```

```
[Out] Integral(1/((2*x + 1)**(3/2)*(5*x**2 + 3*x + 2)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)^3(2\,x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^3*(2*x + 1)^(3/2)), x)
```

3.2330 $\int \frac{x^{9/2}}{\left(a+bx+cx^2\right)^3}\,dx$

Optimal. Leaf size=389

$$\frac{3\left(-\frac{44a^2bc^2-11ab^3c+b^5}{\sqrt{b^2-4ac}}+28a^2c^2-9ab^2c+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}c^{5/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\left(\frac{44a^2bc^2-11ab^3c+b^5}{\sqrt{b^2-4ac}}+28a^2c^2-9ab^2c+b^4\right)\tan^{-1}\left(\cdots\right.}{4\sqrt{2}c^{5/2}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (-3*b*(b^2 - 8*a*c)*Sqrt[x])/(4*c^2*(b^2 - 4*a*c)^2) + (x^(7/2)*(2*a + b*x)
)/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (x^(3/2)*(a*(b^2 - 28*a*c) + b*(b
^2 - 16*a*c)*x))/(4*c*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (3*(b^4 - 9*a*b^
2*c + 28*a^2*c^2 - (b^5 - 11*a*b^3*c + 44*a^2*b*c^2)/Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]*c^(5
/2)*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*(b^4 - 9*a*b^2*c + 28
*a^2*c^2 + (b^5 - 11*a*b^3*c + 44*a^2*b*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]*c^(5/2)*(b^2
 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 1.76296, antiderivative size = 389, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {738, 818, 824, 826, 1166, 205}

$$\frac{3\left(-\frac{44a^2bc^2-11ab^3c+b^5}{\sqrt{b^2-4ac}}+28a^2c^2-9ab^2c+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}c^{5/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\left(\frac{44a^2bc^2-11ab^3c+b^5}{\sqrt{b^2-4ac}}+28a^2c^2-9ab^2c+b^4\right)\tan^{-1}\left(\cdots\right.}{4\sqrt{2}c^{5/2}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(a + b*x + c*x^2)^3,x]
```

```
[Out] (-3*b*(b^2 - 8*a*c)*Sqrt[x])/(4*c^2*(b^2 - 4*a*c)^2) + (x^(7/2)*(2*a + b*x)
)/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (x^(3/2)*(a*(b^2 - 28*a*c) + b*(b
^2 - 16*a*c)*x))/(4*c*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (3*(b^4 - 9*a*b^
2*c + 28*a^2*c^2 - (b^5 - 11*a*b^3*c + 44*a^2*b*c^2)/Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]*c^(5
/2)*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*(b^4 - 9*a*b^2*c + 28
*a^2*c^2 + (b^5 - 11*a*b^3*c + 44*a^2*b*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]*c^(5/2)*(b^2
 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{9/2}}{\left(a+bx+cx^2\right)^3}\,dx = \frac{x^{7/2}(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{\int \frac{x^{5/2}\left(7a+\frac{bx}{2}\right)}{\left(a+bx+cx^2\right)^2}dx}{2\left(b^2-4ac\right)}$$

$$= \frac{x^{7/2}(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^{3/2}\left(a\left(b^2-28ac\right)+b\left(b^2-16ac\right)x\right)}{4c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{\int \frac{\sqrt{x}\left(\frac{3}{4}a\left(b^2-28ac\right)+\frac{3}{4}b\left(b^2-\right.\right.}{a+bx+cx^2}}{2c\left(b^2-4ac\right)^2}$$

$$= -\frac{3b\left(b^2-8ac\right)\sqrt{x}}{4c^2\left(b^2-4ac\right)^2} + \frac{x^{7/2}(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^{3/2}\left(a\left(b^2-28ac\right)+b\left(b^2-16ac\right)x\right)}{4c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - $$

$$= -\frac{3b\left(b^2-8ac\right)\sqrt{x}}{4c^2\left(b^2-4ac\right)^2} + \frac{x^{7/2}(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^{3/2}\left(a\left(b^2-28ac\right)+b\left(b^2-16ac\right)x\right)}{4c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - $$

$$= -\frac{3b\left(b^2-8ac\right)\sqrt{x}}{4c^2\left(b^2-4ac\right)^2} + \frac{x^{7/2}(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^{3/2}\left(a\left(b^2-28ac\right)+b\left(b^2-16ac\right)x\right)}{4c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + $$

$$= -\frac{3b\left(b^2-8ac\right)\sqrt{x}}{4c^2\left(b^2-4ac\right)^2} + \frac{x^{7/2}(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^{3/2}\left(a\left(b^2-28ac\right)+b\left(b^2-16ac\right)x\right)}{4c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + $$

Mathematica [A] time = 1.59104, size = 518, normalized size = 1.33

$$\frac{\frac{x^{11/2}\left(12a^2c^2-25ab^2c-16abc^2x+7b^3cx+7b^4\right)}{2a\left(4ac-b^2\right)(a+x(b+cx))} + \frac{-\frac{6a^2b\sqrt{x}\left(b^2-8ac\right)}{c^2} + \frac{3\sqrt{2}a^2\left(\frac{\left(28a^2c^2\sqrt{b^2-4ac}-44a^2bc^2+b^4\sqrt{b^2-4ac}+11ab^3c-9ab^2c\sqrt{b^2-4ac}-b^5\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}} + \frac{(28a\ldots}{\ldots}\right)}{c^{5/2}\sqrt{b^2-4ac}}}{\ldots}}{2a\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(a + b*x + c*x^2)^3,x]
```

```
[Out] ((x^(11/2)*(b^2 - 2*a*c + b*c*x))/(a + x*(b + c*x))^2 + (x^(11/2)*(7*b^4 -
25*a*b^2*c + 12*a^2*c^2 + 7*b^3*c*x - 16*a*b*c^2*x))/(2*a*(-b^2 + 4*a*c)*(a
 + x*(b + c*x))) + ((-6*a^2*b*(b^2 - 8*a*c)*Sqrt[x])/c^2 + (2*a^2*(b^2 - 28
*a*c)*x^(3/2))/c + 24*a^2*b*x^(5/2) + 6*a*(-3*b^2 + 4*a*c)*x^(7/2) + 2*b*(7
*b^2 - 16*a*c)*x^(9/2) + (3*Sqrt[2]*a^2*(((-b^5 + 11*a*b^3*c - 44*a^2*b*c^2
 + b^4*Sqrt[b^2 - 4*a*c] - 9*a*b^2*c*Sqrt[b^2 - 4*a*c] + 28*a^2*c^2*Sqrt[b^
2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
Sqrt[b - Sqrt[b^2 - 4*a*c]] + ((b^5 - 11*a*b^3*c + 44*a^2*b*c^2 + b^4*Sqrt[
b^2 - 4*a*c] - 9*a*b^2*c*Sqrt[b^2 - 4*a*c] + 28*a^2*c^2*Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/Sqrt[b + Sqr
t[b^2 - 4*a*c]]))/(c^(5/2)*Sqrt[b^2 - 4*a*c]))/(4*a*(b^2 - 4*a*c)))/(2*a*(b
^2 - 4*a*c))
```

Maple [B] time = 0.203, size = 1166, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^2+b*x+a)^3,x)
```

```
[Out] 2*(-1/8*(44*a^2*c^2-37*a*b^2*c+5*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c*x^(7/2)+
1/8*b*(4*a^2*c^2+20*a*b^2*c-3*b^4)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(5/2)-1
/8*a/c^2*(28*a^2*c^2-49*a*b^2*c+6*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(3/2)+3
/8*b*a^2*(8*a*c-b^2)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(1/2))/(c*x^2+b*x+a)^
2-21/2/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctanh(x^(1/2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2+27/8/c/(1
6*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
x^(1/2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b^2-3/8/c^2/(16*a^2*
c^2-8*a*b^2*c+b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x^(1/2
)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4+33/2/(16*a^2*c^2-8*a*b^2
*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
h(x^(1/2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*b-33/8/c/(16*a^2
*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctanh(x^(1/2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b^3+3/
8/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctanh(x^(1/2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))*b^5+21/2/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctan(x^(1/2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2-27/8/
c/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(x^(1/2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b^2+3/8/c^2/(16*a^2
*c^2-8*a*b^2*c+b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x^(1/2)
*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4+33/2/(16*a^2*c^2-8*a*b^2*c
+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x^
(1/2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*b-33/8/c/(16*a^2*c^2-
8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(x^(1/2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b^3+3/8/c^2/(1
6*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctan(x^(1/2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\left(b^{3}c-8\,abc^{2}\right)x^{\frac{9}{2}}+\left(b^{4}-11\,ab^{2}c-44\,a^{2}c^{2}\right)x^{\frac{7}{2}}+2\left(ab^{3}-22\,a^{2}bc\right)x^{\frac{5}{2}}+\left(a^{2}b^{2}-28\,a^{3}c\right)}{4\left(a^{2}b^{4}c-8\,a^{3}b^{2}c^{2}+16\,a^{4}c^{3}+\left(b^{4}c^{3}-8\,ab^{2}c^{4}+16\,a^{2}c^{5}\right)x^{4}+2\left(b^{5}c^{2}-8\,ab^{3}c^{3}+16\,a^{2}bc^{4}\right)x^{3}+\left(b^{6}c-6\,ab^{4}c^{2}+32\,a^{3}c^{4}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(3*(b^3*c - 8*a*b*c^2)*x^(9/2) + (b^4 - 11*a*b^2*c - 44*a^2*c^2)*x^(7/2
) + 2*(a*b^3 - 22*a^2*b*c)*x^(5/2) + (a^2*b^2 - 28*a^3*c)*x^(3/2))/(a^2*b^4
*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4
+ 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^3 + (b^6*c - 6*a*b^4*c^2 + 32*
a^3*c^4)*x^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x) + integrate(-3
/8*((b^3 - 8*a*b*c)*x^(3/2) + (a*b^2 - 28*a^2*c)*sqrt(x))/(a*b^4*c - 8*a^2*
b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c -
8*a*b^3*c^2 + 16*a^2*b*c^3)*x), x)
```

Fricas [B] time = 7.06918, size = 9743, normalized size = 25.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(3*sqrt(1/2)*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + (b^4*c^4 - 8*
a*b^2*c^5 + 16*a^2*c^6)*x^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^3
+ (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^2 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 +
 16*a^3*b*c^4)*x)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c
^3 + 1680*a^4*b*c^4 + (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*
b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^
2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 1
60*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(
b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*
c^9 - 1024*a^5*c^10))*log(27/2*sqrt(1/2)*(b^13 - 31*a*b^11*c + 413*a^2*b^9*
c^2 - 3012*a^3*b^7*c^3 + 12496*a^4*b^5*c^4 - 27584*a^5*b^3*c^5 + 25088*a^6*
b*c^6 - (b^14*c^5 - 30*a*b^12*c^6 + 416*a^2*b^10*c^7 - 3360*a^3*b^8*c^8 + 1
6640*a^4*b^6*c^9 - 49664*a^5*b^4*c^10 + 81920*a^6*b^2*c^11 - 57344*a^7*c^12
)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^
4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*
a^4*b^2*c^14 - 1024*a^5*c^15)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 -
 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*
c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 22*a*
b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*
a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024
*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 +
 1280*a^4*b^2*c^9 - 1024*a^5*c^10)) + 27*(21*a^2*b^8 - 447*a^3*b^6*c + 4189
*a^4*b^4*c^2 - 19208*a^5*b^2*c^3 + 38416*a^6*c^4)*sqrt(x)) - 3*sqrt(1/2)*(a
^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c
^6)*x^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^3 + (b^6*c^2 - 6*a*b^4
*c^3 + 32*a^3*c^5)*x^2 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x)*sq
rt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4
+ (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b
^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3
*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 64
0*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*
c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)
)*log(-27/2*sqrt(1/2)*(b^13 - 31*a*b^11*c + 413*a^2*b^9*c^2 - 3012*a^3*b^7*
c^3 + 12496*a^4*b^5*c^4 - 27584*a^5*b^3*c^5 + 25088*a^6*b*c^6 - (b^14*c^5 -
 30*a*b^12*c^6 + 416*a^2*b^10*c^7 - 3360*a^3*b^8*c^8 + 16640*a^4*b^6*c^9 -
49664*a^5*b^4*c^10 + 81920*a^6*b^2*c^11 - 57344*a^7*c^12)*sqrt((b^8 - 22*a*
b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*
a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024
*a^5*c^15)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 +
1680*a^4*b*c^4 + (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c
^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4
*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^
2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*
c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 -
 1024*a^5*c^10)) + 27*(21*a^2*b^8 - 447*a^3*b^6*c + 4189*a^4*b^4*c^2 - 1920
8*a^5*b^2*c^3 + 38416*a^6*c^4)*sqrt(x)) + 3*sqrt(1/2)*(a^2*b^4*c^2 - 8*a^3*
b^2*c^3 + 16*a^4*c^4 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^4 + 2*(b^5*c^
3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^3 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*
x^2 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x)*sqrt(-(b^9 - 21*a*b^7
*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (b^10*c^5 - 20*a*
b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c
^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4
*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 12
80*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c
^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))*log(27/2*sqrt(1/2
```

```
)*(b^13 - 31*a*b^11*c + 413*a^2*b^9*c^2 - 3012*a^3*b^7*c^3 + 12496*a^4*b^5*
c^4 - 27584*a^5*b^3*c^5 + 25088*a^6*b*c^6 + (b^14*c^5 - 30*a*b^12*c^6 + 416
*a^2*b^10*c^7 - 3360*a^3*b^8*c^8 + 16640*a^4*b^6*c^9 - 49664*a^5*b^4*c^10 +
 81920*a^6*b^2*c^11 - 57344*a^7*c^12)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*
c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2
*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))*sqrt(-(
b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (b^
10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^
9 - 1024*a^5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*
c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3
*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 +
 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)) + 2
7*(21*a^2*b^8 - 447*a^3*b^6*c + 4189*a^4*b^4*c^2 - 19208*a^5*b^2*c^3 + 3841
6*a^6*c^4)*sqrt(x)) - 3*sqrt(1/2)*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4
 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16
*a^2*b*c^5)*x^3 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^2 + 2*(a*b^5*c^2 -
 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2
 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^
6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 22*
a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 2
0*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 10
24*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8
 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))*log(-27/2*sqrt(1/2)*(b^13 - 31*a*b^11
*c + 413*a^2*b^9*c^2 - 3012*a^3*b^7*c^3 + 12496*a^4*b^5*c^4 - 27584*a^5*b^3
*c^5 + 25088*a^6*b*c^6 + (b^14*c^5 - 30*a*b^12*c^6 + 416*a^2*b^10*c^7 - 336
0*a^3*b^8*c^8 + 16640*a^4*b^6*c^9 - 49664*a^5*b^4*c^10 + 81920*a^6*b^2*c^11
 - 57344*a^7*c^12)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*
c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3
*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))*sqrt(-(b^9 - 21*a*b^7*c +
189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (b^10*c^5 - 20*a*b^8*c
^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*
sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)
/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^
4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 -
640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)) + 27*(21*a^2*b^8 - 447
*a^3*b^6*c + 4189*a^4*b^4*c^2 - 19208*a^5*b^2*c^3 + 38416*a^6*c^4)*sqrt(x))
 + 2*(3*a^2*b^3 - 24*a^3*b*c + (5*b^4*c - 37*a*b^2*c^2 + 44*a^2*c^3)*x^3 +
(3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*x^2 + (6*a*b^4 - 49*a^2*b^2*c + 28*a^3*c
^2)*x)*sqrt(x))/(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + (b^4*c^4 - 8*a*
b^2*c^5 + 16*a^2*c^6)*x^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^3 +
(b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^2 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 1
6*a^3*b*c^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**2+b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.2331 $\int \frac{1}{x^{3/2}(a+bx+cx^2)^3}\,dx$

Optimal. Leaf size=458

$$\frac{36a^2c^2+bcx\left(5b^2-32ac\right)-35ab^2c+5b^4}{4a^2\sqrt{x}\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{3\sqrt{c}\left(\sqrt{b^2-4ac}\left(60a^2c^2-37ab^2c+5b^4\right)+124a^2bc^2-47ab^3c+5b^5\right)\tan^{-1}}{4\sqrt{2}a^3\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] (-3*(5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(4*a^3*(b^2 - 4*a*c)^2*Sqrt[x]) + (b^2
- 2*a*c + b*c*x)/(2*a*(b^2 - 4*a*c)*Sqrt[x]*(a + b*x + c*x^2)^2) + (5*b^4 -
 35*a*b^2*c + 36*a^2*c^2 + b*c*(5*b^2 - 32*a*c)*x)/(4*a^2*(b^2 - 4*a*c)^2*S
qrt[x]*(a + b*x + c*x^2)) - (3*Sqrt[c]*(5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2
+ Sqrt[b^2 - 4*a*c]*(5*b^4 - 37*a*b^2*c + 60*a^2*c^2))*ArcTan[(Sqrt[2]*Sqrt
[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]*a^3*(b^2 - 4*a*c)^(5/
2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[c]*(5*b^5 - 47*a*b^3*c + 124*a^2*
b*c^2 - 5*b^4*Sqrt[b^2 - 4*a*c] + 37*a*b^2*c*Sqrt[b^2 - 4*a*c] - 60*a^2*c^2
*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2 - 4*
a*c]]])/(4*Sqrt[2]*a^3*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 9.72157, antiderivative size = 458, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {740, 822, 828, 826, 1166, 205}

$$\frac{36a^2c^2+bcx\left(5b^2-32ac\right)-35ab^2c+5b^4}{4a^2\sqrt{x}\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{3\sqrt{c}\left(\sqrt{b^2-4ac}\left(60a^2c^2-37ab^2c+5b^4\right)+124a^2bc^2-47ab^3c+5b^5\right)\tan^{-1}}{4\sqrt{2}a^3\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x + c*x^2)^3),x]
```

```
[Out] (-3*(5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(4*a^3*(b^2 - 4*a*c)^2*Sqrt[x]) + (b^2
- 2*a*c + b*c*x)/(2*a*(b^2 - 4*a*c)*Sqrt[x]*(a + b*x + c*x^2)^2) + (5*b^4 -
 35*a*b^2*c + 36*a^2*c^2 + b*c*(5*b^2 - 32*a*c)*x)/(4*a^2*(b^2 - 4*a*c)^2*S
qrt[x]*(a + b*x + c*x^2)) - (3*Sqrt[c]*(5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2
+ Sqrt[b^2 - 4*a*c]*(5*b^4 - 37*a*b^2*c + 60*a^2*c^2))*ArcTan[(Sqrt[2]*Sqrt
[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]*a^3*(b^2 - 4*a*c)^(5/
2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[c]*(5*b^5 - 47*a*b^3*c + 124*a^2*
b*c^2 - 5*b^4*Sqrt[b^2 - 4*a*c] + 37*a*b^2*c*Sqrt[b^2 - 4*a*c] - 60*a^2*c^2
*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2 - 4*
a*c]]])/(4*Sqrt[2]*a^3*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+bx+cx^2\right)^3}\,dx = \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\sqrt{x}\left(a+bx+cx^2\right)^2} - \frac{\int \frac{\frac{1}{2}\left(-5b^2+18ac\right)-\frac{7bcx}{2}}{x^{3/2}\left(a+bx+cx^2\right)^2}\,dx}{2a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\sqrt{x}\left(a+bx+cx^2\right)^2} + \frac{5b^4-35ab^2c+36a^2c^2+bc\left(5b^2-32ac\right)x}{4a^2\left(b^2-4ac\right)^2\sqrt{x}\left(a+bx+cx^2\right)} + \frac{\int \frac{3}{4}\left(5b^2-12a\cdots\right.}{\cdots}$$

$$= -\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{4a^3\left(b^2-4ac\right)^2\sqrt{x}} + \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\sqrt{x}\left(a+bx+cx^2\right)^2} + \frac{5b^4-35ab^2c+36a^2c^2+b\cdots}{4a^2\left(b^2-4ac\right)^2\sqrt{x}\left(a\cdots\right.}$$

$$= -\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{4a^3\left(b^2-4ac\right)^2\sqrt{x}} + \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\sqrt{x}\left(a+bx+cx^2\right)^2} + \frac{5b^4-35ab^2c+36a^2c^2+b\cdots}{4a^2\left(b^2-4ac\right)^2\sqrt{x}\left(a\cdots\right.}$$

$$= -\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{4a^3\left(b^2-4ac\right)^2\sqrt{x}} + \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\sqrt{x}\left(a+bx+cx^2\right)^2} + \frac{5b^4-35ab^2c+36a^2c^2+b\cdots}{4a^2\left(b^2-4ac\right)^2\sqrt{x}\left(a\cdots\right.}$$

$$= -\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{4a^3\left(b^2-4ac\right)^2\sqrt{x}} + \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\sqrt{x}\left(a+bx+cx^2\right)^2} + \frac{5b^4-35ab^2c+36a^2c^2+b\cdots}{4a^2\left(b^2-4ac\right)^2\sqrt{x}\left(a\cdots\right.}$$

Mathematica [A] time = 1.26227, size = 440, normalized size = 0.96

$$\frac{\frac{-36a^2c^2+35ab^2c+32abc^2x-5b^3cx-5b^4}{2a\sqrt{x}\left(4ac-b^2\right)\left(a+x(b+cx)\right)} - \frac{\frac{3\sqrt{c}\left(\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}+\left(5b^2-12ac\right)\left(b^2-5ac\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{3\sqrt{c}\left(60a^2c^2\sqrt{b^2-4ac}-124a^2bc^2+5b^4\sqrt{b^2-4ac}+47ab^3c-37ab\cdots\right.}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4a\cdots}}}}{2a^2\left(b^2-4ac\right)}}{2a\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x + c*x^2)^3),x]
```

```
[Out] ((b^2 - 2*a*c + b*c*x)/(Sqrt[x]*(a + x*(b + c*x))^2) + (-5*b^4 + 35*a*b^2*c
 - 36*a^2*c^2 - 5*b^3*c*x + 32*a*b*c^2*x)/(2*a*(-b^2 + 4*a*c)*Sqrt[x]*(a +
x*(b + c*x))) - ((3*(5*b^2 - 12*a*c)*(b^2 - 5*a*c))/Sqrt[x] + (3*Sqrt[c]*((
5*b^2 - 12*a*c)*(b^2 - 5*a*c) + (b*(5*b^4 - 47*a*b^2*c + 124*a^2*c^2))/Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[c]*(-5*b^5 + 47*a*b^3*c
- 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*c] - 37*a*b^2*c*Sqrt[b^2 - 4*a*c] +
60*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/
(2*a^2*(b^2 - 4*a*c)))/(2*a*(b^2 - 4*a*c))
```

Maple [B] time = 0.201, size = 1571, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^2+b*x+a)^3,x)
```

```
[Out] -13/a/(c*x^2+b*x+a)^2*c^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(7/2)-27/(c*x^2+b*x+
a)^2*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(1/2)*c^2+93/2/a/(16*a^2*c^2-8*a*b^2*c+
b^4)*c^3/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
nh(x^(1/2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b+15/8/a^3/(16*a^2*
c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctan(x^(1/2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^5+15/8/a
^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctanh(x^(1/2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*b^5-141/8/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x^(1/2)*c*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*b^3-141/8/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(-4*a*c+b
^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x^(1/2)*c*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3+93/2/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c^
3/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x^(1/2
)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b-9/4/a^2/(c*x^2+b*x+a)^2*b^5
/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(1/2)-7/4/a^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a
*b^2*c+b^4)*x^(3/2)*b^6-45/2/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c^3*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x^(1/2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2))+47/4/a^2/(c*x^2+b*x+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(7/2)
*b^2-34/a/(c*x^2+b*x+a)^2*c^3*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(5/2)+45/2/a/(
16*a^2*c^2-8*a*b^2*c+b^4)*c^3*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tanh(x^(1/2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+99/4/a^2/(c*x^2+b
*x+a)^2*c^2*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(5/2)-25/4/a/(c*x^2+b*x+a)^2/(
16*a^2*c^2-8*a*b^2*c+b^4)*x^(3/2)*b^2*c^2+43/4/a^2/(c*x^2+b*x+a)^2/(16*a^2*
c^2-8*a*b^2*c+b^4)*x^(3/2)*b^4*c+33/2/a/(c*x^2+b*x+a)^2*b^3/(16*a^2*c^2-8*a
*b^2*c+b^4)*x^(1/2)*c-7/4/a^3/(c*x^2+b*x+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4
)*x^(7/2)*b^4-7/2/a^3/(c*x^2+b*x+a)^2*c*b^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(5
/2)-2/a^3/x^(1/2)+111/8/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctan(x^(1/2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))*b^2+15/8/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctanh(x^(1/2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*b^4-15/8/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(x^(1/2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4-
17/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(3/2)*c^3-111/8/a^2/(16*a^2
*c^2-8*a*b^2*c+b^4)*c^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x
^(1/2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 17.034, size = 11857, normalized size = 25.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^2+b*x+a)^3,x, algorithm="fricas")

[Out] -1/8*(3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^5 + 2*(a^3*
b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^4 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6
*c^3)*x^3 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^2 + (a^5*b^4 - 8*a^6
*b^2*c + 16*a^7*c^2)*x)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 1
5015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10 - 20*a^8
*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12
*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*
c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 -
 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 -
1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*
c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5))*log(27/2*sqrt(1/2)*(125*b^17 - 37
75*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623534*a^4*b^9*c^
4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b^3*c^7 + 13248
00*a^8*b*c^8 - (5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*a^1
0*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*c^6
 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*sqrt((625*b^12 - 12250*a*b^10*c +
 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b
^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640
*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))*sqrt(-(25*b^11 - 495*a
*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a
^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 +
1280*a^11*b^2*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*
a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5
+ 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b
^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 16
0*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)) - 27
*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^
7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*sqrt(x)) - 3*sqrt(1/2)*((a^3*b^4*
c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^5 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a
^5*b*c^3)*x^4 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*x^3 + 2*(a^4*b^5 - 8*a
^5*b^3*c + 16*a^6*b*c^2)*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x)*sqrt
(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4
*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 6
40*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 - 12250
*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 3
12300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^
6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10 -
 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 10
24*a^12*c^5))*log(-27/2*sqrt(1/2)*(125*b^17 - 3775*a*b^15*c + 49360*a^2*b^1
3*c^2 - 362733*a^3*b^11*c^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7
146736*a^6*b^5*c^6 - 5684672*a^7*b^3*c^7 + 1324800*a^8*b*c^8 - (5*a^7*b^16
- 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*a^10*b^10*c^3 + 68640*a^11*b^8
*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 12
2880*a^15*c^8)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310
*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^
14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b
^2*c^4 - 1024*a^19*c^5)))*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 -
 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10 - 20*a
^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^
12*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^
6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10
 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4
- 1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^
4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)) - 27*(4125*b^10*c^4 - 77825*a*b
^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 8
10000*a^5*c^9)*sqrt(x)) + 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^
5*c^4)*x^5 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^4 + (a^3*b^6 -
```

```
6*a^4*b^4*c + 32*a^6*c^3)*x^3 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^
2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x)*sqrt(-(25*b^11 - 495*a*b^9*c +
3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5
- (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11
*b^2*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c
^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a
^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 +
1280*a^18*b^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6
*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5))*log(27/2*sqrt
(1/2)*(125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3
+ 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672
*a^7*b^3*c^7 + 1324800*a^8*b*c^8 + (5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*
b^12*c^2 - 14960*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 +
 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*sqrt((625*b^1
2 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^
4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 16
0*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))*sq
rt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a
^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 -
 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 - 122
50*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 -
 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*
b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10
 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 -
1024*a^12*c^5)) - 27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6
- 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*sqrt(x)) - 3*
sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^5 + 2*(a^3*b^5*c -
8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^4 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*x^
3 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^2 + (a^5*b^4 - 8*a^6*b^2*c +
 16*a^7*c^2)*x)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3
*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10 - 20*a^8*b^8*c +
 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*sq
rt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 59
1886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15
*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^1
9*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 12
80*a^11*b^2*c^4 - 1024*a^12*c^5))*log(-27/2*sqrt(1/2)*(125*b^17 - 3775*a*b^
15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623534*a^4*b^9*c^4 - 446
3140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b^3*c^7 + 1324800*a^8*
b*c^8 + (5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*a^10*b^10*
c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*c^6 - 3235
84*a^14*b^2*c^7 + 122880*a^15*c^8)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*
a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5
+ 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b
^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))*sqrt(-(25*b^11 - 495*a*b^9*c
+ 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^
5 - (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^
11*b^2*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8
*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625
*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3
+ 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b
^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)) - 27*(4125*
b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 283
5000*a^4*b^2*c^8 - 810000*a^5*c^9)*sqrt(x)) + 2*(8*a^2*b^4 - 64*a^3*b^2*c +
 128*a^4*c^2 + 3*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*x^4 + (30*b^5*c -
227*a*b^3*c^2 + 392*a^2*b*c^3)*x^3 + (15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2
+ 324*a^3*c^3)*x^2 + (25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2)*x)*sqrt(x))
/((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^5 + 2*(a^3*b^5*c - 8*a^4*b^3
*c^2 + 16*a^5*b*c^3)*x^4 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*x^3 + 2*(a^
```

```
4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c
^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**2+b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.2332 $\int \frac{3-x+x^2}{\sqrt[3]{x}} \, dx$

Optimal. Leaf size=28

$$\frac{3x^{8/3}}{8} - \frac{3x^{5/3}}{5} + \frac{9x^{2/3}}{2}$$

```
[Out] (9*x^(2/3))/2 - (3*x^(5/3))/5 + (3*x^(8/3))/8
```

Rubi [A] time = 0.0044893, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {14}

$$\frac{3x^{8/3}}{8} - \frac{3x^{5/3}}{5} + \frac{9x^{2/3}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + x^2)/x^(1/3),x]
```

```
[Out] (9*x^(2/3))/2 - (3*x^(5/3))/5 + (3*x^(8/3))/8
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{3-x+x^2}{\sqrt[3]{x}} \, dx &= \int \left(\frac{3}{\sqrt[3]{x}} - x^{2/3} + x^{5/3}\right) dx \\ &= \frac{9x^{2/3}}{2} - \frac{3x^{5/3}}{5} + \frac{3x^{8/3}}{8}\end{aligned}$$

Mathematica [A] time = 0.0049692, size = 19, normalized size = 0.68

$$\frac{3}{40}x^{2/3}\left(5x^2-8x+60\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + x^2)/x^(1/3),x]
```

```
[Out] (3*x^(2/3)*(60 - 8*x + 5*x^2))/40
```

Maple [A] time = 0.038, size = 16, normalized size = 0.6

$$\frac{15\,x^2 - 24\,x + 180}{40} x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x+3)/x^(1/3),x)
```

```
[Out] 3/40*x^(2/3)*(5*x^2-8*x+60)
```

Maxima [A] time = 1.0002, size = 22, normalized size = 0.79

$$\frac{3}{8}x^{\frac{8}{3}} - \frac{3}{5}x^{\frac{5}{3}} + \frac{9}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+3)/x^(1/3),x, algorithm="maxima")
```

```
[Out] 3/8*x^(8/3) - 3/5*x^(5/3) + 9/2*x^(2/3)
```

Fricas [A] time = 2.182, size = 45, normalized size = 1.61

$$\frac{3}{40}\left(5x^{2} - 8x + 60\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+3)/x^(1/3),x, algorithm="fricas")
```

```
[Out] 3/40*(5*x^2 - 8*x + 60)*x^(2/3)
```

Sympy [A] time = 1.96746, size = 24, normalized size = 0.86

$$\frac{3x^{\frac{8}{3}}}{8} - \frac{3x^{\frac{5}{3}}}{5} + \frac{9x^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x+3)/x**(1/3),x)
```

```
[Out] 3*x**(8/3)/8 - 3*x**(5/3)/5 + 9*x**(2/3)/2
```

Giac [A] time = 1.891, size = 22, normalized size = 0.79

$$\frac{3}{8}x^{\frac{8}{3}} - \frac{3}{5}x^{\frac{5}{3}} + \frac{9}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+3)/x^(1/3),x, algorithm="giac")
```

```
[Out] 3/8*x^(8/3) - 3/5*x^(5/3) + 9/2*x^(2/3)
```

3.2333 $\int (d+ex)^3\sqrt{a+bx+cx^2}\,dx$

Optimal. Leaf size=248

$$\frac{e\left(a+bx+cx^2\right)^{3/2}\left(-2ce(16ae+75bd)+35b^2e^2+42cex(2cd-be)+192c^2d^2\right)}{240c^3}+\frac{(b+2cx)\sqrt{a+bx+cx^2}(2cd-be)\left(-\right.}{128c}$$

```
[Out] ((2*c*d - b*e)*(16*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(4*b*d + 3*a*e))*(b + 2*c*x)
*Sqrt[a + b*x + c*x^2])/(128*c^4) + (e*(d + e*x)^2*(a + b*x + c*x^2)^(3/2))
/(5*c) + (e*(192*c^2*d^2 + 35*b^2*e^2 - 2*c*e*(75*b*d + 16*a*e) + 42*c*e*(2
*c*d - b*e)*x)*(a + b*x + c*x^2)^(3/2))/(240*c^3) - ((b^2 - 4*a*c)*(2*c*d -
 b*e)*(16*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(4*b*d + 3*a*e))*ArcTanh[(b + 2*c*x)/
(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(9/2))
```

Rubi [A] time = 0.254903, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {742, 779, 612, 621, 206}

$$\frac{e\left(a+bx+cx^2\right)^{3/2}\left(-2ce(16ae+75bd)+35b^2e^2+42cex(2cd-be)+192c^2d^2\right)}{240c^3}+\frac{(b+2cx)\sqrt{a+bx+cx^2}(2cd-be)\left(-\right.}{128c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((2*c*d - b*e)*(16*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(4*b*d + 3*a*e))*(b + 2*c*x)
*Sqrt[a + b*x + c*x^2])/(128*c^4) + (e*(d + e*x)^2*(a + b*x + c*x^2)^(3/2))
/(5*c) + (e*(192*c^2*d^2 + 35*b^2*e^2 - 2*c*e*(75*b*d + 16*a*e) + 42*c*e*(2
*c*d - b*e)*x)*(a + b*x + c*x^2)^(3/2))/(240*c^3) - ((b^2 - 4*a*c)*(2*c*d -
 b*e)*(16*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(4*b*d + 3*a*e))*ArcTanh[(b + 2*c*x)/
(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(9/2))
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^3\sqrt{a+bx+cx^2}\,dx = \frac{e(d+ex)^2\left(a+bx+cx^2\right)^{3/2}}{5c} + \frac{\int (d+ex)\left(\frac{1}{2}\left(10cd^2-e(3bd+4ae)\right)+\frac{7}{2}e(2cd-be)x\right)\sqrt{a+}}{5c}$$

$$= \frac{e(d+ex)^2\left(a+bx+cx^2\right)^{3/2}}{5c} + \frac{e\left(192c^2d^2+35b^2e^2-2ce(75bd+16ae)+42ce(2cd-be)x\right)}{240c^3}$$

$$= \frac{(2cd-be)\left(16c^2d^2+7b^2e^2-4ce(4bd+3ae)\right)(b+2cx)\sqrt{a+bx+cx^2}}{128c^4} + \frac{e(d+ex)^2\left(a+bx\right.}{5c}$$

$$= \frac{(2cd-be)\left(16c^2d^2+7b^2e^2-4ce(4bd+3ae)\right)(b+2cx)\sqrt{a+bx+cx^2}}{128c^4} + \frac{e(d+ex)^2\left(a+bx\right.}{5c}$$

$$= \frac{(2cd-be)\left(16c^2d^2+7b^2e^2-4ce(4bd+3ae)\right)(b+2cx)\sqrt{a+bx+cx^2}}{128c^4} + \frac{e(d+ex)^2\left(a+bx\right.}{5c}$$

Mathematica [A] time = 0.349463, size = 206, normalized size = 0.83

$$\frac{\frac{5(2cd-be)\left(-4ce(3ae+4bd)+7b^2e^2+16c^2d^2\right)\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}-\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{256c^{7/2}} + \frac{e(a+x(b+cx))^{3/2}\left(-2ce(16ae+75bd+21bex)+35b^2e^2+1\right.}{48c^2}}{5c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (e*(d + e*x)^2*(a + x*(b + c*x))^(3/2) + (e*(a + x*(b + c*x))^(3/2)*(35*b^2
*e^2 + 12*c^2*d*(16*d + 7*e*x) - 2*c*e*(75*b*d + 16*a*e + 21*b*e*x)))/(48*c
^2) + (5*(2*c*d - b*e)*(16*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(4*b*d + 3*a*e))*(2*
Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*
x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(256*c^(7/2)))/(5*c)
```

Maple [B] time = 0.052, size = 795, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -7/40*e^3*b/c^2*x*(c*x^2+b*x+a)^(3/2)-7/64*e^3*b^3/c^3*x*(c*x^2+b*x+a)^(1/2
)+15/64*d*e^2*b^3/c^3*(c*x^2+b*x+a)^(1/2)-15/128*d*e^2*b^4/c^(7/2)*ln((1/2*
b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-3/8*d*e^2*a^2/c^(3/2)*ln((1/2*b+c*x)/c^
(1/2)+(c*x^2+b*x+a)^(1/2))-3/8*d^2*e*b^2/c^2*(c*x^2+b*x+a)^(1/2)+3/16*d^2*e
*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/2*d^3*x*(c*x^2+b
*x+a)^(1/2)+3/16*e^3*b/c^2*a*x*(c*x^2+b*x+a)^(1/2)-3/4*d^2*e*b/c*x*(c*x^2+b
*x+a)^(1/2)-3/8*d*e^2*a/c*x*(c*x^2+b*x+a)^(1/2)-3/16*d*e^2*a/c^2*(c*x^2+b*x
+a)^(1/2)*b+9/16*d*e^2*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/
2))*a+3/4*d*e^2*x*(c*x^2+b*x+a)^(3/2)/c-5/8*d*e^2*b/c^2*(c*x^2+b*x+a)^(3/2)
-5/32*e^3*b^3/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+3/32*e^
3*b^2/c^3*a*(c*x^2+b*x+a)^(1/2)+3/16*e^3*b/c^(5/2)*a^2*ln((1/2*b+c*x)/c^(1/
2)+(c*x^2+b*x+a)^(1/2))+7/48*e^3*b^2/c^3*(c*x^2+b*x+a)^(3/2)-7/128*e^3*b^4/
c^4*(c*x^2+b*x+a)^(1/2)+7/256*e^3*b^5/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))-2/15*e^3*a/c^2*(c*x^2+b*x+a)^(3/2)+d^2*e*(c*x^2+b*x+a)^(3/2)
/c+1/5*e^3*x^2*(c*x^2+b*x+a)^(3/2)/c+1/4*d^3/c*(c*x^2+b*x+a)^(1/2)*b-3/4*d^
2*e*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+15/32*d*e^2*b^2
/c^2*x*(c*x^2+b*x+a)^(1/2)+1/2*d^3/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*
x+a)^(1/2))*a-1/8*d^3/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b
^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.11841, size = 1762, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(32*(b^2*c^3 - 4*a*c^4)*d^3 - 48*(b^3*c^2 - 4*a*b*c^3)*d^2*e +
 6*(5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*d*e^2 - (7*b^5 - 40*a*b^3*c + 48*a
^2*b*c^2)*e^3)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x
+ a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(384*c^5*e^3*x^4 + 480*b*c^4*d^3 - 24
0*(3*b^2*c^3 - 8*a*c^4)*d^2*e + 30*(15*b^3*c^2 - 52*a*b*c^3)*d*e^2 - (105*b
^4*c - 460*a*b^2*c^2 + 256*a^2*c^3)*e^3 + 48*(30*c^5*d*e^2 + b*c^4*e^3)*x^3
 + 8*(240*c^5*d^2*e + 30*b*c^4*d*e^2 - (7*b^2*c^3 - 16*a*c^4)*e^3)*x^2 + 2*
(480*c^5*d^3 + 240*b*c^4*d^2*e - 30*(5*b^2*c^3 - 12*a*c^4)*d*e^2 + (35*b^3*
c^2 - 116*a*b*c^3)*e^3)*x)*sqrt(c*x^2 + b*x + a))/c^5, 1/3840*(15*(32*(b^2*
c^3 - 4*a*c^4)*d^3 - 48*(b^3*c^2 - 4*a*b*c^3)*d^2*e + 6*(5*b^4*c - 24*a*b^2
*c^2 + 16*a^2*c^3)*d*e^2 - (7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*e^3)*sqrt(-c
)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x +
a*c)) + 2*(384*c^5*e^3*x^4 + 480*b*c^4*d^3 - 240*(3*b^2*c^3 - 8*a*c^4)*d^2*
e + 30*(15*b^3*c^2 - 52*a*b*c^3)*d*e^2 - (105*b^4*c - 460*a*b^2*c^2 + 256*a
```

```
^2*c^3)*e^3 + 48*(30*c^5*d*e^2 + b*c^4*e^3)*x^3 + 8*(240*c^5*d^2*e + 30*b*c
^4*d*e^2 - (7*b^2*c^3 - 16*a*c^4)*e^3)*x^2 + 2*(480*c^5*d^3 + 240*b*c^4*d^2
*e - 30*(5*b^2*c^3 - 12*a*c^4)*d*e^2 + (35*b^3*c^2 - 116*a*b*c^3)*e^3)*x)*s
qrt(c*x^2 + b*x + a))/c^5]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d+ex)^3 \sqrt{a+bx+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**3*sqrt(a + b*x + c*x**2), x)
```

Giac [A] time = 1.95457, size = 513, normalized size = 2.07

$$\frac{1}{1920}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\left(8\,xe^3+\frac{30\,c^4de^2+bc^3e^3}{c^4}\right)x+\frac{240\,c^4d^2e+30\,bc^3de^2-7\,b^2c^2e^3+16\,ac^3e^3}{c^4}\right)x+\frac{480\,c^4d^3+24\cdots}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*x*e^3 + (30*c^4*d*e^2 + b*c^3*e^3)
/c^4)*x + (240*c^4*d^2*e + 30*b*c^3*d*e^2 - 7*b^2*c^2*e^3 + 16*a*c^3*e^3)/c
^4)*x + (480*c^4*d^3 + 240*b*c^3*d^2*e - 150*b^2*c^2*d*e^2 + 360*a*c^3*d*e^
2 + 35*b^3*c*e^3 - 116*a*b*c^2*e^3)/c^4)*x + (480*b*c^3*d^3 - 720*b^2*c^2*d
^2*e + 1920*a*c^3*d^2*e + 450*b^3*c*d*e^2 - 1560*a*b*c^2*d*e^2 - 105*b^4*e^
3 + 460*a*b^2*c*e^3 - 256*a^2*c^2*e^3)/c^4) + 1/256*(32*b^2*c^3*d^3 - 128*a
*c^4*d^3 - 48*b^3*c^2*d^2*e + 192*a*b*c^3*d^2*e + 30*b^4*c*d*e^2 - 144*a*b^
2*c^2*d*e^2 + 96*a^2*c^3*d*e^2 - 7*b^5*e^3 + 40*a*b^3*c*e^3 - 48*a^2*b*c^2*
e^3)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2)
```

3.2334 $\int (d+ex)^2\sqrt{a+bx+cx^2}\,dx$

Optimal. Leaf size=191

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}\left(-4ce(ae+4bd)+5b^2e^2+16c^2d^2\right)}{64c^3}-\frac{\left(b^2-4ac\right)\left(-4ce(ae+4bd)+5b^2e^2+16c^2d^2\right)\tanh^{-1}\left(\frac{}{2}\right.}{128c^{7/2}}$$

```
[Out] ((16*c^2*d^2 + 5*b^2*e^2 - 4*c*e*(4*b*d + a*e))*(b + 2*c*x)*Sqrt[a + b*x +
c*x^2])/(64*c^3) + (5*e*(2*c*d - b*e)*(a + b*x + c*x^2)^(3/2))/(24*c^2) + (
e*(d + e*x)*(a + b*x + c*x^2)^(3/2))/(4*c) - ((b^2 - 4*a*c)*(16*c^2*d^2 + 5
*b^2*e^2 - 4*c*e*(4*b*d + a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x
 + c*x^2])])/(128*c^(7/2))
```

Rubi [A] time = 0.234412, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {742, 640, 612, 621, 206}

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}\left(-4ce(ae+4bd)+5b^2e^2+16c^2d^2\right)}{64c^3}-\frac{\left(b^2-4ac\right)\left(-4ce(ae+4bd)+5b^2e^2+16c^2d^2\right)\tanh^{-1}\left(\frac{}{2}\right.}{128c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((16*c^2*d^2 + 5*b^2*e^2 - 4*c*e*(4*b*d + a*e))*(b + 2*c*x)*Sqrt[a + b*x +
c*x^2])/(64*c^3) + (5*e*(2*c*d - b*e)*(a + b*x + c*x^2)^(3/2))/(24*c^2) + (
e*(d + e*x)*(a + b*x + c*x^2)^(3/2))/(4*c) - ((b^2 - 4*a*c)*(16*c^2*d^2 + 5
*b^2*e^2 - 4*c*e*(4*b*d + a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x
 + c*x^2])])/(128*c^(7/2))
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^2\sqrt{a+bx+cx^2}\,dx = \frac{e(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4c}+\frac{\int\left(\frac{1}{2}\left(8cd^2-2e\left(\frac{3bd}{2}+ae\right)\right)+\frac{5}{2}e(2cd-be)x\right)\sqrt{a+bx+cx^2}}{4c}$$

$$=\frac{5e(2cd-be)\left(a+bx+cx^2\right)^{3/2}}{24c^2}+\frac{e(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4c}+\frac{\left(-\frac{5}{2}be(2cd-be)+c\left(8cd^2\right.\right.}{}$$

$$=\frac{\left(16c^2d^2+5b^2e^2-4ce(4bd+ae)\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3}+\frac{5e(2cd-be)\left(a+bx+cx^2\right)^{3/2}}{24c^2}$$

$$=\frac{\left(16c^2d^2+5b^2e^2-4ce(4bd+ae)\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3}+\frac{5e(2cd-be)\left(a+bx+cx^2\right)^{3/2}}{24c^2}$$

$$=\frac{\left(16c^2d^2+5b^2e^2-4ce(4bd+ae)\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3}+\frac{5e(2cd-be)\left(a+bx+cx^2\right)^{3/2}}{24c^2}$$

Mathematica [A] time = 0.179127, size = 162, normalized size = 0.85

$$\frac{\frac{\left(-4ce(ae+4bd)+5b^2e^2+16c^2d^2\right)\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}-\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{32c^{5/2}}+\frac{5e(a+x(b+cx))^{3/2}(2cd-be)}{6c}+e(d+ex)(a+x(b+cx))^{3/2}}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((5*e*(2*c*d - b*e)*(a + x*(b + c*x))^(3/2))/(6*c) + e*(d + e*x)*(a + x*(b
+ c*x))^(3/2) + ((16*c^2*d^2 + 5*b^2*e^2 - 4*c*e*(4*b*d + a*e))*(2*Sqrt[c]*
(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + x*(b + c*x)])]))/(32*c^(5/2)))/(4*c)
```

Maple [B] time = 0.05, size = 484, normalized size = 2.5

$$\frac{e^2x}{4c}\left(cx^2+bx+a\right)^{\frac{3}{2}}-\frac{5be^2}{24c^2}\left(cx^2+bx+a\right)^{\frac{3}{2}}+\frac{5b^2e^2x}{32c^2}\sqrt{cx^2+bx+a}+\frac{5b^3e^2}{64c^3}\sqrt{cx^2+bx+a}+\frac{3b^2e^2a}{16}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/4*e^2*x*(c*x^2+b*x+a)^(3/2)/c-5/24*e^2*b/c^2*(c*x^2+b*x+a)^(3/2)+5/32*e^2
*b^2/c^2*x*(c*x^2+b*x+a)^(1/2)+5/64*e^2*b^3/c^3*(c*x^2+b*x+a)^(1/2)+3/16*e^
```

```
2*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-5/128*e^2*b^4/c
^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/8*e^2*a/c*x*(c*x^2+b*x
+a)^(1/2)-1/16*e^2*a/c^2*(c*x^2+b*x+a)^(1/2)*b-1/8*e^2*a^2/c^(3/2)*ln((1/2*
b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+2/3*d*e*(c*x^2+b*x+a)^(3/2)/c-1/2*d*e*b
/c*x*(c*x^2+b*x+a)^(1/2)-1/4*d*e*b^2/c^2*(c*x^2+b*x+a)^(1/2)-1/2*d*e*b/c^(3
/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+1/8*d*e*b^3/c^(5/2)*ln((1
/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/2*d^2*x*(c*x^2+b*x+a)^(1/2)+1/4*d^
2/c*(c*x^2+b*x+a)^(1/2)*b+1/2*d^2/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x
+a)^(1/2))*a-1/8*d^2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^
2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.61507, size = 1137, normalized size = 5.95

$$\left[\frac{3\left(16\left(b^2c^2-4\,ac^3\right)d^2-16\left(b^3c-4\,abc^2\right)de+\left(5\,b^4-24\,ab^2c+16\,a^2c^2\right)e^2\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+bx}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(16*(b^2*c^2 - 4*a*c^3)*d^2 - 16*(b^3*c - 4*a*b*c^2)*d*e + (5*b^4
 - 24*a*b^2*c + 16*a^2*c^2)*e^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4
*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*c^4*e^2*x^3 + 4
8*b*c^3*d^2 - 16*(3*b^2*c^2 - 8*a*c^3)*d*e + (15*b^3*c - 52*a*b*c^2)*e^2 +
8*(16*c^4*d*e + b*c^3*e^2)*x^2 + 2*(48*c^4*d^2 + 16*b*c^3*d*e - (5*b^2*c^2
- 12*a*c^3)*e^2)*x)*sqrt(c*x^2 + b*x + a))/c^4, 1/384*(3*(16*(b^2*c^2 - 4*a
*c^3)*d^2 - 16*(b^3*c - 4*a*b*c^2)*d*e + (5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*
e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^
2 + b*c*x + a*c)) + 2*(48*c^4*e^2*x^3 + 48*b*c^3*d^2 - 16*(3*b^2*c^2 - 8*a*
c^3)*d*e + (15*b^3*c - 52*a*b*c^2)*e^2 + 8*(16*c^4*d*e + b*c^3*e^2)*x^2 + 2
*(48*c^4*d^2 + 16*b*c^3*d*e - (5*b^2*c^2 - 12*a*c^3)*e^2)*x)*sqrt(c*x^2 + b
*x + a))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d+ex)^2\sqrt{a+bx+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**2*sqrt(a + b*x + c*x**2), x)
```

Giac [A] time = 1.68407, size = 317, normalized size = 1.66

$$\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\,xe^2+\frac{16\,c^3de+bc^2e^2}{c^3}\right)x+\frac{48\,c^3d^2+16\,bc^2de-5\,b^2ce^2+12\,ac^2e^2}{c^3}\right)x+\frac{48\,bc^2d^2-48\,b^2cde+1}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*x*e^2 + (16*c^3*d*e + b*c^2*e^2)/c^3)*
x + (48*c^3*d^2 + 16*b*c^2*d*e - 5*b^2*c*e^2 + 12*a*c^2*e^2)/c^3)*x + (48*b
*c^2*d^2 - 48*b^2*c*d*e + 128*a*c^2*d*e + 15*b^3*e^2 - 52*a*b*c*e^2)/c^3) +
 1/128*(16*b^2*c^2*d^2 - 64*a*c^3*d^2 - 16*b^3*c*d*e + 64*a*b*c^2*d*e + 5*b
^4*e^2 - 24*a*b^2*c*e^2 + 16*a^2*c^2*e^2)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

3.2335 $\int (d+ex)\sqrt{a+bx+cx^2}\,dx$

Optimal. Leaf size=115

$$-\frac{(b^2-4ac)(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}}+\frac{(b+2cx)\sqrt{a+bx+cx^2}(2cd-be)}{8c^2}+\frac{e\left(a+bx+cx^2\right)^{3/2}}{3c}$$

```
[Out] ((2*c*d - b*e)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(8*c^2) + (e*(a + b*x + c
*x^2)^(3/2))/(3*c) - ((b^2 - 4*a*c)*(2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(5/2))
```

Rubi [A] time = 0.0428137, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {640, 612, 621, 206}

$$-\frac{(b^2-4ac)(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}}+\frac{(b+2cx)\sqrt{a+bx+cx^2}(2cd-be)}{8c^2}+\frac{e\left(a+bx+cx^2\right)^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*Sqrt[a + b*x + c*x^2],x]
```


```
[Out] ((2*c*d - b*e)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(8*c^2) + (e*(a + b*x + c
*x^2)^(3/2))/(3*c) - ((b^2 - 4*a*c)*(2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(5/2))
```


Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)\sqrt{a+bx+cx^2}\,dx = \frac{e\left(a+bx+cx^2\right)^{3/2}}{3c} + \frac{(2cd-be)\int \sqrt{a+bx+cx^2}\,dx}{2c}$$
$$= \frac{(2cd-be)(b+2cx)\sqrt{a+bx+cx^2}}{8c^2} + \frac{e\left(a+bx+cx^2\right)^{3/2}}{3c} - \frac{\left(\left(b^2-4ac\right)(2cd-be)\right)\int \frac{1}{\sqrt{a+bx+}}}{16c^2}$$
$$= \frac{(2cd-be)(b+2cx)\sqrt{a+bx+cx^2}}{8c^2} + \frac{e\left(a+bx+cx^2\right)^{3/2}}{3c} - \frac{\left(\left(b^2-4ac\right)(2cd-be)\right)\text{Subst}\left(\int\right.}{8c^2}$$
$$= \frac{(2cd-be)(b+2cx)\sqrt{a+bx+cx^2}}{8c^2} + \frac{e\left(a+bx+cx^2\right)^{3/2}}{3c} - \frac{\left(b^2-4ac\right)(2cd-be)\tanh^{-1}\left(\frac{}{2\sqrt{}}\right.}{16c^{5/2}}$$

Mathematica [A] time = 0.105502, size = 114, normalized size = 0.99

$$\frac{2\sqrt{c}\sqrt{a+x(b+cx)}\left(4c(2ae+cx(3d+2ex))-3b^2e+2bc(3d+ex)\right)+3\left(b^2-4ac\right)(be-2cd)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{48c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-3*b^2*e + 2*b*c*(3*d + e*x) + 4*c*(2*a*e
 + c*x*(3*d + 2*e*x))) + 3*(b^2 - 4*a*c)*(-2*c*d + b*e)*ArcTanh[(b + 2*c*x)
/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(48*c^(5/2))
```

Maple [B] time = 0.046, size = 229, normalized size = 2.

$$\frac{e}{3c}\left(cx^2+bx+a\right)^{\frac{3}{2}} - \frac{bxe}{4c}\sqrt{cx^2+bx+a} - \frac{b^2e}{8c^2}\sqrt{cx^2+bx+a} - \frac{aeb}{4}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)c^{-\frac{3}{2}} + \frac{eb^3}{16}\ln\left(\left(\frac{b}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/3*e*(c*x^2+b*x+a)^(3/2)/c-1/4*e*b/c*x*(c*x^2+b*x+a)^(1/2)-1/8*e*b^2/c^2*(
c*x^2+b*x+a)^(1/2)-1/4*e*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/
2))*a+1/16*e*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/2*d*
x*(c*x^2+b*x+a)^(1/2)+1/4*d/c*(c*x^2+b*x+a)^(1/2)*b+1/2*d/c^(1/2)*ln((1/2*b
+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/8*d/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(
c*x^2+b*x+a)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.42144, size = 683, normalized size = 5.94

$$\left[\frac{3\left(2\left(b^2c-4\,ac^2\right)d-\left(b^3-4\,abc\right)e\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right)+4\left(8\,c^3ex^2+\cdots\right.}{96\,c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(2*(b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*sqrt(c)*log(-8*c^2*x^2
 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4
*(8*c^3*e*x^2 + 6*b*c^2*d - (3*b^2*c - 8*a*c^2)*e + 2*(6*c^3*d + b*c^2*e)*x
)*sqrt(c*x^2 + b*x + a))/c^3, 1/48*(3*(2*(b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b
*c)*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*
x^2 + b*c*x + a*c)) + 2*(8*c^3*e*x^2 + 6*b*c^2*d - (3*b^2*c - 8*a*c^2)*e +
2*(6*c^3*d + b*c^2*e)*x)*sqrt(c*x^2 + b*x + a))/c^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d+ex)\sqrt{a+bx+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)*sqrt(a + b*x + c*x**2), x)
```

Giac [A] time = 1.17386, size = 174, normalized size = 1.51

$$\frac{1}{24}\sqrt{cx^2+bx+a}\left(2\left(4\,xe+\frac{6\,c^2d+bce}{c^2}\right)x+\frac{6\,bcd-3\,b^2e+8\,ace}{c^2}\right)+\frac{\left(2\,b^2cd-8\,ac^2d-b^3e+4\,abce\right)\log\left(\left|-2\left(\sqrt{c}x-\cdots\right.\right.\right.}{16\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + b*x + a)*(2*(4*x*e + (6*c^2*d + b*c*e)/c^2)*x + (6*b*c*d
- 3*b^2*e + 8*a*c*e)/c^2) + 1/16*(2*b^2*c*d - 8*a*c^2*d - b^3*e + 4*a*b*c*e
)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(5/2)
```

3.2336 $\int \sqrt{a+bx+cx^2}\,dx$

Optimal. Leaf size=75

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}$$

```
[Out] ((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c
*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2))
```

Rubi [A] time = 0.0196978, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {612, 621, 206}

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c
*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2))
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+bx+cx^2}\,dx &= \frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{8c}\\ &= \frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{4c}\\ &= \frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.064003, size = 71, normalized size = 0.95

$$\frac{(b+2cx)\sqrt{a+x(b+cx)}}{4c}-\frac{(b^2-4ac)\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)}{8c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((b + 2*c*x)*Sqrt[a + x*(b + c*x)])/(4*c) - ((b^2 - 4*a*c)*Log[b + 2*c*x +
2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(8*c^(3/2))
```

Maple [A] time = 0.042, size = 89, normalized size = 1.2

$$\frac{2\,cx+b}{4\,c}\sqrt{cx^2+bx+a}+\frac{a}{2}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)\frac{1}{\sqrt{c}}-\frac{b^2}{8}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/4*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c+1/2/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))*a-1/8/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*
b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.43896, size = 435, normalized size = 5.8

$$\left[-\frac{(b^2-4\,ac)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right)-4\left(2\,c^2x+bc\right)\sqrt{cx^2+bx+a}}{16\,c^2},\ \frac{(b^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((b^2 - 4*a*c)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2
 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(2*c^2*x + b*c)*sqrt(c*x^2 + b
*x + a))/c^2, 1/8*((b^2 - 4*a*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*
(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(2*c^2*x + b*c)*sqrt(c*x^
2 + b*x + a))/c^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+bx+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2), x)
```

Giac [A] time = 1.17612, size = 92, normalized size = 1.23

$$\frac{1}{4}\sqrt{cx^2+bx+a}\left(2x+\frac{b}{c}\right)+\frac{\left(b^2-4ac\right)\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{8c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^2 + b*x + a)*(2*x + b/c) + 1/8*(b^2 - 4*a*c)*log(abs(-2*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(3/2)
```

3.2337 $\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=152

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}e^2}+\frac{\sqrt{a+bx+cx^2}}{e}$$

```
[Out] Sqrt[a + b*x + c*x^2]/e - ((2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqr
t[a + b*x + c*x^2])])/(2*Sqrt[c]*e^2) + (Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTan
h[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b
*x + c*x^2])])/e^2
```

Rubi [A] time = 0.167402, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {734, 843, 621, 206, 724}

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}e^2}+\frac{\sqrt{a+bx+cx^2}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(d + e*x),x]
```

```
[Out] Sqrt[a + b*x + c*x^2]/e - ((2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqr
t[a + b*x + c*x^2])])/(2*Sqrt[c]*e^2) + (Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTan
h[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b
*x + c*x^2])])/e^2
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx &= \frac{\sqrt{a+bx+cx^2}}{e} - \frac{\int \frac{bd-2ae+(2cd-be)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{2e} \\
&= \frac{\sqrt{a+bx+cx^2}}{e} - \frac{(2cd-be)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2e^2} - \frac{(e(bd-2ae)-d(2cd-be))\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{2e^2} \\
&= \frac{\sqrt{a+bx+cx^2}}{e} - \frac{(2cd-be)\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{e^2} - \frac{\left(2\left(cd^2-bde+ae^2\right)\right)\mathrm{Subst}\left(\int \frac{\ldots}{4cd\ldots}\right.}{\ldots} \\
&= \frac{\sqrt{a+bx+cx^2}}{e} - \frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}e^2} + \frac{\sqrt{cd^2-bde+ae^2}\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)\ldots}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx\ldots}}\right.}{e^2}
\end{aligned}$$

Mathematica [A] time = 0.211649, size = 145, normalized size = 0.95

$$\frac{-2\sqrt{e(ae-bd)+cd^2}\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)+\frac{(be-2cd)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}}+2e\sqrt{a+x(b+cx)}}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x),x]
```

```
[Out] (2*e*Sqrt[a + x*(b + c*x)] + ((-2*c*d + b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]
*Sqrt[a + x*(b + c*x)])])/Sqrt[c] - 2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*ArcTan
h[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt
[a + x*(b + c*x)])])/(2*e^2)
```

Maple [B] time = 0.331, size = 715, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d),x)
```

```
[Out] 1/e*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/2/e
*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x
)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b-1/e^2*ln((1/2*(b*e-2*c*d)/e+(d/
e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)
^(1/2))*c^(1/2)*d-1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*
```

```
d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2
*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a+1/e^2/(
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*
(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x
)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b*d-1/e^3/((a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)
/e^2)^(1/2))/(d/e+x))*c*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 14.026, size = 2244, normalized size = 14.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(c*x^2 + b*x + a)*c*e - (2*c*d - b*e)*sqrt(c)*log(-8*c^2*x^2 -
8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 2*sq
rt(c*d^2 - b*d*e + a*e^2)*c*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2
- (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e +
a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2
 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/(c*e^2),
 1/2*(2*sqrt(c*x^2 + b*x + a)*c*e + (2*c*d - b*e)*sqrt(-c)*arctan(1/2*sqrt(
c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + sqrt(c*d^2
 - b*d*e + a*e^2)*c*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2
*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e + a*e^2)*s
qrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b
*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/(c*e^2), 1/4*(4*
sqrt(c*x^2 + b*x + a)*c*e + 4*sqrt(-c*d^2 + b*d*e - a*e^2)*c*arctan(-1/2*sq
rt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*
e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b
*c*d^2 - b^2*d*e + a*b*e^2)*x)) - (2*c*d - b*e)*sqrt(c)*log(-8*c^2*x^2 - 8*
b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c))/(c*e^2)
, 1/2*(2*sqrt(c*x^2 + b*x + a)*c*e + 2*sqrt(-c*d^2 + b*d*e - a*e^2)*c*arcta
n(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2
*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)
*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)) + (2*c*d - b*e)*sqrt(-c)*arctan(1/
2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)))/(c*e
^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx + cx^2}}{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.2338 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=160

$$-\frac{(2cd-be)\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{2e^2\sqrt{ae^2-bde+cd^2}}-\frac{\sqrt{a+bx+cx^2}}{e(d+ex)}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{e^2}$$

```
[Out] -(Sqrt[a + b*x + c*x^2]/(e*(d + e*x))) + (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + b*x + c*x^2])])/e^2 - ((2*c*d - b*e)*ArcTanh[(b*d - 2*a*e +
(2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(2
*e^2*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rubi [A] time = 0.126827, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {732, 843, 621, 206, 724}

$$-\frac{(2cd-be)\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{2e^2\sqrt{ae^2-bde+cd^2}}-\frac{\sqrt{a+bx+cx^2}}{e(d+ex)}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(d + e*x)^2,x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(e*(d + e*x))) + (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + b*x + c*x^2])])/e^2 - ((2*c*d - b*e)*ArcTanh[(b*d - 2*a*e +
(2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(2
*e^2*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2}\,dx &= -\frac{\sqrt{a+bx+cx^2}}{e(d+ex)}+\frac{\int \frac{b+2cx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{2e}\\ &= -\frac{\sqrt{a+bx+cx^2}}{e(d+ex)}+\frac{c\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{e^2}-\frac{(2cd-be)\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{2e^2}\\ &= -\frac{\sqrt{a+bx+cx^2}}{e(d+ex)}+\frac{(2c)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{e^2}+\frac{(2cd-be)\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx\right.}{e^2}\\ &= -\frac{\sqrt{a+bx+cx^2}}{e(d+ex)}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{e^2}-\frac{(2cd-be)\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{2e^2\sqrt{cd^2-bde+ae^2}}\end{aligned}$$

Mathematica [A] time = 0.186628, size = 152, normalized size = 0.95

$$\frac{\frac{(2cd-be)\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{2\sqrt{e(ae-bd)+cd^2}}-\frac{e\sqrt{a+x(b+cx)}}{d+ex}+\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x)^2,x]
```

```
[Out] (-((e*Sqrt[a + x*(b + c*x)])/(d + e*x)) + Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + x*(b + c*x)])] + ((2*c*d - b*e)*ArcTanh[(-(b*d) + 2*a*e - 2*
c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/(
2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]))/e^2
```

Maple [B] time = 0.229, size = 1519, normalized size = 9.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)^2,x)
```

```
[Out] -1/(a*e^2-b*d*e+c*d^2)/(d/e+x)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*
d*e+c*d^2)/e^2)^(3/2)+1/(a*e^2-b*d*e+c*d^2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e
+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b-1/e/(a*e^2-b*d*e+c*d^2)*((d/e+x)^2*c+(
b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*c*d-1/e/(a*e^2-b*d*e+c*
```

```
d^2)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d
/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d*b+1/e^2/(a*e^2-b*d*e+c*d^2)
*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x
)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(3/2)*d^2-1/2/(a*e^2-b*d*e+c*d^2)/((a*e
^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e
+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a*b+1/e/(a*e^2-b*d*e+c*d^2)/((a*e^2-
b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)
+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^
2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a*c*d+1/2/e/(a*e^2-b*d*e+c*d^2)/((a*e^2
-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x
)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e
^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b^2*d-3/2/e^2/(a*e^2-b*d*e+c*d^2)/((a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/
e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(
a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b*d^2*c+1/e^3/(a*e^2-b*d*e+c*d^2)/(
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*
(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x
)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*c^2*d^3+c/(a*e^2-b*d*e+c*d^2)*((
d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x+c^(1/2)/(
a*e^2-b*d*e+c*d^2)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b
*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 20.7297, size = 3155, normalized size = 19.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(c*d^3 - b*d^2*e + a*d*e^2 + (c*d^2*e - b*d*e^2 + a*e^3)*x)*sqrt(c)
*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(
c) - 4*a*c) - (2*c*d^2 - b*d*e + (2*c*d*e - b*e^2)*x)*sqrt(c*d^2 - b*d*e +
a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*
d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b
*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2
 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) - 4*(c*d^2*e - b*d*e^2 + a*e^3
)*sqrt(c*x^2 + b*x + a))/(c*d^3*e^2 - b*d^2*e^3 + a*d*e^4 + (c*d^2*e^3 - b*
d*e^4 + a*e^5)*x), -1/2*((2*c*d^2 - b*d*e + (2*c*d*e - b*e^2)*x)*sqrt(-c*d^
2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*
x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*
d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)) - (c*d^3 -
 b*d^2*e + a*d*e^2 + (c*d^2*e - b*d*e^2 + a*e^3)*x)*sqrt(c)*log(-8*c^2*x^2
- 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 2*
```

```
(c*d^2*e - b*d*e^2 + a*e^3)*sqrt(c*x^2 + b*x + a))/(c*d^3*e^2 - b*d^2*e^3 +
 a*d*e^4 + (c*d^2*e^3 - b*d*e^4 + a*e^5)*x), -1/4*(4*(c*d^3 - b*d^2*e + a*d
*e^2 + (c*d^2*e - b*d*e^2 + a*e^3)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x
+ a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + (2*c*d^2 - b*d*e + (2*
c*d*e - b*e^2)*x)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 -
(b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqr
t(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)
*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x
 + d^2)) + 4*(c*d^2*e - b*d*e^2 + a*e^3)*sqrt(c*x^2 + b*x + a))/(c*d^3*e^2
- b*d^2*e^3 + a*d*e^4 + (c*d^2*e^3 - b*d*e^4 + a*e^5)*x), -1/2*((2*c*d^2 -
b*d*e + (2*c*d*e - b*e^2)*x)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(
-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*
x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*
d^2 - b^2*d*e + a*b*e^2)*x)) + 2*(c*d^3 - b*d^2*e + a*d*e^2 + (c*d^2*e - b*
d*e^2 + a*e^3)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqr
t(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(c*d^2*e - b*d*e^2 + a*e^3)*sqrt(c*x^2 +
 b*x + a))/(c*d^3*e^2 - b*d^2*e^3 + a*d*e^4 + (c*d^2*e^3 - b*d*e^4 + a*e^5)
*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx + cx^2}}{(d + ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)**2,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(d + e*x)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.2339 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3} dx$

Optimal. Leaf size=153

$$\frac{\sqrt{a+bx+cx^2}(-2ae+x(2cd-be)+bd)}{4(d+ex)^2(ae^2-bde+cd^2)}-\frac{(b^2-4ac)\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{8(ae^2-bde+cd^2)^{3/2}}$$

```
[Out] ((b*d - 2*a*e + (2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(4*(c*d^2 - b*d*e +
 a*e^2)*(d + e*x)^2) - ((b^2 - 4*a*c)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*
x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(8*(c*d^2 - b*d*
e + a*e^2)^(3/2))
```

Rubi [A] time = 0.0979662, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {720, 724, 206}

$$\frac{\sqrt{a+bx+cx^2}(-2ae+x(2cd-be)+bd)}{4(d+ex)^2(ae^2-bde+cd^2)}-\frac{(b^2-4ac)\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{8(ae^2-bde+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(d + e*x)^3,x]
```

```
[Out] ((b*d - 2*a*e + (2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(4*(c*d^2 - b*d*e +
 a*e^2)*(d + e*x)^2) - ((b^2 - 4*a*c)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*
x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(8*(c*d^2 - b*d*
e + a*e^2)^(3/2))
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx = \frac{(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{4\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{\left(b^2-4ac\right)\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{8\left(cd^2-bde+ae^2\right)}$$

$$= \frac{(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{4\left(cd^2-bde+ae^2\right)(d+ex)^2} + \frac{\left(b^2-4ac\right)\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-b}{\sqrt{a+bx+cx^2}}\right.}{4\left(cd^2-bde+ae^2\right)}$$

$$= \frac{(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{4\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{8\left(cd^2-bde+ae^2\right)^{3/2}}$$

Mathematica [A] time = 0.170832, size = 149, normalized size = 0.97

$$\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{8\left(e(ae-bd)+cd^2\right)^{3/2}} + \frac{\sqrt{a+x(b+cx)}(-2ae+b(d-ex)+2cdx)}{4(d+ex)^2\left(e(ae-bd)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x)^3,x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(-2*a*e + 2*c*d*x + b*(d - e*x)))/(4*(c*d^2 + e*(-(b
*d) + a*e))*(d + e*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x
 + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/(8*(c*
d^2 + e*(-(b*d) + a*e))^(3/2))
```

Maple [B] time = 0.229, size = 3269, normalized size = 21.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)^3,x)
```

```
[Out] 1/2/e^2*c/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-
b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((
d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b
*d-1/2/(a*e^2-b*d*e+c*d^2)^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b
*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d
/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a*
b*c*d+1/2/e/(a*e^2-b*d*e+c*d^2)^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*
e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x
))*a*c^2*d^2+5/8/e/(a*e^2-b*d*e+c*d^2)^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln
((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))
/(d/e+x))*b^2*d^2*c-1/e^2/(a*e^2-b*d*e+c*d^2)^2/((a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*
d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)
^(1/2))/(d/e+x))*b*d^3*c^2+1/2/e*c/(a*e^2-b*d*e+c*d^2)*((d/e+x)^2*c+(b*e-2*
c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/4*e/(a*e^2-b*d*e+c*d^2)^2*(
(d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2-1/2/e/
(a*e^2-b*d*e+c*d^2)/(d/e+x)^2*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d
*e+c*d^2)/e^2)^(3/2)-1/4*e/(a*e^2-b*d*e+c*d^2)^2*c^(1/2)*ln((1/2*(b*e-2*c*d
```

```
)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^
2)/e^2)^(1/2))*a*b-3/4/e/(a*e^2-b*d*e+c*d^2)^2*ln((1/2*(b*e-2*c*d)/e+(d/e+x
)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1
/2))*c^(3/2)*d^2*b-1/2/e*c/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d
^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^
(1/2))/(d/e+x))*a-1/2/e^3*c^2/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+
c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2))/(d/e+x))*d^2+1/8*e/(a*e^2-b*d*e+c*d^2)^2/((a*e^2-b*d*e+c*d^2)/e^2
)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e
^2)^(1/2))/(d/e+x))*a*b^2+1/2/e^3/(a*e^2-b*d*e+c*d^2)^2/((a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-
b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d
^2)/e^2)^(1/2))/(d/e+x))*c^3*d^4-1/4*e/(a*e^2-b*d*e+c*d^2)^2*c*((d/e+x)^2*c
+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b-1/2/e/(a*e^2-b*d*
e+c*d^2)^2*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*c^2*d^2-1/8/(a*e^2-b*d*e+c*d^2)^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(
a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e
+x))*b^3*d+1/2/(a*e^2-b*d*e+c*d^2)^2*c^(3/2)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*
c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
))*a*d-1/2/e^2*c^(3/2)/(a*e^2-b*d*e+c*d^2)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)
/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))
*d+1/2/e^2/(a*e^2-b*d*e+c*d^2)^2*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+(
(d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(5/2)*d
^3+1/4/e*c^(1/2)/(a*e^2-b*d*e+c*d^2)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/
2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*b+3/4
/(a*e^2-b*d*e+c*d^2)^2*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^
2)/e^2)^(1/2)*b*c*d+1/4/(a*e^2-b*d*e+c*d^2)^2*ln((1/2*(b*e-2*c*d)/e+(d/e+x)
*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/
2))*c^(1/2)*d*b^2-1/2/(a*e^2-b*d*e+c*d^2)^2/(d/e+x)*((d/e+x)^2*c+(b*e-2*c*d
)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)*c*d+1/4*e/(a*e^2-b*d*e+c*d^2)^2/
(d/e+x)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)*b
+1/2/(a*e^2-b*d*e+c*d^2)^2*c^2*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*x*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 11.5019, size = 1972, normalized size = 12.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(((b^2 - 4*a*c)*e^2*x^2 + 2*(b^2 - 4*a*c)*d*e*x + (b^2 - 4*a*c)*d^2)
*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2
 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e +
 a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^
2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) - 4*(b*c
*d^3 + 3*a*b*d*e^2 - 2*a^2*e^3 - (b^2 + 2*a*c)*d^2*e + (2*c^2*d^3 - 3*b*c*d
^2*e - a*b*e^3 + (b^2 + 2*a*c)*d*e^2)*x)*sqrt(c*x^2 + b*x + a))/(c^2*d^6 -
2*b*c*d^5*e - 2*a*b*d^3*e^3 + a^2*d^2*e^4 + (b^2 + 2*a*c)*d^4*e^2 + (c^2*d^
4*e^2 - 2*b*c*d^3*e^3 - 2*a*b*d*e^5 + a^2*e^6 + (b^2 + 2*a*c)*d^2*e^4)*x^2
+ 2*(c^2*d^5*e - 2*b*c*d^4*e^2 - 2*a*b*d^2*e^4 + a^2*d*e^5 + (b^2 + 2*a*c)*
d^3*e^3)*x), -1/8*(((b^2 - 4*a*c)*e^2*x^2 + 2*(b^2 - 4*a*c)*d*e*x + (b^2 -
4*a*c)*d^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e -
a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b
*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a
*b*e^2)*x)) - 2*(b*c*d^3 + 3*a*b*d*e^2 - 2*a^2*e^3 - (b^2 + 2*a*c)*d^2*e +
(2*c^2*d^3 - 3*b*c*d^2*e - a*b*e^3 + (b^2 + 2*a*c)*d*e^2)*x)*sqrt(c*x^2 + b
*x + a))/(c^2*d^6 - 2*b*c*d^5*e - 2*a*b*d^3*e^3 + a^2*d^2*e^4 + (b^2 + 2*a*
c)*d^4*e^2 + (c^2*d^4*e^2 - 2*b*c*d^3*e^3 - 2*a*b*d*e^5 + a^2*e^6 + (b^2 +
2*a*c)*d^2*e^4)*x^2 + 2*(c^2*d^5*e - 2*b*c*d^4*e^2 - 2*a*b*d^2*e^4 + a^2*d*
e^5 + (b^2 + 2*a*c)*d^3*e^3)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx + cx^2}}{(d + ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)**3,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(d + e*x)**3, x)
```

Giac [B] time = 1.265, size = 926, normalized size = 6.05

$$-\frac{(b^2 - 4\,ac)\arctan\left(-\frac{\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)e + \sqrt{c}d}{\sqrt{-cd^2 + bde - ae^2}}\right)}{4\left(cd^2 - bde + ae^2\right)\sqrt{-cd^2 + bde - ae^2}} + \frac{8\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)^3 c^2 d^2 e + 8\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)^2 c^{\frac{5}{2}} d^3 + 8\left(\sqrt{c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/4*(b^2 - 4*a*c)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)
*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*
d*e - a*e^2)) + 1/4*(8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*d^2*e + 8*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*d^3 + 8*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*b*c^2*d^3 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*d*e^
2 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*d^2*e - 8*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))*a*c^2*d^2*e + 2*b^2*c^(3/2)*d^3 - 5*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^2*b^2*sqrt(c)*d*e^2 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^2*a*c^(3/2)*d*e^2 - b^3*sqrt(c)*d^2*e - 4*a*b*c^(3/2)*d^2*e + (sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^3*b^2*e^3 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3
```

```
*a*c*e^3 - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*d*e^2 + 4*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*a*b*c*d*e^2 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2
*a*b*sqrt(c)*e^3 + a*b^2*sqrt(c)*d*e^2 + 4*a^2*c^(3/2)*d*e^2 + (sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))*a*b^2*e^3 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a
^2*c*e^3)/((c*d^2*e^2 - b*d*e^3 + a*e^4)*((sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^2*e + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c)*d + b*d - a*e)^2)
```

3.2340 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=215

$$-\frac{\left(b^2-4ac\right)(2cd-be)\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{16\left(ae^2-bde+cd^2\right)^{5/2}}-\frac{e\left(a+bx+cx^2\right)^{3/2}}{3(d+ex)^3\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{a+bx+cx^2}(2cd-be)(-2ae+}{8(d+ex)^2\left(ae^2-bde\right.}$$

```
[Out] ((2*c*d - b*e)*(b*d - 2*a*e + (2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(8*(c
*d^2 - b*d*e + a*e^2)^2*(d + e*x)^2) - (e*(a + b*x + c*x^2)^(3/2))/(3*(c*d^
2 - b*d*e + a*e^2)*(d + e*x)^3) - ((b^2 - 4*a*c)*(2*c*d - b*e)*ArcTanh[(b*d
 - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c
*x^2])])/(16*(c*d^2 - b*d*e + a*e^2)^(5/2))
```

Rubi [A] time = 0.147503, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {730, 720, 724, 206}

$$-\frac{\left(b^2-4ac\right)(2cd-be)\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{16\left(ae^2-bde+cd^2\right)^{5/2}}-\frac{e\left(a+bx+cx^2\right)^{3/2}}{3(d+ex)^3\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{a+bx+cx^2}(2cd-be)(-2ae+}{8(d+ex)^2\left(ae^2-bde\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(d + e*x)^4,x]
```

```
[Out] ((2*c*d - b*e)*(b*d - 2*a*e + (2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(8*(c
*d^2 - b*d*e + a*e^2)^2*(d + e*x)^2) - (e*(a + b*x + c*x^2)^(3/2))/(3*(c*d^
2 - b*d*e + a*e^2)*(d + e*x)^3) - ((b^2 - 4*a*c)*(2*c*d - b*e)*ArcTanh[(b*d
 - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c
*x^2])])/(16*(c*d^2 - b*d*e + a*e^2)^(5/2))
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^4}\,dx = -\frac{e\left(a+bx+cx^2\right)^{3/2}}{3\left(cd^2-bde+ae^2\right)(d+ex)^3}+\frac{(2cd-be)\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx}{2\left(cd^2-bde+ae^2\right)}$$

$$= \frac{(2cd-be)(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{8\left(cd^2-bde+ae^2\right)^2(d+ex)^2}-\frac{e\left(a+bx+cx^2\right)^{3/2}}{3\left(cd^2-bde+ae^2\right)(d+ex)^3}-\frac{\left((b^2-4ac)\right.}{}$$

$$= \frac{(2cd-be)(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{8\left(cd^2-bde+ae^2\right)^2(d+ex)^2}-\frac{e\left(a+bx+cx^2\right)^{3/2}}{3\left(cd^2-bde+ae^2\right)(d+ex)^3}+\frac{\left((b^2-4ac)\right.}{}$$

$$= \frac{(2cd-be)(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{8\left(cd^2-bde+ae^2\right)^2(d+ex)^2}-\frac{e\left(a+bx+cx^2\right)^{3/2}}{3\left(cd^2-bde+ae^2\right)(d+ex)^3}-\frac{(b^2-4ac)}{}$$

Mathematica [A] time = 0.433355, size = 206, normalized size = 0.96

$$\frac{3(2cd-be)\left(\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{8\left(e(ae-bd)+cd^2\right)^{3/2}}+\frac{\sqrt{a+x(b+cx)}(-2ae+b(d-ex)+2cdx)}{4(d+ex)^2\left(e(ae-bd)+cd^2\right)}\right)-\frac{2e(a+x(b+cx))^{3/2}}{(d+ex)^3}}{6\left(e(ae-bd)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x)^4,x]
```

```
[Out] ((-2*e*(a + x*(b + c*x))^(3/2))/(d + e*x)^3 + 3*(2*c*d - b*e)*((Sqrt[a + x*
(b + c*x)]*(-2*a*e + 2*c*d*x + b*(d - e*x)))/(4*(c*d^2 + e*(-(b*d) + a*e))*
(d + e*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2
*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)]])/(8*(c*d^2 + e*(-(b
*d) + a*e))^(3/2))))/(6*(c*d^2 + e*(-(b*d) + a*e)))
```

Maple [B] time = 0.233, size = 4844, normalized size = 22.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)^4,x)
```

```
[Out] 1/2*e/(a*e^2-b*d*e+c*d^2)^3/(d/e+x)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e
^2-b*d*e+c*d^2)/e^2)^(3/2)*c*d*b-3/4/(a*e^2-b*d*e+c*d^2)^3/((a*e^2-b*d*e+c*
```

d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e
^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+
c*d^2)/e^2)^(1/2))/(d/e+x))*a*b*c^2*d^2-1/2/e/(a*e^2-b*d*e+c*d^2)^2*c^2/((a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d
/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a*d-1/2*e/(a*e^2-b*d*e+c*d^2)^3*c^
2*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*d-1
/2*e/(a*e^2-b*d*e+c*d^2)^3*c^(3/2)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)
+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*a*d*b-5
/4/e^2/(a*e^2-b*d*e+c*d^2)^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b
*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d
/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*c^
3*d^4*b+1/2/e/(a*e^2-b*d*e+c*d^2)^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(
a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e
+x))*a*c^3*d^3+9/8/e/(a*e^2-b*d*e+c*d^2)^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/
e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
))/(d/e+x))*b^2*d^3*c^2-1/4/e/(a*e^2-b*d*e+c*d^2)^2*c/((a*e^2-b*d*e+c*d^2)/
e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2))/(d/e+x))*b^2*d+3/4/e^2/(a*e^2-b*d*e+c*d^2)^2*c^2/((a*e^2-b*d*
e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*
d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b*d^2-1/2/(a*e^2-b*d*e+c*d^2)^3/(d/e+x)*((d
/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)*c^2*d^2+1/2/
(a*e^2-b*d*e+c*d^2)^3*c^(5/2)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/
e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*a*d^2-1/2/e/
(a*e^2-b*d*e+c*d^2)^3*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)*c^3*d^3-1/2/e^2/(a*e^2-b*d*e+c*d^2)^2*c^(5/2)*ln((1/2*(b*e-2*c
*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*
d^2)/e^2)^(1/2))*d^2+1/2/e^2/(a*e^2-b*d*e+c*d^2)^3*ln((1/2*(b*e-2*c*d)/e+(d
/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2
)^(1/2))*c^(7/2)*d^4+1/2/(a*e^2-b*d*e+c*d^2)^3*c^3*((d/e+x)^2*c+(b*e-2*c*d)
/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*d^2+1/2/e/(a*e^2-b*d*e+c*d^2)^2
c^2((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d-1/
8*e^2/(a*e^2-b*d*e+c*d^2)^3/(d/e+x)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e
^2-b*d*e+c*d^2)/e^2)^(3/2)*b^2+1/(a*e^2-b*d*e+c*d^2)^3*((d/e+x)^2*c+(b*e-2*
c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c^2*d^2+3/8*e/(a*e^2-b*d*e+
c*d^2)^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e
-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d
)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a*b^2*c*d+5/8/(a*e^2-b
*d*e+c*d^2)^3*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*
c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(3/2)*d^2*b^2+1/2/e^3/(a*e
^2-b*d*e+c*d^2)^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)
/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(
b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*c^4*d^5-1/8*e
/(a*e^2-b*d*e+c*d^2)^3*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*
c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d*b^3+1/8*e
^2/(a*e^2-b*d*e+c*d^2)^3*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*
d^2)/e^2)^(1/2)*b^3-1/3/e^2/(a*e^2-b*d*e+c*d^2)/(d/e+x)^3*((d/e+x)^2*c+(b*e
-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)+1/4/(a*e^2-b*d*e+c*d^2)^2/
(d/e+x)^2*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)
*b-1/4/(a*e^2-b*d*e+c*d^2)^2*c*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*b-1/8/(a*e^2-b*d*e+c*d^2)^2*c^(1/2)*ln((1/2*(b*e-2*c*
d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d
^2)/e^2)^(1/2))*b^2+1/8*e^2/(a*e^2-b*d*e+c*d^2)^3*c*((d/e+x)^2*c+(b*e-2*c*d
)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b^2-7/16/(a*e^2-b*d*e+c*d^2)^3
/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/

```
e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e
+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b^3*d^2*c+1/4/(a*e^2-b*d*e+c*d
^2)^2*c/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-
2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)
/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a*b-1/2/e^3/(a*e^2-b*d*
e+c*d^2)^2*c^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^
2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e
-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*d^3-5/8*e/(a*e^2
-b*d*e+c*d^2)^3*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)*b^2*c*d-1/16*e^2/(a*e^2-b*d*e+c*d^2)^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/
2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^
2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(
1/2))/(d/e+x))*a*b^3-1/2/e/(a*e^2-b*d*e+c*d^2)^2/(d/e+x)^2*((d/e+x)^2*c+(b*
e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)*c*d-1/e/(a*e^2-b*d*e+c*d^
2)^3*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d
/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(5/2)*d^3*b+1/8*e^2/(a*e^2-b*d*e+c*
d^2)^3*c^(1/2)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2
*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*a*b^2+1/2/e/(a*e^2-b*d*e+c*
d^2)^2*c^(3/2)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2
*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*d*b+1/16*e/(a*e^2-b*d*e+c*d
^2)^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*
c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e
*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b^4*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 49.9147, size = 4016, normalized size = 18.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(3*(2*(b^2*c - 4*a*c^2)*d^4 - (b^3 - 4*a*b*c)*d^3*e + (2*(b^2*c - 4*a
*c^2)*d*e^3 - (b^3 - 4*a*b*c)*e^4)*x^3 + 3*(2*(b^2*c - 4*a*c^2)*d^2*e^2 - (
b^3 - 4*a*b*c)*d*e^3)*x^2 + 3*(2*(b^2*c - 4*a*c^2)*d^3*e - (b^3 - 4*a*b*c)*
d^2*e^2)*x)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 +
 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^
2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) -
2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2
)) + 4*(6*b*c^2*d^5 + 22*a^2*b*d*e^4 - 8*a^3*e^5 - (9*b^2*c + 20*a*c^2)*d^4
*e + (3*b^3 + 40*a*b*c)*d^3*e^2 - (17*a*b^2 + 28*a^2*c)*d^2*e^3 + (4*c^3*d^
4*e - 8*b*c^2*d^3*e^2 + (7*b^2*c - 4*a*c^2)*d^2*e^3 - (3*b^3 - 4*a*b*c)*d*e
^4 + (3*a*b^2 - 8*a^2*c)*e^5)*x^2 + 2*(6*c^3*d^5 - 13*b*c^2*d^4*e + 11*b^2*
c*d^3*e^2 - a^2*b*e^5 - 2*(2*b^3 + a*b*c)*d^2*e^3 + (5*a*b^2 - 6*a^2*c)*d*e
^4)*x)*sqrt(c*x^2 + b*x + a))/(c^3*d^9 - 3*b*c^2*d^8*e - 3*a^2*b*d^4*e^5 +
```

```
a^3*d^3*e^6 + 3*(b^2*c + a*c^2)*d^7*e^2 - (b^3 + 6*a*b*c)*d^6*e^3 + 3*(a*b^
2 + a^2*c)*d^5*e^4 + (c^3*d^6*e^3 - 3*b*c^2*d^5*e^4 - 3*a^2*b*d*e^8 + a^3*e
^9 + 3*(b^2*c + a*c^2)*d^4*e^5 - (b^3 + 6*a*b*c)*d^3*e^6 + 3*(a*b^2 + a^2*c
)*d^2*e^7)*x^3 + 3*(c^3*d^7*e^2 - 3*b*c^2*d^6*e^3 - 3*a^2*b*d^2*e^7 + a^3*d
*e^8 + 3*(b^2*c + a*c^2)*d^5*e^4 - (b^3 + 6*a*b*c)*d^4*e^5 + 3*(a*b^2 + a^2
*c)*d^3*e^6)*x^2 + 3*(c^3*d^8*e - 3*b*c^2*d^7*e^2 - 3*a^2*b*d^3*e^6 + a^3*d
^2*e^7 + 3*(b^2*c + a*c^2)*d^6*e^3 - (b^3 + 6*a*b*c)*d^5*e^4 + 3*(a*b^2 + a
^2*c)*d^4*e^5)*x), -1/48*(3*(2*(b^2*c - 4*a*c^2)*d^4 - (b^3 - 4*a*b*c)*d^3*
e + (2*(b^2*c - 4*a*c^2)*d*e^3 - (b^3 - 4*a*b*c)*e^4)*x^3 + 3*(2*(b^2*c - 4
*a*c^2)*d^2*e^2 - (b^3 - 4*a*b*c)*d*e^3)*x^2 + 3*(2*(b^2*c - 4*a*c^2)*d^3*e
 - (b^3 - 4*a*b*c)*d^2*e^2)*x)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqr
t(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e
)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*
c*d^2 - b^2*d*e + a*b*e^2)*x)) - 2*(6*b*c^2*d^5 + 22*a^2*b*d*e^4 - 8*a^3*e^
5 - (9*b^2*c + 20*a*c^2)*d^4*e + (3*b^3 + 40*a*b*c)*d^3*e^2 - (17*a*b^2 + 2
8*a^2*c)*d^2*e^3 + (4*c^3*d^4*e - 8*b*c^2*d^3*e^2 + (7*b^2*c - 4*a*c^2)*d^2
*e^3 - (3*b^3 - 4*a*b*c)*d*e^4 + (3*a*b^2 - 8*a^2*c)*e^5)*x^2 + 2*(6*c^3*d^
5 - 13*b*c^2*d^4*e + 11*b^2*c*d^3*e^2 - a^2*b*e^5 - 2*(2*b^3 + a*b*c)*d^2*e
^3 + (5*a*b^2 - 6*a^2*c)*d*e^4)*x)*sqrt(c*x^2 + b*x + a))/(c^3*d^9 - 3*b*c^
2*d^8*e - 3*a^2*b*d^4*e^5 + a^3*d^3*e^6 + 3*(b^2*c + a*c^2)*d^7*e^2 - (b^3
+ 6*a*b*c)*d^6*e^3 + 3*(a*b^2 + a^2*c)*d^5*e^4 + (c^3*d^6*e^3 - 3*b*c^2*d^5
*e^4 - 3*a^2*b*d*e^8 + a^3*e^9 + 3*(b^2*c + a*c^2)*d^4*e^5 - (b^3 + 6*a*b*c
)*d^3*e^6 + 3*(a*b^2 + a^2*c)*d^2*e^7)*x^3 + 3*(c^3*d^7*e^2 - 3*b*c^2*d^6*e
^3 - 3*a^2*b*d^2*e^7 + a^3*d*e^8 + 3*(b^2*c + a*c^2)*d^5*e^4 - (b^3 + 6*a*b
*c)*d^4*e^5 + 3*(a*b^2 + a^2*c)*d^3*e^6)*x^2 + 3*(c^3*d^8*e - 3*b*c^2*d^7*e
^2 - 3*a^2*b*d^3*e^6 + a^3*d^2*e^7 + 3*(b^2*c + a*c^2)*d^6*e^3 - (b^3 + 6*a
*b*c)*d^5*e^4 + 3*(a*b^2 + a^2*c)*d^4*e^5)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx + cx^2}}{(d + ex)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(d + e*x)**4, x)
```

Giac [B] time = 1.58178, size = 2631, normalized size = 12.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -1/8*(2*b^2*c*d - 8*a*c^2*d - b^3*e + 4*a*b*c*e)*arctan(-((sqrt(c)*x - sqrt
(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/((c^2*d^4 -
 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4)*sqrt(-c
*d^2 + b*d*e - a*e^2)) + 1/24*(48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*c^(
7/2)*d^4*e + 32*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^4*d^5 + 16*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^3*b*c^3*d^4*e + 48*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^2*b*c^(7/2)*d^5 - 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b*c^(5/2
```

)*d^3*e^2 - 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c^(5/2)*d^4*e - 48
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(7/2)*d^4*e + 24*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))*b^2*c^3*d^5 - 84*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3
*b^2*c^2*d^3*e^2 - 112*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c^3*d^3*e^2
- 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*c^2*d^4*e - 48*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))*a*b*c^3*d^4*e + 4*b^3*c^(5/2)*d^5 + 78*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^4*b^2*c^(3/2)*d^2*e^3 - 24*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^4*a*c^(5/2)*d^2*e^3 - 6*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^3*
c^(3/2)*d^3*e^2 - 72*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*c^(5/2)*d^3*
e^2 - 4*b^4*c^(3/2)*d^4*e - 12*a*b^2*c^(5/2)*d^4*e + 6*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^5*b^2*c*d*e^4 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*
c^2*d*e^4 + 74*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^3*c*d^2*e^3 + 120*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*c^2*d^2*e^3 + 12*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))*b^4*c*d^3*e^2 - 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*
b^2*c^2*d^3*e^2 + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c^3*d^3*e^2 -
15*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^3*sqrt(c)*d*e^4 - 36*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^4*a*b*c^(3/2)*d*e^4 + 24*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^2*b^4*sqrt(c)*d^2*e^3 + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2
*a*b^2*c^(3/2)*d^2*e^3 + 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*c^(5
/2)*d^2*e^3 + 3*b^5*sqrt(c)*d^3*e^2 - 2*a*b^3*c^(3/2)*d^3*e^2 + 24*a^2*b*c^
(5/2)*d^3*e^2 - 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*e^5 + 12*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c*e^5 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^3*b^4*d*e^4 - 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^2*c*d*e^
4 + 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*c^2*d*e^4 + 3*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))*b^5*d^2*e^3 - 18*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)*a*b^3*c*d^2*e^3 + 120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*c^2*d^2*e
^3 + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^(3/2)*e^5 - 72*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^2*a*b^3*sqrt(c)*d*e^4 - 48*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^2*a^2*b*c^(3/2)*d*e^4 - 6*a*b^4*sqrt(c)*d^2*e^3 + 18*a^2*b^
2*c^(3/2)*d^2*e^3 - 8*a^3*c^(5/2)*d^2*e^3 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^3*a*b^3*e^5 + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c*e^5 -
 6*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^4*d*e^4 - 30*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*a^2*b^2*c*d*e^4 - 72*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a
^3*c^2*d*e^4 + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^2*sqrt(c)*e^5
 + 3*a^2*b^3*sqrt(c)*d*e^4 - 28*a^3*b*c^(3/2)*d*e^4 + 3*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*a^2*b^3*e^5 + 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b
*c*e^5 + 16*a^4*c^(3/2)*e^5)/((c^2*d^4*e^2 - 2*b*c*d^3*e^3 + b^2*d^2*e^4 +
2*a*c*d^2*e^4 - 2*a*b*d*e^5 + a^2*e^6)*((sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^2*e + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c)*d + b*d - a*e)^3)

3.2341 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^5}\,dx$

Optimal. Leaf size=308

$$\frac{\sqrt{a+bx+cx^2}\left(-4ce(ae+4bd)+5b^2e^2+16c^2d^2\right)(-2ae+x(2cd-be)+bd)}{64(d+ex)^2\left(ae^2-bde+cd^2\right)^3}-\frac{\left(b^2-4ac\right)\left(-4ce(ae+4bd)+5b^2e^2+16c^2\right.}{128\left(ae^2-bde\right.}$$

```
[Out] ((16*c^2*d^2 + 5*b^2*e^2 - 4*c*e*(4*b*d + a*e))*(b*d - 2*a*e + (2*c*d - b*e
)*x)*Sqrt[a + b*x + c*x^2])/(64*(c*d^2 - b*d*e + a*e^2)^3*(d + e*x)^2) - (e
*(a + b*x + c*x^2)^(3/2))/(4*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^4) - (5*e*(2
*c*d - b*e)*(a + b*x + c*x^2)^(3/2))/(24*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x
)^3) - ((b^2 - 4*a*c)*(16*c^2*d^2 + 5*b^2*e^2 - 4*c*e*(4*b*d + a*e))*ArcTan
h[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b
*x + c*x^2])])/(128*(c*d^2 - b*d*e + a*e^2)^(7/2))
```

Rubi [A] time = 0.415327, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {744, 806, 720, 724, 206}

$$\frac{\sqrt{a+bx+cx^2}\left(-4ce(ae+4bd)+5b^2e^2+16c^2d^2\right)(-2ae+x(2cd-be)+bd)}{64(d+ex)^2\left(ae^2-bde+cd^2\right)^3}-\frac{\left(b^2-4ac\right)\left(-4ce(ae+4bd)+5b^2e^2+16c^2\right.}{128\left(ae^2-bde\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(d + e*x)^5,x]
```

```
[Out] ((16*c^2*d^2 + 5*b^2*e^2 - 4*c*e*(4*b*d + a*e))*(b*d - 2*a*e + (2*c*d - b*e
)*x)*Sqrt[a + b*x + c*x^2])/(64*(c*d^2 - b*d*e + a*e^2)^3*(d + e*x)^2) - (e
*(a + b*x + c*x^2)^(3/2))/(4*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^4) - (5*e*(2
*c*d - b*e)*(a + b*x + c*x^2)^(3/2))/(24*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x
)^3) - ((b^2 - 4*a*c)*(16*c^2*d^2 + 5*b^2*e^2 - 4*c*e*(4*b*d + a*e))*ArcTan
h[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b
*x + c*x^2])])/(128*(c*d^2 - b*d*e + a*e^2)^(7/2))
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
```

```
2*p + 3], 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^5}\,dx = -\frac{e\left(a+bx+cx^2\right)^{3/2}}{4\left(cd^2-bde+ae^2\right)(d+ex)^4} - \frac{\int \frac{\left(\frac{1}{2}(-8cd+5be)+cex\right)\sqrt{a+bx+cx^2}}{(d+ex)^4}\,dx}{4\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{e\left(a+bx+cx^2\right)^{3/2}}{4\left(cd^2-bde+ae^2\right)(d+ex)^4} - \frac{5e(2cd-be)\left(a+bx+cx^2\right)^{3/2}}{24\left(cd^2-bde+ae^2\right)^2(d+ex)^3} + \frac{\left(16c^2d^2+5b^2e^2-4ce(4bd\right.}{16\left(cd^2-bde\right.}$$

$$= \frac{\left(16c^2d^2+5b^2e^2-4ce(4bd+ae)\right)(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{64\left(cd^2-bde+ae^2\right)^3(d+ex)^2} - \frac{e\left(a+bx+cx\right.}{4\left(cd^2-bde+ae^2\right)}$$

$$= \frac{\left(16c^2d^2+5b^2e^2-4ce(4bd+ae)\right)(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{64\left(cd^2-bde+ae^2\right)^3(d+ex)^2} - \frac{e\left(a+bx+cx\right.}{4\left(cd^2-bde+ae^2\right)}$$

$$= \frac{\left(16c^2d^2+5b^2e^2-4ce(4bd+ae)\right)(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{64\left(cd^2-bde+ae^2\right)^3(d+ex)^2} - \frac{e\left(a+bx+cx\right.}{4\left(cd^2-bde+ae^2\right)}$$

Mathematica [A] time = 0.694276, size = 276, normalized size = 0.9

$$\frac{3\left(-2ce(ae+4bd)+\frac{5b^2e^2}{2}+8c^2d^2\right)\left(\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{8\left(e(ae-bd)+cd^2\right)^{3/2}}+\frac{\sqrt{a+x(b+cx)}(-2ae+b(d-ex)+2cdx)}{4(d+ex)^2\left(e(ae-bd)+cd^2\right)}\right)-\frac{6e(a+x(b+cx))^{3/2}\left(e(\right.}{(d+ex)^4}}{24\left(e(ae-bd)+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x)^5,x]
```

```
[Out] ((-6*e*(c*d^2 + e*(-(b*d) + a*e))*(a + x*(b + c*x))^(3/2))/(d + e*x)^4 - (5
*e*(2*c*d - b*e)*(a + x*(b + c*x))^(3/2))/(d + e*x)^3 + 3*(8*c^2*d^2 + (5*b
^2*e^2)/2 - 2*c*e*(4*b*d + a*e))*((Sqrt[a + x*(b + c*x)]*(-2*a*e + 2*c*d*x
+ b*(d - e*x)))/(4*(c*d^2 + e*(-(b*d) + a*e))*(d + e*x)^2) + ((b^2 - 4*a*c)
*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e
)]*Sqrt[a + x*(b + c*x)])])/(8*(c*d^2 + e*(-(b*d) + a*e))^(3/2))))/(24*(c*d
^2 + e*(-(b*d) + a*e))^2)
```

Maple [B] time = 0.238, size = 7991, normalized size = 25.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)^5,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 21.6727, size = 4086, normalized size = 13.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] 1/384*(2*sqrt(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + b*e/(x*e + d) - b*d
*e/(x*e + d)^2 + a*e^2/(x*e + d)^2)*(2*(4*((2*c^3*d^5*e^6*sgn(1/(x*e + d))
- 5*b*c^2*d^4*e^7*sgn(1/(x*e + d)) + 4*b^2*c*d^3*e^8*sgn(1/(x*e + d)) + 4*a
*c^2*d^3*e^8*sgn(1/(x*e + d)) - b^3*d^2*e^9*sgn(1/(x*e + d)) - 6*a*b*c*d^2*
e^9*sgn(1/(x*e + d)) + 2*a*b^2*d*e^10*sgn(1/(x*e + d)) + 2*a^2*c*d*e^10*sgn
(1/(x*e + d)) - a^2*b*e^11*sgn(1/(x*e + d)))/(c^4*d^8*e^8 - 4*b*c^3*d^7*e^9
 + 6*b^2*c^2*d^6*e^10 + 4*a*c^3*d^6*e^10 - 4*b^3*c*d^5*e^11 - 12*a*b*c^2*d^
5*e^11 + b^4*d^4*e^12 + 12*a*b^2*c*d^4*e^12 + 6*a^2*c^2*d^4*e^12 - 4*a*b^3*
d^3*e^13 - 12*a^2*b*c*d^3*e^13 + 6*a^2*b^2*d^2*e^14 + 4*a^3*c*d^2*e^14 - 4*
a^3*b*d*e^15 + a^4*e^16) - 6*(c^3*d^6*e^7*sgn(1/(x*e + d)) - 3*b*c^2*d^5*e^
8*sgn(1/(x*e + d)) + 3*b^2*c*d^4*e^9*sgn(1/(x*e + d)) + 3*a*c^2*d^4*e^9*sgn
(1/(x*e + d)) - b^3*d^3*e^10*sgn(1/(x*e + d)) - 6*a*b*c*d^3*e^10*sgn(1/(x*e
 + d)) + 3*a*b^2*d^2*e^11*sgn(1/(x*e + d)) + 3*a^2*c*d^2*e^11*sgn(1/(x*e +
d)) - 3*a^2*b*d*e^12*sgn(1/(x*e + d)) + a^3*e^13*sgn(1/(x*e + d)))*e^(-1)/(
(c^4*d^8*e^8 - 4*b*c^3*d^7*e^9 + 6*b^2*c^2*d^6*e^10 + 4*a*c^3*d^6*e^10 - 4*
b^3*c*d^5*e^11 - 12*a*b*c^2*d^5*e^11 + b^4*d^4*e^12 + 12*a*b^2*c*d^4*e^12 +
 6*a^2*c^2*d^4*e^12 - 4*a*b^3*d^3*e^13 - 12*a^2*b*c*d^3*e^13 + 6*a^2*b^2*d^
2*e^14 + 4*a^3*c*d^2*e^14 - 4*a^3*b*d*e^15 + a^4*e^16)*(x*e + d)))*e^(-1)/(
x*e + d) + (8*c^3*d^4*e^5*sgn(1/(x*e + d)) - 16*b*c^2*d^3*e^6*sgn(1/(x*e +
d)) + 13*b^2*c*d^2*e^7*sgn(1/(x*e + d)) - 4*a*c^2*d^2*e^7*sgn(1/(x*e + d))
- 5*b^3*d*e^8*sgn(1/(x*e + d)) + 4*a*b*c*d*e^8*sgn(1/(x*e + d)) + 5*a*b^2*e
^9*sgn(1/(x*e + d)) - 12*a^2*c*e^9*sgn(1/(x*e + d)))/(c^4*d^8*e^8 - 4*b*c^3
*d^7*e^9 + 6*b^2*c^2*d^6*e^10 + 4*a*c^3*d^6*e^10 - 4*b^3*c*d^5*e^11 - 12*a*
b*c^2*d^5*e^11 + b^4*d^4*e^12 + 12*a*b^2*c*d^4*e^12 + 6*a^2*c^2*d^4*e^12 -
4*a*b^3*d^3*e^13 - 12*a^2*b*c*d^3*e^13 + 6*a^2*b^2*d^2*e^14 + 4*a^3*c*d^2*e
^14 - 4*a^3*b*d*e^15 + a^4*e^16))*e^(-1)/(x*e + d) + (16*c^3*d^3*e^4*sgn(1/
(x*e + d)) - 24*b*c^2*d^2*e^5*sgn(1/(x*e + d)) + 38*b^2*c*d*e^6*sgn(1/(x*e
+ d)) - 104*a*c^2*d*e^6*sgn(1/(x*e + d)) - 15*b^3*e^7*sgn(1/(x*e + d)) + 52
*a*b*c*e^7*sgn(1/(x*e + d)))/(c^4*d^8*e^8 - 4*b*c^3*d^7*e^9 + 6*b^2*c^2*d^6
*e^10 + 4*a*c^3*d^6*e^10 - 4*b^3*c*d^5*e^11 - 12*a*b*c^2*d^5*e^11 + b^4*d^4
*e^12 + 12*a*b^2*c*d^4*e^12 + 6*a^2*c^2*d^4*e^12 - 4*a*b^3*d^3*e^13 - 12*a^
2*b*c*d^3*e^13 + 6*a^2*b^2*d^2*e^14 + 4*a^3*c*d^2*e^14 - 4*a^3*b*d*e^15 + a
^4*e^16)) + 3*(16*b^2*c^2*d^2*sgn(1/(x*e + d)) - 64*a*c^3*d^2*sgn(1/(x*e +
d)) - 16*b^3*c*d*e*sgn(1/(x*e + d)) + 64*a*b*c^2*d*e*sgn(1/(x*e + d)) + 5*b
^4*e^2*sgn(1/(x*e + d)) - 24*a*b^2*c*e^2*sgn(1/(x*e + d)) + 16*a^2*c^2*e^2*
sgn(1/(x*e + d)))*sqrt(c*d^2 - b*d*e + a*e^2)*log(abs(2*(c*d^2 - b*d*e + a*
e^2)*(sqrt(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + b*e/(x*e + d) - b*d*e/
(x*e + d)^2 + a*e^2/(x*e + d)^2) + sqrt(c*d^2*e^2 - b*d*e^3 + a*e^4)*e^(-1)
/(x*e + d)) - sqrt(c*d^2 - b*d*e + a*e^2)*(2*c*d - b*e)))/(c^5*d^10*e^2 - 5
*b*c^4*d^9*e^3 + 10*b^2*c^3*d^8*e^4 + 5*a*c^4*d^8*e^4 - 10*b^3*c^2*d^7*e^5
- 20*a*b*c^3*d^7*e^5 + 5*b^4*c*d^6*e^6 + 30*a*b^2*c^2*d^6*e^6 + 10*a^2*c^3*
d^6*e^6 - b^5*d^5*e^7 - 20*a*b^3*c*d^5*e^7 - 30*a^2*b*c^2*d^5*e^7 + 5*a*b^4
*d^4*e^8 + 30*a^2*b^2*c*d^4*e^8 + 10*a^3*c^2*d^4*e^8 - 10*a^2*b^3*d^3*e^9 -
 20*a^3*b*c*d^3*e^9 + 10*a^3*b^2*d^2*e^10 + 5*a^4*c*d^2*e^10 - 5*a^4*b*d*e^
11 + a^5*e^12) - (32*c^(9/2)*d^5 - 80*b*c^(7/2)*d^4*e + 124*b^2*c^(5/2)*d^3
*e^2 - 176*a*c^(7/2)*d^3*e^2 + 48*sqrt(c*d^2 - b*d*e + a*e^2)*b^2*c^2*d^2*e
^2*log(abs(2*c^(3/2)*d^2 - 2*b*sqrt(c)*d*e - 2*sqrt(c*d^2 - b*d*e + a*e^2)*
c*d + 2*a*sqrt(c)*e^2 + sqrt(c*d^2 - b*d*e + a*e^2)*b*e)) - 192*sqrt(c*d^2
- b*d*e + a*e^2)*a*c^3*d^2*e^2*log(abs(2*c^(3/2)*d^2 - 2*b*sqrt(c)*d*e - 2*
sqrt(c*d^2 - b*d*e + a*e^2)*c*d + 2*a*sqrt(c)*e^2 + sqrt(c*d^2 - b*d*e + a*
e^2)*b*e)) - 106*b^3*c^(3/2)*d^2*e^3 + 264*a*b*c^(5/2)*d^2*e^3 - 48*sqrt(c*
```

d^2 - b*d*e + a*e^2)*b^3*c*d*e^3*log(abs(2*c^(3/2)*d^2 - 2*b*sqrt(c)*d*e - 2*sqrt(c*d^2 - b*d*e + a*e^2)*c*d + 2*a*sqrt(c)*e^2 + sqrt(c*d^2 - b*d*e + a*e^2)*b*e)) + 192*sqrt(c*d^2 - b*d*e + a*e^2)*a*b*c^2*d*e^3*log(abs(2*c^(3/2)*d^2 - 2*b*sqrt(c)*d*e - 2*sqrt(c*d^2 - b*d*e + a*e^2)*c*d + 2*a*sqrt(c)*e^2 + sqrt(c*d^2 - b*d*e + a*e^2)*b*e)) + 30*b^4*sqrt(c)*d*e^4 - 28*a*b^2*c^(3/2)*d*e^4 - 208*a^2*c^(5/2)*d*e^4 + 15*sqrt(c*d^2 - b*d*e + a*e^2)*b^4*e^4*log(abs(2*c^(3/2)*d^2 - 2*b*sqrt(c)*d*e - 2*sqrt(c*d^2 - b*d*e + a*e^2)*c*d + 2*a*sqrt(c)*e^2 + sqrt(c*d^2 - b*d*e + a*e^2)*b*e)) - 72*sqrt(c*d^2 - b*d*e + a*e^2)*a*b^2*c*e^4*log(abs(2*c^(3/2)*d^2 - 2*b*sqrt(c)*d*e - 2*sqrt(c*d^2 - b*d*e + a*e^2)*c*d + 2*a*sqrt(c)*e^2 + sqrt(c*d^2 - b*d*e + a*e^2)*b*e)) + 48*sqrt(c*d^2 - b*d*e + a*e^2)*a^2*c^2*e^4*log(abs(2*c^(3/2)*d^2 - 2*b*sqrt(c)*d*e - 2*sqrt(c*d^2 - b*d*e + a*e^2)*c*d + 2*a*sqrt(c)*e^2 + sqrt(c*d^2 - b*d*e + a*e^2)*b*e)) - 30*a*b^3*sqrt(c)*e^5 + 104*a^2*b*c^(3/2)*e^5)*sgn(1/(x*e + d))/(c^5*d^10*e^4 - 5*b*c^4*d^9*e^5 + 10*b^2*c^3*d^8*e^6 + 5*a*c^4*d^8*e^6 - 10*b^3*c^2*d^7*e^7 - 20*a*b*c^3*d^7*e^7 + 5*b^4*c*d^6*e^8 + 30*a*b^2*c^2*d^6*e^8 + 10*a^2*c^3*d^6*e^8 - b^5*d^5*e^9 - 20*a*b^3*c*d^5*e^9 - 30*a^2*b*c^2*d^5*e^9 + 5*a*b^4*d^4*e^10 + 30*a^2*b^2*c*d^4*e^10 + 10*a^3*c^2*d^4*e^10 - 10*a^2*b^3*d^3*e^11 - 20*a^3*b*c*d^3*e^11 + 10*a^3*b^2*d^2*e^12 + 5*a^4*c*d^2*e^12 - 5*a^4*b*d*e^13 + a^5*e^14))*e^2

3.2342 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^6}\,dx$

Optimal. Leaf size=402

$$-\frac{e\left(a+bx+cx^2\right)^{3/2}\left(-4ce(8ae+27bd)+35b^2e^2+108c^2d^2\right)}{240(d+ex)^3\left(ae^2-bde+cd^2\right)^3}+\frac{\sqrt{a+bx+cx^2}(2cd-be)\left(-4ce(3ae+4bd)+7b^2e^2+16\right.}{128(d+ex)^2\left(ae^2-bde+c\right.}$$

```
[Out] ((2*c*d - b*e)*(16*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(4*b*d + 3*a*e))*(b*d - 2*a*
e + (2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(128*(c*d^2 - b*d*e + a*e^2)^4*
(d + e*x)^2) - (e*(a + b*x + c*x^2)^(3/2))/(5*(c*d^2 - b*d*e + a*e^2)*(d +
e*x)^5) - (7*e*(2*c*d - b*e)*(a + b*x + c*x^2)^(3/2))/(40*(c*d^2 - b*d*e +
a*e^2)^2*(d + e*x)^4) - (e*(108*c^2*d^2 + 35*b^2*e^2 - 4*c*e*(27*b*d + 8*a*
e))*(a + b*x + c*x^2)^(3/2))/(240*(c*d^2 - b*d*e + a*e^2)^3*(d + e*x)^3) -
((b^2 - 4*a*c)*(2*c*d - b*e)*(16*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(4*b*d + 3*a*e
))*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*S
qrt[a + b*x + c*x^2])])/(256*(c*d^2 - b*d*e + a*e^2)^(9/2))
```

Rubi [A] time = 0.524729, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {744, 834, 806, 720, 724, 206}

$$-\frac{e\left(a+bx+cx^2\right)^{3/2}\left(-4ce(8ae+27bd)+35b^2e^2+108c^2d^2\right)}{240(d+ex)^3\left(ae^2-bde+cd^2\right)^3}+\frac{\sqrt{a+bx+cx^2}(2cd-be)\left(-4ce(3ae+4bd)+7b^2e^2+16\right.}{128(d+ex)^2\left(ae^2-bde+c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(d + e*x)^6,x]
```

```
[Out] ((2*c*d - b*e)*(16*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(4*b*d + 3*a*e))*(b*d - 2*a*
e + (2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(128*(c*d^2 - b*d*e + a*e^2)^4*
(d + e*x)^2) - (e*(a + b*x + c*x^2)^(3/2))/(5*(c*d^2 - b*d*e + a*e^2)*(d +
e*x)^5) - (7*e*(2*c*d - b*e)*(a + b*x + c*x^2)^(3/2))/(40*(c*d^2 - b*d*e +
a*e^2)^2*(d + e*x)^4) - (e*(108*c^2*d^2 + 35*b^2*e^2 - 4*c*e*(27*b*d + 8*a*
e))*(a + b*x + c*x^2)^(3/2))/(240*(c*d^2 - b*d*e + a*e^2)^3*(d + e*x)^3) -
((b^2 - 4*a*c)*(2*c*d - b*e)*(16*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(4*b*d + 3*a*e
))*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*S
qrt[a + b*x + c*x^2])])/(256*(c*d^2 - b*d*e + a*e^2)^(9/2))
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
```

```
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^6}\,dx = -\frac{e\left(a+bx+cx^2\right)^{3/2}}{5\left(cd^2-bde+ae^2\right)(d+ex)^5} - \frac{\int \frac{\left(\frac{1}{2}(-10cd+7be)+2cex\right)\sqrt{a+bx+cx^2}}{(d+ex)^5}dx}{5\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{e\left(a+bx+cx^2\right)^{3/2}}{5\left(cd^2-bde+ae^2\right)(d+ex)^5} - \frac{7e(2cd-be)\left(a+bx+cx^2\right)^{3/2}}{40\left(cd^2-bde+ae^2\right)^2(d+ex)^4} + \frac{\int \frac{\left(\frac{1}{4}\left(80c^2d^2+35b^2e^2-2ce(47bd+16\right.\right.}{(d+e}}{20\left(cd^2-b\right.}$$

$$= -\frac{e\left(a+bx+cx^2\right)^{3/2}}{5\left(cd^2-bde+ae^2\right)(d+ex)^5} - \frac{7e(2cd-be)\left(a+bx+cx^2\right)^{3/2}}{40\left(cd^2-bde+ae^2\right)^2(d+ex)^4} - \frac{e\left(108c^2d^2+35b^2e^2-4ce(2\right.}{240\left(cd^2-bde\right.}$$

$$= \frac{(2cd-be)\left(16c^2d^2+7b^2e^2-4ce(4bd+3ae)\right)(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{128\left(cd^2-bde+ae^2\right)^4(d+ex)^2} - \frac{e\left(\right.}{5\left(cd^2-\right.}$$

$$= \frac{(2cd-be)\left(16c^2d^2+7b^2e^2-4ce(4bd+3ae)\right)(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{128\left(cd^2-bde+ae^2\right)^4(d+ex)^2} - \frac{e\left(\right.}{5\left(cd^2-\right.}$$

$$= \frac{(2cd-be)\left(16c^2d^2+7b^2e^2-4ce(4bd+3ae)\right)(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{128\left(cd^2-bde+ae^2\right)^4(d+ex)^2} - \frac{e\left(\right.}{5\left(cd^2-\right.}$$

Mathematica [A] time = 2.89185, size = 367, normalized size = 0.91

$$-\frac{-\frac{\frac{15}{4}(2cd-be)\left(-4ce(3ae+4bd)+7b^2e^2+16c^2d^2\right)\left(\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{8\left(e(ae-bd)+cd^2\right)^{3/2}}+\frac{\sqrt{a+x(b+cx)}(-2ae+b(d-ex)+2cdx)}{4(d+ex)^2\left(e(ae-bd)+cd^2\right)}\right)-\frac{e(a+x(b+cx))^{3/2}\left(-4ce(8ae+27bd)+35b^2\right.}{2(d+ex)^3}}{24\left(e(ae-bd)+cd^2\right)^2}}{5\left(e(ae-bd)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x)^6,x]
```

```
[Out] -((e*(a + x*(b + c*x))^(3/2))/(d + e*x)^5 + (7*e*(2*c*d - b*e)*(a + x*(b +
c*x))^(3/2))/(8*(c*d^2 + e*(-(b*d) + a*e))*(d + e*x)^4) - (-(e*(108*c^2*d^2
 + 35*b^2*e^2 - 4*c*e*(27*b*d + 8*a*e))*(a + x*(b + c*x))^(3/2))/(2*(d + e*
x)^3) + (15*(2*c*d - b*e)*(16*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(4*b*d + 3*a*e))*
((Sqrt[a + x*(b + c*x)]*(-2*a*e + 2*c*d*x + b*(d - e*x)))/(4*(c*d^2 + e*(-(
b*d) + a*e))*(d + e*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*
x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/(8*(c
*d^2 + e*(-(b*d) + a*e))^(3/2))))/4)/(24*(c*d^2 + e*(-(b*d) + a*e))^2))/(5*
(c*d^2 + e*(-(b*d) + a*e)))
```

Maple [B] time = 0.242, size = 10791, normalized size = 26.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)^6,x)
```

[Out] result too large to display

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^6,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.94078, size = 11410, normalized size = 28.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^6,x, algorithm="giac")
```

```
[Out] -1/128*(32*b^2*c^3*d^3 - 128*a*c^4*d^3 - 48*b^3*c^2*d^2*e + 192*a*b*c^3*d^2
*e + 30*b^4*c*d*e^2 - 144*a*b^2*c^2*d*e^2 + 96*a^2*c^3*d*e^2 - 7*b^5*e^3 +
40*a*b^3*c*e^3 - 48*a^2*b*c^2*e^3)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/((c^4*d^8 - 4*b*c^3*d^7*e
 + 6*b^2*c^2*d^6*e^2 + 4*a*c^3*d^6*e^2 - 4*b^3*c*d^5*e^3 - 12*a*b*c^2*d^5*e
^3 + b^4*d^4*e^4 + 12*a*b^2*c*d^4*e^4 + 6*a^2*c^2*d^4*e^4 - 4*a*b^3*d^3*e^5
 - 12*a^2*b*c*d^3*e^5 + 6*a^2*b^2*d^2*e^6 + 4*a^3*c*d^2*e^6 - 4*a^3*b*d*e^7
 + a^4*e^8)*sqrt(-c*d^2 + b*d*e - a*e^2)) + 1/1920*(7680*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^6*c^(13/2)*d^8*e + 3072*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
```

))^5*c^7*d^9 + 9216*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^6*d^8*e + 768
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b*c^(13/2)*d^9 - 30720*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^6*b*c^(11/2)*d^7*e^2 - 3840*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^4*b^2*c^(11/2)*d^8*e - 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^4*a*c^(13/2)*d^8*e + 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*c^6*d
^9 - 50048*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c^5*d^7*e^2 - 57856*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*c^6*d^7*e^2 - 11520*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^3*b^3*c^5*d^8*e - 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^3*a*b*c^6*d^8*e + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^3*c^(11/
2)*d^9 + 70720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^2*c^(9/2)*d^6*e^3 -
67840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*c^(11/2)*d^6*e^3 - 17600*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^4*b^3*c^(9/2)*d^7*e^2 - 113920*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^4*a*b*c^(11/2)*d^7*e^2 - 7200*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^2*b^4*c^(9/2)*d^8*e - 11520*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^2*a*b^2*c^(11/2)*d^8*e + 960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^4*c^
5*d^9 + 15040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^2*c^4*d^5*e^4 - 60160
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*c^5*d^5*e^4 + 129280*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^5*b^3*c^4*d^6*e^3 - 1024*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^5*a*b*c^5*d^6*e^3 + 14080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^
4*c^4*d^7*e^2 - 90880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^2*c^5*d^7*e
^2 + 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*c^6*d^7*e^2 - 1920*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))*b^5*c^4*d^8*e - 3840*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*a*b^3*c^5*d^8*e + 96*b^5*c^(9/2)*d^9 + 4320*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^8*b^2*c^(7/2)*d^4*e^5 - 17280*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^8*a*c^(9/2)*d^4*e^5 - 52000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
6*b^3*c^(7/2)*d^5*e^4 - 7040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b*c^(9
/2)*d^5*e^4 + 81920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^4*c^(7/2)*d^6*e
^3 + 114240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^2*c^(9/2)*d^6*e^3 + 1
67680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^(11/2)*d^6*e^3 + 13760*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^5*c^(7/2)*d^7*e^2 - 37760*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^2*a*b^3*c^(9/2)*d^7*e^2 + 23040*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^2*a^2*b*c^(11/2)*d^7*e^2 - 192*b^6*c^(7/2)*d^8*e - 480*a*
b^4*c^(9/2)*d^8*e + 480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^2*c^3*d^3*e
^6 - 1920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*c^4*d^3*e^6 - 20320*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^7*b^3*c^3*d^4*e^5 + 81280*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^7*a*b*c^4*d^4*e^5 - 120680*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^5*b^4*c^3*d^5*e^4 - 122240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^
2*c^4*d^5*e^4 + 226432*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*c^5*d^5*e^
4 + 14080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^5*c^3*d^6*e^3 + 88320*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*c^4*d^6*e^3 + 281600*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^3*a^2*b*c^5*d^6*e^3 + 4280*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*b^6*c^3*d^7*e^2 - 8480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^4
*c^4*d^7*e^2 + 11520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^2*c^5*d^7*e^
2 - 6480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^3*c^(5/2)*d^3*e^6 + 25920*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b*c^(7/2)*d^3*e^6 + 7260*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^6*b^4*c^(5/2)*d^4*e^5 + 59680*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^6*a*b^2*c^(7/2)*d^4*e^5 + 182720*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^6*a^2*c^(9/2)*d^4*e^5 - 85780*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^4*b^5*c^(5/2)*d^5*e^4 - 237120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^
3*c^(7/2)*d^5*e^4 + 63040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b*c^(9/
2)*d^5*e^4 - 6340*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^6*c^(5/2)*d^6*e^3
 + 22000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^4*c^(7/2)*d^6*e^3 + 1766
40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^2*c^(9/2)*d^6*e^3 - 7680*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*c^(11/2)*d^6*e^3 + 476*b^7*c^(5/2)*d
^7*e^2 - 848*a*b^5*c^(7/2)*d^7*e^2 + 1920*a^2*b^3*c^(9/2)*d^7*e^2 - 720*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^3*c^2*d^2*e^7 + 2880*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^9*a*b*c^3*d^2*e^7 + 10740*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^7*b^4*c^2*d^3*e^6 - 56480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^2
*c^3*d^3*e^6 + 54080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*c^4*d^3*e^6

+ 47944*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^5*c^2*d^4*e^5 + 167520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^3*c^3*d^4*e^5 - 17920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b*c^4*d^4*e^5 - 25220*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^6*c^2*d^5*e^4 - 124960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^4*c^3*d^5*e^4 - 239360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^2*c^4*d^5*e^4 - 160000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*c^5*d^5*e^4 - 3080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^7*c^2*d^6*e^3 + 480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^5*c^3*d^6*e^3 + 49280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^3*c^4*d^6*e^3 - 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b*c^5*d^6*e^3 + 4050*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^4*c^(3/2)*d^2*e^7 - 19440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^2*c^(5/2)*d^2*e^7 + 12960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*c^(7/2)*d^2*e^7 + 9310*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^5*c^(3/2)*d^3*e^6 - 46960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^3*c^(5/2)*d^3*e^6 - 176160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b*c^(7/2)*d^3*e^6 + 35330*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^6*c^(3/2)*d^4*e^5 + 244660*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^4*c^(5/2)*d^4*e^5 + 32960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^2*c^(7/2)*d^4*e^5 - 178880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*c^(9/2)*d^4*e^5 - 1750*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^7*c^(3/2)*d^5*e^4 - 13120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^5*c^(5/2)*d^5*e^4 - 187840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^3*c^(7/2)*d^5*e^4 - 216960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b*c^(9/2)*d^5*e^4 - 380*b^8*c^(3/2)*d^6*e^3 - 332*a*b^6*c^(5/2)*d^6*e^3 + 5200*a^2*b^4*c^(7/2)*d^6*e^3 - 1920*a^3*b^2*c^(9/2)*d^6*e^3 + 450*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^4*c*d*e^8 - 2160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^2*c^2*d*e^8 + 1440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*c^3*d*e^8 - 1190*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^5*c*d^2*e^7 + 12080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^3*c^2*d^2*e^7 - 29280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b*c^3*d^2*e^7 - 4658*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^6*c*d^3*e^6 - 80020*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^4*c^2*d^3*e^6 - 155840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^2*c^3*d^3*e^6 - 120640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*c^4*d^3*e^6 + 10510*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^7*c*d^4*e^5 + 120280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^5*c^2*d^4*e^5 + 200320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^3*c^3*d^4*e^5 + 42240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b*c^4*d^4*e^5 + 600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^8*c*d^5*e^4 + 5380*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^6*c^2*d^5*e^4 - 46840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^4*c^3*d^5*e^4 - 98880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^2*c^4*d^5*e^4 + 1920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*c^5*d^5*e^4 - 945*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^5*sqrt(c)*d*e^8 + 5400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^3*c^(3/2)*d*e^8 - 6480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b*c^(5/2)*d*e^8 - 3430*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^6*sqrt(c)*d^2*e^7 + 4900*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^4*c^(3/2)*d^2*e^7 + 93120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b^2*c^(5/2)*d^2*e^7 - 16320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^3*c^(7/2)*d^2*e^7 - 4480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^7*sqrt(c)*d^3*e^6 - 101890*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^5*c^(3/2)*d^3*e^6 - 179920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^3*c^(5/2)*d^3*e^6 + 56160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b*c^(7/2)*d^3*e^6 + 1470*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^8*sqrt(c)*d^4*e^5 + 16760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^6*c^(3/2)*d^4*e^5 + 112660*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^4*c^(5/2)*d^4*e^5 + 212960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^2*c^(7/2)*d^4*e^5 + 89920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*c^(9/2)*d^4*e^5 + 105*b^9*sqrt(c)*d^5*e^4 + 990*a*b^7*c^(3/2)*d^5*e^4 - 3244*a^2*b^5*c^(5/2)*d^5*e^4 - 15200*a^3*b^3*c^(7/2)*d^5*e^4 + 960*a^4*b*c^(9/2)*d^5*e^4 - 105*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^5*e^9 + 600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^3*c*e^9 - 720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b*c^2*e^9 - 490*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^6*d*e^8 + 700*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^4*c*d*e^

8 + 6720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^2*c^2*d*e^8 - 6720*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*c^3*d*e^8 - 896*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^5*b^7*d^2*e^7 + 5938*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*
a*b^5*c*d^2*e^7 + 105040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^3*c^2*
d^2*e^7 + 132000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b*c^3*d^2*e^7 -
790*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^8*d^3*e^6 - 47320*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^3*a*b^6*c*d^3*e^6 - 160420*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^3*a^2*b^4*c^2*d^3*e^6 - 79200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^3*a^3*b^2*c^3*d^3*e^6 + 93120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*c
^4*d^3*e^6 + 105*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^9*d^4*e^5 - 1950*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^7*c*d^4*e^5 + 16960*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))*a^2*b^5*c^2*d^4*e^5 + 96640*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*a^3*b^3*c^3*d^4*e^5 + 85120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*
b*c^4*d^4*e^5 + 3430*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^5*sqrt(c)*d*
e^8 - 19600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b^3*c^(3/2)*d*e^8 - 7
200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^3*b*c^(5/2)*d*e^8 + 8960*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^6*sqrt(c)*d^2*e^7 + 113920*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^4*a^2*b^4*c^(3/2)*d^2*e^7 + 51200*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^4*a^3*b^2*c^(5/2)*d^2*e^7 + 71680*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^4*a^4*c^(7/2)*d^2*e^7 - 8250*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^2*a*b^7*sqrt(c)*d^3*e^6 - 46750*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a
^2*b^5*c^(3/2)*d^3*e^6 - 154800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b
^3*c^(5/2)*d^3*e^6 - 40160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b*c^(7
/2)*d^3*e^6 - 420*a*b^8*sqrt(c)*d^4*e^5 - 90*a^2*b^6*c^(3/2)*d^4*e^5 + 1142
0*a^3*b^4*c^(5/2)*d^4*e^5 + 20320*a^4*b^2*c^(7/2)*d^4*e^5 - 192*a^5*c^(9/2)
*d^4*e^5 + 490*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^5*e^9 - 2800*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^3*c*e^9 + 3360*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^7*a^3*b*c^2*e^9 + 1792*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5
*a*b^6*d*e^8 - 6400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^4*c*d*e^8 -
92160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^2*c^2*d*e^8 + 15360*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*c^3*d*e^8 + 2370*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^3*a*b^7*d^2*e^7 + 72310*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^3*a^2*b^5*c*d^2*e^7 + 71280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^3*
c^2*d^2*e^7 + 3680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b*c^3*d^2*e^7
- 420*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^8*d^3*e^6 - 990*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))*a^2*b^6*c*d^3*e^6 - 49820*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))*a^3*b^4*c^2*d^3*e^6 - 74400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a
^4*b^2*c^3*d^3*e^6 - 24640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*c^4*d^3*
e^6 + 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*c^(5/2)*e^9 - 4480*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^5*sqrt(c)*d*e^8 - 71680*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^4*a^3*b^3*c^(3/2)*d*e^8 - 33280*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^4*a^4*b*c^(5/2)*d*e^8 + 15930*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^2*a^2*b^6*sqrt(c)*d^2*e^7 + 55340*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^2*a^3*b^4*c^(3/2)*d^2*e^7 + 58880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
2*a^4*b^2*c^(5/2)*d^2*e^7 - 20800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5
*c^(7/2)*d^2*e^7 + 630*a^2*b^7*sqrt(c)*d^3*e^6 - 3150*a^3*b^5*c^(3/2)*d^3*e
^6 - 16000*a^4*b^3*c^(5/2)*d^3*e^6 - 11936*a^5*b*c^(7/2)*d^3*e^6 - 896*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^5*e^9 + 5120*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^5*a^3*b^3*c*e^9 + 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5
*a^4*b*c^2*e^9 - 2370*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^6*d*e^8 -
44700*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^4*c*d*e^8 - 17920*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^2*c^2*d*e^8 - 13760*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^3*a^5*c^3*d*e^8 + 630*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)*a^2*b^7*d^2*e^7 + 8670*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^5*c*d^2*
e^7 + 43360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^3*c^2*d^2*e^7 + 16160
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b*c^3*d^2*e^7 + 24320*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^4*a^4*b^2*c^(3/2)*e^9 + 2560*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^4*a^5*c^(5/2)*e^9 - 12990*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^2*a^3*b^5*sqrt(c)*d*e^8 - 28720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*

b^3*c^(3/2)*d*e^8 + 160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b*c^(5/2)*d*e^8 - 420*a^3*b^6*sqrt(c)*d^2*e^7 + 5790*a^4*b^4*c^(3/2)*d^2*e^7 + 9008*a^5*b^2*c^(5/2)*d^2*e^7 + 2656*a^6*c^(7/2)*d^2*e^7 + 790*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^5*e^9 + 9200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^3*c*e^9 + 12000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b*c^2*e^9 - 420*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^6*d*e^8 - 9570*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^4*c*d*e^8 - 13520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^2*c^2*d*e^8 + 3680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*c^3*d*e^8 + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^4*sqrt(c)*e^9 + 5120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^2*c^(3/2)*e^9 + 2560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*c^(5/2)*e^9 + 105*a^4*b^5*sqrt(c)*d*e^8 - 4440*a^5*b^3*c^(3/2)*d*e^8 - 816*a^6*b*c^(5/2)*d*e^8 + 105*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^5*e^9 + 3240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^3*c*e^9 + 720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b*c^2*e^9 + 1280*a^6*b^2*c^(3/2)*e^9 - 512*a^7*c^(5/2)*e^9)/((c^4*d^8*e^2 - 4*b*c^3*d^7*e^3 + 6*b^2*c^2*d^6*e^4 + 4*a*c^3*d^6*e^4 - 4*b^3*c*d^5*e^5 - 12*a*b*c^2*d^5*e^5 + b^4*d^4*e^6 + 12*a*b^2*c*d^4*e^6 + 6*a^2*c^2*d^4*e^6 - 4*a*b^3*d^3*e^7 - 12*a^2*b*c*d^3*e^7 + 6*a^2*b^2*d^2*e^8 + 4*a^3*c*d^2*e^8 - 4*a^3*b*d*e^9 + a^4*e^10)*((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*e + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c)*d + b*d - a*e)^5)

3.2343 $\int (d+ex)^3 \left(a+bx+cx^2\right)^{3/2} dx$

Optimal. Leaf size=321

$$\frac{e\left(a+bx+cx^2\right)^{5/2}\left(-2ce(8ae+49bd)+21b^2e^2+30cex(2cd-be)+128c^2d^2\right)}{280c^3}+\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}(2cd-be)(}{128c^4}$$

```
[Out] (-3*(b^2 - 4*a*c)*(2*c*d - b*e)*(8*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(2*b*d + a*e
))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(1024*c^5) + ((2*c*d - b*e)*(8*c^2*d^
2 + 3*b^2*e^2 - 4*c*e*(2*b*d + a*e))*(b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(
128*c^4) + (e*(d + e*x)^2*(a + b*x + c*x^2)^(5/2))/(7*c) + (e*(128*c^2*d^2
+ 21*b^2*e^2 - 2*c*e*(49*b*d + 8*a*e) + 30*c*e*(2*c*d - b*e)*x)*(a + b*x +
c*x^2)^(5/2))/(280*c^3) + (3*(b^2 - 4*a*c)^2*(2*c*d - b*e)*(8*c^2*d^2 + 3*b
^2*e^2 - 4*c*e*(2*b*d + a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x +
 c*x^2])])/(2048*c^(11/2))
```

Rubi [A] time = 0.34696, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {742, 779, 612, 621, 206}

$$\frac{e\left(a+bx+cx^2\right)^{5/2}\left(-2ce(8ae+49bd)+21b^2e^2+30cex(2cd-be)+128c^2d^2\right)}{280c^3}+\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}(2cd-be)(}{128c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*x + c*x^2)^(3/2),x]
```


```
[Out] (-3*(b^2 - 4*a*c)*(2*c*d - b*e)*(8*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(2*b*d + a*e
))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(1024*c^5) + ((2*c*d - b*e)*(8*c^2*d^
2 + 3*b^2*e^2 - 4*c*e*(2*b*d + a*e))*(b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(
128*c^4) + (e*(d + e*x)^2*(a + b*x + c*x^2)^(5/2))/(7*c) + (e*(128*c^2*d^2
+ 21*b^2*e^2 - 2*c*e*(49*b*d + 8*a*e) + 30*c*e*(2*c*d - b*e)*x)*(a + b*x +
c*x^2)^(5/2))/(280*c^3) + (3*(b^2 - 4*a*c)^2*(2*c*d - b*e)*(8*c^2*d^2 + 3*b
^2*e^2 - 4*c*e*(2*b*d + a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x +
 c*x^2])])/(2048*c^(11/2))
```


Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^3\left(a+bx+cx^2\right)^{3/2}\,dx = \frac{e(d+ex)^2\left(a+bx+cx^2\right)^{5/2}}{7c} + \frac{\int (d+ex)\left(\frac{1}{2}\left(14cd^2-e(5bd+4ae)\right)+\frac{9}{2}e(2cd-be)x\right)(a\cdots}{7c}$$

$$= \frac{e(d+ex)^2\left(a+bx+cx^2\right)^{5/2}}{7c} + \frac{e\left(128c^2d^2+21b^2e^2-2ce(49bd+8ae)+30ce(2cd-be)\cdots\right.}{280c^3}$$

$$= \frac{(2cd-be)\left(8c^2d^2+3b^2e^2-4ce(2bd+ae)\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{128c^4} + \frac{e(d+ex)^2\left(a\cdots\right.}{7\cdots}$$

$$= -\frac{3\left(b^2-4ac\right)(2cd-be)\left(8c^2d^2+3b^2e^2-4ce(2bd+ae)\right)(b+2cx)\sqrt{a+bx+cx^2}}{1024c^5} + \frac{(2c\cdots}{\cdots}$$

$$= -\frac{3\left(b^2-4ac\right)(2cd-be)\left(8c^2d^2+3b^2e^2-4ce(2bd+ae)\right)(b+2cx)\sqrt{a+bx+cx^2}}{1024c^5} + \frac{(2c\cdots}{\cdots}$$

$$= -\frac{3\left(b^2-4ac\right)(2cd-be)\left(8c^2d^2+3b^2e^2-4ce(2bd+ae)\right)(b+2cx)\sqrt{a+bx+cx^2}}{1024c^5} + \frac{(2c\cdots}{\cdots}$$

Mathematica [A] time = 0.527782, size = 231, normalized size = 0.72

$$\frac{\frac{7(2cd-be)\left(-4ce(ae+2bd)+3b^2e^2+8c^2d^2\right)\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}\left(4c\left(5a+2cx^2\right)-3b^2+8bcx\right)+3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{2048c^{9/2}} + \frac{e(a+x(b+cx))^{5/2}\left(-2ce(8ae+\cdots\right.}{\cdots}}{7c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (e*(d + e*x)^2*(a + x*(b + c*x))^(5/2) + (e*(a + x*(b + c*x))^(5/2)*(21*b^2
*e^2 + 4*c^2*d*(32*d + 15*e*x) - 2*c*e*(49*b*d + 8*a*e + 15*b*e*x)))/(40*c^
2) + (7*(2*c*d - b*e)*(8*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(2*b*d + a*e))*(2*Sqrt
[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)]*(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^
2)) + 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)
```

```
])]))/(2048*c^(9/2)))/(7*c)
```

Maple [B] time = 0.052, size = 1437, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 7/32*d*e^2*b^2/c^2*x*(c*x^2+b*x+a)^(3/2)-21/256*d*e^2*b^4/c^3*(c*x^2+b*x+a)
^(1/2)*x+3/32*e^3*b/c^2*a^2*(c*x^2+b*x+a)^(1/2)*x-3/32*d*e^2*a^2/c^2*(c*x^2
+b*x+a)^(1/2)*b+3/8*d*e^2*b^2/c^2*(c*x^2+b*x+a)^(1/2)*x*a-9/16*d^2*e*b/c*(c
*x^2+b*x+a)^(1/2)*x*a+3/16*d*e^2*b^3/c^3*(c*x^2+b*x+a)^(1/2)*a+27/64*d*e^2*
b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2-45/256*d*e^2*b^
4/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/8*d*e^2*a/c*x*(c*
x^2+b*x+a)^(3/2)+1/4*d^3*x*(c*x^2+b*x+a)^(3/2)+7/64*d*e^2*b^3/c^3*(c*x^2+b*
x+a)^(3/2)-21/512*d*e^2*b^5/c^4*(c*x^2+b*x+a)^(1/2)+21/1024*d*e^2*b^6/c^(9/
2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-3/16*d*e^2*a^3/c^(3/2)*ln((1
/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-3/16*d^2*e*b^2/c^2*(c*x^2+b*x+a)^(3/
2)+9/128*d^2*e*b^4/c^3*(c*x^2+b*x+a)^(1/2)-9/256*d^2*e*b^5/c^(7/2)*ln((1/2*
b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-3/32*d^3/c*(c*x^2+b*x+a)^(1/2)*x*b^2+3/
16*d^3/c*(c*x^2+b*x+a)^(1/2)*b*a-3/16*d^3/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c
*x^2+b*x+a)^(1/2))*b^2*a+3/5*d^2*e*(c*x^2+b*x+a)^(5/2)/c+3/40*e^3*b^2/c^3*(
c*x^2+b*x+a)^(5/2)-3/128*e^3*b^4/c^4*(c*x^2+b*x+a)^(3/2)+9/1024*e^3*b^6/c^5
*(c*x^2+b*x+a)^(1/2)-2/35*e^3*a/c^2*(c*x^2+b*x+a)^(5/2)+1/7*e^3*x^2*(c*x^2+
b*x+a)^(5/2)/c-9/2048*e^3*b^7/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)
^(1/2))+1/8*d^3/c*(c*x^2+b*x+a)^(3/2)*b+3/8*d^3*(c*x^2+b*x+a)^(1/2)*x*a-3/6
4*d^3/c^2*(c*x^2+b*x+a)^(1/2)*b^3+3/8*d^3/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c
*x^2+b*x+a)^(1/2))*a^2+3/128*d^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+
a)^(1/2))*b^4+21/512*e^3*b^5/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(
1/2))*a+3/32*e^3*b/c^(5/2)*a^3*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+
1/32*e^3*b^2/c^3*a*(c*x^2+b*x+a)^(3/2)-3/32*e^3*b^3/c^3*(c*x^2+b*x+a)^(1/2)
*x*a+1/16*e^3*b/c^2*a*x*(c*x^2+b*x+a)^(3/2)-15/128*e^3*b^3/c^(7/2)*ln((1/2*
b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2-1/16*d*e^2*a/c^2*(c*x^2+b*x+a)^(3/2
)*b-9/32*d^2*e*b^2/c^2*(c*x^2+b*x+a)^(1/2)*a-9/16*d^2*e*b/c^(3/2)*ln((1/2*b
+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+9/32*d^2*e*b^3/c^(5/2)*ln((1/2*b+c*x
)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-3/16*d*e^2*a^2/c*(c*x^2+b*x+a)^(1/2)*x-3/8
*d^2*e*b/c*x*(c*x^2+b*x+a)^(3/2)+9/64*d^2*e*b^3/c^2*(c*x^2+b*x+a)^(1/2)*x-7
/20*d*e^2*b/c^2*(c*x^2+b*x+a)^(5/2)+1/2*d*e^2*x*(c*x^2+b*x+a)^(5/2)/c+3/64*
e^3*b^2/c^3*a^2*(c*x^2+b*x+a)^(1/2)-3/28*e^3*b/c^2*x*(c*x^2+b*x+a)^(5/2)-3/
64*e^3*b^3/c^3*x*(c*x^2+b*x+a)^(3/2)+9/512*e^3*b^5/c^4*(c*x^2+b*x+a)^(1/2)*
x-3/64*e^3*b^4/c^4*(c*x^2+b*x+a)^(1/2)*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.96766, size = 3075, normalized size = 9.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/143360*(105*(16*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d^3 - 24*(b^5*c^2 -
 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^2*e + 2*(7*b^6*c - 60*a*b^4*c^2 + 144*a^2*b^
2*c^3 - 64*a^3*c^4)*d*e^2 - (3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b
*c^3)*e^3)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)
*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(5120*c^7*e^3*x^6 + 1280*(14*c^7*d*e^2 +
5*b*c^6*e^3)*x^5 + 128*(168*c^7*d^2*e + 182*b*c^6*d*e^2 + (b^2*c^5 + 64*a*c
^6)*e^3)*x^4 - 560*(3*b^3*c^4 - 20*a*b*c^5)*d^3 + 168*(15*b^4*c^3 - 100*a*b
^2*c^4 + 128*a^2*c^5)*d^2*e - 14*(105*b^5*c^2 - 760*a*b^3*c^3 + 1296*a^2*b*
c^4)*d*e^2 + (315*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4)
*e^3 + 16*(560*c^7*d^3 + 1848*b*c^6*d^2*e + 14*(3*b^2*c^5 + 140*a*c^6)*d*e^
2 - (9*b^3*c^4 - 44*a*b*c^5)*e^3)*x^3 + 8*(1680*b*c^6*d^3 + 168*(b^2*c^5 +
32*a*c^6)*d^2*e - 14*(7*b^3*c^4 - 36*a*b*c^5)*d*e^2 + (21*b^4*c^3 - 124*a*b
^2*c^4 + 128*a^2*c^5)*e^3)*x^2 + 2*(560*(b^2*c^5 + 20*a*c^6)*d^3 - 168*(5*b
^3*c^4 - 28*a*b*c^5)*d^2*e + 14*(35*b^4*c^3 - 216*a*b^2*c^4 + 240*a^2*c^5)*
d*e^2 - (105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)*e^3)*x)*sqrt(c*x^2 +
 b*x + a))/c^6, -1/71680*(105*(16*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d^3
- 24*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^2*e + 2*(7*b^6*c - 60*a*b^4*c
^2 + 144*a^2*b^2*c^3 - 64*a^3*c^4)*d*e^2 - (3*b^7 - 28*a*b^5*c + 80*a^2*b^3
*c^2 - 64*a^3*b*c^3)*e^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x
+ b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(5120*c^7*e^3*x^6 + 1280*(14*c^7
*d*e^2 + 5*b*c^6*e^3)*x^5 + 128*(168*c^7*d^2*e + 182*b*c^6*d*e^2 + (b^2*c^5
 + 64*a*c^6)*e^3)*x^4 - 560*(3*b^3*c^4 - 20*a*b*c^5)*d^3 + 168*(15*b^4*c^3
- 100*a*b^2*c^4 + 128*a^2*c^5)*d^2*e - 14*(105*b^5*c^2 - 760*a*b^3*c^3 + 12
96*a^2*b*c^4)*d*e^2 + (315*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048
*a^3*c^4)*e^3 + 16*(560*c^7*d^3 + 1848*b*c^6*d^2*e + 14*(3*b^2*c^5 + 140*a*
c^6)*d*e^2 - (9*b^3*c^4 - 44*a*b*c^5)*e^3)*x^3 + 8*(1680*b*c^6*d^3 + 168*(b
^2*c^5 + 32*a*c^6)*d^2*e - 14*(7*b^3*c^4 - 36*a*b*c^5)*d*e^2 + (21*b^4*c^3
- 124*a*b^2*c^4 + 128*a^2*c^5)*e^3)*x^2 + 2*(560*(b^2*c^5 + 20*a*c^6)*d^3 -
 168*(5*b^3*c^4 - 28*a*b*c^5)*d^2*e + 14*(35*b^4*c^3 - 216*a*b^2*c^4 + 240*
a^2*c^5)*d*e^2 - (105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)*e^3)*x)*sqr
t(c*x^2 + b*x + a))/c^6]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d+ex)^3 \left(a+bx+cx^2\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x)**3*(a + b*x + c*x**2)**(3/2), x)
```

Giac [B] time = 1.15233, size = 965, normalized size = 3.01

$$\frac{1}{35840}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\left(4\,cxe^3+\frac{14\,c^7de^2+5\,bc^6e^3}{c^6}\right)x+\frac{168\,c^7d^2e+182\,bc^6de^2+b^2c^5e^3+64\,ac^6e^3}{c^6}\right)x+\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/35840*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(4*c*x*e^3 + (14*c^7*d*e^2 +
5*b*c^6*e^3)/c^6)*x + (168*c^7*d^2*e + 182*b*c^6*d*e^2 + b^2*c^5*e^3 + 64*a
*c^6*e^3)/c^6)*x + (560*c^7*d^3 + 1848*b*c^6*d^2*e + 42*b^2*c^5*d*e^2 + 196
0*a*c^6*d*e^2 - 9*b^3*c^4*e^3 + 44*a*b*c^5*e^3)/c^6)*x + (1680*b*c^6*d^3 +
168*b^2*c^5*d^2*e + 5376*a*c^6*d^2*e - 98*b^3*c^4*d*e^2 + 504*a*b*c^5*d*e^2
 + 21*b^4*c^3*e^3 - 124*a*b^2*c^4*e^3 + 128*a^2*c^5*e^3)/c^6)*x + (560*b^2*
c^5*d^3 + 11200*a*c^6*d^3 - 840*b^3*c^4*d^2*e + 4704*a*b*c^5*d^2*e + 490*b^
4*c^3*d*e^2 - 3024*a*b^2*c^4*d*e^2 + 3360*a^2*c^5*d*e^2 - 105*b^5*c^2*e^3 +
 728*a*b^3*c^3*e^3 - 1168*a^2*b*c^4*e^3)/c^6)*x - (1680*b^3*c^4*d^3 - 11200
*a*b*c^5*d^3 - 2520*b^4*c^3*d^2*e + 16800*a*b^2*c^4*d^2*e - 21504*a^2*c^5*d
^2*e + 1470*b^5*c^2*d*e^2 - 10640*a*b^3*c^3*d*e^2 + 18144*a^2*b*c^4*d*e^2 -
 315*b^6*c*e^3 + 2520*a*b^4*c^2*e^3 - 5488*a^2*b^2*c^3*e^3 + 2048*a^3*c^4*e
^3)/c^6) - 3/2048*(16*b^4*c^3*d^3 - 128*a*b^2*c^4*d^3 + 256*a^2*c^5*d^3 - 2
4*b^5*c^2*d^2*e + 192*a*b^3*c^3*d^2*e - 384*a^2*b*c^4*d^2*e + 14*b^6*c*d*e^
2 - 120*a*b^4*c^2*d*e^2 + 288*a^2*b^2*c^3*d*e^2 - 128*a^3*c^4*d*e^2 - 3*b^7
*e^3 + 28*a*b^5*c*e^3 - 80*a^2*b^3*c^2*e^3 + 64*a^3*b*c^3*e^3)*log(abs(-2*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(11/2)
```

3.2344 $\int (d+ex)^2 \left(a+bx+cx^2\right)^{3/2} \, dx$

Optimal. Leaf size=257

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(-4ce(ae+6bd)+7b^2e^2+24c^2d^2\right)}{192c^3}-\frac{\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}\left(-4ce(ae+6bd)+7b^2\right.}{512c^4}$$

```
[Out] -((b^2 - 4*a*c)*(24*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(6*b*d + a*e))*(b + 2*c*x)*
Sqrt[a + b*x + c*x^2])/(512*c^4) + ((24*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(6*b*d
+ a*e))*(b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(192*c^3) + (7*e*(2*c*d - b*e)
*(a + b*x + c*x^2)^(5/2))/(60*c^2) + (e*(d + e*x)*(a + b*x + c*x^2)^(5/2))/
(6*c) + ((b^2 - 4*a*c)^2*(24*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(6*b*d + a*e))*Arc
Tanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*c^(9/2))
```

Rubi [A] time = 0.332935, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {742, 640, 612, 621, 206}

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(-4ce(ae+6bd)+7b^2e^2+24c^2d^2\right)}{192c^3}-\frac{\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}\left(-4ce(ae+6bd)+7b^2\right.}{512c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] -((b^2 - 4*a*c)*(24*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(6*b*d + a*e))*(b + 2*c*x)*
Sqrt[a + b*x + c*x^2])/(512*c^4) + ((24*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(6*b*d
+ a*e))*(b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(192*c^3) + (7*e*(2*c*d - b*e)
*(a + b*x + c*x^2)^(5/2))/(60*c^2) + (e*(d + e*x)*(a + b*x + c*x^2)^(5/2))/
(6*c) + ((b^2 - 4*a*c)^2*(24*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(6*b*d + a*e))*Arc
Tanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*c^(9/2))
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
```

```
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^2 \left(a+bx+cx^2\right)^{3/2} dx = \frac{e(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6c} + \frac{\int \left(\frac{1}{2}\left(12cd^2-2e\left(\frac{5bd}{2}+ae\right)\right)+\frac{7}{2}e(2cd-be)x\right)\left(a+bx \cdots\right.}{6c}$$

$$= \frac{7e(2cd-be)\left(a+bx+cx^2\right)^{5/2}}{60c^2} + \frac{e(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6c} + \frac{\left(-\frac{7}{2}be(2cd-be)+c \cdots\right.}{\cdots}$$

$$= \frac{\left(24c^2d^2+7b^2e^2-4ce(6bd+ae)\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{192c^3} + \frac{7e(2cd-be)\left(a+bx \cdots\right.}{60c^2}$$

$$= -\frac{\left(b^2-4ac\right)\left(24c^2d^2+7b^2e^2-4ce(6bd+ae)\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4} + \frac{\left(24c^2d^2+ \cdots\right.}{\cdots}$$

$$= -\frac{\left(b^2-4ac\right)\left(24c^2d^2+7b^2e^2-4ce(6bd+ae)\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4} + \frac{\left(24c^2d^2+ \cdots\right.}{\cdots}$$

$$= -\frac{\left(b^2-4ac\right)\left(24c^2d^2+7b^2e^2-4ce(6bd+ae)\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4} + \frac{\left(24c^2d^2+ \cdots\right.}{\cdots}$$

Mathematica [A] time = 0.338076, size = 188, normalized size = 0.73

$$\frac{\frac{\left(-4ce(ae+6bd)+7b^2e^2+24c^2d^2\right)\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}\left(4c\left(5a+2cx^2\right)-3b^2+8bcx\right)+3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{512c^{7/2}}+\frac{7e(a+x(b+cx))^{5/2}(2cd-be)}{10c}+e\cdots}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] ((7*e*(2*c*d - b*e)*(a + x*(b + c*x))^(5/2))/(10*c) + e*(d + e*x)*(a + x*(b
 + c*x))^(5/2) + ((24*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(6*b*d + a*e))*(2*Sqrt[c]
*(b + 2*c*x)*Sqrt[a + x*(b + c*x)]*(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2))
 + 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]
))/(512*c^(7/2)))/(6*c)
```

Maple [B] time = 0.05, size = 922, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 3/16*d*e*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-3/8*d*e*
b/c*(c*x^2+b*x+a)^(1/2)*x*a+1/4*d^2*x*(c*x^2+b*x+a)^(3/2)-3/32*d^2/c*(c*x^2
+b*x+a)^(1/2)*x*b^2+3/16*d^2/c*(c*x^2+b*x+a)^(1/2)*b*a+9/64*e^2*b^2/c^(5/2)
*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2-15/256*e^2*b^4/c^(7/2)*ln(
(1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-7/256*e^2*b^4/c^3*(c*x^2+b*x+a)^
(1/2)*x+1/16*e^2*b^3/c^3*(c*x^2+b*x+a)^(1/2)*a+7/96*e^2*b^2/c^2*x*(c*x^2+b*
x+a)^(3/2)-1/48*e^2*a/c^2*(c*x^2+b*x+a)^(3/2)*b-1/16*e^2*a^2/c*(c*x^2+b*x+a
)^(1/2)*x-1/32*e^2*a^2/c^2*(c*x^2+b*x+a)^(1/2)*b-3/16*d^2/c^(3/2)*ln((1/2*b
+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2*a-1/8*d*e*b^2/c^2*(c*x^2+b*x+a)^(3/2
)+3/64*d*e*b^4/c^3*(c*x^2+b*x+a)^(1/2)-3/128*d*e*b^5/c^(7/2)*ln((1/2*b+c*x)
/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/24*e^2*a/c*x*(c*x^2+b*x+a)^(3/2)+3/32*d*e*b
^3/c^2*(c*x^2+b*x+a)^(1/2)*x+3/128*d^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^
2+b*x+a)^(1/2))*b^4-7/512*e^2*b^5/c^4*(c*x^2+b*x+a)^(1/2)+1/8*d^2/c*(c*x^2+
b*x+a)^(3/2)*b+3/8*d^2*(c*x^2+b*x+a)^(1/2)*x*a-3/64*d^2/c^2*(c*x^2+b*x+a)^(
1/2)*b^3+3/8*d^2/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2-3/
16*d*e*b^2/c^2*(c*x^2+b*x+a)^(1/2)*a-3/8*d*e*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/
2)+(c*x^2+b*x+a)^(1/2))*a^2+1/8*e^2*b^2/c^2*(c*x^2+b*x+a)^(1/2)*x*a-1/4*d*e
*b/c*x*(c*x^2+b*x+a)^(3/2)+2/5*d*e*(c*x^2+b*x+a)^(5/2)/c+7/1024*e^2*b^6/c^(
9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/16*e^2*a^3/c^(3/2)*ln((1
/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/6*e^2*x*(c*x^2+b*x+a)^(5/2)/c-7/60
*e^2*b/c^2*(c*x^2+b*x+a)^(5/2)+7/192*e^2*b^3/c^3*(c*x^2+b*x+a)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.05072, size = 2044, normalized size = 7.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(24*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2 - 24*(b^5*c - 8*
a*b^3*c^2 + 16*a^2*b*c^3)*d*e + (7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*
a^3*c^3)*e^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x +
 a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(1280*c^6*e^2*x^5 + 128*(24*c^6*d*e +
13*b*c^5*e^2)*x^4 + 16*(120*c^6*d^2 + 264*b*c^5*d*e + (3*b^2*c^4 + 140*a*c^
5)*e^2)*x^3 - 120*(3*b^3*c^3 - 20*a*b*c^4)*d^2 + 24*(15*b^4*c^2 - 100*a*b^2
*c^3 + 128*a^2*c^4)*d*e - (105*b^5*c - 760*a*b^3*c^2 + 1296*a^2*b*c^3)*e^2
+ 8*(360*b*c^5*d^2 + 24*(b^2*c^4 + 32*a*c^5)*d*e - (7*b^3*c^3 - 36*a*b*c^4)
*e^2)*x^2 + 2*(120*(b^2*c^4 + 20*a*c^5)*d^2 - 24*(5*b^3*c^3 - 28*a*b*c^4)*d
*e + (35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*e^2)*x)*sqrt(c*x^2 + b*x +
```

```
a))/c^5, -1/15360*(15*(24*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2 - 24*(b^
5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e + (7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c
^2 - 64*a^3*c^3)*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)
*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(1280*c^6*e^2*x^5 + 128*(24*c^6*d*e
+ 13*b*c^5*e^2)*x^4 + 16*(120*c^6*d^2 + 264*b*c^5*d*e + (3*b^2*c^4 + 140*a*
c^5)*e^2)*x^3 - 120*(3*b^3*c^3 - 20*a*b*c^4)*d^2 + 24*(15*b^4*c^2 - 100*a*b
^2*c^3 + 128*a^2*c^4)*d*e - (105*b^5*c - 760*a*b^3*c^2 + 1296*a^2*b*c^3)*e^
2 + 8*(360*b*c^5*d^2 + 24*(b^2*c^4 + 32*a*c^5)*d*e - (7*b^3*c^3 - 36*a*b*c^
4)*e^2)*x^2 + 2*(120*(b^2*c^4 + 20*a*c^5)*d^2 - 24*(5*b^3*c^3 - 28*a*b*c^4)
*d*e + (35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*e^2)*x)*sqrt(c*x^2 + b*x
+ a))/c^5]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d+ex)^2 \left(a+bx+cx^2\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x)**2*(a + b*x + c*x**2)**(3/2), x)
```

Giac [B] time = 1.13443, size = 625, normalized size = 2.43

$$\frac{1}{7680}\,\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\,cxe^2+\frac{24\,c^6de+13\,bc^5e^2}{c^5}\right)x+\frac{120\,c^6d^2+264\,bc^5de+3\,b^2c^4e^2+140\,ac^5e^2}{c^5}\right)x+\frac{36}{}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*c*x*e^2 + (24*c^6*d*e + 13*b*c
^5*e^2)/c^5)*x + (120*c^6*d^2 + 264*b*c^5*d*e + 3*b^2*c^4*e^2 + 140*a*c^5*e
^2)/c^5)*x + (360*b*c^5*d^2 + 24*b^2*c^4*d*e + 768*a*c^5*d*e - 7*b^3*c^3*e^
2 + 36*a*b*c^4*e^2)/c^5)*x + (120*b^2*c^4*d^2 + 2400*a*c^5*d^2 - 120*b^3*c^
3*d*e + 672*a*b*c^4*d*e + 35*b^4*c^2*e^2 - 216*a*b^2*c^3*e^2 + 240*a^2*c^4*
e^2)/c^5)*x - (360*b^3*c^3*d^2 - 2400*a*b*c^4*d^2 - 360*b^4*c^2*d*e + 2400*
a*b^2*c^3*d*e - 3072*a^2*c^4*d*e + 105*b^5*c*e^2 - 760*a*b^3*c^2*e^2 + 1296
*a^2*b*c^3*e^2)/c^5) - 1/1024*(24*b^4*c^2*d^2 - 192*a*b^2*c^3*d^2 + 384*a^2
*c^4*d^2 - 24*b^5*c*d*e + 192*a*b^3*c^2*d*e - 384*a^2*b*c^3*d*e + 7*b^6*e^2
 - 60*a*b^4*c*e^2 + 144*a^2*b^2*c^2*e^2 - 64*a^3*c^3*e^2)*log(abs(-2*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2)
```

3.2345 $\int (d+ex)\left(a+bx+cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=161

$$-\frac{3\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}(2cd-be)}{128c^3}+\frac{3\left(b^2-4ac\right)^2(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}}+\frac{(b+2cx)\left(a+bx+c\right.}{16c^2}$$

```
[Out] (-3*(b^2 - 4*a*c)*(2*c*d - b*e)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(128*c^3
) + ((2*c*d - b*e)*(b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(16*c^2) + (e*(a +
b*x + c*x^2)^(5/2))/(5*c) + (3*(b^2 - 4*a*c)^2*(2*c*d - b*e)*ArcTanh[(b + 2
*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(7/2))
```

Rubi [A] time = 0.0623502, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {640, 612, 621, 206}

$$-\frac{3\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}(2cd-be)}{128c^3}+\frac{3\left(b^2-4ac\right)^2(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}}+\frac{(b+2cx)\left(a+bx+c\right.}{16c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-3*(b^2 - 4*a*c)*(2*c*d - b*e)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(128*c^3
) + ((2*c*d - b*e)*(b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(16*c^2) + (e*(a +
b*x + c*x^2)^(5/2))/(5*c) + (3*(b^2 - 4*a*c)^2*(2*c*d - b*e)*ArcTanh[(b + 2
*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(7/2))
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)\left(a+bx+cx^2\right)^{3/2}\,dx = \frac{e\left(a+bx+cx^2\right)^{5/2}}{5c} + \frac{(2cd-be)\int \left(a+bx+cx^2\right)^{3/2}\,dx}{2c}$$

$$= \frac{(2cd-be)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{16c^2} + \frac{e\left(a+bx+cx^2\right)^{5/2}}{5c} - \frac{\left(3\left(b^2-4ac\right)(2cd-be\right.}{32\ldots}$$

$$= -\frac{3\left(b^2-4ac\right)(2cd-be)(b+2cx)\sqrt{a+bx+cx^2}}{128c^3} + \frac{(2cd-be)(b+2cx)\left(a+bx+cx^2\ldots\right.}{16c^2}$$

$$= -\frac{3\left(b^2-4ac\right)(2cd-be)(b+2cx)\sqrt{a+bx+cx^2}}{128c^3} + \frac{(2cd-be)(b+2cx)\left(a+bx+cx^2\ldots\right.}{16c^2}$$

$$= -\frac{3\left(b^2-4ac\right)(2cd-be)(b+2cx)\sqrt{a+bx+cx^2}}{128c^3} + \frac{(2cd-be)(b+2cx)\left(a+bx+cx^2\ldots\right.}{16c^2}$$

Mathematica [A] time = 0.297728, size = 144, normalized size = 0.89

$$\frac{5(2cd-be)\left(\frac{3\left(b^2-4ac\right)\left(\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}\right)}{128c^{5/2}}+\frac{(b+2cx)(a+x(b+cx))^{3/2}}{8c}\right)+2e(a+x(b+cx))^{5/2}}{10c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*e*(a + x*(b + c*x))^(5/2) + 5*(2*c*d - b*e)*(((b + 2*c*x)*(a + x*(b + c*
x))^(3/2))/(8*c) + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b +
 c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]
)]))/(128*c^(5/2))))/(10*c)
```

Maple [B] time = 0.045, size = 469, normalized size = 2.9

$$\frac{e}{5\,c}\left(cx^2+bx+a\right)^{\frac{5}{2}}-\frac{bxe}{8\,c}\left(cx^2+bx+a\right)^{\frac{3}{2}}-\frac{b^2e}{16\,c^2}\left(cx^2+bx+a\right)^{\frac{3}{2}}-\frac{3\,bxea}{16\,c}\sqrt{cx^2+bx+a}+\frac{3\,eb^3x}{64\,c^2}\sqrt{cx^2+bx+a}-\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 1/5*e*(c*x^2+b*x+a)^(5/2)/c-1/8*e*b/c*x*(c*x^2+b*x+a)^(3/2)-1/16*e*b^2/c^2*
(c*x^2+b*x+a)^(3/2)-3/16*e*b/c*(c*x^2+b*x+a)^(1/2)*x*a+3/64*e*b^3/c^2*(c*x^
2+b*x+a)^(1/2)*x-3/32*e*b^2/c^2*(c*x^2+b*x+a)^(1/2)*a+3/128*e*b^4/c^3*(c*x^
2+b*x+a)^(1/2)-3/16*e*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
*a^2+3/32*e*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-3/256
*e*b^5/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/4*d*x*(c*x^2+b
*x+a)^(3/2)+1/8*d/c*(c*x^2+b*x+a)^(3/2)*b+3/8*d*(c*x^2+b*x+a)^(1/2)*x*a-3/3
2*d/c*(c*x^2+b*x+a)^(1/2)*x*b^2+3/16*d/c*(c*x^2+b*x+a)^(1/2)*b*a-3/64*d/c^2
*(c*x^2+b*x+a)^(1/2)*b^3+3/8*d/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)
^(1/2))*a^2-3/16*d/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2*
a+3/128*d/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.06053, size = 1215, normalized size = 7.55

$$\left[-\frac{15\left(2\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)d-\left(b^5-8\,ab^3c+16\,a^2bc^2\right)e\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2560*(15*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d - (b^5 - 8*a*b^3*c + 1
6*a^2*b*c^2)*e)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x
 + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(128*c^5*e*x^4 + 16*(10*c^5*d + 11*b
*c^4*e)*x^3 + 8*(30*b*c^4*d + (b^2*c^3 + 32*a*c^4)*e)*x^2 - 10*(3*b^3*c^2 -
 20*a*b*c^3)*d + (15*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3)*e + 2*(10*(b^2*c^
3 + 20*a*c^4)*d - (5*b^3*c^2 - 28*a*b*c^3)*e)*x)*sqrt(c*x^2 + b*x + a))/c^4
, -1/1280*(15*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d - (b^5 - 8*a*b^3*c +
16*a^2*b*c^2)*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt
(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(128*c^5*e*x^4 + 16*(10*c^5*d + 11*b*c^4*
e)*x^3 + 8*(30*b*c^4*d + (b^2*c^3 + 32*a*c^4)*e)*x^2 - 10*(3*b^3*c^2 - 20*a
*b*c^3)*d + (15*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3)*e + 2*(10*(b^2*c^3 + 2
0*a*c^4)*d - (5*b^3*c^2 - 28*a*b*c^3)*e)*x)*sqrt(c*x^2 + b*x + a))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d+ex)\left(a+bx+cx^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x)*(a + b*x + c*x**2)**(3/2), x)
```

Giac [A] time = 1.12276, size = 356, normalized size = 2.21

$$\frac{1}{640}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\,cxe+\frac{10\,c^5d+11\,bc^4e}{c^4}\right)x+\frac{30\,bc^4d+b^2c^3e+32\,ac^4e}{c^4}\right)x+\frac{10\,b^2c^3d+200\,ac^4d-5\,b^3c^2e+2}{c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/640*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*c*x*e + (10*c^5*d + 11*b*c^4*e)/c^4
)*x + (30*b*c^4*d + b^2*c^3*e + 32*a*c^4*e)/c^4)*x + (10*b^2*c^3*d + 200*a*
c^4*d - 5*b^3*c^2*e + 28*a*b*c^3*e)/c^4)*x - (30*b^3*c^2*d - 200*a*b*c^3*d
- 15*b^4*c*e + 100*a*b^2*c^2*e - 128*a^2*c^3*e)/c^4) - 3/256*(2*b^4*c*d - 1
6*a*b^2*c^2*d + 32*a^2*c^3*d - b^5*e + 8*a*b^3*c*e - 16*a^2*b*c^2*e)*log(ab
s(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

3.2346 $\int \left(a+bx+cx^2\right)^{3/2} dx$

Optimal. Leaf size=112

$$-\frac{3\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^2}+\frac{3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{5/2}}+\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}$$

```
[Out] (-3*(b^2 - 4*a*c)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(64*c^2) + ((b + 2*c*x
)*(a + b*x + c*x^2)^(3/2))/(8*c) + (3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(
2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(5/2))
```

Rubi [A] time = 0.0331895, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {612, 621, 206}

$$-\frac{3\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^2}+\frac{3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{5/2}}+\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-3*(b^2 - 4*a*c)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(64*c^2) + ((b + 2*c*x
)*(a + b*x + c*x^2)^(3/2))/(8*c) + (3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(
2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(5/2))
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^{3/2} dx = \frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c} - \frac{\left(3\left(b^2-4ac\right)\right)\int \sqrt{a+bx+cx^2}\,dx}{16c}$$
$$= -\frac{3\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^2} + \frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c} + \frac{\left(3\left(b^2-4ac\right)^2\right)\int \frac{}{\sqrt{a+}}}{128c^2}$$
$$= -\frac{3\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^2} + \frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c} + \frac{\left(3\left(b^2-4ac\right)^2\right)\text{Subs}}{}$$
$$= -\frac{3\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^2} + \frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c} + \frac{3\left(b^2-4ac\right)^2 \tanh^{-1}}{128c^{5/2}}$$

Mathematica [A] time = 0.355323, size = 114, normalized size = 1.02

$$\frac{\sqrt{a+x(b+cx)}\left(2(b+2cx)\left(4c\left(5a+2cx^2\right)-3b^2+8bcx\right)-\frac{3\left(b^2-4ac\right)^{3/2}\sin^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}}\right)}{128c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(2*(b + 2*c*x)*(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^
2)) - (3*(b^2 - 4*a*c)^(3/2)*ArcSin[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[(c
*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]))/(128*c^2)
```

Maple [B] time = 0.044, size = 201, normalized size = 1.8

$$\frac{2\,cx+b}{8\,c}\left(cx^2+bx+a\right)^{\frac{3}{2}} + \frac{3\,ax}{8}\sqrt{cx^2+bx+a} - \frac{3\,b^2x}{32\,c}\sqrt{cx^2+bx+a} + \frac{3\,ab}{16\,c}\sqrt{cx^2+bx+a} - \frac{3\,b^3}{64\,c^2}\sqrt{cx^2+bx+a} + \frac{3}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 1/8*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c+3/8*(c*x^2+b*x+a)^(1/2)*x*a-3/32/c*(c*x
^2+b*x+a)^(1/2)*x*b^2+3/16/c*(c*x^2+b*x+a)^(1/2)*b*a-3/64/c^2*(c*x^2+b*x+a)
^(1/2)*b^3+3/8/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2-3/16
/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*b^2+3/128/c^(5/2)*ln
((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.73169, size = 657, normalized size = 5.87

$$\left[\frac{3\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right)+4\left(16\,c^4x^3+24\,bc^3x^2-3\,b\right.}{256\,c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/256*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x -
 b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(16*c^4*x^3
 + 24*b*c^3*x^2 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x)*sqrt(c*x
^2 + b*x + a))/c^3, -1/128*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-c)*arcta
n(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) -
 2*(16*c^4*x^3 + 24*b*c^3*x^2 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^
3)*x)*sqrt(c*x^2 + b*x + a))/c^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+bx+cx^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2), x)
```

Giac [A] time = 1.13939, size = 166, normalized size = 1.48

$$\frac{1}{64}\sqrt{cx^2+bx+a}\left(2\left(4\,(2\,cx+3\,b)x+\frac{b^2c^2+20\,ac^3}{c^3}\right)x-\frac{3\,b^3c-20\,abc^2}{c^3}\right)-\frac{3\left(b^4-8\,ab^2c+16\,a^2c^2\right)\log\left(\left|-2\left(\sqrt{c}x-\sqrt{c}\right.\right.\right.}{128\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/64*sqrt(c*x^2 + b*x + a)*(2*(4*(2*c*x + 3*b)*x + (b^2*c^2 + 20*a*c^3)/c^3
)*x - (3*b^3*c - 20*a*b*c^2)/c^3) - 3/128*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*lo
g(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(5/2)
```

3.2347 $\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=252

$$\frac{\sqrt{a+bx+cx^2}\left(-2ce(5bd-4ae)+b^2e^2-2cex(2cd-be)+8c^2d^2\right)}{8ce^3}-\frac{(2cd-be)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)\tanh}{16c^{3/2}e^4}$$

```
[Out] ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*Sqrt
[a + b*x + c*x^2])/(8*c*e^3) + (a + b*x + c*x^2)^(3/2)/(3*e) - ((2*c*d - b*
e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*e^4) + ((c*d^2 - b*d*e + a*e^2)^(
3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]
*Sqrt[a + b*x + c*x^2])])/e^4
```

Rubi [A] time = 0.34288, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {734, 814, 843, 621, 206, 724}

$$\frac{\sqrt{a+bx+cx^2}\left(-2ce(5bd-4ae)+b^2e^2-2cex(2cd-be)+8c^2d^2\right)}{8ce^3}-\frac{(2cd-be)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)\tanh}{16c^{3/2}e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*Sqrt
[a + b*x + c*x^2])/(8*c*e^3) + (a + b*x + c*x^2)^(3/2)/(3*e) - ((2*c*d - b*
e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*e^4) + ((c*d^2 - b*d*e + a*e^2)^(
3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]
*Sqrt[a + b*x + c*x^2])])/e^4
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
```

```
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx = \frac{\left(a+bx+cx^2\right)^{3/2}}{3e} - \frac{\int \frac{(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{d+ex}\,dx}{2e}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce^3} + \frac{\left(a+bx+cx^2\right)^{3/2}}{3e} + \frac{\int \frac{1}{2}\left(4ce(\ldots\right.}{\ldots}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce^3} + \frac{\left(a+bx+cx^2\right)^{3/2}}{3e} + \frac{\left(cd^2-\ldots\right.}{\ldots}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce^3} + \frac{\left(a+bx+cx^2\right)^{3/2}}{3e} - \frac{\left(2\left(cd^2\ldots\right.\right.}{\ldots}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce^3} + \frac{\left(a+bx+cx^2\right)^{3/2}}{3e} - \frac{(2cd-\ldots}{\ldots}$$

Mathematica [A] time = 0.453537, size = 236, normalized size = 0.94

$$\frac{2\sqrt{c}\left(e\sqrt{a+x(b+cx)}\left(2ce(16ae-15bd+7bex)+3b^2e^2+4c^2\left(6d^2-3dex+2e^2x^2\right)\right)-24c\left(e(ae-bd)+cd^2\right)^{3/2}\tanh^{-1}\left(\ldots\right.\right.}{48c^{3/2}e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] (-3*(2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 + 4*c*e*(-2*b*d + 3*a*e))*ArcTanh[(b
 + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*(e*Sqrt[a + x*(b +
 c*x)]*(3*b^2*e^2 + 2*c*e*(-15*b*d + 16*a*e + 7*b*e*x) + 4*c^2*(6*d^2 - 3*d
*e*x + 2*e^2*x^2)) - 24*c*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*ArcTanh[(-(b*d)
+ 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b
+ c*x)])]))/(48*c^(3/2)*e^4)
```

Maple [B] time = 0.225, size = 1946, normalized size = 7.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d),x)
```

```
[Out] 1/3/e*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)+1/4
/e*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b-1/
2/e^2*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c
*d+1/8/e/c*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*b^2-5/4/e^2*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*b*d+3/4/e/c^(1/2)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*
c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*a*b-3/2/e^2*ln((1/2
*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2
-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d*a-1/16/e/c^(3/2)*ln((1/2*(b*e-2*c*d)/e+
(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e
^2)^(1/2))*b^3-3/8/e^2*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*
c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b^2*d+1/e*(
(d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*a+1/e^3*((
d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*c*d^2+3/2/e
^3*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e
+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d^2*b-1/e^4*ln((1/2*(b*e-2*c*d)
/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2))*c^(3/2)*d^3-1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-
b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((
d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a
^2+2/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e
-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d
)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a*b*d-2/e^3/((a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2
*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-
b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a*c*d^2-1/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*
d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)
^(1/2))/(d/e+x))*b^2*d^2+2/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2
-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(
(d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*
b*d^3*c-1/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2
+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-
2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*c^2*d^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx + cx^2\right)^{\frac{3}{2}}}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)/(d + e*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.2348 $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=227

$$\frac{3\left(-4ce(2bd-ae)+b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}e^4}-\frac{3(2cd-be)\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{2e^4}$$

```
[Out] (-3*(4*c*d - 3*b*e - 2*c*e*x)*Sqrt[a + b*x + c*x^2])/(4*e^3) - (a + b*x + c
*x^2)^(3/2)/(e*(d + e*x)) + (3*(8*c^2*d^2 + b^2*e^2 - 4*c*e*(2*b*d - a*e))*
ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*e^4) - (
3*(2*c*d - b*e)*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d -
 b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(2*e^4)
```

Rubi [A] time = 0.274755, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {732, 814, 843, 621, 206, 724}

$$\frac{3\left(-4ce(2bd-ae)+b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}e^4}-\frac{3(2cd-be)\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{2e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x)^2,x]
```

```
[Out] (-3*(4*c*d - 3*b*e - 2*c*e*x)*Sqrt[a + b*x + c*x^2])/(4*e^3) - (a + b*x + c
*x^2)^(3/2)/(e*(d + e*x)) + (3*(8*c^2*d^2 + b^2*e^2 - 4*c*e*(2*b*d - a*e))*
ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*e^4) - (
3*(2*c*d - b*e)*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d -
 b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(2*e^4)
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
```

```
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(d+ex)^2}\,dx = -\frac{\left(a+bx+cx^2\right)^{3/2}}{e(d+ex)} + \frac{3\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{d+ex}\,dx}{2e}$$

$$= -\frac{3(4cd-3be-2cex)\sqrt{a+bx+cx^2}}{4e^3} - \frac{\left(a+bx+cx^2\right)^{3/2}}{e(d+ex)} - \frac{3\int \frac{c\left(3b^2de+4acde-4b\left(cd^2+ae^2\right)\right)-c\left(8c^2d^2+b^2\right.}{(d+ex)\sqrt{a+bx+cx^2}}}{8ce^3}$$

$$= -\frac{3(4cd-3be-2cex)\sqrt{a+bx+cx^2}}{4e^3} - \frac{\left(a+bx+cx^2\right)^{3/2}}{e(d+ex)} - \frac{\left(3(2cd-be)\left(cd^2-bde+ae^2\right)\right)\int \frac{}{(d+}}{2e^4}$$

$$= -\frac{3(4cd-3be-2cex)\sqrt{a+bx+cx^2}}{4e^3} - \frac{\left(a+bx+cx^2\right)^{3/2}}{e(d+ex)} + \frac{\left(3(2cd-be)\left(cd^2-bde+ae^2\right)\right)\mathrm{Subs}}{}$$

$$= -\frac{3(4cd-3be-2cex)\sqrt{a+bx+cx^2}}{4e^3} - \frac{\left(a+bx+cx^2\right)^{3/2}}{e(d+ex)} + \frac{3\left(8c^2d^2+b^2e^2-4ce(2bd-ae)\right)\tan}{8\sqrt{c}e^4}$$

Mathematica [A] time = 0.440843, size = 214, normalized size = 0.94

$$\frac{\frac{3\left(4ce(ae-2bd)+b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}} + 12(2cd-be)\sqrt{e(ae-bd)+cd^2}\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right) - \frac{8e^3(a+x(b+cx))^{3/}}{d+ex}}{8e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x)^2,x]
```

```
[Out] (6*e*(-4*c*d + 3*b*e + 2*c*e*x)*Sqrt[a + x*(b + c*x)] - (8*e^3*(a + x*(b +
c*x))^(3/2))/(d + e*x) + (3*(8*c^2*d^2 + b^2*e^2 + 4*c*e*(-2*b*d + a*e))*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + 12*(2*c*d -
 b*e)*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*
e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/(8*e^4)
```

Maple [B] time = 0.231, size = 3450, normalized size = 15.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d)^2,x)
```

```
[Out] -9/e^2/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d
*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e
+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a*b*
d^2*c-9/2/e/(a*e^2-b*d*e+c*d^2)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((
d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d*
a*b-3/2/e/(a*e^2-b*d*e+c*d^2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*x*b*c*d+6/e^3/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)
/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b
*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^
2)/e^2)^(1/2))/(d/e+x))*a*c^2*d^3+6/e^3/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c
*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e
+c*d^2)/e^2)^(1/2))/(d/e+x))*b^2*d^3*c+3/e/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*
e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*
d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a^2*c*d+3/e/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*
((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a*b^2*d-15/2/e^4/(a*e^2-b*d*e+c*d^2)/((a*e
^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e
+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b*d^4*c^2+9/4/(a*e^2-b*d*e+c*d^2)*((
d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*a*b+c/(a*e^
2-b*d*e+c*d^2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^
(3/2)*x+3/2*c^(1/2)/(a*e^2-b*d*e+c*d^2)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^
(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*a^
2-1/(a*e^2-b*d*e+c*d^2)/(d/e+x)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b
*d*e+c*d^2)/e^2)^(5/2)+1/(a*e^2-b*d*e+c*d^2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/
e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)*b-3/2/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d
*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a^2*b+3/8/(a*e^2-b*d*e+c*d^2)/c^(1/2)*ln((1/
2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^
2-b*d*e+c*d^2)/e^2)^(1/2))*a*b^2-9/4/e/(a*e^2-b*d*e+c*d^2)*((d/e+x)^2*c+(b*
e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*d-3/e^3/(a*e^2-b*d*e+
c*d^2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*c^
2*d^3-1/e/(a*e^2-b*d*e+c*d^2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d
*e+c*d^2)/e^2)^(3/2)*c*d+3/e^4/(a*e^2-b*d*e+c*d^2)*ln((1/2*(b*e-2*c*d)/e+(d
/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2
)^(1/2))*c^(5/2)*d^4+3/2*c/(a*e^2-b*d*e+c*d^2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(
d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*a+9/2/e^2/(a*e^2-b*d*e+c*d^2)*ln((1
/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e
^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(3/2)*d^2*a+3/2/e^2/(a*e^2-b*d*e+c*d^2)*((d/e
```

```
+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d^2+21/4
/e^2/(a*e^2-b*d*e+c*d^2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*
d^2)/e^2)^(1/2)*b*d^2*c-6/e^3/(a*e^2-b*d*e+c*d^2)*ln((1/2*(b*e-2*c*d)/e+(d/
e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)
^(1/2))*c^(3/2)*d^3*b-3/2/e^2/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+
c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2))/(d/e+x))*b^3*d^2+3/e^5/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)
/e^2)^(1/2))/(d/e+x))*c^3*d^5-3/8/e/(a*e^2-b*d*e+c*d^2)*ln((1/2*(b*e-2*c*d)
/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2))/c^(1/2)*b^3*d+27/8/e^2/(a*e^2-b*d*e+c*d^2)*ln((1/2*(b*e-2*c*d
)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^
2)/e^2)^(1/2))*c^(1/2)*d^2*b^2-3/e/(a*e^2-b*d*e+c*d^2)*((d/e+x)^2*c+(b*e-2*
c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*a*c*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 142.45, size = 3494, normalized size = 15.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(8*c^2*d^3 - 8*b*c*d^2*e + (b^2 + 4*a*c)*d*e^2 + (8*c^2*d^2*e - 8*
b*c*d*e^2 + (b^2 + 4*a*c)*e^3)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 -
4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 12*(2*c^2*d^2 - b*c*
d*e + (2*c^2*d*e - b*c*e^2)*x)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e -
 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2
)*x^2 - 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e +
(2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*
x^2 + 2*d*e*x + d^2)) + 4*(2*c^2*e^3*x^2 - 12*c^2*d^2*e + 9*b*c*d*e^2 - 4*a
*c*e^3 - (6*c^2*d*e^2 - 5*b*c*e^3)*x)*sqrt(c*x^2 + b*x + a))/(c*e^5*x + c*d
*e^4), -1/16*(24*(2*c^2*d^2 - b*c*d*e + (2*c^2*d*e - b*c*e^2)*x)*sqrt(-c*d^
2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*
x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*
d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)) - 3*(8*c^2
*d^3 - 8*b*c*d^2*e + (b^2 + 4*a*c)*d*e^2 + (8*c^2*d^2*e - 8*b*c*d*e^2 + (b^
2 + 4*a*c)*e^3)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 +
b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(2*c^2*e^3*x^2 - 12*c^2*d^2*e + 9
*b*c*d*e^2 - 4*a*c*e^3 - (6*c^2*d*e^2 - 5*b*c*e^3)*x)*sqrt(c*x^2 + b*x + a)
)/(c*e^5*x + c*d*e^4), -1/8*(3*(8*c^2*d^3 - 8*b*c*d^2*e + (b^2 + 4*a*c)*d*e
^2 + (8*c^2*d^2*e - 8*b*c*d*e^2 + (b^2 + 4*a*c)*e^3)*x)*sqrt(-c)*arctan(1/2
*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 6*(2
```

```
*c^2*d^2 - b*c*d*e + (2*c^2*d*e - b*c*e^2)*x)*sqrt(c*d^2 - b*d*e + a*e^2)*l
og((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b
^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*
(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c
)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(2*c^2*e^3*x^2 - 12*c^2*d^2*e + 9*
b*c*d*e^2 - 4*a*c*e^3 - (6*c^2*d*e^2 - 5*b*c*e^3)*x)*sqrt(c*x^2 + b*x + a))
/(c*e^5*x + c*d*e^4), -1/8*(12*(2*c^2*d^2 - b*c*d*e + (2*c^2*d*e - b*c*e^2)
*x)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*s
qrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a
^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*
x)) + 3*(8*c^2*d^3 - 8*b*c*d^2*e + (b^2 + 4*a*c)*d*e^2 + (8*c^2*d^2*e - 8*b
*c*d*e^2 + (b^2 + 4*a*c)*e^3)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*
(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(2*c^2*e^3*x^2 - 12*c^2*d
^2*e + 9*b*c*d*e^2 - 4*a*c*e^3 - (6*c^2*d*e^2 - 5*b*c*e^3)*x)*sqrt(c*x^2 +
b*x + a))/(c*e^5*x + c*d*e^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.2349 $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)^3}\,dx$

Optimal. Leaf size=236

$$\frac{3\left(-4ce(2bd-ae)+b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{8e^4\sqrt{ae^2-bde+cd^2}}+\frac{3\sqrt{a+bx+cx^2}(-be+4cd+2cex)}{4e^3(d+ex)}-\frac{3\sqrt{c}(2cd-be)}{}$$

```
[Out] (3*(4*c*d - b*e + 2*c*e*x)*Sqrt[a + b*x + c*x^2])/(4*e^3*(d + e*x)) - (a +
b*x + c*x^2)^(3/2)/(2*e*(d + e*x)^2) - (3*Sqrt[c]*(2*c*d - b*e)*ArcTanh[(b
+ 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*e^4) + (3*(8*c^2*d^2 + b^2*
e^2 - 4*c*e*(2*b*d - a*e))*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[
c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(8*e^4*Sqrt[c*d^2 - b*d*e +
 a*e^2])
```

Rubi [A] time = 0.252069, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {732, 812, 843, 621, 206, 724}

$$\frac{3\left(-4ce(2bd-ae)+b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{8e^4\sqrt{ae^2-bde+cd^2}}+\frac{3\sqrt{a+bx+cx^2}(-be+4cd+2cex)}{4e^3(d+ex)}-\frac{3\sqrt{c}(2cd-be)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x)^3,x]
```



```
[Out] (3*(4*c*d - b*e + 2*c*e*x)*Sqrt[a + b*x + c*x^2])/(4*e^3*(d + e*x)) - (a +
b*x + c*x^2)^(3/2)/(2*e*(d + e*x)^2) - (3*Sqrt[c]*(2*c*d - b*e)*ArcTanh[(b
+ 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*e^4) + (3*(8*c^2*d^2 + b^2*
e^2 - 4*c*e*(2*b*d - a*e))*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[
c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(8*e^4*Sqrt[c*d^2 - b*d*e +
 a*e^2])
```


Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
```

```
+ 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(d+ex)^3}\,dx = -\frac{\left(a+bx+cx^2\right)^{3/2}}{2e(d+ex)^2}+\frac{3\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{(d+ex)^2}\,dx}{4e}$$

$$=\frac{3(4cd-be+2cex)\sqrt{a+bx+cx^2}}{4e^3(d+ex)}-\frac{\left(a+bx+cx^2\right)^{3/2}}{2e(d+ex)^2}-\frac{3\int \frac{4bcd-b^2e-4ace+4c(2cd-be)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{8e^3}$$

$$=\frac{3(4cd-be+2cex)\sqrt{a+bx+cx^2}}{4e^3(d+ex)}-\frac{\left(a+bx+cx^2\right)^{3/2}}{2e(d+ex)^2}-\frac{(3c(2cd-be))\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2e^4}+\frac{(3(\ldots}{}$$

$$=\frac{3(4cd-be+2cex)\sqrt{a+bx+cx^2}}{4e^3(d+ex)}-\frac{\left(a+bx+cx^2\right)^{3/2}}{2e(d+ex)^2}-\frac{(3c(2cd-be))\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\ldots\right.}{e^4}$$

$$=\frac{3(4cd-be+2cex)\sqrt{a+bx+cx^2}}{4e^3(d+ex)}-\frac{\left(a+bx+cx^2\right)^{3/2}}{2e(d+ex)^2}-\frac{3\sqrt{c}(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right.}{2e^4}$$

Mathematica [A] time = 0.990464, size = 311, normalized size = 1.32

$$\frac{\frac{3\left(-\frac{2e\sqrt{a+x(b+cx)}\left(-ce(2ae-5bd+bex)-b^2e^2+2c^2d(ex-2d)\right)}{e(ae-bd)+cd^2}-\frac{\left(4ce(ae-2bd)+b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{\sqrt{e(ae-bd)+cd^2}}-4\sqrt{c}(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{2e^3}+\frac{3(a\ldots}{(d\ldots}}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x)^3,x]
```

```
[Out] ((-2*(a + x*(b + c*x))^(3/2))/(d + e*x)^2 + (3*(2*c*d - b*e)*(a + x*(b + c*
x))^(3/2))/((c*d^2 + e*(-(b*d) + a*e))*(d + e*x)) + (3*((-2*e*Sqrt[a + x*(b
 + c*x)]*(-(b^2*e^2) + 2*c^2*d*(-2*d + e*x) - c*e*(-5*b*d + 2*a*e + b*e*x))
)/(c*d^2 + e*(-(b*d) + a*e)) - 4*Sqrt[c]*(2*c*d - b*e)*ArcTanh[(b + 2*c*x)/
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - ((8*c^2*d^2 + b^2*e^2 + 4*c*e*(-2*b*d
+ a*e))*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d
) + a*e)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d^2 + e*(-(b*d) + a*e)]))/(2*e^3)
)/(4*e)
```

Maple [B] time = 0.236, size = 7299, normalized size = 30.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d)^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 179.376, size = 5808, normalized size = 24.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(12*(2*c^2*d^5 - 3*b*c*d^4*e - a*b*d^2*e^3 + (b^2 + 2*a*c)*d^3*e^2 +
 (2*c^2*d^3*e^2 - 3*b*c*d^2*e^3 - a*b*e^5 + (b^2 + 2*a*c)*d*e^4)*x^2 + 2*(2
*c^2*d^4*e - 3*b*c*d^3*e^2 - a*b*d*e^4 + (b^2 + 2*a*c)*d^2*e^3)*x)*sqrt(c)*
log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c
) - 4*a*c) - 3*(8*c^2*d^4 - 8*b*c*d^3*e + (b^2 + 4*a*c)*d^2*e^2 + (8*c^2*d^
2*e^2 - 8*b*c*d*e^3 + (b^2 + 4*a*c)*e^4)*x^2 + 2*(8*c^2*d^3*e - 8*b*c*d^2*e
^2 + (b^2 + 4*a*c)*d*e^3)*x)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8
*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*
x^2 - 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2
*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^
2 + 2*d*e*x + d^2)) - 4*(12*c^2*d^4*e - 15*b*c*d^3*e^2 - a*b*d*e^4 - 2*a^2*
e^5 + (3*b^2 + 10*a*c)*d^2*e^3 + 4*(c^2*d^2*e^3 - b*c*d*e^4 + a*c*e^5)*x^2
```

```
+ (18*c^2*d^3*e^2 - 23*b*c*d^2*e^3 - 5*a*b*e^5 + (5*b^2 + 18*a*c)*d*e^4)*x)
*sqrt(c*x^2 + b*x + a))/(c*d^4*e^4 - b*d^3*e^5 + a*d^2*e^6 + (c*d^2*e^6 - b
*d*e^7 + a*e^8)*x^2 + 2*(c*d^3*e^5 - b*d^2*e^6 + a*d*e^7)*x), 1/16*(24*(2*c
^2*d^5 - 3*b*c*d^4*e - a*b*d^2*e^3 + (b^2 + 2*a*c)*d^3*e^2 + (2*c^2*d^3*e^2
 - 3*b*c*d^2*e^3 - a*b*e^5 + (b^2 + 2*a*c)*d*e^4)*x^2 + 2*(2*c^2*d^4*e - 3*
b*c*d^3*e^2 - a*b*d*e^4 + (b^2 + 2*a*c)*d^2*e^3)*x)*sqrt(-c)*arctan(1/2*sqr
t(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 3*(8*c^2
*d^4 - 8*b*c*d^3*e + (b^2 + 4*a*c)*d^2*e^2 + (8*c^2*d^2*e^2 - 8*b*c*d*e^3 +
 (b^2 + 4*a*c)*e^4)*x^2 + 2*(8*c^2*d^3*e - 8*b*c*d^2*e^2 + (b^2 + 4*a*c)*d*
e^3)*x)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a
*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 -
b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4
*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) +
 4*(12*c^2*d^4*e - 15*b*c*d^3*e^2 - a*b*d*e^4 - 2*a^2*e^5 + (3*b^2 + 10*a*c
)*d^2*e^3 + 4*(c^2*d^2*e^3 - b*c*d*e^4 + a*c*e^5)*x^2 + (18*c^2*d^3*e^2 - 2
3*b*c*d^2*e^3 - 5*a*b*e^5 + (5*b^2 + 18*a*c)*d*e^4)*x)*sqrt(c*x^2 + b*x + a
))/(c*d^4*e^4 - b*d^3*e^5 + a*d^2*e^6 + (c*d^2*e^6 - b*d*e^7 + a*e^8)*x^2 +
 2*(c*d^3*e^5 - b*d^2*e^6 + a*d*e^7)*x), 1/8*(3*(8*c^2*d^4 - 8*b*c*d^3*e +
(b^2 + 4*a*c)*d^2*e^2 + (8*c^2*d^2*e^2 - 8*b*c*d*e^3 + (b^2 + 4*a*c)*e^4)*x
^2 + 2*(8*c^2*d^3*e - 8*b*c*d^2*e^2 + (b^2 + 4*a*c)*d*e^3)*x)*sqrt(-c*d^2 +
 b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x +
 a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2
 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)) - 6*(2*c^2*d^
5 - 3*b*c*d^4*e - a*b*d^2*e^3 + (b^2 + 2*a*c)*d^3*e^2 + (2*c^2*d^3*e^2 - 3*
b*c*d^2*e^3 - a*b*e^5 + (b^2 + 2*a*c)*d*e^4)*x^2 + 2*(2*c^2*d^4*e - 3*b*c*d
^3*e^2 - a*b*d*e^4 + (b^2 + 2*a*c)*d^2*e^3)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b
*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 2*(12*c
^2*d^4*e - 15*b*c*d^3*e^2 - a*b*d*e^4 - 2*a^2*e^5 + (3*b^2 + 10*a*c)*d^2*e^
3 + 4*(c^2*d^2*e^3 - b*c*d*e^4 + a*c*e^5)*x^2 + (18*c^2*d^3*e^2 - 23*b*c*d^
2*e^3 - 5*a*b*e^5 + (5*b^2 + 18*a*c)*d*e^4)*x)*sqrt(c*x^2 + b*x + a))/(c*d^
4*e^4 - b*d^3*e^5 + a*d^2*e^6 + (c*d^2*e^6 - b*d*e^7 + a*e^8)*x^2 + 2*(c*d^
3*e^5 - b*d^2*e^6 + a*d*e^7)*x), 1/8*(3*(8*c^2*d^4 - 8*b*c*d^3*e + (b^2 + 4
*a*c)*d^2*e^2 + (8*c^2*d^2*e^2 - 8*b*c*d*e^3 + (b^2 + 4*a*c)*e^4)*x^2 + 2*(
8*c^2*d^3*e - 8*b*c*d^2*e^2 + (b^2 + 4*a*c)*d*e^3)*x)*sqrt(-c*d^2 + b*d*e -
 a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d
 - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d
*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)) + 12*(2*c^2*d^5 - 3*b
*c*d^4*e - a*b*d^2*e^3 + (b^2 + 2*a*c)*d^3*e^2 + (2*c^2*d^3*e^2 - 3*b*c*d^2
*e^3 - a*b*e^5 + (b^2 + 2*a*c)*d*e^4)*x^2 + 2*(2*c^2*d^4*e - 3*b*c*d^3*e^2
- a*b*d*e^4 + (b^2 + 2*a*c)*d^2*e^3)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*
x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(12*c^2*d^4*e - 15
*b*c*d^3*e^2 - a*b*d*e^4 - 2*a^2*e^5 + (3*b^2 + 10*a*c)*d^2*e^3 + 4*(c^2*d^
2*e^3 - b*c*d*e^4 + a*c*e^5)*x^2 + (18*c^2*d^3*e^2 - 23*b*c*d^2*e^3 - 5*a*b
*e^5 + (5*b^2 + 18*a*c)*d*e^4)*x)*sqrt(c*x^2 + b*x + a))/(c*d^4*e^4 - b*d^3
*e^5 + a*d^2*e^6 + (c*d^2*e^6 - b*d*e^7 + a*e^8)*x^2 + 2*(c*d^3*e^5 - b*d^2
*e^6 + a*d*e^7)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.2350 $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)^4} \, dx$

Optimal. Leaf size=307

$$-\frac{\sqrt{a+bx+cx^2}\left(ex\left(-4ce(3bd-2ae)+b^2e^2+12c^2d^2\right)-2cde(3bd-2ae)-be^2(bd-2ae)+8c^2d^3\right)}{8e^3(d+ex)^2\left(ae^2-bde+cd^2\right)}-\frac{(2cd-be)\left(-4ce\right.}{}$$

```
[Out] -((8*c^2*d^3 - b*e^2*(b*d - 2*a*e) - 2*c*d*e*(3*b*d - 2*a*e) + e*(12*c^2*d^
2 + b^2*e^2 - 4*c*e*(3*b*d - 2*a*e))*x)*Sqrt[a + b*x + c*x^2])/(8*e^3*(c*d^
2 - b*d*e + a*e^2)*(d + e*x)^2) - (a + b*x + c*x^2)^(3/2)/(3*e*(d + e*x)^3)
 + (c^(3/2)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/e^4 - (
(2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b*d -
2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^
2])])/(16*e^4*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rubi [A] time = 0.355388, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {732, 810, 843, 621, 206, 724}

$$-\frac{\sqrt{a+bx+cx^2}\left(ex\left(-4ce(3bd-2ae)+b^2e^2+12c^2d^2\right)-2cde(3bd-2ae)-be^2(bd-2ae)+8c^2d^3\right)}{8e^3(d+ex)^2\left(ae^2-bde+cd^2\right)}-\frac{(2cd-be)\left(-4ce\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x)^4,x]
```


```
[Out] -((8*c^2*d^3 - b*e^2*(b*d - 2*a*e) - 2*c*d*e*(3*b*d - 2*a*e) + e*(12*c^2*d^
2 + b^2*e^2 - 4*c*e*(3*b*d - 2*a*e))*x)*Sqrt[a + b*x + c*x^2])/(8*e^3*(c*d^
2 - b*d*e + a*e^2)*(d + e*x)^2) - (a + b*x + c*x^2)^(3/2)/(3*e*(d + e*x)^3)
 + (c^(3/2)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/e^4 - (
(2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b*d -
2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^
2])])/(16*e^4*(c*d^2 - b*d*e + a*e^2)^(3/2))
```


Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
```

```
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(d+ex)^4}\,dx = -\frac{\left(a+bx+cx^2\right)^{3/2}}{3e(d+ex)^3} + \frac{\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx}{2e}$$

$$= -\frac{\left(8c^2d^3 - be^2(bd-2ae) - 2cde(3bd-2ae) + e\left(12c^2d^2 + b^2e^2 - 4ce(3bd-2ae)\right)x\right)\sqrt{a+bx+c}}{8e^3\left(cd^2-bde+ae^2\right)(d+ex)^2}$$

$$= -\frac{\left(8c^2d^3 - be^2(bd-2ae) - 2cde(3bd-2ae) + e\left(12c^2d^2 + b^2e^2 - 4ce(3bd-2ae)\right)x\right)\sqrt{a+bx+c}}{8e^3\left(cd^2-bde+ae^2\right)(d+ex)^2}$$

$$= -\frac{\left(8c^2d^3 - be^2(bd-2ae) - 2cde(3bd-2ae) + e\left(12c^2d^2 + b^2e^2 - 4ce(3bd-2ae)\right)x\right)\sqrt{a+bx+c}}{8e^3\left(cd^2-bde+ae^2\right)(d+ex)^2}$$

$$= -\frac{\left(8c^2d^3 - be^2(bd-2ae) - 2cde(3bd-2ae) + e\left(12c^2d^2 + b^2e^2 - 4ce(3bd-2ae)\right)x\right)\sqrt{a+bx+c}}{8e^3\left(cd^2-bde+ae^2\right)(d+ex)^2}$$

Mathematica [A] time = 1.71053, size = 421, normalized size = 1.37

$$\frac{\frac{3\left(\frac{2(a+x(b+cx))^{3/2}\left(4ce(bd-2ae)+b^2e^2-4c^2d^2\right)}{d+ex}+\frac{2\sqrt{a+x(b+cx)}\left(2c^2e(2ae(2ex-3d)+bd(7d-2ex))-bce^2(-10ae+5bd+bex)-b^3e^3+4c^3d^2(ex-2d)\right)}{e^2}+\frac{(2cd-be)\left(4ce(3ae-2bd)-b^2e^2+8c^2d^2\right)\sqrt{e(ae-b\cdots}}{\cdots}\cdots\right)}{8\left(e(ae-bd)+cd^2\right)^2}\cdots}{6e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x)^4,x]
```

```
[Out] ((-2*(a + x*(b + c*x))^(3/2))/(d + e*x)^3 + (3*(2*c*d - b*e)*(a + x*(b + c*
x))^(3/2))/(2*(c*d^2 + e*(-(b*d) + a*e))*(d + e*x)^2) + (3*((2*(-4*c^2*d^2
+ b^2*e^2 + 4*c*e*(b*d - 2*a*e))*(a + x*(b + c*x))^(3/2))/(d + e*x) + (2*Sq
rt[a + x*(b + c*x)]*(-(b^3*e^3) + 4*c^3*d^2*(-2*d + e*x) - b*c*e^2*(5*b*d -
 10*a*e + b*e*x) + 2*c^2*e*(b*d*(7*d - 2*e*x) + 2*a*e*(-3*d + 2*e*x))))/e^2
 + (16*c^(3/2)*(c*d^2 + e*(-(b*d) + a*e))^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + x*(b + c*x)])] + (2*c*d - b*e)*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*(8*c
^2*d^2 - b^2*e^2 + 4*c*e*(-2*b*d + 3*a*e))*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*
x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/e^3))
/(8*(c*d^2 + e*(-(b*d) + a*e))^2))/(6*e)
```

Maple [B] time = 0.246, size = 10401, normalized size = 33.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d)^4,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)**4,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.2351 $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)^5}\,dx$

Optimal. Leaf size=225

$$-\frac{3(b^2-4ac)\sqrt{a+bx+cx^2}(-2ae+x(2cd-be)+bd)}{64(d+ex)^2(ae^2-bde+cd^2)^2}+\frac{3(b^2-4ac)^2\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{128(ae^2-bde+cd^2)^{5/2}}+\frac{(a+bx+cx}{8(d+}$$

```
[Out] (-3*(b^2 - 4*a*c)*(b*d - 2*a*e + (2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(6
4*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^2) + ((b*d - 2*a*e + (2*c*d - b*e)*x)
*(a + b*x + c*x^2)^(3/2))/(8*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^4) + (3*(b^2
 - 4*a*c)^2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e +
 a*e^2]*Sqrt[a + b*x + c*x^2])])/(128*(c*d^2 - b*d*e + a*e^2)^(5/2))
```

Rubi [A] time = 0.150275, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {720, 724, 206}

$$-\frac{3(b^2-4ac)\sqrt{a+bx+cx^2}(-2ae+x(2cd-be)+bd)}{64(d+ex)^2(ae^2-bde+cd^2)^2}+\frac{3(b^2-4ac)^2\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{128(ae^2-bde+cd^2)^{5/2}}+\frac{(a+bx+cx}{8(d+}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x)^5,x]
```

```
[Out] (-3*(b^2 - 4*a*c)*(b*d - 2*a*e + (2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(6
4*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^2) + ((b*d - 2*a*e + (2*c*d - b*e)*x)
*(a + b*x + c*x^2)^(3/2))/(8*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^4) + (3*(b^2
 - 4*a*c)^2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e +
 a*e^2]*Sqrt[a + b*x + c*x^2])])/(128*(c*d^2 - b*d*e + a*e^2)^(5/2))
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(d+ex)^5}\,dx = \frac{(bd-2ae+(2cd-be)x)\left(a+bx+cx^2\right)^{3/2}}{8\left(cd^2-bde+ae^2\right)(d+ex)^4} - \frac{\left(3\left(b^2-4ac\right)\right)\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx}{16\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{3\left(b^2-4ac\right)(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{64\left(cd^2-bde+ae^2\right)^2(d+ex)^2} + \frac{(bd-2ae+(2cd-be)x)\left(a+bx+cx^2\right)}{8\left(cd^2-bde+ae^2\right)(d+ex)^4}$$

$$= -\frac{3\left(b^2-4ac\right)(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{64\left(cd^2-bde+ae^2\right)^2(d+ex)^2} + \frac{(bd-2ae+(2cd-be)x)\left(a+bx+cx^2\right)}{8\left(cd^2-bde+ae^2\right)(d+ex)^4}$$

$$= -\frac{3\left(b^2-4ac\right)(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{64\left(cd^2-bde+ae^2\right)^2(d+ex)^2} + \frac{(bd-2ae+(2cd-be)x)\left(a+bx+cx^2\right)}{8\left(cd^2-bde+ae^2\right)(d+ex)^4}$$

Mathematica [A] time = 0.768824, size = 222, normalized size = 0.99

$$-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{8\left(e(ae-bd)+cd^2\right)^{3/2}}+\frac{\sqrt{a+x(b+cx)}(-2ae+b(d-ex)+2cdx)}{4(d+ex)^2\left(e(ae-bd)+cd^2\right)}\right)+\frac{2(a+x(b+cx))^{3/2}(2ae-bd+bex-2cdx)}{(d+ex)^4}}{16\left(e(ae-bd)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x)^5,x]
```

```
[Out] -((2*(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)*(a + x*(b + c*x))^(3/2))/(d + e*x)^
4 + 3*(b^2 - 4*a*c)*((Sqrt[a + x*(b + c*x)]*(-2*a*e + 2*c*d*x + b*(d - e*x)
))/(4*(c*d^2 + e*(-(b*d) + a*e))*(d + e*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b
*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x
*(b + c*x)])])/(8*(c*d^2 + e*(-(b*d) + a*e))^(3/2))))/(16*(c*d^2 + e*(-(b*d
) + a*e)))
```

Maple [B] time = 0.24, size = 15932, normalized size = 70.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d)^5,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 18.765, size = 3803, normalized size = 16.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] 1/128*(2*sqrt(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + b*e/(x*e + d) - b*d
*e/(x*e + d)^2 + a*e^2/(x*e + d)^2)*(2*(4*(3*(2*c^4*d^7*e^19*sgn(1/(x*e + d
)) - 7*b*c^3*d^6*e^20*sgn(1/(x*e + d)) + 9*b^2*c^2*d^5*e^21*sgn(1/(x*e + d)
) + 6*a*c^3*d^5*e^21*sgn(1/(x*e + d)) - 5*b^3*c*d^4*e^22*sgn(1/(x*e + d)) -
 15*a*b*c^2*d^4*e^22*sgn(1/(x*e + d)) + b^4*d^3*e^23*sgn(1/(x*e + d)) + 12*
a*b^2*c*d^3*e^23*sgn(1/(x*e + d)) + 6*a^2*c^2*d^3*e^23*sgn(1/(x*e + d)) - 3
*a*b^3*d^2*e^24*sgn(1/(x*e + d)) - 9*a^2*b*c*d^2*e^24*sgn(1/(x*e + d)) + 3*
a^2*b^2*d*e^25*sgn(1/(x*e + d)) + 2*a^3*c*d*e^25*sgn(1/(x*e + d)) - a^3*b*e
^26*sgn(1/(x*e + d)))/(c^4*d^8*e^8 - 4*b*c^3*d^7*e^9 + 6*b^2*c^2*d^6*e^10 +
 4*a*c^3*d^6*e^10 - 4*b^3*c*d^5*e^11 - 12*a*b*c^2*d^5*e^11 + b^4*d^4*e^12 +
 12*a*b^2*c*d^4*e^12 + 6*a^2*c^2*d^4*e^12 - 4*a*b^3*d^3*e^13 - 12*a^2*b*c*d
^3*e^13 + 6*a^2*b^2*d^2*e^14 + 4*a^3*c*d^2*e^14 - 4*a^3*b*d*e^15 + a^4*e^16
) - 2*(c^4*d^8*e^20*sgn(1/(x*e + d)) - 4*b*c^3*d^7*e^21*sgn(1/(x*e + d)) +
6*b^2*c^2*d^6*e^22*sgn(1/(x*e + d)) + 4*a*c^3*d^6*e^22*sgn(1/(x*e + d)) - 4
*b^3*c*d^5*e^23*sgn(1/(x*e + d)) - 12*a*b*c^2*d^5*e^23*sgn(1/(x*e + d)) + b
^4*d^4*e^24*sgn(1/(x*e + d)) + 12*a*b^2*c*d^4*e^24*sgn(1/(x*e + d)) + 6*a^2
*c^2*d^4*e^24*sgn(1/(x*e + d)) - 4*a*b^3*d^3*e^25*sgn(1/(x*e + d)) - 12*a^2
*b*c*d^3*e^25*sgn(1/(x*e + d)) + 6*a^2*b^2*d^2*e^26*sgn(1/(x*e + d)) + 4*a^
3*c*d^2*e^26*sgn(1/(x*e + d)) - 4*a^3*b*d*e^27*sgn(1/(x*e + d)) + a^4*e^28*
```

```
sgn(1/(x*e + d)))*e^(-1)/((c^4*d^8*e^8 - 4*b*c^3*d^7*e^9 + 6*b^2*c^2*d^6*e^
10 + 4*a*c^3*d^6*e^10 - 4*b^3*c*d^5*e^11 - 12*a*b*c^2*d^5*e^11 + b^4*d^4*e^
12 + 12*a*b^2*c*d^4*e^12 + 6*a^2*c^2*d^4*e^12 - 4*a*b^3*d^3*e^13 - 12*a^2*b
*c*d^3*e^13 + 6*a^2*b^2*d^2*e^14 + 4*a^3*c*d^2*e^14 - 4*a^3*b*d*e^15 + a^4*
e^16)*(x*e + d)))*e^(-1)/(x*e + d) - (24*c^4*d^6*e^18*sgn(1/(x*e + d)) - 72
*b*c^3*d^5*e^19*sgn(1/(x*e + d)) + 73*b^2*c^2*d^4*e^20*sgn(1/(x*e + d)) + 6
8*a*c^3*d^4*e^20*sgn(1/(x*e + d)) - 26*b^3*c*d^3*e^21*sgn(1/(x*e + d)) - 13
6*a*b*c^2*d^3*e^21*sgn(1/(x*e + d)) + b^4*d^2*e^22*sgn(1/(x*e + d)) + 70*a*
b^2*c*d^2*e^22*sgn(1/(x*e + d)) + 64*a^2*c^2*d^2*e^22*sgn(1/(x*e + d)) - 2*
a*b^3*d*e^23*sgn(1/(x*e + d)) - 64*a^2*b*c*d*e^23*sgn(1/(x*e + d)) + a^2*b^
2*e^24*sgn(1/(x*e + d)) + 20*a^3*c*e^24*sgn(1/(x*e + d)))/(c^4*d^8*e^8 - 4*
b*c^3*d^7*e^9 + 6*b^2*c^2*d^6*e^10 + 4*a*c^3*d^6*e^10 - 4*b^3*c*d^5*e^11 -
12*a*b*c^2*d^5*e^11 + b^4*d^4*e^12 + 12*a*b^2*c*d^4*e^12 + 6*a^2*c^2*d^4*e^
12 - 4*a*b^3*d^3*e^13 - 12*a^2*b*c*d^3*e^13 + 6*a^2*b^2*d^2*e^14 + 4*a^3*c*
d^2*e^14 - 4*a^3*b*d*e^15 + a^4*e^16))*e^(-1)/(x*e + d) + (16*c^4*d^5*e^17*
sgn(1/(x*e + d)) - 40*b*c^3*d^4*e^18*sgn(1/(x*e + d)) + 26*b^2*c^2*d^3*e^19
*sgn(1/(x*e + d)) + 56*a*c^3*d^3*e^19*sgn(1/(x*e + d)) + b^3*c*d^2*e^20*sgn
(1/(x*e + d)) - 84*a*b*c^2*d^2*e^20*sgn(1/(x*e + d)) - 3*b^4*d*e^21*sgn(1/(
x*e + d)) + 22*a*b^2*c*d*e^21*sgn(1/(x*e + d)) + 40*a^2*c^2*d*e^21*sgn(1/(x
*e + d)) + 3*a*b^3*e^22*sgn(1/(x*e + d)) - 20*a^2*b*c*e^22*sgn(1/(x*e + d))
)/(c^4*d^8*e^8 - 4*b*c^3*d^7*e^9 + 6*b^2*c^2*d^6*e^10 + 4*a*c^3*d^6*e^10 -
4*b^3*c*d^5*e^11 - 12*a*b*c^2*d^5*e^11 + b^4*d^4*e^12 + 12*a*b^2*c*d^4*e^12
 + 6*a^2*c^2*d^4*e^12 - 4*a*b^3*d^3*e^13 - 12*a^2*b*c*d^3*e^13 + 6*a^2*b^2*
d^2*e^14 + 4*a^3*c*d^2*e^14 - 4*a^3*b*d*e^15 + a^4*e^16)) - 3*(b^4*e^14*sgn
(1/(x*e + d)) - 8*a*b^2*c*e^14*sgn(1/(x*e + d)) + 16*a^2*c^2*e^14*sgn(1/(x*
e + d)))*sqrt(c*d^2 - b*d*e + a*e^2)*log(abs(2*(c*d^2 - b*d*e + a*e^2)*(sqr
t(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + b*e/(x*e + d) - b*d*e/(x*e + d)
^2 + a*e^2/(x*e + d)^2) + sqrt(c*d^2*e^2 - b*d*e^3 + a*e^4)*e^(-1)/(x*e + d
)) - sqrt(c*d^2 - b*d*e + a*e^2)*(2*c*d - b*e)))/(c^4*d^8*e - 4*b*c^3*d^7*e
^2 + 6*b^2*c^2*d^6*e^3 + 4*a*c^3*d^6*e^3 - 4*b^3*c*d^5*e^4 - 12*a*b*c^2*d^5
*e^4 + b^4*d^4*e^5 + 12*a*b^2*c*d^4*e^5 + 6*a^2*c^2*d^4*e^5 - 4*a*b^3*d^3*e
^6 - 12*a^2*b*c*d^3*e^6 + 6*a^2*b^2*d^2*e^7 + 4*a^3*c*d^2*e^7 - 4*a^3*b*d*e
^8 + a^4*e^9) - (32*c^(9/2)*d^5*e^9 - 80*b*c^(7/2)*d^4*e^10 + 52*b^2*c^(5/2
)*d^3*e^11 + 112*a*c^(7/2)*d^3*e^11 + 2*b^3*c^(3/2)*d^2*e^12 - 168*a*b*c^(5
/2)*d^2*e^12 - 6*b^4*sqrt(c)*d*e^13 + 44*a*b^2*c^(3/2)*d*e^13 + 80*a^2*c^(5
/2)*d*e^13 - 3*sqrt(c*d^2 - b*d*e + a*e^2)*b^4*e^13*log(abs(2*c^(3/2)*d^2 -
 2*b*sqrt(c)*d*e - 2*sqrt(c*d^2 - b*d*e + a*e^2)*c*d + 2*a*sqrt(c)*e^2 + sq
rt(c*d^2 - b*d*e + a*e^2)*b*e)) + 24*sqrt(c*d^2 - b*d*e + a*e^2)*a*b^2*c*e^
13*log(abs(2*c^(3/2)*d^2 - 2*b*sqrt(c)*d*e - 2*sqrt(c*d^2 - b*d*e + a*e^2)*
c*d + 2*a*sqrt(c)*e^2 + sqrt(c*d^2 - b*d*e + a*e^2)*b*e)) - 48*sqrt(c*d^2 -
 b*d*e + a*e^2)*a^2*c^2*e^13*log(abs(2*c^(3/2)*d^2 - 2*b*sqrt(c)*d*e - 2*sq
rt(c*d^2 - b*d*e + a*e^2)*c*d + 2*a*sqrt(c)*e^2 + sqrt(c*d^2 - b*d*e + a*e^
2)*b*e)) + 6*a*b^3*sqrt(c)*e^14 - 40*a^2*b*c^(3/2)*e^14)*sgn(1/(x*e + d))/(
c^4*d^8 - 4*b*c^3*d^7*e + 6*b^2*c^2*d^6*e^2 + 4*a*c^3*d^6*e^2 - 4*b^3*c*d^5
*e^3 - 12*a*b*c^2*d^5*e^3 + b^4*d^4*e^4 + 12*a*b^2*c*d^4*e^4 + 6*a^2*c^2*d^
4*e^4 - 4*a*b^3*d^3*e^5 - 12*a^2*b*c*d^3*e^5 + 6*a^2*b^2*d^2*e^6 + 4*a^3*c*
d^2*e^6 - 4*a^3*b*d*e^7 + a^4*e^8))*e^2
```

3.2352 $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)^6} dx$

Optimal. Leaf size=296

$$-\frac{3(b^2-4ac)\sqrt{a+bx+cx^2}(2cd-be)(-2ae+x(2cd-be)+bd)}{128(d+ex)^2(ae^2-bde+cd^2)^3}+\frac{3(b^2-4ac)^2(2cd-be)\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{256(ae^2-bde+cd^2)^{7/2}}$$

```
[Out] (-3*(b^2 - 4*a*c)*(2*c*d - b*e)*(b*d - 2*a*e + (2*c*d - b*e)*x)*Sqrt[a + b*
x + c*x^2])/(128*(c*d^2 - b*d*e + a*e^2)^3*(d + e*x)^2) + ((2*c*d - b*e)*(b
*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(3/2))/(16*(c*d^2 - b*d*e +
 a*e^2)^2*(d + e*x)^4) - (e*(a + b*x + c*x^2)^(5/2))/(5*(c*d^2 - b*d*e + a*
e^2)*(d + e*x)^5) + (3*(b^2 - 4*a*c)^2*(2*c*d - b*e)*ArcTanh[(b*d - 2*a*e +
 (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(
256*(c*d^2 - b*d*e + a*e^2)^(7/2))
```

Rubi [A] time = 0.234795, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {730, 720, 724, 206}

$$-\frac{3(b^2-4ac)\sqrt{a+bx+cx^2}(2cd-be)(-2ae+x(2cd-be)+bd)}{128(d+ex)^2(ae^2-bde+cd^2)^3}+\frac{3(b^2-4ac)^2(2cd-be)\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{256(ae^2-bde+cd^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x)^6,x]
```

```
[Out] (-3*(b^2 - 4*a*c)*(2*c*d - b*e)*(b*d - 2*a*e + (2*c*d - b*e)*x)*Sqrt[a + b*
x + c*x^2])/(128*(c*d^2 - b*d*e + a*e^2)^3*(d + e*x)^2) + ((2*c*d - b*e)*(b
*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(3/2))/(16*(c*d^2 - b*d*e +
 a*e^2)^2*(d + e*x)^4) - (e*(a + b*x + c*x^2)^(5/2))/(5*(c*d^2 - b*d*e + a*
e^2)*(d + e*x)^5) + (3*(b^2 - 4*a*c)^2*(2*c*d - b*e)*ArcTanh[(b*d - 2*a*e +
 (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(
256*(c*d^2 - b*d*e + a*e^2)^(7/2))
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(d+ex)^6}\,dx = -\frac{e\left(a+bx+cx^2\right)^{5/2}}{5\left(cd^2-bde+ae^2\right)(d+ex)^5}+\frac{(2cd-be)\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(d+ex)^5}\,dx}{2\left(cd^2-bde+ae^2\right)}$$

$$=\frac{(2cd-be)(bd-2ae+(2cd-be)x)\left(a+bx+cx^2\right)^{3/2}}{16\left(cd^2-bde+ae^2\right)^2(d+ex)^4}-\frac{e\left(a+bx+cx^2\right)^{5/2}}{5\left(cd^2-bde+ae^2\right)(d+ex)^5}-\frac{\left(3\left(b^2-\right.\right.}{}$$

$$=-\frac{3\left(b^2-4ac\right)(2cd-be)(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{128\left(cd^2-bde+ae^2\right)^3(d+ex)^2}+\frac{(2cd-be)(bd-2ae+(2cd-}{16\left(cd^2-bde+ae^2\right.}$$

$$=-\frac{3\left(b^2-4ac\right)(2cd-be)(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{128\left(cd^2-bde+ae^2\right)^3(d+ex)^2}+\frac{(2cd-be)(bd-2ae+(2cd-}{16\left(cd^2-bde+ae^2\right.}$$

$$=-\frac{3\left(b^2-4ac\right)(2cd-be)(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{128\left(cd^2-bde+ae^2\right)^3(d+ex)^2}+\frac{(2cd-be)(bd-2ae+(2cd-}{16\left(cd^2-bde+ae^2\right.}$$

Mathematica [A] time = 1.25351, size = 275, normalized size = 0.93

$$-\frac{(2cd-be)\left(3\left(b^2-4ac\right)\left(\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{8\left(e(ae-bd)+cd^2\right)^{3/2}}+\frac{\sqrt{a+x(b+cx)}(-2ae+b(d-ex)+2cdx)}{4(d+ex)^2\left(e(ae-bd)+cd^2\right)}\right)+\frac{2(a+x(b+cx))^{3/2}(2ae-bd+bex-2cdx)}{(d+ex)^4}\right.}{32\left(e(ae-bd)+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x)^6,x]
```

```
[Out] -(e*(a + x*(b + c*x))^(5/2))/(5*(c*d^2 + e*(-(b*d) + a*e))*(d + e*x)^5) - (
(2*c*d - b*e)*((2*(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)*(a + x*(b + c*x))^(3/2
))/(d + e*x)^4 + 3*(b^2 - 4*a*c)*((Sqrt[a + x*(b + c*x)]*(-2*a*e + 2*c*d*x
+ b*(d - e*x)))/(4*(c*d^2 + e*(-(b*d) + a*e))*(d + e*x)^2) + ((b^2 - 4*a*c)
*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e
)]*Sqrt[a + x*(b + c*x)])])/(8*(c*d^2 + e*(-(b*d) + a*e))^(3/2)))))/(32*(c*
d^2 + e*(-(b*d) + a*e))^2)
```

Maple [B] time = 0.245, size = 20477, normalized size = 69.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d)^6,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^6,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 4.63398, size = 11048, normalized size = 37.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^6,x, algorithm="giac")
```

[Out] 3/128*(2*b^4*c*d - 16*a*b^2*c^2*d + 32*a^2*c^3*d - b^5*e + 8*a*b^3*c*e - 16*a^2*b*c^2*e)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/((c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6)*sqrt(-c*d^2 + b*d*e - a*e^2)) + 1/640*(2560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*c^(13/2)*d^8*e + 1024*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*c^7*d^9 + 2560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*c^6*d^7*e^2 + 3072*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^6*d^8*e + 2560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b*c^(13/2)*d^9 + 1280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*c^(11/2)*d^6*e^3 - 1280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b*c^(11/2)*d^7*e^2 - 1280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*c^(11/2)*d^8*e - 2560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*c^(13/2)*d^8*e + 2560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*c^6*d^9 - 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b*c^5*d^6*e^3 - 7936*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c^5*d^7*e^2 - 512*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*c^6*d^7*e^2 - 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^3*c^5*d^8*e - 5120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*c^6*d^8*e + 1280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^3*c^(11/2)*d^9 - 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b*c^(9/2)*d^5*e^4 - 6400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^2*c^(9/2)*d^6*e^3 + 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*c^(11/2)*d^6*e^3 - 6400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^3*c^(9/2)*d^7*e^2 + 8960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b*c^(11/2)*d^7*e^2 - 2400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^4*c^(9/2)*d^8*e - 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^2*c^(11/2)*d^8*e + 320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^4*c^5*d^9 - 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^2*c^4*d^5*e^4 + 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*c^5*d^5*e^4 - 1280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*c^4*d^6*e^3 + 24832*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c^5*d^6*e^3 - 1600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^4*c^4*d^7*e^2 + 17920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^2*c^5*d^7*e^2 + 2560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*c^6*d^7*e^2 - 640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^5*c^4*d^8*e - 1280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^3*c^5*d^8*e + 32*b^5*c^(9/2)*d^9 + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^2*c^(7/2)*d^4*e^5 + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*c^(9/2)*d^4*e^5 + 1280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^3*c^(7/2)*d^5*e^4 + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b*c^(9/2)*d^5*e^4 + 2400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^4*c^(7/2)*d^6*e^3 + 19200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^2*c^(9/2)*d^6*e^3 - 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^(11/2)*d^6*e^3 + 160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^5*c^(7/2)*d^7*e^2 + 12800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^3*c^(9/2)*d^7*e^2 + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b*c^(11/2)*d^7*e^2 - 64*b^6*c^(7/2)*d^8*e - 160*a*b^4*c^(9/2)*d^8*e + 8960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^3*c^3*d^4*e^5 - 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b*c^4*d^4*e^5 + 4280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^4*c^3*d^5*e^4 - 18880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^2*c^4*d^5*e^4 - 25216*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*c^5*d^5*e^4 + 2080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^5*c^3*d^6*e^3 - 24320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c^5*d^6*e^3 + 120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^6*c^3*d^7*e^2 + 4000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^4*c^4*d^7*e^2 + 1920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^2*c^5*d^7*e^2 - 1280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^3*c^(5/2)*d^3*e^6 - 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b*c^(7/2)*d^3*e^6 + 7420*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^4*c^(5/2)*d^4*e^5 - 3040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^2*c^(7/2)*d^4*e^5 - 16960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*c^(9/2)*d^4*e^5 + 2860*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^5*c^(5/2)*d^5*e^4 - 21600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^3*c^(7/2)*d^5*e^4 - 40000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b*c^(9/2)*d^5*e^4 + 860*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^6*c^(5/2)*d^6*e^3 - 5200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^4*c^(7/2)*d^6*e^3 - 24000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 *a^2*b^2*c^(9/2)*d^6*e^3 - 1280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*c^(11/2)*d^6*e^3 + 12*b^7*c^(5/2)*d^7*e^2 + 464*a*b^5*c^(7/2)*d^7*e^2 + 320*a^2*b^3*c^(9/2)*d^7*e^2 - 4780*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^4*c^2*d^3*e^6 - 7840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^2*c^3*d^3*e^6 - 7360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*c^4*d^3*e^6 + 1448*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^5*c^2*d^4*e^5 + 8640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^3*c^3*d^4*e^5 + 12160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b*c^4*d^4*e^5 + 540*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^6*c^2*d^5*e^4 - 7520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^4*c^3*d^5*e^4 - 13120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^2*c^4*d^5*e^4 + 12800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*c^5*d^5*e^4 + 200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^7*c^2*d^6*e^3 - 1920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^5*c^3*d^6*e^3 - 9600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^3*c^4*d^6*e^3 - 1280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b*c^5*d^6*e^3 - 270*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^4*c^(3/2)*d^2*e^7 + 6000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^2*c^(5/2)*d^2*e^7 - 480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*c^(7/2)*d^2*e^7 - 5330*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^5*c^(3/2)*d^3*e^6 - 9840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^3*c^(5/2)*d^3*e^6 + 8160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b*c^(7/2)*d^3*e^6 - 1390*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^6*c^(3/2)*d^4*e^5 + 9620*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^4*c^(5/2)*d^4*e^5 + 37120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^2*c^(7/2)*d^4*e^5 + 37440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*c^(9/2)*d^4*e^5 - 230*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^7*c^(3/2)*d^5*e^4 - 320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^5*c^(5/2)*d^5*e^4 + 5920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^3*c^(7/2)*d^5*e^4 + 23040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b*c^(9/2)*d^5*e^4 + 20*b^8*c^(3/2)*d^6*e^3 - 204*a*b^6*c^(5/2)*d^6*e^3 - 1360*a^2*b^4*c^(7/2)*d^6*e^3 - 320*a^3*b^2*c^(9/2)*d^6*e^3 - 30*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^4*c*d*e^8 + 240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^2*c^2*d*e^8 - 480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*c^3*d*e^8 + 330*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^5*c*d^2*e^7 + 8880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^3*c^2*d^2*e^7 + 9120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b*c^3*d^2*e^7 - 2626*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^6*c*d^3*e^6 - 6260*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^4*c^2*d^3*e^6 - 21760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^2*c^3*d^3*e^6 + 29120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*c^4*d^3*e^6 - 930*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^7*c*d^4*e^5 + 4760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^5*c^2*d^4*e^5 + 14240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^3*c^3*d^4*e^5 + 42880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b*c^4*d^4*e^5 - 120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^8*c*d^5*e^4 + 260*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^6*c^2*d^5*e^4 + 3880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^4*c^3*d^5*e^4 + 12800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^2*c^4*d^5*e^4 + 640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*c^5*d^5*e^4 + 135*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^5*sqrt(c)*d*e^8 - 1080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^3*c^(3/2)*d*e^8 - 1680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b*c^(5/2)*d*e^8 + 490*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^6*sqrt(c)*d^2*e^7 + 12420*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^4*c^(3/2)*d^2*e^7 - 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b^2*c^(5/2)*d^2*e^7 + 18240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^3*c^(7/2)*d^2*e^7 - 640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^7*sqrt(c)*d^3*e^6 + 1390*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^5*c^(3/2)*d^3*e^6 - 41840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^3*c^(5/2)*d^3*e^6 - 2080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b*c^(7/2)*d^3*e^6 - 210*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^8*sqrt(c)*d^4*e^5 + 1720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^6*c^(3/2)*d^4*e^5 - 2380*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^4*c^(5/2)*d^4*e^5 + 10720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^2*c^(7/2)*d^4*e^5 - 7360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*c^(9/2)*d^4*e^5 - 15*b^9*sqrt(c)*d^5*e^4 + 30*a*b^7*c^(3/2)*d^5*e^4 + 532*a^2*b^5*c^(5/2)*d^5*e^4 + 2240*a^3*b^3*c^(7/2)*d^5*e^4 + 320*a

^4*b*c^(9/2)*d^5*e^4 + 15*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^5*e^9 - 1
20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^3*c*e^9 + 240*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^9*a^2*b*c^2*e^9 + 70*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^7*b^6*d*e^8 - 420*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^4*c*d*e^8 - 1
1520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^2*c^2*d*e^8 + 4800*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^7*a^3*c^3*d*e^8 + 128*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^5*b^7*d^2*e^7 + 9026*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^
5*c*d^2*e^7 + 1520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^3*c^2*d^2*e^
7 + 11040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b*c^3*d^2*e^7 - 70*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^3*b^8*d^3*e^6 + 2280*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^3*a*b^6*c*d^3*e^6 - 20420*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^3*a^2*b^4*c^2*d^3*e^6 - 20320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^
2*c^3*d^3*e^6 - 24640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*c^4*d^3*e^6
 - 15*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^9*d^4*e^5 + 450*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))*a*b^7*c*d^4*e^5 - 2080*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*a^2*b^5*c^2*d^4*e^5 - 1120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^3
*c^3*d^4*e^5 - 8960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b*c^4*d^4*e^5 +
 1280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^3*c^(5/2)*e^9 - 490*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^6*a*b^5*sqrt(c)*d*e^8 - 11440*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^6*a^2*b^3*c^(3/2)*d*e^8 - 1440*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^6*a^3*b*c^(5/2)*d*e^8 + 2560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
4*a*b^6*sqrt(c)*d^2*e^7 + 5120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^
4*c^(3/2)*d^2*e^7 + 30720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^2*c^(
5/2)*d^2*e^7 - 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*c^(7/2)*d^2*
e^7 + 630*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^7*sqrt(c)*d^3*e^6 - 443
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^5*c^(3/2)*d^3*e^6 - 2480*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^3*c^(5/2)*d^3*e^6 - 22240*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^2*a^4*b*c^(7/2)*d^3*e^6 + 60*a*b^8*sqrt(c)*d^4*e
^5 - 330*a^2*b^6*c^(3/2)*d^4*e^5 - 260*a^3*b^4*c^(5/2)*d^4*e^5 - 2560*a^4*b
^2*c^(7/2)*d^4*e^5 - 64*a^5*c^(9/2)*d^4*e^5 - 70*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^7*a*b^5*e^9 + 560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^3*c
e^9 + 2720(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b*c^2*e^9 - 256*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^6*d*e^8 - 8960*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^5*a^2*b^4*c*d*e^8 - 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5
*a^4*c^3*d*e^8 + 210*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^7*d^2*e^7 +
230*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^5*c*d^2*e^7 + 26480*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^3*c^2*d^2*e^7 + 6240*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^3*a^4*b*c^3*d^2*e^7 + 60*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*a*b^8*d^3*e^6 - 750*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^6*c*d^3*
e^6 + 2820*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^4*c^2*d^3*e^6 - 5760*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^2*c^3*d^3*e^6 + 2880*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))*a^5*c^4*d^3*e^6 + 5120*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^6*a^3*b^2*c^(3/2)*e^9 - 3200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a
^2*b^5*sqrt(c)*d*e^8 - 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^3*c
^(3/2)*d*e^8 - 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b*c^(5/2)*d*e
^8 - 630*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^6*sqrt(c)*d^2*e^7 + 71
80*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^4*c^(3/2)*d^2*e^7 + 7360*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^2*c^(5/2)*d^2*e^7 + 11200*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^2*a^5*c^(7/2)*d^2*e^7 - 90*a^2*b^7*sqrt(c)*d^3*
e^6 + 610*a^3*b^5*c^(3/2)*d^3*e^6 - 560*a^4*b^3*c^(5/2)*d^3*e^6 + 1568*a^5*
b*c^(7/2)*d^3*e^6 + 128*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^5*e^9 +
 2560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^3*c*e^9 + 3840*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^5*a^4*b*c^2*e^9 - 210*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^3*a^2*b^6*d*e^8 - 3580*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b
^4*c*d*e^8 - 13760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^2*c^2*d*e^8
+ 5440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*c^3*d*e^8 - 90*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))*a^2*b^7*d^2*e^7 + 750*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))*a^3*b^5*c*d^2*e^7 - 400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^3
*c^2*d^2*e^7 + 6880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b*c^3*d^2*e^7 +

1280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^4*sqrt(c)*e^9 + 2560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b^2*c^(3/2)*e^9 + 2560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*c^(5/2)*e^9 + 210*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^5*sqrt(c)*d*e^8 - 6800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^3*c^(3/2)*d*e^8 - 3040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b*c^(5/2)*d*e^8 + 60*a^3*b^6*sqrt(c)*d^2*e^7 - 450*a^4*b^4*c^(3/2)*d^2*e^7 + 976*a^5*b^2*c^(5/2)*d^2*e^7 - 288*a^6*c^(7/2)*d^2*e^7 + 70*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^5*e^9 + 2000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^3*c*e^9 + 2400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b*c^2*e^9 + 60*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^6*d*e^8 - 450*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^4*c*d*e^8 - 1840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^2*c^2*d*e^8 - 2080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*c^3*d*e^8 + 2560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^2*c^(3/2)*e^9 - 15*a^4*b^5*sqrt(c)*d*e^8 + 120*a^5*b^3*c^(3/2)*d*e^8 - 752*a^6*b*c^(5/2)*d*e^8 - 15*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^5*e^9 + 120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^3*c*e^9 + 1040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b*c^2*e^9 + 256*a^7*c^(5/2)*e^9)/((c^3*d^6*e^4 - 3*b*c^2*d^5*e^5 + 3*b^2*c*d^4*e^6 + 3*a*c^2*d^4*e^6 - b^3*d^3*e^7 - 6*a*b*c*d^3*e^7 + 3*a*b^2*d^2*e^8 + 3*a^2*c*d^2*e^8 - 3*a^2*b*d*e^9 + a^3*e^10)*((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*e + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c)*d + b*d - a*e)^5)

3.2353 $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)^7} dx$

Optimal. Leaf size=409

$$\frac{(a+bx+cx^2)^{3/2}\left(-4ce(ae+6bd)+7b^2e^2+24c^2d^2\right)(-2ae+x(2cd-be)+bd)}{192(d+ex)^4\left(ae^2-bde+cd^2\right)^3}-\frac{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(-4ce(ae+6b\right.}{512(d+ex)^{2}}$$

```
[Out] -((b^2 - 4*a*c)*(24*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(6*b*d + a*e))*(b*d - 2*a*e
 + (2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(512*(c*d^2 - b*d*e + a*e^2)^4*(
d + e*x)^2) + ((24*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(6*b*d + a*e))*(b*d - 2*a*e
+ (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(3/2))/(192*(c*d^2 - b*d*e + a*e^2)^3*
(d + e*x)^4) - (e*(a + b*x + c*x^2)^(5/2))/(6*(c*d^2 - b*d*e + a*e^2)*(d +
e*x)^6) - (7*e*(2*c*d - b*e)*(a + b*x + c*x^2)^(5/2))/(60*(c*d^2 - b*d*e +
a*e^2)^2*(d + e*x)^5) + ((b^2 - 4*a*c)^2*(24*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(6
*b*d + a*e))*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e
+ a*e^2]*Sqrt[a + b*x + c*x^2])])/(1024*(c*d^2 - b*d*e + a*e^2)^(9/2))
```

Rubi [A] time = 0.595926, antiderivative size = 409, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {744, 806, 720, 724, 206}

$$\frac{(a+bx+cx^2)^{3/2}\left(-4ce(ae+6bd)+7b^2e^2+24c^2d^2\right)(-2ae+x(2cd-be)+bd)}{192(d+ex)^4\left(ae^2-bde+cd^2\right)^3}-\frac{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(-4ce(ae+6b\right.}{512(d+ex)^{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x)^7,x]
```

```
[Out] -((b^2 - 4*a*c)*(24*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(6*b*d + a*e))*(b*d - 2*a*e
 + (2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(512*(c*d^2 - b*d*e + a*e^2)^4*(
d + e*x)^2) + ((24*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(6*b*d + a*e))*(b*d - 2*a*e
+ (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(3/2))/(192*(c*d^2 - b*d*e + a*e^2)^3*
(d + e*x)^4) - (e*(a + b*x + c*x^2)^(5/2))/(6*(c*d^2 - b*d*e + a*e^2)*(d +
e*x)^6) - (7*e*(2*c*d - b*e)*(a + b*x + c*x^2)^(5/2))/(60*(c*d^2 - b*d*e +
a*e^2)^2*(d + e*x)^5) + ((b^2 - 4*a*c)^2*(24*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(6
*b*d + a*e))*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e
+ a*e^2]*Sqrt[a + b*x + c*x^2])])/(1024*(c*d^2 - b*d*e + a*e^2)^(9/2))
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
```

```
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(d+ex)^7}\,dx = -\frac{e\left(a+bx+cx^2\right)^{5/2}}{6\left(cd^2-bde+ae^2\right)(d+ex)^6} - \frac{\int \frac{\left(\frac{1}{2}(-12cd+7be)+cex\right)\left(a+bx+cx^2\right)^{3/2}}{(d+ex)^6}\,dx}{6\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{e\left(a+bx+cx^2\right)^{5/2}}{6\left(cd^2-bde+ae^2\right)(d+ex)^6} - \frac{7e(2cd-be)\left(a+bx+cx^2\right)^{5/2}}{60\left(cd^2-bde+ae^2\right)^2(d+ex)^5} + \frac{\left(24c^2d^2+7b^2e^2-4ce(6b\right.}{24\left(cd^2-\right.}$$

$$= \frac{\left(24c^2d^2+7b^2e^2-4ce(6bd+ae)\right)(bd-2ae+(2cd-be)x)\left(a+bx+cx^2\right)^{3/2}}{192\left(cd^2-bde+ae^2\right)^3(d+ex)^4} - \frac{e\left(a+bx\right.}{6\left(cd^2-bde+\right.}$$

$$= -\frac{\left(b^2-4ac\right)\left(24c^2d^2+7b^2e^2-4ce(6bd+ae)\right)(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{512\left(cd^2-bde+ae^2\right)^4(d+ex)^2} + \frac{\left(24c\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(24c^2d^2+7b^2e^2-4ce(6bd+ae)\right)(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{512\left(cd^2-bde+ae^2\right)^4(d+ex)^2} + \frac{\left(24c\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(24c^2d^2+7b^2e^2-4ce(6bd+ae)\right)(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{512\left(cd^2-bde+ae^2\right)^4(d+ex)^2} + \frac{\left(24c\right.}{}$$

Mathematica [A] time = 1.86606, size = 349, normalized size = 0.85

$$-\frac{\left(-2ce(ae+6bd)+\frac{7b^2e^2}{2}+12c^2d^2\right)\left(3\left(b^2-4ac\right)\left(\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{8\left(e(ae-bd)+cd^2\right)^{3/2}}+\frac{\sqrt{a+x(b+cx)}(-2ae+b(d-ex)+2cdx)}{4(d+ex)^2\left(e(ae-bd)+cd^2\right)}\right)+\frac{2(a+\cdots}{\cdots}\right)}{192\left(e(ae-bd)+cd^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x)^7,x]
```

```
[Out] -(e*(a + x*(b + c*x))^(5/2))/(6*(c*d^2 + e*(-(b*d) + a*e))*(d + e*x)^6) - (
7*e*(2*c*d - b*e)*(a + x*(b + c*x))^(5/2))/(60*(c*d^2 + e*(-(b*d) + a*e))^2
*(d + e*x)^5) - ((12*c^2*d^2 + (7*b^2*e^2)/2 - 2*c*e*(6*b*d + a*e))*((2*(-(
b*d) + 2*a*e - 2*c*d*x + b*e*x)*(a + x*(b + c*x))^(3/2))/(d + e*x)^4 + 3*(b
^2 - 4*a*c)*((Sqrt[a + x*(b + c*x)]*(-2*a*e + 2*c*d*x + b*(d - e*x)))/(4*(c
*d^2 + e*(-(b*d) + a*e))*(d + e*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b*d) + 2*
a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*
x)])])/(8*(c*d^2 + e*(-(b*d) + a*e))^(3/2)))))/(192*(c*d^2 + e*(-(b*d) + a*
e))^3)
```

Maple [B] time = 0.269, size = 28629, normalized size = 70.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d)^7,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^7,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)**7,x)
```

```
[Out] Timed out
```

Giac [B] time = 89.3846, size = 19012, normalized size = 46.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^7,x, algorithm="giac")
```

```
[Out] 1/512*(24*b^4*c^2*d^2 - 192*a*b^2*c^3*d^2 + 384*a^2*c^4*d^2 - 24*b^5*c*d*e
+ 192*a*b^3*c^2*d*e - 384*a^2*b*c^3*d*e + 7*b^6*e^2 - 60*a*b^4*c*e^2 + 144*
a^2*b^2*c^2*e^2 - 64*a^3*c^3*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x +
a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/((c^4*d^8 - 4*b*c^3*d^7*e
+ 6*b^2*c^2*d^6*e^2 + 4*a*c^3*d^6*e^2 - 4*b^3*c*d^5*e^3 - 12*a*b*c^2*d^5*e^
3 + b^4*d^4*e^4 + 12*a*b^2*c*d^4*e^4 + 6*a^2*c^2*d^4*e^4 - 4*a*b^3*d^3*e^5
- 12*a^2*b*c*d^3*e^5 + 6*a^2*b^2*d^2*e^6 + 4*a^3*c*d^2*e^6 - 4*a^3*b*d*e^7
+ a^4*e^8)*sqrt(-c*d^2 + b*d*e - a*e^2)) + 1/7680*(24576*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^7*c^8*d^10*e + 8192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6
*c^(17/2)*d^11 + 30720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*c^(15/2)*d^9*e
^2 + 40960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b*c^(15/2)*d^10*e + 24576*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^8*d^11 + 20480*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^9*c^7*d^8*e^3 - 24576*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^5*a*c^8*d^10*e + 30720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*c^(15/2)*
d^11 - 46080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b*c^(13/2)*d^8*e^3 - 101
376*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^2*c^(13/2)*d^9*e^2 - 4096*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^6*a*c^(15/2)*d^9*e^2 - 46080*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^4*b^3*c^(13/2)*d^10*e - 61440*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^4*a*b*c^(15/2)*d^10*e + 20480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^3*b^3*c^7*d^11 - 81920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b*c^6*d^7*e
^4 - 119808*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^2*c^6*d^8*e^3 + 110592*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*c^7*d^8*e^3 - 119808*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^5*b^3*c^6*d^9*e^2 + 110592*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^5*a*b*c^7*d^9*e^2 - 43520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b
^4*c^6*d^10*e - 61440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^2*c^7*d^10*
e + 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^4*c^(13/2)*d^11 - 122880*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^2*c^(11/2)*d^7*e^4 + 122880*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^8*a*c^(13/2)*d^7*e^4 - 55296*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^6*b^3*c^(11/2)*d^8*e^3 + 405504*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^6*a*b*c^(13/2)*d^8*e^3 - 51840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^4*b^4*c^(11/2)*d^9*e^2 + 276480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*
b^2*c^(13/2)*d^9*e^2 + 30720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^(1
5/2)*d^9*e^2 - 18432*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^5*c^(11/2)*d^1
0*e - 30720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^3*c^(13/2)*d^10*e + 1
```

536*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^5*c^6*d^11 + 122880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^2*c^5*d^6*e^5 + 81920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*c^6*d^6*e^5 - 12288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^3 *c^5*d^7*e^4 + 49152*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b*c^6*d^7*e^4 + 41472*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^4*c^5*d^8*e^3 + 414720*(sqr t(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^2*c^6*d^8*e^3 - 110592*(sqrt(c)*x - s qrt(c*x^2 + b*x + a))^5*a^2*c^7*d^8*e^3 - 3840*(sqrt(c)*x - sqrt(c*x^2 + b* x + a))^3*b^5*c^5*d^9*e^2 + 266240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a* b^3*c^6*d^9*e^2 + 61440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c^7*d^9 *e^2 - 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^6*c^5*d^10*e - 7680*(sqrt (c)*x - sqrt(c*x^2 + b*x + a))*a*b^4*c^6*d^10*e + 128*b^6*c^(11/2)*d^11 + 3 37920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^3*c^(9/2)*d^6*e^5 - 61440*(sq rt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b*c^(11/2)*d^6*e^5 + 100800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^4*c^(9/2)*d^7*e^4 - 450048*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^2*c^(11/2)*d^7*e^4 - 549888*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*c^(13/2)*d^7*e^4 + 60480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^5*c^(9/2)*d^8*e^3 + 69120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a* b^3*c^(11/2)*d^8*e^3 - 414720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b*c ^(13/2)*d^8*e^3 + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^6*c^(9/2)*d^ 9*e^2 + 126720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^4*c^(11/2)*d^9*e^2 + 46080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^2*c^(13/2)*d^9*e^2 - 3 20*b^7*c^(9/2)*d^10*e - 768*a*b^5*c^(11/2)*d^10*e - 81920*(sqrt(c)*x - sqrt (c*x^2 + b*x + a))^9*b^3*c^4*d^5*e^6 - 245760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b*c^5*d^5*e^6 + 336960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^4 *c^4*d^6*e^5 + 93696*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^2*c^5*d^6*e^ 5 - 605184*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*c^6*d^6*e^5 + 87360*(s qrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^5*c^4*d^7*e^4 - 600576*(sqrt(c)*x - s qrt(c*x^2 + b*x + a))^5*a*b^3*c^5*d^7*e^4 - 1207296*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b*c^6*d^7*e^4 + 33920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^6*c^4*d^8*e^3 - 130560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^4*c^ 5*d^8*e^3 - 537600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^2*c^6*d^8*e^ 3 - 20480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*c^7*d^8*e^3 + 1152*(sqr t(c)*x - sqrt(c*x^2 + b*x + a))*b^7*c^4*d^9*e^2 + 29952*(sqrt(c)*x - sqrt(c *x^2 + b*x + a))*a*b^5*c^5*d^9*e^2 + 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^3*c^6*d^9*e^2 - 317520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^4* c^(7/2)*d^5*e^6 - 224640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^2*c^(9/2)*d^5*e^6 - 472320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*c^(11/2)*d^5*e ^6 + 95424*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^5*c^(7/2)*d^6*e^5 + 6988 80*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^3*c^(9/2)*d^6*e^5 - 193536*(sq rt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b*c^(11/2)*d^6*e^5 + 15120*(sqrt(c)* x - sqrt(c*x^2 + b*x + a))^4*b^6*c^(7/2)*d^7*e^4 - 228480*(sqrt(c)*x - sqrt (c*x^2 + b*x + a))^4*a*b^4*c^(9/2)*d^7*e^4 - 864000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^2*c^(11/2)*d^7*e^4 + 245760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*c^(13/2)*d^7*e^4 + 11136*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^7*c^(7/2)*d^8*e^3 - 88704*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^5 *c^(9/2)*d^8*e^3 - 322560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^3*c^(11/2)*d^8*e^3 - 30720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b*c^(13/2)* d^8*e^3 + 96*b^8*c^(7/2)*d^9*e^2 + 2816*a*b^6*c^(9/2)*d^9*e^2 + 1920*a^2*b^ 4*c^(11/2)*d^9*e^2 + 2720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^4*c^3*d^4 *e^7 + 387840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^2*c^4*d^4*e^7 - 161 280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*c^5*d^4*e^7 - 419328*(sqrt(c) *x - sqrt(c*x^2 + b*x + a))^7*b^5*c^3*d^5*e^6 - 368640*(sqrt(c)*x - sqrt(c* x^2 + b*x + a))^7*a*b^3*c^4*d^5*e^6 - 73728*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b*c^5*d^5*e^6 - 71808*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^6* c^3*d^6*e^5 + 822720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^4*c^4*d^6*e^ 5 + 907776*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^2*c^5*d^6*e^5 + 1219 584*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*c^6*d^6*e^5 - 18080*(sqrt(c)* x - sqrt(c*x^2 + b*x + a))^3*b^7*c^3*d^7*e^4 + 42240*(sqrt(c)*x - sqrt(c*x^ 2 + b*x + a))^3*a*b^5*c^4*d^7*e^4 - 145920*(sqrt(c)*x - sqrt(c*x^2 + b*x +

a))^3*a^2*b^3*c^5*d^7*e^4 + 573440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^
3*b*c^6*d^7*e^4 + 2112*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^8*c^3*d^8*e^3
- 21888*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^6*c^4*d^8*e^3 - 92160*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^4*c^5*d^8*e^3 - 15360*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))*a^3*b^2*c^6*d^8*e^3 - 3960*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^10*b^4*c^(5/2)*d^3*e^8 + 31680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
10*a*b^2*c^(7/2)*d^3*e^8 - 63360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^2
*c^(9/2)*d^3*e^8 + 99480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^5*c^(5/2)*
d^4*e^7 + 617280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^3*c^(7/2)*d^4*e^
7 + 455040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b*c^(9/2)*d^4*e^7 - 24
2840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^6*c^(5/2)*d^5*e^6 - 593760*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^4*c^(7/2)*d^5*e^6 - 1011840*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^6*a^2*b^2*c^(9/2)*d^5*e^6 + 1564160*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^6*a^3*c^(11/2)*d^5*e^6 - 64440*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^4*b^7*c^(5/2)*d^6*e^5 + 437280*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^4*a*b^5*c^(7/2)*d^6*e^5 + 670080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^4*a^2*b^3*c^(9/2)*d^6*e^5 + 2188800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
4*a^3*b*c^(11/2)*d^6*e^5 - 12384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^8*
c^(5/2)*d^7*e^4 + 54816*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^6*c^(7/2)
*d^7*e^4 + 89280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^4*c^(9/2)*d^7*
e^4 + 476160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^2*c^(11/2)*d^7*e^4
 + 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*c^(13/2)*d^7*e^4 + 176*b
^9*c^(5/2)*d^8*e^3 - 1920*a*b^7*c^(7/2)*d^8*e^3 - 10176*a^2*b^5*c^(9/2)*d^8
*e^3 - 2560*a^3*b^3*c^(11/2)*d^8*e^3 - 360*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^11*b^4*c^2*d^2*e^9 + 2880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a*b^2*
c^3*d^2*e^9 - 5760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^2*c^4*d^2*e^9 +
 15720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^5*c^2*d^3*e^8 - 207680*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^3*c^3*d^3*e^8 + 5760*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^9*a^2*b*c^4*d^3*e^8 + 170520*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^7*b^6*c^2*d^4*e^7 + 846240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a
*b^4*c^3*d^4*e^7 + 5760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^2*c^4*d
^4*e^7 + 1328640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*c^5*d^4*e^7 - 47
400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^7*c^2*d^5*e^6 - 362592*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^5*a*b^5*c^3*d^5*e^6 - 2436480*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^5*a^2*b^3*c^4*d^5*e^6 + 1033728*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^5*a^3*b*c^5*d^5*e^6 - 16320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^3*b^8*c^2*d^6*e^5 + 129920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^6*c^3
*d^6*e^5 - 52800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^4*c^4*d^6*e^5
+ 1374720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^2*c^5*d^6*e^5 - 18944
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*c^6*d^6*e^5 - 3120*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))*b^9*c^2*d^7*e^4 + 14496*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*a*b^7*c^3*d^7*e^4 + 40896*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*
b^5*c^4*d^7*e^4 + 168960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^3*c^5*d^
7*e^4 + 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b*c^6*d^7*e^4 + 3960*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*b^5*c^(3/2)*d^2*e^9 - 31680*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^10*a*b^3*c^(5/2)*d^2*e^9 + 63360*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^10*a^2*b*c^(7/2)*d^2*e^9 + 6390*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^8*b^6*c^(3/2)*d^3*e^8 - 333120*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^8*a*b^4*c^(5/2)*d^3*e^8 - 836640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^8*a^2*b^2*c^(7/2)*d^3*e^8 + 526080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a
^3*c^(9/2)*d^3*e^8 + 115328*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^7*c^(3/
2)*d^4*e^7 + 793248*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^5*c^(5/2)*d^4
e^7 + 480000(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b^3*c^(7/2)*d^4*e^7
 + 739840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^3*b*c^(9/2)*d^4*e^7 + 144
60*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^8*c^(3/2)*d^5*e^6 - 81720*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^6*c^(5/2)*d^5*e^6 - 1728000*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^4*a^2*b^4*c^(7/2)*d^5*e^6 - 792960*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^4*a^3*b^2*c^(9/2)*d^5*e^6 - 1198080*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^4*a^4*c^(11/2)*d^5*e^6 + 600*(sqrt(c)*x - sqrt(c*x^2 + b*

x + a))^2*b^9*c^(3/2)*d^6*e^5 + 26928*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2
*a*b^7*c^(5/2)*d^6*e^5 - 185472*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b
^5*c^(7/2)*d^6*e^5 + 357120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^3*c
^(9/2)*d^6*e^5 - 337920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b*c^(11/2
)*d^6*e^5 - 290*b^10*c^(3/2)*d^7*e^4 + 1272*a*b^8*c^(5/2)*d^7*e^4 + 4752*a^
2*b^6*c^(7/2)*d^7*e^4 + 21760*a^3*b^4*c^(9/2)*d^7*e^4 + 3840*a^4*b^2*c^(11/
2)*d^7*e^4 + 360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b^5*c*d*e^10 - 2880
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a*b^3*c^2*d*e^10 + 5760*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^11*a^2*b*c^3*d*e^10 - 3140*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^9*b^6*c*d^2*e^9 + 30120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*
a*b^4*c^2*d^2*e^9 + 32640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^2*c^3
*d^2*e^9 + 161920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*c^4*d^2*e^9 - 1
3896*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^7*c*d^3*e^8 - 436416*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^7*a*b^5*c^2*d^3*e^8 - 842880*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^7*a^2*b^3*c^3*d^3*e^8 - 552960*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^7*a^3*b*c^4*d^3*e^8 + 38784*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*
b^8*c*d^4*e^7 + 308424*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^6*c^2*d^4*
e^7 + 1435680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^4*c^3*d^4*e^7 + 1
192320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^2*c^4*d^4*e^7 - 1374720*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*c^5*d^4*e^7 + 9440*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^3*b^9*c*d^5*e^6 - 14160*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^3*a*b^7*c^2*d^5*e^6 - 454080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a
^2*b^5*c^3*d^5*e^6 - 684800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^3*c
^4*d^5*e^6 - 1827840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b*c^5*d^5*e^
6 + 900*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^10*c*d^6*e^5 + 5040*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*a*b^8*c^2*d^6*e^5 - 64128*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))*a^2*b^6*c^3*d^6*e^5 + 37440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))*a^3*b^4*c^4*d^6*e^5 - 192000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^2
*c^5*d^6*e^5 - 3072*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*c^6*d^6*e^5 - 1
155*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*b^6*sqrt(c)*d*e^10 + 9900*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^10*a*b^4*c^(3/2)*d*e^10 - 23760*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^10*a^2*b^2*c^(5/2)*d*e^10 + 10560*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^10*a^3*c^(7/2)*d*e^10 - 5355*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^8*b^7*sqrt(c)*d^2*e^9 + 27540*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8
*a*b^5*c^(3/2)*d^2*e^9 + 497520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b
^3*c^(5/2)*d^2*e^9 - 60480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^3*b*c^(7
/2)*d^2*e^9 - 9702*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^8*sqrt(c)*d^3*e^
8 - 403352*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^6*c^(3/2)*d^3*e^8 - 69
3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b^4*c^(5/2)*d^3*e^8 - 756480
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^3*b^2*c^(7/2)*d^3*e^8 - 951040*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*c^(9/2)*d^3*e^8 + 6930*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^4*b^9*sqrt(c)*d^4*e^7 - 3120*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^4*a*b^7*c^(3/2)*d^4*e^7 + 997200*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^4*a^2*b^5*c^(5/2)*d^4*e^7 + 1948800*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^4*a^3*b^3*c^(7/2)*d^4*e^7 - 441600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
4*a^4*b*c^(9/2)*d^4*e^7 + 1785*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^10*s
qrt(c)*d^5*e^6 - 10980*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^8*c^(3/2)*
d^5*e^6 - 14184*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^6*c^(5/2)*d^5*e
^6 - 26400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^4*c^(7/2)*d^5*e^6 -
996480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^2*c^(9/2)*d^5*e^6 + 1059
84*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*c^(11/2)*d^5*e^6 + 105*b^11*sq
rt(c)*d^6*e^5 + 620*a*b^9*c^(3/2)*d^6*e^5 - 7512*a^2*b^7*c^(5/2)*d^6*e^5 +
2592*a^3*b^5*c^(7/2)*d^6*e^5 - 35200*a^4*b^3*c^(9/2)*d^6*e^5 - 1536*a^5*b*c
^(11/2)*d^6*e^5 - 105*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b^6*e^11 + 900
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a*b^4*c*e^11 - 2160*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^11*a^2*b^2*c^2*e^11 + 960*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^11*a^3*c^3*e^11 - 595*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^7*d*e
^10 + 3060*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^5*c*d*e^10 + 4080*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^3*c^2*d*e^10 - 109120*(sqrt(c)*x -

sqrt(c*x^2 + b*x + a))^9*a^3*b*c^3*d*e^10 - 1386*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^8*d^2*e^9 + 21024*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^6*c*d^2*e^9 + 638640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^4*c^2*d^2*e^9 + 466560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b^2*c^3*d^2*e^9 - 172800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*c^4*d^2*e^9 - 1686*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^9*d^3*e^8 - 183000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^7*c*d^3*e^8 - 545904*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^5*c^2*d^3*e^8 - 1482240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^3*c^3*d^3*e^8 - 103680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b*c^4*d^3*e^8 + 595*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^10*d^4*e^7 - 31380*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^8*c*d^4*e^7 + 280200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^6*c^2*d^4*e^7 + 965600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^4*c^3*d^4*e^7 + 873600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^2*c^4*d^4*e^7 + 775680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*c^5*d^4*e^7 + 105*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^11*d^5*e^6 - 4140*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^9*c*d^5*e^6 + 11160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^7*c^2*d^5*e^6 + 66720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^5*c^3*d^5*e^6 - 236160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^3*c^4*d^5*e^6 + 115200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b*c^5*d^5*e^6 + 5355*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^6*sqrt(c)*d*e^10 - 45900*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b^4*c^(3/2)*d*e^10 - 197040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^3*b^2*c^(5/2)*d*e^10 - 18240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^4*c^(7/2)*d*e^10 + 19404*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^7*sqrt(c)*d^2*e^9 + 542784*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b^5*c^(3/2)*d^2*e^9 + 374720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^3*b^3*c^(5/2)*d^2*e^9 + 821760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*b*c^(7/2)*d^2*e^9 - 36150*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^8*sqrt(c)*d^3*e^8 - 162060*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^6*c^(3/2)*d^3*e^8 - 1591200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^4*c^(5/2)*d^3*e^8 - 619200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b^2*c^(7/2)*d^3*e^8 + 744960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*c^(9/2)*d^3*e^8 - 7140*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^9*sqrt(c)*d^4*e^7 + 44640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^7*c^(3/2)*d^4*e^7 + 95520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^5*c^(5/2)*d^4*e^7 + 518400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^3*c^(7/2)*d^4*e^7 + 898560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b*c^(9/2)*d^4*e^7 - 525*a*b^10*sqrt(c)*d^5*e^6 + 1320*a^2*b^8*c^(3/2)*d^5*e^6 + 11704*a^3*b^6*c^(5/2)*d^5*e^6 - 22320*a^4*b^4*c^(7/2)*d^5*e^6 + 30720*a^5*b^2*c^(9/2)*d^5*e^6 + 256*a^6*c^(11/2)*d^5*e^6 + 595*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^6*e^11 - 5100*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^4*c*e^11 + 12240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*b^2*c^2*e^11 + 15040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^4*c^3*e^11 + 2772*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^7*d*e^10 - 19008*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^5*c*d*e^10 - 484800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b^3*c^2*d*e^10 + 99840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*b*c^3*d*e^10 + 5058*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^8*d^2*e^9 + 292248*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^6*c*d^2*e^9 + 462480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^4*c^2*d^2*e^9 + 748800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b^2*c^3*d^2*e^9 + 449280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^5*c^4*d^2*e^9 - 2380*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^9*d^3*e^8 + 11880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^7*c*d^3*e^8 - 645120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^5*c^2*d^3*e^8 - 944000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^3*c^3*d^3*e^8 - 61440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b*c^4*d^3*e^8 - 525*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^10*d^4*e^7 + 8460*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^8*c*d^4*e^7 - 33960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^6*c^2*d^4*e^7 + 66240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^4*c^3*d^4*e^7 + 337536*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^2*c^4*d^4*e^7 - 21504*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*c^5*d^4*e^7 + 76800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^4*b*c^(5/2)*e^11 -

9702*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b^6*sqrt(c)*d*e^10 - 367400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^3*b^4*c^(3/2)*d*e^10 - 300960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*b^2*c^(5/2)*d*e^10 + 51840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^5*c^(7/2)*d*e^10 + 66870*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^7*sqrt(c)*d^2*e^9 + 337080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^5*c^(3/2)*d^2*e^9 + 928800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b^3*c^(5/2)*d^2*e^9 + 5760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b*c^(7/2)*d^2*e^9 + 10710*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^8*sqrt(c)*d^3*e^8 - 104760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^6*c^(3/2)*d^3*e^8 - 265080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^4*c^(5/2)*d^3*e^8 - 556992*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^2*c^(7/2)*d^3*e^8 - 258432*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*c^(9/2)*d^3*e^8 + 1050*a^2*b^9*sqrt(c)*d^4*e^7 - 5680*a^3*b^7*c^(3/2)*d^4*e^7 - 3240*a^4*b^5*c^(5/2)*d^4*e^7 + 42048*a^5*b^3*c^(7/2)*d^4*e^7 - 11392*a^6*b*c^(9/2)*d^4*e^7 - 1386*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^6*e^11 + 11880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b^4*c*e^11 + 97440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*b^2*c^2*e^11 + 24960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^5*c^3*e^11 - 5058*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^7*d*e^10 - 190632*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^5*c*d*e^10 - 305760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b^3*c^2*d*e^10 - 293760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^5*b*c^3*d*e^10 + 3570*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^8*d^2*e^9 + 58240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^6*c*d^2*e^9 + 646200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^4*c^2*d^2*e^9 + 275520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b^2*c^3*d^2*e^9 - 149120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*c^4*d^2*e^9 + 1050*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^9*d^3*e^8 - 10440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^7*c*d^3*e^8 + 18120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^5*c^2*d^3*e^8 - 195264*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^3*c^3*d^3*e^8 - 215424*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b*c^4*d^3*e^8 + 112640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*b^3*c^(3/2)*e^11 + 61440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^5*b*c^(5/2)*e^11 - 53010*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^6*sqrt(c)*d*e^10 - 247800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b^4*c^(3/2)*d*e^10 - 280800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b^2*c^(5/2)*d*e^10 - 59520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^6*c^(7/2)*d*e^10 - 7140*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^7*sqrt(c)*d^2*e^9 + 147240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^5*c^(3/2)*d^2*e^9 + 252288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^3*c^(5/2)*d^2*e^9 + 163968*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*b*c^(7/2)*d^2*e^9 - 1050*a^3*b^8*sqrt(c)*d^3*e^8 + 7270*a^4*b^6*c^(3/2)*d^3*e^8 - 11232*a^5*b^4*c^(5/2)*d^3*e^8 - 45600*a^6*b^2*c^(7/2)*d^3*e^8 + 1792*a^7*c^(9/2)*d^3*e^8 + 1686*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^6*e^11 + 42600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b^4*c*e^11 + 128160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^5*b^2*c^2*e^11 + 24960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^6*c^3*e^11 - 2380*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^7*d*e^10 - 73800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^5*c*d*e^10 - 309120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b^3*c^2*d*e^10 + 30080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*b*c^3*d*e^10 - 1050*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^8*d^2*e^9 + 8460*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^6*c*d^2*e^9 + 27720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^4*c^2*d^2*e^9 + 144768*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b^2*c^3*d^2*e^9 + 56448*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7*c^4*d^2*e^9 + 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b^5*sqrt(c)*e^11 + 61440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b^3*c^(3/2)*e^11 + 92160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^6*b*c^(5/2)*e^11 + 1785*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^6*sqrt(c)*d*e^10 - 107460*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^4*c^(3/2)*d*e^10 - 95376*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*b^2*c^(5/2)*d*e^10 + 20544*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^7*c^(7/2)*d*e^10 + 525*a^4*b^7*sqrt(c)*d^2*e^9 - 4140*a^5*b^5*c^(3/2)*d^2*e^9 + 17136*a^6*b^3*c^(5/2)*d^2*e^9 + 25536*a^7*b*c^(7/2)*d^2*e^9 +

595*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^6*e^11 + 25620*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b^4*c*e^11 + 58320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*b^2*c^2*e^11 + 15040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^7*c^3*e^11 + 525*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^7*d*e^10 - 4140*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^5*c*d*e^10 - 38160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b^3*c^2*d*e^10 - 35904*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7*b*c^3*d*e^10 + 30720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*b^3*c^(3/2)*e^11 + 12288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^7*b*c^(5/2)*e^11 - 105*a^5*b^6*sqrt(c)*d*e^10 + 900*a^6*b^4*c^(3/2)*d*e^10 - 11376*a^7*b^2*c^(5/2)*d*e^10 - 5184*a^8*c^(7/2)*d*e^10 - 105*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^6*e^11 + 900*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b^4*c*e^11 + 13200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7*b^2*c^2*e^11 + 960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^8*c^3*e^11 + 3072*a^8*b*c^(5/2)*e^11)/((c^4*d^8*e^4 - 4*b*c^3*d^7*e^5 + 6*b^2*c^2*d^6*e^6 + 4*a*c^3*d^6*e^6 - 4*b^3*c*d^5*e^7 - 12*a*b*c^2*d^5*e^7 + b^4*d^4*e^8 + 12*a*b^2*c*d^4*e^8 + 6*a^2*c^2*d^4*e^8 - 4*a*b^3*d^3*e^9 - 12*a^2*b*c*d^3*e^9 + 6*a^2*b^2*d^2*e^10 + 4*a^3*c*d^2*e^10 - 4*a^3*b*d*e^11 + a^4*e^12)*((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*e + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c)*d + b*d - a*e)^6)

3.2354 $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)^8}\,dx$

Optimal. Leaf size=510

$$-\frac{e\left(a+bx+cx^2\right)^{5/2}\left(-4ce(4ae+17bd)+21b^2e^2+68c^2d^2\right)}{280(d+ex)^5\left(ae^2-bde+cd^2\right)^3}+\frac{\left(a+bx+cx^2\right)^{3/2}(2cd-be)\left(-4ce(ae+2bd)+3b^2e^2+8c^2d\right.}{128(d+ex)^4\left(ae^2-bde+cd^2\right)^4}$$

```
[Out] (-3*(b^2 - 4*a*c)*(2*c*d - b*e)*(8*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(2*b*d + a*e
))*(b*d - 2*a*e + (2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(1024*(c*d^2 - b*
d*e + a*e^2)^5*(d + e*x)^2) + ((2*c*d - b*e)*(8*c^2*d^2 + 3*b^2*e^2 - 4*c*e
*(2*b*d + a*e))*(b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(3/2))/(1
28*(c*d^2 - b*d*e + a*e^2)^4*(d + e*x)^4) - (e*(a + b*x + c*x^2)^(5/2))/(7*
(c*d^2 - b*d*e + a*e^2)*(d + e*x)^7) - (3*e*(2*c*d - b*e)*(a + b*x + c*x^2)
^(5/2))/(28*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^6) - (e*(68*c^2*d^2 + 21*b^
2*e^2 - 4*c*e*(17*b*d + 4*a*e))*(a + b*x + c*x^2)^(5/2))/(280*(c*d^2 - b*d*
e + a*e^2)^3*(d + e*x)^5) + (3*(b^2 - 4*a*c)^2*(2*c*d - b*e)*(8*c^2*d^2 + 3
*b^2*e^2 - 4*c*e*(2*b*d + a*e))*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*
Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(2048*(c*d^2 - b*d*e +
 a*e^2)^(11/2))
```

Rubi [A] time = 0.765908, antiderivative size = 510, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {744, 834, 806, 720, 724, 206}

$$-\frac{e\left(a+bx+cx^2\right)^{5/2}\left(-4ce(4ae+17bd)+21b^2e^2+68c^2d^2\right)}{280(d+ex)^5\left(ae^2-bde+cd^2\right)^3}+\frac{\left(a+bx+cx^2\right)^{3/2}(2cd-be)\left(-4ce(ae+2bd)+3b^2e^2+8c^2d\right.}{128(d+ex)^4\left(ae^2-bde+cd^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x)^8,x]
```

```
[Out] (-3*(b^2 - 4*a*c)*(2*c*d - b*e)*(8*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(2*b*d + a*e
))*(b*d - 2*a*e + (2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(1024*(c*d^2 - b*
d*e + a*e^2)^5*(d + e*x)^2) + ((2*c*d - b*e)*(8*c^2*d^2 + 3*b^2*e^2 - 4*c*e
*(2*b*d + a*e))*(b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(3/2))/(1
28*(c*d^2 - b*d*e + a*e^2)^4*(d + e*x)^4) - (e*(a + b*x + c*x^2)^(5/2))/(7*
(c*d^2 - b*d*e + a*e^2)*(d + e*x)^7) - (3*e*(2*c*d - b*e)*(a + b*x + c*x^2)
^(5/2))/(28*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^6) - (e*(68*c^2*d^2 + 21*b^
2*e^2 - 4*c*e*(17*b*d + 4*a*e))*(a + b*x + c*x^2)^(5/2))/(280*(c*d^2 - b*d*
e + a*e^2)^3*(d + e*x)^5) + (3*(b^2 - 4*a*c)^2*(2*c*d - b*e)*(8*c^2*d^2 + 3
*b^2*e^2 - 4*c*e*(2*b*d + a*e))*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*
Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(2048*(c*d^2 - b*d*e +
 a*e^2)^(11/2))
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
```

```
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(d+ex)^8}\,dx = -\frac{e\left(a+bx+cx^2\right)^{5/2}}{7\left(cd^2-bde+ae^2\right)(d+ex)^7} - \frac{\int \frac{\left(\frac{1}{2}(-14cd+9be)+2cex\right)\left(a+bx+cx^2\right)^{3/2}}{(d+ex)^7}dx}{7\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{e\left(a+bx+cx^2\right)^{5/2}}{7\left(cd^2-bde+ae^2\right)(d+ex)^7} - \frac{3e(2cd-be)\left(a+bx+cx^2\right)^{5/2}}{28\left(cd^2-bde+ae^2\right)^2(d+ex)^6} + \frac{\int \frac{\left(\frac{3}{4}(56c^2d^2+21b^2e^2-2ce(31bd+8ae\right.}{(d+e}}{42\left(cd^2-b\right.}$$

$$= -\frac{e\left(a+bx+cx^2\right)^{5/2}}{7\left(cd^2-bde+ae^2\right)(d+ex)^7} - \frac{3e(2cd-be)\left(a+bx+cx^2\right)^{5/2}}{28\left(cd^2-bde+ae^2\right)^2(d+ex)^6} - \frac{e\left(68c^2d^2+21b^2e^2-4ce(17b\right.}{280\left(cd^2-bde+\right.}$$

$$= \frac{(2cd-be)\left(8c^2d^2+3b^2e^2-4ce(2bd+ae)\right)(bd-2ae+(2cd-be)x)\left(a+bx+cx^2\right)^{3/2}}{128\left(cd^2-bde+ae^2\right)^4(d+ex)^4} - \frac{e\left(\right.}{7\left(cd^2-\right.}$$

$$= -\frac{3\left(b^2-4ac\right)(2cd-be)\left(8c^2d^2+3b^2e^2-4ce(2bd+ae)\right)(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{1024\left(cd^2-bde+ae^2\right)^5(d+ex)^2}$$

$$= -\frac{3\left(b^2-4ac\right)(2cd-be)\left(8c^2d^2+3b^2e^2-4ce(2bd+ae)\right)(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{1024\left(cd^2-bde+ae^2\right)^5(d+ex)^2}$$

$$= -\frac{3\left(b^2-4ac\right)(2cd-be)\left(8c^2d^2+3b^2e^2-4ce(2bd+ae)\right)(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{1024\left(cd^2-bde+ae^2\right)^5(d+ex)^2}$$

Mathematica [A] time = 6.06938, size = 687, normalized size = 1.35

$$-\frac{(a+x(b+cx))^{3/2}\left(-\frac{-\frac{\left(a+bx+cx^2\right)^{5/2}\left(\frac{3}{4}e\left(-2ce(8ae+31bd)+21b^2e^2+56c^2d^2\right)+\frac{9}{2}cde(2cd-be)\right)}{5(d+ex)^5\left(ae^2-bde+cd^2\right)} - \frac{\left(b\left(\frac{3}{4}e\left(-2ce(8ae+31bd)+21b^2e^2+56c^2d^2\right)-\frac{9}{2}cde(2cd-be)\right)-2\left(\frac{3}{4}cd\left(-2ce(8ae+31bd)+\right.\right.\right.}{}}{6}\right.}{7\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x)^8,x]
```

```
[Out] -(e*(a + b*x + c*x^2)*(a + x*(b + c*x))^(3/2))/(7*(c*d^2 - b*d*e + a*e^2)*(
d + e*x)^7) - ((a + x*(b + c*x))^(3/2)*(-((-2*c*d*e + (e*(-14*c*d + 9*b*e))
/2)*(a + b*x + c*x^2)^(5/2))/(6*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^6) - (-((
(9*c*d*e*(2*c*d - b*e))/2 + (3*e*(56*c^2*d^2 + 21*b^2*e^2 - 2*c*e*(31*b*d +
 8*a*e)))/4)*(a + b*x + c*x^2)^(5/2))/(5*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^
5) - ((-2*((-9*a*c*e^2*(2*c*d - b*e))/2 + (3*c*d*(56*c^2*d^2 + 21*b^2*e^2 -
 2*c*e*(31*b*d + 8*a*e)))/4) + b*((-9*c*d*e*(2*c*d - b*e))/2 + (3*e*(56*c^2
*d^2 + 21*b^2*e^2 - 2*c*e*(31*b*d + 8*a*e)))/4))*(((b*d - 2*a*e + (2*c*d -
b*e)*x)*(a + b*x + c*x^2)^(3/2))/(8*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^4) -
(3*(b^2 - 4*a*c)*(((b*d - 2*a*e + (2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(
```

```
4*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b*d) + 2
*a*e - (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2
])])/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*(4*c*d^2 - 4*b*d*e + 4*a*e^2))))/(16*(c
*d^2 - b*d*e + a*e^2))))/(2*(c*d^2 - b*d*e + a*e^2)))/(6*(c*d^2 - b*d*e + a
*e^2))))/(7*(c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2)^(3/2))
```

Maple [B] time = 0.275, size = 35234, normalized size = 69.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d)^8,x)

[Out] result too large to display
```

Maple [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^8,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^8,x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)**8,x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^8,x, algorithm="giac")
```

```
[Out] Timed out
```

3.2355 $\int (d+ex)^3 \left(a+bx+cx^2\right)^{5/2} dx$

Optimal. Leaf size=400

$$\frac{e\left(a+bx+cx^2\right)^{7/2}\left(-2ce(32ae+243bd)+99b^2e^2+154cex(2cd-be)+640c^2d^2\right)}{2016c^3}+\frac{(b+2cx)\left(a+bx+cx^2\right)^{5/2}(2cd-$$

```
[Out] (5*(b^2 - 4*a*c)^2*(2*c*d - b*e)*(32*c^2*d^2 + 11*b^2*e^2 - 4*c*e*(8*b*d +
3*a*e))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(32768*c^6) - (5*(b^2 - 4*a*c)*(
2*c*d - b*e)*(32*c^2*d^2 + 11*b^2*e^2 - 4*c*e*(8*b*d + 3*a*e))*(b + 2*c*x)*
(a + b*x + c*x^2)^(3/2))/(12288*c^5) + ((2*c*d - b*e)*(32*c^2*d^2 + 11*b^2*
e^2 - 4*c*e*(8*b*d + 3*a*e))*(b + 2*c*x)*(a + b*x + c*x^2)^(5/2))/(768*c^4)
 + (e*(d + e*x)^2*(a + b*x + c*x^2)^(7/2))/(9*c) + (e*(640*c^2*d^2 + 99*b^2
*e^2 - 2*c*e*(243*b*d + 32*a*e) + 154*c*e*(2*c*d - b*e)*x)*(a + b*x + c*x^2
)^(7/2))/(2016*c^3) - (5*(b^2 - 4*a*c)^3*(2*c*d - b*e)*(32*c^2*d^2 + 11*b^2
*e^2 - 4*c*e*(8*b*d + 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x +
 c*x^2])])/(65536*c^(13/2))
```

Rubi [A] time = 0.43993, antiderivative size = 400, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {742, 779, 612, 621, 206}

$$\frac{e\left(a+bx+cx^2\right)^{7/2}\left(-2ce(32ae+243bd)+99b^2e^2+154cex(2cd-be)+640c^2d^2\right)}{2016c^3}+\frac{(b+2cx)\left(a+bx+cx^2\right)^{5/2}(2cd-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*x + c*x^2)^(5/2),x]
```


```
[Out] (5*(b^2 - 4*a*c)^2*(2*c*d - b*e)*(32*c^2*d^2 + 11*b^2*e^2 - 4*c*e*(8*b*d +
3*a*e))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(32768*c^6) - (5*(b^2 - 4*a*c)*(
2*c*d - b*e)*(32*c^2*d^2 + 11*b^2*e^2 - 4*c*e*(8*b*d + 3*a*e))*(b + 2*c*x)*
(a + b*x + c*x^2)^(3/2))/(12288*c^5) + ((2*c*d - b*e)*(32*c^2*d^2 + 11*b^2*
e^2 - 4*c*e*(8*b*d + 3*a*e))*(b + 2*c*x)*(a + b*x + c*x^2)^(5/2))/(768*c^4)
 + (e*(d + e*x)^2*(a + b*x + c*x^2)^(7/2))/(9*c) + (e*(640*c^2*d^2 + 99*b^2
*e^2 - 2*c*e*(243*b*d + 32*a*e) + 154*c*e*(2*c*d - b*e)*x)*(a + b*x + c*x^2
)^(7/2))/(2016*c^3) - (5*(b^2 - 4*a*c)^3*(2*c*d - b*e)*(32*c^2*d^2 + 11*b^2
*e^2 - 4*c*e*(8*b*d + 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x +
 c*x^2])])/(65536*c^(13/2))
```


Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
```

```
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^3 \left(a+bx+cx^2\right)^{5/2} dx = \frac{e(d+ex)^2\left(a+bx+cx^2\right)^{7/2}}{9c} + \frac{\int (d+ex)\left(\frac{1}{2}\left(18cd^2-e(7bd+4ae)\right)+\frac{11}{2}e(2cd-be)x\right)\left(\right.}{9c}$$

$$= \frac{e(d+ex)^2\left(a+bx+cx^2\right)^{7/2}}{9c} + \frac{e\left(640c^2d^2+99b^2e^2-2ce(243bd+32ae)+154ce(2cd-\right.}{2016c^3}$$

$$= \frac{(2cd-be)\left(32c^2d^2+11b^2e^2-4ce(8bd+3ae)\right)(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{768c^4} + \frac{e(d+ex)^2}{}$$

$$= -\frac{5\left(b^2-4ac\right)(2cd-be)\left(32c^2d^2+11b^2e^2-4ce(8bd+3ae)\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{12288c^5}$$

$$= \frac{5\left(b^2-4ac\right)^2(2cd-be)\left(32c^2d^2+11b^2e^2-4ce(8bd+3ae)\right)(b+2cx)\sqrt{a+bx+cx^2}}{32768c^6} -$$

$$= \frac{5\left(b^2-4ac\right)^2(2cd-be)\left(32c^2d^2+11b^2e^2-4ce(8bd+3ae)\right)(b+2cx)\sqrt{a+bx+cx^2}}{32768c^6} -$$

$$= \frac{5\left(b^2-4ac\right)^2(2cd-be)\left(32c^2d^2+11b^2e^2-4ce(8bd+3ae)\right)(b+2cx)\sqrt{a+bx+cx^2}}{32768c^6} -$$

Mathematica [A] time = 0.821426, size = 280, normalized size = 0.7

$$\frac{\frac{3(2cd-be)\left(-4ce(3ae+8bd)+11b^2e^2+32c^2d^2\right)\left(256c^{5/2}(b+2cx)(a+x(b+cx))^{5/2}-5\left(b^2-4ac\right)\left(16c^{3/2}(b+2cx)(a+x(b+cx))^{3/2}-3\left(b^2-4ac\right)\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}-\left(b^2-4a\right.\right.\right.\right.}{65536c^{11/2}}}{9c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (e*(d + e*x)^2*(a + x*(b + c*x))^(7/2) + (e*(a + x*(b + c*x))^(7/2)*(99*b^2
*e^2 + 4*c^2*d*(160*d + 77*e*x) - 2*c*e*(243*b*d + 32*a*e + 77*b*e*x)))/(22
4*c^2) + (3*(2*c*d - b*e)*(32*c^2*d^2 + 11*b^2*e^2 - 4*c*e*(8*b*d + 3*a*e))
*(256*c^(5/2)*(b + 2*c*x)*(a + x*(b + c*x))^(5/2) - 5*(b^2 - 4*a*c)*(16*c^(
3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*Sqrt[c]*(b +
2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]
*Sqrt[a + x*(b + c*x)])]))))/(65536*c^(11/2)))/(9*c)
```

Maple [B] time = 0.056, size = 2294, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 1/6*d^3*x*(c*x^2+b*x+a)^(5/2)-5/128*d*e^2*a^2/c^2*(c*x^2+b*x+a)^(3/2)*b-15/
128*d*e^2*a^3/c*(c*x^2+b*x+a)^(1/2)*x+45/128*d*e^2*b^2/c^(5/2)*ln((1/2*b+c*
x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^3+125/4096*e^3*b^5/c^4*(c*x^2+b*x+a)^(1/2
)*x*a+5/128*e^3*b/c^2*a^2*(c*x^2+b*x+a)^(3/2)*x+9/64*d*e^2*b^2/c^2*x*(c*x^2
+b*x+a)^(5/2)-45/1024*d*e^2*b^4/c^3*(c*x^2+b*x+a)^(3/2)*x+25/256*d*e^2*b^3/
c^3*(c*x^2+b*x+a)^(3/2)*a+135/8192*d*e^2*b^6/c^4*(c*x^2+b*x+a)^(1/2)*x+165/
1024*d*e^2*b^3/c^3*(c*x^2+b*x+a)^(1/2)*a^2-285/4096*d*e^2*b^5/c^4*(c*x^2+b*
x+a)^(1/2)*a+55/65536*e^3*b^9/c^(13/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)
^(1/2))-2/63*e^3*a/c^2*(c*x^2+b*x+a)^(7/2)+55/12288*e^3*b^6/c^5*(c*x^2+b*x+
a)^(3/2)-55/32768*e^3*b^8/c^6*(c*x^2+b*x+a)^(1/2)+11/224*e^3*b^2/c^3*(c*x^2
+b*x+a)^(7/2)-11/768*e^3*b^4/c^4*(c*x^2+b*x+a)^(5/2)+1/12*d^3/c*(c*x^2+b*x+
a)^(5/2)*b+5/24*d^3*(c*x^2+b*x+a)^(3/2)*x*a-5/192*d^3/c^2*(c*x^2+b*x+a)^(3/
2)*b^3+5/16*d^3*(c*x^2+b*x+a)^(1/2)*x*a^2+5/512*d^3/c^3*(c*x^2+b*x+a)^(1/2)
*b^5+5/16*d^3/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^3-5/102
4*d^3/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^6+3/7*d^2*e*(c*
x^2+b*x+a)^(7/2)/c-11/144*e^3*b/c^2*x*(c*x^2+b*x+a)^(7/2)-11/384*e^3*b^3/c^
3*x*(c*x^2+b*x+a)^(5/2)+1/9*e^3*x^2*(c*x^2+b*x+a)^(7/2)/c-285/2048*d*e^2*b^
4/c^3*(c*x^2+b*x+a)^(1/2)*x*a+25/128*d*e^2*b^2/c^2*(c*x^2+b*x+a)^(3/2)*x*a+
15/64*d^2*e*b^3/c^2*(c*x^2+b*x+a)^(1/2)*x*a+165/512*d*e^2*b^2/c^2*(c*x^2+b*
x+a)^(1/2)*x*a^2-5/16*d^2*e*b/c*(c*x^2+b*x+a)^(3/2)*x*a-15/32*d^2*e*b/c*(c*
x^2+b*x+a)^(1/2)*x*a^2+15/256*d^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x
+a)^(1/2))*b^4*a-135/32768*d*e^2*b^8/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))-15/128*d*e^2*a^4/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))+3/8*d*e^2*x*(c*x^2+b*x+a)^(7/2)/c-45/2048*d*e^2*b^5/c^4*(c*x^2+b*x
+a)^(3/2)+135/16384*d*e^2*b^7/c^5*(c*x^2+b*x+a)^(1/2)+125/8192*e^3*b^6/c^5*
(c*x^2+b*x+a)^(1/2)*a+1/64*e^3*b^2/c^3*a*(c*x^2+b*x+a)^(5/2)+5/256*e^3*b^2/
c^3*a^2*(c*x^2+b*x+a)^(3/2)+15/512*e^3*b^2/c^3*a^3*(c*x^2+b*x+a)^(1/2)-1/8*
d^2*e*b^2/c^2*(c*x^2+b*x+a)^(5/2)+5/128*d^2*e*b^4/c^3*(c*x^2+b*x+a)^(3/2)-1
5/1024*d^2*e*b^6/c^4*(c*x^2+b*x+a)^(1/2)+15/2048*d^2*e*b^7/c^(9/2)*ln((1/2*
b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-27/112*d*e^2*b/c^2*(c*x^2+b*x+a)^(7/2)+
9/128*d*e^2*b^3/c^3*(c*x^2+b*x+a)^(5/2)-5/96*d^3/c*(c*x^2+b*x+a)^(3/2)*x*b^
2+5/48*d^3/c*(c*x^2+b*x+a)^(3/2)*b*a+5/256*d^3/c^2*(c*x^2+b*x+a)^(1/2)*x*b^
4+5/32*d^3/c*(c*x^2+b*x+a)^(1/2)*b*a^2-5/64*d^3/c^2*(c*x^2+b*x+a)^(1/2)*b^3
*a-15/64*d^3/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2*a^2+55
/6144*e^3*b^5/c^4*(c*x^2+b*x+a)^(3/2)*x+105/2048*e^3*b^5/c^(9/2)*ln((1/2*b+
c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2-45/4096*e^3*b^7/c^(11/2)*ln((1/2*b+c*
x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-25/256*e^3*b^3/c^(7/2)*ln((1/2*b+c*x)/c^(
1/2)+(c*x^2+b*x+a)^(1/2))*a^3+15/256*e^3*b/c^(5/2)*a^4*ln((1/2*b+c*x)/c^(1/
2)+(c*x^2+b*x+a)^(1/2))-35/1536*e^3*b^4/c^4*(c*x^2+b*x+a)^(3/2)*a-55/16384*
e^3*b^7/c^5*(c*x^2+b*x+a)^(1/2)*x-85/2048*e^3*b^4/c^4*(c*x^2+b*x+a)^(1/2)*a
^2-15/256*d*e^2*a^3/c^2*(c*x^2+b*x+a)^(1/2)*b-1/16*d*e^2*a/c*x*(c*x^2+b*x+a
```

```
)^(5/2)-1/32*d*e^2*a/c^2*(c*x^2+b*x+a)^(5/2)*b-5/64*d*e^2*a^2/c*(c*x^2+b*x+
a)^(3/2)*x-5/32*d^3/c*(c*x^2+b*x+a)^(1/2)*x*a*b^2-85/1024*e^3*b^3/c^3*(c*x^
2+b*x+a)^(1/2)*x*a^2-35/768*e^3*b^3/c^3*(c*x^2+b*x+a)^(3/2)*x*a-15/512*d^2*
e*b^5/c^3*(c*x^2+b*x+a)^(1/2)*x-15/64*d^2*e*b^2/c^2*(c*x^2+b*x+a)^(1/2)*a^2
+15/128*d^2*e*b^4/c^3*(c*x^2+b*x+a)^(1/2)*a-15/32*d^2*e*b/c^(3/2)*ln((1/2*b
+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^3+45/128*d^2*e*b^3/c^(5/2)*ln((1/2*b+c
*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2-45/512*d^2*e*b^5/c^(7/2)*ln((1/2*b+c*x
)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+15/256*e^3*b/c^2*a^3*(c*x^2+b*x+a)^(1/2)*x
-1/4*d^2*e*b/c*x*(c*x^2+b*x+a)^(5/2)+5/64*d^2*e*b^3/c^2*(c*x^2+b*x+a)^(3/2)
*x-5/32*d^2*e*b^2/c^2*(c*x^2+b*x+a)^(3/2)*a+1/32*e^3*b/c^2*a*x*(c*x^2+b*x+a
)^(5/2)-225/1024*d*e^2*b^4/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/
2))*a^2+105/2048*d*e^2*b^6/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/
2))*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 6.85473, size = 4959, normalized size = 12.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")

[Out] [-1/8257536*(315*(64*(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)
*d^3 - 96*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^2*e +
6*(9*b^8*c - 112*a*b^6*c^2 + 480*a^2*b^4*c^3 - 768*a^3*b^2*c^4 + 256*a^4*c^
5)*d*e^2 - (11*b^9 - 144*a*b^7*c + 672*a^2*b^5*c^2 - 1280*a^3*b^3*c^3 + 768
*a^4*b*c^4)*e^3)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*
x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(229376*c^9*e^3*x^8 + 14336*(54*c^9
*d*e^2 + 37*b*c^8*e^3)*x^7 + 1024*(864*c^9*d^2*e + 1782*b*c^8*d*e^2 + (309*
b^2*c^7 + 608*a*c^8)*e^3)*x^6 + 256*(1344*c^9*d^3 + 8352*b*c^8*d^2*e + 18*(
243*b^2*c^7 + 476*a*c^8)*d*e^2 + (5*b^3*c^6 + 3012*a*b*c^7)*e^3)*x^5 + 128*
(6720*b*c^8*d^3 + 288*(37*b^2*c^7 + 72*a*c^8)*d^2*e + 18*(3*b^3*c^6 + 1228*
a*b*c^7)*d*e^2 - (11*b^4*c^5 - 84*a*b^2*c^6 - 3840*a^2*c^7)*e^3)*x^4 + 1344
*(15*b^5*c^4 - 160*a*b^3*c^5 + 528*a^2*b*c^6)*d^3 - 288*(105*b^6*c^3 - 1120
*a*b^4*c^4 + 3696*a^2*b^2*c^5 - 3072*a^3*c^6)*d^2*e + 18*(945*b^7*c^2 - 105
00*a*b^5*c^3 + 37744*a^2*b^3*c^4 - 42432*a^3*b*c^5)*d*e^2 - (3465*b^8*c - 4
0740*a*b^6*c^2 + 162288*a^2*b^4*c^3 - 234432*a^3*b^2*c^4 + 65536*a^4*c^5)*e
^3 + 16*(1344*(27*b^2*c^7 + 52*a*c^8)*d^3 + 288*(3*b^3*c^6 + 788*a*b*c^7)*d
^2*e - 18*(27*b^4*c^5 - 216*a*b^2*c^6 - 6608*a^2*c^7)*d*e^2 + (99*b^5*c^4 -
 856*a*b^3*c^5 + 1968*a^2*b*c^6)*e^3)*x^3 + 8*(1344*(b^3*c^6 + 156*a*b*c^7)
*d^3 - 288*(7*b^4*c^5 - 60*a*b^2*c^6 - 1152*a^2*c^7)*d^2*e + 18*(63*b^5*c^4
 - 568*a*b^3*c^5 + 1392*a^2*b*c^6)*d*e^2 - (231*b^6*c^3 - 2232*a*b^4*c^4 +
6384*a^2*b^2*c^5 - 4096*a^3*c^6)*e^3)*x^2 - 2*(1344*(5*b^4*c^5 - 48*a*b^2*c
^6 - 528*a^2*c^7)*d^3 - 288*(35*b^5*c^4 - 336*a*b^3*c^5 + 912*a^2*b*c^6)*d^
2*e + 18*(315*b^6*c^3 - 3164*a*b^4*c^4 + 9552*a^2*b^2*c^5 - 6720*a^3*c^6)*d
```

```
*e^2 - (1155*b^7*c^2 - 12348*a*b^5*c^3 + 42192*a^2*b^3*c^4 - 44096*a^3*b*c^
5)*e^3)*x)*sqrt(c*x^2 + b*x + a))/c^7, 1/4128768*(315*(64*(b^6*c^3 - 12*a*b
^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*d^3 - 96*(b^7*c^2 - 12*a*b^5*c^3 + 48
*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^2*e + 6*(9*b^8*c - 112*a*b^6*c^2 + 480*a^2*b
^4*c^3 - 768*a^3*b^2*c^4 + 256*a^4*c^5)*d*e^2 - (11*b^9 - 144*a*b^7*c + 672
*a^2*b^5*c^2 - 1280*a^3*b^3*c^3 + 768*a^4*b*c^4)*e^3)*sqrt(-c)*arctan(1/2*s
qrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(229
376*c^9*e^3*x^8 + 14336*(54*c^9*d*e^2 + 37*b*c^8*e^3)*x^7 + 1024*(864*c^9*d
^2*e + 1782*b*c^8*d*e^2 + (309*b^2*c^7 + 608*a*c^8)*e^3)*x^6 + 256*(1344*c^
9*d^3 + 8352*b*c^8*d^2*e + 18*(243*b^2*c^7 + 476*a*c^8)*d*e^2 + (5*b^3*c^6
+ 3012*a*b*c^7)*e^3)*x^5 + 128*(6720*b*c^8*d^3 + 288*(37*b^2*c^7 + 72*a*c^8
)*d^2*e + 18*(3*b^3*c^6 + 1228*a*b*c^7)*d*e^2 - (11*b^4*c^5 - 84*a*b^2*c^6
- 3840*a^2*c^7)*e^3)*x^4 + 1344*(15*b^5*c^4 - 160*a*b^3*c^5 + 528*a^2*b*c^6
)*d^3 - 288*(105*b^6*c^3 - 1120*a*b^4*c^4 + 3696*a^2*b^2*c^5 - 3072*a^3*c^6
)*d^2*e + 18*(945*b^7*c^2 - 10500*a*b^5*c^3 + 37744*a^2*b^3*c^4 - 42432*a^3
*b*c^5)*d*e^2 - (3465*b^8*c - 40740*a*b^6*c^2 + 162288*a^2*b^4*c^3 - 234432
*a^3*b^2*c^4 + 65536*a^4*c^5)*e^3 + 16*(1344*(27*b^2*c^7 + 52*a*c^8)*d^3 +
288*(3*b^3*c^6 + 788*a*b*c^7)*d^2*e - 18*(27*b^4*c^5 - 216*a*b^2*c^6 - 6608
*a^2*c^7)*d*e^2 + (99*b^5*c^4 - 856*a*b^3*c^5 + 1968*a^2*b*c^6)*e^3)*x^3 +
8*(1344*(b^3*c^6 + 156*a*b*c^7)*d^3 - 288*(7*b^4*c^5 - 60*a*b^2*c^6 - 1152*
a^2*c^7)*d^2*e + 18*(63*b^5*c^4 - 568*a*b^3*c^5 + 1392*a^2*b*c^6)*d*e^2 - (
231*b^6*c^3 - 2232*a*b^4*c^4 + 6384*a^2*b^2*c^5 - 4096*a^3*c^6)*e^3)*x^2 -
2*(1344*(5*b^4*c^5 - 48*a*b^2*c^6 - 528*a^2*c^7)*d^3 - 288*(35*b^5*c^4 - 33
6*a*b^3*c^5 + 912*a^2*b*c^6)*d^2*e + 18*(315*b^6*c^3 - 3164*a*b^4*c^4 + 955
2*a^2*b^2*c^5 - 6720*a^3*c^6)*d*e^2 - (1155*b^7*c^2 - 12348*a*b^5*c^3 + 421
92*a^2*b^3*c^4 - 44096*a^3*b*c^5)*e^3)*x)*sqrt(c*x^2 + b*x + a))/c^7]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d + ex)^3 \left(a + bx + cx^2\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Integral((d + e*x)**3*(a + b*x + c*x**2)**(5/2), x)
```

Giac [B] time = 1.17846, size = 1566, normalized size = 3.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2064384*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(2*(4*(14*(16*c^2*x*e^3 + (54*c
^10*d*e^2 + 37*b*c^9*e^3)/c^8)*x + (864*c^10*d^2*e + 1782*b*c^9*d*e^2 + 309
*b^2*c^8*e^3 + 608*a*c^9*e^3)/c^8)*x + (1344*c^10*d^3 + 8352*b*c^9*d^2*e +
4374*b^2*c^8*d*e^2 + 8568*a*c^9*d*e^2 + 5*b^3*c^7*e^3 + 3012*a*b*c^8*e^3)/c
^8)*x + (6720*b*c^9*d^3 + 10656*b^2*c^8*d^2*e + 20736*a*c^9*d^2*e + 54*b^3*
c^7*d*e^2 + 22104*a*b*c^8*d*e^2 - 11*b^4*c^6*e^3 + 84*a*b^2*c^7*e^3 + 3840*
a^2*c^8*e^3)/c^8)*x + (36288*b^2*c^8*d^3 + 69888*a*c^9*d^3 + 864*b^3*c^7*d^
2*e + 226944*a*b*c^8*d^2*e - 486*b^4*c^6*d*e^2 + 3888*a*b^2*c^7*d*e^2 + 118
944*a^2*c^8*d*e^2 + 99*b^5*c^5*e^3 - 856*a*b^3*c^6*e^3 + 1968*a^2*b*c^7*e^3
```

)/c^8)*x + (1344*b^3*c^7*d^3 + 209664*a*b*c^8*d^3 - 2016*b^4*c^6*d^2*e + 17280*a*b^2*c^7*d^2*e + 331776*a^2*c^8*d^2*e + 1134*b^5*c^5*d*e^2 - 10224*a*b^3*c^6*d*e^2 + 25056*a^2*b*c^7*d*e^2 - 231*b^6*c^4*e^3 + 2232*a*b^4*c^5*e^3 - 6384*a^2*b^2*c^6*e^3 + 4096*a^3*c^7*e^3)/c^8)*x - (6720*b^4*c^6*d^3 - 64512*a*b^2*c^7*d^3 - 709632*a^2*c^8*d^3 - 10080*b^5*c^5*d^2*e + 96768*a*b^3*c^6*d^2*e - 262656*a^2*b*c^7*d^2*e + 5670*b^6*c^4*d*e^2 - 56952*a*b^4*c^5*d*e^2 + 171936*a^2*b^2*c^6*d*e^2 - 120960*a^3*c^7*d*e^2 - 1155*b^7*c^3*e^3 + 12348*a*b^5*c^4*e^3 - 42192*a^2*b^3*c^5*e^3 + 44096*a^3*b*c^6*e^3)/c^8)*x + (20160*b^5*c^5*d^3 - 215040*a*b^3*c^6*d^3 + 709632*a^2*b*c^7*d^3 - 30240*b^6*c^4*d^2*e + 322560*a*b^4*c^5*d^2*e - 1064448*a^2*b^2*c^6*d^2*e + 884736*a^3*c^7*d^2*e + 17010*b^7*c^3*d*e^2 - 189000*a*b^5*c^4*d*e^2 + 679392*a^2*b^3*c^5*d*e^2 - 763776*a^3*b*c^6*d*e^2 - 3465*b^8*c^2*e^3 + 40740*a*b^6*c^3*e^3 - 162288*a^2*b^4*c^4*e^3 + 234432*a^3*b^2*c^5*e^3 - 65536*a^4*c^6*e^3)/c^8) + 5/65536*(64*b^6*c^3*d^3 - 768*a*b^4*c^4*d^3 + 3072*a^2*b^2*c^5*d^3 - 4096*a^3*c^6*d^3 - 96*b^7*c^2*d^2*e + 1152*a*b^5*c^3*d^2*e - 4608*a^2*b^3*c^4*d^2*e + 6144*a^3*b*c^5*d^2*e + 54*b^8*c*d*e^2 - 672*a*b^6*c^2*d*e^2 + 2880*a^2*b^4*c^3*d*e^2 - 4608*a^3*b^2*c^4*d*e^2 + 1536*a^4*c^5*d*e^2 - 11*b^9*e^3 + 144*a*b^7*c*e^3 - 672*a^2*b^5*c^2*e^3 + 1280*a^3*b^3*c^3*e^3 - 768*a^4*b*c^4*e^3)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(13/2)

3.2356 $\int (d+ex)^2 \left(a+bx+cx^2\right)^{5/2} dx$

Optimal. Leaf size=323

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{5/2}\left(-4ce(ae+8bd)+9b^2e^2+32c^2d^2\right)}{384c^3}-\frac{5\left(b^2-4ac\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(-4ce(ae+8b\right.}{6144c^4}$$

```
[Out] (5*(b^2 - 4*a*c)^2*(32*c^2*d^2 + 9*b^2*e^2 - 4*c*e*(8*b*d + a*e))*(b + 2*c*
x)*Sqrt[a + b*x + c*x^2])/(16384*c^5) - (5*(b^2 - 4*a*c)*(32*c^2*d^2 + 9*b^
2*e^2 - 4*c*e*(8*b*d + a*e))*(b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(6144*c^4
) + ((32*c^2*d^2 + 9*b^2*e^2 - 4*c*e*(8*b*d + a*e))*(b + 2*c*x)*(a + b*x +
c*x^2)^(5/2))/(384*c^3) + (9*e*(2*c*d - b*e)*(a + b*x + c*x^2)^(7/2))/(112*
c^2) + (e*(d + e*x)*(a + b*x + c*x^2)^(7/2))/(8*c) - (5*(b^2 - 4*a*c)^3*(32
*c^2*d^2 + 9*b^2*e^2 - 4*c*e*(8*b*d + a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(32768*c^(11/2))
```

Rubi [A] time = 0.43986, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {742, 640, 612, 621, 206}

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{5/2}\left(-4ce(ae+8bd)+9b^2e^2+32c^2d^2\right)}{384c^3}-\frac{5\left(b^2-4ac\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(-4ce(ae+8b\right.}{6144c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*x + c*x^2)^(5/2),x]
```


```
[Out] (5*(b^2 - 4*a*c)^2*(32*c^2*d^2 + 9*b^2*e^2 - 4*c*e*(8*b*d + a*e))*(b + 2*c*
x)*Sqrt[a + b*x + c*x^2])/(16384*c^5) - (5*(b^2 - 4*a*c)*(32*c^2*d^2 + 9*b^
2*e^2 - 4*c*e*(8*b*d + a*e))*(b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(6144*c^4
) + ((32*c^2*d^2 + 9*b^2*e^2 - 4*c*e*(8*b*d + a*e))*(b + 2*c*x)*(a + b*x +
c*x^2)^(5/2))/(384*c^3) + (9*e*(2*c*d - b*e)*(a + b*x + c*x^2)^(7/2))/(112*
c^2) + (e*(d + e*x)*(a + b*x + c*x^2)^(7/2))/(8*c) - (5*(b^2 - 4*a*c)^3*(32
*c^2*d^2 + 9*b^2*e^2 - 4*c*e*(8*b*d + a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(32768*c^(11/2))
```


Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^2\left(a+bx+cx^2\right)^{5/2}\,dx=\frac{e(d+ex)\left(a+bx+cx^2\right)^{7/2}}{8c}+\frac{\int\left(\frac{1}{2}\left(16cd^2-2e\left(\frac{7bd}{2}+ae\right)\right)+\frac{9}{2}e(2cd-be)x\right)\left(a+bx+\right.}{8c}$$

$$=\frac{9e(2cd-be)\left(a+bx+cx^2\right)^{7/2}}{112c^2}+\frac{e(d+ex)\left(a+bx+cx^2\right)^{7/2}}{8c}+\frac{\left(-\frac{9}{2}be(2cd-be)+c\left(16\right.\right.}{}$$

$$=\frac{\left(32c^2d^2+9b^2e^2-4ce(8bd+ae)\right)(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{384c^3}+\frac{9e(2cd-be)\left(a+bx+\right.}{112c^2}$$

$$=-\frac{5\left(b^2-4ac\right)\left(32c^2d^2+9b^2e^2-4ce(8bd+ae)\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{6144c^4}+\frac{\left(32c^2d^2\right.}{}$$

$$=\frac{5\left(b^2-4ac\right)^2\left(32c^2d^2+9b^2e^2-4ce(8bd+ae)\right)(b+2cx)\sqrt{a+bx+cx^2}}{16384c^5}-\frac{5\left(b^2-4ac\right)}{}$$

$$=\frac{5\left(b^2-4ac\right)^2\left(32c^2d^2+9b^2e^2-4ce(8bd+ae)\right)(b+2cx)\sqrt{a+bx+cx^2}}{16384c^5}-\frac{5\left(b^2-4ac\right)}{}$$

$$=\frac{5\left(b^2-4ac\right)^2\left(32c^2d^2+9b^2e^2-4ce(8bd+ae)\right)(b+2cx)\sqrt{a+bx+cx^2}}{16384c^5}-\frac{5\left(b^2-4ac\right)}{}$$

Mathematica [A] time = 0.452208, size = 238, normalized size = 0.74

$$-\frac{\frac{\left(2ce(ae+8bd)-\frac{9b^2e^2}{2}-16c^2d^2\right)\left(256c^{5/2}(b+2cx)(a+x(b+cx))^{5/2}-5\left(b^2-4ac\right)\left(16c^{3/2}(b+2cx)(a+x(b+cx))^{3/2}-3\left(b^2-4ac\right)\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}-\left(b^2-4ac\right)\tanh^{-1}\left(\right.\right.\right.\right.}{6144c^{9/2}}}{8c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] ((9*e*(2*c*d - b*e)*(a + x*(b + c*x))^(7/2))/(14*c) + e*(d + e*x)*(a + x*(b
 + c*x))^(7/2) - ((-16*c^2*d^2 - (9*b^2*e^2)/2 + 2*c*e*(8*b*d + a*e))*(256*
c^(5/2)*(b + 2*c*x)*(a + x*(b + c*x))^(5/2) - 5*(b^2 - 4*a*c)*(16*c^(3/2)*(
b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x)
*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[
```

```
a + x*(b + c*x)])]))))/(6144*c^(9/2)))/(8*c)
```

Maple [B] time = 0.05, size = 1517, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 15/64*d*e*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2-15/25
6*d*e*b^5/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-15/1024*e^2
*b^4/c^3*(c*x^2+b*x+a)^(3/2)*x+25/768*e^2*b^3/c^3*(c*x^2+b*x+a)^(3/2)*a+45/
8192*e^2*b^6/c^4*(c*x^2+b*x+a)^(1/2)*x+35/2048*e^2*b^6/c^(9/2)*ln((1/2*b+c*
x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-75/1024*e^2*b^4/c^(7/2)*ln((1/2*b+c*x)/c^
(1/2)+(c*x^2+b*x+a)^(1/2))*a^2-5/96*d^2/c*(c*x^2+b*x+a)^(3/2)*x*b^2-5/128*e
^2*a^3/c*(c*x^2+b*x+a)^(1/2)*x-5/256*e^2*a^3/c^2*(c*x^2+b*x+a)^(1/2)*b-1/48
*e^2*a/c*x*(c*x^2+b*x+a)^(5/2)-1/96*e^2*a/c^2*(c*x^2+b*x+a)^(5/2)*b-1/12*d*
e*b^2/c^2*(c*x^2+b*x+a)^(5/2)+5/192*d*e*b^4/c^3*(c*x^2+b*x+a)^(3/2)-5/512*d
*e*b^6/c^4*(c*x^2+b*x+a)^(1/2)+5/1024*d*e*b^7/c^(9/2)*ln((1/2*b+c*x)/c^(1/2
)+(c*x^2+b*x+a)^(1/2))-5/192*e^2*a^2/c*(c*x^2+b*x+a)^(3/2)*x-5/384*e^2*a^2/
c^2*(c*x^2+b*x+a)^(3/2)*b+55/1024*e^2*b^3/c^3*(c*x^2+b*x+a)^(1/2)*a^2-95/40
96*e^2*b^5/c^4*(c*x^2+b*x+a)^(1/2)*a+3/64*e^2*b^2/c^2*x*(c*x^2+b*x+a)^(5/2)
-5/64*d^2/c^2*(c*x^2+b*x+a)^(1/2)*b^3*a-15/64*d^2/c^(3/2)*ln((1/2*b+c*x)/c^
(1/2)+(c*x^2+b*x+a)^(1/2))*b^2*a^2+15/256*d^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2
)+(c*x^2+b*x+a)^(1/2))*b^4*a+5/48*d^2/c*(c*x^2+b*x+a)^(3/2)*b*a+15/128*e^2*
b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^3+2/7*d*e*(c*x^2+
b*x+a)^(7/2)/c-15/2048*e^2*b^5/c^4*(c*x^2+b*x+a)^(3/2)+45/16384*e^2*b^7/c^5
*(c*x^2+b*x+a)^(1/2)+1/12*d^2/c*(c*x^2+b*x+a)^(5/2)*b+5/24*d^2*(c*x^2+b*x+a
)^(3/2)*x*a-5/192*d^2/c^2*(c*x^2+b*x+a)^(3/2)*b^3+5/16*d^2*(c*x^2+b*x+a)^(1
/2)*x*a^2+5/512*d^2/c^3*(c*x^2+b*x+a)^(1/2)*b^5+5/16*d^2/c^(1/2)*ln((1/2*b+
c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^3-5/1024*d^2/c^(7/2)*ln((1/2*b+c*x)/c^(
1/2)+(c*x^2+b*x+a)^(1/2))*b^6-95/2048*e^2*b^4/c^3*(c*x^2+b*x+a)^(1/2)*x*a+2
5/384*e^2*b^2/c^2*(c*x^2+b*x+a)^(3/2)*x*a+5/64*d*e*b^4/c^3*(c*x^2+b*x+a)^(1
/2)*a-45/32768*e^2*b^8/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
+1/8*e^2*x*(c*x^2+b*x+a)^(7/2)/c-5/128*e^2*a^4/c^(3/2)*ln((1/2*b+c*x)/c^(1/
2)+(c*x^2+b*x+a)^(1/2))-9/112*e^2*b/c^2*(c*x^2+b*x+a)^(7/2)+3/128*e^2*b^3/c
^3*(c*x^2+b*x+a)^(5/2)-1/6*d*e*b/c*x*(c*x^2+b*x+a)^(5/2)+1/6*d^2*x*(c*x^2+b
*x+a)^(5/2)+5/32*d*e*b^3/c^2*(c*x^2+b*x+a)^(1/2)*x*a+5/256*d^2/c^2*(c*x^2+b
*x+a)^(1/2)*x*b^4+5/32*d^2/c*(c*x^2+b*x+a)^(1/2)*b*a^2+5/96*d*e*b^3/c^2*(c*
x^2+b*x+a)^(3/2)*x-5/48*d*e*b^2/c^2*(c*x^2+b*x+a)^(3/2)*a-5/256*d*e*b^5/c^3
*(c*x^2+b*x+a)^(1/2)*x-5/32*d*e*b^2/c^2*(c*x^2+b*x+a)^(1/2)*a^2-5/32*d^2/c*
(c*x^2+b*x+a)^(1/2)*x*a*b^2+55/512*e^2*b^2/c^2*(c*x^2+b*x+a)^(1/2)*x*a^2-5/
16*d*e*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^3-5/16*d*e*b
/c*(c*x^2+b*x+a)^(1/2)*x*a^2-5/24*d*e*b/c*(c*x^2+b*x+a)^(3/2)*x*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.13733, size = 3314, normalized size = 10.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/1376256*(105*(32*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*
d^2 - 32*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d*e + (9*b^
8 - 112*a*b^6*c + 480*a^2*b^4*c^2 - 768*a^3*b^2*c^3 + 256*a^4*c^4)*e^2)*sqr
t(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*s
qrt(c) - 4*a*c) + 4*(43008*c^8*e^2*x^7 + 3072*(32*c^8*d*e + 33*b*c^7*e^2)*x
^6 + 256*(224*c^8*d^2 + 928*b*c^7*d*e + (243*b^2*c^6 + 476*a*c^7)*e^2)*x^5
+ 128*(1120*b*c^7*d^2 + 32*(37*b^2*c^6 + 72*a*c^7)*d*e + (3*b^3*c^5 + 1228*
a*b*c^6)*e^2)*x^4 + 16*(224*(27*b^2*c^6 + 52*a*c^7)*d^2 + 32*(3*b^3*c^5 + 7
88*a*b*c^6)*d*e - (27*b^4*c^4 - 216*a*b^2*c^5 - 6608*a^2*c^6)*e^2)*x^3 + 22
4*(15*b^5*c^3 - 160*a*b^3*c^4 + 528*a^2*b*c^5)*d^2 - 32*(105*b^6*c^2 - 1120
*a*b^4*c^3 + 3696*a^2*b^2*c^4 - 3072*a^3*c^5)*d*e + (945*b^7*c - 10500*a*b^
5*c^2 + 37744*a^2*b^3*c^3 - 42432*a^3*b*c^4)*e^2 + 8*(224*(b^3*c^5 + 156*a*
b*c^6)*d^2 - 32*(7*b^4*c^4 - 60*a*b^2*c^5 - 1152*a^2*c^6)*d*e + (63*b^5*c^3
 - 568*a*b^3*c^4 + 1392*a^2*b*c^5)*e^2)*x^2 - 2*(224*(5*b^4*c^4 - 48*a*b^2*
c^5 - 528*a^2*c^6)*d^2 - 32*(35*b^5*c^3 - 336*a*b^3*c^4 + 912*a^2*b*c^5)*d*
e + (315*b^6*c^2 - 3164*a*b^4*c^3 + 9552*a^2*b^2*c^4 - 6720*a^3*c^5)*e^2)*x
)*sqrt(c*x^2 + b*x + a))/c^6, 1/688128*(105*(32*(b^6*c^2 - 12*a*b^4*c^3 + 4
8*a^2*b^2*c^4 - 64*a^3*c^5)*d^2 - 32*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3
 - 64*a^3*b*c^4)*d*e + (9*b^8 - 112*a*b^6*c + 480*a^2*b^4*c^2 - 768*a^3*b^2
*c^3 + 256*a^4*c^4)*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x +
 b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(43008*c^8*e^2*x^7 + 3072*(32*c^8
*d*e + 33*b*c^7*e^2)*x^6 + 256*(224*c^8*d^2 + 928*b*c^7*d*e + (243*b^2*c^6
+ 476*a*c^7)*e^2)*x^5 + 128*(1120*b*c^7*d^2 + 32*(37*b^2*c^6 + 72*a*c^7)*d*
e + (3*b^3*c^5 + 1228*a*b*c^6)*e^2)*x^4 + 16*(224*(27*b^2*c^6 + 52*a*c^7)*d
^2 + 32*(3*b^3*c^5 + 788*a*b*c^6)*d*e - (27*b^4*c^4 - 216*a*b^2*c^5 - 6608*
a^2*c^6)*e^2)*x^3 + 224*(15*b^5*c^3 - 160*a*b^3*c^4 + 528*a^2*b*c^5)*d^2 -
32*(105*b^6*c^2 - 1120*a*b^4*c^3 + 3696*a^2*b^2*c^4 - 3072*a^3*c^5)*d*e + (
945*b^7*c - 10500*a*b^5*c^2 + 37744*a^2*b^3*c^3 - 42432*a^3*b*c^4)*e^2 + 8*
(224*(b^3*c^5 + 156*a*b*c^6)*d^2 - 32*(7*b^4*c^4 - 60*a*b^2*c^5 - 1152*a^2*
c^6)*d*e + (63*b^5*c^3 - 568*a*b^3*c^4 + 1392*a^2*b*c^5)*e^2)*x^2 - 2*(224*
(5*b^4*c^4 - 48*a*b^2*c^5 - 528*a^2*c^6)*d^2 - 32*(35*b^5*c^3 - 336*a*b^3*c
^4 + 912*a^2*b*c^5)*d*e + (315*b^6*c^2 - 3164*a*b^4*c^3 + 9552*a^2*b^2*c^4
- 6720*a^3*c^5)*e^2)*x)*sqrt(c*x^2 + b*x + a))/c^6]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d+ex)^2 \left(a+bx+cx^2\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Integral((d + e*x)**2*(a + b*x + c*x**2)**(5/2), x)
```

Giac [B] time = 1.16733, size = 1035, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/344064*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(2*(12*(14*c^2*x*e^2 + (32*c^9*d
*e + 33*b*c^8*e^2)/c^7)*x + (224*c^9*d^2 + 928*b*c^8*d*e + 243*b^2*c^7*e^2
+ 476*a*c^8*e^2)/c^7)*x + (1120*b*c^8*d^2 + 1184*b^2*c^7*d*e + 2304*a*c^8*d
*e + 3*b^3*c^6*e^2 + 1228*a*b*c^7*e^2)/c^7)*x + (6048*b^2*c^7*d^2 + 11648*a
*c^8*d^2 + 96*b^3*c^6*d*e + 25216*a*b*c^7*d*e - 27*b^4*c^5*e^2 + 216*a*b^2*
c^6*e^2 + 6608*a^2*c^7*e^2)/c^7)*x + (224*b^3*c^6*d^2 + 34944*a*b*c^7*d^2 -
 224*b^4*c^5*d*e + 1920*a*b^2*c^6*d*e + 36864*a^2*c^7*d*e + 63*b^5*c^4*e^2
- 568*a*b^3*c^5*e^2 + 1392*a^2*b*c^6*e^2)/c^7)*x - (1120*b^4*c^5*d^2 - 1075
2*a*b^2*c^6*d^2 - 118272*a^2*c^7*d^2 - 1120*b^5*c^4*d*e + 10752*a*b^3*c^5*d
*e - 29184*a^2*b*c^6*d*e + 315*b^6*c^3*e^2 - 3164*a*b^4*c^4*e^2 + 9552*a^2*
b^2*c^5*e^2 - 6720*a^3*c^6*e^2)/c^7)*x + (3360*b^5*c^4*d^2 - 35840*a*b^3*c^
5*d^2 + 118272*a^2*b*c^6*d^2 - 3360*b^6*c^3*d*e + 35840*a*b^4*c^4*d*e - 118
272*a^2*b^2*c^5*d*e + 98304*a^3*c^6*d*e + 945*b^7*c^2*e^2 - 10500*a*b^5*c^3
*e^2 + 37744*a^2*b^3*c^4*e^2 - 42432*a^3*b*c^5*e^2)/c^7) + 5/32768*(32*b^6*
c^2*d^2 - 384*a*b^4*c^3*d^2 + 1536*a^2*b^2*c^4*d^2 - 2048*a^3*c^5*d^2 - 32*
b^7*c*d*e + 384*a*b^5*c^2*d*e - 1536*a^2*b^3*c^3*d*e + 2048*a^3*b*c^4*d*e +
 9*b^8*e^2 - 112*a*b^6*c*e^2 + 480*a^2*b^4*c^2*e^2 - 768*a^3*b^2*c^3*e^2 +
256*a^4*c^4*e^2)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b
))/c^(11/2)
```

3.2357 $\int (d+ex)\left(a+bx+cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=207

$$-\frac{5\left(b^2-4ac\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}(2cd-be)}{384c^3}+\frac{5\left(b^2-4ac\right)^2(b+2cx)\sqrt{a+bx+cx^2}(2cd-be)}{1024c^4}-\frac{5\left(b^2-4ac\right)^3(2cd}{}$$

```
[Out] (5*(b^2 - 4*a*c)^2*(2*c*d - b*e)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(1024*c
^4) - (5*(b^2 - 4*a*c)*(2*c*d - b*e)*(b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(
384*c^3) + ((2*c*d - b*e)*(b + 2*c*x)*(a + b*x + c*x^2)^(5/2))/(24*c^2) + (
e*(a + b*x + c*x^2)^(7/2))/(7*c) - (5*(b^2 - 4*a*c)^3*(2*c*d - b*e)*ArcTanh
[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2048*c^(9/2))
```

Rubi [A] time = 0.0885637, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {640, 612, 621, 206}

$$-\frac{5\left(b^2-4ac\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}(2cd-be)}{384c^3}+\frac{5\left(b^2-4ac\right)^2(b+2cx)\sqrt{a+bx+cx^2}(2cd-be)}{1024c^4}-\frac{5\left(b^2-4ac\right)^3(2cd}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (5*(b^2 - 4*a*c)^2*(2*c*d - b*e)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(1024*c
^4) - (5*(b^2 - 4*a*c)*(2*c*d - b*e)*(b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(
384*c^3) + ((2*c*d - b*e)*(b + 2*c*x)*(a + b*x + c*x^2)^(5/2))/(24*c^2) + (
e*(a + b*x + c*x^2)^(7/2))/(7*c) - (5*(b^2 - 4*a*c)^3*(2*c*d - b*e)*ArcTanh
[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2048*c^(9/2))
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)\left(a+bx+cx^2\right)^{5/2}\,dx = \frac{e\left(a+bx+cx^2\right)^{7/2}}{7c}+\frac{(2cd-be)\int \left(a+bx+cx^2\right)^{5/2}\,dx}{2c}$$

$$=\frac{(2cd-be)(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{24c^2}+\frac{e\left(a+bx+cx^2\right)^{7/2}}{7c}-\frac{\left(5\left(b^2-4ac\right)(2cd-be\right.}{4}$$

$$=-\frac{5\left(b^2-4ac\right)(2cd-be)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{384c^3}+\frac{(2cd-be)(b+2cx)\left(a+bx+\right.}{24c^2}$$

$$=\frac{5\left(b^2-4ac\right)^2(2cd-be)(b+2cx)\sqrt{a+bx+cx^2}}{1024c^4}-\frac{5\left(b^2-4ac\right)(2cd-be)(b+2cx)\left(a\right.}{384c^3}$$

$$=\frac{5\left(b^2-4ac\right)^2(2cd-be)(b+2cx)\sqrt{a+bx+cx^2}}{1024c^4}-\frac{5\left(b^2-4ac\right)(2cd-be)(b+2cx)\left(a\right.}{384c^3}$$

$$=\frac{5\left(b^2-4ac\right)^2(2cd-be)(b+2cx)\sqrt{a+bx+cx^2}}{1024c^4}-\frac{5\left(b^2-4ac\right)(2cd-be)(b+2cx)\left(a\right.}{384c^3}$$

Mathematica [A] time = 0.278456, size = 180, normalized size = 0.87

$$\frac{(2cd-be)\left(256c^{5/2}(b+2cx)(a+x(b+cx))^{5/2}-5\left(b^2-4ac\right)\left(16c^{3/2}(b+2cx)(a+x(b+cx))^{3/2}-3\left(b^2-4ac\right)\left(2\sqrt{c}(b+\right.\right.\right.}{6144c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (e*(a + x*(b + c*x))^(7/2))/(7*c) + ((2*c*d - b*e)*(256*c^(5/2)*(b + 2*c*x)
*(a + x*(b + c*x))^(5/2) - 5*(b^2 - 4*a*c)*(16*c^(3/2)*(b + 2*c*x)*(a + x*(
b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*
x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])
)))/(6144*c^(9/2))
```

Maple [B] time = 0.047, size = 807, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 1/7*e*(c*x^2+b*x+a)^(7/2)/c+5/64*e*b^3/c^2*(c*x^2+b*x+a)^(1/2)*x*a+5/384*e*
b^4/c^3*(c*x^2+b*x+a)^(3/2)+5/512*d/c^3*(c*x^2+b*x+a)^(1/2)*b^5+5/16*d/c^(1
/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^3-5/1024*d/c^(7/2)*ln((1/
2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^6+5/24*d*(c*x^2+b*x+a)^(3/2)*x*a-5/
192*d/c^2*(c*x^2+b*x+a)^(3/2)*b^3+1/12*d/c*(c*x^2+b*x+a)^(5/2)*b-5/1024*e*b
^6/c^4*(c*x^2+b*x+a)^(1/2)+5/2048*e*b^7/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))-1/24*e*b^2/c^2*(c*x^2+b*x+a)^(5/2)+5/16*d*(c*x^2+b*x+a)^(1
/2)*x*a^2-5/32*d/c*(c*x^2+b*x+a)^(1/2)*x*a*b^2-5/48*e*b/c*(c*x^2+b*x+a)^(3/
2)*x*a-5/32*e*b/c*(c*x^2+b*x+a)^(1/2)*x*a^2+1/6*d*x*(c*x^2+b*x+a)^(5/2)+5/3
```

```
2*d/c*(c*x^2+b*x+a)^(1/2)*b*a^2-5/64*d/c^2*(c*x^2+b*x+a)^(1/2)*b^3*a-5/32*e
*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^3-1/12*e*b/c*x*(c*
x^2+b*x+a)^(5/2)+15/128*e*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^
(1/2))*a^2-15/512*e*b^5/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
*a+5/128*e*b^4/c^3*(c*x^2+b*x+a)^(1/2)*a-5/64*e*b^2/c^2*(c*x^2+b*x+a)^(1/2)
*a^2-5/96*e*b^2/c^2*(c*x^2+b*x+a)^(3/2)*a-5/512*e*b^5/c^3*(c*x^2+b*x+a)^(1/
2)*x+5/192*e*b^3/c^2*(c*x^2+b*x+a)^(3/2)*x+5/256*d/c^2*(c*x^2+b*x+a)^(1/2)*
x*b^4-5/96*d/c*(c*x^2+b*x+a)^(3/2)*x*b^2-15/64*d/c^(3/2)*ln((1/2*b+c*x)/c^(
1/2)+(c*x^2+b*x+a)^(1/2))*b^2*a^2+15/256*d/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(
c*x^2+b*x+a)^(1/2))*b^4*a+5/48*d/c*(c*x^2+b*x+a)^(3/2)*b*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.28142, size = 1974, normalized size = 9.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/86016*(105*(2*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*d - (
b^7 - 12*a*b^5*c + 48*a^2*b^3*c^2 - 64*a^3*b*c^3)*e)*sqrt(c)*log(-8*c^2*x^2
 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4
*(3072*c^7*e*x^6 + 256*(14*c^7*d + 29*b*c^6*e)*x^5 + 128*(70*b*c^6*d + (37*
b^2*c^5 + 72*a*c^6)*e)*x^4 + 16*(14*(27*b^2*c^5 + 52*a*c^6)*d + (3*b^3*c^4
+ 788*a*b*c^5)*e)*x^3 + 8*(14*(b^3*c^4 + 156*a*b*c^5)*d - (7*b^4*c^3 - 60*a
*b^2*c^4 - 1152*a^2*c^5)*e)*x^2 + 14*(15*b^5*c^2 - 160*a*b^3*c^3 + 528*a^2*
b*c^4)*d - (105*b^6*c - 1120*a*b^4*c^2 + 3696*a^2*b^2*c^3 - 3072*a^3*c^4)*e
 - 2*(14*(5*b^4*c^3 - 48*a*b^2*c^4 - 528*a^2*c^5)*d - (35*b^5*c^2 - 336*a*b
^3*c^3 + 912*a^2*b*c^4)*e)*x)*sqrt(c*x^2 + b*x + a))/c^5, 1/43008*(105*(2*(
b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*d - (b^7 - 12*a*b^5*c +
 48*a^2*b^3*c^2 - 64*a^3*b*c^3)*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a
)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(3072*c^7*e*x^6 + 256*(
14*c^7*d + 29*b*c^6*e)*x^5 + 128*(70*b*c^6*d + (37*b^2*c^5 + 72*a*c^6)*e)*x
^4 + 16*(14*(27*b^2*c^5 + 52*a*c^6)*d + (3*b^3*c^4 + 788*a*b*c^5)*e)*x^3 +
8*(14*(b^3*c^4 + 156*a*b*c^5)*d - (7*b^4*c^3 - 60*a*b^2*c^4 - 1152*a^2*c^5)
*e)*x^2 + 14*(15*b^5*c^2 - 160*a*b^3*c^3 + 528*a^2*b*c^4)*d - (105*b^6*c -
1120*a*b^4*c^2 + 3696*a^2*b^2*c^3 - 3072*a^3*c^4)*e - 2*(14*(5*b^4*c^3 - 48
*a*b^2*c^4 - 528*a^2*c^5)*d - (35*b^5*c^2 - 336*a*b^3*c^3 + 912*a^2*b*c^4)*
e)*x)*sqrt(c*x^2 + b*x + a))/c^5]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d+ex)\left(a+bx+cx^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Integral((d + e*x)*(a + b*x + c*x**2)**(5/2), x)
```

Giac [B] time = 1.15808, size = 601, normalized size = 2.9

$$\frac{1}{21504}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(2\left(12\,c^2xe+\frac{14\,c^8d+29\,bc^7e}{c^6}\right)x+\frac{70\,bc^7d+37\,b^2c^6e+72\,ac^7e}{c^6}\right)x+\frac{378\,b^2c^6d+728}{}\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/21504*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(2*(12*c^2*x*e + (14*c^8*d + 29*b
*c^7*e)/c^6)*x + (70*b*c^7*d + 37*b^2*c^6*e + 72*a*c^7*e)/c^6)*x + (378*b^2
*c^6*d + 728*a*c^7*d + 3*b^3*c^5*e + 788*a*b*c^6*e)/c^6)*x + (14*b^3*c^5*d
+ 2184*a*b*c^6*d - 7*b^4*c^4*e + 60*a*b^2*c^5*e + 1152*a^2*c^6*e)/c^6)*x -
(70*b^4*c^4*d - 672*a*b^2*c^5*d - 7392*a^2*c^6*d - 35*b^5*c^3*e + 336*a*b^3
*c^4*e - 912*a^2*b*c^5*e)/c^6)*x + (210*b^5*c^3*d - 2240*a*b^3*c^4*d + 7392
*a^2*b*c^5*d - 105*b^6*c^2*e + 1120*a*b^4*c^3*e - 3696*a^2*b^2*c^4*e + 3072
*a^3*c^5*e)/c^6) + 5/2048*(2*b^6*c*d - 24*a*b^4*c^2*d + 96*a^2*b^2*c^3*d -
128*a^3*c^4*d - b^7*e + 12*a*b^5*c*e - 48*a^2*b^3*c^2*e + 64*a^3*b*c^3*e)*l
og(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2)
```

3.2358 $\int \left(a+bx+cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=149

$$\frac{5\left(b^2-4ac\right)^2(b+2cx)\sqrt{a+bx+cx^2}}{512c^3}-\frac{5\left(b^2-4ac\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{192c^2}-\frac{5\left(b^2-4ac\right)^3\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{1024c^{7/2}}+$$

```
[Out] (5*(b^2 - 4*a*c)^2*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(512*c^3) - (5*(b^2 -
 4*a*c)*(b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(192*c^2) + ((b + 2*c*x)*(a +
b*x + c*x^2)^(5/2))/(12*c) - (5*(b^2 - 4*a*c)^3*ArcTanh[(b + 2*c*x)/(2*Sqrt
[c]*Sqrt[a + b*x + c*x^2])])/(1024*c^(7/2))
```

Rubi [A] time = 0.0503224, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {612, 621, 206}

$$\frac{5\left(b^2-4ac\right)^2(b+2cx)\sqrt{a+bx+cx^2}}{512c^3}-\frac{5\left(b^2-4ac\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{192c^2}-\frac{5\left(b^2-4ac\right)^3\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{1024c^{7/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2),x]
```


```
[Out] (5*(b^2 - 4*a*c)^2*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(512*c^3) - (5*(b^2 -
 4*a*c)*(b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(192*c^2) + ((b + 2*c*x)*(a +
b*x + c*x^2)^(5/2))/(12*c) - (5*(b^2 - 4*a*c)^3*ArcTanh[(b + 2*c*x)/(2*Sqrt
[c]*Sqrt[a + b*x + c*x^2])])/(1024*c^(7/2))
```


Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^{5/2} dx = \frac{(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{12c} - \frac{\left(5\left(b^2-4ac\right)\right)\int \left(a+bx+cx^2\right)^{3/2} dx}{24c}$$
$$= -\frac{5\left(b^2-4ac\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{192c^2} + \frac{(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{12c} + \frac{\left(5\left(b^2-4ac\right)^2\right)\int}{128}$$
$$= \frac{5\left(b^2-4ac\right)^2(b+2cx)\sqrt{a+bx+cx^2}}{512c^3} - \frac{5\left(b^2-4ac\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{192c^2} + \frac{(b+2cx)}{}$$
$$= \frac{5\left(b^2-4ac\right)^2(b+2cx)\sqrt{a+bx+cx^2}}{512c^3} - \frac{5\left(b^2-4ac\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{192c^2} + \frac{(b+2cx)}{}$$
$$= \frac{5\left(b^2-4ac\right)^2(b+2cx)\sqrt{a+bx+cx^2}}{512c^3} - \frac{5\left(b^2-4ac\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{192c^2} + \frac{(b+2cx)}{}$$

Mathematica [A] time = 0.580177, size = 162, normalized size = 1.09

$$\frac{\sqrt{a+x(b+cx)}\left(2(b+2cx)\left(16c^2\left(33a^2+26acx^2+8c^2x^4\right)+8b^2c\left(11cx^2-20a\right)+32bc^2x\left(13a+8cx^2\right)-40b^3cx+15b\right.\right.}{3072c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(2*(b + 2*c*x)*(15*b^4 - 40*b^3*c*x + 32*b*c^2*x*(13
*a + 8*c*x^2) + 8*b^2*c*(-20*a + 11*c*x^2) + 16*c^2*(33*a^2 + 26*a*c*x^2 +
8*c^2*x^4)) + (15*(b^2 - 4*a*c)^(5/2)*ArcSin[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]
)/Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]))/(3072*c^3)
```

Maple [B] time = 0.044, size = 360, normalized size = 2.4

$$\frac{2\,cx+b}{12\,c}\left(cx^2+bx+a\right)^{\frac{5}{2}}+\frac{5\,ax}{24}\left(cx^2+bx+a\right)^{\frac{3}{2}}-\frac{5\,b^2x}{96\,c}\left(cx^2+bx+a\right)^{\frac{3}{2}}+\frac{5\,ab}{48\,c}\left(cx^2+bx+a\right)^{\frac{3}{2}}-\frac{5\,b^3}{192\,c^2}\left(cx^2+bx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 1/12*(2*c*x+b)*(c*x^2+b*x+a)^(5/2)/c+5/24*(c*x^2+b*x+a)^(3/2)*x*a-5/96/c*(c
*x^2+b*x+a)^(3/2)*x*b^2+5/48/c*(c*x^2+b*x+a)^(3/2)*b*a-5/192/c^2*(c*x^2+b*x
+a)^(3/2)*b^3+5/16*(c*x^2+b*x+a)^(1/2)*x*a^2-5/32/c*(c*x^2+b*x+a)^(1/2)*x*a
*b^2+5/256/c^2*(c*x^2+b*x+a)^(1/2)*x*b^4+5/32/c*(c*x^2+b*x+a)^(1/2)*b*a^2-5
/64/c^2*(c*x^2+b*x+a)^(1/2)*b^3*a+5/512/c^3*(c*x^2+b*x+a)^(1/2)*b^5+5/16/c^
(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^3-15/64/c^(3/2)*ln((1/2
*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*b^2+15/256/c^(5/2)*ln((1/2*b+c*x)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^4*a-5/1024/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c
*x^2+b*x+a)^(1/2))*b^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.73123, size = 1000, normalized size = 6.71

$$\left[-\frac{15\left(b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right)-4\left(256\,c^6x^5\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6144*(15*(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(c)*log(-
8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4
*a*c) - 4*(256*c^6*x^5 + 640*b*c^5*x^4 + 15*b^5*c - 160*a*b^3*c^2 + 528*a^2
*b*c^3 + 16*(27*b^2*c^4 + 52*a*c^5)*x^3 + 8*(b^3*c^3 + 156*a*b*c^4)*x^2 - 2
*(5*b^4*c^2 - 48*a*b^2*c^3 - 528*a^2*c^4)*x)*sqrt(c*x^2 + b*x + a))/c^4, 1/
3072*(15*(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-c)*arctan(1
/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*
(256*c^6*x^5 + 640*b*c^5*x^4 + 15*b^5*c - 160*a*b^3*c^2 + 528*a^2*b*c^3 + 1
6*(27*b^2*c^4 + 52*a*c^5)*x^3 + 8*(b^3*c^3 + 156*a*b*c^4)*x^2 - 2*(5*b^4*c^
2 - 48*a*b^2*c^3 - 528*a^2*c^4)*x)*sqrt(c*x^2 + b*x + a))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+bx+cx^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(5/2), x)
```

Giac [A] time = 1.14355, size = 281, normalized size = 1.89

$$\frac{1}{1536}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(2\,c^2x+5\,bc\right)x+\frac{27\,b^2c^5+52\,ac^6}{c^5}\right)x+\frac{b^3c^4+156\,abc^5}{c^5}\right)x-\frac{5\,b^4c^3-48\,ab^2c^4-528\,a^2c^5}{c^5}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/1536*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(2*c^2*x + 5*b*c)*x + (27*b^2*c^5
 + 52*a*c^6)/c^5)*x + (b^3*c^4 + 156*a*b*c^5)/c^5)*x - (5*b^4*c^3 - 48*a*b^2
*c^4 - 528*a^2*c^5)/c^5)*x + (15*b^5*c^2 - 160*a*b^3*c^3 + 528*a^2*b*c^4)/c
```

```
^5) + 5/1024*(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*log(abs(-2*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

3.2359 $\int \frac{\left(a+bx+cx^2\right)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=459

$$\frac{\sqrt{a+bx+cx^2}\left(8c^2e^2\left(16a^2e^2-39abde+22b^2d^2\right)-2cex(2cd-be)\left(-4ce(4bd-7ae)-3b^2e^2+16c^2d^2\right)-2b^2ce^3(5bd-14\right.}{128c^2e^5}$$

```
[Out] ((128*c^4*d^4 - 3*b^4*e^4 - 2*b^2*c*e^3*(5*b*d - 14*a*e) - 32*c^3*d^2*e*(9*
b*d - 8*a*e) + 8*c^2*e^2*(22*b^2*d^2 - 39*a*b*d*e + 16*a^2*e^2) - 2*c*e*(2*
c*d - b*e)*(16*c^2*d^2 - 3*b^2*e^2 - 4*c*e*(4*b*d - 7*a*e))*x)*Sqrt[a + b*x
 + c*x^2])/(128*c^2*e^5) + ((16*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(11*b*d - 8*a*e
) - 6*c*e*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(3/2))/(48*c*e^3) + (a + b*x +
 c*x^2)^(5/2)/(5*e) - ((2*c*d - b*e)*(128*c^4*d^4 + 3*b^4*e^4 + 8*b^2*c*e^3
*(2*b*d - 5*a*e) - 64*c^3*d^2*e*(4*b*d - 5*a*e) + 16*c^2*e^2*(7*b^2*d^2 - 2
0*a*b*d*e + 15*a^2*e^2))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^
2])])/(256*c^(5/2)*e^6) + ((c*d^2 - b*d*e + a*e^2)^(5/2)*ArcTanh[(b*d - 2*a
*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])
])/e^6
```

Rubi [A] time = 0.749641, antiderivative size = 459, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {734, 814, 843, 621, 206, 724}

$$\frac{\sqrt{a+bx+cx^2}\left(8c^2e^2\left(16a^2e^2-39abde+22b^2d^2\right)-2cex(2cd-be)\left(-4ce(4bd-7ae)-3b^2e^2+16c^2d^2\right)-2b^2ce^3(5bd-14\right.}{128c^2e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/(d + e*x),x]
```



```
[Out] ((128*c^4*d^4 - 3*b^4*e^4 - 2*b^2*c*e^3*(5*b*d - 14*a*e) - 32*c^3*d^2*e*(9*
b*d - 8*a*e) + 8*c^2*e^2*(22*b^2*d^2 - 39*a*b*d*e + 16*a^2*e^2) - 2*c*e*(2*
c*d - b*e)*(16*c^2*d^2 - 3*b^2*e^2 - 4*c*e*(4*b*d - 7*a*e))*x)*Sqrt[a + b*x
 + c*x^2])/(128*c^2*e^5) + ((16*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(11*b*d - 8*a*e
) - 6*c*e*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(3/2))/(48*c*e^3) + (a + b*x +
 c*x^2)^(5/2)/(5*e) - ((2*c*d - b*e)*(128*c^4*d^4 + 3*b^4*e^4 + 8*b^2*c*e^3
*(2*b*d - 5*a*e) - 64*c^3*d^2*e*(4*b*d - 5*a*e) + 16*c^2*e^2*(7*b^2*d^2 - 2
0*a*b*d*e + 15*a^2*e^2))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^
2])])/(256*c^(5/2)*e^6) + ((c*d^2 - b*d*e + a*e^2)^(5/2)*ArcTanh[(b*d - 2*a
*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])
])/e^6
```


Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{5/2}}{d+ex}dx = \frac{\left(a+bx+cx^2\right)^{5/2}}{5e} - \frac{\int \frac{(bd-2ae+(2cd-be)x)\left(a+bx+cx^2\right)^{3/2}}{d+ex}dx}{2e}$$

$$= \frac{\left(16c^2d^2+3b^2e^2-2ce(11bd-8ae)-6ce(2cd-be)x\right)\left(a+bx+cx^2\right)^{3/2}}{48ce^3} + \frac{\left(a+bx+cx^2\right)^{5/2}}{5e} + \int$$

$$= \frac{\left(128c^4d^4-3b^4e^4-2b^2ce^3(5bd-14ae)-32c^3d^2e(9bd-8ae)+8c^2e^2\left(22b^2d^2-39abde+16a^2e\right.\right.}{128c^2e^5}$$

$$= \frac{\left(128c^4d^4-3b^4e^4-2b^2ce^3(5bd-14ae)-32c^3d^2e(9bd-8ae)+8c^2e^2\left(22b^2d^2-39abde+16a^2e\right.\right.}{128c^2e^5}$$

$$= \frac{\left(128c^4d^4-3b^4e^4-2b^2ce^3(5bd-14ae)-32c^3d^2e(9bd-8ae)+8c^2e^2\left(22b^2d^2-39abde+16a^2e\right.\right.}{128c^2e^5}$$

$$= \frac{\left(128c^4d^4-3b^4e^4-2b^2ce^3(5bd-14ae)-32c^3d^2e(9bd-8ae)+8c^2e^2\left(22b^2d^2-39abde+16a^2e\right.\right.}{128c^2e^5}$$

Mathematica [A] time = 1.2842, size = 440, normalized size = 0.96

$$-\frac{\frac{2e\sqrt{a+x(b+cx)}\left(-4c^2e^2\left(32a^2e^2+2abe(7ex-39d)+b^2d(44d-5ex)\right)+2b^2ce^3(-14ae+5bd+3bex)+16c^3de(ae(7ex-16d)+6bd(3d-ex))+3b^4e^4-64c^4d^3(2d-ex)\right)}{c^2} + \frac{(2cd-\cdots}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/(d + e*x),x]
```

```
[Out] (a + x*(b + c*x))^(5/2)/(5*e) + ((a + x*(b + c*x))^(3/2)*(3*b^2*e^2 + 4*c^2
*d*(4*d - 3*e*x) + 2*c*e*(-11*b*d + 8*a*e + 3*b*e*x)))/(48*c*e^3) - ((2*e*S
qrt[a + x*(b + c*x)]*(3*b^4*e^4 - 64*c^4*d^3*(2*d - e*x) + 2*b^2*c*e^3*(5*b
*d - 14*a*e + 3*b*e*x) - 4*c^2*e^2*(32*a^2*e^2 + b^2*d*(44*d - 5*e*x) + 2*a
*b*e*(-39*d + 7*e*x)) + 16*c^3*d*e*(6*b*d*(3*d - e*x) + a*e*(-16*d + 7*e*x)
)))/c^2 + ((2*c*d - b*e)*(128*c^4*d^4 + 3*b^4*e^4 + 8*b^2*c*e^3*(2*b*d - 5*
a*e) - 64*c^3*d^2*e*(4*b*d - 5*a*e) + 16*c^2*e^2*(7*b^2*d^2 - 20*a*b*d*e +
15*a^2*e^2))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(5/2
) + 256*(c*d^2 + e*(-(b*d) + a*e))^(5/2)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x
+ b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])]])/(256*e^
6)
```

Maple [B] time = 0.23, size = 4126, normalized size = 9.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(e*x+d),x)
```

```
[Out] 1/5/e*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(5/2)+1/3
/e*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)*a+1/e*
((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*a^2+5/16/
e^3*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/
e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b^3*d^2+6/e^4/((a*e^2-b*d*e+c*
```

d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e
^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+
c*d^2)/e^2)^(1/2))/(d/e+x))*a*b*d^3*c-1/e^7/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
ln((2(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)
/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/
2))/(d/e+x))*c^3*d^6-1/2/e^4*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*
e+c*d^2)/e^2)^(1/2)*x*c^2*d^3-9/4/e^4*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*d^3*c+2/e^3*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+
x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*a*c*d^2-15/8/e^2*ln((1/2*(b*e-2*c*d)/e+(d
/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2
)^(1/2))*c^(1/2)*d*a^2+5/128/e^2/c^(3/2)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c
^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*b
^4*d-5/2/e^4*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c
d)/e(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(3/2)*d^3*a+5/2/e^5*ln((1/2
*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2
-b*d*e+c*d^2)/e^2)^(1/2))*c^(3/2)*d^4*b-15/8/e^4*ln((1/2*(b*e-2*c*d)/e+(d/e
+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^
(1/2))*c^(1/2)*d^3*b^2+15/16/e/c^(1/2)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(
1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*a^2
*b-5/32/e/c^(3/2)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*
e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*b^3*a-1/4/e^2*((d/e+x)^2
*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)*x*c*d+7/16/e*((d/e+
x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*a*b+1/e^4/((a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d
/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b^3*d^3-3/64/e/c*((d/e+x)^2*c+(b*e
-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b^3-5/32/e^2*((d/e+x)^2*
c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b^2*d+7/32/e/c*((d
/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*a-39/16/
e^2*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*a*d
-5/64/e^2/c*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/
2)*b^3*d+3/256/e/c^(5/2)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^
2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*b^5+1/e^5*((d/e+x
)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*c^2*d^4-1/e/((a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/
e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(
a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a^3-1/e^6*ln((1/2*(b*e-2*c*d)/e+(d/
e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)
^(1/2))*c^(5/2)*d^5+1/3/e^3*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e
+c*d^2)/e^2)^(3/2)*c*d^2-7/8/e^2*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-
b*d*e+c*d^2)/e^2)^(1/2)*x*a*c*d+3/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2
*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^
(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d
/e+x))*a^2*b*d+3/e^6/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d
^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*
c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b*d^5*c^2-
3/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*
c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e
*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a*c^2*d^4+1/8/e*((d/e+x)^
2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)*x*b+1/16/e/c*((d/e
+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)*b^2-11/24/e^2*
((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)*b*d-3/128
/e/c^2*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^
4+11/8/e^3*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*b^2*d^2-3/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e
^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*
e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a^2*c*d^2+3/4/e
^3*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c*
d^2-15/16/e^2/c^(1/2)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c

```
+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*a*b^2*d+15/4/e^3*ln(
(1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d^2*a*b-3/e^5/((a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*
e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/
e^2)^(1/2))/(d/e+x))*b^2*d^4*c-3/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*
(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/
e+x))*a*b^2*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.2360 $\int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=388

$$\frac{5\left(48c^2e^2\left(a^2e^2-4abde+3b^2d^2\right)-8b^2ce^3(2bd-3ae)-64c^3d^2e(4bd-3ae)-b^4e^4+128c^4d^4\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{3/2}e^6} -$$

```
[Out] (-5*(64*c^3*d^3 - b^3*e^3 + 4*b*c*e^2*(12*b*d - 11*a*e) - 16*c^2*d*e*(7*b*d
 - 4*a*e) - 2*c*e*(16*c^2*d^2 + b^2*e^2 - 4*c*e*(4*b*d - 3*a*e))*x)*Sqrt[a
+ b*x + c*x^2])/(64*c*e^5) - (5*(8*c*d - 7*b*e - 6*c*e*x)*(a + b*x + c*x^2)
^(3/2))/(24*e^3) - (a + b*x + c*x^2)^(5/2)/(e*(d + e*x)) + (5*(128*c^4*d^4
- b^4*e^4 - 8*b^2*c*e^3*(2*b*d - 3*a*e) - 64*c^3*d^2*e*(4*b*d - 3*a*e) + 48
*c^2*e^2*(3*b^2*d^2 - 4*a*b*d*e + a^2*e^2))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(128*c^(3/2)*e^6) - (5*(2*c*d - b*e)*(c*d^2 - b*d*
e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*
d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(2*e^6)
```

Rubi [A] time = 0.663218, antiderivative size = 388, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {732, 814, 843, 621, 206, 724}

$$\frac{5\left(48c^2e^2\left(a^2e^2-4abde+3b^2d^2\right)-8b^2ce^3(2bd-3ae)-64c^3d^2e(4bd-3ae)-b^4e^4+128c^4d^4\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{3/2}e^6} -$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/(d + e*x)^2,x]
```

```
[Out] (-5*(64*c^3*d^3 - b^3*e^3 + 4*b*c*e^2*(12*b*d - 11*a*e) - 16*c^2*d*e*(7*b*d
 - 4*a*e) - 2*c*e*(16*c^2*d^2 + b^2*e^2 - 4*c*e*(4*b*d - 3*a*e))*x)*Sqrt[a
+ b*x + c*x^2])/(64*c*e^5) - (5*(8*c*d - 7*b*e - 6*c*e*x)*(a + b*x + c*x^2)
^(3/2))/(24*e^3) - (a + b*x + c*x^2)^(5/2)/(e*(d + e*x)) + (5*(128*c^4*d^4
- b^4*e^4 - 8*b^2*c*e^3*(2*b*d - 3*a*e) - 64*c^3*d^2*e*(4*b*d - 3*a*e) + 48
*c^2*e^2*(3*b^2*d^2 - 4*a*b*d*e + a^2*e^2))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(128*c^(3/2)*e^6) - (5*(2*c*d - b*e)*(c*d^2 - b*d*
e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*
d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(2*e^6)
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
```

```
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(d+ex)^2}\,dx = -\frac{\left(a+bx+cx^2\right)^{5/2}}{e(d+ex)} + \frac{5\int \frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx}{2e}$$

$$= -\frac{5(8cd-7be-6cex)\left(a+bx+cx^2\right)^{3/2}}{24e^3} - \frac{\left(a+bx+cx^2\right)^{5/2}}{e(d+ex)} - \frac{5\int \frac{\left(c\left(7b^2de+4acde-8b\left(cd^2+ae^2\right)\right)-c\left(16c^2 \ldots\right.\right.}{d+ \ldots}}{16c \ldots}$$

$$= -\frac{5\left(64c^3d^3-b^3e^3+4bce^2(12bd-11ae)-16c^2de(7bd-4ae)-2ce\left(16c^2d^2+b^2e^2-4ce(4bd-3 \ldots\right.\right.}{64ce^5}$$

$$= -\frac{5\left(64c^3d^3-b^3e^3+4bce^2(12bd-11ae)-16c^2de(7bd-4ae)-2ce\left(16c^2d^2+b^2e^2-4ce(4bd-3 \ldots\right.\right.}{64ce^5}$$

$$= -\frac{5\left(64c^3d^3-b^3e^3+4bce^2(12bd-11ae)-16c^2de(7bd-4ae)-2ce\left(16c^2d^2+b^2e^2-4ce(4bd-3 \ldots\right.\right.}{64ce^5}$$

$$= -\frac{5\left(64c^3d^3-b^3e^3+4bce^2(12bd-11ae)-16c^2de(7bd-4ae)-2ce\left(16c^2d^2+b^2e^2-4ce(4bd-3 \ldots\right.\right.}{64ce^5}$$

Mathematica [A] time = 1.17093, size = 370, normalized size = 0.95

$$5\left(\left(48c^2e^2\left(a^2e^2-4abde+3b^2d^2\right)-8b^2ce^3(2bd-3ae)-64c^3d^2e(4bd-3ae)-b^4e^4+128c^4d^4\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/(d + e*x)^2,x]
```

```
[Out] (5*(-8*c*d + 7*b*e + 6*c*e*x)*(a + x*(b + c*x))^(3/2))/(24*e^3) - (a + x*(b
 + c*x))^(5/2)/(e*(d + e*x)) + (5*((128*c^4*d^4 - b^4*e^4 - 8*b^2*c*e^3*(2*
b*d - 3*a*e) - 64*c^3*d^2*e*(4*b*d - 3*a*e) + 48*c^2*e^2*(3*b^2*d^2 - 4*a*b
*d*e + a^2*e^2))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2
*Sqrt[c]*(e*Sqrt[a + x*(b + c*x)]*(b^3*e^3 + 32*c^3*d^2*(-2*d + e*x) + 2*b*
c*e^2*(-24*b*d + 22*a*e + b*e*x) + 8*c^2*e*(2*b*d*(7*d - 2*e*x) + a*e*(-8*d
 + 3*e*x))) + 32*c*(2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*ArcTanh[(
-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a
+ x*(b + c*x)])])))/(128*c^(3/2)*e^6)
```

Maple [B] time = 0.233, size = 6711, normalized size = 17.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(e*x+d)^2,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.2361 $\int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)^3} dx$

Optimal. Leaf size=331

$$\frac{5\sqrt{a+bx+cx^2}\left(-4ce(5bd-ae)+5b^2e^2-4cex(2cd-be)+16c^2d^2\right)}{8e^5}-\frac{5(2cd-be)\left(-4ce(4bd-3ae)+b^2e^2+16c^2d^2\right)}{16\sqrt{c}e^6}$$

```
[Out] (5*(16*c^2*d^2 + 5*b^2*e^2 - 4*c*e*(5*b*d - a*e) - 4*c*e*(2*c*d - b*e)*x)*S
qrt[a + b*x + c*x^2])/(8*e^5) + (5*(8*c*d - 3*b*e + 2*c*e*x)*(a + b*x + c*x
^2)^(3/2))/(12*e^3*(d + e*x)) - (a + b*x + c*x^2)^(5/2)/(2*e*(d + e*x)^2) -
 (5*(2*c*d - b*e)*(16*c^2*d^2 + b^2*e^2 - 4*c*e*(4*b*d - 3*a*e))*ArcTanh[(b
 + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*Sqrt[c]*e^6) + (5*Sqrt[c*
d^2 - b*d*e + a*e^2]*(16*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(4*b*d - a*e))*ArcTanh
[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*
x + c*x^2])])/(8*e^6)
```

Rubi [A] time = 0.548146, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {732, 812, 814, 843, 621, 206, 724}

$$\frac{5\sqrt{a+bx+cx^2}\left(-4ce(5bd-ae)+5b^2e^2-4cex(2cd-be)+16c^2d^2\right)}{8e^5}-\frac{5(2cd-be)\left(-4ce(4bd-3ae)+b^2e^2+16c^2d^2\right)}{16\sqrt{c}e^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/(d + e*x)^3,x]
```


```
[Out] (5*(16*c^2*d^2 + 5*b^2*e^2 - 4*c*e*(5*b*d - a*e) - 4*c*e*(2*c*d - b*e)*x)*S
qrt[a + b*x + c*x^2])/(8*e^5) + (5*(8*c*d - 3*b*e + 2*c*e*x)*(a + b*x + c*x
^2)^(3/2))/(12*e^3*(d + e*x)) - (a + b*x + c*x^2)^(5/2)/(2*e*(d + e*x)^2) -
 (5*(2*c*d - b*e)*(16*c^2*d^2 + b^2*e^2 - 4*c*e*(4*b*d - 3*a*e))*ArcTanh[(b
 + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*Sqrt[c]*e^6) + (5*Sqrt[c*
d^2 - b*d*e + a*e^2]*(16*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(4*b*d - a*e))*ArcTanh
[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*
x + c*x^2])])/(8*e^6)
```


Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
```

```
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(d+ex)^3}\,dx = -\frac{\left(a+bx+cx^2\right)^{5/2}}{2e(d+ex)^2} + \frac{5\int \frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{(d+ex)^2}\,dx}{4e}$$

$$= \frac{5(8cd-3be+2cex)\left(a+bx+cx^2\right)^{3/2}}{12e^3(d+ex)} - \frac{\left(a+bx+cx^2\right)^{5/2}}{2e(d+ex)^2} - \frac{5\int \frac{\left(8bcd-3b^2e-4ace+8c(2cd-be)x\right)\sqrt{a+}}{d+ex}}{8e^3}$$

$$= \frac{5\left(16c^2d^2+5b^2e^2-4ce(5bd-ae)-4ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8e^5} + \frac{5(8cd-3be+2cex)\,(}{12e^3(d+}$$

$$= \frac{5\left(16c^2d^2+5b^2e^2-4ce(5bd-ae)-4ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8e^5} + \frac{5(8cd-3be+2cex)\,(}{12e^3(d+}$$

$$= \frac{5\left(16c^2d^2+5b^2e^2-4ce(5bd-ae)-4ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8e^5} + \frac{5(8cd-3be+2cex)\,(}{12e^3(d+}$$

$$= \frac{5\left(16c^2d^2+5b^2e^2-4ce(5bd-ae)-4ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8e^5} + \frac{5(8cd-3be+2cex)\,(}{12e^3(d+}$$

Mathematica [A] time = 1.97118, size = 445, normalized size = 1.34

$$\frac{5\left(\frac{-2c^2e\sqrt{a+x(b+cx)}\left(e(ae-bd)+cd^2\right)\left(4ce(ae-5bd+bex)+5b^2e^2+8c^2d(2d-ex)\right)+2c^2\left(4ce(ae-4bd)+3b^2e^2+16c^2d^2\right)\left(e(ae-bd)+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)+c^{3/2}(2cd-be)(e(}{4c^2e^5}\right.}{e(bd-ae)-cd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/(d + e*x)^3,x]
```

```
[Out] ((-2*(a + x*(b + c*x))^(5/2))/(d + e*x)^2 + (5*(2*c*d - b*e)*(a + x*(b + c*
x))^(5/2))/((c*d^2 + e*(-(b*d) + a*e))*(d + e*x)) + (5*(-((a + x*(b + c*x))
^(3/2)*(3*b^2*e^2 + 2*c^2*d*(4*d - 3*e*x) + c*e*(-11*b*d + 2*a*e + 3*b*e*x)
))/(3*e^2) + (-2*c^2*e*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[a + x*(b + c*x)]*(5*
b^2*e^2 + 8*c^2*d*(2*d - e*x) + 4*c*e*(-5*b*d + a*e + b*e*x)) + c^(3/2)*(2*
c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e))*(16*c^2*d^2 + b^2*e^2 + 4*c*e*(-4*b*d
 + 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*c^2*(
16*c^2*d^2 + 3*b^2*e^2 + 4*c*e*(-4*b*d + a*e))*(c*d^2 + e*(-(b*d) + a*e))^(
3/2)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) +
 a*e)]*Sqrt[a + x*(b + c*x)])])/(4*c^2*e^5)))/(-(c*d^2) + e*(b*d - a*e)))/(
4*e)
```

Maple [B] time = 0.237, size = 14002, normalized size = 42.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(e*x+d)^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.12558, size = 1968, normalized size = 5.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 5/4*(16*c^3*d^4 - 32*b*c^2*d^3*e + 19*b^2*c*d^2*e^2 + 20*a*c^2*d^2*e^2 - 3*
b^3*d*e^3 - 20*a*b*c*d*e^3 + 3*a*b^2*e^4 + 4*a^2*c*e^4)*arctan(-((sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))*e^(-
6)/sqrt(-c*d^2 + b*d*e - a*e^2) + 5/16*(32*c^3*d^3 - 48*b*c^2*d^2*e + 18*b^
2*c*d*e^2 + 24*a*c^2*d*e^2 - b^3*e^3 - 12*a*b*c*e^3)*e^(-6)*log(abs(2*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/sqrt(c) + 1/24*sqrt(c*x^2 + b*
x + a)*(2*(4*c^2*x*e^(-3) - (18*c^4*d*e^14 - 13*b*c^3*e^15)*e^(-18)/c^2)*x
+ (144*c^4*d^2*e^13 - 162*b*c^3*d*e^14 + 33*b^2*c^2*e^15 + 56*a*c^3*e^15)*e
^(-18)/c^2) + 1/4*(40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^3*d^4*e + 72*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(7/2)*d^5 - 120*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^2*b*c^(5/2)*d^4*e + 72*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*b*c^3*d^5 - 80*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c^2*d^3*e^2 - 124*(
```

```
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c^2*d^4*e - 104*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))*a*c^3*d^4*e + 18*b^2*c^(5/2)*d^5 + 51*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^2*b^2*c^(3/2)*d^3*e^2 + 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^2*a*c^(5/2)*d^3*e^2 - 27*b^3*c^(3/2)*d^4*e - 52*a*b*c^(5/2)*d^4*e + 49*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*c*d^2*e^3 + 44*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^3*a*c^2*d^2*e^3 + 59*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b
^3*c*d^3*e^2 + 244*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c^2*d^3*e^2 - 3*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^3*sqrt(c)*d^2*e^3 + 12*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^2*a*b*c^(3/2)*d^2*e^3 + 9*b^4*sqrt(c)*d^3*e^2 + 95*
a*b^2*c^(3/2)*d^3*e^2 + 36*a^2*c^(5/2)*d^3*e^2 - 9*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^3*b^3*d*e^4 - 44*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*c*d*
e^4 - 7*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^4*d^2*e^3 - 127*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))*a*b^2*c*d^2*e^3 - 100*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*a^2*c^2*d^2*e^3 - 21*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^2*sqrt
(c)*d*e^4 - 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*c^(3/2)*d*e^4 - 34
*a*b^3*sqrt(c)*d^2*e^3 - 104*a^2*b*c^(3/2)*d^2*e^3 + 9*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^3*a*b^2*e^5 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*c
*e^5 + 14*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^3*d*e^4 + 64*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))*a^2*b*c*d*e^4 + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^2*a^2*b*sqrt(c)*e^5 + 41*a^2*b^2*sqrt(c)*d*e^4 + 36*a^3*c^(3/2)*d*e^4 -
 7*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^2*e^5 + 4*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))*a^3*c*e^5 - 16*a^3*b*sqrt(c)*e^5)*e^(-6)/((sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^2*e + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c)*d +
b*d - a*e)^2
```

3.2362 $\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=337

$$-\frac{5\sqrt{a+bx+cx^2}\left(-4ce(3bd-ae)+b^2e^2+4cex(2cd-be)+16c^2d^2\right)}{8e^5(d+ex)}+\frac{5\sqrt{c}\left(-4ce(4bd-ae)+3b^2e^2+16c^2d^2\right)\tanh^{-1}\left(\frac{}{2}\right.}{8e^6}$$

```
[Out] (-5*(16*c^2*d^2 + b^2*e^2 - 4*c*e*(3*b*d - a*e) + 4*c*e*(2*c*d - b*e)*x)*Sq
rt[a + b*x + c*x^2])/(8*e^5*(d + e*x)) + (5*(4*c*d - b*e + 2*c*e*x)*(a + b*
x + c*x^2)^(3/2))/(12*e^3*(d + e*x)^2) - (a + b*x + c*x^2)^(5/2)/(3*e*(d +
e*x)^3) + (5*Sqrt[c]*(16*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(4*b*d - a*e))*ArcTanh
[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*e^6) - (5*(2*c*d - b*e)
*(16*c^2*d^2 + b^2*e^2 - 4*c*e*(4*b*d - 3*a*e))*ArcTanh[(b*d - 2*a*e + (2*c
*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(16*e^
6*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rubi [A] time = 0.42885, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {732, 812, 843, 621, 206, 724}

$$-\frac{5\sqrt{a+bx+cx^2}\left(-4ce(3bd-ae)+b^2e^2+4cex(2cd-be)+16c^2d^2\right)}{8e^5(d+ex)}+\frac{5\sqrt{c}\left(-4ce(4bd-ae)+3b^2e^2+16c^2d^2\right)\tanh^{-1}\left(\frac{}{2}\right.}{8e^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/(d + e*x)^4,x]
```



```
[Out] (-5*(16*c^2*d^2 + b^2*e^2 - 4*c*e*(3*b*d - a*e) + 4*c*e*(2*c*d - b*e)*x)*Sq
rt[a + b*x + c*x^2])/(8*e^5*(d + e*x)) + (5*(4*c*d - b*e + 2*c*e*x)*(a + b*
x + c*x^2)^(3/2))/(12*e^3*(d + e*x)^2) - (a + b*x + c*x^2)^(5/2)/(3*e*(d +
e*x)^3) + (5*Sqrt[c]*(16*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(4*b*d - a*e))*ArcTanh
[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*e^6) - (5*(2*c*d - b*e)
*(16*c^2*d^2 + b^2*e^2 - 4*c*e*(4*b*d - 3*a*e))*ArcTanh[(b*d - 2*a*e + (2*c
*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(16*e^
6*Sqrt[c*d^2 - b*d*e + a*e^2])
```


Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
```

```
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(d+ex)^4}\,dx = -\frac{\left(a+bx+cx^2\right)^{5/2}}{3e(d+ex)^3}+\frac{5\int \frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{(d+ex)^3}\,dx}{6e}$$

$$= \frac{5(4cd-be+2cex)\left(a+bx+cx^2\right)^{3/2}}{12e^3(d+ex)^2}-\frac{\left(a+bx+cx^2\right)^{5/2}}{3e(d+ex)^3}-\frac{5\int \frac{\left(2\left(4bcd-b^2e-4ace\right)+8c(2cd-be)x\right)\sqrt{a+}}{(d+ex)^2}}{16e^3}$$

$$= -\frac{5\left(16c^2d^2+b^2e^2-4ce(3bd-ae)+4ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8e^5(d+ex)}+\frac{5(4cd-be+2cex)\left(a\right.}{12e^3(d+e}$$

$$= -\frac{5\left(16c^2d^2+b^2e^2-4ce(3bd-ae)+4ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8e^5(d+ex)}+\frac{5(4cd-be+2cex)\left(a\right.}{12e^3(d+e}$$

$$= -\frac{5\left(16c^2d^2+b^2e^2-4ce(3bd-ae)+4ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8e^5(d+ex)}+\frac{5(4cd-be+2cex)\left(a\right.}{12e^3(d+e}$$

$$= -\frac{5\left(16c^2d^2+b^2e^2-4ce(3bd-ae)+4ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8e^5(d+ex)}+\frac{5(4cd-be+2cex)\left(a\right.}{12e^3(d+e}$$

Mathematica [A] time = 3.40337, size = 559, normalized size = 1.66

$$5\left(-\frac{(a+x(b+cx))^{5/2}\left(4ce(2ae-3bd)+b^2e^2+12c^2d^2\right)}{2(d+ex)}+\frac{(a+x(b+cx))^{3/2}\left(2c^2e(2ae(2ex-3d)+3bd(5d-2ex))+bce^2(10ae-15bd+bex)+b^3e^3-4c^3d^2(4d-3ex)\right)}{2e^2}+\frac{3\left(2e\sqrt{a+x(b+cx)}\left(e(ae-bd)+cd^2\right)\left(4c^2e(ae(ex\cdots\right.\right.}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/(d + e*x)^4,x]
```

```
[Out] ((-2*(a + x*(b + c*x))^(5/2))/(d + e*x)^3 + (5*(2*c*d - b*e)*(a + x*(b + c*
x))^(5/2))/(2*(c*d^2 + e*(-(b*d) + a*e))*(d + e*x)^2) + (5*(-((12*c^2*d^2 +
 b^2*e^2 + 4*c*e*(-3*b*d + 2*a*e))*(a + x*(b + c*x))^(5/2))/(2*(d + e*x)) +
 ((a + x*(b + c*x))^(3/2)*(b^3*e^3 - 4*c^3*d^2*(4*d - 3*e*x) + b*c*e^2*(-15
*b*d + 10*a*e + b*e*x) + 2*c^2*e*(3*b*d*(5*d - 2*e*x) + 2*a*e*(-3*d + 2*e*x
))))/(2*e^2) + (3*(2*e*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[a + x*(b + c*x)]*(b^
3*e^3 + 8*c^3*d^2*(-2*d + e*x) + b*c*e^2*(-13*b*d + 8*a*e + b*e*x) + 4*c^2*
e*(b*d*(7*d - 2*e*x) + a*e*(-3*d + e*x))) + 2*Sqrt[c]*(16*c^2*d^2 + 3*b^2*e
^2 + 4*c*e*(-4*b*d + a*e))*(c*d^2 + e*(-(b*d) + a*e))^2*ArcTanh[(b + 2*c*x)
/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + (2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*
e))^(3/2)*(16*c^2*d^2 + b^2*e^2 + 4*c*e*(-4*b*d + 3*a*e))*ArcTanh[(-(b*d) +
 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b +
 c*x)])]))/(4*e^5)))/(2*(c*d^2 + e*(-(b*d) + a*e))^2))/(6*e)
```

Maple [B] time = 0.238, size = 18718, normalized size = 55.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(e*x+d)^4,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(e*x+d)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 27.1968, size = 2862, normalized size = 8.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -5/8*(32*c^3*d^3 - 48*b*c^2*d^2*e + 18*b^2*c*d*e^2 + 24*a*c^2*d*e^2 - b^3*e
^3 - 12*a*b*c*e^3)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)
*d)/sqrt(-c*d^2 + b*d*e - a*e^2))*e^(-6)/sqrt(-c*d^2 + b*d*e - a*e^2) - 5/8
*(16*c^3*d^2 - 16*b*c^2*d*e + 3*b^2*c*e^2 + 4*a*c^2*e^2)*e^(-6)*log(abs(2*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/sqrt(c) + 1/4*(2*c^2*x*e^(
-4) - (16*c^3*d*e^10 - 9*b*c^2*e^11)*e^(-15)/c)*sqrt(c*x^2 + b*x + a) - 1/2
4*(1680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*c^4*d^4*e + 1504*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^3*c^(9/2)*d^5 + 480*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^5*c^(7/2)*d^3*e^2 - 400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c^(7/2
)*d^4*e + 2256*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^4*d^5 - 2160*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^4*b*c^3*d^3*e^2 - 2412*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^2*b^2*c^3*d^4*e - 2832*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2
*a*c^4*d^4*e + 1128*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c^(7/2)*d^5 - 7
20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^(5/2)*d^2*e^3 - 1308*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^3*b^2*c^(5/2)*d^3*e^2 - 1808*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^3*a*c^(7/2)*d^3*e^2 - 1272*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*b^3*c^(5/2)*d^4*e - 2832*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c^(7/
2)*d^4*e + 188*b^3*c^3*d^5 + 666*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*
c^2*d^2*e^3 + 216*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*c^3*d^2*e^3 + 462
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^3*c^2*d^3*e^2 + 2952*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^2*a*b*c^3*d^3*e^2 - 188*b^4*c^2*d^4*e - 708*a*b^2*c^
3*d^4*e + 306*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c^(3/2)*d*e^4 + 216
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*c^(5/2)*d*e^4 + 574*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^3*b^3*c^(3/2)*d^2*e^3 + 3144*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^3*a*b*c^(5/2)*d^2*e^3 + 324*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*b^4*c^(3/2)*d^3*e^2 + 3420*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*c^(5/
2)*d^3*e^2 + 1776*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c^(7/2)*d^3*e^2 -
 21*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^3*c*d*e^4 + 324*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^4*a*b*c^2*d*e^4 + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^2*b^4*c*d^2*e^3 + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^2*c^2*d^
2*e^3 + 33*b^5*c*d^3*e^2 + 746*a*b^3*c^2*d^3*e^2 + 888*a^2*b*c^3*d^3*e^2 -
33*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*sqrt(c)*e^5 - 108*(sqrt(c)*x -
```

```
 sqrt(c*x^2 + b*x + a))^5*a*b*c^(3/2)*e^5 - 40*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^3*b^4*sqrt(c)*d*e^4 - 912*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b
^2*c^(3/2)*d*e^4 - 672*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*c^(5/2)*d*
e^4 - 15*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^5*sqrt(c)*d^2*e^3 - 774*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*a*b^3*c^(3/2)*d^2*e^3 - 2664*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*a^2*b*c^(5/2)*d^2*e^3 - 144*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^4*a*b^2*c*e^5 - 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^
2*e^5 - 168*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^3*c*d*e^4 - 1008*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b*c^2*d*e^4 - 114*a*b^4*c*d^2*e^3 - 1
050*a^2*b^2*c^2*d^2*e^3 - 376*a^3*c^3*d^2*e^3 + 40*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^3*a*b^3*sqrt(c)*e^5 + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*
a^2*b*c^(3/2)*e^5 + 30*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^4*sqrt(c)*d*
e^4 + 486*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^2*c^(3/2)*d*e^4 + 456*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*c^(5/2)*d*e^4 + 144*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^2*a^2*b^2*c*e^5 + 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^2*a^3*c^2*e^5 + 129*a^2*b^3*c*d*e^4 + 604*a^3*b*c^2*d*e^4 - 15*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*a^2*b^3*sqrt(c)*e^5 - 36*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))*a^3*b*c^(3/2)*e^5 - 48*a^3*b^2*c*e^5 - 112*a^4*c^2*e^5)*e^(-6)
/(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*e + 2*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))*sqrt(c)*d + b*d - a*e)^3*sqrt(c))
```

3.2363 $\int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)^5}\,dx$

Optimal. Leaf size=492

$$\frac{5\left(48c^2e^2\left(a^2e^2-4abde+3b^2d^2\right)-8b^2ce^3(2bd-3ae)-64c^3d^2e(4bd-3ae)-b^4e^4+128c^4d^4\right)\tanh^{-1}\left(\frac{-2ae+x(2cd-be)}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-}}\right.}{128e^6\left(ae^2-bde+cd^2\right)^{3/2}}$$

```
[Out] (5*(64*c^3*d^3 + b^3*e^3 + 4*b*c*e^2*(4*b*d - 5*a*e) - 16*c^2*d*e*(5*b*d -
4*a*e) + 2*c*e*(16*c^2*d^2 + b^2*e^2 - 4*c*e*(4*b*d - 3*a*e))*x)*Sqrt[a + b
*x + c*x^2])/(64*e^5*(c*d^2 - b*d*e + a*e^2)*(d + e*x)) - (5*(16*c^2*d^3 -
b*e^2*(b*d - 4*a*e) - 4*c*d*e*(3*b*d - a*e) + 3*e*(8*c^2*d^2 + b^2*e^2 - 4*
c*e*(2*b*d - a*e))*x)*(a + b*x + c*x^2)^(3/2))/(96*e^3*(c*d^2 - b*d*e + a*e
^2)*(d + e*x)^3) - (a + b*x + c*x^2)^(5/2)/(4*e*(d + e*x)^4) - (5*c^(3/2)*(
2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*e^6
) + (5*(128*c^4*d^4 - b^4*e^4 - 8*b^2*c*e^3*(2*b*d - 3*a*e) - 64*c^3*d^2*e*
(4*b*d - 3*a*e) + 48*c^2*e^2*(3*b^2*d^2 - 4*a*b*d*e + a^2*e^2))*ArcTanh[(b*
d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x +
c*x^2])])/(128*e^6*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rubi [A] time = 0.734763, antiderivative size = 492, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {732, 810, 812, 843, 621, 206, 724}

$$\frac{5\left(48c^2e^2\left(a^2e^2-4abde+3b^2d^2\right)-8b^2ce^3(2bd-3ae)-64c^3d^2e(4bd-3ae)-b^4e^4+128c^4d^4\right)\tanh^{-1}\left(\frac{-2ae+x(2cd-be)}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-}}\right.}{128e^6\left(ae^2-bde+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/(d + e*x)^5,x]
```

```
[Out] (5*(64*c^3*d^3 + b^3*e^3 + 4*b*c*e^2*(4*b*d - 5*a*e) - 16*c^2*d*e*(5*b*d -
4*a*e) + 2*c*e*(16*c^2*d^2 + b^2*e^2 - 4*c*e*(4*b*d - 3*a*e))*x)*Sqrt[a + b
*x + c*x^2])/(64*e^5*(c*d^2 - b*d*e + a*e^2)*(d + e*x)) - (5*(16*c^2*d^3 -
b*e^2*(b*d - 4*a*e) - 4*c*d*e*(3*b*d - a*e) + 3*e*(8*c^2*d^2 + b^2*e^2 - 4*
c*e*(2*b*d - a*e))*x)*(a + b*x + c*x^2)^(3/2))/(96*e^3*(c*d^2 - b*d*e + a*e
^2)*(d + e*x)^3) - (a + b*x + c*x^2)^(5/2)/(4*e*(d + e*x)^4) - (5*c^(3/2)*(
2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*e^6
) + (5*(128*c^4*d^4 - b^4*e^4 - 8*b^2*c*e^3*(2*b*d - 3*a*e) - 64*c^3*d^2*e*
(4*b*d - 3*a*e) + 48*c^2*e^2*(3*b^2*d^2 - 4*a*b*d*e + a^2*e^2))*ArcTanh[(b*
d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x +
c*x^2])])/(128*e^6*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(d+ex)^5}\,dx = -\frac{\left(a+bx+cx^2\right)^{5/2}}{4e(d+ex)^4} + \frac{5\int \frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{(d+ex)^4}\,dx}{8e}$$

$$= -\frac{5\left(16c^2d^3 - be^2(bd-4ae) - 4cde(3bd-ae) + 3e\left(8c^2d^2 + b^2e^2 - 4ce(2bd-ae)\right)x\right)\left(a+bx\right.}{96e^3\left(cd^2-bde+ae^2\right)(d+ex)^3}$$

$$= \frac{5\left(64c^3d^3 + b^3e^3 + 4bce^2(4bd-5ae) - 16c^2de(5bd-4ae) + 2ce\left(16c^2d^2 + b^2e^2 - 4ce(4bd-3\right.\right.}{64e^5\left(cd^2-bde+ae^2\right)(d+ex)}$$

$$= \frac{5\left(64c^3d^3 + b^3e^3 + 4bce^2(4bd-5ae) - 16c^2de(5bd-4ae) + 2ce\left(16c^2d^2 + b^2e^2 - 4ce(4bd-3\right.\right.}{64e^5\left(cd^2-bde+ae^2\right)(d+ex)}$$

$$= \frac{5\left(64c^3d^3 + b^3e^3 + 4bce^2(4bd-5ae) - 16c^2de(5bd-4ae) + 2ce\left(16c^2d^2 + b^2e^2 - 4ce(4bd-3\right.\right.}{64e^5\left(cd^2-bde+ae^2\right)(d+ex)}$$

$$= \frac{5\left(64c^3d^3 + b^3e^3 + 4bce^2(4bd-5ae) - 16c^2de(5bd-4ae) + 2ce\left(16c^2d^2 + b^2e^2 - 4ce(4bd-3\right.\right.}{64e^5\left(cd^2-bde+ae^2\right)(d+ex)}$$

Mathematica [A] time = 3.9276, size = 593, normalized size = 1.21

$$\frac{2e\sqrt{a+x(b+cx)}\left(2ce^2\left(-4a^2e^2\left(11d^2+20dex+27e^2x^2\right)-2abe\left(169d^2ex+45d^3+191de^2x^2+139e^3x^3\right)+b^2d\left(435d^2ex+120d^3+566de^2x^2+323e^3x^3\right)\right)+e^3\left(8a^2be^2(d-17ex\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/(d + e*x)^5,x]
```

```
[Out] ((2*e*Sqrt[a + x*(b + c*x)]*(16*c^3*d^2*(60*d^4 + 210*d^3*e*x + 260*d^2*e^2
*x^2 + 125*d*e^3*x^3 + 12*e^4*x^4) + e^3*(-48*a^3*e^3 + 8*a^2*b*e^2*(d - 17
*e*x) + 2*a*b^2*e*(5*d^2 + 18*d*e*x - 59*e^2*x^2) + b^3*(15*d^3 + 55*d^2*e*
x + 73*d*e^2*x^2 - 15*e^3*x^3)) + 2*c*e^2*(-4*a^2*e^2*(11*d^2 + 20*d*e*x +
27*e^2*x^2) - 2*a*b*e*(45*d^3 + 169*d^2*e*x + 191*d*e^2*x^2 + 139*e^3*x^3)
+ b^2*d*(120*d^3 + 435*d^2*e*x + 566*d*e^2*x^2 + 323*e^3*x^3)) - 8*c^2*e*(-
(a*e*(100*d^4 + 355*d^3*e*x + 448*d^2*e^2*x^2 + 235*d*e^3*x^3 + 24*e^4*x^4)
) + b*d*(150*d^4 + 530*d^3*e*x + 665*d^2*e^2*x^2 + 327*d*e^3*x^3 + 24*e^4*x
^4))))/((c*d^2 + e*(-(b*d) + a*e))*(d + e*x)^4) - 960*c^(3/2)*(2*c*d - b*e)
*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - (15*(128*c^4*d^4
- b^4*e^4 - 8*b^2*c*e^3*(2*b*d - 3*a*e) - 64*c^3*d^2*e*(4*b*d - 3*a*e) + 48
*c^2*e^2*(3*b^2*d^2 - 4*a*b*d*e + a^2*e^2))*ArcTanh[(-(b*d) + 2*a*e - 2*c*d
*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/(c*d
^2 + e*(-(b*d) + a*e))^(3/2))/(384*e^6)
```

Maple [B] time = 0.252, size = 28635, normalized size = 58.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(e*x+d)^5,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(e*x+d)**5,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.2364 $\int \frac{\sqrt{-2-3x+5x^2}}{x} \, dx$

Optimal. Leaf size=88

$$\sqrt{5x^2-3x-2}+\sqrt{2}\tan^{-1}\left(\frac{3x+4}{2\sqrt{2}\sqrt{5x^2-3x-2}}\right)+\frac{3\tanh^{-1}\left(\frac{3-10x}{2\sqrt{5}\sqrt{5x^2-3x-2}}\right)}{2\sqrt{5}}$$

```
[Out] Sqrt[-2 - 3*x + 5*x^2] + Sqrt[2]*ArcTan[(4 + 3*x)/(2*Sqrt[2]*Sqrt[-2 - 3*x
+ 5*x^2])] + (3*ArcTanh[(3 - 10*x)/(2*Sqrt[5]*Sqrt[-2 - 3*x + 5*x^2])])/(2*
Sqrt[5])
```

Rubi [A] time = 0.0487672, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {734, 843, 621, 206, 724, 204}

$$\sqrt{5x^2-3x-2}+\sqrt{2}\tan^{-1}\left(\frac{3x+4}{2\sqrt{2}\sqrt{5x^2-3x-2}}\right)+\frac{3\tanh^{-1}\left(\frac{3-10x}{2\sqrt{5}\sqrt{5x^2-3x-2}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-2 - 3*x + 5*x^2]/x,x]
```

```
[Out] Sqrt[-2 - 3*x + 5*x^2] + Sqrt[2]*ArcTan[(4 + 3*x)/(2*Sqrt[2]*Sqrt[-2 - 3*x
+ 5*x^2])] + (3*ArcTanh[(3 - 10*x)/(2*Sqrt[5]*Sqrt[-2 - 3*x + 5*x^2])])/(2*
Sqrt[5])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-2-3x+5x^2}}{x}\,dx &= \sqrt{-2-3x+5x^2}-\frac{1}{2}\int \frac{4+3x}{x\sqrt{-2-3x+5x^2}}\,dx\\ &= \sqrt{-2-3x+5x^2}-\frac{3}{2}\int \frac{1}{\sqrt{-2-3x+5x^2}}\,dx-2\int \frac{1}{x\sqrt{-2-3x+5x^2}}\,dx\\ &= \sqrt{-2-3x+5x^2}-3\,\text{Subst}\left(\int \frac{1}{20-x^2}\,dx,x,\frac{-3+10x}{\sqrt{-2-3x+5x^2}}\right)+4\,\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx,x,\frac{}{\sqrt{}}\right.\\ &= \sqrt{-2-3x+5x^2}+\sqrt{2}\tan^{-1}\left(\frac{4+3x}{2\sqrt{2}\sqrt{-2-3x+5x^2}}\right)+\frac{3\tanh^{-1}\left(\frac{3-10x}{2\sqrt{5}\sqrt{-2-3x+5x^2}}\right)}{2\sqrt{5}}\end{aligned}$$

Mathematica [A] time = 0.0359788, size = 84, normalized size = 0.95

$$\sqrt{5x^2-3x-2}-\sqrt{2}\tan^{-1}\left(\frac{-3x-4}{2\sqrt{10x^2-6x-4}}\right)-\frac{3\tanh^{-1}\left(\frac{10x-3}{2\sqrt{5}\sqrt{5x^2-3x-2}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-2 - 3*x + 5*x^2]/x,x]
```

```
[Out] Sqrt[-2 - 3*x + 5*x^2] - Sqrt[2]*ArcTan[(-4 - 3*x)/(2*Sqrt[-4 - 6*x + 10*x^
2])] - (3*ArcTanh[(-3 + 10*x)/(2*Sqrt[5]*Sqrt[-2 - 3*x + 5*x^2])])/(2*Sqrt[
5])
```

Maple [A] time = 0.042, size = 71, normalized size = 0.8

$$\sqrt{5\,x^2-3\,x-2}-\frac{3\sqrt{5}}{10}\ln\left(\frac{\sqrt{5}}{5}\left(-\frac{3}{2}+5\,x\right)+\sqrt{5\,x^2-3\,x-2}\right)-\sqrt{2}\arctan\left(\frac{(-3\,x-4)\sqrt{2}}{4}\frac{1}{\sqrt{5\,x^2-3\,x-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2-3*x-2)^(1/2)/x,x)
```

```
[Out] (5*x^2-3*x-2)^(1/2)-3/10*ln(1/5*(-3/2+5*x)*5^(1/2)+(5*x^2-3*x-2)^(1/2))*5^(
1/2)-2^(1/2)*arctan(1/4*(-3*x-4)*2^(1/2)/(5*x^2-3*x-2)^(1/2))
```

Maxima [A] time = 1.51522, size = 81, normalized size = 0.92

$$\sqrt{2}\arcsin\left(\frac{3x}{7|x|}+\frac{4}{7|x|}\right)-\frac{3}{10}\sqrt{5}\log\left(2\sqrt{5}\sqrt{5x^2-3x-2}+10x-3\right)+\sqrt{5x^2-3x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2-3*x-2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(2)*arcsin(3/7*x/abs(x) + 4/7/abs(x)) - 3/10*sqrt(5)*log(2*sqrt(5)*sqrt
(5*x^2 - 3*x - 2) + 10*x - 3) + sqrt(5*x^2 - 3*x - 2)
```

Fricas [A] time = 2.96689, size = 232, normalized size = 2.64

$$\sqrt{2}\arctan\left(\frac{\sqrt{2}(3x+4)}{4\sqrt{5x^2-3x-2}}\right)+\frac{3}{20}\sqrt{5}\log\left(-4\sqrt{5}\sqrt{5x^2-3x-2}(10x-3)+200x^2-120x-31\right)+\sqrt{5x^2-3x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2-3*x-2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] sqrt(2)*arctan(1/4*sqrt(2)*(3*x + 4)/sqrt(5*x^2 - 3*x - 2)) + 3/20*sqrt(5)*
log(-4*sqrt(5)*sqrt(5*x^2 - 3*x - 2)*(10*x - 3) + 200*x^2 - 120*x - 31) + s
qrt(5*x^2 - 3*x - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{(x-1)(5x+2)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2-3*x-2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt((x - 1)*(5*x + 2))/x, x)
```

Giac [A] time = 1.10228, size = 104, normalized size = 1.18

$$-2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{5}x-\sqrt{5x^2-3x-2}\right)\right)+\frac{3}{10}\sqrt{5}\log\left(\left|-10\sqrt{5}x+3\sqrt{5}+10\sqrt{5x^2-3x-2}\right|\right)+\sqrt{5x^2-3x-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2-3*x-2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(5)*x - sqrt(5*x^2 - 3*x - 2))) + 3/10*
sqrt(5)*log(abs(-10*sqrt(5)*x + 3*sqrt(5) + 10*sqrt(5*x^2 - 3*x - 2))) + sq
rt(5*x^2 - 3*x - 2)
```

3.2365 $\int \frac{\sqrt{2-x-x^2}}{x^2}\,dx$

Optimal. Leaf size=68

$$-\frac{\sqrt{-x^2-x+2}}{x}+\frac{\tanh^{-1}\left(\frac{4-x}{2\sqrt{2}\sqrt{-x^2-x+2}}\right)}{2\sqrt{2}}+\sin^{-1}\left(\frac{1}{3}(-2x-1)\right)$$

```
[Out] -(Sqrt[2 - x - x^2]/x) + ArcSin[(-1 - 2*x)/3] + ArcTanh[(4 - x)/(2*Sqrt[2]*
Sqrt[2 - x - x^2])]/(2*Sqrt[2])
```

Rubi [A] time = 0.0406899, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {732, 843, 619, 216, 724, 206}

$$-\frac{\sqrt{-x^2-x+2}}{x}+\frac{\tanh^{-1}\left(\frac{4-x}{2\sqrt{2}\sqrt{-x^2-x+2}}\right)}{2\sqrt{2}}+\sin^{-1}\left(\frac{1}{3}(-2x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - x - x^2]/x^2,x]
```


```
[Out] -(Sqrt[2 - x - x^2]/x) + ArcSin[(-1 - 2*x)/3] + ArcTanh[(4 - x)/(2*Sqrt[2]*
Sqrt[2 - x - x^2])]/(2*Sqrt[2])
```


Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2-x-x^2}}{x^2}\,dx &= -\frac{\sqrt{2-x-x^2}}{x}+\frac{1}{2}\int \frac{-1-2x}{x\sqrt{2-x-x^2}}\,dx\\ &= -\frac{\sqrt{2-x-x^2}}{x}-\frac{1}{2}\int \frac{1}{x\sqrt{2-x-x^2}}\,dx-\int \frac{1}{\sqrt{2-x-x^2}}\,dx\\ &= -\frac{\sqrt{2-x-x^2}}{x}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx,x,-1-2x\right)+\text{Subst}\left(\int \frac{1}{8-x^2}\,dx,x,\frac{4-x}{\sqrt{2-x-x^2}}\right)\\ &= -\frac{\sqrt{2-x-x^2}}{x}+\sin^{-1}\left(\frac{1}{3}(-1-2x)\right)+\frac{\tanh^{-1}\left(\frac{4-x}{2\sqrt{2}\sqrt{2-x-x^2}}\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0339038, size = 68, normalized size = 1.

$$-\frac{\sqrt{-x^2-x+2}}{x}+\frac{\tanh^{-1}\left(\frac{4-x}{2\sqrt{2}\sqrt{-x^2-x+2}}\right)}{2\sqrt{2}}+\sin^{-1}\left(\frac{1}{3}(-2x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - x - x^2]/x^2,x]
```

```
[Out] -(Sqrt[2 - x - x^2]/x) + ArcSin[(-1 - 2*x)/3] + ArcTanh[(4 - x)/(2*Sqrt[2]*
Sqrt[2 - x - x^2])]/(2*Sqrt[2])
```

Maple [A] time = 0.043, size = 88, normalized size = 1.3

$$-\frac{1}{2x}\left(-x^2-x+2\right)^{\frac{3}{2}}-\frac{1}{4}\sqrt{-x^2-x+2}-\arcsin\left(\frac{1}{3}+\frac{2x}{3}\right)+\frac{\sqrt{2}}{4}\text{Artanh}\left(\frac{(4-x)\sqrt{2}}{4}\frac{1}{\sqrt{-x^2-x+2}}\right)+\frac{-1-2x}{4}\sqrt{-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2-x+2)^(1/2)/x^2,x)
```

```
[Out] -1/2/x*(-x^2-x+2)^(3/2)-1/4*(-x^2-x+2)^(1/2)-arcsin(1/3+2/3*x)+1/4*arctanh(
1/4*(4-x)*2^(1/2)/(-x^2-x+2)^(1/2))*2^(1/2)+1/4*(-1-2*x)*(-x^2-x+2)^(1/2)
```

Maxima [A] time = 1.53581, size = 80, normalized size = 1.18

$$\frac{1}{4}\sqrt{2}\log\left(\frac{2\sqrt{2}\sqrt{-x^2-x+2}}{|x|}+\frac{4}{|x|}-1\right)-\frac{\sqrt{-x^2-x+2}}{x}+\arcsin\left(-\frac{2}{3}x-\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-x+2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log(2*sqrt(2)*sqrt(-x^2 - x + 2)/abs(x) + 4/abs(x) - 1) - sqrt(
-x^2 - x + 2)/x + arcsin(-2/3*x - 1/3)
```

Fricas [A] time = 2.62183, size = 232, normalized size = 3.41

$$\frac{\sqrt{2}x\log\left(-\frac{4\sqrt{2}\sqrt{-x^2-x+2}(x-4)+7x^2+16x-32}{x^2}\right)+8x\arctan\left(\frac{\sqrt{-x^2-x+2}(2x+1)}{2(x^2+x-2)}\right)-8\sqrt{-x^2-x+2}}{8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-x+2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*x*log(-(4*sqrt(2)*sqrt(-x^2 - x + 2)*(x - 4) + 7*x^2 + 16*x -
32)/x^2) + 8*x*arctan(1/2*sqrt(-x^2 - x + 2)*(2*x + 1)/(x^2 + x - 2)) - 8*s
qrt(-x^2 - x + 2))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-(x-1)(x+2)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2-x+2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 2))/x**2, x)
```

Giac [B] time = 1.10625, size = 227, normalized size = 3.34

$$-\frac{1}{4}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+\frac{2\left(2\sqrt{-x^2-x+2}-3\right)}{2x+1}+6\right|}{\left|4\sqrt{2}+\frac{2\left(2\sqrt{-x^2-x+2}-3\right)}{2x+1}+6\right|}\right)+\frac{6\left(\frac{3\left(2\sqrt{-x^2-x+2}-3\right)}{2x+1}+1\right)}{\frac{6\left(2\sqrt{-x^2-x+2}-3\right)}{2x+1}+\frac{\left(2\sqrt{-x^2-x+2}-3\right)^2}{(2x+1)^2}+1}-\arcsin\left(\frac{2}{3}x+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-x+2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*log(abs(-4*sqrt(2) + 2*(2*sqrt(-x^2 - x + 2) - 3)/(2*x + 1) +
6)/abs(4*sqrt(2) + 2*(2*sqrt(-x^2 - x + 2) - 3)/(2*x + 1) + 6)) + 6*(3*(2*s
qrt(-x^2 - x + 2) - 3)/(2*x + 1) + 1)/(6*(2*sqrt(-x^2 - x + 2) - 3)/(2*x +
1) + (2*sqrt(-x^2 - x + 2) - 3)^2/(2*x + 1)^2 + 1) - arcsin(2/3*x + 1/3)
```

3.2366 $\int (1+x)^3\sqrt{2+2x+x^2}\,dx$

Optimal. Leaf size=38

$$\frac{1}{5}(x+1)^2\left(x^2+2x+2\right)^{3/2}-\frac{2}{15}\left(x^2+2x+2\right)^{3/2}$$

```
[Out] (-2*(2 + 2*x + x^2)^(3/2))/15 + ((1 + x)^2*(2 + 2*x + x^2)^(3/2))/5
```

Rubi [A] time = 0.0118175, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {692, 629}

$$\frac{1}{5}(x+1)^2\left(x^2+2x+2\right)^{3/2}-\frac{2}{15}\left(x^2+2x+2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^3*Sqrt[2 + 2*x + x^2],x]
```

```
[Out] (-2*(2 + 2*x + x^2)^(3/2))/15 + ((1 + x)^2*(2 + 2*x + x^2)^(3/2))/5
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (1+x)^3\sqrt{2+2x+x^2}\,dx &= \frac{1}{5}(1+x)^2\left(2+2x+x^2\right)^{3/2}-\frac{2}{5}\int (1+x)\sqrt{2+2x+x^2}\,dx\\ &= -\frac{2}{15}\left(2+2x+x^2\right)^{3/2}+\frac{1}{5}(1+x)^2\left(2+2x+x^2\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.0093214, size = 26, normalized size = 0.68

$$\frac{1}{15}\left(x^2+2x+2\right)^{3/2}\left(3x^2+6x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^3*Sqrt[2 + 2*x + x^2],x]
```

```
[Out] ((2 + 2*x + x^2)^(3/2)*(1 + 6*x + 3*x^2))/15
```

Maple [A] time = 0.04, size = 23, normalized size = 0.6

$$\frac{3x^2+6x+1}{15}\left(x^2+2x+2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^3*(x^2+2*x+2)^(1/2),x)
```

```
[Out] 1/15*(x^2+2*x+2)^(3/2)*(3*x^2+6*x+1)
```

Maxima [A] time = 1.51231, size = 55, normalized size = 1.45

$$\frac{1}{5}\left(x^2+2x+2\right)^{\frac{3}{2}}x^2+\frac{2}{5}\left(x^2+2x+2\right)^{\frac{3}{2}}x+\frac{1}{15}\left(x^2+2x+2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3*(x^2+2*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*(x^2 + 2*x + 2)^(3/2)*x^2 + 2/5*(x^2 + 2*x + 2)^(3/2)*x + 1/15*(x^2 + 2
*x + 2)^(3/2)
```

Fricas [A] time = 2.33861, size = 85, normalized size = 2.24

$$\frac{1}{15}\left(3x^4+12x^3+19x^2+14x+2\right)\sqrt{x^2+2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3*(x^2+2*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*x^4 + 12*x^3 + 19*x^2 + 14*x + 2)*sqrt(x^2 + 2*x + 2)
```

Sympy [B] time = 0.259986, size = 85, normalized size = 2.24

$$\frac{x^4\sqrt{x^2+2x+2}}{5}+\frac{4x^3\sqrt{x^2+2x+2}}{5}+\frac{19x^2\sqrt{x^2+2x+2}}{15}+\frac{14x\sqrt{x^2+2x+2}}{15}+\frac{2\sqrt{x^2+2x+2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**3*(x**2+2*x+2)**(1/2),x)
```

```
[Out] x**4*sqrt(x**2 + 2*x + 2)/5 + 4*x**3*sqrt(x**2 + 2*x + 2)/5 + 19*x**2*sqrt(
x**2 + 2*x + 2)/15 + 14*x*sqrt(x**2 + 2*x + 2)/15 + 2*sqrt(x**2 + 2*x + 2)/
15
```

Giac [A] time = 1.08866, size = 38, normalized size = 1.

$$\frac{1}{15}\left(\left(\left(3\left(x+4\right)x+19\right)x+14\right)x+2\right)\sqrt{x^2+2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3*(x^2+2*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*(((3*(x + 4)*x + 19)*x + 14)*x + 2)*sqrt(x^2 + 2*x + 2)
```

3.2367 $\int (-2+3x)\sqrt{8+12x+9x^2}\,dx$

Optimal. Leaf size=54

$$\frac{1}{9}\left(9x^2+12x+8\right)^{3/2}-\frac{2}{3}(3x+2)\sqrt{9x^2+12x+8}-\frac{8}{3}\sinh^{-1}\left(\frac{3x}{2}+1\right)$$

```
[Out] (-2*(2 + 3*x)*Sqrt[8 + 12*x + 9*x^2])/3 + (8 + 12*x + 9*x^2)^(3/2)/9 - (8*A
rcSinh[1 + (3*x)/2])/3
```

Rubi [A] time = 0.0180984, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {640, 612, 619, 215}

$$\frac{1}{9}\left(9x^2+12x+8\right)^{3/2}-\frac{2}{3}(3x+2)\sqrt{9x^2+12x+8}-\frac{8}{3}\sinh^{-1}\left(\frac{3x}{2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 3*x)*Sqrt[8 + 12*x + 9*x^2],x]
```

```
[Out] (-2*(2 + 3*x)*Sqrt[8 + 12*x + 9*x^2])/3 + (8 + 12*x + 9*x^2)^(3/2)/9 - (8*A
rcSinh[1 + (3*x)/2])/3
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int (-2+3x)\sqrt{8+12x+9x^2}\,dx = \frac{1}{9}\left(8+12x+9x^2\right)^{3/2} - 4\int \sqrt{8+12x+9x^2}\,dx$$
$$= -\frac{2}{3}(2+3x)\sqrt{8+12x+9x^2} + \frac{1}{9}\left(8+12x+9x^2\right)^{3/2} - 8\int \frac{1}{\sqrt{8+12x+9x^2}}\,dx$$
$$= -\frac{2}{3}(2+3x)\sqrt{8+12x+9x^2} + \frac{1}{9}\left(8+12x+9x^2\right)^{3/2} - \frac{2}{9}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{144}}}\,dx, x, 1$$
$$= -\frac{2}{3}(2+3x)\sqrt{8+12x+9x^2} + \frac{1}{9}\left(8+12x+9x^2\right)^{3/2} - \frac{8}{3}\sinh^{-1}\left(1+\frac{3x}{2}\right)$$

Mathematica [A] time = 0.0203861, size = 40, normalized size = 0.74

$$\frac{1}{9}\left(\left(9x^2-6x-4\right)\sqrt{9x^2+12x+8} - 24\sinh^{-1}\left(\frac{3x}{2}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 3*x)*Sqrt[8 + 12*x + 9*x^2],x]
```

```
[Out] ((-4 - 6*x + 9*x^2)*Sqrt[8 + 12*x + 9*x^2] - 24*ArcSinh[1 + (3*x)/2])/9
```

Maple [A] time = 0.052, size = 43, normalized size = 0.8

$$-\frac{18x+12}{9}\sqrt{9x^2+12x+8} - \frac{8}{3}\text{Arcsinh}\left(1+\frac{3x}{2}\right) + \frac{1}{9}\left(9x^2+12x+8\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+3*x)*(9*x^2+12*x+8)^(1/2),x)
```

```
[Out] -1/9*(18*x+12)*(9*x^2+12*x+8)^(1/2)-8/3*arcsinh(1+3/2*x)+1/9*(9*x^2+12*x+8)
^(3/2)
```

Maxima [A] time = 1.49423, size = 70, normalized size = 1.3

$$\frac{1}{9}\left(9x^2+12x+8\right)^{\frac{3}{2}} - 2\sqrt{9x^2+12x+8}\,x - \frac{4}{3}\sqrt{9x^2+12x+8} - \frac{8}{3}\,\text{arsinh}\left(\frac{3}{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+3*x)*(9*x^2+12*x+8)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*(9*x^2 + 12*x + 8)^(3/2) - 2*sqrt(9*x^2 + 12*x + 8)*x - 4/3*sqrt(9*x^2
+ 12*x + 8) - 8/3*arcsinh(3/2*x + 1)
```

Fricas [A] time = 2.26475, size = 123, normalized size = 2.28

$$\frac{1}{9}\sqrt{9x^2+12x+8}\left(9x^2-6x-4\right) + \frac{8}{3}\log\left(-3x+\sqrt{9x^2+12x+8}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+3*x)*(9*x^2+12*x+8)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(9*x^2 + 12*x + 8)*(9*x^2 - 6*x - 4) + 8/3*log(-3*x + sqrt(9*x^2 +
12*x + 8) - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (3x-2)\sqrt{9x^2+12x+8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+3*x)*(9*x**2+12*x+8)**(1/2),x)
```

```
[Out] Integral((3*x - 2)*sqrt(9*x**2 + 12*x + 8), x)
```

Giac [A] time = 1.0945, size = 61, normalized size = 1.13

$$\frac{1}{9}\left(3\left(3x-2\right)x-4\right)\sqrt{9x^2+12x+8}+\frac{8}{3}\log\left(-3x+\sqrt{9x^2+12x+8}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+3*x)*(9*x^2+12*x+8)^(1/2),x, algorithm="giac")
```

```
[Out] 1/9*(3*(3*x - 2)*x - 4)*sqrt(9*x^2 + 12*x + 8) + 8/3*log(-3*x + sqrt(9*x^2
+ 12*x + 8) - 2)
```

3.2368 $\int (7-2x)\sqrt{9+16x-4x^2}\,dx$

Optimal. Leaf size=56

$$\frac{1}{6}\left(-4x^2+16x+9\right)^{3/2}-\frac{3}{2}(2-x)\sqrt{-4x^2+16x+9}-\frac{75}{4}\sin^{-1}\left(\frac{2(2-x)}{5}\right)$$

```
[Out] (-3*(2 - x)*Sqrt[9 + 16*x - 4*x^2])/2 + (9 + 16*x - 4*x^2)^(3/2)/6 - (75*Ar
cSin[(2*(2 - x))/5])/4
```

Rubi [A] time = 0.0171845, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {640, 612, 619, 216}

$$\frac{1}{6}\left(-4x^2+16x+9\right)^{3/2}-\frac{3}{2}(2-x)\sqrt{-4x^2+16x+9}-\frac{75}{4}\sin^{-1}\left(\frac{2(2-x)}{5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(7 - 2*x)*Sqrt[9 + 16*x - 4*x^2],x]
```

```
[Out] (-3*(2 - x)*Sqrt[9 + 16*x - 4*x^2])/2 + (9 + 16*x - 4*x^2)^(3/2)/6 - (75*Ar
cSin[(2*(2 - x))/5])/4
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (7-2x)\sqrt{9+16x-4x^2}\,dx = \frac{1}{6}\left(9+16x-4x^2\right)^{3/2}+3\int \sqrt{9+16x-4x^2}\,dx$$
$$= -\frac{3}{2}(2-x)\sqrt{9+16x-4x^2}+\frac{1}{6}\left(9+16x-4x^2\right)^{3/2}+\frac{75}{2}\int \frac{1}{\sqrt{9+16x-4x^2}}\,dx$$
$$= -\frac{3}{2}(2-x)\sqrt{9+16x-4x^2}+\frac{1}{6}\left(9+16x-4x^2\right)^{3/2}-\frac{15}{16}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{400}}}\,dx,x,16-8x\right)$$
$$= -\frac{3}{2}(2-x)\sqrt{9+16x-4x^2}+\frac{1}{6}\left(9+16x-4x^2\right)^{3/2}-\frac{75}{4}\sin^{-1}\left(\frac{2(2-x)}{5}\right)$$

Mathematica [A] time = 0.0264351, size = 43, normalized size = 0.77

$$-\frac{1}{6}\sqrt{-4x^2+16x+9}\left(4x^2-25x+9\right)-\frac{75}{4}\sin^{-1}\left(\frac{4}{5}-\frac{2x}{5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 - 2*x)*Sqrt[9 + 16*x - 4*x^2],x]
```

```
[Out] -(Sqrt[9 + 16*x - 4*x^2]*(9 - 25*x + 4*x^2))/6 - (75*ArcSin[4/5 - (2*x)/5])
/4
```

Maple [A] time = 0.046, size = 43, normalized size = 0.8

$$-\frac{-24x+48}{16}\sqrt{-4x^2+16x+9}+\frac{75}{4}\arcsin\left(-\frac{4}{5}+\frac{2x}{5}\right)+\frac{1}{6}\left(-4x^2+16x+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7-2*x)*(-4*x^2+16*x+9)^(1/2),x)
```

```
[Out] -3/16*(-8*x+16)*(-4*x^2+16*x+9)^(1/2)+75/4*arcsin(-4/5+2/5*x)+1/6*(-4*x^2+1
6*x+9)^(3/2)
```

Maxima [A] time = 1.49151, size = 70, normalized size = 1.25

$$\frac{1}{6}\left(-4x^2+16x+9\right)^{\frac{3}{2}}+\frac{3}{2}\sqrt{-4x^2+16x+9}x-3\sqrt{-4x^2+16x+9}-\frac{75}{4}\arcsin\left(-\frac{2}{5}x+\frac{4}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7-2*x)*(-4*x^2+16*x+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(-4*x^2 + 16*x + 9)^(3/2) + 3/2*sqrt(-4*x^2 + 16*x + 9)*x - 3*sqrt(-4*x
^2 + 16*x + 9) - 75/4*arcsin(-2/5*x + 4/5)
```

Fricas [A] time = 2.39584, size = 135, normalized size = 2.41

$$-\frac{1}{6}\left(4x^2-25x+9\right)\sqrt{-4x^2+16x+9}-\frac{75}{2}\arctan\left(\frac{\sqrt{-4x^2+16x+9}-3}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7-2*x)*(-4*x^2+16*x+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(4*x^2 - 25*x + 9)*sqrt(-4*x^2 + 16*x + 9) - 75/2*arctan(1/2*(sqrt(-4*
x^2 + 16*x + 9) - 3)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int 2x\sqrt{-4x^2+16x+9}\,dx-\int -7\sqrt{-4x^2+16x+9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7-2*x)*(-4*x**2+16*x+9)**(1/2),x)
```

```
[Out] -Integral(2*x*sqrt(-4*x**2 + 16*x + 9), x) - Integral(-7*sqrt(-4*x**2 + 16*
x + 9), x)
```

Giac [A] time = 1.08761, size = 43, normalized size = 0.77

$$-\frac{1}{6}\left((4x-25)x+9\right)\sqrt{-4x^2+16x+9}+\frac{75}{4}\arcsin\left(\frac{2}{5}x-\frac{4}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7-2*x)*(-4*x^2+16*x+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*((4*x - 25)*x + 9)*sqrt(-4*x^2 + 16*x + 9) + 75/4*arcsin(2/5*x - 4/5)
```

3.2369 $\int \frac{\sqrt{-1-x+x^2}}{1+x} dx$

Optimal. Leaf size=61

$$\sqrt{x^2-x-1}+\frac{3}{2}\tanh^{-1}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right)+\tanh^{-1}\left(\frac{3x+1}{2\sqrt{x^2-x-1}}\right)$$

```
[Out] Sqrt[-1 - x + x^2] + (3*ArcTanh[(1 - 2*x)/(2*Sqrt[-1 - x + x^2])])/2 + ArcT
anh[(1 + 3*x)/(2*Sqrt[-1 - x + x^2])]
```

Rubi [A] time = 0.0425961, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {734, 843, 621, 206, 724}

$$\sqrt{x^2-x-1}+\frac{3}{2}\tanh^{-1}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right)+\tanh^{-1}\left(\frac{3x+1}{2\sqrt{x^2-x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 - x + x^2]/(1 + x),x]
```

```
[Out] Sqrt[-1 - x + x^2] + (3*ArcTanh[(1 - 2*x)/(2*Sqrt[-1 - x + x^2])])/2 + ArcT
anh[(1 + 3*x)/(2*Sqrt[-1 - x + x^2])]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1-x+x^2}}{1+x}\,dx &= \sqrt{-1-x+x^2}-\frac{1}{2}\int \frac{1+3x}{(1+x)\sqrt{-1-x+x^2}}\,dx\\ &= \sqrt{-1-x+x^2}-\frac{3}{2}\int \frac{1}{\sqrt{-1-x+x^2}}\,dx+\int \frac{1}{(1+x)\sqrt{-1-x+x^2}}\,dx\\ &= \sqrt{-1-x+x^2}-2\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{-1-3x}{\sqrt{-1-x+x^2}}\right)-3\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{-1+2x}{\sqrt{-1-x+}}\right.\\ &= \sqrt{-1-x+x^2}+\frac{3}{2}\tanh^{-1}\left(\frac{1-2x}{2\sqrt{-1-x+x^2}}\right)+\tanh^{-1}\left(\frac{1+3x}{2\sqrt{-1-x+x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0165095, size = 63, normalized size = 1.03

$$\sqrt{x^2-x-1}-\tanh^{-1}\left(\frac{-3x-1}{2\sqrt{x^2-x-1}}\right)-\frac{3}{2}\tanh^{-1}\left(\frac{2x-1}{2\sqrt{x^2-x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 - x + x^2]/(1 + x),x]
```

```
[Out] Sqrt[-1 - x + x^2] - ArcTanh[(-1 - 3*x)/(2*Sqrt[-1 - x + x^2])] - (3*ArcTan
h[(-1 + 2*x)/(2*Sqrt[-1 - x + x^2])])/2
```

Maple [A] time = 0.042, size = 54, normalized size = 0.9

$$\sqrt{(1+x)^2-2-3\,x}-\frac{3}{2}\ln\left(-\frac{1}{2}+x+\sqrt{(1+x)^2-2-3\,x}\right)-\text{Artanh}\left(\frac{-1-3\,x}{2}\frac{1}{\sqrt{(1+x)^2-2-3\,x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x-1)^(1/2)/(1+x),x)
```

```
[Out] ((1+x)^2-2-3*x)^(1/2)-3/2*ln(-1/2+x+((1+x)^2-2-3*x)^(1/2))-arctanh(1/2*(-1-
3*x)/((1+x)^2-2-3*x)^(1/2))
```

Maxima [A] time = 0.993341, size = 84, normalized size = 1.38

$$\sqrt{x^2-x-1}-\frac{3}{2}\log\left(2\,x+2\,\sqrt{x^2-x-1}-1\right)-\log\left(\frac{2\,\sqrt{x^2-x-1}}{|x+1|}+\frac{2}{|x+1|}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x-1)^(1/2)/(1+x),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 - x - 1) - 3/2*log(2*x + 2*sqrt(x^2 - x - 1) - 1) - log(2*sqrt(x^2
 - x - 1)/abs(x + 1) + 2/abs(x + 1) - 3)
```

Fricas [A] time = 2.37564, size = 169, normalized size = 2.77

$$\sqrt{x^2-x-1}-\log\left(-x+\sqrt{x^2-x-1}\right)+\log\left(-x+\sqrt{x^2-x-1}-2\right)+\frac{3}{2}\log\left(-2x+2\sqrt{x^2-x-1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x-1)^(1/2)/(1+x),x, algorithm="fricas")
```

```
[Out] sqrt(x^2 - x - 1) - log(-x + sqrt(x^2 - x - 1)) + log(-x + sqrt(x^2 - x - 1
) - 2) + 3/2*log(-2*x + 2*sqrt(x^2 - x - 1) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^2-x-1}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x-1)**(1/2)/(1+x),x)
```

```
[Out] Integral(sqrt(x**2 - x - 1)/(x + 1), x)
```

Giac [A] time = 1.08727, size = 90, normalized size = 1.48

$$\sqrt{x^2-x-1}-\log\left(\left|-x+\sqrt{x^2-x-1}\right|\right)+\log\left(\left|-x+\sqrt{x^2-x-1}-2\right|\right)+\frac{3}{2}\log\left(\left|-2x+2\sqrt{x^2-x-1}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x-1)^(1/2)/(1+x),x, algorithm="giac")
```

```
[Out] sqrt(x^2 - x - 1) - log(abs(-x + sqrt(x^2 - x - 1))) + log(abs(-x + sqrt(x^
2 - x - 1) - 2)) + 3/2*log(abs(-2*x + 2*sqrt(x^2 - x - 1) + 1))
```

3.2370 $\int \frac{\sqrt{-1-x+x^2}}{1-x}\,dx$

Optimal. Leaf size=65

$$-\sqrt{x^2-x-1}-\tan^{-1}\left(\frac{3-x}{2\sqrt{x^2-x-1}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right)$$

```
[Out] -Sqrt[-1 - x + x^2] - ArcTan[(3 - x)/(2*Sqrt[-1 - x + x^2])] + ArcTanh[(1 -
 2*x)/(2*Sqrt[-1 - x + x^2])]/2
```

Rubi [A] time = 0.0428585, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {734, 843, 621, 206, 724, 204}

$$-\sqrt{x^2-x-1}-\tan^{-1}\left(\frac{3-x}{2\sqrt{x^2-x-1}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 - x + x^2]/(1 - x),x]
```

```
[Out] -Sqrt[-1 - x + x^2] - ArcTan[(3 - x)/(2*Sqrt[-1 - x + x^2])] + ArcTanh[(1 -
 2*x)/(2*Sqrt[-1 - x + x^2])]/2
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1-x+x^2}}{1-x}\,dx &= -\sqrt{-1-x+x^2}+\frac{1}{2}\int \frac{-3+x}{(1-x)\sqrt{-1-x+x^2}}\,dx\\ &= -\sqrt{-1-x+x^2}-\frac{1}{2}\int \frac{1}{\sqrt{-1-x+x^2}}\,dx-\int \frac{1}{(1-x)\sqrt{-1-x+x^2}}\,dx\\ &= -\sqrt{-1-x+x^2}+2\,\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx,x,\frac{3-x}{\sqrt{-1-x+x^2}}\right)-\text{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{-1+2x}{\sqrt{-1-x+x^2}}\right)\\ &= -\sqrt{-1-x+x^2}-\tan^{-1}\left(\frac{3-x}{2\sqrt{-1-x+x^2}}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{-1+2x}{2\sqrt{-1-x+x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0137811, size = 65, normalized size = 1.

$$-\sqrt{x^2-x-1}-\tan^{-1}\left(\frac{3-x}{2\sqrt{x^2-x-1}}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{2x-1}{2\sqrt{x^2-x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 - x + x^2]/(1 - x),x]
```

```
[Out] -Sqrt[-1 - x + x^2] - ArcTan[(3 - x)/(2*Sqrt[-1 - x + x^2])] - ArcTanh[(-1
+ 2*x)/(2*Sqrt[-1 - x + x^2])]/2
```

Maple [A] time = 0.045, size = 46, normalized size = 0.7

$$-\sqrt{(-1+x)^2-2+x}-\frac{1}{2}\ln\left(-\frac{1}{2}+x+\sqrt{(-1+x)^2-2+x}\right)+\arctan\left(\frac{-3+x}{2}\frac{1}{\sqrt{(-1+x)^2-2+x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x-1)^(1/2)/(1-x),x)
```

```
[Out] -((-1+x)^2-2+x)^(1/2)-1/2*ln(-1/2+x+((-1+x)^2-2+x)^(1/2))+arctan(1/2*(-3+x)
/((-1+x)^2-2+x)^(1/2))
```

Maxima [A] time = 1.51435, size = 78, normalized size = 1.2

$$-\sqrt{x^2-x-1}+\arcsin\left(\frac{\sqrt{5}x}{5\,|x-1|}-\frac{3\,\sqrt{5}}{5\,|x-1|}\right)-\frac{1}{2}\,\log\left(2\,x+2\,\sqrt{x^2-x-1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x-1)^(1/2)/(1-x),x, algorithm="maxima")
```

```
[Out] -sqrt(x^2 - x - 1) + arcsin(1/5*sqrt(5)*x/abs(x - 1) - 3/5*sqrt(5)/abs(x -
1)) - 1/2*log(2*x + 2*sqrt(x^2 - x - 1) - 1)
```

Fricas [A] time = 2.48509, size = 136, normalized size = 2.09

$$-\sqrt{x^2-x-1}+2\arctan\left(-x+\sqrt{x^2-x-1}+1\right)+\frac{1}{2}\log\left(-2x+2\sqrt{x^2-x-1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x-1)^(1/2)/(1-x),x, algorithm="fricas")
```

```
[Out] -sqrt(x^2 - x - 1) + 2*arctan(-x + sqrt(x^2 - x - 1) + 1) + 1/2*log(-2*x +
2*sqrt(x^2 - x - 1) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{x^2-x-1}}{x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x-1)**(1/2)/(1-x),x)
```

```
[Out] -Integral(sqrt(x**2 - x - 1)/(x - 1), x)
```

Giac [A] time = 1.10441, size = 70, normalized size = 1.08

$$-\sqrt{x^2-x-1}+2\arctan\left(-x+\sqrt{x^2-x-1}+1\right)+\frac{1}{2}\log\left(\left|-2x+2\sqrt{x^2-x-1}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x-1)^(1/2)/(1-x),x, algorithm="giac")
```

```
[Out] -sqrt(x^2 - x - 1) + 2*arctan(-x + sqrt(x^2 - x - 1) + 1) + 1/2*log(abs(-2*
x + 2*sqrt(x^2 - x - 1) + 1))
```

3.2371 $\int \frac{x^6}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=261

$$-\frac{\left(7b\left(528a^2c^2-680ab^2c+165b^4\right)-2cx\left(400a^2c^2-1176ab^2c+385b^4\right)\right)\sqrt{a+bx+cx^2}}{2560c^6}+\frac{\left(1680a^2b^2c^2-320a^3c^3-1260\right.}{1}$$

```
[Out] -(b*(77*b^2 - 156*a*c)*x^2*Sqrt[a + b*x + c*x^2])/(320*c^4) + ((99*b^2 - 10
0*a*c)*x^3*Sqrt[a + b*x + c*x^2])/(480*c^3) - (11*b*x^4*Sqrt[a + b*x + c*x^
2])/(60*c^2) + (x^5*Sqrt[a + b*x + c*x^2])/(6*c) - ((7*b*(165*b^4 - 680*a*b
^2*c + 528*a^2*c^2) - 2*c*(385*b^4 - 1176*a*b^2*c + 400*a^2*c^2)*x)*Sqrt[a
+ b*x + c*x^2])/(2560*c^6) + ((231*b^6 - 1260*a*b^4*c + 1680*a^2*b^2*c^2 -
320*a^3*c^3)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*
c^(13/2))
```

Rubi [A] time = 0.38183, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {742, 832, 779, 621, 206}

$$-\frac{\left(7b\left(528a^2c^2-680ab^2c+165b^4\right)-2cx\left(400a^2c^2-1176ab^2c+385b^4\right)\right)\sqrt{a+bx+cx^2}}{2560c^6}+\frac{\left(1680a^2b^2c^2-320a^3c^3-1260\right.}{1}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] -(b*(77*b^2 - 156*a*c)*x^2*Sqrt[a + b*x + c*x^2])/(320*c^4) + ((99*b^2 - 10
0*a*c)*x^3*Sqrt[a + b*x + c*x^2])/(480*c^3) - (11*b*x^4*Sqrt[a + b*x + c*x^
2])/(60*c^2) + (x^5*Sqrt[a + b*x + c*x^2])/(6*c) - ((7*b*(165*b^4 - 680*a*b
^2*c + 528*a^2*c^2) - 2*c*(385*b^4 - 1176*a*b^2*c + 400*a^2*c^2)*x)*Sqrt[a
+ b*x + c*x^2])/(2560*c^6) + ((231*b^6 - 1260*a*b^4*c + 1680*a^2*b^2*c^2 -
320*a^3*c^3)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*
c^(13/2))
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
```

```
|| IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^6}{\sqrt{a+bx+cx^2}}\,dx = \frac{x^5\sqrt{a+bx+cx^2}}{6c} + \frac{\int \frac{x^4\left(-5a-\frac{11bx}{2}\right)}{\sqrt{a+bx+cx^2}}\,dx}{6c}$$

$$= -\frac{11bx^4\sqrt{a+bx+cx^2}}{60c^2} + \frac{x^5\sqrt{a+bx+cx^2}}{6c} + \frac{\int \frac{x^3\left(22ab+\frac{1}{4}\left(99b^2-100ac\right)x\right)}{\sqrt{a+bx+cx^2}}\,dx}{30c^2}$$

$$= \frac{\left(99b^2-100ac\right)x^3\sqrt{a+bx+cx^2}}{480c^3} - \frac{11bx^4\sqrt{a+bx+cx^2}}{60c^2} + \frac{x^5\sqrt{a+bx+cx^2}}{6c} + \frac{\int \frac{x^2\left(-\frac{3}{4}a\left(99b^2-10\ldots\right.\right.}{\sqrt{\ldots}}}{\ldots}$$

$$= -\frac{b\left(77b^2-156ac\right)x^2\sqrt{a+bx+cx^2}}{320c^4} + \frac{\left(99b^2-100ac\right)x^3\sqrt{a+bx+cx^2}}{480c^3} - \frac{11bx^4\sqrt{a+bx+cx^2}}{60c^2} \ldots$$

$$= -\frac{b\left(77b^2-156ac\right)x^2\sqrt{a+bx+cx^2}}{320c^4} + \frac{\left(99b^2-100ac\right)x^3\sqrt{a+bx+cx^2}}{480c^3} - \frac{11bx^4\sqrt{a+bx+cx^2}}{60c^2} \ldots$$

$$= -\frac{b\left(77b^2-156ac\right)x^2\sqrt{a+bx+cx^2}}{320c^4} + \frac{\left(99b^2-100ac\right)x^3\sqrt{a+bx+cx^2}}{480c^3} - \frac{11bx^4\sqrt{a+bx+cx^2}}{60c^2} \ldots$$

$$= -\frac{b\left(77b^2-156ac\right)x^2\sqrt{a+bx+cx^2}}{320c^4} + \frac{\left(99b^2-100ac\right)x^3\sqrt{a+bx+cx^2}}{480c^3} - \frac{11bx^4\sqrt{a+bx+cx^2}}{60c^2} \ldots$$

Mathematica [A] time = 0.322678, size = 263, normalized size = 1.01

$$\frac{8a^2c\left(-2268b^2cx+1785b^3-618bc^2x^2+100c^3x^3\right)+48a^3c^2(50cx-231b)+a\left(5376b^3c^2x^2-1728b^2c^3x^3+16590b^4cx\ldots\right.}{7680c^6\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (48*a^3*c^2*(-231*b + 50*c*x) + 8*a^2*c*(1785*b^3 - 2268*b^2*c*x - 618*b*c^
2*x^2 + 100*c^3*x^3) + a*(-3465*b^5 + 16590*b^4*c*x + 5376*b^3*c^2*x^2 - 17
28*b^2*c^3*x^3 + 736*b*c^4*x^4 - 320*c^5*x^5) + x*(-3465*b^6 - 1155*b^5*c*x
 + 462*b^4*c^2*x^2 - 264*b^3*c^3*x^3 + 176*b^2*c^4*x^4 - 128*b*c^5*x^5 + 12
80*c^6*x^6))/(7680*c^6*Sqrt[a + x*(b + c*x)]) + ((231*b^6 - 1260*a*b^4*c +
1680*a^2*b^2*c^2 - 320*a^3*c^3)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(
b + c*x)])])/(1024*c^(13/2))
```

Maple [A] time = 0.049, size = 394, normalized size = 1.5

$$\frac{x^5}{6c}\sqrt{cx^2+bx+a}-\frac{11\,bx^4}{60\,c^2}\sqrt{cx^2+bx+a}+\frac{33\,b^2x^3}{160\,c^3}\sqrt{cx^2+bx+a}-\frac{77\,b^3x^2}{320\,c^4}\sqrt{cx^2+bx+a}+\frac{77\,b^4x}{256\,c^5}\sqrt{cx^2+bx+a}-\frac{231}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/6*x^5*(c*x^2+b*x+a)^(1/2)/c-11/60*b*x^4*(c*x^2+b*x+a)^(1/2)/c^2+33/160*b^
2/c^3*x^3*(c*x^2+b*x+a)^(1/2)-77/320*b^3/c^4*x^2*(c*x^2+b*x+a)^(1/2)+77/256
*b^4/c^5*x*(c*x^2+b*x+a)^(1/2)-231/512*b^5/c^6*(c*x^2+b*x+a)^(1/2)+231/1024
*b^6/c^(13/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-315/256*b^4/c^(11
/2)*a*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+119/64*b^3/c^5*a*(c*x^2+b
*x+a)^(1/2)-147/160*b^2/c^4*a*x*(c*x^2+b*x+a)^(1/2)+105/64*b^2/c^(9/2)*a^2*
ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+39/80*b/c^3*a*x^2*(c*x^2+b*x+a)
^(1/2)-231/160*b/c^4*a^2*(c*x^2+b*x+a)^(1/2)-5/24*a/c^2*x^3*(c*x^2+b*x+a)^(
1/2)+5/16*a^2/c^3*x*(c*x^2+b*x+a)^(1/2)-5/16*a^3/c^(7/2)*ln((1/2*b+c*x)/c^(
1/2)+(c*x^2+b*x+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.9075, size = 1075, normalized size = 4.12

$$\left[-\frac{15\left(231\,b^6-1260\,ab^4c+1680\,a^2b^2c^2-320\,a^3c^3\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right)-}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(231*b^6 - 1260*a*b^4*c + 1680*a^2*b^2*c^2 - 320*a^3*c^3)*sqr
t(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*s
qrt(c) - 4*a*c) - 4*(1280*c^6*x^5 - 1408*b*c^5*x^4 - 3465*b^5*c + 14280*a*b
```

```
^3*c^2 - 11088*a^2*b*c^3 + 16*(99*b^2*c^4 - 100*a*c^5)*x^3 - 24*(77*b^3*c^3
 - 156*a*b*c^4)*x^2 + 6*(385*b^4*c^2 - 1176*a*b^2*c^3 + 400*a^2*c^4)*x)*sqr
t(c*x^2 + b*x + a))/c^7, -1/15360*(15*(231*b^6 - 1260*a*b^4*c + 1680*a^2*b^
2*c^2 - 320*a^3*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*
sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(1280*c^6*x^5 - 1408*b*c^5*x^4 - 3465
*b^5*c + 14280*a*b^3*c^2 - 11088*a^2*b*c^3 + 16*(99*b^2*c^4 - 100*a*c^5)*x^
3 - 24*(77*b^3*c^3 - 156*a*b*c^4)*x^2 + 6*(385*b^4*c^2 - 1176*a*b^2*c^3 + 4
00*a^2*c^4)*x)*sqrt(c*x^2 + b*x + a))/c^7]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(x**6/sqrt(a + b*x + c*x**2), x)
```

Giac [A] time = 1.16743, size = 281, normalized size = 1.08

$$\frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8x\left(\frac{10x}{c}-\frac{11b}{c^2}\right)+\frac{99b^2c^3-100ac^4}{c^6}\right)x-\frac{3\left(77b^3c^2-156abc^3\right)}{c^6}\right)x+\frac{3\left(385b^4c-1176a\right.}{c^6}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*x*(10*x/c - 11*b/c^2) + (99*b^2*c^
3 - 100*a*c^4)/c^6)*x - 3*(77*b^3*c^2 - 156*a*b*c^3)/c^6)*x + 3*(385*b^4*c
- 1176*a*b^2*c^2 + 400*a^2*c^3)/c^6)*x - 21*(165*b^5 - 680*a*b^3*c + 528*a^
2*b*c^2)/c^6) - 1/1024*(231*b^6 - 1260*a*b^4*c + 1680*a^2*b^2*c^2 - 320*a^3
*c^3)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(13/2)
```

3.2372 $\int \frac{x^5}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=202

$$\frac{\left(1024a^2c^2-14bcx\left(45b^2-92ac\right)-2940ab^2c+945b^4\right)\sqrt{a+bx+cx^2}}{1920c^5}-\frac{b\left(240a^2c^2-280ab^2c+63b^4\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+}}\right.}{256c^{11/2}}$$

```
[Out] ((63*b^2 - 64*a*c)*x^2*Sqrt[a + b*x + c*x^2])/(240*c^3) - (9*b*x^3*Sqrt[a +
 b*x + c*x^2])/(40*c^2) + (x^4*Sqrt[a + b*x + c*x^2])/(5*c) + ((945*b^4 - 2
940*a*b^2*c + 1024*a^2*c^2 - 14*b*c*(45*b^2 - 92*a*c)*x)*Sqrt[a + b*x + c*x
^2])/(1920*c^5) - (b*(63*b^4 - 280*a*b^2*c + 240*a^2*c^2)*ArcTanh[(b + 2*c*
x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(11/2))
```

Rubi [A] time = 0.2165, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {742, 832, 779, 621, 206}

$$\frac{\left(1024a^2c^2-14bcx\left(45b^2-92ac\right)-2940ab^2c+945b^4\right)\sqrt{a+bx+cx^2}}{1920c^5}-\frac{b\left(240a^2c^2-280ab^2c+63b^4\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+}}\right.}{256c^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((63*b^2 - 64*a*c)*x^2*Sqrt[a + b*x + c*x^2])/(240*c^3) - (9*b*x^3*Sqrt[a +
 b*x + c*x^2])/(40*c^2) + (x^4*Sqrt[a + b*x + c*x^2])/(5*c) + ((945*b^4 - 2
940*a*b^2*c + 1024*a^2*c^2 - 14*b*c*(45*b^2 - 92*a*c)*x)*Sqrt[a + b*x + c*x
^2])/(1920*c^5) - (b*(63*b^4 - 280*a*b^2*c + 240*a^2*c^2)*ArcTanh[(b + 2*c*
x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(11/2))
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5}{\sqrt{a+bx+cx^2}}\,dx = \frac{x^4\sqrt{a+bx+cx^2}}{5c} + \frac{\int \frac{x^3\left(-4a-\frac{9bx}{2}\right)}{\sqrt{a+bx+cx^2}}\,dx}{5c}$$

$$= -\frac{9bx^3\sqrt{a+bx+cx^2}}{40c^2} + \frac{x^4\sqrt{a+bx+cx^2}}{5c} + \frac{\int \frac{x^2\left(\frac{27ab}{2}+\frac{1}{4}\left(63b^2-64ac\right)x\right)}{\sqrt{a+bx+cx^2}}\,dx}{20c^2}$$

$$= \frac{\left(63b^2-64ac\right)x^2\sqrt{a+bx+cx^2}}{240c^3} - \frac{9bx^3\sqrt{a+bx+cx^2}}{40c^2} + \frac{x^4\sqrt{a+bx+cx^2}}{5c} + \frac{\int \frac{x\left(-\frac{1}{2}a\left(63b^2-64ac\right)-\right.}{\sqrt{a+bx}}}{60}$$

$$= \frac{\left(63b^2-64ac\right)x^2\sqrt{a+bx+cx^2}}{240c^3} - \frac{9bx^3\sqrt{a+bx+cx^2}}{40c^2} + \frac{x^4\sqrt{a+bx+cx^2}}{5c} + \frac{\left(945b^4-2940ab^2\right.}{}$$

$$= \frac{\left(63b^2-64ac\right)x^2\sqrt{a+bx+cx^2}}{240c^3} - \frac{9bx^3\sqrt{a+bx+cx^2}}{40c^2} + \frac{x^4\sqrt{a+bx+cx^2}}{5c} + \frac{\left(945b^4-2940ab^2\right.}{}$$

$$= \frac{\left(63b^2-64ac\right)x^2\sqrt{a+bx+cx^2}}{240c^3} - \frac{9bx^3\sqrt{a+bx+cx^2}}{40c^2} + \frac{x^4\sqrt{a+bx+cx^2}}{5c} + \frac{\left(945b^4-2940ab^2\right.}{}$$

Mathematica [A] time = 0.207688, size = 213, normalized size = 1.05

$$\frac{4a^2c\left(-735b^2+578bcx+128c^2x^2\right)+1024a^3c^2+a\left(-1148b^2c^2x^2-3570b^3cx+945b^4+344bc^3x^3-128c^4x^4\right)+3x\left(-\right.}{1920c^5\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (1024*a^3*c^2 + 4*a^2*c*(-735*b^2 + 578*b*c*x + 128*c^2*x^2) + a*(945*b^4 -
 3570*b^3*c*x - 1148*b^2*c^2*x^2 + 344*b*c^3*x^3 - 128*c^4*x^4) + 3*x*(315*
b^5 + 105*b^4*c*x - 42*b^3*c^2*x^2 + 24*b^2*c^3*x^3 - 16*b*c^4*x^4 + 128*c^
5*x^5))/(1920*c^5*Sqrt[a + x*(b + c*x)]) - (b*(63*b^4 - 280*a*b^2*c + 240*a
^2*c^2)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(256*c^(11/
2))
```

Maple [A] time = 0.048, size = 290, normalized size = 1.4

$$\frac{x^4}{5c}\sqrt{cx^2+bx+a}-\frac{9\,bx^3}{40\,c^2}\sqrt{cx^2+bx+a}+\frac{21\,b^2x^2}{80\,c^3}\sqrt{cx^2+bx+a}-\frac{21\,b^3x}{64\,c^4}\sqrt{cx^2+bx+a}+\frac{63\,b^4}{128\,c^5}\sqrt{cx^2+bx+a}-\frac{63\,b^5}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/5*x^4*(c*x^2+b*x+a)^(1/2)/c-9/40*b*x^3*(c*x^2+b*x+a)^(1/2)/c^2+21/80*b^2/
c^3*x^2*(c*x^2+b*x+a)^(1/2)-21/64*b^3/c^4*x*(c*x^2+b*x+a)^(1/2)+63/128*b^4/
c^5*(c*x^2+b*x+a)^(1/2)-63/256*b^5/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b
*x+a)^(1/2))+35/32*b^3/c^(9/2)*a*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)
)-49/32*b^2/c^4*a*(c*x^2+b*x+a)^(1/2)+161/240*b/c^3*a*x*(c*x^2+b*x+a)^(1/2)
-15/16*b/c^(7/2)*a^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-4/15*a/c^2
*x^2*(c*x^2+b*x+a)^(1/2)+8/15*a^2/c^3*(c*x^2+b*x+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.74862, size = 848, normalized size = 4.2

$$\left[\frac{15\left(63\,b^5-280\,ab^3c+240\,a^2bc^2\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right)+4\left(384\,c^5x^4-432\right.}{7680\,c^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(63*b^5 - 280*a*b^3*c + 240*a^2*b*c^2)*sqrt(c)*log(-8*c^2*x^2 -
 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(
384*c^5*x^4 - 432*b*c^4*x^3 + 945*b^4*c - 2940*a*b^2*c^2 + 1024*a^2*c^3 + 8
*(63*b^2*c^3 - 64*a*c^4)*x^2 - 14*(45*b^3*c^2 - 92*a*b*c^3)*x)*sqrt(c*x^2 +
 b*x + a))/c^6, 1/3840*(15*(63*b^5 - 280*a*b^3*c + 240*a^2*b*c^2)*sqrt(-c)*
arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*
c)) + 2*(384*c^5*x^4 - 432*b*c^4*x^3 + 945*b^4*c - 2940*a*b^2*c^2 + 1024*a^
2*c^3 + 8*(63*b^2*c^3 - 64*a*c^4)*x^2 - 14*(45*b^3*c^2 - 92*a*b*c^3)*x)*sqr
t(c*x^2 + b*x + a))/c^6]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(a + b*x + c*x**2), x)
```

Giac [A] time = 1.10125, size = 217, normalized size = 1.07

$$\frac{1}{1920}\sqrt{cx^2+bx+a}\left(2\left(4\left(6x\left(\frac{8x}{c}-\frac{9b}{c^2}\right)+\frac{63b^2c^2-64ac^3}{c^5}\right)x-\frac{7\left(45b^3c-92abc^2\right)}{c^5}\right)x+\frac{945b^4-2940ab^2c+1024a^2c^2}{c^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*x*(8*x/c - 9*b/c^2) + (63*b^2*c^2 - 6
4*a*c^3)/c^5)*x - 7*(45*b^3*c - 92*a*b*c^2)/c^5)*x + (945*b^4 - 2940*a*b^2*
c + 1024*a^2*c^2)/c^5) + 1/256*(63*b^5 - 280*a*b^3*c + 240*a^2*b*c^2)*log(a
bs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(11/2)
```

3.2373 $\int \frac{(d+ex)^4}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=296

$$\frac{\left(48c^2e^2\left(a^2e^2+8abde+6b^2d^2\right)-40b^2ce^3(3ae+4bd)-128c^3d^2e(3ae+2bd)+35b^4e^4+128c^4d^4\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{9/2}}$$

```
[Out] (7*e*(2*c*d - b*e)*(d + e*x)^2*Sqrt[a + b*x + c*x^2])/(24*c^2) + (e*(d + e*
x)^3*Sqrt[a + b*x + c*x^2])/(4*c) + (e*(608*c^3*d^3 - 105*b^3*e^3 + 20*b*c*
e^2*(24*b*d + 11*a*e) - 8*c^2*d*e*(101*b*d + 64*a*e) + 2*c*e*(104*c^2*d^2 +
 35*b^2*e^2 - 4*c*e*(26*b*d + 9*a*e))*x)*Sqrt[a + b*x + c*x^2])/(192*c^4) +
 ((128*c^4*d^4 + 35*b^4*e^4 - 128*c^3*d^2*e*(2*b*d + 3*a*e) - 40*b^2*c*e^3*
(4*b*d + 3*a*e) + 48*c^2*e^2*(6*b^2*d^2 + 8*a*b*d*e + a^2*e^2))*ArcTanh[(b
+ 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(9/2))
```

Rubi [A] time = 0.377292, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {742, 832, 779, 621, 206}

$$\frac{\left(48c^2e^2\left(a^2e^2+8abde+6b^2d^2\right)-40b^2ce^3(3ae+4bd)-128c^3d^2e(3ae+2bd)+35b^4e^4+128c^4d^4\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/Sqrt[a + b*x + c*x^2],x]
```


```
[Out] (7*e*(2*c*d - b*e)*(d + e*x)^2*Sqrt[a + b*x + c*x^2])/(24*c^2) + (e*(d + e*
x)^3*Sqrt[a + b*x + c*x^2])/(4*c) + (e*(608*c^3*d^3 - 105*b^3*e^3 + 20*b*c*
e^2*(24*b*d + 11*a*e) - 8*c^2*d*e*(101*b*d + 64*a*e) + 2*c*e*(104*c^2*d^2 +
 35*b^2*e^2 - 4*c*e*(26*b*d + 9*a*e))*x)*Sqrt[a + b*x + c*x^2])/(192*c^4) +
 ((128*c^4*d^4 + 35*b^4*e^4 - 128*c^3*d^2*e*(2*b*d + 3*a*e) - 40*b^2*c*e^3*
(4*b*d + 3*a*e) + 48*c^2*e^2*(6*b^2*d^2 + 8*a*b*d*e + a^2*e^2))*ArcTanh[(b
+ 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(9/2))
```


Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
```

```
|| IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^4}{\sqrt{a+bx+cx^2}}\,dx = \frac{e(d+ex)^3\sqrt{a+bx+cx^2}}{4c} + \frac{\int \frac{(d+ex)^2\left(\frac{1}{2}\left(8cd^2-e(bd+6ae)\right)+\frac{7}{2}e(2cd-be)x\right)}{\sqrt{a+bx+cx^2}}\,dx}{4c}$$

$$= \frac{7e(2cd-be)(d+ex)^2\sqrt{a+bx+cx^2}}{24c^2} + \frac{e(d+ex)^3\sqrt{a+bx+cx^2}}{4c} + \frac{\int \frac{(d+ex)\left(\frac{1}{4}\left(48c^2d^3+7be^2(bd+4ae)-\right.\right.}{}}{}$$

$$= \frac{7e(2cd-be)(d+ex)^2\sqrt{a+bx+cx^2}}{24c^2} + \frac{e(d+ex)^3\sqrt{a+bx+cx^2}}{4c} + \frac{e\left(608c^3d^3-105b^3e^3+20bc\right.}{}$$

$$= \frac{7e(2cd-be)(d+ex)^2\sqrt{a+bx+cx^2}}{24c^2} + \frac{e(d+ex)^3\sqrt{a+bx+cx^2}}{4c} + \frac{e\left(608c^3d^3-105b^3e^3+20bc\right.}{}$$

$$= \frac{7e(2cd-be)(d+ex)^2\sqrt{a+bx+cx^2}}{24c^2} + \frac{e(d+ex)^3\sqrt{a+bx+cx^2}}{4c} + \frac{e\left(608c^3d^3-105b^3e^3+20bc\right.}{}$$

Mathematica [A] time = 0.476546, size = 352, normalized size = 1.19

$$\frac{e\left(-4a^2ce^2(2c(64d+9ex)-55be)+a\left(10b^2ce^2(48d+29ex)-105b^3e^3+4bc^2e\left(-216d^2-208dex+23e^2x^2\right)+8c^3\left(72d\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (e*(-4*a^2*c*e^2*(-55*b*e + 2*c*(64*d + 9*e*x)) + a*(-105*b^3*e^3 + 10*b^2*
c*e^2*(48*d + 29*e*x) + 4*b*c^2*e*(-216*d^2 - 208*d*e*x + 23*e^2*x^2) + 8*c
^3*(96*d^3 + 72*d^2*e*x - 32*d*e^2*x^2 - 3*e^3*x^3)) + x*(b + c*x)*(-105*b^
3*e^3 + 10*b^2*c*e^2*(48*d + 7*e*x) - 8*b*c^2*e*(108*d^2 + 40*d*e*x + 7*e^2
*x^2) + 16*c^3*(48*d^3 + 36*d^2*e*x + 16*d*e^2*x^2 + 3*e^3*x^3))))/(192*c^4
```

```
*Sqrt[a + x*(b + c*x)]) + ((128*c^4*d^4 + 35*b^4*e^4 - 128*c^3*d^2*e*(2*b*d
 + 3*a*e) - 40*b^2*c*e^3*(4*b*d + 3*a*e) + 48*c^2*e^2*(6*b^2*d^2 + 8*a*b*d*
e + a^2*e^2))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(128*
c^(9/2))
```

Maple [B] time = 0.055, size = 627, normalized size = 2.1

$$\frac{e^4x^3}{4c}\sqrt{cx^2+bx+a}-\frac{7\,be^4x^2}{24\,c^2}\sqrt{cx^2+bx+a}+\frac{35\,b^2e^4x}{96\,c^3}\sqrt{cx^2+bx+a}-\frac{35\,e^4b^3}{64\,c^4}\sqrt{cx^2+bx+a}+\frac{35\,b^4e^4}{128}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/4*e^4*x^3/c*(c*x^2+b*x+a)^(1/2)-7/24*e^4*b/c^2*x^2*(c*x^2+b*x+a)^(1/2)+35
/96*e^4*b^2/c^3*x*(c*x^2+b*x+a)^(1/2)-35/64*e^4*b^3/c^4*(c*x^2+b*x+a)^(1/2)
+35/128*e^4*b^4/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-15/16*e
^4*b^2/c^(7/2)*a*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+55/48*e^4*b/c^
3*a*(c*x^2+b*x+a)^(1/2)-3/8*e^4*a/c^2*x*(c*x^2+b*x+a)^(1/2)+3/8*e^4*a^2/c^(
5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+4/3*d*e^3*x^2/c*(c*x^2+b*x
+a)^(1/2)-5/3*d*e^3*b/c^2*x*(c*x^2+b*x+a)^(1/2)+5/2*d*e^3*b^2/c^3*(c*x^2+b*
x+a)^(1/2)-5/4*d*e^3*b^3/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)
)+3*d*e^3*b/c^(5/2)*a*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-8/3*d*e^3
*a/c^2*(c*x^2+b*x+a)^(1/2)+3*d^2*e^2*x/c*(c*x^2+b*x+a)^(1/2)-9/2*d^2*e^2*b/
c^2*(c*x^2+b*x+a)^(1/2)+9/4*d^2*e^2*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))-3*d^2*e^2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(
1/2))+4*d^3*e/c*(c*x^2+b*x+a)^(1/2)-2*d^3*e*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2
)+(c*x^2+b*x+a)^(1/2))+d^4*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1
/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.24645, size = 1368, normalized size = 4.62

$$\left[\frac{3\left(128\,c^4d^4-256\,bc^3d^3e+96\left(3\,b^2c^2-4\,ac^3\right)d^2e^2-32\left(5\,b^3c-12\,abc^2\right)de^3+\left(35\,b^4-120\,ab^2c+48\,a^2c^2\right)e^4\right)\sqrt{c}\log\left(-8\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(128*c^4*d^4 - 256*b*c^3*d^3*e + 96*(3*b^2*c^2 - 4*a*c^3)*d^2*e^2
 - 32*(5*b^3*c - 12*a*b*c^2)*d*e^3 + (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*e^
4)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x
+ b)*sqrt(c) - 4*a*c) + 4*(48*c^4*e^4*x^3 + 768*c^4*d^3*e - 864*b*c^3*d^2*e
^2 + 32*(15*b^2*c^2 - 16*a*c^3)*d*e^3 - 5*(21*b^3*c - 44*a*b*c^2)*e^4 + 8*(
32*c^4*d*e^3 - 7*b*c^3*e^4)*x^2 + 2*(288*c^4*d^2*e^2 - 160*b*c^3*d*e^3 + (3
5*b^2*c^2 - 36*a*c^3)*e^4)*x)*sqrt(c*x^2 + b*x + a))/c^5, -1/384*(3*(128*c^
4*d^4 - 256*b*c^3*d^3*e + 96*(3*b^2*c^2 - 4*a*c^3)*d^2*e^2 - 32*(5*b^3*c -
12*a*b*c^2)*d*e^3 + (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*e^4)*sqrt(-c)*arcta
n(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) -
 2*(48*c^4*e^4*x^3 + 768*c^4*d^3*e - 864*b*c^3*d^2*e^2 + 32*(15*b^2*c^2 - 1
6*a*c^3)*d*e^3 - 5*(21*b^3*c - 44*a*b*c^2)*e^4 + 8*(32*c^4*d*e^3 - 7*b*c^3*
e^4)*x^2 + 2*(288*c^4*d^2*e^2 - 160*b*c^3*d*e^3 + (35*b^2*c^2 - 36*a*c^3)*e
^4)*x)*sqrt(c*x^2 + b*x + a))/c^5]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^4}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**4/sqrt(a + b*x + c*x**2), x)
```

Giac [A] time = 1.11382, size = 373, normalized size = 1.26

$$\frac{1}{192}\sqrt{cx^2+bx+a}\,\Bigg(2\bigg(4x\Big(\frac{6xe^4}{c}+\frac{32c^3de^3-7bc^2e^4}{c^4}\Big)+\frac{288c^3d^2e^2-160bc^2de^3+35b^2ce^4-36ac^2e^4}{c^4}\bigg)x+\frac{768c^3d^3e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*x*(6*x*e^4/c + (32*c^3*d*e^3 - 7*b*c^2*e^
4)/c^4) + (288*c^3*d^2*e^2 - 160*b*c^2*d*e^3 + 35*b^2*c*e^4 - 36*a*c^2*e^4)
/c^4)*x + (768*c^3*d^3*e - 864*b*c^2*d^2*e^2 + 480*b^2*c*d*e^3 - 512*a*c^2*
d*e^3 - 105*b^3*e^4 + 220*a*b*c*e^4)/c^4) - 1/128*(128*c^4*d^4 - 256*b*c^3*
d^3*e + 288*b^2*c^2*d^2*e^2 - 384*a*c^3*d^2*e^2 - 160*b^3*c*d*e^3 + 384*a*b
*c^2*d*e^3 + 35*b^4*e^4 - 120*a*b^2*c*e^4 + 48*a^2*c^2*e^4)*log(abs(-2*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2)
```

3.2374 $\int \frac{(d+ex)^3}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=174

$$\frac{e\sqrt{a+bx+cx^2}\left(-2ce(8ae+27bd)+15b^2e^2+10cex(2cd-be)+64c^2d^2\right)}{24c^3}+\frac{(2cd-be)\left(-4ce(3ae+2bd)+5b^2e^2+8c^2d^2\right)}{16c^{7/2}}$$

```
[Out] (e*(d + e*x)^2*Sqrt[a + b*x + c*x^2])/(3*c) + (e*(64*c^2*d^2 + 15*b^2*e^2 -
 2*c*e*(27*b*d + 8*a*e) + 10*c*e*(2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(2
4*c^3) + ((2*c*d - b*e)*(8*c^2*d^2 + 5*b^2*e^2 - 4*c*e*(2*b*d + 3*a*e))*Arc
Tanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(7/2))
```

Rubi [A] time = 0.151003, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {742, 779, 621, 206}

$$\frac{e\sqrt{a+bx+cx^2}\left(-2ce(8ae+27bd)+15b^2e^2+10cex(2cd-be)+64c^2d^2\right)}{24c^3}+\frac{(2cd-be)\left(-4ce(3ae+2bd)+5b^2e^2+8c^2d^2\right)}{16c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/Sqrt[a + b*x + c*x^2],x]
```


```
[Out] (e*(d + e*x)^2*Sqrt[a + b*x + c*x^2])/(3*c) + (e*(64*c^2*d^2 + 15*b^2*e^2 -
 2*c*e*(27*b*d + 8*a*e) + 10*c*e*(2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(2
4*c^3) + ((2*c*d - b*e)*(8*c^2*d^2 + 5*b^2*e^2 - 4*c*e*(2*b*d + 3*a*e))*Arc
Tanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(7/2))
```


Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^3}{\sqrt{a+bx+cx^2}}\,dx = \frac{e(d+ex)^2\sqrt{a+bx+cx^2}}{3c} + \frac{\int \frac{(d+ex)\left(\frac{1}{2}\left(6cd^2-e(bd+4ae)\right)+\frac{5}{2}e(2cd-be)x\right)}{\sqrt{a+bx+cx^2}}\,dx}{3c}$$

$$= \frac{e(d+ex)^2\sqrt{a+bx+cx^2}}{3c} + \frac{e\left(64c^2d^2+15b^2e^2-2ce(27bd+8ae)+10ce(2cd-be)x\right)\sqrt{a+bx-}}{24c^3}$$

$$= \frac{e(d+ex)^2\sqrt{a+bx+cx^2}}{3c} + \frac{e\left(64c^2d^2+15b^2e^2-2ce(27bd+8ae)+10ce(2cd-be)x\right)\sqrt{a+bx-}}{24c^3}$$

$$= \frac{e(d+ex)^2\sqrt{a+bx+cx^2}}{3c} + \frac{e\left(64c^2d^2+15b^2e^2-2ce(27bd+8ae)+10ce(2cd-be)x\right)\sqrt{a+bx-}}{24c^3}$$

Mathematica [A] time = 0.224165, size = 210, normalized size = 1.21

$$\frac{e\left(-16a^2ce^2+a\left(15b^2e^2-2bce(27d+13ex)+4c^2\left(18d^2+9dex-2e^2x^2\right)\right)+x(b+cx)\left(15b^2e^2-2bce(27d+5ex)+4c^2\right.}{24c^3\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (e*(-16*a^2*c*e^2 + a*(15*b^2*e^2 - 2*b*c*e*(27*d + 13*e*x) + 4*c^2*(18*d^2
 + 9*d*e*x - 2*e^2*x^2)) + x*(b + c*x)*(15*b^2*e^2 - 2*b*c*e*(27*d + 5*e*x)
 + 4*c^2*(18*d^2 + 9*d*e*x + 2*e^2*x^2))))/(24*c^3*Sqrt[a + x*(b + c*x)]) +
 ((2*c*d - b*e)*(8*c^2*d^2 + 5*b^2*e^2 - 4*c*e*(2*b*d + 3*a*e))*ArcTanh[(b
+ 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(16*c^(7/2))
```

Maple [B] time = 0.05, size = 366, normalized size = 2.1

$$\frac{e^3x^2}{3c}\sqrt{cx^2+bx+a} - \frac{5be^3x}{12c^2}\sqrt{cx^2+bx+a} + \frac{5b^2e^3}{8c^3}\sqrt{cx^2+bx+a} - \frac{5b^3e^3}{16}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)c^{-\frac{7}{2}} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/3*e^3*x^2/c*(c*x^2+b*x+a)^(1/2)-5/12*e^3*b/c^2*x*(c*x^2+b*x+a)^(1/2)+5/8*
e^3*b^2/c^3*(c*x^2+b*x+a)^(1/2)-5/16*e^3*b^3/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))+3/4*e^3*b/c^(5/2)*a*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x
+a)^(1/2))-2/3*e^3*a/c^2*(c*x^2+b*x+a)^(1/2)+3/2*d*e^2*x/c*(c*x^2+b*x+a)^(1
/2)-9/4*d*e^2*b/c^2*(c*x^2+b*x+a)^(1/2)+9/8*d*e^2*b^2/c^(5/2)*ln((1/2*b+c*x
)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-3/2*d*e^2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(
c*x^2+b*x+a)^(1/2))+3*d^2*e/c*(c*x^2+b*x+a)^(1/2)-3/2*d^2*e*b/c^(3/2)*ln((1
/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+d^3*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*
```

```
x+a)^(1/2))/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.60892, size = 890, normalized size = 5.11

$$\left[\frac{3\left(16\,c^3d^3-24\,bc^2d^2e+6\left(3\,b^2c-4\,ac^2\right)de^2-\left(5\,b^3-12\,abc\right)e^3\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}(2\,cx+b\right.}{96\,c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(16*c^3*d^3 - 24*b*c^2*d^2*e + 6*(3*b^2*c - 4*a*c^2)*d*e^2 - (5*b^
3 - 12*a*b*c)*e^3)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 +
b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(8*c^3*e^3*x^2 + 72*c^3*d^2*e - 5
4*b*c^2*d*e^2 + (15*b^2*c - 16*a*c^2)*e^3 + 2*(18*c^3*d*e^2 - 5*b*c^2*e^3)*
x)*sqrt(c*x^2 + b*x + a))/c^4, -1/48*(3*(16*c^3*d^3 - 24*b*c^2*d^2*e + 6*(3
*b^2*c - 4*a*c^2)*d*e^2 - (5*b^3 - 12*a*b*c)*e^3)*sqrt(-c)*arctan(1/2*sqrt(
c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(8*c^3*e
^3*x^2 + 72*c^3*d^2*e - 54*b*c^2*d*e^2 + (15*b^2*c - 16*a*c^2)*e^3 + 2*(18*
c^3*d*e^2 - 5*b*c^2*e^3)*x)*sqrt(c*x^2 + b*x + a))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^3}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**3/sqrt(a + b*x + c*x**2), x)
```

Giac [A] time = 1.13047, size = 230, normalized size = 1.32

$$\frac{1}{24}\sqrt{cx^2+bx+a}\left(2\,x\left(\frac{4\,xe^3}{c}+\frac{18\,c^2de^2-5\,bce^3}{c^3}\right)+\frac{72\,c^2d^2e-54\,bcde^2+15\,b^2e^3-16\,ace^3}{c^3}\right)-\frac{\left(16\,c^3d^3-24\,bc^2d^2e+18\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + b*x + a)*(2*x*(4*x*e^3/c + (18*c^2*d*e^2 - 5*b*c*e^3)/c^3
) + (72*c^2*d^2*e - 54*b*c*d*e^2 + 15*b^2*e^3 - 16*a*c*e^3)/c^3) - 1/16*(16
*c^3*d^3 - 24*b*c^2*d^2*e + 18*b^2*c*d*e^2 - 24*a*c^2*d*e^2 - 5*b^3*e^3 + 1
2*a*b*c*e^3)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c
^(7/2)
```

3.2375 $\int \frac{(d+ex)^2}{\sqrt{a+bx+cx^2}} dx$

Optimal. Leaf size=127

$$\frac{\left(-4ce(ae+2bd)+3b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}+\frac{3e\sqrt{a+bx+cx^2}(2cd-be)}{4c^2}+\frac{e(d+ex)\sqrt{a+bx+cx^2}}{2c}$$

```
[Out] (3*e*(2*c*d - b*e)*Sqrt[a + b*x + c*x^2])/(4*c^2) + (e*(d + e*x)*Sqrt[a + b
*x + c*x^2])/(2*c) + ((8*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(2*b*d + a*e))*ArcTanh
[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2))
```

Rubi [A] time = 0.116312, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {742, 640, 621, 206}

$$\frac{\left(-4ce(ae+2bd)+3b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}+\frac{3e\sqrt{a+bx+cx^2}(2cd-be)}{4c^2}+\frac{e(d+ex)\sqrt{a+bx+cx^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (3*e*(2*c*d - b*e)*Sqrt[a + b*x + c*x^2])/(4*c^2) + (e*(d + e*x)*Sqrt[a + b
*x + c*x^2])/(2*c) + ((8*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(2*b*d + a*e))*ArcTanh
[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2))
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{e(d+ex)\sqrt{a+bx+cx^2}}{2c} + \frac{\int \frac{\frac{1}{2}\left(4cd^2-e(bd+2ae)\right)+\frac{3}{2}e(2cd-be)x}{\sqrt{a+bx+cx^2}}\,dx}{2c}$$

$$= \frac{3e(2cd-be)\sqrt{a+bx+cx^2}}{4c^2} + \frac{e(d+ex)\sqrt{a+bx+cx^2}}{2c} + \frac{\left(-\frac{3}{2}be(2cd-be)+c\left(4cd^2-e(bd+2a\right.\right.}{4c^2}$$

$$= \frac{3e(2cd-be)\sqrt{a+bx+cx^2}}{4c^2} + \frac{e(d+ex)\sqrt{a+bx+cx^2}}{2c} + \frac{\left(-\frac{3}{2}be(2cd-be)+c\left(4cd^2-e(bd+2a\right.\right.}{2c}$$

$$= \frac{3e(2cd-be)\sqrt{a+bx+cx^2}}{4c^2} + \frac{e(d+ex)\sqrt{a+bx+cx^2}}{2c} + \frac{\left(8c^2d^2+3b^2e^2-4ce(2bd+ae)\right)\tanh}{8c^{5/2}}$$

Mathematica [A] time = 0.120862, size = 103, normalized size = 0.81

$$\frac{\left(-4ce(ae+2bd)+3b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{8c^{5/2}} + \frac{e\sqrt{a+x(b+cx)}(-3be+8cd+2cex)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (e*(8*c*d - 3*b*e + 2*c*e*x)*Sqrt[a + x*(b + c*x)])/(4*c^2) + ((8*c^2*d^2 +
 3*b^2*e^2 - 4*c*e*(2*b*d + a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x
*(b + c*x)])])/(8*c^(5/2))
```

Maple [A] time = 0.049, size = 198, normalized size = 1.6

$$\frac{e^2x}{2c}\sqrt{cx^2+bx+a} - \frac{3be^2}{4c^2}\sqrt{cx^2+bx+a} + \frac{3b^2e^2}{8}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)c^{-\frac{5}{2}} - \frac{ae^2}{2}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/2*e^2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*e^2*b/c^2*(c*x^2+b*x+a)^(1/2)+3/8*e^2*b
^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2*e^2*a/c^(3/2)*ln
((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+2*d*e/c*(c*x^2+b*x+a)^(1/2)-d*e*b
/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+d^2*ln((1/2*b+c*x)/c^(
1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.52757, size = 582, normalized size = 4.58

$$\left[-\frac{\left(8\,c^2d^2 - 8\,bcde + \left(3\,b^2 - 4\,ac\right)e^2\right)\sqrt{c}\log\left(-8\,c^2x^2 - 8\,bcx - b^2 + 4\,\sqrt{cx^2 + bx + a}(2\,cx + b)\sqrt{c} - 4\,ac\right) - 4\left(2\,c^2e^2x + 8\,c\right.}{16\,c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((8*c^2*d^2 - 8*b*c*d*e + (3*b^2 - 4*a*c)*e^2)*sqrt(c)*log(-8*c^2*x^
2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) -
4*(2*c^2*e^2*x + 8*c^2*d*e - 3*b*c*e^2)*sqrt(c*x^2 + b*x + a))/c^3, -1/8*((
8*c^2*d^2 - 8*b*c*d*e + (3*b^2 - 4*a*c)*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2
 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(2*c^2*e^2*x
+ 8*c^2*d*e - 3*b*c*e^2)*sqrt(c*x^2 + b*x + a))/c^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^2}{\sqrt{a + bx + cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**2/sqrt(a + b*x + c*x**2), x)
```

Giac [A] time = 1.12988, size = 142, normalized size = 1.12

$$\frac{1}{4}\,\sqrt{cx^2 + bx + a}\left(\frac{2\,xe^2}{c} + \frac{8\,cde - 3\,be^2}{c^2}\right) - \frac{\left(8\,c^2d^2 - 8\,bcde + 3\,b^2e^2 - 4\,ace^2\right)\log\left(\left|-2\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)\sqrt{c} - b\right|\right)}{8\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^2 + b*x + a)*(2*x*e^2/c + (8*c*d*e - 3*b*e^2)/c^2) - 1/8*(8*c^
2*d^2 - 8*b*c*d*e + 3*b^2*e^2 - 4*a*c*e^2)*log(abs(-2*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*sqrt(c) - b))/c^(5/2)
```

3.2376 $\int \frac{d+ex}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=68

$$\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}}+\frac{e\sqrt{a+bx+cx^2}}{c}$$

```
[Out] (e*Sqrt[a + b*x + c*x^2])/c + ((2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]
*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2))
```

Rubi [A] time = 0.0242194, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {640, 621, 206}

$$\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}}+\frac{e\sqrt{a+bx+cx^2}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/Sqrt[a + b*x + c*x^2],x]
```


```
[Out] (e*Sqrt[a + b*x + c*x^2])/c + ((2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]
*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2))
```


Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex}{\sqrt{a+bx+cx^2}}\,dx &= \frac{e\sqrt{a+bx+cx^2}}{c}+\frac{(2cd-be)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2c}\\ &= \frac{e\sqrt{a+bx+cx^2}}{c}+\frac{(2cd-be)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{c}\\ &= \frac{e\sqrt{a+bx+cx^2}}{c}+\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0794227, size = 66, normalized size = 0.97

$$\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{2c^{3/2}}+\frac{e\sqrt{a+x(b+cx)}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (e*Sqrt[a + x*(b + c*x)])/c + ((2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]
*Sqrt[a + x*(b + c*x)])])/(2*c^(3/2))
```

Maple [A] time = 0.045, size = 81, normalized size = 1.2

$$\frac{e}{c}\sqrt{cx^2+bx+a}-\frac{be}{2}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)c^{-\frac{3}{2}}+d\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] e*(c*x^2+b*x+a)^(1/2)/c-1/2*e*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))+d*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.32628, size = 400, normalized size = 5.88

$$\left[\frac{4\sqrt{cx^2+bx+a}ce-(2cd-be)\sqrt{c}\log\left(-8c^2x^2-8bcx-b^2+4\sqrt{cx^2+bx+a}(2cx+b)\sqrt{c}-4ac\right)}{4c^2}, \frac{2\sqrt{cx^2+bx+a}ce-}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(c*x^2 + b*x + a)*c*e - (2*c*d - b*e)*sqrt(c)*log(-8*c^2*x^2 -
8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c))/c^2,
1/2*(2*sqrt(c*x^2 + b*x + a)*c*e - (2*c*d - b*e)*sqrt(-c)*arctan(1/2*sqrt(c
*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)))/c^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex}{\sqrt{a + bx + cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)/sqrt(a + b*x + c*x**2), x)
```

Giac [A] time = 1.10093, size = 88, normalized size = 1.29

$$\frac{\sqrt{cx^2 + bx + a}e}{c} - \frac{(2\,cd - be)\log\left(\left|-2\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)\sqrt{c} - b\right|\right)}{2\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(c*x^2 + b*x + a)*e/c - 1/2*(2*c*d - b*e)*log(abs(-2*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))*sqrt(c) - b))/c^(3/2)
```

3.2377 $\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=36

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}$$

```
[Out] ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/Sqrt[c]
```

Rubi [A] time = 0.0105089, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/Sqrt[c]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.0223763, size = 34, normalized size = 0.94

$$\frac{\log\left(2\sqrt{c}\sqrt{a+bx+cx^2}+b+2cx\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*x + c*x^2],x]

[Out] Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]]/Sqrt[c]
```

Maple [A] time = 0.043, size = 30, normalized size = 0.8

$$\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(1/2),x)

[Out] ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 2.48991, size = 263, normalized size = 7.31

$$\left[\frac{\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right)}{2\,\sqrt{c}}, -\frac{\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)}{c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")

[Out] [1/2*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*s
qrt(c) - 4*a*c)/sqrt(c), -sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x
+ b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c))/c]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*x + c*x**2), x)
```

Giac [A] time = 1.10836, size = 49, normalized size = 1.36

$$-\frac{\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/sqrt(c)
```

3.2378 $\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=79

$$\frac{\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\sqrt{ae^2-bde+cd^2}}$$

```
[Out] ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt
[a + b*x + c*x^2])]/Sqrt[c*d^2 - b*d*e + a*e^2]
```

Rubi [A] time = 0.0386044, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {724, 206}

$$\frac{\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\sqrt{ae^2-bde+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt
[a + b*x + c*x^2])]/Sqrt[c*d^2 - b*d*e + a*e^2]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)\right)\\ &= \frac{\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{cd^2-bde+ae^2}}\end{aligned}$$

Mathematica [A] time = 0.0365629, size = 78, normalized size = 0.99

$$-\frac{\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{\sqrt{e(ae-bd)+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*
e)]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d^2 + e*(-(b*d) + a*e)])
```

Maple [B] time = 0.228, size = 157, normalized size = 2.

$$-\frac{1}{e}\ln\left(\left(2\frac{ae^2-bde+cd^2}{e^2}+\frac{be-2cd}{e}\left(\frac{d}{e}+x\right)+2\sqrt{\frac{ae^2-bde+cd^2}{e^2}}\sqrt{\left(\frac{d}{e}+x\right)^2c+\frac{be-2cd}{e}\left(\frac{d}{e}+x\right)+\frac{ae^2-bde+cd^2}{e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c
*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*
(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.07362, size = 743, normalized size = 9.41

$$\left[\frac{\log\left(\frac{8\,abde-8\,a^2e^2-(b^2+4\,ac)d^2-(8\,c^2d^2-8\,bcde+(b^2+4\,ac)e^2)x^2-4\,\sqrt{cd^2-bde+ae^2}\sqrt{cx^2+bx+a}(bd-2\,ae+(2\,cd-be)x)-2\,(4\,bcd^2+4\,abe^2-(3\,b^2+4\,ac)de)x}{e^2x^2+2\,dex+d^2}\right)}{2\,\sqrt{cd^2-bde+ae^2}},\ \sqrt{-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*
e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x
 + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 +
 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2))/sqrt(c*d^2 - b*d*e + a*e^2), sqr
t(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x
^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2
+ (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x))/(c*
d^2 - b*d*e + a*e^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [A] time = 1.10382, size = 97, normalized size = 1.23

$$\frac{2\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\sqrt{-cd^2+bde-ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2 +
 b*d*e - a*e^2))/sqrt(-c*d^2 + b*d*e - a*e^2)
```

3.2379 $\int \frac{1}{(d+ex)^2\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=134

$$\frac{(2cd-be)\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{2\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{e\sqrt{a+bx+cx^2}}{(d+ex)\left(ae^2-bde+cd^2\right)}$$

```
[Out] -((e*Sqrt[a + b*x + c*x^2])/((c*d^2 - b*d*e + a*e^2)*(d + e*x))) + ((2*c*d
- b*e)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^
2]*Sqrt[a + b*x + c*x^2])])/(2*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rubi [A] time = 0.0784632, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {730, 724, 206}

$$\frac{(2cd-be)\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{2\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{e\sqrt{a+bx+cx^2}}{(d+ex)\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((e*Sqrt[a + b*x + c*x^2])/((c*d^2 - b*d*e + a*e^2)*(d + e*x))) + ((2*c*d
- b*e)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^
2]*Sqrt[a + b*x + c*x^2])])/(2*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\sqrt{a+bx+cx^2}}\,dx = -\frac{e\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(d+ex)} + \frac{(2cd-be)\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{2(cd^2-bde+ae^2)}$$

$$= -\frac{e\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(d+ex)} - \frac{(2cd-be)\,\mathrm{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-}{\sqrt{a+bx+cx^2}}\right.}{cd^2-bde+ae^2}$$

$$= -\frac{e\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(d+ex)} + \frac{(2cd-be)\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{2(cd^2-bde+ae^2)^{3/2}}$$

Mathematica [A] time = 0.108025, size = 131, normalized size = 0.98

$$-\frac{e\sqrt{a+x(b+cx)}}{(d+ex)(e(ae-bd)+cd^2)} - \frac{(2cd-be)\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{2(e(ae-bd)+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((e*Sqrt[a + x*(b + c*x)])/((c*d^2 + e*(-(b*d) + a*e))*(d + e*x))) - ((2*c
*d - b*e)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b
*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/(2*(c*d^2 + e*(-(b*d) + a*e))^(3/2))
```

Maple [B] time = 0.228, size = 432, normalized size = 3.2

$$-\frac{1}{ae^2-bde+cd^2}\sqrt{\left(\frac{d}{e}+x\right)^2 c+\frac{be-2cd}{e}\left(\frac{d}{e}+x\right)+\frac{ae^2-bde+cd^2}{e^2}}\left(\frac{d}{e}+x\right)^{-1}+\frac{b}{2ae^2-2bde+2cd^2}\ln\left(\left(2\frac{ae^2-bc}{e}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/(a*e^2-b*d*e+c*d^2)/(d/e+x)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)+1/2/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/
2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^
2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(
1/2))/(d/e+x))*b-1/e/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln
((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))
/(d/e+x))*c*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [B] time = 6.00199, size = 1397, normalized size = 10.43

$$\left[-\frac{\left(2\,cd^2 - bde + \left(2\,cde - be^2\right)x\right)\sqrt{cd^2 - bde + ae^2}\log\left(\frac{8\,abde-8\,a^2e^2-\left(b^2+4\,ac\right)d^2-\left(8\,c^2d^2-8\,bcde+\left(b^2+4\,ac\right)e^2\right)x^2+4\,\sqrt{cd^2-bde+ae^2}\sqrt{cx^2+bx+\cdots}}{e^2x^2+2\,dex+d^2}\right.}{4\left(c^2d^5 - 2\,bcd^4e - 2\,abd^2e^3 + a^2de^4 + \left(b^2 + 2\,ac\right)d^3e^2 + \left(c^2d^4e - 2\,bcd^3e^2 \cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")

[Out] [-1/4*((2*c*d^2 - b*d*e + (2*c*d*e - b*e^2)*x)*sqrt(c*d^2 - b*d*e + a*e^2)*
log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (
b^2 + 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)
*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*
c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) + 4*(c*d^2*e - b*d*e^2 + a*e^3)*sqrt(
c*x^2 + b*x + a))/(c^2*d^5 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + a^2*d*e^4 + (b^2
 + 2*a*c)*d^3*e^2 + (c^2*d^4*e - 2*b*c*d^3*e^2 - 2*a*b*d*e^4 + a^2*e^5 + (b
^2 + 2*a*c)*d^2*e^3)*x), 1/2*((2*c*d^2 - b*d*e + (2*c*d*e - b*e^2)*x)*sqrt(
-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2
 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 +
(c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)) - 2*(
c*d^2*e - b*d*e^2 + a*e^3)*sqrt(c*x^2 + b*x + a))/(c^2*d^5 - 2*b*c*d^4*e -
2*a*b*d^2*e^3 + a^2*d*e^4 + (b^2 + 2*a*c)*d^3*e^2 + (c^2*d^4*e - 2*b*c*d^3*
e^2 - 2*a*b*d*e^4 + a^2*e^5 + (b^2 + 2*a*c)*d^2*e^3)*x)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d + ex\right)^2\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)**2/(c*x**2+b*x+a)**(1/2),x)

[Out] Integral(1/((d + e*x)**2*sqrt(a + b*x + c*x**2)), x)

Giac [B] time = 18.7616, size = 859, normalized size = 6.41

$$\frac{\left(2\,c^{\frac{3}{2}}d^2 - 2\,b\sqrt{c}de + 2\,\sqrt{cd^2 - bde + ae^2}cd\log\left(\left|2\,c^{\frac{3}{2}}d^2 - 2\,b\sqrt{c}de - 2\,\sqrt{cd^2 - bde + ae^2}cd + 2\,a\sqrt{c}e^2 + \sqrt{cd^2 - bde + ae^2}be \cdots\right.\right.\right.}{2\left(c^2d^4 - 2\,bcd^3e + b^2d^2 \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")

[Out] 1/2*(2*c^(3/2)*d^2 - 2*b*sqrt(c)*d*e + 2*sqrt(c*d^2 - b*d*e + a*e^2)*c*d*lo
g(abs(2*c^(3/2)*d^2 - 2*b*sqrt(c)*d*e - 2*sqrt(c*d^2 - b*d*e + a*e^2)*c*d +

```
 2*a*sqrt(c)*e^2 + sqrt(c*d^2 - b*d*e + a*e^2)*b*e)) - sqrt(c*d^2 - b*d*e +
 a*e^2)*b*e*log(abs(2*c^(3/2)*d^2 - 2*b*sqrt(c)*d*e - 2*sqrt(c*d^2 - b*d*e
+ a*e^2)*c*d + 2*a*sqrt(c)*e^2 + sqrt(c*d^2 - b*d*e + a*e^2)*b*e)) + 2*a*sq
rt(c)*e^2)*sgn(1/(x*e + d))/(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^
2*e^2 - 2*a*b*d*e^3 + a^2*e^4) - 1/2*sqrt(c*d^2 - b*d*e + a*e^2)*(2*c*d*e -
 b*e^2)*log(abs(2*(c*d^2 - b*d*e + a*e^2)*(sqrt(c - 2*c*d/(x*e + d) + c*d^2
/(x*e + d)^2 + b*e/(x*e + d) - b*d*e/(x*e + d)^2 + a*e^2/(x*e + d)^2) + sqr
t(c*d^2*e^2 - b*d*e^3 + a*e^4)*e^(-1)/(x*e + d)) - sqrt(c*d^2 - b*d*e + a*e
^2)*(2*c*d - b*e)))/((c^2*d^4*e - 2*b*c*d^3*e^2 + b^2*d^2*e^3 + 2*a*c*d^2*e
^3 - 2*a*b*d*e^4 + a^2*e^5)*sgn(1/(x*e + d))) - sqrt(c - 2*c*d/(x*e + d) +
c*d^2/(x*e + d)^2 + b*e/(x*e + d) - b*d*e/(x*e + d)^2 + a*e^2/(x*e + d)^2)/
(c*d^2*sgn(1/(x*e + d)) - b*d*e*sgn(1/(x*e + d)) + a*e^2*sgn(1/(x*e + d)))
```

3.2380 $\int \frac{1}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=209

$$\frac{\left(-4ce(ae+2bd)+3b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{8\left(ae^2-bde+cd^2\right)^{5/2}}-\frac{3e\sqrt{a+bx+cx^2}(2cd-be)}{4(d+ex)\left(ae^2-bde+cd^2\right)^2}-\frac{e\sqrt{a+bx+cx^2}}{2(d+ex)^2\left(ae^2-bde\right.}$$

```
[Out] -(e*Sqrt[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^2) - (3*e*(
2*c*d - b*e)*Sqrt[a + b*x + c*x^2])/(4*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x))
 + ((8*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(2*b*d + a*e))*ArcTanh[(b*d - 2*a*e + (2
*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(8*(
c*d^2 - b*d*e + a*e^2)^(5/2))
```

Rubi [A] time = 0.218934, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {744, 806, 724, 206}

$$\frac{\left(-4ce(ae+2bd)+3b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{8\left(ae^2-bde+cd^2\right)^{5/2}}-\frac{3e\sqrt{a+bx+cx^2}(2cd-be)}{4(d+ex)\left(ae^2-bde+cd^2\right)^2}-\frac{e\sqrt{a+bx+cx^2}}{2(d+ex)^2\left(ae^2-bde\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*Sqrt[a + b*x + c*x^2]),x]
```


```
[Out] -(e*Sqrt[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^2) - (3*e*(
2*c*d - b*e)*Sqrt[a + b*x + c*x^2])/(4*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x))
 + ((8*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(2*b*d + a*e))*ArcTanh[(b*d - 2*a*e + (2
*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(8*(
c*d^2 - b*d*e + a*e^2)^(5/2))
```


Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx = -\frac{e\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{\int \frac{\frac{1}{2}(-4cd+3be)+cex}{(d+ex)^2\sqrt{a+bx+cx^2}}\,dx}{2\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{e\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{3e(2cd-be)\sqrt{a+bx+cx^2}}{4\left(cd^2-bde+ae^2\right)^2(d+ex)} + \frac{\left(8c^2d^2+3b^2e^2-4ce\right.}{8\left(cd\right.}$$

$$= -\frac{e\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{3e(2cd-be)\sqrt{a+bx+cx^2}}{4\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{\left(8c^2d^2+3b^2e^2-4ce\right.}{}$$

$$= -\frac{e\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{3e(2cd-be)\sqrt{a+bx+cx^2}}{4\left(cd^2-bde+ae^2\right)^2(d+ex)} + \frac{\left(8c^2d^2+3b^2e^2-4ce\right.}{}$$

Mathematica [A] time = 0.304092, size = 205, normalized size = 0.98

$$-\frac{\left(-4ce(ae+2bd)+3b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{8\left(e(ae-bd)+cd^2\right)^{5/2}} - \frac{3e\sqrt{a+x(b+cx)}(2cd-be)}{4(d+ex)\left(e(ae-bd)+cd^2\right)^2} - \frac{e\sqrt{a+x(}}{2(d+ex)^2\left(e(ae\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(e*Sqrt[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e))*(d + e*x)^2) - (3*
e*(2*c*d - b*e)*Sqrt[a + x*(b + c*x)])/(4*(c*d^2 + e*(-(b*d) + a*e))^2*(d +
 e*x)) - ((8*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(2*b*d + a*e))*ArcTanh[(-(b*d) + 2
*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c
*x)])])/(8*(c*d^2 + e*(-(b*d) + a*e))^(5/2))
```

Maple [B] time = 0.237, size = 959, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/2/e/(a*e^2-b*d*e+c*d^2)/(d/e+x)^2*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)+3/4*e/(a*e^2-b*d*e+c*d^2)^2/(d/e+x)*((d/e+x)^2*
```

```
c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b-3/2/(a*e^2-b*d*e+c
*d^2)^2/(d/e+x)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)*c*d-3/8*e/(a*e^2-b*d*e+c*d^2)^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((
2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(
d/e+x))*b^2+3/2/(a*e^2-b*d*e+c*d^2)^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2
*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^
(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d
/e+x))*b*c*d-3/2/e/(a*e^2-b*d*e+c*d^2)^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln
((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))
/(d/e+x))*c^2*d^2+1/2/e*c/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/
2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^
2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(
1/2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 19.7656, size = 2838, normalized size = 13.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((8*c^2*d^4 - 8*b*c*d^3*e + (3*b^2 - 4*a*c)*d^2*e^2 + (8*c^2*d^2*e^2
 - 8*b*c*d*e^3 + (3*b^2 - 4*a*c)*e^4)*x^2 + 2*(8*c^2*d^3*e - 8*b*c*d^2*e^2
+ (3*b^2 - 4*a*c)*d*e^3)*x)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*
a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x
^2 + 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*
c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2
 + 2*d*e*x + d^2)) + 4*(8*c^2*d^4*e - 13*b*c*d^3*e^2 - 7*a*b*d*e^4 + 2*a^2*
e^5 + 5*(b^2 + 2*a*c)*d^2*e^3 + 3*(2*c^2*d^3*e^2 - 3*b*c*d^2*e^3 - a*b*e^5
+ (b^2 + 2*a*c)*d*e^4)*x)*sqrt(c*x^2 + b*x + a))/(c^3*d^8 - 3*b*c^2*d^7*e -
 3*a^2*b*d^3*e^5 + a^3*d^2*e^6 + 3*(b^2*c + a*c^2)*d^6*e^2 - (b^3 + 6*a*b*c
)*d^5*e^3 + 3*(a*b^2 + a^2*c)*d^4*e^4 + (c^3*d^6*e^2 - 3*b*c^2*d^5*e^3 - 3*
a^2*b*d*e^7 + a^3*e^8 + 3*(b^2*c + a*c^2)*d^4*e^4 - (b^3 + 6*a*b*c)*d^3*e^5
 + 3*(a*b^2 + a^2*c)*d^2*e^6)*x^2 + 2*(c^3*d^7*e - 3*b*c^2*d^6*e^2 - 3*a^2*
b*d^2*e^6 + a^3*d*e^7 + 3*(b^2*c + a*c^2)*d^5*e^3 - (b^3 + 6*a*b*c)*d^4*e^4
 + 3*(a*b^2 + a^2*c)*d^3*e^5)*x), 1/8*((8*c^2*d^4 - 8*b*c*d^3*e + (3*b^2 -
4*a*c)*d^2*e^2 + (8*c^2*d^2*e^2 - 8*b*c*d*e^3 + (3*b^2 - 4*a*c)*e^4)*x^2 +
2*(8*c^2*d^3*e - 8*b*c*d^2*e^2 + (3*b^2 - 4*a*c)*d*e^3)*x)*sqrt(-c*d^2 + b*
d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)
*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 -
b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)) - 2*(8*c^2*d^4*e
 - 13*b*c*d^3*e^2 - 7*a*b*d*e^4 + 2*a^2*e^5 + 5*(b^2 + 2*a*c)*d^2*e^3 + 3*(
```

```
2*c^2*d^3*e^2 - 3*b*c*d^2*e^3 - a*b*e^5 + (b^2 + 2*a*c)*d*e^4)*x)*sqrt(c*x^
2 + b*x + a))/(c^3*d^8 - 3*b*c^2*d^7*e - 3*a^2*b*d^3*e^5 + a^3*d^2*e^6 + 3*
(b^2*c + a*c^2)*d^6*e^2 - (b^3 + 6*a*b*c)*d^5*e^3 + 3*(a*b^2 + a^2*c)*d^4*e
^4 + (c^3*d^6*e^2 - 3*b*c^2*d^5*e^3 - 3*a^2*b*d*e^7 + a^3*e^8 + 3*(b^2*c +
a*c^2)*d^4*e^4 - (b^3 + 6*a*b*c)*d^3*e^5 + 3*(a*b^2 + a^2*c)*d^2*e^6)*x^2 +
 2*(c^3*d^7*e - 3*b*c^2*d^6*e^2 - 3*a^2*b*d^2*e^6 + a^3*d*e^7 + 3*(b^2*c +
a*c^2)*d^5*e^3 - (b^3 + 6*a*b*c)*d^4*e^4 + 3*(a*b^2 + a^2*c)*d^3*e^5)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**2+b*x+a)**(1/2),x)

[Out] Integral(1/((d + e*x)**3*sqrt(a + b*x + c*x**2)), x)
```

Giac [B] time = 1.20098, size = 1060, normalized size = 5.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")

[Out] 1/4*(8*c^2*d^2 - 8*b*c*d*e + 3*b^2*e^2 - 4*a*c*e^2)*arctan(-((sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/((c^2*d^
4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4)*sqrt
(-c*d^2 + b*d*e - a*e^2)) - 1/4*(8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^
2*d^2*e + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*d^3 - 24*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*d^2*e + 24*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))*b*c^2*d^3 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*d*e^2
- 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*d^2*e - 40*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))*a*c^2*d^2*e + 6*b^2*c^(3/2)*d^3 + 9*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^2*b^2*sqrt(c)*d*e^2 - 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^2*a*c^(3/2)*d*e^2 - 3*b^3*sqrt(c)*d^2*e - 20*a*b*c^(3/2)*d^2*e + 3*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*e^3 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^3*a*c*e^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*d*e^2 + 28*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*d*e^2 + 11*a*b^2*sqrt(c)*d*e^2 + 12*a^
2*c^(3/2)*d*e^2 - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*e^3 - 4*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*e^3 - 8*a^2*b*sqrt(c)*e^3)/((c^2*d^4 -
 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4)*((sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^2*e + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*s
qrt(c)*d + b*d - a*e)^2)
```

3.2381 $\int \frac{1}{(d+ex)^4\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=293

$$-\frac{e\sqrt{a+bx+cx^2}\left(-4ce(4ae+11bd)+15b^2e^2+44c^2d^2\right)}{24(d+ex)\left(ae^2-bde+cd^2\right)^3}+\frac{(2cd-be)\left(-4ce(3ae+2bd)+5b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{-2ae+}{2\sqrt{a+bx+}}\right)}{16\left(ae^2-bde+cd^2\right)^{7/2}}$$

```
[Out] -(e*Sqrt[a + b*x + c*x^2])/(3*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^3) - (5*e*(
2*c*d - b*e)*Sqrt[a + b*x + c*x^2])/(12*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)
^2) - (e*(44*c^2*d^2 + 15*b^2*e^2 - 4*c*e*(11*b*d + 4*a*e))*Sqrt[a + b*x +
c*x^2])/(24*(c*d^2 - b*d*e + a*e^2)^3*(d + e*x)) + ((2*c*d - b*e)*(8*c^2*d^
2 + 5*b^2*e^2 - 4*c*e*(2*b*d + 3*a*e))*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)
*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(16*(c*d^2 - b*
d*e + a*e^2)^(7/2))
```

Rubi [A] time = 0.359475, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {744, 834, 806, 724, 206}

$$-\frac{e\sqrt{a+bx+cx^2}\left(-4ce(4ae+11bd)+15b^2e^2+44c^2d^2\right)}{24(d+ex)\left(ae^2-bde+cd^2\right)^3}+\frac{(2cd-be)\left(-4ce(3ae+2bd)+5b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{-2ae+}{2\sqrt{a+bx+}}\right)}{16\left(ae^2-bde+cd^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^4*Sqrt[a + b*x + c*x^2]),x]
```


```
[Out] -(e*Sqrt[a + b*x + c*x^2])/(3*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^3) - (5*e*(
2*c*d - b*e)*Sqrt[a + b*x + c*x^2])/(12*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)
^2) - (e*(44*c^2*d^2 + 15*b^2*e^2 - 4*c*e*(11*b*d + 4*a*e))*Sqrt[a + b*x +
c*x^2])/(24*(c*d^2 - b*d*e + a*e^2)^3*(d + e*x)) + ((2*c*d - b*e)*(8*c^2*d^
2 + 5*b^2*e^2 - 4*c*e*(2*b*d + 3*a*e))*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)
*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(16*(c*d^2 - b*
d*e + a*e^2)^(7/2))
```


Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
```

```
IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^4\sqrt{a+bx+cx^2}}\,dx = -\frac{e\sqrt{a+bx+cx^2}}{3\left(cd^2-bde+ae^2\right)(d+ex)^3} - \frac{\int \frac{\frac{1}{2}(-6cd+5be)+2cex}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx}{3\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{e\sqrt{a+bx+cx^2}}{3\left(cd^2-bde+ae^2\right)(d+ex)^3} - \frac{5e(2cd-be)\sqrt{a+bx+cx^2}}{12\left(cd^2-bde+ae^2\right)^2(d+ex)^2} + \frac{\int \frac{\frac{1}{4}\left(24c^2d^2+15b^2e^2-2c \cdots\right.}{(d+ex \cdots}}{6\left(cd^2 \cdots\right.}$$

$$= -\frac{e\sqrt{a+bx+cx^2}}{3\left(cd^2-bde+ae^2\right)(d+ex)^3} - \frac{5e(2cd-be)\sqrt{a+bx+cx^2}}{12\left(cd^2-bde+ae^2\right)^2(d+ex)^2} - \frac{e\left(44c^2d^2+15b^2e^2 \cdots\right.}{24\left(cd^2 \cdots\right.}$$

$$= -\frac{e\sqrt{a+bx+cx^2}}{3\left(cd^2-bde+ae^2\right)(d+ex)^3} - \frac{5e(2cd-be)\sqrt{a+bx+cx^2}}{12\left(cd^2-bde+ae^2\right)^2(d+ex)^2} - \frac{e\left(44c^2d^2+15b^2e^2 \cdots\right.}{24\left(cd^2 \cdots\right.}$$

$$= -\frac{e\sqrt{a+bx+cx^2}}{3\left(cd^2-bde+ae^2\right)(d+ex)^3} - \frac{5e(2cd-be)\sqrt{a+bx+cx^2}}{12\left(cd^2-bde+ae^2\right)^2(d+ex)^2} - \frac{e\left(44c^2d^2+15b^2e^2 \cdots\right.}{24\left(cd^2 \cdots\right.}$$

Mathematica [A] time = 0.673365, size = 288, normalized size = 0.98

$$-\frac{\frac{e\sqrt{a+x(b+cx)}\left(-4ce(4ae+11bd)+15b^2e^2+44c^2d^2\right)}{4(d+ex)\left(e(ae-bd)+cd^2\right)} + \frac{3(2cd-be)\left(-4ce(3ae+2bd)+5b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{8\left(e(ae-bd)+cd^2\right)^{3/2}} + \frac{2e\sqrt{a+x(b+cx)}\left(e(ae-b \cdots\right.}{(d+ex)^3}}{6\left(e(ae-bd)+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^4*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((2*e*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[a + x*(b + c*x)])/(d + e*x)^3 + (5*e
*(2*c*d - b*e)*Sqrt[a + x*(b + c*x)])/(2*(d + e*x)^2) + (e*(44*c^2*d^2 + 15
*b^2*e^2 - 4*c*e*(11*b*d + 4*a*e))*Sqrt[a + x*(b + c*x)])/(4*(c*d^2 + e*(-(
b*d) + a*e))*(d + e*x)) + (3*(2*c*d - b*e)*(8*c^2*d^2 + 5*b^2*e^2 - 4*c*e*(
2*b*d + 3*a*e))*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 +
e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/(8*(c*d^2 + e*(-(b*d) + a*e))^(3
/2)))/(6*(c*d^2 + e*(-(b*d) + a*e))^2)
```

Maple [B] time = 0.236, size = 1665, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^4/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/3/e^2/(a*e^2-b*d*e+c*d^2)/(d/e+x)^3*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+5/12/(a*e^2-b*d*e+c*d^2)^2/(d/e+x)^2*((d/e+x)
^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b-5/6/e/(a*e^2-b*
d*e+c*d^2)^2/(d/e+x)^2*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^
2)/e^2)^(1/2)*c*d-5/8*e^2/(a*e^2-b*d*e+c*d^2)^3/(d/e+x)*((d/e+x)^2*c+(b*e-2
*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2+5/2*e/(a*e^2-b*d*e+c*d^2
)^3/(d/e+x)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/
2)*b*c*d-5/2/(a*e^2-b*d*e+c*d^2)^3/(d/e+x)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+
x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*c^2*d^2+5/16*e^2/(a*e^2-b*d*e+c*d^2)^3/((
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(
d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)
+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b^3-15/8*e/(a*e^2-b*d*e+c*d^2)^3/
((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e
*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+
x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b^2*c*d+15/4/(a*e^2-b*d*e+c*d^2
)^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*
d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(
d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b*c^2*d^2-5/2/e/(a*e^2-b*d*
e+c*d^2)^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b
*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c
*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*c^3*d^3-3/4/(a*e^2-b
*d*e+c*d^2)^2*c/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e
^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*
e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b+3/2/e/(a*e^2-
b*d*e+c*d^2)^2*c^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2
)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+
(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*d+2/3*c/(a*e
^2-b*d*e+c*d^2)^2/(d/e+x)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c
*d^2)/e^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [B] time = 97.8904, size = 5632, normalized size = 19.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(16*c^3*d^6 - 24*b*c^2*d^5*e + 6*(3*b^2*c - 4*a*c^2)*d^4*e^2 - (5*
b^3 - 12*a*b*c)*d^3*e^3 + (16*c^3*d^3*e^3 - 24*b*c^2*d^2*e^4 + 6*(3*b^2*c -
 4*a*c^2)*d*e^5 - (5*b^3 - 12*a*b*c)*e^6)*x^3 + 3*(16*c^3*d^4*e^2 - 24*b*c^
2*d^3*e^3 + 6*(3*b^2*c - 4*a*c^2)*d^2*e^4 - (5*b^3 - 12*a*b*c)*d*e^5)*x^2 +
 3*(16*c^3*d^5*e - 24*b*c^2*d^4*e^2 + 6*(3*b^2*c - 4*a*c^2)*d^3*e^3 - (5*b^
3 - 12*a*b*c)*d^2*e^4)*x)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^
2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2
 - 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*
d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 +
 2*d*e*x + d^2)) - 4*(72*c^3*d^6*e - 162*b*c^2*d^5*e^2 - 34*a^2*b*d*e^6 + 8
*a^3*e^7 + (123*b^2*c + 92*a*c^2)*d^4*e^3 - (33*b^3 + 136*a*b*c)*d^3*e^4 +
(59*a*b^2 + 28*a^2*c)*d^2*e^5 + (44*c^3*d^4*e^3 - 88*b*c^2*d^3*e^4 + (59*b^
2*c + 28*a*c^2)*d^2*e^5 - (15*b^3 + 28*a*b*c)*d*e^6 + (15*a*b^2 - 16*a^2*c)
*e^7)*x^2 + 2*(54*c^3*d^5*e^2 - 113*b*c^2*d^4*e^3 - 5*a^2*b*e^7 + (79*b^2*c
 + 48*a*c^2)*d^3*e^4 - 2*(10*b^3 + 29*a*b*c)*d^2*e^5 + (25*a*b^2 - 6*a^2*c)
*d*e^6)*x)*sqrt(c*x^2 + b*x + a))/(c^4*d^11 - 4*b*c^3*d^10*e - 4*a^3*b*d^4*
e^7 + a^4*d^3*e^8 + 2*(3*b^2*c^2 + 2*a*c^3)*d^9*e^2 - 4*(b^3*c + 3*a*b*c^2)
*d^8*e^3 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^7*e^4 - 4*(a*b^3 + 3*a^2*b*c)*d
^6*e^5 + 2*(3*a^2*b^2 + 2*a^3*c)*d^5*e^6 + (c^4*d^8*e^3 - 4*b*c^3*d^7*e^4 -
 4*a^3*b*d*e^10 + a^4*e^11 + 2*(3*b^2*c^2 + 2*a*c^3)*d^6*e^5 - 4*(b^3*c + 3
*a*b*c^2)*d^5*e^6 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^4*e^7 - 4*(a*b^3 + 3*a
^2*b*c)*d^3*e^8 + 2*(3*a^2*b^2 + 2*a^3*c)*d^2*e^9)*x^3 + 3*(c^4*d^9*e^2 - 4
*b*c^3*d^8*e^3 - 4*a^3*b*d^2*e^9 + a^4*d*e^10 + 2*(3*b^2*c^2 + 2*a*c^3)*d^7
*e^4 - 4*(b^3*c + 3*a*b*c^2)*d^6*e^5 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^5*e
^6 - 4*(a*b^3 + 3*a^2*b*c)*d^4*e^7 + 2*(3*a^2*b^2 + 2*a^3*c)*d^3*e^8)*x^2 +
 3*(c^4*d^10*e - 4*b*c^3*d^9*e^2 - 4*a^3*b*d^3*e^8 + a^4*d^2*e^9 + 2*(3*b^2
*c^2 + 2*a*c^3)*d^8*e^3 - 4*(b^3*c + 3*a*b*c^2)*d^7*e^4 + (b^4 + 12*a*b^2*c
 + 6*a^2*c^2)*d^6*e^5 - 4*(a*b^3 + 3*a^2*b*c)*d^5*e^6 + 2*(3*a^2*b^2 + 2*a^
3*c)*d^4*e^7)*x), 1/48*(3*(16*c^3*d^6 - 24*b*c^2*d^5*e + 6*(3*b^2*c - 4*a*c
^2)*d^4*e^2 - (5*b^3 - 12*a*b*c)*d^3*e^3 + (16*c^3*d^3*e^3 - 24*b*c^2*d^2*e
^4 + 6*(3*b^2*c - 4*a*c^2)*d*e^5 - (5*b^3 - 12*a*b*c)*e^6)*x^3 + 3*(16*c^3*
d^4*e^2 - 24*b*c^2*d^3*e^3 + 6*(3*b^2*c - 4*a*c^2)*d^2*e^4 - (5*b^3 - 12*a*
b*c)*d*e^5)*x^2 + 3*(16*c^3*d^5*e - 24*b*c^2*d^4*e^2 + 6*(3*b^2*c - 4*a*c^2
)*d^3*e^3 - (5*b^3 - 12*a*b*c)*d^2*e^4)*x)*sqrt(-c*d^2 + b*d*e - a*e^2)*arc
tan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e +
(2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^
2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)) - 2*(72*c^3*d^6*e - 162*b*c^2*d^
5*e^2 - 34*a^2*b*d*e^6 + 8*a^3*e^7 + (123*b^2*c + 92*a*c^2)*d^4*e^3 - (33*b
^3 + 136*a*b*c)*d^3*e^4 + (59*a*b^2 + 28*a^2*c)*d^2*e^5 + (44*c^3*d^4*e^3 -
 88*b*c^2*d^3*e^4 + (59*b^2*c + 28*a*c^2)*d^2*e^5 - (15*b^3 + 28*a*b*c)*d*e
^6 + (15*a*b^2 - 16*a^2*c)*e^7)*x^2 + 2*(54*c^3*d^5*e^2 - 113*b*c^2*d^4*e^3
 - 5*a^2*b*e^7 + (79*b^2*c + 48*a*c^2)*d^3*e^4 - 2*(10*b^3 + 29*a*b*c)*d^2*
e^5 + (25*a*b^2 - 6*a^2*c)*d*e^6)*x)*sqrt(c*x^2 + b*x + a))/(c^4*d^11 - 4*b
*c^3*d^10*e - 4*a^3*b*d^4*e^7 + a^4*d^3*e^8 + 2*(3*b^2*c^2 + 2*a*c^3)*d^9*e
^2 - 4*(b^3*c + 3*a*b*c^2)*d^8*e^3 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^7*e^4
 - 4*(a*b^3 + 3*a^2*b*c)*d^6*e^5 + 2*(3*a^2*b^2 + 2*a^3*c)*d^5*e^6 + (c^4*d
^8*e^3 - 4*b*c^3*d^7*e^4 - 4*a^3*b*d*e^10 + a^4*e^11 + 2*(3*b^2*c^2 + 2*a*c
```

```
^3)*d^6*e^5 - 4*(b^3*c + 3*a*b*c^2)*d^5*e^6 + (b^4 + 12*a*b^2*c + 6*a^2*c^2
)*d^4*e^7 - 4*(a*b^3 + 3*a^2*b*c)*d^3*e^8 + 2*(3*a^2*b^2 + 2*a^3*c)*d^2*e^9
)*x^3 + 3*(c^4*d^9*e^2 - 4*b*c^3*d^8*e^3 - 4*a^3*b*d^2*e^9 + a^4*d*e^10 + 2
*(3*b^2*c^2 + 2*a*c^3)*d^7*e^4 - 4*(b^3*c + 3*a*b*c^2)*d^6*e^5 + (b^4 + 12*
a*b^2*c + 6*a^2*c^2)*d^5*e^6 - 4*(a*b^3 + 3*a^2*b*c)*d^4*e^7 + 2*(3*a^2*b^2
 + 2*a^3*c)*d^3*e^8)*x^2 + 3*(c^4*d^10*e - 4*b*c^3*d^9*e^2 - 4*a^3*b*d^3*e^
8 + a^4*d^2*e^9 + 2*(3*b^2*c^2 + 2*a*c^3)*d^8*e^3 - 4*(b^3*c + 3*a*b*c^2)*d
^7*e^4 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^6*e^5 - 4*(a*b^3 + 3*a^2*b*c)*d^5
*e^6 + 2*(3*a^2*b^2 + 2*a^3*c)*d^4*e^7)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)^4\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**4/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x)**4*sqrt(a + b*x + c*x**2)), x)
```

Giac [B] time = 1.33898, size = 2807, normalized size = 9.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(16*c^3*d^3 - 24*b*c^2*d^2*e + 18*b^2*c*d*e^2 - 24*a*c^2*d*e^2 - 5*b^3*
e^3 + 12*a*b*c*e^3)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c
)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/((c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*
e^2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3
*a^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6)*sqrt(-c*d^2 + b*d*e - a*e^2)) - 1
/24*(240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*c^(7/2)*d^4*e + 352*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^3*c^4*d^5 + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^5*c^3*d^3*e^2 - 400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c^3*d^4*e +
 528*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(7/2)*d^5 - 360*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^4*b*c^(5/2)*d^3*e^2 - 756*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^2*b^2*c^(5/2)*d^4*e - 816*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*
a*c^(7/2)*d^4*e + 264*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c^3*d^5 - 72*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^2*d^2*e^3 + 204*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^3*b^2*c^2*d^3*e^2 - 656*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^3*a*c^3*d^3*e^2 - 336*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*c^2*d^4*e
 - 816*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c^3*d^4*e + 44*b^3*c^(5/2)*d
^5 + 270*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*c^(3/2)*d^2*e^3 - 360*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*c^(5/2)*d^2*e^3 + 498*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^2*b^3*c^(3/2)*d^3*e^2 + 648*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^2*a*b*c^(5/2)*d^3*e^2 - 44*b^4*c^(3/2)*d^4*e - 204*a*b^2*c^(5/2)*
d^4*e + 54*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c*d*e^4 - 72*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^5*a*c^2*d*e^4 + 34*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^3*b^3*c*d^2*e^3 + 264*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*c^2*
d^2*e^3 + 180*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^4*c*d^3*e^2 + 900*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*c^2*d^3*e^2 + 480*(sqrt(c)*x - sqrt(c*
```

x^2 + b*x + a))*a^2*c^3*d^3*e^2 - 75*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*
b^3*sqrt(c)*d*e^4 + 180*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b*c^(3/2)*d
e^4 - 120(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^4*sqrt(c)*d^2*e^3 - 432*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^2*c^(3/2)*d^2*e^3 + 288*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^2*a^2*c^(5/2)*d^2*e^3 + 15*b^5*sqrt(c)*d^3*e^2 +
206*a*b^3*c^(3/2)*d^3*e^2 + 240*a^2*b*c^(5/2)*d^3*e^2 - 15*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^5*b^3*e^5 + 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a
*b*c*e^5 - 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^4*d*e^4 - 48*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^3*a*b^2*c*d*e^4 + 192*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^3*a^2*c^2*d*e^4 - 33*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^5*d^2
e^3 - 450(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^3*c*d^2*e^3 - 432*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*c^2*d^2*e^3 + 120*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^2*a*b^3*sqrt(c)*d*e^4 - 78*a*b^4*sqrt(c)*d^2*e^3 - 222*a^2*b
^2*c^(3/2)*d^2*e^3 - 88*a^3*c^(5/2)*d^2*e^3 + 40*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^3*a*b^3*e^5 - 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c*e^
5 + 66*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^4*d*e^4 + 306*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*a^2*b^2*c*d*e^4 - 120*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))*a^3*c^2*d*e^4 - 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*c^(3/2)*e
^5 + 111*a^2*b^3*sqrt(c)*d*e^4 + 28*a^3*b*c^(3/2)*d*e^4 - 33*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*a^2*b^3*e^5 - 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
a^3*b*c*e^5 - 48*a^3*b^2*sqrt(c)*e^5 + 32*a^4*c^(3/2)*e^5)/((c^3*d^6 - 3*b*
c^2*d^5*e + 3*b^2*c*d^4*e^2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a*b*c*d^3*e
^3 + 3*a*b^2*d^2*e^4 + 3*a^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6)*((sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^2*e + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqr
t(c)*d + b*d - a*e)^3)

3.2382 $\int \frac{(d+ex)^4}{(a+bx+cx^2)^{3/2}} dx$

Optimal. Leaf size=286

$$\frac{e\sqrt{a+bx+cx^2}\left(2cex\left(-4ce(3ae+2bd)+5b^2e^2+8c^2d^2\right)-8c^2de(16ae+5bd)+4bce^2(13ae+12bd)-15b^3e^3+32c^3d^3\right)}{4c^3\left(b^2-4ac\right)}+$$

```
[Out] (-2*(d + e*x)^3*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*
x + c*x^2]) + (2*e*(2*c*d - b*e)*(d + e*x)^2*Sqrt[a + b*x + c*x^2])/(c*(b^2
 - 4*a*c)) + (e*(32*c^3*d^3 - 15*b^3*e^3 + 4*b*c*e^2*(12*b*d + 13*a*e) - 8*
c^2*d*e*(5*b*d + 16*a*e) + 2*c*e*(8*c^2*d^2 + 5*b^2*e^2 - 4*c*e*(2*b*d + 3*
a*e))*x)*Sqrt[a + b*x + c*x^2])/(4*c^3*(b^2 - 4*a*c)) + (3*e^2*(16*c^2*d^2
+ 5*b^2*e^2 - 4*c*e*(4*b*d + a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
b*x + c*x^2])])/(8*c^(7/2))
```

Rubi [A] time = 0.317787, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {738, 832, 779, 621, 206}

$$\frac{e\sqrt{a+bx+cx^2}\left(2cex\left(-4ce(3ae+2bd)+5b^2e^2+8c^2d^2\right)-8c^2de(16ae+5bd)+4bce^2(13ae+12bd)-15b^3e^3+32c^3d^3\right)}{4c^3\left(b^2-4ac\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a + b*x + c*x^2)^(3/2),x]
```


```
[Out] (-2*(d + e*x)^3*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*
x + c*x^2]) + (2*e*(2*c*d - b*e)*(d + e*x)^2*Sqrt[a + b*x + c*x^2])/(c*(b^2
 - 4*a*c)) + (e*(32*c^3*d^3 - 15*b^3*e^3 + 4*b*c*e^2*(12*b*d + 13*a*e) - 8*
c^2*d*e*(5*b*d + 16*a*e) + 2*c*e*(8*c^2*d^2 + 5*b^2*e^2 - 4*c*e*(2*b*d + 3*
a*e))*x)*Sqrt[a + b*x + c*x^2])/(4*c^3*(b^2 - 4*a*c)) + (3*e^2*(16*c^2*d^2
+ 5*b^2*e^2 - 4*c*e*(4*b*d + a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
b*x + c*x^2])])/(8*c^(7/2))
```


Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^4}{\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2(d+ex)^3(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} - \frac{2\int \frac{(d+ex)^2(-3e(bd-2ae)-3e(2cd-be)x)}{\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac}$$

$$= -\frac{2(d+ex)^3(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{2e(2cd-be)(d+ex)^2\sqrt{a+bx+cx^2}}{c\left(b^2-4ac\right)} - \frac{2\int \frac{(d+ex)\left(-\frac{3}{2}e(\right.}{}}{}$$

$$= -\frac{2(d+ex)^3(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{2e(2cd-be)(d+ex)^2\sqrt{a+bx+cx^2}}{c\left(b^2-4ac\right)} + \frac{e\left(32c^3d^3-15\right.}{}$$

$$= -\frac{2(d+ex)^3(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{2e(2cd-be)(d+ex)^2\sqrt{a+bx+cx^2}}{c\left(b^2-4ac\right)} + \frac{e\left(32c^3d^3-15\right.}{}$$

$$= -\frac{2(d+ex)^3(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{2e(2cd-be)(d+ex)^2\sqrt{a+bx+cx^2}}{c\left(b^2-4ac\right)} + \frac{e\left(32c^3d^3-15\right.}{}$$

Mathematica [A] time = 0.451202, size = 316, normalized size = 1.1

$$2\sqrt{c}\left(4bc\left(-13a^2e^4+ace^2\left(12d^2+40dex-5e^2x^2\right)+2c^2d^3(d-4ex)\right)+8c^2\left(a^2e^3(16d+3ex)+ace\left(-12d^2ex-8d^3+8a\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[c]*(15*b^4*e^4*x + b^3*e^3*(15*a*e + c*x*(-48*d + 5*e*x)) + 4*b*c*(
-13*a^2*e^4 + 2*c^2*d^3*(d - 4*e*x) + a*c*e^2*(12*d^2 + 40*d*e*x - 5*e^2*x^
```

```
2)) - 2*b^2*c*e^2*(a*e*(24*d + 31*e*x) + c*x*(-24*d^2 + 8*d*e*x + e^2*x^2))
 + 8*c^2*(2*c^2*d^4*x + a^2*e^3*(16*d + 3*e*x) + a*c*e*(-8*d^3 - 12*d^2*e*x
 + 8*d*e^2*x^2 + e^3*x^3))) - 3*(b^2 - 4*a*c)*e^2*(16*c^2*d^2 + 5*b^2*e^2 -
 4*c*e*(4*b*d + a*e))*Sqrt[a + x*(b + c*x)]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + x*(b + c*x)])])/(8*c^(7/2)*(-b^2 + 4*a*c)*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.054, size = 913, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 3*d^2*e^2*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-8*d^3*e*b/(4*a*c-b^2)/(c*
x^2+b*x+a)^(1/2)*x-4*d^3*e*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+16*d*e^3*a
/c*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-4*d^3*e/c/(c*x^2+b*x+a)^(1/2)+3/2*e^
4*a/c^2*x/(c*x^2+b*x+a)^(1/2)+15/16*e^4*b^5/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(
1/2)-13/4*e^4*b/c^3*a/(c*x^2+b*x+a)^(1/2)-6*d^2*e^2*x/c/(c*x^2+b*x+a)^(1/2)
+3*d^2*e^2*b/c^2/(c*x^2+b*x+a)^(1/2)-6*d*e^3*b/c^(5/2)*ln((1/2*b+c*x)/c^(1/
2)+(c*x^2+b*x+a)^(1/2))+4*d*e^3*x^2/c/(c*x^2+b*x+a)^(1/2)-3*d*e^3*b^2/c^3/(
c*x^2+b*x+a)^(1/2)+8*d*e^3*a/c^2/(c*x^2+b*x+a)^(1/2)-5/4*e^4*b/c^2*x^2/(c*x
^2+b*x+a)^(1/2)-15/8*e^4*b^2/c^3*x/(c*x^2+b*x+a)^(1/2)-13/2*e^4*b^2/c^2*a/(
4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-6*d*e^3*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^
(1/2)*x+8*d*e^3*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+6*d^2*e^2*b^2/c/(
4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+2*d^4*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^
(1/2)+1/2*e^4*x^3/c/(c*x^2+b*x+a)^(1/2)+15/16*e^4*b^3/c^4/(c*x^2+b*x+a)^(1/
2)+15/8*e^4*b^2/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-3/2*e^4
*a/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+6*d^2*e^2/c^(3/2)*ln
((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+15/8*e^4*b^4/c^3/(4*a*c-b^2)/(c*x
^2+b*x+a)^(1/2)*x-13/4*e^4*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+6*d*e^
3*b/c^2*x/(c*x^2+b*x+a)^(1/2)-3*d*e^3*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/
2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.10687, size = 2488, normalized size = 8.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*(16*(a*b^2*c^2 - 4*a^2*c^3)*d^2*e^2 - 16*(a*b^3*c - 4*a^2*b*c^2)*
d*e^3 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*e^4 + (16*(b^2*c^3 - 4*a*c^4)
*d^2*e^2 - 16*(b^3*c^2 - 4*a*b*c^3)*d*e^3 + (5*b^4*c - 24*a*b^2*c^2 + 16*a^
2*c^3)*e^4)*x^2 + (16*(b^3*c^2 - 4*a*b*c^3)*d^2*e^2 - 16*(b^4*c - 4*a*b^2*c
^2)*d*e^3 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*e^4)*x)*sqrt(c)*log(-8*c^2*
x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c)
+ 4*(8*b*c^4*d^4 - 64*a*c^4*d^3*e + 48*a*b*c^3*d^2*e^2 - 2*(b^2*c^3 - 4*a*c
^4)*e^4*x^3 - 16*(3*a*b^2*c^2 - 8*a^2*c^3)*d*e^3 + (15*a*b^3*c - 52*a^2*b*c
^2)*e^4 - (16*(b^2*c^3 - 4*a*c^4)*d*e^3 - 5*(b^3*c^2 - 4*a*b*c^3)*e^4)*x^2
+ (16*c^5*d^4 - 32*b*c^4*d^3*e + 48*(b^2*c^3 - 2*a*c^4)*d^2*e^2 - 16*(3*b^3
*c^2 - 10*a*b*c^3)*d*e^3 + (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*e^4)*x)*s
qrt(c*x^2 + b*x + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*c^6)*x^2 + (b
^3*c^4 - 4*a*b*c^5)*x), -1/8*(3*(16*(a*b^2*c^2 - 4*a^2*c^3)*d^2*e^2 - 16*(a
*b^3*c - 4*a^2*b*c^2)*d*e^3 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*e^4 + (
16*(b^2*c^3 - 4*a*c^4)*d^2*e^2 - 16*(b^3*c^2 - 4*a*b*c^3)*d*e^3 + (5*b^4*c
- 24*a*b^2*c^2 + 16*a^2*c^3)*e^4)*x^2 + (16*(b^3*c^2 - 4*a*b*c^3)*d^2*e^2 -
 16*(b^4*c - 4*a*b^2*c^2)*d*e^3 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*e^4)*
x)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2
+ b*c*x + a*c)) + 2*(8*b*c^4*d^4 - 64*a*c^4*d^3*e + 48*a*b*c^3*d^2*e^2 - 2*
(b^2*c^3 - 4*a*c^4)*e^4*x^3 - 16*(3*a*b^2*c^2 - 8*a^2*c^3)*d*e^3 + (15*a*b^
3*c - 52*a^2*b*c^2)*e^4 - (16*(b^2*c^3 - 4*a*c^4)*d*e^3 - 5*(b^3*c^2 - 4*a*
b*c^3)*e^4)*x^2 + (16*c^5*d^4 - 32*b*c^4*d^3*e + 48*(b^2*c^3 - 2*a*c^4)*d^2
*e^2 - 16*(3*b^3*c^2 - 10*a*b*c^3)*d*e^3 + (15*b^4*c - 62*a*b^2*c^2 + 24*a^
2*c^3)*e^4)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4
*a*c^6)*x^2 + (b^3*c^4 - 4*a*b*c^5)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^4}{\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x)**4/(a + b*x + c*x**2)**(3/2), x)
```

Giac [A] time = 1.17574, size = 509, normalized size = 1.78

$$\frac{\left(\left(\frac{2\left(b^2c^2e^4-4ac^3e^4\right)x}{b^2c^3-4ac^4}+\frac{16b^2c^2de^3-64ac^3de^3-5b^3ce^4+20abc^2e^4}{b^2c^3-4ac^4}\right)x-\frac{16c^4d^4-32bc^3d^3e+48b^2c^2d^2e^2-96ac^3d^2e^2-48b^3cde^3+160abc^2de^3+15b^4e^4-62ab}{b^2c^3-4ac^4}\right.}{4\sqrt{cx^2+bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(((2*(b^2*c^2*e^4 - 4*a*c^3*e^4)*x/(b^2*c^3 - 4*a*c^4) + (16*b^2*c^2*d*
e^3 - 64*a*c^3*d*e^3 - 5*b^3*c*e^4 + 20*a*b*c^2*e^4)/(b^2*c^3 - 4*a*c^4))*x
 - (16*c^4*d^4 - 32*b*c^3*d^3*e + 48*b^2*c^2*d^2*e^2 - 96*a*c^3*d^2*e^2 - 4
8*b^3*c*d*e^3 + 160*a*b*c^2*d*e^3 + 15*b^4*e^4 - 62*a*b^2*c*e^4 + 24*a^2*c^
2*e^4)/(b^2*c^3 - 4*a*c^4))*x - (8*b*c^3*d^4 - 64*a*c^3*d^3*e + 48*a*b*c^2*
d^2*e^2 - 48*a*b^2*c*d*e^3 + 128*a^2*c^2*d*e^3 + 15*a*b^3*e^4 - 52*a^2*b*c*
```

```
e^4)/(b^2*c^3 - 4*a*c^4))/sqrt(c*x^2 + b*x + a) - 3/8*(16*c^2*d^2*e^2 - 16*
b*c*d*e^3 + 5*b^2*e^4 - 4*a*c*e^4)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))*sqrt(c) - b))/c^(7/2)
```

3.2383 $\int \frac{(d+ex)^3}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=177

$$\frac{e\sqrt{a+bx+cx^2}\left(-2ce(4ae+3bd)+3b^2e^2+2cex(2cd-be)+8c^2d^2\right)}{c^2\left(b^2-4ac\right)}-\frac{2(d+ex)^2(-2ae+x(2cd-be)+bd)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{3e^2(2cd$$

```
[Out] (-2*(d + e*x)^2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*
x + c*x^2]) + (e*(8*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(3*b*d + 4*a*e) + 2*c*e*(2*
c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(c^2*(b^2 - 4*a*c)) + (3*e^2*(2*c*d -
b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(5/2))
```

Rubi [A] time = 0.177996, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {738, 779, 621, 206}

$$\frac{e\sqrt{a+bx+cx^2}\left(-2ce(4ae+3bd)+3b^2e^2+2cex(2cd-be)+8c^2d^2\right)}{c^2\left(b^2-4ac\right)}-\frac{2(d+ex)^2(-2ae+x(2cd-be)+bd)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{3e^2(2cd$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + b*x + c*x^2)^(3/2),x]
```


```
[Out] (-2*(d + e*x)^2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*
x + c*x^2]) + (e*(8*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(3*b*d + 4*a*e) + 2*c*e*(2*
c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(c^2*(b^2 - 4*a*c)) + (3*e^2*(2*c*d -
b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(5/2))
```


Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2(d+ex)^2(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} - \frac{2\int \frac{(d+ex)(-2e(bd-2ae)-2e(2cd-be)x)}{\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac}$$

$$= -\frac{2(d+ex)^2(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{e\left(8c^2d^2+3b^2e^2-2ce(3bd+4ae)+2ce(2cd-be)x\right)\sqrt{a}}{c^2\left(b^2-4ac\right)}$$

$$= -\frac{2(d+ex)^2(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{e\left(8c^2d^2+3b^2e^2-2ce(3bd+4ae)+2ce(2cd-be)x\right)\sqrt{a}}{c^2\left(b^2-4ac\right)}$$

$$= -\frac{2(d+ex)^2(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{e\left(8c^2d^2+3b^2e^2-2ce(3bd+4ae)+2ce(2cd-be)x\right)\sqrt{a}}{c^2\left(b^2-4ac\right)}$$

Mathematica [A] time = 0.418983, size = 196, normalized size = 1.11

$$\frac{\frac{2\sqrt{c}\left(4c\left(2a^2e^3+ace\left(-3d^2-3dex+e^2x^2\right)+c^2d^3x\right)-b^2e^2(3ae+cx(ex-6d))+2bc\left(ae^2(3d+5ex)+cd^2(d-3ex)\right)-3b^3e^3x\right)}{\sqrt{a+x(b+cx)}}+3e^2\left(b^2-4ac\right)(be-2cd)\tanh^{-1}\left(\frac{}{2\sqrt{}}\right.}{2c^{5/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(-3*b^3*e^3*x - b^2*e^2*(3*a*e + c*x*(-6*d + e*x)) + 2*b*c*(c*d
^2*(d - 3*e*x) + a*e^2*(3*d + 5*e*x)) + 4*c*(2*a^2*e^3 + c^2*d^3*x + a*c*e*
(-3*d^2 - 3*d*e*x + e^2*x^2))))/Sqrt[a + x*(b + c*x)] + 3*(b^2 - 4*a*c)*e^2
*(-2*c*d + b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(2*
c^(5/2)*(-b^2 + 4*a*c))
```

Maple [B] time = 0.05, size = 541, normalized size = 3.1

$$\frac{e^3x^2}{c}\frac{1}{\sqrt{cx^2+bx+a}}+\frac{3\,be^3x}{2\,c^2}\frac{1}{\sqrt{cx^2+bx+a}}-\frac{3\,e^3b^2}{4\,c^3}\frac{1}{\sqrt{cx^2+bx+a}}-\frac{3\,b^3e^3x}{2\,c^2\left(4\,ac-b^2\right)}\frac{1}{\sqrt{cx^2+bx+a}}-\frac{3\,e^3b^4}{4\,c^3\left(4\,ac-b^2\right)\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] e^3*x^2/c/(c*x^2+b*x+a)^(1/2)+3/2*e^3*b/c^2*x/(c*x^2+b*x+a)^(1/2)-3/4*e^3*b
^2/c^3/(c*x^2+b*x+a)^(1/2)-3/2*e^3*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*
x-3/4*e^3*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-3/2*e^3*b/c^(5/2)*ln((1/2
*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+2*e^3*a/c^2/(c*x^2+b*x+a)^(1/2)+4*e^3*
```

```
a/c*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+2*e^3*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+
b*x+a)^(1/2)-3*d*e^2*x/c/(c*x^2+b*x+a)^(1/2)+3/2*d*e^2*b/c^2/(c*x^2+b*x+a)^
(1/2)+3*d*e^2*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+3/2*d*e^2*b^3/c^2/(4*
a*c-b^2)/(c*x^2+b*x+a)^(1/2)+3*d*e^2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))-3*d^2*e/c/(c*x^2+b*x+a)^(1/2)-6*d^2*e*b/(4*a*c-b^2)/(c*x^2+b*
x+a)^(1/2)*x-3*d^2*e*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+2*d^3*(2*c*x+b)/
(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.84532, size = 1574, normalized size = 8.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(3*(2*(a*b^2*c - 4*a^2*c^2)*d*e^2 - (a*b^3 - 4*a^2*b*c)*e^3 + (2*(b^2
*c^2 - 4*a*c^3)*d*e^2 - (b^3*c - 4*a*b*c^2)*e^3)*x^2 + (2*(b^3*c - 4*a*b*c^
2)*d*e^2 - (b^4 - 4*a*b^2*c)*e^3)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2
 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(2*b*c^3*d^3 -
12*a*c^3*d^2*e + 6*a*b*c^2*d*e^2 - (b^2*c^2 - 4*a*c^3)*e^3*x^2 - (3*a*b^2*c
 - 8*a^2*c^2)*e^3 + (4*c^4*d^3 - 6*b*c^3*d^2*e + 6*(b^2*c^2 - 2*a*c^3)*d*e^
2 - (3*b^3*c - 10*a*b*c^2)*e^3)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^3 - 4*a^
2*c^4 + (b^2*c^4 - 4*a*c^5)*x^2 + (b^3*c^3 - 4*a*b*c^4)*x), -1/2*(3*(2*(a*b
^2*c - 4*a^2*c^2)*d*e^2 - (a*b^3 - 4*a^2*b*c)*e^3 + (2*(b^2*c^2 - 4*a*c^3)*
d*e^2 - (b^3*c - 4*a*b*c^2)*e^3)*x^2 + (2*(b^3*c - 4*a*b*c^2)*d*e^2 - (b^4
- 4*a*b^2*c)*e^3)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*
sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(2*b*c^3*d^3 - 12*a*c^3*d^2*e + 6*a*b
*c^2*d*e^2 - (b^2*c^2 - 4*a*c^3)*e^3*x^2 - (3*a*b^2*c - 8*a^2*c^2)*e^3 + (4
*c^4*d^3 - 6*b*c^3*d^2*e + 6*(b^2*c^2 - 2*a*c^3)*d*e^2 - (3*b^3*c - 10*a*b*
c^2)*e^3)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a
*c^5)*x^2 + (b^3*c^3 - 4*a*b*c^4)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^3}{\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x)**3/(a + b*x + c*x**2)**(3/2), x)
```

Giac [A] time = 1.14462, size = 315, normalized size = 1.78

$$\frac{\left(\frac{\left(b^2ce^3-4\,ac^2e^3\right)x}{b^2c^2-4\,ac^3}-\frac{4\,c^3d^3-6\,bc^2d^2e+6\,b^2cde^2-12\,ac^2de^2-3\,b^3e^3+10\,abce^3}{b^2c^2-4\,ac^3}\right)x-\frac{2\,bc^2d^3-12\,ac^2d^2e+6\,abcde^2-3\,ab^2e^3+8\,a^2ce^3}{b^2c^2-4\,ac^3}}{\sqrt{cx^2+bx+a}}-\frac{3\left(2\,cde^2-be^3\right)\log\left(\left|\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] (((b^2*c*e^3 - 4*a*c^2*e^3)*x/(b^2*c^2 - 4*a*c^3) - (4*c^3*d^3 - 6*b*c^2*d^
2*e + 6*b^2*c*d*e^2 - 12*a*c^2*d*e^2 - 3*b^3*e^3 + 10*a*b*c*e^3)/(b^2*c^2 -
 4*a*c^3))*x - (2*b*c^2*d^3 - 12*a*c^2*d^2*e + 6*a*b*c*d*e^2 - 3*a*b^2*e^3
+ 8*a^2*c*e^3)/(b^2*c^2 - 4*a*c^3))/sqrt(c*x^2 + b*x + a) - 3/2*(2*c*d*e^2
- b*e^3)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(5/
2)
```

3.2384 $\int \frac{(d+ex)^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=129

$$\frac{2e\sqrt{a+bx+cx^2}(2cd-be)}{c\left(b^2-4ac\right)}-\frac{2(d+ex)(-2ae+x(2cd-be)+bd)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{e^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

```
[Out] (-2*(d + e*x)*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2]) + (2*e*(2*c*d - b*e)*Sqrt[a + b*x + c*x^2])/(c*(b^2 - 4*a*c)) + (
e^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/c^(3/2)
```

Rubi [A] time = 0.0796547, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {738, 640, 621, 206}

$$\frac{2e\sqrt{a+bx+cx^2}(2cd-be)}{c\left(b^2-4ac\right)}-\frac{2(d+ex)(-2ae+x(2cd-be)+bd)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{e^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*(d + e*x)*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2]) + (2*e*(2*c*d - b*e)*Sqrt[a + b*x + c*x^2])/(c*(b^2 - 4*a*c)) + (
e^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/c^(3/2)
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2(d+ex)(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} - \frac{2\int \frac{-e(bd-2ae)-e(2cd-be)x}{\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac}$$

$$= -\frac{2(d+ex)(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{2e(2cd-be)\sqrt{a+bx+cx^2}}{c\left(b^2-4ac\right)} + \frac{e^2\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{c}$$

$$= -\frac{2(d+ex)(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{2e(2cd-be)\sqrt{a+bx+cx^2}}{c\left(b^2-4ac\right)} + \frac{\left(2e^2\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x,\right.}{c}$$

$$= -\frac{2(d+ex)(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{2e(2cd-be)\sqrt{a+bx+cx^2}}{c\left(b^2-4ac\right)} + \frac{e^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

Mathematica [A] time = 0.25025, size = 127, normalized size = 0.98

$$\frac{\frac{2\sqrt{c}\left(abe^2-2ace(2d+ex)+b^2e^2x+bcd(d-2ex)+2c^2d^2x\right)}{\sqrt{a+x(b+cx)}} - e^2\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{3/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(a*b*e^2 + 2*c^2*d^2*x + b^2*e^2*x + b*c*d*(d - 2*e*x) - 2*a*c*
e*(2*d + e*x)))/Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*e^2*ArcTanh[(b + 2*c*
x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(c^(3/2)*(-b^2 + 4*a*c))
```

Maple [B] time = 0.047, size = 264, normalized size = 2.1

$$-\frac{e^2x}{c}\frac{1}{\sqrt{cx^2+bx+a}} + \frac{e^2b}{2c^2}\frac{1}{\sqrt{cx^2+bx+a}} + \frac{b^2e^2x}{c\left(4ac-b^2\right)}\frac{1}{\sqrt{cx^2+bx+a}} + \frac{b^3e^2}{2c^2\left(4ac-b^2\right)}\frac{1}{\sqrt{cx^2+bx+a}} + e^2\ln\left(\left(\frac{b}{2}+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -e^2*x/c/(c*x^2+b*x+a)^(1/2)+1/2*e^2*b/c^2/(c*x^2+b*x+a)^(1/2)+e^2*b^2/c/(4
*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+1/2*e^2*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(
1/2)+e^2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-2*d*e/c/(c*x^2
+b*x+a)^(1/2)-4*d*e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-2*d*e*b^2/c/(4*a*c-
b^2)/(c*x^2+b*x+a)^(1/2)+2*d^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.52869, size = 995, normalized size = 7.71

$$\left[\frac{\left(\left(b^2c-4\,ac^2\right)e^2x^2+\left(b^3-4\,abc\right)e^2x+\left(ab^2-4\,a^2c\right)e^2\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-\right.}{2\left(ab^2c^2-4\,a^2c^3+\left(b^2c^3-4\,ac^4\right)x^2+\left(b^3c\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^2*c - 4*a*c^2)*e^2*x^2 + (b^3 - 4*a*b*c)*e^2*x + (a*b^2 - 4*a^2*c
)*e^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*
c*x + b)*sqrt(c) - 4*a*c) - 4*(b*c^2*d^2 - 4*a*c^2*d*e + a*b*c*e^2 + (2*c^3
*d^2 - 2*b*c^2*d*e + (b^2*c - 2*a*c^2)*e^2)*x)*sqrt(c*x^2 + b*x + a))/(a*b^
2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x), -((
(b^2*c - 4*a*c^2)*e^2*x^2 + (b^3 - 4*a*b*c)*e^2*x + (a*b^2 - 4*a^2*c)*e^2)*
sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b
*c*x + a*c)) + 2*(b*c^2*d^2 - 4*a*c^2*d*e + a*b*c*e^2 + (2*c^3*d^2 - 2*b*c^
2*d*e + (b^2*c - 2*a*c^2)*e^2)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2
*c^3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^2}{\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x)**2/(a + b*x + c*x**2)**(3/2), x)
```

Giac [A] time = 1.15416, size = 178, normalized size = 1.38

$$-\frac{2\left(\frac{\left(2\,c^2d^2-2\,bcde+b^2e^2-2\,ace^2\right)x}{b^2c-4\,ac^2}+\frac{bcd^2-4\,acde+abe^2}{b^2c-4\,ac^2}\right)}{\sqrt{cx^2+bx+a}}-\frac{e^2\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^2*d^2 - 2*b*c*d*e + b^2*e^2 - 2*a*c*e^2)*x/(b^2*c - 4*a*c^2) + (b*
c*d^2 - 4*a*c*d*e + a*b*e^2)/(b^2*c - 4*a*c^2))/sqrt(c*x^2 + b*x + a) - e^2
*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(3/2)
```

3.2385 $\int \frac{d+ex}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=45

$$-\frac{2(-2ae+x(2cd-be)+bd)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.0110652, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {636}

$$-\frac{2(-2ae+x(2cd-be)+bd)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{d+ex}{\left(a+bx+cx^2\right)^{3/2}}\,dx=-\frac{2(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

Mathematica [A] time = 0.198597, size = 43, normalized size = 0.96

$$\frac{4ae-2bd+2bex-4cdx}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*b*d + 4*a*e - 4*c*d*x + 2*b*e*x)/((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)])
```

Maple [A] time = 0.042, size = 45, normalized size = 1.

$$-2\,\frac{bxe-2\,cdx+2\,ae-bd}{\sqrt{cx^2+bx+a}\left(4\,ac-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -2/(c*x^2+b*x+a)^(1/2)*(b*e*x-2*c*d*x+2*a*e-b*d)/(4*a*c-b^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.44906, size = 162, normalized size = 3.6

$$-\frac{2\sqrt{cx^2+bx+a}(bd-2\,ae+(2\,cd-be)x)}{ab^2-4\,a^2c+\left(b^2c-4\,ac^2\right)x^2+\left(b^3-4\,abc\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*b^2 - 4*a^2*c +
 (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{d+ex}{\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x)/(a + b*x + c*x**2)**(3/2), x)
```

Giac [A] time = 1.11703, size = 77, normalized size = 1.71

$$-\frac{2\left(\frac{(2\,cd-be)x}{b^2-4\,ac}+\frac{bd-2\,ae}{b^2-4\,ac}\right)}{\sqrt{cx^2+bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c*d - b*e)*x/(b^2 - 4*a*c) + (b*d - 2*a*e)/(b^2 - 4*a*c))/sqrt(c*x^2
 + b*x + a)
```

3.2386 $\int \frac{1}{(a+bx+cx^2)^{3/2}} \, dx$

Optimal. Leaf size=32

$$-\frac{2(b+2cx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.0042317, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {613}

$$-\frac{2(b+2cx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(-3/2),x]
```

```
[Out] (-2*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx+cx^2)^{3/2}} \, dx = -\frac{2(b+2cx)}{(b^2-4ac)\sqrt{a+bx+cx^2}}$$

Mathematica [A] time = 0.0172564, size = 31, normalized size = 0.97

$$-\frac{2(b+2cx)}{(b^2-4ac)\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(-3/2),x]
```

```
[Out] (-2*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)])
```

Maple [A] time = 0.043, size = 33, normalized size = 1.

$$2\,\frac{2\,cx+b}{\sqrt{cx^2+bx+a}\,(4\,ac-b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)/(4*a*c-b^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.76706, size = 135, normalized size = 4.22

$$-\frac{2\sqrt{cx^2+bx+a}(2\,cx+b)}{ab^2-4\,a^2c+\left(b^2c-4\,ac^2\right)x^2+\left(b^3-4\,abc\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)/(a*b^2 - 4*a^2*c + (b^2*c - 4*a*c^2)*x
^2 + (b^3 - 4*a*b*c)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(-3/2), x)
```

Giac [A] time = 1.13739, size = 55, normalized size = 1.72

$$-\frac{2\left(\frac{2\,cx}{b^2-4\,ac}+\frac{b}{b^2-4\,ac}\right)}{\sqrt{cx^2+bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(2*c*x/(b^2 - 4*a*c) + b/(b^2 - 4*a*c))/sqrt(c*x^2 + b*x + a)
```

3.2387 $\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=155

$$\frac{e^2\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{2\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

```
[Out] (-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2 -
b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) + (e^2*ArcTanh[(b*d - 2*a*e + (2*c*d
- b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(c*d^2 -
b*d*e + a*e^2)^(3/2)
```

Rubi [A] time = 0.100444, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {740, 12, 724, 206}

$$\frac{e^2\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{2\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2 -
b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) + (e^2*ArcTanh[(b*d - 2*a*e + (2*c*d
- b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(c*d^2 -
b*d*e + a*e^2)^(3/2)
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} - \frac{2\int -\frac{\left(b^2-4ac\right)e^2}{2(d+ex)\sqrt{a+bx+cx^2}}\,dx}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} + \frac{e^2\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} - \frac{\left(2e^2\right)\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, -\right.}{cd^2-bde+ae^2}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} + \frac{e^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\left(cd^2-bde+ae^2\right)^{3/2}}$$

Mathematica [A] time = 0.30375, size = 162, normalized size = 1.05

$$\frac{\frac{e^2\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{\left(e(ae-bd)+cd^2\right)^{3/2}} + \frac{4c(ae+cdx)-2b^2e+2bc(d-ex)}{\sqrt{a+x(b+cx)}\left(e(ae-bd)+cd^2\right)}}{4ac-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] ((-2*b^2*e + 4*c*(a*e + c*d*x) + 2*b*c*(d - e*x))/((c*d^2 + e*(-(b*d) + a*e
))*Sqrt[a + x*(b + c*x)]) + ((b^2 - 4*a*c)*e^2*ArcTanh[(-(b*d) + 2*a*e - 2*
c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/(
c*d^2 + e*(-(b*d) + a*e))^(3/2))/(-b^2 + 4*a*c)
```

Maple [B] time = 0.229, size = 603, normalized size = 3.9

$$\frac{e}{ae^2-bde+cd^2}\frac{1}{\sqrt{\left(\frac{d}{e}+x\right)^2c+\frac{be-2\,cd}{e}\left(\frac{d}{e}+x\right)+\frac{ae^2-bde+cd^2}{e^2}}} - 2\frac{bxec}{\left(ae^2-bde+cd^2\right)\left(4\,ac-b^2\right)}\frac{1}{\sqrt{\left(\frac{d}{e}+x\right)^2c+\frac{be-2\,cd}{e}\left(\frac{d}{e}+x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] e/(a*e^2-b*d*e+c*d^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)-2*e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e
*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c+4/(a*e^2-b*d*e+c*d^2)/(4*a*c-
b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^
2*d-e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2+2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)
```

```
^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d-e/(a*e^2-b*
d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b
*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c
*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 9.39779, size = 2768, normalized size = 17.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^2*c - 4*a*c^2)*e^2*x^2 + (b^3 - 4*a*b*c)*e^2*x + (a*b^2 - 4*a^2*c
)*e^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*
c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b
*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*
b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) -
4*(b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e + (b^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a
^2*c)*e^3 + (2*c^3*d^3 - 3*b*c^2*d^2*e - a*b*c*e^3 + (b^2*c + 2*a*c^2)*d*e^
2)*x)*sqrt(c*x^2 + b*x + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*
a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 -
 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*
(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2
*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^2 + ((b^3*c
^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*
a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*
e^4)*x), (((b^2*c - 4*a*c^2)*e^2*x^2 + (b^3 - 4*a*b*c)*e^2*x + (a*b^2 - 4*a
^2*c)*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a
*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*
d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*
b*e^2)*x)) - 2*(b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e + (b^3 - a*b*c)*d*e^2 -
 (a*b^2 - 2*a^2*c)*e^3 + (2*c^3*d^3 - 3*b*c^2*d^2*e - a*b*c*e^3 + (b^2*c +
2*a*c^2)*d*e^2)*x)*sqrt(c*x^2 + b*x + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*
(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 -
 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*
c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3
)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*
x^2 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2
*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3
- 4*a^3*b*c)*e^4)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral(1/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

Giac [B] time = 1.12036, size = 603, normalized size = 3.89

$$-\frac{2\left(\frac{\left(2c^3d^3-3bc^2d^2e+b^2cde^2+2ac^2de^2-abce^3\right)x}{b^2c^2d^4-4ac^3d^4-2b^3cd^3e+8abc^2d^3e+b^4d^2e^2-2ab^2cd^2e^2-8a^2c^2d^2e^2-2ab^3de^3+8a^2bcde^3+a^2b^2e^4-4a^3ce^4}+\frac{bc^2d^3-2b^2cd^2e+2ac^2}{b^2c^2d^4-4ac^3d^4-2b^3cd^3e+8abc^2d^3e+b^4d^2e^2-2}\right.}{\sqrt{cx^2+bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^3*d^3 - 3*b*c^2*d^2*e + b^2*c*d*e^2 + 2*a*c^2*d*e^2 - a*b*c*e^3)*x
/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2
 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3
+ a^2*b^2*e^4 - 4*a^3*c*e^4) + (b*c^2*d^3 - 2*b^2*c*d^2*e + 2*a*c^2*d^2*e +
 b^3*d*e^2 - a*b*c*d*e^2 - a*b^2*e^3 + 2*a^2*c*e^3)/(b^2*c^2*d^4 - 4*a*c^3*
d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8
*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*
e^4))/sqrt(c*x^2 + b*x + a) + 2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))*e^2/((c*d^2 - b*d*e + a*e^2)
*sqrt(-c*d^2 + b*d*e - a*e^2))
```

3.2388 $\int \frac{1}{(d+ex)^2\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=255

$$-\frac{e\sqrt{a+bx+cx^2}\left(-4ce(2ae+bd)+3b^2e^2+4c^2d^2\right)}{\left(b^2-4ac\right)(d+ex)\left(ae^2-bde+cd^2\right)^2}-\frac{2\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{\left(b^2-4ac\right)(d+ex)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}+\frac{3e^2(2cd-$$

```
[Out] (-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2 -
b*d*e + a*e^2)*(d + e*x)*Sqrt[a + b*x + c*x^2]) - (e*(4*c^2*d^2 + 3*b^2*e^2
 - 4*c*e*(b*d + 2*a*e))*Sqrt[a + b*x + c*x^2])/((b^2 - 4*a*c)*(c*d^2 - b*d*
e + a*e^2)^2*(d + e*x)) + (3*e^2*(2*c*d - b*e)*ArcTanh[(b*d - 2*a*e + (2*c*
d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(2*(c*d
^2 - b*d*e + a*e^2)^(5/2))
```

Rubi [A] time = 0.275376, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {740, 806, 724, 206}

$$-\frac{e\sqrt{a+bx+cx^2}\left(-4ce(2ae+bd)+3b^2e^2+4c^2d^2\right)}{\left(b^2-4ac\right)(d+ex)\left(ae^2-bde+cd^2\right)^2}-\frac{2\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{\left(b^2-4ac\right)(d+ex)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}+\frac{3e^2(2cd-$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2 -
b*d*e + a*e^2)*(d + e*x)*Sqrt[a + b*x + c*x^2]) - (e*(4*c^2*d^2 + 3*b^2*e^2
 - 4*c*e*(b*d + 2*a*e))*Sqrt[a + b*x + c*x^2])/((b^2 - 4*a*c)*(c*d^2 - b*d*
e + a*e^2)^2*(d + e*x)) + (3*e^2*(2*c*d - b*e)*ArcTanh[(b*d - 2*a*e + (2*c*
d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(2*(c*d
^2 - b*d*e + a*e^2)^(5/2))
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)\sqrt{a+bx+cx^2}} - \frac{2\int \frac{\frac{1}{2}e\left(2bcd-3b^2e+8ace\right)+ce(2cd-be)x}{(d+ex)^2\sqrt{a+bx+cx^2}}}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)\sqrt{a+bx+cx^2}} - \frac{e\left(4c^2d^2+3b^2e^2-4ce(bd+2a\right.}{\left(b^2-4ac\right)\left(cd^2-bde+a\right.}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)\sqrt{a+bx+cx^2}} - \frac{e\left(4c^2d^2+3b^2e^2-4ce(bd+2a\right.}{\left(b^2-4ac\right)\left(cd^2-bde+a\right.}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)\sqrt{a+bx+cx^2}} - \frac{e\left(4c^2d^2+3b^2e^2-4ce(bd+2a\right.}{\left(b^2-4ac\right)\left(cd^2-bde+a\right.}$$

Mathematica [A] time = 0.406901, size = 250, normalized size = 0.98

$$\frac{2\left(-\frac{e\sqrt{a+x(b+cx)}\left(-4ce(2ae+bd)+3b^2e^2+4c^2d^2\right)}{2(d+ex)\left(e(ae-bd)+cd^2\right)}+\frac{3e^2\left(b^2-4ac\right)(be-2cd)\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{4\left(e(ae-bd)+cd^2\right)^{3/2}}+\frac{-2c(ae+cdx)+b^2e+bc(ex-d)}{(d+ex)\sqrt{a+x(b+cx)}}\right)}{\left(b^2-4ac\right)\left(e(ae-bd)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(-(e*(4*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(b*d + 2*a*e))*Sqrt[a + x*(b + c*x)]
)/(2*(c*d^2 + e*(-(b*d) + a*e))*(d + e*x)) + (b^2*e - 2*c*(a*e + c*d*x) + b
*c*(-d + e*x))/((d + e*x)*Sqrt[a + x*(b + c*x)]) + (3*(b^2 - 4*a*c)*e^2*(-2
*c*d + b*e)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-
(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/(4*(c*d^2 + e*(-(b*d) + a*e))^(3/2))
))/((b^2 - 4*a*c)*(c*d^2 + e*(-(b*d) + a*e)))
```

Maple [B] time = 0.267, size = 1313, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -1/(a*e^2-b*d*e+c*d^2)/(d/e+x)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)-3/2*e^2/(a*e^2-b*d*e+c*d^2)^2/((d/e+x)^2*c+(b*e-2*c*d
)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b+3*e/(a*e^2-b*d*e+c*d^2)^2/((d/
e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*c*d+3*e^2/(a*
e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*x*b^2*c-12*e/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)/((d/e+
x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c^2*d+12/(a
*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2)*x*c^3*d^2+3/2*e^2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)/
((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^3-6*e/(
a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-
b*d*e+c*d^2)/e^2)^(1/2)*b^2*c*d+6/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)/((d/e+x
)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c^2*d^2+3/2*e^
2/(a*e^2-b*d*e+c*d^2)^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+
c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)
^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b-3*e/(
a*e^2-b*d*e+c*d^2)^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d
^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*
c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*c*d-8*c^2/
(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2)*x-4*c/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(
b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 21.7937, size = 6063, normalized size = 23.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(3*(2*(a*b^2*c - 4*a^2*c^2)*d^2*e^2 - (a*b^3 - 4*a^2*b*c)*d*e^3 + (2*
(b^2*c^2 - 4*a*c^3)*d*e^3 - (b^3*c - 4*a*b*c^2)*e^4)*x^3 + (2*(b^2*c^2 - 4*
a*c^3)*d^2*e^2 + (b^3*c - 4*a*b*c^2)*d*e^3 - (b^4 - 4*a*b^2*c)*e^4)*x^2 + (
2*(b^3*c - 4*a*b*c^2)*d^2*e^2 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*e^3 - (a*b^
3 - 4*a^2*b*c)*e^4)*x)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e
^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 +
4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d -
 b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*
d*e*x + d^2)) + 4*(2*b*c^3*d^5 - 2*(3*b^2*c^2 - 4*a*c^3)*d^4*e + 6*(b^3*c -
 2*a*b*c^2)*d^3*e^2 - (2*b^4 - 3*a*b^2*c - 4*a^2*c^2)*d^2*e^3 + (a*b^3 - 2*
a^2*b*c)*d*e^4 + (a^2*b^2 - 4*a^3*c)*e^5 + (4*c^4*d^4*e - 8*b*c^3*d^3*e^2 +
```

```
(7*b^2*c^2 - 4*a*c^3)*d^2*e^3 - (3*b^3*c - 4*a*b*c^2)*d*e^4 + (3*a*b^2*c -
8*a^2*c^2)*e^5)*x^2 + (4*c^4*d^5 - 6*b*c^3*d^4*e + 8*a*c^3*d^3*e^2 + (5*b^
3*c - 16*a*b*c^2)*d^2*e^3 - (3*b^4 - 8*a*b^2*c - 4*a^2*c^2)*d*e^4 + (3*a*b^
3 - 10*a^2*b*c)*e^5)*x)*sqrt(c*x^2 + b*x + a))/((a*b^2*c^3 - 4*a^2*c^4)*d^7
 - 3*(a*b^3*c^2 - 4*a^2*b*c^3)*d^6*e + 3*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c
^3)*d^5*e^2 - (a*b^5 + 2*a^2*b^3*c - 24*a^3*b*c^2)*d^4*e^3 + 3*(a^2*b^4 - 3
*a^3*b^2*c - 4*a^4*c^2)*d^3*e^4 - 3*(a^3*b^3 - 4*a^4*b*c)*d^2*e^5 + (a^4*b^
2 - 4*a^5*c)*d*e^6 + ((b^2*c^4 - 4*a*c^5)*d^6*e - 3*(b^3*c^3 - 4*a*b*c^4)*d
^5*e^2 + 3*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*d^4*e^3 - (b^5*c + 2*a*b^3*c
^2 - 24*a^2*b*c^3)*d^3*e^4 + 3*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^
5 - 3*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^6 + (a^3*b^2*c - 4*a^4*c^2)*e^7)*x^3 +
((b^2*c^4 - 4*a*c^5)*d^7 - 2*(b^3*c^3 - 4*a*b*c^4)*d^6*e + 3*(a*b^2*c^3 - 4
*a^2*c^4)*d^5*e^2 + (2*b^5*c - 11*a*b^3*c^2 + 12*a^2*b*c^3)*d^4*e^3 - (b^6
- a*b^4*c - 15*a^2*b^2*c^2 + 12*a^3*c^3)*d^3*e^4 + 3*(a*b^5 - 4*a^2*b^3*c)*
d^2*e^5 - (3*a^2*b^4 - 13*a^3*b^2*c + 4*a^4*c^2)*d*e^6 + (a^3*b^3 - 4*a^4*b
*c)*e^7)*x^2 + ((b^3*c^3 - 4*a*b*c^4)*d^7 - (3*b^4*c^2 - 13*a*b^2*c^3 + 4*a
^2*c^4)*d^6*e + 3*(b^5*c - 4*a*b^3*c^2)*d^5*e^2 - (b^6 - a*b^4*c - 15*a^2*b
^2*c^2 + 12*a^3*c^3)*d^4*e^3 + (2*a*b^5 - 11*a^2*b^3*c + 12*a^3*b*c^2)*d^3*
e^4 + 3*(a^3*b^2*c - 4*a^4*c^2)*d^2*e^5 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^6 + (
a^4*b^2 - 4*a^5*c)*e^7)*x), 1/2*(3*(2*(a*b^2*c - 4*a^2*c^2)*d^2*e^2 - (a*b^
3 - 4*a^2*b*c)*d*e^3 + (2*(b^2*c^2 - 4*a*c^3)*d*e^3 - (b^3*c - 4*a*b*c^2)*e
^4)*x^3 + (2*(b^2*c^2 - 4*a*c^3)*d^2*e^2 + (b^3*c - 4*a*b*c^2)*d*e^3 - (b^4
 - 4*a*b^2*c)*e^4)*x^2 + (2*(b^3*c - 4*a*b*c^2)*d^2*e^2 - (b^4 - 6*a*b^2*c
+ 8*a^2*c^2)*d*e^3 - (a*b^3 - 4*a^2*b*c)*e^4)*x)*sqrt(-c*d^2 + b*d*e - a*e^
2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*
a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e +
a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)) - 2*(2*b*c^3*d^5 - 2*(3*b^
2*c^2 - 4*a*c^3)*d^4*e + 6*(b^3*c - 2*a*b*c^2)*d^3*e^2 - (2*b^4 - 3*a*b^2*c
 - 4*a^2*c^2)*d^2*e^3 + (a*b^3 - 2*a^2*b*c)*d*e^4 + (a^2*b^2 - 4*a^3*c)*e^5
 + (4*c^4*d^4*e - 8*b*c^3*d^3*e^2 + (7*b^2*c^2 - 4*a*c^3)*d^2*e^3 - (3*b^3*
c - 4*a*b*c^2)*d*e^4 + (3*a*b^2*c - 8*a^2*c^2)*e^5)*x^2 + (4*c^4*d^5 - 6*b*
c^3*d^4*e + 8*a*c^3*d^3*e^2 + (5*b^3*c - 16*a*b*c^2)*d^2*e^3 - (3*b^4 - 8*a
*b^2*c - 4*a^2*c^2)*d*e^4 + (3*a*b^3 - 10*a^2*b*c)*e^5)*x)*sqrt(c*x^2 + b*x
 + a))/((a*b^2*c^3 - 4*a^2*c^4)*d^7 - 3*(a*b^3*c^2 - 4*a^2*b*c^3)*d^6*e + 3
*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^5*e^2 - (a*b^5 + 2*a^2*b^3*c - 24*
a^3*b*c^2)*d^4*e^3 + 3*(a^2*b^4 - 3*a^3*b^2*c - 4*a^4*c^2)*d^3*e^4 - 3*(a^3
*b^3 - 4*a^4*b*c)*d^2*e^5 + (a^4*b^2 - 4*a^5*c)*d*e^6 + ((b^2*c^4 - 4*a*c^5
)*d^6*e - 3*(b^3*c^3 - 4*a*b*c^4)*d^5*e^2 + 3*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^
2*c^4)*d^4*e^3 - (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*d^3*e^4 + 3*(a*b^4*c
- 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^5 - 3*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^6 +
(a^3*b^2*c - 4*a^4*c^2)*e^7)*x^3 + ((b^2*c^4 - 4*a*c^5)*d^7 - 2*(b^3*c^3 -
4*a*b*c^4)*d^6*e + 3*(a*b^2*c^3 - 4*a^2*c^4)*d^5*e^2 + (2*b^5*c - 11*a*b^3*
c^2 + 12*a^2*b*c^3)*d^4*e^3 - (b^6 - a*b^4*c - 15*a^2*b^2*c^2 + 12*a^3*c^3)
*d^3*e^4 + 3*(a*b^5 - 4*a^2*b^3*c)*d^2*e^5 - (3*a^2*b^4 - 13*a^3*b^2*c + 4*
a^4*c^2)*d*e^6 + (a^3*b^3 - 4*a^4*b*c)*e^7)*x^2 + ((b^3*c^3 - 4*a*b*c^4)*d^
7 - (3*b^4*c^2 - 13*a*b^2*c^3 + 4*a^2*c^4)*d^6*e + 3*(b^5*c - 4*a*b^3*c^2)*
d^5*e^2 - (b^6 - a*b^4*c - 15*a^2*b^2*c^2 + 12*a^3*c^3)*d^4*e^3 + (2*a*b^5
- 11*a^2*b^3*c + 12*a^3*b*c^2)*d^3*e^4 + 3*(a^3*b^2*c - 4*a^4*c^2)*d^2*e^5
- 2*(a^3*b^3 - 4*a^4*b*c)*d*e^6 + (a^4*b^2 - 4*a^5*c)*e^7)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.2389 $\int \frac{1}{(d+ex)^3\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=371

$$-\frac{e\sqrt{a+bx+cx^2}(2cd-be)\left(-4ce(13ae+2bd)+15b^2e^2+8c^2d^2\right)}{4\left(b^2-4ac\right)(d+ex)\left(ae^2-bde+cd^2\right)^3}-\frac{e\sqrt{a+bx+cx^2}\left(-4ce(3ae+2bd)+5b^2e^2+8c^2d^2\right)}{2\left(b^2-4ac\right)(d+ex)^2\left(ae^2-bde+cd^2\right)^2}+$$

```
[Out] (-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2 -
b*d*e + a*e^2)*(d + e*x)^2*Sqrt[a + b*x + c*x^2]) - (e*(8*c^2*d^2 + 5*b^2*e
^2 - 4*c*e*(2*b*d + 3*a*e))*Sqrt[a + b*x + c*x^2])/(2*(b^2 - 4*a*c)*(c*d^2
- b*d*e + a*e^2)^2*(d + e*x)^2) - (e*(2*c*d - b*e)*(8*c^2*d^2 + 15*b^2*e^2
- 4*c*e*(2*b*d + 13*a*e))*Sqrt[a + b*x + c*x^2])/(4*(b^2 - 4*a*c)*(c*d^2 -
b*d*e + a*e^2)^3*(d + e*x)) + (3*e^2*(16*c^2*d^2 + 5*b^2*e^2 - 4*c*e*(4*b*d
 + a*e))*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*
e^2]*Sqrt[a + b*x + c*x^2])])/(8*(c*d^2 - b*d*e + a*e^2)^(7/2))
```

Rubi [A] time = 0.475524, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {740, 834, 806, 724, 206}

$$-\frac{e\sqrt{a+bx+cx^2}(2cd-be)\left(-4ce(13ae+2bd)+15b^2e^2+8c^2d^2\right)}{4\left(b^2-4ac\right)(d+ex)\left(ae^2-bde+cd^2\right)^3}-\frac{e\sqrt{a+bx+cx^2}\left(-4ce(3ae+2bd)+5b^2e^2+8c^2d^2\right)}{2\left(b^2-4ac\right)(d+ex)^2\left(ae^2-bde+cd^2\right)^2}+$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + b*x + c*x^2)^(3/2)),x]
```



```
[Out] (-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2 -
b*d*e + a*e^2)*(d + e*x)^2*Sqrt[a + b*x + c*x^2]) - (e*(8*c^2*d^2 + 5*b^2*e
^2 - 4*c*e*(2*b*d + 3*a*e))*Sqrt[a + b*x + c*x^2])/(2*(b^2 - 4*a*c)*(c*d^2
- b*d*e + a*e^2)^2*(d + e*x)^2) - (e*(2*c*d - b*e)*(8*c^2*d^2 + 15*b^2*e^2
- 4*c*e*(2*b*d + 13*a*e))*Sqrt[a + b*x + c*x^2])/(4*(b^2 - 4*a*c)*(c*d^2 -
b*d*e + a*e^2)^3*(d + e*x)) + (3*e^2*(16*c^2*d^2 + 5*b^2*e^2 - 4*c*e*(4*b*d
 + a*e))*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*
e^2]*Sqrt[a + b*x + c*x^2])])/(8*(c*d^2 - b*d*e + a*e^2)^(7/2))
```


Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
```

```
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)^2\sqrt{a+bx+cx^2}} - \frac{2\int \frac{\frac{1}{2}e\left(4bcd-5b^2e+12ace\right)+2ce(2\cdots}{(d+ex)^3\sqrt{a+bx+cx^2}}}{\left(b^2-4ac\right)\left(cd^2-bde\cdots\right.}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)^2\sqrt{a+bx+cx^2}} - \frac{e\left(8c^2d^2+5b^2e^2-4ce(2b\cdots\right.}{2\left(b^2-4ac\right)\left(cd^2-b\cdots\right.}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)^2\sqrt{a+bx+cx^2}} - \frac{e\left(8c^2d^2+5b^2e^2-4ce(2b\cdots\right.}{2\left(b^2-4ac\right)\left(cd^2-b\cdots\right.}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)^2\sqrt{a+bx+cx^2}} - \frac{e\left(8c^2d^2+5b^2e^2-4ce(2b\cdots\right.}{2\left(b^2-4ac\right)\left(cd^2-b\cdots\right.}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)^2\sqrt{a+bx+cx^2}} - \frac{e\left(8c^2d^2+5b^2e^2-4ce(2b\cdots\right.}{2\left(b^2-4ac\right)\left(cd^2-b\cdots\right.}$$

Mathematica [A] time = 1.41918, size = 353, normalized size = 0.95

$$\frac{2\left(-\frac{e\sqrt{a+x(b+cx)}\left(-4ce(3ae+2bd)+5b^2e^2+8c^2d^2\right)}{4(d+ex)^2\left(e(ae-bd)+cd^2\right)}+\frac{1}{16}e\left(-\frac{2\sqrt{a+x(b+cx)}(2cd-be)\left(-4ce(13ae+2bd)+15b^2e^2+8c^2d^2\right)}{(d+ex)\left(e(ae-bd)+cd^2\right)^2}-\frac{3e\left(b^2-4ac\right)\left(-4ce(ae+4bd)+5b^2e^2+\cdots\right.}{\left(e(a\cdots\right.}\right.\right.}{\left(b^2-4ac\right)\left(e(ae-bd)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(-(e*(8*c^2*d^2 + 5*b^2*e^2 - 4*c*e*(2*b*d + 3*a*e))*Sqrt[a + x*(b + c*x
)])/(4*(c*d^2 + e*(-(b*d) + a*e))*(d + e*x)^2) + (b^2*e - 2*c*(a*e + c*d*x)
 + b*c*(-d + e*x))/((d + e*x)^2*Sqrt[a + x*(b + c*x)]) + (e*((-2*(2*c*d - b
*e)*(8*c^2*d^2 + 15*b^2*e^2 - 4*c*e*(2*b*d + 13*a*e))*Sqrt[a + x*(b + c*x)]
)/((c*d^2 + e*(-(b*d) + a*e))^2*(d + e*x)) - (3*(b^2 - 4*a*c)*e*(16*c^2*d^2
 + 5*b^2*e^2 - 4*c*e*(4*b*d + a*e))*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e
*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/(c*d^2 + e*(
-(b*d) + a*e))^(5/2)))/16))/((b^2 - 4*a*c)*(c*d^2 + e*(-(b*d) + a*e)))
```

Maple [B] time = 0.236, size = 2380, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -15/8*e^3/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e
+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^4-45*e/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^
2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c^
3*d^2+45/2*e^2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e
*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b^2*c^2*d-45/2*e/(a*e^2-b*d*e+c*d
^2)^3/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*b^2*c^2*d^2+15/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)/((d/e+x)^2*c+(b*e
-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c^3*d^3+15/2*e^2/(a*e^2-
b*d*e+c*d^2)^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^
2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e
-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b*c*d+3/2*e*c/(a
*e^2-b*d*e+c*d^2)^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^
2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c
+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))-1/2/e/(a*e^
2-b*d*e+c*d^2)/(d/e+x)^2/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*
d^2)/e^2)^(1/2)+15/8*e^3/(a*e^2-b*d*e+c*d^2)^3/((d/e+x)^2*c+(b*e-2*c*d)/e*(
d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2-3/2*e*c/(a*e^2-b*d*e+c*d^2)^2/((d
/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+5/4*e/(a*e^2
-b*d*e+c*d^2)^2/(d/e+x)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d
^2)/e^2)^(1/2)*b+15/2*e/(a*e^2-b*d*e+c*d^2)^3/((d/e+x)^2*c+(b*e-2*c*d)/e*(d
/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*c^2*d^2-13/(a*e^2-b*d*e+c*d^2)^2*c^2/(
4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*b*d+13*e/(a*e^2-b*d*e+c*d^2)^2*c^2/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e
*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b-26/(a*e^2-b*d*e+c*d^2)^2*c^3/(4
*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
*x*d-15/4*e^3/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*
(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b^3*c+30/(a*e^2-b*d*e+c*d^2)^3/(4*
a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
x*c^4*d^3+45/4*e^2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*
d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^3*c*d-15/8*e^3/(a*e^2-b*d*e+c
*d^2)^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-
2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)
/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b^2-5/2/(a*e^2-b*d*e+c*
d^2)^2/(d/e+x)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^
(1/2)*c*d-15/2*e^2/(a*e^2-b*d*e+c*d^2)^3/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)
+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d-15/2*e/(a*e^2-b*d*e+c*d^2)^3/((a*e^2-
```

```
b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)
+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^
2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*c^2*d^2+13/2*e/(a*e^2-b*d*e+c*d^2)^2*c/
(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/
2)*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 92.8938, size = 11780, normalized size = 31.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")

[Out] [-1/16*(3*(16*(a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - 16*(a*b^3*c - 4*a^2*b*c^2)*
d^3*e^3 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*d^2*e^4 + (16*(b^2*c^3 - 4*
a*c^4)*d^2*e^4 - 16*(b^3*c^2 - 4*a*b*c^3)*d*e^5 + (5*b^4*c - 24*a*b^2*c^2 +
 16*a^2*c^3)*e^6)*x^4 + (32*(b^2*c^3 - 4*a*c^4)*d^3*e^3 - 16*(b^3*c^2 - 4*a
*b*c^3)*d^2*e^4 - 2*(3*b^4*c - 8*a*b^2*c^2 - 16*a^2*c^3)*d*e^5 + (5*b^5 - 2
4*a*b^3*c + 16*a^2*b*c^2)*e^6)*x^3 + (16*(b^2*c^3 - 4*a*c^4)*d^4*e^2 + 16*(
b^3*c^2 - 4*a*b*c^3)*d^3*e^3 - 3*(9*b^4*c - 40*a*b^2*c^2 + 16*a^2*c^3)*d^2*
e^4 + 2*(5*b^5 - 32*a*b^3*c + 48*a^2*b*c^2)*d*e^5 + (5*a*b^4 - 24*a^2*b^2*c
 + 16*a^3*c^2)*e^6)*x^2 + (16*(b^3*c^2 - 4*a*b*c^3)*d^4*e^2 - 16*(b^4*c - 6
*a*b^2*c^2 + 8*a^2*c^3)*d^3*e^3 + (5*b^5 - 56*a*b^3*c + 144*a^2*b*c^2)*d^2*
e^4 + 2*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*d*e^5)*x)*sqrt(c*d^2 - b*d*e
+ a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*
c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 +
 b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b
^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) + 4*(8*b*c^4*d^7 - 16*(2*b^2
*c^3 - 3*a*c^4)*d^6*e + 16*(3*b^3*c^2 - 7*a*b*c^3)*d^5*e^2 - 32*(b^4*c - 3*
a*b^2*c^2 + a^2*c^3)*d^4*e^3 + (8*b^5 - 33*a*b^3*c + 44*a^2*b*c^2)*d^3*e^4
+ (a*b^4 + 14*a^2*b^2*c - 88*a^3*c^2)*d^2*e^5 - 11*(a^2*b^3 - 4*a^3*b*c)*d*
e^6 + 2*(a^3*b^2 - 4*a^4*c)*e^7 + (16*c^5*d^5*e^2 - 40*b*c^4*d^4*e^3 + 2*(3
1*b^2*c^3 - 44*a*c^4)*d^3*e^4 - (53*b^3*c^2 - 132*a*b*c^3)*d^2*e^5 + (15*b^
4*c - 14*a*b^2*c^2 - 104*a^2*c^3)*d*e^6 - (15*a*b^3*c - 52*a^2*b*c^2)*e^7)*
x^3 + (32*c^5*d^6*e - 72*b*c^4*d^5*e^2 + 80*(b^2*c^3 - a*c^4)*d^4*e^3 - (27
*b^3*c^2 - 28*a*b*c^3)*d^3*e^4 - 2*(14*b^4*c - 73*a*b^2*c^2 + 68*a^2*c^3)*d
^2*e^5 + (15*b^5 - 49*a*b^3*c - 20*a^2*b*c^2)*d*e^6 - (15*a*b^4 - 62*a^2*b^
2*c + 24*a^3*c^2)*e^7)*x^2 + (16*c^5*d^7 - 24*b*c^4*d^6*e - 16*(b^2*c^3 - 4
*a*c^4)*d^5*e^2 + 80*(b^3*c^2 - 3*a*b*c^3)*d^4*e^3 - (81*b^4*c - 274*a*b^2*
c^2 + 40*a^2*c^3)*d^3*e^4 + (25*b^5 - 63*a*b^3*c - 76*a^2*b*c^2)*d^2*e^5 -
2*(10*a*b^4 - 47*a^2*b^2*c + 44*a^3*c^2)*d*e^6 - 5*(a^2*b^3 - 4*a^3*b*c)*e^
7)*x)*sqrt(c*x^2 + b*x + a))/((a*b^2*c^4 - 4*a^2*c^5)*d^10 - 4*(a*b^3*c^3 -
 4*a^2*b*c^4)*d^9*e + 2*(3*a*b^4*c^2 - 10*a^2*b^2*c^3 - 8*a^3*c^4)*d^8*e^2
- 4*(a*b^5*c - a^2*b^3*c^2 - 12*a^3*b*c^3)*d^7*e^3 + (a*b^6 + 8*a^2*b^4*c -
```

42*a^3*b^2*c^2 - 24*a^4*c^3)*d^6*e^4 - 4*(a^2*b^5 - a^3*b^3*c - 12*a^4*b*c
^2)*d^5*e^5 + 2*(3*a^3*b^4 - 10*a^4*b^2*c - 8*a^5*c^2)*d^4*e^6 - 4*(a^4*b^3
 - 4*a^5*b*c)*d^3*e^7 + (a^5*b^2 - 4*a^6*c)*d^2*e^8 + ((b^2*c^5 - 4*a*c^6)*
d^8*e^2 - 4*(b^3*c^4 - 4*a*b*c^5)*d^7*e^3 + 2*(3*b^4*c^3 - 10*a*b^2*c^4 - 8
*a^2*c^5)*d^6*e^4 - 4*(b^5*c^2 - a*b^3*c^3 - 12*a^2*b*c^4)*d^5*e^5 + (b^6*c
 + 8*a*b^4*c^2 - 42*a^2*b^2*c^3 - 24*a^3*c^4)*d^4*e^6 - 4*(a*b^5*c - a^2*b^
3*c^2 - 12*a^3*b*c^3)*d^3*e^7 + 2*(3*a^2*b^4*c - 10*a^3*b^2*c^2 - 8*a^4*c^3
)*d^2*e^8 - 4*(a^3*b^3*c - 4*a^4*b*c^2)*d*e^9 + (a^4*b^2*c - 4*a^5*c^2)*e^1
0)*x^4 + (2*(b^2*c^5 - 4*a*c^6)*d^9*e - 7*(b^3*c^4 - 4*a*b*c^5)*d^8*e^2 + 8
*(b^4*c^3 - 3*a*b^2*c^4 - 4*a^2*c^5)*d^7*e^3 - 2*(b^5*c^2 + 6*a*b^3*c^3 - 4
0*a^2*b*c^4)*d^6*e^4 - 2*(b^6*c - 10*a*b^4*c^2 + 18*a^2*b^2*c^3 + 24*a^3*c^
4)*d^5*e^5 + (b^7 - 34*a^2*b^3*c^2 + 72*a^3*b*c^3)*d^4*e^6 - 4*(a*b^6 - 4*a
^2*b^4*c - 2*a^3*b^2*c^2 + 8*a^4*c^3)*d^3*e^7 + 2*(3*a^2*b^5 - 14*a^3*b^3*c
 + 8*a^4*b*c^2)*d^2*e^8 - 2*(2*a^3*b^4 - 9*a^4*b^2*c + 4*a^5*c^2)*d*e^9 + (
a^4*b^3 - 4*a^5*b*c)*e^10)*x^3 + ((b^2*c^5 - 4*a*c^6)*d^10 - 2*(b^3*c^4 - 4
*a*b*c^5)*d^9*e - (2*b^4*c^3 - 13*a*b^2*c^4 + 20*a^2*c^5)*d^8*e^2 + 8*(b^5*
c^2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d^7*e^3 - (7*b^6*c - 22*a*b^4*c^2 - 34*a^2
*b^2*c^3 + 40*a^3*c^4)*d^6*e^4 + 2*(b^7 + 4*a*b^5*c - 38*a^2*b^3*c^2 + 24*a
^3*b*c^3)*d^5*e^5 - (7*a*b^6 - 22*a^2*b^4*c - 34*a^3*b^2*c^2 + 40*a^4*c^3)*
d^4*e^6 + 8*(a^2*b^5 - 5*a^3*b^3*c + 4*a^4*b*c^2)*d^3*e^7 - (2*a^3*b^4 - 13
*a^4*b^2*c + 20*a^5*c^2)*d^2*e^8 - 2*(a^4*b^3 - 4*a^5*b*c)*d*e^9 + (a^5*b^2
 - 4*a^6*c)*e^10)*x^2 + ((b^3*c^4 - 4*a*b*c^5)*d^10 - 2*(2*b^4*c^3 - 9*a*b^
2*c^4 + 4*a^2*c^5)*d^9*e + 2*(3*b^5*c^2 - 14*a*b^3*c^3 + 8*a^2*b*c^4)*d^8*e
^2 - 4*(b^6*c - 4*a*b^4*c^2 - 2*a^2*b^2*c^3 + 8*a^3*c^4)*d^7*e^3 + (b^7 - 3
4*a^2*b^3*c^2 + 72*a^3*b*c^3)*d^6*e^4 - 2*(a*b^6 - 10*a^2*b^4*c + 18*a^3*b^
2*c^2 + 24*a^4*c^3)*d^5*e^5 - 2*(a^2*b^5 + 6*a^3*b^3*c - 40*a^4*b*c^2)*d^4*
e^6 + 8*(a^3*b^4 - 3*a^4*b^2*c - 4*a^5*c^2)*d^3*e^7 - 7*(a^4*b^3 - 4*a^5*b*
c)*d^2*e^8 + 2*(a^5*b^2 - 4*a^6*c)*d*e^9)*x), 1/8*(3*(16*(a*b^2*c^2 - 4*a^2
*c^3)*d^4*e^2 - 16*(a*b^3*c - 4*a^2*b*c^2)*d^3*e^3 + (5*a*b^4 - 24*a^2*b^2*
c + 16*a^3*c^2)*d^2*e^4 + (16*(b^2*c^3 - 4*a*c^4)*d^2*e^4 - 16*(b^3*c^2 - 4
*a*b*c^3)*d*e^5 + (5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*e^6)*x^4 + (32*(b^2
*c^3 - 4*a*c^4)*d^3*e^3 - 16*(b^3*c^2 - 4*a*b*c^3)*d^2*e^4 - 2*(3*b^4*c - 8
*a*b^2*c^2 - 16*a^2*c^3)*d*e^5 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*e^6)*x
^3 + (16*(b^2*c^3 - 4*a*c^4)*d^4*e^2 + 16*(b^3*c^2 - 4*a*b*c^3)*d^3*e^3 - 3
*(9*b^4*c - 40*a*b^2*c^2 + 16*a^2*c^3)*d^2*e^4 + 2*(5*b^5 - 32*a*b^3*c + 48
*a^2*b*c^2)*d*e^5 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*e^6)*x^2 + (16*(b
^3*c^2 - 4*a*b*c^3)*d^4*e^2 - 16*(b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d^3*e^3
 + (5*b^5 - 56*a*b^3*c + 144*a^2*b*c^2)*d^2*e^4 + 2*(5*a*b^4 - 24*a^2*b^2*c
 + 16*a^3*c^2)*d*e^5)*x)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^
2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a
*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 -
 b^2*d*e + a*b*e^2)*x)) - 2*(8*b*c^4*d^7 - 16*(2*b^2*c^3 - 3*a*c^4)*d^6*e +
 16*(3*b^3*c^2 - 7*a*b*c^3)*d^5*e^2 - 32*(b^4*c - 3*a*b^2*c^2 + a^2*c^3)*d^
4*e^3 + (8*b^5 - 33*a*b^3*c + 44*a^2*b*c^2)*d^3*e^4 + (a*b^4 + 14*a^2*b^2*c
 - 88*a^3*c^2)*d^2*e^5 - 11*(a^2*b^3 - 4*a^3*b*c)*d*e^6 + 2*(a^3*b^2 - 4*a^
4*c)*e^7 + (16*c^5*d^5*e^2 - 40*b*c^4*d^4*e^3 + 2*(31*b^2*c^3 - 44*a*c^4)*d
^3*e^4 - (53*b^3*c^2 - 132*a*b*c^3)*d^2*e^5 + (15*b^4*c - 14*a*b^2*c^2 - 10
4*a^2*c^3)*d*e^6 - (15*a*b^3*c - 52*a^2*b*c^2)*e^7)*x^3 + (32*c^5*d^6*e - 7
2*b*c^4*d^5*e^2 + 80*(b^2*c^3 - a*c^4)*d^4*e^3 - (27*b^3*c^2 - 28*a*b*c^3)*
d^3*e^4 - 2*(14*b^4*c - 73*a*b^2*c^2 + 68*a^2*c^3)*d^2*e^5 + (15*b^5 - 49*a
*b^3*c - 20*a^2*b*c^2)*d*e^6 - (15*a*b^4 - 62*a^2*b^2*c + 24*a^3*c^2)*e^7)*
x^2 + (16*c^5*d^7 - 24*b*c^4*d^6*e - 16*(b^2*c^3 - 4*a*c^4)*d^5*e^2 + 80*(b
^3*c^2 - 3*a*b*c^3)*d^4*e^3 - (81*b^4*c - 274*a*b^2*c^2 + 40*a^2*c^3)*d^3*e
^4 + (25*b^5 - 63*a*b^3*c - 76*a^2*b*c^2)*d^2*e^5 - 2*(10*a*b^4 - 47*a^2*b^
2*c + 44*a^3*c^2)*d*e^6 - 5*(a^2*b^3 - 4*a^3*b*c)*e^7)*x)*sqrt(c*x^2 + b*x
 + a))/((a*b^2*c^4 - 4*a^2*c^5)*d^10 - 4*(a*b^3*c^3 - 4*a^2*b*c^4)*d^9*e + 2
*(3*a*b^4*c^2 - 10*a^2*b^2*c^3 - 8*a^3*c^4)*d^8*e^2 - 4*(a*b^5*c - a^2*b^3*
c^2 - 12*a^3*b*c^3)*d^7*e^3 + (a*b^6 + 8*a^2*b^4*c - 42*a^3*b^2*c^2 - 24*a^
4*c^3)*d^6*e^4 - 4*(a^2*b^5 - a^3*b^3*c - 12*a^4*b*c^2)*d^5*e^5 + 2*(3*a^3*

```
b^4 - 10*a^4*b^2*c - 8*a^5*c^2)*d^4*e^6 - 4*(a^4*b^3 - 4*a^5*b*c)*d^3*e^7 +
 (a^5*b^2 - 4*a^6*c)*d^2*e^8 + ((b^2*c^5 - 4*a*c^6)*d^8*e^2 - 4*(b^3*c^4 -
4*a*b*c^5)*d^7*e^3 + 2*(3*b^4*c^3 - 10*a*b^2*c^4 - 8*a^2*c^5)*d^6*e^4 - 4*(
b^5*c^2 - a*b^3*c^3 - 12*a^2*b*c^4)*d^5*e^5 + (b^6*c + 8*a*b^4*c^2 - 42*a^2
*b^2*c^3 - 24*a^3*c^4)*d^4*e^6 - 4*(a*b^5*c - a^2*b^3*c^2 - 12*a^3*b*c^3)*d
^3*e^7 + 2*(3*a^2*b^4*c - 10*a^3*b^2*c^2 - 8*a^4*c^3)*d^2*e^8 - 4*(a^3*b^3*
c - 4*a^4*b*c^2)*d*e^9 + (a^4*b^2*c - 4*a^5*c^2)*e^10)*x^4 + (2*(b^2*c^5 -
4*a*c^6)*d^9*e - 7*(b^3*c^4 - 4*a*b*c^5)*d^8*e^2 + 8*(b^4*c^3 - 3*a*b^2*c^4
 - 4*a^2*c^5)*d^7*e^3 - 2*(b^5*c^2 + 6*a*b^3*c^3 - 40*a^2*b*c^4)*d^6*e^4 -
2*(b^6*c - 10*a*b^4*c^2 + 18*a^2*b^2*c^3 + 24*a^3*c^4)*d^5*e^5 + (b^7 - 34*
a^2*b^3*c^2 + 72*a^3*b*c^3)*d^4*e^6 - 4*(a*b^6 - 4*a^2*b^4*c - 2*a^3*b^2*c^
2 + 8*a^4*c^3)*d^3*e^7 + 2*(3*a^2*b^5 - 14*a^3*b^3*c + 8*a^4*b*c^2)*d^2*e^8
 - 2*(2*a^3*b^4 - 9*a^4*b^2*c + 4*a^5*c^2)*d*e^9 + (a^4*b^3 - 4*a^5*b*c)*e^
10)*x^3 + ((b^2*c^5 - 4*a*c^6)*d^10 - 2*(b^3*c^4 - 4*a*b*c^5)*d^9*e - (2*b^
4*c^3 - 13*a*b^2*c^4 + 20*a^2*c^5)*d^8*e^2 + 8*(b^5*c^2 - 5*a*b^3*c^3 + 4*a
^2*b*c^4)*d^7*e^3 - (7*b^6*c - 22*a*b^4*c^2 - 34*a^2*b^2*c^3 + 40*a^3*c^4)*
d^6*e^4 + 2*(b^7 + 4*a*b^5*c - 38*a^2*b^3*c^2 + 24*a^3*b*c^3)*d^5*e^5 - (7*
a*b^6 - 22*a^2*b^4*c - 34*a^3*b^2*c^2 + 40*a^4*c^3)*d^4*e^6 + 8*(a^2*b^5 -
5*a^3*b^3*c + 4*a^4*b*c^2)*d^3*e^7 - (2*a^3*b^4 - 13*a^4*b^2*c + 20*a^5*c^2
)*d^2*e^8 - 2*(a^4*b^3 - 4*a^5*b*c)*d*e^9 + (a^5*b^2 - 4*a^6*c)*e^10)*x^2 +
 ((b^3*c^4 - 4*a*b*c^5)*d^10 - 2*(2*b^4*c^3 - 9*a*b^2*c^4 + 4*a^2*c^5)*d^9*
e + 2*(3*b^5*c^2 - 14*a*b^3*c^3 + 8*a^2*b*c^4)*d^8*e^2 - 4*(b^6*c - 4*a*b^4
*c^2 - 2*a^2*b^2*c^3 + 8*a^3*c^4)*d^7*e^3 + (b^7 - 34*a^2*b^3*c^2 + 72*a^3*
b*c^3)*d^6*e^4 - 2*(a*b^6 - 10*a^2*b^4*c + 18*a^3*b^2*c^2 + 24*a^4*c^3)*d^5
*e^5 - 2*(a^2*b^5 + 6*a^3*b^3*c - 40*a^4*b*c^2)*d^4*e^6 + 8*(a^3*b^4 - 3*a^
4*b^2*c - 4*a^5*c^2)*d^3*e^7 - 7*(a^4*b^3 - 4*a^5*b*c)*d^2*e^8 + 2*(a^5*b^2
 - 4*a^6*c)*d*e^9)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)^3 \left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral(1/((d + e*x)**3*(a + b*x + c*x**2)**(3/2)), x)
```

Giac [B] time = 1.45875, size = 3475, normalized size = 9.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^7*d^9 - 9*b*c^6*d^8*e + 18*b^2*c^5*d^7*e^2 - 21*b^3*c^4*d^6*e^3 +
15*b^4*c^3*d^5*e^4 + 6*a*b^2*c^4*d^5*e^4 - 12*a^2*c^5*d^5*e^4 - 6*b^5*c^2*d
^4*e^5 - 15*a*b^3*c^3*d^4*e^5 + 30*a^2*b*c^4*d^4*e^5 + b^6*c*d^3*e^6 + 12*a
*b^4*c^2*d^3*e^6 - 18*a^2*b^2*c^3*d^3*e^6 - 16*a^3*c^4*d^3*e^6 - 3*a*b^5*c*
d^2*e^7 - 3*a^2*b^3*c^2*d^2*e^7 + 24*a^3*b*c^3*d^2*e^7 + 3*a^2*b^4*c*d*e^8
- 6*a^3*b^2*c^2*d*e^8 - 6*a^4*c^3*d*e^8 - a^3*b^3*c*e^9 + 3*a^4*b*c^2*e^9)*
x/(b^2*c^6*d^12 - 4*a*c^7*d^12 - 6*b^3*c^5*d^11*e + 24*a*b*c^6*d^11*e + 15*
```

```
b^4*c^4*d^10*e^2 - 54*a*b^2*c^5*d^10*e^2 - 24*a^2*c^6*d^10*e^2 - 20*b^5*c^3
*d^9*e^3 + 50*a*b^3*c^4*d^9*e^3 + 120*a^2*b*c^5*d^9*e^3 + 15*b^6*c^2*d^8*e^
4 - 225*a^2*b^2*c^4*d^8*e^4 - 60*a^3*c^5*d^8*e^4 - 6*b^7*c*d^7*e^5 - 36*a*b
^5*c^2*d^7*e^5 + 180*a^2*b^3*c^3*d^7*e^5 + 240*a^3*b*c^4*d^7*e^5 + b^8*d^6*
e^6 + 26*a*b^6*c*d^6*e^6 - 30*a^2*b^4*c^2*d^6*e^6 - 340*a^3*b^2*c^3*d^6*e^6
 - 80*a^4*c^4*d^6*e^6 - 6*a*b^7*d^5*e^7 - 36*a^2*b^5*c*d^5*e^7 + 180*a^3*b^
3*c^2*d^5*e^7 + 240*a^4*b*c^3*d^5*e^7 + 15*a^2*b^6*d^4*e^8 - 225*a^4*b^2*c^
2*d^4*e^8 - 60*a^5*c^3*d^4*e^8 - 20*a^3*b^5*d^3*e^9 + 50*a^4*b^3*c*d^3*e^9
+ 120*a^5*b*c^2*d^3*e^9 + 15*a^4*b^4*d^2*e^10 - 54*a^5*b^2*c*d^2*e^10 - 24*
a^6*c^2*d^2*e^10 - 6*a^5*b^3*d*e^11 + 24*a^6*b*c*d*e^11 + a^6*b^2*e^12 - 4*
a^7*c*e^12) + (b*c^6*d^9 - 6*b^2*c^5*d^8*e + 6*a*c^6*d^8*e + 15*b^3*c^4*d^7
*e^2 - 24*a*b*c^5*d^7*e^2 - 20*b^4*c^3*d^6*e^3 + 34*a*b^2*c^4*d^6*e^3 + 16*
a^2*c^5*d^6*e^3 + 15*b^5*c^2*d^5*e^4 - 15*a*b^3*c^3*d^5*e^4 - 54*a^2*b*c^4*
d^5*e^4 - 6*b^6*c*d^4*e^5 - 9*a*b^4*c^2*d^4*e^5 + 66*a^2*b^2*c^3*d^4*e^5 +
12*a^3*c^4*d^4*e^5 + b^7*d^3*e^6 + 11*a*b^5*c*d^3*e^6 - 31*a^2*b^3*c^2*d^3*
e^6 - 32*a^3*b*c^3*d^3*e^6 - 3*a*b^6*d^2*e^7 + 30*a^3*b^2*c^2*d^2*e^7 + 3*a
^2*b^5*d*e^8 - 9*a^3*b^3*c*d*e^8 - 3*a^4*b*c^2*d*e^8 - a^3*b^4*e^9 + 4*a^4*
b^2*c*e^9 - 2*a^5*c^2*e^9)/(b^2*c^6*d^12 - 4*a*c^7*d^12 - 6*b^3*c^5*d^11*e
+ 24*a*b*c^6*d^11*e + 15*b^4*c^4*d^10*e^2 - 54*a*b^2*c^5*d^10*e^2 - 24*a^2*
c^6*d^10*e^2 - 20*b^5*c^3*d^9*e^3 + 50*a*b^3*c^4*d^9*e^3 + 120*a^2*b*c^5*d^
9*e^3 + 15*b^6*c^2*d^8*e^4 - 225*a^2*b^2*c^4*d^8*e^4 - 60*a^3*c^5*d^8*e^4 -
 6*b^7*c*d^7*e^5 - 36*a*b^5*c^2*d^7*e^5 + 180*a^2*b^3*c^3*d^7*e^5 + 240*a^3
*b*c^4*d^7*e^5 + b^8*d^6*e^6 + 26*a*b^6*c*d^6*e^6 - 30*a^2*b^4*c^2*d^6*e^6
- 340*a^3*b^2*c^3*d^6*e^6 - 80*a^4*c^4*d^6*e^6 - 6*a*b^7*d^5*e^7 - 36*a^2*b
^5*c*d^5*e^7 + 180*a^3*b^3*c^2*d^5*e^7 + 240*a^4*b*c^3*d^5*e^7 + 15*a^2*b^6
*d^4*e^8 - 225*a^4*b^2*c^2*d^4*e^8 - 60*a^5*c^3*d^4*e^8 - 20*a^3*b^5*d^3*e^
9 + 50*a^4*b^3*c*d^3*e^9 + 120*a^5*b*c^2*d^3*e^9 + 15*a^4*b^4*d^2*e^10 - 54
*a^5*b^2*c*d^2*e^10 - 24*a^6*c^2*d^2*e^10 - 6*a^5*b^3*d*e^11 + 24*a^6*b*c*d
*e^11 + a^6*b^2*e^12 - 4*a^7*c*e^12))/sqrt(c*x^2 + b*x + a) + 3/4*(16*c^2*d
^2*e^2 - 16*b*c*d*e^3 + 5*b^2*e^4 - 4*a*c*e^4)*arctan(-((sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/((c^3*d^6 - 3
*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a*b*c*d^
3*e^3 + 3*a*b^2*d^2*e^4 + 3*a^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6)*sqrt(-
c*d^2 + b*d*e - a*e^2)) - 1/4*(56*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(
5/2)*d^3*e^2 + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*d^2*e^3 + 56*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*d^3*e^2 - 48*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^2*b*c^(3/2)*d^2*e^3 + 14*b^2*c^(3/2)*d^3*e^2 - 24*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^3*b*c*d*e^4 - 44*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))*b^2*c*d^2*e^3 - 88*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*d^2*e^3 + 1
3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*d*e^4 - 28*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*d*e^4 - 7*b^3*sqrt(c)*d^2*e^3 - 44*a*b*
c^(3/2)*d^2*e^3 + 7*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*e^5 - 4*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*e^5 + 9*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*b^3*d*e^4 + 60*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*d*e^4 + 8*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*e^5 + 23*a*b^2*sqrt(c)*d*e^
4 + 28*a^2*c^(3/2)*d*e^4 - 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*e^5
- 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*e^5 - 16*a^2*b*sqrt(c)*e^5)/(
(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3
- 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3
*e^6)*((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*e + 2*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))*sqrt(c)*d + b*d - a*e)^2)
```

3.2390 $\int \frac{1}{(d+ex)^4\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=519

$$-\frac{e\sqrt{a+bx+cx^2}\left(4c^2e^2\left(64a^2e^2+332abde+119b^2d^2\right)-20b^2ce^3(23ae+19bd)-16c^3d^2e(83ae+12bd)+105b^4e^4+96\right.}{24\left(b^2-4ac\right)(d+ex)\left(ae^2-bde+cd^2\right)^4}$$

```
[Out] (-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2 -
b*d*e + a*e^2)*(d + e*x)^3*Sqrt[a + b*x + c*x^2]) - (e*(12*c^2*d^2 + 7*b^2*
e^2 - 4*c*e*(3*b*d + 4*a*e))*Sqrt[a + b*x + c*x^2])/(3*(b^2 - 4*a*c)*(c*d^2
 - b*d*e + a*e^2)^2*(d + e*x)^3) - (e*(2*c*d - b*e)*(24*c^2*d^2 + 35*b^2*e^
2 - 4*c*e*(6*b*d + 29*a*e))*Sqrt[a + b*x + c*x^2])/(12*(b^2 - 4*a*c)*(c*d^2
 - b*d*e + a*e^2)^3*(d + e*x)^2) - (e*(96*c^4*d^4 + 105*b^4*e^4 - 20*b^2*c*
e^3*(19*b*d + 23*a*e) - 16*c^3*d^2*e*(12*b*d + 83*a*e) + 4*c^2*e^2*(119*b^2
*d^2 + 332*a*b*d*e + 64*a^2*e^2))*Sqrt[a + b*x + c*x^2])/(24*(b^2 - 4*a*c)*
(c*d^2 - b*d*e + a*e^2)^4*(d + e*x)) + (5*e^2*(2*c*d - b*e)*(16*c^2*d^2 + 7
*b^2*e^2 - 4*c*e*(4*b*d + 3*a*e))*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(
2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(16*(c*d^2 - b*d*e +
 a*e^2)^(9/2))
```

Rubi [A] time = 0.857626, antiderivative size = 519, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {740, 834, 806, 724, 206}

$$-\frac{e\sqrt{a+bx+cx^2}\left(4c^2e^2\left(64a^2e^2+332abde+119b^2d^2\right)-20b^2ce^3(23ae+19bd)-16c^3d^2e(83ae+12bd)+105b^4e^4+96\right.}{24\left(b^2-4ac\right)(d+ex)\left(ae^2-bde+cd^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^4*(a + b*x + c*x^2)^(3/2)),x]
```


```
[Out] (-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2 -
b*d*e + a*e^2)*(d + e*x)^3*Sqrt[a + b*x + c*x^2]) - (e*(12*c^2*d^2 + 7*b^2*
e^2 - 4*c*e*(3*b*d + 4*a*e))*Sqrt[a + b*x + c*x^2])/(3*(b^2 - 4*a*c)*(c*d^2
 - b*d*e + a*e^2)^2*(d + e*x)^3) - (e*(2*c*d - b*e)*(24*c^2*d^2 + 35*b^2*e^
2 - 4*c*e*(6*b*d + 29*a*e))*Sqrt[a + b*x + c*x^2])/(12*(b^2 - 4*a*c)*(c*d^2
 - b*d*e + a*e^2)^3*(d + e*x)^2) - (e*(96*c^4*d^4 + 105*b^4*e^4 - 20*b^2*c*
e^3*(19*b*d + 23*a*e) - 16*c^3*d^2*e*(12*b*d + 83*a*e) + 4*c^2*e^2*(119*b^2
*d^2 + 332*a*b*d*e + 64*a^2*e^2))*Sqrt[a + b*x + c*x^2])/(24*(b^2 - 4*a*c)*
(c*d^2 - b*d*e + a*e^2)^4*(d + e*x)) + (5*e^2*(2*c*d - b*e)*(16*c^2*d^2 + 7
*b^2*e^2 - 4*c*e*(4*b*d + 3*a*e))*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(
2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(16*(c*d^2 - b*d*e +
 a*e^2)^(9/2))
```


Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
```

```
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^4 \left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)^3\sqrt{a+bx+cx^2}} - \frac{2\int \frac{\frac{1}{2}e\left(6bcd-7b^2e+16ace\right)+3ce(2}{(d+ex)^4\sqrt{a+bx+cx^2}}}{\left(b^2-4ac\right)\left(cd^2-bde-\right.}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)^3\sqrt{a+bx+cx^2}} - \frac{e\left(12c^2d^2+7b^2e^2-4ce(3b\right.}{3\left(b^2-4ac\right)\left(cd^2-b\right.}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)^3\sqrt{a+bx+cx^2}} - \frac{e\left(12c^2d^2+7b^2e^2-4ce(3b\right.}{3\left(b^2-4ac\right)\left(cd^2-b\right.}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)^3\sqrt{a+bx+cx^2}} - \frac{e\left(12c^2d^2+7b^2e^2-4ce(3b\right.}{3\left(b^2-4ac\right)\left(cd^2-b\right.}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)^3\sqrt{a+bx+cx^2}} - \frac{e\left(12c^2d^2+7b^2e^2-4ce(3b\right.}{3\left(b^2-4ac\right)\left(cd^2-b\right.}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)^3\sqrt{a+bx+cx^2}} - \frac{e\left(12c^2d^2+7b^2e^2-4ce(3b\right.}{3\left(b^2-4ac\right)\left(cd^2-b\right.}$$

Mathematica [A] time = 2.56423, size = 486, normalized size = 0.94

$$\frac{2\left(\frac{e\left(-\frac{2\sqrt{a+x(b+cx)}\left(4c^2e^2\left(64a^2e^2+332abde+119b^2d^2\right)-20b^2ce^3(23ae+19bd)-16c^3d^2e(83ae+12bd)+105b^4e^4+96c^4d^4\right)}{(d+ex)\left(e(ae-bd)+cd^2\right)}-\frac{4\sqrt{a+x(b+cx)}(2cd-be)\left(-4ce(29ae+6bd)+35b^2e^2+24c^2d^2\right)}{(d+ex)^2}+\frac{15e(}{}\right.}{96\left(e(ae-bd)+cd^2\right)^2}\right.}{\left(b^2-4ac\right)\left(e(\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^4*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(-(e*(12*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(3*b*d + 4*a*e))*Sqrt[a + x*(b + c*
x)])/(6*(c*d^2 + e*(-(b*d) + a*e))*(d + e*x)^3) + (b^2*e - 2*c*(a*e + c*d*x
) + b*c*(-d + e*x))/((d + e*x)^3*Sqrt[a + x*(b + c*x)]) + (e*((-4*(2*c*d -
b*e)*(24*c^2*d^2 + 35*b^2*e^2 - 4*c*e*(6*b*d + 29*a*e))*Sqrt[a + x*(b + c*x
)])/(d + e*x)^2 - (2*(96*c^4*d^4 + 105*b^4*e^4 - 20*b^2*c*e^3*(19*b*d + 23*
a*e) - 16*c^3*d^2*e*(12*b*d + 83*a*e) + 4*c^2*e^2*(119*b^2*d^2 + 332*a*b*d*
e + 64*a^2*e^2))*Sqrt[a + x*(b + c*x)])/((c*d^2 + e*(-(b*d) + a*e))*(d + e*
x)) + (15*(b^2 - 4*a*c)*e*(-2*c*d + b*e)*(16*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(4
*b*d + 3*a*e))*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e
*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/(c*d^2 + e*(-(b*d) + a*e))^(3/2))
)/(96*(c*d^2 + e*(-(b*d) + a*e))^2)))/((b^2 - 4*a*c)*(c*d^2 + e*(-(b*d) + a
*e)))
```

Maple [B] time = 0.244, size = 3823, normalized size = 7.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d)^4/(c*x^2+b*x+a)^(3/2),x)

[Out] 35/2*e/(a*e^2-b*d*e+c*d^2)^4/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*c^3*d^3+35/16*e^4/(a*e^2-b*d*e+c*d^2)^4/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^5+35/16*e^4/(a*e^2-b*d*e+c*d^2)^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b^3+15/4*e^2/(a*e^2-b*d*e+c*d^2)^3*c/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b-15/2*e/(a*e^2-b*d*e+c*d^2)^3*c^2/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d+32/3*c^3/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x-35/24*e^2/(a*e^2-b*d*e+c*d^2)^3/(d/e+x)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2+105*e^2/(a*e^2-b*d*e+c*d^2)^4/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b^2*c^3*d^2-140*e/(a*e^2-b*d*e+c*d^2)^4/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^4*d^3*b+230/3*e/(a*e^2-b*d*e+c*d^2)^3*c^3/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*d-35*e^3/(a*e^2-b*d*e+c*d^2)^4/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b^3*c^2*d+16/3*c^2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b-35/6/(a*e^2-b*d*e+c*d^2)^3/(d/e+x)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*c^2*d^2+4/3*c/(a*e^2-b*d*e+c*d^2)^2/(d/e+x)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+7/12/(a*e^2-b*d*e+c*d^2)^2/(d/e+x)^2/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b-35/16*e^4/(a*e^2-b*d*e+c*d^2)^4/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^3-1/3/e^2/(a*e^2-b*d*e+c*d^2)/(d/e+x)^3/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+15/2*e/(a*e^2-b*d*e+c*d^2)^3*c^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*d+105/8*e^3/(a*e^2-b*d*e+c*d^2)^4/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*c*d+70/(a*e^2-b*d*e+c*d^2)^4/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^5*d^4+35/(a*e^2-b*d*e+c*d^2)^4/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c^4*d^4-230/3/(a*e^2-b*d*e+c*d^2)^3*c^4/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*d^2-115/3/(a*e^2-b*d*e+c*d^2)^3*c^3/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*d^2-15/4*e^2/(a*e^2-b*d*e+c*d^2)^3*c/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b+35/6*e/(a*e^2-b*d*e+c*d^2)^3/(d/e+x)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d-105/8*e^3/(a*e^2-b*d*e+c*d^2)^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b^2*c*d+35/8*e^4/(a*e^2-b*d*e+c*d^2)^4/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b^4*c+105/4*e^2/(a*e^2-b*d*e+c*d^2)^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*c^2*d^2*b+115/3*e/(a*e^2-b*d*e+c*d^2)^3*c^2/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*d-70*e/(a*e^2-b*d*e+c*d^2)^4/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*c^3*d^3-115/6*e^2/(a*e^2-b*d*e+c*d^2)^3*c^2/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b^2-35/2*e^3/(a*e^2-b*d*e+c*d^2)^4/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d

```
/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^4*c*d+105/2*e^2/(a*e^2-b*d*e+c*d^2)^
4/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*b^3*c^2*d^2-35/2*e/(a*e^2-b*d*e+c*d^2)^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/
2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^
2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(
1/2))/(d/e+x))*c^3*d^3-115/12*e^2/(a*e^2-b*d*e+c*d^2)^3*c/(4*a*c-b^2)/((d/e
+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^3-7/6/e/(a*e
^2-b*d*e+c*d^2)^2/(d/e+x)^2/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*c*d-105/4*e^2/(a*e^2-b*d*e+c*d^2)^4/((d/e+x)^2*c+(b*e-2*
c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*c^2*d^2*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**4/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.71959, size = 6974, normalized size = 13.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

[Out] -2*((2*c^9*d^12 - 12*b*c^8*d^11*e + 34*b^2*c^7*d^10*e^2 - 4*a*c^8*d^10*e^2 - 60*b^3*c^6*d^9*e^3 + 20*a*b*c^7*d^9*e^3 + 71*b^4*c^5*d^8*e^4 - 28*a*b^2*c^6*d^8*e^4 - 34*a^2*c^7*d^8*e^4 - 56*b^5*c^4*d^7*e^5 - 8*a*b^3*c^5*d^7*e^5 + 136*a^2*b*c^6*d^7*e^5 + 28*b^6*c^3*d^6*e^6 + 56*a*b^4*c^4*d^6*e^6 - 196*a^2*b^2*c^5*d^6*e^6 - 56*a^3*c^6*d^6*e^6 - 8*b^7*c^2*d^5*e^7 - 56*a*b^5*c^3*d^5*e^7 + 112*a^2*b^3*c^4*d^5*e^7 + 168*a^3*b*c^5*d^5*e^7 + b^8*c*d^4*e^8 + 24*a*b^6*c^2*d^4*e^8 - 4*a^2*b^4*c^3*d^4*e^8 - 176*a^3*b^2*c^4*d^4*e^8 - 34*a^4*c^5*d^4*e^8 - 4*a*b^7*c*d^3*e^9 - 20*a^2*b^5*c^2*d^3*e^9 + 72*a^3*b^3*c^3*d^3*e^9 + 68*a^4*b*c^4*d^3*e^9 + 6*a^2*b^6*c*d^2*e^10 - 4*a^3*b^4*c^2*d^2*e^10 - 46*a^4*b^2*c^3*d^2*e^10 - 4*a^5*c^4*d^2*e^10 - 4*a^3*b^5*c*d*e^11 + 12*a^4*b^3*c^2*d*e^11 + 4*a^5*b*c^3*d*e^11 + a^4*b^4*c*e^12 - 4*a^5*b^2*c^2*e^12 + 2*a^6*c^3*e^12)*x/(b^2*c^8*d^16 - 4*a*c^9*d^16 - 8*b^3*c^7*d^15*e + 32*a*b*c^8*d^15*e + 28*b^4*c^6*d^14*e^2 - 104*a*b^2*c^7*d^14*e^2 - 32*a^2*c^8*d^14*e^2 - 56*b^5*c^5*d^13*e^3 + 168*a*b^3*c^6*d^13*e^3 + 224*a^2*b*c^7*d^13*e^3 + 70*b^6*c^4*d^12*e^4 - 112*a*b^4*c^5*d^12*e^4 - 644*a^2*b^2*c^6*d^12*e^4 - 112*a^3*c^7*d^12*e^4 - 56*b^7*c^3*d^11*e^5 - 56*a*b^5*c^4*d^11*e^5 + 952*a^2*b^3*c^5*d^11*e^5 + 672*a^3*b*c^6*d^11*e^5 + 28*b^8*c^2*d^10*e^6 + 168*a*b^6*c^3*d^10*e^6 - 700*a^2*b^4*c^4*d^10*e^6 - 1624*a^3*b^2*c^5*d^10*e^6 - 224*a^4*c^6*d^10*e^6 - 8*b^9*c*d^9*e^7 - 136*a*b^7*c^2*d^9*e^7 + 112*a^2*b^5*c^3*d^9*e^7 + 1960*a^3*b^3*c^4*d^9*e^7 + 1120*a^4*b*c^5*d^9*e^7 + b^10*d^8*e^8 + 52*a*b^8*c*d^8*e^8 + 196*a^2*b^6*c^2*d^8*e^8 - 1120*a^3*b^4*c^3*d^8*e^8 - 2170*a^4*b^2*c^4*d^8*e^8 - 280*a^5*c^5*d^8*e^8 - 8*a*b^9*d^7*e^9 - 136*a^2*b^7*c*d^7*e^9 + 112*a^3*b^5*c^2*d^7*e^9 + 1960*a^4*b^3*c^3*d^7*e^9 + 1120*a^5*b*c^4*d^7*e^9 + 28*a^2*b^8*d^6*e^10 + 168*a^3*b^6*c*d^6*e^10 - 700*a^4*b^4*c^2*d^6*e^10 - 1624*a^5*b^2*c^3*d^6*e^10 - 224*a^6*c^4*d^6*e^10 - 56*a^3*b^7*d^5*e^11 - 56*a^4*b^5*c*d^5*e^11 + 952*a^5*b^3*c^2*d^5*e^11 + 672*a^6*b*c^3*d^5*e^11 + 70*a^4*b^6*d^4*e^12 - 112*a^5*b^4*c*d^4*e^12 - 644*a^6*b^2*c^2*d^4*e^12 - 112*a^7*c^3*d^4*e^12 - 56*a^5*b^5*d^3*e^13 + 168*a^6*b^3*c*d^3*e^13 + 224*a^7*b*c^2*d^3*e^13 + 28*a^6*b^4*d^2*e^14 - 104*a^7*b^2*c*d^2*e^14 - 32*a^8*c^2*d^2*e^14 - 8*a^7*b^3*d*e^15 + 32*a^8*b*c*d*e^15 + a^8*b^2*e^16 - 4*a^9*c*e^16) + (b*c^8*d^12 - 8*b^2*c^7*d^11*e + 8*a*c^8*d^11*e + 28*b^3*c^6*d^10*e^2 - 46*a*b*c^7*d^10*e^2 - 56*b^4*c^5*d^9*e^3 + 108*a*b^2*c^6*d^9*e^3 + 24*a^2*c^7*d^9*e^3 + 70*b^5*c^4*d^8*e^4 - 125*a*b^3*c^5*d^8*e^4 - 125*a^2*b*c^6*d^8*e^4 - 56*b^6*c^3*d^7*e^5 + 56*a*b^4*c^4*d^7*e^5 + 272*a^2*b^2*c^5*d^7*e^5 + 16*a^3*c^6*d^7*e^5 + 28*b^7*c^2*d^6*e^6 + 28*a*b^5*c^3*d^6*e^6 - 308*a^2*b^3*c^4*d^6*e^6 - 84*a^3*b*c^5*d^6*e^6 - 8*b^8*c*d^5*e^7 - 48*a*b^6*c^2*d^5*e^7 + 176*a^2*b^4*c^3*d^5*e^7 + 184*a^3*b^2*c^4*d^5*e^7 - 16*a^4*c^5*d^5*e^7 + b^9*d^4*e^8 + 23*a*b^7*c*d^4*e^8 - 29*a^2*b^5*c^2*d^4*e^8 - 198*a^3*b^3*c^3*d^4*e^8 + 23*a^4*b*c^4*d^4*e^8 - 4*a*b^8*d^3*e^9 - 16*a^2*b^6*c*d^3*e^9 + 96*a^3*b^4*c^2*d^3*e^9 + 24*a^4*b^2*c^3*d^3*e^9 - 24*a^5*c^4*d^3*e^9 + 6*a^2*b^7*d^2*e^10 - 10*a^3*b^5*c*d^2*e^10 - 48*a^4*b^3*c^2*d^2*e^10 + 34*a^5*b*c^3*d^2*e^10 - 4*a^3*b^6*d*e^11 + 16*a^4*b^4*c*d*e^11 - 4*a^5*b^2*c^2*d*e^11 - 8*a^6*c^3*d*e^11 + a^4*b^5*e^12 - 5*a^5*b^3*c*e^12 + 5*a^6*b*c^2*e^12)/(b^2*c^8*d^16 - 4*a*c^9*d^16 - 8*b^3*c^7*d^15*e + 32*a*b*c^8*d^15*e + 28*b^4*c^6*d^14*e^2 - 104*a*b^2*c^7*d^14*e^2 - 32*a^2*c^8*d^14*e^2 - 56*b^5*c^5*d^13*e^3 + 168*a*b^3*c^6*d^13*e^3 + 224*a^2*b*c^7*d^13*e^3 + 70*b^6*c^4*d^12*e^4 - 112*a*b^4*c^5*d^12*e^4 - 644*a^2*b^2*c^6*d^12*e^4 - 112*a^3*c^7*d^12*e^4 - 56*b^7*c^3*d^11*e^5 - 56*a*b^5*c^4*d^11*e^5 + 952*a^2*b^3*c^5*d^11*e^5 + 672*a^3*b*c^6*d^11*e^5 + 28*b^8*c^2*d^10*e^6 + 168*a*b^6*c^3*d^10*e^6 - 700*a^2*b^4*c^4*d^10*e^6 - 1624*a^3*b^2*c^5*d^10*e^6 - 224*a^4*c^6*d^10*e^6 - 8*b^9*c*d^9*e^7 - 136*a*b^7*c^2*d^9*e^7 + 112*a^2*b^5*c^3*d^9*e^7 + 1960*a^3*b^3*c^4*d^9*e^7 + 1120*a^4*b*c^5*d^9*e^7 + b^10*d^8*e^8 + 52*a*b^8*c*d^8*e^8 + 196*a^2*b^6*c^2*d^8*e^8 - 1120*a^3*b^4*c^3*d^8*e^8 - 2170*a^4*b^2*c^4*d^8*e^8 - 280*a^5*c^5*d^8*e^8 - 8*a*b^9*d^7*e^9 - 136*a^2*b^7*c*d^7*e^9 + 112*a^3*b^5*c^2*d^7*e^9 + 1960*a^4*b^3*c^3*d^7*e^9 + 1120*a^5*b*c^4*d^7*e^9 + 28*a^2*b^8*d^6*e^10 + 168*a^3*b^6*c*d^6*e^10 - 700*a^4*b^4*c^2*d^6*e^10 - 1624*a^5*b^2*c^3*d^6*e^10 - 224*a^6*c^4*d^6*e^10 - 56*a^3*b^7*d^5*e^11 - 56*a^4*b^5*c*d^5*e^11 + 952*a^5*b^3*c^2*d^5*e^11 + 672*a^6*b*c^3*d^5*e^11 + 70*a^4*b^6*d^4*e^12

- 112*a^5*b^4*c*d^4*e^12 - 644*a^6*b^2*c^2*d^4*e^12 - 112*a^7*c^3*d^4*e^12 - 56*a^5*b^5*d^3*e^13 + 168*a^6*b^3*c*d^3*e^13 + 224*a^7*b*c^2*d^3*e^13 + 28*a^6*b^4*d^2*e^14 - 104*a^7*b^2*c*d^2*e^14 - 32*a^8*c^2*d^2*e^14 - 8*a^7*b^3*d*e^15 + 32*a^8*b*c*d*e^15 + a^8*b^2*e^16 - 4*a^9*c*e^16))/sqrt(c*x^2 + b*x + a) + 5/8*(32*c^3*d^3*e^2 - 48*b*c^2*d^2*e^3 + 30*b^2*c*d*e^4 - 24*a*c^2*d*e^4 - 7*b^3*e^5 + 12*a*b*c*e^5)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/((c^4*d^8 - 4*b*c^3*d^7*e + 6*b^2*c^2*d^6*e^2 + 4*a*c^3*d^6*e^2 - 4*b^3*c*d^5*e^3 - 12*a*b*c^2*d^5*e^3 + b^4*d^4*e^4 + 12*a*b^2*c*d^4*e^4 + 6*a^2*c^2*d^4*e^4 - 4*a*b^3*d^3*e^5 - 12*a^2*b*c*d^3*e^5 + 6*a^2*b^2*d^2*e^6 + 4*a^3*c*d^2*e^6 - 4*a^3*b*d*e^7 + a^4*e^8)*sqrt(-c*d^2 + b*d*e - a*e^2)) - 1/24*(1504*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^4*d^5*e^2 + 1296*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*c^(7/2)*d^4*e^3 + 2256*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(7/2)*d^5*e^2 + 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*c^3*d^3*e^4 - 1168*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c^3*d^4*e^3 + 1128*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c^3*d^5*e^2 - 1872*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b*c^(5/2)*d^3*e^4 - 2892*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c^(5/2)*d^4*e^3 - 3216*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(7/2)*d^4*e^3 + 188*b^3*c^(5/2)*d^5*e^2 - 432*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^2*d^2*e^5 - 60*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*c^2*d^3*e^4 - 2576*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c^3*d^3*e^4 - 1368*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*c^2*d^4*e^3 - 3216*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c^3*d^4*e^3 + 1098*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*c^(3/2)*d^2*e^5 - 936*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*c^(5/2)*d^2*e^5 + 1470*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^3*c^(3/2)*d^3*e^4 + 2568*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*c^(5/2)*d^3*e^4 - 188*b^4*c^(3/2)*d^4*e^3 - 804*a*b^2*c^(5/2)*d^4*e^3 + 258*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c*d*e^6 - 168*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*c^2*d*e^6 + 430*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^3*c*d^2*e^5 + 1992*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*c^2*d^2*e^5 + 612*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^4*c*d^3*e^4 + 3516*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*c^2*d^3*e^4 + 1968*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c^3*d^3*e^4 - 237*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^3*sqrt(c)*d*e^6 + 516*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b*c^(3/2)*d*e^6 - 264*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^4*sqrt(c)*d^2*e^5 - 1008*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^2*c^(3/2)*d^2*e^5 + 1152*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*c^(5/2)*d^2*e^5 + 57*b^5*sqrt(c)*d^3*e^4 + 794*a*b^3*c^(3/2)*d^3*e^4 + 984*a^2*b*c^(5/2)*d^3*e^4 - 57*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*e^7 + 84*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c*e^7 - 136*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^4*d*e^6 - 720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^2*c*d*e^6 + 480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*c^2*d*e^6 - 87*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^5*d^2*e^5 - 1494*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^3*c*d^2*e^5 - 1800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*c^2*d^2*e^5 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^2*sqrt(c)*e^7 + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^(3/2)*e^7 + 120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^3*sqrt(c)*d*e^6 - 432*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b*c^(3/2)*d*e^6 - 258*a*b^4*sqrt(c)*d^2*e^5 - 906*a^2*b^2*c^(3/2)*d^2*e^5 - 376*a^3*c^(5/2)*d^2*e^5 + 136*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*e^7 - 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c*e^7 + 174*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^4*d*e^6 + 918*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^2*c*d*e^6 - 312*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*c^2*d*e^6 + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^2*sqrt(c)*e^7 - 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*c^(3/2)*e^7 + 345*a^2*b^3*sqrt(c)*d*e^6 + 220*a^3*b*c^(3/2)*d*e^6 - 87*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^3*e^7 - 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b*c*e^7 - 144*a^3*b^2*sqrt(c)*e^7 + 80*a^4*c^(3/2)*e^7)/((c^4*d^8 - 4*b*c^3*d^7*e + 6*b^2*c^2*d^6*e^2 + 4*a*c^3*d^6*e^2 - 4*b^3*c*d^5*e^3 - 12*a*b*c^2*d^5*e^3 + b^4*d^4*e^4 + 12*a*b^2*c*d^4*e^4 + 6*a^2*c^2*d^4*e^4 - 4*a*b^3*d^3*e^5 - 12*a^2*b*c*d^3*e^5 + 6*a^2*b^2*d^2*e^6 + 4*a^3*c*d^2*e^6 - 4*a^3*b*d*e^7

```
+ a^4*e^8)*((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*e + 2*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*sqrt(c)*d + b*d - a*e)^3)
```

3.2391 $\int \frac{(d+ex)^5}{\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=389

$$-\frac{e\sqrt{a+bx+cx^2}\left(8c^2e^2\left(-16a^2e^2-25abde+b^2d^2\right)+2cex(2cd-be)\left(-4ce(2bd-7ae)-5b^2e^2+8c^2d^2\right)+10b^2ce^3(10a\right.}{3c^3\left(b^2-4ac\right)^2}$$

```
[Out] (-2*(d + e*x)^4*(b*d - 2*a*e + (2*c*d - b*e)*x))/(3*(b^2 - 4*a*c)*(a + b*x
+ c*x^2)^(3/2)) - (8*(d + e*x)^2*(8*a^2*c*e^3 - 2*b*c*d*(c*d^2 + 3*a*e^2) +
 b^2*(3*c*d^2*e - a*e^3) - (2*c*d - b*e)*(2*c^2*d^2 - b^2*e^2 - 2*c*e*(b*d
- 3*a*e))*x))/(3*c*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2]) - (e*(64*c^4*d^4
- 15*b^4*e^4 - 16*c^3*d^2*e*(7*b*d - 16*a*e) + 10*b^2*c*e^3*(3*b*d + 10*a*e
) + 8*c^2*e^2*(b^2*d^2 - 25*a*b*d*e - 16*a^2*e^2) + 2*c*e*(2*c*d - b*e)*(8*
c^2*d^2 - 5*b^2*e^2 - 4*c*e*(2*b*d - 7*a*e))*x)*Sqrt[a + b*x + c*x^2])/(3*c
^3*(b^2 - 4*a*c)^2) + (5*e^4*(2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + b*x + c*x^2])])/(2*c^(7/2))
```

Rubi [A] time = 0.465186, antiderivative size = 389, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {738, 818, 779, 621, 206}

$$-\frac{e\sqrt{a+bx+cx^2}\left(8c^2e^2\left(-16a^2e^2-25abde+b^2d^2\right)+2cex(2cd-be)\left(-4ce(2bd-7ae)-5b^2e^2+8c^2d^2\right)+10b^2ce^3(10a\right.}{3c^3\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^5/(a + b*x + c*x^2)^(5/2),x]
```


```
[Out] (-2*(d + e*x)^4*(b*d - 2*a*e + (2*c*d - b*e)*x))/(3*(b^2 - 4*a*c)*(a + b*x
+ c*x^2)^(3/2)) - (8*(d + e*x)^2*(8*a^2*c*e^3 - 2*b*c*d*(c*d^2 + 3*a*e^2) +
 b^2*(3*c*d^2*e - a*e^3) - (2*c*d - b*e)*(2*c^2*d^2 - b^2*e^2 - 2*c*e*(b*d
- 3*a*e))*x))/(3*c*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2]) - (e*(64*c^4*d^4
- 15*b^4*e^4 - 16*c^3*d^2*e*(7*b*d - 16*a*e) + 10*b^2*c*e^3*(3*b*d + 10*a*e
) + 8*c^2*e^2*(b^2*d^2 - 25*a*b*d*e - 16*a^2*e^2) + 2*c*e*(2*c*d - b*e)*(8*
c^2*d^2 - 5*b^2*e^2 - 4*c*e*(2*b*d - 7*a*e))*x)*Sqrt[a + b*x + c*x^2])/(3*c
^3*(b^2 - 4*a*c)^2) + (5*e^4*(2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + b*x + c*x^2])])/(2*c^(7/2))
```


Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
```

```
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^5}{\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^4(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\int \frac{(d+ex)^3\left(2\left(2cd^2-3bde+4ae^2\right)-2e(2cd-be)x\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx}{3\left(b^2-4ac\right)}$$

$$= -\frac{2(d+ex)^4(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{8(d+ex)^2\left(8a^2ce^3-2bcd\left(cd^2+3ae^2\right)+b^2\left(3cd^2e-ae^3 \right.\right.}{3c\left(b^2-4ac\right)^2\sqrt{a}}$$

$$= -\frac{2(d+ex)^4(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{8(d+ex)^2\left(8a^2ce^3-2bcd\left(cd^2+3ae^2\right)+b^2\left(3cd^2e-ae^3 \right.\right.}{3c\left(b^2-4ac\right)^2\sqrt{a}}$$

$$= -\frac{2(d+ex)^4(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{8(d+ex)^2\left(8a^2ce^3-2bcd\left(cd^2+3ae^2\right)+b^2\left(3cd^2e-ae^3 \right.\right.}{3c\left(b^2-4ac\right)^2\sqrt{a}}$$

$$= -\frac{2(d+ex)^4(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{8(d+ex)^2\left(8a^2ce^3-2bcd\left(cd^2+3ae^2\right)+b^2\left(3cd^2e-ae^3 \right.\right.}{3c\left(b^2-4ac\right)^2\sqrt{a}}$$

Mathematica [A] time = 1.89061, size = 566, normalized size = 1.46

$$\frac{-2b^3c\left(15a^2e^4(d+7ex)+2ace^4x^2(37ex-45d)+c^2d^2\left(15d^2ex+d^3-30de^2x^2-10e^3x^3\right)\right)+4b^2c\left(3a^2ce^4x(35d+4ex)-2 \right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^5/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (15*b^6*e^5*x^2 + 10*b^5*e^4*x*(3*a*e + c*x*(-3*d + 2*e*x)) + b^4*e^4*(15*a
^2*e + c^2*x^3*(-40*d + 3*e*x) - 30*a*c*x*(2*d + 3*e*x)) - 2*b^3*c*(15*a^2*
e^4*(d + 7*e*x) + 2*a*c*e^4*x^2*(-45*d + 37*e*x) + c^2*d^2*(d^3 + 15*d^2*e*
x - 30*d*e^2*x^2 - 10*e^3*x^3)) + 8*b*c^2*(2*c^3*d^4*x^2*(3*d - 5*e*x) + a^
3*e^4*(25*d + 39*e*x) + 3*a*c^2*d^2*(d^3 - 5*d^2*e*x + 10*d*e^2*x^2 - 10*e^
3*x^3) + 4*a^2*c*e^2*(5*d^3 - 15*d^2*e*x + 8*e^3*x^3)) + 4*b^2*c*(-25*a^3*e
^5 + 3*a^2*c*e^4*x*(35*d + 4*e*x) + c^3*d^3*x*(3*d^2 - 30*d*e*x + 10*e^2*x^
2) + a*c^2*e*(-5*d^4 + 60*d^3*e*x - 30*d^2*e^2*x^2 + 70*d*e^3*x^3 - 6*e^4*x
^4)) + 16*c^2*(8*a^4*e^5 + 2*c^4*d^5*x^3 + a*c^3*d^3*x*(3*d^2 + 10*e^2*x^2)
 + a^3*c*e^3*(-20*d^2 - 15*d*e*x + 12*e^2*x^2) + a^2*c^2*e*(-5*d^4 - 30*d^2
*e^2*x^2 - 20*d*e^3*x^3 + 3*e^4*x^4)))/(3*c^3*(b^2 - 4*a*c)^2*(a + x*(b + c
*x))^(3/2)) + (5*e^4*(2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
x*(b + c*x)])])/(2*c^(7/2))
```

Maple [B] time = 0.058, size = 2395, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 32/3*d^5*c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+5/2*e^5*b/c^3*x/(c*x^2+b*x
+a)^(1/2)+4*e^5*a/c^2*x^2/(c*x^2+b*x+a)^(3/2)-10*d^2*e^3*b/c*a/(4*a*c-b^2)/
(c*x^2+b*x+a)^(3/2)*x+20*d*e^4*b^2/c*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+
5/2*d*e^4*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x-5/3*d^4*e/c/(c*x^2+b*
x+a)^(3/2)+2/3*d^5/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*b+8/3*e^5*a^2/c^3/(c*x^2
+b*x+a)^(3/2)+e^5*x^4/c/(c*x^2+b*x+a)^(3/2)-5/2*e^5*b/c^(7/2)*ln((1/2*b+c*x
)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+5/96*e^5*b^4/c^5/(c*x^2+b*x+a)^(3/2)-5/4*e^5
*b^2/c^4/(c*x^2+b*x+a)^(1/2)+5*d*e^4/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))-5/24*d*e^4*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x-5/3*d*e^
4*b^4/c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+4*e^5*a^2/c^2*b/(4*a*c-b^2)/(
c*x^2+b*x+a)^(3/2)*x-19/12*e^5*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x-
80*d^2*e^3*b*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-40*d^2*e^3*b^2/c*a/(4*a*
c-b^2)^2/(c*x^2+b*x+a)^(1/2)+5/4*d*e^4*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^
(3/2)+10*d*e^4*b^3/c^2*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)+160/3*d^3*e^2*a*
c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-80/3*d^4*e*b*c/(4*a*c-b^2)^2/(c*x^2+b
*x+a)^(1/2)*x-40/3*d^4*e*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)+4/3*d^5/(4*a
*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*c-10*d^2*e^3*x^2/c/(c*x^2+b*x+a)^(3/2)+5/12*d
^2*e^3*b^2/c^3/(c*x^2+b*x+a)^(3/2)-20/3*d^2*e^3*a/c^2/(c*x^2+b*x+a)^(3/2)-5
*d^3*e^2*x/c/(c*x^2+b*x+a)^(3/2)+5/6*d^3*e^2*b/c^2/(c*x^2+b*x+a)^(3/2)-5/3*
d*e^4*x^3/c/(c*x^2+b*x+a)^(3/2)-5/48*d*e^4*b^3/c^4/(c*x^2+b*x+a)^(3/2)-5*d*
e^4/c^2*x/(c*x^2+b*x+a)^(1/2)+5/2*d*e^4/c^3*b/(c*x^2+b*x+a)^(1/2)+5/6*e^5*b
/c^2*x^3/(c*x^2+b*x+a)^(3/2)-5/4*e^5*b^2/c^3*x^2/(c*x^2+b*x+a)^(3/2)+16/3*d
^5*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*b-5/4*e^5*b^4/c^4/(4*a*c-b^2)/(c*x^2
+b*x+a)^(1/2)+5/3*d^3*e^2*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+10/3*d^3*
e^2*a/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*b-38/3*e^5*b^3/c^2*a/(4*a*c-b^2)^2/
(c*x^2+b*x+a)^(1/2)*x+32*e^5*a^2/c*b/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+5*
d*e^4/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+5/6*d^2*e^3*b^3/c^2/(4*a*c-
b^2)/(c*x^2+b*x+a)^(3/2)*x+20/3*d^2*e^3*b^3/c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(
1/2)*x-5*d^2*e^3*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)-5/16*e^5*b^3/c^4
*x/(c*x^2+b*x+a)^(3/2)+5/96*e^5*b^6/c^5/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+5/1
2*e^5*b^6/c^4/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)-e^5*b^2/c^4*a/(c*x^2+b*x+a)
```

```
^(3/2)+5/6*d^3*e^2*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+40/3*d^3*e^2*b^2
/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+20/3*d^3*e^2*b^3/c/(4*a*c-b^2)^2/(c*x^
2+b*x+a)^(1/2)+20/3*d^3*e^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x-5/6*d*e^4*b
^5/c^3/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)+5/3*d*e^4*b/c^3*a/(c*x^2+b*x+a)^(3
/2)+80/3*d^3*e^2*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*b-10/3*d^4*e*b/(4*a*c-
b^2)/(c*x^2+b*x+a)^(3/2)*x-5/3*d^4*e*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+
5/2*d*e^4*b/c^2*x^2/(c*x^2+b*x+a)^(3/2)+5/8*d*e^4*b^2/c^3*x/(c*x^2+b*x+a)^(
3/2)-5/48*d*e^4*b^5/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+5/2*d*e^4/c^3*b^3/(
4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-5/2*d^2*e^3*b/c^2*x/(c*x^2+b*x+a)^(3/2)+5/12
*d^2*e^3*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+10/3*d^2*e^3*b^4/c^2/(4*a*
c-b^2)^2/(c*x^2+b*x+a)^(1/2)-19/24*e^5*b^4/c^4*a/(4*a*c-b^2)/(c*x^2+b*x+a)^
(3/2)-19/3*e^5*b^4/c^3*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)-5/2*e^5*b^3/c^3/
(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+16*e^5*a^2/c^2*b^2/(4*a*c-b^2)^2/(c*x^2+b
*x+a)^(1/2)+e^5*a/c^3*b*x/(c*x^2+b*x+a)^(3/2)+5/6*e^5*b^5/c^3/(4*a*c-b^2)^2
/(c*x^2+b*x+a)^(1/2)*x+2*e^5*a^2/c^3*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+5/
48*e^5*b^5/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 15.039, size = 4651, normalized size = 11.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(15*(2*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d*e^4 - (a^2*b^5 - 8
*a^3*b^3*c + 16*a^4*b*c^2)*e^5 + (2*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d*
e^4 - (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e^5)*x^4 + 2*(2*(b^5*c^2 - 8*a
*b^3*c^3 + 16*a^2*b*c^4)*d*e^4 - (b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*e^5
)*x^3 + (2*(b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*d*e^4 - (b^7 - 6*a*b^5*c + 32
*a^3*b*c^3)*e^5)*x^2 + 2*(2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d*e^4
- (a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*e^5)*x)*sqrt(c)*log(-8*c^2*x^2 - 8
*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(16
0*a^2*b*c^4*d^3*e^2 - 320*a^3*c^4*d^2*e^3 + 3*(b^4*c^3 - 8*a*b^2*c^4 + 16*a
^2*c^5)*e^5*x^4 - 2*(b^3*c^4 - 12*a*b*c^5)*d^5 - 20*(a*b^2*c^4 + 4*a^2*c^5)
*d^4*e - 10*(3*a^2*b^3*c^2 - 20*a^3*b*c^3)*d*e^4 + (15*a^2*b^4*c - 100*a^3*
b^2*c^2 + 128*a^4*c^3)*e^5 + 4*(8*c^7*d^5 - 20*b*c^6*d^4*e + 10*(b^2*c^5 +
4*a*c^6)*d^3*e^2 + 5*(b^3*c^4 - 12*a*b*c^5)*d^2*e^3 - 10*(b^4*c^3 - 7*a*b^2
*c^4 + 8*a^2*c^5)*d*e^4 + (5*b^5*c^2 - 37*a*b^3*c^3 + 64*a^2*b*c^4)*e^5)*x^
3 + 3*(16*b*c^6*d^5 - 40*b^2*c^5*d^4*e + 20*(b^3*c^4 + 4*a*b*c^5)*d^3*e^2 -
 40*(a*b^2*c^4 + 4*a^2*c^5)*d^2*e^3 - 10*(b^5*c^2 - 6*a*b^3*c^3)*d*e^4 + (5
*b^6*c - 30*a*b^4*c^2 + 16*a^2*b^2*c^3 + 64*a^3*c^4)*e^5)*x^2 + 6*(40*a*b^2
*c^4*d^3*e^2 - 80*a^2*b*c^4*d^2*e^3 + 2*(b^2*c^5 + 4*a*c^6)*d^5 - 5*(b^3*c^
4 + 4*a*b*c^5)*d^4*e - 10*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 4*a^3*c^4)*d*e^4 + (
5*a*b^5*c - 35*a^2*b^3*c^2 + 52*a^3*b*c^3)*e^5)*x)*sqrt(c*x^2 + b*x + a))/(
```

```
a^2*b^4*c^4 - 8*a^3*b^2*c^5 + 16*a^4*c^6 + (b^4*c^6 - 8*a*b^2*c^7 + 16*a^2*
c^8)*x^4 + 2*(b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*x^3 + (b^6*c^4 - 6*a*b^
4*c^5 + 32*a^3*c^7)*x^2 + 2*(a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*x),
-1/6*(15*(2*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d*e^4 - (a^2*b^5 - 8*a
^3*b^3*c + 16*a^4*b*c^2)*e^5 + (2*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d*e^
4 - (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e^5)*x^4 + 2*(2*(b^5*c^2 - 8*a*b
^3*c^3 + 16*a^2*b*c^4)*d*e^4 - (b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*e^5)*
x^3 + (2*(b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*d*e^4 - (b^7 - 6*a*b^5*c + 32*a
^3*b*c^3)*e^5)*x^2 + 2*(2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d*e^4 -
(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*e^5)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x
^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(160*a^2*b*
c^4*d^3*e^2 - 320*a^3*c^4*d^2*e^3 + 3*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*
e^5*x^4 - 2*(b^3*c^4 - 12*a*b*c^5)*d^5 - 20*(a*b^2*c^4 + 4*a^2*c^5)*d^4*e -
 10*(3*a^2*b^3*c^2 - 20*a^3*b*c^3)*d*e^4 + (15*a^2*b^4*c - 100*a^3*b^2*c^2
+ 128*a^4*c^3)*e^5 + 4*(8*c^7*d^5 - 20*b*c^6*d^4*e + 10*(b^2*c^5 + 4*a*c^6)
*d^3*e^2 + 5*(b^3*c^4 - 12*a*b*c^5)*d^2*e^3 - 10*(b^4*c^3 - 7*a*b^2*c^4 + 8
*a^2*c^5)*d*e^4 + (5*b^5*c^2 - 37*a*b^3*c^3 + 64*a^2*b*c^4)*e^5)*x^3 + 3*(1
6*b*c^6*d^5 - 40*b^2*c^5*d^4*e + 20*(b^3*c^4 + 4*a*b*c^5)*d^3*e^2 - 40*(a*b
^2*c^4 + 4*a^2*c^5)*d^2*e^3 - 10*(b^5*c^2 - 6*a*b^3*c^3)*d*e^4 + (5*b^6*c -
 30*a*b^4*c^2 + 16*a^2*b^2*c^3 + 64*a^3*c^4)*e^5)*x^2 + 6*(40*a*b^2*c^4*d^3
*e^2 - 80*a^2*b*c^4*d^2*e^3 + 2*(b^2*c^5 + 4*a*c^6)*d^5 - 5*(b^3*c^4 + 4*a*
b*c^5)*d^4*e - 10*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 4*a^3*c^4)*d*e^4 + (5*a*b^5*
c - 35*a^2*b^3*c^2 + 52*a^3*b*c^3)*e^5)*x)*sqrt(c*x^2 + b*x + a))/(a^2*b^4*
c^4 - 8*a^3*b^2*c^5 + 16*a^4*c^6 + (b^4*c^6 - 8*a*b^2*c^7 + 16*a^2*c^8)*x^4
 + 2*(b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*x^3 + (b^6*c^4 - 6*a*b^4*c^5 +
32*a^3*c^7)*x^2 + 2*(a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5/(c*x**2+b*x+a)**(5/2),x)

[Out] Timed out
```

Giac [B] time = 1.18102, size = 1064, normalized size = 2.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")

[Out] 1/3*((((3*(b^4*c^2*e^5 - 8*a*b^2*c^3*e^5 + 16*a^2*c^4*e^5)*x/(b^4*c^3 - 8*a
*b^2*c^4 + 16*a^2*c^5) + 4*(8*c^6*d^5 - 20*b*c^5*d^4*e + 10*b^2*c^4*d^3*e^2
 + 40*a*c^5*d^3*e^2 + 5*b^3*c^3*d^2*e^3 - 60*a*b*c^4*d^2*e^3 - 10*b^4*c^2*d
*e^4 + 70*a*b^2*c^3*d*e^4 - 80*a^2*c^4*d*e^4 + 5*b^5*c*e^5 - 37*a*b^3*c^2*e
^5 + 64*a^2*b*c^3*e^5)/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))*x + 3*(16*b*c^
5*d^5 - 40*b^2*c^4*d^4*e + 20*b^3*c^3*d^3*e^2 + 80*a*b*c^4*d^3*e^2 - 40*a*b
^2*c^3*d^2*e^3 - 160*a^2*c^4*d^2*e^3 - 10*b^5*c*d*e^4 + 60*a*b^3*c^2*d*e^4
+ 5*b^6*e^5 - 30*a*b^4*c*e^5 + 16*a^2*b^2*c^2*e^5 + 64*a^3*c^3*e^5)/(b^4*c^
3 - 8*a*b^2*c^4 + 16*a^2*c^5))*x + 6*(2*b^2*c^4*d^5 + 8*a*c^5*d^5 - 5*b^3*c
```

```
^3*d^4*e - 20*a*b*c^4*d^4*e + 40*a*b^2*c^3*d^3*e^2 - 80*a^2*b*c^3*d^2*e^3 -
 10*a*b^4*c*d*e^4 + 70*a^2*b^2*c^2*d*e^4 - 40*a^3*c^3*d*e^4 + 5*a*b^5*e^5 -
 35*a^2*b^3*c*e^5 + 52*a^3*b*c^2*e^5)/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))
*x - (2*b^3*c^3*d^5 - 24*a*b*c^4*d^5 + 20*a*b^2*c^3*d^4*e + 80*a^2*c^4*d^4*
e - 160*a^2*b*c^3*d^3*e^2 + 320*a^3*c^3*d^2*e^3 + 30*a^2*b^3*c*d*e^4 - 200*
a^3*b*c^2*d*e^4 - 15*a^2*b^4*e^5 + 100*a^3*b^2*c*e^5 - 128*a^4*c^2*e^5)/(b^
4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))/(c*x^2 + b*x + a)^(3/2) - 5/2*(2*c*d*e^4
 - b*e^5)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7
/2)
```

3.2392 $\int \frac{(d+ex)^4}{\left(a+bx+cx^2\right)^{5/2}} \, dx$

Optimal. Leaf size=295

$$-\frac{2e\sqrt{a+bx+cx^2}(2cd-be)\left(-4ce(2bd-5ae)-3b^2e^2+8c^2d^2\right)}{3c^2\left(b^2-4ac\right)^2}+\frac{4(d+ex)\left(x(2cd-be)\left(-4ce(bd-2ae)-b^2e^2+4c^2d\right.\right.}{3c\left(b^2\right.}$$

```
[Out] (-2*(d + e*x)^3*(b*d - 2*a*e + (2*c*d - b*e)*x))/(3*(b^2 - 4*a*c)*(a + b*x
+ c*x^2)^(3/2)) + (4*(d + e*x)*(4*b*c*d*(c*d^2 + 3*a*e^2) - 4*a*c*e*(c*d^2
+ 3*a*e^2) - b^2*(5*c*d^2*e - a*e^3) + (2*c*d - b*e)*(4*c^2*d^2 - b^2*e^2 -
 4*c*e*(b*d - 2*a*e))*x))/(3*c*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2]) - (2*
e*(2*c*d - b*e)*(8*c^2*d^2 - 3*b^2*e^2 - 4*c*e*(2*b*d - 5*a*e))*Sqrt[a + b*
x + c*x^2])/(3*c^2*(b^2 - 4*a*c)^2) + (e^4*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + b*x + c*x^2])])/c^(5/2)
```

Rubi [A] time = 0.344603, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {738, 818, 640, 621, 206}

$$-\frac{2e\sqrt{a+bx+cx^2}(2cd-be)\left(-4ce(2bd-5ae)-3b^2e^2+8c^2d^2\right)}{3c^2\left(b^2-4ac\right)^2}+\frac{4(d+ex)\left(x(2cd-be)\left(-4ce(bd-2ae)-b^2e^2+4c^2d\right.\right.}{3c\left(b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^3*(b*d - 2*a*e + (2*c*d - b*e)*x))/(3*(b^2 - 4*a*c)*(a + b*x
+ c*x^2)^(3/2)) + (4*(d + e*x)*(4*b*c*d*(c*d^2 + 3*a*e^2) - 4*a*c*e*(c*d^2
+ 3*a*e^2) - b^2*(5*c*d^2*e - a*e^3) + (2*c*d - b*e)*(4*c^2*d^2 - b^2*e^2 -
 4*c*e*(b*d - 2*a*e))*x))/(3*c*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2]) - (2*
e*(2*c*d - b*e)*(8*c^2*d^2 - 3*b^2*e^2 - 4*c*e*(2*b*d - 5*a*e))*Sqrt[a + b*
x + c*x^2])/(3*c^2*(b^2 - 4*a*c)^2) + (e^4*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + b*x + c*x^2])])/c^(5/2)
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
```

```
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^4}{\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^3(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\int \frac{(d+ex)^2\left(4cd^2-e(5bd-6ae)-e(2cd-be)x\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx}{3\left(b^2-4ac\right)}$$

$$= -\frac{2(d+ex)^3(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} + \frac{4(d+ex)\left(4bcd\left(cd^2+3ae^2\right)-4ace\left(cd^2+3ae^2\right)-b^2\left(5\right.\right.}{3c\left(b^2-4ac\right)^2}$$

$$= -\frac{2(d+ex)^3(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} + \frac{4(d+ex)\left(4bcd\left(cd^2+3ae^2\right)-4ace\left(cd^2+3ae^2\right)-b^2\left(5\right.\right.}{3c\left(b^2-4ac\right)^2}$$

$$= -\frac{2(d+ex)^3(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} + \frac{4(d+ex)\left(4bcd\left(cd^2+3ae^2\right)-4ace\left(cd^2+3ae^2\right)-b^2\left(5\right.\right.}{3c\left(b^2-4ac\right)^2}$$

$$= -\frac{2(d+ex)^3(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} + \frac{4(d+ex)\left(4bcd\left(cd^2+3ae^2\right)-4ace\left(cd^2+3ae^2\right)-b^2\left(5\right.\right.}{3c\left(b^2-4ac\right)^2}$$

Mathematica [A] time = 0.862558, size = 397, normalized size = 1.35

$$\frac{e^4\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{5/2}} - \frac{2\left(b^3\left(3a^2e^4-18ace^4x^2+c^2d\left(12d^2ex+d^3-18de^2x^2-4e^3x^3\right)\right)-2b^2c\left(21a^2e^4x+2ace\left(18d^2\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*(3*b^5*e^4*x^2 + 2*b^4*e^4*x*(3*a + 2*c*x^2) + b^3*(3*a^2*e^4 - 18*a*c*
e^4*x^2 + c^2*d*(d^3 + 12*d^2*e*x - 18*d*e^2*x^2 - 4*e^3*x^3)) - 4*b*c*(5*a
^3*e^4 + 2*c^3*d^3*x^2*(3*d - 4*e*x) + 12*a^2*c*d*e^2*(d - 2*e*x) + 3*a*c^2
*d*(d^3 - 4*d^2*e*x + 6*d*e^2*x^2 - 4*e^3*x^3)) + 8*c^2*(-2*c^3*d^4*x^3 + a
^3*e^3*(8*d + 3*e*x) - 3*a*c^2*d^2*x*(d^2 + 2*e^2*x^2) + 4*a^2*c*e*(d^3 + 3
*d*e^2*x^2 + e^3*x^3)) - 2*b^2*c*(21*a^2*e^4*x + 3*c^2*d^2*x*(d^2 - 8*d*e*x
 + 2*e^2*x^2) + 2*a*c*e*(-2*d^3 + 18*d^2*e*x - 6*d*e^2*x^2 + 7*e^3*x^3))))/
(3*c^2*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^(3/2)) + (e^4*ArcTanh[(b + 2*c*x)/
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(5/2)
```

Maple [B] time = 0.053, size = 1550, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 8*d^2*e^2*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+4*d^2*e^2*b^3/c/(4*a*c-b^
2)^2/(c*x^2+b*x+a)^(1/2)-1/24*e^4*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x
-1/3*e^4*b^4/c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-4*d*e^3*b/c*a/(4*a*c-b
^2)/(c*x^2+b*x+a)^(3/2)*x+e^4/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^
(1/2))-1/3*e^4*x^3/c/(c*x^2+b*x+a)^(3/2)-1/48*e^4*b^3/c^4/(c*x^2+b*x+a)^(3/
2)-e^4/c^2*x/(c*x^2+b*x+a)^(1/2)+1/2*e^4/c^3*b/(c*x^2+b*x+a)^(1/2)-4/3*d^3*
e/c/(c*x^2+b*x+a)^(3/2)+2/3*d^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*b-4*d*e^3*x
^2/c/(c*x^2+b*x+a)^(3/2)+1/6*d*e^3*b^2/c^3/(c*x^2+b*x+a)^(3/2)-32*d*e^3*b*a
/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+1/2*e^4*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b
*x+a)^(3/2)*x+4*e^4*b^2/c*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-16*d*e^3*b^
2/c*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)+d^2*e^2*b^2/c/(4*a*c-b^2)/(c*x^2+b*
x+a)^(3/2)*x-2*d*e^3*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)-32/3*d^3*e*b
^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)+4/3*d^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2
)*x*c-3*d^2*e^2*x/c/(c*x^2+b*x+a)^(3/2)+1/2*d^2*e^2*b/c^2/(c*x^2+b*x+a)^(3/
2)-8/3*d*e^3*a/c^2/(c*x^2+b*x+a)^(3/2)+1/2*e^4*b/c^2*x^2/(c*x^2+b*x+a)^(3/2
)+1/8*e^4*b^2/c^3*x/(c*x^2+b*x+a)^(3/2)-1/48*e^4*b^5/c^4/(4*a*c-b^2)/(c*x^2
+b*x+a)^(3/2)-1/6*e^4*b^5/c^3/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)+1/3*e^4*b/c
^3*a/(c*x^2+b*x+a)^(3/2)+1/2*e^4/c^3*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+32
/3*d^4*c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+16/3*d^4*c/(4*a*c-b^2)^2/(c*
x^2+b*x+a)^(1/2)*b+2*d^2*e^2*a/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*b+32*d^2*e
^2*a*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-64/3*d^3*e*b*c/(4*a*c-b^2)^2/(c*
x^2+b*x+a)^(1/2)*x+1/3*d*e^3*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+8/3*
d*e^3*b^3/c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+1/2*d^2*e^2*b^3/c^2/(4*a*c-
b^2)/(c*x^2+b*x+a)^(3/2)+1/4*e^4*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+
2*e^4*b^3/c^2*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)+4*d^2*e^2*a/(4*a*c-b^2)/(
c*x^2+b*x+a)^(3/2)*x+16*d^2*e^2*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*b-8/3*d
^3*e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+4/3*d*e^3*b^4/c^2/(4*a*c-b^2)^2/(c
*x^2+b*x+a)^(1/2)-d*e^3*b/c^2*x/(c*x^2+b*x+a)^(3/2)+1/6*d*e^3*b^4/c^3/(4*a*
c-b^2)/(c*x^2+b*x+a)^(3/2)+e^4/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-4/
3*d^3*e*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 11.0915, size = 3106, normalized size = 10.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^4*x^4 + 2*(b^5*c - 8*a*b^3*
c^2 + 16*a^2*b*c^3)*e^4*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*e^4*x^2 + 2*(a
*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*e^4*x + (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*
c^2)*e^4)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*
(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*a^2*b*c^3*d^2*e^2 - 64*a^3*c^3*d*e^3 -
 (b^3*c^3 - 12*a*b*c^4)*d^4 - 8*(a*b^2*c^3 + 4*a^2*c^4)*d^3*e - (3*a^2*b^3*
c - 20*a^3*b*c^2)*e^4 + 4*(4*c^6*d^4 - 8*b*c^5*d^3*e + 3*(b^2*c^4 + 4*a*c^5
)*d^2*e^2 + (b^3*c^3 - 12*a*b*c^4)*d*e^3 - (b^4*c^2 - 7*a*b^2*c^3 + 8*a^2*c
^4)*e^4)*x^3 + 3*(8*b*c^5*d^4 - 16*b^2*c^4*d^3*e + 6*(b^3*c^3 + 4*a*b*c^4)*
d^2*e^2 - 8*(a*b^2*c^3 + 4*a^2*c^4)*d*e^3 - (b^5*c - 6*a*b^3*c^2)*e^4)*x^2
+ 6*(12*a*b^2*c^3*d^2*e^2 - 16*a^2*b*c^3*d*e^3 + (b^2*c^4 + 4*a*c^5)*d^4 -
2*(b^3*c^3 + 4*a*b*c^4)*d^3*e - (a*b^4*c - 7*a^2*b^2*c^2 + 4*a^3*c^3)*e^4)*
x)*sqrt(c*x^2 + b*x + a))/(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4*
c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^4 + 2*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c
^6)*x^3 + (b^6*c^3 - 6*a*b^4*c^4 + 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 - 8*a^2*b
^3*c^4 + 16*a^3*b*c^5)*x), -1/3*(3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^
4*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^4*x^3 + (b^6 - 6*a*b^4*c +
 32*a^3*c^3)*e^4*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*e^4*x + (a^2*
b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*e^4)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x +
 a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(48*a^2*b*c^3*d^2*e^2
 - 64*a^3*c^3*d*e^3 - (b^3*c^3 - 12*a*b*c^4)*d^4 - 8*(a*b^2*c^3 + 4*a^2*c^4
)*d^3*e - (3*a^2*b^3*c - 20*a^3*b*c^2)*e^4 + 4*(4*c^6*d^4 - 8*b*c^5*d^3*e +
 3*(b^2*c^4 + 4*a*c^5)*d^2*e^2 + (b^3*c^3 - 12*a*b*c^4)*d*e^3 - (b^4*c^2 -
7*a*b^2*c^3 + 8*a^2*c^4)*e^4)*x^3 + 3*(8*b*c^5*d^4 - 16*b^2*c^4*d^3*e + 6*(
b^3*c^3 + 4*a*b*c^4)*d^2*e^2 - 8*(a*b^2*c^3 + 4*a^2*c^4)*d*e^3 - (b^5*c - 6
*a*b^3*c^2)*e^4)*x^2 + 6*(12*a*b^2*c^3*d^2*e^2 - 16*a^2*b*c^3*d*e^3 + (b^2*
c^4 + 4*a*c^5)*d^4 - 2*(b^3*c^3 + 4*a*b*c^4)*d^3*e - (a*b^4*c - 7*a^2*b^2*c
^2 + 4*a^3*c^3)*e^4)*x)*sqrt(c*x^2 + b*x + a))/(a^2*b^4*c^3 - 8*a^3*b^2*c^4
 + 16*a^4*c^5 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^4 + 2*(b^5*c^4 - 8*a
*b^3*c^5 + 16*a^2*b*c^6)*x^3 + (b^6*c^3 - 6*a*b^4*c^4 + 32*a^3*c^6)*x^2 + 2
*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15889, size = 741, normalized size = 2.51

$$2\left(\left(\left(\frac{4\left(4c^5d^4-8bc^4d^3e+3b^2c^3d^2e^2+12ac^4d^2e^2+b^3c^2de^3-12abc^3de^3-b^4ce^4+7ab^2c^2e^4-8a^2c^3e^4\right)x}{b^4c^2-8ab^2c^3+16a^2c^4}+\frac{3\left(8bc^4d^4-16b^2c^3d^3e+6b^3c^2d^2e^2+24abc^3d^2e^2-8ab^2 \cdots\right.}{b^4c^2-8ab^2c^3+16a^2c \cdots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(((4*(4*c^5*d^4 - 8*b*c^4*d^3*e + 3*b^2*c^3*d^2*e^2 + 12*a*c^4*d^2*e^2
+ b^3*c^2*d*e^3 - 12*a*b*c^3*d*e^3 - b^4*c*e^4 + 7*a*b^2*c^2*e^4 - 8*a^2*c^
3*e^4)*x/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4) + 3*(8*b*c^4*d^4 - 16*b^2*c^3
*d^3*e + 6*b^3*c^2*d^2*e^2 + 24*a*b*c^3*d^2*e^2 - 8*a*b^2*c^2*d*e^3 - 32*a^
2*c^3*d*e^3 - b^5*e^4 + 6*a*b^3*c*e^4)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)
)*x + 6*(b^2*c^3*d^4 + 4*a*c^4*d^4 - 2*b^3*c^2*d^3*e - 8*a*b*c^3*d^3*e + 12
*a*b^2*c^2*d^2*e^2 - 16*a^2*b*c^2*d*e^3 - a*b^4*e^4 + 7*a^2*b^2*c*e^4 - 4*a
^3*c^2*e^4)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))*x - (b^3*c^2*d^4 - 12*a*b
*c^3*d^4 + 8*a*b^2*c^2*d^3*e + 32*a^2*c^3*d^3*e - 48*a^2*b*c^2*d^2*e^2 + 64
*a^3*c^2*d*e^3 + 3*a^2*b^3*e^4 - 20*a^3*b*c*e^4)/(b^4*c^2 - 8*a*b^2*c^3 + 1
6*a^2*c^4))/(c*x^2 + b*x + a)^(3/2) - e^4*log(abs(-2*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))*sqrt(c) - b))/c^(5/2)
```

3.2393 $\int \frac{(d+ex)^3}{(a+bx+cx^2)^{5/2}}\,dx$

Optimal. Leaf size=118

$$\frac{16\left(ae^2-bde+cd^2\right)(-2ae+x(2cd-be)+bd)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}-\frac{2(d+ex)^2(-2ae+x(2cd-be)+bd)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

```
[Out] (-2*(d + e*x)^2*(b*d - 2*a*e + (2*c*d - b*e)*x))/(3*(b^2 - 4*a*c)*(a + b*x
+ c*x^2)^(3/2)) + (16*(c*d^2 - b*d*e + a*e^2)*(b*d - 2*a*e + (2*c*d - b*e)*
x))/(3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.0416106, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {722, 636}

$$\frac{16\left(ae^2-bde+cd^2\right)(-2ae+x(2cd-be)+bd)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}-\frac{2(d+ex)^2(-2ae+x(2cd-be)+bd)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^2*(b*d - 2*a*e + (2*c*d - b*e)*x))/(3*(b^2 - 4*a*c)*(a + b*x
+ c*x^2)^(3/2)) + (16*(c*d^2 - b*d*e + a*e^2)*(b*d - 2*a*e + (2*c*d - b*e)*
x))/(3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2])
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^3}{\left(a+bx+cx^2\right)^{5/2}}\,dx &= -\frac{2(d+ex)^2(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}-\frac{\left(8\left(cd^2-bde+ae^2\right)\right)\int \frac{d+ex}{\left(a+bx+cx^2\right)^{3/2}}\,dx}{3\left(b^2-4ac\right)}\\ &= -\frac{2(d+ex)^2(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}+\frac{16\left(cd^2-bde+ae^2\right)(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}\end{aligned}$$

Mathematica [A] time = 0.816332, size = 190, normalized size = 1.61

$$\frac{2\left(12b(d-ex)\left(2a^2e^2+ac(d-ex)^2+2c^2d^2x^2\right)-8\left(3a^2ce\left(d^2+e^2x^2\right)+2a^3e^3-3ac^2dx\left(d^2+e^2x^2\right)-2c^3d^3x^3\right)-6b^2\left(\right.}{3\left(b^2-4ac\right)^2(a+x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(-6*b^2*(a*e - c*d*x)*(d^2 - 6*d*e*x + e^2*x^2) + b^3*(-d^3 - 9*d^2*e*x
+ 9*d*e^2*x^2 + e^3*x^3) + 12*b*(d - e*x)*(2*a^2*e^2 + 2*c^2*d^2*x^2 + a*c*
(d - e*x)^2) - 8*(2*a^3*e^3 - 2*c^3*d^3*x^3 + 3*a^2*c*e*(d^2 + e^2*x^2) - 3
*a*c^2*d*x*(d^2 + e^2*x^2))))/(3*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^(3/2))
```

Maple [B] time = 0.047, size = 296, normalized size = 2.5

$$-\frac{24\,abce^3x^3-48\,ac^2de^2x^3-2\,b^3e^3x^3-12\,b^2cde^2x^3+48\,bc^2d^2ex^3-32\,c^3d^3x^3+48\,a^2ce^3x^2+12\,ab^2e^3x^2-72\,abcde^2x^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] -2/3/(c*x^2+b*x+a)^(3/2)*(12*a*b*c*e^3*x^3-24*a*c^2*d*e^2*x^3-b^3*e^3*x^3-6
*b^2*c*d*e^2*x^3+24*b*c^2*d^2*e*x^3-16*c^3*d^3*x^3+24*a^2*c*e^3*x^2+6*a*b^2
*e^3*x^2-36*a*b*c*d*e^2*x^2-9*b^3*d*e^2*x^2+36*b^2*c*d^2*e*x^2-24*b*c^2*d^3
*x^2+24*a^2*b*e^3*x-36*a*b^2*d*e^2*x+36*a*b*c*d^2*e*x-24*a*c^2*d^3*x+9*b^3*
d^2*e*x-6*b^2*c*d^3*x+16*a^3*e^3-24*a^2*b*d*e^2+24*a^2*c*d^2*e+6*a*b^2*d^2*
e-12*a*b*c*d^3+b^3*d^3)/(16*a^2*c^2-8*a*b^2*c+b^4)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 8.75005, size = 783, normalized size = 6.64

$$\frac{2\left(24\,a^2bde^2-16\,a^3e^3-\left(b^3-12\,abc\right)d^3-6\left(ab^2+4\,a^2c\right)d^2e+\left(16\,c^3d^3-24\,bc^2d^2e+6\left(b^2c+4\,ac^2\right)de^2+\left(b^3-12\,abc\right.\right.}{3\left(a^2b^4-8\,a^3b^2c+16\,a^4c^2+\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)x^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(24*a^2*b*d*e^2 - 16*a^3*e^3 - (b^3 - 12*a*b*c)*d^3 - 6*(a*b^2 + 4*a^2*
c)*d^2*e + (16*c^3*d^3 - 24*b*c^2*d^2*e + 6*(b^2*c + 4*a*c^2)*d*e^2 + (b^3
```

```
- 12*a*b*c)*e^3)*x^3 + 3*(8*b*c^2*d^3 - 12*b^2*c*d^2*e + 3*(b^3 + 4*a*b*c)*
d*e^2 - 2*(a*b^2 + 4*a^2*c)*e^3)*x^2 + 3*(12*a*b^2*d*e^2 - 8*a^2*b*e^3 + 2*
(b^2*c + 4*a*c^2)*d^3 - 3*(b^3 + 4*a*b*c)*d^2*e)*x)*sqrt(c*x^2 + b*x + a)/(
a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x
^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 6*a*b^4*c + 32*a^3
*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.12298, size = 474, normalized size = 4.02

$$\frac{\left(\left(\frac{\left(16\,c^3d^3-24\,bc^2d^2e+6\,b^2cde^2+24\,ac^2de^2+b^3e^3-12\,abce^3\right)x}{b^4c^2-8\,ab^2c^3+16\,a^2c^4}+\frac{3\left(8\,bc^2d^3-12\,b^2cd^2e+3\,b^3de^2+12\,abcde^2-2\,ab^2e^3-8\,a^2ce^3\right)}{b^4c^2-8\,ab^2c^3+16\,a^2c^4}\right)x+\frac{3\left(2\,b^2cd^3+8\,ac^2d^3-3\,b^3d^2e-12\cdots\right.}{b^4c^2-8\,ab^2c\cdots}\right.}{3\left(cx^2+bx+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((((16*c^3*d^3 - 24*b*c^2*d^2*e + 6*b^2*c*d*e^2 + 24*a*c^2*d*e^2 + b^3*
e^3 - 12*a*b*c*e^3)*x/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4) + 3*(8*b*c^2*d^3
 - 12*b^2*c*d^2*e + 3*b^3*d*e^2 + 12*a*b*c*d*e^2 - 2*a*b^2*e^3 - 8*a^2*c*e^
3)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))*x + 3*(2*b^2*c*d^3 + 8*a*c^2*d^3 -
 3*b^3*d^2*e - 12*a*b*c*d^2*e + 12*a*b^2*d*e^2 - 8*a^2*b*e^3)/(b^4*c^2 - 8*
a*b^2*c^3 + 16*a^2*c^4))*x - (b^3*d^3 - 12*a*b*c*d^3 + 6*a*b^2*d^2*e + 24*a
^2*c*d^2*e - 24*a^2*b*d*e^2 + 16*a^3*e^3)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c
^4))/(c*x^2 + b*x + a)^(3/2)
```

3.2394 $\int \frac{(d+ex)^2}{(a+bx+cx^2)^{5/2}} dx$

Optimal. Leaf size=98

$$\frac{8(2cd-be)(-2ae+x(2cd-be)+bd)}{3(b^2-4ac)^2\sqrt{a+bx+cx^2}}-\frac{2(b+2cx)(d+ex)^2}{3(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

```
[Out] (-2*(b + 2*c*x)*(d + e*x)^2)/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) + (8
*(2*c*d - b*e)*(b*d - 2*a*e + (2*c*d - b*e)*x))/(3*(b^2 - 4*a*c)^2*Sqrt[a +
 b*x + c*x^2])
```

Rubi [A] time = 0.0374289, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {728, 636}

$$\frac{8(2cd-be)(-2ae+x(2cd-be)+bd)}{3(b^2-4ac)^2\sqrt{a+bx+cx^2}}-\frac{2(b+2cx)(d+ex)^2}{3(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*x + c*x^2)^(5/2),x]
```


```
[Out] (-2*(b + 2*c*x)*(d + e*x)^2)/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) + (8
*(2*c*d - b*e)*(b*d - 2*a*e + (2*c*d - b*e)*x))/(3*(b^2 - 4*a*c)^2*Sqrt[a +
 b*x + c*x^2])
```


Rule 728

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] + Dist[(m*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(
d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2}{(a+bx+cx^2)^{5/2}}dx &= -\frac{2(b+2cx)(d+ex)^2}{3(b^2-4ac)(a+bx+cx^2)^{3/2}}-\frac{(4(2cd-be))\int \frac{d+ex}{(a+bx+cx^2)^{3/2}}dx}{3(b^2-4ac)}\\ &= -\frac{2(b+2cx)(d+ex)^2}{3(b^2-4ac)(a+bx+cx^2)^{3/2}}+\frac{8(2cd-be)(bd-2ae+(2cd-be)x)}{3(b^2-4ac)^2\sqrt{a+bx+cx^2}}\end{aligned}$$

Mathematica [A] time = 0.730552, size = 167, normalized size = 1.7

$$\frac{2\left(4b\left(2a^2e^2+3ac(d-ex)^2+2c^2dx^2(3d-2ex)\right)+8c\left(-2a^2de+acx\left(3d^2+e^2x^2\right)+2c^2d^2x^3\right)+b^2\left(2cx\left(3d^2-12dex+e^2x\right.\right.\right.}{3\left(b^2-4ac\right)^2(a+x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(-(b^3*(d^2 + 6*d*e*x - 3*e^2*x^2)) + 4*b*(2*a^2*e^2 + 2*c^2*d*x^2*(3*d
- 2*e*x) + 3*a*c*(d - e*x)^2) + 8*c*(-2*a^2*d*e + 2*c^2*d^2*x^3 + a*c*x*(3*
d^2 + e^2*x^2)) + b^2*(-4*a*e*(d - 3*e*x) + 2*c*x*(3*d^2 - 12*d*e*x + e^2*x
^2))))/(3*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^(3/2))
```

Maple [B] time = 0.047, size = 215, normalized size = 2.2

$$\frac{16\,ac^2e^2x^3+4\,b^2ce^2x^3-32\,bc^2dex^3+32\,c^3d^2x^3+24\,abce^2x^2+6\,b^3e^2x^2-48\,b^2cdex^2+48\,bc^2d^2x^2+24\,ab^2e^2x-48\,abcd}{48\,a^2c^2-24\,acb^2+3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 2/3/(c*x^2+b*x+a)^(3/2)*(8*a*c^2*e^2*x^3+2*b^2*c*e^2*x^3-16*b*c^2*d*e*x^3+1
6*c^3*d^2*x^3+12*a*b*c*e^2*x^2+3*b^3*e^2*x^2-24*b^2*c*d*e*x^2+24*b*c^2*d^2*
x^2+12*a*b^2*e^2*x-24*a*b*c*d*e*x+24*a*c^2*d^2*x-6*b^3*d*e*x+6*b^2*c*d^2*x+
8*a^2*b*e^2-16*a^2*c*d*e-4*a*b^2*d*e+12*a*b*c*d^2-b^3*d^2)/(16*a^2*c^2-8*a*
b^2*c+b^4)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 8.82085, size = 644, normalized size = 6.57

$$\frac{2\left(8\,a^2be^2+2\left(8\,c^3d^2-8\,bc^2de+\left(b^2c+4\,ac^2\right)e^2\right)x^3-\left(b^3-12\,abc\right)d^2-4\left(ab^2+4\,a^2c\right)de+3\left(8\,bc^2d^2-8\,b^2cde+\left(b^3+4\right.\right.\right.}{3\left(a^2b^4-8\,a^3b^2c+16\,a^4c^2+\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)x^4+2\left(b^5c-8\,ab^3c^2+16\,a^2bc^3\right)x^3+\left(b^6-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(8*a^2*b*e^2 + 2*(8*c^3*d^2 - 8*b*c^2*d*e + (b^2*c + 4*a*c^2)*e^2)*x^3
- (b^3 - 12*a*b*c)*d^2 - 4*(a*b^2 + 4*a^2*c)*d*e + 3*(8*b*c^2*d^2 - 8*b^2*c
```

```
*d*e + (b^3 + 4*a*b*c)*e^2)*x^2 + 6*(2*a*b^2*e^2 + (b^2*c + 4*a*c^2)*d^2 -
(b^3 + 4*a*b*c)*d*e)*x)*sqrt(c*x^2 + b*x + a)/(a^2*b^4 - 8*a^3*b^2*c + 16*a
^4*c^2 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b^3*c^2
+ 16*a^2*b*c^3)*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b^5 - 8*a^2
*b^3*c + 16*a^3*b*c^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.11447, size = 387, normalized size = 3.95

$$\frac{\left(\left(\frac{2\left(8c^3d^2-8bc^2de+b^2ce^2+4ac^2e^2\right)x}{b^4c^2-8ab^2c^3+16a^2c^4}+\frac{3\left(8bc^2d^2-8b^2cde+b^3e^2+4abce^2\right)}{b^4c^2-8ab^2c^3+16a^2c^4}\right)x+\frac{6\left(b^2cd^2+4ac^2d^2-b^3de-4abcde+2ab^2e^2\right)}{b^4c^2-8ab^2c^3+16a^2c^4}\right)x-\frac{b^3d^2-12abcd^2+4ab^2de+}{b^4c^2-8ab^2c^3+}}{3\left(cx^2+bx+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(((2*(8*c^3*d^2 - 8*b*c^2*d*e + b^2*c*e^2 + 4*a*c^2*e^2)*x/(b^4*c^2 - 8
*a*b^2*c^3 + 16*a^2*c^4) + 3*(8*b*c^2*d^2 - 8*b^2*c*d*e + b^3*e^2 + 4*a*b*c
*e^2)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))*x + 6*(b^2*c*d^2 + 4*a*c^2*d^2
- b^3*d*e - 4*a*b*c*d*e + 2*a*b^2*e^2)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)
)*x - (b^3*d^2 - 12*a*b*c*d^2 + 4*a*b^2*d*e + 16*a^2*c*d*e - 8*a^2*b*e^2)/(
b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))/(c*x^2 + b*x + a)^(3/2)
```

3.2395 $\int \frac{d+ex}{\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=91

$$\frac{8(b+2cx)(2cd-be)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}-\frac{2(-2ae+x(2cd-be)+bd)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

```
[Out] (-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/
2)) + (8*(2*c*d - b*e)*(b + 2*c*x))/(3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2
])
```

Rubi [A] time = 0.0213004, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {638, 613}

$$\frac{8(b+2cx)(2cd-be)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}-\frac{2(-2ae+x(2cd-be)+bd)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/
2)) + (8*(2*c*d - b*e)*(b + 2*c*x))/(3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2
])
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex}{\left(a+bx+cx^2\right)^{5/2}}\,dx &= -\frac{2(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}-\frac{(4(2cd-be))\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}}\,dx}{3\left(b^2-4ac\right)}\\ &= -\frac{2(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}+\frac{8(2cd-be)(b+2cx)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}\end{aligned}$$

Mathematica [A] time = 0.12243, size = 110, normalized size = 1.21

$$-\frac{2\left(8c\left(a^2e-3acdx-2c^2dx^3\right)+2b^2(ae+3cx(2ex-d))+4bc\left(2cx^2(ex-3d)-3a(d-ex)\right)+b^3(d+3ex)\right)}{3\left(b^2-4ac\right)^2(a+x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*(b^3*(d + 3*e*x) + 8*c*(a^2*e - 3*a*c*d*x - 2*c^2*d*x^3) + 4*b*c*(-3*a*
(d - e*x) + 2*c*x^2*(-3*d + e*x)) + 2*b^2*(a*e + 3*c*x*(-d + 2*e*x))))/(3*(
b^2 - 4*a*c)^2*(a + x*(b + c*x))^(3/2))
```

Maple [A] time = 0.045, size = 131, normalized size = 1.4

$$-\frac{16\,bc^2ex^3-32\,c^3dx^3+24\,b^2cex^2-48\,bc^2dx^2+24\,abcex-48\,ac^2dx+6\,b^3ex-12\,b^2cdx+16\,a^2ce+4\,ab^2e-24\,cabd}{48\,a^2c^2-24\,acb^2+3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] -2/3/(c*x^2+b*x+a)^(3/2)*(8*b*c^2*e*x^3-16*c^3*d*x^3+12*b^2*c*e*x^2-24*b*c^
2*d*x^2+12*a*b*c*e*x-24*a*c^2*d*x+3*b^3*e*x-6*b^2*c*d*x+8*a^2*c*e+2*a*b^2*e
-12*a*b*c*d+b^3*d)/(16*a^2*c^2-8*a*b^2*c+b^4)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 8.44065, size = 521, normalized size = 5.73

$$\frac{2\left(8\left(2\,c^3d-bc^2e\right)x^3+12\left(2\,bc^2d-b^2ce\right)x^2-\left(b^3-12\,abc\right)d-2\left(ab^2+4\,a^2c\right)e+3\left(2\left(b^2c+4\,ac^2\right)d-\left(b^3+4\right.\right.\right.}{3\left(a^2b^4-8\,a^3b^2c+16\,a^4c^2+\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)x^4+2\left(b^5c-8\,ab^3c^2+16\,a^2bc^3\right)x^3+\left(b^6-6\,ab^4c+32\,a^3c^3\right)x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(8*(2*c^3*d - b*c^2*e)*x^3 + 12*(2*b*c^2*d - b^2*c*e)*x^2 - (b^3 - 12*a
*b*c)*d - 2*(a*b^2 + 4*a^2*c)*e + 3*(2*(b^2*c + 4*a*c^2)*d - (b^3 + 4*a*b*c
)*e)*x)*sqrt(c*x^2 + b*x + a)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^4*c^
2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*
x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*
b*c^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.12858, size = 306, normalized size = 3.36

$$\frac{\left(4\left(\frac{2\left(2c^3d-bc^2e\right)x}{b^4c^2-8ab^2c^3+16a^2c^4}+\frac{3\left(2bc^2d-b^2ce\right)}{b^4c^2-8ab^2c^3+16a^2c^4}\right)x+\frac{3\left(2b^2cd+8ac^2d-b^3e-4abce\right)}{b^4c^2-8ab^2c^3+16a^2c^4}\right)x-\frac{b^3d-12abcd+2ab^2e+8a^2ce}{b^4c^2-8ab^2c^3+16a^2c^4}}{3\left(cx^2+bx+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((4*(2*(2*c^3*d - b*c^2*e)*x/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4) + 3*(
2*b*c^2*d - b^2*c*e)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))*x + 3*(2*b^2*c*d
 + 8*a*c^2*d - b^3*e - 4*a*b*c*e)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))*x -
 (b^3*d - 12*a*b*c*d + 2*a*b^2*e + 8*a^2*c*e)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a
^2*c^4))/(c*x^2 + b*x + a)^(3/2)
```

3.2396 $\int \frac{1}{\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=70

$$\frac{16c(b+2cx)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}-\frac{2(b+2cx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

```
[Out] (-2*(b + 2*c*x))/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) + (16*c*(b + 2*c
*x))/(3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.0118909, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {614, 613}

$$\frac{16c(b+2cx)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}-\frac{2(b+2cx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(-5/2),x]
```

```
[Out] (-2*(b + 2*c*x))/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) + (16*c*(b + 2*c
*x))/(3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx+cx^2\right)^{5/2}}\,dx &= -\frac{2(b+2cx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}-\frac{(8c)\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}}\,dx}{3\left(b^2-4ac\right)}\\ &= -\frac{2(b+2cx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}+\frac{16c(b+2cx)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0285332, size = 57, normalized size = 0.81

$$\frac{2(b+2cx)\left(4c\left(3a+2cx^2\right)-b^2+8bcx\right)}{3\left(b^2-4ac\right)^2(a+x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(-5/2),x]
```

```
[Out] (2*(b + 2*c*x)*(-b^2 + 8*b*c*x + 4*c*(3*a + 2*c*x^2)))/(3*(b^2 - 4*a*c)^2*(
a + x*(b + c*x))^(3/2))
```

Maple [A] time = 0.043, size = 78, normalized size = 1.1

$$\frac{32\,c^3x^3+48\,bc^2x^2+48\,ac^2x+12\,b^2cx+24\,abc-2\,b^3}{48\,a^2c^2-24\,acb^2+3\,b^4}\left(cx^2+bx+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 2/3/(c*x^2+b*x+a)^(3/2)*(16*c^3*x^3+24*b*c^2*x^2+24*a*c^2*x+6*b^2*c*x+12*a*
b*c-b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.88551, size = 406, normalized size = 5.8

$$\frac{2\left(16\,c^3x^3+24\,bc^2x^2-b^3+12\,abc+6\left(b^2c+4\,ac^2\right)x\right)\sqrt{cx^2+bx+a}}{3\left(a^2b^4-8\,a^3b^2c+16\,a^4c^2+\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)x^4+2\left(b^5c-8\,ab^3c^2+16\,a^2bc^3\right)x^3+\left(b^6-6\,ab^4c+32\,a^3c^3\right)x^2+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(16*c^3*x^3 + 24*b*c^2*x^2 - b^3 + 12*a*b*c + 6*(b^2*c + 4*a*c^2)*x)*sq
rt(c*x^2 + b*x + a)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^4*c^2 - 8*a*b^
2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6
 - 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx+cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(-5/2), x)
```

Giac [B] time = 1.1344, size = 194, normalized size = 2.77

$$\frac{2\left(2\left(4\left(\frac{2c^3x}{b^4-8ab^2c+16a^2c^2}+\frac{3bc^2}{b^4-8ab^2c+16a^2c^2}\right)x+\frac{3\left(b^2c+4ac^2\right)}{b^4-8ab^2c+16a^2c^2}\right)x-\frac{b^3-12abc}{b^4-8ab^2c+16a^2c^2}\right)}{3\left(cx^2+bx+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(2*(4*(2*c^3*x/(b^4 - 8*a*b^2*c + 16*a^2*c^2) + 3*b*c^2/(b^4 - 8*a*b^2*
c + 16*a^2*c^2))*x + 3*(b^2*c + 4*a*c^2)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))*x
- (b^3 - 12*a*b*c)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))/(c*x^2 + b*x + a)^(3/2)
```

3.2397 $\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=310

$$-\frac{2\left(-cx(2cd-be)\left(-4ce(2bd-5ae)-3b^2e^2+8c^2d^2\right)-\left(2ace+b^2(-e)+bcd\right)\left(-4ce(bd-3ae)-3b^2e^2+8c^2d^2\right)+4ace(2c\right.}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)^2}$$

```
[Out] (-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/(3*(b^2 - 4*a*c)*(c*d^2
- b*d*e + a*e^2)*(a + b*x + c*x^2)^(3/2)) - (2*(4*a*c*e*(2*c*d - b*e)^2 - (
b*c*d - b^2*e + 2*a*c*e)*(8*c^2*d^2 - 3*b^2*e^2 - 4*c*e*(b*d - 3*a*e)) - c*
(2*c*d - b*e)*(8*c^2*d^2 - 3*b^2*e^2 - 4*c*e*(2*b*d - 5*a*e))*x))/(3*(b^2 -
 4*a*c)^2*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[a + b*x + c*x^2]) + (e^4*ArcTanh[(
b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x
+ c*x^2])])/(c*d^2 - b*d*e + a*e^2)^(5/2)
```

Rubi [A] time = 0.27333, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {740, 822, 12, 724, 206}

$$-\frac{2\left(-cx(2cd-be)\left(-4ce(2bd-5ae)-3b^2e^2+8c^2d^2\right)-\left(2ace+b^2(-e)+bcd\right)\left(-4ce(bd-3ae)-3b^2e^2+8c^2d^2\right)+4ace(2c\right.}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + b*x + c*x^2)^(5/2)),x]
```

```
[Out] (-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/(3*(b^2 - 4*a*c)*(c*d^2
- b*d*e + a*e^2)*(a + b*x + c*x^2)^(3/2)) - (2*(4*a*c*e*(2*c*d - b*e)^2 - (
b*c*d - b^2*e + 2*a*c*e)*(8*c^2*d^2 - 3*b^2*e^2 - 4*c*e*(b*d - 3*a*e)) - c*
(2*c*d - b*e)*(8*c^2*d^2 - 3*b^2*e^2 - 4*c*e*(2*b*d - 5*a*e))*x))/(3*(b^2 -
 4*a*c)^2*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[a + b*x + c*x^2]) + (e^4*ArcTanh[(
b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x
+ c*x^2])])/(c*d^2 - b*d*e + a*e^2)^(5/2)
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
```

```
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(8c^2d^2-3b^2e^2-4ce(bd-3ae)\right)+2ce(2\cdots}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}}{3\left(b^2-4ac\right)\left(cd^2-bde+\cdots\right.}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\left(4ace(2cd-be)^2-\left(bcd-b^2e\cdots\right.\right.}{\cdots}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\left(4ace(2cd-be)^2-\left(bcd-b^2e\cdots\right.\right.}{\cdots}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\left(4ace(2cd-be)^2-\left(bcd-b^2e\cdots\right.\right.}{\cdots}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\left(4ace(2cd-be)^2-\left(bcd-b^2e\cdots\right.\right.}{\cdots}$$

Mathematica [A] time = 0.503178, size = 309, normalized size = 1.

$$\frac{2\left(8c^2\left(3a^2e^3+5acde^2x+2c^2d^3x\right)+2b^2ce\left(cd(ex-6d)-11ae^2\right)+4bc^2\left(5ae^2(d-ex)+2cd^2(d-3ex)\right)+b^3ce^2(d+3e\cdots\right.}{3\left(b^2-4ac\right)^2\sqrt{a+x(b+cx)}\left(e(ae-bd)+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + b*x + c*x^2)^(5/2)),x]
```

```
[Out] (-2*(-(b^2*e) + 2*c*(a*e + c*d*x) + b*c*(d - e*x)))/(3*(b^2 - 4*a*c)*(c*d^2
 + e*(-(b*d) + a*e))*(a + x*(b + c*x))^(3/2)) + (2*(3*b^4*e^3 + b^3*c*e^2*(
d + 3*e*x) + 8*c^2*(3*a^2*e^3 + 2*c^2*d^3*x + 5*a*c*d*e^2*x) + 4*b*c^2*(2*c
*d^2*(d - 3*e*x) + 5*a*e^2*(d - e*x)) + 2*b^2*c*e*(-11*a*e^2 + c*d*(-6*d +
e*x))))/(3*(b^2 - 4*a*c)^2*(c*d^2 + e*(-(b*d) + a*e))^2*Sqrt[a + x*(b + c*x
)]) - (e^4*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(
b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/(c*d^2 + e*(-(b*d) + a*e))^(5/2)
```

Maple [B] time = 0.23, size = 1408, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 1/3*e/(a*e^2-b*d*e+c*d^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c
*d^2)/e^2)^(3/2)-2/3*e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*
c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)*x*c*b+4/3/(a*e^2-b*d*e+c*d^2)
/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3
/2)*x*c^2*d-1/3*e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/
e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)*b^2+2/3/(a*e^2-b*d*e+c*d^2)/(4*a*c
-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)*b*c
*d-16/3*e/(a*e^2-b*d*e+c*d^2)*c^2/(4*a*c-b^2)^2/((d/e+x)^2*c+(b*e-2*c*d)/e*
(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b+32/3/(a*e^2-b*d*e+c*d^2)*c^3/(4*
a*c-b^2)^2/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*x*d-8/3*e/(a*e^2-b*d*e+c*d^2)*c/(4*a*c-b^2)^2/((d/e+x)^2*c+(b*e-2*c*d)/e*
(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2+16/3/(a*e^2-b*d*e+c*d^2)*c^2/(4*
a*c-b^2)^2/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*b*d+e^3/(a*e^2-b*d*e+c*d^2)^2/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2)-2*e^3/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)/((d/e+x)^2*c
+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c+4*e^2/(a*e^2-b*
d*e+c*d^2)^2/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*
d^2)/e^2)^(1/2)*x*c^2*d-e^3/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)/((d/e+x)^2*c+
(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2+2*e^2/(a*e^2-b*d*e
+c*d^2)^2/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)*b*c*d-e^3/(a*e^2-b*d*e+c*d^2)^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1
/2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 27.2546, size = 9372, normalized size = 30.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^4*x^4 + 2*(b^5*c - 8*a*b^3*
c^2 + 16*a^2*b*c^3)*e^4*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*e^4*x^2 + 2*(a
*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*e^4*x + (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*
c^2)*e^4)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4
*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2
- b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*
(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2))
 - 4*((b^3*c^3 - 12*a*b*c^4)*d^5 - (3*b^4*c^2 - 34*a*b^2*c^3 + 8*a^2*c^4)*d
^4*e + (3*b^5*c - 27*a*b^3*c^2 - 20*a^2*b*c^3)*d^3*e^2 - (b^6 - 66*a^2*b^2*
c^2 + 40*a^3*c^3)*d^2*e^3 + (5*a*b^5 - 34*a^2*b^3*c + 16*a^3*b*c^2)*d*e^4 -
 4*(a^2*b^4 - 7*a^3*b^2*c + 8*a^4*c^2)*e^5 - (16*c^6*d^5 - 40*b*c^5*d^4*e +
 2*(13*b^2*c^4 + 28*a*c^5)*d^3*e^2 + (b^3*c^3 - 84*a*b*c^4)*d^2*e^3 - (3*b^
4*c^2 - 22*a*b^2*c^3 - 40*a^2*c^4)*d*e^4 + (3*a*b^3*c^2 - 20*a^2*b*c^3)*e^5
)*x^3 - 3*(8*b*c^5*d^5 - 20*b^2*c^4*d^4*e + (13*b^3*c^3 + 28*a*b*c^4)*d^3*e
^2 + (b^4*c^2 - 46*a*b^2*c^3 + 8*a^2*c^4)*d^2*e^3 - (2*b^5*c - 15*a*b^3*c^2
 - 12*a^2*b*c^3)*d*e^4 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 4*a^3*c^3)*e^5)*x^2 -
 3*(2*(b^2*c^4 + 4*a*c^5)*d^5 - 5*(b^3*c^3 + 4*a*b*c^4)*d^4*e + (3*b^4*c^2
+ 22*a*b^2*c^3 + 24*a^2*c^4)*d^3*e^2 + (b^5*c - 17*a*b^3*c^2 - 28*a^2*b*c^3
)*d^2*e^3 - (b^6 - 6*a*b^4*c - 8*a^2*b^2*c^2 - 16*a^3*c^3)*d*e^4 + (a*b^5 -
 6*a^2*b^3*c)*e^5)*x)*sqrt(c*x^2 + b*x + a))/((a^2*b^4*c^3 - 8*a^3*b^2*c^4
+ 16*a^4*c^5)*d^6 - 3*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^5*e +
3*(a^2*b^6*c - 7*a^3*b^4*c^2 + 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4*e^2 - (a^2*b
^7 - 2*a^3*b^5*c - 32*a^4*b^3*c^2 + 96*a^5*b*c^3)*d^3*e^3 + 3*(a^3*b^6 - 7*
a^4*b^4*c + 8*a^5*b^2*c^2 + 16*a^6*c^3)*d^2*e^4 - 3*(a^4*b^5 - 8*a^5*b^3*c
+ 16*a^6*b*c^2)*d*e^5 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*e^6 + ((b^4*c^
5 - 8*a*b^2*c^6 + 16*a^2*c^7)*d^6 - 3*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6
)*d^5*e + 3*(b^6*c^3 - 7*a*b^4*c^4 + 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^4*e^2 -
(b^7*c^2 - 2*a*b^5*c^3 - 32*a^2*b^3*c^4 + 96*a^3*b*c^5)*d^3*e^3 + 3*(a*b^6*
c^2 - 7*a^2*b^4*c^3 + 8*a^3*b^2*c^4 + 16*a^4*c^5)*d^2*e^4 - 3*(a^2*b^5*c^2
- 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d*e^5 + (a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a
^5*c^4)*e^6)*x^4 + 2*((b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*d^6 - 3*(b^6*c
^3 - 8*a*b^4*c^4 + 16*a^2*b^2*c^5)*d^5*e + 3*(b^7*c^2 - 7*a*b^5*c^3 + 8*a^2
*b^3*c^4 + 16*a^3*b*c^5)*d^4*e^2 - (b^8*c - 2*a*b^6*c^2 - 32*a^2*b^4*c^3 +
96*a^3*b^2*c^4)*d^3*e^3 + 3*(a*b^7*c - 7*a^2*b^5*c^2 + 8*a^3*b^3*c^3 + 16*a
^4*b*c^4)*d^2*e^4 - 3*(a^2*b^6*c - 8*a^3*b^4*c^2 + 16*a^4*b^2*c^3)*d*e^5 +
(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*e^6)*x^3 + ((b^6*c^3 - 6*a*b^4*c
^4 + 32*a^3*c^6)*d^6 - 3*(b^7*c^2 - 6*a*b^5*c^3 + 32*a^3*b*c^5)*d^5*e + 3*(
b^8*c - 5*a*b^6*c^2 - 6*a^2*b^4*c^3 + 32*a^3*b^2*c^4 + 32*a^4*c^5)*d^4*e^2
- (b^9 - 36*a^2*b^5*c^2 + 32*a^3*b^3*c^3 + 192*a^4*b*c^4)*d^3*e^3 + 3*(a*b^
8 - 5*a^2*b^6*c - 6*a^3*b^4*c^2 + 32*a^4*b^2*c^3 + 32*a^5*c^4)*d^2*e^4 - 3*
(a^2*b^7 - 6*a^3*b^5*c + 32*a^5*b*c^3)*d*e^5 + (a^3*b^6 - 6*a^4*b^4*c + 32*
a^6*c^3)*e^6)*x^2 + 2*((a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d^6 - 3*(
a*b^6*c^2 - 8*a^2*b^4*c^3 + 16*a^3*b^2*c^4)*d^5*e + 3*(a*b^7*c - 7*a^2*b^5*
c^2 + 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^4*e^2 - (a*b^8 - 2*a^2*b^6*c - 32*a^3
*b^4*c^2 + 96*a^4*b^2*c^3)*d^3*e^3 + 3*(a^2*b^7 - 7*a^3*b^5*c + 8*a^4*b^3*c
^2 + 16*a^5*b*c^3)*d^2*e^4 - 3*(a^3*b^6 - 8*a^4*b^4*c + 16*a^5*b^2*c^2)*d*e
^5 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*e^6)*x), 1/3*(3*((b^4*c^2 - 8*a
*b^2*c^3 + 16*a^2*c^4)*e^4*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^4
*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*e^4*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16
*a^3*b*c^2)*e^4*x + (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*e^4)*sqrt(-c*d^2 +
 b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x +
 a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2
 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)) - 2*((b^3*c^3
 - 12*a*b*c^4)*d^5 - (3*b^4*c^2 - 34*a*b^2*c^3 + 8*a^2*c^4)*d^4*e + (3*b^5*
c - 27*a*b^3*c^2 - 20*a^2*b*c^3)*d^3*e^2 - (b^6 - 66*a^2*b^2*c^2 + 40*a^3*c
```

```
^3)*d^2*e^3 + (5*a*b^5 - 34*a^2*b^3*c + 16*a^3*b*c^2)*d*e^4 - 4*(a^2*b^4 -
7*a^3*b^2*c + 8*a^4*c^2)*e^5 - (16*c^6*d^5 - 40*b*c^5*d^4*e + 2*(13*b^2*c^4
 + 28*a*c^5)*d^3*e^2 + (b^3*c^3 - 84*a*b*c^4)*d^2*e^3 - (3*b^4*c^2 - 22*a*b
^2*c^3 - 40*a^2*c^4)*d*e^4 + (3*a*b^3*c^2 - 20*a^2*b*c^3)*e^5)*x^3 - 3*(8*b
*c^5*d^5 - 20*b^2*c^4*d^4*e + (13*b^3*c^3 + 28*a*b*c^4)*d^3*e^2 + (b^4*c^2
- 46*a*b^2*c^3 + 8*a^2*c^4)*d^2*e^3 - (2*b^5*c - 15*a*b^3*c^2 - 12*a^2*b*c^
3)*d*e^4 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 4*a^3*c^3)*e^5)*x^2 - 3*(2*(b^2*c^4
 + 4*a*c^5)*d^5 - 5*(b^3*c^3 + 4*a*b*c^4)*d^4*e + (3*b^4*c^2 + 22*a*b^2*c^3
 + 24*a^2*c^4)*d^3*e^2 + (b^5*c - 17*a*b^3*c^2 - 28*a^2*b*c^3)*d^2*e^3 - (b
^6 - 6*a*b^4*c - 8*a^2*b^2*c^2 - 16*a^3*c^3)*d*e^4 + (a*b^5 - 6*a^2*b^3*c)*
e^5)*x)*sqrt(c*x^2 + b*x + a))/((a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5)*
d^6 - 3*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^5*e + 3*(a^2*b^6*c -
 7*a^3*b^4*c^2 + 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4*e^2 - (a^2*b^7 - 2*a^3*b^5
*c - 32*a^4*b^3*c^2 + 96*a^5*b*c^3)*d^3*e^3 + 3*(a^3*b^6 - 7*a^4*b^4*c + 8*
a^5*b^2*c^2 + 16*a^6*c^3)*d^2*e^4 - 3*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2
)*d*e^5 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*e^6 + ((b^4*c^5 - 8*a*b^2*c^
6 + 16*a^2*c^7)*d^6 - 3*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*d^5*e + 3*(b
^6*c^3 - 7*a*b^4*c^4 + 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^4*e^2 - (b^7*c^2 - 2*a
*b^5*c^3 - 32*a^2*b^3*c^4 + 96*a^3*b*c^5)*d^3*e^3 + 3*(a*b^6*c^2 - 7*a^2*b^
4*c^3 + 8*a^3*b^2*c^4 + 16*a^4*c^5)*d^2*e^4 - 3*(a^2*b^5*c^2 - 8*a^3*b^3*c^
3 + 16*a^4*b*c^4)*d*e^5 + (a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^6)*x
^4 + 2*((b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*d^6 - 3*(b^6*c^3 - 8*a*b^4*c
^4 + 16*a^2*b^2*c^5)*d^5*e + 3*(b^7*c^2 - 7*a*b^5*c^3 + 8*a^2*b^3*c^4 + 16*
a^3*b*c^5)*d^4*e^2 - (b^8*c - 2*a*b^6*c^2 - 32*a^2*b^4*c^3 + 96*a^3*b^2*c^4
)*d^3*e^3 + 3*(a*b^7*c - 7*a^2*b^5*c^2 + 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^2*
e^4 - 3*(a^2*b^6*c - 8*a^3*b^4*c^2 + 16*a^4*b^2*c^3)*d*e^5 + (a^3*b^5*c - 8
*a^4*b^3*c^2 + 16*a^5*b*c^3)*e^6)*x^3 + ((b^6*c^3 - 6*a*b^4*c^4 + 32*a^3*c^
6)*d^6 - 3*(b^7*c^2 - 6*a*b^5*c^3 + 32*a^3*b*c^5)*d^5*e + 3*(b^8*c - 5*a*b^
6*c^2 - 6*a^2*b^4*c^3 + 32*a^3*b^2*c^4 + 32*a^4*c^5)*d^4*e^2 - (b^9 - 36*a^
2*b^5*c^2 + 32*a^3*b^3*c^3 + 192*a^4*b*c^4)*d^3*e^3 + 3*(a*b^8 - 5*a^2*b^6*
c - 6*a^3*b^4*c^2 + 32*a^4*b^2*c^3 + 32*a^5*c^4)*d^2*e^4 - 3*(a^2*b^7 - 6*a
^3*b^5*c + 32*a^5*b*c^3)*d*e^5 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*e^6)*
x^2 + 2*((a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d^6 - 3*(a*b^6*c^2 - 8*
a^2*b^4*c^3 + 16*a^3*b^2*c^4)*d^5*e + 3*(a*b^7*c - 7*a^2*b^5*c^2 + 8*a^3*b^
3*c^3 + 16*a^4*b*c^4)*d^4*e^2 - (a*b^8 - 2*a^2*b^6*c - 32*a^3*b^4*c^2 + 96*
a^4*b^2*c^3)*d^3*e^3 + 3*(a^2*b^7 - 7*a^3*b^5*c + 8*a^4*b^3*c^2 + 16*a^5*b*
c^3)*d^2*e^4 - 3*(a^3*b^6 - 8*a^4*b^4*c + 16*a^5*b^2*c^2)*d*e^5 + (a^4*b^5
- 8*a^5*b^3*c + 16*a^6*b*c^2)*e^6)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Integral(1/((d + e*x)*(a + b*x + c*x**2)**(5/2)), x)
```

Giac [B] time = 1.74411, size = 4895, normalized size = 15.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")

[Out] 2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2 +
 b*d*e - a*e^2))*e^4/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2
- 2*a*b*d*e^3 + a^2*e^4)*sqrt(-c*d^2 + b*d*e - a*e^2)) + 1/3*((((16*c^11*d^
15 - 120*b*c^10*d^14*e + 386*b^2*c^9*d^13*e^2 + 136*a*c^10*d^13*e^2 - 689*b
^3*c^8*d^12*e^3 - 884*a*b*c^9*d^12*e^3 + 732*b^4*c^7*d^11*e^4 + 2412*a*b^2*
c^8*d^11*e^4 + 480*a^2*c^9*d^11*e^4 - 451*b^5*c^6*d^10*e^5 - 3542*a*b^3*c^7
*d^10*e^5 - 2640*a^2*b*c^8*d^10*e^5 + 130*b^6*c^5*d^9*e^6 + 2950*a*b^4*c^6*
d^9*e^6 + 5910*a^2*b^2*c^7*d^9*e^6 + 920*a^3*c^8*d^9*e^6 + 9*b^7*c^4*d^8*e^
7 - 1296*a*b^5*c^5*d^8*e^7 - 6795*a^2*b^3*c^6*d^8*e^7 - 4140*a^3*b*c^7*d^8*
e^7 - 16*b^8*c^3*d^7*e^8 + 184*a*b^6*c^4*d^7*e^8 + 4080*a^2*b^4*c^5*d^7*e^8
 + 7240*a^3*b^2*c^6*d^7*e^8 + 1040*a^4*c^7*d^7*e^8 + 3*b^9*c^2*d^6*e^9 + 58
*a*b^7*c^3*d^6*e^9 - 1050*a^2*b^5*c^4*d^6*e^9 - 6020*a^3*b^3*c^5*d^6*e^9 -
3640*a^4*b*c^6*d^6*e^9 - 18*a*b^8*c^2*d^5*e^10 - 30*a^2*b^6*c^3*d^5*e^10 +
2220*a^3*b^4*c^4*d^5*e^10 + 4590*a^4*b^2*c^5*d^5*e^10 + 696*a^5*c^6*d^5*e^1
0 + 45*a^2*b^7*c^2*d^4*e^11 - 160*a^3*b^5*c^3*d^4*e^11 - 2375*a^4*b^3*c^4*d
^4*e^11 - 1740*a^5*b*c^5*d^4*e^11 - 60*a^3*b^6*c^2*d^3*e^12 + 340*a^4*b^4*c
^3*d^3*e^12 + 1356*a^5*b^2*c^4*d^3*e^12 + 256*a^6*c^5*d^3*e^12 + 45*a^4*b^5
*c^2*d^2*e^13 - 294*a^5*b^3*c^3*d^2*e^13 - 384*a^6*b*c^4*d^2*e^13 - 18*a^5*
b^4*c^2*d*e^14 + 122*a^6*b^2*c^3*d*e^14 + 40*a^7*c^4*d*e^14 + 3*a^6*b^3*c^2
*e^15 - 20*a^7*b*c^3*e^15)*x/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4) + 3*(8*b*
c^10*d^15 - 60*b^2*c^9*d^14*e + 193*b^3*c^8*d^13*e^2 + 68*a*b*c^9*d^13*e^2
- 344*b^4*c^7*d^12*e^3 - 446*a*b^2*c^8*d^12*e^3 + 8*a^2*c^9*d^12*e^3 + 363*
b^5*c^6*d^11*e^4 + 1230*a*b^3*c^7*d^11*e^4 + 192*a^2*b*c^8*d^11*e^4 - 218*b
^6*c^5*d^10*e^5 - 1828*a*b^4*c^6*d^10*e^5 - 1224*a^2*b^2*c^7*d^10*e^5 + 48*
a^3*c^8*d^10*e^5 + 55*b^7*c^4*d^9*e^6 + 1540*a*b^5*c^5*d^9*e^6 + 2915*a^2*b
^3*c^6*d^9*e^6 + 220*a^3*b*c^7*d^9*e^6 + 12*b^8*c^3*d^8*e^7 - 678*a*b^6*c^4
*d^8*e^7 - 3510*a^2*b^4*c^5*d^8*e^7 - 1650*a^3*b^2*c^6*d^8*e^7 + 120*a^4*c^
7*d^8*e^7 - 11*b^9*c^2*d^7*e^8 + 86*a*b^7*c^3*d^7*e^8 + 2202*a^2*b^5*c^4*d^
7*e^8 + 3380*a^3*b^3*c^5*d^7*e^8 + 40*a^4*b*c^6*d^7*e^8 + 2*b^10*c*d^6*e^9
+ 40*a*b^8*c^2*d^6*e^9 - 592*a^2*b^6*c^3*d^6*e^9 - 3120*a^3*b^4*c^4*d^6*e^9
 - 1180*a^4*b^2*c^5*d^6*e^9 + 160*a^5*c^6*d^6*e^9 - 12*a*b^9*c*d^5*e^10 - 2
1*a^2*b^7*c^2*d^5*e^10 + 1272*a^3*b^5*c^3*d^5*e^10 + 2055*a^4*b^3*c^4*d^5*e
^10 - 132*a^5*b*c^5*d^5*e^10 + 30*a^2*b^8*c*d^4*e^11 - 110*a^3*b^6*c^2*d^4*
e^11 - 1300*a^4*b^4*c^3*d^4*e^11 - 450*a^5*b^2*c^4*d^4*e^11 + 120*a^6*c^5*d
^4*e^11 - 40*a^3*b^7*c*d^3*e^12 + 235*a^4*b^5*c^2*d^3*e^12 + 638*a^5*b^3*c^
3*d^3*e^12 - 112*a^6*b*c^4*d^3*e^12 + 30*a^4*b^6*c*d^2*e^13 - 204*a^5*b^4*c
^2*d^2*e^13 - 96*a^6*b^2*c^3*d^2*e^13 + 48*a^7*c^4*d^2*e^13 - 12*a^5*b^5*c*
d*e^14 + 85*a^6*b^3*c^2*d*e^14 - 28*a^7*b*c^3*d*e^14 + 2*a^6*b^4*c*e^15 - 1
4*a^7*b^2*c^2*e^15 + 8*a^8*c^3*e^15)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))*
x + 3*(2*b^2*c^9*d^15 + 8*a*c^10*d^15 - 15*b^3*c^8*d^14*e - 60*a*b*c^9*d^14
*e + 48*b^4*c^7*d^13*e^2 + 212*a*b^2*c^8*d^13*e^2 + 64*a^2*c^9*d^13*e^2 - 8
4*b^5*c^6*d^12*e^3 - 472*a*b^3*c^7*d^12*e^3 - 408*a^2*b*c^8*d^12*e^3 + 84*b
^6*c^5*d^11*e^4 + 726*a*b^4*c^6*d^11*e^4 + 1158*a^2*b^2*c^7*d^11*e^4 + 216*
a^3*c^8*d^11*e^4 - 42*b^7*c^4*d^10*e^5 - 772*a*b^5*c^5*d^10*e^5 - 1961*a^2*
b^3*c^6*d^10*e^5 - 1140*a^3*b*c^7*d^10*e^5 + 530*a*b^6*c^4*d^9*e^6 + 2220*a
^2*b^4*c^5*d^9*e^6 + 2560*a^3*b^2*c^6*d^9*e^6 + 400*a^4*c^7*d^9*e^6 + 12*b^
9*c^2*d^8*e^7 - 192*a*b^7*c^3*d^8*e^7 - 1719*a^2*b^5*c^4*d^8*e^7 - 3270*a^3
*b^3*c^5*d^8*e^7 - 1680*a^4*b*c^6*d^8*e^7 - 6*b^10*c*d^7*e^8 + 2*a*b^8*c^2*
d^7*e^8 + 838*a^2*b^6*c^3*d^7*e^8 + 2700*a^3*b^4*c^4*d^7*e^8 + 2830*a^4*b^2
*c^5*d^7*e^8 + 440*a^5*c^6*d^7*e^8 + b^11*d^6*e^9 + 24*a*b^9*c*d^6*e^9 - 18
3*a^2*b^7*c^2*d^6*e^9 - 1496*a^3*b^5*c^3*d^6*e^9 - 2545*a^4*b^3*c^4*d^6*e^9
 - 1380*a^5*b*c^5*d^6*e^9 - 6*a*b^10*d^5*e^10 - 24*a^2*b^8*c*d^5*e^10 + 480
*a^3*b^6*c^2*d^5*e^10 + 1440*a^4*b^4*c^3*d^5*e^10 + 1572*a^5*b^2*c^4*d^5*e^
10 + 288*a^6*c^5*d^5*e^10 + 15*a^2*b^9*d^4*e^11 - 30*a^3*b^7*c*d^4*e^11 - 5
50*a^4*b^5*c^2*d^4*e^11 - 860*a^5*b^3*c^3*d^4*e^11 - 600*a^6*b*c^4*d^4*e^11
 - 20*a^3*b^8*d^3*e^12 + 90*a^4*b^6*c*d^3*e^12 + 318*a^5*b^4*c^2*d^3*e^12 +
 362*a^6*b^2*c^3*d^3*e^12 + 104*a^7*c^4*d^3*e^12 + 15*a^4*b^7*d^2*e^13 - 84
```

```
*a^5*b^5*c*d^2*e^13 - 87*a^6*b^3*c^2*d^2*e^13 - 108*a^7*b*c^3*d^2*e^13 - 6*
a^5*b^6*d*e^14 + 36*a^6*b^4*c*d*e^14 + 8*a^7*b^2*c^2*d*e^14 + 16*a^8*c^3*d*
e^14 + a^6*b^5*e^15 - 6*a^7*b^3*c*e^15)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4
))*x - (b^3*c^8*d^15 - 12*a*b*c^9*d^15 - 8*b^4*c^7*d^14*e + 94*a*b^2*c^8*d^
14*e - 8*a^2*c^9*d^14*e + 28*b^5*c^6*d^13*e^2 - 312*a*b^3*c^7*d^13*e^2 - 40
*a^2*b*c^8*d^13*e^2 - 56*b^6*c^5*d^12*e^3 + 560*a*b^4*c^6*d^12*e^3 + 496*a^
2*b^2*c^7*d^12*e^3 - 80*a^3*c^8*d^12*e^3 + 70*b^7*c^4*d^11*e^4 - 560*a*b^5*
c^5*d^11*e^4 - 1649*a^2*b^3*c^6*d^11*e^4 + 156*a^3*b*c^7*d^11*e^4 - 56*b^8*
c^3*d^10*e^5 + 252*a*b^6*c^4*d^10*e^5 + 2726*a^2*b^4*c^5*d^10*e^5 + 738*a^3
*b^2*c^6*d^10*e^5 - 312*a^4*c^7*d^10*e^5 + 28*b^9*c^2*d^9*e^6 + 56*a*b^7*c^
3*d^9*e^6 - 2447*a^2*b^5*c^4*d^9*e^6 - 3150*a^3*b^3*c^5*d^9*e^6 + 960*a^4*b
*c^6*d^9*e^6 - 8*b^10*c*d^8*e^7 - 128*a*b^8*c^2*d^8*e^7 + 1060*a^2*b^6*c^3*
d^8*e^7 + 4820*a^3*b^4*c^4*d^8*e^7 - 100*a^4*b^2*c^5*d^8*e^7 - 640*a^5*c^6*
d^8*e^7 + b^11*d^7*e^8 + 60*a*b^9*c*d^7*e^8 - 31*a^2*b^7*c^2*d^7*e^8 - 3520
*a^3*b^5*c^3*d^7*e^8 - 2865*a^4*b^3*c^4*d^7*e^8 + 1900*a^5*b*c^5*d^7*e^8 -
10*a*b^10*d^6*e^9 - 146*a^2*b^8*c*d^6*e^9 + 1034*a^3*b^6*c^2*d^6*e^9 + 4180
*a^4*b^4*c^3*d^6*e^9 - 1150*a^5*b^2*c^4*d^6*e^9 - 760*a^6*c^5*d^6*e^9 + 39*
a^2*b^9*d^5*e^10 + 82*a^3*b^7*c*d^5*e^10 - 2198*a^4*b^5*c^2*d^5*e^10 - 1484
*a^5*b^3*c^3*d^5*e^10 + 1848*a^6*b*c^4*d^5*e^10 - 80*a^3*b^8*d^4*e^11 + 240
*a^4*b^6*c*d^4*e^11 + 1952*a^5*b^4*c^2*d^4*e^11 - 936*a^6*b^2*c^3*d^4*e^11
- 528*a^7*c^4*d^4*e^11 + 95*a^4*b^7*d^3*e^12 - 512*a^5*b^5*c*d^3*e^12 - 607
*a^6*b^3*c^2*d^3*e^12 + 900*a^7*b*c^3*d^3*e^12 - 66*a^5*b^6*d^2*e^13 + 430*
a^6*b^4*c*d^2*e^13 - 194*a^7*b^2*c^2*d^2*e^13 - 200*a^8*c^3*d^2*e^13 + 25*a
^6*b^5*d*e^14 - 174*a^7*b^3*c*d*e^14 + 176*a^8*b*c^2*d*e^14 - 4*a^7*b^4*e^1
5 + 28*a^8*b^2*c*e^15 - 32*a^9*c^2*e^15)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4))/(c*x^2 + b*x + a)^(3/2)
```

3.2398 $\int \frac{1}{(d+ex)^2\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=473

$$\frac{e\sqrt{a+bx+cx^2}\left(4c^2e^2\left(-32a^2e^2-36abde+3b^2d^2\right)+20b^2ce^3(5ae+bd)-16c^3d^2e(4bd-9ae)-15b^4e^4+32c^4d^4\right)}{3\left(b^2-4ac\right)^2(d+ex)\left(ae^2-bde+cd^2\right)^3}-\frac{2}{}$$

```
[Out] (-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/(3*(b^2 - 4*a*c)*(c*d^2
- b*d*e + a*e^2)*(d + e*x)*(a + b*x + c*x^2)^(3/2)) - (2*(6*a*c*e*(2*c*d -
b*e)^2 - (b*c*d - b^2*e + 2*a*c*e)*(8*c^2*d^2 - 5*b^2*e^2 - 2*c*e*(b*d - 8*
a*e)) - c*(2*c*d - b*e)*(8*c^2*d^2 - 5*b^2*e^2 - 4*c*e*(2*b*d - 7*a*e))*x))
/(3*(b^2 - 4*a*c)^2*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)*Sqrt[a + b*x + c*x^
2]) + (e*(32*c^4*d^4 - 15*b^4*e^4 - 16*c^3*d^2*e*(4*b*d - 9*a*e) + 20*b^2*c
*e^3*(b*d + 5*a*e) + 4*c^2*e^2*(3*b^2*d^2 - 36*a*b*d*e - 32*a^2*e^2))*Sqrt[
a + b*x + c*x^2])/(3*(b^2 - 4*a*c)^2*(c*d^2 - b*d*e + a*e^2)^3*(d + e*x)) +
 (5*e^4*(2*c*d - b*e)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2
 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(2*(c*d^2 - b*d*e + a*e^2)^(7/2)
)
```

Rubi [A] time = 0.614158, antiderivative size = 473, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {740, 822, 806, 724, 206}

$$\frac{e\sqrt{a+bx+cx^2}\left(4c^2e^2\left(-32a^2e^2-36abde+3b^2d^2\right)+20b^2ce^3(5ae+bd)-16c^3d^2e(4bd-9ae)-15b^4e^4+32c^4d^4\right)}{3\left(b^2-4ac\right)^2(d+ex)\left(ae^2-bde+cd^2\right)^3}-\frac{2}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + b*x + c*x^2)^(5/2)),x]
```


```
[Out] (-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/(3*(b^2 - 4*a*c)*(c*d^2
- b*d*e + a*e^2)*(d + e*x)*(a + b*x + c*x^2)^(3/2)) - (2*(6*a*c*e*(2*c*d -
b*e)^2 - (b*c*d - b^2*e + 2*a*c*e)*(8*c^2*d^2 - 5*b^2*e^2 - 2*c*e*(b*d - 8*
a*e)) - c*(2*c*d - b*e)*(8*c^2*d^2 - 5*b^2*e^2 - 4*c*e*(2*b*d - 7*a*e))*x))
/(3*(b^2 - 4*a*c)^2*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)*Sqrt[a + b*x + c*x^
2]) + (e*(32*c^4*d^4 - 15*b^4*e^4 - 16*c^3*d^2*e*(4*b*d - 9*a*e) + 20*b^2*c
*e^3*(b*d + 5*a*e) + 4*c^2*e^2*(3*b^2*d^2 - 36*a*b*d*e - 32*a^2*e^2))*Sqrt[
a + b*x + c*x^2])/(3*(b^2 - 4*a*c)^2*(c*d^2 - b*d*e + a*e^2)^3*(d + e*x)) +
 (5*e^4*(2*c*d - b*e)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2
 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(2*(c*d^2 - b*d*e + a*e^2)^(7/2)
)
```


Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(8c^2d^2-5b^2e^2-2ce(bd-8ae)}{(d+ex)^2\left(a+bx+cx\right.}}{3\left(b^2-4ac\right)\left(cd^2-b\right.}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\left(6ace(2cd-be)^2-\left(bcd\right.\right.}{}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\left(6ace(2cd-be)^2-\left(bcd\right.\right.}{}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\left(6ace(2cd-be)^2-\left(bcd\right.\right.}{}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\left(6ace(2cd-be)^2-\left(bcd\right.\right.}{}$$

Mathematica [A] time = 1.93774, size = 482, normalized size = 1.02

$$\frac{2\left(\frac{e\sqrt{a+x(b+cx)}\left(-4c^2e^2\left(32a^2e^2+36abde-3b^2d^2\right)+20b^2ce^3(5ae+bd)-16c^3d^2e(4bd-9ae)-15b^4e^4+32c^4d^4\right)}{2\left(b^2-4ac\right)(d+ex)\left(e(ae-bd)+cd^2\right)^2}+\frac{-8c^2\left(4a^2e^3-acde(d-7ex)+2c^2d^3x\right)+2b^2ce\left(16ae^2\right.}{\left(b^2-4ac\right)(d}\right.}{3\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + b*x + c*x^2)^(5/2)),x]
```

```
[Out] (2*((e*(32*c^4*d^4 - 15*b^4*e^4 - 16*c^3*d^2*e*(4*b*d - 9*a*e) + 20*b^2*c*e
^3*(b*d + 5*a*e) - 4*c^2*e^2*(-3*b^2*d^2 + 36*a*b*d*e + 32*a^2*e^2))*Sqrt[a
 + x*(b + c*x)])/(2*(b^2 - 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))^2*(d + e*x)) +
 (b^2*e - 2*c*(a*e + c*d*x) + b*c*(-d + e*x))/((d + e*x)*(a + x*(b + c*x))^
(3/2)) + (-5*b^4*e^3 + b^3*c*e^2*(3*d - 5*e*x) - 8*c^2*(4*a^2*e^3 + 2*c^2*d
^3*x - a*c*d*e*(d - 7*e*x)) - 4*b*c^2*(a*e^2*(9*d - 7*e*x) + 2*c*d^2*(d - 3
*e*x)) + 2*b^2*c*e*(16*a*e^2 + c*d*(5*d + e*x)))/((b^2 - 4*a*c)*(-(c*d^2) +
 e*(b*d - a*e))*(d + e*x)*Sqrt[a + x*(b + c*x)]) + (15*(b^2 - 4*a*c)*e^4*(-
2*c*d + b*e)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(
-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/(4*(c*d^2 + e*(-(b*d) + a*e))^(5/2)
)))/(3*(b^2 - 4*a*c)*(c*d^2 + e*(-(b*d) + a*e)))
```

Maple [B] time = 0.239, size = 2765, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] -5/2*e^4/(a*e^2-b*d*e+c*d^2)^3/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*b-5/6*e^2/(a*e^2-b*d*e+c*d^2)^2/((d/e+x)^2*c+(b*e-2*c
*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)*b+5/3*e/(a*e^2-b*d*e+c*d^2)^2/
((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)*c*d-20/3*
e/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e
^2-b*d*e+c*d^2)/e^2)^(3/2)*x*c^2*b*d-20*e^3/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^
2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c^
2*d-160/3*e/(a*e^2-b*d*e+c*d^2)^2*c^3/(4*a*c-b^2)^2/((d/e+x)^2*c+(b*e-2*c*d
)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*d-1/(a*e^2-b*d*e+c*d^2)/(d/e
+x)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)+5/6*e
^2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*
e^2-b*d*e+c*d^2)/e^2)^(3/2)*b^3+5*e^3/(a*e^2-b*d*e+c*d^2)^3/((d/e+x)^2*c+(b
*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*c*d-16/3*c^2/(a*e^2-b*d*
e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)
/e^2)^(3/2)*x-8/3*c/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d
)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)*b+5/2*e^4/(a*e^2-b*d*e+c*d^2)^3/
(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/
2)*b^3+5/2*e^4/(a*e^2-b*d*e+c*d^2)^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*
(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/
e+x))*b-128/3*c^3/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^2/((d/e+x)^2*c+(b*e-2*c*d
)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x-64/3*c^2/(a*e^2-b*d*e+c*d^2)/(
4*a*c-b^2)^2/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)*b+20/3*e^2/(a*e^2-b*d*e+c*d^2)^2*c/(4*a*c-b^2)^2/((d/e+x)^2*c+(b*e-2*c*
```

```
d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^3-5*e^3/(a*e^2-b*d*e+c*d^2)^3
/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/
e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e
+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*c*d+80/3/(a*e^2-b*d*e+c*d^2)^2
*c^3/(4*a*c-b^2)^2/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)*b*d^2+20/3/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*
c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)*x*c^3*d^2+10/3/(a*e^2-b*d*e+c
*d^2)^2/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/
e^2)^(3/2)*b*c^2*d^2+160/3/(a*e^2-b*d*e+c*d^2)^2*c^4/(4*a*c-b^2)^2/((d/e+x)
^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*d^2-10*e^3/(a*e
^2-b*d*e+c*d^2)^3/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*b^2*c*d+10*e^2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)/((d/e
+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c^2*d^2+5/3*
e^2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a
*e^2-b*d*e+c*d^2)/e^2)^(3/2)*x*c*b^2-10/3*e/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^
2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)*b^2*c*
d+40/3*e^2/(a*e^2-b*d*e+c*d^2)^2*c^2/(4*a*c-b^2)^2/((d/e+x)^2*c+(b*e-2*c*d)
/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b^2-80/3*e/(a*e^2-b*d*e+c*d^2)^
2*c^2/(4*a*c-b^2)^2/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/
e^2)^(1/2)*b^2*d+5*e^4/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-
2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b^2*c+20*e^2/(a*e^2-b*d*e
+c*d^2)^3/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)*x*c^3*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 102.847, size = 18303, normalized size = 38.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(15*(2*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^2*e^4 - (a^2*b^5 -
 8*a^3*b^3*c + 16*a^4*b*c^2)*d*e^5 + (2*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5
)*d*e^5 - (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e^6)*x^5 + (2*(b^4*c^3 - 8
*a*b^2*c^4 + 16*a^2*c^5)*d^2*e^4 + 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)
*d*e^5 - 2*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*e^6)*x^4 + (4*(b^5*c^2 -
8*a*b^3*c^3 + 16*a^2*b*c^4)*d^2*e^4 + 4*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3
*c^4)*d*e^5 - (b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*e^6)*x^3 + (2*(b^6*c - 6*a*b
^4*c^2 + 32*a^3*c^4)*d^2*e^4 - (b^7 - 10*a*b^5*c + 32*a^2*b^3*c^2 - 32*a^3*
b*c^3)*d*e^5 - 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*e^6)*x^2 + (4*(a*b^
5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2*e^4 - 2*(a*b^6 - 9*a^2*b^4*c + 24*a
^3*b^2*c^2 - 16*a^4*c^3)*d*e^5 - (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*e^6
)*x)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)
*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^2 - b*d
```

```
*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*
c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) + 4*
(2*(b^3*c^4 - 12*a*b*c^5)*d^7 - 8*(b^4*c^3 - 11*a*b^2*c^4 + 4*a^2*c^5)*d^6*
e + 4*(3*b^5*c^2 - 26*a*b^3*c^3 - 4*a^2*b*c^4)*d^5*e^2 - 8*(b^6*c - 3*a*b^4
*c^2 - 32*a^2*b^2*c^3 + 32*a^3*c^4)*d^4*e^3 + 2*(b^7 + 16*a*b^5*c - 155*a^2
*b^3*c^2 + 180*a^3*b*c^3)*d^3*e^4 - (16*a*b^6 - 91*a^2*b^4*c + 16*a^3*b^2*c
^2 + 176*a^4*c^3)*d^2*e^5 + (11*a^2*b^5 - 76*a^3*b^3*c + 112*a^4*b*c^2)*d*e
^6 + 3*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*e^7 - (32*c^7*d^6*e - 96*b*c^6*
d^5*e^2 + 4*(19*b^2*c^5 + 44*a*c^6)*d^4*e^3 + 8*(b^3*c^4 - 44*a*b*c^5)*d^3*
e^4 - (35*b^4*c^3 - 256*a*b^2*c^4 - 16*a^2*c^5)*d^2*e^5 + (15*b^5*c^2 - 80*
a*b^3*c^3 - 16*a^2*b*c^4)*d*e^6 - (15*a*b^4*c^2 - 100*a^2*b^2*c^3 + 128*a^3
*c^4)*e^7)*x^4 - 2*(16*c^7*d^7 - 24*b*c^6*d^6*e - 2*(17*b^2*c^5 - 44*a*c^6)
*d^5*e^2 + (61*b^3*c^4 - 44*a*b*c^5)*d^4*e^3 - 4*(b^4*c^3 + 49*a*b^2*c^4 -
32*a^2*c^5)*d^3*e^4 - 2*(15*b^5*c^2 - 121*a*b^3*c^3 + 88*a^2*b*c^4)*d^2*e^5
 + (15*b^6*c - 90*a*b^4*c^2 + 38*a^2*b^2*c^3 + 56*a^3*c^4)*d*e^6 - 3*(5*a*b
^5*c - 35*a^2*b^3*c^2 + 52*a^3*b*c^3)*e^7)*x^3 - 3*(16*b*c^6*d^7 - 4*(11*b^
2*c^5 - 4*a*c^6)*d^6*e + 2*(13*b^3*c^4 + 20*a*b*c^5)*d^5*e^2 + 8*(2*b^4*c^3
 - 17*a*b^2*c^4 + 16*a^2*c^5)*d^4*e^3 - 2*(7*b^5*c^2 - 34*a*b^3*c^3 + 64*a^
2*b*c^4)*d^3*e^4 - (5*b^6*c - 42*a*b^4*c^2 + 28*a^2*b^2*c^3 - 48*a^3*c^4)*d
^2*e^5 + (5*b^7 - 30*a*b^5*c + 18*a^2*b^3*c^2 + 8*a^3*b*c^3)*d*e^6 - (5*a*b
^6 - 30*a^2*b^4*c + 16*a^3*b^2*c^2 + 64*a^4*c^3)*e^7)*x^2 - 2*(6*(b^2*c^5 +
 4*a*c^6)*d^7 - (19*b^3*c^4 + 60*a*b*c^5)*d^6*e + 16*(b^4*c^3 + 4*a*b^2*c^4
 + 7*a^2*c^5)*d^5*e^2 + 2*(3*b^5*c^2 - 46*a*b^3*c^3 - 44*a^2*b*c^4)*d^4*e^3
 - 2*(7*b^6*c - 51*a*b^4*c^2 + 61*a^2*b^2*c^3 - 76*a^3*c^4)*d^3*e^4 + (5*b^
7 - 43*a*b^5*c + 125*a^2*b^3*c^2 - 156*a^3*b*c^3)*d^2*e^5 + (5*a*b^6 - 32*a
^2*b^4*c + 20*a^3*b^2*c^2 + 64*a^4*c^3)*d*e^6 - 2*(5*a^2*b^5 - 37*a^3*b^3*c
 + 64*a^4*b*c^2)*e^7)*x)*sqrt(c*x^2 + b*x + a))/((a^2*b^4*c^4 - 8*a^3*b^2*c
^5 + 16*a^4*c^6)*d^9 - 4*(a^2*b^5*c^3 - 8*a^3*b^3*c^4 + 16*a^4*b*c^5)*d^8*e
 + 2*(3*a^2*b^6*c^2 - 22*a^3*b^4*c^3 + 32*a^4*b^2*c^4 + 32*a^5*c^5)*d^7*e^2
 - 4*(a^2*b^7*c - 5*a^3*b^5*c^2 - 8*a^4*b^3*c^3 + 48*a^5*b*c^4)*d^6*e^3 + (
a^2*b^8 + 4*a^3*b^6*c - 74*a^4*b^4*c^2 + 144*a^5*b^2*c^3 + 96*a^6*c^4)*d^5*
e^4 - 4*(a^3*b^7 - 5*a^4*b^5*c - 8*a^5*b^3*c^2 + 48*a^6*b*c^3)*d^4*e^5 + 2*
(3*a^4*b^6 - 22*a^5*b^4*c + 32*a^6*b^2*c^2 + 32*a^7*c^3)*d^3*e^6 - 4*(a^5*b
^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*d^2*e^7 + (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*
c^2)*d*e^8 + ((b^4*c^6 - 8*a*b^2*c^7 + 16*a^2*c^8)*d^8*e - 4*(b^5*c^5 - 8*a
*b^3*c^6 + 16*a^2*b*c^7)*d^7*e^2 + 2*(3*b^6*c^4 - 22*a*b^4*c^5 + 32*a^2*b^2
*c^6 + 32*a^3*c^7)*d^6*e^3 - 4*(b^7*c^3 - 5*a*b^5*c^4 - 8*a^2*b^3*c^5 + 48*
a^3*b*c^6)*d^5*e^4 + (b^8*c^2 + 4*a*b^6*c^3 - 74*a^2*b^4*c^4 + 144*a^3*b^2*
c^5 + 96*a^4*c^6)*d^4*e^5 - 4*(a*b^7*c^2 - 5*a^2*b^5*c^3 - 8*a^3*b^3*c^4 +
48*a^4*b*c^5)*d^3*e^6 + 2*(3*a^2*b^6*c^2 - 22*a^3*b^4*c^3 + 32*a^4*b^2*c^4
+ 32*a^5*c^5)*d^2*e^7 - 4*(a^3*b^5*c^2 - 8*a^4*b^3*c^3 + 16*a^5*b*c^4)*d*e^
8 + (a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*e^9)*x^5 + ((b^4*c^6 - 8*a*b
^2*c^7 + 16*a^2*c^8)*d^9 - 2*(b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*d^8*e -
 2*(b^6*c^4 - 10*a*b^4*c^5 + 32*a^2*b^2*c^6 - 32*a^3*c^7)*d^7*e^2 + 4*(2*b^
7*c^3 - 17*a*b^5*c^4 + 40*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^6*e^3 - (7*b^8*c^2
- 44*a*b^6*c^3 + 10*a^2*b^4*c^4 + 240*a^3*b^2*c^5 - 96*a^4*c^6)*d^5*e^4 + 2
*(b^9*c + 2*a*b^7*c^2 - 64*a^2*b^5*c^3 + 160*a^3*b^3*c^4)*d^4*e^5 - 2*(4*a*
b^8*c - 23*a^2*b^6*c^2 - 10*a^3*b^4*c^3 + 160*a^4*b^2*c^4 - 32*a^5*c^5)*d^3
*e^6 + 4*(3*a^2*b^7*c - 23*a^3*b^5*c^2 + 40*a^4*b^3*c^3 + 16*a^5*b*c^4)*d^2
*e^7 - (8*a^3*b^6*c - 65*a^4*b^4*c^2 + 136*a^5*b^2*c^3 - 16*a^6*c^4)*d*e^8
+ 2*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*e^9)*x^4 + (2*(b^5*c^5 - 8*a
*b^3*c^6 + 16*a^2*b*c^7)*d^9 - (7*b^6*c^4 - 58*a*b^4*c^5 + 128*a^2*b^2*c^6
- 32*a^3*c^7)*d^8*e + 8*(b^7*c^3 - 8*a*b^5*c^4 + 16*a^2*b^3*c^5)*d^7*e^2 -
2*(b^8*c^2 - 4*a*b^6*c^3 - 20*a^2*b^4*c^4 + 96*a^3*b^2*c^5 - 64*a^4*c^6)*d^
6*e^3 - 2*(b^9*c - 10*a*b^7*c^2 + 38*a^2*b^5*c^3 - 80*a^3*b^3*c^4 + 96*a^4*
b*c^5)*d^5*e^4 + (b^10 - 2*a*b^8*c - 26*a^2*b^6*c^2 + 60*a^3*b^4*c^3 + 192*
a^5*c^5)*d^4*e^5 - 4*(a*b^9 - 6*a^2*b^7*c + 4*a^3*b^5*c^2 + 64*a^5*b*c^4)*d
^3*e^6 + 2*(3*a^2*b^8 - 20*a^3*b^6*c + 20*a^4*b^4*c^2 + 32*a^5*b^2*c^3 + 64
*a^6*c^4)*d^2*e^7 - 2*(2*a^3*b^7 - 13*a^4*b^5*c + 8*a^5*b^3*c^2 + 48*a^6*b*
```

c^3)*d*e^8 + (a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*e^9)*x^3 + ((b^6*c^4 - 6*a*b^4*c^5 + 32*a^3*c^7)*d^9 - 2*(2*b^7*c^3 - 13*a*b^5*c^4 + 8*a^2*b^3*c^5 + 48*a^3*b*c^6)*d^8*e + 2*(3*b^8*c^2 - 20*a*b^6*c^3 + 20*a^2*b^4*c^4 + 32*a^3*b^2*c^5 + 64*a^4*c^6)*d^7*e^2 - 4*(b^9*c - 6*a*b^7*c^2 + 4*a^2*b^5*c^3 + 64*a^4*b*c^5)*d^6*e^3 + (b^10 - 2*a*b^8*c - 26*a^2*b^6*c^2 + 60*a^3*b^4*c^3 + 192*a^5*c^5)*d^5*e^4 - 2*(a*b^9 - 10*a^2*b^7*c + 38*a^3*b^5*c^2 - 80*a^4*b^3*c^3 + 96*a^5*b*c^4)*d^4*e^5 - 2*(a^2*b^8 - 4*a^3*b^6*c - 20*a^4*b^4*c^2 + 96*a^5*b^2*c^3 - 64*a^6*c^4)*d^3*e^6 + 8*(a^3*b^7 - 8*a^4*b^5*c + 16*a^5*b^3*c^2)*d^2*e^7 - (7*a^4*b^6 - 58*a^5*b^4*c + 128*a^6*b^2*c^2 - 32*a^7*c^3)*d*e^8 + 2*(a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*e^9)*x^2 + (2*(a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*d^9 - (8*a*b^6*c^3 - 65*a^2*b^4*c^4 + 136*a^3*b^2*c^5 - 16*a^4*c^6)*d^8*e + 4*(3*a*b^7*c^2 - 23*a^2*b^5*c^3 + 40*a^3*b^3*c^4 + 16*a^4*b*c^5)*d^7*e^2 - 2*(4*a*b^8*c - 23*a^2*b^6*c^2 - 10*a^3*b^4*c^3 + 160*a^4*b^2*c^4 - 32*a^5*c^5)*d^6*e^3 + 2*(a*b^9 + 2*a^2*b^7*c - 64*a^3*b^5*c^2 + 160*a^4*b^3*c^3)*d^5*e^4 - (7*a^2*b^8 - 44*a^3*b^6*c + 10*a^4*b^4*c^2 + 240*a^5*b^2*c^3 - 96*a^6*c^4)*d^4*e^5 + 4*(2*a^3*b^7 - 17*a^4*b^5*c + 40*a^5*b^3*c^2 - 16*a^6*b*c^3)*d^3*e^6 - 2*(a^4*b^6 - 10*a^5*b^4*c + 32*a^6*b^2*c^2 - 32*a^7*c^3)*d^2*e^7 - 2*(a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*d*e^8 + (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)*e^9)*x), 1/6*(15*(2*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^2*e^4 - (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d*e^5 + (2*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d*e^5 - (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e^6)*x^5 + (2*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d^2*e^4 + 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d*e^5 - 2*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*e^6)*x^4 + (4*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^2*e^4 + 4*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^5 - (b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*e^6)*x^3 + (2*(b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*d^2*e^4 - (b^7 - 10*a*b^5*c + 32*a^2*b^3*c^2 - 32*a^3*b*c^3)*d*e^5 - 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*e^6)*x^2 + (4*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2*e^4 - 2*(a*b^6 - 9*a^2*b^4*c + 24*a^3*b^2*c^2 - 16*a^4*c^3)*d*e^5 - (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*e^6)*x)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)) - 2*(2*(b^3*c^4 - 12*a*b*c^5)*d^7 - 8*(b^4*c^3 - 11*a*b^2*c^4 + 4*a^2*c^5)*d^6*e + 4*(3*b^5*c^2 - 26*a*b^3*c^3 - 4*a^2*b*c^4)*d^5*e^2 - 8*(b^6*c - 3*a*b^4*c^2 - 32*a^2*b^2*c^3 + 32*a^3*c^4)*d^4*e^3 + 2*(b^7 + 16*a*b^5*c - 155*a^2*b^3*c^2 + 180*a^3*b*c^3)*d^3*e^4 - (16*a*b^6 - 91*a^2*b^4*c + 16*a^3*b^2*c^2 + 176*a^4*c^3)*d^2*e^5 + (11*a^2*b^5 - 76*a^3*b^3*c + 112*a^4*b*c^2)*d*e^6 + 3*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*e^7 - (32*c^7*d^6*e - 96*b*c^6*d^5*e^2 + 4*(19*b^2*c^5 + 44*a*c^6)*d^4*e^3 + 8*(b^3*c^4 - 44*a*b*c^5)*d^3*e^4 - (35*b^4*c^3 - 256*a*b^2*c^4 - 16*a^2*c^5)*d^2*e^5 + (15*b^5*c^2 - 80*a*b^3*c^3 - 16*a^2*b*c^4)*d*e^6 - (15*a*b^4*c^2 - 100*a^2*b^2*c^3 + 128*a^3*c^4)*e^7)*x^4 - 2*(16*c^7*d^7 - 24*b*c^6*d^6*e - 2*(17*b^2*c^5 - 44*a*c^6)*d^5*e^2 + (61*b^3*c^4 - 44*a*b*c^5)*d^4*e^3 - 4*(b^4*c^3 + 49*a*b^2*c^4 - 32*a^2*c^5)*d^3*e^4 - 2*(15*b^5*c^2 - 121*a*b^3*c^3 + 88*a^2*b*c^4)*d^2*e^5 + (15*b^6*c - 90*a*b^4*c^2 + 38*a^2*b^2*c^3 + 56*a^3*c^4)*d*e^6 - 3*(5*a*b^5*c - 35*a^2*b^3*c^2 + 52*a^3*b*c^3)*e^7)*x^3 - 3*(16*b*c^6*d^7 - 4*(11*b^2*c^5 - 4*a*c^6)*d^6*e + 2*(13*b^3*c^4 + 20*a*b*c^5)*d^5*e^2 + 8*(2*b^4*c^3 - 17*a*b^2*c^4 + 16*a^2*c^5)*d^4*e^3 - 2*(7*b^5*c^2 - 34*a*b^3*c^3 + 64*a^2*b*c^4)*d^3*e^4 - (5*b^6*c - 42*a*b^4*c^2 + 28*a^2*b^2*c^3 - 48*a^3*c^4)*d^2*e^5 + (5*b^7 - 30*a*b^5*c + 18*a^2*b^3*c^2 + 8*a^3*b*c^3)*d*e^6 - (5*a*b^6 - 30*a^2*b^4*c + 16*a^3*b^2*c^2 + 64*a^4*c^3)*e^7)*x^2 - 2*(6*(b^2*c^5 + 4*a*c^6)*d^7 - (19*b^3*c^4 + 60*a*b*c^5)*d^6*e + 16*(b^4*c^3 + 4*a*b^2*c^4 + 7*a^2*c^5)*d^5*e^2 + 2*(3*b^5*c^2 - 46*a*b^3*c^3 - 44*a^2*b*c^4)*d^4*e^3 - 2*(7*b^6*c - 51*a*b^4*c^2 + 61*a^2*b^2*c^3 - 76*a^3*c^4)*d^3*e^4 + (5*b^7 - 43*a*b^5*c + 125*a^2*b^3*c^2 - 156*a^3*b*c^3)*d^2*e^5 + (5*a*b^6 - 32*a^2*b^4*c + 20*a^3*b^2*c^2 + 64*a^4*c^3)*d*e^6 - 2*(5*a^2*b^5 - 37*a^3*b^3*c + 64*a^4*b*c^2)*e^7)*x)*sqrt(c*x^2 + b*x + a))/((a^2*b^4*c^4 - 8*a^3*b^2*c^5 + 16*a^4*c^6)*d^9 - 4*(a^2*b^5*c^3 - 8*a^3*b^3*c^4 + 16*a^4*b*c^5)*d^8*e + 2*(3

```
*a^2*b^6*c^2 - 22*a^3*b^4*c^3 + 32*a^4*b^2*c^4 + 32*a^5*c^5)*d^7*e^2 - 4*(a
^2*b^7*c - 5*a^3*b^5*c^2 - 8*a^4*b^3*c^3 + 48*a^5*b*c^4)*d^6*e^3 + (a^2*b^8
 + 4*a^3*b^6*c - 74*a^4*b^4*c^2 + 144*a^5*b^2*c^3 + 96*a^6*c^4)*d^5*e^4 - 4
*(a^3*b^7 - 5*a^4*b^5*c - 8*a^5*b^3*c^2 + 48*a^6*b*c^3)*d^4*e^5 + 2*(3*a^4*
b^6 - 22*a^5*b^4*c + 32*a^6*b^2*c^2 + 32*a^7*c^3)*d^3*e^6 - 4*(a^5*b^5 - 8*
a^6*b^3*c + 16*a^7*b*c^2)*d^2*e^7 + (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)*d*
e^8 + ((b^4*c^6 - 8*a*b^2*c^7 + 16*a^2*c^8)*d^8*e - 4*(b^5*c^5 - 8*a*b^3*c^
6 + 16*a^2*b*c^7)*d^7*e^2 + 2*(3*b^6*c^4 - 22*a*b^4*c^5 + 32*a^2*b^2*c^6 +
32*a^3*c^7)*d^6*e^3 - 4*(b^7*c^3 - 5*a*b^5*c^4 - 8*a^2*b^3*c^5 + 48*a^3*b*c
^6)*d^5*e^4 + (b^8*c^2 + 4*a*b^6*c^3 - 74*a^2*b^4*c^4 + 144*a^3*b^2*c^5 + 9
6*a^4*c^6)*d^4*e^5 - 4*(a*b^7*c^2 - 5*a^2*b^5*c^3 - 8*a^3*b^3*c^4 + 48*a^4*
b*c^5)*d^3*e^6 + 2*(3*a^2*b^6*c^2 - 22*a^3*b^4*c^3 + 32*a^4*b^2*c^4 + 32*a^
5*c^5)*d^2*e^7 - 4*(a^3*b^5*c^2 - 8*a^4*b^3*c^3 + 16*a^5*b*c^4)*d*e^8 + (a^
4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*e^9)*x^5 + ((b^4*c^6 - 8*a*b^2*c^7
 + 16*a^2*c^8)*d^9 - 2*(b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*d^8*e - 2*(b^6
*c^4 - 10*a*b^4*c^5 + 32*a^2*b^2*c^6 - 32*a^3*c^7)*d^7*e^2 + 4*(2*b^7*c^3 -
 17*a*b^5*c^4 + 40*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^6*e^3 - (7*b^8*c^2 - 44*a*
b^6*c^3 + 10*a^2*b^4*c^4 + 240*a^3*b^2*c^5 - 96*a^4*c^6)*d^5*e^4 + 2*(b^9*c
 + 2*a*b^7*c^2 - 64*a^2*b^5*c^3 + 160*a^3*b^3*c^4)*d^4*e^5 - 2*(4*a*b^8*c -
 23*a^2*b^6*c^2 - 10*a^3*b^4*c^3 + 160*a^4*b^2*c^4 - 32*a^5*c^5)*d^3*e^6 +
4*(3*a^2*b^7*c - 23*a^3*b^5*c^2 + 40*a^4*b^3*c^3 + 16*a^5*b*c^4)*d^2*e^7 -
(8*a^3*b^6*c - 65*a^4*b^4*c^2 + 136*a^5*b^2*c^3 - 16*a^6*c^4)*d*e^8 + 2*(a^
4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*e^9)*x^4 + (2*(b^5*c^5 - 8*a*b^3*c^
6 + 16*a^2*b*c^7)*d^9 - (7*b^6*c^4 - 58*a*b^4*c^5 + 128*a^2*b^2*c^6 - 32*a^
3*c^7)*d^8*e + 8*(b^7*c^3 - 8*a*b^5*c^4 + 16*a^2*b^3*c^5)*d^7*e^2 - 2*(b^8*
c^2 - 4*a*b^6*c^3 - 20*a^2*b^4*c^4 + 96*a^3*b^2*c^5 - 64*a^4*c^6)*d^6*e^3 -
 2*(b^9*c - 10*a*b^7*c^2 + 38*a^2*b^5*c^3 - 80*a^3*b^3*c^4 + 96*a^4*b*c^5)*
d^5*e^4 + (b^10 - 2*a*b^8*c - 26*a^2*b^6*c^2 + 60*a^3*b^4*c^3 + 192*a^5*c^5
)*d^4*e^5 - 4*(a*b^9 - 6*a^2*b^7*c + 4*a^3*b^5*c^2 + 64*a^5*b*c^4)*d^3*e^6
+ 2*(3*a^2*b^8 - 20*a^3*b^6*c + 20*a^4*b^4*c^2 + 32*a^5*b^2*c^3 + 64*a^6*c^
4)*d^2*e^7 - 2*(2*a^3*b^7 - 13*a^4*b^5*c + 8*a^5*b^3*c^2 + 48*a^6*b*c^3)*d*
e^8 + (a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*e^9)*x^3 + ((b^6*c^4 - 6*a*b^4*c
^5 + 32*a^3*c^7)*d^9 - 2*(2*b^7*c^3 - 13*a*b^5*c^4 + 8*a^2*b^3*c^5 + 48*a^3
*b*c^6)*d^8*e + 2*(3*b^8*c^2 - 20*a*b^6*c^3 + 20*a^2*b^4*c^4 + 32*a^3*b^2*c
^5 + 64*a^4*c^6)*d^7*e^2 - 4*(b^9*c - 6*a*b^7*c^2 + 4*a^2*b^5*c^3 + 64*a^4*
b*c^5)*d^6*e^3 + (b^10 - 2*a*b^8*c - 26*a^2*b^6*c^2 + 60*a^3*b^4*c^3 + 192*
a^5*c^5)*d^5*e^4 - 2*(a*b^9 - 10*a^2*b^7*c + 38*a^3*b^5*c^2 - 80*a^4*b^3*c^
3 + 96*a^5*b*c^4)*d^4*e^5 - 2*(a^2*b^8 - 4*a^3*b^6*c - 20*a^4*b^4*c^2 + 96*
a^5*b^2*c^3 - 64*a^6*c^4)*d^3*e^6 + 8*(a^3*b^7 - 8*a^4*b^5*c + 16*a^5*b^3*c
^2)*d^2*e^7 - (7*a^4*b^6 - 58*a^5*b^4*c + 128*a^6*b^2*c^2 - 32*a^7*c^3)*d*e
^8 + 2*(a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*e^9)*x^2 + (2*(a*b^5*c^4 - 8*
a^2*b^3*c^5 + 16*a^3*b*c^6)*d^9 - (8*a*b^6*c^3 - 65*a^2*b^4*c^4 + 136*a^3*b
^2*c^5 - 16*a^4*c^6)*d^8*e + 4*(3*a*b^7*c^2 - 23*a^2*b^5*c^3 + 40*a^3*b^3*c
^4 + 16*a^4*b*c^5)*d^7*e^2 - 2*(4*a*b^8*c - 23*a^2*b^6*c^2 - 10*a^3*b^4*c^3
 + 160*a^4*b^2*c^4 - 32*a^5*c^5)*d^6*e^3 + 2*(a*b^9 + 2*a^2*b^7*c - 64*a^3*
b^5*c^2 + 160*a^4*b^3*c^3)*d^5*e^4 - (7*a^2*b^8 - 44*a^3*b^6*c + 10*a^4*b^4
*c^2 + 240*a^5*b^2*c^3 - 96*a^6*c^4)*d^4*e^5 + 4*(2*a^3*b^7 - 17*a^4*b^5*c
+ 40*a^5*b^3*c^2 - 16*a^6*b*c^3)*d^3*e^6 - 2*(a^4*b^6 - 10*a^5*b^4*c + 32*a
^6*b^2*c^2 - 32*a^7*c^3)*d^2*e^7 - 2*(a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)
*d*e^8 + (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)*e^9)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2399 $\int \frac{1}{(d+ex)^3\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=621

$$\frac{e\sqrt{a+bx+cx^2}(2cd-be)\left(-16c^2e^2\left(81a^2e^2+28abde+b^2d^2\right)+40b^2ce^3(19ae+2bd)-64c^3d^2e(2bd-7ae)-105b^4e^4+\right.}{12\left(b^2-4ac\right)^2(d+ex)\left(ae^2-bde+cd^2\right)^4}$$

```
[Out] (-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/(3*(b^2 - 4*a*c)*(c*d^2
- b*d*e + a*e^2)*(d + e*x)^2*(a + b*x + c*x^2)^(3/2)) - (2*(8*a*c*e*(2*c*d
- b*e)^2 - (b*c*d - b^2*e + 2*a*c*e)*(8*c^2*d^2 - 7*b^2*e^2 + 20*a*c*e^2) -
 c*(2*c*d - b*e)*(8*c^2*d^2 - 7*b^2*e^2 - 4*c*e*(2*b*d - 9*a*e))*x))/(3*(b^
2 - 4*a*c)^2*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^2*Sqrt[a + b*x + c*x^2]) +
 (e*(64*c^4*d^4 - 35*b^4*e^4 - 128*c^3*d^2*e*(b*d - 3*a*e) - 48*a*c^2*e^3*(
8*b*d + 5*a*e) + 8*b^2*c*e^3*(8*b*d + 27*a*e))*Sqrt[a + b*x + c*x^2])/(6*(b
^2 - 4*a*c)^2*(c*d^2 - b*d*e + a*e^2)^3*(d + e*x)^2) + (e*(2*c*d - b*e)*(64
*c^4*d^4 - 105*b^4*e^4 - 64*c^3*d^2*e*(2*b*d - 7*a*e) + 40*b^2*c*e^3*(2*b*d
 + 19*a*e) - 16*c^2*e^2*(b^2*d^2 + 28*a*b*d*e + 81*a^2*e^2))*Sqrt[a + b*x +
 c*x^2])/(12*(b^2 - 4*a*c)^2*(c*d^2 - b*d*e + a*e^2)^4*(d + e*x)) + (5*e^4*
(24*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(6*b*d + a*e))*ArcTanh[(b*d - 2*a*e + (2*c*
d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(8*(c*d
^2 - b*d*e + a*e^2)^(9/2))
```

Rubi [A] time = 1.04109, antiderivative size = 621, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {740, 822, 834, 806, 724, 206}

$$\frac{e\sqrt{a+bx+cx^2}(2cd-be)\left(-16c^2e^2\left(81a^2e^2+28abde+b^2d^2\right)+40b^2ce^3(19ae+2bd)-64c^3d^2e(2bd-7ae)-105b^4e^4+\right.}{12\left(b^2-4ac\right)^2(d+ex)\left(ae^2-bde+cd^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + b*x + c*x^2)^(5/2)),x]
```


```
[Out] (-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/(3*(b^2 - 4*a*c)*(c*d^2
- b*d*e + a*e^2)*(d + e*x)^2*(a + b*x + c*x^2)^(3/2)) - (2*(8*a*c*e*(2*c*d
- b*e)^2 - (b*c*d - b^2*e + 2*a*c*e)*(8*c^2*d^2 - 7*b^2*e^2 + 20*a*c*e^2) -
 c*(2*c*d - b*e)*(8*c^2*d^2 - 7*b^2*e^2 - 4*c*e*(2*b*d - 9*a*e))*x))/(3*(b^
2 - 4*a*c)^2*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^2*Sqrt[a + b*x + c*x^2]) +
 (e*(64*c^4*d^4 - 35*b^4*e^4 - 128*c^3*d^2*e*(b*d - 3*a*e) - 48*a*c^2*e^3*(
8*b*d + 5*a*e) + 8*b^2*c*e^3*(8*b*d + 27*a*e))*Sqrt[a + b*x + c*x^2])/(6*(b
^2 - 4*a*c)^2*(c*d^2 - b*d*e + a*e^2)^3*(d + e*x)^2) + (e*(2*c*d - b*e)*(64
*c^4*d^4 - 105*b^4*e^4 - 64*c^3*d^2*e*(2*b*d - 7*a*e) + 40*b^2*c*e^3*(2*b*d
 + 19*a*e) - 16*c^2*e^2*(b^2*d^2 + 28*a*b*d*e + 81*a^2*e^2))*Sqrt[a + b*x +
 c*x^2])/(12*(b^2 - 4*a*c)^2*(c*d^2 - b*d*e + a*e^2)^4*(d + e*x)) + (5*e^4*
(24*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(6*b*d + a*e))*ArcTanh[(b*d - 2*a*e + (2*c*
d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(8*(c*d
^2 - b*d*e + a*e^2)^(9/2))
```


Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
```

```
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)^2\left(a+bx+cx^2\right)^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(8c^2d^2-7b^2e^2+20ace\cdots\right.}{(d+ex)^3\left(a+bx\cdots\right.}}{3\left(b^2-4ac\right)\left(cd^2\cdots\right.}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)^2\left(a+bx+cx^2\right)^{3/2}} - \frac{2\left(8ace(2cd-be)^2-\cdots\right.}{\cdots}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)^2\left(a+bx+cx^2\right)^{3/2}} - \frac{2\left(8ace(2cd-be)^2-\cdots\right.}{\cdots}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)^2\left(a+bx+cx^2\right)^{3/2}} - \frac{2\left(8ace(2cd-be)^2-\cdots\right.}{\cdots}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)^2\left(a+bx+cx^2\right)^{3/2}} - \frac{2\left(8ace(2cd-be)^2-\cdots\right.}{\cdots}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)^2\left(a+bx+cx^2\right)^{3/2}} - \frac{2\left(8ace(2cd-be)^2-\cdots\right.}{\cdots}$$

Mathematica [A] time = 3.53391, size = 626, normalized size = 1.01

$$2\left(\frac{-8c^2\left(5a^2e^3+acde(9ex-2d)+2c^2d^3x\right)+2b^2ce\left(21ae^2+cd(4d+3ex)\right)-4bc^2\left(ae^2(13d-9ex)+2cd^2(d-3ex)\right)+7b^3ce^2(d-ex)-7b^4e^3}{\left(b^2-4ac\right)(d+ex)^2\sqrt{a+x(b+cx)}\left(e(bd-ae)-cd^2\right)} + \frac{e\left(\frac{2\sqrt{a+x(b+cx)}(2cd-be)\left(-16c^2e^2\cdots\right.}{\cdots}\right.}{\cdots}\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a + b*x + c*x^2)^(5/2)),x]
```

```
[Out] (2*((b^2*e - 2*c*(a*e + c*d*x) + b*c*(-d + e*x))/((d + e*x)^2*(a + x*(b + c
*x))^(3/2)) + (-7*b^4*e^3 + 7*b^3*c*e^2*(d - e*x) - 4*b*c^2*(a*e^2*(13*d -
9*e*x) + 2*c*d^2*(d - 3*e*x)) + 2*b^2*c*e*(21*a*e^2 + c*d*(4*d + 3*e*x)) -
8*c^2*(5*a^2*e^3 + 2*c^2*d^3*x + a*c*d*e*(-2*d + 9*e*x)))/((b^2 - 4*a*c)*(-
(c*d^2) + e*(b*d - a*e))*(d + e*x)^2*Sqrt[a + x*(b + c*x)]) + (e*((4*(64*c^
4*d^4 - 35*b^4*e^4 - 128*c^3*d^2*e*(b*d - 3*a*e) - 48*a*c^2*e^3*(8*b*d + 5*
a*e) + 8*b^2*c*e^3*(8*b*d + 27*a*e))*Sqrt[a + x*(b + c*x)])/(d + e*x)^2 + (
2*(2*c*d - b*e)*(64*c^4*d^4 - 105*b^4*e^4 - 64*c^3*d^2*e*(2*b*d - 7*a*e) +
40*b^2*c*e^3*(2*b*d + 19*a*e) - 16*c^2*e^2*(b^2*d^2 + 28*a*b*d*e + 81*a^2*e
^2))*Sqrt[a + x*(b + c*x)])/((c*d^2 + e*(-(b*d) + a*e))*(d + e*x)) - (15*(b
^2 - 4*a*c)^2*e^3*(24*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(6*b*d + a*e))*ArcTanh[(-
(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a +
 x*(b + c*x)])])/(c*d^2 + e*(-(b*d) + a*e))^(3/2)))/(16*(b^2 - 4*a*c)*(c*d^
2 + e*(-(b*d) + a*e))^2)))/(3*(b^2 - 4*a*c)*(c*d^2 + e*(-(b*d) + a*e)))
```

Maple [B] time = 0.241, size = 4942, normalized size = 8.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] -35*e/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+
(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)*x*c^3*b*d^2-280*e/(a*e^2-b*d*e+c*d^2)^3*c^4/
(4*a*c-b^2)^2/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*x*d^2*b+140*e^2/(a*e^2-b*d*e+c*d^2)^3*c^3/(4*a*c-b^2)^2/((d/e+x)^2*c+(
b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b^2*d-35/8*e^5/(a*e^2
-b*d*e+c*d^2)^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e
^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*
e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b^2+35/6*e/(a*e
^2-b*d*e+c*d^2)^3/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^
2)^(3/2)*c^2*d^2+7/4*e/(a*e^2-b*d*e+c*d^2)^2/(d/e+x)/((d/e+x)^2*c+(b*e-2*c*
d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)*b-35/24*e^3/(a*e^2-b*d*e+c*d^2)
^3/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^
(3/2)*b^4+5/2*e^3*c/(a*e^2-b*d*e+c*d^2)^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*l
n((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
)/(d/e+x))+35/2*e^3/(a*e^2-b*d*e+c*d^2)^4/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x
)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*c^2*d^2-35/8*e^5/(a*e^2-b*d*e+c*d^2)^4/(4*
a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
b^4-7/2/(a*e^2-b*d*e+c*d^2)^2/(d/e+x)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a
*e^2-b*d*e+c*d^2)/e^2)^(3/2)*c*d+35/2*e^2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)
/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)*x*c^2*b^
2*d+105/2*e^4/(a*e^2-b*d*e+c*d^2)^4/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*
(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b^2*c^2*d-105*e^3/(a*e^2-b*d*e+c*d
^2)^4/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*x*b*c^3*d^2+35/24*e^3/(a*e^2-b*d*e+c*d^2)^3/((d/e+x)^2*c+(b*e-2*c*
d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)*b^2+35/8*e^5/(a*e^2-b*d*e+c*d^2
)^4/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2-5
/6*e*c/(a*e^2-b*d*e+c*d^2)^2/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*
e+c*d^2)/e^2)^(3/2)-5/2*e^3*c/(a*e^2-b*d*e+c*d^2)^3/((d/e+x)^2*c+(b*e-2*c*d
)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/2/e/(a*e^2-b*d*e+c*d^2)/(d/e+x
)^2/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)+35/2*
e^4/(a*e^2-b*d*e+c*d^2)^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*
e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+
x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b*c*d
-35/12*e^3/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/
e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)*x*c*b^3-35/4*e^5/(a*e^2-b*d*e+c*d^2)^4/
(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/
2)*x*b^3*c+70*e^2/(a*e^2-b*d*e+c*d^2)^4/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d
)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^4*d^3+11*e/(a*e^2-b*d*e+c*d^
2)^2*c^2/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)
/e^2)^(3/2)*x*b+105/4*e^4/(a*e^2-b*d*e+c*d^2)^4/(4*a*c-b^2)/((d/e+x)^2*c+(b
*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^3*c*d-105/2*e^3/(a*e^2
-b*d*e+c*d^2)^4/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*b^2*c^2*d^2+35*e^2/(a*e^2-b*d*e+c*d^2)^4/(4*a*c-b^2)/((d
/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c^3*d^3-35
/2*e/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(
a*e^2-b*d*e+c*d^2)/e^2)^(3/2)*b^2*c^2*d^2-70/3*e^3/(a*e^2-b*d*e+c*d^2)^3*c^
2/(4*a*c-b^2)^2/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)*x*b^3+5*e^3*c^2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-
2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b-10*e^2*c^3/(a*e^2-b*d*e
+c*d^2)^3/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2
```

```
)/e^2)^(1/2)*x*d-5*e^2*c^2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)/((d/e+x)^2*c+(
b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*d+70*e^2/(a*e^2-b*d*e
+c*d^2)^3*c^2/(4*a*c-b^2)^2/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*b^3*d-140*e/(a*e^2-b*d*e+c*d^2)^3*c^3/(4*a*c-b^2)^2/((d/
e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*d^2+88*e/
(a*e^2-b*d*e+c*d^2)^2*c^3/(4*a*c-b^2)^2/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b+35/4*e^2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^
2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)*b^3*c*
d+560/3/(a*e^2-b*d*e+c*d^2)^3*c^5/(4*a*c-b^2)^2/((d/e+x)^2*c+(b*e-2*c*d)/e*
(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*d^3-22/(a*e^2-b*d*e+c*d^2)^2*c^3/(
4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2
)*x*d-11/(a*e^2-b*d*e+c*d^2)^2*c^2/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(
d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)*b*d-176/(a*e^2-b*d*e+c*d^2)^2*c^4/(4*
a*c-b^2)^2/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*x*d-88/(a*e^2-b*d*e+c*d^2)^2*c^3/(4*a*c-b^2)^2/((d/e+x)^2*c+(b*e-2*c*d)/e
*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*d+11/2*e/(a*e^2-b*d*e+c*d^2)^2*c/
(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/
2)*b^2+44*e/(a*e^2-b*d*e+c*d^2)^2*c^2/(4*a*c-b^2)^2/((d/e+x)^2*c+(b*e-2*c*d
)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2-35/3*e^3/(a*e^2-b*d*e+c*d^2)
^3*c/(4*a*c-b^2)^2/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)*b^4-35/2*e^3/(a*e^2-b*d*e+c*d^2)^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1
/2))/(d/e+x))*c^2*d^2-35/2*e^4/(a*e^2-b*d*e+c*d^2)^4/((d/e+x)^2*c+(b*e-2*c*
d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*c*d*b+5/2*e^3*c/(a*e^2-b*d*e+c*
d^2)^3/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)*b^2+70/3/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*
d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)*x*c^4*d^3+280/3/(a*e^2-b*d*e+c*
d^2)^3*c^4/(4*a*c-b^2)^2/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*
d^2)/e^2)^(1/2)*b*d^3+35/3/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)/((d/e+x)^2*c+(
b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)*b*c^3*d^3-35/6*e^2/(a*e
^2-b*d*e+c*d^2)^3/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^
2)^(3/2)*c*d*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**2+b*x+a)**(5/2),x)

[Out] Timed out
```

Giac [B] time = 17.856, size = 16020, normalized size = 25.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")

[Out] 5/4*(24*c^2*d^2*e^4 - 24*b*c*d*e^5 + 7*b^2*e^6 - 4*a*c*e^6)*arctan(-((sqrt(
c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/
((c^4*d^8 - 4*b*c^3*d^7*e + 6*b^2*c^2*d^6*e^2 + 4*a*c^3*d^6*e^2 - 4*b^3*c*d
^5*e^3 - 12*a*b*c^2*d^5*e^3 + b^4*d^4*e^4 + 12*a*b^2*c*d^4*e^4 + 6*a^2*c^2*
d^4*e^4 - 4*a*b^3*d^3*e^5 - 12*a^2*b*c*d^3*e^5 + 6*a^2*b^2*d^2*e^6 + 4*a^3*
c*d^2*e^6 - 4*a^3*b*d*e^7 + a^4*e^8)*sqrt(-c*d^2 + b*d*e - a*e^2)) + 1/3*((
((16*c^19*d^29 - 232*b*c^18*d^28*e + 1548*b^2*c^17*d^27*e^2 + 304*a*c^18*d^
27*e^2 - 6282*b^3*c^16*d^26*e^3 - 4104*a*b*c^17*d^26*e^3 + 17220*b^4*c^15*d
^25*e^4 + 25572*a*b^2*c^16*d^25*e^4 + 2208*a^2*c^17*d^25*e^4 - 33315*b^5*c^
14*d^24*e^5 - 97350*a*b^3*c^15*d^24*e^5 - 27600*a^2*b*c^16*d^24*e^5 + 45608
*b^6*c^13*d^23*e^6 + 252264*a*b^4*c^14*d^23*e^6 + 159144*a^2*b^2*c^15*d^23*
e^6 + 8608*a^3*c^16*d^23*e^6 - 41492*b^7*c^12*d^22*e^7 - 468096*a*b^5*c^13*
d^22*e^7 - 560556*a^2*b^3*c^14*d^22*e^7 - 98992*a^3*b*c^15*d^22*e^7 + 17424
*b^8*c^11*d^21*e^8 + 634040*a*b^6*c^12*d^21*e^8 + 1344816*a^2*b^4*c^13*d^21
*e^8 + 524568*a^3*b^2*c^14*d^21*e^8 + 19888*a^4*c^15*d^21*e^8 + 14135*b^9*c
^10*d^20*e^9 - 620334*a*b^7*c^11*d^20*e^9 - 2315082*a^2*b^5*c^12*d^20*e^9 -
 1696772*a^3*b^3*c^13*d^20*e^9 - 208824*a^4*b*c^14*d^20*e^9 - 34628*b^10*c^
9*d^19*e^10 + 409860*a*b^8*c^10*d^19*e^10 + 2924460*a^2*b^6*c^11*d^19*e^10
+ 3736040*a^3*b^4*c^12*d^19*e^10 + 1011780*a^4*b^2*c^13*d^19*e^10 + 25872*a
^5*c^14*d^19*e^10 + 35910*b^11*c^8*d^18*e^11 - 132088*a*b^9*c^9*d^18*e^11 -
 2704878*a^2*b^7*c^10*d^18*e^11 - 5898816*a^3*b^5*c^11*d^18*e^11 - 2999150*
a^4*b^3*c^12*d^18*e^11 - 245784*a^5*b*c^13*d^18*e^11 - 24540*b^12*c^7*d^17*
e^12 - 57420*a*b^10*c^8*d^17*e^12 + 1762992*a^2*b^8*c^9*d^17*e^12 + 6826644
*a^3*b^6*c^10*d^17*e^12 + 6064740*a^4*b^4*c^11*d^17*e^12 + 1093356*a^5*b^2*
c^12*d^17*e^12 + 8448*a^6*c^13*d^17*e^12 + 11883*b^13*c^6*d^16*e^13 + 10822
2*a*b^11*c^7*d^16*e^13 - 702372*a^2*b^9*c^8*d^16*e^13 - 5776056*a^3*b^7*c^9
*d^16*e^13 - 8797041*a^4*b^5*c^10*d^16*e^13 - 3026034*a^5*b^3*c^11*d^16*e^1
3 - 71808*a^6*b*c^12*d^16*e^13 - 4080*b^14*c^5*d^15*e^14 - 75888*a*b^12*c^6
*d^15*e^14 + 44880*a^2*b^10*c^7*d^15*e^14 + 3446784*a^3*b^8*c^8*d^15*e^14 +
 9317088*a^4*b^6*c^9*d^15*e^14 + 5789520*a^5*b^4*c^10*d^15*e^14 + 350064*a^
6*b^2*c^11*d^15*e^14 - 35904*a^7*c^12*d^15*e^14 + 952*b^15*c^4*d^14*e^15 +
32640*a*b^13*c^5*d^14*e^15 + 144840*a^2*b^11*c^6*d^14*e^15 - 1286560*a^3*b^
9*c^7*d^14*e^15 - 7135920*a^4*b^7*c^8*d^14*e^15 - 7970688*a^5*b^5*c^9*d^14*
e^15 - 1189320*a^6*b^3*c^10*d^14*e^15 + 269280*a^7*b*c^11*d^14*e^15 - 136*b
^16*c^3*d^13*e^16 - 8976*a*b^14*c^4*d^13*e^16 - 102816*a^2*b^12*c^5*d^13*e^
16 + 146608*a^3*b^10*c^6*d^13*e^16 + 3769920*a^4*b^8*c^7*d^13*e^16 + 791683
2*a^5*b^6*c^8*d^13*e^16 + 2764608*a^6*b^4*c^9*d^13*e^16 - 780912*a^7*b^2*c^
```

10*d^13*e^16 - 80784*a^8*c^11*d^13*e^16 + 9*b^17*c^2*d^12*e^17 + 1462*a*b^1
5*c^3*d^12*e^17 + 36414*a^2*b^13*c^4*d^12*e^17 + 129948*a^3*b^11*c^5*d^12*e
^17 - 1191190*a^4*b^9*c^6*d^12*e^17 - 5513508*a^5*b^7*c^7*d^12*e^17 - 42882
84*a^6*b^5*c^8*d^12*e^17 + 991848*a^7*b^3*c^9*d^12*e^17 + 525096*a^8*b*c^10
*d^12*e^17 - 108*a*b^16*c^2*d^11*e^18 - 7044*a^2*b^14*c^3*d^11*e^18 - 79912
*a^3*b^12*c^4*d^11*e^18 + 89628*a^4*b^10*c^5*d^11*e^18 + 2500344*a^5*b^8*c^
6*d^11*e^18 + 4359432*a^6*b^6*c^7*d^11*e^18 - 121968*a^7*b^4*c^8*d^11*e^18
- 1365804*a^8*b^2*c^9*d^11*e^18 - 93104*a^9*c^10*d^11*e^18 + 594*a^2*b^15*c
^2*d^10*e^19 + 19888*a^3*b^13*c^3*d^10*e^19 + 90486*a^4*b^11*c^4*d^10*e^19
- 595320*a^5*b^9*c^5*d^10*e^19 - 2798004*a^6*b^7*c^6*d^10*e^19 - 1254528*a^
7*b^5*c^7*d^10*e^19 + 1735866*a^8*b^3*c^8*d^10*e^19 + 512072*a^9*b*c^9*d^10
*e^19 - 1980*a^3*b^14*c^2*d^9*e^20 - 35860*a^4*b^12*c^3*d^9*e^20 - 8844*a^5
*b^10*c^4*d^9*e^20 + 1021680*a^6*b^8*c^5*d^9*e^20 + 1661880*a^7*b^6*c^6*d^9
*e^20 - 924660*a^8*b^4*c^7*d^9*e^20 - 1106820*a^9*b^2*c^8*d^9*e^20 - 69344*
a^10*c^9*d^9*e^20 + 4455*a^4*b^13*c^2*d^8*e^21 + 41382*a^5*b^11*c^3*d^8*e^2
1 - 138468*a^6*b^9*c^4*d^8*e^21 - 957528*a^7*b^7*c^5*d^8*e^21 - 193941*a^8*
b^5*c^6*d^8*e^21 + 1140150*a^9*b^3*c^7*d^8*e^21 + 312048*a^10*b*c^8*d^8*e^2
1 - 7128*a^5*b^12*c^2*d^7*e^22 - 26664*a^6*b^10*c^3*d^7*e^22 + 234432*a^7*b
^8*c^4*d^7*e^22 + 488664*a^8*b^6*c^5*d^7*e^22 - 479160*a^9*b^4*c^6*d^7*e^22
- 528792*a^10*b^2*c^7*d^7*e^22 - 34656*a^11*c^8*d^7*e^22 + 8316*a^6*b^11*c
^2*d^6*e^23 + 528*a^7*b^9*c^3*d^6*e^23 - 206316*a^8*b^7*c^4*d^6*e^23 - 5913
6*a^9*b^5*c^5*d^6*e^23 + 394548*a^10*b^3*c^6*d^6*e^23 + 121296*a^11*b*c^7*d
^6*e^23 - 7128*a^7*b^10*c^2*d^5*e^24 + 17424*a^8*b^8*c^3*d^5*e^24 + 106568*
a^9*b^6*c^4*d^5*e^24 - 92400*a^10*b^4*c^5*d^5*e^24 - 148008*a^11*b^2*c^6*d^
5*e^24 - 11312*a^12*c^7*d^5*e^24 + 4455*a^8*b^9*c^2*d^4*e^25 - 18590*a^9*b^
7*c^3*d^4*e^25 - 27258*a^10*b^5*c^4*d^4*e^25 + 66780*a^11*b^3*c^5*d^4*e^25
+ 28280*a^12*b*c^6*d^4*e^25 - 1980*a^9*b^8*c^2*d^3*e^26 + 10604*a^10*b^6*c^
3*d^3*e^26 - 1656*a^11*b^4*c^4*d^3*e^26 - 21156*a^12*b^2*c^5*d^3*e^26 - 219
2*a^13*c^6*d^3*e^26 + 594*a^10*b^7*c^2*d^2*e^27 - 3648*a^11*b^5*c^3*d^2*e^2
7 + 3454*a^12*b^3*c^4*d^2*e^27 + 3288*a^13*b*c^5*d^2*e^27 - 108*a^11*b^6*c^
2*d*e^28 + 716*a^12*b^4*c^3*d*e^28 - 972*a^13*b^2*c^4*d*e^28 - 192*a^14*c^5
*d*e^28 + 9*a^12*b^5*c^2*e^29 - 62*a^13*b^3*c^3*e^29 + 96*a^14*b*c^4*e^29)*
x/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4) + 3*(8*b*c^18*d^29 - 116*b^2*c^17*d^
28*e + 774*b^3*c^16*d^27*e^2 + 152*a*b*c^17*d^27*e^2 - 3136*b^4*c^15*d^26*e
^3 - 2092*a*b^2*c^16*d^26*e^3 + 80*a^2*c^17*d^26*e^3 + 8545*b^5*c^14*d^25*e
^4 + 13306*a*b^3*c^15*d^25*e^4 + 64*a^2*b*c^16*d^25*e^4 - 16266*b^6*c^13*d^
24*e^5 - 51748*a*b^4*c^14*d^24*e^5 - 8008*a^2*b^2*c^15*d^24*e^5 + 944*a^3*c
^16*d^24*e^5 + 21356*b^7*c^12*d^23*e^6 + 137008*a*b^5*c^13*d^23*e^6 + 62068
*a^2*b^3*c^14*d^23*e^6 - 7024*a^3*b*c^15*d^23*e^6 - 17072*b^8*c^11*d^22*e^7
- 259528*a*b^6*c^12*d^22*e^7 - 252472*a^2*b^4*c^13*d^22*e^7 + 10552*a^3*b^
2*c^14*d^22*e^7 + 5088*a^4*c^15*d^22*e^7 + 1947*b^9*c^10*d^21*e^8 + 357962*
a*b^7*c^11*d^21*e^8 + 666094*a^2*b^5*c^12*d^21*e^8 + 79420*a^3*b^3*c^13*d^2
1*e^8 - 46024*a^4*b*c^14*d^21*e^8 + 16390*b^10*c^9*d^20*e^9 - 354552*a*b^8*
c^10*d^20*e^9 - 1232352*a^2*b^6*c^11*d^20*e^9 - 504944*a^3*b^4*c^12*d^20*e^
9 + 168740*a^4*b^2*c^13*d^20*e^9 + 16544*a^5*c^14*d^20*e^9 - 27082*b^11*c^8
*d^19*e^10 + 233376*a*b^9*c^9*d^19*e^10 + 1650066*a^2*b^7*c^10*d^19*e^10 +
1490192*a^3*b^5*c^11*d^19*e^10 - 266750*a^4*b^3*c^12*d^19*e^10 - 152504*a^5
*b*c^13*d^19*e^10 + 25776*b^12*c^7*d^18*e^11 - 68156*a*b^10*c^8*d^18*e^11 -
1601556*a^2*b^8*c^9*d^18*e^11 - 2810412*a^3*b^6*c^10*d^18*e^11 - 121880*a^
4*b^4*c^11*d^18*e^11 + 608828*a^5*b^2*c^12*d^18*e^11 + 36080*a^6*c^13*d^18*
e^11 - 17033*b^13*c^6*d^17*e^12 - 45650*a*b^11*c^7*d^17*e^12 + 1086844*a^2*
b^9*c^8*d^17*e^12 + 3675936*a^3*b^7*c^9*d^17*e^12 + 1487739*a^4*b^5*c^10*d^
17*e^12 - 1323762*a^5*b^3*c^11*d^17*e^12 - 320496*a^6*b*c^12*d^17*e^12 + 81
14*b^14*c^5*d^16*e^13 + 74252*a*b^12*c^6*d^16*e^13 - 448888*a^2*b^10*c^7*d^
16*e^13 - 3400320*a^3*b^8*c^8*d^16*e^13 - 3465462*a^4*b^6*c^9*d^16*e^13 + 1
499388*a^5*b^4*c^10*d^16*e^13 + 1247136*a^6*b^2*c^11*d^16*e^13 + 55440*a^7*
c^12*d^16*e^13 - 2760*b^15*c^4*d^15*e^14 - 51104*a*b^13*c^5*d^15*e^14 + 323
92*a^2*b^11*c^6*d^15*e^14 + 2171488*a^3*b^9*c^7*d^15*e^14 + 4691280*a^4*b^7
*c^8*d^15*e^14 - 182688*a^5*b^5*c^9*d^15*e^14 - 2728968*a^6*b^3*c^10*d^15*e

^14 - 461472*a^7*b*c^11*d^15*e^14 + 640*b^16*c^3*d^14*e^15 + 21872*a*b^14*c
^4*d^14*e^15 + 93392*a^2*b^12*c^5*d^14*e^15 - 860816*a^3*b^10*c^6*d^14*e^15
 - 4160640*a^4*b^8*c^7*d^14*e^15 - 2186976*a^5*b^6*c^8*d^14*e^15 + 3502224*
a^6*b^4*c^9*d^14*e^15 + 1675344*a^7*b^2*c^10*d^14*e^15 + 61248*a^8*c^11*d^1
4*e^15 - 91*b^17*c^2*d^13*e^16 - 6002*a*b^15*c^3*d^13*e^16 - 67562*a^2*b^13
*c^4*d^13*e^16 + 115436*a^3*b^11*c^5*d^13*e^16 + 2414610*a^4*b^9*c^6*d^13*e
^16 + 3711180*a^5*b^7*c^7*d^13*e^16 - 2237004*a^6*b^5*c^8*d^13*e^16 - 34137
84*a^7*b^3*c^9*d^13*e^16 - 469128*a^8*b*c^10*d^13*e^16 + 6*b^18*c*d^12*e^17
 + 976*a*b^16*c^2*d^12*e^17 + 24128*a^2*b^14*c^3*d^12*e^17 + 79712*a^3*b^12
*c^4*d^12*e^17 - 820952*a^4*b^10*c^5*d^12*e^17 - 3232416*a^5*b^8*c^6*d^12*e
^17 - 340032*a^6*b^6*c^7*d^12*e^17 + 4124736*a^7*b^4*c^8*d^12*e^17 + 154664
4*a^8*b^2*c^9*d^12*e^17 + 48576*a^9*c^10*d^12*e^17 - 72*a*b^17*c*d^11*e^18
 - 4686*a^2*b^15*c^2*d^11*e^18 - 52000*a^3*b^13*c^3*d^11*e^18 + 78886*a^4*b^
11*c^4*d^11*e^18 + 1641112*a^5*b^9*c^5*d^11*e^18 + 1958220*a^6*b^7*c^6*d^11
*e^18 - 2710752*a^7*b^5*c^7*d^11*e^18 - 2813910*a^8*b^3*c^8*d^11*e^18 - 338
008*a^9*b*c^9*d^11*e^18 + 396*a^2*b^16*c*d^10*e^19 + 13156*a^3*b^14*c^2*d^1
0*e^19 + 55880*a^4*b^12*c^3*d^10*e^19 - 423676*a^5*b^10*c^4*d^10*e^19 - 169
5672*a^6*b^8*c^5*d^10*e^19 + 398904*a^7*b^6*c^6*d^10*e^19 + 2972112*a^8*b^4
*c^7*d^10*e^19 + 990220*a^9*b^2*c^8*d^10*e^19 + 26928*a^10*c^9*d^10*e^19 -
1320*a^3*b^15*c*d^9*e^20 - 23485*a^4*b^13*c^2*d^9*e^20 + 3190*a^5*b^11*c^3*
d^9*e^20 + 692956*a^6*b^9*c^4*d^9*e^20 + 786456*a^7*b^7*c^5*d^9*e^20 - 1667
193*a^8*b^5*c^6*d^9*e^20 - 1552650*a^9*b^3*c^7*d^9*e^20 - 169312*a^10*b*c^8
*d^9*e^20 + 2970*a^4*b^14*c*d^8*e^21 + 26532*a^5*b^12*c^2*d^8*e^21 - 104016
*a^6*b^10*c^3*d^8*e^21 - 613536*a^7*b^8*c^4*d^8*e^21 + 222618*a^8*b^6*c^5*d
^8*e^21 + 1348820*a^9*b^4*c^6*d^8*e^21 + 432344*a^10*b^2*c^7*d^8*e^21 + 968
0*a^11*c^8*d^8*e^21 - 4752*a^5*b^13*c*d^7*e^22 - 15972*a^6*b^11*c^2*d^7*e^2
2 + 165264*a^7*b^9*c^3*d^7*e^22 + 270996*a^8*b^7*c^4*d^7*e^22 - 565136*a^9*
b^5*c^5*d^7*e^22 - 561836*a^10*b^3*c^6*d^7*e^22 - 56048*a^11*b*c^7*d^7*e^22
 + 5544*a^6*b^12*c*d^6*e^23 - 1848*a^7*b^10*c^2*d^6*e^23 - 139920*a^8*b^8*c
^3*d^6*e^23 + 15048*a^9*b^6*c^4*d^6*e^23 + 371624*a^10*b^4*c^5*d^6*e^23 + 1
23128*a^11*b^2*c^6*d^6*e^23 + 1760*a^12*c^7*d^6*e^23 - 4752*a^7*b^11*c*d^5*
e^24 + 13563*a^8*b^9*c^2*d^5*e^24 + 68090*a^9*b^7*c^3*d^5*e^24 - 93698*a^10
*b^5*c^4*d^5*e^24 - 125924*a^11*b^3*c^5*d^5*e^24 - 10936*a^12*b*c^6*d^5*e^2
4 + 2970*a^8*b^10*c*d^4*e^25 - 13640*a^9*b^8*c^2*d^4*e^25 - 14080*a^10*b^6*
c^3*d^4*e^25 + 55472*a^11*b^4*c^4*d^4*e^25 + 21052*a^12*b^2*c^5*d^4*e^25 -
96*a^13*c^6*d^4*e^25 - 1320*a^9*b^9*c*d^3*e^26 + 7634*a^10*b^7*c^2*d^3*e^26
 - 3632*a^11*b^5*c^3*d^3*e^26 - 15602*a^12*b^3*c^4*d^3*e^26 - 904*a^13*b*c^
5*d^3*e^26 + 396*a^10*b^8*c*d^2*e^27 - 2604*a^11*b^6*c^2*d^2*e^27 + 3208*a^
12*b^4*c^3*d^2*e^27 + 1892*a^13*b^2*c^4*d^2*e^27 - 112*a^14*c^5*d^2*e^27 -
72*a^11*b^7*c*d*e^28 + 509*a^12*b^5*c^2*d*e^28 - 830*a^13*b^3*c^3*d*e^28 +
16*a^14*b*c^4*d*e^28 + 6*a^12*b^6*c*e^29 - 44*a^13*b^4*c^2*e^29 + 80*a^14*b
^2*c^3*e^29 - 16*a^15*c^4*e^29)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))*x + 3
*(2*b^2*c^17*d^29 + 8*a*c^18*d^29 - 29*b^3*c^16*d^28*e - 116*a*b*c^17*d^28*
e + 192*b^4*c^15*d^27*e^2 + 824*a*b^2*c^16*d^27*e^2 + 128*a^2*c^17*d^27*e^2
 - 760*b^5*c^14*d^26*e^3 - 3856*a*b^3*c^15*d^26*e^3 - 1648*a^2*b*c^16*d^26*
e^3 + 1960*b^6*c^13*d^25*e^4 + 13330*a*b^4*c^14*d^25*e^4 + 10114*a^2*b^2*c^
15*d^25*e^4 + 840*a^3*c^16*d^25*e^4 - 3276*b^7*c^12*d^24*e^5 - 35668*a*b^5*
c^13*d^24*e^5 - 40033*a^2*b^3*c^14*d^24*e^5 - 9556*a^3*b*c^15*d^24*e^5 + 29
12*b^8*c^11*d^23*e^6 + 74744*a*b^6*c^12*d^23*e^6 + 116560*a^2*b^4*c^13*d^23
*e^6 + 50816*a^3*b^2*c^14*d^23*e^6 + 3008*a^4*c^15*d^23*e^6 + 1144*b^9*c^10
*d^22*e^7 - 121712*a*b^7*c^11*d^22*e^7 - 267100*a^2*b^5*c^12*d^22*e^7 - 171
608*a^3*b^3*c^13*d^22*e^7 - 29504*a^4*b*c^14*d^22*e^7 - 8580*b^10*c^9*d^21*
e^8 + 150326*a*b^8*c^10*d^21*e^8 + 494186*a^2*b^6*c^11*d^21*e^8 + 426052*a^
3*b^4*c^12*d^21*e^8 + 131450*a^4*b^2*c^13*d^21*e^8 + 6248*a^5*c^14*d^21*e^8
 + 15730*b^11*c^8*d^20*e^9 - 133100*a*b^9*c^9*d^20*e^9 - 735075*a^2*b^7*c^1
0*d^20*e^9 - 850520*a^3*b^5*c^11*d^20*e^9 - 362945*a^4*b^3*c^12*d^20*e^9 -
49060*a^5*b*c^13*d^20*e^9 - 18304*b^12*c^7*d^19*e^10 + 70532*a*b^10*c^8*d^1
9*e^10 + 859056*a^2*b^8*c^9*d^19*e^10 + 1418912*a^3*b^6*c^10*d^19*e^10 + 74
0960*a^4*b^4*c^11*d^19*e^10 + 159544*a^5*b^2*c^12*d^19*e^10 + 6336*a^6*c^13

*d^19*e^10 + 15288*b^13*c^6*d^18*e^11 + 1840*a*b^11*c^7*d^18*e^11 - 758450*
a^2*b^9*c^8*d^18*e^11 - 1957692*a^3*b^7*c^9*d^18*e^11 - 1293116*a^4*b^5*c^1
0*d^18*e^11 - 278168*a^5*b^3*c^11*d^18*e^11 - 24112*a^6*b*c^12*d^18*e^11 -
9464*b^14*c^5*d^17*e^12 - 44258*a*b^12*c^6*d^17*e^12 + 468886*a^2*b^10*c^7*
d^17*e^12 + 2149356*a^3*b^8*c^8*d^17*e^12 + 2050158*a^4*b^6*c^9*d^17*e^12 +
329934*a^5*b^4*c^10*d^17*e^12 - 46662*a^6*b^2*c^11*d^17*e^12 - 2904*a^7*c^
12*d^17*e^12 + 4340*b^15*c^4*d^16*e^13 + 46916*a*b^13*c^5*d^16*e^13 - 15946
3*a^2*b^11*c^6*d^16*e^13 - 1786884*a^3*b^9*c^7*d^16*e^13 - 2793945*a^4*b^7*
c^8*d^16*e^13 - 533676*a^5*b^5*c^9*d^16*e^13 + 454443*a^6*b^3*c^10*d^16*e^1
3 + 80124*a^7*b*c^11*d^16*e^13 - 1440*b^16*c^3*d^15*e^14 - 28880*a*b^14*c^4
*d^15*e^14 - 22112*a^2*b^12*c^5*d^15*e^14 + 1048960*a^3*b^10*c^6*d^15*e^14
+ 2988480*a^4*b^8*c^7*d^15*e^14 + 1254000*a^5*b^6*c^8*d^15*e^14 - 1145760*a
^6*b^4*c^9*d^15*e^14 - 508992*a^7*b^2*c^10*d^15*e^14 - 21120*a^8*c^11*d^15*
e^14 + 328*b^17*c^2*d^14*e^15 + 11728*a*b^15*c^3*d^14*e^15 + 62552*a^2*b^13
*c^4*d^14*e^15 - 369296*a^3*b^11*c^5*d^14*e^15 - 2332880*a^4*b^9*c^6*d^14*e
^15 - 2231328*a^5*b^7*c^7*d^14*e^15 + 1342440*a^6*b^5*c^8*d^14*e^15 + 15380
64*a^7*b^3*c^9*d^14*e^15 + 219648*a^8*b*c^10*d^14*e^15 - 46*b^18*c*d^13*e^1
6 - 3118*a*b^16*c^2*d^13*e^16 - 38210*a^2*b^14*c^3*d^13*e^16 + 19676*a^3*b^
12*c^4*d^13*e^16 + 1232198*a^4*b^10*c^5*d^13*e^16 + 2569116*a^5*b^8*c^6*d^1
3*e^16 - 407484*a^6*b^6*c^7*d^13*e^16 - 2625480*a^7*b^4*c^8*d^13*e^16 - 919
578*a^8*b^2*c^9*d^13*e^16 - 37224*a^9*c^10*d^13*e^16 + 3*b^19*d^12*e^17 + 4
96*a*b^17*c*d^12*e^17 + 12811*a^2*b^15*c^2*d^12*e^17 + 53552*a^3*b^13*c^3*d
^12*e^17 - 372179*a^4*b^11*c^4*d^12*e^17 - 1894508*a^5*b^9*c^5*d^12*e^17 -
960366*a^6*b^7*c^6*d^12*e^17 + 2580336*a^7*b^5*c^7*d^12*e^17 + 2072169*a^8*
b^3*c^8*d^12*e^17 + 290532*a^9*b*c^9*d^12*e^17 - 36*a*b^18*d^11*e^18 - 2400
*a^2*b^16*c*d^11*e^18 - 28768*a^3*b^14*c^2*d^11*e^18 + 14720*a^4*b^12*c^3*d
^11*e^18 + 854128*a^5*b^10*c^4*d^11*e^18 + 1488960*a^6*b^8*c^5*d^11*e^18 -
1197504*a^7*b^6*c^6*d^11*e^18 - 2751936*a^8*b^4*c^7*d^11*e^18 - 942040*a^9*
b^2*c^8*d^11*e^18 - 39424*a^10*c^9*d^11*e^18 + 198*a^2*b^17*d^10*e^19 + 682
0*a^3*b^15*c*d^10*e^19 + 34540*a^4*b^13*c^2*d^10*e^19 - 189640*a^5*b^11*c^3
*d^10*e^19 - 1011780*a^6*b^9*c^4*d^10*e^19 - 222552*a^7*b^7*c^5*d^10*e^19 +
2135760*a^8*b^5*c^6*d^10*e^19 + 1650880*a^9*b^3*c^7*d^10*e^19 + 243760*a^1
0*b*c^8*d^10*e^19 - 660*a^3*b^16*d^9*e^20 - 12430*a^4*b^14*c*d^9*e^20 - 886
6*a^5*b^12*c^2*d^9*e^20 + 352594*a^6*b^10*c^3*d^9*e^20 + 635976*a^7*b^8*c^4
*d^9*e^20 - 797148*a^8*b^6*c^5*d^9*e^20 - 1680690*a^9*b^4*c^6*d^9*e^20 - 61
6858*a^10*b^2*c^7*d^9*e^20 - 28072*a^11*c^8*d^9*e^20 + 1485*a^4*b^15*d^8*e^
21 + 14652*a^5*b^13*c*d^8*e^21 - 41349*a^6*b^11*c^2*d^8*e^21 - 353100*a^7*b
^9*c^3*d^8*e^21 - 74382*a^8*b^7*c^4*d^8*e^21 + 962324*a^9*b^5*c^5*d^8*e^21
+ 820061*a^10*b^3*c^6*d^8*e^21 + 136004*a^11*b*c^7*d^8*e^21 - 2376*a^5*b^14
*d^7*e^22 - 10032*a^6*b^12*c*d^7*e^22 + 77088*a^7*b^10*c^2*d^7*e^22 + 20169
6*a^8*b^8*c^3*d^7*e^22 - 232232*a^9*b^6*c^4*d^7*e^22 - 604208*a^10*b^4*c^5*
d^7*e^22 - 259136*a^11*b^2*c^6*d^7*e^22 - 13632*a^12*c^7*d^7*e^22 + 2772*a^
6*b^13*d^6*e^23 + 1320*a^7*b^11*c*d^6*e^23 - 71544*a^8*b^9*c^2*d^6*e^23 - 4
4880*a^9*b^7*c^3*d^6*e^23 + 228404*a^10*b^5*c^4*d^6*e^23 + 244424*a^11*b^3*
c^5*d^6*e^23 + 49472*a^12*b*c^6*d^6*e^23 - 2376*a^7*b^12*d^5*e^24 + 4950*a^
8*b^10*c*d^5*e^24 + 39710*a^9*b^8*c^2*d^5*e^24 - 22990*a^10*b^6*c^3*d^5*e^2
4 - 116540*a^11*b^4*c^4*d^5*e^24 - 65506*a^12*b^2*c^5*d^5*e^24 - 4360*a^13*
c^6*d^5*e^24 + 1485*a^8*b^11*d^4*e^25 - 5720*a^9*b^9*c*d^4*e^25 - 12287*a^1
0*b^7*c^2*d^4*e^25 + 23528*a^11*b^5*c^3*d^4*e^25 + 38197*a^12*b^3*c^4*d^4*e
^25 + 10804*a^13*b*c^5*d^4*e^25 - 660*a^9*b^10*d^3*e^26 + 3344*a^10*b^8*c*d
^3*e^26 + 992*a^11*b^6*c^2*d^3*e^26 - 9440*a^12*b^4*c^3*d^3*e^26 - 8344*a^1
3*b^2*c^4*d^3*e^26 - 832*a^14*c^5*d^3*e^26 + 198*a^10*b^9*d^2*e^27 - 1164*a
^11*b^7*c*d^2*e^27 + 676*a^12*b^5*c^2*d^2*e^27 + 2152*a^13*b^3*c^3*d^2*e^27
+ 1136*a^14*b*c^4*d^2*e^27 - 36*a^11*b^8*d*e^28 + 230*a^12*b^6*c*d*e^28 -
238*a^13*b^4*c^2*d*e^28 - 290*a^14*b^2*c^3*d*e^28 - 72*a^15*c^4*d*e^28 + 3*
a^12*b^7*e^29 - 20*a^13*b^5*c*e^29 + 25*a^14*b^3*c^2*e^29 + 20*a^15*b*c^3*e
^29)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))*x - (b^3*c^16*d^29 - 12*a*b*c^17
*d^29 - 16*b^4*c^15*d^28*e + 186*a*b^2*c^16*d^28*e - 24*a^2*c^17*d^28*e + 1
20*b^5*c^14*d^27*e^2 - 1328*a*b^3*c^15*d^27*e^2 + 144*a^2*b*c^16*d^27*e^2 -

560*b^6*c^13*d^26*e^3 + 5760*a*b^4*c^14*d^26*e^3 + 672*a^2*b^2*c^15*d^26*e
^3 - 544*a^3*c^16*d^26*e^3 + 1820*b^7*c^12*d^25*e^4 - 16800*a*b^5*c^13*d^25
*e^4 - 10875*a^2*b^3*c^14*d^25*e^4 + 5812*a^3*b*c^15*d^25*e^4 - 4368*b^8*c^
11*d^24*e^5 + 34216*a*b^6*c^12*d^24*e^5 + 58206*a^2*b^4*c^13*d^24*e^5 - 245
58*a^3*b^2*c^14*d^24*e^5 - 4600*a^4*c^15*d^24*e^5 + 8008*b^9*c^10*d^23*e^6
- 48048*a*b^7*c^11*d^23*e^6 - 186372*a^2*b^5*c^12*d^23*e^6 + 39032*a^3*b^3*
c^13*d^23*e^6 + 50688*a^4*b*c^14*d^23*e^6 - 11440*b^10*c^9*d^22*e^7 + 41184
*a*b^8*c^10*d^22*e^7 + 405648*a^2*b^6*c^11*d^22*e^7 + 73360*a^3*b^4*c^12*d^
22*e^7 - 242448*a^4*b^2*c^13*d^22*e^7 - 21504*a^5*c^14*d^22*e^7 + 12870*b^1
1*c^8*d^21*e^8 - 5720*a*b^9*c^9*d^21*e^8 - 627033*a^2*b^7*c^10*d^21*e^8 - 5
42784*a^3*b^5*c^11*d^21*e^8 + 633941*a^4*b^3*c^12*d^21*e^8 + 227172*a^5*b*c
^13*d^21*e^8 - 11440*b^12*c^7*d^20*e^9 - 42900*a*b^10*c^8*d^20*e^9 + 688710
*a^2*b^8*c^9*d^20*e^9 + 1451186*a^3*b^6*c^10*d^20*e^9 - 866052*a^4*b^4*c^11
*d^20*e^9 - 1059102*a^5*b^2*c^12*d^20*e^9 - 64504*a^6*c^13*d^20*e^9 + 8008*
b^13*c^6*d^19*e^10 + 75504*a*b^11*c^7*d^19*e^10 - 507342*a^2*b^9*c^8*d^19*e
^10 - 2406404*a^3*b^7*c^9*d^19*e^10 + 102300*a^4*b^5*c^10*d^19*e^10 + 28150
32*a^5*b^3*c^11*d^19*e^10 + 635536*a^6*b*c^12*d^19*e^10 - 4368*b^14*c^5*d^1
8*e^11 - 75712*a*b^12*c^6*d^18*e^11 + 188496*a^2*b^10*c^7*d^18*e^11 + 27149
76*a^3*b^8*c^8*d^18*e^11 + 2038784*a^4*b^6*c^9*d^18*e^11 - 4505952*a^5*b^4*
c^10*d^18*e^11 - 2767248*a^6*b^2*c^11*d^18*e^11 - 133408*a^7*c^12*d^18*e^11
 + 1820*b^15*c^4*d^17*e^12 + 52416*a*b^13*c^5*d^17*e^12 + 66387*a^2*b^11*c^
6*d^17*e^12 - 2086700*a^3*b^9*c^7*d^17*e^12 - 4439259*a^4*b^7*c^8*d^17*e^12
 + 3860208*a^5*b^5*c^9*d^17*e^12 + 6919209*a^6*b^3*c^10*d^17*e^12 + 1205028
*a^7*b*c^11*d^17*e^12 - 560*b^16*c^3*d^16*e^13 - 26040*a*b^14*c^4*d^16*e^13
 - 151350*a^2*b^12*c^5*d^16*e^13 + 994070*a^3*b^10*c^6*d^16*e^13 + 5238288*
a^4*b^8*c^7*d^16*e^13 + 7326*a^5*b^6*c^8*d^16*e^13 - 10685070*a^6*b^4*c^9*d
^16*e^13 - 4787046*a^7*b^2*c^10*d^16*e^13 - 197208*a^8*c^11*d^16*e^13 + 120
*b^17*c^2*d^15*e^14 + 9200*a*b^15*c^3*d^15*e^14 + 113640*a^2*b^13*c^4*d^15*
e^14 - 155568*a^3*b^11*c^5*d^15*e^14 - 3907376*a^4*b^9*c^6*d^15*e^14 - 4489
056*a^5*b^7*c^7*d^15*e^14 + 9827928*a^6*b^5*c^8*d^15*e^14 + 10874160*a^7*b^
3*c^9*d^15*e^14 + 1609344*a^8*b*c^10*d^15*e^14 - 16*b^18*c*d^14*e^15 - 2208
*a*b^16*c^2*d^14*e^15 - 51264*a^2*b^14*c^3*d^14*e^15 - 152288*a^3*b^12*c^4*
d^14*e^15 + 1774080*a^4*b^10*c^5*d^14*e^15 + 6025536*a^5*b^8*c^6*d^14*e^15
- 3729792*a^6*b^6*c^7*d^14*e^15 - 15241248*a^7*b^4*c^8*d^14*e^15 - 5724576*
a^8*b^2*c^9*d^14*e^15 - 211200*a^9*c^10*d^14*e^15 + b^19*d^13*e^16 + 324*a*
b^17*c*d^13*e^16 + 14625*a^2*b^15*c^2*d^13*e^16 + 131192*a^3*b^13*c^3*d^13*
e^16 - 334497*a^4*b^11*c^4*d^13*e^16 - 4234956*a^5*b^9*c^5*d^13*e^16 - 2639
274*a^6*b^7*c^6*d^13*e^16 + 12912768*a^7*b^5*c^7*d^13*e^16 + 11515779*a^8*b
^3*c^8*d^13*e^16 + 1534236*a^9*b*c^9*d^13*e^16 - 22*a*b^18*d^12*e^17 - 2454
*a^2*b^16*c*d^12*e^17 - 50010*a^3*b^14*c^2*d^12*e^17 - 116468*a^4*b^12*c^3*
d^12*e^17 + 1652046*a^5*b^10*c^4*d^12*e^17 + 4616172*a^6*b^8*c^5*d^12*e^17
- 5238156*a^7*b^6*c^6*d^12*e^17 - 14093640*a^8*b^4*c^7*d^12*e^17 - 4796946*
a^9*b^2*c^8*d^12*e^17 - 162888*a^10*c^9*d^12*e^17 + 186*a^2*b^17*d^11*e^18
+ 10108*a^3*b^15*c*d^11*e^18 + 95796*a^4*b^13*c^2*d^11*e^18 - 227448*a^5*b^
11*c^3*d^11*e^18 - 2897180*a^6*b^9*c^4*d^11*e^18 - 1101672*a^7*b^7*c^5*d^11
*e^18 + 10234224*a^8*b^5*c^6*d^11*e^18 + 8337472*a^9*b^3*c^7*d^11*e^18 + 10
36464*a^10*b*c^8*d^11*e^18 - 880*a^3*b^16*d^10*e^19 - 25872*a^4*b^14*c*d^10
*e^19 - 86592*a^5*b^12*c^2*d^10*e^19 + 858176*a^6*b^10*c^3*d^10*e^19 + 2534
400*a^7*b^8*c^4*d^10*e^19 - 3470544*a^8*b^6*c^5*d^10*e^19 - 8593024*a^9*b^4
*c^6*d^10*e^19 - 2792064*a^10*b^2*c^7*d^10*e^19 - 87648*a^11*c^8*d^10*e^19
+ 2695*a^4*b^15*d^9*e^20 + 43032*a^5*b^13*c*d^9*e^20 - 37719*a^6*b^11*c^2*d
^9*e^20 - 1258532*a^7*b^9*c^3*d^9*e^20 - 574794*a^8*b^7*c^4*d^9*e^20 + 5016
000*a^9*b^5*c^5*d^9*e^20 + 4081231*a^10*b^3*c^6*d^9*e^20 + 480348*a^11*b*c^
7*d^9*e^20 - 5742*a^5*b^14*d^8*e^21 - 45078*a^6*b^12*c*d^8*e^21 + 220770*a^
7*b^10*c^2*d^8*e^21 + 996336*a^8*b^8*c^3*d^8*e^21 - 1171676*a^9*b^6*c^4*d^8
*e^21 - 3397086*a^10*b^4*c^5*d^8*e^21 - 1092234*a^11*b^2*c^6*d^8*e^21 - 301
84*a^12*c^7*d^8*e^21 + 8844*a^6*b^13*d^7*e^22 + 22968*a^7*b^11*c*d^7*e^22 -
 312840*a^8*b^9*c^2*d^7*e^22 - 327184*a^9*b^7*c^3*d^7*e^22 + 1465068*a^10*b
^5*c^4*d^7*e^22 + 1303512*a^11*b^3*c^5*d^7*e^22 + 141184*a^12*b*c^6*d^7*e^2

2 - 10032*a^7*b^12*d^6*e^23 + 9504*a^8*b^10*c*d^6*e^23 + 246048*a^9*b^8*c^2
*d^6*e^23 - 151184*a^10*b^6*c^3*d^6*e^23 - 829680*a^11*b^4*c^4*d^6*e^23 - 2
68368*a^12*b^2*c^5*d^6*e^23 - 4864*a^13*c^6*d^6*e^23 + 8415*a^8*b^11*d^5*e^
24 - 28556*a^9*b^9*c*d^5*e^24 - 108141*a^10*b^7*c^2*d^5*e^24 + 226368*a^11*
b^5*c^3*d^5*e^24 + 255335*a^12*b^3*c^4*d^5*e^24 + 21132*a^13*b*c^5*d^5*e^24
 - 5170*a^9*b^10*d^4*e^25 + 26334*a^10*b^8*c*d^4*e^25 + 14202*a^11*b^6*c^2*
d^4*e^25 - 114404*a^12*b^4*c^3*d^4*e^25 - 36618*a^13*b^2*c^4*d^4*e^25 + 600
*a^14*c^5*d^4*e^25 + 2266*a^10*b^9*d^3*e^26 - 14196*a^11*b^7*c*d^3*e^26 + 1
1580*a^12*b^5*c^2*d^3*e^26 + 28472*a^13*b^3*c^3*d^3*e^26 + 48*a^14*b*c^4*d^
3*e^26 - 672*a^11*b^8*d^2*e^27 + 4736*a^12*b^6*c*d^2*e^27 - 7200*a^13*b^4*c
^2*d^2*e^27 - 2448*a^14*b^2*c^3*d^2*e^27 + 416*a^15*c^4*d^2*e^27 + 121*a^12
*b^7*d*e^28 - 912*a^13*b^5*c*d*e^28 + 1731*a^14*b^3*c^2*d*e^28 - 308*a^15*b
*c^3*d*e^28 - 10*a^13*b^6*e^29 + 78*a^14*b^4*c*e^29 - 162*a^15*b^2*c^2*e^29
 + 56*a^16*c^3*e^29)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))/(c*x^2 + b*x + a
)^(3/2) - 1/4*(88*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*d^3*e^4 + 4
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*d^2*e^5 + 88*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))*b*c^2*d^3*e^4 - 72*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*
b*c^(3/2)*d^2*e^5 + 22*b^2*c^(3/2)*d^3*e^4 - 40*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^3*b*c*d*e^6 - 68*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*d^2*e^5
 - 136*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*d^2*e^5 + 17*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*d*e^6 - 44*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^2*a*c^(3/2)*d*e^6 - 11*b^3*sqrt(c)*d^2*e^5 - 68*a*b*c^(3/2)*d^2*e^
5 + 11*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*e^7 - 4*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^3*a*c*e^7 + 13*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*d*
e^6 + 92*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*d*e^6 + 16*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*e^7 + 35*a*b^2*sqrt(c)*d*e^6 + 44*a^2*
c^(3/2)*d*e^6 - 13*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*e^7 - 4*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*e^7 - 24*a^2*b*sqrt(c)*e^7)/((c^4*d^8 -
 4*b*c^3*d^7*e + 6*b^2*c^2*d^6*e^2 + 4*a*c^3*d^6*e^2 - 4*b^3*c*d^5*e^3 - 12
*a*b*c^2*d^5*e^3 + b^4*d^4*e^4 + 12*a*b^2*c*d^4*e^4 + 6*a^2*c^2*d^4*e^4 - 4
*a*b^3*d^3*e^5 - 12*a^2*b*c*d^3*e^5 + 6*a^2*b^2*d^2*e^6 + 4*a^3*c*d^2*e^6 -
 4*a^3*b*d*e^7 + a^4*e^8)*((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*e + 2*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c)*d + b*d - a*e)^2)

3.2400 $\int \frac{3+x}{\sqrt{5-4x-x^2}} \, dx$

Optimal. Leaf size=29

$$-\sqrt{-x^2-4x+5}-\sin^{-1}\left(\frac{1}{3}(-x-2)\right)$$

```
[Out] -Sqrt[5 - 4*x - x^2] - ArcSin[(-2 - x)/3]
```

Rubi [A] time = 0.0129247, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {640, 619, 216}

$$-\sqrt{-x^2-4x+5}-\sin^{-1}\left(\frac{1}{3}(-x-2)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + x)/Sqrt[5 - 4*x - x^2],x]
```

```
[Out] -Sqrt[5 - 4*x - x^2] - ArcSin[(-2 - x)/3]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{3+x}{\sqrt{5-4x-x^2}} \, dx &= -\sqrt{5-4x-x^2}+\int \frac{1}{\sqrt{5-4x-x^2}} \, dx\\
&= -\sqrt{5-4x-x^2}-\frac{1}{6} \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{36}}} \, dx,x,-4-2x\right)\\
&= -\sqrt{5-4x-x^2}-\sin^{-1}\left(\frac{1}{3}(-2-x)\right)
\end{aligned}$$

Mathematica [A] time = 0.0081645, size = 25, normalized size = 0.86

$$\sin^{-1}\left(\frac{x+2}{3}\right)-\sqrt{-x^2-4x+5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x)/Sqrt[5 - 4*x - x^2],x]
```

```
[Out] -Sqrt[5 - 4*x - x^2] + ArcSin[(2 + x)/3]
```

Maple [A] time = 0.044, size = 22, normalized size = 0.8

$$\arcsin\left(\frac{2}{3}+\frac{x}{3}\right)-\sqrt{-x^{2}-4x+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+x)/(-x^2-4*x+5)^(1/2),x)
```

```
[Out] arcsin(2/3+1/3*x)-(-x^2-4*x+5)^(1/2)
```

Maxima [A] time = 1.48641, size = 31, normalized size = 1.07

$$-\sqrt{-x^{2}-4x+5}-\arcsin\left(-\frac{1}{3}x-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(-x^2-4*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 - 4*x + 5) - arcsin(-1/3*x - 2/3)
```

Fricas [B] time = 2.34188, size = 105, normalized size = 3.62

$$-\sqrt{-x^{2}-4x+5}-\arctan\left(\frac{\sqrt{-x^{2}-4x+5}(x+2)}{x^{2}+4x-5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(-x^2-4*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 - 4*x + 5) - arctan(sqrt(-x^2 - 4*x + 5)*(x + 2)/(x^2 + 4*x - 5)
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+3}{\sqrt{-(x-1)(x+5)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(-x**2-4*x+5)**(1/2),x)
```

```
[Out] Integral((x + 3)/sqrt(-(x - 1)*(x + 5)), x)
```

Giac [A] time = 1.51581, size = 28, normalized size = 0.97

$$-\sqrt{-x^2-4\,x+5}+\arcsin\left(\frac{1}{3}\,x+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(-x^2-4*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 - 4*x + 5) + arcsin(1/3*x + 2/3)
```

3.2401 $\int \frac{5-4x}{\sqrt{-8+12x-4x^2}}\,dx$

Optimal. Leaf size=25

$$\sqrt{-4x^2+12x-8}+\frac{1}{2}\sin^{-1}(3-2x)$$

```
[Out] Sqrt[-8 + 12*x - 4*x^2] + ArcSin[3 - 2*x]/2
```

Rubi [A] time = 0.0107881, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {640, 619, 216}

$$\sqrt{-4x^2+12x-8}+\frac{1}{2}\sin^{-1}(3-2x)$$

Antiderivative was successfully verified.

```
[In] Int[(5 - 4*x)/Sqrt[-8 + 12*x - 4*x^2],x]
```

```
[Out] Sqrt[-8 + 12*x - 4*x^2] + ArcSin[3 - 2*x]/2
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{5-4x}{\sqrt{-8+12x-4x^2}}\,dx &= \sqrt{-8+12x-4x^2}-\int \frac{1}{\sqrt{-8+12x-4x^2}}\,dx\\ &= \sqrt{-8+12x-4x^2}+\frac{1}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{16}}}\,dx,x,12-8x\right)\\ &= \sqrt{-8+12x-4x^2}+\frac{1}{2}\sin^{-1}(3-2x)\end{aligned}$$

Mathematica [A] time = 0.009147, size = 25, normalized size = 1.

$$\sqrt{-4x^2+12x-8}+\frac{1}{2}\sin^{-1}(3-2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - 4*x)/Sqrt[-8 + 12*x - 4*x^2],x]
```

```
[Out] Sqrt[-8 + 12*x - 4*x^2] + ArcSin[3 - 2*x]/2
```

Maple [A] time = 0.045, size = 24, normalized size = 1.

$$-\frac{\arcsin\left(-3+2x\right)}{2}+2\sqrt{-x^{2}+3x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*(5-4*x)/(-x^2+3*x-2)^(1/2),x)
```

```
[Out] -1/2*arcsin(-3+2*x)+2*(-x^2+3*x-2)^(1/2)
```

Maxima [A] time = 1.44415, size = 31, normalized size = 1.24

$$2\sqrt{-x^{2}+3x-2}-\frac{1}{2}\arcsin\left(2x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(5-4*x)/(-x^2+3*x-2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(-x^2 + 3*x - 2) - 1/2*arcsin(2*x - 3)
```

Fricas [B] time = 2.39347, size = 120, normalized size = 4.8

$$2\sqrt{-x^{2}+3x-2}+\frac{1}{2}\arctan\left(\frac{\sqrt{-x^{2}+3x-2}(2x-3)}{2\left(x^{2}-3x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(5-4*x)/(-x^2+3*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(-x^2 + 3*x - 2) + 1/2*arctan(1/2*sqrt(-x^2 + 3*x - 2)*(2*x - 3)/(x^2
 - 3*x + 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{4x}{\sqrt{-x^{2}+3x-2}}\,dx+\int-\frac{5}{\sqrt{-x^{2}+3x-2}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(5-4*x)/(-x**2+3*x-2)**(1/2),x)
```

```
[Out] -(Integral(4*x/sqrt(-x**2 + 3*x - 2), x) + Integral(-5/sqrt(-x**2 + 3*x - 2
), x))/2
```

Giac [A] time = 1.58267, size = 31, normalized size = 1.24

$$2\sqrt{-x^2+3x-2}-\frac{1}{2}\arcsin(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(5-4*x)/(-x^2+3*x-2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(-x^2 + 3*x - 2) - 1/2*arcsin(2*x - 3)
```

3.2402 $\int \frac{3+2x}{\sqrt{5+2x+x^2}} \, dx$

Optimal. Leaf size=23

$$2\sqrt{x^2+2x+5}+\sinh^{-1}\left(\frac{x+1}{2}\right)$$

```
[Out] 2*Sqrt[5 + 2*x + x^2] + ArcSinh[(1 + x)/2]
```

Rubi [A] time = 0.0110523, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {640, 619, 215}

$$2\sqrt{x^2+2x+5}+\sinh^{-1}\left(\frac{x+1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x)/Sqrt[5 + 2*x + x^2],x]

[Out] 2*Sqrt[5 + 2*x + x^2] + ArcSinh[(1 + x)/2]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{3+2x}{\sqrt{5+2x+x^2}} \, dx &= 2\sqrt{5+2x+x^2}+\int \frac{1}{\sqrt{5+2x+x^2}} \, dx \\
&= 2\sqrt{5+2x+x^2}+\frac{1}{4} \operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{16}}} \, dx,x,2+2x\right) \\
&= 2\sqrt{5+2x+x^2}+\sinh^{-1}\left(\frac{1+x}{2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0064274, size = 25, normalized size = 1.09

$$2\sqrt{x^2+2x+5}+\sinh^{-1}\left(\frac{1}{4}(2x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x)/Sqrt[5 + 2*x + x^2],x]
```

```
[Out] 2*Sqrt[5 + 2*x + x^2] + ArcSinh[(2 + 2*x)/4]
```

Maple [A] time = 0.044, size = 20, normalized size = 0.9

$$\operatorname{Arcsinh}\left(\frac{1}{2}+\frac{x}{2}\right)+2\sqrt{x^2+2x+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+2*x)/(x^2+2*x+5)^(1/2),x)
```

```
[Out] arcsinh(1/2+1/2*x)+2*(x^2+2*x+5)^(1/2)
```

Maxima [A] time = 1.47463, size = 26, normalized size = 1.13

$$2\sqrt{x^2+2x+5}+\operatorname{arsinh}\left(\frac{1}{2}x+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(x^2+2*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x^2 + 2*x + 5) + arcsinh(1/2*x + 1/2)
```

Fricas [A] time = 2.50454, size = 80, normalized size = 3.48

$$2\sqrt{x^2+2x+5}-\log\left(-x+\sqrt{x^2+2x+5}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(x^2+2*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x^2 + 2*x + 5) - log(-x + sqrt(x^2 + 2*x + 5) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x+3}{\sqrt{x^2+2x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(x**2+2*x+5)**(1/2),x)
```

```
[Out] Integral((2*x + 3)/sqrt(x**2 + 2*x + 5), x)
```

Giac [A] time = 1.46904, size = 42, normalized size = 1.83

$$2\sqrt{x^2+2x+5}-\log\left(-x+\sqrt{x^2+2x+5}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(x^2+2*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x^2 + 2*x + 5) - log(-x + sqrt(x^2 + 2*x + 5) - 1)
```

3.2403 $\int \frac{-1+x}{\sqrt{3-4x+x^2}}\,dx$

Optimal. Leaf size=34

$$\sqrt{x^2-4x+3}-\tanh^{-1}\left(\frac{2-x}{\sqrt{x^2-4x+3}}\right)$$

```
[Out] Sqrt[3 - 4*x + x^2] - ArcTanh[(2 - x)/Sqrt[3 - 4*x + x^2]]
```

Rubi [A] time = 0.009237, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {640, 621, 206}

$$\sqrt{x^2-4x+3}-\tanh^{-1}\left(\frac{2-x}{\sqrt{x^2-4x+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/Sqrt[3 - 4*x + x^2],x]
```

```
[Out] Sqrt[3 - 4*x + x^2] - ArcTanh[(2 - x)/Sqrt[3 - 4*x + x^2]]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x}{\sqrt{3-4x+x^2}}\,dx &= \sqrt{3-4x+x^2}+\int \frac{1}{\sqrt{3-4x+x^2}}\,dx\\
&= \sqrt{3-4x+x^2}+2\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{-4+2x}{\sqrt{3-4x+x^2}}\right)\\
&= \sqrt{3-4x+x^2}-\tanh^{-1}\left(\frac{2-x}{\sqrt{3-4x+x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0191737, size = 30, normalized size = 0.88

$$\sqrt{x^2-4x+3}+\tanh^{-1}\left(\frac{x-2}{\sqrt{x^2-4x+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/Sqrt[3 - 4*x + x^2],x]
```

```
[Out] Sqrt[3 - 4*x + x^2] + ArcTanh[(-2 + x)/Sqrt[3 - 4*x + x^2]]
```

Maple [A] time = 0.043, size = 26, normalized size = 0.8

$$\ln\left(x-2+\sqrt{x^2-4x+3}\right)+\sqrt{x^2-4x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/(x^2-4*x+3)^(1/2),x)
```

```
[Out] ln(x-2+(x^2-4*x+3)^(1/2))+(x^2-4*x+3)^(1/2)
```

Maxima [A] time = 0.994683, size = 39, normalized size = 1.15

$$\sqrt{x^2-4x+3}+\log\left(2x+2\sqrt{x^2-4x+3}-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2-4*x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 - 4*x + 3) + log(2*x + 2*sqrt(x^2 - 4*x + 3) - 4)
```

Fricas [A] time = 2.29784, size = 77, normalized size = 2.26

$$\sqrt{x^2-4x+3}-\log\left(-x+\sqrt{x^2-4x+3}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2-4*x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x^2 - 4*x + 3) - log(-x + sqrt(x^2 - 4*x + 3) + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x-1}{\sqrt{(x-3)(x-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x**2-4*x+3)**(1/2),x)
```

```
[Out] Integral((x - 1)/sqrt((x - 3)*(x - 1)), x)
```

Giac [A] time = 1.38208, size = 41, normalized size = 1.21

$$\sqrt{x^2-4x+3}-\log\left(\left|-x+\sqrt{x^2-4x+3}+2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2-4*x+3)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x^2 - 4*x + 3) - log(abs(-x + sqrt(x^2 - 4*x + 3) + 2))
```

3.2404 $\int \frac{1}{(1-x)\sqrt{-4+2x+x^2}} dx$

Optimal. Leaf size=19

$$\tan^{-1}\left(\frac{3-2x}{\sqrt{x^2+2x-4}}\right)$$

```
[Out] ArcTan[(3 - 2*x)/Sqrt[-4 + 2*x + x^2]]
```

Rubi [A] time = 0.0118567, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {724, 204}

$$\tan^{-1}\left(\frac{3-2x}{\sqrt{x^2+2x-4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)*Sqrt[-4 + 2*x + x^2]),x]
```

```
[Out] ArcTan[(3 - 2*x)/Sqrt[-4 + 2*x + x^2]]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)\sqrt{-4+2x+x^2}} dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{-4-x^2} dx, x, \frac{6-4x}{\sqrt{-4+2x+x^2}}\right)\right)\\ &= \tan^{-1}\left(\frac{3-2x}{\sqrt{-4+2x+x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0059554, size = 22, normalized size = 1.16

$$\tan^{-1}\left(\frac{6-4x}{2\sqrt{x^2+2x-4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)*Sqrt[-4 + 2*x + x^2]),x]
```

[Out] ArcTan[(6 - 4*x)/(2*Sqrt[-4 + 2*x + x^2])]

Maple [A] time = 0.044, size = 23, normalized size = 1.2

$$-\arctan\left(\frac{-6+4x}{2}\frac{1}{\sqrt{(-1+x)^2-5+4x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1-x)/(x^2+2*x-4)^(1/2),x)

[Out] -arctan(1/2*(-6+4*x)/((-1+x)^2-5+4*x)^(1/2))

Maxima [A] time = 1.49182, size = 36, normalized size = 1.89

$$-\arcsin\left(\frac{2\sqrt{5}x}{5|x-1|}-\frac{3\sqrt{5}}{5|x-1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-x)/(x^2+2*x-4)^(1/2),x, algorithm="maxima")

[Out] -arcsin(2/5*sqrt(5)*x/abs(x - 1) - 3/5*sqrt(5)/abs(x - 1))

Fricas [A] time = 2.03564, size = 55, normalized size = 2.89

$$-2\arctan\left(-x+\sqrt{x^2+2x-4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-x)/(x^2+2*x-4)^(1/2),x, algorithm="fricas")

[Out] -2*arctan(-x + sqrt(x^2 + 2*x - 4) + 1)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{1}{x\sqrt{x^2+2x-4}-\sqrt{x^2+2x-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-x)/(x**2+2*x-4)**(1/2),x)

[Out] -Integral(1/(x*sqrt(x**2 + 2*x - 4) - sqrt(x**2 + 2*x - 4)), x)

Giac [A] time = 1.38457, size = 24, normalized size = 1.26

$$-2\arctan\left(-x+\sqrt{x^2+2x-4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)/(x^2+2*x-4)^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(-x + sqrt(x^2 + 2*x - 4) + 1)
```

3.2405 $\int \frac{1}{(-2+x)\sqrt{3-4x+x^2}} \, dx$

Optimal. Leaf size=13

$$\tan^{-1}\left(\sqrt{x^2-4x+3}\right)$$

```
[Out] ArcTan[Sqrt[3 - 4*x + x^2]]
```

Rubi [A] time = 0.008268, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {688, 203}

$$\tan^{-1}\left(\sqrt{x^2-4x+3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 + x)*Sqrt[3 - 4*x + x^2]),x]
```

```
[Out] ArcTan[Sqrt[3 - 4*x + x^2]]
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(-2+x)\sqrt{3-4x+x^2}} \, dx &= 4 \operatorname{Subst}\left(\int \frac{1}{4+4x^2} \, dx, x, \sqrt{3-4x+x^2}\right) \\
&= \tan^{-1}\left(\sqrt{3-4x+x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0037221, size = 12, normalized size = 0.92

$$\tan^{-1}\left(\sqrt{(x-2)^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-2 + x)*Sqrt[3 - 4*x + x^2]),x]
```

```
[Out] ArcTan[Sqrt[-1 + (-2 + x)^2]]
```

Maple [A] time = 0.044, size = 13, normalized size = 1.

$$-\arctan\left(\frac{1}{\sqrt{(-2+x)^2-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2+x)/(x^2-4*x+3)^(1/2),x)
```

```
[Out] -arctan(1/((-2+x)^2-1)^(1/2))
```

Maxima [A] time = 1.5334, size = 12, normalized size = 0.92

$$-\arcsin\left(\frac{1}{|x-2|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(x^2-4*x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(1/abs(x - 2))
```

Fricas [A] time = 2.05789, size = 54, normalized size = 4.15

$$2\arctan\left(-x+\sqrt{x^2-4x+3}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(x^2-4*x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*arctan(-x + sqrt(x^2 - 4*x + 3) + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x-3)(x-1)}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(x**2-4*x+3)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x - 3)*(x - 1))*(x - 2)), x)
```

Giac [A] time = 1.48421, size = 24, normalized size = 1.85

$$2\arctan\left(-x+\sqrt{x^2-4x+3}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(x^2-4*x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-x + sqrt(x^2 - 4*x + 3) + 2)
```

3.2406 $\int \frac{1+x}{(2+3x+x^2)^{3/2}} \, dx$

Optimal. Leaf size=17

$$\frac{2(x+1)}{\sqrt{x^2+3x+2}}$$

```
[Out] (2*(1 + x))/Sqrt[2 + 3*x + x^2]
```

Rubi [A] time = 0.0045294, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {636}

$$\frac{2(x+1)}{\sqrt{x^2+3x+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(2 + 3*x + x^2)^(3/2),x]
```

```
[Out] (2*(1 + x))/Sqrt[2 + 3*x + x^2]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1+x}{(2+3x+x^2)^{3/2}} \, dx = \frac{2(1+x)}{\sqrt{2+3x+x^2}}$$

Mathematica [A] time = 0.0395123, size = 19, normalized size = 1.12

$$\frac{2\sqrt{x^2+3x+2}}{x+2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(2 + 3*x + x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[2 + 3*x + x^2])/(2 + x)
```

Maple [A] time = 0.041, size = 21, normalized size = 1.2

$$2\,\frac{(1+x)^2(2+x)}{(x^2+3\,x+2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(x^2+3*x+2)^(3/2),x)
```

```
[Out] 2*(1+x)^2*(2+x)/(x^2+3*x+2)^(3/2)
```

Maxima [A] time = 0.96237, size = 35, normalized size = 2.06

$$\frac{2x}{\sqrt{x^2+3x+2}}+\frac{2}{\sqrt{x^2+3x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+3*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*x/sqrt(x^2 + 3*x + 2) + 2/sqrt(x^2 + 3*x + 2)
```

Fricas [A] time = 2.01473, size = 55, normalized size = 3.24

$$\frac{2\left(x+\sqrt{x^2+3x+2}+2\right)}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+3*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(x + sqrt(x^2 + 3*x + 2) + 2)/(x + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x+1}{\left((x+1)(x+2)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**2+3*x+2)**(3/2),x)
```

```
[Out] Integral((x + 1)/((x + 1)*(x + 2))**(3/2), x)
```

Giac [A] time = 1.27919, size = 26, normalized size = 1.53

$$\frac{2}{x-\sqrt{x^2+3x+2}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+3*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/(x - sqrt(x^2 + 3*x + 2) + 2)
```

3.2407 $\int \frac{1}{(d+ex)\sqrt{\frac{b^2}{4c}+bx+cx^2}}\,dx$

Optimal. Leaf size=96

$$\frac{2(b+2cx)\log(b+2cx)}{\sqrt{\frac{b^2}{c}+4bx+4cx^2}(2cd-be)}-\frac{2(b+2cx)\log(d+ex)}{\sqrt{\frac{b^2}{c}+4bx+4cx^2}(2cd-be)}$$

```
[Out] (2*(b + 2*c*x)*Log[b + 2*c*x])/((2*c*d - b*e)*Sqrt[b^2/c + 4*b*x + 4*c*x^2]
) - (2*(b + 2*c*x)*Log[d + e*x])/((2*c*d - b*e)*Sqrt[b^2/c + 4*b*x + 4*c*x^
2])
```

Rubi [A] time = 0.034538, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {646, 36, 31}

$$\frac{2(b+2cx)\log(b+2cx)}{\sqrt{\frac{b^2}{c}+4bx+4cx^2}(2cd-be)}-\frac{2(b+2cx)\log(d+ex)}{\sqrt{\frac{b^2}{c}+4bx+4cx^2}(2cd-be)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[b^2/(4*c) + b*x + c*x^2]),x]
```

```
[Out] (2*(b + 2*c*x)*Log[b + 2*c*x])/((2*c*d - b*e)*Sqrt[b^2/c + 4*b*x + 4*c*x^2]
) - (2*(b + 2*c*x)*Log[d + e*x])/((2*c*d - b*e)*Sqrt[b^2/c + 4*b*x + 4*c*x^
2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{(d+ex)\sqrt{\frac{b^2}{4c}+bx+cx^2}}\,dx = \frac{\left(\frac{b}{2}+cx\right)\int \frac{1}{\left(\frac{b}{2}+cx\right)(d+ex)}\,dx}{\sqrt{\frac{b^2}{4c}+bx+cx^2}}$$
$$= \frac{\left(2c\left(\frac{b}{2}+cx\right)\right)\int \frac{1}{\frac{b}{2}+cx}\,dx}{(2cd-be)\sqrt{\frac{b^2}{4c}+bx+cx^2}} - \frac{\left(2e\left(\frac{b}{2}+cx\right)\right)\int \frac{1}{d+ex}\,dx}{(2cd-be)\sqrt{\frac{b^2}{4c}+bx+cx^2}}$$
$$= \frac{2(b+2cx)\log(b+2cx)}{(2cd-be)\sqrt{\frac{b^2}{c}+4bx+4cx^2}} - \frac{2(b+2cx)\log(d+ex)}{(2cd-be)\sqrt{\frac{b^2}{c}+4bx+4cx^2}}$$

Mathematica [A] time = 0.0317777, size = 51, normalized size = 0.53

$$\frac{2(b+2cx)(\log(b+2cx)-\log(d+ex))}{\sqrt{\frac{(b+2cx)^2}{c}}(2cd-be)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[b^2/(4*c) + b*x + c*x^2]),x]
```

```
[Out] (2*(b + 2*c*x)*(Log[b + 2*c*x] - Log[d + e*x]))/((2*c*d - b*e)*Sqrt[(b + 2*
c*x)^2/c])
```

Maple [A] time = 0.187, size = 58, normalized size = 0.6

$$2\,\frac{(2\,cx+b)(\ln(ex+d)-\ln(2\,cx+b))}{be-2\,cd}\frac{1}{\sqrt{\frac{4\,c^2x^2+4\,bcx+b^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/(e*x+d)/(b^2/c+4*b*x+4*c*x^2)^(1/2),x)
```

```
[Out] 2*(2*c*x+b)*(ln(e*x+d)-ln(2*c*x+b))/((4*c^2*x^2+4*b*c*x+b^2)/c)^(1/2)/(b*e-
2*c*d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(e*x+d)/(1/c*b^2+4*b*x+4*c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.18694, size = 626, normalized size = 6.52

$$\left[-\frac{2\sqrt{c}\log\left(\frac{16c^3e^2x^3+4bc^2d^2+b^3e^2+16\left(c^3de+bc^2e^2\right)x^2+2\left(4c^3d^2+4bc^2de+3b^2ce^2\right)x+\left(4c^2d^2-b^2e^2+4\left(2c^2de-bce^2\right)x\right)\sqrt{c}\sqrt{\frac{4c^2x^2+4bcx+b^2}{c}}}{4c^2ex^3+b^2d+4\left(c^2d+bce\right)x^2+\left(4bcd+b^2e\right)x}\right)}{2cd-be}, -\frac{4\sqrt{-c}\,\mathrm{ar}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(e*x+d)/(1/c*b^2+4*b*x+4*c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-2*sqrt(c)*log((16*c^3*e^2*x^3 + 4*b*c^2*d^2 + b^3*e^2 + 16*(c^3*d*e + b*c
^2*e^2)*x^2 + 2*(4*c^3*d^2 + 4*b*c^2*d*e + 3*b^2*c*e^2)*x + (4*c^2*d^2 - b^
2*e^2 + 4*(2*c^2*d*e - b*c*e^2)*x)*sqrt(c)*sqrt((4*c^2*x^2 + 4*b*c*x + b^2)
/c))/(4*c^2*e*x^3 + b^2*d + 4*(c^2*d + b*c*e)*x^2 + (4*b*c*d + b^2*e)*x))/(
2*c*d - b*e), -4*sqrt(-c)*arctan(-(4*c*e*x + 2*c*d + b*e)*sqrt(-c)*sqrt((4*
c^2*x^2 + 4*b*c*x + b^2)/c)/(2*b*c*d - b^2*e + 2*(2*c^2*d - b*c*e)*x))/(2*c
*d - b*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$2\int\frac{1}{d\sqrt{\frac{b^2}{c}+4bx+4cx^2}+ex\sqrt{\frac{b^2}{c}+4bx+4cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(e*x+d)/(1/c*b**2+4*b*x+4*c*x**2)**(1/2),x)
```

```
[Out] 2*Integral(1/(d*sqrt(b**2/c + 4*b*x + 4*c*x**2) + e*x*sqrt(b**2/c + 4*b*x +
 4*c*x**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(e*x+d)/(1/c*b^2+4*b*x+4*c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.2408 $\int \frac{1}{\left(\frac{be}{2c}+ex\right)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=56

$$\frac{2\sqrt{c}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{e\sqrt{b^2-4ac}}$$

```
[Out] (2*Sqrt[c]*ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c]])/(Sq
rt[b^2 - 4*a*c]*e)
```

Rubi [A] time = 0.0530509, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {688, 205}

$$\frac{2\sqrt{c}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{e\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(((b*e)/(2*c) + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*Sqrt[c]*ArcTan[(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c]])/(Sq
rt[b^2 - 4*a*c]*e)
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(\frac{be}{2c}+ex\right)\sqrt{a+bx+cx^2}}\,dx = (4c)\,\text{Subst}\left(\int \frac{1}{b^2e-4ace+4cex^2}\,dx, x, \sqrt{a+bx+cx^2}\right)$$
$$= \frac{2\sqrt{c}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ace}}$$

Mathematica [A] time = 0.040288, size = 55, normalized size = 0.98

$$\frac{2\sqrt{c}\tan^{-1}\left(\frac{2\sqrt{c}\sqrt{a+x(b+cx)}}{\sqrt{b^2-4ac}}\right)}{e\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(((b*e)/(2*c) + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*Sqrt[c]*ArcTan[(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])/Sqrt[b^2 - 4*a*c]])/(Sq
rt[b^2 - 4*a*c]*e)
```

Maple [B] time = 0.193, size = 98, normalized size = 1.8

$$-2\frac{1}{e}\ln\left(\left(1/2\,\frac{4\,ac-b^2}{c}+1/2\,\sqrt{\frac{4\,ac-b^2}{c}}\sqrt{4\,\left(x+1/2\,\frac{b}{c}\right)^2c+\frac{4\,ac-b^2}{c}}\right)\left(x+1/2\,\frac{b}{c}\right)^{-1}\right)\frac{1}{\sqrt{\frac{4\,ac-b^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/2*b*e/c+e*x)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -2/e/((4*a*c-b^2)/c)^(1/2)*ln((1/2*(4*a*c-b^2)/c+1/2*((4*a*c-b^2)/c)^(1/2)*
(4*(x+1/2*b/c)^2*c+(4*a*c-b^2)/c)^(1/2))/(x+1/2*b/c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/2*b*e/c+e*x)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.29252, size = 385, normalized size = 6.88

$$\left[\frac{\sqrt{-\frac{c}{b^2-4\,ac}}\log\left(-\frac{4\,c^2x^2+4\,bcx-b^2+8\,ac+4\,\sqrt{cx^2+bx+a}\left(b^2-4\,ac\right)\sqrt{-\frac{c}{b^2-4\,ac}}}{4\,c^2x^2+4\,bcx+b^2}\right)}{e},\ \frac{2\,\sqrt{\frac{c}{b^2-4\,ac}}\arctan\left(-\frac{\sqrt{cx^2+bx+a}\left(b^2-4\,ac\right)\sqrt{\frac{c}{b^2-4\,ac}}}{2\left(c^2x^2+bcx+ac\right)}\right)}{e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/2*b*e/c+e*x)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(-c/(b^2 - 4*a*c))*log(-(4*c^2*x^2 + 4*b*c*x - b^2 + 8*a*c + 4*sqrt(c*
x^2 + b*x + a)*(b^2 - 4*a*c)*sqrt(-c/(b^2 - 4*a*c)))/(4*c^2*x^2 + 4*b*c*x +
 b^2))/e, 2*sqrt(c/(b^2 - 4*a*c))*arctan(-1/2*sqrt(c*x^2 + b*x + a)*(b^2 -
4*a*c)*sqrt(c/(b^2 - 4*a*c))/(c^2*x^2 + b*c*x + a*c))/e]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2c\int\frac{1}{b\sqrt{a+bx+cx^2}+2cx\sqrt{a+bx+cx^2}}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/2*b*e/c+e*x)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] 2*c*Integral(1/(b*sqrt(a + b*x + c*x**2) + 2*c*x*sqrt(a + b*x + c*x**2)), x
)/e
```

Giac [A] time = 1.14539, size = 88, normalized size = 1.57

$$\frac{4\,c\arctan\left(-\frac{2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)c+b\sqrt{c}}{\sqrt{b^2c-4\,ac^2}}\right)e^{(-1)}}{\sqrt{b^2c-4\,ac^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/2*b*e/c+e*x)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 4*c*arctan(-(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*c + b*sqrt(c))/sqrt(b^2*
c - 4*a*c^2))*e^(-1)/sqrt(b^2*c - 4*a*c^2)
```

3.2409 $\int \frac{1}{(d+ex)\sqrt{\frac{-cd^2+bde}{e^2}+bx+cx^2}}\,dx$

Optimal. Leaf size=48

$$\frac{2e\sqrt{-\frac{d(cd-be)}{e^2}+bx+cx^2}}{(d+ex)(2cd-be)}$$

```
[Out] (2*e*Sqrt[-((d*(c*d - b*e))/e^2) + b*x + c*x^2])/((2*c*d - b*e)*(d + e*x))
```

Rubi [A] time = 0.0268307, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {650}

$$\frac{2e\sqrt{-\frac{d(cd-be)}{e^2}+bx+cx^2}}{(d+ex)(2cd-be)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[(-(c*d^2) + b*d*e)/e^2 + b*x + c*x^2]),x]
```

```
[Out] (2*e*Sqrt[-((d*(c*d - b*e))/e^2) + b*x + c*x^2])/((2*c*d - b*e)*(d + e*x))
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\sqrt{\frac{-cd^2+bde}{e^2}+bx+cx^2}}\,dx = \frac{2e\sqrt{-\frac{d(cd-be)}{e^2}+bx+cx^2}}{(2cd-be)(d+ex)}$$

Mathematica [A] time = 0.0566954, size = 45, normalized size = 0.94

$$-\frac{2e\sqrt{\frac{(d+ex)(be-cd+cex)}{e^2}}}{(d+ex)(be-2cd)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[(-(c*d^2) + b*d*e)/e^2 + b*x + c*x^2]),x]
```

```
[Out] (-2*e*Sqrt[((d + e*x)*(-(c*d) + b*e + c*e*x))/e^2])/((-2*c*d + b*e)*(d + e*
x))
```

Maple [A] time = 0.19, size = 59, normalized size = 1.2

$$-2\frac{cex+be-cd}{e(be-2cd)}\frac{1}{\sqrt{\frac{ce^2x^2+be^2x+bde-cd^2}{e^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/((b*d*e-c*d^2)/e^2+b*x+c*x^2)^(1/2),x)
```

```
[Out] -2*(c*e*x+b*e-c*d)/e/(b*e-2*c*d)/((c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)/e^2)^(1/2
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/((b*d*e-c*d^2)/e^2+b*x+c*x^2)^(1/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.79547, size = 126, normalized size = 2.62

$$\frac{2e\sqrt{\frac{ce^2x^2+be^2x-cd^2+bde}{e^2}}}{2cd^2-bde+(2cde-be^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/((b*d*e-c*d^2)/e^2+b*x+c*x^2)^(1/2),x, algorithm="frica
s")
```

```
[Out] 2*e*sqrt((c*e^2*x^2 + b*e^2*x - c*d^2 + b*d*e)/e^2)/(2*c*d^2 - b*d*e + (2*c
*d*e - b*e^2)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(\frac{d}{e}+x\right)\left(b-\frac{cd}{e}+cx\right)}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/((b*d*e-c*d**2)/e**2+b*x+c*x**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((d/e + x)*(b - c*d/e + c*x))*(d + e*x)), x)
```

<u>Giac</u> [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/((b*d*e-c*d^2)/e^2+b*x+c*x^2)^(1/2),x, algorithm="giac"
)

[Out] Exception raised: NotImplementedError
```

3.2410 $\int \frac{1}{\left(\frac{be}{2c}+ex\right)\sqrt{\frac{b^2}{4c}+bx+cx^2}}\,dx$

Optimal. Leaf size=27

$$-\frac{2}{e\sqrt{\frac{b^2}{c}+4bx+4cx^2}}$$

```
[Out] -2/(e*Sqrt[b^2/c + 4*b*x + 4*c*x^2])
```

Rubi [A] time = 0.0284133, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {643, 629}

$$-\frac{2}{e\sqrt{\frac{b^2}{c}+4bx+4cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(((b*e)/(2*c) + e*x)*Sqrt[b^2/(4*c) + b*x + c*x^2]),x]
```

```
[Out] -2/(e*Sqrt[b^2/c + 4*b*x + 4*c*x^2])
```

Rule 643

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{be}{2c}+ex\right)\sqrt{\frac{b^2}{4c}+bx+cx^2}}\,dx &= \frac{c\int \frac{\frac{be}{2c}+ex}{\left(\frac{b^2}{4c}+bx+cx^2\right)^{3/2}}\,dx}{e^2}\\ &= -\frac{2}{e\sqrt{\frac{b^2}{c}+4bx+4cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0128009, size = 21, normalized size = 0.78

$$-\frac{2}{e\sqrt{\frac{(b+2cx)^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(((b*e)/(2*c) + e*x)*Sqrt[b^2/(4*c) + b*x + c*x^2]),x]
```

```
[Out] -2/(e*Sqrt[(b + 2*c*x)^2/c])
```

Maple [A] time = 0.153, size = 29, normalized size = 1.1

$$-2\,{\frac {1}{e}}{\frac {1}{\sqrt {{\frac {4\,{c}^{2}{x}^{2}+4\,bcx+{b}^{2}}{c}}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/(1/2*b*e/c+e*x)/(b^2/c+4*b*x+4*c*x^2)^(1/2),x)
```

```
[Out] -2/((4*c^2*x^2+4*b*c*x+b^2)/c)^(1/2)/e
```

Maxima [A] time = 0.984441, size = 45, normalized size = 1.67

$$-\frac{2}{2\, e^{2} x \sqrt{\frac{c}{e^{2}}} + \frac{b e^{2} \sqrt{\frac{c}{e^{2}}}}{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(1/2*b*e/c+e*x)/(1/c*b^2+4*b*x+4*c*x^2)^(1/2),x, algorithm="max
ima")
```

```
[Out] -2/(2*e^2*x*sqrt(c/e^2) + b*e^2*sqrt(c/e^2)/c)
```

Fricas [A] time = 1.96891, size = 103, normalized size = 3.81

$$-\frac{2\, c \sqrt{\frac{4\, c^{2} x^{2} + 4\, b c x + b^{2}}{c}}}{4\, c^{2} e x^{2} + 4\, b c e x + b^{2} e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(1/2*b*e/c+e*x)/(1/c*b^2+4*b*x+4*c*x^2)^(1/2),x, algorithm="fri
cas")
```

```
[Out] -2*c*sqrt((4*c^2*x^2 + 4*b*c*x + b^2)/c)/(4*c^2*e*x^2 + 4*b*c*e*x + b^2*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{4 c \int \frac{1}{b \sqrt{\frac{b^{2}}{c} + 4 b x + 4 c x^{2}} + 2 c x \sqrt{\frac{b^{2}}{c} + 4 b x + 4 c x^{2}}}\, dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(1/2*b*e/c+e*x)/(1/c*b**2+4*b*x+4*c*x**2)**(1/2),x)
```

```
[Out] 4*c*Integral(1/(b*sqrt(b**2/c + 4*b*x + 4*c*x**2) + 2*c*x*sqrt(b**2/c + 4*b
*x + 4*c*x**2)), x)/e
```

Giac [A] time = 1.09872, size = 59, normalized size = 2.19

$$\frac{4\,\sqrt{c}e^{(-1)}}{\left(2\,\sqrt{c}x - \sqrt{4\,cx^2 + 4\,bx + \frac{b^2}{c}}\right)\sqrt{c} + b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(1/2*b*e/c+e*x)/(1/c*b^2+4*b*x+4*c*x^2)^(1/2),x, algorithm="gia
c")
```

```
[Out] 4*sqrt(c)*e^(-1)/((2*sqrt(c)*x - sqrt(4*c*x^2 + 4*b*x + b^2/c))*sqrt(c) + b
)
```

3.2411 $\int \frac{x}{\sqrt{2+4x+3x^2}}\,dx$

Optimal. Leaf size=40

$$\frac{1}{3}\sqrt{3x^2+4x+2}-\frac{2\sinh^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{3\sqrt{3}}$$

```
[Out] Sqrt[2 + 4*x + 3*x^2]/3 - (2*ArcSinh[(2 + 3*x)/Sqrt[2]])/(3*Sqrt[3])
```

Rubi [A] time = 0.0177499, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {640, 619, 215}

$$\frac{1}{3}\sqrt{3x^2+4x+2}-\frac{2\sinh^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[2 + 4*x + 3*x^2],x]
```

```
[Out] Sqrt[2 + 4*x + 3*x^2]/3 - (2*ArcSinh[(2 + 3*x)/Sqrt[2]])/(3*Sqrt[3])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{2+4x+3x^2}}\,dx &= \frac{1}{3}\sqrt{2+4x+3x^2}-\frac{2}{3}\int \frac{1}{\sqrt{2+4x+3x^2}}\,dx\\ &= \frac{1}{3}\sqrt{2+4x+3x^2}-\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{8}}}\,dx,x,4+6x\right)}{3\sqrt{6}}\\ &= \frac{1}{3}\sqrt{2+4x+3x^2}-\frac{2\sinh^{-1}\left(\frac{2+3x}{\sqrt{2}}\right)}{3\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0123403, size = 40, normalized size = 1.

$$\frac{1}{9}\left(3\sqrt{3x^2+4x+2}-2\sqrt{3}\sinh^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[2 + 4*x + 3*x^2],x]
```

```
[Out] (3*Sqrt[2 + 4*x + 3*x^2] - 2*Sqrt[3]*ArcSinh[(2 + 3*x)/Sqrt[2]])/9
```

Maple [A] time = 0.043, size = 30, normalized size = 0.8

$$\frac{1}{3}\sqrt{3\,x^2+4\,x+2}-\frac{2\,\sqrt{3}}{9}\mathrm{Arcsinh}\left(\frac{3\,\sqrt{2}}{2}\left(x+\frac{2}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3*x^2+4*x+2)^(1/2),x)
```

```
[Out] 1/3*(3*x^2+4*x+2)^(1/2)-2/9*3^(1/2)*arcsinh(3/2*2^(1/2)*(x+2/3))
```

Maxima [A] time = 1.47642, size = 42, normalized size = 1.05

$$-\frac{2}{9}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\,\sqrt{2}(3\,x+2)\right)+\frac{1}{3}\,\sqrt{3\,x^2+4\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^2+4*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*arcsinh(1/2*sqrt(2)*(3*x + 2)) + 1/3*sqrt(3*x^2 + 4*x + 2)
```

Fricas [A] time = 2.02219, size = 142, normalized size = 3.55

$$\frac{1}{9}\,\sqrt{3}\log\left(\sqrt{3}\sqrt{3\,x^2+4\,x+2}(3\,x+2)-9\,x^2-12\,x-5\right)+\frac{1}{3}\,\sqrt{3\,x^2+4\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^2+4*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(3)*log(sqrt(3)*sqrt(3*x^2 + 4*x + 2)*(3*x + 2) - 9*x^2 - 12*x - 5)
 + 1/3*sqrt(3*x^2 + 4*x + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{3x^2+4x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x**2+4*x+2)**(1/2),x)
```

```
[Out] Integral(x/sqrt(3*x**2 + 4*x + 2), x)
```

Giac [A] time = 1.17975, size = 65, normalized size = 1.62

$$\frac{2}{9}\sqrt{3}\log\left(-\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+4x+2}\right)-2\right)+\frac{1}{3}\sqrt{3x^2+4x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^2+4*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*sqrt(3)*log(-sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 4*x + 2)) - 2) + 1/3*sqr
t(3*x^2 + 4*x + 2)
```

3.2412 $\int \frac{x}{\sqrt{2+4x-3x^2}}\,dx$

Optimal. Leaf size=40

$$-\frac{1}{3}\sqrt{-3x^2+4x+2}-\frac{2\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{3\sqrt{3}}$$

```
[Out] -Sqrt[2 + 4*x - 3*x^2]/3 - (2*ArcSin[(2 - 3*x)/Sqrt[10]])/(3*Sqrt[3])
```

Rubi [A] time = 0.0225357, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {640, 619, 216}

$$-\frac{1}{3}\sqrt{-3x^2+4x+2}-\frac{2\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[2 + 4*x - 3*x^2],x]
```

```
[Out] -Sqrt[2 + 4*x - 3*x^2]/3 - (2*ArcSin[(2 - 3*x)/Sqrt[10]])/(3*Sqrt[3])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{2+4x-3x^2}}\,dx &= -\frac{1}{3}\sqrt{2+4x-3x^2}+\frac{2}{3}\int \frac{1}{\sqrt{2+4x-3x^2}}\,dx\\ &= -\frac{1}{3}\sqrt{2+4x-3x^2}-\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{40}}}\,dx,x,4-6x\right)}{3\sqrt{30}}\\ &= -\frac{1}{3}\sqrt{2+4x-3x^2}-\frac{2\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{3\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0162402, size = 40, normalized size = 1.

$$\frac{1}{9}\left(-3\sqrt{-3x^2+4x+2}-2\sqrt{3}\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[2 + 4*x - 3*x^2],x]
```

```
[Out] (-3*Sqrt[2 + 4*x - 3*x^2] - 2*Sqrt[3]*ArcSin[(2 - 3*x)/Sqrt[10]])/9
```

Maple [A] time = 0.043, size = 30, normalized size = 0.8

$$-\frac{1}{3}\sqrt{-3x^2+4x+2}+\frac{2\sqrt{3}}{9}\arcsin\left(\frac{3\sqrt{10}}{10}\left(x-\frac{2}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-3*x^2+4*x+2)^(1/2),x)
```

```
[Out] -1/3*(-3*x^2+4*x+2)^(1/2)+2/9*3^(1/2)*arcsin(3/10*10^(1/2)*(x-2/3))
```

Maxima [A] time = 1.51413, size = 42, normalized size = 1.05

$$-\frac{2}{9}\sqrt{3}\arcsin\left(-\frac{1}{10}\sqrt{10}(3x-2)\right)-\frac{1}{3}\sqrt{-3x^2+4x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x^2+4*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*arcsin(-1/10*sqrt(10)*(3*x - 2)) - 1/3*sqrt(-3*x^2 + 4*x + 2)
```

Fricas [A] time = 2.03475, size = 154, normalized size = 3.85

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3x^2+4x+2}(3x-2)}{3(3x^2-4x-2)}\right)-\frac{1}{3}\sqrt{-3x^2+4x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x^2+4*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(-3*x^2 + 4*x + 2)*(3*x - 2)/(3*x^2 - 4
*x - 2)) - 1/3*sqrt(-3*x^2 + 4*x + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{-3x^2+4x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x**2+4*x+2)**(1/2),x)
```

```
[Out] Integral(x/sqrt(-3*x**2 + 4*x + 2), x)
```

Giac [A] time = 1.15499, size = 42, normalized size = 1.05

$$\frac{2}{9}\sqrt{3}\arcsin\left(\frac{1}{10}\sqrt{10}(3x-2)\right)-\frac{1}{3}\sqrt{-3x^2+4x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x^2+4*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*sqrt(3)*arcsin(1/10*sqrt(10)*(3*x - 2)) - 1/3*sqrt(-3*x^2 + 4*x + 2)
```

3.2413 $\int \frac{x}{\sqrt{2+5x+3x^2}}\,dx$

Optimal. Leaf size=57

$$\frac{1}{3}\sqrt{3x^2+5x+2}-\frac{5\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x+2}}\right)}{6\sqrt{3}}$$

```
[Out] Sqrt[2 + 5*x + 3*x^2]/3 - (5*ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[2 + 5*x + 3*
x^2])])/(6*Sqrt[3])
```

Rubi [A] time = 0.014573, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {640, 621, 206}

$$\frac{1}{3}\sqrt{3x^2+5x+2}-\frac{5\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x+2}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[2 + 5*x + 3*x^2],x]
```

```
[Out] Sqrt[2 + 5*x + 3*x^2]/3 - (5*ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[2 + 5*x + 3*
x^2])])/(6*Sqrt[3])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{2+5x+3x^2}}\,dx &= \frac{1}{3}\sqrt{2+5x+3x^2}-\frac{5}{6}\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx\\ &= \frac{1}{3}\sqrt{2+5x+3x^2}-\frac{5}{3}\operatorname{Subst}\left(\int \frac{1}{12-x^2}\,dx,x,\frac{5+6x}{\sqrt{2+5x+3x^2}}\right)\\ &= \frac{1}{3}\sqrt{2+5x+3x^2}-\frac{5\tanh^{-1}\left(\frac{5+6x}{2\sqrt{3}\sqrt{2+5x+3x^2}}\right)}{6\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0186334, size = 52, normalized size = 0.91

$$\frac{1}{18}\left(6\sqrt{3x^2+5x+2}-5\sqrt{3}\tanh^{-1}\left(\frac{6x+5}{2\sqrt{9x^2+15x+6}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[2 + 5*x + 3*x^2],x]
```

```
[Out] (6*Sqrt[2 + 5*x + 3*x^2] - 5*Sqrt[3]*ArcTanh[(5 + 6*x)/(2*Sqrt[6 + 15*x + 9
*x^2])])/18
```

Maple [A] time = 0.041, size = 45, normalized size = 0.8

$$\frac{1}{3}\sqrt{3\,x^2+5\,x+2}-\frac{5\,\sqrt{3}}{18}\ln\left(\frac{\sqrt{3}}{3}\left(\frac{5}{2}+3\,x\right)+\sqrt{3\,x^2+5\,x+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3*x^2+5*x+2)^(1/2),x)
```

```
[Out] 1/3*(3*x^2+5*x+2)^(1/2)-5/18*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x+2)^(1/2))*
3^(1/2)
```

Maxima [A] time = 1.57609, size = 58, normalized size = 1.02

$$-\frac{5}{18}\,\sqrt{3}\log\left(2\,\sqrt{3}\sqrt{3\,x^2+5\,x+2}+6\,x+5\right)+\frac{1}{3}\,\sqrt{3\,x^2+5\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -5/18*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x + 2) + 6*x + 5) + 1/3*sqrt(3*x
^2 + 5*x + 2)
```

Fricas [A] time = 2.08253, size = 151, normalized size = 2.65

$$\frac{5}{36}\,\sqrt{3}\log\left(-4\,\sqrt{3}\sqrt{3\,x^2+5\,x+2}(6\,x+5)+72\,x^2+120\,x+49\right)+\frac{1}{3}\,\sqrt{3\,x^2+5\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 5/36*sqrt(3)*log(-4*sqrt(3)*sqrt(3*x^2 + 5*x + 2)*(6*x + 5) + 72*x^2 + 120*
x + 49) + 1/3*sqrt(3*x^2 + 5*x + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x+1)(3x+2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x**2+5*x+2)**(1/2),x)
```

```
[Out] Integral(x/sqrt((x + 1)*(3*x + 2)), x)
```

Giac [A] time = 1.16071, size = 66, normalized size = 1.16

$$\frac{5}{18}\,\sqrt{3}\log\left(\left|-2\,\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+5\,x+2}\right)-5\right|\right)+\frac{1}{3}\,\sqrt{3\,x^2+5\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 5/18*sqrt(3)*log(abs(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 5*x + 2)) - 5)) +
 1/3*sqrt(3*x^2 + 5*x + 2)
```

3.2414 $\int \frac{x}{\sqrt{2+5x-3x^2}}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{3}\sqrt{-3x^2+5x+2}-\frac{5\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{6\sqrt{3}}$$

```
[Out] -Sqrt[2 + 5*x - 3*x^2]/3 - (5*ArcSin[(5 - 6*x)/7])/(6*Sqrt[3])
```

Rubi [A] time = 0.0127859, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {640, 619, 216}

$$-\frac{1}{3}\sqrt{-3x^2+5x+2}-\frac{5\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[2 + 5*x - 3*x^2],x]

[Out] -Sqrt[2 + 5*x - 3*x^2]/3 - (5*ArcSin[(5 - 6*x)/7])/(6*Sqrt[3])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{2+5x-3x^2}}\,dx &= -\frac{1}{3}\sqrt{2+5x-3x^2}+\frac{5}{6}\int \frac{1}{\sqrt{2+5x-3x^2}}\,dx\\ &= -\frac{1}{3}\sqrt{2+5x-3x^2}-\frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{49}}}\,dx,x,5-6x\right)}{42\sqrt{3}}\\ &= -\frac{1}{3}\sqrt{2+5x-3x^2}-\frac{5\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{6\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0089675, size = 38, normalized size = 1.

$$-\frac{1}{3}\sqrt{-3x^2+5x+2}-\frac{5\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[2 + 5*x - 3*x^2],x]
```

```
[Out] -Sqrt[2 + 5*x - 3*x^2]/3 - (5*ArcSin[(5 - 6*x)/7])/(6*Sqrt[3])
```

Maple [A] time = 0.04, size = 27, normalized size = 0.7

$$\frac{5\sqrt{3}}{18}\arcsin\left(-\frac{5}{7}+\frac{6x}{7}\right)-\frac{1}{3}\sqrt{-3x^2+5x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-3*x^2+5*x+2)^(1/2),x)
```

```
[Out] 5/18*arcsin(-5/7+6/7*x)*3^(1/2)-1/3*(-3*x^2+5*x+2)^(1/2)
```

Maxima [A] time = 1.44108, size = 35, normalized size = 0.92

$$-\frac{5}{18}\sqrt{3}\arcsin\left(-\frac{6}{7}x+\frac{5}{7}\right)-\frac{1}{3}\sqrt{-3x^2+5x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -5/18*sqrt(3)*arcsin(-6/7*x + 5/7) - 1/3*sqrt(-3*x^2 + 5*x + 2)
```

Fricas [B] time = 2.49295, size = 155, normalized size = 4.08

$$-\frac{5}{18}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3x^2+5x+2}(6x-5)}{6(3x^2-5x-2)}\right)-\frac{1}{3}\sqrt{-3x^2+5x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -5/18*sqrt(3)*arctan(1/6*sqrt(3)*sqrt(-3*x^2 + 5*x + 2)*(6*x - 5)/(3*x^2 -
5*x - 2)) - 1/3*sqrt(-3*x^2 + 5*x + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{-(x-2)(3x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x**2+5*x+2)**(1/2),x)
```

```
[Out] Integral(x/sqrt(-(x - 2)*(3*x + 1)), x)
```

Giac [A] time = 1.1524, size = 35, normalized size = 0.92

$$\frac{5}{18}\sqrt{3}\arcsin\left(\frac{6}{7}x-\frac{5}{7}\right)-\frac{1}{3}\sqrt{-3x^2+5x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 5/18*sqrt(3)*arcsin(6/7*x - 5/7) - 1/3*sqrt(-3*x^2 + 5*x + 2)
```

3.2415 $\int \frac{x}{\sqrt{-2+4x+3x^2}}\,dx$

Optimal. Leaf size=54

$$\frac{1}{3}\sqrt{3x^2+4x-2}-\frac{2\tanh^{-1}\left(\frac{3x+2}{\sqrt{3}\sqrt{3x^2+4x-2}}\right)}{3\sqrt{3}}$$

```
[Out] Sqrt[-2 + 4*x + 3*x^2]/3 - (2*ArcTanh[(2 + 3*x)/(Sqrt[3]*Sqrt[-2 + 4*x + 3*
x^2])])/(3*Sqrt[3])
```

Rubi [A] time = 0.015042, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {640, 621, 206}

$$\frac{1}{3}\sqrt{3x^2+4x-2}-\frac{2\tanh^{-1}\left(\frac{3x+2}{\sqrt{3}\sqrt{3x^2+4x-2}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[-2 + 4*x + 3*x^2],x]
```

```
[Out] Sqrt[-2 + 4*x + 3*x^2]/3 - (2*ArcTanh[(2 + 3*x)/(Sqrt[3]*Sqrt[-2 + 4*x + 3*
x^2])])/(3*Sqrt[3])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{-2+4x+3x^2}}\,dx &= \frac{1}{3}\sqrt{-2+4x+3x^2}-\frac{2}{3}\int \frac{1}{\sqrt{-2+4x+3x^2}}\,dx\\ &= \frac{1}{3}\sqrt{-2+4x+3x^2}-\frac{4}{3}\text{Subst}\left(\int \frac{1}{12-x^2}\,dx,x,\frac{4+6x}{\sqrt{-2+4x+3x^2}}\right)\\ &= \frac{1}{3}\sqrt{-2+4x+3x^2}-\frac{2\tanh^{-1}\left(\frac{2+3x}{\sqrt{3}\sqrt{-2+4x+3x^2}}\right)}{3\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0167977, size = 49, normalized size = 0.91

$$\frac{1}{9}\left(3\sqrt{3x^2+4x-2}-2\sqrt{3}\tanh^{-1}\left(\frac{3x+2}{\sqrt{9x^2+12x-6}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[-2 + 4*x + 3*x^2],x]
```

```
[Out] (3*Sqrt[-2 + 4*x + 3*x^2] - 2*Sqrt[3]*ArcTanh[(2 + 3*x)/Sqrt[-6 + 12*x + 9*
x^2]])/9
```

Maple [A] time = 0.041, size = 45, normalized size = 0.8

$$\frac{1}{3}\sqrt{3\,x^2+4\,x-2}-\frac{2\,\sqrt{3}}{9}\ln\left(\frac{(2+3\,x)\,\sqrt{3}}{3}+\sqrt{3\,x^2+4\,x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3*x^2+4*x-2)^(1/2),x)
```

```
[Out] 1/3*(3*x^2+4*x-2)^(1/2)-2/9*ln(1/3*(2+3*x)*3^(1/2)+(3*x^2+4*x-2)^(1/2))*3^(
1/2)
```

Maxima [A] time = 1.48325, size = 58, normalized size = 1.07

$$-\frac{2}{9}\,\sqrt{3}\log\left(2\,\sqrt{3}\sqrt{3\,x^2+4\,x-2}+6\,x+4\right)+\frac{1}{3}\,\sqrt{3\,x^2+4\,x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^2+4*x-2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 4*x - 2) + 6*x + 4) + 1/3*sqrt(3*x^
2 + 4*x - 2)
```

Fricas [A] time = 2.45922, size = 143, normalized size = 2.65

$$\frac{1}{9}\,\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+4\,x-2}(3\,x+2)+9\,x^2+12\,x-1\right)+\frac{1}{3}\,\sqrt{3\,x^2+4\,x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^2+4*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 4*x - 2)*(3*x + 2) + 9*x^2 + 12*x - 1
) + 1/3*sqrt(3*x^2 + 4*x - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{3x^2+4x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x**2+4*x-2)**(1/2),x)
```

```
[Out] Integral(x/sqrt(3*x**2 + 4*x - 2), x)
```

Giac [A] time = 1.17741, size = 66, normalized size = 1.22

$$\frac{2}{9}\sqrt{3}\log\left(\left|-\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+4x-2}\right)-2\right|\right)+\frac{1}{3}\sqrt{3x^2+4x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^2+4*x-2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*sqrt(3)*log(abs(-sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 4*x - 2)) - 2)) + 1/
3*sqrt(3*x^2 + 4*x - 2)
```

3.2416 $\int \frac{x}{\sqrt{-2+4x-3x^2}}\,dx$

Optimal. Leaf size=54

$$-\frac{1}{3}\sqrt{-3x^2+4x-2}-\frac{2\tan^{-1}\left(\frac{2-3x}{\sqrt{3}\sqrt{-3x^2+4x-2}}\right)}{3\sqrt{3}}$$

```
[Out] -Sqrt[-2 + 4*x - 3*x^2]/3 - (2*ArcTan[(2 - 3*x)/(Sqrt[3]*Sqrt[-2 + 4*x - 3*
x^2])])/(3*Sqrt[3])
```

Rubi [A] time = 0.0152533, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {640, 621, 204}

$$-\frac{1}{3}\sqrt{-3x^2+4x-2}-\frac{2\tan^{-1}\left(\frac{2-3x}{\sqrt{3}\sqrt{-3x^2+4x-2}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[-2 + 4*x - 3*x^2],x]
```

```
[Out] -Sqrt[-2 + 4*x - 3*x^2]/3 - (2*ArcTan[(2 - 3*x)/(Sqrt[3]*Sqrt[-2 + 4*x - 3*
x^2])])/(3*Sqrt[3])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{-2+4x-3x^2}}\,dx &= -\frac{1}{3}\sqrt{-2+4x-3x^2}+\frac{2}{3}\int \frac{1}{\sqrt{-2+4x-3x^2}}\,dx\\
&= -\frac{1}{3}\sqrt{-2+4x-3x^2}+\frac{4}{3}\operatorname{Subst}\left(\int \frac{1}{-12-x^2}\,dx, x, \frac{4-6x}{\sqrt{-2+4x-3x^2}}\right)\\
&= -\frac{1}{3}\sqrt{-2+4x-3x^2}-\frac{2\tan^{-1}\left(\frac{2-3x}{\sqrt{3}\sqrt{-2+4x-3x^2}}\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.0189733, size = 49, normalized size = 0.91

$$\frac{1}{9}\left(-3\sqrt{-3x^2+4x-2}-2\sqrt{3}\tan^{-1}\left(\frac{2-3x}{\sqrt{-9x^2+12x-6}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[-2 + 4*x - 3*x^2],x]
```

```
[Out] (-3*Sqrt[-2 + 4*x - 3*x^2] - 2*Sqrt[3]*ArcTan[(2 - 3*x)/Sqrt[-6 + 12*x - 9*
x^2]])/9
```

Maple [A] time = 0.043, size = 41, normalized size = 0.8

$$-\frac{1}{3}\sqrt{-3\,x^2+4\,x-2}+\frac{2\,\sqrt{3}}{9}\arctan\left(\sqrt{3}\left(x-\frac{2}{3}\right)\frac{1}{\sqrt{-3\,x^2+4\,x-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-3*x^2+4*x-2)^(1/2),x)
```

```
[Out] -1/3*(-3*x^2+4*x-2)^(1/2)+2/9*3^(1/2)*arctan(3^(1/2)*(x-2/3)/(-3*x^2+4*x-2)
^(1/2))
```

Maxima [C] time = 1.4471, size = 42, normalized size = 0.78

$$-\frac{2}{9}i\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\,\sqrt{2}(3\,x-2)\right)-\frac{1}{3}\,\sqrt{-3\,x^2+4\,x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x^2+4*x-2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/9*I*sqrt(3)*arcsinh(1/2*sqrt(2)*(3*x - 2)) - 1/3*sqrt(-3*x^2 + 4*x - 2)
```

Fricas [C] time = 2.05362, size = 230, normalized size = 4.26

$$-\frac{1}{9}i\,\sqrt{3}\log\left(\frac{2i\,\sqrt{3}\sqrt{-3\,x^2+4\,x-2}-6\,x+4}{x}\right)+\frac{1}{9}i\,\sqrt{3}\log\left(\frac{-2i\,\sqrt{3}\sqrt{-3\,x^2+4\,x-2}-6\,x+4}{x}\right)-\frac{1}{3}\,\sqrt{-3\,x^2+4\,x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x^2+4*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*I*sqrt(3)*log((2*I*sqrt(3)*sqrt(-3*x^2 + 4*x - 2) - 6*x + 4)/x) + 1/9*
I*sqrt(3)*log((-2*I*sqrt(3)*sqrt(-3*x^2 + 4*x - 2) - 6*x + 4)/x) - 1/3*sqrt
(-3*x^2 + 4*x - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-3x^2 + 4x - 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x**2+4*x-2)**(1/2),x)
```

```
[Out] Integral(x/sqrt(-3*x**2 + 4*x - 2), x)
```

Giac [C] time = 1.11854, size = 42, normalized size = 0.78

$$-\frac{2}{9} i \sqrt{3} \arcsin\left(\frac{1}{2} \sqrt{2}(3 i x - 2 i)\right) - \frac{1}{3} \sqrt{-3 x^2 + 4 x - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x^2+4*x-2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/9*I*sqrt(3)*arcsin(1/2*sqrt(2)*(3*I*x - 2*I)) - 1/3*sqrt(-3*x^2 + 4*x -
2)
```

3.2417 $\int \frac{x}{\sqrt{-2+5x+3x^2}} dx$

Optimal. Leaf size=57

$$\frac{1}{3}\sqrt{3x^2+5x-2}-\frac{5\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x-2}}\right)}{6\sqrt{3}}$$

```
[Out] Sqrt[-2 + 5*x + 3*x^2]/3 - (5*ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[-2 + 5*x +
3*x^2])])/(6*Sqrt[3])
```

Rubi [A] time = 0.0145413, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {640, 621, 206}

$$\frac{1}{3}\sqrt{3x^2+5x-2}-\frac{5\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x-2}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[-2 + 5*x + 3*x^2],x]
```

```
[Out] Sqrt[-2 + 5*x + 3*x^2]/3 - (5*ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[-2 + 5*x +
3*x^2])])/(6*Sqrt[3])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{-2+5x+3x^2}}\,dx &= \frac{1}{3}\sqrt{-2+5x+3x^2}-\frac{5}{6}\int \frac{1}{\sqrt{-2+5x+3x^2}}\,dx\\
&= \frac{1}{3}\sqrt{-2+5x+3x^2}-\frac{5}{3}\operatorname{Subst}\left(\int \frac{1}{12-x^2}\,dx,x,\frac{5+6x}{\sqrt{-2+5x+3x^2}}\right)\\
&= \frac{1}{3}\sqrt{-2+5x+3x^2}-\frac{5\tanh^{-1}\left(\frac{5+6x}{2\sqrt{3}\sqrt{-2+5x+3x^2}}\right)}{6\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.018374, size = 52, normalized size = 0.91

$$\frac{1}{18}\left(6\sqrt{3x^2+5x-2}-5\sqrt{3}\tanh^{-1}\left(\frac{6x+5}{2\sqrt{9x^2+15x-6}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[-2 + 5*x + 3*x^2],x]
```

```
[Out] (6*Sqrt[-2 + 5*x + 3*x^2] - 5*Sqrt[3]*ArcTanh[(5 + 6*x)/(2*Sqrt[-6 + 15*x +
 9*x^2])])/18
```

Maple [A] time = 0.043, size = 45, normalized size = 0.8

$$\frac{1}{3}\sqrt{3\,x^2+5\,x-2}-\frac{5\,\sqrt{3}}{18}\ln\left(\frac{\sqrt{3}}{3}\left(\frac{5}{2}+3\,x\right)+\sqrt{3\,x^2+5\,x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3*x^2+5*x-2)^(1/2),x)
```

```
[Out] 1/3*(3*x^2+5*x-2)^(1/2)-5/18*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x-2)^(1/2))*
3^(1/2)
```

Maxima [A] time = 1.45138, size = 58, normalized size = 1.02

$$-\frac{5}{18}\,\sqrt{3}\log\left(2\,\sqrt{3}\sqrt{3\,x^2+5\,x-2}+6\,x+5\right)+\frac{1}{3}\,\sqrt{3\,x^2+5\,x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^2+5*x-2)^(1/2),x, algorithm="maxima")
```

```
[Out] -5/18*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x - 2) + 6*x + 5) + 1/3*sqrt(3*x
^2 + 5*x - 2)
```

Fricas [A] time = 2.01611, size = 150, normalized size = 2.63

$$\frac{5}{36}\,\sqrt{3}\log\left(-4\,\sqrt{3}\sqrt{3\,x^2+5\,x-2}(6\,x+5)+72\,x^2+120\,x+1\right)+\frac{1}{3}\,\sqrt{3\,x^2+5\,x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^2+5*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 5/36*sqrt(3)*log(-4*sqrt(3)*sqrt(3*x^2 + 5*x - 2)*(6*x + 5) + 72*x^2 + 120*
x + 1) + 1/3*sqrt(3*x^2 + 5*x - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x+2)(3x-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x**2+5*x-2)**(1/2),x)
```

```
[Out] Integral(x/sqrt((x + 2)*(3*x - 1)), x)
```

Giac [A] time = 1.11637, size = 66, normalized size = 1.16

$$\frac{5}{18}\sqrt{3}\log\left(\left|-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+5x-2}\right)-5\right|\right)+\frac{1}{3}\sqrt{3x^2+5x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^2+5*x-2)^(1/2),x, algorithm="giac")
```

```
[Out] 5/18*sqrt(3)*log(abs(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 5*x - 2)) - 5)) +
 1/3*sqrt(3*x^2 + 5*x - 2)
```

3.2418 $\int \frac{x}{\sqrt{-2+5x-3x^2}} dx$

Optimal. Leaf size=34

$$-\frac{1}{3}\sqrt{-3x^2+5x-2}-\frac{5\sin^{-1}(5-6x)}{6\sqrt{3}}$$

```
[Out] -Sqrt[-2 + 5*x - 3*x^2]/3 - (5*ArcSin[5 - 6*x])/(6*Sqrt[3])
```

Rubi [A] time = 0.0091497, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {640, 619, 216}

$$-\frac{1}{3}\sqrt{-3x^2+5x-2}-\frac{5\sin^{-1}(5-6x)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[-2 + 5*x - 3*x^2],x]
```

```
[Out] -Sqrt[-2 + 5*x - 3*x^2]/3 - (5*ArcSin[5 - 6*x])/(6*Sqrt[3])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{-2+5x-3x^2}}\,dx &= -\frac{1}{3}\sqrt{-2+5x-3x^2}+\frac{5}{6}\int \frac{1}{\sqrt{-2+5x-3x^2}}\,dx\\ &= -\frac{1}{3}\sqrt{-2+5x-3x^2}-\frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx,x,5-6x\right)}{6\sqrt{3}}\\ &= -\frac{1}{3}\sqrt{-2+5x-3x^2}-\frac{5\sin^{-1}(5-6x)}{6\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0087704, size = 34, normalized size = 1.

$$-\frac{1}{3}\sqrt{-3x^2+5x-2}-\frac{5\sin^{-1}(5-6x)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[-2 + 5*x - 3*x^2],x]
```

```
[Out] -Sqrt[-2 + 5*x - 3*x^2]/3 - (5*ArcSin[5 - 6*x])/(6*Sqrt[3])
```

Maple [A] time = 0.044, size = 27, normalized size = 0.8

$$\frac{5\arcsin\left(-5+6x\right)\sqrt{3}}{18}-\frac{1}{3}\sqrt{-3x^{2}+5x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-3*x^2+5*x-2)^(1/2),x)
```

```
[Out] 5/18*arcsin(-5+6*x)*3^(1/2)-1/3*(-3*x^2+5*x-2)^(1/2)
```

Maxima [A] time = 1.41195, size = 35, normalized size = 1.03

$$\frac{5}{18}\sqrt{3}\arcsin\left(6x-5\right)-\frac{1}{3}\sqrt{-3x^{2}+5x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x^2+5*x-2)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/18*sqrt(3)*arcsin(6*x - 5) - 1/3*sqrt(-3*x^2 + 5*x - 2)
```

Fricas [B] time = 2.09634, size = 155, normalized size = 4.56

$$-\frac{5}{18}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3x^{2}+5x-2}(6x-5)}{6\left(3x^{2}-5x+2\right)}\right)-\frac{1}{3}\sqrt{-3x^{2}+5x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x^2+5*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] -5/18*sqrt(3)*arctan(1/6*sqrt(3)*sqrt(-3*x^2 + 5*x - 2)*(6*x - 5)/(3*x^2 -
5*x + 2)) - 1/3*sqrt(-3*x^2 + 5*x - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{-\left(x-1\right)\left(3x-2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x**2+5*x-2)**(1/2),x)
```

```
[Out] Integral(x/sqrt(-(x - 1)*(3*x - 2)), x)
```

Giac [A] time = 1.12041, size = 35, normalized size = 1.03

$$\frac{5}{18}\sqrt{3}\arcsin\left(6x-5\right)-\frac{1}{3}\sqrt{-3x^2+5x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x^2+5*x-2)^(1/2),x, algorithm="giac")
```

```
[Out] 5/18*sqrt(3)*arcsin(6*x - 5) - 1/3*sqrt(-3*x^2 + 5*x - 2)
```

3.2419 $\int \frac{1}{x\sqrt{4+12x+9x^2}}\,dx$

Optimal. Leaf size=27

$$-\frac{(3x+2)\tanh^{-1}(3x+1)}{\sqrt{9x^2+12x+4}}$$

```
[Out] -(((2 + 3*x)*ArcTanh[1 + 3*x])/Sqrt[4 + 12*x + 9*x^2])
```

Rubi [B] time = 0.0123664, antiderivative size = 55, normalized size of antiderivative = 2.04, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {646, 36, 29, 31}

$$\frac{(3x+2)\log(x)}{2\sqrt{9x^2+12x+4}}-\frac{(3x+2)\log(3x+2)}{2\sqrt{9x^2+12x+4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[4 + 12*x + 9*x^2]),x]
```

```
[Out] ((2 + 3*x)*Log[x])/(2*Sqrt[4 + 12*x + 9*x^2]) - ((2 + 3*x)*Log[2 + 3*x])/(2
*Sqrt[4 + 12*x + 9*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{4+12x+9x^2}}\,dx = \frac{(6+9x)\int \frac{1}{x(6+9x)}\,dx}{\sqrt{4+12x+9x^2}}$$
$$= \frac{(6+9x)\int \frac{1}{x}\,dx}{6\sqrt{4+12x+9x^2}} - \frac{(3(6+9x))\int \frac{1}{6+9x}\,dx}{2\sqrt{4+12x+9x^2}}$$
$$= \frac{(2+3x)\log(x)}{2\sqrt{4+12x+9x^2}} - \frac{(2+3x)\log(2+3x)}{2\sqrt{4+12x+9x^2}}$$

Mathematica [A] time = 0.011526, size = 31, normalized size = 1.15

$$\frac{(3x+2)(\log(x)-\log(3x+2))}{2\sqrt{(3x+2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[4 + 12*x + 9*x^2]),x]
```

```
[Out] ((2 + 3*x)*(Log[x] - Log[2 + 3*x]))/(2*Sqrt[(2 + 3*x)^2])
```

Maple [A] time = 0.106, size = 28, normalized size = 1.

$$\frac{(2+3x)(\ln(x)-\ln(2+3x))}{2}\frac{1}{\sqrt{(2+3x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((2+3*x)^2)^(1/2),x)
```

```
[Out] 1/2*(2+3*x)*(ln(x)-ln(2+3*x))/((2+3*x)^2)^(1/2)
```

Maxima [A] time = 1.46949, size = 32, normalized size = 1.19

$$-\frac{1}{2}(-1)^{12x+8}\log\left(\frac{12x}{|x|}+\frac{8}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((2+3*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(-1)^(12*x + 8)*log(12*x/abs(x) + 8/abs(x))
```

Fricas [A] time = 2.03214, size = 43, normalized size = 1.59

$$-\frac{1}{2}\log(3x+2)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((2+3*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log(3*x + 2) + 1/2*log(x)
```

Sympy [A] time = 0.127484, size = 12, normalized size = 0.44

$$\frac{\log(x)}{2} - \frac{\log\left(x + \frac{2}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((2+3*x)**2)**(1/2),x)
```

```
[Out] log(x)/2 - log(x + 2/3)/2
```

Giac [A] time = 1.10481, size = 28, normalized size = 1.04

$$-\frac{1}{2}\left(\log\left(|3x+2|\right) - \log\left(|x|\right)\right)\mathrm{sgn}\left(3x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((2+3*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(log(abs(3*x + 2)) - log(abs(x)))*sgn(3*x + 2)
```

3.2420 $\int \frac{1}{x\sqrt{4-12x+9x^2}} dx$

Optimal. Leaf size=27

$$-\frac{(2-3x)\tanh^{-1}(1-3x)}{\sqrt{9x^2-12x+4}}$$

```
[Out] -(((2 - 3*x)*ArcTanh[1 - 3*x])/Sqrt[4 - 12*x + 9*x^2])
```

Rubi [B] time = 0.0157698, antiderivative size = 55, normalized size of antiderivative = 2.04, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {646, 36, 29, 31}

$$\frac{(2-3x)\log(x)}{2\sqrt{9x^2-12x+4}}-\frac{(2-3x)\log(2-3x)}{2\sqrt{9x^2-12x+4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[4 - 12*x + 9*x^2]),x]
```

```
[Out] -((2 - 3*x)*Log[2 - 3*x])/(2*Sqrt[4 - 12*x + 9*x^2]) + ((2 - 3*x)*Log[x])/(
2*Sqrt[4 - 12*x + 9*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{4-12x+9x^2}}\,dx = \frac{(-6+9x)\int \frac{1}{x(-6+9x)}\,dx}{\sqrt{4-12x+9x^2}}$$
$$= -\frac{(-6+9x)\int \frac{1}{x}\,dx}{6\sqrt{4-12x+9x^2}} + \frac{(3(-6+9x))\int \frac{1}{-6+9x}\,dx}{2\sqrt{4-12x+9x^2}}$$
$$= -\frac{(2-3x)\log(2-3x)}{2\sqrt{4-12x+9x^2}} + \frac{(2-3x)\log(x)}{2\sqrt{4-12x+9x^2}}$$

Mathematica [A] time = 0.0158614, size = 31, normalized size = 1.15

$$\frac{(3x-2)(\log(2-3x)-\log(x))}{2\sqrt{(2-3x)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[4 - 12*x + 9*x^2]),x]
```

```
[Out] ((-2 + 3*x)*(Log[2 - 3*x] - Log[x]))/(2*Sqrt[(2 - 3*x)^2])
```

Maple [A] time = 0.106, size = 28, normalized size = 1.

$$-\frac{(-2+3x)(\ln(x)-\ln(-2+3x))}{2}\frac{1}{\sqrt{(-2+3x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((-2+3*x)^2)^(1/2),x)
```

```
[Out] -1/2*(-2+3*x)*(ln(x)-ln(-2+3*x))/((-2+3*x)^2)^(1/2)
```

Maxima [A] time = 1.41986, size = 32, normalized size = 1.19

$$-\frac{1}{2}(-1)^{-12x+8}\log\left(-\frac{12x}{|x|}+\frac{8}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((-2+3*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(-1)^(-12*x + 8)*log(-12*x/abs(x) + 8/abs(x))
```

Fricas [A] time = 2.08787, size = 42, normalized size = 1.56

$$\frac{1}{2}\log(3x-2)-\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((-2+3*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log(3*x - 2) - 1/2*log(x)
```

Sympy [A] time = 0.118993, size = 12, normalized size = 0.44

$$-\frac{\log(x)}{2}+\frac{\log\left(x-\frac{2}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((-2+3*x)**2)**(1/2),x)
```

```
[Out] -log(x)/2 + log(x - 2/3)/2
```

Giac [A] time = 1.12745, size = 28, normalized size = 1.04

$$\frac{1}{2}\left(\log\left(|3x-2|\right)-\log\left(|x|\right)\right)\operatorname{sgn}\left(3x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((-2+3*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(log(abs(3*x - 2)) - log(abs(x)))*sgn(3*x - 2)
```

3.2421 $\int \frac{1}{x\sqrt{-4+12x-9x^2}}\,dx$

Optimal. Leaf size=27

$$-\frac{(2-3x)\tanh^{-1}(1-3x)}{\sqrt{-9x^2+12x-4}}$$

```
[Out] -(((2 - 3*x)*ArcTanh[1 - 3*x])/Sqrt[-4 + 12*x - 9*x^2])
```

Rubi [B] time = 0.0129094, antiderivative size = 55, normalized size of antiderivative = 2.04, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {646, 36, 31, 29}

$$\frac{(2-3x)\log(x)}{2\sqrt{-9x^2+12x-4}}-\frac{(2-3x)\log(2-3x)}{2\sqrt{-9x^2+12x-4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-4 + 12*x - 9*x^2]),x]
```

```
[Out] -((2 - 3*x)*Log[2 - 3*x])/(2*Sqrt[-4 + 12*x - 9*x^2]) + ((2 - 3*x)*Log[x])/
(2*Sqrt[-4 + 12*x - 9*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{-4+12x-9x^2}}\,dx = \frac{(6-9x)\int \frac{1}{(6-9x)x}\,dx}{\sqrt{-4+12x-9x^2}}$$
$$= \frac{(6-9x)\int \frac{1}{x}\,dx}{6\sqrt{-4+12x-9x^2}} + \frac{(3(6-9x))\int \frac{1}{6-9x}\,dx}{2\sqrt{-4+12x-9x^2}}$$
$$= -\frac{(2-3x)\log(2-3x)}{2\sqrt{-4+12x-9x^2}} + \frac{(2-3x)\log(x)}{2\sqrt{-4+12x-9x^2}}$$

Mathematica [A] time = 0.0100771, size = 33, normalized size = 1.22

$$\frac{(3x-2)(\log(2-3x)-\log(x))}{2\sqrt{-(2-3x)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-4 + 12*x - 9*x^2]),x]
```

```
[Out] ((-2 + 3*x)*(Log[2 - 3*x] - Log[x]))/(2*Sqrt[-(2 - 3*x)^2])
```

Maple [A] time = 0.103, size = 30, normalized size = 1.1

$$-\frac{(-2+3x)(\ln(x)-\ln(-2+3x))}{2}\frac{1}{\sqrt{-(-2+3x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-(-2+3*x)^2)^(1/2),x)
```

```
[Out] -1/2*(-2+3*x)*(ln(x)-ln(-2+3*x))/(-(-2+3*x)^2)^(1/2)
```

Maxima [C] time = 1.46814, size = 32, normalized size = 1.19

$$-\frac{1}{2}i(-1)^{-12x+8}\log\left(-\frac{12x}{|x|}+\frac{8}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-(-2+3*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*I*(-1)^(-12*x + 8)*log(-12*x/abs(x) + 8/abs(x))
```

Fricas [C] time = 2.09051, size = 49, normalized size = 1.81

$$-\frac{1}{2}i\log\left(x-\frac{2}{3}\right)+\frac{1}{2}i\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-(-2+3*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*I*log(x - 2/3) + 1/2*I*log(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{-(3x-2)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-(-2+3*x)**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(3*x - 2)**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-(-2+3*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] undef
```

3.2422 $\int \frac{1}{x\sqrt{-4-12x-9x^2}}\,dx$

Optimal. Leaf size=27

$$-\frac{(3x+2)\tanh^{-1}(3x+1)}{\sqrt{-9x^2-12x-4}}$$

```
[Out] -(((2 + 3*x)*ArcTanh[1 + 3*x])/Sqrt[-4 - 12*x - 9*x^2])
```

Rubi [B] time = 0.0137119, antiderivative size = 55, normalized size of antiderivative = 2.04, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {646, 36, 31, 29}

$$\frac{(3x+2)\log(x)}{2\sqrt{-9x^2-12x-4}}-\frac{(3x+2)\log(3x+2)}{2\sqrt{-9x^2-12x-4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-4 - 12*x - 9*x^2]),x]
```

```
[Out] ((2 + 3*x)*Log[x])/(2*Sqrt[-4 - 12*x - 9*x^2]) - ((2 + 3*x)*Log[2 + 3*x])/(
2*Sqrt[-4 - 12*x - 9*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{-4-12x-9x^2}}\,dx = -\left(-\frac{(-6-9x)\int \frac{1}{(-6-9x)x}\,dx}{\sqrt{-4-12x-9x^2}}\right)$$
$$= -\frac{(3(-6-9x))\int \frac{1}{-6-9x}\,dx}{2\sqrt{-4-12x-9x^2}} + -\frac{(-6-9x)\int \frac{1}{x}\,dx}{6\sqrt{-4-12x-9x^2}}$$
$$= \frac{(2+3x)\log(x)}{2\sqrt{-4-12x-9x^2}} - \frac{(2+3x)\log(2+3x)}{2\sqrt{-4-12x-9x^2}}$$

Mathematica [A] time = 0.0071819, size = 33, normalized size = 1.22

$$\frac{(3x+2)(\log(x)-\log(3x+2))}{2\sqrt{-(3x+2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-4 - 12*x - 9*x^2]),x]
```

```
[Out] ((2 + 3*x)*(Log[x] - Log[2 + 3*x]))/(2*Sqrt[-(2 + 3*x)^2])
```

Maple [A] time = 0.103, size = 30, normalized size = 1.1

$$\frac{(2+3x)(\ln(x)-\ln(2+3x))}{2}\frac{1}{\sqrt{-(2+3x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-(2+3*x)^2)^(1/2),x)
```

```
[Out] 1/2*(2+3*x)*(ln(x)-ln(2+3*x))/(-(2+3*x)^2)^(1/2)
```

Maxima [C] time = 1.51444, size = 32, normalized size = 1.19

$$-\frac{1}{2}i\,(-1)^{12x+8}\log\left(\frac{12x}{|x|}+\frac{8}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-(2+3*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*I*(-1)^(12*x + 8)*log(12*x/abs(x) + 8/abs(x))
```

Fricas [C] time = 2.00307, size = 47, normalized size = 1.74

$$\frac{1}{2}i\log\left(x+\frac{2}{3}\right)-\frac{1}{2}i\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-(2+3*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*I*log(x + 2/3) - 1/2*I*log(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{-(3x+2)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-(2+3*x)**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(3*x + 2)**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-(2+3*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] undef
```

3.2423 $\int \frac{1}{x\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=68

$$\frac{\log(x)(a+bx)}{a\sqrt{a^2+2abx+b^2x^2}}-\frac{(a+bx)\log(a+bx)}{a\sqrt{a^2+2abx+b^2x^2}}$$

```
[Out] ((a + b*x)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - ((a + b*x)*Log[a + b
*x])/(a*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0224197, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {646, 36, 29, 31}

$$\frac{\log(x)(a+bx)}{a\sqrt{a^2+2abx+b^2x^2}}-\frac{(a+bx)\log(a+bx)}{a\sqrt{a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

```
[Out] ((a + b*x)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x + b^2*x^2]) - ((a + b*x)*Log[a + b
*x])/(a*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{\left(ab+b^2x\right)\int \frac{1}{x\left(ab+b^2x\right)}\,dx}{\sqrt{a^2+2abx+b^2x^2}}$$
$$= \frac{\left(ab+b^2x\right)\int \frac{1}{x}\,dx}{ab\sqrt{a^2+2abx+b^2x^2}} - \frac{\left(b\left(ab+b^2x\right)\right)\int \frac{1}{ab+b^2x}\,dx}{a\sqrt{a^2+2abx+b^2x^2}}$$
$$= \frac{(a+bx)\log(x)}{a\sqrt{a^2+2abx+b^2x^2}} - \frac{(a+bx)\log(a+bx)}{a\sqrt{a^2+2abx+b^2x^2}}$$

Mathematica [A] time = 0.0156588, size = 31, normalized size = 0.46

$$\frac{(a+bx)(\log(x)-\log(a+bx))}{a\sqrt{(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

```
[Out] ((a + b*x)*(Log[x] - Log[a + b*x]))/(a*Sqrt[(a + b*x)^2])
```

Maple [A] time = 0.16, size = 30, normalized size = 0.4

$$\frac{(bx+a)\left(\ln(x)-\ln(bx+a)\right)}{a}\frac{1}{\sqrt{(bx+a)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x+a)^2)^(1/2),x)
```

```
[Out] (b*x+a)*(ln(x)-ln(b*x+a))/((b*x+a)^2)^(1/2)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.02874, size = 38, normalized size = 0.56

$$-\frac{\log(bx+a)-\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(log(b*x + a) - log(x))/a
```

Sympy [A] time = 0.159831, size = 10, normalized size = 0.15

$$\frac{\log(x) - \log\left(\frac{a}{b} + x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)**2)**(1/2),x)
```

```
[Out] (log(x) - log(a/b + x))/a
```

Giac [A] time = 1.09696, size = 38, normalized size = 0.56

$$-\left(\frac{\log(|bx+a|)}{a} - \frac{\log(|x|)}{a}\right)\mathrm{sgn}(bx+a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(log(abs(b*x + a))/a - log(abs(x))/a)*sgn(b*x + a)
```

3.2424 $\int \frac{1}{x\sqrt{a^2-2abx+b^2x^2}}\,dx$

Optimal. Leaf size=71

$$\frac{\log(x)(a-bx)}{a\sqrt{a^2-2abx+b^2x^2}}-\frac{(a-bx)\log(a-bx)}{a\sqrt{a^2-2abx+b^2x^2}}$$

```
[Out] ((a - b*x)*Log[x])/(a*Sqrt[a^2 - 2*a*b*x + b^2*x^2]) - ((a - b*x)*Log[a - b
*x])/(a*Sqrt[a^2 - 2*a*b*x + b^2*x^2])
```

Rubi [A] time = 0.0241426, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {646, 36, 29, 31}

$$\frac{\log(x)(a-bx)}{a\sqrt{a^2-2abx+b^2x^2}}-\frac{(a-bx)\log(a-bx)}{a\sqrt{a^2-2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a^2 - 2*a*b*x + b^2*x^2]),x]
```

```
[Out] ((a - b*x)*Log[x])/(a*Sqrt[a^2 - 2*a*b*x + b^2*x^2]) - ((a - b*x)*Log[a - b
*x])/(a*Sqrt[a^2 - 2*a*b*x + b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a^2-2abx+b^2x^2}}\,dx = \frac{\left(-ab+b^2x\right)\int \frac{1}{x\left(-ab+b^2x\right)}\,dx}{\sqrt{a^2-2abx+b^2x^2}}$$
$$= -\frac{\left(-ab+b^2x\right)\int \frac{1}{x}\,dx}{ab\sqrt{a^2-2abx+b^2x^2}} + \frac{\left(b\left(-ab+b^2x\right)\right)\int \frac{1}{-ab+b^2x}\,dx}{a\sqrt{a^2-2abx+b^2x^2}}$$
$$= \frac{(a-bx)\log(x)}{a\sqrt{a^2-2abx+b^2x^2}} - \frac{(a-bx)\log(a-bx)}{a\sqrt{a^2-2abx+b^2x^2}}$$

Mathematica [A] time = 0.0186234, size = 34, normalized size = 0.48

$$\frac{(a-bx)(\log(x)-\log(a-bx))}{a\sqrt{(a-bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a^2 - 2*a*b*x + b^2*x^2]),x]

[Out] ((a - b*x)*(Log[x] - Log[a - b*x]))/(a*Sqrt[(a - b*x)^2])
```

Maple [A] time = 0.238, size = 37, normalized size = 0.5

$$-\frac{(bx-a)\left(\ln(x)-\ln(bx-a)\right)}{a}\frac{1}{\sqrt{(bx-a)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x-a)^2)^(1/2),x)

[Out] -(b*x-a)*(ln(x)-ln(b*x-a))/((b*x-a)^2)^(1/2)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x-a)^2)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 2.08486, size = 36, normalized size = 0.51

$$\frac{\log(bx-a)-\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x-a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (log(b*x - a) - log(x))/a
```

Sympy [A] time = 0.174644, size = 10, normalized size = 0.14

$$\frac{-\log(x) + \log\left(-\frac{a}{b} + x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x-a)**2)**(1/2),x)
```

```
[Out] (-log(x) + log(-a/b + x))/a
```

Giac [A] time = 1.08825, size = 42, normalized size = 0.59

$$\left(\frac{\log(|bx - a|)}{a} - \frac{\log(|x|)}{a}\right)\operatorname{sgn}(bx - a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x-a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] (log(abs(b*x - a))/a - log(abs(x))/a)*sgn(b*x - a)
```

3.2425 $\int \frac{1}{x\sqrt{-a^2+2abx-b^2x^2}}\,dx$

Optimal. Leaf size=77

$$\frac{\log(x)(a-bx)}{a\sqrt{-a^2+2abx-b^2x^2}}-\frac{(a-bx)\log(a-bx)}{a\sqrt{-a^2+2abx-b^2x^2}}$$

```
[Out] ((a - b*x)*Log[x])/(a*Sqrt[-a^2 + 2*a*b*x - b^2*x^2]) - ((a - b*x)*Log[a -
b*x])/(a*Sqrt[-a^2 + 2*a*b*x - b^2*x^2])
```

Rubi [A] time = 0.0240125, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {646, 36, 29, 31}

$$\frac{\log(x)(a-bx)}{a\sqrt{-a^2+2abx-b^2x^2}}-\frac{(a-bx)\log(a-bx)}{a\sqrt{-a^2+2abx-b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-a^2 + 2*a*b*x - b^2*x^2]),x]
```

```
[Out] ((a - b*x)*Log[x])/(a*Sqrt[-a^2 + 2*a*b*x - b^2*x^2]) - ((a - b*x)*Log[a -
b*x])/(a*Sqrt[-a^2 + 2*a*b*x - b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{-a^2+2abx-b^2x^2}}\,dx = \frac{\left(ab-b^2x\right)\int \frac{1}{x\left(ab-b^2x\right)}\,dx}{\sqrt{-a^2+2abx-b^2x^2}}$$
$$= \frac{\left(ab-b^2x\right)\int \frac{1}{x}\,dx}{ab\sqrt{-a^2+2abx-b^2x^2}} + \frac{\left(b\left(ab-b^2x\right)\right)\int \frac{1}{ab-b^2x}\,dx}{a\sqrt{-a^2+2abx-b^2x^2}}$$
$$= \frac{(a-bx)\log(x)}{a\sqrt{-a^2+2abx-b^2x^2}} - \frac{(a-bx)\log(a-bx)}{a\sqrt{-a^2+2abx-b^2x^2}}$$

Mathematica [A] time = 0.0129045, size = 36, normalized size = 0.47

$$\frac{(a-bx)(\log(x)-\log(a-bx))}{a\sqrt{-(a-bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-a^2 + 2*a*b*x - b^2*x^2]),x]
```

```
[Out] ((a - b*x)*(Log[x] - Log[a - b*x]))/(a*Sqrt[-(a - b*x)^2])
```

Maple [A] time = 0.194, size = 39, normalized size = 0.5

$$-\frac{(bx-a)\left(\ln(x)-\ln(bx-a)\right)}{a}\frac{1}{\sqrt{-(bx-a)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-(b*x-a)^2)^(1/2),x)
```

```
[Out] -(b*x-a)*(ln(x)-ln(b*x-a))/(-(b*x-a)^2)^(1/2)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-(b*x-a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.09324, size = 161, normalized size = 2.09

$$-\sqrt{-\frac{1}{a^2}}\log\left(\frac{i\,a^2\sqrt{-\frac{1}{a^2}}+2\,bx-a}{2\,b}\right)+\sqrt{-\frac{1}{a^2}}\log\left(\frac{-i\,a^2\sqrt{-\frac{1}{a^2}}+2\,bx-a}{2\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-(b*x-a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-1/a^2)*log(1/2*(I*a^2*sqrt(-1/a^2) + 2*b*x - a)/b) + sqrt(-1/a^2)*lo
g(1/2*(-I*a^2*sqrt(-1/a^2) + 2*b*x - a)/b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{-\left(-a+bx\right)^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-(b*x-a)**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(-a + b*x)**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-(b*x-a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] undef
```

3.2426 $\int \frac{1}{x\sqrt{-a^2-2abx-b^2x^2}}\,dx$

Optimal. Leaf size=74

$$\frac{\log(x)(a+bx)}{a\sqrt{-a^2-2abx-b^2x^2}}-\frac{(a+bx)\log(a+bx)}{a\sqrt{-a^2-2abx-b^2x^2}}$$

```
[Out] ((a + b*x)*Log[x])/(a*Sqrt[-a^2 - 2*a*b*x - b^2*x^2]) - ((a + b*x)*Log[a +
b*x])/(a*Sqrt[-a^2 - 2*a*b*x - b^2*x^2])
```

Rubi [A] time = 0.0264302, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {646, 36, 29, 31}

$$\frac{\log(x)(a+bx)}{a\sqrt{-a^2-2abx-b^2x^2}}-\frac{(a+bx)\log(a+bx)}{a\sqrt{-a^2-2abx-b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-a^2 - 2*a*b*x - b^2*x^2]),x]
```

```
[Out] ((a + b*x)*Log[x])/(a*Sqrt[-a^2 - 2*a*b*x - b^2*x^2]) - ((a + b*x)*Log[a +
b*x])/(a*Sqrt[-a^2 - 2*a*b*x - b^2*x^2])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{-a^2-2abx-b^2x^2}}\,dx = -\left(-\frac{\left(-ab-b^2x\right)\int \frac{1}{x\left(-ab-b^2x\right)}\,dx}{\sqrt{-a^2-2abx-b^2x^2}}\right)$$
$$= -\frac{\left(-ab-b^2x\right)\int \frac{1}{x}\,dx}{ab\sqrt{-a^2-2abx-b^2x^2}} + -\frac{\left(b\left(-ab-b^2x\right)\right)\int \frac{1}{-ab-b^2x}\,dx}{a\sqrt{-a^2-2abx-b^2x^2}}$$
$$= \frac{(a+bx)\log(x)}{a\sqrt{-a^2-2abx-b^2x^2}} - \frac{(a+bx)\log(a+bx)}{a\sqrt{-a^2-2abx-b^2x^2}}$$

Mathematica [A] time = 0.01265, size = 33, normalized size = 0.45

$$\frac{(a+bx)(\log(x)-\log(a+bx))}{a\sqrt{-(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-a^2 - 2*a*b*x - b^2*x^2]),x]
```

```
[Out] ((a + b*x)*(Log[x] - Log[a + b*x]))/(a*Sqrt[-(a + b*x)^2])
```

Maple [A] time = 0.192, size = 32, normalized size = 0.4

$$\frac{(bx+a)(\ln(x)-\ln(bx+a))}{a}\frac{1}{\sqrt{-(bx+a)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-(b*x+a)^2)^(1/2),x)
```

```
[Out] (b*x+a)*(ln(x)-ln(b*x+a))/(-(b*x+a)^2)^(1/2)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.07598, size = 161, normalized size = 2.18

$$-\sqrt{-\frac{1}{a^2}}\log\left(\frac{i\,a^2\sqrt{-\frac{1}{a^2}}+2\,bx+a}{2\,b}\right)+\sqrt{-\frac{1}{a^2}}\log\left(\frac{-i\,a^2\sqrt{-\frac{1}{a^2}}+2\,bx+a}{2\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-1/a^2)*log(1/2*(I*a^2*sqrt(-1/a^2) + 2*b*x + a)/b) + sqrt(-1/a^2)*lo
g(1/2*(-I*a^2*sqrt(-1/a^2) + 2*b*x + a)/b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{-(a+bx)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-(b*x+a)**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(a + b*x)**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] undef
```

3.2427 $\int x\sqrt{3-2x-x^2}\,dx$

Optimal. Leaf size=52

$$-\frac{1}{3}\left(-x^2-2x+3\right)^{3/2}-\frac{1}{2}(x+1)\sqrt{-x^2-2x+3}+2\sin^{-1}\left(\frac{1}{2}(-x-1)\right)$$

```
[Out] -((1 + x)*Sqrt[3 - 2*x - x^2])/2 - (3 - 2*x - x^2)^(3/2)/3 + 2*ArcSin[(-1 -
 x)/2]
```

Rubi [A] time = 0.0137335, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {640, 612, 619, 216}

$$-\frac{1}{3}\left(-x^2-2x+3\right)^{3/2}-\frac{1}{2}(x+1)\sqrt{-x^2-2x+3}+2\sin^{-1}\left(\frac{1}{2}(-x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[3 - 2*x - x^2],x]
```


```
[Out] -((1 + x)*Sqrt[3 - 2*x - x^2])/2 - (3 - 2*x - x^2)^(3/2)/3 + 2*ArcSin[(-1 -
 x)/2]
```


Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int x\sqrt{3-2x-x^2}\,dx = -\frac{1}{3}\left(3-2x-x^2\right)^{3/2} - \int \sqrt{3-2x-x^2}\,dx$$
$$= -\frac{1}{2}(1+x)\sqrt{3-2x-x^2} - \frac{1}{3}\left(3-2x-x^2\right)^{3/2} - 2\int \frac{1}{\sqrt{3-2x-x^2}}\,dx$$
$$= -\frac{1}{2}(1+x)\sqrt{3-2x-x^2} - \frac{1}{3}\left(3-2x-x^2\right)^{3/2} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{16}}}\,dx, x, -2-2x\right)$$
$$= -\frac{1}{2}(1+x)\sqrt{3-2x-x^2} - \frac{1}{3}\left(3-2x-x^2\right)^{3/2} + 2\sin^{-1}\left(\frac{1}{2}(-1-x)\right)$$

Mathematica [A] time = 0.0174071, size = 37, normalized size = 0.71

$$\frac{1}{6}\sqrt{-x^2-2x+3}\left(2x^2+x-9\right) - 2\sin^{-1}\left(\frac{x+1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[3 - 2*x - x^2],x]
```

```
[Out] (Sqrt[3 - 2*x - x^2]*(-9 + x + 2*x^2))/6 - 2*ArcSin[(1 + x)/2]
```

Maple [A] time = 0.042, size = 43, normalized size = 0.8

$$-\frac{1}{3}\left(-x^2-2x+3\right)^{\frac{3}{2}} + \frac{-2x-2}{4}\sqrt{-x^2-2x+3} - 2\arcsin\left(1/2+x/2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^2-2*x+3)^(1/2),x)
```

```
[Out] -1/3*(-x^2-2*x+3)^(3/2)+1/4*(-2*x-2)*(-x^2-2*x+3)^(1/2)-2*arcsin(1/2+1/2*x)
```

Maxima [A] time = 1.49706, size = 70, normalized size = 1.35

$$-\frac{1}{3}\left(-x^2-2x+3\right)^{\frac{3}{2}} - \frac{1}{2}\sqrt{-x^2-2x+3}x - \frac{1}{2}\sqrt{-x^2-2x+3} + 2\arcsin\left(-\frac{1}{2}x-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2-2*x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-x^2 - 2*x + 3)^(3/2) - 1/2*sqrt(-x^2 - 2*x + 3)*x - 1/2*sqrt(-x^2 -
2*x + 3) + 2*arcsin(-1/2*x - 1/2)
```

Fricas [A] time = 2.08042, size = 134, normalized size = 2.58

$$\frac{1}{6}\left(2x^2+x-9\right)\sqrt{-x^2-2x+3} + 2\arctan\left(\frac{\sqrt{-x^2-2x+3}(x+1)}{x^2+2x-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2-2*x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^2 + x - 9)*sqrt(-x^2 - 2*x + 3) + 2*arctan(sqrt(-x^2 - 2*x + 3)*(x
 + 1)/(x^2 + 2*x - 3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{-(x-1)(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**2-2*x+3)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-(x - 1)*(x + 3)), x)
```

Giac [A] time = 1.09121, size = 43, normalized size = 0.83

$$\frac{1}{6}\left((2x+1)x-9\right)\sqrt{-x^2-2x+3}-2\arcsin\left(\frac{1}{2}x+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2-2*x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*((2*x + 1)*x - 9)*sqrt(-x^2 - 2*x + 3) - 2*arcsin(1/2*x + 1/2)
```

3.2428 $\int x\sqrt{8+2x-x^2}\,dx$

Optimal. Leaf size=56

$$-\frac{1}{3}\left(-x^2+2x+8\right)^{3/2}-\frac{1}{2}(1-x)\sqrt{-x^2+2x+8}-\frac{9}{2}\sin^{-1}\left(\frac{1-x}{3}\right)$$

```
[Out] -((1 - x)*Sqrt[8 + 2*x - x^2])/2 - (8 + 2*x - x^2)^(3/2)/3 - (9*ArcSin[(1 - x)/3])/2
```

Rubi [A] time = 0.0137811, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {640, 612, 619, 216}

$$-\frac{1}{3}\left(-x^2+2x+8\right)^{3/2}-\frac{1}{2}(1-x)\sqrt{-x^2+2x+8}-\frac{9}{2}\sin^{-1}\left(\frac{1-x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[8 + 2*x - x^2],x]
```

```
[Out] -((1 - x)*Sqrt[8 + 2*x - x^2])/2 - (8 + 2*x - x^2)^(3/2)/3 - (9*ArcSin[(1 - x)/3])/2
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x\sqrt{8+2x-x^2}\,dx &= -\frac{1}{3}\left(8+2x-x^2\right)^{3/2}+\int \sqrt{8+2x-x^2}\,dx\\
&= -\frac{1}{2}(1-x)\sqrt{8+2x-x^2}-\frac{1}{3}\left(8+2x-x^2\right)^{3/2}+\frac{9}{2}\int \frac{1}{\sqrt{8+2x-x^2}}dx\\
&= -\frac{1}{2}(1-x)\sqrt{8+2x-x^2}-\frac{1}{3}\left(8+2x-x^2\right)^{3/2}-\frac{3}{4}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{36}}}dx,x,2-2x\right)\\
&= -\frac{1}{2}(1-x)\sqrt{8+2x-x^2}-\frac{1}{3}\left(8+2x-x^2\right)^{3/2}-\frac{9}{2}\sin^{-1}\left(\frac{1-x}{3}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0196784, size = 42, normalized size = 0.75

$$
\frac{1}{6}\left(\sqrt{-x^2+2x+8}\left(2x^2-x-19\right)-27\sin^{-1}\left(\frac{1}{3}-\frac{x}{3}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[8 + 2*x - x^2],x]
```

```
[Out] (Sqrt[8 + 2*x - x^2]*(-19 - x + 2*x^2) - 27*ArcSin[1/3 - x/3])/6
```

Maple [A] time = 0.041, size = 43, normalized size = 0.8

$$
-\frac{1}{3}\left(-x^2+2x+8\right)^{\frac{3}{2}}-\frac{2-2x}{4}\sqrt{-x^2+2x+8}+\frac{9}{2}\arcsin\left(-\frac{1}{3}+\frac{x}{3}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^2+2*x+8)^(1/2),x)
```

```
[Out] -1/3*(-x^2+2*x+8)^(3/2)-1/4*(2-2*x)*(-x^2+2*x+8)^(1/2)+9/2*arcsin(-1/3+1/3*
x)
```

Maxima [A] time = 1.47808, size = 70, normalized size = 1.25

$$
-\frac{1}{3}\left(-x^2+2x+8\right)^{\frac{3}{2}}+\frac{1}{2}\sqrt{-x^2+2x+8}x-\frac{1}{2}\sqrt{-x^2+2x+8}-\frac{9}{2}\arcsin\left(-\frac{1}{3}x+\frac{1}{3}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+2*x+8)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-x^2 + 2*x + 8)^(3/2) + 1/2*sqrt(-x^2 + 2*x + 8)*x - 1/2*sqrt(-x^2 +
2*x + 8) - 9/2*arcsin(-1/3*x + 1/3)
```

Fricas [A] time = 2.03887, size = 138, normalized size = 2.46

$$
\frac{1}{6}\left(2x^2-x-19\right)\sqrt{-x^2+2x+8}-\frac{9}{2}\arctan\left(\frac{\sqrt{-x^2+2x+8}(x-1)}{x^2-2x-8}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+2*x+8)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^2 - x - 19)*sqrt(-x^2 + 2*x + 8) - 9/2*arctan(sqrt(-x^2 + 2*x + 8)
*(x - 1)/(x^2 - 2*x - 8))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{-(x-4)(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**2+2*x+8)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-(x - 4)*(x + 2)), x)
```

Giac [A] time = 1.09454, size = 43, normalized size = 0.77

$$\frac{1}{6}\left((2\,x-1)x-19\right)\sqrt{-x^2+2\,x+8}+\frac{9}{2}\,\arcsin\left(\frac{1}{3}\,x-\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+2*x+8)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*((2*x - 1)*x - 19)*sqrt(-x^2 + 2*x + 8) + 9/2*arcsin(1/3*x - 1/3)
```

3.2429 $\int x\sqrt{4+2x+x^2}\,dx$

Optimal. Leaf size=50

$$\frac{1}{3}\left(x^2+2x+4\right)^{3/2}-\frac{1}{2}(x+1)\sqrt{x^2+2x+4}-\frac{3}{2}\sinh^{-1}\left(\frac{x+1}{\sqrt{3}}\right)$$

```
[Out] -((1 + x)*Sqrt[4 + 2*x + x^2])/2 + (4 + 2*x + x^2)^(3/2)/3 - (3*ArcSinh[(1
+ x)/Sqrt[3]])/2
```

Rubi [A] time = 0.0154104, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {640, 612, 619, 215}

$$\frac{1}{3}\left(x^2+2x+4\right)^{3/2}-\frac{1}{2}(x+1)\sqrt{x^2+2x+4}-\frac{3}{2}\sinh^{-1}\left(\frac{x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[4 + 2*x + x^2],x]
```

```
[Out] -((1 + x)*Sqrt[4 + 2*x + x^2])/2 + (4 + 2*x + x^2)^(3/2)/3 - (3*ArcSinh[(1
+ x)/Sqrt[3]])/2
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{4+2x+x^2}\,dx &= \frac{1}{3}\left(4+2x+x^2\right)^{3/2}-\int \sqrt{4+2x+x^2}\,dx\\ &= -\frac{1}{2}(1+x)\sqrt{4+2x+x^2}+\frac{1}{3}\left(4+2x+x^2\right)^{3/2}-\frac{3}{2}\int \frac{1}{\sqrt{4+2x+x^2}}\,dx\\ &= -\frac{1}{2}(1+x)\sqrt{4+2x+x^2}+\frac{1}{3}\left(4+2x+x^2\right)^{3/2}-\frac{1}{4}\sqrt{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{12}}}\,dx,x,2+2x\right)\\ &= -\frac{1}{2}(1+x)\sqrt{4+2x+x^2}+\frac{1}{3}\left(4+2x+x^2\right)^{3/2}-\frac{3}{2}\sinh^{-1}\left(\frac{1+x}{\sqrt{3}}\right)\end{aligned}$$

Mathematica [A] time = 0.0191345, size = 38, normalized size = 0.76

$$\frac{1}{6}\left(\sqrt{x^2+2x+4}\left(2x^2+x+5\right)-9\sinh^{-1}\left(\frac{x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[4 + 2*x + x^2],x]
```

```
[Out] (Sqrt[4 + 2*x + x^2]*(5 + x + 2*x^2) - 9*ArcSinh[(1 + x)/Sqrt[3]])/6
```

Maple [A] time = 0.042, size = 42, normalized size = 0.8

$$\frac{1}{3}\left(x^2+2x+4\right)^{\frac{3}{2}}-\frac{2x+2}{4}\sqrt{x^2+2x+4}-\frac{3}{2}\text{Arcsinh}\left(\frac{(1+x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+2*x+4)^(1/2),x)
```

```
[Out] 1/3*(x^2+2*x+4)^(3/2)-1/4*(2*x+2)*(x^2+2*x+4)^(1/2)-3/2*arcsinh(1/3*(1+x)*3
^(1/2))
```

Maxima [A] time = 1.59189, size = 66, normalized size = 1.32

$$\frac{1}{3}\left(x^2+2x+4\right)^{\frac{3}{2}}-\frac{1}{2}\sqrt{x^2+2x+4}x-\frac{1}{2}\sqrt{x^2+2x+4}-\frac{3}{2}\,\text{arsinh}\left(\frac{1}{3}\sqrt{3}(x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+2*x+4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(x^2 + 2*x + 4)^(3/2) - 1/2*sqrt(x^2 + 2*x + 4)*x - 1/2*sqrt(x^2 + 2*x
+ 4) - 3/2*arcsinh(1/3*sqrt(3)*(x + 1))
```

Fricas [A] time = 1.96893, size = 109, normalized size = 2.18

$$\frac{1}{6}\left(2x^2+x+5\right)\sqrt{x^2+2x+4}+\frac{3}{2}\log\left(-x+\sqrt{x^2+2x+4}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+2*x+4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^2 + x + 5)*sqrt(x^2 + 2*x + 4) + 3/2*log(-x + sqrt(x^2 + 2*x + 4)
- 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{x^2+2x+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+2*x+4)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x**2 + 2*x + 4), x)
```

Giac [A] time = 1.10617, size = 54, normalized size = 1.08

$$\frac{1}{6}\left((2x+1)x+5\right)\sqrt{x^2+2x+4}+\frac{3}{2}\log\left(-x+\sqrt{x^2+2x+4}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+2*x+4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*((2*x + 1)*x + 5)*sqrt(x^2 + 2*x + 4) + 3/2*log(-x + sqrt(x^2 + 2*x + 4
) - 1)
```

3.2430 $\int \frac{1}{x\sqrt{2+4x+3x^2}}\,dx$

Optimal. Leaf size=31

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{2}(x+1)}{\sqrt{3x^2+4x+2}}\right)}{\sqrt{2}}$$

```
[Out] -(ArcTanh[(Sqrt[2]*(1 + x))/Sqrt[2 + 4*x + 3*x^2]]/Sqrt[2])
```

Rubi [A] time = 0.0117503, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {724, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{2}(x+1)}{\sqrt{3x^2+4x+2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[2 + 4*x + 3*x^2]),x]

[Out] -(ArcTanh[(Sqrt[2]*(1 + x))/Sqrt[2 + 4*x + 3*x^2]]/Sqrt[2])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{2+4x+3x^2}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{8-x^2}\,dx, x, \frac{4+4x}{\sqrt{2+4x+3x^2}}\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{2}(1+x)}{\sqrt{2+4x+3x^2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0082974, size = 28, normalized size = 0.9

$$-\frac{\tanh^{-1}\left(\frac{x+1}{\sqrt{\frac{3x^2}{2}+2x+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[2 + 4*x + 3*x^2]),x]
```

```
[Out] -(ArcTanh[(1 + x)/Sqrt[1 + 2*x + (3*x^2)/2]]/Sqrt[2])
```

Maple [A] time = 0.042, size = 29, normalized size = 0.9

$$-\frac{\sqrt{2}}{2}\operatorname{Artanh}\left(\frac{(4+4x)\sqrt{2}}{4}\frac{1}{\sqrt{3x^2+4x+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(3*x^2+4*x+2)^(1/2),x)
```

```
[Out] -1/2*2^(1/2)*arctanh(1/4*(4+4*x)*2^(1/2)/(3*x^2+4*x+2)^(1/2))
```

Maxima [A] time = 1.56766, size = 32, normalized size = 1.03

$$-\frac{1}{2}\sqrt{2}\operatorname{arsinh}\left(\frac{\sqrt{2}x}{|x|}+\frac{\sqrt{2}}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2+4*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arcsinh(sqrt(2)*x/abs(x) + sqrt(2)/abs(x))
```

Fricas [A] time = 2.00629, size = 111, normalized size = 3.58

$$\frac{1}{4}\sqrt{2}\log\left(\frac{2\sqrt{2}\sqrt{3x^2+4x+2}(x+1)-5x^2-8x-4}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2+4*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((2*sqrt(2)*sqrt(3*x^2 + 4*x + 2)*(x + 1) - 5*x^2 - 8*x - 4)
/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\sqrt{3x^2+4x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x**2+4*x+2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(3*x**2 + 4*x + 2)), x)
```

Giac [B] time = 1.12745, size = 81, normalized size = 2.61

$$-\frac{1}{2}\sqrt{2}\log\left(-\sqrt{3}x+\sqrt{2}+\sqrt{3x^2+4x+2}\right)+\frac{1}{2}\sqrt{2}\log\left(\left|-\sqrt{3}x-\sqrt{2}+\sqrt{3x^2+4x+2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2+4*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(-sqrt(3)*x + sqrt(2) + sqrt(3*x^2 + 4*x + 2)) + 1/2*sqrt(2
)*log(abs(-sqrt(3)*x - sqrt(2) + sqrt(3*x^2 + 4*x + 2)))
```

3.2431 $\int \frac{1}{x\sqrt{2+4x-3x^2}}\,dx$

Optimal. Leaf size=31

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{2}(x+1)}{\sqrt{-3x^2+4x+2}}\right)}{\sqrt{2}}$$

```
[Out] -(ArcTanh[(Sqrt[2]*(1 + x))/Sqrt[2 + 4*x - 3*x^2]]/Sqrt[2])
```

Rubi [A] time = 0.0109847, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {724, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{2}(x+1)}{\sqrt{-3x^2+4x+2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[2 + 4*x - 3*x^2]),x]
```

```
[Out] -(ArcTanh[(Sqrt[2]*(1 + x))/Sqrt[2 + 4*x - 3*x^2]]/Sqrt[2])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{2+4x-3x^2}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{8-x^2}\,dx, x, \frac{4+4x}{\sqrt{2+4x-3x^2}}\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{2}(1+x)}{\sqrt{2+4x-3x^2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0076599, size = 28, normalized size = 0.9

$$-\frac{\tanh^{-1}\left(\frac{x+1}{\sqrt{-\frac{3x^2}{2}+2x+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[2 + 4*x - 3*x^2]),x]
```

```
[Out] -(ArcTanh[(1 + x)/Sqrt[1 + 2*x - (3*x^2)/2]]/Sqrt[2])
```

Maple [A] time = 0.042, size = 29, normalized size = 0.9

$$-\frac{\sqrt{2}}{2}\mathrm{Artanh}\left(\frac{(4+4x)\sqrt{2}}{4}\frac{1}{\sqrt{-3x^2+4x+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-3*x^2+4*x+2)^(1/2),x)
```

```
[Out] -1/2*2^(1/2)*arctanh(1/4*(4+4*x)*2^(1/2)/(-3*x^2+4*x+2)^(1/2))
```

Maxima [A] time = 1.49362, size = 47, normalized size = 1.52

$$-\frac{1}{2}\sqrt{2}\log\left(\frac{2\sqrt{2}\sqrt{-3x^2+4x+2}}{|x|}+\frac{4}{|x|}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x^2+4*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*log(2*sqrt(2)*sqrt(-3*x^2 + 4*x + 2)/abs(x) + 4/abs(x) + 4)
```

Fricas [A] time = 2.06355, size = 111, normalized size = 3.58

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{2\sqrt{2}\sqrt{-3x^2+4x+2}(x+1)+x^2-8x-4}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x^2+4*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(2*sqrt(2)*sqrt(-3*x^2 + 4*x + 2)*(x + 1) + x^2 - 8*x - 4)
/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{-3x^2+4x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x**2+4*x+2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-3*x**2 + 4*x + 2)), x)
```

Giac [B] time = 1.26917, size = 132, normalized size = 4.26

$$-\frac{1}{6}\sqrt{6}\sqrt{3}\log\left(\frac{\left|-14\sqrt{10}-14\sqrt{6}+\frac{28\left(\sqrt{3}\sqrt{-3x^2+4x+2}-\sqrt{10}\right)}{3x-2}\right|}{\left|-14\sqrt{10}+14\sqrt{6}+\frac{28\left(\sqrt{3}\sqrt{-3x^2+4x+2}-\sqrt{10}\right)}{3x-2}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x^2+4*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(6)*sqrt(3)*log(abs(-14*sqrt(10) - 14*sqrt(6) + 28*(sqrt(3)*sqrt(-
3*x^2 + 4*x + 2) - sqrt(10))/(3*x - 2))/abs(-14*sqrt(10) + 14*sqrt(6) + 28*
(sqrt(3)*sqrt(-3*x^2 + 4*x + 2) - sqrt(10))/(3*x - 2)))
```

3.2432 $\int \frac{1}{x\sqrt{2+5x+3x^2}}\,dx$

Optimal. Leaf size=36

$$-\frac{\tanh^{-1}\left(\frac{5x+4}{2\sqrt{2}\sqrt{3x^2+5x+2}}\right)}{\sqrt{2}}$$

```
[Out] -(ArcTanh[(4 + 5*x)/(2*Sqrt[2]*Sqrt[2 + 5*x + 3*x^2])]/Sqrt[2])
```

Rubi [A] time = 0.0110107, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {724, 206}

$$-\frac{\tanh^{-1}\left(\frac{5x+4}{2\sqrt{2}\sqrt{3x^2+5x+2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[2 + 5*x + 3*x^2]),x]
```

```
[Out] -(ArcTanh[(4 + 5*x)/(2*Sqrt[2]*Sqrt[2 + 5*x + 3*x^2])]/Sqrt[2])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{2+5x+3x^2}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{8-x^2}\,dx, x, \frac{4+5x}{\sqrt{2+5x+3x^2}}\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{4+5x}{2\sqrt{2}\sqrt{2+5x+3x^2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0102785, size = 31, normalized size = 0.86

$$-\frac{\tanh^{-1}\left(\frac{5x+4}{2\sqrt{6x^2+10x+4}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[2 + 5*x + 3*x^2]),x]
```

```
[Out] -(ArcTanh[(4 + 5*x)/(2*Sqrt[4 + 10*x + 6*x^2])]/Sqrt[2])
```

Maple [A] time = 0.041, size = 29, normalized size = 0.8

$$-\frac{\sqrt{2}}{2}\operatorname{Artanh}\left(\frac{(4+5x)\sqrt{2}}{4}\frac{1}{\sqrt{3x^2+5x+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(3*x^2+5*x+2)^(1/2),x)
```

```
[Out] -1/2*arctanh(1/4*(4+5*x)*2^(1/2)/(3*x^2+5*x+2)^(1/2))*2^(1/2)
```

Maxima [A] time = 1.52742, size = 47, normalized size = 1.31

$$-\frac{1}{2}\sqrt{2}\log\left(\frac{2\sqrt{2}\sqrt{3x^2+5x+2}}{|x|}+\frac{4}{|x|}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*log(2*sqrt(2)*sqrt(3*x^2 + 5*x + 2)/abs(x) + 4/abs(x) + 5)
```

Fricas [A] time = 2.16917, size = 119, normalized size = 3.31

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{4\sqrt{2}\sqrt{3x^2+5x+2}(5x+4)-49x^2-80x-32}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(4*sqrt(2)*sqrt(3*x^2 + 5*x + 2)*(5*x + 4) - 49*x^2 - 80*x
 - 32)/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\sqrt{(x+1)(3x+2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x**2+5*x+2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt((x + 1)*(3*x + 2))), x)
```

Giac [B] time = 1.13422, size = 82, normalized size = 2.28

$$-\frac{1}{2}\sqrt{2}\log\left(\left|-\sqrt{3}x+\sqrt{2}+\sqrt{3x^2+5x+2}\right|\right)+\frac{1}{2}\sqrt{2}\log\left(\left|-\sqrt{3}x-\sqrt{2}+\sqrt{3x^2+5x+2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(abs(-sqrt(3)*x + sqrt(2) + sqrt(3*x^2 + 5*x + 2))) + 1/2*s
qrt(2)*log(abs(-sqrt(3)*x - sqrt(2) + sqrt(3*x^2 + 5*x + 2)))
```

3.2433 $\int \frac{1}{x\sqrt{2+5x-3x^2}}\,dx$

Optimal. Leaf size=36

$$-\frac{\tanh^{-1}\left(\frac{5x+4}{2\sqrt{2}\sqrt{-3x^2+5x+2}}\right)}{\sqrt{2}}$$

```
[Out] -(ArcTanh[(4 + 5*x)/(2*Sqrt[2]*Sqrt[2 + 5*x - 3*x^2])]/Sqrt[2])
```

Rubi [A] time = 0.0126804, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {724, 206}

$$-\frac{\tanh^{-1}\left(\frac{5x+4}{2\sqrt{2}\sqrt{-3x^2+5x+2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[2 + 5*x - 3*x^2]),x]
```

```
[Out] -(ArcTanh[(4 + 5*x)/(2*Sqrt[2]*Sqrt[2 + 5*x - 3*x^2])]/Sqrt[2])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{2+5x-3x^2}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{8-x^2}\,dx, x, \frac{4+5x}{\sqrt{2+5x-3x^2}}\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{4+5x}{2\sqrt{2}\sqrt{2+5x-3x^2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0103498, size = 31, normalized size = 0.86

$$-\frac{\tanh^{-1}\left(\frac{5x+4}{2\sqrt{-6x^2+10x+4}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[2 + 5*x - 3*x^2]),x]
```

```
[Out] -(ArcTanh[(4 + 5*x)/(2*Sqrt[4 + 10*x - 6*x^2])]/Sqrt[2])
```

Maple [A] time = 0.04, size = 29, normalized size = 0.8

$$-\frac{\sqrt{2}}{2}\mathrm{Artanh}\left(\frac{(4+5x)\sqrt{2}}{4}\frac{1}{\sqrt{-3x^2+5x+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-3*x^2+5*x+2)^(1/2),x)
```

```
[Out] -1/2*arctanh(1/4*(4+5*x)*2^(1/2)/(-3*x^2+5*x+2)^(1/2))*2^(1/2)
```

Maxima [A] time = 1.50353, size = 47, normalized size = 1.31

$$-\frac{1}{2}\sqrt{2}\log\left(\frac{2\sqrt{2}\sqrt{-3x^2+5x+2}}{|x|}+\frac{4}{|x|}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*log(2*sqrt(2)*sqrt(-3*x^2 + 5*x + 2)/abs(x) + 4/abs(x) + 5)
```

Fricas [A] time = 2.1122, size = 116, normalized size = 3.22

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{4\sqrt{2}\sqrt{-3x^2+5x+2}(5x+4)-x^2-80x-32}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(4*sqrt(2)*sqrt(-3*x^2 + 5*x + 2)*(5*x + 4) - x^2 - 80*x -
 32)/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{-(x-2)(3x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x**2+5*x+2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(x - 2)*(3*x + 1))), x)
```

Giac [B] time = 1.20105, size = 113, normalized size = 3.14

$$-\frac{1}{6}\sqrt{6}\sqrt{3}\log\left(\frac{\left|-4\sqrt{6}+\frac{10\left(2\sqrt{3}\sqrt{-3x^2+5x+2}-7\right)}{6x-5}-14\right|}{\left|4\sqrt{6}+\frac{10\left(2\sqrt{3}\sqrt{-3x^2+5x+2}-7\right)}{6x-5}-14\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(6)*sqrt(3)*log(abs(-4*sqrt(6) + 10*(2*sqrt(3)*sqrt(-3*x^2 + 5*x +
 2) - 7)/(6*x - 5) - 14)/abs(4*sqrt(6) + 10*(2*sqrt(3)*sqrt(-3*x^2 + 5*x +
2) - 7)/(6*x - 5) - 14))
```

3.2434 $\int \frac{1}{x\sqrt{-2+4x+3x^2}}\,dx$

Optimal. Leaf size=33

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}(1-x)}{\sqrt{3x^2+4x-2}}\right)}{\sqrt{2}}$$

```
[Out] -(ArcTan[(Sqrt[2]*(1 - x))/Sqrt[-2 + 4*x + 3*x^2]]/Sqrt[2])
```

Rubi [A] time = 0.0106804, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {724, 204}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}(1-x)}{\sqrt{3x^2+4x-2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-2 + 4*x + 3*x^2]),x]
```

```
[Out] -(ArcTan[(Sqrt[2]*(1 - x))/Sqrt[-2 + 4*x + 3*x^2]]/Sqrt[2])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{-2+4x+3x^2}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, \frac{-4+4x}{\sqrt{-2+4x+3x^2}}\right)\right)\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{2}(1-x)}{\sqrt{-2+4x+3x^2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0087675, size = 27, normalized size = 0.82

$$\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\frac{3x^2}{2}+2x-1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-2 + 4*x + 3*x^2]),x]
```

```
[Out] ArcTan[(-1 + x)/Sqrt[-1 + 2*x + (3*x^2)/2]]/Sqrt[2]
```

Maple [A] time = 0.042, size = 29, normalized size = 0.9

$$\frac{\sqrt{2}}{2}\arctan\left(\frac{(-4+4x)\sqrt{2}}{4}\frac{1}{\sqrt{3x^2+4x-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(3*x^2+4*x-2)^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*arctan(1/4*(-4+4*x)*2^(1/2)/(3*x^2+4*x-2)^(1/2))
```

Maxima [A] time = 1.54439, size = 35, normalized size = 1.06

$$\frac{1}{2}\sqrt{2}\arcsin\left(\frac{\sqrt{10}x}{5|x|}-\frac{\sqrt{10}}{5|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2+4*x-2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arcsin(1/5*sqrt(10)*x/abs(x) - 1/5*sqrt(10)/abs(x))
```

Fricas [A] time = 2.06923, size = 80, normalized size = 2.42

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}(x-1)}{\sqrt{3x^2+4x-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2+4*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(sqrt(2)*(x - 1)/sqrt(3*x^2 + 4*x - 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\sqrt{3x^2+4x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x**2+4*x-2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(3*x**2 + 4*x - 2)), x)
```

Giac [A] time = 1.13588, size = 41, normalized size = 1.24

$$\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{3}x-\sqrt{3x^2+4x-2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2+4*x-2)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(3)*x - sqrt(3*x^2 + 4*x - 2)))
```

3.2435 $\int \frac{1}{x\sqrt{-2+4x-3x^2}}\,dx$

Optimal. Leaf size=33

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}(1-x)}{\sqrt{-3x^2+4x-2}}\right)}{\sqrt{2}}$$

```
[Out] -(ArcTan[(Sqrt[2]*(1 - x))/Sqrt[-2 + 4*x - 3*x^2]]/Sqrt[2])
```

Rubi [A] time = 0.011465, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {724, 204}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}(1-x)}{\sqrt{-3x^2+4x-2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-2 + 4*x - 3*x^2]),x]
```

```
[Out] -(ArcTan[(Sqrt[2]*(1 - x))/Sqrt[-2 + 4*x - 3*x^2]]/Sqrt[2])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{-2+4x-3x^2}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, \frac{-4+4x}{\sqrt{-2+4x-3x^2}}\right)\right)\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{2}(1-x)}{\sqrt{-2+4x-3x^2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0077565, size = 27, normalized size = 0.82

$$\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{-\frac{3x^2}{2}+2x-1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-2 + 4*x - 3*x^2]),x]
```

```
[Out] ArcTan[(-1 + x)/Sqrt[-1 + 2*x - (3*x^2)/2]]/Sqrt[2]
```

Maple [A] time = 0.043, size = 29, normalized size = 0.9

$$\frac{\sqrt{2}}{2}\arctan\left(\frac{(-4+4x)\sqrt{2}}{4}\frac{1}{\sqrt{-3x^2+4x-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-3*x^2+4*x-2)^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*arctan(1/4*(-4+4*x)*2^(1/2)/(-3*x^2+4*x-2)^(1/2))
```

Maxima [C] time = 1.46616, size = 34, normalized size = 1.03

$$\frac{1}{2}i\sqrt{2}\,\mathrm{arsinh}\left(\frac{\sqrt{2}x}{|x|}-\frac{\sqrt{2}}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x^2+4*x-2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*I*sqrt(2)*arcsinh(sqrt(2)*x/abs(x) - sqrt(2)/abs(x))
```

Fricas [B] time = 2.00073, size = 178, normalized size = 5.39

$$\frac{1}{4}\sqrt{-2}\log\left(\frac{\sqrt{-2}\sqrt{-3x^2+4x-2}+2x-2}{x}\right)-\frac{1}{4}\sqrt{-2}\log\left(-\frac{\sqrt{-2}\sqrt{-3x^2+4x-2}-2x+2}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x^2+4*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-2)*log((sqrt(-2)*sqrt(-3*x^2 + 4*x - 2) + 2*x - 2)/x) - 1/4*sqrt(
-2)*log(-(sqrt(-2)*sqrt(-3*x^2 + 4*x - 2) - 2*x + 2)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\sqrt{-3x^2+4x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x**2+4*x-2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-3*x**2 + 4*x - 2)), x)
```

Giac [C] time = 1.24401, size = 158, normalized size = 4.79

$$-\frac{2i\sqrt{3}\log\left(192i\sqrt{6}+192i\sqrt{2}+\frac{384\left(\sqrt{3}\sqrt{-3x^2+4x-2}+i\sqrt{2}\right)}{3x-2}\right)}{\sqrt{6}+\sqrt{2}}+\frac{2i\sqrt{3}\log\left(-192i\sqrt{6}+192i\sqrt{2}+\frac{384\left(\sqrt{3}\sqrt{-3x^2+4x-2}+i\sqrt{2}\right)}{3x-2}\right)}{\sqrt{6}-\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x^2+4*x-2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*I*sqrt(3)*log(192*I*sqrt(6) + 192*I*sqrt(2) + 384*(sqrt(3)*sqrt(-3*x^2 +
 4*x - 2) + I*sqrt(2))/(3*x - 2))/(sqrt(6) + sqrt(2)) + 2*I*sqrt(3)*log(-19
2*I*sqrt(6) + 192*I*sqrt(2) + 384*(sqrt(3)*sqrt(-3*x^2 + 4*x - 2) + I*sqrt(
2))/(3*x - 2))/(sqrt(6) - sqrt(2))
```

3.2436 $\int \frac{1}{x\sqrt{-2+5x+3x^2}}\,dx$

Optimal. Leaf size=36

$$-\frac{\tan^{-1}\left(\frac{4-5x}{2\sqrt{2}\sqrt{3x^2+5x-2}}\right)}{\sqrt{2}}$$

```
[Out] -(ArcTan[(4 - 5*x)/(2*Sqrt[2]*Sqrt[-2 + 5*x + 3*x^2])]/Sqrt[2])
```

Rubi [A] time = 0.0108348, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {724, 204}

$$-\frac{\tan^{-1}\left(\frac{4-5x}{2\sqrt{2}\sqrt{3x^2+5x-2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-2 + 5*x + 3*x^2]),x]

[Out] -(ArcTan[(4 - 5*x)/(2*Sqrt[2]*Sqrt[-2 + 5*x + 3*x^2])]/Sqrt[2])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{-2+5x+3x^2}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, \frac{-4+5x}{\sqrt{-2+5x+3x^2}}\right)\right)\\ &= -\frac{\tan^{-1}\left(\frac{4-5x}{2\sqrt{2}\sqrt{-2+5x+3x^2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0092225, size = 30, normalized size = 0.83

$$\frac{\tan^{-1}\left(\frac{5x-4}{2\sqrt{6x^2+10x-4}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-2 + 5*x + 3*x^2]),x]
```

```
[Out] ArcTan[(-4 + 5*x)/(2*Sqrt[-4 + 10*x + 6*x^2])]/Sqrt[2]
```

Maple [A] time = 0.048, size = 29, normalized size = 0.8

$$\frac{\sqrt{2}}{2}\arctan\left(\frac{(5x-4)\sqrt{2}}{4}\frac{1}{\sqrt{3x^2+5x-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(3*x^2+5*x-2)^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*arctan(1/4*(5*x-4)*2^(1/2)/(3*x^2+5*x-2)^(1/2))
```

Maxima [A] time = 1.51143, size = 27, normalized size = 0.75

$$\frac{1}{2}\sqrt{2}\arcsin\left(\frac{5x}{7|x|}-\frac{4}{7|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2+5*x-2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arcsin(5/7*x/abs(x) - 4/7/abs(x))
```

Fricas [A] time = 2.05587, size = 88, normalized size = 2.44

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}(5x-4)}{4\sqrt{3x^2+5x-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2+5*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(1/4*sqrt(2)*(5*x - 4)/sqrt(3*x^2 + 5*x - 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\sqrt{(x+2)(3x-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x**2+5*x-2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt((x + 2)*(3*x - 1))), x)
```

Giac [A] time = 1.11749, size = 41, normalized size = 1.14

$$\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{3}x-\sqrt{3x^2+5x-2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2+5*x-2)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(3)*x - sqrt(3*x^2 + 5*x - 2)))
```

3.2437 $\int \frac{1}{x\sqrt{-2+5x-3x^2}}\,dx$

Optimal. Leaf size=36

$$-\frac{\tan^{-1}\left(\frac{4-5x}{2\sqrt{2}\sqrt{-3x^2+5x-2}}\right)}{\sqrt{2}}$$

```
[Out] -(ArcTan[(4 - 5*x)/(2*Sqrt[2]*Sqrt[-2 + 5*x - 3*x^2])]/Sqrt[2])
```

Rubi [A] time = 0.0108753, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {724, 204}

$$-\frac{\tan^{-1}\left(\frac{4-5x}{2\sqrt{2}\sqrt{-3x^2+5x-2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-2 + 5*x - 3*x^2]),x]
```

```
[Out] -(ArcTan[(4 - 5*x)/(2*Sqrt[2]*Sqrt[-2 + 5*x - 3*x^2])]/Sqrt[2])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{-2+5x-3x^2}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, \frac{-4+5x}{\sqrt{-2+5x-3x^2}}\right)\right)\\ &= -\frac{\tan^{-1}\left(\frac{4-5x}{2\sqrt{2}\sqrt{-2+5x-3x^2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0097152, size = 30, normalized size = 0.83

$$\frac{\tan^{-1}\left(\frac{5x-4}{2\sqrt{-6x^2+10x-4}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-2 + 5*x - 3*x^2]),x]
```

```
[Out] ArcTan[(-4 + 5*x)/(2*Sqrt[-4 + 10*x - 6*x^2])]/Sqrt[2]
```

Maple [A] time = 0.042, size = 29, normalized size = 0.8

$$\frac{\sqrt{2}}{2}\arctan\left(\frac{(5x-4)\sqrt{2}}{4}\frac{1}{\sqrt{-3x^2+5x-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-3*x^2+5*x-2)^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*arctan(1/4*(5*x-4)*2^(1/2)/(-3*x^2+5*x-2)^(1/2))
```

Maxima [A] time = 1.49993, size = 27, normalized size = 0.75

$$\frac{1}{2}\sqrt{2}\arcsin\left(\frac{5x}{|x|}-\frac{4}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x^2+5*x-2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arcsin(5*x/abs(x) - 4/abs(x))
```

Fricas [A] time = 2.03531, size = 115, normalized size = 3.19

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-3x^2+5x-2}(5x-4)}{4\left(3x^2-5x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x^2+5*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(1/4*sqrt(2)*sqrt(-3*x^2 + 5*x - 2)*(5*x - 4)/(3*x^2 - 5
*x + 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\sqrt{-(x-1)(3x-2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x**2+5*x-2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(x - 1)*(3*x - 2))), x)
```

Giac [A] time = 1.13379, size = 59, normalized size = 1.64

$$-\frac{1}{3}\sqrt{6}\sqrt{3}\arctan\left(\frac{1}{12}\sqrt{6}\left(\frac{5\left(2\sqrt{3}\sqrt{-3x^2+5x-2}-1\right)}{6x-5}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x^2+5*x-2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(6)*sqrt(3)*arctan(1/12*sqrt(6)*(5*(2*sqrt(3)*sqrt(-3*x^2 + 5*x -
2) - 1)/(6*x - 5) - 1))
```

3.2438 $\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx$

Optimal. Leaf size=57

$$\frac{3\sqrt{x^2+x+1}}{4x}-\frac{\sqrt{x^2+x+1}}{2x^2}+\frac{1}{8}\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)$$

```
[Out] -Sqrt[1 + x + x^2]/(2*x^2) + (3*Sqrt[1 + x + x^2])/(4*x) + ArcTanh[(2 + x)/
(2*Sqrt[1 + x + x^2])]/8
```

Rubi [A] time = 0.0240948, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {744, 806, 724, 206}

$$\frac{3\sqrt{x^2+x+1}}{4x}-\frac{\sqrt{x^2+x+1}}{2x^2}+\frac{1}{8}\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[1 + x + x^2]),x]
```

```
[Out] -Sqrt[1 + x + x^2]/(2*x^2) + (3*Sqrt[1 + x + x^2])/(4*x) + ArcTanh[(2 + x)/
(2*Sqrt[1 + x + x^2])]/8
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx &= -\frac{\sqrt{1+x+x^2}}{2x^2}-\frac{1}{2}\int \frac{\frac{3}{2}+x}{x^2\sqrt{1+x+x^2}}\,dx\\ &= -\frac{\sqrt{1+x+x^2}}{2x^2}+\frac{3\sqrt{1+x+x^2}}{4x}-\frac{1}{8}\int \frac{1}{x\sqrt{1+x+x^2}}\,dx\\ &= -\frac{\sqrt{1+x+x^2}}{2x^2}+\frac{3\sqrt{1+x+x^2}}{4x}+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{2+x}{\sqrt{1+x+x^2}}\right)\\ &= -\frac{\sqrt{1+x+x^2}}{2x^2}+\frac{3\sqrt{1+x+x^2}}{4x}+\frac{1}{8}\tanh^{-1}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0175404, size = 43, normalized size = 0.75

$$\frac{1}{8}\left(\frac{2\sqrt{x^2+x+1}(3x-2)}{x^2}+\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[1 + x + x^2]),x]
```

```
[Out] ((2*(-2 + 3*x)*Sqrt[1 + x + x^2])/x^2 + ArcTanh[(2 + x)/(2*Sqrt[1 + x + x^2
])])/8
```

Maple [A] time = 0.042, size = 44, normalized size = 0.8

$$\frac{1}{8}\text{Artanh}\left(\frac{2+x}{2}\frac{1}{\sqrt{x^2+x+1}}\right)-\frac{1}{2x^2}\sqrt{x^2+x+1}+\frac{3}{4x}\sqrt{x^2+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^2+x+1)^(1/2),x)
```

```
[Out] 1/8*arctanh(1/2*(2+x)/(x^2+x+1)^(1/2))-1/2*(x^2+x+1)^(1/2)/x^2+3/4*(x^2+x+1
)^(1/2)/x
```

Maxima [A] time = 1.48826, size = 68, normalized size = 1.19

$$\frac{3\sqrt{x^2+x+1}}{4x}-\frac{\sqrt{x^2+x+1}}{2x^2}+\frac{1}{8}\,\text{arsinh}\left(\frac{\sqrt{3}x}{3|x|}+\frac{2\sqrt{3}}{3|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^2+x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/4*sqrt(x^2 + x + 1)/x - 1/2*sqrt(x^2 + x + 1)/x^2 + 1/8*arcsinh(1/3*sqrt(
3)*x/abs(x) + 2/3*sqrt(3)/abs(x))
```

Fricas [A] time = 2.04662, size = 169, normalized size = 2.96

$$\frac{x^2\log\left(-x+\sqrt{x^2+x+1}+1\right)-x^2\log\left(-x+\sqrt{x^2+x+1}-1\right)+6x^2+2\sqrt{x^2+x+1}(3x-2)}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^2+x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(x^2*log(-x + sqrt(x^2 + x + 1) + 1) - x^2*log(-x + sqrt(x^2 + x + 1) -
 1) + 6*x^2 + 2*sqrt(x^2 + x + 1)*(3*x - 2))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^3\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**2+x+1)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x**2 + x + 1)), x)
```

Giac [A] time = 1.10904, size = 113, normalized size = 1.98

$$\frac{\left(x-\sqrt{x^2+x+1}\right)^3+9x-9\sqrt{x^2+x+1}+8}{4\left(\left(x-\sqrt{x^2+x+1}\right)^2-1\right)^2}+\frac{1}{8}\log\left(\left|-x+\sqrt{x^2+x+1}+1\right|\right)-\frac{1}{8}\log\left(\left|-x+\sqrt{x^2+x+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^2+x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*((x - sqrt(x^2 + x + 1))^3 + 9*x - 9*sqrt(x^2 + x + 1) + 8)/((x - sqrt(
x^2 + x + 1))^2 - 1)^2 + 1/8*log(abs(-x + sqrt(x^2 + x + 1) + 1)) - 1/8*log
(abs(-x + sqrt(x^2 + x + 1) - 1))
```

3.2439 $\int \left(\frac{1}{x}-\frac{1}{x\sqrt{1+bx+cx^2}}\right) dx$

Optimal. Leaf size=23

$$\log\left(-2\sqrt{bx+cx^2+1}-bx-2\right)$$

```
[Out] Log[-2 - b*x - 2*Sqrt[1 + b*x + c*x^2]]
```

Rubi [A] time = 0.0146442, antiderivative size = 27, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {724, 206}

$$\tanh^{-1}\left(\frac{bx+2}{2\sqrt{bx+cx^2+1}}\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1) - 1/(x*Sqrt[1 + b*x + c*x^2]),x]
```

```
[Out] ArcTanh[(2 + b*x)/(2*Sqrt[1 + b*x + c*x^2])] + Log[x]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \left(\frac{1}{x}-\frac{1}{x\sqrt{1+bx+cx^2}}\right) dx &= \log(x)-\int \frac{1}{x\sqrt{1+bx+cx^2}}\,dx\\ &= \log(x)+2\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{2+bx}{\sqrt{1+bx+cx^2}}\right)\\ &= \tanh^{-1}\left(\frac{2+bx}{2\sqrt{1+bx+cx^2}}\right)+\log(x)\end{aligned}$$

Mathematica [A] time = 0.0549276, size = 27, normalized size = 1.17

$$\tanh^{-1}\left(\frac{bx+2}{2\sqrt{bx+cx^2+1}}\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1) - 1/(x*Sqrt[1 + b*x + c*x^2]),x]
```

```
[Out] ArcTanh[(2 + b*x)/(2*Sqrt[1 + b*x + c*x^2])] + Log[x]
```

Maple [A] time = 0.041, size = 24, normalized size = 1.

$$\operatorname{Artanh}\left(\frac{bx+2}{2}\frac{1}{\sqrt{cx^2+bx+1}}\right)+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x-1/x/(c*x^2+b*x+1)^(1/2),x)
```

```
[Out] arctanh(1/2*(b*x+2)/(c*x^2+b*x+1)^(1/2))+ln(x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x-1/x/(c*x^2+b*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.07543, size = 73, normalized size = 3.17

$$\log(x)-\log\left(-\frac{bx-2\sqrt{cx^2+bx+1}+2}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x-1/x/(c*x^2+b*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] log(x) - log(-(b*x - 2*sqrt(c*x^2 + b*x + 1) + 2)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{bx+cx^2+1}-1}{x\sqrt{bx+cx^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x-1/x/(c*x**2+b*x+1)**(1/2),x)
```

```
[Out] Integral((sqrt(b*x + c*x**2 + 1) - 1)/(x*sqrt(b*x + c*x**2 + 1)), x)
```

Giac [B] time = 1.14853, size = 68, normalized size = 2.96

$$\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+bx+1}+1\right|\right)-\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+bx+1}-1\right|\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x-1/x/(c*x^2+b*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] log(abs(-sqrt(c)*x + sqrt(c*x^2 + b*x + 1) + 1)) - log(abs(-sqrt(c)*x + sqr
t(c*x^2 + b*x + 1) - 1)) + log(abs(x))
```

3.2440 $\int (dx)^{5/2}\sqrt{a+bx+cx^2}\,dx$

Optimal. Leaf size=504

$$-\frac{\sqrt[4]{a}d^3\sqrt{x}\left(42a^2c^2-72ab^2c+\sqrt{a}b\sqrt{c}\left(8b^2-27ac\right)+16b^4\right)\left(\sqrt{a}+\sqrt{cx}\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{cx}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\right.\right.}{315c^{15/4}\sqrt{dx}\sqrt{a+bx+cx^2}}$$

```
[Out] (-4*(8*b^4 - 36*a*b^2*c + 21*a^2*c^2)*d^3*x*Sqrt[a + b*x + c*x^2])/(315*c^(
7/2)*Sqrt[d*x]*(Sqrt[a] + Sqrt[c]*x)) + (2*d^2*Sqrt[d*x]*(b*(8*b^2 + 3*a*c)
 + 3*c*(8*b^2 - 7*a*c)*x)*Sqrt[a + b*x + c*x^2])/(315*c^3) - (4*b*d^2*Sqrt[
d*x]*(a + b*x + c*x^2)^(3/2))/(21*c^2) + (2*d*(d*x)^(3/2)*(a + b*x + c*x^2)
^(3/2))/(9*c) + (4*a^(1/4)*(8*b^4 - 36*a*b^2*c + 21*a^2*c^2)*d^3*Sqrt[x]*(S
qrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*Ellipti
cE[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(315*
c^(15/4)*Sqrt[d*x]*Sqrt[a + b*x + c*x^2]) - (a^(1/4)*(16*b^4 - 72*a*b^2*c +
 42*a^2*c^2 + Sqrt[a]*b*Sqrt[c]*(8*b^2 - 27*a*c))*d^3*Sqrt[x]*(Sqrt[a] + Sq
rt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan
[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(315*c^(15/4)*Sq
rt[d*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.645228, antiderivative size = 504, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {742, 832, 814, 841, 839, 1197, 1103, 1195}

$$-\frac{4d^3x\left(21a^2c^2-36ab^2c+8b^4\right)\sqrt{a+bx+cx^2}}{315c^{7/2}\sqrt{dx}\left(\sqrt{a}+\sqrt{cx}\right)}-\frac{\sqrt[4]{a}d^3\sqrt{x}\left(42a^2c^2-72ab^2c+\sqrt{a}b\sqrt{c}\left(8b^2-27ac\right)+16b^4\right)\left(\sqrt{a}+\sqrt{cx}\right.}{315c^{15/4}\sqrt{dx}\sqrt{a+bx+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*Sqrt[a + b*x + c*x^2],x]
```


```
[Out] (-4*(8*b^4 - 36*a*b^2*c + 21*a^2*c^2)*d^3*x*Sqrt[a + b*x + c*x^2])/(315*c^(
7/2)*Sqrt[d*x]*(Sqrt[a] + Sqrt[c]*x)) + (2*d^2*Sqrt[d*x]*(b*(8*b^2 + 3*a*c)
 + 3*c*(8*b^2 - 7*a*c)*x)*Sqrt[a + b*x + c*x^2])/(315*c^3) - (4*b*d^2*Sqrt[
d*x]*(a + b*x + c*x^2)^(3/2))/(21*c^2) + (2*d*(d*x)^(3/2)*(a + b*x + c*x^2)
^(3/2))/(9*c) + (4*a^(1/4)*(8*b^4 - 36*a*b^2*c + 21*a^2*c^2)*d^3*Sqrt[x]*(S
qrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*Ellipti
cE[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(315*
c^(15/4)*Sqrt[d*x]*Sqrt[a + b*x + c*x^2]) - (a^(1/4)*(16*b^4 - 72*a*b^2*c +
 42*a^2*c^2 + Sqrt[a]*b*Sqrt[c]*(8*b^2 - 27*a*c))*d^3*Sqrt[x]*(Sqrt[a] + Sq
rt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan
[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(315*c^(15/4)*Sq
rt[d*x]*Sqrt[a + b*x + c*x^2])
```


Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
```

```
raticQ[a, b, c, d, e, m, p, x]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 841

```
Int[((f_) + (g_.)*(x_))/(Sqrt[(e_)*(x_)]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_
)^2]), x_Symbol] :> Dist[Sqrt[x]/Sqrt[e*x], Int[(f + g*x)/(Sqrt[x]*Sqrt[a +
 b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0]
```

Rule 839

```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Dist[2, Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
```

```
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int (dx)^{5/2}\sqrt{a+bx+cx^2}\,dx = \frac{2d(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9c} + \frac{2\int \sqrt{dx}\left(-\frac{3ad^2}{2}-3bd^2x\right)\sqrt{a+bx+cx^2}\,dx}{9c}$$

$$= -\frac{4bd^2\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{21c^2} + \frac{2d(dx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9c} + \frac{4\int \frac{\left(\frac{3}{2}abd^3+\frac{3}{4}\left(8b^2-7ac\right)d^3x\right)\sqrt{a+b}}{\sqrt{dx}}}{63c^2}$$

$$= \frac{2d^2\sqrt{dx}\left(b\left(8b^2+3ac\right)+3c\left(8b^2-7ac\right)x\right)\sqrt{a+bx+cx^2}}{315c^3} - \frac{4bd^2\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{21c^2} +$$

$$= \frac{2d^2\sqrt{dx}\left(b\left(8b^2+3ac\right)+3c\left(8b^2-7ac\right)x\right)\sqrt{a+bx+cx^2}}{315c^3} - \frac{4bd^2\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{21c^2} +$$

$$= \frac{2d^2\sqrt{dx}\left(b\left(8b^2+3ac\right)+3c\left(8b^2-7ac\right)x\right)\sqrt{a+bx+cx^2}}{315c^3} - \frac{4bd^2\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{21c^2} +$$

$$= \frac{2d^2\sqrt{dx}\left(b\left(8b^2+3ac\right)+3c\left(8b^2-7ac\right)x\right)\sqrt{a+bx+cx^2}}{315c^3} - \frac{4bd^2\sqrt{dx}\left(a+bx+cx^2\right)^{3/2}}{21c^2} +$$

$$= -\frac{4\left(8b^4-36ab^2c+21a^2c^2\right)d^3x\sqrt{a+bx+cx^2}}{315c^{7/2}\sqrt{dx}\left(\sqrt{a}+\sqrt{c}x\right)} + \frac{2d^2\sqrt{dx}\left(b\left(8b^2+3ac\right)+3c\left(8b^2-7ac\right)\right.}{315c^3}$$

Mathematica [C] time = 3.53395, size = 594, normalized size = 1.18

$$(dx)^{5/2}\left(-\frac{ix\left(21a^2c^2\sqrt{b^2-4ac}-48a^2bc^2+8b^4\sqrt{b^2-4ac}+44ab^3c-36ab^2c\sqrt{b^2-4ac}-8b^5\right)\sqrt{\frac{4a}{x\left(\sqrt{b^2-4ac}+b\right)}+2}\sqrt{\frac{-x\sqrt{b^2-4ac}+2a+bx}{bx-x\sqrt{b^2-4ac}}}\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{a}{\sqrt{b^2-4ac}}}}{\sqrt{x}}\right.\right.}{\sqrt{\frac{a}{\sqrt{b^2-4ac}+b}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((d*x)^(5/2)*((-4*(8*b^4 - 36*a*b^2*c + 21*a^2*c^2)*(a + x*(b + c*x)))/Sqrt
[x] + 2*c*Sqrt[x]*(a + x*(b + c*x))*(8*b^3 - 6*b^2*c*x + b*c*(-27*a + 5*c*x
^2) + 7*c^2*x*(2*a + 5*c*x^2)) + (I*(8*b^4 - 36*a*b^2*c + 21*a^2*c^2)*(-b +
 Sqrt[b^2 - 4*a*c])*Sqrt[2 + (4*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x*Sqrt[(2*a
 + b*x - Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticE[I*ArcS
inh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a
*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[a/(b + Sqrt[b^2 - 4*a*c])] - (I*(-8*b^5
 + 44*a*b^3*c - 48*a^2*b*c^2 + 8*b^4*Sqrt[b^2 - 4*a*c] - 36*a*b^2*c*Sqrt[b^
2 - 4*a*c] + 21*a^2*c^2*Sqrt[b^2 - 4*a*c])*Sqrt[2 + (4*a)/((b + Sqrt[b^2 -
4*a*c])*x)]*x*Sqrt[(2*a + b*x - Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*
```

```
c]*x)]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x
]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[a/(b + Sqrt[b^2
- 4*a*c])]))/(315*c^4*x^(5/2)*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.26, size = 2062, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/315*d^2*(d*x)^(1/2)*(70*x^6*c^6+80*x^5*b*c^5+84*((-b-2*c*x+(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*Ellipti
cF(((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*
((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2))*((b+2*c*x+(-4*a*c+b^2)^(
1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*a^3*c^3-117*((-b-2*c*x+(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticF
(((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((
b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2))*((b+2*c*x+(-4*a*c+b^2)^(1/
2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*a^2*b^2*c^2+27*((-b-2*c*x+(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*Elliptic
F(((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*(
(b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2))*((b+2*c*x+(-4*a*c+b^2)^(1
/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-4*a*c+b^2)^(1/2)*a^2*b*c^2+24*((-b-2*c
*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*x/(b+(-4*a*c+b^2)^(1/2
)))^(1/2)*EllipticF(((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(
1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2))*((b+2*c
*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*a*b^4*c-8*((-b-2*c*x+(
-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*x/(b+(-4*a*c+b^2)^(1/2)))^
(1/2)*EllipticF(((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
,1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2))*((b+2*c*x+(
-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-4*a*c+b^2)^(1/2)*a*b^3*c
-168*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*x/(b+(-4*
a*c+b^2)^(1/2)))^(1/2)*EllipticE(((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b
^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(
1/2))*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*a^3*c^3+3
30*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*x/(b+(-4*a*
c+b^2)^(1/2)))^(1/2)*EllipticE(((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2
)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/
2))*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*a^2*b^2*c^2
+42*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*x/(b+(-4*a
*c+b^2)^(1/2)))^(1/2)*EllipticE(((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1
/2))*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-4*a*c+b^
2)^(1/2)*a^2*b*c^2-136*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(
1/2)*(-c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(((b+2*c*x+(-4*a*c+b^2)^(
1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4
*a*c+b^2)^(1/2))^(1/2))*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)
))^(1/2)*a*b^4*c-72*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2
)*(-c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(((b+2*c*x+(-4*a*c+b^2)^(1/2
))/(b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*
c+b^2)^(1/2))^(1/2))*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^
(1/2)*(-4*a*c+b^2)^(1/2)*a*b^3*c+16*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)^(1/2))^(1/2)*(-c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(((b+2*c*x+(
-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2))*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a
```

```
*c+b^2)^(1/2)))^(1/2)*b^6+16*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1
/2))^(1/2)*(-c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(((b+2*c*x+(-4*a*c+
b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2
))/(-4*a*c+b^2)^(1/2))^(1/2))*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)
^(1/2)))^(1/2)*(-4*a*c+b^2)^(1/2)*b^5+98*x^4*a*c^5-2*x^4*b^2*c^4-16*x^3*a*b
*c^4+4*x^3*b^3*c^3+28*x^2*a^2*c^4-66*x^2*a*b^2*c^3+16*x^2*b^4*c^2-54*x*a^2*
b*c^3+16*x*a*b^3*c^2)/x/(c*x^2+b*x+a)^(1/2)/c^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + bx + a}\,(dx)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(d*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^2 + bx + a}\sqrt{dx}d^2x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(d*x)*d^2*x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{5}{2}}\sqrt{a + bx + cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d*x)**(5/2)*sqrt(a + b*x + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + bx + a}\,(dx)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(d*x)^(5/2), x)
```

3.2441 $\int (d+ex)^{3/2}\sqrt{a+bx+cx^2}\,dx$

Optimal. Leaf size=581

$$\frac{4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\left(-ce(5ae+3bd)+2b^2e^2+3c^2d^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{b^2 \cdots}}{\cdots}\right)\cdots\right)}{105c^3e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] (2*Sqrt[d + e*x]*(3*c^2*d^2 - 4*b^2*e^2 + c*e*(9*b*d - 5*a*e) + 12*c*e*(2*c
*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(105*c^2*e) + (2*e*Sqrt[d + e*x]*(a + b
*x + c*x^2)^(3/2))/(7*c) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*(3*c^2*
d^2 + 8*b^2*e^2 - c*e*(3*b*d + 29*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x +
c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)])/(105*c^3*e^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2
- 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (4*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2
- b*d*e + a*e^2)*(3*c^2*d^2 + 2*b^2*e^2 - c*e*(3*b*d + 5*a*e))*Sqrt[(c*(d +
 e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b
^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^
2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e)])/(105*c^3*e^2*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.868146, antiderivative size = 581, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {742, 814, 843, 718, 424, 419}

$$\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(ce(9bd-5ae)-4b^2e^2+12cex(2cd-be)+3c^2d^2\right)}{105c^2e}+\frac{4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ae^2-bde+c\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2],x]
```


```
[Out] (2*Sqrt[d + e*x]*(3*c^2*d^2 - 4*b^2*e^2 + c*e*(9*b*d - 5*a*e) + 12*c*e*(2*c
*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(105*c^2*e) + (2*e*Sqrt[d + e*x]*(a + b
*x + c*x^2)^(3/2))/(7*c) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*(3*c^2*
d^2 + 8*b^2*e^2 - c*e*(3*b*d + 29*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x +
c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)])/(105*c^3*e^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2
- 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (4*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2
- b*d*e + a*e^2)*(3*c^2*d^2 + 2*b^2*e^2 - c*e*(3*b*d + 5*a*e))*Sqrt[(c*(d +
 e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b
^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^
2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e)])/(105*c^3*e^2*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```


Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int (d+ex)^{3/2}\sqrt{a+bx+cx^2}\,dx = \frac{2e\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}{7c} + \frac{2\int \frac{\left(\frac{1}{2}\left(7cd^2-e(3bd+ae)\right)+2e(2cd-be)x\right)\sqrt{a+bx+cx^2}}{\sqrt{d+ex}}dx}{7c}$$

$$= \frac{2\sqrt{d+ex}\left(3c^2d^2-4b^2e^2+ce(9bd-5ae)+12ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{105c^2e} + \frac{2e\sqrt{d+ex}}{}$$

$$= \frac{2\sqrt{d+ex}\left(3c^2d^2-4b^2e^2+ce(9bd-5ae)+12ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{105c^2e} + \frac{2e\sqrt{d+ex}}{}$$

$$= \frac{2\sqrt{d+ex}\left(3c^2d^2-4b^2e^2+ce(9bd-5ae)+12ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{105c^2e} + \frac{2e\sqrt{d+ex}}{}$$

$$= \frac{2\sqrt{d+ex}\left(3c^2d^2-4b^2e^2+ce(9bd-5ae)+12ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{105c^2e} + \frac{2e\sqrt{d+ex}}{}$$

Mathematica [C] time = 13.0666, size = 5328, normalized size = 9.17

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.398, size = 6516, normalized size = 11.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d+ex)^{\frac{3}{2}}\sqrt{a+bx+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)*sqrt(a + b*x + c*x**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2442 $\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\,dx$

Optimal. Leaf size=513

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2cd-be)\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2e\sqrt{b^2-4ac}}{2cd-e\left(\sqrt{b^2-4ac}\right.}\right.}{15c^2e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(15*c*e) + (2*(d + e
*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*e) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c^2*
d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2
))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sq
rt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2
- 4*a*c])*e)])/(15*c^2*e^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)
*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4
*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*c^2*e^2*Sqrt[d + e*x]*Sq
rt[a + b*x + c*x^2])
```

Rubi [A] time = 0.49166, antiderivative size = 513, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {734, 832, 843, 718, 424, 419}

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2cd-be)\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{15c^2e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2],x]
```


```
[Out] (-2*(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(15*c*e) + (2*(d + e
*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*e) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c^2*
d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2
))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sq
rt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2
- 4*a*c])*e)])/(15*c^2*e^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)
*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4
*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*c^2*e^2*Sqrt[d + e*x]*Sq
rt[a + b*x + c*x^2])
```


Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
```

```
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\,dx = \frac{2(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5e} - \frac{\int \frac{\sqrt{d+ex}(bd-2ae+(2cd-be)x)}{\sqrt{a+bx+cx^2}}dx}{5e}$$

$$= -\frac{2(2cd-be)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15ce} + \frac{2(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5e} - \frac{2\int \frac{\frac{1}{2}(bcd^2+b^2de-8acde+ab}{\sqrt{d+e}}}{}$$

$$= -\frac{2(2cd-be)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15ce} + \frac{2(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5e} + \frac{\left((2cd-be)\left(cd^2-bde\right.\right.}{}$$

$$= -\frac{2(2cd-be)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15ce} + \frac{2(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5e} - \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}\left(c^2d^2\right.\right.}{}$$

$$= -\frac{2(2cd-be)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15ce} + \frac{2(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5e} - \frac{2\sqrt{2}\sqrt{b^2-4ac}\left(c^2d^2+\right.}{}$$

Mathematica [C] time = 10.7821, size = 697, normalized size = 1.36

$$\frac{i(d+ex)\sqrt{1-\frac{2\left(e(ae-bd)+cd^2\right)}{(d+ex)\left(\sqrt{e^2(b^2-4ac)}-be+2cd\right)}}\sqrt{\frac{4\left(e(ae-bd)+cd^2\right)}{(d+ex)\left(\sqrt{e^2(b^2-4ac)}+be-2cd\right)}+2}\left(\left(c\left(ae^2\left(3\sqrt{e^2(b^2-4ac)}+8cd\right)-cd^2\sqrt{e^2(b^2-4ac)}\right)-b^2e^2\left(\sqrt{e^2(b^2-4ac)}+2cd\right)+bce\left(d\sqrt{e^2(b^2-4\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((-4*e^2*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*(a + x*(b + c*x)))/Sqrt[d
+ e*x] + 2*c*e^2*Sqrt[d + e*x]*(a + x*(b + c*x))*(b*e + c*(d + 3*e*x)) + (I
*(d + e*x)*Sqrt[1 - (2*(c*d^2 + e*(-(b*d) + a*e)))/((2*c*d - b*e + Sqrt[(b^
2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[2 + (4*(c*d^2 + e*(-(b*d) + a*e)))/((-2*c
*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*((2*c*d - b*e + Sqrt[(b^2 -
 4*a*c)*e^2])*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*EllipticE[I*ArcSinh[(
Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2
])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*
e + Sqrt[(b^2 - 4*a*c)*e^2]))] + (b^3*e^3 - b^2*e^2*(2*c*d + Sqrt[(b^2 - 4*
a*c)*e^2]) + b*c*e*(-4*a*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2]) + c*(-(c*d^2*Sqrt
[(b^2 - 4*a*c)*e^2]) + a*e^2*(8*c*d + 3*Sqrt[(b^2 - 4*a*c)*e^2])))*Elliptic
F[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2
 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2]
)/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]))/Sqrt[(c*d^2 + e*(-(b*d) + a*e
))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/(15*c^2*e^3*Sqrt[a + x*(b + c
*x)])
```

Maple [B] time = 0.349, size = 4356, normalized size = 8.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/15*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2*(3*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*
c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+
e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2
)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/
2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*
c+b^2)^(1/2)))^(1/2))*b^3*d*e^3+12*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2
)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b
^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e
-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*
d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2
)))^(1/2))*a^2*c*e^4+4*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)
)^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^
(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/
2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-
(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*c
^3*d^4+2*a*b*c*d*e^3+2*x*b*c^2*d^2*e^2+4*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2
)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4
*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/
2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*
e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2
)^(1/2)))^(1/2))*a*b^2*e^4-4*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-
2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1
/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d
))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1
/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1
/2))*b^3*d*e^3-3*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2
)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*
(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*Ell
ipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4
*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b^2*e
^4-2*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c
*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+
(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/
2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1
/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*
a*c*d*e^3+3*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*
(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b
+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*Elliptic
F(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+
b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)
^(1/2)*b*c*d^2*e^2-12*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))
^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(
1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2
)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(
e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*
b*c*d*e^3+8*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*
(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b
+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*Elliptic
E(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+
b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b*c*d*e^3+
6*x^4*c^3*e^4+10*x^2*b*c^2*d*e^3-12*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/
```

```
2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+
b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*
e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c
*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/
2)))^(1/2))*a^2*c*e^4+12*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*
d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2))
)^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(
1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),
(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))
*a*c^2*d^2*e^2-8*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2
)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*
(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*Ell
ipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4
*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c^2*d
^2*e^2+8*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b
-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*
c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2
^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2
)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*c*d^2*e^2-3
*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(
-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*
a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(
-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+
b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*c*d^2*e^2-2^(1/2)*(
-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2
)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(
1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c
/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)
/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*d*e^3-2*2^
(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*
a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c
+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e
*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e
-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*c^2*d^3
*e-8*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c
*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+
(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/
2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1
/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c^2*d^3*e+2^(1/2)
*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b
^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)
^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)
*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*
d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*a*b*e^4+8*x^
3*b*c^2*e^4+8*x^3*c^3*d*e^3+6*x^2*a*c^2*e^4+2*x^2*b^2*c*e^4+2*x^2*c^3*d^2*e
^2+2*x*a*b*c*e^4+8*x*a*c^2*d*e^3+2*x*b^2*c*d*e^3+2*a*c^2*d^2*e^2)/(c*e*x^3+
b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)/e^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + bx + a}\sqrt{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^2 + bx + a}\sqrt{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d + ex}\sqrt{a + bx + cx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*sqrt(a + b*x + c*x**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2443 $\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=444

$$\frac{4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2e\sqrt{b^2-4ac}}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)}{3ce^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}-\cdots$$

```
[Out] (2*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(3*e) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*
(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*El
lipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt
[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c*e
^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c
*x^2]) + (4*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d +
e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e)])/(3*c*e^2*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.307537, antiderivative size = 444, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {734, 843, 718, 424, 419}

$$\frac{4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{3ce^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}-\frac{\sqrt{2}\sqrt{b^2-\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(3*e) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*
(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*El
lipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt
[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c*e
^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c
*x^2]) + (4*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d +
e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e)])/(3*c*e^2*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
```

```
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3e} - \frac{\int \frac{bd-2ae+(2cd-be)x}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx}{3e}$$

$$= \frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3e} - \frac{(2cd-be)\int \frac{\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx}{3e^2} + \frac{\left(2\left(cd^2-bde+ae^2\right)\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,d}{3e^2}$$

$$= \frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3e} - \frac{\left(\sqrt{2}\sqrt{b^2-4ac}(2cd-be)\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{b}}{2cd-be}}}{\sqrt{1-}}\right.}{3ce^2\sqrt{\frac{c(d+ex)}{2cd-be-\sqrt{b^2-4ace}}}\sqrt{a+bx+cx^2}}$$

$$= \frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3e} - \frac{\sqrt{2}\sqrt{b^2-4ac}(2cd-be)\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{3ce^2\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 8.48868, size = 936, normalized size = 2.11

$$\sqrt{d+ex}\left(4(a+x(b+cx))e^2+\frac{(d+ex)\left(\frac{4(be-2cd)\sqrt{\frac{cd^2+e(ae-bd)}{-2cd+be+\sqrt{(b^2-4ac)e^2}}}(a+x(b+cx))e^2}{(d+ex)^2}+\frac{i\sqrt{2}(2cd-be)\left(2cd-be+\sqrt{(b^2-4ac)e^2}\right)\sqrt{\frac{-2ae^2+2cdxe+\sqrt{(b^2-4ac)e^2}xe+b(d-ex)e+\cdots}{\left(2cd-be+\sqrt{(b^2-4ac)e^2}\right)(d+ex\cdots}}}{\cdots}\right)}{\cdots}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x],x]
```

```
[Out] (Sqrt[d + e*x]*(4*e^2*(a + x*(b + c*x)) + ((d + e*x)*((4*e^2*(-2*c*d + b*e)
*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*
(a + x*(b + c*x)))/(d + e*x)^2 + (I*Sqrt[2]*(2*c*d - b*e)*(2*c*d - b*e + Sq
rt[(b^2 - 4*a*c)*e^2])*Sqrt[(-2*a*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] + 2*c*d*e
*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(d - e*x))/((2*c*d - b*e + Sqrt[(b^2
 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*a*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] - 2*c
*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(-d + e*x))/((-2*c*d + b*e + Sqr
t[(b^2 - 4*a*c)*e^2])*(d + e*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2
- b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]],
 -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*
c)*e^2]))])/Sqrt[d + e*x] - (I*Sqrt[2]*(b^2*e^2 - 4*a*c*e^2 + 2*c*d*Sqrt[(b
^2 - 4*a*c)*e^2] - b*e*Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(-2*a*e^2 + d*Sqrt[(b^
2 - 4*a*c)*e^2] + 2*c*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(d - e*x))/
((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*a*e^2 + d*Sqrt
[(b^2 - 4*a*c)*e^2] - 2*c*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(-d + e
*x))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*EllipticF[I*ArcS
inh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c
)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d
 - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x]))/(c*Sqrt[(c*d^2 + e*(-(
b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])))/(6*e^3*Sqrt[a + x
*(b + c*x)])
```

Maple [B] time = 0.481, size = 1854, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -2/3*(c*x^2+b*x+a)^(1/2)*(e*x+d)^(1/2)/c*(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^
2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-
4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1
/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b
*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^
```

```
2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*a*e^3-2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b
^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(
-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(
1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+
b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b
^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*b*d*e^2+2^(1/2)*(-(e*x+d)*c/(e*(-4*a*
c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+
e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2
)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/
2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*
c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*c*d^2*e+2^(1/2)*(-(e*x+d)*c/(e*(-4
*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b
*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+
b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^
(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4
*a*c+b^2)^(1/2)))^(1/2))*a*b*e^3-2*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2
)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b
^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e
-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*
d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2
)))^(1/2))*a*c*d*e^2-2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^
(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1
/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)
*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e
*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2
*d*e^2+3*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b
-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*
c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2
^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2
)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*d^2*e-2*2^(
1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a
*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+
b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*
x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-
2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*c^2*d^3-x^3*c^2*e^3-x^2*b*c
*e^3-x^2*c^2*d*e^2-x*a*c*e^3-x*b*c*d*e^2-a*d*e^2*c)/(c*e*x^3+b*e*x^2+c*d*x^
2+a*e*x+b*d*x+a*d)/e^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{\sqrt{ex + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/sqrt(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2 + bx + a}}{\sqrt{ex + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)/sqrt(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx + cx^2}}{\sqrt{d + ex}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/sqrt(d + e*x), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2444 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=419

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2cd-be)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2e\sqrt{b^2-4ac}}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)}{ce^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}+2\sqrt{2}\cdots$$

```
[Out] (-2*Sqrt[a + b*x + c*x^2])/(e*Sqrt[d + e*x]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]
*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSi
n[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqr
t[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*Sqrt[(c*(d + e
*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[
2]*Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^
2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcS
in[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sq
rt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e^2*Sqrt[d + e*
x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.266012, antiderivative size = 419, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {732, 843, 718, 424, 419}

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2cd-be)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}+2\sqrt{2}\sqrt{b^2-4\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(d + e*x)^(3/2),x]
```


```
[Out] (-2*Sqrt[a + b*x + c*x^2])/(e*Sqrt[d + e*x]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]
*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSi
n[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqr
t[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*Sqrt[(c*(d + e
*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[
2]*Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^
2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcS
in[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sq
rt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e^2*Sqrt[d + e*
x]*Sqrt[a + b*x + c*x^2])
```


Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
```

```
|| LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^{3/2}}\,dx = -\frac{2\sqrt{a+bx+cx^2}}{e\sqrt{d+ex}} + \frac{\int \frac{b+2cx}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx}{e}$$

$$= -\frac{2\sqrt{a+bx+cx^2}}{e\sqrt{d+ex}} + \frac{(2c)\int \frac{\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx}{e^2} - \frac{(2cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx}{e^2}$$

$$= -\frac{2\sqrt{a+bx+cx^2}}{e\sqrt{d+ex}} + \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{b^2-4ac}ex^2}{2cd-be-\sqrt{b^2-4ac}e}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{b+\cdots}}{\cdots}\right.}{e^2\sqrt{\frac{c(d+ex)}{2cd-be-\sqrt{b^2-4ac}e}}\sqrt{a+bx+cx^2}}$$

$$= -\frac{2\sqrt{a+bx+cx^2}}{e\sqrt{d+ex}} + \frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4a\cdots}\right.}\right.}{e^2\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 4.69933, size = 865, normalized size = 2.06

$$\frac{4cx^2e^2+4(a+bx)e^2-2(a+x(b+cx))e^2-\frac{i\sqrt{2}\left(2cd-be+\sqrt{(b^2-4ac)e^2}\right)(d+ex)^{3/2}\sqrt{\frac{-2ae^2+2cdxe+\sqrt{(b^2-4ac)e^2}xe+b(d-ex)e+d\sqrt{(b^2-4ac)e^2}}{\left(2cd-be+\sqrt{(b^2-4ac)e^2}\right)(d+ex)}}\sqrt{\frac{2ae^2-2cd\cdots}{\cdots}}\cdots}{\sqrt{\frac{cd^2+\cdots}{-2cd+be+\cdots}}\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x)^(3/2),x]
```

```
[Out] (4*c*e^2*x^2 + 4*e^2*(a + b*x) - 2*e^2*(a + x*(b + c*x)) - (I*Sqrt[2]*(2*c*
d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x)^(3/2)*Sqrt[(-2*a*e^2 + d*Sqrt[
(b^2 - 4*a*c)*e^2] + 2*c*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(d - e*x
))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*a*e^2 + d*S
qrt[(b^2 - 4*a*c)*e^2] - 2*c*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(-d
+ e*x))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*EllipticE[I*A
rcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*
a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*
c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2
*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])] + (I*Sqrt[2]*Sqrt[(b^2 - 4*a*c)*e^2]
*(d + e*x)^(3/2)*Sqrt[(-2*a*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] + 2*c*d*e*x + e
*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(d - e*x))/((2*c*d - b*e + Sqrt[(b^2 - 4*a
*c)*e^2])*(d + e*x))]*Sqrt[(2*a*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] - 2*c*d*e*x
 + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(-d + e*x))/((-2*c*d + b*e + Sqrt[(b^2
 - 4*a*c)*e^2])*(d + e*x))]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*
e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2
*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2
]))])/Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^
2])])/(e^3*Sqrt[d + e*x]*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.427, size = 1595, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)^(3/2),x)
```

```
[Out] (c*x^2+b*x+a)^(1/2)*(e*x+d)^(1/2)*(4*2^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/
(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/
(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c*e^2*(-(e*x+d)*c/(e*(-4*a*c+b^2
)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4
*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/
2)+b*e-2*c*d))^(1/2)-2^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^
(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4
*a*c+b^2)^(1/2)))^(1/2))*b^2*e^2*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*
d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2))
)^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(
1/2)-2^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)
)^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2))
)^(1/2))*b*e^2*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2
*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*
x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(-4*a*c+b^2)^
```

```
(1/2)+2*2^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c
*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/
2)))^(1/2))*c*d*e*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-
b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2
*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(-4*a*c+b^
2)^(1/2)-4*2^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-
2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^
(1/2)))^(1/2))*a*c*e^2*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*
(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e
*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)+4*2^(
1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),
(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))
*b*c*d*e*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(
-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*
a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)-4*2^(1/2)*EllipticE
(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b
^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*c^2*d^2*(-(e*
x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1
/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2)
)/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)-2*c^2*e^2*x^2-2*b*c*e^2*x-2*a*c*e
^2)/c/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)/e^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{(ex + d)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/(e*x + d)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2 + bx + a}\sqrt{ex + d}}{e^2x^2 + 2\,dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx + cx^2}}{(d + ex)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(d + e*x)**(3/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2445 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^{5/2}} dx$

Optimal. Leaf size=497

$$\frac{4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2e\sqrt{b^2-4ac}}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)}{3e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+e}}{3e}$$

```
[Out] (-2*Sqrt[a + b*x + c*x^2])/(3*e*(d + e*x)^(3/2)) + (2*(2*c*d - b*e)*Sqrt[a
+ b*x + c*x^2])/(3*e*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x]) - (Sqrt[2]*Sqrt
[b^2 - 4*a*c]*(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
- 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e)])/(3*e^2*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqr
t[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (4*Sqrt[2]*Sqrt[b^2 - 4*a*c]*S
qrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x +
 c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqr
t[b^2 - 4*a*c])*e)])/(3*e^2*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.393871, antiderivative size = 497, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {732, 834, 843, 718, 424, 419}

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2cd-be)E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{3e^2\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}+\frac{4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+c}{b^2-4ac}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(d + e*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[a + b*x + c*x^2])/(3*e*(d + e*x)^(3/2)) + (2*(2*c*d - b*e)*Sqrt[a
+ b*x + c*x^2])/(3*e*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x]) - (Sqrt[2]*Sqrt
[b^2 - 4*a*c]*(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
- 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e)])/(3*e^2*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqr
t[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (4*Sqrt[2]*Sqrt[b^2 - 4*a*c]*S
qrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x +
 c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqr
t[b^2 - 4*a*c])*e)])/(3*e^2*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
```

```
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^{5/2}}\,dx = -\frac{2\sqrt{a+bx+cx^2}}{3e(d+ex)^{3/2}} + \frac{\int \frac{b+2cx}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx}{3e}$$

$$= -\frac{2\sqrt{a+bx+cx^2}}{3e(d+ex)^{3/2}} + \frac{2(2cd-be)\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} - \frac{2\int \frac{\frac{1}{2}c(bd-2ae)+\frac{1}{2}c(2cd-be)x}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx}{3e\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{2\sqrt{a+bx+cx^2}}{3e(d+ex)^{3/2}} + \frac{2(2cd-be)\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} + \frac{(2c)\int \frac{1}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx}{3e^2} - \frac{(c(2cd-be))\int \frac{}{\sqrt{a}}}{3e^2\left(cd^2-bde\right.}$$

$$= -\frac{2\sqrt{a+bx+cx^2}}{3e(d+ex)^{3/2}} + \frac{2(2cd-be)\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} - \frac{\left(\sqrt{2}\sqrt{b^2-4ac}(2cd-be)\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right.}{3e^2\left(cd^2-bde+ae^2\right)\sqrt{}}$$

$$= -\frac{2\sqrt{a+bx+cx^2}}{3e(d+ex)^{3/2}} + \frac{2(2cd-be)\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} - \frac{\sqrt{2}\sqrt{b^2-4ac}(2cd-be)\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}}{3e^2\left(cd^2-bde+ae^2\right)\sqrt{\frac{}{2cd}}}$$

Mathematica [C] time = 10.244, size = 1088, normalized size = 2.19

$$\sqrt{d+ex}\sqrt{a+x(b+cx)}\left(-\frac{2(be-2cd)}{3e\left(cd^2-bed+ae^2\right)(d+ex)} - \frac{2}{3e(d+ex)^2}\right) - \frac{(d+ex)^{3/2}\sqrt{a+x(b+cx)}\left(-4(be-2cd)\sqrt{\frac{cd^2}{-2cd+b}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x)^(5/2),x]
```

```
[Out] Sqrt[d + e*x]*Sqrt[a + x*(b + c*x)]*(-2/(3*e*(d + e*x)^2) - (2*(-2*c*d + b*
e))/(3*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x))) - ((d + e*x)^(3/2)*Sqrt[a + x*
(b + c*x)]*(-4*(-2*c*d + b*e)*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e
 + Sqrt[(b^2 - 4*a*c)*e^2])]*(c*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e
*x) + (a*e)/(d + e*x)))/(d + e*x)) + (I*Sqrt[2]*(-2*c*d + b*e)*(2*c*d - b*e
 + Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2*a*e^2)/(d +
e*x) - 2*c*d*(-1 + d/(d + e*x)) + b*e*(-1 + (2*d)/(d + e*x)))/(2*c*d - b*e
+ Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] + (2*a*e^2)/(d +
e*x) + 2*c*d*(-1 + d/(d + e*x)) + b*(e - (2*d*e)/(d + e*x)))/(-2*c*d + b*e
+ Sqrt[(b^2 - 4*a*c)*e^2])]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*
e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2
*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2
]))])/Sqrt[d + e*x] - (I*Sqrt[2]*(-(b^2*e^2) + 4*a*c*e^2 - 2*c*d*Sqrt[(b^2
- 4*a*c)*e^2] + b*e*Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2]
- (2*a*e^2)/(d + e*x) - 2*c*d*(-1 + d/(d + e*x)) + b*e*(-1 + (2*d)/(d + e*x
```

```
)))/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2]
+ (2*a*e^2)/(d + e*x) + 2*c*d*(-1 + d/(d + e*x)) + b*(e - (2*d*e)/(d + e*x)
))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*EllipticF[I*ArcSinh[(Sqrt[2]*S
qrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt
[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[
(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x]))/(6*e^3*(c*d^2 - b*d*e + a*e^2)*Sqrt[
(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[a
 + b*x + c*x^2]*Sqrt[((d + e*x)^2*(c*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(
d + e*x) + (a*e)/(d + e*x)))/(d + e*x)))/e^2])
```

Maple [B] time = 0.366, size = 3645, normalized size = 7.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)^(5/2),x)
```

```
[Out] -2/3*(x^3*b*c*e^4-2*x^3*c^2*d*e^3+x^2*a*c*e^4-x^2*c^2*d^2*e^2+2*x*a*b*e^4-2
^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2
),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2
))*b^2*d^2*e^2*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2
*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*
x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)-2*x^2*b*c*d*e
^3+2^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^
(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^
(1/2))*x*a*b*e^4*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b
-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*
c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)-2^(1/2)*Ell
ipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4
*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*x*b^2*d
*e^3*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a
*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+
b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)-2*2^(1/2)*EllipticE(2^(
1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^
(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*x*c^2*d^3*e*(-(e*
x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1
/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2)
)/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)+2^(1/2)*EllipticF(2^(1/2)*(-(e*x+
d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*
c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*x*a*e^4*(-4*a*c+b^2)^(1/2)*(-
(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)
^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1
/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)+2^(1/2)*EllipticE(2^(1/2)*(-(e
*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e
-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b*d*e^3*(-(e*x+d)*c/(e*(
-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d
-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*
c+b^2)^(1/2)+b*e-2*c*d))^(1/2)-2*2^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(
-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c
*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c*d^2*e^2*(-(e*x+d)*c/(e*(-4*a*c+b^2
)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4
*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/
2)+b*e-2*c*d))^(1/2)+3*2^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2
)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(
-4*a*c+b^2)^(1/2)))^(1/2))*b*c*d^3*e*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-
2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1
```

```
/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d
))^(1/2)+2^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*
c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1
/2)))^(1/2))*a*d*e^3*(-4*a*c+b^2)^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b
*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)
^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*
c*d))^(1/2)-2^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e
-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)
^(1/2)))^(1/2))*b*d^2*e^2*(-4*a*c+b^2)^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1
/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c
+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b
*e-2*c*d))^(1/2)+2^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2
)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c
+b^2)^(1/2)))^(1/2))*c*d^3*e*(-4*a*c+b^2)^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)
^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*
a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2
)+b*e-2*c*d))^(1/2)-2*2^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)
^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-
4*a*c+b^2)^(1/2)))^(1/2))*x*a*c*d*e^3*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e
-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(
1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*
d))^(1/2)+3*2^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e
-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)
^(1/2)))^(1/2))*x*b*c*d^2*e^2*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))
^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(
1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2
)+x^2*b^2*e^4-2^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b
*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^
2)^(1/2)))^(1/2))*x*b*d*e^3*(-4*a*c+b^2)^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^
(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a
*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)
+b*e-2*c*d))^(1/2)+2^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1
/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a
*c+b^2)^(1/2)))^(1/2))*x*c*d^2*e^2*(-4*a*c+b^2)^(1/2)*(-(e*x+d)*c/(e*(-4*a*
c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+
e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2
)^(1/2)+b*e-2*c*d))^(1/2)-2*x*a*c*d*e^3-x*b*c*d^2*e^2+a^2*e^4-2*2^(1/2)*Ell
ipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4
*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*c^2*d^4
*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b
^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)
^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)-a*c*d^2*e^2)/(c*x^2+b*x+a)^
(1/2)/(a*e^2-b*d*e+c*d^2)/e^3/(e*x+d)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{(ex + d)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/(e*x + d)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+bx+a}\sqrt{ex+d}}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2
*e*x + d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a+bx+cx^2}}{(d+ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)**(5/2),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(d + e*x)**(5/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2446 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=617

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2cd-be)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2e\sqrt{b^2-4ac}}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)}{15e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}+\frac{4\sqrt{a+bx}}{15}$$

```
[Out] (-2*Sqrt[a + b*x + c*x^2])/(5*e*(d + e*x)^(5/2)) + (2*(2*c*d - b*e)*Sqrt[a
+ b*x + c*x^2])/(15*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(3/2)) + (4*(c^2*d^
2 + b^2*e^2 - c*e*(b*d + 3*a*e))*Sqrt[a + b*x + c*x^2])/(15*e*(c*d^2 - b*d*
e + a*e^2)^2*Sqrt[d + e*x]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c^2*d^2 + b^2*e
^2 - c*e*(b*d + 3*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4
*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*
a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
)])/(15*e^2*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2
*c*d - b*e)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-(
(c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 -
 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c
*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*e^2*(c*d^2 - b*d*e + a*e^2)*Sqrt[d +
e*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.625272, antiderivative size = 617, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {732, 834, 843, 718, 424, 419}

$$\frac{4\sqrt{a+bx+cx^2}\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)}{15e\sqrt{d+ex}\left(ae^2-bde+cd^2\right)^2}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)E\left(\text{si}\right.}{15e^2\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)^2\sqrt{\frac{}{2cd-}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(d + e*x)^(7/2),x]
```

```
[Out] (-2*Sqrt[a + b*x + c*x^2])/(5*e*(d + e*x)^(5/2)) + (2*(2*c*d - b*e)*Sqrt[a
+ b*x + c*x^2])/(15*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(3/2)) + (4*(c^2*d^
2 + b^2*e^2 - c*e*(b*d + 3*a*e))*Sqrt[a + b*x + c*x^2])/(15*e*(c*d^2 - b*d*
e + a*e^2)^2*Sqrt[d + e*x]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c^2*d^2 + b^2*e
^2 - c*e*(b*d + 3*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4
*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*
a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
)])/(15*e^2*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2
*c*d - b*e)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-(
(c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 -
 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c
*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*e^2*(c*d^2 - b*d*e + a*e^2)*Sqrt[d +
```

```
e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^{7/2}}\,dx = -\frac{2\sqrt{a+bx+cx^2}}{5e(d+ex)^{5/2}} + \frac{\int \frac{b+2cx}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx}{5e}$$

$$= -\frac{2\sqrt{a+bx+cx^2}}{5e(d+ex)^{5/2}} + \frac{2(2cd-be)\sqrt{a+bx+cx^2}}{15e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(-bcd+2b^2e-6ace\right)-\frac{1}{2}c(2cd-be)x}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx}{15e\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{2\sqrt{a+bx+cx^2}}{5e(d+ex)^{5/2}} + \frac{2(2cd-be)\sqrt{a+bx+cx^2}}{15e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} + \frac{4\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)\sqrt{a+bx+c}}{15e\left(cd^2-bde+ae^2\right)^2\sqrt{d+ex}}$$

$$= -\frac{2\sqrt{a+bx+cx^2}}{5e(d+ex)^{5/2}} + \frac{2(2cd-be)\sqrt{a+bx+cx^2}}{15e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} + \frac{4\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)\sqrt{a+bx+c}}{15e\left(cd^2-bde+ae^2\right)^2\sqrt{d+ex}}$$

$$= -\frac{2\sqrt{a+bx+cx^2}}{5e(d+ex)^{5/2}} + \frac{2(2cd-be)\sqrt{a+bx+cx^2}}{15e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} + \frac{4\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)\sqrt{a+bx+c}}{15e\left(cd^2-bde+ae^2\right)^2\sqrt{d+ex}}$$

$$= -\frac{2\sqrt{a+bx+cx^2}}{5e(d+ex)^{5/2}} + \frac{2(2cd-be)\sqrt{a+bx+cx^2}}{15e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} + \frac{4\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)\sqrt{a+bx+c}}{15e\left(cd^2-bde+ae^2\right)^2\sqrt{d+ex}}$$

Mathematica [C] time = 12.7572, size = 3493, normalized size = 5.66

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x)^(7/2),x]
```

```
[Out] Sqrt[d + e*x]*Sqrt[a + x*(b + c*x)]*(-2/(5*e*(d + e*x)^3) - (2*(-2*c*d + b*
e))/(15*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^2) - (4*(-(c^2*d^2) + b*c*d*e -
 b^2*e^2 + 3*a*c*e^2))/(15*e*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x))) - (2*c*S
qrt[a + x*(b + c*x)]*((2*(c^2*d^2 - b*c*d*e + b^2*e^2 - 3*a*c*e^2)*(d + e*x
)^(3/2)*(c + (c*d^2)/(d + e*x)^2 - (b*d*e)/(d + e*x)^2 + (a*e^2)/(d + e*x)^
2 - (2*c*d)/(d + e*x) + (b*e)/(d + e*x)))/(c*Sqrt[((d + e*x)^2*(c*(-1 + d/(
d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)))/e^2])
 - ((c*d^2 - b*d*e + a*e^2)*(d + e*x)*Sqrt[c + (c*d^2)/(d + e*x)^2 - (b*d*e
)/(d + e*x)^2 + (a*e^2)/(d + e*x)^2 - (2*c*d)/(d + e*x) + (b*e)/(d + e*x)]*
((I*c^2*d^2*(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*Sqrt[1 - (2*(c*d^2 -
b*d*e + a*e^2))/((2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*Sqrt
[1 - (2*(c*d^2 - b*d*e + a*e^2))/((2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])
*(d + e*x))]*(EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(
2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*c*d - b*e -
Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])] - Ell
ipticF[I*ArcSinh[(Sqrt[2]*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqr
t[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*c*d - b*e - Sqrt[b^2*e^2 - 4*
a*c*e^2])/(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])]))/(Sqrt[2]*(c*d^2 - b*
d*e + a*e^2)*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4
*a*c*e^2]))]*Sqrt[c + (c*d^2 - b*d*e + a*e^2)/(d + e*x)^2 + (-2*c*d + b*e)/
(d + e*x)]) - (I*b*c*d*e*(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*Sqrt[1 -
```

```
(2*(c*d^2 - b*d*e + a*e^2))/((2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])*(d
+ e*x))]*Sqrt[1 - (2*(c*d^2 - b*d*e + a*e^2))/((2*c*d - b*e + Sqrt[b^2*e^2
- 4*a*c*e^2])*(d + e*x))]*(EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[-((c*d^2 - b*d
*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2
*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c
*e^2])] - EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*
d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*c*d - b*e - Sqrt
[b^2*e^2 - 4*a*c*e^2])/(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])]))/(Sqrt[2
]*(c*d^2 - b*d*e + a*e^2)*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqr
t[b^2*e^2 - 4*a*c*e^2]))]*Sqrt[c + (c*d^2 - b*d*e + a*e^2)/(d + e*x)^2 + (-
2*c*d + b*e)/(d + e*x)]) + (I*b^2*e^2*(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e
^2])*Sqrt[1 - (2*(c*d^2 - b*d*e + a*e^2))/((2*c*d - b*e - Sqrt[b^2*e^2 - 4*
a*c*e^2])*(d + e*x))]*Sqrt[1 - (2*(c*d^2 - b*d*e + a*e^2))/((2*c*d - b*e +
Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*(EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[-
((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[
d + e*x]], (2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*c*d - b*e + Sqrt[b^
2*e^2 - 4*a*c*e^2])] - EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[-((c*d^2 - b*d*e +
 a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*c*d
 - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2
])]))/(Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*
d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))]*Sqrt[c + (c*d^2 - b*d*e + a*e^2)/(d
+ e*x)^2 + (-2*c*d + b*e)/(d + e*x)]) - ((3*I)*a*c*e^2*(2*c*d - b*e + Sqrt[
b^2*e^2 - 4*a*c*e^2])*Sqrt[1 - (2*(c*d^2 - b*d*e + a*e^2))/((2*c*d - b*e -
Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*Sqrt[1 - (2*(c*d^2 - b*d*e + a*e^2))
/((2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*(EllipticE[I*ArcSin
h[(Sqrt[2]*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a
*c*e^2]))])/Sqrt[d + e*x]], (2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*c*
d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])] - EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[-
((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[
d + e*x]], (2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*c*d - b*e + Sqrt[b^
2*e^2 - 4*a*c*e^2])]))/(Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*Sqrt[-((c*d^2 - b*d
*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))]*Sqrt[c + (c*d^2 - b
*d*e + a*e^2)/(d + e*x)^2 + (-2*c*d + b*e)/(d + e*x)]) + (I*Sqrt[2]*c^2*d*S
qrt[1 - (2*(c*d^2 - b*d*e + a*e^2))/((2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^
2])*(d + e*x))]*Sqrt[1 - (2*(c*d^2 - b*d*e + a*e^2))/((2*c*d - b*e + Sqrt[b
^2*e^2 - 4*a*c*e^2])*(d + e*x))]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[-((c*d^2
 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x
]], (2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*c*d - b*e + Sqrt[b^2*e^2 -
 4*a*c*e^2])])/(Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2
- 4*a*c*e^2]))]*Sqrt[c + (c*d^2 - b*d*e + a*e^2)/(d + e*x)^2 + (-2*c*d + b*
e)/(d + e*x)]) - (I*b*c*e*Sqrt[1 - (2*(c*d^2 - b*d*e + a*e^2))/((2*c*d - b*
e - Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*Sqrt[1 - (2*(c*d^2 - b*d*e + a*e
^2))/((2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*EllipticF[I*Arc
Sinh[(Sqrt[2]*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 -
4*a*c*e^2]))])/Sqrt[d + e*x]], (2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2
*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])])/(Sqrt[2]*Sqrt[-((c*d^2 - b*d*e +
a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))]*Sqrt[c + (c*d^2 - b*d*e
+ a*e^2)/(d + e*x)^2 + (-2*c*d + b*e)/(d + e*x)])))/(c*Sqrt[((d + e*x)^2*(c
*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*
x)))/e^2])))/(15*e^3*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[a + b*x + c*x^2])
```

Maple [B] time = 0.424, size = 12980, normalized size = 21.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)^(7/2),x)
```

[Out] result too large to display

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{(ex + d)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/(e*x + d)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2 + bx + a}\sqrt{ex + d}}{e^4x^4 + 4\,de^3x^3 + 6\,d^2e^2x^2 + 4\,d^3ex + d^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2
*e^2*x^2 + 4*d^3*e*x + d^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx + cx^2}}{(d + ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)**(7/2),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(d + e*x)**(7/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2447 $\int (d+ex)^{3/2} \left(a+bx+cx^2\right)^{3/2} dx$

Optimal. Leaf size=816

$$\frac{2e\sqrt{d+ex}\left(cx^2+bx+a\right)^{5/2}}{11c}+\frac{2\sqrt{d+ex}\left(c^2d^2-6b^2e^2+ce(13bd-3ae)+14ce(2cd-be)x\right)\left(cx^2+bx+a\right)^{3/2}}{231c^2e}+\frac{2\sqrt{d+}}{}$$

```
[Out] (2*Sqrt[d + e*x]*(8*c^4*d^4 + 8*b^4*e^4 - c^3*d^2*e*(19*b*d - 42*a*e) - b^2
*c*e^3*(19*b*d + 21*a*e) + 3*c^2*e^2*(2*b^2*d^2 + 17*a*b*d*e - 10*a^2*e^2)
- 3*c*e*(2*c*d - b*e)*(c^2*d^2 + 8*b^2*e^2 - c*e*(b*d + 31*a*e))*x)*Sqrt[a
+ b*x + c*x^2])/(1155*c^3*e^3) + (2*Sqrt[d + e*x]*(c^2*d^2 - 6*b^2*e^2 + c*
e*(13*b*d - 3*a*e) + 14*c*e*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(3/2))/(231*
c^2*e) + (2*e*Sqrt[d + e*x]*(a + b*x + c*x^2)^(5/2))/(11*c) - (8*Sqrt[2]*Sq
rt[b^2 - 4*a*c]*(2*c*d - b*e)*(c^2*d^2 - 2*b^2*e^2 - c*e*(b*d - 9*a*e))*(c^
2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x
^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/
Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^
2 - 4*a*c])*e)])/(1155*c^4*e^4*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 -
4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 -
b*d*e + a*e^2)*(16*c^4*d^4 - 8*b^4*e^4 - 4*c^3*d^2*e*(8*b*d - 21*a*e) + b^2
*c*e^3*(13*b*d + 51*a*e) + 3*c^2*e^2*(b^2*d^2 - 28*a*b*d*e - 20*a^2*e^2))*S
qrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x +
 c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqr
t[b^2 - 4*a*c])*e)])/(1155*c^4*e^4*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 2.83596, antiderivative size = 816, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {742, 814, 843, 718, 424, 419}

$$\frac{2e\sqrt{d+ex}\left(cx^2+bx+a\right)^{5/2}}{11c}+\frac{2\sqrt{d+ex}\left(c^2d^2-6b^2e^2+ce(13bd-3ae)+14ce(2cd-be)x\right)\left(cx^2+bx+a\right)^{3/2}}{231c^2e}+\frac{2\sqrt{d+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(8*c^4*d^4 + 8*b^4*e^4 - c^3*d^2*e*(19*b*d - 42*a*e) - b^2
*c*e^3*(19*b*d + 21*a*e) + 3*c^2*e^2*(2*b^2*d^2 + 17*a*b*d*e - 10*a^2*e^2)
- 3*c*e*(2*c*d - b*e)*(c^2*d^2 + 8*b^2*e^2 - c*e*(b*d + 31*a*e))*x)*Sqrt[a
+ b*x + c*x^2])/(1155*c^3*e^3) + (2*Sqrt[d + e*x]*(c^2*d^2 - 6*b^2*e^2 + c*
e*(13*b*d - 3*a*e) + 14*c*e*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(3/2))/(231*
c^2*e) + (2*e*Sqrt[d + e*x]*(a + b*x + c*x^2)^(5/2))/(11*c) - (8*Sqrt[2]*Sq
rt[b^2 - 4*a*c]*(2*c*d - b*e)*(c^2*d^2 - 2*b^2*e^2 - c*e*(b*d - 9*a*e))*(c^
2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x
^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/
Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^
```

```
2 - 4*a*c])*e)])/(1155*c^4*e^4*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 -
4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 -
b*d*e + a*e^2)*(16*c^4*d^4 - 8*b^4*e^4 - 4*c^3*d^2*e*(8*b*d - 21*a*e) + b^2
*c*e^3*(13*b*d + 51*a*e) + 3*c^2*e^2*(b^2*d^2 - 28*a*b*d*e - 20*a^2*e^2))*S
qrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x +
 c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqr
t[b^2 - 4*a*c])*e)])/(1155*c^4*e^4*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int (d+ex)^{3/2}\left(a+bx+cx^2\right)^{3/2}\,dx=\frac{2e\sqrt{d+ex}\left(a+bx+cx^2\right)^{5/2}}{11c}+\frac{2\int\frac{\left(\frac{1}{2}\left(11cd^2-e(5bd+ae)\right)+3e(2cd-be)x\right)\left(a+bx+cx^2\right)^{3/2}}{\sqrt{d+ex}}\,dx}{11c}$$

$$=\frac{2\sqrt{d+ex}\left(c^2d^2-6b^2e^2+ce(13bd-3ae)+14ce(2cd-be)x\right)\left(a+bx+cx^2\right)^{3/2}}{231c^2e}+\cdots$$

$$=\frac{2\sqrt{d+ex}\left(8c^4d^4+8b^4e^4-c^3d^2e(19bd-42ae)-b^2ce^3(19bd+21ae)+3c^2e^2\left(2b^2\cdots\right.\right.}{11\cdots}$$

$$=\frac{2\sqrt{d+ex}\left(8c^4d^4+8b^4e^4-c^3d^2e(19bd-42ae)-b^2ce^3(19bd+21ae)+3c^2e^2\left(2b^2\cdots\right.\right.}{11\cdots}$$

$$=\frac{2\sqrt{d+ex}\left(8c^4d^4+8b^4e^4-c^3d^2e(19bd-42ae)-b^2ce^3(19bd+21ae)+3c^2e^2\left(2b^2\cdots\right.\right.}{11\cdots}$$

$$=\frac{2\sqrt{d+ex}\left(8c^4d^4+8b^4e^4-c^3d^2e(19bd-42ae)-b^2ce^3(19bd+21ae)+3c^2e^2\left(2b^2\cdots\right.\right.}{11\cdots}$$

Mathematica [C] time = 13.9613, size = 10848, normalized size = 13.29

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^(3/2)*(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.407, size = 11933, normalized size = 14.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx + a\right)^{\frac{3}{2}} (ex + d)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)*(e*x + d)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(cex^3 + (cd + be)x^2 + ad + (bd + ae)x\right)\sqrt{cx^2 + bx + a}\sqrt{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*e*x^3 + (c*d + b*e)*x^2 + a*d + (b*d + a*e)*x)*sqrt(c*x^2 + b*x
 + a)*sqrt(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d + ex)^{\frac{3}{2}} \left(a + bx + cx^2\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)*(a + b*x + c*x**2)**(3/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2448 $\int \sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=712

$$\frac{8\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(2cd-be)\left(ae^2-bde+cd^2\right)\left(-2ce(bd-3ae)-b^2e^2+2c^2d^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\right.}{315c^3e^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] (2*Sqrt[d + e*x]*(8*c^3*d^3 - 4*b^3*e^3 - 3*c^2*d*e*(5*b*d - 8*a*e) + 3*b*c
*e^2*(b*d + 3*a*e) - 6*c*e*(c^2*d^2 + 2*b^2*e^2 - c*e*(b*d + 7*a*e))*x)*Sqr
t[a + b*x + c*x^2])/(315*c^2*e^3) - (2*(2*c*d - b*e)*Sqrt[d + e*x]*(a + b*x
 + c*x^2)^(3/2))/(21*c*e) + (2*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(3/2))/(9*
e) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(16*c^4*d^4 - 8*b^4*e^4 - 4*c^3*d^2*e*(8*b*
d - 15*a*e) + b^2*c*e^3*(7*b*d + 57*a*e) + 3*c^2*e^2*(3*b^2*d^2 - 20*a*b*d*
e - 28*a^2*e^2))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]
*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/S
qrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(31
5*c^3*e^4*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a +
b*x + c*x^2]) + (8*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*(c*d^2 - b*d*e +
 a*e^2)*(2*c^2*d^2 - b^2*e^2 - 2*c*e*(b*d - 3*a*e))*Sqrt[(c*(d + e*x))/(2*c
*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)
)]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]
/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(
315*c^3*e^4*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 1.27277, antiderivative size = 712, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {734, 832, 814, 843, 718, 424, 419}

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(3c^2e^2\left(-28a^2e^2-20abde+3b^2d^2\right)+b^2ce^3(57ae+7bd)-4c^3d^2e(8bd-15ae)-\right.}{315c^3e^4\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(a + b*x + c*x^2)^(3/2),x]
```


```
[Out] (2*Sqrt[d + e*x]*(8*c^3*d^3 - 4*b^3*e^3 - 3*c^2*d*e*(5*b*d - 8*a*e) + 3*b*c
*e^2*(b*d + 3*a*e) - 6*c*e*(c^2*d^2 + 2*b^2*e^2 - c*e*(b*d + 7*a*e))*x)*Sqr
t[a + b*x + c*x^2])/(315*c^2*e^3) - (2*(2*c*d - b*e)*Sqrt[d + e*x]*(a + b*x
 + c*x^2)^(3/2))/(21*c*e) + (2*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(3/2))/(9*
e) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(16*c^4*d^4 - 8*b^4*e^4 - 4*c^3*d^2*e*(8*b*
d - 15*a*e) + b^2*c*e^3*(7*b*d + 57*a*e) + 3*c^2*e^2*(3*b^2*d^2 - 20*a*b*d*
e - 28*a^2*e^2))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]
*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/S
qrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(31
5*c^3*e^4*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a +
b*x + c*x^2]) + (8*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*(c*d^2 - b*d*e +
 a*e^2)*(2*c^2*d^2 - b^2*e^2 - 2*c*e*(b*d - 3*a*e))*Sqrt[(c*(d + e*x))/(2*c
```

```
*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)
)]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]
/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(
315*c^3*e^4*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}dx = \frac{2(d+ex)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9e} - \frac{\int \sqrt{d+ex}(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}\,d}{3e}$$

$$= -\frac{2(2cd-be)\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}{21ce} + \frac{2(d+ex)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9e} - \frac{2\int \frac{\left(\frac{1}{2}\left(bcd\right.\right.}{}}{}$$

$$= \frac{2\sqrt{d+ex}\left(8c^3d^3-4b^3e^3-3c^2de(5bd-8ae)+3bce^2(bd+3ae)-6ce\left(c^2d^2+2b^2e^2\right.\right.}{315c^2e^3}$$

$$= \frac{2\sqrt{d+ex}\left(8c^3d^3-4b^3e^3-3c^2de(5bd-8ae)+3bce^2(bd+3ae)-6ce\left(c^2d^2+2b^2e^2\right.\right.}{315c^2e^3}$$

$$= \frac{2\sqrt{d+ex}\left(8c^3d^3-4b^3e^3-3c^2de(5bd-8ae)+3bce^2(bd+3ae)-6ce\left(c^2d^2+2b^2e^2\right.\right.}{315c^2e^3}$$

$$= \frac{2\sqrt{d+ex}\left(8c^3d^3-4b^3e^3-3c^2de(5bd-8ae)+3bce^2(bd+3ae)-6ce\left(c^2d^2+2b^2e^2\right.\right.}{315c^2e^3}$$

Mathematica [C] time = 13.6839, size = 7541, normalized size = 10.59

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d + e*x]*(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.332, size = 9177, normalized size = 12.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (cx^2 + bx + a)^{\frac{3}{2}} \sqrt{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)*sqrt(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((cx^2 + bx + a)^{\frac{3}{2}} \sqrt{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^(3/2)*sqrt(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d + ex}\left(a + bx + cx^2\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*(a + b*x + c*x**2)**(3/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2449 $\int \frac{(a+bx+cx^2)^{3/2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=579

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\left(-4ce(4bd-5ae)-b^2e^2+16c^2d^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\sin^{-1}\left(\right.\right.}{35c^2e^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] (2*Sqrt[d + e*x]*(8*c^2*d^2 + b^2*e^2 - c*e*(11*b*d - 10*a*e) - 3*c*e*(2*c*
d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(35*c*e^3) + (2*Sqrt[d + e*x]*(a + b*x +
 c*x^2)^(3/2))/(7*e) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*(4*c^2*d^
2 - b^2*e^2 - 4*c*e*(b*d - 2*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2
))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sq
rt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2
- 4*a*c])*e)])/(35*c^2*e^4*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*
e + a*e^2)*(16*c^2*d^2 - b^2*e^2 - 4*c*e*(4*b*d - 5*a*e))*Sqrt[(c*(d + e*x)
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4
*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e)])/(35*c^2*e^4*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.672329, antiderivative size = 579, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {734, 814, 843, 718, 424, 419}

$$\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(-ce(11bd-10ae)+b^2e^2-3cex(2cd-be)+8c^2d^2\right)}{35ce^3}+\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ae^2-bde\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/Sqrt[d + e*x],x]
```


```
[Out] (2*Sqrt[d + e*x]*(8*c^2*d^2 + b^2*e^2 - c*e*(11*b*d - 10*a*e) - 3*c*e*(2*c*
d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(35*c*e^3) + (2*Sqrt[d + e*x]*(a + b*x +
 c*x^2)^(3/2))/(7*e) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*(4*c^2*d^
2 - b^2*e^2 - 4*c*e*(b*d - 2*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2
))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sq
rt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2
- 4*a*c])*e)])/(35*c^2*e^4*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*
e + a*e^2)*(16*c^2*d^2 - b^2*e^2 - 4*c*e*(4*b*d - 5*a*e))*Sqrt[(c*(d + e*x)
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4
*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e)])/(35*c^2*e^4*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}{7e} - \frac{3\int \frac{(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{\sqrt{d+ex}}dx}{7e}$$

$$= \frac{2\sqrt{d+ex}\left(8c^2d^2+b^2e^2-ce(11bd-10ae)-3ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{35ce^3} + \frac{2\sqrt{d+ex}\,(a+}{7e}$$

$$= \frac{2\sqrt{d+ex}\left(8c^2d^2+b^2e^2-ce(11bd-10ae)-3ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{35ce^3} + \frac{2\sqrt{d+ex}\,(a+}{7e}$$

$$= \frac{2\sqrt{d+ex}\left(8c^2d^2+b^2e^2-ce(11bd-10ae)-3ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{35ce^3} + \frac{2\sqrt{d+ex}\,(a+}{7e}$$

$$= \frac{2\sqrt{d+ex}\left(8c^2d^2+b^2e^2-ce(11bd-10ae)-3ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{35ce^3} + \frac{2\sqrt{d+ex}\,(a+}{7e}$$

Mathematica [C] time = 13.1402, size = 5338, normalized size = 9.22

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/Sqrt[d + e*x],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.314, size = 6516, normalized size = 11.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{3}{2}}}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^(1/2),x, algorithm="maxima")

[Out] integrate((c*x^2 + b*x + a)^(3/2)/sqrt(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+bx+a\right)^{\frac{3}{2}}}{\sqrt{ex+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^(1/2),x, algorithm="fricas")

[Out] integral((c*x^2 + b*x + a)^(3/2)/sqrt(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx+cx^2\right)^{\frac{3}{2}}}{\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)**(1/2),x)

[Out] Integral((a + b*x + c*x**2)**(3/2)/sqrt(d + e*x), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^(1/2),x, algorithm="giac")

[Out] Timed out
```

3.2450 $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=515

$$-\frac{16\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2cd-be)\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2e}{2cd-e(}\right.}{5ce^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*Sqrt[d + e*x]*(8*c*d - 7*b*e - 6*c*e*x)*Sqrt[a + b*x + c*x^2])/(5*e^3)
- (2*(a + b*x + c*x^2)^(3/2))/(e*Sqrt[d + e*x]) + (Sqrt[2]*Sqrt[b^2 - 4*a*c
]*(16*c^2*d^2 + b^2*e^2 - 4*c*e*(4*b*d - 3*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a
 + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e)])/(5*c*e^4*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (16*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(
2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[Ar
cSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*
Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(5*c*e^4*Sqrt[d
+ e*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.521268, antiderivative size = 515, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {732, 814, 843, 718, 424, 419}

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(-4ce(4bd-3ae)+b^2e^2+16c^2d^2\right)E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{5ce^4\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x)^(3/2),x]
```


```
[Out] (-2*Sqrt[d + e*x]*(8*c*d - 7*b*e - 6*c*e*x)*Sqrt[a + b*x + c*x^2])/(5*e^3)
- (2*(a + b*x + c*x^2)^(3/2))/(e*Sqrt[d + e*x]) + (Sqrt[2]*Sqrt[b^2 - 4*a*c
]*(16*c^2*d^2 + b^2*e^2 - 4*c*e*(4*b*d - 3*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a
 + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e)])/(5*c*e^4*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (16*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(
2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[Ar
cSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*
Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(5*c*e^4*Sqrt[d
+ e*x]*Sqrt[a + b*x + c*x^2])
```


Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = -\frac{2\left(a+bx+cx^2\right)^{3/2}}{e\sqrt{d+ex}} + \frac{3\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{\sqrt{d+ex}}dx}{e}$$

$$= -\frac{2\sqrt{d+ex}(8cd-7be-6cex)\sqrt{a+bx+cx^2}}{5e^3} - \frac{2\left(a+bx+cx^2\right)^{3/2}}{e\sqrt{d+ex}} - \frac{2\int \frac{\frac{1}{2}c\left(7b^2de+4acde-8b\left(cd^2 \cdots\right.\right.}{\sqrt{\cdots}}}{\cdots}$$

$$= -\frac{2\sqrt{d+ex}(8cd-7be-6cex)\sqrt{a+bx+cx^2}}{5e^3} - \frac{2\left(a+bx+cx^2\right)^{3/2}}{e\sqrt{d+ex}} - \frac{\left(8(2cd-be)\left(cd^2-bde \cdots\right.\right.}{\cdots}$$

$$= -\frac{2\sqrt{d+ex}(8cd-7be-6cex)\sqrt{a+bx+cx^2}}{5e^3} - \frac{2\left(a+bx+cx^2\right)^{3/2}}{e\sqrt{d+ex}} + \frac{\left(\sqrt{2}\sqrt{b^2-4ac}\left(16c^2d^2 \cdots\right.\right.}{\cdots}$$

$$= -\frac{2\sqrt{d+ex}(8cd-7be-6cex)\sqrt{a+bx+cx^2}}{5e^3} - \frac{2\left(a+bx+cx^2\right)^{3/2}}{e\sqrt{d+ex}} + \frac{\sqrt{2}\sqrt{b^2-4ac}\left(16c^2d^2 + \cdots\right.}{\cdots}$$

Mathematica [C] time = 10.8256, size = 732, normalized size = 1.42

$$-\frac{i(d+ex)\sqrt{1-\frac{2\left(e(ae-bd)+cd^2\right)}{(d+ex)\left(\sqrt{e^2\left(b^2-4ac\right)}-be+2cd\right)}}\sqrt{\frac{4\left(e(ae-bd)+cd^2\right)}{(d+ex)\left(\sqrt{e^2\left(b^2-4ac\right)}+be-2cd\right)}+2}\left(\left(\sqrt{e^2\left(b^2-4ac\right)}-be+2cd\right)\left(4ce(3ae-4bd)+b^2e^2+16c^2d^2\right)E\left(i\sinh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{cd^2-bed+a \cdots}{-2cd+be+\sqrt{\left(b^2 \cdots\right.}}}}{\sqrt{d+ex}}\right.\right.\right.\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x)^(3/2),x]
```

```
[Out] ((4*e^2*(16*c^2*d^2 + b^2*e^2 + 4*c*e*(-4*b*d + 3*a*e))*(a + x*(b + c*x)))/
(c*Sqrt[d + e*x]) + (4*e^2*(a + x*(b + c*x))*(e*(7*b*d - 5*a*e + 2*b*e*x) +
 c*(-8*d^2 - 2*d*e*x + e^2*x^2)))/Sqrt[d + e*x] - (I*(d + e*x)*Sqrt[1 - (2*
(c*d^2 + e*(-(b*d) + a*e)))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e
*x))]*Sqrt[2 + (4*(c*d^2 + e*(-(b*d) + a*e)))/((-2*c*d + b*e + Sqrt[(b^2 -
4*a*c)*e^2])*(d + e*x))]*((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(16*c^2*d
^2 + b^2*e^2 + 4*c*e*(-4*b*d + 3*a*e))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c
*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e
*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 -
 4*a*c)*e^2]))] - (-(b^3*e^3) + b^2*e^2*(2*c*d + Sqrt[(b^2 - 4*a*c)*e^2]) +
 4*b*(a*c*e^3 - 4*c*d*e*Sqrt[(b^2 - 4*a*c)*e^2]) + 4*c*(4*c*d^2*Sqrt[(b^2 -
 4*a*c)*e^2] + a*e^2*(-2*c*d + 3*Sqrt[(b^2 - 4*a*c)*e^2])))*EllipticF[I*Arc
Sinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*
c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*
d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]))/(c*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(
-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]))/(10*e^5*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.331, size = 4364, normalized size = 8.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d)^(3/2),x)
```

```
[Out] 2/5*(c*x^2+b*x+a)^(1/2)*(e*x+d)^(1/2)*(3*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2
)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4
*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/
2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*
e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2
)^(1/2)))^(1/2))*b^3*d*e^3+12*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e
-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(
1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*
d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(
1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(
1/2))*a^2*c*e^4-16*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1
/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2
)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*E
llipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(
-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*c^3*d
^4+7*a*b*c*d*e^3-8*x*b*c^2*d^2*e^2-2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2
)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b
^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e
-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*
d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2
)))^(1/2))*a*b^2*e^4+2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^
(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1
/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)
*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e
*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b^3
*d*e^3-3*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b
-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*
c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2
^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2
)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b^2*e^4+8*2^(
1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a
*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+
b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*
x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-
2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*a*c*d*e^
3-12*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c
*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+
(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/
2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1
/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*
b*c*d^2*e^2-12*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*
(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e
*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*Ellip
ticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a
*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b*c*d*e
^3+28*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*
c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x
+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1
/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(
1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b*c*d*e^3+x^4*c^
3*e^4+5*x^2*b*c^2*d*e^3-12*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*
```

```
c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2
)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))
^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2
),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2
))*a^2*c*e^4+12*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)
*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(
e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*Elli
pticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*
a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c^2*d^
2*e^2-28*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b
-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*
c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2
^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2
)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c^2*d^2*e^2-1
7*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+
(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4
*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*
(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)
+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*c*d^2*e^2-3*2^(1/2
)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+
b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2
)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d
)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c
*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*c*d^2*e^2+4*2^(1/2)*(-(e*x
+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/
2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))
/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(
-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c
*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*d*e^3+8*2^(1/2)
*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b
^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)
^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)
*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*
d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*c^2*d^3*e+32
*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(
-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*
a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(
-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+
b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c^2*d^3*e-4*2^(1/2)*(
-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2
)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(
1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c
/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)
/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*a*b*e^4+3*x^3*
b*c^2*e^4-2*x^3*c^3*d*e^3-4*x^2*a*c^2*e^4+2*x^2*b^2*c*e^4-8*x^2*c^3*d^2*e^2
-5*a^2*c*e^4-3*x*a*b*c*e^4-2*x*a*c^2*d*e^3+7*x*b^2*c*d*e^3-8*a*c^2*d^2*e^2)
/c/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)/e^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)/(e*x + d)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(cx^2+bx+a\right)^{\frac{3}{2}}\sqrt{ex+d}}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^(3/2)*sqrt(e*x + d)/(e^2*x^2 + 2*d*e*x + d^2), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx+cx^2\right)^{\frac{3}{2}}}{\left(d+ex\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)**(3/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)/(d + e*x)**(3/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2451 $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=499

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(-4ce(4bd-ae)+3b^2e^2+16c^2d^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\ldots\right.}{3ce^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] (2*(8*c*d - 3*b*e + 2*c*e*x)*Sqrt[a + b*x + c*x^2])/(3*e^3*Sqrt[d + e*x]) -
 (2*(a + b*x + c*x^2)^(3/2))/(3*e*(d + e*x)^(3/2)) - (8*Sqrt[2]*Sqrt[b^2 -
4*a*c]*(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*
c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c
]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])
/(3*e^4*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*
x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(16*c^2*d^2 + 3*b^2*e^2 - 4*c*e*
(4*b*d - a*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt
[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c*e^4*Sqrt[d + e*x]*Sqrt[a + b*x +
c*x^2])
```

Rubi [A] time = 0.452133, antiderivative size = 499, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {732, 812, 843, 718, 424, 419}

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(-4ce(4bd-ae)+3b^2e^2+16c^2d^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-\ldots}}{2cd-\left(b+\sqrt{\ldots}\right.}\right.}{3ce^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x)^(5/2),x]
```

```
[Out] (2*(8*c*d - 3*b*e + 2*c*e*x)*Sqrt[a + b*x + c*x^2])/(3*e^3*Sqrt[d + e*x]) -
 (2*(a + b*x + c*x^2)^(3/2))/(3*e*(d + e*x)^(3/2)) - (8*Sqrt[2]*Sqrt[b^2 -
4*a*c]*(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*
c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c
]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])
/(3*e^4*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*
x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(16*c^2*d^2 + 3*b^2*e^2 - 4*c*e*
(4*b*d - a*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt
[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c*e^4*Sqrt[d + e*x]*Sqrt[a + b*x +
c*x^2])
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)^{5/2}}dx = -\frac{2(a+bx+cx^2)^{3/2}}{3e(d+ex)^{3/2}} + \frac{\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{(d+ex)^{3/2}}dx}{e}$$

$$= \frac{2(8cd-3be+2cex)\sqrt{a+bx+cx^2}}{3e^3\sqrt{d+ex}} - \frac{2(a+bx+cx^2)^{3/2}}{3e(d+ex)^{3/2}} - \frac{2\int \frac{\frac{1}{2}(8bcd-3b^2e-4ace)+4c(2cd-be)x}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}dx}{3e^3}$$

$$= \frac{2(8cd-3be+2cex)\sqrt{a+bx+cx^2}}{3e^3\sqrt{d+ex}} - \frac{2(a+bx+cx^2)^{3/2}}{3e(d+ex)^{3/2}} - \frac{(8c(2cd-be))\int \frac{\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}dx}{3e^4} + \cdots$$

$$= \frac{2(8cd-3be+2cex)\sqrt{a+bx+cx^2}}{3e^3\sqrt{d+ex}} - \frac{2(a+bx+cx^2)^{3/2}}{3e(d+ex)^{3/2}} - \frac{\left(8\sqrt{2}\sqrt{b^2-4ac}(2cd-be)\sqrt{d+e\cdots}\right.}{3\cdots}$$

$$= \frac{2(8cd-3be+2cex)\sqrt{a+bx+cx^2}}{3e^3\sqrt{d+ex}} - \frac{2(a+bx+cx^2)^{3/2}}{3e(d+ex)^{3/2}} - \frac{8\sqrt{2}\sqrt{b^2-4ac}(2cd-be)\sqrt{d+ex\cdots}}{3e^4\cdots}$$

Mathematica [C] time = 8.97314, size = 978, normalized size = 1.96

$$\sqrt{d+ex}\left(\frac{2(a+x(b+cx))\left(c\left(8d^2+10exd+e^2x^2\right)-e(3bd+ae+4bex)\right)}{e^3(d+ex)^2} - \frac{(d+ex)\left(-\frac{16(be-2cd)\sqrt{\frac{cd^2+e(ae-bd)}{-2cd+be+\sqrt{(b^2-4ac)e^2}}}(a+x(b+cx))e^2}{(d+ex)^2} - \frac{4i\sqrt{2}(2cd-be)\left(2cd-be+\sqrt{(b^2-4ac)e^2}\right)\cdots}{\cdots}\right)}{\cdots}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x)^(5/2),x]
```

```
[Out] (Sqrt[d + e*x]*((2*(a + x*(b + c*x))*(-(e*(3*b*d + a*e + 4*b*e*x)) + c*(8*d
^2 + 10*d*e*x + e^2*x^2)))/(e^3*(d + e*x)^2) - ((d + e*x)*((-16*e^2*(-2*c*d
 + b*e)*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*
e^2])]*(a + x*(b + c*x)))/(d + e*x)^2 - ((4*I)*Sqrt[2]*(2*c*d - b*e)*(2*c*d
 - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(-2*a*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2
] + 2*c*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(d - e*x))/((2*c*d - b*e
+ Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*a*e^2 + d*Sqrt[(b^2 - 4*a*c)
*e^2] - 2*c*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(-d + e*x))/((-2*c*d
+ b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*S
qrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt
[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[
```

```
(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] + (I*Sqrt[2]*(b^2*e^2 - 4*a*c*e^2 + 8*
c*d*Sqrt[(b^2 - 4*a*c)*e^2] - 4*b*e*Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(-2*a*e^2
 + d*Sqrt[(b^2 - 4*a*c)*e^2] + 2*c*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*
e*(d - e*x))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*a
*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] - 2*c*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x
+ b*e*(-d + e*x))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Ell
ipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt
[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)
*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x]))/(e^5*Sqrt
[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])))/(3
*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.395, size = 5874, normalized size = 11.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}{(ex + d)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)/(e*x + d)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(cx^2 + bx + a\right)^{\frac{3}{2}}\sqrt{ex + d}}{e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^(3/2)*sqrt(e*x + d)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d
^2*e*x + d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx+cx^2)^{\frac{3}{2}}}{(d+ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)**(5/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)/(d + e*x)**(5/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2452 $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=578

$$-\frac{16\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(2cd-be)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2e\sqrt{b^2-4ac}}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)}{5e^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}-\frac{2\sqrt{a+}\cdots}{\cdots}$$

```
[Out] (-2*(8*c^2*d^3 + a*b*e^3 - c*d*e*(7*b*d - 4*a*e) + e*(10*c^2*d^2 + b^2*e^2
- 2*c*e*(5*b*d - 3*a*e))*x)*Sqrt[a + b*x + c*x^2])/(5*e^3*(c*d^2 - b*d*e +
a*e^2)*(d + e*x)^(3/2)) - (2*(a + b*x + c*x^2)^(3/2))/(5*e*(d + e*x)^(5/2))
 + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(16*c^2*d^2 + b^2*e^2 - 4*c*e*(4*b*d - 3*a*e)
)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcS
in[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sq
rt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(5*e^4*(c*d^2 - b*
d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a
 + b*x + c*x^2]) - (16*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*Sqrt[(c*(d +
 e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b
^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^
2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e)])/(5*e^4*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.518893, antiderivative size = 578, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {732, 810, 843, 718, 424, 419}

$$-\frac{2\sqrt{a+bx+cx^2}\left(ex\left(-2ce(5bd-3ae)+b^2e^2+10c^2d^2\right)-cde(7bd-4ae)+abe^3+8c^2d^3\right)}{5e^3(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a\cdots}{\cdots}}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x)^(7/2),x]
```


```
[Out] (-2*(8*c^2*d^3 + a*b*e^3 - c*d*e*(7*b*d - 4*a*e) + e*(10*c^2*d^2 + b^2*e^2
- 2*c*e*(5*b*d - 3*a*e))*x)*Sqrt[a + b*x + c*x^2])/(5*e^3*(c*d^2 - b*d*e +
a*e^2)*(d + e*x)^(3/2)) - (2*(a + b*x + c*x^2)^(3/2))/(5*e*(d + e*x)^(5/2))
 + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(16*c^2*d^2 + b^2*e^2 - 4*c*e*(4*b*d - 3*a*e)
)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcS
in[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sq
rt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(5*e^4*(c*d^2 - b*
d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a
 + b*x + c*x^2]) - (16*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*Sqrt[(c*(d +
 e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b
^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^
2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e)])/(5*e^4*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx = -\frac{2\left(a+bx+cx^2\right)^{3/2}}{5e(d+ex)^{5/2}} + \frac{3\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{(d+ex)^{5/2}}\,dx}{5e}$$

$$= -\frac{2\left(8c^2d^3+abe^3-cde(7bd-4ae)+e\left(10c^2d^2+b^2e^2-2ce(5bd-3ae)\right)x\right)\sqrt{a+bx+cx^2}}{5e^3\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} - \frac{2(}{}$$

$$= -\frac{2\left(8c^2d^3+abe^3-cde(7bd-4ae)+e\left(10c^2d^2+b^2e^2-2ce(5bd-3ae)\right)x\right)\sqrt{a+bx+cx^2}}{5e^3\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} - \frac{2(}{}$$

$$= -\frac{2\left(8c^2d^3+abe^3-cde(7bd-4ae)+e\left(10c^2d^2+b^2e^2-2ce(5bd-3ae)\right)x\right)\sqrt{a+bx+cx^2}}{5e^3\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} - \frac{2(}{}$$

$$= -\frac{2\left(8c^2d^3+abe^3-cde(7bd-4ae)+e\left(10c^2d^2+b^2e^2-2ce(5bd-3ae)\right)x\right)\sqrt{a+bx+cx^2}}{5e^3\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} - \frac{2(}{}$$

Mathematica [C] time = 12.7933, size = 3506, normalized size = 6.07

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x)^(7/2),x]
```

```
[Out] (Sqrt[d + e*x]*(a + x*(b + c*x))^(3/2)*((-2*(c*d^2 - b*d*e + a*e^2))/(5*e^3
*(d + e*x)^3) + (4*(2*c*d - b*e))/(5*e^3*(d + e*x)^2) - (2*(11*c^2*d^2 - 11
*b*c*d*e + b^2*e^2 + 7*a*c*e^2))/(5*e^3*(c*d^2 - b*d*e + a*e^2)*(d + e*x)))
)/(a + b*x + c*x^2) - (2*c*(a + x*(b + c*x))^(3/2)*(((-16*c^2*d^2 + 16*b*c*
d*e - b^2*e^2 - 12*a*c*e^2)*(d + e*x)^(3/2)*(c + (c*d^2)/(d + e*x)^2 - (b*d
*e)/(d + e*x)^2 + (a*e^2)/(d + e*x)^2 - (2*c*d)/(d + e*x) + (b*e)/(d + e*x)
))/(c*Sqrt[((d + e*x)^2*(c*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) +
 (a*e)/(d + e*x)))/(d + e*x)))/e^2]) + ((c*d^2 - b*d*e + a*e^2)*(d + e*x)*S
qrt[c + (c*d^2)/(d + e*x)^2 - (b*d*e)/(d + e*x)^2 + (a*e^2)/(d + e*x)^2 - (
2*c*d)/(d + e*x) + (b*e)/(d + e*x)]*(((4*I)*Sqrt[2]*c^2*d^2*(2*c*d - b*e +
Sqrt[b^2*e^2 - 4*a*c*e^2])*Sqrt[1 - (2*(c*d^2 - b*d*e + a*e^2))/((2*c*d - b
*e - Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*Sqrt[1 - (2*(c*d^2 - b*d*e + a*
e^2))/((2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*(EllipticE[I*A
rcSinh[(Sqrt[2]*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2
- 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/
(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])] - EllipticF[I*ArcSinh[(Sqrt[2]*S
qrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/
Sqrt[d + e*x]], (2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*c*d - b*e + Sq
rt[b^2*e^2 - 4*a*c*e^2])]))/((c*d^2 - b*d*e + a*e^2)*Sqrt[-((c*d^2 - b*d*e
+ a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])*Sqrt[c + (c*d^2 - b*d*
e + a*e^2)/(d + e*x)^2 + (-2*c*d + b*e)/(d + e*x)]) - ((4*I)*Sqrt[2]*b*c*d*
e*(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*Sqrt[1 - (2*(c*d^2 - b*d*e + a*
e^2))/((2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*Sqrt[1 - (2*(c
*d^2 - b*d*e + a*e^2))/((2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x)
)]*(EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*
```

```
e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*c*d - b*e - Sqrt[b^2*e
^2 - 4*a*c*e^2])/(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])] - EllipticF[I*A
rcSinh[(Sqrt[2]*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2
- 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/
(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])]))/((c*d^2 - b*d*e + a*e^2)*Sqrt[
-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))]*Sqrt[
c + (c*d^2 - b*d*e + a*e^2)/(d + e*x)^2 + (-2*c*d + b*e)/(d + e*x)]) + ((I/
2)*b^2*e^2*(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*Sqrt[1 - (2*(c*d^2 - b
*d*e + a*e^2))/((2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*Sqrt[
1 - (2*(c*d^2 - b*d*e + a*e^2))/((2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*
(d + e*x))]*(EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2
*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*c*d - b*e - S
qrt[b^2*e^2 - 4*a*c*e^2])/(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])] - Elli
pticF[I*ArcSinh[(Sqrt[2]*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt
[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*c*d - b*e - Sqrt[b^2*e^2 - 4*a
*c*e^2])/(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])]))/(Sqrt[2]*(c*d^2 - b*d
*e + a*e^2)*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*
a*c*e^2]))]*Sqrt[c + (c*d^2 - b*d*e + a*e^2)/(d + e*x)^2 + (-2*c*d + b*e)/(
d + e*x)]) + ((3*I)*Sqrt[2]*a*c*e^2*(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2
])*Sqrt[1 - (2*(c*d^2 - b*d*e + a*e^2))/((2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*
c*e^2])*(d + e*x))]*Sqrt[1 - (2*(c*d^2 - b*d*e + a*e^2))/((2*c*d - b*e + Sq
rt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*(EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[-((
c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d
+ e*x]], (2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*c*d - b*e + Sqrt[b^2*
e^2 - 4*a*c*e^2])] - EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[-((c*d^2 - b*d*e + a
*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*c*d -
 b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])
]))/((c*d^2 - b*d*e + a*e^2)*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e -
Sqrt[b^2*e^2 - 4*a*c*e^2]))]*Sqrt[c + (c*d^2 - b*d*e + a*e^2)/(d + e*x)^2 +
 (-2*c*d + b*e)/(d + e*x)]) + ((8*I)*Sqrt[2]*c^2*d*Sqrt[1 - (2*(c*d^2 - b*d
*e + a*e^2))/((2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*Sqrt[1
- (2*(c*d^2 - b*d*e + a*e^2))/((2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*(d
 + e*x))]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*
d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*c*d - b*e - Sqrt
[b^2*e^2 - 4*a*c*e^2])/(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])])/(Sqrt[-(
(c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))]*Sqrt[c
+ (c*d^2 - b*d*e + a*e^2)/(d + e*x)^2 + (-2*c*d + b*e)/(d + e*x)]) - ((4*I)
*Sqrt[2]*b*c*e*Sqrt[1 - (2*(c*d^2 - b*d*e + a*e^2))/((2*c*d - b*e - Sqrt[b^
2*e^2 - 4*a*c*e^2])*(d + e*x))]*Sqrt[1 - (2*(c*d^2 - b*d*e + a*e^2))/((2*c*
d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*EllipticF[I*ArcSinh[(Sqrt[
2]*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])
)])/Sqrt[d + e*x]], (2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*c*d - b*e
+ Sqrt[b^2*e^2 - 4*a*c*e^2])])/(Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e
 - Sqrt[b^2*e^2 - 4*a*c*e^2]))]*Sqrt[c + (c*d^2 - b*d*e + a*e^2)/(d + e*x)^
2 + (-2*c*d + b*e)/(d + e*x)])))/(c*Sqrt[((d + e*x)^2*(c*(-1 + d/(d + e*x))
^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)))/e^2])))/(5*e^5
*(c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2)^(3/2))
```

Maple [B] time = 0.376, size = 12946, normalized size = 22.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d)^(7/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{3}{2}}}{\left(ex+d\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)/(e*x + d)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+bx+a\right)^{\frac{3}{2}}\sqrt{ex+d}}{e^4x^4+4\,de^3x^3+6\,d^2e^2x^2+4\,d^3ex+d^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^(3/2)*sqrt(e*x + d)/(e^4*x^4 + 4*d*e^3*x^3 + 6*d
^2*e^2*x^2 + 4*d^3*e*x + d^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx+cx^2\right)^{\frac{3}{2}}}{\left(d+ex\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)**(7/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)/(d + e*x)**(7/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2453 $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)^{9/2}}\,dx$

Optimal. Leaf size=721

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(-4ce(4bd-5ae)-b^2e^2+16c^2d^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\right.}{35e^4\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

```
[Out] (4*(2*c*d - b*e)*(4*c^2*d^2 - b^2*e^2 - 4*c*e*(b*d - 2*a*e))*Sqrt[a + b*x +
 c*x^2])/(35*e^3*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[d + e*x]) - (2*(8*c^2*d^3 -
 c*d*e*(5*b*d - 4*a*e) - b*e^2*(2*b*d - 3*a*e) + e*(14*c^2*d^2 + b^2*e^2 -
2*c*e*(7*b*d - 5*a*e))*x)*Sqrt[a + b*x + c*x^2])/(35*e^3*(c*d^2 - b*d*e + a
*e^2)*(d + e*x)^(5/2)) - (2*(a + b*x + c*x^2)^(3/2))/(7*e*(d + e*x)^(7/2))
- (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*(4*c^2*d^2 - b^2*e^2 - 4*c*e*(
b*d - 2*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*El
lipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt
[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(35*e^
4*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(16*c^2*d^2
- b^2*e^2 - 4*c*e*(4*b*d - 5*a*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^
2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcS
in[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sq
rt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(35*e^4*(c*d^2 - b
*d*e + a*e^2)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.881375, antiderivative size = 721, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {732, 810, 834, 843, 718, 424, 419}

$$-\frac{2\sqrt{a+bx+cx^2}\left(ex\left(-2ce(7bd-5ae)+b^2e^2+14c^2d^2\right)-cde(5bd-4ae)-be^2(2bd-3ae)+8c^2d^3\right)}{35e^3(d+ex)^{5/2}\left(ae^2-bde+cd^2\right)}+\frac{4\sqrt{a+bx+cx^2}}{35\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x)^(9/2),x]
```


```
[Out] (4*(2*c*d - b*e)*(4*c^2*d^2 - b^2*e^2 - 4*c*e*(b*d - 2*a*e))*Sqrt[a + b*x +
 c*x^2])/(35*e^3*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[d + e*x]) - (2*(8*c^2*d^3 -
 c*d*e*(5*b*d - 4*a*e) - b*e^2*(2*b*d - 3*a*e) + e*(14*c^2*d^2 + b^2*e^2 -
2*c*e*(7*b*d - 5*a*e))*x)*Sqrt[a + b*x + c*x^2])/(35*e^3*(c*d^2 - b*d*e + a
*e^2)*(d + e*x)^(5/2)) - (2*(a + b*x + c*x^2)^(3/2))/(7*e*(d + e*x)^(7/2))
- (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*(4*c^2*d^2 - b^2*e^2 - 4*c*e*(
b*d - 2*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*El
lipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt
[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(35*e^
4*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(16*c^2*d^2
```

```
- b^2*e^2 - 4*c*e*(4*b*d - 5*a*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^
2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcS
in[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sq
rt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(35*e^4*(c*d^2 - b
*d*e + a*e^2)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
```

```
b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(d+ex)^{9/2}}\,dx = -\frac{2\left(a+bx+cx^2\right)^{3/2}}{7e(d+ex)^{7/2}} + \frac{3\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{(d+ex)^{7/2}}\,dx}{7e}$$

$$= -\frac{2\left(8c^2d^3 - cde(5bd-4ae) - be^2(2bd-3ae) + e\left(14c^2d^2 + b^2e^2 - 2ce(7bd-5ae)\right)x\right)\sqrt{a+b}}{35e^3\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}}$$

$$= \frac{4(2cd-be)\left(4c^2d^2 - b^2e^2 - 4ce(bd-2ae)\right)\sqrt{a+bx+cx^2}}{35e^3\left(cd^2-bde+ae^2\right)^2\sqrt{d+ex}} - \frac{2\left(8c^2d^3 - cde(5bd-4ae) - be^2\right.}{}$$

$$= \frac{4(2cd-be)\left(4c^2d^2 - b^2e^2 - 4ce(bd-2ae)\right)\sqrt{a+bx+cx^2}}{35e^3\left(cd^2-bde+ae^2\right)^2\sqrt{d+ex}} - \frac{2\left(8c^2d^3 - cde(5bd-4ae) - be^2\right.}{}$$

$$= \frac{4(2cd-be)\left(4c^2d^2 - b^2e^2 - 4ce(bd-2ae)\right)\sqrt{a+bx+cx^2}}{35e^3\left(cd^2-bde+ae^2\right)^2\sqrt{d+ex}} - \frac{2\left(8c^2d^3 - cde(5bd-4ae) - be^2\right.}{}$$

$$= \frac{4(2cd-be)\left(4c^2d^2 - b^2e^2 - 4ce(bd-2ae)\right)\sqrt{a+bx+cx^2}}{35e^3\left(cd^2-bde+ae^2\right)^2\sqrt{d+ex}} - \frac{2\left(8c^2d^3 - cde(5bd-4ae) - be^2\right.}{}$$

Mathematica [C] time = 13.3999, size = 5469, normalized size = 7.59

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x)^(9/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.441, size = 25722, normalized size = 35.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d)^(9/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}{(ex + d)^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)/(e*x + d)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2 + bx + a\right)^{\frac{3}{2}}\sqrt{ex + d}}{e^5x^5 + 5\,de^4x^4 + 10\,d^2e^3x^3 + 10\,d^3e^2x^2 + 5\,d^4ex + d^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^(3/2)*sqrt(e*x + d)/(e^5*x^5 + 5*d*e^4*x^4 + 10*
d^2*e^3*x^3 + 10*d^3*e^2*x^2 + 5*d^4*e*x + d^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2454 $\int \sqrt{dx}\left(a+bx+cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=616

$$-\frac{\sqrt[4]{a}d\sqrt{x}\left(\sqrt{a}b\sqrt{c}\left(708a^2c^2-241ab^2c+24b^4\right)+2\left(951a^2b^2c^2-924a^3c^3-268ab^4c+24b^6\right)\right)\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}x\right)^2}}\text{Elli}}{9009c^{15/4}\sqrt{dx}\sqrt{a+bx+cx^2}}$$

```
[Out] (-4*(24*b^6 - 268*a*b^4*c + 951*a^2*b^2*c^2 - 924*a^3*c^3)*d*x*Sqrt[a + b*x
 + c*x^2])/(9009*c^(7/2)*Sqrt[d*x]*(Sqrt[a] + Sqrt[c]*x)) + (2*Sqrt[d*x]*(b
*(24*b^4 - 151*a*b^2*c + 108*a^2*c^2) + 3*c*(24*b^4 - 181*a*b^2*c + 308*a^2
*c^2)*x)*Sqrt[a + b*x + c*x^2])/(9009*c^3) - (10*Sqrt[d*x]*(3*b*(6*b^2 - 19
*a*c) + 14*c*(3*b^2 - 11*a*c)*x)*(a + b*x + c*x^2)^(3/2))/(9009*c^2) + (10*
b*Sqrt[d*x]*(a + b*x + c*x^2)^(5/2))/(143*c) + (2*(d*x)^(3/2)*(a + b*x + c*
x^2)^(5/2))/(13*d) + (4*a^(1/4)*(24*b^6 - 268*a*b^4*c + 951*a^2*b^2*c^2 - 9
24*a^3*c^3)*d*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a]
 + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqr
t[a]*Sqrt[c]))/4])/(9009*c^(15/4)*Sqrt[d*x]*Sqrt[a + b*x + c*x^2]) - (a^(1/
4)*(Sqrt[a]*b*Sqrt[c]*(24*b^4 - 241*a*b^2*c + 708*a^2*c^2) + 2*(24*b^6 - 26
8*a*b^4*c + 951*a^2*b^2*c^2 - 924*a^3*c^3))*d*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x)
*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4
)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(9009*c^(15/4)*Sqrt[d*x]
*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.959471, antiderivative size = 616, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {734, 832, 814, 841, 839, 1197, 1103, 1195}

$$\frac{2\sqrt{dx}\left(3cx\left(308a^2c^2-181ab^2c+24b^4\right)+b\left(108a^2c^2-151ab^2c+24b^4\right)\right)\sqrt{a+bx+cx^2}}{9009c^3}-\frac{4dx\left(951a^2b^2c^2-924a^3c^3-26\right.}{9009c^{7/2}\sqrt{dx}\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a + b*x + c*x^2)^(5/2),x]
```


```
[Out] (-4*(24*b^6 - 268*a*b^4*c + 951*a^2*b^2*c^2 - 924*a^3*c^3)*d*x*Sqrt[a + b*x
 + c*x^2])/(9009*c^(7/2)*Sqrt[d*x]*(Sqrt[a] + Sqrt[c]*x)) + (2*Sqrt[d*x]*(b
*(24*b^4 - 151*a*b^2*c + 108*a^2*c^2) + 3*c*(24*b^4 - 181*a*b^2*c + 308*a^2
*c^2)*x)*Sqrt[a + b*x + c*x^2])/(9009*c^3) - (10*Sqrt[d*x]*(3*b*(6*b^2 - 19
*a*c) + 14*c*(3*b^2 - 11*a*c)*x)*(a + b*x + c*x^2)^(3/2))/(9009*c^2) + (10*
b*Sqrt[d*x]*(a + b*x + c*x^2)^(5/2))/(143*c) + (2*(d*x)^(3/2)*(a + b*x + c*
x^2)^(5/2))/(13*d) + (4*a^(1/4)*(24*b^6 - 268*a*b^4*c + 951*a^2*b^2*c^2 - 9
24*a^3*c^3)*d*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a]
 + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqr
t[a]*Sqrt[c]))/4])/(9009*c^(15/4)*Sqrt[d*x]*Sqrt[a + b*x + c*x^2]) - (a^(1/
4)*(Sqrt[a]*b*Sqrt[c]*(24*b^4 - 241*a*b^2*c + 708*a^2*c^2) + 2*(24*b^6 - 26
8*a*b^4*c + 951*a^2*b^2*c^2 - 924*a^3*c^3))*d*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x)
*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4
)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(9009*c^(15/4)*Sqrt[d*x]
*Sqrt[a + b*x + c*x^2])
```


Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 841

```
Int[((f_) + (g_.)*(x_))/(Sqrt[(e_)*(x_)]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_
)^2]), x_Symbol] :> Dist[Sqrt[x]/Sqrt[e*x], Int[(f + g*x)/(Sqrt[x]*Sqrt[a +
 b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0]
```

Rule 839

```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Dist[2, Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
```

```
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \sqrt{dx}\left(a+bx+cx^2\right)^{5/2}dx = \frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{5/2}}{13d} - \frac{5\int \sqrt{dx}(-2ad-bdx)\left(a+bx+cx^2\right)^{3/2}dx}{13d}$$

$$= \frac{10b\sqrt{dx}\left(a+bx+cx^2\right)^{5/2}}{143c} + \frac{2(dx)^{3/2}\left(a+bx+cx^2\right)^{5/2}}{13d} - \frac{10\int \frac{\left(\frac{1}{2}abd^2+\left(3b^2-11ac\right)d^2x\right)\left(a+bx+cx^2\right)}{\sqrt{dx}}}{143cd}$$

$$= -\frac{10\sqrt{dx}\left(3b\left(6b^2-19ac\right)+14c\left(3b^2-11ac\right)x\right)\left(a+bx+cx^2\right)^{3/2}}{9009c^2} + \frac{10b\sqrt{dx}\left(a+bx+cx^2\right)}{143c}$$

$$= \frac{2\sqrt{dx}\left(b\left(24b^4-151ab^2c+108a^2c^2\right)+3c\left(24b^4-181ab^2c+308a^2c^2\right)x\right)\sqrt{a+bx+cx^2}}{9009c^3} -$$

$$= \frac{2\sqrt{dx}\left(b\left(24b^4-151ab^2c+108a^2c^2\right)+3c\left(24b^4-181ab^2c+308a^2c^2\right)x\right)\sqrt{a+bx+cx^2}}{9009c^3} -$$

$$= \frac{2\sqrt{dx}\left(b\left(24b^4-151ab^2c+108a^2c^2\right)+3c\left(24b^4-181ab^2c+308a^2c^2\right)x\right)\sqrt{a+bx+cx^2}}{9009c^3} -$$

$$= \frac{2\sqrt{dx}\left(b\left(24b^4-151ab^2c+108a^2c^2\right)+3c\left(24b^4-181ab^2c+308a^2c^2\right)x\right)\sqrt{a+bx+cx^2}}{9009c^3} -$$

$$= -\frac{4\left(24b^6-268ab^4c+951a^2b^2c^2-924a^3c^3\right)dx\sqrt{a+bx+cx^2}}{9009c^{7/2}\sqrt{dx}\left(\sqrt{a}+\sqrt{c}x\right)} + \frac{2\sqrt{dx}\left(b\left(24b^4-151ab^2c\right.\right.}{}$$

Mathematica [C] time = 4.28974, size = 708, normalized size = 1.15

$$\sqrt{dx}\left(\frac{ix\left(1192a^2b^3c^2-951a^2b^2c^2\sqrt{b^2-4ac}+924a^3c^3\sqrt{b^2-4ac}-1632a^3bc^3-24b^6\sqrt{b^2-4ac}-292ab^5c+268ab^4c\sqrt{b^2-4ac}+24b^7\right)\sqrt{\frac{4a}{x\left(\sqrt{b^2-4ac}+b\right)}+2}\sqrt{\frac{-x\sqrt{b^2-4ac}+2a+bx}{bx-x\sqrt{b^2-4ac}}}}{\sqrt{\frac{a}{\sqrt{b^2-4ac}+b}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d*x]*((-4*(24*b^6 - 268*a*b^4*c + 951*a^2*b^2*c^2 - 924*a^3*c^3)*(a +
 x*(b + c*x)))/Sqrt[x] + 2*c*Sqrt[x]*(a + x*(b + c*x))*(24*b^5 - 18*b^4*c*x
```

```
 + b^3*c*(-241*a + 15*c*x^2) + 3*b^2*c^2*x*(54*a + 371*c*x^2) + 77*c^3*x*(3
1*a^2 + 28*a*c*x^2 + 9*c^2*x^4) + b*c^2*(708*a^2 + 3071*a*c*x^2 + 1701*c^2*
x^4)) + (I*(24*b^6 - 268*a*b^4*c + 951*a^2*b^2*c^2 - 924*a^3*c^3)*(-b + Sqr
t[b^2 - 4*a*c])*Sqrt[2 + (4*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x*Sqrt[(2*a + b
*x - Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticE[I*ArcSinh[
(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])
/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[a/(b + Sqrt[b^2 - 4*a*c])] + (I*(24*b^7 - 2
92*a*b^5*c + 1192*a^2*b^3*c^2 - 1632*a^3*b*c^3 - 24*b^6*Sqrt[b^2 - 4*a*c] +
 268*a*b^4*c*Sqrt[b^2 - 4*a*c] - 951*a^2*b^2*c^2*Sqrt[b^2 - 4*a*c] + 924*a^
3*c^3*Sqrt[b^2 - 4*a*c])*Sqrt[2 + (4*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x*Sqrt
[(2*a + b*x - Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticF[I
*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2
- 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]))/(9009
*c^4*Sqrt[x]*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.286, size = 2810, normalized size = 4.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] -1/9009*(d*x)^(1/2)*(-1902*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))
/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^
(1/2))^(1/2)*(-c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(((b+2*c*x+(-4*a*
c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(1/2))^(1/2))*a^2*b^3*c^2+536*(-4*a*c+b^2)^(1/2)*((b+2*c*
x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)
^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*Ellip
ticE(((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2
)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2))*a*b^5*c+1848*(-4*a*c+b
^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*((-b-
2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*x/(b+(-4*a*c+b^2)^(
1/2)))^(1/2)*EllipticE(((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))
)^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2))*a^3*
b*c^3+6*x^4*b^4*c^4-2256*x^5*b^3*c^5-5698*x^6*a*c^7-9086*x^4*a^2*c^6-4774*x
^2*a^3*c^5-4788*x^7*b*c^7-12*x^3*b^5*c^3-48*x^2*b^6*c^2-5628*x^6*b^2*c^6+36
96*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*((-b-2*c*x+(
-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*x/(b+(-4*a*c+b^2)^(1/2)))^
(1/2)*EllipticF(((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
,1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2))*a^4*c^4-739
2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*((-b-2*c*x+(-
4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*x/(b+(-4*a*c+b^2)^(1/2)))^(
1/2)*EllipticE(((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2),
1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2))*a^4*c^4+831*
((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*((-b-2*c*x+(-4*
a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/
2)*EllipticF(((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2),1/
2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2))*a^2*b^4*c^2-40
46*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*((-b-2*c*x+(
-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*x/(b+(-4*a*c+b^2)^(1/2)))^
(1/2)*EllipticE(((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
,1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2))*a^2*b^4*c^2
+9456*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*((-b-2*c*
x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*x/(b+(-4*a*c+b^2)^(1/2)
))^(1/2)*EllipticE(((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1
```

```
/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2))*a^3*b^2*
c^3-72*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*((-b-2*c
*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*x/(b+(-4*a*c+b^2)^(1/2
)))^(1/2)*EllipticF(((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(
1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2))*a*b^6*c
-3096*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*((-b-2*c*
x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*x/(b+(-4*a*c+b^2)^(1/2)
))^(1/2)*EllipticF(((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1
/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2))*a^3*b^2*
c^3+728*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*((-b-2*
c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*x/(b+(-4*a*c+b^2)^(1/
2)))^(1/2)*EllipticE(((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^
(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2))*a*b^6*
c+708*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2
)))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*x/(b
+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*
a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/
2))^(1/2))*a^3*b*c^3-241*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(
b+(-4*a*c+b^2)^(1/2)))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1
/2))^(1/2)*(-c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(((b+2*c*x+(-4*a*c+
b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2
))/(-4*a*c+b^2)^(1/2))^(1/2))*a^2*b^3*c^2-48*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(
-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*Elliptic
E(((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*(
(b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2))*b^7-1386*x^8*c^8-13856*x^
5*a*b*c^6-8692*x^4*a*b^2*c^5-12332*x^3*a^2*b*c^5+128*x^3*a*b^3*c^4-1740*x^2
*a^2*b^2*c^4+518*x^2*a*b^4*c^3-1416*x*a^3*b*c^4+482*x*a^2*b^3*c^3-48*x*a*b^
5*c^2-48*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*((-b-2
*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*x/(b+(-4*a*c+b^2)^(1
/2)))^(1/2)*EllipticE(((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))
^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2))*b^8+2
4*(-4*a*c+b^2)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^
(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*x/(b+(-4
*a*c+b^2)^(1/2)))^(1/2)*EllipticF(((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+
b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^
(1/2))*a*b^5*c)/(c*x^2+b*x+a)^(1/2)/c^5/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx + a\right)^{\frac{5}{2}} \sqrt{dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)*sqrt(d*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(c^2x^4 + 2\,bcx^3 + 2\,abx + \left(b^2 + 2\,ac\right)x^2 + a^2\right)\sqrt{cx^2 + bx + a}\sqrt{dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*sqrt(c*x
^2 + b*x + a)*sqrt(d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx}\left(a+bx+cx^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Integral(sqrt(d*x)*(a + b*x + c*x**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{\frac{5}{2}}\sqrt{dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)*sqrt(d*x), x)
```

3.2455 $\int \frac{(a+bx+cx^2)^{5/2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=847

$$\frac{2\sqrt{d+ex}\left(cx^2+bx+a\right)^{5/2}}{11e}+\frac{10\sqrt{d+ex}\left(16c^2d^2+3b^2e^2-ce(23bd-18ae)-7ce(2cd-be)x\right)\left(cx^2+bx+a\right)^{3/2}}{693ce^3}+\frac{2\sqrt{d+}}{}$$

```
[Out] (2*Sqrt[d + e*x]*(128*c^4*d^4 - 4*b^4*e^4 - 4*c^3*d^2*e*(76*b*d - 69*a*e) -
 b^2*c*e^3*(7*b*d - 27*a*e) + 3*c^2*e^2*(65*b^2*d^2 - 124*a*b*d*e + 60*a^2*
e^2) - 12*c*e*(2*c*d - b*e)*(4*c^2*d^2 - b^2*e^2 - 4*c*e*(b*d - 2*a*e))*x)*
Sqrt[a + b*x + c*x^2])/(693*c^2*e^5) + (10*Sqrt[d + e*x]*(16*c^2*d^2 + 3*b^
2*e^2 - c*e*(23*b*d - 18*a*e) - 7*c*e*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(3
/2))/(693*c*e^3) + (2*Sqrt[d + e*x]*(a + b*x + c*x^2)^(5/2))/(11*e) - (Sqrt
[2]*Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*(128*c^4*d^4 + 8*b^4*e^4 + b^2*c*e^3*(2
9*b*d - 93*a*e) - 4*c^3*d^2*e*(64*b*d - 93*a*e) + 3*c^2*e^2*(33*b^2*d^2 - 1
24*a*b*d*e + 124*a^2*e^2))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
- 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e)])/(693*c^3*e^6*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]
*Sqrt[a + b*x + c*x^2]) + (4*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e
^2)*(128*c^4*d^4 + 2*b^4*e^4 - 4*c^3*d^2*e*(64*b*d - 69*a*e) + b^2*c*e^3*(5
*b*d - 21*a*e) + 3*c^2*e^2*(41*b^2*d^2 - 92*a*b*d*e + 60*a^2*e^2))*Sqrt[(c*
(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqr
t[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e)])/(693*c^3*e^6*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 2.70071, antiderivative size = 847, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {734, 814, 843, 718, 424, 419}

$$\frac{2\sqrt{d+ex}\left(cx^2+bx+a\right)^{5/2}}{11e}+\frac{10\sqrt{d+ex}\left(16c^2d^2+3b^2e^2-ce(23bd-18ae)-7ce(2cd-be)x\right)\left(cx^2+bx+a\right)^{3/2}}{693ce^3}+\frac{2\sqrt{d+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/Sqrt[d + e*x],x]
```



```
[Out] (2*Sqrt[d + e*x]*(128*c^4*d^4 - 4*b^4*e^4 - 4*c^3*d^2*e*(76*b*d - 69*a*e) -
 b^2*c*e^3*(7*b*d - 27*a*e) + 3*c^2*e^2*(65*b^2*d^2 - 124*a*b*d*e + 60*a^2*
e^2) - 12*c*e*(2*c*d - b*e)*(4*c^2*d^2 - b^2*e^2 - 4*c*e*(b*d - 2*a*e))*x)*
Sqrt[a + b*x + c*x^2])/(693*c^2*e^5) + (10*Sqrt[d + e*x]*(16*c^2*d^2 + 3*b^
2*e^2 - c*e*(23*b*d - 18*a*e) - 7*c*e*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(3
/2))/(693*c*e^3) + (2*Sqrt[d + e*x]*(a + b*x + c*x^2)^(5/2))/(11*e) - (Sqrt
[2]*Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*(128*c^4*d^4 + 8*b^4*e^4 + b^2*c*e^3*(2
9*b*d - 93*a*e) - 4*c^3*d^2*e*(64*b*d - 93*a*e) + 3*c^2*e^2*(33*b^2*d^2 - 1
```

```
24*a*b*d*e + 124*a^2*e^2))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
- 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e)])/(693*c^3*e^6*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]
*Sqrt[a + b*x + c*x^2]) + (4*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e
^2)*(128*c^4*d^4 + 2*b^4*e^4 - 4*c^3*d^2*e*(64*b*d - 69*a*e) + b^2*c*e^3*(5
*b*d - 21*a*e) + 3*c^2*e^2*(41*b^2*d^2 - 92*a*b*d*e + 60*a^2*e^2))*Sqrt[(c*
(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqr
t[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e)])/(693*c^3*e^6*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
```

```
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{5/2}}{\sqrt{d+ex}}dx = \frac{2\sqrt{d+ex}\left(a+bx+cx^2\right)^{5/2}}{11e} - \frac{5\int \frac{(bd-2ae+(2cd-be)x)\left(a+bx+cx^2\right)^{3/2}}{\sqrt{d+ex}}dx}{11e}$$

$$= \frac{10\sqrt{d+ex}\left(16c^2d^2+3b^2e^2-ce(23bd-18ae)-7ce(2cd-be)x\right)\left(a+bx+cx^2\right)^{3/2}}{693ce^3} + \frac{2\sqrt{d+ex}\,(}{}$$

$$= \frac{2\sqrt{d+ex}\left(128c^4d^4-4b^4e^4-4c^3d^2e(76bd-69ae)-b^2ce^3(7bd-27ae)+3c^2e^2\left(65b^2d^2-124al\right.\right.}{693c^2e^5}$$

$$= \frac{2\sqrt{d+ex}\left(128c^4d^4-4b^4e^4-4c^3d^2e(76bd-69ae)-b^2ce^3(7bd-27ae)+3c^2e^2\left(65b^2d^2-124al\right.\right.}{693c^2e^5}$$

$$= \frac{2\sqrt{d+ex}\left(128c^4d^4-4b^4e^4-4c^3d^2e(76bd-69ae)-b^2ce^3(7bd-27ae)+3c^2e^2\left(65b^2d^2-124al\right.\right.}{693c^2e^5}$$

$$= \frac{2\sqrt{d+ex}\left(128c^4d^4-4b^4e^4-4c^3d^2e(76bd-69ae)-b^2ce^3(7bd-27ae)+3c^2e^2\left(65b^2d^2-124al\right.\right.}{693c^2e^5}$$

Mathematica [C] time = 13.9757, size = 10879, normalized size = 12.84

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/Sqrt[d + e*x],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.401, size = 12152, normalized size = 14.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(e*x+d)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)^{\frac{5}{2}}}{\sqrt{ex + d}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)/sqrt(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(c^2x^4 + 2\,bcx^3 + 2\,abx + (b^2 + 2\,ac)x^2 + a^2)\sqrt{cx^2 + bx + a}}{\sqrt{ex + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*sqrt(c*x
^2 + b*x + a)/sqrt(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2456 $\int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)^{3/2}} dx$

Optimal. Leaf size=716

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2cd-be)\left(ae^2-bde+cd^2\right)\left(-4ce(32bd-33ae)-b^2e^2+128c^2d^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{Ellipt}}{63c^2e^6\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*Sqrt[d + e*x]*(128*c^3*d^3 - b^3*e^3 + 3*b*c*e^2*(37*b*d - 36*a*e) - 12
*c^2*d*e*(20*b*d - 11*a*e) - 3*c*e*(32*c^2*d^2 + b^2*e^2 - 4*c*e*(8*b*d - 7
*a*e))*x)*Sqrt[a + b*x + c*x^2])/(63*c*e^5) - (10*Sqrt[d + e*x]*(16*c*d - 1
5*b*e - 14*c*e*x)*(a + b*x + c*x^2)^(3/2))/(63*e^3) - (2*(a + b*x + c*x^2)^
(5/2))/(e*Sqrt[d + e*x]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(128*c^4*d^4 - b^4*
e^4 - 4*c^3*d^2*e*(64*b*d - 57*a*e) - b^2*c*e^3*(7*b*d - 15*a*e) + 3*c^2*e^
2*(45*b^2*d^2 - 76*a*b*d*e + 28*a^2*e^2))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x
+ c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sq
rt[b^2 - 4*a*c])*e)])/(63*c^2*e^6*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*d
 - b*e)*(c*d^2 - b*d*e + a*e^2)*(128*c^2*d^2 - b^2*e^2 - 4*c*e*(32*b*d - 33
*a*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a
 + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e)])/(63*c^2*e^6*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2
])
```

Rubi [A] time = 1.141, antiderivative size = 716, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {732, 814, 843, 718, 424, 419}

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(3c^2e^2\left(28a^2e^2-76abde+45b^2d^2\right)-b^2ce^3(7bd-15ae)-4c^3d^2e(64bd-57ae)-b^4e\right.}{63c^2e^6\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/(d + e*x)^(3/2),x]
```


```
[Out] (-2*Sqrt[d + e*x]*(128*c^3*d^3 - b^3*e^3 + 3*b*c*e^2*(37*b*d - 36*a*e) - 12
*c^2*d*e*(20*b*d - 11*a*e) - 3*c*e*(32*c^2*d^2 + b^2*e^2 - 4*c*e*(8*b*d - 7
*a*e))*x)*Sqrt[a + b*x + c*x^2])/(63*c*e^5) - (10*Sqrt[d + e*x]*(16*c*d - 1
5*b*e - 14*c*e*x)*(a + b*x + c*x^2)^(3/2))/(63*e^3) - (2*(a + b*x + c*x^2)^
(5/2))/(e*Sqrt[d + e*x]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(128*c^4*d^4 - b^4*
e^4 - 4*c^3*d^2*e*(64*b*d - 57*a*e) - b^2*c*e^3*(7*b*d - 15*a*e) + 3*c^2*e^
2*(45*b^2*d^2 - 76*a*b*d*e + 28*a^2*e^2))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x
+ c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sq
rt[b^2 - 4*a*c])*e)])/(63*c^2*e^6*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2
```

```
- 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*d
 - b*e)*(c*d^2 - b*d*e + a*e^2)*(128*c^2*d^2 - b^2*e^2 - 4*c*e*(32*b*d - 33
*a*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a
 + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e)])/(63*c^2*e^6*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2
])
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(d+ex)^{3/2}}\,dx = -\frac{2\left(a+bx+cx^2\right)^{5/2}}{e\sqrt{d+ex}} + \frac{5\int \frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{\sqrt{d+ex}}\,dx}{e}$$

$$= -\frac{10\sqrt{d+ex}(16cd-15be-14cex)\left(a+bx+cx^2\right)^{3/2}}{63e^3} - \frac{2\left(a+bx+cx^2\right)^{5/2}}{e\sqrt{d+ex}} - \frac{10\int \frac{\left(\frac{1}{2}c\left(15b^2de+4acd\right.\right.}{}}{}$$

$$= -\frac{2\sqrt{d+ex}\left(128c^3d^3-b^3e^3+3bce^2(37bd-36ae)-12c^2de(20bd-11ae)-3ce\left(32c^2d^2+b^2e^2-\right.\right.}{63ce^5}$$

$$= -\frac{2\sqrt{d+ex}\left(128c^3d^3-b^3e^3+3bce^2(37bd-36ae)-12c^2de(20bd-11ae)-3ce\left(32c^2d^2+b^2e^2-\right.\right.}{63ce^5}$$

$$= -\frac{2\sqrt{d+ex}\left(128c^3d^3-b^3e^3+3bce^2(37bd-36ae)-12c^2de(20bd-11ae)-3ce\left(32c^2d^2+b^2e^2-\right.\right.}{63ce^5}$$

$$= -\frac{2\sqrt{d+ex}\left(128c^3d^3-b^3e^3+3bce^2(37bd-36ae)-12c^2de(20bd-11ae)-3ce\left(32c^2d^2+b^2e^2-\right.\right.}{63ce^5}$$

Mathematica [C] time = 13.7321, size = 7946, normalized size = 11.1

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/(d + e*x)^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.351, size = 9187, normalized size = 12.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(e*x+d)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx + a\right)^{\frac{5}{2}}}{(ex + d)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)/(e*x + d)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c^2x^4 + 2\,bcx^3 + 2\,abx + \left(b^2 + 2\,ac\right)x^2 + a^2\right)\sqrt{cx^2 + bx + a}\sqrt{ex + d}}{e^2x^2 + 2\,dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*sqrt(c*x
^2 + b*x + a)*sqrt(e*x + d)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx + cx^2\right)^{\frac{5}{2}}}{(d + ex)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(e*x+d)**(3/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(5/2)/(d + e*x)**(3/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2457 $\int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=622

$$\frac{4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\left(-4ce(32bd-5ae)+27b^2e^2+128c^2d^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\sin^{-1}\right.}{21ce^6\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] (2*Sqrt[d + e*x]*(128*c^2*d^2 + 51*b^2*e^2 - 4*c*e*(44*b*d - 5*a*e) - 48*c*
e*(2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(21*e^5) + (10*(16*c*d - 7*b*e +
2*c*e*x)*(a + b*x + c*x^2)^(3/2))/(21*e^3*Sqrt[d + e*x]) - (2*(a + b*x + c*
x^2)^(5/2))/(3*e*(d + e*x)^(3/2)) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*d - b*e
)*(128*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(32*b*d - 29*a*e))*Sqrt[d + e*x]*Sqrt[-(
(c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 -
 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c
*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(21*c*e^6*Sqrt[(c*(d + e*x))/(2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (4*Sqrt[2]*Sqrt[b^2 - 4*a
*c]*(c*d^2 - b*d*e + a*e^2)*(128*c^2*d^2 + 27*b^2*e^2 - 4*c*e*(32*b*d - 5*a
*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a +
 b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e)])/(21*c*e^6*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.805415, antiderivative size = 622, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {732, 812, 814, 843, 718, 424, 419}

$$\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(-4ce(44bd-5ae)+51b^2e^2-48cex(2cd-be)+128c^2d^2\right)}{21e^5}+\frac{4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ae^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/(d + e*x)^(5/2),x]
```


```
[Out] (2*Sqrt[d + e*x]*(128*c^2*d^2 + 51*b^2*e^2 - 4*c*e*(44*b*d - 5*a*e) - 48*c*
e*(2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(21*e^5) + (10*(16*c*d - 7*b*e +
2*c*e*x)*(a + b*x + c*x^2)^(3/2))/(21*e^3*Sqrt[d + e*x]) - (2*(a + b*x + c*
x^2)^(5/2))/(3*e*(d + e*x)^(3/2)) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*d - b*e
)*(128*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(32*b*d - 29*a*e))*Sqrt[d + e*x]*Sqrt[-(
(c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 -
 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c
*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(21*c*e^6*Sqrt[(c*(d + e*x))/(2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (4*Sqrt[2]*Sqrt[b^2 - 4*a
*c]*(c*d^2 - b*d*e + a*e^2)*(128*c^2*d^2 + 27*b^2*e^2 - 4*c*e*(32*b*d - 5*a
*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a +
 b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b
```

```
 + Sqrt[b^2 - 4*a*c])*e)])/(21*c*e^6*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = -\frac{2\left(a+bx+cx^2\right)^{5/2}}{3e(d+ex)^{3/2}} + \frac{5\int \frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx}{3e}$$

$$= \frac{10(16cd-7be+2cex)\left(a+bx+cx^2\right)^{3/2}}{21e^3\sqrt{d+ex}} - \frac{2\left(a+bx+cx^2\right)^{5/2}}{3e(d+ex)^{3/2}} - \frac{10\int \frac{\left(\frac{1}{2}\left(16bcd-7b^2e-4ace\right)+8c(2cd-be\cdots}{\sqrt{d+ex}}}{7e^3}$$

$$= \frac{2\sqrt{d+ex}\left(128c^2d^2+51b^2e^2-4ce(44bd-5ae)-48ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{21e^5} + \frac{10(16cd-\cdots}{\cdots}$$

$$= \frac{2\sqrt{d+ex}\left(128c^2d^2+51b^2e^2-4ce(44bd-5ae)-48ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{21e^5} + \frac{10(16cd-\cdots}{\cdots}$$

$$= \frac{2\sqrt{d+ex}\left(128c^2d^2+51b^2e^2-4ce(44bd-5ae)-48ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{21e^5} + \frac{10(16cd-\cdots}{\cdots}$$

$$= \frac{2\sqrt{d+ex}\left(128c^2d^2+51b^2e^2-4ce(44bd-5ae)-48ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{21e^5} + \frac{10(16cd-\cdots}{\cdots}$$

Mathematica [C] time = 13.5183, size = 5407, normalized size = 8.69

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/(d + e*x)^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.398, size = 12847, normalized size = 20.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(e*x+d)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{5}{2}}}{\left(ex+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)/(e*x + d)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\,ac\right)x^2+a^2\right)\sqrt{cx^2+bx+a}\sqrt{ex+d}}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*sqrt(c*x
^2 + b*x + a)*sqrt(e*x + d)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2458 $\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=603

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(2cd-be)\left(-4ce(32bd-17ae)+15b^2e^2+128c^2d^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{b}\cdots}{}\right.\right.}{15ce^6\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*(128*c^2*d^2 + 15*b^2*e^2 - 4*c*e*(28*b*d - 9*a*e) + 16*c*e*(2*c*d - b*
e)*x)*Sqrt[a + b*x + c*x^2])/(15*e^5*Sqrt[d + e*x]) + (2*(16*c*d - 5*b*e +
6*c*e*x)*(a + b*x + c*x^2)^(3/2))/(15*e^3*(d + e*x)^(3/2)) - (2*(a + b*x +
c*x^2)^(5/2))/(5*e*(d + e*x)^(5/2)) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(128*c^2
*d^2 + 23*b^2*e^2 - 4*c*e*(32*b*d - 9*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*
x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e)])/(15*e^6*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*d -
 b*e)*(128*c^2*d^2 + 15*b^2*e^2 - 4*c*e*(32*b*d - 17*a*e))*Sqrt[(c*(d + e*x
))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e)])/(15*c*e^6*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.689455, antiderivative size = 603, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {732, 812, 843, 718, 424, 419}

$$-\frac{2\sqrt{a+bx+cx^2}\left(-4ce(28bd-9ae)+15b^2e^2+16cex(2cd-be)+128c^2d^2\right)}{15e^5\sqrt{d+ex}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(2cd-be)\left(-4\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/(d + e*x)^(7/2),x]
```

```
[Out] (-2*(128*c^2*d^2 + 15*b^2*e^2 - 4*c*e*(28*b*d - 9*a*e) + 16*c*e*(2*c*d - b*
e)*x)*Sqrt[a + b*x + c*x^2])/(15*e^5*Sqrt[d + e*x]) + (2*(16*c*d - 5*b*e +
6*c*e*x)*(a + b*x + c*x^2)^(3/2))/(15*e^3*(d + e*x)^(3/2)) - (2*(a + b*x +
c*x^2)^(5/2))/(5*e*(d + e*x)^(5/2)) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(128*c^2
*d^2 + 23*b^2*e^2 - 4*c*e*(32*b*d - 9*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*
x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e)])/(15*e^6*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*d -
 b*e)*(128*c^2*d^2 + 15*b^2*e^2 - 4*c*e*(32*b*d - 17*a*e))*Sqrt[(c*(d + e*x
))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
```

```
*e)])/(15*c*e^6*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(d+ex)^{7/2}}\,dx = -\frac{2\left(a+bx+cx^2\right)^{5/2}}{5e(d+ex)^{5/2}} + \frac{\int \frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{(d+ex)^{5/2}}\,dx}{e}$$

$$= \frac{2(16cd-5be+6cex)\left(a+bx+cx^2\right)^{3/2}}{15e^3(d+ex)^{3/2}} - \frac{2\left(a+bx+cx^2\right)^{5/2}}{5e(d+ex)^{5/2}} - \frac{2\int \frac{\left(\frac{1}{2}\left(16bcd-5b^2e-12ace\right)+8c(2cd-be)x\cdots}{(d+ex)^{3/2}}}{5e^3}$$

$$= -\frac{2\left(128c^2d^2+15b^2e^2-4ce(28bd-9ae)+16ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{15e^5\sqrt{d+ex}} + \frac{2(16cd-5be+6c\cdots}{15e^3(\cdots}$$

$$= -\frac{2\left(128c^2d^2+15b^2e^2-4ce(28bd-9ae)+16ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{15e^5\sqrt{d+ex}} + \frac{2(16cd-5be+6c\cdots}{15e^3(\cdots}$$

$$= -\frac{2\left(128c^2d^2+15b^2e^2-4ce(28bd-9ae)+16ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{15e^5\sqrt{d+ex}} + \frac{2(16cd-5be+6c\cdots}{15e^3(\cdots}$$

$$= -\frac{2\left(128c^2d^2+15b^2e^2-4ce(28bd-9ae)+16ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{15e^5\sqrt{d+ex}} + \frac{2(16cd-5be+6c\cdots}{15e^3(\cdots}$$

Mathematica [C] time = 13.2677, size = 8961, normalized size = 14.86

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/(d + e*x)^(7/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.378, size = 14453, normalized size = 24.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(e*x+d)^(7/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{5}{2}}}{(ex+d)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)/(e*x + d)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2x^4 + 2\,bcx^3 + 2\,abx + \left(b^2 + 2\,ac\right)x^2 + a^2\right)\sqrt{cx^2 + bx + a}\sqrt{ex + d}}{e^4x^4 + 4\,de^3x^3 + 6\,d^2e^2x^2 + 4\,d^3ex + d^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*sqrt(c*x
^2 + b*x + a)*sqrt(e*x + d)/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*
e*x + d^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(e*x+d)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2459 $\int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)^{9/2}}\,dx$

Optimal. Leaf size=731

$$\frac{4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(-4ce(32bd-5ae)+27b^2e^2+128c^2d^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\right.}{21e^6\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] (2*c*(128*c^2*d^3 - 4*c*d*e*(44*b*d - 29*a*e) + 3*b*e^2*(17*b*d - 16*a*e) +
 e*(32*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(8*b*d - 5*a*e))*x)*Sqrt[a + b*x + c*x^2
])/(21*e^5*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x]) - (2*(16*c^2*d^3 + 3*a*b*
e^3 - c*d*e*(13*b*d - 4*a*e) + e*(22*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(11*b*d -
5*a*e))*x)*(a + b*x + c*x^2)^(3/2))/(21*e^3*(c*d^2 - b*d*e + a*e^2)*(d + e*
x)^(5/2)) - (2*(a + b*x + c*x^2)^(5/2))/(7*e*(d + e*x)^(7/2)) - (Sqrt[2]*Sq
rt[b^2 - 4*a*c]*(2*c*d - b*e)*(128*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(32*b*d - 29
*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE
[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (
-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(21*e^6*(c*d^
2 - b*d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*
Sqrt[a + b*x + c*x^2]) + (4*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(128*c^2*d^2 + 27*b^2
*e^2 - 4*c*e*(32*b*d - 5*a*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 -
4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b
^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(21*e^6*Sqrt[d + e*x]*
Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.887041, antiderivative size = 731, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {732, 810, 812, 843, 718, 424, 419}

$$-\frac{2\left(a+bx+cx^2\right)^{3/2}\left(ex\left(-2ce(11bd-5ae)+3b^2e^2+22c^2d^2\right)-cde(13bd-4ae)+3abe^3+16c^2d^3\right)}{21e^3(d+ex)^{5/2}\left(ae^2-bde+cd^2\right)}+\frac{2c\sqrt{a+bx+cx^2}\left(e\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/(d + e*x)^(9/2),x]
```


```
[Out] (2*c*(128*c^2*d^3 - 4*c*d*e*(44*b*d - 29*a*e) + 3*b*e^2*(17*b*d - 16*a*e) +
 e*(32*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(8*b*d - 5*a*e))*x)*Sqrt[a + b*x + c*x^2
])/(21*e^5*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x]) - (2*(16*c^2*d^3 + 3*a*b*
e^3 - c*d*e*(13*b*d - 4*a*e) + e*(22*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(11*b*d -
5*a*e))*x)*(a + b*x + c*x^2)^(3/2))/(21*e^3*(c*d^2 - b*d*e + a*e^2)*(d + e*
x)^(5/2)) - (2*(a + b*x + c*x^2)^(5/2))/(7*e*(d + e*x)^(7/2)) - (Sqrt[2]*Sq
rt[b^2 - 4*a*c]*(2*c*d - b*e)*(128*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(32*b*d - 29
*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE
[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (
-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(21*e^6*(c*d^
```

```
2 - b*d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*
Sqrt[a + b*x + c*x^2]) + (4*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(128*c^2*d^2 + 27*b^2
*e^2 - 4*c*e*(32*b*d - 5*a*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 -
4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b
^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(21*e^6*Sqrt[d + e*x]*
Sqrt[a + b*x + c*x^2])
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
```

```
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(d+ex)^{9/2}}\,dx = -\frac{2\left(a+bx+cx^2\right)^{5/2}}{7e(d+ex)^{7/2}}+\frac{5\int \frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx}{7e}$$

$$= -\frac{2\left(16c^2d^3+3abe^3-cde(13bd-4ae)+e\left(22c^2d^2+3b^2e^2-2ce(11bd-5ae)\right)x\right)\left(a+bx+cx^2\right)}{21e^3\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}}$$

$$= \frac{2c\left(128c^2d^3-4cde(44bd-29ae)+3be^2(17bd-16ae)+e\left(32c^2d^2+3b^2e^2-4ce(8bd-5ae)\right)x\right)}{21e^5\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

$$= \frac{2c\left(128c^2d^3-4cde(44bd-29ae)+3be^2(17bd-16ae)+e\left(32c^2d^2+3b^2e^2-4ce(8bd-5ae)\right)x\right)}{21e^5\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

$$= \frac{2c\left(128c^2d^3-4cde(44bd-29ae)+3be^2(17bd-16ae)+e\left(32c^2d^2+3b^2e^2-4ce(8bd-5ae)\right)x\right)}{21e^5\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

$$= \frac{2c\left(128c^2d^3-4cde(44bd-29ae)+3be^2(17bd-16ae)+e\left(32c^2d^2+3b^2e^2-4ce(8bd-5ae)\right)x\right)}{21e^5\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

Mathematica [C] time = 13.3417, size = 5482, normalized size = 7.5

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/(d + e*x)^(9/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.456, size = 25728, normalized size = 35.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(e*x+d)^(9/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)^{\frac{5}{2}}}{(ex + d)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)/(e*x + d)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c^2x^4 + 2\,bcx^3 + 2\,abx + \left(b^2 + 2\,ac\right)x^2 + a^2\right)\sqrt{cx^2 + bx + a}\sqrt{ex + d}}{e^5x^5 + 5\,de^4x^4 + 10\,d^2e^3x^3 + 10\,d^3e^2x^2 + 5\,d^4ex + d^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*sqrt(c*x
^2 + b*x + a)*sqrt(e*x + d)/(e^5*x^5 + 5*d*e^4*x^4 + 10*d^2*e^3*x^3 + 10*d^
3*e^2*x^2 + 5*d^4*e*x + d^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(e*x+d)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2460 $\int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)^{11/2}}\,dx$

Optimal. Leaf size=923

$$-\frac{2\left(cx^2+bx+a\right)^{5/2}}{9e(d+ex)^{9/2}}-\frac{2\left(16c^2d^3-ce(11bd-4ae)d-be^2(2bd-5ae)+e\left(26c^2d^2+3b^2e^2-2ce(13bd-7ae)\right)x\right)\left(cx^2+\right.}{63e^3\left(cd^2-bed+ae^2\right)(d+ex)^{7/2}}$$

```
[Out] (-2*(128*c^4*d^5 - 2*a*b^3*e^5 - 4*c^3*d^3*e*(60*b*d - 49*a*e) - b*c*e^3*(b
^2*d^2 + 9*a*b*d*e - 24*a^2*e^2) + 3*c^2*d*e^2*(37*b^2*d^2 - 52*a*b*d*e + 1
2*a^2*e^2) + e*(160*c^4*d^4 - 2*b^4*e^4 - 4*c^3*d^2*e*(80*b*d - 69*a*e) - b
^2*c*e^3*(11*b*d - 27*a*e) + 3*c^2*e^2*(57*b^2*d^2 - 92*a*b*d*e + 28*a^2*e^
2))*x)*Sqrt[a + b*x + c*x^2])/(63*e^5*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^(
3/2)) - (2*(16*c^2*d^3 - b*e^2*(2*b*d - 5*a*e) - c*d*e*(11*b*d - 4*a*e) + e
*(26*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(13*b*d - 7*a*e))*x)*(a + b*x + c*x^2)^(3/
2))/(63*e^3*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(7/2)) - (2*(a + b*x + c*x^2)
^(5/2))/(9*e*(d + e*x)^(9/2)) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(128*c^4*d^4 -
 b^4*e^4 - 4*c^3*d^2*e*(64*b*d - 57*a*e) - b^2*c*e^3*(7*b*d - 15*a*e) + 3*c
^2*e^2*(45*b^2*d^2 - 76*a*b*d*e + 28*a^2*e^2))*Sqrt[d + e*x]*Sqrt[-((c*(a +
 b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e)])/(63*e^6*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[(c*(d + e
*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[
2]*Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*(128*c^2*d^2 - b^2*e^2 - 4*c*e*(32*b*d -
 33*a*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c
*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4
*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e)])/(63*e^6*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*
x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 1.18827, antiderivative size = 923, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {732, 810, 843, 718, 424, 419}

$$-\frac{2\left(cx^2+bx+a\right)^{5/2}}{9e(d+ex)^{9/2}}-\frac{2\left(16c^2d^3-ce(11bd-4ae)d-be^2(2bd-5ae)+e\left(26c^2d^2+3b^2e^2-2ce(13bd-7ae)\right)x\right)\left(cx^2+\right.}{63e^3\left(cd^2-bed+ae^2\right)(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/(d + e*x)^(11/2),x]
```


```
[Out] (-2*(128*c^4*d^5 - 2*a*b^3*e^5 - 4*c^3*d^3*e*(60*b*d - 49*a*e) - b*c*e^3*(b
^2*d^2 + 9*a*b*d*e - 24*a^2*e^2) + 3*c^2*d*e^2*(37*b^2*d^2 - 52*a*b*d*e + 1
2*a^2*e^2) + e*(160*c^4*d^4 - 2*b^4*e^4 - 4*c^3*d^2*e*(80*b*d - 69*a*e) - b
^2*c*e^3*(11*b*d - 27*a*e) + 3*c^2*e^2*(57*b^2*d^2 - 92*a*b*d*e + 28*a^2*e^
2))*x)*Sqrt[a + b*x + c*x^2])/(63*e^5*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^(
3/2)) - (2*(16*c^2*d^3 - b*e^2*(2*b*d - 5*a*e) - c*d*e*(11*b*d - 4*a*e) + e
```

```
*(26*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(13*b*d - 7*a*e))*x)*(a + b*x + c*x^2)^(3/
2))/(63*e^3*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(7/2)) - (2*(a + b*x + c*x^2)
^(5/2))/(9*e*(d + e*x)^(9/2)) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(128*c^4*d^4 -
 b^4*e^4 - 4*c^3*d^2*e*(64*b*d - 57*a*e) - b^2*c*e^3*(7*b*d - 15*a*e) + 3*c
^2*e^2*(45*b^2*d^2 - 76*a*b*d*e + 28*a^2*e^2))*Sqrt[d + e*x]*Sqrt[-((c*(a +
 b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e)])/(63*e^6*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[(c*(d + e
*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[
2]*Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*(128*c^2*d^2 - b^2*e^2 - 4*c*e*(32*b*d -
 33*a*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c
*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4
*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e)])/(63*e^6*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*
x]*Sqrt[a + b*x + c*x^2])
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(d+ex)^{11/2}}\,dx = -\frac{2\left(a+bx+cx^2\right)^{5/2}}{9e(d+ex)^{9/2}} + \frac{5\int \frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{(d+ex)^{9/2}}\,dx}{9e}$$

$$= -\frac{2\left(16c^2d^3 - be^2(2bd-5ae) - cde(11bd-4ae) + e\left(26c^2d^2+3b^2e^2-2ce(13bd-7ae)\right)x\right)\left(a\right.}{63e^3\left(cd^2-bde+ae^2\right)(d+ex)^{7/2}}$$

$$= -\frac{2\left(128c^4d^5 - 2ab^3e^5 - 4c^3d^3e(60bd-49ae) - bce^3\left(b^2d^2+9abde-24a^2e^2\right) + 3c^2de^2\left(37b^2\right.\right.}{}$$

$$= -\frac{2\left(128c^4d^5 - 2ab^3e^5 - 4c^3d^3e(60bd-49ae) - bce^3\left(b^2d^2+9abde-24a^2e^2\right) + 3c^2de^2\left(37b^2\right.\right.}{}$$

$$= -\frac{2\left(128c^4d^5 - 2ab^3e^5 - 4c^3d^3e(60bd-49ae) - bce^3\left(b^2d^2+9abde-24a^2e^2\right) + 3c^2de^2\left(37b^2\right.\right.}{}$$

$$= -\frac{2\left(128c^4d^5 - 2ab^3e^5 - 4c^3d^3e(60bd-49ae) - bce^3\left(b^2d^2+9abde-24a^2e^2\right) + 3c^2de^2\left(37b^2\right.\right.}{}$$

Mathematica [C] time = 14.0085, size = 8108, normalized size = 8.78

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/(d + e*x)^(11/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.576, size = 44994, normalized size = 48.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(e*x+d)^(11/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{5}{2}}}{\left(ex+d\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)/(e*x + d)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\,ac\right)x^2+a^2\right)\sqrt{cx^2+bx+a}\sqrt{ex+d}}{e^6x^6+6\,de^5x^5+15\,d^2e^4x^4+20\,d^3e^3x^3+15\,d^4e^2x^2+6\,d^5ex+d^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^(11/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*sqrt(c*x
^2 + b*x + a)*sqrt(e*x + d)/(e^6*x^6 + 6*d*e^5*x^5 + 15*d^2*e^4*x^4 + 20*d^
3*e^3*x^3 + 15*d^4*e^2*x^2 + 6*d^5*e*x + d^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(e*x+d)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)^(11/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2461 $\int \frac{(d+ex)^{7/2}}{\sqrt{a+bx+cx^2}} dx$

Optimal. Leaf size=600

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\left(-ce(25ae+71bd)+24b^2e^2+71c^2d^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\sin^{-1}\right.}{105c^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] (2*e*(71*c^2*d^2 + 24*b^2*e^2 - c*e*(71*b*d + 25*a*e))*Sqrt[d + e*x]*Sqrt[a
 + b*x + c*x^2])/(105*c^3) + (12*e*(2*c*d - b*e)*(d + e*x)^(3/2)*Sqrt[a + b
*x + c*x^2])/(35*c^2) + (2*e*(d + e*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(7*c) +
 (8*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*(11*c^2*d^2 + 6*b^2*e^2 - c*e*(
11*b*d + 13*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))
]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/
Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(1
05*c^4*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x
 + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*(71*c^2*d
^2 + 24*b^2*e^2 - c*e*(71*b*d + 25*a*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Elliptic
F[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]],
(-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(105*c^4*Sqr
t[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.848873, antiderivative size = 600, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {742, 832, 843, 718, 424, 419}

$$\frac{2e\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(-ce(25ae+71bd)+24b^2e^2+71c^2d^2\right)}{105c^3}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\left(-ce(25\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/Sqrt[a + b*x + c*x^2],x]
```


```
[Out] (2*e*(71*c^2*d^2 + 24*b^2*e^2 - c*e*(71*b*d + 25*a*e))*Sqrt[d + e*x]*Sqrt[a
 + b*x + c*x^2])/(105*c^3) + (12*e*(2*c*d - b*e)*(d + e*x)^(3/2)*Sqrt[a + b
*x + c*x^2])/(35*c^2) + (2*e*(d + e*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(7*c) +
 (8*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*(11*c^2*d^2 + 6*b^2*e^2 - c*e*(
11*b*d + 13*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))
]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/
Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(1
05*c^4*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x
 + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*(71*c^2*d
^2 + 24*b^2*e^2 - c*e*(71*b*d + 25*a*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Elliptic
F[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]],
(-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(105*c^4*Sqr
```

```
t[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^{7/2}}{\sqrt{a+bx+cx^2}}\,dx = \frac{2e(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7c} + \frac{2\int \frac{(d+ex)^{3/2}\left(\frac{1}{2}\left(7cd^2-e(bd+5ae)\right)+3e(2cd-be)x\right)}{\sqrt{a+bx+cx^2}}dx}{7c}$$

$$= \frac{12e(2cd-be)(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{35c^2} + \frac{2e(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7c} + \frac{4\int \frac{\sqrt{d+ex}\left(\frac{1}{4}\left(35c^2d^3+6be^2(bd+\right.\right.}{}}{}$$

$$= \frac{2e\left(71c^2d^2+24b^2e^2-ce(71bd+25ae)\right)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{105c^3} + \frac{12e(2cd-be)(d+ex)^{3/2}\sqrt{a+bx}}{35c^2}$$

$$= \frac{2e\left(71c^2d^2+24b^2e^2-ce(71bd+25ae)\right)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{105c^3} + \frac{12e(2cd-be)(d+ex)^{3/2}\sqrt{a+bx}}{35c^2}$$

$$= \frac{2e\left(71c^2d^2+24b^2e^2-ce(71bd+25ae)\right)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{105c^3} + \frac{12e(2cd-be)(d+ex)^{3/2}\sqrt{a+bx}}{35c^2}$$

$$= \frac{2e\left(71c^2d^2+24b^2e^2-ce(71bd+25ae)\right)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{105c^3} + \frac{12e(2cd-be)(d+ex)^{3/2}\sqrt{a+bx}}{35c^2}$$

Mathematica [C] time = 13.1642, size = 5340, normalized size = 8.9

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.341, size = 6947, normalized size = 11.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{7}{2}}}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(7/2)/sqrt(c*x^2 + b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e^{3} x^{3} + 3\, d e^{2} x^{2} + 3\, d^{2} e x + d^{3}\right) \sqrt{e x + d}}{\sqrt{c x^{2} + b x + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(e*x + d)/sqrt(c*x^2
 + b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2462 $\int \frac{(d+ex)^{5/2}}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=509

$$-\frac{8\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2cd-be)\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2e\sqrt{b^2-4ac}}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)}{15c^3\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] (8*e*(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(15*c^2) + (2*e*(d
+ e*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*c) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(23*
c^2*d^2 + 8*b^2*e^2 - c*e*(23*b*d + 9*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*
x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e)])/(15*c^3*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (8*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*d -
 b*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sq
rt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^
2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*c^3*Sqrt[d + e*x]*S
qrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.541135, antiderivative size = 509, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {742, 832, 843, 718, 424, 419}

$$-\frac{8\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2cd-be)\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{15c^3\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (8*e*(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(15*c^2) + (2*e*(d
+ e*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*c) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(23*
c^2*d^2 + 8*b^2*e^2 - c*e*(23*b*d + 9*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*
x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e)])/(15*c^3*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (8*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*d -
 b*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sq
rt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^
2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*c^3*Sqrt[d + e*x]*S
qrt[a + b*x + c*x^2])
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
```

```
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\sqrt{a+bx+cx^2}}dx = \frac{2e(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5c} + \frac{2\int \frac{\sqrt{d+ex}\left(\frac{1}{2}\left(5cd^2-e(bd+3ae)\right)+2e(2cd-be)x\right)}{\sqrt{a+bx+cx^2}}dx}{5c}$$

$$= \frac{8e(2cd-be)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15c^2} + \frac{2e(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5c} + \frac{4\int \frac{\frac{1}{4}\left(15c^2d^3+4be^2(bd+ae)-cde(11b\ldots\right.}{\sqrt{d\ldots}}}{\ldots}$$

$$= \frac{8e(2cd-be)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15c^2} + \frac{2e(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5c} - \frac{\left(4(2cd-be)\left(cd^2-bde+ae^2\ldots\right.\right.}{15c^2}$$

$$= \frac{8e(2cd-be)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15c^2} + \frac{2e(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5c} + \frac{\left(\sqrt{2}\sqrt{b^2-4ac}\left(23c^2d^2+8b^2\ldots\right.\right.}{\ldots}$$

$$= \frac{8e(2cd-be)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15c^2} + \frac{2e(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5c} + \frac{\sqrt{2}\sqrt{b^2-4ac}\left(23c^2d^2+8b^2e^2\ldots\right.}{\ldots}$$

Mathematica [C] time = 9.08862, size = 868, normalized size = 1.71

$$\frac{2\sqrt{cx^2+bx+a}\left(\left(23c^2d^2+8b^2e^2-ce(23bd+9ae)\right)\left(c\left(\frac{d}{d+ex}-1\right)^2+\frac{e\left(-\frac{db}{d+ex}+b+\frac{ae}{d+ex}\right)}{d+ex}\right)-\frac{i\sqrt{1-\frac{2\left(cd^2+e(ae-bd)\right)}{\left(2cd-be+\sqrt{(b^2-4ac)e^2}\right)(d+ex)}}\sqrt{1-\frac{2\left(cd^2\ldots\right.}{\left(-2cd+be+\sqrt{\ldots}\right.}}\ldots}{\ldots}\right)}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[d + e*x]*((-2*e*(-11*c*d + 4*b*e))/(15*c^2) + (2*e^2*x)/(5*c))*(a + b
*x + c*x^2))/Sqrt[a + x*(b + c*x)] + (2*(d + e*x)^(3/2)*Sqrt[a + b*x + c*x^
2]*((23*c^2*d^2 + 8*b^2*e^2 - c*e*(23*b*d + 9*a*e))*(c*(-1 + d/(d + e*x))^2
 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)) - ((I/2)*Sqrt[1 -
 (2*(c*d^2 + e*(-(b*d) + a*e)))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d
 + e*x))]*Sqrt[1 + (2*(c*d^2 + e*(-(b*d) + a*e)))/((-2*c*d + b*e + Sqrt[(b^
2 - 4*a*c)*e^2])*(d + e*x))]*((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(23*c
^2*d^2 + 8*b^2*e^2 - c*e*(23*b*d + 9*a*e))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqr
t[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d
 + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b
^2 - 4*a*c)*e^2]))] + (-30*c^3*d^3 + 8*b^2*e^2*(b*e - Sqrt[(b^2 - 4*a*c)*e^
2]) - c^2*d*(-45*b*d*e - 34*a*e^2 + 23*d*Sqrt[(b^2 - 4*a*c)*e^2]) + c*e*(-3
1*b^2*d*e - 17*a*b*e^2 + 23*b*d*Sqrt[(b^2 - 4*a*c)*e^2] + 9*a*e*Sqrt[(b^2 -
 4*a*c)*e^2]))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-
2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e +
Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]))/(Sqrt[
```

```
2]*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])
]*Sqrt[d + e*x]))))/(15*c^3*e*Sqrt[a + x*(b + c*x)]*Sqrt[((d + e*x)^2*(c*(-1
 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)))
/e^2])
```

Maple [B] time = 0.33, size = 4786, normalized size = 9.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -2/15*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^3*(6*2^(1/2)*(-(e*x+d)*c/(e*(-4*a
*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e
+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^
2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1
/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a
*c+b^2)^(1/2)))^(1/2))*b^3*d*e^3+9*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2
)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b
^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e
-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*
d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2
)))^(1/2))*a^2*c*e^4+23*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d
))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))
^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1
/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(
-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*
c^3*d^4+4*a*b*c*d*e^3-11*x*b*c^2*d^2*e^2+8*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b
^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(
-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(
1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+
b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b
^2)^(1/2)))^(1/2))*a*b^2*e^4-8*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*
e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^
(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c
*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^
(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^
(1/2))*b^3*d*e^3-6*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1
/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2
)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*E
llipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(
-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b^2
*e^4-4*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2
*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*
x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(
1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^
(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2
)*a*c*d*e^3+6*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(
e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*Ellipt
icF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*
c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^
2)^(1/2)*b*c*d^2*e^2+6*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)
)^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^
(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/
2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-
```

```
(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a
*b*c*d*e^3-14*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(
e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*Ellipt
icE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*
c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b*c*d*e^
3-3*x^4*c^3*e^4-10*x^2*b*c^2*d*e^3-9*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1
/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c
+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b
*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*
c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1
/2)))^(1/2))*a^2*c*e^4-15*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c
*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)
))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^
(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)
,(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)
)*c^3*d^4-6*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*
(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b
+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*Elliptic
F(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+
b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c^2*d^2*e^
2+14*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c
*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+
(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/
2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1
/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c^2*d^2*e^2+31*2^
(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*
a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c
+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e
*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e
-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*c*d^2*e^2-21*2^(1/2)*(
-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2
)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(
1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c
/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)
/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*c*d^2*e^2-2*2^(1/2)*(-(e*x+d)
*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))
/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e
*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*
a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-
b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*d*e^3-4*2^(1/2)*(-
(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)
^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1
/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/
(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/
(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*c^2*d^3*e-46*2^
(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*
a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c
+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e
*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e
-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c^2*d^3*e+2*2^(1/2)*(-(e
*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(
1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2
))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e
*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2
*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*a*b*e^4+x^3*b*c^2
*e^4-14*x^3*c^3*d*e^3-3*x^2*a*c^2*e^4+4*x^2*b^2*c*e^4-11*x^2*c^3*d^2*e^2+30
*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(
-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*
```

```
a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(
-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+
b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c^2*d^3*e+4*x*a*b*c*e
^4-14*x*a*c^2*d*e^3+4*x*b^2*c*d*e^3-11*a*c^2*d^2*e^2)/e/(c*e*x^3+b*e*x^2+c*
d*x^2+a*e*x+b*d*x+a*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/sqrt(c*x^2 + b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(e^2x^2+2\,dex+d^2\right)\sqrt{ex+d}}{\sqrt{cx^2+bx+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*sqrt(e*x + d)/sqrt(c*x^2 + b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^{\frac{5}{2}}}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**(5/2)/sqrt(a + b*x + c*x**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2463 $\int \frac{(d+ex)^{3/2}}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=439

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2e\sqrt{b^2-4ac}}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)}{3c^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}+$$

```
[Out] (2*e*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(3*c) + (2*Sqrt[2]*Sqrt[b^2 - 4*a
*c]*(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))
]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/
Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3
*c^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x +
 c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d
+ e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(
b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b
^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*
a*c])*e)])/(3*c^2*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.320669, antiderivative size = 439, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {742, 843, 718, 424, 419}

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{3c^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}+\frac{2\sqrt{2}\sqrt{b^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/Sqrt[a + b*x + c*x^2],x]
```


```
[Out] (2*e*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(3*c) + (2*Sqrt[2]*Sqrt[b^2 - 4*a
*c]*(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))
]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/
Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3
*c^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x +
 c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d
+ e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(
b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b
^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*
a*c])*e)])/(3*c^2*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```


Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\sqrt{a+bx+cx^2}}\,dx = \frac{2e\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3c} + \frac{2\int \frac{\frac{1}{2}\left(3cd^2-e(bd+ae)\right)+e(2cd-be)x}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx}{3c}$$

$$= \frac{2e\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3c} + \frac{(2(2cd-be))\int \frac{\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx}{3c} - \frac{\left(cd^2-bde+ae^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}}{3c}$$

$$= \frac{2e\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3c} + \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}(2cd-be)\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2}{2cd}}}{\sqrt{}}\right.}{3c^2\sqrt{\frac{c(d+ex)}{2cd-be-\sqrt{b^2-4ace}}}\sqrt{a+bx+cx^2}}$$

$$= \frac{2e\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3c} + \frac{2\sqrt{2}\sqrt{b^2-4ac}(2cd-be)\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4a}}{\sqrt{b^2-4}}}}{\sqrt{2}}\right.\right.}{3c^2\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 5.65868, size = 609, normalized size = 1.39

$$2\sqrt{d+ex}\left(ce^2(a+x(b+cx))+(d+ex)\left(\frac{2e^2(a+x(b+cx))(2cd-be)}{(d+ex)^2}+\frac{i\sqrt{1-\frac{2\left(e(ae-bd)+cd^2\right)}{(d+ex)\left(\sqrt{e^2(b^2-4ac)}-be+2cd\right)}}\sqrt{\frac{2\left(e(ae-bd)+cd^2\right)}{(d+ex)\left(\sqrt{e^2(b^2-4ac)}+be-2cd\right)}+1}\left(c\left(2d\sqrt{e^2(b^2-\right.\right.}{}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[d + e*x]*(c*e^2*(a + x*(b + c*x)) + (d + e*x)*((2*e^2*(2*c*d - b*e)
*(a + x*(b + c*x)))/(d + e*x)^2 + (I*Sqrt[1 - (2*(c*d^2 + e*(-(b*d) + a*e))
)/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[1 + (2*(c*d^2 +
 e*(-(b*d) + a*e)))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*(
(-2*c*d + b*e)*(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*EllipticE[I*ArcSinh[
(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^
2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b
*e + Sqrt[(b^2 - 4*a*c)*e^2]))] + (3*c^2*d^2 + b*e*(b*e - Sqrt[(b^2 - 4*a*c
)*e^2]) + c*(-3*b*d*e - a*e^2 + 2*d*Sqrt[(b^2 - 4*a*c)*e^2]))*EllipticF[I*A
rcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*
a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*
c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]))/(Sqrt[2]*Sqrt[(c*d^2 + e*(-(b*d) +
 a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[d + e*x]))))/(3*c^2*e
*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.31, size = 2926, normalized size = 6.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/3*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)*(3*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^
2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-
4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1
/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b
*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^
2)^(1/2)))^(1/2))*a*b*e^3-6*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2
*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/
2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)
)^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/
2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/
2))*a*c*d*e^2-2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(
e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*Ellipt
icF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*
c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^
2)^(1/2)*a*e^3-3*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2
)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*
(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*Ell
```

```
ipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4
*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*d*e
^2+9*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c
*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+
(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/
2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1
/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*d^2*e+2^(1/2)*(
-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2
)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(
1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c
/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)
/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*b*d*e^2-6*2^(1
/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*
c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b
^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x
+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2
*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*c^2*d^3-2^(1/2)*(-(e*x+d)*c/
(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2
*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-
4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c
+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e
+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*c*d^2*e-4*2^(1/2)*(-(e*x+
d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2
))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/
(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-
4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*
d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b*e^3+8*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*
c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+
e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2
)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/
2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*
c+b^2)^(1/2)))^(1/2))*a*c*d*e^2+4*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)
+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^
2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-
2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d
))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)
))^(1/2))*b^2*d*e^2-12*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)
)^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^
(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/
2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-
(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b
*c*d^2*e+8*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(
-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+
2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE
(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b
^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*c^2*d^3-2*x^3
*c^2*e^3-2*x^2*b*c*e^3-2*x^2*c^2*d*e^2-2*x*a*c*e^3-2*x*b*c*d*e^2-2*a*d*e^2*
c)/c^2/e/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/sqrt(c*x^2 + b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cx^2+bx+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^(3/2)/sqrt(c*x^2 + b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^{\frac{3}{2}}}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)/sqrt(a + b*x + c*x**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2464 $\int \frac{\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=188

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

```
[Out] (Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
- 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e)])/(c*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a +
b*x + c*x^2])
```

Rubi [A] time = 0.0686928, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {718, 424}

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/Sqrt[a + b*x + c*x^2],x]
```


```
[Out] (Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
- 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e)])/(c*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a +
b*x + c*x^2])
```


Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx = \frac{\left(\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{b^2-4ac}ex^2}{2cd-be-\sqrt{b^2-4ac}e}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)}{c\sqrt{\frac{c(d+ex)}{2cd-be-\sqrt{b^2-4ac}e}}\sqrt{a+bx+cx^2}}$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 0.787024, size = 365, normalized size = 1.94

$$\frac{i\left(e\left(\sqrt{b^2-4ac}-b\right)+2cd\right)\sqrt{\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{e\left(\sqrt{b^2-4ac}+b\right)-2cd}}\sqrt{1-\frac{2c(d+ex)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}\left(E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{\left(b+\sqrt{b^2-4ac}\right)e-2cd}}\sqrt{d+ex}\right)\middle|\frac{2cd-\left(b+\sqrt{b^2\cdots}\right.}{2cd+\left(\sqrt{b^2-\cdots}\right.}\right.\right.}{\sqrt{2}ce\sqrt{a+x(b+cx)}\sqrt{\frac{c}{e\left(\sqrt{b^2-4ac}+b\right)-2cd}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (I*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*Sqrt[(e*(b + Sqrt[b^2 - 4*a*c] + 2*
c*x))/(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d
 + (-b + Sqrt[b^2 - 4*a*c])*e)]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(-2*c*d
 + (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[d + e*x]], (2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e)/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)] - EllipticF[I*ArcSinh[Sqrt[2]
*Sqrt[c/(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[d + e*x]], (2*c*d - (b +
 Sqrt[b^2 - 4*a*c])*e)/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)]))/(Sqrt[2]*c*e
*Sqrt[c/(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.306, size = 747, normalized size = 4.

$$\frac{\sqrt{2}}{2\,e\left(cex^3+bex^2+cdx^2+aex+bdx+ad\right)c^2}\sqrt{ex+d}\sqrt{cx^2+bx+a}\left(e\sqrt{-4\,ac+b^2}+be-2\,cd\right)\sqrt{-c\,(ex+d)\left(e\sqrt{-4\,ac+b\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/2*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)*(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)*2^(1
/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*
c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b
^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(EllipticF(2^(1/2)*(-(e*
x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-
2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*e*b-2*d*EllipticF(2^(1/2)*(
-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+
```

```
b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*c-EllipticF(2^(1/2)*(-(
e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*
e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*e*(-4*a*c+b^2)^(1/2)-Elli
pticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*
a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b*e+2*El
lipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-
4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*c*d+El
lipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-
4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*
c+b^2)^(1/2)*e)/e/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/sqrt(c*x^2 + b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex+d}}{\sqrt{cx^2+bx+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)/sqrt(c*x^2 + b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/sqrt(a + b*x + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)/sqrt(c*x^2 + b*x + a), x)
```

3.2465 $\int \frac{1}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=189

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2e\sqrt{b^2-4ac}}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)}{c\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*
a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2
 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*Sqrt[d + e*x]*Sqrt[a
+ b*x + c*x^2])
```

Rubi [A] time = 0.0711155, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {718, 419}

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*
a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2
 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*Sqrt[d + e*x]*Sqrt[a
+ b*x + c*x^2])
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}dx = \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}\sqrt{\frac{c(d+ex)}{2cd-be-\sqrt{b^2-4ac}e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{b^2-4ac}ex^2}{2cd-be-\sqrt{b^2-4ac}e}}}dx, x, \frac{\sqrt{b}}{}\right.}{c\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$$= \frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right.}{c\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 0.694371, size = 308, normalized size = 1.63

$$\frac{i(d+ex)\sqrt{2-\frac{4\left(e(ae-bd)+cd^2\right)}{(d+ex)\left(\sqrt{e^2(b^2-4ac)}-be+2cd\right)}}\sqrt{\frac{2\left(e(ae-bd)+cd^2\right)}{(d+ex)\left(\sqrt{e^2(b^2-4ac)}+be-2cd\right)}+1}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{ae^2-bde+cd^2}{\sqrt{e^2(b^2-4ac)}+be-2cd}}}{\sqrt{d+ex}}\right), -\frac{\sqrt{e^2(b^2-4ac)}}{\sqrt{e^2(b^2-4ac)}}\right.}{e\sqrt{a+x(b+cx)}\sqrt{\frac{e(ae-bd)+cd^2}{\sqrt{e^2(b^2-4ac)}+be-2cd}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (I*(d + e*x)*Sqrt[2 - (4*(c*d^2 + e*(-(b*d) + a*e)))/((2*c*d - b*e + Sqrt[(
b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[1 + (2*(c*d^2 + e*(-(b*d) + a*e)))/((-2
*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*EllipticF[I*ArcSinh[(Sqrt
[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])
/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e +
Sqrt[(b^2 - 4*a*c)*e^2]))])/(e*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*
e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[a + x*(b + c*x)])
```

Maple [A] time = 0.306, size = 287, normalized size = 1.5

$$\frac{\sqrt{2}}{ce\left(cex^3+bex^2+cdx^2+aex+bdx+ad\right)}\left(-e\sqrt{-4\,ac+b^2}-be+2\,cd\right)\text{EllipticF}\left(\sqrt{2}\sqrt{-c\,(ex+d)\left(e\sqrt{-4\,ac+b^2}+be-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] (-e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)/c*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*
c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*
e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+
b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*
(-4*a*c+b^2)^(1/2)))^(1/2)*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*
c*d))^(1/2)/e*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*
e*x+b*d*x+a*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^2+bx+a}\sqrt{ex+d}}{cex^3+(cd+be)x^2+ad+(bd+ae)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(c*e*x^3 + (c*d + b*e)*x^2 + a
*d + (b*d + a*e)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)), x)
```

3.2466 $\int \frac{1}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=248

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2e\sqrt{a+bx+cx^2}}{\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

```
[Out] (-2*e*Sqrt[a + b*x + c*x^2])/((c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x]) + (Sqr
t[2]*Sqrt[b^2 - 4*a*c]*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*
a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a
*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)
])/((c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e)]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.129045, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {744, 21, 718, 424}

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2e\sqrt{a+bx+cx^2}}{\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-2*e*Sqrt[a + b*x + c*x^2])/((c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x]) + (Sqr
t[2]*Sqrt[b^2 - 4*a*c]*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*
a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a
*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)
])/((c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e)]*Sqrt[a + b*x + c*x^2])
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = -\frac{2e\sqrt{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} - \frac{2\int \frac{-\frac{cd}{2}-\frac{cex}{2}}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{2e\sqrt{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} + \frac{c\int \frac{\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{2e\sqrt{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} + \frac{\left(\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\cdots}}{\cdots}\cdots\right)}{\left(cd^2-bde+ae^2\right)\sqrt{\frac{c(d+ex)}{2cd-be-\sqrt{b^2-4ac}e}}\cdots}$$

$$= -\frac{2e\sqrt{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} + \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-\cdots}}{\sqrt{b^2\cdots}}}}{\sqrt{2}\cdots}\right)\cdots\right)}{\left(cd^2-bde+ae^2\right)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{\cdots}}$$

Mathematica [C] time = 0.793041, size = 408, normalized size = 1.65

$$\frac{-\frac{4e^2(a+x(b+cx))}{\sqrt{d+ex}} + \frac{i\sqrt{2}\left(e\left(\sqrt{b^2-4ac}-b\right)+2cd\right)\sqrt{\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{e\left(\sqrt{b^2-4ac}+b\right)-2cd}}\sqrt{1-\frac{2c(d+ex)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}\left(E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{\left(b+\sqrt{b^2-4ac}\right)e-2cd}}\sqrt{d+ex}\right)\middle|\frac{2cd-\left(b+\sqrt{b^2-4ac}\right)e}{2cd+\left(\sqrt{b^2-4ac}-b\right)e}\right)-\text{Ellip}\cdots\right)}{\sqrt{\frac{c}{e\left(\sqrt{b^2-4ac}+b\right)-2cd}}}}{2e\sqrt{a+x(b+cx)}\left(e(ae-bd)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((-4*e^2*(a + x*(b + c*x)))/Sqrt[d + e*x] + (I*Sqrt[2]*(2*c*d + (-b + Sqrt[
b^2 - 4*a*c])*e)*Sqrt[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*d + (b + Sq
rt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d + (-b + Sqrt[b^2 - 4*a
*c])*e)]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(-2*c*d + (b + Sqrt[b^2 - 4*a*
c])*e)]*Sqrt[d + e*x]], (2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d + (-b +
Sqrt[b^2 - 4*a*c])*e)] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(-2*c*d + (b +
Sqrt[b^2 - 4*a*c])*e)]*Sqrt[d + e*x]], (2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)/
(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)]))/Sqrt[c/(-2*c*d + (b + Sqrt[b^2 - 4*
a*c])*e)])/(2*e*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.334, size = 1365, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 2*(2^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^
(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^
(1/2))*a*e^2*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c
*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+
(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)-2^(1/2)*Ellipti
cF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c
+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b*d*e*(-(e*
x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1
/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2)
)/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)+2^(1/2)*EllipticF(2^(1/2)*(-(e*x+
d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*
c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*c*d^2*(-(e*x+d)*c/(e*(-4*a*c+
b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*
(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^
(1/2)+b*e-2*c*d))^(1/2)-2^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^
2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*
(-4*a*c+b^2)^(1/2)))^(1/2))*a*e^2*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c
*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)
))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^
(1/2)+2^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d
))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)
))^(1/2))*b*d*e*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-
2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c
*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)-2^(1/2)*Elli
pticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*
a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*c*d^2*(-
(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)
^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1
/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)-c*e^2*x^2-x*b*e^2-a*e^2)*(c*x^
2+b*x+a)^(1/2)*(e*x+d)^(1/2)/e/(a*e^2-b*d*e+c*d^2)/(c*e*x^3+b*e*x^2+c*d*x^2
+a*e*x+b*d*x+a*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}(ex + d)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+bx+a}\sqrt{ex+d}}{ce^2x^4+\left(2\,cde+be^2\right)x^3+ad^2+\left(cd^2+2\,bde+ae^2\right)x^2+\left(bd^2+2\,ade\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(c*e^2*x^4 + (2*c*d*e + b*e^2)
*x^3 + a*d^2 + (c*d^2 + 2*b*d*e + a*e^2)*x^2 + (b*d^2 + 2*a*d*e)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)^{\frac{3}{2}}\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x)**(3/2)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2467 $\int \frac{1}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=523

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2e\sqrt{b^2-4ac}}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)}{3\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}+\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{d}\ldots}{\ldots}$$

```
[Out] (-2*e*Sqrt[a + b*x + c*x^2])/(3*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(3/2)) -
(4*e*(2*c*d - b*e)*Sqrt[a + b*x + c*x^2])/(3*(c*d^2 - b*d*e + a*e^2)^2*Sqrt
[d + e*x]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[
-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2
 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2
*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[(c*(d
 + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*S
qrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4
*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*(c*d^2 - b*d*e + a*e^2)*S
qrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.412085, antiderivative size = 523, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {744, 834, 843, 718, 424, 419}

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{3\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}+\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{\ldots}}{3\sqrt{a+\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(5/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-2*e*Sqrt[a + b*x + c*x^2])/(3*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(3/2)) -
(4*e*(2*c*d - b*e)*Sqrt[a + b*x + c*x^2])/(3*(c*d^2 - b*d*e + a*e^2)^2*Sqrt
[d + e*x]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[
-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2
 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2
*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[(c*(d
 + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*S
qrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4
*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*(c*d^2 - b*d*e + a*e^2)*S
qrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
```

```
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx = -\frac{2e\sqrt{a+bx+cx^2}}{3(cd^2-bde+ae^2)(d+ex)^{3/2}} - \frac{2\int \frac{\frac{1}{2}(-3cd+2be)+\frac{cex}{2}}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx}{3(cd^2-bde+ae^2)}$$

$$= -\frac{2e\sqrt{a+bx+cx^2}}{3(cd^2-bde+ae^2)(d+ex)^{3/2}} - \frac{4e(2cd-be)\sqrt{a+bx+cx^2}}{3(cd^2-bde+ae^2)^2\sqrt{d+ex}} + \frac{4\int \frac{\frac{1}{4}c(3cd^2-e(bd+ae))+\frac{1}{2}\cdots}{\sqrt{d+ex}\sqrt{a+bx\cdots}}}{3(cd^2-bde+\cdots}$$

$$= -\frac{2e\sqrt{a+bx+cx^2}}{3(cd^2-bde+ae^2)(d+ex)^{3/2}} - \frac{4e(2cd-be)\sqrt{a+bx+cx^2}}{3(cd^2-bde+ae^2)^2\sqrt{d+ex}} + \frac{(2c(2cd-be))\int \frac{\sqrt{\cdots}}{\sqrt{a+\cdots}}}{3(cd^2-bde+a\cdots}$$

$$= -\frac{2e\sqrt{a+bx+cx^2}}{3(cd^2-bde+ae^2)(d+ex)^{3/2}} - \frac{4e(2cd-be)\sqrt{a+bx+cx^2}}{3(cd^2-bde+ae^2)^2\sqrt{d+ex}} + \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}(2cd\cdots\right.}{\cdots}$$

$$= -\frac{2e\sqrt{a+bx+cx^2}}{3(cd^2-bde+ae^2)(d+ex)^{3/2}} - \frac{4e(2cd-be)\sqrt{a+bx+cx^2}}{3(cd^2-bde+ae^2)^2\sqrt{d+ex}} + \frac{2\sqrt{2}\sqrt{b^2-4ac}(2cd-\cdots}{3\cdots}$$

Mathematica [C] time = 7.74221, size = 643, normalized size = 1.23

$$2\left(\frac{i(d+ex)^{5/2}\sqrt{1-\frac{2(e(ae-bd)+cd^2)}{(d+ex)\left(\sqrt{e^2(b^2-4ac)}-be+2cd\right)}}\sqrt{\frac{2(e(ae-bd)+cd^2)}{(d+ex)\left(\sqrt{e^2(b^2-4ac)}+be-2cd\right)}+1}\left(\left(c\left(2d\sqrt{e^2(b^2-4ac)}-ae^2-3bde\right)+be\left(be-\sqrt{e^2(b^2-4ac)}\right)+3c^2d^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{2}\cdots}{\cdots}\right.\right.\right.}{\sqrt{2}\sqrt{\frac{e(ae-bd)+cd^2}{\sqrt{e^2(b^2-4ac)}+be\cdots}}}\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(5/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*(2*e^2*(2*c*d - b*e)*(d + e*x)*(a + x*(b + c*x)) + e^2*(a + x*(b + c*x))
*(-(c*d*(5*d + 4*e*x)) + e*(3*b*d - a*e + 2*b*e*x)) + (I*(d + e*x)^(5/2)*Sq
rt[1 - (2*(c*d^2 + e*(-(b*d) + a*e)))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^
2])*(d + e*x))]*Sqrt[1 + (2*(c*d^2 + e*(-(b*d) + a*e)))/((-2*c*d + b*e + Sq
rt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*((-2*c*d + b*e)*(2*c*d - b*e + Sqrt[(b^2
 - 4*a*c)*e^2])*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(
-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e +
 Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] + (3*c^
2*d^2 + b*e*(b*e - Sqrt[(b^2 - 4*a*c)*e^2]) + c*(-3*b*d*e - a*e^2 + 2*d*Sqr
t[(b^2 - 4*a*c)*e^2]))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a
*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d
+ b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])
)/(Sqrt[2]*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*
c)*e^2])])))/(3*e*(c*d^2 + e*(-(b*d) + a*e))^2*(d + e*x)^(3/2)*Sqrt[a + x*(
b + c*x)])
```

Maple [B] time = 0.39, size = 5824, normalized size = 11.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}(ex + d)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2 + bx + a}\sqrt{ex + d}}{ce^3x^5 + \left(3\,cde^2 + be^3\right)x^4 + ad^3 + \left(3\,cd^2e + 3\,bde^2 + ae^3\right)x^3 + \left(cd^3 + 3\,bd^2e + 3\,ade^2\right)x^2 + \left(bd^3 + 3\,ad^2e\right)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(c*e^3*x^5 + (3*c*d*e^2 + b*e^
3)*x^4 + a*d^3 + (3*c*d^2*e + 3*b*d*e^2 + a*e^3)*x^3 + (c*d^3 + 3*b*d^2*e +
 3*a*d*e^2)*x^2 + (b*d^3 + 3*a*d^2*e)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex)^{\frac{5}{2}}\sqrt{a + bx + cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(5/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x)**(5/2)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2468 $\int \frac{1}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=629

$$-\frac{8\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2cd-be)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2e\sqrt{b^2-4ac}}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)}{15\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)^2}-\frac{2e\sqrt{a\cdots}}{\cdots}$$

```
[Out] (-2*e*Sqrt[a + b*x + c*x^2])/(5*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(5/2)) -
(8*e*(2*c*d - b*e)*Sqrt[a + b*x + c*x^2])/(15*(c*d^2 - b*d*e + a*e^2)^2*(d
+ e*x)^(3/2)) - (2*e*(23*c^2*d^2 + 8*b^2*e^2 - c*e*(23*b*d + 9*a*e))*Sqrt[a
 + b*x + c*x^2])/(15*(c*d^2 - b*d*e + a*e^2)^3*Sqrt[d + e*x]) + (Sqrt[2]*Sq
rt[b^2 - 4*a*c]*(23*c^2*d^2 + 8*b^2*e^2 - c*e*(23*b*d + 9*a*e))*Sqrt[d + e*
x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b +
Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*
c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*(c*d^2 - b*d*e + a*e^2)^3*S
qrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2
]) - (8*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*Sqrt[(c*(d + e*x))/(2*c*d -
 (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*E
llipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqr
t[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*(
c*d^2 - b*d*e + a*e^2)^2*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.686662, antiderivative size = 629, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {744, 834, 843, 718, 424, 419}

$$-\frac{2e\sqrt{a+bx+cx^2}\left(-ce(9ae+23bd)+8b^2e^2+23c^2d^2\right)}{15\sqrt{d+ex}\left(ae^2-bde+cd^2\right)^3}+\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(-ce(9ae+23bd)+8b^2\cdots\right.}{15\sqrt{a+bx+cx^2}\left(ae^2-bde\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(7/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-2*e*Sqrt[a + b*x + c*x^2])/(5*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(5/2)) -
(8*e*(2*c*d - b*e)*Sqrt[a + b*x + c*x^2])/(15*(c*d^2 - b*d*e + a*e^2)^2*(d
+ e*x)^(3/2)) - (2*e*(23*c^2*d^2 + 8*b^2*e^2 - c*e*(23*b*d + 9*a*e))*Sqrt[a
 + b*x + c*x^2])/(15*(c*d^2 - b*d*e + a*e^2)^3*Sqrt[d + e*x]) + (Sqrt[2]*Sq
rt[b^2 - 4*a*c]*(23*c^2*d^2 + 8*b^2*e^2 - c*e*(23*b*d + 9*a*e))*Sqrt[d + e*
x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b +
Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*
c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*(c*d^2 - b*d*e + a*e^2)^3*S
qrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2
]) - (8*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*Sqrt[(c*(d + e*x))/(2*c*d -
 (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*E
llipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqr
t[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*(
```

```
c*d^2 - b*d*e + a*e^2)^2*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx = -\frac{2e\sqrt{a+bx+cx^2}}{5\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} - \frac{2\int \frac{\frac{1}{2}(-5cd+4be)+\frac{3cex}{2}}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}dx}{5\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{2e\sqrt{a+bx+cx^2}}{5\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} - \frac{8e(2cd-be)\sqrt{a+bx+cx^2}}{15\left(cd^2-bde+ae^2\right)^2(d+ex)^{3/2}} + \frac{4\int \frac{\frac{1}{4}(15c^2d^2+8b}{(}}{15(}$$

$$= -\frac{2e\sqrt{a+bx+cx^2}}{5\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} - \frac{8e(2cd-be)\sqrt{a+bx+cx^2}}{15\left(cd^2-bde+ae^2\right)^2(d+ex)^{3/2}} - \frac{2e\left(23c^2d^2+8\right.}{15(}$$

$$= -\frac{2e\sqrt{a+bx+cx^2}}{5\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} - \frac{8e(2cd-be)\sqrt{a+bx+cx^2}}{15\left(cd^2-bde+ae^2\right)^2(d+ex)^{3/2}} - \frac{2e\left(23c^2d^2+8\right.}{15(}$$

$$= -\frac{2e\sqrt{a+bx+cx^2}}{5\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} - \frac{8e(2cd-be)\sqrt{a+bx+cx^2}}{15\left(cd^2-bde+ae^2\right)^2(d+ex)^{3/2}} - \frac{2e\left(23c^2d^2+8\right.}{15(}$$

$$= -\frac{2e\sqrt{a+bx+cx^2}}{5\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} - \frac{8e(2cd-be)\sqrt{a+bx+cx^2}}{15\left(cd^2-bde+ae^2\right)^2(d+ex)^{3/2}} - \frac{2e\left(23c^2d^2+8\right.}{15(}$$

Mathematica [C] time = 9.09226, size = 983, normalized size = 1.56

$$2\sqrt{cx^2+bx+a}\left(\left(23c^2d^2+8b^2e^2-ce(23bd+9ae)\right)\left(c\left(\frac{d}{d+ex}-1\right)^2+\frac{e\left(-\frac{db}{d+ex}+b+\frac{ae}{d+ex}\right)}{d+ex}\right)-\frac{i\sqrt{1-\frac{2\left(cd^2+e(ae-bd)\right)}{\left(2cd-be+\sqrt{\left(b^2-4ac\right)e^2}\right)(d+ex)}}\sqrt{\frac{}{\left(-2cd+b\right.}}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(7/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[d + e*x]*(a + b*x + c*x^2)*((-2*e)/(5*(c*d^2 - b*d*e + a*e^2)*(d + e*
x)^3) + (8*e*(-2*c*d + b*e))/(15*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^2) + (
2*e*(-23*c^2*d^2 + 23*b*c*d*e - 8*b^2*e^2 + 9*a*c*e^2))/(15*(c*d^2 - b*d*e
+ a*e^2)^3*(d + e*x))))/Sqrt[a + x*(b + c*x)] + (2*(d + e*x)^(3/2)*Sqrt[a +
 b*x + c*x^2]*((23*c^2*d^2 + 8*b^2*e^2 - c*e*(23*b*d + 9*a*e))*(c*(-1 + d/(
d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)) - ((I/
2)*Sqrt[1 - (2*(c*d^2 + e*(-(b*d) + a*e)))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*
c)*e^2])*(d + e*x))]*Sqrt[1 + (2*(c*d^2 + e*(-(b*d) + a*e)))/((-2*c*d + b*e
 + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*
e^2])*(23*c^2*d^2 + 8*b^2*e^2 - c*e*(23*b*d + 9*a*e))*EllipticE[I*ArcSinh[(
Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2
```

```
])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*
e + Sqrt[(b^2 - 4*a*c)*e^2]))] + (-30*c^3*d^3 + 8*b^2*e^2*(b*e - Sqrt[(b^2
- 4*a*c)*e^2]) - c^2*d*(-45*b*d*e - 34*a*e^2 + 23*d*Sqrt[(b^2 - 4*a*c)*e^2]
) + c*e*(-31*b^2*d*e - 17*a*b*e^2 + 23*b*d*Sqrt[(b^2 - 4*a*c)*e^2] + 9*a*e*
Sqrt[(b^2 - 4*a*c)*e^2]))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e
+ a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c
*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])
)]))/(Sqrt[2]*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4
*a*c)*e^2])]*Sqrt[d + e*x])))/(15*e*(c*d^2 - b*d*e + a*e^2)^3*Sqrt[a + x*(b
 + c*x)]*Sqrt[((d + e*x)^2*(c*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x
) + (a*e)/(d + e*x)))/(d + e*x)))/e^2])
```

Maple [B] time = 0.412, size = 14311, normalized size = 22.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}(ex + d)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2 + bx + a}\sqrt{ex + d}}{ce^4x^6 + \left(4\,cde^3 + be^4\right)x^5 + ad^4 + \left(6\,cd^2e^2 + 4\,bde^3 + ae^4\right)x^4 + 2\left(2\,cd^3e + 3\,bd^2e^2 + 2\,ade^3\right)x^3 + \left(cd^4 + 4\,bd^3e + \right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(c*e^4*x^6 + (4*c*d*e^3 + b*e^
4)*x^5 + a*d^4 + (6*c*d^2*e^2 + 4*b*d*e^3 + a*e^4)*x^4 + 2*(2*c*d^3*e + 3*b
*d^2*e^2 + 2*a*d*e^3)*x^3 + (c*d^4 + 4*b*d^3*e + 6*a*d^2*e^2)*x^2 + (b*d^4
+ 4*a*d^3*e)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex)^{\frac{7}{2}}\sqrt{a + bx + cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(7/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x)**(7/2)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2469 $\int \frac{(d+ex)^{7/2}}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=641

$$-\frac{4\sqrt{2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\left(-ce(5ae+3bd)+2b^2e^2+3c^2d^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{3c^3\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*(d + e*x)^(5/2)*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a
+ b*x + c*x^2]) + (4*e*(3*c^2*d^2 + 2*b^2*e^2 - c*e*(3*b*d + 5*a*e))*Sqrt[d
 + e*x]*Sqrt[a + b*x + c*x^2])/(3*c^2*(b^2 - 4*a*c)) + (2*e*(2*c*d - b*e)*(
d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(c*(b^2 - 4*a*c)) + (Sqrt[2]*(2*c*d -
 b*e)*(3*c^2*d^2 + 8*b^2*e^2 - c*e*(3*b*d + 29*a*e))*Sqrt[d + e*x]*Sqrt[-((
c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 -
4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c^3*Sqrt[b^2 - 4*a*c]*Sqrt[(c*(d + e*x)
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (4*Sqrt[2]*
(c*d^2 - b*d*e + a*e^2)*(3*c^2*d^2 + 2*b^2*e^2 - c*e*(3*b*d + 5*a*e))*Sqrt[
(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x
^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/
Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^
2 - 4*a*c])*e)])/(3*c^3*Sqrt[b^2 - 4*a*c]*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^
2])
```

Rubi [A] time = 0.949103, antiderivative size = 641, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {738, 832, 843, 718, 424, 419}

$$\frac{4e\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(-ce(5ae+3bd)+2b^2e^2+3c^2d^2\right)}{3c^2\left(b^2-4ac\right)}-\frac{4\sqrt{2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\left(-ce(5ae+3bd)+2b^2\right.}{3c^3\sqrt{b^2-4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/(a + b*x + c*x^2)^(3/2),x]
```


```
[Out] (-2*(d + e*x)^(5/2)*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a
+ b*x + c*x^2]) + (4*e*(3*c^2*d^2 + 2*b^2*e^2 - c*e*(3*b*d + 5*a*e))*Sqrt[d
 + e*x]*Sqrt[a + b*x + c*x^2])/(3*c^2*(b^2 - 4*a*c)) + (2*e*(2*c*d - b*e)*(
d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(c*(b^2 - 4*a*c)) + (Sqrt[2]*(2*c*d -
 b*e)*(3*c^2*d^2 + 8*b^2*e^2 - c*e*(3*b*d + 29*a*e))*Sqrt[d + e*x]*Sqrt[-((
c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 -
4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c^3*Sqrt[b^2 - 4*a*c]*Sqrt[(c*(d + e*x)
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (4*Sqrt[2]*
(c*d^2 - b*d*e + a*e^2)*(3*c^2*d^2 + 2*b^2*e^2 - c*e*(3*b*d + 5*a*e))*Sqrt[
(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x
^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/
```

```
Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^
2 - 4*a*c])*e)])/(3*c^3*Sqrt[b^2 - 4*a*c]*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^
2])
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^{7/2}}{\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2(d+ex)^{5/2}(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} - \frac{2\int \frac{(d+ex)^{3/2}\left(-\frac{5}{2}e(bd-2ae)-\frac{5}{2}e(2cd-be)x\right)}{\sqrt{a+bx+cx^2}}dx}{b^2-4ac}$$

$$= -\frac{2(d+ex)^{5/2}(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{2e(2cd-be)(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{c\left(b^2-4ac\right)} - \frac{4\int \frac{\sqrt{d+ex}\left(-\frac{5}{4}e(\ldots\right.}{\ldots}}{\ldots}$$

$$= -\frac{2(d+ex)^{5/2}(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{4e\left(3c^2d^2+2b^2e^2-ce(3bd+5ae)\right)\sqrt{d+ex}\sqrt{a+bx+\ldots}}{3c^2\left(b^2-4ac\right)}$$

$$= -\frac{2(d+ex)^{5/2}(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{4e\left(3c^2d^2+2b^2e^2-ce(3bd+5ae)\right)\sqrt{d+ex}\sqrt{a+bx+\ldots}}{3c^2\left(b^2-4ac\right)}$$

$$= -\frac{2(d+ex)^{5/2}(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{4e\left(3c^2d^2+2b^2e^2-ce(3bd+5ae)\right)\sqrt{d+ex}\sqrt{a+bx+\ldots}}{3c^2\left(b^2-4ac\right)}$$

$$= -\frac{2(d+ex)^{5/2}(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{4e\left(3c^2d^2+2b^2e^2-ce(3bd+5ae)\right)\sqrt{d+ex}\sqrt{a+bx+\ldots}}{3c^2\left(b^2-4ac\right)}$$

Mathematica [C] time = 12.9804, size = 5433, normalized size = 8.48

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(7/2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.404, size = 6486, normalized size = 10.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{7}{2}}}{\left(cx^2+bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(7/2)/(c*x^2 + b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e^{3}x^{3} + 3\,de^{2}x^{2} + 3\,d^{2}ex + d^{3}\right)\sqrt{cx^{2} + bx + a}\sqrt{ex + d}}{c^{2}x^{4} + 2\,bcx^{3} + 2\,abx + \left(b^{2} + 2\,ac\right)x^{2} + a^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(c*x^2 + b*x + a)*sq
rt(e*x + d)/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2470 $\int \frac{(d+ex)^{5/2}}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=533

$$-\frac{2\sqrt{2}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2cd-be)\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2e\sqrt{b^2-4ac}}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)}{c^2\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{a+bx+cx^2}}+$$

```
[Out] (-2*(d + e*x)^(3/2)*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a
+ b*x + c*x^2]) + (2*e*(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(
c*(b^2 - 4*a*c)) + (2*Sqrt[2]*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*Sqrt[
d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt
[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2
- 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c^2*Sqrt[b^2 - 4*a*c]*Sq
rt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]
) - (2*Sqrt[2]*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*
c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c
))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]
]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/
(c^2*Sqrt[b^2 - 4*a*c]*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.5121, antiderivative size = 533, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {738, 832, 843, 718, 424, 419}

$$-\frac{2\sqrt{2}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2cd-be)\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c^2\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{a+bx+cx^2}}+\frac{2\sqrt{2}\sqrt{d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(a + b*x + c*x^2)^(3/2),x]
```


```
[Out] (-2*(d + e*x)^(3/2)*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a
+ b*x + c*x^2]) + (2*e*(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(
c*(b^2 - 4*a*c)) + (2*Sqrt[2]*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*Sqrt[
d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt
[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2
- 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c^2*Sqrt[b^2 - 4*a*c]*Sq
rt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]
) - (2*Sqrt[2]*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*
c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c
))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]
]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/
(c^2*Sqrt[b^2 - 4*a*c]*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```


Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2(d+ex)^{3/2}(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} - \frac{2\int \frac{\sqrt{d+ex}\left(-\frac{3}{2}e(bd-2ae)-\frac{3}{2}e(2cd-be)x\right)}{\sqrt{a+bx+cx^2}}dx}{b^2-4ac}$$

$$= -\frac{2(d+ex)^{3/2}(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{2e(2cd-be)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{c\left(b^2-4ac\right)} - 4\int \frac{-\frac{3}{4}e\left(bcd^2+b^2de-\right.}{}$$

$$= -\frac{2(d+ex)^{3/2}(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{2e(2cd-be)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{c\left(b^2-4ac\right)} - \frac{\left((2cd-be)\left(cd^2\right.\right.}{}$$

$$= -\frac{2(d+ex)^{3/2}(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{2e(2cd-be)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{c\left(b^2-4ac\right)} + \frac{\left(2\sqrt{2}\left(c^2d^2+b^2\right.\right.}{}$$

$$= -\frac{2(d+ex)^{3/2}(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{2e(2cd-be)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{c\left(b^2-4ac\right)} + \frac{2\sqrt{2}\left(c^2d^2+b^2e\right.}{}$$

Mathematica [C] time = 11.8235, size = 732, normalized size = 1.37

$$\frac{i(d+ex)\sqrt{1-\frac{2\left(e(ae-bd)+cd^2\right)}{(d+ex)\left(\sqrt{e^2\left(b^2-4ac\right)}-be+2cd\right)}}\sqrt{\frac{4\left(e(ae-bd)+cd^2\right)}{(d+ex)\left(\sqrt{e^2\left(b^2-4ac\right)}+be-2cd\right)}+2}\left(c\left(ae^2\left(3\sqrt{e^2\left(b^2-4ac\right)}+8cd\right)-cd^2\sqrt{e^2\left(b^2-4ac\right)}\right)-b^2e^2\left(\sqrt{e^2\left(b^2-4ac\right)}+2cd\right)+bce\left(d\sqrt{e^2\left(b^2-4\right.\right.}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] ((4*e*(-(c^2*d^2) - b^2*e^2 + c*e*(b*d + 3*a*e))*(a + x*(b + c*x)))/(c*Sqrt
[d + e*x]) + 2*Sqrt[d + e*x]*(a*b*e^2 + 2*c^2*d^2*x + b^2*e^2*x + b*c*d*(d
- 2*e*x) - 2*a*c*e*(2*d + e*x)) + (I*(d + e*x)*Sqrt[1 - (2*(c*d^2 + e*(-(b*
d) + a*e)))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[2 + (
4*(c*d^2 + e*(-(b*d) + a*e)))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d
+ e*x))]*((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(c^2*d^2 + b^2*e^2 - c*e*
(b*d + 3*a*e))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-
2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e +
Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] + (b^3*e
^3 - b^2*e^2*(2*c*d + Sqrt[(b^2 - 4*a*c)*e^2]) + b*c*e*(-4*a*e^2 + d*Sqrt[(
b^2 - 4*a*c)*e^2]) + c*(-(c*d^2*Sqrt[(b^2 - 4*a*c)*e^2]) + a*e^2*(8*c*d + 3
*Sqrt[(b^2 - 4*a*c)*e^2])))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*
e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2
*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2
]))]))/(c*e*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a
*c)*e^2])]))/(c*(-b^2 + 4*a*c)*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.368, size = 4352, normalized size = 8.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] (e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)*(3*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1
/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c
+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b
*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*
c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1
/2)))^(1/2))*b^3*d*e^3+12*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c
*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)
))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^
(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)
,(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)
)*a^2*c*e^4+4*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(
e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*Ellipt
icE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*
c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*c^3*d^4+2*
a*b*c*d*e^3-2*x*b*c^2*d^2*e^2+4*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b
*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)
^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*
c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))
^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))
^(1/2))*a*b^2*e^4-4*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(
1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/
2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*
EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*
(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b^3*
d*e^3-3*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-
2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c
*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^
(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)
^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b^2*e^4-2*2^(1
/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*
c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b
^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x
+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2
*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*a*c*d*e^3
+3*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x
+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-
4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)
*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2
)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*b*
c*d^2*e^2-12*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e
*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(
b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*Ellipti
cF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c
+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b*c*d*e^3
+8*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x
+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-
4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)
*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2
)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b*c*d*e^3+4*x*c^3*d
```

```
^3*e-4*x^2*b*c^2*d*e^3-12*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c
*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)
))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^
(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)
,(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)
)*a^2*c*e^4+12*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*
(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e
*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*Ellip
ticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a
*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c^2*d^2
*e^2-8*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2
*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*
x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(
1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^
(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c^2*d^2*e^2+8*2
^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4
*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*
c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(
e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*
e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*c*d^2*e^2-3*2^(1/2)*(
-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2
)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(
1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c
/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)
/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*c*d^2*e^2-2^(1/2)*(-(e*x+d)*c
/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(
2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(
-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*
c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*
e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*d*e^3-2*2^(1/2)*(-(e
*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(
1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2
))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e
*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2
*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*c^2*d^3*e-8*2^(1/
2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c
+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^
2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+
d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*
c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c^2*d^3*e+2^(1/2)*(-(e*x+d)
*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))
/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e
*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*
a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-
b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*a*b*e^4-4*x^2*a*c^2*e^
4+2*x^2*b^2*c*e^4+4*x^2*c^3*d^2*e^2+2*x*a*b*c*e^4-12*x*a*c^2*d*e^3+2*x*b^2*
c*d*e^3-8*a*c^2*d^2*e^2+2*b*c^2*d^3*e)/c^2/e/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x
+b*d*x+a*d)/(4*a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\left(cx^2+bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/(c*x^2 + b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e^2x^2 + 2\,dex + d^2\right)\sqrt{cx^2 + bx + a}\sqrt{ex + d}}{c^2x^4 + 2\,bcx^3 + 2\,abx + \left(b^2 + 2\,ac\right)x^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(c^2
*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2471 $\int \frac{(d+ex)^{3/2}}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=457

$$-\frac{4\sqrt{2}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2e\sqrt{b^2-4ac}}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)}{c\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{a+bx+cx^2}}-\frac{2\sqrt{d+ex}(}{(b^2-}$$

```
[Out] (-2*Sqrt[d + e*x]*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a +
b*x + c*x^2]) + (Sqrt[2]*(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c
*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)])/(c*Sqrt[b^2 - 4*a*c]*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sq
rt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (4*Sqrt[2]*(c*d^2 - b*d*e + a
*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a
+ b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e)])/(c*Sqrt[b^2 - 4*a*c]*Sqrt[d + e*x]*Sqrt[a + b*x
 + c*x^2])
```

Rubi [A] time = 0.329154, antiderivative size = 457, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {738, 843, 718, 424, 419}

$$-\frac{4\sqrt{2}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{a+bx+cx^2}}-\frac{2\sqrt{d+ex}(-2ae+}{(b^2-4ac)\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a +
b*x + c*x^2]) + (Sqrt[2]*(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c
*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)])/(c*Sqrt[b^2 - 4*a*c]*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sq
rt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (4*Sqrt[2]*(c*d^2 - b*d*e + a
*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a
+ b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e)])/(c*Sqrt[b^2 - 4*a*c]*Sqrt[d + e*x]*Sqrt[a + b*x
 + c*x^2])
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
```

```
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} - \frac{2\int \frac{-\frac{1}{2}e(bd-2ae)-\frac{1}{2}e(2cd-be)x}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac}$$

$$= -\frac{2\sqrt{d+ex}(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{(2cd-be)\int \frac{\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac} - \frac{\left(2\left(cd^2-bde+ae^2\right)\right)\int \frac{\cdots}{\sqrt{d+e\cdots}}}{b^2-4ac}$$

$$= -\frac{2\sqrt{d+ex}(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\left(\sqrt{2}(2cd-be)\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{\cdots}}{2cd-b\cdots}}}{\sqrt{1\cdots}}\cdots\right.}{c\sqrt{b^2-4ac}\sqrt{\frac{c(d+ex)}{2cd-be-\sqrt{b^2-4ac}e}}\sqrt{a+b\cdots}}$$

$$= -\frac{2\sqrt{d+ex}(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\sqrt{2}(2cd-be)\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2\cdots}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\cdots\right.\right.}{c\sqrt{b^2-4ac}\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx\cdots}}$$

Mathematica [C] time = 8.41013, size = 964, normalized size = 2.11

$$\sqrt{d+ex}\left(\frac{4(-bd-2cxd+2ae+bex)}{b^2-4ac} + \frac{(d+ex)\left(\frac{4(be-2cd)\sqrt{\frac{cd^2+e(ae-bd)}{-2cd+be+\sqrt{\left(b^2-4ac\right)e^2}}}(a+x(b+cx))e^2}{(d+ex)^2} + \frac{i\sqrt{2}(2cd-be)\left(2cd-be+\sqrt{\left(b^2-4ac\right)e^2}\right)\sqrt{\frac{-2ae^2+2cdxe+\sqrt{\left(b^2-4ac\right)e^2}xe+b(d-ex)e+d\cdots}{\left(2cd-be+\sqrt{\left(b^2-4ac\right)e^2}\right)(d+ex)}}\cdots}{\cdots}\right)}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d + e*x]*((4*(-(b*d) + 2*a*e - 2*c*d*x + b*e*x))/(b^2 - 4*a*c) + ((d
+ e*x)*((4*e^2*(-2*c*d + b*e)*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e
 + Sqrt[(b^2 - 4*a*c)*e^2])]*(a + x*(b + c*x)))/(d + e*x)^2 + (I*Sqrt[2]*(2
*c*d - b*e)*(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(-2*a*e^2 + d*Sqrt
[(b^2 - 4*a*c)*e^2] + 2*c*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(d - e*
x))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*a*e^2 + d*
Sqrt[(b^2 - 4*a*c)*e^2] - 2*c*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(-d
 + e*x))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*EllipticE[I*
ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4
*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2
*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] - (I*Sqrt[2]*(b^2*e^
2 - 4*a*c*e^2 + 2*c*d*Sqrt[(b^2 - 4*a*c)*e^2] - b*e*Sqrt[(b^2 - 4*a*c)*e^2]
)*Sqrt[(-2*a*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] + 2*c*d*e*x + e*Sqrt[(b^2 - 4*
```

```
a*c)*e^2]*x + b*e*(d - e*x))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d +
e*x))]*Sqrt[(2*a*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] - 2*c*d*e*x + e*Sqrt[(b^2
- 4*a*c)*e^2]*x + b*e*(-d + e*x))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])
*(d + e*x))]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*
c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sq
rt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d +
e*x]))/(c*(-b^2 + 4*a*c)*e*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e +
Sqrt[(b^2 - 4*a*c)*e^2])])))/(2*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.345, size = 1863, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -2*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)*(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(
1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*
c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+
b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2
*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(
1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*a*e^3-2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^
(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a
*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)
+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-
2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^
(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*b*d*e^2+2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^
2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-
4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1
/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b
*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^
2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*c*d^2*e+2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c
+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e
*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)
^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2
)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c
+b^2)^(1/2)))^(1/2))*a*b*e^3-2*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*
e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^
(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c
*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^
(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^
(1/2))*a*c*d*e^2-2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2
)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*
(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*Ell
ipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4
*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*d*e
^2+3*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c
*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+
(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/
2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1
/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*d^2*e-2*2^(1/2)
*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b
^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)
^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)
*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*
d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*c^2*d^3+x^2*b*c*e^3-2*x^2*c^2*d
```

```
*e^2+2*x*a*c*e^3-2*x*c^2*d^2*e+2*a*d*e^2*c-b*c*d^2*e)/c/e/(c*e*x^3+b*e*x^2+
c*d*x^2+a*e*x+b*d*x+a*d)/(4*a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cx^2+bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(c*x^2 + b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}}{c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\,ac\right)x^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)/(c^2*x^4 + 2*b*c*x^3 + 2*a*b
*x + (b^2 + 2*a*c)*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2472 $\int \frac{\sqrt{d+ex}}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=426

$$-\frac{2\sqrt{2}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2cd-be)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2e\sqrt{b^2-4ac}}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)}{c\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{a+bx+cx^2}}-\frac{2(b+2cx)\sqrt{}}{\left(b^2-4ac\right)\sqrt{a+}}$$

```
[Out] (-2*(b + 2*c*x)*Sqrt[d + e*x])/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (2*S
qrt[2]*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE
[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (
-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[b^2 - 4
*a*c]*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x
+ c*x^2]) - (2*Sqrt[2]*(2*c*d - b*e)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[Ar
cSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*
Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*Sqrt[b^2 - 4*
a*c]*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.249196, antiderivative size = 426, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {736, 843, 718, 424, 419}

$$-\frac{2(b+2cx)\sqrt{d+ex}}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2\sqrt{2}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2cd-be)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)}\right)}{c\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(a + b*x + c*x^2)^(3/2),x]
```



```
[Out] (-2*(b + 2*c*x)*Sqrt[d + e*x])/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (2*S
qrt[2]*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE
[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (
-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[b^2 - 4
*a*c]*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x
+ c*x^2]) - (2*Sqrt[2]*(2*c*d - b*e)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[Ar
cSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*
Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*Sqrt[b^2 - 4*
a*c]*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```


Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] - Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)
*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p + 1
```

```
), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 0] && (L
tQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c,
d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2(b+2cx)\sqrt{d+ex}}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{2\int \frac{\frac{be}{2}+cex}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac}$$

$$= -\frac{2(b+2cx)\sqrt{d+ex}}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{(2c)\int \frac{\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac} - \frac{(2cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac}$$

$$= -\frac{2(b+2cx)\sqrt{d+ex}}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\left(2\sqrt{2}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\mathrm{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{b^2-4ac}ex^2}{2cd-be-\sqrt{b^2-4ac}e}}}{\sqrt{1-x^2}}\,dx, x,\right.}{\sqrt{b^2-4ac}\sqrt{\frac{c(d+ex)}{2cd-be-\sqrt{b^2-4ac}e}}\sqrt{a+bx+cx^2}}$$

$$= -\frac{2(b+2cx)\sqrt{d+ex}}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{2\sqrt{2}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle| -\frac{2\sqrt{b^2}}{2cd-\left(b+\right.}\right.}{\sqrt{b^2-4ac}\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 8.54278, size = 996, normalized size = 2.34

$$\frac{(d+ex)^{3/2}\left(cx^2+bx+a\right)^{3/2}\left(-4\sqrt{\frac{cd^2+e(ae-bd)}{-2cd+be+\sqrt{\left(b^2-4ac\right)e^2}}}\left(c\left(\frac{d}{d+ex}-1\right)^2+\frac{e\left(-\frac{db}{d+ex}+b+\frac{ae}{d+ex}\right)}{d+ex}\right)+\frac{i\sqrt{2}\left(2cd-be+\sqrt{\left(b^2-4ac\right)e^2}\right)\sqrt{\frac{-\frac{2ae^2}{d+ex}+b(}{}}}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*(b + 2*c*x)*Sqrt[d + e*x]*(a + b*x + c*x^2))/((b^2 - 4*a*c)*(a + x*(b +
 c*x))^(3/2)) + ((d + e*x)^(3/2)*(a + b*x + c*x^2)^(3/2)*(-4*Sqrt[(c*d^2 +
e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(c*(-1 + d/(d +
 e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)) + (I*Sqrt
[2]*(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] -
 (2*a*e^2)/(d + e*x) - 2*c*d*(-1 + d/(d + e*x)) + b*e*(-1 + (2*d)/(d + e*x)
))/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] +
 (2*a*e^2)/(d + e*x) + 2*c*d*(-1 + d/(d + e*x)) + b*(e - (2*d*e)/(d + e*x))
)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*EllipticE[I*ArcSinh[(Sqrt[2]*Sq
rt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[
d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(
b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] - (I*Sqrt[2]*Sqrt[(b^2 - 4*a*c)*e^2]*Sq
rt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2*a*e^2)/(d + e*x) - 2*c*d*(-1 + d/(d + e*x)
) + b*e*(-1 + (2*d)/(d + e*x)))/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sq
rt[(Sqrt[(b^2 - 4*a*c)*e^2] + (2*a*e^2)/(d + e*x) + 2*c*d*(-1 + d/(d + e*x)
) + b*(e - (2*d*e)/(d + e*x)))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*El
lipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqr
```

```
t[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c
)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x]))/((-b^2 +
 4*a*c)*e*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c
)*e^2])]*(a + x*(b + c*x))^(3/2)*Sqrt[((d + e*x)^2*(c*(-1 + d/(d + e*x))^2
+ (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)))/e^2])
```

Maple [B] time = 0.36, size = 1612, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] (-4*2^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))
^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))
^(1/2))*a*c*e^2*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-
2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c
*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)+2^(1/2)*Elli
pticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*
a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*e^2*
(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^
2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^
(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)+2^(1/2)*EllipticF(2^(1/2)*(-
(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b
*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b*e^2*(-(e*x+d)*c/(e*(-4
*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b
*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+
b^2)^(1/2)+b*e-2*c*d))^(1/2)*(-4*a*c+b^2)^(1/2)-2*2^(1/2)*EllipticF(2^(1/2)
*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2
)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*c*d*e*(-(e*x+d)*c/(e*
(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*
d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a
*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(-4*a*c+b^2)^(1/2)+4*2^(1/2)*EllipticE(2^(1
/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(
1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c*e^2*(-(e*x+d)*
c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/
(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*
(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)-4*2^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*
c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d
)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*d*e*(-(e*x+d)*c/(e*(-4*a*c+b
^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(
-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(
1/2)+b*e-2*c*d))^(1/2)+4*2^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b
^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e
*(-4*a*c+b^2)^(1/2)))^(1/2))*c^2*d^2*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-
2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1
/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d
))^(1/2)+4*c^2*e^2*x^2+2*b*c*e^2*x+4*x*c^2*d*e+2*b*c*d*e)*(e*x+d)^(1/2)*(c*
x^2+b*x+a)^(1/2)/c/e/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)/(4*a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\left(cx^2+bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(c*x^2 + b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+bx+a}\sqrt{ex+d}}{c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\,ac\right)x^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x
 + (b^2 + 2*a*c)*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{d+ex}}{\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/(a + b*x + c*x**2)**(3/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2473 $\int \frac{1}{\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=480

$$-\frac{4\sqrt{2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2e\sqrt{b^2-4ac}}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)}{\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{a+bx+cx^2}}-\frac{2\sqrt{d+ex}\left(2ace+b^2(-e)+c\right.}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(\right.}$$

[Out] (-2*Sqrt[d + e*x]*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) + (Sqrt[2]*(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (4*Sqrt[2]*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[b^2 - 4*a*c]*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])

Rubi [A] time = 0.320468, antiderivative size = 480, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {740, 843, 718, 424, 419}

$$-\frac{2\sqrt{d+ex}\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{2}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(2cd-be)E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b}}{2cd-(b+}\right.}{\sqrt{b^2-4ac}\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[d + e*x]*(a + b*x + c*x^2)^(3/2)),x]

[Out] (-2*Sqrt[d + e*x]*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) + (Sqrt[2]*(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (4*Sqrt[2]*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[b^2 - 4*a*c]*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])

Rule 740

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e

```
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} - \frac{2\int \frac{-\frac{1}{2}ce(bd-2ae)-\frac{1}{2}ce(2cd-be)x}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{2\sqrt{d+ex}\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} - \frac{(2c)\int \frac{1}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac} + \frac{(c(2cd-be}{\left(b^2-4ac\right)}$$

$$= -\frac{2\sqrt{d+ex}\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} + \frac{\left(\sqrt{2}(2cd-be)\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(cd^2-bde+a\right.}$$

$$= -\frac{2\sqrt{d+ex}\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} + \frac{\sqrt{2}(2cd-be)\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E}{\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)}$$

Mathematica [C] time = 8.92943, size = 976, normalized size = 2.03

$$\sqrt{d+ex}\left(-4eb^2+4c(d-ex)b+8c(ae+cdx)-\frac{(d+ex)\left(-\frac{4(be-2cd)\sqrt{\frac{cd^2+e(ae-bd)}{-2cd+be+\sqrt{\left(b^2-4ac\right)e^2}}}(a+x(b+cx))e^2}{(d+ex)^2}-\frac{i\sqrt{2}(2cd-be)\left(2cd-be+\sqrt{\left(b^2-4ac\right)e^2}\right)\sqrt{\frac{-2ae^2+2cdxe-}{(}}}{}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d + e*x]*(-4*b^2*e + 8*c*(a*e + c*d*x) + 4*b*c*(d - e*x) - ((d + e*x)
*((-4*e^2*(-2*c*d + b*e)*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sq
rt[(b^2 - 4*a*c)*e^2])]*(a + x*(b + c*x)))/(d + e*x)^2 - (I*Sqrt[2]*(2*c*d
- b*e)*(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(-2*a*e^2 + d*Sqrt[(b^2
 - 4*a*c)*e^2] + 2*c*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(d - e*x))/(
(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*a*e^2 + d*Sqrt[
(b^2 - 4*a*c)*e^2] - 2*c*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(-d + e*
x))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*EllipticE[I*ArcSi
nh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)
*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d
- b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] + (I*Sqrt[2]*(b^2*e^2 - 4
*a*c*e^2 + 2*c*d*Sqrt[(b^2 - 4*a*c)*e^2] - b*e*Sqrt[(b^2 - 4*a*c)*e^2])*Sqr
t[(-2*a*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] + 2*c*d*e*x + e*Sqrt[(b^2 - 4*a*c)*
```

```
e^2]*x + b*e*(d - e*x))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))
]*Sqrt[(2*a*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] - 2*c*d*e*x + e*Sqrt[(b^2 - 4*a
*c)*e^2]*x + b*e*(-d + e*x))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d +
 e*x))]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d +
 b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b
^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x])
)/(e*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2
])])))/(2*(b^2 - 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.369, size = 1894, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -2*(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*
x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(
-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2
)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/
2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*a
*e^3-2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c
*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+
(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/
2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1
/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*
b*d*e^2+2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-
2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c
*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^
(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)
^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/
2)*c*d^2*e+2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(
-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+
2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE
(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b
^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b*e^3-2*2^(
1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a
*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+
b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*
x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-
2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c*d*e^2-2^(1/2)*(-(e*x+d)
*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))
/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e
*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*
a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-
b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*d*e^2+3*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*
c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+
e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2
)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/
2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*
c+b^2)^(1/2)))^(1/2))*b*c*d^2*e-2*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)
+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^
2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-
2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d
))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)
))^(1/2))*c^2*d^3+x^2*b*c*e^3-2*x^2*c^2*d*e^2-2*x*a*c*e^3+x*b^2*e^3-2*x*c^2
```

```
*d^2*e-2*a*d*e^2*c+b^2*d*e^2-b*c*d^2*e)*(c*x^2+b*x+a)^(1/2)*(e*x+d)^(1/2)/e
/(4*a*c-b^2)/(a*e^2-b*d*e+c*d^2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2 + bx + a\right)^{\frac{3}{2}} \sqrt{ex + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(3/2)*sqrt(e*x + d)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2 + bx + a}\sqrt{ex + d}}{c^2ex^5 + \left(c^2d + 2\,bce\right)x^4 + \left(2\,bcd + \left(b^2 + 2\,ac\right)e\right)x^3 + a^2d + \left(2\,abe + \left(b^2 + 2\,ac\right)d\right)x^2 + \left(2\,abd + a^2e\right)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(c^2*e*x^5 + (c^2*d + 2*b*c*e)
*x^4 + (2*b*c*d + (b^2 + 2*a*c)*e)*x^3 + a^2*d + (2*a*b*e + (b^2 + 2*a*c)*d
)*x^2 + (2*a*b*d + a^2*e)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d + ex}\left(a + bx + cx^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral(1/(sqrt(d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2474 $\int \frac{1}{(d+ex)^{3/2}\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=607

$$-\frac{2\sqrt{2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(2cd-be)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2e\sqrt{b^2-4ac}}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)}{\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}-\frac{4e\sqrt{a+bx+cx}}{\left(b^2-4ac\right)}$$

[Out] (-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]) - (4*e*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*Sqrt[a + b*x + c*x^2])/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[d + e*x]) + (2*Sqrt[2]*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*(2*c*d - b*e)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])

Rubi [A] time = 0.528559, antiderivative size = 607, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {740, 834, 843, 718, 424, 419}

$$-\frac{4e\sqrt{a+bx+cx^2}\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)}{\left(b^2-4ac\right)\sqrt{d+ex}\left(ae^2-bde+cd^2\right)^2}+\frac{2\sqrt{2}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)E\left(\sin^{-1}\right.}{\sqrt{b^2-4ac}\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)^2\sqrt{}}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)^(3/2)*(a + b*x + c*x^2)^(3/2)),x]

[Out] (-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]) - (4*e*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*Sqrt[a + b*x + c*x^2])/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[d + e*x]) + (2*Sqrt[2]*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*(2*c*d - b*e)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x]*Sqrt[a +

```
b*x + c*x^2])
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{d+ex}\sqrt{a+bx+cx^2}} - \frac{2\int \frac{\frac{1}{2}e\left(bcd-2b^2e+6ace\right)+\frac{1}{2}ce(2c}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}}{\left(b^2-4ac\right)\left(cd^2-bde+\right.}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{d+ex}\sqrt{a+bx+cx^2}} - \frac{4e\left(c^2d^2+b^2e^2-ce(bd+\right.}{\left(b^2-4ac\right)\left(cd^2-bde\right.}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{d+ex}\sqrt{a+bx+cx^2}} - \frac{4e\left(c^2d^2+b^2e^2-ce(bd+\right.}{\left(b^2-4ac\right)\left(cd^2-bde\right.}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{d+ex}\sqrt{a+bx+cx^2}} - \frac{4e\left(c^2d^2+b^2e^2-ce(bd+\right.}{\left(b^2-4ac\right)\left(cd^2-bde\right.}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{d+ex}\sqrt{a+bx+cx^2}} - \frac{4e\left(c^2d^2+b^2e^2-ce(bd+\right.}{\left(b^2-4ac\right)\left(cd^2-bde\right.}$$

Mathematica [C] time = 10.9627, size = 784, normalized size = 1.29

$$-\frac{i(d+ex)\sqrt{1-\frac{2\left(e(ae-bd)+cd^2\right)}{(d+ex)\left(\sqrt{e^2\left(b^2-4ac\right)}-be+2cd\right)}}\sqrt{\frac{4\left(e(ae-bd)+cd^2\right)}{(d+ex)\left(\sqrt{e^2\left(b^2-4ac\right)}+be-2cd\right)}+2}\left(\left(c\left(ae^2\left(3\sqrt{e^2\left(b^2-4ac\right)}+8cd\right)-cd^2\sqrt{e^2\left(b^2-4ac\right)}\right)-b^2e^2\left(\sqrt{e^2\left(b^2-4ac\right)}+2cd\right)+bce\left(d\sqrt{e^2}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] ((-4*e*(-(c^2*d^2) - b^2*e^2 + c*e*(b*d + 3*a*e))*(a + x*(b + c*x)))/Sqrt[d
 + e*x] - (2*((b^2 - 4*a*c)*e^3*(a + x*(b + c*x)) + (d + e*x)*(b^3*e^2 + b^
2*c*e*(-2*d + e*x) + b*c*(-3*a*e^2 + c*d*(d - 2*e*x)) + 2*c^2*(c*d^2*x + a*
e*(2*d - e*x)))))/Sqrt[d + e*x] - (I*(d + e*x)*Sqrt[1 - (2*(c*d^2 + e*(-(b*
d) + a*e)))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[2 + (
4*(c*d^2 + e*(-(b*d) + a*e)))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d
+ e*x))]*((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(c^2*d^2 + b^2*e^2 - c*e*
(b*d + 3*a*e))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-
2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e +
Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] + (b^3*e
^3 - b^2*e^2*(2*c*d + Sqrt[(b^2 - 4*a*c)*e^2]) + b*c*e*(-4*a*e^2 + d*Sqrt[(
b^2 - 4*a*c)*e^2]) + c*(-(c*d^2*Sqrt[(b^2 - 4*a*c)*e^2]) + a*e^2*(8*c*d + 3
*Sqrt[(b^2 - 4*a*c)*e^2])))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*
e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2
*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2
]))]))/(e*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c
)*e^2])]))/((b^2 - 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))^2*Sqrt[a + x*(b + c*x)
```

```
])
```

Maple [B] time = 0.386, size = 4415, normalized size = 7.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] (3*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x
+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-
4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)
*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2
)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b^3*d*e^3+12*2^(1/2)*
(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^
2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^
(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*
c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d
)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a^2*c*e^4+4*2^(1/2)*(-(e*x+d)*c/
(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2
*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-
4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c
+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e
+e*(-4*a*c+b^2)^(1/2)))^(1/2))*c^3*d^4-6*a*b*c*d*e^3-2*x*b*c^2*d^2*e^2+4*2^
(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*
a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c
+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e
*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e
-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b^2*e^4-4*2^(1/2)*(-(e*x
+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/
2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))
/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(
-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c
*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b^3*d*e^3-3*2^(1/2)*(-(e*x+d)*c/(e*(-4
*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b
*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+
b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^
(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4
*a*c+b^2)^(1/2)))^(1/2))*a*b^2*e^4-4*b^2*c*d^2*e^2-2*2^(1/2)*(-(e*x+d)*c/(e
*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c
*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*
a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b
^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e
*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*a*c*d*e^3+3*2^(1/2)*(-(e*x+
d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2
))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/
(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-
4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*
d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*b*c*d^2*e^2-12*2^(1/
2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c
+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^
2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+
d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*
c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b*c*d*e^3+8*2^(1/2)*(-(e*x+
d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2
))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/
```

```
(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-
4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*
d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b*c*d*e^3+4*x*c^3*d^3*e-4*x^2*b*c^2*d
*e^3-12*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-
2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c
*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^
(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)
^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a^2*c*e^4+2*a*b^
2*e^4+12*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b
-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*
c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2
^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2
)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c^2*d^2*e^2-8
*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(
-4*a*c+b^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*
a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(
-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+
b*e-2*c*d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c^2*d^2*e^2+8*2^(1/2)
*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b
^2)^(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)
^(1/2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)
*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*
d)/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*c*d^2*e^2-3*2^(1/2)*(-(e*x+
d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2
))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/
(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-
4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*
d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*c*d^2*e^2-2^(1/2)*(-(e*x+d)*c/(e*(-
4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*d-
b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c
+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)
^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*(-
4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*d*e^3-2*2^(1/2)*(-(e*x+d)*
c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/
(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*
(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a
*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b
*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*c^2*d^3*e-8*2^(1/2)*(-(
e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^
(1/2))/(2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/
2))/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(
e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(
2*c*d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c^2*d^3*e+2^(1/2)*(-(e*x+d)*c/(e*
(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*(e*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*
d-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(e*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a
*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^
2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(2*c*d-b*e+e*
(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*a*b*e^4-12*x^2*a*c^2*e^4+4*x
^2*b^2*c*e^4+4*x^2*c^3*d^2*e^2-8*a^2*c*e^4+2*b^3*d*e^3+4*x*b^3*e^4-14*x*a*b
*c*e^4+4*x*a*c^2*d*e^3-2*x*b^2*c*d*e^3+8*a*c^2*d^2*e^2+2*b*c^2*d^3*e)*(c*x^
2+b*x+a)^(1/2)*(e*x+d)^(1/2)/e/(4*a*c-b^2)/(a*e^2-b*d*e+c*d^2)^2/(c*e*x^3+b
*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2 + bx + a\right)^{\frac{3}{2}} (ex + d)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(3/2)*(e*x + d)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+bx+a}\sqrt{ex+d}}{c^2e^2x^6+2\left(c^2de+bce^2\right)x^5+\left(c^2d^2+4\,bcde+\left(b^2+2\,ac\right)e^2\right)x^4+a^2d^2+2\left(bcd^2+abe^2+\left(b^2+2\,ac\right)de\right)x^3+\left(4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(c^2*e^2*x^6 + 2*(c^2*d*e + b*
c*e^2)*x^5 + (c^2*d^2 + 4*b*c*d*e + (b^2 + 2*a*c)*e^2)*x^4 + a^2*d^2 + 2*(b
*c*d^2 + a*b*e^2 + (b^2 + 2*a*c)*d*e)*x^3 + (4*a*b*d*e + a^2*e^2 + (b^2 + 2
*a*c)*d^2)*x^2 + 2*(a*b*d^2 + a^2*d*e)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(d+ex\right)^{\frac{3}{2}}\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral(1/((d + e*x)**(3/2)*(a + b*x + c*x**2)**(3/2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2475 $\int \frac{1}{(d+ex)^{5/2}\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=744

$$-\frac{4\sqrt{2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-ce(5ae+3bd)+2b^2e^2+3c^2d^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2e\sqrt{b^2-4ac}}{2cd-e\left(\sqrt{b^2-4ac}\right.}\right.}{3\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)^2}$$

```
[Out] (-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2 -
b*d*e + a*e^2)*(d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]) - (4*e*(3*c^2*d^2 + 2
*b^2*e^2 - c*e*(3*b*d + 5*a*e))*Sqrt[a + b*x + c*x^2])/(3*(b^2 - 4*a*c)*(c*
d^2 - b*d*e + a*e^2)^2*(d + e*x)^(3/2)) - (2*e*(2*c*d - b*e)*(3*c^2*d^2 + 8
*b^2*e^2 - c*e*(3*b*d + 29*a*e))*Sqrt[a + b*x + c*x^2])/(3*(b^2 - 4*a*c)*(c
*d^2 - b*d*e + a*e^2)^3*Sqrt[d + e*x]) + (Sqrt[2]*(2*c*d - b*e)*(3*c^2*d^2
+ 8*b^2*e^2 - c*e*(3*b*d + 29*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^
2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/S
qrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e)])/(3*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)^3*Sqrt[(c*(d +
 e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (4*Sqr
t[2]*(3*c^2*d^2 + 2*b^2*e^2 - c*e*(3*b*d + 5*a*e))*Sqrt[(c*(d + e*x))/(2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))
]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/
Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3
*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[d + e*x]*Sqrt[a + b*x + c
*x^2])
```

Rubi [A] time = 0.911197, antiderivative size = 744, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {740, 834, 843, 718, 424, 419}

$$-\frac{4e\sqrt{a+bx+cx^2}\left(-ce(5ae+3bd)+2b^2e^2+3c^2d^2\right)}{3\left(b^2-4ac\right)(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)^2}-\frac{2e\sqrt{a+bx+cx^2}(2cd-be)\left(-ce(29ae+3bd)+8b^2e^2+3c^2d^2\right)}{3\left(b^2-4ac\right)\sqrt{d+ex}\left(ae^2-bde+cd^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(5/2)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2 -
b*d*e + a*e^2)*(d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]) - (4*e*(3*c^2*d^2 + 2
*b^2*e^2 - c*e*(3*b*d + 5*a*e))*Sqrt[a + b*x + c*x^2])/(3*(b^2 - 4*a*c)*(c*
d^2 - b*d*e + a*e^2)^2*(d + e*x)^(3/2)) - (2*e*(2*c*d - b*e)*(3*c^2*d^2 + 8
*b^2*e^2 - c*e*(3*b*d + 29*a*e))*Sqrt[a + b*x + c*x^2])/(3*(b^2 - 4*a*c)*(c
*d^2 - b*d*e + a*e^2)^3*Sqrt[d + e*x]) + (Sqrt[2]*(2*c*d - b*e)*(3*c^2*d^2
+ 8*b^2*e^2 - c*e*(3*b*d + 29*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^
2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/S
qrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e)])/(3*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)^3*Sqrt[(c*(d +
 e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (4*Sqr
```

```
t[2]*(3*c^2*d^2 + 2*b^2*e^2 - c*e*(3*b*d + 5*a*e))*Sqrt[(c*(d + e*x))/(2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))
]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/
Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3
*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[d + e*x]*Sqrt[a + b*x + c
*x^2])
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
```

```
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(d+ex)^{5/2}\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}\sqrt{a+bx+cx^2}} - \frac{2\int \frac{\frac{1}{2}e\left(3bcd-4b^2e+10ace\right)+\frac{3}{2}\cdots}{(d+ex)^{5/2}\sqrt{a+bx\cdots}}}{\left(b^2-4ac\right)\left(cd^2-b\cdots\right.}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}\sqrt{a+bx+cx^2}} - \frac{4e\left(3c^2d^2+2b^2e^2-ce(\cdots\right.}{3\left(b^2-4ac\right)\left(cd^2-\cdots\right.}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}\sqrt{a+bx+cx^2}} - \frac{4e\left(3c^2d^2+2b^2e^2-ce(\cdots\right.}{3\left(b^2-4ac\right)\left(cd^2-\cdots\right.}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}\sqrt{a+bx+cx^2}} - \frac{4e\left(3c^2d^2+2b^2e^2-ce(\cdots\right.}{3\left(b^2-4ac\right)\left(cd^2-\cdots\right.}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}\sqrt{a+bx+cx^2}} - \frac{4e\left(3c^2d^2+2b^2e^2-ce(\cdots\right.}{3\left(b^2-4ac\right)\left(cd^2-\cdots\right.}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}\sqrt{a+bx+cx^2}} - \frac{4e\left(3c^2d^2+2b^2e^2-ce(\cdots\right.}{3\left(b^2-4ac\right)\left(cd^2-\cdots\right.}$$

Mathematica [C] time = 13.3263, size = 5565, normalized size = 7.48

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(5/2)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.436, size = 12895, normalized size = 17.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{3}{2}}(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(3/2)*(e*x + d)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2}}{c^2e^3x^7+\left(3\,c^2de^2+2\,bce^3\right)x^6+\left(3\,c^2d^2e+6\,bcde^2+\left(b^2+2\,ac\right)e^3\right)x^5+a^2d^3+\left(c^2d^3+6\,bcd^2e+2\,abe^3+3\left(b^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(c^2*e^3*x^7 + (3*c^2*d*e^2 +
2*b*c*e^3)*x^6 + (3*c^2*d^2*e + 6*b*c*d*e^2 + (b^2 + 2*a*c)*e^3)*x^5 + a^2*
d^3 + (c^2*d^3 + 6*b*c*d^2*e + 2*a*b*e^3 + 3*(b^2 + 2*a*c)*d*e^2)*x^4 + (2*
b*c*d^3 + 6*a*b*d*e^2 + a^2*e^3 + 3*(b^2 + 2*a*c)*d^2*e)*x^3 + (6*a*b*d^2*e
 + 3*a^2*d*e^2 + (b^2 + 2*a*c)*d^3)*x^2 + (2*a*b*d^3 + 3*a^2*d^2*e)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)^{\frac{5}{2}}\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(5/2)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral(1/((d + e*x)**(5/2)*(a + b*x + c*x**2)**(3/2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2476 $\int \frac{(d+ex)^{7/2}}{\left(a+bx+cx^2\right)^{5/2}} dx$

Optimal. Leaf size=659

$$\frac{2\sqrt{2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\left(-4ce(4bd-5ae)-b^2e^2+16c^2d^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{3c^2\left(b^2-4ac\right)^{3/2}\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*(d + e*x)^(5/2)*(b*d - 2*a*e + (2*c*d - b*e)*x))/(3*(b^2 - 4*a*c)*(a +
b*x + c*x^2)^(3/2)) + (2*Sqrt[d + e*x]*(8*b*c*d*(c*d^2 + 3*a*e^2) - 4*a*c*e
*(3*c*d^2 + 5*a*e^2) - b^2*(9*c*d^2*e - a*e^3) + (2*c*d - b*e)*(8*c^2*d^2 -
 b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*x))/(3*c*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c
*x^2]) - (2*Sqrt[2]*(2*c*d - b*e)*(4*c^2*d^2 - b^2*e^2 - 4*c*e*(b*d - 2*a*e
))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[Arc
Sin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*S
qrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c^2*(b^2 - 4*a
*c)^(3/2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a +
b*x + c*x^2]) + (2*Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*(16*c^2*d^2 - b^2*e^2 -
4*c*e*(4*b*d - 5*a*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a
*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c^2*(b^2 - 4*a*c)^(3/2)*Sqr
t[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.810508, antiderivative size = 659, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {738, 818, 843, 718, 424, 419}

$$\frac{2\sqrt{d+ex}\left(x(2cd-be)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)+b^2\left(-\left(9cd^2e-ae^3\right)\right)+8bcd\left(3ae^2+cd^2\right)-4ace\left(5ae^2+3cd\right.\right.}{3c\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(7/2)/(a + b*x + c*x^2)^(5/2),x]
```


```
[Out] (-2*(d + e*x)^(5/2)*(b*d - 2*a*e + (2*c*d - b*e)*x))/(3*(b^2 - 4*a*c)*(a +
b*x + c*x^2)^(3/2)) + (2*Sqrt[d + e*x]*(8*b*c*d*(c*d^2 + 3*a*e^2) - 4*a*c*e
*(3*c*d^2 + 5*a*e^2) - b^2*(9*c*d^2*e - a*e^3) + (2*c*d - b*e)*(8*c^2*d^2 -
 b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*x))/(3*c*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c
*x^2]) - (2*Sqrt[2]*(2*c*d - b*e)*(4*c^2*d^2 - b^2*e^2 - 4*c*e*(b*d - 2*a*e
))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[Arc
Sin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*S
qrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c^2*(b^2 - 4*a
*c)^(3/2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a +
b*x + c*x^2]) + (2*Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*(16*c^2*d^2 - b^2*e^2 -
4*c*e*(4*b*d - 5*a*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b +
```

```
 Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a
*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c^2*(b^2 - 4*a*c)^(3/2)*Sqr
t[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
```

```
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^{7/2}}{\left(a+bx+cx^2\right)^{5/2}}dx = -\frac{2(d+ex)^{5/2}(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\int \frac{(d+ex)^{3/2}\left(\frac{1}{2}\left(8cd^2-9bde+10ae^2\right)-\frac{1}{2}e(2cd-be)x\right)}{\left(a+bx+cx^2\right)^{3/2}}dx}{3\left(b^2-4ac\right)}$$

$$= -\frac{2(d+ex)^{5/2}(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}\left(8bcd\left(cd^2+3ae^2\right)-4ace\left(3cd^2+5ae^2\right)-\right.}{3c\left(b^2-\right.}$$

$$= -\frac{2(d+ex)^{5/2}(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}\left(8bcd\left(cd^2+3ae^2\right)-4ace\left(3cd^2+5ae^2\right)-\right.}{3c\left(b^2-\right.}$$

$$= -\frac{2(d+ex)^{5/2}(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}\left(8bcd\left(cd^2+3ae^2\right)-4ace\left(3cd^2+5ae^2\right)-\right.}{3c\left(b^2-\right.}$$

$$= -\frac{2(d+ex)^{5/2}(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}\left(8bcd\left(cd^2+3ae^2\right)-4ace\left(3cd^2+5ae^2\right)-\right.}{3c\left(b^2-\right.}$$

Mathematica [C] time = 14.3428, size = 5598, normalized size = 8.49

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^(7/2)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.474, size = 19258, normalized size = 29.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(7/2)/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{7}{2}}}{\left(cx^2+bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(7/2)/(c*x^2 + b*x + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\sqrt{cx^2+bx+a}\sqrt{ex+d}}{c^3x^6+3\,bc^2x^5+3\left(b^2c+ac^2\right)x^4+3\,a^2bx+\left(b^3+6\,abc\right)x^3+a^3+3\left(ab^2+a^2c\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(c*x^2 + b*x + a)*sq
rt(e*x + d)/(c^3*x^6 + 3*b*c^2*x^5 + 3*(b^2*c + a*c^2)*x^4 + 3*a^2*b*x + (b
^3 + 6*a*b*c)*x^3 + a^3 + 3*(a*b^2 + a^2*c)*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(7/2)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(7/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2477 $\int \frac{(d+ex)^{5/2}}{(a+bx+cx^2)^{5/2}}\,dx$

Optimal. Leaf size=590

$$\frac{16\sqrt{2}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2cd-be)\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2e\sqrt{b^2-4ac}}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)}{3c\left(b^2-4ac\right)^{3/2}\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*(d + e*x)^(3/2)*(b*d - 2*a*e + (2*c*d - b*e)*x))/(3*(b^2 - 4*a*c)*(a +
b*x + c*x^2)^(3/2)) - (2*Sqrt[d + e*x]*(7*b^2*d*e + 4*a*c*d*e - 8*b*(c*d^2
+ a*e^2) - (16*c^2*d^2 + b^2*e^2 - 4*c*e*(4*b*d - 3*a*e))*x))/(3*(b^2 - 4*a
*c)^2*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*(16*c^2*d^2 + b^2*e^2 - 4*c*e*(4*b*
d - 3*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Elli
pticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2
]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c*(b^
2 - 4*a*c)^(3/2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sq
rt[a + b*x + c*x^2]) + (16*Sqrt[2]*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*Sq
rt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x +
c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)])/(3*c*(b^2 - 4*a*c)^(3/2)*Sqrt[d + e*x]*Sqrt[a + b*x + c
*x^2])
```

Rubi [A] time = 0.587674, antiderivative size = 590, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {738, 820, 843, 718, 424, 419}

$$-\frac{2\sqrt{d+ex}\left(-x\left(-4ce(4bd-3ae)+b^2e^2+16c^2d^2\right)-8b\left(ae^2+cd^2\right)+4acde+7b^2de\right)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}-\frac{\sqrt{2}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(-4\right.}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(a + b*x + c*x^2)^(5/2),x]
```


```
[Out] (-2*(d + e*x)^(3/2)*(b*d - 2*a*e + (2*c*d - b*e)*x))/(3*(b^2 - 4*a*c)*(a +
b*x + c*x^2)^(3/2)) - (2*Sqrt[d + e*x]*(7*b^2*d*e + 4*a*c*d*e - 8*b*(c*d^2
+ a*e^2) - (16*c^2*d^2 + b^2*e^2 - 4*c*e*(4*b*d - 3*a*e))*x))/(3*(b^2 - 4*a
*c)^2*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*(16*c^2*d^2 + b^2*e^2 - 4*c*e*(4*b*
d - 3*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Elli
pticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2
]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c*(b^
2 - 4*a*c)^(3/2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sq
rt[a + b*x + c*x^2]) + (16*Sqrt[2]*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*Sq
rt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x +
c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)])/(3*c*(b^2 - 4*a*c)^(3/2)*Sqrt[d + e*x]*Sqrt[a + b*x + c
```

```
*x^2])
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*
(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p +
 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*Simp[g*
(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(
m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (I
ntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\int \frac{\sqrt{d+ex}\left(\frac{1}{2}\left(8cd^2-7bde+6ae^2\right)+\frac{1}{2}e(2cd-be)x\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx}{3\left(b^2-4ac\right)}$$

$$= -\frac{2(d+ex)^{3/2}(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\sqrt{d+ex}\left(7b^2de+4acde-8b\left(cd^2+ae^2\right)-\left(16c^2d\right.\right.}{3\left(b^2-4ac\right)^2\sqrt{a+bx+c}}$$

$$= -\frac{2(d+ex)^{3/2}(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\sqrt{d+ex}\left(7b^2de+4acde-8b\left(cd^2+ae^2\right)-\left(16c^2d\right.\right.}{3\left(b^2-4ac\right)^2\sqrt{a+bx+c}}$$

$$= -\frac{2(d+ex)^{3/2}(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\sqrt{d+ex}\left(7b^2de+4acde-8b\left(cd^2+ae^2\right)-\left(16c^2d\right.\right.}{3\left(b^2-4ac\right)^2\sqrt{a+bx+c}}$$

$$= -\frac{2(d+ex)^{3/2}(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\sqrt{d+ex}\left(7b^2de+4acde-8b\left(cd^2+ae^2\right)-\left(16c^2d\right.\right.}{3\left(b^2-4ac\right)^2\sqrt{a+bx+c}}$$

<u>Mathematica</u> [C] time = 13.0834, size = 3577, normalized size = 6.06

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d + e*x]*(a + b*x + c*x^2)^3*((2*(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + 2*c
^2*d^2*x - 2*b*c*d*e*x + b^2*e^2*x - 2*a*c*e^2*x))/(3*c*(-b^2 + 4*a*c)*(a +
 b*x + c*x^2)^2) + (2*(8*b*c^2*d^2 - 9*b^2*c*d*e + 4*a*c^2*d*e + b^3*e^2 +
4*a*b*c*e^2 + 16*c^3*d^2*x - 16*b*c^2*d*e*x + b^2*c*e^2*x + 12*a*c^2*e^2*x)
)/(3*c*(-b^2 + 4*a*c)^2*(a + b*x + c*x^2))))/(a + x*(b + c*x))^(5/2) + ((a
+ b*x + c*x^2)^(5/2)*((-2*(16*c^2*d^2 - 16*b*c*d*e + b^2*e^2 + 12*a*c*e^2)*
(d + e*x)^(3/2)*(c + (c*d^2)/(d + e*x)^2 - (b*d*e)/(d + e*x)^2 + (a*e^2)/(d
 + e*x)^2 - (2*c*d)/(d + e*x) + (b*e)/(d + e*x)))/(c*Sqrt[((d + e*x)^2*(c*(
-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)
))/e^2]) + (2*(c*d^2 - b*d*e + a*e^2)*(d + e*x)*Sqrt[c + (c*d^2)/(d + e*x)^
2 - (b*d*e)/(d + e*x)^2 + (a*e^2)/(d + e*x)^2 - (2*c*d)/(d + e*x) + (b*e)/(
d + e*x)]*(((4*I)*Sqrt[2]*c^2*d^2*(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])
*Sqrt[1 - (2*(c*d^2 - b*d*e + a*e^2))/((2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*
e^2])*(d + e*x))]*Sqrt[1 - (2*(c*d^2 - b*d*e + a*e^2))/((2*c*d - b*e + Sqrt
[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*(EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[-((c*
d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d +
e*x]], (2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*c*d - b*e + Sqrt[b^2*e^
2 - 4*a*c*e^2])] - EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[-((c*d^2 - b*d*e + a*e
^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*c*d - b
*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])])
)/((c*d^2 - b*d*e + a*e^2)*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sq
rt[b^2*e^2 - 4*a*c*e^2]))]*Sqrt[c + (c*d^2 - b*d*e + a*e^2)/(d + e*x)^2 + (
-2*c*d + b*e)/(d + e*x)]) - ((4*I)*Sqrt[2]*b*c*d*e*(2*c*d - b*e + Sqrt[b^2*
```

```
e^2 - 4*a*c*e^2])*Sqrt[1 - (2*(c*d^2 - b*d*e + a*e^2))/((2*c*d - b*e - Sqrt
[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*Sqrt[1 - (2*(c*d^2 - b*d*e + a*e^2))/((2
*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*(EllipticE[I*ArcSinh[(S
qrt[2]*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e
^2]))])/Sqrt[d + e*x]], (2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*c*d -
b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])] - EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[-((c*
d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d +
e*x]], (2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*c*d - b*e + Sqrt[b^2*e^
2 - 4*a*c*e^2])]))/((c*d^2 - b*d*e + a*e^2)*Sqrt[-((c*d^2 - b*d*e + a*e^2)/
(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))]*Sqrt[c + (c*d^2 - b*d*e + a*e^2
)/(d + e*x)^2 + (-2*c*d + b*e)/(d + e*x)]) + ((I/2)*b^2*e^2*(2*c*d - b*e +
Sqrt[b^2*e^2 - 4*a*c*e^2])*Sqrt[1 - (2*(c*d^2 - b*d*e + a*e^2))/((2*c*d - b
*e - Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*Sqrt[1 - (2*(c*d^2 - b*d*e + a*
e^2))/((2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*(EllipticE[I*A
rcSinh[(Sqrt[2]*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2
- 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/
(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])] - EllipticF[I*ArcSinh[(Sqrt[2]*S
qrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/
Sqrt[d + e*x]], (2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*c*d - b*e + Sq
rt[b^2*e^2 - 4*a*c*e^2])]))/(Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*Sqrt[-((c*d^2
- b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))]*Sqrt[c + (c*d^
2 - b*d*e + a*e^2)/(d + e*x)^2 + (-2*c*d + b*e)/(d + e*x)]) + ((3*I)*Sqrt[2
]*a*c*e^2*(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*Sqrt[1 - (2*(c*d^2 - b*
d*e + a*e^2))/((2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*Sqrt[1
 - (2*(c*d^2 - b*d*e + a*e^2))/((2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*(
d + e*x))]*(EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*
c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*c*d - b*e - Sq
rt[b^2*e^2 - 4*a*c*e^2])/(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])] - Ellip
ticF[I*ArcSinh[(Sqrt[2]*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[
b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*
c*e^2])/(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])]))/((c*d^2 - b*d*e + a*e^
2)*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])
)]*Sqrt[c + (c*d^2 - b*d*e + a*e^2)/(d + e*x)^2 + (-2*c*d + b*e)/(d + e*x)]
) + ((8*I)*Sqrt[2]*c^2*d*Sqrt[1 - (2*(c*d^2 - b*d*e + a*e^2))/((2*c*d - b*e
 - Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*Sqrt[1 - (2*(c*d^2 - b*d*e + a*e^
2))/((2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*EllipticF[I*ArcS
inh[(Sqrt[2]*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4
*a*c*e^2]))])/Sqrt[d + e*x]], (2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*
c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])])/(Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2
*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))]*Sqrt[c + (c*d^2 - b*d*e + a*e^2)/
(d + e*x)^2 + (-2*c*d + b*e)/(d + e*x)]) - ((4*I)*Sqrt[2]*b*c*e*Sqrt[1 - (2
*(c*d^2 - b*d*e + a*e^2))/((2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e
*x))]*Sqrt[1 - (2*(c*d^2 - b*d*e + a*e^2))/((2*c*d - b*e + Sqrt[b^2*e^2 - 4
*a*c*e^2])*(d + e*x))]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[-((c*d^2 - b*d*e +
 a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*c*d
 - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2
])])/(Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^
2]))]*Sqrt[c + (c*d^2 - b*d*e + a*e^2)/(d + e*x)^2 + (-2*c*d + b*e)/(d + e*
x)])))/(c*Sqrt[((d + e*x)^2*(c*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*
x) + (a*e)/(d + e*x)))/(d + e*x)))/e^2])))/(3*(-b^2 + 4*a*c)^2*e*(a + x*(b
+ c*x))^(5/2))
```

Maple [B] time = 0.437, size = 12990, normalized size = 22.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{(cx^2+bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/(c*x^2 + b*x + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e^2x^2+2\,dex+d^2)\sqrt{cx^2+bx+a}\sqrt{ex+d}}{c^3x^6+3\,bc^2x^5+3\,(b^2c+ac^2)x^4+3\,a^2bx+(b^3+6\,abc)x^3+a^3+3\,(ab^2+a^2c)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(c^3
*x^6 + 3*b*c^2*x^5 + 3*(b^2*c + a*c^2)*x^4 + 3*a^2*b*x + (b^3 + 6*a*b*c)*x^
3 + a^3 + 3*(a*b^2 + a^2*c)*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2478 $\int \frac{(d+ex)^{3/2}}{(a+bx+cx^2)^{5/2}}\,dx$

Optimal. Leaf size=542

$$\frac{2\sqrt{2}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(-4ce(4bd-ae)+3b^2e^2+16c^2d^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2e\sqrt{b^2-4ac}}{2cd-e\left(\sqrt{b^2-4ac}+b\right.}\right.}{3c\left(b^2-4ac\right)^{3/2}\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*Sqrt[d + e*x]*(b*d - 2*a*e + (2*c*d - b*e)*x))/(3*(b^2 - 4*a*c)*(a + b*
x + c*x^2)^(3/2)) + (2*Sqrt[d + e*x]*(8*b*c*d - 5*b^2*e + 4*a*c*e + 8*c*(2*
c*d - b*e)*x))/(3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2]) - (8*Sqrt[2]*(2*c*
d - b*e)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellipti
cE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]],
 (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*(b^2 - 4
*a*c)^(3/2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a
+ b*x + c*x^2]) + (2*Sqrt[2]*(16*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(4*b*d - a*e))
*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x
 + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2
*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)])/(3*c*(b^2 - 4*a*c)^(3/2)*Sqrt[d + e*x]*Sqrt[a + b*x
+ c*x^2])
```

Rubi [A] time = 0.563703, antiderivative size = 542, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {738, 822, 843, 718, 424, 419}

$$\frac{2\sqrt{2}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(-4ce(4bd-ae)+3b^2e^2+16c^2d^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{3c\left(b^2-4ac\right)^{3/2}\sqrt{d+ex}\sqrt{a+bx+cx^2}}-2\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(a + b*x + c*x^2)^(5/2),x]
```


```
[Out] (-2*Sqrt[d + e*x]*(b*d - 2*a*e + (2*c*d - b*e)*x))/(3*(b^2 - 4*a*c)*(a + b*
x + c*x^2)^(3/2)) + (2*Sqrt[d + e*x]*(8*b*c*d - 5*b^2*e + 4*a*c*e + 8*c*(2*
c*d - b*e)*x))/(3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2]) - (8*Sqrt[2]*(2*c*
d - b*e)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellipti
cE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]],
 (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*(b^2 - 4
*a*c)^(3/2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a
+ b*x + c*x^2]) + (2*Sqrt[2]*(16*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(4*b*d - a*e))
*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x
 + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2
*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)])/(3*c*(b^2 - 4*a*c)^(3/2)*Sqrt[d + e*x]*Sqrt[a + b*x
+ c*x^2])
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2\sqrt{d+ex}(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(8cd^2-5bde+2ae^2\right)+\frac{3}{2}e(2cd-be)x}{\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}dx}{3\left(b^2-4ac\right)}$$

$$= -\frac{2\sqrt{d+ex}(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}\left(8bcd-5b^2e+4ace+8c(2cd-be)x\right)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}} + \frac{4\int \cdots}{\cdots}$$

$$= -\frac{2\sqrt{d+ex}(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}\left(8bcd-5b^2e+4ace+8c(2cd-be)x\right)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}} - \frac{(8c(2c\cdots}{\cdots}$$

$$= -\frac{2\sqrt{d+ex}(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}\left(8bcd-5b^2e+4ace+8c(2cd-be)x\right)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}} - \frac{\left(8\sqrt{2}(\cdots\right.}{\cdots}$$

$$= -\frac{2\sqrt{d+ex}(bd-2ae+(2cd-be)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} + \frac{2\sqrt{d+ex}\left(8bcd-5b^2e+4ace+8c(2cd-be)x\right)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}} - \frac{8\sqrt{2}(2\cdots}{\cdots}$$

Mathematica [C] time = 10.3074, size = 1141, normalized size = 2.11

$$\frac{\sqrt{d+ex}\left(cx^2+bx+a\right)^3\left(\frac{2(-bd-2cxd+2ae+bex)}{3\left(b^2-4ac\right)\left(cx^2+bx+a\right)^2}-\frac{2\left(5eb^2-8cdb+8cexb-4ace-16c^2dx\right)}{3\left(b^2-4ac\right)^2\left(cx^2+bx+a\right)}\right)}{(a+x(b+cx))^{5/2}} - \frac{(d+ex)^{3/2}\left(cx^2+bx+a\right)^{5/2}\left(-16(be-2cd)\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d + e*x]*(a + b*x + c*x^2)^3*((2*(-(b*d) + 2*a*e - 2*c*d*x + b*e*x))/
(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (2*(-8*b*c*d + 5*b^2*e - 4*a*c*e -
16*c^2*d*x + 8*b*c*e*x))/(3*(b^2 - 4*a*c)^2*(a + b*x + c*x^2))))/(a + x*(b
+ c*x))^(5/2) - ((d + e*x)^(3/2)*(a + b*x + c*x^2)^(5/2)*(-16*(-2*c*d + b*e
)*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]
*(c*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d +
 e*x)) + ((4*I)*Sqrt[2]*(-2*c*d + b*e)*(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^
2])*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2*a*e^2)/(d + e*x) - 2*c*d*(-1 + d/(d
+ e*x)) + b*e*(-1 + (2*d)/(d + e*x)))/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2
])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] + (2*a*e^2)/(d + e*x) + 2*c*d*(-1 + d/(d
+ e*x)) + b*(e - (2*d*e)/(d + e*x)))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2
])]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e
 + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 -
```

```
4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] - (I
*Sqrt[2]*(-(b^2*e^2) + 4*a*c*e^2 - 8*c*d*Sqrt[(b^2 - 4*a*c)*e^2] + 4*b*e*Sq
rt[(b^2 - 4*a*c)*e^2])*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2*a*e^2)/(d + e*x)
- 2*c*d*(-1 + d/(d + e*x)) + b*e*(-1 + (2*d)/(d + e*x)))/(2*c*d - b*e + Sqr
t[(b^2 - 4*a*c)*e^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] + (2*a*e^2)/(d + e*x)
+ 2*c*d*(-1 + d/(d + e*x)) + b*(e - (2*d*e)/(d + e*x)))/(-2*c*d + b*e + Sqr
t[(b^2 - 4*a*c)*e^2])]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a
*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d
+ b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])
/Sqrt[d + e*x]))/(3*(-b^2 + 4*a*c)^2*e*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*
c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(a + x*(b + c*x))^(5/2)*Sqrt[((d + e*
x)^2*(c*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/
(d + e*x)))/e^2])
```

Maple [B] time = 0.383, size = 8889, normalized size = 16.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cx^2+bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(c*x^2 + b*x + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}}{c^3x^6+3\,bc^2x^5+3\left(b^2c+ac^2\right)x^4+3\,a^2bx+\left(b^3+6\,abc\right)x^3+a^3+3\left(ab^2+a^2c\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)/(c^3*x^6 + 3*b*c^2*x^5 + 3*(
b^2*c + a*c^2)*x^4 + 3*a^2*b*x + (b^3 + 6*a*b*c)*x^3 + a^3 + 3*(a*b^2 + a^2
*c)*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2479 $\int \frac{\sqrt{d+ex}}{(a+bx+cx^2)^{5/2}}\,dx$

Optimal. Leaf size=605

$$\frac{16\sqrt{2}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2cd-be)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2e\sqrt{b^2-4ac}}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)}{3\left(b^2-4ac\right)^{3/2}\sqrt{d+ex}\sqrt{a+bx+cx^2}}-\frac{2\sqrt{d+ex}\left(-cx\left(\right.\right.}{}$$

```
[Out] (-2*(b + 2*c*x)*Sqrt[d + e*x])/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) -
(2*Sqrt[d + e*x]*(9*b^2*c*d*e - 4*a*c^2*d*e - b^3*e^2 - 4*b*c*(2*c*d^2 + a*
e^2) - c*(16*c^2*d^2 + b^2*e^2 - 4*c*e*(4*b*d - 3*a*e))*x))/(3*(b^2 - 4*a*c
)^2*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*(16*c^2*d^2 +
 b^2*e^2 - 4*c*e*(4*b*d - 3*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqr
t[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e)])/(3*(b^2 - 4*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d + e
*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (16*Sqrt
[2]*(2*c*d - b*e)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*S
qrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*(b^2 - 4*a*c)^(3/2)*Sqrt[d + e*x
]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.575615, antiderivative size = 605, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {736, 822, 843, 718, 424, 419}

$$-\frac{2\sqrt{d+ex}\left(-cx\left(-4ce(4bd-3ae)+b^2e^2+16c^2d^2\right)-4bc\left(ae^2+2cd^2\right)-4ac^2de+9b^2cde+b^3\left(-e^2\right)\right)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{2}\sqrt{d+ex}\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*(b + 2*c*x)*Sqrt[d + e*x])/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) -
(2*Sqrt[d + e*x]*(9*b^2*c*d*e - 4*a*c^2*d*e - b^3*e^2 - 4*b*c*(2*c*d^2 + a*
e^2) - c*(16*c^2*d^2 + b^2*e^2 - 4*c*e*(4*b*d - 3*a*e))*x))/(3*(b^2 - 4*a*c
)^2*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*(16*c^2*d^2 +
 b^2*e^2 - 4*c*e*(4*b*d - 3*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqr
t[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e)])/(3*(b^2 - 4*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d + e
*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (16*Sqrt
[2]*(2*c*d - b*e)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*S
qrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*(b^2 - 4*a*c)^(3/2)*Sqrt[d + e*x
```

```
]*Sqrt[a + b*x + c*x^2])
```

Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] - Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)
*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p + 1
), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 0] && (L
tQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c,
d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2(b+2cx)\sqrt{d+ex}}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} + \frac{2\int \frac{-4cd+\frac{be}{2}-3cex}{\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}\,dx}{3\left(b^2-4ac\right)}$$

$$= -\frac{2(b+2cx)\sqrt{d+ex}}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\sqrt{d+ex}\left(9b^2cde-4ac^2de-b^3e^2-4bc\left(2cd^2+ae^2\right)-c\left(\right.\right.}{3\left(b^2-4ac\right)^2\left(cd^2-bde+ae^2\right)\sqrt{a}}$$

$$= -\frac{2(b+2cx)\sqrt{d+ex}}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\sqrt{d+ex}\left(9b^2cde-4ac^2de-b^3e^2-4bc\left(2cd^2+ae^2\right)-c\left(\right.\right.}{3\left(b^2-4ac\right)^2\left(cd^2-bde+ae^2\right)\sqrt{a}}$$

$$= -\frac{2(b+2cx)\sqrt{d+ex}}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\sqrt{d+ex}\left(9b^2cde-4ac^2de-b^3e^2-4bc\left(2cd^2+ae^2\right)-c\left(\right.\right.}{3\left(b^2-4ac\right)^2\left(cd^2-bde+ae^2\right)\sqrt{a}}$$

$$= -\frac{2(b+2cx)\sqrt{d+ex}}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\sqrt{d+ex}\left(9b^2cde-4ac^2de-b^3e^2-4bc\left(2cd^2+ae^2\right)-c\left(\right.\right.}{3\left(b^2-4ac\right)^2\left(cd^2-bde+ae^2\right)\sqrt{a}}$$

Mathematica [C] time = 12.6291, size = 3560, normalized size = 5.88

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d + e*x]*(a + b*x + c*x^2)^3*((-2*(b + 2*c*x))/(3*(b^2 - 4*a*c)*(a +
b*x + c*x^2)^2) + (2*(8*b*c^2*d^2 - 9*b^2*c*d*e + 4*a*c^2*d*e + b^3*e^2 + 4
*a*b*c*e^2 + 16*c^3*d^2*x - 16*b*c^2*d*e*x + b^2*c*e^2*x + 12*a*c^2*e^2*x))
/(3*(-b^2 + 4*a*c)^2*(c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2))))/(a + x*(b
 + c*x))^(5/2) + (2*c*(a + b*x + c*x^2)^(5/2)*(((-16*c^2*d^2 + 16*b*c*d*e -
 b^2*e^2 - 12*a*c*e^2)*(d + e*x)^(3/2)*(c + (c*d^2)/(d + e*x)^2 - (b*d*e)/(
d + e*x)^2 + (a*e^2)/(d + e*x)^2 - (2*c*d)/(d + e*x) + (b*e)/(d + e*x)))/(c
*Sqrt[((d + e*x)^2*(c*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e
)/(d + e*x)))/(d + e*x)))/e^2]) + ((c*d^2 - b*d*e + a*e^2)*(d + e*x)*Sqrt[c
 + (c*d^2)/(d + e*x)^2 - (b*d*e)/(d + e*x)^2 + (a*e^2)/(d + e*x)^2 - (2*c*d
)/(d + e*x) + (b*e)/(d + e*x)]*(((4*I)*Sqrt[2]*c^2*d^2*(2*c*d - b*e + Sqrt[
b^2*e^2 - 4*a*c*e^2])*Sqrt[1 - (2*(c*d^2 - b*d*e + a*e^2))/((2*c*d - b*e -
Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*Sqrt[1 - (2*(c*d^2 - b*d*e + a*e^2))
/((2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*(EllipticE[I*ArcSin
h[(Sqrt[2]*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a
*c*e^2]))])/Sqrt[d + e*x]], (2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*c*
d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])] - EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[-
((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[
d + e*x]], (2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*c*d - b*e + Sqrt[b^
2*e^2 - 4*a*c*e^2])]))/((c*d^2 - b*d*e + a*e^2)*Sqrt[-((c*d^2 - b*d*e + a*e
^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))]*Sqrt[c + (c*d^2 - b*d*e + a
```

```
*e^2)/(d + e*x)^2 + (-2*c*d + b*e)/(d + e*x)]) - ((4*I)*Sqrt[2]*b*c*d*e*(2*
c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*Sqrt[1 - (2*(c*d^2 - b*d*e + a*e^2))
/((2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*Sqrt[1 - (2*(c*d^2
- b*d*e + a*e^2))/((2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*(E
llipticE[I*ArcSinh[(Sqrt[2]*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - S
qrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*c*d - b*e - Sqrt[b^2*e^2 -
4*a*c*e^2])/(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])] - EllipticF[I*ArcSin
h[(Sqrt[2]*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a
*c*e^2]))])/Sqrt[d + e*x]], (2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*c*
d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])]))/((c*d^2 - b*d*e + a*e^2)*Sqrt[-((c*
d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))]*Sqrt[c + (
c*d^2 - b*d*e + a*e^2)/(d + e*x)^2 + (-2*c*d + b*e)/(d + e*x)]) + ((I/2)*b^
2*e^2*(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*Sqrt[1 - (2*(c*d^2 - b*d*e
+ a*e^2))/((2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*Sqrt[1 - (
2*(c*d^2 - b*d*e + a*e^2))/((2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*(d +
e*x))]*(EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d
- b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*c*d - b*e - Sqrt[b
^2*e^2 - 4*a*c*e^2])/(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])] - EllipticF
[I*ArcSinh[(Sqrt[2]*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*
e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^
2])/(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])]))/(Sqrt[2]*(c*d^2 - b*d*e +
a*e^2)*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e
^2]))]*Sqrt[c + (c*d^2 - b*d*e + a*e^2)/(d + e*x)^2 + (-2*c*d + b*e)/(d + e
*x)]) + ((3*I)*Sqrt[2]*a*c*e^2*(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*Sq
rt[1 - (2*(c*d^2 - b*d*e + a*e^2))/((2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2
])*(d + e*x))]*Sqrt[1 - (2*(c*d^2 - b*d*e + a*e^2))/((2*c*d - b*e + Sqrt[b^
2*e^2 - 4*a*c*e^2])*(d + e*x))]*(EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[-((c*d^2
 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x
]], (2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*c*d - b*e + Sqrt[b^2*e^2 -
 4*a*c*e^2])] - EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[-((c*d^2 - b*d*e + a*e^2)
/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*c*d - b*e
- Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])]))/(
(c*d^2 - b*d*e + a*e^2)*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[
b^2*e^2 - 4*a*c*e^2]))]*Sqrt[c + (c*d^2 - b*d*e + a*e^2)/(d + e*x)^2 + (-2*
c*d + b*e)/(d + e*x)]) + ((8*I)*Sqrt[2]*c^2*d*Sqrt[1 - (2*(c*d^2 - b*d*e +
a*e^2))/((2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*Sqrt[1 - (2*
(c*d^2 - b*d*e + a*e^2))/((2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*
x))]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b
*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*c*d - b*e - Sqrt[b^2*
e^2 - 4*a*c*e^2])/(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])])/(Sqrt[-((c*d^
2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))]*Sqrt[c + (c*
d^2 - b*d*e + a*e^2)/(d + e*x)^2 + (-2*c*d + b*e)/(d + e*x)]) - ((4*I)*Sqrt
[2]*b*c*e*Sqrt[1 - (2*(c*d^2 - b*d*e + a*e^2))/((2*c*d - b*e - Sqrt[b^2*e^2
 - 4*a*c*e^2])*(d + e*x))]*Sqrt[1 - (2*(c*d^2 - b*d*e + a*e^2))/((2*c*d - b
*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*EllipticF[I*ArcSinh[(Sqrt[2]*Sq
rt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/S
qrt[d + e*x]], (2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*c*d - b*e + Sqr
t[b^2*e^2 - 4*a*c*e^2])])/(Sqrt[-((c*d^2 - b*d*e + a*e^2)/(2*c*d - b*e - Sq
rt[b^2*e^2 - 4*a*c*e^2]))]*Sqrt[c + (c*d^2 - b*d*e + a*e^2)/(d + e*x)^2 + (
-2*c*d + b*e)/(d + e*x)])))/(c*Sqrt[((d + e*x)^2*(c*(-1 + d/(d + e*x))^2 +
(e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)))/e^2])))/(3*(-b^2 +
4*a*c)^2*e*(c*d^2 - b*d*e + a*e^2)*(a + x*(b + c*x))^(5/2))
```

Maple [B] time = 0.407, size = 13071, normalized size = 21.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\left(cx^2+bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(c*x^2 + b*x + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+bx+a}\sqrt{ex+d}}{c^3x^6+3\,bc^2x^5+3\left(b^2c+ac^2\right)x^4+3\,a^2bx+\left(b^3+6\,abc\right)x^3+a^3+3\left(ab^2+a^2c\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(c^3*x^6 + 3*b*c^2*x^5 + 3*(b^
2*c + a*c^2)*x^4 + 3*a^2*b*x + (b^3 + 6*a*b*c)*x^3 + a^3 + 3*(a*b^2 + a^2*c
)*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2480 $\int \frac{1}{\sqrt{d+ex}\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=725

$$\frac{2\sqrt{2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-4ce(4bd-5ae)-b^2e^2+16c^2d^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,\mathrm{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2e\sqrt{b^2-4ac}}{2cd-e\left(\sqrt{b^2-4ac}+b\right.}\right.}{3\left(b^2-4ac\right)^{3/2}\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

```
[Out] (-2*Sqrt[d + e*x]*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/(3*(b^2 -
4*a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2)^(3/2)) - (2*Sqrt[d + e*x]*
(3*a*c*e*(2*c*d - b*e)^2 - (b*c*d - b^2*e + 2*a*c*e)*(8*c^2*d^2 - 2*b^2*e^2
 - 5*c*e*(b*d - 2*a*e)) - 2*c*(2*c*d - b*e)*(4*c^2*d^2 - b^2*e^2 - 4*c*e*(b
*d - 2*a*e))*x))/(3*(b^2 - 4*a*c)^2*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[a + b*x
+ c*x^2]) - (2*Sqrt[2]*(2*c*d - b*e)*(4*c^2*d^2 - b^2*e^2 - 4*c*e*(b*d - 2*
a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[
ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-
2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*(b^2 - 4*a*
c)^(3/2)*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^
2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*(16*c^2*d^2 - b^2*e^2 -
 4*c*e*(4*b*d - 5*a*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*
a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*(b^2 - 4*a*c)^(3/2)*(c*d^2
 - b*d*e + a*e^2)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.735936, antiderivative size = 725, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {740, 822, 843, 718, 424, 419}

$$-\frac{2\sqrt{d+ex}\left(-2cx(2cd-be)\left(-4ce(bd-2ae)-b^2e^2+4c^2d^2\right)-\left(2ace+b^2(-e)+bcd\right)\left(-5ce(bd-2ae)-2b^2e^2+8c^2d^2\right)+3}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(a + b*x + c*x^2)^(5/2)),x]
```


```
[Out] (-2*Sqrt[d + e*x]*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/(3*(b^2 -
4*a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2)^(3/2)) - (2*Sqrt[d + e*x]*
(3*a*c*e*(2*c*d - b*e)^2 - (b*c*d - b^2*e + 2*a*c*e)*(8*c^2*d^2 - 2*b^2*e^2
 - 5*c*e*(b*d - 2*a*e)) - 2*c*(2*c*d - b*e)*(4*c^2*d^2 - b^2*e^2 - 4*c*e*(b
*d - 2*a*e))*x))/(3*(b^2 - 4*a*c)^2*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[a + b*x
+ c*x^2]) - (2*Sqrt[2]*(2*c*d - b*e)*(4*c^2*d^2 - b^2*e^2 - 4*c*e*(b*d - 2*
a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[
ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-
2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*(b^2 - 4*a*
c)^(3/2)*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^
2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*(16*c^2*d^2 - b^2*e^2 -
 4*c*e*(4*b*d - 5*a*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
```

```
*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*
a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*(b^2 - 4*a*c)^(3/2)*(c*d^2
 - b*d*e + a*e^2)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
```

```
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2\sqrt{d+ex}\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(8c^2d^2-2b^2e^2-5ce(bd-2ae)\right)+\frac{3}{2}ce(2cd-}{\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right.}$$

$$= -\frac{2\sqrt{d+ex}\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\sqrt{d+ex}\left(3ace(2cd-be)^2-\left(bcd\right.\right.}{}$$

$$= -\frac{2\sqrt{d+ex}\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\sqrt{d+ex}\left(3ace(2cd-be)^2-\left(bcd\right.\right.}{}$$

$$= -\frac{2\sqrt{d+ex}\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\sqrt{d+ex}\left(3ace(2cd-be)^2-\left(bcd\right.\right.}{}$$

$$= -\frac{2\sqrt{d+ex}\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\sqrt{d+ex}\left(3ace(2cd-be)^2-\left(bcd\right.\right.}{}$$

Mathematica [C] time = 13.0773, size = 5566, normalized size = 7.68

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[d + e*x]*(a + b*x + c*x^2)^(5/2)),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.464, size = 19400, normalized size = 26.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{5}{2}}\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(5/2)*sqrt(e*x + d)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+bx+a}\sqrt{ex+d}}{c^3ex^7+\left(c^3d+3\,bc^2e\right)x^6+3\left(bc^2d+\left(b^2c+ac^2\right)e\right)x^5+\left(3\left(b^2c+ac^2\right)d+\left(b^3+6\,abc\right)e\right)x^4+a^3d+\left(\left(b^3+6\,a\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(c^3*e*x^7 + (c^3*d + 3*b*c^2*
e)*x^6 + 3*(b*c^2*d + (b^2*c + a*c^2)*e)*x^5 + (3*(b^2*c + a*c^2)*d + (b^3
+ 6*a*b*c)*e)*x^4 + a^3*d + ((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*x^3 +
 3*(a^2*b*e + (a*b^2 + a^2*c)*d)*x^2 + (3*a^2*b*d + a^3*e)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2481 $\int \frac{1}{(d+ex)^{3/2}(a+bx+cx^2)^{5/2}}\,dx$

Optimal. Leaf size=918

$$-\frac{\sqrt{2}\sqrt{d+ex}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)\left(16c^4d^4-4c^3e(8bd-15ae)d^2-8b^4e^4+b^2ce^3(7bd\right.}{3\left(b^2-4ac\right)^{3/2}\left(cd^2-bed+ae^2\right)^3\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2+bx+a}}$$

```
[Out] (-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/(3*(b^2 - 4*a*c)*(c*d^2
- b*d*e + a*e^2)*Sqrt[d + e*x]*(a + b*x + c*x^2)^(3/2)) - (2*(5*a*c*e*(2*c*
d - b*e)^2 - (b*c*d - b^2*e + 2*a*c*e)*(8*c^2*d^2 - 4*b^2*e^2 - c*e*(3*b*d
- 14*a*e)) - 4*c*(2*c*d - b*e)*(2*c^2*d^2 - b^2*e^2 - 2*c*e*(b*d - 3*a*e))*
x))/(3*(b^2 - 4*a*c)^2*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[d + e*x]*Sqrt[a + b*x
 + c*x^2]) + (2*e*(16*c^4*d^4 - 8*b^4*e^4 - 4*c^3*d^2*e*(8*b*d - 15*a*e) +
b^2*c*e^3*(7*b*d + 57*a*e) + 3*c^2*e^2*(3*b^2*d^2 - 20*a*b*d*e - 28*a^2*e^2
))*Sqrt[a + b*x + c*x^2])/(3*(b^2 - 4*a*c)^2*(c*d^2 - b*d*e + a*e^2)^3*Sqrt
[d + e*x]) - (Sqrt[2]*(16*c^4*d^4 - 8*b^4*e^4 - 4*c^3*d^2*e*(8*b*d - 15*a*e
) + b^2*c*e^3*(7*b*d + 57*a*e) + 3*c^2*e^2*(3*b^2*d^2 - 20*a*b*d*e - 28*a^2
*e^2))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE
[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (
-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*(b^2 - 4*a
*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)^3*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (8*Sqrt[2]*(2*c*d - b*e)*(2*c^2*d
^2 - b^2*e^2 - 2*c*e*(b*d - 3*a*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[Arc
Sin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*S
qrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*(b^2 - 4*a*c)^
(3/2)*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 1.46332, antiderivative size = 918, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {740, 822, 834, 843, 718, 424, 419}

$$-\frac{\sqrt{2}\sqrt{d+ex}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)\left(16c^4d^4-4c^3e(8bd-15ae)d^2-8b^4e^4+b^2ce^3(7bd\right.}{3\left(b^2-4ac\right)^{3/2}\left(cd^2-bed+ae^2\right)^3\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2+bx+a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*(a + b*x + c*x^2)^(5/2)),x]
```



```
[Out] (-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/(3*(b^2 - 4*a*c)*(c*d^2
- b*d*e + a*e^2)*Sqrt[d + e*x]*(a + b*x + c*x^2)^(3/2)) - (2*(5*a*c*e*(2*c*
d - b*e)^2 - (b*c*d - b^2*e + 2*a*c*e)*(8*c^2*d^2 - 4*b^2*e^2 - c*e*(3*b*d
- 14*a*e)) - 4*c*(2*c*d - b*e)*(2*c^2*d^2 - b^2*e^2 - 2*c*e*(b*d - 3*a*e))*
x))/(3*(b^2 - 4*a*c)^2*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[d + e*x]*Sqrt[a + b*x
 + c*x^2]) + (2*e*(16*c^4*d^4 - 8*b^4*e^4 - 4*c^3*d^2*e*(8*b*d - 15*a*e) +
b^2*c*e^3*(7*b*d + 57*a*e) + 3*c^2*e^2*(3*b^2*d^2 - 20*a*b*d*e - 28*a^2*e^2
))*Sqrt[a + b*x + c*x^2])/(3*(b^2 - 4*a*c)^2*(c*d^2 - b*d*e + a*e^2)^3*Sqrt
```

```
[d + e*x]) - (Sqrt[2]*(16*c^4*d^4 - 8*b^4*e^4 - 4*c^3*d^2*e*(8*b*d - 15*a*e
) + b^2*c*e^3*(7*b*d + 57*a*e) + 3*c^2*e^2*(3*b^2*d^2 - 20*a*b*d*e - 28*a^2
*e^2))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE
[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (
-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*(b^2 - 4*a
*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)^3*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (8*Sqrt[2]*(2*c*d - b*e)*(2*c^2*d
^2 - b^2*e^2 - 2*c*e*(b*d - 3*a*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[Arc
Sin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*S
qrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*(b^2 - 4*a*c)^
(3/2)*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\left(a+bx+cx^2\right)^{5/2}}dx = -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(8c^2d^2-3bcde-4b^2e^2+14\cdots\right.}{(d+ex)^{3/2}\left(a+bx\cdots\right.}}{3\left(b^2-4ac\right)\left(cd^2\cdots\right.}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}} - \frac{2\left(5ace(2cd-be)^2-\left(bc\cdots\right.\right.}{\cdots}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}} - \frac{2\left(5ace(2cd-be)^2-\left(bc\cdots\right.\right.}{\cdots}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}} - \frac{2\left(5ace(2cd-be)^2-\left(bc\cdots\right.\right.}{\cdots}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}} - \frac{2\left(5ace(2cd-be)^2-\left(bc\cdots\right.\right.}{\cdots}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}} - \frac{2\left(5ace(2cd-be)^2-\left(bc\cdots\right.\right.}{\cdots}$$

Mathematica [C] time = 14.0416, size = 7870, normalized size = 8.57

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x)^(3/2)*(a + b*x + c*x^2)^(5/2)),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.517, size = 27157, normalized size = 29.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{5}{2}}(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(5/2)*(e*x + d)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{}{c^3e^2x^8+\left(2\,c^3de+3\,bc^2e^2\right)x^7+\left(c^3d^2+6\,bc^2de+3\left(b^2c+ac^2\right)e^2\right)x^6+\left(3\,bc^2d^2+6\left(b^2c+ac^2\right)de+\left(b^3+6\,a\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(c^3*e^2*x^8 + (2*c^3*d*e + 3*
b*c^2*e^2)*x^7 + (c^3*d^2 + 6*b*c^2*d*e + 3*(b^2*c + a*c^2)*e^2)*x^6 + (3*b
*c^2*d^2 + 6*(b^2*c + a*c^2)*d*e + (b^3 + 6*a*b*c)*e^2)*x^5 + a^3*d^2 + (3*
(b^2*c + a*c^2)*d^2 + 2*(b^3 + 6*a*b*c)*d*e + 3*(a*b^2 + a^2*c)*e^2)*x^4 +
(3*a^2*b*e^2 + (b^3 + 6*a*b*c)*d^2 + 6*(a*b^2 + a^2*c)*d*e)*x^3 + (6*a^2*b*
d*e + a^3*e^2 + 3*(a*b^2 + a^2*c)*d^2)*x^2 + (3*a^2*b*d^2 + 2*a^3*d*e)*x),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)^{\frac{3}{2}}\left(a+bx+cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Integral(1/((d + e*x)**(3/2)*(a + b*x + c*x**2)**(5/2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2482 $\int \frac{\sqrt{3+5x}}{\sqrt{2+5x-12x^2}}\,dx$

Optimal. Leaf size=30

$$-\frac{1}{3}\sqrt{19}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|\frac{55}{76}\right)$$

```
[Out] -(Sqrt[19]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], 55/76])/3
```

Rubi [A] time = 0.02086, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {718, 424}

$$-\frac{1}{3}\sqrt{19}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|\frac{55}{76}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 + 5*x]/Sqrt[2 + 5*x - 12*x^2],x]

[Out] -(Sqrt[19]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], 55/76])/3
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{3+5x}}{\sqrt{2+5x-12x^2}}\,dx &= -\left(\frac{1}{3}\sqrt{19}\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{55x^2}{76}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{16-24x}}{\sqrt{22}}\right)\right)\\ &= -\frac{1}{3}\sqrt{19}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|\frac{55}{76}\right)\end{aligned}$$

Mathematica [B] time = 0.0766456, size = 86, normalized size = 2.87

$$\frac{\sqrt{19}\sqrt{-4x-1}\sqrt{2-3x}\left(\text{EllipticF}\left(\sin^{-1}\left(\frac{2\sqrt{5x+3}}{\sqrt{7}}\right),\frac{21}{76}\right)-E\left(\sin^{-1}\left(\frac{2\sqrt{5x+3}}{\sqrt{7}}\right)|\frac{21}{76}\right)\right)}{3\sqrt{-12x^2+5x+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 + 5*x]/Sqrt[2 + 5*x - 12*x^2],x]
```

```
[Out] (Sqrt[19]*Sqrt[-1 - 4*x]*Sqrt[2 - 3*x]*(-EllipticE[ArcSin[(2*Sqrt[3 + 5*x])
/Sqrt[7]], 21/76] + EllipticF[ArcSin[(2*Sqrt[3 + 5*x])/Sqrt[7]], 21/76]))/(
3*Sqrt[2 + 5*x - 12*x^2])
```

Maple [B] time = 0.119, size = 77, normalized size = 2.6

$$-\frac{\sqrt{57}}{6840\,x^{2}-2850\,x-1140}\left(\mathrm{EllipticF}\left(\frac{1}{19}\sqrt{171+285\,x},\frac{2\,\sqrt{399}}{21}\right)-\mathrm{EllipticE}\left(\frac{1}{19}\sqrt{171+285\,x},\frac{2\,\sqrt{399}}{21}\right)\right)\sqrt{190-285\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+5*x)^(1/2)/(-12*x^2+5*x+2)^(1/2),x)
```

```
[Out] -1/570*(EllipticF(1/19*(171+285*x)^(1/2),2/21*399^(1/2))-EllipticE(1/19*(17
1+285*x)^(1/2),2/21*399^(1/2)))*(190-285*x)^(1/2)*(-140*x-35)^(1/2)*57^(1/2
)*(-12*x^2+5*x+2)^(1/2)/(12*x^2-5*x-2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{5\,x+3}}{\sqrt{-12\,x^{2}+5\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+5*x)^(1/2)/(-12*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(5*x + 3)/sqrt(-12*x^2 + 5*x + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-12\,x^{2}+5\,x+2}\sqrt{5\,x+3}}{12\,x^{2}-5\,x-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+5*x)^(1/2)/(-12*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-12*x^2 + 5*x + 2)*sqrt(5*x + 3)/(12*x^2 - 5*x - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{5x+3}}{\sqrt{-(3x-2)(4x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+5*x)**(1/2)/(-12*x**2+5*x+2)**(1/2),x)
```

```
[Out] Integral(sqrt(5*x + 3)/sqrt(-(3*x - 2)*(4*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{5\,x+3}}{\sqrt{-12\,x^2+5\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+5*x)^(1/2)/(-12*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(5*x + 3)/sqrt(-12*x^2 + 5*x + 2), x)
```

3.2483 $\int (d+ex)^2 \left(a+bx+cx^2\right)^{4/3} dx$

Optimal. Leaf size=638

$$\frac{\sqrt[3]{2}3^{3/4}\sqrt{2+\sqrt{3}}\left(b^2-4ac\right)^2\left(\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{a+bx+cx^2}\right)\sqrt{\frac{-2^{2/3}\sqrt[3]{c}\sqrt[3]{b^2-4ac}\sqrt[3]{a+bx+cx^2}+\left(b^2-4ac\right)^{2/3}+2\sqrt[3]{2}c^{2/3}\left(a+bx+cx^2\right)^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{a+bx+cx^2}\right)^2}}\left(\right.}{935c^{13/3}(b+2cx)\sqrt{\frac{\sqrt[3]{b^2-4ac}\left(\sqrt[3]{b^2-4ac}+2^{2/3}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{b^2-4ac}+2^{2/3}\right.}}}$$

```
[Out] (-3*(b^2 - 4*a*c)*(17*c^2*d^2 + 5*b^2*e^2 - c*e*(17*b*d + 3*a*e))*(b + 2*c*
x)*(a + b*x + c*x^2)^(1/3))/(935*c^4) + (3*(17*c^2*d^2 + 5*b^2*e^2 - c*e*(1
7*b*d + 3*a*e))*(b + 2*c*x)*(a + b*x + c*x^2)^(4/3))/(374*c^3) + (15*e*(2*c
*d - b*e)*(a + b*x + c*x^2)^(7/3))/(119*c^2) + (3*e*(d + e*x)*(a + b*x + c*
x^2)^(7/3))/(17*c) + (2^(1/3)*3^(3/4)*Sqrt[2 + Sqrt[3]]*(b^2 - 4*a*c)^2*(17
*c^2*d^2 + 5*b^2*e^2 - c*e*(17*b*d + 3*a*e))*((b^2 - 4*a*c)^(1/3) + 2^(2/3)
*c^(1/3)*(a + b*x + c*x^2)^(1/3))*Sqrt[((b^2 - 4*a*c)^(2/3) - 2^(2/3)*c^(1/
3)*(b^2 - 4*a*c)^(1/3)*(a + b*x + c*x^2)^(1/3) + 2*2^(1/3)*c^(2/3)*(a + b*x
 + c*x^2)^(2/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a +
b*x + c*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b^2 - 4*a*c)^(1/3)
+ 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/
3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))], -7 - 4*Sqrt[3]])/(935*c^(13
/3)*(b + 2*c*x)*Sqrt[((b^2 - 4*a*c)^(1/3)*((b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^
(1/3)*(a + b*x + c*x^2)^(1/3)))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3
)*c^(1/3)*(a + b*x + c*x^2)^(1/3))^2])
```

Rubi [A] time = 1.07187, antiderivative size = 638, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {742, 640, 623, 321, 218}

$$\frac{3(b+2cx)\left(a+bx+cx^2\right)^{4/3}\left(-ce(3ae+17bd)+5b^2e^2+17c^2d^2\right)}{374c^3}-\frac{3\left(b^2-4ac\right)(b+2cx)\sqrt[3]{a+bx+cx^2}\left(-ce(3ae+17bd)\right.}{935c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*x + c*x^2)^(4/3),x]
```


```
[Out] (-3*(b^2 - 4*a*c)*(17*c^2*d^2 + 5*b^2*e^2 - c*e*(17*b*d + 3*a*e))*(b + 2*c*
x)*(a + b*x + c*x^2)^(1/3))/(935*c^4) + (3*(17*c^2*d^2 + 5*b^2*e^2 - c*e*(1
7*b*d + 3*a*e))*(b + 2*c*x)*(a + b*x + c*x^2)^(4/3))/(374*c^3) + (15*e*(2*c
*d - b*e)*(a + b*x + c*x^2)^(7/3))/(119*c^2) + (3*e*(d + e*x)*(a + b*x + c*
x^2)^(7/3))/(17*c) + (2^(1/3)*3^(3/4)*Sqrt[2 + Sqrt[3]]*(b^2 - 4*a*c)^2*(17
*c^2*d^2 + 5*b^2*e^2 - c*e*(17*b*d + 3*a*e))*((b^2 - 4*a*c)^(1/3) + 2^(2/3)
*c^(1/3)*(a + b*x + c*x^2)^(1/3))*Sqrt[((b^2 - 4*a*c)^(2/3) - 2^(2/3)*c^(1/
3)*(b^2 - 4*a*c)^(1/3)*(a + b*x + c*x^2)^(1/3) + 2*2^(1/3)*c^(2/3)*(a + b*x
 + c*x^2)^(2/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a +
b*x + c*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b^2 - 4*a*c)^(1/3)
+ 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/
3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))], -7 - 4*Sqrt[3]])/(935*c^(13
/3)*(b + 2*c*x)*Sqrt[((b^2 - 4*a*c)^(1/3)*((b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^
(1/3)*(a + b*x + c*x^2)^(1/3)))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3
```

```
)*c^(1/3)*(a + b*x + c*x^2)^(1/3))^2])
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int (d+ex)^2\left(a+bx+cx^2\right)^{4/3}dx = \frac{3e(d+ex)\left(a+bx+cx^2\right)^{7/3}}{17c} + \frac{3\int\left(\frac{1}{3}\left(17cd^2-3e\left(\frac{7bd}{3}+ae\right)\right)+\frac{10}{3}e(2cd-be)x\right)\left(a+b\right.}{17c}$$

$$= \frac{15e(2cd-be)\left(a+bx+cx^2\right)^{7/3}}{119c^2} + \frac{3e(d+ex)\left(a+bx+cx^2\right)^{7/3}}{17c} + \frac{\left(3\left(-\frac{10}{3}be(2cd-be)+\right.\right.}{}$$

$$= \frac{15e(2cd-be)\left(a+bx+cx^2\right)^{7/3}}{119c^2} + \frac{3e(d+ex)\left(a+bx+cx^2\right)^{7/3}}{17c} + \frac{\left(9\left(-\frac{10}{3}be(2cd-be)+\right.\right.}{}$$

$$= \frac{3\left(17c^2d^2+5b^2e^2-ce(17bd+3ae)\right)(b+2cx)\left(a+bx+cx^2\right)^{4/3}}{374c^3} + \frac{15e(2cd-be)\left(a+b\right.}{119c^2}$$

$$= -\frac{3\left(b^2-4ac\right)\left(17c^2d^2+5b^2e^2-ce(17bd+3ae)\right)(b+2cx)\sqrt[3]{a+bx+cx^2}}{935c^4} + \frac{3\left(17c^2d^2\right.}{}$$

$$= -\frac{3\left(b^2-4ac\right)\left(17c^2d^2+5b^2e^2-ce(17bd+3ae)\right)(b+2cx)\sqrt[3]{a+bx+cx^2}}{935c^4} + \frac{3\left(17c^2d^2\right.}{}$$

Mathematica [C] time = 0.383738, size = 164, normalized size = 0.26

$$\frac{3(a+x(b+cx))^{4/3}\left(\frac{14\sqrt[3]{2}(b+2cx)\left(-ce(3ae+17bd)+5b^2e^2+17c^2d^2\right){}_2F_1\left(-\frac{4}{3},\frac{1}{2};\frac{3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{3c^2\left(-\frac{c(a+x(b+cx))}{b^2-4ac}\right)^{4/3}} - \frac{160e(a+x(b+cx))(be-2cd)}{c} + 224e(d+ex)(a+x(b+c\right.}{3808c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*x + c*x^2)^(4/3),x]
```

```
[Out] (3*(a + x*(b + c*x))^(4/3)*((-160*e*(-2*c*d + b*e)*(a + x*(b + c*x)))/c + 2
24*e*(d + e*x)*(a + x*(b + c*x)) + (14*2^(1/3)*(17*c^2*d^2 + 5*b^2*e^2 - c*
e*(17*b*d + 3*a*e))*(b + 2*c*x)*Hypergeometric2F1[-4/3, 1/2, 3/2, (b + 2*c*
x)^2/(b^2 - 4*a*c)])/(3*c^2*(-((c*(a + x*(b + c*x)))/(b^2 - 4*a*c)))^(4/3))
))/(3808*c)
```

Maple [F] time = 1.145, size = 0, normalized size = 0.

$$\int (ex+d)^2\left(cx^2+bx+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+b*x+a)^(4/3),x)
```

```
[Out] int((e*x+d)^2*(c*x^2+b*x+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{\frac{4}{3}}(ex+d)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(4/3)*(e*x + d)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ce^2x^4 + \left(2\,cde + be^2\right)x^3 + ad^2 + \left(cd^2 + 2\,bde + ae^2\right)x^2 + \left(bd^2 + 2\,ade\right)x\right)\left(cx^2 + bx + a\right)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((c*e^2*x^4 + (2*c*d*e + b*e^2)*x^3 + a*d^2 + (c*d^2 + 2*b*d*e + a*
e^2)*x^2 + (b*d^2 + 2*a*d*e)*x)*(c*x^2 + b*x + a)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d + ex)^2 \left(a + bx + cx^2\right)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+b*x+a)**(4/3),x)
```

```
[Out] Integral((d + e*x)**2*(a + b*x + c*x**2)**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx + a\right)^{\frac{4}{3}} (ex + d)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(4/3)*(e*x + d)^2, x)
```

3.2484 $\int (d+ex)\left(a+bx+cx^2\right)^{4/3}\,dx$

Optimal. Leaf size=539

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\left(b^2-4ac\right)^2\left(\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{a+bx+cx^2}\right)\sqrt{\frac{-2^{2/3}\sqrt[3]{c}\sqrt[3]{b^2-4ac}\sqrt[3]{a+bx+cx^2}+\left(b^2-4ac\right)^{2/3}+2\sqrt[3]{2}c^{2/3}\left(a+bx+cx^2\right)^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{a+bx+cx^2}\right)^2}}(2cd\cdots}{55\,2^{2/3}c^{10/3}(b+2cx)\sqrt{\frac{\sqrt[3]{b^2-4ac}\left(\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{a+bx+cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{a+bx+cx^2}\right)^2}}}$$

```
[Out] (-3*(b^2 - 4*a*c)*(2*c*d - b*e)*(b + 2*c*x)*(a + b*x + c*x^2)^(1/3))/(110*c
^3) + (3*(2*c*d - b*e)*(b + 2*c*x)*(a + b*x + c*x^2)^(4/3))/(44*c^2) + (3*e
*(a + b*x + c*x^2)^(7/3))/(14*c) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*(b^2 - 4*a*c)
^2*(2*c*d - b*e)*((b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(
1/3))*Sqrt[((b^2 - 4*a*c)^(2/3) - 2^(2/3)*c^(1/3)*(b^2 - 4*a*c)^(1/3)*(a +
b*x + c*x^2)^(1/3) + 2*2^(1/3)*c^(2/3)*(a + b*x + c*x^2)^(2/3))/((1 + Sqrt[
3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))^2]*Ellip
ticF[ArcSin[((1 - Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x +
 c*x^2)^(1/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*
x + c*x^2)^(1/3))], -7 - 4*Sqrt[3]])/(55*2^(2/3)*c^(10/3)*(b + 2*c*x)*Sqrt[
((b^2 - 4*a*c)^(1/3)*((b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^
2)^(1/3)))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x +
c*x^2)^(1/3))^2])
```

Rubi [A] time = 0.51375, antiderivative size = 539, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {640, 623, 321, 218}

$$-\frac{3\left(b^2-4ac\right)(b+2cx)\sqrt[3]{a+bx+cx^2}(2cd-be)}{110c^3}+\frac{3^{3/4}\sqrt{2+\sqrt{3}}\left(b^2-4ac\right)^2\left(\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{a+bx+cx^2}\right)\sqrt{\frac{-2^{2/3}\sqrt[3]{\cdots}}{\cdots}}}{55\,2^{2/3}c^{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*x + c*x^2)^(4/3),x]
```


```
[Out] (-3*(b^2 - 4*a*c)*(2*c*d - b*e)*(b + 2*c*x)*(a + b*x + c*x^2)^(1/3))/(110*c
^3) + (3*(2*c*d - b*e)*(b + 2*c*x)*(a + b*x + c*x^2)^(4/3))/(44*c^2) + (3*e
*(a + b*x + c*x^2)^(7/3))/(14*c) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*(b^2 - 4*a*c)
^2*(2*c*d - b*e)*((b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(
1/3))*Sqrt[((b^2 - 4*a*c)^(2/3) - 2^(2/3)*c^(1/3)*(b^2 - 4*a*c)^(1/3)*(a +
b*x + c*x^2)^(1/3) + 2*2^(1/3)*c^(2/3)*(a + b*x + c*x^2)^(2/3))/((1 + Sqrt[
3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))^2]*Ellip
ticF[ArcSin[((1 - Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x +
 c*x^2)^(1/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*
x + c*x^2)^(1/3))], -7 - 4*Sqrt[3]])/(55*2^(2/3)*c^(10/3)*(b + 2*c*x)*Sqrt[
((b^2 - 4*a*c)^(1/3)*((b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^
2)^(1/3)))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x +
c*x^2)^(1/3))^2])
```


Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int (d+ex)\left(a+bx+cx^2\right)^{4/3}dx = \frac{3e\left(a+bx+cx^2\right)^{7/3}}{14c}+\frac{(2cd-be)\int\left(a+bx+cx^2\right)^{4/3}dx}{2c}$$

$$=\frac{3e\left(a+bx+cx^2\right)^{7/3}}{14c}+\frac{\left(3(2cd-be)\sqrt{(b+2cx)^2}\right)\text{Subst}\left(\int\frac{x^6}{\sqrt{b^2-4ac+4cx^3}}dx,x,\sqrt[3]{a+b\right.}{2c(b+2cx)}$$

$$=\frac{3(2cd-be)(b+2cx)\left(a+bx+cx^2\right)^{4/3}}{44c^2}+\frac{3e\left(a+bx+cx^2\right)^{7/3}}{14c}-\frac{\left(3\left(b^2-4ac\right)(2cd-\right.}{}$$

$$=-\frac{3\left(b^2-4ac\right)(2cd-be)(b+2cx)\sqrt[3]{a+bx+cx^2}}{110c^3}+\frac{3(2cd-be)(b+2cx)\left(a+bx+cx\right.}{44c^2}$$

$$=-\frac{3\left(b^2-4ac\right)(2cd-be)(b+2cx)\sqrt[3]{a+bx+cx^2}}{110c^3}+\frac{3(2cd-be)(b+2cx)\left(a+bx+cx\right.}{44c^2}$$

Mathematica [C] time = 0.222329, size = 113, normalized size = 0.21

$$\frac{(a+x(b+cx))^{4/3}\left(48c^2e(a+x(b+cx))-\frac{7\sqrt[3]{2}c(b+2cx)(be-2cd)\,{}_2F_1\left(-\frac{4}{3},\frac{1}{2};\frac{3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{\left(-\frac{c(a+x(b+cx))}{b^2-4ac}\right)^{4/3}}\right)}{224c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*x + c*x^2)^(4/3),x]
```

```
[Out] ((a + x*(b + c*x))^(4/3)*(48*c^2*e*(a + x*(b + c*x)) - (7*2^(1/3)*c*(-2*c*d
 + b*e)*(b + 2*c*x)*Hypergeometric2F1[-4/3, 1/2, 3/2, (b + 2*c*x)^2/(b^2 -
4*a*c)])/(-((c*(a + x*(b + c*x)))/(b^2 - 4*a*c))^(4/3)))/(224*c^3)
```

Maple [F] time = 1.012, size = 0, normalized size = 0.

$$\int (ex+d)\left(cx^2+bx+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+b*x+a)^(4/3),x)
```

```
[Out] int((e*x+d)*(c*x^2+b*x+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{\frac{4}{3}}(ex+d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(4/3)*(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cex^3+(cd+be)x^2+ad+(bd+ae)x\right)\left(cx^2+bx+a\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((c*e*x^3 + (c*d + b*e)*x^2 + a*d + (b*d + a*e)*x)*(c*x^2 + b*x + a
)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d+ex)\left(a+bx+cx^2\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+b*x+a)**(4/3),x)
```

```
[Out] Integral((d + e*x)*(a + b*x + c*x**2)**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{\frac{4}{3}}(ex+d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(4/3)*(e*x + d), x)
```

3.2485 $\int \left(a+bx+cx^2\right)^{4/3}\,dx$

Optimal. Leaf size=490

$$\frac{\sqrt[3]{2}3^{3/4}\sqrt{2+\sqrt{3}}\left(b^2-4ac\right)^2\left(\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{a+bx+cx^2}\right)\sqrt{\frac{-2^{2/3}\sqrt[3]{c}\sqrt[3]{b^2-4ac}\sqrt[3]{a+bx+cx^2}+\left(b^2-4ac\right)^{2/3}+2\sqrt[3]{2}c^{2/3}\left(a+bx+cx^2\right)^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{a+bx+cx^2}\right)^2}}\,\mathrm{El}}{55c^{7/3}(b+2cx)\sqrt{\frac{\sqrt[3]{b^2-4ac}\left(\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{a+bx+cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{a+bx+cx^2}\right)^2}}}$$

```
[Out] (-3*(b^2 - 4*a*c)*(b + 2*c*x)*(a + b*x + c*x^2)^(1/3))/(55*c^2) + (3*(b + 2
*c*x)*(a + b*x + c*x^2)^(4/3))/(22*c) + (2^(1/3)*3^(3/4)*Sqrt[2 + Sqrt[3]]*
(b^2 - 4*a*c)^2*((b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1
/3))*Sqrt[((b^2 - 4*a*c)^(2/3) - 2^(2/3)*c^(1/3)*(b^2 - 4*a*c)^(1/3)*(a + b
*x + c*x^2)^(1/3) + 2*2^(1/3)*c^(2/3)*(a + b*x + c*x^2)^(2/3))/((1 + Sqrt[3
])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x +
c*x^2)^(1/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x
 + c*x^2)^(1/3))], -7 - 4*Sqrt[3]])/(55*c^(7/3)*(b + 2*c*x)*Sqrt[((b^2 - 4*
a*c)^(1/3)*((b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3)))
/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/
3))^2])
```

Rubi [A] time = 0.390992, antiderivative size = 490, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {623, 321, 218}

$$-\frac{3\left(b^2-4ac\right)(b+2cx)\sqrt[3]{a+bx+cx^2}}{55c^2}+\frac{\sqrt[3]{2}3^{3/4}\sqrt{2+\sqrt{3}}\left(b^2-4ac\right)^2\left(\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{a+bx+cx^2}\right)\sqrt{\frac{-2^{2/3}\sqrt[3]{c}\sqrt[3]{b^2-4a}}{(1}}}{55c^{7/3}(b+2cx)\sqrt{\frac{\sqrt[3]{}}{(}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(4/3),x]
```

```
[Out] (-3*(b^2 - 4*a*c)*(b + 2*c*x)*(a + b*x + c*x^2)^(1/3))/(55*c^2) + (3*(b + 2
*c*x)*(a + b*x + c*x^2)^(4/3))/(22*c) + (2^(1/3)*3^(3/4)*Sqrt[2 + Sqrt[3]]*
(b^2 - 4*a*c)^2*((b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1
/3))*Sqrt[((b^2 - 4*a*c)^(2/3) - 2^(2/3)*c^(1/3)*(b^2 - 4*a*c)^(1/3)*(a + b
*x + c*x^2)^(1/3) + 2*2^(1/3)*c^(2/3)*(a + b*x + c*x^2)^(2/3))/((1 + Sqrt[3
])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x +
c*x^2)^(1/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x
 + c*x^2)^(1/3))], -7 - 4*Sqrt[3]])/(55*c^(7/3)*(b + 2*c*x)*Sqrt[((b^2 - 4*
a*c)^(1/3)*((b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3)))
/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/
3))^2])
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
```

```
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^{4/3} dx = \frac{\left(3\sqrt{(b+2cx)^2}\right) \text{Subst}\left(\int \frac{x^6}{\sqrt{b^2-4ac+4cx^3}} dx, x, \sqrt[3]{a+bx+cx^2}\right)}{b+2cx}$$

$$= \frac{3(b+2cx)\left(a+bx+cx^2\right)^{4/3}}{22c} - \frac{\left(6\left(b^2-4ac\right)\sqrt{(b+2cx)^2}\right) \text{Subst}\left(\int \frac{x^3}{\sqrt{b^2-4ac+4cx^3}} dx, x, \sqrt[3]{a+\right.}{11c(b+2cx)}$$

$$= -\frac{3\left(b^2-4ac\right)(b+2cx)\sqrt[3]{a+bx+cx^2}}{55c^2} + \frac{3(b+2cx)\left(a+bx+cx^2\right)^{4/3}}{22c} + \frac{\left(3\left(b^2-4ac\right)^2\sqrt{(b+\right.}{}$$

$$= -\frac{3\left(b^2-4ac\right)(b+2cx)\sqrt[3]{a+bx+cx^2}}{55c^2} + \frac{3(b+2cx)\left(a+bx+cx^2\right)^{4/3}}{22c} + \frac{\sqrt[3]{2}3^{3/4}\sqrt{2+\sqrt{3}}\left(b^2\right.}{}$$

Mathematica [C] time = 0.059461, size = 95, normalized size = 0.19

$$-\frac{\left(b^2-4ac\right)(b+2cx)\sqrt[3]{a+x(b+cx)}\,{}_2F_1\left(-\frac{4}{3},\frac{1}{2};\frac{3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{8\ 2^{2/3}c^2\sqrt[3]{\frac{c(a+x(b+cx))}{4ac-b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(4/3),x]
```

```
[Out] -((b^2 - 4*a*c)*(b + 2*c*x)*(a + x*(b + c*x))^(1/3)*Hypergeometric2F1[-4/3,
 1/2, 3/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(8*2^(2/3)*c^2*((c*(a + x*(b + c*x
)))/(-b^2 + 4*a*c))^(1/3))
```

Maple [F] time = 2.244, size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(4/3),x)
```

```
[Out] int((c*x^2+b*x+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(cx^2+bx+a\right)^{\frac{4}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^(4/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+bx+cx^2\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(4/3),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(4/3), x)
```

3.2486 $\int \frac{(a+bx+cx^2)^{4/3}}{d+ex}\,dx$

Optimal. Leaf size=180

$$\frac{3\left(a+bx+cx^2\right)^{4/3} F_1\left(-\frac{8}{3};-\frac{4}{3},-\frac{4}{3};-\frac{5}{3};\frac{2cd-\left(b-\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{c}}{2(d+ex)}\right)}{\sqrt[3]{2}e\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{4/3}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{4/3}}$$

```
[Out] (3*(a + b*x + c*x^2)^(4/3)*AppellF1[-8/3, -4/3, -4/3, -5/3, (2*c*d - (b - S
qrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x)), (2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c)
/(2*(d + e*x))]])/(2^(1/3)*e*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*
x)))^(4/3)*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(4/3))
```

Rubi [A] time = 0.198309, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {758, 133}

$$\frac{3\left(a+bx+cx^2\right)^{4/3} F_1\left(-\frac{8}{3};-\frac{4}{3},-\frac{4}{3};-\frac{5}{3};\frac{2cd-\left(b-\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{c}}{2(d+ex)}\right)}{\sqrt[3]{2}e\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{4/3}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(4/3)/(d + e*x),x]
```

```
[Out] (3*(a + b*x + c*x^2)^(4/3)*AppellF1[-8/3, -4/3, -4/3, -5/3, (2*c*d - (b - S
qrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x)), (2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c)
/(2*(d + e*x))]])/(2^(1/3)*e*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*
x)))^(4/3)*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(4/3))
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, -Dist[((1/(d + e*x))^(2*p)*(a + b*
x + c*x^2)^p)/(e*((e*(b - q + 2*c*x))/(2*c*(d + e*x)))^p*((e*(b + q + 2*c*x
))/(2*c*(d + e*x)))^p), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - (e*(b -
q))/(2*c))*x, x]^p*Simp[1 - (d - (e*(b + q))/(2*c))*x, x]^p, x], x, 1/(d +
e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m,
0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{4/3}}{d+ex}\,dx = -\frac{\left(4\,2^{2/3}\left(\frac{1}{d+ex}\right)^{8/3}(a+bx+cx^2)^{4/3}\right)\text{Subst}\left(\int \frac{\left(1-\frac{1}{2}\left(2d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{c}\right)x\right)^{4/3}\left(1-\frac{1}{2}\left(2d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{c}\right)x\right)^{4/3}}{x^{11/3}}\,dx,\right.}{e\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{4/3}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{4/3}}$$

$$= \frac{3(a+bx+cx^2)^{4/3}F_1\left(-\frac{8}{3};-\frac{4}{3},-\frac{4}{3};-\frac{5}{3};\frac{2cd-\left(b-\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{c}}{2(d+ex)}\right)}{\sqrt[3]{2}e\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{4/3}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{4/3}}$$

Mathematica [F] time = 1.30788, size = 0, normalized size = 0.

$$\int \frac{(a+bx+cx^2)^{4/3}}{d+ex}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*x + c*x^2)^(4/3)/(d + e*x),x]
```

```
[Out] Integrate[(a + b*x + c*x^2)^(4/3)/(d + e*x), x]
```

Maple [F] time = 1.302, size = 0, normalized size = 0.

$$\int \frac{1}{ex+d}\left(cx^2+bx+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(4/3)/(e*x+d),x)
```

```
[Out] int((c*x^2+b*x+a)^(4/3)/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{4}{3}}}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(4/3)/(e*x + d), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx + cx^2\right)^{\frac{4}{3}}}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(4/3)/(e*x+d),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(4/3)/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx + a\right)^{\frac{4}{3}}}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(4/3)/(e*x + d), x)
```

3.2487 $\int \frac{(a+bx+cx^2)^{4/3}}{(d+ex)^2}\,dx$

Optimal. Leaf size=189

$$\frac{12\ 2^{2/3}\left(a+bx+cx^2\right)^{4/3}F_1\left(-\frac{5}{3};-\frac{4}{3},-\frac{4}{3};-\frac{2}{3};\frac{2cd-\left(b-\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{c}}{2(d+ex)}\right)}{5e(d+ex)\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{4/3}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{4/3}}$$

```
[Out] (12*2^(2/3)*(a + b*x + c*x^2)^(4/3)*AppellF1[-5/3, -4/3, -4/3, -2/3, (2*c*d
 - (b - Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x)), (2*d - ((b + Sqrt[b^2 - 4*a*
c])*e)/c)/(2*(d + e*x))]])/(5*e*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d +
 e*x)))^(4/3)*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(4/3)*(d
+ e*x))
```

Rubi [A] time = 0.0908302, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {758, 133}

$$\frac{12\ 2^{2/3}\left(a+bx+cx^2\right)^{4/3}F_1\left(-\frac{5}{3};-\frac{4}{3},-\frac{4}{3};-\frac{2}{3};\frac{2cd-\left(b-\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{c}}{2(d+ex)}\right)}{5e(d+ex)\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{4/3}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(4/3)/(d + e*x)^2,x]
```


```
[Out] (12*2^(2/3)*(a + b*x + c*x^2)^(4/3)*AppellF1[-5/3, -4/3, -4/3, -2/3, (2*c*d
 - (b - Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x)), (2*d - ((b + Sqrt[b^2 - 4*a*
c])*e)/c)/(2*(d + e*x))]])/(5*e*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d +
 e*x)))^(4/3)*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(4/3)*(d
+ e*x))
```


Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, -Dist[((1/(d + e*x))^(2*p)*(a + b*
x + c*x^2)^p)/(e*((e*(b - q + 2*c*x))/(2*c*(d + e*x)))^p*((e*(b + q + 2*c*x
))/(2*c*(d + e*x)))^p), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - (e*(b -
q))/(2*c))*x, x]^p*Simp[1 - (d - (e*(b + q))/(2*c))*x, x]^p, x], x, 1/(d +
e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m,
0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{4/3}}{(d+ex)^2}\,dx = -\frac{\left(4\ 2^{2/3}\left(\frac{1}{d+ex}\right)^{8/3}\left(a+bx+cx^2\right)^{4/3}\right)\operatorname{Subst}\left(\int \frac{\left(1-\frac{1}{2}\left(2d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{c}\right)x\right)^{4/3}\left(1-\frac{1}{2}\left(2d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{c}\right)x\right)^{4/3}}{x^{8/3}}\right.}{e\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{4/3}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{4/3}}$$

$$= \frac{12\ 2^{2/3}\left(a+bx+cx^2\right)^{4/3}F_1\left(-\frac{5}{3};-\frac{4}{3},-\frac{4}{3};-\frac{2}{3};\frac{2cd-\left(b-\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{c}}{2(d+ex)}\right)}{5e\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{4/3}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{4/3}(d+ex)}$$

Mathematica [F] time = 0.899644, size = 0, normalized size = 0.

$$\int \frac{\left(a+bx+cx^2\right)^{4/3}}{(d+ex)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*x + c*x^2)^(4/3)/(d + e*x)^2,x]
```

```
[Out] Integrate[(a + b*x + c*x^2)^(4/3)/(d + e*x)^2, x]
```

Maple [F] time = 1.816, size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)^2}\left(cx^2+bx+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(4/3)/(e*x+d)^2,x)
```

```
[Out] int((c*x^2+b*x+a)^(4/3)/(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{4}{3}}}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(4/3)/(e*x + d)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(4/3)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{4}{3}}}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(4/3)/(e*x + d)^2, x)
```

3.2488 $\int \frac{\left(a+bx+cx^2\right)^{4/3}}{(d+ex)^3}\,dx$

Optimal. Leaf size=187

$$\frac{6\,2^{2/3}\left(a+bx+cx^2\right)^{4/3}F_1\left(-\frac{2}{3};-\frac{4}{3},-\frac{4}{3};\frac{1}{3};\frac{2cd-\left(b-\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{c}}{2(d+ex)}\right)}{e(d+ex)^2\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{4/3}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{4/3}}$$

```
[Out] (6*2^(2/3)*(a + b*x + c*x^2)^(4/3)*AppellF1[-2/3, -4/3, -4/3, 1/3, (2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x)), (2*d - ((b + Sqrt[b^2 - 4*a*c]
)*e)/c)/(2*(d + e*x))]])/(e*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x
)))^(4/3)*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(4/3)*(d + e*
x)^2)
```

Rubi [A] time = 0.0885369, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {758, 133}

$$\frac{6\,2^{2/3}\left(a+bx+cx^2\right)^{4/3}F_1\left(-\frac{2}{3};-\frac{4}{3},-\frac{4}{3};\frac{1}{3};\frac{2cd-\left(b-\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{c}}{2(d+ex)}\right)}{e(d+ex)^2\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{4/3}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(4/3)/(d + e*x)^3,x]
```

```
[Out] (6*2^(2/3)*(a + b*x + c*x^2)^(4/3)*AppellF1[-2/3, -4/3, -4/3, 1/3, (2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x)), (2*d - ((b + Sqrt[b^2 - 4*a*c]
)*e)/c)/(2*(d + e*x))]])/(e*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x
)))^(4/3)*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(4/3)*(d + e*
x)^2)
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, -Dist[((1/(d + e*x))^(2*p)*(a + b*
x + c*x^2)^p)/(e*((e*(b - q + 2*c*x))/(2*c*(d + e*x)))^p*((e*(b + q + 2*c*x
))/(2*c*(d + e*x)))^p), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - (e*(b -
q))/(2*c))*x, x]^p*Simp[1 - (d - (e*(b + q))/(2*c))*x, x]^p, x], x, 1/(d +
e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m,
0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{4/3}}{(d+ex)^3}\,dx = -\frac{\left(4\ 2^{2/3}\left(\frac{1}{d+ex}\right)^{8/3}\left(a+bx+cx^2\right)^{4/3}\right)\text{Subst}\left(\int \frac{\left(1-\frac{1}{2}\left(2d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{c}\right)x\right)^{4/3}\left(1-\frac{1}{2}\left(2d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{c}\right)x\right)^{4/3}}{x^{5/3}}\,dx,\right.}{e\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{4/3}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{4/3}}$$

$$= \frac{6\ 2^{2/3}\left(a+bx+cx^2\right)^{4/3}F_1\left(-\frac{2}{3};-\frac{4}{3},-\frac{4}{3};\frac{1}{3};\frac{2cd-\left(b-\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{c}}{2(d+ex)}\right)}{e\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{4/3}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{4/3}(d+ex)^2}$$

Mathematica [F] time = 0.876346, size = 0, normalized size = 0.

$$\int \frac{\left(a+bx+cx^2\right)^{4/3}}{(d+ex)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*x + c*x^2)^(4/3)/(d + e*x)^3,x]
```

```
[Out] Integrate[(a + b*x + c*x^2)^(4/3)/(d + e*x)^3, x]
```

Maple [F] time = 1.074, size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)^3}\left(cx^2+bx+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(4/3)/(e*x+d)^3,x)
```

```
[Out] int((c*x^2+b*x+a)^(4/3)/(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{4}{3}}}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(4/3)/(e*x + d)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(4/3)/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx + a\right)^{\frac{4}{3}}}{(ex + d)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(4/3)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(4/3)/(e*x + d)^3, x)
```

3.2489 $\int \frac{(d+ex)^3}{(a+bx+cx^2)^{7/3}}\,dx$

Optimal. Leaf size=1224

result too large to display

```
[Out] (-3*(d + e*x)^2*(b*d - 2*a*e + (2*c*d - b*e)*x))/(4*(b^2 - 4*a*c)*(a + b*x
+ c*x^2)^(4/3)) + (3*(10*b*c*d*(c*d^2 + 3*a*e^2) - 8*a*c*e*(2*c*d^2 + 3*a*e
^2) - b^2*(11*c*d^2*e - a*e^3) + (2*c*d - b*e)*(10*c^2*d^2 - b^2*e^2 - 2*c*
e*(5*b*d - 7*a*e))*x))/(4*c*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^(1/3)) - (3*(
2*c*d - b*e)*(5*c^2*d^2 - b^2*e^2 - c*e*(5*b*d - 9*a*e))*(b + 2*c*x))/(2*2^
(1/3)*c^(5/3)*(b^2 - 4*a*c)^2*((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*
c^(1/3)*(a + b*x + c*x^2)^(1/3))) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*(2*c*d - b
*e)*(5*c^2*d^2 - b^2*e^2 - c*e*(5*b*d - 9*a*e))*((b^2 - 4*a*c)^(1/3) + 2^(2
/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))*Sqrt[((b^2 - 4*a*c)^(2/3) - 2^(2/3)*c^
(1/3)*(b^2 - 4*a*c)^(1/3)*(a + b*x + c*x^2)^(1/3) + 2*2^(1/3)*c^(2/3)*(a +
b*x + c*x^2)^(2/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a
 + b*x + c*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt[3])*(b^2 - 4*a*c)^(1/
3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^
(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))], -7 - 4*Sqrt[3]])/(4*2^(1
/3)*c^(5/3)*(b^2 - 4*a*c)^(5/3)*(b + 2*c*x)*Sqrt[((b^2 - 4*a*c)^(1/3)*((b^2
 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3)))/((1 + Sqrt[3])*
(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))^2]) - (3^(3/
4)*(2*c*d - b*e)*(5*c^2*d^2 - b^2*e^2 - c*e*(5*b*d - 9*a*e))*((b^2 - 4*a*c)
^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))*Sqrt[((b^2 - 4*a*c)^(2/3)
 - 2^(2/3)*c^(1/3)*(b^2 - 4*a*c)^(1/3)*(a + b*x + c*x^2)^(1/3) + 2*2^(1/3)*
c^(2/3)*(a + b*x + c*x^2)^(2/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/
3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b^2
 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))/((1 + Sqrt[3])*(
b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))], -7 - 4*Sqrt
[3]])/(2^(5/6)*c^(5/3)*(b^2 - 4*a*c)^(5/3)*(b + 2*c*x)*Sqrt[((b^2 - 4*a*c)^
(1/3)*((b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3)))/((1
+ Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))^2
])
```

Rubi [A] time = 1.55613, antiderivative size = 1224, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {738, 777, 623, 303, 218, 1877}

$$-\frac{3(bd-2ae+(2cd-be)x)(d+ex)^2}{4\left(b^2-4ac\right)\left(cx^2+bx+a\right)^{4/3}}+\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}(2cd-be)\left(5c^2d^2-b^2e^2-ce(5bd-9ae)\right)\left(\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{cx^2 \cdots}\right.}{4\sqrt[3]{2}c^{5/3}\left(b^2-4 \cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + b*x + c*x^2)^(7/3),x]
```


```
[Out] (-3*(d + e*x)^2*(b*d - 2*a*e + (2*c*d - b*e)*x))/(4*(b^2 - 4*a*c)*(a + b*x
+ c*x^2)^(4/3)) + (3*(10*b*c*d*(c*d^2 + 3*a*e^2) - 8*a*c*e*(2*c*d^2 + 3*a*e
^2) - b^2*(11*c*d^2*e - a*e^3) + (2*c*d - b*e)*(10*c^2*d^2 - b^2*e^2 - 2*c*
e*(5*b*d - 7*a*e))*x))/(4*c*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^(1/3)) - (3*(
2*c*d - b*e)*(5*c^2*d^2 - b^2*e^2 - c*e*(5*b*d - 9*a*e))*(b + 2*c*x))/(2*2^
```

```
(1/3)*c^(5/3)*(b^2 - 4*a*c)^2*((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*
c^(1/3)*(a + b*x + c*x^2)^(1/3))) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*(2*c*d - b
*e)*(5*c^2*d^2 - b^2*e^2 - c*e*(5*b*d - 9*a*e))*((b^2 - 4*a*c)^(1/3) + 2^(2
/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))*Sqrt[((b^2 - 4*a*c)^(2/3) - 2^(2/3)*c^
(1/3)*(b^2 - 4*a*c)^(1/3)*(a + b*x + c*x^2)^(1/3) + 2*2^(1/3)*c^(2/3)*(a +
b*x + c*x^2)^(2/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a
 + b*x + c*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt[3])*(b^2 - 4*a*c)^(1/
3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^
(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))], -7 - 4*Sqrt[3]])/(4*2^(1
/3)*c^(5/3)*(b^2 - 4*a*c)^(5/3)*(b + 2*c*x)*Sqrt[((b^2 - 4*a*c)^(1/3)*((b^2
 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3)))/((1 + Sqrt[3])*
(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))^2]) - (3^(3/
4)*(2*c*d - b*e)*(5*c^2*d^2 - b^2*e^2 - c*e*(5*b*d - 9*a*e))*((b^2 - 4*a*c)
^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))*Sqrt[((b^2 - 4*a*c)^(2/3)
 - 2^(2/3)*c^(1/3)*(b^2 - 4*a*c)^(1/3)*(a + b*x + c*x^2)^(1/3) + 2*2^(1/3)*
c^(2/3)*(a + b*x + c*x^2)^(2/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/
3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b^2
 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))/((1 + Sqrt[3])*(
b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))], -7 - 4*Sqrt
[3]])/(2^(5/6)*c^(5/3)*(b^2 - 4*a*c)^(5/3)*(b + 2*c*x)*Sqrt[((b^2 - 4*a*c)^
(1/3)*((b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3)))/((1
+ Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))^2
])
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 777

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p +
1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(
2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(a+bx+cx^2\right)^{7/3}}\,dx = -\frac{3(d+ex)^2(bd-2ae+(2cd-be)x)}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{4/3}} - \frac{3\int \frac{(d+ex)\left(\frac{1}{3}\left(10cd^2-11bde+12ae^2\right)-\frac{1}{3}e(2cd-be)x\right)}{\left(a+bx+cx^2\right)^{4/3}}\,dx}{4\left(b^2-4ac\right)}$$

$$= -\frac{3(d+ex)^2(bd-2ae+(2cd-be)x)}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{4/3}} + \frac{3\left(10bcd\left(cd^2+3ae^2\right)-8ace\left(2cd^2+3ae^2\right)-b^2\left(11cd^2e\right.\right.}{4c\left(b^2-4ac\right)^2}$$

$$= -\frac{3(d+ex)^2(bd-2ae+(2cd-be)x)}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{4/3}} + \frac{3\left(10bcd\left(cd^2+3ae^2\right)-8ace\left(2cd^2+3ae^2\right)-b^2\left(11cd^2e\right.\right.}{4c\left(b^2-4ac\right)^2}$$

$$= -\frac{3(d+ex)^2(bd-2ae+(2cd-be)x)}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{4/3}} + \frac{3\left(10bcd\left(cd^2+3ae^2\right)-8ace\left(2cd^2+3ae^2\right)-b^2\left(11cd^2e\right.\right.}{4c\left(b^2-4ac\right)^2}$$

$$= -\frac{3(d+ex)^2(bd-2ae+(2cd-be)x)}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{4/3}} + \frac{3\left(10bcd\left(cd^2+3ae^2\right)-8ace\left(2cd^2+3ae^2\right)-b^2\left(11cd^2e\right.\right.}{4c\left(b^2-4ac\right)^2}$$

Mathematica [C] time = 1.25734, size = 446, normalized size = 0.36

$$\frac{3c\left(b^2\left(a^2e^3+ace\left(-9d^2+42dex-11e^2x^2\right)+c^2dx\left(8d^2-45dex+6e^2x^2\right)\right)+2bc\left(a^2e^2(15d-19ex)+ac\left(-21d^2ex+7d^3+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + b*x + c*x^2)^(7/3),x]
```

```
[Out] (3*c*(b^4*e^3*x^2 + b^2*(a^2*e^3 + a*c*e*(-9*d^2 + 42*d*e*x - 11*e^2*x^2) +
 c^2*d*x*(8*d^2 - 45*d*e*x + 6*e^2*x^2)) + 4*c*(-6*a^3*e^3 + 5*c^3*d^3*x^3
```

```
+ a^2*c*e*(-6*d^2 + 3*d*e*x - 8*e^2*x^2) + a*c^2*d*x*(7*d^2 + 9*e^2*x^2)) +
 2*b*c*(a^2*e^2*(15*d - 19*e*x) + 15*c^2*d^2*x^2*(d - e*x) + a*c*(7*d^3 - 2
1*d^2*e*x + 27*d*e^2*x^2 - 9*e^3*x^3)) + b^3*(2*a*e^3*x - c*(d^3 + 12*d^2*e
*x - 9*d*e^2*x^2 - 2*e^3*x^3))) - 2^(2/3)*(2*c*d - b*e)*(b + 2*c*x)*((c*(a
+ x*(b + c*x)))/(-b^2 + 4*a*c))^(1/3)*(9*a^2*c*e^2 + (5*c^2*d^2 - 5*b*c*d*e
 - b^2*e^2)*x*(b + c*x) + a*(-(b^2*e^2) + b*c*e*(-5*d + 9*e*x) + c^2*(5*d^2
 + 9*e^2*x^2)))*Hypergeometric2F1[1/3, 1/2, 3/2, (b + 2*c*x)^2/(b^2 - 4*a*c
)])/(4*c^2*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^(4/3))
```

Maple [F] time = 1.211, size = 0, normalized size = 0.

$$\int (ex+d)^3 \left(cx^2+bx+a\right)^{-\frac{7}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^2+b*x+a)^(7/3),x)
```

```
[Out] int((e*x+d)^3/(c*x^2+b*x+a)^(7/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3}{\left(cx^2+bx+a\right)^{\frac{7}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3/(c*x^2 + b*x + a)^(7/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\left(cx^2+bx+a\right)^{\frac{2}{3}}}{c^3x^6+3\,bc^2x^5+3\left(b^2c+ac^2\right)x^4+3\,a^2bx+\left(b^3+6\,abc\right)x^3+a^3+3\left(ab^2+a^2c\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^(7/3),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(c*x^2 + b*x + a)^(2/3)/
(c^3*x^6 + 3*b*c^2*x^5 + 3*(b^2*c + a*c^2)*x^4 + 3*a^2*b*x + (b^3 + 6*a*b*c
)*x^3 + a^3 + 3*(a*b^2 + a^2*c)*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**2+b*x+a)**(7/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3}{\left(cx^2+bx+a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3/(c*x^2 + b*x + a)^(7/3), x)
```

3.2490 $\int \frac{(d+ex)^2}{\left(a+bx+cx^2\right)^{7/3}}\,dx$

Optimal. Leaf size=1153

result too large to display

```
[Out] (-3*(d + e*x)*(b*d - 2*a*e + (2*c*d - b*e)*x))/(4*(b^2 - 4*a*c)*(a + b*x +
c*x^2)^(4/3)) - (3*(4*b^2*d*e + 4*a*c*d*e - 5*b*(c*d^2 + a*e^2) - (10*c^2*d
^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*x))/(2*(b^2 - 4*a*c)^2*(a + b*x + c*x
^2)^(1/3)) - (3*(10*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*(b + 2*c*x))
/(2*2^(1/3)*c^(2/3)*(b^2 - 4*a*c)^2*((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^
(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*(10*
c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*((b^2 - 4*a*c)^(1/3) + 2^(2/3)*c
^(1/3)*(a + b*x + c*x^2)^(1/3))*Sqrt[((b^2 - 4*a*c)^(2/3) - 2^(2/3)*c^(1/3)
*(b^2 - 4*a*c)^(1/3)*(a + b*x + c*x^2)^(1/3) + 2*2^(1/3)*c^(2/3)*(a + b*x +
 c*x^2)^(2/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*
x + c*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt[3])*(b^2 - 4*a*c)^(1/3) +
2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3)
 + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))], -7 - 4*Sqrt[3]])/(4*2^(1/3)*c
^(2/3)*(b^2 - 4*a*c)^(5/3)*(b + 2*c*x)*Sqrt[((b^2 - 4*a*c)^(1/3)*((b^2 - 4*
a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3)))/((1 + Sqrt[3])*(b^2
- 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))^2]) - (3^(3/4)*(1
0*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*((b^2 - 4*a*c)^(1/3) + 2^(2/3)
*c^(1/3)*(a + b*x + c*x^2)^(1/3))*Sqrt[((b^2 - 4*a*c)^(2/3) - 2^(2/3)*c^(1/
3)*(b^2 - 4*a*c)^(1/3)*(a + b*x + c*x^2)^(1/3) + 2*2^(1/3)*c^(2/3)*(a + b*x
 + c*x^2)^(2/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a +
b*x + c*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b^2 - 4*a*c)^(1/3)
+ 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/
3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))], -7 - 4*Sqrt[3]])/(2^(5/6)*c
^(2/3)*(b^2 - 4*a*c)^(5/3)*(b + 2*c*x)*Sqrt[((b^2 - 4*a*c)^(1/3)*((b^2 - 4*
a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3)))/((1 + Sqrt[3])*(b^2
- 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))^2])
```

Rubi [A] time = 1.36771, antiderivative size = 1153, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {738, 638, 623, 303, 218, 1877}

$$-\frac{3\left(10c^2d^2+b^2e^2-2ce(5bd-3ae)\right)(b+2cx)}{2\sqrt[3]{2}c^{2/3}\left(b^2-4ac\right)^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{cx^2+bx+a}\right)}-\frac{3\left(4deb^2-5\left(cd^2+ae^2\right)b+4acde-\left(10c^2d^2+\right.\right.}{2\left(b^2-4ac\right)^2\sqrt[3]{cx^2+bx+}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*x + c*x^2)^(7/3),x]
```

```
[Out] (-3*(d + e*x)*(b*d - 2*a*e + (2*c*d - b*e)*x))/(4*(b^2 - 4*a*c)*(a + b*x +
c*x^2)^(4/3)) - (3*(4*b^2*d*e + 4*a*c*d*e - 5*b*(c*d^2 + a*e^2) - (10*c^2*d
^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*x))/(2*(b^2 - 4*a*c)^2*(a + b*x + c*x
^2)^(1/3)) - (3*(10*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*(b + 2*c*x))
/(2*2^(1/3)*c^(2/3)*(b^2 - 4*a*c)^2*((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^
(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*(10*
c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*((b^2 - 4*a*c)^(1/3) + 2^(2/3)*c
```

```
^(1/3)*(a + b*x + c*x^2)^(1/3))*Sqrt[((b^2 - 4*a*c)^(2/3) - 2^(2/3)*c^(1/3)
*(b^2 - 4*a*c)^(1/3)*(a + b*x + c*x^2)^(1/3) + 2*2^(1/3)*c^(2/3)*(a + b*x +
 c*x^2)^(2/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*
x + c*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt[3])*(b^2 - 4*a*c)^(1/3) +
2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3)
 + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))], -7 - 4*Sqrt[3]])/(4*2^(1/3)*c
^(2/3)*(b^2 - 4*a*c)^(5/3)*(b + 2*c*x)*Sqrt[((b^2 - 4*a*c)^(1/3)*((b^2 - 4*
a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3)))/((1 + Sqrt[3])*(b^2
- 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))^2]) - (3^(3/4)*(1
0*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*a*e))*((b^2 - 4*a*c)^(1/3) + 2^(2/3)
*c^(1/3)*(a + b*x + c*x^2)^(1/3))*Sqrt[((b^2 - 4*a*c)^(2/3) - 2^(2/3)*c^(1/
3)*(b^2 - 4*a*c)^(1/3)*(a + b*x + c*x^2)^(1/3) + 2*2^(1/3)*c^(2/3)*(a + b*x
 + c*x^2)^(2/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a +
b*x + c*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b^2 - 4*a*c)^(1/3)
+ 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/
3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))], -7 - 4*Sqrt[3]])/(2^(5/6)*c
^(2/3)*(b^2 - 4*a*c)^(5/3)*(b + 2*c*x)*Sqrt[((b^2 - 4*a*c)^(1/3)*((b^2 - 4*
a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3)))/((1 + Sqrt[3])*(b^2
- 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))^2])
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
```

```
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(a+bx+cx^2\right)^{7/3}}\,dx = -\frac{3(d+ex)(bd-2ae+(2cd-be)x)}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{4/3}} - \frac{3\int \frac{\frac{2}{3}\left(5cd^2-e(4bd-3ae)\right)+\frac{2}{3}e(2cd-be)x}{\left(a+bx+cx^2\right)^{4/3}}\,dx}{4\left(b^2-4ac\right)}$$

$$= -\frac{3(d+ex)(bd-2ae+(2cd-be)x)}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{4/3}} - \frac{3\left(4b^2de+4acde-5b\left(cd^2+ae^2\right)-\left(10c^2d^2+b^2e^2-\right.\right.}{2\left(b^2-4ac\right)^2\sqrt[3]{a+bx+cx^2}}$$

$$= -\frac{3(d+ex)(bd-2ae+(2cd-be)x)}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{4/3}} - \frac{3\left(4b^2de+4acde-5b\left(cd^2+ae^2\right)-\left(10c^2d^2+b^2e^2-\right.\right.}{2\left(b^2-4ac\right)^2\sqrt[3]{a+bx+cx^2}}$$

$$= -\frac{3(d+ex)(bd-2ae+(2cd-be)x)}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{4/3}} - \frac{3\left(4b^2de+4acde-5b\left(cd^2+ae^2\right)-\left(10c^2d^2+b^2e^2-\right.\right.}{2\left(b^2-4ac\right)^2\sqrt[3]{a+bx+cx^2}}$$

$$= -\frac{3(d+ex)(bd-2ae+(2cd-be)x)}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{4/3}} - \frac{3\left(4b^2de+4acde-5b\left(cd^2+ae^2\right)-\left(10c^2d^2+b^2e^2-\right.\right.}{2\left(b^2-4ac\right)^2\sqrt[3]{a+bx+cx^2}}$$

Mathematica [C] time = 0.849296, size = 342, normalized size = 0.3

$$3c\left(2b\left(5a^2e^2+ac\left(7d^2-14dex+9e^2x^2\right)+5c^2dx^2(3d-2ex)\right)+4c\left(a^2e(ex-4d)+acx\left(7d^2+3e^2x^2\right)+5c^2d^2x^3\right)+b^2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*x + c*x^2)^(7/3),x]
```

```
[Out] (3*c*(-(b^3*(d^2 + 8*d*e*x - 3*e^2*x^2)) + b^2*(2*a*e*(-3*d + 7*e*x) + 2*c*
x*(4*d^2 - 15*d*e*x + e^2*x^2)) + 4*c*(5*c^2*d^2*x^3 + a^2*e*(-4*d + e*x) +
 a*c*x*(7*d^2 + 3*e^2*x^2)) + 2*b*(5*a^2*e^2 + 5*c^2*d*x^2*(3*d - 2*e*x) +
a*c*(7*d^2 - 14*d*e*x + 9*e^2*x^2))) - 2^(2/3)*(b + 2*c*x)*((c*(a + x*(b +
c*x)))/(-b^2 + 4*a*c))^(1/3)*(6*a^2*c*e^2 + (10*c^2*d^2 - 10*b*c*d*e + b^2*
e^2)*x*(b + c*x) + a*(b^2*e^2 + 2*b*c*e*(-5*d + 3*e*x) + 2*c^2*(5*d^2 + 3*e
^2*x^2)))*Hypergeometric2F1[1/3, 1/2, 3/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(4
*c*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^(4/3))
```

Maple [F] time = 1.213, size = 0, normalized size = 0.

$$\int (ex+d)^2 \left(cx^2+bx+a\right)^{-\frac{7}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^2+b*x+a)^(7/3),x)
```

```
[Out] int((e*x+d)^2/(c*x^2+b*x+a)^(7/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2}{\left(cx^2+bx+a\right)^{\frac{7}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2/(c*x^2 + b*x + a)^(7/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e^2x^2+2\,dex+d^2\right)\left(cx^2+bx+a\right)^{\frac{2}{3}}}{c^3x^6+3\,bc^2x^5+3\left(b^2c+ac^2\right)x^4+3\,a^2bx+\left(b^3+6\,abc\right)x^3+a^3+3\left(ab^2+a^2c\right)x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^(7/3),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(c*x^2 + b*x + a)^(2/3)/(c^3*x^6 + 3*b*c
^2*x^5 + 3*(b^2*c + a*c^2)*x^4 + 3*a^2*b*x + (b^3 + 6*a*b*c)*x^3 + a^3 + 3*
(a*b^2 + a^2*c)*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**2+b*x+a)**(7/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2}{\left(cx^2+bx+a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2/(c*x^2 + b*x + a)^(7/3), x)
```

3.2491 $\int \frac{d+ex}{(a+bx+cx^2)^{7/3}}\,dx$

Optimal. Leaf size=1043

result too large to display

```
[Out] (-3*(b*d - 2*a*e + (2*c*d - b*e)*x))/(4*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(4/
3)) + (15*(2*c*d - b*e)*(b + 2*c*x))/(4*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^(
1/3)) - (15*c^(1/3)*(2*c*d - b*e)*(b + 2*c*x))/(2*2^(1/3)*(b^2 - 4*a*c)^2*(
(1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3)
)) + (15*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(2*c*d - b*e)*((b^2 - 4*a*c)^(1/
3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))*Sqrt[((b^2 - 4*a*c)^(2/3) - 2
^(2/3)*c^(1/3)*(b^2 - 4*a*c)^(1/3)*(a + b*x + c*x^2)^(1/3) + 2*2^(1/3)*c^(2
/3)*(a + b*x + c*x^2)^(2/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c
^(1/3)*(a + b*x + c*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt[3])*(b^2 - 4
*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))/((1 + Sqrt[3])*(b^2
- 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))], -7 - 4*Sqrt[3]]
)/(4*2^(1/3)*(b^2 - 4*a*c)^(5/3)*(b + 2*c*x)*Sqrt[((b^2 - 4*a*c)^(1/3)*((b^
2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3)))/((1 + Sqrt[3])
*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))^2]) - (5*3^
(3/4)*c^(1/3)*(2*c*d - b*e)*((b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x
 + c*x^2)^(1/3))*Sqrt[((b^2 - 4*a*c)^(2/3) - 2^(2/3)*c^(1/3)*(b^2 - 4*a*c)^
(1/3)*(a + b*x + c*x^2)^(1/3) + 2*2^(1/3)*c^(2/3)*(a + b*x + c*x^2)^(2/3))/
((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3
))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)
*(a + b*x + c*x^2)^(1/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1
/3)*(a + b*x + c*x^2)^(1/3))], -7 - 4*Sqrt[3]])/(2^(5/6)*(b^2 - 4*a*c)^(5/3
)*(b + 2*c*x)*Sqrt[((b^2 - 4*a*c)^(1/3)*((b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1
/3)*(a + b*x + c*x^2)^(1/3)))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*
c^(1/3)*(a + b*x + c*x^2)^(1/3))^2])
```

Rubi [A] time = 0.98868, antiderivative size = 1043, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {638, 623, 325, 303, 218, 1877}

$$\frac{15(2cd-be)(b+2cx)}{4\left(b^2-4ac\right)^2\sqrt[3]{cx^2+bx+a}}-\frac{15\sqrt[3]{c}(2cd-be)(b+2cx)}{2\sqrt[3]{2}\left(b^2-4ac\right)^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{cx^2+bx+a}\right)}-\frac{3(bd-2ae+(2cd-be}{4\left(b^2-4ac\right)\left(cx^2+bx+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*x + c*x^2)^(7/3),x]
```


```
[Out] (-3*(b*d - 2*a*e + (2*c*d - b*e)*x))/(4*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(4/
3)) + (15*(2*c*d - b*e)*(b + 2*c*x))/(4*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^(
1/3)) - (15*c^(1/3)*(2*c*d - b*e)*(b + 2*c*x))/(2*2^(1/3)*(b^2 - 4*a*c)^2*(
(1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3)
)) + (15*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(2*c*d - b*e)*((b^2 - 4*a*c)^(1/
3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))*Sqrt[((b^2 - 4*a*c)^(2/3) - 2
^(2/3)*c^(1/3)*(b^2 - 4*a*c)^(1/3)*(a + b*x + c*x^2)^(1/3) + 2*2^(1/3)*c^(2
/3)*(a + b*x + c*x^2)^(2/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c
^(1/3)*(a + b*x + c*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt[3])*(b^2 - 4
*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))/((1 + Sqrt[3])*(b^2
```

```
- 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))], -7 - 4*Sqrt[3]]
)/(4*2^(1/3)*(b^2 - 4*a*c)^(5/3)*(b + 2*c*x)*Sqrt[((b^2 - 4*a*c)^(1/3)*((b^
2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3)))/((1 + Sqrt[3])
*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))^2]) - (5*3^
(3/4)*c^(1/3)*(2*c*d - b*e)*((b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x
 + c*x^2)^(1/3))*Sqrt[((b^2 - 4*a*c)^(2/3) - 2^(2/3)*c^(1/3)*(b^2 - 4*a*c)^
(1/3)*(a + b*x + c*x^2)^(1/3) + 2*2^(1/3)*c^(2/3)*(a + b*x + c*x^2)^(2/3))/
((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3
))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)
*(a + b*x + c*x^2)^(1/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1
/3)*(a + b*x + c*x^2)^(1/3))], -7 - 4*Sqrt[3]])/(2^(5/6)*(b^2 - 4*a*c)^(5/3
)*(b + 2*c*x)*Sqrt[((b^2 - 4*a*c)^(1/3)*((b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1
/3)*(a + b*x + c*x^2)^(1/3)))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*
c^(1/3)*(a + b*x + c*x^2)^(1/3))^2])
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
```

```
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{d+ex}{\left(a+bx+cx^2\right)^{7/3}}\,dx = -\frac{3(bd-2ae+(2cd-be)x)}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{4/3}} - \frac{(5(2cd-be))\int \frac{1}{\left(a+bx+cx^2\right)^{4/3}}\,dx}{4\left(b^2-4ac\right)}$$

$$= -\frac{3(bd-2ae+(2cd-be)x)}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{4/3}} - \frac{\left(15(2cd-be)\sqrt{(b+2cx)^2}\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{b^2-4ac+4cx^3}}\,dx, x, \sqrt[3]{a\cdots}\right.}{4\left(b^2-4ac\right)(b+2cx)}$$

$$= -\frac{3(bd-2ae+(2cd-be)x)}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{4/3}} + \frac{15(2cd-be)(b+2cx)}{4\left(b^2-4ac\right)^2\sqrt[3]{a+bx+cx^2}} - \frac{\left(15c(2cd-be)\sqrt{(b+2cx)^2}\right)\text{S}\cdots}{2\left(b^2\cdots\right.}$$

$$= -\frac{3(bd-2ae+(2cd-be)x)}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{4/3}} + \frac{15(2cd-be)(b+2cx)}{4\left(b^2-4ac\right)^2\sqrt[3]{a+bx+cx^2}} - \frac{\left(15c^{2/3}(2cd-be)\sqrt{(b+2cx)^2}\right)\cdots}{2\cdots}$$

$$= -\frac{3(bd-2ae+(2cd-be)x)}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{4/3}} + \frac{15(2cd-be)(b+2cx)}{4\left(b^2-4ac\right)^2\sqrt[3]{a+bx+cx^2}} - \frac{15\sqrt[3]{c}(2cd\cdots}{2\sqrt[3]{2}\left(b^2-4ac\right)^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{b\cdots}\right.}$$

Mathematica [C] time = 0.53009, size = 200, normalized size = 0.19

$$\frac{3\left(5\,2^{2/3}\left(-\sqrt{b^2-4ac}+b+2cx\right)\sqrt[3]{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}(be-2cd)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};\frac{-b-2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)+\frac{4\left(b^2-4ac\right)(2ae-bd+bex-2cdx)}{a+x(b+cx)}+20(b\cdots\right.}{16\left(b^2-4ac\right)^2\sqrt[3]{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*x + c*x^2)^(7/3),x]
```

```
[Out] (3*(20*(2*c*d - b*e)*(b + 2*c*x) + (4*(b^2 - 4*a*c)*(-(b*d) + 2*a*e - 2*c*d
*x + b*e*x))/(a + x*(b + c*x)) + 5*2^(2/3)*(-2*c*d + b*e)*(b - Sqrt[b^2 - 4
*a*c] + 2*c*x)*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c])^(1/3)*Hy
pergeometric2F1[1/3, 2/3, 5/3, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/(2*Sqrt[b^2
 - 4*a*c])]))/(16*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^(1/3))
```

Maple [F] time = 1.058, size = 0, normalized size = 0.

$$\int (ex+d)\left(cx^2+bx+a\right)^{-\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^2+b*x+a)^(7/3),x)
```

```
[Out] int((e*x+d)/(c*x^2+b*x+a)^(7/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex+d}{\left(cx^2+bx+a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)/(c*x^2 + b*x + a)^(7/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+bx+a\right)^{\frac{2}{3}}(ex+d)}{c^3x^6+3\,bc^2x^5+3\left(b^2c+ac^2\right)x^4+3\,a^2bx+\left(b^3+6\,abc\right)x^3+a^3+3\left(ab^2+a^2c\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^(7/3),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^(2/3)*(e*x + d)/(c^3*x^6 + 3*b*c^2*x^5 + 3*(b^2*
c + a*c^2)*x^4 + 3*a^2*b*x + (b^3 + 6*a*b*c)*x^3 + a^3 + 3*(a*b^2 + a^2*c)*
x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**2+b*x+a)**(7/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex+d}{\left(cx^2+bx+a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)/(c*x^2 + b*x + a)^(7/3), x)
```

3.2492 $\int \frac{1}{\left(a+bx+cx^2\right)^{7/3}}\,dx$

Optimal. Leaf size=993

result too large to display

```
[Out] (-3*(b + 2*c*x))/(4*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(4/3)) + (15*c*(b + 2*c
*x))/(2*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^(1/3)) - (15*c^(4/3)*(b + 2*c*x))
/(2^(1/3)*(b^2 - 4*a*c)^2*((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1
/3)*(a + b*x + c*x^2)^(1/3))) + (15*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(4/3)*((b^2
 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))*Sqrt[((b^2 - 4*a
*c)^(2/3) - 2^(2/3)*c^(1/3)*(b^2 - 4*a*c)^(1/3)*(a + b*x + c*x^2)^(1/3) + 2
*2^(1/3)*c^(2/3)*(a + b*x + c*x^2)^(2/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3
) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt
[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))/((1 + S
qrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))], -7
 - 4*Sqrt[3]])/(2*2^(1/3)*(b^2 - 4*a*c)^(5/3)*(b + 2*c*x)*Sqrt[((b^2 - 4*a*
c)^(1/3)*((b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3)))/(
(1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3)
)^2]) - (5*2^(1/6)*3^(3/4)*c^(4/3)*((b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(
a + b*x + c*x^2)^(1/3))*Sqrt[((b^2 - 4*a*c)^(2/3) - 2^(2/3)*c^(1/3)*(b^2 -
4*a*c)^(1/3)*(a + b*x + c*x^2)^(1/3) + 2*2^(1/3)*c^(2/3)*(a + b*x + c*x^2)^
(2/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^
2)^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*
c^(1/3)*(a + b*x + c*x^2)^(1/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/
3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))], -7 - 4*Sqrt[3]])/((b^2 - 4*a*c)^(5/3)
*(b + 2*c*x)*Sqrt[((b^2 - 4*a*c)^(1/3)*((b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/
3)*(a + b*x + c*x^2)^(1/3)))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c
^(1/3)*(a + b*x + c*x^2)^(1/3))^2])
```

Rubi [A] time = 0.953901, antiderivative size = 993, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {623, 325, 303, 218, 1877}

$$\frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{cx^2+bx+a}\right)\sqrt{\frac{\left(b^2-4ac\right)^{2/3}-2^{2/3}\sqrt[3]{c}\sqrt[3]{cx^2+bx+a}\sqrt[3]{b^2-4ac}+2\sqrt[3]{2}c^{2/3}\left(cx^2+bx+a\right)^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{cx^2+bx+a}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)}\right)\right)}{2\sqrt[3]{2}\left(b^2-4ac\right)^{5/3}(b+2cx)\sqrt{\frac{\sqrt[3]{b^2-4ac}\left(\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{cx^2+bx+a}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{b^2-4ac}+2^{2/3}\sqrt[3]{c}\sqrt[3]{cx^2+bx+a}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(-7/3),x]
```

```
[Out] (-3*(b + 2*c*x))/(4*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(4/3)) + (15*c*(b + 2*c
*x))/(2*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^(1/3)) - (15*c^(4/3)*(b + 2*c*x))
/(2^(1/3)*(b^2 - 4*a*c)^2*((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1
/3)*(a + b*x + c*x^2)^(1/3))) + (15*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(4/3)*((b^2
 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))*Sqrt[((b^2 - 4*a
*c)^(2/3) - 2^(2/3)*c^(1/3)*(b^2 - 4*a*c)^(1/3)*(a + b*x + c*x^2)^(1/3) + 2
*2^(1/3)*c^(2/3)*(a + b*x + c*x^2)^(2/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3
) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt
[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))/((1 + S
qrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))], -7
 - 4*Sqrt[3]])/(2*2^(1/3)*(b^2 - 4*a*c)^(5/3)*(b + 2*c*x)*Sqrt[((b^2 - 4*a*
```

```
c)^(1/3)*((b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3)))/(
(1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^2)^(1/3)
)^2]) - (5*2^(1/6)*3^(3/4)*c^(4/3)*((b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(
a + b*x + c*x^2)^(1/3))*Sqrt[((b^2 - 4*a*c)^(2/3) - 2^(2/3)*c^(1/3)*(b^2 -
4*a*c)^(1/3)*(a + b*x + c*x^2)^(1/3) + 2*2^(1/3)*c^(2/3)*(a + b*x + c*x^2)^
(2/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/3)*(a + b*x + c*x^
2)^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*
c^(1/3)*(a + b*x + c*x^2)^(1/3))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/
3)*c^(1/3)*(a + b*x + c*x^2)^(1/3))], -7 - 4*Sqrt[3]])/((b^2 - 4*a*c)^(5/3)
*(b + 2*c*x)*Sqrt[((b^2 - 4*a*c)^(1/3)*((b^2 - 4*a*c)^(1/3) + 2^(2/3)*c^(1/
3)*(a + b*x + c*x^2)^(1/3)))/((1 + Sqrt[3])*(b^2 - 4*a*c)^(1/3) + 2^(2/3)*c
^(1/3)*(a + b*x + c*x^2)^(1/3))^2])
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx+cx^2\right)^{7/3}}\,dx = \frac{\left(3\sqrt{(b+2cx)^2}\right)\text{Subst}\left(\int \frac{1}{x^5\sqrt{b^2-4ac+4cx^3}}\,dx, x, \sqrt[3]{a+bx+cx^2}\right)}{b+2cx}$$

$$= -\frac{3(b+2cx)}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{4/3}} - \frac{\left(15c\sqrt{(b+2cx)^2}\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{b^2-4ac+4cx^3}}\,dx, x, \sqrt[3]{a+bx+cx\right.}{2\left(b^2-4ac\right)(b+2cx)}$$

$$= -\frac{3(b+2cx)}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{4/3}} + \frac{15c(b+2cx)}{2\left(b^2-4ac\right)^2\sqrt[3]{a+bx+cx^2}} - \frac{\left(15c^2\sqrt{(b+2cx)^2}\right)\text{Subst}\left(\int\right.}{\left(b^2-4ac\right.}$$

$$= -\frac{3(b+2cx)}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{4/3}} + \frac{15c(b+2cx)}{2\left(b^2-4ac\right)^2\sqrt[3]{a+bx+cx^2}} - \frac{\left(15c^{5/3}\sqrt{(b+2cx)^2}\right)\text{Subst}\left(\int\right.}{2^{2/3}\left(\right.}$$

$$= -\frac{3(b+2cx)}{4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{4/3}} + \frac{15c(b+2cx)}{2\left(b^2-4ac\right)^2\sqrt[3]{a+bx+cx^2}} - \frac{15c^{4/3}(}{\sqrt[3]{2}\left(b^2-4ac\right)^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{b^2}\right.}$$

Mathematica [C] time = 0.20328, size = 138, normalized size = 0.14

$$\frac{(b+2cx)\left(3\sqrt[3]{2}\left(2c\left(7a+5cx^2\right)-b^2+10bcx\right)-20c(a+x(b+cx))\sqrt[3]{\frac{c(a+x(b+cx))}{4ac-b^2}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)\right)}{4\sqrt[3]{2}\left(b^2-4ac\right)^2(a+x(b+cx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(-7/3),x]
```

```
[Out] ((b + 2*c*x)*(3*2^(1/3)*(-b^2 + 10*b*c*x + 2*c*(7*a + 5*c*x^2)) - 20*c*(a +
 x*(b + c*x))*((c*(a + x*(b + c*x)))/(-b^2 + 4*a*c))^(1/3)*Hypergeometric2F
1[1/3, 1/2, 3/2, (b + 2*c*x)^2/(b^2 - 4*a*c)]))/(4*2^(1/3)*(b^2 - 4*a*c)^2*
(a + x*(b + c*x))^(4/3))
```

Maple [F] time = 2.279, size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{-\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(7/3),x)
```

```
[Out] int(1/(c*x^2+b*x+a)^(7/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(-7/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+bx+a\right)^{\frac{2}{3}}}{c^3x^6+3\,bc^2x^5+3\left(b^2c+ac^2\right)x^4+3\,a^2bx+\left(b^3+6\,abc\right)x^3+a^3+3\left(ab^2+a^2c\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(7/3),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^(2/3)/(c^3*x^6 + 3*b*c^2*x^5 + 3*(b^2*c + a*c^2)
*x^4 + 3*a^2*b*x + (b^3 + 6*a*b*c)*x^3 + a^3 + 3*(a*b^2 + a^2*c)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx+cx^2\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(7/3),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(-7/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(-7/3), x)
```

3.2493 $\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{7/3}}\,dx$

Optimal. Leaf size=182

$$-\frac{3\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{7/3}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{7/3}F_1\left(\frac{14}{3};\frac{7}{3},\frac{7}{3};\frac{17}{3};\frac{2cd-\left(b-\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{c}}{2(d+ex)}\right)}{224\ 2^{2/3}e\left(a+bx+cx^2\right)^{7/3}}$$

```
[Out] (-3*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(7/3)*((e*(b + Sqrt
[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(7/3)*AppellF1[14/3, 7/3, 7/3, 17/3,
 (2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x)), (2*d - ((b + Sqrt[b^2
 - 4*a*c])*e)/c)/(2*(d + e*x))])/(224*2^(2/3)*e*(a + b*x + c*x^2)^(7/3))
```

Rubi [A] time = 0.109792, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {758, 133}

$$-\frac{3\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{7/3}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{7/3}F_1\left(\frac{14}{3};\frac{7}{3},\frac{7}{3};\frac{17}{3};\frac{2cd-\left(b-\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{c}}{2(d+ex)}\right)}{224\ 2^{2/3}e\left(a+bx+cx^2\right)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + b*x + c*x^2)^(7/3)),x]
```

```
[Out] (-3*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(7/3)*((e*(b + Sqrt
[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(7/3)*AppellF1[14/3, 7/3, 7/3, 17/3,
 (2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x)), (2*d - ((b + Sqrt[b^2
 - 4*a*c])*e)/c)/(2*(d + e*x))])/(224*2^(2/3)*e*(a + b*x + c*x^2)^(7/3))
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, -Dist[((1/(d + e*x))^(2*p)*(a + b*
x + c*x^2)^p)/(e*((e*(b - q + 2*c*x))/(2*c*(d + e*x)))^p*((e*(b + q + 2*c*x
))/(2*c*(d + e*x)))^p), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - (e*(b -
q))/(2*c))*x, x]^p*Simp[1 - (d - (e*(b + q))/(2*c))*x, x]^p, x], x, 1/(d +
e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m,
0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{7/3}}\,dx = -\frac{\left(\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{7/3}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{7/3}\right)\text{Subst}\left(\int \frac{x^{11/3}}{\left(1-\frac{1}{2}\left(2d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{c}\right)x\right)^{7/3}\left(1-\frac{1}{2}\left(2d\cdots\right.\right.}\right.}{16\,2^{2/3}e\left(\frac{1}{d+ex}\right)^{14/3}\left(a+bx+cx^2\right)^{7/3}}$$

$$= -\frac{3\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{7/3}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{7/3} F_1\left(\frac{14}{3};\frac{7}{3},\frac{7}{3};\frac{17}{3};\frac{2cd-\left(b-\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2d-\frac{\left(b+\sqrt{b\cdots}\right.}{}}{2(d+\cdots}\right.}{224\,2^{2/3}e\left(a+bx+cx^2\right)^{7/3}}$$

Mathematica [A] time = 0.667002, size = 180, normalized size = 0.99

$$-\frac{3\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{7/3}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{7/3}F_1\left(\frac{14}{3};\frac{7}{3},\frac{7}{3};\frac{17}{3};\frac{2cd-\left(b+\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2cd-be+\sqrt{b^2-4ac}e}{2cd+2cex}\right)}{224\,2^{2/3}e(a+x(b+cx))^{7/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x)*(a + b*x + c*x^2)^(7/3)),x]
```

```
[Out] (-3*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(7/3)*((e*(b + Sqrt
[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(7/3)*AppellF1[14/3, 7/3, 7/3, 17/3,
 (2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x)), (2*c*d - b*e + Sqrt[b
^2 - 4*a*c]*e)/(2*c*d + 2*c*e*x)])/(224*2^(2/3)*e*(a + x*(b + c*x))^(7/3))
```

Maple [F] time = 1.304, size = 0, normalized size = 0.

$$\int \frac{1}{ex+d}\left(cx^2+bx+a\right)^{-\frac{7}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+b*x+a)^(7/3),x)
```

```
[Out] int(1/(e*x+d)/(c*x^2+b*x+a)^(7/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{7}{3}}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(7/3)*(e*x + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(7/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+b*x+a)**(7/3),x)
```

```
[Out] Integral(1/((d + e*x)*(a + b*x + c*x**2)**(7/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{7}{3}}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(7/3)*(e*x + d)), x)
```

3.2494 $\int \frac{1}{(d+ex)^2(a+bx+cx^2)^{7/3}}\,dx$

Optimal. Leaf size=189

$$-\frac{3\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{7/3}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{7/3}F_1\left(\frac{17}{3};\frac{7}{3},\frac{7}{3};\frac{20}{3};\frac{2cd-\left(b-\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{c}}{2(d+ex)}\right)}{272\ 2^{2/3}e(d+ex)\left(a+bx+cx^2\right)^{7/3}}$$

```
[Out] (-3*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(7/3)*((e*(b + Sqrt
[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(7/3)*AppellF1[17/3, 7/3, 7/3, 20/3,
 (2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x)), (2*d - ((b + Sqrt[b^2
 - 4*a*c])*e)/c)/(2*(d + e*x))])/(272*2^(2/3)*e*(d + e*x)*(a + b*x + c*x^2)
^(7/3))
```

Rubi [A] time = 0.0913116, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {758, 133}

$$-\frac{3\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{7/3}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{7/3}F_1\left(\frac{17}{3};\frac{7}{3},\frac{7}{3};\frac{20}{3};\frac{2cd-\left(b-\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{c}}{2(d+ex)}\right)}{272\ 2^{2/3}e(d+ex)\left(a+bx+cx^2\right)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + b*x + c*x^2)^(7/3)),x]
```

```
[Out] (-3*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(7/3)*((e*(b + Sqrt
[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(7/3)*AppellF1[17/3, 7/3, 7/3, 20/3,
 (2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x)), (2*d - ((b + Sqrt[b^2
 - 4*a*c])*e)/c)/(2*(d + e*x))])/(272*2^(2/3)*e*(d + e*x)*(a + b*x + c*x^2)
^(7/3))
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, -Dist[((1/(d + e*x))^(2*p)*(a + b*
x + c*x^2)^p)/(e*((e*(b - q + 2*c*x))/(2*c*(d + e*x)))^p*((e*(b + q + 2*c*x
))/(2*c*(d + e*x)))^p), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - (e*(b -
q))/(2*c))*x, x]^p*Simp[1 - (d - (e*(b + q))/(2*c))*x, x]^p, x], x, 1/(d +
e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m,
0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+bx+cx^2\right)^{7/3}}\,dx = -\frac{\left(\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{7/3}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{7/3}\right)\text{Subst}\left(\int \frac{x^{14/3}}{\left(1-\frac{1}{2}\left(2d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{c}\right)x\right)^{7/3}\left(1-\frac{1}{2}\left(2d-\right.\right.}\right.}{16\ 2^{2/3}e\left(\frac{1}{d+ex}\right)^{14/3}\left(a+bx+cx^2\right)^{7/3}}$$

$$= -\frac{3\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{7/3}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{7/3}F_1\left(\frac{17}{3};\frac{7}{3},\frac{7}{3};\frac{20}{3};\frac{2cd-\left(b-\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2d-\frac{\left(b+\sqrt{b^2-4\right.}}{c}}{2(d+ex)}\right.}{272\ 2^{2/3}e(d+ex)\left(a+bx+cx^2\right)^{7/3}}$$

Mathematica [A] time = 1.0946, size = 190, normalized size = 1.01

$$-\frac{3e^3\sqrt[3]{\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}}\sqrt[3]{\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}}F_1\left(\frac{17}{3};\frac{7}{3},\frac{7}{3};\frac{20}{3};\frac{2cd-\left(b+\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2cd-be+\sqrt{b^2-4ac}e}{2cd+2cex}\right)}{17\ 2^{2/3}c^2(d+ex)^5\sqrt[3]{a+x(b+cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x)^2*(a + b*x + c*x^2)^(7/3)),x]
```

```
[Out] (-3*e^3*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(1/3)*((e*(b +
Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(1/3)*AppellF1[17/3, 7/3, 7/3, 2
0/3, (2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x)), (2*c*d - b*e + Sq
rt[b^2 - 4*a*c]*e)/(2*c*d + 2*c*e*x)])/(17*2^(2/3)*c^2*(d + e*x)^5*(a + x*(
b + c*x))^(1/3))
```

Maple [F] time = 1.274, size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)^2}\left(cx^2+bx+a\right)^{-\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^2+b*x+a)^(7/3),x)
```

```
[Out] int(1/(e*x+d)^2/(c*x^2+b*x+a)^(7/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{7}{3}}(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x+a)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(7/3)*(e*x + d)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x+a)^(7/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**2+b*x+a)**(7/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2 + bx + a\right)^{\frac{7}{3}}(ex + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(7/3)*(e*x + d)^2), x)
```

3.2495 $\int \frac{1}{(d+ex)^3\left(a+bx+cx^2\right)^{7/3}}\,dx$

Optimal. Leaf size=189

$$-\frac{3\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{7/3}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{7/3}F_1\left(\frac{20}{3};\frac{7}{3},\frac{7}{3};\frac{23}{3};\frac{2cd-\left(b-\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{c}}{2(d+ex)}\right)}{320\ 2^{2/3}e(d+ex)^2\left(a+bx+cx^2\right)^{7/3}}$$

```
[Out] (-3*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(7/3)*((e*(b + Sqrt
[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(7/3)*AppellF1[20/3, 7/3, 7/3, 23/3,
 (2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x)), (2*d - ((b + Sqrt[b^2
 - 4*a*c])*e)/c)/(2*(d + e*x))])/(320*2^(2/3)*e*(d + e*x)^2*(a + b*x + c*x^
2)^(7/3))
```

Rubi [A] time = 0.0902785, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {758, 133}

$$-\frac{3\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{7/3}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{7/3}F_1\left(\frac{20}{3};\frac{7}{3},\frac{7}{3};\frac{23}{3};\frac{2cd-\left(b-\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{c}}{2(d+ex)}\right)}{320\ 2^{2/3}e(d+ex)^2\left(a+bx+cx^2\right)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + b*x + c*x^2)^(7/3)),x]
```


```
[Out] (-3*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(7/3)*((e*(b + Sqrt
[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(7/3)*AppellF1[20/3, 7/3, 7/3, 23/3,
 (2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x)), (2*d - ((b + Sqrt[b^2
 - 4*a*c])*e)/c)/(2*(d + e*x))])/(320*2^(2/3)*e*(d + e*x)^2*(a + b*x + c*x^
2)^(7/3))
```


Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, -Dist[((1/(d + e*x))^(2*p)*(a + b*
x + c*x^2)^p)/(e*((e*(b - q + 2*c*x))/(2*c*(d + e*x)))^p*((e*(b + q + 2*c*x
))/(2*c*(d + e*x)))^p), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - (e*(b -
q))/(2*c))*x, x]^p*Simp[1 - (d - (e*(b + q))/(2*c))*x, x]^p, x], x, 1/(d +
e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m,
0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(a+bx+cx^2\right)^{7/3}}\,dx = -\frac{\left(\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{7/3}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{7/3}\right)\text{Subst}\left(\int \frac{x^{17/3}}{\left(1-\frac{1}{2}\left(2d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{c}\right)x\right)^{7/3}\left(1-\frac{1}{2}\left(2\right.\right.}\right.}{16\,2^{2/3}e\left(\frac{1}{d+ex}\right)^{14/3}\left(a+bx+cx^2\right)^{7/3}}$$

$$= -\frac{3\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{7/3}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{7/3}F_1\left(\frac{20}{3};\frac{7}{3},\frac{7}{3};\frac{23}{3};\frac{2cd-\left(b-\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2d-\frac{(b+\sqrt{}}{}}{2(d}\right.}{320\,2^{2/3}e(d+ex)^2\left(a+bx+cx^2\right)^{7/3}}$$

Mathematica [A] time = 2.1661, size = 190, normalized size = 1.01

$$-\frac{3e^3\sqrt[3]{\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}}\sqrt[3]{\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}}F_1\left(\frac{20}{3};\frac{7}{3},\frac{7}{3};\frac{23}{3};\frac{2cd-\left(b+\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2cd-be+\sqrt{b^2-4ac}e}{2cd+2cex}\right)}{20\,2^{2/3}c^2(d+ex)^6\sqrt[3]{a+x(b+cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x)^3*(a + b*x + c*x^2)^(7/3)),x]
```

```
[Out] (-3*e^3*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(1/3)*((e*(b +
Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(1/3)*AppellF1[20/3, 7/3, 7/3, 2
3/3, (2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x)), (2*c*d - b*e + Sq
rt[b^2 - 4*a*c]*e)/(2*c*d + 2*c*e*x)]])/(20*2^(2/3)*c^2*(d + e*x)^6*(a + x*(
b + c*x))^(1/3))
```

Maple [F] time = 1.21, size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)^3}\left(cx^2+bx+a\right)^{-\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*x^2+b*x+a)^(7/3),x)
```

```
[Out] int(1/(e*x+d)^3/(c*x^2+b*x+a)^(7/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{7}{3}}(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x+a)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(7/3)*(e*x + d)^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x+a)^(7/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**2+b*x+a)**(7/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2 + bx + a\right)^{\frac{7}{3}} (ex + d)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(7/3)*(e*x + d)^3), x)
```

3.2496 $\int \frac{1}{(d+ex)\sqrt[3]{c^2d^2-bcde+b^2e^2+3bce^2x+3c^2e^2x^2}}\,dx$

Optimal. Leaf size=242

$$\frac{\log\left(-3ce^2\sqrt[3]{2cd-be}\sqrt[3]{b^2e^2-bcde+3bce^2x+c^2d^2+3c^2e^2x^2}+3ce^2(cd-be)-3c^2e^3x\right)}{2e(2cd-be)^{2/3}}-\frac{\tan^{-1}\left(\frac{2(-be+cd-}{\sqrt{3}\sqrt[3]{2cd-be}\sqrt[3]{b^2e^2-bcde+3}}\right.}{\sqrt{3}e(2cd-}$$

```
[Out] -(ArcTan[1/Sqrt[3] + (2*(c*d - b*e - c*e*x))/(Sqrt[3]*(2*c*d - b*e)^(1/3)*(
c^2*d^2 - b*c*d*e + b^2*e^2 + 3*b*c*e^2*x + 3*c^2*e^2*x^2)^(1/3))]/(Sqrt[3]
*e*(2*c*d - b*e)^(2/3))) - Log[d + e*x]/(2*e*(2*c*d - b*e)^(2/3)) + Log[3*c
*e^2*(c*d - b*e) - 3*c^2*e^3*x - 3*c*e^2*(2*c*d - b*e)^(1/3)*(c^2*d^2 - b*c
*d*e + b^2*e^2 + 3*b*c*e^2*x + 3*c^2*e^2*x^2)^(1/3)]/(2*e*(2*c*d - b*e)^(2/
3))
```

Rubi [A] time = 0.139311, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.019, Rules used = {750}

$$\frac{\log\left(-3ce^2\sqrt[3]{2cd-be}\sqrt[3]{b^2e^2-bcde+3bce^2x+c^2d^2+3c^2e^2x^2}+3ce^2(cd-be)-3c^2e^3x\right)}{2e(2cd-be)^{2/3}}-\frac{\tan^{-1}\left(\frac{2(-be+cd-}{\sqrt{3}\sqrt[3]{2cd-be}\sqrt[3]{b^2e^2-bcde+3}}\right.}{\sqrt{3}e(2cd-}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(c^2*d^2 - b*c*d*e + b^2*e^2 + 3*b*c*e^2*x + 3*c^2*e^2*x^2
)^(1/3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] + (2*(c*d - b*e - c*e*x))/(Sqrt[3]*(2*c*d - b*e)^(1/3)*(
c^2*d^2 - b*c*d*e + b^2*e^2 + 3*b*c*e^2*x + 3*c^2*e^2*x^2)^(1/3))]/(Sqrt[3]
*e*(2*c*d - b*e)^(2/3))) - Log[d + e*x]/(2*e*(2*c*d - b*e)^(2/3)) + Log[3*c
*e^2*(c*d - b*e) - 3*c^2*e^3*x - 3*c*e^2*(2*c*d - b*e)^(1/3)*(c^2*d^2 - b*c
*d*e + b^2*e^2 + 3*b*c*e^2*x + 3*c^2*e^2*x^2)^(1/3)]/(2*e*(2*c*d - b*e)^(2/
3))
```

Rule 750

```
Int[1/(((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)), x_Sy
mbol] :> With[{q = Rt[3*c*e^2*(2*c*d - b*e), 3]}, -Simp[(Sqrt[3]*c*e*ArcTan
[1/Sqrt[3] + (2*(c*d - b*e - c*e*x))/(Sqrt[3]*q*(a + b*x + c*x^2)^(1/3))])/
q^2, x] + (-Simp[(3*c*e*Log[d + e*x])/(2*q^2), x] + Simp[(3*c*e*Log[c*d - b
*e - c*e*x - q*(a + b*x + c*x^2)^(1/3)])/(2*q^2), x])] /; FreeQ[{a, b, c, d
, e}, x] && NeQ[2*c*d - b*e, 0] && EqQ[c^2*d^2 - b*c*d*e + b^2*e^2 - 3*a*c*
e^2, 0] && PosQ[c*e^2*(2*c*d - b*e)]
```

Rubi steps

$$\int \frac{1}{(d+ex)\sqrt[3]{c^2d^2-bcde+b^2e^2+3bce^2x+3c^2e^2x^2}}\,dx=-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2(cd-be-cex)}{\sqrt{3}\sqrt[3]{2cd-be}\sqrt[3]{c^2d^2-bcde+b^2e^2+3bce^2x+3c^2e^2x^2}}\right)}{\sqrt{3}e(2cd-be)^{2/3}}-\frac{\log}{2e(2c}$$

Mathematica [C] time = 0.491314, size = 317, normalized size = 1.31

$$-\frac{\sqrt[3]{3}\sqrt[3]{\frac{-\sqrt{3}\sqrt{-c^2e^2(be-2cd)^2}+3bce^2+6c^2e^2x}{c^2e(d+ex)}}\sqrt[3]{\frac{\sqrt{3}\sqrt{-c^2e^2(be-2cd)^2}+3bce^2+6c^2e^2x}{c^2e(d+ex)}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};-\frac{-6dec^2+3be^2c+\sqrt{3}\sqrt{-c^2e^2(be-2cd)^2}}{6c^2e(d+ex)},\frac{6dec^2-3be^2c+\sqrt{}}{6c^2e}\right.}{2\,2^{2/3}e\sqrt[3]{b^2e^2+bce(3ex-d)+c^2\left(d^2+3e^2x^2\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x)*(c^2*d^2 - b*c*d*e + b^2*e^2 + 3*b*c*e^2*x + 3*c^2*e
^2*x^2)^(1/3)),x]
```

```
[Out] -(3^(1/3)*((3*b*c*e^2 - Sqrt[3]*Sqrt[-(c^2*e^2*(-2*c*d + b*e)^2)] + 6*c^2*e
^2*x)/(c^2*e*(d + e*x)))^(1/3)*((3*b*c*e^2 + Sqrt[3]*Sqrt[-(c^2*e^2*(-2*c*d
 + b*e)^2)] + 6*c^2*e^2*x)/(c^2*e*(d + e*x)))^(1/3)*AppellF1[2/3, 1/3, 1/3,
 5/3, -(-6*c^2*d*e + 3*b*c*e^2 + Sqrt[3]*Sqrt[-(c^2*e^2*(-2*c*d + b*e)^2)])
/(6*c^2*e*(d + e*x)), (6*c^2*d*e - 3*b*c*e^2 + Sqrt[3]*Sqrt[-(c^2*e^2*(-2*c
*d + b*e)^2)])/(6*c^2*e*(d + e*x))])/(2*2^(2/3)*e*(b^2*e^2 + b*c*e*(-d + 3*
e*x) + c^2*(d^2 + 3*e^2*x^2))^(1/3))
```

Maple [F] time = 1.477, size = 0, normalized size = 0.

$$\int \frac{1}{ex+d}\frac{1}{\sqrt[3]{3\,c^2e^2x^2+3\,bce^2x+b^2e^2-bcde+c^2d^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(3*c^2*e^2*x^2+3*b*c*e^2*x+b^2*e^2-b*c*d*e+c^2*d^2)^(1/3),x)
```

```
[Out] int(1/(e*x+d)/(3*c^2*e^2*x^2+3*b*c*e^2*x+b^2*e^2-b*c*d*e+c^2*d^2)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,c^2e^2x^2+3\,bce^2x+c^2d^2-bcde+b^2e^2\right)^{\frac{1}{3}}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(3*c^2*e^2*x^2+3*b*c*e^2*x+b^2*e^2-b*c*d*e+c^2*d^2)^(1/
3),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*c^2*e^2*x^2 + 3*b*c*e^2*x + c^2*d^2 - b*c*d*e + b^2*e^2)^(1
/3)*(e*x + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(3*c^2*e^2*x^2+3*b*c*e^2*x+b^2*e^2-b*c*d*e+c^2*d^2)^(1/
3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)\sqrt[3]{b^2e^2-bcde+3bce^2x+c^2d^2+3c^2e^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(3*c**2*e**2*x**2+3*b*c*e**2*x+b**2*e**2-b*c*d*e+c**2*d
**2)**(1/3),x)
```

```
[Out] Integral(1/((d + e*x)*(b**2*e**2 - b*c*d*e + 3*b*c*e**2*x + c**2*d**2 + 3*c
**2*e**2*x**2)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,c^2e^2x^2+3\,bce^2x+c^2d^2-bcde+b^2e^2\right)^{\frac{1}{3}}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(3*c^2*e^2*x^2+3*b*c*e^2*x+b^2*e^2-b*c*d*e+c^2*d^2)^(1/
3),x, algorithm="giac")
```

```
[Out] integrate(1/((3*c^2*e^2*x^2 + 3*b*c*e^2*x + c^2*d^2 - b*c*d*e + b^2*e^2)^(1
/3)*(e*x + d)), x)
```

3.2497 $\int \frac{(2+3x)^3}{\sqrt[3]{52-54x+27x^2}}\,dx$

Optimal. Leaf size=635

$$\frac{50\,5^{5/6}\left(30-\sqrt[3]{10}\sqrt[3]{(54x-54)^2+2700}\right)\sqrt{\frac{10^{2/3}\left((54x-54)^2+2700\right)^{2/3}+30\sqrt[3]{10}\sqrt[3]{(54x-54)^2+2700}+900}{\left(30\left(1-\sqrt{3}\right)-\sqrt[3]{10}\sqrt[3]{(54x-54)^2+2700}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{30\left(1+\sqrt{3}\right)-\sqrt[3]{10}\sqrt[3]{(}}{30\left(1-\sqrt{3}\right)-\sqrt[3]{10}\sqrt[3]{(}}\right.\right.}{189\,3^{3/4}(1-x)\sqrt{-\frac{30-\sqrt[3]{10}\sqrt[3]{(54x-54)^2+2700}}{\left(30\left(1-\sqrt{3}\right)-\sqrt[3]{10}\sqrt[3]{(54x-54)^2+2700}\right)^2}}}$$

```
[Out] ((2 + 3*x)^2*(52 - 54*x + 27*x^2)^(2/3))/30 + ((27 + 8*x)*(52 - 54*x + 27*x
^2)^(2/3))/7 + (9000*5^(1/3)*(1 - x))/(7*(30*(1 - Sqrt[3]) - 10^(1/3)*(2700
 + (-54 + 54*x)^2)^(1/3))) - (25*5^(5/6)*Sqrt[2 + Sqrt[3]]*(30 - 10^(1/3)*(
2700 + (-54 + 54*x)^2)^(1/3))*Sqrt[(900 + 30*10^(1/3)*(2700 + (-54 + 54*x)^
2)^(1/3) + 10^(2/3)*(2700 + (-54 + 54*x)^2)^(2/3))/(30*(1 - Sqrt[3]) - 10^(
1/3)*(2700 + (-54 + 54*x)^2)^(1/3))^2]*EllipticE[ArcSin[(30*(1 + Sqrt[3]) -
 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700
 + (-54 + 54*x)^2)^(1/3))], -7 + 4*Sqrt[3]])/(189*Sqrt[2]*3^(1/4)*(1 - x)*S
qrt[-((30 - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^
(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))^2)]) + (50*5^(5/6)*(30 - 10^(1/3)*(270
0 + (-54 + 54*x)^2)^(1/3))*Sqrt[(900 + 30*10^(1/3)*(2700 + (-54 + 54*x)^2)^
(1/3) + 10^(2/3)*(2700 + (-54 + 54*x)^2)^(2/3))/(30*(1 - Sqrt[3]) - 10^(1/3
)*(2700 + (-54 + 54*x)^2)^(1/3))^2]*EllipticF[ArcSin[(30*(1 + Sqrt[3]) - 10
^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700 +
(-54 + 54*x)^2)^(1/3))], -7 + 4*Sqrt[3]])/(189*3^(3/4)*(1 - x)*Sqrt[-((30 -
 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700
 + (-54 + 54*x)^2)^(1/3))^2)])
```

Rubi [A] time = 0.732842, antiderivative size = 635, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {742, 779, 619, 235, 304, 219, 1879}

$$\frac{1}{30}\left(27x^2-54x+52\right)^{2/3}(3x+2)^2+\frac{1}{7}(8x+27)\left(27x^2-54x+52\right)^{2/3}+\frac{9000\sqrt[3]{5}(1-x)}{7\left(30\left(1-\sqrt{3}\right)-\sqrt[3]{10}\sqrt[3]{(54x-54)^2+2700}\right)}+\frac{50}{}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)^3/(52 - 54*x + 27*x^2)^(1/3),x]
```

```
[Out] ((2 + 3*x)^2*(52 - 54*x + 27*x^2)^(2/3))/30 + ((27 + 8*x)*(52 - 54*x + 27*x
^2)^(2/3))/7 + (9000*5^(1/3)*(1 - x))/(7*(30*(1 - Sqrt[3]) - 10^(1/3)*(2700
 + (-54 + 54*x)^2)^(1/3))) - (25*5^(5/6)*Sqrt[2 + Sqrt[3]]*(30 - 10^(1/3)*(
2700 + (-54 + 54*x)^2)^(1/3))*Sqrt[(900 + 30*10^(1/3)*(2700 + (-54 + 54*x)^
2)^(1/3) + 10^(2/3)*(2700 + (-54 + 54*x)^2)^(2/3))/(30*(1 - Sqrt[3]) - 10^(
1/3)*(2700 + (-54 + 54*x)^2)^(1/3))^2]*EllipticE[ArcSin[(30*(1 + Sqrt[3]) -
 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700
 + (-54 + 54*x)^2)^(1/3))], -7 + 4*Sqrt[3]])/(189*Sqrt[2]*3^(1/4)*(1 - x)*S
qrt[-((30 - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^
(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))^2)]) + (50*5^(5/6)*(30 - 10^(1/3)*(270
0 + (-54 + 54*x)^2)^(1/3))*Sqrt[(900 + 30*10^(1/3)*(2700 + (-54 + 54*x)^2)^
(1/3) + 10^(2/3)*(2700 + (-54 + 54*x)^2)^(2/3))/(30*(1 - Sqrt[3]) - 10^(1/3
)*(2700 + (-54 + 54*x)^2)^(1/3))^2]*EllipticF[ArcSin[(30*(1 + Sqrt[3]) - 10
```

```
^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700 +
(-54 + 54*x)^2)^(1/3))], -7 + 4*Sqrt[3]])/(189*3^(3/4)*(1 - x)*Sqrt[-((30 -
 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700
 + (-54 + 54*x)^2)^(1/3))^2)])
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
```

```
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(2+3x)^3}{\sqrt[3]{52-54x+27x^2}}\,dx = \frac{1}{30}(2+3x)^2\left(52-54x+27x^2\right)^{2/3}+\frac{1}{90}\int \frac{(2+3x)(-360+2160x)}{\sqrt[3]{52-54x+27x^2}}\,dx$$
$$= \frac{1}{30}(2+3x)^2\left(52-54x+27x^2\right)^{2/3}+\frac{1}{7}(27+8x)\left(52-54x+27x^2\right)^{2/3}+\frac{500}{7}\int \frac{1}{\sqrt[3]{52-54x+}}$$
$$= \frac{1}{30}(2+3x)^2\left(52-54x+27x^2\right)^{2/3}+\frac{1}{7}(27+8x)\left(52-54x+27x^2\right)^{2/3}+\frac{1}{189}\left(50\sqrt[3]{5}\right)\text{Subst}\Bigg(\int$$
$$= \frac{1}{30}(2+3x)^2\left(52-54x+27x^2\right)^{2/3}+\frac{1}{7}(27+8x)\left(52-54x+27x^2\right)^{2/3}+\frac{\left(250\sqrt[3]{5}\sqrt{(-54+54x}\right.}{}$$
$$= \frac{1}{30}(2+3x)^2\left(52-54x+27x^2\right)^{2/3}+\frac{1}{7}(27+8x)\left(52-54x+27x^2\right)^{2/3}-\frac{\left(250\sqrt[3]{5}\sqrt{(-54+54x}\right.}{}$$
$$= \frac{1}{30}(2+3x)^2\left(52-54x+27x^2\right)^{2/3}+\frac{1}{7}(27+8x)\left(52-54x+27x^2\right)^{2/3}+\frac{9000\sqrt[3]{}}{7\left(30-30\sqrt{3}-\sqrt[3]{10}\sqrt[3]{}\right.}$$

Mathematica [C] time = 0.035177, size = 59, normalized size = 0.09

$$\frac{1}{210}\left(3000\sqrt[3]{5}(x-1)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{27}{25}(x-1)^2\right)+\left(27x^2-54x+52\right)^{2/3}\left(63x^2+324x+838\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)^3/(52 - 54*x + 27*x^2)^(1/3),x]
```

```
[Out] ((52 - 54*x + 27*x^2)^(2/3)*(838 + 324*x + 63*x^2) + 3000*5^(1/3)*(-1 + x)*
Hypergeometric2F1[1/3, 1/2, 3/2, (-27*(-1 + x)^2)/25])/210
```

Maple [F] time = 3.566, size = 0, normalized size = 0.

$$\int (2+3x)^3\frac{1}{\sqrt[3]{27x^2-54x+52}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^3/(27*x^2-54*x+52)^(1/3),x)
```

```
[Out] int((2+3*x)^3/(27*x^2-54*x+52)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x+2)^3}{(27x^2-54x+52)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^3/(27*x^2-54*x+52)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((3*x + 2)^3/(27*x^2 - 54*x + 52)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{27x^3+54x^2+36x+8}{(27x^2-54x+52)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^3/(27*x^2-54*x+52)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((27*x^3 + 54*x^2 + 36*x + 8)/(27*x^2 - 54*x + 52)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x+2)^3}{\sqrt[3]{27x^2-54x+52}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**3/(27*x**2-54*x+52)**(1/3),x)
```

```
[Out] Integral((3*x + 2)**3/(27*x**2 - 54*x + 52)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x+2)^3}{(27x^2-54x+52)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^3/(27*x^2-54*x+52)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((3*x + 2)^3/(27*x^2 - 54*x + 52)^(1/3), x)
```

3.2498 $\int \frac{(2+3x)^2}{\sqrt[3]{52-54x+27x^2}}\,dx$

Optimal. Leaf size=628

$$\frac{5\,5^{5/6}\left(30-\sqrt[3]{10}\sqrt[3]{(54x-54)^2+2700}\right)\sqrt{\frac{10^{2/3}\left((54x-54)^2+2700\right)^{2/3}+30\sqrt[3]{10}\sqrt[3]{(54x-54)^2+2700}+900}{\left(30\left(1-\sqrt{3}\right)-\sqrt[3]{10}\sqrt[3]{(54x-54)^2+2700}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{30\left(1+\sqrt{3}\right)-\sqrt[3]{10}\sqrt[3]{(5}}{30\left(1-\sqrt{3}\right)-\sqrt[3]{10}\sqrt[3]{(5}}\right.\right.}{63\,3^{3/4}\sqrt{-\frac{30-\sqrt[3]{10}\sqrt[3]{(54x-54)^2+2700}}{\left(30\left(1-\sqrt{3}\right)-\sqrt[3]{10}\sqrt[3]{(54x-54)^2+2700}\right)^2}}(1-x)}$$

```
[Out] (25*(52 - 54*x + 27*x^2)^(2/3))/42 + ((2 + 3*x)*(52 - 54*x + 27*x^2)^(2/3))
/21 + (2700*5^(1/3)*(1 - x))/(7*(30*(1 - Sqrt[3]) - 10^(1/3)*(2700 + (-54 +
 54*x)^2)^(1/3))) - (5*5^(5/6)*Sqrt[2 + Sqrt[3]]*(30 - 10^(1/3)*(2700 + (-5
4 + 54*x)^2)^(1/3))*Sqrt[(900 + 30*10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3) +
 10^(2/3)*(2700 + (-54 + 54*x)^2)^(2/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700
 + (-54 + 54*x)^2)^(1/3))^2]*EllipticE[ArcSin[(30*(1 + Sqrt[3]) - 10^(1/3)*
(2700 + (-54 + 54*x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700 + (-54 +
54*x)^2)^(1/3))], -7 + 4*Sqrt[3]])/(126*Sqrt[2]*3^(1/4)*(1 - x)*Sqrt[-((30
- 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(270
0 + (-54 + 54*x)^2)^(1/3))^2)]) + (5*5^(5/6)*(30 - 10^(1/3)*(2700 + (-54 +
54*x)^2)^(1/3))*Sqrt[(900 + 30*10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3) + 10^
(2/3)*(2700 + (-54 + 54*x)^2)^(2/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700 + (
-54 + 54*x)^2)^(1/3))^2]*EllipticF[ArcSin[(30*(1 + Sqrt[3]) - 10^(1/3)*(270
0 + (-54 + 54*x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54*x
)^2)^(1/3))], -7 + 4*Sqrt[3]])/(63*3^(3/4)*(1 - x)*Sqrt[-((30 - 10^(1/3)*(2
700 + (-54 + 54*x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54
*x)^2)^(1/3))^2)])
```

Rubi [A] time = 0.541443, antiderivative size = 628, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {742, 12, 640, 619, 235, 304, 219, 1879}

$$\frac{1}{21}(3x+2)\left(27x^2-54x+52\right)^{2/3}+\frac{25}{42}\left(27x^2-54x+52\right)^{2/3}+\frac{2700\sqrt[3]{5}(1-x)}{7\left(30\left(1-\sqrt{3}\right)-\sqrt[3]{10}\sqrt[3]{(54x-54)^2+2700}\right)}+\frac{5\,5^{5/6}\left(30-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)^2/(52 - 54*x + 27*x^2)^(1/3),x]
```

```
[Out] (25*(52 - 54*x + 27*x^2)^(2/3))/42 + ((2 + 3*x)*(52 - 54*x + 27*x^2)^(2/3))
/21 + (2700*5^(1/3)*(1 - x))/(7*(30*(1 - Sqrt[3]) - 10^(1/3)*(2700 + (-54 +
 54*x)^2)^(1/3))) - (5*5^(5/6)*Sqrt[2 + Sqrt[3]]*(30 - 10^(1/3)*(2700 + (-5
4 + 54*x)^2)^(1/3))*Sqrt[(900 + 30*10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3) +
 10^(2/3)*(2700 + (-54 + 54*x)^2)^(2/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700
 + (-54 + 54*x)^2)^(1/3))^2]*EllipticE[ArcSin[(30*(1 + Sqrt[3]) - 10^(1/3)*
(2700 + (-54 + 54*x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700 + (-54 +
54*x)^2)^(1/3))], -7 + 4*Sqrt[3]])/(126*Sqrt[2]*3^(1/4)*(1 - x)*Sqrt[-((30
- 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(270
0 + (-54 + 54*x)^2)^(1/3))^2)]) + (5*5^(5/6)*(30 - 10^(1/3)*(2700 + (-54 +
54*x)^2)^(1/3))*Sqrt[(900 + 30*10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3) + 10^
(2/3)*(2700 + (-54 + 54*x)^2)^(2/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700 + (
-54 + 54*x)^2)^(1/3))^2]*EllipticF[ArcSin[(30*(1 + Sqrt[3]) - 10^(1/3)*(270
```

```
0 + (-54 + 54*x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54*x
)^2)^(1/3))], -7 + 4*Sqrt[3]])/(63*3^(3/4)*(1 - x)*Sqrt[-((30 - 10^(1/3)*(2
700 + (-54 + 54*x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54
*x)^2)^(1/3))^2)])
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
```

```
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(2+3x)^2}{\sqrt[3]{52-54x+27x^2}}\,dx = \frac{1}{21}(2+3x)\left(52-54x+27x^2\right)^{2/3} + \frac{1}{63}\int \frac{1350x}{\sqrt[3]{52-54x+27x^2}}\,dx$$

$$= \frac{1}{21}(2+3x)\left(52-54x+27x^2\right)^{2/3} + \frac{150}{7}\int \frac{x}{\sqrt[3]{52-54x+27x^2}}\,dx$$

$$= \frac{25}{42}\left(52-54x+27x^2\right)^{2/3} + \frac{1}{21}(2+3x)\left(52-54x+27x^2\right)^{2/3} + \frac{150}{7}\int \frac{1}{\sqrt[3]{52-54x+27x^2}}\,dx$$

$$= \frac{25}{42}\left(52-54x+27x^2\right)^{2/3} + \frac{1}{21}(2+3x)\left(52-54x+27x^2\right)^{2/3} + \frac{1}{63}\left(5\sqrt[3]{5}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1+\frac{x^2}{2700}}}\right.$$

$$= \frac{25}{42}\left(52-54x+27x^2\right)^{2/3} + \frac{1}{21}(2+3x)\left(52-54x+27x^2\right)^{2/3} + \frac{\left(25\sqrt{3}\sqrt[3]{5}\sqrt{(-54+54x)^2}\right)\text{Sub}}{7(-5}$$

$$= \frac{25}{42}\left(52-54x+27x^2\right)^{2/3} + \frac{1}{21}(2+3x)\left(52-54x+27x^2\right)^{2/3} - \frac{\left(25\sqrt{3}\sqrt[3]{5}\sqrt{(-54+54x)^2}\right)\text{Sub}}{7(-5}$$

$$= \frac{25}{42}\left(52-54x+27x^2\right)^{2/3} + \frac{1}{21}(2+3x)\left(52-54x+27x^2\right)^{2/3} + \frac{2700\sqrt[3]{5}(1-x)}{7\left(30-30\sqrt{3}-\sqrt[3]{10}\sqrt[3]{2700+(}\right.}$$

Mathematica [C] time = 0.0241067, size = 54, normalized size = 0.09

$$\frac{1}{42}\left(180\sqrt[3]{5}(x-1)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{27}{25}(x-1)^2\right)+\left(27x^2-54x+52\right)^{2/3}(6x+29)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)^2/(52 - 54*x + 27*x^2)^(1/3),x]
```

```
[Out] ((29 + 6*x)*(52 - 54*x + 27*x^2)^(2/3) + 180*5^(1/3)*(-1 + x)*Hypergeometri
c2F1[1/3, 1/2, 3/2, (-27*(-1 + x)^2)/25])/42
```

Maple [F] time = 1.437, size = 0, normalized size = 0.

$$\int (2+3x)^2 \frac{1}{\sqrt[3]{27x^2-54x+52}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^2/(27*x^2-54*x+52)^(1/3),x)
```

```
[Out] int((2+3*x)^2/(27*x^2-54*x+52)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x+2)^2}{(27x^2-54x+52)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2/(27*x^2-54*x+52)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((3*x + 2)^2/(27*x^2 - 54*x + 52)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{9x^2+12x+4}{(27x^2-54x+52)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2/(27*x^2-54*x+52)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((9*x^2 + 12*x + 4)/(27*x^2 - 54*x + 52)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x+2)^2}{\sqrt[3]{27x^2-54x+52}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**2/(27*x**2-54*x+52)**(1/3),x)
```

```
[Out] Integral((3*x + 2)**2/(27*x**2 - 54*x + 52)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x+2)^2}{(27x^2-54x+52)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2/(27*x^2-54*x+52)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((3*x + 2)^2/(27*x^2 - 54*x + 52)^(1/3), x)
```

3.2499 $\int \frac{2+3x}{\sqrt[3]{52-54x+27x^2}}\,dx$

Optimal. Leaf size=603

$$\frac{5^{5/6}\left(30-\sqrt[3]{10}\sqrt[3]{(54x-54)^2+2700}\right)\sqrt{\frac{10^{2/3}\left((54x-54)^2+2700\right)^{2/3}+30\sqrt[3]{10}\sqrt[3]{(54x-54)^2+2700}+900}{\left(30\left(1-\sqrt{3}\right)-\sqrt[3]{10}\sqrt[3]{(54x-54)^2+2700}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{30\left(1+\sqrt{3}\right)-\sqrt[3]{10}\sqrt[3]{(54x-}}{30\left(1-\sqrt{3}\right)-\sqrt[3]{10}\sqrt[3]{(54x-}}\right.\right.}{54\,3^{3/4}\sqrt{-\frac{30-\sqrt[3]{10}\sqrt[3]{(54x-54)^2+2700}}{\left(30\left(1-\sqrt{3}\right)-\sqrt[3]{10}\sqrt[3]{(54x-54)^2+2700}\right)^2}}(1-x)}$$

```
[Out] (52 - 54*x + 27*x^2)^(2/3)/12 + (90*5^(1/3)*(1 - x))/(30*(1 - Sqrt[3]) - 10
^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3)) - (5^(5/6)*Sqrt[2 + Sqrt[3]]*(30 - 10
^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))*Sqrt[(900 + 30*10^(1/3)*(2700 + (-54
+ 54*x)^2)^(1/3) + 10^(2/3)*(2700 + (-54 + 54*x)^2)^(2/3))/(30*(1 - Sqrt[3]
) - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))^2]*EllipticE[ArcSin[(30*(1 + Sq
rt[3]) - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^(1/
3)*(2700 + (-54 + 54*x)^2)^(1/3))], -7 + 4*Sqrt[3]])/(108*Sqrt[2]*3^(1/4)*(
1 - x)*Sqrt[-((30 - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))/(30*(1 - Sqrt[3
]) - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))^2)]) + (5^(5/6)*(30 - 10^(1/3)
*(2700 + (-54 + 54*x)^2)^(1/3))*Sqrt[(900 + 30*10^(1/3)*(2700 + (-54 + 54*x
)^2)^(1/3) + 10^(2/3)*(2700 + (-54 + 54*x)^2)^(2/3))/(30*(1 - Sqrt[3]) - 10
^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))^2]*EllipticF[ArcSin[(30*(1 + Sqrt[3])
 - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(27
00 + (-54 + 54*x)^2)^(1/3))], -7 + 4*Sqrt[3]])/(54*3^(3/4)*(1 - x)*Sqrt[-((
30 - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(
2700 + (-54 + 54*x)^2)^(1/3))^2)])
```

Rubi [A] time = 0.485279, antiderivative size = 603, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {640, 619, 235, 304, 219, 1879}

$$\frac{1}{12}\left(27x^2-54x+52\right)^{2/3}+\frac{90\sqrt[3]{5}(1-x)}{30\left(1-\sqrt{3}\right)-\sqrt[3]{10}\sqrt[3]{(54x-54)^2+2700}}+\frac{5^{5/6}\left(30-\sqrt[3]{10}\sqrt[3]{(54x-54)^2+2700}\right)\sqrt{\frac{10^{2/3}\left((54x-54\right.}{\left(30\right.}}}{54\,3^{3/}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)/(52 - 54*x + 27*x^2)^(1/3),x]
```


```
[Out] (52 - 54*x + 27*x^2)^(2/3)/12 + (90*5^(1/3)*(1 - x))/(30*(1 - Sqrt[3]) - 10
^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3)) - (5^(5/6)*Sqrt[2 + Sqrt[3]]*(30 - 10
^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))*Sqrt[(900 + 30*10^(1/3)*(2700 + (-54
+ 54*x)^2)^(1/3) + 10^(2/3)*(2700 + (-54 + 54*x)^2)^(2/3))/(30*(1 - Sqrt[3]
) - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))^2]*EllipticE[ArcSin[(30*(1 + Sq
rt[3]) - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^(1/
3)*(2700 + (-54 + 54*x)^2)^(1/3))], -7 + 4*Sqrt[3]])/(108*Sqrt[2]*3^(1/4)*(
1 - x)*Sqrt[-((30 - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))/(30*(1 - Sqrt[3
]) - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))^2)]) + (5^(5/6)*(30 - 10^(1/3)
*(2700 + (-54 + 54*x)^2)^(1/3))*Sqrt[(900 + 30*10^(1/3)*(2700 + (-54 + 54*x
)^2)^(1/3) + 10^(2/3)*(2700 + (-54 + 54*x)^2)^(2/3))/(30*(1 - Sqrt[3]) - 10
^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))^2]*EllipticF[ArcSin[(30*(1 + Sqrt[3])
 - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(27
00 + (-54 + 54*x)^2)^(1/3))], -7 + 4*Sqrt[3]])/(54*3^(3/4)*(1 - x)*Sqrt[-((
```

```
30 - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(
2700 + (-54 + 54*x)^2)^(1/3))^2)])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{2+3x}{\sqrt[3]{52-54x+27x^2}}\,dx = \frac{1}{12}\left(52-54x+27x^2\right)^{2/3}+5\int \frac{1}{\sqrt[3]{52-54x+27x^2}}\,dx$$
$$= \frac{1}{12}\left(52-54x+27x^2\right)^{2/3}+\frac{1}{54}\sqrt[3]{5}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1+\frac{x^2}{2700}}}\,dx,x,-54+54x\right)$$
$$= \frac{1}{12}\left(52-54x+27x^2\right)^{2/3}+\frac{\left(5\sqrt[3]{5}\sqrt{(-54+54x)^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx,x,\frac{\sqrt[3]{2700+(-54+54x)^2}}{3\ 10^{2/3}}\right)}{2\sqrt{3}(-54+54x)}$$
$$= \frac{1}{12}\left(52-54x+27x^2\right)^{2/3}-\frac{\left(5\sqrt[3]{5}\sqrt{(-54+54x)^2}\right)\text{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx,x,\frac{\sqrt[3]{2700+(-54+54x)^2}}{3\ 10^{2/3}}\right)}{2\sqrt{3}(-54+54x)}+ \cdots$$
$$= \frac{1}{12}\left(52-54x+27x^2\right)^{2/3}+\frac{90\sqrt[3]{5}(1-x)}{30-30\sqrt{3}-\sqrt[3]{10}\sqrt[3]{2700+(-54+54x)^2}}-\frac{5^{5/6}\sqrt{2+\sqrt{3}}\left(30-\sqrt[3]{1}\cdots\right.}{\cdots}$$

Mathematica [C] time = 0.0112111, size = 47, normalized size = 0.08

$$\sqrt[3]{5}(x-1)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{27}{25}(x-1)^2\right)+\frac{1}{12}\left(27x^2-54x+52\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)/(52 - 54*x + 27*x^2)^(1/3),x]
```

```
[Out] (52 - 54*x + 27*x^2)^(2/3)/12 + 5^(1/3)*(-1 + x)*Hypergeometric2F1[1/3, 1/2
, 3/2, (-27*(-1 + x)^2)/25]
```

Maple [F] time = 1.403, size = 0, normalized size = 0.

$$\int (2+3x)\frac{1}{\sqrt[3]{27x^2-54x+52}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)/(27*x^2-54*x+52)^(1/3),x)
```

```
[Out] int((2+3*x)/(27*x^2-54*x+52)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x+2}{\left(27x^2-54x+52\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(27*x^2-54*x+52)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((3*x + 2)/(27*x^2 - 54*x + 52)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{3\,x+2}{\left(27\,x^2-54\,x+52\right)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(27*x^2-54*x+52)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((3*x + 2)/(27*x^2 - 54*x + 52)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x+2}{\sqrt[3]{27x^2-54x+52}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(27*x**2-54*x+52)**(1/3),x)
```

```
[Out] Integral((3*x + 2)/(27*x**2 - 54*x + 52)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x+2}{\left(27\,x^2-54\,x+52\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(27*x^2-54*x+52)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((3*x + 2)/(27*x^2 - 54*x + 52)^(1/3), x)
```

3.2500 $\int \frac{1}{(2+3x)\sqrt[3]{52-54x+27x^2}}\,dx$

Optimal. Leaf size=108

$$\frac{\log\left(-27\sqrt[3]{10}\sqrt[3]{27x^2-54x+52}-81x+216\right)}{6\ 10^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}(8-3x)}{\sqrt{3}\sqrt[3]{5}\sqrt[3]{27x^2-54x+52}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}10^{2/3}}-\frac{\log(3x+2)}{6\ 10^{2/3}}$$

```
[Out] -ArcTan[1/Sqrt[3] + (2^(2/3)*(8 - 3*x))/(Sqrt[3]*5^(1/3)*(52 - 54*x + 27*x^
2)^(1/3))]/(3*Sqrt[3]*10^(2/3)) - Log[2 + 3*x]/(6*10^(2/3)) + Log[216 - 81*
x - 27*10^(1/3)*(52 - 54*x + 27*x^2)^(1/3)]/(6*10^(2/3))
```

Rubi [A] time = 0.0188848, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {750}

$$\frac{\log\left(-27\sqrt[3]{10}\sqrt[3]{27x^2-54x+52}-81x+216\right)}{6\ 10^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}(8-3x)}{\sqrt{3}\sqrt[3]{5}\sqrt[3]{27x^2-54x+52}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}10^{2/3}}-\frac{\log(3x+2)}{6\ 10^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + 3*x)*(52 - 54*x + 27*x^2)^(1/3)),x]
```

```
[Out] -ArcTan[1/Sqrt[3] + (2^(2/3)*(8 - 3*x))/(Sqrt[3]*5^(1/3)*(52 - 54*x + 27*x^
2)^(1/3))]/(3*Sqrt[3]*10^(2/3)) - Log[2 + 3*x]/(6*10^(2/3)) + Log[216 - 81*
x - 27*10^(1/3)*(52 - 54*x + 27*x^2)^(1/3)]/(6*10^(2/3))
```

Rule 750

```
Int[1/(((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)), x_Sy
mbol] :> With[{q = Rt[3*c*e^2*(2*c*d - b*e), 3]}, -Simp[(Sqrt[3]*c*e*ArcTan
[1/Sqrt[3] + (2*(c*d - b*e - c*e*x))/(Sqrt[3]*q*(a + b*x + c*x^2)^(1/3))])/
q^2, x] + (-Simp[(3*c*e*Log[d + e*x])/(2*q^2), x] + Simp[(3*c*e*Log[c*d - b
*e - c*e*x - q*(a + b*x + c*x^2)^(1/3)])/(2*q^2), x])] /; FreeQ[{a, b, c, d
, e}, x] && NeQ[2*c*d - b*e, 0] && EqQ[c^2*d^2 - b*c*d*e + b^2*e^2 - 3*a*c*
e^2, 0] && PosQ[c*e^2*(2*c*d - b*e)]
```

Rubi steps

$$\int \frac{1}{(2+3x)\sqrt[3]{52-54x+27x^2}}\,dx=-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2^{2/3}(8-3x)}{\sqrt{3}\sqrt[3]{5}\sqrt[3]{52-54x+27x^2}}\right)}{3\sqrt{3}10^{2/3}}-\frac{\log(2+3x)}{6\ 10^{2/3}}+\frac{\log\left(216-81x-27\sqrt[3]{10}\sqrt[3]{52-54x+27x^2}\right)}{6\ 10^{2/3}}$$

Mathematica [C] time = 0.0770016, size = 126, normalized size = 1.17

$$-\frac{\sqrt[3]{\frac{9x-5i\sqrt{3}-9}{3x+2}}\sqrt[3]{\frac{9x+5i\sqrt{3}-9}{3x+2}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{15-5i\sqrt{3}}{9x+6},\frac{15+5i\sqrt{3}}{9x+6}\right)}{2\ 3^{2/3}\sqrt[3]{27x^2-54x+52}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 + 3*x)*(52 - 54*x + 27*x^2)^(1/3)),x]
```

```
[Out] -(((-9 - (5*I)*Sqrt[3] + 9*x)/(2 + 3*x))^(1/3)*((-9 + (5*I)*Sqrt[3] + 9*x)/
(2 + 3*x))^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, (15 - (5*I)*Sqrt[3])/(6 + 9*x
), (15 + (5*I)*Sqrt[3])/(6 + 9*x)])/(2*3^(2/3)*(52 - 54*x + 27*x^2)^(1/3))
```

Maple [F] time = 1.619, size = 0, normalized size = 0.

$$\int \frac{1}{2+3x}\frac{1}{\sqrt[3]{27x^{2}-54x+52}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*x)/(27*x^2-54*x+52)^(1/3),x)
```

```
[Out] int(1/(2+3*x)/(27*x^2-54*x+52)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(27x^{2}-54x+52\right)^{\frac{1}{3}}(3x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(27*x^2-54*x+52)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((27*x^2 - 54*x + 52)^(1/3)*(3*x + 2)), x)
```

Fricas [B] time = 22.4995, size = 663, normalized size = 6.14

$$-\frac{1}{90}\cdot 100^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{100^{\frac{1}{6}}\left(2\cdot 100^{\frac{2}{3}}\sqrt{3}\left(27x^{2}-54x+52\right)^{\frac{2}{3}}(3x-8)+100^{\frac{1}{3}}\sqrt{3}\left(27x^{3}+54x^{2}+36x+8\right)+20\sqrt{3}(2\right.}{90\left(9x^{3}-162x^{2}+372x-344\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(27*x^2-54*x+52)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/90*100^(1/6)*sqrt(3)*arctan(1/90*100^(1/6)*(2*100^(2/3)*sqrt(3)*(27*x^2
- 54*x + 52)^(2/3)*(3*x - 8) + 100^(1/3)*sqrt(3)*(27*x^3 + 54*x^2 + 36*x +
8) + 20*sqrt(3)*(27*x^2 - 54*x + 52)^(1/3)*(9*x^2 - 48*x + 64))/(9*x^3 - 16
2*x^2 + 372*x - 344)) - 1/1800*100^(2/3)*log((100^(2/3)*(27*x^2 - 54*x + 52
)^(2/3) + 100^(1/3)*(9*x^2 - 48*x + 64) - 10*(27*x^2 - 54*x + 52)^(1/3)*(3*
x - 8))/(9*x^2 + 12*x + 4)) + 1/900*100^(2/3)*log((100^(1/3)*(3*x - 8) + 10
*(27*x^2 - 54*x + 52)^(1/3))/(3*x + 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x+2)\sqrt[3]{27x^2-54x+52}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(27*x**2-54*x+52)**(1/3),x)
```

```
[Out] Integral(1/((3*x + 2)*(27*x**2 - 54*x + 52)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(27\,x^2-54\,x+52\right)^{\frac{1}{3}}(3\,x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(27*x^2-54*x+52)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((27*x^2 - 54*x + 52)^(1/3)*(3*x + 2)), x)
```

3.2501 $\int \frac{1}{(2+3x)^2 \sqrt[3]{52-54x+27x^2}} \, dx$

Optimal. Leaf size=719

$$\frac{\left(30-\sqrt[3]{10}\sqrt[3]{(54x-54)^2+2700}\right)\sqrt{\frac{10^{2/3}\left((54x-54)^2+2700\right)^{2/3}+30\sqrt[3]{10}\sqrt[3]{(54x-54)^2+2700}+900}{\left(30(1-\sqrt{3})-\sqrt[3]{10}\sqrt[3]{(54x-54)^2+2700}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{30(1+\sqrt{3})-\sqrt[3]{10}\sqrt[3]{(54x-5}}{30(1-\sqrt{3})-\sqrt[3]{10}\sqrt[3]{(54x-5}}\right.\right.}{5400\ 3^{3/4}\sqrt[6]{5}\sqrt{-\frac{30-\sqrt[3]{10}\sqrt[3]{(54x-54)^2+2700}}{\left(30(1-\sqrt{3})-\sqrt[3]{10}\sqrt[3]{(54x-54)^2+2700}\right)^2}}(1-x)}$$

```
[Out] -(52 - 54*x + 27*x^2)^(2/3)/(300*(2 + 3*x)) + (9*(1 - x))/(10*5^(2/3)*(30*(
1 - Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))) - ArcTan[1/Sqrt[3]
+ (2^(2/3)*(8 - 3*x))/(Sqrt[3]*5^(1/3)*(52 - 54*x + 27*x^2)^(1/3))]/(30*Sqr
t[3]*10^(2/3)) - (Sqrt[2 + Sqrt[3]]*(30 - 10^(1/3)*(2700 + (-54 + 54*x)^2)^
(1/3))*Sqrt[(900 + 30*10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3) + 10^(2/3)*(27
00 + (-54 + 54*x)^2)^(2/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54*
x)^2)^(1/3))^2]*EllipticE[ArcSin[(30*(1 + Sqrt[3]) - 10^(1/3)*(2700 + (-54
+ 54*x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3
))], -7 + 4*Sqrt[3]])/(10800*Sqrt[2]*3^(1/4)*5^(1/6)*(1 - x)*Sqrt[-((30 - 1
0^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700 +
 (-54 + 54*x)^2)^(1/3))^2)]) + ((30 - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3
))*Sqrt[(900 + 30*10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3) + 10^(2/3)*(2700 +
 (-54 + 54*x)^2)^(2/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54*x)^2
)^(1/3))^2]*EllipticF[ArcSin[(30*(1 + Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54
*x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))],
 -7 + 4*Sqrt[3]])/(5400*3^(3/4)*5^(1/6)*(1 - x)*Sqrt[-((30 - 10^(1/3)*(2700
 + (-54 + 54*x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54*x)
^2)^(1/3))^2)]) - Log[2 + 3*x]/(60*10^(2/3)) + Log[216 - 81*x - 27*10^(1/3)
*(52 - 54*x + 27*x^2)^(1/3)]/(60*10^(2/3))
```

Rubi [A] time = 0.605284, antiderivative size = 719, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {744, 843, 619, 235, 304, 219, 1879, 750}

$$-\frac{\left(27x^2-54x+52\right)^{2/3}}{300(3x+2)}+\frac{\log\left(-27\sqrt[3]{10}\sqrt[3]{27x^2-54x+52}-81x+216\right)}{60\ 10^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}(8-3x)}{\sqrt{3}\sqrt[3]{5}\sqrt[3]{27x^2-54x+52}}+\frac{1}{\sqrt{3}}\right)}{30\sqrt{3}10^{2/3}}+\frac{}{10\ 5^{2/3}\ (}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + 3*x)^2*(52 - 54*x + 27*x^2)^(1/3)),x]
```


```
[Out] -(52 - 54*x + 27*x^2)^(2/3)/(300*(2 + 3*x)) + (9*(1 - x))/(10*5^(2/3)*(30*(
1 - Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))) - ArcTan[1/Sqrt[3]
+ (2^(2/3)*(8 - 3*x))/(Sqrt[3]*5^(1/3)*(52 - 54*x + 27*x^2)^(1/3))]/(30*Sqr
t[3]*10^(2/3)) - (Sqrt[2 + Sqrt[3]]*(30 - 10^(1/3)*(2700 + (-54 + 54*x)^2)^
(1/3))*Sqrt[(900 + 30*10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3) + 10^(2/3)*(27
00 + (-54 + 54*x)^2)^(2/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54*
x)^2)^(1/3))^2]*EllipticE[ArcSin[(30*(1 + Sqrt[3]) - 10^(1/3)*(2700 + (-54
+ 54*x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3
))], -7 + 4*Sqrt[3]])/(10800*Sqrt[2]*3^(1/4)*5^(1/6)*(1 - x)*Sqrt[-((30 - 1
0^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700 +
 (-54 + 54*x)^2)^(1/3))^2)]) + ((30 - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3
```

```
))*Sqrt[(900 + 30*10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3) + 10^(2/3)*(2700 +
 (-54 + 54*x)^2)^(2/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54*x)^2
)^(1/3))^2]*EllipticF[ArcSin[(30*(1 + Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54
*x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))],
 -7 + 4*Sqrt[3]])/(5400*3^(3/4)*5^(1/6)*(1 - x)*Sqrt[-((30 - 10^(1/3)*(2700
 + (-54 + 54*x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54*x)
^2)^(1/3))^2)]) - Log[2 + 3*x]/(60*10^(2/3)) + Log[216 - 81*x - 27*10^(1/3)
*(52 - 54*x + 27*x^2)^(1/3)]/(60*10^(2/3))
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
```

```
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 750

```
Int[1/(((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)), x_Sy
mbol] :> With[{q = Rt[3*c*e^2*(2*c*d - b*e), 3]}, -Simp[(Sqrt[3]*c*e*ArcTan
[1/Sqrt[3] + (2*(c*d - b*e - c*e*x))/(Sqrt[3]*q*(a + b*x + c*x^2)^(1/3))])/
q^2, x] + (-Simp[(3*c*e*Log[d + e*x])/(2*q^2), x] + Simp[(3*c*e*Log[c*d - b
*e - c*e*x - q*(a + b*x + c*x^2)^(1/3)])/(2*q^2), x])] /; FreeQ[{a, b, c, d
, e}, x] && NeQ[2*c*d - b*e, 0] && EqQ[c^2*d^2 - b*c*d*e + b^2*e^2 - 3*a*c*
e^2, 0] && PosQ[c*e^2*(2*c*d - b*e)]
```

Rubi steps

$$\int \frac{1}{(2+3x)^2\sqrt[3]{52-54x+27x^2}}\,dx = -\frac{\left(52-54x+27x^2\right)^{2/3}}{300(2+3x)} - \frac{1}{900}\int \frac{-108-27x}{(2+3x)\sqrt[3]{52-54x+27x^2}}\,dx$$

$$= -\frac{\left(52-54x+27x^2\right)^{2/3}}{300(2+3x)} + \frac{1}{100}\int \frac{1}{\sqrt[3]{52-54x+27x^2}}\,dx + \frac{1}{10}\int \frac{1}{(2+3x)\sqrt[3]{52-5\ldots}}$$

$$= -\frac{\left(52-54x+27x^2\right)^{2/3}}{300(2+3x)} - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2^{2/3}(8-3x)}{\sqrt{3}\sqrt[3]{5}\sqrt[3]{52-54x+27x^2}}\right)}{30\sqrt{3}10^{2/3}} - \frac{\log(2+3x)}{60\ 10^{2/3}} + \frac{\log\left(2\ldots\right.}{}$$

$$= -\frac{\left(52-54x+27x^2\right)^{2/3}}{300(2+3x)} - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2^{2/3}(8-3x)}{\sqrt{3}\sqrt[3]{5}\sqrt[3]{52-54x+27x^2}}\right)}{30\sqrt{3}10^{2/3}} - \frac{\log(2+3x)}{60\ 10^{2/3}} + \frac{\log\left(2\ldots\right.}{}$$

$$= -\frac{\left(52-54x+27x^2\right)^{2/3}}{300(2+3x)} - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2^{2/3}(8-3x)}{\sqrt{3}\sqrt[3]{5}\sqrt[3]{52-54x+27x^2}}\right)}{30\sqrt{3}10^{2/3}} - \frac{\log(2+3x)}{60\ 10^{2/3}} + \frac{\log\left(2\ldots\right.}{}$$

$$= -\frac{\left(52-54x+27x^2\right)^{2/3}}{300(2+3x)} + \frac{9(1-x)}{10\ 5^{2/3}\left(30-30\sqrt{3}-\sqrt[3]{10}\sqrt[3]{2700+(-54+54x)^2}\right)} - \frac{\tan\ldots}{\ldots}$$

Mathematica [C] time = 0.284888, size = 243, normalized size = 0.34

$$\frac{-100\sqrt[3]{3}(3x+2)\sqrt[3]{\frac{9x-5i\sqrt{3}-9}{3x+2}}\sqrt[3]{\frac{9x+5i\sqrt{3}-9}{3x+2}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{15-5i\sqrt{3}}{9x+6},\frac{15+5i\sqrt{3}}{9x+6}\right)+\sqrt[3]{3}10^{2/3}\sqrt[3]{-9ix+5\sqrt{3}+9i}(3x+2)\left(3\sqrt{3}x-\ldots\right.}{6000(3x+2)\sqrt[3]{27x^2-54x+52}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 + 3*x)^2*(52 - 54*x + 27*x^2)^(1/3)),x]
```

```
[Out] (-20*(52 - 54*x + 27*x^2) - 100*3^(1/3)*(2 + 3*x)*((-9 - (5*I)*Sqrt[3] + 9*
x)/(2 + 3*x))^(1/3)*((-9 + (5*I)*Sqrt[3] + 9*x)/(2 + 3*x))^(1/3)*AppellF1[2
```

```
/3, 1/3, 1/3, 5/3, (15 - (5*I)*Sqrt[3])/(6 + 9*x), (15 + (5*I)*Sqrt[3])/(6
+ 9*x)] + 3^(1/3)*10^(2/3)*(9*I + 5*Sqrt[3] - (9*I)*x)^(1/3)*(2 + 3*x)*(-5*
I - 3*Sqrt[3] + 3*Sqrt[3]*x)*Hypergeometric2F1[1/3, 2/3, 5/3, (-9*I + 5*Sqr
t[3] + (9*I)*x)/(10*Sqrt[3])])/(6000*(2 + 3*x)*(52 - 54*x + 27*x^2)^(1/3))
```

Maple [F] time = 1.412, size = 0, normalized size = 0.

$$\int \frac{1}{\left(2+3x\right)^{2}}\frac{1}{\sqrt[3]{27x^{2}-54x+52}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*x)^2/(27*x^2-54*x+52)^(1/3),x)
```

```
[Out] int(1/(2+3*x)^2/(27*x^2-54*x+52)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(27x^{2}-54x+52\right)^{\frac{1}{3}}\left(3x+2\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)^2/(27*x^2-54*x+52)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((27*x^2 - 54*x + 52)^(1/3)*(3*x + 2)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(27x^{2}-54x+52\right)^{\frac{2}{3}}}{243x^{4}-162x^{3}-72x^{2}+408x+208},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)^2/(27*x^2-54*x+52)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((27*x^2 - 54*x + 52)^(2/3)/(243*x^4 - 162*x^3 - 72*x^2 + 408*x + 2
08), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3x+2\right)^{2}\sqrt[3]{27x^{2}-54x+52}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)**2/(27*x**2-54*x+52)**(1/3),x)
```

```
[Out] Integral(1/((3*x + 2)**2*(27*x**2 - 54*x + 52)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(27\,x^2-54\,x+52\right)^{\frac{1}{3}}(3\,x+2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)^2/(27*x^2-54*x+52)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((27*x^2 - 54*x + 52)^(1/3)*(3*x + 2)^2), x)
```

3.2502 $\int \frac{1}{(2+3x)^3 \sqrt[3]{52-54x+27x^2}}\,dx$

Optimal. Leaf size=744

$$\frac{\left(30-\sqrt[3]{10}\sqrt[3]{(54x-54)^2+2700}\right)\sqrt{\frac{10^{2/3}\left((54x-54)^2+2700\right)^{2/3}+30\sqrt[3]{10}\sqrt[3]{(54x-54)^2+2700}+900}{\left(30\left(1-\sqrt{3}\right)-\sqrt[3]{10}\sqrt[3]{(54x-54)^2+2700}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{30\left(1+\sqrt{3}\right)-\sqrt[3]{10}\sqrt[3]{(54x-54)^2}}{30\left(1-\sqrt{3}\right)-\sqrt[3]{10}\sqrt[3]{(54x-54)^2}}\right.\right.}{27000\;3^{3/4}\sqrt[6]{5}\sqrt{-\frac{30-\sqrt[3]{10}\sqrt[3]{(54x-54)^2+2700}}{\left(30\left(1-\sqrt{3}\right)-\sqrt[3]{10}\sqrt[3]{(54x-54)^2+2700}\right)^2}(1-x)}}$$

```
[Out] -(52 - 54*x + 27*x^2)^(2/3)/(600*(2 + 3*x)^2) - (52 - 54*x + 27*x^2)^(2/3)/
(1500*(2 + 3*x)) + (9*(1 - x))/(50*5^(2/3)*(30*(1 - Sqrt[3]) - 10^(1/3)*(27
00 + (-54 + 54*x)^2)^(1/3))) - ArcTan[1/Sqrt[3] + (2^(2/3)*(8 - 3*x))/(Sqrt
[3]*5^(1/3)*(52 - 54*x + 27*x^2)^(1/3))]/(300*Sqrt[3]*10^(2/3)) - (Sqrt[2 +
 Sqrt[3]]*(30 - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))*Sqrt[(900 + 30*10^(
1/3)*(2700 + (-54 + 54*x)^2)^(1/3) + 10^(2/3)*(2700 + (-54 + 54*x)^2)^(2/3)
)/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))^2]*EllipticE[
ArcSin[(30*(1 + Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))/(30*(1 -
 Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))], -7 + 4*Sqrt[3]])/(540
00*Sqrt[2]*3^(1/4)*5^(1/6)*(1 - x)*Sqrt[-((30 - 10^(1/3)*(2700 + (-54 + 54*
x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))^2)
]) + ((30 - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))*Sqrt[(900 + 30*10^(1/3)
*(2700 + (-54 + 54*x)^2)^(1/3) + 10^(2/3)*(2700 + (-54 + 54*x)^2)^(2/3))/(3
0*(1 - Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))^2]*EllipticF[ArcS
in[(30*(1 + Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))/(30*(1 - Sqr
t[3]) - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))], -7 + 4*Sqrt[3]])/(27000*3
^(3/4)*5^(1/6)*(1 - x)*Sqrt[-((30 - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))
/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))^2)]) - Log[2 +
 3*x]/(600*10^(2/3)) + Log[216 - 81*x - 27*10^(1/3)*(52 - 54*x + 27*x^2)^(1
/3)]/(600*10^(2/3))
```

Rubi [A] time = 0.674776, antiderivative size = 744, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {744, 834, 843, 619, 235, 304, 219, 1879, 750}

$$-\frac{\left(27x^2-54x+52\right)^{2/3}}{1500(3x+2)}-\frac{\left(27x^2-54x+52\right)^{2/3}}{600(3x+2)^2}+\frac{\log\left(-27\sqrt[3]{10}\sqrt[3]{27x^2-54x+52}-81x+216\right)}{600\;10^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}(8-3x)}{\sqrt{3}\sqrt[3]{5}\sqrt[3]{27x^2-54x+}}\right.}{300\sqrt{3}10^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + 3*x)^3*(52 - 54*x + 27*x^2)^(1/3)),x]
```


```
[Out] -(52 - 54*x + 27*x^2)^(2/3)/(600*(2 + 3*x)^2) - (52 - 54*x + 27*x^2)^(2/3)/
(1500*(2 + 3*x)) + (9*(1 - x))/(50*5^(2/3)*(30*(1 - Sqrt[3]) - 10^(1/3)*(27
00 + (-54 + 54*x)^2)^(1/3))) - ArcTan[1/Sqrt[3] + (2^(2/3)*(8 - 3*x))/(Sqrt
[3]*5^(1/3)*(52 - 54*x + 27*x^2)^(1/3))]/(300*Sqrt[3]*10^(2/3)) - (Sqrt[2 +
 Sqrt[3]]*(30 - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))*Sqrt[(900 + 30*10^(
1/3)*(2700 + (-54 + 54*x)^2)^(1/3) + 10^(2/3)*(2700 + (-54 + 54*x)^2)^(2/3)
)/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))^2]*EllipticE[
ArcSin[(30*(1 + Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))/(30*(1 -
 Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))], -7 + 4*Sqrt[3]])/(540
00*Sqrt[2]*3^(1/4)*5^(1/6)*(1 - x)*Sqrt[-((30 - 10^(1/3)*(2700 + (-54 + 54*
```

```
x)^2)^(1/3))/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))^2)
]) + ((30 - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))*Sqrt[(900 + 30*10^(1/3)
*(2700 + (-54 + 54*x)^2)^(1/3) + 10^(2/3)*(2700 + (-54 + 54*x)^2)^(2/3))/(3
0*(1 - Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))^2]*EllipticF[ArcS
in[(30*(1 + Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))/(30*(1 - Sqr
t[3]) - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))], -7 + 4*Sqrt[3]])/(27000*3
^(3/4)*5^(1/6)*(1 - x)*Sqrt[-((30 - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))
/(30*(1 - Sqrt[3]) - 10^(1/3)*(2700 + (-54 + 54*x)^2)^(1/3))^2)]) - Log[2 +
 3*x]/(600*10^(2/3)) + Log[216 - 81*x - 27*10^(1/3)*(52 - 54*x + 27*x^2)^(1
/3)]/(600*10^(2/3))
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 750

```
Int[1/(((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)), x_Sy
mbol] :> With[{q = Rt[3*c*e^2*(2*c*d - b*e), 3]}, -Simp[(Sqrt[3]*c*e*ArcTan
[1/Sqrt[3] + (2*(c*d - b*e - c*e*x))/(Sqrt[3]*q*(a + b*x + c*x^2)^(1/3))])/
q^2, x] + (-Simp[(3*c*e*Log[d + e*x])/(2*q^2), x] + Simp[(3*c*e*Log[c*d - b
*e - c*e*x - q*(a + b*x + c*x^2)^(1/3)])/(2*q^2), x])] /; FreeQ[{a, b, c, d
, e}, x] && NeQ[2*c*d - b*e, 0] && EqQ[c^2*d^2 - b*c*d*e + b^2*e^2 - 3*a*c*
e^2, 0] && PosQ[c*e^2*(2*c*d - b*e)]
```

Rubi steps

$$\int \frac{1}{(2+3x)^3\sqrt[3]{52-54x+27x^2}}\,dx = -\frac{\left(52-54x+27x^2\right)^{2/3}}{600(2+3x)^2} - \frac{\int \frac{-324+54x}{(2+3x)^2\sqrt[3]{52-54x+27x^2}}\,dx}{1800}$$

$$= -\frac{\left(52-54x+27x^2\right)^{2/3}}{600(2+3x)^2} - \frac{\left(52-54x+27x^2\right)^{2/3}}{1500(2+3x)} + \frac{\int \frac{22680+9720x}{(2+3x)\sqrt[3]{52-54x+27x^2}}\,dx}{1620000}$$

$$= -\frac{\left(52-54x+27x^2\right)^{2/3}}{600(2+3x)^2} - \frac{\left(52-54x+27x^2\right)^{2/3}}{1500(2+3x)} + \frac{1}{500}\int \frac{1}{\sqrt[3]{52-54x+27x^2}}\,dx +$$

$$= -\frac{\left(52-54x+27x^2\right)^{2/3}}{600(2+3x)^2} - \frac{\left(52-54x+27x^2\right)^{2/3}}{1500(2+3x)} - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2^{2/3}(8-3x)}{\sqrt{3}\sqrt[3]{5}\sqrt[3]{52-54x+27x^2}}\right)}{300\sqrt{3}10^{2/3}}$$

$$= -\frac{\left(52-54x+27x^2\right)^{2/3}}{600(2+3x)^2} - \frac{\left(52-54x+27x^2\right)^{2/3}}{1500(2+3x)} - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2^{2/3}(8-3x)}{\sqrt{3}\sqrt[3]{5}\sqrt[3]{52-54x+27x^2}}\right)}{300\sqrt{3}10^{2/3}}$$

$$= -\frac{\left(52-54x+27x^2\right)^{2/3}}{600(2+3x)^2} - \frac{\left(52-54x+27x^2\right)^{2/3}}{1500(2+3x)} - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2^{2/3}(8-3x)}{\sqrt{3}\sqrt[3]{5}\sqrt[3]{52-54x+27x^2}}\right)}{300\sqrt{3}10^{2/3}}$$

$$= -\frac{\left(52-54x+27x^2\right)^{2/3}}{600(2+3x)^2} - \frac{\left(52-54x+27x^2\right)^{2/3}}{1500(2+3x)} + \frac{9(1-x)}{50\,5^{2/3}\left(30-30\sqrt{3}-\sqrt[3]{10}\sqrt[3]{27}\right.}$$

Mathematica [C] time = 0.297046, size = 233, normalized size = 0.31

$$\frac{-150\sqrt[3]{3}\sqrt[3]{\frac{9x-5i\sqrt{3}-9}{3x+2}}\sqrt[3]{\frac{9x+5i\sqrt{3}-9}{3x+2}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{15-5i\sqrt{3}}{9x+6},\frac{15+5i\sqrt{3}}{9x+6}\right) + 3^{5/6}10^{2/3}\sqrt[3]{-9ix+5\sqrt{3}+9i}\left(9x-5i\sqrt{3}-9\right)\,{}_2F_1\left(\frac{1}{3},\right.}{90000\sqrt[3]{27x^2-54x+52}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 + 3*x)^3*(52 - 54*x + 27*x^2)^(1/3)),x]
```

```
[Out] ((-90*(3 + 2*x)*(52 - 54*x + 27*x^2))/(2 + 3*x)^2 - 150*3^(1/3)*((-9 - (5*I
)*Sqrt[3] + 9*x)/(2 + 3*x))^(1/3)*((-9 + (5*I)*Sqrt[3] + 9*x)/(2 + 3*x))^(1
/3)*AppellF1[2/3, 1/3, 1/3, 5/3, (15 - (5*I)*Sqrt[3])/(6 + 9*x), (15 + (5*I
)*Sqrt[3])/(6 + 9*x)] + 3^(5/6)*10^(2/3)*(9*I + 5*Sqrt[3] - (9*I)*x)^(1/3)*
(-9 - (5*I)*Sqrt[3] + 9*x)*Hypergeometric2F1[1/3, 2/3, 5/3, (-9*I + 5*Sqrt[
3] + (9*I)*x)/(10*Sqrt[3])])/(90000*(52 - 54*x + 27*x^2)^(1/3))
```

Maple [F] time = 1.249, size = 0, normalized size = 0.

$$\int \frac{1}{(2+3x)^3}\frac{1}{\sqrt[3]{27x^2-54x+52}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*x)^3/(27*x^2-54*x+52)^(1/3),x)
```

[Out] int(1/(2+3*x)^3/(27*x^2-54*x+52)^(1/3),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(27\,x^2-54\,x+52\right)^{\frac{1}{3}}(3\,x+2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)^3/(27*x^2-54*x+52)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((27*x^2 - 54*x + 52)^(1/3)*(3*x + 2)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(27\,x^2-54\,x+52\right)^{\frac{2}{3}}}{729\,x^5-540\,x^3+1080\,x^2+1440\,x+416},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)^3/(27*x^2-54*x+52)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((27*x^2 - 54*x + 52)^(2/3)/(729*x^5 - 540*x^3 + 1080*x^2 + 1440*x
+ 416), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x+2)^3\sqrt[3]{27x^2-54x+52}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)**3/(27*x**2-54*x+52)**(1/3),x)
```

```
[Out] Integral(1/((3*x + 2)**3*(27*x**2 - 54*x + 52)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(27\,x^2-54\,x+52\right)^{\frac{1}{3}}(3\,x+2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)^3/(27*x^2-54*x+52)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((27*x^2 - 54*x + 52)^(1/3)*(3*x + 2)^3), x)
```

3.2503 $\int \frac{(2+3x)^3}{\sqrt[3]{28+54x+27x^2}}\,dx$

Optimal. Leaf size=589

$$\frac{4\left(6-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}\right)\sqrt{\frac{\left(27x^2+54x+28\right)^{2/3}+\sqrt[3]{27x^2+54x+28}+1}{\left(6\left(1-\sqrt{3}\right)-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{6\left(1+\sqrt{3}\right)-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}}{6\left(1-\sqrt{3}\right)-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}}\right),4\sqrt{3}-7}{63\;3^{3/4}(x+1)\sqrt{-\frac{6-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}}{\left(6\left(1-\sqrt{3}\right)-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}\right)^2}}}$$

```
[Out] ((2 + 3*x)^2*(28 + 54*x + 27*x^2)^(2/3))/30 - ((1 + 8*x)*(28 + 54*x + 27*x^
2)^(2/3))/35 + (72*(1 + x))/(7*(6*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x
)^2)^(1/3))) - (Sqrt[2*(2 + Sqrt[3])]*(6 - 2^(1/3)*(108 + (54 + 54*x)^2)^(1
/3))*Sqrt[(1 + (28 + 54*x + 27*x^2)^(1/3) + (28 + 54*x + 27*x^2)^(2/3))/(6*
(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))^2]*EllipticE[ArcSin[(6
*(1 + Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))/(6*(1 - Sqrt[3]) - 2^
(1/3)*(108 + (54 + 54*x)^2)^(1/3))], -7 + 4*Sqrt[3]])/(63*3^(1/4)*(1 + x)*S
qrt[-((6 - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))/(6*(1 - Sqrt[3]) - 2^(1/3)*
(108 + (54 + 54*x)^2)^(1/3))^2)]) + (4*(6 - 2^(1/3)*(108 + (54 + 54*x)^2)^(
1/3))*Sqrt[(1 + (28 + 54*x + 27*x^2)^(1/3) + (28 + 54*x + 27*x^2)^(2/3))/(6
*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))^2]*EllipticF[ArcSin[(
6*(1 + Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))/(6*(1 - Sqrt[3]) - 2
^(1/3)*(108 + (54 + 54*x)^2)^(1/3))], -7 + 4*Sqrt[3]])/(63*3^(3/4)*(1 + x)*
Sqrt[-((6 - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))/(6*(1 - Sqrt[3]) - 2^(1/3)
*(108 + (54 + 54*x)^2)^(1/3))^2)])
```

Rubi [A] time = 0.56319, antiderivative size = 589, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {742, 779, 619, 235, 304, 219, 1879}

$$\frac{1}{30}\left(27x^2+54x+28\right)^{2/3}(3x+2)^2-\frac{1}{35}(8x+1)\left(27x^2+54x+28\right)^{2/3}+\frac{4\left(6-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}\right)\sqrt{\frac{\left(27x^2+54x+28\right.}{\left(6\left(1-\sqrt{3}\right)-\sqrt[3]{\ }\right.}}}{63\;3^{3/4}(x+}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)^3/(28 + 54*x + 27*x^2)^(1/3),x]
```

```
[Out] ((2 + 3*x)^2*(28 + 54*x + 27*x^2)^(2/3))/30 - ((1 + 8*x)*(28 + 54*x + 27*x^
2)^(2/3))/35 + (72*(1 + x))/(7*(6*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x
)^2)^(1/3))) - (Sqrt[2*(2 + Sqrt[3])]*(6 - 2^(1/3)*(108 + (54 + 54*x)^2)^(1
/3))*Sqrt[(1 + (28 + 54*x + 27*x^2)^(1/3) + (28 + 54*x + 27*x^2)^(2/3))/(6*
(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))^2]*EllipticE[ArcSin[(6
*(1 + Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))/(6*(1 - Sqrt[3]) - 2^
(1/3)*(108 + (54 + 54*x)^2)^(1/3))], -7 + 4*Sqrt[3]])/(63*3^(1/4)*(1 + x)*S
qrt[-((6 - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))/(6*(1 - Sqrt[3]) - 2^(1/3)*
(108 + (54 + 54*x)^2)^(1/3))^2)]) + (4*(6 - 2^(1/3)*(108 + (54 + 54*x)^2)^(
1/3))*Sqrt[(1 + (28 + 54*x + 27*x^2)^(1/3) + (28 + 54*x + 27*x^2)^(2/3))/(6
*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))^2]*EllipticF[ArcSin[(
6*(1 + Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))/(6*(1 - Sqrt[3]) - 2
^(1/3)*(108 + (54 + 54*x)^2)^(1/3))], -7 + 4*Sqrt[3]])/(63*3^(3/4)*(1 + x)*
Sqrt[-((6 - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))/(6*(1 - Sqrt[3]) - 2^(1/3)
```

```
*(108 + (54 + 54*x)^2)^(1/3))^2)])
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
```

```
EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(2+3x)^3}{\sqrt[3]{28+54x+27x^2}}\,dx = \frac{1}{30}(2+3x)^2\left(28+54x+27x^2\right)^{2/3} + \frac{1}{90}\int \frac{(-360-432x)(2+3x)}{\sqrt[3]{28+54x+27x^2}}\,dx$$

$$= \frac{1}{30}(2+3x)^2\left(28+54x+27x^2\right)^{2/3} - \frac{1}{35}(1+8x)\left(28+54x+27x^2\right)^{2/3} - \frac{4}{7}\int \frac{1}{\sqrt[3]{28+54x+}}$$

$$= \frac{1}{30}(2+3x)^2\left(28+54x+27x^2\right)^{2/3} - \frac{1}{35}(1+8x)\left(28+54x+27x^2\right)^{2/3} - \frac{2}{189}\,\text{Subst}\left(\int \frac{}{\sqrt[3]{1}}\right.$$

$$= \frac{1}{30}(2+3x)^2\left(28+54x+27x^2\right)^{2/3} - \frac{1}{35}(1+8x)\left(28+54x+27x^2\right)^{2/3} - \frac{\left(2\sqrt{(54+54x)^2}\right)\text{S}}{}$$

$$= \frac{1}{30}(2+3x)^2\left(28+54x+27x^2\right)^{2/3} - \frac{1}{35}(1+8x)\left(28+54x+27x^2\right)^{2/3} + \frac{\left(2\sqrt{(54+54x)^2}\right)\text{S}}{}$$

$$= \frac{1}{30}(2+3x)^2\left(28+54x+27x^2\right)^{2/3} - \frac{1}{35}(1+8x)\left(28+54x+27x^2\right)^{2/3} + \frac{12(1+}{7\left(1-\sqrt{3}-\sqrt[3]{28+}\right.}$$

Mathematica [C] time = 0.031316, size = 53, normalized size = 0.09

$$\frac{1}{210}\left(27x^2+54x+28\right)^{2/3}\left(63x^2+36x+22\right) - \frac{4}{7}(x+1)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-27(x+1)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)^3/(28 + 54*x + 27*x^2)^(1/3),x]
```

```
[Out] ((28 + 54*x + 27*x^2)^(2/3)*(22 + 36*x + 63*x^2))/210 - (4*(1 + x)*Hypergeo
metric2F1[1/3, 1/2, 3/2, -27*(1 + x)^2])/7
```

Maple [F] time = 3.254, size = 0, normalized size = 0.

$$\int (2+3x)^3 \frac{1}{\sqrt[3]{27x^2+54x+28}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^3/(27*x^2+54*x+28)^(1/3),x)
```

```
[Out] int((2+3*x)^3/(27*x^2+54*x+28)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x+2)^3}{\left(27x^2+54x+28\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^3/(27*x^2+54*x+28)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((3*x + 2)^3/(27*x^2 + 54*x + 28)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{27\,x^3+54\,x^2+36\,x+8}{\left(27\,x^2+54\,x+28\right)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^3/(27*x^2+54*x+28)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((27*x^3 + 54*x^2 + 36*x + 8)/(27*x^2 + 54*x + 28)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(3x+2)^3}{\sqrt[3]{27x^2+54x+28}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**3/(27*x**2+54*x+28)**(1/3),x)
```

```
[Out] Integral((3*x + 2)**3/(27*x**2 + 54*x + 28)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(3\,x+2)^3}{\left(27\,x^2+54\,x+28\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^3/(27*x^2+54*x+28)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((3*x + 2)^3/(27*x^2 + 54*x + 28)^(1/3), x)
```

3.2504 $\int \frac{(2+3x)^2}{\sqrt[3]{28+54x+27x^2}}\,dx$

Optimal. Leaf size=585

$$-\frac{2\left(6-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}\right)\sqrt{\frac{(27x^2+54x+28)^{2/3}+\sqrt[3]{27x^2+54x+28}+1}{\left(6(1-\sqrt{3})-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{6(1+\sqrt{3})-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}}{6(1-\sqrt{3})-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}}\right),4\sqrt{3}-\right.}{21\ 3^{3/4}\sqrt{-\frac{6-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}}{\left(6(1-\sqrt{3})-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}\right)^2}}(x+1)}$$

```
[Out] (-5*(28 + 54*x + 27*x^2)^(2/3))/42 + ((2 + 3*x)*(28 + 54*x + 27*x^2)^(2/3))
/21 - (108*(1 + x))/(7*(6*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/
3))) + (Sqrt[2 + Sqrt[3]]*(6 - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))*Sqrt[(1
 + (28 + 54*x + 27*x^2)^(1/3) + (28 + 54*x + 27*x^2)^(2/3))/(6*(1 - Sqrt[3]
) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))^2]*EllipticE[ArcSin[(6*(1 + Sqrt[3
]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))/(6*(1 - Sqrt[3]) - 2^(1/3)*(108 +
 (54 + 54*x)^2)^(1/3))], -7 + 4*Sqrt[3]])/(21*Sqrt[2]*3^(1/4)*(1 + x)*Sqrt[
-((6 - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))/(6*(1 - Sqrt[3]) - 2^(1/3)*(108
 + (54 + 54*x)^2)^(1/3))^2)]) - (2*(6 - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3)
)*Sqrt[(1 + (28 + 54*x + 27*x^2)^(1/3) + (28 + 54*x + 27*x^2)^(2/3))/(6*(1
- Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))^2]*EllipticF[ArcSin[(6*(1
 + Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))/(6*(1 - Sqrt[3]) - 2^(1/
3)*(108 + (54 + 54*x)^2)^(1/3))], -7 + 4*Sqrt[3]])/(21*3^(3/4)*(1 + x)*Sqrt
[-((6 - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))/(6*(1 - Sqrt[3]) - 2^(1/3)*(10
8 + (54 + 54*x)^2)^(1/3))^2)])
```

Rubi [A] time = 0.485929, antiderivative size = 585, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {742, 640, 619, 235, 304, 219, 1879}

$$\frac{1}{21}(3x+2)\left(27x^2+54x+28\right)^{2/3}-\frac{5}{42}\left(27x^2+54x+28\right)^{2/3}-\frac{2\left(6-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}\right)\sqrt{\frac{(27x^2+54x+28)^{2/3}+\sqrt[3]{27x^2+\cdots}}{\left(6(1-\sqrt{3})-\sqrt[3]{2}\sqrt[3]{(54x+54)\cdots}\right.}}}{21\ 3^{3/4}\sqrt{-\frac{6-\sqrt[3]{2}\sqrt[3]{\cdots}}{\left(6(1-\sqrt{3})-\sqrt[3]{\cdots}\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)^2/(28 + 54*x + 27*x^2)^(1/3),x]
```

```
[Out] (-5*(28 + 54*x + 27*x^2)^(2/3))/42 + ((2 + 3*x)*(28 + 54*x + 27*x^2)^(2/3))
/21 - (108*(1 + x))/(7*(6*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/
3))) + (Sqrt[2 + Sqrt[3]]*(6 - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))*Sqrt[(1
 + (28 + 54*x + 27*x^2)^(1/3) + (28 + 54*x + 27*x^2)^(2/3))/(6*(1 - Sqrt[3]
) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))^2]*EllipticE[ArcSin[(6*(1 + Sqrt[3
]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))/(6*(1 - Sqrt[3]) - 2^(1/3)*(108 +
 (54 + 54*x)^2)^(1/3))], -7 + 4*Sqrt[3]])/(21*Sqrt[2]*3^(1/4)*(1 + x)*Sqrt[
-((6 - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))/(6*(1 - Sqrt[3]) - 2^(1/3)*(108
 + (54 + 54*x)^2)^(1/3))^2)]) - (2*(6 - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3)
)*Sqrt[(1 + (28 + 54*x + 27*x^2)^(1/3) + (28 + 54*x + 27*x^2)^(2/3))/(6*(1
- Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))^2]*EllipticF[ArcSin[(6*(1
 + Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))/(6*(1 - Sqrt[3]) - 2^(1/
3)*(108 + (54 + 54*x)^2)^(1/3))], -7 + 4*Sqrt[3]])/(21*3^(3/4)*(1 + x)*Sqrt
[-((6 - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))/(6*(1 - Sqrt[3]) - 2^(1/3)*(10
```

```
8 + (54 + 54*x)^2)^(1/3))^2)])
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(2+3x)^2}{\sqrt[3]{28+54x+27x^2}}\,dx = \frac{1}{21}(2+3x)\left(28+54x+27x^2\right)^{2/3} + \frac{1}{63}\int \frac{-216-270x}{\sqrt[3]{28+54x+27x^2}}\,dx$$

$$= -\frac{5}{42}\left(28+54x+27x^2\right)^{2/3} + \frac{1}{21}(2+3x)\left(28+54x+27x^2\right)^{2/3} + \frac{6}{7}\int \frac{1}{\sqrt[3]{28+54x+27x^2}}\,d$$

$$= -\frac{5}{42}\left(28+54x+27x^2\right)^{2/3} + \frac{1}{21}(2+3x)\left(28+54x+27x^2\right)^{2/3} + \frac{1}{63}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1+\frac{x^2}{108}}}\,d\right.$$

$$= -\frac{5}{42}\left(28+54x+27x^2\right)^{2/3} + \frac{1}{21}(2+3x)\left(28+54x+27x^2\right)^{2/3} + \frac{\left(\sqrt{3}\sqrt{(54+54x)^2}\right)\text{Subst}}{7}$$

$$= -\frac{5}{42}\left(28+54x+27x^2\right)^{2/3} + \frac{1}{21}(2+3x)\left(28+54x+27x^2\right)^{2/3} - \frac{\left(\sqrt{3}\sqrt{(54+54x)^2}\right)\text{Subst}}{7}$$

$$= -\frac{5}{42}\left(28+54x+27x^2\right)^{2/3} + \frac{1}{21}(2+3x)\left(28+54x+27x^2\right)^{2/3} - \frac{18(1+x)}{7\left(1-\sqrt{3}-\sqrt[3]{28+54x+}\right.}$$

Mathematica [C] time = 0.0184156, size = 47, normalized size = 0.08

$$\frac{1}{42}\left(36(x+1)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-27(x+1)^2\right)+\left(27x^2+54x+28\right)^{2/3}(6x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)^2/(28 + 54*x + 27*x^2)^(1/3),x]
```

```
[Out] ((-1 + 6*x)*(28 + 54*x + 27*x^2)^(2/3) + 36*(1 + x)*Hypergeometric2F1[1/3,
1/2, 3/2, -27*(1 + x)^2])/42
```

Maple [F] time = 1.506, size = 0, normalized size = 0.

$$\int (2+3x)^2 \frac{1}{\sqrt[3]{27x^2+54x+28}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^2/(27*x^2+54*x+28)^(1/3),x)
```

```
[Out] int((2+3*x)^2/(27*x^2+54*x+28)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x+2)^2}{\left(27x^2+54x+28\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2/(27*x^2+54*x+28)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((3*x + 2)^2/(27*x^2 + 54*x + 28)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{9\,x^2+12\,x+4}{\left(27\,x^2+54\,x+28\right)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2/(27*x^2+54*x+28)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((9*x^2 + 12*x + 4)/(27*x^2 + 54*x + 28)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(3x+2)^2}{\sqrt[3]{27x^2+54x+28}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**2/(27*x**2+54*x+28)**(1/3),x)
```

```
[Out] Integral((3*x + 2)**2/(27*x**2 + 54*x + 28)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(3\,x+2)^2}{\left(27\,x^2+54\,x+28\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2/(27*x^2+54*x+28)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((3*x + 2)^2/(27*x^2 + 54*x + 28)^(1/3), x)
```

3.2505 $\int \frac{2+3x}{\sqrt[3]{28+54x+27x^2}}\,dx$

Optimal. Leaf size=560

$$\frac{\left(6-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}\right)\sqrt{\frac{(27x^2+54x+28)^{2/3}+\sqrt[3]{27x^2+54x+28}+1}{\left(6(1-\sqrt{3})-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{6(1+\sqrt{3})-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}}{6(1-\sqrt{3})-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}}\right),4\sqrt{3}-7\right)}{9\,3^{3/4}\sqrt{-\frac{6-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}}{\left(6(1-\sqrt{3})-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}\right)^2}}(x+1)}$$

```
[Out] (28 + 54*x + 27*x^2)^(2/3)/12 + (18*(1 + x))/(6*(1 - Sqrt[3]) - 2^(1/3)*(10
8 + (54 + 54*x)^2)^(1/3)) - (Sqrt[2 + Sqrt[3]]*(6 - 2^(1/3)*(108 + (54 + 54
*x)^2)^(1/3))*Sqrt[(1 + (28 + 54*x + 27*x^2)^(1/3) + (28 + 54*x + 27*x^2)^(
2/3))/(6*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))^2]*EllipticE[
ArcSin[(6*(1 + Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))/(6*(1 - Sqrt
[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))], -7 + 4*Sqrt[3]])/(18*Sqrt[2]*
3^(1/4)*(1 + x)*Sqrt[-((6 - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))/(6*(1 - Sq
rt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))^2)]) + ((6 - 2^(1/3)*(108 + (
54 + 54*x)^2)^(1/3))*Sqrt[(1 + (28 + 54*x + 27*x^2)^(1/3) + (28 + 54*x + 27
*x^2)^(2/3))/(6*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))^2]*Ell
ipticF[ArcSin[(6*(1 + Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))/(6*(1
 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^
(3/4)*(1 + x)*Sqrt[-((6 - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))/(6*(1 - Sqrt
[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))^2)])
```

Rubi [A] time = 0.378452, antiderivative size = 560, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {640, 619, 235, 304, 219, 1879}

$$\frac{1}{12}\left(27x^2+54x+28\right)^{2/3}+\frac{\left(6-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}\right)\sqrt{\frac{(27x^2+54x+28)^{2/3}+\sqrt[3]{27x^2+54x+28}+1}{\left(6(1-\sqrt{3})-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}\right)^2}}F\left(\sin^{-1}\left(\frac{6(1+\sqrt{3})-\sqrt[3]{2}\sqrt[3]{(54x+54}}{6(1-\sqrt{3})-\sqrt[3]{2}\sqrt[3]{(54x+54}}\right.\right.}{9\,3^{3/4}\sqrt{-\frac{6-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}}{\left(6(1-\sqrt{3})-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}\right)^2}}(x+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)/(28 + 54*x + 27*x^2)^(1/3),x]
```

```
[Out] (28 + 54*x + 27*x^2)^(2/3)/12 + (18*(1 + x))/(6*(1 - Sqrt[3]) - 2^(1/3)*(10
8 + (54 + 54*x)^2)^(1/3)) - (Sqrt[2 + Sqrt[3]]*(6 - 2^(1/3)*(108 + (54 + 54
*x)^2)^(1/3))*Sqrt[(1 + (28 + 54*x + 27*x^2)^(1/3) + (28 + 54*x + 27*x^2)^(
2/3))/(6*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))^2]*EllipticE[
ArcSin[(6*(1 + Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))/(6*(1 - Sqrt
[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))], -7 + 4*Sqrt[3]])/(18*Sqrt[2]*
3^(1/4)*(1 + x)*Sqrt[-((6 - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))/(6*(1 - Sq
rt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))^2)]) + ((6 - 2^(1/3)*(108 + (
54 + 54*x)^2)^(1/3))*Sqrt[(1 + (28 + 54*x + 27*x^2)^(1/3) + (28 + 54*x + 27
*x^2)^(2/3))/(6*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))^2]*Ell
ipticF[ArcSin[(6*(1 + Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))/(6*(1
 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^
(3/4)*(1 + x)*Sqrt[-((6 - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))/(6*(1 - Sqrt
[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))^2)])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{2+3x}{\sqrt[3]{28+54x+27x^2}}\,dx = \frac{1}{12}\left(28+54x+27x^2\right)^{2/3} - \int \frac{1}{\sqrt[3]{28+54x+27x^2}}\,dx$$
$$= \frac{1}{12}\left(28+54x+27x^2\right)^{2/3} - \frac{1}{54}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1+\frac{x^2}{108}}}\,dx, x, 54+54x\right)$$
$$= \frac{1}{12}\left(28+54x+27x^2\right)^{2/3} - \frac{\sqrt{(54+54x)^2}\,\text{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{28+54x+27x^2}\right)}{2\sqrt{3}(54+54x)}$$
$$= \frac{1}{12}\left(28+54x+27x^2\right)^{2/3} + \frac{\sqrt{(54+54x)^2}\,\text{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{28+54x+27x^2}\right)}{2\sqrt{3}(54+54x)} - \frac{(\sqrt{}}{}$$
$$= \frac{1}{12}\left(28+54x+27x^2\right)^{2/3} + \frac{3(1+x)}{1-\sqrt{3}-\sqrt[3]{28+54x+27x^2}} - \frac{\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{28+54x+27x\right.}}{}$$

Mathematica [C] time = 0.007192, size = 41, normalized size = 0.07

$$\frac{1}{12}\left(27x^2+54x+28\right)^{2/3} - (x+1)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-27(x+1)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)/(28 + 54*x + 27*x^2)^(1/3),x]
```

```
[Out] (28 + 54*x + 27*x^2)^(2/3)/12 - (1 + x)*Hypergeometric2F1[1/3, 1/2, 3/2, -2
7*(1 + x)^2]
```

Maple [F] time = 1.478, size = 0, normalized size = 0.

$$\int (2+3x)\frac{1}{\sqrt[3]{27x^2+54x+28}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)/(27*x^2+54*x+28)^(1/3),x)
```

```
[Out] int((2+3*x)/(27*x^2+54*x+28)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x+2}{\left(27x^2+54x+28\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(27*x^2+54*x+28)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((3*x + 2)/(27*x^2 + 54*x + 28)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{3x+2}{\left(27x^2+54x+28\right)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(27*x^2+54*x+28)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((3*x + 2)/(27*x^2 + 54*x + 28)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x+2}{\sqrt[3]{27x^2+54x+28}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(27*x**2+54*x+28)**(1/3),x)
```

```
[Out] Integral((3*x + 2)/(27*x**2 + 54*x + 28)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x+2}{\left(27x^2+54x+28\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(27*x^2+54*x+28)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((3*x + 2)/(27*x^2 + 54*x + 28)^(1/3), x)
```

3.2506 $\int \frac{1}{(2+3x)\sqrt[3]{28+54x+27x^2}}\,dx$

Optimal. Leaf size=103

$$\frac{\log\left(27\sqrt[3]{2}\sqrt[3]{27x^2+54x+28}-81x-108\right)}{6\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}(3x+4)}{\sqrt{3}\sqrt[3]{27x^2+54x+28}}+\frac{1}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\log(3x+2)}{6\ 2^{2/3}}$$

```
[Out] -ArcTan[1/Sqrt[3] + (2^(2/3)*(4 + 3*x))/(Sqrt[3]*(28 + 54*x + 27*x^2)^(1/3)
)]/(3*2^(2/3)*Sqrt[3]) - Log[2 + 3*x]/(6*2^(2/3)) + Log[-108 - 81*x + 27*2^
(1/3)*(28 + 54*x + 27*x^2)^(1/3)]/(6*2^(2/3))
```

Rubi [A] time = 0.0187359, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {752}

$$\frac{\log\left(27\sqrt[3]{2}\sqrt[3]{27x^2+54x+28}-81x-108\right)}{6\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}(3x+4)}{\sqrt{3}\sqrt[3]{27x^2+54x+28}}+\frac{1}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\log(3x+2)}{6\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + 3*x)*(28 + 54*x + 27*x^2)^(1/3)),x]
```

```
[Out] -ArcTan[1/Sqrt[3] + (2^(2/3)*(4 + 3*x))/(Sqrt[3]*(28 + 54*x + 27*x^2)^(1/3)
)]/(3*2^(2/3)*Sqrt[3]) - Log[2 + 3*x]/(6*2^(2/3)) + Log[-108 - 81*x + 27*2^
(1/3)*(28 + 54*x + 27*x^2)^(1/3)]/(6*2^(2/3))
```

Rule 752

```
Int[1/(((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)), x_Sy
mbol] :> With[{q = Rt[-3*c*e^2*(2*c*d - b*e), 3]}, -Simp[(Sqrt[3]*c*e*ArcTa
n[1/Sqrt[3] - (2*(c*d - b*e - c*e*x))/(Sqrt[3]*q*(a + b*x + c*x^2)^(1/3))])
/q^2, x] + (-Simp[(3*c*e*Log[d + e*x])/(2*q^2), x] + Simp[(3*c*e*Log[c*d -
b*e - c*e*x + q*(a + b*x + c*x^2)^(1/3)])/(2*q^2), x])] /; FreeQ[{a, b, c,
d, e}, x] && NeQ[2*c*d - b*e, 0] && EqQ[c^2*d^2 - b*c*d*e + b^2*e^2 - 3*a*c
*e^2, 0] && NegQ[c*e^2*(2*c*d - b*e)]
```

Rubi steps

$$\int \frac{1}{(2+3x)\sqrt[3]{28+54x+27x^2}}\,dx=-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2^{2/3}(4+3x)}{\sqrt{3}\sqrt[3]{28+54x+27x^2}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\log(2+3x)}{6\ 2^{2/3}}+\frac{\log\left(-108-81x+27\sqrt[3]{2}\sqrt[3]{28+5\right.}{6\ 2^{2/3}}$$

Mathematica [C] time = 0.0767898, size = 127, normalized size = 1.23

$$-\frac{\sqrt[3]{\frac{9x-i\sqrt{3}+9}{3x+2}}\sqrt[3]{\frac{9x+i\sqrt{3}+9}{3x+2}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};-\frac{3+i\sqrt{3}}{9x+6},\frac{-3+i\sqrt{3}}{9x+6}\right)}{2\ 3^{2/3}\sqrt[3]{27x^2+54x+28}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 + 3*x)*(28 + 54*x + 27*x^2)^(1/3)),x]
```

```
[Out] -(((9 - I*Sqrt[3] + 9*x)/(2 + 3*x))^(1/3)*((9 + I*Sqrt[3] + 9*x)/(2 + 3*x))
^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, -((3 + I*Sqrt[3])/(6 + 9*x)), (-3 + I*S
qrt[3])/(6 + 9*x)])/(2*3^(2/3)*(28 + 54*x + 27*x^2)^(1/3))
```

Maple [F] time = 1.678, size = 0, normalized size = 0.

$$\int \frac{1}{2+3x}\frac{1}{\sqrt[3]{27x^2+54x+28}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*x)/(27*x^2+54*x+28)^(1/3),x)
```

```
[Out] int(1/(2+3*x)/(27*x^2+54*x+28)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(27x^2+54x+28\right)^{\frac{1}{3}}(3x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(27*x^2+54*x+28)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((27*x^2 + 54*x + 28)^(1/3)*(3*x + 2)), x)
```

Fricas [B] time = 25.4418, size = 626, normalized size = 6.08

$$-\frac{1}{18}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{4^{\frac{1}{6}}\left(2\cdot 4^{\frac{2}{3}}\sqrt{3}\left(27x^2+54x+28\right)^{\frac{2}{3}}(3x+4)+4^{\frac{1}{3}}\sqrt{3}\left(27x^3+54x^2+36x+8\right)-4\sqrt{3}\left(27x^2+54x+\right.\right.}{18\left(9x^3+54x^2+84x+40\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(27*x^2+54*x+28)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/18*4^(1/6)*sqrt(3)*arctan(1/18*4^(1/6)*(2*4^(2/3)*sqrt(3)*(27*x^2 + 54*x
 + 28)^(2/3)*(3*x + 4) + 4^(1/3)*sqrt(3)*(27*x^3 + 54*x^2 + 36*x + 8) - 4*s
qrt(3)*(27*x^2 + 54*x + 28)^(1/3)*(9*x^2 + 24*x + 16))/(9*x^3 + 54*x^2 + 84
*x + 40)) - 1/72*4^(2/3)*log((4^(2/3)*(27*x^2 + 54*x + 28)^(2/3) + 4^(1/3)*
(9*x^2 + 24*x + 16) + 2*(27*x^2 + 54*x + 28)^(1/3)*(3*x + 4))/(9*x^2 + 12*x
 + 4)) + 1/36*4^(2/3)*log((4^(1/3)*(3*x + 4) - 2*(27*x^2 + 54*x + 28)^(1/3)
)/(3*x + 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x+2)\sqrt[3]{27x^2+54x+28}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(27*x**2+54*x+28)**(1/3),x)
```

```
[Out] Integral(1/((3*x + 2)*(27*x**2 + 54*x + 28)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(27\,x^2+54\,x+28\right)^{\frac{1}{3}}(3\,x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(27*x^2+54*x+28)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((27*x^2 + 54*x + 28)^(1/3)*(3*x + 2)), x)
```

3.2507 $\int \frac{1}{(2+3x)^2 \sqrt[3]{28+54x+27x^2}} \, dx$

Optimal. Leaf size=671

$$-\frac{\left(6-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}\right)\sqrt{\frac{\left(27x^2+54x+28\right)^{2/3}+\sqrt[3]{27x^2+54x+28}+1}{\left(6\left(1-\sqrt{3}\right)-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{6\left(1+\sqrt{3}\right)-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}}{6\left(1-\sqrt{3}\right)-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}}\right),4\sqrt{3}-7\right)}{36\ 3^{3/4}\sqrt{-\frac{6-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}}{\left(6\left(1-\sqrt{3}\right)-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}\right)^2}}(x+1)}-$$

```
[Out] -(28 + 54*x + 27*x^2)^(2/3)/(12*(2 + 3*x)) - (9*(1 + x))/(2*(6*(1 - Sqrt[3]
) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))) + ArcTan[1/Sqrt[3] + (2^(2/3)*(4
+ 3*x))/(Sqrt[3]*(28 + 54*x + 27*x^2)^(1/3))]/(6*2^(2/3)*Sqrt[3]) + (Sqrt[2
 + Sqrt[3]]*(6 - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))*Sqrt[(1 + (28 + 54*x
+ 27*x^2)^(1/3) + (28 + 54*x + 27*x^2)^(2/3))/(6*(1 - Sqrt[3]) - 2^(1/3)*(1
08 + (54 + 54*x)^2)^(1/3))^2]*EllipticE[ArcSin[(6*(1 + Sqrt[3]) - 2^(1/3)*(
108 + (54 + 54*x)^2)^(1/3))/(6*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2
)^(1/3))], -7 + 4*Sqrt[3]])/(72*Sqrt[2]*3^(1/4)*(1 + x)*Sqrt[-((6 - 2^(1/3)
*(108 + (54 + 54*x)^2)^(1/3))/(6*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)
^2)^(1/3))^2)]) - ((6 - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))*Sqrt[(1 + (28
+ 54*x + 27*x^2)^(1/3) + (28 + 54*x + 27*x^2)^(2/3))/(6*(1 - Sqrt[3]) - 2^(
1/3)*(108 + (54 + 54*x)^2)^(1/3))^2]*EllipticF[ArcSin[(6*(1 + Sqrt[3]) - 2^
(1/3)*(108 + (54 + 54*x)^2)^(1/3))/(6*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 +
54*x)^2)^(1/3))], -7 + 4*Sqrt[3]])/(36*3^(3/4)*(1 + x)*Sqrt[-((6 - 2^(1/3)*
(108 + (54 + 54*x)^2)^(1/3))/(6*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^
2)^(1/3))^2)]) + Log[2 + 3*x]/(12*2^(2/3)) - Log[-108 - 81*x + 27*2^(1/3)*(
28 + 54*x + 27*x^2)^(1/3)]/(12*2^(2/3))
```

Rubi [A] time = 0.594944, antiderivative size = 671, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {744, 12, 843, 619, 235, 304, 219, 1879, 752}

$$-\frac{\left(27x^2+54x+28\right)^{2/3}}{12(3x+2)}-\frac{\log\left(27\sqrt[3]{2}\sqrt[3]{27x^2+54x+28}-81x-108\right)}{12\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}(3x+4)}{\sqrt{3}\sqrt[3]{27x^2+54x+28}}+\frac{1}{\sqrt{3}}\right)}{6\ 2^{2/3}\sqrt{3}}-\frac{\left(6-\sqrt[3]{2}\sqrt[3]{(54x+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + 3*x)^2*(28 + 54*x + 27*x^2)^(1/3)),x]
```


```
[Out] -(28 + 54*x + 27*x^2)^(2/3)/(12*(2 + 3*x)) - (9*(1 + x))/(2*(6*(1 - Sqrt[3]
) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))) + ArcTan[1/Sqrt[3] + (2^(2/3)*(4
+ 3*x))/(Sqrt[3]*(28 + 54*x + 27*x^2)^(1/3))]/(6*2^(2/3)*Sqrt[3]) + (Sqrt[2
 + Sqrt[3]]*(6 - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))*Sqrt[(1 + (28 + 54*x
+ 27*x^2)^(1/3) + (28 + 54*x + 27*x^2)^(2/3))/(6*(1 - Sqrt[3]) - 2^(1/3)*(1
08 + (54 + 54*x)^2)^(1/3))^2]*EllipticE[ArcSin[(6*(1 + Sqrt[3]) - 2^(1/3)*(
108 + (54 + 54*x)^2)^(1/3))/(6*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2
)^(1/3))], -7 + 4*Sqrt[3]])/(72*Sqrt[2]*3^(1/4)*(1 + x)*Sqrt[-((6 - 2^(1/3)
*(108 + (54 + 54*x)^2)^(1/3))/(6*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)
^2)^(1/3))^2)]) - ((6 - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))*Sqrt[(1 + (28
+ 54*x + 27*x^2)^(1/3) + (28 + 54*x + 27*x^2)^(2/3))/(6*(1 - Sqrt[3]) - 2^(
1/3)*(108 + (54 + 54*x)^2)^(1/3))^2]*EllipticF[ArcSin[(6*(1 + Sqrt[3]) - 2^
(1/3)*(108 + (54 + 54*x)^2)^(1/3))/(6*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 +
```

```
54*x)^2)^(1/3))], -7 + 4*Sqrt[3]])/(36*3^(3/4)*(1 + x)*Sqrt[-((6 - 2^(1/3)*
(108 + (54 + 54*x)^2)^(1/3))/(6*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^
2)^(1/3))^2)]) + Log[2 + 3*x]/(12*2^(2/3)) - Log[-108 - 81*x + 27*2^(1/3)*(
28 + 54*x + 27*x^2)^(1/3)]/(12*2^(2/3))
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 752

```
Int[1/(((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)), x_Sy
mbol] :> With[{q = Rt[-3*c*e^2*(2*c*d - b*e), 3]}, -Simp[(Sqrt[3]*c*e*ArcTa
n[1/Sqrt[3] - (2*(c*d - b*e - c*e*x))/(Sqrt[3]*q*(a + b*x + c*x^2)^(1/3))])
/q^2, x] + (-Simp[(3*c*e*Log[d + e*x])/(2*q^2), x] + Simp[(3*c*e*Log[c*d -
b*e - c*e*x + q*(a + b*x + c*x^2)^(1/3)])/(2*q^2), x])] /; FreeQ[{a, b, c,
d, e}, x] && NeQ[2*c*d - b*e, 0] && EqQ[c^2*d^2 - b*c*d*e + b^2*e^2 - 3*a*c
*e^2, 0] && NegQ[c*e^2*(2*c*d - b*e)]
```

Rubi steps

$$\int \frac{1}{(2+3x)^2\sqrt[3]{28+54x+27x^2}}\,dx = -\frac{\left(28+54x+27x^2\right)^{2/3}}{12(2+3x)} - \frac{1}{36}\int -\frac{27x}{(2+3x)\sqrt[3]{28+54x+27x^2}}\,dx$$

$$= -\frac{\left(28+54x+27x^2\right)^{2/3}}{12(2+3x)} + \frac{3}{4}\int \frac{x}{(2+3x)\sqrt[3]{28+54x+27x^2}}\,dx$$

$$= -\frac{\left(28+54x+27x^2\right)^{2/3}}{12(2+3x)} + \frac{1}{4}\int \frac{1}{\sqrt[3]{28+54x+27x^2}}\,dx - \frac{1}{2}\int \frac{1}{(2+3x)\sqrt[3]{28+54x+2}}$$

$$= -\frac{\left(28+54x+27x^2\right)^{2/3}}{12(2+3x)} + \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2^{2/3}(4+3x)}{\sqrt{3}\sqrt[3]{28+54x+27x^2}}\right)}{6\ 2^{2/3}\sqrt{3}} + \frac{\log(2+3x)}{12\ 2^{2/3}} - \frac{\log\left(-108\right.}{}$$

$$= -\frac{\left(28+54x+27x^2\right)^{2/3}}{12(2+3x)} + \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2^{2/3}(4+3x)}{\sqrt{3}\sqrt[3]{28+54x+27x^2}}\right)}{6\ 2^{2/3}\sqrt{3}} + \frac{\log(2+3x)}{12\ 2^{2/3}} - \frac{\log\left(-108\right.}{}$$

$$= -\frac{\left(28+54x+27x^2\right)^{2/3}}{12(2+3x)} + \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2^{2/3}(4+3x)}{\sqrt{3}\sqrt[3]{28+54x+27x^2}}\right)}{6\ 2^{2/3}\sqrt{3}} + \frac{\log(2+3x)}{12\ 2^{2/3}} - \frac{\log\left(-108\right.}{}$$

$$= -\frac{\left(28+54x+27x^2\right)^{2/3}}{12(2+3x)} - \frac{3(1+x)}{4\left(1-\sqrt{3}-\sqrt[3]{28+54x+27x^2}\right)} + \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2^{2/3}(4+3}{\sqrt{3}\sqrt[3]{28+54x}}\right.}{6\ 2^{2/3}\sqrt{3}}$$

Mathematica [C] time = 0.252011, size = 240, normalized size = 0.36

$$\frac{4\sqrt[3]{3}(3x+2)\sqrt[3]{\frac{9x-i\sqrt{3}+9}{3x+2}}\sqrt[3]{\frac{9x+i\sqrt{3}+9}{3x+2}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};-\frac{3+i\sqrt{3}}{9x+6},\frac{-3+i\sqrt{3}}{9x+6}\right)+2^{2/3}\sqrt[3]{3}\sqrt[3]{-9ix+\sqrt{3}-9i}(3x+2)\left(3\sqrt{3}x+3\sqrt{3}-i\right)\,{}_2F}{48(3x+2)\sqrt[3]{27x^2+54x+28}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 + 3*x)^2*(28 + 54*x + 27*x^2)^(1/3)),x]
```

```
[Out] (-4*(28 + 54*x + 27*x^2) + 4*3^(1/3)*(2 + 3*x)*((9 - I*Sqrt[3] + 9*x)/(2 +
3*x))^(1/3)*((9 + I*Sqrt[3] + 9*x)/(2 + 3*x))^(1/3)*AppellF1[2/3, 1/3, 1/3,
 5/3, -((3 + I*Sqrt[3])/(6 + 9*x)), (-3 + I*Sqrt[3])/(6 + 9*x)] + 2^(2/3)*3
^(1/3)*(-9*I + Sqrt[3] - (9*I)*x)^(1/3)*(2 + 3*x)*(-I + 3*Sqrt[3] + 3*Sqrt[
3]*x)*Hypergeometric2F1[1/3, 2/3, 5/3, (9*I + Sqrt[3] + (9*I)*x)/(2*Sqrt[3]
)])/(48*(2 + 3*x)*(28 + 54*x + 27*x^2)^(1/3))
```

Maple [F] time = 1.715, size = 0, normalized size = 0.

$$\int \frac{1}{(2+3x)^2}\frac{1}{\sqrt[3]{27x^2+54x+28}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*x)^2/(27*x^2+54*x+28)^(1/3),x)
```

```
[Out] int(1/(2+3*x)^2/(27*x^2+54*x+28)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(27x^2+54x+28\right)^{\frac{1}{3}}(3x+2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)^2/(27*x^2+54*x+28)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((27*x^2 + 54*x + 28)^(1/3)*(3*x + 2)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(27x^2+54x+28\right)^{\frac{2}{3}}}{243x^4+810x^3+1008x^2+552x+112}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)^2/(27*x^2+54*x+28)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((27*x^2 + 54*x + 28)^(2/3)/(243*x^4 + 810*x^3 + 1008*x^2 + 552*x +
 112), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x+2)^2\sqrt[3]{27x^2+54x+28}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)**2/(27*x**2+54*x+28)**(1/3),x)
```

```
[Out] Integral(1/((3*x + 2)**2*(27*x**2 + 54*x + 28)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(27\,x^2+54\,x+28\right)^{\frac{1}{3}}(3\,x+2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)^2/(27*x^2+54*x+28)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((27*x^2 + 54*x + 28)^(1/3)*(3*x + 2)^2), x)
```

3.2508 $\int \frac{1}{(2+3x)^3 \sqrt[3]{28+54x+27x^2}} \, dx$

Optimal. Leaf size=696

$$\frac{\left(6-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}\right)\sqrt{\frac{(27x^2+54x+28)^{2/3}+\sqrt[3]{27x^2+54x+28}+1}{\left(6(1-\sqrt{3})-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{6(1+\sqrt{3})-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}}{6(1-\sqrt{3})-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}}\right),4\sqrt{3}-7\right)}{36\, 3^{3/4}\sqrt{-\frac{6-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}}{\left(6(1-\sqrt{3})-\sqrt[3]{2}\sqrt[3]{(54x+54)^2+108}\right)^2}}(x+1)}$$

```
[Out] -(28 + 54*x + 27*x^2)^(2/3)/(24*(2 + 3*x)^2) + (28 + 54*x + 27*x^2)^(2/3)/(
12*(2 + 3*x)) + (9*(1 + x))/(2*(6*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x
)^2)^(1/3))) - ArcTan[1/Sqrt[3] + (2^(2/3)*(4 + 3*x))/(Sqrt[3]*(28 + 54*x +
 27*x^2)^(1/3))]/(12*2^(2/3)*Sqrt[3]) - (Sqrt[2 + Sqrt[3]]*(6 - 2^(1/3)*(10
8 + (54 + 54*x)^2)^(1/3))*Sqrt[(1 + (28 + 54*x + 27*x^2)^(1/3) + (28 + 54*x
 + 27*x^2)^(2/3))/(6*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))^2
]*EllipticE[ArcSin[(6*(1 + Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))/
(6*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))], -7 + 4*Sqrt[3]])/
(72*Sqrt[2]*3^(1/4)*(1 + x)*Sqrt[-((6 - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3)
)/(6*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))^2)]) + ((6 - 2^(1
/3)*(108 + (54 + 54*x)^2)^(1/3))*Sqrt[(1 + (28 + 54*x + 27*x^2)^(1/3) + (28
 + 54*x + 27*x^2)^(2/3))/(6*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(
1/3))^2]*EllipticF[ArcSin[(6*(1 + Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^
(1/3))/(6*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))], -7 + 4*Sqr
t[3]])/(36*3^(3/4)*(1 + x)*Sqrt[-((6 - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))
/(6*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))^2)]) - Log[2 + 3*x
]/(24*2^(2/3)) + Log[-108 - 81*x + 27*2^(1/3)*(28 + 54*x + 27*x^2)^(1/3)]/(
24*2^(2/3))
```

Rubi [A] time = 0.595902, antiderivative size = 696, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {744, 834, 843, 619, 235, 304, 219, 1879, 752}

$$\frac{(27x^2+54x+28)^{2/3}}{12(3x+2)}-\frac{(27x^2+54x+28)^{2/3}}{24(3x+2)^2}+\frac{\log\left(27\sqrt[3]{2}\sqrt[3]{27x^2+54x+28}-81x-108\right)}{24\, 2^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}(3x+4)}{\sqrt{3}\sqrt[3]{27x^2+54x+28}}\right.}{12\, 2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + 3*x)^3*(28 + 54*x + 27*x^2)^(1/3)),x]
```

```
[Out] -(28 + 54*x + 27*x^2)^(2/3)/(24*(2 + 3*x)^2) + (28 + 54*x + 27*x^2)^(2/3)/(
12*(2 + 3*x)) + (9*(1 + x))/(2*(6*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x
)^2)^(1/3))) - ArcTan[1/Sqrt[3] + (2^(2/3)*(4 + 3*x))/(Sqrt[3]*(28 + 54*x +
 27*x^2)^(1/3))]/(12*2^(2/3)*Sqrt[3]) - (Sqrt[2 + Sqrt[3]]*(6 - 2^(1/3)*(10
8 + (54 + 54*x)^2)^(1/3))*Sqrt[(1 + (28 + 54*x + 27*x^2)^(1/3) + (28 + 54*x
 + 27*x^2)^(2/3))/(6*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))^2
]*EllipticE[ArcSin[(6*(1 + Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))/
(6*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))], -7 + 4*Sqrt[3]])/
(72*Sqrt[2]*3^(1/4)*(1 + x)*Sqrt[-((6 - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3)
)/(6*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))^2)]) + ((6 - 2^(1
/3)*(108 + (54 + 54*x)^2)^(1/3))*Sqrt[(1 + (28 + 54*x + 27*x^2)^(1/3) + (28
 + 54*x + 27*x^2)^(2/3))/(6*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(
```

```
1/3))^2]*EllipticF[ArcSin[(6*(1 + Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^
(1/3))/(6*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))], -7 + 4*Sqr
t[3]])/(36*3^(3/4)*(1 + x)*Sqrt[-((6 - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))
/(6*(1 - Sqrt[3]) - 2^(1/3)*(108 + (54 + 54*x)^2)^(1/3))^2)]) - Log[2 + 3*x
]/(24*2^(2/3)) + Log[-108 - 81*x + 27*2^(1/3)*(28 + 54*x + 27*x^2)^(1/3)]/(
24*2^(2/3))
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
```

```
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 752

```
Int[1/(((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)), x_Sy
mbol] :> With[{q = Rt[-3*c*e^2*(2*c*d - b*e), 3]}, -Simp[(Sqrt[3]*c*e*ArcTa
n[1/Sqrt[3] - (2*(c*d - b*e - c*e*x))/(Sqrt[3]*q*(a + b*x + c*x^2)^(1/3))])
/q^2, x] + (-Simp[(3*c*e*Log[d + e*x])/(2*q^2), x] + Simp[(3*c*e*Log[c*d -
b*e - c*e*x + q*(a + b*x + c*x^2)^(1/3)])/(2*q^2), x])] /; FreeQ[{a, b, c,
d, e}, x] && NeQ[2*c*d - b*e, 0] && EqQ[c^2*d^2 - b*c*d*e + b^2*e^2 - 3*a*c
*e^2, 0] && NegQ[c*e^2*(2*c*d - b*e)]
```

Rubi steps

$$\int \frac{1}{(2+3x)^3\sqrt[3]{28+54x+27x^2}}\,dx = -\frac{\left(28+54x+27x^2\right)^{2/3}}{24(2+3x)^2} - \frac{1}{72}\int \frac{108+54x}{(2+3x)^2\sqrt[3]{28+54x+27x^2}}\,dx$$

$$= -\frac{\left(28+54x+27x^2\right)^{2/3}}{24(2+3x)^2} + \frac{\left(28+54x+27x^2\right)^{2/3}}{12(2+3x)} + \frac{\int \frac{-648-1944x}{(2+3x)\sqrt[3]{28+54x+27x^2}}\,dx}{2592}$$

$$= -\frac{\left(28+54x+27x^2\right)^{2/3}}{24(2+3x)^2} + \frac{\left(28+54x+27x^2\right)^{2/3}}{12(2+3x)} - \frac{1}{4}\int \frac{1}{\sqrt[3]{28+54x+27x^2}}\,dx + \frac{1}{4}$$

$$= -\frac{\left(28+54x+27x^2\right)^{2/3}}{24(2+3x)^2} + \frac{\left(28+54x+27x^2\right)^{2/3}}{12(2+3x)} - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2^{2/3}(4+3x)}{\sqrt{3}\sqrt[3]{28+54x+27x^2}}\right)}{12\ 2^{2/3}\sqrt{3}}$$

$$= -\frac{\left(28+54x+27x^2\right)^{2/3}}{24(2+3x)^2} + \frac{\left(28+54x+27x^2\right)^{2/3}}{12(2+3x)} - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2^{2/3}(4+3x)}{\sqrt{3}\sqrt[3]{28+54x+27x^2}}\right)}{12\ 2^{2/3}\sqrt{3}}$$

$$= -\frac{\left(28+54x+27x^2\right)^{2/3}}{24(2+3x)^2} + \frac{\left(28+54x+27x^2\right)^{2/3}}{12(2+3x)} - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2^{2/3}(4+3x)}{\sqrt{3}\sqrt[3]{28+54x+27x^2}}\right)}{12\ 2^{2/3}\sqrt{3}}$$

$$= -\frac{\left(28+54x+27x^2\right)^{2/3}}{24(2+3x)^2} + \frac{\left(28+54x+27x^2\right)^{2/3}}{12(2+3x)} + \frac{3(1+x)}{4\left(1-\sqrt{3}-\sqrt[3]{28+54x+27x^2}\right.}$$

Mathematica [C] time = 0.234778, size = 233, normalized size = 0.33

$$\frac{-54\sqrt[3]{3}\sqrt[3]{\frac{9x-i\sqrt{3}+9}{3x+2}}\sqrt[3]{\frac{9x+i\sqrt{3}+9}{3x+2}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};-\frac{3+i\sqrt{3}}{9x+6},\frac{-3+i\sqrt{3}}{9x+6}\right)+9i2^{2/3}3^{5/6}\sqrt[3]{-9ix+\sqrt{3}-9i}\left(9ix+\sqrt{3}+9i\right){}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};\frac{9ix-}{}\right.}{1296\sqrt[3]{27x^2+54x+28}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 + 3*x)^3*(28 + 54*x + 27*x^2)^(1/3)),x]
```

```
[Out] ((162*(1 + 2*x)*(28 + 54*x + 27*x^2))/(2 + 3*x)^2 - 54*3^(1/3)*((9 - I*Sqrt
[3] + 9*x)/(2 + 3*x))^(1/3)*((9 + I*Sqrt[3] + 9*x)/(2 + 3*x))^(1/3)*AppellF
1[2/3, 1/3, 1/3, 5/3, -((3 + I*Sqrt[3])/(6 + 9*x)), (-3 + I*Sqrt[3])/(6 + 9
*x)] + (9*I)*2^(2/3)*3^(5/6)*(-9*I + Sqrt[3] - (9*I)*x)^(1/3)*(9*I + Sqrt[3
] + (9*I)*x)*Hypergeometric2F1[1/3, 2/3, 5/3, (9*I + Sqrt[3] + (9*I)*x)/(2*
Sqrt[3])])/(1296*(28 + 54*x + 27*x^2)^(1/3))
```

Maple [F] time = 1.77, size = 0, normalized size = 0.

$$\int \frac{1}{\left(2+3x\right)^{3}}\frac{1}{\sqrt[3]{27x^{2}+54x+28}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*x)^3/(27*x^2+54*x+28)^(1/3),x)
```

```
[Out] int(1/(2+3*x)^3/(27*x^2+54*x+28)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(27x^{2}+54x+28\right)^{\frac{1}{3}}\left(3x+2\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)^3/(27*x^2+54*x+28)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((27*x^2 + 54*x + 28)^(1/3)*(3*x + 2)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(27x^{2}+54x+28\right)^{\frac{2}{3}}}{729x^{5}+2916x^{4}+4644x^{3}+3672x^{2}+1440x+224},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)^3/(27*x^2+54*x+28)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((27*x^2 + 54*x + 28)^(2/3)/(729*x^5 + 2916*x^4 + 4644*x^3 + 3672*x
^2 + 1440*x + 224), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x+2)^3\sqrt[3]{27x^2+54x+28}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)**3/(27*x**2+54*x+28)**(1/3),x)
```

```
[Out] Integral(1/((3*x + 2)**3*(27*x**2 + 54*x + 28)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(27\,x^2+54\,x+28\right)^{\frac{1}{3}}(3\,x+2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)^3/(27*x^2+54*x+28)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((27*x^2 + 54*x + 28)^(1/3)*(3*x + 2)^3), x)
```

3.2509 $\int \frac{1}{(d+ex)\sqrt[3]{-c^2d^2+bcde+2b^2e^2+9bce^2x+9c^2e^2x^2}}\,dx$

Optimal. Leaf size=564

$$-\frac{\log(d+ex)\sqrt[3]{3c^2e^2x-ce(cd-2be)}\sqrt[3]{ce(be+cd)+3c^2e^2x}}{2\sqrt[3]{2}c^{2/3}e^{5/3}(2cd-be)^{2/3}\sqrt[3]{-(cd-2be)(be+cd)+9bce^2x+9c^2e^2x^2}}+\frac{3\sqrt[3]{3c^2e^2x-ce(cd-2be)}\sqrt[3]{ce(be+cd)+3c^2e^2x}\log}{4\sqrt[3]{2}c^{2/3}e^{5/3}(2cd-be)^{2/3}\sqrt[3]{-(cd}$$

```
[Out] -(Sqrt[3]*(-(c*e*(c*d - 2*b*e)) + 3*c^2*e^2*x)^(1/3)*(c*e*(c*d + b*e) + 3*c
^2*e^2*x)^(1/3)*ArcTan[1/Sqrt[3] - (2^(1/3)*(-(c*e*(c*d - 2*b*e)) + 3*c^2*e
^2*x)^(2/3))/(Sqrt[3]*c^(1/3)*e^(1/3)*(2*c*d - b*e)^(1/3)*(c*e*(c*d + b*e)
+ 3*c^2*e^2*x)^(1/3))])/(2*2^(1/3)*c^(2/3)*e^(5/3)*(2*c*d - b*e)^(2/3)*(-((
c*d - 2*b*e)*(c*d + b*e)) + 9*b*c*e^2*x + 9*c^2*e^2*x^2)^(1/3)) - ((-(c*e*(
c*d - 2*b*e)) + 3*c^2*e^2*x)^(1/3)*(c*e*(c*d + b*e) + 3*c^2*e^2*x)^(1/3)*Lo
g[d + e*x])/(2*2^(1/3)*c^(2/3)*e^(5/3)*(2*c*d - b*e)^(2/3)*(-((c*d - 2*b*e)
*(c*d + b*e)) + 9*b*c*e^2*x + 9*c^2*e^2*x^2)^(1/3)) + (3*(-(c*e*(c*d - 2*b*
e)) + 3*c^2*e^2*x)^(1/3)*(c*e*(c*d + b*e) + 3*c^2*e^2*x)^(1/3)*Log[-(((3/2)
^(1/3)*(-(c*e*(c*d - 2*b*e)) + 3*c^2*e^2*x)^(2/3))/(c^(1/3)*e^(1/3)*(2*c*d
- b*e)^(1/3))) - 6^(1/3)*(c*e*(c*d + b*e) + 3*c^2*e^2*x)^(1/3)])/(4*2^(1/3)
*c^(2/3)*e^(5/3)*(2*c*d - b*e)^(2/3)*(-((c*d - 2*b*e)*(c*d + b*e)) + 9*b*c*
e^2*x + 9*c^2*e^2*x^2)^(1/3))
```

Rubi [A] time = 0.405284, antiderivative size = 564, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 53, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {755, 123}

$$-\frac{\log(d+ex)\sqrt[3]{3c^2e^2x-ce(cd-2be)}\sqrt[3]{ce(be+cd)+3c^2e^2x}}{2\sqrt[3]{2}c^{2/3}e^{5/3}(2cd-be)^{2/3}\sqrt[3]{-(cd-2be)(be+cd)+9bce^2x+9c^2e^2x^2}}+\frac{3\sqrt[3]{3c^2e^2x-ce(cd-2be)}\sqrt[3]{ce(be+cd)+3c^2e^2x}\log}{4\sqrt[3]{2}c^{2/3}e^{5/3}(2cd-be)^{2/3}\sqrt[3]{-(cd}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(-(c^2*d^2) + b*c*d*e + 2*b^2*e^2 + 9*b*c*e^2*x + 9*c^2*e^
2*x^2)^(1/3)),x]
```


```
[Out] -(Sqrt[3]*(-(c*e*(c*d - 2*b*e)) + 3*c^2*e^2*x)^(1/3)*(c*e*(c*d + b*e) + 3*c
^2*e^2*x)^(1/3)*ArcTan[1/Sqrt[3] - (2^(1/3)*(-(c*e*(c*d - 2*b*e)) + 3*c^2*e
^2*x)^(2/3))/(Sqrt[3]*c^(1/3)*e^(1/3)*(2*c*d - b*e)^(1/3)*(c*e*(c*d + b*e)
+ 3*c^2*e^2*x)^(1/3))])/(2*2^(1/3)*c^(2/3)*e^(5/3)*(2*c*d - b*e)^(2/3)*(-((
c*d - 2*b*e)*(c*d + b*e)) + 9*b*c*e^2*x + 9*c^2*e^2*x^2)^(1/3)) - ((-(c*e*(
c*d - 2*b*e)) + 3*c^2*e^2*x)^(1/3)*(c*e*(c*d + b*e) + 3*c^2*e^2*x)^(1/3)*Lo
g[d + e*x])/(2*2^(1/3)*c^(2/3)*e^(5/3)*(2*c*d - b*e)^(2/3)*(-((c*d - 2*b*e)
*(c*d + b*e)) + 9*b*c*e^2*x + 9*c^2*e^2*x^2)^(1/3)) + (3*(-(c*e*(c*d - 2*b*
e)) + 3*c^2*e^2*x)^(1/3)*(c*e*(c*d + b*e) + 3*c^2*e^2*x)^(1/3)*Log[-(((3/2)
^(1/3)*(-(c*e*(c*d - 2*b*e)) + 3*c^2*e^2*x)^(2/3))/(c^(1/3)*e^(1/3)*(2*c*d
- b*e)^(1/3))) - 6^(1/3)*(c*e*(c*d + b*e) + 3*c^2*e^2*x)^(1/3)])/(4*2^(1/3)
*c^(2/3)*e^(5/3)*(2*c*d - b*e)^(2/3)*(-((c*d - 2*b*e)*(c*d + b*e)) + 9*b*c*
e^2*x + 9*c^2*e^2*x^2)^(1/3))
```


Rule 755

```
Int[1/(((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)), x_Sy
mbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[((b + q + 2*c*x)^(1/3)*(b - q
+ 2*c*x)^(1/3))/(a + b*x + c*x^2)^(1/3), Int[1/((d + e*x)*(b + q + 2*c*x)^(
1/3)*(b - q + 2*c*x)^(1/3)), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^
```

```
2 - 4*a*c, 0] && EqQ[c^2*d^2 - b*c*d*e - 2*b^2*e^2 + 9*a*c*e^2, 0]
```

Rule 123

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)*((e_.) + (f_.)*(x_))
^(1/3)), x_Symbol] :> With[{q = Rt[(b*(b*e - a*f))/(b*c - a*d)^2, 3]}, -Sim
p[Log[a + b*x]/(2*q*(b*c - a*d)), x] + (-Simp[(Sqrt[3]*ArcTan[1/Sqrt[3] + (
2*q*(c + d*x)^(2/3))/(Sqrt[3]*(e + f*x)^(1/3))])/(2*q*(b*c - a*d)), x] + Si
mp[(3*Log[q*(c + d*x)^(2/3) - (e + f*x)^(1/3)])/(4*q*(b*c - a*d)), x])] /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - b*c*f - a*d*f, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\sqrt[3]{-c^2d^2+bcde+2b^2e^2+9bce^2x+9c^2e^2x^2}}\,dx = \frac{\left(\sqrt[3]{9bce^2-3ce(2cd-be)+18c^2e^2x}\sqrt[3]{9bce^2+3ce(2cd-be)\cdots}\right.}{\sqrt[3]{-c^2d^2+bcde\cdots}}$$

$$= -\frac{\sqrt{3}\sqrt[3]{-ce(cd-2be)+3c^2e^2x}\sqrt[3]{ce(cd+be)+3c^2e^2x}\tan^{-1}\cdots}{2\sqrt[3]{2}c^{2/3}e^{5/3}(2cd-be)^{2/3}\sqrt[3]{-(cd-2be)(cd+\cdots}}$$

Mathematica [C] time = 0.422211, size = 290, normalized size = 0.51

$$-\frac{\sqrt[3]{3}\sqrt[3]{\frac{-\sqrt{c^2e^2(be-2cd)^2}+3bce^2+6c^2e^2x}{c^2e(d+ex)}}\sqrt[3]{\frac{\sqrt{c^2e^2(be-2cd)^2}+3bce^2+6c^2e^2x}{c^2e(d+ex)}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};-\frac{-6dec^2+3be^2c+\sqrt{c^2e^2(be-2cd)^2}}{6c^2e(d+ex)},\frac{6dec^2-3be^2c+\sqrt{c^2e^2(be-2\cdots}}{6c^2e(d+ex)}\cdots\right)}{2\,2^{2/3}e\sqrt[3]{2b^2e^2+bce(d+9ex)+c^2\left(-\left(d^2-9e^2x^2\right)\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x)*(-(c^2*d^2) + b*c*d*e + 2*b^2*e^2 + 9*b*c*e^2*x + 9*
c^2*e^2*x^2)^(1/3)),x]
```

```
[Out] -(3^(1/3)*((3*b*c*e^2 - Sqrt[c^2*e^2*(-2*c*d + b*e)^2] + 6*c^2*e^2*x)/(c^2*
e*(d + e*x)))^(1/3)*((3*b*c*e^2 + Sqrt[c^2*e^2*(-2*c*d + b*e)^2] + 6*c^2*e^
2*x)/(c^2*e*(d + e*x)))^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, -(-6*c^2*d*e + 3
*b*c*e^2 + Sqrt[c^2*e^2*(-2*c*d + b*e)^2])/(6*c^2*e*(d + e*x)), (6*c^2*d*e
- 3*b*c*e^2 + Sqrt[c^2*e^2*(-2*c*d + b*e)^2])/(6*c^2*e*(d + e*x))])/(2*2^(2
/3)*e*(2*b^2*e^2 + b*c*e*(d + 9*e*x) - c^2*(d^2 - 9*e^2*x^2))^(1/3))
```

Maple [F] time = 1.313, size = 0, normalized size = 0.

$$\int \frac{1}{ex+d}\frac{1}{\sqrt[3]{9\,c^2e^2x^2+9\,bce^2x+2\,b^2e^2+bcde-c^2d^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(9*c^2*e^2*x^2+9*b*c*e^2*x+2*b^2*e^2+b*c*d*e-c^2*d^2)^(1/3),x
)
```

```
[Out] int(1/(e*x+d)/(9*c^2*e^2*x^2+9*b*c*e^2*x+2*b^2*e^2+b*c*d*e-c^2*d^2)^(1/3),x
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(9\,c^2e^2x^2+9\,bce^2x-c^2d^2+bcde+2\,b^2e^2\right)^{\frac{1}{3}}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(9*c^2*e^2*x^2+9*b*c*e^2*x+2*b^2*e^2+b*c*d*e-c^2*d^2)^(
1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((9*c^2*e^2*x^2 + 9*b*c*e^2*x - c^2*d^2 + b*c*d*e + 2*b^2*e^2)^
(1/3)*(e*x + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(9*c^2*e^2*x^2+9*b*c*e^2*x+2*b^2*e^2+b*c*d*e-c^2*d^2)^(
1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{(be+cd+3cex)\,(2be-cd+3cex)}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(9*c**2*e**2*x**2+9*b*c*e**2*x+2*b**2*e**2+b*c*d*e-c**2
*d**2)**(1/3),x)
```

```
[Out] Integral(1/(((b*e + c*d + 3*c*e*x)*(2*b*e - c*d + 3*c*e*x))**(1/3)*(d + e*x
)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(9\,c^2e^2x^2+9\,bce^2x-c^2d^2+bcde+2\,b^2e^2\right)^{\frac{1}{3}}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(9*c^2*e^2*x^2+9*b*c*e^2*x+2*b^2*e^2+b*c*d*e-c^2*d^2)^(
1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((9*c^2*e^2*x^2 + 9*b*c*e^2*x - c^2*d^2 + b*c*d*e + 2*b^2*e^2)^
(1/3)*(e*x + d)), x)
```

3.2510 $\int (d+ex)^3 \sqrt[4]{a+bx+cx^2}\,dx$

Optimal. Leaf size=374

$$-\frac{\left(b^2-4ac\right)^{5/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)(2cd-be)\left(-4ce(6ae+7bd)+13b^2e^2+28c^2d^2\right)\text{EllipticF}\left(\right.}{336\sqrt{2}c^{17/4}(b+2cx)}$$

```
[Out] ((2*c*d - b*e)*(28*c^2*d^2 + 13*b^2*e^2 - 4*c*e*(7*b*d + 6*a*e))*(b + 2*c*x
)*(a + b*x + c*x^2)^(1/4))/(168*c^4) + (2*e*(d + e*x)^2*(a + b*x + c*x^2)^(
5/4))/(9*c) + (e*(616*c^2*d^2 + 117*b^2*e^2 - 2*c*e*(243*b*d + 56*a*e) + 13
0*c*e*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(5/4))/(630*c^3) - ((b^2 - 4*a*c)^
(5/4)*(2*c*d - b*e)*(28*c^2*d^2 + 13*b^2*e^2 - 4*c*e*(7*b*d + 6*a*e))*Sqrt[
(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^
2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*E
llipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(
1/4)], 1/2])/(336*Sqrt[2]*c^(17/4)*(b + 2*c*x))
```

Rubi [A] time = 0.518445, antiderivative size = 374, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {742, 779, 612, 623, 220}

$$\frac{e\left(a+bx+cx^2\right)^{5/4}\left(-2ce(56ae+243bd)+117b^2e^2+130cex(2cd-be)+616c^2d^2\right)}{630c^3}+\frac{(b+2cx)\sqrt[4]{a+bx+cx^2}(2cd-be)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*x + c*x^2)^(1/4),x]
```


```
[Out] ((2*c*d - b*e)*(28*c^2*d^2 + 13*b^2*e^2 - 4*c*e*(7*b*d + 6*a*e))*(b + 2*c*x
)*(a + b*x + c*x^2)^(1/4))/(168*c^4) + (2*e*(d + e*x)^2*(a + b*x + c*x^2)^(
5/4))/(9*c) + (e*(616*c^2*d^2 + 117*b^2*e^2 - 2*c*e*(243*b*d + 56*a*e) + 13
0*c*e*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(5/4))/(630*c^3) - ((b^2 - 4*a*c)^
(5/4)*(2*c*d - b*e)*(28*c^2*d^2 + 13*b^2*e^2 - 4*c*e*(7*b*d + 6*a*e))*Sqrt[
(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^
2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*E
llipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(
1/4)], 1/2])/(336*Sqrt[2]*c^(17/4)*(b + 2*c*x))
```


Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
```

```
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int (d+ex)^3 \sqrt[4]{a+bx+cx^2}\,dx = \frac{2e(d+ex)^2\left(a+bx+cx^2\right)^{5/4}}{9c} + \frac{2\int (d+ex)\left(\frac{1}{4}\left(18cd^2-5bde-8ae^2\right)+\frac{13}{4}e(2cd-be)x\right)\sqrt[4]{}}{9c}$$

$$= \frac{2e(d+ex)^2\left(a+bx+cx^2\right)^{5/4}}{9c} + \frac{e\left(616c^2d^2+117b^2e^2-2ce(243bd+56ae)+130ce(2cd-\right.}{630c^3}$$

$$= \frac{(2cd-be)\left(28c^2d^2+13b^2e^2-4ce(7bd+6ae)\right)(b+2cx)\sqrt[4]{a+bx+cx^2}}{168c^4} + \frac{2e(d+ex)^2\left(a+\right.}{9c}$$

$$= \frac{(2cd-be)\left(28c^2d^2+13b^2e^2-4ce(7bd+6ae)\right)(b+2cx)\sqrt[4]{a+bx+cx^2}}{168c^4} + \frac{2e(d+ex)^2\left(a+\right.}{9c}$$

$$= \frac{(2cd-be)\left(28c^2d^2+13b^2e^2-4ce(7bd+6ae)\right)(b+2cx)\sqrt[4]{a+bx+cx^2}}{168c^4} + \frac{2e(d+ex)^2\left(a+\right.}{9c}$$

Mathematica [A] time = 0.924641, size = 235, normalized size = 0.63

$$15(2cd-be)\left(-4ce(6ae+7bd)+13b^2e^2+28c^2d^2\right)\left(2c(b+2cx)(a+x(b+cx))-\sqrt{2}\left(b^2-4ac\right)^{3/2}\left(\frac{c(a+x(b+cx))}{4ac-b^2}\right)^{3/4}\text{Elliptic}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*x + c*x^2)^(1/4),x]
```

```
[Out] (1120*c^4*e*(d + e*x)^2*(a + x*(b + c*x))^2 + 8*c^2*e*(a + x*(b + c*x))^2*(
117*b^2*e^2 + 4*c^2*d*(154*d + 65*e*x) - 2*c*e*(243*b*d + 56*a*e + 65*b*e*x
)) + 15*(2*c*d - b*e)*(28*c^2*d^2 + 13*b^2*e^2 - 4*c*e*(7*b*d + 6*a*e))*(2*
```

```
c*(b + 2*c*x)*(a + x*(b + c*x)) - Sqrt[2]*(b^2 - 4*a*c)^(3/2)*((c*(a + x*(b
 + c*x)))/(-b^2 + 4*a*c))^(3/4)*EllipticF[ArcSin[(b + 2*c*x)/Sqrt[b^2 - 4*a
*c]]/2, 2]))/(5040*c^5*(a + x*(b + c*x))^(3/4))
```

Maple [F] time = 1.073, size = 0, normalized size = 0.

$$\int (ex + d)^3 \sqrt[4]{cx^2 + bx + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x+a)^(1/4),x)
```

```
[Out] int((e*x+d)^3*(c*x^2+b*x+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx + a\right)^{\frac{1}{4}} (ex + d)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(1/4)*(e*x + d)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3\right)\left(cx^2 + bx + a\right)^{\frac{1}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(c*x^2 + b*x + a)^(1/4),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d + ex)^3 \sqrt[4]{a + bx + cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x+a)**(1/4),x)
```

```
[Out] Integral((d + e*x)**3*(a + b*x + c*x**2)**(1/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx + a\right)^{\frac{1}{4}} (ex + d)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(1/4)*(e*x + d)^3, x)
```

3.2511 $\int (d+ex)^2 \sqrt[4]{a+bx+cx^2}\,dx$

Optimal. Leaf size=319

$$-\frac{\left(b^2-4ac\right)^{5/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)\left(-4ce(2ae+7bd)+9b^2e^2+28c^2d^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt{}}{}\right.\right.}{168\sqrt{2}c^{13/4}(b+2cx)}$$

```
[Out] ((28*c^2*d^2 + 9*b^2*e^2 - 4*c*e*(7*b*d + 2*a*e))*(b + 2*c*x)*(a + b*x + c*
x^2)^(1/4))/(84*c^3) + (9*e*(2*c*d - b*e)*(a + b*x + c*x^2)^(5/4))/(35*c^2)
 + (2*e*(d + e*x)*(a + b*x + c*x^2)^(5/4))/(7*c) - ((b^2 - 4*a*c)^(5/4)*(28
*c^2*d^2 + 9*b^2*e^2 - 4*c*e*(7*b*d + 2*a*e))*Sqrt[(b + 2*c*x)^2/((b^2 - 4*
a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*
Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[
2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(168*Sqrt[2
]*c^(13/4)*(b + 2*c*x))
```

Rubi [A] time = 0.423134, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {742, 640, 612, 623, 220}

$$\frac{(b+2cx)\sqrt[4]{a+bx+cx^2}\left(-4ce(2ae+7bd)+9b^2e^2+28c^2d^2\right)}{84c^3}-\frac{\left(b^2-4ac\right)^{5/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+\right.}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*x + c*x^2)^(1/4),x]
```

```
[Out] ((28*c^2*d^2 + 9*b^2*e^2 - 4*c*e*(7*b*d + 2*a*e))*(b + 2*c*x)*(a + b*x + c*
x^2)^(1/4))/(84*c^3) + (9*e*(2*c*d - b*e)*(a + b*x + c*x^2)^(5/4))/(35*c^2)
 + (2*e*(d + e*x)*(a + b*x + c*x^2)^(5/4))/(7*c) - ((b^2 - 4*a*c)^(5/4)*(28
*c^2*d^2 + 9*b^2*e^2 - 4*c*e*(7*b*d + 2*a*e))*Sqrt[(b + 2*c*x)^2/((b^2 - 4*
a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*
Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[
2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(168*Sqrt[2
]*c^(13/4)*(b + 2*c*x))
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int (d+ex)^2 \sqrt[4]{a+bx+cx^2}\,dx = \frac{2e(d+ex)\left(a+bx+cx^2\right)^{5/4}}{7c} + \frac{2\int \left(\frac{1}{4}\left(14cd^2-4e\left(\frac{5bd}{4}+ae\right)\right)+\frac{9}{4}e(2cd-be)x\right)\sqrt[4]{a+bx+\cdots}}{7c}$$

$$= \frac{9e(2cd-be)\left(a+bx+cx^2\right)^{5/4}}{35c^2} + \frac{2e(d+ex)\left(a+bx+cx^2\right)^{5/4}}{7c} + \frac{\left(-\frac{9}{4}be(2cd-be)+\frac{1}{2}c\left(14\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(28c^2d^2+9b^2e^2-4ce(7bd+2ae)\right)(b+2cx)\sqrt[4]{a+bx+cx^2}}{84c^3} + \frac{9e(2cd-be)\left(a+bx+cx^2\right)^{5\cdots}}{35c^2}$$

$$= \frac{\left(28c^2d^2+9b^2e^2-4ce(7bd+2ae)\right)(b+2cx)\sqrt[4]{a+bx+cx^2}}{84c^3} + \frac{9e(2cd-be)\left(a+bx+cx^2\right)^{5\cdots}}{35c^2}$$

$$= \frac{\left(28c^2d^2+9b^2e^2-4ce(7bd+2ae)\right)(b+2cx)\sqrt[4]{a+bx+cx^2}}{84c^3} + \frac{9e(2cd-be)\left(a+bx+cx^2\right)^{5\cdots}}{35c^2}$$

Mathematica [A] time = 0.459999, size = 191, normalized size = 0.6

$$\frac{\frac{5\left(-4ce(2ae+7bd)+9b^2e^2+28c^2d^2\right)\left(2c(b+2cx)(a+x(b+cx))-\sqrt{2}\left(b^2-4ac\right)^{3/2}\left(\frac{c(a+x(b+cx))}{4ac-b^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right),2\right)\right)}{4c^2} - 54e(a+x(b+cx))^2(be-2c\cdots}{210c^2(a+x(b+cx))^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*x + c*x^2)^(1/4),x]
```

```
[Out] (-54*e*(-2*c*d + b*e)*(a + x*(b + c*x))^2 + 60*c*e*(d + e*x)*(a + x*(b + c*
x))^2 + (5*(28*c^2*d^2 + 9*b^2*e^2 - 4*c*e*(7*b*d + 2*a*e))*(2*c*(b + 2*c*x
)*(a + x*(b + c*x)) - Sqrt[2]*(b^2 - 4*a*c)^(3/2)*((c*(a + x*(b + c*x)))/(-
b^2 + 4*a*c))^(3/4)*EllipticF[ArcSin[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]/2, 2]))
/(4*c^2))/(210*c^2*(a + x*(b + c*x))^(3/4))
```

Maple [F] time = 0.978, size = 0, normalized size = 0.

$$\int (ex+d)^2 \sqrt[4]{cx^2+bx+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+b*x+a)^(1/4),x)
```

```
[Out] int((e*x+d)^2*(c*x^2+b*x+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{\frac{1}{4}}(ex+d)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(1/4)*(e*x + d)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^2+2\,dex+d^2\right)\left(cx^2+bx+a\right)^{\frac{1}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(c*x^2 + b*x + a)^(1/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d+ex)^2 \sqrt[4]{a+bx+cx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+b*x+a)**(1/4),x)
```

```
[Out] Integral((d + e*x)**2*(a + b*x + c*x**2)**(1/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{\frac{1}{4}}(ex+d)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(1/4)*(e*x + d)^2, x)
```

3.2512 $\int (d+ex)\sqrt[4]{a+bx+cx^2}\,dx$

Optimal. Leaf size=241

$$-\frac{\left(b^2-4ac\right)^{5/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)(2cd-be)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{a+bx+cx^2}}{\sqrt[4]{b^2-4ac}}\right),\frac{1}{2}\right)}{12\sqrt{2}c^{9/4}(b+2cx)}+\frac{(b+2}{}$$

```
[Out] ((2*c*d - b*e)*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4))/(6*c^2) + (2*e*(a + b*x
 + c*x^2)^(5/4))/(5*c) - ((b^2 - 4*a*c)^(5/4)*(2*c*d - b*e)*Sqrt[(b + 2*c*x
)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c]
)^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2
*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2
])/(12*Sqrt[2]*c^(9/4)*(b + 2*c*x))
```

Rubi [A] time = 0.183802, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {640, 612, 623, 220}

$$-\frac{\left(b^2-4ac\right)^{5/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)(2cd-be)F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{cx^2+bx+a}}{\sqrt[4]{b^2-4ac}}\right)|\frac{1}{2}\right)}{12\sqrt{2}c^{9/4}(b+2cx)}+\frac{(b+2cx)\sqrt[4]{a+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*x + c*x^2)^(1/4),x]
```

```
[Out] ((2*c*d - b*e)*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4))/(6*c^2) + (2*e*(a + b*x
 + c*x^2)^(5/4))/(5*c) - ((b^2 - 4*a*c)^(5/4)*(2*c*d - b*e)*Sqrt[(b + 2*c*x
)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c]
)^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2
*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2
])/(12*Sqrt[2]*c^(9/4)*(b + 2*c*x))
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int (d+ex)\sqrt[4]{a+bx+cx^2}\,dx = \frac{2e\left(a+bx+cx^2\right)^{5/4}}{5c} + \frac{(2cd-be)\int \sqrt[4]{a+bx+cx^2}\,dx}{2c}$$

$$= \frac{(2cd-be)(b+2cx)\sqrt[4]{a+bx+cx^2}}{6c^2} + \frac{2e\left(a+bx+cx^2\right)^{5/4}}{5c} - \frac{\left(\left(b^2-4ac\right)(2cd-be)\right)\int \frac{}{(a+bx+}}{24c^2}$$

$$= \frac{(2cd-be)(b+2cx)\sqrt[4]{a+bx+cx^2}}{6c^2} + \frac{2e\left(a+bx+cx^2\right)^{5/4}}{5c} - \frac{\left(\left(b^2-4ac\right)(2cd-be)\sqrt{(b+2c}\right.}{}$$

$$= \frac{(2cd-be)(b+2cx)\sqrt[4]{a+bx+cx^2}}{6c^2} + \frac{2e\left(a+bx+cx^2\right)^{5/4}}{5c} - \frac{\left(b^2-4ac\right)^{5/4}(2cd-be)\sqrt{\frac{}{(b^2-4a}}}{}$$

Mathematica [A] time = 0.258339, size = 140, normalized size = 0.58

$$\frac{5(2cd-be)\left(2c(b+2cx)(a+x(b+cx))-\sqrt{2}\left(b^2-4ac\right)^{3/2}\left(\frac{c(a+x(b+cx))}{4ac-b^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right),2\right)\right)+24c^2e(a+x}{60c^3(a+x(b+cx))^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*x + c*x^2)^(1/4),x]
```

```
[Out] (24*c^2*e*(a + x*(b + c*x))^2 + 5*(2*c*d - b*e)*(2*c*(b + 2*c*x)*(a + x*(b
+ c*x)) - Sqrt[2]*(b^2 - 4*a*c)^(3/2)*((c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)
)^(3/4)*EllipticF[ArcSin[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]/2, 2]))/(60*c^3*(a
+ x*(b + c*x))^(3/4))
```

Maple [F] time = 0.99, size = 0, normalized size = 0.

$$\int (ex+d)\sqrt[4]{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+b*x+a)^(1/4),x)
```

```
[Out] int((e*x+d)*(c*x^2+b*x+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{\frac{1}{4}}(ex+d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(1/4)*(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2+bx+a\right)^{\frac{1}{4}}(ex+d), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^(1/4)*(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d+ex)\sqrt[4]{a+bx+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+b*x+a)**(1/4),x)
```

```
[Out] Integral((d + e*x)*(a + b*x + c*x**2)**(1/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{\frac{1}{4}}(ex+d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(1/4)*(e*x + d), x)
```

3.2513 $\int \sqrt[4]{a+bx+cx^2}\,dx$

Optimal. Leaf size=201

$$\frac{(b+2cx)\sqrt[4]{a+bx+cx^2}}{3c}-\frac{\left(b^2-4ac\right)^{5/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{a+bx+cx^2}}{\sqrt[4]{b^2-4ac}}\right),\right.}{6\sqrt{2}c^{5/4}(b+2cx)}$$

```
[Out] ((b + 2*c*x)*(a + b*x + c*x^2)^(1/4))/(3*c) - ((b^2 - 4*a*c)^(5/4)*Sqrt[(b
+ 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 -
 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*Elli
pticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4
)], 1/2])/(6*Sqrt[2]*c^(5/4)*(b + 2*c*x))
```

Rubi [A] time = 0.131318, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {612, 623, 220}

$$\frac{(b+2cx)\sqrt[4]{a+bx+cx^2}}{3c}-\frac{\left(b^2-4ac\right)^{5/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{cx^2+bx+a}}{\sqrt[4]{b^2-4ac}}\right)|\frac{1}{2}\right)}{6\sqrt{2}c^{5/4}(b+2cx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(1/4),x]
```

```
[Out] ((b + 2*c*x)*(a + b*x + c*x^2)^(1/4))/(3*c) - ((b^2 - 4*a*c)^(5/4)*Sqrt[(b
+ 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 -
 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*Elli
pticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4
)], 1/2])/(6*Sqrt[2]*c^(5/4)*(b + 2*c*x))
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \sqrt[4]{a+bx+cx^2}\,dx = \frac{(b+2cx)\sqrt[4]{a+bx+cx^2}}{3c} - \frac{\left(b^2-4ac\right)\int \frac{1}{\left(a+bx+cx^2\right)^{3/4}}\,dx}{12c}$$
$$= \frac{(b+2cx)\sqrt[4]{a+bx+cx^2}}{3c} - \frac{\left(\left(b^2-4ac\right)\sqrt{(b+2cx)^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b^2-4ac+4cx^4}}\,dx, x, \sqrt[4]{a+bx+cx^2}\right)}{3c(b+2cx)}$$
$$= \frac{(b+2cx)\sqrt[4]{a+bx+cx^2}}{3c} - \frac{\left(b^2-4ac\right)^{5/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)F\left(2\tan^{-1}\right.}{6\sqrt{2}c^{5/4}(b+2cx)}$$

Mathematica [A] time = 0.242979, size = 100, normalized size = 0.5

$$\frac{\sqrt[4]{a+x(b+cx)}\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\,\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right),2\right)}{\sqrt[4]{\frac{c(a+x(b+cx))}{4ac-b^2}}}+2(b+2cx)\right)}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(1/4),x]
```

```
[Out] ((a + x*(b + c*x))^(1/4)*(2*(b + 2*c*x) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*Ellipt
icF[ArcSin[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]/2, 2])/((c*(a + x*(b + c*x)))/(-b
^2 + 4*a*c))^(1/4)))/(6*c)
```

Maple [F] time = 2.113, size = 0, normalized size = 0.

$$\int \sqrt[4]{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/4),x)
```

```
[Out] int((c*x^2+b*x+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2+bx+a\right)^{\frac{1}{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^(1/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[4]{a+bx+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/4),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(1/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(1/4), x)
```

3.2514 $\int \frac{\sqrt[4]{a+bx+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=881

$$-\frac{\left(4ac-b^2\right)^{3/4}\sqrt[4]{cd^2-bed+ae^2}\left(-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{4ac-b^2}\sqrt{e}\sqrt[4]{1-\frac{(b+2cx)^2}{b^2-4ac}}}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{cd^2-bed+ae^2}}\right)}{c^{3/4}e^{3/2}\left(cx^2+bx+a\right)^{3/4}}-\frac{\left(4ac-b^2\right)^{3/4}\sqrt[4]{cd^2-bed+ae^2}\left(-\frac{c(cx^2}{b^2}\right.}{c^{3/4}e^{3/2}\left(cx\right.}$$

```
[Out] (2*(a + b*x + c*x^2)^(1/4))/e - ((-b^2 + 4*a*c)^(3/4)*(c*d^2 - b*d*e + a*e^
2)^(1/4)*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)*ArcTan[((-b^2 + 4*a
*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)
*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(c^(3/4)*e^(3/2)*(a + b*x + c*x^2)^(3/4))
 - ((-b^2 + 4*a*c)^(3/4)*(c*d^2 - b*d*e + a*e^2)^(1/4)*(-((c*(a + b*x + c*x
^2))/(b^2 - 4*a*c)))^(3/4)*ArcTanh[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b +
2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/
4))])/(c^(3/4)*e^(3/2)*(a + b*x + c*x^2)^(3/4)) - ((b^2 - 4*a*c)^(1/4)*(2*c
*d - b*e)*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x +
c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[
b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/
(b^2 - 4*a*c)^(1/4)], 1/2])/(Sqrt[2]*c^(1/4)*e^2*(b + 2*c*x)) - ((b^2 - 4*a
*c)*(2*c*d - b*e)*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c)))^(3/4)*EllipticPi[-(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[
c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -
1])/(Sqrt[2]*c*e^2*(b + 2*c*x)*(a + b*x + c*x^2)^(3/4)) - ((b^2 - 4*a*c)*(2
*c*d - b*e)*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2
 - 4*a*c)))^(3/4)*EllipticPi[(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 -
 b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(Sq
rt[2]*c*e^2*(b + 2*c*x)*(a + b*x + c*x^2)^(3/4))
```

Rubi [A] time = 2.63761, antiderivative size = 881, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.773, Rules used = {734, 843, 623, 220, 749, 748, 747, 401, 108, 409, 1213, 537, 444, 63, 212, 208, 205}

$$-\frac{\left(4ac-b^2\right)^{3/4}\sqrt[4]{cd^2-bed+ae^2}\left(-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{4ac-b^2}\sqrt{e}\sqrt[4]{1-\frac{(b+2cx)^2}{b^2-4ac}}}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{cd^2-bed+ae^2}}\right)}{c^{3/4}e^{3/2}\left(cx^2+bx+a\right)^{3/4}}-\frac{\left(4ac-b^2\right)^{3/4}\sqrt[4]{cd^2-bed+ae^2}\left(-\frac{c(cx^2}{b^2}\right.}{c^{3/4}e^{3/2}\left(cx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(1/4)/(d + e*x),x]
```

```
[Out] (2*(a + b*x + c*x^2)^(1/4))/e - ((-b^2 + 4*a*c)^(3/4)*(c*d^2 - b*d*e + a*e^
2)^(1/4)*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)*ArcTan[((-b^2 + 4*a
*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)
*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(c^(3/4)*e^(3/2)*(a + b*x + c*x^2)^(3/4))
 - ((-b^2 + 4*a*c)^(3/4)*(c*d^2 - b*d*e + a*e^2)^(1/4)*(-((c*(a + b*x + c*x
^2))/(b^2 - 4*a*c)))^(3/4)*ArcTanh[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b +
2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/
4))])/(c^(3/4)*e^(3/2)*(a + b*x + c*x^2)^(3/4)) - ((b^2 - 4*a*c)^(1/4)*(2*c
*d - b*e)*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x +
c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[
b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/
```

```
(b^2 - 4*a*c)^(1/4)], 1/2])/(Sqrt[2]*c^(1/4)*e^2*(b + 2*c*x)) - ((b^2 - 4*a
*c)*(2*c*d - b*e)*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c)))^(3/4)*EllipticPi[-(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[
c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -
1])/(Sqrt[2]*c*e^2*(b + 2*c*x)*(a + b*x + c*x^2)^(3/4)) - ((b^2 - 4*a*c)*(2
*c*d - b*e)*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2
 - 4*a*c)))^(3/4)*EllipticPi[(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 -
 b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(Sq
rt[2]*c*e^2*(b + 2*c*x)*(a + b*x + c*x^2)^(3/4))
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 749

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[(a + b*x + c*x^2)^p/(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^p, I
nt[(-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) - (c^2*x^2)/(b^2 - 4*a*c
))^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&  !GtQ[4*a - b^2/c
, 0] && IntegerQ[4*p]
```

Rule 748

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[1/((-4*c)/(b^2 - 4*a*c))^p, Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c),
 x]^p/Simp[2*c*d - b*e + e*x, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && GtQ[4*a - b^2/c, 0] && IntegerQ[4*p]
```

Rule 747

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
```

```
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[-((b*x^2)/a)]/(2*x), Subst[Int[1/(Sqrt[-((b*x)/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 108

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - (d*e
)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx+cx^2}}{d+ex}\,dx = \frac{2\sqrt[4]{a+bx+cx^2}}{e} - \frac{\int \frac{bd-2ae+(2cd-be)x}{(d+ex)\left(a+bx+cx^2\right)^{3/4}}\,dx}{2e}$$

$$= \frac{2\sqrt[4]{a+bx+cx^2}}{e} - \frac{(2cd-be)\int \frac{1}{\left(a+bx+cx^2\right)^{3/4}}\,dx}{2e^2} - \frac{(e(bd-2ae)-d(2cd-be))\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/4}}}{2e^2}$$

$$= \frac{2\sqrt[4]{a+bx+cx^2}}{e} - \frac{\left(2(2cd-be)\sqrt{(b+2cx)^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{b^2-4ac+4cx^4}}\,dx, x, \sqrt[4]{a+bx+cx^2}\right)}{e^2(b+2cx)} - \frac{\left((e\right.}{}$$

$$= \frac{2\sqrt[4]{a+bx+cx^2}}{e} - \frac{\sqrt[4]{b^2-4ac}(2cd-be)\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)F\left(2\tan^{-1}\right.}{\sqrt{2}\sqrt[4]{c}e^2(b+2cx)}$$

$$= \frac{2\sqrt[4]{a+bx+cx^2}}{e} - \frac{\sqrt[4]{b^2-4ac}(2cd-be)\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)F\left(2\tan^{-1}\right.}{\sqrt{2}\sqrt[4]{c}e^2(b+2cx)}$$

$$= \frac{2\sqrt[4]{a+bx+cx^2}}{e} - \frac{\sqrt[4]{b^2-4ac}(2cd-be)\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)F\left(2\tan^{-1}\right.}{\sqrt{2}\sqrt[4]{c}e^2(b+2cx)}$$

$$= \frac{2\sqrt[4]{a+bx+cx^2}}{e} - \frac{\sqrt[4]{b^2-4ac}(2cd-be)\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)F\left(2\tan^{-1}\right.}{\sqrt{2}\sqrt[4]{c}e^2(b+2cx)}$$

$$= \frac{2\sqrt[4]{a+bx+cx^2}}{e} - \frac{\sqrt[4]{b^2-4ac}(2cd-be)\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)F\left(2\tan^{-1}\right.}{\sqrt{2}\sqrt[4]{c}e^2(b+2cx)}$$

$$= \frac{2\sqrt[4]{a+bx+cx^2}}{e} - \frac{\left(-b^2+4ac\right)^{3/4}\sqrt[4]{cd^2-bde+ae^2}\left(-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{-b^2+4ac}\sqrt{e}\sqrt[4]{1-\frac{(b+2c}{b^2-4}}}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{cd^2-bde+ae^2}}\right.}{c^{3/4}e^{3/2}\left(a+bx+cx^2\right)^{3/4}}$$

$$= \frac{2\sqrt[4]{a+bx+cx^2}}{e} - \frac{\left(-b^2+4ac\right)^{3/4}\sqrt[4]{cd^2-bde+ae^2}\left(-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{-b^2+4ac}\sqrt{e}\sqrt[4]{1-\frac{(b+2c}{b^2-4}}}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{cd^2-bde+ae^2}}\right.}{c^{3/4}e^{3/2}\left(a+bx+cx^2\right)^{3/4}}$$

Mathematica [A] time = 2.27345, size = 617, normalized size = 0.7

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\left(\frac{c(a+x(b+cx))}{4ac-b^2}\right)^{3/4}(be-2cd)\operatorname{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right),2\right)}{c} + \frac{\left(4ac-b^2\right)^{3/4}\left(\frac{c(a+x(b+cx))}{4ac-b^2}\right)^{3/4}\left(-\sqrt{2}\sqrt[4]{c}\sqrt{e}(b+2cx)\sqrt[4]{e(ae-bd)+cd^2}\left(\tan^{-1}\left(\frac{\sqrt{e}\sqrt[4]{4ac-b^2}\sqrt[4]{}}{\sqrt[4]{c}\sqrt[4]{e(ae}}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(1/4)/(d + e*x),x]
```

```
[Out] (2*e*(a + x*(b + c*x)) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(-2*c*d + b*e)*((c*(a +
 x*(b + c*x)))/(-b^2 + 4*a*c))^(3/4)*EllipticF[ArcSin[(b + 2*c*x)/Sqrt[b^2
- 4*a*c]]/2, 2])/c + ((-b^2 + 4*a*c)^(3/4)*((c*(a + x*(b + c*x)))/(-b^2 + 4
*a*c))^(3/4)*(-(Sqrt[2]*c^(1/4)*Sqrt[e]*(c*d^2 + e*(-(b*d) + a*e))^(1/4)*(b
 + 2*c*x)*(ArcTan[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*((c*(a + x*(b + c*x)))/(-b^
2 + 4*a*c))^(1/4))/(c^(1/4)*(c*d^2 + e*(-(b*d) + a*e))^(1/4))] + ArcTanh[((
-b^2 + 4*a*c)^(1/4)*Sqrt[e]*((c*(a + x*(b + c*x)))/(-b^2 + 4*a*c))^(1/4))/(
c^(1/4)*(c*d^2 + e*(-(b*d) + a*e))^(1/4))])) + (-b^2 + 4*a*c)^(1/4)*(-2*c*d
 + b*e)*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*EllipticPi[-(Sqrt[-b^2 + 4*a*c]*e
)/(2*Sqrt[c]*Sqrt[c*d^2 + e*(-(b*d) + a*e)]), -ArcSin[Sqrt[2]*((c*(a + x*(b
 + c*x)))/(-b^2 + 4*a*c))^(1/4)], -1] + (-b^2 + 4*a*c)^(1/4)*(-2*c*d + b*e)
*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*EllipticPi[(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqr
t[c]*Sqrt[c*d^2 + e*(-(b*d) + a*e)]), -ArcSin[Sqrt[2]*((c*(a + x*(b + c*x))
)/(-b^2 + 4*a*c))^(1/4)], -1]))/(Sqrt[2]*c*(b + 2*c*x)))/(e^2*(a + x*(b + c
*x))^(3/4))
```

Maple [F] time = 1.296, size = 0, normalized size = 0.

$$\int \frac{1}{ex+d}\sqrt[4]{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/4)/(e*x+d),x)
```

```
[Out] int((c*x^2+b*x+a)^(1/4)/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{1}{4}}}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/4)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(1/4)/(e*x + d), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/4)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[4]{a + bx + cx^2}}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/4)/(e*x+d),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(1/4)/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)^{\frac{1}{4}}}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/4)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(1/4)/(e*x + d), x)
```

3.2515 $\int \frac{\sqrt[4]{a+bx+cx^2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=944

$$\frac{\left(b^2-4ac\right)\sqrt{\frac{(b+2cx)^2}{b^2-4ac}}\left(-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}\right)^{3/4}\Pi\left(-\frac{\sqrt{4ac-b^2}e}{2\sqrt{c}\sqrt{cd^2-bed+ae^2}};\sin^{-1}\left(\sqrt[4]{1-\frac{(b+2cx)^2}{b^2-4ac}}\right)\middle|-1\right)(2cd-be)^2}{4\sqrt{2}ce^2\left(cd^2-bed+ae^2\right)(b+2cx)\left(cx^2+bx+a\right)^{3/4}}+\frac{\left(b^2-4ac\right)\sqrt{\frac{(b+2cx)^2}{b^2-4ac}}\left(\cdots\right.}{\cdots}$$

```
[Out] -((a + b*x + c*x^2)^(1/4)/(e*(d + e*x))) + ((-b^2 + 4*a*c)^(3/4)*(2*c*d - b
*e)*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)*ArcTan[((-b^2 + 4*a*c)^(
1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d
^2 - b*d*e + a*e^2)^(1/4))]])/(4*c^(3/4)*e^(3/2)*(c*d^2 - b*d*e + a*e^2)^(3/
4)*(a + b*x + c*x^2)^(3/4)) + ((-b^2 + 4*a*c)^(3/4)*(2*c*d - b*e)*(-((c*(a
+ b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)*ArcTanh[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]
*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e +
 a*e^2)^(1/4))]])/(4*c^(3/4)*e^(3/2)*(c*d^2 - b*d*e + a*e^2)^(3/4)*(a + b*x
+ c*x^2)^(3/4)) + (c^(3/4)*(b^2 - 4*a*c)^(1/4)*Sqrt[(b + 2*c*x)^2/((b^2 - 4
*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2
*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt
[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(Sqrt[2]*e
^2*(b + 2*c*x)) + ((b^2 - 4*a*c)*(2*c*d - b*e)^2*Sqrt[(b + 2*c*x)^2/(b^2 -
4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)*EllipticPi[-(Sqrt[-b
^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (b + 2*
c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(4*Sqrt[2]*c*e^2*(c*d^2 - b*d*e + a*e^2)
*(b + 2*c*x)*(a + b*x + c*x^2)^(3/4)) + ((b^2 - 4*a*c)*(2*c*d - b*e)^2*Sqrt
[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4
)*EllipticPi[(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2])
, ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(4*Sqrt[2]*c*e^2*(c
*d^2 - b*d*e + a*e^2)*(b + 2*c*x)*(a + b*x + c*x^2)^(3/4))
```

Rubi [A] time = 2.02592, antiderivative size = 944, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.773, Rules used = {732, 843, 623, 220, 749, 748, 747, 401, 108, 409, 1213, 537, 444, 63, 212, 208, 205}

$$\frac{\left(b^2-4ac\right)\sqrt{\frac{(b+2cx)^2}{b^2-4ac}}\left(-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}\right)^{3/4}\Pi\left(-\frac{\sqrt{4ac-b^2}e}{2\sqrt{c}\sqrt{cd^2-bed+ae^2}};\sin^{-1}\left(\sqrt[4]{1-\frac{(b+2cx)^2}{b^2-4ac}}\right)\middle|-1\right)(2cd-be)^2}{4\sqrt{2}ce^2\left(cd^2-bed+ae^2\right)(b+2cx)\left(cx^2+bx+a\right)^{3/4}}+\frac{\left(b^2-4ac\right)\sqrt{\frac{(b+2cx)^2}{b^2-4ac}}\left(\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(1/4)/(d + e*x)^2,x]
```

```
[Out] -((a + b*x + c*x^2)^(1/4)/(e*(d + e*x))) + ((-b^2 + 4*a*c)^(3/4)*(2*c*d - b
*e)*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)*ArcTan[((-b^2 + 4*a*c)^(
1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d
^2 - b*d*e + a*e^2)^(1/4))]])/(4*c^(3/4)*e^(3/2)*(c*d^2 - b*d*e + a*e^2)^(3/
4)*(a + b*x + c*x^2)^(3/4)) + ((-b^2 + 4*a*c)^(3/4)*(2*c*d - b*e)*(-((c*(a
+ b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)*ArcTanh[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]
*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e +
 a*e^2)^(1/4))]])/(4*c^(3/4)*e^(3/2)*(c*d^2 - b*d*e + a*e^2)^(3/4)*(a + b*x
+ c*x^2)^(3/4)) + (c^(3/4)*(b^2 - 4*a*c)^(1/4)*Sqrt[(b + 2*c*x)^2/((b^2 - 4
```

```
*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2
*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt
[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(Sqrt[2]*e
^2*(b + 2*c*x)) + ((b^2 - 4*a*c)*(2*c*d - b*e)^2*Sqrt[(b + 2*c*x)^2/(b^2 -
4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)*EllipticPi[-(Sqrt[-b
^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (b + 2*
c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(4*Sqrt[2]*c*e^2*(c*d^2 - b*d*e + a*e^2)
*(b + 2*c*x)*(a + b*x + c*x^2)^(3/4)) + ((b^2 - 4*a*c)*(2*c*d - b*e)^2*Sqrt
[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4
)*EllipticPi[(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2])
, ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(4*Sqrt[2]*c*e^2*(c
*d^2 - b*d*e + a*e^2)*(b + 2*c*x)*(a + b*x + c*x^2)^(3/4))
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 749

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[(a + b*x + c*x^2)^p/(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^p, I
nt[(-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) - (c^2*x^2)/(b^2 - 4*a*c
))^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&  !GtQ[4*a - b^2/c
, 0] && IntegerQ[4*p]
```

Rule 748

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[1/((-4*c)/(b^2 - 4*a*c))^p, Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c),
 x]^p/Simp[2*c*d - b*e + e*x, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && GtQ[4*a - b^2/c, 0] && IntegerQ[4*p]
```

Rule 747

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[-((b*x^2)/a)]/(2*x), Subst[Int[1/(Sqrt[-((b*x)/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 108

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - (d*e
)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
```

```
x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx+cx^2}}{(d+ex)^2}\,dx = -\frac{\sqrt[4]{a+bx+cx^2}}{e(d+ex)} + \frac{\int \frac{b+2cx}{(d+ex)\left(a+bx+cx^2\right)^{3/4}}\,dx}{4e}$$

$$= -\frac{\sqrt[4]{a+bx+cx^2}}{e(d+ex)} + \frac{c\int \frac{1}{\left(a+bx+cx^2\right)^{3/4}}\,dx}{2e^2} - \frac{(2cd-be)\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/4}}\,dx}{4e^2}$$

$$= -\frac{\sqrt[4]{a+bx+cx^2}}{e(d+ex)} + \frac{\left(2c\sqrt{(b+2cx)^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b^2-4ac+4cx^4}}\,dx, x, \sqrt[4]{a+bx+cx^2}\right)}{e^2(b+2cx)} - \frac{\left((2cd-be)\left(-\right.\right.}{}$$

$$= -\frac{\sqrt[4]{a+bx+cx^2}}{e(d+ex)} + \frac{c^{3/4}\sqrt[4]{b^2-4ac}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{a-}}{\sqrt[4]{b^2-}}\right.\right.}{\sqrt{2}e^2(b+2cx)}$$

$$= -\frac{\sqrt[4]{a+bx+cx^2}}{e(d+ex)} + \frac{c^{3/4}\sqrt[4]{b^2-4ac}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{a-}}{\sqrt[4]{b^2-}}\right.\right.}{\sqrt{2}e^2(b+2cx)}$$

$$= -\frac{\sqrt[4]{a+bx+cx^2}}{e(d+ex)} + \frac{c^{3/4}\sqrt[4]{b^2-4ac}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{a-}}{\sqrt[4]{b^2-}}\right.\right.}{\sqrt{2}e^2(b+2cx)}$$

$$= -\frac{\sqrt[4]{a+bx+cx^2}}{e(d+ex)} + \frac{c^{3/4}\sqrt[4]{b^2-4ac}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{a-}}{\sqrt[4]{b^2-}}\right.\right.}{\sqrt{2}e^2(b+2cx)}$$

$$= -\frac{\sqrt[4]{a+bx+cx^2}}{e(d+ex)} + \frac{c^{3/4}\sqrt[4]{b^2-4ac}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{a-}}{\sqrt[4]{b^2-}}\right.\right.}{\sqrt{2}e^2(b+2cx)}$$

$$= -\frac{\sqrt[4]{a+bx+cx^2}}{e(d+ex)} + \frac{\left(-b^2+4ac\right)^{3/4}(2cd-be)\left(-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{-b^2+4ac}\sqrt{e}\sqrt[4]{1-\frac{(b+2cx)^2}{b^2-4ac}}}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{cd^2-bde+ae^2}}\right)}{4c^{3/4}e^{3/2}\left(cd^2-bde+ae^2\right)^{3/4}\left(a+bx+cx^2\right)^{3/4}} + \frac{\left(-b\right.}{}$$

$$= -\frac{\sqrt[4]{a+bx+cx^2}}{e(d+ex)} + \frac{\left(-b^2+4ac\right)^{3/4}(2cd-be)\left(-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{-b^2+4ac}\sqrt{e}\sqrt[4]{1-\frac{(b+2cx)^2}{b^2-4ac}}}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{cd^2-bde+ae^2}}\right)}{4c^{3/4}e^{3/2}\left(cd^2-bde+ae^2\right)^{3/4}\left(a+bx+cx^2\right)^{3/4}} + \frac{\left(-b\right.}{}$$

Mathematica [A] time = 2.48449, size = 646, normalized size = 0.68

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\left(\frac{c(a+x(b+cx))}{4ac-b^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right),2\right) + \frac{\left(4ac-b^2\right)^{3/4}\left(\frac{c(a+x(b+cx))}{4ac-b^2}\right)^{3/4}(be-2cd)\left(-\sqrt{2}\sqrt[4]{c}\sqrt{e}(b+2cx)\sqrt[4]{e(ae-bd)+cd^2}\left(\text{ta}\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(1/4)/(d + e*x)^2,x]
```

```
[Out] ((-2*e*(a + x*(b + c*x)))/(d + e*x) + 2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*((c*(a +
x*(b + c*x)))/(-b^2 + 4*a*c))^(3/4)*EllipticF[ArcSin[(b + 2*c*x)/Sqrt[b^2 -
 4*a*c]]/2, 2] + ((-b^2 + 4*a*c)^(3/4)*(-2*c*d + b*e)*((c*(a + x*(b + c*x))
)/(-b^2 + 4*a*c))^(3/4)*(-(Sqrt[2]*c^(1/4)*Sqrt[e]*(c*d^2 + e*(-(b*d) + a*e
))^(1/4)*(b + 2*c*x)*(ArcTan[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*((c*(a + x*(b +
c*x)))/(-b^2 + 4*a*c))^(1/4))/(c^(1/4)*(c*d^2 + e*(-(b*d) + a*e))^(1/4))] +
 ArcTanh[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*((c*(a + x*(b + c*x)))/(-b^2 + 4*a*c
))^(1/4))/(c^(1/4)*(c*d^2 + e*(-(b*d) + a*e))^(1/4))])) + (-b^2 + 4*a*c)^(1
/4)*(-2*c*d + b*e)*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*EllipticPi[-(Sqrt[-b^2
 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 + e*(-(b*d) + a*e)]), -ArcSin[Sqrt[2]*((
c*(a + x*(b + c*x)))/(-b^2 + 4*a*c))^(1/4)], -1] + (-b^2 + 4*a*c)^(1/4)*(-2
*c*d + b*e)*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*EllipticPi[(Sqrt[-b^2 + 4*a*c
]*e)/(2*Sqrt[c]*Sqrt[c*d^2 + e*(-(b*d) + a*e)]), -ArcSin[Sqrt[2]*((c*(a + x
*(b + c*x)))/(-b^2 + 4*a*c))^(1/4)], -1]))/(2*Sqrt[2]*c*(c*d^2 + e*(-(b*d)
+ a*e))*(b + 2*c*x)))/(2*e^2*(a + x*(b + c*x))^(3/4))
```

Maple [F] time = 1.272, size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)^2} \sqrt[4]{cx^2+bx+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/4)/(e*x+d)^2,x)
```

```
[Out] int((c*x^2+b*x+a)^(1/4)/(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2+bx+a)^{\frac{1}{4}}}{(ex+d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/4)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(1/4)/(e*x + d)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/4)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[4]{a + bx + cx^2}}{(d + ex)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/4)/(e*x+d)**2,x)
```

```
[Out] Integral((a + b*x + c*x**2)**(1/4)/(d + e*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx + a\right)^{\frac{1}{4}}}{(ex + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/4)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(1/4)/(e*x + d)^2, x)
```

3.2516 $\int (d+ex)^3 \left(a+bx+cx^2\right)^{3/4} dx$

Optimal. Leaf size=703

$$-\frac{\left(b^2-4ac\right)^{7/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)(2cd-be)\left(-4ce(2ae+3bd)+5b^2e^2+12c^2d^2\right)\text{EllipticF}\left(2\right.}{160\sqrt{2}c^{19/4}(b+2cx)}$$

```
[Out] ((2*c*d - b*e)*(12*c^2*d^2 + 5*b^2*e^2 - 4*c*e*(3*b*d + 2*a*e))*(b + 2*c*x)
*(a + b*x + c*x^2)^(3/4))/(120*c^4) + (2*e*(d + e*x)^2*(a + b*x + c*x^2)^(7
/4))/(11*c) + (e*(312*c^2*d^2 + 55*b^2*e^2 - 2*c*e*(121*b*d + 24*a*e) + 70*
c*e*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(7/4))/(462*c^3) - (Sqrt[b^2 - 4*a*c
]*(2*c*d - b*e)*(12*c^2*d^2 + 5*b^2*e^2 - 4*c*e*(3*b*d + 2*a*e))*(b + 2*c*x
)*(a + b*x + c*x^2)^(1/4))/(80*c^(9/2)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])/Sqrt[b^2 - 4*a*c])) + ((b^2 - 4*a*c)^(7/4)*(2*c*d - b*e)*(12*c^2*d^2 + 5
*b^2*e^2 - 4*c*e*(3*b*d + 2*a*e))*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2
*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt
[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticE[2*ArcTan[(Sqrt[2]*c^(1/4)*(
a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(80*Sqrt[2]*c^(19/4)*(b
 + 2*c*x)) - ((b^2 - 4*a*c)^(7/4)*(2*c*d - b*e)*(12*c^2*d^2 + 5*b^2*e^2 - 4
*c*e*(3*b*d + 2*a*e))*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqr
t[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c
*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*
x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(160*Sqrt[2]*c^(19/4)*(b + 2*c*x))
```

Rubi [A] time = 0.866685, antiderivative size = 703, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {742, 779, 612, 623, 305, 220, 1196}

$$\frac{e\left(a+bx+cx^2\right)^{7/4}\left(-2ce(24ae+121bd)+55b^2e^2+70cex(2cd-be)+312c^2d^2\right)}{462c^3}+\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/4}(2cd-be}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*x + c*x^2)^(3/4),x]
```

```
[Out] ((2*c*d - b*e)*(12*c^2*d^2 + 5*b^2*e^2 - 4*c*e*(3*b*d + 2*a*e))*(b + 2*c*x)
*(a + b*x + c*x^2)^(3/4))/(120*c^4) + (2*e*(d + e*x)^2*(a + b*x + c*x^2)^(7
/4))/(11*c) + (e*(312*c^2*d^2 + 55*b^2*e^2 - 2*c*e*(121*b*d + 24*a*e) + 70*
c*e*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(7/4))/(462*c^3) - (Sqrt[b^2 - 4*a*c
]*(2*c*d - b*e)*(12*c^2*d^2 + 5*b^2*e^2 - 4*c*e*(3*b*d + 2*a*e))*(b + 2*c*x
)*(a + b*x + c*x^2)^(1/4))/(80*c^(9/2)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])/Sqrt[b^2 - 4*a*c])) + ((b^2 - 4*a*c)^(7/4)*(2*c*d - b*e)*(12*c^2*d^2 + 5
*b^2*e^2 - 4*c*e*(3*b*d + 2*a*e))*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2
*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt
[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticE[2*ArcTan[(Sqrt[2]*c^(1/4)*(
a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(80*Sqrt[2]*c^(19/4)*(b
 + 2*c*x)) - ((b^2 - 4*a*c)^(7/4)*(2*c*d - b*e)*(12*c^2*d^2 + 5*b^2*e^2 - 4
*c*e*(3*b*d + 2*a*e))*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqr
t[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c
*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*
```

```
x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(160*Sqrt[2]*c^(19/4)*(b + 2*c*x))
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int (d+ex)^3 \left(a+bx+cx^2\right)^{3/4} dx = \frac{2e(d+ex)^2 \left(a+bx+cx^2\right)^{7/4}}{11c} + \frac{2\int (d+ex)\left(\frac{1}{4}\left(22cd^2-7bde-8ae^2\right)+\frac{15}{4}e(2cd-b\right.}{11c}$$

$$= \frac{2e(d+ex)^2 \left(a+bx+cx^2\right)^{7/4}}{11c} + \frac{e\left(312c^2d^2+55b^2e^2-2ce(121bd+24ae)+70ce(2c\right.}{462c^3}$$

$$= \frac{(2cd-be)\left(12c^2d^2+5b^2e^2-4ce(3bd+2ae)\right)(b+2cx)\left(a+bx+cx^2\right)^{3/4}}{120c^4} + \frac{2e(d+e}{}$$

$$= \frac{(2cd-be)\left(12c^2d^2+5b^2e^2-4ce(3bd+2ae)\right)(b+2cx)\left(a+bx+cx^2\right)^{3/4}}{120c^4} + \frac{2e(d+e}{}$$

$$= \frac{(2cd-be)\left(12c^2d^2+5b^2e^2-4ce(3bd+2ae)\right)(b+2cx)\left(a+bx+cx^2\right)^{3/4}}{120c^4} + \frac{2e(d+e}{}$$

$$= \frac{(2cd-be)\left(12c^2d^2+5b^2e^2-4ce(3bd+2ae)\right)(b+2cx)\left(a+bx+cx^2\right)^{3/4}}{120c^4} + \frac{2e(d+e}{}$$

Mathematica [C] time = 0.636513, size = 234, normalized size = 0.33

$$\frac{77(b+2cx)(2cd-be)\left(-4ce(2ae+3bd)+5b^2e^2+12c^2d^2\right)\left(8c(a+x(b+cx))-3\sqrt{2}\left(b^2-4ac\right)\sqrt[4]{\frac{c(a+x(b+cx))}{4ac-b^2}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\right.\right.}{739}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*x + c*x^2)^(3/4),x]
```

```
[Out] (13440*c^4*e*(d + e*x)^2*(a + x*(b + c*x))^2 + 160*c^2*e*(a + x*(b + c*x))^
2*(55*b^2*e^2 + 4*c^2*d*(78*d + 35*e*x) - 2*c*e*(121*b*d + 24*a*e + 35*b*e*
x)) + 77*(2*c*d - b*e)*(12*c^2*d^2 + 5*b^2*e^2 - 4*c*e*(3*b*d + 2*a*e))*(b
+ 2*c*x)*(8*c*(a + x*(b + c*x)) - 3*Sqrt[2]*(b^2 - 4*a*c)*((c*(a + x*(b + c
*x)))/(-b^2 + 4*a*c))^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (b + 2*c*x)^2/
(b^2 - 4*a*c)]))/(73920*c^5*(a + x*(b + c*x))^(1/4))
```

Maple [F] time = 1.04, size = 0, normalized size = 0.

$$\int (ex+d)^3 \left(cx^2+bx+a\right)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x+a)^(3/4),x)
```

```
[Out] int((e*x+d)^3*(c*x^2+b*x+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{\frac{3}{4}} (ex+d)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/4)*(e*x + d)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\left(cx^2+bx+a\right)^{\frac{3}{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(c*x^2 + b*x + a)^(3/4),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(d+ex)^3\left(a+bx+cx^2\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x+a)**(3/4),x)
```

```
[Out] Integral((d + e*x)**3*(a + b*x + c*x**2)**(3/4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^(3/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.2517 $\int (d+ex)^2 \left(a+bx+cx^2\right)^{3/4} dx$

Optimal. Leaf size=630

$$-\frac{\left(b^2-4ac\right)^{7/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)\left(-4ce(2ae+9bd)+11b^2e^2+36c^2d^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\right.\right.}{240\sqrt{2}c^{15/4}(b+2cx)}$$

```
[Out] ((36*c^2*d^2 + 11*b^2*e^2 - 4*c*e*(9*b*d + 2*a*e))*(b + 2*c*x)*(a + b*x + c
*x^2)^(3/4))/(180*c^3) + (11*e*(2*c*d - b*e)*(a + b*x + c*x^2)^(7/4))/(63*c
^2) + (2*e*(d + e*x)*(a + b*x + c*x^2)^(7/4))/(9*c) - (Sqrt[b^2 - 4*a*c]*(3
6*c^2*d^2 + 11*b^2*e^2 - 4*c*e*(9*b*d + 2*a*e))*(b + 2*c*x)*(a + b*x + c*x^
2)^(1/4))/(120*c^(7/2)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*
a*c])) + ((b^2 - 4*a*c)^(7/4)*(36*c^2*d^2 + 11*b^2*e^2 - 4*c*e*(9*b*d + 2*a
*e))*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 -
 4*a*c])*EllipticE[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2
- 4*a*c)^(1/4)], 1/2])/(120*Sqrt[2]*c^(15/4)*(b + 2*c*x)) - ((b^2 - 4*a*c)^
(7/4)*(36*c^2*d^2 + 11*b^2*e^2 - 4*c*e*(9*b*d + 2*a*e))*Sqrt[(b + 2*c*x)^2/
((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)
]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*Arc
Tan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(
240*Sqrt[2]*c^(15/4)*(b + 2*c*x))
```

Rubi [A] time = 0.848154, antiderivative size = 630, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {742, 640, 612, 623, 305, 220, 1196}

$$-\frac{\sqrt{b^2-4ac}(b+2cx)\sqrt[4]{a+bx+cx^2}\left(-4ce(2ae+9bd)+11b^2e^2+36c^2d^2\right)}{120c^{7/2}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)}+\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/4}\left(-4ce(2ae+9b\right.}{180c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*x + c*x^2)^(3/4),x]
```


```
[Out] ((36*c^2*d^2 + 11*b^2*e^2 - 4*c*e*(9*b*d + 2*a*e))*(b + 2*c*x)*(a + b*x + c
*x^2)^(3/4))/(180*c^3) + (11*e*(2*c*d - b*e)*(a + b*x + c*x^2)^(7/4))/(63*c
^2) + (2*e*(d + e*x)*(a + b*x + c*x^2)^(7/4))/(9*c) - (Sqrt[b^2 - 4*a*c]*(3
6*c^2*d^2 + 11*b^2*e^2 - 4*c*e*(9*b*d + 2*a*e))*(b + 2*c*x)*(a + b*x + c*x^
2)^(1/4))/(120*c^(7/2)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*
a*c])) + ((b^2 - 4*a*c)^(7/4)*(36*c^2*d^2 + 11*b^2*e^2 - 4*c*e*(9*b*d + 2*a
*e))*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 -
 4*a*c])*EllipticE[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2
- 4*a*c)^(1/4)], 1/2])/(120*Sqrt[2]*c^(15/4)*(b + 2*c*x)) - ((b^2 - 4*a*c)^
(7/4)*(36*c^2*d^2 + 11*b^2*e^2 - 4*c*e*(9*b*d + 2*a*e))*Sqrt[(b + 2*c*x)^2/
((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)
]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*Arc
Tan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(
240*Sqrt[2]*c^(15/4)*(b + 2*c*x))
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int (d+ex)^2 \left(a+bx+cx^2\right)^{3/4} dx = \frac{2e(d+ex)\left(a+bx+cx^2\right)^{7/4}}{9c} + \frac{2\int \left(\frac{1}{4}\left(18cd^2-4e\left(\frac{7bd}{4}+ae\right)\right)+\frac{11}{4}e(2cd-be)x\right)\left(a\right.}{9c}$$

$$= \frac{11e(2cd-be)\left(a+bx+cx^2\right)^{7/4}}{63c^2} + \frac{2e(d+ex)\left(a+bx+cx^2\right)^{7/4}}{9c} + \frac{\left(-\frac{11}{4}be(2cd-be)\right.}{}$$

$$= \frac{\left(36c^2d^2+11b^2e^2-4ce(9bd+2ae)\right)(b+2cx)\left(a+bx+cx^2\right)^{3/4}}{180c^3} + \frac{11e(2cd-be)\left(a\right.}{63c^2}$$

$$= \frac{\left(36c^2d^2+11b^2e^2-4ce(9bd+2ae)\right)(b+2cx)\left(a+bx+cx^2\right)^{3/4}}{180c^3} + \frac{11e(2cd-be)\left(a\right.}{63c^2}$$

$$= \frac{\left(36c^2d^2+11b^2e^2-4ce(9bd+2ae)\right)(b+2cx)\left(a+bx+cx^2\right)^{3/4}}{180c^3} + \frac{11e(2cd-be)\left(a\right.}{63c^2}$$

$$= \frac{\left(36c^2d^2+11b^2e^2-4ce(9bd+2ae)\right)(b+2cx)\left(a+bx+cx^2\right)^{3/4}}{180c^3} + \frac{11e(2cd-be)\left(a\right.}{63c^2}$$

Mathematica [C] time = 0.327816, size = 196, normalized size = 0.31

$$\frac{-\frac{7(b+2cx)\left(ce(2ae+9bd)-\frac{11b^2e^2}{4}-9c^2d^2\right)\left(8c(a+x(b+cx))-3\sqrt{2}\left(b^2-4ac\right)\sqrt[4]{\frac{c(a+x(b+cx))}{4ac-b^2}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)\right)}{40c^2\sqrt[4]{a+x(b+cx)}}+11e(a+x(b+cx))^{7/4}(2cd-be)+1}{63c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*x + c*x^2)^(3/4),x]
```

```
[Out] (11*e*(2*c*d - b*e)*(a + x*(b + c*x))^(7/4) + 14*c*e*(d + e*x)*(a + x*(b +
c*x))^(7/4) - (7*(-9*c^2*d^2 - (11*b^2*e^2)/4 + c*e*(9*b*d + 2*a*e))*(b + 2
*c*x)*(8*c*(a + x*(b + c*x)) - 3*Sqrt[2]*(b^2 - 4*a*c)*((c*(a + x*(b + c*x)
))/(-b^2 + 4*a*c))^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (b + 2*c*x)^2/(b^
2 - 4*a*c)]))/(40*c^2*(a + x*(b + c*x))^(1/4)))/(63*c^2)
```

Maple [F] time = 1.011, size = 0, normalized size = 0.

$$\int (ex+d)^2 \left(cx^2+bx+a\right)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+b*x+a)^(3/4),x)
```

```
[Out] int((e*x+d)^2*(c*x^2+b*x+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{\frac{3}{4}} (ex+d)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/4)*(e*x + d)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^2 + 2\,dex + d^2\right)\left(cx^2 + bx + a\right)^{\frac{3}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(c*x^2 + b*x + a)^(3/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d + ex)^2 \left(a + bx + cx^2\right)^{\frac{3}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+b*x+a)**(3/4),x)
```

```
[Out] Integral((d + e*x)**2*(a + b*x + c*x**2)**(3/4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(3/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.2518 $\int (d+ex)\left(a+bx+cx^2\right)^{3/4}\,dx$

Optimal. Leaf size=510

$$-\frac{3\left(b^2-4ac\right)^{7/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)(2cd-be)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{a+bx+cx^2}}{\sqrt[4]{b^2-4ac}}\right),\frac{1}{2}\right)}{40\sqrt{2}c^{11/4}(b+2cx)}-\frac{3\sqrt{b}\cdots}{\cdots}$$

```
[Out] ((2*c*d - b*e)*(b + 2*c*x)*(a + b*x + c*x^2)^(3/4))/(10*c^2) + (2*e*(a + b*
x + c*x^2)^(7/4))/(7*c) - (3*Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*(b + 2*c*x)*(a
 + b*x + c*x^2)^(1/4))/(20*c^(5/2)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/S
qrt[b^2 - 4*a*c])) + (3*(b^2 - 4*a*c)^(7/4)*(2*c*d - b*e)*Sqrt[(b + 2*c*x)^
2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^
2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticE[2*A
rcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])
/(20*Sqrt[2]*c^(11/4)*(b + 2*c*x)) - (3*(b^2 - 4*a*c)^(7/4)*(2*c*d - b*e)*S
qrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqr
t[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c
])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*
c)^(1/4)], 1/2])/(40*Sqrt[2]*c^(11/4)*(b + 2*c*x))
```

Rubi [A] time = 0.438951, antiderivative size = 510, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {640, 612, 623, 305, 220, 1196}

$$-\frac{3\sqrt{b^2-4ac}(b+2cx)\sqrt[4]{a+bx+cx^2}(2cd-be)}{20c^{5/2}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)}-\frac{3\left(b^2-4ac\right)^{7/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)(2cd-be\cdots}{40\sqrt{2}c^{11/4}(b+2cx)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*x + c*x^2)^(3/4),x]
```

```
[Out] ((2*c*d - b*e)*(b + 2*c*x)*(a + b*x + c*x^2)^(3/4))/(10*c^2) + (2*e*(a + b*
x + c*x^2)^(7/4))/(7*c) - (3*Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*(b + 2*c*x)*(a
 + b*x + c*x^2)^(1/4))/(20*c^(5/2)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/S
qrt[b^2 - 4*a*c])) + (3*(b^2 - 4*a*c)^(7/4)*(2*c*d - b*e)*Sqrt[(b + 2*c*x)^
2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^
2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticE[2*A
rcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])
/(20*Sqrt[2]*c^(11/4)*(b + 2*c*x)) - (3*(b^2 - 4*a*c)^(7/4)*(2*c*d - b*e)*S
qrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqr
t[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c
])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*
c)^(1/4)], 1/2])/(40*Sqrt[2]*c^(11/4)*(b + 2*c*x))
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int (d+ex)\left(a+bx+cx^2\right)^{3/4}\,dx = \frac{2e\left(a+bx+cx^2\right)^{7/4}}{7c}+\frac{(2cd-be)\int \left(a+bx+cx^2\right)^{3/4}\,dx}{2c}$$

$$= \frac{(2cd-be)(b+2cx)\left(a+bx+cx^2\right)^{3/4}}{10c^2}+\frac{2e\left(a+bx+cx^2\right)^{7/4}}{7c}-\frac{\left(3\left(b^2-4ac\right)(2cd-be)\right)}{40c^2}$$

$$= \frac{(2cd-be)(b+2cx)\left(a+bx+cx^2\right)^{3/4}}{10c^2}+\frac{2e\left(a+bx+cx^2\right)^{7/4}}{7c}-\frac{\left(3\left(b^2-4ac\right)(2cd-be)\sqrt{\ }\right.}{}$$

$$= \frac{(2cd-be)(b+2cx)\left(a+bx+cx^2\right)^{3/4}}{10c^2}+\frac{2e\left(a+bx+cx^2\right)^{7/4}}{7c}-\frac{\left(3\left(b^2-4ac\right)^{3/2}(2cd-b\right.}{}$$

$$= \frac{(2cd-be)(b+2cx)\left(a+bx+cx^2\right)^{3/4}}{10c^2}+\frac{2e\left(a+bx+cx^2\right)^{7/4}}{7c}-\frac{3\sqrt{b^2-4ac}(2cd-be)(b}{20c^{5/2}\left(1+\frac{2\sqrt{c}}{}\right.}$$

Mathematica [C] time = 0.242966, size = 141, normalized size = 0.28

$$\frac{(b+2cx)(2cd-be)\left(8c(a+x(b+cx))-3\sqrt{2}\left(b^2-4ac\right)\sqrt[4]{\frac{c(a+x(b+cx))}{4ac-b^2}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)\right)}{80c^3\sqrt[4]{a+x(b+cx)}}+\frac{2e(a+x(b+cx))^{7/4}}{7c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*x + c*x^2)^(3/4),x]
```

```
[Out] (2*e*(a + x*(b + c*x))^(7/4))/(7*c) + ((2*c*d - b*e)*(b + 2*c*x)*(8*c*(a +
x*(b + c*x)) - 3*Sqrt[2]*(b^2 - 4*a*c)*((c*(a + x*(b + c*x)))/(-b^2 + 4*a*c
))^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (b + 2*c*x)^2/(b^2 - 4*a*c)]))/(8
0*c^3*(a + x*(b + c*x))^(1/4))
```

Maple [F] time = 1.049, size = 0, normalized size = 0.

$$\int (ex+d)\left(cx^2+bx+a\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+b*x+a)^(3/4),x)
```

```
[Out] int((e*x+d)*(c*x^2+b*x+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{\frac{3}{4}}(ex+d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/4)*(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(cx^2+bx+a\right)^{\frac{3}{4}}(ex+d),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^(3/4)*(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d+ex)\left(a+bx+cx^2\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+b*x+a)**(3/4),x)
```

```
[Out] Integral((d + e*x)*(a + b*x + c*x**2)**(3/4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(3/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.2519 $\int \left(a+bx+cx^2\right)^{3/4} dx$

Optimal. Leaf size=452

$$-\frac{3\left(b^2-4ac\right)^{7/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{a+bx+cx^2}}{\sqrt[4]{b^2-4ac}}\right),\frac{1}{2}\right)}{20\sqrt{2}c^{7/4}(b+2cx)}-\frac{3\sqrt{b^2-4ac}(b}{10c^{3/2}\left(\frac{2}{}\right.}$$

```
[Out] ((b + 2*c*x)*(a + b*x + c*x^2)^(3/4))/(5*c) - (3*Sqrt[b^2 - 4*a*c]*(b + 2*c
*x)*(a + b*x + c*x^2)^(1/4))/(10*c^(3/2)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x
^2])/Sqrt[b^2 - 4*a*c])) + (3*(b^2 - 4*a*c)^(7/4)*Sqrt[(b + 2*c*x)^2/((b^2
- 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 +
 (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticE[2*ArcTan[(S
qrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(10*Sqr
t[2]*c^(7/4)*(b + 2*c*x)) - (3*(b^2 - 4*a*c)^(7/4)*Sqrt[(b + 2*c*x)^2/((b^2
 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1
+ (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(
Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(20*Sq
rt[2]*c^(7/4)*(b + 2*c*x))
```

Rubi [A] time = 0.362471, antiderivative size = 452, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {612, 623, 305, 220, 1196}

$$-\frac{3\sqrt{b^2-4ac}(b+2cx)\sqrt[4]{a+bx+cx^2}}{10c^{3/2}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)}-\frac{3\left(b^2-4ac\right)^{7/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{}}{\sqrt[4]{b^2}}\right.\right.}{20\sqrt{2}c^{7/4}(b+2cx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/4),x]
```

```
[Out] ((b + 2*c*x)*(a + b*x + c*x^2)^(3/4))/(5*c) - (3*Sqrt[b^2 - 4*a*c]*(b + 2*c
*x)*(a + b*x + c*x^2)^(1/4))/(10*c^(3/2)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x
^2])/Sqrt[b^2 - 4*a*c])) + (3*(b^2 - 4*a*c)^(7/4)*Sqrt[(b + 2*c*x)^2/((b^2
- 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 +
 (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticE[2*ArcTan[(S
qrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(10*Sqr
t[2]*c^(7/4)*(b + 2*c*x)) - (3*(b^2 - 4*a*c)^(7/4)*Sqrt[(b + 2*c*x)^2/((b^2
 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1
+ (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(
Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(20*Sq
rt[2]*c^(7/4)*(b + 2*c*x))
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^{3/4}\,dx = \frac{(b+2cx)\left(a+bx+cx^2\right)^{3/4}}{5c}-\frac{\left(3\left(b^2-4ac\right)\right)\int \frac{1}{\sqrt[4]{a+bx+cx^2}}\,dx}{20c}$$

$$=\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/4}}{5c}-\frac{\left(3\left(b^2-4ac\right)\sqrt{(b+2cx)^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b^2-4ac+4cx^4}}\,dx,x,\sqrt[4]{a+bx+}\right.}{5c(b+2cx)}$$

$$=\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/4}}{5c}-\frac{\left(3\left(b^2-4ac\right)^{3/2}\sqrt{(b+2cx)^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b^2-4ac+4cx^4}}\,dx,x,\sqrt[4]{a+b}\right.}{10c^{3/2}(b+2cx)}$$

$$=\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/4}}{5c}-\frac{3\sqrt{b^2-4ac}(b+2cx)\sqrt[4]{a+bx+cx^2}}{10c^{3/2}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}+\frac{3\left(b^2-4ac\right)^{7/4}\sqrt{\frac{(b\cdots}{\left(b^2-4ac\right)\left(1+\cdots\right.}}}{\cdots}$$

Mathematica [C] time = 0.0950361, size = 119, normalized size = 0.26

$$\frac{(b+2cx)(a+x(b+cx))^{3/4}\left(3\sqrt{2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)+8\left(\frac{c(a+x(b+cx))}{4ac-b^2}\right)^{3/4}\right)}{40c\left(\frac{c(a+x(b+cx))}{4ac-b^2}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/4),x]
```

```
[Out] ((b + 2*c*x)*(a + x*(b + c*x))^(3/4)*(8*((c*(a + x*(b + c*x)))/(-b^2 + 4*a*
c))^(3/4) + 3*Sqrt[2]*Hypergeometric2F1[1/4, 1/2, 3/2, (b + 2*c*x)^2/(b^2 -
```

```
4*a*c)]))/(40*c*((c*(a + x*(b + c*x)))/(-b^2 + 4*a*c))^(3/4))
```

Maple [F] time = 2.293, size = 0, normalized size = 0.

$$\int \left(cx^2 + bx + a\right)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/4),x)
```

```
[Out] int((c*x^2+b*x+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx + a\right)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2 + bx + a\right)^{\frac{3}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^(3/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + bx + cx^2\right)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/4),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx + a\right)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/4), x)
```

3.2520 $\int \frac{\left(a+bx+cx^2\right)^{3/4}}{d+ex}\,dx$

Optimal. Leaf size=1209

result too large to display

```
[Out] (2*(a + b*x + c*x^2)^(3/4))/(3*e) - ((2*c*d - b*e)*(b + 2*c*x)*(a + b*x + c
*x^2)^(1/4))/(Sqrt[c]*Sqrt[b^2 - 4*a*c]*e^2*(1 + (2*Sqrt[c]*Sqrt[a + b*x +
c*x^2])/Sqrt[b^2 - 4*a*c])) + ((-b^2 + 4*a*c)^(1/4)*(c*d^2 - b*d*e + a*e^2)
^(3/4)*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(1/4)*ArcTan[((-b^2 + 4*a*c
)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(
c*d^2 - b*d*e + a*e^2)^(1/4))]])/(c^(1/4)*e^(5/2)*(a + b*x + c*x^2)^(1/4)) -
 ((-b^2 + 4*a*c)^(1/4)*(c*d^2 - b*d*e + a*e^2)^(3/4)*(-((c*(a + b*x + c*x^2
))/(b^2 - 4*a*c)))^(1/4)*ArcTanh[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*
c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4)
)]])/(c^(1/4)*e^(5/2)*(a + b*x + c*x^2)^(1/4)) + ((b^2 - 4*a*c)^(3/4)*(2*c*d
 - b*e)*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*
x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^
2 - 4*a*c])*EllipticE[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b
^2 - 4*a*c)^(1/4)], 1/2])/(Sqrt[2]*c^(3/4)*e^2*(b + 2*c*x)) - ((b^2 - 4*a*c
)^(3/4)*(2*c*d - b*e)*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqr
t[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c
*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*
x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(2*Sqrt[2]*c^(3/4)*e^2*(b + 2*c*x))
 - (Sqrt[-b^2 + 4*a*c]*(2*c*d - b*e)*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[(b +
2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(1/4)*Elli
pticPi[-(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2]), Arc
Sin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(Sqrt[2]*Sqrt[c]*e^3*(b
+ 2*c*x)*(a + b*x + c*x^2)^(1/4)) + (Sqrt[-b^2 + 4*a*c]*(2*c*d - b*e)*Sqrt[
c*d^2 - b*d*e + a*e^2]*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c
*x^2))/(b^2 - 4*a*c)))^(1/4)*EllipticPi[(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*S
qrt[c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)
], -1])/(Sqrt[2]*Sqrt[c]*e^3*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4))
```

Rubi [A] time = 2.51291, antiderivative size = 1209, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 18, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {734, 843, 623, 305, 220, 1196, 749, 748, 746, 399, 490, 1213, 537, 444, 63, 298, 205, 208}

$$-\frac{(2cd-be)\sqrt[4]{cx^2+bx+a}(b+2cx)}{\sqrt{c}\sqrt{b^2-4ac}e^2\left(\frac{2\sqrt{c}\sqrt{cx^2+bx+a}}{\sqrt{b^2-4ac}}+1\right)}+\frac{\sqrt[4]{4ac-b^2}\left(cd^2-bed+ae^2\right)^{3/4}\sqrt[4]{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt[4]{4ac-b^2}\sqrt{e}\sqrt[4]{1-\frac{(b+2cx)^2}{b^2-4ac}}}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{cd^2-bed+ae^2}}\right)}{\sqrt[4]{c}e^{5/2}\sqrt[4]{cx^2+bx+a}}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/4)/(d + e*x),x]
```

```
[Out] (2*(a + b*x + c*x^2)^(3/4))/(3*e) - ((2*c*d - b*e)*(b + 2*c*x)*(a + b*x + c
*x^2)^(1/4))/(Sqrt[c]*Sqrt[b^2 - 4*a*c]*e^2*(1 + (2*Sqrt[c]*Sqrt[a + b*x +
c*x^2])/Sqrt[b^2 - 4*a*c])) + ((-b^2 + 4*a*c)^(1/4)*(c*d^2 - b*d*e + a*e^2)
^(3/4)*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(1/4)*ArcTan[((-b^2 + 4*a*c
)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(
c*d^2 - b*d*e + a*e^2)^(1/4))]])/(c^(1/4)*e^(5/2)*(a + b*x + c*x^2)^(1/4)) -
```

```
((-b^2 + 4*a*c)^(1/4)*(c*d^2 - b*d*e + a*e^2)^(3/4)*(-((c*(a + b*x + c*x^2
))/(b^2 - 4*a*c)))^(1/4)*ArcTanh[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*
c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4)
)])/(c^(1/4)*e^(5/2)*(a + b*x + c*x^2)^(1/4)) + ((b^2 - 4*a*c)^(3/4)*(2*c*d
 - b*e)*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*
x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^
2 - 4*a*c])*EllipticE[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b
^2 - 4*a*c)^(1/4)], 1/2])/(Sqrt[2]*c^(3/4)*e^2*(b + 2*c*x)) - ((b^2 - 4*a*c
)^(3/4)*(2*c*d - b*e)*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqr
t[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c
*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*
x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(2*Sqrt[2]*c^(3/4)*e^2*(b + 2*c*x))
 - (Sqrt[-b^2 + 4*a*c]*(2*c*d - b*e)*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[(b +
2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(1/4)*Elli
pticPi[-(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2]), Arc
Sin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(Sqrt[2]*Sqrt[c]*e^3*(b
+ 2*c*x)*(a + b*x + c*x^2)^(1/4)) + (Sqrt[-b^2 + 4*a*c]*(2*c*d - b*e)*Sqrt[
c*d^2 - b*d*e + a*e^2]*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c
*x^2))/(b^2 - 4*a*c)))^(1/4)*EllipticPi[(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*S
qrt[c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)
], -1])/(Sqrt[2]*Sqrt[c]*e^3*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4))
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 749

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[(a + b*x + c*x^2)^p/(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^p, I
nt[(-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) - (c^2*x^2)/(b^2 - 4*a*c
))^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&  !GtQ[4*a - b^2/c
, 0] && IntegerQ[4*p]
```

Rule 748

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[1/((-4*c)/(b^2 - 4*a*c))^p, Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c),
 x]^p/Simp[2*c*d - b*e + e*x, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && GtQ[4*a - b^2/c, 0] && IntegerQ[4*p]
```

Rule 746

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/4}}{d+ex}\,dx = \frac{2\left(a+bx+cx^2\right)^{3/4}}{3e} - \frac{\int \frac{bd-2ae+(2cd-be)x}{(d+ex)\sqrt[4]{a+bx+cx^2}}\,dx}{2e}$$

$$= \frac{2\left(a+bx+cx^2\right)^{3/4}}{3e} - \frac{(2cd-be)\int \frac{1}{\sqrt[4]{a+bx+cx^2}}\,dx}{2e^2} - \frac{(e(bd-2ae)-d(2cd-be))\int \frac{1}{(d+ex)\sqrt[4]{a+bx+c}}}{2e^2}$$

$$= \frac{2\left(a+bx+cx^2\right)^{3/4}}{3e} - \frac{\left(2(2cd-be)\sqrt{(b+2cx)^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b^2-4ac+4cx^4}}\,dx, x, \sqrt[4]{a+bx+cx^2}\right)}{e^2(b+2cx)}$$

$$= \frac{2\left(a+bx+cx^2\right)^{3/4}}{3e} - \frac{\left(\sqrt{b^2-4ac}(2cd-be)\sqrt{(b+2cx)^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b^2-4ac+4cx^4}}\,dx, x, \sqrt[4]{a+b}\right.}{\sqrt{c}e^2(b+2cx)}$$

$$= \frac{2\left(a+bx+cx^2\right)^{3/4}}{3e} - \frac{(2cd-be)(b+2cx)\sqrt[4]{a+bx+cx^2}}{\sqrt{c}\sqrt{b^2-4ac}e^2\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)} + \frac{\left(b^2-4ac\right)^{3/4}(2cd-be)\sqrt{\frac{}{\left(b^2-4ac\right)}}}{}$$

$$= \frac{2\left(a+bx+cx^2\right)^{3/4}}{3e} - \frac{(2cd-be)(b+2cx)\sqrt[4]{a+bx+cx^2}}{\sqrt{c}\sqrt{b^2-4ac}e^2\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)} + \frac{\left(b^2-4ac\right)^{3/4}(2cd-be)\sqrt{\frac{}{\left(b^2-4ac\right)}}}{}$$

$$= \frac{2\left(a+bx+cx^2\right)^{3/4}}{3e} - \frac{(2cd-be)(b+2cx)\sqrt[4]{a+bx+cx^2}}{\sqrt{c}\sqrt{b^2-4ac}e^2\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)} + \frac{\left(b^2-4ac\right)^{3/4}(2cd-be)\sqrt{\frac{}{\left(b^2-4ac\right)}}}{}$$

$$= \frac{2\left(a+bx+cx^2\right)^{3/4}}{3e} - \frac{(2cd-be)(b+2cx)\sqrt[4]{a+bx+cx^2}}{\sqrt{c}\sqrt{b^2-4ac}e^2\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)} + \frac{\left(b^2-4ac\right)^{3/4}(2cd-be)\sqrt{\frac{}{\left(b^2-4ac\right)}}}{}$$

$$= \frac{2\left(a+bx+cx^2\right)^{3/4}}{3e} - \frac{(2cd-be)(b+2cx)\sqrt[4]{a+bx+cx^2}}{\sqrt{c}\sqrt{b^2-4ac}e^2\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)} + \frac{\sqrt[4]{-b^2+4ac}\left(cd^2-bde+ae^2\right)^{3/4}}{\sqrt[4]{c}e^5}$$

Mathematica [C] time = 0.42403, size = 180, normalized size = 0.15

$$\frac{4\sqrt{2}(a+x(b+cx))^{3/4}F_1\left(-\frac{3}{2};-\frac{3}{4},-\frac{3}{4};-\frac{1}{2};\frac{2cd-\left(b+\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2cd-be+\sqrt{b^2-4ac}e}{2cd+2cex}\right)}{3e\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{3/4}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)^(3/4)/(d + e*x),x]
```

```
[Out] (4*Sqrt[2]*(a + x*(b + c*x))^(3/4)*AppellF1[-3/2, -3/4, -3/4, -1/2, (2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x)), (2*c*d - b*e + Sqrt[b^2 - 4*a
*c]*e)/(2*c*d + 2*c*e*x)])/(3*e*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d
+ e*x)))^(3/4)*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(3/4))
```

Maple [F] time = 1.278, size = 0, normalized size = 0.

$$\int \frac{1}{ex+d}\left(cx^2+bx+a\right)^{\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/4)/(e*x+d),x)
```

```
[Out] int((c*x^2+b*x+a)^(3/4)/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{3}{4}}}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/4)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/4)/(e*x + d), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/4)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx+cx^2\right)^{\frac{3}{4}}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/4)/(e*x+d),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/4)/(d + e*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/4)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.2521 $\int \frac{\left(a+bx+cx^2\right)^{3/4}}{(d+ex)^2}\,dx$

Optimal. Leaf size=1220

result too large to display

```
[Out] -((a + b*x + c*x^2)^(3/4)/(e*(d + e*x))) + (3*Sqrt[c]*(b + 2*c*x)*(a + b*x
+ c*x^2)^(1/4))/(Sqrt[b^2 - 4*a*c]*e^2*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])/Sqrt[b^2 - 4*a*c])) - (3*(-b^2 + 4*a*c)^(1/4)*(2*c*d - b*e)*(-((c*(a + b
*x + c*x^2))/(b^2 - 4*a*c)))^(1/4)*ArcTan[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1
- (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e
^2)^(1/4))])/(4*c^(1/4)*e^(5/2)*(c*d^2 - b*d*e + a*e^2)^(1/4)*(a + b*x + c*
x^2)^(1/4)) + (3*(-b^2 + 4*a*c)^(1/4)*(2*c*d - b*e)*(-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c)))^(1/4)*ArcTanh[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c
*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))
])/(4*c^(1/4)*e^(5/2)*(c*d^2 - b*d*e + a*e^2)^(1/4)*(a + b*x + c*x^2)^(1/4)
) - (3*c^(1/4)*(b^2 - 4*a*c)^(3/4)*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (
2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqr
t[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticE[2*ArcTan[(Sqrt[2]*c^(1/4)*
(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(Sqrt[2]*e^2*(b + 2*c*
x)) + (3*c^(1/4)*(b^2 - 4*a*c)^(3/4)*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 +
 (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*S
qrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4
)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(2*Sqrt[2]*e^2*(b +
2*c*x)) + (3*Sqrt[-b^2 + 4*a*c]*(2*c*d - b*e)^2*Sqrt[(b + 2*c*x)^2/(b^2 - 4
*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(1/4)*EllipticPi[-(Sqrt[-b^
2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c
*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(4*Sqrt[2]*Sqrt[c]*e^3*Sqrt[c*d^2 - b*d*e
 + a*e^2]*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4)) - (3*Sqrt[-b^2 + 4*a*c]*(2*c
*d - b*e)^2*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2
 - 4*a*c)))^(1/4)*EllipticPi[(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 -
 b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(4*
Sqrt[2]*Sqrt[c]*e^3*Sqrt[c*d^2 - b*d*e + a*e^2]*(b + 2*c*x)*(a + b*x + c*x^
2)^(1/4))
```

Rubi [A] time = 2.17902, antiderivative size = 1220, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 18, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {732, 843, 623, 305, 220, 1196, 749, 748, 746, 399, 490, 1213, 537, 444, 63, 298, 205, 208}

$$\frac{3\sqrt{4ac-b^2}\sqrt{\frac{(b+2cx)^2}{b^2-4ac}}\sqrt[4]{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\Pi\left(-\frac{\sqrt{4ac-b^2}e}{2\sqrt{c}\sqrt{cd^2-bed+ae^2}};\sin^{-1}\left(\sqrt[4]{1-\frac{(b+2cx)^2}{b^2-4ac}}\right)\middle|-1\right)(2cd-be)^2}{4\sqrt{2}\sqrt{c}e^3\sqrt{cd^2-bed+ae^2}(b+2cx)\sqrt[4]{cx^2+bx+a}}-\frac{3\sqrt{4ac-b^2}\sqrt{\frac{(b+2cx)^2}{b^2-4ac}}\sqrt[4]{\cdots}}{4\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/4)/(d + e*x)^2,x]
```


```
[Out] -((a + b*x + c*x^2)^(3/4)/(e*(d + e*x))) + (3*Sqrt[c]*(b + 2*c*x)*(a + b*x
+ c*x^2)^(1/4))/(Sqrt[b^2 - 4*a*c]*e^2*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])/Sqrt[b^2 - 4*a*c])) - (3*(-b^2 + 4*a*c)^(1/4)*(2*c*d - b*e)*(-((c*(a + b
*x + c*x^2))/(b^2 - 4*a*c)))^(1/4)*ArcTan[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1
- (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e
```

```
^2)^(1/4))])/(4*c^(1/4)*e^(5/2)*(c*d^2 - b*d*e + a*e^2)^(1/4)*(a + b*x + c*
x^2)^(1/4)) + (3*(-b^2 + 4*a*c)^(1/4)*(2*c*d - b*e)*(-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c)))^(1/4)*ArcTanh[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c
*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))
])/(4*c^(1/4)*e^(5/2)*(c*d^2 - b*d*e + a*e^2)^(1/4)*(a + b*x + c*x^2)^(1/4)
) - (3*c^(1/4)*(b^2 - 4*a*c)^(3/4)*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (
2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqr
t[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticE[2*ArcTan[(Sqrt[2]*c^(1/4)*
(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(Sqrt[2]*e^2*(b + 2*c*
x)) + (3*c^(1/4)*(b^2 - 4*a*c)^(3/4)*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 +
 (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*S
qrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4
)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(2*Sqrt[2]*e^2*(b +
2*c*x)) + (3*Sqrt[-b^2 + 4*a*c]*(2*c*d - b*e)^2*Sqrt[(b + 2*c*x)^2/(b^2 - 4
*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(1/4)*EllipticPi[-(Sqrt[-b^
2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c
*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(4*Sqrt[2]*Sqrt[c]*e^3*Sqrt[c*d^2 - b*d*e
 + a*e^2]*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4)) - (3*Sqrt[-b^2 + 4*a*c]*(2*c
*d - b*e)^2*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2
 - 4*a*c)))^(1/4)*EllipticPi[(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 -
 b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(4*
Sqrt[2]*Sqrt[c]*e^3*Sqrt[c*d^2 - b*d*e + a*e^2]*(b + 2*c*x)*(a + b*x + c*x^
2)^(1/4))
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 749

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[(a + b*x + c*x^2)^p/(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^p, I
nt[(-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) - (c^2*x^2)/(b^2 - 4*a*c
))^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&  !GtQ[4*a - b^2/c
, 0] && IntegerQ[4*p]
```

Rule 748

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[1/((-4*c)/(b^2 - 4*a*c))^p, Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c),
 x]^p/Simp[2*c*d - b*e + e*x, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && GtQ[4*a - b^2/c, 0] && IntegerQ[4*p]
```

Rule 746

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
```

```
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/4}}{(d+ex)^2}\,dx = -\frac{\left(a+bx+cx^2\right)^{3/4}}{e(d+ex)} + \frac{3\int \frac{b+2cx}{(d+ex)\sqrt[4]{a+bx+cx^2}}\,dx}{4e}$$

$$= -\frac{\left(a+bx+cx^2\right)^{3/4}}{e(d+ex)} + \frac{(3c)\int \frac{1}{\sqrt[4]{a+bx+cx^2}}\,dx}{2e^2} - \frac{(3(2cd-be))\int \frac{1}{(d+ex)\sqrt[4]{a+bx+cx^2}}\,dx}{4e^2}$$

$$= -\frac{\left(a+bx+cx^2\right)^{3/4}}{e(d+ex)} + \frac{\left(6c\sqrt{(b+2cx)^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b^2-4ac+4cx^4}}\,dx, x, \sqrt[4]{a+bx+cx^2}\right)}{e^2(b+2cx)} - \frac{\left(3(2cd-\right.}{}$$

$$= -\frac{\left(a+bx+cx^2\right)^{3/4}}{e(d+ex)} + \frac{\left(3\sqrt{c}\sqrt{b^2-4ac}\sqrt{(b+2cx)^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b^2-4ac+4cx^4}}\,dx, x, \sqrt[4]{a+bx+cx^2}\right)}{e^2(b+2cx)}$$

$$= -\frac{\left(a+bx+cx^2\right)^{3/4}}{e(d+ex)} + \frac{3\sqrt{c}(b+2cx)\sqrt[4]{a+bx+cx^2}}{\sqrt{b^2-4ac}e^2\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)} - \frac{3\sqrt[4]{c}\left(b^2-4ac\right)^{3/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+c}}{\sqrt{b^2-4ac}}\right.}}}{}$$

$$= -\frac{\left(a+bx+cx^2\right)^{3/4}}{e(d+ex)} + \frac{3\sqrt{c}(b+2cx)\sqrt[4]{a+bx+cx^2}}{\sqrt{b^2-4ac}e^2\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)} - \frac{3\sqrt[4]{c}\left(b^2-4ac\right)^{3/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+c}}{\sqrt{b^2-4ac}}\right.}}}{}$$

$$= -\frac{\left(a+bx+cx^2\right)^{3/4}}{e(d+ex)} + \frac{3\sqrt{c}(b+2cx)\sqrt[4]{a+bx+cx^2}}{\sqrt{b^2-4ac}e^2\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)} - \frac{3\sqrt[4]{c}\left(b^2-4ac\right)^{3/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+c}}{\sqrt{b^2-4ac}}\right.}}}{}$$

$$= -\frac{\left(a+bx+cx^2\right)^{3/4}}{e(d+ex)} + \frac{3\sqrt{c}(b+2cx)\sqrt[4]{a+bx+cx^2}}{\sqrt{b^2-4ac}e^2\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)} - \frac{3\sqrt[4]{c}\left(b^2-4ac\right)^{3/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+c}}{\sqrt{b^2-4ac}}\right.}}}{}$$

$$= -\frac{\left(a+bx+cx^2\right)^{3/4}}{e(d+ex)} + \frac{3\sqrt{c}(b+2cx)\sqrt[4]{a+bx+cx^2}}{\sqrt{b^2-4ac}e^2\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)} - \frac{3\sqrt[4]{-b^2+4ac}(2cd-be)\sqrt[4]{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\ \text{t}}{4\sqrt[4]{c}e^{5/2}\sqrt[4]{cd^2-bde+ae^2}}$$

Mathematica [C] time = 0.33911, size = 185, normalized size = 0.15

$$\frac{4\sqrt{2}(a+x(b+cx))^{3/4}F_1\left(-\frac{1}{2};-\frac{3}{4},-\frac{3}{4};\frac{1}{2};\frac{2cd-\left(b+\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2cd-be+\sqrt{b^2-4ace}}{2cd+2cex}\right)}{e(d+ex)\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{3/4}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)^(3/4)/(d + e*x)^2,x]
```

```
[Out] (4*Sqrt[2]*(a + x*(b + c*x))^(3/4)*AppellF1[-1/2, -3/4, -3/4, 1/2, (2*c*d -
 (b + Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x)), (2*c*d - b*e + Sqrt[b^2 - 4*a*
c]*e)/(2*c*d + 2*c*e*x)])/(e*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e
*x)))^(3/4)*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(3/4)*(d +
e*x))
```

Maple [F] time = 1.217, size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex+d\right)^{2}} \left(c\,x^{2}+bx+a\right)^{\frac{3}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/4)/(e*x+d)^2,x)
```

```
[Out] int((c*x^2+b*x+a)^(3/4)/(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2}+bx+a\right)^{\frac{3}{4}}}{\left(ex+d\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/4)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/4)/(e*x + d)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/4)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx+cx^{2}\right)^{\frac{3}{4}}}{\left(d+ex\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/4)/(e*x+d)**2,x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/4)/(d + e*x)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/4)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.2522 $\int (d+ex)^3 \left(a+bx+cx^2\right)^{5/4} dx$

Optimal. Leaf size=448

$$\frac{5\left(b^2-4ac\right)^{9/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)(2cd-be)\left(-4ce(6ae+11bd)+17b^2e^2+44c^2d^2\right)\text{EllipticF}}{14784\sqrt{2}c^{21/4}(b+2cx)}$$

```
[Out] (-5*(b^2 - 4*a*c)*(2*c*d - b*e)*(44*c^2*d^2 + 17*b^2*e^2 - 4*c*e*(11*b*d +
6*a*e))*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4))/(7392*c^5) + ((2*c*d - b*e)*(4
4*c^2*d^2 + 17*b^2*e^2 - 4*c*e*(11*b*d + 6*a*e))*(b + 2*c*x)*(a + b*x + c*x
^2)^(5/4))/(616*c^4) + (2*e*(d + e*x)^2*(a + b*x + c*x^2)^(9/4))/(13*c) + (
e*(1320*c^2*d^2 + 221*b^2*e^2 - 2*c*e*(507*b*d + 88*a*e) + 306*c*e*(2*c*d -
 b*e)*x)*(a + b*x + c*x^2)^(9/4))/(2574*c^3) + (5*(b^2 - 4*a*c)^(9/4)*(2*c*
d - b*e)*(44*c^2*d^2 + 17*b^2*e^2 - 4*c*e*(11*b*d + 6*a*e))*Sqrt[(b + 2*c*x
)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c]
)^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2
*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2
])/(14784*Sqrt[2]*c^(21/4)*(b + 2*c*x))
```

Rubi [A] time = 0.559006, antiderivative size = 448, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {742, 779, 612, 623, 220}

$$-\frac{5\left(b^2-4ac\right)(b+2cx)\sqrt[4]{a+bx+cx^2}(2cd-be)\left(-4ce(6ae+11bd)+17b^2e^2+44c^2d^2\right)}{7392c^5}+\frac{e\left(a+bx+cx^2\right)^{9/4}\left(-2ce(88a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*x + c*x^2)^(5/4),x]
```


```
[Out] (-5*(b^2 - 4*a*c)*(2*c*d - b*e)*(44*c^2*d^2 + 17*b^2*e^2 - 4*c*e*(11*b*d +
6*a*e))*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4))/(7392*c^5) + ((2*c*d - b*e)*(4
4*c^2*d^2 + 17*b^2*e^2 - 4*c*e*(11*b*d + 6*a*e))*(b + 2*c*x)*(a + b*x + c*x
^2)^(5/4))/(616*c^4) + (2*e*(d + e*x)^2*(a + b*x + c*x^2)^(9/4))/(13*c) + (
e*(1320*c^2*d^2 + 221*b^2*e^2 - 2*c*e*(507*b*d + 88*a*e) + 306*c*e*(2*c*d -
 b*e)*x)*(a + b*x + c*x^2)^(9/4))/(2574*c^3) + (5*(b^2 - 4*a*c)^(9/4)*(2*c*
d - b*e)*(44*c^2*d^2 + 17*b^2*e^2 - 4*c*e*(11*b*d + 6*a*e))*Sqrt[(b + 2*c*x
)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c]
)^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2
*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2
])/(14784*Sqrt[2]*c^(21/4)*(b + 2*c*x))
```


Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int (d+ex)^3\left(a+bx+cx^2\right)^{5/4}dx = \frac{2e(d+ex)^2\left(a+bx+cx^2\right)^{9/4}}{13c}+\frac{2\int (d+ex)\left(\frac{1}{4}\left(26cd^2-9bde-8ae^2\right)+\frac{17}{4}e(2cd-be)x\right.}{13c}$$

$$= \frac{2e(d+ex)^2\left(a+bx+cx^2\right)^{9/4}}{13c}+\frac{e\left(1320c^2d^2+221b^2e^2-2ce(507bd+88ae)+306ce(2\right.}{2574c^3}$$

$$= \frac{(2cd-be)\left(44c^2d^2+17b^2e^2-4ce(11bd+6ae)\right)(b+2cx)\left(a+bx+cx^2\right)^{5/4}}{616c^4}+\frac{2e(d+ex}{}$$

$$= -\frac{5\left(b^2-4ac\right)(2cd-be)\left(44c^2d^2+17b^2e^2-4ce(11bd+6ae)\right)(b+2cx)\sqrt[4]{a+bx+cx^2}}{7392c^5}$$

$$= -\frac{5\left(b^2-4ac\right)(2cd-be)\left(44c^2d^2+17b^2e^2-4ce(11bd+6ae)\right)(b+2cx)\sqrt[4]{a+bx+cx^2}}{7392c^5}$$

$$= -\frac{5\left(b^2-4ac\right)(2cd-be)\left(44c^2d^2+17b^2e^2-4ce(11bd+6ae)\right)(b+2cx)\sqrt[4]{a+bx+cx^2}}{7392c^5}$$

Mathematica [A] time = 1.17966, size = 286, normalized size = 0.64

$$39(2cd-be)\left(-4ce(6ae+11bd)+17b^2e^2+44c^2d^2\right)\left(5\sqrt{2}\left(b^2-4ac\right)^{5/2}\left(\frac{c(a+x(b+cx))}{4ac-b^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right),2\right)+$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*x + c*x^2)^(5/4),x]
```

```
[Out] (88704*c^5*e*(d + e*x)^2*(a + x*(b + c*x))^3 + 224*c^3*e*(a + x*(b + c*x))^
3*(221*b^2*e^2 + 12*c^2*d*(110*d + 51*e*x) - 2*c*e*(507*b*d + 88*a*e + 153*
b*e*x)) + 39*(2*c*d - b*e)*(44*c^2*d^2 + 17*b^2*e^2 - 4*c*e*(11*b*d + 6*a*e
))*(2*c*(b + 2*c*x)*(32*a^2*c + a*(-5*b^2 + 44*b*c*x + 44*c^2*x^2) + x*(-5*
b^3 + 7*b^2*c*x + 24*b*c^2*x^2 + 12*c^3*x^3)) + 5*Sqrt[2]*(b^2 - 4*a*c)^(5/
2)*((c*(a + x*(b + c*x)))/(-b^2 + 4*a*c))^(3/4)*EllipticF[ArcSin[(b + 2*c*x
)/Sqrt[b^2 - 4*a*c]]/2, 2]))/(576576*c^6*(a + x*(b + c*x))^(3/4))
```

Maple [F] time = 1.085, size = 0, normalized size = 0.

$$\int (ex+d)^3 \left(cx^2+bx+a\right)^{\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x+a)^(5/4),x)
```

```
[Out] int((e*x+d)^3*(c*x^2+b*x+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{\frac{5}{4}} (ex+d)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/4)*(e*x + d)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(ce^3x^5+\left(3\,cde^2+be^3\right)x^4+ad^3+\left(3\,cd^2e+3\,bde^2+ae^3\right)x^3+\left(cd^3+3\,bd^2e+3\,ade^2\right)x^2+\left(bd^3+3\,ad^2e\right)x\right)(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((c*e^3*x^5 + (3*c*d*e^2 + b*e^3)*x^4 + a*d^3 + (3*c*d^2*e + 3*b*d*
e^2 + a*e^3)*x^3 + (c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*x^2 + (b*d^3 + 3*a*d^2*e
)*x)*(c*x^2 + b*x + a)^(1/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d+ex)^3 \left(a+bx+cx^2\right)^{\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x+a)**(5/4),x)
```

```
[Out] Integral((d + e*x)**3*(a + b*x + c*x**2)**(5/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx + a\right)^{\frac{5}{4}} (ex + d)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/4)*(e*x + d)^3, x)
```

3.2523 $\int (d+ex)^2 \left(a+bx+cx^2\right)^{5/4} dx$

Optimal. Leaf size=384

$$\frac{5\left(b^2-4ac\right)^{9/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)\left(-4ce(2ae+11bd)+13b^2e^2+44c^2d^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\right.\right.}{7392\sqrt{2}c^{17/4}(b+2cx)}$$

```
[Out] (-5*(b^2 - 4*a*c)*(44*c^2*d^2 + 13*b^2*e^2 - 4*c*e*(11*b*d + 2*a*e))*(b + 2
*c*x)*(a + b*x + c*x^2)^(1/4))/(3696*c^4) + ((44*c^2*d^2 + 13*b^2*e^2 - 4*c
*e*(11*b*d + 2*a*e))*(b + 2*c*x)*(a + b*x + c*x^2)^(5/4))/(308*c^3) + (13*e
*(2*c*d - b*e)*(a + b*x + c*x^2)^(9/4))/(99*c^2) + (2*e*(d + e*x)*(a + b*x
+ c*x^2)^(9/4))/(11*c) + (5*(b^2 - 4*a*c)^(9/4)*(44*c^2*d^2 + 13*b^2*e^2 -
4*c*e*(11*b*d + 2*a*e))*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*S
qrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x +
 c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x +
c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(7392*Sqrt[2]*c^(17/4)*(b + 2*c*x
))
```

Rubi [A] time = 0.541275, antiderivative size = 384, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {742, 640, 612, 623, 220}

$$-\frac{5\left(b^2-4ac\right)(b+2cx)\sqrt[4]{a+bx+cx^2}\left(-4ce(2ae+11bd)+13b^2e^2+44c^2d^2\right)}{3696c^4}+\frac{(b+2cx)\left(a+bx+cx^2\right)^{5/4}\left(-4ce(2ae+\right.}{308c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*x + c*x^2)^(5/4),x]
```


```
[Out] (-5*(b^2 - 4*a*c)*(44*c^2*d^2 + 13*b^2*e^2 - 4*c*e*(11*b*d + 2*a*e))*(b + 2
*c*x)*(a + b*x + c*x^2)^(1/4))/(3696*c^4) + ((44*c^2*d^2 + 13*b^2*e^2 - 4*c
*e*(11*b*d + 2*a*e))*(b + 2*c*x)*(a + b*x + c*x^2)^(5/4))/(308*c^3) + (13*e
*(2*c*d - b*e)*(a + b*x + c*x^2)^(9/4))/(99*c^2) + (2*e*(d + e*x)*(a + b*x
+ c*x^2)^(9/4))/(11*c) + (5*(b^2 - 4*a*c)^(9/4)*(44*c^2*d^2 + 13*b^2*e^2 -
4*c*e*(11*b*d + 2*a*e))*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*S
qrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x +
 c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x +
c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(7392*Sqrt[2]*c^(17/4)*(b + 2*c*x
))
```


Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int (d+ex)^2\left(a+bx+cx^2\right)^{5/4}\,dx = \frac{2e(d+ex)\left(a+bx+cx^2\right)^{9/4}}{11c}+\frac{2\int\left(\frac{1}{4}\left(22cd^2-4e\left(\frac{9bd}{4}+ae\right)\right)+\frac{13}{4}e(2cd-be)x\right)\left(a+b\right.}{11c}$$

$$=\frac{13e(2cd-be)\left(a+bx+cx^2\right)^{9/4}}{99c^2}+\frac{2e(d+ex)\left(a+bx+cx^2\right)^{9/4}}{11c}+\frac{\left(-\frac{13}{4}be(2cd-be)+\frac{1}{2}\right.}{}$$

$$=\frac{\left(44c^2d^2+13b^2e^2-4ce(11bd+2ae)\right)(b+2cx)\left(a+bx+cx^2\right)^{5/4}}{308c^3}+\frac{13e(2cd-be)\left(a+b\right.}{99c^2}$$

$$=-\frac{5\left(b^2-4ac\right)\left(44c^2d^2+13b^2e^2-4ce(11bd+2ae)\right)(b+2cx)\sqrt[4]{a+bx+cx^2}}{3696c^4}+\frac{\left(44c^2d^2\right.}{}$$

$$=-\frac{5\left(b^2-4ac\right)\left(44c^2d^2+13b^2e^2-4ce(11bd+2ae)\right)(b+2cx)\sqrt[4]{a+bx+cx^2}}{3696c^4}+\frac{\left(44c^2d^2\right.}{}$$

$$=-\frac{5\left(b^2-4ac\right)\left(44c^2d^2+13b^2e^2-4ce(11bd+2ae)\right)(b+2cx)\sqrt[4]{a+bx+cx^2}}{3696c^4}+\frac{\left(44c^2d^2\right.}{}$$

Mathematica [A] time = 0.604829, size = 233, normalized size = 0.61

$$\frac{2\left(-\frac{\left(ce(2ae+11bd)-\frac{13b^2e^2}{4}-11c^2d^2\right)\left(24c^2(b+2cx)(a+x(b+cx))^2-5\left(b^2-4ac\right)\left(2c(b+2cx)(a+x(b+cx))-\sqrt{2}\left(b^2-4ac\right)^{3/2}\left(\frac{c(a+x(b+cx))}{4ac-b^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right),\right.\right.\right.}{336c^4(a+x(b+cx))^{3/4}}\right.}{11c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*x + c*x^2)^(5/4),x]
```

```
[Out] (2*((13*e*(2*c*d - b*e)*(a + x*(b + c*x))^(9/4))/(18*c) + e*(d + e*x)*(a +
x*(b + c*x))^(9/4) - ((-11*c^2*d^2 - (13*b^2*e^2)/4 + c*e*(11*b*d + 2*a*e))
*(24*c^2*(b + 2*c*x)*(a + x*(b + c*x))^2 - 5*(b^2 - 4*a*c)*(2*c*(b + 2*c*x)
*(a + x*(b + c*x)) - Sqrt[2]*(b^2 - 4*a*c)^(3/2)*((c*(a + x*(b + c*x)))/(-b
^2 + 4*a*c))^(3/4)*EllipticF[ArcSin[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]/2, 2]))
/(336*c^4*(a + x*(b + c*x))^(3/4))))/(11*c)
```

Maple [F] time = 1.01, size = 0, normalized size = 0.

$$\int (ex+d)^2 \left(cx^2+bx+a\right)^{\frac{5}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+b*x+a)^(5/4),x)
```

```
[Out] int((e*x+d)^2*(c*x^2+b*x+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{\frac{5}{4}}(ex+d)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/4)*(e*x + d)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(ce^2x^4+\left(2\,cde+be^2\right)x^3+ad^2+\left(cd^2+2\,bde+ae^2\right)x^2+\left(bd^2+2\,ade\right)x\right)\left(cx^2+bx+a\right)^{\frac{1}{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((c*e^2*x^4 + (2*c*d*e + b*e^2)*x^3 + a*d^2 + (c*d^2 + 2*b*d*e + a*
e^2)*x^2 + (b*d^2 + 2*a*d*e)*x)*(c*x^2 + b*x + a)^(1/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d+ex)^2 \left(a+bx+cx^2\right)^{\frac{5}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+b*x+a)**(5/4),x)
```

```
[Out] Integral((d + e*x)**2*(a + b*x + c*x**2)**(5/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx + a\right)^{\frac{5}{4}} (ex + d)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/4)*(e*x + d)^2, x)
```

3.2524 $\int (d+ex)\left(a+bx+cx^2\right)^{5/4}\,dx$

Optimal. Leaf size=285

$$\frac{5\left(b^2-4ac\right)^{9/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)(2cd-be)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{a+bx+cx^2}}{\sqrt[4]{b^2-4ac}}\right),\frac{1}{2}\right)}{336\sqrt{2}c^{13/4}(b+2cx)}-\frac{5\left(b^2\right.}{}$$

```
[Out] (-5*(b^2 - 4*a*c)*(2*c*d - b*e)*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4))/(168*c
^3) + ((2*c*d - b*e)*(b + 2*c*x)*(a + b*x + c*x^2)^(5/4))/(14*c^2) + (2*e*(
a + b*x + c*x^2)^(9/4))/(9*c) + (5*(b^2 - 4*a*c)^(9/4)*(2*c*d - b*e)*Sqrt[(
b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2
 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*El
lipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1
/4)], 1/2])/(336*Sqrt[2]*c^(13/4)*(b + 2*c*x))
```

Rubi [A] time = 0.212182, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {640, 612, 623, 220}

$$-\frac{5\left(b^2-4ac\right)(b+2cx)\sqrt[4]{a+bx+cx^2}(2cd-be)}{168c^3}+\frac{5\left(b^2-4ac\right)^{9/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)(2cd-b}{336\sqrt{2}c^{13/4}(b+2cx)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*x + c*x^2)^(5/4),x]
```

```
[Out] (-5*(b^2 - 4*a*c)*(2*c*d - b*e)*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4))/(168*c
^3) + ((2*c*d - b*e)*(b + 2*c*x)*(a + b*x + c*x^2)^(5/4))/(14*c^2) + (2*e*(
a + b*x + c*x^2)^(9/4))/(9*c) + (5*(b^2 - 4*a*c)^(9/4)*(2*c*d - b*e)*Sqrt[(
b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2
 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*El
lipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1
/4)], 1/2])/(336*Sqrt[2]*c^(13/4)*(b + 2*c*x))
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
```

```
- 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
<= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int (d+ex)\left(a+bx+cx^2\right)^{5/4}dx = \frac{2e\left(a+bx+cx^2\right)^{9/4}}{9c} + \frac{(2cd-be)\int \left(a+bx+cx^2\right)^{5/4}dx}{2c}$$
$$= \frac{(2cd-be)(b+2cx)\left(a+bx+cx^2\right)^{5/4}}{14c^2} + \frac{2e\left(a+bx+cx^2\right)^{9/4}}{9c} - \frac{\left(5\left(b^2-4ac\right)(2cd-be)\right)}{56c^2}$$
$$= -\frac{5\left(b^2-4ac\right)(2cd-be)(b+2cx)\sqrt[4]{a+bx+cx^2}}{168c^3} + \frac{(2cd-be)(b+2cx)\left(a+bx+cx^2\right)^{5/4}}{14c^2}$$
$$= -\frac{5\left(b^2-4ac\right)(2cd-be)(b+2cx)\sqrt[4]{a+bx+cx^2}}{168c^3} + \frac{(2cd-be)(b+2cx)\left(a+bx+cx^2\right)^{5/4}}{14c^2}$$
$$= -\frac{5\left(b^2-4ac\right)(2cd-be)(b+2cx)\sqrt[4]{a+bx+cx^2}}{168c^3} + \frac{(2cd-be)(b+2cx)\left(a+bx+cx^2\right)^{5/4}}{14c^2}$$

Mathematica [A] time = 0.348044, size = 175, normalized size = 0.61

$$\frac{(2cd-be)\left(24c^2(b+2cx)(a+x(b+cx))^2-5\left(b^2-4ac\right)\left(2c(b+2cx)(a+x(b+cx))-\sqrt{2}\left(b^2-4ac\right)^{3/2}\left(\frac{c(a+x(b+cx))}{4ac-b^2}\right)^{3/4}\text{Ell}\right.\right.}{336c^4(a+x(b+cx))^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*x + c*x^2)^(5/4),x]
```

```
[Out] (2*e*(a + x*(b + c*x))^(9/4))/(9*c) + ((2*c*d - b*e)*(24*c^2*(b + 2*c*x)*(a
 + x*(b + c*x))^2 - 5*(b^2 - 4*a*c)*(2*c*(b + 2*c*x)*(a + x*(b + c*x)) - Sq
rt[2]*(b^2 - 4*a*c)^(3/2)*((c*(a + x*(b + c*x)))/(-b^2 + 4*a*c))^(3/4)*Elli
pticF[ArcSin[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]/2, 2])))/(336*c^4*(a + x*(b + c
*x))^(3/4))
```

Maple [F] time = 1.121, size = 0, normalized size = 0.

$$\int (ex+d)\left(cx^2+bx+a\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+b*x+a)^(5/4),x)
```

```
[Out] int((e*x+d)*(c*x^2+b*x+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{\frac{5}{4}}(ex+d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/4)*(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cex^3+(cd+be)x^2+ad+(bd+ae)x\right)\left(cx^2+bx+a\right)^{\frac{1}{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((c*e*x^3 + (c*d + b*e)*x^2 + a*d + (b*d + a*e)*x)*(c*x^2 + b*x + a
)^(1/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d+ex)\left(a+bx+cx^2\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+b*x+a)**(5/4),x)
```

```
[Out] Integral((d + e*x)*(a + b*x + c*x**2)**(5/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{\frac{5}{4}}(ex+d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/4)*(e*x + d), x)
```

3.2525 $\int \left(a+bx+cx^2\right)^{5/4} dx$

Optimal. Leaf size=236

$$\frac{5\left(b^2-4ac\right)^{9/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{a+bx+cx^2}}{\sqrt[4]{b^2-4ac}}\right),\frac{1}{2}\right)}{168\sqrt{2}c^{9/4}(b+2cx)}-\frac{5\left(b^2-4ac\right)(b+2c}{84c}$$

```
[Out] (-5*(b^2 - 4*a*c)*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4))/(84*c^2) + ((b + 2*c
*x)*(a + b*x + c*x^2)^(5/4))/(7*c) + (5*(b^2 - 4*a*c)^(9/4)*Sqrt[(b + 2*c*x
)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c]
)^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2
*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2
])/(168*Sqrt[2]*c^(9/4)*(b + 2*c*x))
```

Rubi [A] time = 0.157169, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {612, 623, 220}

$$-\frac{5\left(b^2-4ac\right)(b+2cx)\sqrt[4]{a+bx+cx^2}}{84c^2}+\frac{5\left(b^2-4ac\right)^{9/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{cx^2}}{\sqrt[4]{b^2-4}}\right.\right.}{168\sqrt{2}c^{9/4}(b+2cx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/4),x]
```

```
[Out] (-5*(b^2 - 4*a*c)*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4))/(84*c^2) + ((b + 2*c
*x)*(a + b*x + c*x^2)^(5/4))/(7*c) + (5*(b^2 - 4*a*c)^(9/4)*Sqrt[(b + 2*c*x
)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c]
)^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2
*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2
])/(168*Sqrt[2]*c^(9/4)*(b + 2*c*x))
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^{5/4} dx = \frac{(b+2cx)\left(a+bx+cx^2\right)^{5/4}}{7c} - \frac{\left(5\left(b^2-4ac\right)\right)\int \sqrt[4]{a+bx+cx^2}\,dx}{28c}$$

$$= -\frac{5\left(b^2-4ac\right)(b+2cx)\sqrt[4]{a+bx+cx^2}}{84c^2} + \frac{(b+2cx)\left(a+bx+cx^2\right)^{5/4}}{7c} + \frac{\left(5\left(b^2-4ac\right)^2\right)\int \frac{}{(a+b}}{336c^2}$$

$$= -\frac{5\left(b^2-4ac\right)(b+2cx)\sqrt[4]{a+bx+cx^2}}{84c^2} + \frac{(b+2cx)\left(a+bx+cx^2\right)^{5/4}}{7c} + \frac{\left(5\left(b^2-4ac\right)^2\sqrt{(b+}\right.}{}$$

$$= -\frac{5\left(b^2-4ac\right)(b+2cx)\sqrt[4]{a+bx+cx^2}}{84c^2} + \frac{(b+2cx)\left(a+bx+cx^2\right)^{5/4}}{7c} + \frac{5\left(b^2-4ac\right)^{9/4}\sqrt{\frac{}{(b^2-}}}{}$$

Mathematica [A] time = 0.377877, size = 125, normalized size = 0.53

$$\frac{\sqrt[4]{a+x(b+cx)}\left(2(b+2cx)\left(4c\left(8a+3cx^2\right)-5b^2+12bcx\right)-\frac{5\sqrt{2}\left(b^2-4ac\right)^{3/2}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right),2\right)}{\sqrt[4]{\frac{c(a+x(b+cx))}{4ac-b^2}}}\right)}{168c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/4),x]
```

```
[Out] ((a + x*(b + c*x))^(1/4)*(2*(b + 2*c*x)*(-5*b^2 + 12*b*c*x + 4*c*(8*a + 3*c
*x^2)) - (5*Sqrt[2]*(b^2 - 4*a*c)^(3/2)*EllipticF[ArcSin[(b + 2*c*x)/Sqrt[b
^2 - 4*a*c]]/2, 2])/((c*(a + x*(b + c*x)))/(-b^2 + 4*a*c))^(1/4)))/(168*c^2
)
```

Maple [F] time = 2.196, size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/4),x)
```

```
[Out] int((c*x^2+b*x+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2 + bx + a\right)^{\frac{5}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^(5/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + bx + cx^2\right)^{\frac{5}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/4),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(5/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx + a\right)^{\frac{5}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/4), x)
```

3.2526 $\int \frac{(a+bx+cx^2)^{5/4}}{d+ex}\,dx$

Optimal. Leaf size=1014

result too large to display

```
[Out] ((12*c^2*d^2 + b^2*e^2 - 2*c*e*(7*b*d - 6*a*e) - 2*c*e*(2*c*d - b*e)*x)*(a
+ b*x + c*x^2)^(1/4))/(6*c*e^3) + (2*(a + b*x + c*x^2)^(5/4))/(5*e) - ((-b^
2 + 4*a*c)^(3/4)*(c*d^2 - b*d*e + a*e^2)^(5/4)*(-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c)))^(3/4)*ArcTan[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/
(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(c^
(3/4)*e^(7/2)*(a + b*x + c*x^2)^(3/4)) - ((-b^2 + 4*a*c)^(3/4)*(c*d^2 - b*d
*e + a*e^2)^(5/4)*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)*ArcTanh[((
-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[
2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(c^(3/4)*e^(7/2)*(a + b*x + c*x
^2)^(3/4)) - ((b^2 - 4*a*c)^(1/4)*(2*c*d - b*e)*(12*c^2*d^2 - b^2*e^2 - 4*c
*e*(3*b*d - 4*a*e))*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[
a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x
^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^
2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(12*Sqrt[2]*c^(5/4)*e^4*(b + 2*c*x))
- ((b^2 - 4*a*c)*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[(b + 2*c*x)^2/(
b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)*EllipticPi[-(S
qrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (
b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(Sqrt[2]*c*e^4*(b + 2*c*x)*(a + b*
x + c*x^2)^(3/4)) - ((b^2 - 4*a*c)*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*Sq
rt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3
/4)*EllipticPi[(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2
]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(Sqrt[2]*c*e^4*(b
 + 2*c*x)*(a + b*x + c*x^2)^(3/4))
```

Rubi [A] time = 2.6939, antiderivative size = 1014, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 18, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {734, 814, 843, 623, 220, 749, 748, 747, 401, 108, 409, 1213, 537, 444, 63, 212, 208, 205}

$$-\frac{(4ac-b^2)^{3/4}\left(-\frac{c(cx^2+bx+a)}{b^2-4ac}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{4ac-b^2}\sqrt{e}\sqrt[4]{1-\frac{(b+2cx)^2}{b^2-4ac}}}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{cd^2-bed+ae^2}}\right)(cd^2-bed+ae^2)^{5/4}}{c^{3/4}e^{7/2}(cx^2+bx+a)^{3/4}}-\frac{(4ac-b^2)^{3/4}\left(-\frac{c(cx^2+bx+a)}{b^2-4ac}\right)^{3/4}\tanh}{c^{3/4}e^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/4)/(d + e*x),x]
```

```
[Out] ((12*c^2*d^2 + b^2*e^2 - 2*c*e*(7*b*d - 6*a*e) - 2*c*e*(2*c*d - b*e)*x)*(a
+ b*x + c*x^2)^(1/4))/(6*c*e^3) + (2*(a + b*x + c*x^2)^(5/4))/(5*e) - ((-b^
2 + 4*a*c)^(3/4)*(c*d^2 - b*d*e + a*e^2)^(5/4)*(-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c)))^(3/4)*ArcTan[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/
(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(c^
(3/4)*e^(7/2)*(a + b*x + c*x^2)^(3/4)) - ((-b^2 + 4*a*c)^(3/4)*(c*d^2 - b*d
*e + a*e^2)^(5/4)*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)*ArcTanh[((
-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[
2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(c^(3/4)*e^(7/2)*(a + b*x + c*x
^2)^(3/4)) - ((b^2 - 4*a*c)^(1/4)*(2*c*d - b*e)*(12*c^2*d^2 - b^2*e^2 - 4*c
*e*(3*b*d - 4*a*e))*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[
```

```
a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x
^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^
2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(12*Sqrt[2]*c^(5/4)*e^4*(b + 2*c*x))
- ((b^2 - 4*a*c)*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[(b + 2*c*x)^2/(
b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)*EllipticPi[-(S
qrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (
b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(Sqrt[2]*c*e^4*(b + 2*c*x)*(a + b*
x + c*x^2)^(3/4)) - ((b^2 - 4*a*c)*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*Sq
rt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3
/4)*EllipticPi[(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2
]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(Sqrt[2]*c*e^4*(b
 + 2*c*x)*(a + b*x + c*x^2)^(3/4))
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 749

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[(a + b*x + c*x^2)^p/(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^p, I
nt[(-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) - (c^2*x^2)/(b^2 - 4*a*c
))^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&  !GtQ[4*a - b^2/c
, 0] && IntegerQ[4*p]
```

Rule 748

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[1/((-4*c)/(b^2 - 4*a*c))^p, Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c),
 x]^p/Simp[2*c*d - b*e + e*x, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && GtQ[4*a - b^2/c, 0] && IntegerQ[4*p]
```

Rule 747

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[-((b*x^2)/a)]/(2*x), Subst[Int[1/(Sqrt[-((b*x)/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 108

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - (d*e
)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{5/4}}{d+ex}\,dx = \frac{2\left(a+bx+cx^2\right)^{5/4}}{5e} - \frac{\int \frac{(bd-2ae+(2cd-be)x)\sqrt[4]{a+bx+cx^2}}{d+ex}\,dx}{2e}$$

$$= \frac{\left(12c^2d^2+b^2e^2-2ce(7bd-6ae)-2ce(2cd-be)x\right)\sqrt[4]{a+bx+cx^2}}{6ce^3} + \frac{2\left(a+bx+cx^2\right)^{5/4}}{5e} + \cdots$$

$$= \frac{\left(12c^2d^2+b^2e^2-2ce(7bd-6ae)-2ce(2cd-be)x\right)\sqrt[4]{a+bx+cx^2}}{6ce^3} + \frac{2\left(a+bx+cx^2\right)^{5/4}}{5e} + \cdots$$

$$= \frac{\left(12c^2d^2+b^2e^2-2ce(7bd-6ae)-2ce(2cd-be)x\right)\sqrt[4]{a+bx+cx^2}}{6ce^3} + \frac{2\left(a+bx+cx^2\right)^{5/4}}{5e} - \cdots$$

$$= \frac{\left(12c^2d^2+b^2e^2-2ce(7bd-6ae)-2ce(2cd-be)x\right)\sqrt[4]{a+bx+cx^2}}{6ce^3} + \frac{2\left(a+bx+cx^2\right)^{5/4}}{5e} - \cdots$$

$$= \frac{\left(12c^2d^2+b^2e^2-2ce(7bd-6ae)-2ce(2cd-be)x\right)\sqrt[4]{a+bx+cx^2}}{6ce^3} + \frac{2\left(a+bx+cx^2\right)^{5/4}}{5e} - \cdots$$

$$= \frac{\left(12c^2d^2+b^2e^2-2ce(7bd-6ae)-2ce(2cd-be)x\right)\sqrt[4]{a+bx+cx^2}}{6ce^3} + \frac{2\left(a+bx+cx^2\right)^{5/4}}{5e} - \cdots$$

$$= \frac{\left(12c^2d^2+b^2e^2-2ce(7bd-6ae)-2ce(2cd-be)x\right)\sqrt[4]{a+bx+cx^2}}{6ce^3} + \frac{2\left(a+bx+cx^2\right)^{5/4}}{5e} - \cdots$$

$$= \frac{\left(12c^2d^2+b^2e^2-2ce(7bd-6ae)-2ce(2cd-be)x\right)\sqrt[4]{a+bx+cx^2}}{6ce^3} + \frac{2\left(a+bx+cx^2\right)^{5/4}}{5e} - \cdots$$

$$= \frac{\left(12c^2d^2+b^2e^2-2ce(7bd-6ae)-2ce(2cd-be)x\right)\sqrt[4]{a+bx+cx^2}}{6ce^3} + \frac{2\left(a+bx+cx^2\right)^{5/4}}{5e} - \cdots$$

$$= \frac{\left(12c^2d^2+b^2e^2-2ce(7bd-6ae)-2ce(2cd-be)x\right)\sqrt[4]{a+bx+cx^2}}{6ce^3} + \frac{2\left(a+bx+cx^2\right)^{5/4}}{5e} - \cdots$$

Mathematica [A] time = 4.87183, size = 705, normalized size = 0.7

$$\frac{\left(\frac{c(a+x(b+cx))}{4ac-b^2}\right)^{3/4}\left(-\sqrt{b^2-4ac}(be-2cd)\left(4ce(3bd-4ae)+b^2e^2-12c^2d^2\right)\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right),2\right)-\frac{6c\left(4ac-b^2\right)^{3/4}\left(e(ae\cdots\right.}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/4)/(d + e*x),x]
```

```
[Out] (2*(a + x*(b + c*x))^(5/4))/(5*e) + ((a + x*(b + c*x))^(1/4)*(b^2*e^2 + 4*c
^2*d*(3*d - e*x) + 2*c*e*(-7*b*d + 6*a*e + b*e*x)))/(6*c*e^3) + (((c*(a + x
*(b + c*x)))/(-b^2 + 4*a*c))^(3/4)*(-(Sqrt[b^2 - 4*a*c]*(-2*c*d + b*e)*(-12
*c^2*d^2 + b^2*e^2 + 4*c*e*(3*b*d - 4*a*e))*EllipticF[ArcSin[(b + 2*c*x)/Sq
rt[b^2 - 4*a*c]]/2, 2]) - (6*c*(-b^2 + 4*a*c)^(3/4)*(c*d^2 + e*(-(b*d) + a*
e))*(Sqrt[2]*c^(1/4)*Sqrt[e]*(c*d^2 + e*(-(b*d) + a*e))^(1/4)*(b + 2*c*x)*(
ArcTan[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*((c*(a + x*(b + c*x)))/(-b^2 + 4*a*c))
^(1/4))/(c^(1/4)*(c*d^2 + e*(-(b*d) + a*e))^(1/4))] + ArcTanh[((-b^2 + 4*a*
c)^(1/4)*Sqrt[e]*((c*(a + x*(b + c*x)))/(-b^2 + 4*a*c))^(1/4))/(c^(1/4)*(c*
d^2 + e*(-(b*d) + a*e))^(1/4))]) - (-b^2 + 4*a*c)^(1/4)*(-2*c*d + b*e)*Sqrt
[(b + 2*c*x)^2/(b^2 - 4*a*c)]*EllipticPi[-(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]
*Sqrt[c*d^2 + e*(-(b*d) + a*e)]), -ArcSin[Sqrt[2]*((c*(a + x*(b + c*x)))/(-
b^2 + 4*a*c))^(1/4)], -1] - (-b^2 + 4*a*c)^(1/4)*(-2*c*d + b*e)*Sqrt[(b + 2
*c*x)^2/(b^2 - 4*a*c)]*EllipticPi[(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*
d^2 + e*(-(b*d) + a*e)]), -ArcSin[Sqrt[2]*((c*(a + x*(b + c*x)))/(-b^2 + 4*
a*c))^(1/4)], -1]))/(b + 2*c*x)))/(6*Sqrt[2]*c^2*e^4*(a + x*(b + c*x))^(3/4
))
```

Maple [F] time = 1.305, size = 0, normalized size = 0.

$$\int \frac{1}{ex+d}\left(cx^2+bx+a\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/4)/(e*x+d),x)
```

```
[Out] int((c*x^2+b*x+a)^(5/4)/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{5}{4}}}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/4)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/4)/(e*x + d), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/4)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx + cx^2\right)^{\frac{5}{4}}}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/4)/(e*x+d),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(5/4)/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx + a\right)^{\frac{5}{4}}}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/4)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/4)/(e*x + d), x)
```

3.2527 $\int \frac{(a+bx+cx^2)^{5/4}}{(d+ex)^2} dx$

Optimal. Leaf size=975

$$\frac{5(b^2-4ac)\sqrt{\frac{(b+2cx)^2}{b^2-4ac}}\left(-\frac{c(cx^2+bx+a)}{b^2-4ac}\right)^{3/4}\Pi\left(-\frac{\sqrt{4ac-b^2}e}{2\sqrt{c}\sqrt{cd^2-bed+ae^2}};\sin^{-1}\left(\sqrt[4]{1-\frac{(b+2cx)^2}{b^2-4ac}}\right)\middle|-1\right)(2cd-be)^2}{4\sqrt{2}ce^4(b+2cx)(cx^2+bx+a)^{3/4}}+\frac{5(b^2-4ac)\sqrt{\frac{(b+2c}{b^2-4}}}{}$$

[Out] (-5*(3*c*d - 2*b*e - c*e*x)*(a + b*x + c*x^2)^(1/4))/(3*e^3) - (a + b*x + c*x^2)^(5/4)/(e*(d + e*x)) + (5*(-b^2 + 4*a*c)^(3/4)*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)^(1/4)*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)*ArcTan[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(4*c^(3/4)*e^(7/2)*(a + b*x + c*x^2)^(3/4)) + (5*(-b^2 + 4*a*c)^(3/4)*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)^(1/4)*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)*ArcTanh[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(4*c^(3/4)*e^(7/2)*(a + b*x + c*x^2)^(3/4)) + (5*(b^2 - 4*a*c)^(1/4)*(6*c^2*d^2 + b^2*e^2 - 2*c*e*(3*b*d - a*e))*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(6*Sqrt[2]*c^(1/4)*e^4*(b + 2*c*x)) + (5*(b^2 - 4*a*c)*(2*c*d - b*e)^2*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)*EllipticPi[-(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(4*Sqrt[2]*c*e^4*(b + 2*c*x)*(a + b*x + c*x^2)^(3/4)) + (5*(b^2 - 4*a*c)*(2*c*d - b*e)^2*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)*EllipticPi[(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(4*Sqrt[2]*c*e^4*(b + 2*c*x)*(a + b*x + c*x^2)^(3/4))

Rubi [A] time = 2.32835, antiderivative size = 975, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 18, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {732, 814, 843, 623, 220, 749, 748, 747, 401, 108, 409, 1213, 537, 444, 63, 212, 208, 205}

$$\frac{5(b^2-4ac)\sqrt{\frac{(b+2cx)^2}{b^2-4ac}}\left(-\frac{c(cx^2+bx+a)}{b^2-4ac}\right)^{3/4}\Pi\left(-\frac{\sqrt{4ac-b^2}e}{2\sqrt{c}\sqrt{cd^2-bed+ae^2}};\sin^{-1}\left(\sqrt[4]{1-\frac{(b+2cx)^2}{b^2-4ac}}\right)\middle|-1\right)(2cd-be)^2}{4\sqrt{2}ce^4(b+2cx)(cx^2+bx+a)^{3/4}}+\frac{5(b^2-4ac)\sqrt{\frac{(b+2c}{b^2-4}}}{}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2)^(5/4)/(d + e*x)^2,x]

[Out] (-5*(3*c*d - 2*b*e - c*e*x)*(a + b*x + c*x^2)^(1/4))/(3*e^3) - (a + b*x + c*x^2)^(5/4)/(e*(d + e*x)) + (5*(-b^2 + 4*a*c)^(3/4)*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)^(1/4)*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)*ArcTan[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(4*c^(3/4)*e^(7/2)*(a + b*x + c*x^2)^(3/4)) + (5*(-b^2 + 4*a*c)^(3/4)*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)^(1/4)*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)*ArcTanh[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/

```
4)*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(4*c^(3/4)*e^(7/2)*(a + b*x + c*x^2)^(3
/4)) + (5*(b^2 - 4*a*c)^(1/4)*(6*c^2*d^2 + b^2*e^2 - 2*c*e*(3*b*d - a*e))*S
qrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqr
t[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c
])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*
c)^(1/4)], 1/2])/(6*Sqrt[2]*c^(1/4)*e^4*(b + 2*c*x)) + (5*(b^2 - 4*a*c)*(2*
c*d - b*e)^2*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c)))^(3/4)*EllipticPi[-(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2
 - b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(
4*Sqrt[2]*c*e^4*(b + 2*c*x)*(a + b*x + c*x^2)^(3/4)) + (5*(b^2 - 4*a*c)*(2*
c*d - b*e)^2*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c)))^(3/4)*EllipticPi[(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2
- b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(4
*Sqrt[2]*c*e^4*(b + 2*c*x)*(a + b*x + c*x^2)^(3/4))
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 749

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[(a + b*x + c*x^2)^p/(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^p, I
nt[(-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) - (c^2*x^2)/(b^2 - 4*a*c
))^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&  !GtQ[4*a - b^2/c
, 0] && IntegerQ[4*p]
```

Rule 748

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[1/((-4*c)/(b^2 - 4*a*c))^p, Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c),
 x]^p/Simp[2*c*d - b*e + e*x, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && GtQ[4*a - b^2/c, 0] && IntegerQ[4*p]
```

Rule 747

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[-((b*x^2)/a)]/(2*x), Subst[Int[1/(Sqrt[-((b*x)/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 108

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - (d*e
)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{5/4}}{(d+ex)^2}\,dx = -\frac{\left(a+bx+cx^2\right)^{5/4}}{e(d+ex)} + \frac{5\int \frac{(b+2cx)\sqrt[4]{a+bx+cx^2}}{d+ex}\,dx}{4e}$$

$$= -\frac{5(3cd-2be-cex)\sqrt[4]{a+bx+cx^2}}{3e^3} - \frac{\left(a+bx+cx^2\right)^{5/4}}{e(d+ex)} - \frac{5\int \frac{c\left(2b^2de+4acde-3b\left(cd^2+ae^2\right)\right)-c\left(6c^2d^2+b^2e^2 \ldots}{(d+ex)\left(a+bx+cx^2\right)^{3/4}}}{12ce^3}$$

$$= -\frac{5(3cd-2be-cex)\sqrt[4]{a+bx+cx^2}}{3e^3} - \frac{\left(a+bx+cx^2\right)^{5/4}}{e(d+ex)} - \frac{\left(5(2cd-be)\left(cd^2-bde+ae^2\right)\right)\int \frac{\ldots}{(d+e \ldots}}{4e^4}$$

$$= -\frac{5(3cd-2be-cex)\sqrt[4]{a+bx+cx^2}}{3e^3} - \frac{\left(a+bx+cx^2\right)^{5/4}}{e(d+ex)} + \frac{\left(5\left(6c^2d^2+b^2e^2-2ce(3bd-ae)\right)\sqrt{(b \ldots}}{\ldots}$$

$$= -\frac{5(3cd-2be-cex)\sqrt[4]{a+bx+cx^2}}{3e^3} - \frac{\left(a+bx+cx^2\right)^{5/4}}{e(d+ex)} + \frac{5\sqrt[4]{b^2-4ac}\left(6c^2d^2+b^2e^2-2ce(3bd - \ldots\right.}{\ldots}$$

$$= -\frac{5(3cd-2be-cex)\sqrt[4]{a+bx+cx^2}}{3e^3} - \frac{\left(a+bx+cx^2\right)^{5/4}}{e(d+ex)} + \frac{5\sqrt[4]{b^2-4ac}\left(6c^2d^2+b^2e^2-2ce(3bd - \ldots\right.}{\ldots}$$

$$= -\frac{5(3cd-2be-cex)\sqrt[4]{a+bx+cx^2}}{3e^3} - \frac{\left(a+bx+cx^2\right)^{5/4}}{e(d+ex)} + \frac{5\sqrt[4]{b^2-4ac}\left(6c^2d^2+b^2e^2-2ce(3bd - \ldots\right.}{\ldots}$$

$$= -\frac{5(3cd-2be-cex)\sqrt[4]{a+bx+cx^2}}{3e^3} - \frac{\left(a+bx+cx^2\right)^{5/4}}{e(d+ex)} + \frac{5\sqrt[4]{b^2-4ac}\left(6c^2d^2+b^2e^2-2ce(3bd - \ldots\right.}{\ldots}$$

$$= -\frac{5(3cd-2be-cex)\sqrt[4]{a+bx+cx^2}}{3e^3} - \frac{\left(a+bx+cx^2\right)^{5/4}}{e(d+ex)} + \frac{5\sqrt[4]{b^2-4ac}\left(6c^2d^2+b^2e^2-2ce(3bd - \ldots\right.}{\ldots}$$

$$= -\frac{5(3cd-2be-cex)\sqrt[4]{a+bx+cx^2}}{3e^3} - \frac{\left(a+bx+cx^2\right)^{5/4}}{e(d+ex)} + \frac{5\left(-b^2+4ac\right)^{3/4}(2cd-be)\sqrt[4]{cd^2-bd \ldots}}{4c^{3/4} \ldots}$$

$$= -\frac{5(3cd-2be-cex)\sqrt[4]{a+bx+cx^2}}{3e^3} - \frac{\left(a+bx+cx^2\right)^{5/4}}{e(d+ex)} + \frac{5\left(-b^2+4ac\right)^{3/4}(2cd-be)\sqrt[4]{cd^2-bd \ldots}}{4c^{3/4} \ldots}$$

Mathematica [A] time = 4.71552, size = 663, normalized size = 0.68

$$5\left(\frac{c(a+x(b+cx))}{4ac-b^2}\right)^{3/4}\left(\sqrt{b^2-4ac}\left(2ce(ae-3bd)+b^2e^2+6c^2d^2\right)\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right),2\right)+\frac{3\left(4ac-b^2\right)^{3/4}(be-2cd)\left(-\sqrt{2}\sqrt[4]{c}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/4)/(d + e*x)^2,x]
```

```
[Out] (5*(-3*c*d + 2*b*e + c*e*x)*(a + x*(b + c*x))^(1/4))/(3*e^3) - (a + x*(b +
c*x))^(5/4)/(e*(d + e*x)) + (5*((c*(a + x*(b + c*x)))/(-b^2 + 4*a*c))^(3/4)
*(Sqrt[b^2 - 4*a*c]*(6*c^2*d^2 + b^2*e^2 + 2*c*e*(-3*b*d + a*e))*EllipticF[
ArcSin[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]/2, 2] + (3*(-b^2 + 4*a*c)^(3/4)*(-2*c
*d + b*e)*(-(Sqrt[2]*c^(1/4)*Sqrt[e]*(c*d^2 + e*(-(b*d) + a*e))^(1/4)*(b +
2*c*x)*(ArcTan[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*((c*(a + x*(b + c*x)))/(-b^2 +
 4*a*c))^(1/4))/(c^(1/4)*(c*d^2 + e*(-(b*d) + a*e))^(1/4))] + ArcTanh[((-b^
2 + 4*a*c)^(1/4)*Sqrt[e]*((c*(a + x*(b + c*x)))/(-b^2 + 4*a*c))^(1/4))/(c^(
1/4)*(c*d^2 + e*(-(b*d) + a*e))^(1/4))])) + (-b^2 + 4*a*c)^(1/4)*(-2*c*d +
b*e)*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*EllipticPi[-(Sqrt[-b^2 + 4*a*c]*e)/(
2*Sqrt[c]*Sqrt[c*d^2 + e*(-(b*d) + a*e)]), -ArcSin[Sqrt[2]*((c*(a + x*(b +
c*x)))/(-b^2 + 4*a*c))^(1/4)], -1] + (-b^2 + 4*a*c)^(1/4)*(-2*c*d + b*e)*Sq
rt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*EllipticPi[(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c
]*Sqrt[c*d^2 + e*(-(b*d) + a*e)]), -ArcSin[Sqrt[2]*((c*(a + x*(b + c*x)))/(
-b^2 + 4*a*c))^(1/4)], -1]))/(4*(b + 2*c*x))))/(3*Sqrt[2]*c*e^4*(a + x*(b +
 c*x))^(3/4))
```

Maple [F] time = 1.625, size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)^2}\left(cx^2+bx+a\right)^{\frac{5}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/4)/(e*x+d)^2,x)
```

```
[Out] int((c*x^2+b*x+a)^(5/4)/(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{5}{4}}}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/4)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/4)/(e*x + d)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/4)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/4)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{5}{4}}}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/4)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/4)/(e*x + d)^2, x)
```

3.2528 $\int \frac{(d+ex)^3}{\sqrt[4]{a+bx+cx^2}}\,dx$

Optimal. Leaf size=637

$$\frac{\left(b^2-4ac\right)^{3/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)(2cd-be)\left(-4ce(6ae+5bd)+11b^2e^2+20c^2d^2\right)\text{EllipticF}\left(2\right.}{40\sqrt{2}c^{15/4}(b+2cx)}$$

```
[Out] (2*e*(d + e*x)^2*(a + b*x + c*x^2)^(3/4))/(7*c) + (e*(360*c^2*d^2 + 77*b^2*
e^2 - 2*c*e*(147*b*d + 40*a*e) + 66*c*e*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^
(3/4))/(210*c^3) + ((2*c*d - b*e)*(20*c^2*d^2 + 11*b^2*e^2 - 4*c*e*(5*b*d +
 6*a*e))*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4))/(20*c^(7/2)*Sqrt[b^2 - 4*a*c]
*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])) - ((b^2 - 4*a*c
)^(3/4)*(2*c*d - b*e)*(20*c^2*d^2 + 11*b^2*e^2 - 4*c*e*(5*b*d + 6*a*e))*Sqr
t[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[
b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])
*EllipticE[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)
^(1/4)], 1/2])/(20*Sqrt[2]*c^(15/4)*(b + 2*c*x)) + ((b^2 - 4*a*c)^(3/4)*(2*
c*d - b*e)*(20*c^2*d^2 + 11*b^2*e^2 - 4*c*e*(5*b*d + 6*a*e))*Sqrt[(b + 2*c*
x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c
])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[
2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/
2])/(40*Sqrt[2]*c^(15/4)*(b + 2*c*x))
```

Rubi [A] time = 0.727933, antiderivative size = 637, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {742, 779, 623, 305, 220, 1196}

$$\frac{e\left(a+bx+cx^2\right)^{3/4}\left(-2ce(40ae+147bd)+77b^2e^2+66cex(2cd-be)+360c^2d^2\right)}{210c^3}+\frac{(b+2cx)\sqrt[4]{a+bx+cx^2}(2cd-be)\left(\right.}{20c^{7/2}\sqrt{b^2-4ac}\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + b*x + c*x^2)^(1/4),x]
```


```
[Out] (2*e*(d + e*x)^2*(a + b*x + c*x^2)^(3/4))/(7*c) + (e*(360*c^2*d^2 + 77*b^2*
e^2 - 2*c*e*(147*b*d + 40*a*e) + 66*c*e*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^
(3/4))/(210*c^3) + ((2*c*d - b*e)*(20*c^2*d^2 + 11*b^2*e^2 - 4*c*e*(5*b*d +
 6*a*e))*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4))/(20*c^(7/2)*Sqrt[b^2 - 4*a*c]
*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])) - ((b^2 - 4*a*c
)^(3/4)*(2*c*d - b*e)*(20*c^2*d^2 + 11*b^2*e^2 - 4*c*e*(5*b*d + 6*a*e))*Sqr
t[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[
b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])
*EllipticE[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)
^(1/4)], 1/2])/(20*Sqrt[2]*c^(15/4)*(b + 2*c*x)) + ((b^2 - 4*a*c)^(3/4)*(2*
c*d - b*e)*(20*c^2*d^2 + 11*b^2*e^2 - 4*c*e*(5*b*d + 6*a*e))*Sqrt[(b + 2*c*
x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c
])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[
2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/
2])/(40*Sqrt[2]*c^(15/4)*(b + 2*c*x))
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(d+ex)^3}{\sqrt[4]{a+bx+cx^2}}\,dx = \frac{2e(d+ex)^2\left(a+bx+cx^2\right)^{3/4}}{7c} + \frac{2\int \frac{(d+ex)\left(\frac{1}{4}(14cd^2-3bde-8ae^2)+\frac{11}{4}e(2cd-be)x\right)}{\sqrt[4]{a+bx+cx^2}}dx}{7c}$$

$$= \frac{2e(d+ex)^2\left(a+bx+cx^2\right)^{3/4}}{7c} + \frac{e\left(360c^2d^2+77b^2e^2-2ce(147bd+40ae)+66ce(2cd-be)x\right)\left(\right.}{210c^3}$$

$$= \frac{2e(d+ex)^2\left(a+bx+cx^2\right)^{3/4}}{7c} + \frac{e\left(360c^2d^2+77b^2e^2-2ce(147bd+40ae)+66ce(2cd-be)x\right)\left(\right.}{210c^3}$$

$$= \frac{2e(d+ex)^2\left(a+bx+cx^2\right)^{3/4}}{7c} + \frac{e\left(360c^2d^2+77b^2e^2-2ce(147bd+40ae)+66ce(2cd-be)x\right)\left(\right.}{210c^3}$$

$$= \frac{2e(d+ex)^2\left(a+bx+cx^2\right)^{3/4}}{7c} + \frac{e\left(360c^2d^2+77b^2e^2-2ce(147bd+40ae)+66ce(2cd-be)x\right)\left(\right.}{210c^3}$$

Mathematica [C] time = 0.306702, size = 211, normalized size = 0.33

$$\frac{\frac{21(b+2cx)\sqrt[4]{\frac{c(a+x(b+cx))}{4ac-b^2}}(2cd-be)\left(-4ce(6ae+5bd)+11b^2e^2+20c^2d^2\right){}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{8\sqrt{2}c^3} + \frac{e(a+x(b+cx))\left(-2ce(40ae+147bd+33bex)+77b^2e^2+12c^2d(30d+11\right.}{2c^2}}{105c\sqrt[4]{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + b*x + c*x^2)^(1/4),x]
```

```
[Out] (30*e*(d + e*x)^2*(a + x*(b + c*x)) + (e*(a + x*(b + c*x))*(77*b^2*e^2 + 12
*c^2*d*(30*d + 11*e*x) - 2*c*e*(147*b*d + 40*a*e + 33*b*e*x)))/(2*c^2) + (2
1*(2*c*d - b*e)*(20*c^2*d^2 + 11*b^2*e^2 - 4*c*e*(5*b*d + 6*a*e))*(b + 2*c*
x)*((c*(a + x*(b + c*x)))/(-b^2 + 4*a*c))^(1/4)*Hypergeometric2F1[1/4, 1/2,
 3/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(8*Sqrt[2]*c^3))/(105*c*(a + x*(b + c*x
))^(1/4))
```

Maple [F] time = 0.994, size = 0, normalized size = 0.

$$\int (ex+d)^3 \frac{1}{\sqrt[4]{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^2+b*x+a)^(1/4),x)
```

```
[Out] int((e*x+d)^3/(c*x^2+b*x+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3}{\left(cx^2+bx+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3/(c*x^2 + b*x + a)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^{3}x^{3} + 3\,de^{2}x^{2} + 3\,d^{2}ex + d^{3}}{\left(cx^{2} + bx + a\right)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)/(c*x^2 + b*x + a)^(1/4),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^{3}}{\sqrt[4]{a + bx + cx^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**2+b*x+a)**(1/4),x)
```

```
[Out] Integral((d + e*x)**3/(a + b*x + c*x**2)**(1/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^{3}}{\left(cx^{2} + bx + a\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3/(c*x^2 + b*x + a)^(1/4), x)
```

3.2529 $\int \frac{(d+ex)^2}{\sqrt[4]{a+bx+cx^2}}\,dx$

Optimal. Leaf size=573

$$\frac{\left(b^2-4ac\right)^{3/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)\left(-4ce(2ae+5bd)+7b^2e^2+20c^2d^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{}}{}\right.\right.}{20\sqrt{2}c^{11/4}(b+2cx)}$$

```
[Out] (7*e*(2*c*d - b*e)*(a + b*x + c*x^2)^(3/4))/(15*c^2) + (2*e*(d + e*x)*(a +
b*x + c*x^2)^(3/4))/(5*c) + ((20*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(5*b*d + 2*a*e
))*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4))/(10*c^(5/2)*Sqrt[b^2 - 4*a*c]*(1 +
(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])) - ((b^2 - 4*a*c)^(3/4
)*(20*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(5*b*d + 2*a*e))*Sqrt[(b + 2*c*x)^2/((b^2
 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1
+ (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticE[2*ArcTan[(
Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(10*Sq
rt[2]*c^(11/4)*(b + 2*c*x)) + ((b^2 - 4*a*c)^(3/4)*(20*c^2*d^2 + 7*b^2*e^2
- 4*c*e*(5*b*d + 2*a*e))*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x
+ c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x +
 c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(20*Sqrt[2]*c^(11/4)*(b + 2*c*x)
)
```

Rubi [A] time = 0.725086, antiderivative size = 573, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {742, 640, 623, 305, 220, 1196}

$$\frac{(b+2cx)\sqrt[4]{a+bx+cx^2}\left(-4ce(2ae+5bd)+7b^2e^2+20c^2d^2\right)}{10c^{5/2}\sqrt{b^2-4ac}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)}+\frac{\left(b^2-4ac\right)^{3/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*x + c*x^2)^(1/4),x]
```


```
[Out] (7*e*(2*c*d - b*e)*(a + b*x + c*x^2)^(3/4))/(15*c^2) + (2*e*(d + e*x)*(a +
b*x + c*x^2)^(3/4))/(5*c) + ((20*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(5*b*d + 2*a*e
))*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4))/(10*c^(5/2)*Sqrt[b^2 - 4*a*c]*(1 +
(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])) - ((b^2 - 4*a*c)^(3/4
)*(20*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(5*b*d + 2*a*e))*Sqrt[(b + 2*c*x)^2/((b^2
 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1
+ (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticE[2*ArcTan[(
Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(10*Sq
rt[2]*c^(11/4)*(b + 2*c*x)) + ((b^2 - 4*a*c)^(3/4)*(20*c^2*d^2 + 7*b^2*e^2
- 4*c*e*(5*b*d + 2*a*e))*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x
+ c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x +
 c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(20*Sqrt[2]*c^(11/4)*(b + 2*c*x)
)
```


Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\sqrt[4]{a+bx+cx^2}}\,dx = \frac{2e(d+ex)\left(a+bx+cx^2\right)^{3/4}}{5c} + \frac{2\int \frac{\frac{1}{4}\left(10cd^2-4e\left(\frac{3bd}{4}+ae\right)\right)+\frac{7}{4}e(2cd-be)x}{\sqrt[4]{a+bx+cx^2}}dx}{5c}$$

$$= \frac{7e(2cd-be)\left(a+bx+cx^2\right)^{3/4}}{15c^2} + \frac{2e(d+ex)\left(a+bx+cx^2\right)^{3/4}}{5c} + \frac{\left(-\frac{7}{4}be(2cd-be)+\frac{1}{2}c\left(10cd^2-\right.\right.}{5}$$

$$= \frac{7e(2cd-be)\left(a+bx+cx^2\right)^{3/4}}{15c^2} + \frac{2e(d+ex)\left(a+bx+cx^2\right)^{3/4}}{5c} + \frac{\left(4\left(-\frac{7}{4}be(2cd-be)+\frac{1}{2}c\left(10cd\right.\right.\right.}{}$$

$$= \frac{7e(2cd-be)\left(a+bx+cx^2\right)^{3/4}}{15c^2} + \frac{2e(d+ex)\left(a+bx+cx^2\right)^{3/4}}{5c} + \frac{\left(2\sqrt{b^2-4ac}\left(-\frac{7}{4}be(2cd-be)\right.\right.}{}$$

$$= \frac{7e(2cd-be)\left(a+bx+cx^2\right)^{3/4}}{15c^2} + \frac{2e(d+ex)\left(a+bx+cx^2\right)^{3/4}}{5c} + \frac{\left(20c^2d^2+7b^2e^2-4ce(5bd+2\right.}{10c^{5/2}\sqrt{b^2-4ac}\left(1\right.}$$

Mathematica [C] time = 0.206822, size = 165, normalized size = 0.29

$$\frac{\frac{3(b+2cx)\sqrt[4]{\frac{c(a+x(b+cx))}{4ac-b^2}}\left(-4ce(2ae+5bd)+7b^2e^2+20c^2d^2\right){}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{2\sqrt{2}c^2} - \frac{14e(a+x(b+cx))(be-2cd)}{c} + 12e(d+ex)(a+x(b+cx))}{30c\sqrt[4]{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*x + c*x^2)^(1/4),x]
```

```
[Out] ((-14*e*(-2*c*d + b*e)*(a + x*(b + c*x)))/c + 12*e*(d + e*x)*(a + x*(b + c*
x)) + (3*(20*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(5*b*d + 2*a*e))*(b + 2*c*x)*((c*(
a + x*(b + c*x)))/(-b^2 + 4*a*c))^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (b
 + 2*c*x)^2/(b^2 - 4*a*c)])/(2*Sqrt[2]*c^2))/(30*c*(a + x*(b + c*x))^(1/4))
```

Maple [F] time = 0.954, size = 0, normalized size = 0.

$$\int (ex+d)^2 \frac{1}{\sqrt[4]{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^2+b*x+a)^(1/4),x)
```

```
[Out] int((e*x+d)^2/(c*x^2+b*x+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2}{\left(cx^2+bx+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2/(c*x^2 + b*x + a)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^2x^2 + 2\,dex + d^2}{\left(cx^2 + bx + a\right)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)/(c*x^2 + b*x + a)^(1/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^2}{\sqrt[4]{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**2+b*x+a)**(1/4),x)
```

```
[Out] Integral((d + e*x)**2/(a + b*x + c*x**2)**(1/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^2}{\left(cx^2 + bx + a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2/(c*x^2 + b*x + a)^(1/4), x)
```

3.2530 $\int \frac{d+ex}{\sqrt[4]{a+bx+cx^2}}\,dx$

Optimal. Leaf size=469

$$\frac{\left(b^2-4ac\right)^{3/4}\sqrt{\frac{(b+2cx)^2}{(b^2-4ac)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)(2cd-be)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{a+bx+cx^2}}{\sqrt[4]{b^2-4ac}}\right),\frac{1}{2}\right)}{2\sqrt{2}c^{7/4}(b+2cx)}+\frac{(b+2c\ldots}{c^{3/2}\sqrt{b\ldots}}$$

```
[Out] (2*e*(a + b*x + c*x^2)^(3/4))/(3*c) + ((2*c*d - b*e)*(b + 2*c*x)*(a + b*x +
 c*x^2)^(1/4))/(c^(3/2)*Sqrt[b^2 - 4*a*c]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*
x^2])/Sqrt[b^2 - 4*a*c])) - ((b^2 - 4*a*c)^(3/4)*(2*c*d - b*e)*Sqrt[(b + 2*
c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a
*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*Elliptic
E[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)],
1/2])/(Sqrt[2]*c^(7/4)*(b + 2*c*x)) + ((b^2 - 4*a*c)^(3/4)*(2*c*d - b*e)*Sq
rt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt
[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c]
)*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c
)^(1/4)], 1/2])/(2*Sqrt[2]*c^(7/4)*(b + 2*c*x))
```

Rubi [A] time = 0.381251, antiderivative size = 469, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {640, 623, 305, 220, 1196}

$$\frac{(b+2cx)\sqrt[4]{a+bx+cx^2}(2cd-be)}{c^{3/2}\sqrt{b^2-4ac}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)}+\frac{\left(b^2-4ac\right)^{3/4}\sqrt{\frac{(b+2cx)^2}{(b^2-4ac)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)(2cd-be)F\left(2\tan^{-1}\left(\ldots\right.\right.}{2\sqrt{2}c^{7/4}(b+2cx)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*x + c*x^2)^(1/4),x]
```


```
[Out] (2*e*(a + b*x + c*x^2)^(3/4))/(3*c) + ((2*c*d - b*e)*(b + 2*c*x)*(a + b*x +
 c*x^2)^(1/4))/(c^(3/2)*Sqrt[b^2 - 4*a*c]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*
x^2])/Sqrt[b^2 - 4*a*c])) - ((b^2 - 4*a*c)^(3/4)*(2*c*d - b*e)*Sqrt[(b + 2*
c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a
*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*Elliptic
E[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)],
1/2])/(Sqrt[2]*c^(7/4)*(b + 2*c*x)) + ((b^2 - 4*a*c)^(3/4)*(2*c*d - b*e)*Sq
rt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt
[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c]
)*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c
)^(1/4)], 1/2])/(2*Sqrt[2]*c^(7/4)*(b + 2*c*x))
```


Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{d+ex}{\sqrt[4]{a+bx+cx^2}}\,dx = \frac{2e\left(a+bx+cx^2\right)^{3/4}}{3c} + \frac{(2cd-be)\int \frac{1}{\sqrt[4]{a+bx+cx^2}}\,dx}{2c}$$

$$= \frac{2e\left(a+bx+cx^2\right)^{3/4}}{3c} + \frac{\left(2(2cd-be)\sqrt{(b+2cx)^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b^2-4ac+4cx^4}}\,dx, x, \sqrt[4]{a+bx+cx^2}\right)}{c(b+2cx)}$$

$$= \frac{2e\left(a+bx+cx^2\right)^{3/4}}{3c} + \frac{\left(\sqrt{b^2-4ac}(2cd-be)\sqrt{(b+2cx)^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b^2-4ac+4cx^4}}\,dx, x, \sqrt[4]{a+bx+\ldots}\right.}{c^{3/2}(b+2cx)}$$

$$= \frac{2e\left(a+bx+cx^2\right)^{3/4}}{3c} + \frac{(2cd-be)(b+2cx)\sqrt[4]{a+bx+cx^2}}{c^{3/2}\sqrt{b^2-4ac}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)} - \frac{\left(b^2-4ac\right)^{3/4}(2cd-be)\sqrt{\frac{(b+2c\ldots}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{\ldots}}{\ldots}\right.}}\ldots}{\ldots}$$

Mathematica [C] time = 0.122807, size = 112, normalized size = 0.24

$$\frac{3\sqrt{2}(b+2cx)\sqrt[4]{\frac{c(a+x(b+cx))}{4ac-b^2}}(2cd-be)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)+8ce(a+x(b+cx))}{12c^2\sqrt[4]{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*x + c*x^2)^(1/4),x]
```

```
[Out] (8*c*e*(a + x*(b + c*x)) + 3*Sqrt[2]*(2*c*d - b*e)*(b + 2*c*x)*((c*(a + x*(
b + c*x)))/(-b^2 + 4*a*c))^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (b + 2*c*
x)^2/(b^2 - 4*a*c)])/(12*c^2*(a + x*(b + c*x))^(1/4))
```

Maple [F] time = 1.037, size = 0, normalized size = 0.

$$\int (ex+d)\frac{1}{\sqrt[4]{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^2+b*x+a)^(1/4),x)
```

```
[Out] int((e*x+d)/(c*x^2+b*x+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex+d}{(cx^2+bx+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)/(c*x^2 + b*x + a)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{ex+d}{(cx^2+bx+a)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)/(c*x^2 + b*x + a)^(1/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d+ex}{\sqrt[4]{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**2+b*x+a)**(1/4),x)
```

```
[Out] Integral((d + e*x)/(a + b*x + c*x**2)**(1/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex+d}{\left(cx^2+bx+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)/(c*x^2 + b*x + a)^(1/4), x)
```

3.2531 $\int \frac{1}{\sqrt[4]{a+bx+cx^2}}\,dx$

Optimal. Leaf size=418

$$\frac{\left(b^2-4ac\right)^{3/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{a+bx+cx^2}}{\sqrt[4]{b^2-4ac}}\right),\frac{1}{2}\right)}{\sqrt{2}c^{3/4}(b+2cx)}-\frac{\sqrt{2}\left(b^2-4ac\right)^{3/4}}{}$$

```
[Out] (2*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4))/(Sqrt[c]*Sqrt[b^2 - 4*a*c]*(1 + (2*
Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])) - (Sqrt[2]*(b^2 - 4*a*c)
^(3/4)*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x
^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2
 - 4*a*c])*EllipticE[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^
2 - 4*a*c)^(1/4)], 1/2])/(c^(3/4)*(b + 2*c*x)) + ((b^2 - 4*a*c)^(3/4)*Sqrt[
(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^
2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*E
llipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(
1/4)], 1/2])/(Sqrt[2]*c^(3/4)*(b + 2*c*x))
```

Rubi [A] time = 0.305418, antiderivative size = 418, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {623, 305, 220, 1196}

$$\frac{\left(b^2-4ac\right)^{3/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{cx^2+bx+a}}{\sqrt[4]{b^2-4ac}}\right)|\frac{1}{2}\right)}{\sqrt{2}c^{3/4}(b+2cx)}-\frac{\sqrt{2}\left(b^2-4ac\right)^{3/4}\sqrt{\frac{}{\left(b^2-4ac\right)}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(-1/4),x]
```


```
[Out] (2*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4))/(Sqrt[c]*Sqrt[b^2 - 4*a*c]*(1 + (2*
Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])) - (Sqrt[2]*(b^2 - 4*a*c)
^(3/4)*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x
^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2
 - 4*a*c])*EllipticE[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^
2 - 4*a*c)^(1/4)], 1/2])/(c^(3/4)*(b + 2*c*x)) + ((b^2 - 4*a*c)^(3/4)*Sqrt[
(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^
2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*E
llipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(
1/4)], 1/2])/(Sqrt[2]*c^(3/4)*(b + 2*c*x))
```


Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+bx+cx^2}}\,dx = \frac{\left(4\sqrt{(b+2cx)^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b^2-4ac+4cx^4}}\,dx, x, \sqrt[4]{a+bx+cx^2}\right)}{b+2cx}$$

$$= \frac{\left(2\sqrt{b^2-4ac}\sqrt{(b+2cx)^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b^2-4ac+4cx^4}}\,dx, x, \sqrt[4]{a+bx+cx^2}\right)}{\sqrt{c}(b+2cx)} - \frac{\left(2\sqrt{b^2-4ac}\sqrt{(b+2cx)^2}\right.}{}$$

$$= \frac{2(b+2cx)\sqrt[4]{a+bx+cx^2}}{\sqrt{c}\sqrt{b^2-4ac}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)} - \frac{\sqrt{2}\left(b^2-4ac\right)^{3/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)E}{c^{3/4}(b+2cx)}$$

Mathematica [C] time = 0.0333465, size = 84, normalized size = 0.2

$$\frac{(b+2cx)\sqrt[4]{\frac{c(a+x(b+cx))}{4ac-b^2}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)}{\sqrt{2}c\sqrt[4]{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(-1/4),x]
```

```
[Out] ((b + 2*c*x)*((c*(a + x*(b + c*x)))/(-b^2 + 4*a*c))^(1/4)*Hypergeometric2F1
[1/4, 1/2, 3/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(Sqrt[2]*c*(a + x*(b + c*x))^
(1/4))
```

Maple [F] time = 2.052, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(1/4),x)
```

```
[Out] int(1/(c*x^2+b*x+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(-1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\left(cx^2+bx+a\right)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^(-1/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(1/4),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(-1/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(-1/4), x)
```

3.2532 $\int \frac{1}{(d+ex)\sqrt[4]{a+bx+cx^2}}\,dx$

Optimal. Leaf size=733

$$\frac{\sqrt[4]{4ac-b^2}\sqrt[4]{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{e}\sqrt[4]{4ac-b^2}\sqrt[4]{1-\frac{(b+2cx)^2}{b^2-4ac}}}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{ae^2-bde+cd^2}}\right)}{\sqrt[4]{c}\sqrt{e}\sqrt[4]{a+bx+cx^2}\sqrt[4]{ae^2-bde+cd^2}}-\frac{\sqrt[4]{4ac-b^2}\sqrt[4]{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt[4]{4ac-b^2}\sqrt[4]{1-\frac{(b+2cx)^2}{b^2-4ac}}}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{ae^2-bde+cd^2}}\right)}{\sqrt[4]{c}\sqrt{e}\sqrt[4]{a+bx+cx^2}\sqrt[4]{ae^2-bde+cd^2}}-\frac{\sqrt{4ac-}}{}$$

```
[Out] ((-b^2 + 4*a*c)^(1/4)*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(1/4)*ArcTan
[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sq
rt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(c^(1/4)*Sqrt[e]*(c*d^2 - b*
d*e + a*e^2)^(1/4)*(a + b*x + c*x^2)^(1/4)) - ((-b^2 + 4*a*c)^(1/4)*(-((c*(
a + b*x + c*x^2))/(b^2 - 4*a*c)))^(1/4)*ArcTanh[((-b^2 + 4*a*c)^(1/4)*Sqrt[
e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e
 + a*e^2)^(1/4))])/(c^(1/4)*Sqrt[e]*(c*d^2 - b*d*e + a*e^2)^(1/4)*(a + b*x
+ c*x^2)^(1/4)) - (Sqrt[-b^2 + 4*a*c]*(2*c*d - b*e)*Sqrt[(b + 2*c*x)^2/(b^2
 - 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(1/4)*EllipticPi[-(Sqrt
[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (b +
 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(Sqrt[2]*Sqrt[c]*e*Sqrt[c*d^2 - b*d*e
 + a*e^2]*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4)) + (Sqrt[-b^2 + 4*a*c]*(2*c*d
 - b*e)*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 - 4
*a*c)))^(1/4)*EllipticPi[(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*d
*e + a*e^2]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(Sqrt[2
]*Sqrt[c]*e*Sqrt[c*d^2 - b*d*e + a*e^2]*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4)
)
```

Rubi [A] time = 1.55996, antiderivative size = 733, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {749, 748, 746, 399, 490, 1213, 537, 444, 63, 298, 205, 208}

$$\frac{\sqrt[4]{4ac-b^2}\sqrt[4]{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{e}\sqrt[4]{4ac-b^2}\sqrt[4]{1-\frac{(b+2cx)^2}{b^2-4ac}}}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{ae^2-bde+cd^2}}\right)}{\sqrt[4]{c}\sqrt{e}\sqrt[4]{a+bx+cx^2}\sqrt[4]{ae^2-bde+cd^2}}-\frac{\sqrt[4]{4ac-b^2}\sqrt[4]{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt[4]{4ac-b^2}\sqrt[4]{1-\frac{(b+2cx)^2}{b^2-4ac}}}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{ae^2-bde+cd^2}}\right)}{\sqrt[4]{c}\sqrt{e}\sqrt[4]{a+bx+cx^2}\sqrt[4]{ae^2-bde+cd^2}}-\frac{\sqrt{4ac-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + b*x + c*x^2)^(1/4)),x]
```

```
[Out] ((-b^2 + 4*a*c)^(1/4)*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(1/4)*ArcTan
[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sq
rt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(c^(1/4)*Sqrt[e]*(c*d^2 - b*
d*e + a*e^2)^(1/4)*(a + b*x + c*x^2)^(1/4)) - ((-b^2 + 4*a*c)^(1/4)*(-((c*(
a + b*x + c*x^2))/(b^2 - 4*a*c)))^(1/4)*ArcTanh[((-b^2 + 4*a*c)^(1/4)*Sqrt[
e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e
 + a*e^2)^(1/4))])/(c^(1/4)*Sqrt[e]*(c*d^2 - b*d*e + a*e^2)^(1/4)*(a + b*x
+ c*x^2)^(1/4)) - (Sqrt[-b^2 + 4*a*c]*(2*c*d - b*e)*Sqrt[(b + 2*c*x)^2/(b^2
 - 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(1/4)*EllipticPi[-(Sqrt
[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (b +
 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(Sqrt[2]*Sqrt[c]*e*Sqrt[c*d^2 - b*d*e
 + a*e^2]*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4)) + (Sqrt[-b^2 + 4*a*c]*(2*c*d
 - b*e)*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 - 4
*a*c)))^(1/4)*EllipticPi[(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*d
*e + a*e^2]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(Sqrt[2
```

```
]*Sqrt[c]*e*Sqrt[c*d^2 - b*d*e + a*e^2]*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4)
)
```

Rule 749

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[(a + b*x + c*x^2)^p/(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^p, I
nt[(-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) - (c^2*x^2)/(b^2 - 4*a*c
))^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&  !GtQ[4*a - b^2/c
, 0] && IntegerQ[4*p]
```

Rule 748

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[1/((-4*c)/(b^2 - 4*a*c))^p, Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c),
 x]^p/Simp[2*c*d - b*e + e*x, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && GtQ[4*a - b^2/c, 0] && IntegerQ[4*p]
```

Rule 746

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)\sqrt[4]{a+bx+cx^2}}\,dx = \frac{\sqrt[4]{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\int \frac{1}{(d+ex)\sqrt[4]{-\frac{ac}{b^2-4ac}-\frac{bcx}{b^2-4ac}-\frac{c^2x^2}{b^2-4ac}}}\,dx}{\sqrt[4]{a+bx+cx^2}}$$

$$= \frac{\left(\sqrt{2}\sqrt[4]{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\left(-\frac{c(2cd-be)}{b^2-4ac}+ex\right)\sqrt[4]{1-\frac{(b^2-4ac)x^2}{c^2}}}\,dx, x, -\frac{bc}{b^2-4ac}-\frac{2c^2x}{b^2-4ac}\right)}{\sqrt[4]{a+bx+cx^2}}$$

$$= -\frac{\left(\sqrt{2}e\sqrt[4]{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{x}{\sqrt[4]{1-\frac{(b^2-4ac)x^2}{c^2}}\left(\frac{c^2(2cd-be)^2}{(b^2-4ac)^2}-e^2x^2\right)}\,dx, x, -\frac{bc}{b^2-4ac}-\frac{2c^2x}{b^2-4ac}\right)}{\sqrt[4]{a+bx+cx^2}} - \ldots$$

$$= -\frac{\left(e\sqrt[4]{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{(b^2-4ac)x}{c^2}}\left(\frac{c^2(2cd-be)^2}{(b^2-4ac)^2}-e^2x\right)}\,dx, x, \left(-\frac{bc}{b^2-4ac}-\frac{2c^2x}{b^2-4ac}\right)^2\right)}{\sqrt{2}\sqrt[4]{a+bx+cx^2}} - \ldots$$

$$= \frac{\left(2\sqrt{2}c^2e\sqrt[4]{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{x^2}{-\frac{c^2e^2}{b^2-4ac}+\frac{c^2(2cd-be)^2}{(b^2-4ac)^2}+\frac{c^2e^2x^4}{b^2-4ac}}\,dx, x, \sqrt[4]{1-\frac{(b+2cx)^2}{b^2-4ac}}\right)}{(b^2-4ac)\sqrt[4]{a+bx+cx^2}} - \ldots$$

$$= \frac{\left(\sqrt{2}\left(-b^2+4ac\right)^{3/2}\sqrt[4]{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{2\sqrt{c}\sqrt{cd^2-bde+ae^2}-\sqrt{-b^2+4ac}ex^2}\,dx, x, \sqrt[4]{1-\ldots}\right.}{(b^2-4ac)\sqrt[4]{a+bx+cx^2}}$$

$$= \frac{\sqrt[4]{-b^2+4ac}\sqrt[4]{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt[4]{-b^2+4ac}\sqrt{e}\sqrt[4]{1-\frac{(b+2cx)^2}{b^2-4ac}}}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{cd^2-bde+ae^2}}\right)}{\sqrt[4]{c}\sqrt{e}\sqrt[4]{cd^2-bde+ae^2}\sqrt[4]{a+bx+cx^2}} - \frac{\sqrt[4]{-b^2+4ac}\sqrt[4]{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\ldots}{\sqrt[4]{c}\sqrt{e}\sqrt[4]{cd^2-bde\ldots}}$$

Mathematica [C] time = 0.326815, size = 178, normalized size = 0.24

$$-\frac{\sqrt{2}\sqrt[4]{\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}}\sqrt[4]{\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}}F_1\left(\frac{1}{2};\frac{1}{4},\frac{1}{4};\frac{3}{2};\frac{2cd-\left(b+\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2cd-be+\sqrt{b^2-4ac}e}{2cd+2cex}\right)}{e\sqrt[4]{a+x(b+cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x)*(a + b*x + c*x^2)^(1/4)),x]
```

```
[Out] -((Sqrt[2]*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(1/4)*((e*(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(1/4)*AppellF1[1/2, 1/4, 1/4,
 3/2, (2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x)), (2*c*d - b*e + S
qrt[b^2 - 4*a*c]*e)/(2*c*d + 2*c*e*x)])/(e*(a + x*(b + c*x))^(1/4)))
```

Maple [F] time = 1.245, size = 0, normalized size = 0.

$$\int \frac{1}{ex+d}\frac{1}{\sqrt[4]{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+b*x+a)^(1/4),x)
```

```
[Out] int(1/(e*x+d)/(c*x^2+b*x+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{1}{4}}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(1/4)*(e*x + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)\sqrt[4]{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+b*x+a)**(1/4),x)
```

```
[Out] Integral(1/((d + e*x)*(a + b*x + c*x**2)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{1}{4}}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(1/4)*(e*x + d)), x)
```

3.2533 $\int \frac{1}{(d+ex)^2 \sqrt[4]{a+bx+cx^2}}\,dx$

Optimal. Leaf size=1280

result too large to display

```
[Out] -((e*(a + b*x + c*x^2)^(3/4))/((c*d^2 - b*d*e + a*e^2)*(d + e*x))) + (Sqrt[
c]*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4))/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e +
 a*e^2)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])) + ((-b^2
 + 4*a*c)^(1/4)*(2*c*d - b*e)*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(1/4
)*ArcTan[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1
/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(4*c^(1/4)*Sqrt[e]*(
c*d^2 - b*d*e + a*e^2)^(5/4)*(a + b*x + c*x^2)^(1/4)) - ((-b^2 + 4*a*c)^(1/
4)*(2*c*d - b*e)*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(1/4)*ArcTanh[((-
b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2
]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(4*c^(1/4)*Sqrt[e]*(c*d^2 - b*d*
e + a*e^2)^(5/4)*(a + b*x + c*x^2)^(1/4)) - (c^(1/4)*(b^2 - 4*a*c)^(3/4)*Sq
rt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt
[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c]
)*EllipticE[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c
)^(1/4)], 1/2])/(Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*(b + 2*c*x)) + (c^(1/4)*(b
^2 - 4*a*c)^(3/4)*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a
+ b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)
^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(2*Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*(b +
 2*c*x)) - (Sqrt[-b^2 + 4*a*c]*(2*c*d - b*e)^2*Sqrt[(b + 2*c*x)^2/(b^2 - 4*
a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(1/4)*EllipticPi[-(Sqrt[-b^2
 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c*
x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(4*Sqrt[2]*Sqrt[c]*e*(c*d^2 - b*d*e + a*e^
2)^(3/2)*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4)) + (Sqrt[-b^2 + 4*a*c]*(2*c*d
- b*e)^2*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 -
4*a*c)))^(1/4)*EllipticPi[(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*
d*e + a*e^2]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(4*Sqr
t[2]*Sqrt[c]*e*(c*d^2 - b*d*e + a*e^2)^(3/2)*(b + 2*c*x)*(a + b*x + c*x^2)^
(1/4))
```

Rubi [A] time = 2.3938, antiderivative size = 1280, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 18, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {744, 843, 623, 305, 220, 1196, 749, 748, 746, 399, 490, 1213, 537, 444, 63, 298, 205, 208}

$$-\frac{\sqrt{4ac-b^2}\sqrt{\frac{(b+2cx)^2}{b^2-4ac}}\sqrt[4]{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\,\Pi\left(-\frac{\sqrt{4ac-b^2}e}{2\sqrt{c}\sqrt{cd^2-bed+ae^2}};\sin^{-1}\left(\sqrt[4]{1-\frac{(b+2cx)^2}{b^2-4ac}}\right)\middle|-1\right)(2cd-be)^2}{4\sqrt{2}\sqrt{c}e\left(cd^2-bed+ae^2\right)^{3/2}(b+2cx)\sqrt[4]{cx^2+bx+a}}+\frac{\sqrt{4ac-b^2}\sqrt{\frac{(b+2cx)^2}{b^2-4ac}}\sqrt[4]{-}}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + b*x + c*x^2)^(1/4)),x]
```

```
[Out] -((e*(a + b*x + c*x^2)^(3/4))/((c*d^2 - b*d*e + a*e^2)*(d + e*x))) + (Sqrt[
c]*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4))/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e +
 a*e^2)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])) + ((-b^2
 + 4*a*c)^(1/4)*(2*c*d - b*e)*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(1/4
)*ArcTan[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1
```

```
/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(4*c^(1/4)*Sqrt[e]*(
c*d^2 - b*d*e + a*e^2)^(5/4)*(a + b*x + c*x^2)^(1/4)) - ((-b^2 + 4*a*c)^(1/
4)*(2*c*d - b*e)*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(1/4)*ArcTanh[((-
b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2
]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(4*c^(1/4)*Sqrt[e]*(c*d^2 - b*d*
e + a*e^2)^(5/4)*(a + b*x + c*x^2)^(1/4)) - (c^(1/4)*(b^2 - 4*a*c)^(3/4)*Sq
rt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt
[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c]
)*EllipticE[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c
)^(1/4)], 1/2])/(Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*(b + 2*c*x)) + (c^(1/4)*(b
^2 - 4*a*c)^(3/4)*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a
+ b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)
^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(2*Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*(b +
 2*c*x)) - (Sqrt[-b^2 + 4*a*c]*(2*c*d - b*e)^2*Sqrt[(b + 2*c*x)^2/(b^2 - 4*
a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(1/4)*EllipticPi[-(Sqrt[-b^2
 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c*
x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(4*Sqrt[2]*Sqrt[c]*e*(c*d^2 - b*d*e + a*e^
2)^(3/2)*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4)) + (Sqrt[-b^2 + 4*a*c]*(2*c*d
- b*e)^2*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 -
4*a*c)))^(1/4)*EllipticPi[(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*
d*e + a*e^2]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(4*Sqr
t[2]*Sqrt[c]*e*(c*d^2 - b*d*e + a*e^2)^(3/2)*(b + 2*c*x)*(a + b*x + c*x^2)^
(1/4))
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 749

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[(a + b*x + c*x^2)^p/(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^p, I
nt[(-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) - (c^2*x^2)/(b^2 - 4*a*c
))^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&  !GtQ[4*a - b^2/c
, 0] && IntegerQ[4*p]
```

Rule 748

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[1/((-4*c)/(b^2 - 4*a*c))^p, Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c),
 x]^p/Simp[2*c*d - b*e + e*x, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && GtQ[4*a - b^2/c, 0] && IntegerQ[4*p]
```

Rule 746

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
```

```
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\sqrt[4]{a+bx+cx^2}}\,dx = -\frac{e\left(a+bx+cx^2\right)^{3/4}}{\left(cd^2-bde+ae^2\right)(d+ex)} - \frac{\int \frac{\frac{1}{4}(-4cd+be)-\frac{cex}{2}}{(d+ex)\sqrt[4]{a+bx+cx^2}}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{e\left(a+bx+cx^2\right)^{3/4}}{\left(cd^2-bde+ae^2\right)(d+ex)} + \frac{c\int \frac{1}{\sqrt[4]{a+bx+cx^2}}\,dx}{2\left(cd^2-bde+ae^2\right)} + \frac{(2cd-be)\int \frac{1}{(d+ex)\sqrt[4]{a+bx+cx^2}}\,dx}{4\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{e\left(a+bx+cx^2\right)^{3/4}}{\left(cd^2-bde+ae^2\right)(d+ex)} + \frac{\left(2c\sqrt{(b+2cx)^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b^2-4ac+4cx^4}}\,dx, x, \sqrt[4]{a+bx+cx\ldots}\right.}{\left(cd^2-bde+ae^2\right)(b+2cx)}$$

$$= -\frac{e\left(a+bx+cx^2\right)^{3/4}}{\left(cd^2-bde+ae^2\right)(d+ex)} + \frac{\left(\sqrt{c}\sqrt{b^2-4ac}\sqrt{(b+2cx)^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b^2-4ac+4cx^4}}\,dx, x, \sqrt[4]{\ldots}\right.}{\left(cd^2-bde+ae^2\right)(b+2cx)}$$

$$= -\frac{e\left(a+bx+cx^2\right)^{3/4}}{\left(cd^2-bde+ae^2\right)(d+ex)} + \frac{\sqrt{c}(b+2cx)\sqrt[4]{a+bx+cx^2}}{\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)} - \frac{\sqrt[4]{c}\left(b^2-\ldots\right.}{\ldots}$$

$$= -\frac{e\left(a+bx+cx^2\right)^{3/4}}{\left(cd^2-bde+ae^2\right)(d+ex)} + \frac{\sqrt{c}(b+2cx)\sqrt[4]{a+bx+cx^2}}{\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)} - \frac{\sqrt[4]{c}\left(b^2-\ldots\right.}{\ldots}$$

$$= -\frac{e\left(a+bx+cx^2\right)^{3/4}}{\left(cd^2-bde+ae^2\right)(d+ex)} + \frac{\sqrt{c}(b+2cx)\sqrt[4]{a+bx+cx^2}}{\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)} - \frac{\sqrt[4]{c}\left(b^2-\ldots\right.}{\ldots}$$

$$= -\frac{e\left(a+bx+cx^2\right)^{3/4}}{\left(cd^2-bde+ae^2\right)(d+ex)} + \frac{\sqrt{c}(b+2cx)\sqrt[4]{a+bx+cx^2}}{\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)} - \frac{\sqrt[4]{c}\left(b^2-\ldots\right.}{\ldots}$$

$$= -\frac{e\left(a+bx+cx^2\right)^{3/4}}{\left(cd^2-bde+ae^2\right)(d+ex)} + \frac{\sqrt{c}(b+2cx)\sqrt[4]{a+bx+cx^2}}{\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)} + \frac{\sqrt[4]{-b^2+\ldots}}{\ldots}$$

Mathematica [C] time = 0.462766, size = 187, normalized size = 0.15

$$-\frac{\sqrt{2}\sqrt[4]{\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}}\sqrt[4]{\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}}F_1\left(\frac{3}{2};\frac{1}{4},\frac{1}{4};\frac{5}{2};\frac{2cd-\left(b+\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2cd-be+\sqrt{b^2-4ac}e}{2cd+2cex}\right)}{3e(d+ex)\sqrt[4]{a+x(b+cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x)^2*(a + b*x + c*x^2)^(1/4)),x]
```

```
[Out] -(Sqrt[2]*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(1/4)*((e*(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(1/4)*AppellF1[3/2, 1/4, 1/4,
5/2, (2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x)), (2*c*d - b*e + Sq
rt[b^2 - 4*a*c]*e)/(2*c*d + 2*c*e*x)])/(3*e*(d + e*x)*(a + x*(b + c*x))^(1/
4))
```

Maple [F] time = 1.189, size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex+d\right)^{2}}\frac{1}{\sqrt[4]{cx^{2}+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^2+b*x+a)^(1/4),x)
```

```
[Out] int(1/(e*x+d)^2/(c*x^2+b*x+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^{2}+bx+a\right)^{\frac{1}{4}}\left(ex+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(1/4)*(e*x + d)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x+a)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+ex\right)^{2}\sqrt[4]{a+bx+cx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**2+b*x+a)**(1/4),x)
```

```
[Out] Integral(1/((d + e*x)**2*(a + b*x + c*x**2)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{1}{4}}(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(1/4)*(e*x + d)^2), x)
```

3.2534 $\int \frac{1}{(d+ex)^3\sqrt[4]{a+bx+cx^2}}\,dx$

Optimal. Leaf size=1465

result too large to display

```
[Out] -(e*(a + b*x + c*x^2)^(3/4))/(2*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^2) - (5*e
*(2*c*d - b*e)*(a + b*x + c*x^2)^(3/4))/(8*(c*d^2 - b*d*e + a*e^2)^2*(d + e
*x)) + (5*Sqrt[c]*(2*c*d - b*e)*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4))/(8*Sqr
t[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x
^2])/Sqrt[b^2 - 4*a*c])) + ((-b^2 + 4*a*c)^(1/4)*(12*c^2*d^2 + 5*b^2*e^2 -
4*c*e*(3*b*d + 2*a*e))*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(1/4)*ArcTa
n[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(S
qrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(32*c^(1/4)*Sqrt[e]*(c*d^2
- b*d*e + a*e^2)^(9/4)*(a + b*x + c*x^2)^(1/4)) - ((-b^2 + 4*a*c)^(1/4)*(12
*c^2*d^2 + 5*b^2*e^2 - 4*c*e*(3*b*d + 2*a*e))*(-((c*(a + b*x + c*x^2))/(b^2
 - 4*a*c)))^(1/4)*ArcTanh[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/
(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(32
*c^(1/4)*Sqrt[e]*(c*d^2 - b*d*e + a*e^2)^(9/4)*(a + b*x + c*x^2)^(1/4)) - (
5*c^(1/4)*(b^2 - 4*a*c)^(3/4)*(2*c*d - b*e)*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*
c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sq
rt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticE[2*ArcTan[(Sqrt[2]
*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(8*Sqrt[2]*(c
*d^2 - b*d*e + a*e^2)^2*(b + 2*c*x)) + (5*c^(1/4)*(b^2 - 4*a*c)^(3/4)*(2*c*
d - b*e)*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c
*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b
^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(
b^2 - 4*a*c)^(1/4)], 1/2])/(16*Sqrt[2]*(c*d^2 - b*d*e + a*e^2)^2*(b + 2*c*x
)) - (Sqrt[-b^2 + 4*a*c]*(2*c*d - b*e)*(12*c^2*d^2 + 5*b^2*e^2 - 4*c*e*(3*b
*d + 2*a*e))*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c)))^(1/4)*EllipticPi[-(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2
 - b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(
32*Sqrt[2]*Sqrt[c]*e*(c*d^2 - b*d*e + a*e^2)^(5/2)*(b + 2*c*x)*(a + b*x + c
*x^2)^(1/4)) + (Sqrt[-b^2 + 4*a*c]*(2*c*d - b*e)*(12*c^2*d^2 + 5*b^2*e^2 -
4*c*e*(3*b*d + 2*a*e))*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c
*x^2))/(b^2 - 4*a*c)))^(1/4)*EllipticPi[(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*S
qrt[c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)
], -1])/(32*Sqrt[2]*Sqrt[c]*e*(c*d^2 - b*d*e + a*e^2)^(5/2)*(b + 2*c*x)*(a
+ b*x + c*x^2)^(1/4))
```

Rubi [A] time = 3.21763, antiderivative size = 1465, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 19, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.864, Rules used = {744, 834, 843, 623, 305, 220, 1196, 749, 748, 746, 399, 490, 1213, 537, 444, 63, 298, 205, 208}

$$-\frac{5(2cd-be)\left(cx^2+bx+a\right)^{3/4}e}{8\left(cd^2-bed+ae^2\right)^2(d+ex)}-\frac{\left(cx^2+bx+a\right)^{3/4}e}{2\left(cd^2-bed+ae^2\right)(d+ex)^2}-\frac{5\sqrt[4]{c}\left(b^2-4ac\right)^{3/4}(2cd-be)\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(\frac{2\sqrt{c}\sqrt{cx^2+bx+a}}{\sqrt{b^2-4ac}}+1\right)^2}}\cdots}{8\sqrt{2}\left(cd^2-bed+\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + b*x + c*x^2)^(1/4)),x]
```

```
[Out] -(e*(a + b*x + c*x^2)^(3/4))/(2*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^2) - (5*e
*(2*c*d - b*e)*(a + b*x + c*x^2)^(3/4))/(8*(c*d^2 - b*d*e + a*e^2)^2*(d + e
*x)) + (5*Sqrt[c]*(2*c*d - b*e)*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4))/(8*Sqr
t[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x
^2])/Sqrt[b^2 - 4*a*c])) + ((-b^2 + 4*a*c)^(1/4)*(12*c^2*d^2 + 5*b^2*e^2 -
4*c*e*(3*b*d + 2*a*e))*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(1/4)*ArcTa
n[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(S
qrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(32*c^(1/4)*Sqrt[e]*(c*d^2
- b*d*e + a*e^2)^(9/4)*(a + b*x + c*x^2)^(1/4)) - ((-b^2 + 4*a*c)^(1/4)*(12
*c^2*d^2 + 5*b^2*e^2 - 4*c*e*(3*b*d + 2*a*e))*(-((c*(a + b*x + c*x^2))/(b^2
 - 4*a*c)))^(1/4)*ArcTanh[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/
(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(32
*c^(1/4)*Sqrt[e]*(c*d^2 - b*d*e + a*e^2)^(9/4)*(a + b*x + c*x^2)^(1/4)) - (
5*c^(1/4)*(b^2 - 4*a*c)^(3/4)*(2*c*d - b*e)*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*
c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sq
rt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticE[2*ArcTan[(Sqrt[2]
*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(8*Sqrt[2]*(c
*d^2 - b*d*e + a*e^2)^2*(b + 2*c*x)) + (5*c^(1/4)*(b^2 - 4*a*c)^(3/4)*(2*c*
d - b*e)*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c
*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b
^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(
b^2 - 4*a*c)^(1/4)], 1/2])/(16*Sqrt[2]*(c*d^2 - b*d*e + a*e^2)^2*(b + 2*c*x
)) - (Sqrt[-b^2 + 4*a*c]*(2*c*d - b*e)*(12*c^2*d^2 + 5*b^2*e^2 - 4*c*e*(3*b
*d + 2*a*e))*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c)))^(1/4)*EllipticPi[-(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2
 - b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(
32*Sqrt[2]*Sqrt[c]*e*(c*d^2 - b*d*e + a*e^2)^(5/2)*(b + 2*c*x)*(a + b*x + c
*x^2)^(1/4)) + (Sqrt[-b^2 + 4*a*c]*(2*c*d - b*e)*(12*c^2*d^2 + 5*b^2*e^2 -
4*c*e*(3*b*d + 2*a*e))*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c
*x^2))/(b^2 - 4*a*c)))^(1/4)*EllipticPi[(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*S
qrt[c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)
], -1])/(32*Sqrt[2]*Sqrt[c]*e*(c*d^2 - b*d*e + a*e^2)^(5/2)*(b + 2*c*x)*(a
+ b*x + c*x^2)^(1/4))
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
```

```
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 749

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[(a + b*x + c*x^2)^p/(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^p, I
nt[(-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) - (c^2*x^2)/(b^2 - 4*a*c
))^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&  !GtQ[4*a - b^2/c
, 0] && IntegerQ[4*p]
```

Rule 748

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[1/((-4*c)/(b^2 - 4*a*c))^p, Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c),
 x]^p/Simp[2*c*d - b*e + e*x, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && GtQ[4*a - b^2/c, 0] && IntegerQ[4*p]
```

Rule 746

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
```

```
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\sqrt[4]{a+bx+cx^2}}\,dx = -\frac{e\left(a+bx+cx^2\right)^{3/4}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{\int \frac{\frac{1}{4}(-8cd+5be)+\frac{cex}{2}}{(d+ex)^2\sqrt[4]{a+bx+cx^2}}\,dx}{2\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{e\left(a+bx+cx^2\right)^{3/4}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{5e(2cd-be)\left(a+bx+cx^2\right)^{3/4}}{8\left(cd^2-bde+ae^2\right)^2(d+ex)} + \frac{\int \frac{\frac{1}{16}\left(32c^2d^2+5b^2e^2-2ce \cdots\right.}{(d+ex) \cdots}}{2\left(cd^2- \cdots\right.}$$

$$= -\frac{e\left(a+bx+cx^2\right)^{3/4}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{5e(2cd-be)\left(a+bx+cx^2\right)^{3/4}}{8\left(cd^2-bde+ae^2\right)^2(d+ex)} + \frac{(5c(2cd-be))\int \frac{\cdots}{\sqrt[4]{a \cdots}}}{16\left(cd^2-bde+ \cdots\right.}$$

$$= -\frac{e\left(a+bx+cx^2\right)^{3/4}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{5e(2cd-be)\left(a+bx+cx^2\right)^{3/4}}{8\left(cd^2-bde+ae^2\right)^2(d+ex)} + \frac{\left(5c(2cd-be)\sqrt{(b+ \cdots}\right.}{\cdots}$$

$$= -\frac{e\left(a+bx+cx^2\right)^{3/4}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{5e(2cd-be)\left(a+bx+cx^2\right)^{3/4}}{8\left(cd^2-bde+ae^2\right)^2(d+ex)} + \frac{\left(5\sqrt{c}\sqrt{b^2-4ac}(2cd \cdots\right.}{\cdots}$$

$$= -\frac{e\left(a+bx+cx^2\right)^{3/4}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{5e(2cd-be)\left(a+bx+cx^2\right)^{3/4}}{8\left(cd^2-bde+ae^2\right)^2(d+ex)} + \frac{5\sqrt{c}(2cd-be \cdots}{8\sqrt{b^2-4ac}\left(cd^2- \cdots\right.}$$

$$= -\frac{e\left(a+bx+cx^2\right)^{3/4}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{5e(2cd-be)\left(a+bx+cx^2\right)^{3/4}}{8\left(cd^2-bde+ae^2\right)^2(d+ex)} + \frac{5\sqrt{c}(2cd-be \cdots}{8\sqrt{b^2-4ac}\left(cd^2- \cdots\right.}$$

$$= -\frac{e\left(a+bx+cx^2\right)^{3/4}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{5e(2cd-be)\left(a+bx+cx^2\right)^{3/4}}{8\left(cd^2-bde+ae^2\right)^2(d+ex)} + \frac{5\sqrt{c}(2cd-be \cdots}{8\sqrt{b^2-4ac}\left(cd^2- \cdots\right.}$$

$$= -\frac{e\left(a+bx+cx^2\right)^{3/4}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{5e(2cd-be)\left(a+bx+cx^2\right)^{3/4}}{8\left(cd^2-bde+ae^2\right)^2(d+ex)} + \frac{5\sqrt{c}(2cd-be \cdots}{8\sqrt{b^2-4ac}\left(cd^2- \cdots\right.}$$

$$= -\frac{e\left(a+bx+cx^2\right)^{3/4}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{5e(2cd-be)\left(a+bx+cx^2\right)^{3/4}}{8\left(cd^2-bde+ae^2\right)^2(d+ex)} + \frac{5\sqrt{c}(2cd-be \cdots}{8\sqrt{b^2-4ac}\left(cd^2- \cdots\right.}$$

Mathematica [C] time = 0.559102, size = 187, normalized size = 0.13

$$-\frac{\sqrt{2}\sqrt[4]{\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}}\sqrt[4]{\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}}F_1\left(\frac{5}{2};\frac{1}{4},\frac{1}{4};\frac{7}{2};\frac{2cd-\left(b+\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2cd-be+\sqrt{b^2-4ac}e}{2cd+2cex}\right)}{5e(d+ex)^2\sqrt[4]{a+x(b+cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x)^3*(a + b*x + c*x^2)^(1/4)),x]
```

```
[Out] -(Sqrt[2]*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(1/4)*((e*(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(1/4)*AppellF1[5/2, 1/4, 1/4,
7/2, (2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x)), (2*c*d - b*e + Sq
rt[b^2 - 4*a*c]*e)/(2*c*d + 2*c*e*x)])/(5*e*(d + e*x)^2*(a + x*(b + c*x))^(
1/4))
```

Maple [F] time = 1.189, size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)^3}\frac{1}{\sqrt[4]{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*x^2+b*x+a)^(1/4),x)
```

```
[Out] int(1/(e*x+d)^3/(c*x^2+b*x+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{1}{4}}(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(1/4)*(e*x + d)^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x+a)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)^3\sqrt[4]{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**2+b*x+a)**(1/4),x)
```

```
[Out] Integral(1/((d + e*x)**3*(a + b*x + c*x**2)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{1}{4}}(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(1/4)*(e*x + d)^3), x)
```

3.2535 $\int \frac{(d+ex)^3}{\left(a+bx+cx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=307

$$\frac{\sqrt[4]{b^2-4ac}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)(2cd-be)\left(-4ce(2ae+bd)+3b^2e^2+4c^2d^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\cdots\right.\right.}{4\sqrt{2}c^{13/4}(b+2cx)}$$

```
[Out] (2*e*(d + e*x)^2*(a + b*x + c*x^2)^(1/4))/(5*c) + (e*(56*c^2*d^2 + 15*b^2*e
^2 - 2*c*e*(25*b*d + 8*a*e) + 6*c*e*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(1/4
))/(10*c^3) + ((b^2 - 4*a*c)^(1/4)*(2*c*d - b*e)*(4*c^2*d^2 + 3*b^2*e^2 - 4
*c*e*(b*d + 2*a*e))*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[
a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x
^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^
2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(4*Sqrt[2]*c^(13/4)*(b + 2*c*x))
```

Rubi [A] time = 0.329917, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {742, 779, 623, 220}

$$\frac{e\sqrt[4]{a+bx+cx^2}\left(-2ce(8ae+25bd)+15b^2e^2+6cex(2cd-be)+56c^2d^2\right)}{10c^3}+\frac{\sqrt[4]{b^2-4ac}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+\cdots}}{\sqrt{b^2-4ac}}\right.\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + b*x + c*x^2)^(3/4),x]
```

```
[Out] (2*e*(d + e*x)^2*(a + b*x + c*x^2)^(1/4))/(5*c) + (e*(56*c^2*d^2 + 15*b^2*e
^2 - 2*c*e*(25*b*d + 8*a*e) + 6*c*e*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(1/4
))/(10*c^3) + ((b^2 - 4*a*c)^(1/4)*(2*c*d - b*e)*(4*c^2*d^2 + 3*b^2*e^2 - 4
*c*e*(b*d + 2*a*e))*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[
a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x
^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^
2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(4*Sqrt[2]*c^(13/4)*(b + 2*c*x))
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
```

```
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(a+bx+cx^2\right)^{3/4}}dx = \frac{2e(d+ex)^2\sqrt[4]{a+bx+cx^2}}{5c} + \frac{2\int \frac{(d+ex)\left(\frac{1}{4}\left(10cd^2-e(bd+8ae)\right)+\frac{9}{4}e(2cd-be)x\right)}{\left(a+bx+cx^2\right)^{3/4}}dx}{5c}$$

$$= \frac{2e(d+ex)^2\sqrt[4]{a+bx+cx^2}}{5c} + \frac{e\left(56c^2d^2+15b^2e^2-2ce(25bd+8ae)+6ce(2cd-be)x\right)\sqrt[4]{a+b}}{10c^3}$$

$$= \frac{2e(d+ex)^2\sqrt[4]{a+bx+cx^2}}{5c} + \frac{e\left(56c^2d^2+15b^2e^2-2ce(25bd+8ae)+6ce(2cd-be)x\right)\sqrt[4]{a+b}}{10c^3}$$

$$= \frac{2e(d+ex)^2\sqrt[4]{a+bx+cx^2}}{5c} + \frac{e\left(56c^2d^2+15b^2e^2-2ce(25bd+8ae)+6ce(2cd-be)x\right)\sqrt[4]{a+b}}{10c^3}$$

Mathematica [A] time = 0.424407, size = 244, normalized size = 0.79

$$\frac{2ce\left(-16a^2ce^2+a\left(15b^2e^2-2bce(25d+11ex)+4c^2\left(15d^2+5dex-3e^2x^2\right)\right)+x(b+cx)\left(15b^2e^2-2bce(25d+3ex)+4\right.}{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + b*x + c*x^2)^(3/4),x]
```

```
[Out] (2*c*e*(-16*a^2*c*e^2 + a*(15*b^2*e^2 - 2*b*c*e*(25*d + 11*e*x) + 4*c^2*(15
*d^2 + 5*d*e*x - 3*e^2*x^2)) + x*(b + c*x)*(15*b^2*e^2 - 2*b*c*e*(25*d + 3*
e*x) + 4*c^2*(15*d^2 + 5*d*e*x + e^2*x^2))) - 5*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(
-2*c*d + b*e)*(4*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(b*d + 2*a*e))*((c*(a + x*(b +
 c*x)))/(-b^2 + 4*a*c))^(3/4)*EllipticF[ArcSin[(b + 2*c*x)/Sqrt[b^2 - 4*a*c
]]/2, 2])/(20*c^4*(a + x*(b + c*x))^(3/4))
```

Maple [F] time = 0.95, size = 0, normalized size = 0.

$$\int (ex+d)^3\left(cx^2+bx+a\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^2+b*x+a)^(3/4),x)
```

```
[Out] int((e*x+d)^3/(c*x^2+b*x+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3}{(cx^2+bx+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3/(c*x^2 + b*x + a)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3}{(cx^2+bx+a)^{\frac{3}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)/(c*x^2 + b*x + a)^(3/4),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^3}{(a+bx+cx^2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**2+b*x+a)**(3/4),x)
```

```
[Out] Integral((d + e*x)**3/(a + b*x + c*x**2)**(3/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3}{(cx^2+bx+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^(3/4),x, algorithm="giac")

[Out] integrate((e*x + d)^3/(c*x^2 + b*x + a)^(3/4), x)
```

3.2536 $\int \frac{(d+ex)^2}{\left(a+bx+cx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=262

$$\frac{\sqrt[4]{b^2-4ac}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)\left(-4ce(2ae+3bd)+5b^2e^2+12c^2d^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{a+bx}}{\sqrt[4]{b^2-4a}}\right.\right.}{6\sqrt{2}c^{9/4}(b+2cx)}$$

```
[Out] (5*e*(2*c*d - b*e)*(a + b*x + c*x^2)^(1/4))/(3*c^2) + (2*e*(d + e*x)*(a + b
*x + c*x^2)^(1/4))/(3*c) + ((b^2 - 4*a*c)^(1/4)*(12*c^2*d^2 + 5*b^2*e^2 - 4
*c*e*(3*b*d + 2*a*e))*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqr
t[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c
*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*
x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(6*Sqrt[2]*c^(9/4)*(b + 2*c*x))
```

Rubi [A] time = 0.254795, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {742, 640, 623, 220}

$$\frac{\sqrt[4]{b^2-4ac}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)\left(-4ce(2ae+3bd)+5b^2e^2+12c^2d^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{cx^2+bx+a}}{\sqrt[4]{b^2-4ac}}\right)|\frac{1}{2}\right.}{6\sqrt{2}c^{9/4}(b+2cx)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*x + c*x^2)^(3/4),x]
```

```
[Out] (5*e*(2*c*d - b*e)*(a + b*x + c*x^2)^(1/4))/(3*c^2) + (2*e*(d + e*x)*(a + b
*x + c*x^2)^(1/4))/(3*c) + ((b^2 - 4*a*c)^(1/4)*(12*c^2*d^2 + 5*b^2*e^2 - 4
*c*e*(3*b*d + 2*a*e))*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqr
t[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c
*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*
x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(6*Sqrt[2]*c^(9/4)*(b + 2*c*x))
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(a+bx+cx^2\right)^{3/4}}\,dx = \frac{2e(d+ex)\sqrt[4]{a+bx+cx^2}}{3c} + \frac{2\int \frac{\frac{1}{4}\left(6cd^2-e(bd+4ae)\right)+\frac{5}{4}e(2cd-be)x}{\left(a+bx+cx^2\right)^{3/4}}\,dx}{3c}$$

$$= \frac{5e(2cd-be)\sqrt[4]{a+bx+cx^2}}{3c^2} + \frac{2e(d+ex)\sqrt[4]{a+bx+cx^2}}{3c} + \frac{\left(-\frac{5}{4}be(2cd-be)+\frac{1}{2}c\left(6cd^2-e(bc\right.\right.}{3c^2}$$

$$= \frac{5e(2cd-be)\sqrt[4]{a+bx+cx^2}}{3c^2} + \frac{2e(d+ex)\sqrt[4]{a+bx+cx^2}}{3c} + \frac{\left(4\left(-\frac{5}{4}be(2cd-be)+\frac{1}{2}c\left(6cd^2-e\right.\right.\right.}{}$$

$$= \frac{5e(2cd-be)\sqrt[4]{a+bx+cx^2}}{3c^2} + \frac{2e(d+ex)\sqrt[4]{a+bx+cx^2}}{3c} + \frac{\sqrt[4]{b^2-4ac}\left(12c^2d^2+5b^2e^2-4ce(3\right.}{}$$

Mathematica [A] time = 0.265212, size = 150, normalized size = 0.57

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\left(\frac{c(a+x(b+cx))}{4ac-b^2}\right)^{3/4}\left(-4ce(2ae+3bd)+5b^2e^2+12c^2d^2\right)\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right),2\right)+2ce(a+x(b+cx)}{6c^3(a+x(b+cx))^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*x + c*x^2)^(3/4),x]
```

```
[Out] (2*c*e*(a + x*(b + c*x))*(-5*b*e + 2*c*(6*d + e*x)) + Sqrt[2]*Sqrt[b^2 - 4*
a*c]*(12*c^2*d^2 + 5*b^2*e^2 - 4*c*e*(3*b*d + 2*a*e))*((c*(a + x*(b + c*x))
)/(-b^2 + 4*a*c))^(3/4)*EllipticF[ArcSin[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]/2,
2])/(6*c^3*(a + x*(b + c*x))^(3/4))
```

Maple [F] time = 0.944, size = 0, normalized size = 0.

$$\int (ex+d)^2\left(cx^2+bx+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^2+b*x+a)^(3/4),x)
```

```
[Out] int((e*x+d)^2/(c*x^2+b*x+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2}{(cx^2+bx+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2/(c*x^2 + b*x + a)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{e^2x^2+2\,dex+d^2}{(cx^2+bx+a)^{\frac{3}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)/(c*x^2 + b*x + a)^(3/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^2}{(a+bx+cx^2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**2+b*x+a)**(3/4),x)
```

```
[Out] Integral((d + e*x)**2/(a + b*x + c*x**2)**(3/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2}{(cx^2+bx+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2/(c*x^2 + b*x + a)^(3/4), x)
```

3.2537 $\int \frac{d+ex}{\left(a+bx+cx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=200

$$\frac{\sqrt[4]{b^2-4ac}\sqrt{\frac{(b+2cx)^2}{(b^2-4ac)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)(2cd-be)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{a+bx+cx^2}}{\sqrt[4]{b^2-4ac}}\right),\frac{1}{2}\right)}{\sqrt{2}c^{5/4}(b+2cx)}+\frac{2e\sqrt[4]{a+bx}}{c}$$

```
[Out] (2*e*(a + b*x + c*x^2)^(1/4))/c + ((b^2 - 4*a*c)^(1/4)*(2*c*d - b*e)*Sqrt[(
b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2
 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*El
lipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1
/4)], 1/2])/(Sqrt[2]*c^(5/4)*(b + 2*c*x))
```

Rubi [A] time = 0.142158, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {640, 623, 220}

$$\frac{\sqrt[4]{b^2-4ac}\sqrt{\frac{(b+2cx)^2}{(b^2-4ac)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)(2cd-be)F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{cx^2+bx+a}}{\sqrt[4]{b^2-4ac}}\right)|\frac{1}{2}\right)}{\sqrt{2}c^{5/4}(b+2cx)}+\frac{2e\sqrt[4]{a+bx+cx^2}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*x + c*x^2)^(3/4),x]
```

```
[Out] (2*e*(a + b*x + c*x^2)^(1/4))/c + ((b^2 - 4*a*c)^(1/4)*(2*c*d - b*e)*Sqrt[(
b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2
 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*El
lipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1
/4)], 1/2])/(Sqrt[2]*c^(5/4)*(b + 2*c*x))
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{d+ex}{\left(a+bx+cx^2\right)^{3/4}}\,dx = \frac{2e\sqrt[4]{a+bx+cx^2}}{c} + \frac{(2cd-be)\int \frac{1}{\left(a+bx+cx^2\right)^{3/4}}\,dx}{2c}$$
$$= \frac{2e\sqrt[4]{a+bx+cx^2}}{c} + \frac{\left(2(2cd-be)\sqrt{(b+2cx)^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b^2-4ac+4cx^4}}\,dx, x, \sqrt[4]{a+bx+cx^2}\right)}{c(b+2cx)}$$
$$= \frac{2e\sqrt[4]{a+bx+cx^2}}{c} + \frac{\sqrt[4]{b^2-4ac}(2cd-be)\sqrt{\frac{(b+2cx)^2}{(b^2-4ac)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)F\left(2\tan^{-1}\right.}{\sqrt{2}c^{5/4}(b+2cx)}$$

Mathematica [A] time = 0.172171, size = 111, normalized size = 0.56

$$\frac{2ce(a+x(b+cx))-\sqrt{2}\sqrt{b^2-4ac}\left(\frac{c(a+x(b+cx))}{4ac-b^2}\right)^{3/4}(be-2cd)\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right),2\right)}{c^2(a+x(b+cx))^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*x + c*x^2)^(3/4),x]

[Out] (2*c*e*(a + x*(b + c*x)) - Sqrt[2]*Sqrt[b^2 - 4*a*c]*(-2*c*d + b*e)*((c*(a
+ x*(b + c*x)))/(-b^2 + 4*a*c))^(3/4)*EllipticF[ArcSin[(b + 2*c*x)/Sqrt[b^2
 - 4*a*c]]/2, 2])/(c^2*(a + x*(b + c*x))^(3/4))
```

Maple [F] time = 0.999, size = 0, normalized size = 0.

$$\int (ex+d)\left(cx^2+bx+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^2+b*x+a)^(3/4),x)

[Out] int((e*x+d)/(c*x^2+b*x+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex+d}{\left(cx^2+bx+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^(3/4),x, algorithm="maxima")

[Out] integrate((e*x + d)/(c*x^2 + b*x + a)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex+d}{\left(cx^2+bx+a\right)^{\frac{3}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)/(c*x^2 + b*x + a)^(3/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d+ex}{\left(a+bx+cx^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**2+b*x+a)**(3/4),x)
```

```
[Out] Integral((d + e*x)/(a + b*x + c*x**2)**(3/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex+d}{\left(cx^2+bx+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)/(c*x^2 + b*x + a)^(3/4), x)
```

3.2538 $\int \frac{1}{\left(a+bx+cx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=170

$$\frac{\sqrt{2}\sqrt[4]{b^2-4ac}\sqrt{\frac{(b+2cx)^2}{(b^2-4ac)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{a+bx+cx^2}}{\sqrt[4]{b^2-4ac}}\right),\frac{1}{2}\right)}{\sqrt[4]{c}(b+2cx)}$$

```
[Out] (Sqrt[2]*(b^2 - 4*a*c)^(1/4)*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt
[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a +
b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b
*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(c^(1/4)*(b + 2*c*x))
```

Rubi [A] time = 0.111645, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {623, 220}

$$\frac{\sqrt{2}\sqrt[4]{b^2-4ac}\sqrt{\frac{(b+2cx)^2}{(b^2-4ac)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{cx^2+bx+a}}{\sqrt[4]{b^2-4ac}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}(b+2cx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(-3/4),x]
```

```
[Out] (Sqrt[2]*(b^2 - 4*a*c)^(1/4)*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt
[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a +
b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b
*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(c^(1/4)*(b + 2*c*x))
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx+cx^2\right)^{3/4}}\,dx &= \frac{\left(4\sqrt{(b+2cx)^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b^2-4ac+4cx^4}}\,dx,x,\sqrt[4]{a+bx+cx^2}\right)}{b+2cx}\\ &= \frac{\sqrt{2}\sqrt[4]{b^2-4ac}\sqrt{\frac{(b+2cx)^2}{(b^2-4ac)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{a+bx+cx^2}}{\sqrt[4]{b^2-4ac}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}(b+2cx)}\end{aligned}$$

Mathematica [A] time = 0.0456623, size = 88, normalized size = 0.52

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\left(\frac{c(a+x(b+cx))}{4ac-b^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right),2\right)}{c(a+x(b+cx))^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(-3/4),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*((c*(a + x*(b + c*x)))/(-b^2 + 4*a*c))^(3/4)*E
llipticF[ArcSin[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]/2, 2])/(c*(a + x*(b + c*x))^
(3/4))
```

Maple [F] time = 2.028, size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(3/4),x)
```

```
[Out] int(1/(c*x^2+b*x+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(-3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(cx^2+bx+a\right)^{\frac{3}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^(-3/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx+cx^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(3/4),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(-3/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(-3/4), x)
```

3.2539 $\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=709

$$-\frac{\sqrt{e}\left(4ac-b^2\right)^{3/4}\left(-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{e}\sqrt[4]{4ac-b^2}\sqrt[4]{1-\frac{(b+2cx)^2}{b^2-4ac}}}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{ae^2-bde+cd^2}}\right)}{c^{3/4}\left(a+bx+cx^2\right)^{3/4}\left(ae^2-bde+cd^2\right)^{3/4}}-\frac{\sqrt{e}\left(4ac-b^2\right)^{3/4}\left(-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt[4]{4ac-b^2}}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{a}}\right.}{c^{3/4}\left(a+bx+cx^2\right)^{3/4}\left(ae^2-bde+cd^2\right)^{3/4}}$$

```
[Out] -(((-b^2 + 4*a*c)^(3/4)*Sqrt[e]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3
/4)*ArcTan[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^
(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(c^(3/4)*(c*d^2 -
b*d*e + a*e^2)^(3/4)*(a + b*x + c*x^2)^(3/4))) - ((-b^2 + 4*a*c)^(3/4)*Sqrt
[e]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)*ArcTanh[((-b^2 + 4*a*c)^
(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*
d^2 - b*d*e + a*e^2)^(1/4))])/(c^(3/4)*(c*d^2 - b*d*e + a*e^2)^(3/4)*(a + b
*x + c*x^2)^(3/4)) - ((b^2 - 4*a*c)*(2*c*d - b*e)*Sqrt[(b + 2*c*x)^2/(b^2 -
 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)*EllipticPi[-(Sqrt[-
b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (b + 2
*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(Sqrt[2]*c*(c*d^2 - b*d*e + a*e^2)*(b +
 2*c*x)*(a + b*x + c*x^2)^(3/4)) - ((b^2 - 4*a*c)*(2*c*d - b*e)*Sqrt[(b + 2
*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)*Ellip
ticPi[(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2]), ArcSi
n[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(Sqrt[2]*c*(c*d^2 - b*d*e
+ a*e^2)*(b + 2*c*x)*(a + b*x + c*x^2)^(3/4))
```

Rubi [A] time = 1.57429, antiderivative size = 709, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {749, 748, 747, 401, 108, 409, 1213, 537, 444, 63, 212, 208, 205}

$$-\frac{\sqrt{e}\left(4ac-b^2\right)^{3/4}\left(-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{e}\sqrt[4]{4ac-b^2}\sqrt[4]{1-\frac{(b+2cx)^2}{b^2-4ac}}}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{ae^2-bde+cd^2}}\right)}{c^{3/4}\left(a+bx+cx^2\right)^{3/4}\left(ae^2-bde+cd^2\right)^{3/4}}-\frac{\sqrt{e}\left(4ac-b^2\right)^{3/4}\left(-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt[4]{4ac-b^2}}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{a}}\right.}{c^{3/4}\left(a+bx+cx^2\right)^{3/4}\left(ae^2-bde+cd^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + b*x + c*x^2)^(3/4)),x]
```

```
[Out] -(((-b^2 + 4*a*c)^(3/4)*Sqrt[e]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3
/4)*ArcTan[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^
(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(c^(3/4)*(c*d^2 -
b*d*e + a*e^2)^(3/4)*(a + b*x + c*x^2)^(3/4))) - ((-b^2 + 4*a*c)^(3/4)*Sqrt
[e]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)*ArcTanh[((-b^2 + 4*a*c)^
(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*
d^2 - b*d*e + a*e^2)^(1/4))])/(c^(3/4)*(c*d^2 - b*d*e + a*e^2)^(3/4)*(a + b
*x + c*x^2)^(3/4)) - ((b^2 - 4*a*c)*(2*c*d - b*e)*Sqrt[(b + 2*c*x)^2/(b^2 -
 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)*EllipticPi[-(Sqrt[-
b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (b + 2
*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(Sqrt[2]*c*(c*d^2 - b*d*e + a*e^2)*(b +
 2*c*x)*(a + b*x + c*x^2)^(3/4)) - ((b^2 - 4*a*c)*(2*c*d - b*e)*Sqrt[(b + 2
*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)*Ellip
ticPi[(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2]), ArcSi
n[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(Sqrt[2]*c*(c*d^2 - b*d*e
```

```
+ a*e^2)*(b + 2*c*x)*(a + b*x + c*x^2)^(3/4))
```

Rule 749

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[(a + b*x + c*x^2)^p/(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^p, I
nt[(-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) - (c^2*x^2)/(b^2 - 4*a*c
))^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&  !GtQ[4*a - b^2/c
, 0] && IntegerQ[4*p]
```

Rule 748

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[1/((-4*c)/(b^2 - 4*a*c))^p, Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c),
 x]^p/Simp[2*c*d - b*e + e*x, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && GtQ[4*a - b^2/c, 0] && IntegerQ[4*p]
```

Rule 747

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[-((b*x^2)/a)]/(2*x), Subst[Int[1/(Sqrt[-((b*x)/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 108

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - (d*e
)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/4}}\,dx = \frac{\left(-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}\right)^{3/4}\int \frac{1}{(d+ex)\left(-\frac{ac}{b^2-4ac}-\frac{bcx}{b^2-4ac}-\frac{c^2x^2}{b^2-4ac}\right)^{3/4}}\,dx}{\left(a+bx+cx^2\right)^{3/4}}$$

$$= \frac{\left(2\sqrt{2}\left(-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(-\frac{c(2cd-be)}{b^2-4ac}+ex\right)\left(1-\frac{\left(b^2-4ac\right)x^2}{c^2}\right)^{3/4}}\,dx, x, -\frac{bc}{b^2-4ac}-\frac{2c^2x}{b^2-4ac}\right)}{\left(a+bx+cx^2\right)^{3/4}}$$

$$= -\frac{\left(2\sqrt{2}e\left(-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}\right)^{3/4}\right)\text{Subst}\left(\int \frac{x}{\left(1-\frac{\left(b^2-4ac\right)x^2}{c^2}\right)^{3/4}\left(\frac{c^2(2cd-be)^2}{\left(b^2-4ac\right)^2}-e^2x^2\right)}\,dx, x, -\frac{bc}{b^2-4ac}-\frac{2c^2x}{b^2-4ac}\right.}{\left(a+bx+cx^2\right)^{3/4}}$$

$$= -\frac{\left(\sqrt{2}e\left(-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{\left(b^2-4ac\right)x}{c^2}\right)^{3/4}\left(\frac{c^2(2cd-be)^2}{\left(b^2-4ac\right)^2}-e^2x\right)}\,dx, x, \left(-\frac{bc}{b^2-4ac}-\frac{2c^2x}{b^2-4ac}\right)^2\right.}{\left(a+bx+cx^2\right)^{3/4}}$$

$$= \frac{\left(4\sqrt{2}c^2e\left(-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{-\frac{c^2e^2}{b^2-4ac}+\frac{c^2(2cd-be)^2}{\left(b^2-4ac\right)^2}+\frac{c^2e^2x^4}{b^2-4ac}}\,dx, x, \sqrt[4]{1-\frac{(b+2cx)^2}{b^2-4ac}}\right)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/4}} + \frac{\left(4\sqrt{\cdots}\right.}{}$$

$$= \frac{\left(\sqrt{2}\left(b^2-4ac\right)e\left(-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{2\sqrt{c}\sqrt{cd^2-bde+ae^2}-\sqrt{-b^2+4ac}ex^2}\,dx, x, \sqrt[4]{1-\frac{(b+\cdots}{b^2-\cdots}}\right.}{\sqrt{c}\sqrt{cd^2-bde+ae^2}\left(a+bx+cx^2\right)^{3/4}}$$

$$= -\frac{\left(-b^2+4ac\right)^{3/4}\sqrt{e}\left(-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{-b^2+4ac}\sqrt{e}\sqrt[4]{1-\frac{(b+2cx)^2}{b^2-4ac}}}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{cd^2-bde+ae^2}}\right)}{c^{3/4}\left(cd^2-bde+ae^2\right)^{3/4}\left(a+bx+cx^2\right)^{3/4}} - \frac{\left(-b^2+4ac\right)^{3/4}\sqrt{e}\cdots}{c^{3/4}\left(\cdots\right.}$$

$$= -\frac{\left(-b^2+4ac\right)^{3/4}\sqrt{e}\left(-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{-b^2+4ac}\sqrt{e}\sqrt[4]{1-\frac{(b+2cx)^2}{b^2-4ac}}}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{cd^2-bde+ae^2}}\right)}{c^{3/4}\left(cd^2-bde+ae^2\right)^{3/4}\left(a+bx+cx^2\right)^{3/4}} - \frac{\left(-b^2+4ac\right)^{3/4}\sqrt{e}\cdots}{c^{3/4}\left(\cdots\right.}$$

Mathematica [A] time = 1.74005, size = 533, normalized size = 0.75

$$\frac{\sqrt[4]{a+x(b+cx)}\left(-\sqrt{2}\sqrt[4]{c}\sqrt{e}(b+2cx)\sqrt[4]{e(ae-bd)+cd^2}\left(\tan^{-1}\left(\frac{\sqrt{e}\sqrt[4]{4ac-b^2}\sqrt[4]{\frac{c(a+x(b+cx))}{4ac-b^2}}}{\sqrt[4]{c}\sqrt[4]{e(ae-bd)+cd^2}}\right)+\tanh^{-1}\left(\frac{\sqrt{e}\sqrt[4]{4ac-b^2}\sqrt[4]{\frac{c(a+x(b+cx))}{4ac-b^2}}}{\sqrt[4]{c}\sqrt[4]{e(ae-bd)+cd^2}}\right)\right)+\cdots}{\sqrt{2}\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + b*x + c*x^2)^(3/4)),x]
```

```
[Out] ((a + x*(b + c*x))^(1/4)*(-(Sqrt[2]*c^(1/4)*Sqrt[e]*(c*d^2 + e*(-(b*d) + a*
e))^(1/4)*(b + 2*c*x)*(ArcTan[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*((c*(a + x*(b +
 c*x)))/(-b^2 + 4*a*c))^(1/4))/(c^(1/4)*(c*d^2 + e*(-(b*d) + a*e))^(1/4))]
+ ArcTanh[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*((c*(a + x*(b + c*x)))/(-b^2 + 4*a*
c))^(1/4))/(c^(1/4)*(c*d^2 + e*(-(b*d) + a*e))^(1/4))])) + (-b^2 + 4*a*c)^(
1/4)*(-2*c*d + b*e)*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*EllipticPi[-(Sqrt[-b^
2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 + e*(-(b*d) + a*e)]), -ArcSin[Sqrt[2]*(
(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c))^(1/4)], -1] + (-b^2 + 4*a*c)^(1/4)*(-
2*c*d + b*e)*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*EllipticPi[(Sqrt[-b^2 + 4*a*
c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 + e*(-(b*d) + a*e)]), -ArcSin[Sqrt[2]*((c*(a +
x*(b + c*x)))/(-b^2 + 4*a*c))^(1/4)], -1]))/(Sqrt[2]*(-b^2 + 4*a*c)^(1/4)*(
c*d^2 + e*(-(b*d) + a*e))*(b + 2*c*x)*((c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)
)^(1/4))
```

Maple [F] time = 1.242, size = 0, normalized size = 0.

$$\int \frac{1}{ex+d}\left(cx^{2}+bx+a\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+b*x+a)^(3/4),x)
```

```
[Out] int(1/(e*x+d)/(c*x^2+b*x+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^{2}+bx+a\right)^{\frac{3}{4}}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(3/4)*(e*x + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)\left(a+bx+cx^{2}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+b*x+a)**(3/4),x)
```

```
[Out] Integral(1/((d + e*x)*(a + b*x + c*x**2)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2 + bx + a\right)^{\frac{3}{4}} (ex + d)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(3/4)*(e*x + d)), x)
```

3.2540 $\int \frac{1}{(d+ex)^2\left(a+bx+cx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=970

$$-\frac{3\left(b^2-4ac\right)\sqrt{\frac{(b+2cx)^2}{b^2-4ac}}\left(-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}\right)^{3/4}\Pi\left(-\frac{\sqrt{4ac-b^2}e}{2\sqrt{c}\sqrt{cd^2-bed+ae^2}};\sin^{-1}\left(\sqrt[4]{1-\frac{(b+2cx)^2}{b^2-4ac}}\right)\middle|-1\right)(2cd-be)^2}{4\sqrt{2}c\left(cd^2-bed+ae^2\right)^2(b+2cx)\left(cx^2+bx+a\right)^{3/4}}-\frac{3\left(b^2-4ac\right)\sqrt{}}{}$$

```
[Out] -((e*(a + b*x + c*x^2)^(1/4))/((c*d^2 - b*d*e + a*e^2)*(d + e*x))) - (3*(-b
^2 + 4*a*c)^(3/4)*Sqrt[e]*(2*c*d - b*e)*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a
*c)))^(3/4)*ArcTan[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 -
4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(4*c^(3/4)
*(c*d^2 - b*d*e + a*e^2)^(7/4)*(a + b*x + c*x^2)^(3/4)) - (3*(-b^2 + 4*a*c)
^(3/4)*Sqrt[e]*(2*c*d - b*e)*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)
*ArcTanh[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1
/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(4*c^(3/4)*(c*d^2 -
b*d*e + a*e^2)^(7/4)*(a + b*x + c*x^2)^(3/4)) - (c^(3/4)*(b^2 - 4*a*c)^(1/4
)*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/
Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*
a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4
*a*c)^(1/4)], 1/2])/(Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*(b + 2*c*x)) - (3*(b^2
 - 4*a*c)*(2*c*d - b*e)^2*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x
+ c*x^2))/(b^2 - 4*a*c)))^(3/4)*EllipticPi[-(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[
c]*Sqrt[c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(
1/4)], -1])/(4*Sqrt[2]*c*(c*d^2 - b*d*e + a*e^2)^2*(b + 2*c*x)*(a + b*x + c
*x^2)^(3/4)) - (3*(b^2 - 4*a*c)*(2*c*d - b*e)^2*Sqrt[(b + 2*c*x)^2/(b^2 - 4
*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)*EllipticPi[(Sqrt[-b^2
 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c*
x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(4*Sqrt[2]*c*(c*d^2 - b*d*e + a*e^2)^2*(b
+ 2*c*x)*(a + b*x + c*x^2)^(3/4))
```

Rubi [A] time = 2.01385, antiderivative size = 970, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.773, Rules used = {744, 843, 623, 220, 749, 748, 747, 401, 108, 409, 1213, 537, 444, 63, 212, 208, 205}

$$-\frac{3\left(b^2-4ac\right)\sqrt{\frac{(b+2cx)^2}{b^2-4ac}}\left(-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}\right)^{3/4}\Pi\left(-\frac{\sqrt{4ac-b^2}e}{2\sqrt{c}\sqrt{cd^2-bed+ae^2}};\sin^{-1}\left(\sqrt[4]{1-\frac{(b+2cx)^2}{b^2-4ac}}\right)\middle|-1\right)(2cd-be)^2}{4\sqrt{2}c\left(cd^2-bed+ae^2\right)^2(b+2cx)\left(cx^2+bx+a\right)^{3/4}}-\frac{3\left(b^2-4ac\right)\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + b*x + c*x^2)^(3/4)),x]
```


```
[Out] -((e*(a + b*x + c*x^2)^(1/4))/((c*d^2 - b*d*e + a*e^2)*(d + e*x))) - (3*(-b
^2 + 4*a*c)^(3/4)*Sqrt[e]*(2*c*d - b*e)*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a
*c)))^(3/4)*ArcTan[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 -
4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(4*c^(3/4)
*(c*d^2 - b*d*e + a*e^2)^(7/4)*(a + b*x + c*x^2)^(3/4)) - (3*(-b^2 + 4*a*c)
^(3/4)*Sqrt[e]*(2*c*d - b*e)*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)
*ArcTanh[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1
/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(4*c^(3/4)*(c*d^2 -
```

```
b*d*e + a*e^2)^(7/4)*(a + b*x + c*x^2)^(3/4)) - (c^(3/4)*(b^2 - 4*a*c)^(1/4
)*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/
Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*
a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4
*a*c)^(1/4)], 1/2])/(Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*(b + 2*c*x)) - (3*(b^2
 - 4*a*c)*(2*c*d - b*e)^2*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x
+ c*x^2))/(b^2 - 4*a*c)))^(3/4)*EllipticPi[-(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[
c]*Sqrt[c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(
1/4)], -1])/(4*Sqrt[2]*c*(c*d^2 - b*d*e + a*e^2)^2*(b + 2*c*x)*(a + b*x + c
*x^2)^(3/4)) - (3*(b^2 - 4*a*c)*(2*c*d - b*e)^2*Sqrt[(b + 2*c*x)^2/(b^2 - 4
*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)*EllipticPi[(Sqrt[-b^2
 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c*
x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(4*Sqrt[2]*c*(c*d^2 - b*d*e + a*e^2)^2*(b
+ 2*c*x)*(a + b*x + c*x^2)^(3/4))
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 749

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[(a + b*x + c*x^2)^p/(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^p, I
nt[(-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) - (c^2*x^2)/(b^2 - 4*a*c
))^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&  !GtQ[4*a - b^2/c
, 0] && IntegerQ[4*p]
```

Rule 748

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[1/((-4*c)/(b^2 - 4*a*c))^p, Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c),
```

```
 x]^p/Simp[2*c*d - b*e + e*x, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && GtQ[4*a - b^2/c, 0] && IntegerQ[4*p]
```

Rule 747

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[-((b*x^2)/a)]/(2*x), Subst[Int[1/(Sqrt[-((b*x)/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 108

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - (d*e
)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+bx+cx^2\right)^{3/4}}\,dx = -\frac{e\sqrt[4]{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(d+ex)} - \frac{\int \frac{\frac{1}{4}(-4cd+3be)+\frac{cex}{2}}{(d+ex)\left(a+bx+cx^2\right)^{3/4}}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{e\sqrt[4]{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(d+ex)} - \frac{c\int \frac{1}{\left(a+bx+cx^2\right)^{3/4}}\,dx}{2\left(cd^2-bde+ae^2\right)} + \frac{(3(2cd-be))\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)}}{4\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{e\sqrt[4]{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(d+ex)} - \frac{\left(2c\sqrt{(b+2cx)^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b^2-4ac+4cx^4}}\,dx, x, \sqrt[4]{a+b\right.}{\left(cd^2-bde+ae^2\right)(b+2cx)}$$

$$= -\frac{e\sqrt[4]{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(d+ex)} - \frac{c^{3/4}\sqrt[4]{b^2-4ac}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+b}}{\sqrt{b^2-4}}\right.}{\sqrt{2}\left(cd^2-bde+ae^2\right)(b}$$

$$= -\frac{e\sqrt[4]{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(d+ex)} - \frac{c^{3/4}\sqrt[4]{b^2-4ac}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+b}}{\sqrt{b^2-4}}\right.}{\sqrt{2}\left(cd^2-bde+ae^2\right)(b}$$

$$= -\frac{e\sqrt[4]{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(d+ex)} - \frac{c^{3/4}\sqrt[4]{b^2-4ac}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+b}}{\sqrt{b^2-4}}\right.}{\sqrt{2}\left(cd^2-bde+ae^2\right)(b}$$

$$= -\frac{e\sqrt[4]{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(d+ex)} - \frac{c^{3/4}\sqrt[4]{b^2-4ac}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+b}}{\sqrt{b^2-4}}\right.}{\sqrt{2}\left(cd^2-bde+ae^2\right)(b}$$

$$= -\frac{e\sqrt[4]{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(d+ex)} - \frac{c^{3/4}\sqrt[4]{b^2-4ac}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+b}}{\sqrt{b^2-4}}\right.}{\sqrt{2}\left(cd^2-bde+ae^2\right)(b}$$

$$= -\frac{e\sqrt[4]{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(d+ex)} - \frac{3\left(-b^2+4ac\right)^{3/4}\sqrt{e}(2cd-be)\left(-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}\right)^{3/4}\tan^{-}}{4c^{3/4}\left(cd^2-bde+ae^2\right)^{7/4}\left(a+bx+\right.}$$

$$= -\frac{e\sqrt[4]{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(d+ex)} - \frac{3\left(-b^2+4ac\right)^{3/4}\sqrt{e}(2cd-be)\left(-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}\right)^{3/4}\tan^{-}}{4c^{3/4}\left(cd^2-bde+ae^2\right)^{7/4}\left(a+bx+\right.}$$

Mathematica [A] time = 3.03752, size = 657, normalized size = 0.68

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\left(\frac{c(a+x(b+cx))}{4ac-b^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right),2\right)+\frac{3\left(4ac-b^2\right)^{3/4}\left(\frac{c(a+x(b+cx))}{4ac-b^2}\right)^{3/4}(be-2cd)\left(-\sqrt{2}\sqrt[4]{c}\sqrt{e}(b+2cx)\sqrt[4]{e(ae-bd)+cd^2}\,\text{ta}\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + b*x + c*x^2)^(3/4)),x]
```

```
[Out] ((e*(a + x*(b + c*x)))/(d + e*x) + Sqrt[2]*Sqrt[b^2 - 4*a*c]*((c*(a + x*(b
+ c*x)))/(-b^2 + 4*a*c))^(3/4)*EllipticF[ArcSin[(b + 2*c*x)/Sqrt[b^2 - 4*a*
c]]/2, 2] + (3*(-b^2 + 4*a*c)^(3/4)*(-2*c*d + b*e)*((c*(a + x*(b + c*x)))/(
-b^2 + 4*a*c))^(3/4)*(-(Sqrt[2]*c^(1/4)*Sqrt[e]*(c*d^2 + e*(-(b*d) + a*e))^
(1/4)*(b + 2*c*x)*(ArcTan[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*((c*(a + x*(b + c*x
)))/(-b^2 + 4*a*c))^(1/4))/(c^(1/4)*(c*d^2 + e*(-(b*d) + a*e))^(1/4))] + Ar
cTanh[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*((c*(a + x*(b + c*x)))/(-b^2 + 4*a*c))^
(1/4))/(c^(1/4)*(c*d^2 + e*(-(b*d) + a*e))^(1/4))])) + (-b^2 + 4*a*c)^(1/4)
*(-2*c*d + b*e)*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*EllipticPi[-(Sqrt[-b^2 +
4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 + e*(-(b*d) + a*e)]), -ArcSin[Sqrt[2]*((c*(
a + x*(b + c*x)))/(-b^2 + 4*a*c))^(1/4)], -1] + (-b^2 + 4*a*c)^(1/4)*(-2*c*
d + b*e)*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*EllipticPi[(Sqrt[-b^2 + 4*a*c]*e
)/(2*Sqrt[c]*Sqrt[c*d^2 + e*(-(b*d) + a*e)]), -ArcSin[Sqrt[2]*((c*(a + x*(b
 + c*x)))/(-b^2 + 4*a*c))^(1/4)], -1]))/(4*Sqrt[2]*c*(c*d^2 + e*(-(b*d) + a
*e))*(b + 2*c*x)))/((-(c*d^2) + e*(b*d - a*e))*(a + x*(b + c*x))^(3/4))
```

Maple [F] time = 1.252, size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)^2}\left(cx^2+bx+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^2+b*x+a)^(3/4),x)
```

```
[Out] int(1/(e*x+d)^2/(c*x^2+b*x+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{3}{4}}(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(3/4)*(e*x + d)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x+a)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)^2\left(a+bx+cx^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**2+b*x+a)**(3/4),x)
```

```
[Out] Integral(1/((d + e*x)**2*(a + b*x + c*x**2)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{3}{4}}(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(3/4)*(e*x + d)^2), x)
```

3.2541 $\int \frac{1}{(d+ex)^3\left(a+bx+cx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=1134

result too large to display

```
[Out] -(e*(a + b*x + c*x^2)^(1/4))/(2*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^2) - (7*e
*(2*c*d - b*e)*(a + b*x + c*x^2)^(1/4))/(8*(c*d^2 - b*d*e + a*e^2)^2*(d + e
*x)) - (3*(-b^2 + 4*a*c)^(3/4)*Sqrt[e]*(20*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(5*b
*d + 2*a*e))*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)*ArcTan[((-b^2 +
 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(
1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))]])/(32*c^(3/4)*(c*d^2 - b*d*e + a*e^2)^(
11/4)*(a + b*x + c*x^2)^(3/4)) - (3*(-b^2 + 4*a*c)^(3/4)*Sqrt[e]*(20*c^2*d^
2 + 7*b^2*e^2 - 4*c*e*(5*b*d + 2*a*e))*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*
c)))^(3/4)*ArcTanh[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 -
4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))]])/(32*c^(3/4
)*(c*d^2 - b*d*e + a*e^2)^(11/4)*(a + b*x + c*x^2)^(3/4)) - (7*c^(3/4)*(b^2
 - 4*a*c)^(1/4)*(2*c*d - b*e)*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a +
 b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a +
b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(8*Sqrt[2]*(c*d^2 - b*d*e +
 a*e^2)^2*(b + 2*c*x)) - (3*(b^2 - 4*a*c)*(2*c*d - b*e)*(20*c^2*d^2 + 7*b^2
*e^2 - 4*c*e*(5*b*d + 2*a*e))*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a +
b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)*EllipticPi[-(Sqrt[-b^2 + 4*a*c]*e)/(2*S
qrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c
))^(1/4)], -1]])/(32*Sqrt[2]*c*(c*d^2 - b*d*e + a*e^2)^3*(b + 2*c*x)*(a + b*
x + c*x^2)^(3/4)) - (3*(b^2 - 4*a*c)*(2*c*d - b*e)*(20*c^2*d^2 + 7*b^2*e^2
- 4*c*e*(5*b*d + 2*a*e))*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x +
 c*x^2))/(b^2 - 4*a*c)))^(3/4)*EllipticPi[(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]
*Sqrt[c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/
4)], -1]])/(32*Sqrt[2]*c*(c*d^2 - b*d*e + a*e^2)^3*(b + 2*c*x)*(a + b*x + c*
x^2)^(3/4))
```

Rubi [A] time = 2.54877, antiderivative size = 1134, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 18, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {744, 834, 843, 623, 220, 749, 748, 747, 401, 108, 409, 1213, 537, 444, 63, 212, 208, 205}

$$-\frac{7(2cd-be)\sqrt[4]{cx^2+bx+ae}}{8\left(cd^2-bed+ae^2\right)^2(d+ex)}-\frac{\sqrt[4]{cx^2+bx+ae}}{2\left(cd^2-bed+ae^2\right)(d+ex)^2}-\frac{3\left(4ac-b^2\right)^{3/4}\left(20c^2d^2+7b^2e^2-4ce(5bd+2ae)\right)\left(-\frac{c(cx^2\cdots}{b^2\cdots}\right.}{32c^{3/4}\left(cd^2-bed+ae^2\right)^{11/4}(c\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + b*x + c*x^2)^(3/4)),x]
```

```
[Out] -(e*(a + b*x + c*x^2)^(1/4))/(2*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^2) - (7*e
*(2*c*d - b*e)*(a + b*x + c*x^2)^(1/4))/(8*(c*d^2 - b*d*e + a*e^2)^2*(d + e
*x)) - (3*(-b^2 + 4*a*c)^(3/4)*Sqrt[e]*(20*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(5*b
*d + 2*a*e))*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)*ArcTan[((-b^2 +
 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(
1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))]])/(32*c^(3/4)*(c*d^2 - b*d*e + a*e^2)^(
11/4)*(a + b*x + c*x^2)^(3/4)) - (3*(-b^2 + 4*a*c)^(3/4)*Sqrt[e]*(20*c^2*d^
2 + 7*b^2*e^2 - 4*c*e*(5*b*d + 2*a*e))*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*
```

```
c)))^(3/4)*ArcTanh[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 -
4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(32*c^(3/4
)*(c*d^2 - b*d*e + a*e^2)^(11/4)*(a + b*x + c*x^2)^(3/4)) - (7*c^(3/4)*(b^2
 - 4*a*c)^(1/4)*(2*c*d - b*e)*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a +
 b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a +
b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(8*Sqrt[2]*(c*d^2 - b*d*e +
 a*e^2)^2*(b + 2*c*x)) - (3*(b^2 - 4*a*c)*(2*c*d - b*e)*(20*c^2*d^2 + 7*b^2
*e^2 - 4*c*e*(5*b*d + 2*a*e))*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a +
b*x + c*x^2))/(b^2 - 4*a*c)))^(3/4)*EllipticPi[-(Sqrt[-b^2 + 4*a*c]*e)/(2*S
qrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c
))^(1/4)], -1])/(32*Sqrt[2]*c*(c*d^2 - b*d*e + a*e^2)^3*(b + 2*c*x)*(a + b*
x + c*x^2)^(3/4)) - (3*(b^2 - 4*a*c)*(2*c*d - b*e)*(20*c^2*d^2 + 7*b^2*e^2
- 4*c*e*(5*b*d + 2*a*e))*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x +
 c*x^2))/(b^2 - 4*a*c)))^(3/4)*EllipticPi[(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]
*Sqrt[c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/
4)], -1])/(32*Sqrt[2]*c*(c*d^2 - b*d*e + a*e^2)^3*(b + 2*c*x)*(a + b*x + c*
x^2)^(3/4))
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
```

```
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 749

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[(a + b*x + c*x^2)^p/(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^p, I
nt[(-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) - (c^2*x^2)/(b^2 - 4*a*c
))^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&  !GtQ[4*a - b^2/c
, 0] && IntegerQ[4*p]
```

Rule 748

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[1/((-4*c)/(b^2 - 4*a*c))^p, Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c),
 x]^p/Simp[2*c*d - b*e + e*x, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && GtQ[4*a - b^2/c, 0] && IntegerQ[4*p]
```

Rule 747

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[-((b*x^2)/a)]/(2*x), Subst[Int[1/(Sqrt[-((b*x)/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 108

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - (d*e
)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(a+bx+cx^2\right)^{3/4}}\,dx = -\frac{e\sqrt[4]{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{\int \frac{\frac{1}{4}(-8cd+7be)+\frac{3cex}{2}}{(d+ex)^2\left(a+bx+cx^2\right)^{3/4}}\,dx}{2\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{e\sqrt[4]{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{7e(2cd-be)\sqrt[4]{a+bx+cx^2}}{8\left(cd^2-bde+ae^2\right)^2(d+ex)} + \frac{\int \frac{\frac{1}{16}\left(32c^2d^2+21b^2e^2-2ce(\right.}{(d+ex)\left(a\right.}}{2\left(cd^2-\right.}$$

$$= -\frac{e\sqrt[4]{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{7e(2cd-be)\sqrt[4]{a+bx+cx^2}}{8\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{(7c(2cd-be))\int \frac{}{\left(a+bx\right.}}{16\left(cd^2-bde+\right.}$$

$$= -\frac{e\sqrt[4]{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{7e(2cd-be)\sqrt[4]{a+bx+cx^2}}{8\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{\left(7c(2cd-be)\sqrt{(b+2\right.}{4}$$

$$= -\frac{e\sqrt[4]{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{7e(2cd-be)\sqrt[4]{a+bx+cx^2}}{8\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{7c^{3/4}\sqrt[4]{b^2-4ac}(2cd-}{}$$

$$= -\frac{e\sqrt[4]{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{7e(2cd-be)\sqrt[4]{a+bx+cx^2}}{8\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{7c^{3/4}\sqrt[4]{b^2-4ac}(2cd-}{}$$

$$= -\frac{e\sqrt[4]{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{7e(2cd-be)\sqrt[4]{a+bx+cx^2}}{8\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{7c^{3/4}\sqrt[4]{b^2-4ac}(2cd-}{}$$

$$= -\frac{e\sqrt[4]{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{7e(2cd-be)\sqrt[4]{a+bx+cx^2}}{8\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{7c^{3/4}\sqrt[4]{b^2-4ac}(2cd-}{}$$

$$= -\frac{e\sqrt[4]{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{7e(2cd-be)\sqrt[4]{a+bx+cx^2}}{8\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{7c^{3/4}\sqrt[4]{b^2-4ac}(2cd-}{}$$

$$= -\frac{e\sqrt[4]{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{7e(2cd-be)\sqrt[4]{a+bx+cx^2}}{8\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{3\left(-b^2+4ac\right)^{3/4}\sqrt{e}\,(}{}$$

$$= -\frac{e\sqrt[4]{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{7e(2cd-be)\sqrt[4]{a+bx+cx^2}}{8\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{3\left(-b^2+4ac\right)^{3/4}\sqrt{e}\,(}{}$$

Mathematica [A] time = 6.23953, size = 1696, normalized size = 1.5

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a + b*x + c*x^2)^(3/4)),x]
```

```
[Out] -(e*(a + b*x + c*x^2))/(2*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^2*(a + x*(b + c
*x))^(3/4)) - ((a + b*x + c*x^2)^(3/4)*((((-3*c*d*e)/2 + (e*(-8*c*d + 7*b*e
))/4)*(a + b*x + c*x^2)^(1/4))/((-(c*d^2) + b*d*e - a*e^2)*(d + e*x)) + ((7
*Sqrt[b^2 - 4*a*c]*(c^2/((b^2 - 4*a*c)*((b^2*c^2)/(b^2 - 4*a*c)^2 - (4*a*c^
3)/(b^2 - 4*a*c)^2)))^(3/4)*(2*c*d - b*e)*(-((c*(a + b*x + c*x^2))/(b^2 - 4
*a*c)))^(3/4)*EllipticF[ArcSin[(Sqrt[b^2 - 4*a*c]*(-((b*c)/(b^2 - 4*a*c)) -
 (2*c^2*x)/(b^2 - 4*a*c)))/c]/2, 2])/(2*Sqrt[2]*(a + b*x + c*x^2)^(3/4)) +
(2*Sqrt[2]*(c^2/((b^2 - 4*a*c)*((b^2*c^2)/(b^2 - 4*a*c)^2 - (4*a*c^3)/(b^2
- 4*a*c)^2)))^(3/4)*((7*c*d*e*(2*c*d - b*e))/8 + (e*(32*c^2*d^2 + 21*b^2*e^
2 - 2*c*e*(23*b*d + 12*a*e)))/16)*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^
(3/4)*(-2*e*((c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4)*ArcTan[((-b^2 + 4*a*c)^
(1/4)*Sqrt[e]*(1 - (-((b*c)/(b^2 - 4*a*c)) - (2*c^2*x)/(b^2 - 4*a*c))^2/((b
^2*c^2)/(b^2 - 4*a*c)^2 - (4*a*c^3)/(b^2 - 4*a*c)^2))^(1/4))/(Sqrt[2]*c^(1/
4)*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(Sqrt[2]*(-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(
e^2 - ((-2*c^2*d)/(b^2 - 4*a*c) + (b*c*e)/(b^2 - 4*a*c))^2/((b^2*c^2)/(b^2
- 4*a*c)^2 - (4*a*c^3)/(b^2 - 4*a*c)^2))) + (c^(1/4)*(c*d^2 - b*d*e + a*e^2
)^(1/4)*ArcTanh[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (-((b*c)/(b^2 - 4*a*c))
- (2*c^2*x)/(b^2 - 4*a*c))^2/((b^2*c^2)/(b^2 - 4*a*c)^2 - (4*a*c^3)/(b^2 -
4*a*c)^2))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(Sqrt[2
]*(-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(e^2 - ((-2*c^2*d)/(b^2 - 4*a*c) + (b*c*e)/(
b^2 - 4*a*c))^2/((b^2*c^2)/(b^2 - 4*a*c)^2 - (4*a*c^3)/(b^2 - 4*a*c)^2))))
+ ((-2*c^2*d)/(b^2 - 4*a*c) + (b*c*e)/(b^2 - 4*a*c))*(-((Sqrt[(-((b*c)/(b^2
 - 4*a*c)) - (2*c^2*x)/(b^2 - 4*a*c))^2/((b^2*c^2)/(b^2 - 4*a*c)^2 - (4*a*c
^3)/(b^2 - 4*a*c)^2)]*EllipticPi[-(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*
d^2 - b*d*e + a*e^2]), -ArcSin[(1 - (-((b*c)/(b^2 - 4*a*c)) - (2*c^2*x)/(b^
2 - 4*a*c))^2/((b^2*c^2)/(b^2 - 4*a*c)^2 - (4*a*c^3)/(b^2 - 4*a*c)^2))^(1/4
)], -1])/((e^2 - ((-2*c^2*d)/(b^2 - 4*a*c) + (b*c*e)/(b^2 - 4*a*c))^2/((b^2
*c^2)/(b^2 - 4*a*c)^2 - (4*a*c^3)/(b^2 - 4*a*c)^2))*(-((b*c)/(b^2 - 4*a*c))
 - (2*c^2*x)/(b^2 - 4*a*c)))) - (Sqrt[(-((b*c)/(b^2 - 4*a*c)) - (2*c^2*x)/(
b^2 - 4*a*c))^2/((b^2*c^2)/(b^2 - 4*a*c)^2 - (4*a*c^3)/(b^2 - 4*a*c)^2)]*El
lipticPi[(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2]), -A
rcSin[(1 - (-((b*c)/(b^2 - 4*a*c)) - (2*c^2*x)/(b^2 - 4*a*c))^2/((b^2*c^2)/
(b^2 - 4*a*c)^2 - (4*a*c^3)/(b^2 - 4*a*c)^2))^(1/4)], -1])/((e^2 - ((-2*c^2
*d)/(b^2 - 4*a*c) + (b*c*e)/(b^2 - 4*a*c))^2/((b^2*c^2)/(b^2 - 4*a*c)^2 - (
4*a*c^3)/(b^2 - 4*a*c)^2))*(-((b*c)/(b^2 - 4*a*c)) - (2*c^2*x)/(b^2 - 4*a*c
))))))/(e*(a + b*x + c*x^2)^(3/4))))/(-(c*d^2) + b*d*e - a*e^2)))/(2*(c*d^2
 - b*d*e + a*e^2)*(a + x*(b + c*x))^(3/4))
```

Maple [F] time = 1.217, size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)^3}\left(cx^2+bx+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*x^2+b*x+a)^(3/4),x)
```

```
[Out] int(1/(e*x+d)^3/(c*x^2+b*x+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{3}{4}}(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(3/4)*(e*x + d)^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x+a)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)^3\left(a+bx+cx^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**2+b*x+a)**(3/4),x)
```

```
[Out] Integral(1/((d + e*x)**3*(a + b*x + c*x**2)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{3}{4}}(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^2+b*x+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(3/4)*(e*x + d)^3), x)
```

3.2542 $\int \frac{(d+ex)^3}{\left(a+bx+cx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=662

$$\frac{\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)(2cd-be)\left(-4ce(6ae+bd)+7b^2e^2+4c^2d^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{a+\cdots}}{\sqrt[4]{b^2-4\cdots}}\right.\right.}{2\sqrt{2}c^{11/4}\sqrt[4]{b^2-4ac}(b+2cx)}$$

```
[Out] (-4*(d + e*x)^2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*(a + b*x +
c*x^2)^(1/4)) + (2*e*(24*c^2*d^2 + 7*b^2*e^2 - 2*c*e*(9*b*d + 8*a*e) + 6*c*
e*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(3/4))/(3*c^2*(b^2 - 4*a*c)) + ((2*c*d
 - b*e)*(4*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(b*d + 6*a*e))*(b + 2*c*x)*(a + b*x
+ c*x^2)^(1/4))/(c^(5/2)*(b^2 - 4*a*c)^(3/2)*(1 + (2*Sqrt[c]*Sqrt[a + b*x +
 c*x^2])/Sqrt[b^2 - 4*a*c])) - ((2*c*d - b*e)*(4*c^2*d^2 + 7*b^2*e^2 - 4*c*
e*(b*d + 6*a*e))*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a +
 b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2]
)/Sqrt[b^2 - 4*a*c])*EllipticE[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^
(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(Sqrt[2]*c^(11/4)*(b^2 - 4*a*c)^(1/4)*(b
 + 2*c*x)) + ((2*c*d - b*e)*(4*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(b*d + 6*a*e))*S
qrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqr
t[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c
])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*
c)^(1/4)], 1/2])/(2*Sqrt[2]*c^(11/4)*(b^2 - 4*a*c)^(1/4)*(b + 2*c*x))
```

Rubi [A] time = 0.7245, antiderivative size = 662, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {738, 779, 623, 305, 220, 1196}

$$\frac{2e\left(a+bx+cx^2\right)^{3/4}\left(-2ce(8ae+9bd)+7b^2e^2+6cex(2cd-be)+24c^2d^2\right)}{3c^2\left(b^2-4ac\right)}+\frac{(b+2cx)\sqrt[4]{a+bx+cx^2}(2cd-be)\left(-4ce(6\cdots\right.}{c^{5/2}\left(b^2-4ac\right)^{3/2}\left(\frac{2\sqrt{c}\sqrt{a+b\cdots}}{\sqrt{b^2-\cdots}}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + b*x + c*x^2)^(5/4),x]
```


```
[Out] (-4*(d + e*x)^2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*(a + b*x +
c*x^2)^(1/4)) + (2*e*(24*c^2*d^2 + 7*b^2*e^2 - 2*c*e*(9*b*d + 8*a*e) + 6*c*
e*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(3/4))/(3*c^2*(b^2 - 4*a*c)) + ((2*c*d
 - b*e)*(4*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(b*d + 6*a*e))*(b + 2*c*x)*(a + b*x
+ c*x^2)^(1/4))/(c^(5/2)*(b^2 - 4*a*c)^(3/2)*(1 + (2*Sqrt[c]*Sqrt[a + b*x +
 c*x^2])/Sqrt[b^2 - 4*a*c])) - ((2*c*d - b*e)*(4*c^2*d^2 + 7*b^2*e^2 - 4*c*
e*(b*d + 6*a*e))*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a +
 b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2]
)/Sqrt[b^2 - 4*a*c])*EllipticE[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^
(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(Sqrt[2]*c^(11/4)*(b^2 - 4*a*c)^(1/4)*(b
 + 2*c*x)) + ((2*c*d - b*e)*(4*c^2*d^2 + 7*b^2*e^2 - 4*c*e*(b*d + 6*a*e))*S
qrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqr
t[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c
])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*
c)^(1/4)], 1/2])/(2*Sqrt[2]*c^(11/4)*(b^2 - 4*a*c)^(1/4)*(b + 2*c*x))
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(a+bx+cx^2\right)^{5/4}}\,dx = -\frac{4(d+ex)^2(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt[4]{a+bx+cx^2}} - \frac{4\int \frac{(d+ex)\left(\frac{1}{2}\left(-2cd^2-3bde+8ae^2\right)-\frac{5}{2}e(2cd-be)x\right)}{\sqrt[4]{a+bx+cx^2}}dx}{b^2-4ac}$$

$$= -\frac{4(d+ex)^2(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt[4]{a+bx+cx^2}} + \frac{2e\left(24c^2d^2+7b^2e^2-2ce(9bd+8ae)+6ce(2cd-be)\right.}{3c^2\left(b^2-4ac\right)}$$

$$= -\frac{4(d+ex)^2(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt[4]{a+bx+cx^2}} + \frac{2e\left(24c^2d^2+7b^2e^2-2ce(9bd+8ae)+6ce(2cd-be)\right.}{3c^2\left(b^2-4ac\right)}$$

$$= -\frac{4(d+ex)^2(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt[4]{a+bx+cx^2}} + \frac{2e\left(24c^2d^2+7b^2e^2-2ce(9bd+8ae)+6ce(2cd-be)\right.}{3c^2\left(b^2-4ac\right)}$$

$$= -\frac{4(d+ex)^2(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt[4]{a+bx+cx^2}} + \frac{2e\left(24c^2d^2+7b^2e^2-2ce(9bd+8ae)+6ce(2cd-be)\right.}{3c^2\left(b^2-4ac\right)}$$

Mathematica [C] time = 0.570505, size = 247, normalized size = 0.37

$$\frac{3\sqrt{2}(b+2cx)\sqrt[4]{\frac{c(a+x(b+cx))}{4ac-b^2}}(2cd-be)\left(-4ce(6ae+bd)+7b^2e^2+4c^2d^2\right)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)-8c\left(4c\left(4a^2e^3+ace\left(-9a\right.\right.\right.}{12c^3\left(b^2-4ac\right)\sqrt[4]{a+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + b*x + c*x^2)^(5/4),x]
```

```
[Out] (-8*c*(-7*b^3*e^3*x - b^2*e^2*(7*a*e + c*x*(-18*d + e*x)) + 2*b*c*(3*c*d^2*
(d - 3*e*x) + a*e^2*(9*d + 11*e*x)) + 4*c*(4*a^2*e^3 + 3*c^2*d^3*x + a*c*e*
(-9*d^2 - 9*d*e*x + e^2*x^2))) + 3*Sqrt[2]*(2*c*d - b*e)*(4*c^2*d^2 + 7*b^2
*e^2 - 4*c*e*(b*d + 6*a*e))*(b + 2*c*x)*((c*(a + x*(b + c*x)))/(-b^2 + 4*a*
c))^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(1
2*c^3*(b^2 - 4*a*c)*(a + x*(b + c*x))^(1/4))
```

Maple [F] time = 2.526, size = 0, normalized size = 0.

$$\int (ex+d)^3\left(cx^2+bx+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^2+b*x+a)^(5/4),x)
```

```
[Out] int((e*x+d)^3/(c*x^2+b*x+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3}{\left(cx^2+bx+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3/(c*x^2 + b*x + a)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3\right)\left(cx^2 + bx + a\right)^{\frac{3}{4}}}{c^2x^4 + 2\,bcx^3 + 2\,abx + \left(b^2 + 2\,ac\right)x^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(c*x^2 + b*x + a)^(3/4)/
(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^3}{\left(a + bx + cx^2\right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**2+b*x+a)**(5/4),x)
```

```
[Out] Integral((d + e*x)**3/(a + b*x + c*x**2)**(5/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^3}{\left(cx^2 + bx + a\right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^2+b*x+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3/(c*x^2 + b*x + a)^(5/4), x)
```

3.2543 $\int \frac{(d+ex)^2}{(a+bx+cx^2)^{5/4}} dx$

Optimal. Leaf size=594

$$\frac{\sqrt{\frac{(b+2cx)^2}{(b^2-4ac)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)\left(-4ce(2ae+bd)+3b^2e^2+4c^2d^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{a+bx+cx^2}}{\sqrt[4]{b^2-4ac}}\right),\frac{1}{2}\right)}{\sqrt{2}c^{7/4}\sqrt[4]{b^2-4ac}(b+2cx)}$$

```
[Out] (-4*(d + e*x)*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*(a + b*x + c*
x^2)^(1/4)) + (4*e*(2*c*d - b*e)*(a + b*x + c*x^2)^(3/4))/(c*(b^2 - 4*a*c))
 + (2*(4*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(b*d + 2*a*e))*(b + 2*c*x)*(a + b*x +
c*x^2)^(1/4))/(c^(3/2)*(b^2 - 4*a*c)^(3/2)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c
*x^2])/Sqrt[b^2 - 4*a*c])) - (Sqrt[2]*(4*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(b*d +
 2*a*e))*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c
*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b
^2 - 4*a*c])*EllipticE[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(
b^2 - 4*a*c)^(1/4)], 1/2])/(c^(7/4)*(b^2 - 4*a*c)^(1/4)*(b + 2*c*x)) + ((4*
c^2*d^2 + 3*b^2*e^2 - 4*c*e*(b*d + 2*a*e))*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c
)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*
c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(Sqrt[2]*c^(7/
4)*(b^2 - 4*a*c)^(1/4)*(b + 2*c*x))
```

Rubi [A] time = 0.736935, antiderivative size = 594, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {738, 640, 623, 305, 220, 1196}

$$\frac{2(b+2cx)\sqrt[4]{a+bx+cx^2}\left(-4ce(2ae+bd)+3b^2e^2+4c^2d^2\right)}{c^{3/2}\left(b^2-4ac\right)^{3/2}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)}+\frac{\sqrt{\frac{(b+2cx)^2}{(b^2-4ac)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)\left(-4ce(2ae+\right.}{\sqrt{2}c^{7/4}\sqrt[4]{b^2-4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*x + c*x^2)^(5/4),x]
```

```
[Out] (-4*(d + e*x)*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*(a + b*x + c*
x^2)^(1/4)) + (4*e*(2*c*d - b*e)*(a + b*x + c*x^2)^(3/4))/(c*(b^2 - 4*a*c))
 + (2*(4*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(b*d + 2*a*e))*(b + 2*c*x)*(a + b*x +
c*x^2)^(1/4))/(c^(3/2)*(b^2 - 4*a*c)^(3/2)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c
*x^2])/Sqrt[b^2 - 4*a*c])) - (Sqrt[2]*(4*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(b*d +
 2*a*e))*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c
*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b
^2 - 4*a*c])*EllipticE[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(
b^2 - 4*a*c)^(1/4)], 1/2])/(c^(7/4)*(b^2 - 4*a*c)^(1/4)*(b + 2*c*x)) + ((4*
c^2*d^2 + 3*b^2*e^2 - 4*c*e*(b*d + 2*a*e))*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c
)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*
c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(Sqrt[2]*c^(7/
4)*(b^2 - 4*a*c)^(1/4)*(b + 2*c*x))
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(d+ex)^2}{(a+bx+cx^2)^{5/4}}\,dx = -\frac{4(d+ex)(bd-2ae+(2cd-be)x)}{(b^2-4ac)\sqrt[4]{a+bx+cx^2}} - \frac{4\int \frac{\frac{1}{2}\left(-2cd^2-2e\left(\frac{bd}{2}-2ae\right)\right)-\frac{3}{2}e(2cd-be)x}{\sqrt[4]{a+bx+cx^2}}dx}{b^2-4ac}$$

$$= -\frac{4(d+ex)(bd-2ae+(2cd-be)x)}{(b^2-4ac)\sqrt[4]{a+bx+cx^2}} + \frac{4e(2cd-be)(a+bx+cx^2)^{3/4}}{c(b^2-4ac)} + \frac{(4c^2d^2+3b^2e^2-4ce \cdots}{c(\cdots}$$

$$= -\frac{4(d+ex)(bd-2ae+(2cd-be)x)}{(b^2-4ac)\sqrt[4]{a+bx+cx^2}} + \frac{4e(2cd-be)(a+bx+cx^2)^{3/4}}{c(b^2-4ac)} + \frac{(4(4c^2d^2+3b^2e^2- \cdots}{\cdots}$$

$$= -\frac{4(d+ex)(bd-2ae+(2cd-be)x)}{(b^2-4ac)\sqrt[4]{a+bx+cx^2}} + \frac{4e(2cd-be)(a+bx+cx^2)^{3/4}}{c(b^2-4ac)} + \frac{(2(4c^2d^2+3b^2e^2- \cdots}{\cdots}$$

$$= -\frac{4(d+ex)(bd-2ae+(2cd-be)x)}{(b^2-4ac)\sqrt[4]{a+bx+cx^2}} + \frac{4e(2cd-be)(a+bx+cx^2)^{3/4}}{c(b^2-4ac)} + \frac{2(4c^2d^2+3b^2e^2-4 \cdots}{c^{3/2}(b^2- \cdots}$$

Mathematica [C] time = 0.336608, size = 177, normalized size = 0.3

$$\frac{\sqrt{2}(b+2cx)\sqrt[4]{\frac{c(a+x(b+cx))}{4ac-b^2}}\left(-4ce(2ae+bd)+3b^2e^2+4c^2d^2\right)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)-8c\left(abe^2-2ace(2d+ex)+b^2e^2x+ \cdots\right.}{2c^2(b^2-4ac)\sqrt[4]{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*x + c*x^2)^(5/4),x]
```

```
[Out] (-8*c*(a*b*e^2 + 2*c^2*d^2*x + b^2*e^2*x + b*c*d*(d - 2*e*x) - 2*a*c*e*(2*d
 + e*x)) + Sqrt[2]*(4*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(b*d + 2*a*e))*(b + 2*c*x
)*((c*(a + x*(b + c*x)))/(-b^2 + 4*a*c))^(1/4)*Hypergeometric2F1[1/4, 1/2,
3/2, (b + 2*c*x)^2/(b^2 - 4*a*c)])/(2*c^2*(b^2 - 4*a*c)*(a + x*(b + c*x))^(
1/4))
```

Maple [F] time = 1.151, size = 0, normalized size = 0.

$$\int (ex+d)^2\left(cx^2+bx+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^2+b*x+a)^(5/4),x)
```

```
[Out] int((e*x+d)^2/(c*x^2+b*x+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2}{(cx^2+bx+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2/(c*x^2 + b*x + a)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e^2x^2 + 2\,dex + d^2\right)\left(cx^2 + bx + a\right)^{\frac{3}{4}}}{c^2x^4 + 2\,bcx^3 + 2\,abx + \left(b^2 + 2\,ac\right)x^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(c*x^2 + b*x + a)^(3/4)/(c^2*x^4 + 2*b*c
*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^2}{\left(a + bx + cx^2\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**2+b*x+a)**(5/4),x)
```

```
[Out] Integral((d + e*x)**2/(a + b*x + c*x**2)**(5/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^2}{\left(cx^2 + bx + a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^2+b*x+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2/(c*x^2 + b*x + a)^(5/4), x)
```

3.2544 $\int \frac{d+ex}{\left(a+bx+cx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=490

$$\frac{\sqrt{2}\sqrt{\frac{(b+2cx)^2}{(b^2-4ac)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)(2cd-be)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{a+bx+cx^2}}{\sqrt[4]{b^2-4ac}}\right),\frac{1}{2}\right)}{c^{3/4}\sqrt[4]{b^2-4ac}(b+2cx)}-2\sqrt{2}\sqrt{\frac{(b+\cdots}{(b^2-4ac)\left(\frac{2\sqrt{c}\cdots}{}\right.}}\cdots$$

```
[Out] (-4*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)^(1/4)
) + (4*(2*c*d - b*e)*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4))/(Sqrt[c]*(b^2 - 4
*a*c)^(3/2)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])) - (2
*Sqrt[2]*(2*c*d - b*e)*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sq
rt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x +
c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticE[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c
*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(c^(3/4)*(b^2 - 4*a*c)^(1/4)*(b +
2*c*x)) + (Sqrt[2]*(2*c*d - b*e)*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*
Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[
a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a
 + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(c^(3/4)*(b^2 - 4*a*c)^(
1/4)*(b + 2*c*x))
```

Rubi [A] time = 0.396542, antiderivative size = 490, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {638, 623, 305, 220, 1196}

$$\frac{\sqrt{2}\sqrt{\frac{(b+2cx)^2}{(b^2-4ac)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)(2cd-be)F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{cx^2+bx+a}}{\sqrt[4]{b^2-4ac}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt[4]{b^2-4ac}(b+2cx)}-2\sqrt{2}\sqrt{\frac{(b+2cx)^2}{(b^2-4ac)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+\cdots\right.}}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*x + c*x^2)^(5/4),x]
```


```
[Out] (-4*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)^(1/4)
) + (4*(2*c*d - b*e)*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4))/(Sqrt[c]*(b^2 - 4
*a*c)^(3/2)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])) - (2
*Sqrt[2]*(2*c*d - b*e)*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sq
rt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x +
c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticE[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c
*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(c^(3/4)*(b^2 - 4*a*c)^(1/4)*(b +
2*c*x)) + (Sqrt[2]*(2*c*d - b*e)*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*
Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[
a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a
 + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(c^(3/4)*(b^2 - 4*a*c)^(
1/4)*(b + 2*c*x))
```


Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
```

```
NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{d+ex}{\left(a+bx+cx^2\right)^{5/4}}\,dx = -\frac{4(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt[4]{a+bx+cx^2}} + \frac{(2(2cd-be))\int \frac{1}{\sqrt[4]{a+bx+cx^2}}\,dx}{b^2-4ac}$$

$$= -\frac{4(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt[4]{a+bx+cx^2}} + \frac{\left(8(2cd-be)\sqrt{(b+2cx)^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b^2-4ac+4cx^4}}\,dx, x, \sqrt[4]{a+bx+\right.}{\left(b^2-4ac\right)(b+2cx)}$$

$$= -\frac{4(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt[4]{a+bx+cx^2}} + \frac{\left(4(2cd-be)\sqrt{(b+2cx)^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b^2-4ac+4cx^4}}\,dx, x, \sqrt[4]{a+bx+\right.}{\sqrt{c}\sqrt{b^2-4ac}(b+2cx)}$$

$$= -\frac{4(bd-2ae+(2cd-be)x)}{\left(b^2-4ac\right)\sqrt[4]{a+bx+cx^2}} + \frac{4(2cd-be)(b+2cx)\sqrt[4]{a+bx+cx^2}}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)} - \frac{2\sqrt{2}(2cd-be)\sqrt{\frac{(b}{\left(b^2-4ac\right)\left(1+\right.}}}{}$$

Mathematica [C] time = 0.229875, size = 167, normalized size = 0.34

$$-\frac{2\left(2^{3/4}\left(-\sqrt{b^2-4ac}+b+2cx\right)\sqrt[4]{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}(be-2cd)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{-b-2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)+6c(-2ae+b(d-ex)+2cdx)\right)}{3c\left(b^2-4ac\right)\sqrt[4]{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*x + c*x^2)^(5/4),x]
```

```
[Out] (-2*(6*c*(-2*a*e + 2*c*d*x + b*(d - e*x)) + 2^(3/4)*(-2*c*d + b*e)*(b - Sqr
t[b^2 - 4*a*c] + 2*c*x)*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c])
^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/(2
*Sqrt[b^2 - 4*a*c])]))/(3*c*(b^2 - 4*a*c)*(a + x*(b + c*x))^(1/4))
```

Maple [F] time = 0.957, size = 0, normalized size = 0.

$$\int (ex+d)\left(cx^2+bx+a\right)^{-\frac{5}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^2+b*x+a)^(5/4),x)
```

```
[Out] int((e*x+d)/(c*x^2+b*x+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex+d}{\left(cx^2+bx+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)/(c*x^2 + b*x + a)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+bx+a\right)^{\frac{3}{4}}(ex+d)}{c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\,ac\right)x^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^(3/4)*(e*x + d)/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x +
 (b^2 + 2*a*c)*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d+ex}{\left(a+bx+cx^2\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**2+b*x+a)**(5/4),x)
```

```
[Out] Integral((d + e*x)/(a + b*x + c*x**2)**(5/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex+d}{\left(cx^2+bx+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^2+b*x+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)/(c*x^2 + b*x + a)^(5/4), x)
```

3.2545 $\int \frac{1}{(a+bx+cx^2)^{5/4}}\,dx$

Optimal. Leaf size=451

$$\frac{2\sqrt{2}\sqrt[4]{c}\sqrt{\frac{(b+2cx)^2}{(b^2-4ac)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{a+bx+cx^2}}{\sqrt[4]{b^2-4ac}}\right),\frac{1}{2}\right)}{\sqrt[4]{b^2-4ac}(b+2cx)}-\frac{4(b+2cx)}{(b^2-4ac)\sqrt[4]{a+bx+c\ldots}}$$

```
[Out] (-4*(b + 2*c*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)^(1/4)) + (8*Sqrt[c]*(b +
2*c*x)*(a + b*x + c*x^2)^(1/4))/((b^2 - 4*a*c)^(3/2)*(1 + (2*Sqrt[c]*Sqrt[a
 + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])) - (4*Sqrt[2]*c^(1/4)*Sqrt[(b + 2*c*x)^
2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^
2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticE[2*A
rcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])
/((b^2 - 4*a*c)^(1/4)*(b + 2*c*x)) + (2*Sqrt[2]*c^(1/4)*Sqrt[(b + 2*c*x)^2/
((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)
]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*Arc
Tan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(
(b^2 - 4*a*c)^(1/4)*(b + 2*c*x))
```

Rubi [A] time = 0.360722, antiderivative size = 451, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {614, 623, 305, 220, 1196}

$$-\frac{4(b+2cx)}{(b^2-4ac)\sqrt[4]{a+bx+cx^2}}+\frac{8\sqrt{c}(b+2cx)\sqrt[4]{a+bx+cx^2}}{(b^2-4ac)^{3/2}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)}+\frac{2\sqrt{2}\sqrt[4]{c}\sqrt{\frac{(b+2cx)^2}{(b^2-4ac)\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)^2}}\left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}+1\right)F\ldots}{\sqrt[4]{b^2-4ac}(b+2cx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(-5/4),x]
```

```
[Out] (-4*(b + 2*c*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)^(1/4)) + (8*Sqrt[c]*(b +
2*c*x)*(a + b*x + c*x^2)^(1/4))/((b^2 - 4*a*c)^(3/2)*(1 + (2*Sqrt[c]*Sqrt[a
 + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])) - (4*Sqrt[2]*c^(1/4)*Sqrt[(b + 2*c*x)^
2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^
2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticE[2*A
rcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])
/((b^2 - 4*a*c)^(1/4)*(b + 2*c*x)) + (2*Sqrt[2]*c^(1/4)*Sqrt[(b + 2*c*x)^2/
((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)
]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*Arc
Tan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(
(b^2 - 4*a*c)^(1/4)*(b + 2*c*x))
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\left(a+bx+cx^2\right)^{5/4}}\,dx = -\frac{4(b+2cx)}{\left(b^2-4ac\right)\sqrt[4]{a+bx+cx^2}}+\frac{(4c)\int \frac{1}{\sqrt[4]{a+bx+cx^2}}\,dx}{b^2-4ac}$$

$$= -\frac{4(b+2cx)}{\left(b^2-4ac\right)\sqrt[4]{a+bx+cx^2}}+\frac{\left(16c\sqrt{(b+2cx)^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b^2-4ac+4cx^4}}\,dx,x,\sqrt[4]{a+bx+cx^2}\right)}{\left(b^2-4ac\right)(b+2cx)}$$

$$= -\frac{4(b+2cx)}{\left(b^2-4ac\right)\sqrt[4]{a+bx+cx^2}}+\frac{\left(8\sqrt{c}\sqrt{(b+2cx)^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b^2-4ac+4cx^4}}\,dx,x,\sqrt[4]{a+bx+cx^2}\right)}{\sqrt{b^2-4ac}(b+2cx)} -$$

$$= -\frac{4(b+2cx)}{\left(b^2-4ac\right)\sqrt[4]{a+bx+cx^2}}+\frac{8\sqrt{c}(b+2cx)\sqrt[4]{a+bx+cx^2}}{\left(b^2-4ac\right)^{3/2}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}-\frac{4\sqrt{2}\sqrt[4]{c}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right.}}}{}$$

Mathematica [C] time = 0.0708589, size = 100, normalized size = 0.22

$$-\frac{2\sqrt{2}(b+2cx)\left(\sqrt{2}-\sqrt[4]{\frac{c(a+x(b+cx))}{4ac-b^2}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{(b+2cx)^2}{b^2-4ac}\right)\right)}{\left(b^2-4ac\right)\sqrt[4]{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(-5/4),x]
```

```
[Out] (-2*Sqrt[2]*(b + 2*c*x)*(Sqrt[2] - ((c*(a + x*(b + c*x)))/(-b^2 + 4*a*c))^(
1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (b + 2*c*x)^2/(b^2 - 4*a*c)]))/((b^2
- 4*a*c)*(a + x*(b + c*x))^(1/4))
```

Maple [F] time = 2.167, size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(5/4),x)
```

```
[Out] int(1/(c*x^2+b*x+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(-5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+bx+a\right)^{\frac{3}{4}}}{c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\,ac\right)x^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^(3/4)/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*
a*c)*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx+cx^2\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(5/4),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(-5/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(-5/4), x)
```

3.2546 $\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=1299

result too large to display

```
[Out] (-4*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2 -
b*d*e + a*e^2)*(a + b*x + c*x^2)^(1/4)) + (4*Sqrt[c]*(2*c*d - b*e)*(b + 2*c
*x)*(a + b*x + c*x^2)^(1/4))/((b^2 - 4*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)*(
1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])) + ((-b^2 + 4*a*c)
^(1/4)*e^(3/2)*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(1/4)*ArcTan[((-b^2
 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c
^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(5
/4)*(a + b*x + c*x^2)^(1/4)) - ((-b^2 + 4*a*c)^(1/4)*e^(3/2)*(-((c*(a + b*x
 + c*x^2))/(b^2 - 4*a*c)))^(1/4)*ArcTanh[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 -
 (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^
2)^(1/4))])/(c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(5/4)*(a + b*x + c*x^2)^(1/4))
 - (2*Sqrt[2]*c^(1/4)*(2*c*d - b*e)*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 +
(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sq
rt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticE[2*ArcTan[(Sqrt[2]*c^(1/4)
*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/((b^2 - 4*a*c)^(1/4)*
(c*d^2 - b*d*e + a*e^2)*(b + 2*c*x)) + (Sqrt[2]*c^(1/4)*(2*c*d - b*e)*Sqrt[
(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^
2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*E
llipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(
1/4)], 1/2])/((b^2 - 4*a*c)^(1/4)*(c*d^2 - b*d*e + a*e^2)*(b + 2*c*x)) - (S
qrt[-b^2 + 4*a*c]*e*(2*c*d - b*e)*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(
a + b*x + c*x^2))/(b^2 - 4*a*c)))^(1/4)*EllipticPi[-(Sqrt[-b^2 + 4*a*c]*e)/
(2*Sqrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4
*a*c))^(1/4)], -1])/(Sqrt[2]*Sqrt[c]*(c*d^2 - b*d*e + a*e^2)^(3/2)*(b + 2*c
*x)*(a + b*x + c*x^2)^(1/4)) + (Sqrt[-b^2 + 4*a*c]*e*(2*c*d - b*e)*Sqrt[(b
+ 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(1/4)*El
lipticPi[(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2]), Ar
cSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(Sqrt[2]*Sqrt[c]*(c*d^2
 - b*d*e + a*e^2)^(3/2)*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4))
```

Rubi [A] time = 2.51762, antiderivative size = 1299, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 18, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {740, 843, 623, 305, 220, 1196, 749, 748, 746, 399, 490, 1213, 537, 444, 63, 298, 205, 208}

$$\frac{\sqrt[4]{4ac-b^2}\sqrt[4]{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt[4]{4ac-b^2}\sqrt{e}\sqrt[4]{1-\frac{(b+2cx)^2}{b^2-4ac}}}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{cd^2-bed+ae^2}}\right)e^{3/2}}{\sqrt[4]{c}\left(cd^2-bed+ae^2\right)^{5/4}\sqrt[4]{cx^2+bx+a}}-\frac{\sqrt[4]{4ac-b^2}\sqrt[4]{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\tanh^{-1}\left(\frac{\sqrt[4]{4ac-b^2}\sqrt{e}\sqrt[4]{1-\frac{(b+2cx)^2}{b^2-4ac}}}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{cd^2-bed+ae^2}}\right)e^{3/2}}{\sqrt[4]{c}\left(cd^2-bed+ae^2\right)^{5/4}\sqrt[4]{cx^2+bx+a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + b*x + c*x^2)^(5/4)),x]
```

```
[Out] (-4*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2 -
b*d*e + a*e^2)*(a + b*x + c*x^2)^(1/4)) + (4*Sqrt[c]*(2*c*d - b*e)*(b + 2*c
*x)*(a + b*x + c*x^2)^(1/4))/((b^2 - 4*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)*(
1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])) + ((-b^2 + 4*a*c)
```

```
^(1/4)*e^(3/2)*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(1/4)*ArcTan[((-b^2
 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c
^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))]])/(c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(5
/4)*(a + b*x + c*x^2)^(1/4)) - ((-b^2 + 4*a*c)^(1/4)*e^(3/2)*(-((c*(a + b*x
 + c*x^2))/(b^2 - 4*a*c)))^(1/4)*ArcTanh[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 -
 (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^
2)^(1/4))]])/(c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(5/4)*(a + b*x + c*x^2)^(1/4))
 - (2*Sqrt[2]*c^(1/4)*(2*c*d - b*e)*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 +
(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sq
rt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticE[2*ArcTan[(Sqrt[2]*c^(1/4)
*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/((b^2 - 4*a*c)^(1/4)*
(c*d^2 - b*d*e + a*e^2)*(b + 2*c*x)) + (Sqrt[2]*c^(1/4)*(2*c*d - b*e)*Sqrt[
(b + 2*c*x)^2/((b^2 - 4*a*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^
2 - 4*a*c])^2)]*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*E
llipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(
1/4)], 1/2])/((b^2 - 4*a*c)^(1/4)*(c*d^2 - b*d*e + a*e^2)*(b + 2*c*x)) - (S
qrt[-b^2 + 4*a*c]*e*(2*c*d - b*e)*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(
a + b*x + c*x^2))/(b^2 - 4*a*c)))^(1/4)*EllipticPi[-(Sqrt[-b^2 + 4*a*c]*e)/
(2*Sqrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4
*a*c))^(1/4)], -1])/(Sqrt[2]*Sqrt[c]*(c*d^2 - b*d*e + a*e^2)^(3/2)*(b + 2*c
*x)*(a + b*x + c*x^2)^(1/4)) + (Sqrt[-b^2 + 4*a*c]*e*(2*c*d - b*e)*Sqrt[(b
+ 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(1/4)*El
lipticPi[(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2]), Ar
cSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(Sqrt[2]*Sqrt[c]*(c*d^2
 - b*d*e + a*e^2)^(3/2)*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4))
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 749

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[(a + b*x + c*x^2)^p/(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^p, I
nt[(-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) - (c^2*x^2)/(b^2 - 4*a*c
))^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&  !GtQ[4*a - b^2/c
, 0] && IntegerQ[4*p]
```

Rule 748

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[1/((-4*c)/(b^2 - 4*a*c))^p, Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c),
 x]^p/Simp[2*c*d - b*e + e*x, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && GtQ[4*a - b^2/c, 0] && IntegerQ[4*p]
```

Rule 746

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{5/4}}\,dx = -\frac{4\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt[4]{a+bx+cx^2}} - \frac{4\int \frac{\frac{1}{4}\left(-4c^2d^2-b^2e^2+2ce(bd+2ae)\right)-\frac{1}{2}ce(2cd-b\cdots}{(d+ex)\sqrt[4]{a+bx+cx^2}}}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)}\cdots$$

$$= -\frac{4\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt[4]{a+bx+cx^2}} + \frac{e^2\int \frac{1}{(d+ex)\sqrt[4]{a+bx+cx^2}}\,dx}{cd^2-bde+ae^2} + \frac{(2c(2cd-\cdots}{\left(b^2-4ac\cdots}$$

$$= -\frac{4\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt[4]{a+bx+cx^2}} + \frac{\left(8c(2cd-be)\sqrt{(b+2cx)^2}\right)\text{Subst}\left(\int\cdots\right.}{\left(b^2-4ac\right)\left(cd^2-b\cdots\right.}$$

$$= -\frac{4\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt[4]{a+bx+cx^2}} + \frac{\left(4\sqrt{c}(2cd-be)\sqrt{(b+2cx)^2}\right)\text{Subst}\cdots}{\sqrt{b^2-4ac}\left(cd^2-\cdots\right.}$$

$$= -\frac{4\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt[4]{a+bx+cx^2}} + \frac{4\sqrt{c}(2cd-be)(b+2cx)\sqrt[4]{a+b\cdots}}{\left(b^2-4ac\right)^{3/2}\left(cd^2-bde+ae^2\right)\left(1+\cdots\right.}$$

$$= -\frac{4\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt[4]{a+bx+cx^2}} + \frac{4\sqrt{c}(2cd-be)(b+2cx)\sqrt[4]{a+b\cdots}}{\left(b^2-4ac\right)^{3/2}\left(cd^2-bde+ae^2\right)\left(1+\cdots\right.}$$

$$= -\frac{4\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt[4]{a+bx+cx^2}} + \frac{4\sqrt{c}(2cd-be)(b+2cx)\sqrt[4]{a+b\cdots}}{\left(b^2-4ac\right)^{3/2}\left(cd^2-bde+ae^2\right)\left(1+\cdots\right.}$$

$$= -\frac{4\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt[4]{a+bx+cx^2}} + \frac{4\sqrt{c}(2cd-be)(b+2cx)\sqrt[4]{a+b\cdots}}{\left(b^2-4ac\right)^{3/2}\left(cd^2-bde+ae^2\right)\left(1+\cdots\right.}$$

$$= -\frac{4\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt[4]{a+bx+cx^2}} + \frac{4\sqrt{c}(2cd-be)(b+2cx)\sqrt[4]{a+b\cdots}}{\left(b^2-4ac\right)^{3/2}\left(cd^2-bde+ae^2\right)\left(1+\cdots\right.}$$

Mathematica [C] time = 0.55087, size = 180, normalized size = 0.14

$$-\frac{\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{5/4}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{5/4}F_1\left(\frac{5}{2};\frac{5}{4},\frac{5}{4};\frac{7}{2};\frac{2cd-\left(b+\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2cd-be+\sqrt{b^2-4ace}}{2cd+2cex}\right)}{10\sqrt{2}e(a+x(b+cx))^{5/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x)*(a + b*x + c*x^2)^(5/4)),x]
```

```
[Out] -(((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(5/4)*((e*(b + Sqrt[b
^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(5/4)*AppellF1[5/2, 5/4, 5/4, 7/2, (2*
c*d - (b + Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x)), (2*c*d - b*e + Sqrt[b^2 -
 4*a*c]*e)/(2*c*d + 2*c*e*x)])/(10*Sqrt[2]*e*(a + x*(b + c*x))^(5/4))
```

Maple [F] time = 1.271, size = 0, normalized size = 0.

$$\int \frac{1}{ex+d}\left(cx^2+bx+a\right)^{-\frac{5}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+b*x+a)^(5/4),x)
```

```
[Out] int(1/(e*x+d)/(c*x^2+b*x+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{5}{4}}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(5/4)*(e*x + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(5/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+b*x+a)**(5/4),x)
```

```
[Out] Integral(1/((d + e*x)*(a + b*x + c*x**2)**(5/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2 + bx + a\right)^{\frac{5}{4}}(ex + d)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(5/4)*(e*x + d)), x)
```

3.2547 $\int \frac{1}{(d+ex)^2\left(a+bx+cx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=1485

result too large to display

```
[Out] (-4*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2 -
b*d*e + a*e^2)*(d + e*x)*(a + b*x + c*x^2)^(1/4)) - (e*(8*c^2*d^2 + 5*b^2*e
^2 - 4*c*e*(2*b*d + 3*a*e))*(a + b*x + c*x^2)^(3/4))/((b^2 - 4*a*c)*(c*d^2
- b*d*e + a*e^2)^2*(d + e*x)) + (Sqrt[c]*(8*c^2*d^2 + 5*b^2*e^2 - 4*c*e*(2*
b*d + 3*a*e))*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4))/((b^2 - 4*a*c)^(3/2)*(c*
d^2 - b*d*e + a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*
a*c])) + (5*(-b^2 + 4*a*c)^(1/4)*e^(3/2)*(2*c*d - b*e)*(-((c*(a + b*x + c*x
^2))/(b^2 - 4*a*c)))^(1/4)*ArcTan[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2
*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4
))])/(4*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(9/4)*(a + b*x + c*x^2)^(1/4)) - (5
*(-b^2 + 4*a*c)^(1/4)*e^(3/2)*(2*c*d - b*e)*(-((c*(a + b*x + c*x^2))/(b^2 -
 4*a*c)))^(1/4)*ArcTanh[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b
^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(4*c^
(1/4)*(c*d^2 - b*d*e + a*e^2)^(9/4)*(a + b*x + c*x^2)^(1/4)) - (c^(1/4)*(8*
c^2*d^2 + 5*b^2*e^2 - 4*c*e*(2*b*d + 3*a*e))*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a
*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*S
qrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticE[2*ArcTan[(Sqrt[2
]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(Sqrt[2]*(b^
2 - 4*a*c)^(1/4)*(c*d^2 - b*d*e + a*e^2)^2*(b + 2*c*x)) + (c^(1/4)*(8*c^2*d
^2 + 5*b^2*e^2 - 4*c*e*(2*b*d + 3*a*e))*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(
1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(
1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(2*Sqrt[2]*(b^2 -
 4*a*c)^(1/4)*(c*d^2 - b*d*e + a*e^2)^2*(b + 2*c*x)) - (5*Sqrt[-b^2 + 4*a*c
]*e*(2*c*d - b*e)^2*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^
2))/(b^2 - 4*a*c)))^(1/4)*EllipticPi[-(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqr
t[c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)],
 -1])/(4*Sqrt[2]*Sqrt[c]*(c*d^2 - b*d*e + a*e^2)^(5/2)*(b + 2*c*x)*(a + b*x
 + c*x^2)^(1/4)) + (5*Sqrt[-b^2 + 4*a*c]*e*(2*c*d - b*e)^2*Sqrt[(b + 2*c*x)
^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(1/4)*EllipticPi
[(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2]), ArcSin[(1
- (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(4*Sqrt[2]*Sqrt[c]*(c*d^2 - b*d
*e + a*e^2)^(5/2)*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4))
```

Rubi [A] time = 3.17006, antiderivative size = 1485, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 19, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.864, Rules used = {740, 834, 843, 623, 305, 220, 1196, 749, 748, 746, 399, 490, 1213, 537, 444, 63, 298, 205, 208}

$$-\frac{5\sqrt{4ac-b^2}e\sqrt{\frac{(b+2cx)^2}{b^2-4ac}}\sqrt[4]{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\Pi\left(-\frac{\sqrt{4ac-b^2}e}{2\sqrt{c}\sqrt{cd^2-bed+ae^2}};\sin^{-1}\left(\sqrt[4]{1-\frac{(b+2cx)^2}{b^2-4ac}}\right)\middle|-1\right)(2cd-be)^2}{4\sqrt{2}\sqrt{c}\left(cd^2-bed+ae^2\right)^{5/2}(b+2cx)\sqrt[4]{cx^2+bx+a}}+\frac{5\sqrt{4ac-b^2}e\sqrt{\frac{(b+2cx)^2}{b^2-4ac}}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + b*x + c*x^2)^(5/4)),x]
```

```
[Out] (-4*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2 -
b*d*e + a*e^2)*(d + e*x)*(a + b*x + c*x^2)^(1/4)) - (e*(8*c^2*d^2 + 5*b^2*e
^2 - 4*c*e*(2*b*d + 3*a*e))*(a + b*x + c*x^2)^(3/4))/((b^2 - 4*a*c)*(c*d^2
- b*d*e + a*e^2)^2*(d + e*x)) + (Sqrt[c]*(8*c^2*d^2 + 5*b^2*e^2 - 4*c*e*(2*
b*d + 3*a*e))*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4))/((b^2 - 4*a*c)^(3/2)*(c*
d^2 - b*d*e + a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*
a*c])) + (5*(-b^2 + 4*a*c)^(1/4)*e^(3/2)*(2*c*d - b*e)*(-((c*(a + b*x + c*x
^2))/(b^2 - 4*a*c)))^(1/4)*ArcTan[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2
*c*x)^2/(b^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4
))])/(4*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(9/4)*(a + b*x + c*x^2)^(1/4)) - (5
*(-b^2 + 4*a*c)^(1/4)*e^(3/2)*(2*c*d - b*e)*(-((c*(a + b*x + c*x^2))/(b^2 -
 4*a*c)))^(1/4)*ArcTanh[((-b^2 + 4*a*c)^(1/4)*Sqrt[e]*(1 - (b + 2*c*x)^2/(b
^2 - 4*a*c))^(1/4))/(Sqrt[2]*c^(1/4)*(c*d^2 - b*d*e + a*e^2)^(1/4))])/(4*c^
(1/4)*(c*d^2 - b*d*e + a*e^2)^(9/4)*(a + b*x + c*x^2)^(1/4)) - (c^(1/4)*(8*
c^2*d^2 + 5*b^2*e^2 - 4*c*e*(2*b*d + 3*a*e))*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a
*c)*(1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*S
qrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticE[2*ArcTan[(Sqrt[2
]*c^(1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(Sqrt[2]*(b^
2 - 4*a*c)^(1/4)*(c*d^2 - b*d*e + a*e^2)^2*(b + 2*c*x)) + (c^(1/4)*(8*c^2*d
^2 + 5*b^2*e^2 - 4*c*e*(2*b*d + 3*a*e))*Sqrt[(b + 2*c*x)^2/((b^2 - 4*a*c)*(
1 + (2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])^2)]*(1 + (2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])/Sqrt[b^2 - 4*a*c])*EllipticF[2*ArcTan[(Sqrt[2]*c^(
1/4)*(a + b*x + c*x^2)^(1/4))/(b^2 - 4*a*c)^(1/4)], 1/2])/(2*Sqrt[2]*(b^2 -
 4*a*c)^(1/4)*(c*d^2 - b*d*e + a*e^2)^2*(b + 2*c*x)) - (5*Sqrt[-b^2 + 4*a*c
]*e*(2*c*d - b*e)^2*Sqrt[(b + 2*c*x)^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^
2))/(b^2 - 4*a*c)))^(1/4)*EllipticPi[-(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqr
t[c*d^2 - b*d*e + a*e^2]), ArcSin[(1 - (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)],
 -1])/(4*Sqrt[2]*Sqrt[c]*(c*d^2 - b*d*e + a*e^2)^(5/2)*(b + 2*c*x)*(a + b*x
 + c*x^2)^(1/4)) + (5*Sqrt[-b^2 + 4*a*c]*e*(2*c*d - b*e)^2*Sqrt[(b + 2*c*x)
^2/(b^2 - 4*a*c)]*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(1/4)*EllipticPi
[(Sqrt[-b^2 + 4*a*c]*e)/(2*Sqrt[c]*Sqrt[c*d^2 - b*d*e + a*e^2]), ArcSin[(1
- (b + 2*c*x)^2/(b^2 - 4*a*c))^(1/4)], -1])/(4*Sqrt[2]*Sqrt[c]*(c*d^2 - b*d
*e + a*e^2)^(5/2)*(b + 2*c*x)*(a + b*x + c*x^2)^(1/4))
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 749

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[(a + b*x + c*x^2)^p/(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^p, I
nt[(-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) - (c^2*x^2)/(b^2 - 4*a*c
))^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&  !GtQ[4*a - b^2/c
, 0] && IntegerQ[4*p]
```

Rule 748

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[1/((-4*c)/(b^2 - 4*a*c))^p, Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c),
 x]^p/Simp[2*c*d - b*e + e*x, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && GtQ[4*a - b^2/c, 0] && IntegerQ[4*p]
```

Rule 746

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
```

```
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+bx+cx^2\right)^{5/4}}\,dx = -\frac{4\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)\sqrt[4]{a+bx+cx^2}} - \frac{4\int \frac{\frac{1}{4}\left(-4c^2d^2-5b^2e^2+6ce(bd+2ae)\right)+\frac{1}{2}}{(d+ex)^2\sqrt[4]{a+bx+cx^2}}}{\left(b^2-4ac\right)\left(cd^2-bde+\right.}$$

$$= -\frac{4\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)\sqrt[4]{a+bx+cx^2}} - \frac{e\left(8c^2d^2+5b^2e^2-4ce(2bd+3\right.}{\left(b^2-4ac\right)\left(cd^2-bde+\right.}$$

$$= -\frac{4\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)\sqrt[4]{a+bx+cx^2}} - \frac{e\left(8c^2d^2+5b^2e^2-4ce(2bd+3\right.}{\left(b^2-4ac\right)\left(cd^2-bde+\right.}$$

$$= -\frac{4\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)\sqrt[4]{a+bx+cx^2}} - \frac{e\left(8c^2d^2+5b^2e^2-4ce(2bd+3\right.}{\left(b^2-4ac\right)\left(cd^2-bde+\right.}$$

$$= -\frac{4\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)\sqrt[4]{a+bx+cx^2}} - \frac{e\left(8c^2d^2+5b^2e^2-4ce(2bd+3\right.}{\left(b^2-4ac\right)\left(cd^2-bde+\right.}$$

$$= -\frac{4\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)\sqrt[4]{a+bx+cx^2}} - \frac{e\left(8c^2d^2+5b^2e^2-4ce(2bd+3\right.}{\left(b^2-4ac\right)\left(cd^2-bde+\right.}$$

$$= -\frac{4\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)\sqrt[4]{a+bx+cx^2}} - \frac{e\left(8c^2d^2+5b^2e^2-4ce(2bd+3\right.}{\left(b^2-4ac\right)\left(cd^2-bde+\right.}$$

$$= -\frac{4\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)\sqrt[4]{a+bx+cx^2}} - \frac{e\left(8c^2d^2+5b^2e^2-4ce(2bd+3\right.}{\left(b^2-4ac\right)\left(cd^2-bde+\right.}$$

$$= -\frac{4\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)\sqrt[4]{a+bx+cx^2}} - \frac{e\left(8c^2d^2+5b^2e^2-4ce(2bd+3\right.}{\left(b^2-4ac\right)\left(cd^2-bde+\right.}$$

$$= -\frac{4\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)\sqrt[4]{a+bx+cx^2}} - \frac{e\left(8c^2d^2+5b^2e^2-4ce(2bd+3\right.}{\left(b^2-4ac\right)\left(cd^2-bde+\right.}$$

Mathematica [C] time = 0.937171, size = 187, normalized size = 0.13

$$-\frac{\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{5/4}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{5/4}F_1\left(\frac{7}{2};\frac{5}{4},\frac{5}{4};\frac{9}{2};\frac{2cd-\left(b+\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2cd-be+\sqrt{b^2-4ace}}{2cd+2cex}\right)}{14\sqrt{2}e(d+ex)(a+x(b+cx))^{5/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x)^2*(a + b*x + c*x^2)^(5/4)),x]
```

```
[Out] -(((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(5/4)*((e*(b + Sqrt[b
^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^(5/4)*AppellF1[7/2, 5/4, 5/4, 9/2, (2*
c*d - (b + Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x)), (2*c*d - b*e + Sqrt[b^2 -
 4*a*c]*e)/(2*c*d + 2*c*e*x)])/(14*Sqrt[2]*e*(d + e*x)*(a + x*(b + c*x))^(5
/4))
```

Maple [F] time = 1.247, size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)^2}\left(cx^2+bx+a\right)^{-\frac{5}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^2+b*x+a)^(5/4),x)
```

```
[Out] int(1/(e*x+d)^2/(c*x^2+b*x+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{5}{4}}(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(5/4)*(e*x + d)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x+a)^(5/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)^2\left(a+bx+cx^2\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**2+b*x+a)**(5/4),x)
```

```
[Out] Integral(1/((d + e*x)**2*(a + b*x + c*x**2)**(5/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2 + bx + a\right)^{\frac{5}{4}} (ex + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+b*x+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(5/4)*(e*x + d)^2), x)
```

3.2548 $\int \frac{1}{(d+ex)^{3/2}\sqrt[4]{a+bx+cx^2}}\,dx$

Optimal. Leaf size=239

$$\frac{2\left(-\sqrt{b^2-4ac}+b+2cx\right)\sqrt[4]{\frac{\left(\sqrt{b^2-4ac}+b+2cx\right)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}{\left(-\sqrt{b^2-4ac}+b+2cx\right)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};-\frac{4c\sqrt{b^2-4ac}(d+ex)}{\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\left(b+2cx-\sqrt{b^2-4ac}\right)}\right)}{\sqrt{d+ex}\sqrt[4]{a+bx+cx^2}\left(e\sqrt{b^2-4ac}-be+2cd\right)}$$

```
[Out] (2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)*(((2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(b - Sq
rt[b^2 - 4*a*c] + 2*c*x)))^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, (-4*c*Sq
rt[b^2 - 4*a*c]*(d + e*x))/((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(b - Sqrt[b
^2 - 4*a*c] + 2*c*x))])/((2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)*Sqrt[d + e*x]*
(a + b*x + c*x^2)^(1/4))
```

Rubi [A] time = 0.153645, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {726}

$$\frac{2\left(-\sqrt{b^2-4ac}+b+2cx\right)\sqrt[4]{\frac{\left(\sqrt{b^2-4ac}+b+2cx\right)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}{\left(-\sqrt{b^2-4ac}+b+2cx\right)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};-\frac{4c\sqrt{b^2-4ac}(d+ex)}{\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\left(b+2cx-\sqrt{b^2-4ac}\right)}\right)}{\sqrt{d+ex}\sqrt[4]{a+bx+cx^2}\left(e\sqrt{b^2-4ac}-be+2cd\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/4)),x]
```


```
[Out] (2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)*(((2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(b - Sq
rt[b^2 - 4*a*c] + 2*c*x)))^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, (-4*c*Sq
rt[b^2 - 4*a*c]*(d + e*x))/((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(b - Sqrt[b
^2 - 4*a*c] + 2*c*x))])/((2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)*Sqrt[d + e*x]*
(a + b*x + c*x^2)^(1/4))
```


Rule 726

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((b - Rt[b^2 - 4*a*c, 2] + 2*c*x)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^p*Hypergeometric2F1[m + 1, -p, m + 2, (-4*c*Rt[b^2 - 4*a*c, 2]*(
d + e*x))/((2*c*d - b*e - e*Rt[b^2 - 4*a*c, 2])*(b - Rt[b^2 - 4*a*c, 2] + 2
*c*x))])/((m + 1)*(2*c*d - b*e + e*Rt[b^2 - 4*a*c, 2])*(((2*c*d - b*e + e*R
t[b^2 - 4*a*c, 2])*(b + Rt[b^2 - 4*a*c, 2] + 2*c*x))/((2*c*d - b*e - e*Rt[b
^2 - 4*a*c, 2])*(b - Rt[b^2 - 4*a*c, 2] + 2*c*x)))^p), x] /; FreeQ[{a, b, c
, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &
& NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\sqrt[4]{a+bx+cx^2}}\,dx = \frac{2\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt[4]{\frac{\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\left(b-\sqrt{b^2-4ac}+2cx\right)}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};-\frac{4c\sqrt{b^2}}{\left(2cd-\left(b+\sqrt{b^2-4a}\right.\right.}\right.}{\left(2cd-be+\sqrt{b^2-4ac}e\right)\sqrt{d+ex}\sqrt[4]{a+bx+cx^2}}$$

Mathematica [A] time = 0.264665, size = 235, normalized size = 0.98

$$\frac{2\left(-\sqrt{b^2-4ac}+b+2cx\right)\sqrt[4]{\frac{\left(\sqrt{b^2-4ac}+b+2cx\right)\left(e\left(\sqrt{b^2-4ac}-b\right)+2cd\right)}{\left(\sqrt{b^2-4ac}-b-2cx\right)\left(e\left(\sqrt{b^2-4ac}+b\right)-2cd\right)}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};-\frac{4c\sqrt{b^2-4ac}(d+ex)}{\left(\left(b+\sqrt{b^2-4ac}\right)e-2cd\right)\left(-b-2cx+\sqrt{b^2-4ac}\right)}\right)}{\sqrt{d+ex}\sqrt[4]{a+x(b+cx)}\left(e\left(\sqrt{b^2-4ac}-b\right)+2cd\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/4)),x]
```

```
[Out] (2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)*(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*(
b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*(-b +
 Sqrt[b^2 - 4*a*c] - 2*c*x)))^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, (-4*c
*Sqrt[b^2 - 4*a*c]*(d + e*x))/((-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*(-b + S
qrt[b^2 - 4*a*c] - 2*c*x))])/((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*Sqrt[d +
 e*x]*(a + x*(b + c*x))^(1/4))
```

Maple [F] time = 1.284, size = 0, normalized size = 0.

$$\int (ex+d)^{-\frac{3}{2}}\frac{1}{\sqrt[4]{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/4),x)
```

```
[Out] int(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{1}{4}}(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(1/4)*(e*x + d)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+bx+a\right)^{\frac{3}{4}}\sqrt{ex+d}}{ce^2x^4+\left(2\,cde+be^2\right)x^3+ad^2+\left(cd^2+2\,bde+ae^2\right)x^2+\left(bd^2+2\,ade\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^(3/4)*sqrt(e*x + d)/(c*e^2*x^4 + (2*c*d*e + b*e^
2)*x^3 + a*d^2 + (c*d^2 + 2*b*d*e + a*e^2)*x^2 + (b*d^2 + 2*a*d*e)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(d+ex)^{\frac{3}{2}}\sqrt[4]{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(c*x**2+b*x+a)**(1/4),x)
```

```
[Out] Integral(1/((d + e*x)**(3/2)*(a + b*x + c*x**2)**(1/4)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.2549 $\int (d+ex)^m \left(a+bx+cx^2\right)^4 \, dx$

Optimal. Leaf size=485

$$\frac{(d+ex)^{m+5}\left(6c^2e^2\left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4\right)}{e^9(m+5)}+\frac{2(d+ex)^{m+3}\left(\right.}{}$$

```
[Out] ((c*d^2 - b*d*e + a*e^2)^4*(d + e*x)^(1 + m))/(e^9*(1 + m)) - (4*(2*c*d - b
*e)*(c*d^2 - b*d*e + a*e^2)^3*(d + e*x)^(2 + m))/(e^9*(2 + m)) + (2*(c*d^2
- b*d*e + a*e^2)^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e))*(d + e*x)
^(3 + m))/(e^9*(3 + m)) - (4*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*(7*c^2*d
^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e))*(d + e*x)^(4 + m))/(e^9*(4 + m)) + ((70
*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(5*b*d - 3*a*e) - 20*c^3*d^2*e*(7*b*d - 3*
a*e) + 6*c^2*e^2*(15*b^2*d^2 - 10*a*b*d*e + a^2*e^2))*(d + e*x)^(5 + m))/(e
^9*(5 + m)) - (4*c*(2*c*d - b*e)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e)
)*(d + e*x)^(6 + m))/(e^9*(6 + m)) + (2*c^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e
*(7*b*d - a*e))*(d + e*x)^(7 + m))/(e^9*(7 + m)) - (4*c^3*(2*c*d - b*e)*(d
+ e*x)^(8 + m))/(e^9*(8 + m)) + (c^4*(d + e*x)^(9 + m))/(e^9*(9 + m))
```

Rubi [A] time = 0.38575, antiderivative size = 485, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$\frac{(d+ex)^{m+5}\left(6c^2e^2\left(a^2e^2-10abde+15b^2d^2\right)-4b^2ce^3(5bd-3ae)-20c^3d^2e(7bd-3ae)+b^4e^4+70c^4d^4\right)}{e^9(m+5)}+\frac{2(d+ex)^{m+3}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a + b*x + c*x^2)^4,x]
```


```
[Out] ((c*d^2 - b*d*e + a*e^2)^4*(d + e*x)^(1 + m))/(e^9*(1 + m)) - (4*(2*c*d - b
*e)*(c*d^2 - b*d*e + a*e^2)^3*(d + e*x)^(2 + m))/(e^9*(2 + m)) + (2*(c*d^2
- b*d*e + a*e^2)^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e*(7*b*d - a*e))*(d + e*x)
^(3 + m))/(e^9*(3 + m)) - (4*(2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*(7*c^2*d
^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e))*(d + e*x)^(4 + m))/(e^9*(4 + m)) + ((70
*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(5*b*d - 3*a*e) - 20*c^3*d^2*e*(7*b*d - 3*
a*e) + 6*c^2*e^2*(15*b^2*d^2 - 10*a*b*d*e + a^2*e^2))*(d + e*x)^(5 + m))/(e
^9*(5 + m)) - (4*c*(2*c*d - b*e)*(7*c^2*d^2 + b^2*e^2 - c*e*(7*b*d - 3*a*e)
)*(d + e*x)^(6 + m))/(e^9*(6 + m)) + (2*c^2*(14*c^2*d^2 + 3*b^2*e^2 - 2*c*e
*(7*b*d - a*e))*(d + e*x)^(7 + m))/(e^9*(7 + m)) - (4*c^3*(2*c*d - b*e)*(d
+ e*x)^(8 + m))/(e^9*(8 + m)) + (c^4*(d + e*x)^(9 + m))/(e^9*(9 + m))
```


Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^m \left(a+bx+cx^2\right)^4 dx = \int \left(\frac{\left(cd^2-bde+ae^2\right)^4 (d+ex)^m}{e^8} + \frac{4(-2cd+be)\left(cd^2-bde+ae^2\right)^3 (d+ex)^{1+m}}{e^8} + \frac{2}{} \right.$$

$$= \frac{\left(cd^2-bde+ae^2\right)^4 (d+ex)^{1+m}}{e^9(1+m)} - \frac{4(2cd-be)\left(cd^2-bde+ae^2\right)^3 (d+ex)^{2+m}}{e^9(2+m)} + \frac{2\left(cd\right.}{}$$

Mathematica [B] time = 4.38169, size = 1309, normalized size = 2.7

$$(d+ex)^{m+1}\left((a+x(b+cx))^4 - \frac{4(d+ex)(be(m+15)+2c(e(m+8)x-7d))(a+x(b+cx))^3}{e^2(m+8)(m+9)} - \frac{12(d+ex)\left(ce^4(m+2)(m+3)(m+4)(m+5)(a+x(b+cx))^2\left(-28d^2(\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(a + b*x + c*x^2)^4,x]
```

```
[Out] ((d + e*x)^(1 + m)*((a + x*(b + c*x))^4 - (4*(d + e*x)*(a + x*(b + c*x))^3*
(b*e*(15 + m) + 2*c*(-7*d + e*(8 + m)*x)))/(e^2*(8 + m)*(9 + m)) - (12*(d +
 e*x)*(-2*((2*c*d - b*e)*(3 + m)*(840*c^5*d^6 + b^4*e^5*(b*d - a*e)*(8 + 14
*m + 7*m^2 + m^3) - b^2*c*e^4*(2 + 3*m + m^2)*(b^2*d^2*(-16 + m) + 8*a*b*d*
e*(9 + m) - 4*a^2*e^2*(13 + 2*m)) - 40*c^4*d^4*e*(63*b*d + a*e*(-59 + 6*m +
 2*m^2)) + 4*c^3*d^2*e^2*(5*b^2*d^2*(128 + 3*m + m^2) + 20*a*b*d*e*(-59 + 6
*m + 2*m^2) - 2*a^2*e^2*(-259 + 88*m + 34*m^2 + 2*m^3)) - 8*c^2*e^3*(5*b^3*
d^3*(23 + 3*m + m^2) - a*b^2*d^2*e*(313 - m + 2*m^2 + m^3) + a^3*e^3*(-69 +
 58*m + 24*m^2 + 2*m^3) - a^2*b*d*e^2*(-259 + 88*m + 34*m^2 + 2*m^3))) + (2
 + m)*(-1680*c^6*d^6 + b^6*e^6*(12 + 19*m + 8*m^2 + m^3) - b^4*c*e^5*(3 + 4
*m + m^2)*(b*d*(-8 + 3*m) + a*e*(56 + 9*m)) + 80*c^5*d^4*e*(63*b*d + a*e*(-
66 - m + 2*m^2)) + b^2*c^2*e^4*(1 + m)*(3*b^2*d^2*(12 - 13*m + m^2) + 12*a^
2*e^2*(69 + 24*m + 2*m^2) + 8*a*b*d*e*(-39 + 11*m + 3*m^2)) - 4*c^4*d^2*e^2
*(40*a*b*d*e*(-66 - m + 2*m^2) + 5*b^2*d^2*(249 - m + 2*m^2) - 12*a^2*e^2*(
-123 - 11*m + 8*m^2 + m^3)) - 8*c^3*e^3*(-5*b^3*d^3*(39 - m + 2*m^2) + 3*a*
b^2*d^2*e*(207 - 6*m - 2*m^2 + m^3) + 6*a^2*b*d*e^2*(-123 - 11*m + 8*m^2 +
m^3) + 2*a^3*e^3*(192 + 104*m + 18*m^2 + m^3))))*(d + e*x) + e^2*(2 + m)*(3
 + m)*(c*e*(5 + m)*(c*e*(b*d - 2*a*e)*(7 + m)*(14*b*(c*d^2 + a*e^2) + 4*a*c*
d*e*(1 + m) - b^2*d*e*(15 + m)) - (b*d*(5*c*d - 2*b*e) + a*e*(2*c*d*(1 + m)
 - b*e*(2 + m)))*(28*c^2*d^2 - b^2*e^2*(1 + m) + 4*c*e*(-7*b*d + a*e*(8 + m
)))) - (3*c*d - b*e)*(c*e*(2*c*d - b*e)*(7 + m)*(14*b*(c*d^2 + a*e^2) + 4*a
*c*d*e*(1 + m) - b^2*d*e*(15 + m)) - (10*c^2*d^2 - b^2*e^2*(4 + m) + c*e*(b
*d*(-3 + m) + 2*a*e*(6 + m)))*(28*c^2*d^2 - b^2*e^2*(1 + m) + 4*c*e*(-7*b*d
 + a*e*(8 + m)))) + c*e*(4 + m)*(c*e*(2*c*d - b*e)*(7 + m)*(14*b*(c*d^2 + a
*e^2) + 4*a*c*d*e*(1 + m) - b^2*d*e*(15 + m)) - (10*c^2*d^2 - b^2*e^2*(4 +
m) + c*e*(b*d*(-3 + m) + 2*a*e*(6 + m)))*(28*c^2*d^2 - b^2*e^2*(1 + m) + 4*
c*e*(-7*b*d + a*e*(8 + m))))*x)*(a + x*(b + c*x))) + c*e^4*(2 + m)*(3 + m)*
(4 + m)*(5 + m)*(a + x*(b + c*x))^2*(-2*b^3*e^3*(1 + m) - 28*c^3*d^2*(5*d -
 e*(6 + m)*x) - b*c*e^2*(-2*a*e*(81 + 11*m) + b*d*(156 + 17*m + m^2) + b*e*
(6 + 7*m + m^2)*x) + 2*c^2*e*(7*b*d*(d*(21 + m) - 2*e*(6 + m)*x) + 2*a*e*(d
*(-33 + 3*m + m^2) + e*(48 + 14*m + m^2)*x)))))/(c^2*e^8*(2 + m)*(3 + m)*(4
 + m)*(5 + m)*(6 + m)*(7 + m)*(8 + m)*(9 + m))))/(e*(1 + m))
```

Maple [B] time = 0.083, size = 7696, normalized size = 15.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^4,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.32018, size = 13875, normalized size = 28.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

```
[Out] (a^4*d*e^8*m^8 + 40320*c^4*d^9 - 181440*b*c^3*d^8*e - 725760*a^3*b*d^2*e^7
+ 362880*a^4*d*e^8 + 103680*(3*b^2*c^2 + 2*a*c^3)*d^7*e^2 - 241920*(b^3*c +
 3*a*b*c^2)*d^6*e^3 + 72576*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^5*e^4 - 362880
*(a*b^3 + 3*a^2*b*c)*d^4*e^5 + 241920*(3*a^2*b^2 + 2*a^3*c)*d^3*e^6 + (c^4*
e^9*m^8 + 36*c^4*e^9*m^7 + 546*c^4*e^9*m^6 + 4536*c^4*e^9*m^5 + 22449*c^4*e
^9*m^4 + 67284*c^4*e^9*m^3 + 118124*c^4*e^9*m^2 + 109584*c^4*e^9*m + 40320*
c^4*e^9)*x^9 + (181440*b*c^3*e^9 + (c^4*d*e^8 + 4*b*c^3*e^9)*m^8 + 4*(7*c^4
*d*e^8 + 37*b*c^3*e^9)*m^7 + 14*(23*c^4*d*e^8 + 164*b*c^3*e^9)*m^6 + 56*(35
*c^4*d*e^8 + 347*b*c^3*e^9)*m^5 + 7*(967*c^4*d*e^8 + 13948*b*c^3*e^9)*m^4 +
 28*(469*c^4*d*e^8 + 10579*b*c^3*e^9)*m^3 + 36*(363*c^4*d*e^8 + 14584*b*c^3
*e^9)*m^2 + 144*(35*c^4*d*e^8 + 3407*b*c^3*e^9)*m)*x^8 - 4*(a^3*b*d^2*e^7 -
 11*a^4*d*e^8)*m^7 + 2*(51840*(3*b^2*c^2 + 2*a*c^3)*e^9 + (2*b*c^3*d*e^8 +
(3*b^2*c^2 + 2*a*c^3)*e^9)*m^8 - 2*(2*c^4*d^2*e^7 - 30*b*c^3*d*e^8 - 19*(3*
b^2*c^2 + 2*a*c^3)*e^9)*m^7 - 4*(21*c^4*d^2*e^7 - 182*b*c^3*d*e^8 - 151*(3*
b^2*c^2 + 2*a*c^3)*e^9)*m^6 - 14*(50*c^4*d^2*e^7 - 330*b*c^3*d*e^8 - 373*(3
*b^2*c^2 + 2*a*c^3)*e^9)*m^5 - 7*(420*c^4*d^2*e^7 - 2354*b*c^3*d*e^8 - 3817
*(3*b^2*c^2 + 2*a*c^3)*e^9)*m^4 - 28*(232*c^4*d^2*e^7 - 1170*b*c^3*d*e^8 -
2939*(3*b^2*c^2 + 2*a*c^3)*e^9)*m^3 - 36*(196*c^4*d^2*e^7 - 922*b*c^3*d*e^8
 - 4101*(3*b^2*c^2 + 2*a*c^3)*e^9)*m^2 - 144*(20*c^4*d^2*e^7 - 90*b*c^3*d*e
^8 - 967*(3*b^2*c^2 + 2*a*c^3)*e^9)*m)*x^7 - 2*(84*a^3*b*d^2*e^7 - 413*a^4*
d*e^8 - 2*(3*a^2*b^2 + 2*a^3*c)*d^3*e^6)*m^6 + 2*(120960*(b^3*c + 3*a*b*c^2
)*e^9 + ((3*b^2*c^2 + 2*a*c^3)*d*e^8 + 2*(b^3*c + 3*a*b*c^2)*e^9)*m^8 - 2*(
7*b*c^3*d^2*e^7 - 16*(3*b^2*c^2 + 2*a*c^3)*d*e^8 - 39*(b^3*c + 3*a*b*c^2)*e
^9)*m^7 + 4*(7*c^4*d^3*e^6 - 84*b*c^3*d^2*e^7 + 103*(3*b^2*c^2 + 2*a*c^3)*d
*e^8 + 318*(b^3*c + 3*a*b*c^2)*e^9)*m^6 + 2*(210*c^4*d^3*e^6 - 1540*b*c^3*d
^2*e^7 + 1375*(3*b^2*c^2 + 2*a*c^3)*d*e^8 + 5634*(b^3*c + 3*a*b*c^2)*e^9)*m
^5 + (2380*c^4*d^3*e^6 - 13860*b*c^3*d^2*e^7 + 10219*(3*b^2*c^2 + 2*a*c^3)*
d*e^8 + 58938*(b^3*c + 3*a*b*c^2)*e^9)*m^4 + 2*(3150*c^4*d^3*e^6 - 16093*b*
c^3*d^2*e^7 + 10489*(3*b^2*c^2 + 2*a*c^3)*d*e^8 + 92511*(b^3*c + 3*a*b*c^2)
*e^9)*m^3 + 4*(1918*c^4*d^3*e^6 - 9051*b*c^3*d^2*e^7 + 5442*(3*b^2*c^2 + 2*
a*c^3)*d*e^8 + 84307*(b^3*c + 3*a*b*c^2)*e^9)*m^2 + 48*(70*c^4*d^3*e^6 - 31
5*b*c^3*d^2*e^7 + 180*(3*b^2*c^2 + 2*a*c^3)*d*e^8 + 6709*(b^3*c + 3*a*b*c^2
```

)*e^9)*m)*x^6 - 4*(742*a^3*b*d^2*e^7 - 2156*a^4*d*e^8 + 6*(a*b^3 + 3*a^2*b*
c)*d^4*e^5 - 39*(3*a^2*b^2 + 2*a^3*c)*d^3*e^6)*m^5 + (72576*(b^4 + 12*a*b^2
*c + 6*a^2*c^2)*e^9 + (4*(b^3*c + 3*a*b*c^2)*d*e^8 + (b^4 + 12*a*b^2*c + 6*
a^2*c^2)*e^9)*m^8 - 4*(3*(3*b^2*c^2 + 2*a*c^3)*d^2*e^7 - 34*(b^3*c + 3*a*b*
c^2)*d*e^8 - 10*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*e^9)*m^7 + 2*(84*b*c^3*d^3*e
^6 - 162*(3*b^2*c^2 + 2*a*c^3)*d^2*e^7 + 932*(b^3*c + 3*a*b*c^2)*d*e^8 + 33
5*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*e^9)*m^6 - 4*(84*c^4*d^4*e^5 - 798*b*c^3*d
^3*e^6 + 831*(3*b^2*c^2 + 2*a*c^3)*d^2*e^7 - 3304*(b^3*c + 3*a*b*c^2)*d*e^8
 - 1525*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*e^9)*m^5 - (3360*c^4*d^4*e^5 - 21000
*b*c^3*d^3*e^6 + 16380*(3*b^2*c^2 + 2*a*c^3)*d^2*e^7 - 51796*(b^3*c + 3*a*b
*c^2)*d*e^8 - 32773*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*e^9)*m^4 - 4*(2940*c^4*d
^4*e^5 - 15330*b*c^3*d^3*e^6 + 10182*(3*b^2*c^2 + 2*a*c^3)*d^2*e^7 - 27766*
(b^3*c + 3*a*b*c^2)*d*e^8 - 26365*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*e^9)*m^3 -
 12*(1400*c^4*d^4*e^5 - 6636*b*c^3*d^3*e^6 + 4008*(3*b^2*c^2 + 2*a*c^3)*d^2
e^7 - 9928(b^3*c + 3*a*b*c^2)*d*e^8 - 16365*(b^4 + 12*a*b^2*c + 6*a^2*c^2
)*e^9)*m^2 - 144*(56*c^4*d^4*e^5 - 252*b*c^3*d^3*e^6 + 144*(3*b^2*c^2 + 2*a
*c^3)*d^2*e^7 - 336*(b^3*c + 3*a*b*c^2)*d*e^8 - 1325*(b^4 + 12*a*b^2*c + 6*
a^2*c^2)*e^9)*m)*x^5 - (28560*a^3*b*d^2*e^7 - 54649*a^4*d*e^8 - 24*(b^4 + 1
2*a*b^2*c + 6*a^2*c^2)*d^5*e^4 + 840*(a*b^3 + 3*a^2*b*c)*d^4*e^5 - 2500*(3*
a^2*b^2 + 2*a^3*c)*d^3*e^6)*m^4 + (362880*(a*b^3 + 3*a^2*b*c)*e^9 + ((b^4 +
 12*a*b^2*c + 6*a^2*c^2)*d*e^8 + 4*(a*b^3 + 3*a^2*b*c)*e^9)*m^8 - 4*(5*(b^3
*c + 3*a*b*c^2)*d^2*e^7 - 9*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d*e^8 - 41*(a*b^
3 + 3*a^2*b*c)*e^9)*m^7 + 2*(30*(3*b^2*c^2 + 2*a*c^3)*d^3*e^6 - 300*(b^3*c
+ 3*a*b*c^2)*d^2*e^7 + 263*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d*e^8 + 1412*(a*b
^3 + 3*a^2*b*c)*e^9)*m^6 - 4*(210*b*c^3*d^4*e^5 - 345*(3*b^2*c^2 + 2*a*c^3)
*d^3*e^6 + 1730*(b^3*c + 3*a*b*c^2)*d^2*e^7 - 999*(b^4 + 12*a*b^2*c + 6*a^2
*c^2)*d*e^8 - 6626*(a*b^3 + 3*a^2*b*c)*e^9)*m^5 + (1680*c^4*d^5*e^4 - 12600
*b*c^3*d^4*e^5 + 11100*(3*b^2*c^2 + 2*a*c^3)*d^3*e^6 - 38400*(b^3*c + 3*a*b
*c^2)*d^2*e^7 + 16789*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d*e^8 + 147076*(a*b^3
+ 3*a^2*b*c)*e^9)*m^4 + 4*(2520*c^4*d^5*e^4 - 13650*b*c^3*d^4*e^5 + 9375*(3
*b^2*c^2 + 2*a*c^3)*d^3*e^6 - 26345*(b^3*c + 3*a*b*c^2)*d^2*e^7 + 9576*(b^4
 + 12*a*b^2*c + 6*a^2*c^2)*d*e^8 + 122249*(a*b^3 + 3*a^2*b*c)*e^9)*m^3 + 12
*(1540*c^4*d^5*e^4 - 7350*b*c^3*d^4*e^5 + 4470*(3*b^2*c^2 + 2*a*c^3)*d^3*e^
6 - 11150*(b^3*c + 3*a*b*c^2)*d^2*e^7 + 3597*(b^4 + 12*a*b^2*c + 6*a^2*c^2)
*d*e^8 + 78228*(a*b^3 + 3*a^2*b*c)*e^9)*m^2 + 144*(70*c^4*d^5*e^4 - 315*b*c
^3*d^4*e^5 + 180*(3*b^2*c^2 + 2*a*c^3)*d^3*e^6 - 420*(b^3*c + 3*a*b*c^2)*d^
2*e^7 + 126*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d*e^8 + 6499*(a*b^3 + 3*a^2*b*c)
*e^9)*m)*x^4 - 4*(40369*a^3*b*d^2*e^7 - 53669*a^4*d*e^8 + 120*(b^3*c + 3*a*
b*c^2)*d^6*e^3 - 180*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^5*e^4 + 2910*(a*b^3 +
 3*a^2*b*c)*d^4*e^5 - 5265*(3*a^2*b^2 + 2*a^3*c)*d^3*e^6)*m^3 + 2*(120960*(
3*a^2*b^2 + 2*a^3*c)*e^9 + (2*(a*b^3 + 3*a^2*b*c)*d*e^8 + (3*a^2*b^2 + 2*a^
3*c)*e^9)*m^8 - 2*((b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^2*e^7 - 38*(a*b^3 + 3*a
^2*b*c)*d*e^8 - 21*(3*a^2*b^2 + 2*a^3*c)*e^9)*m^7 + 2*(20*(b^3*c + 3*a*b*c^
2)*d^3*e^6 - 33*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^2*e^7 + 592*(a*b^3 + 3*a^2
*b*c)*d*e^8 + 372*(3*a^2*b^2 + 2*a^3*c)*e^9)*m^6 - 2*(60*(3*b^2*c^2 + 2*a*c
^3)*d^4*e^5 - 540*(b^3*c + 3*a*b*c^2)*d^3*e^6 + 427*(b^4 + 12*a*b^2*c + 6*a
^2*c^2)*d^2*e^7 - 4850*(a*b^3 + 3*a^2*b*c)*d*e^8 - 3609*(3*a^2*b^2 + 2*a^3*
c)*e^9)*m^5 + (1680*b*c^3*d^5*e^4 - 2400*(3*b^2*c^2 + 2*a*c^3)*d^4*e^5 + 10
600*(b^3*c + 3*a*b*c^2)*d^3*e^6 - 5430*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^2*e
^7 + 44438*(a*b^3 + 3*a^2*b*c)*d*e^8 + 41619*(3*a^2*b^2 + 2*a^3*c)*e^9)*m^4
 - 4*(840*c^4*d^6*e^3 - 5040*b*c^3*d^5*e^4 + 3750*(3*b^2*c^2 + 2*a*c^3)*d^4
e^5 - 11250(b^3*c + 3*a*b*c^2)*d^3*e^6 + 4322*(b^4 + 12*a*b^2*c + 6*a^2*c
^2)*d^2*e^7 - 27796*(a*b^3 + 3*a^2*b*c)*d*e^8 - 36117*(3*a^2*b^2 + 2*a^3*c)
*e^9)*m^3 - 4*(2520*c^4*d^6*e^3 - 12180*b*c^3*d^5*e^4 + 7500*(3*b^2*c^2 + 2
*a*c^3)*d^4*e^5 - 18940*(b^3*c + 3*a*b*c^2)*d^3*e^6 + 6186*(b^4 + 12*a*b^2*
c + 6*a^2*c^2)*d^2*e^7 - 33954*(a*b^3 + 3*a^2*b*c)*d*e^8 - 72569*(3*a^2*b^2
 + 2*a^3*c)*e^9)*m^2 - 48*(140*c^4*d^6*e^3 - 630*b*c^3*d^5*e^4 + 360*(3*b^2
*c^2 + 2*a*c^3)*d^4*e^5 - 840*(b^3*c + 3*a*b*c^2)*d^3*e^6 + 252*(b^4 + 12*a
*b^2*c + 6*a^2*c^2)*d^2*e^7 - 1260*(a*b^3 + 3*a^2*b*c)*d*e^8 - 6289*(3*a^2*

```
b^2 + 2*a^3*c)*e^9)*m)*x^3 - 4*(133938*a^3*b*d^2*e^7 - 127251*a^4*d*e^8 - 3
60*(3*b^2*c^2 + 2*a*c^3)*d^7*e^2 + 2880*(b^3*c + 3*a*b*c^2)*d^6*e^3 - 2010*
(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^5*e^4 + 19950*(a*b^3 + 3*a^2*b*c)*d^4*e^5
- 24574*(3*a^2*b^2 + 2*a^3*c)*d^3*e^6)*m^2 + 2*(362880*a^3*b*e^9 + (2*a^3*b
*e^9 + (3*a^2*b^2 + 2*a^3*c)*d*e^8)*m^8 + 2*(43*a^3*b*e^9 - 3*(a*b^3 + 3*a^
2*b*c)*d^2*e^7 + 20*(3*a^2*b^2 + 2*a^3*c)*d*e^8)*m^7 + 2*(784*a^3*b*e^9 + 3
*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^3*e^6 - 108*(a*b^3 + 3*a^2*b*c)*d^2*e^7 +
 332*(3*a^2*b^2 + 2*a^3*c)*d*e^8)*m^6 + 2*(7882*a^3*b*e^9 - 60*(b^3*c + 3*a
*b*c^2)*d^4*e^5 + 93*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^3*e^6 - 1560*(a*b^3 +
 3*a^2*b*c)*d^2*e^7 + 2945*(3*a^2*b^2 + 2*a^3*c)*d*e^8)*m^5 + (95018*a^3*b*
e^9 + 360*(3*b^2*c^2 + 2*a*c^3)*d^5*e^4 - 3000*(b^3*c + 3*a*b*c^2)*d^4*e^5
+ 2190*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^3*e^6 - 22860*(a*b^3 + 3*a^2*b*c)*d
^2*e^7 + 29839*(3*a^2*b^2 + 2*a^3*c)*d*e^8)*m^4 - 2*(2520*b*c^3*d^6*e^3 - 1
74307*a^3*b*e^9 - 3240*(3*b^2*c^2 + 2*a*c^3)*d^5*e^4 + 12900*(b^3*c + 3*a*b
*c^2)*d^4*e^5 - 5955*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^3*e^6 + 43797*(a*b^3
+ 3*a^2*b*c)*d^2*e^7 - 42395*(3*a^2*b^2 + 2*a^3*c)*d*e^8)*m^3 + 12*(840*c^4
*d^7*e^2 - 4200*b*c^3*d^6*e^3 + 62511*a^3*b*e^9 + 2670*(3*b^2*c^2 + 2*a*c^3
)*d^5*e^4 - 6950*(b^3*c + 3*a*b*c^2)*d^4*e^5 + 2337*(b^4 + 12*a*b^2*c + 6*a
^2*c^2)*d^3*e^6 - 13197*(a*b^3 + 3*a^2*b*c)*d^2*e^7 + 10058*(3*a^2*b^2 + 2*
a^3*c)*d*e^8)*m^2 + 144*(70*c^4*d^7*e^2 - 315*b*c^3*d^6*e^3 + 5869*a^3*b*e^
9 + 180*(3*b^2*c^2 + 2*a*c^3)*d^5*e^4 - 420*(b^3*c + 3*a*b*c^2)*d^4*e^5 + 1
26*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^3*e^6 - 630*(a*b^3 + 3*a^2*b*c)*d^2*e^7
 + 420*(3*a^2*b^2 + 2*a^3*c)*d*e^8)*m)*x^2 - 48*(420*b*c^3*d^8*e + 20094*a^
3*b*d^2*e^7 - 13827*a^4*d*e^8 - 510*(3*b^2*c^2 + 2*a*c^3)*d^7*e^2 + 1910*(b
^3*c + 3*a*b*c^2)*d^6*e^3 - 825*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^5*e^4 + 56
37*(a*b^3 + 3*a^2*b*c)*d^4*e^5 - 5018*(3*a^2*b^2 + 2*a^3*c)*d^3*e^6)*m + (3
62880*a^4*e^9 + (4*a^3*b*d*e^8 + a^4*e^9)*m^8 + 4*(42*a^3*b*d*e^8 + 11*a^4*
e^9 - (3*a^2*b^2 + 2*a^3*c)*d^2*e^7)*m^7 + 2*(1484*a^3*b*d*e^8 + 413*a^4*e^
9 + 12*(a*b^3 + 3*a^2*b*c)*d^3*e^6 - 78*(3*a^2*b^2 + 2*a^3*c)*d^2*e^7)*m^6
+ 4*(7140*a^3*b*d*e^8 + 2156*a^4*e^9 - 6*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^4
*e^5 + 210*(a*b^3 + 3*a^2*b*c)*d^3*e^6 - 625*(3*a^2*b^2 + 2*a^3*c)*d^2*e^7)
*m^5 + (161476*a^3*b*d*e^8 + 54649*a^4*e^9 + 480*(b^3*c + 3*a*b*c^2)*d^5*e^
4 - 720*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^4*e^5 + 11640*(a*b^3 + 3*a^2*b*c)*
d^3*e^6 - 21060*(3*a^2*b^2 + 2*a^3*c)*d^2*e^7)*m^4 + 4*(133938*a^3*b*d*e^8
+ 53669*a^4*e^9 - 360*(3*b^2*c^2 + 2*a*c^3)*d^6*e^3 + 2880*(b^3*c + 3*a*b*c
^2)*d^5*e^4 - 2010*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^4*e^5 + 19950*(a*b^3 +
3*a^2*b*c)*d^3*e^6 - 24574*(3*a^2*b^2 + 2*a^3*c)*d^2*e^7)*m^3 + 12*(1680*b*
c^3*d^7*e^2 + 80376*a^3*b*d*e^8 + 42417*a^4*e^9 - 2040*(3*b^2*c^2 + 2*a*c^3
)*d^6*e^3 + 7640*(b^3*c + 3*a*b*c^2)*d^5*e^4 - 3300*(b^4 + 12*a*b^2*c + 6*a
^2*c^2)*d^4*e^5 + 22548*(a*b^3 + 3*a^2*b*c)*d^3*e^6 - 20072*(3*a^2*b^2 + 2*
a^3*c)*d^2*e^7)*m^2 - 144*(280*c^4*d^8*e - 1260*b*c^3*d^7*e^2 - 5040*a^3*b*
d*e^8 - 4609*a^4*e^9 + 720*(3*b^2*c^2 + 2*a*c^3)*d^6*e^3 - 1680*(b^3*c + 3*
a*b*c^2)*d^5*e^4 + 504*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^4*e^5 - 2520*(a*b^3
 + 3*a^2*b*c)*d^3*e^6 + 1680*(3*a^2*b^2 + 2*a^3*c)*d^2*e^7)*m)*x)*(e*x + d)
^m/(e^9*m^9 + 45*e^9*m^8 + 870*e^9*m^7 + 9450*e^9*m^6 + 63273*e^9*m^5 + 269
325*e^9*m^4 + 723680*e^9*m^3 + 1172700*e^9*m^2 + 1026576*e^9*m + 362880*e^9
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.19686, size = 17743, normalized size = 36.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^4,x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*c^4*m^8*x^9*e^9 + (x*e + d)^m*c^4*d*m^8*x^8*e^8 + 4*(x*e + d)^
m*b*c^3*m^8*x^8*e^9 + 36*(x*e + d)^m*c^4*m^7*x^9*e^9 + 4*(x*e + d)^m*b*c^3*
d*m^8*x^7*e^8 + 28*(x*e + d)^m*c^4*d*m^7*x^8*e^8 - 8*(x*e + d)^m*c^4*d^2*m^
7*x^7*e^7 + 6*(x*e + d)^m*b^2*c^2*m^8*x^7*e^9 + 4*(x*e + d)^m*a*c^3*m^8*x^7
*e^9 + 148*(x*e + d)^m*b*c^3*m^7*x^8*e^9 + 546*(x*e + d)^m*c^4*m^6*x^9*e^9
+ 6*(x*e + d)^m*b^2*c^2*d*m^8*x^6*e^8 + 4*(x*e + d)^m*a*c^3*d*m^8*x^6*e^8 +
 120*(x*e + d)^m*b*c^3*d*m^7*x^7*e^8 + 322*(x*e + d)^m*c^4*d*m^6*x^8*e^8 -
28*(x*e + d)^m*b*c^3*d^2*m^7*x^6*e^7 - 168*(x*e + d)^m*c^4*d^2*m^6*x^7*e^7
+ 56*(x*e + d)^m*c^4*d^3*m^6*x^6*e^6 + 4*(x*e + d)^m*b^3*c*m^8*x^6*e^9 + 12
*(x*e + d)^m*a*b*c^2*m^8*x^6*e^9 + 228*(x*e + d)^m*b^2*c^2*m^7*x^7*e^9 + 15
2*(x*e + d)^m*a*c^3*m^7*x^7*e^9 + 2296*(x*e + d)^m*b*c^3*m^6*x^8*e^9 + 4536
*(x*e + d)^m*c^4*m^5*x^9*e^9 + 4*(x*e + d)^m*b^3*c*d*m^8*x^5*e^8 + 12*(x*e
+ d)^m*a*b*c^2*d*m^8*x^5*e^8 + 192*(x*e + d)^m*b^2*c^2*d*m^7*x^6*e^8 + 128*
(x*e + d)^m*a*c^3*d*m^7*x^6*e^8 + 1456*(x*e + d)^m*b*c^3*d*m^6*x^7*e^8 + 19
60*(x*e + d)^m*c^4*d*m^5*x^8*e^8 - 36*(x*e + d)^m*b^2*c^2*d^2*m^7*x^5*e^7 -
 24*(x*e + d)^m*a*c^3*d^2*m^7*x^5*e^7 - 672*(x*e + d)^m*b*c^3*d^2*m^6*x^6*e
^7 - 1400*(x*e + d)^m*c^4*d^2*m^5*x^7*e^7 + 168*(x*e + d)^m*b*c^3*d^3*m^6*x
^5*e^6 + 840*(x*e + d)^m*c^4*d^3*m^5*x^6*e^6 - 336*(x*e + d)^m*c^4*d^4*m^5*
x^5*e^5 + (x*e + d)^m*b^4*m^8*x^5*e^9 + 12*(x*e + d)^m*a*b^2*c*m^8*x^5*e^9
+ 6*(x*e + d)^m*a^2*c^2*m^8*x^5*e^9 + 156*(x*e + d)^m*b^3*c*m^7*x^6*e^9 + 4
68*(x*e + d)^m*a*b*c^2*m^7*x^6*e^9 + 3624*(x*e + d)^m*b^2*c^2*m^6*x^7*e^9 +
 2416*(x*e + d)^m*a*c^3*m^6*x^7*e^9 + 19432*(x*e + d)^m*b*c^3*m^5*x^8*e^9 +
 22449*(x*e + d)^m*c^4*m^4*x^9*e^9 + (x*e + d)^m*b^4*d*m^8*x^4*e^8 + 12*(x*
e + d)^m*a*b^2*c*d*m^8*x^4*e^8 + 6*(x*e + d)^m*a^2*c^2*d*m^8*x^4*e^8 + 136*
(x*e + d)^m*b^3*c*d*m^7*x^5*e^8 + 408*(x*e + d)^m*a*b*c^2*d*m^7*x^5*e^8 + 2
472*(x*e + d)^m*b^2*c^2*d*m^6*x^6*e^8 + 1648*(x*e + d)^m*a*c^3*d*m^6*x^6*e^
8 + 9240*(x*e + d)^m*b*c^3*d*m^5*x^7*e^8 + 6769*(x*e + d)^m*c^4*d*m^4*x^8*e
^8 - 20*(x*e + d)^m*b^3*c*d^2*m^7*x^4*e^7 - 60*(x*e + d)^m*a*b*c^2*d^2*m^7*
x^4*e^7 - 972*(x*e + d)^m*b^2*c^2*d^2*m^6*x^5*e^7 - 648*(x*e + d)^m*a*c^3*d
^2*m^6*x^5*e^7 - 6160*(x*e + d)^m*b*c^3*d^2*m^5*x^6*e^7 - 5880*(x*e + d)^m*
c^4*d^2*m^4*x^7*e^7 + 180*(x*e + d)^m*b^2*c^2*d^3*m^6*x^4*e^6 + 120*(x*e +
d)^m*a*c^3*d^3*m^6*x^4*e^6 + 3192*(x*e + d)^m*b*c^3*d^3*m^5*x^5*e^6 + 4760*
(x*e + d)^m*c^4*d^3*m^4*x^6*e^6 - 840*(x*e + d)^m*b*c^3*d^4*m^5*x^4*e^5 - 3
360*(x*e + d)^m*c^4*d^4*m^4*x^5*e^5 + 1680*(x*e + d)^m*c^4*d^5*m^4*x^4*e^4
+ 4*(x*e + d)^m*a*b^3*m^8*x^4*e^9 + 12*(x*e + d)^m*a^2*b*c*m^8*x^4*e^9 + 40
*(x*e + d)^m*b^4*m^7*x^5*e^9 + 480*(x*e + d)^m*a*b^2*c*m^7*x^5*e^9 + 240*(x
*e + d)^m*a^2*c^2*m^7*x^5*e^9 + 2544*(x*e + d)^m*b^3*c*m^6*x^6*e^9 + 7632*(
x*e + d)^m*a*b*c^2*m^6*x^6*e^9 + 31332*(x*e + d)^m*b^2*c^2*m^5*x^7*e^9 + 20
888*(x*e + d)^m*a*c^3*m^5*x^7*e^9 + 97636*(x*e + d)^m*b*c^3*m^4*x^8*e^9 + 6
7284*(x*e + d)^m*c^4*m^3*x^9*e^9 + 4*(x*e + d)^m*a*b^3*d*m^8*x^3*e^8 + 12*(
x*e + d)^m*a^2*b*c*d*m^8*x^3*e^8 + 36*(x*e + d)^m*b^4*d*m^7*x^4*e^8 + 432*(
x*e + d)^m*a*b^2*c*d*m^7*x^4*e^8 + 216*(x*e + d)^m*a^2*c^2*d*m^7*x^4*e^8 +
1864*(x*e + d)^m*b^3*c*d*m^6*x^5*e^8 + 5592*(x*e + d)^m*a*b*c^2*d*m^6*x^5*e
^8 + 16500*(x*e + d)^m*b^2*c^2*d*m^5*x^6*e^8 + 11000*(x*e + d)^m*a*c^3*d*m^
5*x^6*e^8 + 32956*(x*e + d)^m*b*c^3*d*m^4*x^7*e^8 + 13132*(x*e + d)^m*c^4*d
*m^3*x^8*e^8 - 4*(x*e + d)^m*b^4*d^2*m^7*x^3*e^7 - 48*(x*e + d)^m*a*b^2*c*d
^2*m^7*x^3*e^7 - 24*(x*e + d)^m*a^2*c^2*d^2*m^7*x^3*e^7 - 600*(x*e + d)^m*b
^3*c*d^2*m^6*x^4*e^7 - 1800*(x*e + d)^m*a*b*c^2*d^2*m^6*x^4*e^7 - 9972*(x*e
 + d)^m*b^2*c^2*d^2*m^5*x^5*e^7 - 6648*(x*e + d)^m*a*c^3*d^2*m^5*x^5*e^7 -
27720*(x*e + d)^m*b*c^3*d^2*m^4*x^6*e^7 - 12992*(x*e + d)^m*c^4*d^2*m^3*x^7
```

e^7 + 80(x*e + d)^m*b^3*c*d^3*m^6*x^3*e^6 + 240*(x*e + d)^m*a*b*c^2*d^3*m
^6*x^3*e^6 + 4140*(x*e + d)^m*b^2*c^2*d^3*m^5*x^4*e^6 + 2760*(x*e + d)^m*a*
c^3*d^3*m^5*x^4*e^6 + 21000*(x*e + d)^m*b*c^3*d^3*m^4*x^5*e^6 + 12600*(x*e
+ d)^m*c^4*d^3*m^3*x^6*e^6 - 720*(x*e + d)^m*b^2*c^2*d^4*m^5*x^3*e^5 - 480*
(x*e + d)^m*a*c^3*d^4*m^5*x^3*e^5 - 12600*(x*e + d)^m*b*c^3*d^4*m^4*x^4*e^5
 - 11760*(x*e + d)^m*c^4*d^4*m^3*x^5*e^5 + 3360*(x*e + d)^m*b*c^3*d^5*m^4*x
^3*e^4 + 10080*(x*e + d)^m*c^4*d^5*m^3*x^4*e^4 - 6720*(x*e + d)^m*c^4*d^6*m
^3*x^3*e^3 + 6*(x*e + d)^m*a^2*b^2*m^8*x^3*e^9 + 4*(x*e + d)^m*a^3*c*m^8*x^
3*e^9 + 164*(x*e + d)^m*a*b^3*m^7*x^4*e^9 + 492*(x*e + d)^m*a^2*b*c*m^7*x^4
e^9 + 670(x*e + d)^m*b^4*m^6*x^5*e^9 + 8040*(x*e + d)^m*a*b^2*c*m^6*x^5*e
^9 + 4020*(x*e + d)^m*a^2*c^2*m^6*x^5*e^9 + 22536*(x*e + d)^m*b^3*c*m^5*x^6
e^9 + 67608(x*e + d)^m*a*b*c^2*m^5*x^6*e^9 + 160314*(x*e + d)^m*b^2*c^2*m
^4*x^7*e^9 + 106876*(x*e + d)^m*a*c^3*m^4*x^7*e^9 + 296212*(x*e + d)^m*b*c^
3*m^3*x^8*e^9 + 118124*(x*e + d)^m*c^4*m^2*x^9*e^9 + 6*(x*e + d)^m*a^2*b^2*
d*m^8*x^2*e^8 + 4*(x*e + d)^m*a^3*c*d*m^8*x^2*e^8 + 152*(x*e + d)^m*a*b^3*d
*m^7*x^3*e^8 + 456*(x*e + d)^m*a^2*b*c*d*m^7*x^3*e^8 + 526*(x*e + d)^m*b^4*
d*m^6*x^4*e^8 + 6312*(x*e + d)^m*a*b^2*c*d*m^6*x^4*e^8 + 3156*(x*e + d)^m*a
^2*c^2*d*m^6*x^4*e^8 + 13216*(x*e + d)^m*b^3*c*d*m^5*x^5*e^8 + 39648*(x*e +
 d)^m*a*b*c^2*d*m^5*x^5*e^8 + 61314*(x*e + d)^m*b^2*c^2*d*m^4*x^6*e^8 + 408
76*(x*e + d)^m*a*c^3*d*m^4*x^6*e^8 + 65520*(x*e + d)^m*b*c^3*d*m^3*x^7*e^8
 + 13068*(x*e + d)^m*c^4*d*m^2*x^8*e^8 - 12*(x*e + d)^m*a*b^3*d^2*m^7*x^2*e^
7 - 36*(x*e + d)^m*a^2*b*c*d^2*m^7*x^2*e^7 - 132*(x*e + d)^m*b^4*d^2*m^6*x^
3*e^7 - 1584*(x*e + d)^m*a*b^2*c*d^2*m^6*x^3*e^7 - 792*(x*e + d)^m*a^2*c^2*
d^2*m^6*x^3*e^7 - 6920*(x*e + d)^m*b^3*c*d^2*m^5*x^4*e^7 - 20760*(x*e + d)^
m*a*b*c^2*d^2*m^5*x^4*e^7 - 49140*(x*e + d)^m*b^2*c^2*d^2*m^4*x^5*e^7 - 327
60*(x*e + d)^m*a*c^3*d^2*m^4*x^5*e^7 - 64372*(x*e + d)^m*b*c^3*d^2*m^3*x^6*
e^7 - 14112*(x*e + d)^m*c^4*d^2*m^2*x^7*e^7 + 12*(x*e + d)^m*b^4*d^3*m^6*x^
2*e^6 + 144*(x*e + d)^m*a*b^2*c*d^3*m^6*x^2*e^6 + 72*(x*e + d)^m*a^2*c^2*d^
3*m^6*x^2*e^6 + 2160*(x*e + d)^m*b^3*c*d^3*m^5*x^3*e^6 + 6480*(x*e + d)^m*a
*b*c^2*d^3*m^5*x^3*e^6 + 33300*(x*e + d)^m*b^2*c^2*d^3*m^4*x^4*e^6 + 22200*
(x*e + d)^m*a*c^3*d^3*m^4*x^4*e^6 + 61320*(x*e + d)^m*b*c^3*d^3*m^3*x^5*e^6
 + 15344*(x*e + d)^m*c^4*d^3*m^2*x^6*e^6 - 240*(x*e + d)^m*b^3*c*d^4*m^5*x^
2*e^5 - 720*(x*e + d)^m*a*b*c^2*d^4*m^5*x^2*e^5 - 14400*(x*e + d)^m*b^2*c^2
*d^4*m^4*x^3*e^5 - 9600*(x*e + d)^m*a*c^3*d^4*m^4*x^3*e^5 - 54600*(x*e + d)
^m*b*c^3*d^4*m^3*x^4*e^5 - 16800*(x*e + d)^m*c^4*d^4*m^2*x^5*e^5 + 2160*(x*
e + d)^m*b^2*c^2*d^5*m^4*x^2*e^4 + 1440*(x*e + d)^m*a*c^3*d^5*m^4*x^2*e^4 +
 40320*(x*e + d)^m*b*c^3*d^5*m^3*x^3*e^4 + 18480*(x*e + d)^m*c^4*d^5*m^2*x^
4*e^4 - 10080*(x*e + d)^m*b*c^3*d^6*m^3*x^2*e^3 - 20160*(x*e + d)^m*c^4*d^6
*m^2*x^3*e^3 + 20160*(x*e + d)^m*c^4*d^7*m^2*x^2*e^2 + 4*(x*e + d)^m*a^3*b*
m^8*x^2*e^9 + 252*(x*e + d)^m*a^2*b^2*m^7*x^3*e^9 + 168*(x*e + d)^m*a^3*c*m
^7*x^3*e^9 + 2824*(x*e + d)^m*a*b^3*m^6*x^4*e^9 + 8472*(x*e + d)^m*a^2*b*c*
m^6*x^4*e^9 + 6100*(x*e + d)^m*b^4*m^5*x^5*e^9 + 73200*(x*e + d)^m*a*b^2*c*
m^5*x^5*e^9 + 36600*(x*e + d)^m*a^2*c^2*m^5*x^5*e^9 + 117876*(x*e + d)^m*b^
3*c*m^4*x^6*e^9 + 353628*(x*e + d)^m*a*b*c^2*m^4*x^6*e^9 + 493752*(x*e + d)
^m*b^2*c^2*m^3*x^7*e^9 + 329168*(x*e + d)^m*a*c^3*m^3*x^7*e^9 + 525024*(x*e
 + d)^m*b*c^3*m^2*x^8*e^9 + 109584*(x*e + d)^m*c^4*m*x^9*e^9 + 4*(x*e + d)^
m*a^3*b*d*m^8*x*e^8 + 240*(x*e + d)^m*a^2*b^2*d*m^7*x^2*e^8 + 160*(x*e + d)
^m*a^3*c*d*m^7*x^2*e^8 + 2368*(x*e + d)^m*a*b^3*d*m^6*x^3*e^8 + 7104*(x*e +
 d)^m*a^2*b*c*d*m^6*x^3*e^8 + 3996*(x*e + d)^m*b^4*d*m^5*x^4*e^8 + 47952*(x
*e + d)^m*a*b^2*c*d*m^5*x^4*e^8 + 23976*(x*e + d)^m*a^2*c^2*d*m^5*x^4*e^8 +
 51796*(x*e + d)^m*b^3*c*d*m^4*x^5*e^8 + 155388*(x*e + d)^m*a*b*c^2*d*m^4*x
^5*e^8 + 125868*(x*e + d)^m*b^2*c^2*d*m^3*x^6*e^8 + 83912*(x*e + d)^m*a*c^3
*d*m^3*x^6*e^8 + 66384*(x*e + d)^m*b*c^3*d*m^2*x^7*e^8 + 5040*(x*e + d)^m*c
^4*d*m*x^8*e^8 - 12*(x*e + d)^m*a^2*b^2*d^2*m^7*x*e^7 - 8*(x*e + d)^m*a^3*c
*d^2*m^7*x*e^7 - 432*(x*e + d)^m*a*b^3*d^2*m^6*x^2*e^7 - 1296*(x*e + d)^m*a
^2*b*c*d^2*m^6*x^2*e^7 - 1708*(x*e + d)^m*b^4*d^2*m^5*x^3*e^7 - 20496*(x*e
+ d)^m*a*b^2*c*d^2*m^5*x^3*e^7 - 10248*(x*e + d)^m*a^2*c^2*d^2*m^5*x^3*e^7
 - 38400*(x*e + d)^m*b^3*c*d^2*m^4*x^4*e^7 - 115200*(x*e + d)^m*a*b*c^2*d^2*
m^4*x^4*e^7 - 122184*(x*e + d)^m*b^2*c^2*d^2*m^3*x^5*e^7 - 81456*(x*e + d)^

m*a*c^3*d^2*m^3*x^5*e^7 - 72408*(x*e + d)^m*b*c^3*d^2*m^2*x^6*e^7 - 5760*(x
*e + d)^m*c^4*d^2*m*x^7*e^7 + 24*(x*e + d)^m*a*b^3*d^3*m^6*x*e^6 + 72*(x*e
+ d)^m*a^2*b*c*d^3*m^6*x*e^6 + 372*(x*e + d)^m*b^4*d^3*m^5*x^2*e^6 + 4464*(
x*e + d)^m*a*b^2*c*d^3*m^5*x^2*e^6 + 2232*(x*e + d)^m*a^2*c^2*d^3*m^5*x^2*e
^6 + 21200*(x*e + d)^m*b^3*c*d^3*m^4*x^3*e^6 + 63600*(x*e + d)^m*a*b*c^2*d^
3*m^4*x^3*e^6 + 112500*(x*e + d)^m*b^2*c^2*d^3*m^3*x^4*e^6 + 75000*(x*e + d
)^m*a*c^3*d^3*m^3*x^4*e^6 + 79632*(x*e + d)^m*b*c^3*d^3*m^2*x^5*e^6 + 6720*
(x*e + d)^m*c^4*d^3*m*x^6*e^6 - 24*(x*e + d)^m*b^4*d^4*m^5*x*e^5 - 288*(x*e
 + d)^m*a*b^2*c*d^4*m^5*x*e^5 - 144*(x*e + d)^m*a^2*c^2*d^4*m^5*x*e^5 - 600
0*(x*e + d)^m*b^3*c*d^4*m^4*x^2*e^5 - 18000*(x*e + d)^m*a*b*c^2*d^4*m^4*x^2
e^5 - 90000(x*e + d)^m*b^2*c^2*d^4*m^3*x^3*e^5 - 60000*(x*e + d)^m*a*c^3*
d^4*m^3*x^3*e^5 - 88200*(x*e + d)^m*b*c^3*d^4*m^2*x^4*e^5 - 8064*(x*e + d)^
m*c^4*d^4*m*x^5*e^5 + 480*(x*e + d)^m*b^3*c*d^5*m^4*x*e^4 + 1440*(x*e + d)^
m*a*b*c^2*d^5*m^4*x*e^4 + 38880*(x*e + d)^m*b^2*c^2*d^5*m^3*x^2*e^4 + 25920
*(x*e + d)^m*a*c^3*d^5*m^3*x^2*e^4 + 97440*(x*e + d)^m*b*c^3*d^5*m^2*x^3*e^
4 + 10080*(x*e + d)^m*c^4*d^5*m*x^4*e^4 - 4320*(x*e + d)^m*b^2*c^2*d^6*m^3*
x*e^3 - 2880*(x*e + d)^m*a*c^3*d^6*m^3*x*e^3 - 100800*(x*e + d)^m*b*c^3*d^6
*m^2*x^2*e^3 - 13440*(x*e + d)^m*c^4*d^6*m*x^3*e^3 + 20160*(x*e + d)^m*b*c^
3*d^7*m^2*x*e^2 + 20160*(x*e + d)^m*c^4*d^7*m*x^2*e^2 - 40320*(x*e + d)^m*c
^4*d^8*m*x*e + (x*e + d)^m*a^4*m^8*x*e^9 + 172*(x*e + d)^m*a^3*b*m^7*x^2*e^
9 + 4464*(x*e + d)^m*a^2*b^2*m^6*x^3*e^9 + 2976*(x*e + d)^m*a^3*c*m^6*x^3*e
^9 + 26504*(x*e + d)^m*a*b^3*m^5*x^4*e^9 + 79512*(x*e + d)^m*a^2*b*c*m^5*x^
4*e^9 + 32773*(x*e + d)^m*b^4*m^4*x^5*e^9 + 393276*(x*e + d)^m*a*b^2*c*m^4*
x^5*e^9 + 196638*(x*e + d)^m*a^2*c^2*m^4*x^5*e^9 + 370044*(x*e + d)^m*b^3*c
*m^3*x^6*e^9 + 1110132*(x*e + d)^m*a*b*c^2*m^3*x^6*e^9 + 885816*(x*e + d)^m
*b^2*c^2*m^2*x^7*e^9 + 590544*(x*e + d)^m*a*c^3*m^2*x^7*e^9 + 490608*(x*e +
 d)^m*b*c^3*m*x^8*e^9 + 40320*(x*e + d)^m*c^4*x^9*e^9 + (x*e + d)^m*a^4*d*m
^8*e^8 + 168*(x*e + d)^m*a^3*b*d*m^7*x*e^8 + 3984*(x*e + d)^m*a^2*b^2*d*m^6
*x^2*e^8 + 2656*(x*e + d)^m*a^3*c*d*m^6*x^2*e^8 + 19400*(x*e + d)^m*a*b^3*d
*m^5*x^3*e^8 + 58200*(x*e + d)^m*a^2*b*c*d*m^5*x^3*e^8 + 16789*(x*e + d)^m*
b^4*d*m^4*x^4*e^8 + 201468*(x*e + d)^m*a*b^2*c*d*m^4*x^4*e^8 + 100734*(x*e
+ d)^m*a^2*c^2*d*m^4*x^4*e^8 + 111064*(x*e + d)^m*b^3*c*d*m^3*x^5*e^8 + 333
192*(x*e + d)^m*a*b*c^2*d*m^3*x^5*e^8 + 130608*(x*e + d)^m*b^2*c^2*d*m^2*x^
6*e^8 + 87072*(x*e + d)^m*a*c^3*d*m^2*x^6*e^8 + 25920*(x*e + d)^m*b*c^3*d*m
*x^7*e^8 - 4*(x*e + d)^m*a^3*b*d^2*m^7*e^7 - 468*(x*e + d)^m*a^2*b^2*d^2*m^
6*x*e^7 - 312*(x*e + d)^m*a^3*c*d^2*m^6*x*e^7 - 6240*(x*e + d)^m*a*b^3*d^2*
m^5*x^2*e^7 - 18720*(x*e + d)^m*a^2*b*c*d^2*m^5*x^2*e^7 - 10860*(x*e + d)^m
*b^4*d^2*m^4*x^3*e^7 - 130320*(x*e + d)^m*a*b^2*c*d^2*m^4*x^3*e^7 - 65160*(
x*e + d)^m*a^2*c^2*d^2*m^4*x^3*e^7 - 105380*(x*e + d)^m*b^3*c*d^2*m^3*x^4*e
^7 - 316140*(x*e + d)^m*a*b*c^2*d^2*m^3*x^4*e^7 - 144288*(x*e + d)^m*b^2*c^
2*d^2*m^2*x^5*e^7 - 96192*(x*e + d)^m*a*c^3*d^2*m^2*x^5*e^7 - 30240*(x*e +
d)^m*b*c^3*d^2*m*x^6*e^7 + 12*(x*e + d)^m*a^2*b^2*d^3*m^6*e^6 + 8*(x*e + d)
^m*a^3*c*d^3*m^6*e^6 + 840*(x*e + d)^m*a*b^3*d^3*m^5*x*e^6 + 2520*(x*e + d)
^m*a^2*b*c*d^3*m^5*x*e^6 + 4380*(x*e + d)^m*b^4*d^3*m^4*x^2*e^6 + 52560*(x*
e + d)^m*a*b^2*c*d^3*m^4*x^2*e^6 + 26280*(x*e + d)^m*a^2*c^2*d^3*m^4*x^2*e^
6 + 90000*(x*e + d)^m*b^3*c*d^3*m^3*x^3*e^6 + 270000*(x*e + d)^m*a*b*c^2*d^
3*m^3*x^3*e^6 + 160920*(x*e + d)^m*b^2*c^2*d^3*m^2*x^4*e^6 + 107280*(x*e +
d)^m*a*c^3*d^3*m^2*x^4*e^6 + 36288*(x*e + d)^m*b*c^3*d^3*m*x^5*e^6 - 24*(x*
e + d)^m*a*b^3*d^4*m^5*e^5 - 72*(x*e + d)^m*a^2*b*c*d^4*m^5*e^5 - 720*(x*e
+ d)^m*b^4*d^4*m^4*x*e^5 - 8640*(x*e + d)^m*a*b^2*c*d^4*m^4*x*e^5 - 4320*(x
*e + d)^m*a^2*c^2*d^4*m^4*x*e^5 - 51600*(x*e + d)^m*b^3*c*d^4*m^3*x^2*e^5 -
 154800*(x*e + d)^m*a*b*c^2*d^4*m^3*x^2*e^5 - 180000*(x*e + d)^m*b^2*c^2*d^
4*m^2*x^3*e^5 - 120000*(x*e + d)^m*a*c^3*d^4*m^2*x^3*e^5 - 45360*(x*e + d)^
m*b*c^3*d^4*m*x^4*e^5 + 24*(x*e + d)^m*b^4*d^5*m^4*e^4 + 288*(x*e + d)^m*a*
b^2*c*d^5*m^4*e^4 + 144*(x*e + d)^m*a^2*c^2*d^5*m^4*e^4 + 11520*(x*e + d)^m
*b^3*c*d^5*m^3*x*e^4 + 34560*(x*e + d)^m*a*b*c^2*d^5*m^3*x*e^4 + 192240*(x*
e + d)^m*b^2*c^2*d^5*m^2*x^2*e^4 + 128160*(x*e + d)^m*a*c^3*d^5*m^2*x^2*e^4
 + 60480*(x*e + d)^m*b*c^3*d^5*m*x^3*e^4 - 480*(x*e + d)^m*b^3*c*d^6*m^3*e^
3 - 1440*(x*e + d)^m*a*b*c^2*d^6*m^3*e^3 - 73440*(x*e + d)^m*b^2*c^2*d^6*m^

2*x*e^3 - 48960*(x*e + d)^m*a*c^3*d^6*m^2*x*e^3 - 90720*(x*e + d)^m*b*c^3*d^6*m*x^2*e^3 + 4320*(x*e + d)^m*b^2*c^2*d^7*m^2*e^2 + 2880*(x*e + d)^m*a*c^3*d^7*m^2*e^2 + 181440*(x*e + d)^m*b*c^3*d^7*m*x*e^2 - 20160*(x*e + d)^m*b*c^3*d^8*m*e + 40320*(x*e + d)^m*c^4*d^9 + 44*(x*e + d)^m*a^4*m^7*x*e^9 + 3136*(x*e + d)^m*a^3*b*m^6*x^2*e^9 + 43308*(x*e + d)^m*a^2*b^2*m^5*x^3*e^9 + 28872*(x*e + d)^m*a^3*c*m^5*x^3*e^9 + 147076*(x*e + d)^m*a*b^3*m^4*x^4*e^9 + 441228*(x*e + d)^m*a^2*b*c*m^4*x^4*e^9 + 105460*(x*e + d)^m*b^4*m^3*x^5*e^9 + 1265520*(x*e + d)^m*a*b^2*c*m^3*x^5*e^9 + 632760*(x*e + d)^m*a^2*c^2*m^3*x^5*e^9 + 674456*(x*e + d)^m*b^3*c*m^2*x^6*e^9 + 2023368*(x*e + d)^m*a*b*c^2*m^2*x^6*e^9 + 835488*(x*e + d)^m*b^2*c^2*m*x^7*e^9 + 556992*(x*e + d)^m*a*c^3*m*x^7*e^9 + 181440*(x*e + d)^m*b*c^3*x^8*e^9 + 44*(x*e + d)^m*a^4*d*m^7*e^8 + 2968*(x*e + d)^m*a^3*b*d*m^6*x*e^8 + 35340*(x*e + d)^m*a^2*b^2*d*m^5*x^2*e^8 + 23560*(x*e + d)^m*a^3*c*d*m^5*x^2*e^8 + 88876*(x*e + d)^m*a*b^3*d*m^4*x^3*e^8 + 266628*(x*e + d)^m*a^2*b*c*d*m^4*x^3*e^8 + 38304*(x*e + d)^m*b^4*d*m^3*x^4*e^8 + 459648*(x*e + d)^m*a*b^2*c*d*m^3*x^4*e^8 + 229824*(x*e + d)^m*a^2*c^2*d*m^3*x^4*e^8 + 119136*(x*e + d)^m*b^3*c*d*m^2*x^5*e^8 + 357408*(x*e + d)^m*a*b*c^2*d*m^2*x^5*e^8 + 51840*(x*e + d)^m*b^2*c^2*d*m*x^6*e^8 + 34560*(x*e + d)^m*a*c^3*d*m*x^6*e^8 - 168*(x*e + d)^m*a^3*b*d^2*m^6*e^7 - 7500*(x*e + d)^m*a^2*b^2*d^2*m^5*x*e^7 - 5000*(x*e + d)^m*a^3*c*d^2*m^5*x*e^7 - 45720*(x*e + d)^m*a*b^3*d^2*m^4*x^2*e^7 - 137160*(x*e + d)^m*a^2*b*c*d^2*m^4*x^2*e^7 - 34576*(x*e + d)^m*b^4*d^2*m^3*x^3*e^7 - 414912*(x*e + d)^m*a*b^2*c*d^2*m^3*x^3*e^7 - 207456*(x*e + d)^m*a^2*c^2*d^2*m^3*x^3*e^7 - 133800*(x*e + d)^m*b^3*c*d^2*m^2*x^4*e^7 - 401400*(x*e + d)^m*a*b*c^2*d^2*m^2*x^4*e^7 - 62208*(x*e + d)^m*b^2*c^2*d^2*m*x^5*e^7 - 41472*(x*e + d)^m*a*c^3*d^2*m*x^5*e^7 + 468*(x*e + d)^m*a^2*b^2*d^3*m^5*e^6 + 312*(x*e + d)^m*a^3*c*d^3*m^5*e^6 + 11640*(x*e + d)^m*a*b^3*d^3*m^4*x*e^6 + 34920*(x*e + d)^m*a^2*b*c*d^3*m^4*x*e^6 + 23820*(x*e + d)^m*b^4*d^3*m^3*x^2*e^6 + 285840*(x*e + d)^m*a*b^2*c*d^3*m^3*x^2*e^6 + 142920*(x*e + d)^m*a^2*c^2*d^3*m^3*x^2*e^6 + 151520*(x*e + d)^m*b^3*c*d^3*m^2*x^3*e^6 + 454560*(x*e + d)^m*a*b*c^2*d^3*m^2*x^3*e^6 + 77760*(x*e + d)^m*b^2*c^2*d^3*m*x^4*e^6 + 51840*(x*e + d)^m*a*c^3*d^3*m*x^4*e^6 - 840*(x*e + d)^m*a*b^3*d^4*m^4*e^5 - 2520*(x*e + d)^m*a^2*b*c*d^4*m^4*e^5 - 8040*(x*e + d)^m*b^4*d^4*m^3*x*e^5 - 96480*(x*e + d)^m*a*b^2*c*d^4*m^3*x*e^5 - 48240*(x*e + d)^m*a^2*c^2*d^4*m^3*x*e^5 - 166800*(x*e + d)^m*b^3*c*d^4*m^2*x^2*e^5 - 500400*(x*e + d)^m*a*b*c^2*d^4*m^2*x^2*e^5 - 103680*(x*e + d)^m*b^2*c^2*d^4*m*x^3*e^5 - 69120*(x*e + d)^m*a*c^3*d^4*m*x^3*e^5 + 720*(x*e + d)^m*b^4*d^5*m^3*e^4 + 8640*(x*e + d)^m*a*b^2*c*d^5*m^3*e^4 + 4320*(x*e + d)^m*a^2*c^2*d^5*m^3*e^4 + 91680*(x*e + d)^m*b^3*c*d^5*m^2*x*e^4 + 275040*(x*e + d)^m*a*b*c^2*d^5*m^2*x*e^4 + 155520*(x*e + d)^m*b^2*c^2*d^5*m*x^2*e^4 + 103680*(x*e + d)^m*a*c^3*d^5*m*x^2*e^4 - 11520*(x*e + d)^m*b^3*c*d^6*m^2*e^3 - 34560*(x*e + d)^m*a*b*c^2*d^6*m^2*e^3 - 311040*(x*e + d)^m*b^2*c^2*d^6*m*x*e^3 - 207360*(x*e + d)^m*a*c^3*d^6*m*x*e^3 + 73440*(x*e + d)^m*b^2*c^2*d^7*m*e^2 + 48960*(x*e + d)^m*a*c^3*d^7*m*e^2 - 181440*(x*e + d)^m*b*c^3*d^8*e + 826*(x*e + d)^m*a^4*m^6*x*e^9 + 31528*(x*e + d)^m*a^3*b*m^5*x^2*e^9 + 249714*(x*e + d)^m*a^2*b^2*m^4*x^3*e^9 + 166476*(x*e + d)^m*a^3*c*m^4*x^3*e^9 + 488996*(x*e + d)^m*a*b^3*m^3*x^4*e^9 + 1466988*(x*e + d)^m*a^2*b*c*m^3*x^4*e^9 + 196380*(x*e + d)^m*b^4*m^2*x^5*e^9 + 2356560*(x*e + d)^m*a*b^2*c*m^2*x^5*e^9 + 1178280*(x*e + d)^m*a^2*c^2*m^2*x^5*e^9 + 644064*(x*e + d)^m*b^3*c*m*x^6*e^9 + 1932192*(x*e + d)^m*a*b*c^2*m*x^6*e^9 + 311040*(x*e + d)^m*b^2*c^2*x^7*e^9 + 207360*(x*e + d)^m*a*c^3*x^7*e^9 + 826*(x*e + d)^m*a^4*d*m^6*e^8 + 28560*(x*e + d)^m*a^3*b*d*m^5*x*e^8 + 179034*(x*e + d)^m*a^2*b^2*d*m^4*x^2*e^8 + 119356*(x*e + d)^m*a^3*c*d*m^4*x^2*e^8 + 222368*(x*e + d)^m*a*b^3*d*m^3*x^3*e^8 + 667104*(x*e + d)^m*a^2*b*c*d*m^3*x^3*e^8 + 43164*(x*e + d)^m*b^4*d*m^2*x^4*e^8 + 517968*(x*e + d)^m*a*b^2*c*d*m^2*x^4*e^8 + 258984*(x*e + d)^m*a^2*c^2*d*m^2*x^4*e^8 + 48384*(x*e + d)^m*b^3*c*d*m*x^5*e^8 + 145152*(x*e + d)^m*a*b*c^2*d*m*x^5*e^8 - 2968*(x*e + d)^m*a^3*b*d^2*m^5*e^7 - 63180*(x*e + d)^m*a^2*b^2*d^2*m^4*x*e^7 - 42120*(x*e + d)^m*a^3*c*d^2*m^4*x*e^7 - 175188*(x*e + d)^m*a*b^3*d^2*m^3*x^2*e^7 - 525564*(x*e + d)^m*a^2*b*c*d^2*m^3*x^2*e^7 - 49488*(x*e + d)^m*b^4*d^2*m^2*x^3*e^7 - 593856*(x*e + d)^m*a*b^2*c*d^2*m^2*x^3*e^7

- 296928*(x*e + d)^m*a^2*c^2*d^2*m^2*x^3*e^7 - 60480*(x*e + d)^m*b^3*c*d^2
*m*x^4*e^7 - 181440*(x*e + d)^m*a*b*c^2*d^2*m*x^4*e^7 + 7500*(x*e + d)^m*a^
2*b^2*d^3*m^4*e^6 + 5000*(x*e + d)^m*a^3*c*d^3*m^4*e^6 + 79800*(x*e + d)^m*
a*b^3*d^3*m^3*x*e^6 + 239400*(x*e + d)^m*a^2*b*c*d^3*m^3*x*e^6 + 56088*(x*e
+ d)^m*b^4*d^3*m^2*x^2*e^6 + 673056*(x*e + d)^m*a*b^2*c*d^3*m^2*x^2*e^6 +
336528*(x*e + d)^m*a^2*c^2*d^3*m^2*x^2*e^6 + 80640*(x*e + d)^m*b^3*c*d^3*m*
x^3*e^6 + 241920*(x*e + d)^m*a*b*c^2*d^3*m*x^3*e^6 - 11640*(x*e + d)^m*a*b^
3*d^4*m^3*e^5 - 34920*(x*e + d)^m*a^2*b*c*d^4*m^3*e^5 - 39600*(x*e + d)^m*b
^4*d^4*m^2*x*e^5 - 475200*(x*e + d)^m*a*b^2*c*d^4*m^2*x*e^5 - 237600*(x*e +
d)^m*a^2*c^2*d^4*m^2*x*e^5 - 120960*(x*e + d)^m*b^3*c*d^4*m*x^2*e^5 - 3628
80*(x*e + d)^m*a*b*c^2*d^4*m*x^2*e^5 + 8040*(x*e + d)^m*b^4*d^5*m^2*e^4 + 9
6480*(x*e + d)^m*a*b^2*c*d^5*m^2*e^4 + 48240*(x*e + d)^m*a^2*c^2*d^5*m^2*e^
4 + 241920*(x*e + d)^m*b^3*c*d^5*m*x*e^4 + 725760*(x*e + d)^m*a*b*c^2*d^5*m
*x*e^4 - 91680*(x*e + d)^m*b^3*c*d^6*m*e^3 - 275040*(x*e + d)^m*a*b*c^2*d^6
*m*e^3 + 311040*(x*e + d)^m*b^2*c^2*d^7*e^2 + 207360*(x*e + d)^m*a*c^3*d^7*
e^2 + 8624*(x*e + d)^m*a^4*m^5*x*e^9 + 190036*(x*e + d)^m*a^3*b*m^4*x^2*e^9
+ 866808*(x*e + d)^m*a^2*b^2*m^3*x^3*e^9 + 577872*(x*e + d)^m*a^3*c*m^3*x^
3*e^9 + 938736*(x*e + d)^m*a*b^3*m^2*x^4*e^9 + 2816208*(x*e + d)^m*a^2*b*c*
m^2*x^4*e^9 + 190800*(x*e + d)^m*b^4*m*x^5*e^9 + 2289600*(x*e + d)^m*a*b^2*
c*m*x^5*e^9 + 1144800*(x*e + d)^m*a^2*c^2*m*x^5*e^9 + 241920*(x*e + d)^m*b^
3*c*x^6*e^9 + 725760*(x*e + d)^m*a*b*c^2*x^6*e^9 + 8624*(x*e + d)^m*a^4*d*m
^5*e^8 + 161476*(x*e + d)^m*a^3*b*d*m^4*x*e^8 + 508740*(x*e + d)^m*a^2*b^2*
d*m^3*x^2*e^8 + 339160*(x*e + d)^m*a^3*c*d*m^3*x^2*e^8 + 271632*(x*e + d)^m
*a*b^3*d*m^2*x^3*e^8 + 814896*(x*e + d)^m*a^2*b*c*d*m^2*x^3*e^8 + 18144*(x*
e + d)^m*b^4*d*m*x^4*e^8 + 217728*(x*e + d)^m*a*b^2*c*d*m*x^4*e^8 + 108864*
(x*e + d)^m*a^2*c^2*d*m*x^4*e^8 - 28560*(x*e + d)^m*a^3*b*d^2*m^4*e^7 - 294
888*(x*e + d)^m*a^2*b^2*d^2*m^3*x*e^7 - 196592*(x*e + d)^m*a^3*c*d^2*m^3*x*
e^7 - 316728*(x*e + d)^m*a*b^3*d^2*m^2*x^2*e^7 - 950184*(x*e + d)^m*a^2*b*c
*d^2*m^2*x^2*e^7 - 24192*(x*e + d)^m*b^4*d^2*m*x^3*e^7 - 290304*(x*e + d)^m
*a*b^2*c*d^2*m*x^3*e^7 - 145152*(x*e + d)^m*a^2*c^2*d^2*m*x^3*e^7 + 63180*(
x*e + d)^m*a^2*b^2*d^3*m^3*e^6 + 42120*(x*e + d)^m*a^3*c*d^3*m^3*e^6 + 2705
76*(x*e + d)^m*a*b^3*d^3*m^2*x*e^6 + 811728*(x*e + d)^m*a^2*b*c*d^3*m^2*x*e
^6 + 36288*(x*e + d)^m*b^4*d^3*m*x^2*e^6 + 435456*(x*e + d)^m*a*b^2*c*d^3*m
*x^2*e^6 + 217728*(x*e + d)^m*a^2*c^2*d^3*m*x^2*e^6 - 79800*(x*e + d)^m*a*b
^3*d^4*m^2*e^5 - 239400*(x*e + d)^m*a^2*b*c*d^4*m^2*e^5 - 72576*(x*e + d)^m
*b^4*d^4*m*x*e^5 - 870912*(x*e + d)^m*a*b^2*c*d^4*m*x*e^5 - 435456*(x*e + d
)^m*a^2*c^2*d^4*m*x*e^5 + 39600*(x*e + d)^m*b^4*d^5*m*e^4 + 475200*(x*e + d
)^m*a*b^2*c*d^5*m*e^4 + 237600*(x*e + d)^m*a^2*c^2*d^5*m*e^4 - 241920*(x*e
+ d)^m*b^3*c*d^6*e^3 - 725760*(x*e + d)^m*a*b*c^2*d^6*e^3 + 54649*(x*e + d)
^m*a^4*m^4*x*e^9 + 697228*(x*e + d)^m*a^3*b*m^3*x^2*e^9 + 1741656*(x*e + d)
^m*a^2*b^2*m^2*x^3*e^9 + 1161104*(x*e + d)^m*a^3*c*m^2*x^3*e^9 + 935856*(x*
e + d)^m*a*b^3*m*x^4*e^9 + 2807568*(x*e + d)^m*a^2*b*c*m*x^4*e^9 + 72576*(x
*e + d)^m*b^4*x^5*e^9 + 870912*(x*e + d)^m*a*b^2*c*x^5*e^9 + 435456*(x*e +
d)^m*a^2*c^2*x^5*e^9 + 54649*(x*e + d)^m*a^4*d*m^4*e^8 + 535752*(x*e + d)^m
*a^3*b*d*m^3*x*e^8 + 724176*(x*e + d)^m*a^2*b^2*d*m^2*x^2*e^8 + 482784*(x*e
+ d)^m*a^3*c*d*m^2*x^2*e^8 + 120960*(x*e + d)^m*a*b^3*d*m*x^3*e^8 + 362880
*(x*e + d)^m*a^2*b*c*d*m*x^3*e^8 - 161476*(x*e + d)^m*a^3*b*d^2*m^3*e^7 - 7
22592*(x*e + d)^m*a^2*b^2*d^2*m^2*x*e^7 - 481728*(x*e + d)^m*a^3*c*d^2*m^2*
x*e^7 - 181440*(x*e + d)^m*a*b^3*d^2*m*x^2*e^7 - 544320*(x*e + d)^m*a^2*b*c
*d^2*m*x^2*e^7 + 294888*(x*e + d)^m*a^2*b^2*d^3*m^2*e^6 + 196592*(x*e + d)^
m*a^3*c*d^3*m^2*e^6 + 362880*(x*e + d)^m*a*b^3*d^3*m*x*e^6 + 1088640*(x*e +
d)^m*a^2*b*c*d^3*m*x*e^6 - 270576*(x*e + d)^m*a*b^3*d^4*m*e^5 - 811728*(x*
e + d)^m*a^2*b*c*d^4*m*e^5 + 72576*(x*e + d)^m*b^4*d^5*e^4 + 870912*(x*e +
d)^m*a*b^2*c*d^5*e^4 + 435456*(x*e + d)^m*a^2*c^2*d^5*e^4 + 214676*(x*e + d
)^m*a^4*m^3*x*e^9 + 1500264*(x*e + d)^m*a^3*b*m^2*x^2*e^9 + 1811232*(x*e +
d)^m*a^2*b^2*m*x^3*e^9 + 1207488*(x*e + d)^m*a^3*c*m*x^3*e^9 + 362880*(x*e
+ d)^m*a*b^3*x^4*e^9 + 1088640*(x*e + d)^m*a^2*b*c*x^4*e^9 + 214676*(x*e +
d)^m*a^4*d*m^3*e^8 + 964512*(x*e + d)^m*a^3*b*d*m^2*x*e^8 + 362880*(x*e + d
)^m*a^2*b^2*d*m*x^2*e^8 + 241920*(x*e + d)^m*a^3*c*d*m*x^2*e^8 - 535752*(x*

e + d)^m*a^3*b*d^2*m^2*e^7 - 725760*(x*e + d)^m*a^2*b^2*d^2*m*x*e^7 - 48384
0*(x*e + d)^m*a^3*c*d^2*m*x*e^7 + 722592*(x*e + d)^m*a^2*b^2*d^3*m*e^6 + 48
1728*(x*e + d)^m*a^3*c*d^3*m*e^6 - 362880*(x*e + d)^m*a*b^3*d^4*e^5 - 10886
40*(x*e + d)^m*a^2*b*c*d^4*e^5 + 509004*(x*e + d)^m*a^4*m^2*x*e^9 + 1690272
*(x*e + d)^m*a^3*b*m*x^2*e^9 + 725760*(x*e + d)^m*a^2*b^2*x^3*e^9 + 483840*
(x*e + d)^m*a^3*c*x^3*e^9 + 509004*(x*e + d)^m*a^4*d*m^2*e^8 + 725760*(x*e
+ d)^m*a^3*b*d*m*x*e^8 - 964512*(x*e + d)^m*a^3*b*d^2*m*e^7 + 725760*(x*e +
 d)^m*a^2*b^2*d^3*e^6 + 483840*(x*e + d)^m*a^3*c*d^3*e^6 + 663696*(x*e + d)
^m*a^4*m*x*e^9 + 725760*(x*e + d)^m*a^3*b*x^2*e^9 + 663696*(x*e + d)^m*a^4*
d*m*e^8 - 725760*(x*e + d)^m*a^3*b*d^2*e^7 + 362880*(x*e + d)^m*a^4*x*e^9 +
 362880*(x*e + d)^m*a^4*d*e^8)/(m^9*e^9 + 45*m^8*e^9 + 870*m^7*e^9 + 9450*m
^6*e^9 + 63273*m^5*e^9 + 269325*m^4*e^9 + 723680*m^3*e^9 + 1172700*m^2*e^9
+ 1026576*m*e^9 + 362880*e^9)

3.2550 $\int (d+ex)^m \left(a+bx+cx^2\right)^3 \, dx$

Optimal. Leaf size=305

$$\frac{3(d+ex)^{m+3}\left(ae^2-bde+cd^2\right)\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{e^7(m+3)}-\frac{(2cd-be)(d+ex)^{m+4}\left(-2ce(5bd-3ae)+b^2e^2+10c^2\right.}{e^7(m+4)}$$

```
[Out] ((c*d^2 - b*d*e + a*e^2)^3*(d + e*x)^(1 + m))/(e^7*(1 + m)) - (3*(2*c*d - b
*e)*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^(2 + m))/(e^7*(2 + m)) + (3*(c*d^2
- b*d*e + a*e^2)*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^(3 + m
))/(e^7*(3 + m)) - ((2*c*d - b*e)*(10*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*
a*e))*(d + e*x)^(4 + m))/(e^7*(4 + m)) + (3*c*(5*c^2*d^2 + b^2*e^2 - c*e*(5
*b*d - a*e))*(d + e*x)^(5 + m))/(e^7*(5 + m)) - (3*c^2*(2*c*d - b*e)*(d + e
*x)^(6 + m))/(e^7*(6 + m)) + (c^3*(d + e*x)^(7 + m))/(e^7*(7 + m))
```

Rubi [A] time = 0.211511, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$\frac{3(d+ex)^{m+3}\left(ae^2-bde+cd^2\right)\left(-ce(5bd-ae)+b^2e^2+5c^2d^2\right)}{e^7(m+3)}-\frac{(2cd-be)(d+ex)^{m+4}\left(-2ce(5bd-3ae)+b^2e^2+10c^2\right.}{e^7(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a + b*x + c*x^2)^3,x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)^3*(d + e*x)^(1 + m))/(e^7*(1 + m)) - (3*(2*c*d - b
*e)*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^(2 + m))/(e^7*(2 + m)) + (3*(c*d^2
- b*d*e + a*e^2)*(5*c^2*d^2 + b^2*e^2 - c*e*(5*b*d - a*e))*(d + e*x)^(3 + m
))/(e^7*(3 + m)) - ((2*c*d - b*e)*(10*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 3*
a*e))*(d + e*x)^(4 + m))/(e^7*(4 + m)) + (3*c*(5*c^2*d^2 + b^2*e^2 - c*e*(5
*b*d - a*e))*(d + e*x)^(5 + m))/(e^7*(5 + m)) - (3*c^2*(2*c*d - b*e)*(d + e
*x)^(6 + m))/(e^7*(6 + m)) + (c^3*(d + e*x)^(7 + m))/(e^7*(7 + m))
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^m \left(a+bx+cx^2\right)^3 \, dx = \int \left(\frac{\left(cd^2-bde+ae^2\right)^3 (d+ex)^m}{e^6}+\frac{3(-2cd+be)\left(cd^2-bde+ae^2\right)^2 (d+ex)^{1+m}}{e^6}+\frac{3}{}\right.$$

$$=\frac{\left(cd^2-bde+ae^2\right)^3 (d+ex)^{1+m}}{e^7(1+m)}-\frac{3(2cd-be)\left(cd^2-bde+ae^2\right)^2 (d+ex)^{2+m}}{e^7(2+m)}+\frac{3\left(cd\right.}{}$$

Mathematica [A] time = 1.70416, size = 476, normalized size = 1.56

$$(d+ex)^{m+1}\left(\frac{3(d+ex)\left(-2(m+2)(d+ex)\left(2c^2e^2\left(4a^2e^2\left(m^2+10m+24\right)+4abde\left(m^2-2m-33\right)+b^2d^2\left(m^2-2m+57\right)\right)-2b^2ce^3(m+1)(3ae(m+5)+bd(m-3))-8c^3d^2e\left(\right.\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(a + b*x + c*x^2)^3,x]
```

```
[Out] ((d + e*x)^(1 + m)*((a + x*(b + c*x))^3 + (3*(d + e*x)*(2*(2*c*d - b*e)*(c*
d^2 + e*(-(b*d) + a*e))*(3 + m)*(60*c^2*d^2 + b^2*e^2*(2 + 3*m + m^2) - 4*c
*e*(15*b*d + a*e*(-13 + 3*m + m^2))) - 2*(2 + m)*(120*c^4*d^4 + b^4*e^4*(3
+ 4*m + m^2) - 2*b^2*c*e^3*(1 + m)*(b*d*(-3 + m) + 3*a*e*(5 + m)) - 8*c^3*d
^2*e*(30*b*d + a*e*(-33 - 2*m + m^2)) + 2*c^2*e^2*(4*a*b*d*e*(-33 - 2*m + m
^2) + b^2*d^2*(57 - 2*m + m^2) + 4*a^2*e^2*(24 + 10*m + m^2)))*(d + e*x) -
c*e^4*(2 + m)*(3 + m)*(4 + m)*(5 + m)*(a + x*(b + c*x))^2*(b*e*(11 + m) + 2
*c*(-5*d + e*(6 + m)*x)) - 2*e^2*(2 + m)*(3 + m)*(a + x*(b + c*x))*(-(b^3*e
^3*(1 + m)) + 20*c^3*d^2*(-3*d + e*(4 + m)*x) - b*c*e^2*(-2*a*e*(37 + 7*m)
+ b*d*(72 + 13*m + m^2) + b*e*(4 + 5*m + m^2)*x) + 2*c^2*e*(5*b*d*(d*(13 +
m) - 2*e*(4 + m)*x) + 2*a*e*(d*(-13 + 3*m + m^2) + e*(24 + 10*m + m^2)*x)))
))/(c*e^6*(2 + m)*(3 + m)*(4 + m)*(5 + m)*(6 + m)*(7 + m))))/(e*(1 + m))
```

Maple [B] time = 0.059, size = 2927, normalized size = 9.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^3,x)
```

```
[Out] (e*x+d)^(1+m)*(c^3*e^6*m^6*x^6+3*b*c^2*e^6*m^6*x^5+21*c^3*e^6*m^5*x^6+3*a*c
^2*e^6*m^6*x^4+3*b^2*c*e^6*m^6*x^4+66*b*c^2*e^6*m^5*x^5-6*c^3*d*e^5*m^5*x^5
+175*c^3*e^6*m^4*x^6+6*a*b*c*e^6*m^6*x^3+69*a*c^2*e^6*m^5*x^4+b^3*e^6*m^6*x
^3+69*b^2*c*e^6*m^5*x^4-15*b*c^2*d*e^5*m^5*x^4+570*b*c^2*e^6*m^4*x^5-90*c^3
*d*e^5*m^4*x^5+735*c^3*e^6*m^3*x^6+3*a^2*c*e^6*m^6*x^2+3*a*b^2*e^6*m^6*x^2+
144*a*b*c*e^6*m^5*x^3-12*a*c^2*d*e^5*m^5*x^3+621*a*c^2*e^6*m^4*x^4+24*b^3*e
^6*m^5*x^3-12*b^2*c*d*e^5*m^5*x^3+621*b^2*c*e^6*m^4*x^4-255*b*c^2*d*e^5*m^4
*x^4+2460*b*c^2*e^6*m^3*x^5+30*c^3*d^2*e^4*m^4*x^4-510*c^3*d*e^5*m^3*x^5+16
24*c^3*e^6*m^2*x^6+3*a^2*b*e^6*m^6*x+75*a^2*c*e^6*m^5*x^2+75*a*b^2*e^6*m^5*
x^2-18*a*b*c*d*e^5*m^5*x^2+1356*a*b*c*e^6*m^4*x^3-228*a*c^2*d*e^5*m^4*x^3+2
775*a*c^2*e^6*m^3*x^4-3*b^3*d*e^5*m^5*x^2+226*b^3*e^6*m^4*x^3-228*b^2*c*d*e
^5*m^4*x^3+2775*b^2*c*e^6*m^3*x^4+60*b*c^2*d^2*e^4*m^4*x^3-1575*b*c^2*d*e^5
*m^3*x^4+5547*b*c^2*e^6*m^2*x^5+300*c^3*d^2*e^4*m^3*x^4-1350*c^3*d*e^5*m^2*
x^5+1764*c^3*e^6*m*x^6+a^3*e^6*m^6+78*a^2*b*e^6*m^5*x-6*a^2*c*d*e^5*m^5*x+7
41*a^2*c*e^6*m^4*x^2-6*a*b^2*d*e^5*m^5*x+741*a*b^2*e^6*m^4*x^2-378*a*b*c*d*
e^5*m^4*x^2+6336*a*b*c*e^6*m^3*x^3+36*a*c^2*d^2*e^4*m^4*x^2-1572*a*c^2*d*e^
5*m^3*x^3+6432*a*c^2*e^6*m^2*x^4-63*b^3*d*e^5*m^4*x^2+1056*b^3*e^6*m^3*x^3+
36*b^2*c*d^2*e^4*m^4*x^2-1572*b^2*c*d*e^5*m^3*x^3+6432*b^2*c*e^6*m^2*x^4+78
0*b*c^2*d^2*e^4*m^3*x^3-4425*b*c^2*d*e^5*m^2*x^4+6114*b*c^2*e^6*m*x^5-120*c
^3*d^3*e^3*m^3*x^3+1050*c^3*d^2*e^4*m^2*x^4-1644*c^3*d*e^5*m*x^5+720*c^3*e^
6*x^6+27*a^3*e^6*m^5-3*a^2*b*d*e^5*m^5+810*a^2*b*e^6*m^4*x-138*a^2*c*d*e^5*
m^4*x+3657*a^2*c*e^6*m^3*x^2-138*a*b^2*d*e^5*m^4*x+3657*a*b^2*e^6*m^3*x^2+3
6*a*b*c*d^2*e^4*m^4*x-2934*a*b*c*d*e^5*m^3*x^2+15270*a*b*c*e^6*m^2*x^3+576*
a*c^2*d^2*e^4*m^3*x^2-4812*a*c^2*d*e^5*m^2*x^3+7236*a*c^2*e^6*m*x^4+6*b^3*d
^2*e^4*m^4*x-489*b^3*d*e^5*m^3*x^2+2545*b^3*e^6*m^2*x^3+576*b^2*c*d^2*e^4*m
^3*x^2-4812*b^2*c*d*e^5*m^2*x^3+7236*b^2*c*e^6*m*x^4-180*b*c^2*d^3*e^3*m^3*
x^2+3180*b*c^2*d^2*e^4*m^2*x^3-5610*b*c^2*d*e^5*m*x^4+2520*b*c^2*e^6*x^5-72
0*c^3*d^3*e^3*m^2*x^3+1500*c^3*d^2*e^4*m*x^4-720*c^3*d*e^5*x^5+295*a^3*e^6*
m^4-75*a^2*b*d*e^5*m^4+4260*a^2*b*e^6*m^3*x+6*a^2*c*d^2*e^4*m^4-1206*a^2*c*
d*e^5*m^3*x+9336*a^2*c*e^6*m^2*x^2+6*a*b^2*d^2*e^4*m^4-1206*a*b^2*d*e^5*m^3
*x+9336*a*b^2*e^6*m^2*x^2+684*a*b*c*d^2*e^4*m^3*x-10206*a*b*c*d*e^5*m^2*x^2
+17712*a*b*c*e^6*m*x^3-72*a*c^2*d^3*e^3*m^3*x+2988*a*c^2*d^2*e^4*m^2*x^2-64
```

```
80*a*c^2*d*e^5*m*x^3+3024*a*c^2*e^6*x^4+114*b^3*d^2*e^4*m^3*x-1701*b^3*d*e^
5*m^2*x^2+2952*b^3*e^6*m*x^3-72*b^2*c*d^3*e^3*m^3*x+2988*b^2*c*d^2*e^4*m^2*
x^2-6480*b^2*c*d*e^5*m*x^3+3024*b^2*c*e^6*x^4-1800*b*c^2*d^3*e^3*m^2*x^2+49
80*b*c^2*d^2*e^4*m*x^3-2520*b*c^2*d*e^5*x^4+360*c^3*d^4*e^2*m^2*x^2-1320*c^
3*d^3*e^3*m*x^3+720*c^3*d^2*e^4*x^4+1665*a^3*e^6*m^3-735*a^2*b*d*e^5*m^3+11
787*a^2*b*e^6*m^2*x+132*a^2*c*d^2*e^4*m^3-4902*a^2*c*d*e^5*m^2*x+11388*a^2*
c*e^6*m*x^2+132*a*b^2*d^2*e^4*m^3-4902*a*b^2*d*e^5*m^2*x+11388*a*b^2*e^6*m*
x^2-36*a*b*c*d^3*e^3*m^3+4500*a*b*c*d^2*e^4*m^2*x-15192*a*b*c*d*e^5*m*x^2+7
560*a*b*c*e^6*x^3-1008*a*c^2*d^3*e^3*m^2*x+5472*a*c^2*d^2*e^4*m*x^2-3024*a*
c^2*d*e^5*x^3-6*b^3*d^3*e^3*m^3+750*b^3*d^2*e^4*m^2*x-2532*b^3*d*e^5*m*x^2+
1260*b^3*e^6*x^3-1008*b^2*c*d^3*e^3*m^2*x+5472*b^2*c*d^2*e^4*m*x^2-3024*b^2
*c*d*e^5*x^3+360*b*c^2*d^4*e^2*m^2*x-4140*b*c^2*d^3*e^3*m*x^2+2520*b*c^2*d^
2*e^4*x^3+1080*c^3*d^4*e^2*m*x^2-720*c^3*d^3*e^3*x^3+5104*a^3*e^6*m^2-3525*
a^2*b*d*e^5*m^2+15822*a^2*b*e^6*m*x+1074*a^2*c*d^2*e^4*m^2-8868*a^2*c*d*e^5
*m*x+5040*a^2*c*e^6*x^2+1074*a*b^2*d^2*e^4*m^2-8868*a*b^2*d*e^5*m*x+5040*a*
b^2*e^6*x^2-648*a*b*c*d^3*e^3*m^2+11412*a*b*c*d^2*e^4*m*x-7560*a*b*c*d*e^5*
x^2+72*a*c^2*d^4*e^2*m^2-3960*a*c^2*d^3*e^3*m*x+3024*a*c^2*d^2*e^4*x^2-108*
b^3*d^3*e^3*m^2+1902*b^3*d^2*e^4*m*x-1260*b^3*d*e^5*x^2+72*b^2*c*d^4*e^2*m^
2-3960*b^2*c*d^3*e^3*m*x+3024*b^2*c*d^2*e^4*x^2+2880*b*c^2*d^4*e^2*m*x-2520
*b*c^2*d^3*e^3*x^2-720*c^3*d^5*e*m*x+720*c^3*d^4*e^2*x^2+8028*a^3*e^6*m-826
2*a^2*b*d*e^5*m+7560*a^2*b*e^6*x+3828*a^2*c*d^2*e^4*m-5040*a^2*c*d*e^5*x+38
28*a*b^2*d^2*e^4*m-5040*a*b^2*d*e^5*x-3852*a*b*c*d^3*e^3*m+7560*a*b*c*d^2*e
^4*x+936*a*c^2*d^4*e^2*m-3024*a*c^2*d^3*e^3*x-642*b^3*d^3*e^3*m+1260*b^3*d^
2*e^4*x+936*b^2*c*d^4*e^2*m-3024*b^2*c*d^3*e^3*x-360*b*c^2*d^5*e*m+2520*b*c
^2*d^4*e^2*x-720*c^3*d^5*e*x+5040*a^3*e^6-7560*a^2*b*d*e^5+5040*a^2*c*d^2*e
^4+5040*a*b^2*d^2*e^4-7560*a*b*c*d^3*e^3+3024*a*c^2*d^4*e^2-1260*b^3*d^3*e^
3+3024*b^2*c*d^4*e^2-2520*b*c^2*d^5*e+720*c^3*d^6)/e^7/(m^7+28*m^6+322*m^5+
1960*m^4+6769*m^3+13132*m^2+13068*m+5040)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.98024, size = 5573, normalized size = 18.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] (a^3*d*e^6*m^6 + 720*c^3*d^7 - 2520*b*c^2*d^6*e - 7560*a^2*b*d^2*e^5 + 5040
*a^3*d*e^6 + 3024*(b^2*c + a*c^2)*d^5*e^2 - 1260*(b^3 + 6*a*b*c)*d^4*e^3 +
5040*(a*b^2 + a^2*c)*d^3*e^4 + (c^3*e^7*m^6 + 21*c^3*e^7*m^5 + 175*c^3*e^7*
m^4 + 735*c^3*e^7*m^3 + 1624*c^3*e^7*m^2 + 1764*c^3*e^7*m + 720*c^3*e^7)*x^
7 + (2520*b*c^2*e^7 + (c^3*d*e^6 + 3*b*c^2*e^7)*m^6 + 3*(5*c^3*d*e^6 + 22*b
*c^2*e^7)*m^5 + 5*(17*c^3*d*e^6 + 114*b*c^2*e^7)*m^4 + 15*(15*c^3*d*e^6 + 1
64*b*c^2*e^7)*m^3 + (274*c^3*d*e^6 + 5547*b*c^2*e^7)*m^2 + 6*(20*c^3*d*e^6
+ 1019*b*c^2*e^7)*m)*x^6 - 3*(a^2*b*d^2*e^5 - 9*a^3*d*e^6)*m^5 + 3*(1008*(b
```

```
^2*c + a*c^2)*e^7 + (b*c^2*d*e^6 + (b^2*c + a*c^2)*e^7)*m^6 - (2*c^3*d^2*e^
5 - 17*b*c^2*d*e^6 - 23*(b^2*c + a*c^2)*e^7)*m^5 - (20*c^3*d^2*e^5 - 105*b*
c^2*d*e^6 - 207*(b^2*c + a*c^2)*e^7)*m^4 - 5*(14*c^3*d^2*e^5 - 59*b*c^2*d*e
^6 - 185*(b^2*c + a*c^2)*e^7)*m^3 - 2*(50*c^3*d^2*e^5 - 187*b*c^2*d*e^6 - 1
072*(b^2*c + a*c^2)*e^7)*m^2 - 12*(4*c^3*d^2*e^5 - 14*b*c^2*d*e^6 - 201*(b^
2*c + a*c^2)*e^7)*m)*x^5 - (75*a^2*b*d^2*e^5 - 295*a^3*d*e^6 - 6*(a*b^2 + a
^2*c)*d^3*e^4)*m^4 + (1260*(b^3 + 6*a*b*c)*e^7 + (3*(b^2*c + a*c^2)*d*e^6 +
 (b^3 + 6*a*b*c)*e^7)*m^6 - 3*(5*b*c^2*d^2*e^5 - 19*(b^2*c + a*c^2)*d*e^6 -
 8*(b^3 + 6*a*b*c)*e^7)*m^5 + (30*c^3*d^3*e^4 - 195*b*c^2*d^2*e^5 + 393*(b^
2*c + a*c^2)*d*e^6 + 226*(b^3 + 6*a*b*c)*e^7)*m^4 + 3*(60*c^3*d^3*e^4 - 265
*b*c^2*d^2*e^5 + 401*(b^2*c + a*c^2)*d*e^6 + 352*(b^3 + 6*a*b*c)*e^7)*m^3 +
 5*(66*c^3*d^3*e^4 - 249*b*c^2*d^2*e^5 + 324*(b^2*c + a*c^2)*d*e^6 + 509*(b
^3 + 6*a*b*c)*e^7)*m^2 + 18*(10*c^3*d^3*e^4 - 35*b*c^2*d^2*e^5 + 42*(b^2*c
+ a*c^2)*d*e^6 + 164*(b^3 + 6*a*b*c)*e^7)*m)*x^4 - 3*(245*a^2*b*d^2*e^5 - 5
55*a^3*d*e^6 + 2*(b^3 + 6*a*b*c)*d^4*e^3 - 44*(a*b^2 + a^2*c)*d^3*e^4)*m^3
+ (5040*(a*b^2 + a^2*c)*e^7 + ((b^3 + 6*a*b*c)*d*e^6 + 3*(a*b^2 + a^2*c)*e^
7)*m^6 - 3*(4*(b^2*c + a*c^2)*d^2*e^5 - 7*(b^3 + 6*a*b*c)*d*e^6 - 25*(a*b^2
 + a^2*c)*e^7)*m^5 + (60*b*c^2*d^3*e^4 - 192*(b^2*c + a*c^2)*d^2*e^5 + 163*
(b^3 + 6*a*b*c)*d*e^6 + 741*(a*b^2 + a^2*c)*e^7)*m^4 - 3*(40*c^3*d^4*e^3 -
200*b*c^2*d^3*e^4 + 332*(b^2*c + a*c^2)*d^2*e^5 - 189*(b^3 + 6*a*b*c)*d*e^6
 - 1219*(a*b^2 + a^2*c)*e^7)*m^3 - 4*(90*c^3*d^4*e^3 - 345*b*c^2*d^3*e^4 +
456*(b^2*c + a*c^2)*d^2*e^5 - 211*(b^3 + 6*a*b*c)*d*e^6 - 2334*(a*b^2 + a^2
*c)*e^7)*m^2 - 12*(20*c^3*d^4*e^3 - 70*b*c^2*d^3*e^4 + 84*(b^2*c + a*c^2)*d
^2*e^5 - 35*(b^3 + 6*a*b*c)*d*e^6 - 949*(a*b^2 + a^2*c)*e^7)*m)*x^3 - (3525
*a^2*b*d^2*e^5 - 5104*a^3*d*e^6 - 72*(b^2*c + a*c^2)*d^5*e^2 + 108*(b^3 + 6
*a*b*c)*d^4*e^3 - 1074*(a*b^2 + a^2*c)*d^3*e^4)*m^2 + 3*(2520*a^2*b*e^7 + (
a^2*b*e^7 + (a*b^2 + a^2*c)*d*e^6)*m^6 + (26*a^2*b*e^7 - (b^3 + 6*a*b*c)*d^
2*e^5 + 23*(a*b^2 + a^2*c)*d*e^6)*m^5 + (270*a^2*b*e^7 + 12*(b^2*c + a*c^2)
*d^3*e^4 - 19*(b^3 + 6*a*b*c)*d^2*e^5 + 201*(a*b^2 + a^2*c)*d*e^6)*m^4 - (6
0*b*c^2*d^4*e^3 - 1420*a^2*b*e^7 - 168*(b^2*c + a*c^2)*d^3*e^4 + 125*(b^3 +
 6*a*b*c)*d^2*e^5 - 817*(a*b^2 + a^2*c)*d*e^6)*m^3 + (120*c^3*d^5*e^2 - 480
*b*c^2*d^4*e^3 + 3929*a^2*b*e^7 + 660*(b^2*c + a*c^2)*d^3*e^4 - 317*(b^3 +
6*a*b*c)*d^2*e^5 + 1478*(a*b^2 + a^2*c)*d*e^6)*m^2 + 6*(20*c^3*d^5*e^2 - 70
*b*c^2*d^4*e^3 + 879*a^2*b*e^7 + 84*(b^2*c + a*c^2)*d^3*e^4 - 35*(b^3 + 6*a
*b*c)*d^2*e^5 + 140*(a*b^2 + a^2*c)*d*e^6)*m)*x^2 - 6*(60*b*c^2*d^6*e + 137
7*a^2*b*d^2*e^5 - 1338*a^3*d*e^6 - 156*(b^2*c + a*c^2)*d^5*e^2 + 107*(b^3 +
 6*a*b*c)*d^4*e^3 - 638*(a*b^2 + a^2*c)*d^3*e^4)*m + (5040*a^3*e^7 + (3*a^2
*b*d*e^6 + a^3*e^7)*m^6 + 3*(25*a^2*b*d*e^6 + 9*a^3*e^7 - 2*(a*b^2 + a^2*c)
*d^2*e^5)*m^5 + (735*a^2*b*d*e^6 + 295*a^3*e^7 + 6*(b^3 + 6*a*b*c)*d^3*e^4
- 132*(a*b^2 + a^2*c)*d^2*e^5)*m^4 + 3*(1175*a^2*b*d*e^6 + 555*a^3*e^7 - 24
*(b^2*c + a*c^2)*d^4*e^3 + 36*(b^3 + 6*a*b*c)*d^3*e^4 - 358*(a*b^2 + a^2*c)
*d^2*e^5)*m^3 + 2*(180*b*c^2*d^5*e^2 + 4131*a^2*b*d*e^6 + 2552*a^3*e^7 - 46
8*(b^2*c + a*c^2)*d^4*e^3 + 321*(b^3 + 6*a*b*c)*d^3*e^4 - 1914*(a*b^2 + a^2
*c)*d^2*e^5)*m^2 - 36*(20*c^3*d^6*e - 70*b*c^2*d^5*e^2 - 210*a^2*b*d*e^6 -
223*a^3*e^7 + 84*(b^2*c + a*c^2)*d^4*e^3 - 35*(b^3 + 6*a*b*c)*d^3*e^4 + 140
*(a*b^2 + a^2*c)*d^2*e^5)*m)*x)*(e*x + d)^m/(e^7*m^7 + 28*e^7*m^6 + 322*e^7
*m^5 + 1960*e^7*m^4 + 6769*e^7*m^3 + 13132*e^7*m^2 + 13068*e^7*m + 5040*e^7
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.51168, size = 7272, normalized size = 23.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*c^3*m^6*x^7*e^7 + (x*e + d)^m*c^3*d*m^6*x^6*e^6 + 3*(x*e + d)^
m*b*c^2*m^6*x^6*e^7 + 21*(x*e + d)^m*c^3*m^5*x^7*e^7 + 3*(x*e + d)^m*b*c^2*
d*m^6*x^5*e^6 + 15*(x*e + d)^m*c^3*d*m^5*x^6*e^6 - 6*(x*e + d)^m*c^3*d^2*m^
5*x^5*e^5 + 3*(x*e + d)^m*b^2*c*m^6*x^5*e^7 + 3*(x*e + d)^m*a*c^2*m^6*x^5*e
^7 + 66*(x*e + d)^m*b*c^2*m^5*x^6*e^7 + 175*(x*e + d)^m*c^3*m^4*x^7*e^7 + 3
*(x*e + d)^m*b^2*c*d*m^6*x^4*e^6 + 3*(x*e + d)^m*a*c^2*d*m^6*x^4*e^6 + 51*(
x*e + d)^m*b*c^2*d*m^5*x^5*e^6 + 85*(x*e + d)^m*c^3*d*m^4*x^6*e^6 - 15*(x*e
 + d)^m*b*c^2*d^2*m^5*x^4*e^5 - 60*(x*e + d)^m*c^3*d^2*m^4*x^5*e^5 + 30*(x*
e + d)^m*c^3*d^3*m^4*x^4*e^4 + (x*e + d)^m*b^3*m^6*x^4*e^7 + 6*(x*e + d)^m*
a*b*c*m^6*x^4*e^7 + 69*(x*e + d)^m*b^2*c*m^5*x^5*e^7 + 69*(x*e + d)^m*a*c^2
*m^5*x^5*e^7 + 570*(x*e + d)^m*b*c^2*m^4*x^6*e^7 + 735*(x*e + d)^m*c^3*m^3*
x^7*e^7 + (x*e + d)^m*b^3*d*m^6*x^3*e^6 + 6*(x*e + d)^m*a*b*c*d*m^6*x^3*e^6
 + 57*(x*e + d)^m*b^2*c*d*m^5*x^4*e^6 + 57*(x*e + d)^m*a*c^2*d*m^5*x^4*e^6
+ 315*(x*e + d)^m*b*c^2*d*m^4*x^5*e^6 + 225*(x*e + d)^m*c^3*d*m^3*x^6*e^6 -
 12*(x*e + d)^m*b^2*c*d^2*m^5*x^3*e^5 - 12*(x*e + d)^m*a*c^2*d^2*m^5*x^3*e^
5 - 195*(x*e + d)^m*b*c^2*d^2*m^4*x^4*e^5 - 210*(x*e + d)^m*c^3*d^2*m^3*x^5
*e^5 + 60*(x*e + d)^m*b*c^2*d^3*m^4*x^3*e^4 + 180*(x*e + d)^m*c^3*d^3*m^3*x
^4*e^4 - 120*(x*e + d)^m*c^3*d^4*m^3*x^3*e^3 + 3*(x*e + d)^m*a*b^2*m^6*x^3*
e^7 + 3*(x*e + d)^m*a^2*c*m^6*x^3*e^7 + 24*(x*e + d)^m*b^3*m^5*x^4*e^7 + 14
4*(x*e + d)^m*a*b*c*m^5*x^4*e^7 + 621*(x*e + d)^m*b^2*c*m^4*x^5*e^7 + 621*(
x*e + d)^m*a*c^2*m^4*x^5*e^7 + 2460*(x*e + d)^m*b*c^2*m^3*x^6*e^7 + 1624*(x
*e + d)^m*c^3*m^2*x^7*e^7 + 3*(x*e + d)^m*a*b^2*d*m^6*x^2*e^6 + 3*(x*e + d)
^m*a^2*c*d*m^6*x^2*e^6 + 21*(x*e + d)^m*b^3*d*m^5*x^3*e^6 + 126*(x*e + d)^m
*a*b*c*d*m^5*x^3*e^6 + 393*(x*e + d)^m*b^2*c*d*m^4*x^4*e^6 + 393*(x*e + d)^
m*a*c^2*d*m^4*x^4*e^6 + 885*(x*e + d)^m*b*c^2*d*m^3*x^5*e^6 + 274*(x*e + d)
^m*c^3*d*m^2*x^6*e^6 - 3*(x*e + d)^m*b^3*d^2*m^5*x^2*e^5 - 18*(x*e + d)^m*a
*b*c*d^2*m^5*x^2*e^5 - 192*(x*e + d)^m*b^2*c*d^2*m^4*x^3*e^5 - 192*(x*e + d
)^m*a*c^2*d^2*m^4*x^3*e^5 - 795*(x*e + d)^m*b*c^2*d^2*m^3*x^4*e^5 - 300*(x*
e + d)^m*c^3*d^2*m^2*x^5*e^5 + 36*(x*e + d)^m*b^2*c*d^3*m^4*x^2*e^4 + 36*(x
*e + d)^m*a*c^2*d^3*m^4*x^2*e^4 + 600*(x*e + d)^m*b*c^2*d^3*m^3*x^3*e^4 + 3
30*(x*e + d)^m*c^3*d^3*m^2*x^4*e^4 - 180*(x*e + d)^m*b*c^2*d^4*m^3*x^2*e^3
- 360*(x*e + d)^m*c^3*d^4*m^2*x^3*e^3 + 360*(x*e + d)^m*c^3*d^5*m^2*x^2*e^2
 + 3*(x*e + d)^m*a^2*b*m^6*x^2*e^7 + 75*(x*e + d)^m*a*b^2*m^5*x^3*e^7 + 75*
(x*e + d)^m*a^2*c*m^5*x^3*e^7 + 226*(x*e + d)^m*b^3*m^4*x^4*e^7 + 1356*(x*e
 + d)^m*a*b*c*m^4*x^4*e^7 + 2775*(x*e + d)^m*b^2*c*m^3*x^5*e^7 + 2775*(x*e
+ d)^m*a*c^2*m^3*x^5*e^7 + 5547*(x*e + d)^m*b*c^2*m^2*x^6*e^7 + 1764*(x*e +
 d)^m*c^3*m*x^7*e^7 + 3*(x*e + d)^m*a^2*b*d*m^6*x*e^6 + 69*(x*e + d)^m*a*b^
2*d*m^5*x^2*e^6 + 69*(x*e + d)^m*a^2*c*d*m^5*x^2*e^6 + 163*(x*e + d)^m*b^3*
d*m^4*x^3*e^6 + 978*(x*e + d)^m*a*b*c*d*m^4*x^3*e^6 + 1203*(x*e + d)^m*b^2*
c*d*m^3*x^4*e^6 + 1203*(x*e + d)^m*a*c^2*d*m^3*x^4*e^6 + 1122*(x*e + d)^m*b
*c^2*d*m^2*x^5*e^6 + 120*(x*e + d)^m*c^3*d*m*x^6*e^6 - 6*(x*e + d)^m*a*b^2*
d^2*m^5*x*e^5 - 6*(x*e + d)^m*a^2*c*d^2*m^5*x*e^5 - 57*(x*e + d)^m*b^3*d^2*
m^4*x^2*e^5 - 342*(x*e + d)^m*a*b*c*d^2*m^4*x^2*e^5 - 996*(x*e + d)^m*b^2*c
*d^2*m^3*x^3*e^5 - 996*(x*e + d)^m*a*c^2*d^2*m^3*x^3*e^5 - 1245*(x*e + d)^m
*b*c^2*d^2*m^2*x^4*e^5 - 144*(x*e + d)^m*c^3*d^2*m*x^5*e^5 + 6*(x*e + d)^m*
b^3*d^3*m^4*x*e^4 + 36*(x*e + d)^m*a*b*c*d^3*m^4*x*e^4 + 504*(x*e + d)^m*b^
2*c*d^3*m^3*x^2*e^4 + 504*(x*e + d)^m*a*c^2*d^3*m^3*x^2*e^4 + 1380*(x*e + d
```

)^m*b*c^2*d^3*m^2*x^3*e^4 + 180*(x*e + d)^m*c^3*d^3*m*x^4*e^4 - 72*(x*e + d
)^m*b^2*c*d^4*m^3*x*e^3 - 72*(x*e + d)^m*a*c^2*d^4*m^3*x*e^3 - 1440*(x*e +
d)^m*b*c^2*d^4*m^2*x^2*e^3 - 240*(x*e + d)^m*c^3*d^4*m*x^3*e^3 + 360*(x*e +
d)^m*b*c^2*d^5*m^2*x*e^2 + 360*(x*e + d)^m*c^3*d^5*m*x^2*e^2 - 720*(x*e +
d)^m*c^3*d^6*m*x*e + (x*e + d)^m*a^3*m^6*x*e^7 + 78*(x*e + d)^m*a^2*b*m^5*x
^2*e^7 + 741*(x*e + d)^m*a*b^2*m^4*x^3*e^7 + 741*(x*e + d)^m*a^2*c*m^4*x^3*
e^7 + 1056*(x*e + d)^m*b^3*m^3*x^4*e^7 + 6336*(x*e + d)^m*a*b*c*m^3*x^4*e^7
+ 6432*(x*e + d)^m*b^2*c*m^2*x^5*e^7 + 6432*(x*e + d)^m*a*c^2*m^2*x^5*e^7
+ 6114*(x*e + d)^m*b*c^2*m*x^6*e^7 + 720*(x*e + d)^m*c^3*x^7*e^7 + (x*e + d
)^m*a^3*d*m^6*e^6 + 75*(x*e + d)^m*a^2*b*d*m^5*x*e^6 + 603*(x*e + d)^m*a*b^
2*d*m^4*x^2*e^6 + 603*(x*e + d)^m*a^2*c*d*m^4*x^2*e^6 + 567*(x*e + d)^m*b^3
*d*m^3*x^3*e^6 + 3402*(x*e + d)^m*a*b*c*d*m^3*x^3*e^6 + 1620*(x*e + d)^m*b^
2*c*d*m^2*x^4*e^6 + 1620*(x*e + d)^m*a*c^2*d*m^2*x^4*e^6 + 504*(x*e + d)^m*
b*c^2*d*m*x^5*e^6 - 3*(x*e + d)^m*a^2*b*d^2*m^5*e^5 - 132*(x*e + d)^m*a*b^2
*d^2*m^4*x*e^5 - 132*(x*e + d)^m*a^2*c*d^2*m^4*x*e^5 - 375*(x*e + d)^m*b^3*
d^2*m^3*x^2*e^5 - 2250*(x*e + d)^m*a*b*c*d^2*m^3*x^2*e^5 - 1824*(x*e + d)^m
*b^2*c*d^2*m^2*x^3*e^5 - 1824*(x*e + d)^m*a*c^2*d^2*m^2*x^3*e^5 - 630*(x*e
+ d)^m*b*c^2*d^2*m*x^4*e^5 + 6*(x*e + d)^m*a*b^2*d^3*m^4*e^4 + 6*(x*e + d)^
m*a^2*c*d^3*m^4*e^4 + 108*(x*e + d)^m*b^3*d^3*m^3*x*e^4 + 648*(x*e + d)^m*a
*b*c*d^3*m^3*x*e^4 + 1980*(x*e + d)^m*b^2*c*d^3*m^2*x^2*e^4 + 1980*(x*e + d
)^m*a*c^2*d^3*m^2*x^2*e^4 + 840*(x*e + d)^m*b*c^2*d^3*m*x^3*e^4 - 6*(x*e +
d)^m*b^3*d^4*m^3*e^3 - 36*(x*e + d)^m*a*b*c*d^4*m^3*e^3 - 936*(x*e + d)^m*b
^2*c*d^4*m^2*x*e^3 - 936*(x*e + d)^m*a*c^2*d^4*m^2*x*e^3 - 1260*(x*e + d)^m
*b*c^2*d^4*m*x^2*e^3 + 72*(x*e + d)^m*b^2*c*d^5*m^2*e^2 + 72*(x*e + d)^m*a*
c^2*d^5*m^2*e^2 + 2520*(x*e + d)^m*b*c^2*d^5*m*x*e^2 - 360*(x*e + d)^m*b*c^
2*d^6*m*e + 720*(x*e + d)^m*c^3*d^7 + 27*(x*e + d)^m*a^3*m^5*x*e^7 + 810*(x
*e + d)^m*a^2*b*m^4*x^2*e^7 + 3657*(x*e + d)^m*a*b^2*m^3*x^3*e^7 + 3657*(x*
e + d)^m*a^2*c*m^3*x^3*e^7 + 2545*(x*e + d)^m*b^3*m^2*x^4*e^7 + 15270*(x*e
+ d)^m*a*b*c*m^2*x^4*e^7 + 7236*(x*e + d)^m*b^2*c*m*x^5*e^7 + 7236*(x*e + d
)^m*a*c^2*m*x^5*e^7 + 2520*(x*e + d)^m*b*c^2*x^6*e^7 + 27*(x*e + d)^m*a^3*d
*m^5*e^6 + 735*(x*e + d)^m*a^2*b*d*m^4*x*e^6 + 2451*(x*e + d)^m*a*b^2*d*m^3
*x^2*e^6 + 2451*(x*e + d)^m*a^2*c*d*m^3*x^2*e^6 + 844*(x*e + d)^m*b^3*d*m^2
*x^3*e^6 + 5064*(x*e + d)^m*a*b*c*d*m^2*x^3*e^6 + 756*(x*e + d)^m*b^2*c*d*m
*x^4*e^6 + 756*(x*e + d)^m*a*c^2*d*m*x^4*e^6 - 75*(x*e + d)^m*a^2*b*d^2*m^4
e^5 - 1074(x*e + d)^m*a*b^2*d^2*m^3*x*e^5 - 1074*(x*e + d)^m*a^2*c*d^2*m^
3*x*e^5 - 951*(x*e + d)^m*b^3*d^2*m^2*x^2*e^5 - 5706*(x*e + d)^m*a*b*c*d^2*
m^2*x^2*e^5 - 1008*(x*e + d)^m*b^2*c*d^2*m*x^3*e^5 - 1008*(x*e + d)^m*a*c^2
*d^2*m*x^3*e^5 + 132*(x*e + d)^m*a*b^2*d^3*m^3*e^4 + 132*(x*e + d)^m*a^2*c*
d^3*m^3*e^4 + 642*(x*e + d)^m*b^3*d^3*m^2*x*e^4 + 3852*(x*e + d)^m*a*b*c*d^
3*m^2*x*e^4 + 1512*(x*e + d)^m*b^2*c*d^3*m*x^2*e^4 + 1512*(x*e + d)^m*a*c^2
*d^3*m*x^2*e^4 - 108*(x*e + d)^m*b^3*d^4*m^2*e^3 - 648*(x*e + d)^m*a*b*c*d^
4*m^2*e^3 - 3024*(x*e + d)^m*b^2*c*d^4*m*x*e^3 - 3024*(x*e + d)^m*a*c^2*d^4
*m*x*e^3 + 936*(x*e + d)^m*b^2*c*d^5*m*e^2 + 936*(x*e + d)^m*a*c^2*d^5*m*e^
2 - 2520*(x*e + d)^m*b*c^2*d^6*e + 295*(x*e + d)^m*a^3*m^4*x*e^7 + 4260*(x*
e + d)^m*a^2*b*m^3*x^2*e^7 + 9336*(x*e + d)^m*a*b^2*m^2*x^3*e^7 + 9336*(x*e
+ d)^m*a^2*c*m^2*x^3*e^7 + 2952*(x*e + d)^m*b^3*m*x^4*e^7 + 17712*(x*e + d
)^m*a*b*c*m*x^4*e^7 + 3024*(x*e + d)^m*b^2*c*x^5*e^7 + 3024*(x*e + d)^m*a*c
^2*x^5*e^7 + 295*(x*e + d)^m*a^3*d*m^4*e^6 + 3525*(x*e + d)^m*a^2*b*d*m^3*x
e^6 + 4434(x*e + d)^m*a*b^2*d*m^2*x^2*e^6 + 4434*(x*e + d)^m*a^2*c*d*m^2*
x^2*e^6 + 420*(x*e + d)^m*b^3*d*m*x^3*e^6 + 2520*(x*e + d)^m*a*b*c*d*m*x^3*
e^6 - 735*(x*e + d)^m*a^2*b*d^2*m^3*e^5 - 3828*(x*e + d)^m*a*b^2*d^2*m^2*x*
e^5 - 3828*(x*e + d)^m*a^2*c*d^2*m^2*x*e^5 - 630*(x*e + d)^m*b^3*d^2*m*x^2*
e^5 - 3780*(x*e + d)^m*a*b*c*d^2*m*x^2*e^5 + 1074*(x*e + d)^m*a*b^2*d^3*m^2
e^4 + 1074(x*e + d)^m*a^2*c*d^3*m^2*e^4 + 1260*(x*e + d)^m*b^3*d^3*m*x*e^
4 + 7560*(x*e + d)^m*a*b*c*d^3*m*x*e^4 - 642*(x*e + d)^m*b^3*d^4*m*e^3 - 38
52*(x*e + d)^m*a*b*c*d^4*m*e^3 + 3024*(x*e + d)^m*b^2*c*d^5*e^2 + 3024*(x*e
+ d)^m*a*c^2*d^5*e^2 + 1665*(x*e + d)^m*a^3*m^3*x*e^7 + 11787*(x*e + d)^m*
a^2*b*m^2*x^2*e^7 + 11388*(x*e + d)^m*a*b^2*m*x^3*e^7 + 11388*(x*e + d)^m*a
^2*c*m*x^3*e^7 + 1260*(x*e + d)^m*b^3*x^4*e^7 + 7560*(x*e + d)^m*a*b*c*x^4*

```
e^7 + 1665*(x*e + d)^m*a^3*d*m^3*e^6 + 8262*(x*e + d)^m*a^2*b*d*m^2*x*e^6 +
 2520*(x*e + d)^m*a*b^2*d*m*x^2*e^6 + 2520*(x*e + d)^m*a^2*c*d*m*x^2*e^6 -
3525*(x*e + d)^m*a^2*b*d^2*m^2*e^5 - 5040*(x*e + d)^m*a*b^2*d^2*m*x*e^5 - 5
040*(x*e + d)^m*a^2*c*d^2*m*x*e^5 + 3828*(x*e + d)^m*a*b^2*d^3*m*e^4 + 3828
*(x*e + d)^m*a^2*c*d^3*m*e^4 - 1260*(x*e + d)^m*b^3*d^4*e^3 - 7560*(x*e + d
)^m*a*b*c*d^4*e^3 + 5104*(x*e + d)^m*a^3*m^2*x*e^7 + 15822*(x*e + d)^m*a^2*
b*m*x^2*e^7 + 5040*(x*e + d)^m*a*b^2*x^3*e^7 + 5040*(x*e + d)^m*a^2*c*x^3*e
^7 + 5104*(x*e + d)^m*a^3*d*m^2*e^6 + 7560*(x*e + d)^m*a^2*b*d*m*x*e^6 - 82
62*(x*e + d)^m*a^2*b*d^2*m*e^5 + 5040*(x*e + d)^m*a*b^2*d^3*e^4 + 5040*(x*e
 + d)^m*a^2*c*d^3*e^4 + 8028*(x*e + d)^m*a^3*m*x*e^7 + 7560*(x*e + d)^m*a^2
*b*x^2*e^7 + 8028*(x*e + d)^m*a^3*d*m*e^6 - 7560*(x*e + d)^m*a^2*b*d^2*e^5
+ 5040*(x*e + d)^m*a^3*x*e^7 + 5040*(x*e + d)^m*a^3*d*e^6)/(m^7*e^7 + 28*m^
6*e^7 + 322*m^5*e^7 + 1960*m^4*e^7 + 6769*m^3*e^7 + 13132*m^2*e^7 + 13068*m
*e^7 + 5040*e^7)
```

3.2551 $\int (d+ex)^m \left(a+bx+cx^2\right)^2 \, dx$

Optimal. Leaf size=178

$$\frac{(d+ex)^{m+3}\left(-2ce(3bd-ae)+b^2e^2+6c^2d^2\right)}{e^5(m+3)}+\frac{(d+ex)^{m+1}\left(ae^2-bde+cd^2\right)^2}{e^5(m+1)}-\frac{2(2cd-be)(d+ex)^{m+2}\left(ae^2-bde+cd^2\right)}{e^5(m+2)}$$

```
[Out] ((c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^(1 + m))/(e^5*(1 + m)) - (2*(2*c*d - b
*e)*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(2 + m))/(e^5*(2 + m)) + ((6*c^2*d^2
+ b^2*e^2 - 2*c*e*(3*b*d - a*e))*(d + e*x)^(3 + m))/(e^5*(3 + m)) - (2*c*(2
*c*d - b*e)*(d + e*x)^(4 + m))/(e^5*(4 + m)) + (c^2*(d + e*x)^(5 + m))/(e^5
*(5 + m))
```

Rubi [A] time = 0.113784, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$\frac{(d+ex)^{m+3}\left(-2ce(3bd-ae)+b^2e^2+6c^2d^2\right)}{e^5(m+3)}+\frac{(d+ex)^{m+1}\left(ae^2-bde+cd^2\right)^2}{e^5(m+1)}-\frac{2(2cd-be)(d+ex)^{m+2}\left(ae^2-bde+cd^2\right)}{e^5(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^(1 + m))/(e^5*(1 + m)) - (2*(2*c*d - b
*e)*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(2 + m))/(e^5*(2 + m)) + ((6*c^2*d^2
+ b^2*e^2 - 2*c*e*(3*b*d - a*e))*(d + e*x)^(3 + m))/(e^5*(3 + m)) - (2*c*(2
*c*d - b*e)*(d + e*x)^(4 + m))/(e^5*(4 + m)) + (c^2*(d + e*x)^(5 + m))/(e^5
*(5 + m))
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int (d+ex)^m \left(a+bx+cx^2\right)^2 dx &= \int \left(\frac{\left(cd^2-bde+ae^2\right)^2 (d+ex)^m}{e^4}+\frac{2(-2cd+be)\left(cd^2-bde+ae^2\right)(d+ex)^{1+m}}{e^4}+\frac{(6c^2d\right. \\ &= \frac{\left(cd^2-bde+ae^2\right)^2 (d+ex)^{1+m}}{e^5(1+m)}-\frac{2(2cd-be)\left(cd^2-bde+ae^2\right)(d+ex)^{2+m}}{e^5(2+m)}+\frac{(6c^2d^2+\end{aligned}$$

Mathematica [A] time = 0.399677, size = 178, normalized size = 1.

$$\frac{(d+ex)^{m+1}\left(\frac{2(d+ex)\left(\frac{6(2cd-be)\left(e(ae-bd)+cd^2\right)}{m+2}-\frac{(d+ex)\left(4ce(ae(m+4)-3bd)-b^2e^2(m+1)+12c^2d^2\right)}{m+3}\right)}{e^4(m+4)(m+5)}-\frac{2(d+ex)(a+x(b+cx))(be(m+7)-6cd+2ce(m+4)x)}{e^2(m+4)(m+5)}+(a+x(b+\right.}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + m)*((-2*(d + e*x)*(-6*c*d + b*e*(7 + m) + 2*c*e*(4 + m)*x)*
(a + x*(b + c*x)))/(e^2*(4 + m)*(5 + m)) + (a + x*(b + c*x))^2 + (2*(d + e*
x)*((6*(2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e)))/(2 + m) - ((12*c^2*d^2 - b
^2*e^2*(1 + m) + 4*c*e*(-3*b*d + a*e*(4 + m)))*(d + e*x))/(3 + m)))/(e^4*(4
 + m)*(5 + m))))/(e*(1 + m))
```

Maple [B] time = 0.052, size = 822, normalized size = 4.6

$$\frac{(ex+d)^{1+m}\left(c^2e^4m^4x^4+2\,bce^4m^4x^3+10\,c^2e^4m^3x^4+2\,ace^4m^4x^2+b^2e^4m^4x^2+22\,bce^4m^3x^3-4\,c^2de^3m^3x^3+35\,c^2e^4m\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^2,x)
```

```
[Out] (e*x+d)^(1+m)*(c^2*e^4*m^4*x^4+2*b*c*e^4*m^4*x^3+10*c^2*e^4*m^3*x^4+2*a*c*e
^4*m^4*x^2+b^2*e^4*m^4*x^2+22*b*c*e^4*m^3*x^3-4*c^2*d*e^3*m^3*x^3+35*c^2*e^
4*m^2*x^4+2*a*b*e^4*m^4*x+24*a*c*e^4*m^3*x^2+12*b^2*e^4*m^3*x^2-6*b*c*d*e^3
*m^3*x^2+82*b*c*e^4*m^2*x^3-24*c^2*d*e^3*m^2*x^3+50*c^2*e^4*m*x^4+a^2*e^4*m
^4+26*a*b*e^4*m^3*x-4*a*c*d*e^3*m^3*x+98*a*c*e^4*m^2*x^2-2*b^2*d*e^3*m^3*x+
49*b^2*e^4*m^2*x^2-48*b*c*d*e^3*m^2*x^2+122*b*c*e^4*m*x^3+12*c^2*d^2*e^2*m^
2*x^2-44*c^2*d*e^3*m*x^3+24*c^2*e^4*x^4+14*a^2*e^4*m^3-2*a*b*d*e^3*m^3+118*
a*b*e^4*m^2*x-40*a*c*d*e^3*m^2*x+156*a*c*e^4*m*x^2-20*b^2*d*e^3*m^2*x+78*b^
2*e^4*m*x^2+12*b*c*d^2*e^2*m^2*x-102*b*c*d*e^3*m*x^2+60*b*c*e^4*x^3+36*c^2*
d^2*e^2*m*x^2-24*c^2*d*e^3*x^3+71*a^2*e^4*m^2-24*a*b*d*e^3*m^2+214*a*b*e^4*
m*x+4*a*c*d^2*e^2*m^2-116*a*c*d*e^3*m*x+80*a*c*e^4*x^2+2*b^2*d^2*e^2*m^2-58
*b^2*d*e^3*m*x+40*b^2*e^4*x^2+72*b*c*d^2*e^2*m*x-60*b*c*d*e^3*x^2-24*c^2*d^
3*e*m*x+24*c^2*d^2*e^2*x^2+154*a^2*e^4*m-94*a*b*d*e^3*m+120*a*b*e^4*x+36*a*
c*d^2*e^2*m-80*a*c*d*e^3*x+18*b^2*d^2*e^2*m-40*b^2*d*e^3*x-12*b*c*d^3*e*m+6
0*b*c*d^2*e^2*x-24*c^2*d^3*e*x+120*a^2*e^4-120*a*b*d*e^3+80*a*c*d^2*e^2+40*
b^2*d^2*e^2-60*b*c*d^3*e+24*c^2*d^4)/e^5/(m^5+15*m^4+85*m^3+225*m^2+274*m+1
20)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.75326, size = 1897, normalized size = 10.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (a^2*d*e^4*m^4 + 24*c^2*d^5 - 60*b*c*d^4*e - 120*a*b*d^2*e^3 + 120*a^2*d*e^
4 + 40*(b^2 + 2*a*c)*d^3*e^2 + (c^2*e^5*m^4 + 10*c^2*e^5*m^3 + 35*c^2*e^5*m
^2 + 50*c^2*e^5*m + 24*c^2*e^5)*x^5 + (60*b*c*e^5 + (c^2*d*e^4 + 2*b*c*e^5)
*m^4 + 2*(3*c^2*d*e^4 + 11*b*c*e^5)*m^3 + (11*c^2*d*e^4 + 82*b*c*e^5)*m^2 +
 2*(3*c^2*d*e^4 + 61*b*c*e^5)*m)*x^4 - 2*(a*b*d^2*e^3 - 7*a^2*d*e^4)*m^3 +
(40*(b^2 + 2*a*c)*e^5 + (2*b*c*d*e^4 + (b^2 + 2*a*c)*e^5)*m^4 - 4*(c^2*d^2*
e^3 - 4*b*c*d*e^4 - 3*(b^2 + 2*a*c)*e^5)*m^3 - (12*c^2*d^2*e^3 - 34*b*c*d*e
^4 - 49*(b^2 + 2*a*c)*e^5)*m^2 - 2*(4*c^2*d^2*e^3 - 10*b*c*d*e^4 - 39*(b^2
+ 2*a*c)*e^5)*m)*x^3 - (24*a*b*d^2*e^3 - 71*a^2*d*e^4 - 2*(b^2 + 2*a*c)*d^3
*e^2)*m^2 + (120*a*b*e^5 + (2*a*b*e^5 + (b^2 + 2*a*c)*d*e^4)*m^4 - 2*(3*b*c
*d^2*e^3 - 13*a*b*e^5 - 5*(b^2 + 2*a*c)*d*e^4)*m^3 + (12*c^2*d^3*e^2 - 36*b
*c*d^2*e^3 + 118*a*b*e^5 + 29*(b^2 + 2*a*c)*d*e^4)*m^2 + 2*(6*c^2*d^3*e^2 -
 15*b*c*d^2*e^3 + 107*a*b*e^5 + 10*(b^2 + 2*a*c)*d*e^4)*m)*x^2 - 2*(6*b*c*d
^4*e + 47*a*b*d^2*e^3 - 77*a^2*d*e^4 - 9*(b^2 + 2*a*c)*d^3*e^2)*m + (120*a^
2*e^5 + (2*a*b*d*e^4 + a^2*e^5)*m^4 + 2*(12*a*b*d*e^4 + 7*a^2*e^5 - (b^2 +
2*a*c)*d^2*e^3)*m^3 + (12*b*c*d^3*e^2 + 94*a*b*d*e^4 + 71*a^2*e^5 - 18*(b^2
 + 2*a*c)*d^2*e^3)*m^2 - 2*(12*c^2*d^4*e - 30*b*c*d^3*e^2 - 60*a*b*d*e^4 -
77*a^2*e^5 + 20*(b^2 + 2*a*c)*d^2*e^3)*m)*x)*(e*x + d)^m/(e^5*m^5 + 15*e^5*
m^4 + 85*e^5*m^3 + 225*e^5*m^2 + 274*e^5*m + 120*e^5)
```

Sympy [A] time = 11.1325, size = 9853, normalized size = 55.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**2,x)
```

```
[Out] Piecewise((d**m*(a**2*x + a*b*x**2 + 2*a*c*x**3/3 + b**2*x**3/3 + b*c*x**4/
2 + c**2*x**5/5), Eq(e, 0)), (-3*a**2*d**2*e**4/(12*d**6*e**5 + 48*d**5*e**
6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4) - 2*a*b*d*
*3*e**3/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x
**3 + 12*d**2*e**9*x**4) - 8*a*b*d**2*e**4*x/(12*d**6*e**5 + 48*d**5*e**6*x
 + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4) + 8*a*c*d*e**
5*x**3/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x*
*3 + 12*d**2*e**9*x**4) + 2*a*c*e**6*x**4/(12*d**6*e**5 + 48*d**5*e**6*x +
72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4) + 4*b**2*d*e**5*
x**3/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3
 + 12*d**2*e**9*x**4) + b**2*e**6*x**4/(12*d**6*e**5 + 48*d**5*e**6*x + 72*
d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4) + 6*b*c*d*e**5*x**4
/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 1
2*d**2*e**9*x**4) + 12*c**2*d**6*log(d/e + x)/(12*d**6*e**5 + 48*d**5*e**6*
x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4) + 7*c**2*d**
6/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 +
12*d**2*e**9*x**4) + 48*c**2*d**5*e*x*log(d/e + x)/(12*d**6*e**5 + 48*d**5*
e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4) + 16*c*
*2*d**5*e*x/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e*
*8*x**3 + 12*d**2*e**9*x**4) + 72*c**2*d**4*e**2*x**2*log(d/e + x)/(12*d**6
*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e*
*9*x**4) + 48*c**2*d**3*e**3*x**3*log(d/e + x)/(12*d**6*e**5 + 48*d**5*e**6
*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4) - 24*c**2*d
**3*e**3*x**3/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*
e**8*x**3 + 12*d**2*e**9*x**4) + 12*c**2*d**2*e**4*x**4*log(d/e + x)/(12*d*
*6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*
e**9*x**4) - 18*c**2*d**2*e**4*x**4/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**
```

```
4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4), Eq(m, -5)), (-a**2*d*
e**4/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3) - a*b
*d**2*e**3/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3)
 - 3*a*b*d*e**4*x/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**
8*x**3) + 2*a*c*e**5*x**3/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 +
 3*d*e**8*x**3) + b**2*e**5*x**3/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7
*x**2 + 3*d*e**8*x**3) + 6*b*c*d**4*e*log(d/e + x)/(3*d**4*e**5 + 9*d**3*e*
*6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3) + 5*b*c*d**4*e/(3*d**4*e**5 + 9*d*
*3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3) + 18*b*c*d**3*e**2*x*log(d/e
+ x)/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3) + 9*b
*c*d**3*e**2*x/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x
**3) + 18*b*c*d**2*e**3*x**2*log(d/e + x)/(3*d**4*e**5 + 9*d**3*e**6*x + 9*
d**2*e**7*x**2 + 3*d*e**8*x**3) + 6*b*c*d*e**4*x**3*log(d/e + x)/(3*d**4*e*
*5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3) - 6*b*c*d*e**4*x**3/
(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3) - 12*c**2*
d**5*log(d/e + x)/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**
8*x**3) - 10*c**2*d**5/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*
d*e**8*x**3) - 36*c**2*d**4*e*x*log(d/e + x)/(3*d**4*e**5 + 9*d**3*e**6*x +
 9*d**2*e**7*x**2 + 3*d*e**8*x**3) - 18*c**2*d**4*e*x/(3*d**4*e**5 + 9*d**3
*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3) - 36*c**2*d**3*e**2*x**2*log(d/
e + x)/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3) - 1
2*c**2*d**2*e**3*x**3*log(d/e + x)/(3*d**4*e**5 + 9*d**3*e**6*x + 9*d**2*e*
*7*x**2 + 3*d*e**8*x**3) + 12*c**2*d**2*e**3*x**3/(3*d**4*e**5 + 9*d**3*e**
6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3) + 3*c**2*d*e**4*x**4/(3*d**4*e**5 +
 9*d**3*e**6*x + 9*d**2*e**7*x**2 + 3*d*e**8*x**3), Eq(m, -4)), (-a**2*e**4
/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 2*a*b*d*e**3/(2*d**2*e**5 + 4*d
*e**6*x + 2*e**7*x**2) - 4*a*b*e**4*x/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x*
*2) + 4*a*c*d**2*e**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2)
 + 6*a*c*d**2*e**2/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 8*a*c*d*e**3*
x*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 8*a*c*d*e**3*x/(2
*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 4*a*c*e**4*x**2*log(d/e + x)/(2*d*
*2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 2*b**2*d**2*e**2*log(d/e + x)/(2*d**2
*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 3*b**2*d**2*e**2/(2*d**2*e**5 + 4*d*e**
6*x + 2*e**7*x**2) + 4*b**2*d*e**3*x*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x
 + 2*e**7*x**2) + 4*b**2*d*e**3*x/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2)
+ 2*b**2*e**4*x**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) -
12*b*c*d**3*e*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 18*b*
c*d**3*e/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 24*b*c*d**2*e**2*x*log(
d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 24*b*c*d**2*e**2*x/(2*d
**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 12*b*c*d*e**3*x**2*log(d/e + x)/(2*d
**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 4*b*c*e**4*x**3/(2*d**2*e**5 + 4*d*e
**6*x + 2*e**7*x**2) + 12*c**2*d**4*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x
+ 2*e**7*x**2) + 18*c**2*d**4/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 24
*c**2*d**3*e*x*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 24*c
**2*d**3*e*x/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 12*c**2*d**2*e**2*x
**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 4*c**2*d*e**3*x
**3/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + c**2*e**4*x**4/(2*d**2*e**5
+ 4*d*e**6*x + 2*e**7*x**2), Eq(m, -3)), (-3*a**2*e**4/(3*d*e**5 + 3*e**6*x
) + 6*a*b*d*e**3*log(d/e + x)/(3*d*e**5 + 3*e**6*x) + 6*a*b*d*e**3/(3*d*e**
5 + 3*e**6*x) + 6*a*b*e**4*x*log(d/e + x)/(3*d*e**5 + 3*e**6*x) - 12*a*c*d*
*2*e**2*log(d/e + x)/(3*d*e**5 + 3*e**6*x) - 12*a*c*d**2*e**2/(3*d*e**5 + 3
*e**6*x) - 12*a*c*d*e**3*x*log(d/e + x)/(3*d*e**5 + 3*e**6*x) + 6*a*c*e**4*
x**2/(3*d*e**5 + 3*e**6*x) - 6*b**2*d**2*e**2*log(d/e + x)/(3*d*e**5 + 3*e*
*6*x) - 6*b**2*d**2*e**2/(3*d*e**5 + 3*e**6*x) - 6*b**2*d*e**3*x*log(d/e +
x)/(3*d*e**5 + 3*e**6*x) + 3*b**2*e**4*x**2/(3*d*e**5 + 3*e**6*x) + 18*b*c*
d**3*e*log(d/e + x)/(3*d*e**5 + 3*e**6*x) + 18*b*c*d**3*e/(3*d*e**5 + 3*e**
6*x) + 18*b*c*d**2*e**2*x*log(d/e + x)/(3*d*e**5 + 3*e**6*x) - 9*b*c*d*e**3
*x**2/(3*d*e**5 + 3*e**6*x) + 3*b*c*e**4*x**3/(3*d*e**5 + 3*e**6*x) - 12*c*
*2*d**4*log(d/e + x)/(3*d*e**5 + 3*e**6*x) - 12*c**2*d**4/(3*d*e**5 + 3*e**
```

```
6*x) - 12*c**2*d**3*e*x*log(d/e + x)/(3*d*e**5 + 3*e**6*x) + 6*c**2*d**2*e*
*2*x**2/(3*d*e**5 + 3*e**6*x) - 2*c**2*d*e**3*x**3/(3*d*e**5 + 3*e**6*x) +
c**2*e**4*x**4/(3*d*e**5 + 3*e**6*x), Eq(m, -2)), (a**2*log(d/e + x)/e - 2*
a*b*d*log(d/e + x)/e**2 + 2*a*b*x/e + 2*a*c*d**2*log(d/e + x)/e**3 - 2*a*c*
d*x/e**2 + a*c*x**2/e + b**2*d**2*log(d/e + x)/e**3 - b**2*d*x/e**2 + b**2*
x**2/(2*e) - 2*b*c*d**3*log(d/e + x)/e**4 + 2*b*c*d**2*x/e**3 - b*c*d*x**2/
e**2 + 2*b*c*x**3/(3*e) + c**2*d**4*log(d/e + x)/e**5 - c**2*d**3*x/e**4 +
c**2*d**2*x**2/(2*e**3) - c**2*d*x**3/(3*e**2) + c**2*x**4/(4*e), Eq(m, -1)
), (a**2*d*e**4*m**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3
+ 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 14*a**2*d*e**4*m**3*(d + e*x)**m
/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 12
0*e**5) + 71*a**2*d*e**4*m**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e
**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 154*a**2*d*e**4*m*(d +
e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5
*m + 120*e**5) + 120*a**2*d*e**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 8
5*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + a**2*e**5*m**4*x*(d
+ e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e*
*5*m + 120*e**5) + 14*a**2*e**5*m**3*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m*
*4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 71*a**2*e**5*m
**2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2
 + 274*e**5*m + 120*e**5) + 154*a**2*e**5*m*x*(d + e*x)**m/(e**5*m**5 + 15*
e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 120*a**
2*e**5*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m
**2 + 274*e**5*m + 120*e**5) - 2*a*b*d**2*e**3*m**3*(d + e*x)**m/(e**5*m**5
 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 2
4*a*b*d**2*e**3*m**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3
+ 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 94*a*b*d**2*e**3*m*(d + e*x)**m/
(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120
*e**5) - 120*a*b*d**2*e**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5
*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 2*a*b*d*e**4*m**4*x*(d + e
*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*
m + 120*e**5) + 24*a*b*d*e**4*m**3*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4
 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 94*a*b*d*e**4*m*
*2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2
+ 274*e**5*m + 120*e**5) + 120*a*b*d*e**4*m*x*(d + e*x)**m/(e**5*m**5 + 15*
e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 2*a*b*e
**5*m**4*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e
**5*m**2 + 274*e**5*m + 120*e**5) + 26*a*b*e**5*m**3*x**2*(d + e*x)**m/(e**
5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**
5) + 118*a*b*e**5*m**2*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**
5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 214*a*b*e**5*m*x**2*(d +
e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5
*m + 120*e**5) + 120*a*b*e**5*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 +
 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 4*a*c*d**3*e**2*m*
*2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 +
274*e**5*m + 120*e**5) + 36*a*c*d**3*e**2*m*(d + e*x)**m/(e**5*m**5 + 15*e*
*5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 80*a*c*d*
*3*e**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m*
*2 + 274*e**5*m + 120*e**5) - 4*a*c*d**2*e**3*m**3*x*(d + e*x)**m/(e**5*m**
5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) -
36*a*c*d**2*e**3*m**2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m*
*3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 80*a*c*d**2*e**3*m*x*(d + e*x
)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m
+ 120*e**5) + 2*a*c*d*e**4*m**4*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4
 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 20*a*c*d*e**4*m*
*3*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m*
*2 + 274*e**5*m + 120*e**5) + 58*a*c*d*e**4*m**2*x**2*(d + e*x)**m/(e**5*m*
*5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) +
 40*a*c*d*e**4*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3
```

+ 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 2*a*c*e**5*m**4*x**3*(d + e*x)*
*m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m +
120*e**5) + 24*a*c*e**5*m**3*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 +
85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 98*a*c*e**5*m**2*x*
3(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 +
274*e**5*m + 120*e**5) + 156*a*c*e**5*m*x**3*(d + e*x)**m/(e**5*m**5 + 15*e
5*m4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 80*a*c*e
5*x3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m
2 + 274*e5*m + 120*e**5) + 2*b**2*d**3*e**2*m**2*(d + e*x)**m/(e**5*m**
5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) +
18*b**2*d**3*e**2*m*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 +
225*e**5*m**2 + 274*e**5*m + 120*e**5) + 40*b**2*d**3*e**2*(d + e*x)**m/(e
5*m5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e
5) - 2*b2*d**2*e**3*m**3*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*
e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 18*b**2*d**2*e**3*m**2
x(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 +
274*e**5*m + 120*e**5) - 40*b**2*d**2*e**3*m*x*(d + e*x)**m/(e**5*m**5 + 15
*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + b**2*d
*e**4*m**4*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225
*e**5*m**2 + 274*e**5*m + 120*e**5) + 10*b**2*d*e**4*m**3*x**2*(d + e*x)**m
/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 12
0*e**5) + 29*b**2*d*e**4*m**2*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 +
85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 20*b**2*d*e**4*m*x
2*(d + e*x)m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 +
274*e**5*m + 120*e**5) + b**2*e**5*m**4*x**3*(d + e*x)**m/(e**5*m**5 + 15*
e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 12*b**2
*e**5*m**3*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225
*e**5*m**2 + 274*e**5*m + 120*e**5) + 49*b**2*e**5*m**2*x**3*(d + e*x)**m/(
e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*
e**5) + 78*b**2*e**5*m*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**
5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 40*b**2*e**5*x**3*(d + e*
x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m
+ 120*e**5) - 12*b*c*d**4*e*m*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*
e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 60*b*c*d**4*e*(d + e*x
)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m
+ 120*e**5) + 12*b*c*d**3*e**2*m**2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**
4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 60*b*c*d**3*e**
2*m*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2
+ 274*e**5*m + 120*e**5) - 6*b*c*d**2*e**3*m**3*x**2*(d + e*x)**m/(e**5*m*
*5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) -
36*b*c*d**2*e**3*m**2*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**
5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 30*b*c*d**2*e**3*m*x**2*(
d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*
e**5*m + 120*e**5) + 2*b*c*d*e**4*m**4*x**3*(d + e*x)**m/(e**5*m**5 + 15*e*
*5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 16*b*c*d*
e**4*m**3*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*
e**5*m**2 + 274*e**5*m + 120*e**5) + 34*b*c*d*e**4*m**2*x**3*(d + e*x)**m/(
e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*
e**5) + 20*b*c*d*e**4*m*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e*
*5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 2*b*c*e**5*m**4*x**4*(d
+ e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e*
*5*m + 120*e**5) + 22*b*c*e**5*m**3*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*
m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 82*b*c*e**5*
m**2*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*
m**2 + 274*e**5*m + 120*e**5) + 122*b*c*e**5*m*x**4*(d + e*x)**m/(e**5*m**5
+ 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 6
0*b*c*e**5*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225
*e**5*m**2 + 274*e**5*m + 120*e**5) + 24*c**2*d**5*(d + e*x)**m/(e**5*m**5
+ 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 24

```
*c**2*d**4*e*m*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 22
5*e**5*m**2 + 274*e**5*m + 120*e**5) + 12*c**2*d**3*e**2*m**2*x**2*(d + e*x
)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m
+ 120*e**5) + 12*c**2*d**3*e**2*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m*
*4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 4*c**2*d**2*e*
*3*m**3*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e*
*5*m**2 + 274*e**5*m + 120*e**5) - 12*c**2*d**2*e**3*m**2*x**3*(d + e*x)**m
/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 12
0*e**5) - 8*c**2*d**2*e**3*m*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 +
85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + c**2*d*e**4*m**4*x*
*4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 +
274*e**5*m + 120*e**5) + 6*c**2*d*e**4*m**3*x**4*(d + e*x)**m/(e**5*m**5 +
15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 11*c
**2*d*e**4*m**2*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3
+ 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 6*c**2*d*e**4*m*x**4*(d + e*x)**
m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 1
20*e**5) + c**2*e**5*m**4*x**5*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*
e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 10*c**2*e**5*m**3*x**5
*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 27
4*e**5*m + 120*e**5) + 35*c**2*e**5*m**2*x**5*(d + e*x)**m/(e**5*m**5 + 15*
e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 50*c**2
*e**5*m*x**5*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e*
*5*m**2 + 274*e**5*m + 120*e**5) + 24*c**2*e**5*x**5*(d + e*x)**m/(e**5*m**
5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5), T
rue))
```

Giac [B] time = 1.25513, size = 2318, normalized size = 13.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*c^2*m^4*x^5*e^5 + (x*e + d)^m*c^2*d*m^4*x^4*e^4 + 2*(x*e + d)^
m*b*c*m^4*x^4*e^5 + 10*(x*e + d)^m*c^2*m^3*x^5*e^5 + 2*(x*e + d)^m*b*c*d*m^
4*x^3*e^4 + 6*(x*e + d)^m*c^2*d*m^3*x^4*e^4 - 4*(x*e + d)^m*c^2*d^2*m^3*x^3
*e^3 + (x*e + d)^m*b^2*m^4*x^3*e^5 + 2*(x*e + d)^m*a*c*m^4*x^3*e^5 + 22*(x*
e + d)^m*b*c*m^3*x^4*e^5 + 35*(x*e + d)^m*c^2*m^2*x^5*e^5 + (x*e + d)^m*b^2
*d*m^4*x^2*e^4 + 2*(x*e + d)^m*a*c*d*m^4*x^2*e^4 + 16*(x*e + d)^m*b*c*d*m^3
*x^3*e^4 + 11*(x*e + d)^m*c^2*d*m^2*x^4*e^4 - 6*(x*e + d)^m*b*c*d^2*m^3*x^2
*e^3 - 12*(x*e + d)^m*c^2*d^2*m^2*x^3*e^3 + 12*(x*e + d)^m*c^2*d^3*m^2*x^2*
e^2 + 2*(x*e + d)^m*a*b*m^4*x^2*e^5 + 12*(x*e + d)^m*b^2*m^3*x^3*e^5 + 24*(
x*e + d)^m*a*c*m^3*x^3*e^5 + 82*(x*e + d)^m*b*c*m^2*x^4*e^5 + 50*(x*e + d)^
m*c^2*m*x^5*e^5 + 2*(x*e + d)^m*a*b*d*m^4*x*e^4 + 10*(x*e + d)^m*b^2*d*m^3*
x^2*e^4 + 20*(x*e + d)^m*a*c*d*m^3*x^2*e^4 + 34*(x*e + d)^m*b*c*d*m^2*x^3*e
^4 + 6*(x*e + d)^m*c^2*d*m*x^4*e^4 - 2*(x*e + d)^m*b^2*d^2*m^3*x*e^3 - 4*(x
*e + d)^m*a*c*d^2*m^3*x*e^3 - 36*(x*e + d)^m*b*c*d^2*m^2*x^2*e^3 - 8*(x*e +
 d)^m*c^2*d^2*m*x^3*e^3 + 12*(x*e + d)^m*b*c*d^3*m^2*x*e^2 + 12*(x*e + d)^m
*c^2*d^3*m*x^2*e^2 - 24*(x*e + d)^m*c^2*d^4*m*x*e + (x*e + d)^m*a^2*m^4*x*e
^5 + 26*(x*e + d)^m*a*b*m^3*x^2*e^5 + 49*(x*e + d)^m*b^2*m^2*x^3*e^5 + 98*(
x*e + d)^m*a*c*m^2*x^3*e^5 + 122*(x*e + d)^m*b*c*m*x^4*e^5 + 24*(x*e + d)^m
*c^2*x^5*e^5 + (x*e + d)^m*a^2*d*m^4*e^4 + 24*(x*e + d)^m*a*b*d*m^3*x*e^4 +
 29*(x*e + d)^m*b^2*d*m^2*x^2*e^4 + 58*(x*e + d)^m*a*c*d*m^2*x^2*e^4 + 20*(
x*e + d)^m*b*c*d*m*x^3*e^4 - 2*(x*e + d)^m*a*b*d^2*m^3*e^3 - 18*(x*e + d)^m
*b^2*d^2*m^2*x*e^3 - 36*(x*e + d)^m*a*c*d^2*m^2*x*e^3 - 30*(x*e + d)^m*b*c*
d^2*m*x^2*e^3 + 2*(x*e + d)^m*b^2*d^3*m^2*e^2 + 4*(x*e + d)^m*a*c*d^3*m^2*e
```

```
^2 + 60*(x*e + d)^m*b*c*d^3*m*x*e^2 - 12*(x*e + d)^m*b*c*d^4*m*e + 24*(x*e
+ d)^m*c^2*d^5 + 14*(x*e + d)^m*a^2*m^3*x*e^5 + 118*(x*e + d)^m*a*b*m^2*x^2
*e^5 + 78*(x*e + d)^m*b^2*m*x^3*e^5 + 156*(x*e + d)^m*a*c*m*x^3*e^5 + 60*(x
*e + d)^m*b*c*x^4*e^5 + 14*(x*e + d)^m*a^2*d*m^3*e^4 + 94*(x*e + d)^m*a*b*d
*m^2*x*e^4 + 20*(x*e + d)^m*b^2*d*m*x^2*e^4 + 40*(x*e + d)^m*a*c*d*m*x^2*e^
4 - 24*(x*e + d)^m*a*b*d^2*m^2*e^3 - 40*(x*e + d)^m*b^2*d^2*m*x*e^3 - 80*(x
*e + d)^m*a*c*d^2*m*x*e^3 + 18*(x*e + d)^m*b^2*d^3*m*e^2 + 36*(x*e + d)^m*a
*c*d^3*m*e^2 - 60*(x*e + d)^m*b*c*d^4*e + 71*(x*e + d)^m*a^2*m^2*x*e^5 + 21
4*(x*e + d)^m*a*b*m*x^2*e^5 + 40*(x*e + d)^m*b^2*x^3*e^5 + 80*(x*e + d)^m*a
*c*x^3*e^5 + 71*(x*e + d)^m*a^2*d*m^2*e^4 + 120*(x*e + d)^m*a*b*d*m*x*e^4 -
 94*(x*e + d)^m*a*b*d^2*m*e^3 + 40*(x*e + d)^m*b^2*d^3*e^2 + 80*(x*e + d)^m
*a*c*d^3*e^2 + 154*(x*e + d)^m*a^2*m*x*e^5 + 120*(x*e + d)^m*a*b*x^2*e^5 +
154*(x*e + d)^m*a^2*d*m*e^4 - 120*(x*e + d)^m*a*b*d^2*e^3 + 120*(x*e + d)^m
*a^2*x*e^5 + 120*(x*e + d)^m*a^2*d*e^4)/(m^5*e^5 + 15*m^4*e^5 + 85*m^3*e^5
+ 225*m^2*e^5 + 274*m*e^5 + 120*e^5)
```

3.2552 $\int (d+ex)^m \left(a+bx+cx^2\right) dx$

Optimal. Leaf size=82

$$\frac{(d+ex)^{m+1}\left(ae^2-bde+cd^2\right)}{e^3(m+1)}-\frac{(2cd-be)(d+ex)^{m+2}}{e^3(m+2)}+\frac{c(d+ex)^{m+3}}{e^3(m+3)}$$

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(d + e*x)^(1 + m))/(e^3*(1 + m)) - ((2*c*d - b*e)*
(d + e*x)^(2 + m))/(e^3*(2 + m)) + (c*(d + e*x)^(3 + m))/(e^3*(3 + m))
```

Rubi [A] time = 0.0447129, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {698}

$$\frac{(d+ex)^{m+1}\left(ae^2-bde+cd^2\right)}{e^3(m+1)}-\frac{(2cd-be)(d+ex)^{m+2}}{e^3(m+2)}+\frac{c(d+ex)^{m+3}}{e^3(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a + b*x + c*x^2),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(d + e*x)^(1 + m))/(e^3*(1 + m)) - ((2*c*d - b*e)*
(d + e*x)^(2 + m))/(e^3*(2 + m)) + (c*(d + e*x)^(3 + m))/(e^3*(3 + m))
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int (d+ex)^m \left(a+bx+cx^2\right) dx &= \int \left(\frac{\left(cd^2-bde+ae^2\right)(d+ex)^m}{e^2}+\frac{(-2cd+be)(d+ex)^{1+m}}{e^2}+\frac{c(d+ex)^{2+m}}{e^2}\right) dx\\ &= \frac{\left(cd^2-bde+ae^2\right)(d+ex)^{1+m}}{e^3(1+m)}-\frac{(2cd-be)(d+ex)^{2+m}}{e^3(2+m)}+\frac{c(d+ex)^{3+m}}{e^3(3+m)}\end{aligned}$$

Mathematica [A] time = 0.0820852, size = 83, normalized size = 1.01

$$\frac{(d+ex)^{m+1}\left(cd^2-e(bd-ae)\right)}{e^3(m+1)}-\frac{(2cd-be)(d+ex)^{m+2}}{e^3(m+2)}+\frac{c(d+ex)^{m+3}}{e^3(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(a + b*x + c*x^2),x]
```

```
[Out] ((c*d^2 - e*(b*d - a*e))*(d + e*x)^(1 + m))/(e^3*(1 + m)) - ((2*c*d - b*e)*
(d + e*x)^(2 + m))/(e^3*(2 + m)) + (c*(d + e*x)^(3 + m))/(e^3*(3 + m))
```

Maple [A] time = 0.043, size = 135, normalized size = 1.7

$$\frac{(ex+d)^{1+m}\left(ce^2m^2x^2 + be^2m^2x + 3\,ce^2mx^2 + ae^2m^2 + 4\,be^2mx - 2\,cdemx + 2\,ce^2x^2 + 5\,ae^2m - bdem + 3\,xbe^2 - 2\,cde\right.}{e^3\left(m^3 + 6\,m^2 + 11\,m + 6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a),x)
```

```
[Out] (e*x+d)^(1+m)*(c*e^2*m^2*x^2+b*e^2*m^2*x+3*c*e^2*m*x^2+a*e^2*m^2+4*b*e^2*m*
x-2*c*d*e*m*x+2*c*e^2*x^2+5*a*e^2*m-b*d*e*m+3*b*e^2*x-2*c*d*e*x+6*a*e^2-3*b
*d*e+2*c*d^2)/e^3/(m^3+6*m^2+11*m+6)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.59161, size = 416, normalized size = 5.07

$$\frac{\left(ade^2m^2 + 2\,cd^3 - 3\,bd^2e + 6\,ade^2 + \left(ce^3m^2 + 3\,ce^3m + 2\,ce^3\right)x^3 + \left(3\,be^3 + \left(cde^2 + be^3\right)m^2 + \left(cde^2 + 4\,be^3\right)m\right)x^2 - \left(bd\right.\right.}{e^3m^3 + 6\,e^3m^2 + 11\,e^3m + 6\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] (a*d*e^2*m^2 + 2*c*d^3 - 3*b*d^2*e + 6*a*d*e^2 + (c*e^3*m^2 + 3*c*e^3*m + 2
*c*e^3)*x^3 + (3*b*e^3 + (c*d*e^2 + b*e^3)*m^2 + (c*d*e^2 + 4*b*e^3)*m)*x^2
 - (b*d^2*e - 5*a*d*e^2)*m + (6*a*e^3 + (b*d*e^2 + a*e^3)*m^2 - (2*c*d^2*e
- 3*b*d*e^2 - 5*a*e^3)*m)*x)*(e*x + d)^m/(e^3*m^3 + 6*e^3*m^2 + 11*e^3*m +
6*e^3)
```

Sympy [A] time = 2.03763, size = 1416, normalized size = 17.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a),x)
```

```
[Out] Piecewise((d**m*(a*x + b*x**2/2 + c*x**3/3), Eq(e, 0)), (-a*e**2/(2*d**2*e*
*3 + 4*d*e**4*x + 2*e**5*x**2) - b*d*e/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x
**2) - 2*b*e**2*x/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2) + 2*c*d**2*log(d
```

/e + x)/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2) + 3*c*d**2/(2*d**2*e**3 +
4*d*e**4*x + 2*e**5*x**2) + 4*c*d*e*x*log(d/e + x)/(2*d**2*e**3 + 4*d*e**4*
x + 2*e**5*x**2) + 4*c*d*e*x/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2) + 2*c
*e**2*x**2*log(d/e + x)/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2), Eq(m, -3)
), (-a*e**2/(d*e**3 + e**4*x) + b*d*e*log(d/e + x)/(d*e**3 + e**4*x) + b*d*
e/(d*e**3 + e**4*x) + b*e**2*x*log(d/e + x)/(d*e**3 + e**4*x) - 2*c*d**2*lo
g(d/e + x)/(d*e**3 + e**4*x) - 2*c*d**2/(d*e**3 + e**4*x) - 2*c*d*e*x*log(d
/e + x)/(d*e**3 + e**4*x) + c*e**2*x**2/(d*e**3 + e**4*x), Eq(m, -2)), (a*l
og(d/e + x)/e - b*d*log(d/e + x)/e**2 + b*x/e + c*d**2*log(d/e + x)/e**3 -
c*d*x/e**2 + c*x**2/(2*e), Eq(m, -1)), (a*d*e**2*m**2*(d + e*x)**m/(e**3*m*
*3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + 5*a*d*e**2*m*(d + e*x)**m/(e**3*m*
*3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + 6*a*d*e**2*(d + e*x)**m/(e**3*m**3
 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + a*e**3*m**2*x*(d + e*x)**m/(e**3*m**
3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + 5*a*e**3*m*x*(d + e*x)**m/(e**3*m**
3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + 6*a*e**3*x*(d + e*x)**m/(e**3*m**3
+ 6*e**3*m**2 + 11*e**3*m + 6*e**3) - b*d**2*e*m*(d + e*x)**m/(e**3*m**3 +
6*e**3*m**2 + 11*e**3*m + 6*e**3) - 3*b*d**2*e*(d + e*x)**m/(e**3*m**3 + 6*
e**3*m**2 + 11*e**3*m + 6*e**3) + b*d*e**2*m**2*x*(d + e*x)**m/(e**3*m**3 +
 6*e**3*m**2 + 11*e**3*m + 6*e**3) + 3*b*d*e**2*m*x*(d + e*x)**m/(e**3*m**3
 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + b*e**3*m**2*x**2*(d + e*x)**m/(e**3*
m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + 4*b*e**3*m*x**2*(d + e*x)**m/(e*
*3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + 3*b*e**3*x**2*(d + e*x)**m/(e
3*m3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + 2*c*d**3*(d + e*x)**m/(e**3*
m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) - 2*c*d**2*e*m*x*(d + e*x)**m/(e**
3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + c*d*e**2*m**2*x**2*(d + e*x)**
m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + c*d*e**2*m*x**2*(d + e*x
)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + c*e**3*m**2*x**3*(d +
 e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + 3*c*e**3*m*x**3*(
d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + 2*c*e**3*x**3*
(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3), True))

Giac [B] time = 1.16598, size = 477, normalized size = 5.82

$$(xe+d)^m cm^2x^3e^3 + (xe+d)^m cdm^2x^2e^2 + (xe+d)^m bm^2x^2e^3 + 3\,(xe+d)^m cmx^3e^3 + (xe+d)^m bdm^2xe^2 + (xe+d)^m cdmx^2e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^m*(c*x^2+b*x+a),x, algorithm="giac")

[Out] ((x*e + d)^m*c*m^2*x^3*e^3 + (x*e + d)^m*c*d*m^2*x^2*e^2 + (x*e + d)^m*b*m^
2*x^2*e^3 + 3*(x*e + d)^m*c*m*x^3*e^3 + (x*e + d)^m*b*d*m^2*x*e^2 + (x*e +
d)^m*c*d*m*x^2*e^2 - 2*(x*e + d)^m*c*d^2*m*x*e + (x*e + d)^m*a*m^2*x*e^3 +
4*(x*e + d)^m*b*m*x^2*e^3 + 2*(x*e + d)^m*c*x^3*e^3 + (x*e + d)^m*a*d*m^2*e
^2 + 3*(x*e + d)^m*b*d*m*x*e^2 - (x*e + d)^m*b*d^2*m*e + 2*(x*e + d)^m*c*d^
3 + 5*(x*e + d)^m*a*m*x*e^3 + 3*(x*e + d)^m*b*x^2*e^3 + 5*(x*e + d)^m*a*d*m
e^2 - 3(x*e + d)^m*b*d^2*e + 6*(x*e + d)^m*a*x*e^3 + 6*(x*e + d)^m*a*d*e^
2)/(m^3*e^3 + 6*m^2*e^3 + 11*m*e^3 + 6*e^3)

3.2553 $\int \frac{(d+ex)^m}{a+bx+cx^2} dx$

Optimal. Leaf size=191

$$\frac{2c(d+ex)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{(m+1)\sqrt{b^2-4ac}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{2c(d+ex)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{(m+1)\sqrt{b^2-4ac}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}$$

```
[Out] (-2*c*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (2*c*(d + e*x))/
(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[b^2 - 4*a*c]*(2*c*d - (b - Sqrt
[b^2 - 4*a*c])*e)*(1 + m)) + (2*c*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1
+ m, 2 + m, (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[b^2
 - 4*a*c]*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 + m))
```

Rubi [A] time = 0.383565, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {711, 68}

$$\frac{2c(d+ex)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{(m+1)\sqrt{b^2-4ac}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{2c(d+ex)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{(m+1)\sqrt{b^2-4ac}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(a + b*x + c*x^2),x]
```


```
[Out] (-2*c*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (2*c*(d + e*x))/
(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[b^2 - 4*a*c]*(2*c*d - (b - Sqrt
[b^2 - 4*a*c])*e)*(1 + m)) + (2*c*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1
+ m, 2 + m, (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[b^2
 - 4*a*c]*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 + m))
```


Rule 711

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[ExpandIntegrand[(d + e*x)^m, 1/(a + b*x + c*x^2), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[m]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(d+ex)^m}{a+bx+cx^2}\,dx = \int \left(\frac{2c(d+ex)^m}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)} - \frac{2c(d+ex)^m}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}+2cx\right)}\right) dx$$

$$= \frac{(2c)\int \frac{(d+ex)^m}{b-\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(d+ex)^m}{b+\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}}$$

$$= -\frac{2c(d+ex)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{b^2-4ac}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+m)} + \frac{2c(d+ex)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{2c(d\cdot}{2cd-\left(b+\sqrt{\right.}\right.}\right.}{\sqrt{b^2-4ac}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(1}$$

Mathematica [A] time = 0.452013, size = 163, normalized size = 0.85

$$\frac{2c(d+ex)^{m+1}\left(\frac{{}_2F_1\left(1,m+1;m+2;\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)} - \frac{{}_2F_1\left(1,m+1;m+2;\frac{2c(d+ex)}{2cd+\left(\sqrt{b^2-4ac}-b\right)e}\right)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd}\right)}{(m+1)\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(a + b*x + c*x^2),x]
```

```
[Out] (2*c*(d + e*x)^(1 + m)*(-(Hypergeometric2F1[1, 1 + m, 2 + m, (2*c*(d + e*x)
)/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)]/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e
)) + Hypergeometric2F1[1, 1 + m, 2 + m, (2*c*(d + e*x))/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)]/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)))/(Sqrt[b^2 - 4*a*c]*
(1 + m))
```

Maple [F] time = 1.28, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(c*x^2+b*x+a),x)
```

```
[Out] int((e*x+d)^m/(c*x^2+b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/(c*x^2 + b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^m}{cx^2+bx+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m/(c*x^2 + b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m}{a+bx+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(c*x**2+b*x+a),x)
```

```
[Out] Integral((d + e*x)**m/(a + b*x + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(c*x^2 + b*x + a), x)
```

3.2554 $\int \frac{(d+ex)^m}{\left(a+bx+cx^2\right)^2} dx$

Optimal. Leaf size=425

$$\frac{c(d+ex)^{m+1}\left(-2ce\left(dm\sqrt{b^2-4ac}-2ae(1-m)+2bd\right)+be^2m\left(\sqrt{b^2-4ac}+b\right)+4c^2d^2\right){}_2F_1\left(1,m+1;m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-\right.}}\right.}{(m+1)\left(b^2-4ac\right)^{3/2}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)\left(ae^2-bde+cd^2\right)}$$

```
[Out] -(((d + e*x)^(1 + m)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 -
 4*a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2))) + (c*(4*c^2*d^2 + b*(b
+ Sqrt[b^2 - 4*a*c])*e^2*m - 2*c*e*(2*b*d - 2*a*e*(1 - m) + Sqrt[b^2 - 4*a*
c]*d*m))*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (2*c*(d + e*x
))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/((b^2 - 4*a*c)^(3/2)*(2*c*d - (b -
 Sqrt[b^2 - 4*a*c])*e)*(c*d^2 - b*d*e + a*e^2)*(1 + m)) - (c*(e*(2*c*d - b*
e)*m + (4*c^2*d^2 - 4*c*e*(b*d - a*e*(1 - m)) + b^2*e^2*m)/Sqrt[b^2 - 4*a*c
])*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (2*c*(d + e*x))/(2*
c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/((b^2 - 4*a*c)*(2*c*d - (b + Sqrt[b^2 -
4*a*c])*e)*(c*d^2 - b*d*e + a*e^2)*(1 + m))
```

Rubi [A] time = 1.06873, antiderivative size = 425, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {740, 830, 68}

$$\frac{c(d+ex)^{m+1}\left(-2ce\left(dm\sqrt{b^2-4ac}-2ae(1-m)+2bd\right)+be^2m\left(\sqrt{b^2-4ac}+b\right)+4c^2d^2\right){}_2F_1\left(1,m+1;m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-\right.}}\right.}{(m+1)\left(b^2-4ac\right)^{3/2}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(a + b*x + c*x^2)^2,x]
```


```
[Out] -(((d + e*x)^(1 + m)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 -
 4*a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2))) + (c*(4*c^2*d^2 + b*(b
+ Sqrt[b^2 - 4*a*c])*e^2*m - 2*c*e*(2*b*d - 2*a*e*(1 - m) + Sqrt[b^2 - 4*a*
c]*d*m))*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (2*c*(d + e*x
))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/((b^2 - 4*a*c)^(3/2)*(2*c*d - (b -
 Sqrt[b^2 - 4*a*c])*e)*(c*d^2 - b*d*e + a*e^2)*(1 + m)) - (c*(e*(2*c*d - b*
e)*m + (4*c^2*d^2 - 4*c*e*(b*d - a*e*(1 - m)) + b^2*e^2*m)/Sqrt[b^2 - 4*a*c
])*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (2*c*(d + e*x))/(2*
c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/((b^2 - 4*a*c)*(2*c*d - (b + Sqrt[b^2 -
4*a*c])*e)*(c*d^2 - b*d*e + a*e^2)*(1 + m))
```


Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(d+ex)^m}{\left(a+bx+cx^2\right)^2}\,dx = -\frac{(d+ex)^{1+m}\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)} - \frac{\int \frac{(d+ex)^m\left(2c^2d^2+b^2e^2m+ce(2ae(1-m)-bd(2+m))-ce(2c\cdots\right.}{a+bx+cx^2}}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{(d+ex)^{1+m}\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)} - \frac{\int \left(\frac{\left(-ce(2cd-be)m+\frac{c\left(4c^2d^2-4bcde+4ace^2+b^2e^2m-4ace^2m\right)}{\sqrt{b^2-4ac}}\right)(\cdots}{b-\sqrt{b^2-4ac}+2cx}\cdots\right.}{\left(b^2-\cdots\right.}$$

$$= -\frac{(d+ex)^{1+m}\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)} + \frac{\left(c\left(e(2cd-be)m-\frac{4c^2d^2-4ce(bd-ae(1-m))+b^2e^2m\cdots}{\sqrt{b^2-4ac}}\cdots\right.\right.}{\left(b^2-4ac\right)\left(cd^2-bde+\cdots\right.}$$

$$= -\frac{(d+ex)^{1+m}\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)} + \frac{c\left(4c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2m-2ce\left(2b\cdots\right.\right.}{\left(b^2-4ac\right)^{3/2}\cdots}$$

Mathematica [A] time = 1.28903, size = 339, normalized size = 0.8

$$\frac{(d+ex)^{m+1}\left(-\frac{c\left(\frac{4ce(ae(m-1)+bd)-b^2e^2m-4c^2d^2}{\sqrt{b^2-4ac}}+em(2cd-be)\right){}_2F_1\left(1,m+1;m+2;\frac{2c(d+ex)}{2cd+\left(\sqrt{b^2-4ac}-b\right)e}\right)}{(m+1)\left(e\left(\sqrt{b^2-4ac}-b\right)+2cd\right)}-\frac{c\left(\frac{-4ce(ae(m-1)+bd)+b^2e^2m+4c^2d^2}{\sqrt{b^2-4ac}}+em(2cd-be)\right){}_2F_1\left(1,m\cdots\right.}{(m+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\cdots\right.}\cdots\right.}{\left(b^2-4ac\right)\left(e(ae-bd)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(a + b*x + c*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + m)*((b^2*e - 2*c*(a*e + c*d*x) + b*c*(-d + e*x))/(a + x*(b
+ c*x)) - (c*(e*(2*c*d - b*e)*m + (-4*c^2*d^2 + 4*c*e*(b*d + a*e*(-1 + m))
- b^2*e^2*m)/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 + m, 2 + m, (2*c*(d
+ e*x))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)])/((2*c*d + (-b + Sqrt[b^2 - 4
*a*c])*e)*(1 + m)) - (c*(e*(2*c*d - b*e)*m + (4*c^2*d^2 - 4*c*e*(b*d + a*e*
(-1 + m)) + b^2*e^2*m)/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 + m, 2 + m
, (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/((2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)*(1 + m))))/((b^2 - 4*a*c)*(c*d^2 + e*(-(b*d) + a*e)))
```

Maple [F] time = 1.241, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(cx^2+bx+a\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(c*x^2+b*x+a)^2,x)
```

```
[Out] int((e*x+d)^m/(c*x^2+b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(cx^2+bx+a\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/(c*x^2 + b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^m}{c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\,ac\right)x^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a
^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(cx^2+bx+a\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(c*x^2 + b*x + a)^2, x)
```

3.2555 $\int (d+ex)^m \left(a+bx+cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=189

$$\frac{\left(a+bx+cx^2\right)^{5/2}(d+ex)^{m+1}F_1\left(m+1;-\frac{5}{2},-\frac{5}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(m+1)\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{5/2}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{5/2}}$$

```
[Out] ((d + e*x)^(1 + m)*(a + b*x + c*x^2)^(5/2)*AppellF1[1 + m, -5/2, -5/2, 2 +
m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*
c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*(1 - (2*c*(d + e*x))/(2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e))^(5/2)*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e))^(5/2))
```

Rubi [A] time = 0.252348, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {759, 133}

$$\frac{\left(a+bx+cx^2\right)^{5/2}(d+ex)^{m+1}F_1\left(m+1;-\frac{5}{2},-\frac{5}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(m+1)\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{5/2}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] ((d + e*x)^(1 + m)*(a + b*x + c*x^2)^(5/2)*AppellF1[1 + m, -5/2, -5/2, 2 +
m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*
c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*(1 - (2*c*(d + e*x))/(2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e))^(5/2)*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e))^(5/2))
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (d+ex)^m \left(a+bx+cx^2\right)^{5/2} dx = \frac{\left(a+bx+cx^2\right)^{5/2} \operatorname{Subst}\left(\int x^m \left(1-\frac{2cx}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{5/2} \left(1-\frac{2cx}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{5/2} dx,\right.}{e\left(1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}\right)^{5/2} \left(1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}\right)^{5/2}}$$

$$= \frac{(d+ex)^{1+m} \left(a+bx+cx^2\right)^{5/2} F_1\left(1+m;-\frac{5}{2},-\frac{5}{2};2+m;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-\right.}}\right.}{e(1+m)\left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{5/2} \left(1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{5/2}}$$

Mathematica [F] time = 0.121759, size = 0, normalized size = 0.

$$\int (d+ex)^m \left(a+bx+cx^2\right)^{5/2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m*(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] Integrate[(d + e*x)^m*(a + b*x + c*x^2)^(5/2), x]
```

Maple [F] time = 1.225, size = 0, normalized size = 0.

$$\int (ex+d)^m \left(cx^2+bx+a\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{\frac{5}{2}} (ex+d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)*(e*x + d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\,ac\right)x^2+a^2\right)\sqrt{cx^2+bx+a}(ex+d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*sqrt(c*x
^2 + b*x + a)*(e*x + d)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx + a\right)^{\frac{5}{2}} (ex + d)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(5/2)*(e*x + d)^m, x)
```

3.2556 $\int (d+ex)^m \left(a+bx+cx^2\right)^{3/2} dx$

Optimal. Leaf size=189

$$\frac{\left(a+bx+cx^2\right)^{3/2}(d+ex)^{m+1}F_1\left(m+1;-\frac{3}{2},-\frac{3}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(m+1)\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{3/2}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{3/2}}$$

```
[Out] ((d + e*x)^(1 + m)*(a + b*x + c*x^2)^(3/2)*AppellF1[1 + m, -3/2, -3/2, 2 +
m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*
c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*(1 - (2*c*(d + e*x))/(2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e))^(3/2)*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e))^(3/2))
```

Rubi [A] time = 0.106063, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {759, 133}

$$\frac{\left(a+bx+cx^2\right)^{3/2}(d+ex)^{m+1}F_1\left(m+1;-\frac{3}{2},-\frac{3}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(m+1)\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{3/2}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] ((d + e*x)^(1 + m)*(a + b*x + c*x^2)^(3/2)*AppellF1[1 + m, -3/2, -3/2, 2 +
m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*
c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*(1 - (2*c*(d + e*x))/(2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e))^(3/2)*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e))^(3/2))
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (d+ex)^m \left(a+bx+cx^2\right)^{3/2} dx = \frac{\left(a+bx+cx^2\right)^{3/2} \text{Subst}\left(\int x^m \left(1-\frac{2cx}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{3/2} \left(1-\frac{2cx}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{3/2} dx, x, c\right.}{e\left(1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}\right)^{3/2} \left(1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}\right)^{3/2}}$$

$$= \frac{(d+ex)^{1+m} \left(a+bx+cx^2\right)^{3/2} F_1\left(1+m; -\frac{3}{2}, -\frac{3}{2}; 2+m; \frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}, \frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)}\right.}{e(1+m)\left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{3/2} \left(1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{3/2}}$$

Mathematica [F] time = 0.0683525, size = 0, normalized size = 0.

$$\int (d+ex)^m \left(a+bx+cx^2\right)^{3/2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m*(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] Integrate[(d + e*x)^m*(a + b*x + c*x^2)^(3/2), x]
```

Maple [F] time = 1.331, size = 0, normalized size = 0.

$$\int (ex+d)^m \left(cx^2+bx+a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{\frac{3}{2}} (ex+d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)*(e*x + d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2+bx+a\right)^{\frac{3}{2}} (ex+d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^(3/2)*(e*x + d)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx + a\right)^{\frac{3}{2}} (ex + d)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)*(e*x + d)^m, x)
```

3.2557 $\int (d+ex)^m \sqrt{a+bx+cx^2}\,dx$

Optimal. Leaf size=189

$$\frac{\sqrt{a+bx+cx^2}(d+ex)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(m+1)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

```
[Out] ((d + e*x)^(1 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m,
 (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d
 - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)])
```

Rubi [A] time = 0.106671, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {759, 133}

$$\frac{\sqrt{a+bx+cx^2}(d+ex)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(m+1)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((d + e*x)^(1 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m,
 (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d
 - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)])
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (d+ex)^m\sqrt{a+bx+cx^2}\,dx = \frac{\sqrt{a+bx+cx^2}\,\text{Subst}\left(\int x^m\sqrt{1-\frac{2cx}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2cx}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}dx,x,d+ex\right)}{e\sqrt{1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}}\sqrt{1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}}}$$

$$= \frac{(d+ex)^{1+m}\sqrt{a+bx+cx^2}F_1\left(1+m;-\frac{1}{2},-\frac{1}{2};2+m;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(1+m)\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}$$

Mathematica [A] time = 0.214308, size = 207, normalized size = 1.1

$$\frac{\sqrt{a+x(b+cx)}(d+ex)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd+\left(\sqrt{b^2-4ac}-b\right)e}\right)}{e(m+1)\sqrt{\frac{e\left(\sqrt{b^2-4ac}-b-2cx\right)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}\sqrt{\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{e\left(\sqrt{b^2-4ac}+b\right)-2cd}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^m*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((d + e*x)^(1 + m)*Sqrt[a + x*(b + c*x)]*AppellF1[1 + m, -1/2, -1/2, 2 + m,
 (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*
d + (-b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*Sqrt[(e*(-b + Sqrt[b^2 - 4*a*c
] - 2*c*x))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[(e*(b + Sqrt[b^2 - 4
*a*c] + 2*c*x))/(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)])
```

Maple [F] time = 1.208, size = 0, normalized size = 0.

$$\int (ex+d)^m\sqrt{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2+bx+a}(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^2 + bx + a}(ex + d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*(e*x + d)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d + ex)^m \sqrt{a + bx + cx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**m*sqrt(a + b*x + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + bx + a}(ex + d)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^m, x)
```

3.2558 $\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=189

$$\frac{(d+ex)^{m+1}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(m+1)\sqrt{a+bx+cx^2}}$$

```
[Out] ((d + e*x)^(1 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c]
)*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*AppellF
1[1 + m, 1/2, 1/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*
e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*Sqrt[a
 + b*x + c*x^2])
```

Rubi [A] time = 0.106562, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {759, 133}

$$\frac{(d+ex)^{m+1}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(m+1)\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((d + e*x)^(1 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c]
)*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*AppellF
1[1 + m, 1/2, 1/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*
e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*Sqrt[a
 + b*x + c*x^2])
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx = \frac{\left(\sqrt{1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}}\sqrt{1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}}\right)\text{Subst}\left(\int \frac{x^m}{\sqrt{1-\frac{2cx}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2cx}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}dx, x, d+ex\right)}{e\sqrt{a+bx+cx^2}}$$

$$= \frac{(d+ex)^{1+m}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}F_1\left(1+m;\frac{1}{2},\frac{1}{2};2+m;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{}{2cd-}\right.}{e(1+m)\sqrt{a+bx+cx^2}}$$

Mathematica [A] time = 0.193333, size = 207, normalized size = 1.1

$$\frac{(d+ex)^{m+1}\sqrt{\frac{e\left(\sqrt{b^2-4ac}-b-2cx\right)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}\sqrt{\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{e\left(\sqrt{b^2-4ac}+b\right)-2cd}}F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd+\left(\sqrt{b^2-4ac}-b\right)e}\right)}{e(m+1)\sqrt{a+x(b+cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^m/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[(e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c]
)*e)]*Sqrt[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*d + (b + Sqrt[b^2 - 4*
a*c])*e)]*(d + e*x)^(1 + m)*AppellF1[1 + m, 1/2, 1/2, 2 + m, (2*c*(d + e*x)
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d + (-b + Sqrt[
b^2 - 4*a*c])*e)])/(e*(1 + m)*Sqrt[a + x*(b + c*x)])
```

Maple [F] time = 1.22, size = 0, normalized size = 0.

$$\int (ex+d)^m \frac{1}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] int((e*x+d)^m/(c*x^2+b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^m}{\sqrt{cx^2+bx+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**m/sqrt(a + b*x + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

3.2559 $\int \frac{(d+ex)^m}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=189

$$\frac{(d+ex)^{m+1}\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{3/2}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{3/2}F_1\left(m+1;\frac{3}{2},\frac{3}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(m+1)\left(a+bx+cx^2\right)^{3/2}}$$

```
[Out] ((d + e*x)^(1 + m)*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)
)^(3/2)*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))^(3/2)*App
ellF1[1 + m, 3/2, 3/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*
c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*(a
 + b*x + c*x^2)^(3/2))
```

Rubi [A] time = 0.104656, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {759, 133}

$$\frac{(d+ex)^{m+1}\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{3/2}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{3/2}F_1\left(m+1;\frac{3}{2},\frac{3}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(m+1)\left(a+bx+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] ((d + e*x)^(1 + m)*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)
)^(3/2)*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))^(3/2)*App
ellF1[1 + m, 3/2, 3/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*
c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*(a
 + b*x + c*x^2)^(3/2))
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] && !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{(d+ex)^m}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{\left(\left(1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}\right)^{3/2}\left(1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}\right)^{3/2}\right)\mathrm{Subst}\left(\int \frac{x^m}{\left(1-\frac{2cx}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{3/2}\left(1-\frac{2cx}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{3/2}}\right.}{e\left(a+bx+cx^2\right)^{3/2}}$$

$$= \frac{(d+ex)^{1+m}\left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{3/2}\left(1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{3/2}F_1\left(1+m;\frac{3}{2},\frac{3}{2};2+m;\frac{2c(d+}{2cd-\left(b-\sqrt{b}\right.}\right.}{e(1+m)\left(a+bx+cx^2\right)^{3/2}}$$

Mathematica [A] time = 0.375169, size = 267, normalized size = 1.41

$$-\frac{e\left(\sqrt{b^2-4ac}-b-2cx\right)\left(\sqrt{b^2-4ac}+b+2cx\right)(d+ex)^{m+1}\sqrt{\frac{e\left(\sqrt{b^2-4ac}-b-2cx\right)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}\sqrt{\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{e\left(\sqrt{b^2-4ac}+b\right)-2cd}}F_1\left(m+1;\frac{3}{2},\frac{3}{2};m+2;\right.}{4c(m+1)(a+x(b+cx))^{3/2}\left(e(ae-bd)+cd^2\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^m/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] -(e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)*Sqrt[(e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*
x))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)*S
qrt[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e
)]*(d + e*x)^(1 + m)*AppellF1[1 + m, 3/2, 3/2, 2 + m, (2*c*(d + e*x))/(2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d + (-b + Sqrt[b^2 - 4
*a*c])*e)])/(4*c*(c*d^2 + e*(-(b*d) + a*e))*(1 + m)*(a + x*(b + c*x))^(3/2)
)
```

Maple [F] time = 1.202, size = 0, normalized size = 0.

$$\int (ex+d)^m\left(cx^2+bx+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] int((e*x+d)^m/(c*x^2+b*x+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(cx^2+bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/(c*x^2 + b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+bx+a}(ex+d)^m}{c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\,ac\right)x^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*(e*x + d)^m/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x +
 (b^2 + 2*a*c)*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m}{\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x)**m/(a + b*x + c*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(cx^2+bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(c*x^2 + b*x + a)^(3/2), x)
```

3.2560 $\int \frac{(d+ex)^m}{\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=189

$$\frac{(d+ex)^{m+1}\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{5/2}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{5/2}F_1\left(m+1;\frac{5}{2},\frac{5}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(m+1)\left(a+bx+cx^2\right)^{5/2}}$$

```
[Out] ((d + e*x)^(1 + m)*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)
)^(5/2)*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))^(5/2)*App
ellF1[1 + m, 5/2, 5/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*
c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*(a
 + b*x + c*x^2)^(5/2))
```

Rubi [A] time = 0.110178, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {759, 133}

$$\frac{(d+ex)^{m+1}\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{5/2}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{5/2}F_1\left(m+1;\frac{5}{2},\frac{5}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(m+1)\left(a+bx+cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] ((d + e*x)^(1 + m)*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)
)^(5/2)*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))^(5/2)*App
ellF1[1 + m, 5/2, 5/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*
c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*(a
 + b*x + c*x^2)^(5/2))
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{(d+ex)^m}{\left(a+bx+cx^2\right)^{5/2}}\,dx = \frac{\left(\left(1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}\right)^{5/2}\left(1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}\right)^{5/2}\right)\text{Subst}\left(\int \frac{x^m}{\left(1-\frac{2cx}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{5/2}\left(1-\frac{2cx}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{5/2}}\,dx,\right.}{e\left(a+bx+cx^2\right)^{5/2}}$$

$$= \frac{(d+ex)^{1+m}\left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{5/2}\left(1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{5/2}F_1\left(1+m;\frac{5}{2},\frac{5}{2};2+m;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4a\right.}\right.}}{e(1+m)\left(a+bx+cx^2\right)^{5/2}}$$

Mathematica [A] time = 0.354907, size = 225, normalized size = 1.19

$$\frac{e^3(d+ex)^{m+1}\sqrt{\frac{e\left(\sqrt{b^2-4ac}-b-2cx\right)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}\sqrt{\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{e\left(\sqrt{b^2-4ac}+b\right)-2cd}}F_1\left(m+1;\frac{5}{2},\frac{5}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd+\left(\sqrt{b^2-4ac}-b\right)e}\right)}{(m+1)\sqrt{a+x(b+cx)}\left(e(ae-bd)+cd^2\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^m/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (e^3*Sqrt[(e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*d + (-b + Sqrt[b^2 - 4*
a*c])*e)]*Sqrt[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*d + (b + Sqrt[b^2
- 4*a*c])*e)]*(d + e*x)^(1 + m)*AppellF1[1 + m, 5/2, 5/2, 2 + m, (2*c*(d +
e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d + (-b + S
qrt[b^2 - 4*a*c])*e)]])/((c*d^2 + e*(-(b*d) + a*e))^2*(1 + m)*Sqrt[a + x*(b
+ c*x)])
```

Maple [F] time = 1.259, size = 0, normalized size = 0.

$$\int (ex+d)^m\left(cx^2+bx+a\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] int((e*x+d)^m/(c*x^2+b*x+a)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(cx^2+bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/(c*x^2 + b*x + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+bx+a}(ex+d)^m}{c^3x^6+3\,bc^2x^5+3\left(b^2c+ac^2\right)x^4+3\,a^2bx+\left(b^3+6\,abc\right)x^3+a^3+3\left(ab^2+a^2c\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*(e*x + d)^m/(c^3*x^6 + 3*b*c^2*x^5 + 3*(b^2*
c + a*c^2)*x^4 + 3*a^2*b*x + (b^3 + 6*a*b*c)*x^3 + a^3 + 3*(a*b^2 + a^2*c)*
x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(d+ex)^m}{\left(a+bx+cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Integral((d + e*x)**m/(a + b*x + c*x**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ex+d)^m}{\left(cx^2+bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(c*x^2 + b*x + a)^(5/2), x)
```

3.2561 $\int (dx)^m \left(a+bx+cx^2\right)^p \, dx$

Optimal. Leaf size=137

$$\frac{(dx)^{m+1}\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx+cx^2\right)^p F_1\left(m+1;-p,-p;m+2;-\frac{2cx}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)}$$

```
[Out] ((d*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (-2*c*x)/
(b - Sqrt[b^2 - 4*a*c]), (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(d*(1 + m)*(1 +
 (2*c*x)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x)/(b + Sqrt[b^2 - 4*a*c]))^p
)
```

Rubi [A] time = 0.277654, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {759, 133}

$$\frac{(dx)^{m+1}\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx+cx^2\right)^p F_1\left(m+1;-p,-p;m+2;-\frac{2cx}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x + c*x^2)^p,x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (-2*c*x)/
(b - Sqrt[b^2 - 4*a*c]), (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(d*(1 + m)*(1 +
 (2*c*x)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x)/(b + Sqrt[b^2 - 4*a*c]))^p
)
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (dx)^m \left(a+bx+cx^2\right)^p \, dx = \frac{\left(\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx+cx^2\right)^p\right)\text{Subst}\left(\int x^m\left(1+\frac{2cx}{\left(b-\sqrt{b^2-4ac}\right)d}\right)^p\left(1+\right.\right.}{d}$$

$$= \frac{(dx)^{1+m}\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx+cx^2\right)^p F_1\left(1+m;-p,-p;2+m;-\frac{2}{b-\sqrt{}}\right.}{d(1+m)}$$

Mathematica [A] time = 0.271972, size = 160, normalized size = 1.17

$$\frac{x(dx)^m\left(\frac{-\sqrt{b^2-4ac}+b+2cx}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}+b}\right)^{-p}(a+x(b+cx))^pF_1\left(m+1;-p,-p;m+2;-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{\sqrt{b^2-4ac}-b}\right)}{m+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m*(a + b*x + c*x^2)^p,x]
```

```
[Out] (x*(d*x)^m*(a + x*(b + c*x))^p*AppellF1[1 + m, -p, -p, 2 + m, (-2*c*x)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])])/((1 + m)*((b - Sqrt
[b^2 - 4*a*c] + 2*c*x)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] +
 2*c*x)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F] time = 1.112, size = 0, normalized size = 0.

$$\int (dx)^m \left(cx^2+bx+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2+b*x+a)^p,x)
```

```
[Out] int((d*x)^m*(c*x^2+b*x+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^p (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^p*(d*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2+bx+a\right)^p (dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^p*(d*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2+b*x+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx + a\right)^p (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^p*(d*x)^m, x)
```

3.2562 $\int (d+ex)^m \left(a+bx+cx^2\right)^p \, dx$

Optimal. Leaf size=187

$$\frac{(d+ex)^{m+1}\left(a+bx+cx^2\right)^p \left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p} \left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p} F_1\left(m+1;-p,-p;m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},-\frac{}{2}\right.}{e(m+1)}$$

```
[Out] ((d + e*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (2*c*
(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sq
rt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e))^p)
```

Rubi [A] time = 0.110415, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {759, 133}

$$\frac{(d+ex)^{m+1}\left(a+bx+cx^2\right)^p \left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p} \left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p} F_1\left(m+1;-p,-p;m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},-\frac{}{2}\right.}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a + b*x + c*x^2)^p,x]
```

```
[Out] ((d + e*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (2*c*
(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sq
rt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e))^p)
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (d+ex)^m \left(a+bx+cx^2\right)^p dx = \frac{\left(\left(a+bx+cx^2\right)^p \left(1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}\right)^{-p} \left(1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}\right)^{-p}\right) \text{Subst}\left(\int x^m \left(1-\frac{2cx}{2cd-\left(b-\sqrt{b^2\right.}}\right.\right.}{e}$$

$$= \frac{(d+ex)^{1+m} \left(a+bx+cx^2\right)^p \left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{-p} \left(1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{-p} F_1\left(1+m;-\right.}{e(1+m)}$$

Mathematica [A] time = 0.4013, size = 205, normalized size = 1.1

$$\frac{(d+ex)^{m+1}(a+x(b+cx))^p \left(\frac{e\left(\sqrt{b^2-4ac}-b-2cx\right)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd}\right)^{-p} \left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{e\left(\sqrt{b^2-4ac}+b\right)-2cd}\right)^{-p} F_1\left(m+1;-p,-p;m+2;\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd+\left(\sqrt{b^2-4ac}\right.}\right.}{e(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^m*(a + b*x + c*x^2)^p,x]
```

```
[Out] ((d + e*x)^(1 + m)*(a + x*(b + c*x))^p*AppellF1[1 + m, -p, -p, 2 + m, (2*c*
(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d + (-
b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*((e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)
)/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e))^p*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*
x))/(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e))^p)
```

Maple [F] time = 1.395, size = 0, normalized size = 0.

$$\int (ex+d)^m \left(cx^2+bx+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^p,x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^p (ex+d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2+bx+a\right)^p (ex+d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx + a\right)^p (ex + d)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

3.2563 $\int (d+ex)^3 \left(a+bx+cx^2\right)^p \, dx$

Optimal. Leaf size=327

$$-\frac{2^{p-1}(2cd-be)\left(a+bx+cx^2\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(-2ce(3ae+bd(2p+3))+b^2e^2(p+3)+2c^2d^2(2p+3)\right)\,{}_2F_1\left(-p\right.}{c^3(p+1)(2p+3)\sqrt{b^2-4ac}}$$

```
[Out] (e*(d + e*x)^2*(a + b*x + c*x^2)^(1 + p))/(2*c*(2 + p)) - (e*(b*e*(2*c*d -
b*e)*(2 + p)*(3 + p) - 2*c*(3 + 2*p)*(c*d^2*(5 + 2*p) - e*(a*e + b*d*(2 + p
))) - 2*c*e*(2*c*d - b*e)*(1 + p)*(3 + p)*x)*(a + b*x + c*x^2)^(1 + p))/(4*
c^3*(1 + p)*(2 + p)*(3 + 2*p)) - (2^(-1 + p)*(2*c*d - b*e)*(b^2*e^2*(3 + p)
 + 2*c^2*d^2*(3 + 2*p) - 2*c*e*(3*a*e + b*d*(3 + 2*p)))*(-((b - Sqrt[b^2 -
4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x + c*x^2)^(1 + p)*Hype
rgeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2
 - 4*a*c])])/(c^3*Sqrt[b^2 - 4*a*c]*(1 + p)*(3 + 2*p))
```

Rubi [A] time = 0.423452, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {742, 779, 624}

$$-\frac{2^{p-1}(2cd-be)\left(a+bx+cx^2\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(-2ce(3ae+bd(2p+3))+b^2e^2(p+3)+2c^2d^2(2p+3)\right)\,{}_2F_1\left(-p\right.}{c^3(p+1)(2p+3)\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*x + c*x^2)^p,x]
```


```
[Out] (e*(d + e*x)^2*(a + b*x + c*x^2)^(1 + p))/(2*c*(2 + p)) - (e*(b*e*(2*c*d -
b*e)*(2 + p)*(3 + p) - 2*c*(3 + 2*p)*(c*d^2*(5 + 2*p) - e*(a*e + b*d*(2 + p
))) - 2*c*e*(2*c*d - b*e)*(1 + p)*(3 + p)*x)*(a + b*x + c*x^2)^(1 + p))/(4*
c^3*(1 + p)*(2 + p)*(3 + 2*p)) - (2^(-1 + p)*(2*c*d - b*e)*(b^2*e^2*(3 + p)
 + 2*c^2*d^2*(3 + 2*p) - 2*c*e*(3*a*e + b*d*(3 + 2*p)))*(-((b - Sqrt[b^2 -
4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x + c*x^2)^(1 + p)*Hype
rgeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2
 - 4*a*c])])/(c^3*Sqrt[b^2 - 4*a*c]*(1 + p)*(3 + 2*p))
```


Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
```

```
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rubi steps

$$\int (d+ex)^3\left(a+bx+cx^2\right)^p\,dx = \frac{e(d+ex)^2\left(a+bx+cx^2\right)^{1+p}}{2c(2+p)} + \frac{\int (d+ex)\left(2cd^2(2+p)-e(2ae+bd(1+p))+e(2cd-\right.}{2c(2+p)}$$

$$= \frac{e(d+ex)^2\left(a+bx+cx^2\right)^{1+p}}{2c(2+p)} - \frac{e\left(be(2cd-be)(2+p)(3+p)-2c(3+2p)\left(cd^2(5+2\right.\right.}{4c^3}$$

$$= \frac{e(d+ex)^2\left(a+bx+cx^2\right)^{1+p}}{2c(2+p)} - \frac{e\left(be(2cd-be)(2+p)(3+p)-2c(3+2p)\left(cd^2(5+2\right.\right.}{4c^3}$$

Mathematica [C] time = 1.52061, size = 558, normalized size = 1.71

$$\frac{1}{4}(a+x(b+cx))^p\left(6d^2ex^2\left(\frac{-\sqrt{b^2-4ac}+b+2cx}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}+b}\right)^{-p}F_1\left(2;-p,-p;3;-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{}{\sqrt{b^2-}}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^3*(a + b*x + c*x^2)^p,x]
```

```
[Out] ((a + x*(b + c*x))^p*((6*d^2*e*x^2*AppellF1[2, -p, -p, 3, (-2*c*x)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])])/(((b - Sqrt[b^2 - 4*a*c
] + 2*c*x)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(b +
 Sqrt[b^2 - 4*a*c]))^p) + (4*d*e^2*x^3*AppellF1[3, -p, -p, 4, (-2*c*x)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])])/(((b - Sqrt[b^2 - 4
*a*c] + 2*c*x)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/
(b + Sqrt[b^2 - 4*a*c]))^p) + (e^3*x^4*AppellF1[4, -p, -p, 5, (-2*c*x)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])])/(((b - Sqrt[b^2 - 4
*a*c] + 2*c*x)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/
(b + Sqrt[b^2 - 4*a*c]))^p) + (2^(1 + p)*d^3*(b - Sqrt[b^2 - 4*a*c] + 2*c*x
)*Hypergeometric2F1[-p, 1 + p, 2 + p, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/(2*S
qrt[b^2 - 4*a*c])])/(c*(1 + p)*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
4*a*c])^p)))/4
```

Maple [F] time = 1.213, size = 0, normalized size = 0.

$$\int (ex+d)^3\left(cx^2+bx+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x+a)^p,x)
```

```
[Out] int((e*x+d)^3*(c*x^2+b*x+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3 \left(cx^2+bx+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3*(c*x^2 + b*x + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\left(cx^2+bx+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(c*x^2 + b*x + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3 \left(cx^2+bx+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(c*x^2 + b*x + a)^p, x)
```

3.2564 $\int (d+ex)^2 \left(a+bx+cx^2\right)^p \, dx$

Optimal. Leaf size=248

$$-\frac{2^p \left(a+bx+cx^2\right)^{p+1} \left(-\frac{-\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}\right)^{-p-1} \left(-2ce(ae+bd(2p+3))+b^2e^2(p+2)+2c^2d^2(2p+3)\right) {}_2F_1\left(-p,p+1;p+\right.}{c^2(p+1)(2p+3)\sqrt{b^2-4ac}}$$

```
[Out] (e*(2*c*d - b*e)*(2 + p)*(a + b*x + c*x^2)^(1 + p))/(2*c^2*(1 + p)*(3 + 2*p
)) + (e*(d + e*x)*(a + b*x + c*x^2)^(1 + p))/(c*(3 + 2*p)) - (2^p*(b^2*e^2*
(2 + p) + 2*c^2*d^2*(3 + 2*p) - 2*c*e*(a*e + b*d*(3 + 2*p)))*(-((b - Sqrt[b
^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x + c*x^2)^(1 + p)
*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqr
t[b^2 - 4*a*c])])/(c^2*Sqrt[b^2 - 4*a*c]*(1 + p)*(3 + 2*p))
```

Rubi [A] time = 0.209372, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {742, 640, 624}

$$-\frac{2^p \left(a+bx+cx^2\right)^{p+1} \left(-\frac{-\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}\right)^{-p-1} \left(-2ce(ae+bd(2p+3))+b^2e^2(p+2)+2c^2d^2(2p+3)\right) {}_2F_1\left(-p,p+1;p+\right.}{c^2(p+1)(2p+3)\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*x + c*x^2)^p,x]
```


```
[Out] (e*(2*c*d - b*e)*(2 + p)*(a + b*x + c*x^2)^(1 + p))/(2*c^2*(1 + p)*(3 + 2*p
)) + (e*(d + e*x)*(a + b*x + c*x^2)^(1 + p))/(c*(3 + 2*p)) - (2^p*(b^2*e^2*
(2 + p) + 2*c^2*d^2*(3 + 2*p) - 2*c*e*(a*e + b*d*(3 + 2*p)))*(-((b - Sqrt[b
^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x + c*x^2)^(1 + p)
*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqr
t[b^2 - 4*a*c])])/(c^2*Sqrt[b^2 - 4*a*c]*(1 + p)*(3 + 2*p))
```


Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
```

```
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[4*p]
```

Rubi steps

$$\int (d+ex)^2\left(a+bx+cx^2\right)^p\,dx = \frac{e(d+ex)\left(a+bx+cx^2\right)^{1+p}}{c(3+2p)} + \frac{\int \left(cd^2(3+2p)-e(ae+bd(1+p))+e(2cd-be)(2+p)x\right)}{c(3+2p)}$$

$$= \frac{e(2cd-be)(2+p)\left(a+bx+cx^2\right)^{1+p}}{2c^2(1+p)(3+2p)} + \frac{e(d+ex)\left(a+bx+cx^2\right)^{1+p}}{c(3+2p)} + \frac{\left(b^2e^2(2+p)+2c^2\right.}{}$$

$$= \frac{e(2cd-be)(2+p)\left(a+bx+cx^2\right)^{1+p}}{2c^2(1+p)(3+2p)} + \frac{e(d+ex)\left(a+bx+cx^2\right)^{1+p}}{c(3+2p)} - \frac{2^p\left(b^2e^2(2+p)+\right.}{}$$

Mathematica [C] time = 0.811754, size = 414, normalized size = 1.67

$$\frac{1}{6}(a+x(b+cx))^p\left(6dex^2\left(\frac{-\sqrt{b^2-4ac}+b+2cx}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}+b}\right)^{-p}F_1\left(2;-p,-p;3;-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{\sqrt{b^2-4ac}}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^2*(a + b*x + c*x^2)^p,x]
```

```
[Out] ((a + x*(b + c*x))^p*((6*d*e*x^2*AppellF1[2, -p, -p, 3, (-2*c*x)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])])/(((b - Sqrt[b^2 - 4*a*c]
+ 2*c*x)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(b + S
qrt[b^2 - 4*a*c]))^p) + (2*e^2*x^3*AppellF1[3, -p, -p, 4, (-2*c*x)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])])/(((b - Sqrt[b^2 - 4*a*c
] + 2*c*x)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(b +
 Sqrt[b^2 - 4*a*c]))^p) + (3*2^p*d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)*Hyperg
eometric2F1[-p, 1 + p, 2 + p, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/(2*Sqrt[b^2
- 4*a*c])])/(c*(1 + p)*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c])^
p)))/6
```

Maple [F] time = 1.245, size = 0, normalized size = 0.

$$\int (ex+d)^2\left(cx^2+bx+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+b*x+a)^p,x)
```

```
[Out] int((e*x+d)^2*(c*x^2+b*x+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^2\left(cx^2+bx+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2*(c*x^2 + b*x + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^2+2\,dex+d^2\right)\left(cx^2+bx+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(c*x^2 + b*x + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+b*x+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^2\left(cx^2+bx+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(c*x^2 + b*x + a)^p, x)
```

3.2565 $\int (d+ex)\left(a+bx+cx^2\right)^p\,dx$

Optimal. Leaf size=160

$$\frac{e\left(a+bx+cx^2\right)^{p+1}}{2c(p+1)}-\frac{2^p(2cd-be)\left(-\frac{-\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+bx+cx^2\right)^{p+1}\,{}_2F_1\left(-p,p+1;p+2;\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c(p+1)\sqrt{b^2-4ac}}$$

```
[Out] (e*(a + b*x + c*x^2)^(1 + p))/(2*c*(1 + p)) - (2^p*(2*c*d - b*e)*(-((b - Sq
rt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x + c*x^2)^(1
+ p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2
*Sqrt[b^2 - 4*a*c])])/(c*Sqrt[b^2 - 4*a*c]*(1 + p))
```

Rubi [A] time = 0.0451696, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {640, 624}

$$\frac{e\left(a+bx+cx^2\right)^{p+1}}{2c(p+1)}-\frac{2^p(2cd-be)\left(-\frac{-\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+bx+cx^2\right)^{p+1}\,{}_2F_1\left(-p,p+1;p+2;\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c(p+1)\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*x + c*x^2)^p,x]
```

```
[Out] (e*(a + b*x + c*x^2)^(1 + p))/(2*c*(1 + p)) - (2^p*(2*c*d - b*e)*(-((b - Sq
rt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x + c*x^2)^(1
+ p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2
*Sqrt[b^2 - 4*a*c])])/(c*Sqrt[b^2 - 4*a*c]*(1 + p))
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rubi steps

$$\int (d+ex)\left(a+bx+cx^2\right)^p\,dx=\frac{e\left(a+bx+cx^2\right)^{1+p}}{2c(1+p)}+\frac{(2cd-be)\int\left(a+bx+cx^2\right)^p\,dx}{2c}$$

$$=\frac{e\left(a+bx+cx^2\right)^{1+p}}{2c(1+p)}-\frac{2^p(2cd-be)\left(-\frac{b-\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}\right)^{-1-p}\left(a+bx+cx^2\right)^{1+p}\,{}_2F_1\left(-p,1+p\right.}{c\sqrt{b^2-4ac}(1+p)}$$

Mathematica [C] time = 0.384858, size = 268, normalized size = 1.68

$$\frac{1}{2}(a+x(b+cx))^p\left(ex^2\left(\frac{-\sqrt{b^2-4ac}+b+2cx}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}+b}\right)^{-p}F_1\left(2;-p,-p;3;-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{\sqrt{b^2-4a}}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)*(a + b*x + c*x^2)^p,x]
```

```
[Out] ((a + x*(b + c*x))^p*((e*x^2*AppellF1[2, -p, -p, 3, (-2*c*x)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])])/(((b - Sqrt[b^2 - 4*a*c] + 2*
c*x)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(b + Sqrt[
b^2 - 4*a*c]))^p) + (2^p*d*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)*Hypergeometric2F
1[-p, 1 + p, 2 + p, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/(2*Sqrt[b^2 - 4*a*c])]
)/(c*(1 + p)*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c])^p)))/2
```

Maple [F] time = 1.055, size = 0, normalized size = 0.

$$\int (ex+d)\left(cx^2+bx+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+b*x+a)^p,x)
```

```
[Out] int((e*x+d)*(c*x^2+b*x+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)\left(cx^2+bx+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(c*x^2 + b*x + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex+d)\left(cx^2+bx+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)*(c*x^2 + b*x + a)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d+ex)\left(a+bx+cx^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+b*x+a)**p,x)
```

```
[Out] Integral((d + e*x)*(a + b*x + c*x**2)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)\left(cx^2+bx+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(c*x^2 + b*x + a)^p, x)
```

3.2566 $\int \left(a+bx+cx^2\right)^p\,dx$

Optimal. Leaf size=122

$$-\frac{2^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+bx+cx^2\right)^{p+1}{}_2F_1\left(-p,p+1;p+2;\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{(p+1)\sqrt{b^2-4ac}}$$

```
[Out] -((2^(1 + p)*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]))^(-1 - p
)*(a + b*x + c*x^2)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b
^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(1 + p)))
```

Rubi [A] time = 0.0149388, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {624}

$$-\frac{2^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+bx+cx^2\right)^{p+1}{}_2F_1\left(-p,p+1;p+2;\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{(p+1)\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^p,x]
```

```
[Out] -((2^(1 + p)*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]))^(-1 - p
)*(a + b*x + c*x^2)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b
^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(1 + p)))
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^p\,dx=-\frac{2^{1+p}\left(-\frac{b-\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}\right)^{-1-p}\left(a+bx+cx^2\right)^{1+p}{}_2F_1\left(-p,1+p;2+p;\frac{b+\sqrt{b^2-4ac}+2cx}{2\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}(1+p)}$$

Mathematica [A] time = 0.0611328, size = 126, normalized size = 1.03

$$\frac{2^{p-1}\left(-\sqrt{b^2-4ac}+b+2cx\right)\left(\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}\right)^{-p}(a+x(b+cx))^p\,{}_2F_1\left(-p,p+1;p+2;\frac{-b-2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^p,x]
```

```
[Out] (2^(-1 + p)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)*(a + x*(b + c*x))^p*Hypergeomet
ric2F1[-p, 1 + p, 2 + p, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/(2*Sqrt[b^2 - 4*a
*c])])/(c*(1 + p)*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c])^p)
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \left(c x^2+b x+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^p,x)
```

```
[Out] int((c*x^2+b*x+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c x^2+b x+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(c x^2+b x+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b x+c x^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**p,x)
```

```
[Out] Integral((a + b*x + c*x**2)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^p, x)
```

3.2567 $\int \frac{\left(a+bx+cx^2\right)^p}{d+ex}\,dx$

Optimal. Leaf size=184

$$\frac{2^{2p-1}\left(a+bx+cx^2\right)^p\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{-p}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{2cd-\left(b-\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{c}}{2(d+ex)}\right)}{ep}$$

```
[Out] (2^(-1 + 2*p)*(a + b*x + c*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (2*c*d -
(b - Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x)), (2*d - ((b + Sqrt[b^2 - 4*a*c])
*e)/c)/(2*(d + e*x))])/(e*p*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*
x)))^p*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^p)
```

Rubi [A] time = 0.184456, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {758, 133}

$$\frac{2^{2p-1}\left(a+bx+cx^2\right)^p\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{-p}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{2cd-\left(b-\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{c}}{2(d+ex)}\right)}{ep}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^p/(d + e*x),x]
```


```
[Out] (2^(-1 + 2*p)*(a + b*x + c*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (2*c*d -
(b - Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x)), (2*d - ((b + Sqrt[b^2 - 4*a*c])
*e)/c)/(2*(d + e*x))])/(e*p*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*
x)))^p*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^p)
```


Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, -Dist[((1/(d + e*x))^(2*p)*(a + b*
x + c*x^2)^p)/(e*((e*(b - q + 2*c*x))/(2*c*(d + e*x)))^p*((e*(b + q + 2*c*x
))/(2*c*(d + e*x)))^p), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - (e*(b -
q))/(2*c))*x, x]^p*Simp[1 - (d - (e*(b + q))/(2*c))*x, x]^p, x], x, 1/(d +
e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m,
0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^p}{d+ex}\,dx = -\frac{\left(2^{2p}\left(\frac{1}{d+ex}\right)^{2p}\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{-p}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{-p}\left(a+bx+cx^2\right)^p\right)\text{Subst}\left(\int x^{1-2(1+p)}\left(1\right.\right.}{e}$$

$$= \frac{2^{-1+2p}\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{-p}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{-p}\left(a+bx+cx^2\right)^p F_1\left(-2p;-p,-p;1-2p;\frac{2cd-\left(b-\right.}{2c(}\right.}{ep}$$

Mathematica [A] time = 0.262125, size = 182, normalized size = 0.99

$$\frac{2^{2p-1}(a+x(b+cx))^p\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{-p}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{2cd-\left(b+\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2cd-be+\sqrt{b^2-4ace}}{2cd+2cex}\right)}{ep}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)^p/(d + e*x),x]
```

```
[Out] (2^(-1 + 2*p)*(a + x*(b + c*x))^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x)), (2*c*d - b*e + Sqrt[b^2 - 4*a*c
]*e)/(2*c*d + 2*c*e*x)]])/(e*p*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d +
e*x)))^p*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^p)
```

Maple [F] time = 1.313, size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^p/(e*x+d),x)
```

```
[Out] int((c*x^2+b*x+a)^p/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^p/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+bx+a\right)^p}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^p/(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**p/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^2+bx+a\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^p/(e*x + d), x)
```

3.2568 $\int \frac{(a+bx+cx^2)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=196

$$-\frac{4^p\left(a+bx+cx^2\right)^p\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{-p}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{-p}F_1\left(1-2p;-p,-p;2(1-p);\frac{2cd-\left(b-\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{c}}{2(d+ex)}\right)}{e(1-2p)(d+ex)}$$

```
[Out] -((4^p*(a + b*x + c*x^2)^p*AppellF1[1 - 2*p, -p, -p, 2*(1 - p), (2*c*d - (b
 - Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x)), (2*d - ((b + Sqrt[b^2 - 4*a*c])*e
)/c)/(2*(d + e*x))])/(e*(1 - 2*p)*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(
d + e*x)))^p*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^p*(d + e*x
)))
```

Rubi [A] time = 0.109219, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {758, 133}

$$-\frac{4^p\left(a+bx+cx^2\right)^p\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{-p}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{-p}F_1\left(1-2p;-p,-p;2(1-p);\frac{2cd-\left(b-\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{c}}{2(d+ex)}\right)}{e(1-2p)(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^p/(d + e*x)^2,x]
```


```
[Out] -((4^p*(a + b*x + c*x^2)^p*AppellF1[1 - 2*p, -p, -p, 2*(1 - p), (2*c*d - (b
 - Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x)), (2*d - ((b + Sqrt[b^2 - 4*a*c])*e
)/c)/(2*(d + e*x))])/(e*(1 - 2*p)*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(
d + e*x)))^p*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^p*(d + e*x
)))
```


Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, -Dist[((1/(d + e*x))^(2*p)*(a + b*
x + c*x^2)^p)/(e*((e*(b - q + 2*c*x))/(2*c*(d + e*x)))^p*((e*(b + q + 2*c*x
))/(2*c*(d + e*x)))^p), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - (e*(b -
q))/(2*c))*x, x]^p*Simp[1 - (d - (e*(b + q))/(2*c))*x, x]^p, x], x, 1/(d +
e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m,
0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^p}{(d+ex)^2}\,dx = -\frac{\left(2^{2p}\left(\frac{1}{d+ex}\right)^{2p}\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{-p}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{-p}\left(a+bx+cx^2\right)^p\right)\text{Subst}\left(\int x^{2-2(1+p)}\left(1-\right.\right.}{e}$$

$$= -\frac{4^p\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{-p}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{-p}\left(a+bx+cx^2\right)^p F_1\left(1-2p;-p,-p;2(1-p);\frac{2cd-\left(b-\sqrt{\right.}}{2c(d+}\right.}{e(1-2p)(d+ex)}$$

Mathematica [A] time = 0.265387, size = 191, normalized size = 0.97

$$\frac{4^p(a+x(b+cx))^p\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{-p}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{-p}F_1\left(1-2p;-p,-p;2-2p;\frac{2cd-\left(b+\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2cd-be+\sqrt{b^2-4ace}}{2cd+2cex}\right)}{e(2p-1)(d+ex)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)^p/(d + e*x)^2,x]
```

```
[Out] (4^p*(a + x*(b + c*x))^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x)), (2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/(
2*c*d + 2*c*e*x)])/(e*(-1 + 2*p)*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d
 + e*x)))^p*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^p*(d + e*x)
)
```

Maple [F] time = 1.339, size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^p/(e*x+d)^2,x)
```

```
[Out] int((c*x^2+b*x+a)^p/(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^p/(e*x + d)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+bx+a\right)^p}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^p/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**p/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^p/(e*x + d)^2, x)
```

3.2569 $\int \frac{(a+bx+cx^2)^p}{(d+ex)^3}\,dx$

Optimal. Leaf size=200

$$-\frac{2^{2p-1}\left(a+bx+cx^2\right)^p\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{-p}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{-p}F_1\left(2(1-p);-p,-p;3-2p;\frac{2cd-\left(b-\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{c}}{2(d+ex)}\right)}{e(1-p)(d+ex)^2}$$

```
[Out] -((2^(-1 + 2*p)*(a + b*x + c*x^2)^p*AppellF1[2*(1 - p), -p, -p, 3 - 2*p, (2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x)), (2*d - ((b + Sqrt[b^2 -
4*a*c])*e)/c)/(2*(d + e*x))])/(e*(1 - p)*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x
))/(c*(d + e*x)))^p*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^p*(
d + e*x)^2))
```

Rubi [A] time = 0.114028, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {758, 133}

$$-\frac{2^{2p-1}\left(a+bx+cx^2\right)^p\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{-p}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{-p}F_1\left(2(1-p);-p,-p;3-2p;\frac{2cd-\left(b-\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{c}}{2(d+ex)}\right)}{e(1-p)(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^p/(d + e*x)^3,x]
```

```
[Out] -((2^(-1 + 2*p)*(a + b*x + c*x^2)^p*AppellF1[2*(1 - p), -p, -p, 3 - 2*p, (2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x)), (2*d - ((b + Sqrt[b^2 -
4*a*c])*e)/c)/(2*(d + e*x))])/(e*(1 - p)*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x
))/(c*(d + e*x)))^p*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^p*(
d + e*x)^2))
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, -Dist[((1/(d + e*x))^(2*p)*(a + b*
x + c*x^2)^p)/(e*((e*(b - q + 2*c*x))/(2*c*(d + e*x)))^p*((e*(b + q + 2*c*x
))/(2*c*(d + e*x)))^p), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - (e*(b -
q))/(2*c))*x, x]^p*Simp[1 - (d - (e*(b + q))/(2*c))*x, x]^p, x], x, 1/(d +
e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m,
0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^p}{(d+ex)^3}\,dx = -\frac{2^{2p}\left(\frac{1}{d+ex}\right)^{2p}\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{-p}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{-p}\left(a+bx+cx^2\right)^p \text{Subst}\left(\int x^{3-2(1+p)}\left(1 \cdots\right.\right.}{e}$$

$$= -\frac{2^{-1+2p}\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{-p}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{-p}\left(a+bx+cx^2\right)^p F_1\left(2(1-p);-p,-p;3-2p;\cdots\right.}{e(1-p)(d+ex)^2}$$

Mathematica [A] time = 0.385815, size = 193, normalized size = 0.96

$$\frac{2^{2p-1}(a+x(b+cx))^p\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{-p}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{c(d+ex)}\right)^{-p}F_1\left(2-2p;-p,-p;3-2p;\frac{2cd-\left(b+\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2cd-be+\sqrt{b^2-4a\cdots}}{2cd+2cex}\right.}{e(p-1)(d+ex)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)^p/(d + e*x)^3,x]
```

```
[Out] (2^(-1 + 2*p)*(a + x*(b + c*x))^p*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x)), (2*c*d - b*e + Sqrt[b^2 - 4*
a*c]*e)/(2*c*d + 2*c*e*x)]])/(e*(-1 + p)*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)
)/(c*(d + e*x)))^p*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(d + e*x)))^p*(d
 + e*x)^2)
```

Maple [F] time = 1.32, size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^p}{\left(ex+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^p/(e*x+d)^3,x)
```

```
[Out] int((c*x^2+b*x+a)^p/(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^p}{\left(ex+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^p/(e*x + d)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+bx+a\right)^p}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^p/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**p/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^p}{\left(ex+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^p/(e*x + d)^3, x)
```

3.2570 $\int (d+ex)^{3/2} \left(a+bx+cx^2\right)^p \, dx$

Optimal. Leaf size=185

$$\frac{2(d+ex)^{5/2}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(\frac{5}{2};-p,-p;\frac{7}{2};\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{5e}$$

```
[Out] (2*(d + e*x)^(5/2)*(a + b*x + c*x^2)^p*AppellF1[5/2, -p, -p, 7/2, (2*c*(d +
 e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)])/(5*e*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4
*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))^p)
```

Rubi [A] time = 0.150212, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {759, 133}

$$\frac{2(d+ex)^{5/2}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(\frac{5}{2};-p,-p;\frac{7}{2};\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{5e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*(a + b*x + c*x^2)^p,x]
```

```
[Out] (2*(d + e*x)^(5/2)*(a + b*x + c*x^2)^p*AppellF1[5/2, -p, -p, 7/2, (2*c*(d +
 e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)])/(5*e*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4
*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))^p)
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (d+ex)^{3/2}\left(a+bx+cx^2\right)^p\,dx = \frac{\left(\left(a+bx+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}\right)^{-p}\left(1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}\right)^{-p}\right)\text{Subst}\left(\int x^{3/2}\left(1-\frac{2c}{2cd-\left(b-\sqrt{\right.}\right.}\right.\right.}{e}$$

$$= \frac{2(d+ex)^{5/2}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{-p}F_1\left(\frac{5}{2};-p,-\right.}{5e}$$

Mathematica [A] time = 1.44695, size = 239, normalized size = 1.29

$$\frac{2(d+ex)^{5/2}(a+x(b+cx))^p\left(\frac{e\left(e\sqrt{\frac{b^2-4ac}{e^2}}-b-2cx\right)}{e\left(e\sqrt{\frac{b^2-4ac}{e^2}}-b\right)+2cd}\right)^{-p}\left(\frac{e\left(e\sqrt{\frac{b^2-4ac}{e^2}}+b+2cx\right)}{e\left(e\sqrt{\frac{b^2-4ac}{e^2}}+b\right)-2cd}\right)^{-p}F_1\left(\frac{5}{2};-p,-p;\frac{7}{2};\frac{2c(d+ex)}{2cd-e\left(b+\sqrt{\frac{b^2-4ac}{e^2}}e\right)},\frac{2c(d+ex)}{2cd+e\left(\sqrt{\frac{b^2-4ac}{e^2}}e-b\right)}\right)}{5e}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^(3/2)*(a + b*x + c*x^2)^p,x]
```

```
[Out] (2*(d + e*x)^(5/2)*(a + x*(b + c*x))^p*AppellF1[5/2, -p, -p, 7/2, (2*c*(d +
 e*x))/(2*c*d - e*(b + Sqrt[(b^2 - 4*a*c)/e^2]*e)), (2*c*(d + e*x))/(2*c*d
+ e*(-b + Sqrt[(b^2 - 4*a*c)/e^2]*e))]])/(5*e*((e*(-b + Sqrt[(b^2 - 4*a*c)/e
^2]*e - 2*c*x))/(2*c*d + e*(-b + Sqrt[(b^2 - 4*a*c)/e^2]*e)))^p*((e*(b + Sq
rt[(b^2 - 4*a*c)/e^2]*e + 2*c*x))/(-2*c*d + e*(b + Sqrt[(b^2 - 4*a*c)/e^2]*
e)))^p)
```

Maple [F] time = 1.269, size = 0, normalized size = 0.

$$\int (ex+d)^{\frac{3}{2}}\left(cx^2+bx+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(c*x^2+b*x+a)^p,x)
```

```
[Out] int((e*x+d)^(3/2)*(c*x^2+b*x+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^{\frac{3}{2}}\left(cx^2+bx+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*(c*x^2 + b*x + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex+d)^{\frac{3}{2}}\left(cx^2+bx+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^(3/2)*(c*x^2 + b*x + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(c*x**2+b*x+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.2571 $\int \sqrt{d+ex}\left(a+bx+cx^2\right)^p\,dx$

Optimal. Leaf size=185

$$\frac{2(d+ex)^{3/2}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{3e}$$

```
[Out] (2*(d + e*x)^(3/2)*(a + b*x + c*x^2)^p*AppellF1[3/2, -p, -p, 5/2, (2*c*(d +
 e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)])/(3*e*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4
*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))^p)
```

Rubi [A] time = 0.10627, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {759, 133}

$$\frac{2(d+ex)^{3/2}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(a + b*x + c*x^2)^p,x]
```

```
[Out] (2*(d + e*x)^(3/2)*(a + b*x + c*x^2)^p*AppellF1[3/2, -p, -p, 5/2, (2*c*(d +
 e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)])/(3*e*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4
*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))^p)
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \sqrt{d+ex}\left(a+bx+cx^2\right)^p\,dx = \frac{\left(\left(a+bx+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}\right)^{-p}\left(1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}\right)^{-p}\right)\text{Subst}\left(\int\sqrt{x}\left(1-\frac{2}{2cd-\left(b-\right.}\right.\right.}{e}$$

$$= \frac{2(d+ex)^{3/2}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{-p}F_1\left(\frac{3}{2};-p\right.}{3e}$$

Mathematica [A] time = 0.482945, size = 211, normalized size = 1.14

$$\frac{2^{1-2p}(d+ex)^{3/2}(a+x(b+cx))^p\left(\frac{e\left(\sqrt{b^2-4ac}-b-2cx\right)}{4e\left(\sqrt{b^2-4ac}-b\right)+8cd}\right)^{-p}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{e\left(\sqrt{b^2-4ac}+b\right)-2cd}\right)^{-p}F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd+\left(\sqrt{b^2-4ac-}\right.}\right.}{3e}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d + e*x]*(a + b*x + c*x^2)^p,x]
```

```
[Out] (2^(1 - 2*p)*(d + e*x)^(3/2)*(a + x*(b + c*x))^p*AppellF1[3/2, -p, -p, 5/2,
 (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*
d + (-b + Sqrt[b^2 - 4*a*c])*e)])/(3*e*((e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)
)/(8*c*d + 4*(-b + Sqrt[b^2 - 4*a*c])*e))^p*((e*(b + Sqrt[b^2 - 4*a*c] + 2*
c*x))/(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e))^p)
```

Maple [F] time = 1.284, size = 0, normalized size = 0.

$$\int \sqrt{ex+d}\left(cx^2+bx+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(c*x^2+b*x+a)^p,x)
```

```
[Out] int((e*x+d)^(1/2)*(c*x^2+b*x+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex+d}\left(cx^2+bx+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*(c*x^2 + b*x + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{ex+d}\left(cx^2+bx+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*(c*x^2 + b*x + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(c*x**2+b*x+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex+d}\left(cx^2+bx+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(c*x^2 + b*x + a)^p, x)
```

3.2572 $\int \frac{(a+bx+cx^2)^p}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=183

$$\frac{2\sqrt{d+ex}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e}$$

```
[Out] (2*Sqrt[d + e*x]*(a + b*x + c*x^2)^p*AppellF1[1/2, -p, -p, 3/2, (2*c*(d + e
*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqr
t[b^2 - 4*a*c])*e)])/(e*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c
])*e))^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))^p)
```

Rubi [A] time = 0.1046, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {759, 133}

$$\frac{2\sqrt{d+ex}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^p/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[d + e*x]*(a + b*x + c*x^2)^p*AppellF1[1/2, -p, -p, 3/2, (2*c*(d + e
*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqr
t[b^2 - 4*a*c])*e)])/(e*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c
])*e))^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))^p)
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^p}{\sqrt{d+ex}}\,dx = \frac{\left(\left(a+bx+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}\right)^{-p}\left(1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1-\frac{2cx}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^p\left(1-\frac{2c}{2cd-\left(b+\sqrt{\right.}\right.}\right.}{\sqrt{x}}\right.}{e}$$

$$= \frac{2\sqrt{d+ex}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{-p}F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};\frac{2c(}{2cd-(b-}\right.}{e}$$

Mathematica [A] time = 0.370982, size = 208, normalized size = 1.14

$$\frac{2^{1-2p}\sqrt{d+ex}(a+x(b+cx))^p\left(\frac{e\left(\sqrt{b^2-4ac}-b-2cx\right)}{4e\left(\sqrt{b^2-4ac}-b\right)+8cd}\right)^{-p}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{e\left(\sqrt{b^2-4ac}+b\right)-2cd}\right)^{-p}F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd+\left(\sqrt{b^2-4ac}-b\right)e}\right)}{e}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)^p/Sqrt[d + e*x],x]
```

```
[Out] (2^(1 - 2*p)*Sqrt[d + e*x]*(a + x*(b + c*x))^p*AppellF1[1/2, -p, -p, 3/2, (
2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d
+ (-b + Sqrt[b^2 - 4*a*c])*e)]])/(e*((e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(8
*c*d + 4*(-b + Sqrt[b^2 - 4*a*c])*e))^p*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)
)/(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e))^p)
```

Maple [F] time = 1.238, size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^p \frac{1}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^p/(e*x+d)^(1/2),x)
```

```
[Out] int((c*x^2+b*x+a)^p/(e*x+d)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^p}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^p/sqrt(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+bx+a\right)^p}{\sqrt{ex+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^p/sqrt(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx+cx^2\right)^p}{\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**p/(e*x+d)**(1/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)**p/sqrt(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^2+bx+a\right)^p}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^p/sqrt(e*x + d), x)
```

3.2573 $\int \frac{(a+bx+cx^2)^p}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=183

$$-\frac{2\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(-\frac{1}{2};-p,-p;\frac{1}{2};\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e\sqrt{d+ex}}$$

```
[Out] (-2*(a + b*x + c*x^2)^p*AppellF1[-1/2, -p, -p, 1/2, (2*c*(d + e*x))/(2*c*d
- (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e)])/(e*Sqrt[d + e*x]*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a
*c])*e))^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))^p)
```

Rubi [A] time = 0.104666, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {759, 133}

$$-\frac{2\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(-\frac{1}{2};-p,-p;\frac{1}{2};\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^p/(d + e*x)^(3/2),x]
```


```
[Out] (-2*(a + b*x + c*x^2)^p*AppellF1[-1/2, -p, -p, 1/2, (2*c*(d + e*x))/(2*c*d
- (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e)])/(e*Sqrt[d + e*x]*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a
*c])*e))^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))^p)
```


Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^p}{(d+ex)^{3/2}}\,dx = \frac{\left(\left(a+bx+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}\right)^{-p}\left(1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1-\frac{2cx}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^p\left(1-\frac{}{2cd-}\right.}{x^{3/2}}\right.}{e}$$

$$= -\frac{2\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{-p}F_1\left(-\frac{1}{2};-p,-p;\frac{1}{2};\frac{2c(d+}{2cd-\left(b-\sqrt{b}\right.}\right.}{e\sqrt{d+ex}}$$

Mathematica [A] time = 0.406327, size = 209, normalized size = 1.14

$$-\frac{2^{1-2p}(a+x(b+cx))^p\left(\frac{e\left(\sqrt{b^2-4ac}-b-2cx\right)}{4e\left(\sqrt{b^2-4ac}-b\right)+8cd}\right)^{-p}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{e\left(\sqrt{b^2-4ac}+b\right)-2cd}\right)^{-p}F_1\left(-\frac{1}{2};-p,-p;\frac{1}{2};\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd+\left(\sqrt{b^2-4ac}-b\right)e}\right)}{e\sqrt{d+ex}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)^p/(d + e*x)^(3/2),x]
```

```
[Out] -((2^(1 - 2*p)*(a + x*(b + c*x))^p*AppellF1[-1/2, -p, -p, 1/2, (2*c*(d + e*
x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d + (-b + Sqr
t[b^2 - 4*a*c])*e)])/(e*((e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(8*c*d + 4*(-
b + Sqrt[b^2 - 4*a*c])*e))^p*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*d +
 (b + Sqrt[b^2 - 4*a*c])*e))^p*Sqrt[d + e*x]))
```

Maple [F] time = 1.276, size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^p (ex+d)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^p/(e*x+d)^(3/2),x)
```

```
[Out] int((c*x^2+b*x+a)^p/(e*x+d)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^p}{(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^p/(e*x + d)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex+d}\left(cx^2+bx+a\right)^p}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*(c*x^2 + b*x + a)^p/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**p/(e*x+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^p}{(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^p/(e*x + d)^(3/2), x)
```

3.2574 $\int (d+ex)^{-2p}\left(a+bx+cx^2\right)^p\,dx$

Optimal. Leaf size=195

$$\frac{(d+ex)^{1-2p}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(1-2p;-p,-p;2-2p;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\right.}{e(1-2p)}$$

```
[Out] ((d + e*x)^(1 - 2*p)*(a + b*x + c*x^2)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p,
 (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 - 2*p)*(1 - (2*c*(d + e*x))/(2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2
- 4*a*c])*e))^p)
```

Rubi [A] time = 0.105212, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {759, 133}

$$\frac{(d+ex)^{1-2p}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(1-2p;-p,-p;2-2p;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\right.}{e(1-2p)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^p/(d + e*x)^(2*p),x]
```

```
[Out] ((d + e*x)^(1 - 2*p)*(a + b*x + c*x^2)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p,
 (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 - 2*p)*(1 - (2*c*(d + e*x))/(2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2
- 4*a*c])*e))^p)
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (d+ex)^{-2p}\left(a+bx+cx^2\right)^p\,dx = \frac{\left(\left(a+bx+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}\right)^{-p}\left(1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}\right)^{-p}\right)\text{Subst}\left(\int x^{-2p}\left(1-\frac{\cdots}{2cd-\left(b-\cdots\right.}\right.\right.}{e}$$

$$= \frac{(d+ex)^{1-2p}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{-p}F_1\left(1-2p\cdots\right.}{e(1-2p)}$$

Mathematica [A] time = 0.331343, size = 214, normalized size = 1.1

$$-\frac{(d+ex)^{1-2p}(a+x(b+cx))^p\left(\frac{e\left(\sqrt{b^2-4ac}-b-2cx\right)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd}\right)^{-p}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{e\left(\sqrt{b^2-4ac}+b\right)-2cd}\right)^{-p}F_1\left(1-2p;-p,-p;2-2p;\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e},\frac{2c(d+\cdots}{2cd+\left(\sqrt{b^2-\cdots}\right.}\right.}{e(2p-1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)^p/(d + e*x)^(2*p),x]
```

```
[Out] -(((d + e*x)^(1 - 2*p)*(a + x*(b + c*x))^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*
p, (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*
c*d + (-b + Sqrt[b^2 - 4*a*c])*e)])/(e*(-1 + 2*p)*((e*(-b + Sqrt[b^2 - 4*a*
c] - 2*c*x))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e))^p*((e*(b + Sqrt[b^2 - 4*
a*c] + 2*c*x))/(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e))^p))
```

Maple [F] time = 1.26, size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^p}{(ex+d)^{2p}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^p/((e*x+d)^(2*p)),x)
```

```
[Out] int((c*x^2+b*x+a)^p/((e*x+d)^(2*p)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^p}{(ex+d)^{2p}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/((e*x+d)^(2*p)),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^p/(e*x + d)^(2*p), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+bx+a\right)^p}{(ex+d)^{2p}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/((e*x+d)^(2*p)),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^p/(e*x + d)^(2*p), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**p/((e*x+d)**(2*p)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^p}{(ex+d)^{2p}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p/((e*x+d)^(2*p)),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^p/(e*x + d)^(2*p), x)
```

3.2575 $\int (d+ex)^{-1-2p} \left(a+bx+cx^2\right)^p \, dx$

Optimal. Leaf size=190

$$-\frac{(d+ex)^{-2p}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2ep}$$

```
[Out] -((a + b*x + c*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (2*c*(d + e*x))/(2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*
a*c])*e)])/(2*e*p*(d + e*x)^(2*p)*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b
^2 - 4*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)
)^p)
```

Rubi [A] time = 0.105936, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {759, 133}

$$-\frac{(d+ex)^{-2p}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2ep}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(-1 - 2*p)*(a + b*x + c*x^2)^p,x]
```


```
[Out] -((a + b*x + c*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (2*c*(d + e*x))/(2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*
a*c])*e)])/(2*e*p*(d + e*x)^(2*p)*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b
^2 - 4*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)
)^p)
```


Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] && !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (d+ex)^{-1-2p}\left(a+bx+cx^2\right)^p\,dx = \frac{\left(\left(a+bx+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}\right)^{-p}\left(1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}\right)^{-p}\right)\text{Subst}\left(\int x^{-1-2p}\left(1-\right.\right.}{e}$$

$$= -\frac{(d+ex)^{-2p}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{-p}F_1\left(\right.}{2ep}$$

Mathematica [A] time = 0.373094, size = 208, normalized size = 1.09

$$-\frac{(d+ex)^{-2p}(a+x(b+cx))^p\left(\frac{e\left(\sqrt{b^2-4ac}-b-2cx\right)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd}\right)^{-p}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{e\left(\sqrt{b^2-4ac}+b\right)-2cd}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e},\frac{2c(d+e}{2cd+\left(\sqrt{b^2-}\right.}\right.}{2ep}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^(-1 - 2*p)*(a + b*x + c*x^2)^p,x]
```

```
[Out] -((a + x*(b + c*x))^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (2*c*(d + e*x))/(2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d + (-b + Sqrt[b^2 - 4
*a*c])*e)])/(2*e*p*((e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*d + (-b + Sqr
t[b^2 - 4*a*c])*e))^p*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*d + (b + S
qrt[b^2 - 4*a*c])*e))^p*(d + e*x)^(2*p))
```

Maple [F] time = 1.25, size = 0, normalized size = 0.

$$\int (ex+d)^{-1-2p}\left(cx^2+bx+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(-1-2*p)*(c*x^2+b*x+a)^p,x)
```

```
[Out] int((e*x+d)^(-1-2*p)*(c*x^2+b*x+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^p(ex+d)^{-2p-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-1-2*p)*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^p*(e*x + d)^(-2*p - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2+bx+a\right)^p(ex+d)^{-2p-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-1-2*p)*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^p*(e*x + d)^(-2*p - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(-1-2*p)*(c*x**2+b*x+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^p (ex+d)^{-2p-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-1-2*p)*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^p*(e*x + d)^(-2*p - 1), x)
```

3.2576 $\int (d+ex)^{-2-2p}\left(a+bx+cx^2\right)^p\,dx$

Optimal. Leaf size=248

$$\frac{\left(-\sqrt{b^2-4ac}+b+2cx\right)(d+ex)^{-2p-1}\left(a+bx+cx^2\right)^p\left(\frac{\left(\sqrt{b^2-4ac}+b+2cx\right)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}{\left(-\sqrt{b^2-4ac}+b+2cx\right)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}\right)^{-p}{}_2F_1\left(-2p-1,-p;-2p;-\cdots\right)}{(2p+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}$$

```
[Out] ((b - Sqrt[b^2 - 4*a*c] + 2*c*x)*(d + e*x)^(-1 - 2*p)*(a + b*x + c*x^2)^p*H
ypergeometric2F1[-1 - 2*p, -p, -2*p, (-4*c*Sqrt[b^2 - 4*a*c]*(d + e*x))/((2
*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))])/((2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e)*(1 + 2*p)*(((2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(
b - Sqrt[b^2 - 4*a*c] + 2*c*x)))^p)
```

Rubi [A] time = 0.12183, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {726}

$$\frac{\left(-\sqrt{b^2-4ac}+b+2cx\right)(d+ex)^{-2p-1}\left(a+bx+cx^2\right)^p\left(\frac{\left(\sqrt{b^2-4ac}+b+2cx\right)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}{\left(-\sqrt{b^2-4ac}+b+2cx\right)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}\right)^{-p}{}_2F_1\left(-2p-1,-p;-2p;-\cdots\right)}{(2p+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(-2 - 2*p)*(a + b*x + c*x^2)^p,x]
```

```
[Out] ((b - Sqrt[b^2 - 4*a*c] + 2*c*x)*(d + e*x)^(-1 - 2*p)*(a + b*x + c*x^2)^p*H
ypergeometric2F1[-1 - 2*p, -p, -2*p, (-4*c*Sqrt[b^2 - 4*a*c]*(d + e*x))/((2
*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))])/((2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e)*(1 + 2*p)*(((2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(
b - Sqrt[b^2 - 4*a*c] + 2*c*x)))^p)
```

Rule 726

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((b - Rt[b^2 - 4*a*c, 2] + 2*c*x)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^p*Hypergeometric2F1[m + 1, -p, m + 2, (-4*c*Rt[b^2 - 4*a*c, 2]*(
d + e*x))/((2*c*d - b*e - e*Rt[b^2 - 4*a*c, 2])*(b - Rt[b^2 - 4*a*c, 2] + 2
*c*x))])/((m + 1)*(2*c*d - b*e + e*Rt[b^2 - 4*a*c, 2])*(((2*c*d - b*e + e*R
t[b^2 - 4*a*c, 2])*(b + Rt[b^2 - 4*a*c, 2] + 2*c*x))/((2*c*d - b*e - e*Rt[b
^2 - 4*a*c, 2])*(b - Rt[b^2 - 4*a*c, 2] + 2*c*x)))^p), x] /; FreeQ[{a, b, c
, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &
& NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int (d+ex)^{-2-2p}\left(a+bx+cx^2\right)^p\,dx=\frac{\left(b-\sqrt{b^2-4ac}+2cx\right)\left(\frac{\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\left(b-\sqrt{b^2-4ac}+2cx\right)}\right)^{-p}(d+ex)^{-1-2p}\left(a+bx\cdots\right.}{\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\cdots\right.}$$

Mathematica [A] time = 0.375191, size = 242, normalized size = 0.98

$$-\frac{\left(\sqrt{b^2-4ac}+b+2cx\right)(d+ex)^{-2p-1}(a+x(b+cx))^p\left(\frac{\left(\sqrt{b^2-4ac}+b+2cx\right)\left(e\left(\sqrt{b^2-4ac}-b\right)+2cd\right)}{\left(\sqrt{b^2-4ac}-b-2cx\right)\left(e\left(\sqrt{b^2-4ac}+b\right)-2cd\right)}\right)^{-p-1}{}_2F_1\left(-2p-1,-p;-2p;-\frac{}{\left(\left(b+\sqrt{\right.\right.}}\right.}{(2p+1)\left(e\left(\sqrt{b^2-4ac}+b\right)-2cd\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(-2 - 2*p)*(a + b*x + c*x^2)^p,x]
```

```
[Out] -(((b + Sqrt[b^2 - 4*a*c] + 2*c*x)*(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*(
b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*(-b +
 Sqrt[b^2 - 4*a*c] - 2*c*x)))^(-1 - p)*(d + e*x)^(-1 - 2*p)*(a + x*(b + c*x
))^p*Hypergeometric2F1[-1 - 2*p, -p, -2*p, (-4*c*Sqrt[b^2 - 4*a*c]*(d + e*x
))/((-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))]
)/((-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*(1 + 2*p)))
```

Maple [F] time = 1.238, size = 0, normalized size = 0.

$$\int (ex+d)^{-2-2p}\left(cx^2+bx+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(-2-2*p)*(c*x^2+b*x+a)^p,x)
```

```
[Out] int((e*x+d)^(-2-2*p)*(c*x^2+b*x+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^p(ex+d)^{-2p-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-2-2*p)*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^p*(e*x + d)^(-2*p - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(cx^2+bx+a\right)^p(ex+d)^{-2p-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-2-2*p)*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^p*(e*x + d)^(-2*p - 2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(-2-2*p)*(c*x**2+b*x+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx + a\right)^p (ex + d)^{-2p-2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-2-2*p)*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^p*(e*x + d)^(-2*p - 2), x)
```

3.2577 $\int (d+ex)^{-3-2p}\left(a+bx+cx^2\right)^p\,dx$

Optimal. Leaf size=332

$$\frac{\left(-\sqrt{b^2-4ac}+b+2cx\right)(2cd-be)(d+ex)^{-2p-1}\left(a+bx+cx^2\right)^p\left(\frac{\left(\sqrt{b^2-4ac}+b+2cx\right)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}{\left(-\sqrt{b^2-4ac}+b+2cx\right)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}\right)^{-p}{}_2F_1\left(-2p-1,-p;-2\right.}{2(2p+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)\left(ae^2-bde+cd^2\right)}$$

```
[Out] -(e*(a + b*x + c*x^2)^(1 + p))/(2*(c*d^2 - b*d*e + a*e^2)*(1 + p)*(d + e*x)
^(2*(1 + p))) + ((2*c*d - b*e)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)*(d + e*x)^(-
1 - 2*p)*(a + b*x + c*x^2)^p*Hypergeometric2F1[-1 - 2*p, -p, -2*p, (-4*c*Sq
rt[b^2 - 4*a*c]*(d + e*x))/((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(b - Sqrt[b
^2 - 4*a*c] + 2*c*x))])/(2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*(c*d^2 - b*d
*e + a*e^2)*(1 + 2*p)*(((2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*(b + Sqrt[b^2 -
 4*a*c] + 2*c*x))/((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(b - Sqrt[b^2 - 4*a*
c] + 2*c*x)))^p)
```

Rubi [A] time = 0.13574, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {730, 726}

$$\frac{\left(-\sqrt{b^2-4ac}+b+2cx\right)(2cd-be)(d+ex)^{-2p-1}\left(a+bx+cx^2\right)^p\left(\frac{\left(\sqrt{b^2-4ac}+b+2cx\right)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}{\left(-\sqrt{b^2-4ac}+b+2cx\right)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}\right)^{-p}{}_2F_1\left(-2p-1,-p;-2\right.}{2(2p+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(-3 - 2*p)*(a + b*x + c*x^2)^p,x]
```


```
[Out] -(e*(a + b*x + c*x^2)^(1 + p))/(2*(c*d^2 - b*d*e + a*e^2)*(1 + p)*(d + e*x)
^(2*(1 + p))) + ((2*c*d - b*e)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)*(d + e*x)^(-
1 - 2*p)*(a + b*x + c*x^2)^p*Hypergeometric2F1[-1 - 2*p, -p, -2*p, (-4*c*Sq
rt[b^2 - 4*a*c]*(d + e*x))/((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(b - Sqrt[b
^2 - 4*a*c] + 2*c*x))])/(2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*(c*d^2 - b*d
*e + a*e^2)*(1 + 2*p)*(((2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*(b + Sqrt[b^2 -
 4*a*c] + 2*c*x))/((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(b - Sqrt[b^2 - 4*a*
c] + 2*c*x)))^p)
```


Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 726

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((b - Rt[b^2 - 4*a*c, 2] + 2*c*x)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^p*Hypergeometric2F1[m + 1, -p, m + 2, (-4*c*Rt[b^2 - 4*a*c, 2]*(
d + e*x))/((2*c*d - b*e - e*Rt[b^2 - 4*a*c, 2])*(b - Rt[b^2 - 4*a*c, 2] + 2
*c*x))])/((m + 1)*(2*c*d - b*e + e*Rt[b^2 - 4*a*c, 2])*(((2*c*d - b*e + e*R
```

```
t[b^2 - 4*a*c, 2])*(b + Rt[b^2 - 4*a*c, 2] + 2*c*x))/((2*c*d - b*e - e*Rt[b
^2 - 4*a*c, 2])*(b - Rt[b^2 - 4*a*c, 2] + 2*c*x)))^p), x] /; FreeQ[{a, b, c
, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &
& NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int (d+ex)^{-3-2p}\left(a+bx+cx^2\right)^p\,dx = -\frac{e(d+ex)^{-2(1+p)}\left(a+bx+cx^2\right)^{1+p}}{2\left(cd^2-bde+ae^2\right)(1+p)}+\frac{(2cd-be)\int (d+ex)^{-2-2p}\left(a+bx+cx^2\right)^p}{2\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{e(d+ex)^{-2(1+p)}\left(a+bx+cx^2\right)^{1+p}}{2\left(cd^2-bde+ae^2\right)(1+p)}+\frac{(2cd-be)\left(b-\sqrt{b^2-4ac}+2cx\right)\left(\frac{\left(2cd-\left(b-\right.\right.}{\left(2cd-\left(b+\right.\right.}\right.}{}$$

Mathematica [A] time = 0.331777, size = 294, normalized size = 0.89

$$\frac{(d+ex)^{-2(p+1)}(a+x(b+cx))^p\left(\frac{(d+ex)\left(\sqrt{b^2-4ac}+b+2cx\right)(be-2cd)\left(\frac{\left(\sqrt{b^2-4ac}+b+2cx\right)\left(e\left(\sqrt{b^2-4ac}-b\right)+2cd\right)}{\left(\sqrt{b^2-4ac}-b-2cx\right)\left(e\left(\sqrt{b^2-4ac}+b\right)-2cd\right)}\right)^{-p-1}{}_2F_1\left(-2p-1,-p;-2p;-\frac{4c\sqrt{b^2-4ac}\ldots}{\left(\left(b+\sqrt{b^2-4ac}\right)e-2cd\right)\ldots}\right)}{(2p+1)\left(e\left(\sqrt{b^2-4ac}+b\right)-2cd\right)}\ldots\right)}{2\left(e(ae-bd)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(-3 - 2*p)*(a + b*x + c*x^2)^p,x]
```

```
[Out] ((a + x*(b + c*x))^p*(-((e*(a + x*(b + c*x)))/(1 + p)) + ((-2*c*d + b*e)*(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x)*(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*(b + S
qrt[b^2 - 4*a*c] + 2*c*x))/((-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*(-b + Sqrt
[b^2 - 4*a*c] - 2*c*x)))^(-1 - p)*(d + e*x)*Hypergeometric2F1[-1 - 2*p, -p,
 -2*p, (-4*c*Sqrt[b^2 - 4*a*c]*(d + e*x))/((-2*c*d + (b + Sqrt[b^2 - 4*a*c]
)*e)*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))])/((-2*c*d + (b + Sqrt[b^2 - 4*a*c])
*e)*(1 + 2*p))))/(2*(c*d^2 + e*(-(b*d) + a*e))*(d + e*x)^(2*(1 + p)))
```

Maple [F] time = 1.329, size = 0, normalized size = 0.

$$\int (ex+d)^{-3-2p}\left(cx^2+bx+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(-3-2*p)*(c*x^2+b*x+a)^p,x)
```

```
[Out] int((e*x+d)^(-3-2*p)*(c*x^2+b*x+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^p(ex+d)^{-2p-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-3-2*p)*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^p*(e*x + d)^(-2*p - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2+bx+a\right)^p(ex+d)^{-2p-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-3-2*p)*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^p*(e*x + d)^(-2*p - 3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(-3-2*p)*(c*x**2+b*x+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^2+bx+a\right)^p(ex+d)^{-2p-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-3-2*p)*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^p*(e*x + d)^(-2*p - 3), x)
```

3.2578 $\int (d+ex)^{-4-2p}\left(a+bx+cx^2\right)^p\,dx$

Optimal. Leaf size=442

$$\frac{\left(-\sqrt{b^2-4ac}+b+2cx\right)(d+ex)^{-2p-1}\left(a+bx+cx^2\right)^p\left(-2ce(ae+bd(2p+3))+b^2e^2(p+2)+2c^2d^2(2p+3)\right)\left(\frac{\left(\sqrt{b^2-4ac}\right.}{\left(-\sqrt{b^2-4a}\right.}\right.}{2(2p+1)(2p+3)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)\left(ae^2\right.}$$

```
[Out] -((e*(d + e*x)^(-3 - 2*p)*(a + b*x + c*x^2)^(1 + p))/((c*d^2 - b*d*e + a*e^
2)*(3 + 2*p))) - (e*(2*c*d - b*e)*(2 + p)*(a + b*x + c*x^2)^(1 + p))/(2*(c*
d^2 - b*d*e + a*e^2)^2*(1 + p)*(3 + 2*p)*(d + e*x)^(2*(1 + p))) + ((b^2*e^2
*(2 + p) + 2*c^2*d^2*(3 + 2*p) - 2*c*e*(a*e + b*d*(3 + 2*p)))*(b - Sqrt[b^2
 - 4*a*c] + 2*c*x)*(d + e*x)^(-1 - 2*p)*(a + b*x + c*x^2)^p*Hypergeometric2
F1[-1 - 2*p, -p, -2*p, (-4*c*Sqrt[b^2 - 4*a*c]*(d + e*x))/((2*c*d - (b + Sq
rt[b^2 - 4*a*c])*e)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))])/(2*(2*c*d - (b - Sqr
t[b^2 - 4*a*c])*e)*(c*d^2 - b*d*e + a*e^2)^2*(1 + 2*p)*(3 + 2*p)*(((2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*d - (b +
 Sqrt[b^2 - 4*a*c])*e)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)))^p)
```

Rubi [A] time = 0.352557, antiderivative size = 442, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {744, 806, 726}

$$\frac{\left(-\sqrt{b^2-4ac}+b+2cx\right)(d+ex)^{-2p-1}\left(a+bx+cx^2\right)^p\left(-2ce(ae+bd(2p+3))+b^2e^2(p+2)+2c^2d^2(2p+3)\right)\left(\frac{\left(\sqrt{b^2-4ac}\right.}{\left(-\sqrt{b^2-4a}\right.}\right.}{2(2p+1)(2p+3)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)\left(ae^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(-4 - 2*p)*(a + b*x + c*x^2)^p,x]
```


```
[Out] -((e*(d + e*x)^(-3 - 2*p)*(a + b*x + c*x^2)^(1 + p))/((c*d^2 - b*d*e + a*e^
2)*(3 + 2*p))) - (e*(2*c*d - b*e)*(2 + p)*(a + b*x + c*x^2)^(1 + p))/(2*(c*
d^2 - b*d*e + a*e^2)^2*(1 + p)*(3 + 2*p)*(d + e*x)^(2*(1 + p))) + ((b^2*e^2
*(2 + p) + 2*c^2*d^2*(3 + 2*p) - 2*c*e*(a*e + b*d*(3 + 2*p)))*(b - Sqrt[b^2
 - 4*a*c] + 2*c*x)*(d + e*x)^(-1 - 2*p)*(a + b*x + c*x^2)^p*Hypergeometric2
F1[-1 - 2*p, -p, -2*p, (-4*c*Sqrt[b^2 - 4*a*c]*(d + e*x))/((2*c*d - (b + Sq
rt[b^2 - 4*a*c])*e)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))])/(2*(2*c*d - (b - Sqr
t[b^2 - 4*a*c])*e)*(c*d^2 - b*d*e + a*e^2)^2*(1 + 2*p)*(3 + 2*p)*(((2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*d - (b +
 Sqrt[b^2 - 4*a*c])*e)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)))^p)
```


Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 726

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((b - Rt[b^2 - 4*a*c, 2] + 2*c*x)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^p*Hypergeometric2F1[m + 1, -p, m + 2, (-4*c*Rt[b^2 - 4*a*c, 2]*(
d + e*x))/((2*c*d - b*e - e*Rt[b^2 - 4*a*c, 2])*(b - Rt[b^2 - 4*a*c, 2] + 2
*c*x))])/((m + 1)*(2*c*d - b*e + e*Rt[b^2 - 4*a*c, 2])*(((2*c*d - b*e + e*R
t[b^2 - 4*a*c, 2])*(b + Rt[b^2 - 4*a*c, 2] + 2*c*x))/((2*c*d - b*e - e*Rt[b
^2 - 4*a*c, 2])*(b - Rt[b^2 - 4*a*c, 2] + 2*c*x)))^p), x] /; FreeQ[{a, b, c
, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &
& NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int (d+ex)^{-4-2p}\left(a+bx+cx^2\right)^p\,dx = -\frac{e(d+ex)^{-3-2p}\left(a+bx+cx^2\right)^{1+p}}{\left(cd^2-bde+ae^2\right)(3+2p)} - \frac{\int (d+ex)^{-3-2p}(be(2+p)-cd(3+2p)+cex)\left(\right.}{\left(cd^2-bde+ae^2\right)(3+2p)}$$

$$= -\frac{e(d+ex)^{-3-2p}\left(a+bx+cx^2\right)^{1+p}}{\left(cd^2-bde+ae^2\right)(3+2p)} - \frac{e(2cd-be)(2+p)(d+ex)^{-2(1+p)}\left(a+bx+cx^2\right.}{2\left(cd^2-bde+ae^2\right)^2(1+p)(3+2p)}$$

$$= -\frac{e(d+ex)^{-3-2p}\left(a+bx+cx^2\right)^{1+p}}{\left(cd^2-bde+ae^2\right)(3+2p)} - \frac{e(2cd-be)(2+p)(d+ex)^{-2(1+p)}\left(a+bx+cx^2\right.}{2\left(cd^2-bde+ae^2\right)^2(1+p)(3+2p)}$$

Mathematica [A] time = 1.48216, size = 399, normalized size = 0.9

$$-\frac{(d+ex)^{-2p-3}(a+x(b+cx))^p\left(\frac{(d+ex)^2\left(\sqrt{b^2-4ac}+b+2cx\right)\left(-2ce(ae+bd(2p+3))+b^2e^2(p+2)+2c^2d^2(2p+3)\right)\left(\frac{\left(\sqrt{b^2-4ac}+b+2cx\right)\left(e\left(\sqrt{b^2-4ac}-b\right)+2cd\right)}{\left(\sqrt{b^2-4ac}-b-2cx\right)\left(e\left(\sqrt{b^2-4ac}+b\right)-2cd\right)}\right)^{-p-1}{}_2F_1\left(\right.}{(2p+1)\left(e\left(\sqrt{b^2-4ac}+b\right)-2cd\right)\left(e(ae-bd)+cd^2\right)}\right.}{2(2p+3)\left(e(ae-bd)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(-4 - 2*p)*(a + b*x + c*x^2)^p,x]
```

```
[Out] -((d + e*x)^(-3 - 2*p)*(a + x*(b + c*x))^p*(2*e*(a + x*(b + c*x)) + (e*(2*c
*d - b*e)*(2 + p)*(d + e*x)*(a + x*(b + c*x)))/((c*d^2 + e*(-(b*d) + a*e))*
(1 + p)) + ((b^2*e^2*(2 + p) + 2*c^2*d^2*(3 + 2*p) - 2*c*e*(a*e + b*d*(3 +
2*p)))*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)*(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*
e)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*(
-b + Sqrt[b^2 - 4*a*c] - 2*c*x)))^(-1 - p)*(d + e*x)^2*Hypergeometric2F1[-1
 - 2*p, -p, -2*p, (-4*c*Sqrt[b^2 - 4*a*c]*(d + e*x))/((-2*c*d + (b + Sqrt[b
^2 - 4*a*c])*e)*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))])/((-2*c*d + (b + Sqrt[b^
```

```
2 - 4*a*c])*e)*(c*d^2 + e*(-(b*d) + a*e))*(1 + 2*p))))/(2*(c*d^2 + e*(-(b*d
) + a*e))*(3 + 2*p))
```

Maple [F] time = 1.299, size = 0, normalized size = 0.

$$\int (ex+d)^{-4-2p} \left(cx^2+bx+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(-4-2*p)*(c*x^2+b*x+a)^p,x)
```

```
[Out] int((e*x+d)^(-4-2*p)*(c*x^2+b*x+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^p (ex+d)^{-2p-4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-4-2*p)*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^p*(e*x + d)^(-2*p - 4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2+bx+a\right)^p (ex+d)^{-2p-4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-4-2*p)*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^p*(e*x + d)^(-2*p - 4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(-4-2*p)*(c*x**2+b*x+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^p (ex+d)^{-2p-4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-4-2*p)*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^p*(e*x + d)^(-2*p - 4), x)
```

3.2579 $\int (d+ex)^{-5-2p}\left(a+bx+cx^2\right)^p\,dx$

Optimal. Leaf size=577

$$\frac{\left(-\sqrt{b^2-4ac}+b+2cx\right)(2cd-be)(d+ex)^{-2p-1}\left(a+bx+cx^2\right)^p\left(-2ce(3ae+bd(2p+3))+b^2e^2(p+3)+2c^2d^2(2p+3\right.}{4(2p+1)(2p+3)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right.}$$

```
[Out] -(e*(2*c*d - b*e)*(3 + p)*(d + e*x)^(-3 - 2*p)*(a + b*x + c*x^2)^(1 + p))/(
2*(c*d^2 - b*d*e + a*e^2)^2*(2 + p)*(3 + 2*p)) - (e*(b^2*e^2*(6 + 5*p + p^2
) + 2*c^2*d^2*(9 + 8*p + 2*p^2) - 2*c*e*(a*e*(3 + 2*p) + b*d*(9 + 8*p + 2*p
^2)))*(a + b*x + c*x^2)^(1 + p))/(4*(c*d^2 - b*d*e + a*e^2)^3*(1 + p)*(2 +
p)*(3 + 2*p)*(d + e*x)^(2*(1 + p))) - (e*(a + b*x + c*x^2)^(1 + p))/(2*(c*d
^2 - b*d*e + a*e^2)*(2 + p)*(d + e*x)^(2*(2 + p))) + ((2*c*d - b*e)*(b^2*e^
2*(3 + p) + 2*c^2*d^2*(3 + 2*p) - 2*c*e*(3*a*e + b*d*(3 + 2*p)))*(b - Sqrt[
b^2 - 4*a*c] + 2*c*x)*(d + e*x)^(-1 - 2*p)*(a + b*x + c*x^2)^p*Hypergeometr
ic2F1[-1 - 2*p, -p, -2*p, (-4*c*Sqrt[b^2 - 4*a*c]*(d + e*x))/((2*c*d - (b +
 Sqrt[b^2 - 4*a*c])*e)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))])/(4*(2*c*d - (b -
Sqrt[b^2 - 4*a*c])*e)*(c*d^2 - b*d*e + a*e^2)^3*(1 + 2*p)*(3 + 2*p)*(((2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)))^p)
```

Rubi [A] time = 0.867737, antiderivative size = 577, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {744, 836, 806, 726}

$$\frac{\left(-\sqrt{b^2-4ac}+b+2cx\right)(2cd-be)(d+ex)^{-2p-1}\left(a+bx+cx^2\right)^p\left(-2ce(3ae+bd(2p+3))+b^2e^2(p+3)+2c^2d^2(2p+3\right.}{4(2p+1)(2p+3)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(-5 - 2*p)*(a + b*x + c*x^2)^p,x]
```


```
[Out] -(e*(2*c*d - b*e)*(3 + p)*(d + e*x)^(-3 - 2*p)*(a + b*x + c*x^2)^(1 + p))/(
2*(c*d^2 - b*d*e + a*e^2)^2*(2 + p)*(3 + 2*p)) - (e*(b^2*e^2*(6 + 5*p + p^2
) + 2*c^2*d^2*(9 + 8*p + 2*p^2) - 2*c*e*(a*e*(3 + 2*p) + b*d*(9 + 8*p + 2*p
^2)))*(a + b*x + c*x^2)^(1 + p))/(4*(c*d^2 - b*d*e + a*e^2)^3*(1 + p)*(2 +
p)*(3 + 2*p)*(d + e*x)^(2*(1 + p))) - (e*(a + b*x + c*x^2)^(1 + p))/(2*(c*d
^2 - b*d*e + a*e^2)*(2 + p)*(d + e*x)^(2*(2 + p))) + ((2*c*d - b*e)*(b^2*e^
2*(3 + p) + 2*c^2*d^2*(3 + 2*p) - 2*c*e*(3*a*e + b*d*(3 + 2*p)))*(b - Sqrt[
b^2 - 4*a*c] + 2*c*x)*(d + e*x)^(-1 - 2*p)*(a + b*x + c*x^2)^p*Hypergeometr
ic2F1[-1 - 2*p, -p, -2*p, (-4*c*Sqrt[b^2 - 4*a*c]*(d + e*x))/((2*c*d - (b +
 Sqrt[b^2 - 4*a*c])*e)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))])/(4*(2*c*d - (b -
Sqrt[b^2 - 4*a*c])*e)*(c*d^2 - b*d*e + a*e^2)^3*(1 + 2*p)*(3 + 2*p)*(((2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)))^p)
```


Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
```

```
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2
 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p + 3]
, 0] && NeQ[m, -1]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 726

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((b - Rt[b^2 - 4*a*c, 2] + 2*c*x)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^p*Hypergeometric2F1[m + 1, -p, m + 2, (-4*c*Rt[b^2 - 4*a*c, 2]*(
d + e*x))/((2*c*d - b*e - e*Rt[b^2 - 4*a*c, 2])*(b - Rt[b^2 - 4*a*c, 2] + 2
*c*x))])/((m + 1)*(2*c*d - b*e + e*Rt[b^2 - 4*a*c, 2])*(((2*c*d - b*e + e*R
t[b^2 - 4*a*c, 2])*(b + Rt[b^2 - 4*a*c, 2] + 2*c*x))/((2*c*d - b*e - e*Rt[b
^2 - 4*a*c, 2])*(b - Rt[b^2 - 4*a*c, 2] + 2*c*x)))^p), x] /; FreeQ[{a, b, c
, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &
& NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int (d+ex)^{-5-2p}\left(a+bx+cx^2\right)^p\,dx = -\frac{e(d+ex)^{-2(2+p)}\left(a+bx+cx^2\right)^{1+p}}{2\left(cd^2-bde+ae^2\right)(2+p)} - \frac{\int (d+ex)^{-4-2p}(-2cd(2+p)+be(3+p)+2ce\ldots}{2\left(cd^2-bde+ae^2\right)(2+p\ldots}$$

$$= -\frac{e(2cd-be)(3+p)(d+ex)^{-3-2p}\left(a+bx+cx^2\right)^{1+p}}{2\left(cd^2-bde+ae^2\right)^2(2+p)(3+2p)} - \frac{e(d+ex)^{-2(2+p)}\left(a+bx+cx^2\right)\ldots}{2\left(cd^2-bde+ae^2\right)(2+p)\ldots}$$

$$= -\frac{e(2cd-be)(3+p)(d+ex)^{-3-2p}\left(a+bx+cx^2\right)^{1+p}}{2\left(cd^2-bde+ae^2\right)^2(2+p)(3+2p)} - \frac{e\left(b^2e^2\left(6+5p+p^2\right)+2c^2d^2\ldots}{\ldots}$$

$$= -\frac{e(2cd-be)(3+p)(d+ex)^{-3-2p}\left(a+bx+cx^2\right)^{1+p}}{2\left(cd^2-bde+ae^2\right)^2(2+p)(3+2p)} - \frac{e\left(b^2e^2\left(6+5p+p^2\right)+2c^2d^2\ldots}{\ldots}$$

Mathematica [A] time = 5.97402, size = 731, normalized size = 1.27

$$-\frac{(d+ex)^{-2(p+2)}(a+x(b+cx))^p\left(-\frac{(p+3)(d+ex)(be-2cd)\left(\frac{(d+ex)^2\left(\sqrt{b^2-4ac}+b+2cx\right)\left(-2ce(ae+bd(2p+3))+b^2e^2(p+2)+2c^2d^2(2p+3)\right)\left(\frac{\left(\sqrt{b^2-4ac}+b+2cx\right)\left(e\left(\sqrt{b^2-4ac}\cdots\right.\right.}{\left(\sqrt{b^2-4ac}-b-2cx\right)\left(e\left(\sqrt{b^2-4ac}\cdots\right.\right.}\right.}{(2p+1)\left(e\left(\sqrt{b^2-4ac}+b\right)-2cd\right)\left(e(ae-\cdots\right.}\right.}{(2p+3)\left(e\cdots\right.}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(-5 - 2*p)*(a + b*x + c*x^2)^p,x]
```

```
[Out] -((a + x*(b + c*x))^p*(2*e*(a + x*(b + c*x)) + (2*c*(d + e*x)^2*(-((e*(a +
x*(b + c*x)))/(1 + p)) + ((-2*c*d + b*e)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)*((
(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((-2*
c*d + (b + Sqrt[b^2 - 4*a*c])*e)*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)))^(-1 - p
)*(d + e*x)*Hypergeometric2F1[-1 - 2*p, -p, -2*p, (-4*c*Sqrt[b^2 - 4*a*c]*(
d + e*x))/((-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*(-b + Sqrt[b^2 - 4*a*c] - 2
*c*x))])/((-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*(1 + 2*p))))/(c*d^2 + e*(-(b
*d) + a*e)) - ((-2*c*d + b*e)*(3 + p)*(d + e*x)*(2*e*(a + x*(b + c*x)) + (e
*(2*c*d - b*e)*(2 + p)*(d + e*x)*(a + x*(b + c*x)))/((c*d^2 + e*(-(b*d) + a
*e))*(1 + p)) + ((b^2*e^2*(2 + p) + 2*c^2*d^2*(3 + 2*p) - 2*c*e*(a*e + b*d*
(3 + 2*p)))*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)*(((2*c*d + (-b + Sqrt[b^2 - 4*a
*c])*e)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((-2*c*d + (b + Sqrt[b^2 - 4*a*c])
*e)*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)))^(-1 - p)*(d + e*x)^2*Hypergeometric2
F1[-1 - 2*p, -p, -2*p, (-4*c*Sqrt[b^2 - 4*a*c]*(d + e*x))/((-2*c*d + (b + S
qrt[b^2 - 4*a*c])*e)*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))])/((-2*c*d + (b + Sq
rt[b^2 - 4*a*c])*e)*(c*d^2 + e*(-(b*d) + a*e))*(1 + 2*p))))/((c*d^2 + e*(-(
b*d) + a*e))*(3 + 2*p))))/(4*(c*d^2 + e*(-(b*d) + a*e))*(2 + p)*(d + e*x)^(
2*(2 + p)))
```

Maple [F] time = 1.311, size = 0, normalized size = 0.

$$\int (ex+d)^{-5-2p}\left(cx^2+bx+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(-5-2*p)*(c*x^2+b*x+a)^p,x)
```

```
[Out] int((e*x+d)^(-5-2*p)*(c*x^2+b*x+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^p(ex+d)^{-2p-5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-5-2*p)*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^p*(e*x + d)^(-2*p - 5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2 + bx + a\right)^p (ex + d)^{-2p-5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-5-2*p)*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^p*(e*x + d)^(-2*p - 5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(-5-2*p)*(c*x**2+b*x+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx + a\right)^p (ex + d)^{-2p-5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-5-2*p)*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^p*(e*x + d)^(-2*p - 5), x)
```

3.2580 $\int (d+ex)^{-6-2p}\left(a+bx+cx^2\right)^p\,dx$

Optimal. Leaf size=809

$$-\frac{e\left(cx^2+bx+a\right)^{p+1}(d+ex)^{-2p-5}}{\left(cd^2-bed+ae^2\right)(2p+5)}-\frac{e\left(2c^2\left(2p^2+11p+18\right)d^2+b^2e^2\left(p^2+7p+12\right)-2ce\left(3ae(p+2)+bd\left(2p^2+11\right.\right.\right.}{2\left(cd^2-bed+ae^2\right)^3(p+2)(2p+3)(2p+}$$

```
[Out] -((e*(d + e*x)^(-5 - 2*p)*(a + b*x + c*x^2)^(1 + p))/((c*d^2 - b*d*e + a*e^
2)*(5 + 2*p))) - (e*(b^2*e^2*(12 + 7*p + p^2) + 2*c^2*d^2*(18 + 11*p + 2*p^
2) - 2*c*e*(3*a*e*(2 + p) + b*d*(18 + 11*p + 2*p^2)))*(d + e*x)^(-3 - 2*p)*
(a + b*x + c*x^2)^(1 + p))/(2*(c*d^2 - b*d*e + a*e^2)^3*(2 + p)*(3 + 2*p)*(
5 + 2*p)) - (e*(2*c*d - b*e)*(3 + p)*(b^2*e^2*(8 + 6*p + p^2) + 2*c^2*d^2*(
8 + 7*p + 2*p^2) - 2*c*e*(a*e*(8 + 5*p) + b*d*(8 + 7*p + 2*p^2)))*(a + b*x
+ c*x^2)^(1 + p))/(4*(c*d^2 - b*d*e + a*e^2)^4*(1 + p)*(2 + p)*(3 + 2*p)*(5
 + 2*p)*(d + e*x)^(2*(1 + p))) - (e*(2*c*d - b*e)*(4 + p)*(a + b*x + c*x^2)
^(1 + p))/(2*(c*d^2 - b*d*e + a*e^2)^2*(2 + p)*(5 + 2*p)*(d + e*x)^(2*(2 +
p))) + ((b^4*e^4*(12 + 7*p + p^2) + 4*c^4*d^4*(15 + 16*p + 4*p^2) - 8*c^3*d
^2*e*(5 + 2*p)*(3*a*e + b*d*(3 + 2*p)) - 4*b^2*c*e^3*(3 + p)*(3*a*e + b*d*(
5 + 2*p)) + 12*c^2*e^2*(a^2*e^2 + 2*a*b*d*e*(5 + 2*p) + b^2*d^2*(10 + 9*p +
 2*p^2)))*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)*(d + e*x)^(-1 - 2*p)*(a + b*x + c
*x^2)^p*Hypergeometric2F1[-1 - 2*p, -p, -2*p, (-4*c*Sqrt[b^2 - 4*a*c]*(d +
e*x))/((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))
])/(4*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*(c*d^2 - b*d*e + a*e^2)^4*(1 + 2*
p)*(3 + 2*p)*(5 + 2*p)*(((2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*(b + Sqrt[b^2
- 4*a*c] + 2*c*x))/((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(b - Sqrt[b^2 - 4*a
*c] + 2*c*x)))^p)
```

Rubi [A] time = 1.71996, antiderivative size = 809, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {744, 836, 806, 726}

$$-\frac{e\left(cx^2+bx+a\right)^{p+1}(d+ex)^{-2p-5}}{\left(cd^2-bed+ae^2\right)(2p+5)}-\frac{e\left(2c^2\left(2p^2+11p+18\right)d^2+b^2e^2\left(p^2+7p+12\right)-2ce\left(3ae(p+2)+bd\left(2p^2+11\right.\right.\right.}{2\left(cd^2-bed+ae^2\right)^3(p+2)(2p+3)(2p+}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(-6 - 2*p)*(a + b*x + c*x^2)^p,x]
```

```
[Out] -((e*(d + e*x)^(-5 - 2*p)*(a + b*x + c*x^2)^(1 + p))/((c*d^2 - b*d*e + a*e^
2)*(5 + 2*p))) - (e*(b^2*e^2*(12 + 7*p + p^2) + 2*c^2*d^2*(18 + 11*p + 2*p^
2) - 2*c*e*(3*a*e*(2 + p) + b*d*(18 + 11*p + 2*p^2)))*(d + e*x)^(-3 - 2*p)*
(a + b*x + c*x^2)^(1 + p))/(2*(c*d^2 - b*d*e + a*e^2)^3*(2 + p)*(3 + 2*p)*(
5 + 2*p)) - (e*(2*c*d - b*e)*(3 + p)*(b^2*e^2*(8 + 6*p + p^2) + 2*c^2*d^2*(
8 + 7*p + 2*p^2) - 2*c*e*(a*e*(8 + 5*p) + b*d*(8 + 7*p + 2*p^2)))*(a + b*x
+ c*x^2)^(1 + p))/(4*(c*d^2 - b*d*e + a*e^2)^4*(1 + p)*(2 + p)*(3 + 2*p)*(5
 + 2*p)*(d + e*x)^(2*(1 + p))) - (e*(2*c*d - b*e)*(4 + p)*(a + b*x + c*x^2)
^(1 + p))/(2*(c*d^2 - b*d*e + a*e^2)^2*(2 + p)*(5 + 2*p)*(d + e*x)^(2*(2 +
p))) + ((b^4*e^4*(12 + 7*p + p^2) + 4*c^4*d^4*(15 + 16*p + 4*p^2) - 8*c^3*d
^2*e*(5 + 2*p)*(3*a*e + b*d*(3 + 2*p)) - 4*b^2*c*e^3*(3 + p)*(3*a*e + b*d*(
5 + 2*p)) + 12*c^2*e^2*(a^2*e^2 + 2*a*b*d*e*(5 + 2*p) + b^2*d^2*(10 + 9*p +
 2*p^2)))*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)*(d + e*x)^(-1 - 2*p)*(a + b*x + c
*x^2)^p*Hypergeometric2F1[-1 - 2*p, -p, -2*p, (-4*c*Sqrt[b^2 - 4*a*c]*(d +
```

```
e*x))/((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))
])/(4*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*(c*d^2 - b*d*e + a*e^2)^4*(1 + 2*
p)*(3 + 2*p)*(5 + 2*p)*(((2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*(b + Sqrt[b^2
- 4*a*c] + 2*c*x))/((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(b - Sqrt[b^2 - 4*a
*c] + 2*c*x)))^p)
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2
 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p + 3]
, 0] && NeQ[m, -1]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 726

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((b - Rt[b^2 - 4*a*c, 2] + 2*c*x)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^p*Hypergeometric2F1[m + 1, -p, m + 2, (-4*c*Rt[b^2 - 4*a*c, 2]*(
d + e*x))/((2*c*d - b*e - e*Rt[b^2 - 4*a*c, 2])*(b - Rt[b^2 - 4*a*c, 2] + 2
*c*x))])/((m + 1)*(2*c*d - b*e + e*Rt[b^2 - 4*a*c, 2])*(((2*c*d - b*e + e*R
t[b^2 - 4*a*c, 2])*(b + Rt[b^2 - 4*a*c, 2] + 2*c*x))/((2*c*d - b*e - e*Rt[b
^2 - 4*a*c, 2])*(b - Rt[b^2 - 4*a*c, 2] + 2*c*x)))^p), x] /; FreeQ[{a, b, c
, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &
& NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int (d+ex)^{-6-2p}\left(a+bx+cx^2\right)^p\,dx = -\frac{e(d+ex)^{-5-2p}\left(a+bx+cx^2\right)^{1+p}}{\left(cd^2-bde+ae^2\right)(5+2p)} - \frac{\int (d+ex)^{-5-2p}(be(4+p)-cd(5+2p)+3ce \ldots}{\left(cd^2-bde+ae^2\right)(5+2 \ldots}$$

$$= -\frac{e(d+ex)^{-5-2p}\left(a+bx+cx^2\right)^{1+p}}{\left(cd^2-bde+ae^2\right)(5+2p)} - \frac{e(2cd-be)(4+p)(d+ex)^{-2(2+p)}\left(a+bx+c \ldots\right.}{2\left(cd^2-bde+ae^2\right)^2(2+p)(5+2p)}$$

$$= -\frac{e(d+ex)^{-5-2p}\left(a+bx+cx^2\right)^{1+p}}{\left(cd^2-bde+ae^2\right)(5+2p)} - \frac{e\left(b^2e^2\left(12+7p+p^2\right)+2c^2d^2\left(18+11p+2 \ldots\right.\right.}{2\left(cd \ldots\right.}$$

$$= -\frac{e(d+ex)^{-5-2p}\left(a+bx+cx^2\right)^{1+p}}{\left(cd^2-bde+ae^2\right)(5+2p)} - \frac{e\left(b^2e^2\left(12+7p+p^2\right)+2c^2d^2\left(18+11p+2 \ldots\right.\right.}{2\left(cd \ldots\right.}$$

$$= -\frac{e(d+ex)^{-5-2p}\left(a+bx+cx^2\right)^{1+p}}{\left(cd^2-bde+ae^2\right)(5+2p)} - \frac{e\left(b^2e^2\left(12+7p+p^2\right)+2c^2d^2\left(18+11p+2 \ldots\right.\right.}{2\left(cd \ldots\right.}$$

Mathematica [A] time = 6.27372, size = 1577, normalized size = 1.95

$$\frac{e(d+ex)^{1-2(p+3)}\left(cx^2+bx+a\right)^{p+1}}{\left(cd^2-bed+ae^2\right)(1-2(p+3))} + \frac{3c\left(\frac{e(d+ex)^{3-2(p+3)}\left(cx^2+bx+a\right)^{p+1}}{\left(cd^2-bed+ae^2\right)(3-2(p+3))} + \frac{\left(b(cde+(be(p+2)-cd(2p+3))e)-2\left(ace^2+cd(be(p+2)-cd(2p+3))\right)\right)\left(b+2cx-\sqrt{b^2-4a \ldots}\right.}{\ldots}\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(-6 - 2*p)*(a + b*x + c*x^2)^p,x]
```

```
[Out] (e*(d + e*x)^(1 - 2*(3 + p))*(a + b*x + c*x^2)^(1 + p))/((c*d^2 - b*d*e + a
*e^2)*(1 - 2*(3 + p))) + (3*c*((e*(d + e*x)^(3 - 2*(3 + p))*(a + b*x + c*x^
2)^(1 + p))/((c*d^2 - b*d*e + a*e^2)*(3 - 2*(3 + p))) + (-((-(c*d*e) + e*(b
*e*(2 + p) - c*d*(3 + 2*p)))*(d + e*x)^(4 - 2*(3 + p))*(a + b*x + c*x^2)^(1
 + p))/(2*(c*d^2 - b*d*e + a*e^2)*(1 + p)) + ((-2*(a*c*e^2 + c*d*(b*e*(2 +
p) - c*d*(3 + 2*p))) + b*(c*d*e + e*(b*e*(2 + p) - c*d*(3 + 2*p))))*(b - Sq
rt[b^2 - 4*a*c] + 2*c*x)*(d + e*x)^(5 - 2*(3 + p))*(a + b*x + c*x^2)^p*Hype
rgeometric2F1[-p, 5 - 2*(3 + p), 6 - 2*(3 + p), (-4*c*Sqrt[b^2 - 4*a*c]*(d
+ e*x))/((2*c*d - b*e - Sqrt[b^2 - 4*a*c]*e)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x
```

```
))])/(2*(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)*(c*d^2 - b*d*e + a*e^2)*(5 - 2*
(3 + p))*(((2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)*(b + Sqrt[b^2 - 4*a*c] + 2*c
*x))/((2*c*d - b*e - Sqrt[b^2 - 4*a*c]*e)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)))
^p))/((c*d^2 - b*d*e + a*e^2)*(3 - 2*(3 + p)))) + ((-3*c*d*e + e*(b*e*(4 +
p) - c*d*(5 + 2*p)))*((e*(d + e*x)^(2 - 2*(3 + p))*(a + b*x + c*x^2)^(1 + p
))/((c*d^2 - b*d*e + a*e^2)*(2 - 2*(3 + p))) + (2*c*((e*(d + e*x)^(4 - 2*(3
 + p))*(a + b*x + c*x^2)^(1 + p))/((c*d^2 - b*d*e + a*e^2)*(4 - 2*(3 + p)))
 - ((2*c*d - b*e)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)*(d + e*x)^(5 - 2*(3 + p))
*(a + b*x + c*x^2)^p*Hypergeometric2F1[-p, 5 - 2*(3 + p), 6 - 2*(3 + p), (-
4*c*Sqrt[b^2 - 4*a*c]*(d + e*x))/((2*c*d - b*e - Sqrt[b^2 - 4*a*c]*e)*(b -
Sqrt[b^2 - 4*a*c] + 2*c*x))])/(2*(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)*(c*d^2
 - b*d*e + a*e^2)*(5 - 2*(3 + p))*(((2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)*(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*d - b*e - Sqrt[b^2 - 4*a*c]*e)*(b - Sqr
t[b^2 - 4*a*c] + 2*c*x)))^p)) + ((-2*c*d*e + e*(-2*c*d*(2 + p) + b*e*(3 + p
)))*((e*(d + e*x)^(3 - 2*(3 + p))*(a + b*x + c*x^2)^(1 + p))/((c*d^2 - b*d*
e + a*e^2)*(3 - 2*(3 + p))) + (-((-(c*d*e) + e*(b*e*(2 + p) - c*d*(3 + 2*p)
))*(d + e*x)^(4 - 2*(3 + p))*(a + b*x + c*x^2)^(1 + p))/(2*(c*d^2 - b*d*e +
 a*e^2)*(1 + p)) + ((-2*(a*c*e^2 + c*d*(b*e*(2 + p) - c*d*(3 + 2*p))) + b*(
c*d*e + e*(b*e*(2 + p) - c*d*(3 + 2*p))))*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)*(
d + e*x)^(5 - 2*(3 + p))*(a + b*x + c*x^2)^p*Hypergeometric2F1[-p, 5 - 2*(3
 + p), 6 - 2*(3 + p), (-4*c*Sqrt[b^2 - 4*a*c]*(d + e*x))/((2*c*d - b*e - Sq
rt[b^2 - 4*a*c]*e)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))])/(2*(2*c*d - b*e + Sqr
t[b^2 - 4*a*c]*e)*(c*d^2 - b*d*e + a*e^2)*(5 - 2*(3 + p))*(((2*c*d - b*e +
Sqrt[b^2 - 4*a*c]*e)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*d - b*e - Sqrt[
b^2 - 4*a*c]*e)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)))^p))/((c*d^2 - b*d*e + a*e
^2)*(3 - 2*(3 + p)))))/e)/((c*d^2 - b*d*e + a*e^2)*(2 - 2*(3 + p)))))/e)/((
c*d^2 - b*d*e + a*e^2)*(1 - 2*(3 + p)))
```

Maple [F] time = 1.268, size = 0, normalized size = 0.

$$\int (ex+d)^{-6-2p} \left(cx^2+bx+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(-6-2*p)*(c*x^2+b*x+a)^p,x)
```

```
[Out] int((e*x+d)^(-6-2*p)*(c*x^2+b*x+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^p (ex+d)^{-2p-6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-6-2*p)*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^p*(e*x + d)^(-2*p - 6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(cx^2+bx+a\right)^p (ex+d)^{-2p-6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-6-2*p)*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^p*(e*x + d)^(-2*p - 6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(-6-2*p)*(c*x**2+b*x+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx + a\right)^p (ex + d)^{-2p-6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(-6-2*p)*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^p*(e*x + d)^(-2*p - 6), x)
```

3.2581 $\int (d+ex)^m \left(a+bx+cx^2\right)^{-2-\frac{m}{2}} \, dx$

Optimal. Leaf size=440

$$-\frac{\left(-\sqrt{b^2-4ac}+b+2cx\right)(d+ex)^{m+3}\left(a+bx+cx^2\right)^{-\frac{m}{2}-2}\left(4ce(ae-bd(m+1))+b^2e^2m+4c^2d^2(m+1)\right)\left(\frac{\left(\sqrt{b^2-4ac}+b+2cx\right)(}{\left(-\sqrt{b^2-4ac}+b+2cx\right)}\right.}{4(m+1)(m+3)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)\left(ae^2-b\right.}$$

```
[Out] (e*(d + e*x)^(1 + m)*(a + b*x + c*x^2)^(-1 - m/2))/((c*d^2 - b*d*e + a*e^2)
*(1 + m)) + (e*(2*c*d - b*e)*m*(d + e*x)^(2 + m)*(a + b*x + c*x^2)^(-1 - m/
2))/(2*(c*d^2 - b*d*e + a*e^2)^2*(1 + m)*(2 + m)) - ((b^2*e^2*m + 4*c^2*d^2
*(1 + m) + 4*c*e*(a*e - b*d*(1 + m)))*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)*(((2*
c*d - (b - Sqrt[b^2 - 4*a*c])*e)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*d -
 (b + Sqrt[b^2 - 4*a*c])*e)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)))^((4 + m)/2)*(
d + e*x)^(3 + m)*(a + b*x + c*x^2)^(-2 - m/2)*Hypergeometric2F1[3 + m, (4 +
 m)/2, 4 + m, (-4*c*Sqrt[b^2 - 4*a*c]*(d + e*x))/((2*c*d - (b + Sqrt[b^2 -
4*a*c])*e)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))])/(4*(2*c*d - (b - Sqrt[b^2 - 4
*a*c])*e)*(c*d^2 - b*d*e + a*e^2)^2*(1 + m)*(3 + m))
```

Rubi [A] time = 0.385077, antiderivative size = 440, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {744, 806, 726}

$$-\frac{\left(-\sqrt{b^2-4ac}+b+2cx\right)(d+ex)^{m+3}\left(a+bx+cx^2\right)^{-\frac{m}{2}-2}\left(4ce(ae-bd(m+1))+b^2e^2m+4c^2d^2(m+1)\right)\left(\frac{\left(\sqrt{b^2-4ac}+b+2cx\right)(}{\left(-\sqrt{b^2-4ac}+b+2cx\right)}\right.}{4(m+1)(m+3)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)\left(ae^2-b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a + b*x + c*x^2)^(-2 - m/2),x]
```


```
[Out] (e*(d + e*x)^(1 + m)*(a + b*x + c*x^2)^(-1 - m/2))/((c*d^2 - b*d*e + a*e^2)
*(1 + m)) + (e*(2*c*d - b*e)*m*(d + e*x)^(2 + m)*(a + b*x + c*x^2)^(-1 - m/
2))/(2*(c*d^2 - b*d*e + a*e^2)^2*(1 + m)*(2 + m)) - ((b^2*e^2*m + 4*c^2*d^2
*(1 + m) + 4*c*e*(a*e - b*d*(1 + m)))*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)*(((2*
c*d - (b - Sqrt[b^2 - 4*a*c])*e)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*d -
 (b + Sqrt[b^2 - 4*a*c])*e)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)))^((4 + m)/2)*(
d + e*x)^(3 + m)*(a + b*x + c*x^2)^(-2 - m/2)*Hypergeometric2F1[3 + m, (4 +
 m)/2, 4 + m, (-4*c*Sqrt[b^2 - 4*a*c]*(d + e*x))/((2*c*d - (b + Sqrt[b^2 -
4*a*c])*e)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))])/(4*(2*c*d - (b - Sqrt[b^2 - 4
*a*c])*e)*(c*d^2 - b*d*e + a*e^2)^2*(1 + m)*(3 + m))
```


Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 726

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((b - Rt[b^2 - 4*a*c, 2] + 2*c*x)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^p*Hypergeometric2F1[m + 1, -p, m + 2, (-4*c*Rt[b^2 - 4*a*c, 2]*(
d + e*x))/((2*c*d - b*e - e*Rt[b^2 - 4*a*c, 2])*(b - Rt[b^2 - 4*a*c, 2] + 2
*c*x))])/((m + 1)*(2*c*d - b*e + e*Rt[b^2 - 4*a*c, 2])*(((2*c*d - b*e + e*R
t[b^2 - 4*a*c, 2])*(b + Rt[b^2 - 4*a*c, 2] + 2*c*x))/((2*c*d - b*e - e*Rt[b
^2 - 4*a*c, 2])*(b - Rt[b^2 - 4*a*c, 2] + 2*c*x)))^p), x] /; FreeQ[{a, b, c
, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &
& NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int (d+ex)^m\left(a+bx+cx^2\right)^{-2-\frac{m}{2}}\,dx=\frac{e(d+ex)^{1+m}\left(a+bx+cx^2\right)^{-1-\frac{m}{2}}}{\left(cd^2-bde+ae^2\right)(1+m)}+\frac{\int (d+ex)^{1+m}\left(\frac{1}{2}(-bem+2cd(1+m))+cex\right.}{\left(cd^2-bde+ae^2\right)(1+}$$

$$=\frac{e(d+ex)^{1+m}\left(a+bx+cx^2\right)^{-1-\frac{m}{2}}}{\left(cd^2-bde+ae^2\right)(1+m)}+\frac{e(2cd-be)m(d+ex)^{2+m}\left(a+bx+cx^2\right)^{-1-\frac{m}{2}}}{2\left(cd^2-bde+ae^2\right)^2(1+m)(2+m)}$$

$$=\frac{e(d+ex)^{1+m}\left(a+bx+cx^2\right)^{-1-\frac{m}{2}}}{\left(cd^2-bde+ae^2\right)(1+m)}+\frac{e(2cd-be)m(d+ex)^{2+m}\left(a+bx+cx^2\right)^{-1-\frac{m}{2}}}{2\left(cd^2-bde+ae^2\right)^2(1+m)(2+m)}$$

Mathematica [A] time = 3.78267, size = 379, normalized size = 0.86

$$\frac{(d+ex)^{m+1}(a+x(b+cx))^{-\frac{m}{2}-2}\left(\frac{(d+ex)^2\left(\sqrt{b^2-4ac}+b+2cx\right)\left(-4ce(bd(m+1)-ae)+b^2e^2m+4c^2d^2(m+1)\right)\left(\frac{\left(\sqrt{b^2-4ac}+b+2cx\right)\left(e\left(\sqrt{b^2-4ac}-b\right)+2cd\right)}{\left(\sqrt{b^2-4ac}-b-2cx\right)\left(e\left(\sqrt{b^2-4ac}+b\right)-2cd\right)}\right)^{\frac{m+2}{2}}{}_2F_1}{2(m+3)\left(e\left(\sqrt{b^2-4ac}+b\right)-2cd\right)}\right.}{2(m+1)\left(e(ae-bd\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(a + b*x + c*x^2)^(-2 - m/2),x]
```

```
[Out] ((d + e*x)^(1 + m)*(a + x*(b + c*x))^(-2 - m/2)*(2*e*(c*d^2 + e*(-(b*d) + a
*e))*(a + x*(b + c*x)) + (e*(2*c*d - b*e)*m*(d + e*x)*(a + x*(b + c*x)))/(2
 + m) + ((b^2*e^2*m + 4*c^2*d^2*(1 + m) - 4*c*e*(-(a*e) + b*d*(1 + m)))*(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x)*(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*(b + Sq
rt[b^2 - 4*a*c] + 2*c*x))/((-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*(-b + Sqrt[
b^2 - 4*a*c] - 2*c*x)))^((2 + m)/2)*(d + e*x)^2*Hypergeometric2F1[3 + m, (4
```

```
 + m)/2, 4 + m, (-4*c*Sqrt[b^2 - 4*a*c]*(d + e*x))/((-2*c*d + (b + Sqrt[b^2
 - 4*a*c])*e)*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))])/(2*(-2*c*d + (b + Sqrt[b^
2 - 4*a*c])*e)*(3 + m))))/(2*(c*d^2 + e*(-(b*d) + a*e))^2*(1 + m))
```

Maple [F] time = 1.258, size = 0, normalized size = 0.

$$\int (ex+d)^m \left(cx^2+bx+a\right)^{-2-\frac{m}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^(-2-1/2*m),x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^(-2-1/2*m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^{-\frac{1}{2}\,m-2}(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(-2-1/2*m),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(-1/2*m - 2)*(e*x + d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(cx^2+bx+a\right)^{-\frac{1}{2}\,m-2}(ex+d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(-2-1/2*m),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^(-1/2*m - 2)*(e*x + d)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**(-2-1/2*m),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(-2-1/2*m),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.2582 $\int \frac{1}{\sqrt[3]{1+x}\sqrt[3]{1-x+x^2}}\,dx$

Optimal. Leaf size=102

$$\frac{\sqrt[3]{x^3+1}\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{x+1}\sqrt[3]{x^2-x+1}}-\frac{\sqrt[3]{x^3+1}\log\left(\sqrt[3]{x^3+1}-x\right)}{2\sqrt[3]{x+1}\sqrt[3]{x^2-x+1}}$$

```
[Out] ((1 + x^3)^(1/3)*ArcTan[(1 + (2*x)/(1 + x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*(1 +
 x)^(1/3)*(1 - x + x^2)^(1/3)) - ((1 + x^3)^(1/3)*Log[-x + (1 + x^3)^(1/3)]
)/(2*(1 + x)^(1/3)*(1 - x + x^2)^(1/3))
```

Rubi [A] time = 0.0171366, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {713, 239}

$$\frac{\sqrt[3]{x^3+1}\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{x+1}\sqrt[3]{x^2-x+1}}-\frac{\sqrt[3]{x^3+1}\log\left(\sqrt[3]{x^3+1}-x\right)}{2\sqrt[3]{x+1}\sqrt[3]{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)^(1/3)*(1 - x + x^2)^(1/3)),x]
```

```
[Out] ((1 + x^3)^(1/3)*ArcTan[(1 + (2*x)/(1 + x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*(1 +
 x)^(1/3)*(1 - x + x^2)^(1/3)) - ((1 + x^3)^(1/3)*Log[-x + (1 + x^3)^(1/3)]
)/(2*(1 + x)^(1/3)*(1 - x + x^2)^(1/3))
```

Rule 713

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^FracPart[p])/(a*d +
c*e*x^3)^FracPart[p], Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
IGtQ[m - p + 1, 0] &&  !IntegerQ[p]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[3]{1+x}\sqrt[3]{1-x+x^2}}\,dx &= \frac{\sqrt[3]{1+x^3}\int \frac{1}{\sqrt[3]{1+x^3}}\,dx}{\sqrt[3]{1+x}\sqrt[3]{1-x+x^2}}\\ &= \frac{\sqrt[3]{1+x^3}\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{1+x}\sqrt[3]{1-x+x^2}}-\frac{\sqrt[3]{1+x^3}\log\left(-x+\sqrt[3]{1+x^3}\right)}{2\sqrt[3]{1+x}\sqrt[3]{1-x+x^2}}\end{aligned}$$

Mathematica [C] time = 0.0704854, size = 132, normalized size = 1.29

$$\frac{3\sqrt[3]{\frac{-2ix+\sqrt{3}+i}{\sqrt{3}+3i}}\sqrt[3]{\frac{2ix+\sqrt{3}-i}{\sqrt{3}-3i}}(x+1)^{2/3}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{2i(x+1)}{3i+\sqrt{3}},-\frac{2i(x+1)}{-3i+\sqrt{3}}\right)}{2\sqrt[3]{x^2-x+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 + x)^(1/3)*(1 - x + x^2)^(1/3)),x]
```

```
[Out] (3*((I + Sqrt[3] - (2*I)*x)/(3*I + Sqrt[3]))^(1/3)*((-I + Sqrt[3] + (2*I)*x
)/(-3*I + Sqrt[3]))^(1/3)*(1 + x)^(2/3)*AppellF1[2/3, 1/3, 1/3, 5/3, ((2*I)
*(1 + x))/(3*I + Sqrt[3]), ((-2*I)*(1 + x))/(-3*I + Sqrt[3])])/(2*(1 - x +
x^2)^(1/3))
```

Maple [F] time = 1.582, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{1+x}}\frac{1}{\sqrt[3]{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)^(1/3)/(x^2-x+1)^(1/3),x)
```

```
[Out] int(1/(1+x)^(1/3)/(x^2-x+1)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2-x+1\right)^{\frac{1}{3}}(x+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/3)/(x^2-x+1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - x + 1)^(1/3)*(x + 1)^(1/3)), x)
```

Fricas [A] time = 3.30881, size = 333, normalized size = 3.26

$$\frac{1}{3}\sqrt{3}\arctan\left(-\frac{4\sqrt{3}\left(x^2-x+1\right)^{\frac{1}{3}}(x+1)^{\frac{1}{3}}x^2-2\sqrt{3}\left(x^2-x+1\right)^{\frac{2}{3}}(x+1)^{\frac{2}{3}}x+\sqrt{3}\left(x^3+1\right)}{9x^3+1}\right)-\frac{1}{6}\log\left(3\left(x^2-x+1\right)^{\frac{1}{3}}(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/3)/(x^2-x+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(-(4*sqrt(3)*(x^2 - x + 1)^(1/3)*(x + 1)^(1/3)*x^2 - 2*sq
rt(3)*(x^2 - x + 1)^(2/3)*(x + 1)^(2/3)*x + sqrt(3)*(x^3 + 1))/(9*x^3 + 1))
 - 1/6*log(3*(x^2 - x + 1)^(1/3)*(x + 1)^(1/3)*x^2 - 3*(x^2 - x + 1)^(2/3)*
```

```
(x + 1)^(2/3)*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{x+1}\sqrt[3]{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)**(1/3)/(x**2-x+1)**(1/3),x)
```

```
[Out] Integral(1/((x + 1)**(1/3)*(x**2 - x + 1)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2-x+1\right)^{\frac{1}{3}}(x+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/3)/(x^2-x+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - x + 1)^(1/3)*(x + 1)^(1/3)), x)
```

3.2583 $\int \frac{1}{(1+x)^{2/3}\left(1-x+x^2\right)^{2/3}}\,dx$

Optimal. Leaf size=45

$$\frac{x\left(x^3+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-x^3\right)}{(x+1)^{2/3}\left(x^2-x+1\right)^{2/3}}$$

```
[Out] (x*(1 + x^3)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -x^3])/((1 + x)^(2/3)*(
1 - x + x^2)^(2/3))
```

Rubi [A] time = 0.0152849, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {713, 245}

$$\frac{x\left(x^3+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-x^3\right)}{(x+1)^{2/3}\left(x^2-x+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)^(2/3)*(1 - x + x^2)^(2/3)),x]
```

```
[Out] (x*(1 + x^3)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -x^3])/((1 + x)^(2/3)*(
1 - x + x^2)^(2/3))
```

Rule 713

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^FracPart[p])/(a*d +
c*e*x^3)^FracPart[p], Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
IGtQ[m - p + 1, 0] && !IntegerQ[p]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1+x)^{2/3}\left(1-x+x^2\right)^{2/3}}\,dx &= \frac{\left(1+x^3\right)^{2/3}\int \frac{1}{\left(1+x^3\right)^{2/3}}\,dx}{(1+x)^{2/3}\left(1-x+x^2\right)^{2/3}}\\
&= \frac{x\left(1+x^3\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-x^3\right)}{(1+x)^{2/3}\left(1-x+x^2\right)^{2/3}}
\end{aligned}$$

Mathematica [C] time = 0.0740744, size = 143, normalized size = 3.18

$$\frac{3\left(2ix+\sqrt{3}-i\right)\sqrt[3]{x+1}\left(-\frac{\left(\sqrt{3}-3i\right)x+\sqrt{3}+3i}{\left(\sqrt{3}+3i\right)x+\sqrt{3}-3i}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};\frac{4i\sqrt{3}(x+1)}{\left(3i+\sqrt{3}\right)\left(2ix+\sqrt{3}-i\right)}\right)}{\left(\sqrt{3}-3i\right)\left(x^2-x+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)^(2/3)*(1 - x + x^2)^(2/3)),x]
```

```
[Out] (3*(-I + Sqrt[3] + (2*I)*x)*(1 + x)^(1/3)*(-((3*I + Sqrt[3] + (-3*I + Sqrt[
3])*x)/(-3*I + Sqrt[3] + (3*I + Sqrt[3])*x)))^(2/3)*Hypergeometric2F1[1/3,
2/3, 4/3, ((4*I)*Sqrt[3]*(1 + x))/((3*I + Sqrt[3])*(-I + Sqrt[3] + (2*I)*x)
)])/((-3*I + Sqrt[3])*(1 - x + x^2)^(2/3))
```

Maple [F] time = 1.603, size = 0, normalized size = 0.

$$\int (1+x)^{-\frac{2}{3}}\left(x^2-x+1\right)^{-\frac{2}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)^(2/3)/(x^2-x+1)^(2/3),x)
```

```
[Out] int(1/(1+x)^(2/3)/(x^2-x+1)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2-x+1\right)^{\frac{2}{3}}(x+1)^{\frac{2}{3}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(2/3)/(x^2-x+1)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - x + 1)^(2/3)*(x + 1)^(2/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(x^2-x+1\right)^{\frac{1}{3}}(x+1)^{\frac{1}{3}}}{x^3+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(2/3)/(x^2-x+1)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((x^2 - x + 1)^(1/3)*(x + 1)^(1/3)/(x^3 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x+1)^{\frac{2}{3}}\left(x^2-x+1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)**(2/3)/(x**2-x+1)**(2/3),x)
```

```
[Out] Integral(1/((x + 1)**(2/3)*(x**2 - x + 1)**(2/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2-x+1\right)^{\frac{2}{3}}(x+1)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(2/3)/(x^2-x+1)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - x + 1)^(2/3)*(x + 1)^(2/3)), x)
```

3.2584 $\int (1+x)^p \left(1-x+x^2\right)^p \, dx$

Optimal. Leaf size=41

$$x(x+1)^p \left(x^2-x+1\right)^p \left(x^3+1\right)^{-p} {}_2F_1\left(\frac{1}{3},-p;\frac{4}{3};-x^3\right)$$

```
[Out] (x*(1 + x)^p*(1 - x + x^2)^p*Hypergeometric2F1[1/3, -p, 4/3, -x^3])/(1 + x^
3)^p
```

Rubi [A] time = 0.0122855, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {713, 245}

$$x(x+1)^p \left(x^2-x+1\right)^p \left(x^3+1\right)^{-p} {}_2F_1\left(\frac{1}{3},-p;\frac{4}{3};-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^p*(1 - x + x^2)^p,x]
```

```
[Out] (x*(1 + x)^p*(1 - x + x^2)^p*Hypergeometric2F1[1/3, -p, 4/3, -x^3])/(1 + x^
3)^p
```

Rule 713

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^FracPart[p])/(a*d +
c*e*x^3)^FracPart[p], Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
IGtQ[m - p + 1, 0] && !IntegerQ[p]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (1+x)^p \left(1-x+x^2\right)^p \, dx &= \left((1+x)^p \left(1-x+x^2\right)^p \left(1+x^3\right)^{-p}\right) \int \left(1+x^3\right)^p \, dx\\ &= x(1+x)^p \left(1-x+x^2\right)^p \left(1+x^3\right)^{-p} {}_2F_1\left(\frac{1}{3},-p;\frac{4}{3};-x^3\right)\end{aligned}$$

Mathematica [C] time = 0.112215, size = 132, normalized size = 3.22

$$\frac{\left(\frac{-2ix+\sqrt{3}+i}{\sqrt{3}+3i}\right)^{-p} \left(\frac{2ix+\sqrt{3}-i}{\sqrt{3}-3i}\right)^{-p} (x+1)^{p+1} \left(x^2-x+1\right)^p F_1\left(p+1;-p,-p;p+2;\frac{2i(x+1)}{3i+\sqrt{3}},-\frac{2i(x+1)}{-3i+\sqrt{3}}\right)}{p+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x)^p*(1 - x + x^2)^p,x]
```

```
[Out] ((1 + x)^(1 + p)*(1 - x + x^2)^p*AppellF1[1 + p, -p, -p, 2 + p, ((2*I)*(1 +
 x))/(3*I + Sqrt[3]), ((-2*I)*(1 + x))/(-3*I + Sqrt[3])])/((1 + p)*((I + Sq
rt[3] - (2*I)*x)/(3*I + Sqrt[3]))^p*((-I + Sqrt[3] + (2*I)*x)/(-3*I + Sqrt[
3]))^p)
```

Maple [F] time = 1.566, size = 0, normalized size = 0.

$$\int (1+x)^p \left(x^2-x+1\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^p*(x^2-x+1)^p,x)
```

```
[Out] int((1+x)^p*(x^2-x+1)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2-x+1\right)^p (x+1)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^p*(x^2-x+1)^p,x, algorithm="maxima")
```

```
[Out] integrate((x^2 - x + 1)^p*(x + 1)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(x^2-x+1\right)^p (x+1)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^p*(x^2-x+1)^p,x, algorithm="fricas")
```

```
[Out] integral((x^2 - x + 1)^p*(x + 1)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (x+1)^p \left(x^2-x+1\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**p*(x**2-x+1)**p,x)
```

```
[Out] Integral((x + 1)**p*(x**2 - x + 1)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (x^2 - x + 1)^p (x + 1)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^p*(x^2-x+1)^p,x, algorithm="giac")
```

```
[Out] integrate((x^2 - x + 1)^p*(x + 1)^p, x)
```

3.2585 $\int \frac{1}{\sqrt[3]{1-x}\sqrt[3]{1+x+x^2}}\,dx$

Optimal. Leaf size=109

$$\frac{\sqrt[3]{1-x^3}\log\left(\sqrt[3]{1-x^3}+x\right)}{2\sqrt[3]{1-x}\sqrt[3]{x^2+x+1}}-\frac{\sqrt[3]{1-x^3}\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{1-x}\sqrt[3]{x^2+x+1}}$$

```
[Out] -(((1 - x^3)^(1/3)*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*(1
 - x)^(1/3)*(1 + x + x^2)^(1/3))) + ((1 - x^3)^(1/3)*Log[x + (1 - x^3)^(1/3
)])/(2*(1 - x)^(1/3)*(1 + x + x^2)^(1/3))
```

Rubi [A] time = 0.0187458, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {713, 239}

$$\frac{\sqrt[3]{1-x^3}\log\left(\sqrt[3]{1-x^3}+x\right)}{2\sqrt[3]{1-x}\sqrt[3]{x^2+x+1}}-\frac{\sqrt[3]{1-x^3}\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{1-x}\sqrt[3]{x^2+x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(1/3)*(1 + x + x^2)^(1/3)),x]
```

```
[Out] -(((1 - x^3)^(1/3)*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*(1
 - x)^(1/3)*(1 + x + x^2)^(1/3))) + ((1 - x^3)^(1/3)*Log[x + (1 - x^3)^(1/3
)])/(2*(1 - x)^(1/3)*(1 + x + x^2)^(1/3))
```

Rule 713

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^FracPart[p])/(a*d +
c*e*x^3)^FracPart[p], Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
IGtQ[m - p + 1, 0] &&  !IntegerQ[p]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[3]{1-x}\sqrt[3]{1+x+x^2}}\,dx &= \frac{\sqrt[3]{1-x^3}\int \frac{1}{\sqrt[3]{1-x^3}}\,dx}{\sqrt[3]{1-x}\sqrt[3]{1+x+x^2}}\\ &= -\frac{\sqrt[3]{1-x^3}\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{1-x}\sqrt[3]{1+x+x^2}}+\frac{\sqrt[3]{1-x^3}\log\left(x+\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{1-x}\sqrt[3]{1+x+x^2}}\end{aligned}$$

Mathematica [C] time = 0.0650933, size = 132, normalized size = 1.21

$$-\frac{3(1-x)^{2/3}\sqrt[3]{\frac{-2ix+\sqrt{3}-i}{\sqrt{3}-3i}}\sqrt[3]{\frac{2ix+\sqrt{3}+i}{\sqrt{3}+3i}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};-\frac{2i(x-1)}{3i+\sqrt{3}},\frac{2i(x-1)}{-3i+\sqrt{3}}\right)}{2\sqrt[3]{x^2+x+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 - x)^(1/3)*(1 + x + x^2)^(1/3)),x]
```

```
[Out] (-3*(1 - x)^(2/3)*((-I + Sqrt[3] - (2*I)*x)/(-3*I + Sqrt[3]))^(1/3)*((I + S
qrt[3] + (2*I)*x)/(3*I + Sqrt[3]))^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, ((-2*
I)*(-1 + x))/(3*I + Sqrt[3]), ((2*I)*(-1 + x))/(-3*I + Sqrt[3])])/(2*(1 + x
 + x^2)^(1/3))
```

Maple [F] time = 1.691, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{1-x}}\frac{1}{\sqrt[3]{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(1/3)/(x^2+x+1)^(1/3),x)
```

```
[Out] int(1/(1-x)^(1/3)/(x^2+x+1)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+x+1\right)^{\frac{1}{3}}(-x+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/3)/(x^2+x+1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + x + 1)^(1/3)*(-x + 1)^(1/3)), x)
```

Fricas [A] time = 3.56897, size = 339, normalized size = 3.11

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\left(x^2+x+1\right)^{\frac{1}{3}}x^2(-x+1)^{\frac{1}{3}}+2\sqrt{3}\left(x^2+x+1\right)^{\frac{2}{3}}x(-x+1)^{\frac{2}{3}}-\sqrt{3}\left(x^3-1\right)}{9x^3-1}\right)+\frac{1}{6}\log\left(3\left(x^2+x+1\right)^{\frac{1}{3}}x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/3)/(x^2+x+1)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan((4*sqrt(3)*(x^2 + x + 1)^(1/3)*x^2*(-x + 1)^(1/3) + 2*s
qrt(3)*(x^2 + x + 1)^(2/3)*x*(-x + 1)^(2/3) - sqrt(3)*(x^3 - 1))/(9*x^3 - 1
)) + 1/6*log(3*(x^2 + x + 1)^(1/3)*x^2*(-x + 1)^(1/3) + 3*(x^2 + x + 1)^(2/
```

```
3)*x*(-x + 1)^(2/3) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{1-x}\sqrt[3]{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(1/3)/(x**2+x+1)**(1/3),x)
```

```
[Out] Integral(1/((1 - x)**(1/3)*(x**2 + x + 1)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+x+1\right)^{\frac{1}{3}}(-x+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/3)/(x^2+x+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + x + 1)^(1/3)*(-x + 1)^(1/3)), x)
```

3.2586 $\int \frac{1}{(1-x)^{2/3}\left(1+x+x^2\right)^{2/3}} \, dx$

Optimal. Leaf size=45

$$\frac{x\left(1-x^3\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^3\right)}{(1-x)^{2/3}\left(x^2+x+1\right)^{2/3}}$$

```
[Out] (x*(1 - x^3)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, x^3])/((1 - x)^(2/3)*(1
 + x + x^2)^(2/3))
```

Rubi [A] time = 0.016428, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {713, 245}

$$\frac{x\left(1-x^3\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^3\right)}{(1-x)^{2/3}\left(x^2+x+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(2/3)*(1 + x + x^2)^(2/3)),x]
```

```
[Out] (x*(1 - x^3)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, x^3])/((1 - x)^(2/3)*(1
 + x + x^2)^(2/3))
```

Rule 713

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^FracPart[p])/(a*d +
c*e*x^3)^FracPart[p], Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
IGtQ[m - p + 1, 0] &&  !IntegerQ[p]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{2/3}\left(1+x+x^2\right)^{2/3}} \, dx &= \frac{\left(1-x^3\right)^{2/3} \int \frac{1}{\left(1-x^3\right)^{2/3}} \, dx}{(1-x)^{2/3}\left(1+x+x^2\right)^{2/3}}\\ &= \frac{x\left(1-x^3\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^3\right)}{(1-x)^{2/3}\left(1+x+x^2\right)^{2/3}}\end{aligned}$$

Mathematica [C] time = 0.0859381, size = 145, normalized size = 3.22

$$-\frac{3\sqrt[3]{1-x}\left(2ix+\sqrt{3}+i\right)\left(\frac{\left(\sqrt{3}+3i\right)x-\sqrt{3}+3i}{-\left(\sqrt{3}-3i\right)x+\sqrt{3}+3i}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};\frac{4i\sqrt{3}(x-1)}{\left(-3i+\sqrt{3}\right)\left(2ix+\sqrt{3}+i\right)}\right)}{\left(\sqrt{3}+3i\right)\left(x^2+x+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(2/3)*(1 + x + x^2)^(2/3)),x]
```

```
[Out] (-3*(1 - x)^(1/3)*(I + Sqrt[3] + (2*I)*x)*((3*I - Sqrt[3] + (3*I + Sqrt[3])
*x)/(3*I + Sqrt[3] - (-3*I + Sqrt[3])*x))^(2/3)*Hypergeometric2F1[1/3, 2/3,
 4/3, ((4*I)*Sqrt[3]*(-1 + x))/((-3*I + Sqrt[3])*(I + Sqrt[3] + (2*I)*x))])
/((3*I + Sqrt[3])*(1 + x + x^2)^(2/3))
```

Maple [F] time = 1.712, size = 0, normalized size = 0.

$$\int (1-x)^{-\frac{2}{3}}\left(x^2+x+1\right)^{-\frac{2}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(2/3)/(x^2+x+1)^(2/3),x)
```

```
[Out] int(1/(1-x)^(2/3)/(x^2+x+1)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+x+1\right)^{\frac{2}{3}}(-x+1)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(2/3)/(x^2+x+1)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + x + 1)^(2/3)*(-x + 1)^(2/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(x^2+x+1\right)^{\frac{1}{3}}(-x+1)^{\frac{1}{3}}}{x^3-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(2/3)/(x^2+x+1)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(-(x^2 + x + 1)^(1/3)*(-x + 1)^(1/3)/(x^3 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(1-x)^{\frac{2}{3}} \left(x^2+x+1\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(2/3)/(x**2+x+1)**(2/3),x)
```

```
[Out] Integral(1/((1 - x)**(2/3)*(x**2 + x + 1)**(2/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+x+1\right)^{\frac{2}{3}} (-x+1)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(2/3)/(x^2+x+1)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + x + 1)^(2/3)*(-x + 1)^(2/3)), x)
```

3.2587 $\int (1-x)^p \left(1+x+x^2\right)^p \, dx$

Optimal. Leaf size=41

$$x(1-x)^p \left(x^2+x+1\right)^p \left(1-x^3\right)^{-p} {}_2F_1\left(\frac{1}{3},-p;\frac{4}{3};x^3\right)$$

```
[Out] ((1 - x)^p*x*(1 + x + x^2)^p*Hypergeometric2F1[1/3, -p, 4/3, x^3])/(1 - x^3
)^p
```

Rubi [A] time = 0.0122532, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {713, 245}

$$x(1-x)^p \left(x^2+x+1\right)^p \left(1-x^3\right)^{-p} {}_2F_1\left(\frac{1}{3},-p;\frac{4}{3};x^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^p*(1 + x + x^2)^p,x]
```

```
[Out] ((1 - x)^p*x*(1 + x + x^2)^p*Hypergeometric2F1[1/3, -p, 4/3, x^3])/(1 - x^3
)^p
```

Rule 713

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^FracPart[p])/(a*d +
c*e*x^3)^FracPart[p], Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
IGtQ[m - p + 1, 0] && !IntegerQ[p]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (1-x)^p \left(1+x+x^2\right)^p \, dx &= \left((1-x)^p \left(1+x+x^2\right)^p \left(1-x^3\right)^{-p}\right) \int \left(1-x^3\right)^p \, dx\\ &= (1-x)^p x \left(1+x+x^2\right)^p \left(1-x^3\right)^{-p} {}_2F_1\left(\frac{1}{3},-p;\frac{4}{3};x^3\right)\end{aligned}$$

Mathematica [C] time = 0.120741, size = 133, normalized size = 3.24

$$\frac{(x-1)(1-x)^p \left(\frac{-2ix+\sqrt{3}-i}{\sqrt{3}-3i}\right)^{-p} \left(\frac{2ix+\sqrt{3}+i}{\sqrt{3}+3i}\right)^{-p} \left(x^2+x+1\right)^p F_1\left(p+1;-p,-p;p+2;\frac{2i(x-1)}{-3i+\sqrt{3}},-\frac{2i(x-1)}{3i+\sqrt{3}}\right)}{p+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - x)^p*(1 + x + x^2)^p,x]
```

```
[Out] ((1 - x)^p*(-1 + x)*(1 + x + x^2)^p*AppellF1[1 + p, -p, -p, 2 + p, ((2*I)*(
-1 + x))/(-3*I + Sqrt[3]), ((-2*I)*(-1 + x))/(3*I + Sqrt[3])])/((1 + p)*((-
I + Sqrt[3] - (2*I)*x)/(-3*I + Sqrt[3]))^p*((I + Sqrt[3] + (2*I)*x)/(3*I +
Sqrt[3]))^p)
```

Maple [F] time = 1.783, size = 0, normalized size = 0.

$$\int (1-x)^p \left(x^2+x+1\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^p*(x^2+x+1)^p,x)
```

```
[Out] int((1-x)^p*(x^2+x+1)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2+x+1\right)^p (-x+1)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^p*(x^2+x+1)^p,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + x + 1)^p*(-x + 1)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(x^2+x+1\right)^p (-x+1)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^p*(x^2+x+1)^p,x, algorithm="fricas")
```

```
[Out] integral((x^2 + x + 1)^p*(-x + 1)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (1-x)^p \left(x^2+x+1\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**p*(x**2+x+1)**p,x)
```

```
[Out] Integral((1 - x)**p*(x**2 + x + 1)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2 + x + 1\right)^p (-x + 1)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^p*(x^2+x+1)^p,x, algorithm="giac")
```

```
[Out] integrate((x^2 + x + 1)^p*(-x + 1)^p, x)
```

3.2588 $\int \frac{1}{\sqrt[3]{be-cex}\sqrt[3]{b^2+bcx+c^2x^2}}\,dx$

Optimal. Leaf size=196

$$\frac{\sqrt[3]{b^3e-c^3ex^3}\log\left(\sqrt[3]{b^3e-c^3ex^3}+c\sqrt[3]{e}x\right)}{2c\sqrt[3]{e}\sqrt[3]{b^2+bcx+c^2x^2}\sqrt[3]{be-cex}}-\frac{\sqrt[3]{b^3e-c^3ex^3}\tan^{-1}\left(\frac{1-\frac{2c\sqrt[3]{e}x}{\sqrt[3]{b^3e-c^3ex^3}}}{\sqrt{3}}\right)}{\sqrt{3}c\sqrt[3]{e}\sqrt[3]{b^2+bcx+c^2x^2}\sqrt[3]{be-cex}}$$

```
[Out] -(((b^3*e - c^3*e*x^3)^(1/3)*ArcTan[(1 - (2*c*e^(1/3)*x)/(b^3*e - c^3*e*x^3
)^(1/3))/Sqrt[3]])/(Sqrt[3]*c*e^(1/3)*(b*e - c*e*x)^(1/3)*(b^2 + b*c*x + c^
2*x^2)^(1/3))) + ((b^3*e - c^3*e*x^3)^(1/3)*Log[c*e^(1/3)*x + (b^3*e - c^3*
e*x^3)^(1/3)])/(2*c*e^(1/3)*(b*e - c*e*x)^(1/3)*(b^2 + b*c*x + c^2*x^2)^(1/
3))
```

Rubi [A] time = 0.0585334, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {713, 239}

$$\frac{\sqrt[3]{b^3e-c^3ex^3}\log\left(\sqrt[3]{b^3e-c^3ex^3}+c\sqrt[3]{e}x\right)}{2c\sqrt[3]{e}\sqrt[3]{b^2+bcx+c^2x^2}\sqrt[3]{be-cex}}-\frac{\sqrt[3]{b^3e-c^3ex^3}\tan^{-1}\left(\frac{1-\frac{2c\sqrt[3]{e}x}{\sqrt[3]{b^3e-c^3ex^3}}}{\sqrt{3}}\right)}{\sqrt{3}c\sqrt[3]{e}\sqrt[3]{b^2+bcx+c^2x^2}\sqrt[3]{be-cex}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*e - c*e*x)^(1/3)*(b^2 + b*c*x + c^2*x^2)^(1/3)),x]
```

```
[Out] -(((b^3*e - c^3*e*x^3)^(1/3)*ArcTan[(1 - (2*c*e^(1/3)*x)/(b^3*e - c^3*e*x^3
)^(1/3))/Sqrt[3]])/(Sqrt[3]*c*e^(1/3)*(b*e - c*e*x)^(1/3)*(b^2 + b*c*x + c^
2*x^2)^(1/3))) + ((b^3*e - c^3*e*x^3)^(1/3)*Log[c*e^(1/3)*x + (b^3*e - c^3*
e*x^3)^(1/3)])/(2*c*e^(1/3)*(b*e - c*e*x)^(1/3)*(b^2 + b*c*x + c^2*x^2)^(1/
3))
```

Rule 713

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^FracPart[p])/(a*d +
c*e*x^3)^FracPart[p], Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
IGtQ[m - p + 1, 0] &&  !IntegerQ[p]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{be-cex}\sqrt[3]{b^2+bcx+c^2x^2}}\,dx = \frac{\sqrt[3]{b^3e-c^3ex^3}\int \frac{1}{\sqrt[3]{b^3e-c^3ex^3}}\,dx}{\sqrt[3]{be-cex}\sqrt[3]{b^2+bcx+c^2x^2}}$$

$$= -\frac{\sqrt[3]{b^3e-c^3ex^3}\tan^{-1}\left(\frac{1-\frac{2c\sqrt[3]{e}x}{\sqrt[3]{b^3e-c^3ex^3}}}{\sqrt{3}}\right)}{\sqrt{3}c\sqrt[3]{e}\sqrt[3]{be-cex}\sqrt[3]{b^2+bcx+c^2x^2}} + \frac{\sqrt[3]{b^3e-c^3ex^3}\log\left(c\sqrt[3]{e}x+\sqrt[3]{b^3e-c^3ex^3}\right)}{2c\sqrt[3]{e}\sqrt[3]{be-cex}\sqrt[3]{b^2+bcx+c^2x^2}}$$

Mathematica [C] time = 0.190699, size = 241, normalized size = 1.23

$$-\frac{3\sqrt[3]{\frac{-\sqrt{3}\sqrt{-b^2c^2}+bc+2c^2x}{3bc-\sqrt{3}\sqrt{-b^2c^2}}}\sqrt[3]{\frac{\sqrt{3}\sqrt{-b^2c^2}+bc+2c^2x}{\sqrt{3}\sqrt{-b^2c^2}+3bc}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{2c(b-cx)}{3bc+\sqrt{3}\sqrt{-b^2c^2}},\frac{2c(b-cx)}{3bc-\sqrt{3}\sqrt{-b^2c^2}}\right)(e(b-cx))^{2/3}}{2ce\sqrt[3]{b^2+bcx+c^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((b*e - c*e*x)^(1/3)*(b^2 + b*c*x + c^2*x^2)^(1/3)),x]
```

```
[Out] (-3*(e*(b - c*x))^(2/3)*((b*c - Sqrt[3]*Sqrt[-(b^2*c^2)] + 2*c^2*x)/(3*b*c
- Sqrt[3]*Sqrt[-(b^2*c^2)]))^(1/3)*((b*c + Sqrt[3]*Sqrt[-(b^2*c^2)] + 2*c^2
*x)/(3*b*c + Sqrt[3]*Sqrt[-(b^2*c^2)]))^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3,
(2*c*(b - c*x))/(3*b*c + Sqrt[3]*Sqrt[-(b^2*c^2)]), (2*c*(b - c*x))/(3*b*c
- Sqrt[3]*Sqrt[-(b^2*c^2)])])/(2*c*e*(b^2 + b*c*x + c^2*x^2)^(1/3))
```

Maple [F] time = 3.069, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{-cex+be}}\frac{1}{\sqrt[3]{c^2x^2+bcx+b^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c*e*x+b*e)^(1/3)/(c^2*x^2+b*c*x+b^2)^(1/3),x)
```

```
[Out] int(1/(-c*e*x+b*e)^(1/3)/(c^2*x^2+b*c*x+b^2)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c^2x^2+bcx+b^2\right)^{\frac{1}{3}}(-cex+be)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*e*x+b*e)^(1/3)/(c^2*x^2+b*c*x+b^2)^(1/3),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((c^2*x^2 + b*c*x + b^2)^(1/3)*(-c*e*x + b*e)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*e*x+b*e)^(1/3)/(c^2*x^2+b*c*x+b^2)^(1/3),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{-e\left(-b+cx\right)}\sqrt[3]{b^2+bcx+c^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*e*x+b*e)**(1/3)/(c**2*x**2+b*c*x+b**2)**(1/3),x)
```

```
[Out] Integral(1/((-e*(-b + c*x))**(1/3)*(b**2 + b*c*x + c**2*x**2)**(1/3)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*e*x+b*e)^(1/3)/(c^2*x^2+b*c*x+b^2)^(1/3),x, algorithm="giac
")
```

```
[Out] Timed out
```

3.2589 $\int \frac{1}{(be-cex)^{2/3}\left(b^2+bcx+c^2x^2\right)^{2/3}}\,dx$

Optimal. Leaf size=71

$$\frac{x\left(1-\frac{c^3x^3}{b^3}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};\frac{c^3x^3}{b^3}\right)}{\left(b^2+bcx+c^2x^2\right)^{2/3}(be-cex)^{2/3}}$$

```
[Out] (x*(1 - (c^3*x^3)/b^3)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, (c^3*x^3)/b^3
])/((b*e - c*e*x)^(2/3)*(b^2 + b*c*x + c^2*x^2)^(2/3))
```

Rubi [A] time = 0.0391936, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {713, 246, 245}

$$\frac{x\left(1-\frac{c^3x^3}{b^3}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};\frac{c^3x^3}{b^3}\right)}{\left(b^2+bcx+c^2x^2\right)^{2/3}(be-cex)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*e - c*e*x)^(2/3)*(b^2 + b*c*x + c^2*x^2)^(2/3)),x]
```

```
[Out] (x*(1 - (c^3*x^3)/b^3)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, (c^3*x^3)/b^3
])/((b*e - c*e*x)^(2/3)*(b^2 + b*c*x + c^2*x^2)^(2/3))
```

Rule 713

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^FracPart[p])/(a*d +
c*e*x^3)^FracPart[p], Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
IGtQ[m - p + 1, 0] && !IntegerQ[p]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(be-cex)^{2/3}\left(b^2+bcx+c^2x^2\right)^{2/3}}\,dx = \frac{\left(b^3e-c^3ex^3\right)^{2/3}\int \frac{1}{\left(b^3e-c^3ex^3\right)^{2/3}}\,dx}{(be-cex)^{2/3}\left(b^2+bcx+c^2x^2\right)^{2/3}}$$
$$= \frac{\left(1-\frac{c^3x^3}{b^3}\right)^{2/3}\int \frac{1}{\left(1-\frac{c^3x^3}{b^3}\right)^{2/3}}\,dx}{(be-cex)^{2/3}\left(b^2+bcx+c^2x^2\right)^{2/3}}$$
$$= \frac{x\left(1-\frac{c^3x^3}{b^3}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};\frac{c^3x^3}{b^3}\right)}{(be-cex)^{2/3}\left(b^2+bcx+c^2x^2\right)^{2/3}}$$

Mathematica [B] time = 0.217441, size = 258, normalized size = 3.63

$$-\frac{3\left(-\sqrt{3}\sqrt{-b^2}+b+2cx\right)\left(\frac{-\sqrt{3}\sqrt{-b^2}cx+3b^2+\sqrt{3}\sqrt{-b^2}b+3bcx}{\sqrt{3}\sqrt{-b^2}cx+3b^2-\sqrt{3}\sqrt{-b^2}b+3bcx}\right)^{2/3}\sqrt[3]{e(b-cx)}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};\frac{4\sqrt{3}\sqrt{-b^2}(b-cx)}{\left(3b+\sqrt{3}\sqrt{-b^2}\right)\left(-b-2cx+\sqrt{3}\sqrt{-b^2}\right)}\right)}{\left(3b-\sqrt{3}\sqrt{-b^2}\right)ce\left(b^2+bcx+c^2x^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*e - c*e*x)^(2/3)*(b^2 + b*c*x + c^2*x^2)^(2/3)),x]
```

```
[Out] (-3*(e*(b - c*x))^(1/3)*(b - Sqrt[3]*Sqrt[-b^2] + 2*c*x)*((3*b^2 + Sqrt[3]*
b*Sqrt[-b^2] + 3*b*c*x - Sqrt[3]*Sqrt[-b^2]*c*x)/(3*b^2 - Sqrt[3]*b*Sqrt[-b
^2] + 3*b*c*x + Sqrt[3]*Sqrt[-b^2]*c*x))^(2/3)*Hypergeometric2F1[1/3, 2/3,
4/3, (4*Sqrt[3]*Sqrt[-b^2]*(b - c*x))/((3*b + Sqrt[3]*Sqrt[-b^2])*(-b + Sqr
t[3]*Sqrt[-b^2] - 2*c*x))]])/((3*b - Sqrt[3]*Sqrt[-b^2])*c*e*(b^2 + b*c*x +
c^2*x^2)^(2/3))
```

Maple [F] time = 2.985, size = 0, normalized size = 0.

$$\int (-cex+be)^{-\frac{2}{3}}\left(c^2x^2+bcx+b^2\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c*e*x+b*e)^(2/3)/(c^2*x^2+b*c*x+b^2)^(2/3),x)
```

```
[Out] int(1/(-c*e*x+b*e)^(2/3)/(c^2*x^2+b*c*x+b^2)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c^2x^2+bcx+b^2\right)^{\frac{2}{3}}(-cex+be)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*e*x+b*e)^(2/3)/(c^2*x^2+b*c*x+b^2)^(2/3),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((c^2*x^2 + b*c*x + b^2)^(2/3)*(-c*e*x + b*e)^(2/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(c^2x^2+bcx+b^2\right)^{\frac{1}{3}}(-cex+be)^{\frac{1}{3}}}{c^3ex^3-b^3e},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*e*x+b*e)^(2/3)/(c^2*x^2+b*c*x+b^2)^(2/3),x, algorithm="fric
as")
```

```
[Out] integral(-(c^2*x^2 + b*c*x + b^2)^(1/3)*(-c*e*x + b*e)^(1/3)/(c^3*e*x^3 - b
^3*e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(-e(-b+cx))^{\frac{2}{3}}\left(b^2+bcx+c^2x^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*e*x+b*e)**(2/3)/(c**2*x**2+b*c*x+b**2)**(2/3),x)
```

```
[Out] Integral(1/((-e*(-b + c*x))**(2/3)*(b**2 + b*c*x + c**2*x**2)**(2/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(c^2x^2+bcx+b^2\right)^{\frac{2}{3}}(-cex+be)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*e*x+b*e)^(2/3)/(c^2*x^2+b*c*x+b^2)^(2/3),x, algorithm="giac
")
```

```
[Out] integrate(1/((c^2*x^2 + b*c*x + b^2)^(2/3)*(-c*e*x + b*e)^(2/3)), x)
```

3.2590 $\int (be - cex)^p \left(b^2 + bcx + c^2x^2\right)^p \, dx$

Optimal. Leaf size=67

$$x\left(b^2+bcx+c^2x^2\right)^p\left(1-\frac{c^3x^3}{b^3}\right)^{-p}(be-cex)^p\,{}_2F_1\left(\frac{1}{3},-p;\frac{4}{3};\frac{c^3x^3}{b^3}\right)$$

```
[Out] (x*(b*e - c*e*x)^p*(b^2 + b*c*x + c^2*x^2)^p*Hypergeometric2F1[1/3, -p, 4/3
, (c^3*x^3)/b^3])/(1 - (c^3*x^3)/b^3)^p
```

Rubi [A] time = 0.0334323, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {713, 246, 245}

$$x\left(b^2+bcx+c^2x^2\right)^p\left(1-\frac{c^3x^3}{b^3}\right)^{-p}(be-cex)^p\,{}_2F_1\left(\frac{1}{3},-p;\frac{4}{3};\frac{c^3x^3}{b^3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b*e - c*e*x)^p*(b^2 + b*c*x + c^2*x^2)^p,x]
```

```
[Out] (x*(b*e - c*e*x)^p*(b^2 + b*c*x + c^2*x^2)^p*Hypergeometric2F1[1/3, -p, 4/3
, (c^3*x^3)/b^3])/(1 - (c^3*x^3)/b^3)^p
```

Rule 713

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^FracPart[p])/(a*d +
c*e*x^3)^FracPart[p], Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
IGtQ[m - p + 1, 0] && !IntegerQ[p]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (be-cex)^p\left(b^2+bcx+c^2x^2\right)^p\,dx &= \left((be-cex)^p\left(b^2+bcx+c^2x^2\right)^p\left(b^3e-c^3ex^3\right)^{-p}\right)\int\left(b^3e-c^3ex^3\right)^p\,dx\\ &= \left((be-cex)^p\left(b^2+bcx+c^2x^2\right)^p\left(1-\frac{c^3x^3}{b^3}\right)^{-p}\right)\int\left(1-\frac{c^3x^3}{b^3}\right)^p\,dx\\ &= x(be-cex)^p\left(b^2+bcx+c^2x^2\right)^p\left(1-\frac{c^3x^3}{b^3}\right)^{-p}\,{}_2F_1\left(\frac{1}{3},-p;\frac{4}{3};\frac{c^3x^3}{b^3}\right)\end{aligned}$$

Mathematica [C] time = 0.306608, size = 243, normalized size = 3.63

$$\frac{(cx-b)\left(\frac{-\sqrt{3}\sqrt{-b^2c^2}+bc+2c^2x}{3bc-\sqrt{3}\sqrt{-b^2c^2}}\right)^{-p}\left(\frac{\sqrt{3}\sqrt{-b^2c^2}+bc+2c^2x}{\sqrt{3}\sqrt{-b^2c^2}+3bc}\right)^{-p}\left(b^2+bcx+c^2x^2\right)^p F_1\left(p+1;-p,-p;p+2;\frac{2c(b-cx)}{3bc+\sqrt{3}\sqrt{-b^2c^2}},\frac{2c(b-cx)}{3bc-\sqrt{3}\sqrt{-b^2c^2}}\right)}{c(p+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*e - c*e*x)^p*(b^2 + b*c*x + c^2*x^2)^p,x]
```

```
[Out] ((e*(b - c*x))^p*(-b + c*x)*(b^2 + b*c*x + c^2*x^2)^p*AppellF1[1 + p, -p, -
p, 2 + p, (2*c*(b - c*x))/(3*b*c + Sqrt[3]*Sqrt[-(b^2*c^2)]), (2*c*(b - c*x
))/(3*b*c - Sqrt[3]*Sqrt[-(b^2*c^2)])])/(c*(1 + p)*((b*c - Sqrt[3]*Sqrt[-(b
^2*c^2)] + 2*c^2*x)/(3*b*c - Sqrt[3]*Sqrt[-(b^2*c^2)]))^p*((b*c + Sqrt[3]*S
qrt[-(b^2*c^2)] + 2*c^2*x)/(3*b*c + Sqrt[3]*Sqrt[-(b^2*c^2)]))^p)
```

Maple [F] time = 3.016, size = 0, normalized size = 0.

$$\int (-cex+be)^p \left(c^2x^2+bcx+b^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*e*x+b*e)^p*(c^2*x^2+b*c*x+b^2)^p,x)
```

```
[Out] int((-c*e*x+b*e)^p*(c^2*x^2+b*c*x+b^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2x^2+bcx+b^2\right)^p (-cex+be)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e*x+b*e)^p*(c^2*x^2+b*c*x+b^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + b*c*x + b^2)^p*(-c*e*x + b*e)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(c^2x^2+bcx+b^2\right)^p (-cex+be)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e*x+b*e)^p*(c^2*x^2+b*c*x+b^2)^p,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^2 + b*c*x + b^2)^p*(-c*e*x + b*e)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (-e(-b+cx))^p \left(b^2+bcx+c^2x^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e*x+b*e)**p*(c**2*x**2+b*c*x+b**2)**p,x)
```

```
[Out] Integral((-e*(-b + c*x))**p*(b**2 + b*c*x + c**2*x**2)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2x^2+bcx+b^2\right)^p (-cex+be)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*e*x+b*e)^p*(c^2*x^2+b*c*x+b^2)^p,x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + b*c*x + b^2)^p*(-c*e*x + b*e)^p, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
    not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
    )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
    ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
    type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
    ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
    (flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                         #since this estimate of leaf count is bit lower than
```

```
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0], Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1], Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0], Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```